

Impedance shaping for whole-system small-signal stability enhancement via co-operation between Grid and IBR operators

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Statement of Originality

I herewith declare that the work produced here was either conducted by myself or where collaborative work occurred, it is indicated. All information has been attributed to its original source. This thesis has not previously been presented in identical or similar form to any other British or foreign examination board. This work was conducted from October 2019 to March 2024 under the supervision of Prof. Tim C. Green at Imperial College London. The copyright of this thesis rests with the author and is made available under a Creative Commons Attribution Non-Commercial No Derivatives licence. Researchers are free to copy, distribute or transmit the thesis on the condition that they attribute it, that they do not use it for commercial purposes and that they do not alter, transform or build upon it. For any reuse or redistribution, researchers must make clear to others the licence terms of this work.

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Abstract

The transition to an electric grid powered largely by Inverter-Based Resources (IBR) is changing the tools required by an Electricity System Operator (ESO) to ensure stable operation of the power grid. Recent research has revealed that measurable system residues relate changes in apparatus impedance to the damping of the oscillation modes which define the whole-system small-signal stability.

This thesis describes a process where an IBR operator and an ESO co-operate with shared information to guide changes in the impedance of the IBR, to improve the system stability. Three main barriers to the implementation of the proposed process are identified and investigated.

The first barrier investigated is the unknown capability of grid-following IBR to provide the required modification in its impedance spectrum shape. A study is presented assessing the most suitable parameters for impedance shaping and the constraints imposed on the shape-ability by ensuring satisfactory performance of the IBR DC voltage control.

A second barrier is identified as the need to ensure stability of the system when the change in mode damping after the parameter change is different to that initially predicted. A method called the "minimum- $\Delta\rho$ path constraint" is proposed to account for changes in the relationship between the impedance and the parameter in the neighbourhood of a mode to calculate an updated constraint on the allowed parameter change.

The third barrier considered is the effect of measurement noise on the modes and residues estimated with vector fitting. A novel version of vector fitting - referred to as "constrained vector fitting" - is proposed to improve the identification performance.

Taken together, the techniques proposed to address the identified barriers could improve the effectiveness of cooperation between an ESO and IBR operators, such that whole-system, small-signal stability can be more effectively enhanced through alterations of IBR control parameters.

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Abbreviations

BPF	Band-Pass Filter
ESO	Electricity System Operator
FDSIM	Frequency-Domain System Identification Method
HPF	High-Pass Filter
IBR	Inverter Based Resource
LPF	Low-Pass Filter
MIMO	Multiple Input Multiple Output
PEM	Prediction Error Method
PLL	Phase-Locked Loop
PSS	Power System Stabilizer
PWM	Pulse-Width Modulation
SISO	Single Input Single Output
SM	Synchronous Machine
SSO	Sub-Synchronous Oscillations
SVC	Static Var Compensator

Notation

Non-capitalised dq	A variable measured with respect to an IBR internal reference frame (aligned with the PLL)
The colour green in a diagram	Represents an input
The colour red in a diagram	Represents an output

Symbols

σ_{cc}	Current control bandwidth
σ_{dc}	DC control bandwidth
$g_{cc}(s)$	Current control: current reference tracking transfer function (LPF)
$g_{DC}(s)$	DC-link power balance to output current transfer function (LPF)
$g_{PLL}(s)$	PLL - grid phase angle synchronization transfer function (LPF)
$h_{cc}(s)$	Current control: grid voltage disturbance rejection transfer function (HPF)
$h_{DC}(s)$	DC-link power balance to DC-link voltage disturbance rejection transfer function (HPF)
$h_{PLL}(s)$	Grid phase disturbance to internal PLL q axis voltage disturbance rejection transfer function (HPF)
$\mathbf{i}_B(s)$	Vector of deviations in current injected in parallel to the grid (not from the apparatus)
$\mathbf{i}_L(s)$	Vector of deviations in apparatus current injections to the network
ρ	An apparatus parameter
σ_{pll}	PLL bandwidth
$\mathbf{v}_B(s)$	Vector of deviations in bus voltage
$\mathbf{v}_L(s)$	Vector of deviations in voltage across a source inserted between the network buses and the apparatus (current source or voltage source)
$\mathbf{Y}_A(s)$	Block-diagonal transfer function matrix of apparatus 2x2 dq domain admittances - describing the changes in current injected into the network in response to changes in the bus voltages
$Y_{Ak}(s)$	2x2 dq domain admittance transfer function matrix for the apparatus at bus k
$\mathbf{Y}_N(s)$	Networked impedance model: nodal admittance transfer function matrix
$\hat{\mathbf{Y}}(s)$	Networked impedance model: whole-system admittance transfer function matrix
$\hat{\mathbf{Z}}(s)$	Networked impedance model: whole-system impedance transfer function matrix

Symbols (continued)

- $\mathbf{Z}_B(s)$ Bus network impedance transfer function matrix describing the changes in bus voltages in response to changes in the currents injected into the network
- $\mathbf{Z}_L(s)$ Networked impedance model: loop impedance transfer function matrix

1 Introduction

The global build-out of renewable energy resources is rapidly increasing the penetration of Inverter-Based Resources (IBR) in electricity grids [1]. The techniques for ensuring the stable operation of the electricity grid powered by traditional synchronous machines (SM) have been built up over the last 100 years [2], but must now rapidly adapt to both the challenges and opportunities brought by the different characteristics of IBR [3].

Stable operation of an electricity grid can be broken down into several sub-problems [2] but small-signal stability is a necessary condition for overall system stability. Small disturbances to a grid with poor small-signal stability characteristics can cause large voltage/current oscillations, triggering protection systems to take action potentially resulting in de-energisation of large parts of a national electricity grid [4] with potentially serious consequences.

Small-signal stability has normally been assessed with a state-space model of the whole system under consideration, obtained by linearising the system dynamics at a particular operating point. An example of a state-space model is shown in Equations 1.1 and 1.2, relating an input current $i(t)$ to an output voltage $v(t)$ via the system states x . Since the state-space model is a linearised representation of the real non-linear system the variables x , i and v only describe deviations from equilibrium.

$$\frac{dx(t)}{dt} = Ax(t) + Bi(t) \quad (1.1)$$

$$v(t) = Cx(t) + Di(t) \quad (1.2)$$

The state-space model allows the calculation of the system modes closest to instability, finding the system states participating in these modes [5] and guiding the design of stabilizers to improve their damping [6]. The analysis is performed across the expected range of system operating points to ensure that any stabilizing designs are robust to changes in operating points [2, 7, 8].

Building a state-space model for the whole system requires the connection of state-space models of the network and all apparatus [9], which requires knowledge of any internal dynamics which may have impact in the frequency range of interest. This was possible with a grid

composed of a few, large SM since their dynamic models are mainly standardised and determined by the underlying physics [8]. However, IBR dynamics depend largely on the programming of their control logic which manufacturers regard as proprietary information. This makes the formation of the whole-system state-space model difficult using traditional techniques.

Unfortunately, there have now been multiple recorded incidents of sub-synchronous oscillations (SSO) where IBR controllers are suspected or known to be involved. A summary of the events can be found in [10].

A record of one of the incidents is shown in Figure 1.1, which shows a voltage oscillation event on the Scottish electricity transmission network on the 24th August 2021. Two sets of oscillations occurred, approximately 20-25 seconds each and 30 minutes apart, which resulted in users being disconnected from the grid. The beginning of the zoomed in part of the record clearly shows the envelope of the signal growing in an exponential manner, indicating the potential presence of an unstable mode. The occurrences of the SSO phenomenon now motivates research into tools which can analyse small-signal stability without the explicit formation of the system state-space model.

To avoid needing to form state-space models, small-signal stability techniques which utilise impedance models are becoming increasingly popular in the research community and in application [12–14]. Impedance models are transfer functions representing the same dynamics as a state-space model. The state-space model in Equations 1.1 and 1.2 can easily be mapped to a frequency-domain transfer function representation with Equation 1.3, where I is the identity matrix of appropriate size. Equivalently, an admittance model describes the response of output current to variations in input voltage.

$$\frac{v(s)}{i(s)} = G(s) = C(sI - A)^{-1}B + D \quad (1.3)$$

There are an infinite number of state-space representations of any transfer function model so the mapping in Equation 1.3 cannot be reversed - ensuring the confidentiality of controller details if an impedance model of an IBR is shared. Additionally, its frequency response can be directly physically measured [15–17] without requiring access to any internal states or settings. Measuring the impedance/admittance of an IBR is a method to obtain information about the

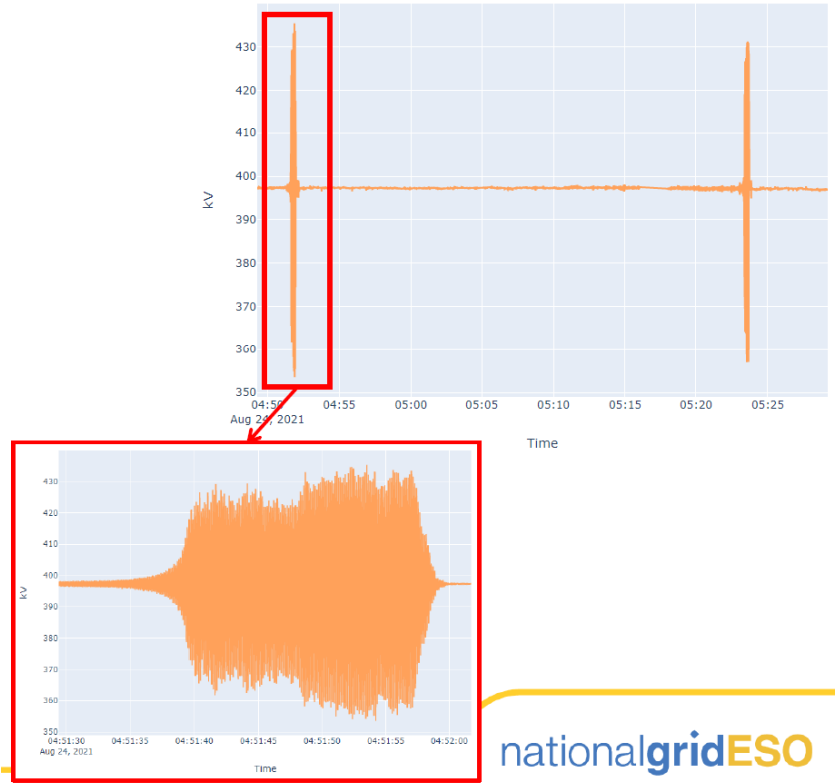


Figure 1.1: Two occurrences of approximately 8 Hz voltage oscillations on the Scottish transmission network, 24th August 2021 [11]. A classic growing exponential is clearly visible in initial phase, indicating a small-signal stability issue. Protection systems operate to contain the magnitude of the oscillations, which prevents the oscillations from continuing to grow exponentially. Some electricity users were tripped off as a result.

dynamics of the IBR without requiring any information from the IBR manufacturer.

If there are no repeated eigenvalues of the state-space 'A' matrix the transfer function can also be described with the pole-residue form in Equation 1.4. Each eigenvalue of the 'A' matrix is represented by λ_i , which are equal to the whole-system oscillation modes. If λ_i is complex, there will exist another mode which is its complex conjugate and together they indicate the presence of oscillations. The variable R_i is called the residue, which is associated with a specific mode λ_i .

$$G(s) = D + \sum_i \frac{1}{s - \lambda_i} R_i \quad (1.4)$$

Another feature of the changing electricity grid is that modern information technology has allowed a huge expansion in the data gathering from the electricity grid [18], resulting in greater

visibility of the status of the grid than ever before. Additionally, since the dynamics of IBR are so configurable there is a much greater range of levers for the grid operators to pull to ensure the grid is stable. The most useful stability techniques of the future will be those that utilise the strengths of IBR and the data available in the "smart grid". Thus, the focus moves from overcoming the stability threats IBR introduce to exploiting their versatility.

1.1 Background: Impedance-based small-signal stability analysis techniques to deal with high penetrations of IBRs

One impedance-based data-driven approach assessing system stability is to use the frequency responses of all the connected IBR (either made available by vendors or measured) to create non-unique state-space apparatus models, then connect those to the (assumed known) state-space model of the grid and perform standard state-space analysis [19,20]. This has proven successful in replicating observed oscillations, but provides limited insight into the changes required to improve the damping of the oscillations since the identified state-space models do not align with any of the actual internal control logic. Additionally, all connected apparatus must be included in the analytical modelling.

If the control structure is assumed to be known (or known to be one of a set), the parameters of the controller have been estimated from frequency response data [21,22] - which if correct allows the formation of a state-space model for the IBR. However, clearly there is no guarantee that all considered IBR will have one of the assumed control structures. Since the small-signal stability is a global property of the interconnected network, any inaccuracies in realising the individual state-space models of any of the IBR can result in misleading overall results.

Another vein of impedance model based stability research has focused on designing each IBR such that its impedance is passive [23–25], since passive systems are known to always be stable when connected. However, passivity is difficult to achieve in practice since the impedance of the IBR close to the fundamental frequency is significantly influenced by its operating conditions [16], which can result in non-passive operation. Passivity has also been shown to be a conservative requirement limiting the performance of the IBR under normal operation, although there has recently been some promising developments in reducing the conservativeness of the passivity theory [26] which could be useful for future decentralised stability guarantees.

Alternatively, describing the IBR and the grid in the form of source/load SISO impedance transfer functions allows the use of the Nyquist criterion [27], commonly described as the impedance-based stability criterion in power electronics research literature [28]. Figure 1.2 shows a diagram of the equivalent circuit required for use of the impedance-based stability criterion. It can be derived that the transfer function relationships between input disturbances and the apparatus terminal voltage/current are equal to Equations 1.5 and 1.6. The impedance-based criterion makes use of the fact that Equations 1.5 and 1.6 effectively represent a feedback loop with the loop gain $Y(s)Z(s)$, so the Nyquist stability criterion based on encirclements of the $-1 + j0$ point by $Y(s)Z(s)$ can be used.

$$v_{\text{out}}(s) = (I + Y(s)Z(s))^{-1} (v_{\text{in}}(s) + Z(s)i_{\text{in}}(s)) \quad (1.5)$$

$$i_{\text{out}}(s) = (I + Y(s)Z(s))^{-1} (Y(s)v_{\text{in}}(s) - i_{\text{in}}(s)) \quad (1.6)$$

The analysis of the loop gain $Y(s)Z(s)$ allows the calculation of gain and phase stability margins as is fundamental to classical control theory. The impedance-based criterion has found widespread application in DC systems [29,30] and AC systems under assumptions of symmetric impedance [31,32]. A symmetric impedance is a 2x2 impedance matrix in the form of Equation 1.7, which allows the non-scalar relationship between real dq currents and voltages to be written as the scalar relationship between complex currents and voltages in Equation 1.8.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} z_d & -z_q \\ z_q & z_d \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (1.7)$$

$$v_d + jv_q = (z_d + jz_q)(i_d + ji_q) \quad (1.8)$$

The application of the impedance-based criterion requires a small-signal model of the rest of the grid. The model of the grid is commonly assumed to be formed of simple resistive-inductive (RL) branches, often with a shunt capacitance at the point of connection to the grid. A "strong" grid has a relatively low value of the RL impedance, whereas a "weak" grid has a relatively high RL value.

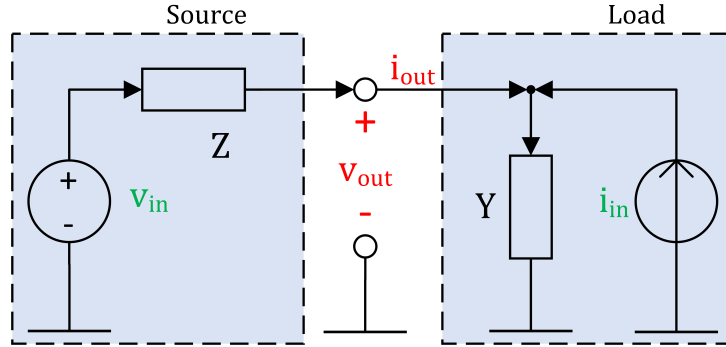


Figure 1.2: Splitting the grid and apparatus into a source/load pair allows the use of the Nyquist stability criterion since the pair form a feedback loop

IBR impedances are often not symmetric and normally require at least two channels to describe them, usually either positive and negative sequence or the dq synchronous reference frame coordinate system [33]. It has been shown [34] that neglecting the off-diagonal dq impedance terms can give misleading stability assessment results, particularly under non-zero steady state reactive power transfer. There have been attempts to manipulate the IBR impedance matrix without neglecting off-diagonal terms to obtain a form which can be still be used with SISO Nyquist analysis [35–37], but there is no clear consensus on the veracity of their stability conclusions.

In the non-SISO case, the generalised Nyquist criterion must be used instead [17]. The determinant or eigenvalues of the loop gain matrix are used as the characteristic loci to consider encirclements of the $-1 + j0$ point, but there are no simple gain and phase margins available since no control gain maps linearly to the characteristic loci.

To consider the interaction between multiple IBRs, it is not sufficient for the "rest of the grid" to be represented by an equivalent RL impedance [38]. The partition of the system into "source" and "load" is no longer straightforward and techniques to deal with this issue require the formation of networked impedance models [39–42].

The first key building block of networked impedance models is the bus impedance transfer function matrix $\mathbf{Z}_{\mathbf{B}}(s)$, which relates the deviations in the bus voltages $\mathbf{v}_{\mathbf{B}}(s)$ to the current injections $\mathbf{i}_{\mathbf{L}}(s)$. Both $\mathbf{v}_{\mathbf{B}}(s)$ and $\mathbf{i}_{\mathbf{L}}(s)$ describe small-signal variations from an operating point and are vectors containing two entries per bus - the d axis variable and the q axis variable. The bus impedance matrix contains all the network dynamics and is shown in Equation 1.9 and Figure 1.3 (a).

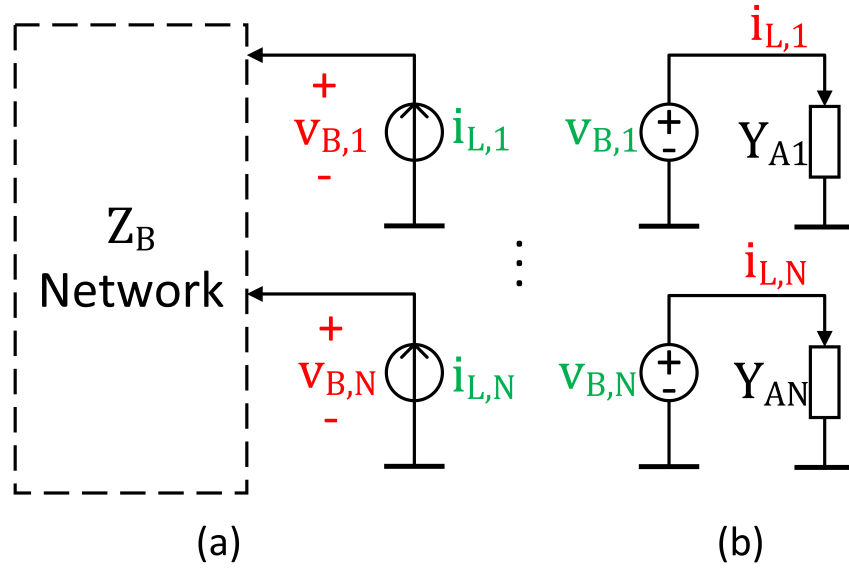


Figure 1.3: Illustration of the equivalent circuits of the building blocks of the networked impedance models: (a) the bus impedance transfer function matrix $\mathbf{Z}_B(s)$ and (b) the block-diagonal apparatus admittance transfer function matrix $\mathbf{Y}_A(s)$. Inputs are indicated with green text, outputs with red text

$$\mathbf{v}_B(s) = \mathbf{Z}_B(s)\mathbf{i}_L(s) \quad (1.9)$$

The second key building block is apparatus admittance transfer function matrix $\mathbf{Y}_A(s)$ relating the changes in current drawn by apparatus $\mathbf{i}_L(s)$ to the variations in bus voltages $\mathbf{v}_B(s)$. The apparatus transfer function matrix is block diagonal and is shown in Equations 1.10 and 1.11. It is illustrated in Figure 1.3 (b).

$$\mathbf{Y}_A(s) = \begin{bmatrix} Y_{A1}(s) & & & \\ & Y_{A2}(s) & & \\ & & \dots & \\ & & & Y_{AN}(s) \end{bmatrix} \quad (1.10)$$

$$\mathbf{i}_L(s) = \mathbf{Y}_A(s)\mathbf{v}_B(s) \quad (1.11)$$

Networked impedance models combine the building blocks into transfer function matrices that allow insight into the dynamics of the grid as a whole. The analysis of small-signal stability with networked impedance models has taken three main paths, illustrated in the three rows of

Table 1.1.

The first approach - called the "generalised Nyquist approach" here - still applies the Nyquist criterion but the loop gain is the MIMO transfer function matrix $\mathbf{G}_V(s)$ or $\mathbf{G}_I(s)$ [43,44]. From inspection of Equations 1.12 and 1.13, it is clear that $\mathbf{G}_V(s)$ represents a voltage-to-voltage gain and $\mathbf{G}_I(s)$ represents a current-to-current gain. Since the loop gains $\mathbf{G}_V(s)$ and $\mathbf{G}_I(s)$ are transfer function matrices the generalised Nyquist criterion is required for stability assessment.

$$\mathbf{G}_V(s) = \mathbf{Z}_B(s)\mathbf{Y}_A(s) \quad (1.12)$$

$$\mathbf{G}_I(s) = \mathbf{Y}_A(s)\mathbf{Z}_B(s) \quad (1.13)$$

The equivalent circuits are illustrated in Table 1.1 (a) for the voltage-to-voltage gain and (b) for the current-to-current gain.

Although phase and gain margins are no longer calculable, the eigenvalues of the loop gain matrix can be calculated over a range of frequencies. Poorly damped modes are said to be identifiable by one of the eigenvalues passing near to the $-1 + j0$ point in the complex plane [43,44]. Participation analysis is applied to the eigenvalue which passes closest to $-1 + j0$, at the frequency at which the closest pass occurs, in order to calculate a frequency-domain participation factor [43,44].

The generalised Nyquist approach requires analytical modelling of all connected apparatus since it is difficult to measure $\mathbf{Y}_A(s)\mathbf{Z}_B(s)$ or $\mathbf{Z}_B(s)\mathbf{Y}_A(s)$ directly. To see this, if $\mathbf{i}_L(s)$ is set then $\mathbf{Z}_B(s)$ could be measured based on the bus voltage measurements $\mathbf{v}_B(s)$. However, in order to set $\mathbf{i}_L(s)$ the admittance $\mathbf{Y}_A(s)$ must be disconnected as otherwise $\mathbf{i}_L(s)$ is set by $\mathbf{Y}_A(s)$. This implies that in order to measure $\mathbf{Z}_B(s)$ or $\mathbf{Y}_A(s)$ online the input signal must be measured rather than set, with measurement noise in both input and output. This networked impedance approach has been implemented in the literature [43,44] with frequency response data obtained from models rather than real measurements.

The second networked impedance approach [45,46] - called the "critical eigenvalue approach" here - makes use of two key transfer function matrices: the nodal admittance matrix $\mathbf{Y}_N(s)$ and the loop impedance matrix $\mathbf{Z}_L(s)$. The nodal admittance matrix gives the total current

$\mathbf{i}_B(s)$ drawn by the apparatus and the network when the nodal voltages $\mathbf{v}_B(s)$ are set and is illustrated in Table 1.1 (c) and Equation 1.14.

The loop impedance matrix represents the voltage $\mathbf{v}_L(s)$ induced across the current source when the currents $\mathbf{i}_L(s)$ injected into the network by the apparatus are set. It is shown in Table 1.1 (d) and Equation 1.15. The double negative is shown in Equation 1.15 to emphasise that the voltage $\mathbf{v}_L(s)$ represents the difference between two non-zero voltages, one of which is due to current flowing in the opposite direction to the direction in which the admittance was originally defined.

$$\mathbf{i}_B(s) = (\mathbf{Z}_B(s)^{-1} + \mathbf{Y}_A(s))\mathbf{v}_B(s) = \mathbf{Y}_N(s)\mathbf{v}_B(s) \quad (1.14)$$

$$\mathbf{v}_L(s) = (\mathbf{Z}_B(s) - -\mathbf{Y}_A(s)^{-1})\mathbf{i}_L(s) = \mathbf{Z}_L(s)\mathbf{i}_L(s) \quad (1.15)$$

It can be shown that the roots of the determinants of the nodal admittance and loop impedance transfer function matrices are equal to the system modes [47,48], so that $\det(\mathbf{Y}_N(\lambda)) = 0$. Since its determinant is zero $\mathbf{Y}_N(\lambda)$ has one zero eigenvalue - referred to as the "critical eigenvalue" in [48] - assuming non-repeated eigenvalues.

Participation analysis can be applied to the critical eigenvalue and the matrix $\mathbf{Y}_N(\lambda)$ to find a frequency domain participation factor for the critical eigenvalue [45,46]. The participation factor derived from the critical eigenvalue approach was mathematically related to the participation factor derived from the generalised Nyquist approach in [49]. Again, it is difficult to measure $\mathbf{Y}_N(s)$ or $\mathbf{Z}_L(s)$ by setting their inputs since the nodal voltages $\mathbf{v}_B(s)$ and current injections $\mathbf{i}_L(s)$ tend to be set by strong sources.

Name	Equivalent Circuit	Corresponding Transfer Function Matrix [Relevant literature]	Name	Equivalent Circuit	Corresponding Transfer Function Matrix [Relevant literature]
(a): Open Loop Voltage Gain		$\mathbf{G}_V(s) = \mathbf{Z}_B(s) \mathbf{Y}_A(s)$	(b): Open Loop Current Gain		$\mathbf{G}_I(s) = \mathbf{Y}_A(s) \mathbf{Z}_B(s)$ [43, 44]
(c): Nodal Admittance		$\mathbf{Y}_N(s) = \mathbf{Z}_B(s)^{-1} + \mathbf{Y}_A(s)$ [39–41, 45, 46]	(d): Loop Impedance		$\mathbf{Z}_L(s) = \mathbf{Z}_B(s) - \mathbf{Y}_A(s)^{-1}$ [39]
(e): Whole-System Admittance		$\hat{\mathbf{Y}}(s) = \mathbf{Y}_A(s)(I + \mathbf{Z}_B(s) \mathbf{Y}_A(s))^{-1}$ [42]	(f): Whole-System Impedance		$\hat{\mathbf{Z}}(s) = \mathbf{Z}_B(s)(I + \mathbf{Y}_A(s) \mathbf{Z}_B(s))^{-1}$ [42]

Table 1.1: Summarised illustrations of the models used in Networked impedance small-signal stability techniques. The inputs are labelled with green text and the outputs with red text.

Additionally, both the generalised Nyquist and critical eigenvalue methods give no exact insight into which parameters should be tuned in order to improve the damping of any poorly damped modes. State-space models have received such widespread use because methods exist for them to guide the process of improving small-signal stability. With full knowledge of the state transition matrix, A , controllers can be synthesized to place the poles at arbitrary positions in the complex plane [50] solving small-signal stability completely. In reality, restrictions on the state measurement, controller order and control effort result in partial pole placement of just a desired subset of the modes [51]. As the dates of the references indicate, state-space system stability has been thoroughly investigated.

The third networked impedance approach utilises the same building blocks but combines them in a manner which can be measured without any disconnection of apparatus. The inputs are taken to be *parallel current injections* or *series voltage injections*, illustrated in Table 1.1 (e) and (f) respectively. The two transfer function matrices are named the whole-system impedance and the whole-system admittance in [42] and are shown in Equations 1.16 and 1.17.

It can be shown that the whole-system impedance and admittance transfer function matrices are the inverses of the nodal admittance and the loop impedance models respectively [48]. Equations 1.16 and 1.17 also neatly show the relationships between the three types of networked impedance models.

$$\begin{aligned}\mathbf{v}_B(s) &= \hat{\mathbf{Z}}(s)\mathbf{i}_B(s) \\ \mathbf{i}_L(s) &= \hat{\mathbf{Y}}(s)\mathbf{v}_L(s) \\ \hat{\mathbf{Z}}(s) &= \mathbf{Z}_B(s)(\mathbf{I} + \mathbf{Y}_A(s)\mathbf{Z}_B(s))^{-1} = \mathbf{Y}_N(s)^{-1} \\ \hat{\mathbf{Y}}(s) &= \mathbf{Y}_A(s)(\mathbf{I} + \mathbf{Z}_B(s)\mathbf{Y}_A(s))^{-1} = \mathbf{Z}_L(s)^{-1}\end{aligned}\tag{1.16}$$

$$\tag{1.17}$$

It was shown in [52] that the residue matrix $\mathbf{R}_{\mathbf{Z}_i}$ of the whole-system impedance defines the sensitivity of the mode λ_i to the apparatus admittance matrix $\mathbf{Y}_A(s)$ according to Equation 1.19. Similarly, the residue of the whole-system admittance defines the sensitivity of the mode to the apparatus impedance. $\langle x, y \rangle$ represents the Frobenius norm.

If a parameter ρ affects only the entry $Y_{mn}(s)$ of the larger apparatus admittance transfer

function matrix $\mathbf{Y}_{\mathbf{A}}(s)$ then the sensitivity of the mode to the parameter can be simplified to the scalar relationship in Equation 1.20. For a small change in the parameter ρ the predicted change in the mode $\Delta\lambda_i$ can be approximately found with Equation 1.21.

$$\hat{\mathbf{Z}}(s) = \sum_i \frac{1}{s - \lambda_i} \mathbf{R}_{\mathbf{Z}i} \quad (1.18)$$

$$\frac{\partial \lambda_i}{\partial \rho} = - \left\langle \mathbf{R}_{\mathbf{Z}i} \cdot \frac{\partial \mathbf{Y}_{\mathbf{A}}(s)}{\partial \rho} \right\rangle \quad (1.19)$$

$$= -R_{Z,nm} \frac{\partial Y_{mn}(s)}{\partial \rho} \quad (1.20)$$

$$\Delta\lambda_i \approx -R_{Z,nm} \frac{\partial Y_{mn}(s)}{\partial \rho} \Delta\rho \quad (1.21)$$

The residue $\mathbf{R}_{\mathbf{Z}i}$ can be estimated from the whole-system impedance frequency response data $\hat{\mathbf{Z}}(j\omega)$ using standard system identification techniques. Since $\hat{\mathbf{Z}}(j\omega)$ can be measured online, the sensitivity of a mode to a change in apparatus parameter can be obtained from measurements without needing to analytically model any of the other apparatus connected to the grid.

The term $\frac{\partial Y_{mn}(s)}{\partial \rho}$ is henceforth referred to as the "admittance-parameter sensitivity" transfer function, to distinguish from the admittance $Y_{mn}(s)$.

Since $\mathbf{Y}_{\mathbf{A}}(s)$ is a 2x2 block diagonal matrix, the required residue in Equation 1.20 can be obtained from a local measurement of the whole-system impedance.

This networked impedance approach is referred to as the "grey-box method" in this thesis as the authors proposed [52], since it does not require full visibility of all details of all apparatus connected to the grid in order to calculate the mode sensitivity. The identification of the transfer function residues from "black-box" frequency response data allows the calculation of the eigenvalue sensitivities which were previously only calculable with a full "white-box" state-space model of the entire system, hence "grey-box". The mathematical relationship between the grey-box approach and the critical eigenvalue approach was revealed in [48].

The "grey-box" relationship forms the foundation of this thesis, the objective of which is to investigate how the grey-box relationship can be effectively utilised with co-operation of the ESO and the IBR operator to improve small-signal stability.

1.1.1 Frequency dynamics in the impedance-based approach

A drawback of an impedance or admittance model is that the model cannot represent any explicit dependence of the grid fundamental frequency on system variables (for example, if the frequency is set by a controller), or vice versa. The steady-state value of the system frequency is allowed to be different from case to case, but traditional impedance models cannot explicitly represent frequency dynamics.

Grid-forming IBR are often programmed with a dynamic relationship between active power and frequency for load-sharing or synchronous machine emulation [53]. Due to this, impedance-based analysis has mainly considered grid-following IBR - or the frequency dynamics has often been ignored if grid-forming IBR are considered [54].

The dynamics of temporary deviations in frequency can be considered by embedding the apparatus "swing frame angle" in the impedance model as a variable [42]. Here, the apparatus "swing frame" refers to the natural internal reference frame for a particular apparatus. For example, the frame locked to the rotor in synchronous machines, or the PLL frame in grid-following IBR, or the frame locked to the voltage generated by a grid-forming IBR. To obtain a linear model the swing frame angle deviations must be linearised with respect to a global reference frame rotating with constant frequency. Therefore, the swing-frame approach can only represent the dynamics of small changes in an apparatus swing frame angle but cannot represent any global changes in the system frequency. A grid-forming IBR modelled in this manner could make small changes to its voltage output angle (and therefore a brief change to its local frequency) but in steady-state its output voltage frequency must be the same as the constant global frequency.

Other recent work [55, 56] has explicitly represented the global frequency dynamics by including the system frequency as an additional variable. This approach is referred to here as the "explicit frequency variable" approach. For example, the system frequency could be set by a grid-forming IBR with an extra output in its impedance model. The extra output allows the dynamics of the dependence of the grid-forming IBR voltage frequency on its output power to be included in its impedance model.

To use the explicit frequency variable approach in a networked impedance model, other component models must take the system frequency as additional input. The extra input allows

the dynamics of the dependence of variables on the system frequency to be included in the overall networked impedance model. These additional dynamics will increase the complexity of the overall model, since they require additional modelling/measurements to be represented accurately. The benefit of the additional modelling is that it does allow the system frequency dynamics - including variations in the global steady-state frequency - to be represented in a small-signal manner.

The explicit frequency variable approach effectively creates a new feedback channel representing the system frequency dynamics and therefore the generalised Nyquist approach can still be used in another form of the impedance-based stability criterion [55]. This was referred to as the "extended impedance-based criterion" in [56].

The original work describing the grey-box method [48, 52] used the swing frame approach in its modelling. Accordingly, the open-source Simplus toolbox [57] which was used to generate the simulations in the grey-box papers [48, 52] also utilizes the swing frame approach. To do this, the whole-system impedance and admittance matrices in Equations 1.16 and 1.17 were defined with each apparatus admittance in $\mathbf{Y}_A(s)$ (Equation 1.10) including embedded swing frame dynamics [42]. The embedding of swing frame dynamics allows the grey-box approach to hold even when parameters which influence the frequency dynamics are used, such as the grid-forming IBR frequency droop in [52]. However - as previously discussed - changes in the global frequency cannot be modelled.

The work presented in this thesis also uses the Simplus toolbox [57] and makes extensive use of the grey-box method [48, 52]. Therefore, the assumption that the global system frequency is constant (at a certain value which can be different case-to-case) is made now and will be inherent in all simulation results presented in this thesis. There has not yet been any discussion in the literature regarding including changes in the global system frequency in the grey-box method, for example by using the explicit frequency variable approach.

1.2 Use-case Scenario for this Research: Grid and IBR operators co-operate to improve small-signal stability

The manner in which a country's electricity grid is operated varies across the world, but there is often an overarching body whose responsibility is to ensure the stable operation of

the grid [58]. In this thesis, the overarching operator will be referred to as the Electricity System Operator (ESO) but should not be taken as a reference to any organisation or country in particular.

It is common that the ESO does not actually own or operate any generation, leading to the common scenario that a generation plant owner, operator and manufacturer are different entities to the ESO. A generation plant owner/operator will be referred to in this thesis as the 'IBR operator'.

In the original paper on the grey-box approach [52] the technique was illustrated with a case study in which it was used to identify that the root cause of an oscillation was that the bandwidth of the current controllers of three IBR had been tuned too low. It was also shown that when their bandwidths were increased, the damping of the mode in question was improved. In order to obtain this result, the whole-system impedance had to be measured at the three busses at which the poorly-tuned IBR were connected, the residues of the mode estimated, and the sensitivities of the apparatus admittances to the current control bandwidths had to be known. In order to identify the root cause, presumably the sensitivity of the admittances to other parameters must also be known in order to compare and identify the largest and hence the root cause.

In the future power grid with high penetration of IBR, there may be oscillation modes which result from interactions between many different apparatus, and a "root-cause" may be difficult to pin down. This is particularly so if there are significant participations from IBR which are unknown to the ESO (for example, the aggregated dynamics of thousands of rooftop solar inverters), or those IBR whose operators are unable/unwilling to share any parameter sensitivities. It is likely that the frequency range of interactions and typical participating states will continue to expand beyond the types of IBR modes of oscillation identified so far [59–62], so a configurable, adaptable method for damping oscillation modes across a wide frequency range is desirable.

The scenario which will be considered in this thesis is that only some IBR operators are willing to share with the ESO more information than a basic impedance spectrum (such as the admittance-parameter sensitivity spectrum) and then go on to participate in shaping their admittance in order to improve the damping of oscillation modes. The unwillingness to assist in improving damping may well apply to IBR which are contributing to the poorly damped oscilla-

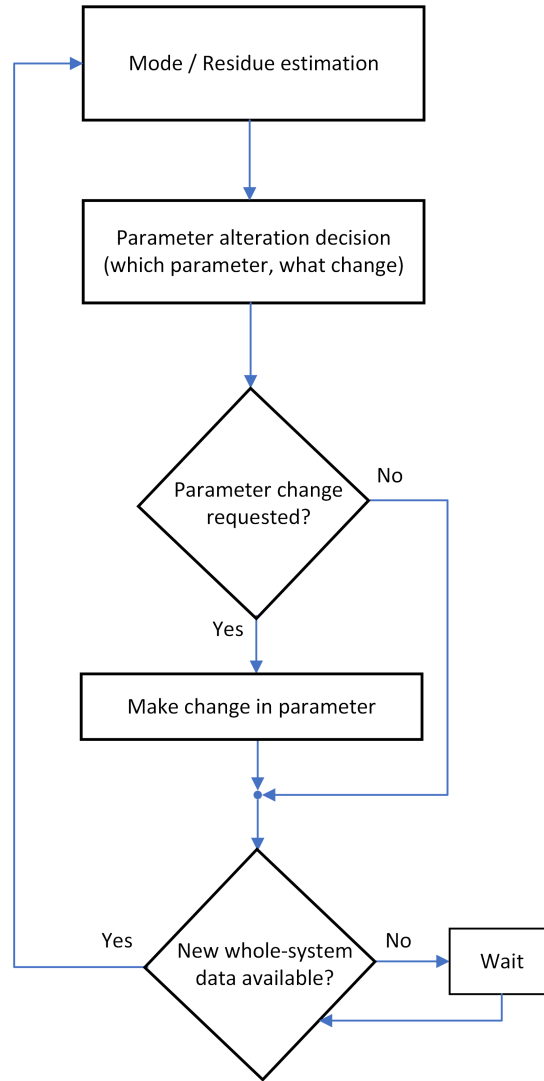


Figure 1.4: The proposed algorithm for application of the grey-box method for iterative small-signal enhancement via impedance shaping.

tions, limiting the usefulness of root-cause analysis. The IBRs of owners who are willing to assist in improving damping are henceforth termed "Active Damping IBR". To take advantage of these active damping IBR, the ESO must be able to measure whole-system impedance/admittance frequency responses at the busses at which the active damping IBR are connected, but not necessarily at any other buses.

The objectives of an IBR operator are normally mainly to do with maximising profit, they will not often be concerned with the whole-system stability as long as they are not affected by outages. Additionally, any re-programming of the IBR will likely involve a monetary cost in the form of paying an engineer or purchasing an additional control system. On the other hand, the objectives of the ESO however are mainly to do with ensuring the stable and secure operation of the power grid [58,63]. Therefore, it may be necessary for the ESO to financially incentivise

an IBR operator to participate in any parameter alteration for the purpose of whole-system stability enhancement.

The proposed co-operative impedance shaping process is illustrated in Figure 1.4. The first step is estimation of the whole-system modes and residues values using measurement data obtained from current or voltage injections at each bus where there is an IBR that can be used for active damping. The next step is the main decision stage, where a decision is made as to the parameter to be altered and to what value. To make this decision, each active damping IBR operator needs to provide the ESO with admittance-parameter sensitivity spectra. If a change is requested, the next step is for the chosen active damping IBR to make the change in parameter. With the changes implemented, injections of current or voltage are made on the new system and data gathered so that the process can be repeated. In this manner the whole-system stability can be improved by iteration.

If more than one Active Damping IBR is to be used then communication is required to identify the best parameter in each IBR to change at each decision stage. However, the principle is the same whether there is one Active Damping IBR or multiple and in this thesis the process will be considered only for one IBR. The process is easily expanded to multiple IBR's with communications present.

Conflicts of interest could arise if an Active Damping IBR operator also had access to controller parameters at another IBR. Then, they could purposefully de-tune the other IBR in order to be paid to re-stabilise the system. The data on patterns of payments and system oscillations should be carefully scrutinised by the ESO in order to detect any suspicious patterns of behaviour.

Grid-following IBR are exclusively considered as the Active Damping IBR in the work presented in this thesis. However, there is nothing preventing the use of grid-forming IBR in the co-operative impedance shaping process as long as the parameters to be changed do not impact the steady-state frequency of the grid-forming voltage output, since this change could not be represented with the swing-frame based approach which the grey-box method is currently based on [42,52].

1.3 Motivations for this thesis

Oscillations caused by the interaction of IBR controls with the grid and other IBR is now recognised as an important problem [10] meriting sustained research effort across the power systems / power electronics / control engineering domains in order to prevent this problem hindering the further roll-out of renewable generation. A process has been proposed in Chapter 1.2 which utilises recent advances in impedance-based small signal stability analysis to systematically improve the damping of small-signal oscillation modes via impedance shaping and co-operation between ESO and IBR operators. The motivation for the research reported in this thesis is that certain practical barriers to implementing the proposed process of enhancing stability are known to exist and these need to be better understood and techniques devised to overcome them. The potential barriers foreseen are listed in this section.

Barrier 1. *Capability of grid-following IBR to provide the impedance shaping required*

The criteria for passivity or Nyquist-based stability place constraints on value of the desired apparatus impedance/admittance over the frequency spectrum. A key benefit of the proposed impedance-shaping process is that the desired change in apparatus impedance/admittance is wholly measurement-based - but this places no inherent constraints on its desired value.

Since the apparatus impedance is to be shaped with a set of alterable parameters, the ranges over which these parameters can be varied without detrimental effect on the IBR's core functions will define the available range of the IBR impedance. Implicit in this is the assumption that the IBR's core functions can be met by a family of parameter settings rather than one unique set of values. Therefore, the allowable ranges of the parameters become a key constraint on the shape-ability of the IBR impedance spectrum and hence how much the damping of whole-system modes can be improved. There is no work in the literature to assess which IBR control parameters are most suitable for impedance shaping for the application of the grey-box method, or on what the fundamental constraints are on the allowable range of their values.

Barrier 2. *Possibility of destabilisation due to variations in $\frac{\partial Y_{mn}}{\partial \rho}(s)$*

The changes made in parameters will move the whole-system modes according to the grey-box relationship in Equation 1.20. However, the value of the term $\frac{\partial Y_{mn}}{\partial \rho}(s)$ in Equation 1.20 will change as the mode λ_i moves in the complex plane. Areas of the frequency spectrum where

$\frac{\partial Y_{mn}}{\partial \rho}(s)$ changes rapidly will result in less accurate predictions of the changes in the modes and could result in destabilisation of the system. The sensitivity $\frac{\partial Y_{mn}}{\partial \rho}(s)$ should be known by the active-damping IBR operator, so it should be possible to account for the shape of $\frac{\partial Y_{mn}}{\partial \rho}(s)$ when calculating the allowable range in parameter changes.

Barrier 3. *Effects of measurement noise on the estimation of the whole-system modes and residues*

The proposed impedance-shaping process requires the estimation of the system residues from measured frequency response data. Since real data will inevitably contain noise, the residues cannot be estimated perfectly and the grey-box method will have to be applied with imperfect information. It is vital that the estimated residue are as accurate as possible to avoid degrading the damping of oscillation modes instead of improving it.

Barrier 4. *Information sharing between ESO and IBR Operators*

Although part of the premise of the proposed scenario is that the active damping IBR(s) are more disposed to co-operate with the ESO than other IBR operators, for the scenario to be realistic the information which needs to be shared between ESO and IBR operators needs to be clearly defined and justified as being no more than necessary. Particularly, proposed solutions for overcoming the aforementioned barriers should consider the information sharing requirements required for their inclusion in the proposed impedance-shaping process.

1.4 Thesis Structure

Chapter 2 reviews literature relevant to small-signal stability enhancement, in addition to mode and residue estimation from measurement data. Subsequently, the state-of-the-art in grid-following converter impedance modelling and impedance shaping is reviewed.

