THE APPLICATION OF STATISTICAL TECHNIQUES TO FAULT LOCATION IN ANALOGUE CIRCUITS

by

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ABSTRACT

The research reported in this thesis is concerned with the location of single soft faults in analogue circuits, assuming that nonfaulty component parameters can have any value within their respective tolerance ranges. The particular situation refers to the location of a faulty component during actual field use, so that it can be removed and replaced, and the circuit thereby restored to normal operation; in these circumstances, it is not necessary to determine the parameter value of the faulty component.

In order to perform this diagnosis, a statistically-based algorithm is developed, which involves three steps. First, in the design stage, by means of computer simulation, component faults are characterized by statistical parameters, and a number of responses are evaluated for each faulty component. Next, still in the design stage, a suitable set of diagnostic responses is selected from the fault simulation results. The selection procedure is based on Analysis of Variance or Eigenanalysis, and the selected set indicates the measurements that must be performed in the field on the faulty circuit. Finally, in the field, from measurements made on the faulty circuit and reference to a previously stored data-base, a calculation is carried out (Mahalanobis distance) to identify the most likely faulty component.

The thesis is structured as follows. Chapter one introduces and briefly describes the fundamental concepts and problems found in the field of fault diagnosis. Chapter

two presents a critical review of contributions to this field. The identification of the particular problem considered in this thesis, and the relevant statistical techniques used in developing the algorithm, are discussed in chapter three. Chapter four describes the fault diagnosis algorithm, and its practical application on a circuit example is demonstrated. Chapter five discusses the application of the method to non-linear circuits. Finally, chapter six summarizes the research contribution and outlines suggestions for further research.

ACKNOWLEDGEMENT

I would like to express my grateful thanks to my supervisor, Professor Robert Spence, for his valued guidance and encouragement throughout the course of the research reported in this thesis.

I also wish to thank my colleagues in the Information Engineering Section for many encouraging discussions and their helpful suggestions.

I am deeply indebted to my wife and friends for continual encouragement and moral support.

I gratefully acknowledge the financial support provided by World University Service.

To

My wife Isabel

and daughters

Marianella & Patricia

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

STATEMENT OF ORIGINALITY

The opinions and techniques not original to the author are acknowledged by specific references. The main contributions reported in this thesis, in the field of fault diagnosis, are deemed to be the following:

(1) The developmemt and implementation of a statistically based algorithm for the location of single soft faults in toleranced analogue circuits (chapters four and five)

(2) The introduction and application of statistical techniques such as Monte Carlo analysis, Analysis of Variance, and Eigenanalysis to the fault diagnosis problems, at different levels of algorithm development (chapters three and four)

(3) An introduction to the different areas of fault diagnosis, identifying those with less research contribution, and an extensive critical review of recent methods, with comments on the domain of application (chapters one and two)

(4) Suggestions for further research of those additional aspects that may need some attention in improving the capabilities of the diagnostic method (chapter six).

CHAPTER ONE

FAULT ANALYSIS - DEFINITIONS AND PROBLEMS

1.1 INTRODUCTION

The technological development and manufacture of many different complex electronic circuits, and the need to fabricate highly reliable products, have increased the emphasis on using automatic testing and the development of techniques associated with it, specially those that can perform around low-cost computer systems. The size and complexity of these circuits have made nearly impossible the use of manual testing. When it is used, it results in a very expensive operation and low productivity (during manufacture) due to the considerable amount of testing involved and the need of experts to perform the testing function; this is especially true in high volume production where testing generally becomes the most expensive function. Under these circumstances, automatic testing is being introduced and it is proving to be the only approach that can significantly lower the production and maintenance costs, and reduce the need for highly qualified personnel.

Manufacturers and maintenance organizations would generally like to perform testing for a variety of reasons and at different levels.

(1) Testing during manufacture: It is basically assumed that during the design process, circuits have been submitted to different levels of testing to determine

any design error, so that a valid design is to be manufactured. The general objectives of testing during this process are the detection of those faults introduced by the manufacturing process itself (e.g., soldering and interconnection errors) as well as those that may arise from the omission of some practical considerations during the design stage . Moreover, design verification is also an important objective during this process. It is expected that manufacturing testing be as comprehensive as possible in order to not only produce and deliver to the field a high quality and reliable final product but also accumulate useful diagnostic information for maintenance purposes.

(2) Testing for specification assurance: This function may form part of manufacturing testing in its final stages. Its main objective is to ensure that the final assembled product meets its required performance specifications. However, it may be that, due to the difficulties and high cost involved in testing the complete system, emphasis be put on performing this testing function on individual circuit boards (partitioning) to ensure that they meet their required specifications such that the whole system meets its own performance specifications. This procedure allows a more exhaustive testing as compared to testing the complete system. Manufactured circuits are tested in order to detect failures caused by either component parameter tolerances or the drift of component parameters from their values during manufacture.

Circuits that fail this testing function may be discarded or returned for service (repair).

(3) Testing for maintenance and repair (Field Testing): The general assumption in this testing area is that, manufactured circuits have been operating properly, so that faults likely to occur would not include those caused by design or manufacturing errors. Instead, faults in the field are often caused by environmental conditions, component ageing and other factors. Field Testing may be divided into two general areas; Fault Detection and Fault Diagnosis.

(3.1) Fault Detection: It is usually a periodic testing procedure that should provide sufficient information to determine whether or not the unit under test is operating within its performance specifications.

(3.2) Fault Diagnosis: From negative results in the detection process, some additional test may be necessary in order to obtain adequate information for maintenance and repair decisions. This information may in the first place be used to locate and replace the particular faulty unit within a system and later to isolate the faulty components within that unit, with subsequent replacement. The information may also be useful to anticipate and prevent faults in other similar circuits.

As far as the research reported in this thesis is concerned, the emphasis is on field testing, particularly on the development of algorithms for the fault diagnosis problem. Within this context, chapter one introduces the general concepts and problems found in this area. Section 2 defines some of the fundamental concepts (definitions). Section 3 outlines the three main fault diagnosis problems; Modelling, Diagnosability, and Testability. Finally, section 4 presents a brief description of the basic elements that may be considered necessary in the implementation of an automatic testing system.

1.2 DEFINITIONS

(1) UNIT UNDER TEST (UUT): It refers to the circuit or system being tested.

(2) FAULT: A physical defect which results in the violation of performance specifications. This is often due to a circuit component parameter lying outside its acceptable tolerance range. Of the different types of faults which are possible, the following are the most important:

(a) Soft fault: It is the fault caused by a continuous change in component parameter value, due to either ageing or environmental conditions, such that an unacceptable value is reached and violation of specifications occurs. This fault is also known as Deviation or Drift fault.

(b) Catastrophic (Hard) fault: The fault caused by a sudden and large deviation of component parameter; usually this is a short or open circuit.

(c) Single fault: A fault involving one component only.

(d) Multiple fault: A fault involving several components simultaneously.

(3) FAULT SIGNATURE: It is the characterization of a fault, usually derived from measurements performed on the UUT, or analytical models.

(4) DETECTION: It is the process of detecting that a fault exists in a circuit or system. The fault could be either a single or multiple one. The cause of the failure is ignored at this stage, the only concern is whether the circuit or system works correctly or not. The detection process is performed either at the manufacturing stage or during maintenance.

(5) LOCATION (ISOLATION): It is the process by which one determines the faulty component(s) or board(s) after a fault has been detected. Location of the fault is required so that repair or replacement may be carried out at a later stage.

(6) IDENTIFICATION (DIAGNOSIS): It is the process of finding the cause of failure, in other words, it identifies the cause of the fault (e.g., the exact value of off-tolerance component parameters). Following a correct diagnosis, steps may be taken to prevent a repeat occurrence of the fault.

(7) TEST: A procedure which detects, locates or identifies faults.

(8) TEST TERMINALS (POINTS): These are external terminals of a UUT where measurements can be made and its behaviour observed. Test terminals can be divided into two groups:

(a) Accessible terminals: These are points where voltages and currents can be applied and/or measured.

(b) Partly accessible terminals: Terminals at which only voltages can be applied and/or measured.

(9) INTERNAL NODES: These are inaccessible nodes within a circuit where neither voltages nor currents can be applied and/or measured.

(10) ON-LINE TEST: Testing procedure where the UUT operation is not interrupted.

(11) OFF-LINE TEST: Testing procedure where the UUT operation is interrupted.

(12) FAULT ANALYSIS (DIAGNOSIS): It refers to the generation of testing procedures for maintenance or repair decisions, that will lead to the determination of components (location and/or identification) that are causing the circuit to violate its performance specifications.

1.3 PROBLEM FORMULATION

Considering that a circuit has failed its performance specifications, two possibilities arise: (a) throwing away, and (b) further test for fault location or identification. This section is concerned with discussing the problems encountered in the second possibility.

1.3.1 FAULT MODELLING

The determination or definition of an effective component parameter fault model is an important problem in fault analysis. Conceptually, any deviation from a component parameter nominal value may be considered a component fault. Catastrophic (open and short circuits) and out-of-tolerance (deviation/soft) faults are the most common. Within the scope of Automatic Test Program Generation, it is advisable to incorporate and model these faults by simply adjusting the values of existing component parameter models, such that no additional models are introduced. However, the fault model must be designed in such a way that the basic topology of the nominal circuit is not altered, especially when catastrophic faults are considered.

(1) Catastrophic faults: In general, these are extreme changes of the component parameter values causing major alterations on the normal operation of a circuit. Theoretically, an open circuit is a zero valued current source, while a short circuit is a zero valued voltage source. In practice, these conditions are difficult to handle and introduce changes in the topology of the

circuit; for that reason, they are considered and modelled as nonideal open or short circuit faults. This involves representing the open circuit as a very low admittance (high impedance), and the short circuit as a very high admittance (low impedance).

A general fault model for lumped component parameters is shown in Fig. 1.1(a). Considering Fig. 1.1(b), a single component parameter of Fig. 1.1(a), ΔY_k should be a large change such that a catastrophic fault (open or short circuit) is approximated by its adequate corresponding admittance value. For a transistor, fault modelling is more difficult, since different fault combinations exist. These fault conditions may be modelled by inserting either externally or internally the required conductance to the existing transistor model. In this way, open circuit may be modelled by inserting low conductance parameter value in series with the transistor terminals, short circuit may be modelled by inserting high conductance parameter value across pairs of transistor terminals. Representations for ideal and practical models [40] for the different fault conditions are shown in Figs. 1.2 and 1.3 respectively.

(2) Soft faults: These faults are produced when the component parameter values of a circuit drift from their nominal values, so that the circuit response deviates outside the acceptable performance specifications. The drift or deviation in parameter values is usually caused by component overstress or

ageing. In passive components, these soft faults are modelled by simply modifying their values (Fig. 1.1). Soft faults in transistors, causing degradation of the DC conditions can be simulated by modifying the values of the model shown in Fig. 1.3. In the AC domain, these faults are usually simulated by appropriately modifying the parameter values of the small signal transistor models of Fig. 1.4 or Fig. 1.5.

1.3.2 DIAGNOSABILITY PROBLEM

The general problem associated with the maintenance of complex circuits is that a circuit fails to perform satisfactorily its functions due to the failure of one or more of its components. Under these circumstances, and based on a limited set of measurements, it is necessary to determine which components have failed. This, in fact, leads to two specific problems: Identification (knowledge of the exact value of the component parameters) and, Location (isolation of the faulty components, without determining their actual parameter values).

(1) Identification: This is a problem in which the determination of the exact parameter values of the faulty circuit's components is required. The determination of these values is not difficult if both the currents through the components and the voltages across the components can be measured. Unfortunately this presents some practical problems within a circuit; for example, all nodes would have to be accessible for measurements, and the current through the components

would have to be measured by breaking the connection at one end of the components. This is a time consuming operation and requires an extremely large number of measurements for the determination of all component parameter values of the faulty circuit.

However, since faulty components affect the overall circuit performance, the identification problem can be formulated as a relationship between the external measured responses and the internal component parameter values. The general relationship can be described by the following nonlinear equation

$$y = f(u,x) \tag{1.1}$$

where y: is the vector of external measurements

u: is the excitation vector signal, and

x: is the vector of internal components to be identified.

It is necessary to have sufficient equations for the unique determination of component parameter values. Therefore, the number of independent measurements required is directly related to the number of possible faulty components (e.g., for a linear circuit, measurements could be performed, either at different test points, or at one point using different test frequencies, provided that there is a unique dependence of component values on the frequency response).

It is generally accepted that, to determine uniquely all component parameter values, all the terminals should be accessible for measurements. Under this condition, the solution for all component parameter values, in the case of linear circuits, can be simplified by solving a set of linear equations. This description is an ideal condition for solving the identification problem. However, in practice, the number of accessible test terminals for measurements is limited to the external ports and some internal nodes, such that, not all the circuit component parameter values can be identified. Instead selective component parameters identification can be carried out, considering the rest of the component parameter values to be known.

Some methods for solving the identification problem are reviewed in chapter two section 1.

(2) Location: This is a problem in which the determination of the exact component parameter values of a faulty circuit is not necessary, to identify the faulty components; instead the interest is in locating the faulty components from a set of measurements and previously stored information concerning the different fault conditions of the circuit under test.

The basic operation is based on comparing the set of measured responses

$$y = \{y_1, y_2, \ldots\ldots, y_m\}^T \qquad (1.2)$$

corresponding to an unknown fault condition of the UUT, to fault patterns stored in a fault dictionary or matrix recognition form

$$R = [\, r_{ij} \,] \quad \begin{array}{l} i = 1,2,\ldots,n \text{ fault conditions} \\ j = 1,2,\ldots,m \text{ responses} \end{array} \qquad (1.3)$$

which contains the information of the n fault conditions, each represented by a pattern of m responses. The objective is to determine the most likely faulty components. This is accomplished by classifying the measured responses y, according to a decision criteria, as one of the fault conditions of matrix R.

Fault conditions are usually simulated before actual test to construct the required matrix R in equation (1.3). The size of this matrix is determined by the number of fault conditions and the number of test points (measurements); both variables being related to achieve an effective diagnosis. The number of fault conditions is very much dependent on the number of circuit components, such that, if a circuit contains N components, there will be at least N fault conditions for single faults (2N for catastrophic faults, considering that, open and short circuits are two different faults in one component). However, if multiple faults are to be considered; e.g., F components fail simultaneously from the total N, then the possible number of fault conditions will be

$N!/(N - F)!\,F!$, which obviously increases drastically the size of matrix R. As far as the number of test points is concerned, this often increases with the number of fault conditions, and is very important in achieving a high confidence level in the location of faulty components.

The situations mentioned above usually tend to increase the cost of testing, especially for large circuits and multiple faults. In order to minimize these problems and reduce testing costs, it is sometimes necessary to accept tests which do not locate faults in specific components, but in groups of components having similar fault patterns. In this way, location of the faulty components is performed by, firstly isolating the group where it is assumed to belong, and then locating the specific faulty components in that group. This is known as a fault tree approach. Some methods for solving the location problem are reviewed in chapter two, section 2.

1.3.3 TESTABILITY PROBLEM

It was mentioned during discussion of the diagnosability problem that, test points, where measurements are performed in a circuit under test, are an important factor in its solution (either identification or location problem). The reason is that, in order to find a unique solution for these problems, it is generally necessary to have at least as many independent measurements as the number of fault conditions that can exist. This situation gives rise to the Testability problem and the question of having access to nodes of the circuit under test.

This problem is generally concerned with identifying those circuit nodes to be regarded as access terminals, where diagnostic test (measurements) can be carried out, providing sufficient information for the identification or location of the faulty components. The selection of these points must be based on the criterion that they should reflect the symptoms of all the fault conditions to be encountered in the circuit under test, such that no additional measurements are necessary. In addition to this, circuit responses taken at these points should provide good discrimination between fault conditions to enable a unique diagnosis of the faulty components. It is generally accepted that more access points are needed for the solution of the identification problem in comparison to the location problem. In the former case, the faulty components are determined by simultaneous solution of a set of independent equations (the number of equations being equal to the number of possible fault conditions); whereas, in the location case, a trade-off situation may be possible between the confidence of isolating the faulty components and the number of access points for measurements.

In the early stages of diagnostic testing development, the testability problem was not much taken into considerarion due to the following two factors: In the first case, it was assumed that all circuit nodes were accessible for measurements, so that a deterministic solution could be obtained for the faulty components; in the other case, especially for circuits in the frequency domain, only the input-output ports were considered to be accessible for measurements such that an approximate solution based on a priori statistical information was possible for the faulty components. However, as circuits increased in size and complexity, testing under these conditions was becoming very costly and time consuming, and in other cases, the assumptions were no longer valid or satisfactory, either because not all circuit nodes were accessible or the two port measurements did not provide sufficient information to reach an adequate solution. Under these circumstances, an intuitive partial solution was first proposed; it consisted in having access to a limited number of internal circuit nodes, where different circuit performances could be measured. Unfortunately, this consideration created some other problems such as the impossibility of diagnosing all faulty components. It was then realised that a more appropriate and comprehensive solution could be achieved by considering testability as one of the design considerations in the early stages of circuit design and development. This idea is being taken up by major circuit designers, and in recent years some theoretical research [37], [41 - 44] has been done in this direction.

1.4 BASIC ELEMENTS OF AN AUTOMATIC TESTING SYSTEM

The discussion in previous sections provided some knowledge of the type of problems created by the development of more complex circuits and high volume production. This meant that a number of manufacturers, maintenance organizations, and researchers concentrated their efforts at developing automatic testing techniques and automatic test equipments (ATE) as realistic approaches to solve these problems.

The integration of techniques, equipments, and other elements related to testing and diagnosis form the basis of an automatic testing system. Thus, although this is not the main interest of the research reported in this thesis, a brief discussion may be considered of interest, so that, the present method be understood within that context. A more or less efficient automatic testing system [45] should at least incorporate the six basic elements (Fig. 1.6) to be described below.

1.4.1 BASIC CIRCUIT DESCRIPTION DATA

This refers to data on circuit interconnection, schematic diagrams, component models. The data is used in computer simulation and test generation.

1.4.2 TEST SIMULATION

Circuit simulation must be based on a circuit model, which is automatically derived from the interconnection list and a library of component models. Subsequently, a test method should be selected, which requires the definition of the type of characteristics to be measured, the description mode of the unit under test, and finally, the characterization of the potential faulty conditions.

(1) Characteristics to be measured: It is necessary to take into account the measurement capabilities provided by the automatic test equipment to be used. The most common characteristics measured are based on harmonic measurements, and time domain measurements.

(2) Description modes: According to the types of signals and the circuit characteristics to be measured, two modes of testing have been adopted. These are the functional description and the structural description.

(2.1) Functional description (Function-oriented point of view): The unit under test is treated as a black box which performs a specific function given in terms of design specifications. Testing consists of checking whether a given sample meets the design specifications or not. Neither the components nor their interconnections are considered in this approach, with the result that it only leads to fault detection. Additional structural information is needed to isolate a fault. Techniques used by this approach are: transfer equations, flowgraphs, and experimental representation relating input-output signals.

(2.2) Structural description (Circuit-oriented point of view): This mode of description assumes exact knowledge of the nominal structure (circuit design and the input and output terminals of the UUT). This approach uses matrix representations, such as nodal and/or branch techniques, hybrid techniques, and state variable techniques.

(3) Fault characterization: Potential faults must be characterized in a continuous or discrete quantity to form a fault signature which will be needed during the diagnosis process. Three important aspects are involved in this process.

(3.1) Type of signatures: Signatures are derived by measuring different characteristics of the unit under test. These could include measurement of the input and output signals of the UUT at different frequencies or time intervals, at all the test points, measurement of the transfer function and/or its coefficients, correlation function between output signals, and cross-correlation function between input and output signals.

(3.2) Signatures' determination: This operation involves the determination of signatures for nominal conditions and potential fault cases to be considered. The information could be obtained from those designers who considered testing in the design process, or it could be derived from analytic models by software simulation.

(3.3) Signature representation: Representation of the signatures is an important factor for the automatic test equipmemt, because this is directly related to the amount of data to be stored. Usually the data is stored as discrete information previously processed by a computer. The data must be a good representation of all the faulty conditions of the UUT. Sometimes a trade-off is necessary in order to decrease the amount of data to be stored.

1.4.3 TEST PROGRAM GENERATION

Complete testing requires the measurement of some parameters or functions of the unit under test. Algorithms or heuristic methods must be developed for the generation of test procedures, from which the presence of any fault can be detected, isolated, and/or identified. Algorithms in a software form should include the following four main functions.

(1) Generation of test signals: This concerns the selection or determination of input signals which will perform a test function such as detection, isolation, and possible identification.

(2) Determination of fault signatures: This is related to the actual processing of the information provided by the measurements in order to derive the fault signatures.

(3) Interpretation of the signatures: It is the process of checking the signatures obtained at the step (2) against the reference data of the UUT, with the aim of detecting a fault and then performing its isolation.

(4) Diagnosis: It is the process of isolating a fault

with subsequent identification. Isolation is necessary when the interest is in replacing the module or faulty component. But sometimes during the manafacturing process, identification is necessary in order to adjust or tune the component.

1.4.4 COMMAND LANGUAGE AND LIBRARY OF TEST DATA

A test command language is necessary to instruct the automatic test equipment in testing any unit under test. A high level language is desirable to manipulate the data and to obtain the maximun information from the UUT. This has the advantage that for any UUT, a documented program could be written, probably in the same language.

The other important factor is the storage of the data representing the fault signatures of a particular UUT. This information must be stored as library test data, accessible when it is required.

1.4.5 AUTOMATIC TEST EQUIPMENT (ATE)

The type of equipment to be used must be suitable to test the UUT with minimum human intervention. It should have adequate capability to ease the modelling and simulation of the UUT, and allow the use of a high level programming language. The test equipment should be able to perform at least the four functions mentioned in Test generation.

1.4.6 DIAGNOSTIC PROCEDURES

This includes developing or adopting algorithms capable of isolating a fault and/or identifying the cause of it, once the UUT is detected as faulty. The algorithms should take into account the functions described in Test simulation and Test generation, with some consideration of the type of ATE to be used.

1.5 SUMMARY

A brief general review of the most common concepts and problems encountered in Fault Analysis is presented and discussed in this chapter. Additionally, what constitutes the basic elements of an automatic testing system is also described to some extent. Apart from these concepts and problems, there are several other more specific ones, which are not introduced here; however, those related to the algorithm for fault location described in this thesis (chapter four) will be explained in chapter three.

Although it is possible to deal with each problem separately, they are interrelated and dependent. It generally happens that when developing a fault diagnosis method, the emphasis is on solving one particular problem, making the assumption that other problems do not exist or they already have some solution. Of the three problems described in this chapter, the diagnosability (Fault diagnosis in Fig. 1.6) is the one that has received more attention from researchers, specially those related to the identification problem.

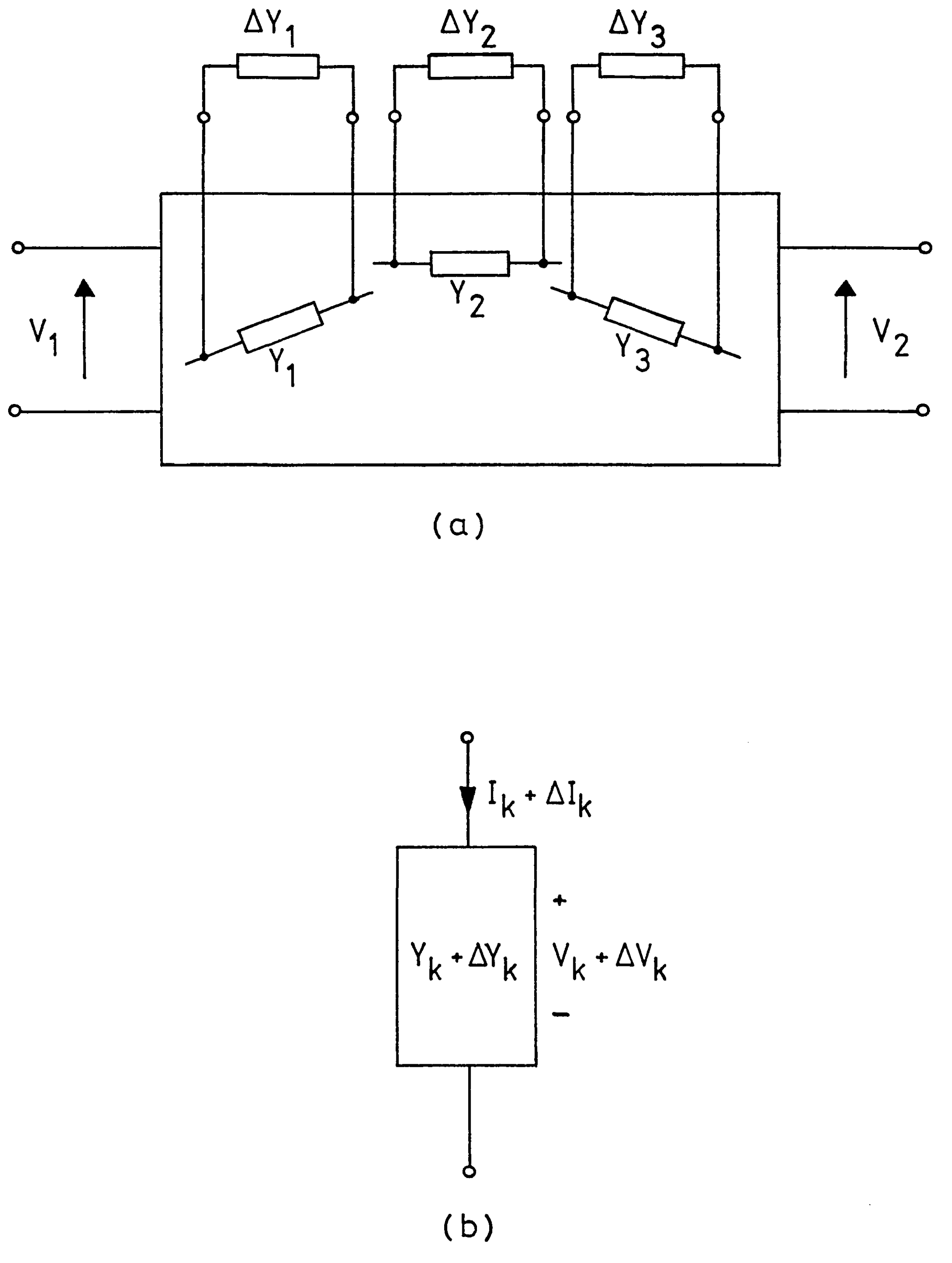


Fig. 1.1 Fault modelling for passive components

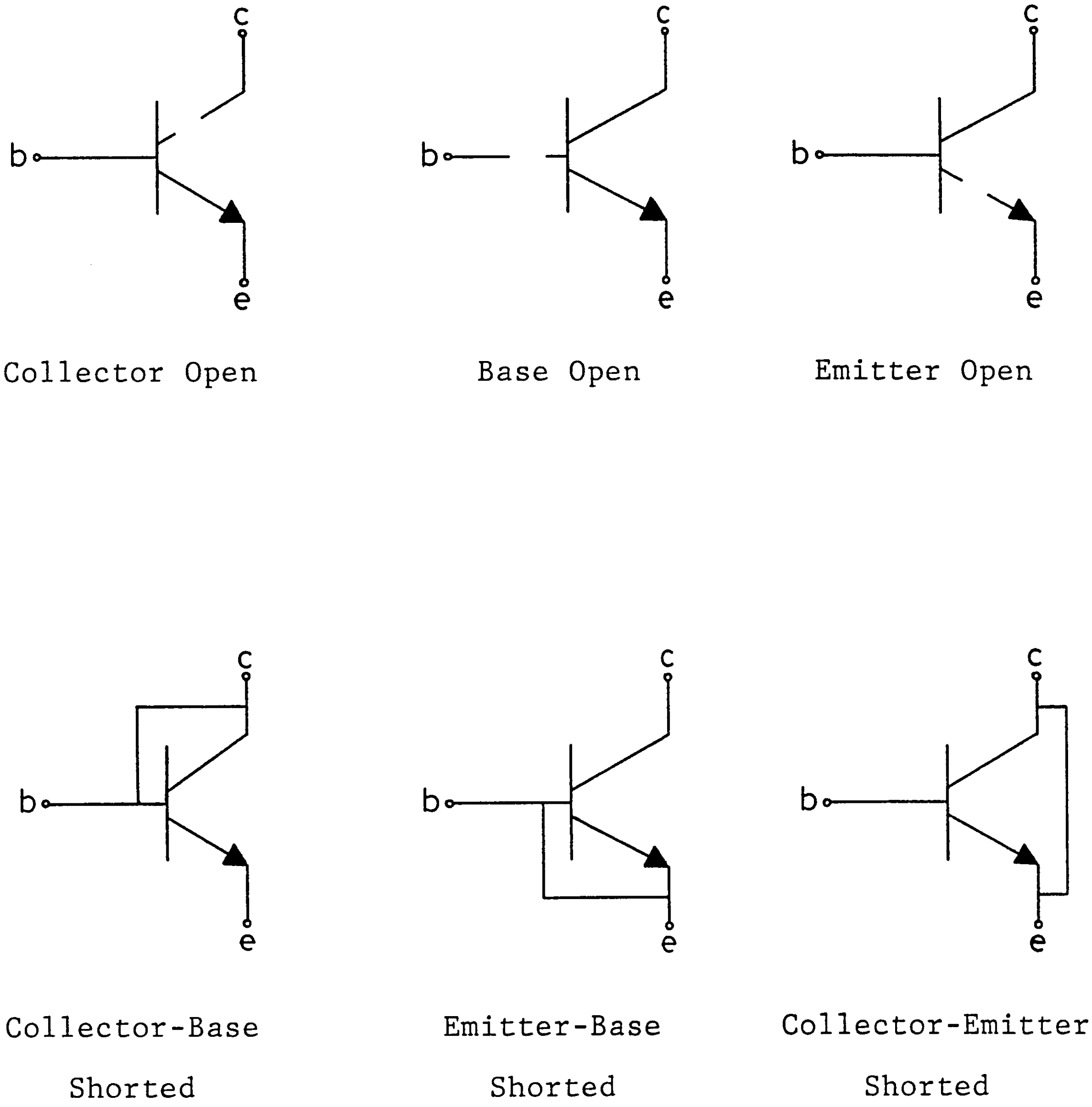
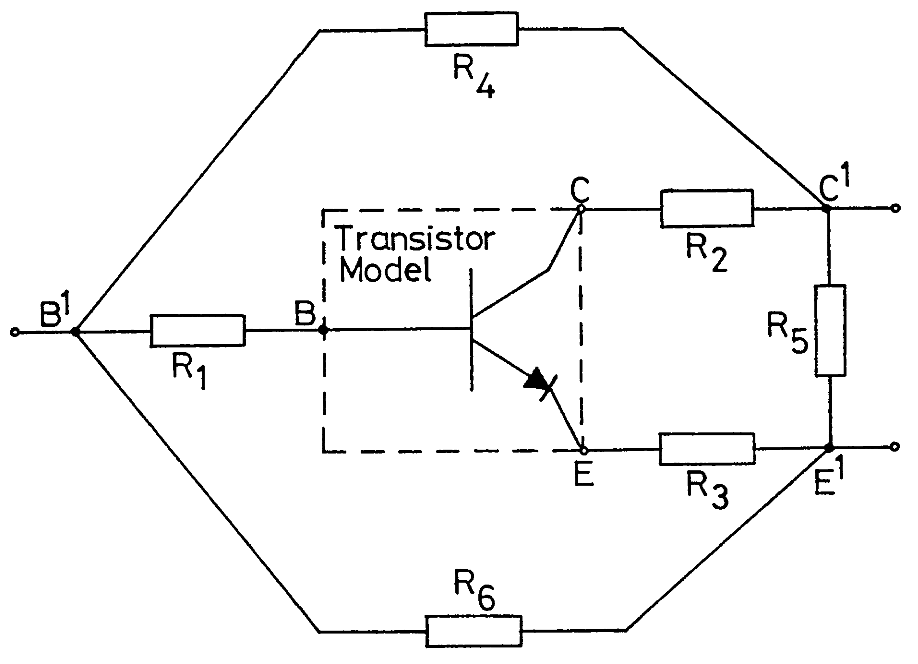


Fig. 1.2 Symbolic representation for ideal catastrophic fault modes for a bipolar transistor



Normal operation $R_1 = R_2 = R_3 = 0.01\ \Omega$

$R_4 = R_5 = R_6 = 100\ M\Omega$

CATASTROPHIC FAULTS

Base Open,	Set	$R_1 = 100\ M\Omega$
Collector Open,	Set	$R_2 = 100\ M\Omega$
Emitter Open,	Set	$R_3 = 100\ M\Omega$
Collector-Base Shorted,	Set	$R_4 = 0.01\ \Omega$
Collector - Emitter Shorted,	Set	$R_5 = 0.01\ \Omega$
Base-Emitter Shorted,	Set	$R_6 = 0.01\ \Omega$

Fig. 1.3 Built-in fault model for a bipolar transistor

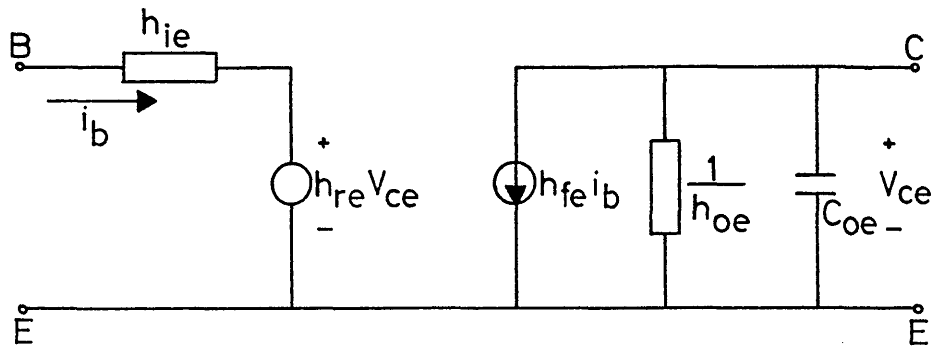


Fig. 1.4 Small - signal hybrid model for a bipolar transistor

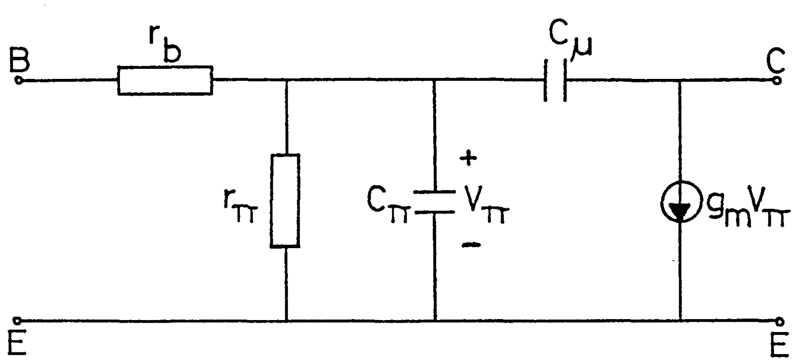


Fig. 1.5 Hybrid - π transistor model

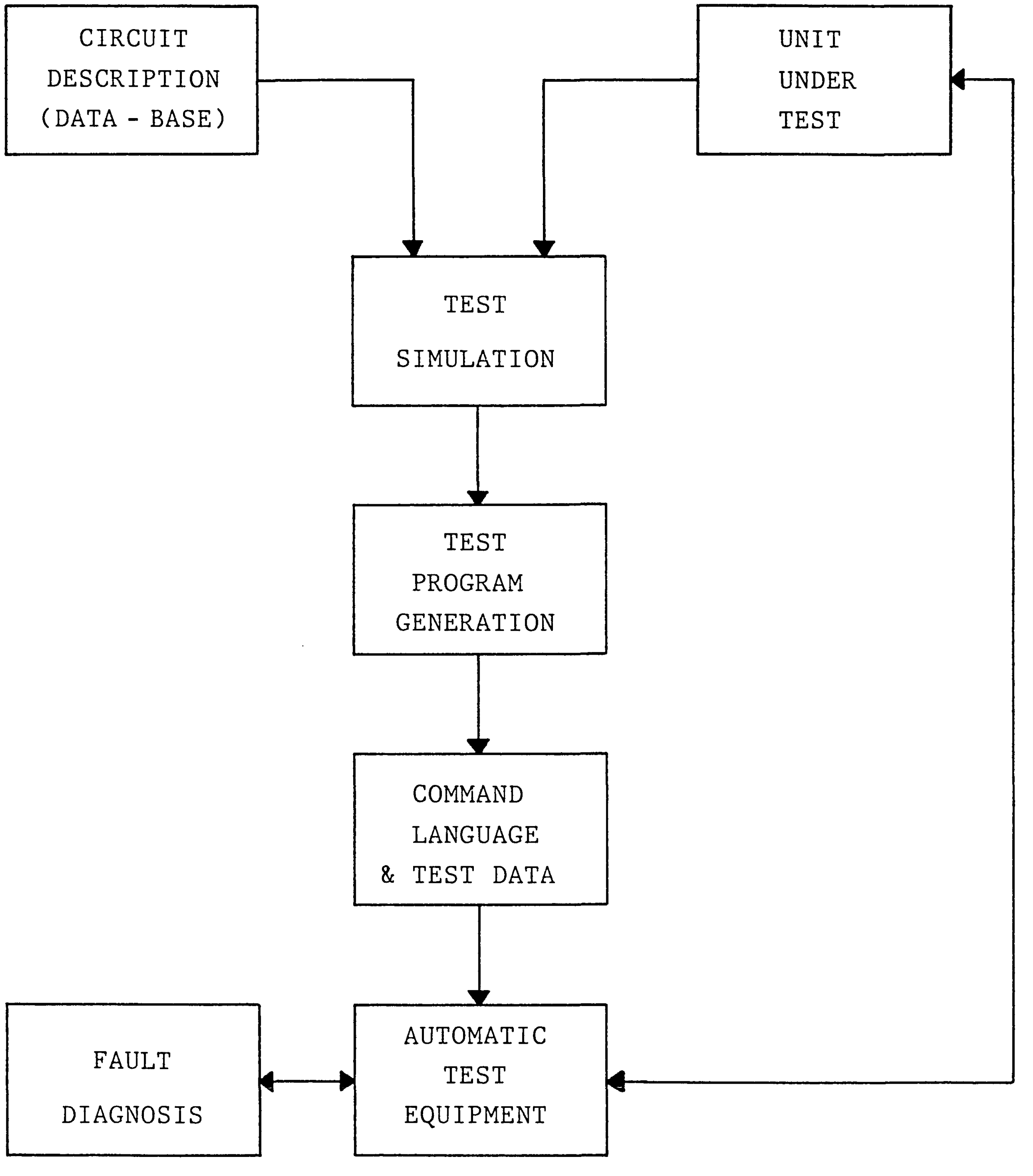


Fig. 1.6 Basic elements of an Automatic Test System

CHAPTER TWO

AUTOMATIC FAULT DIAGNOSIS FOR ANALOGUE CIRCUITS - A CRITICAL REVIEW OF AVAILABLE TECHNIQUES

2.1 INTRODUCTION

The generation of automatic testing procedures for the detection and isolation/identification of faults in electronic circuits has become a real necessity, as the size and complexity of modern circuits are increasing and the manual approach becomes obsolete and very costly. This fact, together with the availability of low cost computer systems, has attracted the interest of many researchers, in developing methods to overcome some of the problems encountered in fault analysis [29 - 30]. The general fault analysis problem for an analogue circuit or system leads to the measurement of a set of externally accessible performances of the unit under test (UUT), from which it is expected to determine its internal parameter values or, alternatively, locate the faulty component(s).

Current techniques available are based on two fundamentally different concepts (A different criterion is used for the classification adopted in Fig. 2.1). The first one is the concept of Component Simulation. In this case, the actual component parameter values of the UUT are calculated from measurements performed at the test points and previously provided information concerning the nominal circuit (e.g., topology and component parameter values). Thus, the faulty components are identified by checking whether their parameter values are within or outside the design tolerance range. The first work using this concept was published by Berkowitz [11], whose approach was mainly based on current measurements. Subsequent approaches based on this concept have not only been concentrated on the solvability problem but also on the use of voltage measurements [2 - 4]. Furthermore, the idea of component parameter values calculation under limited measurements have been developed in recent papers [5], [10], [12], and [31].

The concept of <u>Fault Simulation</u> is used in some approaches, whose basis is the generation of fault dictionaries. The approach consists of simulating all possible component failures, by changing the values of the parameters. Simulated measurements are made at certain test points and a computer sorting program used to build up a table of fault signatures for component parameter values or sets of them, corresponding to each measurement; this table is called a Fault Dictionary and contains the information for all potential fault conditions. In the actual test, the measured characteristics are compared to the fault signatures recorded in the dictionary, such that the fault is isolated as the one which gives the closest match to an entry in the fault dictionary. One of the first works in this direction was published by S. Seshu and R. Waxman [32]. In their approach, a fault dictionary was generated from the evaluation of the transfer function (Gain) with different component parameter values at different test frequencies. Other works using the Fault Simulation concept have used different techniques to generate the dictionaries. Some of these techniques are the Bilinear transformation [33], DC node voltages for nonlinear analog circuits [34], and state space analysis of linear circuits [35]. In more recent approaches, the generation of comprehensive dictionaries has been avoided, instead more sophisticated techniques such as pattern recognition and fuzzy sets [22 - 27] have been used to represent the fault conditions.

In addition to these two groups, the concept of Component connection model [36 - 39] has recently been introduced in approaches dealing with large scale systems. In this case, the fault diagnosis equations are analytically derived expressing the relationship between the external measurable system bahaviour and the internal parameters, so that by measuring these external parameters, a set of nonlinear algebraic equations is solved for the internal parameters to finally locate the faulty component.

Although these concepts could be taken as a classification criterion for existing fault diagnosis methods, Duhamel and Rault [1] suggested some other different criteria. Some of these criteria are: the purpose of testing, mode of representation of the behaviour of the unit under test, mathematical techniques, type and nature of measured responses, and type of excitation signals. For the purpose of this review, the mathematical techniques will be adopted as a classification criterion. On this basis, two categories of methods (Fig. 2.1) will be described -deterministic and non-deterministic- with the emphasis being placed on linear circuits and systems. The non-deterministic methods include approaches based on probabilistic techniques, pattern recognition techniques, and fuzzy set techniques.

2.2 DETERMINISTIC METHODS

The general principle of these methods is based on the calculation of the UUT component parameter values from measurements performed at test terminals and analytic relations that describe the UUT. Appropriate equations are formulated for the faulty unit; these equations are solved by numerical methods to find out the component parameter values. In the case that all the internal nodes are accessible, the values of the component parameters can be found by solving a set of linear equations [2 - 4], and [11]. If this is not the case, the system can be described by a set of nonlinear equations. Navid and Willson [31] have formulated the sufficient conditions for the solution of these equations.

Calculation of all the component parameter values is computationally expensive, and most of the internal nodes have to be accessible to formulate the equations. In order to avoid these problems, some approaches have been developed to deal with selected component parameter identification [5], and [13 - 14]; this requires that some component aparameter values are to be calculated and the others are known or assumed to be known.

2.2.1 ADJOINT CIRCUIT CONCEPT

The general idea of the methods using the adjoint circuit concept [2 - 5] is to obtain a linear relation between the component parameter deviations from their nominal values and test point voltage measurements. A sufficient number of circuit measurements and adjoint circuit simulations are required to obtain a set of linearly independent algebraic equations equal to the number of unknown component parameters. The values of the component parameters are computed by solving the set of linear algebraic equations.

(a) Description

Consider a linear circuit N which has m ports available, and b internal branches whose respective branch voltages and currents are denoted by $\{V_k, I_k\}$. Furthermore, let $\hat{N}$ be the adjoint circuit of N with the same topology but not necessarily the same branch and ports constraints. Under these conditions and assuming that these circuits obey Kirchhoff's laws, by means of Tellegen's theorem [56], the following equation can be derived:

$$\sum_{k=1}^{b} (\hat{V}_k I_k - V_k \hat{I}_k) = \sum_{j=1}^{m} (\hat{V}_{pj} I_{pj} - V_{pj} \hat{I}_{pj}) \qquad (2.1)$$

Next, suppose that one or more faults occurs in the original circuit N so that voltages and currents are changed to new values as follows: $V_k \rightarrow V_k + \Delta V_k$, $I_k \rightarrow I_k + \Delta I_k$; $V_{pj} \rightarrow V_{pj} + \Delta V_{pj}$, $I_{pj} \rightarrow I_{pj} + \Delta I_{pj}$. Introducing these changes in equation (2.1), a new equation is formed from which equation (2.1) can be subtracted to give

$$\sum_{k=1}^{b} (\hat{V}_k \Delta I_k - \Delta V_k \hat{I}_k) = \sum_{j=1}^{m} (\hat{V}_{pj} \Delta I_{pj} - \Delta V_{pj} \hat{I}_{pj}) \quad (2.2)$$

This equation shows the relationship between the voltage and current changes in circuit N, which arise due to the component parameter changes, to the voltages and currents in the adjoint circuit $\hat{N}$.

