

RESEARCH ARTICLE

Strong-motion duration and response scaling of yielding and degrading eccentric structures

Christian Málaga-Chuquitaype

Department of Civil and Environmental Engineering, Imperial College London, London, UK

Correspondence: Christian Málaga-Chuquitaype, Department of Civil and Environmental Engineering, Skemp-ton Building, Imperial College London, London SW7 2AZ, UK.
Email: c.malaga@imperial.ac.uk

Abstract

Plan irregular structures, whose complex response represents a generalisation of the simpler de-coupled motion ascribed to symmetric buildings, make up a large proportion of the failures during major earthquakes. This paper examines the seismic response scaling of degrading and no-degrading eccentric structures subjected to bidirectional earthquake action and its relationship with the duration of the ground motion by means of dimensional and orientational analyses. Structures with reflectionally symmetric stiffness distribution and mass eccentricity subjected to orthogonal pairs of ideal pulses are considered as the fundamental case. The application of Vaschy–Buckingham’s Π -theorem reduces the number of variables governing the peak orthogonal displacements leading to the emergence of remarkable order in the structural response. If orientationally consistent dimensionless parameters are selected, the response becomes self-similar. By contrast, when degradation is introduced, peak inelastic displacements are dramatically affected and the self-similarity in the response is lost immediately after the onset of inelastic deformations. Conversely, if the uniform duration, instead of the period, of the strong motion is adopted as a timescale, a practically self-similar response is observed. This offers unequivocal proof of the fundamental role played by the ground-motion duration in defining the peak displacement response of degrading structures even at small inelastic demands, although its importance increases with increasing deformation levels. Finally, the existence of complete similarities, or similarities of the first kind, are explored and the practical implications of these findings are briefly outlined in the context of real pulse-like ground motions with varying degrees of coherency.

KEYWORDS

bidirectional ground motion, cyclic degradation, dimensional analysis, earthquake displacement demands, ground-motion duration, self-similarity

This is an open access article under the terms of the [Creative Commons Attribution](https://creativecommons.org/licenses/by/4.0/) License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2020 The Authors. *Earthquake Engineering & Structural Dynamics* published by John Wiley & Sons Ltd.

1 | INTRODUCTION

State-of-the-art seismic design and assessment procedures ascribe great importance to the estimation of peak displacements in view of the direct relationship observed between maximum deformations and earthquake damage. This is recognised in current seismic provisions^{1,2} where performance-based methodologies are constructed around rates of exceedance of predefined deformation limits. The significance of displacement estimations, together with the growing number of recorded ground motions and the increasing availability of computational resources able to expedite the execution of large numbers of analyses, has propelled an abundance of statistical studies on the inelastic deformation demands of regular and irregular structures. A good number of these studies have concerned themselves with the identification of trends and correlations, mostly in regular buildings, while true insight into the underlying physical phenomenon has remained limited or hampered by the challenge of presenting large amounts of data in a meaningful way.^{3,4}

In this context, the estimation of seismic demands in eccentric and irregular structures represents a major challenge for earthquake engineers. Plan eccentric structures are more vulnerable to earthquake damage than their regular counterparts^{4–6} due to the additional deformation demands imposed by their torsional motion. This torsion is heightened by the building eccentricity (i.e. the distance between the centre of mass (CM) and centre of resistance) which leads to complex coupling between what would otherwise have been predominantly orthogonal translational modes of deformation.⁷ By contrast, when the CM and the centre of resistance coincide, the lateral-torsional coupling of the building motion under bidirectional loads vanishes and the response of ideally symmetric buildings can be studied with reference to their de-coupled in-plane vibration,⁸ a condition that is rarely attained in practice but is ubiquitous in the academic literature.

Various studies have examined in detail the seismic drift demands in plan-asymmetric structures, for example^{9–11}, including the effects of degradation¹² and ground-motion scaling.¹³ However, a basic lack of agreement subsists in relation to the definition of adequate parameters of the system and its response.¹⁴ To date, most codes recommend an equivalent static procedure that prescribes minimum values of accidental eccentricity and focus on the evaluation of maximum orthogonal responses which are subsequently merged through simple combination rules. The adequacy of codified provisions to represent the complex torsional response of asymmetric buildings has been the subject of numerous assessments, for example^{15–17}, including proposals for the abolition of accidental eccentricity considerations altogether.¹⁸

The seismic response of eccentric buildings is particularly sensitive to the features of the ground motion¹⁹ and issues concerning the selection, scaling and application of ground-motion records deserve careful consideration.²⁰ Although real seismic motion involves three translational and three rotational components, and robust procedures for the estimation of rotational motions have now become available,²¹ it is still customary to neglect the effects of the rotational ground-motion components⁸ and rely on the application of orthogonal pairs of acceleration series.^{11,22,23} In this regard, several researchers have studied the influence of the record-pair orientation on the estimation of structural responses from non-linear response history analysis. Beyer and Bommer,²⁰ for instance, proposed the use of a random orientation as a suitable approximation leading to median structural response estimates. On the other hand, Giannopoulos and Vamvatsikos²⁴ have argued that, even for eccentric structures, using as many different record pairs as possible is more important than accounting for different orientations with a smaller number of ground motions.

Among the main ground-motion parameters, the duration of the excitation and its effects on eccentric structures has received virtually no attention in previous studies. Moreover, until very recently, ground-motion duration was considered immaterial for the estimation of peak displacements in all types of yielding systems^{25–27} and it was deemed significant only for the calculation of cumulative parameters like normalised hysteretic energy.²⁸ Nonetheless, the recent availability of simple non-linear hysteretic models capable of accounting for cyclic degradation^{29,30} has renewed the interest on the influence of ground-motion duration in structural demands. In this regard, the studies of Raghunandan and Liel³¹ and Chandramohan et al.³² were among the first to uncover a fundamental link between ground-motion duration and structural collapse. By analysing the response of 17 archetypical reinforced-concrete buildings subjected to 76 ground-motion records with a broad range of durations, Raghunandan and Liel³¹ found differences of 26–56% between the median collapse capacities obtained with long- and short-duration records. By contrast, Chandramohan et al.³² employed ‘spectrally equivalent’ record sets to isolate the effects of ground-motion duration on a five-story steel moment frame and a reinforced concrete bridge pier. They found that longer records are related to lower median collapse capacities, in the order of 17–29%, in comparison with shorter records. These pioneering studies have been followed by a long line of subsequent works that have sought to re-confirm and re-quantify the statistical differences between deformation response estimates under short- and long-duration earthquakes, sometimes without significant physical insight. However, formal dimensional analysis has not been applied to the study of strong-motion duration effects on peak deformation demands of eccentric structures with or without degradation. Consequently, this paper seeks to employ dimensional and orientational analyses

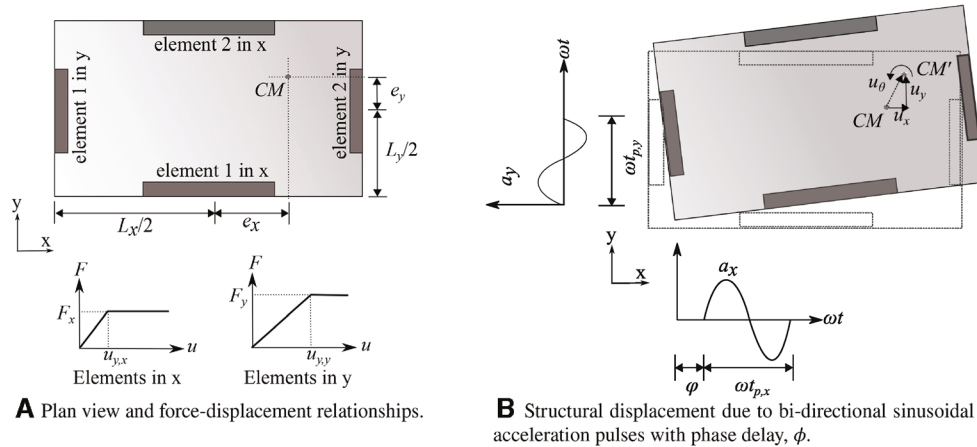


FIGURE 1 Structure with reflectionally symmetric stiffness and eccentric centre of mass [Correction added on September 30, 2020 after online publication: Figure 1b was replaced with the correct version.]

to unveil the fundamental role played by earthquake duration on the response of degrading structures under bidirectional motion.

The application of dimensional analysis to the response of non-linear structures subjected to pulse-like ground motions was introduced by Makris and Black^{33,34} with reference to rigid-plastic, elastic-plastic and bi-linear unidirectional oscillators. They showed that when the structural response is presented in terms of dimensionless Π -terms, remarkable order emerges and the maximum relative dimensionless deformations become self-similar. The approach hinges around the identification of a ground-motion characteristic length scale, $L_g = a_g \omega_g^{-2}$, where a_g and ω_g are the amplitude and circular frequency of the most energetic component of the record. Following this principle, dimensional analysis has been applied to a wide variety of problems in earthquake engineering including rocking structures,^{35,36} steel structures,³⁷ seismic control devices^{38,39} and fragility analysis.^{36,40} Most of these studies have employed records with coherent pulses and clearly identifiable time and length scales⁴¹ due to the difficulties associated with the selection of an adequate set of ground-motion time and length-scale parameters for non-coherent earthquake actions.^{37,42}

This paper revisits the seismic response scaling of non-degrading and degrading structures with various degrees of eccentricity with the aid of dimensional and orientational analyses. The study uncovers the fundamental physical similarities that shape the peak deformation of eccentric structures under earthquake actions and highlights the key role played by the ground-motion duration in the response scaling of degrading systems. After describing the structural models under consideration, the application of Vaschy–Buckingham's Π -theorem^{43,44} and orientational homogeneity principles⁴⁵ to the peak orthogonal displacement response of non-degrading structures is presented. This is followed by the analysis of degrading structures where the property of self-similarity is unveiled only after the uniform duration of the ground motion is introduced as a timescale. Particular attention is given to the emergence of self-similarity in the response of orientationally consistent quantities. The existence of complete similarities in structural strength and eccentricity as well as in the difference between the frequencies and phase of the two orthogonal components of the ground motion is explored next. The paper concludes with a brief discussion on the repercussions of its findings on the seismic analysis of structures under real pulse-like ground motions with varying degrees of coherency.

2 | BIDIRECTIONAL YIELDING STRUCTURES

Figure 1A presents a plan view of the typical generic structure examined in this study. The structural system under consideration comprises four edge lateral-resisting elements, two per each orthogonal direction, connected by means of a rigid diaphragm of dimensions L_x and L_y . Only structures with reflectionally symmetric stiffness and strength distribution are considered herein such that their centre of resistance coincides with the geometric centre of their diaphragm. An eccentric CM is assumed, with eccentricities e_x and e_y along the x and y axes, respectively. The vertical deformation and out-of-plane stiffness of the structural elements is disregarded and their in-plane force-deformation relationships can be fully characterised by their specific strengths, F_x or F_y , and yield displacements, $u_{y,x}$ or $u_{y,y}$, both assumed to act in the direction of the orthogonal x -axis or y -axis, respectively. While this structural model is of an elementary

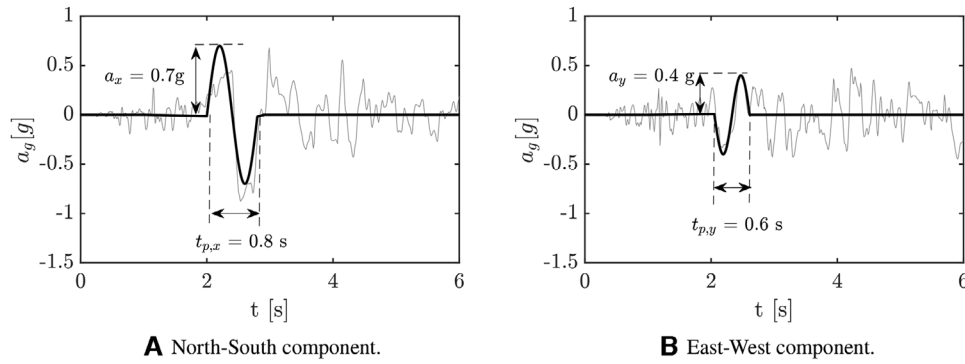


FIGURE 2 Orthogonal components of the acceleration time history recorded during the 1994 Northridge Earthquake, Rinaldi station, together with cycloidal pulses

nature, the problem of the torsional response of buildings under earthquakes is an already complicated matter and the selection of a model used in past studies (see Ref. [13] and references therein) with a small number of input parameters of unambiguous physical meaning is deemed adequate to answer the main question of this paper which is to determine whether ground-motion duration plays any role in the scaling of peak torsional displacements and under which circumstances.