Chapter 3 investigates Barrier 1 by exploring the capabilities of a common grid-following controller architecture to provide a range of impedance shape-ability. Specifically, the best controller parameters for the task are compared via their sensitivity spectra and the performance trade-offs required for altering their values. The grey-box method and the performance trade-offs are demonstrated with two case studies: a Single-Machine Infinite Bus (SMIB) system and a 14 bus network. Both case studies are utilised again in later chapters.

Chapter 4 proposes a novel method to calculate constraints on the maximum allowed change in parameters, accounting for variations in the admittance-parameter sensitivity $\frac{\partial Y_{mn}}{\partial \rho}(s)$. The method addresses the issue raised in Barrier 2.

Chapter 5 examines the impact of noise on the mode and residue estimation and proposes an alteration to a commonly used algorithm to account for the specific characteristics of the whole-system impedance and admittance residue identifications. The work in this chapter is relevant to Barrier 3.

The discussions at the end of Chapters 4 and 5 consider the impact of the proposed methods on the information sharing requirements between the ESO and the IBR operator, to take account of Barrier 4.

Chapter 6 discusses the conclusions of this thesis and possible future work.

1.5 Original contributions

Chapter 3 presents and discusses a study on the the capability of grid-following converter to shape its admittance to improve the damping of SSO modes. The damping effect of virtual impedance/admittance has been documented widely in the literature, however the observation that it may be particularly well suited for application with the grey-box approach is original. The impact of a change in virtual impedance, which is intended to improve whole-system small-signal stability, on the DC link voltage control performance has not been previously deeply discussed. In particular, that the DC performance could be a limiting factor on the grid-following IBR admittance shape-ability has not been mentioned at all in the literature, to the best of the author's knowledge.

Chapter 4 presents a novel constraint on the allowable change in the parameters of an apparatus to ensure small-signal stability. Although it requires making an assumption which is discussed in more detail in Chapter 4, the analysis of the maximum allowed parameter change accounting for admittance spectrum variations is novel. The use of the calculus of variations to account for the non-linear shape in the sensitivity of the admittance to the parameter has not been attempted before.

Chapter 5 presents a novel alteration to the popular open-source "vector fitting" algorithm to tailor it towards better estimation of whole-system impedance and admittance residues under

noisy measurement conditions. The alteration makes use of a previously unreported relationship between the whole-system impedance and admittance residues which were previously thought to be independent. This approach could allow for more accurate prediction of the change in system mode damping when applying the grey-box method.

Together, the work in this thesis investigates practical barriers to the application of the grey-box method which have not been discussed in depth before. The contributions to overcoming these barriers could help future researchers and engineers apply the grey-box method in a real system, to enhance the damping of IBR-driven oscillation modes without more significant investment in new infrastructure.

1.6 Publications

The work in Chapter 5 is in the final stages of preparation for submission to the journal *IET Generation, Transmission & Distribution*, under the title ”*Constrained vector fitting for whole-system impedance and admittance residue estimation from noisy frequency response data*”.

The work in Chapter 3 is under preparation for submission as a case study type paper.

2 Literature Review

This Chapter reviews literature relevant to the studies presented in Chapters 3, 4 and 5.

Section 2.1 reviews literature regarding small-signal stability enhancement techniques in a broader sense than Chapter 1, which focussed on impedance-based techniques. First, Section 2.1.1 reviews the traditional approach to small-signal stability in a mainly synchronous machine-driven system and compares the similarities between the design of Power System Stabilizers (PSS) and the proposed co-operative impedance-shaping process. Then, Section 2.1.2 further reviews work on the optimisation-related methods for tuning the PSS, which are relevant to the proposed impedance-shaping process and provide some context for the work in Chapters 3 and 4 pertaining to Barriers 1 and 2. Finally, Section 2.1.3 reviews other (not impedance-based) approaches for enhancing the stability of mainly IBR-driven SSO.

Section 2.2 reviews literature on system identification techniques for mode and residue estimation. Particularly, the technique known as vector fitting is introduced, which forms the foundation for the work presented in Chapter 5.

Section 2.3 reviews literature on the modelling of grid-following converters, which will be vital to the study presented in Chapter 3.

Section 2.4 reviews literature particularly focused on the IBR small-signal stability technique known as virtual-impedance based control, which will be relevant to the work in Chapters 3 and 4.

2.1 Review of power system small-signal stability enhancement techniques

2.1.1 The traditional approach before deep penetration of IBR

The consensus on small-signal stability before deep penetration of IBR was that oscillation modes involving synchronous machines and other traditional equipment were separable into four main categories [2, 64]:

- Local modes: SMs swinging against each other, normally contained to a small part of the power network
- Inter-area modes: groups of SMs swinging against each other, often across large distances

in national power grids

- Torsional modes: the generator shaft natural modes of oscillation interacting with other equipment controllers or series-capacitor compensated lines.
- Control modes: poorly tuned controllers (AVRs, speed governors etc) interacting with grid equipment or other equipment controls (SVC, IBR, HVDC etc)

To resolve traditional control or torsional oscillation modes normally required re-tuning of controllers or additional control logic (for example notch filters [65]). The resolution of local or inter-area modes has normally been performed with Power System Stabilizers (PSS). PSS are controllers which give a supplementary reference to the AVR of a synchronous machine and they can be tuned to damp low-frequency oscillations and improve the whole-system small-signal stability.

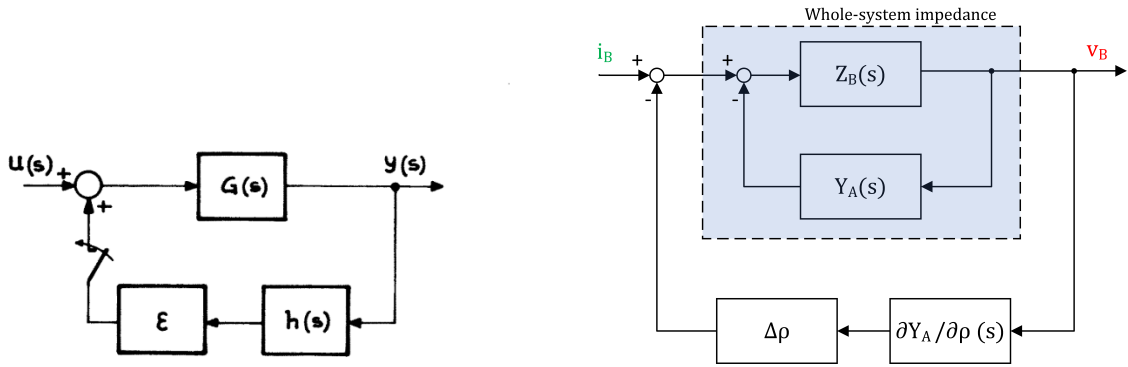
When designing PSS with a full state-space model of the system, the identification of the best location to apply the PSS was often determined in conjunction with residue analysis [66, 67]. Residue analysis was the observation that the sensitivity of a system mode to feedback was defined by the residue of the open-loop transfer function associated with the system mode in question, as shown in Equations 2.1 and 2.2. The transfer functions are illustrated in Figure 2.1a taken from the paper [66].

In Figure 2.1a and Equation 2.2 $h(s)$ represents the controller (the PSS) that is being designed and $G(s)$ the system the controller is acting on. The presence of ε indicates that the relationship in Equation 2.2 is valid for small magnitudes of feedback. Crucially $G(s)$ contains the dynamics of the entire system under consideration, not just the machine the controller is acting upon.

$$G(s) = \sum_i \frac{1}{s - \lambda_i} R_i \quad (2.1)$$

$$\Delta\lambda_i \approx \varepsilon h(\lambda_i) R_i \quad (2.2)$$

R_i represents the residue of the transfer function $G(s)$ corresponding to the mode λ_i . Although it was remarked from the very earliest work that it is technically feasible to extract the frequency response $G(j\omega)$ and hence estimate its residues R_i from physical measurements



(a) Illustration of the residue relationship from [66], where the sensitivity of a mode λ_i of the system $G(s)$ to the feedback loop $\epsilon h(s)$ is defined by Equation 2.2

(b) Illustration of the grey-box approach, where the sensitivity of a mode λ_i of the whole-system impedance $\hat{Z}(s)$ to a change in apparatus admittance $\frac{\partial Y(s)}{\partial \rho} \Delta \rho$ is defined by Equation 1.20

Figure 2.1: Illustration of the similarities of the grey-box relationship to the original form of the residue relationship shown in (a), where the whole-system impedance takes the place of $G(s)$ and the change in an apparatus admittance takes the place of $h(s)$

[66], it was seen as easier to calculate the residues R_i of $G(s)$ from the state-space model of the system.

The larger magnitude of the residue R_i indicates that a smaller control gain is needed to have the desired effect on a mode, therefore the location and signal with the largest magnitude R_i is the most effective and probably most economical choice to apply a PSS [68–70].

To shift a poorly damped oscillation mode λ_i with residue R_i by a distance $-\Delta\sigma_i$ (in purely the negative real direction in the complex plane) would require a controller value of $\epsilon h(\lambda_i) = \frac{-\Delta\sigma_i}{R_i}$ according to Equation 2.2 [66]. Therefore, the choice of desired change $-\Delta\sigma_i$ in each mode λ_i that the controller is designed to achieve puts a constraint on the gain and phase of the controller at the frequency λ_i .

If there are multiple poorly damped modes in the system, it is in general not possible to design a controller that would move all the modes in the purely negative real direction and so a trade-off is required between the increases of damping of the different modes.

The residue-based design described above can be seen to be similar to the grey-box approach, as illustrated in Figures 2.1a and 2.1b. Of course, the types of underlying transfer functions are different: the grey-box method is based on networked impedance models which do not include global frequency dynamics, while the PSS residue tuning method involves transfer functions which do include global frequency dynamics. However, the maths of the residue-based prediction of the movement of the system oscillation modes is identical in both cases: the residue is

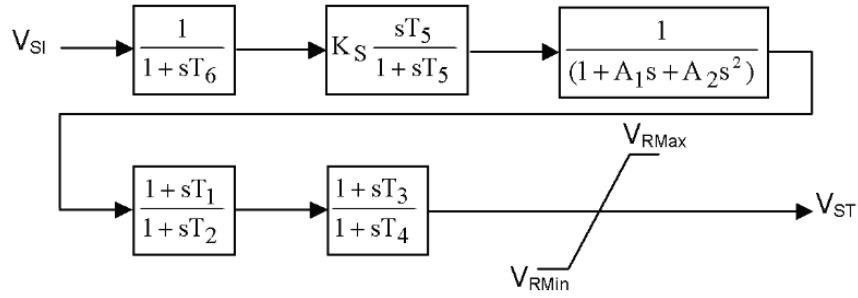


Figure 2.2: The block diagram layout of a classic Power System Stabilizer, showing the voltage transducer, the washout filter, the controller gain and the lead/lag compensation [71]

used in combination with the incremental feedback. In the grey-box approach the incremental feedback is the change in apparatus admittance $\frac{\partial Y_{mn}}{\partial \rho}(\lambda_i)\Delta\rho$, while the classical approach uses the incremental PSS feedback $\varepsilon h(\lambda_i)$.

2.1.2 Review of PSS tuning

The layout of a classic PSS controller is shown in Figure 2.2, where the input signal is normally chosen as one of the generator rotor speed, the bus frequency or the integral of the generator power [2] (these variables are associated with the system frequency dynamics). The washout filter is a high pass filter that prevents changes steady-state frequency from influencing the generator voltage. The gain and the lead-lag compensator blocks provide the required shape of controller transfer function as determined by the design method.

The classic and most commonly used method of picking the PSS parameters is known as the phase compensation approach and it is normally combined with root locus analysis [72–74]. In this approach, the lead/lag characteristics (parameters T_1 , T_2 , T_3 , T_4 in Figure 2.2) of the PSS controller are picked to compensate for the lead/lag characteristics of the closed-loop transfer function $G(s)$ between the AVR reference and the generator speed (noting that $G(s)$ includes all of the rest of the system). In the original implementations, the PSS gain (K_S in in Figure 2.2) was increased until a mode was destabilised and then turned down to one-third of that value for the final selection [73]. However, this approach does not consider inter-area modes.

The other blocks in Figure 2.2 represent the simplified representation of the voltage transducer as a low pass filter with time constant T_6 , a filter to avoid exciting torsional modes with parameters A_1 and A_2 , and a saturation block to protect the equipment.

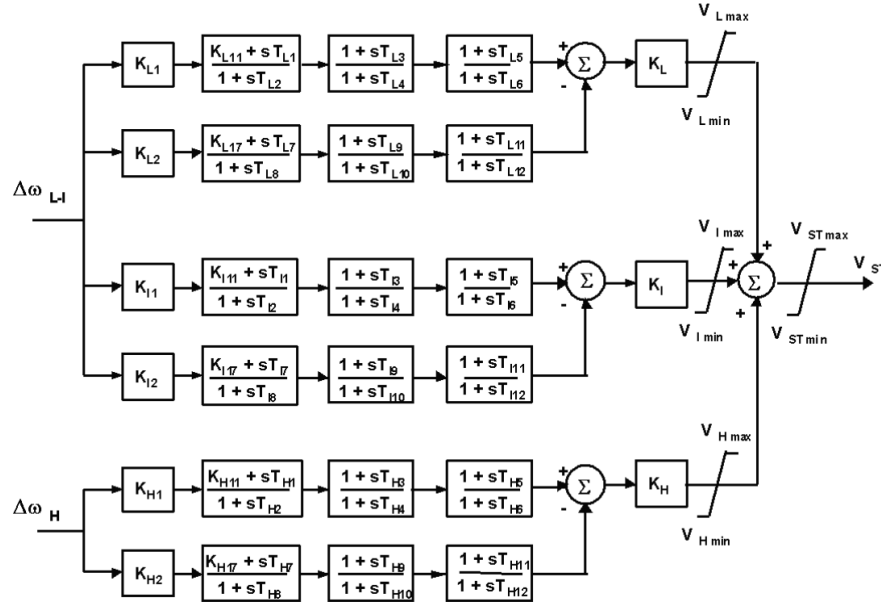


Figure 2.3: A modern multi-band Power System Stabilizer utilizing differently filtered signals for different lead/lag characteristics across the frequency spectrum, allowing more fine tuned interaction with multiple oscillation modes [71]

To design a PSS to damp multiple inter-area modes requires the satisfaction of multiple constraints on the PSS transfer function. One approach is to use a multi-band PSS as shown in Figure 2.3, where each filter applies phase compensation at different frequencies according to the phase of the residues of the modes being considered [75, 76].

Sometimes, additional global signals from a centralized controller are used as inputs to the PSS [77, 78] to improve the damping of inter-area oscillation modes but the time delays in communication links need consideration in the design. The reliability of the communications links also needs to be considered. The global signals often utilize wide-area measurement systems (WAMS).

There has also been work to design centralized damping controllers with classic PSS design techniques and utilizing HVDC converters as the actuator [79]. Similar controller signals (bus frequency input and power output) and similar controller structures (lead/lag blocks, washout filter and gain) are used. Again, centralized control requires reliable and low-delay communication links.

A variety of other methods for PSS control design have been discussed in the literature, such as pole-placement [7, 50] and robust control techniques [80–82]. HVDC-based oscillation damping controllers have also been synthesized from robust control principles [83] but it is noted

that this can result in controller of very high order.

A change in PSS parameter will possibly move different modes in different directions so a common approach is to decide on the parameter change with a form of optimisation to weigh up the different options. Several approaches have been taken when optimising the parameter change:

- Minimise the negative sum of all the mode damping factors with constraints on the allowable range of PSS parameters [84–86].
- Minimise the sum of a weighted combination of the real parts of the modes and negative mode damping factors with constraints on the allowable range of PSS parameters [87–89]
- Optimise the difference between actual mode locations and target mode locations [90]. This approach is basically iterative pole-placement, however it requires pre-determined knowledge of where the poles are desired to be which is much more specific knowledge than simply minimising the negative sum of the damping factors or the real parts of the modes
- Minimise the controller gain with constraints on mode locations in the complex plane [91–93]. This is the complementary approach to the minimising the negative mode damping factor with constraints on the controller parameters.
- Optimise a nonlinear time domain-based objective function based on a nonlinear simulation of the system [94,95]
- Optimise the sum of the squares of the difference between actual and ideal PSS transfer function according to phase compensation based-design [96]

Often the sensitivity of the objective function to the possible change in parameters is known (for example, with the residue relationship). The simplest application of the sensitivity-based approach is to do just one iteration of design, *i.e.*, to use the residues to design the PSS without further iteration [97]. The one-iteration approach may not use the capabilities of the PSS to its full extent, but is simpler than the iterative approach. An iterative approach would re-calculate the sensitivity after a change in parameter and use the updated sensitivity to perform another parameter change. The iterative approach could allow smaller changes in magnitude of the PSS

transfer function compared to a one-step design, so the accuracy of the linearised relationship in Equation 2.2 which the design is based on would be improved.

With knowledge of the objective function and the sensitivity, a simple optimisation algorithm such as maximum descent can be used to guide the optimal change in parameters [91–93, 98–100]. Alternatively, more complex algorithms such as sequential quadratic programming [89] or sequential conic programming [90] have been used.

It may not be feasible to find the sensitivity to PSS parameters for all of the objective functions listed above. For instance, this applies to an objective function based on a time-domain simulation. One way around this is seen in the large body of literature that describes derivative-free optimisation such as some form of genetic algorithm [84, 86, 88, 94] or particle-swarm [87, 95]. However, derivative-free algorithms are more suitable for offline simulation studies since they tend to require calculation of the objective function for a wider range of parameters. If calculating the objective function requires making the parameter change before taking measurements, a derivative-free parameter change could be suggested which results in an unstable system.

The methods for tuning PSS discussed in this Chapter share three common properties which should be taken into account in the implementation of the proposed impedance-shaping process:

- A frequency-targeted approach, to shift the desired modes in the desired directions without excessively impacting modes elsewhere in the frequency spectrum, while ignoring steady-state changes
- Constraints on the predicted changes in mode locations to ensure no individual mode is destabilized
- Incentives to avoid excessive control effort: either including a measure of the control effort in the cost function or explicitly constraining the allowable change in parameters

2.1.3 Other techniques for IBR-driven oscillation mitigation

Since the proliferation of IBR on the electricity grid the consensus on the categorisation of oscillation modes has expanded to include IBR-driven instability modes [101]:

- Fast Interaction IBR-Driven Instability: interactions in the frequency range of tens to

hundreds of Hertz, generally the current-control loop of the IBR interacting with grid resonances or fast control loops in other IBR.

- **Slow Interaction IBR-Driven Instability:** slower interactions mainly driven by the dynamics of the converter outer control loops - the main type of interaction which results in SSO's. For example, under weak grid conditions using a high-bandwidth PLL in a grid-following IBR can lead to unstable interactions with the grid. Also, the outer loop controllers of different IBR can interact [38,61].

An implemented method of resolution for both types of IBR-driven instabilities is to "strengthen" the grid with a device which can perform dynamic voltage regulation to reduce the sensitivity of the grid voltage to the IBR current. Grid-forming IBR can be used [102,103], although their reactive power response to grid voltage deviations can be limited to their rated power. Alternatively, synchronous condensers have been installed in grids around the world to mitigate potential SSO instabilities [104,105].

Synchronous condensers may be more expensive than grid-forming IBR of the same rated capacity, but can exceed their rated reactive power capacity by several times for short durations of time when responding to grid voltage fluctuations [106]. These solutions can be viewed as last-resort mitigations since they involve investing in new infrastructure, the costs of which can run into the hundreds of millions GBP [107].

Another implemented method to resolve IBR-driven instabilities is to identify grid dispatch scenarios which may be vulnerable to the instabilities and disconnect sufficient IBR generation to avoid them. Clearly, this is a sub-optimal method which ESO's are also keen to avoid, but it has also been used in practice [108].

Traditionally, PSS were seen as a better value option for the damping of electro-mechanical synchronous machine oscillations [109] compared to the installation of dedicated equipment (such as an SVC) since PSS were merely "tacked on" to existing equipment and do not require significant investment in additional hardware. In a similar manner, retuning the controller parameters of existing IBR could be seen as a low-cost and low-intervention approach to resolve IBR-driven oscillations without investing in significant new infrastructure.

2.2 Review of oscillation mode and residue identification in power systems

The application of the grey-box approach requires the estimation of the modes and residues from the whole-system measurement data. This section reviews techniques to perform this.

The standard template for system identification techniques is the prediction error method (PEM) [110], where the optimal vector of model parameters $\boldsymbol{\theta}^*$ are estimated based on minimising a norm of the error between the measured data $y(j\omega_i)$ and the estimated model output $\hat{y}(\boldsymbol{\theta}, j\omega_i)$. For the least-squares approach shown in Equation 2.3 the norm used is the 2-norm. The individual data entries $y(j\omega_i)$ are shown in this section for clarity, while the system identification would be applied to the whole measurement data-set $\mathbf{y}$.

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sqrt{\sum_i (y(j\omega_i) - \hat{y}(\boldsymbol{\theta}, j\omega_i))^2} \quad (2.3)$$

For identification of a non-linear system $y(j\omega_i)$ with a linear model $\hat{y}(\boldsymbol{\theta}, j\omega_i)$ the measured data can be broken down as the contributions from the closest linear representation $y^{\text{true}}(j\omega_i)$ plus components from nonlinear distortions $y^{\text{non}}(j\omega_i)$ and measurement noise $n(j\omega_i)$ [111], as illustrated in Equation 2.4.

$$y(j\omega_i) = y^{\text{true}}(j\omega_i) + y^{\text{non}}(j\omega_i) + n(j\omega_i) \quad (2.4)$$

$$y(j\omega_i) \approx y^{\text{true}}(j\omega_i) + n(j\omega_i) \quad (2.5)$$

If the non-linear distortions are negligible the measured data can be approximated as Equation 2.5, consisted of the response of the "true" system $y^{\text{true}}(j\omega_i)$ and the noise $n(j\omega_i)$. The type of model $\hat{y}(\boldsymbol{\theta}, j\omega_i)$ and the method for solving the minimisation in Equation 2.3 are the distinguishing features of different system identification algorithms.

Time-domain PEMs have been widely used in power system applications for mode identification from ring-down time-domain signals: Prony analysis [74], eigensystem realization algorithm (ERA) [112] and matrix pencil (MP) [113] are the three most widely used linear system identification methods [114]. Prony's method has also been applied for residue estimation from

simulation data for PSS design [115], however it has been reported that Prony analysis is quite susceptible to bias with noise [114] in comparison with ERA and MP.

For identification from arbitrary, un-controlled inputs time-domain identification is accepted to be the best method [116] - hence the wide application in the literature using data gathered from ring-down events.

However, if a frequency response spectrum is available (for example to apply impedance-based analysis techniques as discussed in Chapter 1) then frequency domain system identification techniques can be used [116]. Obtaining a frequency response spectrum requires a controllable input with excitation across the frequency spectrum with periodic signals, followed by application of the Fast Fourier Transform (FFT) [117] at each frequency. This type of input is within the scope of the proposed scenario described in Chapter 1, so a frequency-domain approach is taken in this thesis.

Frequency domain system identification methods (FDSIM) estimate the parameters of the transfer function from the frequency response data: they are essentially curve-fitting algorithms. A frequency response resulting from a transfer function is shown in pole-residue form in Equation 2.6 and pole-zero form in Equation 2.7.

$$G(j\omega) = \sum_i \frac{R_i}{j\omega - \lambda_i} \quad (2.6)$$

$$= M \frac{\prod_i j\omega - z_i}{\prod_i j\omega - \lambda_i} \quad (2.7)$$

$$M = \frac{\sum_i R_i \left(\prod_{k \neq i} j\omega - \lambda_k \right)}{\prod_i j\omega - z_i} \quad (2.8)$$

Early implementations of FDSIM identified the transfer function in pole-zero form, which requires an additional constraint to obtain unique results for the poles and zeros since clearly all the poles and zeros could be multiplied by a constant and the transfer function would be unchanged. Fixing any of the poles or zeros to the value '1' was often used as a solution, but if the "true" value of this parameter was actually zero then numerical issues could prevent the convergence of the numerical solution [116].

One of the most effective and widely used FDSIM is accepted [118, 119] to be the open-

source algorithm known as vector fitting [120, 121]. Vector fitting avoided the non-uniqueness problem by fixing a parameter in an intermediate function used in the solution process. The algorithm was later updated from the original version by relaxing the original constraint [122] to a constraint on the norm of the intermediate function which allowed better robustness to noise, followed by another update for faster solutions for non-scalar systems [123].

Vector fitting has been widely applied in the structural and electromagnetic engineering fields. Applications in power systems/power electronics have been mainly for admittance model identification of IBR [20, 21, 124, 125] or a detailed grid model [126] from frequency response data, to generate a state-space model for eigenvalue-based small-signal stability analysis. There has not yet been consideration of the suitability of vector fitting for whole-system impedance mode/residue identification for the application of the grey-box method.

Noise is not so often considered in the literature on vector fitting applied to power systems/power electronics, but when it is the assumption that measurement noise can be modelled as zero-mean Gaussian-distributed white noise is commonly made [127–130].

It is known that the frequency domain representation of zero-mean Gaussian-distributed white noise is as shown in Equation 2.9, a combination of a real and an imaginary random variables with equal variance in real and complex parts, with equal variance σ^2 across the frequency spectrum.

$$n(j\omega_i) = \mathcal{N}(0, \sigma^2) + j\mathcal{N}(0, \sigma^2) \quad (2.9)$$

With actual measurements more accurate noise models can be used. For example, in [131] the background noise frequency spectrum is measured and it is observed that the noise will have larger magnitude at frequencies near problematic harmonic emission frequencies or network resonances, which is used in the determining the minimum amplitude of signal injections in a frequency sweep. Similarly, [132] determines the optimal set of frequencies to perform a simultaneous multi-frequency impedance measurement at, based on a frequency domain representation of the measurement noise.

There have been attempts to improve the convergence of vector fitting with noisy data [133, 134] or to guide the model order selection [135], but not so far any attempt to tailor it for the application of the whole-system impedance/admittance residue estimation.

2.3 Review of Grid-following converter impedance modeling

This section reviews literature on grid-following converter admittance modelling to build a model which will be used in the study in Chapter 3.4. The work in Chapter 3.4 utilises the model to explain the effect of the parameter changes on the IBR admittance, where the parameter changes are made in order to improve the whole-system small-signal stability. Chapter 3 will also consider the impact of the parameter changes on the IBR performance after voltage dips, since changes in parameter may benefit small-signal stability at the cost of degraded transient performance of the internal variable regulation. Additionally, the change in parameters could affect grid code compliance [136], or large-signal stability [137], which could also be taken into account when altering the parameters.

Particular attention is paid in this section to the terminal voltage feed-forward terms in the grid-following converter model, which are often referred to as "virtual impedances" [138]. Since the model derived in this section is an admittance model, the term "virtual admittance" will be used as well. The two terms are often used interchangeably in the literature.

A grid-following voltage-source converter generates a controllable AC voltage using pulse-width modulation (PWM), which induces a current to flow into/out of the grid through the output filter. Figure 2.4 illustrates a typical control layout of a grid-following converter. The current controller sets the PWM voltage in order to track the current references and reject disturbances.

The current references are set by the outer loops, while the Phase-Locked Loop (PLL) synchronizes the converter with the grid voltage. The synchronization ensures that the d axis current mainly generates real power and the q axis current mainly generates reactive power.

The programming of the outer control loops vary and are often either used to track reference values of active and reactive power, or to control the DC capacitor voltage with the active power and inject reactive power in response to grid voltage magnitude variations [139].

It is common in the literature (eg. [13, 140, 141]) to assume a stiff DC voltage source and to just control the output power to a reference value. However, DC-link voltage dynamics are reported to affect sub-synchronous interaction modes [142, 143] so DC-voltage control is considered in this thesis.

The DC link buffers differences in power between the input power from the power source

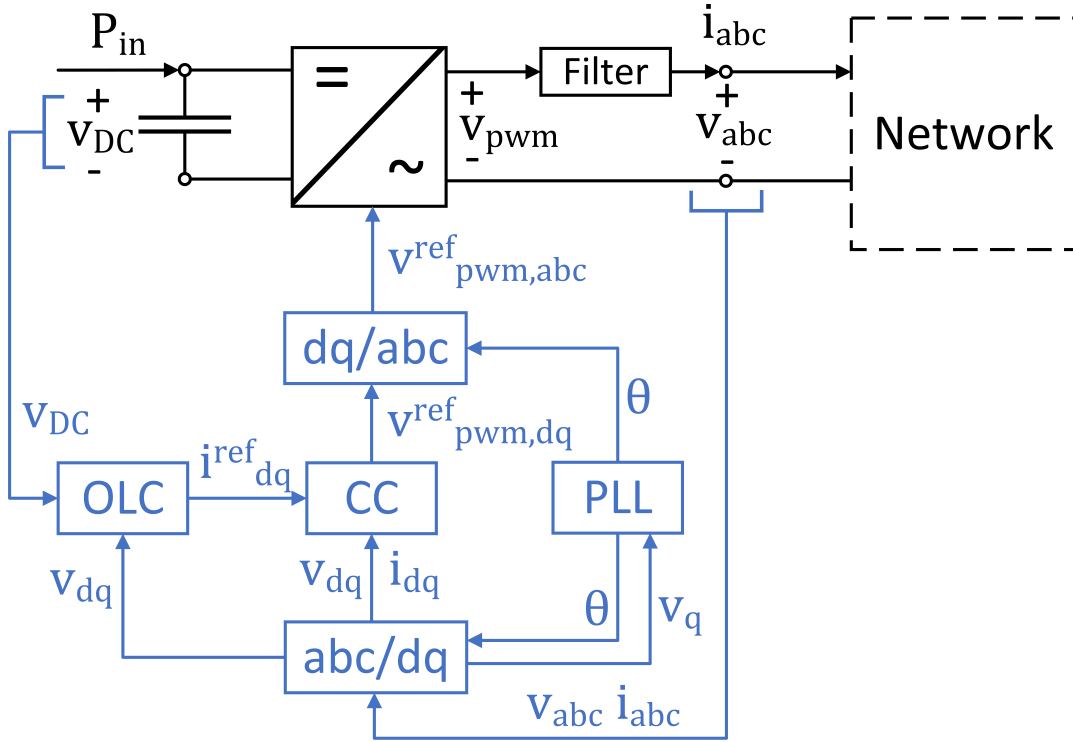


Figure 2.4: Grid-following converter control diagram with the following abbreviations; CC - Current control, OLC - Outer Loop Control, PLL - Phase Locked Loop. Physical quantities are shown in black and signals are shown in blue

via the AC-DC converter and the output power fed to the grid via the DC-AC converter. It is assumed that the input power from the power source is constant - which is a common assumption made in the literature and allows the converter model to not include the machine-side AC-DC converter [23, 59].

Although most practical converters employ an LCL-filter to filter out the PWM switching transients in the output current, for the analysis of sub-synchronous oscillations it is assumed that the filter dynamics can be represented by a simple resistor-inductor pair. When the purpose of the modelling is not do with higher frequency effects such as harmonics or interaction in the time-scale of the current control the L-filter assumption is common [23, 59, 140, 144].

In this thesis non-capitalised dq variables such as v_d, v_q represent variables in the converter internal co-ordinate system, while capitalised DQ variables represent variables in the external co-ordinate system. It is assumed that the terminal voltages in the two co-ordinate systems are initially aligned such that $v_d^0 = v_D^0 = E_0$ and $v_q^0 = v_Q^0 = 0$.

V_{PWM} represents the voltage set at the PWM, since sub-synchronous frequencies are under

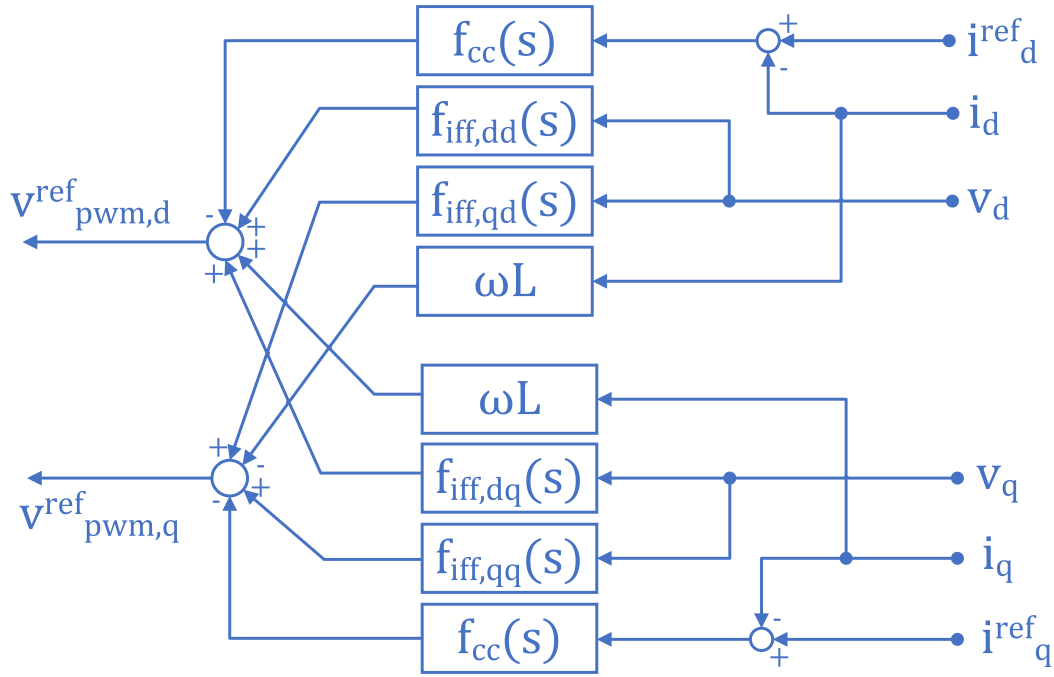


Figure 2.5: Grid-following converter current control diagram showing the current reference tracking controllers $f_{cc}(s)$, the decoupling terms ωL and all four quadrant of inner virtual admittance

consideration the time delay of the PWM is assumed negligible [23, 59, 139]. i represents the filter current, which is taken as positive into the filter from the grid. Finally, v represents the inverter terminal voltage - all variables are illustrated in Figure 2.4.

The admittance model is a linear model, so all variables are assumed to be expressed in terms of deviations from an operating point.

The current controller sets the PWM voltages $V_{PWM,d}$, $V_{PWM,q}$ and is shown linearised in Equation 2.10 and illustrated in Figure 2.5. The PWM voltage is commonly set by a controller $f_{cc}(s)$ acting on the current error plus a decoupling term ωL acting on the measured filter currents i_d, i_q . Here, L is the inductance of the filter. The resistance of the filter is assumed negligible.

$$\begin{aligned} V_{PWM,d} &= -(i_d^{\text{ref}} - i_d)f_{cc}(s) + \omega L i_q + f_{\text{iff},dd}(s)v_d + f_{\text{iff},dq}(s)v_q \\ V_{PWM,q} &= -(i_q^{\text{ref}} - i_q)f_{cc}(s) - \omega L i_d + f_{\text{iff},qd}(s)v_d + f_{\text{iff},qq}(s)v_q \end{aligned} \quad (2.10)$$

Also shown in Equation 2.10 are the terminal voltage feed-forward terms (such as $f_{\text{iff},dd}(s)$) acting on the measured converter terminal voltages v_d, v_q . All four quadrants of feed-forward are included to keep the resulting virtual admittance formulation as general as possible.

The control law in Equation 2.10 can be combined with the dynamics of the filter inductor in Equation 2.11 give the closed loop equations for the current dynamics in Equation 2.12.

$$\begin{aligned} v_d - V_{PWM,d} &= sLi_d + \omega Li_q \\ v_q - V_{PWM,q} &= sLi_q - \omega Li_d \end{aligned} \quad (2.11)$$

$$\begin{aligned} i_d &= g_{cc}(s)i_d^{\text{ref}} + h_{cc}(s)(1 - f_{\text{iff},dd}(s))v_d - h_{cc}(s)f_{\text{iff},dq}(s)v_q \\ i_q &= g_{cc}(s)i_q^{\text{ref}} - h_{cc}(s)f_{\text{iff},qd}(s)v_d + h_{cc}(s)(1 - f_{\text{iff},qq}(s))v_q \end{aligned} \quad (2.12)$$

In Equation 2.12 $g_{cc}(s)$ represents the current reference tracking transfer function, shown in Equation 2.13. $h_{cc}(s)$ represents the grid-voltage disturbance rejection transfer function, shown in more detail in Equation 2.14. When the perfect current control assumption is made $g_{cc}(s) \approx 1$ and $h_{cc}(s) \approx 0$.

$$g_{cc}(s) = \frac{f_{cc}(s)}{sL + f_{cc}(s)} = \frac{\partial i_d}{\partial i_d^{\text{ref}}} = \frac{\partial i_q}{\partial i_q^{\text{ref}}} \quad (2.13)$$

$$h_{cc}(s) = \frac{1}{sL + f_{cc}(s)} = \frac{\partial i_d}{\partial v_d} = \frac{\partial i_q}{\partial v_q} \mid f_{\text{iff},dd}(s), f_{\text{iff},qq}(s) = 0 \quad (2.14)$$

The form of Equation 2.12 is a widely used representation which can be seen in [13, 23, 145, 146], although all four feed-forward terms are not commonly shown (particularly not the $d - q$ and $q - d$ terms $f_{\text{iff},dq}(s)$ and $f_{\text{iff},qd}(s)$).

The expression in Equation 2.11 is given in terms of the internal PLL co-ordinates - it is assumed here that the dynamics of the PLL are negligible in the timescale of the dynamics of the filter inductor. This is a common assumption made in the literature [13, 23, 140, 144] and when decoupling control is used this assumption leads to perfect decoupling in Equation 2.12. The perfect decoupling also requires the assumption of a negligible time delay in the PWM voltage reference tracking.

It can be seen from Equation 2.12 that the feed-forward terms directly influence the dynamics of the relationship between the terminal voltages v_d , v_q and the output currents i_d , i_q . For this reason, the feed-forward terms are often referred to as a "virtual impedance" or, to correspond with the representation in Equation 2.12, as a "virtual admittance". The feed-forward

terms from the terminal voltage to the PWM voltage are further specified as "inner" virtual impedances in [138] and will be referred to as such in this thesis.

The phase locked loop (PLL) uses a dq transformation as a phase detector. The synchronisation angle θ is used to transform the measured external phase voltages v_{abc} to the internal co-ordinate system v_{dq} . The synchronization is implemented by driving v_q to zero with a controller $f_{\text{PLL}}(s)$.

There are multitudes of different kinds of PLL designs reported in the literature [147] but it is shown in [148] that various more complex PLL designs can be reduced to the form of Equation 2.18 under linearisation and the assumption that the grid frequency is constant.

The translation of variables between the coordinate systems is linearised in Equation 2.15 with the initial conditions $v_d^0 = v_D^0 = E_0$ and $v_q^0 = v_Q^0 = 0$.

$$\begin{aligned} v_d &= v_D + v_q^0 \Delta\theta = v_D \\ v_q &= v_Q - v_d^0 \Delta\theta = v_Q - E_0 \Delta\theta \end{aligned} \quad (2.15)$$

Equation 2.16 describes the action of the PLL controller acting on the measured q axis voltage. Utilizing Equation 2.15, the dynamics of $\Delta\theta$ in response to the grid voltage v_Q can be derived giving Equation 2.19.

$$\Delta\omega = f_{\text{PLL}}(s)v_q = s\Delta\theta \quad (2.16)$$

$$\Delta\theta = \frac{1}{s} f_{\text{PLL}}(s) (v_Q - E_0 \Delta\theta) \quad (2.17)$$

$$\Delta\theta = \frac{f_{\text{PLL}}(s)}{s + E_0 f_{\text{PLL}}(s)} v_Q \quad (2.18)$$

$$g_{\text{PLL}}(s) = \frac{\partial \Delta\theta}{\partial v_Q} = \frac{f_{\text{PLL}}(s)}{s + E_0 f_{\text{PLL}}(s)} \quad (2.19)$$

In Equation 2.19 $g_{\text{PLL}}(s)$ represents the PLL angle tracking performance and has the effective form of a low-pass filter, while in Equation 2.20 $h_{\text{PLL}}(s)$ represents how changes in v_Q are transferred to changes in v_q and takes the form of a high-pass filter.

The expression for $\Delta\theta$ in Equation 2.19 combined with Equation 2.15 allows the translation

of the voltages and the currents between co-ordinate systems. The translations are shown in Equations 2.20 and Equation 2.21.

$$\frac{\partial v_q}{\partial v_Q} = (1 - E_0 g_{\text{PLL}}(s)) = \frac{s}{s + E_0 f_{\text{PLL}}(s)} = h_{\text{PLL}}(s) \quad (2.20)$$

$$\begin{aligned} i_D &= i_d - i_q^0 \Delta\theta = i_d + \frac{Q_0}{E_0} g_{\text{PLL}}(s) v_Q \\ i_Q &= i_q + i_d^0 \Delta\theta = i_q + \frac{P_0}{E_0} g_{\text{PLL}}(s) v_Q \end{aligned} \quad (2.21)$$

If the DC-link dynamics are considered the d axis current reference i_d^{ref} is normally used to control the DC-link voltage V_{DC} , since when the PLL is synchronized the d axis current generates mainly real power.

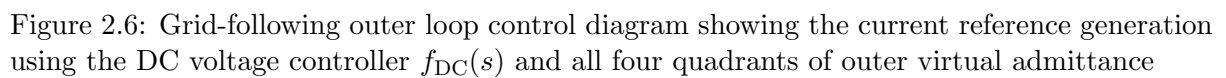
Setting the current-references based on the measured terminal voltage is referred to as an "outer" virtual impedance in [138] and will be referred to as such in this thesis.

Expressions for calculating the current references are shown in Equation 2.22, where $f_{\text{DC}}(s)$ is the controller acting on the DC capacitor voltage deviation and $f_{\text{off,dd}}(s)$, $f_{\text{off,dq}}(s)$, $f_{\text{off,qd}}(s)$ and $f_{\text{off,qq}}(s)$ are feed-forward terms acting on the measured terminal voltage.

$$\begin{aligned} i_d^{\text{ref}} &= f_{\text{DC}}(s) V_{DC} + f_{\text{off,dd}}(s) v_d + f_{\text{off,dq}}(s) v_q \\ i_q^{\text{ref}} &= f_{\text{off,qd}}(s) v_d + f_{\text{off,qq}}(s) v_q \end{aligned} \quad (2.22)$$

The feed-forward term $f_{\text{off,qd}}(s)$ is often used for control of the AC voltage magnitude since it governs the reactive power response to terminal voltage magnitude variations. The other three feed-forward terms are implemented less often but examples of the use of the $f_{\text{off,qq}}(s)$ and $f_{\text{off,dq}}(s)$ can be found in [146, 149, 150] - which will be discussed further in Chapter 2.4. The dd feed-forward $f_{\text{off,dd}}(s)$ is included for completeness.

Including the current control dynamics from Equation 2.12 gives the expression for the currents i_d, i_q considering the outer loops in Equation 2.23.



It can be seen that both the inner and the outer virtual admittances play a similar role in Equation 2.23 so for brevity they are combined into "generalised virtual admittance" terms in Equation 2.24. The simplified representation in Equation 2.25 gives the converter output current in terms of the DC-link voltage and terminal voltage, utilising the generalised virtual admittance terms.

$$\begin{aligned} i_d &= g_{cc}(s)f_{DC}(s)V_{DC} + (h_{cc}(s) + g_{ff,dd}(s))v_d + g_{ff,dq}(s)v_q \\ i_q &= g_{ff,qd}(s)v_d + (h_{cc}(s) + g_{ff,qq}(s))v_q \end{aligned} \quad (2.25)$$

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link from the power source is constant).

The filter losses are neglected in the majority of the literature modelling converter impedances for stability studies (for example [23, 59, 151, 152]) but it is reported that this can cause a loss of accuracy in [139, 142]. The filter losses will only reduce the impact of the change in power balance on the DC voltage so the decision to ignore them is a conservative approximation. The filter losses will not be considered further in this work.

