

A Computational Study of the Stagnation Phase in Inertial Confinement Fusion: Hotspot Energetics, Diagnostics, and Burn Propagation

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Declaration of Originality

The entirety of this thesis is the work of the author, unless explicitly mentioned in the text and with the following contributions from others. The neutron spectrum post-process code was written by Brian Appelbe based on his work on the neutron production spectrum, and was parallelised and given a new input deck by the author. The X-ray imaging post-process code and the radiation loss model described in Chapter 2 was developed by Nicolas Niasse. The kinetic model of alpha particle transport used in Chapter 5 was written for a single test particle by Brian Appelbe based on the work of Mark Sherlock and implemented in Medusa by the author. The Gorgon implementation of the kinetic alpha particle transport model was carried out by Simon Vickers and Brian Appelbe, with some additional tweaks by the author. Gorgon itself is an international, multi-user code originally written by multiple authors at Imperial College.

Abstract

This thesis studies stagnating ICF capsules. The work is theoretical, with numerical modelling carried out primarily using a 3D multi-physics hydrocode, Gorgon. Simulations of NIF-relevant ICF stagnations are generated from the output of a 1D Hydra NIF post-shot simulation of shot N120205, with an inherently 3D perturbation imposed in the velocity field to generate asymmetries.

The simulations are used to develop a detailed phenomenology of the effect of perturbations from spherical symmetry on the energy balance of the hotspot at stagnation. The main effect is an increase in the fluid velocity at the hotspot edge before the time of peak temperature, which results in early thermalisation of kinetic energy and additional compression early in time. The increase of the perturbed target's surface area above $4\pi R^2$ is found to increase the thermal conduction rate. An inefficiency of thermalisation of kinetic energy is caused by asymmetries in the ram pressure. The combination of these effects makes it possible for the yield to be reduced to experimental levels in the simulations purely by perturbation of an all-DT system.

In order to study the energy balance, an accurate analytic model of the hotspot energy source terms in unperturbed targets is developed by taking advantage of the self-similar temperature profile the hotspot converges upon. This is an improvement upon 0D models such as that of Widner [1]. In particular, the thermal conduction loss is significantly lower when the thermal conductivity at the hotspot edge is used, and the 'ideal ignition temperature' is found to be lower when the ion and electron species are in equilibrium.

The diagnostic signatures of hotspot asymmetries are investigated by generating synthetic diagnostic data from the 3D simulations. It is found that stagnation-phase perturbations of mode number $l \gtrsim 6$ with amplitudes large enough to reduce the yield to experimental levels are not visible on the currently available NIF diagnostics, or are able to alias as perturbations of lower mode number. Comparisons are also made between the P0 as measured from X-ray images with the physical hotspot radius, and the neutron spectrum width with the burn-averaged and mass-averaged hotspot temperatures, including the effects of motion Doppler broadening.

The effect of alpha particle transport on the burn process is investigated using the 1D Lagrangian code Medusa by comparing an accurate model of the alpha particle species with models that make various assumptions about the particle kinetics. The fraction of fusion energy deposited within the burning region χ is related empirically to the properties of the hotspot, which allows alpha particle transport to be included as a loss term in the energy balance model. Alpha transport in 3D perturbed hotspots is investigated by post-processing Gorgon data, which finds that perturbed targets have significantly increased fusion energy loss relative to equivalent unperturbed targets.

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Glossary of Symbols

Symbol	Meaning
α	In-flight adiabat
α_{stag}	Stagnation adiabat
γ_α	Rate of hotspot thermal energy change due to alpha particle heating assuming local instantaneous deposition
γ_m	Rate of hotspot thermal energy change due to mechanical work
γ_r	Rate of hotspot thermal energy change due to radiation emission
γ_{TC}	Rate of hotspot thermal energy change due to thermal conduction
χ	Fraction of generated alpha energy deposited outside of the ‘burning region’
χ_{1D}	χ as calculated for a spherically-symmetric hotspot with a given ρR and T
$c_{i,j,k}^n$	$\partial E / \partial T$ (‘heat capacity’) in cell i, j, k at timestep n
c_s	Sound speed
c_{TC}	Thermal scale-length constant in the Widner-type 0D model
δR	Perturbation amplitude defined as $\frac{1}{2}$ range of hotspot radii
$\delta R / R$	δR normalised to hotspot radius
δu_R	Perturbation amplitude defined as $\frac{1}{2}$ range of hotspot-edge radial velocity
$\partial_t W_\parallel$	Test particle slowing-down rate in a Maxwellian background
$\partial_t W_\parallel^2$	Diffusion rate in a Maxwellian background parallel to test particle velocity
$\partial_t W_\perp^2$	Diffusion rate in a Maxwellian background perpendicular to test particle velocity
Δx	Grid spacing in the x -direction
e	Charge of the electron
E	Total hotspot thermal energy
E_α	Birth energy of a DT fusion alpha particle (~ 13.52 MeV)
E_n	Energy of a detected neutron
$\varepsilon_e, \varepsilon_i$	Thermal energy density of the electron (ion) species
ε_{fuel}	Specific thermal energy of the fuel layer
η	Couling efficiency from optical laser light to kinetic energy of the implosion
F_e, F_i	Fraction of alpha particle deposited in electrons (ions) at a given location

$H_m(T_c, q)$	Shorthand for the hypergeometric function ${}_2F_1(3/q, m; 1 + 3/q; (1 - T_R/T_c))$
$\mathcal{H}_{m,n}(T_c, q)$	Shorthand for the hypergeometric function ${}_2F_1(3/2, m/n; 5/2; (1 - (T_R/T_c)^n))$
κ_e, κ_i	Electron (ion) thermal conductivity
κ_P	Planck opacity
L	Thermal scale length of the hotspot
l_α	Typical range of alpha particles in the hotspot
M	Total hotspot mass
M_{tot}	Total remaining mass of the target
m_i	Average ion mass in a DT plasma
n	Timestep number, thermal conductivity exponent
N	Number of spikes in a given single-mode perturbation configuration
P_a	Peak pressure at the X-ray ablation front during the ablation phase
P_e, P_i	Electron (ion) partial pressure
φ	Azimuthal coordinate
$\varphi(x)$	Error function
q, q_i, q_e	Exponent characterising hotspot (ion, electron) temperature profile
$R, \langle R \rangle$	Hotspot radius, defined by (average) radial location of T_R isotherm
ρ_{fuel}	Density of the fuel layer
ρR	Areal density (hotspot or total as indicated in text)
S	Hotspot surface area
σ	Stefan-Boltzmann constant
t	Time elapsed since peak velocity
t_0	Timescale of the Kidder-like stagnation
T_{ba}	Burn-averaged hotspot temperature (weighted by reactivity)
T_{ma}	Mass-averaged hotspot temperature (weighted by density)
T_{va}	Volume-averaged hotspot temperature
T_c	Central hotspot temperature
T_n	Temperature as diagnosed from neutron spectrum
T_R	Hotspot boundary temperature (defined)

T_r	Radiation temperature
u_r	Radial velocity of the fluid (inward negative, outward positive)
$u_R, \langle u_R \rangle$	(Average) radial velocity of the fluid at the hotspot edge
V	Hotspot volume
v_{th}	Thermal speed
W_α	Total hotspot energy change per unit time due to alpha particle energy deposition as calculated by the local instantaneous deposition model
W_m	Total hotspot energy change per unit time due to mechanical work
W_r	Total hotspot energy change per unit time due to radiation emission
W_{TC}	Total hotspot energy change per unit time due to thermal conduction

Chapter 1

Introduction

1.1) The Inertial Confinement approach to fusion

The primary goal of fusion technology is to produce energy by nuclear fusion reactions, in order to generate electricity. It is also of scientific interest due to the extreme physical conditions it provides controlled access to. Fusion of Deuterium and Tritium (DT) ions $D + T \rightarrow {}^4\text{He} + n$ is the most common reaction considered, since the cross-section of the DT fusion reaction is significantly higher than any others for temperatures achievable in the laboratory. Later generations of fusion reactor may use aneutronic reactions such as $p + {}^{11}\text{B} \rightarrow 3 {}^4\text{He}$, in order to eliminate neutron activation of the reactor. Fusion reactions in a sufficiently efficient reactor must be thermonuclear in origin, since beam fusion loses far too much energy by Coulomb scattering. The volumetric reaction rate of a thermonuclear fusion plasma is given by $n_i^2 \langle \sigma v \rangle$, where n_i is the ion number density and $\langle \sigma v \rangle$ is the thermal fusion reactivity, a function only of temperature. For DT fusion, $\langle \sigma v \rangle$ peaks at a temperature of around 66 keV, and systems which are to produce the levels of fusion power required for commercially viable energy generation must be heated to temperature scales of keV. In order to reach such high temperatures efficiently, it is desirable to achieve ‘ignition’, in which the rate of energy loss by the plasma W_{loss} is exceeded by the rate of energy deposition within the plasma by fusion reaction products W_α :

$$W_\alpha > W_{loss}. \quad (1.1.1)$$

It is usually assumed that only the charged reaction products deposit energy within the system, and in the following discussion it will be assumed that each charged reaction product deposits all of its energy. In a spatially uniform system¹, the charged-particle heating rate is given by

$$W_\alpha = \frac{1}{4} E_\alpha n_i^2 \langle \sigma v \rangle V, \quad (1.1.2)$$

where V is the volume of the system and E_α is the energy of the charged fusion products, 3.52 MeV in the form of alpha particles for DT fusion. The right-hand side of the inequality (1.1.1), the energy loss rate, may be written as the ratio of the fluid thermal energy to an energy loss timescale τ

$$W_{loss} = \frac{3n_i e T_i}{\tau} V, \quad (1.1.3)$$

under the assumption that the system is spatially uniform. e is the electronic charge and T_i is in eV, a convention maintained throughout this thesis unless otherwise stated. τ is also known as the ‘confinement time’ of the fusion system. Assuming the system is at the temperature for which $\langle \sigma v \rangle / T_i$ is a maximum, the ignition criterion is

$$n_i \tau > \frac{12e}{E_\alpha} \left(\frac{T_i}{\langle \sigma v \rangle} \right)_{min} = 1.5 \times 10^{20} \text{ s/m}^3 \quad (1.1.4)$$

which sets a minimum condition on the product of n_i and τ necessary to achieve self-heating in a fusion reactor. Assuming instead that the system is at the slightly lower temperature for which $\langle \sigma v \rangle / T_i^2$ is a maximum, a general ignition criterion may be written in terms of the thermal pressure $P = 2n_i e T$:

$$P \tau > \frac{24e^2}{E_\alpha} \left(\frac{T_i^2}{\langle \sigma v \rangle} \right)_{min} = 3.3 \times 10^{21} \text{ keV s/m}^3 \quad (1.1.5)$$

For DT, the minimum of $T_i^2 / \langle \sigma v \rangle$ occurs at approximately 13.5 keV. Both of the conditions (1.1.4) and (1.1.5) are sometimes referred to as the ‘Lawson criterion’ [2], although Lawson’s original paper [3] derives

¹ In the literature the source terms are generally evaluated in volumetric form, which implicitly assumes that the hotspot profiles are uniform. In this work, volume-integrated source terms are used, and volumetric forms from the literature have been multiplied by V in cases where the uniform-hotspot assumption is implied.

an $n_i\tau$ condition for breakeven, not ignition. Breakeven as defined by Lawson is when the electrical energy produced exceeds the energy required to produce the fusing system, which does not require ignition to be satisfied.

Some fusion schemes such as Magnetic Confinement Fusion (MCF), exemplified by the tokamak approach, aim to achieve high values of τ at moderate densities n_i . Inertial Confinement Fusion (ICF) aims to compress DT to high n_i , so that it is not necessary to achieve such high values of τ . Unlike MCF which applies a magnetic field to generate a high τ value, ICF relies on the inertia of the DT plasma and does not apply additional confinement techniques. This means the maximum τ value achievable is of the order of R/c_s , the timescale for hydrodynamic expansion of the system, where R is the size of the (spherical) plasma, and c_s is the sound speed. The $n_i\tau$ parameter therefore becomes $\rho R/m_i c_s$, where ρ is the mass density and m_i is the average ion mass of the DT plasma. A simple calculation of the ρR value required in ICF is carried out by Lindl [4] by writing the rate of change of the fraction of DT which has been burnt as

$$\frac{d\varphi}{dt} = \frac{n_i}{2} (1 - \varphi)^2 \langle \sigma v \rangle, \quad (1.1.6)$$

which can be time-integrated, assuming constant n_i and $\langle \sigma v \rangle$ over the burn duration, to give the total burnup fraction

$$\varphi = \frac{n_i \tau}{n_i \tau - 2/\langle \sigma v \rangle} = \frac{\rho R}{\rho R + 2m_i c_s / \langle \sigma v \rangle}. \quad (1.1.7)$$

Using a temperature of 20 keV, for which equation (1.1.7) agrees well with simulations [4], equation (1.1.7) suggests that a ρR value of 0.3 g/cm^2 is required for 33% burnup. This leads to required mass densities of around 400 g/cm^3 , since the density required to achieve a given ρR scales with $1/M^2$ and masses as low as milligrams must be used in order to keep the yield below laboratory-safe levels. A high areal density ρR is also desirable for the confinement of alpha particle fusion products. Intuitively, one may imagine that an alpha particle interacts with the particles within a cylinder of fixed cross-section as it travels radially outward through the system. The number of particles within that cylinder is proportional to ρR .

The high compression required to generate laboratory-scale yields can be achieved by imploding a spherical DT capsule using a radiation source, commonly either laser beams directly incident on the surface of the capsule ('direct-drive') or first converted to X-rays by interaction with the inside of a high-Z container called the hohlraum ('indirect-drive'). The indirect-drive approach results in a more uniform radiation drive due to the thermal nature of the X-ray field, and allows the energy to be deposited onto the capsule at a higher density due to the higher critical density of the higher-frequency radiation. For these reasons, the indirect-drive approach is probably the most active area of research at present, and is the focus of the two major experimental facilities, the National Ignition Facility (NIF) and the Laser Megajoule (LMJ). Fast- and shock-ignition schemes, in which the compression and heating of the target are achieved by separate processes, are naturally given to the direct-drive approach, and may become the focus of research in the future if the conventional approach to ignition proves too difficult.

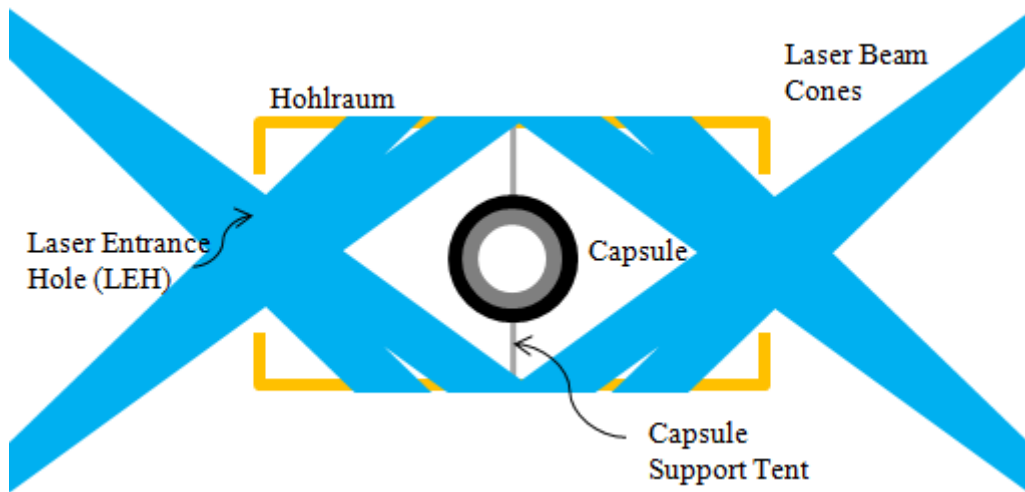


Figure 1.1: Schematic diagram of the hohlraum with laser beams, capsule and support tent shown. Not shown: capsule fill tube, diagnostic windows in the hohlraum, hohlraum gas fill, and LEH 'storm windows'.

The original scheme for ICF [5] involved simultaneous ignition of the entire fuel mass, but it was soon realised that it is more efficient to only heat the centre of the fuel and allow the burn to propagate outward through a dense fuel layer from a central region called the 'hotspot' [6]. The propagation of the burn wave is discussed in detail in Chapter 5. At first the idea was for the hotspot to be formed at the same density as the rest of the capsule. A more efficient configuration is for the hotspot to be formed at the same pressure as the rest of the capsule, with a lower density and higher temperature [7]. A schematic diagram of this ideal

configuration is shown in Figure 1.2. This configuration saves energy because less mass needs to be raised to the ignition temperature, and it increases the confinement time because the ideal ignition hotspot configuration has no pressure gradients to cause fluid expansion. For this reason, ignition capsules are designed with a low-density DT gas fill in the centre, contained within a layer of solid DT, which is surrounded by an ‘ablator’ layer designed for efficient radiation absorption. These targets must be cryogenically cooled to 18 K in order to form the solid DT layer, which is a time-consuming process. Many experiments are carried out with non-cryogenic targets which allow investigation of the physics of the implosion and the production of neutron and X-ray sources with a simpler experimental design.

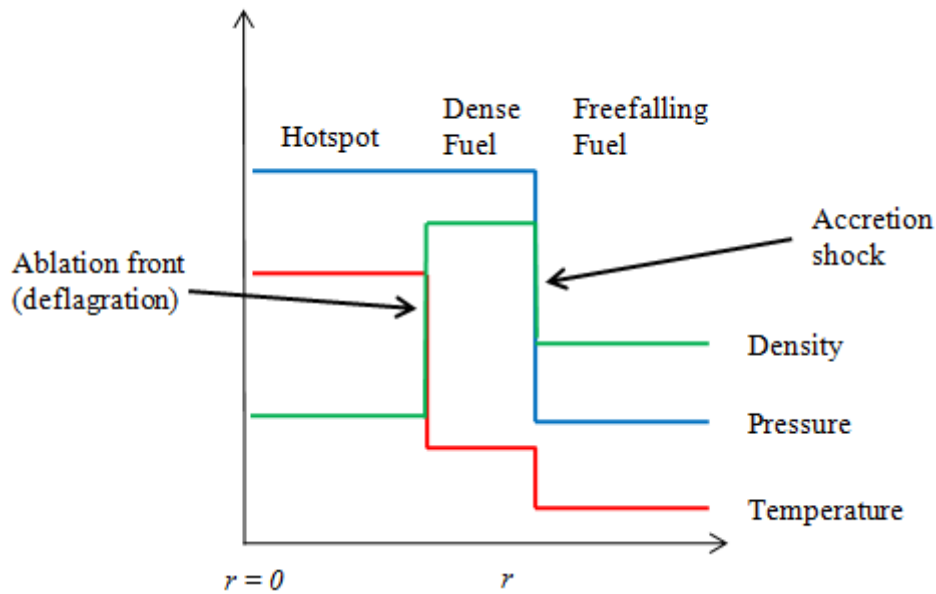


Figure 1.2: Schematic diagram of the ideal isobaric ignition configuration

The implosion of cryogenic ignition targets proceeds as follows. In the initial ‘ablation’ phase, the majority of the target is stationary while an ablation front propagates inward from its surface, driven by the radiation drive. A shock moves ahead of the ablation front, accelerating and compressing the shell as it does so. This shock is driven at a relatively low intensity in order to minimise preheat of the shell. Preheating the shell reduces its compressibility and wastes energy heating it that could be spent compressing it. During the ‘acceleration’ phase, several sudden rises in the radiation temperature drive a series of subsequent shocks through the shell. This also minimises shell heating, since the combined Hugoniot of several weak shocks more closely approximates an isentrope than that of a single strong shock. The pressure jump across these shocks is just below the strong shock limit, to maximise compression for minimum heating. The shocks are

timed to combine into a single strong shock just inside the inner surface of the shell, in order to heat the gas fill. At this point the shell is at its thinnest because it begins to expand inward once the shocks enter the gas fill. The ratio of the shell thickness to the target radius at this point is known as the In Flight Aspect Ratio (IFAR). After ramping to peak intensity, the radiation drive continues until all available laser energy has been deposited into the capsule. After this time, the drive switches off and if the combined shock has not reached the centre the capsule enters the ‘coast’ phase, in which the accelerated material is ‘freefalling’ inward. Once the shock hits the centre it rebounds and the ‘deceleration’ or ‘stagnation’ phase begins, in which the reflected shock decelerates the inward-moving fuel. The reflected shock can be thought of as an accretion, since the material it has passed through is stagnated in the centre at a relatively low velocity, and the freefalling fuel is collecting on the outside of this stagnated material.