Figure 1B depicts the structure under the action of a ground motion described by two orthogonal components aligned in the x and y directions. The system is free to translate and rotate in the x - y plane and the corresponding displacement of the CM is also shown in Figure 1B. Any motion of the system can be reconstructed with reference to the orthogonal translations of the CM (u_x and u_y) and its rotation (u_θ). Moreover, earthquakes with different angles of attack can be resolved into components along two fundamental orthogonal directions. The ground-motion components are represented in the amplitude phase space in Figure 1B and are fully defined by means of their corresponding amplitudes, a_x and a_y , and periods, $t_{p,x}$ and $t_{p,y}$, or their circular frequencies $\omega_{g,x} = 2\pi/t_{p,x}$ and $\omega_{g,y} = 2\pi/t_{p,y}$, as well as the phase lag ϕ . Thus, the current practice of neglecting the rotational component of the earthquake is adopted herein.

Ideal sinusoidal pulses are employed in each orthogonal direction. These pulses are defined as⁴⁶

$$\ddot{u}_{g,y}(t) = \omega_{g,y} \frac{v_{g,y}}{2} \sin(\omega_{g,y}t), \quad 0 \leq t \leq t_{p,y} \quad (1)$$

$$\ddot{u}_{g,x}(t) = \omega_{g,x} \frac{v_{g,x}}{2} \sin(\omega_{g,x}t + \phi), \quad \phi/\omega_{g,x} \leq t \leq t_{p,x} + \phi/\omega_{g,x}, \quad (2)$$

where $v_{g,x}$ and $v_{g,y}$ are the velocity amplitudes that are related to their corresponding acceleration amplitudes as $a_y = \omega_{g,y} \frac{v_{g,y}}{2}$ and $a_x = \omega_{g,x} \frac{v_{g,x}}{2}$, respectively, and $\omega_{g,x}$ and $\omega_{g,y}$ are the cycloidal pulse frequencies. In the case of single pulse sinusoids, the total duration of the pulse is the same as its period.

It should be noted that distinguishable pulses dominate a wide range of earthquake ground motions and can be formally extracted with validated mathematical models.^{33,41} Figure 2 shows the acceleration series of the Northridge earthquake recorded at the Rinaldi station together with sinusoidal pulses defined by their corresponding amplitudes and periods.

The use of fundamental orthogonal directions for both structural response and ground-motion parameters has the advantage of defining an un-ambiguous orientational framework.⁴⁵ Indeed, orientational analysis principles demand that the variables under consideration have a predefined set of orientations instead of orientations that change over time. This also hints to the formal orientational superiority of peak orthogonal displacements as structural demand parameters in contrast with other measures like the maximum geometric mean, whose orientation is time-dependent.^{20,47}

3 | DIMENSIONAL-ORIENTATIONAL ANALYSIS OF ECCENTRIC STRUCTURES

In order to apply dimensional analysis and find general scaling laws that describe the response of the system introduced in the preceding section, it is necessary to define appropriate ground-motion time and length scales (i.e. T_g and L_g). In

TABLE 1 Multiplication table of the unit orientations

	l_0	l_x	l_y	l_z
l_0	l_0	l_x	l_y	l_z
l_x	l_x	l_0	l_z	l_y
l_y	l_y	l_z	l_0	l_x
l_z	l_z	l_y	l_x	l_0

particular, two relevant ground-motion length scales, L_g , can be identified,³³ one per each orthogonal component as

$$L_{g,y} = a_y \omega_{g,y}^{-2} \quad (3)$$

$$L_{g,x} = a_x \omega_{g,x}^{-2}. \quad (4)$$

These length scales reflect the persistence of the earthquake action and are characteristic to it.³³ Likewise, the two natural ground-motion timescales, T_g , are

$$T_{g,y} = t_{p,y} = 2\pi \omega_{g,y}^{-1} \quad (5)$$

$$T_{g,x} = t_{p,x} = 2\pi \omega_{g,x}^{-1}. \quad (6)$$

Therefore, the parameters governing the seismic response of the yielding system described above (Figure 1) are the structural strengths, F_x and F_y ; the yield displacements, $u_{y,x}$ and $u_{y,y}$; the mass of the system, m ; the plan eccentricities, e_x and e_y ; the diaphragm dimensions, L_x and L_y ; the phase lag, ϕ ; the acceleration amplitudes, a_x and a_y ; and the ground-motion circular frequencies, $\omega_{g,x}$ and $\omega_{g,y}$. If a constant viscous damping ($\xi = 5\%$ of the critical value) is assumed in all cases, any response parameter of the system, $u_{response}$, can be expressed as a function of

$$u_{response} = f(a_x, a_y, \omega_{g,x}, \omega_{g,y}, F_x, F_y, m, u_{y,x}, u_{y,y}, e_x, e_y, L_x, L_y, \phi). \quad (7)$$

This results in a group of 15 characteristic variables involving 3 reference dimensions, those of length $[L]$, time $[T]$ and mass $[M]$. The Vaschy–Buckingham Π -theorem^{43,44} dictates that *if an equation involving n variables is dimensionally homogeneous, it can be reduced to a relationship among $n - k$ independent dimensionless Π -products where k is the minimum number of reference dimensions involved*. Therefore, if the characteristic length of the ground motion along the x -axis, $L_{g,x}$, is used for normalisation purposes, Equation (7) is reduced to (15 variables)–(3 reference dimensions)=12 dimensionless Π -terms as follows:

$$\frac{u_{response}}{L_{g,x}} = \Phi\left(\frac{F_x}{mL_{g,x}\omega_{g,x}^2}, \frac{F_y}{mL_{g,x}\omega_{g,x}^2}, \frac{u_{y,x}}{L_{g,x}}, \frac{u_{y,y}}{L_{g,x}}, \frac{e_x}{L_{g,x}}, \frac{e_y}{L_{g,x}}, \frac{L_x}{L_{g,x}}, \frac{L_y}{L_{g,x}}, \frac{L_{g,y}}{L_{g,x}}, \frac{\omega_{g,y}}{\omega_{g,x}}, \phi\right) \quad (8)$$

with $\Pi_{response} = \frac{u_{response}}{L_{g,x}}$, $\Pi_{F,x} = \frac{F_x}{mL_{g,x}\omega_{g,x}^2}$, $\Pi_{F,y} = \frac{F_y}{mL_{g,x}\omega_{g,x}^2}$, $\Pi_{u_{y,x}} = \frac{u_{y,x}}{L_{g,x}}$, $\Pi_{u_{y,y}} = \frac{u_{y,y}}{L_{g,x}}$, $\Pi_{e,x} = \frac{e_x}{L_{g,x}}$, $\Pi_{e,y} = \frac{e_y}{L_{g,x}}$, $\Pi_{L,x} = \frac{L_x}{L_{g,x}}$, $\Pi_{L,y} = \frac{L_y}{L_{g,x}}$, $\Pi_{L_g} = \frac{L_{g,y}}{L_{g,x}}$, $\Pi_{\omega_g} = \frac{\omega_{g,y}}{\omega_{g,x}}$, $\Pi_{\phi} = \phi$. This framework, defined by Equation (8) on the basis of the yield displacements, $u_{y,x}$ and $u_{y,y}$, will be referred to as *Framework U* throughout this paper.

At this stage, it is important to recall that the selection of an appropriate response parameter, $u_{response}$, among the two alternative orthogonal responses, u_x and u_y , can be informed by orientational analysis principles.⁴⁵ To this end, l_x , l_y and l_z designate the unit orientations along the x , y and z axes, respectively; and l_0 represents an orientationless identity element. The multiplication rules of these orientational symbols are summarised in Table 1⁴⁵ where, for example $l_x l_x \doteq l_0$ or $l_x l_y \doteq l_z$, where the symbol $\doteq$ denotes orientational equality.

Besides,

$$l_x \doteq l_x^{-1} \quad (9)$$

$$l_x^{2n+1} \doteq l_x \quad (10)$$

$$l_x^{2n} \doteq l_0. \quad (11)$$

Accordingly, an angle θ in the x - y plane is l_z -oriented while a phase angle is orientationless l_0 . Following this reasoning, the orientations of the characteristic variables governing the response of the bidirectionally acted structure within Framework U presented above (Equation (8)) can be determined as $\frac{F_x}{mL_{g,x}\omega_{g,x}^2} \doteq \frac{l_x}{l_x} \doteq l_0$, $\frac{F_y}{mL_{g,x}\omega_{g,x}^2} \doteq \frac{l_y}{l_x} \doteq l_z$, $\frac{u_{y,x}}{L_{g,x}} \doteq \frac{l_x}{l_x} \doteq l_0$, $\frac{u_{y,y}}{L_{g,x}} \doteq \frac{l_y}{l_x} \doteq l_z$, $\frac{e_x}{L_{g,x}} \doteq \frac{l_x}{l_x} \doteq l_0$, $\frac{e_y}{L_{g,x}} \doteq \frac{l_y}{l_x} \doteq l_z$, $\frac{L_x}{L_{g,x}} \doteq \frac{l_x}{l_x} \doteq l_0$, $\frac{L_y}{L_{g,x}} \doteq \frac{l_y}{l_x} \doteq l_z$, $\frac{L_{g,y}}{L_{g,x}} \doteq \frac{l_y}{l_x} \doteq l_z$, $\frac{\omega_{g,y}}{\omega_{g,x}} \doteq \frac{l_0}{l_0} \doteq l_0$, $\phi \doteq l_0$. And the right-hand side of Equation (8) can be expressed in orientational terms as

$$\begin{aligned} & (\Pi_{F,x}, \Pi_{F,y}, \Pi_{u_{y,x}}, \Pi_{u_{y,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}) \\ & \rightarrow (l_0^{\epsilon_1}, l_z^{\epsilon_2}, l_0^{\epsilon_3}, l_z^{\epsilon_4}, l_0^{\epsilon_5}, l_z^{\epsilon_6}, l_0^{\epsilon_7}, l_z^{\epsilon_8}, l_0^{\epsilon_9}, l_0^{\epsilon_{10}}, l_0^{\epsilon_{11}}) \doteq l_z^{\epsilon_2+\epsilon_4+\epsilon_6+\epsilon_8+\epsilon_9} \doteq l_z, \end{aligned} \quad (12)$$

where, in its most straightforward form, the sum of $\epsilon_2 + \epsilon_4 + \epsilon_6 + \epsilon_8 + \epsilon_9 = 5$, an odd number. Although orientational analysis does not yield a definite value for the unknown exponents, ϵ_i , it is clear from the previous discussion that Equation (8) will be orientationally homogeneous if $u_{response} = u_y$ such that

$$\Pi_{response} = \Pi_{u,y} = \frac{u_y}{L_{g,x}} \doteq \frac{l_y}{l_x} \doteq l_z. \quad (13)$$

This choice of response variable yields a dimensionless–orientationless relationship that, as will be seen later, brings forward the property of self-similarity in the response. On the other hand, if the alternative ground-motion length scale along the y -axis, $L_{g,y}$, is selected for normalisation purposes, orientational consistency would demand the use of $u_{response} = u_x$ within Framework U. It is worth noting that the requirement to have response and ground-motion length parameters of different orthogonal orientations, as revealed by Equation (13), is particular to the set of characteristic variables employed (i.e. Equation (8)). Indeed, it is perfectly possible to use the structural circular frequencies ($\omega_{s,x} = \sqrt{k_x/m}$ and $\omega_{s,y} = \sqrt{k_y/m}$, where k_x and k_y are the stiffnesses of the structural elements aligned in the x and y directions) in lieu of the yield displacements ($u_{y,x}$ and $u_{y,y}$) to fully characterise the structural model of Figure 1. This alternative selection of characteristic parameters, referred throughout this paper as *Framework W*, yields the following dimensionless relationship:

$$\frac{u_{response}}{L_{g,x}} = \Phi \left(\frac{F_x}{mL_{g,x}\omega_{g,x}^2}, \frac{F_y}{mL_{g,x}\omega_{g,x}^2}, \frac{\omega_{s,x}}{\omega_{g,x}}, \frac{\omega_{s,y}}{\omega_{g,x}}, \frac{e_x}{L_{g,x}}, \frac{e_y}{L_{g,x}}, \frac{L_x}{L_{g,x}}, \frac{L_y}{L_{g,x}}, \frac{L_{g,y}}{L_{g,x}}, \frac{\omega_{g,y}}{\omega_{g,x}}, \phi \right). \quad (14)$$