The details of the DC-capacitor dynamics derivation are laid out in Appendix A, the result of which is shown in Equation 2.26.

$$V_{DC} = h_{DC}(s) \left(\left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) v_d + \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) v_q \right) \quad (2.26)$$

$$h_{DC}(s) = \frac{1}{sC_{DC}V_{DC}^0 - E_0 g_{cc}(s) f_{DC}(s)} \quad (2.27)$$

The expression for the DC voltage in Equation 2.26 can be substituted for V_{DC} in Equations 2.28 and 2.29. The substitution in Equation 2.30 is used to give the final form of the analytical admittance expression in Equation 2.31.

$$i_d = g_{cc}(s) f_{DC}(s) V_{DC} + (h_{cc}(s) + g_{ff,dd}(s)) v_d + g_{ff,dq}(s) v_q \quad (2.28)$$

$$i_d = g_{cc}(s) f_{DC}(s) \left(\left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) v_d + \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) v_q \right) + (h_{cc}(s) + g_{ff,dd}(s)) v_d + g_{ff,dq}(s) v_q \quad (2.29)$$

$$g_{DC}(s) = g_{cc}(s) f_{DC}(s) h_{DC}(s) \quad (2.30)$$

$$Y_{dd}(s) = g_{DC}(s) \left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) + h_{cc}(s) + g_{ff,dd}(s)$$

$$Y_{dq}(s) = g_{DC}(s) \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) + g_{ff,dq}(s) \quad (2.31)$$

$$Y_{qd}(s) = g_{ff,qd}(s)$$

$$Y_{qq}(s) = g_{ff,qq}(s) + h_{cc}(s)$$

The internal reference frame is translated into the external frame using Equations 2.15 and 2.21 to give the overall admittance expressions in Equation 2.32.

$$\begin{aligned}
 Y_{DD}(s) &= g_{DC}(s) \left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) + h_{cc}(s) + g_{ff,dd}(s) \\
 Y_{DQ}(s) &= \left(g_{DC}(s) \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) + g_{ff,dq}(s) \right) h_{PLL}(s) \\
 &\quad + \frac{Q_0}{E_0} g_{PLL}(s) \\
 Y_{QD}(s) &= g_{ff,qd}(s) \\
 Y_{QQ}(s) &= (h_{cc}(s) + g_{ff,qq}(s)) h_{PLL}(s) + \frac{P_0}{E_0} g_{PLL}(s)
 \end{aligned} \tag{2.32}$$

The relatively clean formulation in Equation 2.32 results from a number of assumptions listed here for clarity.

- The time delay in implementing the PWM voltage reference is negligible
- The PWM is lossless
- DC-link control is enabled
- The input power from the power source is constant
- The filter dynamics can be represented by an inductor
- The PLL dynamics are negligible within the timeframe of the filter dynamics in the current control, which combined with the lossless, instant PWM leads to perfect current control decoupling.
- The PLL is initially synchronized with the grid voltage
- The power losses in the filter are negligible

It can be seen that the effect of the DC control can be isolated to two quadrants of the admittance $Y_{DD}(s)$ and $Y_{DQ}(s)$ since the assumptions remove the impact of the DC voltage dynamics on the q-axis current. The PLL effect is isolated to the two quadrants $Y_{DQ}(s)$ and $Y_{QQ}(s)$ since the linearisation with the initial condition $v_q^0 = 0$ removes the effect of the PLL on the d-axis voltage.

Under the assumptions used the admittance quadrant $Y_{QD}(s)$ is affected by neither the DC-link dynamics nor the PLL dynamics so it is determined wholly by the feed-forward. Since the q-axis current mainly determines the output reactive power, the feed-forward $g_{ff,qd}(s)$ would contain the dynamics of the desired reactive power response to grid voltage magnitude deviations.

The outer control loops are not standardised across applications and it is possible to set the current control references based on other signals, such as voltage deviations at a remote bus [79]. This case would require communication links and possibly a centralized controller and may involve non-negligible time delays. One of the motivations of the thesis is to investigate the capability of grid-following converters to improve small-signal stability without investing in additional dedicated hardware, so current references based on remote signals will not be considered further in this thesis. Additionally, time-delays greatly complicate the complexity of the small-signal modelling required.

In some applications (particularly microgrids) the active power reference may instead be set by a frequency droop [152]. A frequency-droop would introduce dynamics not fully describable with an impedance model (since the input is frequency) and is out of scope of the grey-box method. Due to this, the frequency droop is not included in the modelling.

The admittance model of a grid-following converter described in this section has been widely used in passivity [153, 154] and Nyquist-based [13, 155] stability analysis. However, there has been little research on the shape-ability of the admittance for the purposes of utilising the grey-box approach.

2.4 Review of virtual impedance based small-signal stability enhancement

This section will review specifically the application of virtual impedance in the converter controller. Chapter 2.3 reviewed the modelling of a grid-following converter and introduced the inner and outer virtual impedance terminology which is used here without further introduction. Virtual impedance is reviewed here in the context of small-signal stability enhancement; while there are other uses reported in the literature (e.g. load-sharing, fault current limitation [138]) these will not be discussed further in this thesis.

The inner virtual impedance has been most commonly analysed in the literature, often used

to improve the disturbance rejection capability of the current-control loop [23] or to damp LCL-filter resonances [153, 156] or torsional resonances [145, 157].

The application of outer virtual impedance [138] for the damping of converter-driven SSO has been less widely discussed. It was reported that feed-forward of the q axis voltage to q axis current reference (a qq outer virtual impedance) improves the stability of the PLL under weak grid conditions [146, 149, 158], particularly when the feed-forward includes an integrator. A dq outer virtual impedance was also shown to improve the PLL weak grid stability [150], but the DC link is not modelled so no consideration can be given to the impact on the DC control.

The qd outer virtual impedance has not often been analysed for small-signal stability enhancement as it mainly used for AC voltage control. It has been reported that too aggressive AC voltage control (i.e. the qd virtual impedance gain is too large or the bandwidth too high) can result in poor small-signal stability, particularly under weak grids [10] or in the presence of other IBR where the interactions of the IBR voltage controllers has not been considered [38, 60].

There has been little discussion of the dd outer virtual impedance without ignoring the DC control. The only work where a dd outer virtual impedance has been considered in conjunction with DC control to the best of the authors knowledge is in [159], where it is used to extend the frequency range of passivity to counteract the non-passive region due to the DC control. However, the simulations performed only consider the qq channel virtual impedance with the DC control disabled so there was no discussion of the impact of the virtual impedance on the DC voltage.

The feed-forward of the active power reference [160], the DC link input power [23] or DC link voltage [146] to the d axis current-reference for improved disturbance rejection is common when the DC link input power is not assumed to be constant (as is assumed here).

The form of the virtual impedance is usually just a gain (i.e. a virtual resistor) but more complex implementations have been used requiring measurement differentiation (a virtual inductor) [153] or an integrator [161]. Most practical implementations of virtual impedance include a low pass filter (LPF) cascaded with the measured terminal voltage in order to avoid amplification of harmonics or unintended interactions with un-modelled higher frequency dynamics.

Much more complex forms of the virtual impedance have been synthesized [162, 163] using robust control techniques to stabilize the PLL, but as is discussed in [146] the order of the virtual

impedance becomes similar to the order of the rest of the converter, and simple feed-forward schemes are shown to work just as well.

Measurement-based adaptive schemes in the literature focus on estimating the grid impedance and use the virtual impedance technique to reshape the IBR impedance based on Nyquist analysis. The grid impedance has been estimated with system identification applied to measurements of responses to injected disturbances [155, 164, 165], or based on ambient data [166, 167].

Both inner [155, 164–166] and outer [167, 168] virtual impedances have been utilised, but again the majority of the literature use the inner virtual impedance.

The impact of the virtual impedance on other aspects of the converter performance has not often been discussed in conjunction with small-signal stability enhancement. There has also not yet been any research on applying virtual impedance for measurement-based impedance shaping to apply the grey-box / residue method, where the residues predict the effect of the change in virtual impedance on the poorly damped oscillation modes.

3 Converter Impedance Shaping Capability

3.1 Chapter Introduction

This chapter explores the capability of a standard grid-following converter architecture to damp SSO modes by shaping its impedance guided by the grey-box method. First, the default IBR controller design is briefly discussed. Then, the requirements placed on the sensitivity of the converter admittance to the change in parameter due to the fundamentals of the grey-box method are discussed. The available parameters are then compared for their ability to shape the converter admittance in the required manner.

Subsequently, the impacts of changing IBR parameters on other aspects of the converter performance such as the PLL synchronization, the DC voltage control or AC terminal voltage regulation are discussed. A case study which improves the damping of a weak grid / PLL SSO mode in a grid-following converter Single-Machine Infinite Bus (SMIB) system is then used to illustrate the grey-box method and the outlined performance trade-offs.

The proposed impedance-shaping process discussed in Chapter 1.2 is intended for damping oscillations present in the system due to the interaction of multiple IBR. Therefore, Chapter 3.6 outlines the fundamentals of influencing the damping of a mode with an IBR which is not the "root-cause" of the oscillation mode. To demonstrate the proposed process in a multi-IBR system an IBR is utilised to improve the damping of an SSO mode caused by the interactions of different IBR in a 14 bus network. Both case studies will be used again in later chapters.

This chapter concludes by discussing the information sharing requirements to account for the impact on performance when choosing the optimal parameter change to make.

The results presented in this Chapter have been generated with the Simplus open-source toolbox [57], with the generic grid-following IBR model updated to include virtual impedance. The frequency-domain bode plots and eigenvalue plots were generated with models derived by a linearisation of the system dynamics in the dq domain.

3.2 Default IBR controller design

Fundamentally, the objectives of IBR design are to export all available power while satisfying the grid code and ensuring internal variables stay within their allowable limits. In the majority

of cases, an IBR will be designed with one set of parameters and fixed control logic structure, which may then be tuned upon commissioning [136] then left in place. The default values of the controllers would be initially designed based on the IBR hardware (for the current controller/DC voltage controller), then tuned based on the grid strength where the IBR is connecting at and the overall power park scheme design (for the PLL and the reactive power/AC voltage control).

In more advanced cases, the IBR may have several "control modes" where the control logic and parameters can be switched between several discrete sets on operator input [169]. However, to regularly alter the values of IBR parameters away from their commissioned values - in the manner described in the proposed impedance-shaping method - may not be a widely available feature in commercial IBR controllers. It may also not be widely encouraged by IBR manufacturers since the parameter alteration could result in the degradation of some other aspect of the IBR performance. Therefore, to provide motivation for altering its control parameters (and making the necessary upfront investment to enable this capability), the IBR operator may need financial compensation as was discussed in Chapter 1.

3.3 Requirements placed on the admittance-parameter sensitivity for the best application of the grey-box method

To recap, the key expression describing the grey-box method is repeated in Equation 3.1 for changes to a 2x2 dq apparatus admittance. The terms $\frac{\partial Y(\lambda_i)}{\partial \rho} \Delta \rho$ are properties of the apparatus design and the parameter choice ρ , whereas the residue terms R_Z are the measured parameter which reflect the dynamics of the whole system. The residues are a property of the whole-system impedance, as shown in Equation 3.2.

Equation 3.1 shows the approximate change in a mode, based on the changes in an apparatus admittance and the whole-system impedance residues at the same bus.

$$\Delta \lambda_i \approx - \left(R_{Z,dd} \frac{\partial Y_{DD}(\lambda_i)}{\partial \rho} + R_{Z,qd} \frac{\partial Y_{DQ}(\lambda_i)}{\partial \rho} + R_{Z,dq} \frac{\partial Y_{QD}(\lambda_i)}{\partial \rho} + R_{Z,qq} \frac{\partial Y_{QQ}(\lambda_i)}{\partial \rho} \right) \Delta \rho \quad (3.1)$$

$$\hat{\mathbf{Z}}(s) = \sum_i \frac{1}{s - \lambda_i} \mathbf{R}_{\mathbf{Z}_i} \quad (3.2)$$

All residue terms are now assumed to be those local to the bus at which the apparatus in question is connected.

The grey-box relationship is based on small-signal analysis, so to maximise its accuracy the parameter change $\Delta\rho$ should not affect the fundamental frequency admittance values. If the fundamental frequency admittance changes due to the parameter change $\Delta\rho$, then the power-flow would be altered and the operating conditions of all the other apparatus in the system would change. The grey-box relationship shown in Equation 3.1 does not take into account the changing of the power-flow so the magnitude of the admittance-parameter sensitivity $|\frac{\partial Y}{\partial \rho}(s)|$ should tend to zero as the frequency tends to zero (the fundamental frequency in the dq reference frame).

Additionally, to limit the bandwidth of frequency response data required the admittance sensitivity $\frac{\partial Y}{\partial \rho}(s)$ should be band-limited. Otherwise, if $|\frac{\partial Y}{\partial \rho}(s)|$ were non-negligible at a frequency higher than the measured frequency response data a high-frequency poorly damped mode could be destabilised as it was not detectable in the available data.

Altogether, the form suggested for $\frac{\partial Y}{\partial \rho}(s)$ by the aforementioned requirements is equivalent to a band-pass filter, where the change in parameter affects neither the fundamental frequency current nor any un-detected high frequency modes. The band-pass frequency range for a specific parameter is hereafter referred to as the "parameter active-damping bandwidth". Ideally, $\frac{\partial Y}{\partial \rho}(s)$ should have minimal change in its magnitude or phase across the active-damping bandwidth to maximise the accuracy of Equation 3.1.

The active-damping bandwidth for a parameter which fulfills the discussed requirements well is illustrated in Figure 3.1.

The design of traditional Power System Stabilizers was also noted in Chapter 2.1.2 to be frequency-targeted in a similar manner.

Any SSO modes which it is desired to improve the damping of with a certain parameter must (1) have at least one sufficiently large residue and (2) be within the active-damping bandwidth.

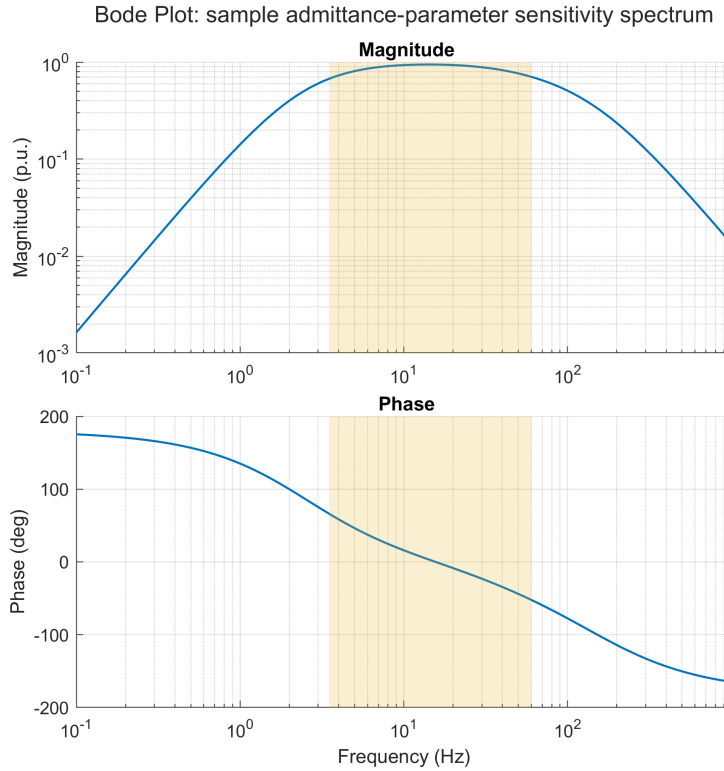


Figure 3.1: An example of a satisfactory admittance-parameter sensitivity frequency response spectrum $\frac{\partial Y}{\partial p}(j\omega)$, with the active-damping bandwidth shown highlighted

A mode which does not fulfill these two conditions could still be damped with a sufficiently large change in parameter (as long as neither the residue nor the admittance-parameter sensitivity are zero), but a very large change in parameter is likely to destabilise a different mode or have other undesirable consequences for the performance of the IBR.

The proposed impedance-shaping process is an iterative process which may require many iterations of residue measurement and parameter alteration. Therefore, the parameters which are available to alter in the active-damping IBR should be able to be altered with minimal effort/cost.

3.4 Admittance shaping

There are three families of parameters in an IBR which can shape the converter admittance - parameters in the converter control logic, the operating conditions or the physical components. This section will describe each and discuss their suitability for use with the grey-box method.

The simplified converter admittance model derived in Chapter 2.3 is repeated for reference in Equation 3.3. The meaning of the variables and functions are outlined in Chapter 2.3 and are

laid out in the Glossary for reference. The transfer functions $g_{\text{PLL}}(s)$ and $h_{\text{PLL}}(s)$ are mainly influenced by the Phase-Locked Loop (PLL) and are described in Equations 2.19 and 2.20. The function $g_{\text{DC}}(s)$ is mainly determined by the DC control, its derivation can be found in Equation 2.30. The function $h_{\text{cc}}(s)$ is determined by the current control and is shown in Equation 2.14, while $g_{\text{ff,dd}}(s)$, $g_{\text{ff,dq}}(s)$, $g_{\text{ff,qd}}(s)$ and $g_{\text{ff,qq}}(s)$ are determined by the virtual admittance and the current control according to Equation 2.24.

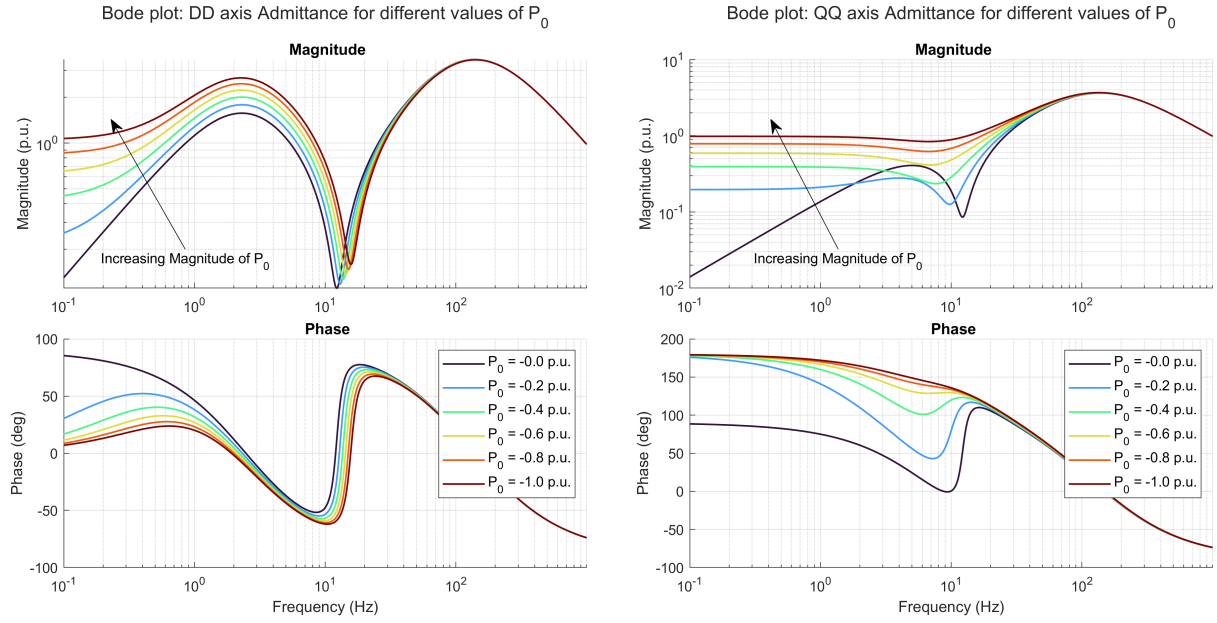
$$\begin{aligned}
 Y_{DD}(s) &= g_{\text{DC}}(s) \left(\frac{P_0}{E_0} + E_0 (h_{\text{cc}}(s) + g_{\text{ff,dd}}(s)) \right) + h_{\text{cc}}(s) + g_{\text{ff,dd}}(s) \\
 Y_{DQ}(s) &= \left(g_{\text{DC}}(s) \left(-\frac{Q_0}{E_0} + E_0 g_{\text{ff,dq}}(s) \right) + g_{\text{ff,dq}}(s) \right) h_{\text{PLL}}(s) \\
 &\quad + \frac{Q_0}{E_0} g_{\text{PLL}}(s) \\
 Y_{QD}(s) &= g_{\text{ff,qd}}(s) \\
 Y_{QQ}(s) &= (h_{\text{cc}}(s) + g_{\text{ff,qq}}(s)) h_{\text{PLL}}(s) + \frac{P_0}{E_0} g_{\text{PLL}}(s)
 \end{aligned} \tag{3.3}$$

3.4.1 Operating conditions

The exact dependence of the converter admittance on operating conditions is complex, particularly the steady-state grid voltage. In [170] it is attempted to decouple the admittance model from the operating conditions and the resulting derivation is arduous. However, the simplifying assumptions used in the converter admittance model developed in Chapter 2.3 allows some basic insights into the dependence on the initial active and reactive power.

$Y_{DD}(s)$, $Y_{DQ}(s)$ and $Y_{QQ}(s)$ all depend on P_0 and Q_0 due to the dynamics of the outer loops. The dependence's of $Y_{DD}(s)$ and $Y_{QQ}(s)$ on P_0 are shown in Equations 3.4 and 3.5 and illustrated in Figure 3.2 (a) and (b) for a range of DC control and PLL bandwidths. The plots show that the admittance spectrum is very dependent on the operating conditions within the bandwidths of the DC control and the PLL. This low-frequency dependence has been confirmed experimentally elsewhere [16]. The sensitivity of $Y_{DQ}(s)$ to Q_0 is shown in Equation 3.6 and can be shown to similarly be dependent on Q_0 within the bandwidths of the DC control and PLL.

$$\frac{\partial Y_{DD}(s)}{\partial P_0} \approx g_{\text{DC}}(s)/E_0 \tag{3.4}$$



(a) Plot of $Y_{DD}(s)$ in Equation 3.3 for variations in P_0 , for a DC bandwidth of 10 Hz

(b) Plot of $Y_{QQ}(s)$ in Equation 3.3 for variations in P_0 , for a PLL bandwidth of 10 Hzs

Figure 3.2: Dependencies of $Y_{DD}(s)$ and $Y_{QQ}(s)$ on P_0 : it can be seen that the frequency range of the sensitivities are mainly within the bandwidths of the DC control and PLL bandwidths (10 and 15 Hz in this case). P_0 was taken as positive into the converter in the derivation in Chapter 2.3.

$$\frac{\partial Y_{QQ}(s)}{\partial P_0} \approx \frac{1}{E_0} g_{PLL}(s) \quad (3.5)$$

$$\frac{\partial Y_{DQ}(s)}{\partial Q_0} \approx (g_{PLL}(s) - g_{DC}(s)h_{PLL}(s)) / E_0 \quad (3.6)$$

Clearly, the operating conditions should not be used to shape the converter admittance since as discussed it will impact all the other apparatus in the network in manner not predicted by the grey-box relationship. However, it is useful to establish the fact that the operating conditions determine the admittance in the low-frequency range, so shaping the low-frequency impedance will be difficult with any other parameters.

3.4.2 Physical components

It is also possible to shape converter admittance by changing physical components. Within the converter, the DC capacitor or filter could be changed. However, the converter control logic is often designed to compensate for the physical component values in order achieve a target closed-loop bandwidth [23]. If it is desired to alter the dynamics of the DC or current control, it would be much easier to change the control logic rather than the physical components

themselves.

Alternatively, components such as filters or shunt capacitors could be switched in or out. Filters are a common solution for damping harmonic resonances, and it could be possible to design filters which have very little impact at the fundamental frequency to fulfill the band-pass requirement.

The admittance of an IBR could be scaled by altering the number of identical IBR connected in parallel. However, this would impact the operating conditions if the initial active or reactive power values of each IBR were not also altered. The sensitivity to scaling up or down an apparatus admittance was already reported as the Layer-2 of the grey-box method [52], although the impact of the scaling changing operating conditions was not mentioned.

Larger interventions such as new, higher voltage transmission lines could also come under the umbrella of new physical components, but is not often viewed in the lense of admittance shaping, rather grid strength improvement. As was discussed in Chapter 2.1, large expensive infrastructure investments such as new transmission lines or synchronous condensers would be last-resort interventions, which the IBR admittance shaping approach is intended to avoid.

The costs of physical changes such as these would be much greater than simply changing the controller programming.

3.4.3 Control parameters: PLL, DC and current control bandwidths

A common technique in the literature for shaping an IBR admittance is to change the bandwidths of one of the main control loops within the converter. This has been applied in the literature for altering the region of passivity of the converter to avoid resonant frequencies [23,159], or when one control loop is clearly identified as part of the root cause of an oscillation mode [48].

The sensitivities of $Y_{DD}(s)$, $Y_{DQ}(s)$ and $Y_{QQ}(s)$ to the DC control and PLL bandwidths σ_{dc} and σ_{pll} are shown in Equations 3.7 to 3.10. It can be seen that all the sensitivities depend on the operating conditions, the feed-forward and the initial bandwidth of the controller being altered. The self-dependence of the bandwidth-parameter sensitivities is contained within the terms like $\frac{\partial g_{DC}(s)}{\partial \sigma_{dc}}$.

Additionally, the sensitivities depend on the current control disturbance rejection transfer

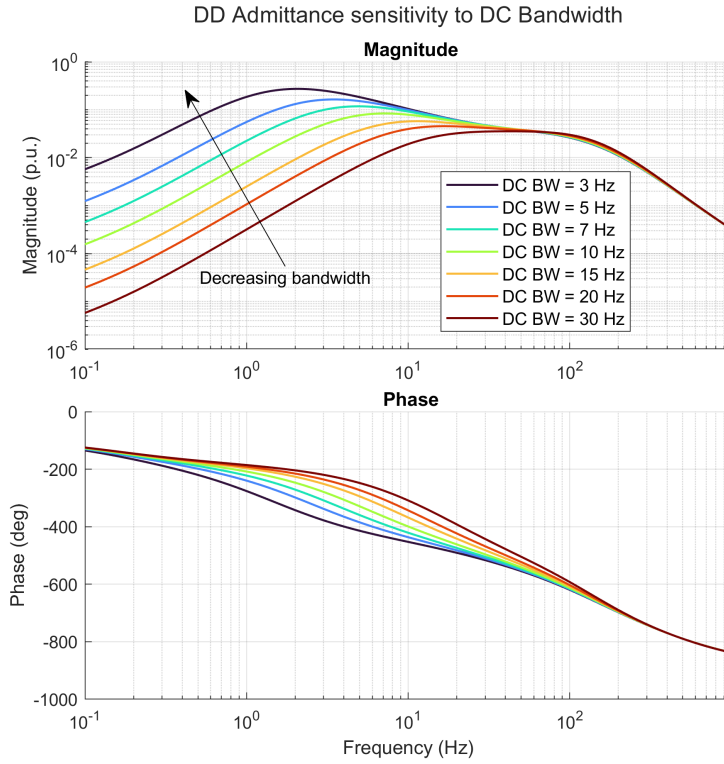


Figure 3.3: The sensitivity of $Y_{DD}(s)$ to changing the DC bandwidth shown in Equation 3.7, plotted for a range of DC control bandwidths. It can be seen that magnitude of the sensitivity is greatest near the DC bandwidth itself. Sensitivities of $Y_{DQ}(s)$ and $Y_{QQ}(s)$ to σ_{pll} and σ_{dc} have similar spectra so are not shown here.

function $h_{cc}(s)$ which will be small at sub-synchronous frequencies - it is assumed to be zero if perfect current control is assumed. Therefore, Equations 3.7 to 3.10 indicate that when P_0 and Q_0 are small and without any feed-forward the sensitivity of the admittance to changing the DC control or PLL bandwidth will be small over the sub-synchronous frequency range.

The sensitivity of $Y_{DD}(s)$ to changing the DC bandwidth is shown in Equation 3.7 and Figure 3.3, confirming that it is largest close to the initial bandwidth of the DC control.

$$\frac{\partial Y_{DD}(s)}{\partial \sigma_{dc}} = \left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) \frac{\partial g_{DC}(s)}{\partial \sigma_{dc}} \quad (3.7)$$

$$\frac{\partial Y_{DQ}(s)}{\partial \sigma_{dc}} = \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) h_{PLL}(s) \frac{\partial g_{DC}(s)}{\partial \sigma_{dc}} \quad (3.8)$$

$$\frac{\partial Y_{DQ}(s)}{\partial \sigma_{pll}} = \left(\frac{Q_0}{E_0} - E_0 g_{DC}(s) \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) + g_{ff,dq}(s) \right) \frac{\partial g_{PLL}(s)}{\partial \sigma_{pll}} \quad (3.9)$$

$$\frac{\partial Y_{QQ}(s)}{\partial \sigma_{pll}} = \left(\frac{P_0}{E_0} - E_0 (h_{cc}(s) + g_{ff,qq}(s)) \right) \frac{\partial g_{PLL}(s)}{\partial \sigma_{pll}} \quad (3.10)$$

It can be seen from Figure 3.3 that shaping the IBR admittance with the DC control or PLL

bandwidths fulfill the band-pass grey-box requirement. However, the active-damping bandwidth is only near the bandwidth of the controller in question, which may limit its usefulness for all SSO modes. Additionally, the effect of changing the controller bandwidths may be small under a low-load operating condition.

The current control is embedded within many of the other functions in Equation 3.3 and therefore will have a complex effect on all four quadrants of the overall admittance. No analytical consideration is given here, but its effect will be considered in the case studies.

3.4.4 Control parameters: Feed-forward

Another option to shape the converter admittance by altering the control logic is to add or modify existing feed-forward control terms.

The feed-forward terms are described in Chapter 2.3 as a "virtual admittance", which when added to the PWM voltage reference in the current control are referred to as an "inner" virtual admittance, $f_{\text{ff,dd}}(s)$ in Equation 2.10. When added to the current reference the feed-forward terms are referred to as an "outer" virtual admittance, for example $f_{\text{off,dd}}(s)$ in Equation 2.24.

Equations 3.11 to 3.14 show the sensitivities of all four admittance quadrants to the relevant quadrant of generalised virtual admittance. Particularly, the solution of Equation 3.11 shows that the DC control acts as a high-pass filter. As discussed in Chapter 3.4.1, the DC control ensures that the admittance near the fundamental frequency is defined by the operating conditions. Accordingly, the DC control rejects any attempt to change the IBR $Y_{DD}(s)$ or $Y_{DQ}(s)$ admittance at frequencies within its control bandwidth. Figure 3.4 (a) illustrates the high-pass filter effect in the sensitivity in Equation 3.11 by plotting its magnitude for a range of DC control bandwidths.

$$\begin{aligned} \frac{\partial Y_{DD}(s)}{\partial g_{\text{ff,dd}}(s)} &= 1 + g_{\text{DC}}(s)E_0 = \frac{sC_{DC}V_{DC}^0 - E_0g_{\text{cc}}(s)f_{\text{DC}}(s) + E_0g_{\text{cc}}(s)f_{\text{DC}}(s)}{sC_{DC}V_{DC}^0 - E_0g_{\text{cc}}(s)f_{\text{DC}}(s)} \\ &= \frac{sC_{DC}V_{DC}^0}{sC_{DC}V_{DC}^0 - E_0g_{\text{cc}}(s)f_{\text{DC}}(s)} \end{aligned} \quad (3.11)$$

The function $h_{\text{PLL}}(s)$ in Equations 3.12 and 3.14 is the transfer function between changes in the Q axis voltage (in the reference frame initially aligned with the IBR terminal voltage) and

the q axis voltage in the internal PLL reference frame. Since the objective of the PLL is to drive the q axis voltage to zero, any virtual admittance which takes as input the measured q channel voltage will have a small output at frequencies within the bandwidth of the PLL. Figure 3.4 (b) highlights the high-pass filter effect of the PLL on the sensitivity of $Y_{QQ}(s)$ to the outer virtual admittance gain.

Since the generalised feed-forward $g_{ff,dq}(s)$ acts on the q axis voltage but affects the d axis current the sensitivity of $Y_{DQ}(s)$ to $g_{ff,dq}(s)$ is impacted by both DC control and PLL high-pass filter effects.

$$\begin{aligned} \frac{\partial Y_{DQ}(s)}{\partial g_{ff,dq}(s)} &= (1 + g_{DC}(s)E_0) h_{PLL}(s) \\ &= \frac{sC_{DC}V_{DC}^0}{sC_{DC}V_{DC}^0 - E_0g_{cc}(s)f_{DC}(s)} \frac{s}{s + E_0f_{PLL}(s)} \end{aligned} \quad (3.12)$$

$$\frac{\partial Y_{QD}(s)}{\partial g_{ff,qd}(s)} = 1 \quad (3.13)$$

$$\frac{\partial Y_{QQ}(s)}{\partial g_{ff,qq}(s)} = h_{PLL}(s) = \frac{s}{s + E_0f_{PLL}(s)} \quad (3.14)$$

The PLL HPF effect is discussed in [158,161] where it is noted that the HPF effect may limit the ability of the virtual admittance to damp weak grid / PLL interaction modes of frequency below that of the PLL bandwidth. For the grey-box application the PLL HPF effect ensures that changes to the qq or the dq virtual admittances do not affect the operating conditions. In [158] an integral term is included in the virtual admittance to allow a larger $\frac{\partial Y}{\partial p}(s)$ at frequencies within the PLL bandwidth, but this could affect the operating conditions since the integral of v_q is not controlled to zero by the PLL. Due to this, pure integrals in the virtual admittance will not be considered in this work.

There is no inherent HPF filter affecting the qd quadrant feed-forward $g_{ff,qd}(s)$ in the admittance model derived, as shown in Equation 3.13. To use an outer virtual admittance $f_{off,qd}(s)$ the d axis voltage should also be filtered though a high-pass filter for the band-pass requirement to hold.

In practical application, the virtual admittance would contain a low-pass filter (LPF) to avoid amplification of harmonics. The LPF bandwidth will therefore set the upper range of the active-damping bandwidth, assuming that the LPF bandwidth is lower than the current control

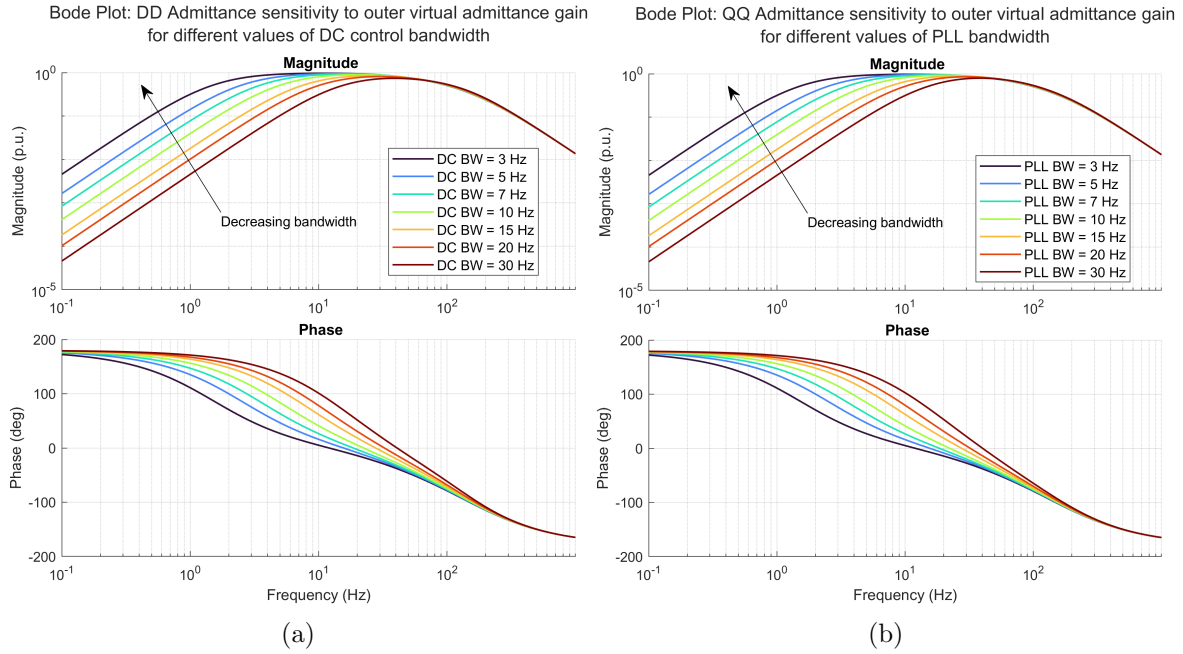


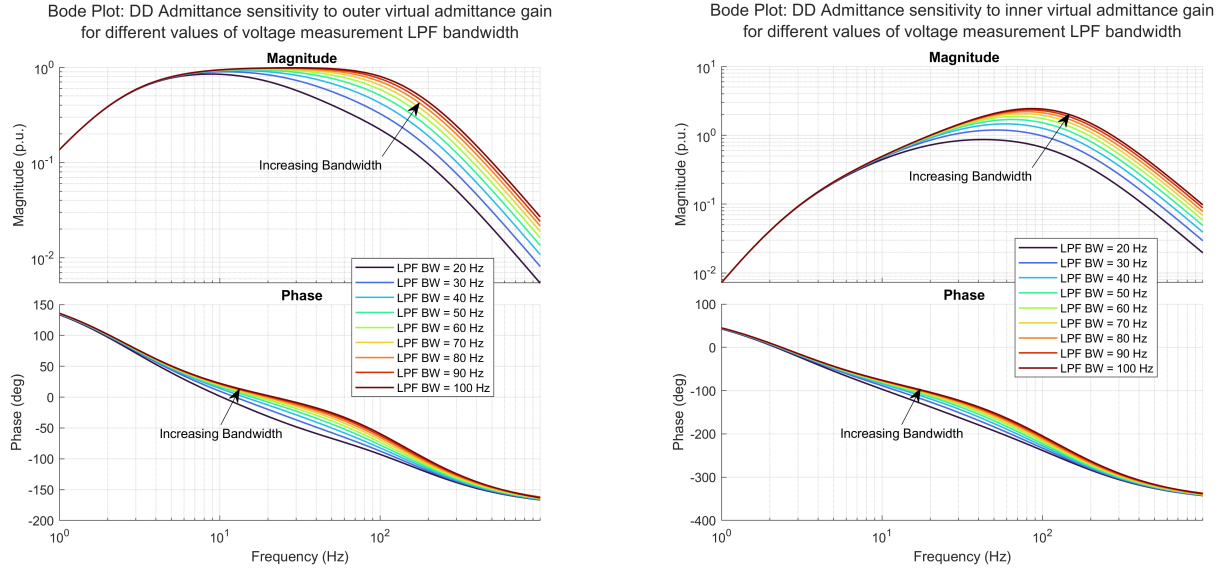
Figure 3.4: The sensitivities of the admittance quadrant $Y_{DD}(s)$ and $Y_{QQ}(s)$ to the corresponding outer virtual admittance gain. It can be seen that the DC control and PLL bandwidths define the lower range of the active-damping bandwidths. A 50 Hz low-pass filter is used on the measured voltage to define the upper edge of the active damping bandwidth

bandwidth. It is now assumed that the form of an arbitrary virtual admittance consists of a gain cascaded with a LPF, as demonstrated in Equations 3.15 and 3.16.

$$f_{\text{off,dd}}(s) = K_{\text{odd}} \frac{\sigma_{\text{off,dd}}}{s + \sigma_{\text{off,dd}}} \quad (3.15)$$

$$f_{\text{iff,dd}}(s) = K_{\text{idd}} \frac{\sigma_{\text{iff,dd}}}{s + \sigma_{\text{iff,dd}}} \quad (3.16)$$

The sensitivity of the $Y_{DD}(s)$ IBR admittance to the inner and outer virtual admittance gains K_{idd} and K_{odd} are plotted in Figure 3.5 for a range of low-pass filter bandwidths $\sigma_{\text{iff,dd}}$ and $\sigma_{\text{off,dd}}$. Figure 3.5 confirms that if the same LPF filter bandwidths are used for inner and outer virtual admittance are used then for most of the active-damping bandwidths the phase difference between the inner and outer admittance sensitivities will be approximately 90 degrees. The 90 degree separation property could be useful for the grey-box approach since it implies that application of both inner and outer virtual admittance could move one mode in an arbitrary direction in the complex plane.



(a) The sensitivity $\frac{\partial Y}{\partial \rho}(s)$ of the DD admittance to the outer virtual admittance gain

(b) The sensitivity $\frac{\partial Y}{\partial \rho}(s)$ of the DD admittance to inner virtual admittance gain

Figure 3.5: The sensitivity of the admittance quadrant $Y_{DD}(s)$ to the inner and outer virtual admittance gains, for a range of voltage measurement LPF bandwidths. It can be seen that the voltage measurement low-pass filter defines the upper edge of the active damping bandwidth. If the same bandwidth LPF is used, it can be seen that the inner virtual admittance sensitivity lags the outer sensitivity by approximately 90 degrees, for frequencies below the current control bandwidth

3.4.5 Parameter choice discussion

It can immediately be stated that changing physical components will be the most costly and time-consuming option. This would mainly be implemented due to legal obligations for e.g. fulfilment of the grid code and would be unlikely to be performed regularly due to its significant financial implications.

The operating conditions are mainly out of the control of the IBR operator, but the ESO could mandate that a certain proportion of a generator's IBR are taken out of service to ensure system stability (indeed this is an approach which is taken by grid operators at the moment [108]). The IBR operators would however expect full compensation for the curtailed output, so this can also be a very expensive option for the ESO.

Therefore, control parameter tuning would be a much cheaper option for regular use for stabilising the system. Several critical IBR parameters may not be allowed to change by the device manufacturer (eg. current control bandwidth, switching frequency), as they may be limited by the IBR hardware or may cause sufficient performance degradation to void the manufacturer warranty or breach the grid code. Others may be stability-critical parameters in

the event of severe disturbances or weak grid operating conditions (eg. PLL bandwidth) which may not be alterable in practice due to the possibility of a weak operating condition. However, virtual admittance parameters allow for shaping the IBR admittance, while not significantly changing the operation of the critical controllers (current control and PLL) away from their design at commissioning.

It can be seen that from Figure 3.5b that the inner virtual admittance may significantly impact modes at higher frequencies if it is used for damping modes in the SSO frequency range. This is due to the peak in the magnitude of the sensitivity which can be seen to be higher than 50 Hz. In comparison, it can be seen from Figure 3.5a that the admittance sensitivity to the outer virtual admittance gain has flat magnitude and small phase lag across much of the sub-synchronous frequency range. The application of outer virtual admittance with a 50 Hz LPF bandwidth shall mainly be used for the case studies in this chapter as it is deemed to have the optimal sensitivity spectrum for providing the admittance shape-ability to damp SSO modes, according to the requirements outlined in Chapter 3.3. If a narrower bandwidth is required, the LPF bandwidth could be lowered.

To further justify the choice of focusing on adjusting the virtual admittance as compared to the controller bandwidths, it could be considered simpler to give supplementary references to the current controller rather than alter the controller bandwidths. It was shown in Equations 3.7 to 3.10 that the sensitivity $\frac{\partial Y}{\partial \rho}(s)$ of the admittance to the controller bandwidths is dependent on the controller bandwidth in question. The sensitivity of the admittance to the virtual admittance gain can be seen from Equations 3.11 to 3.14 (along with Equation 2.24) to be independent of the value of the virtual admittance gain. This independence will improve the accuracy of the grey-box approach shown in Equation 1.21.

Some further comparison with altering the controller bandwidths will be made in the case studies in this chapter, but the case studies in the remainder of this thesis will mainly shape the IBR admittance with the virtual admittance gain parameters. This is due to the discussed advantages of utilizing these parameters: (1) the shape of $\frac{\partial Y}{\partial \rho}(s)$ fulfills the requirements outlined in Chapter 3.3, (2) the sensitivity $\frac{\partial Y}{\partial \rho}(s)$ is approximately independent of ρ and (3) changing controller parameters should be much cheaper and easier than changing any physical components.

3.5 Stability vs Performance trade-offs

The effects of changing controller bandwidths on the performance of the IBR are clear: reducing the PLL bandwidth would impact the performance of the synchronization whereas reducing the DC bandwidth would impact the performance of the DC control. Changing the feed-forward/ virtual admittance terms has less obvious impacts on the performance of the IBR.

The formula used for the generation of the d axis current reference is repeated in Equation 3.17, where the outer virtual admittance terms are represented by $f_{\text{off},dd}(s)$ and $f_{\text{off},dq}(s)$. The small-signal DC voltage dynamics based on the power balance is shown in 3.18, where it can be seen that the operating conditions define the impact of the terminal voltage variations on the DC voltage. Assuming perfect current control, the closed-loop dynamics of the DC control are shown in Equation 3.19.

A more detailed derivation of the DC voltage dynamics including the current control dynamics is given in Appendix A.

$$i_d^{\text{ref}} = f_{\text{DC}}(s)V_{\text{DC}} + f_{\text{off},dd}(s)v_d + f_{\text{off},dq}(s)v_q \quad (3.17)$$

$$sC_{\text{DC}}V_{\text{DC}}^0V_{\text{DC}} = E_0i_d + \frac{P_0}{E_0}v_d - \frac{Q_0}{E_0}v_q \quad (3.18)$$

$$V_{\text{DC}} = \frac{1}{sC_{\text{DC}}V_{\text{DC}}^0 - E_0g_{\text{cc}}(s)f_{\text{DC}}(s)} \left(\left(\frac{P_0}{E_0} + E_0f_{\text{off},dd}(s) \right) v_d + \left(-\frac{Q_0}{E_0} + E_0f_{\text{off},dq}(s) \right) v_q \right) \quad (3.19)$$

The current is taken as positive into the IBR, so if the IBR is operating in inverter mode the value of P_0 is negative. Therefore, a positive dd channel virtual admittance $f_{\text{off},dd}(s)$ can actually improve the performance of the DC control by reducing the impact of disturbances in v_d on the DC voltage. However, if the gain of the virtual admittance $f_{\text{off},dd}(s)$ is larger than P_0/E_0^2 increasing the gain further will degrade the performance of the DC control by increasing the impact of disturbances in v_d on the DC voltage.