Subsequently, the branch constraints of circuit $\hat{N}$ are chosen in such a way that this circuit is the adjoint of circuit N; thus, introducing these constraints in equation (2.2), the required linear relationship is obtained. To solve for the unknown component parameter changes, the generation of a set of independent linear equations is needed, with rank equal to the number of unknown component parameters. This system of equations can be obtained by properly choosing the test conditions in circuit N, so that the values of the unknowns are calculated by a simple matrix inversion.

In [5], it is assumed that circuits N and $\hat{N}$ have only two test ports, so that in the right hand side of equation (2.2), the variable j goes from 1 to 2. Adequate choice of the use of first-order sensitivities reduces equation (2.2) to a new equation

$$\sum_{k=1}^{b} S_{P_k}^{V_{pj}} (1 + \Delta X_k / X_k) \Delta P_k / P_k = \Delta V_{pj} / V_{pj} \quad , \ j=1,2 \quad (2.3)$$

where X_k refers to the kth branch response, P_k is the kth component parameter, ΔV_{pj} is the change in the jth port voltage, and $S_{P_k}^{V_{pj}}$ is the first-order normalized sensitivity of port voltage V_{pj} with respect to P_k. Due to the fact that only two test points are accessible, some ΔX_k's cannot be measured; thus, the term $(1 + \Delta X_k/X_k)\,\Delta P_k\,/\,P_k$ is considered as the unknown, and is represented by ΔZ_k, k ranging from 1 to b. Equation (2.3) may now be written in matrix form.

$$\begin{bmatrix} S_{P_1}^{V_{p1}} & S_{P_2}^{V_{p1}} & \cdots & S_{P_b}^{V_{p1}} \\ S_{P_1}^{V_{p2}} & S_{P_2}^{V_{p2}} & \cdots & S_{P_b}^{V_{p2}} \end{bmatrix} \begin{bmatrix} \Delta Z_1 \\ \vdots \\ \Delta Z_b \end{bmatrix} = \begin{bmatrix} \Delta V_{p1}/V_{p1} \\ \Delta V_{p2}/V_{p2} \end{bmatrix} \tag{2.4}$$

This equation has an infinite number of solutions, but considering the port constraints, the fault can only be isolated if it is assumed that there is a single faulty component and the others are equal to their nominal values, so that $\Delta Z_k \neq 0$ for some k, and $\Delta Z_i = 0$ for $k \neq i$. Therefore, the isolation of the fault is performed by first solving the first equation in (2.4) b times to obtain $\Delta Z_k^{(1)}$, next the procedure is repeated for the second equation in (2.4) to obtain $\Delta Z_k^{(2)}$, and finally, both results are compared forming a difference vector

$$D_k = |\Delta Z_k^{(1)} - \Delta Z_k^{(2)}| \quad k = 1,2,3,\ldots,b \tag{2.5}$$

The faulty component is isolated as the one for which $D_k=0$.

(b) Domain of Application

(1) Linear circuits with possible extension to nonlinear circuits,

(2) Single and multiple faults (number of faults that can be isolated is one less than the number of test points),

(3) Deviation faults,

(4) Isolation and/or Identification of faulty components.

(c) Test Preparation

(1) Test points and measurements selection,

(2) Adjoint circuit simulations,

(3) Conditions required to determine the circuit component parameter values,

(4) First-order sensitivities ([5])

(5) Nominal values and its associated tolerances.

(d) Test Procedure

(1) Measure voltages at the test points,

(2) Form a set of linear equations,

(3) Calculate component parameter values by solving the set of linear equations.

(e) Discussion

Approaches based on the adjoint circuit concept lead to the component parameter values, by solving the set of linear equations. This information not only isolates the faulty components, but also allows for the possibility of tuning if the diagnosis is performed during manufacture;

another advantage is that it is computationally simple and does not require large amount of storage. The disadvantage of this method is that, the branch voltage of the unknown component parameters must be available to generate the set of linear equations. Otherwise if sufficient test measurements are not performed, it is impossible to solve for the component parameter values via linear equations. This problem has been partially solved in [5], using first-order sensitivities at the expense that nonfaulty component parameters must take their nominal values (single fault).

2.2.2 NODAL ANALYSIS METHOD

This well known technique is used to formulate the appropriate system of equations for identification [6 - 10] of all unknown component parameters of an analogue circuit. It consists of nodal voltage measurements using different current excitations to provide the maximum number of independent equations. This method is also valid under "limited measurements" [6] and [10] providing that the circuit is "element-value solvable" [4] and [11 - 12].

(a) Description

Of the approaches mentioned above, all use this nodal analysis technique, but there is a slight difference in the way the component parameter identification is solved and the considerations taken into account. Therefore, only one of them will be described [7]; this being taken as representative of the whole group.

Here a relation is obtained between the node voltages and the node admittance matrix, from which the values of the circuit component parameters are determined. Consider a linear time invariant circuit N with (n + 1) nodes (the (n+1)th node as reference). The basic nodal equation of this circuit is obtained by suitable excitations.

$$[I_1, I_2, \ldots\ldots, I_n]^T = [Y]\,[V_1, V_2, \ldots\ldots, V_n]^T \qquad (2.6)$$

where I_k is the current in the kth node, [Y] is the node admittance matrix, and V_k is the voltage at the kth node. Assuming that [Y] is nonsingular, from (2.6), the following equation is derived

$$[V_1, V_2, \ldots\ldots, V_n]^T = [Y]^{-1}\,[I_1, I_2, \ldots\ldots, I_n]^T \qquad (2.7)$$

Next, suppose that the circuit N is excited by harmonic current vectors (only amplitudes are considered) belonging to the system S of orthonormal vectors of length n. The excitation vector S_k is given as $S_k = [0,0,\ldots,1,\ldots,0,0]^T$, and if the circuit N is excited by S_k, the current vector of equation (2.7) will have a unity current at the kth row. Repeating this procedure for all $S_k \in S$, it is possible to derive a number of equations whose combination gives the following system of equations.

$$\begin{bmatrix} V_1^{(1)} & V_1^{(2)} \ldots\ldots & V_1^{(k)} \ldots\ldots & V_1^{(n)} \\ V_2^{(1)} & V_2^{(2)} \ldots\ldots & V_2^{(k)} \ldots\ldots & V_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ V_i^{(1)} & V_i^{(2)} \ldots\ldots & V_i^{(k)} \ldots\ldots & V_i^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ V_n^{(1)} & V_n^{(2)} \ldots\ldots & V_n^{(k)} \ldots\ldots & V_n^{(k)} \end{bmatrix} = [Y]^{-1}\,[I] = [Z] \qquad (2.8)$$

where [I] is the unit matrix and [V] is the node voltages matrix, whose $V_i^{(k)}$ element is the voltage of the ith node when the kth node is excited by a unit current.

From equation (2.8), [Y] may be derived: $[Y] = [V]^{-1}$. The values of the component parameters are derived from this equation by appropriately decomposing matrix [Y]. If N is a reciprocal circuit, the component parameter values are calculated by the following equations.

$$Z_{ik} = -\frac{1}{y_{ik}} \qquad i,k \in (n-1) \quad i \neq n+1$$

and

$$Z_{i,n+1} = \frac{1}{\sum_{t=1}^{n} y_{it}} \qquad (2.9)$$

where Z_{ik} is the impedance (at the excitation frequency) between nodes i and k, while $Z_{i,n+1}$ is the impedance between node i and the reference node.

(b) Domain of Application

(1) Linear analogue circuits,

(2) Single and multiple faults,

(3) Deviation (Soft) faults,

(4) Isolation and identification.

(c) Test Preparation

(1) Circuit topology,

(2) Nominal component parameter values and their associated tolerances,

(3) Selection of excitation signals and port of excitation,

(4) Ports of measurements.

(d) Test Procedure

(1) Application of excitation signal and node voltage measurements,

(2) Generate the system of linear equations to solve for unknown component parameter values,

(3) Isolate and/or identify the faulty component(s) by comparing actual values to the nominals, taking into consideration their associated tolerances.

(e) Discussion

Application of the nodal analysis technique to fault diagnosis problem provides a useful solution. In contrast with approaches based on the adjoint circuit concept, the fault-free circuit simulation is avoided, but the topology of the circuit must be known and provided to find a unique solution to the component parameter identification problem.

The main drawback of this method is that, all the internal nodes must be available for nodal voltage measurements, so that a system of linear equations can be generated to calculate the unknown component parameter values. However, because of the existence of some redundant measurements, the system of equations can be reduced systematically by checking whether the reduced system provides sufficient information for the component parameters identification [6] and [10].

2.2.3 BILINEAR DEPENDENCE METHOD

The bilinear dependence of network functions on its component parameters is used as a technique for single fault isolation as well as for a limited number of multiple faults [13 - 14]. It is based on voltage measurements with current excitations applied at a single frequency. The number of voltages measured has to be greater than the number of simultaneous faults to be isolated. Sets of linear equations are generated via adjoint circuit simulations for all possible fault combinations for nominal component parameter values; then if there is a fault, the isolation is performed by checking consistency or inconsistency of the sets of equations which are independent of faulty components.

(a) Description

The approaches [13 - 14] have been developed for linear and lumped analogue circuits. The theoretical basis of the method is derived under the assumption that nonfaulty component parameters assume their nominal values, though deviations from nominal within prescribed tolerances may be considered in the implementation of the method. The method leads to the isolation of a single as well as a limited number of simultaneous faults. Its foundation is the bilinear dependence of network functions.

Consider the case of a single fault for which there are two different circuit functions.

$$f_1 = \frac{A_1 + B_1 y}{C_1 + D_1 y} \quad , \qquad f_2 = \frac{A_2 + B_2 y}{C_2 + D_2 y} \tag{2.10}$$

If the two functions depend on y, then each of them can be solved for y. Furthermore, assuming that both are of the same type, then a linear relation is derived.

$$a \cdot f_1 + b \cdot f_2 = c \tag{2.11}$$

where $a \triangleq C_1 B_2 - D_1 A_2$, $b \triangleq A_1 D_2 - B_1 C_2$ and
$c \triangleq A_1 B_2 - B_1 A_2$.

Equation (2.11) gives the relationship between values of f_1 and f_2 for y as faulty and all the other component parameters at nominal. Supposing that the circuit contains p component parameters, similar relationships between f_1 and f_2 can be derived by generating a system of p equations.

$$a^i f_1 + b^i f_2 = c^i \qquad i = 1,2,\ldots\ldots,p \tag{2.12}$$

For nominal conditions, all p equations will be satisfied. If there is a single fault in the circuit, then only the equation which corresponds to the faulty component parameter will be satisfied since all other component parameters are at their nominal values. This situation is clarified if the deviations Δf_1 and Δf_2 are used instead of the actual measured values of f_1 and f_2 such that the system of equations (2.12) becomes a system of homogeneous equations.

$$a^i \Delta f_1 + b^i \Delta f_2 = 0 \qquad i = 1,2,\ldots\ldots,p \qquad (2.13)$$

The actual values of functions f_1 and f_2 are determined by measuring voltages at two testing ports, corresponding to current excitations. Then, from knowledge of nominal values, the Δf_1 and Δf_2 are derived, which in fact corresponds to the voltages measurement deviations ΔV_1^m and ΔV_2^m respectively. The coefficients for equations (2.13) are determined either by measurements on as many p (number of component parameters) different 4-port circuits or by two adjoint circuit simulations.

Of the two suggestions, the latter is adopted for computational convenience. For the first simulation a unit current is applied to the first measurement port and the voltages across all component parameters $\hat{V}_{11}, \hat{V}_{21}, \ldots\ldots, \hat{V}_{p1}$ are calculated; for the second simulation, a unit current is applied to the second measurement port to find $\hat{V}_{12}$, $\hat{V}_{22}$, $\ldots\ldots, \hat{V}_{p2}$. Therefore equation (2.13) is rewritten as follows.

$$\hat{V}_{i2}\Delta V_1^m - \hat{V}_{i1}\Delta V_2^m = 0 \qquad i = 1,2,\ldots\ldots,p \qquad (2.14)$$

These simulations are performed with all the component parameters at nominal values, and the voltages across the component parameters represent trans-impedances.

This approach developed for a single fault has been extended and generalized for several simultaneous faults [14]. Supposing that there are k faults within the circuit. These are represented as external loads of an (n + k) port network, where n is the number of ports of measurement and k the number of ports of fault, with $k \leq n - 1$. For the ports of measurement, the voltages are represented by the vector V^m and the currents by the vector I^m; a similar representation describes the ports of fault where the vector V^x corresponds to the voltages and the vector I^x to the currents.

Making the assumption that the impedance matrix of the (n + k) port network exists, the following equation is formulated.

$$\begin{bmatrix} V^m \\ V^x \end{bmatrix} = \begin{bmatrix} Z_{mm} & Z_{mx} \\ Z_{xm} & Z_{xx} \end{bmatrix} \begin{bmatrix} I^m \\ I^x \end{bmatrix} \qquad (2.15)$$

If the ports of measurement are open- circuit or are excited by independent current sources, the voltage change vector ΔV^m is found from equation (2.15).

$$\Delta V^m = Z_{mx} I^x \qquad (2.16)$$

Z_{mx} is a rectangular matrix with more rows than columns. Taking this fact into consideration the solution of equation (2.16) is (for Z_{mx} as a full column rank matrix)

$$I^x = (Z_{mx}^T Z_{mx})^{-1} Z_{mx}^T \Delta V^m \qquad (2.17)$$

From equations (2.16) and (2.17), the generalized equation corresponding to several simultaneous faults is derived as an extension of (2.14).

$$[Z_{mx}(Z_{mx}^T Z_{mx})^{-1} Z_{mx}^T - 1]\Delta V^m = 0 \qquad (2.18)$$

For the k simultaneous faults, one needs to generate a linear system of equations similar to (2.18) for all possible combinations consisting of k component parameters.

$$(\overline{Z}_{mx}^i - 1)\Delta V^m = 0 \qquad i = 1,2,\ldots\ldots,\binom{p}{k} \qquad (2.19)$$

where $\overline{Z}_{mx} = Z_{mx}(Z_{mx}^T Z_{mx})^{-1} Z_{mx}^T$, and matrices $\overline{Z}_{mx}^i$, as in the case of single fault are calculated by adjoint circuit simulations with all the component parameters at nominal. A unit current is applied to each port of measurement and voltages are calculated across all component parameters. Then, by selecting the values corresponding to the combinations of k parameters, the matrix Z_{mx} is formed.

$$Z_{mx}^{T} = [\hat{V}^{x_1}\ldots\ldots\hat{V}^{x_n}] \qquad (2.20)$$

Therefore, n adjoint circuit simulations are needed in order to obtain the coefficients of equation (2.19) for all possible combinations of k component parameters. If there are k faults, then isolation is performed in the same way as for a single fault, that is, checking the equations (2.19).

(b) Domain of Application

(1) Linear analogue circuits,

(2) Single and multiple faults,

(3) Deviation and catastrophic faults,

(4) Isolation only.

(c) Test Preparation

(1) Selection of excitation port(s) and measurement ports,

(2) Nominal responses at the ports of measurement,

(3) n adjoint circuit simulations, where n is the number of measurement ports,

(4) Formulation of the systems of equations for the k simultaneous faults (eq. 2.19),

(5) Check the existence of the impedance matrix of the (n + k) port circuit.

(d) Test Procedure

(1) Apply a unit current at the port(s) of excitation,

(2) Measure voltages at the ports of measurement and calculate their deviations from nominal responses,

(3) Isolate the faulty components by checking the consistency or inconsistency of equations (2.19). The equation which is satisfied isolates the faulty components.

(e) Discussion

The method described is based on simple and well known techniques such as the adjoint circuit simulation and the bilinear dependence of circuit functions on its component parameters. In contrast with some other deterministic methods, it only leads to the isolation of single as well as multiple soft and catastrophic faults. The number of simultaneous faults that can be isolated is determined by the number of available ports of measurement (usually, the number of faults is one less than the number of ports of measurement).

From the practical point of view, it has the advantages that a limited number of simultaneous faults can be considered, so that the calculation of all component parameter values is avoided, and that all internal nodes do not need to be available for measurements. Another possible advantage is the fact that tolerances on the nonfaulty component parameters may be taken into consideration in the implementation of the method, so that it is possible to deal with more realistic situations; in most cases nonfaulty component parameter values are within a prescribed tolerance range and not at nominal values.

Recently, the same authors, together with A. E. Salama [15] have extended this method to deal with tolerance on the nonfaulty component parameters; they have introduced a two-step algorithm. First, an approximate method is used, which consists of defining an error parameter for every circuit element. This error is related to the deviation in the circuit component parameter value from nominal, so that a system of linear equations is constructed and a linear programming formulation is used to isolate the most likely faulty components. Second, a verification method is used to verify the results of the approximate method; the bilinear approach [13] is extended to compensate for the uncertainty caused by the tolerance of the component parameters. This procedure involves taking more measurements than those sufficient for the zero tolerance case, to construct the system of linear equations that are invariant on faulty component parameters. Then, one can check that the equations are satisfied by searching for a feasible tolerance vector, using the linear programming formulation.

The amount of data to be generated is one of the main limitations of this method, because it (the data) increases rapidly with the factor $\binom{p}{k}$ as the number of simultaneous faults increases (with p being the total number of component parameters, and k the number of simultaneous faults). Thus, one would have to generate p equations for single faults, $\binom{p}{2}$ matrix equations for double faults, $\binom{p}{3}$ matrix equations for three simultaneous faults, etc. Another aspect for consideration is that, in some cases, more than one of the equations (2.19) could be satisfied and isolation would not be unique to a particular combination set of simultaneous faults; this may suggest that more alternative measurements would be needed to achieve unique isolation.

2.2.4 ESTIMATION METHOD

Approaches within this category [16 - 18] do not calculate the exact values of the component parameters, but estimate their actual values. Generally, the deviations of the measured responses from nominal are represented through a set of linear equations $\Delta X = A\,\Delta Y$, where A is the sensitivity matrix, and vectors ΔX and ΔY are the deviations between actual and nominal values for the measured responses and component parameters respectively. Then, the objective is to determine the most likely faulty component from the sensitivity matrix A and the measured responses, according to a factor of merit chosen to characterize the difference between nominal and faulty characteristics. The important feature of this method is that nonfaulty component parameter values can be either at nominal or within their associated tolerances, though near the nominals, otherwise the method may give erroneous results.

(a) Description

The approach developed by C. C. Wu et al [16] will be reviewed, and the other two approaches [17 - 18] are described in the review paper by P. Duhamel and J. C. Rault [1].

The approach proposes a simulation before test algorithm for analogue fault diagnosis, in which a differential-interpolative technique is used to eliminate the ambiguity caused by tolerance effects of the nonfaulty component parameters. In the simulation process, it is assumed that a limited number of components fail simultaneously, with the others remaining at nominal, so that faulty circuits need only be simulated at a discrete set of points along the coordinate axes in the component parameter space. For each simulated circuit its responses and sensitivity matrices are evaluated.

At the testing process, a fault is detected which may be located anywhere in the component parameter space. One of two situations may arise. Firstly, the failure may occur anywhere along the axis with good component parameters at nominal, so that some sort of approximation scheme is required to interpolate between the previous discrete simulations. Secondly, the good component parameter may not be at nominal, but within their associated tolerance, which suggest that the solution of the problem lies near but not necessarily on the coordinate axis. The first order Taylor series is used to approximate the deviation caused by nonfaulty component parameter tolerances.

For a single faulty component, the general region is located by the differential-interpolative approach which uses a classical minimum distance algorithm. Thus, assuming that y is the faulty component parameter vector, f(y) and $J^{-1}(y)$ denote, respectively, the approximated functions for the system responses and the associated inverse sensitivity matrices along the axis, which are derived from the interpolation of the previous simulated data points. Next, if z and m represent the actual faulty component parameter vector and the system responses respectively; then, using the first order Taylor series approximation combined with the interpolation approach, the measured responses could be approximated by the following equation:

$$m = f(y) + J(y)[z - y] \tag{2.21}$$

for those values of y near z. Consequently, the vector of interest [z - y] is written as follows.

$$[z - y] = J^{-1}(y)\,[m - f(y)] \tag{2.22}$$

In order to solve this equation for the faulty component parameter value y in a more suitable way, equation (2.22) could be transformed into a scalar equation, which also eliminates the requirement of storing the inverse sensitivity matrices. This transformation is possible, because the vector [z - y] is perpendicular to the axis at the point y, thus if e_y denotes the unit vector in the direction of the faulty component parameter axis, then equation (2.22) is re-written as

$$0 = e_y^T[z - y] = e_y^T J^{-1}(y)[m - f(y)] \tag{2.23}$$

which can be solved for the faulty component parameter value y. In this case, the only information to be stored are the simulated circuit responses f_i ($i = 1,2,\ldots\ldots,n$ responses), and the vectors $e_y^T J_i^{-1}$ instead of the inverse sensitivity matrices.

(b) Domain of Application

(1) Linear and nonlinear analogue circuits,
(2) Single and multiple faults,
(3) Deviation faults,
(4) Isolation and identification,

(c) Test Preparation

(1) For each faulty component parameter, simulate faulty circuits at discrete points along the coordinate axes, keeping the nonfaulty component parameters at nominal.
(2) For each faulty circuit evaluate the required responses and their sensitivities, storing the results.

(d) Test Procedure

(1) Measure the required testing responses,
(2) Locate the general region of the faulty component parameter by a classical minimum distance algorithm,
(3) Approximate the circuit responses and the associated inverse sensitivity matrix by interpolating the before test simulation data along the coordinate axes of the faulty component parameter,
(4) Calculate the value of the faulty component parameter by solving the appropriate equation. For a single fault, it could be either equation (2.22) or (2.23).

(e) Discussion

The main advantage of the differential-interpolative approach lies in the consideration of the tolerance effect of nonfaulty component parameters, although diagnosis is reliable only for small component parameter deviations, because a first order Taylor series approximation is used. The fact that the pre-test simulations are performed at discrete points on the coordinate axes of the faulty component parameters could also be considered as an advantage, specially for linear circuits, where a very small number of simulations is thought to be needed to approximate by interpolation the response and the associated sensitivity of a faulty circuit.

Although great effort is not involved in the simulation process the disadvantage of the method lies in the storage requirements, because the evaluated responses and their associated inverse sensitivity matrices must be stored for each simulated faulty circuit. Some saving can be achieved by storing the inverse sensitivity matrices in vector form which could be performed by multiplying the matrices by a unit vector in the direction of the axes of the faulty component parameter.

2.2.5 A DETERMINISTIC EXAMPLE

The circuit shown in Fig. 2.2a is taken as an example to illustrate the application of one of the deterministic methods. In this case, the approach developed by L.M. Roytman and M.N.S. Swamy [7] is used to determine the component parameter values from node voltage measurements.

For nominal conditions, the component parameter values are: $R_1 = 2\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_4 = 4\Omega$, $R_5 = 6\Omega$, and all the circuit nodes are assumed to be accessible for measurements. Under faulty conditions, it is necessary to identify the faulty components. This is achieved in the following way. A 1A current source is applied in turn to each of the circuit nodes (Figs. 2.2b - 2.2d), and each time the node voltages are measured, to finally form the node voltages matrix V, of the type of equation (2.8).

$$[V] = \begin{bmatrix} 14/17 & 5/17 & 3/17 \\ 5/17 & 20/17 & 12/17 \\ 3/17 & 12/17 & 48/17 \end{bmatrix}$$

This matrix is then inverted to find the admitance matrix Y.

$$[Y] = [V]^{-1} = \begin{bmatrix} 4/3 & -1/3 & 0 \\ -1/3 & 13/12 & -1/4 \\ 0 & -1/4 & 5/12 \end{bmatrix}$$

The individual component parameter values are then calculated from matrix Y, according to equation (2.9)

$$R_1 = \frac{1}{y_{11} + y_{12} + y_{13}} \qquad R_1 = 1\Omega$$

$$R_2 = -\frac{1}{y_{12}} \qquad R_2 = 3\Omega$$

$$R_3 = \frac{1}{y_{21} + y_{22} + y_{23}} \qquad R_3 = 2\Omega$$

$$R_4 = -\frac{1}{y_{23}} \qquad R_4 = 4\Omega$$

$$R_5 = \frac{1}{y_{31} + y_{32} + y_{33}} \qquad R_5 = 6\Omega$$

Comparing this result with the nominal values, it is clearly seen that component R_1 is the faulty one, because it has deviated 50% of its original nominal value, while the rest has remained unchanged.

This example could be considered typical within the deterministic category in two respects. First, it is based on node voltage measurements. Second, in order to determine the value of each individual component parameter, it is almost always necessary to have access to all circuit nodes. However, the advantage is that, it gives a more clear decision (100% confidence) in the diagnosis of the faulty components.

2.3 NON-DETERMINISTIC METHODS

Approaches based on three different mathematical techniques are classified within this category: Probabilistic or Statistical methods; Pattern Recognition methods; and Fuzzy Set methods. The common characteristic of these methods is that they lead only to the most likely faulty components and do not calculate their actual values; the other important factor is the consideration of the effect of the associated production tolerances of nonfaulty component parameters.

Probabilistic methods consist of determining the probability of occurence, for each component parameter of the circuit, from the measured responses and the associated statistical component parameters distribution.

Pattern recognition methods are based on classification of a response pattern vector corresponding to a specific fault, as belonging to one of several predetermined categories, in which the pattern is known to belong.

Fuzzy set methods consider the overlapping effect produced at the boundaries between the various fault cases (represented as regions), so that belonging to a given region or pattern is weighted by a function whose value lies between 0 and 1.

2.3.1 PROBABILISTIC METHODS

The approaches belonging to this category [19 - 21] determine the probability of fault occurence associated with each individual component parameter from the measured responses and the statistical distribution of the component parameter values. These methods lead to the most likely faulty component with an acceptable diagnosis resolution, even in cases where the number of measurements is small compared to the number of component parameters of the unit under test.

(a) Description

The "Optimum Fault Isolation by Statistical Inference" approach developed by S. Freeman [20] will be described below. The approach is developed for the isolation of catastrophic faults as well as noncatastrophic ones, while the associated tolerances of the nonfaulty component parameters are considered. The isolation is performed by determining the actual probability of each fault based on the values of the measured characteristics.

The unit under test is assumed to have one of a finite number of faults indexed by $j = 1,2,\ldots\ldots,J$, or else to be faultless which is treated as $j = 0$ fault type. The number of measurements performed are indexed by $i = 1,2,\ldots\ldots,I$. The nominal response with no measurement errors is represented by

$$M_i^{(j)} \tag{2.24}$$

corresponding to the ith measurement if the UUT had fault type j.

Next, supposing that the UUT has a fault of type j, then the measured values differ from the nominal values $M_i^{(j)}$ due to departures of component parameter values from nominal values and to measurement errors. The actual measured values is then written as

$$M_i = M_i^{(j)} + \sum_{k=1}^{K} \delta_k S_{ki}^{(j)} + \varepsilon_i^{(j)} \qquad (2.25)$$

where δ_k^* represents the component parameter values deviation from nominal values; $S_{ki}^{(j)}$ is the sensitivity of the ith measurement to the kth component parameter for system j; and $\varepsilon_i^{(j)}$ is the error term which is purely random and affects only the ith measurement. $\varepsilon_i^{(j)}$ includes sampling error, circuit noise, roundoff error, and any other random error having some effect on the measured value.

The terms δ's and ε's are assumed to be statistically independent with a multivariate normal distribution. Their means are supposed to be zero, so that they are represented by their standard deviations ω_k and $\sigma_i^{(j)}$ respectively; then the variances can be written as follows:

$$\mathrm{Var}(\delta_k) = \omega_k^2 \qquad (2.26)$$

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* It is assumed that the values of δ_k are sufficiently small that a linear approximation to the effect of δ_k on M_i is adequate.

$$\mathrm{Var}(\varepsilon_i^{(j)}) = [\sigma_i^{(j)}]^2 \qquad (2.27)$$

The isolation of a fault as being of type j, from a set of measurement $\tilde{M}$ ($\tilde{M}$ has components M_i) is performed by determining the conditional probability Prob($j/\tilde{M}$) which is defined from Bayes' theorem as

$$\mathrm{Prob}(j/\tilde{M}) = \frac{\mathrm{Prob}(\tilde{M}/j)p_j}{\mathrm{Prob}(\tilde{M})} \qquad (2.28)$$

Optimum isolation is achieved by picking the j, for which, equation (2.28) is maximum, in other words, the fault type j which has the smallest probability of being wrong for a particular set of measurements $\tilde{M}$. Prob($\tilde{M}/j$) is the probability of measuring the values M_i ($i=1,2,\ldots\ldots,I$) for a UUT of type j; p_j is the prior probability that the UUT is of type j, and Prob($\tilde{M}$) is the overall probability of measuring $\tilde{M}$.

The Prob($\tilde{M}/j$) in equation (2.28) is evaluated directly from equations (2.25), (2.26), and (2.27), so that in terms of the available information, this probability can be written as follows:

$$\mathrm{Prob}(\tilde{M}/j) = \frac{\exp\{-\frac{1}{2}(\tilde{M} - \tilde{M}^{(j)})[U^{(j)}]^{-1}(\tilde{M} - \tilde{M}^{(j)})\}}{(2\pi)^{I/2}[\det U^{(j)}]^{\frac{1}{2}}} \qquad (2.29)$$

where the vector $\tilde{M}^{(j)}$ has components $M_i^{(j)}$; $[U^{(j)}]^{-1}$ is the matrix inverse of the symmetric matrix $U^{(j)}$, whose elements are:

$$(U^{(j)})_{ir} = \delta_{ir}[\sigma_i^{(j)}]^2 + \sum_{k=1}^{K} S_{ki}^{(j)} S_{kr}^{(j)} \omega_k^2 \qquad (2.30)$$

with δ_{ir} being the Kronecker delta ($r= 1,2,\ldots\ldots,I$). The exponential term of equation (2.29) can be written in terms of individual components of vectors $\tilde{M}$ and $\tilde{M}^{(j)}$, and matrix $[U^{(j)}]^{-1}$.

$$-\tfrac{1}{2} \sum_{i=1}^{I} \sum_{r=1}^{I} (M_i - M_i^{(j)})(M_r - M_r^{(j)})\{[U^{(j)}]^{-1}\}_{ir} \qquad (2.31)$$

Thus, a fault of type j is optimally isolated by minimizing the following expression.

$$\sum_{i=1}^{I} \sum_{r=1}^{I} (M_i - M_i^{(j)})(M_r - M_r^{(j)})\{[U^{(j)}]^{-1}\}_{ir} \qquad (2.32)$$

$$+ \ln[\det U^{(j)}/p_j^2]$$

Finally, the probability that this choice is correct is simply $Prob(j/\tilde{M})$ calculated from equations (2.28) and (2.29)

$$Prob(j/\tilde{M}) = \exp[-\tfrac{1}{2}D_j(\tilde{M})] / \sum_{j=1}^{J} \exp[-\tfrac{1}{2}D_j(\tilde{M})] \qquad (2.33)$$

where $D_j(\tilde{M})$ is equal to equation (2.32)

(b) Domain of Application

(1) Linear analogue circuits,

(2) Single faults considering associated tolerances of nonfaulty component parameters,

(3) Catastrophic and noncatastrophic faults,

(4) Isolation to the most likely faulty component.

(c) Test Preparation

(1) Evaluation of the nominal responses $M_i^{(j)}$,

(2) Definition of all the different fault types j, (j = 1,2,......,J),

(3) Determination of the prior probability p_j of each fault type j,

(4) Probability distribution function of all the component parameter deviations δ_k (k = 1,2,......,K) and the random error term $\varepsilon_i^{(j)}$ (i = 1,2,......,I measurements and j = 1,2,......,J fault types). This also includes knowledge of the means and standard deviations or variances of δ_k and $\varepsilon_i^{(j)}$ (Equations (2.26 and (2.27)),

(5) Calculation of the sensitivities $S_{ki}^{(j)}$ (sensitivity of the ith measurement to the kth component parameter for fault type j),

(6) The following information must be stored: the nominal responses $M_i^{(j)}$; the variance of each component parameter inside its tolerance range ω_k^2; the variance of the random error variable $[\sigma_i^{(j)}]^2$; the prior probability p_j and the sensitivities $S_{ki}^{(j)}$,

(7) No assumption is made about the selection of test points.

(d) Test Procedure

(1) Measurement of the set of M_i response values ($i = 1,2, ..., I$),

(2) Forming the $[U^{(j)}]$ matrix whose components are defined by equation (2.30),

(3) Isolation of the most likely fault type j, performed by evaluation of equations (2.32) and (2.33).

(e) Discussion

In general, probabilistic methods provide a useful solution for the isolation of a single fault, while the associated tolerance of nonfaulty component parameters are considered. Isolation is performed only to the most likely faulty component, and it depends very much on the accuracy of the pre-test data, such as, the prior probability of each fault condition, the statistical model of the tolerance effect of nonfaulty component parameters (represented by means and variances), and in some cases [20] the random error, which usually includes sampling error, circuit noise and roundoff error. The approach described above [20] also evaluates the sensitivity of the responses with respect to the component parameters, during the pre-test data generation. The generation of all this data is cumbersome and constitutes one of the main disadvantages of probabilistic methods. Another disadvantage is the calculation of the determinant and the inversion of the covariance matrix used by approaches [19] and [20].

A common factor used for probabilistic methods is that, identification of the most likely fault is realized by maximizing the posterior probability of a fault associated with each component parameter, which is based on the Bayes' theorem. Approaches [20] and [21] also provide algorithms to select an optimum set of measurements to perform the isolation, so that a limited number of measurement could be used in the calculation of the isolation criterion. Finally, although the approach developed by S. Freeman [20] is applicable to either catastrophic or noncatastrophic faults, the others are restricted to the noncatastrophic.

2.3.2 PATTERN RECOGNITION METHOD

The basic principle outlining this method [22 - 25] is the classification of a pattern vector, corresponding to an unknown fault, as belonging to one of several predetermined fault categories.

Fault signatures of likely faults are usually obtained via simulation of a mathematical model of the UUT or by physically changing component parameter values to impose faults in a breadboard model/hardware UUT or by accumulating past failure data for a UUT already in service. Then comparing measured characteristics to these fault signatures, classification is achieved based on a pattern recognition decision process regarding the membership of the unknown fault characterized by the measured response.

Practical implementation of the method differ mainly in the techniques used to achieve the classification. K. C. Varghese et al [22] and C. Morgan et al [24] used the nearest neighbour rule (NNR) to classify an unknown fault to a group of components and then to a particular component; H. Sriyananda et al [23] (voting technique) correlates the measured responses with the fault signatures, and isolates the faulty component as that giving the highest correlation factor.

The approach developed by J. K. Fidler and R. J. Mack could also be classified within this category. They use an error criterion computed at a set of frequencies, which span the frequencies of maximum sensitivity and discrimination for each component. Thus, measuring the required responses at these frequencies, an accumulated error is formed for each component; these errors are ranked in magnitude, and the faulty component is isolated as the one for which the error is minimum.

(a) Description

The "Simplified ATPS and Analog Fault Location Via a Clustering and Separability Technique" approach developed by K. C. Varghese et al [22] will be described. The approach presents an Automatic test programme generation (ATPG) procedure for analogue system fault location, and a procedure for generating a set of tests for linear circuits using gain and phase measurements from input-output ports. It provides an optimization procedure for selecting the best subset of features for fault diagnosis, which incorporates a confidence level or performance index for subsets of features, this being based on the separability of fault signatures. The separability measure is used to group fault signatures (clustering) with low separability, so that a reduction in the number of fault cases to be considered on-line is achieved. In fault location, a fault is first identified as belonging to a specific group and then the particular faulty component can be identified in that group.

A Fault signature or pattern corresponding to a particular fault is evaluated as the deviation in response from nominal, which is further normalized via a checkout deviation $\pm g_i$ defined for each measurement, and then euclidean normalized; this pattern is represented by the vector $x^* = [x_1^*, x_2^*, \ldots\ldots, x_M^*]^T$, with elements x_i^* representing the normalized value of specific measurements, and the dimension M corresponding to the number of measurements used to represent the pattern. Thus, pattern vectors in the M space representing all likely faults are grouped to form a recognition matrix

$$X = [\ x_{iJ}^*\ ] \quad \begin{array}{l} i=1,2,\ldots,M \text{ measurements} \\ J=1,2,\ldots,N \text{ fault cases} \end{array} \qquad (2.34)$$

Furthermore, this matrix is made sparse by eliminating entries (they are set to zero) which are less than a prescribed threshold, typically set up as (1/3M) of the following expression

$$\text{CSUM} = \sum_{i=1}^{M} x_i^* \tag{2.35}$$

Thus, the final a priori information matrix X becomes sparse with several of its entries set to zero.

The next step concerns the selection of the best feature subset with the subsequent cluster formation for grouping and reducing the number of fault cases. The separability measure between fault cases and the discriminatory information of fault cases are used as bases for feature selection. The separability measure between any two fault cases J_k and J_ℓ is calculated according to the following expression.

$$d(J_k, J_\ell) = \left[\sum_{i=1}^{M} (x_{iJ_k}^* - x_{iJ_\ell}^*)^2 \right]^{\frac{1}{2}} \tag{2.36}$$

which defines N(N - 1)/2 separability spaces for the N fault cases problem and calculates distances that separate each fault case at the selected features. Due to the Euclidean normalization, this separability measure is bounded as $0 \leq d(J_k, J_\ell) \leq 2$, so that it can also be expressed in terms of percentage

$$\text{Percent Separability Measure} = \frac{\text{Separability Measure}}{\text{Maximum Sep. Measure}} \times 100 \tag{2.37}$$

The discriminatory information of fault cases for the ith feature is calculated according to the following expression

$$D_i = \left[\sum_{i=1}^{N-1}(x^*_{iJ} - x^*_{i(J+1)})^2\right]^{\frac{1}{2}} \qquad i=1,2,\ldots.,M \qquad (2.38)$$

Equations (2.36) to (2.38) are used to select the best feature subset for diagnosis purposes. An optimization procedure algorithm is provided in [22] to perform this task.

After the number of measurements has been reduced by the procedure described above, a further reduction in the number of fault cases is achieved by grouping (clustering) fault cases with a separability measure less than or equal to a prescribed threshold α (practically, this is set to 30 per cent of the maximum separability measure). This final recognition matrix containing Q features ($Q < M$) and R groups ($R < N$) is used for on-line diagnosis. Then, an unknown fault represented by the pattern vector $y = [y_1, y_2, \ldots\ldots, y_Q]^T$ is classified as belonging to one of the R groups, using the nearest neighbour rule,

$$d_J = \left[\sum_{i=1}^{Q}(z_{iJ} - y_i)^2\right]^{\frac{1}{2}} \qquad J=1,2,\ldots\ldots,R \qquad (2.39)$$

where z_{iJ} represents the mean value of the fault cases contained in the Jth group. Finally, the pattern vector y is assigned to the group for which d_J is minimum.

(b) Domain of Application

(1) Linear analogue systems with possible extension to nonlinear systems,

(2) Single faults, taking into account production tolerances of nonfaulty component parameters. It could also be extended to multiple faults,

(3) Noncatastrophic (soft) faults,

(4) Isolation to the most likely faulty component.

(c) Test Preparation

(1) Evaluation of the nominal system response vector,

(2) Definition of the N fault cases (generally N/2 is the number of possible faulty components),

(3) Evaluation of the response vector for each of the fault cases forming a deviation fault matrix (deviation is evaluated with respect to the nominal response),

(4) Normalize the previous matrix to form a recognition fault matrix (eq. 2.34),

(5) Make this matrix sparse by eliminating entries which are less than the threshold defined by equation (2.35), ((1/3M)CSUM),

(6) Select the best feature subset by using the optimization algorithm described in [22], which uses equations (2.36) to (2.38),

(7) Form clusters by grouping fault cases with low separability measure,

(8) Form the final recognition matrix to be used for on-line diagnosis to contain Q selected features and R groups.

(d) Test Procedure

(1) Measure the response of the faulty system forming a pattern vector,

(2) Obtain a deviation vector from nominal response with subsequent normalization, to form an euclidean normalized vector,

(3) Calculate distances according to equation (2.39) and classify the pattern vector as belonging to the group for which the distance d_J is minimum.

(e) Discussion

Although the approach described does not consider the effect of the tolerances on nonfaulty component parameters in the generation of the recognition matrix, their effect is taken into account in the actual testing to isolate the faulty component. This effect is also considered in the other approaches [23 - 25], so that the diagnosis process is based on a more realistic situation.

The results have a strong dependency on the structure and on the number of features used to represent the fault cases. This situation leads to the possibility of a trade-off between the accuracy of isolation and the number of features. Furthermore, due to the impossibility of discrimination between certain fault cases, there is always the need to form groups of fault cases, at the expense of losing information about individual fault cases. At the isolation stage, a particular fault is first located in a particular group with a relatively high probability of accuracy, and then the faulty component is located via declustering, but in this case, the probability of correct isolation is lower, in most of the cases.

As well as allowing consideration of the effect of the tolerances of nonfaulty component parameters, this approach can be extended to multiple faults in linear systems and single faults in nonlinear systems [24]. In the latter case, the noise effect is considerable and must be taken into account in improving the probability of correct isolation.

In general, pattern recognition methods provide a useful solution to the diagnosis problem, with the restriction that no component parameter values are calculated, but fewer measurements are needed to perform isolation, as compared to the deterministic methods. Measurements can be taken at the input-output ports only (linear case) and at some internal circuit nodes (nonlinear case).

2.3.3 FUZZY SET METHOD

The modelling of the faulty circuit as a fuzzy system is the main characteristic of the approaches [26 - 27] classified within this method; two factors encountered in fault isolation are thought to be the reasons for this modelling. First, the imprecision and the indeterminacy of the complex structure of faulty circuits makes it impossible to find exact solutions. The imprecision referred to, includes the lack of precision in measurements, usually deteriorated by noise, and, the lack of precision in the criteria for establishing fault conditions. Second, it is not always necessary to seek exact solutions to the isolation problem. From this arises the possibility that a fault does not only belong or not belong to a predetermined fault, but intermediate grades of membership are also possible.

Approaches [26] and [27] present algorithms for the isolation of single soft faults using fuzzy set concepts; these are based on the definition of a fuzzy membership function of a faulty component parameter as a function of response deviations.

(a) Description

The idea developed by S.D. Bedrosian and J.H. Lee [26 - 27] is the introduction of fault isolation algorithms in the context of a fuzzy decision problem. It is suggested that this could be achieved by converting the actual test measurements into fuzzy membership functions using fuzzy set techniques. The fuzzy membership functions for the faulty component parameters are modelled as a function of the faulty response deviations. Fault isolation is performed by using either a fuzzy entropy criterion or a fuzzy distance criterion. A faulty circuit is classified into the fault patterns (fault case) for which either of the above measures is minimum for a given measurement.

Single soft faults are considered while the associated tolerances of all nonfaulty component parameters are taken into account. Measurements are taken on port responses which could include not only the input-output ports, but also some other additional defined ports.

Assuming that there are n single fault cases $(j=1,2,\ldots\ldots,n)$ and m different responses $(i=1,2,\ldots\ldots,m)$ available for a UUT, it is possible to define briefly the functions mentioned above.* The model for the fault membership function involves taking values in the range $[0\ ,1]$ depending on the degree of belonging to a predetermined set (fault case). If there is a fault in an analogue circuit, this usually means that the observed response has deviated from its nominal more than the acceptable tolerance limit α as a result of the component parameter deviation from its nominal value. A circuit is considered faulty as soon as the deviation exceeds its tolerance limit α for which a fuzzy membership 0 is assigned. If the deviation increases to a certain point β, the circuit is said to be definitely faulty and a fuzzy membership 1 is assigned, which remains 1 for deviations beyond β. It is assumed that the degree of fault increases linearly in the transition region between fuzzy membership 0 and 1. A graphical representation is shown in Fig. 2.3, where for a particular circuit response x_i, μ_{x_i} represents the circuit fault membership function as a function of the ith response deviations.

* Detailed information can be found in references [26] and [28].

As is usually the case, the fault in a circuit is due to a specific faulty component parameter p_j (single fault only). Therefore, in a similar manner as above, the model of a component fault membership function, as a function of response deviations is represented by $\mu_{x_{ij}}$, and graphically shown in Fig. 2.4. For each component parameter deviation, three response deviation points γ_1, γ_2, and γ_3 are considered as the response deviations associated respectively with the acceptable tolerance limit of the component parameter, the usual region where it is considered faulty, and its points of extreme value, which can be open or short.

The other function to consider, resulting from the combination of the two previous ones, is the circuit fault membership function due to the jth faulty component. This is graphically shown in Fig. 2.5, represented by $\mu_{c_{ij}}$, and defined as the minimum of the circuit fault membership function μ_{x_i} and the jth component fault membership function $\mu_{x_{ij}}$. The concepts of Fuzzy Expected Value (FEV) and Fuzzy Measure are also defined [26] in connection with the previous definitions for the faulty circuit.

By using these concepts, it is possible to describe the information of the faulty circuit in the context of fuzzy set theory. To perform the actual isolation of the faulty circuit (single fault), fuzzy entropy or fuzzy distance is used. In the case of the fuzzy entropy criterion, the objective is to determine the jth fault case which minimizes the number of incorrect decisions.

$$\min_{j} \sum_{i=1}^{m} J_{ij}(x) \tag{2.40}$$

where $J_{ij}(x)$ is the decision function defined in terms of FEV, and x is the vector of measured responses.

