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THE GROWTH OF HAIL

by

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ABSTRACT.

A hailstone grows by the accretion of water droplets in a supercooled cloud. The temperature of the surface of the hailstone is raised above that of the environment by this process until the rate of release of heat by the freezing droplets is balanced by the rate at which heat is transferred from the stone. The experimentally determined heat transfer coefficients for spheres indicate that once a spherical stone acquires a diameter of the order of centimetres, it is likely to accrete more water than can be frozen. If the size of the stone and conditions in the cloud are such that all the water accreted can be frozen, its growth is said to be "dry", while, in the other circumstance, a film of water covers the surface of the stone and its growth is said to be "wet". Previously it has been supposed that any liquid in excess of that which can be frozen is shed in the wake of the stone. This imposes a considerable restriction on its growth rate and there has therefore been some difficulty in explaining how large hailstones are formed in reasonable times.

Experimental investigations of the factors affecting the nature of the ice accreted during both dry and wet growth have been carried out. The dependence of the density and structure of ice formed in this way on the airspeed, ambient temperature, droplet size and liquid water concentration in the supercooled cloud, and on the size of the accreting object have been determined.

The density of ice deposits formed during dry growth ranged from 0.1 to 0.9 g cm⁻³ depending on the value of the quantity $-r v_0/T_s$, r being the median volume radius of the droplets comprising the cloud, v_0 the speed with which the droplets struck the object and T_s the mean temperature of the accreting surface. The term $r v_0$ is interpreted as being a measure of the force with which the droplets were packed together and T_s the temperature which governed their freezing rate. At high values of $r v_0$ and T_s the droplets were distorted and spread considerably before freezing.

Both of these processes resulted in closer packing and hence higher densities.

It was discovered that the excess water which could not be frozen during wet growth was not shed in the wake of the accreting object but was incorporated into the ice structure, giving a "spongy" deposit. It was estimated that some of these deposits contained at least 50% of unfrozen liquid. It was found also that in this condition the deposits could readily accrete substantial quantities of ice particles, comparable in fact to the concentration of the liquid water present in the supercooled cloud. These two phenomena undoubtedly affect the growth of hail in the wet condition and the restriction formerly thought to be imposed on the growth rate may now be removed.

An important parameter determining the growth rate of a hailstone is its fallspeed. Measurements have been made of the speeds of ice objects resembling hailstones during free fall, and of the drag coefficients of such objects and of natural hailstones. The drag coefficients ranged from those of smooth spheres (about 0.45) to 0.8, depending on the shape and surface characteristics of the stones. A reasonable mean value for the drag coefficients of hailstones with diameters of a few centimetres is 0.6, which implies fallspeeds 15 to 20% less than those of smooth spheres of the same mass and density.

A study has been made of some aspects of two severe storms which occurred during the course of this work. These include a detailed investigation of the distribution of hail at the ground, the characteristics of the hailstones which fell during the storms and estimations of their spatial concentrations. The densities of some 160 stones from one of the storms were all found to be greater than 0.87 g cm^{-3} . A structural analysis of sixty of the stones showed that over 90% had transparent cores which suggested that growth was initiated from frozen drops 1 to 2 mm in diameter. Calculations based on the experimentally determined densities of

ice deposits formed during dry growth, together with considerations of the structure of the collected hailstones, have indicated that the stones were initiated at low levels in the cloud where the temperature was -5 to -10°C . Some of the stones had structures which suggested that during their growth they had alternated between the dry and wet growth regimes, while others apparently spent most of their growth time in the wet condition. The experimentally determined densities also predict with reasonable accuracy the comparatively low densities of soft hail.

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LIST OF SYMBOLS.

a	heat transfer coefficient
A	drop size distribution (median volume diameter 22μ)
b	mass transfer coefficient
B	drop size distribution (median volume diameter 47μ)
c	specific heat ($\text{cal g}^{-1} \text{ } ^\circ\text{C}^{-1}$)
C	drop size distribution (median volume diameter 65μ)
C_D	drag coefficient
d	diameter : of cylinder (cm), of droplet (μ)
d_o	initial diameter of riming cylinder
E	efficiency of catch of an object for droplets
g	acceleration due to gravity (cm sec^{-2})
h	pressure head of water (cm)
k	coefficient of diffusion of water vapour in air ($\text{cm}^2 \text{ sec}^{-1}$)
K	coefficient of thermal conductivity of air ($\text{cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$)
l	length of cylinder (cm)
L_f	latent heat of fusion of water (cal g^{-1})
L_v	latent heat of vaporisation of water or ice (cal g^{-1})
$Q_{(1,2,3)}$	quantity of heat (cal)
r	radius of droplet (μ)
R	radius of spherical hailstone (cm)
Re	Reynolds number
t	time (seconds)
T	ambient temperature ($^\circ\text{C}$)
T_1	initial temperature ($^\circ\text{C}$)
T_s	mean temperature of a riming surface ($^\circ\text{C}$)
v	airspeed (cm sec^{-1})
v_c	critical speed of impact for splashing of droplets (m sec^{-1})
v_o	speed of impact of droplets (m sec^{-1})
V	fallspeed of hailstone (cm sec^{-1} , m sec^{-1})
W	liquid water concentration (g cm^{-3} , g m^{-3})
W_c	critical liquid water concentration (g cm^{-3} , g m^{-3})

LIST OF SYMBOLS.

Δ	difference (in temperature, vapour density)
η	dynamic viscosity of air (poise)
μ	microns
ρ	density (g cm^{-3})
ρ_a	density of air (g cm^{-3})
ρ_1	density of air relative to air at 0°C , 1000 mb pressure
τ	freezing time of droplet

1. INTRODUCTION : THE THEORY OF THE GROWTH OF HAILSTONES.

1.1 Definitions.

The term "hail" is applied to a variety of solid hydrometeors which are precipitated from supercooled clouds. The International Commission for Ice and Snow has recommended the following definitions (International Cloud Atlas, 1956) which broadly categorise the various forms of such precipitation.

1.1.1 Snow pellets (soft hail, graupel)

White and opaque grains of ice. These grains are spherical or sometimes conical with diameters ranging from 1 to 6 mm. They are composed largely of small cloud droplets individually frozen together, have a low density and are readily crushed. They may break up on striking a hard surface.

1.1.2 Ice pellets (grains of ice, small hail)

Transparent or translucent pellets of ice. They are spherical or irregular and have diameters of a few millimetres. There are two main types of this form of precipitation: (a) frozen raindrops or largely melted and refrozen snowflakes, the freezing process usually taking place near the ground, and (b) pellets of snow encased in a thin layer of ice which has formed from the freezing either of droplets intercepted by the pellets or of water resulting from the partial melting of the pellets.

1.1.3 Hail

Small balls or pieces of ice (commonly termed hailstones). Their diameters range from about 5 mm upwards. Falls of hail are generally observed during thunderstorms, a moderately severe storm yielding stones a few centimetres in diameter while a very severe storm may give rise to stones having a maximum dimension of 10 cm or more. Hailstones are composed, almost exclusively, of transparent

ice or of a series of layers of ice alternately transparent and opaque.

The work described in this thesis is primarily concerned with hail or hailstones as defined above (1.1.3) although some of the experimental results obtained are applicable to the other two categories.

1.2 Reynolds' theory and experiments.

Hail, being a widespread natural phenomenon, has aroused considerable interest and innumerable reports of hailstorms may be found in the literature (for example, review articles by Russell 1893, Trabert 1899, Wegener 1924, Hann and Suring 1926, Souter and Emerson 1952, Weickmann 1953 and Mason 1957). However, virtually all of these reports are confined to unusual characteristics either of the hailstones (size and shape) or of the storm itself (severity, lightning displays, damage etc.). Theories of the formation and growth of hail within the parent cloud have also been numerous but necessarily qualitative. Two essentially different growth mechanisms were proposed by early writers and given support by various workers, namely that hail grows either by the aggregation of cloud elements at temperatures below 0°C due to their relative motion or by the condensation of vapour directly on an ice nucleus. In some cases it was thought that the aggregation or condensation was promoted by electrical forces. The discovery by Dufour (1861) that water could exist in the liquid state at temperatures below 0°C gave rise to the possibility that the aggregating particles could be supercooled droplets which would freeze on coming into contact with an ice particle. As will be seen later, it is now evident that condensation can play little or no part in the growth of hailstones, and, at the commencement of the present investigations, it was generally accepted that hail grows predominantly by the accretion of supercooled water droplets (Mason 1957, p. 262).

One of the first reasonable accounts of the growth of cloud

elements was given by Reynolds (1876). He considered that if some of the particles in a cloud attained a larger size than the others, they would fall faster and grow by collecting smaller ones which lay in their path; either raindrops or hailstones would be formed depending on whether the cloud was composed of water or ice. He was led to this view by his observations of conical hail which had "the appearance of being an aggregation of minute particles of ice fitting closely together, but without any crystallisation such as that seen in the snowflake." Their conical shape he attributed to some of the accreted particles overhanging the edge of the lower face so that it continually grew broader, and he explained the observed increase in density towards the base of the stone as being due to an increase in the velocity of fall as it grew in size which forced the particles into a more compact mass. Although Reynolds was aware of Dufour's experiments on the supercooling of water and admitted the possibility of supercooled droplets contributing to the growth of hail, he did not think that such a mechanism would give rise to the structures he observed.

Reynolds carried out some interesting experiments to confirm his ideas. By introducing a stream of plaster of paris into a jet of steam which was directed against a rod, he obtained deposits which were roughly conical with their bases facing the jet. Similar results were obtained by blowing naphthalene vapour against the rod.

In a later paper he describes an apparatus with which he produced conical ice particles (Reynolds, 1877). Water and ether droplets were sprayed side by side, the ether evaporating and chilling the water droplets (Reynolds in fact considered the water droplets to be frozen by this process). The spray was directed against splinters of wood and structures very similar to conical hail were formed. He estimated from the speed of the jet that the larger deposits (slightly more than 1 cm thick) would have

traversed one to three miles of "frozen" spray. Since on humid days he could produce pellets of this size without using the water spray (but simply relying on the water vapour present in the air), he considered that the water concentrations in dense clouds were comparable with the ones he used. From this he deduced that the clouds would have to be one to three miles thick to produce hailstones of comparable size and that this would take at least five to ten minutes.

1.3 Schumann's treatment of the growth of hailstones

Using a number of simplifying assumptions, Schumann (1938) produced the first really quantitative treatment of the growth of hailstones. He sought to answer two difficult questions : (1) how is it possible for a hailstone to remain suspended in the parent cloud for a sufficient length of time for it to acquire the relatively large dimensions (diameters of up to 10 cm) sometimes observed? and (2) how is the heat of fusion of the supercooled droplets, which collect and freeze on the surface of the stone, dissipated?

Schumann assumed that a hailstone grows as a spherical object by accreting all the supercooled droplets in the volume of air it sweeps out. In travelling a distance dS relative to the air, the increment in hailstone radius, dR , is then given by

$$W dS = 4\rho dR \quad (1.1)$$

where W is the liquid water concentration in the cloud and ρ is the density of the added ice. If the total distance travelled is S then, assuming ρ constant,

$$W S = \int_{0.25}^R 4\rho dR = \rho(4R - 1) \quad (1.2)$$

Schumann chose the lower limit of $R = 0.25$ cm for this integral because hailstones of 0.5 cm diameter are of frequent occurrence indicating that conditions favourable for the formation of small sized hail are quite common.

Further, if the corresponding time interval is dt , then

$$4 \rho \frac{dR}{dt} = W \frac{dS}{dt} = WV \quad (1.3)$$

where V is the fallspeed of the stone. Using an average value for the drag coefficient of smooth spheres obtained by Bilham and Relf (1937), Schumann deduced the relation

$$V = 2000 \left(\frac{\rho R}{\rho_1} \right)^{\frac{1}{2}} \quad (1.4)$$

where ρ_1 is the relative density of air with respect to air at 0°C and 1000 mb pressure. Substituting this in eq. 1.3 and assuming an average value for ρ_1 , integrating between the above limits gives

$$500 W t = (\rho \rho_1)^{\frac{1}{2}} (2R^{\frac{1}{2}} - 1) \quad (1.5)$$

where t is the growth time. Finally, if U is the upward speed of the air supporting the stone, the distance Z travelled by the hailstone relative to a fixed level is

$$Z = S - U t \quad (1.6)$$

$$\text{i.e. } Z = \frac{\rho (4R - 1)}{W} - \frac{U (\rho \rho_1)^{\frac{1}{2}} (2R^{\frac{1}{2}} - 1)}{500W} \quad (1.7)$$

Schumann used this equation to determine the dependence of the final radius attained by a spherical stone in falling through a supercooled cloud from a fixed height, on the values of W and U . An example of his computations is shown in Fig. 1.1. On the basis of a number of such examples he concluded that the "conditions essential for the building up of large hailstones are that the cumulus clouds in which the stones are formed must extend to considerable heights, and that there must either be extraordinarily high concentrations of condensed water or there must be extremely violent upcurrents to maintain the hailstone for a sufficient length of time in the environment favourable to its growth."

The concentration of liquid water condensed during adiabatic ascent from a condensation level at 900 mb (about 1 km above the ground) as a function of height in the cloud and temperature at the condensation level is shown in Fig. 1.2. It is seen that the

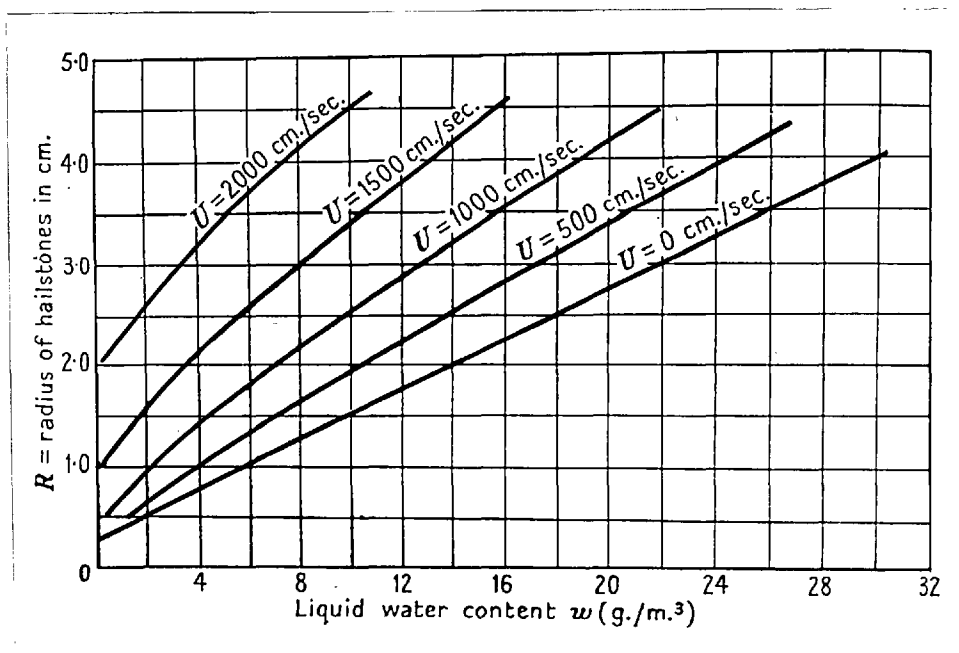


Fig. 1.1 The final radius attained by a hailstone of mean density 0.6 g cm^{-3} falling from a height of 7 km (-20°C) to 4 km (0°C) for different values of the liquid water concentration and updraught velocity. An average value of 0.533 has been taken for ρ_1 .
(from Schumann, 1938).

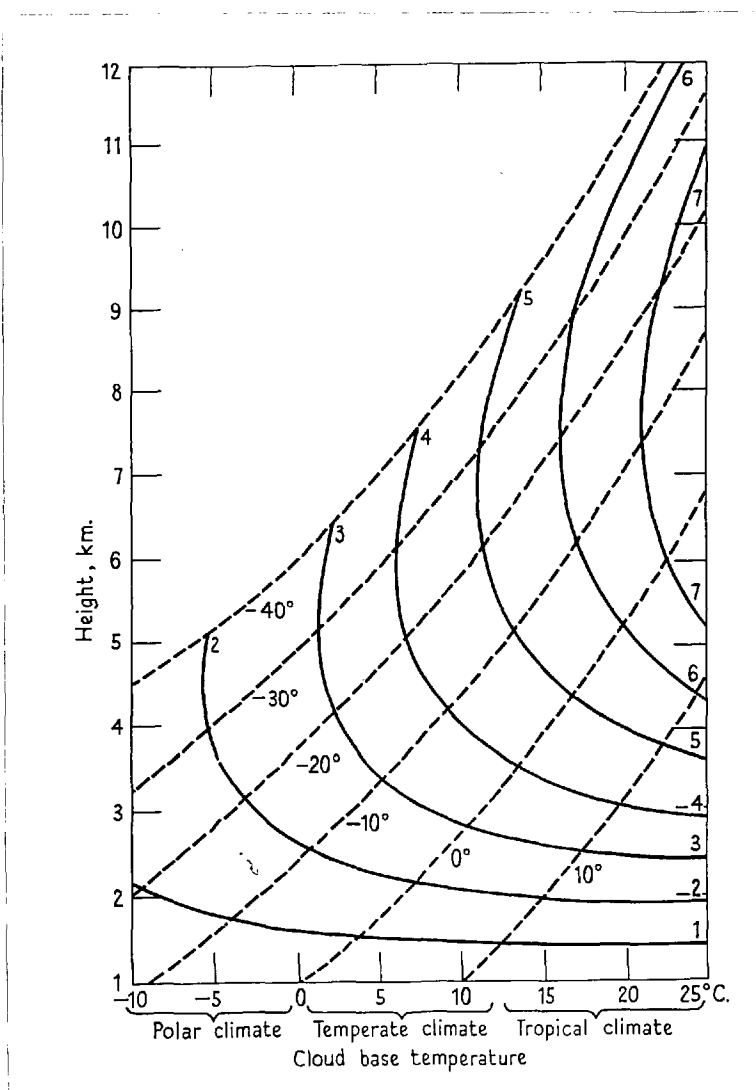


Fig. 1.2 Concentrations (in g m^{-3}) of liquid water condensed during adiabatic ascent from a condensation level at 900 mb (about 1 km above the ground) as a function of height and temperature at the condensation level.

(from Ludlam, Mason 1957. p. XIV)

maximum value attained in this way is 7 or 8 g m^{-3} so that the larger concentrations used by Schumann are excessive. Measurements of the rates of rise of the tops of radar echoes received from moderately severe storms indicate updraughts of up to 30 m sec^{-1} (Ludlam, 1959); in intense storms larger updraughts probably occur. The values for the updraught employed by Schumann are therefore reasonable and his calculations indicate that clouds of moderate liquid water concentrations and updraughts may produce hail of a few centimetres diameter.

In the second part of his paper, Schumann considered the heat economy of the growing hailstone and showed that, under certain conditions, not all of the accreted water can be frozen. The heat liberated by the freezing droplets may be dissipated in three ways: (1) the absorption of heat by the stone (2) the conduction of heat to the air and (3) the evaporation of water vapour from the surface of the stone. Ignoring the first as relatively insignificant, Schumann attempted to derive quantitative expressions for the heat transferred by the other two processes, from which he deduced that, near the 0°C level, the hailstone is definitely unable to assimilate all the liquid water it sweeps up. (The heat and mass transfer coefficients given by Schumann are inexact and his full treatment of this problem is therefore not given here; in Ludlam's theory (see next section) these coefficients are given values which have been experimentally determined.) He also demonstrated clearly that, because the temperature of the surface of a growing stone is raised above that of the environment by the freezing process, evaporation of water vapour takes place; it is readily seen from the tabulated values of the saturated vapour densities over water and ice surfaces that, even at -30°C , the temperature of the surface need only be raised 2.9°C above that of the environment for vapour to be transferred from the stone. Condensation cannot therefore contribute to the growth of a hailstone as suggested by a number of earlier writers; in fact the growth rate is reduced slightly by the mass

transfer to the environment.

1.4 Ludlam's theory of hailstone growth.

The problem of the dissipation of heat from the surface of a growing hailstone has been considered quantitatively by Ludlam (1950, 1958). The release of heat of fusion by the freezing droplets causes the temperature of the surface of the stone to become raised above that of the environment. The rate at which heat is transferred from a ventilated sphere by conduction to the airstream is given by

$$\frac{dQ_1}{dt} = 4\pi R a K \Delta T \quad (1.8)$$

where a is a coefficient whose value varies with the Reynolds number (Re), K is the thermal conductivity of air and ΔT is the difference in temperature between the surface of the sphere and the undisturbed flow. The value of a quoted by Ludlam is that determined experimentally by Kramers (1946), namely

$$a = 1.6 + 0.295 Re^{\frac{1}{2}} \quad (1.9)$$

The rate of transfer of heat due to evaporation of water vapour from the surface is

$$\frac{dQ_2}{dt} = 4\pi R b L_v k \Delta \rho \quad (1.10)$$

where b is a coefficient dependent on the Reynolds number, L_v is the latent heat of vaporisation of water, k is the coefficient of diffusion of water vapour in air and $\Delta \rho$ is the difference between the vapour density at the surface and that in the airstream.

The value of b deduced from experiments by Frössling (1938) is

$$b = 1 + 0.229 Re^{\frac{1}{2}} \quad (1.11)$$

The two transfer coefficients, a and b , are regarded as valid for values of the Reynolds number up to 10^5 which virtually covers the complete range of hailstone sizes. Further, since particles of a few millimetres diameter falling at their terminal speeds have Reynolds numbers greater than 400, and, since the experimentally determined expressions can be regarded only as

approximate when applied to this problem, Ludlam takes $a = b = 0.3 \text{ Re}^{\frac{1}{2}}$.

Now the rate at which the hailstone collects water is $\pi R^2 V W$ (the efficiency of catch of a hailstone for the droplets comprising the cloud may be taken to be unity and the terminal velocities of the droplets may be neglected in comparison with the fallspeed of the stone) so that the rate of liberation of heat is

$$\frac{dQ_3}{dt} = \pi R^2 V W (L_f + T) \quad (1.12)$$

where L_f is the latent heat of fusion of water and T is the temperature of the added water in $^{\circ}\text{C}$ (i.e. ambient temperature). This assumes that all the accreted water shall be just frozen, that is, the surface temperature is just maintained at 0°C , which gives

$$\pi R^2 V W_c (L_f + T) = 4\pi R a (L_v k \Delta \rho - K T) \quad (1.13)$$

where W_c is the "critical" liquid water concentration for this condition. Some heat may be conducted into the interior of the stone; Ludlam has shown, however, that the heat required to raise the temperature of the whole stone by 30°C is equal to that removed by the airstream in only a few seconds, and may therefore be neglected.

When, for given ambient conditions and hailstone radius, the concentration of cloud water is less than the critical value given by this equation, the temperature of the surface of the stone is below 0°C ; all the accreted water is frozen and Ludlam speaks of growth in the "dry" regime; the growth rate is then given by eq. 1.3. On the other hand, when the concentration exceeds W_c , more water is collected than can be frozen, the excess accumulates as a liquid film or is shed in the wake of the stone, and Ludlam refers to growth in the "wet" regime. He now assumes that transparent ice of density about 0.9 g cm^{-3} is deposited beneath the liquid film so that the growth rate is governed by

the rate of transfer of heat to the environment, whence

$$4 \pi R^2 \rho (L_f + T) dR = 4 \pi R a (L_v k \Delta \rho - KT) dt \quad (1.14)$$

Substituting for ρ and a gives

$$\left(\frac{R}{V}\right)^{\frac{1}{2}} \frac{dR}{dt} = \left(\frac{2\rho_a}{\eta}\right)^{\frac{1}{2}} \frac{(L_v k \Delta \rho - KT)}{3(L_f + T)} \quad (1.15)$$

where ρ_a and η are the density and viscosity of air respectively, the expression on the right hand side being a function of height only. Since the two terms on the right hand side of eq. 1.14 are comparable in magnitude, the rate of change of mass due to evaporation is approximately the fraction $L_f + T/2L_v$ of the growth rate. This is only a few percent and is ignored.

In order to carry out computations of the growth of hailstones using eqs. 1.3 and 1.15 it is necessary to specify the conditions within the supercooled cloud. Fig. 1.3 shows the adiabatic values of temperature, air density and liquid water concentration as a function of height in a model cloud with a base at 900 mb, 20°C. Using these values and the two growth equations, Ludlam has obtained Fig. 1.4 which depicts the subsequent growth of two small cloud droplets, frozen at temperatures of -5 and -10°C respectively, in a persistent and constant updraught of 20 m sec⁻¹. The density of the stones during dry growth, which is represented by the dashed portion of the curves, is assumed to be 0.3 g cm⁻³; the stones become wet at about 10 and 11 km respectively, with radii of 0.6 and 0.9 cm, and they finally attain radii at the 0°C level of about 1.2 and 1.5 cm. This is about the maximum size attainable on this basis for, if the updraught is increased to increase the growth time, the stones are carried above the -40°C level. Here all the cloud water is frozen due to either heterogeneous or homogeneous nucleation (Langham and Mason, 1958) and growth ceases. This maximum value is small by comparison with the diameters of 6 to 10 cm with which hailstones occasionally reach the ground. The only way to grow large stones is to increase the time they spend in the

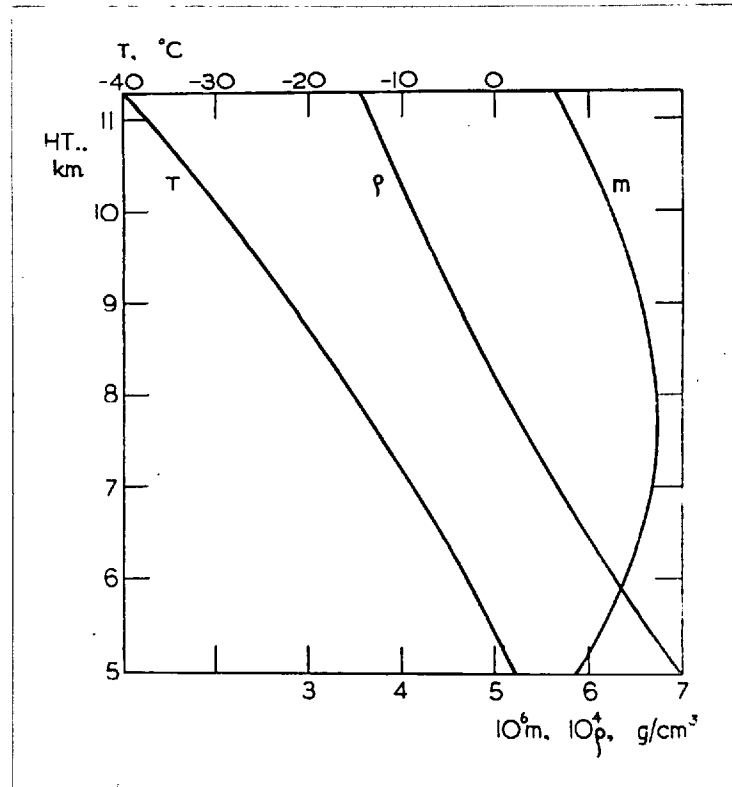


Fig. 1.3 Adiabatic values of temperature (T), air density (ρ) and liquid water concentration (m) as a function of height in the model cloud with base at 900 mb, 20°C.

(from Ludlam, 1958).

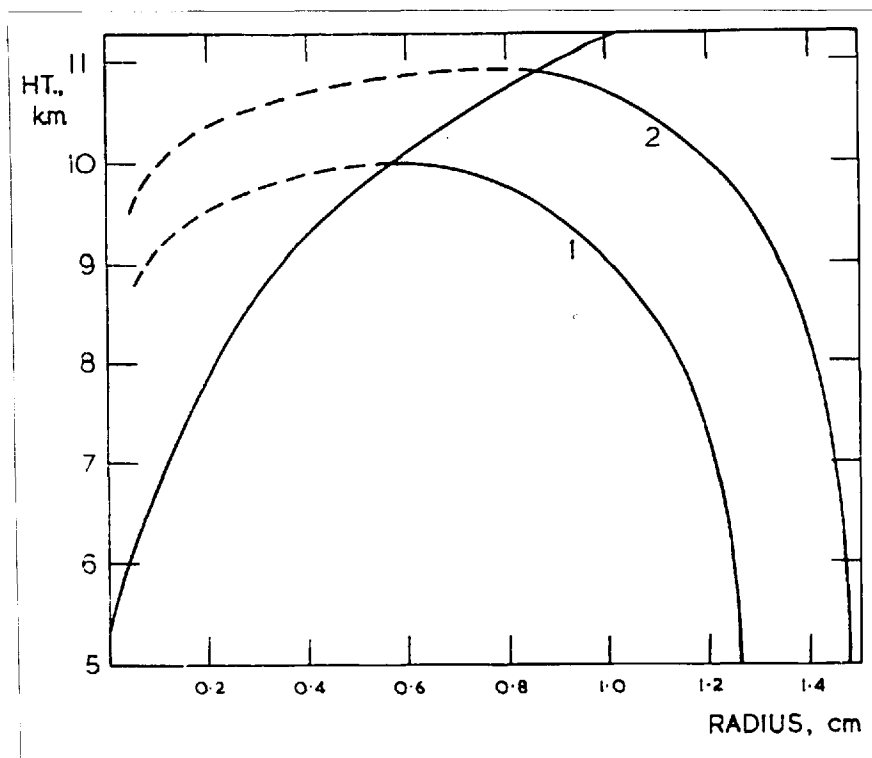


Fig. 1.4 Growth of hailstones in the model cloud containing the adiabatic liquid water content. The curves marked (1), (2) show the growth with $U = \text{constant} = 20 \text{ m sec}^{-1}$, following the freezing of small cloud droplets at -5 and -10°C respectively.

(from Ludlam, 1958)

regions of the cloud between the 0 and -40°C levels, which, Ludlam concludes, is probably brought about by an intermittent updraught.

The restriction imposed on the growth rate of hailstones in the wet growth regime is considerable, particularly in the case of large stones. This is illustrated in Fig. 1.5 which shows the condition for the entry of hailstones into the wet growth phase in the model cloud. The radius of the hailstone is given as a function of the ambient temperature at which the surface is just wet at various liquid water concentrations in an environment whose properties are defined in Fig. 1.3. The isopleth for $W = 6$ corresponds approximately to the adiabatic liquid water concentration. If the position determined on the diagram by the temperature and size is to the left of the appropriate water concentration isopleth, the stone is in the dry growth regime; if the position is on the right hand side, the surface of the stone is wet.

In constructing this diagram, it has been assumed that hailstones fall with the speeds of smooth spheres of density of 0.90 g cm^{-3} . The high value for the density is justified later (section 6.2). It has been found that the drag coefficients of actual stones of markedly aspherical shape are almost double those of spheres of the same mass (section 4.3), but they fall in a preferred attitude in which the rate of accretion on the collecting surface is considerably increased, so that even in these cases it is thought that the critical liquid water concentrations are unlikely to be underestimated by a factor of as much as 1.5 to 2. These curves are therefore approximately applicable to natural hailstones and even in modest liquid water concentrations it is seen that large hailstones must be often in the wet growth regime.

Some computations of the growth times of hailstones, based on a combination of the dry and wet growth equations, have been made by the writer (Fig. 1.6). The liquid water concentration has been taken to be 4 g m^{-3} and the density of the stones 0.90 g cm^{-3} . At temperatures as low as -30°C the stones rapidly enter the wet

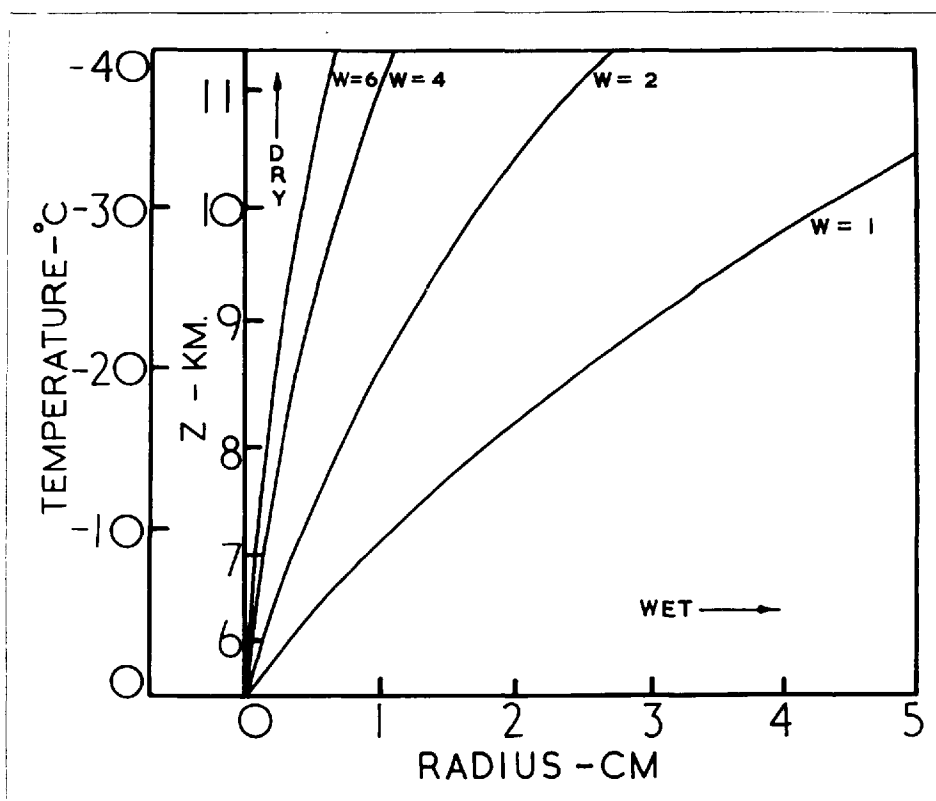


Fig. 1.5 The condition for the entry of hailstones into the wet growth regime in the model cloud.

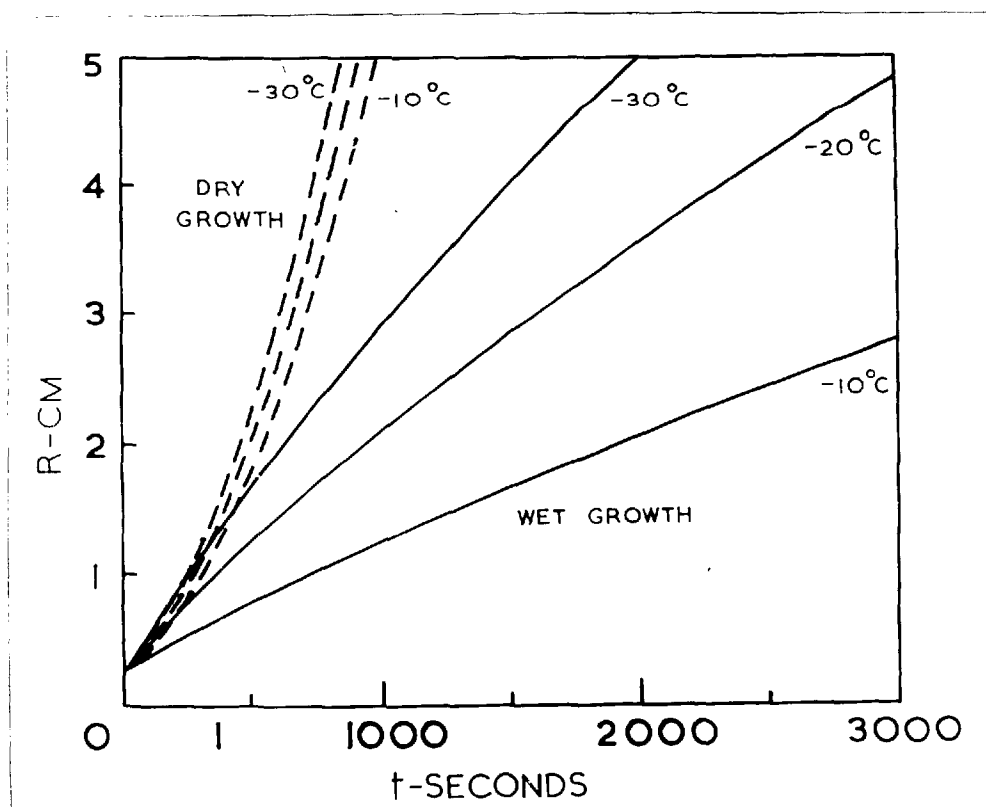


Fig. 1.6 Growth times of hailstones in the model cloud, assuming a liquid water concentration of 4 g m^{-3} , computed from a combination of the dry and wet growth equations. For comparison the curves representing dry growth have been extended (dashed curves).

regime and grow at a comparatively slow rate.

It should be pointed out here that, in establishing eq. 1.15, Ludlam has not considered the possibility that any excess water shed in the wake of the stone could have been warmed above ambient temperature. This gives another heat transfer term to be added to the right hand side of the equation since an additional amount of water $(W_c - W)^T/80$ is frozen if the temperature of the excess water is raised to 0°C also. This effect complicates the wet growth equation considerably.

1.5 The aims of the present investigation.

There are a number of problems which arise from the above theoretical considerations.

(1) A knowledge of the density of the accreted ice is necessary if the growth equations are to be used quantitatively. While it is reasonable to assume that the density of the material added during wet growth is about 0.9 g cm^{-3} , it is not at all obvious what values should be inserted in the dry growth equation since ice formed on objects near the ground in supercooled fogs has a density ranging from 0.1 to 0.9 g cm^{-3} (see section 2.1). The density may be expected to depend on the conditions within the cloud - temperature, liquid water concentration and droplet size - and the fallspeed of the hailstone.

(2) The theory put forward by Ludlam indicates that a hailstone which ultimately reaches a diameter of a few centimetres spends a considerable part of its growth time in the wet condition. The growth rate is thereby restricted and the theory has difficulty in explaining how large stones (6 to 10 cm diameter) can be formed in reasonably short times. (It is thought, on the basis of experience gained in field studies of cumulonimbus, that times of the order of 2,000 seconds, given by the wet growth equation for the growth of large hailstones, are excessive.)

(3) An important parameter in both growth equations is the fall-

speed of the hailstone and, in their computations, Schumann and Ludlam used values appropriate to smooth spheres. As hailstones have irregular surfaces and their shapes are sometimes markedly aspherical, this assumption may not be justified.

It is primarily with these problems that this thesis is concerned; at the time of commencement of the present investigations, no systematic study of them had been made under controlled experimental conditions. In addition some aspects of two severe thunderstorms which gave hail several centimetres across have been investigated, and stones collected from these storms have been subjected to a density and structure analysis.

2. THE DENSITY AND STRUCTURE OF ICE FORMED DURING DRY GROWTH.

2.1 Introduction.

Whenever supercooled droplets strike a cold solid object they freeze and form an ice deposit. Familiar examples of this phenomenon are rime formation during winter fogs and the icing of aircraft and growth of hailstones in supercooled clouds. The ice formed in this way may assume a wide variety of forms and structures and have a wide range of densities (0.1 to 0.9 g cm⁻³). The dependence of these characteristics on the possible parameters involved (wind velocity, ambient temperature, droplet diameter, liquid water concentration in the supercooled cloud, and size and shape of the accreting object) is not yet clearly understood.

A concerted effort on this problem has been made at the Mount Washington Observatory, U.S.A. (see, e.g., the Harvard-Mount Washington Icing Research Report 1946-47). Density measurements were made after exposing a number of coaxial cylinders, rotated to give a uniform deposit, within the orographic cloud frequently formed at the summit, and the results correlated with the various meteorological parameters listed above. Langmuir (1944) derived a set of empirical formulae based on seventeen series of such measurements using a set of six cylinders ranging in size from 0.2 to 7.6 cm (of which five series were later rejected as not fitting the formulae). These gave the rime density as a function of the ambient temperature, droplet radius and impact speed at which the droplets struck the cylinders, the density increasing with increases in each of these quantities. No dependence was found on the rate of accretion. He interpreted these factors as affecting the rate of freezing and impact energy of the accreted droplets and therefore as determining the penetration of freezing droplets into the existing rime structure.

In order to verify his formulae, Langmuir computed the densities of the rime deposits, the measured densities of which had been used in the analysis, and in plotting the computed against the measured values

found deviations from the expected straight line of 0.15 g cm^{-3} . This is a considerable error at low densities and, as Ludlam (1958) suggests, the formulae may be applicable only to conditions peculiar to low-level orographic clouds, since the cloud parameters are undoubtedly interrelated (as Langmuir himself showed). Calculations based on these formulae suggest that densities of 0.2 to 0.6 g cm^{-3} are likely to occur with small hailstones, but as the stones become larger values of 0.4 to 0.8 g cm^{-3} are more probable (Fig. 2.1).

Langmuir's analysis of the rime density data disagreed with an earlier finding by Arenberg (1943) who stated that, to a first approximation, the density of rime was independent of the drop size and was determined only by the rate of accretion and the ambient temperature. On the other hand, Loughborough (1946), also using data obtained at the Observatory, claimed that the density could be expressed simply as a function of the efficiency of catch of the collector.

On the basis of several years' observations made at Mount Washington, Clark (1948) classified rime deposits into two main families, the family of glazes and the family of rimes, the former having a shiny or glazed appearance which the latter lacked. These two kinds were further sub-divided so that there were six main categories: clear ice, with virtually no air entrapped; transparent ice, with moderate amounts of air entrapped in fairly large bubbles; milky ice, with considerable amounts of air enclosed as small bubbles; opaque rime, dull and white, which crumbled rather than cracked; kernel rime, similar in appearance to kernels of corn on a cob, and feathery rime, a deposit similar to kernel rime but of a more open and fragile structure. Clark showed that, between broad limits, the type of icing was related to its density (the density decreasing from 0.9 g cm^{-3} for clear ice to 0.1 g cm^{-3} for feathery rime) and was dependent on the ambient temperature, airspeed, liquid water concentration, drop size and cylinder diameter. The density increased with increases in the airspeed, liquid water concentration and droplet size and with temperature rise, but decreased with increasing cylinder diameter. However, he was

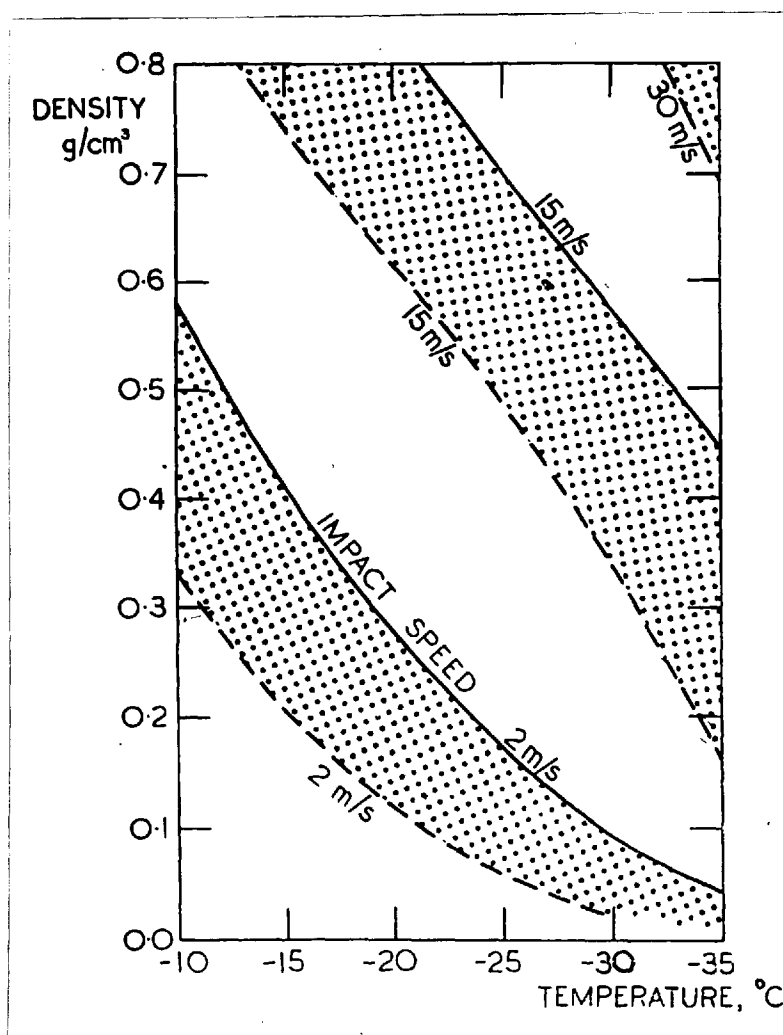


Fig. 2.1 Rime density according to empirical formulae of Langmuir. The curves shown represent rime density as a function of the speed of impact of supercooled droplets of radius $17\text{ }\mu$ (pecked lines) or $45\text{ }\mu$ and of the air temperature. They may also represent the density of the rime formed on hailstones having the corresponding fallspeeds. Fallspeeds of about 2 m sec^{-1} are associated with small stones just large enough to be precipitation; those of 15 m sec^{-1} with stones of diameter approaching 1 cm, and those of 30 m sec^{-1} with stones of radius 1 cm or more.

(from Ludlam, 1958)

unable to determine any explicit relationships between these parameters and the density.

Melcher (1951) has reported experiments in which he sprayed water droplets into a wind tunnel in a cold room, collecting a rime deposit on a stationary rod. He also found that the density of the deposit depended on the ambient temperature, liquid-water concentration and droplet size but did not investigate the effects of the airspeed or rod diameter. His results cover only a very narrow part of the range of conditions encountered in the atmosphere. Using a thermocouple embedded in the rime surface he showed that the temperature of the surface was raised above that of the surrounding air; increasing the water concentration raised the temperature of the rime still higher because of the greater rate of liberation of heat by the freezing droplets. Fraser, Rush and Baxter (1952) found in icing tunnel experiments that the transition from a rime to a glaze formation took place when the temperature of the ice surface neared 0°C .

Because of the uncertainties of the effects of the various cloud parameters on the density of rime and of the general applicability of the quantitative measurements so far made, a further investigation of this problem has been carried out. The effects of the wind velocity, ambient temperature, liquid water concentration, droplet diameter and size of the accreting object on the density, appearance and internal structure of the rime formation have been determined.

2.2 Experimental procedure.

A small wind tunnel with a working section 20 cm square and capable of providing wind speeds up to 13 m sec^{-1} , was set up in a cold room. This room was approximately a cube of side 8 metres. Air temperatures down to -35°C could be produced and continuously recorded from a thermistor situated in an air duct about $1\frac{1}{2}$ metres from the roof. In addition a standardised thermometer was employed to measure the air temperature at the level of the wind tunnel. The temperature indicated by this was 0.5 to 1.5°C lower than that recorded by the

upper instrument.

It was not possible to carry out a series of experiments at exactly the same temperature owing to the technical difficulties in cooling the room and to the slow rise in temperature during the course of individual experiments (which lasted between two and sixty minutes). The ambient temperatures in a series of experiments therefore ranged about the mean value by 1 to 2°C. The temperatures used in the analysis to represent individual experiments are those which were read from the thermometer, and they are considered to be representative of the air temperature in the tunnel to within 1°C (see Appendix 2.1). Experiments were conducted at temperatures of approximately -5, -10, -20 and -30°C; the actual temperature ranges are indicated in Fig. 2.2.

Three airspeeds were employed, the mean values of which were 2.5, 5.8 and 11.8 m sec⁻¹. It was not possible to reproduce speeds exactly, nor was it possible to maintain the same speed during an individual experiment, as the interior surface of the tunnel became rimed. The speeds were measured to $\pm 5\%$ at the beginning and end of each experiment by inserting a small anemometer in the wind tunnel. The actual speeds ranged about the values given above by approximately 10%. A set of coaxial cylinders collected the rime, and these were rotated at the rate of one revolution every two seconds, so that their peripheral speed was negligible compared with the air speeds used.

Water droplets were sprayed into the room from a bank of eight sprays, housed in an insulated box to prevent blockage through freezing. Ordinary tap water was used and the temperature at which this was fed to the sprays was adjusted so that the temperature of the water at the jet was slightly above 0°C. The sprays were directed towards the mouth of the tunnel from three metres away; in this circumstance it may be shown that, on reaching the tunnel, the droplets had cooled practically to the room temperature (see Appendix 2.1). The water concentrations were varied in steps according to the number of sprays in operation, and values up to 6 or 7 g m⁻³ were produced over the range of velocities used. The actual concentration present during an

experiment was computed from the rates of deposition on the cylinders, the airspeed and values of the efficiencies of catch of the cylinders given by Langmuir and Blodgett (1946). A manually operated shutter was installed across the tunnel entrance and the times of exposure of the cylinders to the airstream were measured to within $\pm 1\%$ with a stop-watch.

The drop size distributions produced by the sprays were determined by exposing in the tunnel narrow strips of perspex coated with oil and photographing the droplets caught through a microscope. Corrections were employed to allow for the change of collection efficiency of the strips with droplet radius (Langmuir and Blodgett, 1946). Three drop size distributions were used, two distributions being obtained by using jets of different sizes to produce the sprays, and the third by restricting the amount of water fed to the smaller nozzles. The median drop diameters of these distributions were 3, 7 and 9μ , while the median drop-volume diameters were 22, 47 and 65μ respectively. These distributions will be referred to subsequently as A, B and C respectively. In the analysis of the results the median volume diameters (or radii) have been used. The effective diameter changes slightly from cylinder to cylinder, depending on the efficiency of catch, and this has been taken into account.

The collecting cylinders were thin brass sleeves, mounted on co-axial formers of poor thermal conductivity, with outside diameters of 0.103, 0.634 and 1.433 cm respectively. The corresponding diameters after riming were about 0.6, 1.0 and 1.8 cm, the actual values depending on the conditions of the experiment. It is shown in Appendix 2.2 that the effect on the density of the absorption of heat by the cylinder supports is inconsiderable.

The density of the rime was computed from measurements of the mass and the linear dimensions of the cylindrical deposit. In order to avoid non-uniform deposits near their edges, the two largest cylinders were constructed so that a 3.0 cm length from the middle of each could be removed. In the case of the smallest cylinder the non-uniform

deposits were removed before measurements were made. The radial dimension was obtained by measurement of photographic images of the rimed cylinder projected to eight times natural size. The degree of enlargement was accurately determined and measurements of the diameter could be made to about ± 0.01 cm, depending on the nature of the surface. Where the surface of the deposit was not smooth (as in the case of feathery, kernel and, to some extent, opaque rime) the mean, rather than the overall value of the diameter was measured.

Measurements of the mean temperature of the riming surface were made during the course of the experiments. A thermocouple was attached to an insulated rod of a size comparable to the middle cylinder in such a way that the junction was 0.5 to 1 mm away from its surface. The rod was rotated in the stream of droplets and the temperature determined from a calibrated galvanometer scale. It was found, however, that an ordinary mercury thermometer of the same diameter inserted in the tunnel and rotated, gave values within 1°C of those given by the thermocouple. The procedure then adopted was as follows. The thermometer was placed in the droplet stream and the temperature noted. The ice deposit was then removed, the thermometer set at this temperature and replaced in the tunnel, and the reading observed. This process was repeated until very little change in the reading occurred, an accuracy of $\pm 0.5^{\circ}\text{C}$ being obtained in this way.

Samples of the rime formed under various experimental conditions were examined under a microscope to observe the structure of the deposit and to determine its relation to the density.

2.3 Results and discussion.

2.3.1 Analysis of the data.

Since the deposits were cylindrical the density, ρ , was calculated from the relation

$$\rho = \frac{4m}{\pi(d - d_0)(d + d_0)l} \quad (2.1)$$

where m was the mass of ice deposited on a cylinder of length l whose initial and final diameters were d_0 and d respectively. The mass of rime was determined to within 1%, the length of the cylindrical deposit to 2% and its thickness to about 4 (in the case of the smallest) to 6% (in the case of the largest cylinder). The density values obtained were generally consistent to within $\pm 10\%$. The thicknesses of low density deposits (0.1 to 0.3 g cm^{-3}) were more difficult to determine since their surfaces were irregular and the accuracy of the measurements in this range was consequently ± 10 to 20% . In the cases where the ice deposit was virtually clear it was extremely difficult to separate the individual cylinders for measurement because the deposit was very brittle and it was assumed that the density was nearly that of pure ice, namely, 0.916 g cm^{-3} . Where measurements were possible, the values obtained were within $\pm 0.05 \text{ g cm}^{-3}$ of this value.