It is possible that a practical IBR will be set up with some form of power-balance feed-forward already [23], so increasing the voltage to current reference gain away from its default value will degrade the DC performance to some extent. In the models used in this work no default power-

balance feed-forward is considered, so some initial DC control performance improvement can be seen when applying the dd virtual admittance with non-zero initial active power P_0 .

A similar possibility exists for improving/degrading the DC control performance with the dq channel virtual admittance $f_{\text{off},dq}(s)$, depending on the sign and value of the initial reactive power Q_0 .

The qd and qq virtual admittances affect the reactive power output much more than the active power, so in an inductive grid influence the IBR terminal voltage dynamics. Since the current is taken as positive into the inverter a negative q axis current reference will absorb reactive power from the grid, reducing the terminal voltage magnitude. Therefore, a negative value of the qd virtual admittance gain will tend to damp oscillations in the IBR voltage magnitude (since to a small-signal approximation v_d is equal to the variation in terminal voltage magnitude).

However, the qq virtual admittance $f_{\text{off},qq}(s)$ acts on the q axis internal reference frame voltage v_q . Depending on the disturbance, the qq virtual admittance may not always affect the reactive power output in a manner which reduces the magnitude of the terminal voltage magnitude oscillations.

3.5.1 Case study: SMIB system

The application of the proposed impedance-shaping process is now demonstrated with a simple case study. A grid-following converter SMIB system, illustrated in Figure 3.6a, contains an SSO mode at approximately 23 Hz due to weak grid / PLL interaction. With knowledge of the whole-system residues and the admittance-parameter sensitivity spectra the grey-box method is applied to guide the shaping of an IBR impedance, to improve the damping of the mode. The impact of changing different parameters are compared, both on the system stability and on the IBR performance.

The grid-following converter admittance model developed in Chapter 2.3 was used to make the comparisons of the parameters effects on the admittance and other aspects of the converter performance discussed already in this chapter. The eigenvalue results presented hereafter are obtained from a linearisation of the dq domain dynamics of the SMIB system around operating points obtained from the power-flow analysis, generated by the Simplus open-source toolbox [57]. The system parameters shown in Table 3.1 result in an SSO mode at approximately 23

Hz. The participating states in the SSO are shown in Table 3.2, confirming that the SSO is a case of weak grid / PLL instability.

All time-domain results presented in this chapter are obtained from a three-phase non-linear SIMULINK model, also generated by the Simplus open-source toolbox [57]. The time-domain results show the responses to the same input: a 0.05 p.u. step in the d-axis infinite bus voltage at 0.1 seconds, followed by a 0.05 p.u. step in the q-axis infinite bus voltage at 2.1 seconds. The SMIB SIMULINK model is illustrated in Figure 3.6b, where the network dynamics are modelled with the "Branch" components connecting the relevant buses. The grid-following converter is represented with the "VSI-PLL" component.

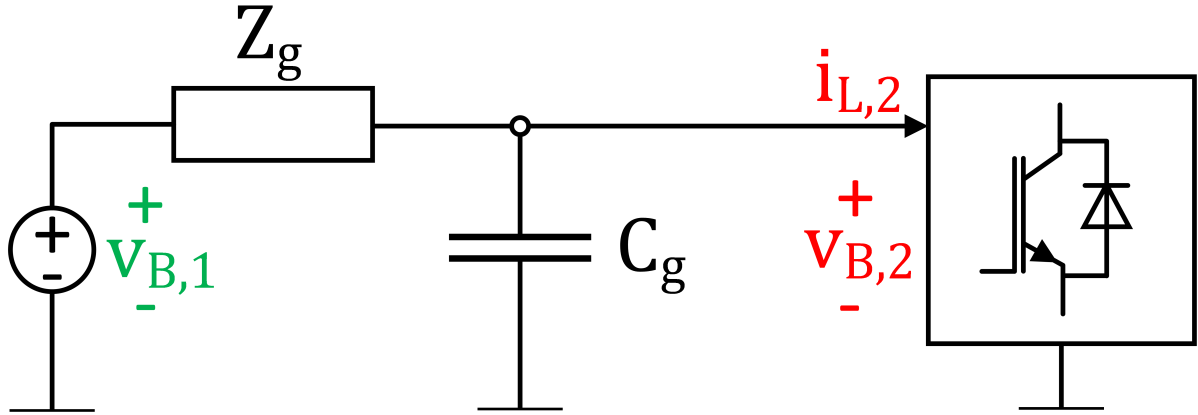
Parameter	Value
Current Control Bandwidth	275 Hz
Filter Impedance	0.05 p.u.
PLL Bandwidth	15 Hz
DC Control Bandwidth	5 Hz
DC Link Capacitor	0.2 p.u.
Initial Output Power	1 p.u.
Grid Impedance	0.5 p.u.
Grid X/R Ratio	5
Virtual Admittance LPF bandwidth	50 Hz

Table 3.1: Summary of the parameters in the grid-following converter Single Machine Infinite Bus case study. The current control bandwidth has been reduced to demonstrate the weak-grid/PLL SSO mode. The DC capacitor size has been reduced to better illustrate the DC voltage deviations..

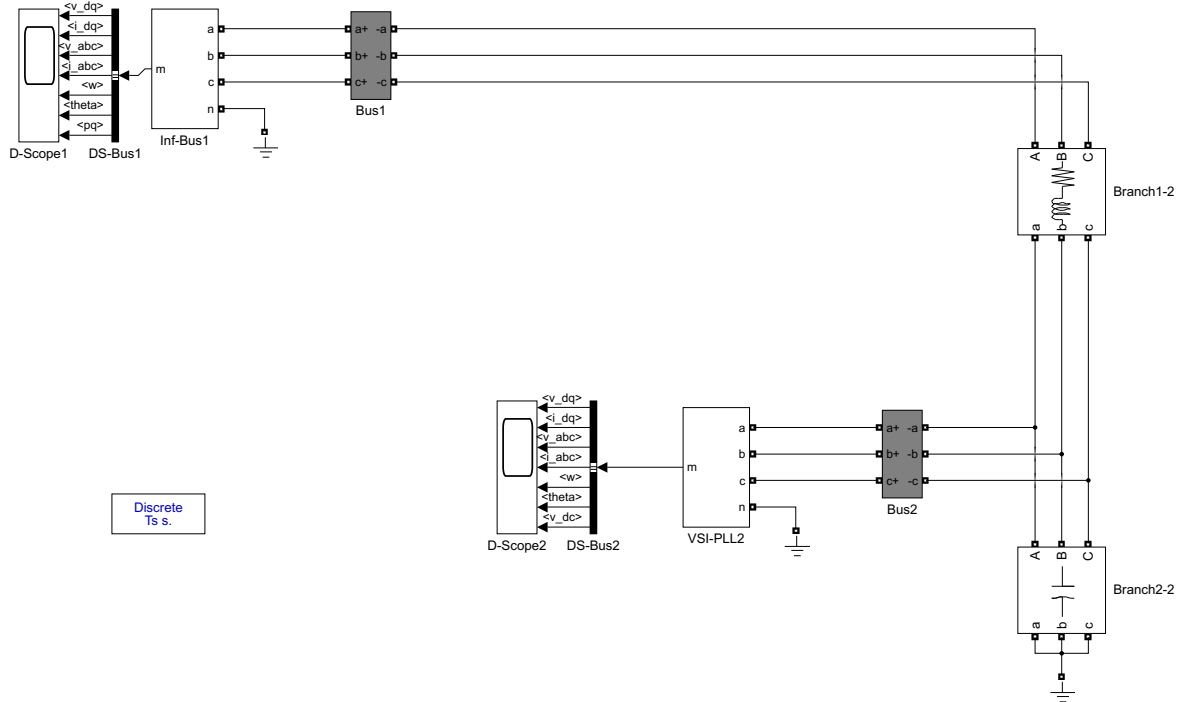
State	Participation Percentage
d-axis Current Reference Error Integral	28%
q-axis Current Reference Error Integral	22%
PLL angle	21%
Q-axis Grid Current	12%
D-axis Grid Current	7%
DC link voltage	7%
q-axis Voltage Integral for PLL	3%

Table 3.2: Participation analysis of the 23 Hz SMIB oscillation mode: $-0.65 + j23.05$

The damping of the SSO can be improved by tuning the controller bandwidths as illustrated in Figure 3.7, guided by the grey-box method. The next predicted location of the mode according to Equation 3.1 is shown with a dashed grey line and grey circle. Increasing the current



(a) Illustration of the grid-following converter Single-Machine Infinite Bus (SMIB) system, where the parameter values can be found in Table 3.1. The voltage $V_{B,2}$ shown represents the IBR terminal voltage, which is plotted in time-domain simulations in this chapter. The voltage source $V_{B,1}$ represents the infinite bus, labelled green to be consistent with previous diagrams labelling inputs with green text. 0.05 p.u. step disturbances to the infinite bus voltage are the inputs used for the time-domain simulations in this chapter.



(b) The SIMULINK model used to generate the SMIB time-domain simulation results. The infinite bus is at Bus 1, while the grid-following IBR model is connected at Bus 2. The network components are modelled with "Branch1-2" and "Branch2-2".

Figure 3.6: Illustrations of the grid-following converter Single-Machine Infinite Bus (SMIB) in (a) simplified diagram form, and (b) the SIMULINK model.

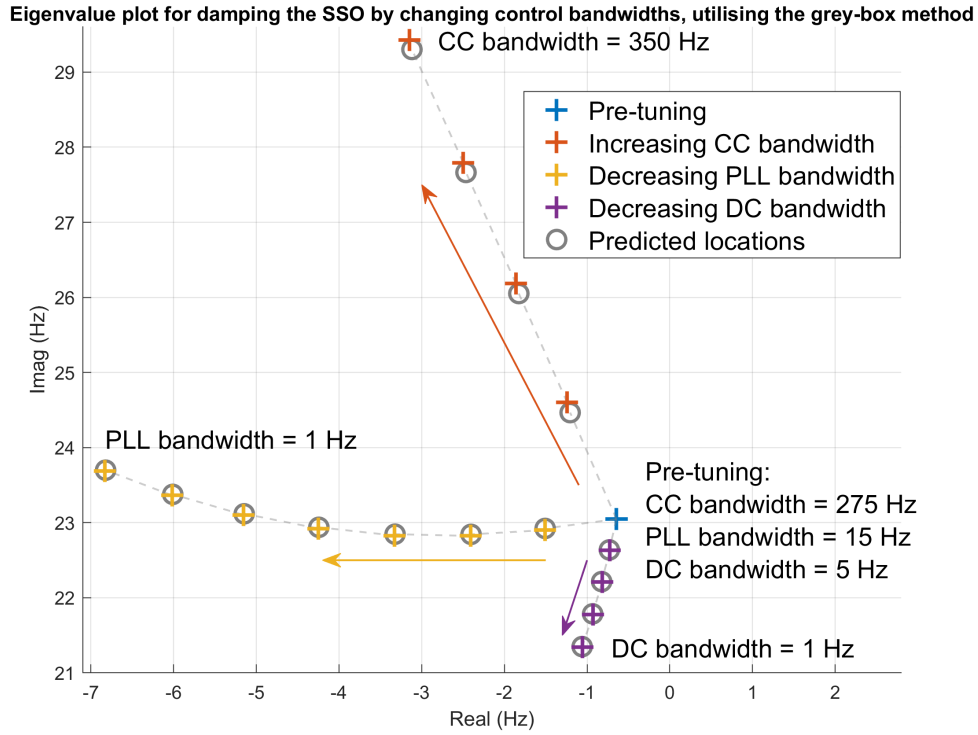


Figure 3.7: Increasing the damping of the weak grid / PLL oscillation mode in the SMIB system by tuning the bandwidths of three control loops: increasing the current control (CC) bandwidth from 275 Hz to 350 Hz in steps of 15 Hz, decreasing the PLL bandwidth from 15 Hz to 1 Hz in steps of 2 Hz, decreasing the DC bandwidth from 5 Hz to 1 Hz in steps of 1 Hz. Also shown are the next predicted locations utilising the grey-box method (plotted with a grey dashed line and grey circles)

control bandwidth increases the mode frequency and slightly increases the damping, whereas decreasing the PLL bandwidth improves the mode damping without significantly changing the mode frequency. Clearly, root-cause analysis based on the grey-box method would point to the PLL tuned too high and/or the current controller being tuned too low for the level of grid impedance, as either of these changes would be effective in improving the mode damping.

Figure 3.7 also shows that decreasing the bandwidth of the DC control barely improves the damping of the mode in this case.

Time-domain simulations illustrating the performance trade-offs resulting from changing the controller bandwidths are plotted in Figure 3.8. Increasing the bandwidth of the current control only improves the converter performance, but clearly the hardware must be capable of supporting such a change. For clarity, the increased bandwidth of the current control is not plotted in Figure 3.8.

Figure 3.8 highlights that decreasing the bandwidth of the PLL degrades the synchronization speed of the PLL, while decreasing the bandwidth of the DC control degrades the disturbance

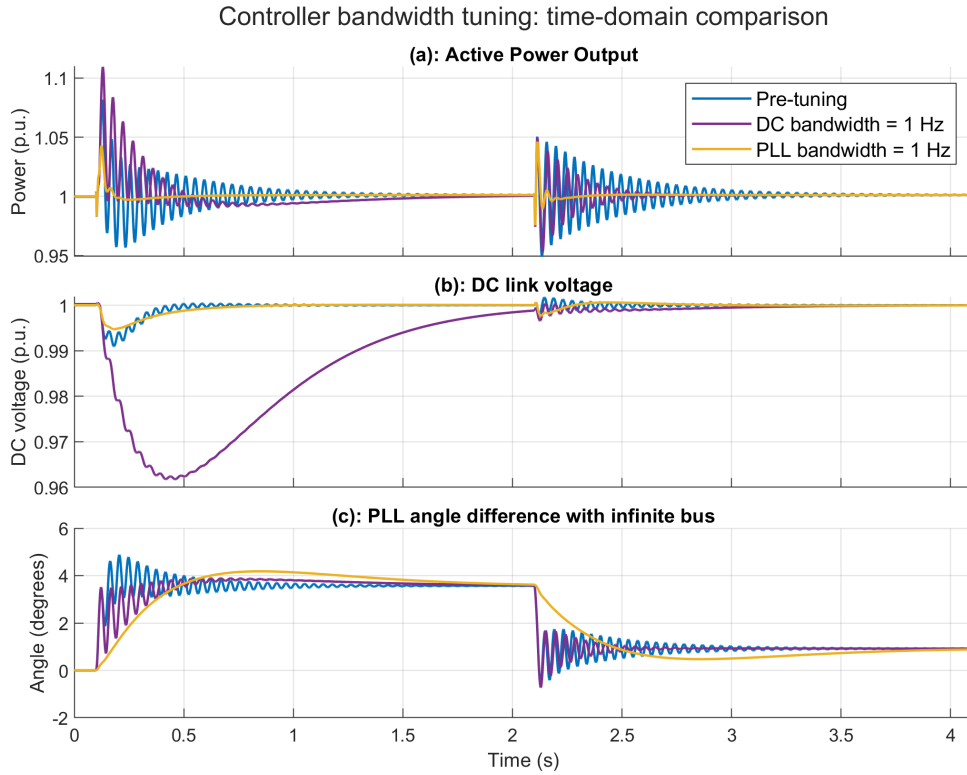


Figure 3.8: Illustration of the trade-offs inherent in decreasing the outer loop bandwidths: decreasing the PLL bandwidth successfully damps the SSO mode but the synchronization speed is slightly reduced, whereas in this case reducing the DC bandwidth does not improve the SSO mode damping and results in reduced DC control performance

rejection capabilities of the DC control and barely improves the damping of the SSO mode. Clearly these are the expected consequences of reducing the bandwidths of the controllers.

It has been discussed in the literature [150,158,161] that it is possible to use an outer virtual admittance to improve the stability of the PLL in a weak grid. The literature has focussed on the application of virtual admittance in the dq [150] and the qq quadrants [158,161]. From the perspective of passivity, these are the main quadrants which can influence the PLL since passivity only considers the IBR admittance in isolation from the grid. However, the grey-box method utilises the closed-loop system characteristics and allows comparison of the damping effects of all four quadrants of virtual admittance.

The root locus of the SSO mode as the feed-forward parameters are tuned are shown in Figure 3.9 (a), along with the predicted location of the mode using the grey-box method. Figure 3.9 confirms the work in the literature that the qq and the dq quadrant feed-forward improves the damping of the weak grid / PLL SSO mode. The dq virtual admittance is less effective and increases the SSO mode frequency more than the mode damping.

A zoomed-out version of Figure 3.9 (a) is plotted in Figure 3.9 (b) and shows that increasing the dq virtual admittance gain past 1 p.u. begins to destabilise another previously well damped low-frequency mode, which participation analysis can reveal consists of interactions between the PLL and DC control. In the literature which utilises the dq virtual admittance [150] there is no DC control enabled so the destabilising interaction may not be encountered. Figure 3.9 (b) also shows that increasing the qq virtual admittance gain past 1.5 p.u. begins to decrease rather than increase the SSO mode damping.

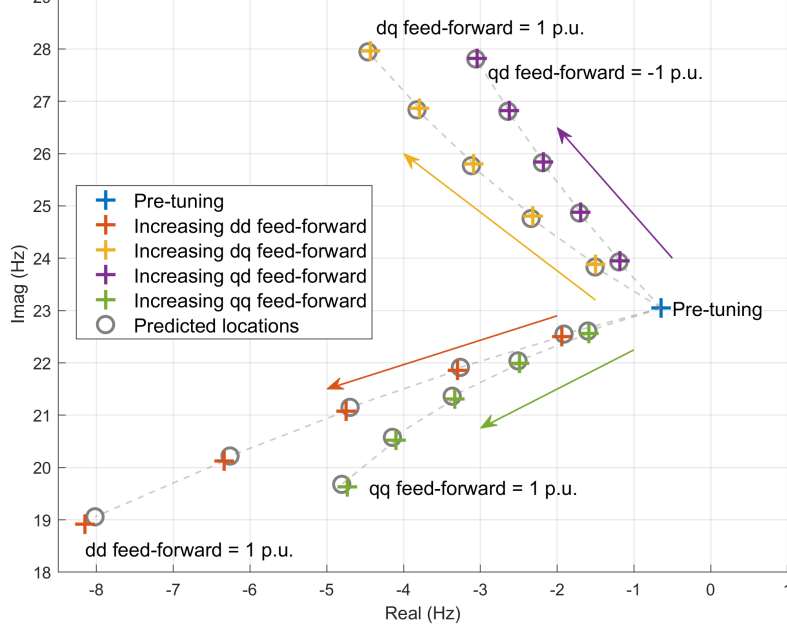
However, the residues associates with the pre-tuning SSO mode also indicate that the dd virtual admittance should also be effective in enhancing the damping of the PLL / weak grid interaction mode. The analysis of the IBR admittance from the perspective of passivity [158,159] cannot identify this possibility since the PLL mainly impacts the passivity of the qq and dq admittance quadrants, where changing the dd virtual admittance would not have a significant impact. The root locus as the dd virtual admittance gain is increased is also shown in Figure 3.9 (a), highlighting that in this case the dd virtual admittance is just as effective at enhancing the SSO mode damping as the qq virtual admittances.

Figure 3.9 (b) shows that that further increasing dd virtual admittance also begins to destabilize a low-frequency mode. Participation analysis of this mode shows it is due to interaction between the feed-forward low-pass filter, the DC control and the PLL dynamics.

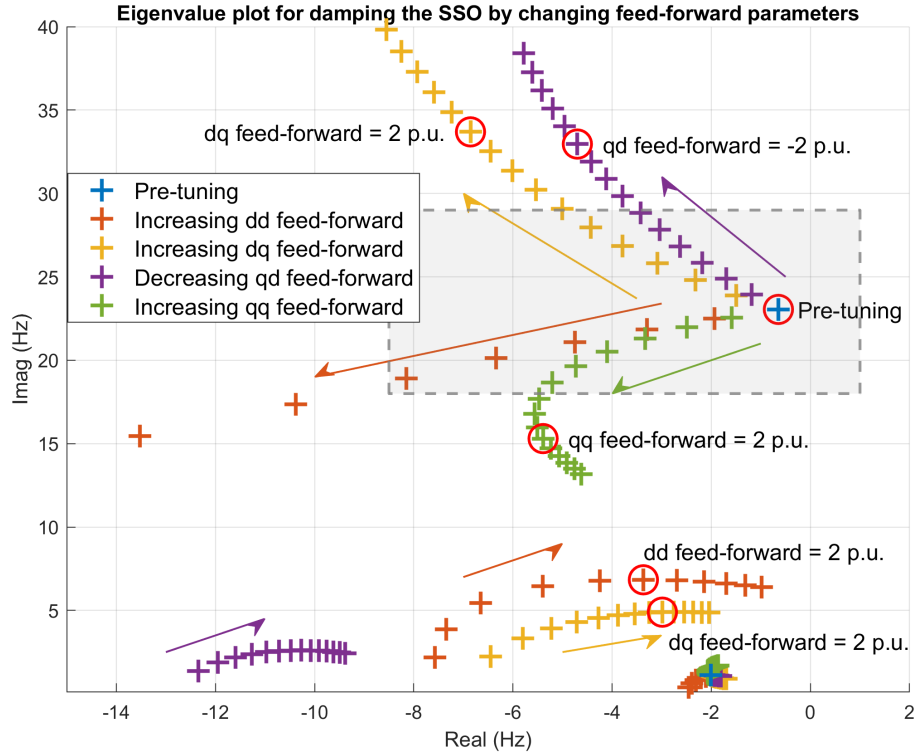
The root locus for tuning the qd quadrant virtual admittance is also included in Figure 3.9, where it can be seen that decreasing the qd virtual admittance has a similar impact on the SSO mode to increasing the dq virtual admittance but does not destabilize a low frequency mode.

None of the virtual admittance applications significantly affect the performance of the PLL - a strength of the virtual admittance approach noted in [158] in comparison to reducing the PLL bandwidth.

Eigenvalue plot for damping the SSO by changing feed-forward parameters, utilising the grey-box method

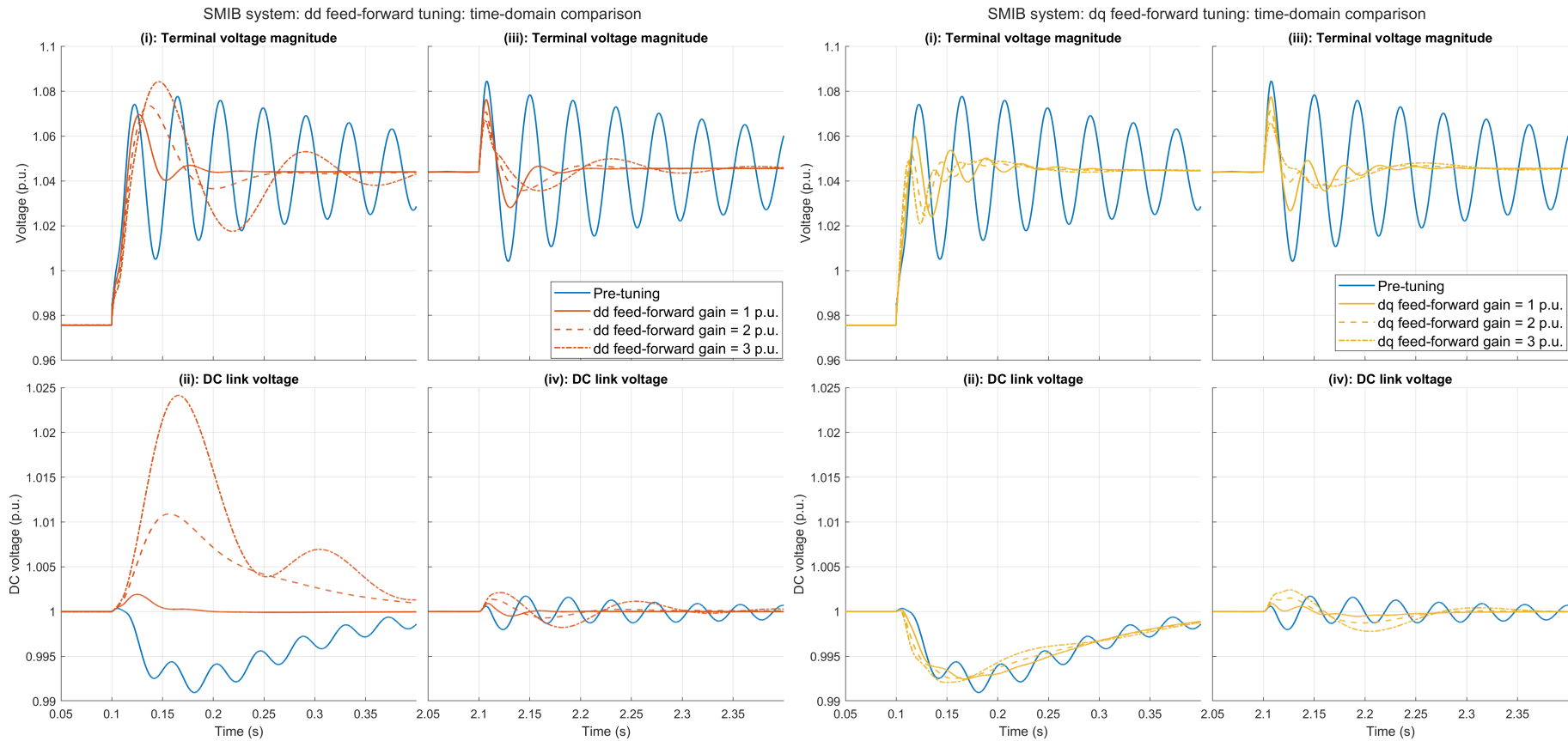


(a) Zoomed in eigenvalue plot



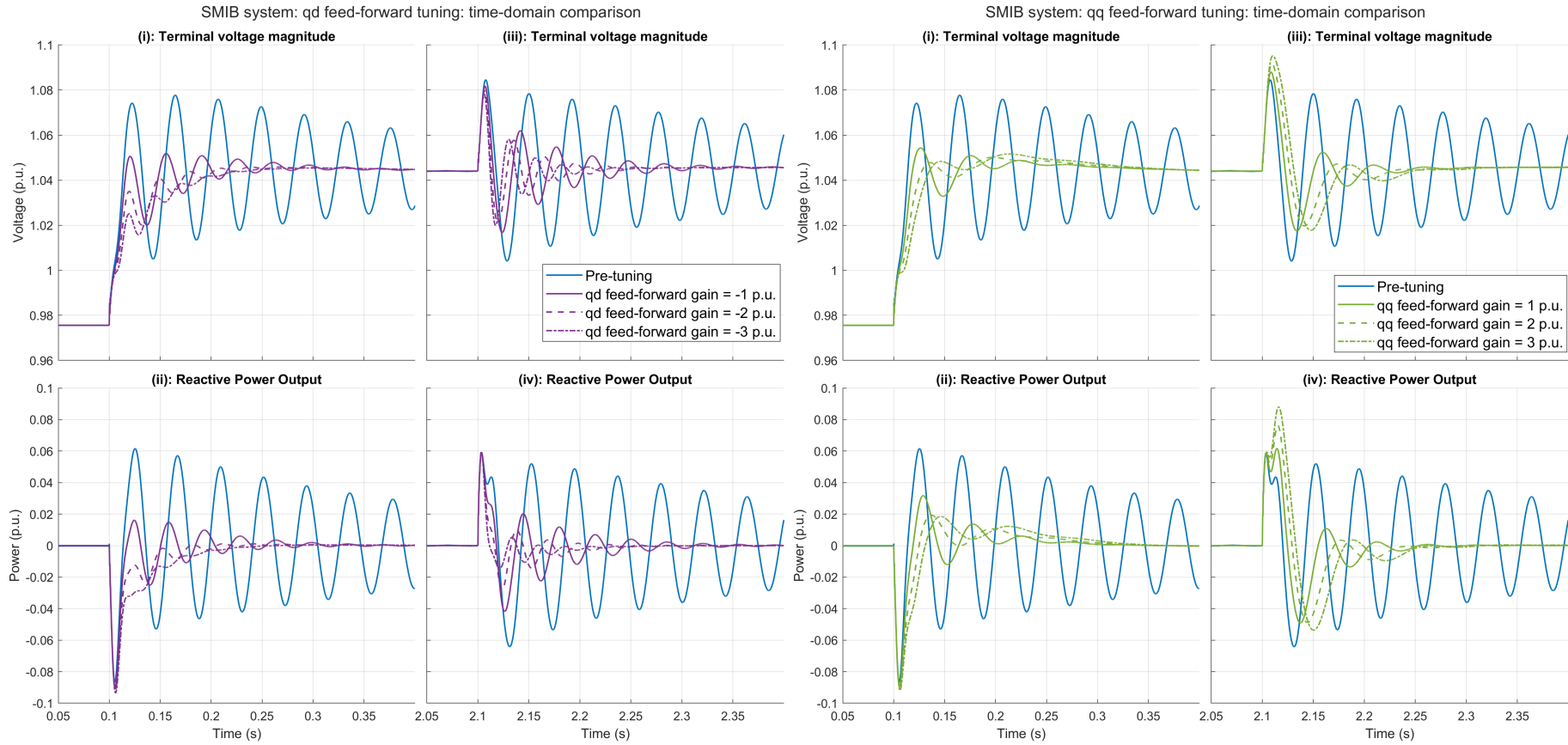
(b) Zoomed out eigenvalue plot - the coverage of (a) is illustrated with the grey box

Figure 3.9: Increasing the damping of the weak grid / PLL oscillation mode in the SMIB system by tuning the feed-forward gains in all four quadrant in steps of 0.2 p.u. from 0 (pre-tuning) to ± 3 p.u. It can be seen that the dd feed-forward is the most effective, and the qd the least effective in improving the damping of the SSO mode. Plot (a) is a zoomed-in version of (b) and only shows up to 1 p.u. The coverage of (a) is highlighted in (b) with a shaded grey box. Also shown in (a) are the predicted next locations of the modes utilising the grey-box method. The predicted locations are not shown in (b) for clarity. The destabilised low-frequency modes when the dd and dq virtual admittances are sufficiently large are visible at the bottom of (b)



(a) Time-domain responses showing the effect of increasing the dd virtual admittance gain from 0 to 3 p.u. in steps of 1 p.u. (b) Time-domain responses showing the effect of increasing the dq virtual admittance gain from 0 to 3 p.u. in steps of 1 p.u.

Figure 3.10: Time domain responses of the IBR terminal voltage magnitude and DC voltage to a step of 0.05 p.u. in the d-axis infinite bus voltage at 0.1 seconds, followed by a 0.05 p.u. q-axis step at 2.1 seconds. The time between 0.4 seconds and 2.05 seconds is omitted for clarity, such that the first column of plots shows the response to the d-axis step and the second column shows the response to the q-axis step. *dd* virtual admittance is applied in (a), while *dq* virtual admittance is applied in (b)



(a) Time-domain responses showing the effect of increasing the qd virtual admittance gain from 0 to 3 p.u. in steps of 1 p.u. (b) Time-domain responses showing the effect of increasing the qq virtual admittance gain from 0 to 3 p.u. in steps of 1 p.u.

Figure 3.11: Time domain responses of the IBR terminal voltage magnitude and reactive power output to a step of 0.05 p.u. in the d-axis infinite bus voltage at 0.1 seconds, followed by a 0.05 p.u. q-axis step at 2.1 seconds. The time between 0.4 seconds and 2.05 seconds is omitted for clarity, such that the first column of plots shows the response to the d-axis step and the second column shows the response to the q-axis step. *qd* virtual admittance is applied in (a), while *qq* virtual admittance is applied in (b)

Time domain simulations are shown in Figures 3.10 and 3.11 for virtual admittance gains of 1, 2 and 3 p.u. The improvement in the SSO mode damping is clearly visible across all utilised quadrants of virtual admittance, but there are varying levels of effectiveness and performance degradation.

The application of the dd axis virtual admittance in Figure 3.10 (a) is effective at damping the SSO mode as can be seen in all plots in 3.10 (a), however the reduced damping of the lower-frequency oscillation mode becomes visible in plots (i) and (ii) when the virtual admittance gain increases to 3 p.u.

The initial active power output is 1 p.u., so a virtual admittance gain value of $K_{\text{odd}} = 1$ p.u. offsets the impact of the d voltage disturbance on the DC voltage as discussed at the start of Chapter 3.5. The initial performance enhancement of the DC control with the dd axis feed-forward is visible in Figure 3.10 (a) (i). Increasing the dd quadrant gain past 1 p.u. then degrades the performance of the DC control during d axis voltage disturbances, as is also visible in Figure 3.10 (a) (i).

The dq virtual admittance is less effective at damping the SSO mode, as the oscillations are still visible with a gain of 1 p.u. in Figure 3.10 (b) (i) and (iii). However, the impact on the DC link voltage performance is reduced compared to the dd virtual admittance, since the q axis voltage disturbance is separated from the d axis disturbance. The presence of the low-frequency oscillation mode is apparent in Figure 3.10 (b) (iv) when the dq gain is increased to 3 p.u.

Figure 3.11 (a) shows the time-domain simulations utilising the qd virtual admittance, where the SSO oscillations are still partially visible even with a gain of up to 3 p.u. However, it can be seen in Figure 3.11 (a) (ii) that the action of the virtual admittance makes the reactive power output act against the voltage magnitude oscillations. The qd virtual admittance therefore acts to reduce the peak magnitude of the first swing in voltage magnitude, as shown in Figure 3.11 (a) (i).

The SSO mode is damped more effectively with the application of the qq virtual admittance shown in Figure 3.11 (b). It can be seen in Figure 3.11 (b) (i) and (ii) that the response of the virtual admittance effectively damps the oscillations in both voltage and reactive power. However, Figure 3.11 (b) (iii) and (iv) are an example of the qq virtual admittance increasing the reactive power output when the voltage is already high, resulting in a larger peak magnitude

in the first swing of the terminal voltage response. The root cause is that the internal IBR q axis voltage is in phase with the voltage magnitude oscillation so the virtual admittance generates more reactive power when voltage magnitude is already high. The damping of the SSO mode is sufficiently improved so it is only the first swing which has larger peak magnitude than the pre-tuning case.

The SMIB case study has demonstrated the capability of grid-following converter to damp a weak grid / PLL SSO with all four quadrants of virtual admittance, without ignoring the DC control as has been common in the literature. The virtual admittances do not degrade the PLL synchronization performance as would be the result of reducing the PLL bandwidth. The performance trade-offs are instead larger deviations in DC-link or IBR terminal voltage in response to disturbances, depending on which quadrant of virtual admittance is utilised. Care should be taken as the dd and dq quadrants of the virtual admittance can provoke low-frequency interactions between the DC control and PLL.

3.6 Multi-IBR systems

The previous section has illustrated the capability of the IBR in a single-machine system to enhance small-signal stability with virtual admittance guided by the grey-box method and discussed the resulting performance trade-offs. The capability of dq and qq virtual admittances to enhance stability in the single machine system has been well discussed in the literature before, but it is less clear whether capability exists for one active damping IBR to improve the damping of a mode which is due to the interaction of multiple apparatus, which the active damping IBR does not have significant state-participation in.

It is known that the state-participation factors in a certain mode also represent the sensitivity of the mode to the corresponding diagonal entry in the state-space 'A' matrix a_{mm} . The calculation of the state-participation factor from the grey-box method and knowledge of the apparatus sensitivity to the corresponding diagonal entry is shown in Equation 3.20, which was reported in [52].

$$\frac{\partial \lambda_i}{\partial a_{mm}} = - \left(R_{Z,dd} \frac{\partial Y_{DD}(\lambda_i)}{\partial a_{mm}} + R_{Z,qd} \frac{\partial Y_{DQ}(\lambda_i)}{\partial a_{mm}} + R_{Z,dq} \frac{\partial Y_{QD}(\lambda_i)}{\partial a_{mm}} + R_{Z,qq} \frac{\partial Y_{QQ}(\lambda_i)}{\partial a_{mm}} \right) \quad (3.20)$$

It can be seen from Equation 3.20 that low state-participation does not imply low mode-sensitivity (residue magnitude), since mode-sensitivity can still be not insignificant if $\frac{\partial Y(s)}{\partial a_{mm}}$ is small. A particularly simple example is a resistance, which is known to influence the damping of oscillation modes but has no state-participation in any modes. Alternatively, the four components of Equation 3.20 could mostly cancel even if none of the magnitudes of $\frac{\partial Y(s)}{\partial a_{mm}}$ or the residues are small.

The distinction between sensitivity and participation helps justify the proposed impedance shaping process, where one active damping IBR can be tuned to dampen the interactions of other IBR's even without having significant state-participation. If there exists a residue with sufficiently large magnitude at the bus the active-damping IBR is connected at the proposed impedance-shaping approach can improve the damping of an SSO mode without having to identify the root-cause at all.

A case study of a multi-IBR system is now presented to demonstrate the capability of grid-following IBR to damp interactions of other IBR on the network.

3.6.1 Case study: 14 bus network

The case study uses the IEEE 14-bus system [171] modified with four grid-following IBR replacing synchronous machines as shown in Figure 3.12. The parameter values of all apparatus in the system can be found in Appendix D, along with the SIMULINK model used. A similar case study is used in [48], where the residue analysis indicates that raising the bandwidth of the current controllers in three of the IBR will improve the damping of the mode.

There is one IBR (Apparatus 12 - shown in red in Figure 3.12) which is poorly tuned, resulting in one very poorly damped SSO at approximately 18.5 Hz. Participation analysis reveals that the SSO mode has 95 % participation from states belonging to Apparatus 6, 12 and 13. For the case study, one converter (Apparatus 11 - shown in green in Figure 3.12) is chosen as the active damping IBR to demonstrate the impedance shaping process. It only has approximately 3% participation in the SSO mode. The size of the DC capacitor belonging to Apparatus 11 size has been reduced from the default in the Simplus toolbox [57] to better illustrate its DC voltage deviations.

Figure 3.13 (a) shows the root locus plot for impedance shaping with the active damping IBR

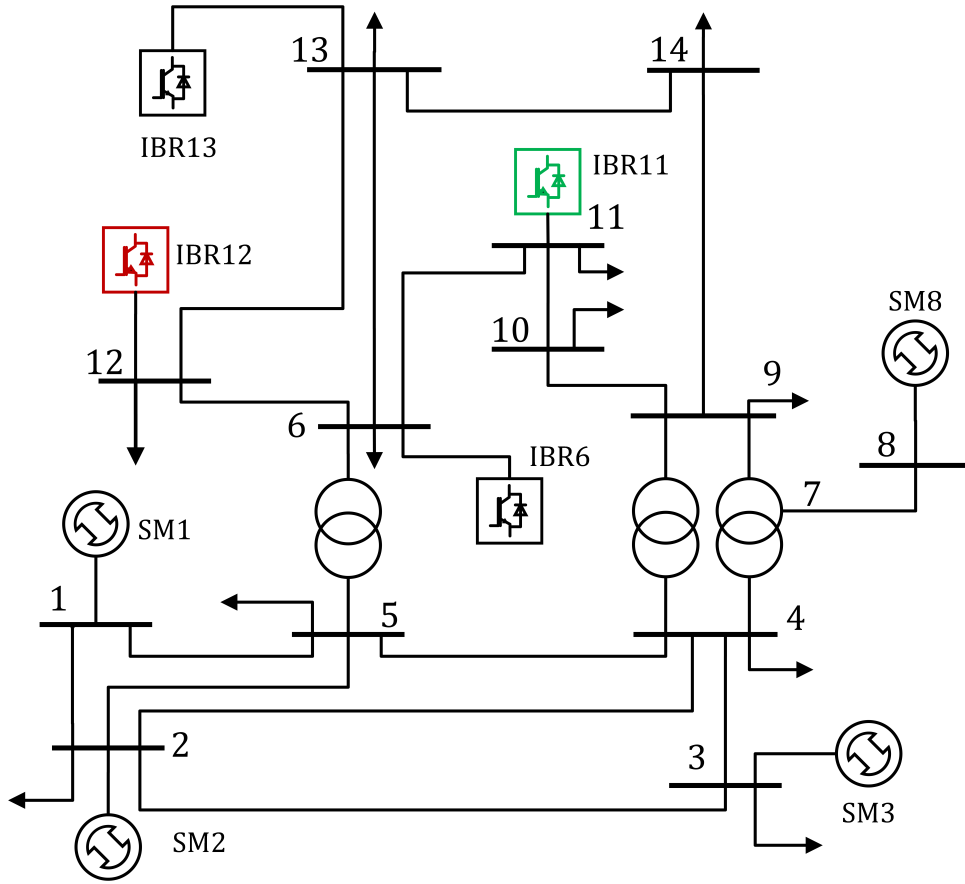


Figure 3.12: IEEE 14 bus system used in the second case study: parameter values and the SIMULINK model can be found in Appendix D. The apparatus shown in red (IBR12) is poorly tuned and the apparatus shown in green (IBR11) will be utilised as the active-damping IBR to enhance the small-signal stability

current control, DC control and the PLL bandwidths. Figure 3.13 highlights that relatively little change in the mode damping is achieved even when the PLL bandwidth is reduced to a very low value (1 Hz), which would significantly degrade the performance of the phase-synchronization.

A greater damping effect can be achieved utilising the virtual impedance approach, as shown in Figure 3.13. The mode is less sensitive to the virtual admittance than in the SMIB case (shown in Figure 3.9), so the same change in the feed-forward parameter results in smaller increases in mode damping. However, as shown in the time-domain results in Figure 3.14 the damping of the oscillations are still noticeably improved in each case.

The same performance trade-offs as discussed in the SMIB system are visible: DC control performance degradation when the dd virtual admittance is increased past 1 p.u., and a greater peak in the first swing of the terminal voltage magnitude when utilising the qq virtual admittance. Again, the qd virtual admittance effectively reduces the magnitude of the voltage

oscillations but is less effective at damping the SSO mode than the dd or qq virtual admittance.

The DC control performance degradation with the dq virtual admittance is visible in Figure 3.13b (ii) is more severe compared to the SMIB case shown in Figure 3.10(b) (iv). The reason for the difference is that the disturbance in the 14 bus case study results in a steady-state phase shift in the IBR terminal voltage of 15 degrees, as opposed to the SMIB case study where the shift is 4 degrees for the infinite bus d -axis voltage disturbance, then 3 degrees for the q axis disturbance. The larger phase shift results in larger disturbance to the internal PLL reference frame q axis voltage and hence the action of the the dq virtual admittance is larger.

The 14-bus simulation study re-affirms the observations made in the single-machine case study, but the virtual admittance is less effective at providing an increase in damping for a similar degradation in performance. A not-insignificant increase in damping is still seen, which is the aim of applying the impedance-shaping process.

3.7 Information sharing requirements to account for the performance impacts in the parameter choice

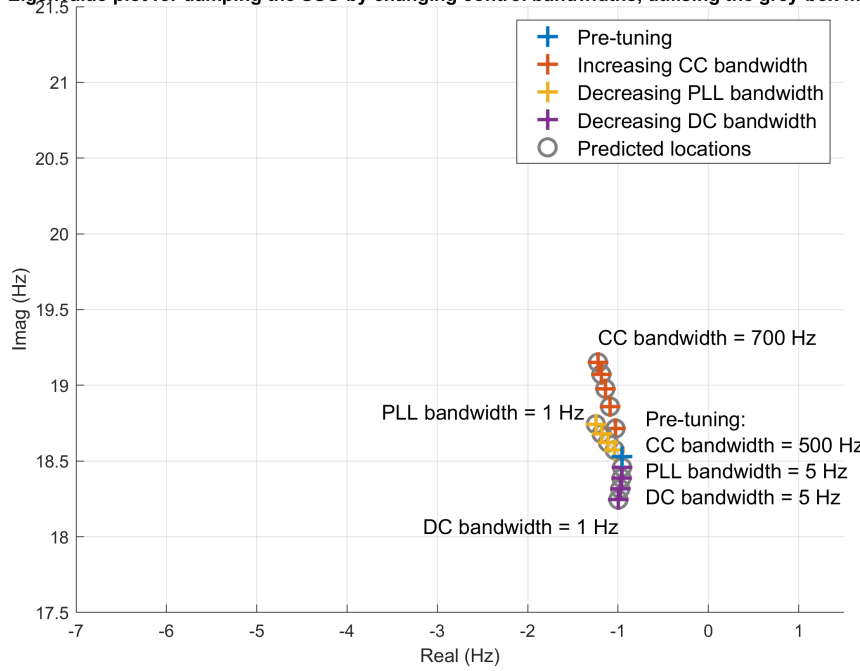
Chapter 2.1.2 reviewed methods for optimising Power System Stabilizer (PSS) parameter tuning to improve small-signal stability, based on linearised mode-parameter sensitivities. The two common methods for respecting the limits of the PSS in the parameter choice were shown to be:

- Set hard constraints on the allowable range of parameters.
- Include a measure of the performance in the optimisation objective function as a "cost" which it is desired to minimise. The optimisation process would likely require a sensitivity of the measure of the performance to each considered parameter change.