A measure of equivalence is defined in the application of fuzzy distance to the problem of isolation. The objective is to determine the jth fault case for which the equivalence between a vector measurement y of the faulty circuit and the preset value of the x_j fault case is minimum.

$$\mu(y \equiv x_j) = 1 - d(y - x_j) \tag{2.41}$$

This can also be interpreted as finding the minimum distance between measurement y and the preset values of vector x_j.

(b) Domain of Application

(1) Linear and nonlinear analogue systems,

(2) Single faults,

(3) Soft and catastrophic faults while the production tolerances of nonfaulty component parameters are considered,

(4) Isolation to the most likely faulty component.

(c) Test Preparation

(1) Test signal generation,

(2) Nominal system port responses evaluation (i=1,2,....,m),

(3) Definition of the n fault cases (j=1,2,....,n) and calculation of the m responses under faulty conditions,

(4) Description of the model for the fuzzy membership functions of the fault cases as a function of response deviations: circuit fault membership function for each ith response (μ_{x_i}); component parameter fault membership function as a function of response deviations ($\mu_{x_{ij}}$); circuit fault membership function due to the jth faulty component parameter ($\mu_{c_{ij}}$).

(d) <u>Test Procedure</u>

(1) Measure the m responses of the faulty circuit,

(2) Calculation of circuit fault membership function as function of the measured data,

(3) Calculation of the fuzzy measure and the fuzzy expected value (FEV) for each ith response and jth fault case,

(4) Calculate the decision criterion (Fuzzy entropy or Fuzzy distance),

(5) Isolation of the faulty component as the one for which the decision criterion used is minimum.

(e) <u>Discussion</u>

The approaches developed by S.D Bedrosian and J.H. Lee [26 - 27] have introduced the concept of fuzzy set theory to the fault analysis problem in analogue systems. This provides the framework which enables one to handle some of the problems usually found in the complex structure of faulty circuits. Thus, the effect of noise, inaccuracy in measurements, and the vagueness in the definition of faulty conditions, can be taken into account, and therefore, the system can be modelled on the basis of fuzzy membership functions.

Fuzzy set techniques are used for detection as well as isolation of faults in linear and nonlinear analogue circuits, while the associated tolerance of all nonfaulty component parameters is considered. Isolation to the most likely faulty component is performed by using either fuzzy entropy or fuzzy distance criterion. Although the method is more suitable to the isolation of soft faults, it can also be applied to the isolation of catastrophic faults, which may be defined as extreme cases of soft faults.

The model for the fuzzy membership functions is the main factor in this method, so that the confidence of the results depends very much on this. This situation suggest that the points at which fuzzy membership 0 and 1 are assigned must be carefully defined; unfortunately, no algorithm is provided in the approaches [26 - 27]. Instead, these points are chosen according to the decision maker's own criteria. Another negative aspect is the assumption that the degree of membership increases linearly in the transition regions, which is not always the case, especially for nonlinear circuits.

2.3.4 A NONDETERMINISTIC EXAMPLE

The circuit shown in Fig. 2.6, used by a number of researchers ([22 - 23] and [25]), is taken as an example for the application of one of the nondeterministic methods. Experiments reported by K.C. Varghese et al [22] are reported here as an illustration of the sort of results that can be obtained using approaches within this category. For nominal conditions, the component parameter values are: $R_1 = R_4 = 1\ M\Omega$; $R_2 = 10\ M\Omega$; $R_3 = 2\ M\Omega$; $C_1 = 0.01\ \mu F$; $C_2 = C_3 = 0.001\ \mu F$, and normal distribution for manufacturing tolerances of 3% of nominal.

Following the steps of the method, first, the 14 fault cases of the circuit are defined and simulated, one at a time, at ±50% deviation of nominal value, while the nonfaulty component are kept at nominal. For each fault case, 56 circuit responses (features) are evaluated in the frequency domain, $f = [10,20,\ldots,100,200,\ldots,1000,2000,\ldots\ldots,10000]^T$ rad/sec; these comprising 28 features for gain and 28 features for phase. Then, the simulated fault cases are grouped and euclidean normalized to form a recognition matrix. At the next step, for a required confidence level (90.4%), a small subset of features is selected by means of the optimization procedure. Finally, the recognition matrix is made sparse, and the number of fault cases reduced by grouping those fault cases having a low separability measure (7 groups are formed). The selected feature set is $[AR10, AR200, AR600, \phi 2000, \phi 8000]^T$, where AR is the amplitude ratio, ϕ is the phase, and the number is the radian frequency. Table 2.2 shows the reduced sparse recognition matrix formed by grouping fault cases with low discrimination; in this case, the groups are represented by the mean value of the fault cases associated with that group.

A simulation of a practical application is made by simulating 100 faulty circuits for each fault case. The component parameter values of the faulty component (single fault) are drawn from normal distribution with nominal mean and 30% standard deviation; the rest of the component parameters are kept at their nominal values. The selected responses are calculated and checked for any failure. If the circuit is faulty, manufacturing tolerances are assigned to all nonfaulty component parameters, drawing their values from their distribution. The selected responses are calculated and checked again. If it is still faulty, the faulty circuit response (feature) is classified (diagnosis) to one of the 7 groups. Furthermore, for correct group classification, location of the particular faulty component (fault case) within that group is performed. In order to see the effect of manufacturing tolerances of nonfaulty component parameters on diagnosis, additional experiments with tolerance variations of 6% of nominal are carried out. Table 2.3 shows the results for the percentage of correct group classification (diagnosis), and the percentage of correct fault case diagnosis (Fig. 2.7) within that group.

In summary, the results show that the percentage of correct diagnosis to fault case level decreases as compared with diagnosis to groups. For example, fault case R_1^+ is 96% correctly diagnosed to the group (R_1^+, R_2^-), but only 66.6% out of the 96% to the correct fault case R_1^+ within that group. The percentage of correct diagnosis for both, groups and fault cases, also falls, as the tolerances of nonfaulty component parameters increase.

2.4 SUMMARY

Approaches classified into the deterministic and non-deterministic categories have been reviewed. Within each category, the approaches are further grouped according to the common technique used in solving the fault diagnosis problem. The review provides information in the context of domain of application, test preparation, and test procedure of the approaches described. This information is summarized in tables 2.4 to 2.6 respectively.

Deterministic methods are more suitable for linear circuits. They calculate or estimate the values of all component parameters by solving the fault diagnosis equations. Consequently, faulty components are identified by direct comparison with the nominal values. In solving this problem, most internal nodes of the unit under test have to be accessible for measurements, which makes these methods impractical for field applications. However, as a way of avoiding this problem, some approaches have been developed to allow identification of selected components, assuming that all the remaining component parameter values are known.

In contrast with deterministic methods, non-deterministic methods can usually be applied to either linear or nonlinear circuits, and the emphasis is not in the component parameter values calculation, but in locating the most likely faulty component(s). Some consideration of the tolerance associated with nonfaulty component parameters is allowed and, for linear circuits, none or very few internal nodes are needed for measurements. The main disadvantage of the approaches within this category is the amount of data generated at the pre-test stage, at which the fault conditions are simulated.

Despite the progress already made in the search for methods of fault diagnosis, there are still areas for further research, especially those involving the manufacturing tolerances associated with nonfaulty component parameters.

The effect on circuit performances of these tolerances greatly complicates the fault diagnosis problem. In that context, what is of particular interest is the component fault modelling and fault simulation problem. This is a real practical problem in analogue circuits and it should be considered in the development of fault diagnosis methods by providing basic techniques. Some studies have been made in this direction [46 - 47]; it is also the area of fault diagnosis problems addressed in this thesis. The problem is further described in chapter three, particularly in section 3, and an algorithm is developed in chapter four.

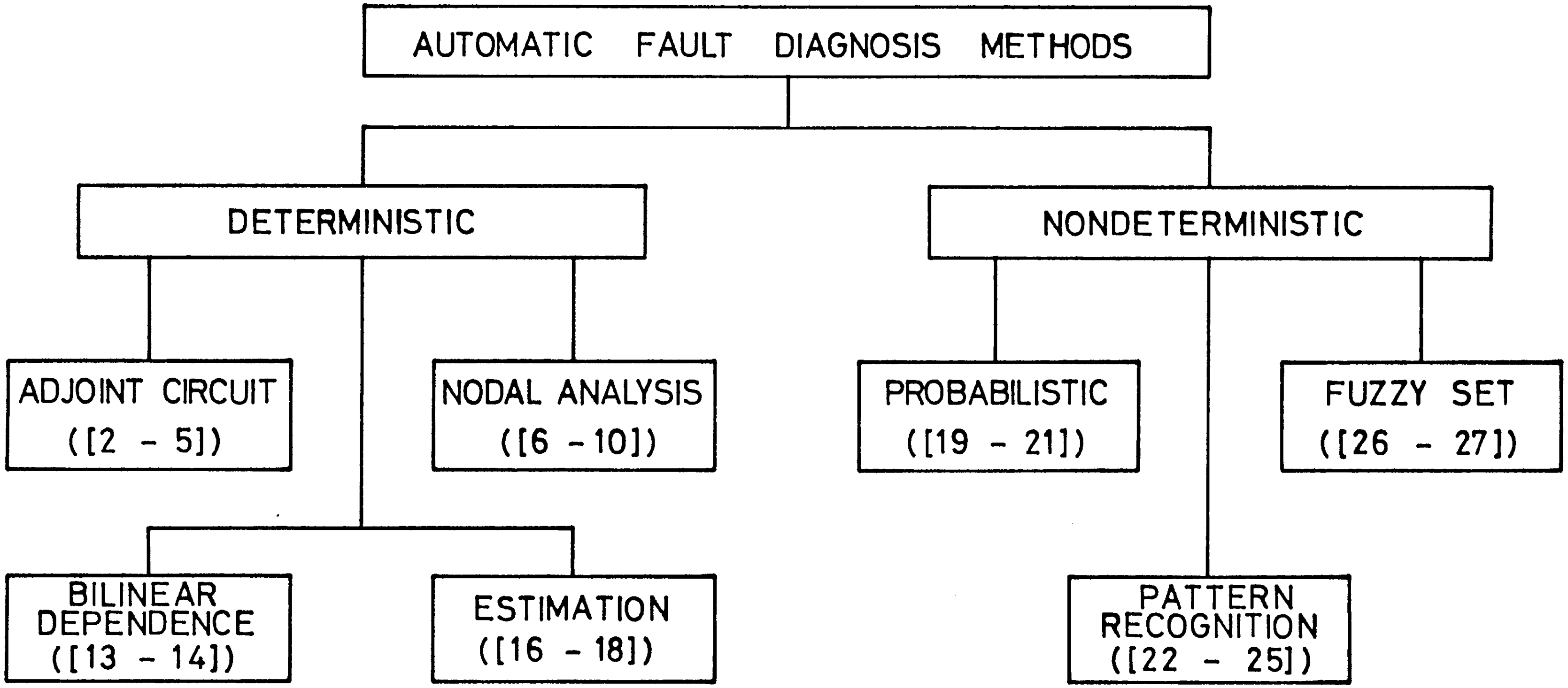


Fig. 2.1 Classification of Automatic Fault Diagnosis methods into Deterministic and Nondeterministic categories. (Numbers refer to the references)

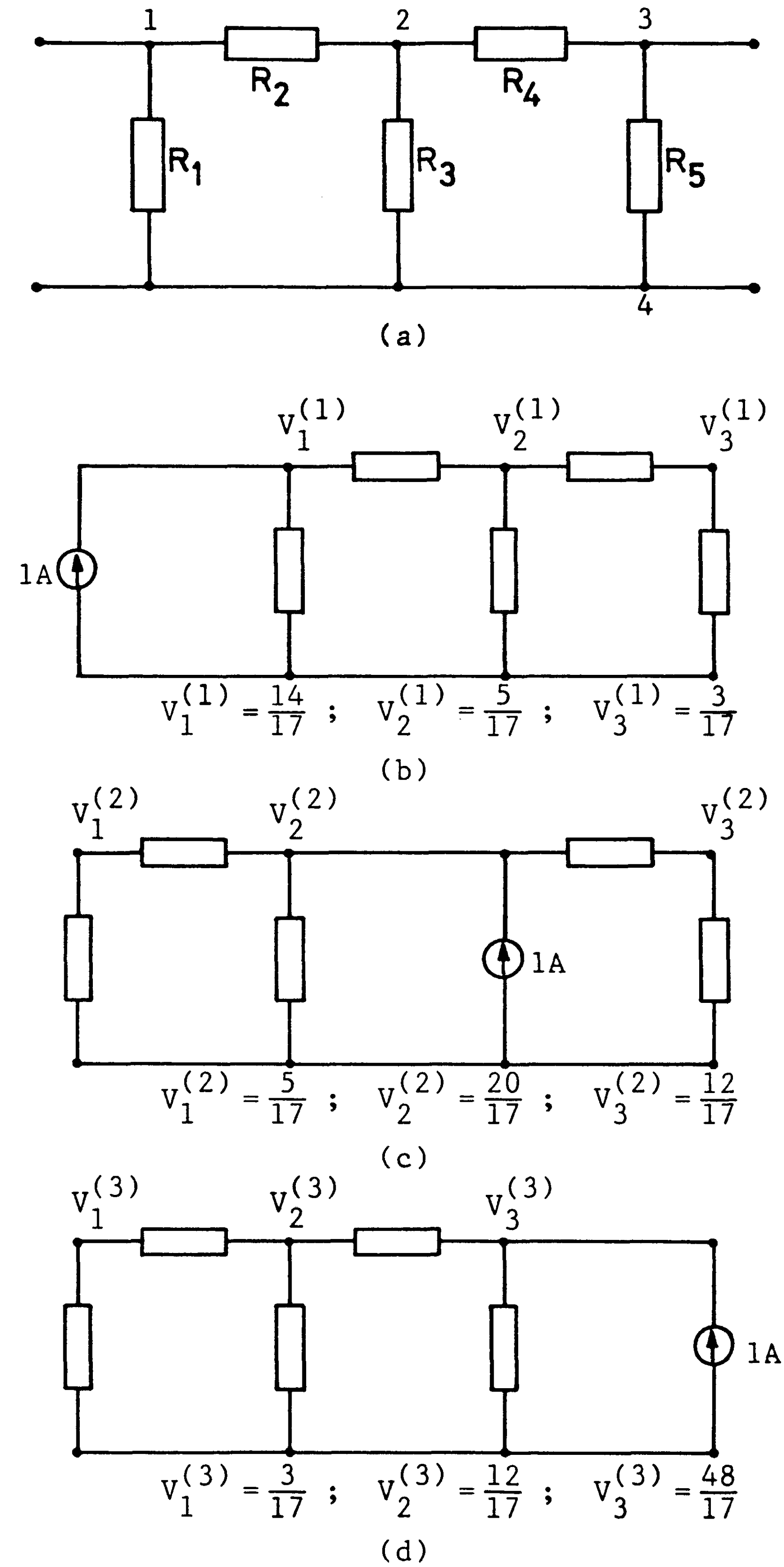


Fig. 2.2 Linear circuit used as an example to illustrate the application of a deterministic method

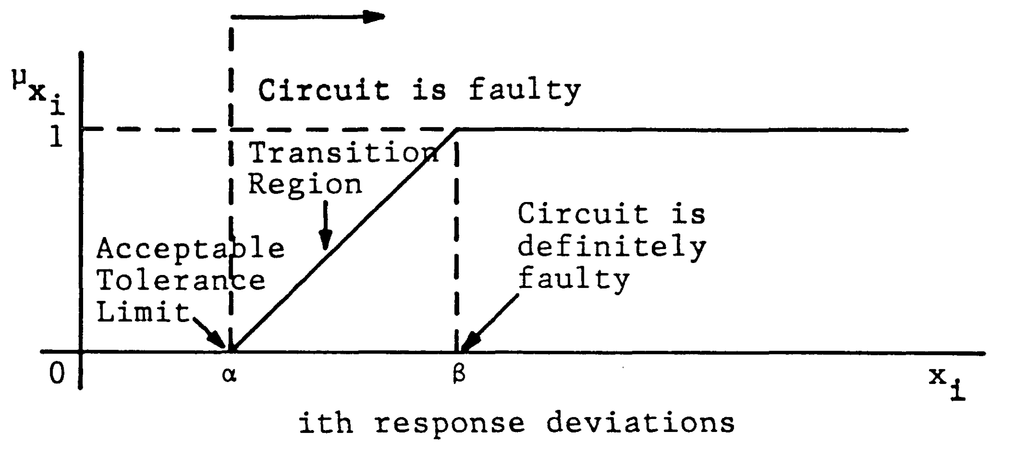


Fig. 2.3 A circuit fault membership function

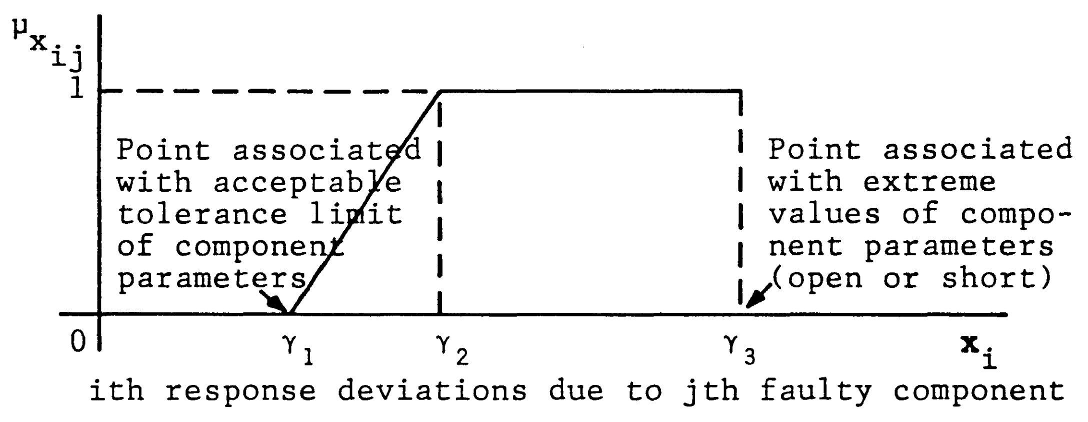


Fig. 2.4 A component fault membership function

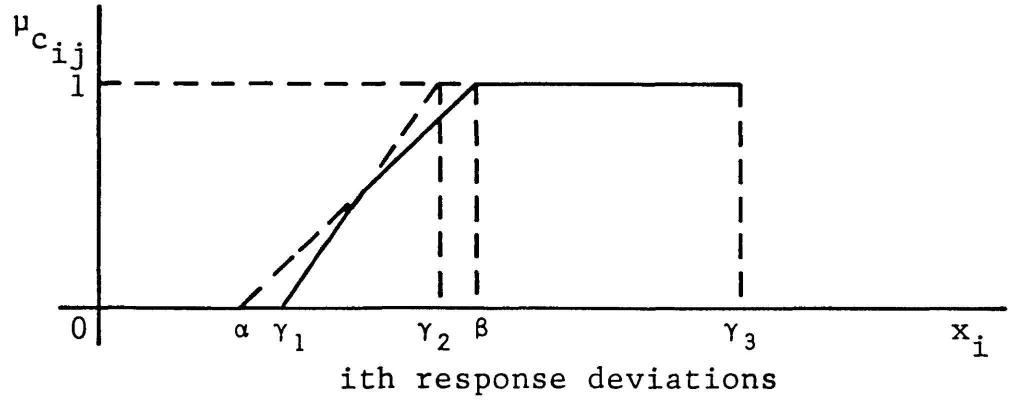
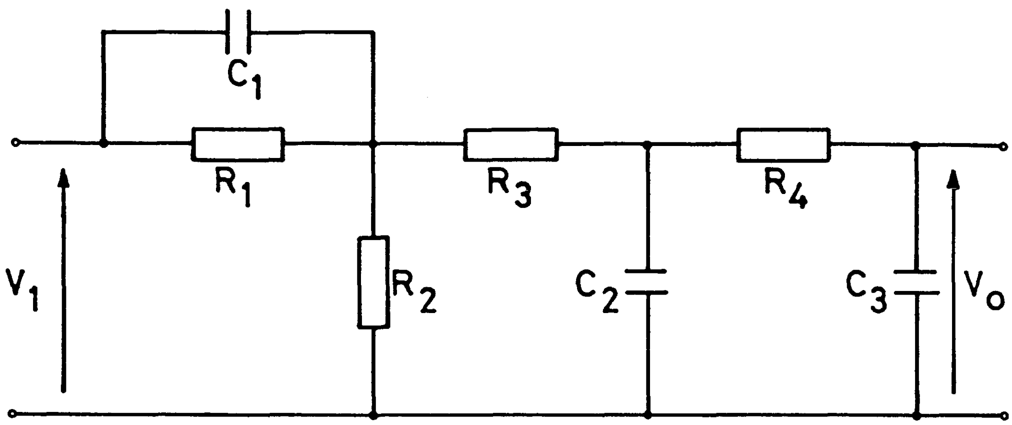


Fig. 2.5 The circuit fault membership function due to the jth faulty component



$R_1 = 1\ M\Omega$ $R_2 = 10\ M\Omega$ $R_3 = 2\ M\Omega$ $R_4 = 1\ M\Omega$

$C_1 = 0.01\ \mu F$ $C_2 = 0.001\ \mu F$ $C_3 = 0.001\ \mu F$

Fig. 2.6 An RC circuit used as an example to illustrate the application of a nondeterministic method

Table 2.1 Sparse recognition matrix for the selected features, for the 14 fault cases of the RC circuit

FAULT	FEATURES				
CASES	AR10	AR200	AR600	ϕ2000	ϕ8000
R_1^-	0.9652	0.2552			
R_1^+	-0.9993				
R_2^-	-0.9856	-0.1668			
R_2^+	0.9888	0.1474			
R_3^-		0.5331	0.8024	0.2492	
R_3^+		-0.7489	-0.6447	-0.1444	
R_4^-		0.1570	0.2284	0.7727	0.5711
R_4^+		-0.2943	-0.3994	-0.7917	-0.3564
C_1^-		-0.9418	-0.3037	0.1385	
C_1^+		0.9812	0.1684	-0.0910	
C_2^-		0.5165	0.5049	0.5773	0.3808
C_2^+		-0.6795	-0.5554	-0.4366	-0.1974
C_3^-		0.5977	0.6775	0.3758	0.2061
C_3^+		-0.7516	-0.6046	-0.2421	

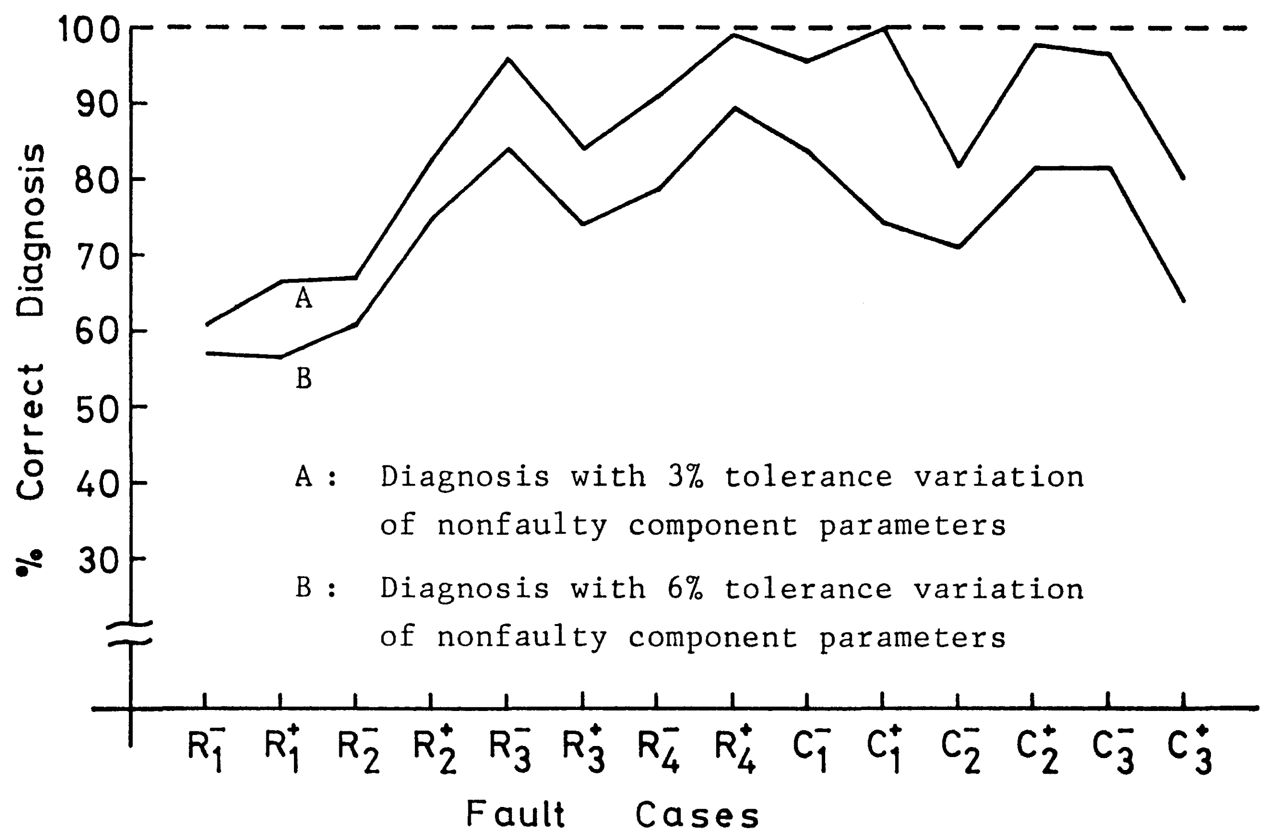


Fig. 2.7 An illustration of the effect, on diagnosis, of different tolerance variations in nonfaulty component parameters

Table 2.2 Reduced sparse recognition matrix for the 7 fault groups of the RC circuit

	FEATURES				
GROUPS	AR10	AR200	AR600	ϕ2000	ϕ8000
(R_1^-, R_2^+)	0.976	0.214			
(R_1^+, R_2^-)	-0.995	-0.101			
(R_3^-, C_2^-, C_3^-)		0.563	0.698	0.387	0.214
$(R_3^+, C_1^-, C_2^+, C_3^+)$		-0.784	-0.579	-0.205	
(R_4^-, C_2^-)		0.362	0.390	0.693	0.486
(R_4^+, C_2^+)		-0.537	-0.509	-0.612	-0.276
(C_1^+)		0.981	0.168		

Table 2.3 Diagnosis results considering tolerance variation for the nonfaulty components

	TOLERANCE VARIATION NONFAULTY COMP. 3%		TOLERANCE VARIATION NONFAULTY COMP. 3%	
GROUPS	% CORRECT GROUP DIAG.	% CORRECT FAULT CASE DIAGNOSIS	% CORRECT GROUP DIAG.	% CORRECT FAULT CASE DIAGNOSIS
R_1^-	97	60.8	72	56.9
R_2^+	98	82.6	84	75.0
R_1^+	96	66.6	78	56.4
R_2^-	97	67.0	87	60.9
R_3^-	100	96.0	100	84.0
C_2^-	100	82.0	100	71.0
C_3^-	100	97.0	99	81.8
R_3^+	100	84.0	100	74.0
C_1^-	100	96.0	93	83.8
C_2^+	100	98.0	99	81.8
C_3^+	100	80.0	97	63.9
R_4^-	100	91.0	95	78.9
C_2^-	100	82.0	100	71.0
R_4^+	100	99.0	96	89.5
C_2^+	100	98.0	99	81.8
C_1^+	100	100.0	74.1	74.1

Table 2.4 Domain of application of fault diagnosis methods

	DOMAIN OF APPLICATION			
METHODS	CIRCUIT TYPE	DIAGNOSIS LEVEL	TYPES OF FAULTS	TEST TERMINAL
ADJOINT CIRCUIT	L	LC , I	SI , M , SF	I/O , AI
NODAL ANALYSIS	L	LC , I	SI , M , SF	I/O , AI
BILINEAR DEPENDENCE	L	LC	SI,M,SF,C	I/O , ST
ESTIMATION	L , NL	LC , I	SI , M , SF	I/O , ST
PROBABILISTIC	L	LC	SI , SF	I/O , ST
PATTERN RECOGNITION	L , NL	LC , LM	SI , SF	I/O , ST
FUZZY SETS	L , NL	LC , LM	SI , SF , C	I/O , ST

L : Linear circuits or systems
NL : Nonlinear circuits or systems
LC : Location at component level
I : Identification
LM : Location at module level
SI : Single faults
M : Multiple faults
SF : Soft (deviation) faults
C : Catastrophic faults
I/O : Input - output terminals (ports)
AI : All internal nodes
ST : Selected test terminals

Table 2.5 Test preparation for the fault diagnosis methods

METHODS	TEST PREPARATION: COMPONENT DATA	CIRCUIT DATA	SIMULATION TECHNIQUE	ANALYSIS
ADJOINT CIRCUIT	NV , T	NC	CS , AD	NA , S
NODAL ANALYSIS	NV , T	NC , TP	CS	NA
BILINEAR DEPENDENCE	NV	NC	FS , AD	FA
ESTIMATION	NV , T	NC	CS , FS	FA , S
PROBABILISTIC	NV , T , SD	NC , TA	FS	FA , S
PATTERN RECOGNITION	NV , T	NC , TA	FS	FA , SE , CL
FUZZY SETS	NV , T	NC , TA	FS	FA , FM

NV : Nominal values of the component parameters
T : Associated tolerance of the component parameters
SD : Statistical distribution of component parameters
NC : Nominal characteristics
TP : Topology of the circuit
TA : Tolerance analysis
CS : Component simulation
AD : Adjoint circuit simulation
FS : Fault simulation
NA : Nodal analysis
S : Sensitivity analysis
FA : Functional analysis
SE : Selection of features by optimization
CL : Clustering analysis
FM : Fuzzy membership analysis

Table 2.6 Test procedure for the fault diagnosis methods

METHODS	TEST PROCEDURE: MEASUREMENT	TEST PROCEDURE: PROCESSING
ADJOINT CIRCUIT	NVO	ES , CV
NODAL ANALYSIS	NVO	ES , CV
BILINEAR DEPENDENCE	PV	ES , EC
ESTIMATION	CP	ES , INT
PROBABILISTIC	CP	TR , AE
PATTERN RECOGNITION	CP	TR
FUZZY SETS	CP	TR , AE

NVO : Nodal voltage measurements
PV : Voltages at the test (ports) terminals
CP : Circuit or system performances
ES : Equation solving
CV : Comparison of component parameter values
EC : Checking the consistency or inconsistency of equations
INT : Interpolation
TR : Table retrieval
AE : Evaluation of algebraic expressions

CHAPTER THREE

STATISTICAL TECHNIQUES FOR THE FAULT ANALYSIS PROBLEM

3.1 INTRODUCTION

A component parameter value may differ from nominal for a variety of reasons, and in so doing may give rise to the possibility that, either during manufacture or in actual field use, the circuit containing that component may fail to satisfy its performance specifications. Such a possibility may be acknowledged in different ways at the different stages of circuit design, manufacture and field use. Thus, at the design stage, tolerance design techniques [48 - 51] are available to minimize the effect, on manufacturing cost, of the inevitable component parameter tolerances. Later, during manufacture, the failure of a circuit to meet its specifications may be compensated by using tuning techniques [52 - 53].

In contrast, diagnostic testing technique are used in dealing with the effect of component parameter changes after manufacture and while the circuit is in field use. Gross changes in the value of component parameters may cause the circuit to violate its performance specifications. In these circumstances, if a fault has occurred, it is useful to be able, rapidly and inexpensively, to perform measurements and undertake calculations necessary to identify the faulty component(s). The latter situation is often referred to as "Fault Diagnosis" (see section 3.2 of chapter one). Of the problems found in this area, the location of faulty components is the main concern of this thesis. Thus, in this chapter, necessary notations are introduced, and the specific problem defined. Additionally, the relevant techniques used in developing the fault location algorithm are discussed to some extent.

3.2 PRELIMINARIES

The terminology described below is necessary to understand and clarify the formulation of the problem and the proposed approach.

(1) TOLERANCE REGION (R_T): In general, a point $P \triangleq \{p_1 p_2 \ldots\ldots p_K\}^T$ is a vector of K elements corresponding to the component parameter values of a circuit. As far as the nominal point is concerned, this is represented as $P^o \triangleq \{p_1^o p_2^o \ldots\ldots p_K^o\}^T$, which is often associated with a set of non-negative tolerances $T \triangleq \{t_1 t_2 \ldots\ldots t_K\}^T \geq 0$. Thus the tolerance region R_T (Fig. 3.1) is defined by

$$R_T \triangleq \{P \mid p_i^o - t_i \leq p_i \leq p_i^o + t_i \ , \ i=1,2,\ldots.,K\} \quad (3.1)$$

(2) ACCEPTABILITY REGION (R_A): Circuits are designed to meet the specifications placed on some responses of interest. If these specifications are mapped from the output space onto the input space (component parameter space), they define a Region of Acceptability R_A (Fig. 3.1). However, depending on the position of the p_i^o's and the size of the t_i's a point P may or may not satisfy these performance specifications. Thus, the Region of Acceptability is generally defined by

$$R_A \triangleq \{P | g_j(P) \geq 0 \ , \quad j=1,2,\ldots,\ell\} \qquad (3.2)$$

where $g_j(P)$ is the circuit performance function, having upper and lower limits ($\underline{g}_j < g_j(P) < \overline{g}_j$), and j is the number of constraint functions. During the design stage, the interest is to make feasible the complete Tolerance Region; in other words $R_T \subseteq R_A$. This condition may be achieved by using computer aided circuit design (CACD) techniques susch as Tolerance Design and Worst-case Design (e.g., Design Centering, and Tolerance Assigment).

(3) FAULTY TOLERANCE REGION (R_F): In the field of fault Analysis it is necessary to modify the concept of a Tolerance Region, to be able to consider the simulation of faults. In this context, for single faults, the tolerance associated with a particular component parameter j, can be extended from its original $\pm t_j$ to $\pm t_j'$ forming, in this way, the Faulty Tolerance regions R_F shown in Fig. 3.1 (These regions may also be called fault case regions, and this is the name to be often used herein). The limits to which the tolerance t_j can be extended must be fixed by the circuit designer or testing engineer during the fault simulation process. If the assumption that $R_T \subsetneq R_A$ is met, there will be no need to sample in that region (R_T). Thus, under this condition, for the jth component parameter, the Faulty Tolerance Region R_F may be defined by

$$R_F = \overline{R_T' \cap R_T} \tag{3.3}$$

which, expressed in terms of component parameters is given by

$$R_F \triangleq \{P \mid [p_i^o - t_i \leq p_i \leq p_i^o + t_i], [p_j^o - t_j' \leq p_j \leq p_j^o - t_j \text{ or } p_j^o + t_j \leq p_j \leq p_j^o + t_j'] \ \forall\, i \neq j \quad i,j=1,2,\ldots,K\} \tag{3.4}$$

and R_T' is defined by

$$R_T' \triangleq \{P \mid [p_i^o - t_i \leq p_i \leq p_i^o + t_i], [p_j^o - t_j' \leq p_j \leq p_j^o + t_j'] \ \forall\, i \neq j \quad i,j=1,2,\ldots,K\} \tag{3.5}$$

(4) SPECIFICATION TEST: It refers to the test performed on a circuit or system to check whether or not the specifications are being met. Performance specifications are usually imposed by customers (this determines the R_A region in component parameter space), so that the decision as to whether a circuit is faulty or not must be based on the result of this test.

(5) DIAGNOSTIC TEST: It concerns the measurements performed after a circuit or system has been detected as faulty, such that from these measurements, and some

additional information about the particular unit, calculations can be carried out which allow the identification of the component or module causing that failure.

3.3 PROBLEM IDENTIFICATION

The problem to be solved is viewed and identified in the component parameter space. In this context, for illustration purposes, the simple but easily generalized case of the circuit Fig. 3.2 is considered; this contains only two components (p_1, p_2) that are subject to failure. The circuit is shown as lying within a box to indicate that there is some relation between the external variables (e.g. I_1, V_1, I_2, V_2) and the circuit's component parameters that constitutes tha desirable attribute of the circuit.

3.3.1 THE FAULT MODELLING PROBLEM IN THE CONTEXT OF MULTIDIMENSIONAL COMPONENT PARAMETER SPACE

A circuit may be described in several ways, but it is felt that a useful representation could be that which is based on the concept of multidimensional component parameter space. For the example of Fig. 3.2, with two potentially faulty components, component parameter space (p_1, p_2) is two dimensional (Fig. 3.3). The nominal condition of the circuit is represented by the point N whose coordinates (p_1^o, p_2^o) are the nominal values of the component parameters. The Tolerance Region R_T describes the region within which the point describing the circuit may lie as a result of the inevitable manufacturing tolerances associated with the component parameters. For uncorrelated component parameters, R_T is rectangular.

Next, consider the specifications placed by the customer on the response of the circuit. For a linear circuit, its frequency domain performance in terms of the magnitude of the transfer impedance (Fig. 3.4) might be specified; for a nonlinear circuit such as a switch, the rise time might be the response of interest. If the specifications are now transformed into component parameter space they define a unique Region of Acceptability R_A (Fig. 3.5); a point within R_A represents a circuit that satisfies the performance specifications. Normally, R_A does not lend itself to simple description.

In the normal course of design, the nominal values of the component parameters, as well as their tolerances, will have been chosen to take account of the existence of tolerances of component parameters. For example, the nominal component parameter values may have been chosen to maximize the manufacturing yield, and/or the tolerances may have been chosen to reduce the unit cost of satisfactory circuits. Whatever the result of such design procedures, if circuits are tested after manufacture and only supplied to customers if they satisfy the specifications, then the circuits that go into field use are represented by the cross-hatched regions in either Figures 3.6(a) or 3.6(b).

After manufacture, changes may occur in the component parameter values for a variety of reasons such as permanent overstress (high temperature, continued overload operation, material stress, etc.). If each component parameter value still remains within its tolerance range then, as is evident from Fig. 3.6(a), the circuit will perform satisfactorily. On the other hand, if the change in one component parameter is such that its value now lies outside the tolerance range, the circuit is described by a point lying outside R_T. The consequencies may not be serious if the point is still within R_A (e.g. point A in Fig. 3.6(a)). However, if the point lies outside R_A (e.g. point B), then the circuit fails its specifications. In this thesis we consider any movement outside R_T which is caused by a <u>single</u> component parameter value moving out of its tolerance range, though not sufficiently to transform the component into an open-circuit or short-circuit. Thus, the regions of component parameter space considered to be potentially liable to lead to unsatisfactory circuit performance are shown in Fig. 3.7. Each cross-hatched rectangular region will be referred to as a <u>fault case</u> (see section 3.2 for its mathematical definition); the limits to these regions are arbitrary and open to choice, but limits of about 0.5 and 1.5 times the relevant nominal value may be typical (Fig. 3.7). The fault cases illustrated in Fig. 3.7 are generally referred to as "soft faults", in contrast to "hard faults" in which component parameter values approximate to zero and infinity.

It is important to realize that, in some circuit designs, the tolerance region will intentionally be chosen to lie partially outside R_A (Fig. 3.6(b)), normally for economic reasons. In this case, the manufacturing process selects only those circuits whose performance is satisfactory, and which are represented by points within R_A. Clearly, a subsequent change in a component parameter value (e.g. to point C) could still result in a circuit within R_T, but exhibiting unsatisfactory performance. The fault diagnosis procedure described in chapter four is equally applicable to this situation, though some particular considerations would have to be taken into account, as follows.

Firstly, fault simulation may be performed in two ways:

(a) As it is described herein, i.e., outside the original tolerance region. This may cause some problems during the diagnosis process, in the case that component parameters have deviated inside its own tolerance range R_T (point C in Fig. 3.6(b)), but causing malfunction. This situation arises from the fact that these faults have not been considered during fault simulation.

(b) A more sophisticated procedure could be implemented to take account these faults. This may consist of first finding a feasible tolerance region (see Fig. 3.8) by means of CACD (e.g. worst-case analysis or Monte Carlo analysis), and then performing fault simulation outside this new tolerance region.

Secondly, there are two possibilities for the repair procedure: One possibility is to use a replacement component having a sufficiently narrow tolerance range. Another is to verify the substitute component before replacement.

3.3.2 THE DIAGNOSTIC RESPONSE PROBLEM

If the variation in a component parameter value causes the point representing the circuit to move outside R_A, thereby leading to unacceptable performances, this movement can be detected by a measurement of the responses (the specification responses) for which specifications exist, a procedure termed the Specification Test. It is then necessary to locate the fault; that is, to identify the component whose parameter value has moved outside its tolerance range. This is achieved by making measurements of diagnostic responses which, in general, need not be identical with the specification responses or even measured at the same terminals. Thus, specifications may be placed on a circuit's input impedance, but its output voltage observed in order to deduce which component is faulty. The diagnostic responses are chosen, as far as possible, to exhibit a unique pattern for each component, thereby easing the decision as to which component is the faulty one.

The selection, at the design stage, of diagnostic measurements to be carried out in the field, is not straightforward, as is illustrated in Fig. 3.9. For the circuit of Fig. 3.9(a), the plots of Fig. 3.9(b) show the values of the two normalized response deviations $\Delta V_0|_{f=10\text{ Hz}}$ and $\Delta V_0|_{f=21.5\text{ Hz}}$ (where $\Delta V_0 = (|V_0|/|V_0^N|) - 1$

and $|V_0^N|$ is the nominal response value) associated with the four fault cases in each of which 20 circuits have been selected at random. In other words, each point in Figure 3.9(b) is associated with a point (i.e. circuit) lying within one of the hatched regions in Fig. 3.7. Obviously, since $|V_0|$ as a function of frequency is determined solely by the RC product, it is not possible to differentiate between changes in R and C. However, if the effect of the same fault cases on the normalized deviation of the magnitude of the input impedance ($\Delta Z_{in} = (|Z_{in}|/Z_{in}^N) - 1$; $|Z_{in}^N|$ is the value at nominal) at 2.15 Hz and 4.64 Hz is examined (Fig. 3.9(c)), the discrimination between fault cases is seen to be much easier. Thus, in selecting suitable diagnostic responses, it may be beneficial to examine a number of possibilities and then select, for economic reasons, as small a subset as possible that will offer good discrimination.

From the theoretical point of view it is entirely plausible that, from a basis of electrical circuit theory, it may eventually be possible to derive a technique for selecting, for any circuit, the diagnostic responses which are optimum with respect to the discrimination between fault cases. However, since the development of such a theory , not only for nonlinear circuits but also for the case where all component parameters are toleranced, appears extremely difficult, it is preferred to select diagnostic responses from a collection proposed by the designer. This approach does not exclude the value of the designer's insight, and has the advantage of universality.

3.4 MONTE CARLO ANALYSIS IN FAULT DIAGNOSIS

It was mentioned in the previous sections that the statistical variations (tolerances) of component parameters produced variations in the behaviour of circuits. These effects may be analysed by different techniques (e.g. sensitivity analysis) but, in order to obtain more insight, a statistical approach very often used is the Monte Carlo analysis. The procedure is based on the simulation and analysis of a number of circuits that are generated according to a prescribed probability distribution of component parameter values, so that the circuit performance distribution may then be found or approximated (Fig. 3.10 shows a basic model of this situation).

The use of the Monte Carlo method requires the generation of random numbers having a known probability distribution function such that, each time a circuit is simulated at random, the component parameters take values according to that distribution. Subsequently, the circuit's performance is calculated and checked against the circuit's performance specifications (Fig. 3.11 illustrates a two dimensional example). In this manner, sampling experiments on a model of the circuit can be carried out rather than on the actual physical circuit. Moreover, the results obtained through this procedure are usually the same as those obtained from observations or measurements performed on the circuit itself.

The most common probability distribution functions (p.d.f.) used in simulation are the uniform and gaussian distributions. The uniform distribution of a random variable x in the interval x_1 and x_2 (Fig. 3.12) is given by

$$\phi(x) = \begin{cases} \dfrac{1}{x_2 - x_1} & x_1 < x < x_2 \\ \\ 0 & \text{otherwise} \end{cases} \tag{3.6}$$

where $\phi(x)$ is the probability distribution function, and the area under ϕ between the intervals x_1 and x_2 is unity. The gaussian distribution for the same variable x (Fig. 3.13) is given by

$$\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\tfrac{1}{2}(x - \bar{x})^2/\sigma^2 \tag{3.7}$$

where $\bar{x}$ is the mean value, σ^2 its variance, and the total area under the curve is also unity. For most manufactured components, this distribution is usually found to be truncated to $\pm 3\sigma$ around the mean value.