Framework W (Equation (14)) is dimensionally equivalent but not orientationally equivalent to Framework U (Equation (8)). In this case, $\Pi_{\omega_{s,x}} = \frac{\omega_{s,x}}{\omega_{g,x}} \doteq l_0$ and $\Pi_{\omega_{s,y}} = \frac{\omega_{s,y}}{\omega_{g,x}} \doteq l_0$, leading to

$$\begin{aligned} & (\Pi_{F,x}, \Pi_{F,y}, \Pi_{\omega_{s,x}}, \Pi_{\omega_{s,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}) \\ & \rightarrow (l_0^{\epsilon_1}, l_z^{\epsilon_2}, l_0^{\epsilon_3}, l_0^{\epsilon_4}, l_0^{\epsilon_5}, l_z^{\epsilon_6}, l_0^{\epsilon_7}, l_z^{\epsilon_8}, l_0^{\epsilon_9}, l_0^{\epsilon_{10}}, l_0^{\epsilon_{11}}) \doteq l_z^{\epsilon_2+\epsilon_6+\epsilon_8+\epsilon_9} \doteq l_0, \end{aligned} \quad (15)$$

where $\epsilon_2 + \epsilon_6 + \epsilon_8 + \epsilon_9 = 4$, an even number; and therefore, the $u_{response}$ selection that renders Framework W (Equation (14)) orientationally consistent is $u_{response} = u_x$:

$$\Pi_{response} = \Pi_{u,x} = \frac{u_x}{L_{g,x}} \doteq \frac{l_x}{l_x} \doteq l_0. \quad (16)$$

Both frameworks, U and W, from Equations (8) and (14), are considered next to illustrate the importance of orientational consistency, a feature that plays a central role in the emergence of self-similarity in degrading systems as will be discussed later.

Undoubtedly, the most notable feature of the dimensional–orientational analysis described above is that it brings forward the property of self-similarity. To illustrate this, Figure 3 shows the response of a bidirectional yielding oscillator with normalised eccentricity $\Pi_{e,x} = \Pi_{e,y} = 0.1\Pi_{L,x}$ (10% accidental eccentricity) following Framework U. Figure 3A depicts the variation of the dimensionless peak displacement product, $\Pi_{u,x} = u_x/L_{g,x}$, against different dimensionless

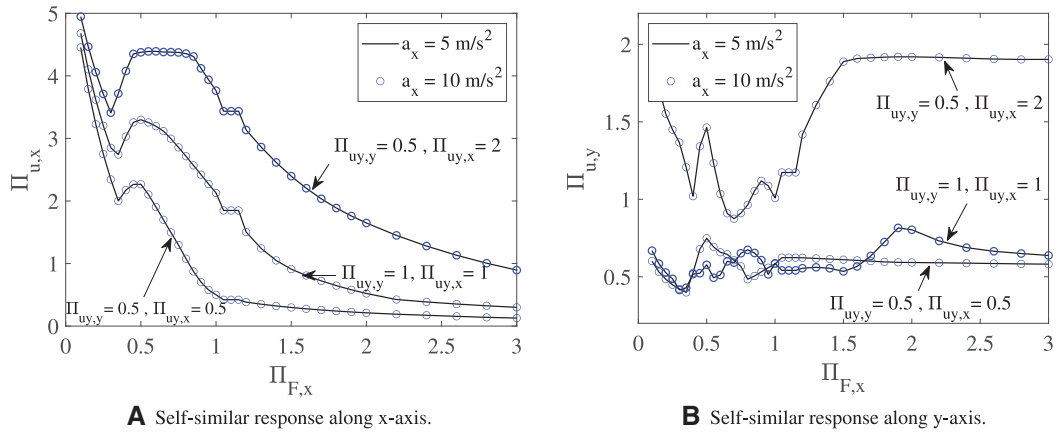


FIGURE 3 Self-similar response of eccentric oscillators subjected to sinusoidal pulse-type excitations of normal directivity
Note: $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{F,y} = 0.2$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$, $\Pi_{e,x} = \Pi_{e,y} = 0.1\Pi_{L,x}$; Framework U.

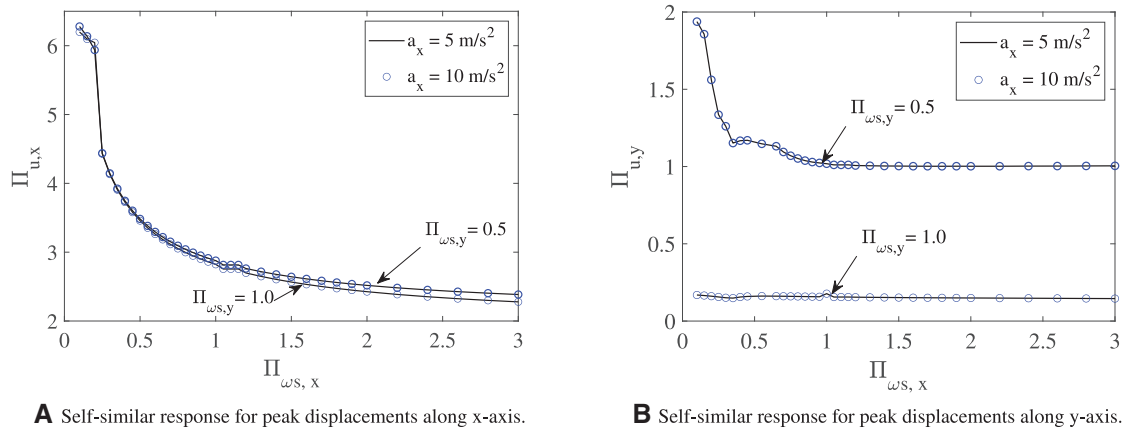


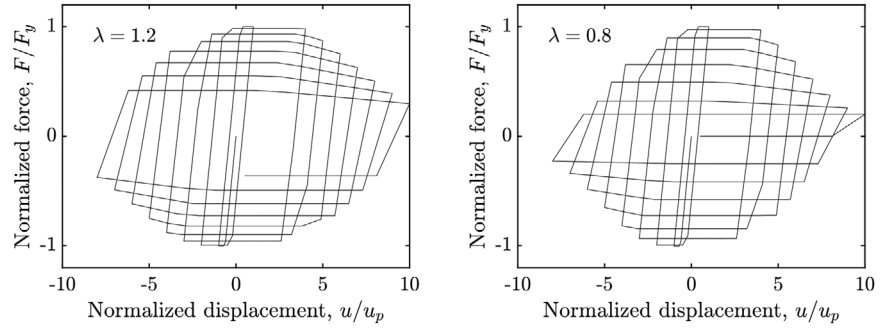
FIGURE 4 Self-similar response spectra of eccentric oscillators subjected to sinusoidal pulse-type excitations
Note: $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{F,x} = \Pi_{F,y} = 0.2$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$, $\Pi_{e,x} = \Pi_{e,y} = 0.1\Pi_{L,x}$; Framework W.

strengths, $\Pi_{F,x} = F_x / m L_{g,x} \omega_{g,x}^2$, while the corresponding plot for $\Pi_{u,y} = u_y / L_{g,x}$ is shown in Figure 3B. Results are presented for different combinations of dimensionless yield displacements, $(\Pi_{u,y,y}, \Pi_{u,y,x})$. A distinctive attribute of Figure 3 is that responses to different excitation intensities (e.g. $a_x = 5 \text{ m/s}^2$ and $a_x = 10 \text{ m/s}^2$) collapse into a single ‘master curve’ when expressed in dimensionless Π -terms. This scale invariance between the maximum relative displacement, the yield displacement, and the acceleration amplitude is known as self-similarity and plays a fundamental role in the understanding and ordering of the response of bidirectionally loaded structures such as those examined herein. In this regard, self-similarity, which is a form of symmetry, is important not only because it allows to reduce the number of independent variables of the problem but rather because it reflects a fundamental property of the phenomena distinguishable from the effects of accidental features of the problem. Notably, this scale symmetry holds even at very large non-linear deformation levels, that is, when $\Pi_{F,x} \ll 1$. It is also worth recalling that the results of Figure 3A (on the left) are dimensionless but not orientationless, while the curves of Figure 3B (on the right) represent both dimensionally and orientationally homogeneous relationships.

An alternative presentation of the response of eccentric systems to bidirectional ground motion is offered in Figure 4 where the self-similar response spectra are shown. To this end, the formulation presented in Equation (14) in terms of structural frequencies (i.e. Framework W) is adopted. Figure 4 plots the dimensionless peak displacements for constant dimensionless structural frequencies in the y direction, $\Pi_{\omega s,y}$, and for varying values of structural frequencies in the x direction, $\Pi_{\omega s,x}$. In this case, Figure 4A (on the left) presents dimensionless–orientationless relationships, while the plots of Figure 4B (on the right) represent dimensionally consistent but orientationally inconsistent relationships. As the frequency ratio (dimensionless $\Pi_{\omega s,x}$ -product) increases from left to right in Figure 4 the structure becomes stiffer along its

FIGURE 5 Degrading hysteretic relationships for different degradation ratios λ

Note: Low degradation (left) and more severe degradation (right).



x-axis. It can be seen from Figure 4A that structures with more flexible x-oriented elements (e.g. $\Pi_{\omega s, x} < 1$), have peak responses that are less dependent on the variation of stiffness in the orthogonal direction (y structural axis), thus the weaker axis dominates the response, a feature that has been suggested in other studies⁷ and is now fully proved.

4 | BIDIRECTIONAL DEGRADING STRUCTURES

The preceding sections have dealt with the response of non-degrading structures for which the effects of cyclic deterioration in stiffness or strength can be neglected. As such, they follow a wealth of previous studies^{9–11,33,34} that have arrived to important findings based on non-degrading structural models. Indeed, non-degrading systems have been used to uncover the decisive property of self-similarity in the response of a variety of structures,^{33,34,37,42} a property that, as has been demonstrated in this paper, also shapes the non-linear behaviour of bidirectionally loaded eccentric structures.

This section deals with the dimensional–orientational analysis of degrading structures and its implications. To this end, the eccentric structure described before (Figure 1A) is modelled in two dimensions in the open-source finite element framework OpenSees⁴⁸ following a key modification of its hysteretic relationships to incorporate degradation effects. More specifically, the unidirectional force-displacement relationships assigned to the edge elements of the plan asymmetric structure of Figure 1A are now replaced with unidirectional degrading relationships of the modified Ibarra–Medina–Krawinkler (IMK) type.^{29,30} The cyclic deterioration of the IMK model is controlled by a rule, initially developed by Rahnama and Krawinkler,⁴⁹ that relies on the hysteretic energy dissipated when the structural component is subjected to cyclic loads. The main assumption of this model is that every dissipating structural component has a reference hysteretic energy dissipation capacity, E_t , regardless of the loading history applied to it. This reference hysteretic energy dissipation capacity is expressed as a multiple of the structural deformation, u_p , and the underlying action force, F_{yield} , at yield. Therefore, for a structural component (or element) oriented in the i direction, with a given degradation rate, λ :

$$E_{t,i} = \lambda \cdot F_{yield,i} \cdot u_{p,i} \text{ or } E_{t,i} = \Lambda \cdot F_{yield,i} \quad (17)$$

with

$$\Lambda = \lambda \cdot u_{p,i}, \quad (18)$$

where Λ denotes the reference cumulative deformation capacity. Typical values of λ can be as low as 0.8³⁰; the lower the λ , the higher the degradation effects. Figure 5 presents examples of degrading hysteretic responses with elastic-plastic backbones obtained by solely varying the deterioration rate within a realistic range.³⁰ It is acknowledged that this degradation model is an idealisation of a very complex phenomenon and that its definition relies on calibration studies, it neglects an explicit consideration of the effects of multi-axial interactions and pinching, and it cannot simulate element shortening or yielding in arbitrary locations as other more sophisticated models.^{50,51} However, the main advantage of deterioration models such as the one adopted herein lies in their simplicity and versatility which allows them to represent a family of deterioration responses from a range of observations.^{30,52} Moreover, since various cyclic deterioration modes can be expressed by a single parameter, λ , this model is well suited for the dimensional analysis carried out herein in an effort to limit the dimensional space of study and obtain fundamental behavioural relationships.