Either of these two approaches could be utilised in the proposed impedance-shaping process to consider the IBR performance. Including a measure of the performance in the objective function could be beneficial as it could indicate situations where a change in parameters improves performance, such as the increase of the dd virtual admittance gain to 1 p.u. in the case study presented in Figure 3.10 (a).

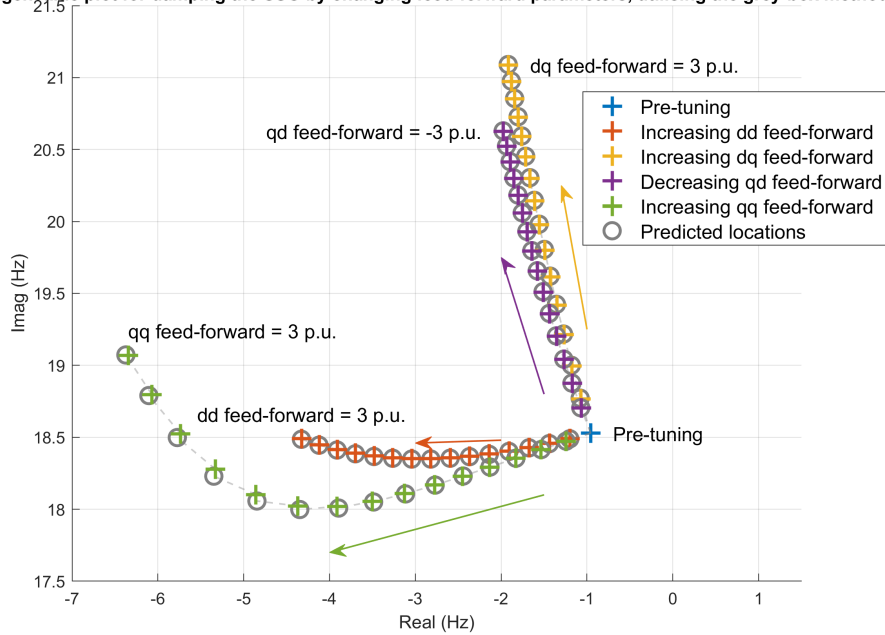
The measure of the DC voltage performance to be used could for example be a norm of the

Eigenvalue plot for damping the SSO by changing control bandwidths, utilising the grey-box method



(a)

Eigenvalue plot for damping the SSO by changing feed-forward parameters, utilising the grey-box method



(b)

Figure 3.13: Improving the damping of an SSO mode in the IEEE 14 bus system system by (a) tuning the bandwidths of three control loops: increasing the current control (CC) bandwidth from 500 Hz to 700 Hz in steps of 40 Hz, decreasing the PLL bandwidth from 5 Hz to 1 Hz in steps of 1 Hz, decreasing the DC bandwidth from 5 Hz to 1 Hz in steps of 1 Hz. (b) shows that the application of the application of the dd and qq virtual admittance are more effective at improving the damping of the mode, whereas the dq and qd virtual admittances increase the mode frequency more than the damping. Also shown are the predicted locations for each step utilising the grey-box method. Both (a) and (b) are shown at the same scale for easier comparison

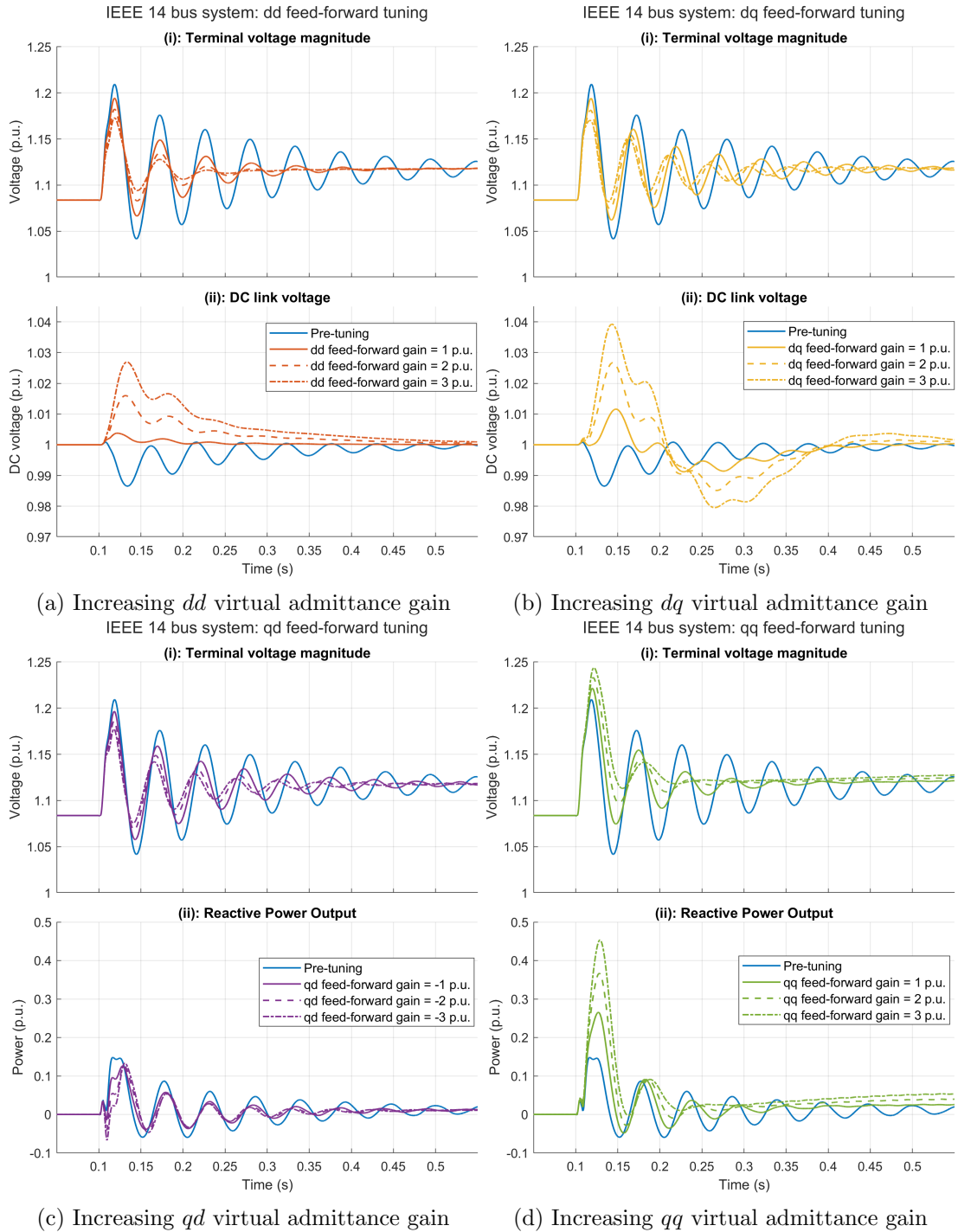


Figure 3.14: Time domain responses of the IBR terminal voltage magnitude, DC link voltage and reactive power output to a 0.7 p.u. step in load at bus 13 at 0.1 seconds. Values of virtual admittance gain are shown at 0, 1, 2 and 3 p.u. for all four quadrants. The discussed degradation of the DC control is visible in (a) (ii) and (b) (ii), whereas the voltage overshoot is visible in (d) (i)

transfer function matrix between the grid voltage and the DC voltage. The two components of the transfer function matrix are shown in Equation 3.19 and are plotted in Figures 3.15a and 3.15b. The shape of these transfer functions could be derived by the IBR operator.

The norm of the transfer functions shown in Figures 3.15a and 3.15b could also be used to calculate the constraints on the allowable range of the parameters by constraining the maximum allowable value of the norm.

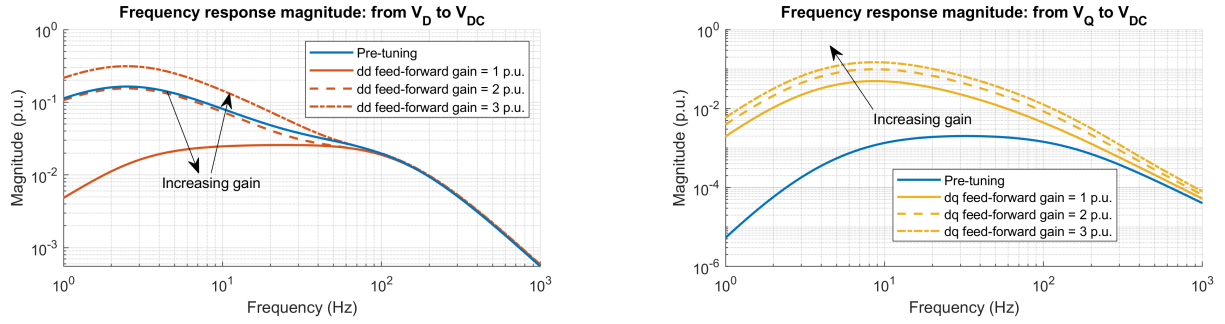
The relationship between the QD and QQ virtual admittance gains and the AC voltage magnitude is more complex and depends heavily on the network configuration. The network configuration could be unknown since the objective of the entire impedance-shaping process is to avoid detailed modelling of the rest of the grid. In this case, it may be simpler to use a measure of the control effort. A simple measure of which in the proposed admittance-shaping process could be a measure proportional to the change in virtual admittance gains.

Many of the PSS tuning works [87–89] use PSS controller gains directly as the control effort measure considered in the optimisation. However, this may also miss situations where the change in a parameter improves an aspect of the IBR performance. For example, the qd virtual admittance was shown to mainly reduce the AC voltage magnitude deviation. In this case, the cost of changing the qq virtual admittance could be weighted more heavily than the qd virtual admittance in the objective function.

With agreement between the ESO and IBR operator the cost-based approach could allow a direct calculation of financial compensation for a certain degree of performance degradation in exchange for small-signal stability enhancement.

However, the cost approach would likely require the sensitivity of the performance measure to the objective to be calculated for each considered parameter change. Additionally, there would be more frequent information sharing required to update the performance measure and the performance sensitivity at each parameter optimisation step in the impedance-shaping process. The role of the alterable parameters and the role of any of the performance measures could be anonymised, which would make the sharing of the performance sensitivity less revealing.

Placing constraints on the allowed range of the parameters could be a simpler approach which requires minimal information sharing after the initial agreement of the constraints.



(a) The gain from the grid voltage V_D to the DC voltage, for an increasing value of dd virtual admittance gain

(b) The gain from the grid voltage V_Q to the DC voltage, for an increasing value of dq virtual admittance gain

Figure 3.15: Frequency response magnitudes of the transfer functions between the IBR terminal voltages and the IBR DC voltage, shown for a selection of the virtual admittance gains. The improvement in DC voltage performance can be seen when at the dd virtual admittance gain = 1 p.u. in (a), otherwise increasing the virtual admittance gain degrades the DC performance

3.8 Chapter Conclusions

This chapter has investigated Barrier 1 by demonstrating the capability of grid-following IBR to provide impedance shape-ability to improve the damping of whole-system small-signal oscillation modes.

The available parameters for impedance shaping were investigated and compared, where the measured IBR terminal voltage to output current reference feed-forward parameters were identified as particularly suitable for the expected frequency range of SSO. Subsequently, two case studies were presented to demonstrate the proposed impedance-shaping process improving the damping of poorly damped oscillation modes.

First, a SMIB system suffering from an SSO due to the well-known phenomenon of weak grid-PLL interaction was analysed. The versatility of the proposed process is demonstrated as feed-forward parameters in all four quadrants are shown to be able to improve the damping of the SSO mode to various degrees of effectiveness. The capability of the dq and qq quadrant feed-forward parameters have already been documented in the literature, but it was demonstrated in this chapter that the dd quadrant feed-forward was in the considered case equally effective.

It was shown that the dd or dq quadrant virtual admittances introduce an input disturbance to the DC control loop and degrade the performance of the DC control. Under operating conditions of non-zero initial output power the dd or dq quadrants can improve performance in the case there is not already some form of load power feed-forward already implemented. Most work

in the literature which analyses outer virtual admittance for small-signal stability enhancement does not consider DC control. This chapter has shown that there can exist sufficient shapeability of the dd quadrant virtual admittance to improve the damping of SSO modes without abandoning DC control, at the cost of degraded DC voltage control performance.

Increasing the outer virtual admittance gains too far can destabilise low-frequency modes of interaction between the PLL and DC control, so the assessment of the effect of the change in virtual admittance gain on the stability of low-frequency modes should be considered.

It was also shown that the initial terminal voltage magnitude deviation in response to disturbances can be magnified with the application of the qq quadrant virtual admittance. Not all disturbances result in this performance degradation: the initial deviations in d and q voltage must have the same sign for the generated reactive power to magnify the initial voltage deviation, even when the damping of the mode is sufficiently improved for the subsequent voltage oscillations to be of smaller magnitude.

A second case study of a 14 bus multi-IBR system was presented. In this case study there is an SSO mode present due to the interaction of three IBR controllers, one of which was poorly tuned. The fundamental premise of the proposed impedance shaping process is demonstrated by utilising the feed-forward parameters of a fourth IBR - initially not significantly participating in the SSO mode - to improve the damping of the SSO mode. In this case it is again shown that utilising the dd or dq feed-forward parameters trade the active damping IBR DC control performance for the enhanced damping of the SSO whole-system mode. Utilizing the qq feed-forward parameter also enhances the damping, at the cost of increased AC voltage magnitude deviation. The damping enhancement is also demonstrated with the qd quadrant parameter, but is shown to be less effective in the considered case study.

Proportionally less damping enhancement is obtained compared to the SMIB case for similar performance degradation. The underlying reason is poorer mode observability/controllability, manifesting as smaller whole-system residues. However, there is a not-insignificant degree of damping enhancement obtained and the performance degradation ensued may be acceptable (particularly if there is compensation for participating in the active-damping).

Finally, methods of accounting for the performance trade-offs when choosing the optimal parameter change to make were discussed and the information-sharing required to implement

them was briefly outlined.

4 The minimum- $\Delta\rho$ path constraint

4.1 Chapter Introduction

This chapter describes a contribution to the application of the co-operative impedance-shaping process described in Chapter 1. In the impedance-shaping process, incremental adjustments in IBR parameters are made with the aim of improving the whole-system small-signal stability. The work presented in Chapter 3 demonstrated that poorly damped modes of oscillation can exist due to interaction of IBR controllers with the grid or with each other. The adjustments in IBR parameters are guided by the "grey-box" method, in order to improve the damping of these modes and hence the system small-signal stability.

The parameters which are considered for change in this chapter are the grid voltage to current-reference feed-forward gains, also described as virtual admittance gains in Chapter 2. These parameters are used here since they were deemed to be most suitable for application with the grey-box method in Chapter 3. However, the work presented in this chapter could also be applied with any other other apparatus parameter, as long as the parameter does not directly influence the global system frequency. As discussed in Chapter 1, the grey-box method was derived assuming a constant global frequency.

It was also discussed in Chapter 2 that there were several constraints placed on the allowed change in parameters. The change in parameters must not unacceptably degrade the performance of the IBR, which was investigated in Chapter 3. Additionally, the range of the allowed change in IBR parameters is also constrained by the need to ensure system small-signal stability after the change in parameters is made. Although maximising the mode damping factors should ensure that the small-signal stability improves, there may be multiple modes to consider and it may be necessary to trade off improving the damping of one mode against degrading the damping of another. In this case, the constraints on the allowed change in parameters need to ensure that all the system modes remain stable. Additionally, the grey-box relationship is a linearised relationship valid at the initial operating conditions. Changing a parameter may result in the residues or admittance-parameter sensitivity changing, which should also be taken into account.

The work in this chapter presents a novel method for calculating constraints on the allowed changes in the apparatus parameters, taking into account the variations of the admittance-

parameter sensitivity in the neighbourhood of each whole-system mode. However, variation in the residue magnitude is not explicitly taken into account, and requires an assumption to be made which could be a subject for future research.

The novel constraint should be calculated by the ESO or IBR operator when choosing which parameter change to make (step "Parameter alteration decision" in Figure 1.4). The extra data which would be required to make use of the novel constraint is discussed in more detail in Chapter 4.6.

4.2 Terminology and fundamentals

The mathematical relationship at the heart of the grey-box approach introduced in Chapter 1 is repeated here in Equation 4.1.

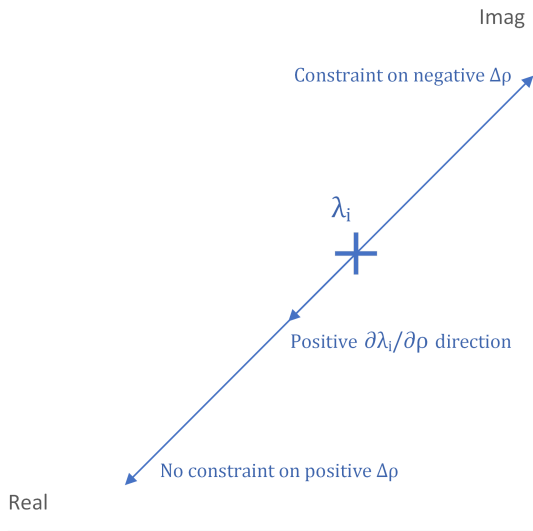
$$\frac{\partial \lambda_i}{\partial \rho} = S_{\lambda_i} = -R_Z \frac{\partial Y}{\partial \rho}(\lambda_i) \quad (4.1)$$

The term $\frac{\partial Y}{\partial \rho}(s)$ is referred to as the admittance-parameter sensitivity, or the "sensitivity" for short. It is important to distinguish the poles of the sensitivity (the zeros of the denominator of $\frac{\partial Y}{\partial \rho}(s)$, like any other transfer function) from the whole-system modes λ_i .

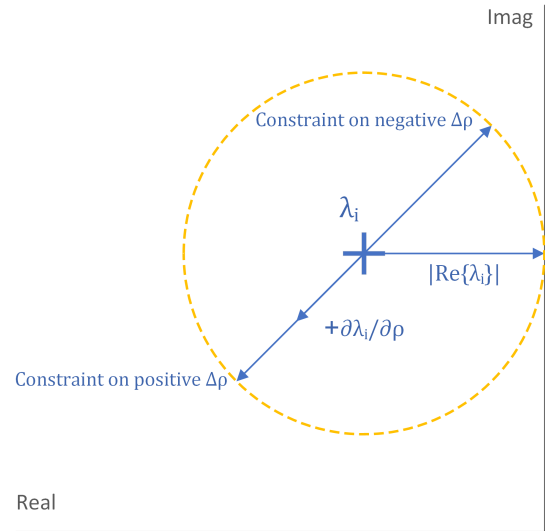
The poles present in the sensitivity $\frac{\partial Y}{\partial \rho}(s)$ are designated p_j . They are referred to as open-loop poles, since connecting the apparatus to the network will move the poles in a similar manner to closing the loop in a classical control system. The whole-system modes are the oscillation modes present in the real interconnected network and the overall objective is to ensure they are all well damped.

4.3 Constraints on parameter step sizes for whole-system stability

There are two main approaches to calculating the stability constraints on the parameter change. The approaches are referred to here as Constraint 1 and Constraint 2 for clarity, they are also illustrated in Figure 4.1. Each mode results in a separate constraint on the allowed change in each parameter in the form of Equation 4.2 or 4.3. The tightest constraint over all the modes defines the overall allowed range of each parameter.



(a) Illustration of the constraint placed on the allowed change in parameter with Constraint 1, for a certain mode λ_i .



(b) Illustration of the constraint placed on the allowed change in parameter with Constraint 2, for a certain mode λ_i .

Figure 4.1

- Constraint 1: all predicted mode locations should be stable, as shown in Equation 4.2. This constraint is widely used in the literature discussed in Chapter 2.1.2, for example [89,172]. Constraint 1 is shown in Figure 4.1a.

$$\Re \left\{ \lambda_i + \frac{\partial \lambda_i}{\partial \rho} \Delta \rho \right\} < 0 \quad \forall \lambda_i \quad (4.2)$$

- Constraint 2: all predicted changes in mode locations should be smaller than the distance between the mode and the imaginary axis, such that if the phase of the $\frac{\partial \lambda_i}{\partial \rho}$ changes the mode will not be destabilized. This constraint is shown in Equation 4.2. This constraint is less widely used, but a similar approach is taken in [173] to calculate the "Impedance Margin Ratio", a measure of small-signal system strength utilizing the grey-box relationship. Constraint 2 is shown in Figure 4.1b.

$$\Re \{ \lambda_i \} + \left| \frac{\partial \lambda_i}{\partial \rho} \Delta \rho \right| < 0 \quad \forall \lambda_i \quad (4.3)$$

It can be seen that Constraint 2 is more restrictive than Constraint 1 because it takes into account the fact that the angle of $\partial \lambda_i / \partial \rho$ may not be reliably known.

4.4 Motivation for an enhanced approach

Equation 4.1 describes the direction and magnitude of movement of a mode in the complex plane in response to a change in parameter. Since the sensitivity term in the grey-box relationship $\frac{\partial Y}{\partial \rho}(\lambda_i)$ is evaluated at the whole-system mode λ_i , the value of $\frac{\partial Y}{\partial \rho}(\lambda_i)$ is dependent on the location of the mode λ_i . The sensitivity $\frac{\partial Y}{\partial \rho}(s)$ may be independent of ρ , but the term $\frac{\partial Y}{\partial \rho}(\lambda_i)$ will still change as ρ is changed and the whole-system mode moves. The residue will also change as ρ is changed. Since each of $\frac{\partial Y}{\partial \rho}(\lambda_i)$ and the residue R_Z have magnitude and phase there are four variables which affect the movement of the mode and which themselves may change as ρ changes. However, all four are assumed to be constant in constraints 1 and 2.

The approach taken in this work is to allow the change in the angle of the residue to be unbounded, but assume the change in magnitude of the residue magnitude to be limited. Estimating how the residue magnitude will change as a parameter changes is currently an unsolved problem, it is recommended that future research could investigate this. Assumption 1 is now made to limit the maximum change in the residue magnitude as ρ changes.

Assumption 1. *There is a residue magnitude $R_{Z\max}$ which describes the maximum magnitude of the residue R_Z over the change in parameter $\rho = \rho_0 + \Delta\rho$*

The new constraint definition to be described in this Chapter will account for the changing angle of the residue phase, and the changing magnitude and angle of the admittance sensitivity. There remains the unknown change in residue magnitude to be accounted for in future research.

4.4.1 Motivating case study: the SMIB system with a tuned band-pass filter

The case to be considered here is a proposal to include in the system, here a single machine (IBR) infinite bus system, a band-pass filter that can be tuned such that when an IBR parameter is changed to move a particular whole-system mode only that mode moves significantly. This approach would introduce additional modes associated with the band-pass filter and these also have to be considered for stability. The band-pass filter could be implemented within the inner or outer virtual admittances instead of the low-pass filter which was utilised in the previous chapter (in Equation 3.15). The implementation of the virtual admittance was described in more detail in Chapters 2.3, 2.4 and 3.4.4.

The case study analysed in this section attempts to improve the damping of the SSO mode

present in the SMIB system described in more detail in Chapter 3. The band-pass filter is tuned sharply such that the virtual admittance will only affect the neighbourhood of the SSO mode. Due to the sharply tuned filter, there are now two poorly damped modes to be considered when making a parameter change. However, the impact on all other modes in the system is negligible.

An initial choice in outer virtual admittance DD gain results in the two modes having the positions indicated by the blue crosses in Figure 4.2. The next proposed step could be a change to the inner DQ virtual admittance gain with the direction of $\frac{\partial \lambda_i}{\partial \rho}$ as shown by the arrows in Figure 4.2. The example change improves the damping of the poorly damped Mode 2 but at the cost of degrading the damping of Mode 1. In order to assess this step it is important to calculate by how much the gain could be increased without destabilising Mode 1.

The magnitude of the sensitivity of the DQ admittance to the gain of the virtual admittance is illustrated with the contour plot in Figure 4.3. The presence of the open-loop pole due to the band-pass filter is visible in the contour plot as the yellow circle (the contour plot magnitude is capped at 3 for clarity).

Figure 4.3 also shows the path that Mode 1 takes as the inner virtual admittance gain proposed in Figure 4.2 is increased. Mode 1 is destabilised at a gain of magnitude 0.73 p.u.

The applications of Constraints 1 and 2 are represented in Figure 4.3. However, both constraints overestimate the safe range of the increase in the parameter - Constraint 1 gives a maximum change in parameter of 1.73 p.u., while Constraint 2 allows a maximum of 0.87 p.u. The calculation of these values is described in Appendix C.

The DQ whole-system impedance residue magnitude associated with the SSO mode as it is destabilised is plotted in Figure 4.4a: it can be seen that the residue magnitude decreases as the virtual admittance gain is increased. If the sensitivity magnitude were to stay constant, Constraint 2 should be sufficient to prevent destabilisation of the mode. However, as shown in Figure 4.4b, the sensitivity magnitude increases significantly due to the open-loop pole.

The increase in the sensitivity magnitude results in the mode destabilising for a smaller change in parameter than predicted by Constraints 1 or 2. Essentially, the open-loop pole provides a "high-speed path" to the imaginary axis. If the mode moves close to this pole of the sensitivity it can result in faster destabilisation than predicted with Constraint 1 or Constraint 2.

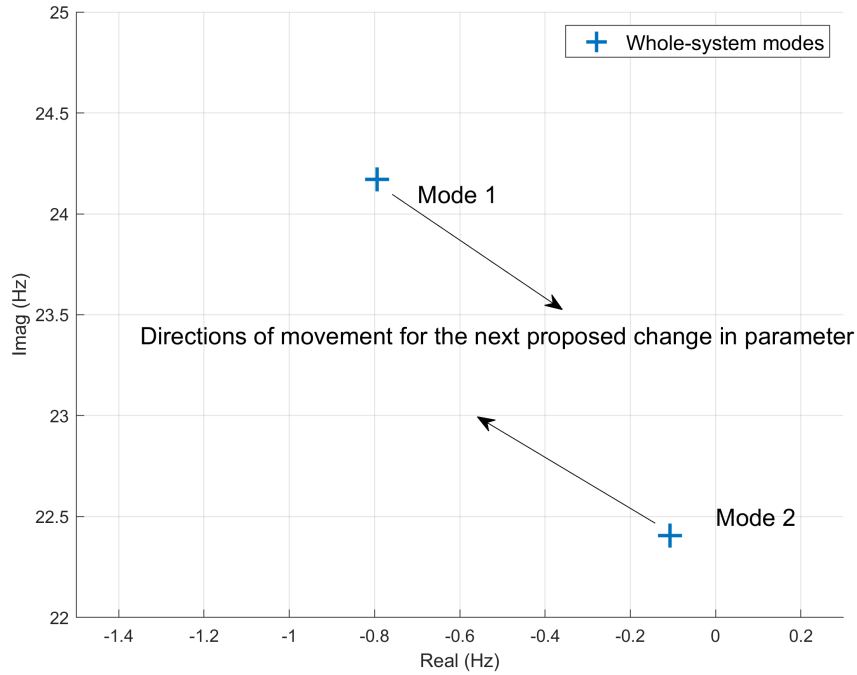


Figure 4.2: The initial condition for the motivating case study: a sharply tuned band-pass filter in the virtual admittance results in two poorly damped modes. The next considered change in parameter improves the damping of the less well damped Mode 2 but degrades the damping of Mode 1. No other system modes are significantly sensitive to the change in parameter considered.

The grey-box approach requires that the operator of the IBR providing active-damping knows the admittance-parameter sensitivity spectrum $\frac{\partial Y}{\partial \rho}(s)$ in order to share its value evaluated at each estimated whole-system mode. With this knowledge, it should be possible to account for the possibility of an increase in the sensitivity due to an open-loop pole. The remainder of this chapter will describe a technique for doing this.

4.5 Proposed stability constraint: The minimum- $\Delta\rho$ path

This section will outline the novel constraint. The problem the constraint considers is: given knowledge of the magnitude of the admittance-parameter sensitivity $\frac{\partial Y}{\partial \rho}(s)$ and assuming Assumption 1 holds, what is the minimum change in parameter $\Delta\rho_{\min}$ which could destabilise a given mode λ_i ?

The phase of $\frac{\partial Y}{\partial \rho}(s)$ is irrelevant since the phase of the residue is unbounded. The unbounded phase of the residue implies that the phase of $\frac{\partial \lambda_i}{\partial \rho}$ can take any value regardless the value of the phase of $\frac{\partial Y}{\partial \rho}(s)$.

The considered problem is similar to a classical mechanics problem: given two points, what

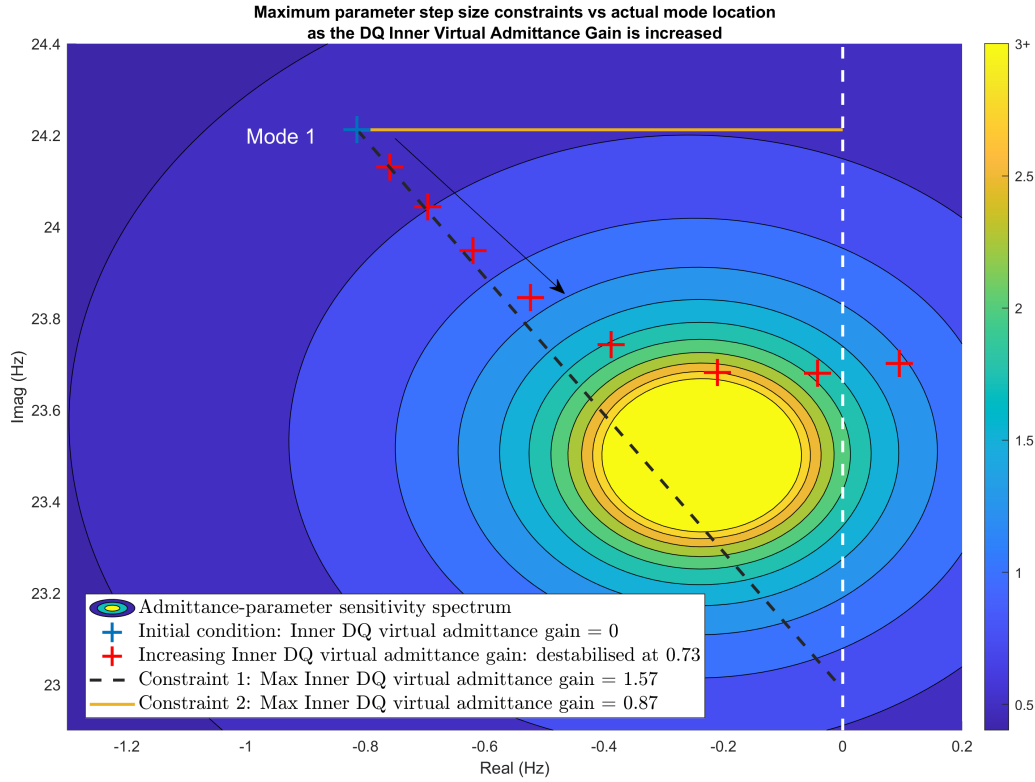
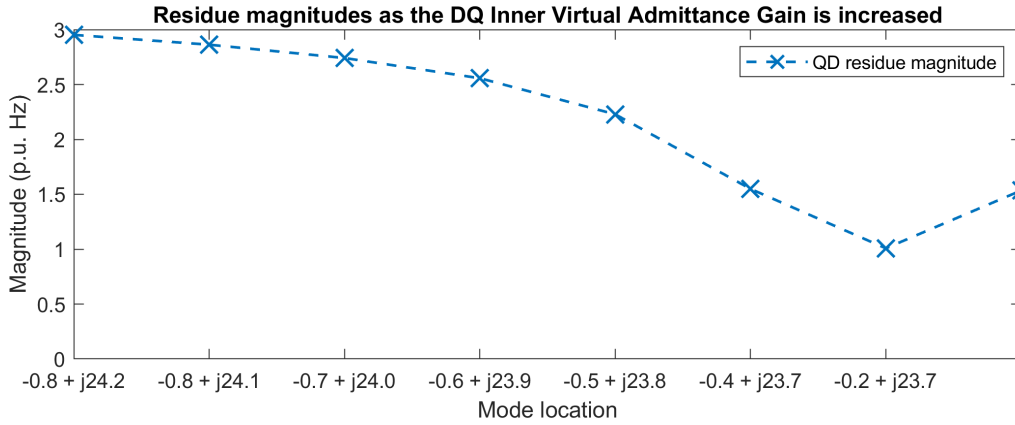


Figure 4.3: Illustration of the motivating case study: the two stability constraints shown in Equations 4.2 and 4.3 fail to predict the destabilisation of the whole-system mode as the virtual admittance gain is increased. The contour in this plot shows the magnitude of the sensitivity of the active-damping IBR admittance to a change in virtual admittance gain - the open-loop pole is clearly visible but the peak is flattened for clarity. Also shown is the movement in the complex plane of an SSO mode (with red crosses) as the virtual admittance gain is increased - the mode is destabilised at a gain value of 0.73 p.u

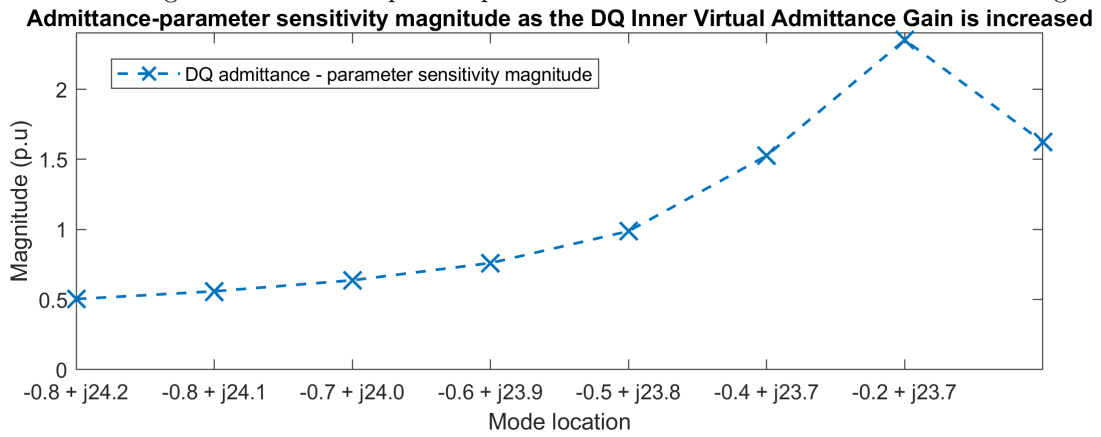
is the quickest path to travel between them? Normally, the answer is the straight line between the two points. However, if the speed of travel varies depending on the path taken (a common example used often is crossing a river), then the problem becomes more difficult to solve.

To apply the analogy to the small-signal stability of oscillation modes the length of a path travelled by a mode in the complex plane should be equivalent to distance. Distance is now denoted with the symbol L .

The independent variable driving the change in the system is the change in the parameter $\Delta\rho$, so it is equivalent to time in the quickest-path analogy. In the spirit of this analogy, Equation 4.1 can be interpreted as showing that the direction of travel of the mode λ_i is set by the phase of S_{λ_i} . The magnitude of S_{λ_i} sets the "speed" of the movement of λ_i in the complex plane, relative to the change in ρ . Equation 4.5 shows the expression for the 'speed' of travel of the mode.



(a) The residue magnitudes at each step in the path of the SSO mode shown with red crosses in Figure 4.3.



(b) The magnitude of the sensitivity of the DQ admittance to the DQ virtual admittance gain at each step in the path of the SSO mode shown with red crosses in Figure 4.3.

Figure 4.4: Plots of (a) the residue magnitude and (b) the sensitivity of the DQ admittance to the DQ virtual admittance gain as the DQ virtual admittance gain is increased in the motivating case study shown in Figure 4.3. The x-labels are given as the mode location at each step, shown with red crosses in Figure 4.3

$$dL = |d\lambda_i| \quad (4.4)$$

$$\frac{dL}{d\rho} = |S_{\lambda_i}| \quad (4.5)$$

If a mode travels along a path P , the length of the line ΔL traced in the complex plane can be found by the integral of $|S_{\lambda_i}|$ with respect to the $\Delta\rho$ which caused it to move, as per Equation 4.6. Accordingly, the change in ρ required to make the mode travel the path P can be found with Equation 4.7.

$$\Delta L = \int_{\rho_0}^{\rho_0 + \Delta\rho} |S_{\lambda_i}| d\rho \quad (4.6)$$

$$\Delta\rho = \int_P \frac{1}{|S_{\lambda_i}|} dL \quad (4.7)$$

The spectrum of the admittance-parameter sensitivity $\frac{\partial Y}{\partial \rho}(s)$ is expected to be known by the operator of the active-damping IBR. Utilizing Assumption 1 that $R_Z \leq R_{Z\max}$, the relationship between the position of the mode and its maximum speed of travel is known and is shown in Equation 4.8. It should therefore be possible to calculate to "quickest" possible path $P_{\min, \text{Im}}$ between the initial mode location λ_i and the imaginary axis, as shown in Equation 4.9.

$$\frac{dL}{d\rho} \leq \frac{1}{\left| R_{Z\max} \frac{\partial Y}{\partial \rho}(\lambda_i) \right|} \quad (4.8)$$

$$P_{\min, \text{Im}} = \arg \min_P \int_P \frac{1}{\left| R_{Z\max} \frac{\partial Y}{\partial \rho}(\lambda_i) \right|} dL \quad (4.9)$$

The key concept defining the proposed constraint is that the allowed change in parameter should be less than the minimum change in parameter $\Delta\rho_{\min}$ which could destabilise the mode if it takes the quickest path to destabilisation. The quickest path is that which requires the minimum time - or rather, the minimum- $\Delta\rho$ - to reach the imaginary axis.

Finding a minimum-time path in classical mechanics is usually solved with the calculus of variations. However, the likely form of $\frac{\partial Y}{\partial \rho}(s)$ will be too complex to admit an analytical solution. Therefore, a conservative analytical solution can be obtained by approximating the magnitude of $\frac{\partial Y}{\partial \rho}(s)$ in the neighbourhood of the mode with a simpler function $\frac{\partial Y}{\partial \rho}_{\text{app}}(s)$, as per Equation 4.10.

$$\frac{\partial Y}{\partial \rho}_{\text{app}}(s) \geq \left| \frac{\partial Y}{\partial \rho}(s) \right| \quad (4.10)$$

Finally, the proposed minimum- $\Delta\rho$ path constraint can be described with Equation 4.11.

- Constraint 3: the change in parameter should be smaller than the minimum required to reach the imaginary axis along the minimum- $\Delta\rho$ path, as shown in Equation 4.11.

$$\Delta\rho < \Delta\rho_{\min} = \frac{1}{R_{Z_{\max}}} \int_{P_{\min, \text{Im}}} \frac{1}{\left| \frac{\partial Y}{\partial \rho}_{\text{app}}(s) \right|} dL \quad \forall \lambda_i \quad (4.11)$$

4.5.1 Calculation of the maximum parameter step and minimum- $\Delta\rho$ path for the motivating case study

Appendix B derives the analytical solution for the minimum- $\Delta\rho$ path, utilising the calculus of variations and an approximation of $\left| \frac{\partial Y}{\partial \rho}(s) \right|$ in the form shown in Equation 4.12. For the case where $\frac{\partial Y}{\partial \rho}(s)$ is dominated by a pole as in the motivating case study, the approximation order n should be -1. The approximation is quite accurate in the neighbourhood of the pole, as is shown in Appendix B.

$$\frac{\partial Y}{\partial \rho}_{\text{app}}(s) = G|s - p_j|^n \quad (4.12)$$

The positions of the mode λ_i and the intercept of the minimum- $\Delta\rho$ path with the imaginary axis z_b are shown in terms of their distance and angle relative to from the open-loop pole p_j in Equations 4.13 and 4.14. The co-ordinates z_a and z_b are illustrated in Figure 4.5.

$$z_a = \lambda_i - p_j = r_a e^{j\theta_a} \quad (4.13)$$

$$z_b = r_b e^{j\theta_b} \quad (4.14)$$

With the co-ordinates described in Equations 4.13 and 4.14 the solutions for the minimum- $\Delta\rho$ required to traverse the minimum- $\Delta\rho$ path are shown in Equations 4.15 and 4.16. The full derivation can be found in Appendix B.

$$(1 - n)(\theta_b - \theta_a) < \pi :$$

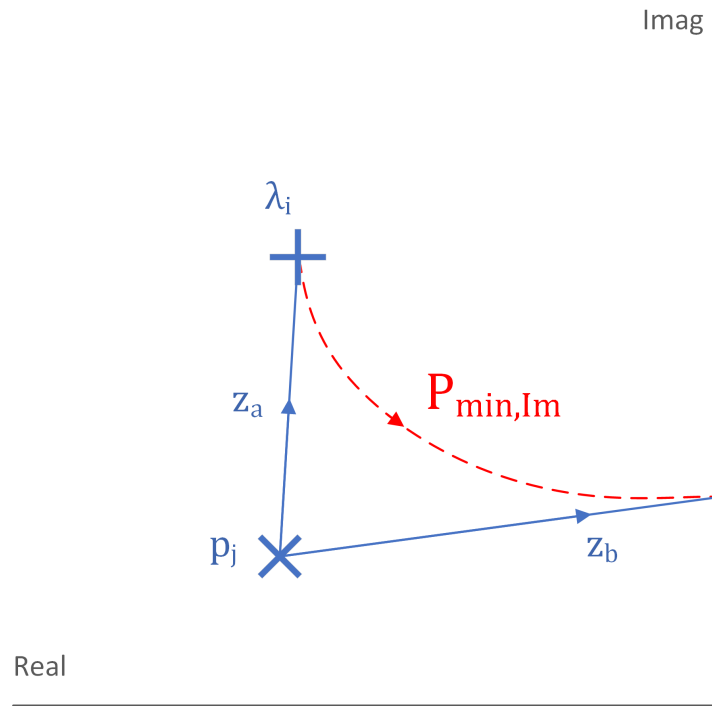


Figure 4.5: The co-ordinate system used in the calculation of the minimum- $\Delta\rho$ path: z_a represents the complex vector between the open-loop pole p_j and whole-system mode λ_i (shown in Equation 4.13), while z_b represents the complex vector between the pole and the intercept of the path of the mode and the imaginary axis (shown in Equation 4.14)

$$\Delta\rho_{\min} = \frac{1}{R_{Z\max}G(1-n)} \sqrt{r_a^{2(1-n)} + r_b^{2(1-n)} - 2r_a^{(1-n)}r_b^{(1-n)}\cos((1-n)(\theta_b - \theta_a))} \quad (4.15)$$

$$(1-n)(\theta_b - \theta_a) > \pi :$$

$$\Delta\rho_{\min} = \frac{1}{R_{Z\max}G(1-n)} \left(r_a^{(1-n)} + r_b^{(1-n)} \right) \quad (4.16)$$

In the case where $n = -1$, if the two points z_a and z_b are less than 90 degrees apart the $\cos(\cdot)$ term in Equation 4.15 results in a problem which cannot be solved analytically. The initial location of the mode is known so Equations 4.15 and 4.16 can be minimized numerically over the locations z_b on the imaginary axis. The calculation of the minimum- $\Delta\rho$ path constraint for the motivating case study is described in more detail in Appendix C.

The minimum- $\Delta\rho$ path for the motivating case study is plotted as Constraint 3 in Figure 4.6. It can be seen that since the two ends of the path are more than 90 degrees apart the result in Equation 4.16 is applied. In this case, the minimum- $\Delta\rho$ path is just the straight lines from the mode to the open-loop pole, to the imaginary axis. In this case the integral in Equation 4.7 for $\Delta\rho_{\min}$ can be easily found as the result in Equation 4.16.