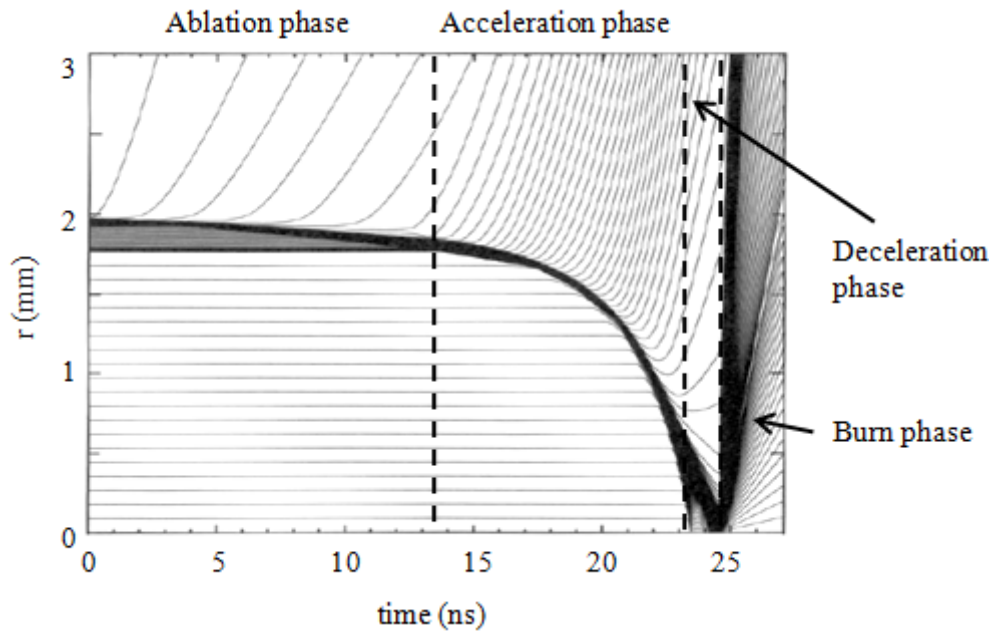


Figure 1.3: (r, t) diagram of a direct-drive implosion to illustrate the various phases. This direct-drive target begins about twice the radial size of the indirect-drive targets used on the NIF, although the time scale is approximately the same. The lines show the trajectories of Lagrangian cells (fluid particles). Reproduced from [8] with modifications.

1.2) Hotspot self-heating and the beam energy required for ignition

The hotspot parameters required to achieve the self-heating condition (1.1.1) can be ascertained by considering the energy source terms acting on the assembled hotspot. Besides the alpha heating rate W_α , the other three main energy source terms acting on the hotspot are the bremsstrahlung radiation emission rate W_r , the thermal conduction rate W_{TC} and the rate of mechanical work done on the hotspot W_m . An analysis of these source terms is outlined by Lindl [4], primarily following the analysis of Widner [1]. W_m is calculated by assuming the fuel compresses the hotspot like a piston moving with velocity u_R :

$$W_m = -PSu_R = -8\pi R^2 \frac{e}{m_i} \rho T u_R \quad (1.2.1)$$

where the negative sign accounts for the fact that, when the hotspot is being compressed, the sign of the radial velocity is negative but the work rate is positive. W_α and W_r are calculated by integrating the volumetric rates over a uniform hotspot, assuming instantaneous local deposition of alpha particle energy and no reabsorption of radiation:

$$W_\alpha = A_\alpha \rho^2 \langle \sigma v \rangle (T) V \quad (1.2.2)$$

$$W_r = A_r \rho^2 T^{1/2} V \quad (1.2.3)$$

with $A_r = 9.615 \times 10^{14} \text{ J m}^3 \text{ kg}^{-2} \text{ s}^{-1} \text{ keV}^{-1/2}$ and $A_\alpha = E_\alpha / 4m_i^2$. In [1] and [8], W_{TC} is calculated² by approximating the ‘thermal scale-length’ $L = c_{TC} R / T$, where c_{TC} is a constant close to unity, giving

$$W_{TC} = \kappa \frac{S}{L} = \frac{A_{TC} S}{\ln \Lambda L} T^{5/2} = \frac{A_{TC}}{\ln \Lambda} c_{TC} 4\pi R T^{7/2} \quad (1.2.4)$$

² Lindl [4] provides an alternative derivation by assuming the heat flux $Q \propto r$, which he gets by setting the thermal conduction loss rate equal to the other source terms. This produces a parabolic temperature profile $T = T_c (1 - r^2/r_f^2)^{2/7}$ with a temperature gradient given by $dT/dr = (4/7)T_c (r/r_f^2) (1 - r^2/r_f^2)^{-5/7}$, which is undefined at the hotspot edge r_f . However, since $T^{5/2} \propto (1 - r^2/r_f^2)^{5/7}$, the product $T^{5/2} dT/dr$ is defined at the hotspot edge, and allows the heat flux at $r = r_f$ to be calculated. The real hotspot temperature does not go to 0 at the hotspot edge, so it may be argued that the hotspot boundary should be taken as a surface with a finite temperature, located at some $r = R < r_f$. The heat flux through this surface may be calculated analytically, mathematically resembling the result for the model profile used in Chapter 3. This calculation is described in Appendix B.

where $S = 4\pi R^2$ is the hotspot surface area and $\kappa = A_{TC} T^{5/2} / \ln \Lambda$ is the thermal conductivity. Dividing each of these by R provides a hotspot heating condition in terms of three parameters, ρR , T and u_R :

$$(\rho R)^2 (A_\alpha \langle \sigma v \rangle - A_r T^{1/2}) / 3 - 2e(\rho R) T u_R / m_i - A_{TC} c_{TC} T^{7/2} / \ln \Lambda > 0 \quad (1.2.5)$$

The value of u_R for which this condition is satisfied is shown as a function of the hotspot ρR and T in Figure 1.4. The hotspot ignites in the yellow/red region, for which $u_R > 0$ and the hotspot does not require compressive work to have a positive heating rate. When the shell must compress the hotspot ($u_R < 0$) in order to achieve a positive heating rate, the hotspot is not ‘self-heating’; the fusion feedback loop is not taking place so ignition cannot be said to have occurred. A result of this analysis is the often-cited rule of thumb for ignition, that the hotspot ion temperature needs to be of the order of 10 keV, and the hotspot ρR must be ≥ 0.3 g/cm³ ($\sim 10^{-0.5}$ g/cm³ for comparison with Figure 1.4). This ρR value happens to be the same as that required for a 33% burnup fraction.

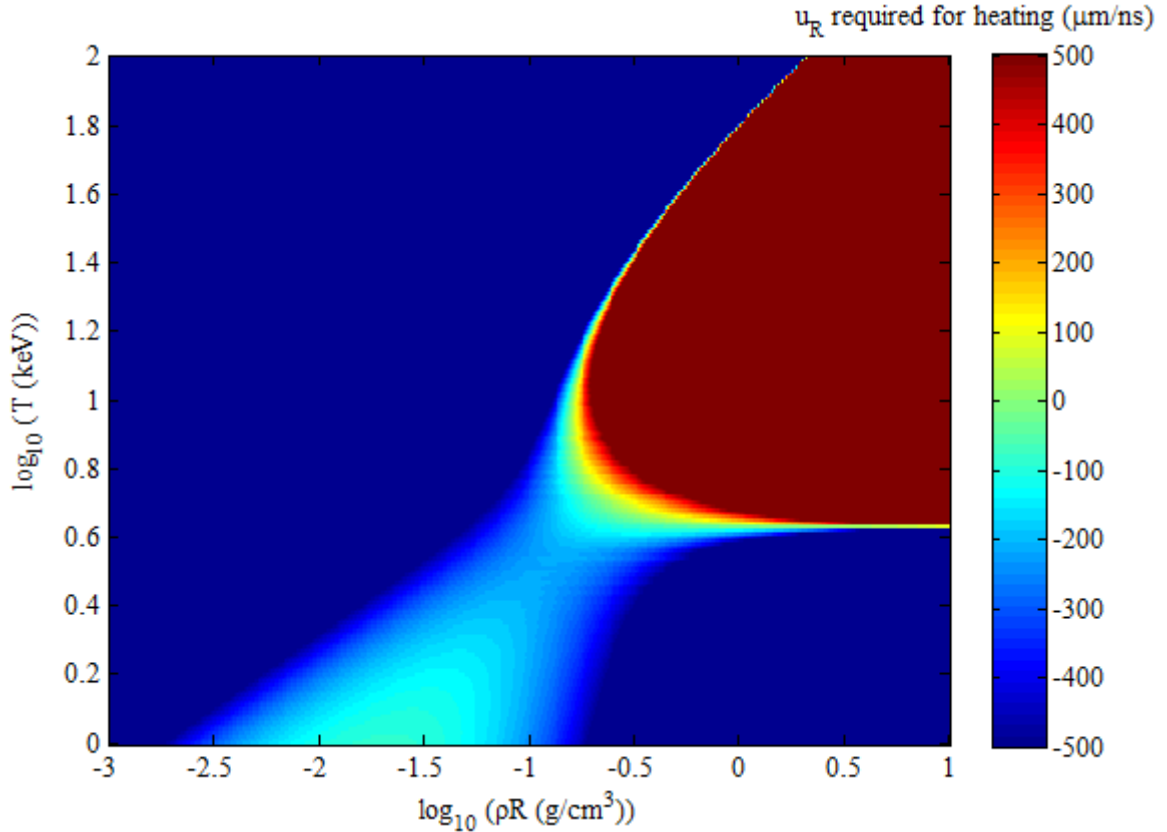


Figure 1.4: Minimum shell radial velocity u_R required for positive rate-of-change of hotspot energy. When the hotspot heats for a positive u_R , self-heating by alpha particle energy deposition is occurring; this constitutes ignition.

Having calculated the hotspot conditions required for ignition, one can find the minimum thermal energy that must be delivered to the hotspot in order to achieve them. A simple argument to calculate this is presented by Murakami and Meyer-ter-Vehn [9]. The total thermal energy of the hotspot at stagnation can be written in terms of the desired hotspot ρR and T according to

$$E \propto (\rho RT)^3 / P^2 \quad (1.2.6)$$

In the isobaric hotspot configuration, the hotspot pressure P is equal to the pressure in the cold fuel. The specific internal energy of the cold fuel ϵ_{fuel} is the Fermi energy multiplied by the adiabat parameter

$$\epsilon_{fuel} \propto \alpha \rho_{fuel}^{2/3} \quad (1.2.7)$$

Assuming all the kinetic energy of the implosion is converted to internal energy of the hotspot at stagnation, the specific internal energy of the fuel at stagnation is given by

$$\epsilon_{fuel} = u_{imp}^2 / 2, \quad (1.2.8)$$

(where u_{imp} is the implosion velocity), which for these purposes should be defined as the peak of E_k / M_{tot} where E_k is the total target kinetic energy and M_{tot} is the total mass. Combining (1.2.8) with (1.2.7), the fuel density at stagnation is

$$\rho_{fuel} \propto u_{imp}^3 \alpha^{-3/2}. \quad (1.2.9)$$

Since the fuel pressure $P \propto \rho_{fuel} \epsilon_{fuel}$, the fuel pressure at stagnation is given by

$$P \propto u_{imp}^5 \alpha^{-3/2} \quad (1.2.10)$$

and by (1.2.6) the thermal energy of the hotspot required to achieve a given ρR and T is given by

$$E \propto (\rho RT)^3 u_{imp}^{-10} \alpha^3, \quad (1.2.11)$$

which suggests an astonishingly high sensitivity to the implosion velocity.

Of course, this very simple derivation omits important details and (1.2.11) does not agree closely with all simulations. A set of computational studies was carried out at the end of the 1990s to establish more

accurately the parameters of the implosion which set the minimum energy required to achieve a given ρR and T , and therefore ignition. Levedahl and Lindl [10] rescaled a set of fluid profiles for a capsule at peak velocity taken from a 1D implosion simulation, in order to generate a database of stagnation-phase simulations to which an ignition energy law of the form $E \propto u_{imp}^a \alpha^b$ could be fit. The rescaling was carried out in a manner that kept the pressure profile unchanged. The ignition energy law produced was $E \propto u_{imp}^{-5.5} \alpha^{1.7}$. Basko and Johner [11] used a different set of peak-velocity profiles, generated from an analytic model of the implosion dynamics, and used a different rescaling procedure to initialise their stagnation-phase simulations. This produced an ignition energy fit much closer to (1.2.11), $E \propto u_{imp}^{-9.1} \alpha^3$. Applying the pressure-preserving scalings of [10] to their analytic profiles, Basko and Johner were able to reproduce the $E \propto u_{imp}^{-5.5} \alpha^{1.7}$ law. Herrmann et al. [12] reconciled the two results by noting that the adiabat relevant to the ignition energy is not the in-flight adiabat α , but the adiabat of the fuel during the stagnation phase, α_{stag} . The dynamics of the stagnation, in particular the accretion shock which increases the internal energy of the fuel it passes through, were found to increase the value of α_{stag} beyond α by a factor related to α , u_{imp} , and the drive pressure P_a at peak velocity. On this basis, a fit was found to a set of 1D simulations which were not rescaled as before, but simulated computationally from the beginning of the implosion using a pressure-drive boundary condition to emulate the effect of the thermal radiation drive. This produced the ignition energy law

$$E(kJ) = 50.8 \alpha^{1.88} \left(\frac{u_{imp}}{300 \text{ } \mu\text{m ns}^{-1}} \right)^{-5.89} \left(\frac{P_a}{100 \text{ Mbar}} \right)^{-0.77}, \quad (1.2.12)$$

or in terms of the stagnation-phase adiabat, $E(kJ) = 2.1 \alpha_{stag}^{2.66} (u_{imp}/300 \text{ } \mu\text{m ns}^{-1})^{-7.21}$. Setting P_a constant, this reproduces the $E \propto u_{imp}^{-5.5} \alpha^{1.7}$ scaling of Levedahl and Lindl [10], and setting $P_a = u_{imp}^5 \alpha^{-3/2}$ the $E \propto u_{imp}^{-9.1} \alpha^3$ scaling of Basko and Johner is recovered. A remarkably simple analytic derivation [13] [14] of the computational result (1.2.12) was discovered shortly after its publication.

The high sensitivity of the beam energy to the implosion velocity (1.2.12) might seem optimistic for ignition. Unfortunately, there is a practical upper limit on the implosion velocity that sets a relatively high ignition beam energy. By considering the implosion as an ablation-driven rocket, Lindl [15] derives an equation for u_{imp} in terms of the radiation temperature, IFAR, and in-flight adiabat

$$u_{imp} \propto \alpha^{3/5} T_r^{0.9} IFAR, \quad (1.2.13)$$

so the limits to the achievable u_{imp} are the adiabat α , the radiation temperature T_r and the in-flight aspect ratio. The adiabat must be kept as low as possible as previously discussed. The radiation temperature is limited by the maximum practical laser intensity, since above laser intensities of the order of 10^{15} W/cm² laser-plasma instabilities begin to severely inhibit the coupling efficiency between the laser and the hohlraum. This also limits the stagnation pressure factor in the ignition energy law (1.2.12). The in-flight aspect ratio is limited (to below ~ 30) because thinner shells have larger density gradients, and therefore larger Rayleigh-Taylor growth rates [8]. As a back-of-the-envelope calculation of the minimum hotspot thermal energy required to ignite an ICF capsule, we take as typical implosion parameters: $T_r = 300$ eV (giving an ablation pressure of 140 Mbar), $u_{imp} = 320$ μ m/ns and $\alpha = 1.4$, leading to a hotspot energy requirement of 50 kJ according to equation (1.2.12). Murakami and Meyer-ter-Vehn [9] calculate the total beam energy required to deliver a given thermal energy to the hotspot in terms of a coupling efficiency η where $E = \eta E_{beam}$. η includes the conversion efficiency from optical laser light to thermal X-rays, the efficiency of transfer of X-ray energy between the hohlraum wall and the target, and the proportion of energy absorbed by the capsule that is lost in the ablated material. Murakami and Meyer-ter-Vehn [9] calculate η to be between 5% and 10%; taking the lower bound of 5% gives a minimum beam energy for ignition of 1 MJ. Both the LMJ and the NIF are capable of producing a laser energy of 1.8 MJ on target, and both are designed with the intention of generating igniting ICF implosions. The LMJ is still under construction at the time of writing, and is expected to be in operation in early 2014. The NIF has been in operation since 2009, with the first cryogenic layered target shot on 29th September 2010. An overview of the effect of target perturbations on the effort to achieve ignition on the NIF will be presented in the next section.

1.3) Sources and role of perturbations in the National Ignition Campaign

The National Ignition Campaign (NIC) ran from the first half of 2009 until the end of 2012 with the aim of producing ignition of an ICF capsule. Fusion yields were lower than expected from simulations and ignition

was not achieved. However, considerable progress was made in the necessary scientific understanding and technical capability to achieve ignition. Various tuning campaigns demonstrated improvements in control of processes of which there was a degree of uncertainty in the known physics, such as shock timing and drive symmetry. Improvements were also made in pre-shot capsule measurements, shot diagnostics, and in understanding the influence on implosion symmetry of hardware such as the capsule support tent and fill tube. The designs of the target, hohlraum, and pulse shape were altered to improve performance. These developments translated into increases in measured yield and areal density ρR of cryogenic ignition-attempt targets. 37 cryogenic layered implosions were carried out in total, and it is estimated that the maximum yield, around 10^{15} neutrons was between 3 and 10 times smaller than that required for ignition. The maximum total ρR measured, around 1.3 g/cm^2 , was large enough for ignition to occur according to simulations. However, the maximum hotspot pressure achieved was 150 Gbar, approximately 40% of the 350 Gbar required for ignition. This splits into a 50% shortfall in hotspot density - 50 g/cm^3 of a required 100 g/cm^3 - and a 15% shortfall in hotspot temperature - 3 keV of a required 3.5 keV [16].

One of the major candidates to explain this discrepancy is larger than expected amplitude of perturbations from spherical symmetry of the target at stagnation. These perturbations are usually split by wavelength into ‘mix’ and ‘shape’ perturbations. Shape usually refers to perturbations with Legendre mode number $l = 2$ or 4 , whereas mix usually refers to short-wavelength perturbations of mode number $l \gtrsim 50$. Perturbations grow by the Richtmeyer-Meshkov (RM) and Rayleigh-Taylor (RT) instabilities, which are unstable on the ablator surface during the ablation/acceleration phase and the ablator/ice interface and inner ice surface (hotspot surface) during the deceleration phase. Some perturbations such as that seeded by the fill tube would also grow without the effect of hydrodynamic instabilities since they generate jets in which the fluid is moving faster than the surrounding fluid [17]. It has also been shown that locally flat regions of the ablator can create jets [18]. Perturbation growth on the outer surfaces is ‘fed through’ to the inner surfaces according to the $\exp(-k|r - r_0|)$ amplitude decay of the RT (where r_0 is the position of maximum instability and k is the wavenumber of the perturbation), which allows the ablation/acceleration phase instabilities to seed the deceleration phase instability. This effect is stronger for longer-wavelength perturbations.

Shape-scale perturbations are caused primarily by asymmetry of the radiation drive. Each half of the hohlraum is illuminated by four cones of laser beams, split into the outer cones at 50° and 44.5° and inner at 30° and 23.5° , as measured from the hohlraum axis. An imbalance in the power of these beams causes long-wavelength perturbations, in particular $l = 2$ (designated ‘P2’) and $l = 4$ (designated ‘P4’) modes. The cylindrical and mirror symmetry of the hohlraum ensures odd-numbered modes or modes with a finite m number are probably much smaller in amplitude. An exception to this is the $l = 1$ (designated ‘P1’) mode, which can be seeded by an effect that occurs during the formation of the DT ice layer in which the ice on one side of the capsule is thicker than at the other. A P4-like perturbation has also been observed resulting from the capsule support tent. 2D X-ray backlighter radiographs taken during the implosion have shown that regions where the tent makes contact with the capsule implode faster than other regions, as if the contact points cause the formation of jets. The effect of shape-scale perturbations on the stagnated hotspot is generally considered to be a decreased efficiency in the conversion of kinetic energy to thermal energy [19]. This is confirmed to be the case for slightly shorter-wavelength perturbations in Chapter 4, and it is also found that a significant increase in thermal conduction losses can result from the increased hotspot surface area.