It was also possible to deduce the liquid water concentration of the droplet cloud, W , from the mass of ice accreted, since

$$m = \frac{EWvt(d + d_0)}{2}l \quad (2.2)$$

where v was the airspeed, t the time of exposure of the cylinder and E the mean efficiency of catch of the cylinder as its diameter changed from d_0 to d . Rather than use the actual values of W which have been obtained from this relation, it was found more convenient to use the parameter EW in the analysis since this was determined directly from the experimental data and the additional errors and complications introduced in the computations of E were avoided. The values of EW obtained are accurate to $\pm 10\%$. (It will be seen later that, for physical reasons, EW is in fact the better parameter to use.)

The experimental results are depicted in Fig. 2.2 (a) - (d), the curves showing the density of rime as a function of EW for each of the temperature ranges concerned. The points are plotted for the droplet

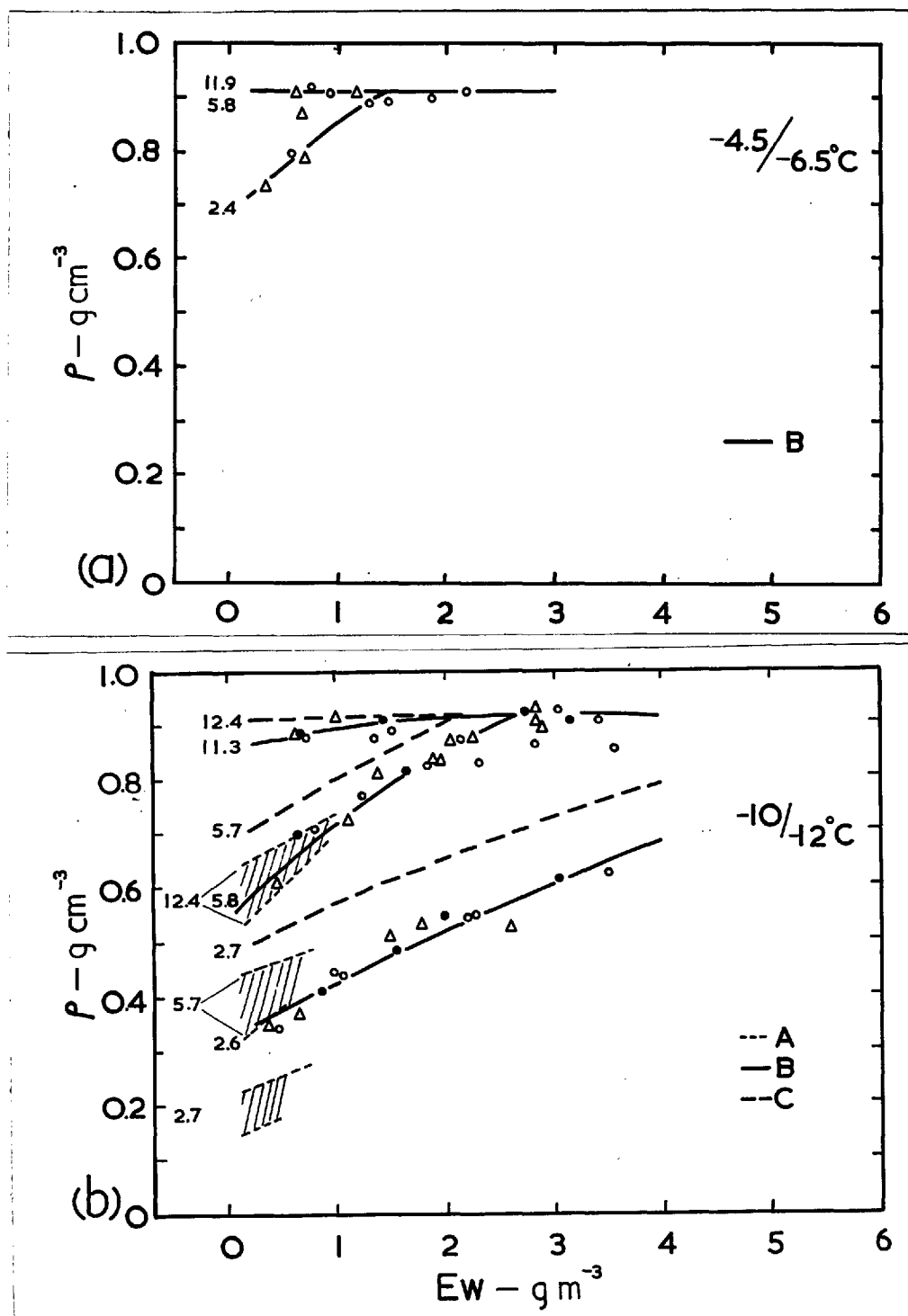


Fig. 2.2 The density, $\rho \text{ g cm}^{-3}$, is given as a function of EW (g m^{-3}) for the appropriate velocities in each of the temperature ranges over which measurements were made. The open circles

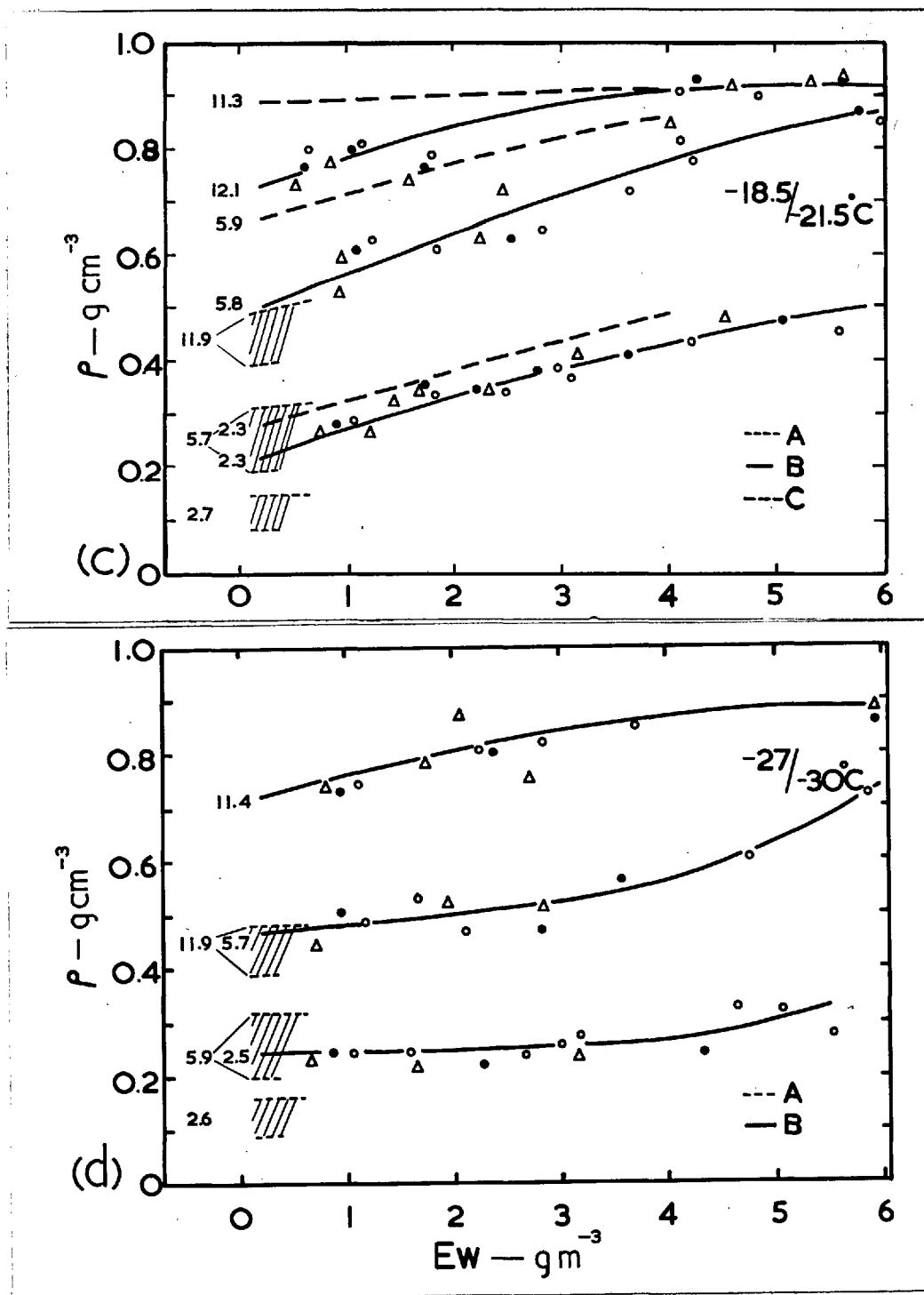


Fig. 2.2 (contd). represent the smallest, the dots the middle and crosses the largest diameter cylinders respectively.

distribution B, the points for the other two distributions being omitted to avoid confusion. These curves summarise over 250 density measurements. As already mentioned it was not possible to reproduce exactly the various experimental conditions nor to maintain them accurately during the course of a single experiment. In drawing these curves (and the curves in subsequent diagrams) account has been taken of the slightly varying conditions appropriate to each plotted point.

In the cloud with the smallest droplets, EW has only small values because the droplets were formed by restricting the flow of water to the sprays; values of EW up to 4 g m^{-3} were used with distribution C. It can be seen that the density increases with velocity increase and temperature rise, with the size of the droplets and with the value of EW. Within the limits of experimental error there is little dependence on the cylinder diameter except for the cloud with the smallest droplets, in which the effect of cylinder size is considerable.

In order to explain these results it is convenient to introduce two physical quantities, the temperature of the riming surface and the speed of impact of the cloud droplets, which are derived from the parameters given above.

2.3.1.1 The temperature of the riming surface.

The heat economy of a riming surface includes heat liberated during freezing and heat transmitted to the environment. If the heat conduction into the interior of the rime deposits is small, these two terms are approximately equal. For this condition Ludlam (1951) has indicated an expression from which the mean surface temperature, T_s , of a rotating cylinder being rimed may be determined, namely

$$\frac{EW v (L_f + T - \sigma T_s)}{\pi} = \frac{0.24 Re^{0.6} [K(T_s - T) + L_v k \Delta \rho]}{d} \quad (2.3)$$

where L_f and L_v are the latent heats of fusion and vaporisation of water and ice respectively, T is the ambient temperature, σ is the specific heat of ice, d is the cylinder diameter, K is the coefficient of conductivity of air, k is the coefficient of diffusion of water vapour in air, v is the airspeed, Re is the Reynolds number and $\Delta\rho$ is the difference in the vapour densities at the surface and in the environment. The expression on the left represents the rate of release of heat per unit area of surface and that on the right the rate at which heat is removed by conduction and evaporation into the air flowing past the cylinder. It is assumed that the heat conduction and mass transfer coefficients are equal; Powell (1940) has shown that the agreement between these two coefficients for ventilated cylinders is within 5 to 10% over the range of Reynolds numbers between 100 and 2×10^4 . The coefficient given in eq. 2.3 does not quite cover the range of conditions of the present experiments (Re from about 400 to 2×10^4) and for values of Re between 400 and 4×10^3 the coefficient $0.24 Re^{0.6}$ is better replaced by $0.615 Re^{0.466}$ (Fishenden and Saunders, 1950 p. 130). However, the use of eq. 2.3 at the lower Reynolds numbers does not introduce a serious error (a maximum of 10%). The equation indicates that the mean temperature of the riming cylinder is a function of two variables, $EW(vd)^{0.4}$ and T , and values of T_s for given conditions have been obtained graphically.

The rate of accretion on the thermometer rotated in the drop-let stream was inferred from the measured rates of accretion on the cylinders and the appropriate mean surface temperature was then deduced from eq. 2.3. A plot of the experimentally determined surface temperature against the corresponding theoretical value is shown in Fig. 2.3. Since the temperature increases due to accretion ranged from 2 to 28°C the agreement is reasonable; some of the scatter may be due to variations in liquid water concentration across the tunnel section, which would have resulted in different accretion rates on the thermometer and on the cylinders.

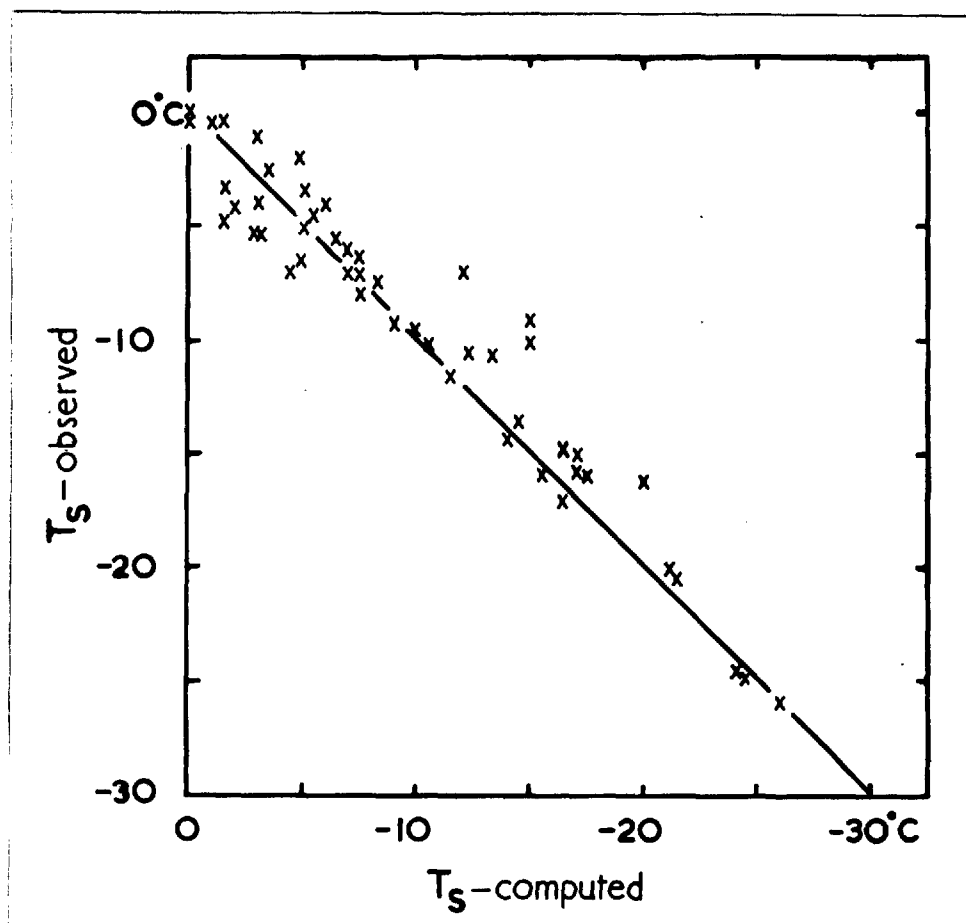


Fig. 2.3 The observed value of the temperature of the riming surface plotted against the value deduced from eq. 2.3.

Values of the mean surface temperature of each of the cylinders corresponding to the average diameters and rates of accretion have been determined for the various experimental conditions from eq. 2.3. This equation, however, is valid only when there is negligible heat absorption by the cylinders and insulating former and the surface temperatures given by this equation for the larger cylinders are possibly somewhat high. Further, Reiher (1925) has shown that the presence on the surface of longitudinal fins or grooves $1/10$ the diameter of the cylinder raises the heat transfer by up to 40% at Reynolds numbers between 10^3 and 10^4 . In the case of rime surfaces of comparable roughness in the present experiments, T_s may be overestimated by 2 or 3°C.

2.3.1.2 The speed of impact of the droplets.

The airspeed measured in the tunnel is not the speed with which the droplets strike the cylinders. The speed of impact of a droplet, v_o , is influenced by its size and the cylinder diameter, and the ratio v_o/v diminishes with a decrease of the former or an increase of the latter.

The values of v_o have been computed for the present experimental conditions using the method given by Langmuir and Blodgett (1946). It must be pointed out, however, that the values of v_o used are those for the drop-volume medians of the distributions striking the foremost edges of the cylinders. Further, as the cylinder diameter changes during riming, the impact velocity also changes. This change is 10% at an air speed of 2.5 m sec^{-1} and 2 to 3% at the higher velocities. Values of v_o can be considered to be only approximate when reference is being made to the impact speed of individual droplets.

The density of the rime deposits was found to be a function of T_s . Fig. 2.4 shows this relationship for distribution B. Over most of the range of values of T_s the relationship is unique; at

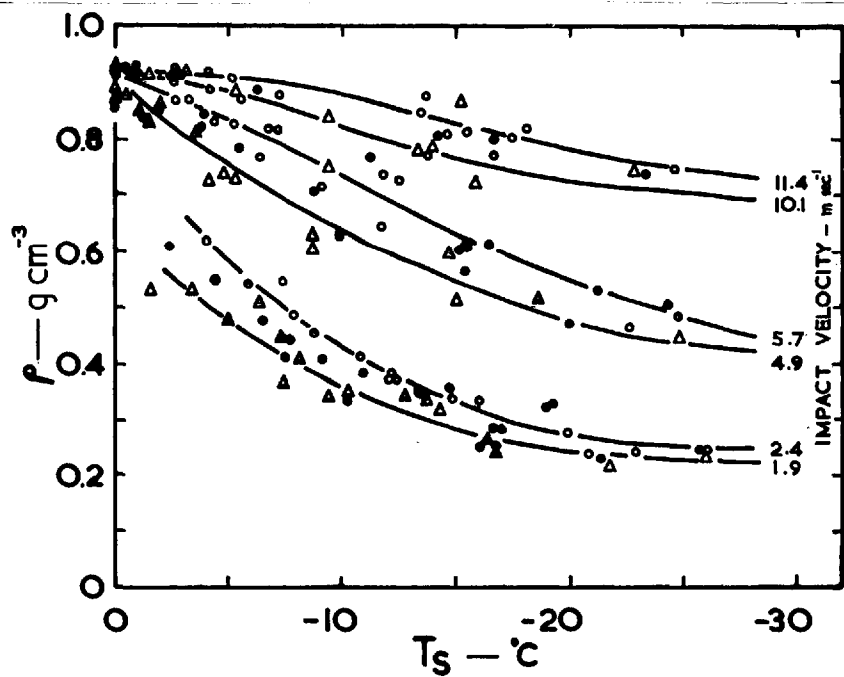


Fig. 2.4 The density measurements for drop distribution B as a function of the mean surface temperature.

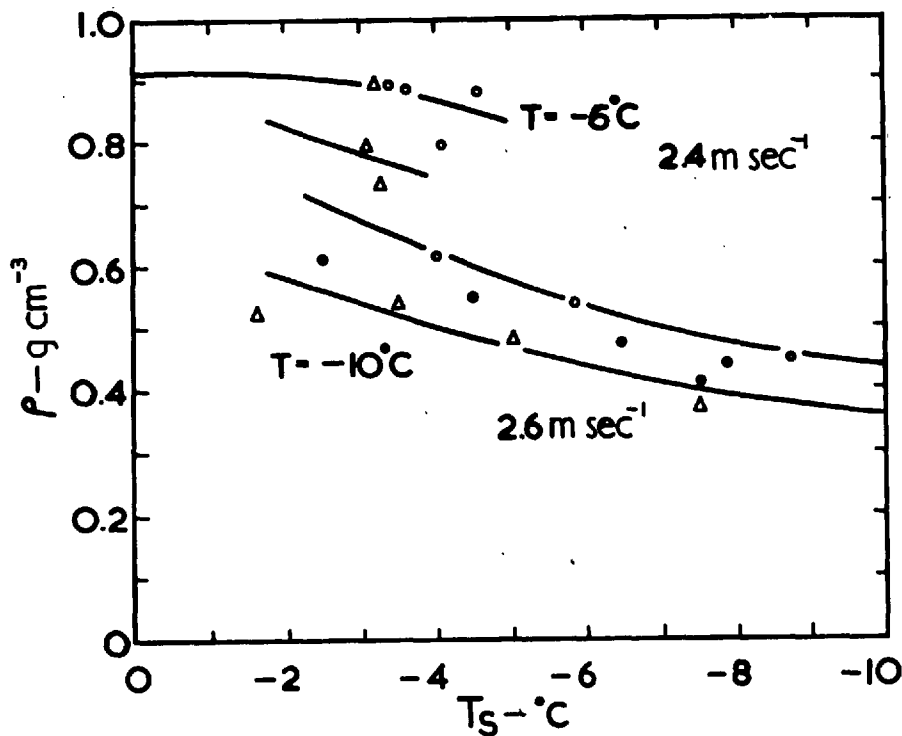


Fig. 2.5 The density measurements for drop size distribution B at high surface temperatures for a velocity of 2.5 m sec^{-1} . There is a definite separation between the two sets of points for ambient temperatures of -6 and -10°C .

temperatures above -5°C and for low velocities, however, there is a dependence on the ambient, as well as on the surface temperature (Fig. 2.5). This also occurs at a velocity of 5.8 m sec^{-1} but the effect is masked because the densities in this temperature region are high. Within the scatter of the points, due partly to experimental errors and partly to errors introduced in computing T_s , it is seen that the density is also dependent on the cylinder diameter. Langmuir (1944) has shown that the decrease in density with increasing cylinder size may be attributed to the fall in impact speed. The curves drawn in Fig. 2.4 are for the smallest and largest cylinder diameters and the appropriate values of the impact speeds, computed for the median drop-volume diameter and for the mean values of the cylinder diameters, are indicated. In the case of the smallest drop size distribution, the impact speed decreases more rapidly with increase in cylinder size and the effect on the density is more marked. This is seen in Fig. 2.6. The flattening of the curves at low temperatures in Figs. 2.4 and 2.6 is to be expected, since when the time of freezing of the droplets is sufficiently rapid for them to behave as virtually "solid" particles on impact, a further reduction in the temperature has little effect.

Using the isopleths drawn in Figs. 2.4 and 2.6, and the data for drop size distribution C, it was found that the density could be expressed as a function of rv_0 , the product of the median drop-volume radius and the impact speed (Fig. 2.7). Again the relationship is complicated at high temperatures because the density depends on the ambient, as well as on the surface temperature. Over the straight line parts of the curves the density is given by $\rho = F(rv_0)^{\alpha}$ where F is a function of T_s . The values of α are sensibly constant, ranging from 0.65 ($T_s = -5^{\circ}\text{C}$) to 0.77 ($T_s = -30^{\circ}\text{C}$). The values of F may be obtained from Fig. 2.6 and for surface temperatures of -5 , -10 , -15 , -20 and -30°C are 0.047, 0.026, 0.017, 0.012 and 0.011; the corresponding values of α are 0.65, 0.67, 0.72, 0.77 and 0.77 respectively. These values of F are given for the following units:

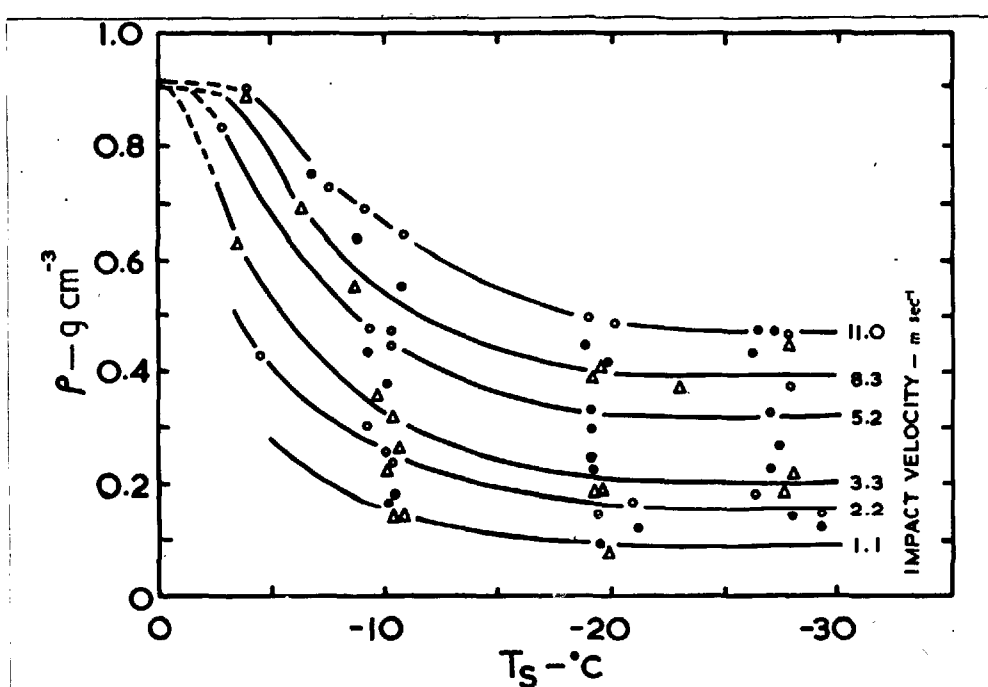


Fig. 2.6 The density measurements for drop distribution A as a function of the surface temperature. (Included in this figure are a few density measurements made at -5°C , which are not shown in Fig. 2.2a.)

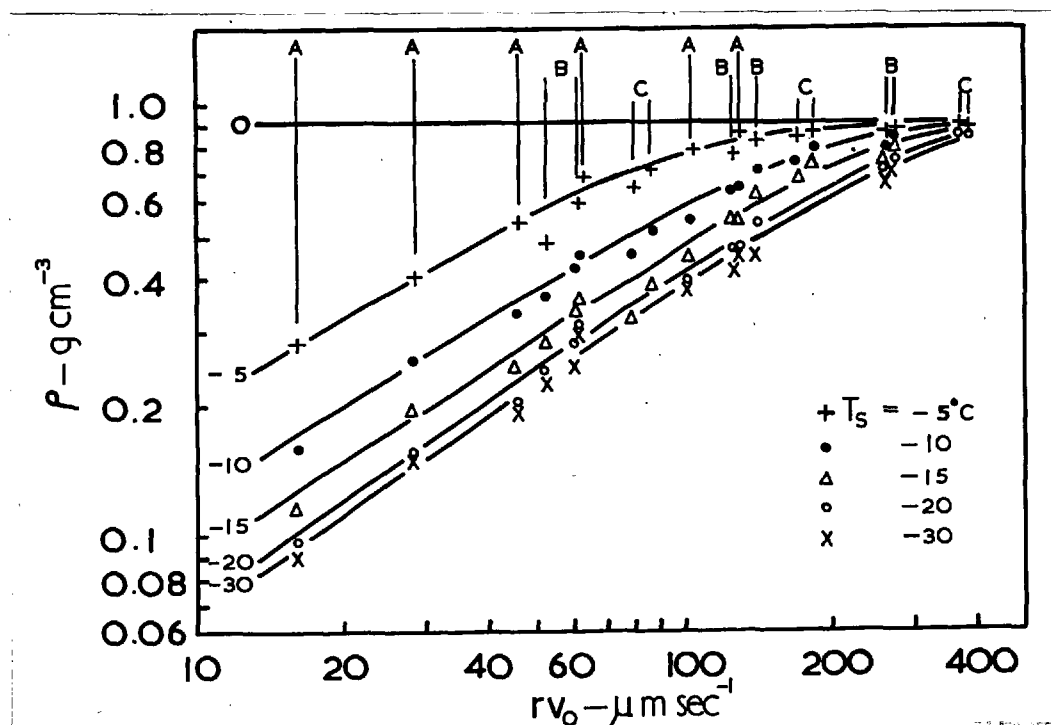


Fig. 2.7 The density is shown as a function of $r v_0$ for fixed values of the surface temperature; the points appropriate to each drop size distribution are indicated.

ρ - g cm⁻³, r - microns, v_o - m sec⁻¹; in C.G.S. units the values of F are 0.88, 0.61, 0.47, 0.42 and 0.39 respectively.

An attempt to determine the relation between F and T_s indicated that F could be expressed as a power of T_s , the value of which was about -0.7. As the magnitude of this was approximately equal to $\propto$, the density was plotted as a function of rv_o/T_s (Fig. 2.8). If the points for $T_s = -30^\circ\text{C}$ are ignored (the density becomes virtually independent of the surface temperature for values below -20°C) it is possible to draw a smooth curve through these points, the lower part of which is a straight line.

Because of the complexity of this diagram it is not possible to differentiate between the three drop size distributions. To show that there is a genuine overlapping of the points for the three distributions, the values of ρ and rv_o/T_s appropriate to points plotted in this diagram are listed in Table 2.1.

This then gives the following relatively simple relation between the density (g cm⁻³) and the drop-volume radius (microns), the impact velocity (m sec⁻¹) and the mean surface temperature ($^\circ\text{C}$):

$$\rho = 0.110 \left(-\frac{r v_o}{T_s} \right)^{0.76} \quad (2.4)$$

In C.G.S. units the constant of proportionality is 3.64. This relation holds for values of $-rv_o/T_s$ between 0.8 and 10 provided T_s is above -20°C . To a first approximation the density is given by eq.2.4 for values of $-rv_o/T_s$ up to 17, beyond which the density is 0.9 g cm⁻³. Presumably the curve also flattens at low values of $-rv_o/T_s$ since the density cannot decrease indefinitely, but there is no indication that it does over the present range of conditions.

The original data may also be plotted in this way (Fig. 2.9). The scatter of the points in this figure is considerably greater than in the previous one, as is to be expected since smoothed density values were used in the latter. The same general trend is shown and it is possible to fit a straight line to the lower part of the curve which is not significantly different from that shown in Fig. 2.8, indicating

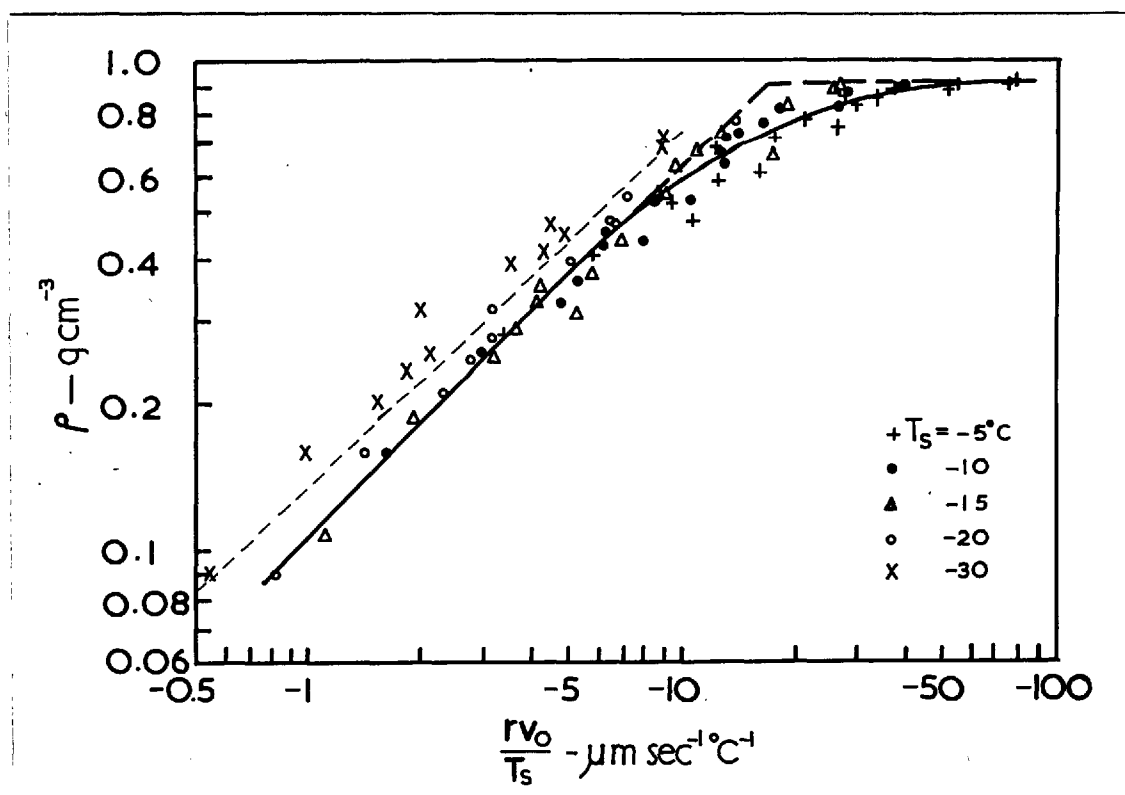


Fig. 2.8 The density as a function of the parameter rv_0/T_s . In drawing the smooth curve, the points for $T_s = -30^\circ\text{C}$ have been ignored.

TABLE 2.1

The values of ρ (right hand side of each column) corresponding to the various values of $-r v_o/T_s$. These have been read from Figs. 2.4 and 2.6 and from curves for distribution C.

Distribution A:												
v_o	1.1		2.2		3.3		5.2		8.3		11.0	
$T_s - 5$	3.3	.28	5.7	.41	9.2	.53	12.5	.68	21	.78	27	.86
-10	1.65	.16	2.9	.26	4.6	.33	6.3	.46	10.4	.54	13	.67
-15	1.10	.11	1.91	.19	3.1	.25	4.2	.36	6.9	.44	8.9	.55
-20	0.82	.09	1.43	.16	2.3	.21	3.1	.32	5.2	.40	6.7	.48
-30	0.55	.09	0.95	.16	1.54	.20	2.1	.32	3.5	.39	4.4	.47
Distribution B:												
v_o	1.9		2.4		4.9		5.7		10.1		11.4	
$T_s - 5$	10.6	.48	12.3	.58	26	.75	29	.83	52	.89	54	.91
-10	5.3	.36	6.2	.43	13	.64	14	.74	26	.82	27	.88
-15	3.5	.29	4.1	.33	8.7	.55	9.6	.64	17.4	.67	18.1	.83
-20	2.7	.25	3.1	.28	6.5	.48	7.2	.55	13.0	.73	13.6	.78
-30	1.77	.23	2.1	.25	4.3	.42	4.8	.45	8.7	.69	9.0	.72
Distribution C:												
v_o	2.1		2.6		4.8		5.6		10.9		11.8	
$T_s - 5$	15.8	.61	17.3	.72	33	.85	37	.88	76	.91	77	.92
-10	7.9	.44	8.6	.53	16.7	.77	18.4	.82	38	.90	39	.92
-15	5.3	.31	5.8	.38	11.1	.69	12.3	.75	25	.88	26	.91

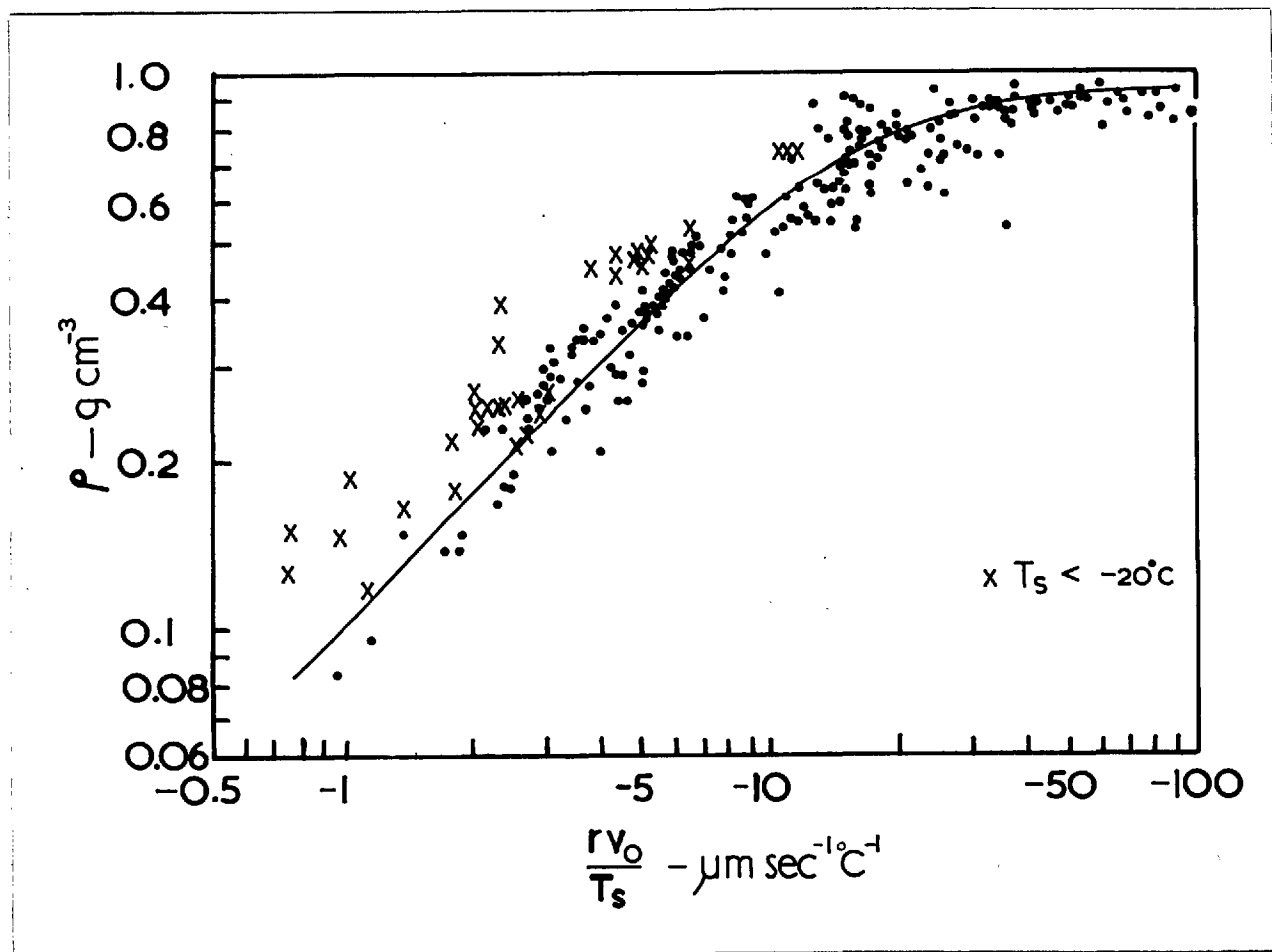


Fig. 2.9 The measured density values as a function of rv_0/T_s .

that the relationship given above is not dependent on the analytical procedure.

2.3.2 Physical interpretation.

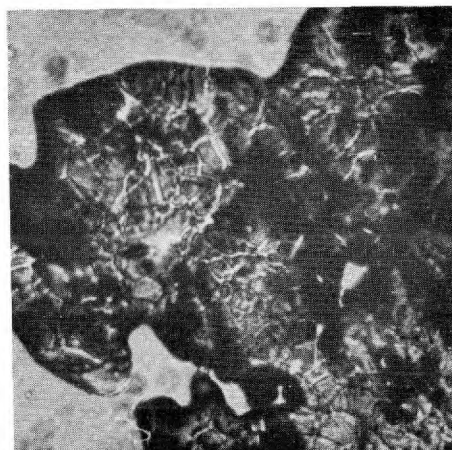
In order to give some physical interpretation of the relationship deduced above, it is helpful to examine the internal structure of the ice deposits formed under various conditions. Plate 2.1 contains a number of microphotographs of rime samples formed from droplets of distribution B at impact speeds of about 11 and 2.5 m sec⁻¹ and at surface temperatures ranging from -4 to -24.5°C. At temperatures of -16.5°C and lower (b, c and f) the droplets tend to freeze individually; even at high speeds, when the droplets are packed very closely, the "units" of the structure are still comparable in size to single droplets.

The effect of impact speed on the rime structure and density can be appreciated by comparing photographs (b) and (f), and (a) and (d). It is seen that, over the whole temperature range, the impact velocity is a factor of considerable importance in the determination of the density. At -16.5°C the density is increased from 0.28 to 0.80 gcm⁻³ when the impact velocity is raised from 2.0 to 11.4 m sec⁻¹; at -4°C the corresponding change is from 0.62 to 0.89 g cm⁻³. If their impact speed is low and their freezing time short, the droplets tend to retain their spherical shape and form an open structure of low density, while for higher impact speeds and longer freezing times they are distorted, and packed more closely. (At an ambient temperature of -30°C some of the larger droplets in the distributions were undoubtedly frozen in the airstream (section 3.3.5) and may have been incorporated into the rime structure.)

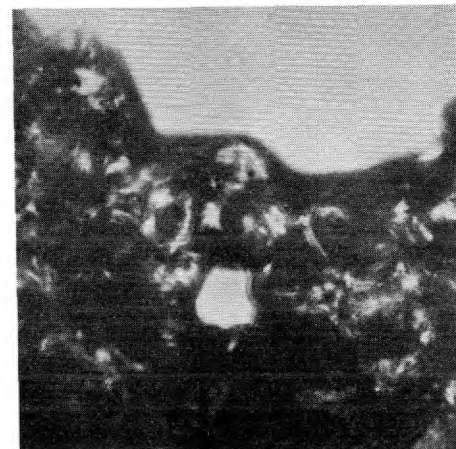
At temperatures about -10°C the droplets begin to merge but individuals are still distinguishable (photograph e). At temperatures close to 0°C the units of the structure are groups of some tens of droplets so fused together that individuals can no longer be discerned (a and d). It is estimated that the units in (d) have some twenty



a, $T_s = -4 : T = -11$
 $V_0 = 11.4 : \rho = .89$



b, $T_s = -16.5 : T = -20.5$
 $V_0 = 11.4 : \rho = .80$

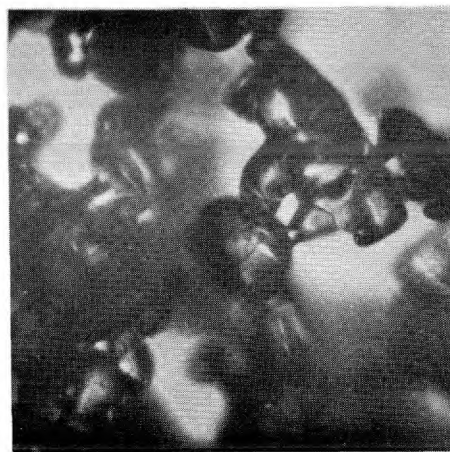


c, $T_s = -24.5 : T = -29.5$
 $V_0 = 11.0 : \rho = .74$

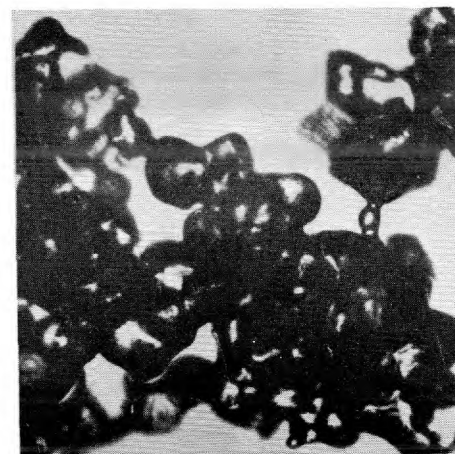
200 μ



d, $T_s = -4 : T = -11.5$
 $V_0 = 2.5 : \rho = .62$



e, $T_s = -8 : T = -12$
 $V_0 = 2.4 : \rho = .44$



f, $T_s = -16.5 : T = -20$
 $V_0 = 2.0 : \rho = .28$

times the volume of those in (e). This amalgamation of the droplets may be due to two physically distinct processes: (1) the spreading of liquid over the underlying ice surface and (2) the partial or complete coalescence of the droplets before they freeze.

The following considerations indicate that, over most of the temperature range, the second of these, namely, droplet coalescence, is unlikely to have a significant effect on the density. The time taken for a droplet to freeze depends on its heat capacity and the rate at which the heat of fusion may be dissipated to its environment. Considering first that it is transferred to the air and assuming the droplet to remain spherical, the rate of transfer is given by the sum of eqs. 1.8 and 1.10, namely

$$\frac{dQ}{dt} = 4 \pi r (a K \Delta T + b L_v k \Delta \rho) \quad (2.5)$$

Since while freezing the temperature of water is 0°C , ΔT is $-T$, where $T^\circ\text{C}$ is the temperature of the air surrounding the droplet; $\Delta \rho$ also is now approximately proportional to T . For an unventilated sphere, a and b are close to unity, so that the minimum rate of heat transfer is approximately ($K = 5.5 \times 10^{-5} \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$, $L_v = 600 \text{ cal g}^{-1}$, $k = 0.21 \text{ cm}^2 \text{ sec}^{-1}$ and $\Delta \rho \approx -2.3 \times 10^{-7} \Delta T \text{ g cm}^{-3}$)

$$\frac{dQ}{dt} \approx 1.1 \times 10^{-3} r \Delta T \quad (2.6)$$

The quantity of heat to be removed is $4.2 r^3 (L_f + T)$ calories, so that the maximum time for the droplet to freeze completely can be estimated as

$$t \approx \frac{3.8 \times 10^3 r^2 (L_f + T)}{\Delta T}$$

For $\Delta T = 10$, $r = 25\mu$, the conditions of Plate 2.1 (e), $t \approx 0.25 \text{ sec}$; for the larger droplets in the distribution concerned ($r = 50\mu$), the time of freezing is approximately 1 second.

However, it is likely that the freezing of the droplet occurs from the outside inwards, an outer shell of ice forming first since the heat of freezing is most readily conducted away from the surface

layer . Mason and Maybank (1960) have studied the freezing of droplets ranging in size from 60μ to 2 mm in diameter. These authors estimate that, even with millimetric droplets, an outer shell forms within 0.1 sec. of the initiation of freezing, even though the total time of freezing is as much as 1 to 2 minutes. From their experimental data it is possible to infer that the smaller droplets freeze in a similar way, and evidence for this manner of freezing has been found in the present experiments (see section 2.7). When a droplet has acquired a hard outer shell, then from the point of view of its effect on the density of the deposit, it may be considered to be "frozen". If the droplets are distorted their surface area, and consequently the heat transfer, is increased. Further, because the thermal conductivities of water and ice are considerably higher than that of air (the thermal conductivities of water, ice and air are 1.5×10^{-3} , 5×10^{-3} and 5.5×10^{-5} cal cm^{-1} sec $^{-1}$ deg $^{-1}$ respectively) transfer of heat first rapidly takes place between the droplet and the ice surface. For these reasons the effective, and possibly the actual, time of freezing of the droplets is probably much less than that estimated above.

The rate of arrival of the droplets at the surface may be deduced in the following way. The number of droplets of radius r per unit volume of air in a liquid water concentration W is $3W/4\pi r^3$ so that the average time between two droplets striking a stationary surface with a separation of centres equal to $2r$ is $r/3Wv$. Because the cylinders are rotating, the area over which accretion is occurring is increased by a factor of π , so that the average time, τ , between two droplets overlapping is greater than $\pi r/3Wv$. If coalescence is to occur this time must be comparable with the freezing times given above. For the conditions of Plate 2.1 (e), τ is 6 seconds (12 seconds for the larger droplets) which is a factor of 30 (15) greater than the maximum time estimated for the droplets to freeze completely. If the liquid water

concentration is increased to that corresponding to photograph (d), τ is reduced to 1.5 secs. The time of freezing of the droplets is probably slightly increased because the surface temperature is raised, but the two times are still not comparable. Thus the increase in density with rise in EW is not due merely to an increase in the flux of droplets, but must be due to the rise in surface temperature. The coalescence of droplets is unlikely to have an appreciable effect on the density except at temperatures close to 0°C .

This means that the fusion of the droplets in Plate 2.1 (e) is due to an enhanced spreading of liquid over the underlying ice surface. This effect also increases as the temperature is raised, since the longer the time of freezing of the droplets the more they spread. This is illustrated by the following investigations. Dufour (1861) carried out some experiments with supercooled water droplets of 3 to 5 mm diameter suspended in a chilled organic liquid. He observed the manner of freezing of a supercooled droplet when it was brought into contact with one which had already frozen. When the temperature was -3 to -4°C , he found that the liquid drop spread out over the frozen one; at -1 to -2°C the liquid droplet spread completely around the frozen particle before finally freezing. Weickmann (1953) reports similar experiments in which droplets were allowed to fall on an ice surface at various temperatures. Droplets (whose temperature is not specified) which fell on an ice surface at temperatures below -5°C solidified individually whereas at higher temperatures they spread over the ice surface forming only slight prominences. At -14°C all droplets of up to 150μ radius solidified as spheres, whereas at -5°C most of them were flattened.

The dependence of the density on the parameter rv_0/T_s can be explained in the following way. It is assumed in the first instance that the droplets remain approximately spherical on freezing; this assumption is valid if the impact speed and the surface temperature are moderately low (see Plate 2.1). When a supercooled droplet arrives at the surface of the deposit it begins to freeze, the

initial freezing occurring over the area of contact between the droplet and the lower surface. If a bond can form quickly and with sufficient strength to overcome the impact momentum of the droplet, then the droplet is held in this position. If the bond is not formed quickly or strongly enough the droplet moves or slips over the surface of the frozen droplet on which it has impinged until it is brought to rest either by the solidification of this first bond or by the formation of bonds with other frozen droplets. In both of these circumstances the structure is more compact and has a higher density. The density may, therefore, be expected to depend on the impact momentum of the droplet, $m v_0$, that is, on $r^3 v_0$. The strength of the bond depends on the area of contact between the droplet and the lower surface and this may reasonably be expected to be proportional to r^2 . The density is, therefore, a function of the ratio of $r^3 v_0$ and r^2 , that is, of $r v_0$.

The density is also dependent on the time of formation of the bond the droplet makes with the underlying ice surface. The longer this time the more compact the structure will be. For the reasons already given, the transfer of heat takes place initially between the droplet and the ice surface so that the thermal gradient governing this process is proportional to T_s (in $^{\circ}\text{C}$). The temperature of the droplets themselves may play some part in determining the rate of freezing but the rate of conduction of heat into the ice is greater than that through the water. To a first approximation the time of formation of the bond is proportional to $1/T_s$; the density is then a function of $r v_0/T_s$.

It is of interest to determine what range of densities would be covered if the droplets do in fact behave as spheres. Fig. 2.10 shows two ways in which spherical droplets may be frozen together. In (a) the droplets are frozen together in chains; the microscopic examination of samples formed from droplets of distributions A and B at airspeeds of 2.5 m sec^{-1} showed similar

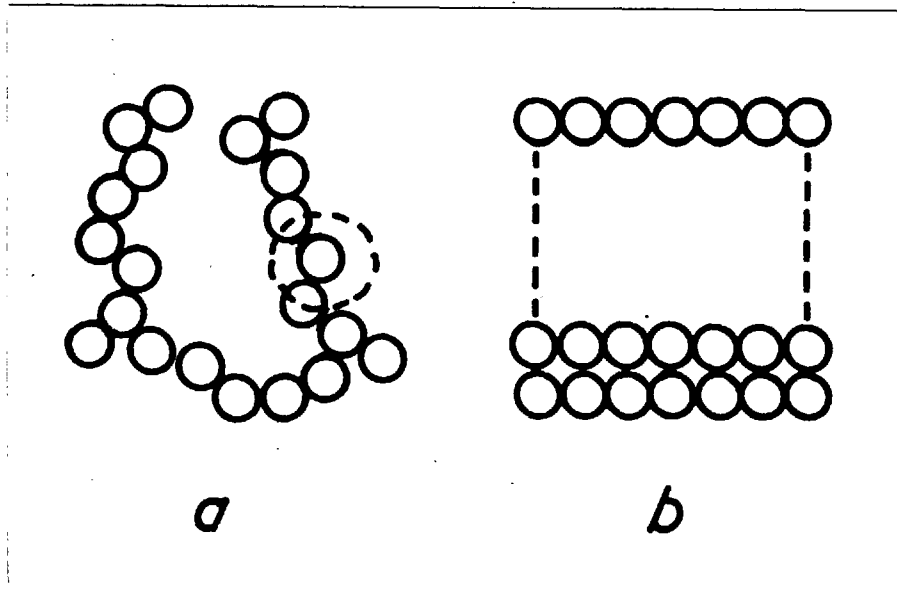


Fig. 2.10 Ways in which spherical droplets may be frozen together.

structures. In the dotted circle there are approximately two whole droplets, so that the density within this region is 0.23 g cm^{-3} (taking the density of the frozen particles to be 0.9 g cm^{-3}). The density throughout the bulk of such a deposit may be somewhat greater or less than this, depending on the way in which it is built up, and densities of about 0.1 g cm^{-3} are not unreasonable on this basis. In (b) (which is not intended to be realistic) there is one droplet in a cube whose side is equal to the diameter of the droplet and the density is 0.48 g cm^{-3} . Hexagonal or cubic close packing would give a density of 0.67 g cm^{-3} .