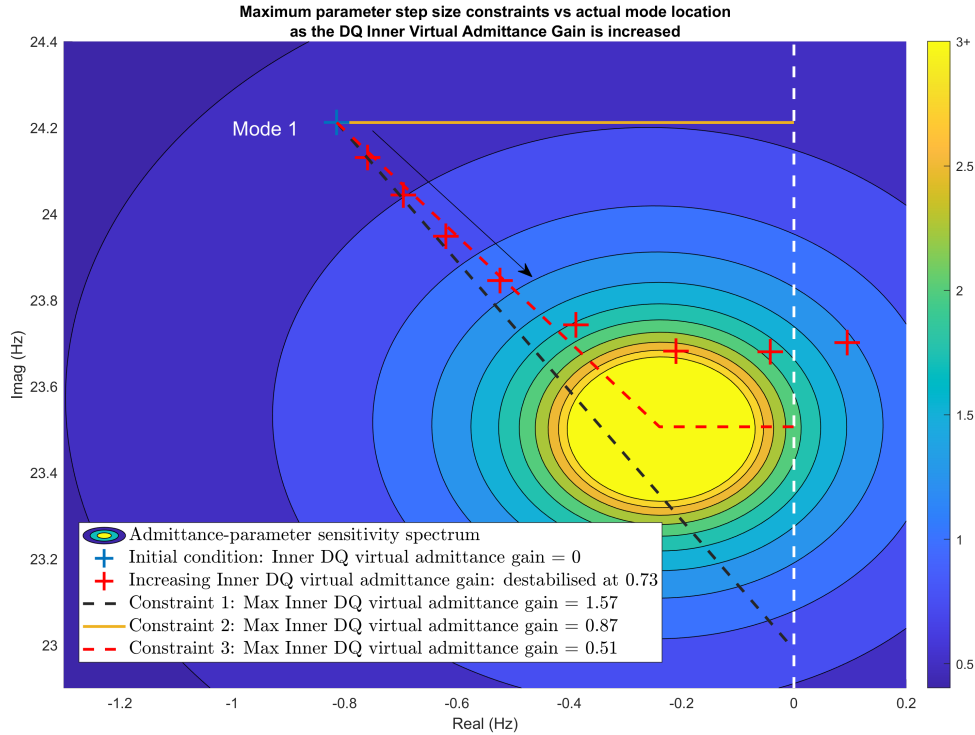


Figure 4.6: The contour in this plot shows the magnitude of the sensitivity of the active-damping IBR admittance to the change in virtual admittance gain. The actual movement in the complex plane of the SSO mode as the virtual admittance gain is increased is shown with red crosses. The proposed minimum- $\Delta\rho$ path constraint accounts for the fact that the sensitivity increases near the open-loop pole. The "quickest" path to the imaginary axis for the mode is mathematically shown to be through the open-loop pole. The minimum- $\Delta\rho$ path constraint allows the calculation of a reduced value for the maximum allowable change in parameter, which would prevent the mode being destabilised.

Constraint 3 is shown in Figure 4.6 to give a smaller maximum allowable $\Delta\rho$ than Constraint 1 or 2. The smaller change would prevent the mode being destabilised before the next residue measurement.

The proposed constraint could allow a sharp band-pass filter to be used in the virtual admittance with better certainty of stability after a change in parameter. This could be useful when it is desired to not impact any other poorly damped modes elsewhere in the complex plane. However, the addition of another poorly damped mode may unnecessarily complicate the parameter selection process.

In the application of all three constraints in Figure 4.6 the changing residue magnitude has not been taken into account, assuming that the maximum residue $R_{Z\max}$ is equal to the initial residue R_Z . A further safety margin to account for the possibility of the residue increasing should be applied, a topic on which further research is recommended.

4.5.2 Simplifications and limitations for other use cases

Equation 4.15 can be solved analytically if the complex vector z_b is aligned with the imaginary axis. Although it is unlikely that the active-damping IBR would be designed to have a pole on the imaginary axis there could be a zero on the imaginary axis. Indeed, zero magnitude of $\frac{\partial Y}{\partial \rho}(s)$ at $0 + j0$ Hz (in the synchronous frame) was a requirement placed on the eligible parameters for admittance shaping discussed in Chapter 3.4. In the case that $\frac{\partial Y}{\partial \rho}(s)$ is mainly shaped by a zero at $0 + j0$ in the neighbourhood of the mode under consideration the intercept z_b with the imaginary axis will have the constant intercept angle $\theta_b = \pi/2$. In this case Equation 4.15 can be minimised analytically for the expression in Equation 4.17. For a zero n will be positive and equivalent to the order of the roll-off towards $0 + j0$. The full derivation is given in Appendix B.

$$\Delta\rho_{\min} = \frac{|\lambda_i|^{1-n} \sin(1-n)(\theta_a + \pi/2)}{R_{Z\max} G(1-n)} \quad (4.17)$$

Equation 4.18 shows the relationship between the proposed minimum- $\Delta\rho$ path constraint and Constraint 2 for the case where $\frac{\partial Y}{\partial \rho}(s)$ is dominated by a zero at $0 + j0$. The proportional de-rating of the minimum- $\Delta\rho$ constraint compared to Constraint 2 is represented by $D(n, \theta_a)$ and its magnitude is plotted in Figure 4.7 for different orders of roll-off and up to a damping factor of 0.4. It is clear that the difference between the constraints is only substantial for higher orders of roll-off for modes with larger damping factors.

$$\Delta\rho_{\min} = \underbrace{\frac{|\Re\{\lambda_i\}|}{R_{Z\max} G |\lambda_i|^n}}_{\text{Constraint 2}} \underbrace{\frac{|\lambda_i| \sin(1-n)(\theta_a + \pi/2)}{(1-n) |\Re\{\lambda_i\}|}}_{D(n, \theta_0)} \quad (4.18)$$

Additionally, if the controllers in the active-damping IBR are tuned to be well-damped then the poles in $\frac{\partial Y}{\partial \rho}(s)$ are on/near the real axis. According to the maximum principle the magnitude of $\frac{\partial Y}{\partial \rho}(s)$ should increase as the damping factor of λ_i increases towards the open-loop poles. This phenomenon is shown in Figure 4.8 by the contour plot increasing in the negative real direction.

With poles mainly near the real axis constraint 2 will tend to be an overestimate of the minimum $\Delta\rho$ required to destabilise the mode since the magnitude of $\frac{\partial Y}{\partial \rho}(s)$ decreases as the

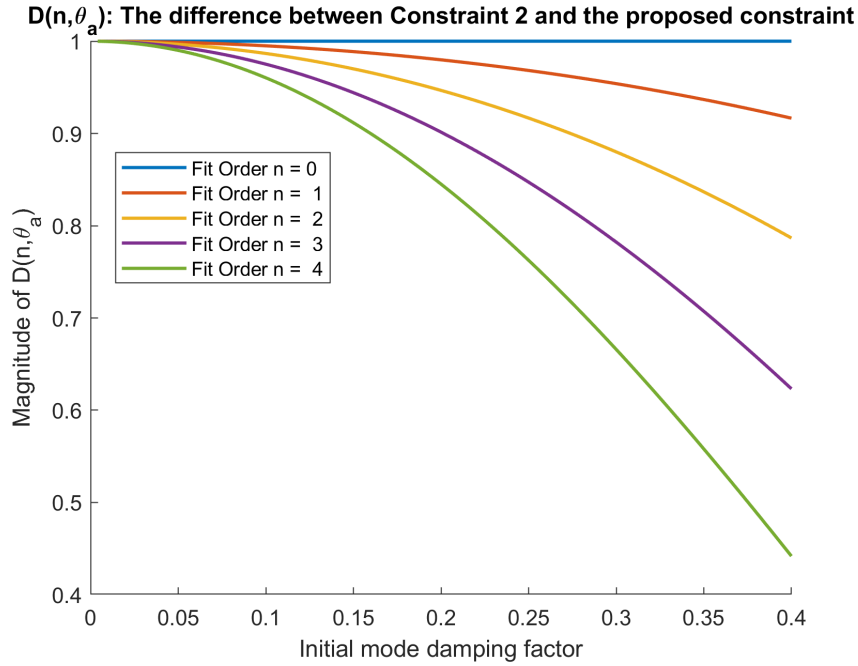


Figure 4.7: Illustration of the proportional difference between the proposed minimum- $\Delta\rho$ path constraint and the previously discussed Constraint 2 for a whole-system mode near a zero at $0 + j0$ of multiplicity n in the sensitivity $\frac{\partial Y}{\partial \rho}(s)$. The difference is represented mathematically by $D(n, \theta_a)$ in Equation 4.18.

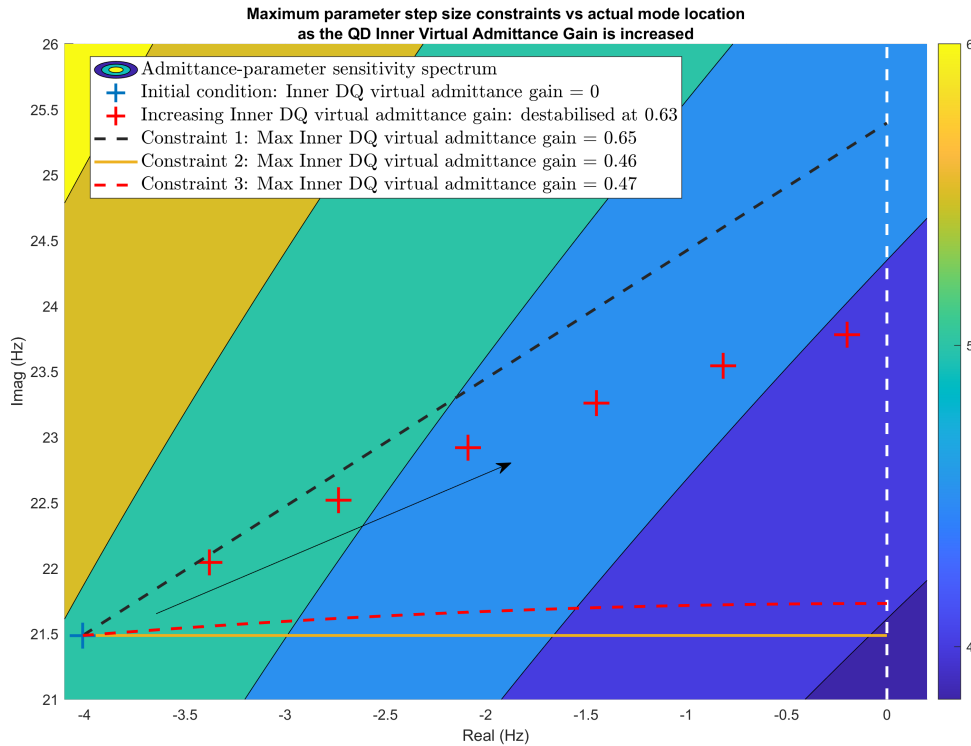


Figure 4.8: A contour plot of the sensitivity of the admittance to the DQ inner virtual admittance gain, without the band-pass filter. The change in the magnitude of the sensitivity in the neighbourhood of the whole-system mode is not significant enough for the minimum- $\Delta\rho$ path to be significantly different to the path assumed in Constraint 2, leading to the similar values for the maximum change in parameter

mode moves towards the imaginary axis. In this case, the value of further overestimating the minimum $\Delta\rho$ required by correcting for the increase in magnitude of $\frac{\partial Y}{\partial \rho}(s)$ as the mode frequency increases away from the zero may be limited.

In the case of a zero with low multiplicity the minimum- $\Delta\rho$ path constraint will mostly be similar to Constraint 2 (as shown in Figure 4.7). Additionally, Constraint 2 should be an overestimate anyway (as demonstrated in Figure 4.8). Due to these two factors the value of the proposed minimum- $\Delta\rho$ path constraint may be limited to situations where the whole-system mode is close to an area of $\frac{\partial Y}{\partial \rho}(s)$ where it substantially increases in magnitude, such as an open-loop pole.

4.6 Information sharing and the multiple quadrant/parameter scenario

If one parameter affects multiple quadrants the proposed constraint can still be applied. The core of the grey-box approach is using the residue-weighted sum of the changes in the admittance quadrants, according to Equation 4.19.

The equivalent shape of $\frac{\partial Y}{\partial \rho}(s)$ to approximate in the minimum- $\Delta\rho$ path constraint can be determined with an "equivalent weighted admittance sensitivity" $\partial Y_{eq}(s)/\partial \rho$, which is the sum of the individual quadrants of $\frac{\partial Y}{\partial \rho}(s)$ weighted by the relevant residue magnitude, as shown in Equation 4.20.

$$\Delta\lambda_i \approx - \left(R_{ZZ,dd} \frac{\partial Y_{DD}(\lambda_i)}{\partial \rho} + R_{ZZ,qd} \frac{\partial Y_{DQ}(\lambda_i)}{\partial \rho} + R_{ZZ,dq} \frac{\partial Y_{QD}(\lambda_i)}{\partial \rho} + R_{ZZ,qq} \frac{\partial Y_{QQ}(\lambda_i)}{\partial \rho} \right) \Delta\rho \quad (4.19)$$

$$\frac{\partial Y_{eq}(s)}{\partial \rho} = \left| R_{ZZ,dd} \frac{\partial Y_{DD}(s)}{\partial \rho} \right| + \left| R_{ZZ,qd} \frac{\partial Y_{DQ}(s)}{\partial \rho} \right| + \left| R_{ZZ,dq} \frac{\partial Y_{QD}(s)}{\partial \rho} \right| + \left| R_{ZZ,qq} \frac{\partial Y_{QQ}(s)}{\partial \rho} \right| \quad (4.20)$$

$$(4.21)$$

To calculate the minimum- $\Delta\rho$ path constraint with the change of multiple parameters simultaneously is also straightforward to implement with the use of a "virtual parameter" α . Here, the virtual parameter simply determines the ratios of the changes in each actual parameter, as their effect together is equivalent to that of changing only the virtual parameter.

The total effect of the virtual parameters can be obtained with the sums over each admittance quadrant and over each parameter ρ_k and for each its proportion relative to the virtual parameter $\Delta\rho_k/\Delta\alpha$. The sum is shown in Equation 4.22.

$$\begin{aligned} \frac{\partial Y_{eq}(s)}{\partial \alpha} = & |R_{ZZ,dd}| \sum_{\rho_k} \left| \frac{\partial Y_{DD}(s)}{\partial \rho_k} \right| \frac{\Delta\rho_k}{\Delta\alpha} + |R_{ZZ,qd}| \sum_{\rho_k} \left| \frac{\partial Y_{DQ}(s)}{\partial \rho_k} \right| \frac{\Delta\rho_k}{\Delta\alpha} \\ & + |R_{ZZ,dq}| \sum_{\rho_k} \left| \frac{\partial Y_{QD}(s)}{\partial \rho_k} \right| \frac{\Delta\rho_k}{\Delta\alpha} + |R_{ZZ,qq}| \sum_{\rho_k} \left| \frac{\partial Y_{QQ}(s)}{\partial \rho_k} \right| \frac{\Delta\rho_k}{\Delta\alpha} \end{aligned} \quad (4.22)$$

The proportions $\Delta\rho_k/\Delta\alpha$ requires the knowledge of the magnitudes in the change of each parameter relative to each other, which is a result of the parameter choice optimisation and will therefore be difficult to use as a constraint on the optimisation.

As a workaround, the minimum- $\Delta\rho$ path constraint can be calculated for each parameter individually first to give initial constraints on the allowed deviations in parameters, then the IBR operator can check that the optimised set of parameter changes fulfill the multi-parameter minimum- $\Delta\rho$ constraint before implementing them. If the proposed change in parameter violates the minimum- $\Delta\rho$ constraint then IBR operator should reduce the individual constraints on the allowed deviations in parameters and the ESO should re-calculate the optimal parameter choice.

The IBR operator calculating the minimum- $\Delta\rho$ path constraint would remove the need to share the approximation of the sensitivity function $\partial Y_{eq}(s)/\partial \alpha$ with the ESO. Sharing the function would be more revealing than just the sensitivities evaluated at the mode values $\frac{\partial Y}{\partial \rho}(\lambda_i)$, as is required for the grey-box approach. However, the ESO would have less oversight of the stability calculations.

This data would come from knowledge of the IBR analytical model, ideally validated with measurements. The data may be beyond the IBR operator's capability to produce so may have to be requested from the IBR manufacturer. Again, the sensitivity spectrum requires less information than the analytical model which could improve the willingness of the manufacturer to share it, although legislation/compensation may be required in order to convince the manufacturer to share the information. The actual parameters could also be anonymised to further reduce any insight gained into the inner working of the IBR - eg. there are alterable parameters

'A' and 'B' which have these certain spectrums of admittance-parameter sensitivities which can be shared with the ESO.

Figure 4.9 illustrates a possible configuration of a parameter optimisation process which utilises the minimum- $\Delta\rho$ path constraint in the manner discussed.

4.7 Chapter Conclusion

This chapter has presented a novel constraint for the allowable range of changes in IBR parameters to ensure small-signal stability of the system is maintained. The constraint accounts for the changes that occur in the angle of the residue and changes in the admittance-parameter sensitivity in the neighbourhood of a whole-system mode. The calculus of variations was utilised to calculate the path which would destabilise the mode with the smallest change in parameter and those smallest changes are taken as the bounds on the parameter change.

The importance was highlighted of considering the presence of open-loop poles, particularly those with a small real part, in the admittance-parameter sensitivity. Such an open-loop pole was shown to provide the whole-system mode a "high-speed path" to destabilisation, the effect of which was not previously taken into account when predicting the maximum deviation of the mode with the grey-box relationship applied in a linear manner.

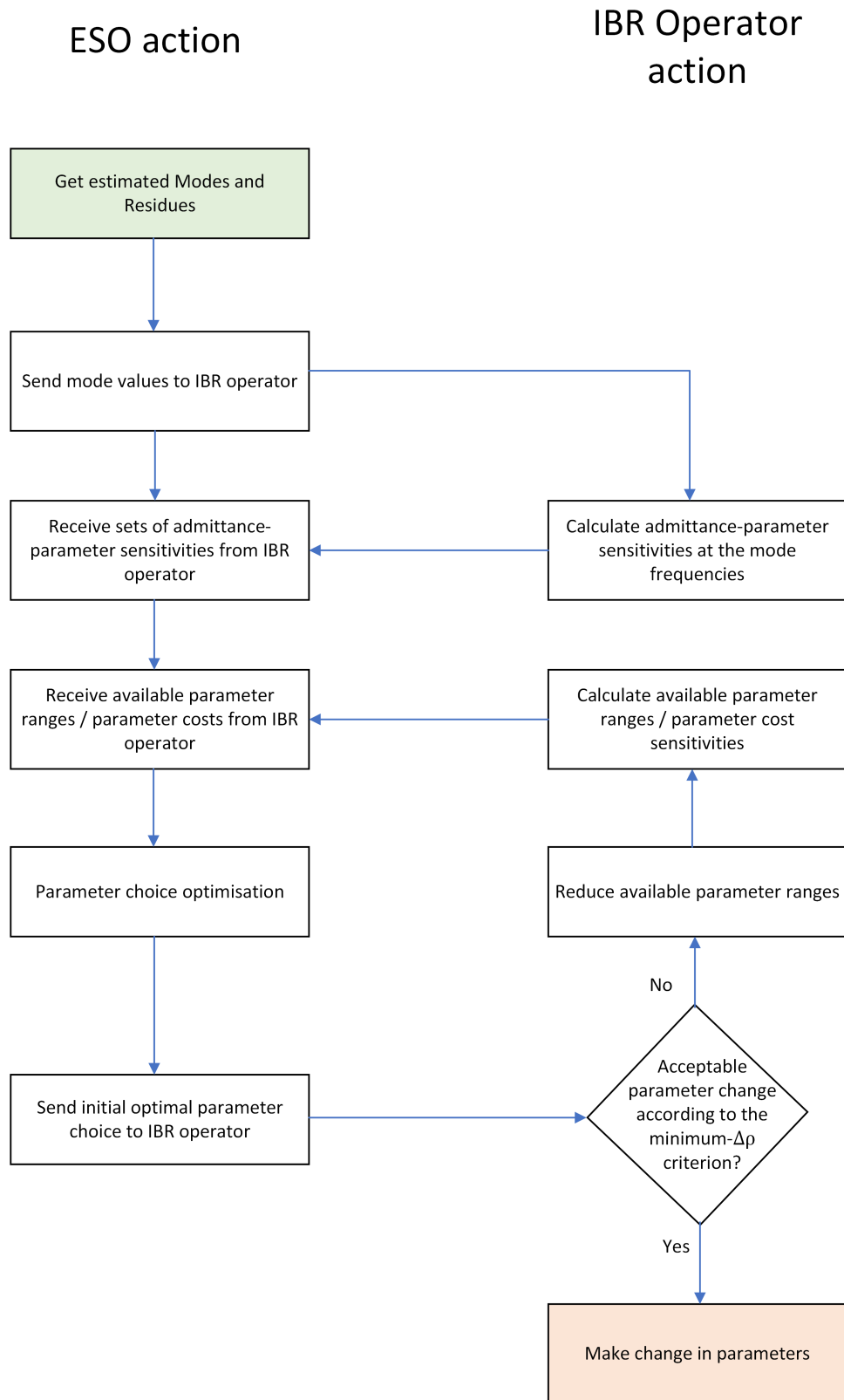


Figure 4.9: The proposed algorithm for performing the parameter choice optimisation, assuming the modes and residues have already been identified. The minimum- $\Delta\rho$ path constraint is utilised to warn for potential instabilities in the case of variations in the admittance-parameter sensitivity in the neighbourhood of each mode

5 Constrained vector fitting

5.1 Chapter Introduction

This chapter presents a study on the effects of measurement noise on the application of vector fitting to extract the whole-system residues from frequency response data. A novel enhancement - called "constrained vector fitting" - is proposed to improve the accuracy of the identified parameters.

In what follows, first, the modelling assumptions and notation are set out. Then, the role that vector fitting has within the proposed impedance shaping method is described and motivations are given for improving the accuracy of the estimated residues.

The algorithm "standard" vector fitting uses to estimate modes and residues is laid out in detail and the enhancements required for constrained vector fitting are described. Finally the results for constrained vector fitting are presented and it is shown that for the case where there is noise present in the whole-system impedance and admittance data, it is possible to improve the accuracy of the estimated residues to a modest degree by implementing the proposed constrained vector fitting.

5.2 Modelling assumptions and notations

The system identification methods discussed in this chapter utilize frequency-domain data, which is also the case for other impedance-based analysis techniques as was discussed in Chapter 1.

The literature review in Chapter 2.2 concludes that measured frequency domain data consists of contributions from the closest linear representation $y^{\text{true}}(j\omega_i)$ plus components from nonlinear distortions $y^{\text{non}}(j\omega_i)$ and measurement noise $n(j\omega_i)$ [111], as shown in Equation 5.1.

$$y(j\omega_i) = y^{\text{true}}(j\omega_i) + y^{\text{non}}(j\omega_i) + n(j\omega_i) \quad (5.1)$$

Obtaining frequency-domain data from time-domain measurements is commonly performed with the Discrete-Fourier Transform (DFT) which is a very well studied process.

The scope of this work is to enhance impedance-based techniques and not to study the details of the time-domain to frequency domain transformation. Non-linear distortions $y^{\text{non}}(j\omega_i)$ are produced by the time-domain to frequency-domain transformation, since the time-domain measurements are produced by the real non-linear system and the frequency domain model is linear. It is therefore assumed here that the non-linear distortions are negligible as shown in Equation 5.2. Negligible non-linear distortions would require a small enough input such that the small-signal approximation is of good accuracy. With a smaller input, however, the impact of noise may be more severe.

$$y(j\omega_i) \approx y^{\text{true}}(j\omega_i) + n(j\omega_i) \quad (5.2)$$

Neglecting the impact of non-linear distortions allows the test frequency-response data to be generated with a linearised model (in this case obtained from the Simplus open-source toolbox [57]). As described in Chapter 2.2, the impact of noise (assumed to be white noise) is modelled by adding a complex number generated by two normal distributions with identical variance across the frequency spectrum. The noise variance is hereafter referred to with the symbol σ^2 .

$$n(j\omega_i) = \mathcal{N}(0, \sigma^2) + j\mathcal{N}(0, \sigma^2) \quad (5.3)$$

As was discussed in Chapter 2.3, the white noise assumption is commonly made in the literature when there is not data available on the likely frequency spectrum of the measurement noise. When making the frequency sweep measurements discussed in this chapter, it is possible that some information would be available to refine the model of the noise beyond the white noise model. For example, as discussed in [131] there could be more noise at certain frequencies due to harmonic sources or network resonances.

In the case of noise magnitude varying across the frequency spectrum, the vector fitting weighting function can be scaled in inverse proportion to the noise variance frequency spectrum in order for the effective impact of the noise to be uniform across the frequency spectrum [111]. In this manner, application-specific knowledge can be utilized to negate the disproportionate impact of eg. harmonic sources on certain frequencies in the system identification process. The

weighting function is described in Equation 5.4 in Chapter 5.3.

The fit order N_O is the number of modes present in the identified model. The frequency samples measured are represented by ω and the total number of samples by N_D .

In general, vectors and matrices will be indicated with bold font. Most variables in this chapter are complex so complex variables will not be highlighted with notation. Residues will be represented with an R , where the sub-script will give further information as to which function they belong to.

The term 'channel' is used as a general term to refer to a quadrant of frequency response data, particularly when there is no need to differentiate between whole-system admittance or whole-system impedance frequency response data.

5.3 Background: The application of system identification in the proposed impedance-shaping process

The estimation of modes and residues from frequency response measurement data is critical to the proposed impedance-shaping process outlined in Chapter 1 and illustrated in Figure 1.4. The system identification algorithm known as "vector fitting" is used for the parameter estimation, since it is open-source and widely used for frequency-domain transfer function identification [174].

Vector fitting is described in more detail in Chapter 5.5.1, but its high-level function can be described as minimising the RMS value of the difference between the frequency domain data $y(j\omega_i)$ and that of the response of the identified model at the same frequencies $\hat{y}(\theta, j\omega_i)$. The unknown model parameters are represented by θ and the optimal choice of the model parameters selected by vector fitting are represented by θ^* . The weighting of each frequency sample in the overall fitting process can be tuned with the parameters W_i to account for non-uniform noise variance. In this work, a uniform noise variance and uniform weighting $W_i = 1$ is used.

$$\theta^* = \arg \min_{\theta} \sqrt{\sum_i^{N_D} W_i |y(j\omega_i) - \hat{y}(\theta, j\omega_i)|^2} \quad (5.4)$$

The difference between the model output and the data is commonly described as the "resid-

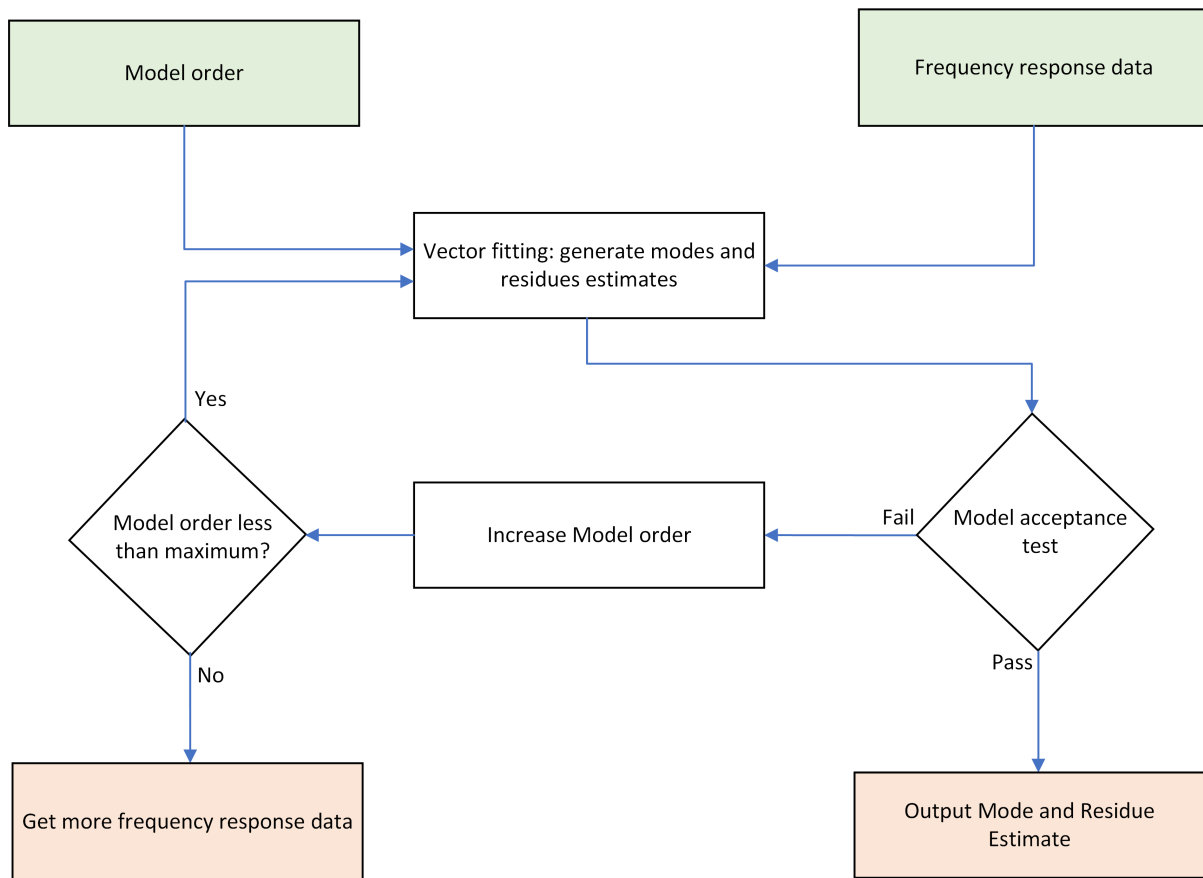


Figure 5.1: The process for applying vector fitting given there is frequency response data available and an initial model order choice (generally the same as the number of peaks in the data [120]). The process inputs are shown in green, whole the outputs are shown with red. The model acceptance test is described with Equation 5.5. The model order is increased until the acceptance test is passed, or more frequency response data needs to be obtained

ual” in system identification literature [111,175], but to avoid confusion with residues of modes, it will be referred to here as the ”fit error” in this work. The fit error is the only gauge the user has of the quality of the identified model.

The process of applying vector fitting to estimate the parameters is illustrated in Figure 5.1. Given a set of frequency response measurements, the user increases the model order used until the fit error is sufficiently small. The number of frequency samples represents the maximum possible model order due to the numerical properties of the vector fitting algorithm.

Noisy data constrains the minimum possible value of the fit error. Chapter 9.4.2 of [111] uses the known variance of the noise to calculate whether there are any model errors present. If the RMS value of the fit error is within the magnitude expected for the known noise variance then the model is as accurate as can be expected. If there is larger than expected fit error RMS, then there could be model errors present.

The noise is assumed to be normally distributed so the expected maximum RMS value of N_D data points can be found with the cumulative Chi-square distribution in Equation 5.5. The expected maximum is given for the 99th percentile probability. Since every data point has a real and an imaginary component, there are $2N_D$ degrees of freedom in the Chi-square distribution. Equation 5.5 only holds if the noise variance σ is constant across the frequency spectrum and a uniform weighting $W_i = 1$ is applied in vector fitting process, both of which are the case in this chapter.

$$\sqrt{\frac{\sum_i^{N_D} |y(j\omega_i) - \hat{y}(\boldsymbol{\theta}, j\omega_i)|^2}{N_D}} \leq \frac{\sigma \chi^2(0.99, 2N_D)}{\sqrt{N_D}} \quad (5.5)$$

Equation 5.5 is used as the model acceptance test in the system identification process illustrated in Figure 5.1. If the model does not pass the acceptance test, then the model order should be increased or more frequency response data should be obtained, as illustrated in Figure 5.1.

Once an acceptable model is obtained the estimated modes and residues can be utilised in the next stage of the proposed impedance-shaping process, as illustrated in Figure 1.4.

All parameter estimation results presented in this chapter come from identified models which have passed the acceptance test in Equation 5.5.

5.4 Motivation for improving the parameter identification accuracy

For the proposed impedance-shaping process, the estimated mode and residue values are used to select and guide changes in parameters to improve the small-signal stability of the system. The accuracy of the identified parameters is of high importance so as to avoid degrading rather than enhancing stability.

Vector fitting is generally very effective at estimating the mode and residue values of very poorly damped modes. Figure 5.2a shows the whole-system impedance and admittance frequency responses for the IEEE 14-bus system, which was introduced in Chapter 3. The frequency responses are those evaluated at bus 11, the location of the IBR which is to shape its admittance to improve the damping of the SSO mode. Noise of standard deviation 0.01 p.u. has been added to the data. The presence of SSO mode 1 is clearly visible in the peak of the

whole-system impedance at a frequency a little less than 20 Hz.

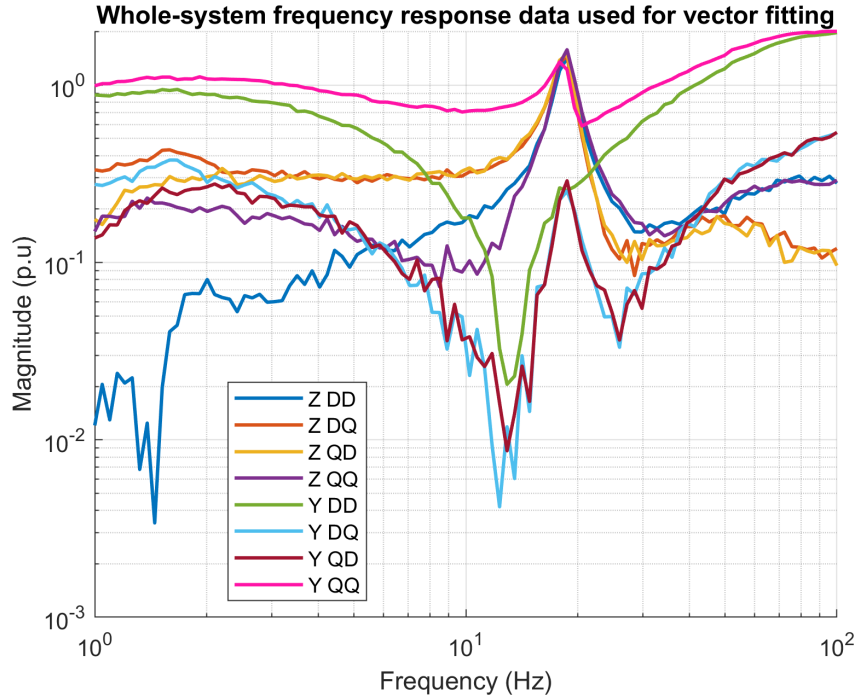
Figure 5.2b shows the results of the application of vector fitting on the frequency response data in Figure 5.2a. It can be seen that the mode is identified (blue circle) accurately with respect to the actual position (the black '+' symbol) at $-1+j18.5$ Hz. Additionally, Figure 5.2b shows the exact predicted location (black triangle) after increasing the DD and QQ outer virtual impedance gains from 0 to 2 p.u. The exact predicted location is shown for the true residue values generated with the linearised model of the 14-bus system. The true residue value does not predict the next location of the mode (the pink '+' sign) exactly. It is slightly inaccurate because the grey-box relationship is a linearised sensitivity and changes as the parameters change.

Also shown in Figure 5.2b are the predicted locations based on the whole-system impedance and the whole-system admittance residues estimated with vector fitting. It can be seen that the next predicted mode location is close to the exact grey-box prediction, which is the best prediction possible. Although there is noise present in the data, clearly the frequency response data is of sufficiently high quality for the application of the grey-box method to improve the system small-signal stability.

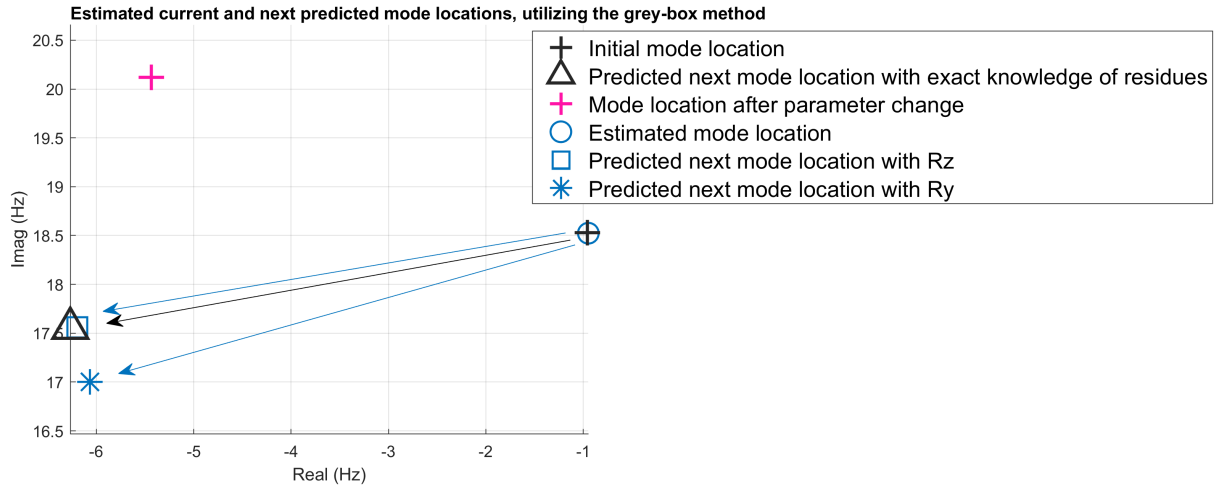
A more accurate exact prediction could be obtained by increasing the parameters in smaller steps as shown in Figures 3.9 and 3.13 in Chapter 3. In practice smaller steps would likely be taken, however the plots used in this chapter are for illustration of the properties of the parameter identification so only one step is shown for clarity.

Figure 5.3a shows the frequency response data obtained after increasing the DD and QQ outer virtual impedance gains from 0 to 2 p.u. Random noise of the same standard deviation 0.01 p.u. has again been added to the data. The peak in the data has been flattened, indicating that the damping of the SSO mode has been improved.

Although the improvement in damping obtained may already be sufficient, it is still useful to know whether there are any further changes in parameters that could be made to improve the small-signal stability further. Figure 5.3b shows the current estimated location of the mode and predicted next location for an increase in the DQ outer virtual admittance gain from 0 to 1 p.u. This time, the mode position is estimated with much poorer accuracy. Additionally, the next predicted location utilizing the estimated whole-system impedance residues is very inaccurate. If only the whole-system impedance were measured, then the estimated parameters



(a) The whole-system frequency response data used for the prediction of the next mode location shown in (b). The legend indicates the quadrant of the data, where a "Z" indicates whole-system impedance and a "Y" indicates whole-system admittance. Normally distributed random noise of standard deviation 0.01 p.u. has been added to frequency response data generated by the linearised model of the IEEE 14 bus system. There are 100 frequency sample points for each quadrant, spaced logarithmically between 1 and 100 Hz.



(b) The results of vector fitting applied to the data shown in (a) - it can be seen that the mode estimate is very accurate. The next predicted mode is plotted with application of the grey-box method for the cases: (i) triangle - if the whole-system residues were known exactly, (ii) square - utilising the whole-system impedance residues estimated with vector fitting applied to the data in (a), (iii) asterisk - utilising the whole-system admittance residues.

Figure 5.2: The prediction of the next location of the SSO mode in the IEEE 14-bus system after the increase of both the virtual admittance DD and QQ gains from 0 to 2 p.u.

could recommend a change in parameter which degrades the damping of the SSO mode rather than improving it.

The most obvious solution is to take more measurements before a change in parameter can be recommended. However, it is still worth making the best use of the finite information available. In particular, the discrepancy between next predicted locations with the whole-system admittance and impedance residues (when they should be identical) indicates that there has been some error in the estimation. The knowledge that both types of residues should predict the same location for the mode is an additional piece of information available to the user which is not being utilized in the system identification process. The utilization of this missed information will be the topic covered in the rest of this chapter.

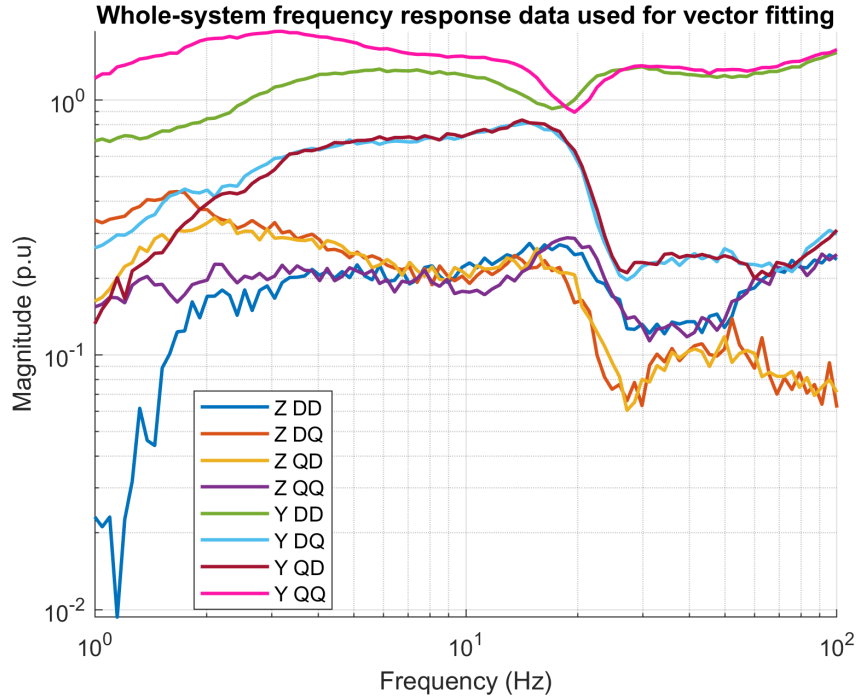
5.5 Constrained vector fitting: physics-informed improvement to estimated residue accuracy

The premise of the constrained vector fitting approach is the grey-box relationship shown in Equations 5.6 and 5.7. Equation 5.6 describes the grey-box relationship utilising the whole-system impedance residue and the sensitivity of the apparatus admittance to a change in parameter. The predicted mode locations utilizing Equation 5.6 are plotted with square markers in Figures 5.2b, 5.3b and 5.6.

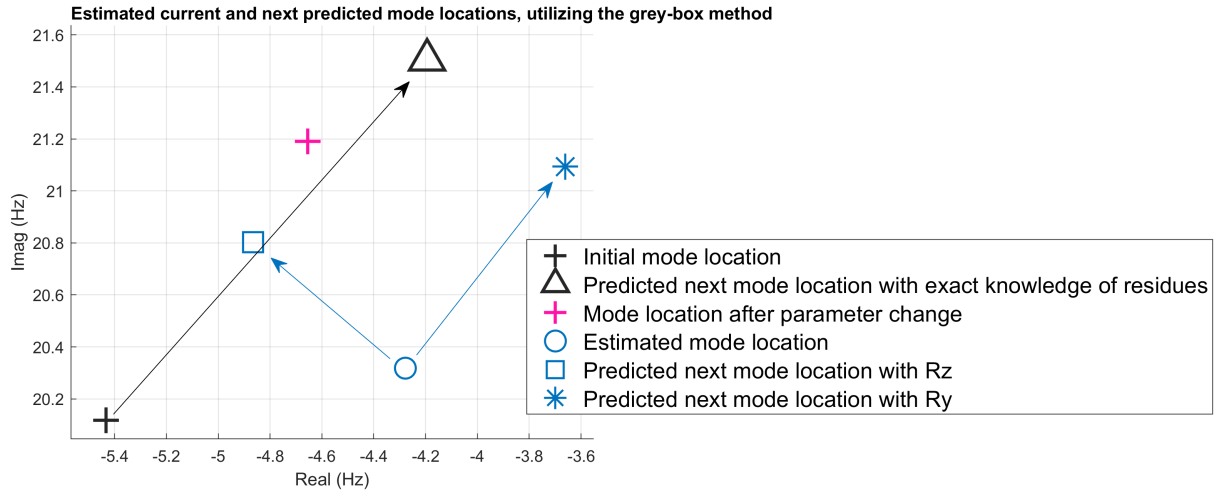
Equation 5.7 shows the grey-box relationship utilizing the whole-system admittance residue and the apparatus impedance sensitivity to the same change in parameter. The terms R_Z and R_Y indicate the whole-system impedance and admittance residues, while the terms $\frac{\partial Y}{\partial \rho}(\lambda_i)$ and $\frac{\partial Z}{\partial \rho}(\lambda_i)$ are the sensitivities of the apparatus admittance and impedance to the change in parameter. The apparatus in question is the one connected at the bus where the current/voltage injections are being made to measure the whole-system frequency responses. The predicted mode locations utilizing Equation 5.6 are plotted with asterisk markers in Figures 5.2b, 5.3b and 5.6.

$$\frac{\partial \lambda_i}{\partial \rho} = -R_Z \frac{\partial Y}{\partial \rho}(\lambda_i) \quad (5.6)$$

$$\frac{\partial \lambda_i}{\partial \rho} = -R_Y \frac{\partial Z}{\partial \rho}(\lambda_i) \quad (5.7)$$



(a) The whole-system frequency response data used for the prediction of the next mode location shown in (b). The legend indicates the quadrant of the data, where a "Z" indicates whole-system impedance and a "Y" indicates whole-system admittance. Normally distributed random noise of standard deviation 0.01 p.u. has been added to frequency response data generated by the linearised model of the IEEE 14 bus system. There are 100 frequency sample points for each quadrant, spaced logarithmically between 1 and 100 Hz.



(b) The results of vector fitting applied to the data shown in (a) - it can be seen that the mode estimate is much less accurate than was shown in Figure 5.2b. The next predicted mode is plotted with application of the grey-box method for the cases: (i) triangle - if the whole-system residues were known exactly, (ii) square - utilising the whole-system impedance residues estimated with vector fitting applied to the data in (a), (iii) asterisk - utilising the estimated whole-system admittance residues.