It follows from the previous discussion that, in the case of degrading systems subjected to bidirectional ground motion with constant viscous damping, $\xi = 0.05$, if an equal degradation rate, λ , is assumed for all elements while keeping all

other hysteretic rule parameters constant, any structural response, $u_{response}$, of the system can be expressed, according to Framework U, as a function of

$$u_{response} = f(a_x, a_y, \omega_{g,x}, \omega_{g,y}, F_x, F_y, m, u_{y,x}, u_{y,y}, e_x, e_y, L_x, L_y, \phi, \lambda). \quad (19)$$

Alternatively, the equivalent relationship following Framework W is

$$u_{response} = f(a_x, a_y, \omega_{g,x}, \omega_{g,y}, F_x, F_y, m, \omega_{s,x}, \omega_{s,y}, e_x, e_y, L_x, L_y, \phi, \lambda). \quad (20)$$

5 | DIMENSIONAL-ORIENTATIONAL ANALYSIS OF DEGRADING STRUCTURES

The fundamental difference between non-degrading and degrading bidirectionally loaded structures is the need to account for the degradation parameter λ in the latter case. This results in a group of 16 characteristic variables involving three reference dimensions $[L, M, T]$, hence, the Vaschy–Buckingham's Π -theorem^{43,44} leads to a reduction of the number of dimensionless Π -products to 13. If, as it was done before for non-degrading systems in Equation (8), the characteristic length of the ground motion along the x-axis ($L_{g,x}$) is used for normalisation purposes, Equation (19) (Framework U) can be re-cast in dimensionless terms as

$$\frac{u_{response}}{L_{g,x}} = \Phi\left(\frac{F_x}{mL_{g,x}\omega_{g,x}^2}, \frac{F_y}{mL_{g,x}\omega_{g,x}^2}, \frac{u_{y,x}}{L_{g,x}}, \frac{u_{y,y}}{L_{g,x}}, \frac{e_x}{L_{g,x}}, \frac{e_y}{L_{g,x}}, \frac{L_x}{L_{g,x}}, \frac{L_y}{L_{g,x}}, \frac{L_{g,y}}{L_{g,x}}, \frac{\omega_{g,y}}{\omega_{g,x}}, \phi, \lambda\right). \quad (21)$$

Or, what is the same:

$$\Pi_{u,y} = \Phi(\Pi_{F,x}, \Pi_{F,y}, \Pi_{u_{y,x}}, \Pi_{u_{y,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}, \Pi_{\Lambda}). \quad (22)$$

Since λ is dimensionless the dimensional homogeneity of Equations (21) and (22) is preserved.

Alternatively, if structural frequencies instead of yield displacements are employed to characterise the structural response (in line with Framework W), Equation (20) can be re-written as

$$\Pi_{u,x} = \Phi(\Pi_{F,x}, \Pi_{F,y}, \Pi_{\omega_{s,x}}, \Pi_{\omega_{s,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}, \Pi_{\Lambda}), \quad (23)$$

which is also dimensionally homogeneous. Moreover, since the degradation ratio λ is orientationless, the orientational consistency of Equations (22) and (23) is also upheld. This can be verified by observing that $\Pi_{\Lambda} = \lambda \doteq l_0^{\epsilon_{12}}$ leading to

$$\begin{aligned} & (\Pi_{F,x}, \Pi_{F,y}, \Pi_{u_{y,x}}, \Pi_{u_{y,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}, \Pi_{\Lambda}) \\ & \rightarrow (l_0^{\epsilon_1}, l_z^{\epsilon_2}, l_0^{\epsilon_3}, l_z^{\epsilon_4}, l_0^{\epsilon_5}, l_z^{\epsilon_6}, l_0^{\epsilon_7}, l_z^{\epsilon_8}, l_0^{\epsilon_9}, l_z^{\epsilon_{10}}, l_0^{\epsilon_{11}}, l_0^{\epsilon_{12}}) \doteq l_z^{\epsilon_2 + \epsilon_4 + \epsilon_6 + \epsilon_8 + \epsilon_9} \end{aligned} \quad (24)$$

in the case of Framework U (Equation (22)), and to

$$\begin{aligned} & (\Pi_{F,x}, \Pi_{F,y}, \Pi_{\omega_{s,x}}, \Pi_{\omega_{s,y}}, \Pi_{e,x}, \Pi_{e,y}, \Pi_{L,x}, \Pi_{L,y}, \Pi_{L_g}, \Pi_{\omega_g}, \Pi_{\phi}, \Pi_{\Lambda}) \\ & \rightarrow (l_0^{\epsilon_1}, l_z^{\epsilon_2}, l_0^{\epsilon_3}, l_0^{\epsilon_4}, l_0^{\epsilon_5}, l_z^{\epsilon_6}, l_0^{\epsilon_7}, l_z^{\epsilon_8}, l_z^{\epsilon_9}, l_0^{\epsilon_{10}}, l_0^{\epsilon_{11}}, l_0^{\epsilon_{12}}) \doteq l_z^{\epsilon_2 + \epsilon_6 + \epsilon_8 + \epsilon_9} \end{aligned} \quad (25)$$

in the case of Framework W (Equation (23)), and noting that both Equations (24) and (25) are orientationally equivalent to Equations (12) and (15), respectively.

The dimensional-orientational analysis presented above is only valid if the parameters governing the structural response are fully considered and unambiguously defined. As highlighted in the introduction, the duration of the excitation has long been considered immaterial for the estimation of peak displacements and is only recently that its role in the quantification of the collapse potential of degrading systems (i.e. large deformation range) has been recognised. In this context, the question of whether the ground-motion duration has any place among the dominant parameters that govern the peak displacement response at lower displacement demand levels is explored herein. This will be done by

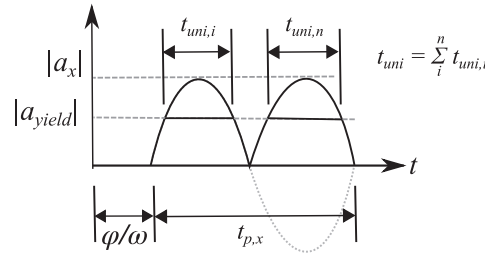
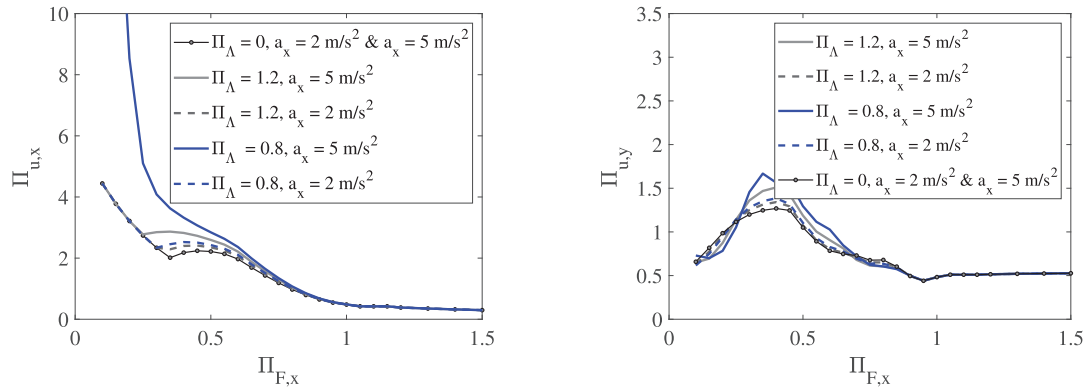


FIGURE 6 Uniform duration, t_{uni} , compared with the total duration, t_p , of the pulse



A Response spectra for peak displacements along x-axis in degrading structures (Framework W).

B Peak displacements along x-axis as a function of dimensionless strength (Framework U).

FIGURE 7 Dimensionless peak displacement for degrading structures with different degradation ratios, Π_Λ , when the total duration of the ground motion, t_p , is used as timescale (i.e. $L_{g,x} = a_x t_{p,x}^2 / 4\pi^2$)

Note: $\Pi_{F,y} = 0.2$, $\Pi_{u,y,x} = \Pi_{u,y,y} = 0.5$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.10\Pi_{L,x}$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_\phi = 0$; Framework U.

explicitly considering the uniform duration of the ground motion (t_{uni}) rather than its period (t_p) as a timescale. In other words, by assuming $T_g = t_{uni}$ instead of $T_g = t_p$.

Using $T_g = t_{uni}$ means that the ground-motion characteristic length, $L_{g,x}$, is now defined as

$$L_{g,x} = \frac{a_x t_{uni,x}^2}{4\pi^2}, \quad (26)$$

where a_x is the amplitude of the ground acceleration along the x-axis (Figure 1). Figure 6 illustrates the definition of t_{uni} and t_p for a particular harmonic pulse plotted in absolute ordinate values. More specifically, the uniform duration, t_{uni} , is defined as the summation of the time intervals during which the absolute acceleration exceeds an acceleration value associated with an equivalent static force that would cause yielding in the structure, a_{yield} . This is because in degrading systems, or at least in those of the type employed in this study, degradation in strength and stiffness is proportional to the dissipated energy through λ which starts to accumulate only after yielding. This renders the parts of the ground motion associated with demands below the plastic limit less relevant and explains the selection of a_{yield} as the appropriate threshold for the quantification of the ground-motion uniform duration.

In the case of yielding but non-degrading systems under cycloidal pulses, the period of the pulse and its total duration coincide, hence the ground-motion duration effects are concealed. However, if the period of the pulse is employed as the ground-motion timescale in degrading structures (i.e. if $T_g = t_p$ as for the yielding systems of the previous section) self-similarity is lost. This is demonstrated in Figures 7 and 8 where $L_{g,x} = a_x t_{p,x}^2 / 4\pi^2$ and different degradation ratios, Π_Λ , are considered. It can be seen from these figures that the peak structural response over different ranges of normalised frequency and strength deviates from each other. That is to say, the response of a degrading system when the period of the ground motion is employed as the timescale is not self-similar.