In Figs. 2.8 and 2.9 it is seen that the straight line representing the lower densities begins to curve at a value between 0.5 and 0.6 g cm^{-3} . The reason for this is that it is difficult to attain ordered close packing when the droplets are being accreted in a random manner. Scott (quoted by Bernal, 1960) has determined the density of irregularly close packed spheres which, for spheres of density 0.9 g cm^{-3} , is 0.57 g cm^{-3} . If the droplets remained spheres this would be about the highest density attainable under accretion conditions unless at high impact speeds the droplets are forced into closer arrays. In any case, densities above 0.7 g cm^{-3} must be due to distortion and spreading of the droplets and will therefore still depend on rv_0 and $1/T_s$.

The assumption that the droplets freeze as spheres and that their freezing (in so far as this determines the density) is governed primarily by the surface temperature does permit a reasonable interpretation of the data for densities below 0.6 to 0.7 g cm^{-3} . Higher densities must be produced by the distortion and spreading of the droplets. The exact nature of the dependence of the density on rv_0/T_s cannot be explained fully on this simple basis and may in fact involve the ways in which the parameters r , v_0 and T_s in this expression are defined since they are only broadly representative of the conditions they describe.

2.4 The effect of growth from the vapour phase on the density of rime.

If the saturated vapour pressure over ice at the surface temperature is less than that over liquid water at the ambient temperature, then deposition directly from the vapour phase occurs, the second term on the right hand side of eq. 2.3 being negative. If the temperature is such that needles or dendrites tend to grow (-3 to -5°C and -12 to -16°C according to Hallet and Mason, 1958) then the density of the deposit may be markedly reduced, particularly at low impact velocities and small drop sizes when the accretion rate and impact energy are small. For this effect to occur, the difference between the temperature at the ice surface and in the saturated air has to be less than 2.9 , 1.9 , 1.1 and 0.7°C for ambient temperatures of -30 , -20 , -10 and -5°C respectively. In the present series of measurements the difference was usually greater than these values for the distributions B and C, but not always for distribution A. At ambient temperatures of -20 and -30°C and for $v = 2.5 \text{ m sec}^{-1}$ at -11°C , computations of T_s indicate that there must always have been some deposition from the vapour for this last distribution.

The rate of deposition from the vapour per unit area is given approximately by

$$\left(\frac{dm}{dt}\right)_v = \frac{0.24 \text{ Re}^{0.6} k \Delta \rho}{d} \quad (2.8)$$

For the conditions under consideration this may be written

$$\left(\frac{dm}{dt}\right)_v = 0.13 v^{0.6} d^{-0.4} \Delta \rho \quad (2.9)$$

which gives maximum rates of deposition between 0.5 and $3 \times 10^{-6} \text{ g cm}^{-2} \text{ sec}^{-1}$. The corresponding minimum experimental accretion rates lay between 0.6 and $4 \times 10^{-5} \text{ g cm}^{-2} \text{ sec}^{-1}$ and therefore the maximum contribution of the vapour phase to the growth of the deposit was about 10%. A more probable figure is between 2 and 5% since the deposition of vapour itself raises T_s slightly and

causes a diminution of the vapour density difference. Even so, this figure would apply to only some 20% of the experiments for the droplets under consideration. In the case of the other two distributions, deposition from the vapour would have contributed a small fraction of a per cent to the deposit in those few cases where it occurred at all.

These estimates, together with the facts that few measurements which could be affected by growth from the vapour phase were made under conditions favourable to the growth of either needles or dendrites, and that no crystalline structure compatible with this form of growth was observed in the subsequent microscopic analysis, permit the conclusion that the present series of density values are representative of growth by accretion only.

2.5 Comparison with other rime density measurements.

As pointed out above, the only attempt to find a quantitative method for estimating the density of ice accreted under given conditions was made by Langmuir (1944). On the basis of 17 sets of measurements made at the Mount Washington Observatory with a six-cylinder collector, Langmuir showed that the density could be computed from a knowledge of the ambient temperature, droplet size and impact speed. He found no dependence on the rate of riming because the clouds in which these measurements were made contained comparatively little water (see below), and the riming rates were so small that the surface temperature of the deposits did not differ appreciably from the ambient temperature. Langmuir showed that the density could be expressed as a power of the impact velocity, and obtained values of this power ranging between 0.7 and 1.4, the lower values of which are consistent with those given in Figs. 2.7 and 2.8.

A comparison between densities computed from Langmuir's formulae for the conditions appropriate to the present experiments and the actual values is shown in Table 2.2. While there is good

TABLE 2.2

Comparison of rime density values computed from Langmuir's formulae (on left hand side of each column) with those of the present investigation. The values given are for the smallest and largest diameter cylinders respectively, it being assumed that T_s is approximately equal to the ambient temperature. The values in brackets were obtained by extrapolation.

mean velocity m sec ⁻¹	A						B					
	2.7		5.8		12.1		2.4		5.8		11.6	
T_s - 5°C	.34	.41	.56	.69	.80	.86	.59	.58	.78	.83	.97	.91
	.28	(.28)	.46	.53	.76	.79	.55	.48	.74	.75	.97	.88
-10	.20	.26	.37	.46	.63	.67	.42	.43	.62	.73	.84	.88
	.12	.16	.28	.33	.52	.55	.38	.36	.58	.64	.82	.82
-20	.07	.16	.18	.32	.38	.49	.18	.28	.35	.55	.58	.78
	.05	.09	.13	.21	.32	.40	.15	.25	.31	.48	.55	.73
-30	<.02	(.16)	<.04	(.32)	.16	(.47)	.04	(.24)	.13	(.44)	.30	(.72)
		(.09)		(.20)	.09	(.39)	.03	(.23)	.11	(.42)	.25	(.69)
- 5°C			C									
	2.5		5.8		11.9							
	.70	.72	.88	.88	1.0	.92						
	.69	.63	.86	.85	1.0	.91						
	.52	.53	.71	.82	.92	.92						
	.50	.45	.70	.77	.91	.90						
	.24	(.30)	.43	(.68)	.65	(.89)						
	.22	(.24)	.39	(.60)	.64	(.86)						

agreement at temperatures of -5 and -10°C , the computed values are lower than the present ones at temperatures of -20 and -30°C . At -30°C they are lower by a factor of between 2 and 10.

The ranges of values of the parameters in Langmuir's data were as follows: temperature, -1.5 to -25°C ; drop-volume diameters, 7 to 20μ ; airspeeds, 13 to 32 m sec^{-1} ; liquid water concentrations, 0.04 to 0.8 g m^{-3} ; cylinder diameters, from 0.3 to 7.8 cm (when rimed). Because of the small size of the droplets and the low values of the liquid water concentrations, the accretion rates were small and there was little difference between the surface temperature of the rime deposits and the ambient temperature. This means that growth from the vapour could have taken place. For the example which Langmuir gives of a set of six-cylinder observations, computations of the surface temperature show that growth from the vapour would have occurred on at least the three biggest cylinders. In the case of the largest cylinder, the ratio of the rate of deposition from the vapour phase to that by accretion would have been about 1 : 4 and in the case of the second largest cylinder, 1 : 16.

The rates of riming in five other of the seventeen sets of data were less, by as much as a factor of 4, than those in the instance given, which suggests that growth from the vapour could have affected the rime density, particularly at low temperatures and low velocities. The extremely low densities deduced from the formulae at -30°C are due to unjustified extrapolation, since the formulae do not predict the flattening of the density versus temperature curves indicated in Figs. 2.4 and 2.6. In comparing the two series of density values it must be remembered that Langmuir's formulae are being extrapolated beyond the ranges of drop sizes and temperatures for which they were derived, that the parameters on which the formulae are based were not independent, and that Langmuir rejected 5 of a total of 17 sets of observations. Without the details of the Mount Washington data it is not possible to give a quantitative assessment of any differences between these

measurements and the present ones.

Loughborough's assertion (Loughborough, 1946) that the density of rime may be expressed simply as a function of the collection efficiency is not borne out by the present data. If the range of the various parameters concerned is small, as was so in the examples he analysed, then it is possible to obtain a fortuitous relationship similar to the one he gave, since E is a measure of v_0 , the impact velocity. In the case of the six-cylinder data, the velocity effect is most marked because the range of cylinder diameters is considerable.

The density measurements made by Melcher (1951) indicate that at an airspeed of 5 m sec^{-1} , 5μ diameter droplets yield a rime deposit of between 0.02 and 0.10 g cm^{-3} depending on the liquid water concentration and the ambient temperature. These values are somewhat lower than those given here, but the droplets Melcher used were smaller. He found that although the density increased with temperature rise, the density actually decreased as the water concentration rose from 0.3 to 1.6 g m^{-3} . In these experiments it was necessary for Melcher to expose the rod in the wind tunnel for between 1 and 6 hours to obtain a measurable deposit, and undoubtedly growth from the vapour was playing at least as important a part as accretion in determining the structure and density of the rime. Melcher used two temperatures for this experiment, -5 and -12°C ; the former is favourable for the growth of needles, the latter for the growth of dendrites. Under these circumstances it is possible that increasing the water content maintains higher vapour density gradients and causes a more rapid deposition of vapour with a subsequent fall in the mean density.

In a separate experiment using larger drops (ranging from 20 to 200μ diameter) Melcher found that the density increased from 0.6 to 0.7 g cm^{-3} as the liquid water content rose from 1.7 to 3.4 g m^{-3} ; the measurements were made at a temperature of -8°C and at a velocity of 5 m sec^{-1} . The corresponding increase in rime

temperature, which he measured with a thermocouple, was from -5 to -3.5°C . He pointed out that the largest drops in this case did not reach the experimental rod, so that it is not possible to compare his measurements directly with the present ones. His values lie between those of distributions A and B and a rise in surface temperature from -5 to -3.5°C results in corresponding increases in the density in both sets of measurements.

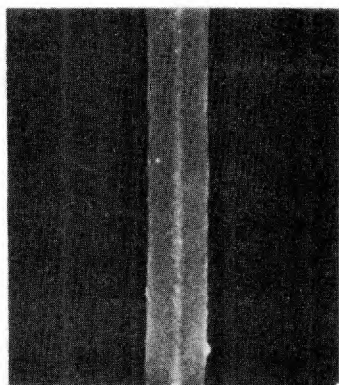
2.6 The nature of ice deposits.

Descriptions of the six main types of ice formed by accretion (clear, transparent and milky ice, and opaque, kernel and feathery rime) have already been given. This broad classification, which involves some personal judgement, is used in the following discussion. Examples of the various types of deposit are shown in Plate 2.2.

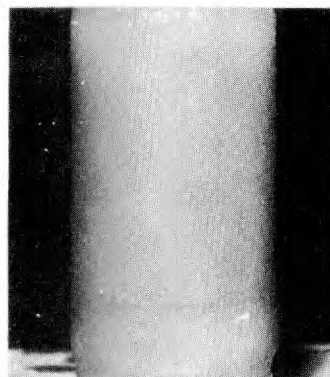
2.6.1 The opacity of the deposits.

The opacity and whiteness of ice deposits are considered to be due primarily to the presence of light-scattering air bubbles of minute size (the order of microns) frozen within individual accreted droplets, rather than to air trapped between frozen droplets. These bubbles are composed of air dissolved in the droplets before they freeze. This view has also been put forward by Coste (1940). It is possible to obtain samples of white opaque rime of high density (0.85 to 0.9 g cm^{-3}) and milky ice which, as a microscopic examination shows, contain virtually no interstitial air. It is possible also to form ice whose opacity is comparable with that formed by accretion, simply by freezing ordinary bulk water quickly. In this case the opacity is caused solely by the air which had been dissolved in the water.

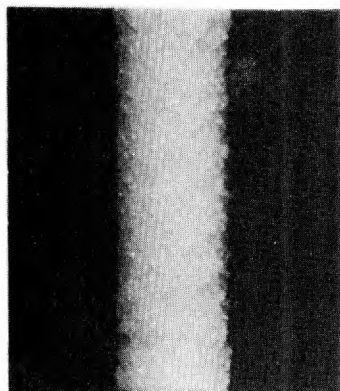
Bubble formation in a liquid is a typical nucleation phenomenon and the number and average size of the bubbles (and hence the opacity of the deposit) depends on the number of nucleation centres



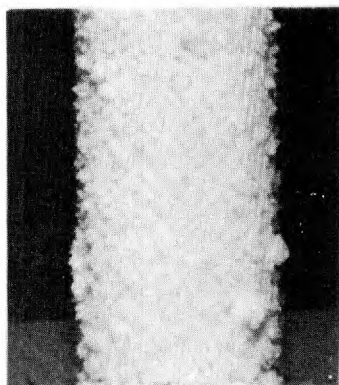
TRANSPARENT
ICE



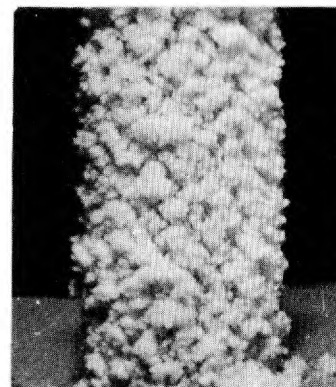
MILKY
ICE



OPAQUE
RIME



KERNEL
RIME



FEATHERY
RIME

Plate 2.2 The appearance of various types of rime and glaze ice deposits.
(It was not possible to satisfactorily reproduce a photograph
of clear ice.)

available at the supersaturation accompanying a given rate of freezing. If the freezing is rapid then large numbers of small bubbles form. Higher supersaturations are also produced when the droplets freeze from the surface inwards. In this circumstance the air coming out of solution in the centre of a freezing droplet is unable to escape through the solid surface. In relatively clean water it is possible that there are comparatively few foreign nuclei capable of initiating bubble formation. Supersaturations may then be reached in freezing droplets at which self-nucleation occurs in a manner analogous to the homogeneous nucleation of supercooled water, giving rise to numerous small bubbles.

The present experiments show that the opacity of the deposit is dependent on the surface temperature. For a given ambient temperature, liquid water concentration and airspeed the surface temperature is a function of $Ed^{0.4}$. For high windspeeds and large drop sizes the efficiency of catch does not fall markedly with cylinder size, so that the surface temperature rises with increasing cylinder diameter. For similar conditions in the present experiments it was often observed that, when the surface temperatures of the cylinders neared 0°C , the ice accreted on the largest cylinder was more transparent than that accreted on the smallest cylinder. In some instances the ice accreted by the smallest cylinder was white opaque rime while that on the largest cylinder was transparent glaze ice. The reason for this must be that at surface temperatures near 0°C , the manner of freezing of the accreted water is more like that of a liquid film, thereby permitting virtually all the dissolved air to escape by diffusion and by the migration of bubbles to the surface. This is brought about by the spreading and coalescence of the droplets.

2.6.2 The appearance of the deposits.

If the impact speeds of the droplets are low the initial places of deposition become favoured regions of growth due to a local increase in the efficiency of catch and at low temperatures the structure has a fragile feathery nature, the droplets freezing individually. An increase in the impact speed or in the droplet size results in the initial places of deposition becoming less favoured and the gaps between the "feathers" tend to be filled, giving the rime an appearance similar to that of "kernels of corn on a cob". At higher impact speeds, temperatures and drop sizes, the deposit has a much more compact structure for the reasons discussed earlier. It no longer has the appearance characteristic of feathery and kernel rime, but is simply an opaque mass. As the surface temperature approaches 0°C there is a change in the texture of the deposit, the droplets fusing together to give the glazed appearance which marks the transition from the family of rimes to that of the glazes.

The dependence of the appearance of the accreted ice on the surface temperature and on the product of impact speed and droplet radius, is indicated in Fig. 2.11. (There is no immediate physical reason for choosing $r v_0$ as the ordinate in this diagram; it is certainly better than using v_0 alone since, in this case, the kernel and feathery regimes are not separated. It would be just as appropriate to use the efficiency of catch, E , but the parameter $r v_0$ is convenient because it also determines the density.) It is seen that the family of glazes is characteristic of a narrow range of surface temperatures close to 0°C . Fraser, Rush and Baxter (1950) also found that the transition from the rime to the glaze family occurred at surface temperatures near 0°C . The slopes of the lines defining the members of the glaze family may possibly be spurious but the higher windspeeds would promote spreading causing the glazed appearance to occur at slightly lower values of T_s .

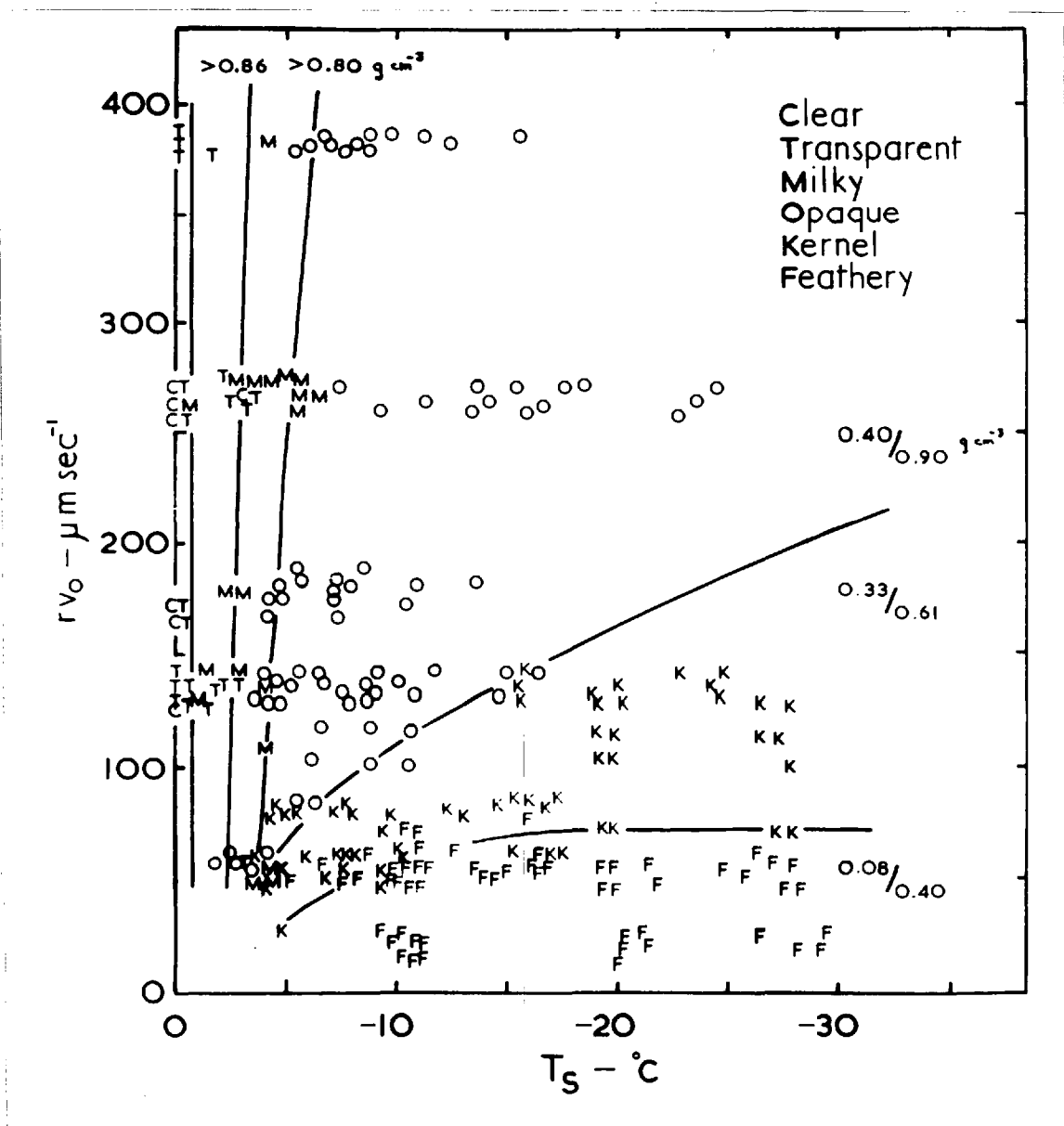


Fig. 2.11 The dependence of the appearance of the ice deposits on the parameter $r v_0$ and the surface temperature. Areas are broadly defined in which the type of icing falls within one or another category. The density ranges obtained for each type of deposit are indicated.

As already indicated, the opacity of the deposits may be attributed primarily to the presence of air previously in solution. It is of interest to ascertain the effect of this dissolved air on the density. There is no experimental information on the solubility of air in supercooled water but extrapolation of the known solubilities suggests that the percentage by volume of dissolved air increases from three to seven as the temperature decreases from 0 to -30°C . (Computations based on the rate of solution of air in water given by Dorsey (1940, p.552) indicate that even the largest droplets employed were over 90% saturated by the time they reached the cylinders.) This implies a density range of between about 0.85 and 0.92 g cm^{-3} depending on the ambient temperature and the amount of air which escapes during freezing. The densities of the glaze family measured in the present experiments ranged from 0.80 g cm^{-3} upwards but this value is likely to be low owing to experimental error. The average density obtained for transparent ice in the present experiments was 0.906 g cm^{-3} (9 cases) while that obtained for milky ice was 0.876 g cm^{-3} (19 cases). It is likely therefore that the density, as well as the opacity, of members of the glaze family is due to the air dissolved in the supercooled droplets. The members of the rime family on the other hand are formed over a much wider range of conditions and this is reflected in their densities; the density ranges of opaque, kernel and feathery rime deposits were 0.40 to 0.90 , 0.33 to 0.61 and 0.08 to 0.40 g cm^{-3} respectively.

2.6.3 The effect on the deposit of the airflow about the accreting object.

In the above experiments a cylinder was used as the accreting object, so that the nature of the airflow, which affects both the transfer of heat to the airstream (and hence T_g) and the speed of impact of the droplets, is known and determined only by the Reynolds number. Further, the rime was evenly deposited on the

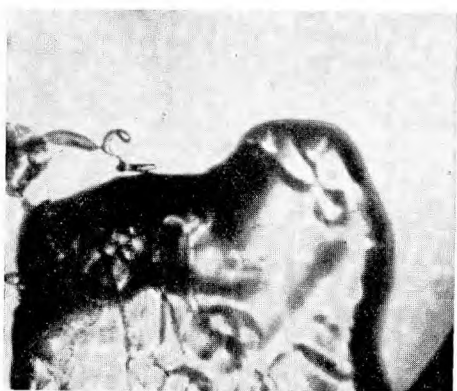
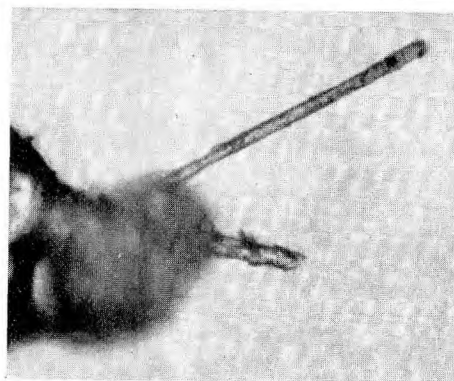
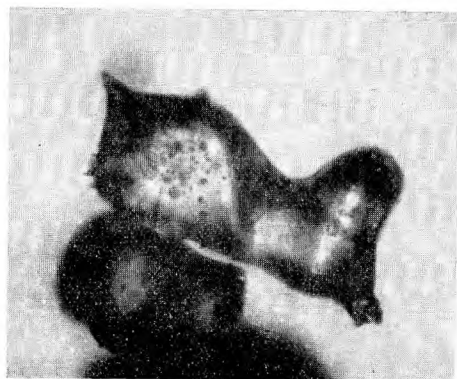
collecting surface. In general, however, the shape of the accreting object and the manner in which the deposit is distributed over the surface will affect, to some extent, its density and structure. The flow around irregularly shaped objects is complicated but, provided T_g and v_o can be obtained, Figs. 2.8 and 2.11 may be used to give an indication of the density and type of deposit to be expected under various conditions.

In some of the experiments carried out at Mount Washington, for example, the collectors (not all of which were cylinders) were held stationary and it was frequently observed that the ice deposit formed on a single collector some centimetres across was transparent or clear at the centre but became milky and then opaque towards its edges (Clark, 1948). The reason for this is that both the rate of deposition (and hence T_g) and impact speed have values which are a maximum at the foremost edge and decrease as the distance from this increases. This is reflected in the change of the nature and density of the deposit across the surface.

2.7 The formation of ice splinters during riming.

On occasions during the riming experiments it was noticed that the ice crystal concentration in the cold room increased two or three orders of magnitude over a period of some minutes, reaching values of the order of $100 L^{-1}$. (The influence of these crystals on the rime density is not considered to be great since the ice crystal concentration was a small fraction of the droplet concentration).

It has been shown (e.g. Mason and Maybank, 1960) that the freezing of droplets from the surface inwards, results under certain conditions in the rupturing of the outer shell with the formation of cracks and spicules; occasionally the droplets shatter completely. Simultaneously ice splinters are ejected, and Mason and Maybank state that, in the last-mentioned case, 100 or more fragments may be produced by one drop. The photographs in Plate 2.3 (a) and (b) indicate that such a manner of freezing occurs also during accretion.



200 μ

Plate 2.3 Photographs of spicules and a splash formed during accretion.

The spicule in (b) is 340μ long and stems from a droplet 80μ in diameter (out of focus). Spicules were observed in rime formed at surface temperatures near freezing and at velocities of 2.5 m sec^{-1} . The former condition agrees with that found by Mason and Maybank; the latter is possibly particular to the present experiments. Higher velocities (6 and 12 m sec^{-1}) result in a close packing of the droplets so that spicules, if formed, are not readily observed. However, because of the distortion of the droplets at these speeds, the shell may be broken up as it forms, so that there would be much less chance of the phenomenon occurring.

The microscopic examination of the rime samples obtained also revealed structures on the surface similar to that shown in Plate 2.3 (c). It is improbable that this represents a chance configuration of droplets and a more plausible interpretation is that it is the splash of a droplet, the remnants of which have frozen in the manner illustrated. The splashing of droplets collected in kerosene has been studied by Rupe (1950) who gives the empirical relation $v_c d = 1785$ from which the critical velocity, $v_c (\text{m sec}^{-1})$ for the splashing of droplets of diameter $d\mu$ may be obtained. At 12 m sec^{-1} , d is 150μ . Splashing on solid surfaces may be anticipated to occur at somewhat lower speeds and smaller drop sizes, and therefore within the range of the present conditions. Such a structure on the riming surface would be very fragile and readily disintegrated, liable to be blown off by the air motion or broken off by the impact of other droplets. Numbers of small ice particles may be introduced into the air in this way.

Similar splash-structures were observed at moderately low temperatures, -10°C and below, and at the higher air speeds, 6 m sec^{-1} and above. Such conditions are reasonable for a minimum speed would be required to cause splashing, while the temperature has to be sufficiently low to freeze the splash as it forms. The values of the temperature and velocity quoted, however, are not necessarily limiting, since experiments were not conducted over a

continuous range of these parameters. There will also be a dependence on the drop size as larger drops tend to shatter more readily.

Rime of low density (0.1 to 0.3 g cm^{-3}) is itself extremely fragile and fragments can readily break away and contribute to the production of ice particles. Such low densities occur at temperatures of about -20°C and lower and at airspeeds of 2.5 m sec^{-1} and less.

It was not possible to estimate quantitatively the relation between the numbers of ice crystals produced and the rate of accretion, as the inner surface of the tunnel and other auxiliary equipment were also being rimed and undoubtedly contributed to the production of the splinters.

Appendix 2.1 : The cooling of the droplets sprayed into the cold room.

According to the technical information supplied with the equipment used to produce the droplet fog in the cold room, the rate of consumption of compressed air, under the conditions of the experiment, was about $0.01 \text{ m}^3 \text{ sec}^{-1}$ and the rate of feed of water 0.1 to 8 g sec^{-1} depending on the number of sprays in operation and the manner of their operation.

The distance between the bank of sprays and the tunnel mouth was 3 metres. Within 1 metre of the sprays the roughly conical jet of droplets had a cross-sectional area greater than 0.1 m^2 and the droplets were moving with a speed of 2.5 m sec^{-1} . The flux of air at this point was therefore of the order of $0.25 \text{ m}^3 \text{ sec}^{-1}$ and the mixing of the room air with the compressed air was in the ratio of $25 : 1$. At the tunnel mouth the cross-sectional area of the droplet stream was greater than 1 m^2 and the rate of movement about 1 m sec^{-1} so that mixing had taken place in the ratio of $100 : 1$.

A thermometer placed in the jet of air within a few centimetres

of the spray nozzle indicated temperatures of -10 to -12°C when the ambient temperature was -20°C . The maximum temperature difference between this air and the room air was therefore about 20°C (the minimum working temperature was -30°C) and, at the mixing ratios given above, the temperature of the air passing into the tunnel would not have differed from the ambient temperature of the room by more than a fraction of a degree. Measurements confirmed this.

When water droplets were also ejected by the sprays, the mean temperature of the droplets within a few centimetres of the spray nozzles was less than 0°C since ice formed on an object placed in the droplet stream. This initial cooling of the water was presumably produced partly by the cooling by expansion of the compressed air as the effective thermal capacities of the water and the air ejected were comparable. The maximum cooling of the water and air mixture required before its entry into the tunnel was therefore 30°C and, at the maximum water concentration of 8 g m^{-3} the temperature of the airstream in the tunnel would be anticipated to be higher than the general room temperature by 1°C at most. In fact, the temperature of the supercooled fog near the tunnel mouth was measured to be 1 to 2°C above room temperature under extreme conditions, but a substantial part of this temperature difference may be attributed to the release of heat as the thermometer bulb became rimed.

Thus both the water droplets and the compressed air were admitted without appreciably raising the temperature of the airstream in the tunnel above that of the room. It has still to be discussed, however, whether the droplets themselves have time to cool to room temperature before reaching the tunnel mouth.

In the last two metres of their path, the droplets are in an environment which, because of the mixing, possessed the properties of room air. They traversed this distance in a time estimated to be 1 second. Relative to the airstream they were falling

downwards at their terminal velocities and, in this circumstance, it is possible to estimate their rates of cooling in the following way.

The rate at which heat is conducted away from a sphere in air has already been given (eq. 2.5) namely

$$\frac{dQ}{dt} = 4 \pi r \left[a K (T_s - T) + b L_v k \Delta \rho \right] \quad (2.10)$$

where T_s is the temperature of the droplet surface and T is the ambient temperature, the other symbols having the same meaning as before.

The heat Q which must be supplied to the particle to change its temperature from T_1 (initial value) to T_s is given by

$$Q = \frac{4}{3} \pi r^3 \rho c (T_s - T_1) \quad (2.11)$$

where ρ and c are respectively the density and specific heat of the particle. The two terms in the bracket on the right hand side of eq. 2.10 are of comparable magnitude and it is convenient to ignore the second. If the time taken for a droplet to cool from an initial temperature T_1 to a temperature T_s is t , then

$$t < \frac{-\rho c r^2}{3 a K} \cdot \ln \frac{T_s - T}{T_1 - T} \quad (2.12)$$

When the droplets cool from 0°C to within 5% of the ambient temperature

$$t < \frac{r^2}{a K} \quad (2.13)$$

For the largest droplets used in any of the distributions (140 μ diameter) a is about 2; at these temperatures K is 5.5×10^{-5} cal $\text{cm}^{-1} \text{sec}^{-1} \text{deg}^{-1}$ and $t < 0.4$ seconds. This is less than the time taken (1 second) for the droplets to reach the tunnel even though an extreme value of the initial temperature difference has been used and the contribution to the cooling by evaporation has been ignored. The enhanced ventilation during the retarding of the droplets from the ejection velocity (some tens of metres per second) to about 2.5 m sec^{-1} in the first metre of their path, has also been neglected. The cooling time is therefore more likely

to be 0.1 to 0.2 seconds.

Appendix 2.2 : The effect on the rime density of absorption of heat by the supports.

In the above analysis and discussion the effect of the absorption of heat by the cylinders, and by the insulating supports on which they were mounted, has been ignored and the density of the deposit has been related solely to parameters which are determined by the conditions within the droplet stream and by the size of the collector. Since, before the onset of riming, the cylinders were at ambient temperature, transfer of heat into the cylinders would have occurred until at least the surface layers became warmed to a temperature approximately equal to T_s . Heat is transferred also by radiation, and by conduction from the cylinders through the supports to the rest of the equipment. The former was estimated to be less than 1% of the total heat transfer and has been ignored; computations showed that the latter also was negligible, particularly as the lower supports themselves were being warmed by the riming process.

The thermal capacity of the smallest cylinder (a 3.5 cm brass pin mounted on the former of the second cylinder) was such that it could have absorbed only a few per cent of the heat liberated by the freezing droplets even for a temperature increase of 20 to 30°C. The largest cylinder could have absorbed about 10% of the heat liberated by the mass of rime deposited on it for these extreme temperature increases.

The rate of heat transfer from unit area of the cylindrical surface is given by the right hand side of eq. 2.3. Since the two terms in brackets are comparable in magnitude, then for $d = 1$ cm and a velocity of 2.5 m sec^{-1} , the rate of conduction of heat to the environment is approximately $1.5 \times 10^{-3} \Delta T \text{ cal cm}^{-2} \text{ sec}^{-1}$; at 12 m sec^{-1} it is a factor of 2.5 greater. The thermal conductivity of the material composing the former was $8 \times 10^{-4} \text{ cal cm}^{-2} \text{ sec}^{-1} \text{ deg}^{-1}$,

so that the rate of transfer of heat into the surface layer of the former may have been comparable with that to the environment. The thermal capacity of the former of the largest cylinder was such that, for a temperature rise of 10°C , the cylinder and former could have absorbed as much as 50% of the heat liberated by the freezing droplets, depending on the mass of ice accreted. The greatest temperature increase occurred with distribution B at 12 m sec^{-1} but since at this velocity the density is high ($>0.8\text{ g cm}^{-3}$) the effect would not have been appreciable. With the lower velocities the temperature increases were not as great and consequently the heat absorption by the cylinders would have been less.

In order to clarify this, some fifty of the measurements covering the whole range of conditions were repeated using only the smallest and largest cylinders, now mounted on thin-walled insulating supports. The heat absorbed by the smallest cylinder could have been again only a few per cent, while that absorbed by the largest cylinder and its support was now only about 10% of the heat liberated by the freezing droplets. Further, the loss of heat through the supports to the rest of the equipment was considerably reduced, by a factor of at least 20.

In Fig. 2.12 the densities obtained in the second series of experiments are shown plotted against the smoothed values of the densities obtained for corresponding conditions in the first series. The two sets of values agree to within the experimental error, proving that any absorption of heat by the supports did not affect the density significantly. In some instances the largest cylinder was chilled to about -80°C and packed with dry ice. The densities obtained under these conditions are also shown in Fig. 2.12 and are some 20 to 40% lower than those obtained without the severe chilling.

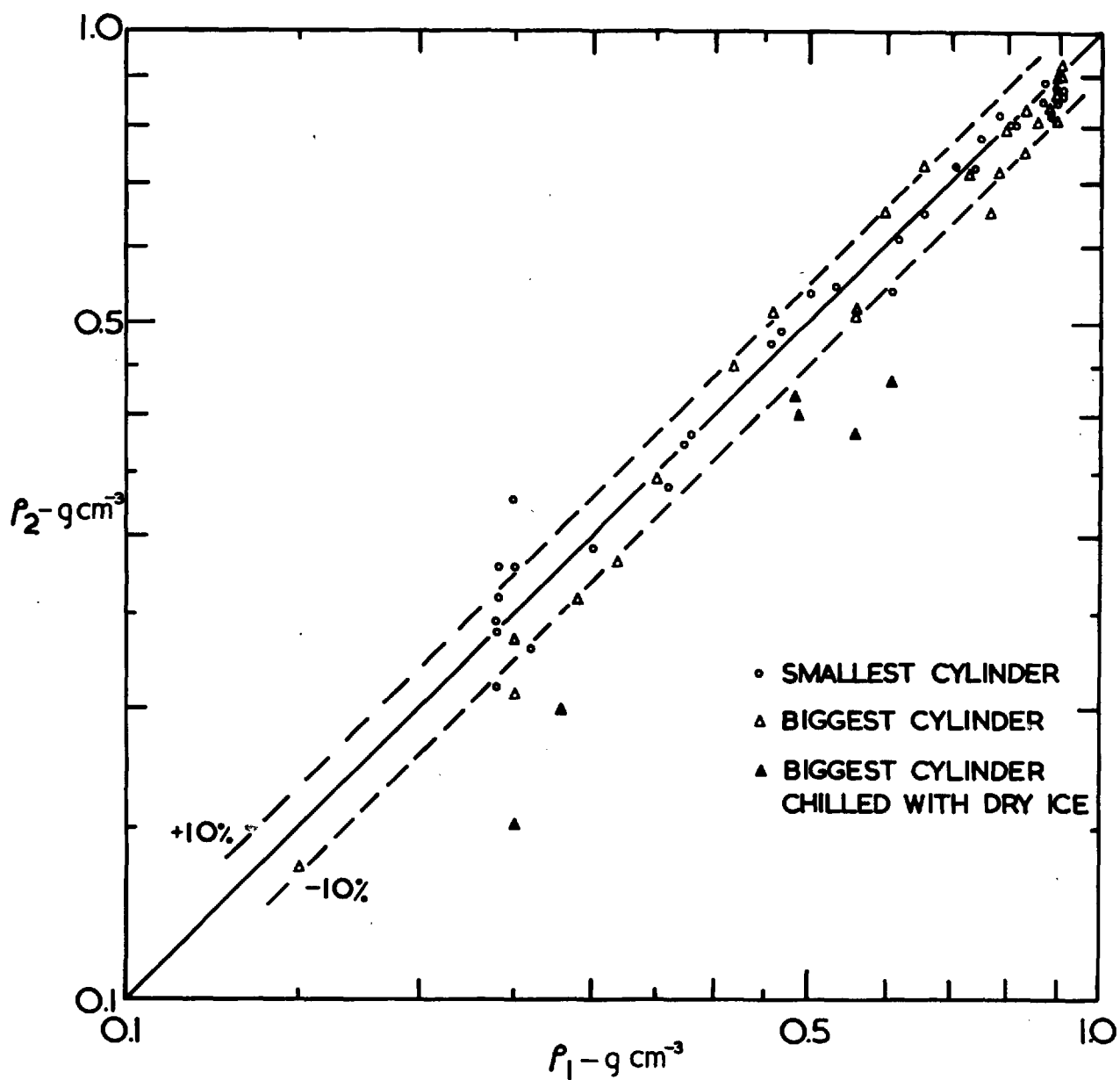


Fig.2.12 The densities obtained using the better insulated cylinders (ρ_2) plotted against the densities obtained for corresponding conditions in the first series of measurements (ρ_1).

In virtually all icing problems there will be some heat transfer between the accreting object and the ice deposited, and this will affect, to a greater or lesser degree, the density and structure of the deposit. Some of the fall in density with increasing cylinder size, shown in Figs. 2.4 and 2.6 may be due to this effect.

3 : ACCRETION UNDER WET GROWTH CONDITIONS.

3.1 Introduction.

It has been shown in section 1.4 that large hailstones are likely to grow in the wet condition, that is, they accrete more liquid water than can be frozen. In this circumstance Ludlam (1958) assumed that the stone grows as a solid mass of density 0.9 g cm^{-3} , the excess water being shed in its wake. This greatly restricts their growth and the theory has difficulty in explaining how large hailstones are produced in a reasonable time.

In the experimental investigation of the factors affecting the density of the deposit during dry growth, it was observed that, when the cloud liquid water concentration exceeded the value for which the cylinder surface became wet, the excess was not shed in the wake but was incorporated into the ice structure, producing a "mushy" or "spongy" deposit which contained a substantial fraction of unfrozen liquid.

A survey of the literature showed that this phenomenon was known to other workers. Fraser, Rush and Baxter (1952) describe icing tunnel experiments which resulted in deposits from which water could be squeezed "as though from a wet sponge". This type of ice has also been observed on aircraft flying through supercooled clouds (Brun, 1957).

It was further noticed during the experiments that ice crystals occasionally present in the airstream became incorporated into rime structures and could be seen glinting in the surface. The number of ice crystals swept up in this way depended on the condition of the surface, and when the surface was wet, appreciable quantities of small ice particles, deliberately introduced into the airstream, were collected. This too has been observed by other investigators working on problems of ice accretion. Clark (1948) describes experiments carried out at the Mount Washington

Observatory, U.S.A., in which objects were exposed in the supercooled cloud at the summit. Snow and fragments of rime were sometimes seen embedded in the accreted ice. Minervin (1956) states that when measurements of liquid water concentration are made by accretion instruments in supercooled clouds, the measured values of the liquid water concentration may exceed the critical value for the instrument because ice crystals become frozen into the newly formed ice.

It is apparent that these two phenomena, the formation of mushy or spongy ice and the accretion of ice particles as well as supercooled droplets could well affect the growth of hail in the wet condition. Gaviola and Fuertes (1947) assert that the "use of snow as a building material (of hailstones) in addition to supercooled water droplets, makes the growth to large sizes easier and explains the concentric layers". Landis (1907) describes large oblate hailstones (about $2\frac{1}{4}$ " by $1\frac{3}{4}$ ") having layers which were "undoubtedly moist snow". Pollard (1936) has also described hailstones with layers which appeared to be "snow frozen hard". These last two interpretations cannot be accepted without reserve, however, as there are varieties of rime ice which could give such impressions.

In order to elucidate these phenomena, the following experiments on ice accretion in mixed clouds were carried out.

3.2 Experimental procedure.

The experimental arrangement was similar to that described in section 2.2, the only difference being that ice particles could also be introduced into the tunnel in addition to the supercooled droplets. This was effected by sieving rime of low density (0.1 to 0.3 g cm^{-3}) through fine gauze situated just above the tunnel mouth (see Fig. 3.1). The mixture of ice particles and supercooled water droplets was then blown over the cylinders, which were rotated horizontally to give more or less uniform deposits. Only

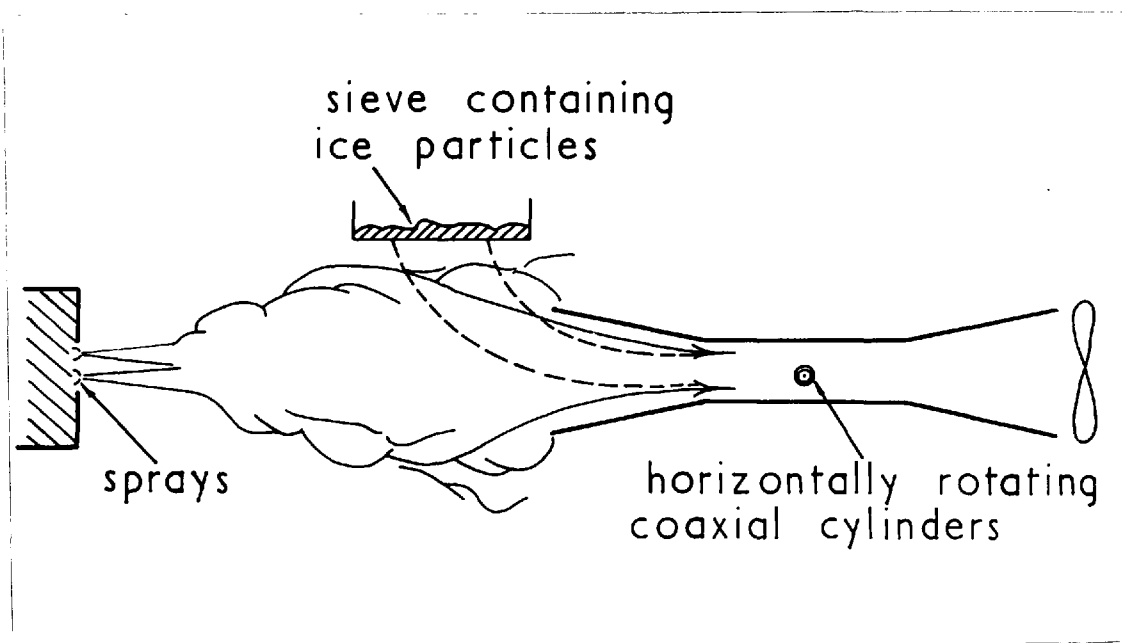


Fig. 3.1 Schematic diagram of experimental arrangement.

temperatures of -12°C and above and airspeeds of 13 m sec^{-1} were used in these experiments in order that the wet condition could be readily attained.

The ice particles were dispensed manually at an approximately uniform rate; the mass of ice used was determined by weighing the amount of ice in the sieve before and after each experiment, the fraction actually entering the tunnel being estimated visually. When the surface of the deposit remained wet even though ice particles were also being accreted, it was possible to obtain an independent measure of the ice concentration, since it could be assumed that all the particles in the volume swept out by the cylinders were being collected. Ice concentrations up to 5 g m^{-3} were produced and estimated to $\pm 30\%$.

The size distribution of the ice particles was determined in the same manner as that of the droplets. Narrow strips of perspex coated with oil were exposed in the tunnel and the particles caught and photographed through a microscope. The number distribution of ice particles was roughly constant from 20 to 300μ and the equivalent median volume diameter was estimated to be 260μ . It was not possible to use larger ice particles as these could not be suspended in the airstream long enough to reach the cylinders.

For given conditions of ambient temperature, airspeed and liquid water concentration, droplets only were sprayed into the tunnel, and accretion permitted to occur on a set of cylinders for a chosen time. A period of two to ten minutes was sufficient to build up a coating a few millimetres thick at the liquid water concentrations and airspeeds used. The experiment was then repeated using another set of cylinders and adding ice particles to the airstream. Finally a third control experiment was performed using droplets only. The three sets of cylinders were photographed and weighings made of sections from individual cylinders. Where the deposits were irregular, the average radial dimension was obtained by assuming that the density of the deposit was 0.90 g cm^{-3} ;

the error introduced by this assumption is insignificant. The agreement between the mass measurements of the first and final experiments was better than 10%.

3.3 Results and discussion.

3.3.1 Experiments using droplets alone.

The critical liquid water concentration, W_c , which just maintains the surface temperature of an accreting cylinder at 0°C is obtained from eq. 2.3 by putting $T_s = 0$, so that

$$\frac{E W_c v (L_f + T)}{\pi} = \frac{0.24 \text{ Re}^{0.6}}{d} (L_v k \Delta p - KT) \quad (3.1)$$

the symbols having the same meaning as before. The flow around the cylinders is assumed to be non-turbulent; if the flow is turbulent then the transfer of heat may be increased by up to 40% (Reiher, 1925).

The values of W_c have been computed from eq. 3.1 for the present experimental conditions and these are shown in Fig. 3.2 plotted against the liquid water concentrations (corrected for values of the collection efficiency less than unity) deduced from the mass accreted on the cylinders. The four conditions of the ice surface indicated in the diagram are: "dry"; "just wet" as determined by a visible sheen on the surface; "wet" when the surface is covered by a liquid film; and "spongy". It can be seen that the liquid which may be retained by the cylinder exceeds the critical value deduced from eq. 3.1 by as much as 7 times.

Eq. 3.1 defines the liquid water concentration in which the surface temperature of the ice deposit on the cylinder is just 0°C . If the water concentration is greater than the critical value then it is possible that an additional amount of water $(W_c - W)T/80$ be frozen if the temperature of the excess water is raised to 0°C also. Further, when spongy, the deposit is readily moulded by the airstream, and becomes irregular and knobbly.

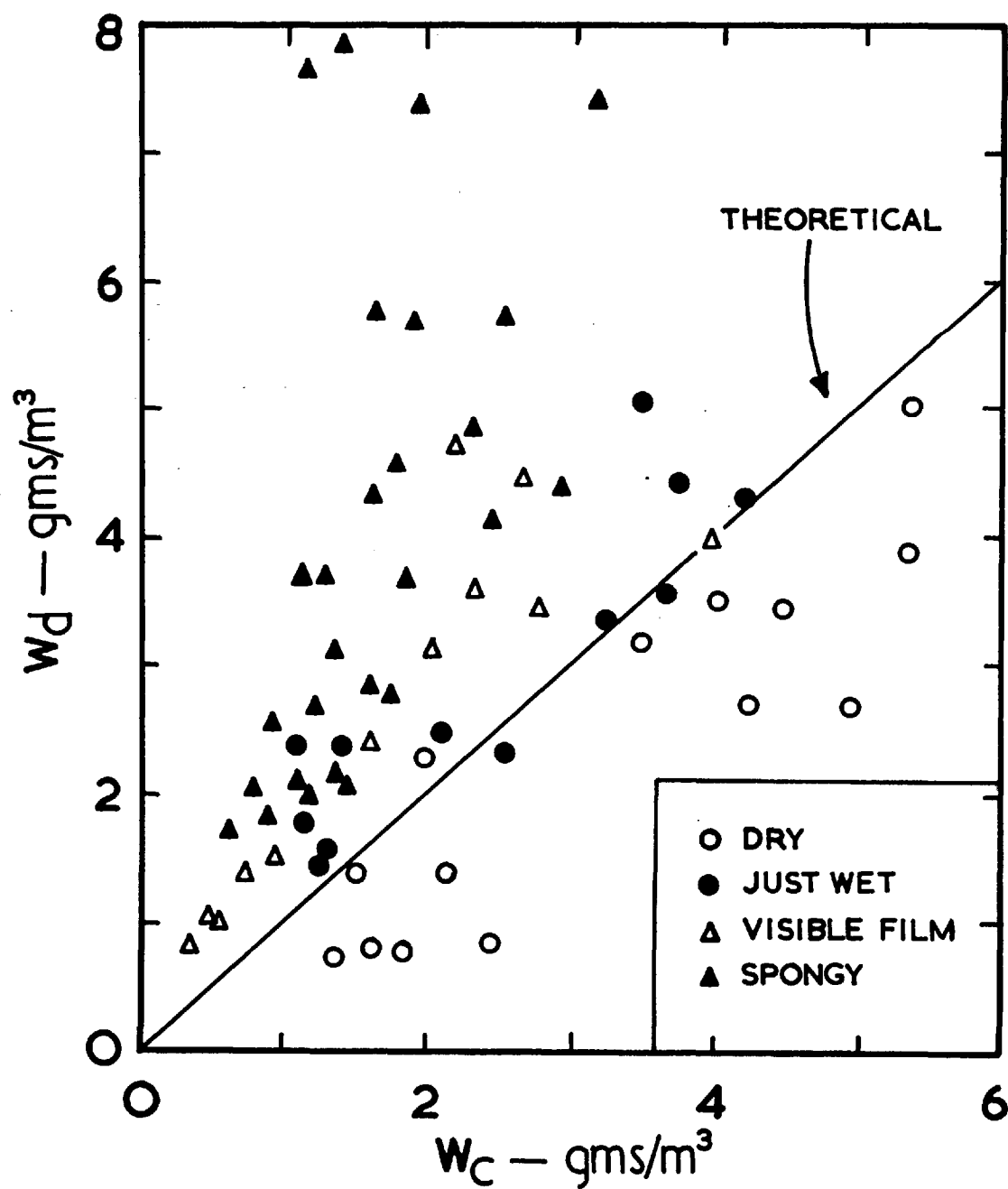


FIG.3.2 The liquid water concentration effectively caught by the cylinders, $W_d \text{ g m}^{-3}$, plotted against the critical liquid water concentration, $W_c \text{ g m}^{-3}$.