Figure 5.3: The prediction of the next location of the SSO mode in the IEEE 14-bus system after the increase of the DQ virtual admittance gain from 0 to 1 p.u. (the DD and QQ virtual admittance gains are already both at 2 p.u., as was considered in Figure 5.2

Previously, the only way to obtain information regarding the whole-system impedance residue would be to measure the whole-system impedance itself. However, Equations 5.6 and 5.7 can be re-arranged for a novel observation: the whole-system impedance and admittance residues are not independent. They are linked by the value of the apparatus admittance. The apparatus in question is the one connected at the bus where the current/voltage injections are being made to measure the whole-system frequency responses. In the context of the proposed impedance-shaping method discussed in Chapter 1 the apparatus would be the active damping IBR which is being utilized to improve the damping of whole-system modes.

$$-R_Z \frac{\partial Y}{\partial \rho}(\lambda_i) = -R_Y \frac{\partial Z}{\partial \rho}(\lambda_i) \quad (5.8)$$

$$R_Z \frac{\partial Y}{\partial \rho}(\lambda_i) = R_Y \frac{\partial Z}{\partial Y}(\lambda_i) \frac{\partial Y}{\partial \rho}(\lambda_i) \quad (5.9)$$

$$R_Z = R_Y \frac{\partial}{\partial Y} \{Y^{-1}\}(\lambda_i) \quad (5.10)$$

$$R_Z = -Y(\lambda_i)^{-2} R_Y \quad (5.11)$$

Estimates of the whole-system impedance and whole-system admittance residues have been shown to be potentially inaccurate if the measurement data is noisy. Equation 5.11 mathematically describes the requirement that the grey-box method utilizing the whole-system impedance residue (Equation 5.6) or the whole-system admittance (Equation 5.7) should give the same predicted change in mode location.

The requirement in Equation 5.11 can be enforced if the apparatus admittance $Y(\lambda_i)$ is known by only allowing vector fitting to estimate values of the residues which satisfy Equation 5.11. Two different sets of measurements can therefore be used to estimate one underlying set of residues, which should improve the accuracy of the results.

Equation 5.11 can be extended to three-phase systems in Equation 5.12, where again a linear relationship between the whole-system impedance and the whole-system admittance residues is obtained. The notation (λ_i) in the apparatus admittance terms is now dropped for brevity so that all admittance terms are evaluated at $s = \lambda_i$. The notation of the residues indicate that for example R_{Z,DD,λ_i} represents the DD whole-system impedance residue associated with the mode λ_i , whereas R_{Y,QQ,λ_i} represents the QQ whole-system admittance residue.

$$\begin{bmatrix} R_{Z,DD,\lambda_i} \\ R_{Z,DQ,\lambda_i} \\ R_{Z,QD,\lambda_i} \\ R_{Z,QQ,\lambda_i} \end{bmatrix} = \frac{1}{(Y_{DD}Y_{QQ} - Y_{DQ}Y_{QD})^2} \begin{bmatrix} -Y_{QQ}^2 & Y_{QD}Y_{QQ} & Y_{DQ}Y_{QQ} & -Y_{DQ}Y_{QD} \\ Y_{DQ}Y_{QQ} & -Y_{DD}Y_{QQ} & -Y_{DQ}^2 & Y_{DD}Y_{DQ} \\ Y_{QD}Y_{QQ} & -Y_{QD}^2 & -Y_{DD}Y_{QD} & Y_{DD}Y_{QD} \\ -Y_{DQ}Y_{QD} & Y_{DD}Y_{QD} & Y_{DD}Y_{DQ} & -Y_{DD}^2 \end{bmatrix} \begin{bmatrix} R_{Y,DD,\lambda_i} \\ R_{Y,DQ,\lambda_i} \\ R_{Y,QD,\lambda_i} \\ R_{Y,QQ,\lambda_i} \end{bmatrix} \quad (5.12)$$

The rest of this section will describe in more detail the incorporation of the constraint in Equation 5.12 into the vector fitting algorithm. First, the standard vector fitting algorithm is briefly described. Descriptions can also be found in the papers describing the method [120, 122, 176] or on the open-source code repository [121], but the description is laid out here for an explanation more specific to the whole-system residue estimation problem.

Subsequently to the description of the standard vector fitting algorithm, a novel method is described for incorporating the constraint. Two variants of the novel method are proposed as two 'levels' of constrained vector fitting.

5.5.1 Description of standard vector fitting

The aim of vector fitting is to estimate the modes and the residues of a transfer function. $\hat{Y}_{DD}(s)$ is used as an example transfer function in Equation 5.13.

$$\hat{Y}_{DD}(s) = \sum_i \frac{R_{YDD,i}}{s - \lambda_i} \quad (5.13)$$

Vector fitting requires an initial guess of the mode locations, the number of which sets the fit order N_O . The set of the initial mode estimates is denoted by $\hat{\lambda}$.

Vector fitting is usually applied in two steps, illustrated in Figure 5.4:

1. Mode estimation: given the initial guess of the modes $\hat{\lambda}$, calculate a new estimate. Iterate until the solution has converged.
2. Residue estimation: Use the final mode estimate to estimate the residue values.

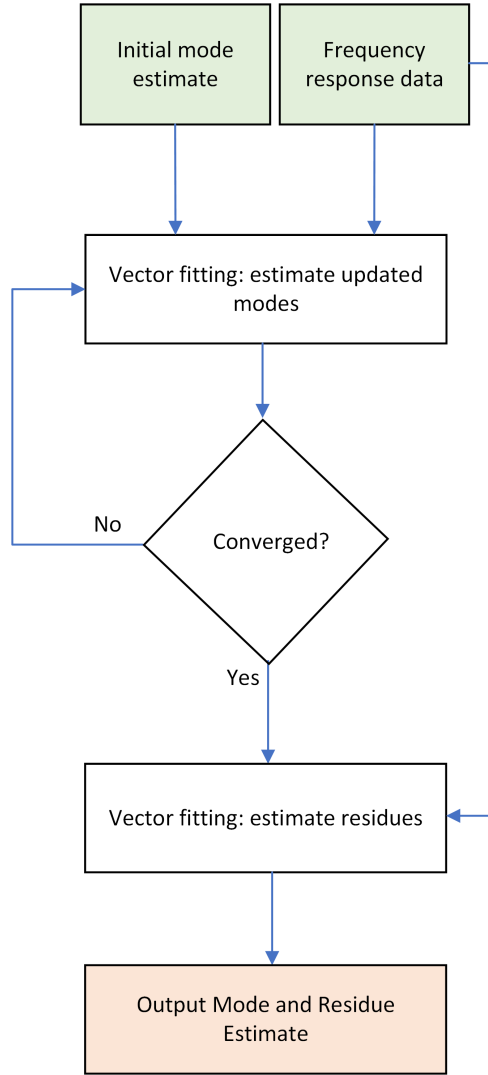


Figure 5.4: The internal processes in vector fitting: given an initial estimate of the modes and the frequency response data the modes are found by solving the mode estimation problem shown in Equations 5.20 and 5.25. The updated mode estimate is then used as the initial mode estimate and the process iterates until a solution is converged upon. The final step is estimating the residues by solving the problem in Equation 5.26. The inputs to the vector fitting process are indicated in green, while the outputs are indicated in red.

Once the mode and residue estimates are complete, the fit error can be calculated to check if it is sufficiently small to pass the acceptance test as previously described in Equation 5.5.

To improve the numerical conditioning of the problem the mode estimation uses two intermediate functions: $f(s)$ shown in Equation 5.14 and the function $g_{YDD}(s)$ shown in Equation 5.15. Both functions have known poles equal to the initial estimate of the modes but unknown residues

$$f(s) = \sum_i^{N_O} \frac{R_{fi}}{s - \hat{\lambda}_i} + d_f = \frac{\prod_i (s - z_{f,i})}{\prod_i (s - \hat{\lambda}_i)} \quad (5.14)$$

$$g_{YDD}(s) = f(s)\hat{Y}_{DD}(s) = \sum_i^{N_O} \frac{R_{gYDD,i}}{s - \hat{\lambda}_i} = \frac{\prod_i (s - z_i)}{\prod_i (s - \hat{\lambda}_i)} \quad (5.15)$$

Solving for the zeros, $z_{f,i}$, of $f(s)$ is equivalent to solving for the poles of $\hat{Y}_{DD}(s)$. This is shown in Equation 5.16, since the zeros $z_{f,i}$ become the denominator of $\hat{Y}_{DD}(s)$ when it is expressed in pole-zero form.

$$\hat{Y}_{DD}(s) = \frac{g_{YDD}(s)}{f(s)} = \frac{\prod_i (s - z_i)}{\prod_i (s - z_{f,i})} \quad (5.16)$$

The zeros $z_{f,i}$ can be found from the vector of residues $\mathbf{R}_f$ of $f(s)$ with Equation 5.17, where $\mathbf{1}$ is a column of ones of the appropriate size. The updated estimate of the modes $\hat{\boldsymbol{\lambda}}_{\text{new}}$ is set as the zeros of $f(s)$.

$$\hat{\boldsymbol{\lambda}}_{\text{new}} = \mathbf{z}_f = \text{eig}\{\text{diag}\{\hat{\boldsymbol{\lambda}}\} - \mathbf{1}d_f^{-1}\mathbf{R}_f^T\} \quad (5.17)$$

The unknown parameters in $f(s)$ are combined together in a vector $\boldsymbol{\rho}_f$ in Equation 5.18. Only the parameters in the vector $\boldsymbol{\rho}_f$ are of interest to find the modes [176], as was shown in Equation 5.17 (since the vector of mode estimates $\hat{\boldsymbol{\lambda}}$ is already known).

$$\boldsymbol{\rho}_f = \begin{bmatrix} \mathbf{R}_f \\ d_f \end{bmatrix} \quad (5.18)$$

The parameter vector $\boldsymbol{\rho}_f$ is found by solving Equation 5.19 in a least-squares manner over every frequency sample point and every channel of data.

$$g_{YDD}(s) - \hat{Y}_{DD}(s)f(s) = 0 \quad (5.19)$$

Equation 5.20 shows the formation of the problem in Equation 5.19 with the unknown

parameters separated from the measured frequency response data. The matrices ψ and ψ_f are used to build the frequency response of the functions with one row for each frequency response sample.

$$\begin{bmatrix} \psi & -\hat{Y}_{DD}(j\omega) \cdot \psi_f \end{bmatrix} \begin{bmatrix} \mathbf{R}_{gYDD} \\ \boldsymbol{\rho}_f \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (5.20)$$

$$\psi = \begin{bmatrix} \frac{1}{s_1 - \hat{\lambda}_1} & \cdots & \frac{1}{s_1 - \hat{\lambda}_{N_O}} \\ \vdots & & \\ \frac{1}{s_{N_D} - \hat{\lambda}_1} & \cdots & \frac{1}{s_{N_D} - \hat{\lambda}_{N_O}} \end{bmatrix} \quad (5.21)$$

$$\psi_f = \begin{bmatrix} \psi & \mathbf{1} \end{bmatrix} \quad (5.22)$$

$$(5.23)$$

A similar problem to Equation 5.20 can be formed for each of the eight channels (four quadrants each of the whole-system impedance and admittance). To reduce the dimension of the problem the QR-decomposition is used on the equivalent expression to Equation 5.20 for every channel [176]. The QR-decomposition for the $\hat{Y}_{DD}(s)$ channel is shown in Equation 5.24.

$$\begin{bmatrix} \mathbf{T}_{11,YDD} & \mathbf{T}_{12,YDD} \\ 0 & \mathbf{T}_{22,YDD} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{gYDD} \\ \boldsymbol{\rho}_f \end{bmatrix} = \mathbf{Q}_{YDD}^T \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (5.24)$$

The sub-matrices $\mathbf{T}_{22}$ from each QR-decomposition are stacked for each channel in Equation 5.25, to give a final problem to solve for the parameter vector $\boldsymbol{\rho}_f$. Equation 5.17 is then used to find the next estimate of the modes.

$$\begin{bmatrix} \mathbf{T}_{22,YDD} \\ \mathbf{T}_{22,YDQ} \\ \mathbf{T}_{22,YQD} \\ \mathbf{T}_{22,YQQ} \\ \mathbf{T}_{33,ZDD} \\ \mathbf{T}_{33,ZDQ} \\ \mathbf{T}_{33,ZQD} \\ \mathbf{T}_{33,ZQQ} \end{bmatrix} \boldsymbol{\rho}_f = 0 \quad (5.25)$$

To estimate the residues, the final estimate of the modes is included in the matrix $\boldsymbol{\psi}$ in Equation 5.26. $\boldsymbol{\psi}$ is formed from the known frequencies of the frequency samples and the mode estimates as shown in Equation 5.21.

The residues are solved for simultaneously in the matrix of column vectors on the left-hand side of Equation 5.26, since each set of residue vectors only impact one channel of frequency data. The matrix $\boldsymbol{\psi}$ has as many rows as there are frequency samples, so Equation 5.26 should be over-determined and the parameters are solved for in a least-squares manner with the Moore-Penrose pseudo-inverse.

$$\boldsymbol{\psi} \begin{bmatrix} \mathbf{R}_{YDD} & \mathbf{R}_{YDQ} & \dots & \mathbf{R}_{ZQD} & \mathbf{R}_{ZQQ} \end{bmatrix} = \begin{bmatrix} \hat{Y}_{DD}(j\omega) & \hat{Y}_{DQ}(j\omega) & \dots & \hat{Z}_{QD}(j\omega) & \hat{Z}_{QQ}(j\omega) \end{bmatrix} \quad (5.26)$$

5.5.2 Details of constrained vector fitting: Level 1

This section will describe the incorporation of the relationship in Equation 5.12 into the vector fitting algorithm. Any estimated mode where the relationship between whole-system admittance and impedance residues in Equation 5.12 is enforced is described as "constrained".

To implement constrained vector fitting it is proposed to use two 'levels' of the constraint. The first level allows standard vector fitting to find the estimated modes $\hat{\boldsymbol{\lambda}}$ as normal then only enforces the constraint in Equation 5.12 during the residue solving step. Figure 5.5 illustrates

both levels of constrained vector fitting, alongside the standard, unconstrained vector vector fitting. It can be seen that constrained requires information regarding the apparatus admittance, which may have to be subject to a data-sharing agreement with the active-damping IBR operator.

In practice, it is not possible to enforce Equation 5.12 for the residues associated with all of the modes. Equation 5.27 requires the value of the mode to evaluate the admittance at. If the mode estimate is too far away from the true value the constraint equations will be incorrect and degrade the accuracy of the solution. Due to this, it is only proposed to constrain the residues associated with a subset of the whole-system modes which can already be determined with reasonable accuracy. The conditions for constraining are initially proposed to be a damping factor less than 0.4.

The subset of the estimated modes to be constrained is notated by $\hat{\lambda}_C$. A similar subscript is used to indicate the associated whole-system residues, for example $\mathbf{R}_{Y,DD,C}$ and $\mathbf{R}_{Z,DD,C}$. The constraint in Equation 5.27 can be stacked for all the constrained residues to form the overall relationship between the constrained whole-system impedance and admittance residues in Equation 5.28. $\mathbf{M}_{\hat{\lambda}_C}$ is a block-diagonal matrix with one sub-matrix for each constrained mode, where each sub-matrix can be found with Equation 5.12 evaluated at each constrained mode.

$$\begin{bmatrix} R_{Z,DD,\hat{\lambda}_i} \\ R_{Z,DQ,\hat{\lambda}_i} \\ R_{Z,QD,\hat{\lambda}_i} \\ R_{Z,QQ,\hat{\lambda}_i} \end{bmatrix} = \mathbf{M}(\hat{\lambda}_i) \begin{bmatrix} R_{Y,DD,\hat{\lambda}_i} \\ R_{Y,DQ,\hat{\lambda}_i} \\ R_{Y,QD,\hat{\lambda}_i} \\ R_{Y,QQ,\hat{\lambda}_i} \end{bmatrix} \quad (5.27)$$

$$\begin{bmatrix} \mathbf{R}_{Z,DD,C} \\ \mathbf{R}_{Z,DQ,C} \\ \mathbf{R}_{Z,QD,C} \\ \mathbf{R}_{Z,QQ,C} \end{bmatrix} = \mathbf{M}_{\hat{\lambda}_C} \begin{bmatrix} \mathbf{R}_{Y,DD,C} \\ \mathbf{R}_{Y,DQ,C} \\ \mathbf{R}_{Y,QD,C} \\ \mathbf{R}_{Y,QQ,C} \end{bmatrix} \quad (5.28)$$

$$\mathbf{M}_{\hat{\lambda}_C} = \text{blkdiag}\{\mathbf{M}(\hat{\lambda}_i)\} \quad \forall \hat{\lambda}_i \in \hat{\lambda}_C \quad (5.29)$$

The rest of the estimated modes which are not to be constrained are represented by $\hat{\lambda}_B$,

with similar sub-scripts for their associated residues.

The frequency sample matrix ψ is split up into one acting on the constrained mode estimates ψ_C and one acting on the unconstrained mode estimates ψ_B , as shown in Equations 5.30 and 5.31. The constrained residues now affect multiple channels of data each, so the residues must be solved for as a column vector.

$$\psi_B = \begin{bmatrix} \frac{1}{s_1 - \hat{\lambda}_1} & \cdots & \frac{1}{s_1 - \hat{\lambda}_i} \\ \vdots & & \\ \frac{1}{s_{N_D} - \hat{\lambda}_1} & \cdots & \frac{1}{s_{N_D} - \hat{\lambda}_i} \end{bmatrix} \quad \forall \hat{\lambda}_i \in \hat{\lambda}_B \quad (5.30)$$

$$\psi_C = \begin{bmatrix} \frac{1}{s_1 - \hat{\lambda}_1} & \cdots & \frac{1}{s_1 - \hat{\lambda}_i} \\ \vdots & & \\ \frac{1}{s_{N_D} - \hat{\lambda}_1} & \cdots & \frac{1}{s_{N_D} - \hat{\lambda}_i} \end{bmatrix} \quad \forall \hat{\lambda}_i \in \hat{\lambda}_C \quad (5.31)$$

$$(5.32)$$

For brevity of notation the frequency sample matrices ψ_B and ψ_C are stacked in block-diagonal form in Equations 5.33 and 5.34. Equation 5.35 now represents the main numerical computation to be performed to solve for the unknown residues in constrained vector fitting level 1.

Only one set of the constrained residues is present in the variable vector. Once the variable vector is solved for, the remaining constrained residues can be found with Equation 5.28.

$$\Psi_B = \text{blkdiag}\{\psi_B, \psi_B, \psi_B, \psi_B\} \quad (5.33)$$

$$\Psi_C = \text{blkdiag}\{\psi_C, \psi_C, \psi_C, \psi_C\} \quad (5.34)$$

$$\begin{bmatrix} \Psi_B & \mathbf{0} & \Psi_C \\ \mathbf{0} & \Psi_B & \Psi_C \mathbf{M}_{\hat{\lambda}_C} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{Y,DD,B} \\ \mathbf{R}_{Y,DQ,B} \\ \mathbf{R}_{Y,QD,B} \\ \mathbf{R}_{Y,QQ,B} \\ \mathbf{R}_{Z,DD,B} \\ \mathbf{R}_{Z,DQ,B} \\ \mathbf{R}_{Z,QD,B} \\ \mathbf{R}_{Z,QQ,B} \\ \mathbf{R}_{Y,DD,C} \\ \mathbf{R}_{Y,DQ,C} \\ \mathbf{R}_{Y,QD,C} \\ \mathbf{R}_{Y,QQ,C} \end{bmatrix} = \begin{bmatrix} \hat{Y}_{DD}(j\omega) \\ \hat{Y}_{DQ}(j\omega) \\ \hat{Y}_{QD}(j\omega) \\ \hat{Y}_{QQ}(j\omega) \\ \hat{Z}_{DD}(j\omega) \\ \hat{Z}_{DQ}(j\omega) \\ \hat{Z}_{QD}(j\omega) \\ \hat{Z}_{QQ}(j\omega) \end{bmatrix} \quad (5.35)$$

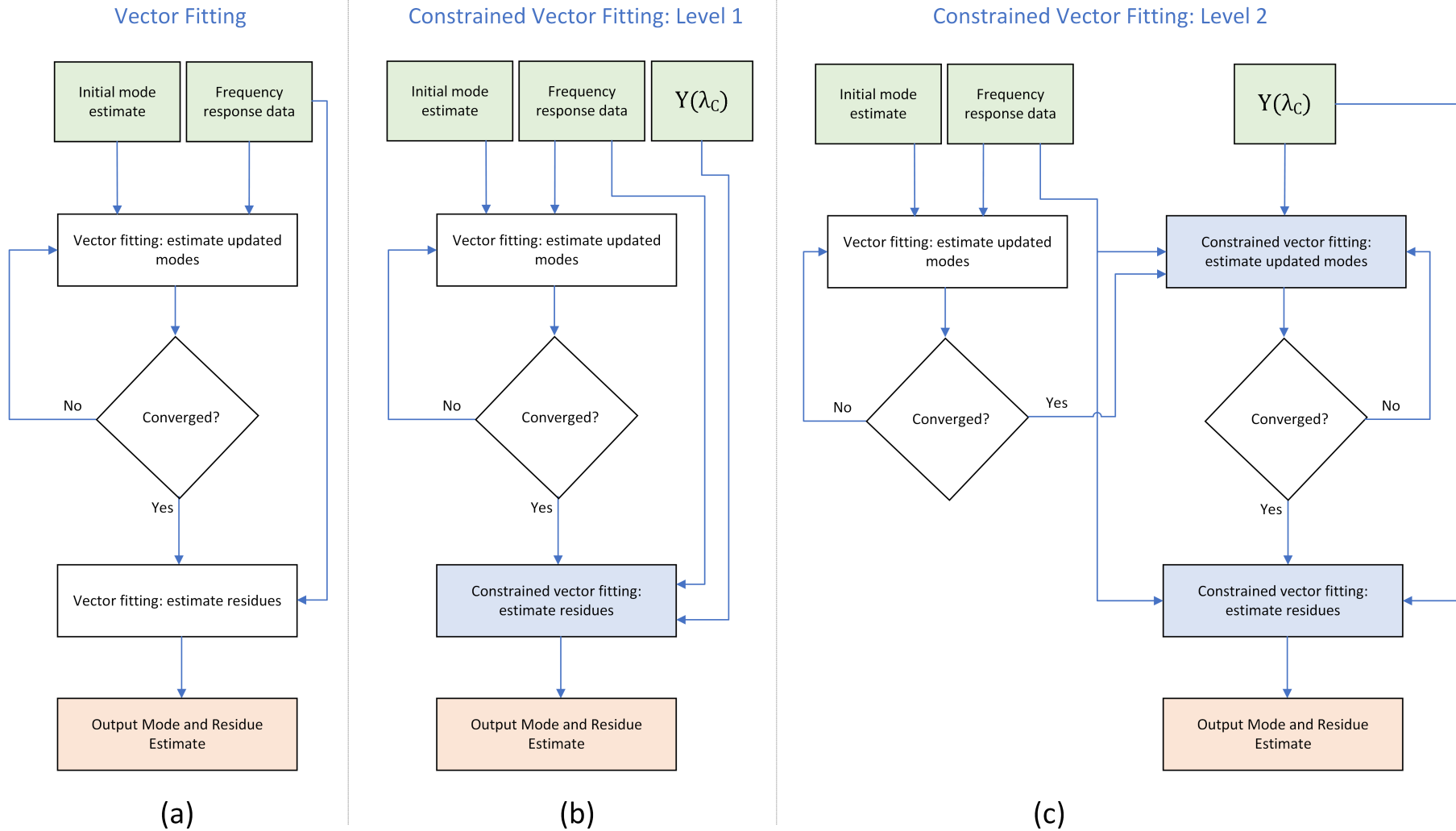


Figure 5.5: The algorithms for (a) standard vector fitting, (b) constrained vector fitting level 1, (c) constrained vector fitting level 2. The initial estimate of the modes and the frequency response data are required in all three processes. Standard vector fitting is described in more detail in Figure 5.4 but is included again here for comparison. Constrained vector fitting level 1 estimates the modes in an identical manner as standard vector fitting, then requires information on the apparatus admittance during the residue solving step. Constrained vector fitting level 2 first estimates the modes using the same process as standard vector fitting, but then performs another set of iterations including the apparatus admittance information in the constraint. An updated set of values of the apparatus admittance is required for each iteration. Subsequently, the residues are estimated using the same process as constrained vector fitting level 1. The inputs to the three processes are indicated in green, while the outputs are indicated in red. The novel constrained vector fitting processes are highlighted in blue.

5.5.3 Details of constrained vector fitting: Level 2

Constrained vector fitting level 2 is now described, where the constraint is enforced during both the mode estimation and the residue estimation steps. Figure 5.5 illustrates both levels of constrained vector fitting alongside the standard, unconstrained vector fitting.

For constrained vector fitting level 2 the constraint equations in Equation 5.12 still require a reasonably accurate estimate of the mode locations, so unconstrained vector fitting is run first to get initial mode estimates.

To start the constrained mode estimation, the QR decomposition method can still be performed as normal for the four channels of whole-system admittance frequency response data. Here, 'normal' means in the manner described in Chapter 5.5.1 with the standard version of vector fitting. The four QR-decomposition steps result in four stacked sub-matrices as previously described in Equation 5.20.

The constraint in Equation 5.27 is split into the constraint on each quadrant of the whole-system impedance residues in Equation 5.36. Equation 5.37 illustrate how the constraint matrices Equation 5.36 determine the DD whole-system impedance residues, based on the whole-system admittance residues.

$$\mathbf{M}_{\hat{\lambda}_C} = \begin{bmatrix} \mathbf{M}_{DD,\hat{\lambda}_C} \\ \mathbf{M}_{DQ,\hat{\lambda}_C} \\ \mathbf{M}_{QD,\hat{\lambda}_C} \\ \mathbf{M}_{QQ,\hat{\lambda}_C} \end{bmatrix} \quad (5.36)$$

$$\mathbf{R}_{Z,DD,C} = \mathbf{M}_{DD,\hat{\lambda}_C} \begin{bmatrix} \mathbf{R}_{Y,DD,C} \\ \mathbf{R}_{Y,DQ,C} \\ \mathbf{R}_{Y,QD,C} \\ \mathbf{R}_{Y,QQ,C} \end{bmatrix} \quad (5.37)$$

For each whole-system impedance channel, the relevant constraint matrix from Equation 5.36 is included in the QR-decomposition step as shown in Equation 5.38. The QR-decomposition step for the $\hat{Z}(j\omega)$ channel is shown in Equations 5.38 and 5.39.

The four QR-decompositions give four more sub-matrices $\mathbf{T}_{33,ZDD}$, $\mathbf{T}_{33,ZDQ}$, $\mathbf{T}_{33,ZQD}$ and $\mathbf{T}_{33,ZQQ}$. These four are stacked with those obtained from the whole-system admittance as shown in Equation 5.40. Finally, Equation 5.40 is solved for $\boldsymbol{\rho}_f$ in a least-squares manner with the Moore-Penrose pseudo-inverse. The next set of estimated modes are found with Equation 5.17.

$$\begin{bmatrix} \Psi_B & \Psi_C \mathbf{M}_{DD, \hat{\lambda}_C} & -\hat{Z}_{DD}(j\omega) \cdot \psi_f \end{bmatrix} \begin{bmatrix} \mathbf{R}_{Z,DD,B} \\ \mathbf{R}_{Y,C} \\ \boldsymbol{\rho}_f \end{bmatrix} = \mathbf{0} \quad (5.38)$$

$$\begin{bmatrix} \mathbf{T}_{11,ZDD} & \mathbf{T}_{12,ZDD} & \mathbf{T}_{13,ZDD} \\ 0 & \mathbf{T}_{22,ZDD} & \mathbf{T}_{23,ZDD} \\ 0 & 0 & \mathbf{T}_{33,ZDD} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{Z,DD,B} \\ \mathbf{R}_{Y,C} \\ \boldsymbol{\rho}_f \end{bmatrix} = \mathbf{Q}_{ZDD}^T \mathbf{0} \quad (5.39)$$

$$\begin{bmatrix} \mathbf{T}_{22,YDD} \\ \mathbf{T}_{22,YDQ} \\ \mathbf{T}_{22,YQD} \\ \mathbf{T}_{22,YQQ} \\ \mathbf{T}_{33,ZDD} \\ \mathbf{T}_{33,ZDQ} \\ \mathbf{T}_{33,ZQD} \\ \mathbf{T}_{33,ZQQ} \end{bmatrix} \boldsymbol{\rho}_f = \mathbf{0} \quad (5.40)$$

After iterative applications of the mode identification step have converged on a set of estimated modes, constrained vector fitting level 1 can be used to estimate the residues.

5.5.4 Results of the application of constrained vector fitting

The results of constrained vector fitting applied to the same noisy data shown in Figure 5.3a is plotted in Figure 5.6. It can be seen that constrained vector fitting gives identical next mode location predictions with the whole-system admittance and impedance residues, indicating the constraint was applied successfully.

Constrained vector fitting level 1 has the same mode estimation results as unconstrained

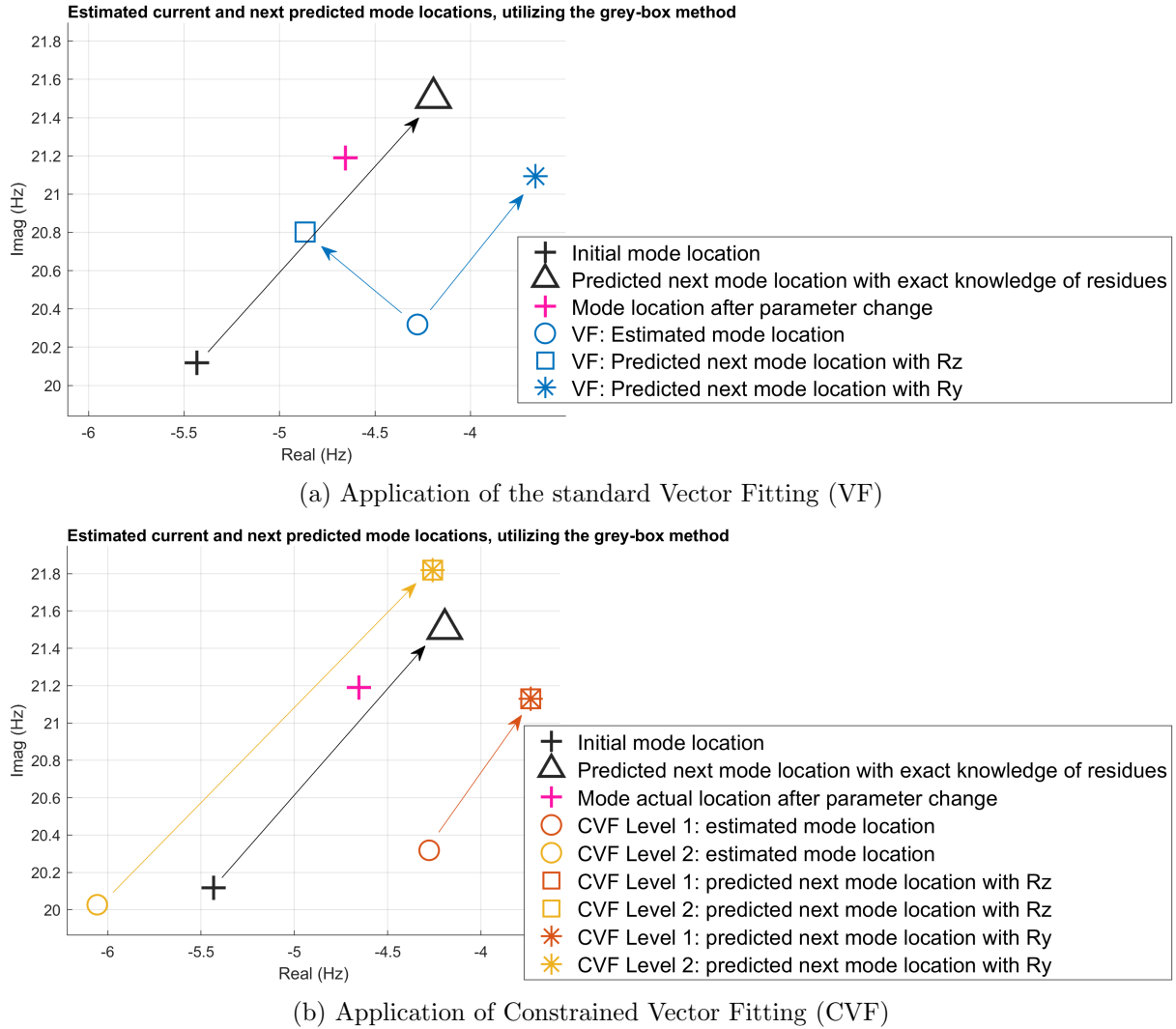
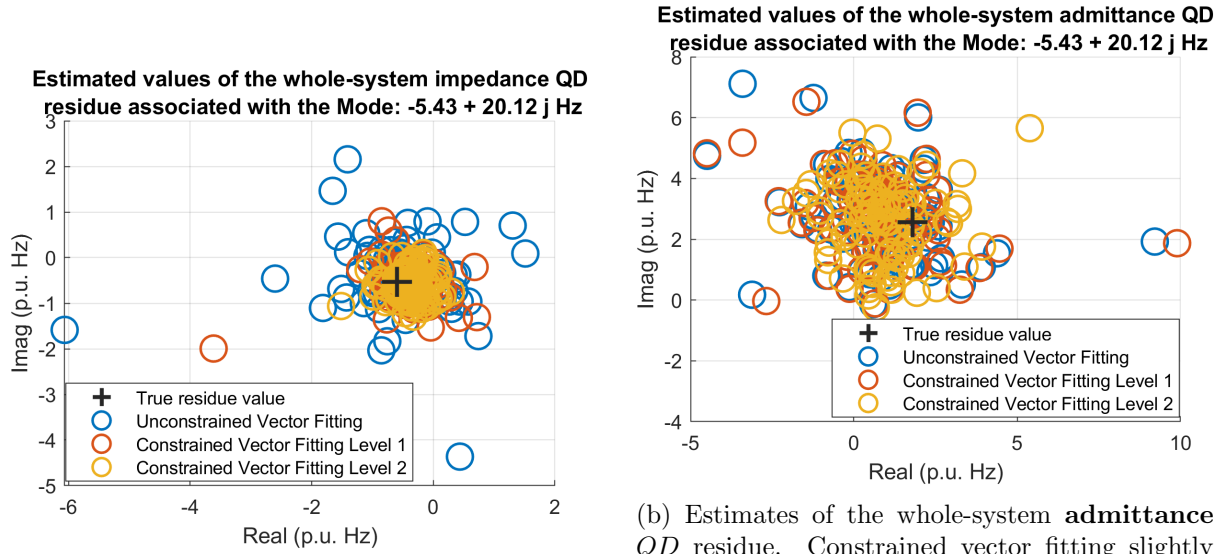


Figure 5.6: Predicting the next location of the SSO mode in the 14-bus system after the increase of the DQ virtual admittance gain from 0 to 1 p.u. The data used was shown in Figure 5.3a. The next predicted mode is plotted for the cases: (i) triangle - if the whole-system residues were known exactly, (ii) square - utilising the estimated whole-system impedance residues, (iii) asterisk - utilising the whole-system admittance residues. It can be seen that constrained vector fitting results in identical predictions with the whole-system impedance and admittance residues, and are in the correct direction.

vector fitting, since the mode estimation step is unchanged. However, the predicted next location of the mode is in the right direction for the predictions utilizing both the impedance and admittance residues. The current and next predicted mode locations estimated by constrained vector fitting level 2 are both more accurate, indicating some promise of the proposed method.

However, Figure 5.6 only shows the result based on one particular sample of noisy data. A Monte-Carlo approach is taken to assess the performance of constrained vector fitting more robustly. The same underlying system as was presented in Figure 5.2 is used with 100 frequency response samples of each quadrant of the whole-system impedance and admittance. 100 fre-



(a) Estimates of the whole-system **impedance** QD residue. It can be seen that constrained vector fitting level 2 significantly improves the average accuracy of the estimated residue compared to unconstrained vector fitting

(b) Estimates of the whole-system **admittance** QD residue. Constrained vector fitting slightly improves the average accuracy of the estimated residue but not to an extent large enough to be determined visibly. However, it is visible that constrained vector fitting level 2 results in fewer outlier residue estimates.

Figure 5.7: Monte-Carlo study results for 100 estimates of the whole-system residues, with every estimate based on a different random sample of noise added to the same frequency response data

quency response data-sets are generated with added normally distributed random noise of the same standard deviation 0.01 p.u. .

The vector fitting results for the estimated whole-system QD impedance and admittance residues are shown in Figure 5.7 (a) and (b). It can be seen that the whole-system impedance residue estimates are significantly more accurate, but the whole-system admittance residues are more similar to the unconstrained vector fitting estimates.

Figure 5.8 illustrates the results for the modes and all of the quadrants of the residues. The mean of the relative error of the 100 residue estimates are plotted for each parameter. Here, the relative error means the magnitude of the distance between the true parameter and the estimated parameter, divided by the true parameter magnitude. It can be seen that constrained vector fitting significantly improves the accuracy of the estimation of the whole-system impedance residues. However, the whole-system admittance residue accuracy is more modestly improved, with the mean relative error reduced by a proportion in the range of 0.06 to 0.09. In general, constrained vector fitting level 2 results in more accurate residue estimates than level 1.

The accuracy of the mode estimate is not significantly affected.

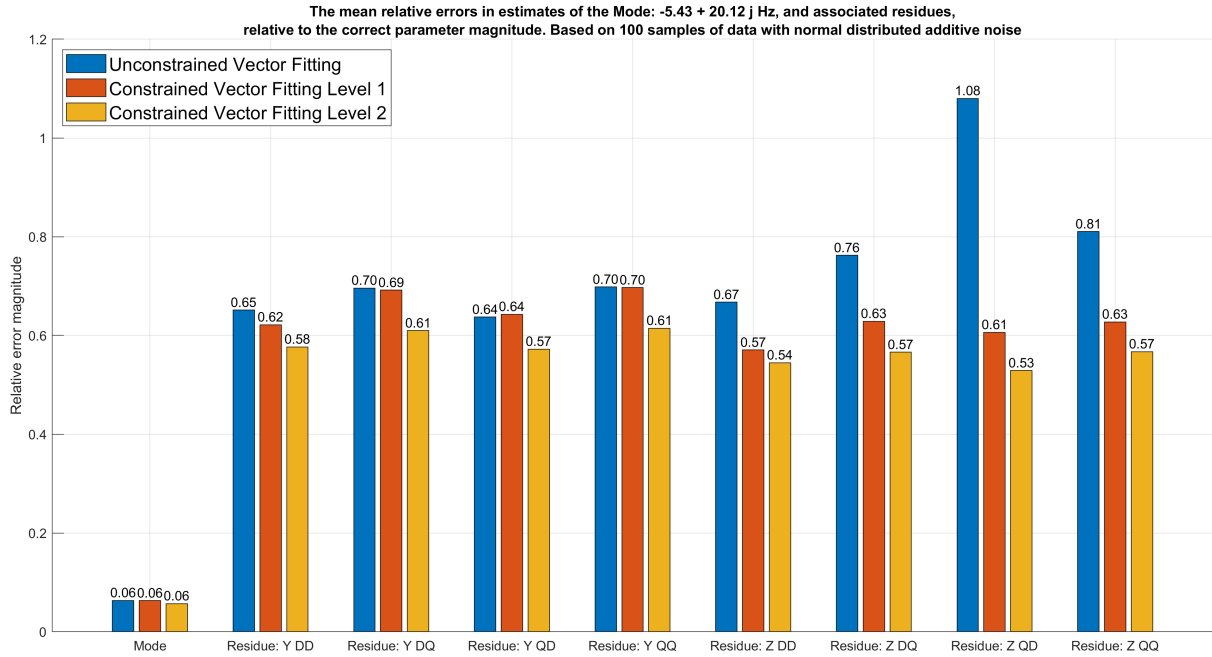


Figure 5.8: Monte-Carlo study results for 100 estimates of the SSO mode and whole-system impedance and admittance residues. In the x-label, a "Z" indicates a whole-system impedance residue and a "Y" indicates a whole-system admittance residue. The y-axis represents the mean of the relative error - the magnitude of the distance between the true parameter value and the estimated parameter value, divided by the magnitude of the true parameter value. The mean is taken over all of the 100 estimated sets of parameters, each of which is based on a different random sample of noise added to the same frequency response data generated by the linearised model. Constrained vector fitting improves the average accuracy of the estimated whole-system residues by 0.06 to 0.09. The accuracy of the whole-system impedance residues are significantly improved.

5.5.5 Data measurement and information sharing requirements for integration of constrained vector fitting into the impedance-shaping process

Both levels of constrained vector fitting require the value of the active-damping IBR admittance evaluated at all the constrained mode values in $\hat{\lambda}_C$. For the proposed impedance shaping process the active-damping IBR in question would already be required to share values of the admittance-parameter sensitivities evaluated at λ_i . Therefore, it is not unreasonable that the active-damping IBR would also be willing to share with the ESO values of the admittance spectrum evaluated at λ_i .

Constrained vector fitting level 1 only requires the admittance once: during the residue estimation step. However, constrained vector fitting level 2 requires the admittance during every iteration of the constrained mode estimation. As a rough estimate learned from application on test data, there are usually 5 - 20 iterations required. The level of constrained vector fitting to

be used would have to be taken into account in any data-sharing agreement.

The grey-box approach can be implemented with either the whole-system impedance or whole-system admittance measurements so measuring both sets of data may seem to be an unnecessary step. However, the act of shaping the apparatus admittance will affect whether it is most effective to predict the next mode location with the whole-system admittance or impedance residue measurement. To demonstrate this, Equation 5.11 is repeated in Equation 5.41.

$$\frac{R_Z}{R_Y} = -Y(\lambda_i)^{-2} \quad (5.41)$$

In Equation 5.41 $Y(\lambda_i)$ is the admittance of the apparatus shaping its admittance. R_Z is the whole system impedance residue associated with the mode λ_i at the bus the apparatus is connected at, while R_Y is the whole-system admittance residue.

Equation 5.41 demonstrates that when the apparatus admittance is small at the SSO mode λ_i the whole-system impedance residue will have a larger magnitude than the whole-system admittance residue. However, after the virtual admittance gains are increased to improve the damping of the SSO mode Equation 5.41 indicates that the whole-system impedance residue will decrease in magnitude compared to the whole-system admittance residue. Therefore, the mode may become more observable in the whole-system admittance data. Due to this phenomenon, it could be advisable to measure both whole-system admittance and impedance to ensure the data with maximum observability of the mode is captured, when applying the proposed impedance-shaping process.

Additionally, the Signal-to-Noise Ratios (SNR) of the different sets of measurements will be important. In the case study presented in this chapter, noise of a constant variance was added to all the whole-system frequency response data. In reality, the impedance and admittance measurements would be obtained with different equipment so the noise may have very different characteristics. However, as a first investigation into the application of constrained vector fitting a constant noise variance was used to keep the case study as simple and general as possible.

It can be seen from Figure 5.3a that the the SNR of the whole-system admittance is superior to that of the whole-system impedance. The difference in SNR is the reason that the accuracy of the whole-system impedance is improved more significantly, as the constraint effectively

allows vector fitting to use the higher-quality whole-system admittance data to determine the whole-system impedance residues. However, there is still a not-insignificant improvement in the whole-system admittance residue estimates as well.

The best solution in practice might be to make more measurements to reduce the effective variance of the noise. However, if both whole-system impedance and admittance data were already being measured and the apparatus admittance were available, only performing unconstrained vector fitting would be letting potential information go unused. This "leaving information on the table" could sacrifice a modest amount of parameter estimation accuracy, as demonstrated in Figure 5.8.

5.6 Limitations of the proposed constrained vector fitting method

When the estimated parameters are already very accurate constrained vector fitting will result in minimal improvement of the residue estimates since the residues will already satisfy the constraint enforced by the proposed algorithm. Figure 5.9 shows the result of another Monte-Carlo analysis applied to the initial state of the IEEE 14 bus system, when the virtual admittance gains are zero. Similar frequency response data was plotted in Figure 5.2a, where the SSO mode is very poorly damped and there is a clear peak in the whole-system impedance frequency response data.

Due to the clear peak in the data the mode and whole-system impedance residue estimates are already very accurate using unconstrained vector fitting. Imposing the constraint results in minimal improvement of the whole-system impedance residue estimation accuracy. The accuracy of the three whole-system admittance residues without a clear peak in the data in Figure 5.2a are significantly improved, since the constraint allows their calculation from the more accurate whole-system impedance residue estimates.