P2 and P4 perturbations caused by beam imbalance can be minimised by tuning the relative intensities of the laser cones, which allows exchange of energy between the cones, thereby tuning the angular distribution of the radiation field on the target [20]. The uniformity of the radiation field is tuned empirically using three types of surrogate capsule. ‘Re-emit’ capsules are made from bismuth, which emits enough radiation when illuminated by the X-ray drive that the distribution of X-rays on the capsule surface during the first few nanoseconds can be diagnosed. This allows tuning of the early part of the drive (particularly the ‘picket’), which sets the symmetry of the first shock. The magnitude of P2 and P4 asymmetries of the shocks can also be diagnosed and tuned using three-axis ‘keyhole’ targets, which consist of a plastic sphere with a gold cone placed with its apex facing the inside surface of the plastic. The sphere and cone are filled with D_2 to simulate the dynamics of the DT ice layer and a VISAR beam is directed down the axis of the cone to measure the shock velocity by reflecting off the shock surface once the shock is within the D_2 [21]. Since the shock speed increase caused by the coalescence of shocks can be measured as a fringe shift in the VISAR trace, the relative timing of shocks can be measured. Initially measured down a single line-of-sight, the

surrogate was upgraded for two and then three lines-of-sight, enabling the relative shock timings at various capsule locations to be detected. Symmetry capsules ('symcaps'), replace the DT ice layer with an equivalent mass of plastic in order to reduce the time required to prepare the shot, and are used to tune the symmetry at stagnation time using X-ray self-emission images as diagnostics. However, it has been shown using simulations that targets may have P2 and P4 perturbations at stagnation yet produce self-emission images that appear round [19]. Despite the improvements in symmetry made by tuning campaigns, shape perturbations are still considered an outstanding barrier to ignition [22].

Mix-scale perturbations are seeded by surface roughness, manufacturing defects such as dust and bumps on the ablator [23], grooves on the inner surface of the DT ice layer [24], and potentially by the fill tube [25]. One of the ways mix can degrade target performance is by injecting ablator dopants into the hotspot, which greatly enhance radiation loss from the hotspot due to their relatively high Z . Ablator mix can be diagnosed by X-ray spectroscopy and by the presence of bright spots in X-ray self-emission images [26]. An anticorrelation between ablator mix and yield has been observed in layered DT implosions [16]. Ablator mix is considered the probable cause of low yield on some shots, but cannot explain the discrepancy between simulated and measured yield on other shots since it brings the hotspot temperature and size below the measured values [22]. This effect of ablator mix is beyond the scope of this thesis, since the entire capsule is modelled as DT.

The effect of cold DT mixing with the hotspot can also reduce yield. This is variously called 'cold fuel mix' or 'thermal mix' when mentioned in the literature. This effect exists for perturbations of any wavelength, but becomes more severe as perturbation wavelength decreases. For a single, short-wavelength perturbation mode, the effect of cold fuel mix is to reduce the hotspot radius to the 'clean radius' R_{clean} which is defined as the distance from the hotspot centre to the spike tips. This concept can be extended to longer wavelengths by considering an 'effective radius' R_{eff} , which is larger than R_{clean} because the hotspot material located between the cold fuel spikes is hot enough to contribute to the yield. Kishony and Shvarts [27] derive ignition conditions as a function of the mode number l and perturbation amplitude ξ , and define R_{eff} in terms of the ratio of the minimum areal densities required for ignition between 1D and perturbed hotspots,

$$\frac{R_{eff}}{R_{1D}} = \frac{(\rho R)_{min}^{1D}}{(\rho R)_{min}^{l,\xi}} \quad (1.3.1)$$

The analysis is extended to a perturbation consisting of multiple mode numbers by defining an ‘effective amplitude’ ξ_{eff} of the multi-mode perturbation which is a quadrature sum of the mode amplitudes of the individual modes. This analysis is carried out on the basis of a 2D self-similar analysis of perturbed targets and validated against 2D simulations. The ‘Kishony-weighted’ effective hotspot radius is included in the metric used to assess the distance from the ignition cliff of a given capsule implosion, the Ignition Threshold Factor (ITF). The ITF is the result of a fit to a large database of 2D simulations [28], and is proportional to $(1 - 1.2\Delta R/R)^4$. The $\Delta R/R$ parameter is an average of the RMS deviation of the hotspot shape, weighted by l according to Kishony and Shvarts’ analysis of $R_{eff}(l)$, and therefore parameterises the effect of cold-fuel mix on target performance.

For a broader overview of the status of and plans for ignition-relevant experiments on the NIF, the reader is referred to the three companion review papers published in June 2011 [28] [29] and [30], the document produced by the May 2012 workshop on the *Science of Fusion Ignition on NIF* [31], and a talk entitled *Progress Towards Ignition on the NIF* given in February 2013 [16]. A complete journal of activities on the NIF, from the beginning of the NIC to its end and beyond, is available on the NIF website [32], with many links to relevant papers.

1.4) Description of the thesis

As described above, perturbations from spherical symmetry of the interface between the hotspot and cold DT fuel are an important barrier to generating a self-heating ICF system. Target designs which ignite in 1D simulations are found not to ignite in the experiment. Part of the explanation is that the energy coupled into the hotspot is reduced by various effects to do with uncertainties in the 1D physics of the drive and the implosion dynamics, but the best available methods to account for these effects still produce 1D simulated yields significantly higher than produced experimentally. Multidimensional simulations suggest large perturbations are present in cryogenic layered implosions, and it is desirable to understand in detail the causes and extent of the hotspot thermal energy reduction they cause. In this thesis, 3D simulations of stagnating ICF

capsules are used to investigate the effects of perturbations on the energy balance of the hotspot, resulting in a full phenomenological description of how perturbations modify the source terms throughout the stagnation. The analysis requires accurate formulae relating the energy source terms to the properties of the hotspot, which are derived for unperturbed hotspots with radially-dependant fluid properties. This is an improvement on the classic Widner-type uniform-hotspot model discussed in Section 1.2 and provides more accurate, although similarly intuitive, forms of the source terms. Modifications made to the hydrodynamics code Gorgon necessary to perform the stagnation-phase simulations, and the generation of the initial conditions, are described in Chapter 2. The energy balance study as a whole is presented in Chapter 3.

Although perturbation amplitudes are expected to be large from multidimensional simulations, experimental diagnosis of the perturbation amplitude at stagnation is difficult to achieve. Diagnosing 0D properties of the stagnated target is simpler, although interpreting these diagnostics in terms of interesting parameters such as the hotspot radius or temperature requires conversion from the experimentally measureable quantities into the useful parameters desired. By generating synthetic diagnostics from stagnation-phase simulations, it is possible to relate the physical configuration of the system to the diagnostic measurables produced. When this is carried out, it is found that perturbations large enough to significantly reduce the yield are not clearly visible on any existing stagnation-phase diagnostic for mode numbers $l \gtrsim 4$, or are able to alias as perturbations of mode number 4 or below. Some of the ways in which diagnostic measures of 0D parameters can be misinterpreted are also identified by the method of producing synthetic diagnostics from simulations. This is the subject of Chapter 4.

In the sub-ignition targets discussed in Chapter 3, fusion energy production is not a major contributor to the hotspot energy balance, and it is sufficient to model alpha particle energy deposition as occurring instantaneously at the location the alpha particles are produced. In igniting targets, and in ‘marginally igniting’ targets which are non-igniting but generate sufficient fusion energy to influence the hydrodynamics, any alpha particle energy transported out of the hotspot effectively becomes an energy loss term. This is investigated in Chapter 5 using a hybrid model of the burning plasma in which the alpha species is treated fully kinetically. The model is used to investigate the role of alpha particle deposition on the dynamics of burning targets, by comparison with other alpha particle transport models that omit various elements of the alpha particle kinetics.

By generating an empirical relationship between the hotspot properties and the fraction of alpha particle energy deposited within the hotspot, the analytic energy balance model of Chapter 3 is modified to include the effects of alpha particle transport. By postprocessing perturbed 3D simulations, it is shown that a perturbed hotspot shape can reduce the fraction of alpha particle energy the hotspot retains, which constitutes another mechanism of energy loss by perturbations.

Chapter 2

Implementing Stagnation-Phase ICF Calculations in Gorgon

This chapter describes the hydrodynamics code, Gorgon, used for the simulations in this work, modifications to it that were necessary to simulate the stagnation phase, and the initial conditions used for the simulations. The simulations begin at the start of the stagnation phase to avoid modelling the radiation drive. Modifications to Gorgon's thermal-conduction solver were necessary to deal with the high thermal conductivities and small spatial scales of the stagnation phase. A finite spatial-grid velocity was also implemented, enabling the grid to track the trajectory of the converging sphere and maintain good spatial resolution. Since the simulations were initialised part-way through the implosion, it was necessary to generate initial conditions that lead to realistic stagnation-phase dynamics. The radial profiles were taken from 1D simulations of NIF targets performed with LLNL's Hydra code [33], with a velocity-field perturbation applied to generate 3D structure. This initial perturbation seeds the deceleration-phase Rayleigh-Taylor instability.

2.1) Gorgon

The simulations in this work are performed using Gorgon, a 3D Eulerian hydrocode. The plasma is assumed to be a fully-ionised Maxwellian ideal gas at this stage of the implosion, and the ions and electrons are treated

as separate species. A Cartesian grid is used for the simulations in this work, which is appropriate for stagnating plasmas because the centre points of spherical or cylindrical grids are difficult to deal with computationally. A common solution to this is to model a wedge-shaped section of the system, not including the centre point. This approach imposes rotational symmetry on the target in such a way that momentum reaching the centre is always met by equal and opposite momentum from the opposite side of the system. Two-dimensional grids suffer a similar imposed symmetry problem. When modelling the global dynamics of a stagnating ICF capsule, it is desirable to avoid any artificial symmetry by modelling all 4π steradians of the capsule, in 3D. Gorgon's Eulerian grid helps with this, as a Lagrangian grid may become highly distorted in the convergent, perturbed flow at stagnation. In the case that global electromagnetic fields in the system are assumed negligible, Gorgon solves the following fluid equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.1.1)$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla (P_i + P_e) \quad (2.1.2)$$

$$\frac{\partial \varepsilon_i}{\partial t} + \nabla \cdot (\varepsilon_i \mathbf{v}) = -P_i \nabla \cdot \mathbf{v} + \nabla \cdot (\kappa_i \nabla T_i) + \frac{dW_{ie}}{dV} + F_i \frac{dW_\alpha}{dV} \quad (2.1.3)$$

$$\frac{\partial \varepsilon_e}{\partial t} + \nabla \cdot (\varepsilon_e \mathbf{v}) = -P_e \nabla \cdot \mathbf{v} + \nabla \cdot (\kappa_e \nabla T_e) - \frac{dW_{ie}}{dV} - \frac{dW_r}{dV} + F_e \frac{dW_\alpha}{dV} \quad (2.1.4)$$

dW_{ie}/dV is the volumetric equilibration rate between the electron and ion species. For non-igniting simulations, fusion energy deposition is assumed to occur instantly at the location that the reaction occurs. Neutron energy deposition is not included. The volumetric rate of fusion energy generated in the form of alpha particles is

$$\frac{dW_\alpha}{dV} = E_\alpha \frac{\rho^2}{4m_i^2} \langle \sigma v \rangle (T_i) \quad (2.1.5)$$

$E_\alpha \sim 3.52$ MeV is the kinetic energy per DT fusion alpha. $\langle \sigma v \rangle (T_i)$ is the DT fusion reactivity; the Bosch-Hale reactivity [34] is used. F_i and F_e are the fraction of alpha particle energy deposited into ions and electrons respectively; the Fraley [6] approximation

$$F_i = \frac{1}{1 + 32 \text{ keV}/T_i} \quad (2.1.6)$$

is used, with $F_e = 1 - F_i$. Alpha particle transport is discussed, and a more sophisticated model used, in Chapter 5.

The volumetric radiation loss term dW_r/dV is calculated using an escape probability model in which the emission from a particular cell is based on the probability of a photon escaping unabsorbed from it and its neighbours. The probability of a photon escaping from cell (i, j, k) is [35]:

$$P_{i,j,k} = \exp\left(-\frac{2}{3}\Delta x \kappa_P\right). \quad (2.1.7)$$

$2\Delta x/3$ is the Cauchy mean chord length for a cell, which is the mean distance travelled for a photon travelling through the cell on a random trajectory (Δx is the grid spacing). κ_P is the Planck opacity of the cell. The radiation loss from cell (i, j, k) is [35]:

$$\frac{dW_r}{dV} = \frac{\pi \Delta x^2 \sigma T_e^4}{\Delta x^3} (1 - P_{i,j,k}) \frac{1}{6} (P_{i-1,j,k} + P_{i+1,j,k} + P_{i,j-1,k} + P_{i,j+1,k} + P_{i,j,k-1} + P_{i,j,k+1}). \quad (2.1.8)$$

In the limit that cell (i, j, k) is optically thick ($P_{i,j,k} = 0$) and the neighbouring cells are optically thin, W_r tends to the volumetric blackbody emission rate of the cell. If a contiguous region is optically thick, the cells on its surface emit with the correct total blackbody emission rate. In the optically thin limit, the $(1 - P_{i,j,k})$ term in (2.1.8) tends to $2\Delta x \kappa_P/3$. Since the Planck opacity $\kappa_P = j/B$ where j is the free-free Bremsstrahlung rate and B is the blackbody emission, (2.1.8) becomes the volumetric Bremsstrahlung emission rate of an optically-thin emitter in this limit.

In some circumstances the temperature gradient may become high enough that $\kappa \nabla T$ is higher than the product of the thermal velocity and the thermal energy density, $v_{th} \epsilon$. It can be shown [8] that in this situation, the slow-moving particles moving up the density gradient are unable to replace the fast-moving particles moving down it, and the distribution function becomes negative in places, which is unphysical. A heat flux of $v_{th} \epsilon$ is known as the ‘free-streaming limit’, and may be interpreted as the total thermal energy of the plasma moving down the temperature gradient at the velocity v_{th} . Atzeni [8] carries out a perturbation analysis of the

Fokker-Planck equation, finding that the heat flux must remain below $0.1v_{th}\varepsilon$, and cites transport simulations of laser ablation from a sphere in which the heat flux asymptotes to $0.06v_{th}\varepsilon$ for high thermal gradients. In our simulations, this effect is modelled by limiting the thermal conductivity to the harmonic mean of $\kappa\nabla T$ and $0.1v_{th}\varepsilon$, allowing a smooth transition between the two regimes.

Gorgon is designed to be a robust, highly scalable parallel code. This is generally achieved by using explicit finite difference methods, with each term in the fluid equations solved separately at each timestep under the operator-splitting assumption. In the case of the fluid motion solver, this is not a restrictive limitation, as the Courant stability condition requires only that the timestep is the minimum time required for a sound wave to cross any cell, which scales linearly with the grid spacing Δx . However, in the case of the thermal conduction, the fully explicit solution scales as Δx^2 , so it is desirable to find an alternative that does not place a stability constraint on the timestep, but scales as well as the explicit solution. This will be the subject of the next section.

2.2) Alternating-Direction Implicit solution of the thermal conduction equation in Gorgon

The explicit solution to parabolic differential equations may be applied to the thermal conduction terms in the energy equations by writing the LHS in terms of temperature:

$$\frac{\partial \varepsilon}{\partial t} = c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T), \quad (2.2.1)$$

where $c = \partial \varepsilon / \partial T$. This is forward-differenced in time and central-differenced in space to get

$$T_{i,j,k}^{n+1} = T_{i,j,k}^n + \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \left(\kappa_{i+1/2,j,k}^n (T_{i+1,j,k}^n - T_{i,j,k}^n) - \kappa_{i-1/2,j,k}^n (T_{i,j,k}^n - T_{i-1,j,k}^n) \right) + \dots \quad (2.2.2)$$

where the timestep number is indicated by n and the x, y, z spatial grid position is indicated by i, j , and k respectively. The ellipsis indicates that the spatial differencing in the y and z directions takes the same form as in the x -direction. To ensure energy conservation one must edge-difference the thermal conductivity so that

the flux into and out of a cell through the same boundary is equal. The stability criterion for this method in three dimensions is [36]:

$$\Delta t \leq \frac{\Delta x^2}{6 \max(\kappa/c)}. \quad (2.2.3)$$

This is limited by the maximum value of κ/c across the entire system, which can reach values of around $20 \text{ m}^2/\text{s}$ in ICF stagnation simulations. The maximum timestep can therefore be as low as $8 \times 10^{-15} \text{ s}$ for a $1 \text{ }\mu\text{m}$ grid spacing, resulting in the need to subcycle the thermal conduction solver hundreds of times per hydro timestep. Fully implicit methods, such as the Euler implicit method or the Crank-Nicholson method, would potentially be an improvement as they are unconditionally stable and do not require subcycling. However, implementing a fully implicit method in Gorgon would require inverting a sparse matrix formed from the entire spatial grid, which would limit the code's parallel scalability. Discussed below are two alternatives which scale well, but were found to have other limitations.

The DuFort-Frankel method uses cell (i, j, k) 's temperature value at the previous timestep, replacing $T_{i,j,k}^n$ with $T_{i,j,k}^{n-1}$ and Δt with $2\Delta t$ in (2.2.2), resulting in

$$T_{i,j,k}^{n+1} = T_{i,j,k}^{n-1} + \frac{2\Delta t}{c_{i,j,k}^n \Delta x^2} \left(\kappa_{i+1/2,j,k}^n (T_{i+1,j,k}^n - T_{i,j,k}^{n-1}) - \kappa_{i-1/2,j,k}^n (T_{i,j,k}^{n-1} - T_{i-1,j,k}^n) \right) + \dots \quad (2.2.4)$$

This produces a theoretically stable explicit method, but in practice the implementation of the method presents complications that limit its robustness. Firstly, the advection of the fluid interferes with the routine by altering the temperature between timesteps. If cells $(i-1, j, k)$, (i, j, k) and $(i+1, j, k)$ are at 200 eV at timestep $n-1$ and advection raises the temperature of cell (i, j, k) to 400 eV at timestep n , cells $(i-1, j, k)$ and $(i+1, j, k)$ will use the value of $T_{i,j,k}^n = 400 \text{ eV}$ and will gain energy, but cell (i, j, k) will use the value of $T_{i,j,k}^{n-1} = 200 \text{ eV}$ and will not lose energy. This problem can be overcome by treating the temperature at the previous timestep, $T_{i,j,k}^{n-1}$, as if it is an intrinsic property of the fluid and advecting it during the advection step. Secondly, the method assumes the timestep is constant. In practice, the total time between steps $n-1$ and $n+1$ is $\Delta t^{n-1/2} + \Delta t^{n+1/2}$, so this must be used in the place of $2\Delta t$ in (2.2.4). The properties of the method change as timestep n becomes less equidistant from steps $n-1$ and $n+1$. As $\Delta t^{n+1/2}/\Delta t^{n-1/2} \rightarrow 0$, $T_{i,j,k}^{n+1}$

is at approximately the same position in time as $T_{i-1,j,k}^n$ and $T_{i+1,j,k}^n$, so the method tends to the Euler implicit method. As $\Delta t^{n-1/2}/\Delta t^{n+1/2} \rightarrow 0$, $T_{i,j,k}^{n-1}$ is at approximately the same position in time as $T_{i-1,j,k}^n$ and $T_{i+1,j,k}^n$, so the method tends to the classic explicit method, which as mentioned above is only conditionally stable. Finally, the temperature $T_{i,j,k}^{n+1}$ only depends on the temperatures in the neighbouring cells, so a high-temperature region a few cells away will not influence the value of $T_{i,j,k}^{n+1}$ no matter how large the timestep; this effectively sets a limit on the propagation speed of thermal waves.

The Saul'yev method is an ‘asymmetric’ semi-implicit method. $T_{i-1,j,k}^n$ in (2.2.2) is replaced with the value at timestep $n + 1$. Since the boundary condition $T_{0,j,k}^{n+1}$ is known by definition, the Saul'yev equation for $T_{1,j,k}^{n+1}$ only contains one unknown, as with an explicit method. Once $T_{1,j,k}^{n+1}$ has been calculated, $T_{2,j,k}^{n+1}$ can be calculated, and so on iterating forward across the grid. The method can also be formulated with $T_{i+1,j,k}^n$ replaced with the value at $n + 1$ instead and the iteration proceeding backward across the grid. Both of these methods are unconditionally stable. Propagating information forward or backward across the grid means that distantly separated cells are able to influence each other for any size of timestep, unlike in the DuFort-Frankel method. However, using the forward or backward versions alone results in a one-way transfer of information. It is possible to combine the two methods by performing the forward method for half a timestep, followed by the backward method for the other half. As well as allowing distant transfer of information in both directions, this combination of the two methods can be shown to be second-order accurate in time, whereas either method used alone is only first-order accurate. In the three-dimensional implementation one of the two neighbouring cells in each dimension is replaced with the value at $n + 1$. This results in eight distinct methods (each beginning at a different corner of the grid) which must be implemented for $1/8^{\text{th}}$ of a timestep each. This 3D method retains unconditional stability and second-order accuracy. It is also relatively simple to parallelise and scales well with processor number. Unfortunately, the Saul'yev method does not conserve energy. During the forward iteration, the flux from cell (i, j, k) into $(i - 1, j, k)$ is calculated based on $T_{i-1,j,k}^{n+1}$ and $T_{i,j,k}^n$, but the flux from cell $(i - 1, j, k)$ into (i, j, k) is calculated based on $T_{i-1,j,k}^n$ and $T_{i,j,k}^n$. As a result, a different amount of energy is deposited in cell (i, j, k) than is removed from cell $(i - 1, j, k)$, so energy is not conserved. This is not fixed by combining the backward and forward methods.