This increases the surface area above that of a cylinder of comparable diameter, promotes turbulence and thus raises the rate of heat transfer. Some heat is absorbed also by the cylinders themselves and by their supports. While this is only a few percent in the case of the smallest cylinder it may be 10% or more in the case of the largest one (see Appendix 2.2). For these reasons the factor of 7 given above does not imply that there was 7 times as much liquid as ice in the deposit; for this ratio a value of 2 or 3 is more likely. This value was roughly confirmed by a visual examination when the deposits were broken up. Using a quantitative calorimetric method, List (1959a) has since determined values of up to 72% for the proportion of unfrozen liquid within similar spongy ice deposits.

Spongy ice was produced at all temperatures down to -12°C , the lowest employed in these experiments. There was, however, less tendency for appreciable spongy ice deposits to be produced at temperatures only just below freezing (-1 to -2°C), and at these temperatures, most of the water was in fact shed in the wake of the cylinders.

3.3.2 Experiments with mixed clouds.

If the accreting object has a wet or spongy surface layer, then it is obvious that an ice particle arriving with a speed of several metres per second can sink into it and be captured. This ice particle can remove the heat liberated by the freezing of a mass of water corresponding to the fraction $-T/160$ of its own mass (the specific heat of ice is $0.5 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}$ and the latent heat of fusion of water 80 cal g^{-1}) and become frozen into the surface.

The experimental data for the smallest cylinder, which absorbs virtually none of the heat liberated by the freezing droplets, are shown in Table 3.1; all the relevant quantities are converted to the corresponding spatial concentrations. Since

TABLE 3.1 : Experimental data for the smallest cylinder.

T is the ambient temperature, v the airspeed, W_c the critical liquid water concentration computed for the mean diameter of the iced cylinder, W the liquid water concentration, W_d the water concentration effectively caught by the cylinder when droplets were used, W_i the concentration in addition to W_d caught when ice particles were used as well as the droplets, and m_i the estimated concentration of the ice particles dispensed.

T (°C)	v m sec ⁻¹	W_c g m ⁻³	W g m ⁻³	W_d g m ⁻³	W_i g m ⁻³	m_i g m ⁻³
-12	13.2	4.0	4.0	4.0	0	2.1
	13.5	3.7	3.6	3.6	0	1.9
- 8	13.6	2.6	2.4	2.4	0	2.2
- 7.5	12.8	2.5	0.9	0.9	0	1.9
			5.8	5.8	1.4	1.4
					1.6	2.0
- 7	12.6	2.5	4.2	4.2	0.9	1.5
					0.6	5.3
	12.8	2.2	1.4	1.4	0	2.7
- 6	12.8	1.9	3.7	3.7	2.6	3.0
	13.0	1.9	7.5	7.5	2.6	2.8
					3.3	5.7
- 5	12.8	1.7	2.8	2.8	1.1	2.9
- 4.5	12.6	1.4	2.2	2.2	1.3	3.2
- 4	13.1	1.4	2.1	2.1	1.0	2.7
- 2	13.3	0.9	2.0	1.4	1.1	1.1
					2.3	2.9
- 1.5	13.3	0.8	7.5	1.4	3.6	1.1
					7.3	2.5

in most cases no appreciable quantity of water was shed into the wake of the cylinder, the water concentration given by the sprays is assumed to be that determined from the mass of the deposit. It was only at temperatures near 0°C that these two quantities differed appreciably. The liquid water concentrations given by the sprays under these latter conditions was estimated from the rate of accretion at lower temperatures and the values given are considered to be correct to $\pm 20\%$.

If the surface was dry (i.e. $W < W_c$) then, although individual ice crystals were incorporated into the rime structure, no appreciable quantities of ice were accreted. For values of $W > W_c$, the maximum quantity of ice accreted depended on the excess water concentration $W - W_c$, as indicated in Fig. 3.3: here it is seen that it was possible to use for accretion approximately 1 g m^{-3} of ice for each 1 g m^{-3} of excess water. There is a dependence on the temperature, the amount of ice which can be accreted increasing with temperature rise. A better parameter to use is the quantity $W - W_c/W_c$ since this is a measure of the ratio of unfrozen to frozen liquid or the "degree of sponginess" of the deposit formed from the water droplets alone. The relationship between W_i and this parameter is shown in Fig. 3.4. While there is still some scatter in the points, this does seem to be a better presentation of the data than Fig. 3.3. The amount of ice which was accreted was approximately 1.4 times the value of $W - W_c/W_c$. In both these diagrams, for the reasons given above, W_c is an underestimate (by a factor up to perhaps 2) of the actual critical liquid water concentrations of the cylinders and this must be taken into account when applying these results.

Plate 3.1 shows the iced cylinders after the experiments providing the data in the last entry of Table 3.1. The cylinders on the left were exposed to 7.5 g m^{-3} of droplets and the effective concentration accreted was 0.8 g m^{-3} ; the centre cylinders were exposed to 1.1 g m^{-3} of ice particles, and those on the right hand

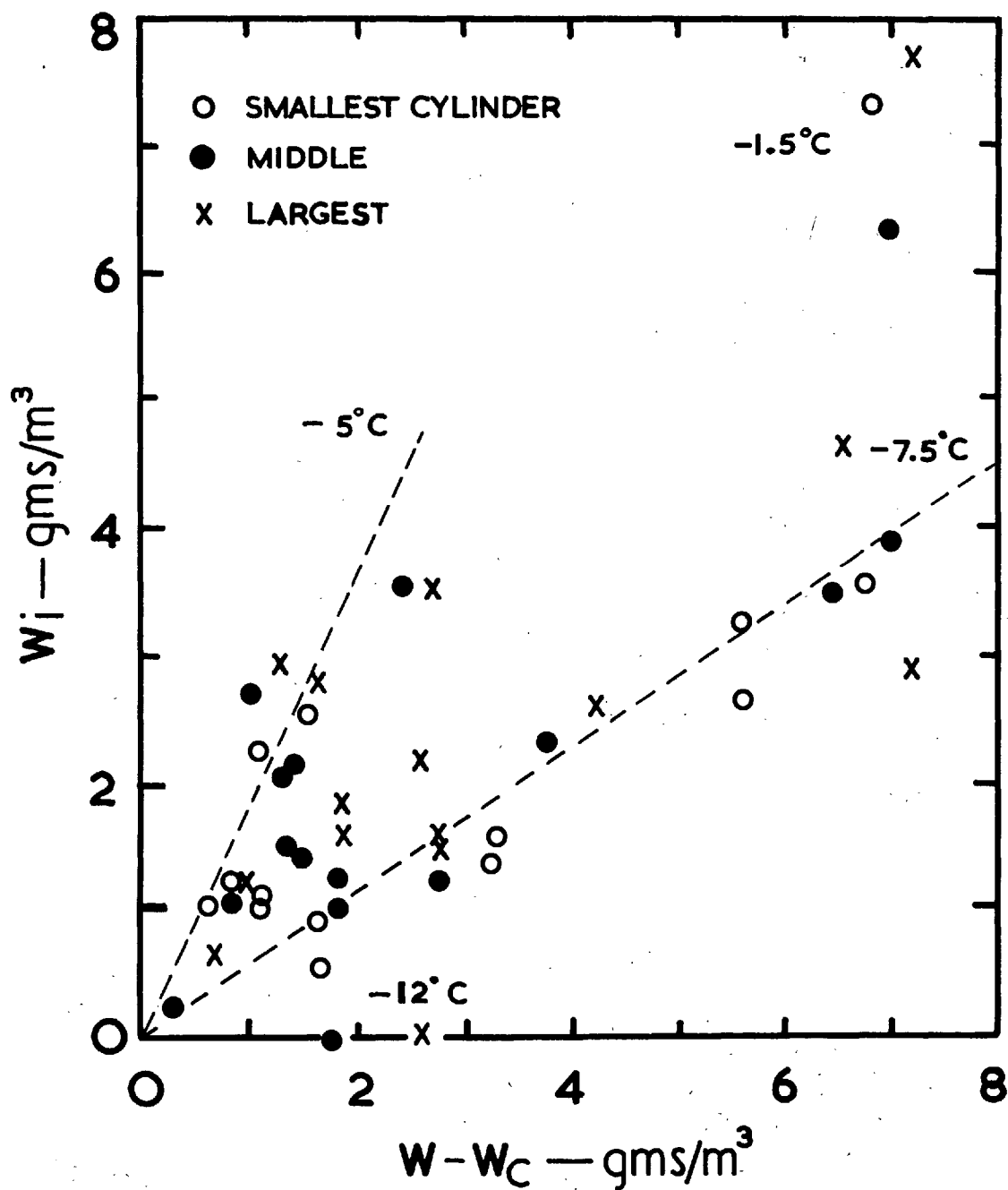


FIG. 3.3 The concentration of ice particles, W_i g m⁻³, accreted in addition to the supercooled droplets, as a function of the excess liquid water concentration, $W - W_c$ g m⁻³.

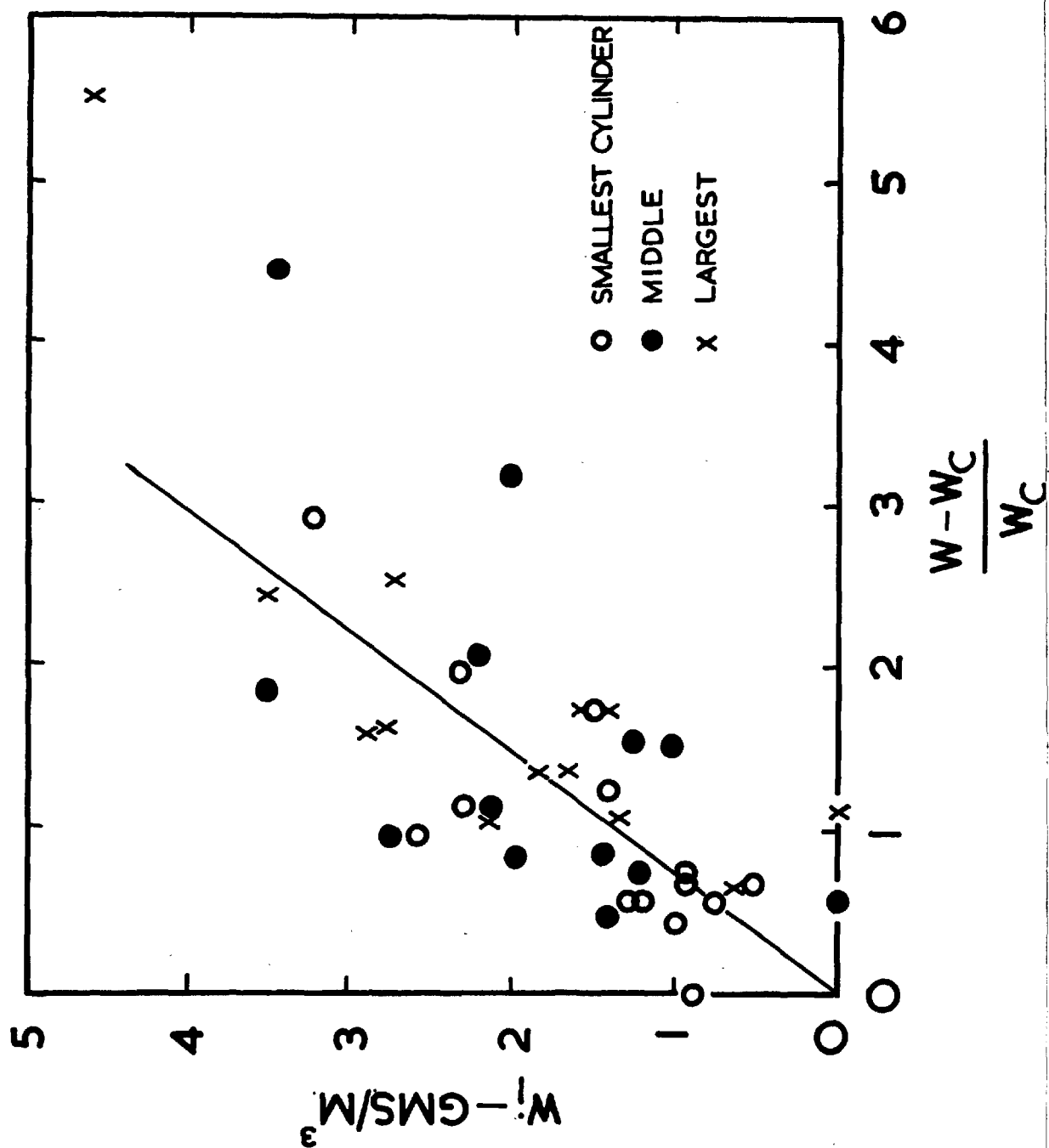


Fig. 3.4 The concentration of ice particles, W_i g m⁻³, accreted in addition to the supercooled droplets, as a function of the quantity $W - W_c/W_c$.

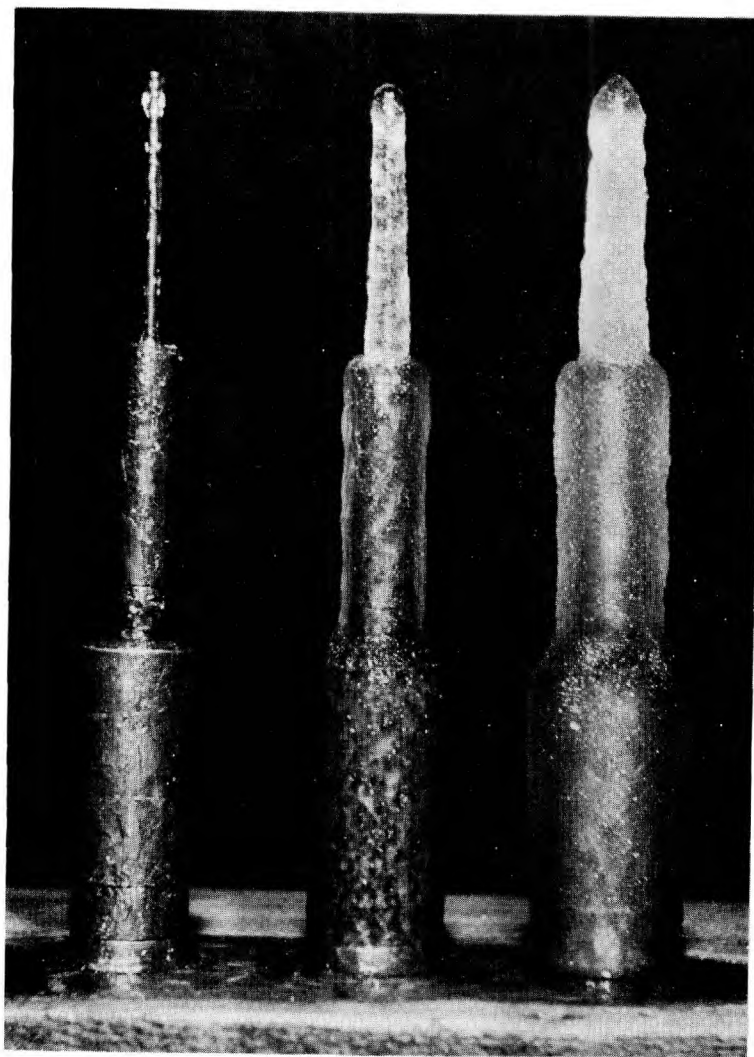


PLATE 3.1 The iced cylinders after the experiments providing the data in the last entry of Table I.

side to 2.5 g m^{-3} of ice, in addition to the droplets. The effective concentration accreted in this last case was 7.8 g m^{-3} . The additional liquid collected was far in excess of the amount which could possibly be frozen by heat transfer to the ice particles, which means that the ice particles themselves influenced the manner in which the liquid water was held within the deposit.

3.3.3 The nature of the accretion process during wet growth.

When all the liquid caught by an object in the supercooled cloud cannot be frozen, a film of water accumulates on the surface. This film has been observed during the experiments and it was estimated that, at the airspeeds used, the thickness was a fraction of a millimetre, $\frac{1}{4}$ mm or less. It was also noted that at the highest temperatures employed, the liquid water composing the film was blown rapidly to the downstream side of the cylinders, where droplets 1 to 2 mm in diameter were being shed into the wake. If the ice continues to grow as a compact mass of density about 0.9 g cm^{-3} as Ludlam (1958) suggests, there is no way in which liquid water can be held within the ice structure and the excess must be blown away.

When ice particles are also accreted, the circumstances can be greatly changed. Hallet (1960) considers that if an ice crystal alights with arbitrary orientation on a water surface which is supercooled, crystal growth occurs both along the surface and into the liquid, the structure in the latter direction being a system of branching dendrites. Provided there is effective nucleation, crystallisation at the surface of the liquid film is favoured, since there the heat is conducted away more readily. When the number of accreted ice particles, and therefore of surface nucleation centres, is relatively large, an interwoven mesh of dendritic ice structures can be supposed to form within the film. This traps excess liquid water so that a mushy or spongy ice layer can develop and be retained if the mechanical strength of the

dendritic system is sufficient.

in air An examination of the crystalline structure of spongy ice deposits (see next section) suggests that, if this is the mechanism by which spongy ice deposits are formed, ice crystal concentrations of the order of 1 L^{-1} are sufficient to ensure the trapping of liquid. Even if some of the droplets themselves did not freeze, incidental splintering from rimed auxiliary equipment could readily produce free ice particles in these concentrations. They are only fractions of a per cent of the droplet concentrations (order 100 cm^{-3}) and would be undetectable in the droplet stream. Nevertheless, their presence may be necessary for the formation of spongy ice deposits.

On the other hand, very little is known about the way in which ice crystallises from the liquid under the conditions of wet growth, when large temperature gradients are maintained by the continuous arrival of supercooled water. It is possible that the ice does not continue to grow outward as a compact mass of high density, but that a crystalline structure similar to that envisaged in the last paragraph penetrates into the liquid film.

In Table 3.1 it can be seen that spongy growth was inhibited at temperatures only just below freezing, and it was not in fact until substantial quantities of ice particles were deliberately introduced into the airstream that considerable deposits were produced. In this case the spongy growth is presumably formed in a different way, the ice particles being frozen together irregularly into an open structure, the interstices of which filled with water.

In the case of wet growth on non-rotating objects, spongy projections can grow in much the same manner as icicles. As liquid flows to the back of the object it tends to stream off any irregularities; since the heat transfer is favoured because of the increased surface area, the irregularities continue to grow outward as partially frozen cylindrical projections. Such structures have been observed by the writer in the laboratory. Plate 3.2

shows a hailstone which, from its appearance, seems to have grown in this manner. Similar protuberances are found on many large hailstones.

There are therefore a number of possible ways in which the nature and structure of accreted ice may be affected during wet growth. Certainly water in excess of that which can be frozen is not always shed into the wake of the object, as formerly supposed; rather, a considerable proportion if not all of the excess can be incorporated into the ice structure and lead to growth rates much greater than previously envisaged. Ice particles, even when present in concentrations comparable with the droplet concentration, are readily incorporated into the deposit and, indeed, may be responsible for modifying the structure in such a way that none of the excess water is shed.

3.3.4 The crystalline structure of ice deposits.

Since ice is anisotropic, its crystalline structure is readily observed by placing a thin slice between crossed polaroids. When viewed by transmitted white light, neighbouring crystals, being differently orientated, give rise to different colours, and are thereby easily distinguished.

Samples of spongy ice were formed on ice rods placed in a stream of supercooled droplets. (Ice rods consisting of large single crystals were used to avoid the possibility of nucleation from many places on a foreign substrate and care was taken to avoid the deposition of other crystals on their surfaces prior to use.) The airspeed, liquid water concentration and temperature in the droplet stream were approximately 10 m sec^{-1} , 8 g m^{-3} and -12°C respectively. The spongy deposit was allowed to freeze completely and a section, 0.3 to 0.35 mm thick, was made by melting it between two parallel metal plates. Plate 3.3(a) shows the appearance of one such section when placed between crossed polaroids. The crystals are 1 to 3 mm long and their

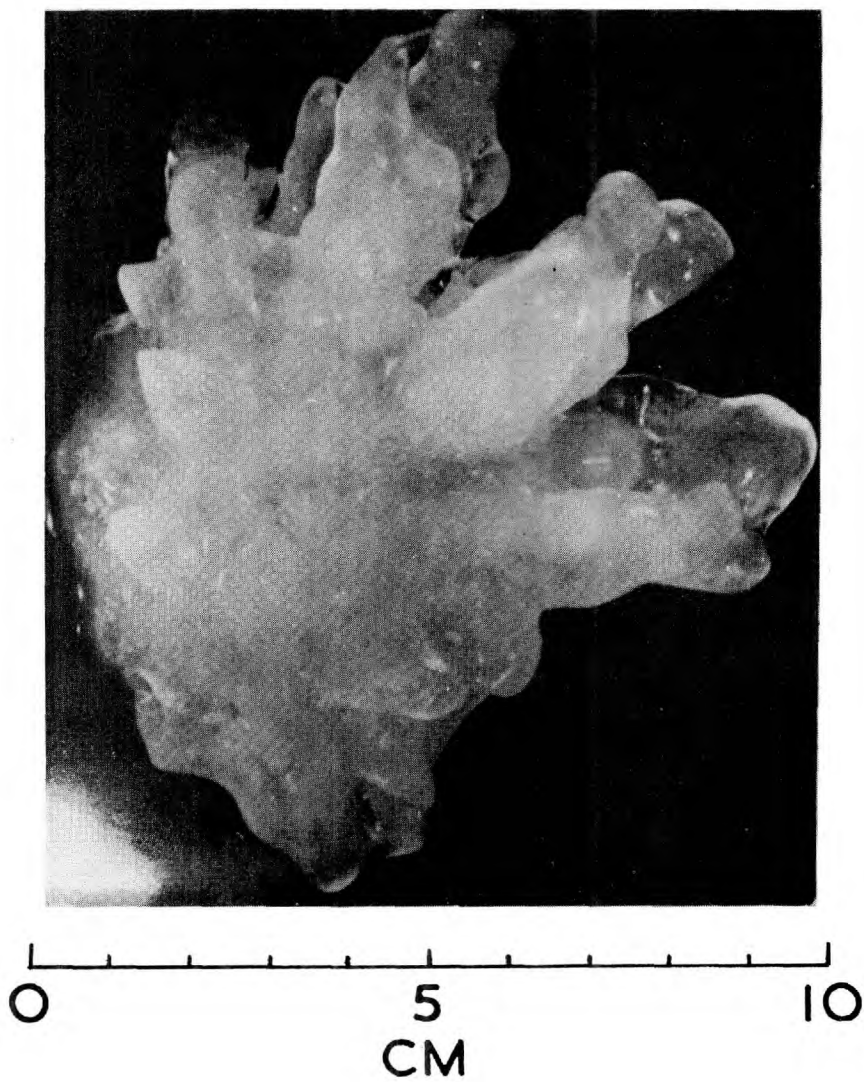


PLATE 3.2 Hailstone which fell at Pretoria, South Africa, November 1949. Photo by courtesy of the South African National Bureau of Standards.

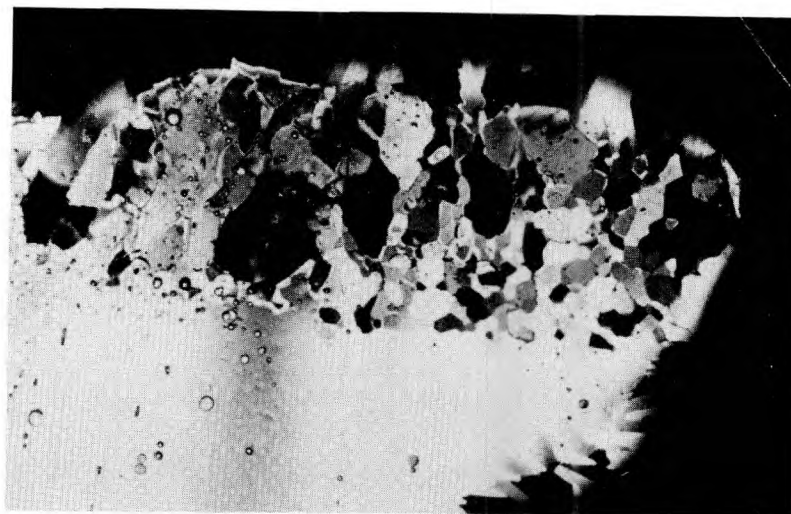
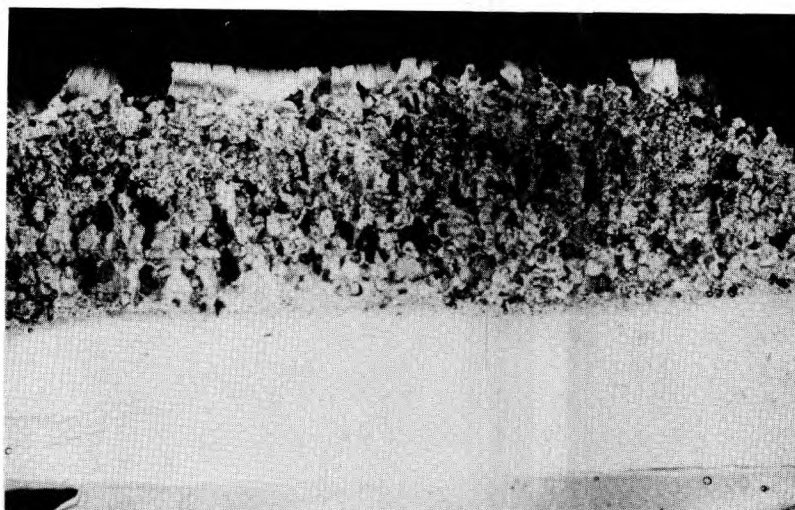
*a**b*

PLATE 3.3 Thin sections of spongy ice viewed
between crossed polaroids.

mean volume is approximately 10^{-2} cm^3 . Assuming that each crystal was grown upon a nucleus accreted with the droplets, this volume implies a spatial concentration of 1 L^{-1} of such nuclei.

The nucleation could not have been caused by natural freezing nuclei present in the air since their effective concentration at -12°C is generally of the order of only 10 m^{-3} ; further the efficiency of catch of the rods for these sub-micron particles would have been very small (less than 0.01). Therefore the crystallisation of the deposit must have been produced by ice particles present in the airstream. These may have arisen during the riming of auxiliary equipment (section 2.7) or by the freezing of droplets caused by foreign nuclei within them (see next section). The ice particle concentration in the room during this experiment was estimated visually to be 1 L^{-1} .

After some 30 minutes the ice particle concentration had increased to 100 L^{-1} and a second experiment was performed under the same conditions, with the result shown in Plate 3.3 (b). The dimension of the crystals is smaller by a factor of about 5, implying an increase in the free air concentration of ice particles of the order of 100, as observed.

This experiment shows that the internal structure of spongy ice formed by accretion can be affected by the presence of ice particles in the supercooled cloud.

3.3.5 The effect produced on the crystalline structure of the deposit by the freezing of droplets in the airstream.

The supercooling of water has been investigated by a number of workers. Bigg (1953), for example, studied the supercooling of groups of water droplets of varying diameters (20μ to 2 cm) suspended at the interface of two immiscible liquids of different densities. He showed that there was a linear relation between the logarithm of the drop diameter and the median freezing temperature for each group. Langham and Mason (1958) have repeated

these experiments using water of differing degrees of purity and they consider that Bigg's curves are typical of nucleation by foreign particles present in the liquid.

From a curve for rainwater given by Langham and Mason, the median freezing temperatures for groups of droplets of different sizes may be obtained, namely -8°C for droplets of diameter 1 cm, -19°C for a diameter of 1 mm, and -31°C for a diameter of 100μ . The nuclei present in the water would cause freezing of the droplets in the airstream in corresponding concentrations (approximately 10^{-2} , 10 and 10^4 L^{-1} respectively for a water concentration of 5 g m^{-3}) so that after they had nucleated accreted water at these three temperatures, individual crystals about 1 cm, 1 mm and 100μ across would be present in the ice deposit. For tap water the temperatures appropriate to these three sizes may be expected to be somewhat higher, for distilled water a few degrees lower.

List (1960) has reported experiments similar to the ones described in the previous section. He has examined between crossed polaroids thin sections of spongy ice, formed in an icing tunnel over a temperature range of -10 to -30°C ; at -10°C single crystals several millimetres long were formed and at -30°C the crystal size was 30 to 50μ . Since he was using ordinary tap water these results are reasonably explained by the above effect and are not due to a real temperature dependence as he indicated (see Appendix 3.1).

It is apparent, therefore, that in normal icing experiments the crystalline structure of the deposits is dependent on the purity of the water used, as well as on other factors, and there is never complete certainty that the droplet stream is free of ice particles. This is an important consideration when applying the results of such experiments to the crystalline structure of hailstones.

3.4 Application to the growth of hailstones.

3.4.1 The concentrations of ice particles in cumulonimbus clouds.

In order to apply the results of the above experiments it is necessary to know what concentrations of ice particles occur in cumulonimbus clouds. The ice particles may be produced in three ways: (1) by heterogeneous nucleation due to the presence of foreign particles (freezing nuclei), (2) by homogeneous nucleation of the cloud droplets, and (3) by various splintering processes such as those discussed in section 2.7. It is possible also that the concentrations are increased at lower levels due to the filtering back of ice particles produced high in the cloud. This, however, is unlikely in the intense cores of the updraughts.

The air in strongly convective parts of the cumulonimbus has been lifted from near the ground so that the freezing nucleus concentrations in the cloud may be expected to be approximately equal to those observed in laboratory cloud chambers. Measurements of the concentration of freezing nuclei at the ground in typical thunderstorm situations have been made by the writer using a large cold box with a sampling capacity of 15 litres (reported by Ludlam, 1959). Ice crystals were formed by the action of the nuclei in a supercooled fog and allowed to settle on a tray containing supercooled sugar solution in which they grew at a controlled rate (Bigg, 1957). Precautions were taken to ensure that nucleation did not occur from frost on the walls of the box and observations were made about noon each day for a six week period. The temperature at which ice crystals in a concentration of 0.1 L^{-1} were observed varied between -12.5 and -18°C , and that corresponding to a concentration of 1 L^{-1} varied between -15 and -20°C . These values agree with other such observations (Mason, 1957 p. 139). At lower temperatures (i.e. higher levels in the cloud) this concentration is substantially increased; at -35°C its value is the order of 10^3 L^{-1} . The median volume diameter of the cloud droplets is

approximately 30μ (see section 6.1) which, on the basis of the experiments of Langham and Mason (1958), means that 50% of the cloud water is frozen at temperatures below -35°C . Below -40°C all the cloud droplets freeze.

The effect of the various multiplication processes on the ice particle concentration is unknown. Some measurements of the concentrations of these particles in cumulonimbus clouds have been made by the Meteorological Research Flight (Jones, 1960). A number of traverses were made through the clouds and imprints of ice particles recorded on strips of aluminium foil. The results of 104 records made on six separate days show that, in the size range $80 - 375\mu$ diameter, there are, on the average, between 10 and 100 particles L^{-1} over the temperature range -5 to -35°C , the number increasing as the temperature decreases; in the size range $375 - 750\mu$ diameter, the concentrations were 1 to 5 particles L^{-1} . The concentrations from one day to another varied by a factor of only 2 to 3 and Jones estimates the corresponding mass concentrations to be about 1.3 g cm^{-3} . However, these observations are probably not applicable to the updraught regions of intense storms.

3.4.2 The growth rates of hailstones.

It has been shown above that in normal laboratory experiments spongy deposits are the typical mode of growth in the wet condition. It has been indicated also that the presence of ice particles in the cloud may be necessary for spongy growth to occur. The crystalline structure of ice deposits grown in this manner shows that, if this is the case, a concentration of ice crystals of the order of 1 L^{-1} is sufficient. This is not necessarily the lower limit however. Spongy growth therefore occurs at temperatures below -20°C ; it may occur also at higher temperatures, depending on whether the various crystal production processes give the required concentration. On the other hand, if the presence of ice crystals is not necessary for the formation of spongy ice, such growth will occur at all

levels in the supercooled cloud except at those where the temperature is close to 0°C .

The updraughts in cumulonimbus clouds are known to increase with height, the maximum values in intense storms occurring near the tropopause, at levels where the temperature is lower than -40°C . The later stages of the growth of large hailstones, in which the bulk of the ice is accreted, must take place above the -20°C level so that, in water concentrations of a few g m^{-3} , the growth will be spongy. The wet growth equation is no longer valid for these stones.

If all the water and ice swept up by a hailstone is retained, and it is assumed that the stone grows as a sphere, then its growth rate is determined by eq. 1.3. It is shown later than an average value for the drag coefficient of hailstones is 0.6 (section 4.3) and that for the density is 0.9 g cm^{-3} (section 6.2) so that

$$v = 62.6 \left(\frac{R}{\rho_a} \right)^{\frac{1}{2}} \quad (3.3)$$

and

$$\frac{dR}{dt} = 17.4 W \left(\frac{R}{\rho_a} \right)^{\frac{1}{2}} \quad (3.4)$$

where ρ_a is the density of air. Taking ρ_a to be constant and integrating gives

$$t = 0.115 \frac{\rho_a^{\frac{1}{2}}}{W} (R_1^{\frac{1}{2}} - R_0^{\frac{1}{2}}) \quad (3.5)$$

where t is the time taken for the hailstone radius to increase from R_0 to R_1 .

Putting $W = 4 \times 10^{-6} \text{ g cm}^{-3}$, $\rho_a = 4 \times 10^{-4} \text{ g cm}^{-3}$ (appropriate to the upper troposphere) and $R_0 = 0.25 \text{ cm}$

$$t = 575 (R_1^{\frac{1}{2}} - 0.5) \quad (3.6)$$

For $R_1 = 5 \text{ cm}$, $t = 1000$ seconds. The corresponding time given by the appropriate combination of the dry and wet growth equations at a

temperature of -30°C is 2000 seconds; at higher temperatures this value is considerably increased (Fig. 1.6).

The value of 1000 seconds deduced here for the time for a hailstone to grow from 0.5 cm to 10 cm diameter seems more reasonable. The answer to the problem of how the growing hailstone remains in the regions of very strong updraught for even this relatively long period can be given only when the detailed dynamical structure of the storm is examined (see section 5.3).

3.4.3 The solidification of hailstones.

As far as is known, large hailstones which arrive at the ground are hard; commonly they are reported to rebound from firm ground as much as 2 or 3 metres into the air. Thus, although a large part of the ice deposited during growth may be spongy, the outer layers of these stones must be frozen completely before their arrival at the ground. The stones may still have liquid inclusions beneath their surface layers but it is unlikely that any observer not seeking them would notice this; stones preserved for examination have generally been stored in cold boxes and would have become completely frozen.

At levels where the temperature is below -35°C at least half of the cloud water is frozen and stones which are in the wet condition at these levels will accrete both the water droplets and ice particles. This produces an at least partially consolidated outer layer. Freezing of the water in the stones occurs either in the cloud tops at temperatures below -40°C where no more liquid is accreted or in the clear air at temperatures below 0°C . Even at levels where the temperature is below -40°C , accretion of ice particles alone can continue until the outer surface is hardened.

The rate at which heat is transferred from a ventilated spherical hailstone is given approximately by the sum of eqs. 1.8 and 1.10. Taking $a = b = 0.3 \text{ Re}^{\frac{1}{2}}$ and V as given by eq. 3.3,

$$\frac{dQ_1}{dt} = 42 \rho_a^{\frac{1}{2}} \eta^{-\frac{1}{2}} R^{7/4} (K \Delta T + L_v k \Delta \rho) \quad (3.7)$$

For the model cloud at the -40°C level, $p = 245$ mb, $\eta = 1.51 \times 10^{-4}$ poise, $\rho_a = 3.66 \times 10^{-4} \text{ g cm}^{-3}$, $k = 0.68 \text{ cm}^2 \text{ sec}^{-1}$, $K = 5.1 \times 10^{-5} \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$ and $L_v = 680 \text{ cal g}^{-1}$ which gives

$$\frac{dQ_1}{dt} = 470 R^{7/4} (5.1 \times 10^{-5} \Delta T + 460 \Delta \rho) \quad (3.8)$$

The values of ΔT and $\Delta \rho$ depend on the temperature of the surface.

However, since the outer layers of the stone freeze first, the heat has to be conducted through these layers if the inner parts of the stone are to be frozen. The thermal conductivity of ice is $5 \times 10^{-3} \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$ so that the rate at which heat is conducted across a frozen outer layer of thickness Δx , is

$$\frac{dQ_2}{dt} = -6.3 \times 10^{-2} R^2 \frac{T_s}{\Delta x} \quad (3.9)$$

where T_s ($^\circ\text{C}$) is the surface temperature of the stone, the unfrozen liquid having a temperature of 0°C . In a quasi-steady state the surface temperature of the stone is such that the two rates of heat transfer given by eqs. 3.7 and 3.9 are equal. For $R = 5$ cm and $\Delta x = 0.5$ cm, this condition is fulfilled when T_s is -6.5°C ; the rate of heat transfer is then 20 cal sec^{-1} . If the hailstone contains 50% water, an additional layer of thickness 0.5 cm will be completely frozen within 4 minutes. The outermost layer must have been frozen in a time less than this so that a completely frozen layer 1 cm in thickness is formed in less than eight minutes. However such giant hailstones are very irregular in shape (see Plate 3.2) and the surface area is considerably greater than that for the equivalent sphere; the heat transfer is therefore increased and this time of solidification correspondingly reduced, possibly by a factor of as much as 2. Similar considerations show that the

outer centimetre of a hailstone having a radius of 3 cm and consisting of 50% water is completely frozen in a comparable time. Smaller stones are likely to contain less water so that again a centimetre layer may be frozen in a few minutes.

3.5 Conclusions.

It is now clear that the assumption that the growth of hailstones in the wet regime is severely restricted by the rate of heat transfer to the environment is incorrect, at least at levels where the temperature is below -15 to -20°C and where it is believed that the largest hail is grown. In these regions of the clouds all the liquid water accreted may be retained and this results in the outer layers of growing hailstones having a spongy texture. It is not certain whether the presence of free ice crystals is essential for this type of growth but it certainly enhances it. At high levels a considerable fraction of the cloud water may be frozen but again this favours rather than hinders the growth of large hail. However, although large hailstones undoubtedly grow with a spongy texture, the outer layers at least can be consolidated or frozen high in the cloud so that they arrive at the ground in an apparently solid state.

Appendix 3.1: The relation between the rate of freezing and the size of single crystals formed in an ice deposit.

The rate at which ice forms in a deposit depends on the rate at which heat may be transferred from the deposit to its surroundings. In the case of ice being formed by the accretion of supercooled water droplets, this is dependent on the ambient temperature. Other factors remaining constant, the ambient temperature can affect the crystal structure of the deposit only in so far as it affects the freezing rate.

In order to determine the relation between the single crystal size and the rate of freezing the following simple

experiment was performed. A number of thin walled hollow spheres were filled with ordinary tap water. Some of these were placed at different levels in a refrigerator; others were lowered into a thermos flask containing liquid air. In the former case the water took an hour or more to freeze completely while in the latter the water could be frozen in a time as short as $1\frac{1}{2}$ minutes (a layer of ice about 1.5 cm thick being formed in this time). Examination of thin sections of these ice samples between crossed polaroids showed that the ones frozen in the refrigerator were composed of single crystals of the order of 1 cm while those rapidly frozen contained crystals about half this size. The extreme rate of freezing thus produced a slight, but certainly not significant, effect. Therefore, if the ambient temperature does affect the crystal structure of spongy ice deposits formed by accretion, it is through some other mechanism, such as that already discussed.

4 : THE FALLSPEEDS OF HAILSTONES.

4.1 Introduction.

A knowledge of the fallspeeds of hailstones is important because they determine the growth rates of the stones and the size of the largest which can be grown in a given updraught. In discussing the growth of hailstones, Schumann (1938) and Ludlam (1958) assume that the fallspeeds are equal to those of smooth spheres but, in reviewing the aerodynamic properties of hailstones, Weickmann (1953) suggests that regular stones have a drag coefficient of about 1 and that stones of irregular shape have drag coefficients as great as 3.

Over the range of Reynolds numbers corresponding to hailstones of diameter greater than 1 cm, the drag coefficient for spheres is approximately constant, having a value of 0.45 ± 0.03 (Goldstein, 1938). At a Reynolds number of about 2×10^5 the boundary layer becomes turbulent and the drag is greatly reduced; this is thought to impose an upper limit of approximately 10 cm on the diameter of hailstones (Bilham and Relf, 1937). However, if Weickmann's suggested values are correct, then the fallspeeds of hailstones may be less than half those of spheres of the same mass.

List (1959b) has recently reported experiments in which he determined the drag coefficients of four hailstones having masses between 18 and 32 g. For an approximately spherical stone he obtained a drag coefficient of about 0.6 while for the others, he obtained values of 0.62 to 0.73. These are considerably lower than the values proposed by Weickmann, but higher than those appropriate to spheres.

Experiments which were carried out to determine the speeds of model hailstones during free fall using a GL-7 lock-follow radar, and the drag coefficients of both model and natural stones, are now described.

4.2 Measurements made during free fall.

4.2.1. Procedure.

Model stones were made by freezing water in hemispherical cups and then cementing the two halves together. Within each hemisphere were frozen a number of strips of aluminium foil (radar "window") to increase the back-scatter cross-section. A stone was placed in a cardboard (non-reflecting) box suspended beneath a 100 g meteorological balloon inflated with hydrogen; a second 10 g balloon, partially inflated with air, was attached to the bottom of the box (Fig. 4.1). A fuse, made by soaking loose string in a concentrated solution of sodium nitrate, was tied to the suspending cord and its length adjusted so that at a height of about 2 km the box containing the ice object was separated from the lifting balloon. The lower balloon then served as a parachute, tipping the box upside down and ejecting the stone.

The stones were observed with a GL-7 lock-follow radar and once the radar had been set on target, there was no difficulty in following both the ascent and descent of the stone, even though the latter was considerably faster. The radar was instrumented for wind-finding and the range, azimuth and elevation of the target were recorded automatically every five seconds.

Of the seventeen stones with which measurements were successfully made, three were virtually spherical, five had axes in the ratio 1 : 1 : 0.7 - 0.9, and the remaining three in the ratio 1 : 1 : 0.43 - 0.52. The surfaces of the stones were relatively smooth with no major projections, but in some instances were made with surface depressions, and similar irregularities, comparable with those found on natural hailstones. The stones were chilled to -40 to -60°C to prevent melting as far as possible, but it is probable that a superficial melting occurred during their fall. The mass, dimensions and the condition of the surface of each stone were recorded prior to each flight.

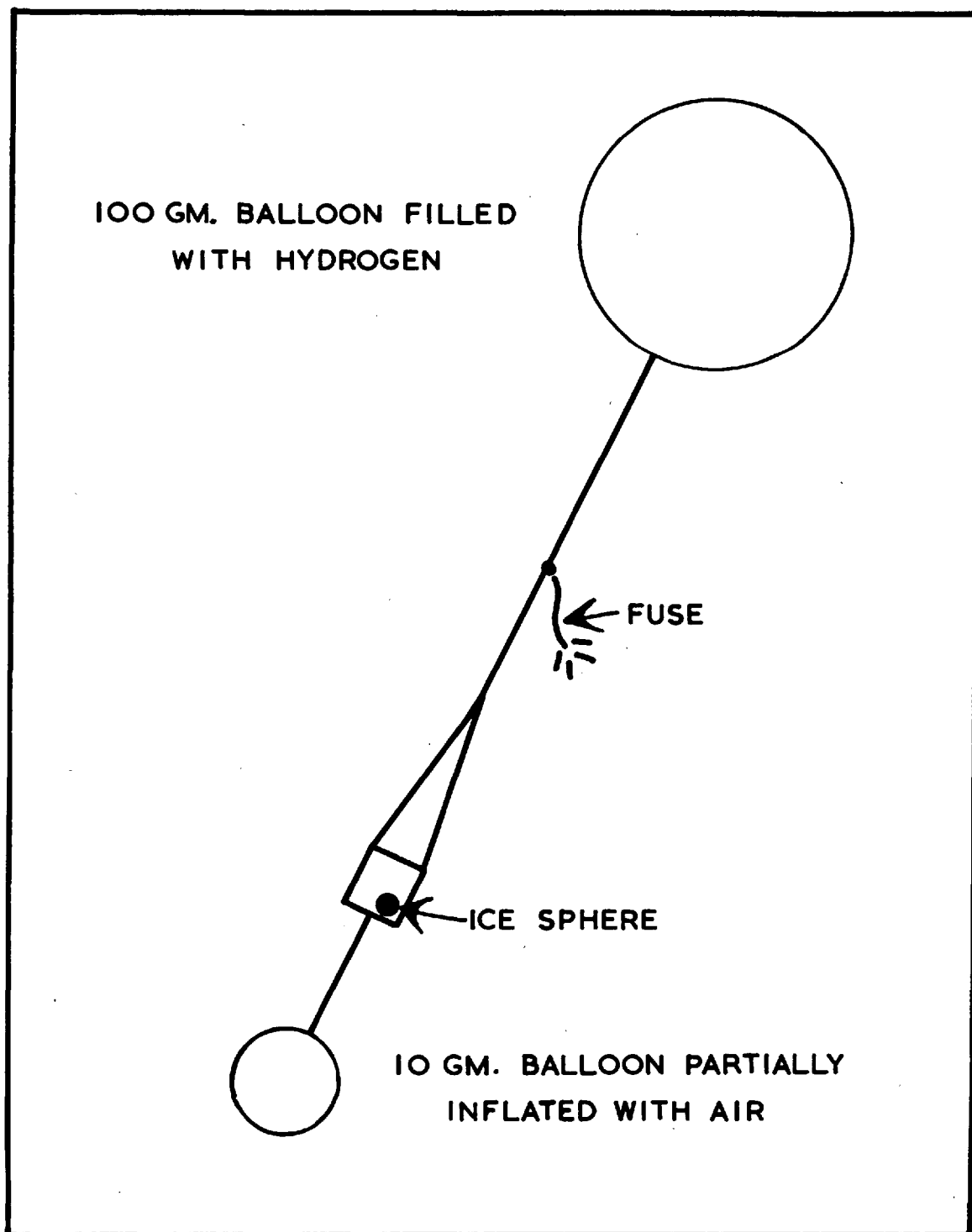


FIG.4.1 Apparatus used to drop the model hailstones in the free fall experiments.

4.2.2 Results.

From the radar records a plot of the height of each stone against time was made, as shown in Fig. 4.2. The accuracy with which the fallspeed was determined from such diagrams was $\pm 1 \text{ m sec}^{-1}$, except in a few instances where the fall of the stone was observed for only a short period or where there was a discontinuity in the height-time relationship, presumably due to a re-orientation of the stone.

Fig. 4.3 shows the measured fallspeed as a function of the mass of the stones; where the error is greater than $\pm 1 \text{ m sec}^{-1}$ the range of values is indicated. Superimposed on this diagram is a curve based on the drag coefficients for spheres of density 0.9 g cm^{-3} , the density of the ice objects used. Most of the points are within $\pm 20\%$ of the curve for spheres and there is no consistent effect produced by the surface irregularities or by small eccentricities. The scatter in the velocities is considered to be due, at least in part, to slightly different orientations of the objects as they fell. The smaller oblate stones, however, had fallspeeds considerably less than spheres of the same mass. The drag coefficient for these stones, calculated from their dimensions, mass and fallspeeds and the air density (deduced from the appropriate aerological sounding), was found to be 0.8. The accuracy of this method of determining the drag coefficient is not great (about $\pm 25\%$) as it was not possible to determine the orientations of the objects during their fall. However, the values obtained suggest that, while the drag coefficients of smooth spherical stones approximate to those accepted for spheres, more oblate shapes have drag coefficients which may be as much as twice those of spheres of the same mass. Accordingly, it was decided to carry out a series of measurements of the drag coefficients of both model and natural hailstones under controlled conditions.

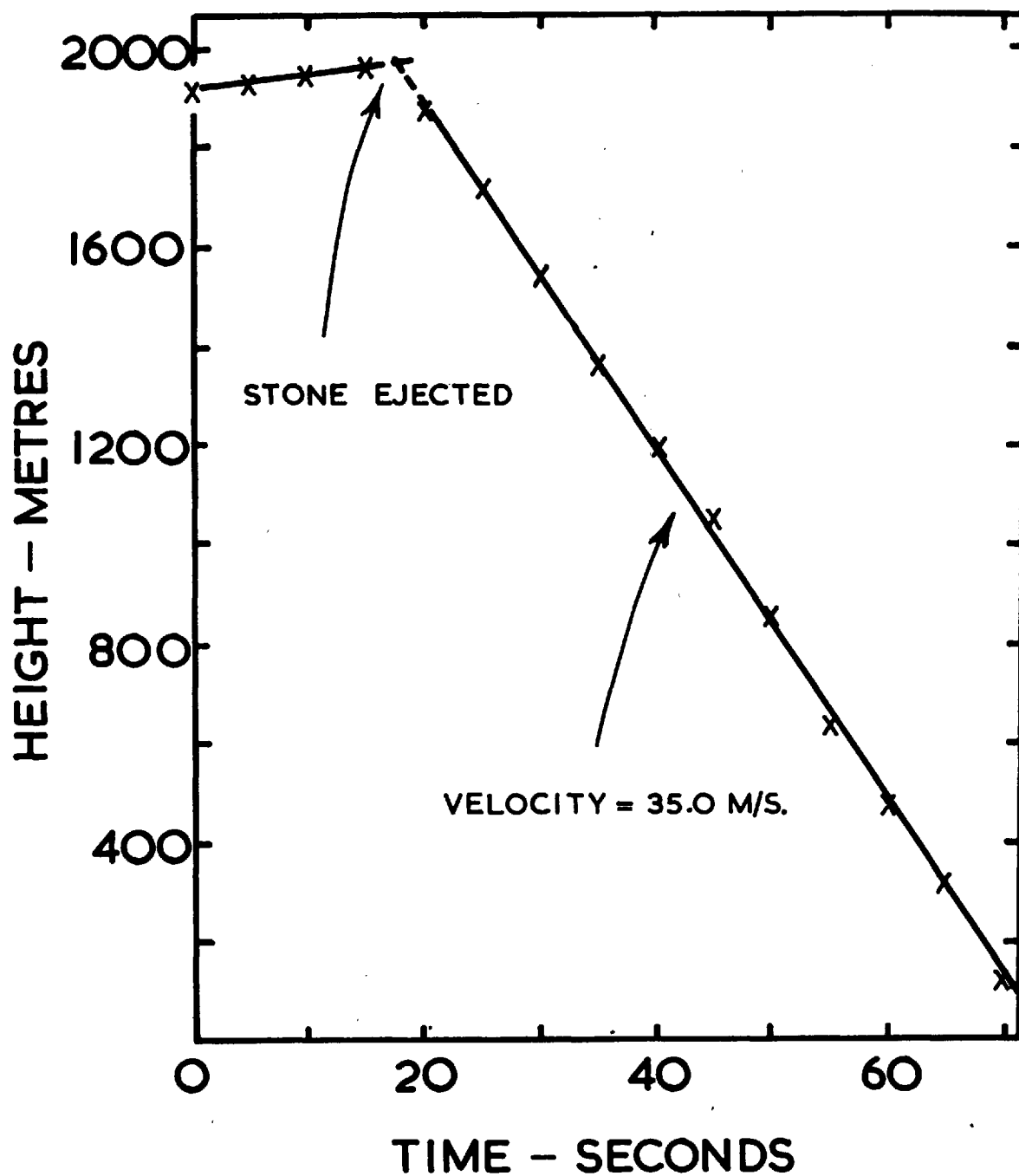


FIG.4.2 Typical example of the height-time trajectory of an ice sphere.