The departure from self-similarity in the response of degrading systems when t_p rather than t_{uni} is considered as a timescale highlights the fundamental dependency of peak deformations on the hysteretic energy potential of the ground

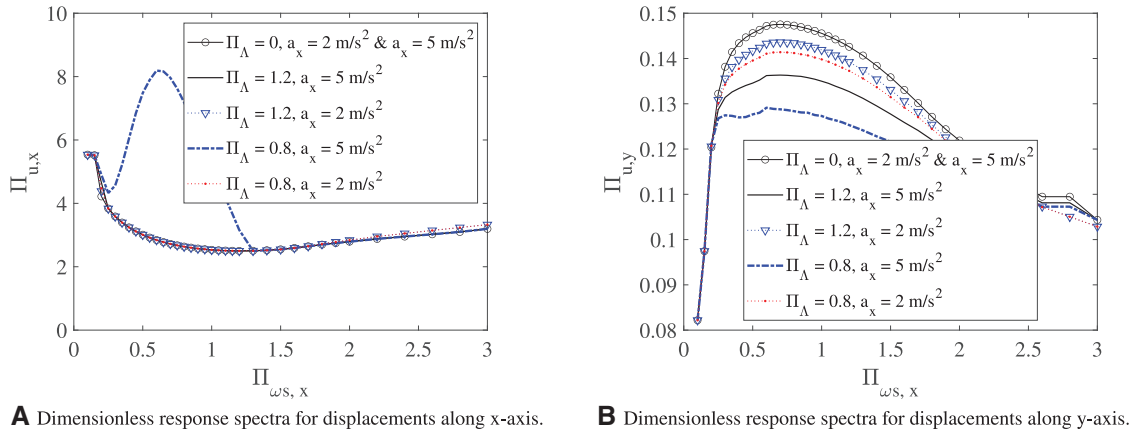


FIGURE 8 Dimensionless response spectra for degrading structures with different degradation ratios, Π_Λ , when the total duration of the ground motion, t_p , is used as timescale (i.e. $L_{g,x} = a_x t_{p,x}^2 / 4\pi^2$)

Note: $\Pi_{F,x} = \Pi_{F,y} = 0.2$, $\Pi_{\omega s,y} = 1.0$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.10\Pi_{L,x}$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_\phi = 0$; Framework W.

motion and not only on its overall persistence. This can be further examined with reference to Figure 7 which shows dimensionless plots of peak deformations against strength (Framework U). A value of $\Pi_{F,x} < 1$ indicates a yielding structure which has experienced a degree of hysteretic energy dissipation. Structures with dimensionless yield displacements of $\Pi_{u,y,y} = \Pi_{u,y,x} = 0.5$ are considered in this plot but the same patterns were observed for other parameter combinations. Figure 7 demonstrates that as soon as yielding takes place, the peak deformations of degrading structures with the same degradation ratio, Π_Λ , but subjected to earthquakes of different intensities start to deviate from each other. Moreover, higher degradation ratios and higher earthquake intensities lead to proportionally larger displacements. Interestingly, for highly non-linear behaviour, $\Pi_{F,x} < 0.27$, the responses tend to re-converge towards a common curve, except for the extreme case of $\Pi_\Lambda = 0.8$ and $a_x = 5 \text{ m/s}^2$. This may be attributed to the fact that very low values of $\Pi_{F,x}$ are associated with larger ratios of a_x/a_{yield} causing t_{uni} to approximate t_p , however, as the demand increases further (e.g. for $\Pi_\Lambda = 0.8$, $\Pi_{F,x} < 0.25$ and $a_x = 5 \text{ m/s}^2$), severe degradation ensues leading to structural instabilities and more pronounced displacements. The departure from self-similarity is appreciated in both peak orthogonal displacements, $\Pi_{u,x}$ and $\Pi_{u,y}$, in Figures 7 and 8.

In general, Figures 7 and 8 support the argument that the period of the ground motion, t_p , is unable to fully characterise the displacement demands when applied to degrading eccentric structures. On the other hand, when the uniform duration of the pulse, t_{uni} , is employed as a timescale (i.e. when $T_g = t_{uni}$) all dimensionless-orientationless curves tend to collapse into a single master curve and approximate self-similarity emerges. This finding is depicted in Figure 9 that shows the evolution of dimensionless $\Pi_{u,x}$ - and $\Pi_{u,y}$ -products for degrading structures with different degradation ratios, Π_Λ , when the uniform duration of the ground motion is used as the timescale (hence $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$). Curves are presented for different dimensionless yield displacements products ($\Pi_{u,y,y} = \Pi_{u,y,x} = 0.5$ in Figure 9A,B, and $\Pi_{u,y,y} = \Pi_{u,y,x} = 1.0$ in Figure 9C,D). It should be noted that unlike t_p , t_{uni} is properly defined only when the ground motion is strong enough to cause structural yielding, namely when $a_x > a_{yield}$ as per Figure 6. This is the reason why only the ranges of $\Pi_{F,x}$ -products where inelastic response is attained are represented in Figure 9. These ranges depend on the particular structural configuration under study, since different structural systems will be associated with different yielding thresholds, and are different for systems with $\Pi_{u,y,y} = \Pi_{u,y,x} = 0.5$ and $\Pi_{u,y,y} = \Pi_{u,y,x} = 1.0$.

It is also important to note that in the case of degrading structures both dimensional and orientational consistency (Figure 9B,D) is required in order to reveal approximate self-similarity in their response. In this respect, it is helpful to recall that when structural yield displacements are employed as independent variables (under Framework U, Equation (23)), orientational homogeneity demands that $u_{response} = u_y$. The dimensionless-orientationless relationships defined in those terms and depicted in Figure 9B,D tend to collapse into a single 'master curve'. The negligible differences encountered in those curves can be attributed to small numerical discrepancies of the degrading model which is notoriously challenging to converge under non-hardening conditions such as those studied herein. By contrast, the orientationally inhomogeneous curves of Figure 9A,C do not collapse into a single curve. Moreover, as noted before, the differences between Π -curves in Figure 9A,C are more significant for structures with more pronounced degrading behaviour subjected to intense ground motions (e.g. $\Pi_\Lambda = 0.8$ with $a_x = 5 \text{ m/s}^2$) which stresses the importance of degradation effects.

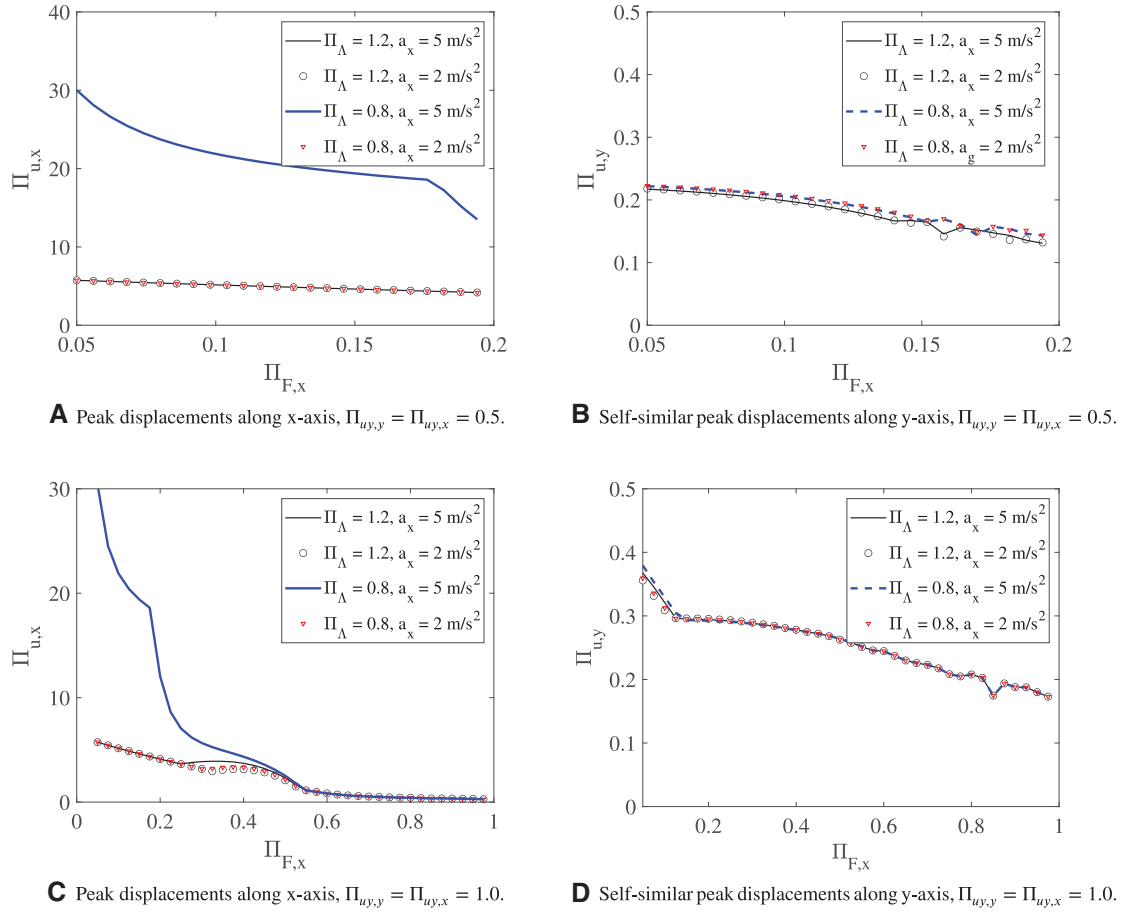


FIGURE 9 Dimensionless peak displacement for degrading structures with different degradation ratios, Π_Λ , when the uniform duration of the ground motion, t_{uni} , is used as timescale (i.e. $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$)

Note: $\Pi_{F,y} = 0.2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.10\Pi_{L,x}$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_\phi = 0$.

6 | ANALYSIS OF COMPLETE SELF-SIMILARITIES

The dimensional–orientational analyses presented above have demonstrated that the emergence of self-similarity in the peak orthogonal displacements of degrading systems is determined by the choice of t_{uni} as a timescale within an orientationally consistent framework. Therefore, the fundamental role played by the ground-motion duration as a timescale governing the scaling of the peak displacement response of degrading and eccentric systems at all demand levels, large and small, has been uncovered. The discovery of self-similarity makes manifest their intermediate asymptotic behaviour,⁵³ or more precisely in this case, it reveals the range of responses in which their dimensionless displacements cease to depend on the intensity of the ground motion. To that end, dimensionless–orientationless relationships of the following form have been formulated:

$$\Pi_{response} = \Phi(\Pi_1, \dots, \Pi_{n-k}), \quad (27)$$

where n is the number of dimensional parameters and k is the number of dimensions involved. However, there are cases in which, without loss of precision, Φ can be replaced by a new function Φ_0 of the form:⁵³

$$\Pi_{response} = \Phi_0(\Pi_1, \dots, \Pi_{i-1}, \Pi_{i+1}, \dots, \Pi_{n-k}). \quad (28)$$

In such cases,

$$\Phi_0 = \lim_{\Pi_i \rightarrow 0/\infty} \Phi \quad (29)$$

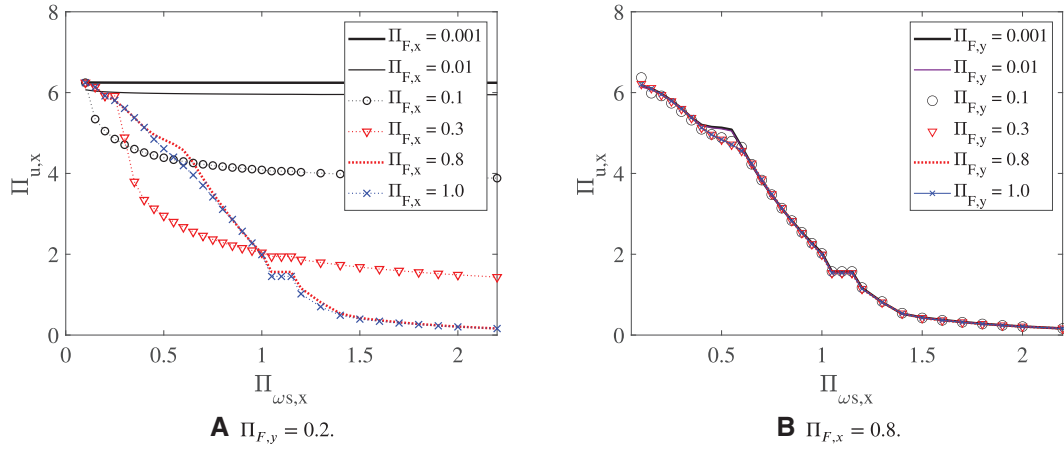


FIGURE 10 Self-similar response spectra for displacements along the x-axis in yielding structures when the total duration of the ground motion, t_p , is used as timescale (i.e. $L_{g,x} = a_x t_{p,x}^2 / 4\pi^2$)

Note: $\Pi_{\omega S,y} = 0.5$, $\Pi_{L,x} = \Pi_{L,y} = 80$, $\Pi_{e,x} = \Pi_{e,y} = 0.10\Pi_{L,x}$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$; Framework W.