A satisfactory solution was found in the form of an Alternating Direction Implicit (ADI) method. The ADI used has all the advantages of the Saul'yev method and can be made to operate in a similar manner, but is not asymmetric and therefore conserves energy. Alternating-Direction Implicit (ADI) methods finite-difference the model equation implicitly in one dimension and explicitly in the other two. The timestep is split into three substeps, with the implicit dimension changed each substep. Depending on the particular difference method used, this can lead to unconditional stability and high order accuracy. For the solution of the thermal conduction equation in Gorgon, we use the Douglas-Brian method [37], [38], which is unconditionally stable and second-order accurate. The Douglas-Brian method as applied to (2.2.1) can be written:

$$T_{i,j,k}^* = T_{i,j,k}^n + \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \left[\frac{1}{2} \delta_i^2(T^*) + \frac{1}{2} \delta_i^2(T^n) + \delta_j^2(T^n) + \delta_k^2(T^n) \right], \quad (2.2.5)$$

$$T_{i,j,k}^{**} = T_{i,j,k}^* + \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \left[\frac{1}{2} \delta_j^2(T^{**}) - \frac{1}{2} \delta_j^2(T^n) \right], \quad (2.2.6)$$

$$T_{i,j,k}^{n+1} = T_{i,j,k}^{**} + \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \left[\frac{1}{2} \delta_k^2(T^{n+1}) - \frac{1}{2} \delta_k^2(T^n) \right], \quad (2.2.7)$$

where the operator δ_i^2 represents a central difference of the thermal flux gradient:

$$\delta_i^2(T^n) = \kappa_{i+1/2,j,k}^n (T_{i+1,j,k}^n - T_{i,j,k}^n) - \kappa_{i-1/2,j,k}^n (T_{i,j,k}^n - T_{i-1,j,k}^n) \quad (2.2.8)$$

and similarly for δ_j^2 and δ_k^2 . $T_{i,j,k}^*$ and $T_{i,j,k}^{**}$ are intermediate values of $T_{i,j,k}^n$, and do not correspond to the physical temperature at any point in time. For a one-dimensional system, the Douglas-Brian method reduces to the Crank-Nicholson method, which applies half an implicit and half an explicit difference. Steps two and three can be considered ‘implicit correction steps’ in which half of the explicit contribution to the original step is subtracted, and a new implicit contribution is added.

Each step of the ADI contains a tridiagonal matrix which must be inverted. For example, step 1 may be written:

$$a_i T_{i-1,j,k}^* + b_i T_{i,j,k}^* + c_i T_{i+1,j,k}^* = d_i \quad (2.2.9)$$

where

$$a_i = -\frac{1}{2} \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \kappa_{i-1/2,j,k}^n \quad (2.2.10)$$

$$b_i = 1 + \frac{1}{2} \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} (\kappa_{i+1/2,j,k}^n + \kappa_{i-1/2,j,k}^n) \quad (2.2.11)$$

$$c_i = -\frac{1}{2} \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \kappa_{i+1/2,j,k}^n \quad (2.2.12)$$

$$d_i = T_{i,j,k}^n + \frac{\Delta t}{c_{i,j,k}^n \Delta x^2} \left(\frac{1}{2} \delta_i^2(T^n) + \delta_j^2(T^n) + \delta_k^2(T^n) \right). \quad (2.2.13)$$

This system is solved using the Thomas algorithm, which inverts a tridiagonal matrix of size N in $5N - 4$ operations. The algorithm consists of a ‘forward sweep’ which replaces the c_i elements with

$$c'_i = \begin{cases} \frac{c_1}{b_1} & i = 1 \\ \frac{c_i}{b_i - c'_{i-1} a_i} & N \geq i > 1 \end{cases} \quad (2.2.14)$$

and the d_i elements with

$$d'_i = \begin{cases} \frac{d_1}{b_1} & i = 1 \\ \frac{d_i - d'_{i-1} a_i}{b_i - c'_{i-1} a_i} & N \geq i > 1 \end{cases}, \quad (2.2.15)$$

and a ‘backward sweep’ which obtains the $T_{i,j,k}^*$ values:

$$T_{i,j,k}^* = \begin{cases} d'_N & i = N \\ d'_i - c'_i T_{i+1,j,k}^* & N > i \geq 1 \end{cases}. \quad (2.2.16)$$

As well as requiring relatively few operations, the Thomas algorithm has the advantage that it is simple and efficient to parallelise. Gorgon’s $N_x \times N_y \times N_z$ cell spatial grid is split into equal subdomains of size $M_x \times M_y \times M_z$ with each subdomain assigned to a different process. We refer to each subdomain/process by its coordinate (I_x, I_y, I_z) within the global grid (Figure 2.1(a)). Taking the first step of the Douglas-Brian ADI as an example, the forward sweep of the Thomas algorithm must be carried out along all N_x cells before the backward sweep can begin. Therefore when the forward sweep reaches the boundary of subdomain I_x , process

I_x must pass $c'_{I_x M_x}$ and $d'_{I_x M_x}$ to process $I_x + 1$. Process I_x must then wait for processes $I_x + 1 \dots N_x/M_x$ to complete the forward and backward sweeps before it can begin the backward sweep. On the backward sweep, $T^*_{I_x M_x + 1, j, k}$ must be passed from process $I_x + 1$ to process I_x . For a one-dimensional grid with $N_y/M_y = 1, N_z/M_z = 1$, process I_x is idle during this time, and the algorithm is no faster than on a single processor. However, for a two- or three-dimensional grid, once process I_x has carried out the forward sweep for row j, k it can begin the forward sweep for row $j + 1, k$ and so on (Figure 2.1(b)). The fraction of the total processing time spent idle by each process during the first step is $1/M_y M_z$, and similarly for the second and third steps.

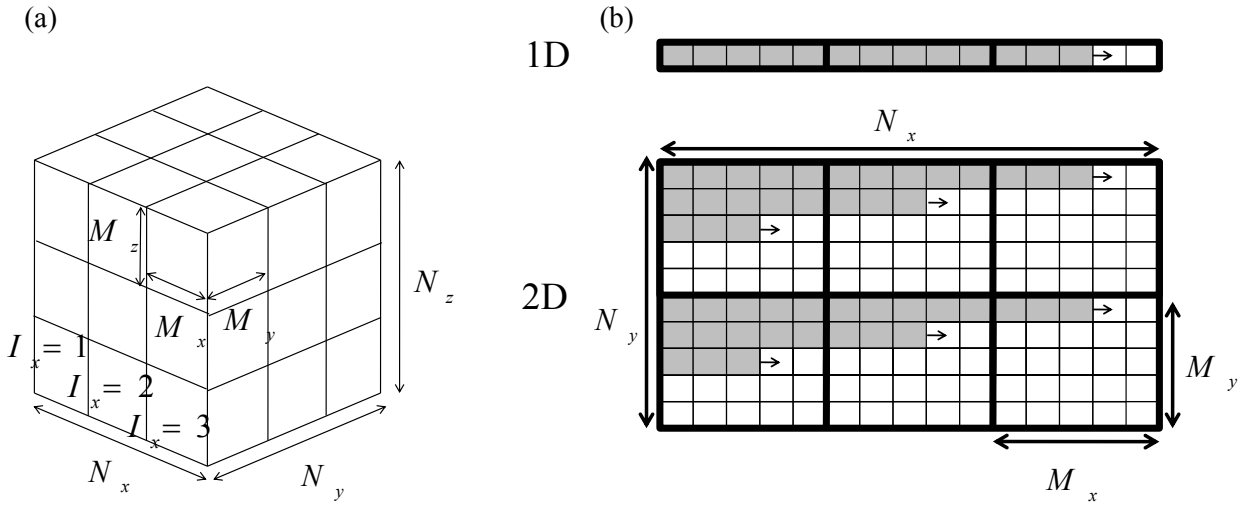


Figure 2.1: (a) Location and size of subdomains within the global domain. (b) Forward-sweep process in 1D and 2D.

In addition to the time spent idle, there is also a time penalty associated with the time spent passing the boundary values between processes. This communication cost consists of a startup cost and a latency cost [39]. The startup cost is independent of the size of the data to be communicated. By combining several rows into one message, the total startup cost incurred can be reduced – if the forward sweep is carried out over blocks of size $m_y m_z$ (Figure 2.2), the total startup cost is reduced by a factor of $M_y M_z / m_y m_z$. Combining rows in this way increases the fraction of time spent idle to $m_y m_z / M_y M_z$, so the block size must be chosen to optimise between time spent idle and communication startup cost.

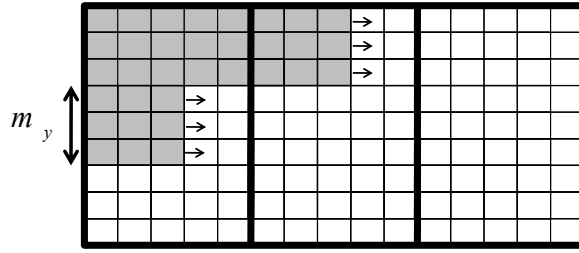


Figure 2.2: Message vectorisation in 2D. Communication times can be reduced by grouping messages together.

An appropriate test problem for the ADI is a typical perturbed ICF simulation near the time of stagnation. It is important to test the ADI on a multidimensional problem because the way the ADI handles each dimension separately might conceivably lead to miscalculation of thermal transport diagonally to the grid. Using a fluid configuration from a stagnation-phase ICF simulation will directly test the ability of the ADI to simulate the problem it is intended to be used for. In the test problem, Gorgon is run as usual up to a given time, at which point the fluid motion and all source terms are switched off, leaving only the thermal conduction running. This prevents other physical effects which may dominate over the thermal conduction masking any problems with the thermal conduction solver. The test is benchmarked against the same simulation run using the explicit model. After running the thermal conduction alone for 100 ps, the profiles along the x -axis produced by each of the methods are identical to within 0.06 eV (Figure 2.3). The difference between the models is larger in the two-dimensional map (Figure 2.4), but is still only 0.48 eV, which is 0.03% of the maximum temperature. The small difference along the x -axis may simply be caused by a difference in the numerical error in the finite-difference approximations; the ADI is second-order accurate in time, whereas the explicit method is first-order accurate, but uses a smaller timestep. The difference in the two-dimensional map is primarily in the z -direction, which suggests that running the implicit part of the z -direction transport on $T_{i,j,k}^{n+1}$ when the x -direction is run on $T_{i,j,k}^*$ could be introducing a small underestimate in the thermal flux. This 0.03% error is far too small to cause a significant difference in the dynamics of the stagnation. The ADI took 1.85 s to complete the first timestep on a 320^3 -cell grid split between 64 processors. Since the ADI is not subcycled, it always takes this time to run on a 64 processor 320^3 -cell grid whatever the

physical conditions. The explicit method took 17.31 s, making the ADI 9.36 times faster for this system at this spatial resolution. The explicit method was subcycled 78 times in the first timestep.

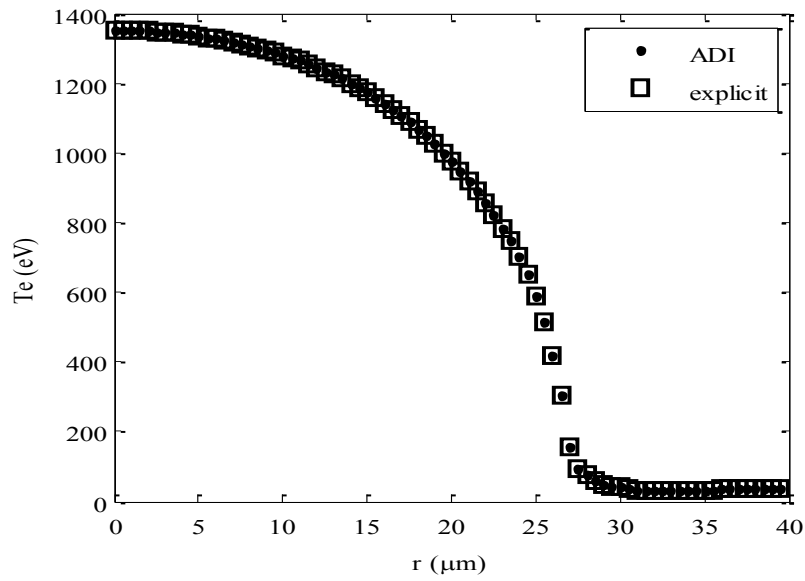


Figure 2.3: Electron temperature profiles along the x-axis generated by the ADI (dots) and explicit (squares) methods.

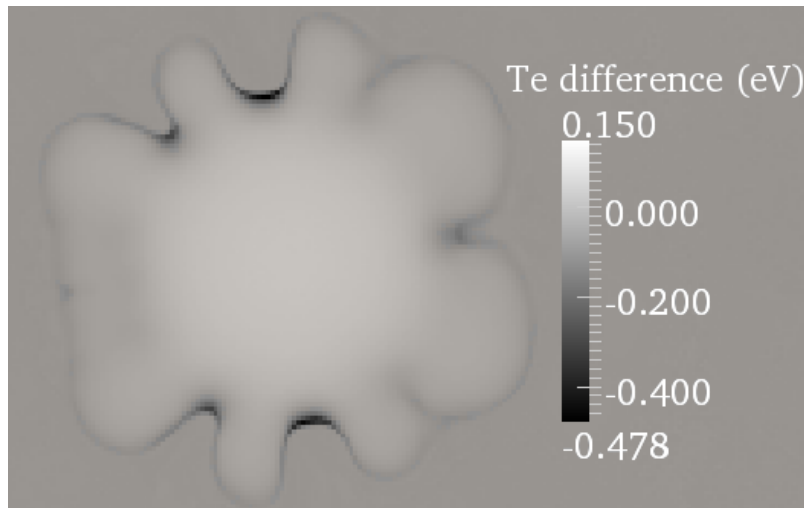


Figure 2.4: Difference in the YZ plane of the electron temperature fields generated by the ADI and the explicit method.

2.3) A sliding Eulerian grid for Gorgon

The stagnating plasmas in this work converge radially by approximately a factor of 5 at peak compression. In a full ICF capsule simulation beginning at the ablation phase, this would be increased to around 30. Since the implosion is spherical, the stagnated volume occupies approximately $1/125$ of the computational grid if the spatial grid spacing Δx is kept constant in time. The efficiency of the simulation can be improved significantly if the spatial grid spacing is reduced with time to track the motion of the plasma. In effect, the grid points are given a finite velocity in the lab frame, $\mathbf{v}_g$. In order to keep Δx constant in space, $\mathbf{v}_g$ must be directed toward the centre of the implosion and increase linearly with the radial position of the grid points:

$$\mathbf{v}_{g_{i,j,k}} = u_g \begin{bmatrix} (x_{i,j,k} - x_c)/(\Delta x N_x) \\ (y_{i,j,k} - y_c)/(\Delta y N_y) \\ (z_{i,j,k} - z_c)/(\Delta z N_z) \end{bmatrix}, \quad (2.3.1)$$

where u_g is the speed at which the outermost grid point moves towards the centre along a given axis and (x_c, y_c, z_c) is the location of the centre. The optimum setting of u_g depends on the simulation – it is desirable to track the motion of the shell, and the low-density material on the outside of the shell must leave the domain for this to occur. However, if u_g is set too high, material that affects the dynamics of the stagnation may end up being lost. In practice, u_g is set heuristically by examining the speed of the implosion in a preliminary low-resolution simulation – one setting acquired this way is usually suitable for a whole set of related simulations. u_g is usually set to the minimum of the average radial velocity and the average radial velocity weighted by fluid density, with a multiplier close to 1 on each. The implementation of the sliding grid was tested by running a sliding-grid simulation on a uniform plasma – as the grid contracts, the density, temperature and momentum of the plasma remain constant. The improvement in resolution achievable using the sliding grid is shown in Figure 2.5. Figure 2.5(a) shows the ion temperature in the YZ plane of perturbed simulation with a fixed grid spacing of $2 \mu\text{m}$, and Figure 2.5(b) shows the same simulation run with an initial grid spacing of $2 \mu\text{m}$ and a finite grid velocity. The grid spacing at the time shown was $0.86 \mu\text{m}$.

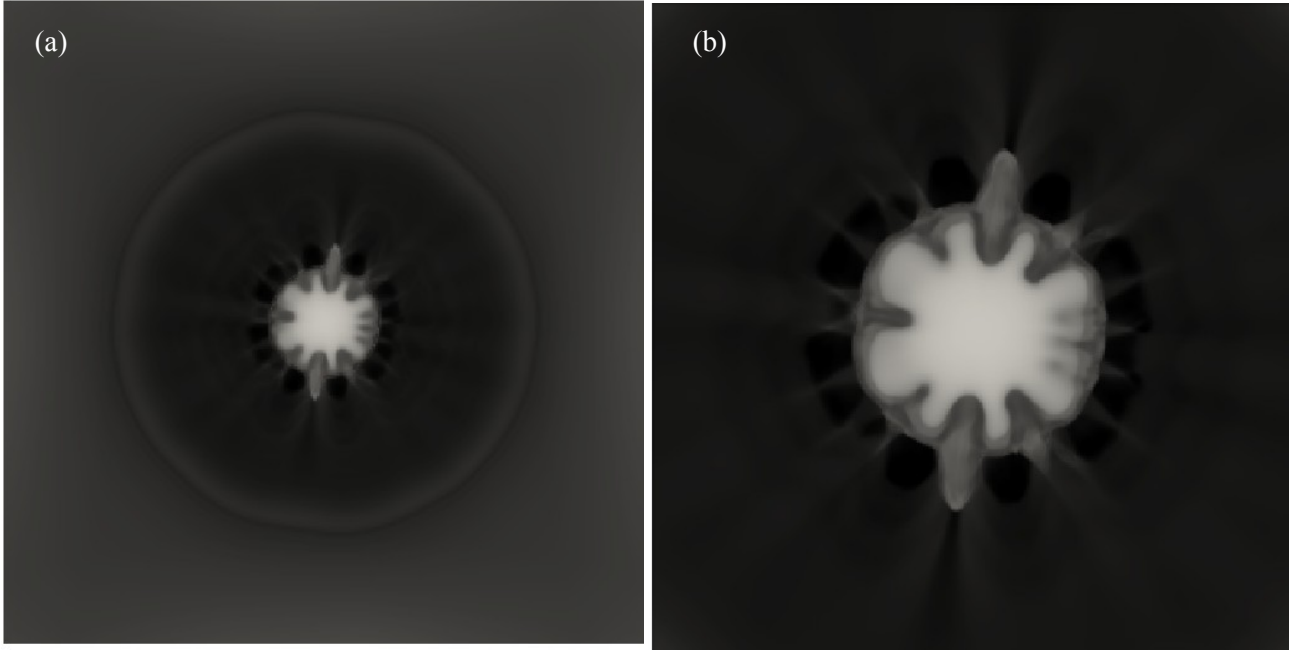


Figure 2.5: Comparison between perturbed stagnations simulated (a) without and (b) with the sliding grid.

2.4) Initial conditions

The simulations in this work begin at the start of the deceleration phase, when the radiation drive has stopped accelerating the target. This point in time is referred to as ‘peak velocity’ since it is the time that the implosion velocity is highest. The initial radial profiles are taken from the output of 1D Lagrangian ‘post-shot’ simulations of the capsule, without the inclusion of the hohlraum, performed at Lawrence Livermore National Laboratory using the Hydra code [33]. Post-shot simulations begin with the initial state of the cryogenic layered target from a particular shot, with a radiation drive applied as a boundary condition. The radiation drive pulse is informed by measurements of the radiation field in the hohlraum made using DANTE, and integrated hohlraum calculations which include the hohlraum and the target. Simulations of surrogate VISAR and conA targets carried out with the radiation pulse from integrated hohlraum calculations do not match the measured shock timings and implosion trajectories from the experimental surrogates. In order to account for this discrepancy, the drive pulse is rescaled in a time-dependant manner such that the simulated VISAR and conA data match the experiment. The drive pulse power is reduced by an average of 15% by the rescaling process [40]. This rescaled drive pulse is used for the post-shot simulations.