The Monte Carlo procedure described above can be used to perform fault simulation during the circuit design process, such that at this stage the critical components and the testing points may be identified. The procedure may allow the designer or the testing engineer to develop a fault diagnosis method, capable of locating faults to the component level. For single faults, Figs. 3.14 and 3.15

illustrate a simple two dimensional case where each component has 2 fault case regions in which simulation is performed. A sampling probability distribution function is assumed for the faulty component parameter and the sampling process takes place only in the regions of interest, as shown in the figures, avoiding in this way unnecessary sampling within the original tolerance region R_T where circuits are assumed to meet their performance specifications.

3.5 ANALYSIS OF VARIANCE

There are situations in which it may be necessary to carry out a comparison of several sample means, in order to find out whether there is a significant difference among them. The most common statistical technique used in performing this operation is the One-Factor Analysis of Variance (ANOVA) [54], which consists of testing a hypothesis of no difference in the population means from which the samples are taken. Thus, assuming that there are K independent population means, the interest is on testing what is known as the null hypothesis

$$H_0 : \mu_1 = \mu_2 = \ldots\ldots = \mu_K \tag{3.8}$$

The testing of this hypothesis requires the analysis of variance, and involves numerical calculations to determine the degree of difference that exists among the K populations sample means. The F-ratio test is the technique usually used, which is defined as the ratio between the variance (s_K^2) of the K sample means and the weighted average variance (s_p^2) of the K sample variances, commonly known as the pooled variance.

$$F = s_K^2 / s_p^2 \tag{3.9}$$

where

$$s_K^2 = \frac{1}{K-1} \sum_{i=1}^{K} n_i(\bar{x}_i - \bar{x})^2 \quad i=1,2,\ldots\ldots,K \tag{3.10}$$

$$s_p^2 = \frac{1}{N-K} \sum_{i=1}^{K} (n_i - 1)s_i^2 \tag{3.11}$$

and N is the total number of samples, $N = \Sigma_{i=1}^{K} n_i$, with n_i being the sample size from the ith population. The parameters $\bar{x}_i$ and $\bar{x}$ are the ith sample mean and the overall mean respectively, and s_i^2 is the sample variance for the ith population. These being defined in equations (3.12), (3.13) and, (3.14) respectively.

$$\bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij} \qquad j=1,2,\ldots\ldots,n_i \tag{3.12}$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{K} n_i \bar{x}_i = \frac{1}{N} \sum_{i=1}^{K} \sum_{j=1}^{n_i} x_{ij} \tag{3.13}$$

$$s_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 \qquad (3.14)$$

For each ith population, the sample variance s_i^2 has $(n_i - 1)$ degrees of freedom, such that the pooled variance s_p^2 has $\Sigma_{i=1}^{K} (n_i - 1)$ degrees of freedom. The variance of the K sample means s_K^2 has $(K - 1)$ degrees of freedom.

The F-ratio test (s_K^2 / s_p^2), regardless of whether the null hypothesis H_0 is true, and due to statistical fluctuation, has an F-distribution. The shape of this distribution depends on the degrees of freedom of s_K^2 (numerator) and s_p^2 (denominator) respectively; Fig. 3.16 illustrates some typical F-distributions. Depending on the numerical value of the F-ratio, it is possible to judge whether H_0 is true or not. In the case the K sample means $\bar{x}_i$ differ widely from one another, the s_K^2 tends to be relatively large as compared with s_p^2, resulting in a high value for the F-ratio. Under this condition, the hypothesis H_0 is probably not true; consequently the hypothesis of equal means will be rejected if F exceeds a critical value associated with a particular level of significance. Figure 3.17 shows an example of the F distribution, when H_0 is true with 2 degrees of freedom in numerator (s_K^2) and 12 degrees of freedom in the denominator of the F-ratio. Testing at 5% level of significance, the critical decision value, corresponding to $F_{0.05}$, is 3.89; therefore, the hypothesis H_0 will be rejected for those F values exceeding this critical value, with only 5% probability of being wrong.

3.5.1 THE ANALYSIS OF VARIANCE TABLE

The calculations described above are, for convenience, usually summarized and laid out in an Analysis of Variance (ANOVA) Table (Tables 3.1 and 3.2 are examples which are referred in detail below). The first and second rows of the table show, respectively, calculations made for the numerator and denominator of the F-ratio. The first column contains the source of variation. The second column contains the sums of squares, commonly denoted as SS. The third column contains the degrees of freedom for each sum of squares; these are the quantities by which the sums of squares SS are divided to obtain the statistical parameters (variances, generally denoted as MS) shown in the fourth column. The fifth column contains the information used in calculating the F-ratio, which should be compared with the tabled F value shown in the sixth column, in order to accept or reject the H_0 hypothesis, with a level of significance α (generally $\alpha = 0.05$). This F value is derived from statistical tables, according to the degrees of freedom of numerator and denominator of the F-ratio.

An additional consideration in the ANOVA Table is the sample size; although it is desirable to have equal sample size n from each population, this is sometimes not possible. Thus, calculations performed with equal sample size n, and unequal sample size n_i (n_i represents the number of samples from the ith population) are laid out in Tables 3.1 and 3.2 respectively.

3.5.2 THE ANOVA APPLICATION IN FAULT DIAGNOSIS

It has been mentioned that in classical statistics, the F-ratio is used to test the hypothesis $H_0 : \mu_1 = \mu_2 = .. = \mu_K$. The reason for testing this hypothesis is not that the K population means may be equal; indeed, they are generally unequal. But from the numerical value of the F-ratio test, it is possible to judge the degree of difference existing among them. Thus, if the hypothesis H_0 is accepted, it usually means that, although they may be unequal, their differences are rather small; whereas, if the hypothesis H_0 is rejected (high F value), it will be because there are some significant differences among the population means.

This last feature of the F-ratio test could well be used in fault diagnosis, to select the circuit's attributes, test points and/or frequencies (AC domain), such that a relatively good discrimination is achieved between all the fault conditions of a circuit under test. The higher the F value, the better the chance that a particular circuit feature may offer a good discrimination, to distinguish one fault condition from another. Feature selection is an important aspect of the fault diagnosis problem because, if a circuit is found to be faulty, the location of the component responsible for that failure should be made on the basis of measuring these circuit performances.

3.6 SUMMARY

The concept of multidimensional component parameter space is introduced. In this context, the terminology to be used in the fault location algorithm has been defined, i.e. Tolerance Region; Acceptability Region; Faulty Tolerance Region; Specification Test; and Diagnostic Test.

Additionally, the Fault Modelling and Diagnostic Response problems have been identified and described. Finally, the statistical techniques of Monte Carlo analysis and the One-Factor Analysis of Variance are suggested as a means of dealing with the problems mentioned above.

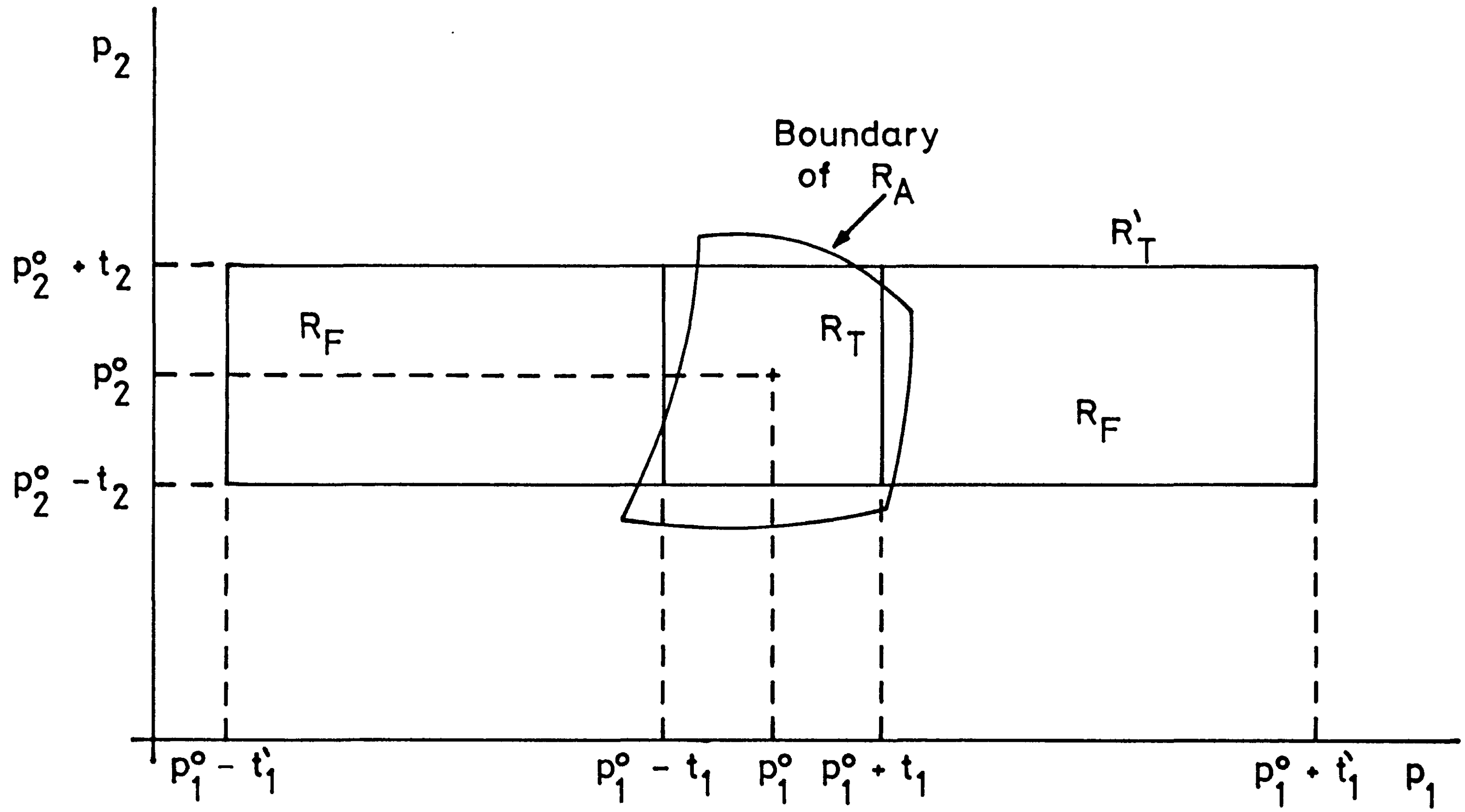


Fig. 3.1 Tolerance regions and the Acceptability region in multidimensional component parameter space

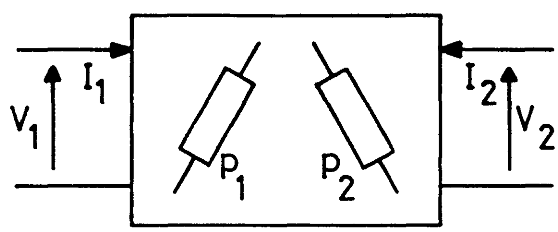


Fig. 3.2 Example circuit containing two components subject to failure

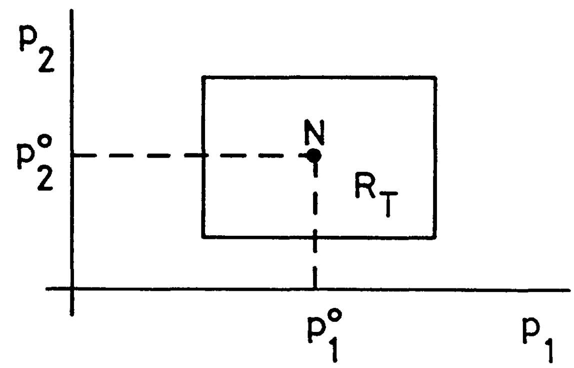


Fig. 3.3 Tolerance region R_T and the nominal circuit N in component parameter space

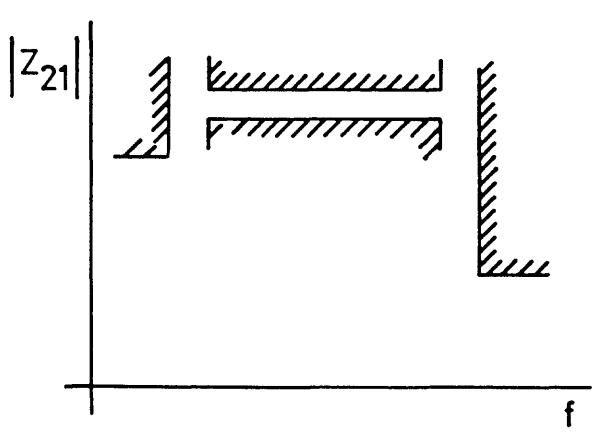


Fig. 3.4 Example of performance specification in the output space (frequency domain)

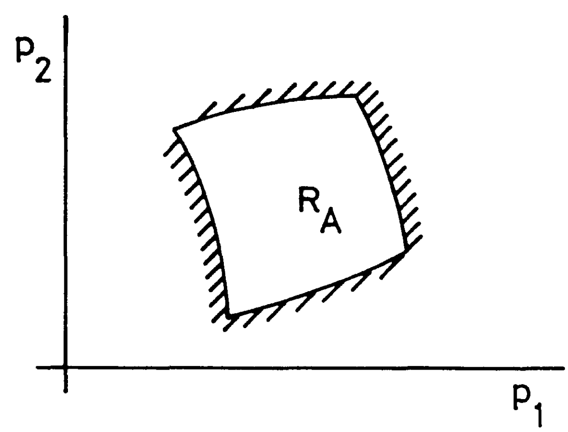


Fig. 3.5 Region of Acceptability R_A in component parameter space

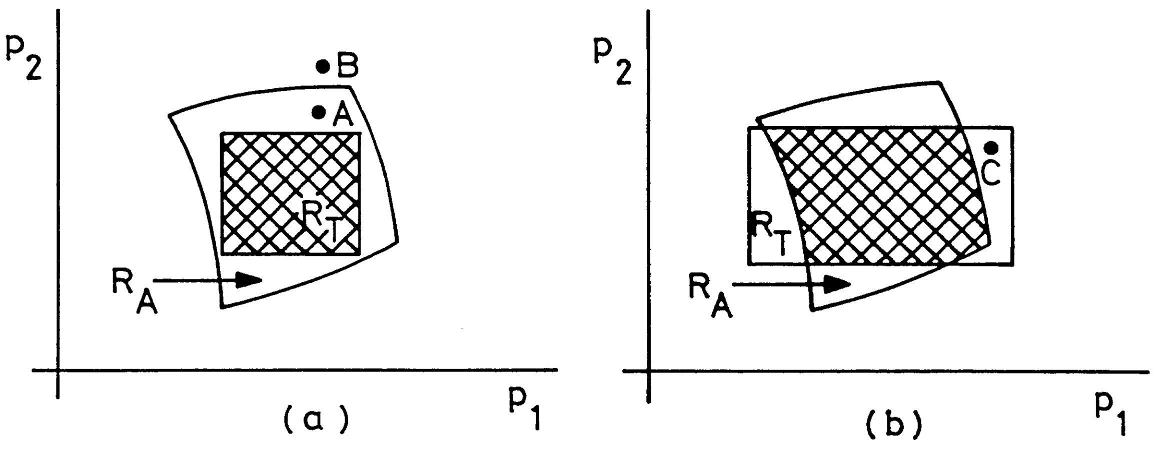


Fig. 3.6 Tolerance region R_T and region of Acceptability R_A in component parameter space

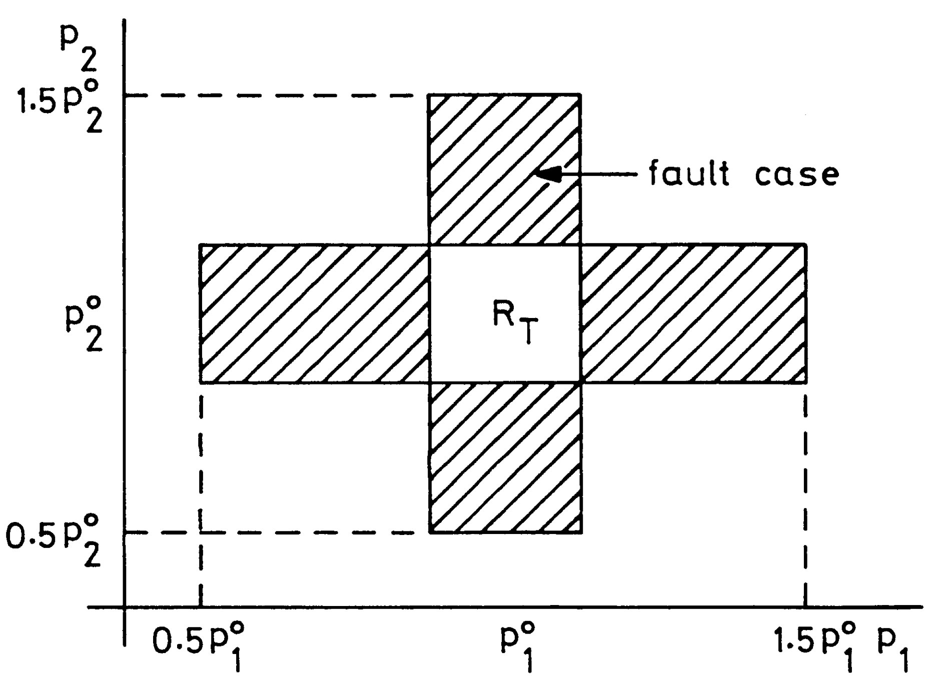


Fig. 3.7 Illustration in component parameter space of the four fault-case regions for a two component circuit

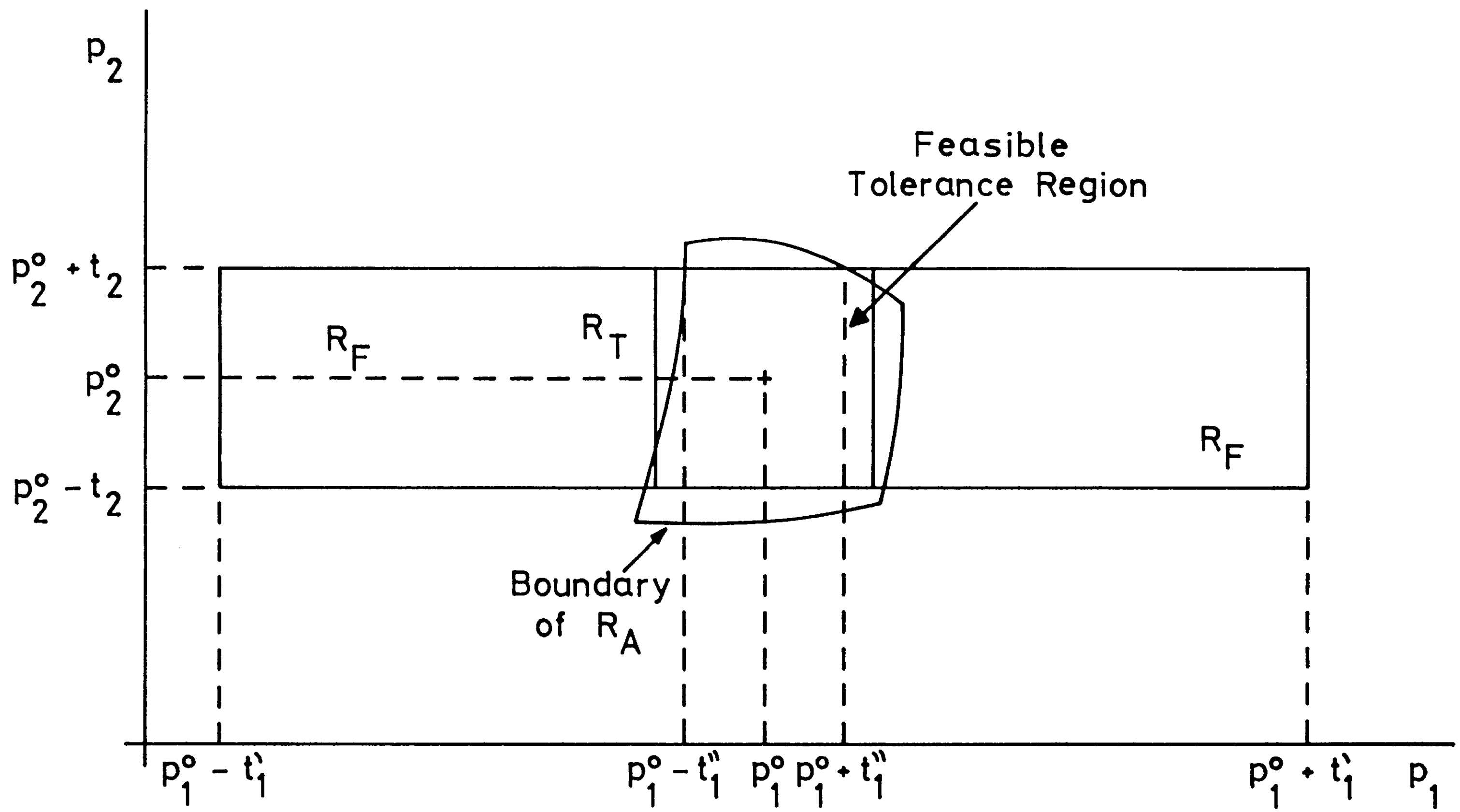


Fig. 3.8 Illustration of possible determination of the feasible tolerance region for the case that R_T region is not a subset of R_A region

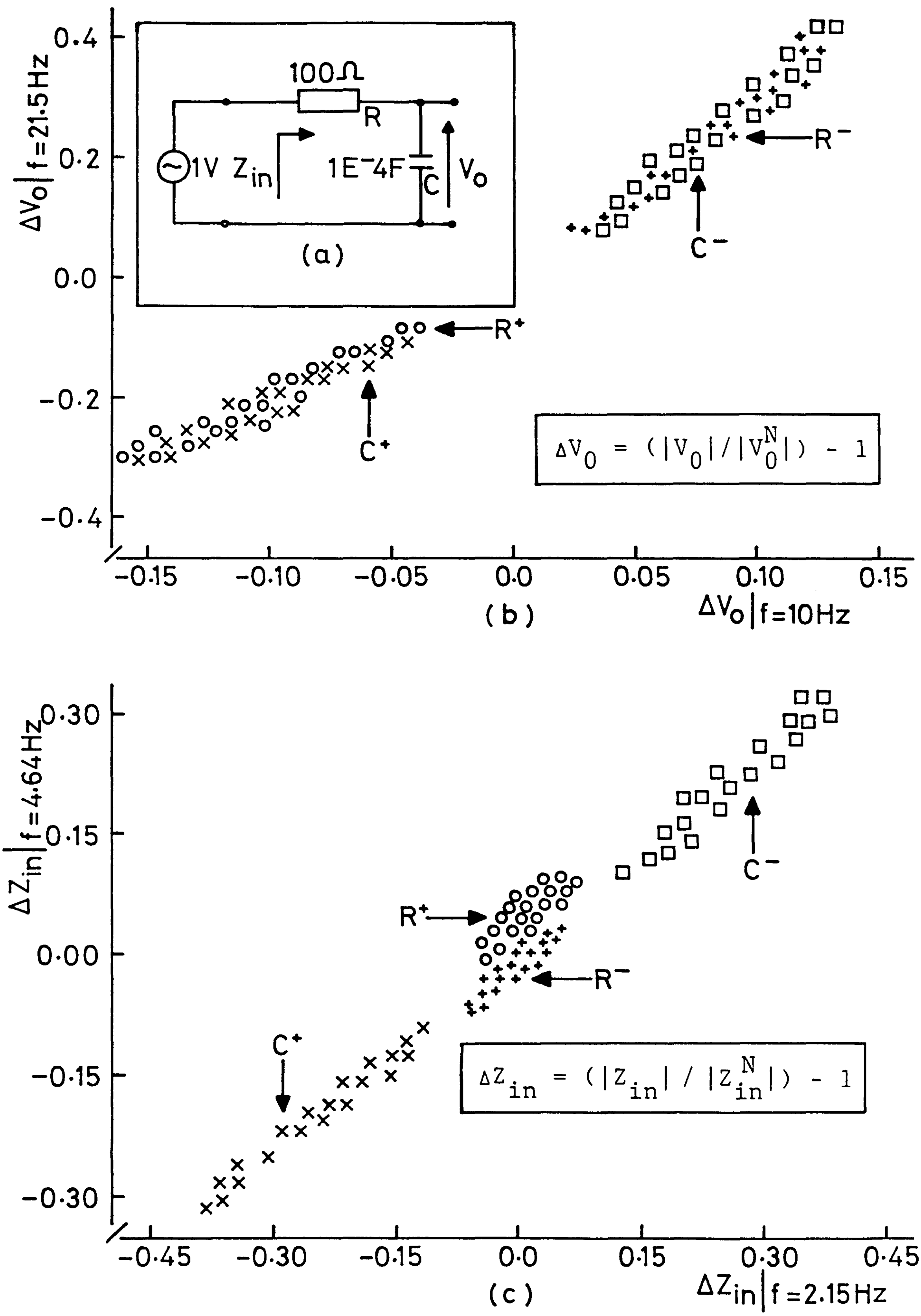


Fig. 3.9 Illustration of the effect, on the discrimination between fault cases, of the selection of different diagnostic responses

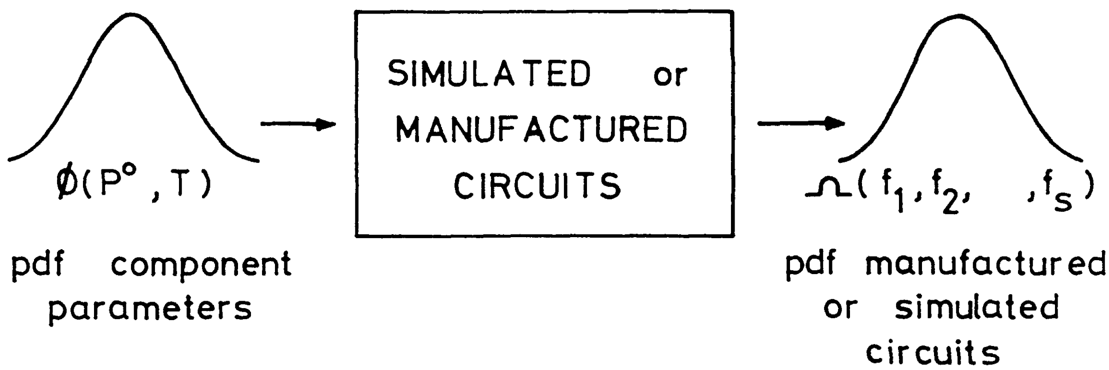


Fig. 3.10 Basic model of the effect of component parameter tolerances on the responses of manufactured circuits

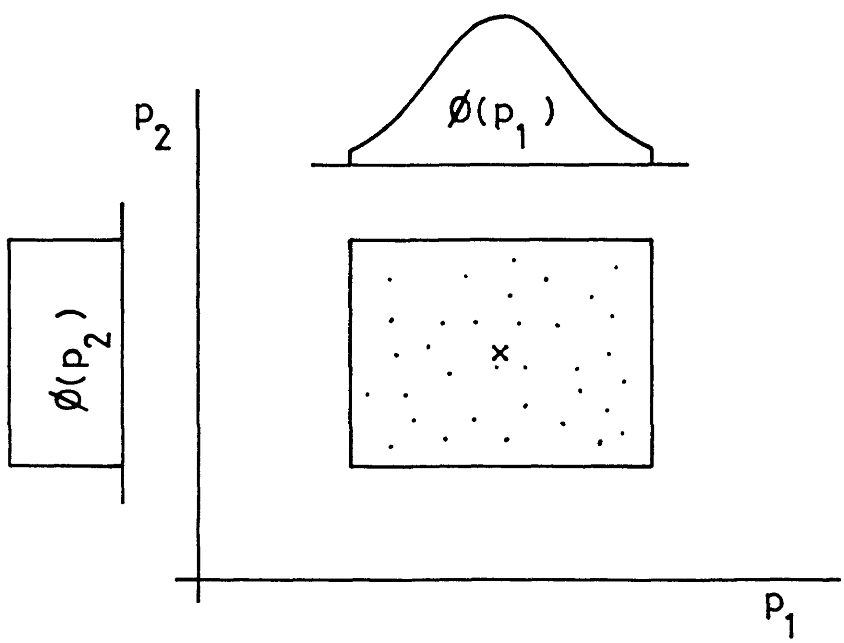


Fig. 3.11 A two dimensional illustration of Monte Carlo analysis, with component parameters taking values according to their probability distribution

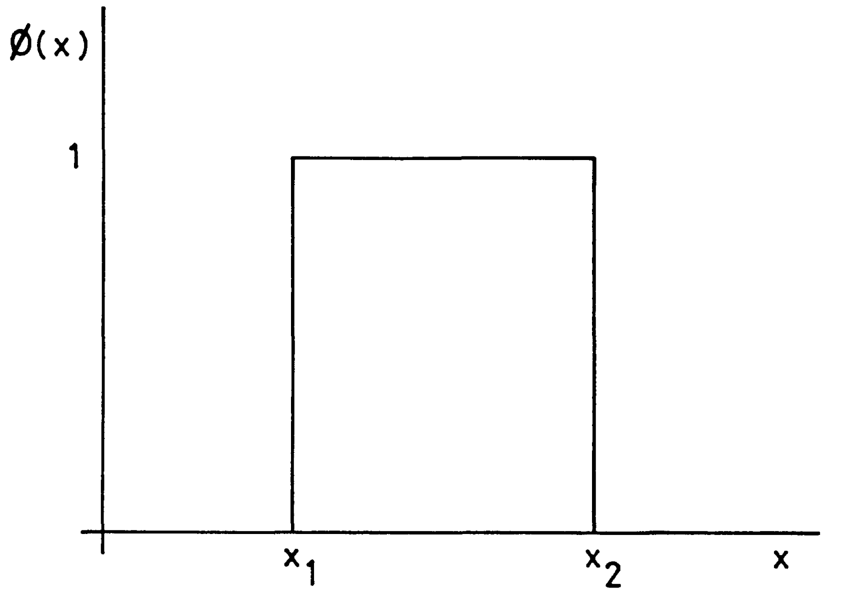


Fig. 3.12 Uniform distribution

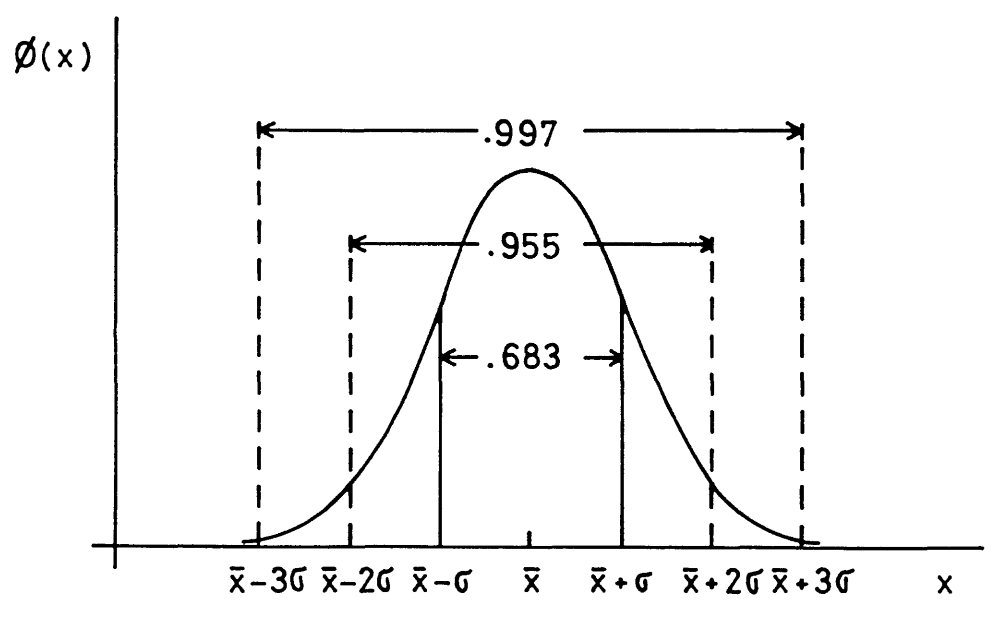


Fig. 3.13 Gaussian distribution

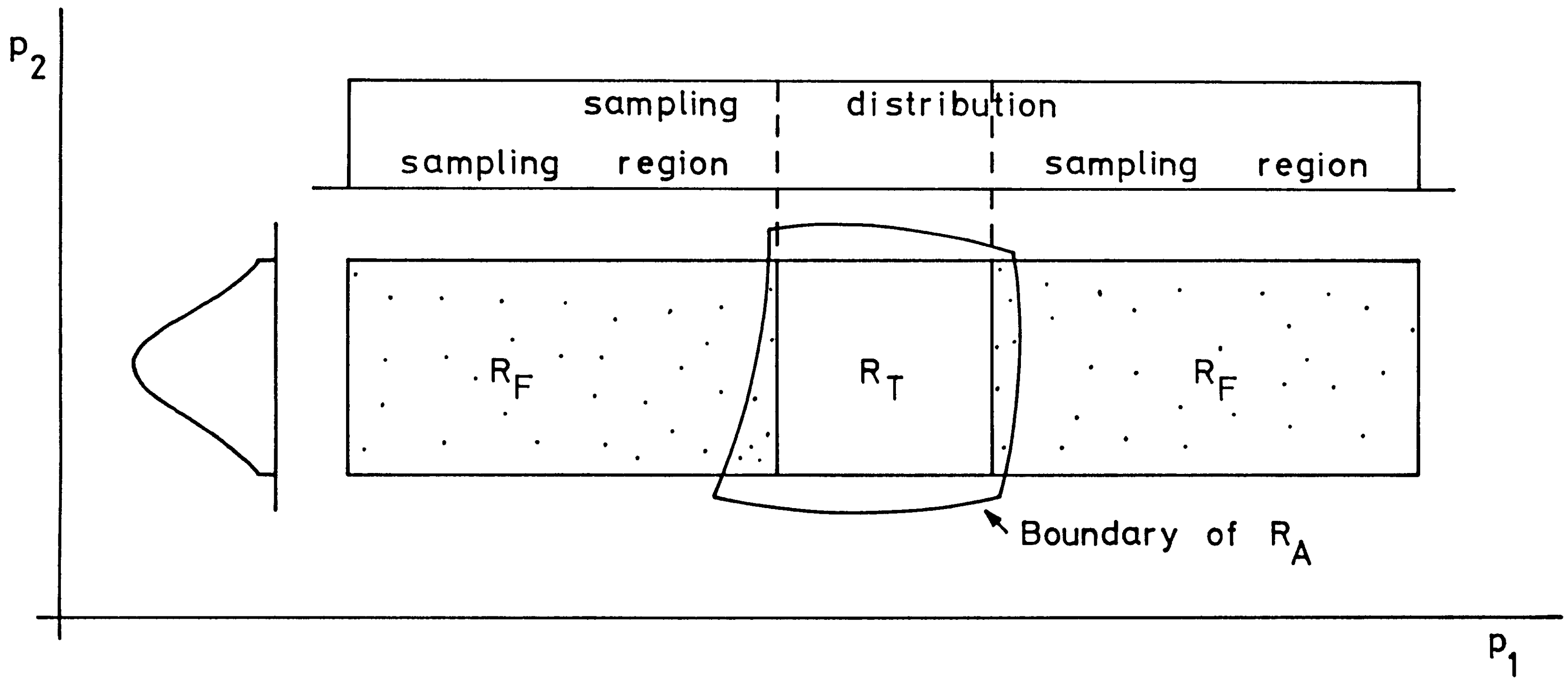


Fig. 3.14 Fault simulation considering component one as faulty. Sampling is performed only at the regions of interest (R_F)

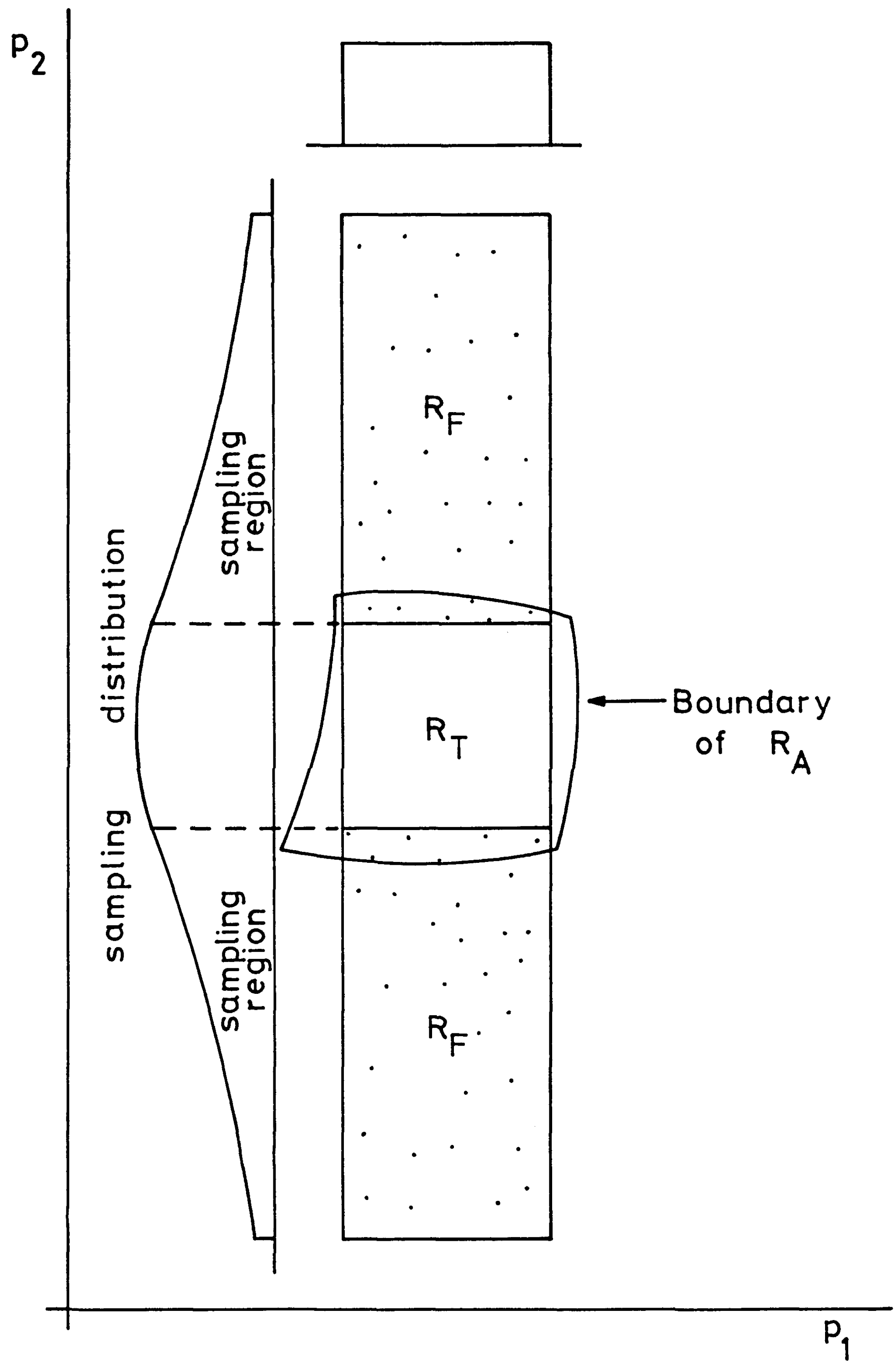


Fig. 3.15 Fault simulation considering component two as faulty. Sampling is performed only at the region of interest (R_F)

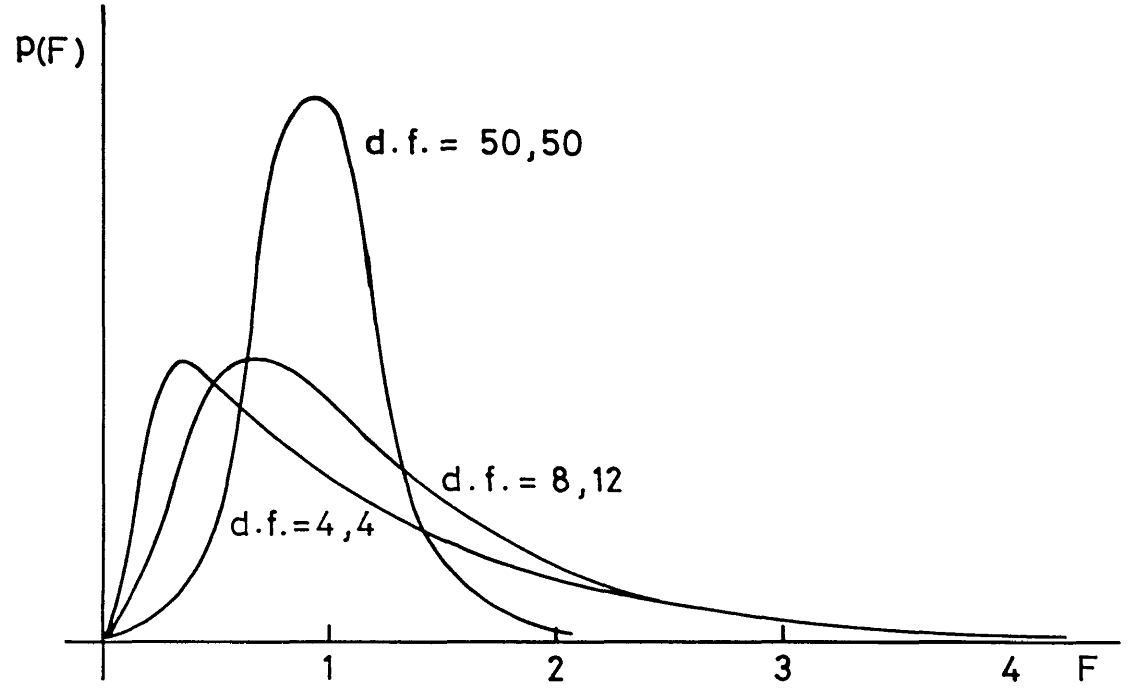


Fig. 3.16 The F distribution with various degrees of freedom in numerator and denominator of the F-ratio (Equation 3.9)

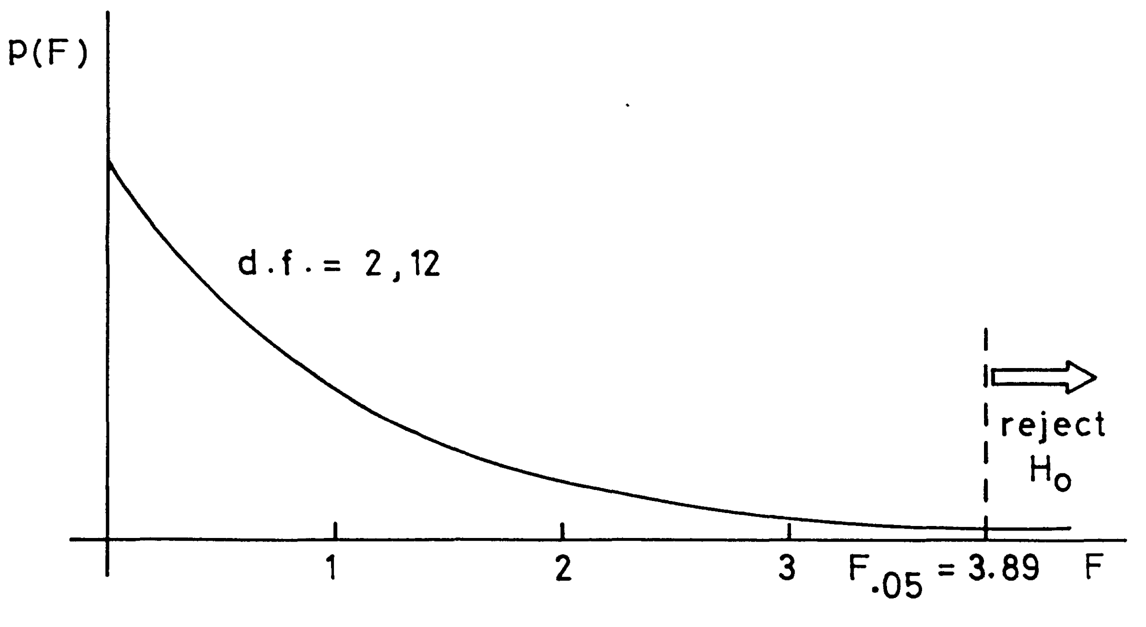


Fig. 3.17 The distribution of F when H_0 is true, with 2 , 12 degrees of freedom

Table 3.1 1-Factor Analysis of Variance table for equal sample sizes

<table>
<tr><th>SOURCE OF VARIATION</th><th>VARIATION SUM OF SQUARES (SS)</th><th>DEGREES OF FREEDOM (d.f)</th><th>VARIANCE MEAN SQUARE (MS)</th><th>CALCULATED F-RATIO</th><th>TABLED F</th></tr>
<tr><td>BETWEEN POPULATIONS: BY DIFFERENCES IN $\bar{x}_i$'s</td><td>$SS_K = n \sum_{i=1}^{K} (\bar{x}_i - \bar{x})^2$</td><td>$K - 1$</td><td>$MS_K = SS_K / (K - 1)$
$= s_K^2$</td><td rowspan="2">$F = \frac{MS_K}{MS_r}$</td><td rowspan="2">$F[1 - \alpha; K - 1; K(n - 1)]$</td></tr>
<tr><td>WITHIN POPULATIONS: DEVIATION ABOUT $\bar{x}_i$ (RESIDUAL)</td><td>$SS_r = \sum_{i=1}^{K} \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2$</td><td>$K(n - 1)$</td><td>$MS_r = SS_r / K(n - 1)$
$= s_p^2$</td></tr>
<tr><td>TOTAL</td><td>$SS_t = \sum_{i=1}^{K} \sum_{j=1}^{n} (x_{ij} - \bar{x})^2$</td><td>$Kn - 1$</td><td></td><td></td><td></td></tr>
</table>

Table 3.2 1-Factor Analysis of Variance table for unequal sample sizes

SOURCE OF VARIATION	VARIATION SUM OF SQUARES (SS)	DEGREES OF FREEDOM (d.f)	VARIANCE MEAN SQUARE (MS)	CALCULATED F-RATIO	TABLED F
BETWEEN POPULATIONS: BY DIFFERENCES IN $\overline{x}_i$'s	$SS_K = \sum_{i=1}^{K} n_i(\overline{x}_i - \overline{x})^2$	$K - 1$	$MS_K = SS_K / (K - 1)$ $= s_K^2$	$F = \dfrac{MS_K}{MS_r}$	$F[1 - \alpha; K - 1; N - K]$
WITHIN POPULATIONS: DEVIATION ABOUT $\overline{x}_i$ (RESIDUAL)	$SS_r = \sum_{i=1}^{K} \sum_{j=1}^{n_i} (x_{ij} - \overline{x}_i)^2$	$\sum_{i=1}^{K} (n_i - 1)$ $= N - K$	$MS_r = SS_r / (N - K)$ $= s_p^2$		
TOTAL	$SS_t = \sum_{i=1}^{K} \sum_{j=1}^{n_i} (x_{ij} - \overline{x})^2$	$\sum_{i=1}^{K} n_i - 1$ $= N - 1$			

CHAPTER FOUR

STATISTICALLY BASED ALGORITHM FOR FAULT LOCATION IN TOLERANCED ANALOGUE CIRCUITS

4.1 INTRODUCTION

Considerable progress has already been made in the search for simple and efficient methods of fault diagnosis. Chapter two presents an extensive review of existing methods which, on the basis of the mathematical techniques employed, have been classified into deterministic and non-deterministic ones, the latter including probabilistic, pattern recognition, and fuzzy set methods. The majority of these approaches belong to the first (deterministic) category (chapter two, section 2), and are based on the principle of solving the equations describing the faulty circuit. Generally, the objective, as mentioned in section 2 of chapter two, is to determine the value of each component parameter, whether or not it is faulty. However, although these deterministic methods may give a more definitive repair decision, the majority are applicable only when all the circuit's internal nodes are accessible to allow diagnostic measurements to be performed. Selective component identification is performed when not all the circuit's internal nodes are accessible but, in this case, non-faulty components are assumed to have their nominal parameter values; in other words, no account is taken of the fact that acceptable component parameters have tolerances associated with them.