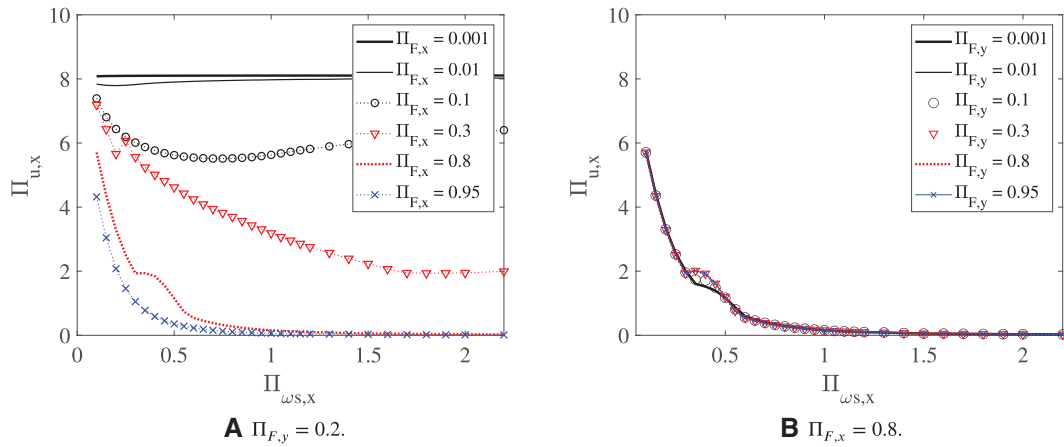


FIGURE 11 Self-similar response spectra for displacements along the x-axis in degrading structures when the uniform duration of the ground motion, t_{uni} , is used as timescale (i.e. $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$)

Note: $\Pi_{\omega S,y} = 0.5$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.10\Pi_{L,x}$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$, $\Pi_{\Lambda} = 1.0$; Framework W.

and Π_i is sufficiently small or large. When this happens, it is said that Π_i 's influence is negligible and that complete self-similarity or similarity of the first kind exists with respect to Π_i . Naturally, this holds only if

$$\lim_{\Pi_i \rightarrow 0/\infty} \Phi \neq 0 \text{ and finite.} \quad (30)$$

This section will examine the complete self-similarities of this kind. For reasons of expository convenience, only the dimensionally and orientationally consistent Framework W, defined in terms of structural frequencies and $\Pi_{u,x}$ displacements will be employed hereinafter.

6.1 | Complete similarity in structural strength

The influence of the dimensionless strength is examined first, with reference to Figures 10 and 11. Figure 10 depicts the results of elastic-plastic non-degrading systems while Figure 11 shows the corresponding graphs for degrading structures. These figures illustrate the evolution of peak dimensionless-orientationless displacements against structural frequency Π -products for different values of dimensionless strength $\Pi_{F,x}$ and $\Pi_{F,y}$. As the frequency ratio $\Pi_{\omega S,x}$ increases, the structure

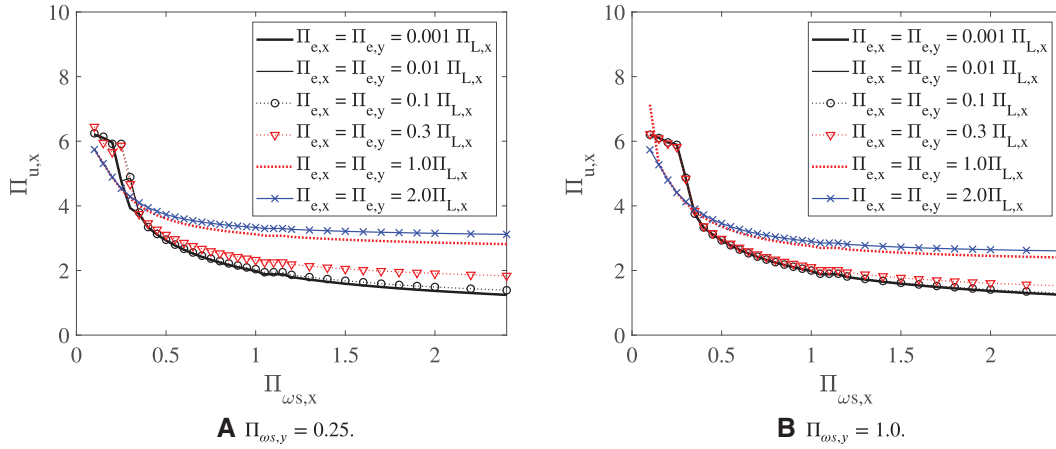


FIGURE 12 Self-similar response spectra for displacements along the x -axis in yielding structures when the total duration of the ground motion, t_p , is used as timescale (i.e. $L_{g,x} = a_x t_{p,x}^2 / 4\pi^2$)

Note: $\Pi_{F,x} = 0.3$, $\Pi_{F,y} = 0.2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$; Framework W.

becomes stiffer along its x -axis. It is evident from these figures that, generally, larger normalised strengths lead to smaller normalised displacements, as expected. The only exceptions are systems with very low strength or those subjected to overwhelmingly large ground-motion intensities (i.e. $\Pi_{F,x} = 0.01$ or less).

Moreover, when the normalised structural strength approaches a very small or a very large value in Figures 10A and 11A, the curves approach a non-zero finite limit. This hints to the existence of two states of complete similarity in the dimensionless strength. One when the strength of the system is sufficiently small in comparison with the intensity of the ground motion ($\Pi_{F,x} \approx 0.001$), and another when the demand of the ground motion is sufficiently small in comparison with the structural strength ($\Pi_{F,x} \approx 1$). What is perhaps more interesting is the practical invariance observed for dimensionless displacements, $\Pi_{u,x}$, with respect to $\Pi_{F,y}$ when the strength along the x -axis is appreciable (i.e. when $\Pi_{F,x} = 0.8$ in Figures 10B and 11B). Recalling that $\Pi_{Lg} = 0.8$ indicates a ground motion whose length scale is 20% longer in the x direction than in the y direction, the practically complete invariance of $\Pi_{u,x}$ with respect to $\Pi_{F,y}$ hints to the peak displacements along the line of action of the strongest ground-motion component being insensitive to the structural strength in the perpendicular axis. In this respect, Figures 10 and 11, or similarly obtained graphs, can be used in the preliminary design of eccentric systems to get the first-level approximations of stiffness and strength ratios and to evaluate potential ranges where the strength of the structure is immaterial to the maximum displacement response of the system.

6.2 | Complete similarity in structural eccentricity

The influence of plan eccentricity on the peak displacement demands of bidirectionally loaded structures is analysed in Figures 12 and 13 which depict self-similar response spectra for yielding and degrading systems, respectively. Different magnitudes of eccentricity are considered together with two levels of structural stiffness expressed as frequency ratios in the y direction, namely $\Pi_{\omega s, y} = 0.25$ and $\Pi_{\omega s, y} = 1.0$. It is clear from these figures that the differences in dimensionless displacement between the two levels of structural stiffness in the x direction are minimal. Besides, more eccentric structures ($\Pi_{e, x} = 0.3 \Pi_{L, x}$ or more) experience larger peak displacements in the direction aligned with the most persistent earthquake component than more regular structures (e.g. $\Pi_{e, x} = 0.01$ for $\Pi_{\omega s, x} > 0.4$ in Figure 12).

On the other hand, for shorter pulses or more flexible structures the relationship is inverted with the pivoting point located at $\Pi_{\omega s, x} \approx 0.4$ and $\Pi_{\omega s, x} \approx 0.75$ for non-degrading and degrading structures, respectively. The difference in the location of such pivoting point in Figures 12 and 13 is caused by the change in timescale, since $t_{uni} < t_p$, rather than denoting other fundamental changes in the response trends. Notably, the response of structures with higher $\Pi_{\omega s, x}$ values is more sensitive to the eccentricity, $\Pi_{e, x}$, whereas more flexible structures with lower $\Pi_{\omega s, x}$ values are more influenced by the structural stiffness.

Importantly, the relationships presented in Figures 12 and 13 tend towards finite values for very large and very small eccentricities indicating the existence of two region of complete similarity with respect to $\Pi_{e, x} = \Pi_{e, y}$. A value of

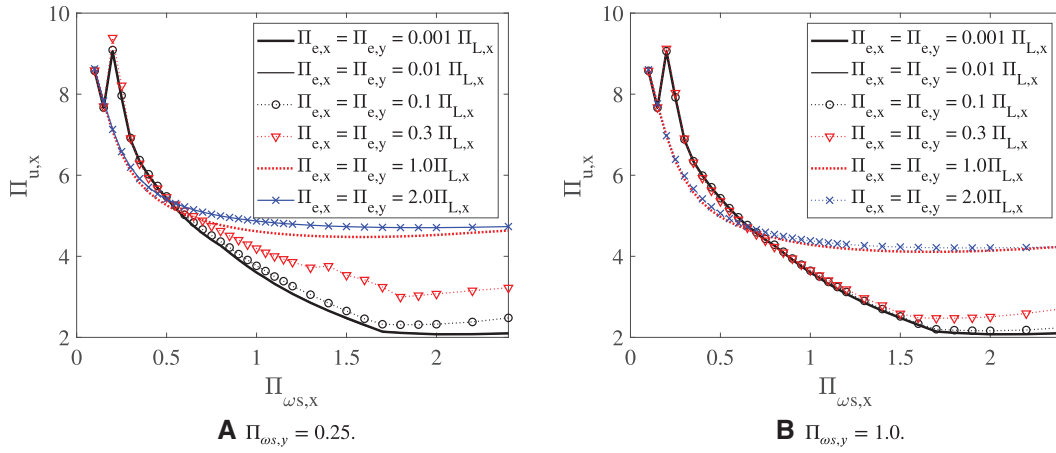


FIGURE 13 Self-similar response spectra for displacements along the x -axis in degrading structures when the uniform duration of the ground motion, t_{uni} , is used as timescale (i.e. $L_{g,x} = \alpha_x t_{uni}^2 / 4\pi^2$)

Note: $\Pi_{F,x} = 0.3$, $\Pi_{F,y} = 0.2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{Lg} = 0.8$, $\Pi_{\omega g} = 0.5$, $\Pi_{\phi} = 0$, $\Pi_{\Lambda} = 1.0$; Framework W.

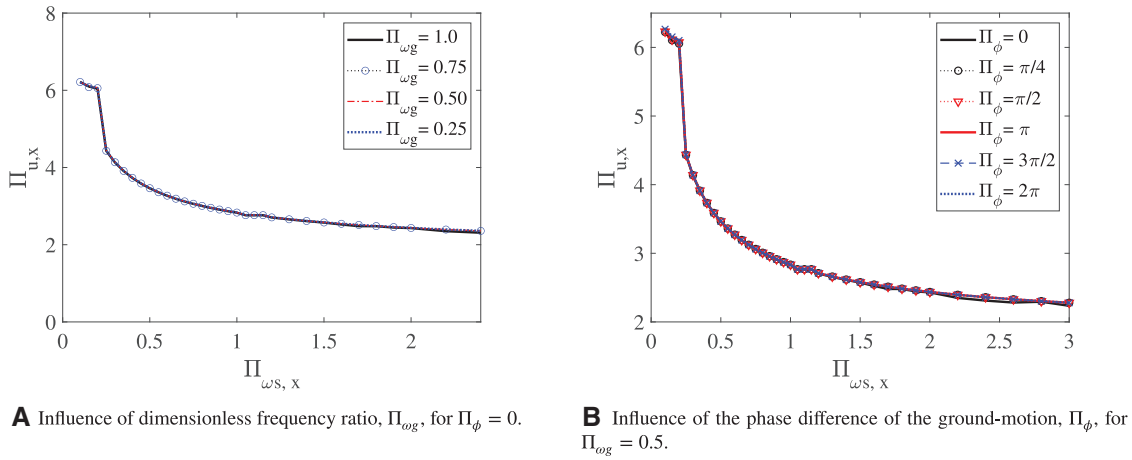


FIGURE 14 Influence of ground-motion phase and frequency on the self-similar response spectra for displacements along the x -axis in yielding structures when the total duration of the ground motion, t_p , is used as timescale (i.e. $L_{g,x} = \alpha_x t_p^2 / 4\pi^2$)

Note: $\Pi_{F,x} = 0.3$, $\Pi_{F,y} = 0.2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.2 \Pi_{L,x}$, $\Pi_{\omega s, y} = 0.5$, $\Pi_{Lg} = 0.8$; Framework W.

$\Pi_{e,x} = \Pi_{e,y} = 2 \Pi_{L,x}$ can represent a very strong torsional response and $\Pi_{e,x} = \Pi_{e,y} = 0.001 \Pi_{L,x}$ stands for ideally symmetric buildings.