The radial profiles used in the simulations in this work are taken from shot N120205, performed in February 2012. N120205 had a relatively high neutron yield (5.6×10^{14}) and down-scattered neutron ratio (4.3%, corresponding to a total capsule ρR of 0.88 g/cm^2). It used a plastic ablator with a silicon dopant, the longest available 3 ns pulse rise time, and 419 TW of laser power in the final pulse. The ‘no-coast’ extension of the final pulse which prevents expansion of the shell during the coast phase was not used on N120205. The peak-velocity radial profiles used for the simulations in this work are shown in Figure 2.6. For the entirety of this thesis, the time of peak velocity, at which the simulations are initialised, is designated $t = 0$ and other times are measured relative to this.

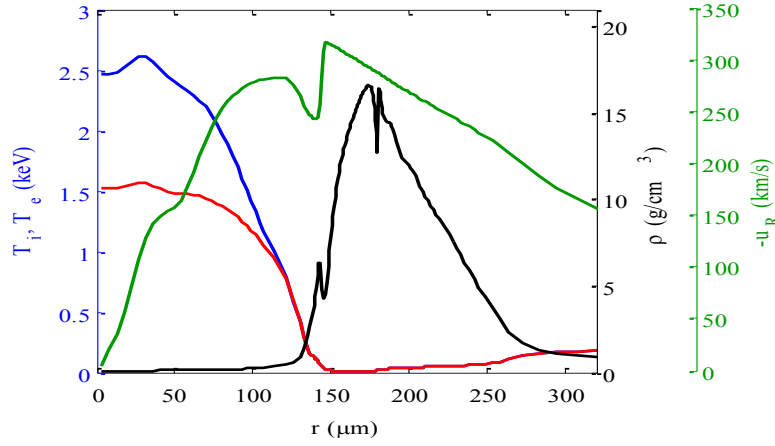


Figure 2.6: The radial profiles of electron temperature (red), ion temperature (blue), radial velocity (green) and density (black) from the post-shot simulation of N120205 used to initialise the simulations in this work.

Since the data we have available from the Hydra simulation is one-dimensional, it is necessary to impose a perturbation on the target at the beginning of the deceleration phase. In Lagrangian codes, a perturbation is often imposed by perturbing the Lagrangian grid [41] – this option is not available in an Eulerian code. A perturbation of the velocity field was found to be the most reliable way of producing a Rayleigh-Taylor-like perturbation in Gorgon. Perturbing the velocity field is equivalent to applying an instantaneous force to the target, and has an advantage over a perturbation of a Lagrangian grid in that the latter does not immediately result in a variation in momentum as would be expected in a Rayleigh-Taylor perturbed system. The velocity field used is taken from Layzer’s analytic study of a cylindrically-symmetric fluid above a vacuum in a gravitational field [42]. The component of the velocity in the axial direction of the

cylinder is given by a zeroth-order Bessel function, exponentially decaying with axial distance from the surface

$$u_z = F(t)e^{-|z|}J_0(r) \quad (2.4.1)$$

and the radial component is given by a first-order Bessel function, exponentially decaying from the surface

$$u_r = F(t)e^{-|z|}J_1(r). \quad (2.4.2)$$

$F(t)$ is a time-dependant amplitude (Figure 2.7(a)). z and r here are the axial and radial coordinates in Layzer's cylindrical coordinate system.

Our initial conditions are intended to reproduce the perturbation generated by the acceleration-phase Rayleigh-Taylor instability (RTI). In the acceleration-phase RTI, the light fluid is on the outside of the target and the denser fluid is radially inward of it. Therefore, the negative z -direction in Layzer's system corresponds to the positive r -direction in our system. In this work, the inward-moving fluid at the centre of the Layzer cylinder is referred to as the 'spike' because it becomes the spike region of the deceleration-phase RTI, and the fluid at the edge of the Layzer cylinder is referred to as the 'bubble' because it becomes the bubble region of the deceleration-phase RTI. The point of maximum density gradient on the outside of the target is chosen for the $z = 0$ position. The r -direction in Layzer's system is translated to the radial coordinate of a cone with its apex at the target centre (Figure 2.7(b)). This is referred to as the 'transverse' direction. The argument of the Bessel functions in (2.4.1) and (2.4.2) is

$$x = \alpha \frac{\Delta\theta}{\theta_{cone}} \quad (2.4.3)$$

where $\alpha \cong 3.8317$ is the first zero of J_0 , $\Delta\theta$ is the angular distance from the centre of the cone, and θ_{cone} is the aperture of the cone, which sets the wavelength of the perturbation (Figure 2.7(b)). The apex of the cone can be set off-centre to produce off-centre spike trajectories. When this is done, the cone is rotated around the point of maximum perturbation, so the distribution of spikes across the target surface remains as unchanged as possible, while their trajectories are relocated.

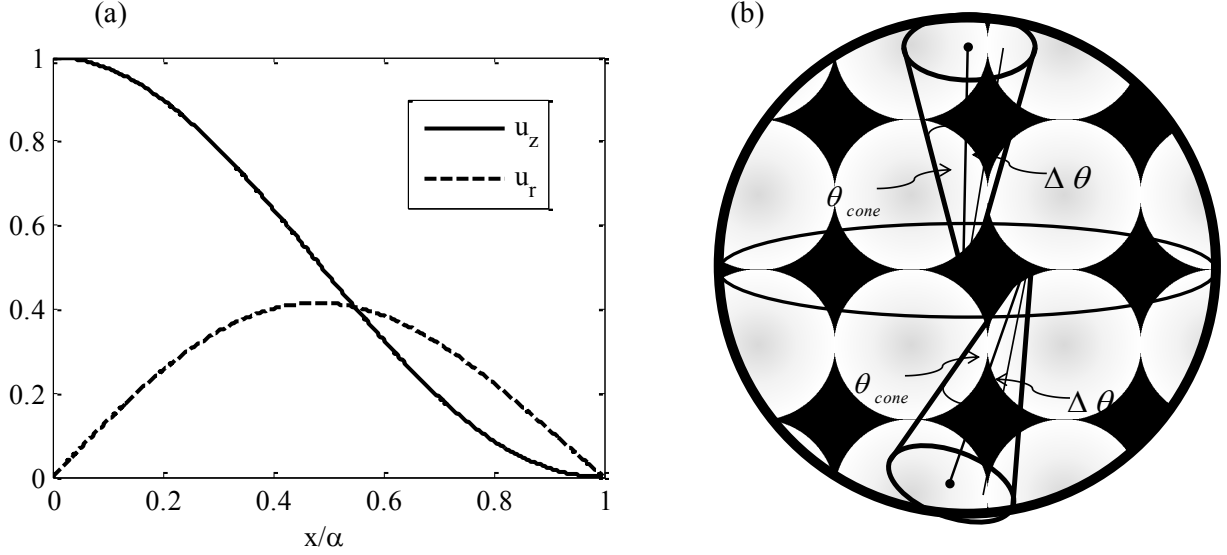


Figure 2.7: (a) Axial (solid) and radial (dashed) components of velocity in Layzer's cylinder. (b) Layzer's cylindrical geometry is mapped onto a cone with its apex at the target centre (as in the cone shown on the North pole) or off-centre (as in the cone shown on the South pole).

In order to produce a quasi-spherically-symmetric perturbation, the Layzer cones are located at each vertex of a geodesic sphere [43] generated from an icosahedron³. A geodesic sphere is an approximation to a sphere generated from a platonic solid by taking the midpoints between each vertex and projecting them out to the same radius as the original vertices. Geodesised icosahedra of order 0, 1 and 2 are shown in Figure 2.8. Geodesic spheres are not perfectly spherically symmetric, because the edges become elongated as the new vertices are moved outward in radius, and edges connecting two new vertices become more elongated than edges connecting a new vertex to an old vertex. The wavelength halves with each order of geodisation, because the aperture of each cone is set to half the angle between the vertices. Orders 0 – 4 have 12, 42, 162, 642 and 2564 vertices. In this work we will refer to perturbations by the number of vertices N , eg. $N = 42$ referring to the first order of geodisation. Multi-mode perturbations can be generated by superimposing any number of individual orders. To avoid vertices from different orders being placed in the same position and producing a coherent interference pattern, the coordinate systems of each order are rotated by a random angle

³ It is common to initialise perturbations in the form of spherical harmonics. While it is possible to reproduce any function defined on (θ, φ) using an infinite number of spherical harmonics, using a finite number of spherical harmonics can introduce spherical asymmetry in the perturbation. In particular, spherical harmonics with $l > 4$ are smaller in magnitude near the poles, and this effect is greater for higher l . This results in the shape of the perturbation depending on the choice of pole location, and can cause long-wavelength jet-like features to appear at the poles where there is less short-wavelength perturbation structure to interfere with them. Additionally, when applying a velocity-field perturbation, spherical harmonics do not provide any obvious way to apply velocity perturbations in the non-radial directions.

around the x , y and z axes of the grid. In the multi-mode case, the amplitude of each individual spike is also randomised by a uniformly distributed random variable between 0 and a maximum amplitude. This is not done in the single-mode case because a varying distribution of spike amplitudes effectively adds a low-wavelength perturbation to the system.

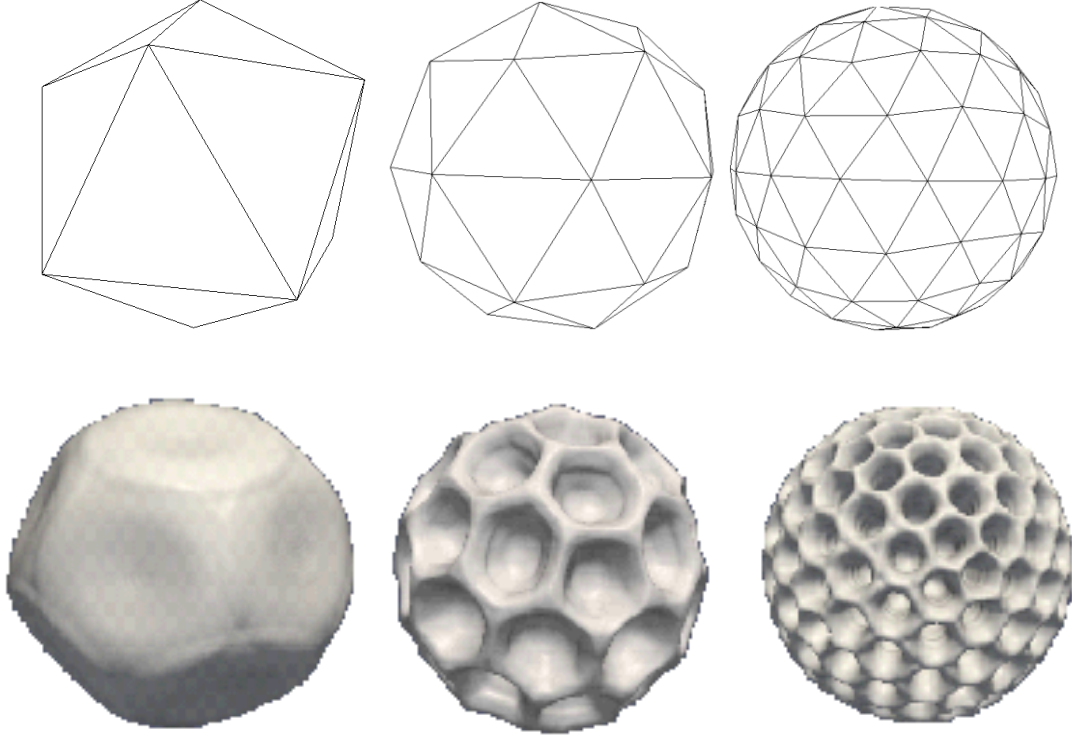


Figure 2.8: Top: positions of vertices of geodesic icosahedra of order 0, 1 and 2 (left to right). Bottom: density contours after 200 ps of perturbed stagnations of order 0, 1 and 2 (left to right) with number of spikes $N = 12$, 42 and 162.

The perturbation generated by this method grows as expected of the deceleration-phase RTI. A simple model of the growth of the perturbation can be constructed based on the observation that the velocity profile in the hotspot is linear with radius. This is justified during the time that the sound speed in the hotspot is greater than the fluid velocity at the hotspot edge, as observed in Section 3.3 (Figure 3.9). It also serves as a working approximation at times when the motion in the hotspot is not entirely subsonic. Following the notation of [8], the radial position of a given Lagrangian coordinate a is given for a system with a linear velocity profile by

$$r(a, \tau) = h(\tau)r(a, t = 0), \quad (2.4.4)$$

where

$$h(\tau) = \sqrt{1 + (t - t_{stag}/t_0)^2}, \quad (2.4.5)$$

and t_{stag} is the time at which the fluid velocity in the hotspot is 0. This is similar to Kidder's self-similar implosion [44], but the positive sign has been taken in the square root in order to produce a decelerating flow. The result is a stagnating flow where t_{stag} is the point of maximum compression, and t_0 is the timescale of the stagnation. We are interested in the growth of the perturbation to the hotspot boundary, so we choose a to be the Lagrangian position of the hotspot boundary. The velocity at this position is given by

$$u_R = R/t_0. \quad (2.4.6)$$

The acceleration of a Lagrangian coordinate is

$$g(t) = \frac{d^2 r(a, t)}{dt^2} = \frac{t_0^2}{((t - t_{stag})^2 + t_0^2)^{3/2}} u_R(t = 0). \quad (2.4.7)$$

This is substituted into the linear RTI growth rate for a simple stratified system of density gradient scale-length L_ρ [45]:

$$\gamma(t) = \sqrt{\frac{Akg(t)}{1 + AkL_\rho}} \quad (2.4.8)$$

to get:

$$\gamma(t) = \sqrt{\frac{Aku_R(0)}{1 + AkL_\rho}} \frac{t_0}{((t - t_{stag})^2 + t_0^2)^{3/4}}. \quad (2.4.9)$$

From the 1D profiles of the unperturbed simulation, it can be observed that L_ρ is on the scale of the hotspot radius, and the Atwood number $A \sim 1$. The product AkL_ρ is therefore approximately $2\pi\sqrt{N}$. Since $N = 12$ even for the longest-wavelength mode, AkL_ρ is sufficiently large that $Aku_R(0)/(1 + AkL_\rho) \rightarrow u_R(0)/L_\rho$, so A and k do not affect the growth rate in the linear regime.

In order to compare this model with the data (Figure 2.9), we take the perturbation amplitude from the code at an arbitrary time t_{lin} during the linear phase and extrapolate by integrating the exponential growth over the intervening times:

$$\delta R(t) = \delta R(t_{lin}) \int_{t_{lin}}^t \exp[\gamma(t' - t_{lin})] dt'. \quad (2.4.10)$$

At a particular time t_{sat} , δR becomes comparable to the wavelength and the perturbation begins to resemble the ‘bubble and spike’ configuration of the nonlinear regime. At this point, the approximate wavelength-independence of the perturbation growth is broken because t_{sat} and the subsequent bubble velocity depends on wavelength. Layzer [42] calculated a bubble velocity of $0.511\sqrt{\lambda g}$ for the planar-geometry case. [46] finds that the growth factor is approximately a factor of two higher in spherically convergent geometry for a stagnating ICF system. In Figure 2.9, t_{sat} is marked by the dashed vertical line, and the growth after this time has been integrated forward using a bubble velocity of $\sqrt{\lambda g}$. This simple model is sufficient to reproduce the behaviour of the perturbation in the simulation, and confirms that the perturbation grows as expected of the Rayleigh-Taylor instability.

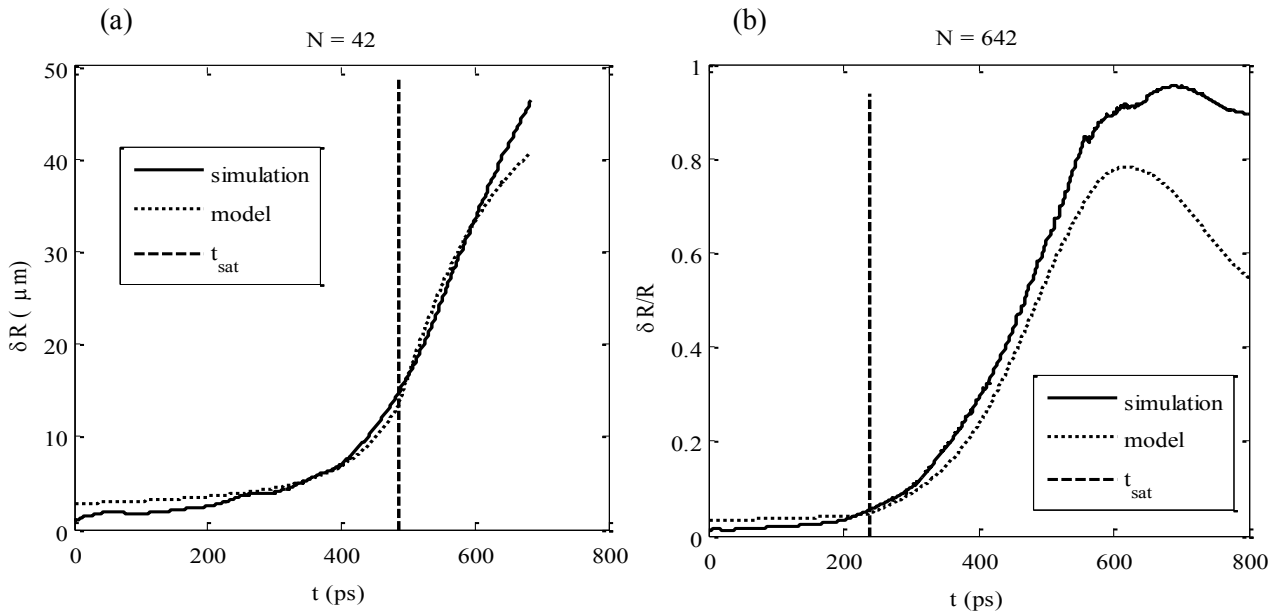


Figure 2.9: Comparison of the growth of the perturbations in the stagnation-phase simulations (solid) with a model based on Rayleigh-Taylor growth theory (dotted). Right: an $N = 42$ perturbation. Perturbation amplitude is parameterised by the range of the hotspot radius, δR . Left: an $N = 642$ perturbation. In this case δR has been normalised to the hotspot radius, R . The perturbation saturates when $\delta R/R \approx 1$.

Chapter 3

Energy Balance of the Stagnating Hotspot

Part of the explanation of the lower-than-expected pressures, and therefore yields, produced in ICF experiments on the NIF is the perturbation of the hotspot shape from spherical symmetry. In order to analyse the effect of perturbations on the hotspot thermal energy, analytic energy source terms are derived for unperturbed targets by taking advantage of the self-similar forms of the radial profiles that the state variables adopt. This model of hotspot energetics is an improvement upon the 0D models developed by Widner and others, and matches very closely the results of unperturbed simulations. The model enables the behaviour of the source terms observed in perturbed hotspots to be explained in terms of how the perturbations affect 0D hotspot parameters such as volume and thermal energy, and how they alter the hotspot profiles from the self-similar forms. Using this approach, a phenomenological understanding of how perturbations reduce the hotspot thermal energy is developed.

3.1) Connection between hotspot thermal energy and yield

In order to analyse the behaviour of the hotspot, it is necessary to use a precise definition of which part of the system constitutes the hotspot. In this work, the hotspot is defined to be the region for which

$$T_i > 1 \text{ keV.} \quad (3.1.1)$$

Given that the pressure in the hotspot is approximately uniform, it is the ion temperature that sets the volumetric fusion reaction rate, since for uniform hotspot pressure P_{HS} , (2.1.5) becomes

$$\frac{dW_\alpha}{dV} \propto \frac{P_{HS}^2}{T_i^2} \langle \sigma v \rangle (T_i), \quad (3.1.2)$$

which is a function only of ion temperature. At 1 keV the reactivity of DT is around 1% of that for 5 keV DT at the same pressure and at 2 keV it is around 4%. The material just within the hotspot is on the threshold of making a significant contribution to the yield, and the material outside of the hotspot is not contributing significantly. Defining the hotspot in terms of a temperature contour also proves to be convenient for the mathematical analysis in this chapter.