By contrast, non-deterministic methods (chapter two, section 3) usually take some account of the normal production variation (i.e. tolerance) of the non-faulty component parameters. The identification of the faulty component is thereby rendered much more difficult than for the deterministic approaches, so that only the most likely faulty component can be located. Probabilistic methods [19 - 21] are based on the principle of determining the probability of each component being faulty on the basis of measurements made at test points, and from knowledge concerning the statistical distributions of the component parameter values: the component with maximum probability is declared as the most likely faulty component. Pattern recognition and fuzzy set methods (e.g. 22) are based on a comparison between diagnostic measurements carried out on the faulty circuit and items within a database formed from the simulation of all possible fault conditions. The simulated fault condition which is "nearest" in some sense to the diagnostic measurements is then judged to be the most likely cause of the circuit malfunction.

The new fault diagnosis method presented in this chapter is within the category of non-deterministic methods. By contrast to many other methods, it offers a valuable degree of universality in three principal respects. First, there is no restriction to linear circuits. Second, the value of any non-faulty component parameter can lie anywhere within its tolerance range. Third, no a priori assumption is made concerning the location, within the circuit, of the nodes at which diagnostic measurements can be made. The viewpoint taken in developing the algorithm also differs somewhat from other contributions in two respects. First, a distinction is made between <u>specification</u>

responses, the limits to which are set by the customer, and the diagnostic responses measured in the field. The two sets of responses need not be identical, the latter being selected solely for efficient fault diagnosis. Second, the problem to be solved is viewed in component parameter space, since it is felt that a better insight may thereby be gained into the problem of fault diagnosis.

However, there are at least two aspects that should be additionally considered in assessing possible limitations associated with the method described. Although the algorithm has been developed for single faults, it seems to be equally appropriate for multiple faults, where the main difficulty is likely to be found in the generation and processing of the amount of fault data. The next consideration refers to the application of the method for large deviations on component parameters. It is thought the method may be suited to "soft" and "hard" faults, but it may not be adequate for faults approximating open and short circuits, specially for nonlinear circuits.

The specific problem addressed herein is most easily illustrated by reference to the scheme shown in Fig. 4.1. Following the realization that the performance of a circuit (A) has violated its specification, some diagnostic measurements (B) are made on the faulty circuit, and compared with information within a data-base (C) to determine which previously simulated component fault is "nearest" in some sense to the diagnostic measurements. The data-base (C) itself is compiled during the design stage of the circuit.

In view of the need to minimize the size of this data-base, it is selected from a much larger data-base (D) to ensure effective fault diagnosis with the minimum number of diagnostic measurements. This larger data-base (D) is generated by the extensive simulation of possible fault cases.

The organization of this chapter is also easily related to the scheme of Fig. 4.1. First, section 2 describes the simulation, during the design phase, of all possible fault cases, and their subsequent statistical characterization. Then, section 3 considers the selection of suitable diagnostic measurements to be made in the field. The use of the diagnostic measurements to identify the faulty component is described in section 4. Following a summary of the algorithm in section 5, illustrative results for a 7-component linear circuit are presented in section 6. Lastly, it should be noted that the problem has already been described and illustrated in section 3 of chapter three, in the context of component parameter space.

4.2 FAULT SIMULATION AND FAULT DATA GENERATION

Consider a circuit containing K components. With each component will be associated two fault cases, corresponding to positive and negative changes, as illustrated in Figures 3.7, 3.14, and 3.15. The fault cases will be indexed by $j = 1,2,\ldots\ldots,2K$. M responses of the circuit are considered to be candidates for selection as diagnostic responses; these will be indexed by $i = 1,2,\ldots,M$.

One or more specification responses can, but need not, constitute some of these M responses.

The value of each response will in general be different from its nominal value, not only because a component may be faulty but also because all component parameters are free to assume any value within their tolerance range. If the nominal values of the M responses are represented by the vector vn:

$$vn = [vn_1, vn_2, \ldots\ldots, vn_M]^T \qquad (4.1)$$

and the response values associated with a fault by x:

$$x = [x_1, x_2, \ldots\ldots\ldots, x_M]^T \qquad (4.2)$$

then a deviation vector characterizing the fault could be formed by simple subtraction. However, since each of the responses might be measured in different units, and might have widely different nominal values, a normalized vector of responses y is instead considered:

$$y = [y_1, y_2, \ldots\ldots\ldots, y_M]^T \qquad (4.3)$$

where

$$y_i = (x_i - vn_i) / vn_i \qquad i = 1, 2, \ldots\ldots, M \qquad (4.4)$$

With each component is associated two fault cases. In the method proposed, each fault case is characterized statistically by a sampling process, in the following way:

for the jth fault case, N_j circuits are randomly generated, with a uniform distribution, within the corresponding region of component parameter space. Each of the N_j circuits is then simulated by means of a conventional circuit analysis package to see if it fails the specifications. If it does, the normalized values of the M responses are computed and recorded. In this way, an $M \times N_j$ matrix of normalized responses will be obtained for the fault case if all N_j samples correspond to failed circuits. If some satisfactory circuits are found, their responses are not recorded, and further samples are taken until a total of N_j failed circuits have been recorded. The procedure is repeated for all 2K fault cases. Thus, the minimum number of circuit simulations involved is $2 \times K \times N_j$.

Each fault case is now characterized statistically on the basis of the M responses associated with the sample circuits representing failed circuits within the fault case. Thus, the jth fault case is represented by three quantities. First, the column vector $\hat{y}_j$, which is the vector of the mean normalized values of the M responses:

$$\hat{y}_j = [\hat{y}_{1j}, \hat{y}_{2j}, \ldots\ldots, \hat{y}_{Mj}]^T \tag{4.5}$$

where

$$\hat{y}_{ij} = \frac{1}{N_j} \sum_{r=1}^{N_j} y_{irj} \tag{4.6}$$

and r ranges over the N_j samples associated with the fault case. As before, i ranges over the responses and j over the fault cases. Second, by the vector v_j^2 characterizing the variances of the M normalized responses:

$$v_j^2 = [v_{1j}^2, v_{2j}^2, \ldots\ldots, v_{Mj}^2]^T \tag{4.7}$$

where

$$v_{ij}^2 = \frac{1}{(N_j - 1)} \sum_{r=1}^{N_j} (y_{irj} - \hat{y}_{ij})^2 \tag{4.8}$$

Third, by the M×M variance-covariance matrix S_j:

$$S_j = \frac{1}{(N_j - 1)} \sum_{r=1}^{N_j} (y_{rj} - \hat{y}_j)(y_{rj} - \hat{y}_j)^T \tag{4.9}$$

where

$$y_{rj} = [y_{1rj}, y_{2rj}, \ldots\ldots, y_{Mrj}]^T \tag{4.10}$$

whose elements indicate the interrelation of the M responses. The matrix S_j is obtained from the M×N_j matrix containing all responses for the jth fault case. To ensure nonsingularity of S_j (matrix inversion) the constraint $N_j \geqq M$ must always be satisfied.

Having obtained the vectors $\hat{y}_j$ and v_j^2, and the matrix S_j, for each fault case, all this information is now combined and placed into matrices Y, V and S in the following way: the M×2K matrix Y contains all the vectors $\hat{y}_j$ as individual columns; thus, the jth column of Y contains the means of the M responses for the jth fault case. The M×2K variance matrix V similarly contains all the vectors v_j^2. The M×M matrix S is termed the pooled covariance matrix. The matrices Y, V and S are defined by:

$$Y = [\ \hat{y}_{ij}\ ] \tag{4.11}$$

$$V = [\ v_{ij}^2\ ] \tag{4.12}$$

$$S = [\ s_{ik}\ ] = \frac{1}{N - 2K} \sum_{j=1}^{2K} (N_j - 1)\, S_j \tag{4.13}$$

where

$$N = \sum_{j=1}^{2K} N_j \tag{4.14}$$

and k ranges from 1 to M responses.

The matrices Y, V and S constitute a statistical characterization of the component faults in terms of their effect on the set of candidate responses. The choice of an acceptably small set of diagnostic responses from the candidate set, on the basis of the information contained within Y, V and S, is the subject of the next section.

4.3 SELECTION OF DIAGNOSTIC RESPONSES

The designers's insight into circuit behaviour, perhaps supported by a sensitivity analysis of the circuit and/or previous fault data, may provide some indication of those circuit performances that can be considered for selection as diagnostic responses. In the field, of course, it is preferred, for economic reasons, to measure as small a number of responses as are necessary for the location of the faulty component with acceptably high confidence. For this reason, the selection of diagnostic responses must be based on the criterion that they should reflect the symptoms of the fault conditions of the circuit, and provide discrimination between them to enable, as far as possible, a unique diagnosis of the faulty component. It is therefore necessary to examine the data (i.e. Y, V and S) characterising the fault cases with the aim of selecting a limited number of diagnostic measurements. Two methods are proposed, and experimental results presented later enable their merit to be assesed.

4.3.1 ANALYSIS OF VARIANCE

The first technique that can be used to reduce the number of diagnostic responses is the 1-Factor Analysis of Variance, which is a numerical measure of the degree to which the means of the 2K fault cases differ for the ith measurement. The measure indicates how large the variance of these means is relative to the average spread (pooled variance) of the 2K individual fault cases. For each ith measurement, the variance of the 2K means (equation 4.11) is first calculated so that, for the M measurements, a vector z^2 is obtained:

$$z^2 = [z_1^2, z_2^2, \ldots\ldots, z_M^2]^T \tag{4.15}$$

where

$$z_i^2 = \frac{1}{(2K - 1)} \sum_{j=1}^{2K} N_j (\hat{y}_{ij} - \bar{y}_i)^2 \qquad (4.16)$$

and

$$\bar{y}_i = \frac{1}{N} \sum_{j=1}^{2K} N_j \hat{y}_{ij} \qquad (4.17)$$

where N is the total number of samples, as defined in equation 4.14. Next, the average spread of the 2K fault cases for each ith measurement is calculated from equation 4.12, resulting in a M-dimensional vector sp^2 defined by:

$$sp^2 = [sp_1^2, sp_2^2, \ldots\ldots, sp_M^2]^T \qquad (4.18)$$

where

$$sp_i^2 = \frac{1}{N - 2K} \sum_{j=1}^{2K} (N_j - 1) v_{ij}^2 \qquad (4.19)$$

For each jth fault case, the population variance v_{ij}^2 has $(N_j - 1)$ degrees of freedom, so that the pooled variance sp_i^2 has $\Sigma_{j=1}^{2K} (N_j - 1)$ degrees of freedom. For the ith measurement the F-ratio z_i^2 / sp_i^2 is calculated, which is the variance of the means divided by the mean of the variances:

$$F = [F_1, F_2, \ldots\ldots, F_M]^T \qquad (4.20)$$

where

$$F_i = z_i^2 / sp_i^2 \qquad (4.21)$$

If for any particular ith measurement, the mean values of the 2K populations (fault cases) differ widely, then z_i^2 will be relatively large compared with sp_i^2 and a high F-ratio will result. As discussed below, the numerical value of the F-ratio for a particular measurement serves as an indication of the discrimination that the measurement might offer between fault cases.

Since the F-ratio vector characterizes the separability of fault case means of the M responses, candidate measurements having a low F-ratio can first be eliminated. But this leaves unanswered the general question of how many candidate measurements should be selected as diagnostic responses, a question which is directly related to the classification of a particular fault into the pre-specified fault cases. In general there is a simple trade-off between the number of diagnostic responses and the probability of diagnostic error (as will be illustrated later for a sample circuit) though its nature has not been established theoretically for the methods discussed herein. However, on the basis of a limited number of experiments, it has been found that a satisfactory result is obtained when the number of diagnostic responses is equal to the number (K) of potentially faulty components within the circuit. In this case the K largest values in the F-ratio vector would identify which of the M candidate responses should be selected as diagnostic responses.

If, in general, R diagnostic responses are selected corresponding to the R highest F values, the matrices Y, V and S are reduced to the matrices DY, DV and DS respectively, simply by selection of the entries corresponding to the R selected diagnostic measurements: i.e. by deletion of the rows and columns associated with the discarded candidate responses:

$$DY = [\ \hat{y}_{hj}\ ] \tag{4.22}$$

$$DV = [\ v^2_{hj}\ ] \tag{4.23}$$

$$DS = [\ S_{h\ell}\ ] \tag{4.24}$$

where h and ℓ range over the R measurements and, as before, j ranges over the 2K fault cases. Matrices DY and DV are of size $R \times 2K$, and the size of the reduced covariance matrix DS is $R \times R$.

4.3.2 EIGENANALYSIS

An alternative method of selecting the diagnostic responses from the available candidates is suggested by the fact that, although the above basis leads to acceptable results (see section 4.6) it does not take into account any interrelation between the M measurements. It is therefore natural to investigate the possible benefits of using the covariance matrices, instead of the variances, to calculate the F-ratio. The $M \times M$ between fault case covariance matrix B is calculated from equation 4.11 according to the expression:

$$B = \frac{1}{2K - 1} \sum_{j=1}^{2K} N_j\,(\hat{y}_j - \bar{y})\,(\hat{y}_j - \bar{y})^T \tag{4.25}$$

where, as before, indices i and j ranges over the measurements and fault cases respectively (see equation 4.17).

$$\bar{y} = [\bar{y}_1, \bar{y}_2, \ldots\ldots, \bar{y}_M]^T \qquad (4.26)$$

The matrix B characterizes the dispersion, in the M-dimensional candidate response space, among the fault case response means.

Covariance, such as the one defined in eqn. 4.9 or eqn. 4.25, measures the closeness of association, generally linear, between the samples of different variables (responses); that is, the extent to which the sample points on a scatter diagram cluster around a sloping straight line. The strength or degree of this relationship is given by a numerical quantity that may be positive, negative, or zero, indicating direct, inverse, and no association at all (independent variables) respectively.

An indication of whether there is an association between a pair of random variables (responses) $[y_1, y_2]$, may be provided, as an illustrative example, in the scatter of N sample points plotted in Fig. 4.2, where the origin of the response axes has been taken at the sample means $[\hat{y}_1, \hat{y}_2]$. It can be seen that the dispersion of the sample points seems to have an ellipsoidal shape orientated in the direction of maximum variation of the sample points. This situation may be described, in the two-dimensional space, by introducing the new sets of orthogonal linear coordinates w_1 and w_2. Considering the major axis w_1 and denoting its angle with the original variables $[y_1, y_2]$ as $[\alpha_1, \alpha_2]$ respectively, its orientation is completely determined by the direction cosines

$$a_{11} = \cos \alpha_1 \qquad a_{21} = \cos \alpha_2 \qquad (4.27)$$

which should be subjected to the constraint $a_{11}^2 + a_{21}^2 = 1$ ($a_1^T a_1 = 1$; $a_1^T = [a_{11}, a_{21}]$), to assure uniqueness of the coefficients. Then the value of the sample points $[y_{i1}, y_{i2}]$ on the new coordinate axis w_1 will be

$$w_{i1} = a_{11}(y_{i1} - \hat{y}_1) + a_{21}(y_{i2} - \hat{y}_2) \qquad (4.28)$$

$$i = 1,2,\ldots\ldots,N \text{ samples}$$

which is in fact the projections of the sample points onto the new coordinate axis w_1. A more comprehensive and appropriate analysis involves finding the eigenvalues and eigenvectors of the sample covariance matrix Q (defined, similarly as in equation 4.9), such that, for obtaining the new coordinate w_1, all that is involved is the determination of the largest eigenvalue of the covariance matrix Q and its associated eigenvectors (coefficients a_{11}, a_{21}). Similarly, to obtain the other new coordinate w_2, it is necessary to determine the next largest eigenvalue and its associated eigenvectors (coefficients a_{12}, a_{22}).

Thus, the purpose of this alternative method is to find linear combinations of the M responses which show high ratios of variance among the 2K fault cases relative to the variance within the 2K fault cases. Then, the M original variables (measurements) denoted by y, are considered to be linearly transformed to $w = a^T y$, where a^T is a new row vector of M elements. The 1-Factor Analysis of Variance for the new variable w then leads to the F-ratio:

$$F_a = a^T B a / a^T S a \qquad (4.29)$$

The maximum value of this ratio [55] is the largest eigenvalue c_1 of the matrix $S^{-1} B$, and the associated solution for the vector a is the eigenvector a_1 of $S^{-1} B$ corresponding to the eigenvalue c_1. Then the F-ratio, as expressed by equation 4.29 is, $F_{a_1} = a_1^T B a_1 / a_1^T S a_1$, which is equal to c_1. This value F_{a_1} corresponds to that linear combination of the M candidate responses that exhibits the largest ratio of variance among the 2K fault cases relative to the variance within the 2K fault cases; additionally, in the M-dimensional response space, the vector a_1 defines the direction in which the 2K fault cases show their maximum separation (Fig. 4.3 illustrates a particular two dimensional example). Similarly, the linear combination w_2 associated with the next largest F-ratio is defined by the eigenvector a_2 corresponding to the next largest eigenvalue, c_2 of $S^{-1} B$. Other linear combinations are found following the same procedure. There will be M eigenvalues associated with the M×M matrix $S^{-1} B$, each associated with a particular candidate response.

In general, the number of positive eigenvalues of $S^{-1} B$ will be the smaller of (2K - 1) and M because, if

$2K < M$, the $2K$ fault case means are contained within a $(2K - 1)$ dimensional hyperplane. Thus if, as is usual, $2K < M$, the number (p) of positive eigenvalues will be less than the number of candidate responses, and a first reduction of the number of candidate responses will thereby have been achieved. If the (p) positive eigenvalues are now ranked in order:

$$c_1 \geqq c_2 \geqq c_3 \geqq \ldots\ldots\ldots \geqq c_p > 0 \qquad (4.30)$$

then the selection of the R largest eigenvalues will achieve a further reduction in the number of candidate responses and identify the R candidate responses to be selected as the diagnostic responses. The corresponding reduction of the matrices Y, V and S to obtain DY, DV and DS proceeds in the same way as for the first method.

Although, in the above discussion, the original responses are considered to be linearly transformed according to $w = a^T y$, it is not necessary to store the relevant eigenvectors (a_1 to a_R) or to perform the transformation either on data generated in the design phase or on the diagnostic responses measured in the field. The sole purpose of the eigenanalysis is to select the diagnostic responses, along which the 2K fault cases show their greatest separation. The diagnostic responses chosen in this way will in general differ from those chosen by the Analysis of Variance method (subsection 4.3.1).

4.4 FAULT LOCATION

Following the detection of a circuit failure in the field, the R selected diagnostic measurements are performed on the faulty circuit. The results of these measurements are then normalized and will be denoted by the vector u:

$$u = [u_1, u_2, \ldots\ldots\ldots, u_R]^T \tag{4.31}$$

Fault diagnosis comprises the classification of the vector u into one of the 2K fault cases characterized by the matrices DY, DV and DS.

Classification is achieved by identifying the fault case which, in R-dimensional response space, is "nearest" in some sense to the point u. Thus, some measure of distance is required. Here, the Mahalanobis distance [55] is used:

$$d_j^2 = (u - \hat{y}_j)^T P (u - \hat{y}_j) \tag{4.32}$$

since it offers a general class of squared distance functions. The matrix P is positive definite to ensure that d_j^2, which is the squared distance between u and the point representing the jth fault case, is greater than or equal to zero. In equation (4.32), $\hat{y}_j = [\hat{y}_{1j}, \hat{y}_{2j}, \ldots, \hat{y}_{Rj}]^T$ is the jth column of the matrix DY. Thus, for example, if P is the R×R unit matrix, then d_j^2 is simply the familiar Euclidean squared distance. The unknown fault represented by the vector u will be classified into the fault case for which d_j^2 is smallest.

Two choices of P will be considered, and are based on the use of the previously stored diagnostic data within DY, DV and DS. In the first choice, P is a diagonal matrix with diagonal elements equal to the reciprocal of the variances of the different measurements:

$$\underline{\text{Method A}}: \quad P = \text{diag}\,[1 / v_j^2\,] \tag{4.33}$$

where v_j^2 is the jth column of matrix DV. This choice of P takes account of the fact that, for the different fault cases, responses possess different variances which should be used to weight the distances (Fig. 4.4 is an example).

In the second choice, denoted method B, the matrix P is chosen to be the inverse of the pooled covariance matrix DS (see equation 4.24):

$$\underline{\text{Method B}}: \quad P = DS^{-1} \tag{4.34}$$

This choice for P takes account of possible interrelations among the responses, as well as differences in the variance of the responses (Fig. 4.5 illustrates a two-dimensional example). Although the use of the pooled covariance matrix DS saves storage and computation effort, it might usefully be restricted to situations in which the fault case dispersion characteristics are not widely different. If this is not the case, the reduced covariance matrix corresponding to the jth fault case can be used to calculate the squared distance.

Whichever basis (method A or B) is used to compute the distance d_j^2 for each fault case, it is suggested that the smallest value of d_j^2 be chosen to identify the most likely fault case.

4.5 IMPLEMENTATION OF THE ALGORITHM

The practical application of the proposed algorithm involves two separate activities, one in the design phase and the other in the field following circuit failure. They are considered in that order.

4.5.1 THE DESIGN PHASE

In the design phase, the nominal circuit must first be characterized, in the following steps:

(1) Take circuit description from designer,

(2) Choose M candidate responses,

(3) Evaluate the M responses for the nominal circuit.

The tolerances on component parameters, and the nature of the fault cases, must now be taken into account in order to accumulate the data on which the selection of diagnostic responses can be based. The following steps are involved (Flowchart in Fig. 4.6):

(1) Note the component parameter tolerances,

(2) Accept performance specifications supplied by the customer,

(3) Choose the tolerance limits defining the fault cases,

(4) Select a fault case. Generate randomly, and with uniform distribution, a component fault. Check the

circuit's performance against specifications; if it fails, calculate the values of the M candidate responses,

(5) Repeat step (4) a sufficient number of times (typically between 30 and 50), and thereby generate and store the appropriate column of Y (equation 4.11) and V (equation 4.12). Also compute the covariance matrix S_j for the fault case, according to equation 4.9,

(6) Repeat steps (4) and (5) for each fault case, thereby obtaining the complete Y and V matrices. Compute the pooled covariance matrix S. According to whether the first or second alternative selection of diagnostic responses is required, proceed to steps (7) or (8) (Flowchart in Fig. 4.7),

(7) To use the first alternative method:

(7.1) With data available in Y and V, evaluate the vectors z^2 and sp^2 according to equations 4.16 and 4.19 respectively, and hence calculate the F-ratio for each response according to equation 4.21,

(7.2) Select the R diagnostic responses which correspond to the R highest F-values: a common value for R is the number of potentially faulty components,

(7.3) Form and store the new matrices DY, DV and DS by retaining only those rows and columns of Y, V, and S associated with the R diagnostic responses,

(8) To use the second alternative method:

(8.1) Carry out an eigenanalysis [57] of the matrix $S^{-1}B$, and record the positive eigenvalue associated with each candidate response,

(8.2) Select the R diagnostic responses which correspond to the R largest eigenvalues: a common value for R is the number of potentially faulty components,

(8.3) Form and store the new matrices DY, DV, and DS by retaining only those rows and columns of Y, V and S associated with the R diagnostic responses.

Whichever of the two alternative methods is used as the basis for fault diagnosis, the diagnostic data contained within DY, DV and DS must be made available in the field to enable the diagnosis to be carried out.

4.5.2 DIAGNOSIS IN THE FIELD

Following the detection of a circuit whose performance does not satisfy the specifications, the following steps will be taken to identify the faulty component (Flowchart in Fig. 4.8):

(1) Measure and normalized the diagnostic responses of the faulty circuit,

(2) Calculate the squared distance (equation 4.32) of the measurement from each fault case according to method A (equation 4.33) or method B (equation 4.34),

(3) The fault case with the smallest squared distance from the diagnostic measurement identifies the faulty component.

4.6 RESULTS

The algorithm described in this chapter has firstly been applied to linear circuits, of which Fig. 4.9 is an example (results in subsection 4.6.1). However, since linearity was not a requirement in the development of the method, an exploration has been carried out to establish its usefulness for nonlinear circuits (results in chapter five, section 3).

4.6.1 AN RC FILTER CIRCUIT EXAMPLE

The circuit of Fig. 4.9 is used as an example to test the proposed algorithm (it has similarly been used by other authors [22, 23 and 25]). In fault-free conditions, each component parameter has a tolerance of ±5% of nominal. A fault case was deemed to occur if the value of one of the component parameters deviated by more than ±5%, but less than ±50% of nominal. Specifications on the response of interest (voltage gain) together with the nominal response are shown in Fig. 4.10. The chosen candidate responses were the magnitudes of the output voltage, the input impedance and the input-output transfer impedance at 10 frequencies equally distribuited logarithmically between 10 Hz and 10,000 Hz. Thus 30 candidate responses were involved. For each fault case, 50 samples associated with faulty circuits were simulated, and the relevant data (matrices Y, V and S) recorded.

For comparison purposes, the selection of diagnostic responses was carried out separately by both methods. For the first approach, seven diagnostic responses were selected:

they comprised the magnitude of the output voltage and transfer impedance each at three frequencies (10, 21.54 and 46.42 Hz) and the magnitude of the input impedance at 10 Hz. Using the second (eigenanalysis) approach a total of seven diagnostic responses was again selected: they comprised the magnitude of the input impedance at the 1st and 3rd frequencies, and the magnitude of output voltage at the 2nd to 4th frequencies inclusive, and at the 6th and 10th frequencies.

In order to test the effectiveness of the diagnostic algorithm, 50 faulty circuits were selected at random in each of the fourteen fault cases, and the diagnostic responses computed. For each faulty circuit, the computed diagnostic responses were then treated as field measurements in the application of the fault diagnosis algorithm. To enable a comparison to be made, both of the P matrices discussed earlier were used to compute the squared distance for each fault case. Additionally, location was also performed by computing the Euclidean squared distance, in order to demonstrate that the use of P as a unit matrix provides no improvement at all in the percentage of correct location, as compared with the two suggested methods.

For the first of the two approaches, the results are presented in Figs. 4.11 and 4.12. The results of Fig. 4.11 correspond to the algorithm as described above. The results of Fig. 4.12 indicate the percentage of cases in which the correct faulty component was associated with either the smallest value of d_j^2 or the next smallest.

For most faulty components, the diagnostic success is high if method B is used to compute squared distance, not so high but still acceptable with method A and, as expected, low with the Euclidean squared distance. The principal cause of uncertainty in the diagnosis of the circuit of Fig. 4.9 arises from the difficulty of discriminating between faults in C_2 and C_3, since their associated fault cases overlap to some extent. The hypothesis that they overlap is strongly supported by the fact that a correct diagnosis occurs in method B if the two smallest squared distances are used to identify the most probable faulty component.

The approach involving an eigenanalysis to select the diagnostic responses led to a greater success in fault diagnosis, as is evidenced in Figs. 4.13 and 4.14. Averaged over all components, the diagnostic success is increased from 78% to 90% using method A to compute distance, and from 94% to 97% using method B, with no apparent improvement using the Euclidean squared distance (71% to 72%). It should be noted that, although the computational effort associated with the eigenanalysis approach is higher in the design stage, it carries no computational disadvantage as far as field diagnosis is concerned.

On the basis of these results, it is concluded that the selection of diagnostic responses should be achieved by the eigenanalysis approach if possible and that, during diagnosis in the field, the use of the pooled covariance matrix DS to compute distance according to equations 4.32

and 4.34 is to be preferred. A comparison of the method with other published methods ([22] and [25]) is provided in Fig. 4.15. This comparison must, however, be interpreted with care, owing to different conditions relevant to each method.

It was stated earlier that the number R of diagnostic responses selected from the candidate responses was entirely at the designer's choice and that, while the matter had not been investigated theoretically, a trade-off had been observed between R and the probability of error in the diagnostic procedure. To obtain some familiarity with the nature of the trade-off, the diagnostic procedure for the circuit of Fig. 4.9 was repeated with choices for R betwwen 2 and 14. The first approach (Analysis of Variance) to the selection of diagnostic responses was used and both methods of computing distance were explored. For each fault case, fifteen faulty circuits were simulated. The results of this investigation, presented in Fig. 4.16, show that as R increases the diagnostic success also increases, though it is always higher, indeed significantly higher, if the classification (by minimum squared distance) is evaluated by the pooled covariance matrix DS.

4.6.2 FAULT LOCATION ERROR v/s COMPONENT PARAMETER DEVIATION.

An exploratory exercise was carried out on the 7 component RC filter circuit of Fig. 4.9, to investigate the behaviour of the method in relation to the degree of error in isolation (wrong location) as component parameter deviations increase from the production tolerance of ±5% to ±50%. A sequential exploration at deviations ±6%, ±9%, ±12%, ±15%, ±20%, ±30%, ±40%, and ±50% on the assumed faulty component was performed, while the nonfaulty component parameters were allowed to take values at random within their manufacturing tolerances of ±5% from nominal. A preliminary exploration was carried out with the specifications on the circuit, but it was observed that it was nearly impossible to find faults for low percentage deviations of component parameters, i.e., ±6% and ±9%. The reason for this is mainly due to the strong influence that the combination of the random variation of nonfaulty component parameters have on the specification responses.

Subsequently, it was decided not to put the specifications on the responses, such that the whole range of percentage deviations mentioned above for the faulty component would be covered. Then 20 simulations for each deviation on the faulty component were performed and the diagnostic responses of both the analysis of variance and the eigenanalysis approach were evaluated. Fault location was carried out by using the diagnostic data-base of both approaches, and the variances (method A) and the pooled covariance matrix (method B) as the weighting factor in the computation of the squared distance d_j^2.

The exploration was performed on a limited number of components, mainly R_1, C_1, R_3, C_2, and C_3, whose previous fault location results did not show a high diagnostic success, specially for components C_2 and C_3. The results, in graphical form, are shown in Figs. 4.17 to 4.26, where it can clearly be seen that, using the diagnostic data-base selected by the eigenanalysis approach, with the pooled covariance matrix as the weighting factor in the distance d_j^2 computation (fault location), leads to a monotonic decrease in the percentage of fault location error (wrong location) as the percentage of parameter deviation on the faulty component increases. A similar result is obtained by making use of the data-base selected by the analysis of variance approach and the pooled covariance matrix as the weighting factor in the distance calculation; the only exception is presented for positive parameter deviation of component C_2. However, the use of variances as the weighting factor in the computation of the squared distance d_j^2 for fault location, does not usually lead to a monotonic decrease in the percentage of fault location error as the percentage of parameter deviations increases on the faulty component.

These results suggest that, for fault cases having a certain amount of overlapping, the use of variance as a weighting factor in the fault location procedure would not necessarily enhance the discriminatory qualities of the selected diagnostic data-base, whereas the opposite appears to be true when using the pooled covariance matrix.

4.6.3 APPLICATION OF THE ALGORITHM TO HARD FAULTS

An experimental study was undertaken to examine the effectiveness of the fault diagnostic algorithm for "hard" faults (these faults do not necessarily approximate open or short circuits) using the previously selected diagnostic data-base for "soft" faults, i.e., the fault data-base selected by the analysis of variance and the eigenanalysis approaches for faults generated in the region of ±5% to ±50% deviation of nominal component parameter value.

A similar procedure for testing the effectiveness of the diagnostic algorithm as in subsection 4.6.1 was followed, but in this case, the 50 faulty circuits for each of the fourteen fault cases were again randomly simulated in the region of ±50% to ±95% deviation of nominal value. The fault location results are presented in Figs. 4.27 and 4.28, where location is performed to the first smallest d_j^2 value. As expected the average percentage of diagnostic success is lower than the results of subsection 4.6.1, but they are considered to be encouraging. This suggests that, using the "soft" faults diagnostic data-base, the algorithm could still be used for locating "hard" faults, on the understanding that the probability of error would be increased. The results also confirm that, even under these conditions, a superior diagnostic success is achieved using method B (eqn. 4.34) in the computation of the squared distance d_j^2.

4.7 SUMMARY

A simple statistically based algorithm has been developed for the location of a single "soft" component fault, under the assumption that nonfaulty component parameter can exhibit normal production tolerances. Tests carried out on a sample circuit have shown that the algorithm can lead to successful diagnosis, particularly if the eigenanalysis approach is used to select diagnostic responses and if the pooled covariance matrix is used to classify field measurements into a fault case; it was also established that using the Euclidean distance for classification leads to no improvement over the two suggested fault location methods. Experimental evidence has been obtained of the general influence of the number of diagnostic responses on the likehood of correct diagnosis. Further experiments have been carried out, with encouraging results, to examine the monotonicity in the fault location error as component parameter deviation increases, and the effectiveness of the algorithm for "hard" faults.

Most of the computational efforts is concentrated in the design phase, during which the Y, V and S data matrices are generated as a result of fault simulation, and then reduced to DY, DV and DS by one of the two methods proposed for selecting the diagnostic responses that must be measured in the field. The actual diagnosis involves the measurement of the diagnostic responses and the calculation of the squared distances. Thus, during fault simulation, if N faulty circuits are to be simulated for each fault case of a circuit containing K components and M responses to evaluate, then the total number of simulations to be performed will be 2KN with 2KNM response calculations. The estimated storage requirements is M values for nominal responses, MN values for the jth fault case, 4KM values corresponding to means and variances, and $2M^2$ values for covariance matrices (In summary: $M(2M + N + 4K + 1)$ values stored in secondary memory). The final stored information will be R (selected diagnostic responses) nominal response values, 2KR mean values, R^2 pooled covariance matrix values and/or 2KR variance values (In summary: $R(R+4K+1)$ values stored as a library diagnostic data-base).

The proposed algorithm, although successful, would undoubtedly benefit from further research. With regard to the choice of candidate responses, it is assumed that they should include a range of circuit functions (e.g., loss, input impedance, transfer impedance) in order to broaden the possibilities of finding responses providing good discrimination among the fault cases. Although in the RC filter circuit example, the choice of test points has been restricted to the input and output terminals, the algorithm could equally be applied, at the design stage, to select test points from among all the circuit nodes, both internal and external. Further study is required of the effect, on diagnostic success, of the number R of diagnostic responses and the distribution of simulated circuit faults, as well as the effect of measurement inacuracy. Finally, the application of the algorithm to nonlinear circuits is discussed in the next chapter.

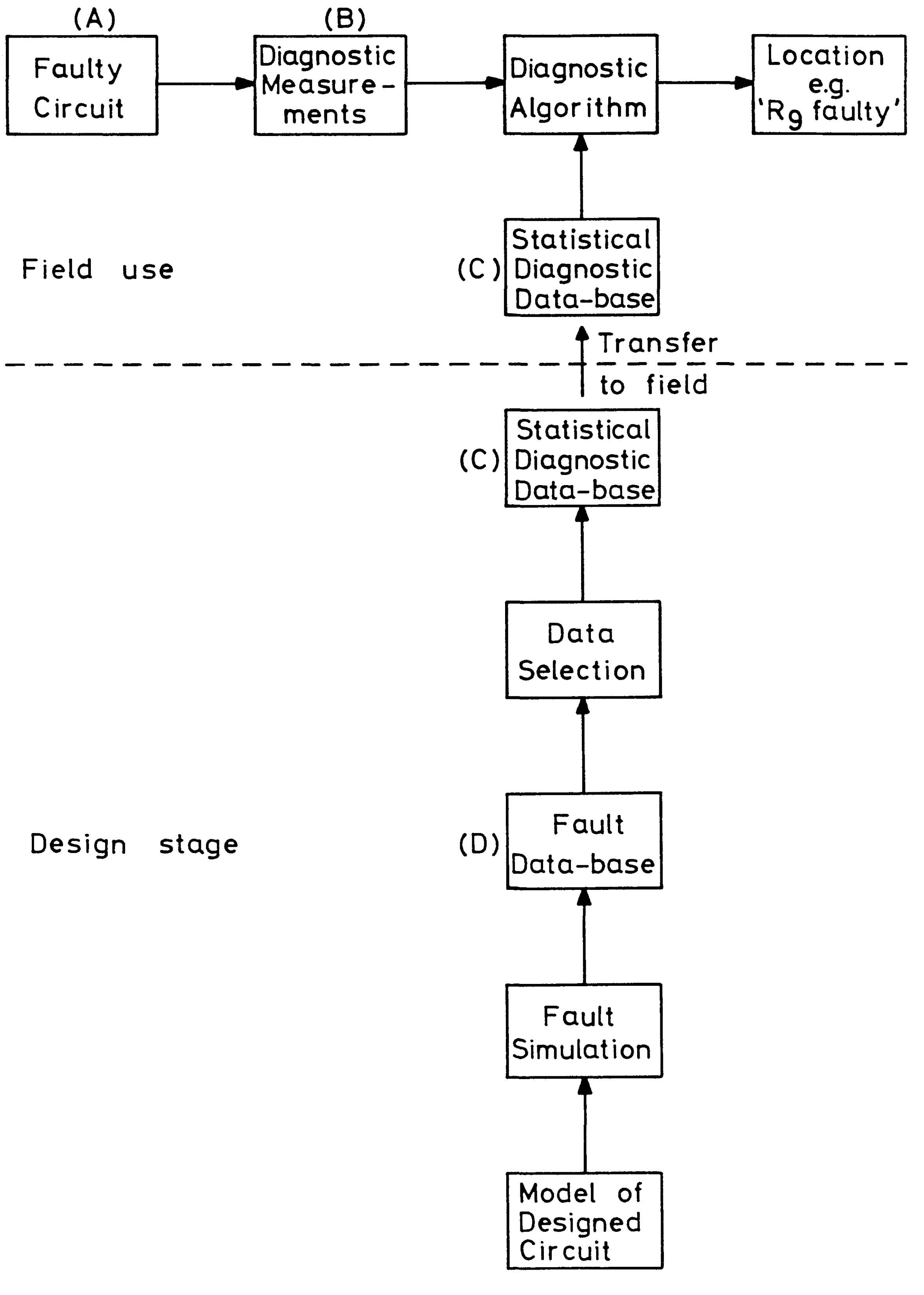


Fig. 4.1 General scheme showing the steps involved in developing the fault location algorithm

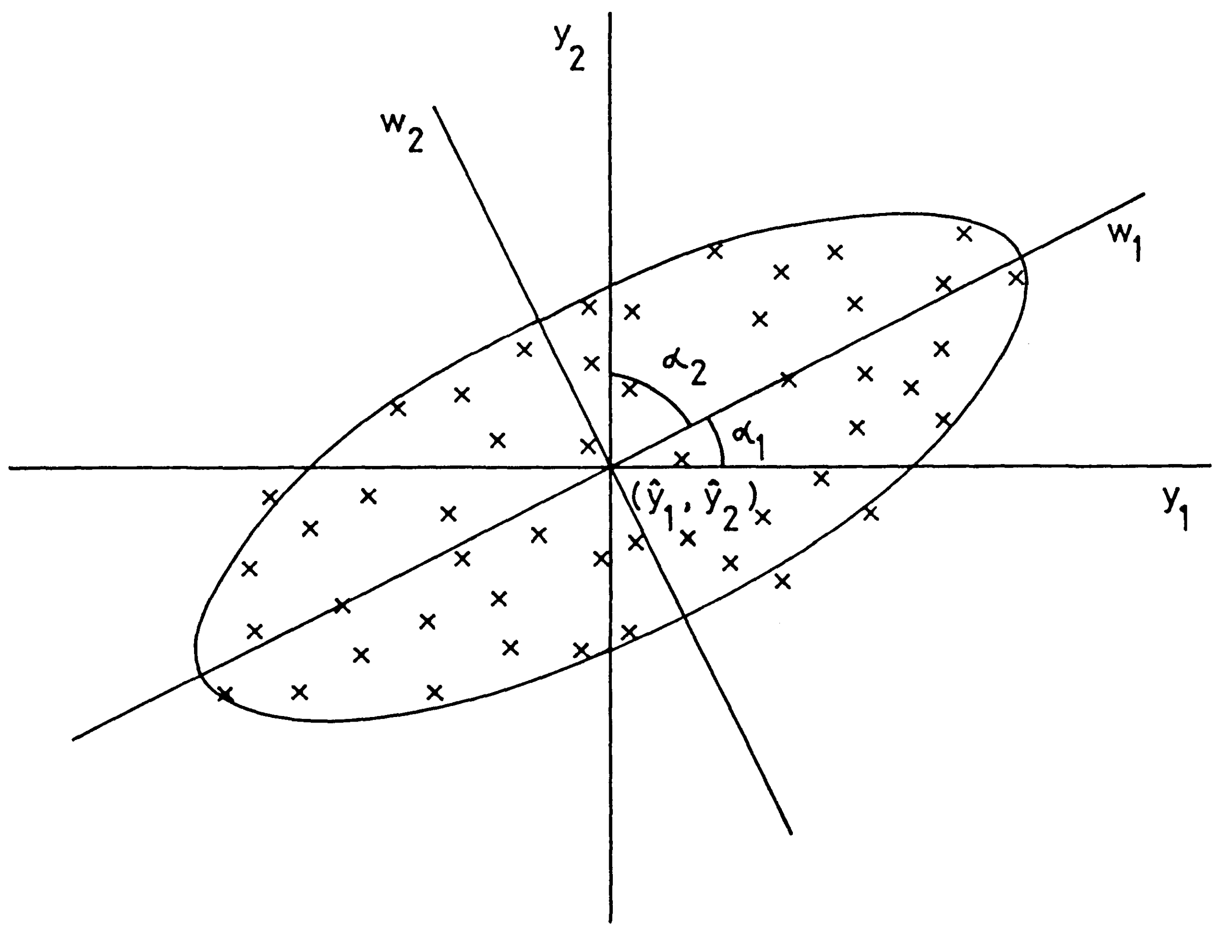


Fig. 4.2 An illustration of the degree of association (covariance) for the random sample points of variables y_1 and y_2 (two dimensional example)