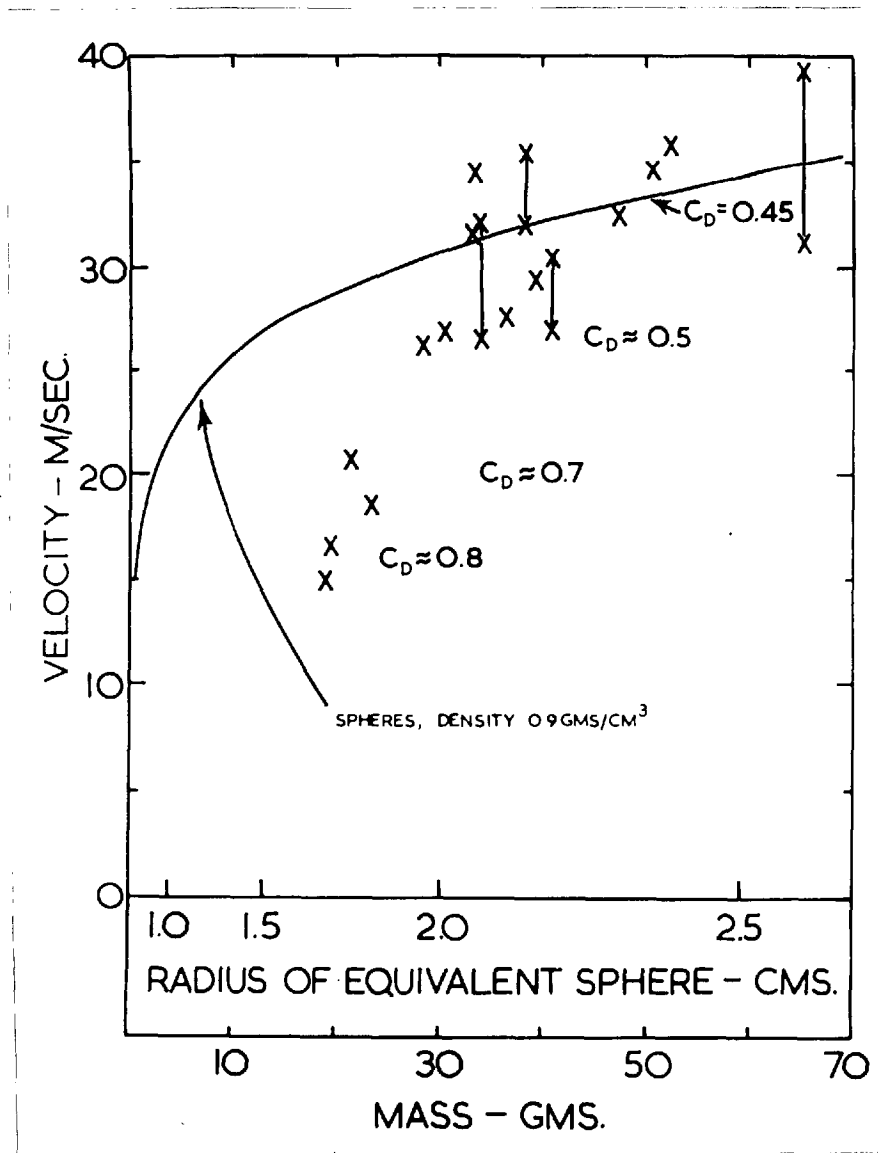


Fig. 4.3 The fallspeed as a function of the mass of the stones. Drag coefficients, estimated from the data, are indicated.

4.3 Measurements of drag coefficients.

4.3.1 Procedure.

The objects to be tested were mounted on a right angled ("sting") support in a wind tunnel which could produce airspeeds between 12 and 40 m sec⁻¹. The tunnel was equipped for making drag measurements with a balance (which could be read to ± 0.5 g) and an anemometer which measured the pressure head from a pitot tube inserted in the airstream (to ± 0.05 mm of water). The room temperature and pressure were recorded and airspeeds could be estimated to $\pm 5\%$ although, for the purpose of determining the drag coefficient, the direct measurement of the pressure head was sufficient (see below). Measurements were made at intervals in wind speed of 5 m sec⁻¹ over the range from 12 to 37 m sec⁻¹.

In order to check the balance calibration and to take into account the effect of the airstream on the exposed portions of the support, tests were carried out using three accurately constructed smooth spheres having diameters of 3.5, 5.1 and 7.5 cm. An independent calibration of the balance alone was made by exerting a known horizontal force on the support; weights were placed on a scale pan hanging vertically and connected to the support by a thread which passed over a virtually frictionless pulley.

The models tested were constructed either of plasticine moulded over a wooden former to give rigidity, or ice formed by freezing water in hollow rubber spheres. In both cases variations in shape were readily produced. A short piece of rod was inserted in each object and this was used to locate it on the support. The object was rotated relative to the airstream by changing the position of this rod and three orientations were employed, differing by 45°. The cross-sectional areas presented to the airstream were obtained photographically.

The shape of the models and nature of the surface irregularities were deliberately varied to ascertain their respective effects on the drag coefficient. The plasticine models were approximately ellipsoidal with the following ratios of axes: 1 : 1 : 1, 1 : 1 : 0.8, 1 : 1 : 0.67, 1 : 1 : 0.46, 1 : 0.84 : 0.77 and 1 : 0.75 : 0.68. The maximum dimension of each was between 5 and 7.5 cm. The models were given smooth surfaces, surface depressions, and projections. The models made of ice had a variety of shapes and sizes in approximately the same range of eccentricities as those of plasticine, but their diameters were smaller, ranging from 3.5 to 6 cm (masses 27 to 185 g); tests were not made on ice objects with projections. The ice models (and the hailstones tested) were coated with a thin plastic film (formed by dipping them into a 1% solution of formvar in ethylene dichloride) and chilled in dry ice prior to each observation to prevent excessive melting. Since large hailstones tend to grow with wet surfaces, observations were made to see if the drag coefficient was affected if the surface of the model was wet.

Measurements were also made on three moderate-sized hailstones having masses of 14.7, 19.6 and 21.8 g, and ratio of axes 1 : 0.89 : 0.70, 1 : 1 : 1, and 1 : 0.77 : 0.45 respectively. (These fell during the thunderstorm of 9/7/59 discussed in section 5.3). It is believed that non-spherical stones fall with their shortest axis vertical. When a hailstone grows with a wet surface the excess water is forced round the sides to the back of the stone so that it grows as an ellipsoid; the front accreting surface is consequently smooth while the back is irregular and knobbly. List (1959b) has carried out similarity experiments in a water tank which confirm this direction of fall. The measurements of the drag coefficients of the oblate hailstones made in the present experiments were confined to airflow in this direction.

4.3.2 Results.

The drag coefficient, C_D , is given by the following relation

$$D = \frac{1}{2} \rho_a v^2 A C_D \quad (4.1)$$

where D is the drag force, ρ_a is the density of air, v is the airspeed and A is the cross-sectional area of the object normal to the airstream. The quantity $\frac{1}{2} \rho_a v^2$ was obtained directly from the manometer measurement since

$$\frac{1}{2} \rho_a v^2 = \rho_w g h \quad (4.2)$$

where ρ_w is the density of water, g is the acceleration due to gravity and h is the pressure head.

4.3.2.1 Tests on smooth spheres.

The measurements of the drag coefficients of spheres were made primarily to calibrate the equipment over the range of conditions of the experiments. The systematic error introduced by the effect of the airstream on the exposed parts of the upright support to the balance was removed by assuming that the drag coefficients for spheres are those given by Goldstein (1938, p.16). The expected drag force appropriate to the conditions of a measurement was deduced from these values, the actual drag force measured and the difference attributed to the effect of the airstream on the support. A mean correction based on the results for all three spheres was then deduced; this amounted to a 10 to 20% reduction in the measured values of the drag force, depending on the airspeed. The drag coefficients of the spheres were measured for the three different diameters over partly coincident ranges of the Reynolds number with a consistency which can be seen in Fig.

4.4. It is considered that the method employed gives values which are generally accurate to better than $\pm 10\%$.

4.3.2.2 Tests on plasticine models.

The values for the drag coefficients for the plasticine models are summarised in Table 4.1. The nature of the surface is described as "smooth" when no deliberate attempt was made to

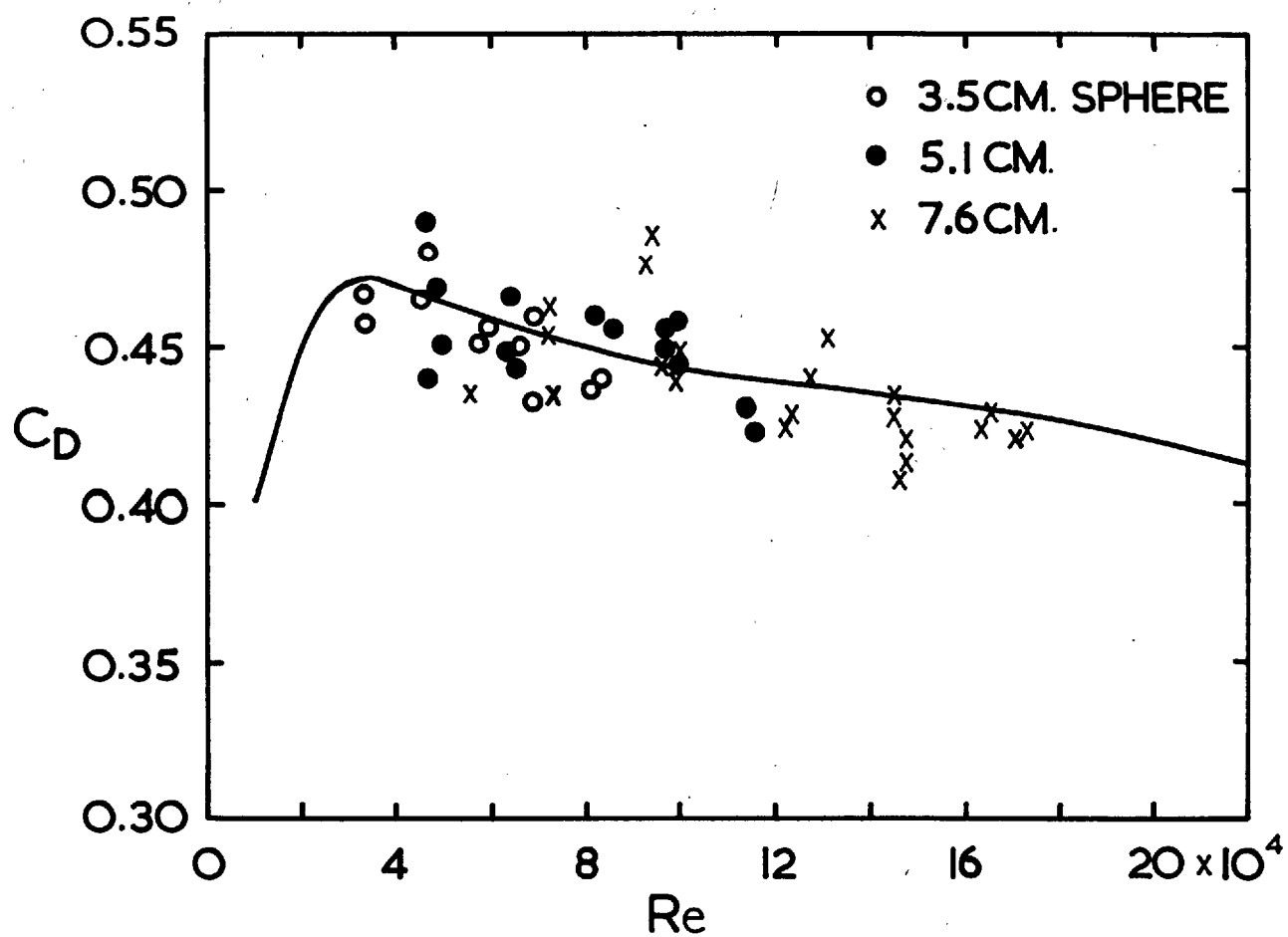


FIG.4.4 The drag coefficients obtained for smooth spheres. The curve shown is that given by Goldstein (1938).

TABLE 4.1 : Results for the plasticine models.

Ratio of Axes	Nature of surface	Orient- ation	Approx. Range of $10^{-4} Re^x$	C_D
1 : 1 : 1	(1) smooth		7 - 17	0.45 - 0.21
	with depressions			0.50 - 0.21
	(2) smooth		5 - 13	0.53 - 0.25
	with projections, 0.5 - 1.0 cm 1.5 - 2.0 cm			0.58 - 0.41 0.64 - 0.52
1 : 1 : 0.8	smooth	0°	7 - 17	0.55 - 0.40
		45		0.51 - 0.44
		90		0.48 - 0.17
	with depressions	0	7 - 17	0.54 - 0.40
1 : 1 : 0.67	smooth	0	7 - 15	0.61 - 0.57
		45		0.60 - 0.53
		90		0.36 - 0.19
	with depressions	0	7 - 15	0.80 - 0.70
1 : 1 : 0.46	smooth	0	7 - 18	0.67 - 0.61
		45		0.61 - 0.56
		90		0.33 - 0.15
	with depressions	0		0.64 - 0.50
	with projections, 0.5 - 1.5 cm	0		0.70 - 0.61
1 : 0.84 : 0.77	smooth	0	7 - 16	0.59 - 0.35
		45		0.52 - 0.41
		90		0.31 - 0.13
	with depressions	0		0.59 - 0.41
1 : 0.75 : 0.68	smooth	0	6 - 14	0.48 - 0.35
		45		0.50 - 0.25
		90		0.39 - 0.16
	with depressions	0		0.70 - 0.35

x the dimension used in determining the Reynolds number of an irregular object was the diameter of the circle of the same area of cross-section.

introduce surface irregularities, as "with depressions" when roughly circular depressions 1 to 2 cm across and 0.75 deep covered the surface, and "with projections" when blunt pointed projections having bases 1 to 1.5 cm across were introduced randomly over the surface approximately 3 to 4 cm apart. The orientation of the ellipsoidal objects is designated 0° when the shortest axis was in the direction of the airstream, and 90° when the longest axis was in this direction. With these and the irregular objects the dimension used to determine the Reynolds number was the diameter of a circle whose area was equal to that presented by the object to the airstream.

The values of the drag coefficient of the plasticine spheres decreased from 0.53 to 0.21 as the Reynolds number increased from 5 to 17×10^4 . In the critical range of Reynolds numbers in which transition to turbulence occurs in the boundary layer, the nature of the surface plays an important part in determining the drag coefficient. The roughness parameter (the ratio of the size of the surface irregularities to the diameter) for the plasticine spheres was estimated to be about 0.02, and the drag coefficients obtained here are consistent with those determined by other workers (see Hoerner 1958, p.3 - 8). A similar effect may occur with the ellipsoidal models, and this must be considered when applying this data to objects with surfaces of different character.

It has been indicated already that hailstones grow with their shortest axis vertical, so that the most relevant data in Table 4.1 are those for an orientation of 0° . As the objects were rotated from 0 to 90° the drag coefficient decreased; this is to be expected as the object then presented a more streamline shape to the airflow. As the objects became more oblate the drag coefficient increased, the values increasing from 0.45 to 0.67 as the shortest axis decreased to a value about one-half that of the other two axes. Shortening a second axis (objects 5 and 6) reduced

the drag coefficient, particularly at high Reynolds numbers.

The effect of surface depressions and protuberances was not consistent throughout the series of measurements. In the case of the third and sixth objects there was a significant increase in the drag coefficient when the surface depressions were introduced, while in the case of the fourth object there was a reduction in the drag coefficient (although in view of the experimental error it is not possible to tell just how significant this decrease was). The drag coefficient of the spheres were increased markedly by the protuberances, the increase being dependent on their size, while with the fourth model, the relatively high drag coefficient was virtually unaffected.

4.3.2.3 Tests on ice models.

The values of the drag coefficient for the ice models are given in Table 4.2; measurements were made only when the shortest axis was in the direction of the airstream. The results show a similar trend to those of the plasticine models, the values increasing as the objects become more oblate. The values for the spheres are comparable with those of plasticine, but do not decrease as rapidly with increasing Reynolds number. This is attributed to differences in surface texture. The results for ellipsoids having similar axis ratios in the two series of measurements are also comparable.

Several observations were made with various ice models to ascertain whether the state of the surface (dry or wet) affected the drag coefficient. No detectable change was observed on any occasion when the surface was allowed to become wet during the course of an experiment.

4.3.2.4 Measurements made with hailstones.

Details of the determinations of the drag coefficients of the three natural hailstones are shown in Table 4.3 together with

TABLE 4.2 : Results for ice models.

Ratio of axes	Nature of surface	Approx. range of 10^{-4} Re	C_D
1 : 1 : 1	smooth	7 - 12	0.54 - 0.41
	smooth	6 - 14	0.53 - 0.46
	minor irregularities	6 - 14	0.47 - 0.50
	smooth	5 - 11	0.62 - 0.51
	smooth	7	0.50
1 : 1 : 0.8	smooth	7	0.54
1 : 0.96 : 0.87	with depressions	7	0.54
1 : 1 : 0.75	smooth	5 - 11	0.53 - 0.54
1 : 1 : 0.88	smooth	6 - 10	0.56 - 0.53
1 : 1 : 0.81	minor irregularities	8 - 16	0.50 - 0.48
1 : 1 : 0.76	smooth : asymmetrical	5 - 13	0.53 - 0.50
1 : 1 : 0.68	asymmetrical with depressions	7 - 12	0.63 - 0.57
1 : 0.91 : 0.64	smooth	6 - 14	0.65 - 0.54
1 : 0.86 : 0.60	with depressions	4 - 8	0.61
1 : 0.80 : 0.57	with depressions	5 - 13	0.62 - 0.55
1 : 0.85 : 0.62	with depressions	5 - 9	0.68 - 0.66

those made by List (1959b). In the present experiments the probable orientation of the stones during free fall was decided by inspection; List gives the drag coefficients for both orientations with the shortest axis in the direction of the airstream. The agreement between the values for the hailstones, those for models of the same shape, and List's values is very reasonable which suggests that it is primarily the shape which determines the drag coefficients of large hailstones.

The fallspeeds given in the last column of Table 4.3 were obtained by plotting the measured values of the drag force against the corresponding airspeeds. The speed appropriate to a drag force equal to the weight of the stone was then determined; the self consistency of the plotted points suggests an accuracy of $\pm 1 \text{ m sec}^{-1}$. The fallspeeds quoted for List's experiments are taken from his paper and are appropriate to the conditions at the Weissfluhjoch Observatory (2,665 m above sea level).

4.4 Summary and conclusions.

The drag coefficients of hailstones range from about 0.45 (those for smooth spheres) to about 0.8, depending mainly on the shape and the Reynolds number, and partly on the nature of the surface of the stone. The values obtained for spherical stones were in general greater than those for smooth spheres, although in some instances comparable values were obtained. A schematic summary of the above measurements is given in Fig. 4.5. It is only approximate, however, and individual objects may have drag coefficients differing appreciably from those indicated; for example, the values obtained for the third plasticine model (1 : 1 : 0.67) when surface irregularities were introduced are higher than those shown in the diagram. The present data indicate that the drag coefficients for hailstones, both of regular and irregular form, are considerably lower than those envisaged by

TABLE 4.3 : Results for hailstones.

Mass (g)	Axes (cm)	velocity (m sec ⁻¹)	10 ⁻⁴ Re	C _D	Fallspeed (m sec ⁻¹)
21.8	5.2, 4.0, 2.35 (1 : 0.77 : 0.45)	11.4	3.4	0.73	at 15 ⁰ C, 1000mb
		15.2	4.5	0.75	
		20.5	6.0	0.68	20 ± 1
		25.3	7.5	0.71	
19.6	3.6 spherical (second orientation)	11.3	2.6	0.61	
		14.7	3.9	0.63	
		20.3	4.7	0.53	25
		25.4	5.9	0.51	
		14.8	3.9	0.59	
		19.8	4.6	0.53	
		25.1	5.9	0.55	
		14.7	3.7, 3.3, 2.8 (1 : 0.89 : 0.70)	11.4	2.6
14.7		14.5	3.3	0.66	21
		20.3	4.6	0.59	
		25.4	5.7	0.66	
		List's values - at the Weissfluhjoch Observatory - 2665 m above sea level.			
24.3	1 : 0.79 : 0.44			0.73	20
				0.65	
32.4	1 : 0.78 : 0.55			0.65	23
				0.70	
18.6	approximately spherical			0.70	23
				0.57	
				0.62	24.5
				0.64	
				0.54	27
				0.53	
23.3	1 : 0.77 : 0.64			0.62	23
				0.62	

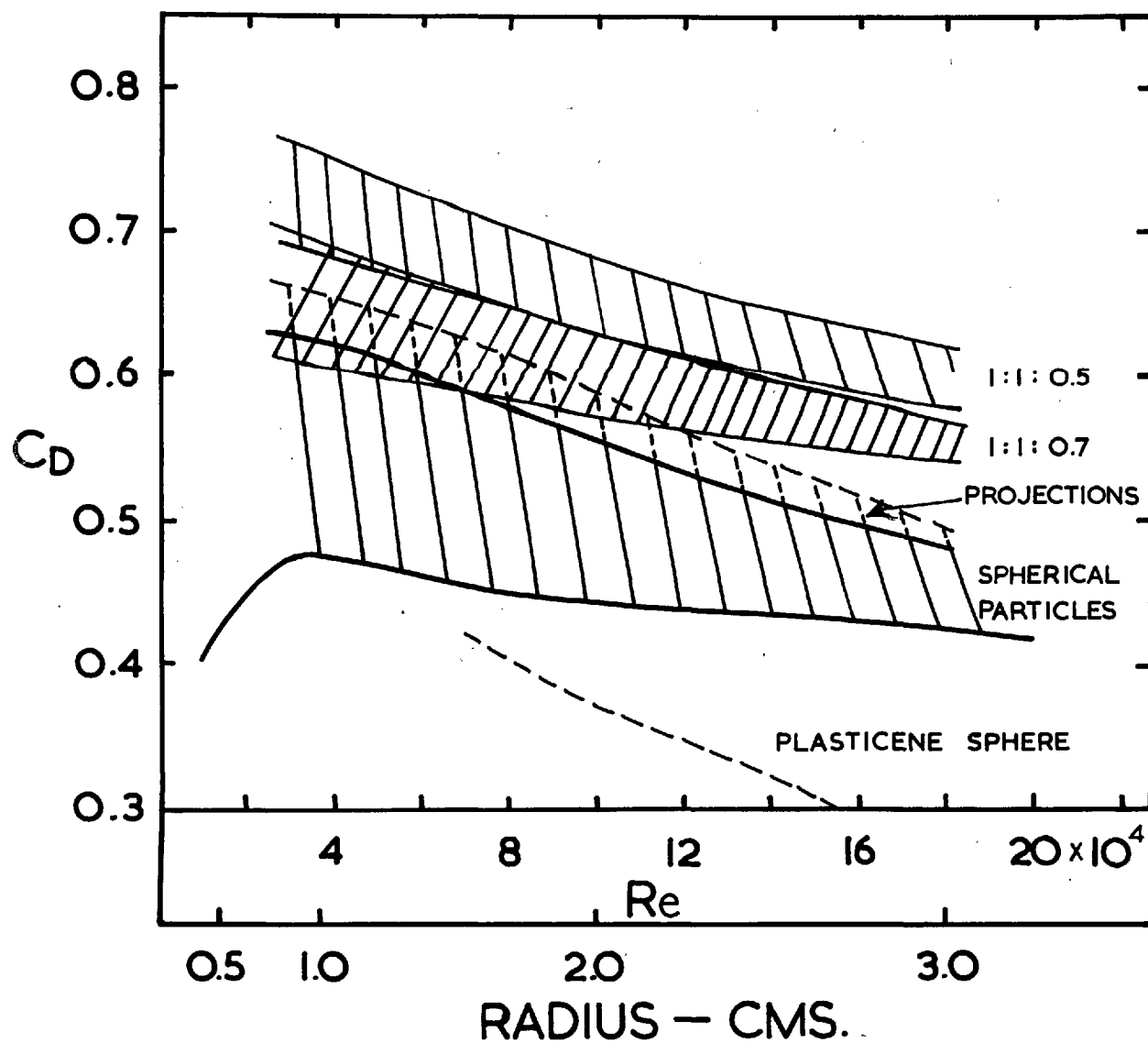


Fig. 4.5 Summary of the measurements. An approximate scale of hailstone radius, obtained by assuming $C_D = 0.6$, $T = -10^\circ\text{C}$ and $p = 600$ mb, is included.

Weickmann (1953). Since large hailstones are often asymmetrical, a reasonable mean value for the drag coefficient of hailstones with diameters greater than 1 cm is 0.6, which implies fallspeeds 15 to 20% less than those of spheres of the same mass.

5 : SOME ASPECTS OF SEVERE HAILSTORMS.

5.1 Introduction.

In the last decade centimetric radar has been used extensively in the study of the distribution, movement, structure and development of precipitating clouds (see, for example, Mason 1957, p. 309 and Battan 1959, p.74). However, because of the complexity both of the precipitation processes and of the relationships between the back-scatter cross-sections of the cloud particles and their size, shape and constituency and the wavelength used, radar studies of the structure and development of severe storms are useful only when some knowledge of the nature and distribution of the precipitation has been obtained independently. This is particularly true in the case of hailstorms. Since in this case it is not possible to ascertain such information by direct observation within clouds, a detailed study of the precipitation at the ground during the development and movement of a storm is invaluable for the interpretation of the radar data.

The study of the distribution of hail at the ground during a severe storm is, however, of interest for its own sake and, during the course of the last 150 years, a number of observer networks have been established for this purpose. The earliest of these was located in Württemberg, Germany and recorded from 1828 to 1887 (see Weickmann, 1953); another extended throughout the whole of Austria and recorded from about 1885 to 1914. The latter was established by Prohaska, who has published accounts of a number of storms investigated in this way (e.g. Prohaska 1900, 1905, 1907, 1914). Similar ground observational networks have been used in conjunction with radar studies by, for example, Ludlam (1959), Donaldson (1958, 1959, 1960) and Douglas and Hitschfeld (1959).

Many interesting results have been obtained in this way. Prohaska established that hail swaths associated with severe travelling storms are typically 10 to 20 km wide while their lengths

may be a few hundreds of kilometres; the rates of travel of the storms ranged up to 68 km hr^{-1} . In one storm he reports hailstones having maximum dimensions of 15 cm and weighing between 0.8 and 1.1 kg (Prohaska, 1905). Donaldson has shown that the occurrence of hail is related to the radar reflectivity and the height of the top of the radar echo, the probability of large hailstones falling at the ground increasing with increases in both of these parameters. A similar conclusion was drawn by Douglas and Hitschfeld. Ludlam has shown that storms take typically 40 to 50 minutes to develop from rain shower clouds to a state in which hailstones of a few centimetres diameter are produced. During this time the level of the tops of the clouds rises progressively at a speed of a few metres per second and the updraughts within the clouds eventually attain values of some tens of metres per second. The large hailstones cease to fall about 10 to 20 minutes after the peak of the cloud development, and although there may be some intermittent smaller hail the precipitation subsequently becomes predominantly rain, but in torrential intensity.

In order to make a detailed study of the distribution of hail at the ground it is necessary to have a high density of observers, preferably previously instructed in some way. (Such instruction as is given usually takes the form of a questionnaire, distributed prior to the working period, which is filled in by the observer at the time of the storm.) The observers are necessarily voluntary and may be unable to make the required observations at the time. On occasions Prohaska had as many as 900 reports for a single thundery day but it is not possible to determine the observer density for a single storm since the area over which these were spread is unknown. Donaldson had a network of some 300 observers located within about 150 miles of his radar site (1 per 60 square miles) while Douglas and Hitschfeld obtained 1 report for each 10 to 15 square miles of storm path.

Voluntary observer networks were established during the

present study for the purpose of reporting and collecting hail. It was found, however, that these networks were too sparse (about 1 observer per 20 square miles) to give information in sufficient detail, so it was decided to visit the regions which suffered the storm and to interrogate directly people who had experienced it. The disadvantage that the observations were not recorded at the time of the storm is outweighed by the advantage of obtaining a much greater number of reports. In the case of storms which were sufficiently severe for the relevant details to be impressed on the persons concerned, this technique was found to be satisfactory, although obviously not ideal. It was found also that a number of people had stored hailstones in deep-freezes and refrigerators and, thereby, samples of stones which fell from various parts of the storm were obtained for analysis.

Two severe storms have been studied. One occurred on September 5, 1958 (the "Horsham storm") and the other on July 9, 1959 (the "Wokingham storm"). The latter storm occurred during an observational programme in which radar was employed more intensively than in any previous investigation. The aspects of this storm summarised here have been combined with the radar data by Atlas and Ludlam (1960) and by Browning and Ludlam (1960) to deduce the distribution of hail and air motion in the storm clouds, thereby throwing new light on the processes of hail growth.

5.2 The Horsham storm of September 5, 1958.

On September 5, 1958 a severe thunderstorm travelled across S.E. England from the Isle of Wight to the London area and then to Suffolk. It produced a very heavy rainfall along its path, and in the neighbourhood of Horsham, Sussex, was accompanied by an intense hailstorm in which hailstones of diameter up to 8 cm fell. Considerable damage was caused both by the hail and by a tornado which developed on the right hand side of the storm (all directions are given in relation to the direction of travel of the storm).

Storm reports were obtained in the manner described above, namely from a voluntary observer network already established and by direct interrogation of persons who experienced it. Additional observations were obtained from the voluntary rainfall observer network of the Meteorological Office and from a response to requests for information published in local newspapers. Information regarding the nature of the precipitation, its time of occurrence and duration, the size of the largest hailstones and their number per unit area, the intensity of the rainfall, the direction and strength of the wind and other such relevant features, was obtained. The observations were located to within 150 metres on the National Grid.

The hailstones were usually described in terms of common objects such as peas, marbles and walnuts; while this leaves something to be desired, this method of typifying the stones was encouraged and the dimensions of the stones inferred from such descriptions. In some cases direct measurements of the dimensions and mass of the stones, and photographs were supplied.

The general distribution of rain and hail thus obtained for this storm is shown in Fig. 5.1. Some radar observations of the storm, obtained from the Meteorological Office, are superimposed on this diagram; unfortunately the radar data are too few to disclose its detailed structure and movement.

It is evident however that the large hail and heaviest precipitation occurred on the right hand side of the storm. The isohyets of rainfall amounts were obtained from the rainfall observer network of the Meteorological Office and are therefore strictly 24 hour totals. Other showers occurred during the course of the day but it is considered that the rainfall indicated fell predominantly from the large storm being discussed. In the severe parts of the storm, the rainfall recorded may be low because the gauges might well have failed to catch all of the hail or rain driven by strong winds.

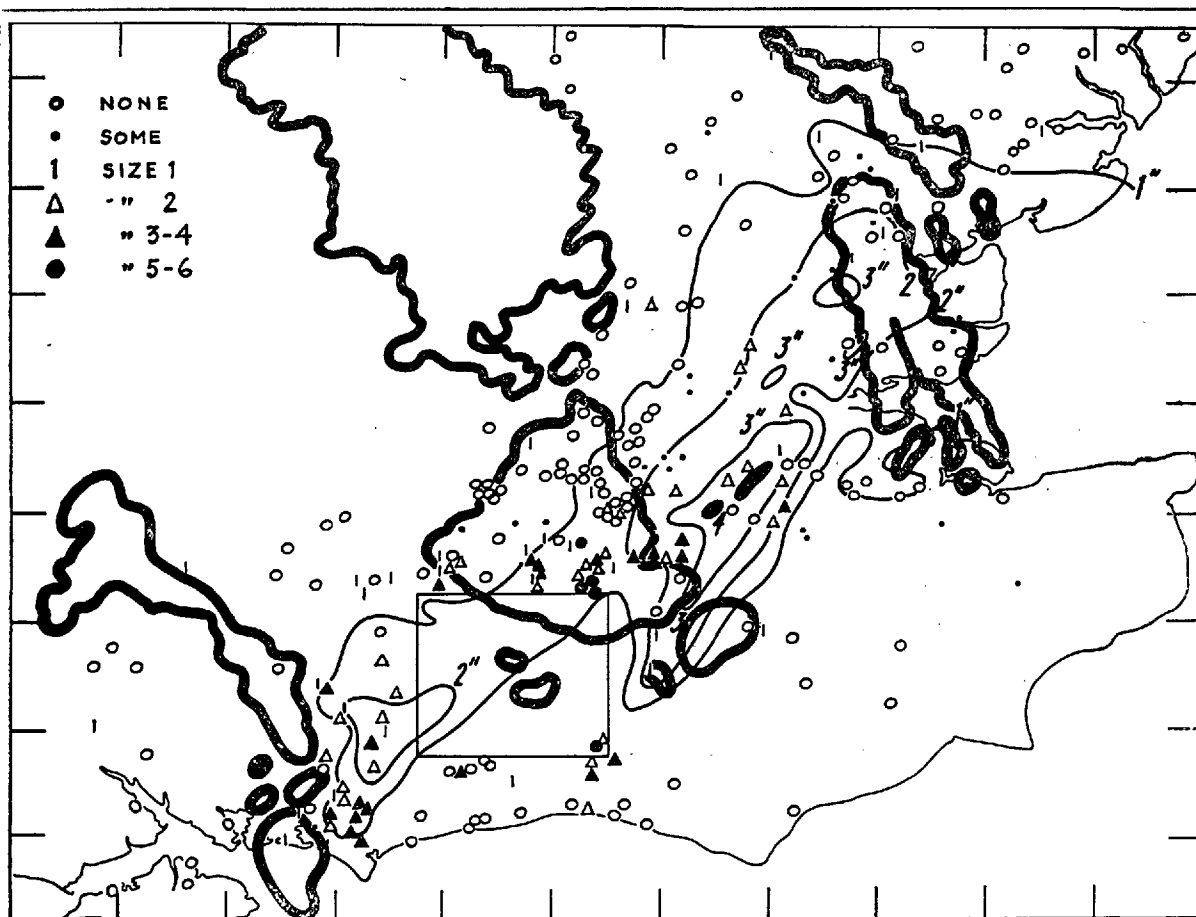


Figure 51 - General distribution of rain and hail, storm of 5 Sept. 1958. The map has markers at 10 km intervals along its edges, and shows the coast-line of S.E. England and three successive positions of the radar echoes from the principal storm (thick lines), at 1600 G.M.T. (left), 1800 (centre), and 2055 (right). The thin lines are isohyets for rainfall amounts of 1, 2, 3 and 4 inches; they refer to the 24-hour period beginning 0900 September 5, but in this period the amounts due to other storms were generally negligible. The region of maximum rainfall lies N.E. of the Horsham area, shown by the rectangle. The symbols show the maximum sizes of hail, according to the following scale:

- 0 - no hail; (a small dot in the diagram indicates some hail of unspecified, probably small size)
- 1 - pea (diameter about 0.6 cm)
- 2 - marble (1.6 cm)
- 3 - walnut (2.5 cm)
- 4 - golf-ball (3.5 cm)
- 5 - hen's egg (4.5 × 5.5 cm)
- 6 - tennis ball (7 cm)
- 7 - half grapefruit (9 cm).

The storm was most intense in the region bounded by the rectangle and a detailed study of the hail distribution in this area was made by the direct interrogation of the inhabitants. Some 240 reports were obtained in an area of approximately 500 km^2 . The distribution of maximum size hail at the ground during this intense period is shown in Fig. 5.2 in which the numbers refer to the scale of hailstone sizes indicated in the legend of Fig. 5.1. It must be stressed that the information plotted refers only to the largest hailstones and, with one exception (discussed in section 5.5), the distribution of sizes at a given location is unknown. The path of the tornado associated with this storm was readily ascertained by the damage it caused.

As shown in the diagram, it is possible to draw a coherent set of isopleths of maximum size of hailstones. In this region the updraughts in the clouds must have been some tens of metres per second to produce hail of the size observed. The time taken for air to traverse the depth of the storm above the 0°C level (about 6 km) at an average speed of 25 m sec^{-1} would have been 4 minutes which is certainly less than the time of 15 to 20 minutes required to grow hailstones of 7 to 8 cm diameter.

Since the horizontal speed of the storm was approximately 50 km hr^{-1} and large hail occurred over a region extending some 15 km, a more or less steady dynamical state must have been established within the storm mass for at least 30 minutes. Even so there still must have been regions where the storm intensified for periods of several minutes, these resulting in corresponding areas over which the largest hailstones were observed to fall. For example, the observer at the position marked x in the middle of Fig. 5.2 reported that large stones fell for about 10 minutes before giving way to light rain, lasting altogether for 15 to 20 minutes. Other observers in the worst part of the storm agreed that the heavy rain and wind squalls were intermittent, and it appears that these and the heavier falls of hail occurred characteristically in periods

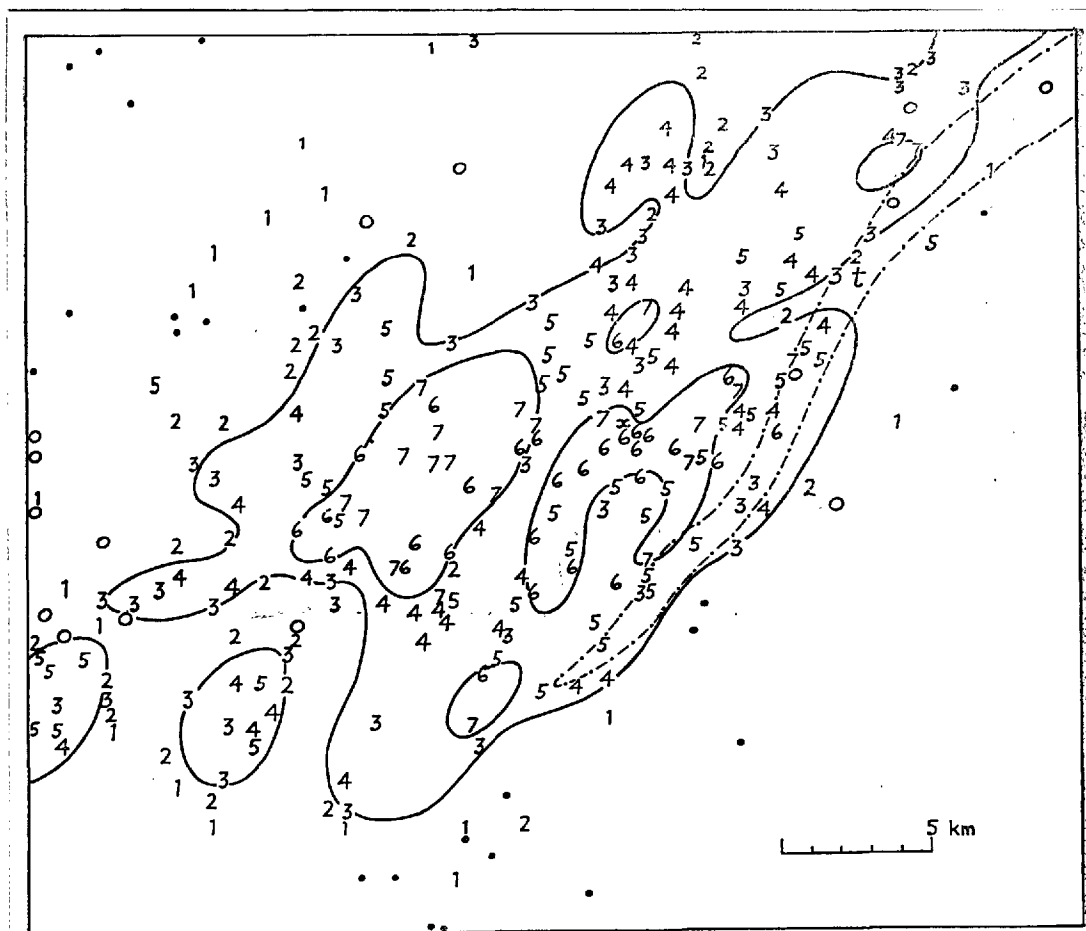


Figure 2 - Detailed distribution of maximum hail size in the Horsham area, shown by rectangle on fig. 1. Here the numbers again represent hail size according to the scale specified in the legend to fig. 1, but now the dots show where it is known that some rain fell without hail. The isopleths within which fell stones of diameter greater than 2.5 cm (size 3) or 7 cm (size 6) are drawn, and the pecked line on the right shows the approximate area within which tree damage by tornado winds occurred: at the point marked 't' a brick-works in an open field was destroyed.

of duration of 5 or 6 minutes.

These regions of intensification are thought to correspond to the columns of intense radar echoes received from beneath individual cloud towers and having a width of 2 to 3 km. Since the speed of travel of the storm was 50 km hr^{-1} , the area at the ground affected by a given hail shower would have been a strip about 5 km long and 2 to 3 km wide. A reasonable interpretation of the isopleth patterns in Fig. 5.2 is that they consist of such strips lying along the direction of movement of individual cloud towers (i.e. in the direction of the wind), overlapping in places so that there large hailstones occur over bigger areas. The storm as a whole, however, propagates by active growth on the right hand flank so that its direction of movement lies across the wind direction by 20 to 30° .

It is apparent from the diagram that investigations aimed at relating the hail- or rain-fall at the ground to the properties of cloud towers or radar echo columns require observations at points not much more than 1 km apart, referred to times accurate to within a minute or two. Such information has not hitherto been obtained.

5.3 The Wokingham storm of July 9, 1959.

The weather at the ground in this second storm was investigated in the same manner. The storm passed inland across the Dorset coast at about 1030 (B.S.T.) and reached East Anglia about 1500. It was most intense over parts of Hampshire and Berkshire where it produced hailstones of maximum dimension up to 6 cm.

The distribution of hail at the ground during this intense period is shown in Fig. 5.3 and isopleths of maximum hail size, based on some 440 observations spread over an area of about 1200 km^2 , have been constructed in the same manner as before. The general features of this distribution are similar to those of the previous storm and the same general considerations apply.

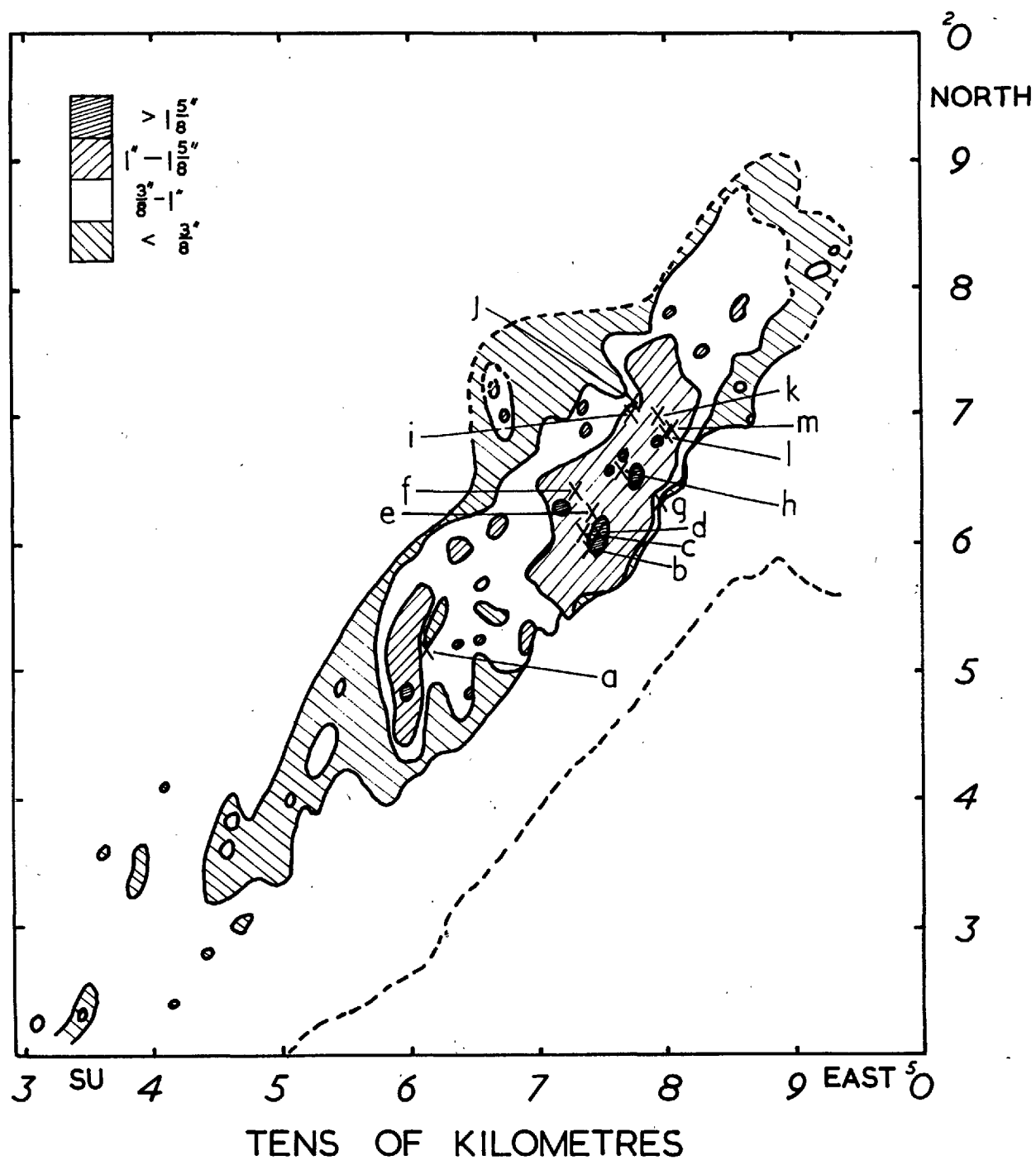


Fig. 5.3 The distribution of hail at the ground for the storm of July 9, 1959; the curves drawn represent the different size ranges indicated on the key. The path of the edge of the radar echo is shown by the dashed line and the letters refer to the positions of the samples of hail collected (see section 5.4.2).

The path of the edge of the radar echo associated with this storm is indicated in the diagram, and in passing from right to left across the storm, the first precipitation encountered is hail. Very little rain occurred on the right hand side of the swath, these relatively small particles being held aloft by the updraught. Even in the hail zone the rain and smaller stones were carried to the left of the swath by the wind so that on the far side there was an extensive rain area. This is apparent also in Fig. 5.1. The hail strips are orientated slightly across the storm for the reason already given.

Using the hail information depicted in Fig. 5.3, in conjunction with the radar observations of the storm, Browning and Ludlam (1960) have been able to construct a dynamical model of the storm during its intense phase. Early in the storm a succession of separate columns of echo associated with intermittently-rising individual cloud towers could be identified. In the intense phase (which corresponds to the central portion of the hail swath) it was no longer possible to distinguish separate echo columns. A more uniform organisation of the echo mass was established and a more or less steady state ensued for a period of some 30 minutes. This change in behaviour is thought to be due to a transition from the previous intermittent updraughts to one large and persistent updraught. The authors point out, however, that small echo turrets occasionally penetrated a few thousand feet above the top of the main echo mass, indicating that the continuous updraught was not quite steady. This is consistent with the interpretation of the distribution of hail at the ground given above. The intensity and top of the radar echo rose steadily during this period, the latter reaching a maximum value of 13 to 14 km in the region where most of the large hail fell.

A distinctive feature of the storm echo in this phase was that in its right hand and central parts the leading precipitation (consisting of large hail) formed a "wall" which had a sharply

defined and upright front face, orientated perpendicularly to the direction of movement of the storm. Browning and Ludlam consider that the downdraught associated with the most intense precipitation of small hail and rain further to the rear of the storm produced a circulation in the lower layers of the atmosphere, effectively scooping aloft potentially warm air which became buoyant and produced updraughts of up to 40 m sec^{-1} . The downdraught therefore played an integral part in the organisation of the cumulonimbus convection; a more or less steady state was established in which the updraught was strongly inclined in the lower, and more nearly upright in the upper troposphere.

Their model of cumulonimbus convection is shown schematically in Fig. 5.4. (This is a slightly modified version of the model which appears in their paper and has been communicated privately to the writer.) This shows the air motion relative to the storm in a vertical section parallel to its direction of movement (from right to left in the diagram). The important feature of this model from the point of view of hail growth, is that, relative to the storm, the stones are moving backwards (from left to right in the diagram). This means that the updraught supporting them is continually changing. Stones which grow in the narrow region in front (on the left) of the central streamline move into regions of continually increasing updraught, while the updraughts supporting the stones growing behind the central streamline are continually diminishing. If a stone grows at such a rate that its fallspeed is always only a few m sec^{-1} less than the magnitude of the updraught, then it will travel throughout the depth of the storm containing supercooled water in a time of the order of 1000 to 1500 seconds. It is these stones which attain maximum size. They are then thrown forward near the cloud top, re-enter the cloud at a lower level after falling through clear air, and form the upright leading edge of the precipitation. The trajectories of all other stones are such that they fall behind this leading edge.

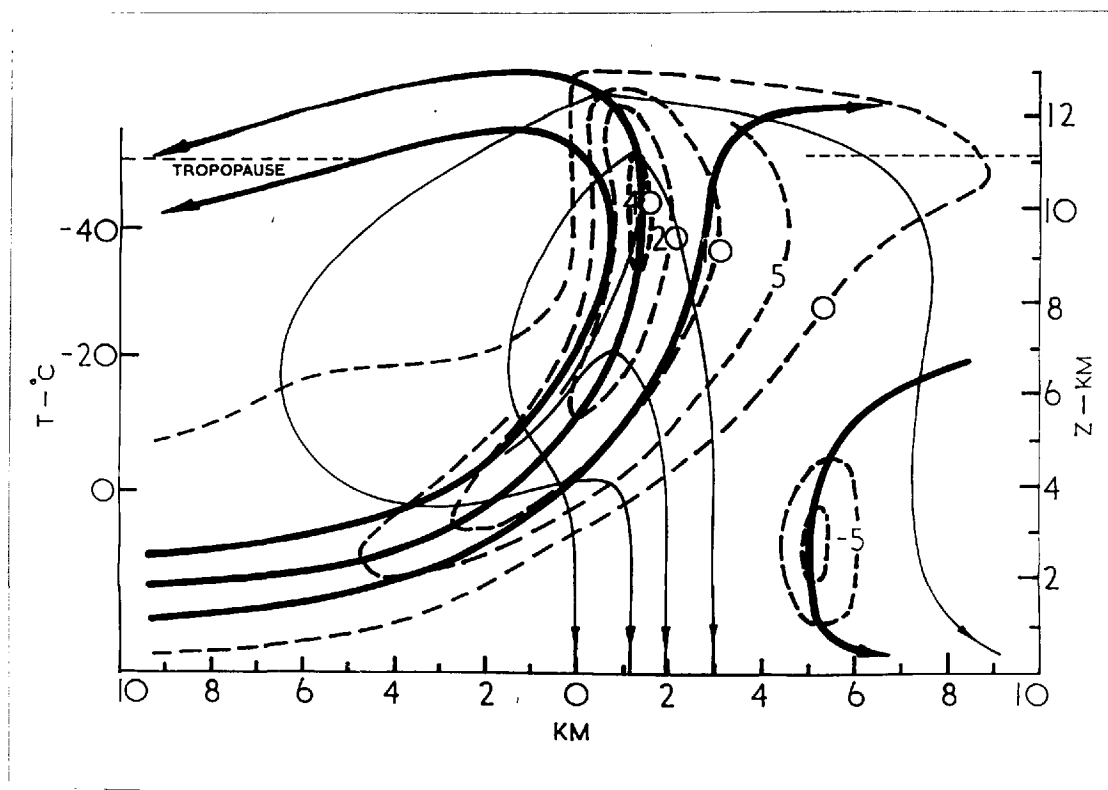


Fig. 5.4 Inferred streamlines (thick lines) and isopleths of vertical velocity (dashed lines, labelled in m sec^{-1}) within a range-height section along the direction of movement of the storm. The wall of precipitation consisting of the largest hailstones (about 2 cm radius) is vertically above 0. Probable trajectories of stones of various sizes are shown (fine lines), the size of a stone being gauged by the updraught velocity at the position at which it commences to fall.