In nonigniting hotspots, the ion temperature remains below 7 keV at all times. The Bosch-Hale reactivity can be fit by $\langle \sigma v \rangle \propto T_i^4$ to within 10% between 2 keV and 7 keV (Figure 3.1). For a hotspot of uniform temperature and density, the rate of fusion energy production may be broken into factors of hotspot thermal energy, mass and volume:

$$W_\alpha \propto \rho^2 T_i^4 V \propto \frac{E^4}{M^2 V} \quad (3.1.3)$$

This is valid under the assumption that ρ and T_i are uniform in the hotspot. It is shown below that it is also approximately valid in the case of the unperturbed stagnation in this work.

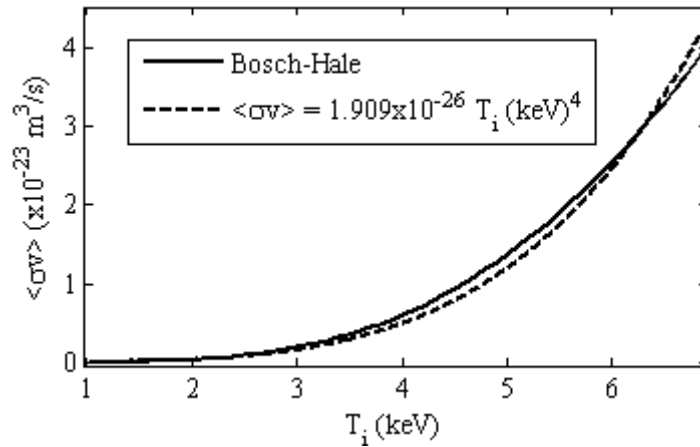


Figure 3.1: T_i^4 fit to the Bosch-Hale DT fusion reactivity.

Figure 3.2 breaks the fusion rate W_α into its factors according to (3.1.3), for the unperturbed stagnation. W_α is shown normalised to its initial value. The factors E^4, M^{-2} and V^{-1} are also shown normalised to their initial values. The dashed line shows the product of the red, green and blue lines, and agrees closely with the fusion rate line, confirming the relationship (3.1.3). It is useful to write the fusion rate in this form because it consists of extrinsic quantities, one of which (thermal energy) can be understood in terms of simple source terms and another (volume) is a property of the hotspot shape and size. When the fusion rate is written this way, it is clear that energy lost from the hotspot and recycled back as enthalpy in ablated mass still results in a reduced fusion rate; if the hotspot thermal energy and volume are held constant and its mass increased by ΔM , the fusion rate will decrease by a factor of $(1 + \Delta M/M)^{l-2}$ as long as the temperature exponent of the fusion cross-section $l > 2$, which is the case for $T_i < 13.5$ keV. The thermal energy of the hotspot is distributed over more mass, so the temperature and therefore reaction rate is lower. In the case that the volume increases due to the ablation, the reduction in reaction rate is greater.

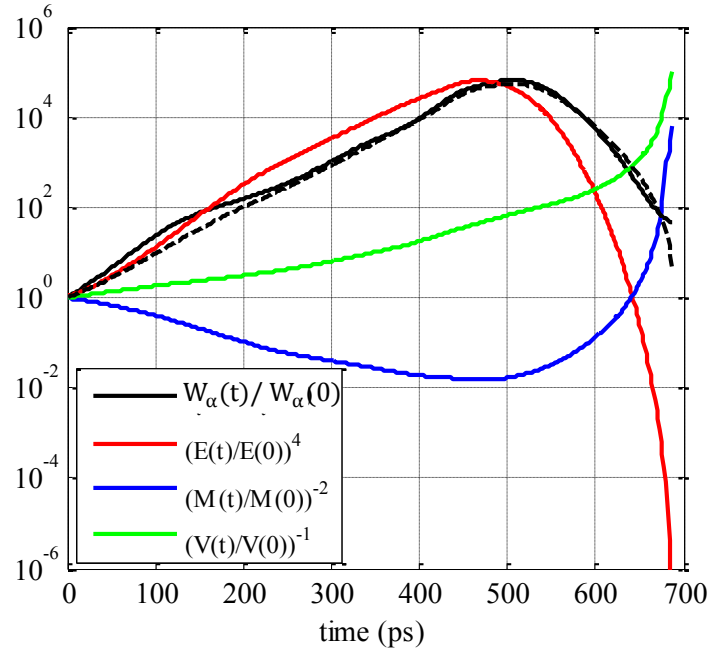


Figure 3.2: DT fusion rate of the unperturbed simulation (black), compared to its energy (red), mass (blue) and volume (green) factors. Each quantity is normalised to its initial value.

Figure 3.2 shows that the time-evolution of the fusion rate is dominated by the total hotspot thermal energy. Therefore in order to analyse the hotspot performance it is useful to link the thermal energy source terms to the hotspot properties such as density, temperature and radius. This is trivial in the case of uniform

profiles of the state variables (ρ, T, u) , but it is not necessarily accurate to make that assumption. This chapter derives volume-integrated source terms for the hotspot profiles observed in simulations. These are used to link the reduction in yield by a perturbation to the perturbed hotspot's properties. First the dynamics of the unperturbed stagnating hotspot and the observed state-variable profiles are discussed.

3.2) Dynamics and state variable profiles in the unperturbed stagnation

During the acceleration phase, the radiation field drives four shocks through the shell, which combine at a point near the gas/shell boundary to produce a strong shock that carries out the initial heating of the hotspot. This shock reflects off the target centre. As it moves outward from the centre, the shock decelerates the imploding fluid and can be thought of as an accretion. The fluid radially outward of it is 'freefalling' inward at the implosion velocity, and collects on the surface of the shocked region, becoming compressed and heated as it does so. When the shock reaches the hotspot boundary, it is partially reflected back into the hotspot. The part of the shock which has been reflected back continues to bounce back and forth between the target centre and the hotspot boundary, becoming weaker each time it does so. This can be seen in the (r, t) diagram of radial velocity $(u_r(r, t))$ (Figure 3.3). The hotspot boundary and the accretion are visible as changes in the gradient of u_r , and are marked on the diagram.

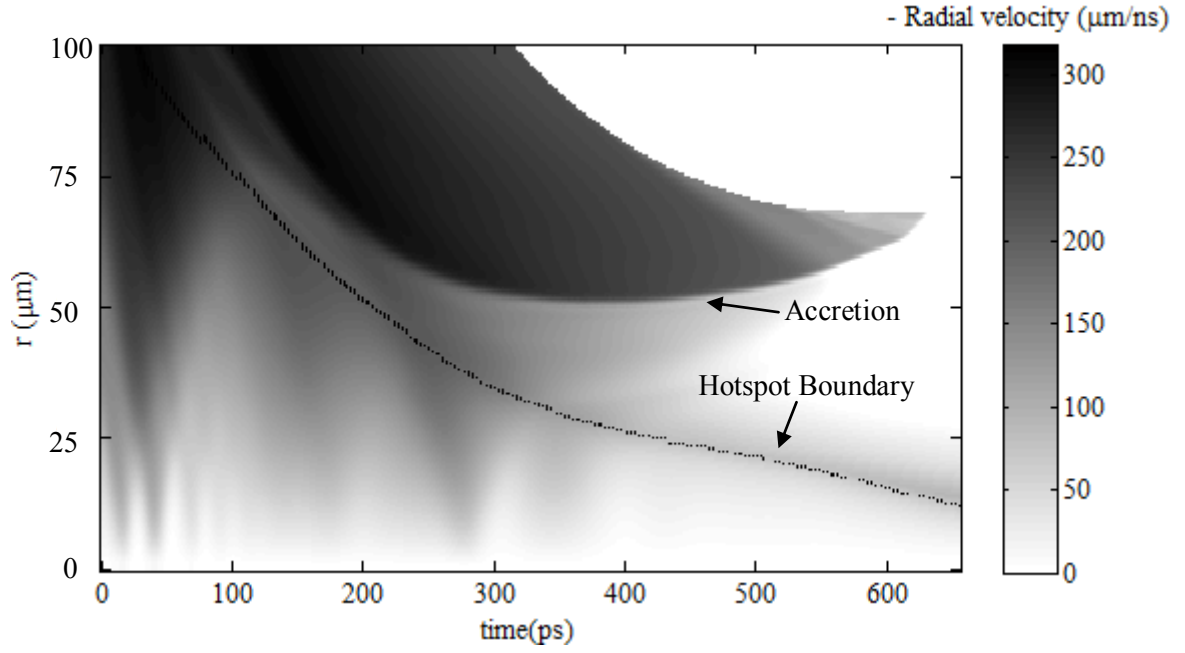


Figure 3.3: Radial velocity of the unperturbed stagnation in (r, t) space. The black contour shows the position of the hotspot radius.

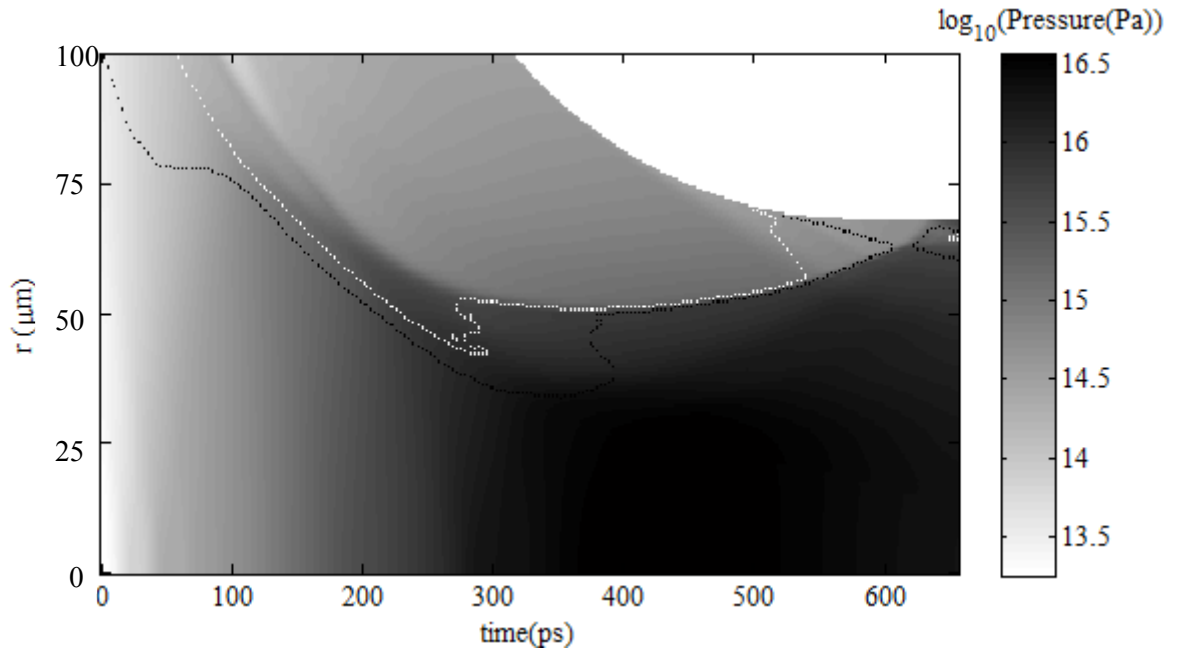


Figure 3.4: Pressure of the unperturbed stagnation in (r, t) space. Contours of Mach number $M = 1$ (white) and $M = 2$ (black) are shown.

The shock-dominated period of the deceleration phase ends around 200 ps, which is around the time the yield rate starts to become significant. Between $t \sim 200$ ps and $t \sim 400$ ps, the velocity profile in the hotspot is approximately linear with radius, as discussed in Section 2.4. This period is designated the ‘continuous deceleration phase’. The large velocity spike at around $t = 280$ ps is due to a shock reflecting off the accretion surface and returning to the hotspot (the reflection occurs at $t \sim 300$ ps at a radius of ~ 70 μm). This perturbs the profile from linearity, but hotspot velocity profile remains approximately linear except in the region of shocks propagating within the hotspot; Figure 3.5(d) shows linear fits to the velocity profile at $t = 100$ ps, 400 ps and 500 ps. The timescale of the Kidder-like stagnation (2.4.4) resulting from a linear velocity profile is given by

$$\tau_0 \propto \frac{R}{u_R} \approx \frac{60 \text{ } \mu\text{m}}{300 \text{ } \mu\text{m/ns}} = 200 \text{ ps} \quad (3.2.1)$$

which agrees with the timescale of our simulation.

The temperature profiles in the unperturbed stagnation are fit accurately by a self-similar power law

$$T(r, t) = T_c(t) - (T_c(t) - T_R) \left(\frac{r}{R(t)} \right)^q \quad (3.2.2)$$

where $T_c(t)$ is the central temperature and $R(t)$ the hotspot radius at time t . $R(t)$ is defined as the point where the ion (and electron) temperature drops below a particular boundary temperature T_R , 1 keV in our simulations. Although T_i and T_e differ in the centre, the equilibration rate on the boundary is fast enough that their 1 keV contours are the same. Figure 3.5(a) shows how the temperature profile exponent for the electrons, q_e , and ions, q_i , vary with time. Electron temperature profiles with (3.2.2) fit to them are shown for various times in Figure 3.5(b).

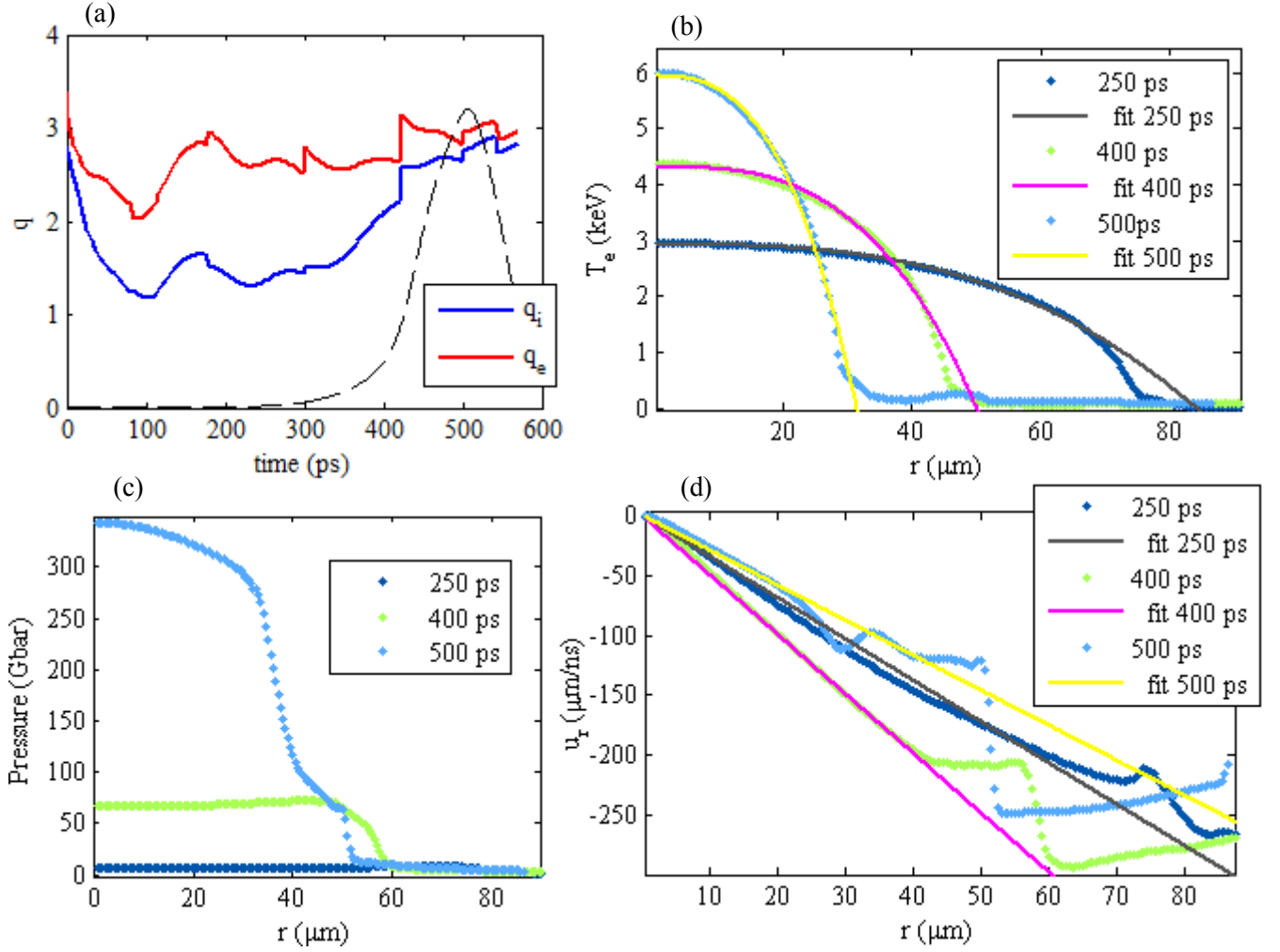


Figure 3.5: (a) Time dependence of the temperature profile exponent q for ions (blue) and electrons (red). The fusion rate pulse is shown for comparison, in arbitrary units. (b) Electron temperature profiles from the unperturbed simulation with the self-similar profile (3.2.2) fit. The time after peak velocity, t , is shown in the legend. (c) Pressure profiles showing the approximate uniformity of the pressure in the hotspot. (d) Radial velocity profiles with linear fits.

During the continuous part of the deceleration phase, the pressure in the hotspot P_{HS} is approximately uniform. At 400 ps the pressure varies by 6% within the hotspot. The density profile follows from $P(r, t) \propto \rho(r, t)T(r, t)$:

$$\rho(r, t) = \frac{\rho_c(t)T_c(t)}{T(r, t)} = \rho_c(t) / \left(1 - \frac{(T_c(t) - T_R)}{T_c(t)} \left(\frac{r}{R(t)} \right)^q \right), \quad (3.2.3)$$

where ρ_c is the central density. It is convenient for calculation to choose either the electron or ion temperature in the formula for ρ . In the centre when $T_i/T_e \approx 3/2.5$ this will introduce a $\sim 10\%$ error in the value of ρ ;

integrated over the whole radius the error is much less since the temperatures are the same at R . We drop the subscripts e and i in our analysis of the hotspot energy balance, using the appropriate species' $T_c(t)$ for each source term.

While this model provides a useful parameterisation of the simulated hotspot, there are some ways in which the simulated profiles differ from it, principally associated with the shocks. The pressure and ion temperature deviate from the model profiles when shocks are present within the hotspot, for example the pressure profile at 500 ps varies by 25% within the hotspot (Figure 3.5(c)). However, at 550 ps the pressure has returned to uniformity to within 2%. Until 400 ps, the ion temperature profile has an inflection point not present in the model profile, and a larger proportion of its thermal energy is concentrated in the centre (Figure 3.6(c)). Shocks arriving in the centre heat the ions preferentially, and the ion thermal conductivity is not high enough to regulate the profile back to (3.2.2). It is most likely that thermal conduction is the mechanism driving the hotspot temperature profile to converge on the form (3.2.2). The model profile (3.2.2) fits that of the central thermal wave present in point explosions when thermal conductivity is included [47], for an isobaric system with typical ICF parameters. In the extreme case of zero thermal conductivity a point explosion has a divergent central temperature, and as thermal conduction becomes more significant less of the thermal energy is concentrated in the centre. The equilibration rate becomes sufficient for the electron thermal conduction to influence the ions around 400 ps, and $q_i \approx q_e$ after this time (Figure 3.5(a)). The model profile (3.2.2) still differs from the data by no more than 4% at 250 ps (and similarly at other times) despite the weaker thermal conduction and the presence of the inflection point on the T_i profile, so it is still valid as a means of parameterising the hotspot.