5.7 Chapter Conclusion

This chapter has presented a novel version of the popular vector fitting algorithm used for system identification that has been tailored for use in the grey-box method. The proposed version, named "constrained vector fitting", utilises a fundamental relationship between the admittance of the active-damping IBR, the whole-system impedance residues and the whole-

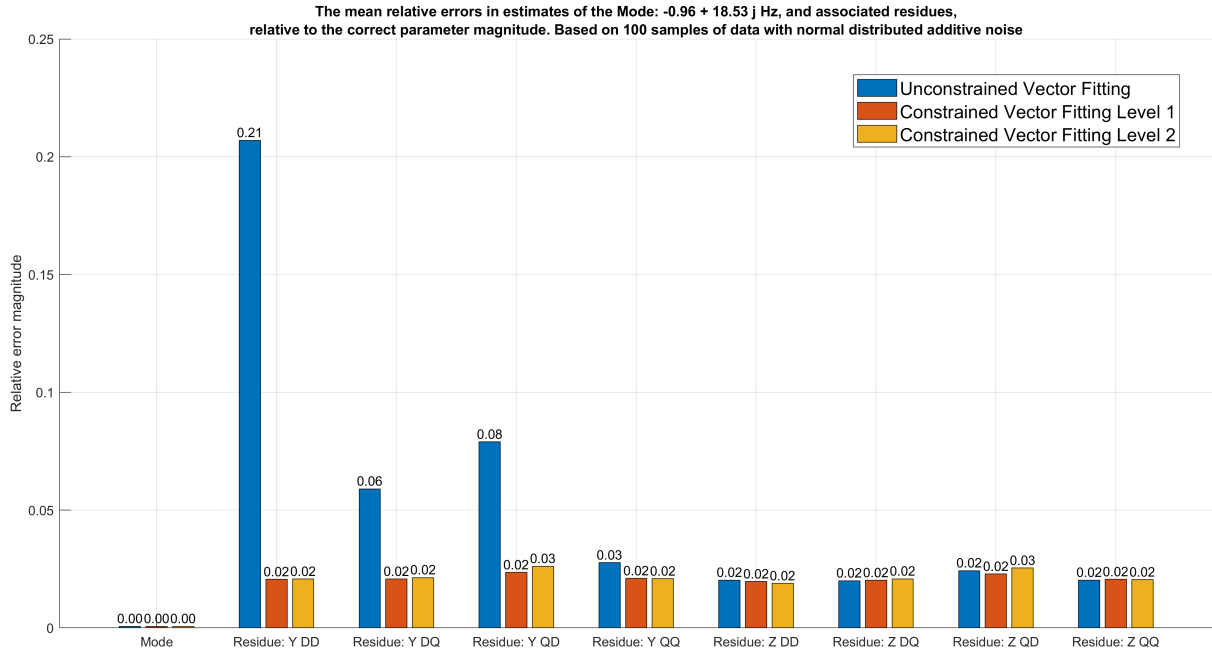


Figure 5.9: Monte-Carlo study results for 100 estimates of the SSO mode in the initial state of the IEEE 14 bus system - an example of its frequency response was plotted in Figure 5.2a. Also estimated are the whole-system impedance and admittance residues. In the x-label, a "Z" indicates a whole-system impedance residue and a "Y" indicates a whole-system admittance residue. The y-axis represents the mean of the relative error - the magnitude of the distance between the true parameter value and the estimated parameter value, divided by the magnitude of the true parameter value. The mean is taken over all of the 100 estimated sets of parameters, each of which is based on a different random sample of noise added to the same frequency response data generated by the linearised model. Constrained vector fitting does not improve the average accuracy of the estimated whole-system impedance residues, since they are already very accurate and already fulfill the constraint. The accuracy of the whole-system admittance residues are significantly improved.

system admittance residues to constrain the allowable range of the residue values.

The alterations to the vector fitting algorithm were laid out alongside the information required for its implementation. It was shown that when the frequency response data is noisy, it is possible to improve the accuracy of the identified residues by implementing the proposed constrained vector fitting. It is noted, however, that in cases where the standard vector fitting already produces very good estimates of the residues (for example, when the associated mode is very poorly damped) then the new algorithm can not offer significant improvement in the accuracy.

6 Conclusions

It is difficult for an electricity system operator (ESO) of an electricity system, such as National Grid ESO in Great Britain, to obtain analytical models of networks with high penetration of inverter-based resources (IBR) because of the proprietary nature and wide variety of the control systems of those IBR. This problem is motivating the study of system stability-enhancement techniques which can effectively utilize measurements, rather than relying solely on modelling. The recent work on the "grey-box" approach has unveiled the relationship between the admittance of an apparatus and the damping of the system oscillation modes, which determine the system small-signal stability. The grey-box approach can guide changes in the parameters of one apparatus to improve the damping of the modes without requiring models of the rest of the apparatus on the network. Instead, measured frequency response data can be used to estimate mode residues which guide the change in parameters.

This thesis has described a possible process where the ESO and an IBR operator co-operate to select the optimal parameters to alter the IBR admittance and improve the small-signal stability of the system. It is envisioned that the IBR operator would have an agreement with the ESO to provide admittance-shaping services and to share some more information than might be required of other apparatus connected to the network.

The objective of this thesis was to investigate three identified barriers to the implementation of the described small-signal stability enhancement process.

6.1 Barrier 1

The first barrier investigated was the unknown capability of a grid-following converter (the most common form of IBR present in the electricity grid to date) to modify its admittance without any unacceptable degradation in its own performance. A study was performed to assess the parameters in a grid-following converter which are suitable for alteration to shape its admittance and to assess their impact on the converter performance.

Two test systems were considered: a Single Machine Infinite Bus (SMIB) system experiencing sub-synchronous oscillations (SSO) due to the well-known weak-grid PLL instability, and a multi-IBR 14-bus system experiencing SSO due to interaction of the IBR. It was shown by simulation that the grid-voltage to current-reference feed-forward parameters (also known as a

"virtual admittance") were effective at improving the damping of the modes in both systems.

However, it was also demonstrated that the DD and DQ components of the virtual admittance could impact the performance of the IBR's DC-link voltage control. The IBR terminal AC voltage magnitude was identified as being impacted by the influence of the QD and QQ components of virtual admittance on reactive power output.

For the ESO to make an optimal choice of parameter value that considers both the desired impact on the mode and the possibly undesired impact on IBR performance requires the ESO to be provided with additional information. The form of the information required depends on whether the performance impacts are to be accounted for in the form of hard constraints on the parameter values, or whether the impact on the performance should be included in the parameter choice optimisation as a "cost".

The cost-based approach could provide a straightforward method of calculating any financial compensation agreed between the ESO and the IBR operator for any performance degradation ensued in the pursuit of improved small-signal stability. However, it would likely require sharing the sensitivity of the cost to each alterable parameter. Sharing only hard constraints on the parameter ranges would be much simpler and require less sharing of information.

6.2 Barrier 2

A second barrier was identified as the need to ensure system stability even if the path a mode takes in response to a change in parameter is different to that predicted by the grey-box approach. This may occur, for instance, because the prediction assumes a constant sensitivity (or gradient) whereas the sensitivity may be changing rapidly in the region in question. Constraints on the allowed changes in parameters should be applied to ensure that the system remains stable. However, in the case that there are large variations in the sensitivity of the IBR admittance to a parameter, it was found that it is possible that a mode could be destabilised with a parameter change that was previously considered allowed.

A method was proposed to account for the variations in the sensitivity when calculating the parameter constraints. The proposed method - named the "minimum- $\Delta\rho$ path constraint" - utilizes the calculus of variations to find the path of the mode which would require the minimum change in parameter to destabilise the system. An example was presented of an IBR with a

sharply tuned band-pass filter within the virtual admittance. Due to the band-pass filter, there was a pole present in the apparatus admittance sensitivity spectrum leading to sensitivities that change rapidly. The pole contributed to the destabilisation of a whole-system mode with a smaller change in parameter than predicted with the grey-box approach. The minimum- $\Delta\rho$ path constraint was shown to be able to account for the presence of the pole and so calculate a tighter constraint that avoids destabilisation of the whole-system mode.

The minimum- $\Delta\rho$ path method requires an approximation for the admittance-parameter sensitivity in the neighbourhood of the whole-system mode, a sensitivity which the IBR operator can determine. However, it also requires a limit on the maximum change in the magnitude of the relevant residue of the whole-system impedance (which is the sensitivity to impedance) as the parameter is changed. A technique to estimate this maximum residue magnitude is not yet available which may limit the utilisation of the minimum- $\Delta\rho$ path constraint as things stand.

Sharing an approximation of the admittance-parameter sensitivity would be more revealing than simply sharing the sensitivity evaluated at discrete points. Accordingly, it may be preferable for the IBR operator to perform the calculation of the minimum- $\Delta\rho$ path constraint and only share the resulting constraints on the allowable parameter ranges with the ESO.

6.3 Barrier 3

The third barrier was identified as the accurate estimation of the residues from the measured frequency response data. To minimise the effect of non-linear distortions, it may be desirable to inject a relatively small input signal to measure the whole-system frequency response. However, small signals will be affected more significantly by measurement noise.

To help improve estimation and allow injection at small amplitude, a novel method named "constrained vector fitting" has been proposed. It improves the accuracy of estimation of mode and residue values from noisy frequency response data. The method utilises a previously unreported fundamental relationship unveiled by the grey-box method - that the residues of the whole-system impedance and of the whole-system admittance associated with a certain mode are not independent of each other. The residues are linked by the admittance of the apparatus connected at the bus where the whole-system frequency responses are measured, evaluated at the mode frequency. This linkage provides an opportunity to improve identification. If the

apparatus admittance in question is known - for example if an IBR operator has an agreement in place to share certain information with the ESO - then both the whole-system impedance and admittance measurements can be used to estimate the same underlying residues. Utilising two different sources of data to estimate the same parameters in this manner is shown to be able to improve the performance of the residue estimation under noisy conditions.

The new estimation method builds on the well-known open-source algorithm "vector fitting", but alters it to take account of the connection between the residues of the whole-system impedance and admittance. The mathematical alterations to the standard vector-fitting algorithm were laid out and an open-source implementation will be published. The connection is embedded in the vector fitting algorithm in the form of a constraint on the allowed values of the estimated residues.

A Monte-Carlo study was performed on the parameter estimation of the SSO mode present in the 14-bus test system, after its damping has been improved with an application of virtual admittance. It was shown that the constrained vector fitting approach gave a modest improvement (in the range of 6 - 9 %) of the average accuracy of the estimated residues.

In the case that standard vector fitting can already estimate the residues very accurately then constrained vector fitting cannot significantly improve the accuracy of the estimated residues since they already fulfil the underlying constraint. For example, the whole-system impedance frequency response of the initial condition of the 14-bus test system has a sharp peak due to the very poorly damped SSO mode. Due to the sharp peak, the residues can already be estimated well with the standard implementation of vector fitting.

Constrained vector fitting requires the values of the admittance of an IBR providing active-damping, evaluated at the whole-system modes. Two different levels of constrained vector fitting were proposed, which require information sharing at different intervals. The two levels could allow some flexibility in the information-sharing agreement between the IBR operator and the ESO, depending on their needs. Again, the IBR operator would only need to share the admittance values at the whole-system modes, not any models of the apparatus. Since the IBR operator is already required to share values of the admittance-parameter sensitivities at the modes, this requirement is not seen as too onerous.

6.4 Discussion and recommendations

The dynamic interaction of IBR controllers in the form of poorly damped oscillations is recognized as a problem which is worsening in grids across the world. So far, grid operators have commonly dealt with the issue by either limiting the total allowed penetration of IBR or disconnecting IBR which are identified as the culprits of oscillations. Some of the worst affected grid operators have also made significant investments in infrastructure to strengthen the grid. These are all expensive approaches which follow traditional methods of grid operation.

There has been relatively little practical application of impedance-based parameter tuning approaches, such as the grey-box method. This can be attributed to the fact that the impedance-based approaches have only recently been applied to large-scale AC power system analysis and they may not be viewed as mature enough for practical application by the ESO. Additionally, utilising the flexibility of IBR control programming allows the real-time shaping of IBR impedances over a large frequency range. This flexibility would not be possible with synchronous machines, and therefore the possibility has not been widely considered previously in the large-scale power systems stability literature.

Also, the parameter-tuning approach requires that the grid operator incentivises apparatus operators to actively alter apparatus parameters, with the aim of whole-system stability enhancement. This would possibly require a market-based approach with legal obligations for parameter tuning when requested, alongside any required information sharing. Properly defining and managing these requirements could be complex for a grid operator to manage.

In the authors opinion, if the political will can be found for the ESO to investigate the application of impedance measurement-based parameter tuning approaches, it could be possible to avoid some future investments in grid infrastructure. Since grid infrastructure investments are ultimately paid for with higher energy bills, keeping the size of these infrastructure investments to the minimum required is good for society in the long run. In the meantime, it should be recommended that forward-looking grid operators gain some practical experience with impedance-based approaches, since they have a promising future in grids with high penetration of IBR.

It should also be recommended that the information sharing regime is specified as early in the process as possible, to avoid insufficient information being gathered by the ESO for effective

application of the parameter tuning approaches. For this reason, the novel methods in this thesis were careful to consider the required information to be shared between the ESO and IBR operators. With a comprehensive agreed regime for information sharing, it may be possible for the novel methods described in this thesis to contribute to overcoming barriers to enhancing whole-system small-signal stability via the shaping of an IBR admittance.

6.5 Future work

This section will suggest topics relating to this thesis which may merit further investigation.

The trade-off between converter DC voltage performance and the whole-system stability when utilizing virtual admittance could be investigated experimentally to establish likely allowable ranges of parameters for common hardware and control setups. The relationship between performance degradation and the required admittance shape-ability could be quantified to investigate the financial incentives required to convince IBR operators to provide stability services. More practically, any impact on eg. manufacturer warranties could be assessed.

The application of similar analysis to HVDC converters could also be investigated: for example, a more realistic DC cable model and source side dynamics could be included for further investigation of the DC voltage dynamics when damping AC-side system oscillations.

The sensitivity of the whole-system residues to a change in parameter, essentially the second order sensitivity of the mode to the parameter, will be vital to investigating Assumption 1. Similarly, further work is required to assess the viability of the grey-box relationship after changing operating conditions. This could be combined with the work in Chapter 4 to give an overall measure of the "danger" posed by a change in parameter, whether a controller parameter or an operating condition. Similarly, the use of the grey-box method with parameters which affect the global frequency is not yet defined and could be a topic for deeper investigation.

The constrained vector fitting algorithm could be tested over a wider variety of frequency responses to robustly gauge its effectiveness over various types of noise and frequency response. Additionally, the impacts of nonlinear distortions in the frequency response data were ignored in this thesis, which could be very useful to assess.

Utilizing time-domain data to estimate the whole-system residues are a possible next step for the implementation of the grey-box approach. Particularly, the mathematics underlying

the constrained vector fitting could be embedded within a time-domain system identification algorithm to skip the frequency domain transformation. Especially, the application of the constrained vector fitting approach with field-measured whole-system impedance/admittance data would be hugely beneficial to assess its viability in practice.

More broadly, it is not clear as to the requirements for policy to incentivise the impedance-based mode damping approach which was the use-case scenario in this work. Particularly, the stability market design which could provide efficient incentives for the IBR operators to alter their parameters real-time in the manner discussed in this thesis. The costs of this approach could be compared against the cost of widespread grid strengthening investments, for examples new overhead lines or synchronous condensers.

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A Appendix: Grid-Following converter DC dynamics derivation

The DC voltage is determined by the energy stored in the capacitor, which is the integral of the power balance: the difference between the power injected into the converter by the grid P_{in} minus the power absorbed by the filter (P_{filter}). The DC-capacitor dynamics can be linearised as shown in Equation A.1, where $P_{\text{capacitor}}$ represents the capacitor power balance [23].

$$\begin{aligned} P_{\text{capacitor}} &= \frac{1}{2} s C_{DC} V_{DC}^2 \\ \Delta P_{\text{capacitor}} &= s C_{DC} V_{DC}^0 V_{DC} = \Delta P_{\text{in}} - \Delta P_{\text{filter}} \end{aligned} \quad (\text{A.1})$$

The power injected by the grid can be linearised as per Equation A.2, neglecting the filter losses.

$$\begin{aligned} P_{\text{in}} &= v_D i_D + v_Q i_Q \\ \Delta P_{\text{in}} &= E_0 i_d + \frac{P_0}{E_0} v_d - \frac{Q_0}{E_0} v_q \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} s C_{DC} V_{DC}^0 V_{DC} &= E_0 i_d + \frac{P_0}{E_0} v_d - \frac{Q_0}{E_0} v_q \\ V_{DC} &= \frac{1}{s C_{DC} V_{DC}^0} \left(\frac{P_0}{E_0} v_d - \frac{Q_0}{E_0} v_q + E_0 i_d \right) \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} E_0 &= E_0 - (R + sL) \frac{P_0}{E_0} \\ &= (R + sL) \frac{Q_0}{E_0} \end{aligned} \quad (\text{A.4})$$

The DC dynamics are laid out in Equation A.5 with the expressions for the currents substituted in. The feedback loop is then embedded in the transfer function $h_{\text{DC}}(s)$, shown in Equation A.8. This gives the transfer functions from the internal PLL voltages to the DC voltage shown in Equation A.7.

$$\begin{aligned} V_{DC} &= \frac{1}{s C_{DC} V_{DC}^0} \left(\frac{P_0}{E_0} v_d - \frac{Q_0}{E_0} v_q + E_0 (g_{cc}(s) f_{\text{DC}}(s) V_{DC} + (h_{cc}(s) + g_{\text{ff},\text{dd}}(s)) v_d \right. \\ &\quad \left. + g_{\text{ff},\text{dq}}(s) v_q) + (g_{\text{ff},\text{qd}}(s) v_d + (h_{cc}(s) + g_{\text{ff},\text{qq}}(s)) v_q) \right) \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned}
 V_{DC} & \left(1 - E_0 g_{cc}(s) f_{DC}(s) \frac{1}{s C_{DC} V_{DC}^0} \right) \\
 &= \frac{1}{s C_{DC} V_{DC}^0} \left(\left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) + g_{ff,qd}(s) \right) v_d \right. \\
 & \quad \left. + \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) + (h_{cc}(s) + g_{ff,qq}(s)) \right) v_q \right)
 \end{aligned} \tag{A.6}$$

$$\begin{aligned}
 V_{DC} &= h_{DC}(s) \left(\left(\frac{P_0}{E_0} + E_0 (h_{cc}(s) + g_{ff,dd}(s)) \right) v_d \right. \\
 & \quad \left. + \left(-\frac{Q_0}{E_0} + E_0 g_{ff,dq}(s) \right) v_q \right)
 \end{aligned} \tag{A.7}$$

$$V_{DC} = G_{VDC,d}(s) v_d + G_{VDC,q}(s) v_q$$

$$h_{DC}(s) = \frac{1}{1 - E_0 g_{cc}(s) f_{DC}(s) \frac{1}{s C_{DC} V_{DC}^0}} \frac{1}{s C_{DC} V_{DC}^0} = \frac{1}{s C_{DC} V_{DC}^0 - E_0 g_{cc}(s) f_{DC}(s)} \tag{A.8}$$

B Appendix: Minimum- $\Delta\rho$ path derivation

The magnitude $\left| R \frac{\partial Y_{mn}}{\partial \rho}(s) \right|$ here represents the "speed" of the movement of the whole-system mode and is now represented by $v(r)$, where r is a positive scalar representing the distance from the centre of the approximation (pole or zero) to a point in the complex plane. The minimum-time $t_{\min}$ required to travel between two points can therefore be calculated as

$$v(r) = Gr^n \quad (\text{B.1})$$

$$t_{\min} = \min_P \oint_P \frac{ds}{v} = \min_{r,\theta} \int \frac{\sqrt{dr^2 + (rd\theta)^2}}{Gr^n} \quad (\text{B.2})$$

Letting r vary as $r(\theta)$, the minimum-time path and $t_{\min}$ can be solved for using the Euler-Lagrange equation. Since $v(r)$ is independent of θ , the Beltrami identity can be used - where C is a constant according to the Beltrami identity. If v depends on θ as well as r , this solution becomes much more difficult. Therefore, only the case of $v(r)$ is considered.

$$t_{\min} = \frac{1}{G} \min_{r(\theta)} \int r(\theta)^{-n} \sqrt{r'(\theta)^2 + r(\theta)^2} d\theta \quad (\text{B.3})$$

$$C = r^{-n} \left((r'^2 + r^2)^{1/2} - r'^2 (r'^2 + r^2)^{-1/2} \right) \quad (\text{B.4})$$

$$C = r^{2-n} (r'^2 + r^2)^{-1/2} \quad (\text{B.5})$$

$$C^2 = r^{2(2-n)} (r'^2 + r^2)^{-1} \quad (\text{B.6})$$

$$r'^2 = \frac{r^{4-2n}}{C^2} - r^2 = r^2 \left(\frac{r^{2-2n}}{C^2} - 1 \right) \quad (\text{B.7})$$

$$\frac{dr}{d\theta} = r \left(\frac{r^{2(1-n)}}{C^2} - 1 \right)^{1/2} \quad (\text{B.8})$$

$$(\text{B.9})$$

As long as the two points are less than 90 degrees apart the integral can be solved:

$$\int d\theta = \int \frac{1}{r \left(\frac{r^{2(1-n)}}{C^2} - 1 \right)^{1/2}} dr \quad (\text{B.10})$$

If $n \neq 1$

$$\theta + k = \frac{1}{1-n} \tan^{-1} \left(\left(\frac{r^{2(1-n)}}{C^2} - 1 \right)^{1/2} \right) \quad (\text{B.11})$$

$$\frac{r^{2(1-n)}}{C^2} = \tan^2((1-n)(\theta + k)) + 1 \quad (\text{B.12})$$

$$r(\theta)^{1-n} = |C| \sec((1-n)(\theta + k)) \quad (\text{B.13})$$

If $n = 1$

$$\theta + k = \frac{\ln(r)}{\left(\frac{1}{C^2} - 1\right)^{1/2}} \quad (\text{B.14})$$

$$r(\theta) = e^{\left(\frac{1}{C^2} - 1\right)^{1/2}(\theta + k)} \quad (\text{B.15})$$

For $n = -1$, as the two ends of the path approach 90 degrees apart $r(\theta)$ approaches infinity at the ends of the path - i.e. the path tends to the straight lines between the ends of the path and the point $r = 0$. In this case the rest of the calculations are simple, so will not be listed here.

If the ends of the path are less than 90 degrees apart: Redefining the constants as $C = |C|$ and $k = (1-n)k$ and using the initial/final conditions θ_a/θ_b gives for $n \neq 1$:

$$r(\theta) = (C \sec((1-n)\theta + k))^{\frac{1}{1-n}} \quad (\text{B.16})$$

$$r'(\theta) = C^{\frac{1}{1-n}} \frac{1-n}{1-n} \sin((1-n)\theta + k) \sec((1-n)\theta + k)^{\frac{1}{1-n}+1} \quad (\text{B.17})$$

$$\begin{aligned} t_{\min} &= \frac{1}{G_{\inf}} \int_{\theta_a}^{\theta_b} (C \sec((1-n)\theta + k))^{\frac{-n}{1-n}} \\ &\quad \left(\left(C^{\frac{1}{1-n}} \sin((1-n)\theta + k) \sec((1-n)\theta + k)^{\frac{1}{1-n}+1} \right)^2 \right. \\ &\quad \left. + (C \sec((1-n)\theta + k))^{\frac{2}{1-n}} \right)^{1/2} d\theta \end{aligned} \quad (\text{B.18})$$

$$\begin{aligned} &= \frac{C^{\frac{-n}{1-n} + \frac{1}{1-n}}}{G_{\inf}} \int_{\theta_a}^{\theta_b} \sec((1-n)\theta + k)^{\frac{-n}{1-n} + \frac{1}{1-n}} \\ &\quad (\tan^2((1-n)\theta + k) + 1^2)^{1/2} d\theta \end{aligned} \quad (\text{B.19})$$

$$= \frac{C}{G_{\inf}} \int_{\theta_a}^{\theta_b} \sec((1-n)\theta + k)^2 d\theta \quad (\text{B.20})$$

$$= \frac{C}{G_{\inf}(1-n)} [\tan((1-n)\theta + k)]_{\theta_a}^{\theta_b} \quad (\text{B.21})$$

The constants C and k can be found from the initial/final conditions and used to find the overall result.

$$C = r_a^{(1-n)} \cos(((1-n))\theta_a + k) \quad (\text{B.22})$$

$$\tan k = \frac{r_b^{(1-n)} \cos(((1-n))\theta_b) - r_a^{(1-n)} \cos(((1-n))\theta_a)}{r_b^{(1-n)} \sin(((1-n))\theta_b) - r_a^{(1-n)} \sin(((1-n))\theta_a)} \quad (\text{B.23})$$

$$t_{\min} = \frac{C}{G(1-n)} (\tan(((1-n))\theta_b + k) - \tan(((1-n))\theta_a + k)) \quad (\text{B.24})$$

For brevity, let $m = 1 - n$. The result for $t_{\min}$ can be simplified using:

$$\tan(m\theta_b + k) - \tan(m\theta_a + k) \quad (\text{B.25})$$

$$= \frac{(\tan(m\theta_b) - \tan(m\theta_a)) (1 + \tan(k)^2)}{(1 - \tan(m\theta_a) \tan(k)) (1 - \tan(m\theta_b) \tan(k))} \quad (\text{B.26})$$

$$C = r_a^m \cos(m\theta_a + k) = r_a^m (\cos(m\theta_a) \cos(k) - \sin(m\theta_a) \sin(k)) \quad (\text{B.27})$$

$$= r_a^m \cos(m\theta_a) \cos(k) (1 - \tan(m\theta_a) \tan(k)) \quad (\text{B.28})$$

$$t_{\min} = \frac{r_a^m \cos(m\theta_a)}{Gm} \frac{\tan(m\theta_b) - \tan(m\theta_a)}{\cos(k) - \tan(m\theta_b) \sin(k)} \quad (\text{B.29})$$

To simplify further, if $\tan(k) = \frac{D}{E}$ then $\sin(k) = \frac{D}{\sqrt{D^2 + E^2}}$ and $\cos k = \frac{E}{\sqrt{D^2 + E^2}}$.

$$t_{\min} = \frac{r_a^m \cos(m\theta_a)}{Gm} \frac{\tan(m\theta_b) - \tan(m\theta_a)}{E - \tan(m\theta_b) D} \sqrt{D^2 + E^2} \quad (\text{B.30})$$

$$D^2 + E^2 = r_a^{2m} + r_b^{2m} - 2r_a^m r_b^m \cos(m\theta_b - m\theta_a) \quad (\text{B.31})$$

$$E - \tan(m\theta_b) D = r_b^m \sin(m\theta_b) - r_a^m \sin(m\theta_a) \quad (\text{B.32})$$

$$- r_b^m \sin(m\theta_b) - r_a^m \cos(m\theta_a) \tan(m\theta_b) \quad (\text{B.33})$$

$$= r_a^m \cos(m\theta_a) (\tan(m\theta_b) - \tan(m\theta_a)) \quad (\text{B.34})$$

$$t_{\min} = \frac{1}{Gm} \sqrt{r_a^{2m} + r_b^{2m} - 2r_a^m r_b^m \cos(m\theta_b - m\theta_a)} \quad (\text{B.35})$$

Equation B.35 gives the general solution for the minimum time required to travel between two points. To solve for the minimum-time path where one of the points (r_a say) can be any location on the imaginary axis requires the boundary condition $\sigma_1/r_a = \sin\theta_a$, where σ_1 is the distance of the local origin from the imaginary axis.

If $m \neq \pm 1$ the $\cos$ term in Equation B.35 will result in a polynomial of higher order than two and hence require a numerical solution to find the minimum-time r_a and path.

However, if the local origin is on the imaginary axis this allows an analytical solution for any value of m (except 0) by aligning the reference frame with the imaginary axis (so $m\theta_a = 0$).

$$\begin{aligned} & \frac{\partial}{\partial r_a} \sqrt{r_b^{2m} + r_a^{2m} - 2r_a^m r_b^m \cos m\theta_b} \\ &= \frac{-2r_b^m \cos(m\theta_b) m r_a^{m-1} + 2m r_a^{2m-1}}{2\sqrt{r_b^{2m} + r_a^{2m} - 2r_a^m r_b^m \cos m\theta_b}} \end{aligned} \quad (\text{B.36})$$

$$= 0 \quad \text{when} \quad (\text{B.37})$$

$$r_a^m = r_b^m \cos m\theta_b \quad (\text{B.38})$$

$$\min_{r_a} t_{\min} = \frac{1}{Gm} \sqrt{r_b^{2m} - 2(r_b^m \cos m\theta_b)^2 + (r_b^m \cos m\theta_b)^2} \quad (\text{B.39})$$

$$= \frac{1}{Gm} \sqrt{r_b^{2m} - (r_b^m \cos m\theta_b)^2} \quad (\text{B.40})$$

$$= \frac{r_b^m \sin m\theta_b}{Gm} \quad (\text{B.41})$$

It can be seen that when $n = 0$ (uniform speed) and $m = 1$ the minimum-time path is the minimum-distance straight line to the imaginary axis, as expected.

The full solution for the minimum-time path to destabilisation of the mode at $|\lambda_i|e^{j\theta_0+j\pi/2}$ is therefore given in Equations B.42 to B.44 (the $\pi/2$ offset is required since the above derivation considers the imaginary axis as $\theta = 0$).

$$r(\theta) = |\lambda_i| \cos((1-n)\theta_0 + k) \sec((1-n)\theta + k)^{\frac{1}{1-n}} \quad (\text{B.42})$$

$$\tan k = \frac{\cos((1-n)\theta_0) - \cos((1-n)\theta_0)^{1-n}}{\sin((1-n)\theta_0)} \quad (\text{B.43})$$

$$t_{\min, \min} = \frac{|\lambda_i|^{1-n} \sin(1-n)\theta_0}{G(1-n)} \quad (\text{B.44})$$

To consider the imaginary axis as $\theta = \pi/2$ instead simply add $\pi/2$ to the θ_0 in Equation B.44.

C Appendix: Constraint calculations for Chapter 4

No correction for the possibility of changing residue is performed in these calculations, essentially assuming that $R_{\max} = R$.

C.1 Constraints 1 and 2

$$\lambda_i = 0.81 + j24.21 \text{ Hz} \quad (\text{C.1})$$

$$R_{QD} = 2.04 - j2.15 \text{ p.u./Hz} \quad (\text{C.2})$$

$$\frac{\partial Y_{DQ}(\lambda_i)}{\partial \rho} = -0.31 + j0.05 \text{ p.u.} \quad (\text{C.3})$$

$$\frac{\partial \lambda_i}{\partial \rho} = -0.52 + j0.78 \text{ Hz} \quad (\text{C.4})$$

$$\text{Constraint 1} = \mathbb{R} \{ \lambda_i \} / \mathbb{R} \left\{ \frac{\partial \lambda_i}{\partial \rho} \right\} = 1.57 \quad (\text{C.5})$$

$$\text{Constraint 2} = \mathbb{R} \{ \lambda_i \} / \left| \frac{\partial \lambda_i}{\partial \rho} \right| = 0.87 \quad (\text{C.6})$$

C.2 Minimum- $\Delta\rho$ path constraint calculation

The residue-scaled admittance-parameter sensitivity is plotted in Figure C.1a for the neighbourhood of the whole-system mode and the open-loop pole. The approximated fit in the neighbourhood of the pole is shown in Figure C.1b. The fit is performed in terms of the distance r rad/s from the approximate pole location at $(-0.24 + j23.51)2\pi$ rad/s. The whole-system mode is at $(-0.81 + j24.21)2\pi$ rad/s. The calculation in Equation C.8 makes use of the formula stated in Equation 4.16, since the minimum-delta rho path is straight to the centre of the open-loop pole, then straight to the imaginary axis.

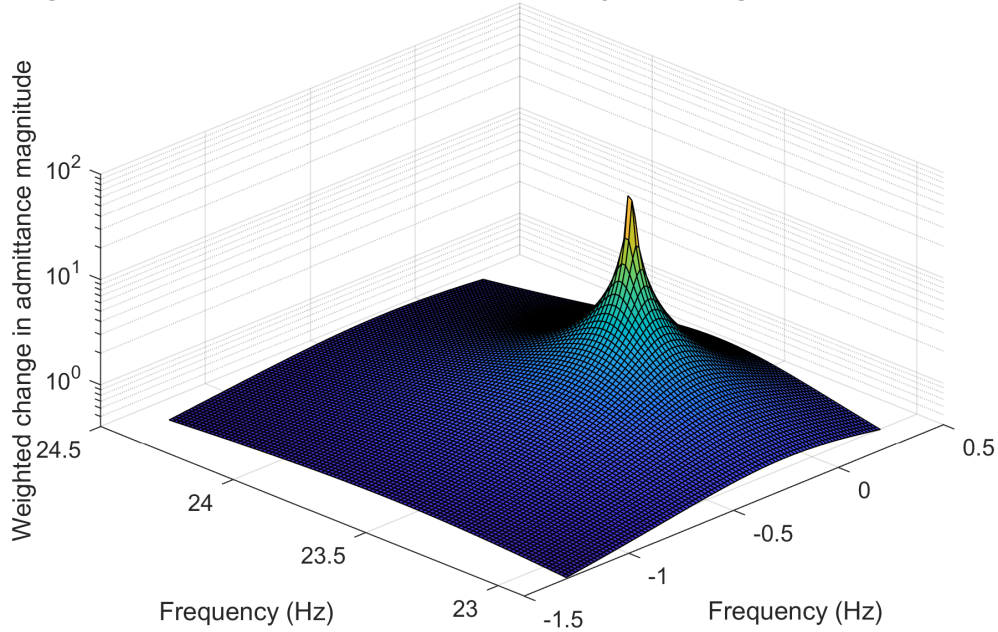
$$\left| R_{QD} \frac{\partial Y_{DQ}}{\partial \rho}(r) \right| \approx 34(r)^{-1} \text{ rad/s} \quad (\text{C.7})$$

$$\Delta\rho_{\min} = (|-0.81 + j24.21 - (-0.24 + j23.51)|^2 + |-0.24|^2)(2\pi)^2 / (2 * 34) \quad (\text{C.8})$$

$$= 0.51 \quad (\text{C.9})$$

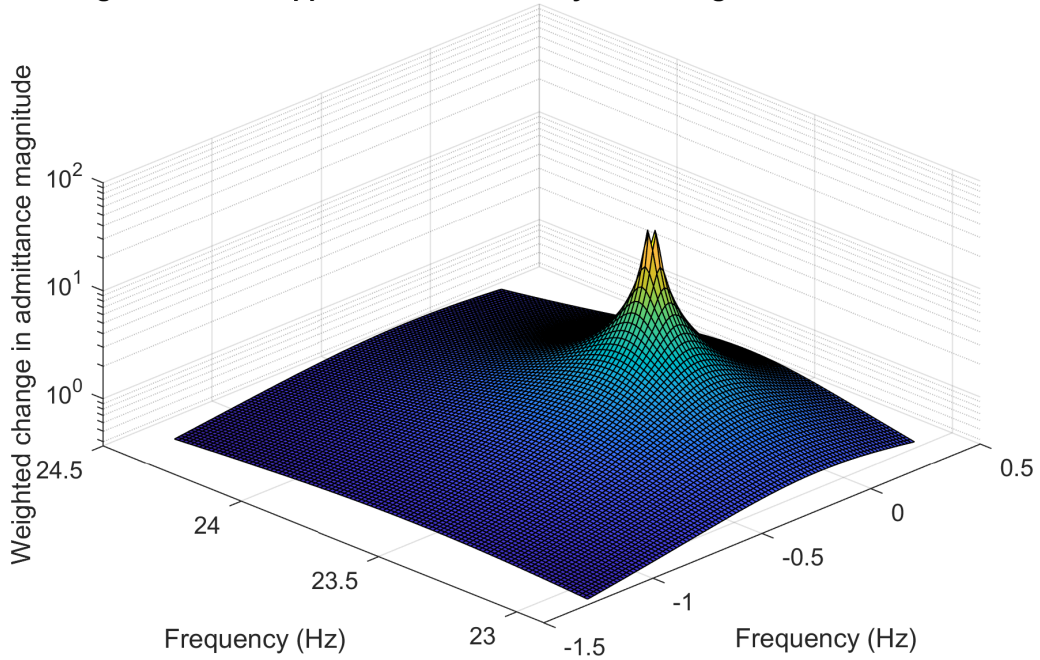
The variation of $\Delta\rho_{\min}$ with the angle of the interception with the imaginary axis, relative to the pole (θ_b) is plotted in Figure C.2.

Magnitude of the admittance-parameter sensitivity in the neighbourhood of the mode



(a)

Magnitude of the approximated sensitivity in the neighbourhood of the mode



(b)

Figure C.1: The admittance-parameter sensitivity and its approximation in the neighborhood of the whole-system mode, where the peak due to the open-loop pole is clearly visible

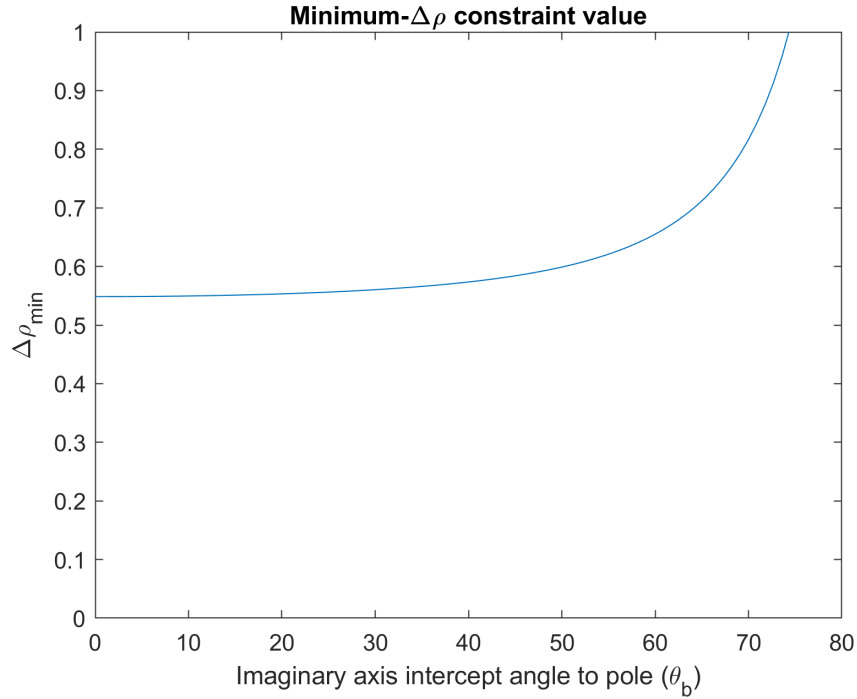


Figure C.2

D Appendix: Modelling data

This section lists modelling data used in this thesis.

In the Simplus toolbox, the grid-following controller PI controllers $f_{cc}(s)$, $f_{DC}(s)$ and $f_{PLL}(s)$ are designed such that [57, 177]:

$$f_{DC}(s) = (V_{DC}C_{DC}\sigma_{dc})(1 + \sigma_{dc}/(4s)) \quad (D.1)$$

$$f_{PLL}(s) = \sigma_{pll}(1 + \sigma_{pll}/(4s)) \quad (D.2)$$

$$f_{cc}(s) = \omega L\sigma_{cc}(1 + \sigma_{cc}/(4s)) \quad (D.3)$$

Where σ_{dc} , σ_{pll} and σ_{cc} are the closed-loop bandwidths in rad/s of the DC, PLL and current controllers respectively.

For the modified 14-bus network used in this case study, the original network comes from the models built by the KIOS centre at the University of Cyprus [171]. It was modified in [48], and modified again here.

Grid-following IBR were inserted at buses 6, 11, 12 and 13 as shown in Figure 3.12. Their

control parameters are shown in Table D.1.

The synchronous machine data can be found in [171].

IBR	ωL	R	C_{DC}	V_{DC}^0	Current control Bandwidth	DC control Bandwidth	PLL Band- width	P_0	Q_0
6	0.03	0.01	1.25	2.5	400 Hz	10 Hz	35 Hz	0	0.12
11	0.03	0.01	0.1	1	500 Hz	5 Hz	5 Hz	-1	0
12	0.03	0.01	1.25	2.5	400 Hz	10 Hz	25 Hz	-0.1	0
13	0.03	0.01	1.25	2.5	500 Hz	10 Hz	20 Hz	-0.1	0

Table D.1: Parameter values for the IBR present in the IEEE test system. The DC capacitor size for IBR 11 has been reduced to better illustrate the DC voltage deviations. Additionally, IBR 12 is 'poorly tuned' by increasing the DC control integral gain to 5x that the standard in the Simplus toolbox [57], and has a 2 p.u. outer qq virtual admittance gain.

The SIMULINK model generated by the SIMPLUS toolbox for the 14-bus system is shown in Figure D.1. The buses are displayed in increasing numerical order in a downwards diagonal from the top left corner to the bottom right corner. The labelled apparatus names correspond to the labels in Figure 3.12 in Chapter 3, where a label starting "IBR" indicates a grid-following converter and a label starting "SM" indicates a synchronous machine. The network dynamics are modelled with the branch components between the buses. The increase in load at bus 13 (which was used as the input disturbance in Chapter 3.6.1) is modelled with the component labelled "Load Step".

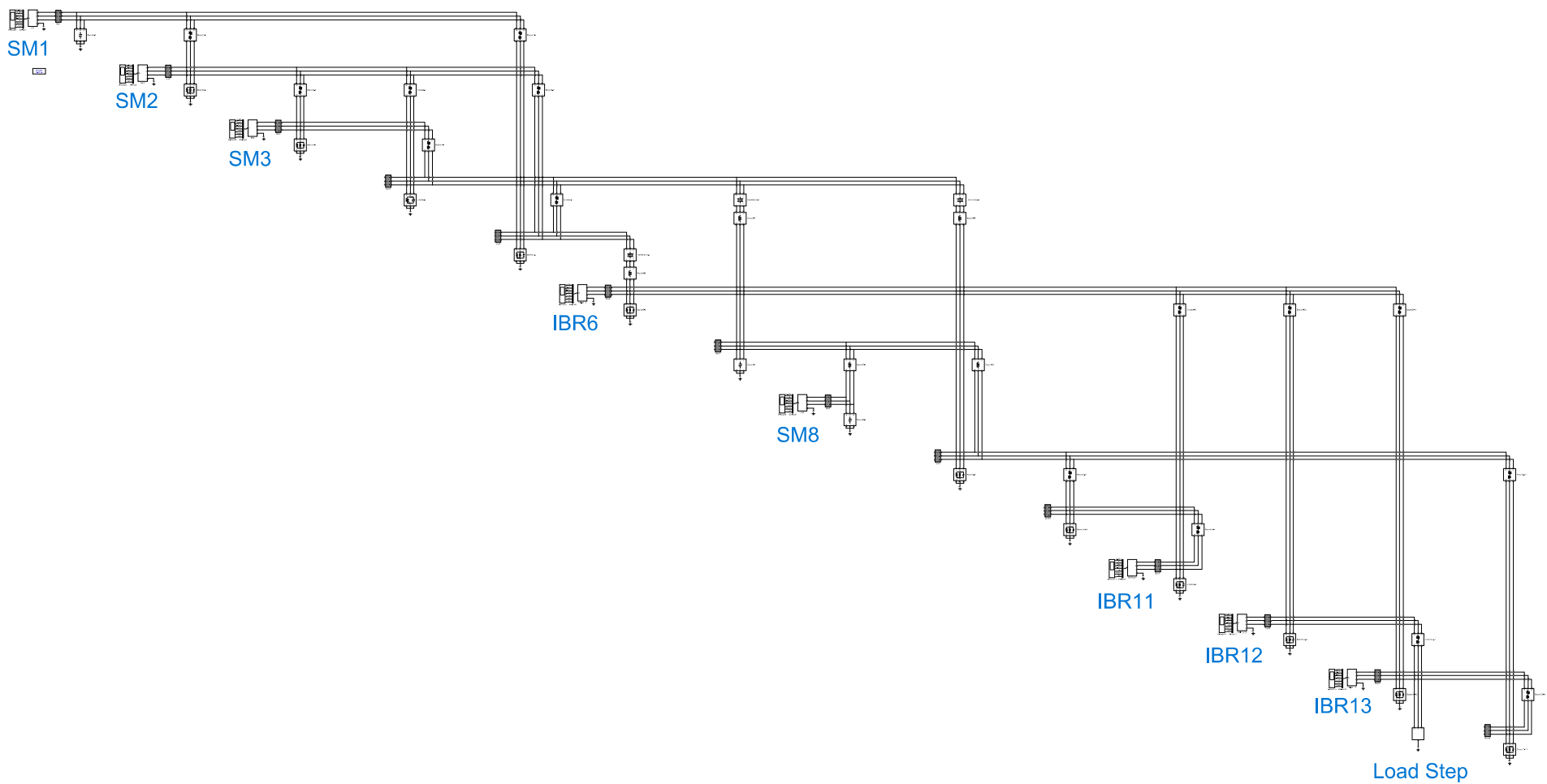


Figure D.1: The SIMULINK model used to generate the 14-bus network time-domain simulation results in Chapter 3.6.1