(from Browning and Ludlam, private communication)

5.4 The general characteristics of the hailstones.

Having taken note of their unusual size and shape, a number of people were able to furnish descriptions of the hailstones which fell in these two storms. In a number of cases the stones had been preserved in refrigerators and were made available for inspection and analysis. It must be pointed out, however, that the samples collected in this way are not truly representative of the complete fall of hail at the corresponding locations, because it is virtually certain that the persons collecting the stones were biased in their selection towards those with distinctive features.

5.4.1 The Horsham storm hailstones.

The largest stones from the Horsham storm were reported to be 7 to 9 cm across. A number of people made measurements of their mass, with results mainly between 100 and 120 g, although one stone of 160 and another of 185 (to within about ± 2) g were recorded. (The equivalent spherical diameter of the last stone was 7.3 cm.) None of the stones inspected was perfectly spherical, as might be expected, although "more or less spherical" was a typical description given. Many were smooth on one side but dimpled on the other, while some were described as flattened discs and others as "irregular" or even "jagged". Some of the stones were cut open to reveal the typical structure of alternately transparent and opaque shells of ice (Plate 5.1 and Fig. 5.5). Of five large stones which were opened (diameters 4 to 5 cm), four had internal cores of transparent ice of diameters $\frac{1}{2}$ to 1 cm, implying early growth in the wet stage, possibly on frozen raindrops. Some of the smaller stones examined also had clear cores. The densities of the stones were obviously high and the densities of the transparent layers must have been virtually that of clear ice.

Because of the lack of adequate facilities at the time, no systematic study of these stones was made.

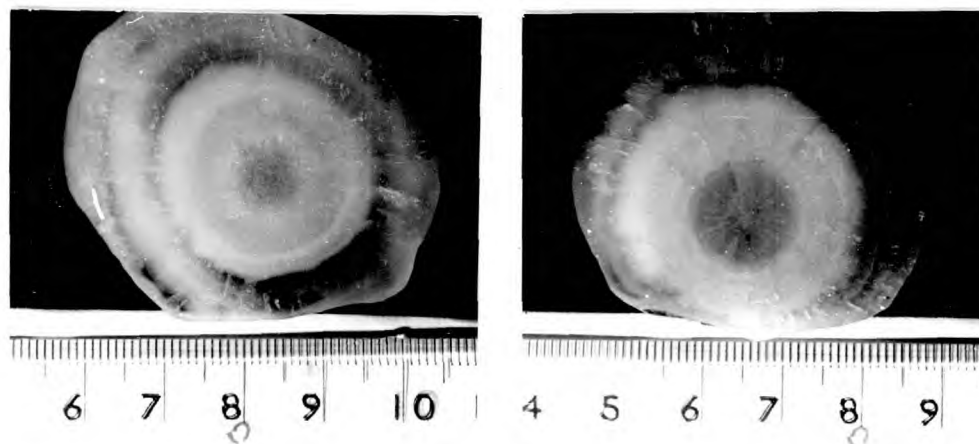


Plate 5.1 Typical hailstones from the Horsham storm, cut roughly through their centres, beside a scale of cm.

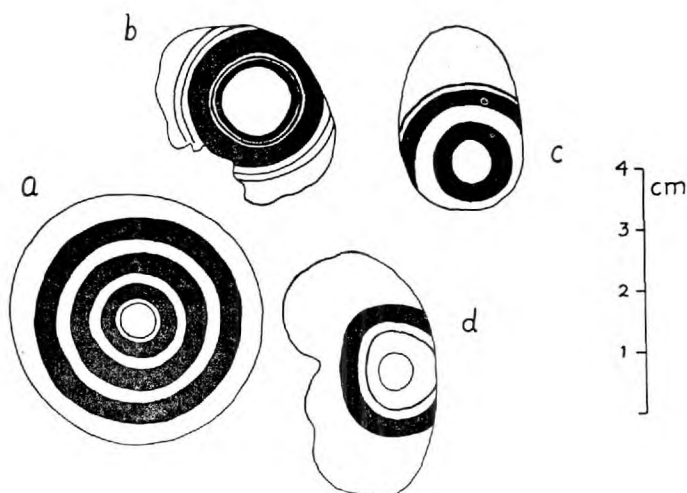


Fig. 5.5 Sections of hailstones showing the alternation of opaque ice (dark bands) and transparent ice (unshaded).

5.4.2 The Wokingham storm hailstones.

In the case of the Wokingham storm a deliberate attempt was made to collect samples of stones for subsequent analysis. A request was made to the ground observer network to store samples of hailstones, and insulated cold boxes chilled with dry ice were carried by the interviewers. In this way thirteen samples, totalling some several hundreds of stones, were obtained, two from voluntary observers and the others from people sufficiently interested to preserve them. The locations of these samples in the hail swath are shown in Fig. 5.3.

The hailstones which fell in the Wokingham storm were in general smaller than those which fell in the Horsham storm. Photographs of two samples of these stones are shown in Plate 5.2. The largest stone collected weighed 21.8 g and was oblate with axes of 5.2, 4.0 and 2.35 cm. While there probably were stones somewhat larger than this, it is certain that their masses were much less than the maximum of 185 g measured in the first storm. A number of the stones have been subjected to a density and structure analysis and these investigations are discussed in sections 6.2 and 6.3. The drag coefficients of three of the largest stones have been determined (section 4.3).

The dimensions of nearly 100 of the stones were measured to determine their general shape characteristics. The values for the ratios of the three principal axes of the stones, determined by inspection, are summarised in Fig. 5.6. It must be pointed out however, that the stones have been modified by melting both during their fall from the freezing level and while lying on the ground prior to preservation. A 1 to 2 mm layer is melted from the outer surface of a hailstone 2 to 3 cm in radius, while a stone of radius 0.5 cm is melted completely, in falling from the 0°C level (see section 5.5). Therefore the conclusions drawn here cannot be applied directly to the problem of the shape of hailstones within the cloud without some reservations. However, at present

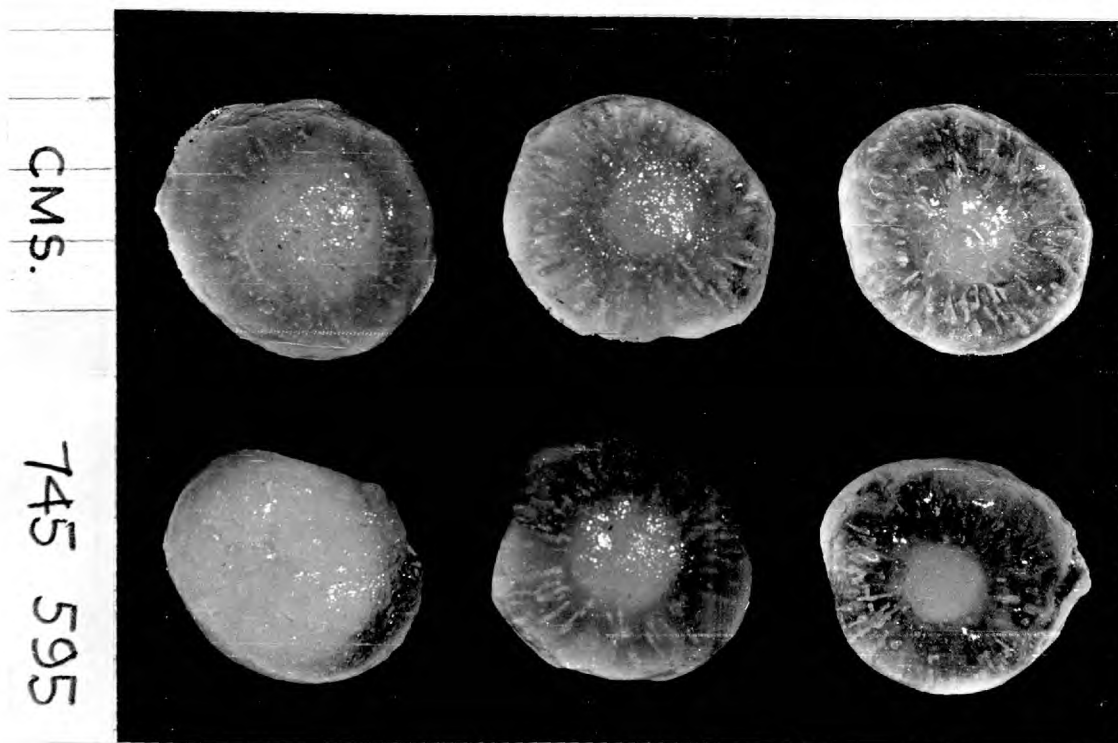
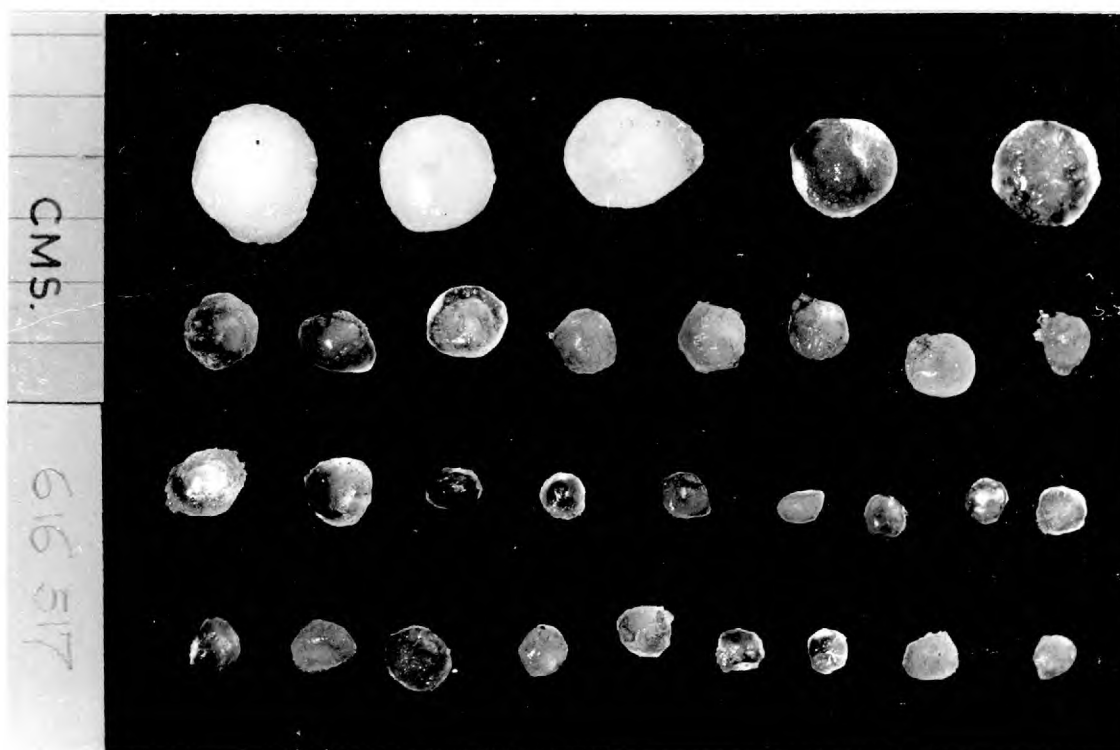


Plate 5.2 Hailstones from positions a (upper photograph) and b in the Wokingham storm hail swath.

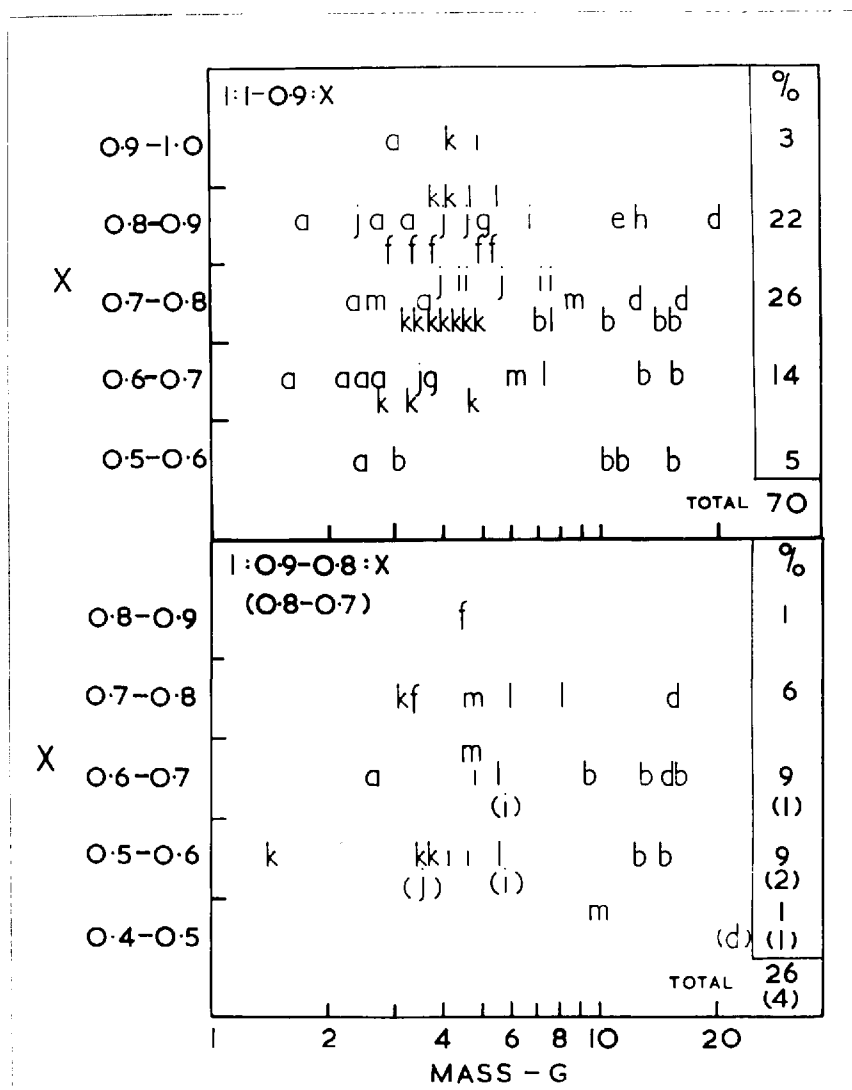


Fig. 5.6 The shape characteristics of the hailstones from the Wokingham storm. The letters represent the positions of the stones in the hail swath.

there is no other way of determining the general shape of hailstones.

It is seen from Fig. 5.6 that 70% of the stones have the two largest axes in the ratio $1 : 1 - 0.9$, that is, they have approximately spherical cross-sections. Their shortest axes however, may be as little as one half that of the largest dimension. Only 3% of the stones have axes in the ratio ranges $1 : 1 - 0.9 : 1 - 0.9$ while 25% have axis ratios of $1 : 1 - 0.8 : 1 - 0.8$. Thus only $\frac{1}{4}$ of the stones can be termed even "roughly spherical". This is so for all the stones since there is no systematic relation between their oblateness and mass. The shape of smaller stones, however, may be appreciably affected by melting.

Harper (1960) has shown that the back-scatter cross-sections of perspex^{*} spheroids with the shortest axis a factor of 0.6 to 0.8 times the longer axes may be as much as 5 times those of the equivalent spheres, depending on the orientation. No measurements are available for ellipsoids but, from these results and Fig. 5.6, it is apparent that the shape of the stones may be an important parameter in determining the magnitude of the radar echo. The effect of shape becomes more important when two radars of different polarisations are used to make simultaneous records of the same storm. Harper has shown that, for the spheroids under discussion, the ratio of signal strengths obtained using vertical and horizontal polarisations is a factor of 6 to 10. This may at least partly explain the difference in signal strength received by different radars used to observe the Wokingham storm (Browning and Ludlam, 1960).

The equation describing the growth of hailstones assumes for simplicity that they remain spherical, and, on the basis of the

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* Perspex has a refractive index approximately equal to that of ice and for some purposes can be used as a convenient substitute in making controlled measurements (see Atlas, Harper, Ludlam and Macklin, 1960).

measurements described here, it might be expected that the growth equation applies only to a relatively small fraction of the stones. However, as the oblateness of the stones increases, the fall speeds decrease (section 4.3) and, for stones of the same mass, the rate of accretion is sensibly constant regardless of their shape. For example, the rates at which the two largest hailstones (21.8 and 19.6 g) would sweep out air in free fall are 1.6 and 1.5 litres per second even though the former is oblate (axis ratios 1 : 0.77 : 0.45) and the latter spherical. Thus the growth equation is approximately applicable to a large proportion of the stones provided the radius given by the equation is taken to be the radius of the sphere of the same mass.

5.5 The spatial concentration of hailstones.

Knowing the spacing of hailstones in a given size range, their velocity and duration of fall, it is possible to deduce the spatial concentration of the stones near the ground. The duration and spacing of the hailstones were generally obtained by interrogation but in some cases the latter could be inferred from observations of damage. (The large stones left pits in lawns, dented iron roofs, broke roof tiles etc. and from an inspection of such damage the average spacing of the stones could be determined.) The velocity of fall was deduced assuming the stones to have a spherical cross-section and a drag coefficient of 0.6.

The concentrations obtained in this way are shown plotted against the dimension of the stone, deduced from its description, in Fig. 5.7. In the case of the smaller stones this is approximately the diameter of the sphere of the same mass; the dimensions of some of the larger irregular stones have been modified so that the dimension, D , is consistently defined as this diameter. It is considered that the concentrations plotted are correct to within a factor of 4. Points are shown for both the Horsham and Wokingham storms and it is seen that there is no essential difference

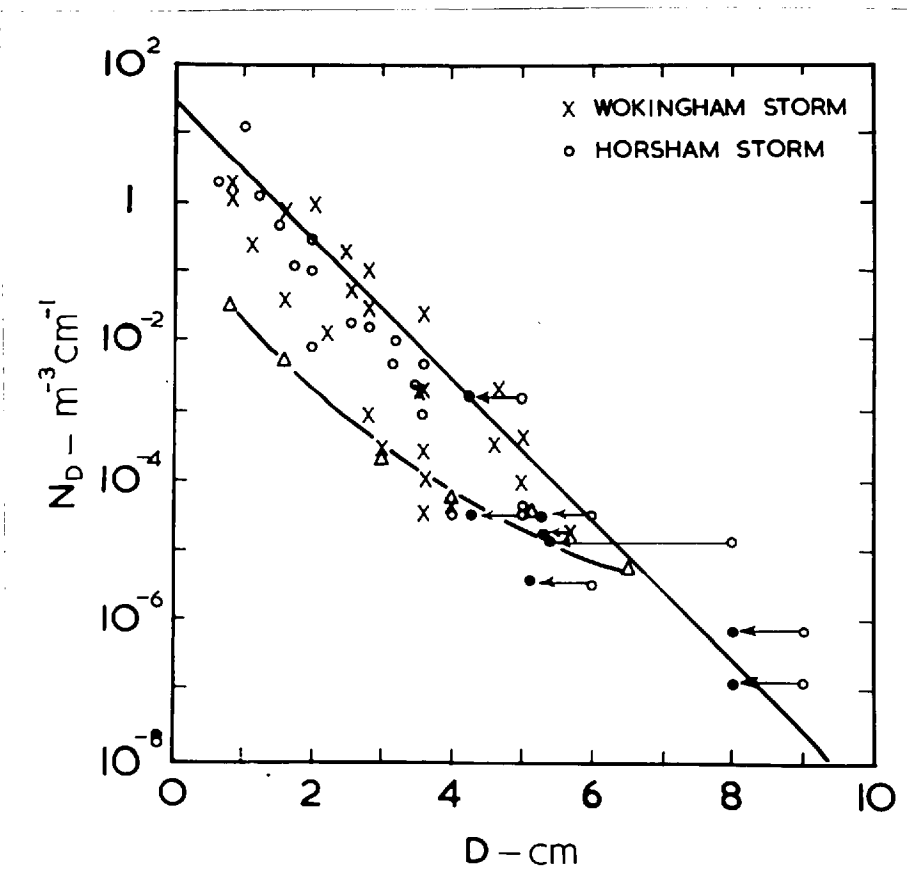


Fig. 5.7 The spatial concentrations of hailstones near the ground. The plotted maximum dimensions of irregular stones have been altered (indicated by arrows) so that D is the diameter of the equivalent sphere.

between the concentrations from the two storms. It is possible to draw a straight line through the upper points in the diagram (making allowance for the possible over-estimation of some of the values) indicating that the maximum spatial concentrations near the ground in each centimetre size range (N_D) of the largest stones to fall at one place during the passage of the storm obeys the exponential law

$$N_D = 20 e^{-2.30D} \quad (5.1)$$

D being the diameter of the stone. Donaldson, Chmela and Shackford (1960) have since obtained some spatial concentrations for less severe storms and their maximum values are 2 m^{-3} and 0.2 m^{-3} for stones of diameters of 1 and 2 cm respectively. These are comparable with those obtained here although Donaldson et al. do not specify the size range appropriate to their concentrations.

In one instance in the Horsham storm it was possible to deduce the size distribution of hail at the ground at a given location. Estimates of the spacing of hailstones of different sizes were made from a photograph of a lawn, partially covered with stones, supplied by an observer at Horsham. The curve appropriate to this distribution lies lower than the line drawn for the maximum concentrations of the largest stones (see Fig. 5.7) and it is probable that the latter is the locus of end points of individual distributions similar to the one obtained here. Atlas and Ludlam (1960) show that there is a considerable sorting of hailstones according to size in the wind field associated with such storms. Stones of different sizes generated at the same position in the cloud are separated by distances of 1 to 2 km at the ground by a cross wind component of 10 to 20 km hr^{-1} and it is therefore extremely difficult to interpret specific size distributions of hail which are obtained at the ground. The smaller stones are displaced to the left of the storm so that places on the right of the storm, where large hail falls, will be deficient

in small stones. Hence it is to be expected that the individual distributions which contain large stones lie lower on the diagram than the curve for the maximum concentrations. The size distribution in the updraught where the stones are grown may be nearer the upper curve.

It is possible to deduce the corresponding concentrations in the cloud by taking into account the change of velocity with air density and the effect of melting. The rate of change of radius due to melting is given by an equation similar to that for wet growth (eq. 1.15), heat now being transferred to the stone from the environment (Ludlam, 1958). For the conditions appropriate to these two storms, the relation between the radius of the stone at the 0°C level, R_0 , and the radius at the ground, R_1 , is given by

$$R_0^{7/4} - R_1^{7/4} = 0.35 \quad (5.2)$$

This indicates that a hailstone of radius 0.5 cm completely melts before reaching the ground. Applying this correction, together with the correction for fallspeed, to the points in Fig. 5.7 the relation

$$N_D = 32 e^{-2.34D} \quad (5.3)$$

is obtained for the concentration of hailstones at the -30°C level.

The mass concentration of this distribution is given by

$$\begin{aligned} W &= \frac{\pi}{6} \rho 32 \int_0^{\infty} D^3 e^{-2.34D} dD \\ &= 3 \text{ g m}^{-3} \end{aligned} \quad (5.4)$$

since the value of the integral is $\Gamma(4)/2.34^4$ where Γ is the gamma function and ρ , the density of the hailstones, is 0.9 g cm^{-3} . This is a reasonable value, although subject to a probable error of a factor of 2.

The distribution of hailstone concentrations represented by

eq. 5.3 is therefore considered to be a reasonable representation of the maximum concentrations likely to be encountered in severe thunderstorms. The rapid decrease of concentration with size must be due to some selection process by which only a very few of the smaller stones produced attain large diameters. This is possibly a result of the dynamical structure of the storm since only relatively few stones would have trajectories which pass through the updraught maximum and along which the growth rate is such that their fallspeeds are always only a few m sec^{-1} less than the magnitude of the updraught.

6. THE DENSITY AND STRUCTURE OF HAIL.

6.1 General considerations.

It is considered that there is no physical distinction between the ice deposited on a rotating cylinder and that deposited on a hailstone both of which are subject to accretion in a supercooled cloud. The density and structure of hail growing in the dry regime are determined by the same factors which determine the density and structure of rime, namely, the temperature of the surface and the droplet size and impact speed. In the wet regime under appropriate conditions hail grows in a spongy state, accreting ice particles as well as supercooled droplets. While the experimental results may not be exactly applicable (in the quantitative sense) to the growth of hailstones because of differences in shape, they do permit general conclusions regarding their density and structure.

6.1.1 The density and structure of soft hail.

A number of measurements have been made of the density of soft hail. Nakaya and Terada (1935) measured the density and fallspeed of such particles formed near the summit of Mount Tokati, Japan. The average density of roughly spherical pellets between 1 and 4 mm diameter was found to be 0.125 g cm^{-3} ; individual particles, however, had densities between 0.04 and 0.24 g cm^{-3} . The fallspeeds ranged from 1.2 m sec^{-1} for a pellet of 1.8 mm diameter to 2.6 m sec^{-1} for one of 5.6 mm diameter. Values of the density of graupel given by Magono (1954) for similar conditions range from 0.04 g cm^{-3} for 1 mm diameter particles to 0.17 g cm^{-3} for 6 mm diameter particles, with a scatter of $\pm 0.035 \text{ g cm}^{-3}$. Arenberg (1941) estimated the density of soft hail pellets to be 0.2 g cm^{-3} or less while Ludlam (1952) gives a value of 0.1 g cm^{-3} for particles slightly less than 1 mm diameter.

All of the above density determinations were made with soft hail pellets which fell during winter showers. In summer weather such particles melt before reaching low ground. List (1958 a)

gives the mean densities of pellets of diameters up to 6 mm which fell at the Weissfluhjoch Observatory (2665 m above sea level) during two summer showers as 0.5 and $0.7 \pm 0.15 \text{ g cm}^{-3}$ respectively.

The densities of ice deposits formed by accretion under conditions appropriate to the formation of soft hail have been estimated from the experimental data given in section 2.3. In order to do this it was necessary to specify the droplet size, liquid water concentration and temperature in the supercooled cloud and the relation between the fallspeeds and Reynolds numbers of the pellets. Nakaya and Terada made their measurements when the ambient temperature was between -8 and -15°C and the pressure about 900 mb; the particles were therefore grown at temperatures and pressures somewhat lower than these. Drop size distributions based on a microscopic examination of rimed snow crystals made by Nakaya and Terada indicated a value of 30μ for the median drop-volume diameter and this value was used in the present calculations.

In a cloud whose base is at -10°C , 900 mb the adiabatic value of the liquid water concentration is less than 1.5 g m^{-3} (Fig. 1.2). Since the fallspeeds of the graupel pellets formed in such clouds are 3 to 4 m sec^{-1} at most, the maximum rise in surface temperature for these conditions is 5 to 10°C . To simplify the calculations, it was assumed that the surface temperature of a growing particle remains constant.

From the relation between the fallspeeds, density and diameters of the pellets given by Nakaya and Terada approximate values of the drag coefficient were determined. These were about 1 for particles of 3 mm diameter and greater but increased for smaller particles (the drag coefficient of spheres also increases from 0.6 to 2 at the Reynolds numbers corresponding to these latter particles). Two sets of calculations have been made. In the first it was assumed that the drag coefficient was constant and equal to 1 (this was necessary to obtain approximate values of the Reynolds numbers involved) and in the second, values of the drag

coefficient of spheres were used where these were greater than 1 for the appropriate Reynolds numbers, while at higher Reynolds numbers a constant value of 1 was assumed. The air densities employed were those appropriate to a cloud whose base is at -10°C , 900 mb.

It was also necessary to postulate an initial particle from which growth commences. Nakaya and Terada consider that the graupel pellets whose density they determined were formed by the riming of dendritic ice crystals and their measurements show that the fallspeeds of these particles would not have been more than 30 to 50 cm sec^{-1} . A frozen droplet of 100μ diameter also falls at a speed of about 30 cm sec^{-1} under these conditions (Gunn and Kinzer, 1949). The density of ice such a particle accretes in a cloud of droplets whose median drop-volume diameter and temperature are 30μ and -15 to -20°C respectively is certainly less than 0.1 g cm^{-3} (Fig. 2.7). An extrapolation of the curves in Fig. 2.7 suggests a value of about 0.05 g cm^{-3} . The present experiments do not indicate whether such a low value is possible but both Langmuir (1944) and Melcher (1951) give rime densities as low as 0.02 g cm^{-3} . The measured densities of graupel pellets are also sometimes as low as 0.04 g cm^{-3} . If the particle grows to a radius of 0.25 mm by accreting ice of density 0.05 g cm^{-3} , its fallspeed is still only about 30 cm sec^{-1} and, at -15 to -20°C , it is still accreting ice of this density. The starting point in these computations was therefore taken to be a particle of radius 0.25 mm and mean density 0.05 g cm^{-3} . At -10°C , similar considerations show that a particle of radius 0.25 mm has a mean density of approximately 0.1 g cm^{-3} .

It was assumed that the pellets remain spherical and that the speed of impact of the droplets is equal to the fallspeed of the pellets. The speeds of impact of the droplets are about 80 to 90% of the fallspeeds so that the error introduced by the latter assumption is not considerable. For values of the density below

0.1 g cm^{-3} linear extrapolations of the curves in Fig. 2.7 have been used.

The results of these computations, carried out in a stepwise manner, are shown in Fig. 6.1 which gives the mean density of a pellet as a function of its radius for values of the surface temperature of -10 , -15 and -20°C respectively. Densities computed for a surface temperature of -30°C are approximately the same as those for -20°C , for, although the $T_s = -30^\circ\text{C}$ isotherm in Fig. 2.7 lies below that for $T_s = -20^\circ\text{C}$, the pellets have slightly higher fallspeeds at the higher level in the cloud. The density values deduced from Nakaya and Terada's measurements and those given by Magono for similar conditions have been included in the diagram, and it is seen that the agreement is very reasonable.

This shows that, in cold-base (winter) clouds, where growth occurs at relatively low levels and temperatures in small liquid water concentrations on frozen cloud droplets or ice crystals, the density of the hail pellets formed is low, generally less than 0.2 g cm^{-3} . The pellets do not grow to a particularly large size because the updraughts in such clouds are weak (a few m sec^{-1}) and the cloud depth small. In this density region, the accreted ice is composed of droplets individually frozen together with little distortion or spreading and the gradation in density throughout these particles would hardly be noticeable. If the temperature of the cloud base is near 0°C then it is possible for low density pellets to acquire a high density layer as they fall through the lower parts of the cloud, as sometimes observed (e.g. Arenberg, 1941).

Similar computations have been carried out for soft hail particles grown in warm-base (summer) clouds and these are discussed in the next section.

6.1.2 The density of hailstones.

It is possible to carry out stepwise computations of the

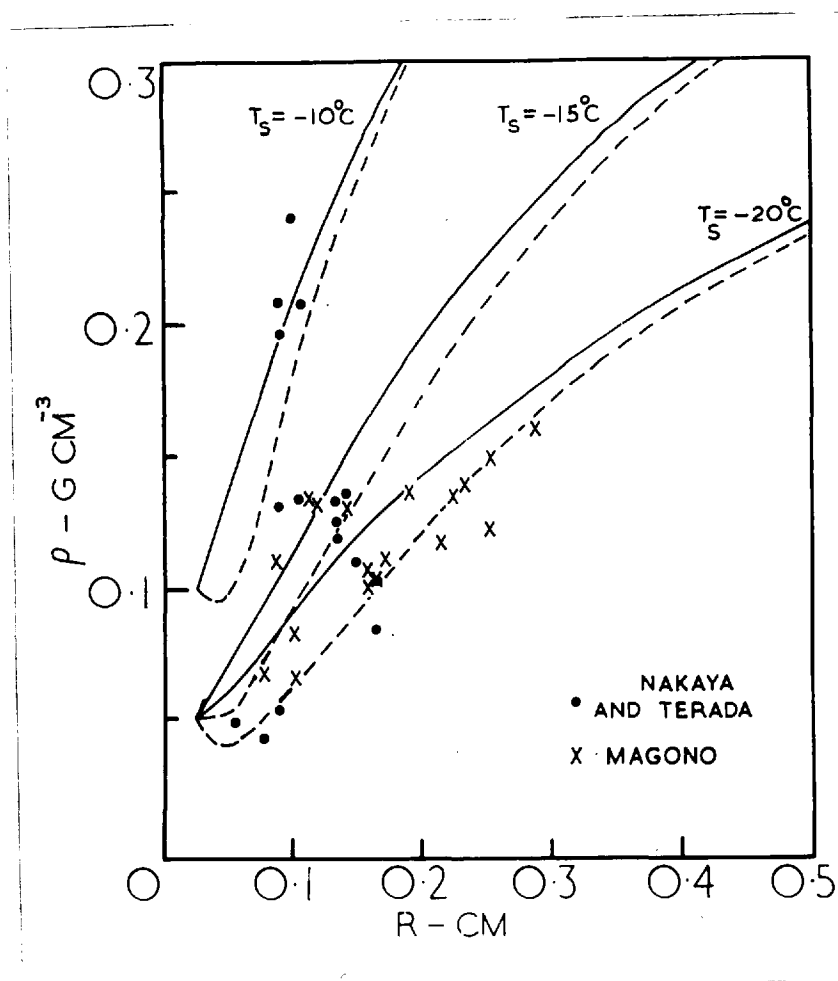


Fig. 6.1 Computed mean densities of soft hail pellets as a function of radius.

density of hailstones in the same manner as above. These are more complicated however because, in the warm-base clouds (base temperatures of 10 to 20°C) in which large hailstones are formed, the liquid water concentrations are likely to be 3 or 4 g m⁻³ and the fallspeeds of the stones higher (since the air density for a given temperature level is considerably lower than in the cold-base clouds). This means that the surface temperature of a stone increases markedly with its size so that the mean density changes relatively rapidly. In these computations it was assumed that hailstones grow as spherical objects at a fixed level in the cloud and that their heat transfer coefficients are equal to those of spheres. When more detailed information concerning the trajectories of the stones within the storm cloud becomes available, it will be possible to carry out more elaborate computations.

In order to make these calculations it is again necessary to specify the median volume radius of the cloud droplets, the water concentration and the relation between the drag coefficient of the stone and its Reynolds number. Because of the difficulties involved no measurements of the drop size distribution or water concentration have been made in the active regions of severe thunderstorms. The adiabatic liquid water concentration in the model cloud (base 20°C, 900 mb) is 6 to 7 g m⁻³. The concentration of nuclei effecting condensation is typically of the order of a few hundreds per cm³ (Ludlam, 1958) which gives average drop radii of 10 to 20 μ . A value of 15 μ has been chosen as this permits a comparison with the previous computations. Since in the initial stages of growth the collection efficiency of the hailstones for droplets of this size is somewhat less than unity (approximately 0.8 to 0.9 depending on the diameter and fallspeed of the stone) and because mixing with air at higher levels or the development of precipitation is likely to reduce the liquid water concentration somewhat below the adiabatic value, it has been

assumed that the stones grow by accreting droplets in a concentration equivalent to 4 g m^{-3} . The relation between the drag coefficient and Reynolds number is not known accurately for small stones and two sets of calculations have been made, one assuming a constant drag coefficient equal to 1, the other assuming values appropriate to smooth spheres. The latter values give an upper limit to the density.

The exact nature of the particles initiating hail growth is at present unknown. List (1958a) for example, considers that all types of hail are initiated by an ice nucleus which first grows by sublimation into a snow crystal and then by accretion into soft hail, from which the hailstone is developed. It is equally possible that hailstones grow from droplets a millimetre or two in diameter which are introduced into the regions of the cloud above the 0°C level and freeze (see below). The calculations have therefore been commenced both from small low density particles and from frozen droplets 1 and 2 mm in diameter. It has been assumed that the ambient temperature remains fixed and the mean surface temperature has been computed in a manner analogous to the computations of the mean surface temperatures of cylinders which are undergoing accretion. No account has been taken of the heat absorbed by the stones.

The mean densities obtained for hailstones under various conditions as a function of the radius are shown in Figs. 6.2 (a) and (b). From a comparison of Figs. 6.1 and 6.2 it is seen that the density of soft hail grown in warm-base clouds is considerably higher than that grown in cold-base clouds, as is to be expected since the fallspeeds and surface temperatures are correspondingly higher. It is interesting, however, that the densities of stones originating from the droplet of radius 0.05 cm are comparable with those originating from the small low density particles. Soft hail pellets of densities greater than 0.4 g cm^{-3} (e.g. List, 1958 c) could well be initiated either way. It is seen also

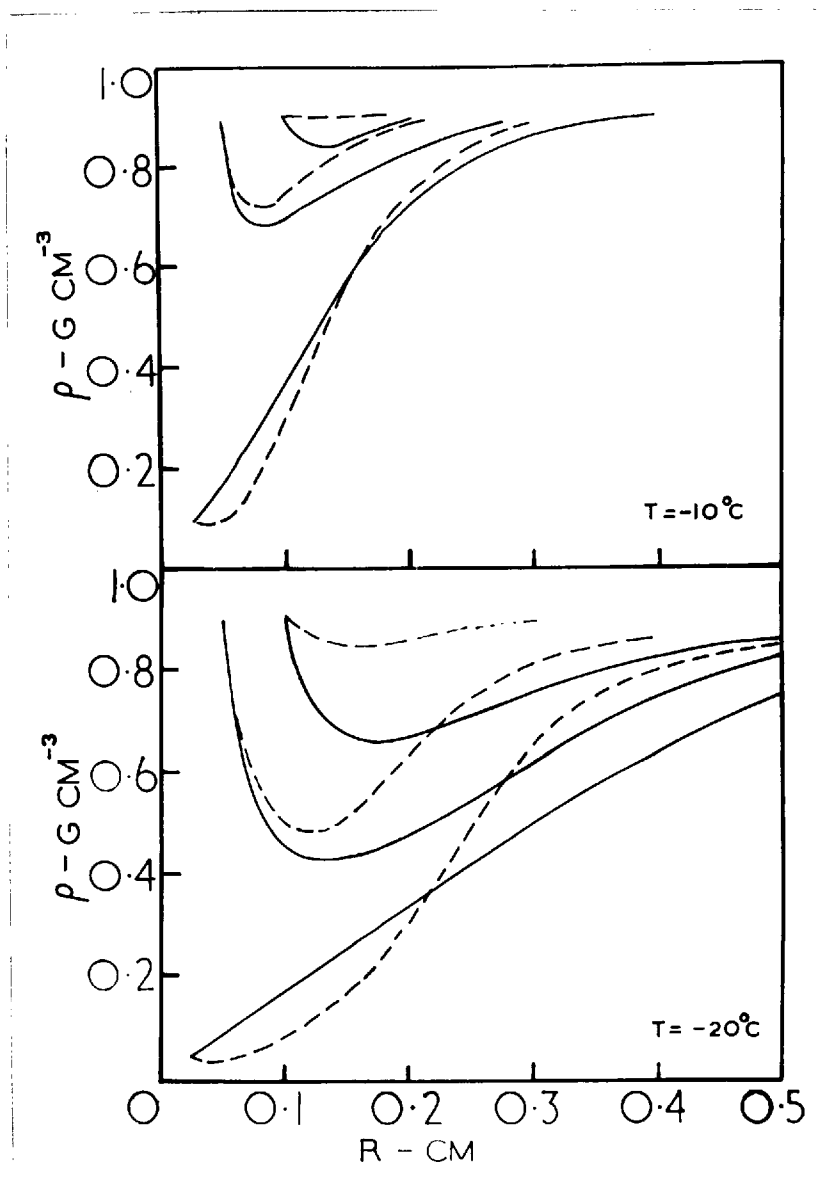


Fig. 6.2 Computed mean densities of hailstones, growing in the model cloud, as a function of radius. It has been assumed that the ambient temperature remains constant at (a) -10°C and (b) -20°C .

that once a hailstone has achieved a diameter of a centimetre or two at these levels its mean density is necessarily high, greater than 0.8 g cm^{-3} . This is due to the relatively rapid increase of the quantity $-v_0/T_s$. The growth of large hailstones is complicated by their entry into the wet regime.

If hailstones grow about or above the level in the cloud where the temperature is -20°C , then they must contain a core of moderately low density rime (0.4 to 0.7 g cm^{-3}) of several millimetres diameter unless the initiating particle was a frozen droplet of at least 1 to 2 mm radius. Even at -10°C the initiating particle must have a radius of at least 0.05 cm if a core of high density is to be formed. These conclusions are relevant to the discussion of hailstone structure in section 6.3.2.

6.2 Measurements of the densities of hailstones.

Despite the innumerable reports of various hailstone characteristics in the literature, there are surprisingly few which describe accurate density measurements. At the time of commencement of the present investigations there was only one such report, namely that of Steyn (1950) who gave the specific gravities of nine stones, 5 to 9 cm diameter, as lying between 0.89 and 0.91. There was also little knowledge of how the density might vary throughout the stones; Mason (1957, p.260), for example, suggested that since hailstones may be composed entirely of clear ice, entirely of opaque ice, or a mixture of both, their mean density may lie anywhere between about 0.1 and 0.9 g cm^{-3} .

Vittori and Di Caporiacco (1959) have since reported very accurate density measurements (some to better than $\pm 0.0005 \text{ g cm}^{-3}$) of forty stones ranging in mass from 0.53 to 13.6 g. The method employed was to weigh the stones first in air and then in chilled ethyl ether whose specific gravity was accurately known. Three

of the stones had densities of 0.80 g cm^{-3} ; the others had values between 0.873 and 0.915 g cm^{-3} .

The densities of some 160 stones which fell from the Wokingham storm were determined by immersing each stone in turn in a series of chilled liquids of varying densities. These liquids were mixtures of paraffin oil and carbon tetrachloride and their specific gravities ranged from 0.850 to 0.910 in steps of 0.010 , these values being measured to ± 0.0005 with an accurately calibrated hydrometer. In some cases a liquid of specific gravity of 0.916 was also used. By observing the velocity of fall or rise of the stones in the liquids, it was possible to estimate their densities to within $\pm 0.003 \text{ g cm}^{-3}$. It must be pointed out, however, that this technique is useful only if the densities are high and there is no significant penetration of the liquid into the stones. It is unlikely that this did occur since any surface pores or bubbles would have been sealed during the storage of the stones by the re-freezing of melt water. (This could, of course, modify the density slightly so that it is doubtful whether very accurate measurements are really justified.) Further, careful observation showed that no air bubbles were liberated when the stones were immersed in the liquids.

Fig. 6.3 shows the density of the stones as a function of their mass, the letters used indicating the position of the stones in the hail swath. It is seen that the densities of the small stones (less than 0.5 g) are all greater than 0.905 g cm^{-3} ; however, these were all obtained from the same place and fell relatively early in the storm. The densities of the stones which fell on the right hand side of the swath (b, c, d, and g's) tend to be higher than those which fell on the left, although such a generalisation must be treated with caution because of the rather arbitrary sampling procedure (see section 5.4).

The high values of the densities obtained here are consistent with those given above. Twenty eight of the stones (mass 0.41

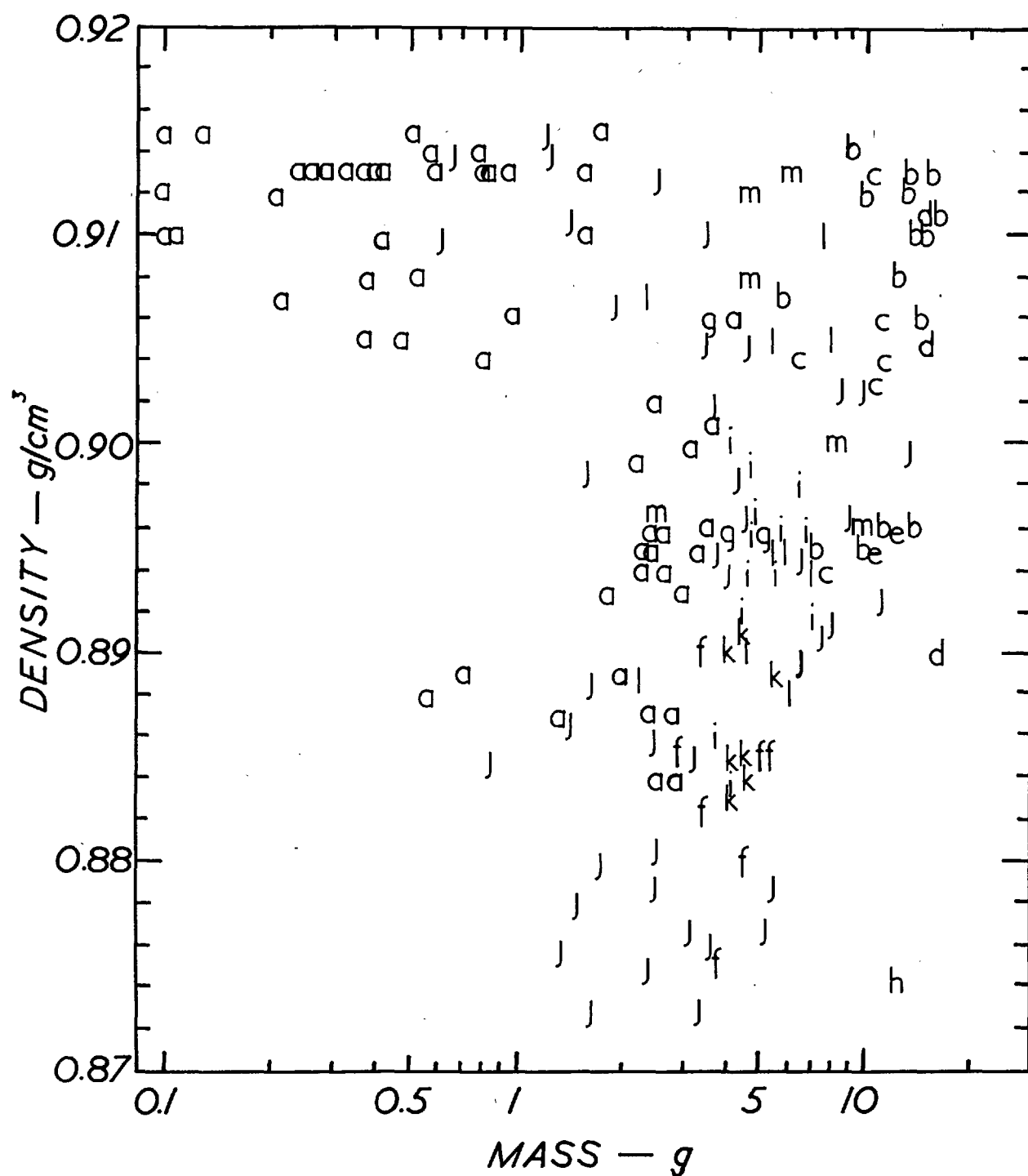


FIG. 6.3 The density of the hailstones as a function of their mass. The letters plotted represent positions in the hail swath.

to 17.3 g) were melted down in a water bath to a diameter of about 5 mm and their densities determined at successive stages. In these experiments more careful measurements were made, and it is considered that the relative density changes observed were determined with an accuracy of $\pm 0.002 \text{ g cm}^{-3}$ or better. In fifteen cases the density increased, in seven cases the density decreased, and in six cases there was no significant change in the density as the diameter decreased. The stones were not selected at random, a disproportionate number with opaque centres being chosen so that these figures are not a true indication of the relative numbers of stones with centres of high density (see section 6.3.2). The increases in density were greater on the average than the decreases, the maximum increase being 0.026 g cm^{-3} while the maximum decrease was 0.012 g cm^{-3} . The average increase in the density was 0.005 g cm^{-3} . While some of the density increases may have been due to the penetration of water into the stones as they melted, the increases (decreases) in density were generally accompanied by increases in the transparency (opacity) of the stones, as is to be expected. It is therefore considered that the trend indicated here is real.

6.3 The structure of hailstones.

6.3.1 The shape of hailstones.

The shape of a hailstone is obviously dependent on the conditions under which it was formed. If the stone falls without rotating and its surface remains dry, then it grows as a cone for the reason given by Reynolds (1876). Conical stones up to 2 cm long have been observed (Weickmann, 1953). On the other hand if a stone growing with a dry surface is being continually rotated in a random manner in the updraught, then it attains an approximately spherical shape. If the surface of the stone becomes wet, the slowly freezing water is driven to

the periphery of the stone where it solidifies so that the stone becomes spheroidal (Weickmann, 1953). If the hailstone is growing with a spongy texture it can be moulded by the airstream in a similar way while if the stone is excessively wet then icicle-like projections are formed as explained in section 3.3.3. Melting between the 0°C level and the ground may affect the shape of the stones to some extent but in the case of stones with radii greater than 2 cm this only affects the outer millimetre or two (Ludlam, 1958).

6.3.2 The internal structure of hailstones.

Hailstones are often composed of alternate layers of clear and opaque ice, with as many as twenty concentric layers, although the number is generally not more than five (Mason, 1957 p.260). Commonly such a structure has been explained by assuming that the updraught supporting the stone is intermittent and that growth takes place at alternately high and low levels in the cloud, which causes different freezing rates of the accreted water (e.g. Brunt, 1941 p. 56). In view of the rather persistent nature of the updraught in the dynamical model of the hail cloud which is emerging from recent studies (Browning and Ludlam, 1960) it seems more reasonable to look for local variations in the environmental conditions as the cause of the formation of the layers.

On the basis of the present observations of hail structure, it is considered that the ice composing many of the opaque rings is characteristic of the glaze rather than the rime family and, on the basis of Clark's classification, would be termed "milky" (section 2.1). It is seen from Fig. 2.11 that members of the glaze family occur over a relatively narrow range of surface temperatures near 0°C . A change in the ambient temperature of three or four degrees is sufficient to produce a marked change in the opacity of the ice accreted by a stone whose surface temperature was initially just being maintained at 0°C . However, the

surface temperature is affected also by the rate of accretion and by the rate of heat transfer to the environment. A change of a few degrees in the surface temperature could be produced by a change in the liquid water concentration of 20 to 25%. Variations of this magnitude are likely to be encountered along the trajectory of a hailstone which crosses the width of the updraught simply because of an irregular distribution of moisture with height in the layer of air which feeds the updraught (Fig. 5.4). A similar change may be produced by an increase in the turbulence of the updraught, the heat transfer coefficients being increased by up to 40% in this way (Reiher, 1925). Such turbulence, however, would have to be on a scale which is comparable to the size of the hailstone and at the present time there is no information concerning this.

Opaque layers can be formed in other ways. Gaviola and Fuertes (1947) suggested that the accretion of snow or ice particles may produce an opaque layer and the experiments described in section 3.3.2 show that this is possible. However, substantial quantities of ice are not likely to occur in strong updraughts at temperatures much higher than -35°C . The freezing of a spongy ice layer in a hailstone causes it to become opaque since then the freezing occurs from the outer layer inwards and any dissolved air is entrapped.

In the case of moderate-sized stones (diameters about 1 to 4 cm) the alternation between clear and opaque layers is reasonably attributed to an oscillation between the wet and dry growth regimes caused by relatively slight changes in the environmental conditions and in their size (and hence fallspeeds). In the outer parts of large stones of diameters greater than 5 cm this is less likely since the stones are wet at nearly all levels in the cloud in a water concentration of only 2 g m^{-3} . The opacity in these parts of the stones must then be due to the freezing of spongy layers (see Plate 3.2) or to the accretion of considerable quantities of

ice particles.

Further evidence of the growth conditions may be obtained from the crystalline structure of hailstones which is readily observed by placing thin sections between crossed polaroids and illuminating them with transmitted white light. Using this technique, Floegel (1872) found that the crystals in the centres of the stones he examined were radial, and he inferred from this observation that the centres were formed first. A systematic study of sections of hailstones a few centimetres in diameter has been made by List (1958 b). Sections about 0.3 mm in thickness were made using a small circular saw in a cold room and were then viewed in the manner described above. The centres of nearly all of the stones exhibited crystals the dimensions of which were of the order of millimetres. These central parts of the stones were typically several millimetres to a centimetre or more across. The crystals in the outer parts were generally smaller and in many cases there was a sharp transition between the central crystals and the outer layers of crystals the size of which was of the order of only 100μ .