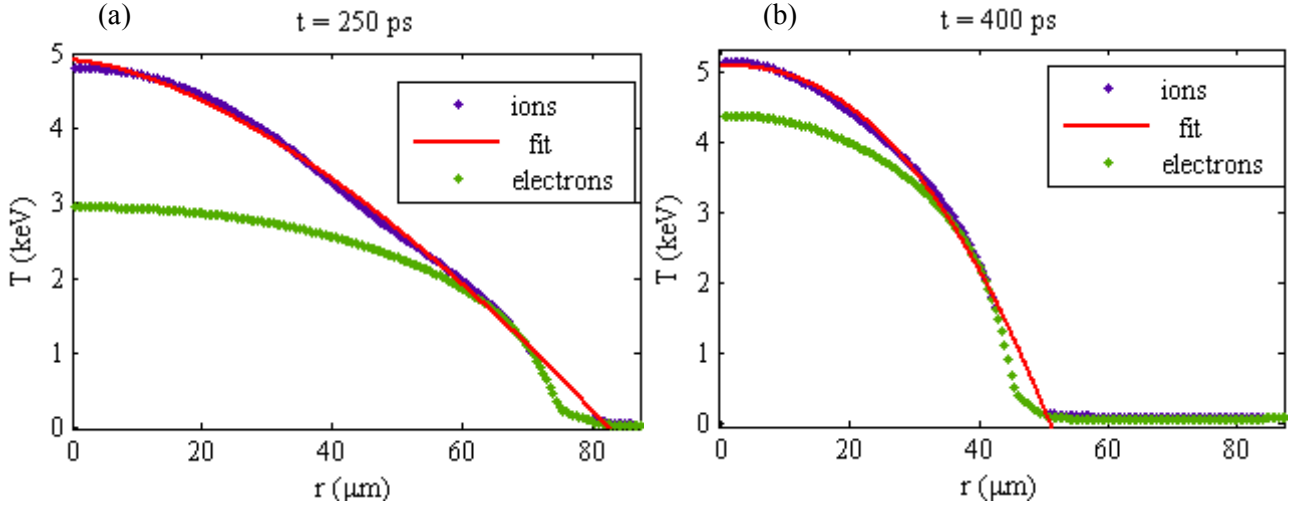


Figure 3.6: Comparison between ion and electron temperature profiles at (a) $t = 250$ ps and (b) $t = 400$ ps. The self-similar temperature profile (3.2.2) is fit to the ion temperature profile.

3.3) Analytic volume-integrated source terms for the 1D profiles

The main energy source terms in the hotspot at stagnation are work, radiation emission, fusion energy deposition, and thermal conduction. The total rate of energy change of the hotspot by each of these processes can be calculated in terms of the hotspot's basic properties such as its radius, central density, and central temperature, by integrating the observed analytic profiles over the hotspot volume.

The mechanical work source term is simply

$$W_m = PSu_R \quad (3.3.1)$$

where $u_R(t)$ is the fluid velocity at the hotspot radius at time t . This is not the same as the rate of change of the hotspot radius, as the hotspot radius is moving relative to the fluid.

The thermal conduction term only depends on the conditions at the surface, according to the divergence theorem:

$$W_{TC} = - \int \nabla \cdot \kappa \nabla T \, dV = - \int \kappa(R) \left. \frac{dT}{dr} \right|_{r=R} dS. \quad (3.3.2)$$

Using $\kappa(R) = A_{TC} T_R^{5/2} / \ln \Lambda$ and $dT/dr|_{r=R} = -q(T_c - T_R)/R$, the integrand is constant over the surface giving

$$W_{TC} = 4\pi R q A_{TC} T_R^{5/2} (T_c - T_R) / \ln \Lambda \quad (3.3.3)$$

When $T_c = T_R$ this is 0; such a system would have a uniform temperature profile so no thermal conduction.

Note that this can be written

$$W_{TC} = \kappa(R) S / L \quad (3.3.4)$$

where $S = 4\pi R^2$ is the hotspot surface area, and a thermal scale length

$$L = R / (q(T_c - T_R)) = -1 / (dT/dr|_{r=R}) \quad (3.3.5)$$

is automatically defined. This provides an exact definition of the ‘factor close to unity’ used by Widner [1] and Atzeni [8], $c_{TC} = q(1 - T_R/T_c)$. With $q_e \approx 3$, the thermal scale length can be a factor of 2 to 3 shorter than it would be under the assumption that $c_{TC} = 1$.

The volumetric bremsstrahlung rate is

$$dW_r/dV = A_r \rho^2 T^{1/2} \quad (3.3.6)$$

where A_r is a constant. Due to the isobaricity of the hotspot this can be written entirely in terms of the temperature profile:

$$dW_r/dV = A_r \rho_c^2 T_c^2 T(r)^{-3/2}. \quad (3.3.7)$$

Assuming the emitted radiation is not reabsorbed, the volume-integrated radiation loss is

$$W_r = 4\pi A_r (T_c \rho_c)^2 \int_0^R (T_c - (T_c - T_R)(r/R)^q)^{-3/2} r^2 dr. \quad (3.3.8)$$

We can calculate this using a general form of the integral,

$$\int_0^R (T_c - (T_c - T_R)(r/R)^q)^{-m} r^2 dr = H_m(T_c, q) \frac{R^3}{3} T_c^{-m}. \quad (3.3.9)$$

$H_m(T_c, q)$ has been adopted here as shorthand for the hypergeometric function ${}_2F_1(3/q, m; 1 + 3/q; (1 - T_R/T_c))$. The hypergeometric function ${}_2F_1$ is given by

$${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \quad (3.3.10)$$

where $(a)_n$ is a Pockhammer symbol, which denotes the product $a(a+1) \dots (a+n-1)$. Applying (3.3.9) to (3.3.8) gives

$$W_r = H_{3/2}(T_c, q) A_r \rho_c^2 T_c^{1/2} V. \quad (3.3.11)$$

The radiation loss rate differs from the uniform value by a ‘profile factor’ $H_{3/2}(T_c, q)$, which depends on the central temperature. The dependence on q is less important as q does not change much during the stagnation, though it is different between the electron and ion temperature profiles. Figure 3.7(a) shows the dependence of $H_{3/2}(T_c, q)$ on T_c for a range of T_c relevant to nonigniting hotspots, for two values of q : $q = 3$, the approximate value of q_e throughout the stagnation, and $q = 1.5$, the approximate value of q_i while the equilibration rate is low. When $T_c = T_R$ the uniform approximation is recovered as expected. For the temperature range of interest, the profile factor $H_{3/2}(T_c, 3)$ is fit to within 2% by $T_c^{0.7}$, with T_c in keV, so the volume-integrated radiation loss term depends on the central temperature as

$$W_r \approx A_r \rho_c^2 T_c (\text{keV})^{1.2} V. \quad (3.3.12)$$

The stronger dependence of W_r on T_c in a nonuniform hotspot relative to a uniform hotspot is due to the higher density of the plasma at larger radii. Increasing the central temperature T_c increases the pressure $P_{HS} \propto \rho_c T_c$ throughout the hotspot. Writing the volumetric bremsstrahlung rate in terms of pressure and density

$$dW_r/dV \propto P_{HS}^{1/2} \rho^{3/2} \quad (3.3.13)$$

it is clear that for a higher central temperature (higher P_{HS}), the bremsstrahlung rate at a radial position r increases by a factor of $\rho(r)^{3/2}/\rho_c^{3/2}$ over the rate in the centre, resulting in a larger W_r increase than would be expected in a uniform hotspot.

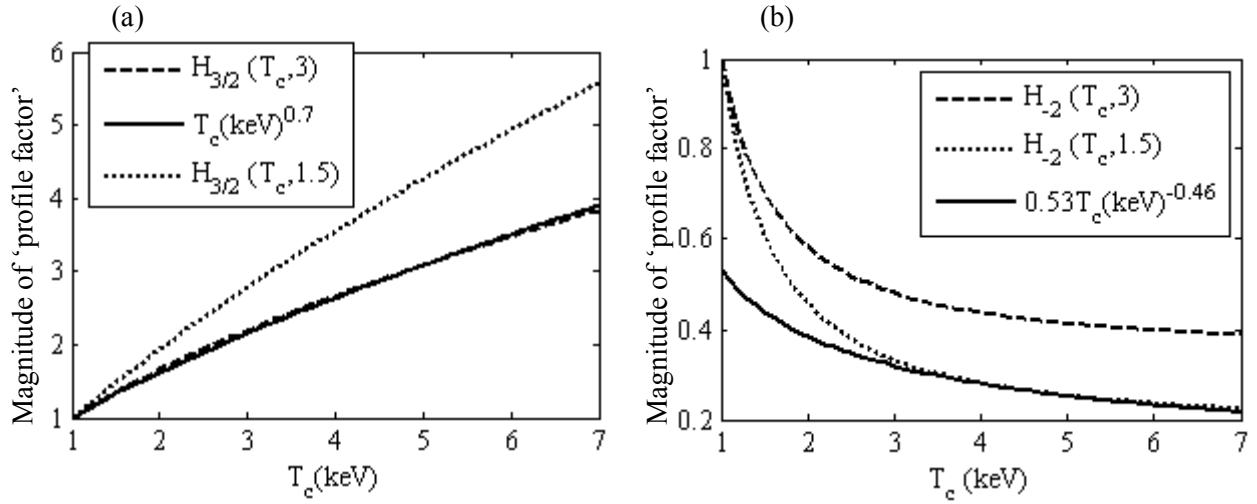


Figure 3.7: (a) Temperature dependence of the radiation emission ‘profile factor’ for $q = 1.5$ and $q = 3$. The $q = 3$ profile factor is fit by $T_c(\text{keV})^{0.7}$. (b) Temperature dependence of the fusion ‘profile factor’ for $q = 1.5$ and $q = 3$. The $q = 1.5$ profile factor is fit by $0.53T_c(\text{keV})^{-0.46}$.

Assuming alpha deposition is instantaneous and local, the alpha deposition source follows in the same manner as the radiation with T^4 in the place of $T^{1/2}$, and a different constant, A_α :

$$W_\alpha = H_{-2}(T_c, q) A_\alpha \rho_c^2 T_c^4 V. \quad (3.3.14)$$

The dependence of the profile factor $H_{-2}(T_c, q)$ on T_c for $q = 3$ and $q = 1.5$ is shown in Figure 3.7(b). There is no useful approximation of the form $H_{-2}(T_c, q) \approx T_c^\nu$ as there was for the radiation emission; the best fits $H_{-2}(T_c, 3) \approx T_c^{-0.59}$ and $H_{-2}(T_c, 1.5) \approx T_c^{-0.93}$ differ from their respective hypergeometric functions by up to 20%. However, for $3.5 \text{ keV} < T_c < 7 \text{ keV}$, $H_{-2}(T_c, 1.5)$ is fit by $0.53T_c(\text{keV})^{-0.46}$ to within 2%. This temperature range is appropriate for the hotspot at the time it is generating most of the fusion reactions. Using this approximation, (3.3.14) becomes

$$W_\alpha = 0.53 A_\alpha \rho_c^2 T_c(\text{keV})^{3.54} V. \quad (3.3.15)$$

An increase of the central temperature results in a smaller increase in W_α than would be expected in a uniform hotspot, which is the opposite of the result for the radiation emission rate.

The volume-integrated source terms derived above can be checked for accuracy by comparing them against the source terms in the simulation, summed numerically at each timestep over all cells for which $T_i > 1 \text{ keV}$. The analytic source terms are calculated from the hotspot parameters (ρ_c, T_c, R, u_R and q) taken from

the simulation at each timestep. Comparing the two therefore numerically tests the validity of equations (3.3.1), (3.3.4), (3.3.11), and (3.3.14). Figure 3.8 compares the simulated (black) and analytic (blue) values. The thermal conduction does not match the data as closely as the others. This is because the model profile (3.2.2), while accurate for the majority of the hotspot, differs most at the hotspot boundary. The Mach number goes above 1 at the hotspot edge (Figure 3.9), causing the fluid to become compressed locally at the boundary. The temperature gradient at the boundary is 1.69 times higher in the simulation than in the model profile at the peak of W_{TC} , which accounts for the discrepancy. The thermal conduction loss rate through the 2 keV contour is fit much better by the analytic model, as the model profile is more accurate at that radius.

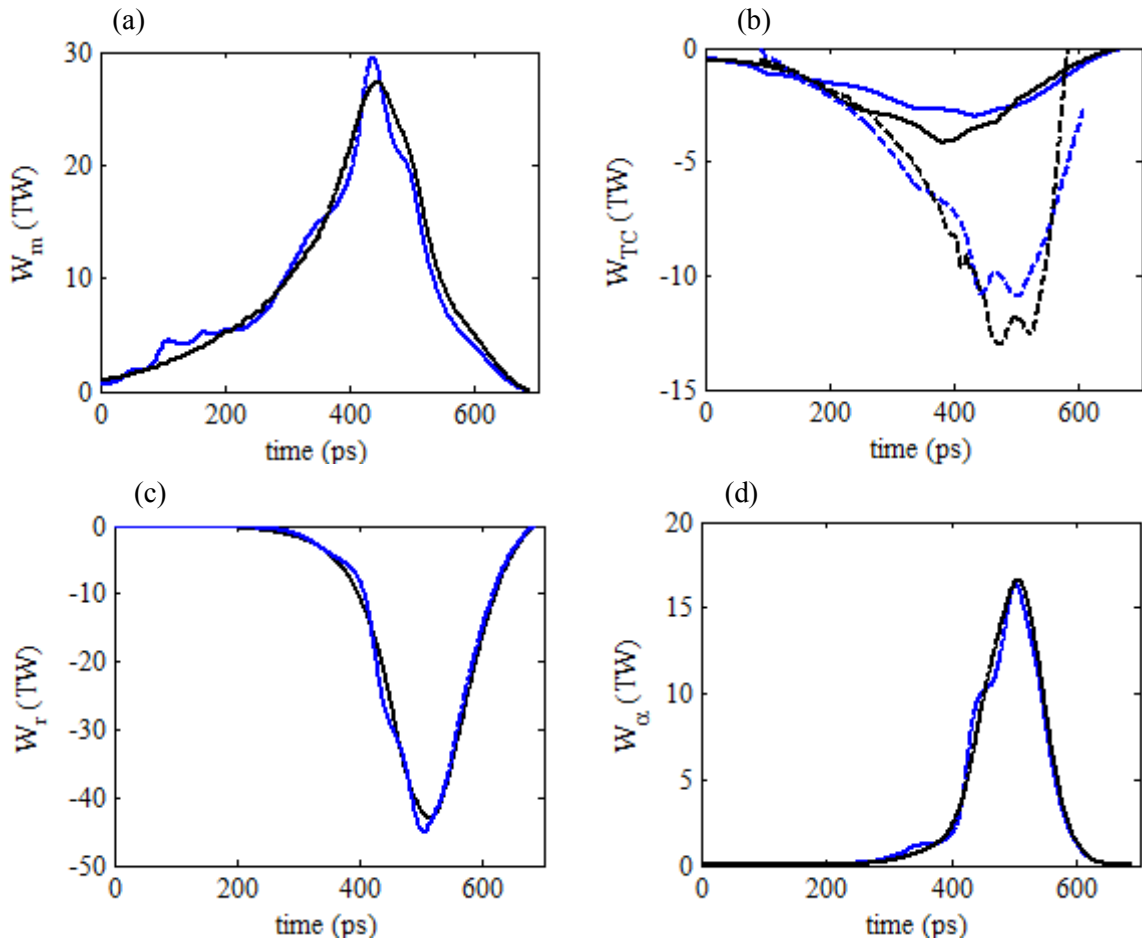


Figure 3.8: Test of the validity of the volume-integrated source terms. Source terms are numerically integrated over the hotspot volume directly from the simulation (black), and calculated from the analytic model using the hotspot parameters from the simulation (blue). (a) Work, (b) thermal conduction, (c) radiation emission, (d) fusion.

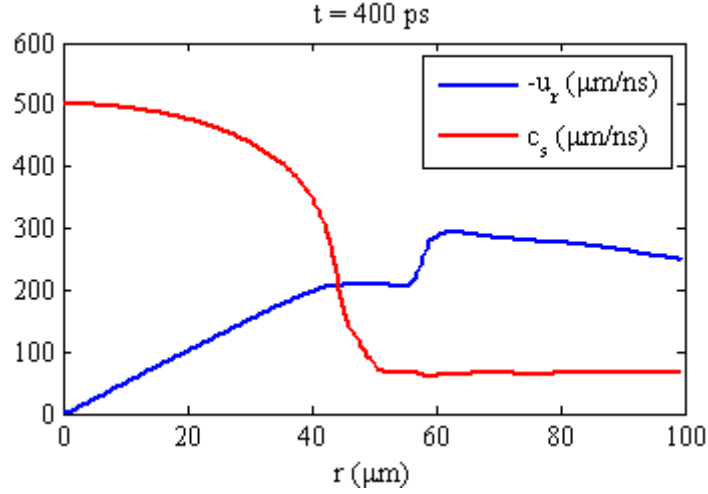


Figure 3.9: Sound speed (red) compared with radial velocity (blue). The Mach number goes above 1 at the hotspot edge.

To make use of the analytic volume-integrated source terms (3.3.1), (3.3.4), (3.3.11), and (3.3.14), it is useful to write them in terms of the hotspot thermal energy, mass and volume. In terms of the central density and temperature, the hotspot thermal energy and mass are:

$$E = 3 \frac{e}{m_i} \rho_c T_c V \quad (3.3.16)$$

where m_i is the average mass of the ions in the DT fluid, and

$$M = H_1(T_c, q) \rho_c V \quad (3.3.17)$$

To convert $H_1(T_c, q)$ into a power-law in terms of T_c , we use the average of $H_1(T_c, 1.5)$ and $H_1(T_c, 3)$ at times when $q_i = 1.5$. As shown in Figure 3.10, this average is fit to within 2% by $T_c(\text{keV})^{0.5}$. Hence the mass is approximately

$$M \approx T_c(\text{keV})^{0.5} \rho_c V \quad (3.3.18)$$

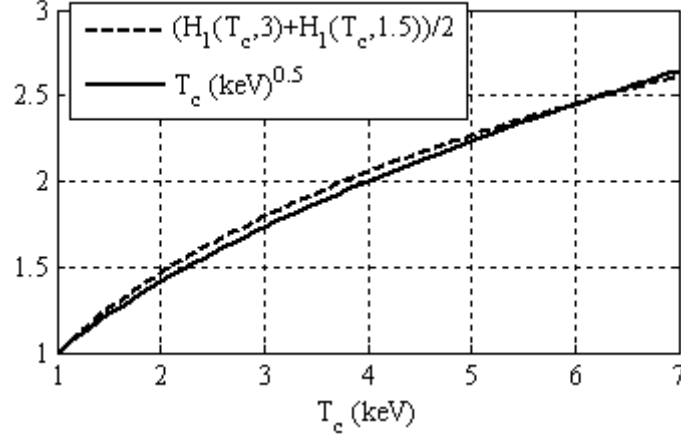


Figure 3.10: Average between the mass profile factors for temperature profiles with $q = 3$ and $q = 1.5$ (dashed). This is fit to within 2% by $T_c(\text{keV})^{0.5}$.

Any product of powers of ρ_c , T_c , and V can be written in terms of M , E and V according to

$$\rho_c^a T_c^b V^c = E^\alpha M^\beta V^\gamma \quad (3.3.19)$$

which when combined with equations (3.3.16) and (3.3.18) gives a set of three simultaneous equations with the solution $\alpha = 2b - a$, $\beta = 2(a - b)$ and $\gamma = c - a$. Therefore the radiation emission source term can be written in terms of E , M and V using $a = 2$, $b = 1.2$ and $c = 1$:

$$W_r \propto \frac{E^{0.4} M^{1.6}}{V} \quad (3.3.20)$$

and for the fusion source term, $a = 2$, $b = 3.54$ and $c = 1$, so

$$W_\alpha \propto \frac{E^{5.08}}{M^{3.08} V}. \quad (3.3.21)$$

For the hotspot pressure, $a = 1$, $b = 1$ and $c = 0$, so $P \propto E/V$ and the work source term is

$$W_m \propto \frac{E}{V} S u_R. \quad (3.3.22)$$

These forms of the energy source terms, along with the forms in terms of ρ_c , T_c , and V , are used in Sections 3.5 and 3.6 for analysing the effect of perturbations on the energy balance of the hotspot at stagnation.

3.4) Energy balance in single-mode perturbations

In this section we examine how a perturbation consisting of a single geodesic mode influences the hotspot energy balance. The differences in the energy source terms between perturbed and unperturbed simulations can be understood in terms of how the perturbation alters the 0D hotspot parameters (E, V, M, R and q), as long as the model derived above matches the observed energy source terms. Where the model does not fit, the change in the source terms can be understood in terms of how the perturbation alters the radial profiles from their assumed forms.

Figure 3.11(a) shows a slice of the ion temperature field from an $N = 42$ perturbed stagnation with a yield of 8.7×10^{14} neutrons, near the bang time ($t = 400$ ps). The state variables and energy source rates $\gamma = W/E$ of the $N = 42$ target are compared to those of an unperturbed simulation in Figure 3.12. The initial velocity perturbation amplitude $\delta u_R/u_R$ was 0.5 in this simulation. In this section, the phrase ‘perturbed simulation’ or ‘perturbed stagnation’ is used to refer to this particular setup unless otherwise stated.