6.3 | Complete similarity in ground-motion frequency and phase

The influence of the phase and frequency ratios is assessed in Figures 14 and 15 corresponding to non-degrading and degrading systems, respectively. The response spectra for different frequency ratios, $\Pi_{\omega g} = \omega_{g,y} / \omega_{g,x} = \{0.25, 0.50, 0.75, 1.0\}$, and different phase angles, $\Pi_{\phi} = \phi = \{0, \pi/4, \pi/2, \pi, 3\pi/2, 2\pi\}$, are presented. It is clear from these figures that when the dimensionless-orientationless response parameter, $\Pi_{u,x}$, is aligned with the more persistent earthquake motion (associated with $\Pi_{Lg} = 0.8$ in this case), the response scaling of yielding non-degrading systems (Figure 14) is fully invariant with respect to phase or frequency variations between the two earthquake components over the whole spectral range. This is in partial accord with the findings of Alexander⁵⁴ who observed that phase difference content played a minor role in the response of asymmetric buildings subjected to synthetically modified earthquake records. However, when degradation is incorporated, as in the case of Figure 15, such response invariance is only observed within the spectral region comprehended between $0.5 < \Pi_{\omega s, x} < 1.5$. The precise limits of this spectral region dependent on the level

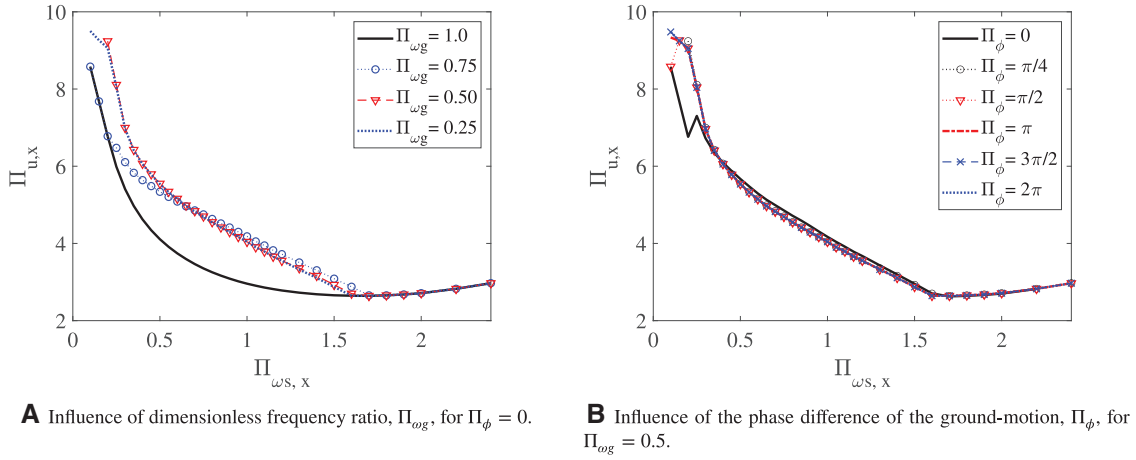


FIGURE 15 Influence of ground-motion phase and frequency on the self-similar response spectra for displacements along the x -axis in degrading structures when the total duration of the ground motion, t_{uni} , is used as timescale (i.e. $L_{g,x} = a_x t_{uni}^2 / 4\pi^2$)

Note: $\Pi_{F,x} = 0.3$, $\Pi_{F,y} = 0.2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.2\Pi_{L,x}$, $\Pi_{\omega s,y} = 0.5$, $\Pi_{Lg} = 0.8$, $\Pi_\Lambda = 1.0$; Framework W.

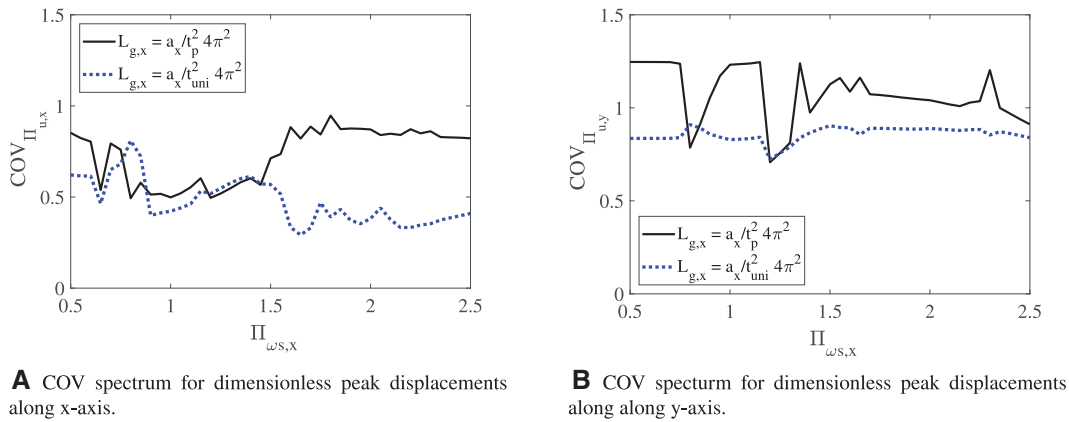


FIGURE 16 Coefficient of variation for response displacements of degrading eccentric oscillators for different ground-motion timescales, t_p and t_{uni}

Note: $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.05\Pi_{L,x}$, $\Pi_{F,x} = \Pi_{F,y} = 0.2$, $\Pi_{\omega s,y} = 0.5$, $\Pi_{Lg} = [0.5, 1.45]$, $\Pi_{\omega g} = [0.87, 1.65]$, $\Pi_\Lambda = 1.0$.

of inelastic behaviour experienced by the structure. The notable deviation observed for $\Pi_{\omega g} = 1.0$ in Figure 15A can be attributed to the markedly elastic response experienced by this system in comparison with the others.

7 | PRACTICAL IMPLICATIONS AND RESPONSE UNDER REAL RECORDS

The dimensional-orientational analyses presented above have unearthed the foundational similarities that govern the response scaling in eccentric structures subjected to bidirectional cycloidal pulses uncovering the fundamental role played by the ground-motion duration. Besides their physical insight, these self-similar relationships can also guide the formulation of predictive models by informing the choice of functional forms and set of parameters to be considered. Perhaps more importantly, given the underlying scale invariance associated with self-similarity, if the dimensionless relationships formulated on the basis of the uniform duration of coherent pulses could be extended to more realistic non-coherent records, lower dispersions in the response estimate are to be expected.^{37,40,55} This is illustrated in Figure 16 which presents the analysis of eccentric systems subjected to a suit of 14 real pulse-like ground-motion pairs with different degrees of coherency. The ground motions employed correspond to the near-field pulse-like record set of the FEMA P695.⁵⁶ These ground motions come from events with an average magnitude of $M_w = 7.0$ and peak ground accelerations varying from 0.22 to 1.43 g with a mean value of 0.6 g. The ground-motion pairs employed represent ranges of $\Pi_{Lg} = [0.5, 1.45]$

and $\Pi_{\omega g} = [0.87, 1.65]$. Further details of these ground motions can be found in Ref. [56]. Both $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$ and $L_{g,y} = a_y t_{uni,y}^2 / 4\pi^2$ are used in Figure 16 for comparison purposes.

The plots of Figure 16 confirm the reduction in the variability associated with the use of t_{uni} as a timescale mentioned above. This reduced variability is expressed as a lower coefficient of variation of dimensionless peak displacements when $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$ is used. This reduction is particularly notable for the high-frequency range in the case of $\Pi_{u,x}$ and can lead to the transition between high-variance and low-variance relationships in the case of $\Pi_{u,y}$. This observation reinforces the importance of incorporating a measure of duration in the description of peak responses of structures exhibiting degradation under real seismic action.

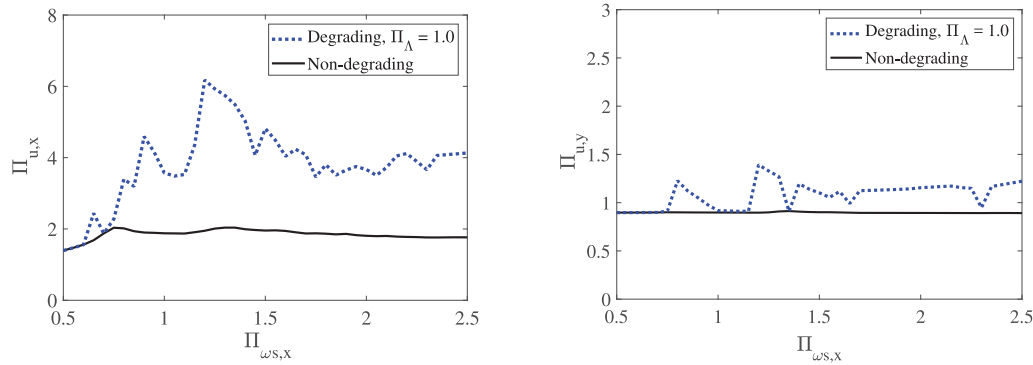
While the construction and validation of generalisable predictive models for non-coherent earthquake records is outside the scope of this study, some challenges associated with the use of realistic earthquake records within a dimensionless-orientationless framework are outlined as follows:

1. The first of such challenges comes from the non-coherent nature of real acceleration histories. Although the simplicity of the sinusoidal pulses employed before helps to focus the examination on fundamental aspects of the response aside from the effects of accidental details of the problem or its boundary conditions; real ground motions contain, besides coherent pulses, a richness of non-coherent components and frequencies that complicate the analysis. Nevertheless, the two basic time and length scales of the ground motion employed above, namely $T_g = t_{uni}$ and $L_{g,x} = a_x t_{uni,x}^2 / 4\pi^2$, where a_x is the peak ground acceleration corresponding to the most energetic pulse within the record,⁴¹ remain fully defined in real accelerograms.
2. A second potential complication springs from the need to quantify the phase differences between the two perpendicular ground-motion components. To this end, at least two approaches can be followed. First, the phase lag can be encompassed by a ground-motion parameter that characterises the difference between the intensities of the two orthogonal ground-motion components, such as the correlation coefficient proposed by Razeinani and Der Kiureghian.⁵⁷ Alternatively, a coherence analysis can be performed between the acceleration series of the two perpendicular components and their phase lag can be chosen as that corresponding to the frequency with the largest maximum squared coherence value. This latter approach has been adopted in this section. It should be noted, however, that in light of the complete similarity on Π_ϕ found before, the precise determination of the phase difference between the two orthogonal components is not expected to play a determinant role in the estimations.
3. Thirdly, and more importantly perhaps, is the issue related to securing specific concurrent values of dimensionless ground-motion frequency, $\Pi_{\omega g}$, and length, Π_{Lg} , parameters. In fact, while amplitude scaling is widely employed in current earthquake engineering practice, frequency scaling involves the addition of synthetic frequency wavelets that raise questions around the physical realisability of the obtained output. These concerns are exacerbated if the uniform duration of the ground motion, t_{uni} , is used as a timescale, since temporal scaling may result in unrealistic waveforms that defy the purpose of using recorded accelerograms in the first place. For these reasons, an approach where carefully selected bins of real ground motions are employed covering a pre-determined range, instead of enforcing a single value of $\Pi_{\omega g}$ and Π_{Lg} , may be recommendable.

In order to illustrate the application of the solutions and approaches proposed above, Figure 17 presents an examination of the influence of degradation on the peak structural response of eccentric systems under real ground motions. The same set of accelerograms described before is used. It can be noted from Figure 17 that while increments of not more than 50% are expected in peak displacements along the y-axis, $\Pi_{u,y}$, twofold and even threefold increments can be observed in the response along the x-axis, $\Pi_{u,x}$, when degradation is considered.

8 | CONCLUSIONS

This paper has used formal dimensional and orientational analyses to unveil the fundamental role played by strong-motion duration on the peak deformation demands of degrading and eccentric structures. First, the application of Vaschy-Buckingham's Π -theorem to the response of bidirectionally loaded eccentric structures uncovered the decisive property of self-similarity in their non-linear response under cycloidal pulses. By contrast, when structural degradation was introduced, peak inelastic displacements were observed to deviate from each other and the self-similarity in the response was lost immediately after the onset of inelastic deformations. This proves that structural degradation affects peak



A Dimensionless spectrum for peak displacements along x-axis.