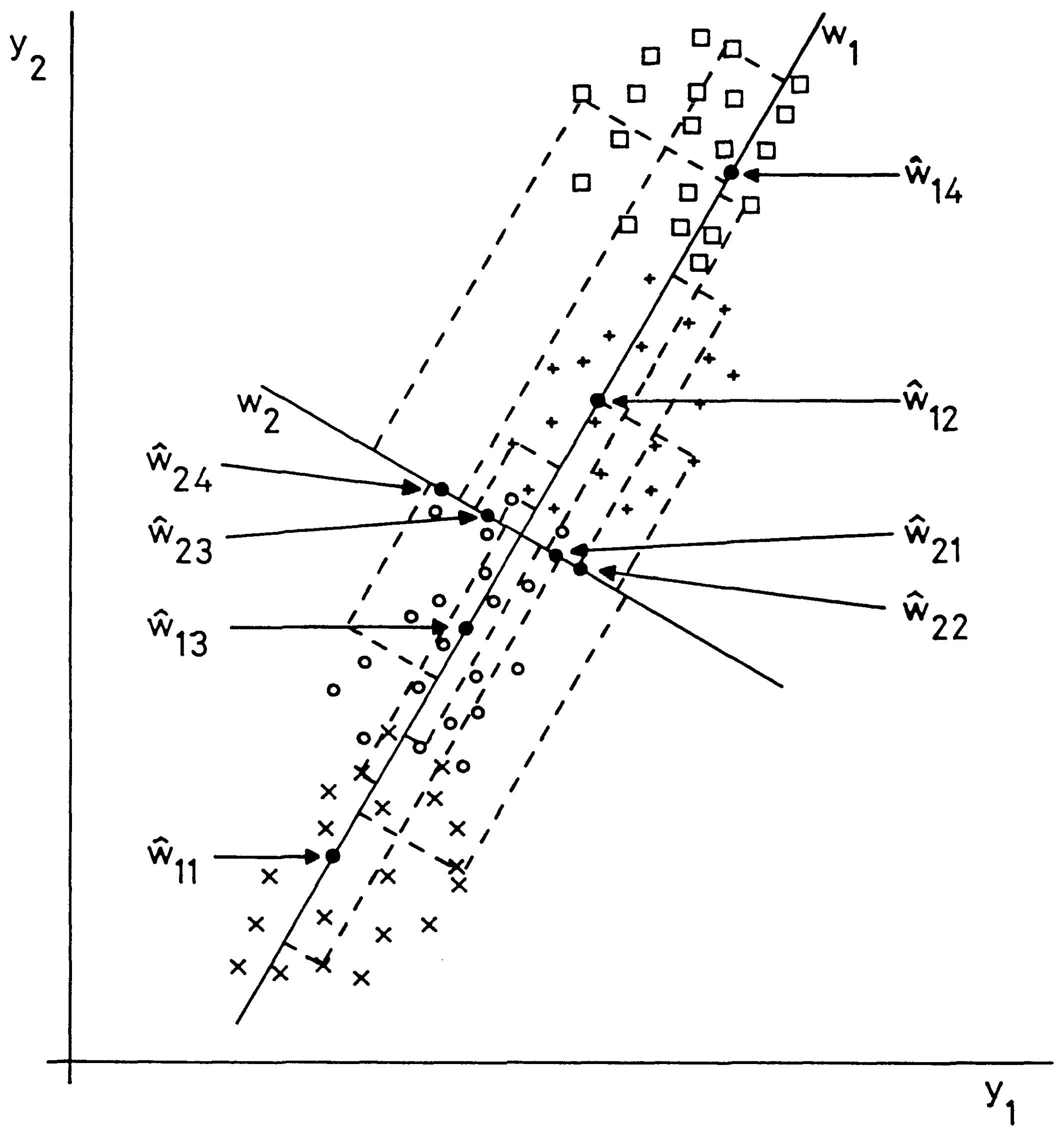


Fig. 4.3 Illustration of the eigenanalysis method for four fault cases, in two dimensional response space. The maximum separation for the fault cases occurs in the direction of response y_2. The eigenanalysis of matrix $S^{-1}B$ for this example will show that the eigenvalue c_1 in the direction y_2 is greater than the eigenvalue c_2 in the direction y_1

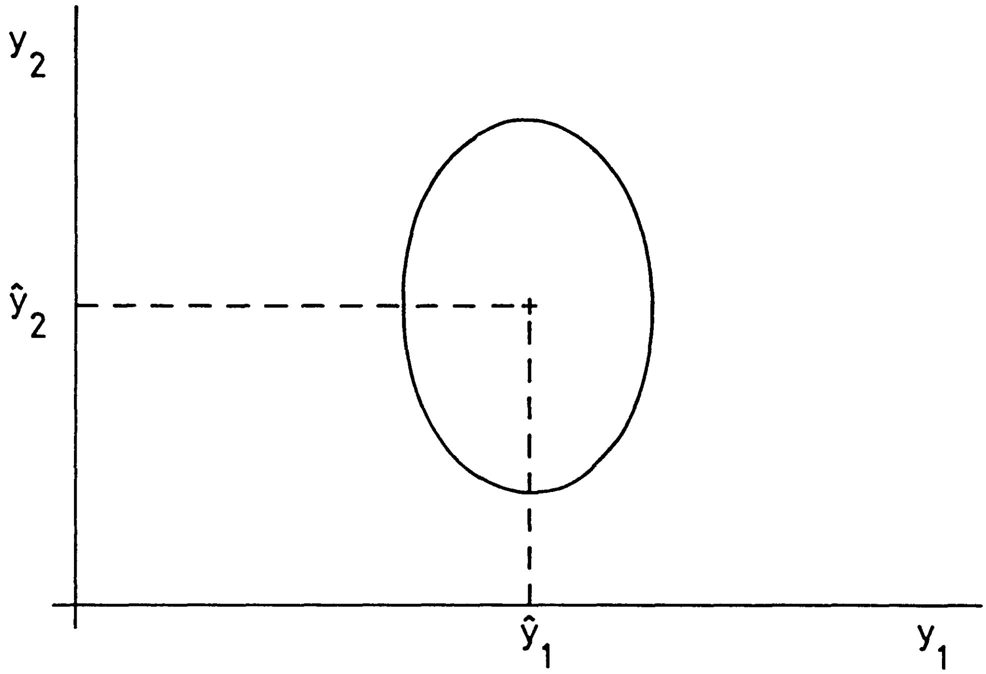


Fig. 4.4 Two dimensional illustration of the squared distance measure with different weights (variances) for the responses

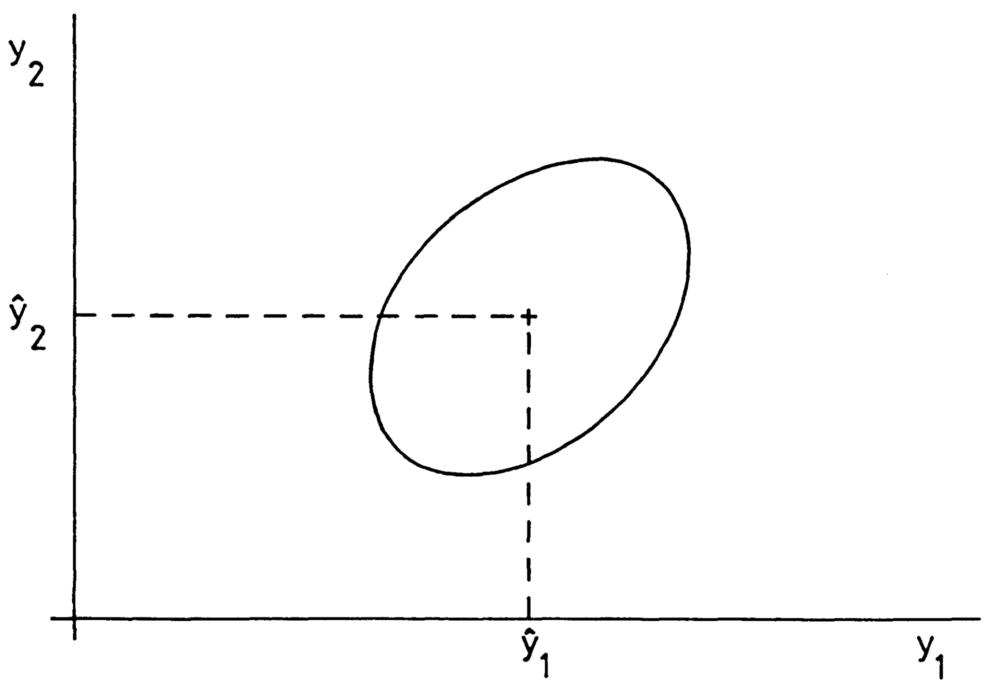


Fig. 4.5 Two dimensional illustration of the general squared distance measure, which is weighted according to the interrelation between the responses as well as their variances

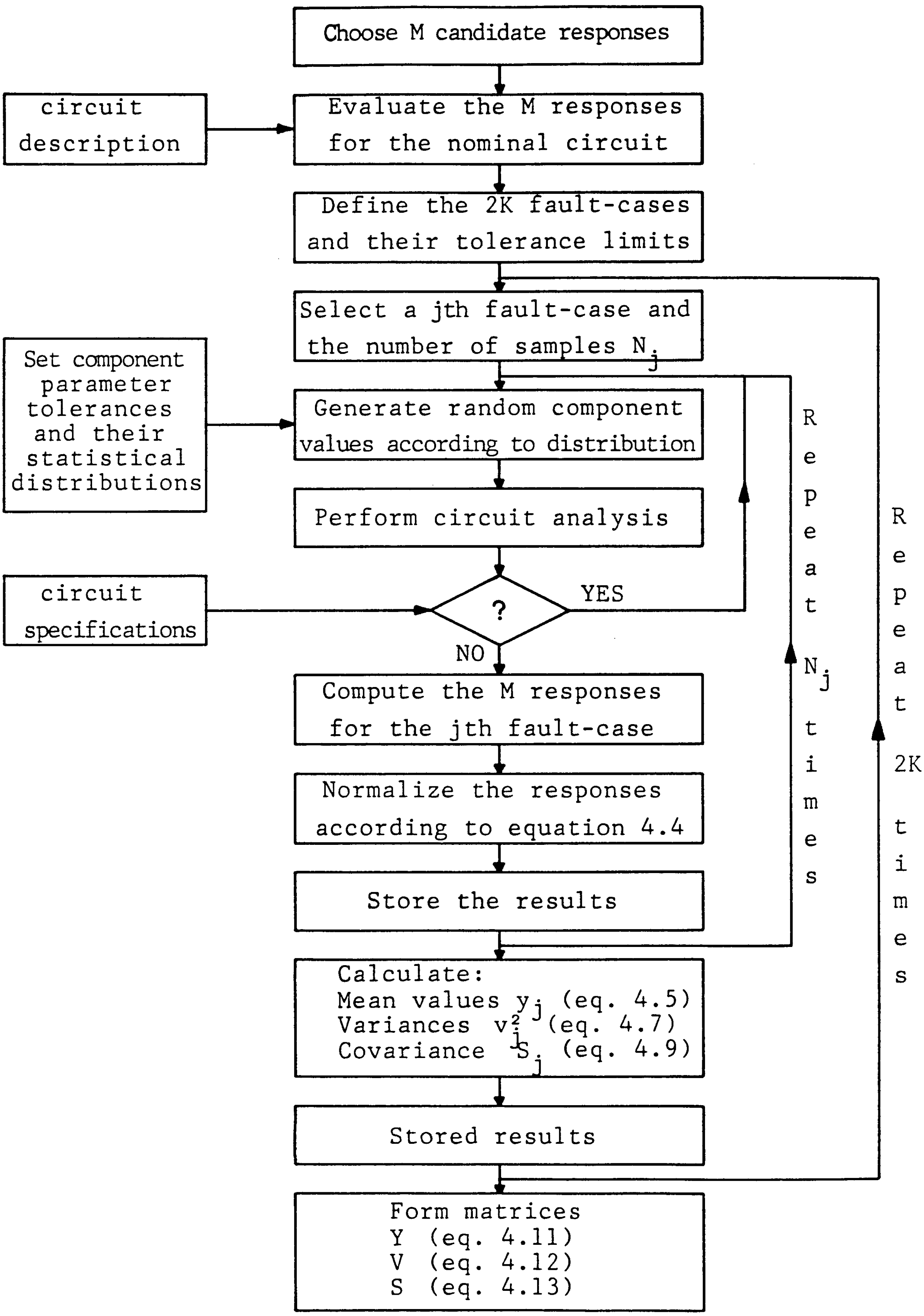


Fig. 4.6 Statistical fault simulation and fault data generation

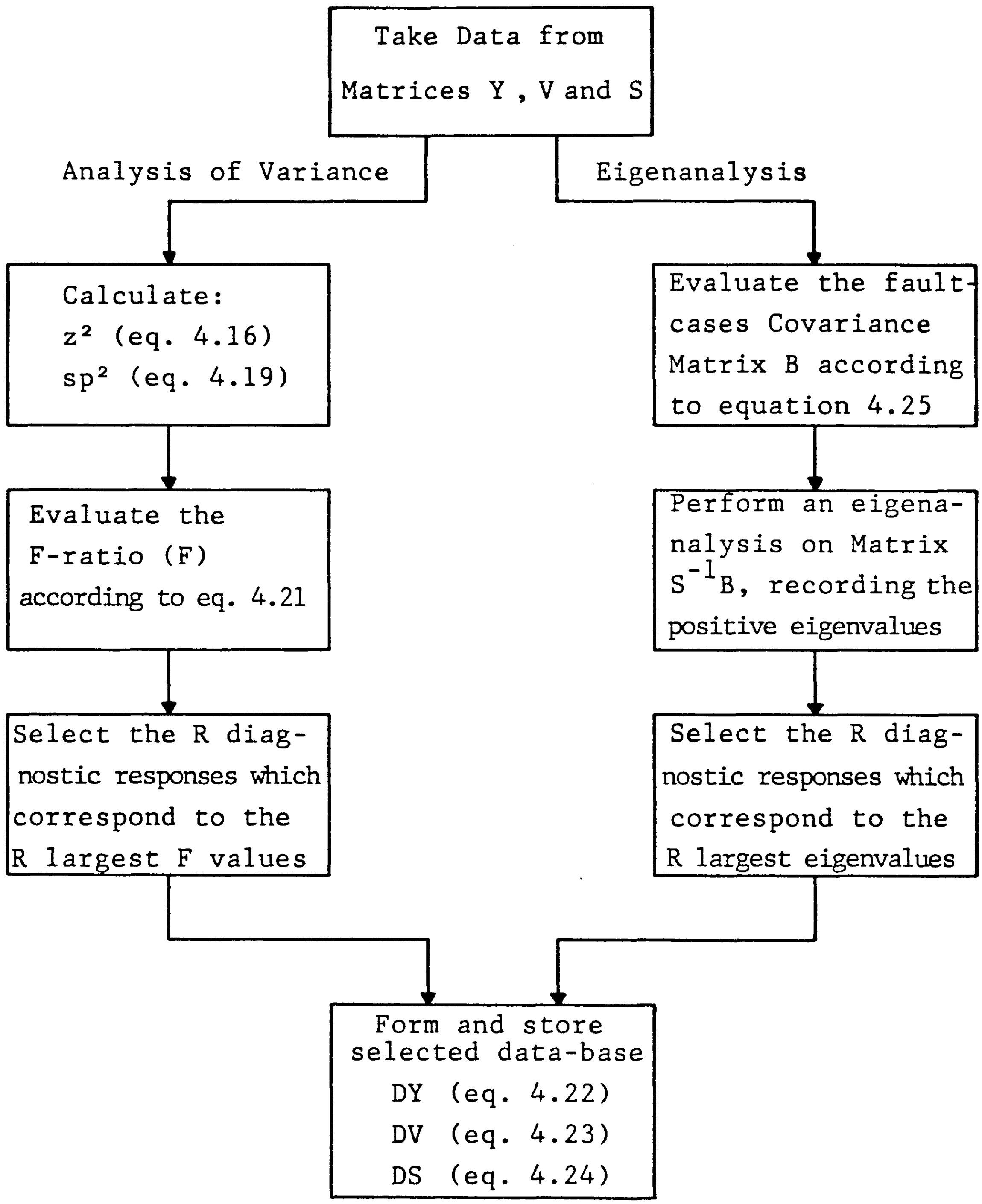


Fig. 4.7 Methods for selecting diagnostic data-base

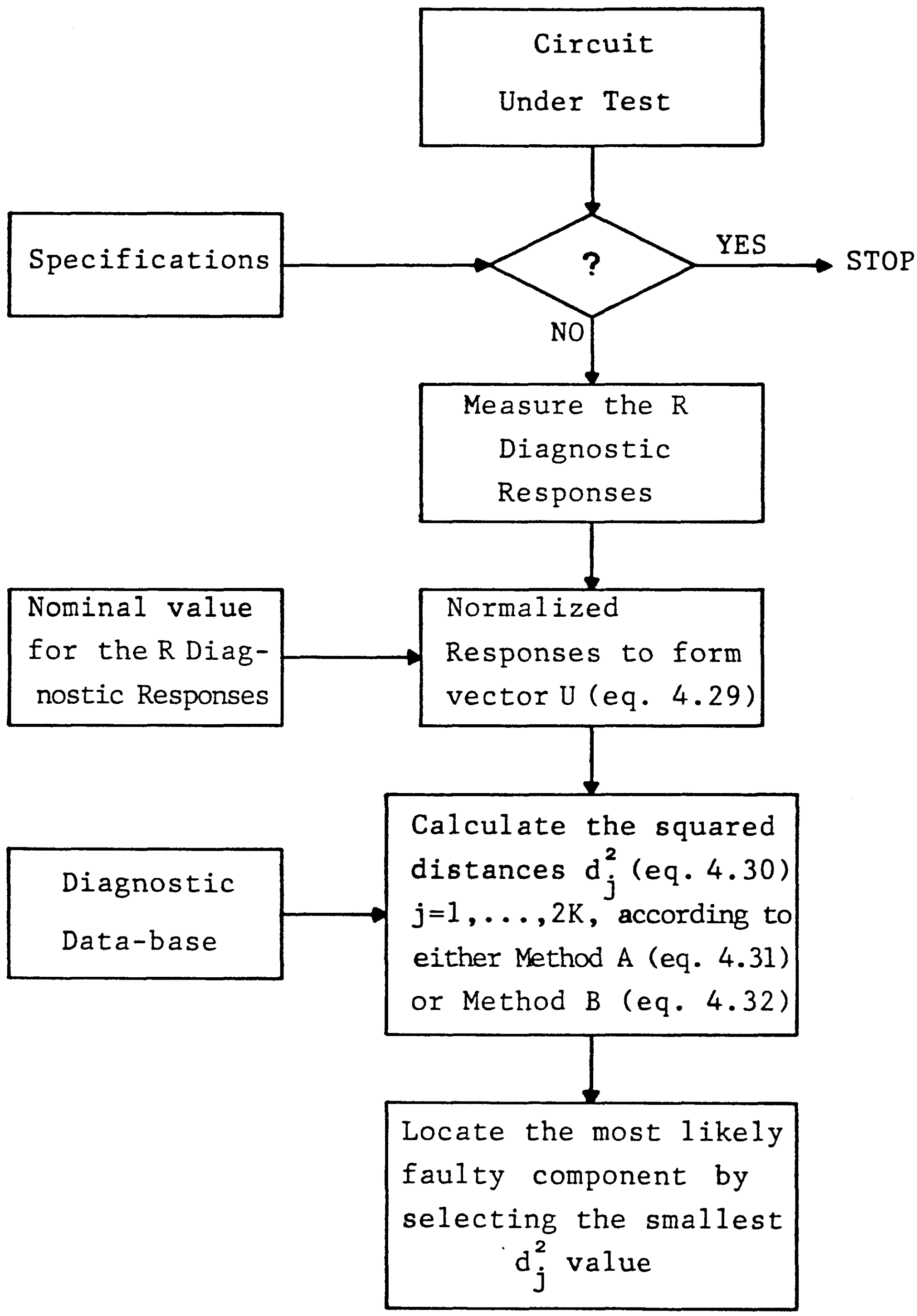
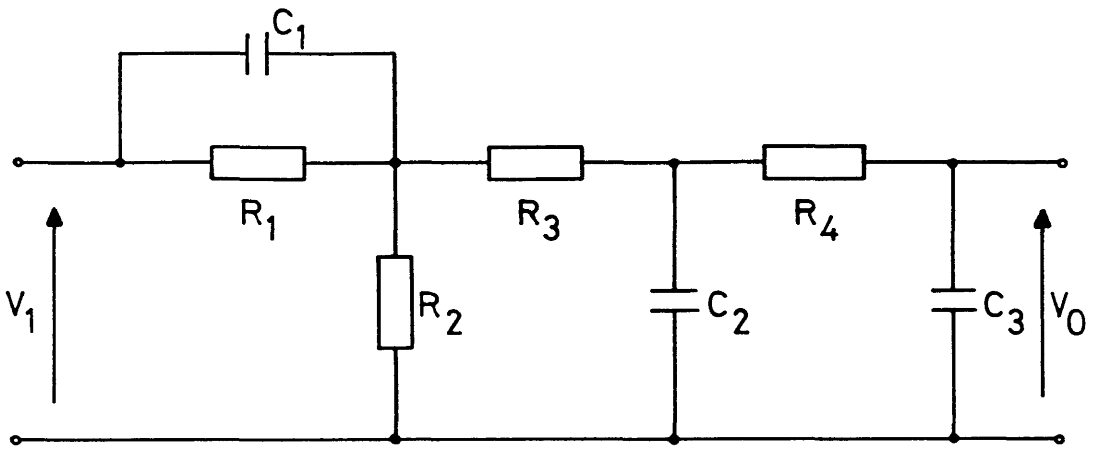


Fig. 4.8 Location of the faulty component in the field



R_1 = 1 MΩ R_2 = 10 MΩ R_3 = 2 MΩ R_4 = 1 MΩ

C_1 = 0.01 μF C_2 = 0.001 μF C_3 = 0.001 μF

Fig. 4.9 An RC filter circuit

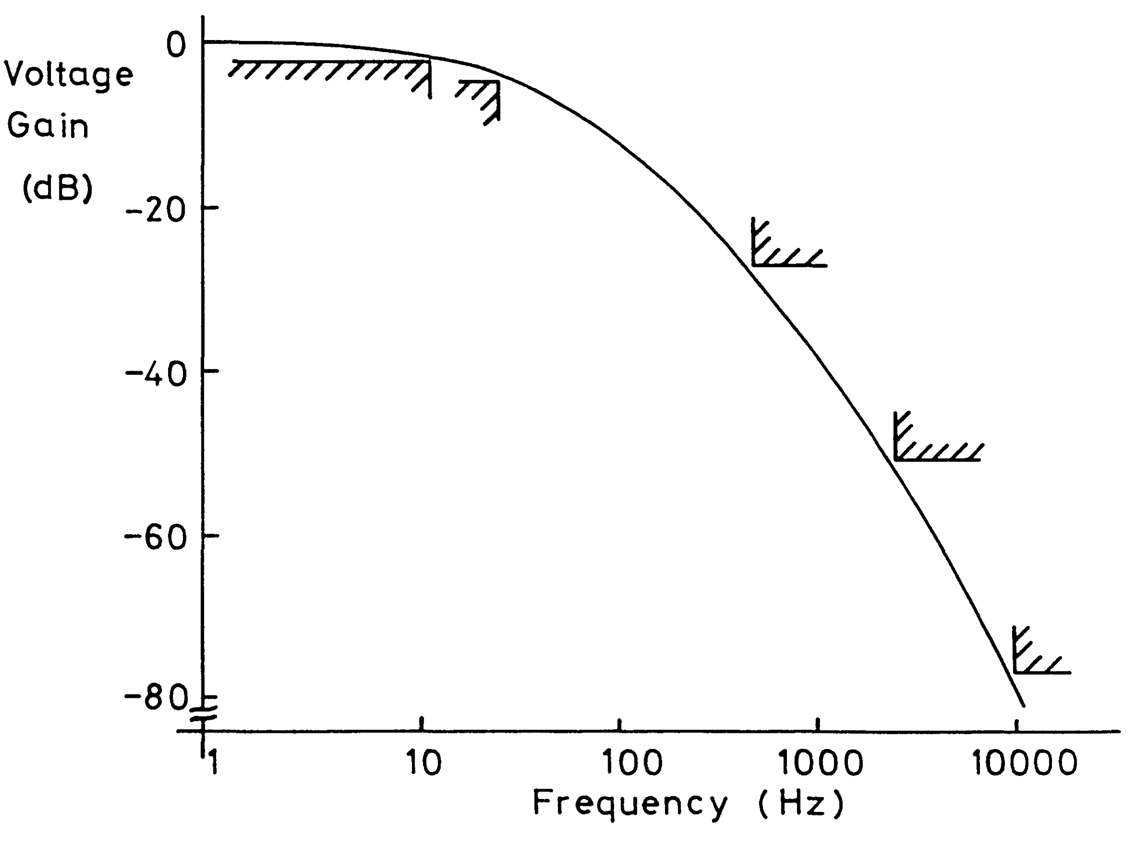


Fig. 4.10 Nominal response and performance specifications for the RC circuit of Fig. 4.8

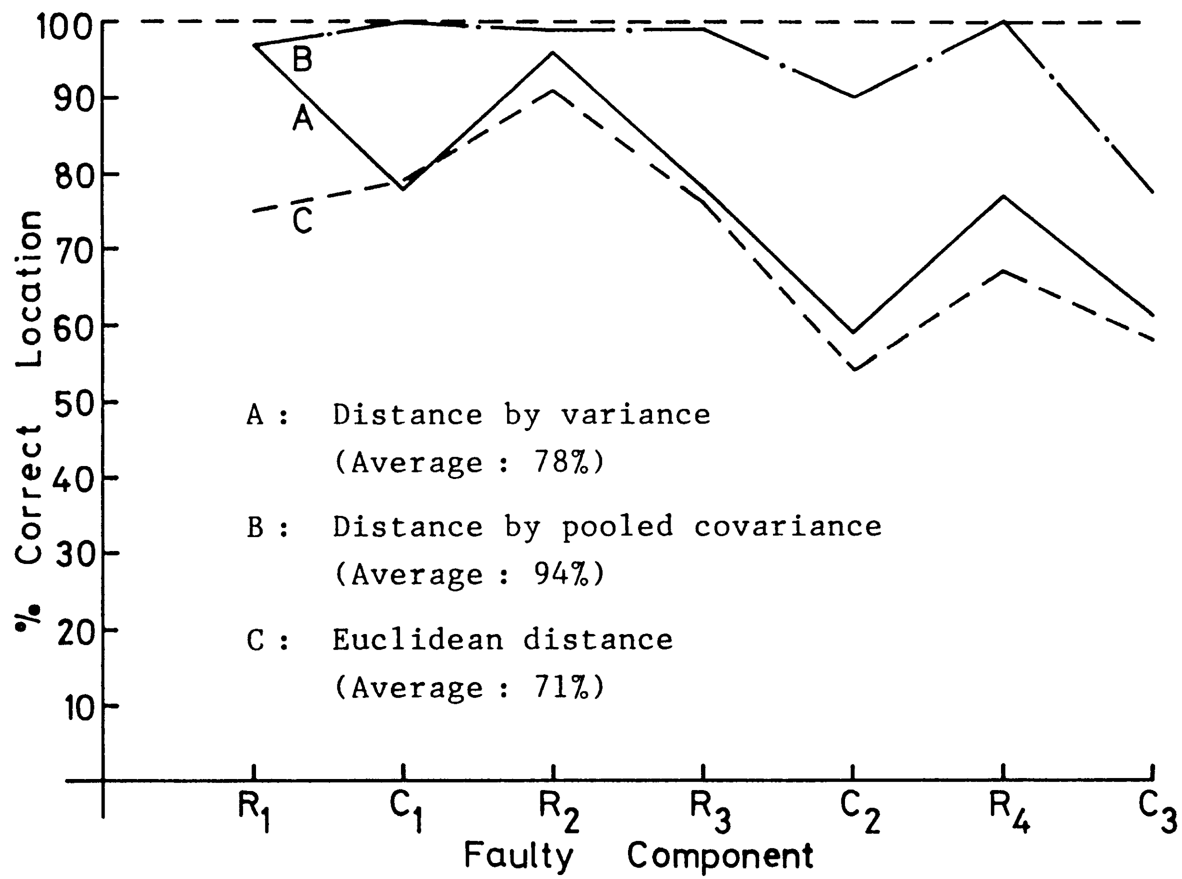


Fig. 4.11 Fault location, using data selected by the analysis of variance approach

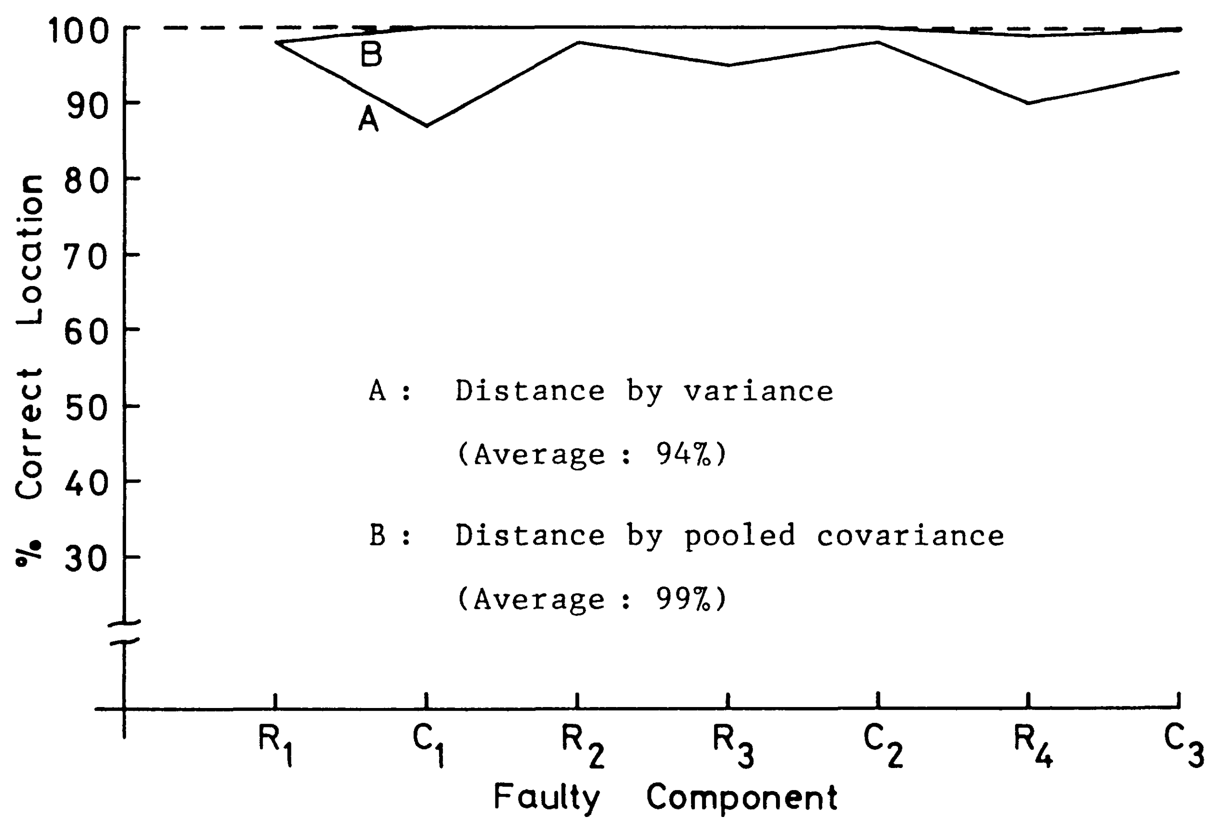


Fig. 4.12 Fault location by either the first or second smallest distance d_j^2, using data selected by the analysis of variance approach

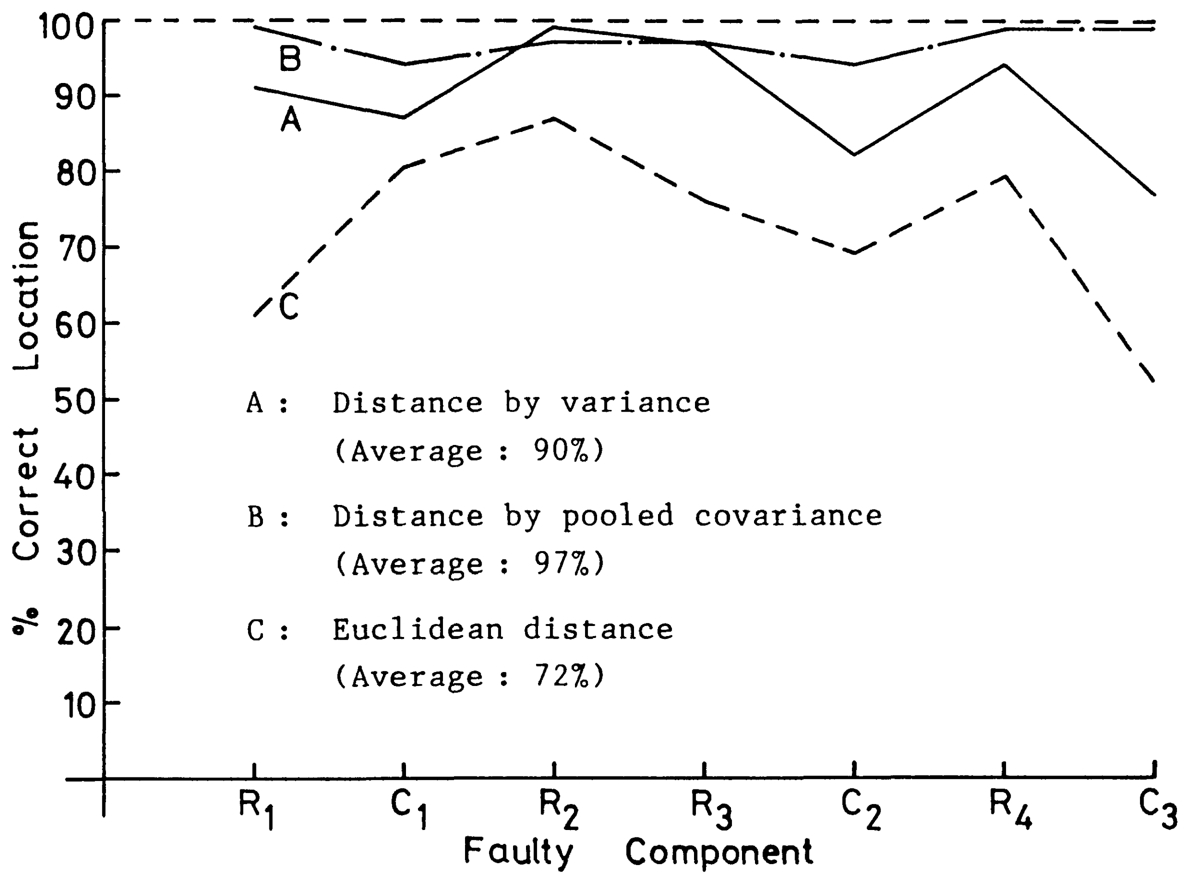


Fig. 4.13 Fault location, using data selected by the eigenanalysis approach

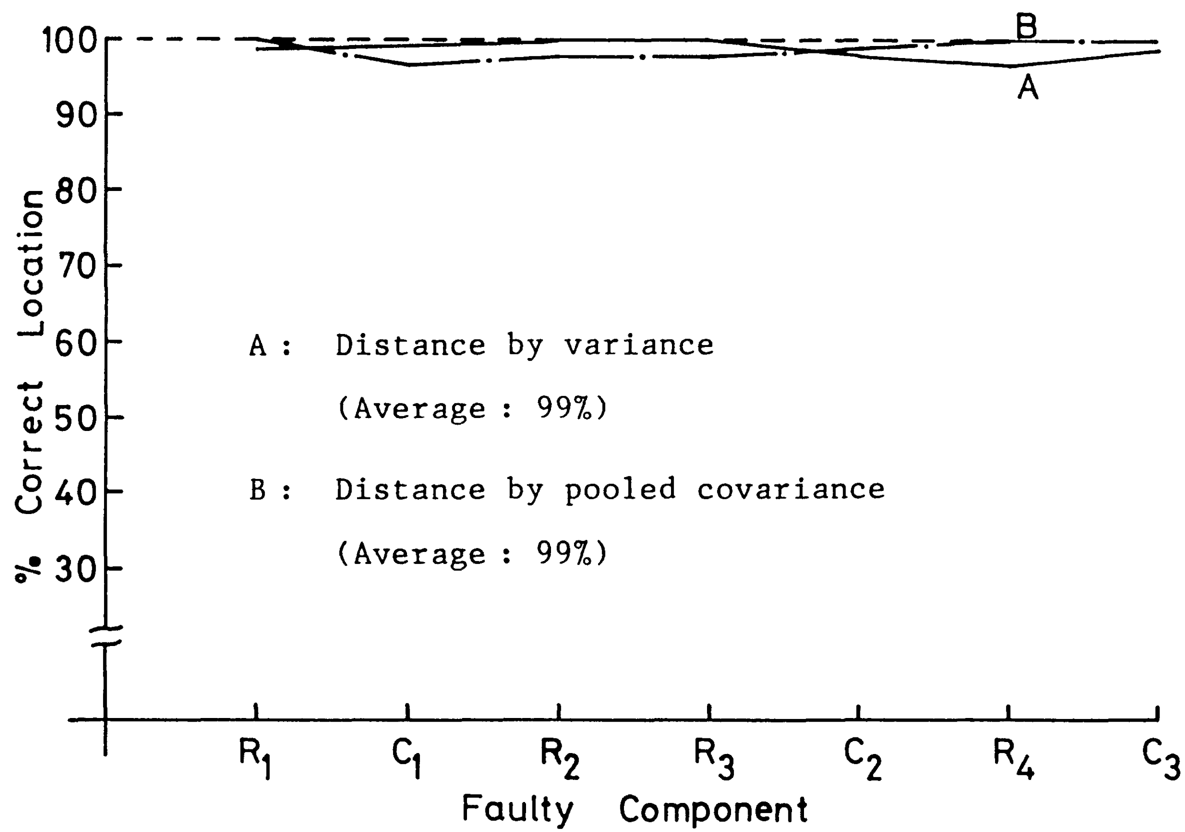


Fig. 4.14 Fault location by either the first or second smallest distance d_j^2, using data selected by the eigenanalysis approach

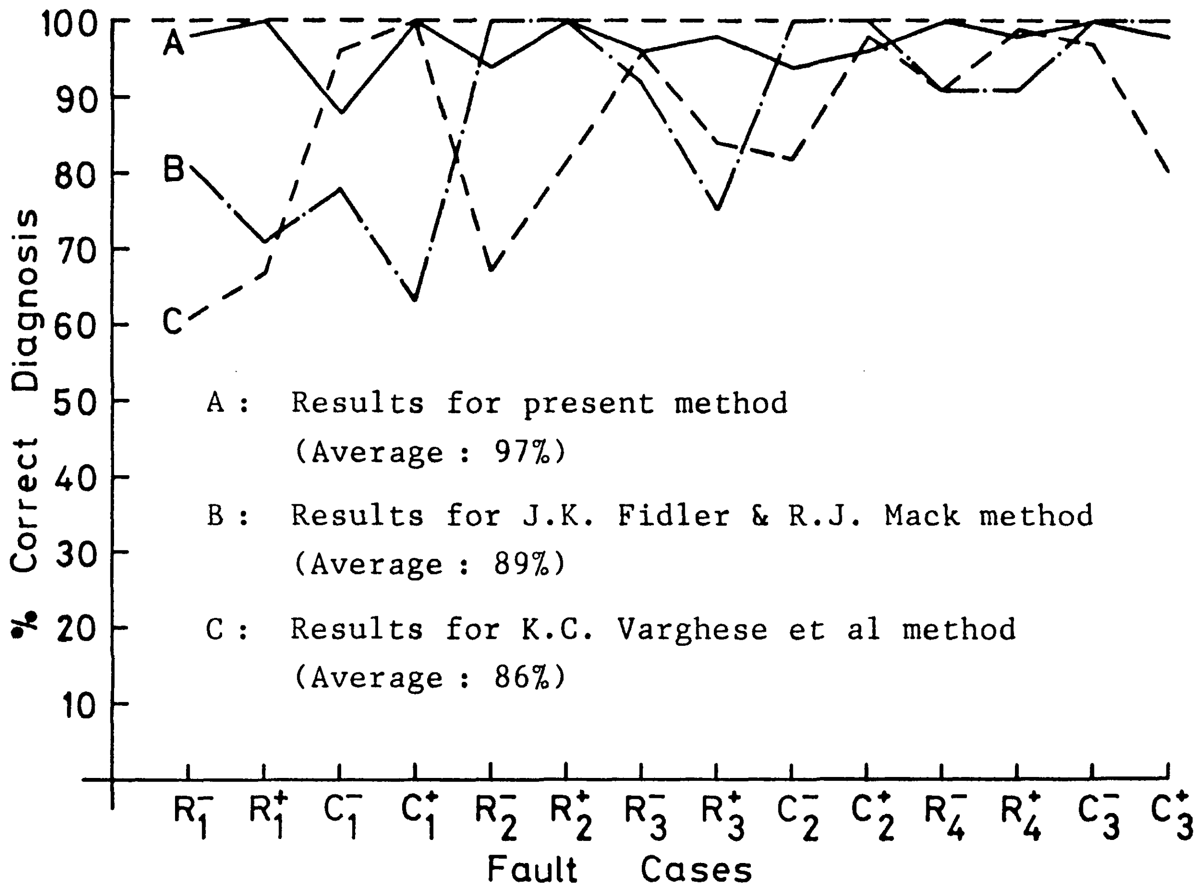


Fig. 4.15 Comparison of results, for the RC circuit, for three different methods

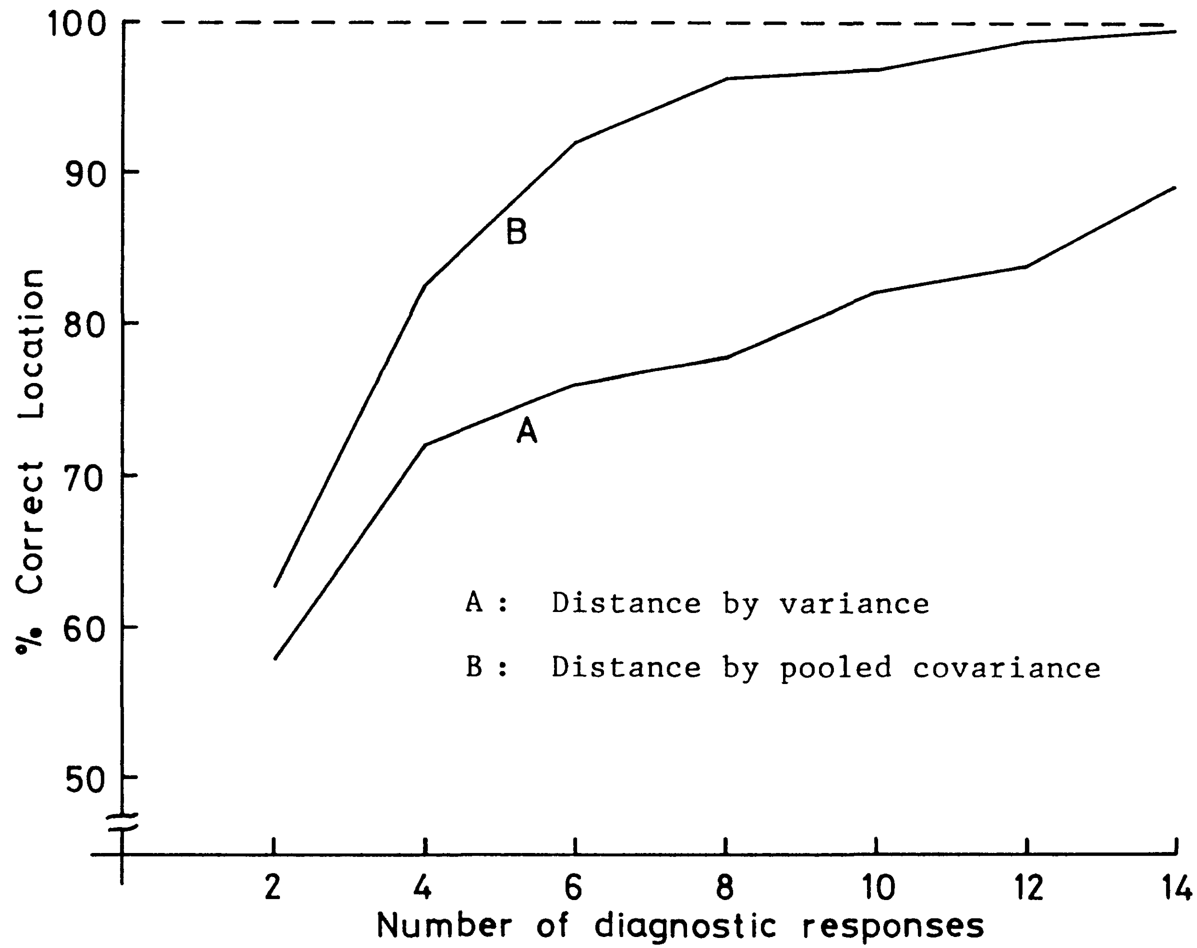
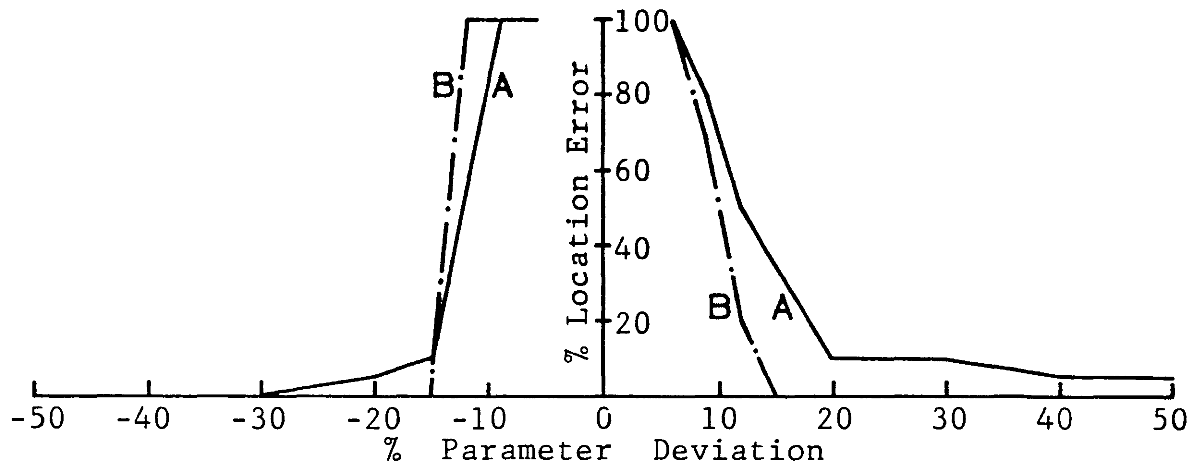
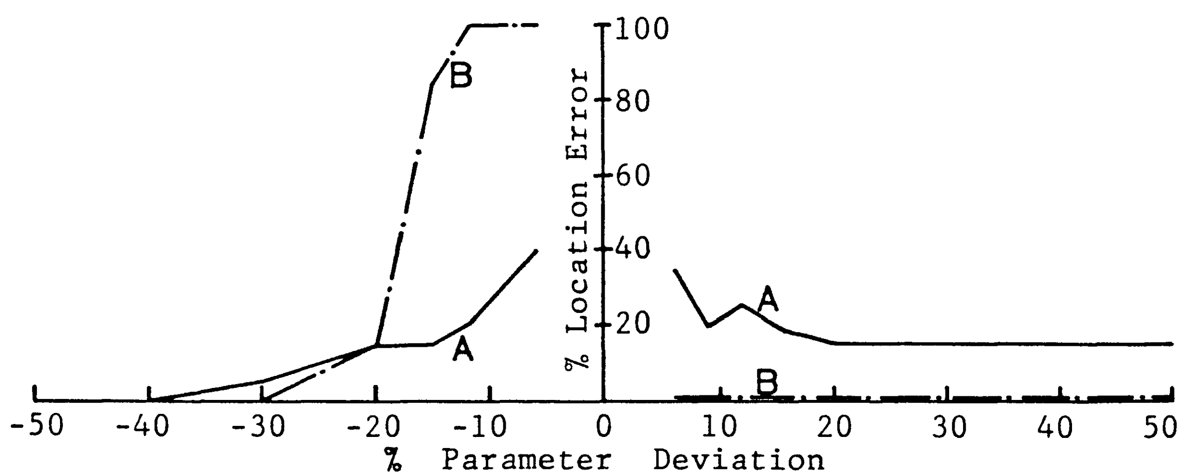