In a number of stones the central crystals appeared to radiate from a point in the form of a cone and List concluded from this that the centre portion was originally a conical soft hail pellet. In order to explain its now obviously high density, List assumed that the pellet had fallen from the level at which it had grown to a level where the temperature was near 0°C . Here it either partially melted (if the temperature was higher than 0°C) or entered the wet growth regime, so that the interstices between the droplets became filled with water. On re-entering the updraught it froze completely forming the high density core of the stone. Not all the central crystals had this appearance; in a number of cases the crystals were randomly arranged, in others the centre consisted of a single crystal several millimetres across.

Schaefer (1960) has made similar observations of thin sections of hailstones (up to 5 cm in maximum dimension) and he found that, in some cases, the central portions of stones were composed of small crystals of the order of a few hundred microns across. In one stone he found as many as six roughly

circular layers of alternately large (i.e. several mm in length) and small crystals. The transition between these layers was

extremely sharp. He suggests that large crystals are formed either during wet growth or by the accretion of raindrops while small crystals are due either to the accretion of ice particles in the supercooled cloud or by formation of low density layers which become filled with water in subsequent wet growth.

It has been shown in section 3.3.4 that the presence of ice particles in the supercooled cloud can affect the crystalline structure of spongy ice deposits: the greater the concentration of particles the smaller the individual crystals. It is also possible that reversion to dry growth gives rise to crystals of small size. On accretion a supercooled droplet commences to freeze as soon as it impinges on the droplets already frozen. If, because of its impact momentum, the droplet is distorted or moved into another position it is probable that the initial ice structure already formed is fractured and a re-orientation of the crystal axes occurs. Ice splinters are also produced at the surface during dry growth by various mechanisms (section 2.7) and these too may effect nucleation. The microscopic examination of the rime samples (Plate 2.1) showed that there were definite boundaries between individual droplets or groups of droplets which could have been associated with crystal re-orientation. Between crossed polaroids such samples did show a fine structure with single crystals of the order of a few hundreds of microns, but it is uncertain whether these were due to nucleation from the ice particles always present in the cold room or whether they were due to nucleation by one of the mechanisms described

above. It was found that small crystals were not necessarily associated with the members of the glaze family. As already suggested the freezing of water in these cases is similar to that of a liquid film which would give rise to a structure similar to that found in frozen bulk water. A transition from a clear ice layer to an opaque ("milky") layer is not necessarily accompanied by a change in crystal structure.

Thin sections of some sixty of the hailstones which fell during the Wokingham storm were prepared by melting the stones between parallel metal plates, care being taken to ensure that the sections were made through the centres of the stones. In the case of ellipsoidal stones, the sections were made parallel to the shortest axis since it is considered that the stones grew with this axis vertical. The stones were examined from time to time during this process and no visible change in the structure was observed. Low density layers would have melted more quickly leaving a hole in the section. The sections were carefully dried on filter paper to avoid any effects produced by the re-freezing of meltwater and then stored on glass slides in a refrigerator. Sections of an approximately uniform thickness of 0.3 mm were obtained in this way.

It was found that over 90% of the sections had transparent centres containing only a few relatively large air bubbles. The sizes and nature of the cores of 35 of the stones, obtained from photographs enlarged to several times natural size, are listed in Table 6.1, from which it is seen that the size of the cores ranged from 1 to 12 mm.

As in the case of List's observations, the centres of all of the stones consisted of relatively large single crystals of the order of millimetres in length. A number of stones had only one crystal at the centre, while in others the crystals radiated from an approximately central point. In only one or two cases was the central portion of the stones conical. It is not necessary,

TABLE 6.1 : The size and nature of the cores of the stones.

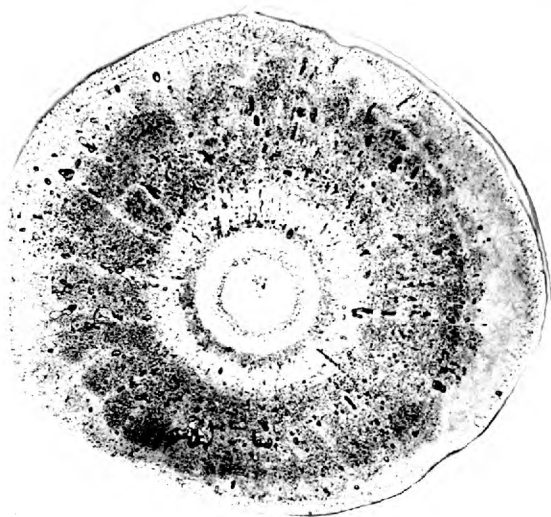
Position	Dimensions of the hail section (mm)	Nature of core (T = transparent C = cloudy)	Size of core (mm)
a	15	T	1.5
	9	C	3 x 2.5
	17 x 15	T	3 x 2
	18 x 16	T	2
	18 x 15	completely transparent	
	13 x 11	completely transparent	
	16 x 12	T	3
b	33 x 32	T	2
	36 x 19	T	12 x 10
	32 x 30	C	2.5
	37 x 16	T	3.5
	35 x 20	C	1
d	33 x 23	T	6
	39 x 23	T	6 x 5
	29	T	3
	32 x 31	T	4
f	22 x 17	T	3 x 2
	21 x 17	T	1.5
	23 x 21	T	2
	28 x 27	T	10
	19 x 16	C	4 x 3
	18 x 17	C	4 x 3
	16 x 15	T	2
g	22 x 20	T	5
	20 x 19	T	1
i	23 x 22	C	3 x 2
	23 x 18	T	3
	19 x 14	T	3 x 2
	23 x 15	T	11 x 10
k	20 x 16	T	4.5
	20 x 14	T	2
	21 x 14	T	10 x 9
	21 x 19	T	8
	20 x 16	T	10
	21 x 16	T	5

The measurements given in this Table were obtained from photographs enlarged to several times natural size.

however, to postulate a soft hail core to explain these as high density conical particles can grow in the same way. Typical examples of sections of stones are shown in Plate 6.1. The stones are labelled (a), (b), (c) and (d) and were obtained from positions d, b, k and f in the hail swath.

It is seen in stones (a) and (b) that the change in bubble structure (i.e. opacity) is not necessarily accompanied by a transition in the size of the crystals, which confirms the laboratory observations mentioned earlier. It has been shown in section 2.6 that clear ice is formed only when the surface temperature is nearly 0°C , and it is thought that, because of their small size and spherical shape, at least the first two opaque rings in (a) were produced by reversions to dry growth. The thick opaque layer, which consists of crystals about 120μ across, may have been due to dry growth, or to wet growth at a level in the cloud where the concentration of ice particles was of the order of 10^3 L^{-1} . Because of the lack of information of the concentrations of ice particles in the updraughts in cumulonimbus clouds it is not possible to distinguish between these two possibilities with any confidence.

The adiabatic value of the liquid water concentration in the Wokingham storm was 3 g m^{-3} at a height of 3.9 km (0°C level) and 4 to 4.6 g m^{-3} at heights above 5 km (-6°C level). The concentrations appropriate to the growing hailstones were likely to be less than this for the reasons already given (section 6.1.2). A diagram has been prepared showing the condition for entry into the wet regime for this storm (Fig. 6.4). (It has been assumed that the density of the stones is 0.9 g cm^{-3} , that the drag coefficient of hail in this size range (0.4 to 3.0 cm diameter) is 0.55 and that the heat transfer coefficients are equal to those of spheres. It has been assumed also that the value of the efficiency of catch of the hailstones for the droplets comprising the cloud is unity. The values for the



(a)



(c)

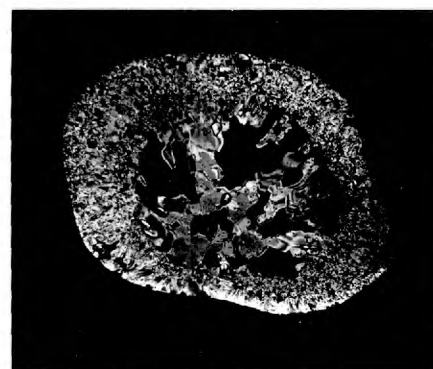
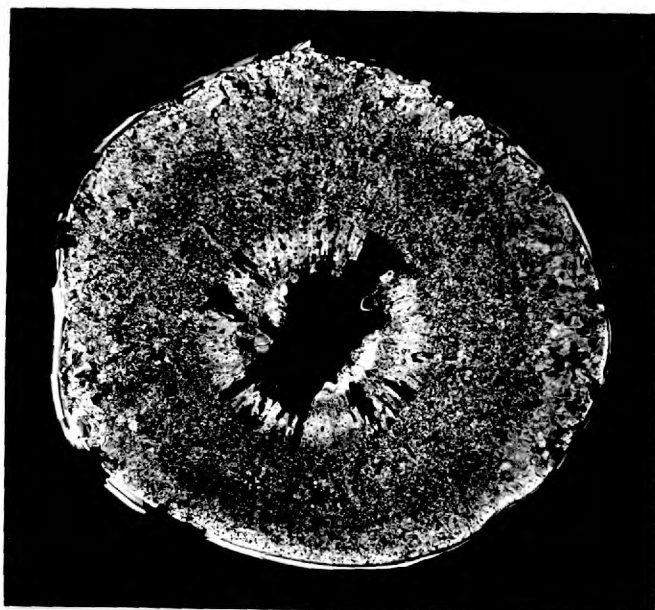
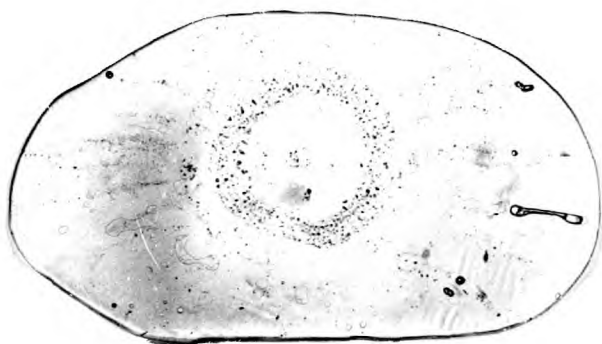


Plate 6.1 Thin sections of hailstones viewed with transmitted white light. The lower photographs were taken with the sections between crossed polaroids.

Magnification : 2.2 x



(b)



(d)

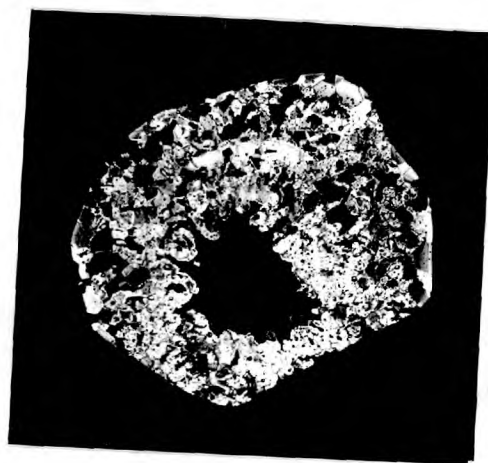
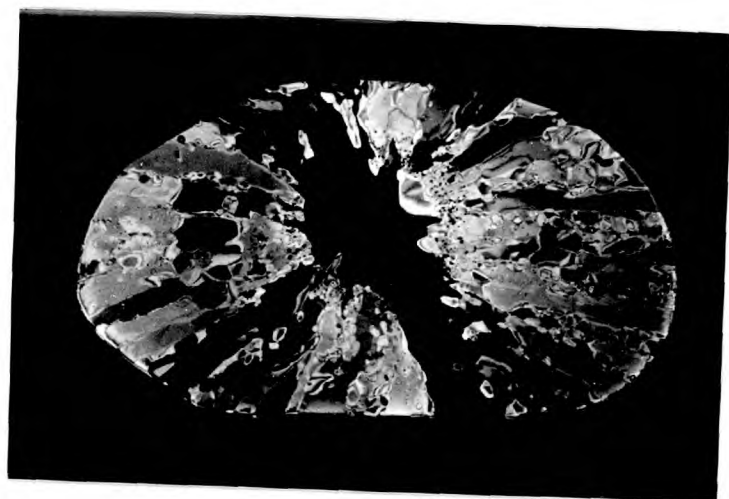


Plate 6.1 (contd.)

Magnification : 2.2 x

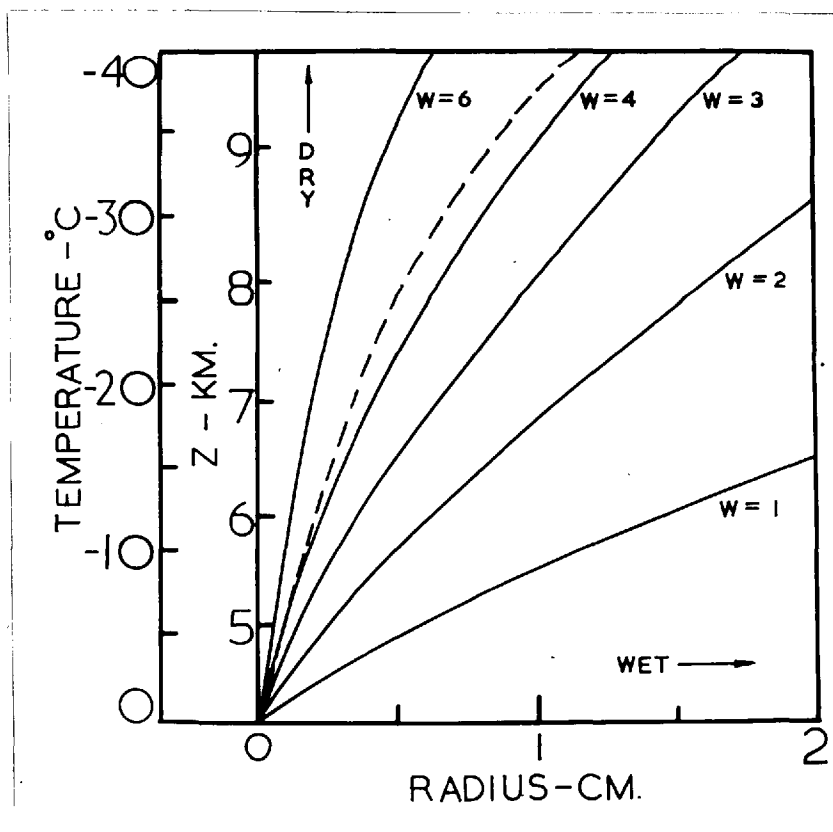


Fig. 6.4 The condition for the entry of hailstones into the wet growth regime in the Wokingham storm. The dashed curve represents the adiabatic liquid water concentration.

hailstones under discussion in a cloud of 30μ diameter droplets would have ranged from 0.8 to 0.9 depending on their size and fallspeed. The actual concentrations in the cloud were therefore greater than the effective critical concentrations by 10 to 20%.)

Using this diagram and the dimensions of the rings where the opacity changes, Table 6.2 has been obtained. This gives the approximate temperatures and hence heights in the cloud at which the changes are likely to occur, depending on whether the effective liquid water concentration was 2, 3 or 4 g m^{-3} . The increments in time for the growth of each layer have been computed from the dry growth equation and from these a mean rate of rise of the stone relative to the ground has been determined. This is the difference between the updraught and the fallspeed of the stone. The latter has also been calculated and from this the updraughts appropriate to the various levels deduced. These are compared with the updraughts in the model of the dynamical structure of the storm given by Browning and Ludlam (Fig. 5.4).

The agreement between the two sets of values for the updraughts is very reasonable. It is particularly interesting that the structure of the stones may be accounted for by any one of the three liquid water concentrations chosen and still be consistent with the dynamical model. However, it is unlikely that an effective liquid water concentration of as much as 4 g m^{-3} occurred at levels where the temperature was as low as -35°C because there it can be anticipated that about half of the condensed water was frozen. Equally it is unlikely that the effective liquid water concentration was lower than 2 g m^{-3} because the temperatures in the early stages of growth would then have been above -5°C , and there would be some difficulty in explaining the nucleation of the initiating droplet. At a radius of 1.3 cm the stone acquired its final clear coat and in this layer the crystals are, on the average, 0.5 mm across. The concentration in the air of ice particles which would have been

TABLE 6.2

The growth of hailstone (a) in the Wokingham storm. T is the temperature at which the stone is just dry in effective critical liquid water concentrations of 2, 3 and 4 g m^{-3} . Z is the height corresponding to T and Δt is the increment of time taken for the stone to traverse the various height intervals. The mean rate of rise of the stone is $\overline{U - V}$ from which U may be obtained if V is known. The values of U deduced in this way are comparable with those in the dynamical model.

R (cm)	T ($^{\circ}\text{C}$)			Z (km)			Δt (sec)		
	W=2	W=3	W=4						
0.24	- 5.8	- 9.1	-13.0	4.88	5.38	6.00	175	113	82
0.37	- 8.3	-12.8	-18.0	5.29	5.97	6.72	102	66	46
0.46	- 9.9	-15.1	-20.8	5.47	6.31	7.13	165	106	75
0.63	-12.6	-19.0	-26.0	5.95	6.89	7.86	520	300	
1.32	-22.5	-33.1	<-40	7.40	8.79				
R (cm)	$\overline{U - V}$ (m sec^{-1})			U (m sec^{-1})			U (model)		
0.24				14	17	21	17	20	23
	2.3	5.2	8.8						
0.37				17	21	25	19	23	28
	1.8	5.2	8.9						
0.46				19	22	28	20	25	29
	2.9	5.5	9.7						
0.63				23	28	32	23	28	34
	2.8	7.0							
1.32				30	41		32	39	

necessary to account for this is approximately 20 L^{-1} . This value is consistent with growth at 2 to 3 g m^{-3} since the layer would have been formed at a temperature between -22 and -33°C , in which temperature range a concentration of ice particles about this value may be anticipated.

At this stage it is thought that the stone crossed the central streamline of the updraught and began to fall. The size of the crystals may now be expected to become larger since the stone is growing wet at levels where the concentration of ice particles is small. However, the stone rapidly accelerates because the updraughts on this side of the central streamline are diminishing so that the size of the stone is not greatly increased. Further, the outer millimetre or two is melted away during the fall of the stone from the 0°C level to the ground. Another spherical stone which fell at the same place in the hail swath has a similar structure which leads to the same general conclusions.

The growth of hailstone (b) is obviously markedly different. There is only one layer which is opaque and this would have occurred at a temperature of -9°C (-15 , -20°C) in a liquid water concentration of 2 g m^{-3} (3 , 4 g m^{-3}). Beyond this layer it appears that the stone has grown in the wet condition and has therefore acquired an ellipsoidal shape (section 6.3.1). There is comparatively little change in the size of the crystals which means that growth must have occurred at levels below that corresponding to -15 to -20°C ; again an effective liquid water concentration of 2 to 3 g m^{-3} is implied if the opaque layer is a reversion to dry growth.

The fallspeed of this stone at the -20°C level, based on its measured mass and dimensions and a drag coefficient of 0.7 (see the first hailstone listed in Table 4.3) would have been 25 m sec^{-1} . This value is comparable with the maximum updraught at this height in the dynamical model (28 m sec^{-1}) so that the stone could have been grown to this size without entering regions of the cloud where the temperature was below -20°C . The effective critical liquid water

concentration required to keep the stone wet at radii greater than that of the opaque ring would have been 3 g m^{-3} .

Stones (c) and (d) show other typical crystalline structures. In (c) there is again a sharp transition in crystal size. The core of this stone is composed of crystals a few millimetres in length which at the centre tend to be orientated in a random manner while the core of (d) is a single crystal over 5 mm across. Because of the radial elongation of many of the larger bubbles, and of the shells of clear ice which are apparent, it is probable that this stone also grew near the wet condition. The crystalline structure of the outer layers is therefore thought to be due to the accretion of ice particles and this implies growth between about the -20 and -25°C levels.

It has been shown that the cores of over 90% of the stones were of transparent ice and that their size ranged from 1 mm to 1 cm diameter (Table 6.1). There are two possible ways in which these could have arisen: (1) by the growth of soft hail which later became modified by wetting near the 0°C level (2) by initial growth on frozen water drops of about 1 to 2 mm diameter at temperatures above -10°C . It is not possible for soft hail pellets to be grown at low levels from small ice particles (of the order of tens of microns) in the strong updraughts because they are rapidly carried aloft. Browning and Ludlam (private communication) have shown that in the Wokingham storm particles grown at high levels could not have re-entered the updraught at lower levels unless their fallspeeds were greater than 10 m sec^{-1} . It is therefore unlikely that the centres of the stones in this storm were composed of soft hail. It is more probable that the stones were grown on water drops of 1 to 2 mm diameter which froze at levels where the temperature was -5 to -10°C .

In the centres of stones (a) and (b) are groups of air bubbles, the size of individuals of which have been determined by enlarging the photographs. The solubility of air in water at 0°C is 3%

by volume (section 2.6), and since this does not change with pressure (Henry's law), it may be inferred that these bubbles could have arisen by the freezing of water droplets less than 2mm in diameter. The bubbles are in fact confined to regions 1.5 mm across. While this does not prove conclusively that these air bubbles did arise in this way, their presence does not conflict with the view that the cores of the stones were in fact frozen water drops of this size. A stepwise computation of the growth of hailstone (a) shows that it had a size of about 1 mm at the 0°C level.

Droplets of 1 to 2 mm diameter are produced near the 0°C level by the wetting and melting of hailstones already falling to the ground. In the ice accretion experiments it was noticed that when the cylinders became excessively wet and water was shed in their wake, droplets of 1 to 2 mm diameter were formed (section 3.3.3). If it is assumed that all the water swept up by a hailstone of radius 0.5 cm in travelling a distance of 0.5 km near the 0°C level is converted to droplets of diameter 1 (2) mm, the number formed is 225 (28) if the liquid water concentration is 3 g m^{-3} . Small hailstones formed early in the storm can therefore produce more than sufficient "nuclei" for the growth of large stones in the intense phase.

It is still necessary, however, for these droplets to be frozen if they are to grow into hailstones. The concentration of natural freezing nuclei effective at temperatures of -5 to -10°C is of the order of 1 m^{-3} (Findeisen and Schulz, 1944) so that a sufficient number would be frozen to account for the observed concentrations of large stones (Fig. 5.7). It is also possible that if spongy growth does occur at all temperatures below -3 to -4°C , then, in this temperature region, the water drops shed in the wake of spongy stones may contain ice and need no external nucleation.

6.4 Summary and conclusions.

Calculations based on the experimentally determined values of the density of ice formed by accretion show that the density of soft hail pellets which reach the ground from cold-weather clouds (bases at about -10°C , 900 mb) are likely to be less than 0.2 to 0.3 g cm^{-3} , as observed. The densities of soft hail pellets which form in warm-weather clouds (base temperatures of 10 to 20°C) are likely to be higher : 0.4 to 0.6 g cm^{-3} for pellets of a few mm diameter. However, because the 0°C level is then rather high (about 4 km) these particles are considerably modified before reaching the ground; any hail pellet of radius less than 0.5 cm is completely melted in a fall from this height.

The calculations show also that the mean density of large hailstones is inevitably high. Once $-r v_0/T_s$ has reached a value of about 20, the density of the accreted ice is greater than 0.8 g cm^{-3} . At temperatures above -20°C in a cloud containing a few g m^{-3} of condensed water, any hailstone greater than 1 to 2 cm in diameter fulfils this condition. If the early growth of the stone occurs at high temperatures the density of the stone is uniformly high throughout and only small density changes are associated with the transitions from transparent to opaque layers.

The growth of hailstones in intense thunderstorms is complicated not only by the factors which affect the nature of the ice accreted but also by the dynamical structure of the storm. It has been shown that the internal structure of the hailstones reflects the conditions encountered inside the cloud at successive stages in their growth, and can therefore provide useful criteria for establishing more detailed dynamical models of hailstorm clouds.

REFERENCES.

- | | | |
|---|------|--|
| Atlas, D. and Ludlam, F.H. | 1960 | Contract AF61 (052) - 254,
Dept. Met., Imperial College,
Tech. Note, No.4. |
| Atlas, D., Harper, W.G.,
Ludlam, F.H. and Macklin,
W.C. | 1960 | Ibid., Tech. Note, No. 2. |
| Arenberg, D.L. | 1941 | Bull. Amer. Met. Soc., <u>22</u> , 113. |
| | 1943 | Blue Hill Observatory, Harvard
Univ., Rep. No. 7. |
| Battan, L.J. | 1959 | Radar Meteorology, Univ. of
Chicago Press. |
| Bernal, J.D. | 1960 | Nature, <u>185</u> , 68. |
| Bigg, E.K. | 1953 | Proc. Phys. Soc. B, <u>66</u> , 688. |
| | 1957 | Tellus, <u>9</u> , 394. |
| Bilham, E.G. and Relf,
E.F. | 1937 | Quart. J. Roy. Met. Soc., <u>63</u> ,
149. |
| Browning, K.A. and Ludlam,
F.H. | 1960 | Contract No. AF61 (052) - 254,
Dept. Met., Imperial College,
Tech. Note, No. 5. |
| Brun, E.A. | 1957 | N.A.T.O. Advisory Group for
Aeronautical Research and
Development, Agardograph 16. |
| Brunt, D. | 1941 | Physical and Dynamical Meteor-
ology, Cambridge University
Press. |
| Clark, V. | 1948 | Harvard-Mount Washington Icing
Research Report, 1946-47, U.S.
A.F. Tech. Rep. No. 5676, 415. |
| Coste, J.H. | 1940 | Quart. J. Roy. Met. Soc., <u>66</u> ,
365. |
| Donaldson, R.J. | 1958 | Amer. Met. Soc., Proc. 7th
Weather Radar Conf. B, 8. |
| | 1959 | J. Met., <u>16</u> , 281. |
| | 1960 | Amer. Met. Soc., Conf. on
Severe Local Storms. |
| Donaldson, R.J., Chmela,
A.C. and Shackford, C.R. | 1959 | Proc. Second Conf. on Cloud
Physics, Woods Hole Oceanograph-
ic Institute. |
| Dorsey, N.E. | 1940 | Properties of Ordinary Water
Substance, Reinhold Publishing Co. |

REFERENCES.

- | | | |
|---|------|--|
| Douglas, R.H. and
Hitschfeld, W. | 1959 | Quart. J. Roy. Met. Soc., <u>85</u> ,
105. |
| Dufour, L. | 1861 | Archiv. des Sci. Phys. et Nat.,
<u>10</u> , 346. |
| Findeisen, W. and Schulz, G. | 1944 | Forsch. u. Erfahr. Reichsamt.
f. Wetterdienst, <u>A</u> , 27. |
| Fishenden, M. and
Saunders, O. A. | 1950 | An Introduction to Heat Trans-
fer, Oxford University Press. |
| Floegel, J.H.L. | 1872 | Poggendorfs Ann. <u>146</u> , 482. |
| Fraser, D., Rush, C.K. and
Baxter, D. | 1952 | N.A.E. Canada, Lab. Rep. LR-32. |
| Frössling, N. | 1938 | Beitr. Geophys., Leipzig, <u>52</u> ,
170. |
| Gaviola, E. and Fuertes,
F.A. | 1947 | J. Met., <u>4</u> , 116. |
| Goldstein, S. (ed.) | 1938 | Modern Developments in Fluid
Dynamics, Clarendon Press,
Vol. 1. |
| Gunn, R. and Kinzer, G.D. | 1949 | J. Met., <u>6</u> , 243. |
| Hallet, J. | 1960 | J. Glac., <u>3</u> , 698. |
| Hallet, J. and Mason, B.J. | 1958 | Proc. Roy. Soc. A, <u>247</u> , 440. |
| Hann, J. and Suring, R. | 1926 | Lehrbuch der Meteorologie,
Verlag Willibald Keller,
Leipzig. |
| Harper, W.G. | 1960 | International Conf. on the
Physics of Clouds, Verona, Italy. |
| Harvard-Mount Washington Icing Research Report, 1946-47, U.S.A.F. | | Tech. Rep. No. 5676., 1948. |
| Hoerner, S.F. | 1958 | Fluid-Dynamic Drag, published
by the author, Library of Congress
Catalog Card Number 57-13009. |
| International Cloud Atlas, World Meteorological Organisation, | | Geneva, 1956. |
| Jones, R.F. | 1960 | Quart. J. Roy. Met. Soc., <u>86</u> , 187. |
| Kramers, H. | 1946 | Physica, <u>12</u> , 61. |
| Landis, D.S. | 1907 | Quart. J. Roy. Met. Soc., <u>33</u> , 63. |
| Langham, E.J. and
Mason, B.J. | 1958 | Proc. Roy. Soc. A, <u>247</u> , 493. |

REFERENCES.

- | | | |
|------------------------------------|-------|---|
| Langmuir, I. | 1944 | General Electric Co.,
Schenectady, N.Y., Rep. on
Contract No. W-33-106-sc-65. |
| Langmuir, I. and
Blodgett, K.B. | 1946 | U.S.A.A.F. Tech. Rep. No.
5418. |
| List, R. | 1958a | Z. angew. Math. Phys., <u>9A</u> , 180. |
| | 1958b | Z. angew. Math. Phys., <u>9A</u> , 217. |
| | 1959a | Helvetica Physica Acta, <u>32</u> , 293. |
| | 1959b | Z. angew. Math. Phys., <u>10</u> , 143. |
| | 1960 | International Conf. on the
Physics of Clouds, Verona, Italy. |
| Ludlam, F.H. | 1950 | Quart. J. Roy. Met. Soc., <u>76</u> , 52. |
| | 1951 | Ibid., <u>77</u> , 663. |
| | 1952 | Ibid., <u>78</u> , 543. |
| | 1958 | Nubila, <u>1</u> , 13. |
| | 1959 | Nubila, <u>2</u> , 7. |
| Loughborough, D.L. | 1946 | B.F. Goodrich & Co.,
Research Report. |
| Magono, C. | 1954 | Sci. Rep. Yokohama Univ., Sec. 1,
No. 3. |
| Mason, B.J. | 1957 | The Physics of Clouds, Clarendon
Press. |
| Mason, B.J. and
Maybank, J. | 1960 | Quart. J. Roy. Met. Soc., <u>86</u> , 176. |
| Melcher, D. | 1951 | Z. angew. Math. Phys., <u>2</u> , 421. |
| Minervin, V.E. | 1957 | Tsent. Aerol. Obs., Moscow,
T. Vyp 17, 15. |
| Nakaya, U. and
Terada, T. | 1935 | J. Fac. Sci., Hokkaido Univ.,
Ser. 2, <u>1</u> , 191. |
| Pollard, A.E. | 1936 | Met. Mag., <u>71</u> , 15. |
| Powell, R.W. | 1940 | Trans. Inst. Chem. Eng., <u>18</u> , 36. |
| Prohaska, K. | 1900 | Met. Zeit. <u>17</u> , 327. |
| | 1905 | Ibid., <u>22</u> , 519 |
| | 1907 | Ibid., <u>24</u> , 193. |
| | 1914 | Ibid., <u>31</u> , 410. |

REFERENCES.

- | | | |
|-----------------------------------|------|--|
| Reiher, H. | 1925 | Forschungsarb. a. d. Geb. d. Ingenieurwes, No. 269. |
| Reynolds, O. | 1876 | Papers on Mechanical and Physical Subjects, Vol. 1, 214, Cambridge University Press, 1900. |
| | 1877 | Ibid, 223. |
| Rupe, J.H. | 1950 | Propulsion Lab., Calif. Inst. Tech., Prog. Rep. No. 4-80. |
| Russell, R. | 1893 | On Hail, Edward Stanford. |
| Schumann, T.E.W. | 1938 | Quart. J. Roy. Met. Soc., <u>64</u> , 3. |
| Souter, R.K. and Emerson, J.B. | 1952 | N.A.C.A., Washington, Tech. Note No. 2734 |
| Schaefer, V.J. | 1960 | Nubila, <u>3</u> , 18. |
| Steyn, K. | 1950 | Public Works of South Africa, <u>10</u> , No. 75. |
| Trabert, W. | 1899 | Met. Zeit., <u>16</u> , 433. |
| Vittori, O. and Di Caporiacco, G. | 1959 | Nubila, <u>2</u> , 52. |
| Wegener, A. | 1924 | Thermodynamik der Atmosphäre, Verlag von Johann Ambrosius Barth, Leipzig. |
| Weickmann, H.K. | 1953 | Thunderstorm Electricity, ed. H.R. Byers, Univ. of Chicago Press, 66. |

RADAR SCATTER BY LARGE HAIL

by

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Summary

Radars with wavelengths of 3.3 and 4.67 cm have been used to measure the back-scatter cross-sections of individual artificial hailstones and their variations as melting occurs. The measurements confirm the cross-sections computed theoretically. An ice sphere is a better scatterer than a liquid water sphere of equal size when its diameter exceeds 0.8 wavelengths; when the diameter lies between 1.2 and 2.3 wavelengths it is better by a factor of 10 or more. This behaviour of ice is supported by scatter measurements on a set of plexiglass (perspex) spheres with refractive index close to that of ice.

When melting begins, the 3.3 cm cross-sections drop sharply toward the all-liquid values; those at 4.67 cm decrease gradually, and sometimes not at all. The rate of decrease of the cross-sections during melting is also more variable at 4.67 cm than at 3.3 cm. Most of the 3.3 cm change appears to occur during the initial growth of the liquid coat to a maximum thickness of about 0.01 cm, while the major drop in 4.67 cm cross-sections occurs well beyond this stage of melting. The implications are that: (1) a water coat of 0.01 cm thickness has a major effect at 3.3 cm and a minor one at 4.67 cm; (2) liquid continues to collect in sub-surface cracks and cavities in the ice and causes a further decrease in the cross-sections after the maximum thickness of water coat has formed. The final cross-sections are generally within 2 db of the all-liquid value after 7 to 10 minutes of melting at 3.3 cm and after 15 to 20 minutes at 4.67 cm. Possible reasons for these results are discussed.

1. Introduction

Recent studies have shown that quantitative measurement of the radar back-scatter from a convective storm provides a useful indication of the presence of hail (Donaldson, 1958, 1959). However, the interpretation of such measurements has been restricted by the lack of adequate data on the back-scatter cross-sections of large hail. Until recently, the only available data were theoretical values calculated by Ryde (1946) from the rigorous Mie theory. These were restricted to ice-spheres of diameter D less than 1.25 times the wavelength, λ ; i.e. to hail smaller than 3.75 cm at a wavelength of 3 cm. Moreover, some authors (Atlas and Donaldson, 1954) had expressed intuitive doubt of their validity since ice with diameter to wavelength ratios D/λ in excess of 0.8 were indicated to scatter significantly better than liquid-water spheres of equal size, while the reverse is true for both smaller and much larger spheres. The back-scatter measurements reported here were therefore undertaken to check and to extend empirically Ryde's theoretical computations, and to determine how the cross-section of a large ice particle varies during the melting process.

After the experimental phase of this study was completed, the desirability of further theoretical computations became apparent. It was coincidental that Herman and Battan (1959) at the University of Arizona were already established to make these on an electronic computer. This work has now been completed and the authors have graciously made the results available to us in advance of their publication. We include a brief comparison with our measurements.

The apparently anomalous behaviour of the back-scatter cross-section of ice with respect to that of water is indicated by the crossing of their

scattering curves in Figure 7. Indeed, since the scattering by metal spheres is represented essentially by an upward displacement of the water curve in Figure 7, until it oscillates around a ratio of radar to geometric cross-sections of $\sigma / (\pi D^2/4) = -1$, it is evident also that moderately large ice spheres have larger back-scatter cross-sections than metal spheres of the same size. To make a comparison, we examine the situation in the regions defined by $D/\lambda \ll 1$, the so-called Rayleigh region, and $D/\lambda \gg 1$, the geometric optics region. Writing $\sigma / (\pi D^2/4) = f_s$, in the Rayleigh region we have

$$f_s = 4 \pi^4 |K|^2 D^4 / \lambda^4 \quad (1)$$

where $K = (m^2 - 1) / (m^2 + 2)$, and m is the complex refractive index. For metal spheres with $m = \infty$ and $|K|^2 = 1$, the cross-section given by (1) is enhanced by $9/4$ (Kerr, 1951, p.452). For very large D/λ it is well known that

$$f_s = (m - 1)^2 / (m + 1)^2 \quad (2)$$

This is simply the power reflection coefficient of a plane surface. Gunn and East (1954) give values of m and $|K|^2$ for water and ice at various wavelengths. For example at 3.21 cm : $m = 7.14 - 2.89 i$ and $|K|^2 = 0.93$ for water at 0°C; $m = 1.78$ and $|K|^2 = 0.197$ for ice. The ratios of the back-scatter from water and metal spheres to those from ice spheres of equal size are then as given in Table 1. The superiority of metal and water to ice in both regions is notable. Confirmation of the theoretical calculations which show the reverse for the intermediate size range is therefore desirable.

Table 1. Ratios of back-scatter cross-section of metal- and water- to those from equal sized ice-spheres for the Rayleigh region ($D/\lambda \ll 1$) and geometric optics ($D/\lambda \gg 1$) (at 0°C and for $\lambda = 3.21$ cm)

	<u>$D/\lambda \ll 1$</u>	<u>$D/\lambda \gg 1$</u>
metal/ice	11.4	12.7
water/ice	4.72	7.81

2. Experimental procedure

Two radars were used in the experiment, the 3.31 cm TPS-10 and the 4.67 cm MPS-4. The targets were raised by a 500 g balloon (tethered by a nylon fishing line) at a range of 1.4 to 1.5 miles, until their echoes could readily be distinguished from ground clutter; their elevations varied between 4° and 13° . Special care had been taken in selecting the balloon launching site to ensure that at these angles there were no detectable ground echoes at the target range.

The targets were placed in fine nylon hair nets and tied to the balloon line. The balloon itself gave a measurable echo, so the targets were suspended at least 150 ft below the balloon. Since the vertical beam-widths of the TPS-10 and the MPS-4 are 0.7° and 0.8° , or 97 and 111 ft respectively at a range of 1.5 miles, the balloon and target echoes could readily be distinguished, particularly as the former was subject to fading. On occasion as many as 3 targets could be suspended from the balloon simultaneously, with separations of about 150 ft.

A 12-inch diameter metal sphere, whose cross-section is equal to its geometric area, was used as a reference in all the measurements. Initially

the reference sphere was raised at the start and end of a series of measurements on test targets. However, it was found that during a protracted series of measurements the intensity of the echo from the standard target varied by as much as a factor of 2, partly because of changes of range due to wind fluctuations. In later measurements the reference target was therefore suspended from the balloon simultaneously with the test target and measurements of its echo power were made immediately before and after, and even during a series of measurements on the test target. These usually agreed to within ± 0.4 db, or even less if the wind was steady. If the test and standard targets were at equal ranges, the cross-section of the test target was simply related to that of the standard target in the ratio of their echo powers. Thus, in principle, the accuracy of the measurements depends mainly on the relative accuracy of the attenuator of the microwave signal generators with which the radars were calibrated. Unfortunately, however, there were other sources of error in the 4.67 cm measurements.

On the TPS-10 radar, the target echo was displayed on an A-scope and the receiver gain adjusted to bring the echo amplitude below saturation. An artificial signal from a calibrated signal generator was displayed immediately beside the true echo and the attenuator adjusted so as to match the two signals in amplitude. This matching could be reproduced to within about ± 0.2 db, the remaining spread (± 0.2 db) in the measurements being attributable to actual echo changes associated with small displacements of the target. Variations in echo power due to motion of the target away from the beam axis were kept very small by manually orientating the antenna to obtain maximum signal on the A-scope prior to each measurement. Since the mean of a series of measurements could be determined reliably

the overall measurement accuracy on the TPS-10 was indeed determined almost solely by the accuracy of the test set attenuator.

On the MPS-4, the echo intensity was determined by reducing the receiver gain control (I.F. bias) until the echo just broke the base line on the A-scope. This threshold gain setting or I.F. bias was always calibrated in terms of echo power before and after a test series, and frequently at intervals during the test, using a TS-620 microwave signal generator. In order to obtain better accuracy than that of the attenuator incorporated in the test set, a precision attenuator was inserted at the output of the signal generator. This attenuator was precisely calibrated by the manufacturer to an accuracy of ± 0.1 db over a range of 45 db.

The threshold gain calibration curve for 4 and 8 August, 1959 is reproduced in Figure 1. The standard error of estimate of the observed points from the smooth curve is ± 0.53 db. Since the technique for obtaining the threshold gain setting for the artificial calibration signal is identical to that for true echoes, this is also the accuracy with which we may expect to reproduce readings from a perfectly steady target echo.

There were, however, two other more significant sources of error in the MPS-4 measurements. The first is that due to the motion of the target. The signal could be maximized readily in azimuth by slight adjustment of the manual slewing control. However, elevation could be controlled only by a switch which directed the elevation motor to drive upwards or downwards. With the narrow vertical beam (0.8°) it was difficult to stop the antenna exactly where desired. The result was a much broader spread in echo intensities than was the case with the TPS-10. Figure 2 shows two typical series of measurements. The spread in the measured echo power is rather

large, but most of the low values must be regarded as 'off-axis' measurements. Since proper orientation of the beam will provide the maximum possible echo, the best estimate of the true echo power must be close to the maximum observed values. However, allowance must be made for possible over-estimation, due to random errors in adjustment and reading of the threshold gain setting, with the standard error of about ± 0.5 db indicated above. It is unlikely, therefore, that a measurement will exceed the true maximum by more than 1 db. Similarly, all values more than 1 db below the true maximum are probably due to orientation errors; some of these may cause the values to be as much as 6 db too low. It is evident from the measurements in Figure 2 that the extreme orientation errors are not usually successive. Since the observer watched the A-scope for maximum signal as he varied the elevation angle, the first few measurements after an elevation change tend to occur near the maximum. The signal may then decrease as the target drifts away from the beam axis. Since the change in signal strength was quite obvious, a new elevation adjustment was then made. The points in Figure 2 therefore tend to cluster within 2 db of the maximum measured value. We estimate the true echo-power level by drawing a line through this cluster in such a manner that out of a total of 10 - 20 measurements, 1 point falls approximately 1 db above and 2 or 3 points fall within 0.5 db above the line. While there is some judgement involved in drawing the line, the same estimate can be repeated by independent analysts to about ± 0.5 db.

The other source of difficulty in the MPS-4 measurements was a severe oscillation in the system response at short ranges, resulting from a faulty component which could not be replaced until the last working day (8 August), and which caused the receiver response to vary by as much as 7 db as

the range changed by only 150 yards. The peaks in the response curve were spaced about 300 yards apart in range. Since the echo pulses were approximately 250 yards wide, the strongest portion of the echo would appear at that range at which the receiver response was a maximum within the 250 yard pulse width. Thus it was not possible to use a series of measurements unless we were reasonably sure that the echoes from both the reference sphere and the unknown target occurred at identical ranges, and preferably on the same peak in receiver response. Roughly half the MPS-4 measurements were discarded on this basis. The oscillation in the receiver sensitivity is probably responsible for part of the scatter in the MPS-4 measurements. However, by using maximum signals for both the unknown and reference targets, the accuracy of the measurements which we have selected as valid is not seriously affected.

3. Results

3.1 Metal spheres

Perhaps the best test of the overall accuracy of our methods arises from the results obtained with a set of accurately machined aluminium spheres 1, 2, and 3 inches in diameter. The cross-sections of such spheres are well established. The data from the TPS-10 are listed in Table 2. The MPS-4 measurements on the metal spheres were unacceptable because when these heavy targets were flown with the reference sphere even a light wind drew the balloon so far from the tethering point that the awkward range fluctuations mentioned above were unavoidable. Individual measurements of the cross-section and their means are listed as an indication of their

Table 2. Measured and theoretical radar cross-sections of metal spheres (TPS-10, 3.31 cm)

Sphere		Cross-sections (cm ²)				Theory	10 Log σ_m / σ_t (db)
		Measured					
		Individual Data		Mean			
Diameter	Area						
D (inches)	r ² (cm ²)						
3	45.6	77.7	76.5	73.3	75.8	55.2	+ 1.37
2	20.5	45.8			45.8	26.5	+ 2.38
1	5.07	10.1	10.2	9.6	9.97	9.28	+ 0.30

reproducibility. The theoretical cross-sections were obtained from Kerr (1951, p.453, fig.6.1). The last column gives the decibel ratio of the average measured cross-section (σ_m) to that indicated by theory (σ_t).

There appears to be a definite tendency for the measured cross-sections to exceed those indicated by theory. In Figure 3, we have plotted the measured cross-sections against the theoretical values. Since all measurements were made with respect to the 12" sphere with $\sigma = 730 \text{ cm}^2$, this point is established on the curve with zero error. If there were no systematic errors in our measurements we should expect the values to fall along the solid curve. The reproducibility of the measurements indicated in Table 2 does suggest some sort of systematic error, although the single measurement on the 2" sphere leaves this point in doubt. The dashed curve in Figure 3 may, therefore, be used as an approximate calibration curve to indicate the true cross-section corresponding to any measured value. Since it is difficult to see how systematic errors of this sort can arise except

as a result of an error in the test-set attenuator, we have placed scales of true and measured attenuation on the top and right side of Figure 3. We shall use this calibration curve later to estimate possible errors in our subsequent results.

3.2 Plexiglass (perspex) spheres

Plexiglass (perspex) has an index of refraction $m = 1.60$. Because this is so close to that of ice, we used a set of machined plexiglass spheres as a control. Twenty such spheres were manufactured with diameters ranging from $5/8$ to 3 inches in $1/8$ inch steps. The results of measurements with these targets are shown in Figure 4.

Most of the points in Figure 4 represent individual measurements. In the case of the TPS-10 (3.31 cm) an "individual" measurement is the mean of a series of perhaps ten observations made in a period of about 15 minutes. The MPS-4 measurements were obtained in the manner already described. The 3.31 cm points for $D/\lambda = 1.53$ and 2.30, corresponding to the 2 and 3 inch diameter spheres, represent the means of 4 and 5 measurements respectively, made during the month of tests. In both cases the individual values fell within ± 0.6 db of the means, indicating good reproducibility. Where there were two measurements made at 4.67 cm, both are plotted. These fall within ± 1.2 db of the mean, except for the two smallest values of D/λ , where the small signal strengths made the measurements rather more inaccurate.

The ringed dots in Figure 4 represent theoretical values of f_s computed from Lowan's tables (1949) for $m = 1.55$. These generally fall slightly below the measured values for plexiglass, as might be expected in this region of D/λ . For comparison we have also superimposed the ice ($m = 1.78$) scattering curve taken from Figure 5. The portion of the ice

curve for $D/\lambda < 1.25$ is due to Ryde (1946). The remainder of the curve is based on our measurements. By reference to both the ice scattering curve and the theoretical values for $m = 1.55$, we have been able to fit quite a reasonable curve to the plexiglass measurements. The dotted portion of the curve in the vicinity of $D/\lambda = 2$ is in doubt because of the lack of data there. However, it seems reasonable to expect another peak of about the indicated magnitude and position in this region.

The back-scatter curve has been drawn to fit the uncorrected TPS-10 data quite closely. However, we have seen in Figure 3 that the measured cross-sections of the metal spheres tended to be slightly too high. We have therefore used the "calibration" curve in Figure 3 to reduce the measured cross-sections of the plexiglass spheres accordingly. The portion of the curve which has been corrected in this manner is shown by the dot-dash curve in Figure 4. This modification of the curve brings it in closer accord with the 4.67 cm measurements, and is in slightly better agreement with the theoretical values for $m = 1.55$. Except for the minimum at $D/\lambda = 1.4$ which seems rather too low, possibly as a result of over-correction, we must assume that the corrected curve is closely representative of the true values of f_s for plexiglass.

Note that the plexiglass scatter curve crosses the ice curve in the vicinity of $D/\lambda = 2$; beyond this point it appears that plexiglass, with smaller refractive index, is a better scatterer than ice. We shall later see a similar cross-over for particles of $m = 2.0$ and 1.78 (Figure 5). Also of interest is the phase shift of the peaks and minima on the plexiglass curve from those for ice, the peaks of one corresponding closely to the minima of the other.

It is striking that the scattering curve reaches a peak value $f_s = 13.9$ at $D/\lambda = 2.22$. This implies that a plexiglass sphere of diameter 7.35 cm at $\lambda = 3.31$ cm has a cross-section of 590 cm², equivalent to that of a metal sphere of diameter 27.4 cm.

3.3 Ice spheres, dry

The ice spheres were manufactured by freezing water in hollow rubber balls of several cm internal diameter, and then cutting the ball away from the ice. Intermediate sizes were obtained by melting larger spheres in a warm water bath. The shapes were not perfectly spherical, but in no case did the diameters of a single stone vary initially by as much as 5%. The spheres were stored in a deep-freeze box and transferred on the day of use to a portable cold box containing dry ice. Since the ice spheres probably attained a temperature of less than -30°C before they were brought into the open air, a thin coating of frost formed on them, which persisted without melting for periods of up to about 15 minutes, so that scattering measurements could readily be made while they were completely dry. Selected measurements are shown in Figure 6. When melting started, the signals usually dropped noticeably. This effect will be discussed in the next section.

The complete results for ice spheres with a dry surface are presented in Figure 5. The TPS-10 measurements have been corrected according to the calibration curve in Figure 3. For comparison, we have plotted the theoretical scattering curve for ice computed by Ryde (1946) for values of D/λ up to 1.25, and the theoretical values furnished by Herman and Battan (1959). A few theoretical values for $m = 2.0$, taken from Lowan's tables, are also shown. The measurements are generally in remarkably good accord

with the theoretical values. Before receiving the data of Herman and Battan we had sketched in a reasonable (dashed) curve which was based solely on the measurements (the portion drawn as a dotted line was in doubt because of absence of measurements). This empirical scatter curve is in excellent agreement with the calculations of Herman and Battan, excepting only those at $D/\lambda = 2.04$ and 2.36 , and we have not altered it. For $D/\lambda < 1.0$, the measured values are generally too high, approaching the theoretical ones for $m = 2.0$. In view of the possible experimental errors, especially for the small values of σ associated with low echo intensities not far above the receiver noise level, there is no evidence to indicate any significant discrepancy between theory and measurement. Rather, we consider the measurements as a good confirmation of the theoretical calculations. Where they differ, we must accept the theoretical values.

It is to be noted that the theoretical scattering cross-sections for $m = 2.0$ exceed those for ice for values of D/λ up to about 1.4 , where they suddenly fall below the ice cross-sections. This behaviour is to be expected from existing knowledge of the total extinction cross-sections for non-absorbing spheres. Since, in this case, the extinction is due entirely to scattering, we may expect the total scattering cross-section and the back-scatter to vary in roughly the same manner with D/λ . However, this is only qualitatively true since the distribution of scattered energy with scattering angle is also a function of D/λ , i.e. the fraction of the energy scattered back per unit solid angle is not constant. Nevertheless, it is interesting that the extinction curves presented by van de Hulst (1957, p.151) show similar crossings, although at smaller values of D/λ . Indeed, the general behaviour of the back-scatter curves for

$m = 2.0, 1.78, 1.6,$ and 1.55 which are shown in Figures 4 and 5, is qualitatively similar to that of the total extinction curves for a similar range of refractive indices.

3.4 Melting ice spheres

Selected back-scattering measurements made on the ice spheres are shown in Figure 6. It is evident that at both wave-lengths at first the signals are fairly steady, but subsequently decrease markedly. This decrease can only be due to the melting process. The 3.3 cm signals drop more sharply than those at 4.67 cm, and tend to approach an asymptotic value, whereas the 4.67 cm signals usually continue to decrease at a fairly rapid rate even after the 3.3 cm signals have become steady.