B Dimensionless spectrum for peak displacements along y-axis.

FIGURE 17 Maximum response displacement of eccentric oscillators subjected to real pulse-like ground motions when $T_g = t_{uni}$, $L_{g,x} = \alpha_x t_{uni,x}^2 / 4\pi^2$, $\Pi_{L,x} = \Pi_{L,y} = 100$, $\Pi_{e,x} = \Pi_{e,y} = 0.05\Pi_{L,x}$, $\Pi_{F,x} = \Pi_{F,y} = 0.2$, $\Pi_{\omega s,y} = 0.5$, $\Pi_{Lg} = [0.5, 1.45]$, $\Pi_{\omega g} = [0.87, 1.65]$

deformations even at lower levels of inelastic demand, although its influence was found to increase with earthquake intensity and degradation severity. Conversely, when the uniform duration, instead of the period, of the strong-motion was adopted as a timescale, a practically self-similar response was reinstated. This constitutes unequivocal proof of the fundamental role played by the ground-motion duration in the peak response of degrading structures.

The study also shows that both dimensional and orientational consistencies are required for the emergence of approximate self-similarity in degrading eccentric structures. On this basis, this paper probes the existence of a number of complete similarities. In particular, complete similarities in structural strength, structural eccentricity, frequency ratio between the ground-motion components and phase lag were discovered. Finally, the challenges brought about by the consideration of real pulse-like earthquake records were highlighted and the potential reduction in the variability associated with the use of the uniform duration of the ground motion, t_{uni} , as a timescale were explored in a case study. These observation reinforced the importance of incorporating a measure of the ground-motion duration on the future formulation of descriptive models for the peak response of degrading systems.

REFERENCES

1. CEN. *Eurocode 8: Design of Structures for Earthquake Resistance—Part 1: General Rules, Seismic Actions and Rules for Buildings*. Brussels: European Committee for Standardization; 2005.
2. ATC. *FEMA P-58, Seismic Performance Assessment of Buildings*. Redwood City, CA: Applied Technology Council; 2012.
3. Pearl J. Causal inference in statistics: an overview. *Stat Surv*. 2009;3:96-146.
4. Anagnostopoulos SA, Kyrkos MT, Stathopoulos KG. Earthquake induced torsion in buildings: critical review and state of the art. *Earthquake Struct*. 2015;8(2):305-377.
5. Rosenblueth E, Meli R. The 1985 Mexico earthquake. *Concr Int*. 1986;8(5):23-34.
6. Esteva L. Earthquake engineering research and practice in Mexico after the 1985 earthquakes. *Bull NZ Natl Soc Earthquake Eng*. 1987;20(3):159-200.
7. Priestley MJN. *Myths and Fallacies in Earthquake Engineering, Revisited*. Vienna: IUSS Press; 2003.
8. Kubo T, Penzien J. Analysis of three-dimensional strong ground motions along principal axes, San Fernando earthquake. *Earthquake Eng Struct Dyn*. 1979;7(3):265-278.
9. Tanabashi R, Kobori T, Kaneta K. *Nonlinear Torsional Vibration Structures Due to an Earthquake*. Kyoto: Disaster Prevention Research Institute, Kyoto University; 1962.
10. De La Llera JC, Chopra AK. Understanding the inelastic seismic behaviour of asymmetric-plan buildings. *Earthquake Eng Struct Dyn*. 1995;24(4):549-572.
11. Erduran E, Ryan KL. Effects of torsion on the behavior of peripheral steel-braced frame systems. *Earthquake Eng Struct Dyn*. 2011;40(5):491-507.
12. Dutta SC. Effect of strength deterioration on inelastic seismic torsional behaviour of asymmetric RC buildings. *Build Environ*. 2001;36(10):1109-1118.
13. Roy R, Thakur P, Chakroborty S. Scaling of ground motions and its implications to plan-asymmetric structures. *Soil Dyn Earthquake Eng*. 2014;57:46-67.
14. Chakroborty S, Roy R. Seismic behavior of horizontally irregular structures: current wisdom and challenges ahead. *ASME Appl Mech Rev*. 2016;68:060802-1-060802-17.

15. Tso WK, Dempsey KM. Seismic torsional provisions for dynamic eccentricity. *Earthquake Eng Struct Dyn*. 1980;8(3):275-289.
16. Chopra AK, Goel RK. Evaluation of torsional provisions in seismic codes. *J Struct Eng*. 1991;117(12):3762-3782.
17. Ghersi A, Rossi PP. Formulation of design eccentricity to reduce ductility demand in asymmetric buildings. *Eng Struct*. 2000;22(7):857-871.
18. Anagnostopoulos SA, Kyrkos MT, Papalymperi A, Plevri E. Should accidental eccentricity be eliminated from Eurocode 8? *Earthquake Struct*. 2015;8(2):463-484.
19. Chakroborty S, Roy R. Role of ground motion characteristics on inelastic seismic response of irregular structures. *J Architect Eng*. 2016;22(1):B4015007.
20. Beyer K, Bommer JJ. Selection and scaling of real accelerograms for bi-directional loading: a review of current practice and code provisions. *J Earthquake Eng*. 2007;11(S1):13-45.
21. Todorovska MI, Trifunac MD, Lee VW, Orbović N. Synthetic earthquake ground motions on an array. *Soil Dyn Earthquake Eng*. 2013;48:234-251.
22. Goel RK. Seismic behaviour of asymmetric buildings with supplemental damping. *Earthquake Eng Struct Dyn*. 2000;29(4):461-480.
23. Lin W-H, Chopra AK. Asymmetric one-storey elastic systems with non-linear viscous and viscoelastic dampers: earthquake response. *Earthquake Eng Struct Dyn*. 2003;32(4):555-577.
24. Giannopoulos D, Vamvatsikos D. Ground motion records for seismic performance assessment: to rotate or not to rotate? *Earthquake Eng Struct Dyn*. 2018;47(12):2410-2425.
25. Cornell CA. Does duration really matter? *Proceedings of the FHWA/NCEER Workshop on the National Representation of Seismic Ground Motion for New and Existing Highway Facilities*, Burlingame, CA; 1997.
26. Hancock J, Bommer JJ. Using spectral matched records to explore the influence of strong-motion duration on inelastic structural response. *Soil Dyn Earthquake Eng*. 2007;27(4):291-299.
27. Iervolino I, Manfredi G, Cosenza E. Ground motion duration effects on nonlinear seismic response. *Earthquake Eng Struct Dyn*. 2006;35(1):21-38.
28. Guo G, Yang D, Liu Y. Duration effect of near-fault pulse-like ground motions and identification of most suitable duration measure. *Bull Earthquake Eng*. 2018;16(11):5095-5119.
29. Ibarra LF, Medina RA, Krawinkler H. Hysteretic models that incorporate strength and stiffness deterioration. *Earthquake Eng Struct Dyn*. 2005;34(12):1489-1511.
30. Lignos DG, Krawinkler H. Deterioration modeling of steel components in support of collapse prediction of steel moment frames under earthquake loading. *J Struct Eng*. 2010;137(11):1291-1302.
31. Raghunandan M, Liel AB. Effect of ground motion duration on earthquake-induced structural collapse. *Struct Saf*. 2013;41:119-133.
32. Chandramohan R, Baker JW, Deierlein GG. Quantifying the influence of ground motion duration on structural collapse capacity using spectrally equivalent records. *Earthquake Spectra*. 2016;32(2):927-950.
33. Makris N, Black CJ. Dimensional analysis of bilinear oscillators under pulse-type excitations. *J Eng Mech*. 2004;130(9):1019-1031.
34. Makris N, Black CJ. Dimensional analysis of rigid-plastic and elastoplastic structures under pulse-type excitations. *J Eng Mech*. 2004;130(9):1006-1018.
35. Dimitrakopoulos EG, DeJong MJ. Revisiting the rocking block: closed-form solutions and similarity laws. *Proc. Math. Phys. Eng. Sci.*. 2012;468(2144):2294-2318.
36. Dimitrakopoulos EG, Paraskeva TS. Dimensionless fragility curves for rocking response to near-fault excitations. *Earthquake Eng Struct Dyn*. 2015;44(12):2015-2033.
37. Málaga-Chuquitaype C. Estimation of peak displacements in steel structures through dimensional analysis and the efficiency of alternative ground-motion time and length scales. *Eng Struct*. 2015;101:264-278.
38. Shu Z, Zhang J, Nagarajaiah S. Dimensional analysis of inelastic structures with negative stiffness and supplemental damping devices. *J Struct Eng*. 2017;143(3):04016184.
39. Thiers-Moggia R, Málaga-Chuquitaype C. Seismic protection of rocking structures with inerters. *Earthquake Eng Struct Dyn*. 2019;48(5):528-547.
40. Málaga-Chuquitaype C, Psaltakis ME, Kampas G, Wu J. Dimensionless fragility analysis of seismic acceleration demands through low-order building models. *Bull Earthquake Eng*. 2019;17(7):3815-3845.
41. Vassiliou MF, Makris N. Estimating time scales and length scales in pulselike earthquake acceleration records with wavelet analysis. *Bull Seismol Soc Am*. 2011;101(2):596-618.
42. Dimitrakopoulos E, Kappos AJ, Makris N. Dimensional analysis of yielding and pounding structures for records without distinct pulses. *Soil Dyn Earthquake Eng*. 2009;29(7):1170-1180.
43. Vaschy A. Sur les lois de similitude en physique. *Ann Electr*. 1892;19:25-28.
44. Buckingham E. Model experiments and the forms of empirical equations. *Trans Am Soc Mech Eng*. 1915;37:263-296.
45. Siano DB. Orientational analysis—a supplement to dimensional analysis-I. *J Franklin Inst B*. 1985;320(6):267-283.
46. Ayre RS. Transient response to step and pulse functions. *Shock Vibration handb*. McGrawHill: 1961;8.1-8.59.
47. Málaga-Chuquitaype C, Bougatsas K. Vector-IM-based assessment of alternative framing systems under bi-directional ground-motion. *Eng Struct*. 2017;132:188-204.
48. McKenna F. OpenSees: a framework for earthquake engineering simulation. *Comput Sci Eng*. 2011;13(4):58-66.
49. Rahnama M, Krawinkler H. *Effects of Soft Soil and Hysteresis Model on Seismic Demands* [Rep. BLUME-108]. Stanford, CA: John A. Blume Earthquake Engineering Center, Stanford University; 1993.

50. Subodh K, Amit K, Maha K, Dimitrios L, Sashi K. Simulating cyclic local buckling-induced softening in steel beam-columns using a nonlocal material model in displacement-based fiber elements. *J Struct Eng*. 2020;146(1):04019174.
51. Sirumbal-Zapata LF, Málaga-Chuquitaype C, Elghazouli AY. A three-dimensional plasticity-damage constitutive model for timber under cyclic loads. *Comput Struct*. 2018;195:47-63.
52. Málaga-Chuquitaype C, Elghazouli AY, Bento R. Rigid-plastic models for the seismic design and assessment of steel framed structures. *Earthquake Eng Struct Dyn*. 2009;38(14):1609-1630.
53. Gratton J. Similarity and self-similarity in fluid dynamics. *Fundam Cosmic Phys*. 1991;15:1-106.
54. Alexander NA. The role of phase difference components of ground motions in the torsional response of asymmetric buildings. *Earthquake Eng Struct Dyn*. 2007;36(10):1385-1406.
55. Karavasilis TL, Makris N, Bazeos N, Beskos DE. Dimensional response analysis of multistory regular steel MRF subjected to pulslike earthquake ground motions. *J Struct Eng*. 2010;136(8):921-932.
56. Council Applied Technology, Agency United States, Federal Emergency Management. *Quantification of Building Seismic Performance Factors*. Washington, DC: US Department of Homeland Security, FEMA; 2009.
57. Rezaeian S, Der Kiureghian A. Simulation of orthogonal horizontal ground motion components for specified earthquake and site characteristics. *Earthquake Eng Struct Dyn*. 2012;41(2):335-353.

How to cite this article: Málaga-Chuquitaype C. Strong-motion duration and response scaling of yielding and degrading eccentric structures. *Earthquake Engng Struct Dyn*. 2021;50:635–654. <https://doi.org/10.1002/eqe.3350>