Fig. 4.16 Trade-off between percentage of correct location and the number of diagnostic responses selected by the analysis of variance approach



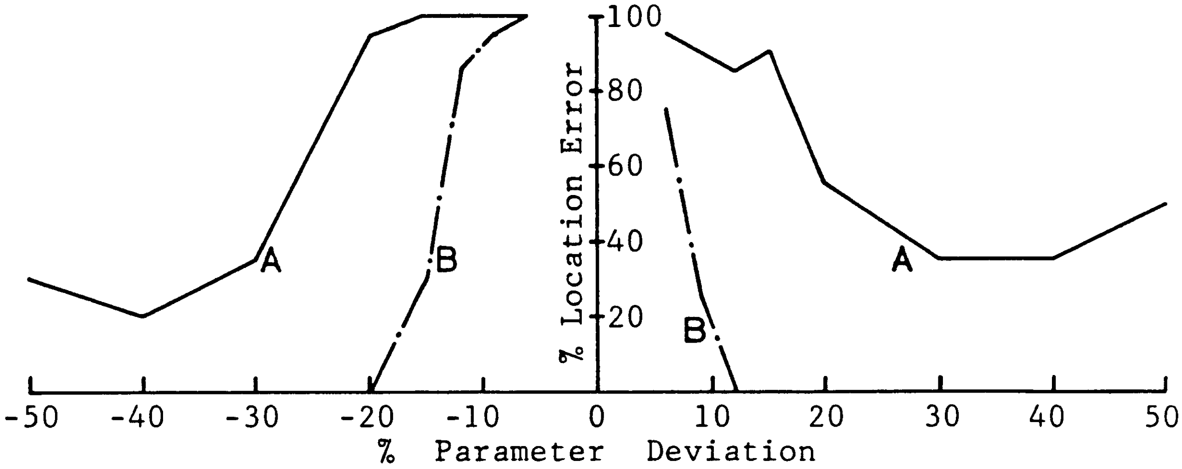
A : Distance by variance B : Distance by pooled covariance

Fig. 4.17 Percentage of fault location error v/s percentage of parameter deviation for component R_1, using data selected by the analysis of variance approach



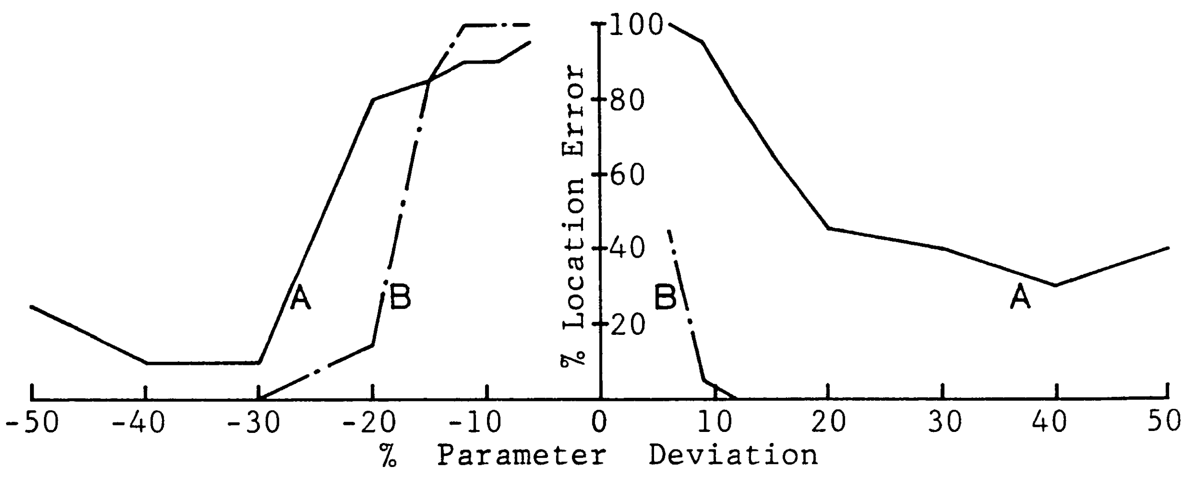
A : Distance by variance B : Distance by pooled covariance

Fig. 4.18 Percentage of fault location error v/s percentage of parameter deviation for component R_1, using data selected by the eigenanalysis approach



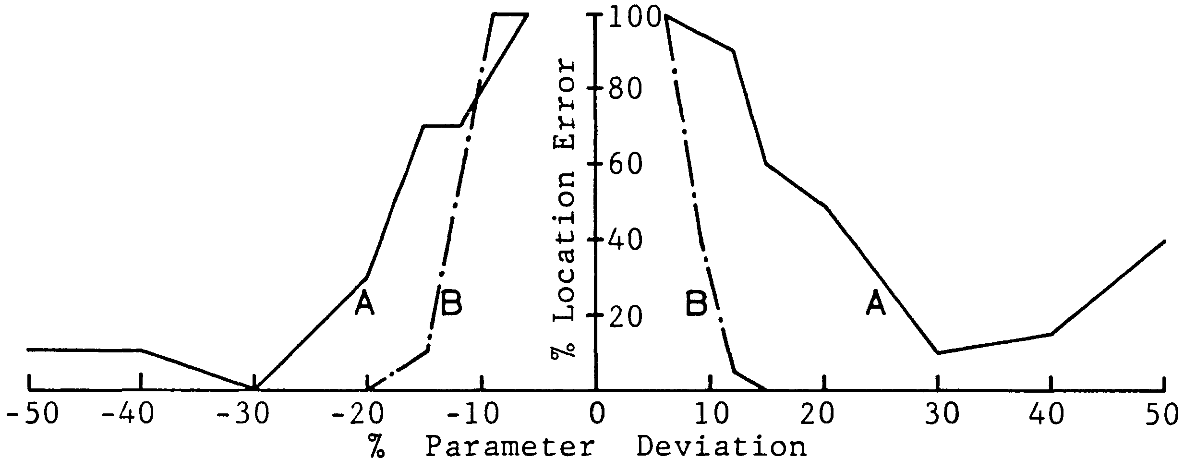
A : Distance by variance B : Distance by pooled covariance

Fig. 4.19 Percentage of fault location error v/s percentage of parameter deviation for component C_1, using data selected by the analysis of variance approach



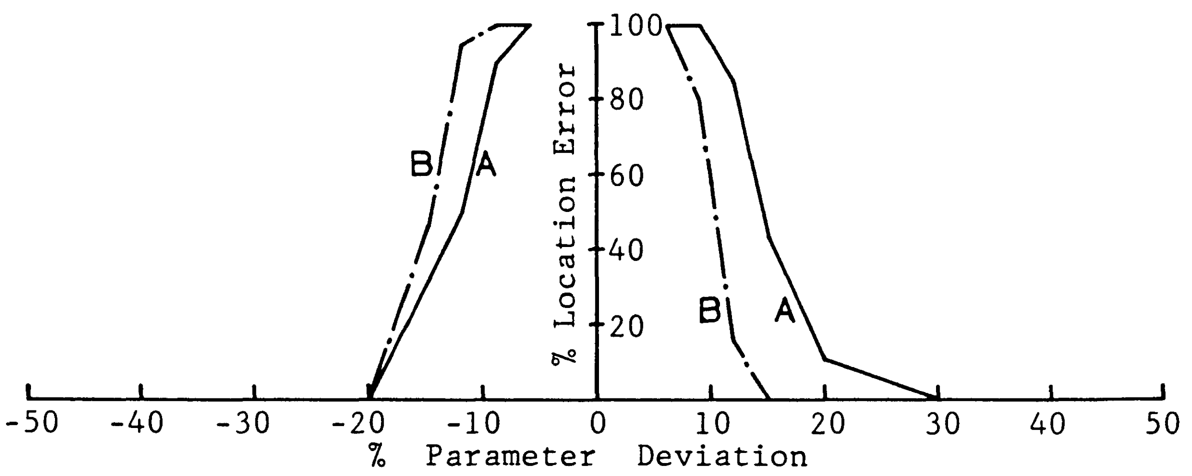
A : Distance by variance B : Distance by pooled covariance

Fig. 4.20 Percentage of fault location error v/s percentage of parameter deviation for component C_1, using data selected by the eigenanalysis approach



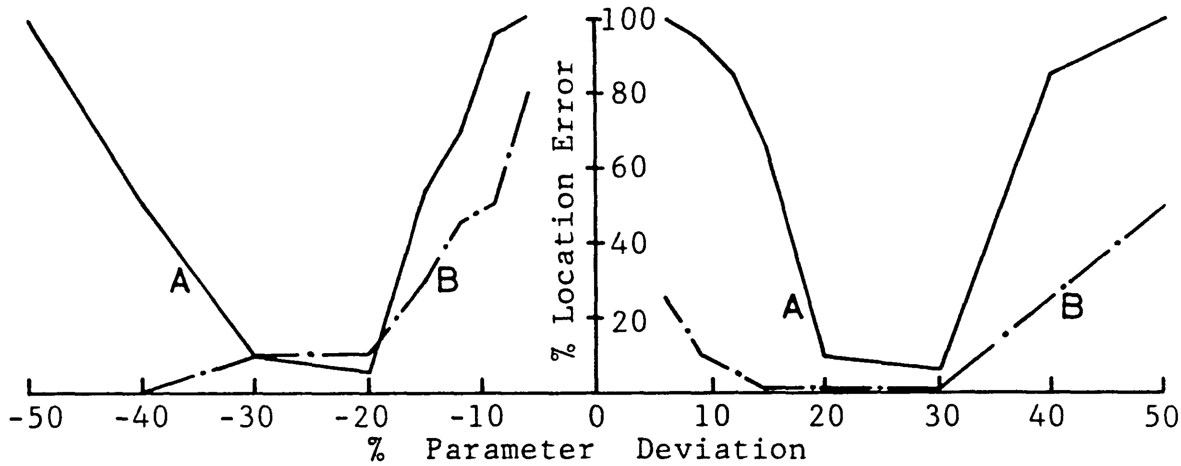
A : Distance by variance   B : Distance by pooled covariance

Fig. 4.21 Percentage of fault location error v/s percentage of parameter deviation for component R_3, using data selected by the analysis of variance approach



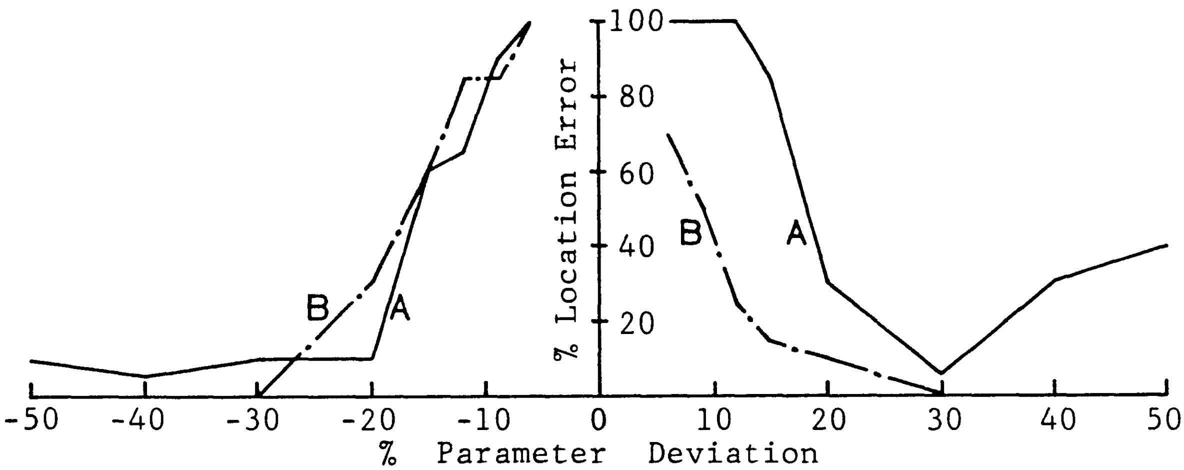
A : Distance by variance   B : Distance by pooled covariance

Fig. 4.22 Percentage of fault location error v/s percentage of parameter deviation for component R_3, using data selected by the eigenanalysis approach



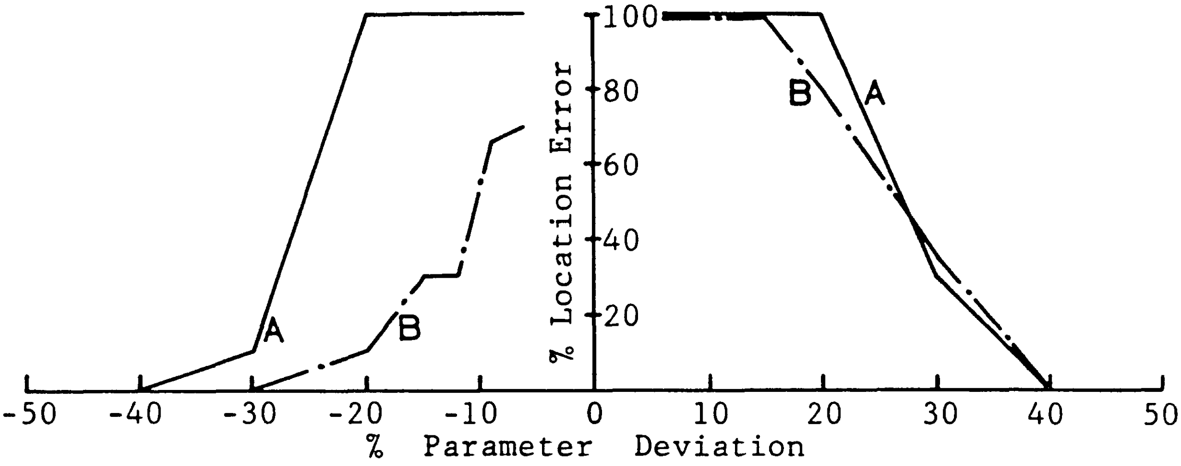
A : Distance by variance   B : Distance by pooled covariance

Fig. 4.23 Percentage of fault location error v/s percentage of parameter deviation for component C_2, using data selected by the analysis of variance approach



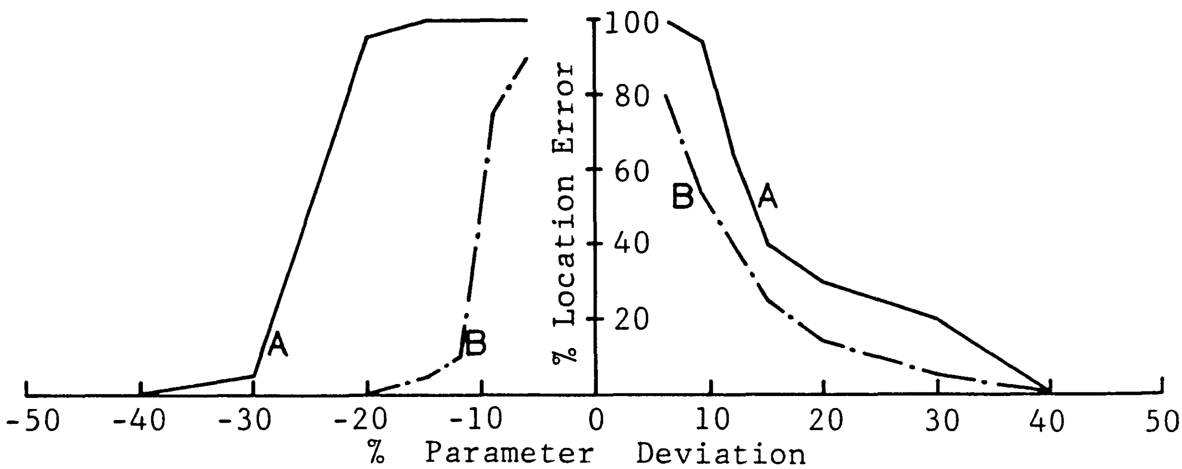
A : Distance by variance   B : Distance by pooled covariance

Fig. 4.24 Percentage of fault location error v/s percentage of parameter deviation for component C_2, using data selected by the eigenanalysis approach



A : Distance by variance   B : Distance by pooled covariance

Fig. 4.25 Percentage of fault location error v/s percentage of parameter deviation for component C_3, using data selected by the analysis of variance approach



A : Distance by variance   B : Distance by pooled covariance

Fig. 4.26 Percentage of fault location error v/s percentage of parameter deviation for component C_3, using data selected by the eigenanalysis approach

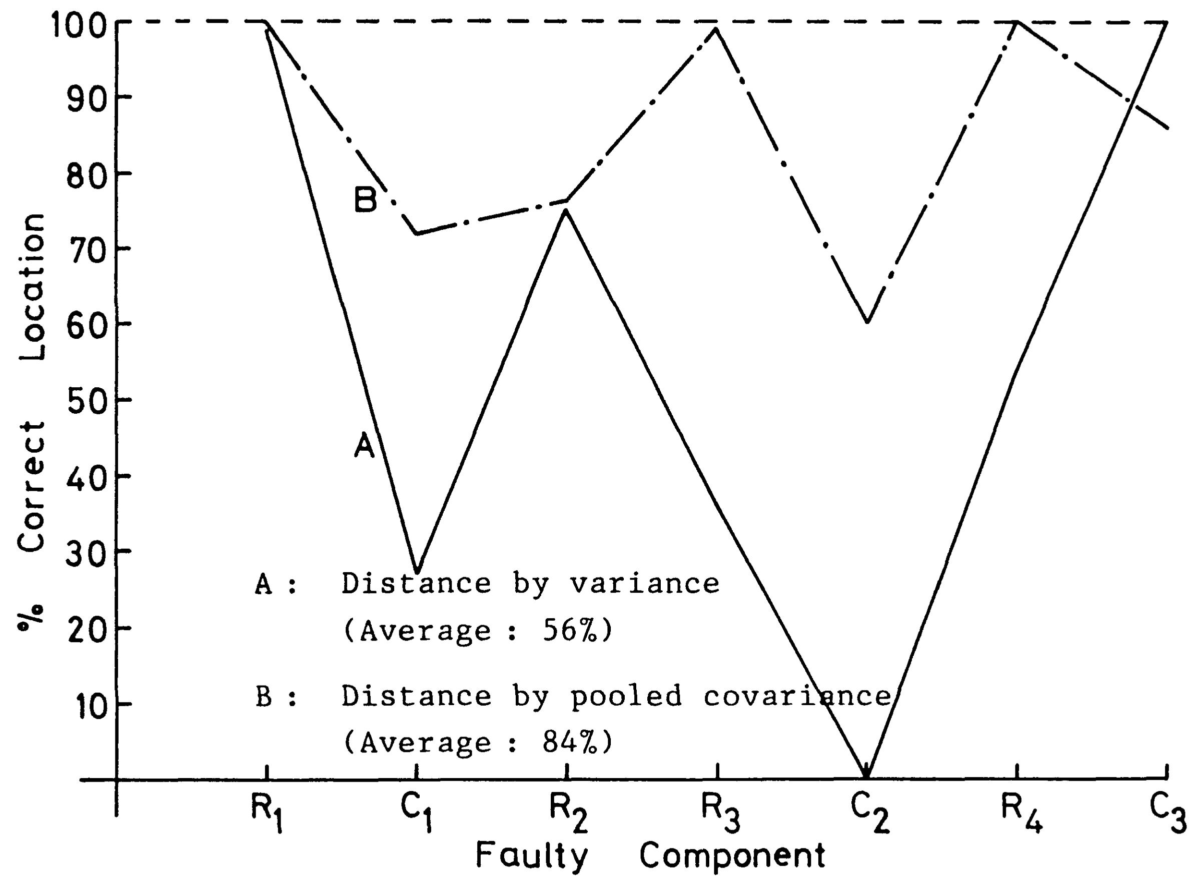


Fig. 4.27 Fault location for hard faults in the regions ±50% to ±95% deviation of nominal value, using data selected by the analysis of variance approach

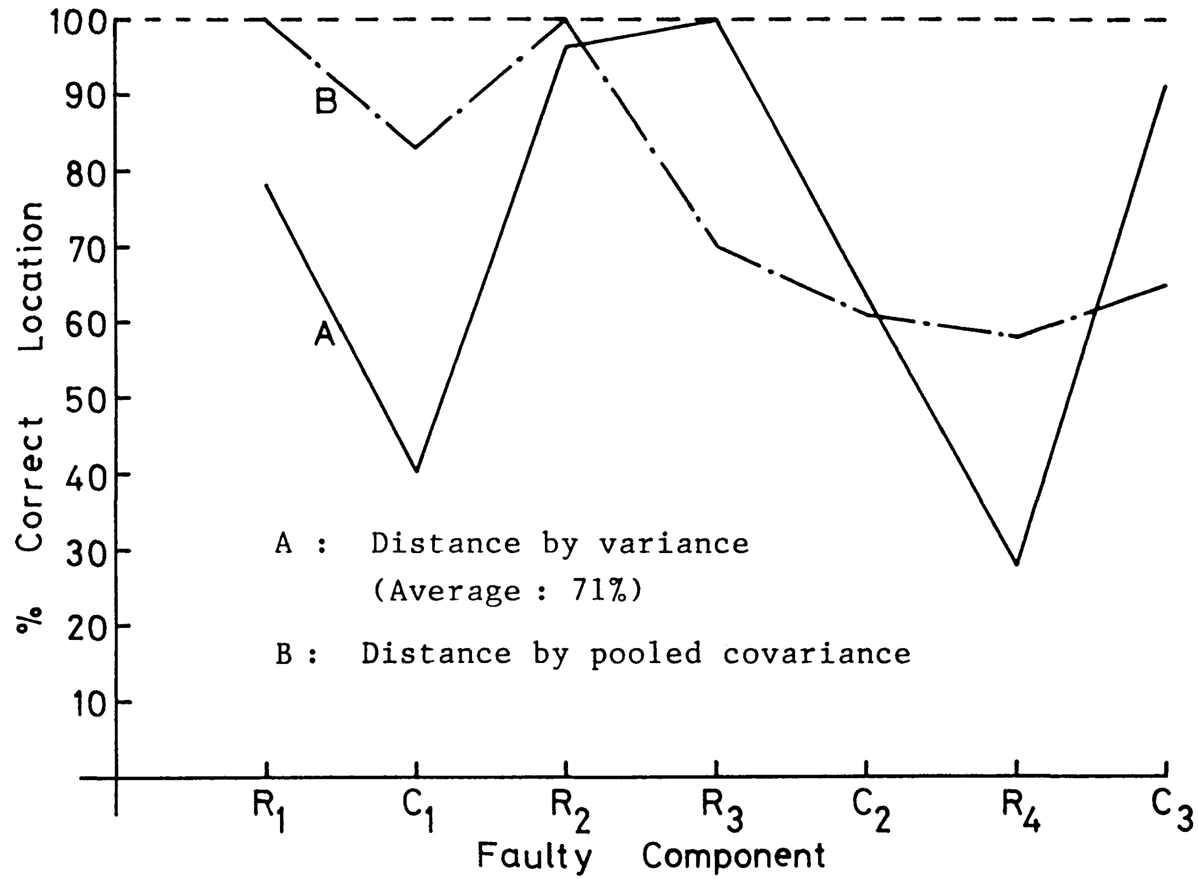


Fig. 4.28 Fault location for hard faults in the regions ±50% to ±95% deviation of nominal value, using data selected by the eigenanalysis approach

CHAPTER FIVE

FAULT LOCATION IN NONLINEAR CIRCUITS

5.1 INTRODUCTION

Most practical electronic circuits are composed of both linear and nonlinear elements; examples of the latter are diodes and transistors. It is therefore worthwhile to consider the application of the algorithms developed in the previous chapter to such nonlinear circuits.

The characterization of a nonlinear element is much more complex than the characterization of linear elements. The difficulty lies precisely in the fact that the volt-ampere characteristic of this type of element is nonlinear and that the small-signal behaviour depends upon the operating point. However, a simple description for the dc behaviour of, for example, a bipolar transistor, can usually be provided by the Ebers-Moll model which is composed of ideal circuit elements such as resistors, diodes and current controlled current sources. The analysis of circuit behaviour is usually performed by an iterative approach, where each nonlinear element is replaced by its linearized equivalent model, and the resulting linear circuit then analyzed until convergence is achieved. This means that, at each iteration, each nonlinear element (diode) of the Ebers-Moll model is replaced by a conductance in parallel with a current source, such that, the conductance value is equal to the slope of the tangent to the diode characteristics. The current source has a value equal to the intercept of this tangent with the current axis.

In fault analysis, this dependency of the behaviour upon the operating point increases the complexity of the diagnosis when compared with the situation associated with linear circuits. For example, if a shift in the operating point causes the circuit to fail, this could be due to a variety of reasons: the fault may be due to a faulty bias resistor, the random variation of nonfaulty component parameter values inside their tolerance ranges, a fault in the nonlinear element itself (i.e., an incorrect value of one or more parameters of the model), or a combination of some of these circumstances. A successful diagnostic procedure should be able to distinguish between these possibilities and isolate the correct faulty component. Additional problems are also introduced in considering the number of fault conditions that can exist for a single transistor. For catastrophic cases, the basic fault conditions are six; three open-circuits at each of the three external nodes; and three short-circuits between paired nodes; furthermore, faults can occur as combinations (multiple faults) of the basic ones and thereby increase the number of fault conditions for a single transistor.

The first problem identified in the previous discussion is that of fault modelling. In principle it would be advantageous to adopt a single model that can accommodate any type of transistor fault condition, such that sufficient fault diagnosis information can be obtained by either performing DC/AC analysis, or in more difficult cases a transient analysis. If such a model could be developed, then fault conditions could be simulated by adjustments of the parameter values of the model. One important aspect to consider in fault modelling is that the basic topology of the circuit should not be violated (especially for catastrophic faults) so that the formation of a singular matrix for the solution procedure of the simulation is avoided.

Unfortunately, transistor fault modelling has proved not to be a simple problem, though some researchers have suggested some particular models. Tucker and McNamee [40] developed a transistor fault model more suited to catastrophic faults. The model incorporated additional components to an existing transistor model such that it is possible, by adjusting the values of these component parameters, to simulate the 6 basic catastrophic faults or any combination of them. The model was created by adding, externally to each node and between nodes, appropriate conductance values. Thus, to simulate an open circuit, the conductance connected in series with the corresponding transistor node is changed to the required low conductance value. For a short circuit, the conductance value across a pair of nodes is changed to high conductance value. Fig. 1.3 * shows an example of this model and the conditions to simulate the catastrophic fault cases. A dc three-terminal model shown in Fig. 5.2 was suggested by Navid and Willson [31]. Their model is based on the dc small signal linear model (Fig. 5.1), assuming that for dc small signals, diodes and transistors operate with reasonable linearity. The transistor fault conditions are then simulated by adjusting the values of parameters r_{be}, r_{bc}, r_{ce}, and β of the three-terminal model. It is

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* Fig. 1.3 is in Chapter one of the thesis, page 42

suggested that this particular model is capable of describing the transistor in either saturation, cutoff or active regions. Another approach, particularly used for soft faults, uses the ac small-signal transistor model (Figs. 1.4 and 1.5)*, whose parameter values are generated by linearizing the circuit's components about their operating point.

The diagnosability problem also tends to be much more complicated as compared to diagnosis in linear circuits, and it is often required to perform dc and ac testing for complete diagnosis. The dc testing takes into consideration the nonlinearity of the circuit, and it is used to determine the bias point. It is also believed that this testing is more appropriate for the diagnosis of catastrophic faults. In contrast, ac testing has often been used for the diagnosis of soft faults, especially for those components such as capacitors that are frequency dependent. In this case, testing is performed on the linearized circuit about its operating point. However, a more convenient fault diagnosis approach should, as mentioned above, include a combination of these two testing procedures, to diagnose those fault conditions that may not be possible to identify using only one of the testing modes.

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* Figs. 1.4 & 1.5 are in Chapter one of thesis, page 43

5.2 SENSITIVITY INFORMATION AS AN AID IN THE A PRIORI SELECTION OF CANDIDATE RESPONSES

Information regarding the relationship between variation in the output of a circuit and the changes of its component parameter values is usually provided by sensitivity analysis. Differential sensitivity (i.e., the partial derivative $\partial F_i / \partial p_j$ where F_i is the ith output response, and p_j is the jth component parameter) is the most common sensitivity measure used, and it gives an indication of how the output responses would change due to variations in component parameters. One important use of this information has been in the prediction of worst-cases; that is, in finding the minimum and maximum limits of the output responses for variation of component parameter values within their tolerance ranges. To achieve this result, the signs of the sensitivities or derivatives are taken into consideration. Thus, the minimum of a given output is achieved by setting to their minimum value all component parameters with positive derivative, and to their maximum value all component parameters with negative derivative. Similarly, the maximum of an output is achieved by reversing the above setting.

In the context of fault diagnosis, the sensitivity information could be used in two particular situations:

(1) An identification of the most sensitive and least sensitive output responses of a circuit (nodes and/or functions) to changes in component parameters, could lead to a priori selection of the testing points at which candidate responses for fault diagnosis should

be evaluated. This information is useful in eliminating the generation of redundant information (output responses with very low sensitivity) in the fault simulation process such that, not only a reduction in the number of candidate responses for fault diagnosis may be achieved, but also a computational saving.

(2) An analysis of the sensitivity matrix may lead to a further elimination of redundant information. This could be achieved by the identification of those circuit responses that show similar effect to variations in the component parameters that are to be considered as faulty.

5.2.1 INVESTIGATION OF THE SENSITIVITY INFORMATION IN NONLINEAR DC CIRCUITS

Since differential sensitivity, as previously mentioned, provides an indication of the effect on circuit responses of random variation in component parameters, it is proposed to explore these effects with the objective of obtaining some preliminary useful information for fault diagnosis procedures.

Considering a nonlinear DC circuit with K component parameters and M output responses, the differential sensitivity of a specific circuit response of interest F_i with respect to a particular component parameter p_j is defined by $\partial F_i / \partial p_j$ or $\Delta F_i / \Delta p_j |_{\Delta p_j \to 0}$. However, for comparison purposes, a more convenient definition of sensitivity is provided by the normalized differential sensitivity defined as

$$S_{p_j}^{F_i} = \frac{\partial F_i}{\partial p_j} \cdot \frac{p_j^o}{F_i^o} \qquad \begin{matrix} i=1,2,\ldots.,M \text{ responses} \\ j=1,2,\ldots.,K \text{ component parameters} \end{matrix} \qquad (5.1)$$

where p_j^o and F_i^o are the nominal values for the jth component parameter and the ith circuit response respectively. A comprehensive sensitivity analysis for a circuit will generate a K×M matrix, containing the required sensitivity information.

$$S = \begin{bmatrix} S_{p_1}^{F_1} & S_{p_1}^{F_2} & \cdots\cdots & S_{p_1}^{F_i} & \cdots\cdot & S_{p_1}^{F_M} \\ S_{p_2}^{F_1} & S_{p_2}^{F_2} & \cdots\cdots & S_{p_2}^{F_i} & \cdots\cdot & S_{p_2}^{F_M} \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ \vdots & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ S_{p_j}^{F_1} & S_{p_j}^{F_2} & \cdots\cdots & S_{p_j}^{F_i} & \cdots\cdot & S_{p_j}^{F_M} \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ \cdot & \cdot & \cdots\cdots & \cdot & \cdots\cdot & \cdot \\ S_{p_K}^{F_1} & S_{p_K}^{F_2} & \cdots\cdots & S_{p_K}^{F_i} & \cdots\cdot & S_{p_K}^{F_M} \end{bmatrix} \qquad (5.2)$$

The use of sensitivity information, i.e., normalized differential sensitivity, has been examined in two particular nonlinear circuits (Figs. 5.3 and 5.4) exhibiting similar characteristics but with different topological configurations. The first circuit example is shown in Fig. 5.3 for which a sensitivity calculation, according to equation 5.1, was carried out, to generate the sensitivity matrix S (eq. 5.2).

The interest was in exploring the effect on circuit responses V_1, V_2, V_3, (nodal voltages), and the current I_{cc} drawn from the supply, of changes in component parameters R_1, R_2, R_3, R_4 and the transistor current gain β. The sensitivity calculation result is presented in Table 5.1, where the columns show the normalized differential sensitivity for a particular circuit response, to variations in component parameters; for example, column 1 shows the effect that component parameters R_1, R_2, R_3, R_4, and β have on the nodal voltage V_1.

The information presented in Table 5.1 is then to be examined with the objective of identifying and eliminating any redundant information. This operation should be performed following some specific guidelines such that the end result be useful for fault diagnosis. In other words, it would achieve a preliminary selection of circuit performances that are to be considered as candidate responses.

An attempt to develop these specific guidelines has been made, and can be summarized in the following steps:

(1) Identify those circuit responses that are insensitive to changes in component parameters. If there are any, eliminate them, because they will provide no useful information for fault diagnosis.

(2) Identify those circuits responses that present similar effect to variations in each of the component parameters. This operation involves a comparison of columns in equation 5.2, in terms of numerical value and the sign of the sensitivities. Only one response is required to

be considered as a candidate response from this group, because the rest would provide redundant information.

(3) Take each component parameter in turn and compare the sensitivity values in the columns identified in step two with the sensitivity values of the remaining circuit responses (columns). When a sensitivity value in one of the columns (say column X) identified in step two is similar to a value in one or more of the remaining columns, it suggests that the particular performance (i.e., associated with column X) is redundant. It can be useful to place a marker by each sensitivity value in the column (e.g., column X) so identified.

(4) From the exercise carried out in the previous step, after each component parameter has been considered, the circuit response(s) (columns) which has the most markers should then be eliminated as the redundant response(s).

These guidelines have been applied in the examination of the sensitivity information provided in Table 5.1. At the first step, no response is eliminated, because all of them are sensitive to changes in component parameters. In the second step, nodal voltages V_1 and V_2 are identified as the responses that exhibit a similar effect to changes in each of the component parameters, i.e., R_1, R_2, R_3, R_4, and β. The identification can clearly be realized from a visual analysis of the columns of Table 5.1; the information is also graphically displayed in Fig. 5.5. It is then clear that one of these responses is providing redundant information and should be eliminated. It can be observed at the third step that the component parameters R_1, R_2, R_3, and β have similar effect on the current I_{cc} and the node voltages V_1 and V_2. The comparison of the sensitivity values between these node voltages (V_1, V_2) and the current I_{cc} is carried out. From this comparison and also from the graphical representation shown in Fig. 5.5, nodal voltage V_2 is isolated as the redundant response because, although changes in component parameters R_1, R_2, R_3, and β have a similar effect on node voltages V_1, V_2 and current I_{cc}, this similarity is more closely associated between V_2 and I_{cc}, rather than V_1 and I_{cc}. Consequently, for this first nonlinear circuit example, nodal voltages V_1 and V_3, and the current I_{cc} drawn from the supply, should be chosen as candidate responses for fault diagnosis, if the sensitivity information is to be used as according to the guidelines established above.

The second nonlinear circuit example is shown in Fig. 5.4, with component parameters R_1, R_2, R_3, and the transistor current gain β. The responses of interest are the nodal voltages V_1, V_2, V_3, and the currents I_{BB} and I_{cc} drawn from the DC voltage sources. The sensitivity matrix S (eq. 5.2) was generated by performing sensitivity calculations according to eq. 5.1. The result is presented in Table 5.2, where, as in the previous example, columns show the normalized differential sensitivity for a particular response to changes in the circuit component parameters.

The analysis of this sensitivity information was carried out following the guidelines previously developed. A simple inspection of the information provided in Table 5.2 shows that all the circuit responses are sensitive to changes in component parameters, so that no response is eliminated at this step. However, it can be noticed that variations in component parameters R_1, R_2, R_3, and β produce a similar effect on nodal voltages V_1 and V_2. Changes in some of these component parameters, i.e., R_1, R_2 and β, also have a similar effect on the current I_{cc}, as compared with the effect on V_1 and V_2, though more closely associated with V_2. Then, from this analysis, aided by the graphical representation displayed in Fig. 5.6, node voltage V_2 is isolated as the response providing redundant information. Therefore, using this information as an aid in the a priori selection of candidate responses for fault diagnosis, responses V_1, V_3, I_{BB}, and I_{cc} should be selected, and V_2 eliminated.

5.3 APPLICATION OF THE ALGORITHM TO NONLINEAR CIRCUITS

Since linearity was intentionally never invoked in the development of the statistically-based diagnostic method, an exploration of its effectiveness for nonlinear circuits was undertaken for the two circuit examples shown in Figs. 5.3 and 5.4. DC testing was considered and the nonlinear circuit's equation formulated from the nodal description of the circuit. Transistors were characterized using the Ebers-Moll model. A solution for the responses of the circuit is found by using iterative techniques such as the Newton-Raphson method based on the analysis of a linearized version of the nonlinear circuit. This means that, at each iteration, the diodes in the Ebers-Moll model of the transistor are replaced by a conductance in parallel with a current source, until convergence to the required solution is achieved.

5.3.1 NONLINEAR CIRCUIT EXAMPLE ONE

The first circuit example chosen for the experiment is shown in Fig. 5.3 together with the specifications put on node voltages V_1 and V_3 as acceptable upper and lower limits of variation. At the outset, it will be recognized as a particularly demanding example, since some of the fault conditions are extremely difficult to differentiate and/or detect, and the results should therefore be interpreted in this light.

All component parameters were assumed subject to ±5% tolerance, with the exception of the transistor current gain (β) whose normal (i.e., acceptable) variation was taken to be ±60% of nominal. The candidate responses were the node voltages V_1, V_2^* and V_3, and the current I_{cc} drawn from the supply. Faults were simulated by taking 50 samples for each of the fault cases (9 fault cases in total). The fault cases include all resistor values lying between ±5% and ±50% of noninal, and β values between -60% and -95% of

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* Although the sensitivity analysis strongly suggests that V_2 should not be chosen as a candidate diagnostic response, it is considered to allow comparison and to check the effectiveness of using sensitivity information in the a priori selection of candidate responses.

nominal. The fault simulation results were recorded in matrices Y, V, and S. Then, only three responses were automatically selected as diagnostic ones. The selection of diagnostic responses was carried out separately by both methods (i.e., Analysis of Variance, and Eigenanalysis); however, for this particular circuit, both methods selected the same responses: voltages V_1 and V_3, and the current I_{cc} drawn from the supply.

In testing the algorithm, 50 faulty circuits were again simulated at random, for each of the 9 fault cases, and the diagnostic responses evaluated. Node voltage V_2, not selected by either of the two methods, and considered to provide redundant information from the sensitivity analysis, was also considered and evaluated, in order to demonstrate that its inclusion as a diagnostic response does not provide any improvement in the fault location success. Fault location was performed by using both of the P matrices discussed in Chapter four (eqs. 4.31 and 4.32), and the results are shown in Figs. 5.7 and 5.8, where location is performed to the first smallest d_j^2 value. The fault location results shown in Fig. 5.7 correspond to the case when node voltage V_2 was included as a diagnostic response. The results presented in Fig. 5.8 correspond to the case of using the selected diagnostic responses (i.e., V_1, V_3, and I_{cc}) in achieving the fault location.

In view of the difficulty of diagnosis imposed by the nature of this circuit (and due principally to overlap among faults in component parameters R_1, R_2, and β), the results are considered to be encouraging and, as in the linear circuit example, the diagnostic success is superior if method B, as compared with method A, is used to compute the square distance d_j^2. An improvement in the location can be realized (Fig. 5.9) if location is performed to either the first or second smallest d_j^2 value (the selected diagnostic responses were used to effect this exercise). The results also establish the validity of using the sensitivity information as an aid in the a priori selection of candidate responses for fault diagnosis. In this example, the inclusion of node voltage V_2 as a diagnostic response provides no improvement in the diagnostic success, as can be seen from the results presented in Figs. 5.7 and 5.8.

5.3.2 NONLINEAR CIRCUIT EXAMPLE TWO

The results of the previous experiment demonstrated that it is nearly impossible to distinguish between faults in the two bias resistors R_1 and R_2 (Fig. 5.3). However, it is possible to investigate another circuit with similar characteristics, by combining both resistors into a single one and having an independent supply for the bias of the base. This generates the circuit shown in Fig. 5.4, which was chosen for the second nonlinear circuit experiment. The specifications were put on node voltages V_1 and V_3 as acceptable upper and lower limits of variation.

In fault-free conditions, the parameters of all resistors were assumed to have a tolerance of ±5% of nominal, and the parameter of the transistor current gain (β) was taken to have a variation of ±60% of nominal. A fault is deemed to have occurred if any of the parameter values of the resistors deviates into the range between ±5% and ±50%, or the β value lies between -60% and -95% of nominal. The chosen candidate responses were the node voltages V_1, V_2^* and V_3, and the currents I_{BB} and I_{cc} drawn from the supplies. Faults were simulated by taking 50 samples for each of the 7 fault cases. The results were recorded in matrices Y, V and S. Then, the selection of diagnostic responses was carried out separately by both methods. Four diagnostic responses were selected by the analysis of variance approach: they included the node voltages V_1, V_2 and V_3, and the current I_{cc}. Use of the eigenanalysis approach also resulted in the selection of four diagnostic responses: they included the node voltages V_1 and V_3, and the currents I_{BB} and I_{cc}.

In testing the effectiveness of the diagnostic algorithm, 50 faulty circuits were again simulated at random in each of the 7 fault case regions, and the diagnostic responses evaluated. For comparison purposes, fault location was performed by making use of both of the P matrices (eqs. 4.31 and 4.32) in the computation of the squared distance d_j^2. Use of diagnostic responses selected by the analysis of variance approach leads to the fault location results

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* Although, as in the previous example, V_2 should be excluded according to the sensitivity analysis (V_2 provides redundant information), it was included as a candidate response in order to demonstrate that its use as a diagnostic response does not provide any improvement in the fault location success.

presented in Fig. 5.10, where successful location of faulty component is associated with the smallest d_j^2 value. The fault location results obtained by using the diagnostic responses selected by the eigenanalysis approach are presented in Fig. 5.11 where, as before, successful location of the faulty component is associated with the smallest d_j^2 value.