All the scattering measurements made during the melting stages are plotted in Figure 7 in the form of f_s against D/λ . In order to compute the latter values, it was necessary to determine the size of the ice as a function of time. Over a small time interval it was assumed that the melting obeyed an approximate relation

$$a_0^2 - a^2 = kt, \quad (3)$$

that the melting commenced at the moment when the 3.3 cm signals decreased, and that at this moment the radius a_0 was that measured before the target was raised. The constant k is then determined by measuring the final diameter of the ice at a known time t after melting begins. Since the spheres did not melt quite uniformly, usually an equivalent radius was determined from a weighing made after the target was lowered to the ground. On some occasions, the final size was determined by measuring three diameters with calipers.

Figure 7 also contains the scattering curves for completely frozen ice spheres (taken from Figure 5) and for liquid water spheres (Aden, 1951).^{*} The most striking feature of the diagram is that melting causes the scattering cross-section to drop sharply from a point near the all-ice curve to a final value lying close to the all-water curve. Since we shall see that Equation (3) slightly overestimates the melting rate initially, the actual sizes must be slightly greater in the first 4 or 5 minutes. We have therefore overestimated $(\sigma/\pi a^2)$ and underestimated (D/λ) in the early stage of the melting. The upper parts of the drawn curves should therefore have a greater slope bringing them even closer to the vertical, but the final points are accurate.

In the cases of two of the three smallest ice spheres in Figure 7, we see that the 3.3 cm cross-sections first fall sharply toward the all-water curve and then rise again, tending towards the peak in the all-water curve at $D/\lambda = 0.7$. Since at 3.3 cm the smallest particle has initially a cross-section 2.6 db above the all-ice curve, it is reasonable that the entire curve for this particle should be lowered by this amount. Such a modification would bring it almost tangent to the water curve.

The behaviour of small particles at 4.67 cm is somewhat erratic and we have drawn only one curve, through the measurements for a single particle, in the region where D/λ lies between 0.5 and 0.8. This is the curve marked A and corresponds to the 3.3 cm curve marked A'.

* Aden's curve for water spheres was computed for a wavelength of 16.23 cm and a temperature of 20°C. However, it is sufficiently accurate for all wavelengths down to 3 cm and temperatures of 0°C for diameters $D \geq 0.3 \lambda$. The ratio of Aden's cross-sections to those at 3.2 cm and 0°C is of the order of the ratio of the asymptotic values given by Equation (2), or $0.64/0.614 = 1.04$.

The actual measurements for this particle are shown in Figure 8. Wetting seems to have begun just before the first measurement was made, since the 3.3 cm cross-sections are falling from the beginning, but the initial value falls on the all-ice curve (A' in Figure 7). In contrast, the 4.67 cm cross-sections rise to a peak and then fall back toward their initial values. In Figure 7, we see that the initial and final values for this particle are some 3 db below the all-ice curve while the peak is on the all-water curve. It may be inferred that the development of a water coat in this region of D/λ may increase the scattering cross-sections of ice-spheres, as might have been expected from the relative positions of the all-ice and all-water curves. Since at 3.3 cm the same sphere is associated with a higher value of D/λ , corresponding to a different position in Figure 7, it is understandable that the scattering cross-section at 3.3 cm can fall while that at 4.67 cm rises. At both wavelengths the initial changes as melting commences are toward the all-water values.

Other measurements at 4.67 cm in this region of D/λ (not shown) showed no significant changes as melting progressed, although at 3.3 cm the cross-sections always fell initially. This is not unreasonable since the scatter in the 4.67 measurements is usually greater than the maximum difference of 3 db between the all-ice and all-water curves found at $D/\lambda = 0.72$. It is odd, however, that the three 4.67 cm observations made at values of D/λ between 0.8 and 0.9 (see Figure 5, all-ice curve) showed no definite tendency to fall as melting began, in contrast to the 3.3 cm measurements. This suggests that the thickness of the water coat which is required to affect the cross-section is greater at 4.67 cm than at 3.3 cm.

Let us turn our attention now to the effects of melting in the region of $D/\lambda > 1$. We note first that there are five spheres in Figure 7 whose final cross-sections fall below the all-water curve. Two of these (E' and F') lie far enough below for the difference to be considered significant. However, it is difficult to see how this can come about. We shall see that the maximum thickness of water coat measured in the laboratory was 10^{-2} cm, corresponding to a change in D/λ of only 0.006 at 3.3 cm. It can be supposed that after such a thickness has been attained the formation and shedding of drips prevents it from being exceeded, so that subsequently the cross-section should correspond to an ice sphere of decreasing size with a more or less constant thickness of water shell. It is surprising that the observed cross-sections continue to decrease so sharply from the all-ice values after the establishment of the water shell, and even more so that they sometimes fall below the all-water curve. This might be explained if a certain thickness of water coat acted as a resonant shell or interference filter serving to trap the radiation which entered it. Alternatively, such extremely low cross-sections may be related to cracks and liquid inclusions in the ice, a phenomenon which we shall discuss later, or to deviations from sphericity. In this regard, it was noted that one diameter of particle E' was 14% smaller than the other two when its dimensions were measured on its return to the ground some 10 minutes after the scattering observations were completed.

In order to assess the effect of progressive melting on the scatter cross-section and better to compare the 3.3 and 4.67 cm observations, the data for the four ice spheres in Figure 6 have been re-plotted in the form shown in Figure 9. The ordinate σ/σ_0 represents the fraction of the initial or dry-state cross-section, while the abscissa Δa , represents

the change in the radius of the ice sphere according to the simple melting law of Equation (3). It was thought that $0.9 \Delta a$ might represent the thickness of the water coat forming on the ice in the very early stage of melting, and that we might therefore be able to determine the water thickness required to substantially reduce the cross-section. Unfortunately this is not so.

We have measured the maximum thickness of liquid coat which is present on a melting sphere in the laboratory by pressing a strip of dyed filter paper along the perimeter of the ice and then rapidly removing it. The mass accumulated was determined by weighing before and after contact, while the collection area was indicated by the dyed area, correction being made for spreading after contact. We have thus found the thickness of the water coat to be only 10^{-2} cm within $\pm 5\%$. According to Equation (3), the time for the formation of such a coat would range from 20 to 60 seconds with the observed values of the coefficient k , which varied from 0.17 to 0.07 cm^2/min respectively. However, we find that Equation (3) overestimates the initial melting rate.

During the field experiments, each ice sphere before its removal from the dry ice chamber attained a uniform temperature between -30° and -60°C . By calculation and by experience at the ground we find that in the 10 or 15 minutes which elapse between its initial weighing at the ground and the beginning of melting aloft (as deduced from the initial drop in radar cross-section), sufficient frost forms upon the sphere to yield a melt-water shell of thickness 1 or 2×10^{-2} cm. It seems likely that the radar signal begins to decrease as these outermost frost surfaces melt.

From experiments made in a wind tunnel, in which ice spheres have been melted under similar conditions, we have found that the temperature even at their centres reaches within several degrees of 0°C before the first drip of melt water forms, irrespective of their initial temperature in the range -30° to -60°C . For this reason the simple law of Equation (3) is reasonably accurate except during the initial melting, when heat is still flowing at a considerable rate through the surface and into the interior. In fact it was found that Equation (3) is closely followed except during the melting away of an outermost shell of thickness about 3×10^{-2} cm during the first four or five minutes. Accordingly, although the radius of the sphere as deduced from (3) is sufficiently accurate for changes of more than about 3×10^{-2} cm, smaller changes are overestimated, and no accurate description of the true thickness of the water shell in this early period is possible.

All we can say with confidence is that the water coat does not exceed 10^{-2} cm in thickness and that the period for the development of such a shell ranges from 1 to 3 minutes. In Figure 6 we see that a substantial drop in the 3.3 cm cross-section occurs within such periods. In all cases, the signals drop by more than 6 db in 3 minutes, but they continue to decrease after the steady state water coat has been attained. At 4.67 cm, the cross-sections decrease more slowly, falling by only 1 or 2 db in 3 minutes in three of the four cases. These are the maximum effects produced by the 10^{-2} cm water shell.

In Figure 9 there is some consistency among the 3.3 cm measurements if we neglect the peculiar results for sphere E. This is the one in Figure 7 for which the cross-section drops sharply below the all-water curve. We have therefore placed a reasonable envelope around the measurements with

the above exception. If we take an indicated $\Delta a = 3 \times 10^{-2}$ cm as a reasonable approximation to the true maximum water coat of 10^{-2} cm, we see that a considerable part of the drop in 3.3 cm cross-section (4.4 to 7.5 db) is due to the formation of the initial water shell, as suggested earlier. When the cross-section has fallen to reach a value on the all-water curve, so that the sphere is then behaving for all practical purposes as an all-water sphere, it can be expected to follow the all-water curve in Figure 7 as the melting proceeds. We have seen this happen with two of the smaller particles in Figure 7; curve D' in Figure 7 exhibits a similar tendency. Neglecting the fluctuations in the all-water curve, σ is roughly proportional to (πa^2) . Thus when an ice sphere reaches a radius of $a = 3.3$ cm, which is roughly the average size of the particles in Figure 9 when $\Delta a = 0.3$ cm, then a further decrease of 0.1 cm in size should correspond to a decrease of 0.2 db in σ . The solid line in Figure 9 indicates this slope. We see that the 3.3 cm cross-section curves do in fact finally tend to have this slope. However, the slope of the σ/σ_0 curves is in general much too great to be explained simply in this way.

The 4.67 cm measurements shown in Figure 9 are much less consistent than those made at 3.3 cm, except in so far as the total change in σ in all four cases is very nearly 10 db. Nowhere in the data, except possibly in curve D, do we see an inflection which may be regarded as the transition between the effect of a thickening shell of water and the effect of decreasing particle size. It is inconceivable that the water coat might in fact be growing thicker beyond an indicated Δa of 0.03 cm. The curves thus emphasise that as the melting proceeds the cross-sections continue to decrease long after the coat of liquid water has been

established with its maximum thickness. We are forced to conclude that the ice particles must somehow be taking on more water well beyond the point at which their water coats have attained a maximum thickness of 10^{-2} cm. We suggest that this is because liquid water progressively accumulates in sub-surface cracks or cavities, either by percolation from the surface or by internal melting. Minute cracks are almost certainly produced in our artificial hailstones during their production because of strains set up by the expansion of the interior, which was the last part to freeze. Their presence may also be inferred from the "cracking" noises which were sometimes heard when the artificial hail was placed in a warm water bath for the purpose of reducing their sizes. On one occasion a water-filled cavity was found near the top of a large ice sphere when it was brought to the ground.

This is the only reasonable explanation which we can offer for the rather odd and inconsistent behaviour of the 4.67 cm scattering cross-sections and their differences from those at 3.3 cm. First of all, since the absorption coefficient of water is greater at 3.3 cm than at 4.67 cm, we expect radiation of the former wavelength to be affected the more strongly by a thin water coat. This is apparent from the relative changes in σ at the two wavelengths as a function of time in Figure 6 and as a function of change of size due to melting in Figure 9. At 3.3 cm the building of the water shell to a maximum thickness of about 10^{-2} cm appears to account for the major reduction in cross-section, while this thickness of coat has a rather small effect at 4.67 cm. At 3.3 cm therefore, what happens below this water shell is of substantially less importance than at 4.67 cm. At the latter wavelength the rate of change of σ appears to

depend largely on how fast water is accumulating in sub-surface cracks or cavities below the outer shell of water. Thus the variations in the number and size of cracks or cavities from one particle to the next could readily account for the wide variations in the 4.67 cm σ/σ_0 curves in Figure 9, which occur despite the similarity in size of the original ice spheres. Indeed, this may also explain why some of the smaller ice particles showed no significant change in scattering cross-section as melting progressed: these may not have contained cracks.

This is not to say that the effect of cracks is negligible at 3.3 cm. In fact, it is the only reasonable means of explaining the continued fall in σ Figure 9, at a rate substantially in excess of the rate of decrease in particle sectional area, after the water coat has reached its maximum thickness. This is evidenced also by the 3.3 cm melting curves marked F' and G' in Figure 7. When the associated spheres diminish to a size such that $D/\lambda = 1.2$, and presumably have acquired a water coat of maximum thickness, G' shows a cross-section 5.6 db greater than F'. This could occur only if sphere F' did in fact contain the greater amount of liquid water. Perhaps extensive liquid-filled cracks or cavities in this sphere are responsible for its cross-section falling so far below the all-water curve.

4. Discussion

Experience has shown that it is hazardous to attempt to apply a simple physical picture to the scattering mechanism. Certainly the rigorous Mie scatter theory is so complex as to defy a simple physical representation. On the other hand, while the theory of geometric optics lends itself to

reasonable physical models, it does not provide exact solutions to the scattering problem in the region of $D \approx \lambda$. In the case of metal spheres, the solutions given by geometric optics and the Mie theory converge with increasing D/λ so that the former is sufficiently accurate for $D/\lambda \geq 6$. Thus, it is not unreasonable to use geometric optics to provide some physical insight into the cause of the large back-scatter cross-section for ice when $D/\lambda > 1$.

Consider first a large metal sphere. Its cross-section is determined essentially by the specular reflection from the first Fresnel zone; i.e. the surface area intercepted by cutting a plane perpendicular to the sphere diameter (along the direction of propagation) a distance $\lambda/4$ in from the surface. The reflection is entirely a surface phenomenon; it is of no consequence whether the sphere is hollow or solid. This is due to its large conductivity, which manifests itself as an extremely large absorption coefficient; i.e. imaginary part of the refractive index. Furthermore, for a large sphere we may remove the half of the sphere obscured from the radar without affecting its cross-section.

Although the absorption coefficient of water in the microwave region is much smaller than that of metal, we find that the back-scatter curve for water spheres is essentially a downward displacement of that for metal by an amount given by Equation (2). It appears therefore that the absorption coefficient of water is sufficiently large to make the reflection essentially a surface phenomenon. Our radar measurements of melting ice spheres confirm this, although the water surface must have some finite thickness. The shorter the wavelength, the higher the absorption coefficient and the thinner the water shell required. Equation (2) indicates that at 3.2 cm 61%

of the incident power is reflected from the front surface (where that surface is perpendicular to the direction of propagation). At other angles the reflected energy is even greater. The 39% of the incident power which enters the surface is rapidly absorbed. At 3.2 cm, the power decays to 10% in a water depth of only 2 mm. Thus only 3.9% of the power incident on the surface penetrates to a depth greater than 2 mm. We must infer from this and especially from our 3.3 cm measurements that it matters little what is contained within the water shell, at least for the larger spheres with which we are concerned. It is apparent also as with metal, that we could remove the back half of the sphere without reducing its radar cross-section. It is the front surface which largely determines the reflection.

On the other hand, ice has a negligible absorption coefficient. The reflection from the front surface of a large ice sphere is $1/8$ of that from an equal water sphere and $1/13$ of that from an equal metal sphere. Thus a major part of the incident energy enters the sphere. None of this energy is absorbed in penetrating the ice. Instead it is partially focused as in a lens and reaches the back surface with an increased power density. The energy partially reflected from the back surface is concentrated further due to its concavity toward the radar. Thus in the case of ice we conclude that the reflection from the back surface is significantly greater than that from the front. Indeed, it is greater by an order of magnitude than the reflection from the convex front surface of a metal sphere. Interference of the phase-shifted large reflection from the back surface with the smaller reflection from the front may be responsible for the humps in the scatter curve. In addition, with larger real refractive index (and no absorption) optimum focusing on the back surface will be

attained with smaller spheres, thus accounting for the shift in the peak of the scatter curve toward lower D/λ which was noted for the refractive indices of 1.55, 1.60, 1.78, and 2.0.

It may be noted that this qualitative description agrees in principle with the theory of the dielectric Luneberg lens (Peeler and Coleman, 1958). By tapering the refractive index with radial distance r in the sphere according to the law $m = [2 - (r/R)^2]^{1/2}$ where R is the radius of the lens, a plane wave incident on the front surface is brought to a perfect focus on the back surface. A small spherical segment of metal placed at the focal point on the back surface then ensures that all the energy is reflected along reciprocal paths. Practical radar targets have been built on this principle whose radar cross-sections are phenomenally greater than their geometric sizes. For example a 3" (7.62 cm) diameter Luneberg lens has a radar cross-section of 3 ft² (2800 cm²) while a 1 ft diameter lens has a cross-section of 700 ft² at 3.2 cm. * By comparison, our results indicate that a 3" ice sphere has a 3.3 cm cross-section of 270 cm². While the analogy between an ice sphere and the Luneberg lens is not rigorous, it does demonstrate the effects of partial focusing in the ice and reflection by the back surface. Since the power reflection from the ice surface is 1/12.7 (Equation 2), the cross-section of our 3" ice sphere might be enhanced by somewhat less than a factor of 12.7 if we were to place a metal cap at the back surface. Then its cross-section would approach that of the 3" Luneberg lens mentioned above, an indication of the efficiency of focusing by the ice.

* Emerson and Cumming, Inc., Canton, Massachusetts.

If our qualitative interpretation is correct, it may be noted that the partially focused energy escaping from the back surface is 12.7 times that reflected. Thus if this surface were largely responsible for the back scatter, we might expect the forward scatter cross-section to be slightly less than 12.7 times the back scatter cross-section. This is supported by reference to Lowan's (1949) computations for refractive indices of $m = 1.55$ and 2.0 , which bracket that of ice. These indicate that a 3" ice sphere would have a forward scatter cross-section of about 3000 cm^2 .

The large back scatter cross-sections of ice spheres with dry surfaces and the effects of wetting which have been reported here have interesting implications in the interpretation of radar observations of hailstorms. However, these are so extensive as to be appropriately left to a future paper.

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References

- Aden, A.L., 1951 : Electromagnetic scattering from metal and water spheres. *J. Appl. Phys.*, **22**, 601.
- Atlas, D. and R.J. Donaldson, 1954 : The feasibility of the identification of hail and severe storms. *Air Force Surveys in Geophysics*, No. 65, Air Force Cambridge Research Center, 91 pp.
- Donaldson, R.J., 1958 : Vertical profiles of radar reflectivity in thunderstorms. *Proc. 7th Wea. Radar Conf.*, Univ. of Miami Nov. 17-20, 1958, p. B8-16. Amer. Meteor. Soc., Boston.
- Donaldson, R.J., 1959 : Analysis of severe convective storms observed by radar -II. *J. Meteor.*, **16**, 281-287.
- Gunn, K.L.S. and T.W.R. East, 1954 : The microwave properties of precipitation particles. *Quart. J. Roy. Met. Soc.*, **80**, 522-545.
- Herman, B.M. and L.J. Battan, 1959 : Calculations of Mie back-scattering of microwaves from ice spheres. *Scientific Rep. No. 12*, *Inst. Atmos. Phys., Univ. of Arizona*, Dec. 31, 1959.
- Kerr, D.E., 1951 : Propagation of short radio waves. *M.I.T. Radiation Laboratory Series*, Vol. 13. McGraw-Hill, N.Y. p.452-453.
- Lowan, A.N., 1949 : Tables of scattering functions for spherical particles. *Applied Mathematics Series*, No. 4. U.S. Nat. Bureau of Standards.
- Peeler, G.D.M. and H.D. Coleman, 1958 : Microwave stepped-index Luneberg lens. *Trans. Inst. Radio Engs., Antennas and Propagation*, AP-6, 202-207.
- Ryde, J.W., 1946 : The attenuation and radar echoes produced at centimetre wavelengths by various meteorological phenomena. *Meteorological Factors in Radio Wave Propagation*, The Physical Society, London.
- van de Hulst, H.C., 1957 : Light scattering by small particles. John Wiley and Sons, Inc., N.Y.

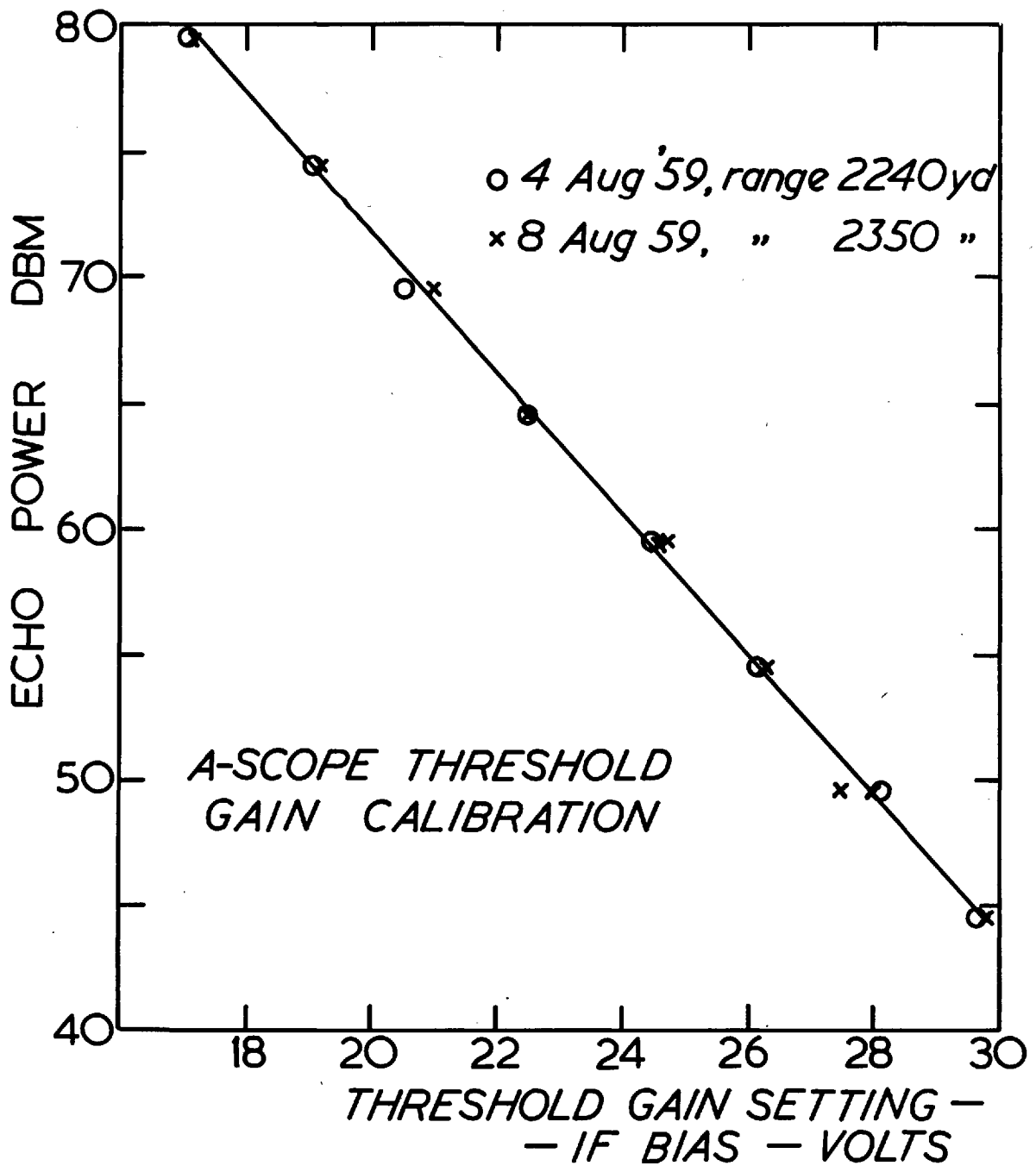


FIG. 1. MPS-4 A-scope threshold gain calibration representing the echo power in decibels below 1 milliwatt which is required for detectability.

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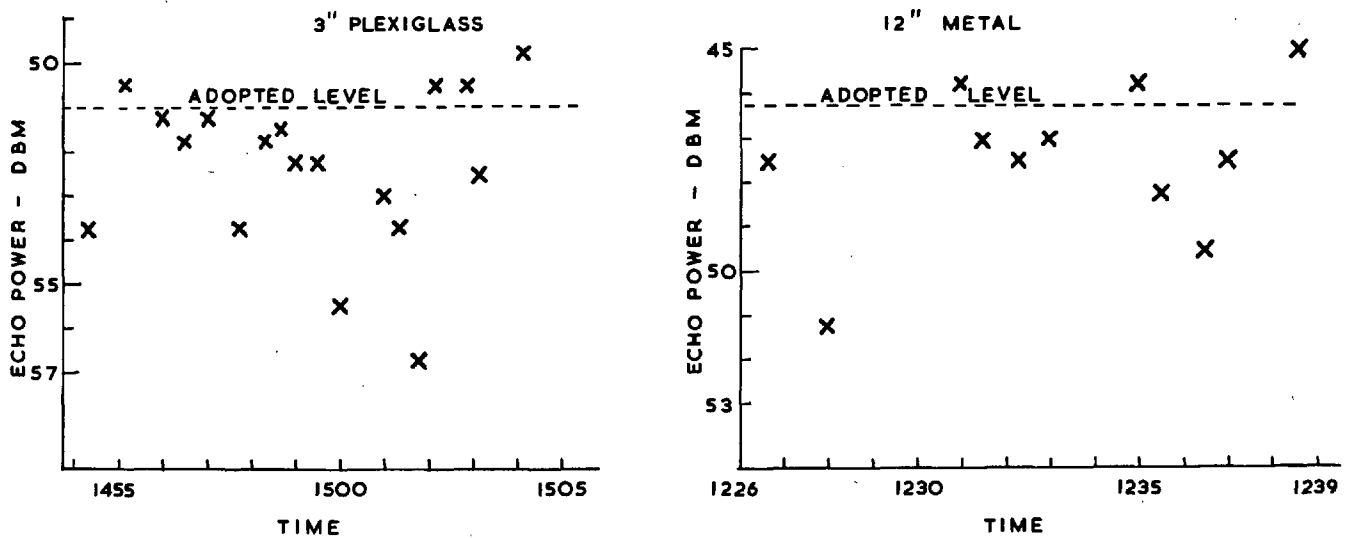


FIG. 2. Sample MPS-4 measurements of echo intensity from the indicated targets in decibels below 1 milliwatt. The dashed lines indicate the levels adopted as the true echo intensities. See text for details.

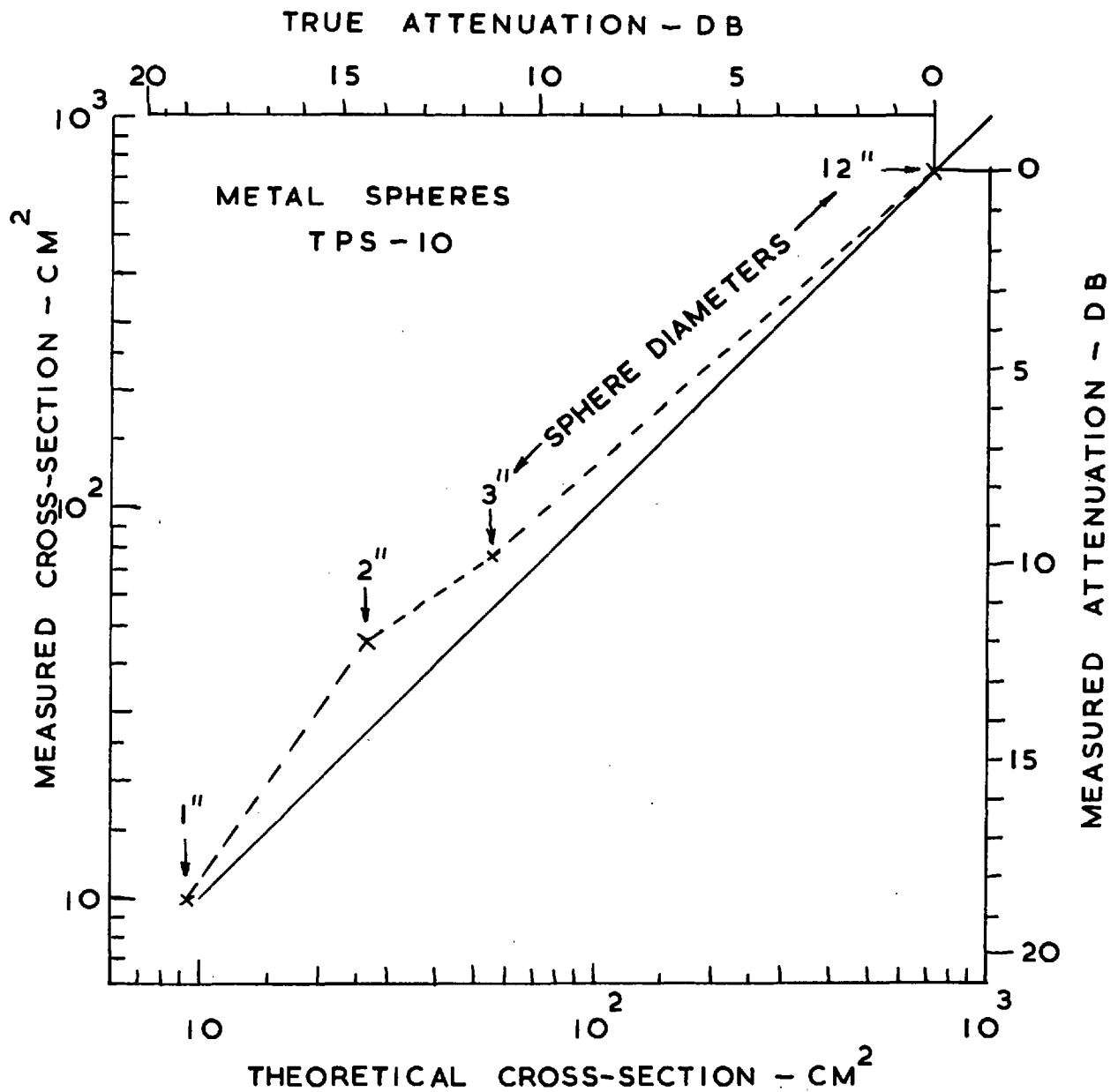


FIG. 3. Measured versus theoretical cross-sections for metal spheres with the 3.31 cm TPS-10.

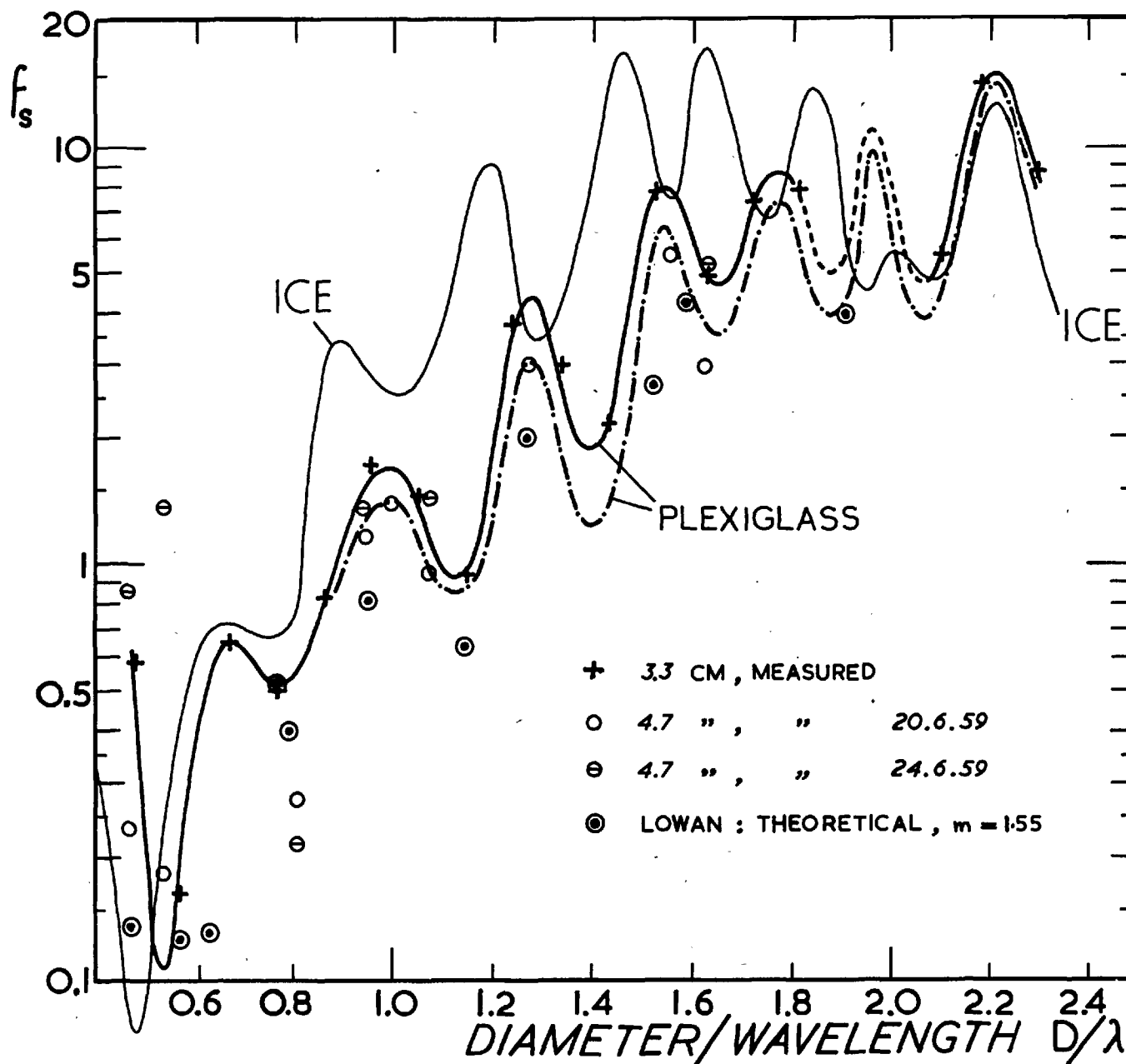


FIG. 4. Normalized back-scatter cross-section f_s of plexiglass spheres ($m = 1.6$) as measured by the TPS-10 and MPS-4. Heavy solid curve is based on TPS-10 measurements before correction. Dot-dash curve is corrected according to "calibration" of Figure 3. The thin solid curve is that for ice taken from Figure 5 while the circled dots are theoretical values for $m = 1.55$ according to Lowan (1949).

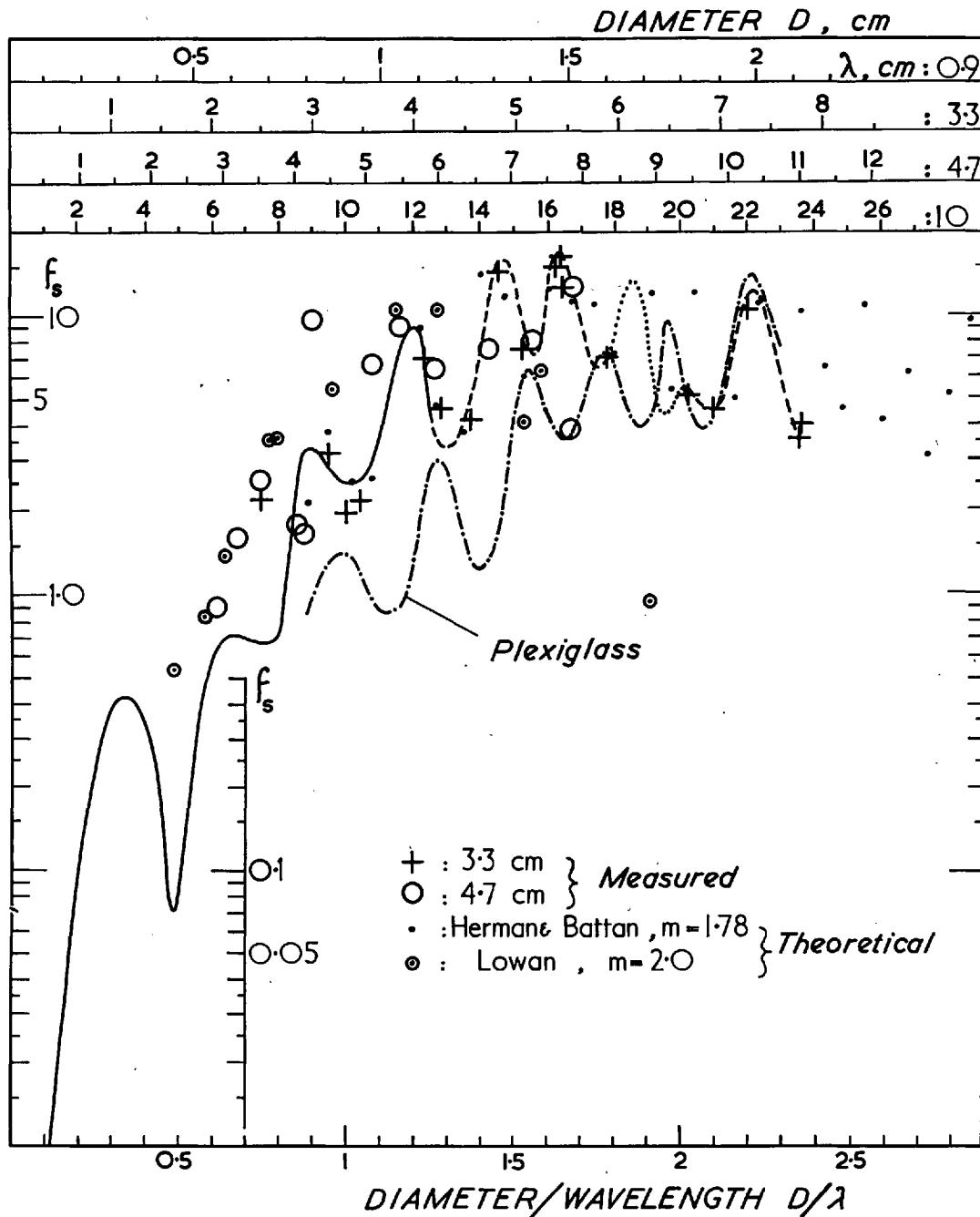


FIG. 5. Normalized back-scatter cross-sections $f_s = \sigma / (\pi D^2/4)$ of ice spheres with $m = 1.78$ as measured by the TPS-10 and MPS-4. The solid part is based on theoretical computations by Ryde (1946). The dashed part is based entirely on the measurements; dotted portion in doubt. Theoretical values computed for ice by Herman and Battan (1959) are shown by dots while corresponding values for $m = 2.0$ are taken from Lowan (1949). Scales at top represent sphere diameters at indicated wavelengths.

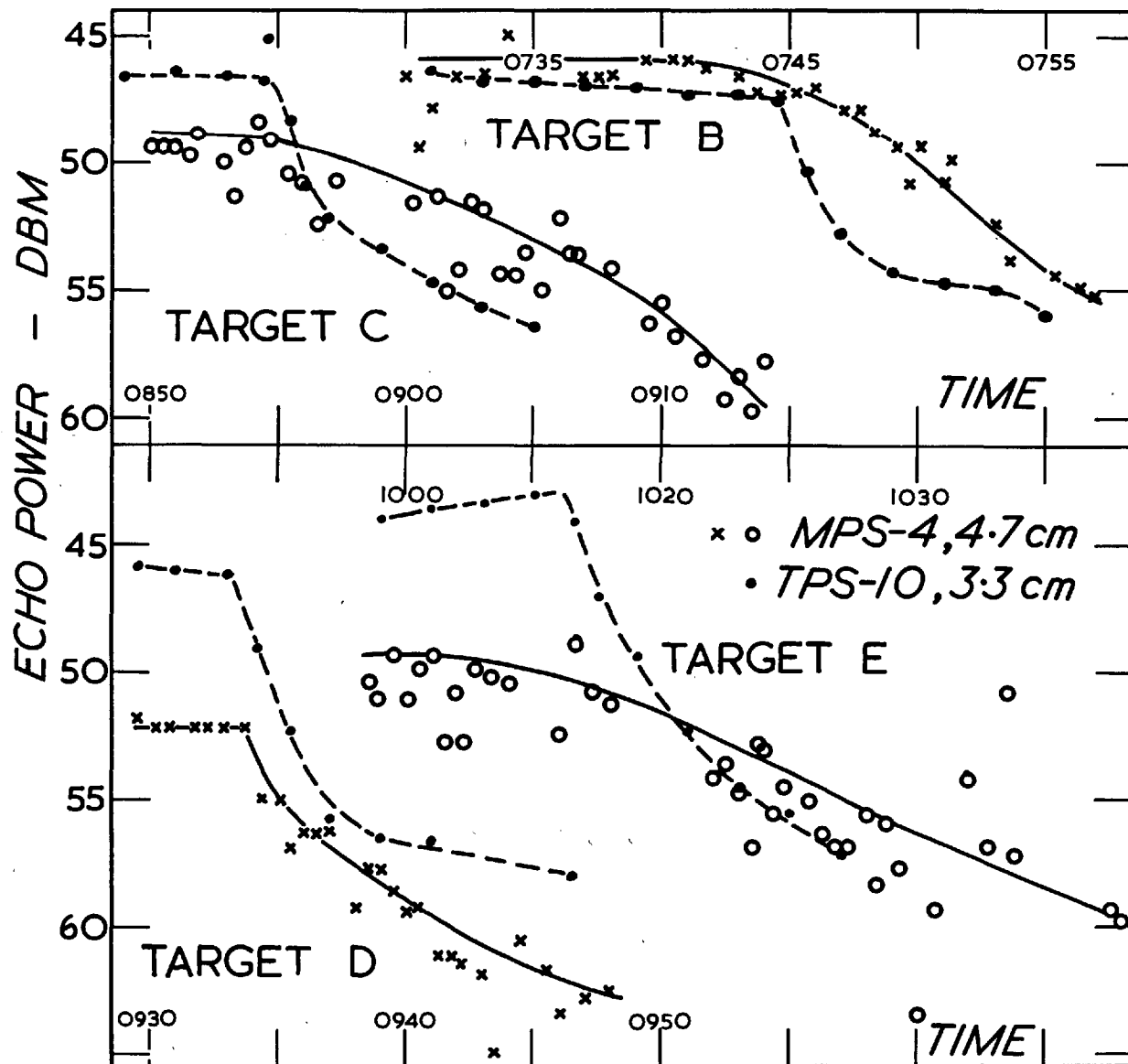


FIG. 6. Simultaneous echo intensity measurements with the TPS-10 and MPS-4 from four large melting ice spheres on August 8, 1959. Initial radii are B - 3.90 cm; C - 3.33 cm; D - 2.95 cm; E - 3.63 cm. These targets correspond to those similarly marked in Figures 7 and 9.

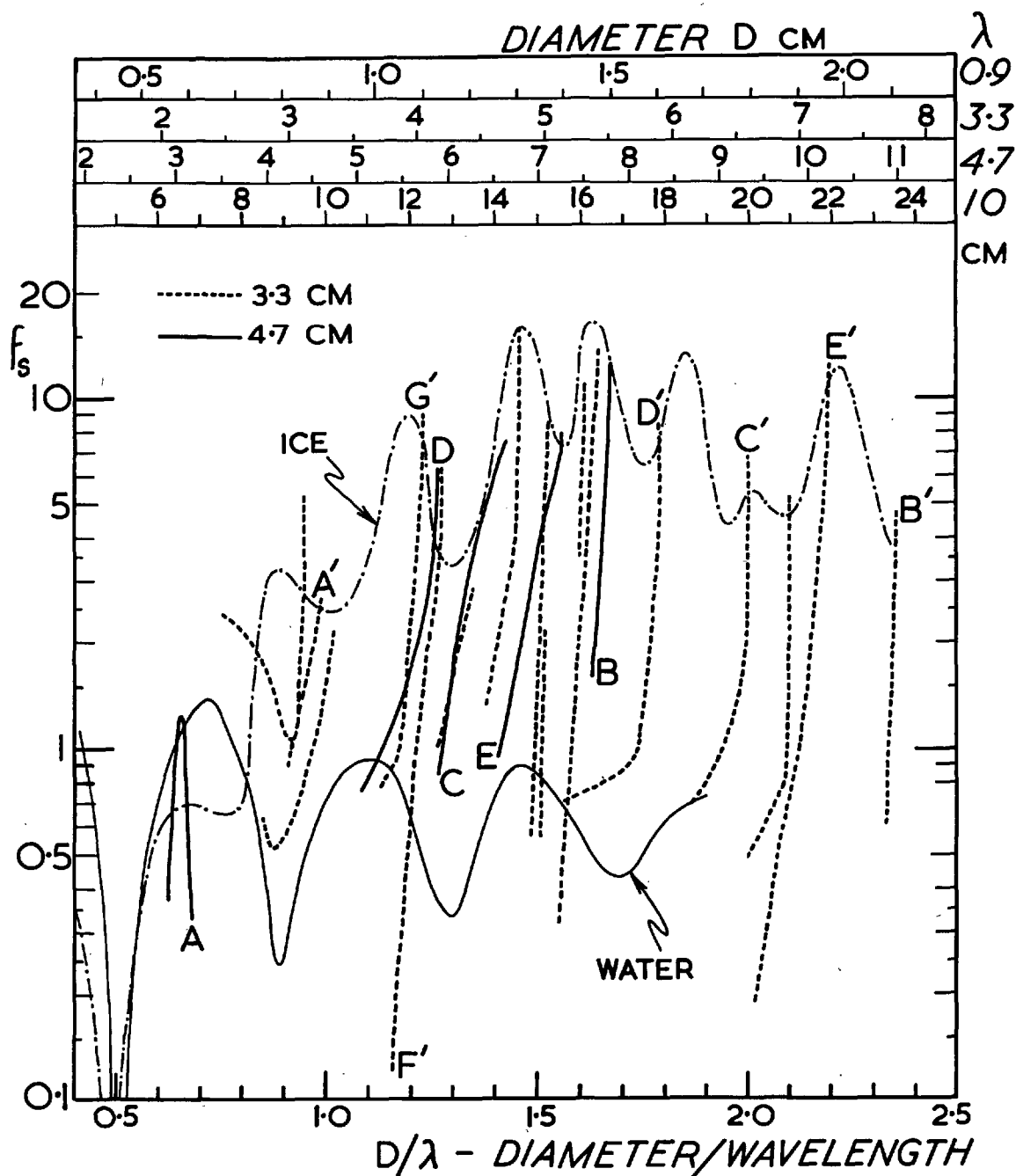
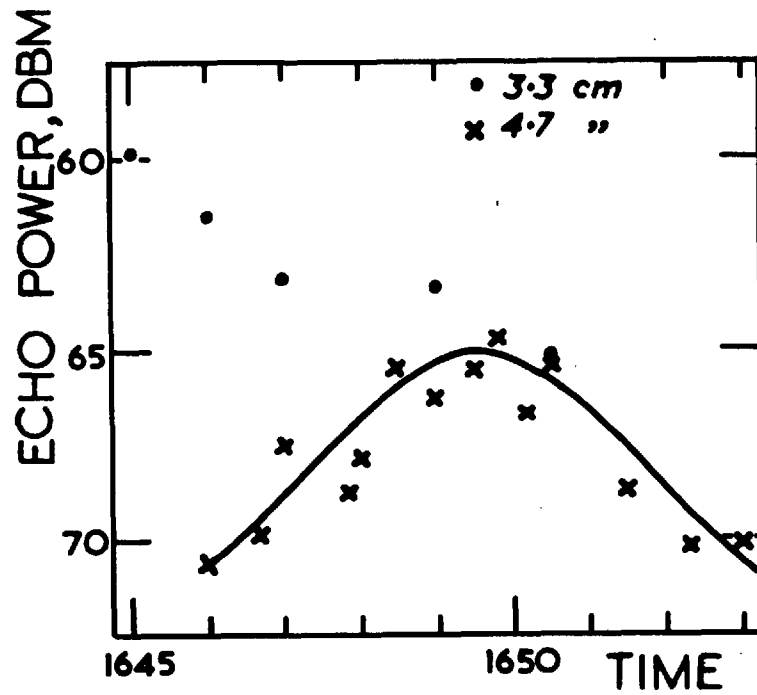


FIG. 7. Variations of the normalized back-scatter cross-sections $f_s = \sigma / (\pi D^2/4)$ during melting at 3.3 and 4.67 cm. The ice curve (dot-dash) is taken from Figure 5. The water curve is after Aden (1951). Curves marked by symbols are those referenced in text. Primed and unprimed symbols are measurements for the same particle at 3.3 and 4.67 cm respectively. Scales at top represent sphere diameters at indicated wavelengths. The water curve is not accurate at a wavelength of 0.9 cm.



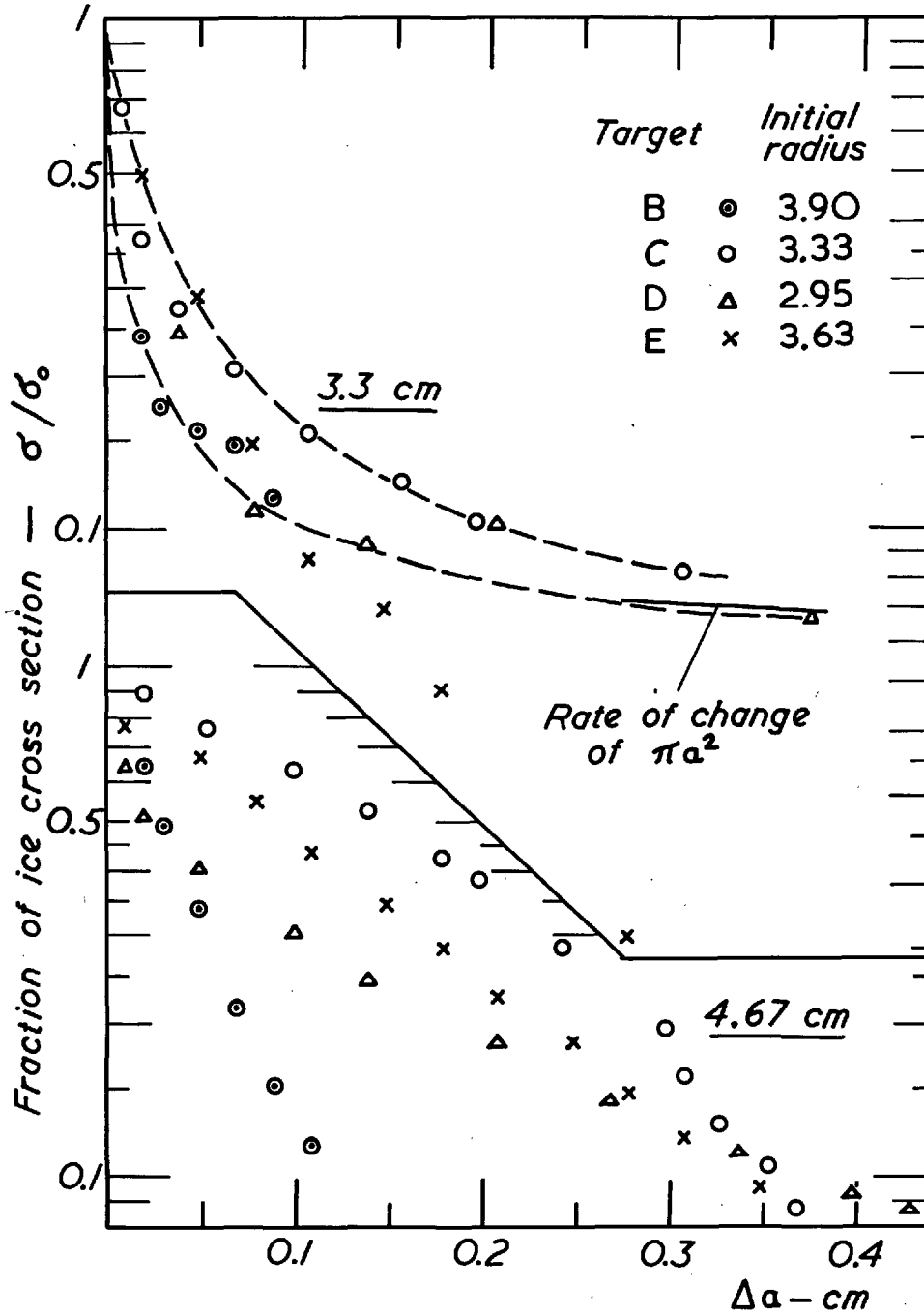


FIG. 9. Measured fractional decrease in the initial (dry) cross-section of a melting ice sphere versus calculated change in its radius. Symbols correspond to particles in Figures 6 and 7.