Based on the above results, it is possible to verify that, even with a limited number of candidate responses for diagnosis, the analysis of variance selects a different set of diagnostic responses as compared with the set of diagnostic responses selected by the eigenanalysis approach. It has also been observed that use of the diagnostic data-base selected by the eigenanalysis method leads to a better fault location result, particularly if method B for computing the squared distance d_j^2 is used. Another interesting issue to consider is the apparent relation existing between the diagnostic responses selected by the eigenanalysis approach and the sensitivity information provided by the sensitivity analysis. Both approaches tend to eliminate the same candidate response; in the selection case, because it does not yield a sufficient degree of discrimination for fault cases and, in the sensitivity case, because it provides redundant information. Finally, consideration of the node voltage V_2 (assumed to provide redundant information from the sensitivity analysis) as another diagnostic response does not contribute to an improvement in the fault location success, as is evidenced in the fault location results presented in Fig. 5.12; here in addition to the diagnostic responses selected by the eigenanalysis method (V_1, V_3, I_{BB} and I_{cc}), node voltage V_2 was also included as a diagnostic response. This outcome establishes the effectiveness of the use of sensitivity information in the a priori selection of candidate responses.

5.4 SUMMARY

The statistically based algorithm has been applied to the location of single "soft" faults in nonlinear DC circuits, under the assumption that nonfaulty component parameters are subject to normal production tolerances. The results of the experiments performed on the circuit examples indicate that the algorithm can confidently be used for the diagnosis of nonlinear circuits. However, as was the case for the linear circuit example, the algorithm seems to be more effective if diagnostic responses are selected by the eigenanalysis approach and fault location is achieved by using the pooled covariance matrix (method B) in the classification procedure.

The usefulness of the sensitivity information as an aid in the a priori selection of candidate responses for diagnosis is also demonstrated, despite the fact that, for these particular circuit examples, the number of candidate responses was limited. It has been established that the inclusion in the diagnostic data-base of a circuit response providing redundant information (as evidenced from the sensitivity analysis) does not produce an improvement in the fault location success. The importance of using sensitivity information can be much appreciated when dealing with large circuits having a large number of candidate responses. For these cases, a preliminary selection and elimination of responses observed to provide redundant information may prove to be advantageous in computational terms, and present a better set of candidate responses from which to select the diagnostic ones.

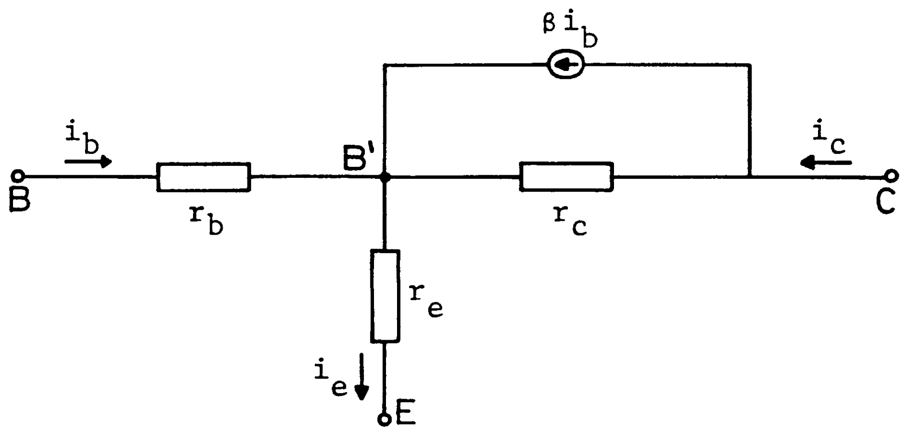
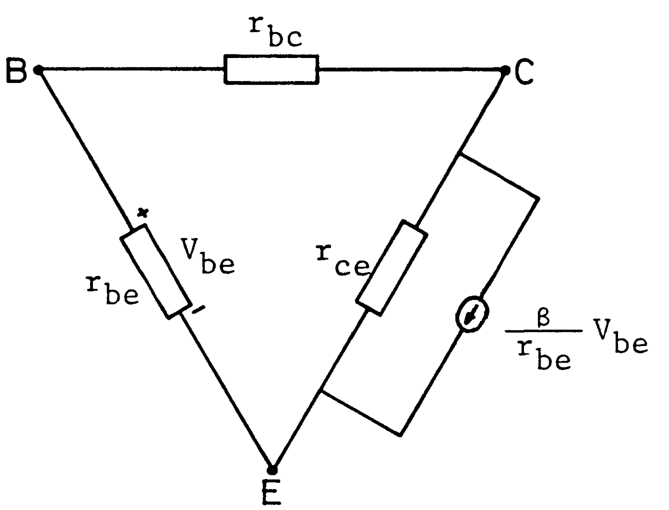


Fig. 5.1 Transistor small-signal model

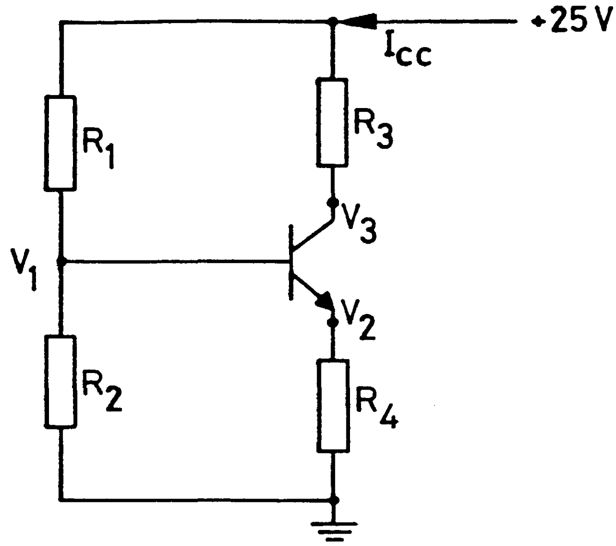


$$r_{be} = \frac{r_c\, r_e\,(1+\beta) + r_b\, r_c + r_b\, r_e}{r_c}$$

$$r_{bc} = \frac{r_c\, r_e\,(1+\beta) + r_b\, r_c + r_b\, r_e}{r_e}$$

$$r_{ce} = \frac{r_c\, r_e\,(1+\beta) + r_b\, r_c + r_b\, r_e}{r_b}$$

Fig. 5.2 Transistor small-signal
three-terminal model



R_1 = 90 KΩ

R_2 = 10 KΩ

R_3 = 5 KΩ

R_4 = 1 KΩ

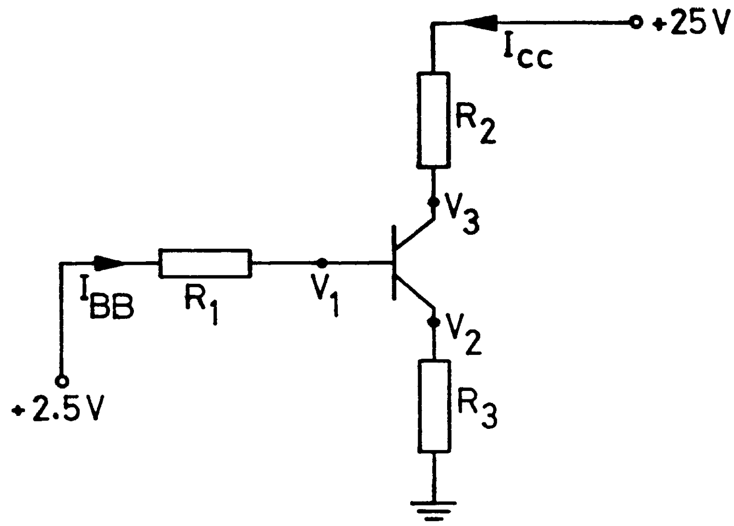
β = 100

Specifications on	V_1 and	V_3
Upper limits :	2.60 V	18.70 V
Lower limits :	2.05 V	15.45 V

Fig. 5.3 Nonlinear circuit example one

Table 5.1 Normalized sensitivity information for nonlinear circuit example one

Compon. Paramet.	V_1	V_2	V_3	I_{cc}
R_1	-0.88	-1.23	0.6	-1.18
R_2	0.83	1.15	-0.56	0.98
R_3	≅ 0.00	≅ 0.00	-0.48	≅ 0.00
R_4	0.056	0.1	0.43	-0.78
β	0.057	0.079	-0.043	0.076



$R_1 = 9$ KΩ

$R_2 = 5$ KΩ

$R_3 = 1$ KΩ

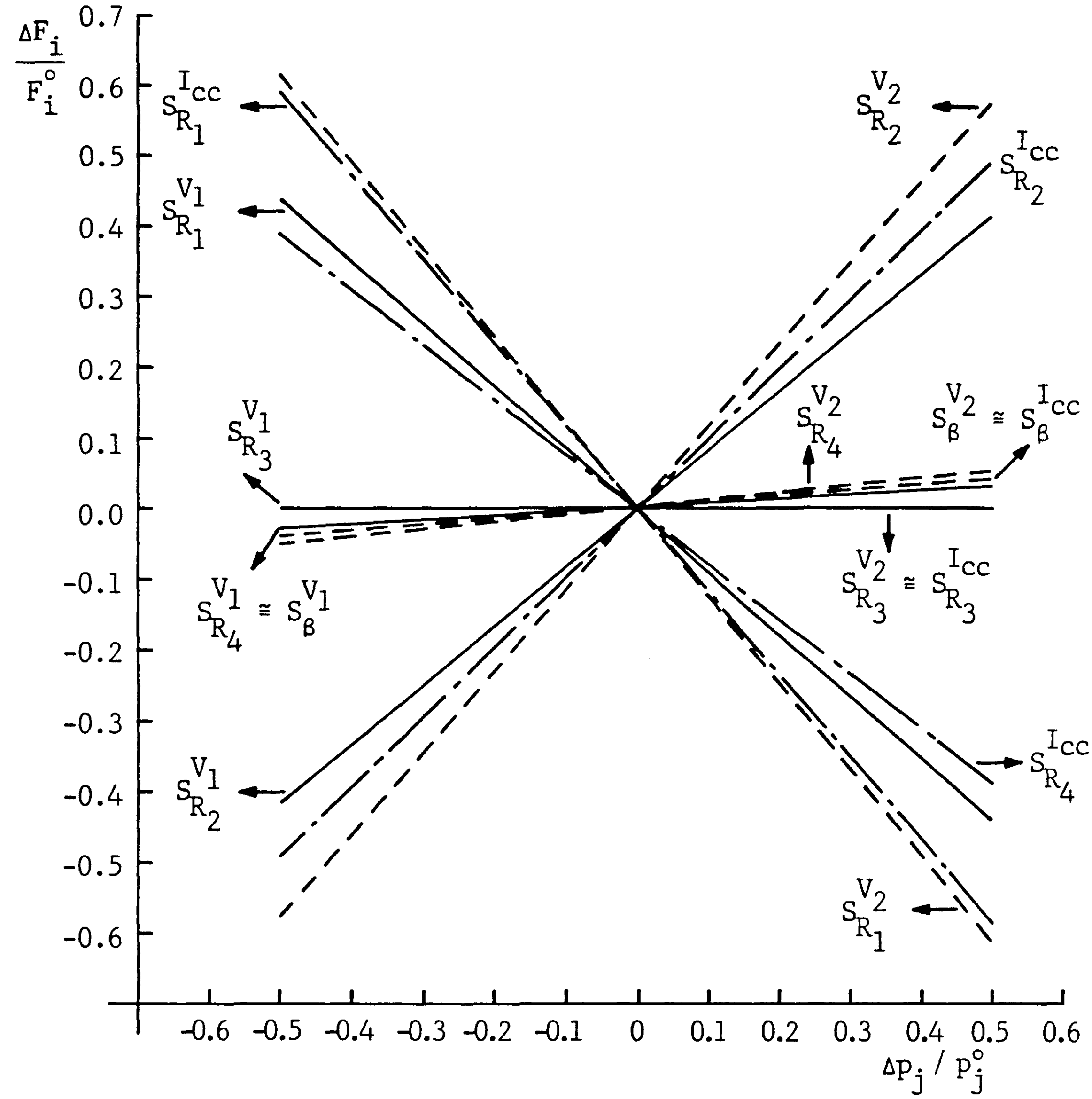
$\beta = 100$

Specifications on	V_1 and	V_3
Upper limits :	2.41 V	18.32 V
Lower limits :	2.17 V	16.11 V

Fig. 5.4 Nonlinear circuit example two

Table 5.2 Normalized sensitivity information for nonlinear circuit example two

Compon. Paramet	V_1	V_2	V_3	I_{BB}	I_{cc}
R_1	-0.0574	-0.0797	0.0386	-0.0797	-0.0797
R_2	≈ 0.00	≈ 0.00	-0.484	≈ 0.00	≈ 0.00
R_3	0.0559	0.105	0.433	-0.895	-0.895
β	0.0568	0.0789	-0.043	-0.910	0.0888

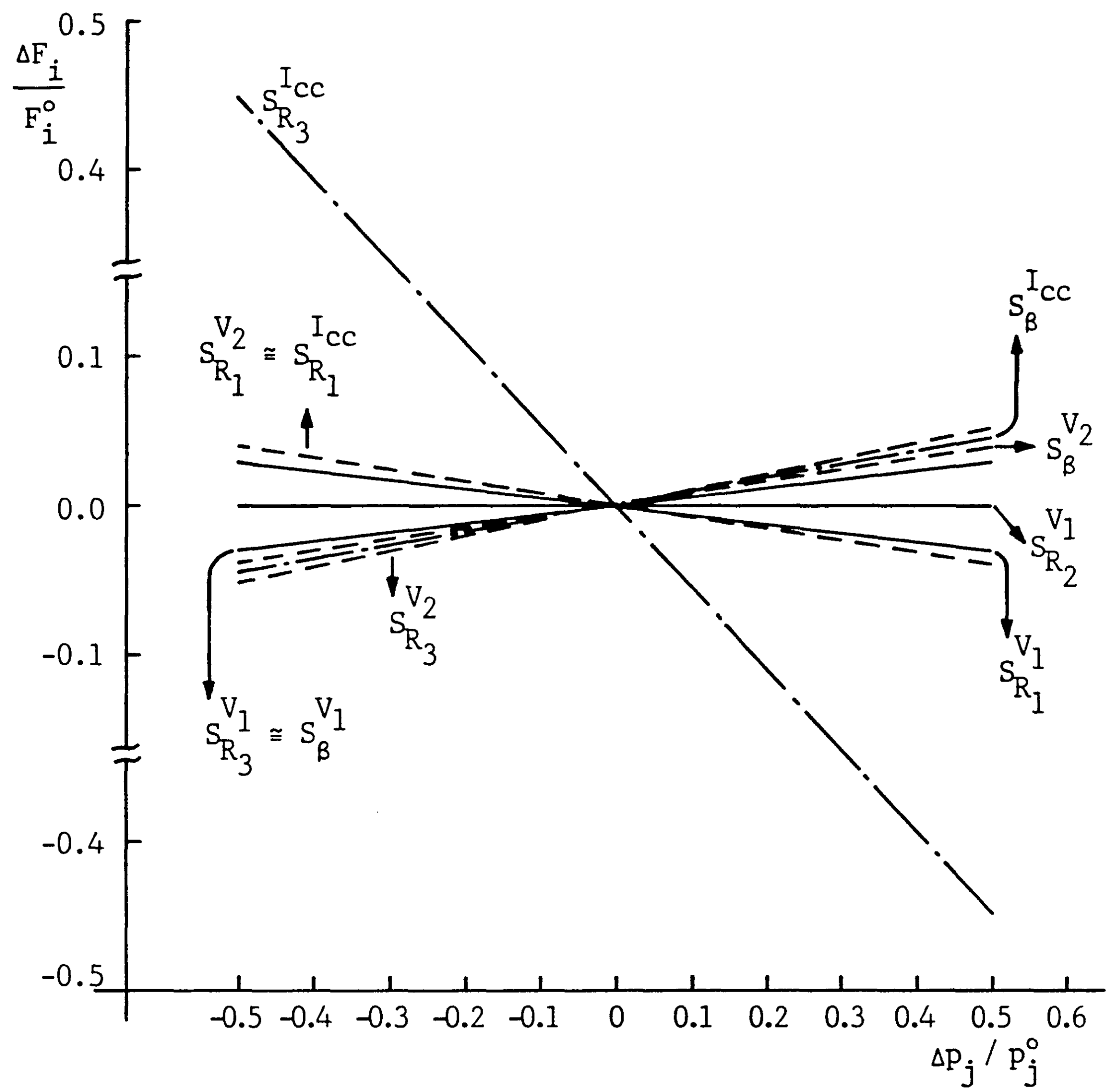


—— Sensitivity of nodal voltage V_1 with respect to each component parameter

– – – Sensitivity of nodal voltage V_2 with respect to each component parameter

—·— Sensitivity of the current I_{cc} with respect to each component parameter

Fig. 5.5 Graphical representation of the normalized differential sensitivity, of nodal voltages V_1 and V_2, and current I_{cc}, with respect to component parameters of the nonlinear circuit of Figure 5.3



——— Sensitivity of nodal voltage V_1 with respect to each component parameter

– – – Sensitivity of nodal voltage V_2 with respect to each component parameter

—·— Sensitivity of the current I_{cc} with respect to each component parameter

Fig. 5.6 Graphical representation of the normalized differential sensitivity, of nodal voltages V_1, V_2 and current I_{cc}, with respect to component parameters of the nonlinear circuit of Figure 5.4

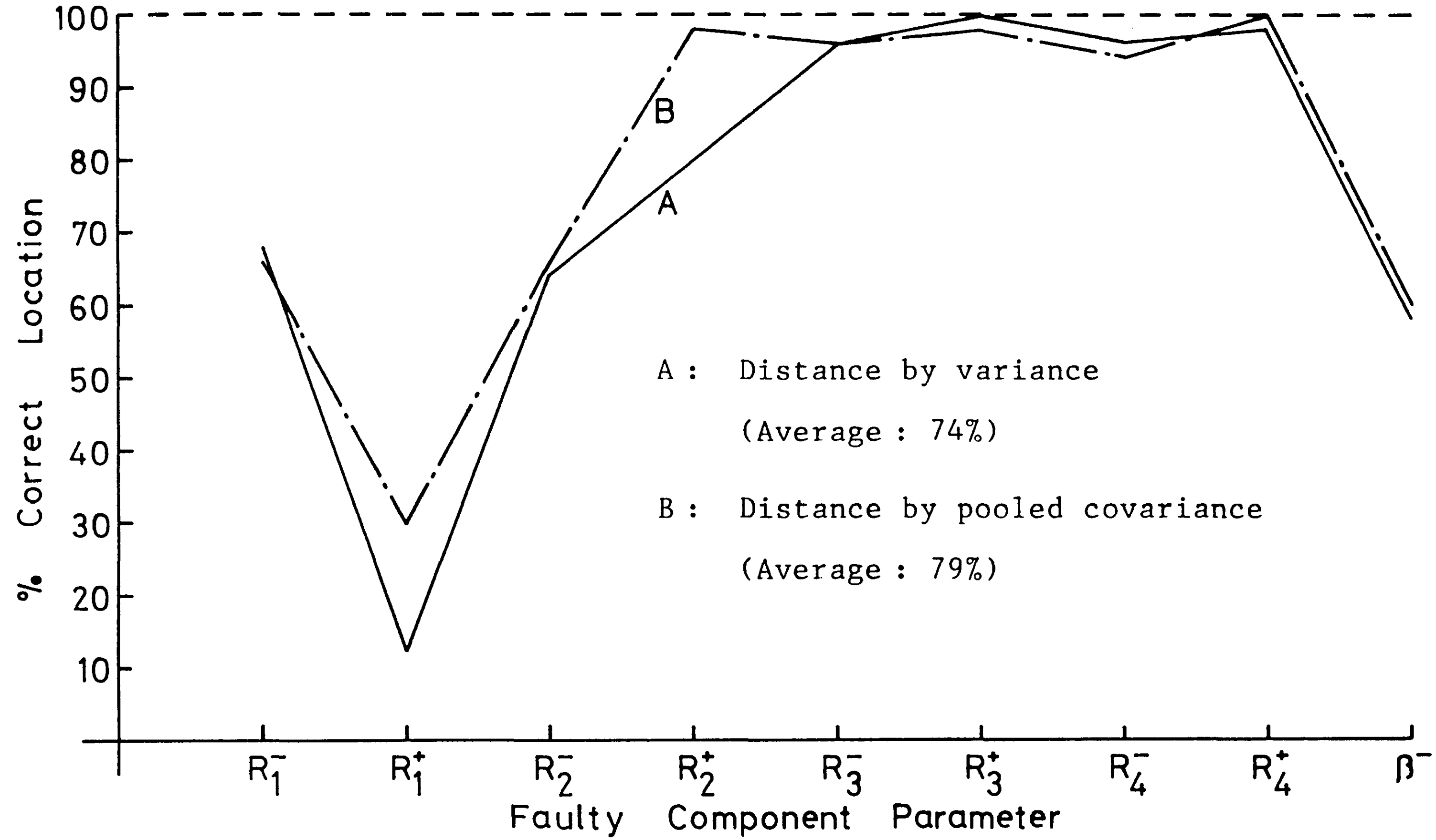


Fig. 5.7 Fault location for the nonlinear circuit of Fig. 5.3, using the selected diagnostic data and the nodal voltage V_2

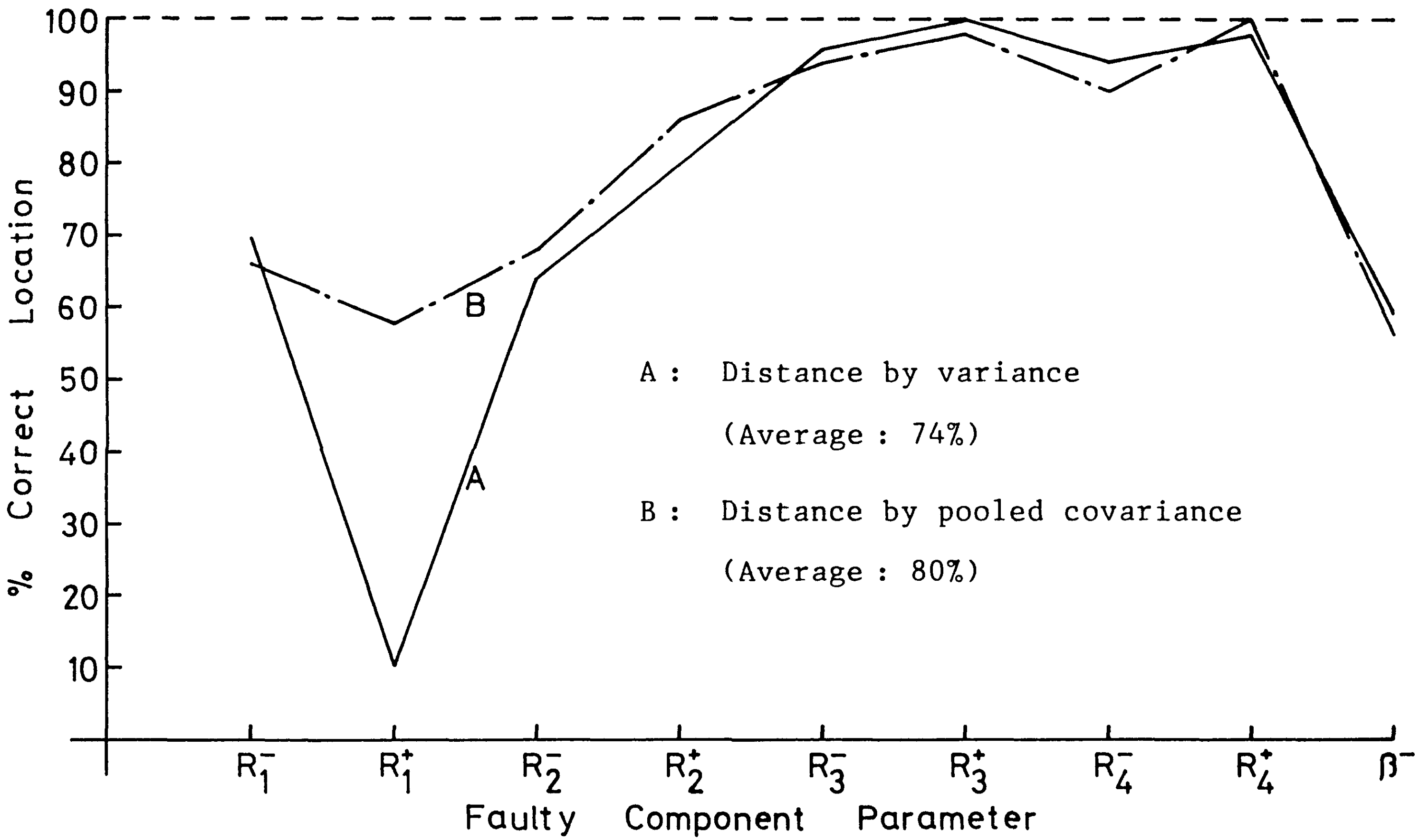


Fig. 5.8 Fault location for the nonlinear circuit of Fig. 5.3, using the selected diagnostic data

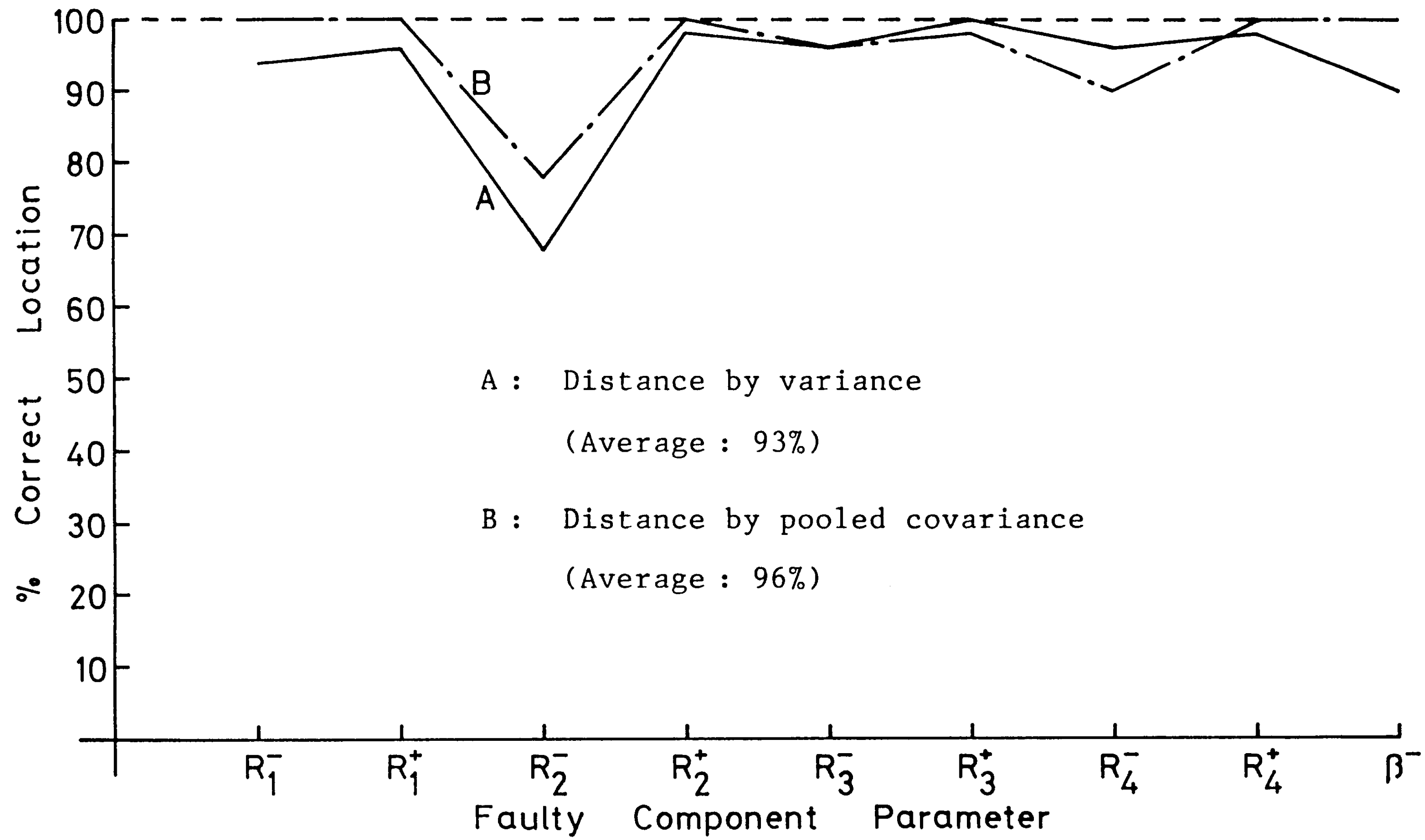


Fig. 5.9 Fault location for the nonlinear circuit of Fig. 5.3 by either the first or second smallest distance d_j^2, using the selected diagnostic data

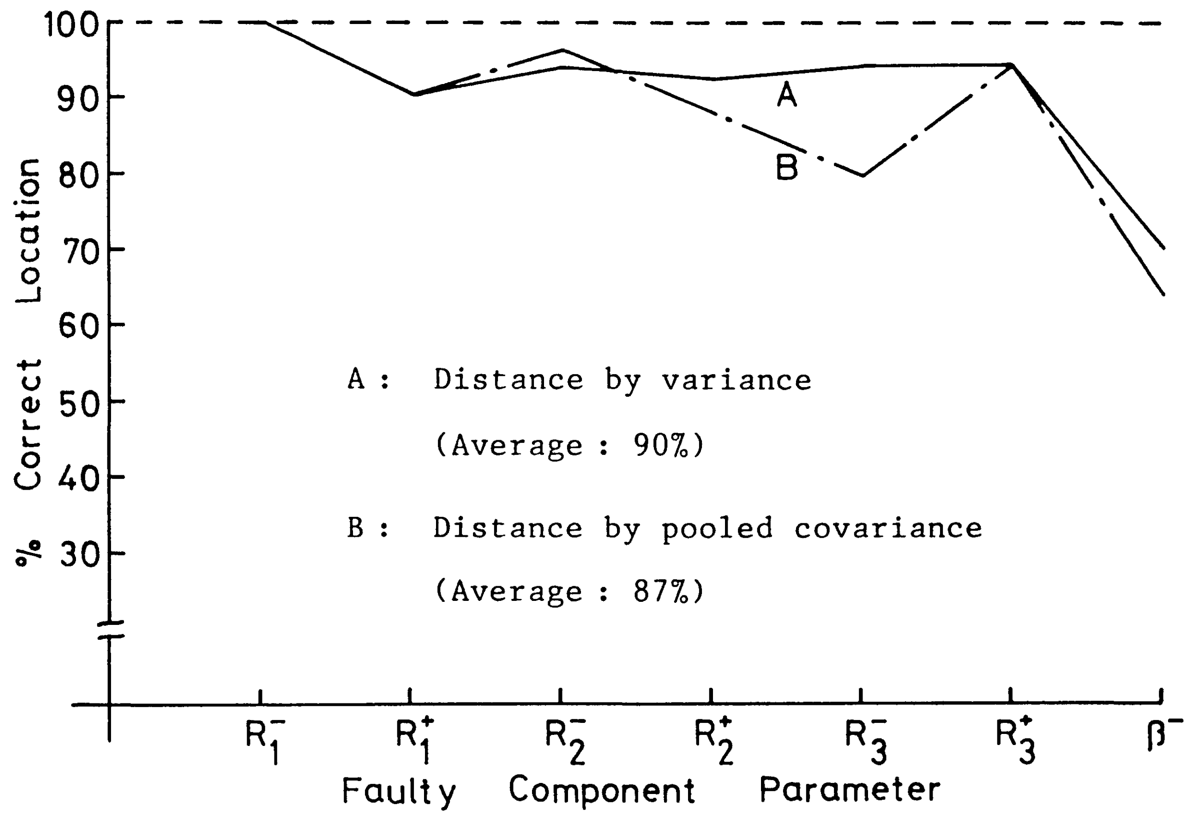


Fig. 5.10 Fault location for the nonlinear circuit of Fig. 5.4, using data selected by the analysis of variance approach

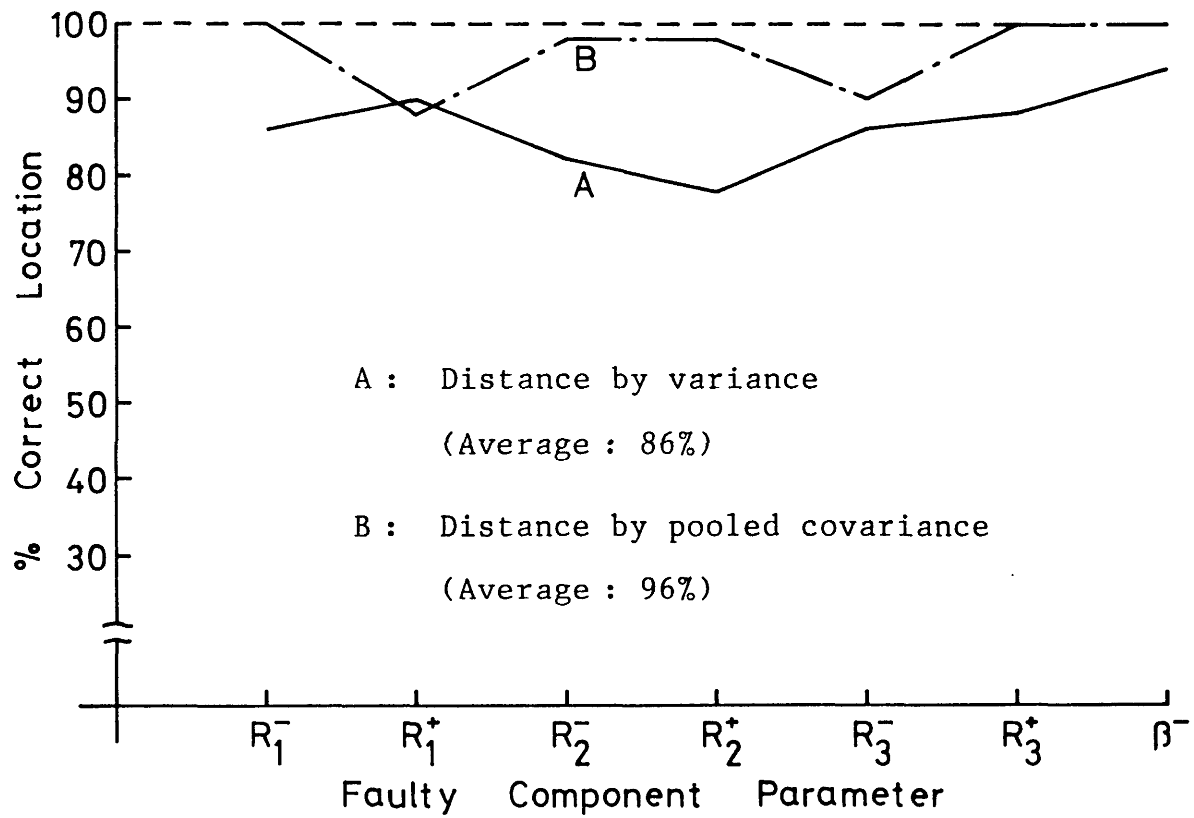


Fig. 5.11 Fault location for the nonlinear circuit of Fig. 5.4, using data selected by the eigenanalysis approach

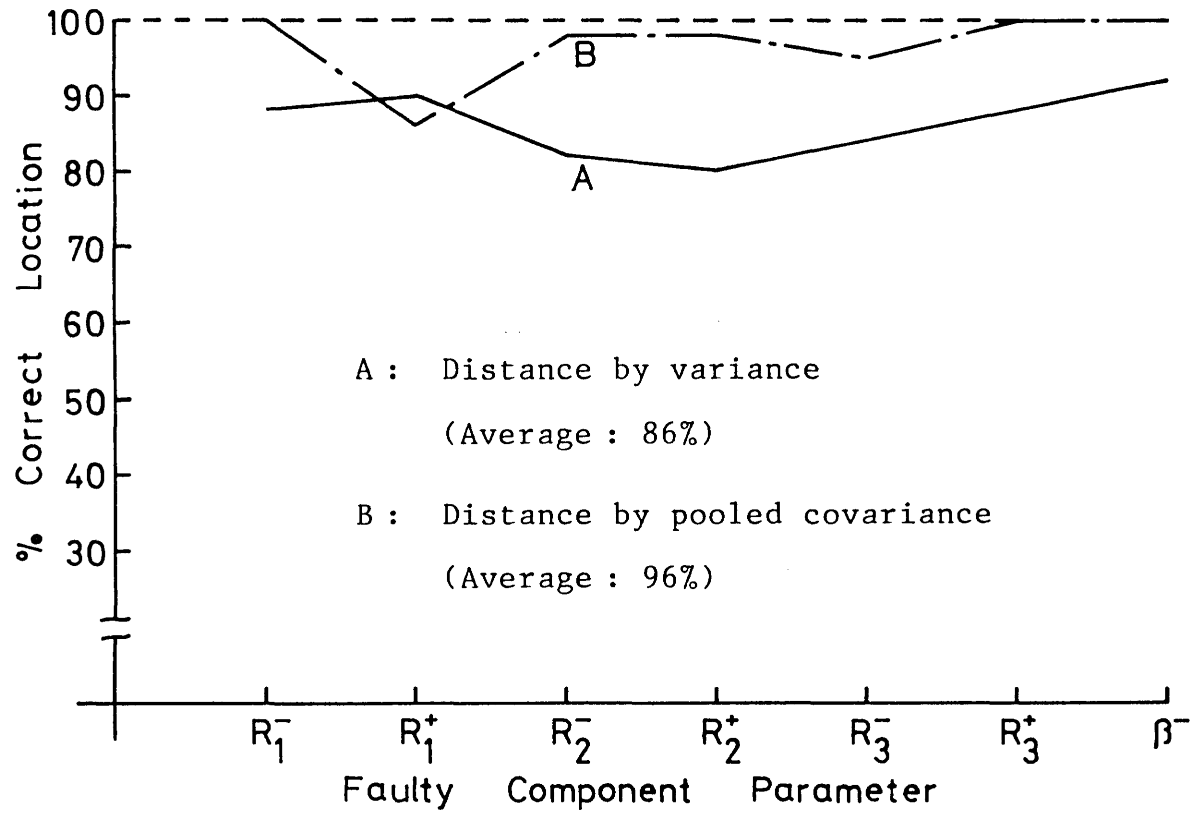


Fig. 5.12 Fault location for the nonlinear circuit of Fig. 5.4, using the diagnostic data selected by the eigenanalysis approach, and the nodal voltage V_2

CHAPTER SIX

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 CONCLUSIONS

The general concern of the research reported in this thesis has been the study (described in chapter one) of fault diagnosis problems in analogue circuits. An extensive review of existing methods for solving these problems is presented in chapter two. On the basis of the mathematical techniques used, these methods have been grouped into two categories: deterministic and nondeterministic methods. It is concluded that, although considerable progress has been made in the search for simple and efficient methods for fault diagnosis, much less effort has been devoted to fault problems involving toleranced fault-free component parameters, and with less successful results due principally to the complexity of the problem. In this context, the main research contribution presented in this thesis is the development of an integrated statistically-based algorithm for the location of single faults in toleranced analogue circuits.

Satisfactory results (chapter four, section 6, and chapter five, section 3) have been obtained from the application of the method on sample circuits. The main features of the algorithm are as follows:

(1) Two distinctive steps are involved in the procedure. First, in the circuit design stage, fault simulation and selection of diagnostic responses are performed with subsequent storage of a data-base representing the fault conditions of the circuit. Second, in the field, circuits are checked against their performance specifications. When they are found to be faulty, the

diagnostic responses are measured and, on the basis of a distance measure, then classified as one of the fault conditions in the data-base. The result is the location of the faulty component causing the circuit to fail.

(2) In Fault Simulation, fault conditions (fault cases) are defined within the context of component parameter space. This allows a clear definition of the areas of tolerance region at which components are at fault.

(3) A clear distinction is made between the specification responses and the diagnostic responses. The first are for detecting a fault in the circuit, and the second for locating the faulty component.

(4) There are no a priori assumptions made concerning the linearity of the circuit, and the location of the nodes at which diagnostic measurements can be made, though the use of sensitivity information (chapter five, section 2) may suggest the location of suitable candidate responses.

(5) Fault-free component parameters can take any value within their manufacturing tolerance ranges.

(6) Other important factors associated with the method are as follows:

(a) Although the algorithm has been tested only on single faults, the method could equally be applied to the case of multiple faults, where the main difficulty would be associated with the large volume of data to be processed (cost of simulation). It should also be mentioned that in some cases (usually for nonlinear circuits) a single fault can encompass related faults, i.e., a faulty bias resistor R could cause an increase in the collector current, resulting in an increase in the current gain (β), giving the appearance that the transistor (β parameter) is also faulty.

(b) The method might not be suitable for wide variations in the value of component parameters approximating open and short circuits, especially for nonlinear circuits, though some satisfactory results have been obtained for large variations in the range of ±95% deviation of nominal.

6.2 SUGGESTIONS FOR FURTHER RESEARCH

6.2.1 SENSITIVITY INFORMATION IN FAULT DIAGNOSIS PROBLEMS

The Monte Carlo approach used to characterize statistically the component fault associated with faulty circuits requires the analysis of N_j circuits for each of the fault cases, with a total of $2KN_j$ circuit analyses if there are 2K fault cases. For large circuits, this approach may become computationally expensive. However, research efforts have been concentrated in reducing the apparent cost disadvantage associated with large Monte Carlo analysis, and recently a new statistical technique [58], based on the exploitation of differential sensitivity information has been developed for this purpose.

In general, differential sensitivity is a measure of the effect on circuit responses of the changes in component parameters. The sensitivity information could be used in fault diagnosis problems in the following particular situations:

(1) The number of Monte Carlo analyses could be reduced in the sense that fewer sample points would need to be taken in each fault case region. If, in addition to evaluating the required performance functions (candidate responses), the differential sensitivity can also be calculated, this information could be used to evaluate responses at other nearby points in the fault case regions by means of first order Taylor series.

(2) A preliminary study of the usefulness of the sensitivity information as an aid in the a priori selection of candidate responses has been carried out for nonlinear DC circuits (chapter five, sections 2 and 3), with encouraging results. However, in order to generally establish its effectiveness, it may be necessary to further investigate its application to circuits in the frequency domain.

6.2.2 NONLINEAR CIRCUITS

It has been mentioned that, for linear circuits, the fault diagnosis problem becomes more complex when the production tolerances for fault-free component parameters are taken into consideration. In the case of nonlinear circuits, this complexity increases even more due to a variety of factors (discussed in chapter five, section 1). It has been observed that, in spite of these complexities, the algorithm could confidently be applied to the diagnosis of nonlinear circuits, since it provided some encouraging results for nonlinear DC circuit examples (chapter five, section 3). It may nevertheless be appropriate to further test the algorithm by considering the nonlinear circuit's ac circuit performance. Thus, the search for appropriate diagnostic responses may be expanded to include both dc and ac responses, which in the end may provide better possibilities of finding responses that exhibit good discrimination for the different fault cases of the circuit.

Another factor to consider is the pooling of covariance matrices. The nonlinear nature of this type of circuit may produce a covariance structure for individual fault cases with such dissimilar dispersions that pooling may become inappropriate in trying to discriminate one fault case from another in the classification (location) process. Instead, it may be better to exploit these dissimilarities to locate a fault to the correct fault case. However, this is not an easy problem, and it creates additional complications in terms of storage and the use of an adequate technique for analyzing the covariance structure.

6.2.3 DIAGNOSTIC SUCCESS

The diagnostic success, as mentioned in chapter four, sections 3 and 6, depends on the discrimination among fault cases and the number of features selected as diagnostic responses. A preliminary heuristic study has been done on this matter, involving the trade-off between the percentage of correct location of faulty components and the number of diagnostic responses. However, although this provides some useful information, there is no clear indication of the overall confidence level on the results. Therefore, it would be apropriate to develop some sort of measure in this respect, which may select an optimum number of diagnostic responses for a required (e.g., 95%) overall confidence level. The importance of this measure comes from the statistical nature of the fault diagnosis method, which based on simulation provides statistical estimates for the fault cases rather than exact results.

6.2.4 INTEGRATED CIRCUITS

A possible extension of the algorithm may be related to the design and/or testing of integrated circuit, particularly in the identification or selection of circuit performances that may be considered of importance in establishing the statistical behaviour of the circuit. It has been observed that changes in circuit responses are dependent on the statistical variation of the process parameters (e.g., sheet resistivities, junction depths, temperature, doping levels, etc.). Another important consideration is the fact these parameters are often correlated, which has usually been used as an aid in reducing the variation of circuit responses and hence obtaining an improved design. The eigenanalysis method may be used to identify those circuit responses that are associated with one or a group of process parameters, and those circuit responses that can discriminate between the influence of individual or groups of parameters. This information could be useful such that, during the manufacturing process, it may be possible to find, by examining the response pattern, the source of variation in the circuit performance.

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