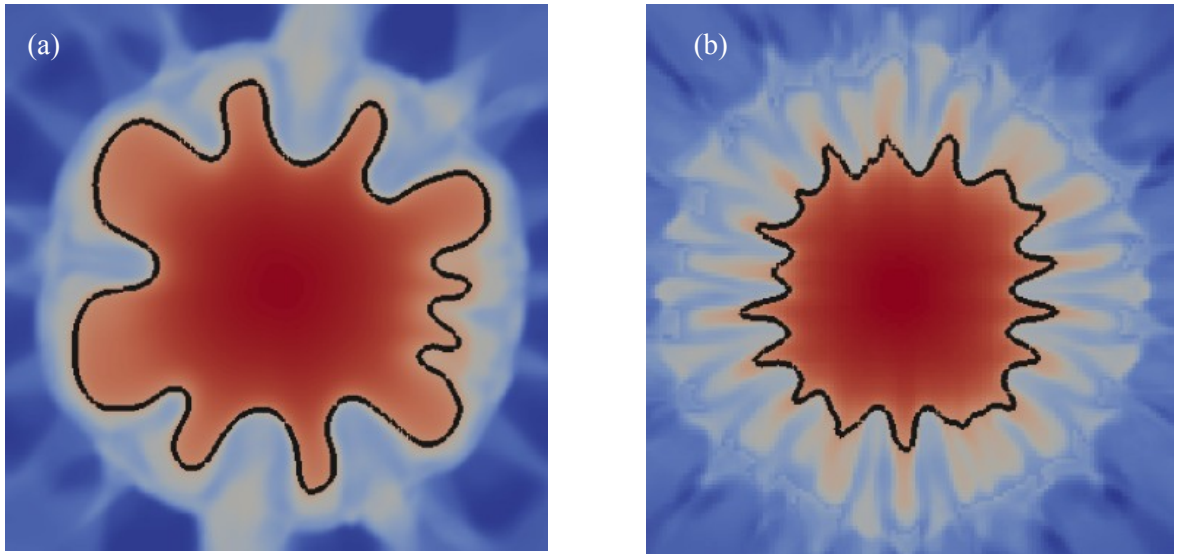


Figure 3.11: Ion temperature slices at $t = 400$ ps for single-mode perturbations with number of nodes (a) $N = 42$ and (b) $N = 162$. The black line shows the $T_i = 1$ keV contour.

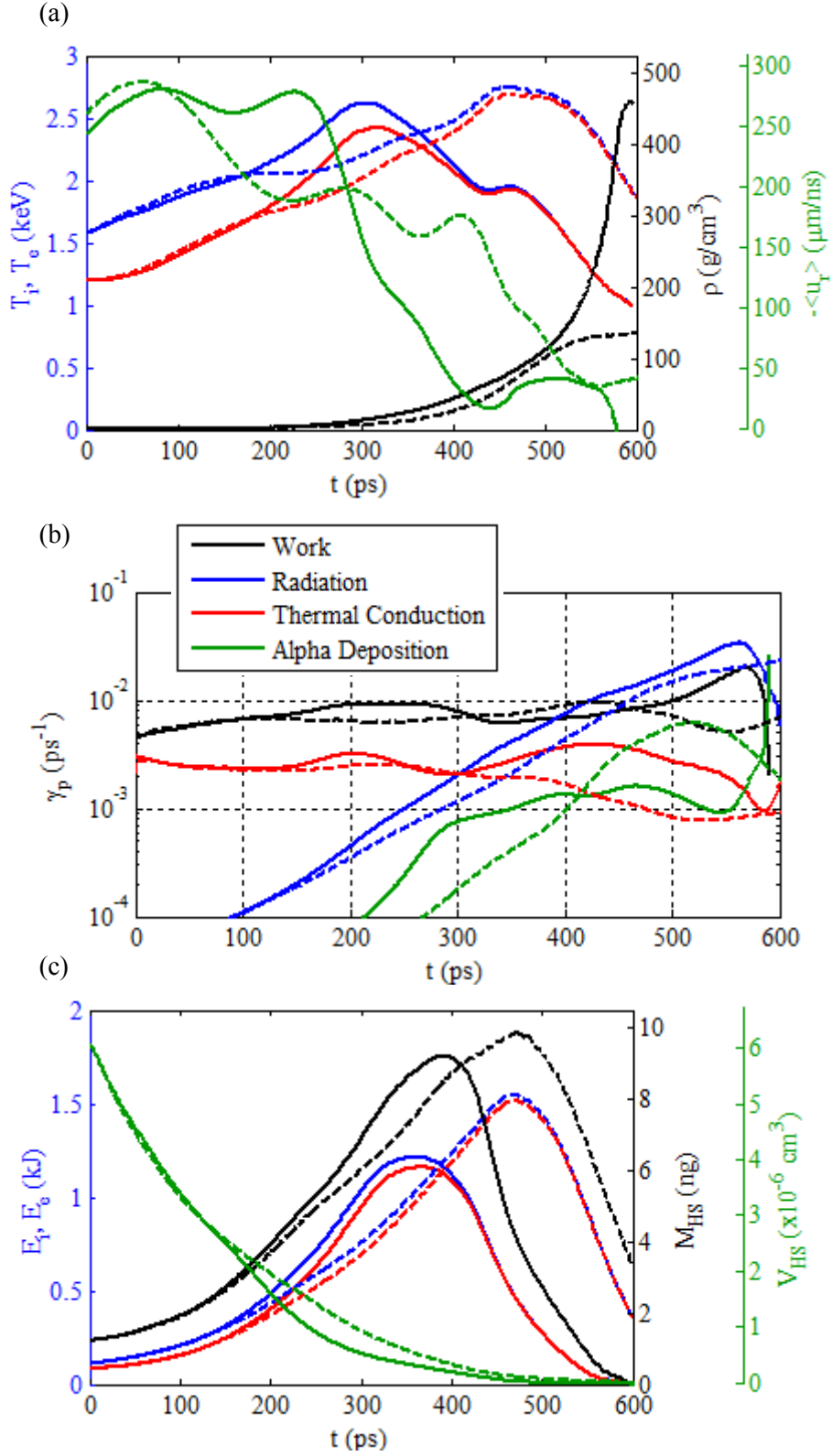


Figure 3.12: Comparison between the $N = 42$ hotspot (solid) and the unperturbed (dashed). (a) Mass-averaged temperature of the ions (blue) and electrons (red), mass-averaged hotspot radial velocity (green) and average density (black). (b) Rates of work (black), radiation emission (blue), thermal conduction (red) and fusion (green). (c) Hotspot thermal energy in ions (blue) and electrons (red), hotspot mass (black) and hotspot volume (green).

The perturbed stagnation has a higher work rate than the unperturbed at around $t = 200$ ps, causing its temperature to peak earlier. By writing (3.3.22) as

$$\gamma_m = \frac{W_m}{E} = \frac{S}{V} \langle u_R \rangle, \quad (3.4.1)$$

we may decompose the work rate increase into two factors; the average radial velocity $\langle u_R \rangle$ at the hotspot surface, and the hotspot's surface area to volume ratio S/V . Both of these factors increases with perturbation amplitude at $t = 200$ ps.

The higher $\langle u_R \rangle$ in the perturbed stagnation at this time can be explained by transverse flow of mass away from the spike regions, which leads to higher-mass bubble regions (Figure 3.13(a)). Consider a bubble and spike as two unequal masses $\langle m \rangle - \delta m$ and $\langle m \rangle + \delta m$ travelling on parallel trajectories with velocity $\langle u_R \rangle + \delta u_R$ and $\langle u_R \rangle - \delta u_R$ respectively (Figure 3.13(b)). If the total kinetic energy is held at a constant $E_k = m u_0^2$ where u_0 is the unperturbed velocity, the perturbed velocity averaged across the surface is given by

$$\frac{\langle u_R \rangle}{u_0} = \frac{\delta m}{m} \frac{\delta u_R}{u_0} + \sqrt{1 - \frac{\delta u_R^2}{u_0^2} \left(1 - \frac{\delta m^2}{m^2}\right)}. \quad (3.4.2)$$

At the beginning of the simulation, $\delta m = 0$, giving $\langle u_R \rangle / u_0 = (1 - \delta u_R^2 / u_0^2)^{1/2}$, so the velocity perturbation reduces $\langle u_R \rangle$ at $t = 0$. As transverse mass flow increases δm , the positive terms in (3.4.2) become finite and enable $\langle u_R \rangle$ to become larger than u_0 .

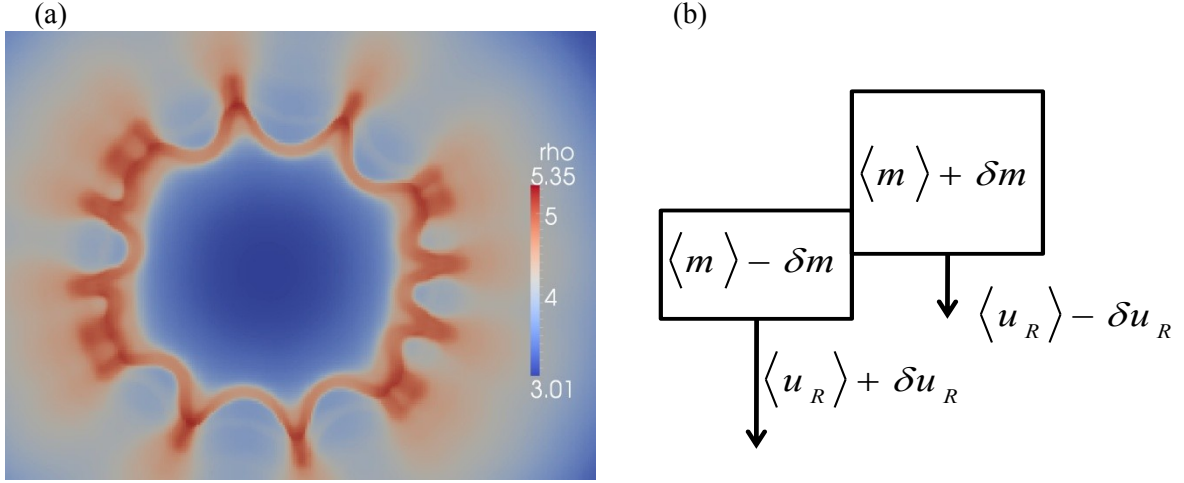


Figure 3.13: (a) Log density slice from the unperturbed hotspot at $t = 200$ ps. (b) Two-component system model of mass and velocity distribution in bubble and spike regions.

The higher surface area to volume ratio of perturbed hotspot is responsible for the remaining 32% of the γ_m increase at $t = 200$ ps. Approximately half of the increase of the surface area to volume ratio S/V is due to the smaller average radius $\langle R \rangle$ of the perturbed hotspot, which is a consequence of the higher $\langle u_R \rangle$ occurring before the temperature peak. The other half is due to the aspherical shape of the hotspot; $\langle R \rangle S/V$ should be constant for objects of the same shape but it has increased by 11% in the perturbed simulation by $t = 200$ ps. In other words, the perturbed hotspot's surface area goes above $4\pi\langle R \rangle^2$ as the perturbation grows. As shown in Figure 3.14, the volume of the single-mode perturbed hotspot remains close to $4\pi\langle R \rangle^3/3$ for the duration of the stagnation, so the aspherical hotspot shape causes a higher S/V .

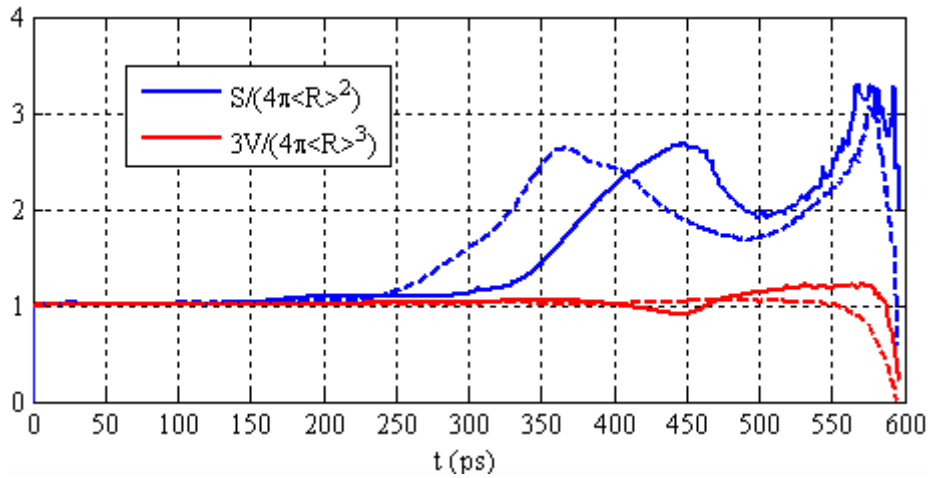


Figure 3.14: Deviation from the values of a sphere of equal radius of: surface area (blue) and volume (red) of single-mode perturbed hotspots with number of nodes $N = 42$ (solid) and $N = 162$ (dashed).

Despite having a higher work rate earlier in time, the perturbed simulation thermalises less kinetic energy in total. The perturbed simulation does 0.71 times as much work on the hotspot as the unperturbed simulation. Some of the leftover kinetic energy is present in the bubble regions after the stagnation has occurred. At $t = 600$ ps, the unperturbed simulation's velocity is close to 0 in the region behind the accretion (Figure 3.15(a)), and a low-density cavity remains in the centre. In the perturbed simulation, the spikes have collided in the centre, and the only low-density regions are at the locations of the bubbles. These regions are expanding outward with a radial velocity of around $200 \mu\text{m/ns}$ at $t = 600$ ps (Figure 3.15(b)). The shell in the bubble regions has a significantly lower ram pressure ρv^2 than the unperturbed shell, but the hotspot thermal pressure is not proportionately reduced because of the work done by the spikes. Instead of heating the hotspot, a portion of the work done by the spikes has been transferred to an outward expansion of the bubbles. The rest of the leftover kinetic energy remains in the spikes until they reach the target centre. The lower rate of work done by the bubble regions, along with the increased energy loss in the hotspot by radiation and thermal conduction, results in a lower thermal pressure in the hotspot than there would be for an unperturbed shell imploding at the velocity of the spikes. Therefore the spikes are not decelerated fully until they collide with each other in the centre, and consequently do less work on the hotspot than an unperturbed shell at the same velocity.

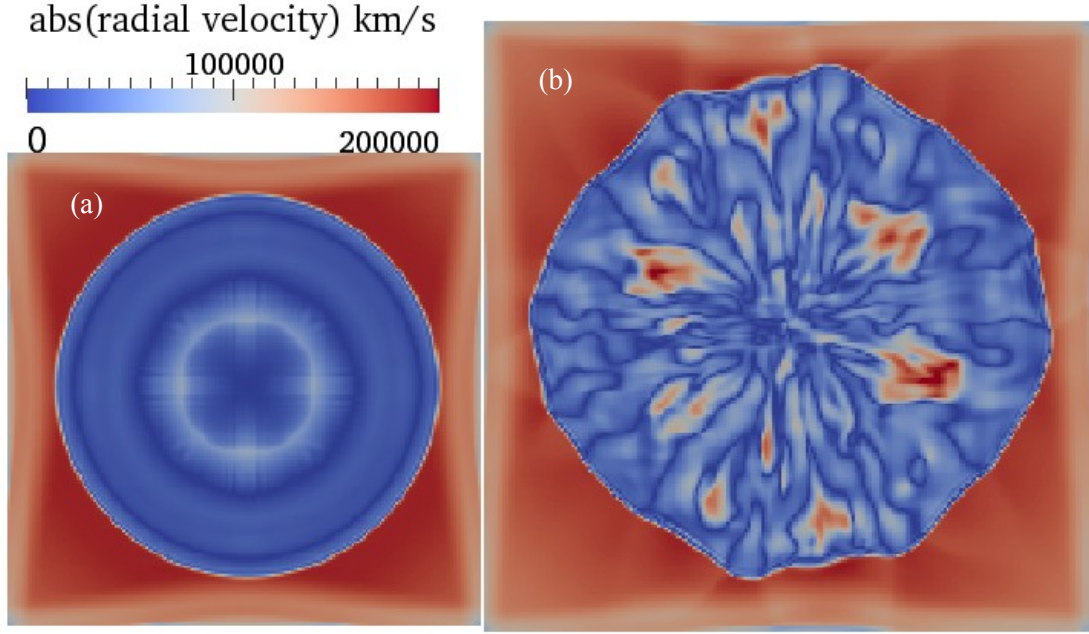


Figure 3.15: Radial velocity amplitude slices at $t = 600$ ps in (a) the unperturbed and (b) the $N = 42$ simulations.

The general phenomenology described above for a simulation with an initial velocity perturbation amplitude $\delta u_R/u_R = 0.5$ applies to simulations with perturbations at least in the range of $0.2 < \delta u_R/u_R < 0.6$. Figure 3.16(a) shows the work rate increase at $t = 200$ ps as a function of the initial perturbation amplitude, broken into factors of the u_R and S/V increases. The dashed line is the product of the u_R and S/V factors, and agrees closely with the work rate taken directly from the code. Figure 3.16(b) shows the influence of the perturbation on the work done on the hotspot and the thermal energy in the shell at $t = 600$ ps. The difference between the work not done on the hotspot and the extra shell thermal energy is approximately constant with perturbation amplitude. As the perturbation amplitude is increased, any additional work not done goes into heating the shell.

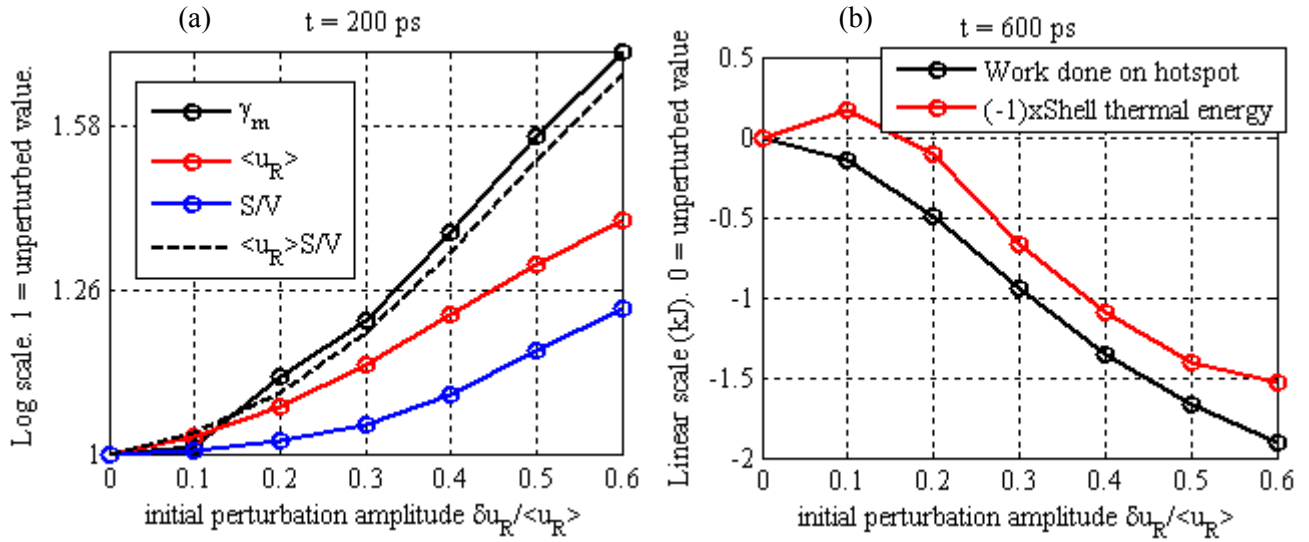


Figure 3.16: (a) Perturbation amplitude dependence of the work rate at $t = 200 \text{ ps}$, broken into factors of average radial velocity at the hotspot surface (red) and hotspot surface-to-volume ratio (blue). (b) Perturbation amplitude dependence at $t = 600 \text{ ps}$ of work done on the hotspot (black) and shell thermal energy (red).

The perturbed stagnation has a greatly increased radiation emission rate $\gamma_r = W_r/E$ for all time (Figure 3.12(b)). Using (3.3.20) the radiation emission rate can be written:

$$\gamma_r \propto \frac{M^{1.6}}{VE^{0.6}}. \quad (3.4.3)$$

The dependence of γ_r on the initial perturbation amplitude is broken into its factors in Figure 3.17(a). The higher radiation emission rate is dominated by the reduction in hotspot volume associated with the perturbation, which results in a higher density. This is a direct consequence of the higher $\langle u_R \rangle$ of the perturbation, which reduces the average hotspot radius $\langle R \rangle$; the hotspot is more compressed in the perturbed case, so its radiation emission rate is higher. $V/\langle R \rangle^3$ remains effectively constant with perturbation amplitude throughout the stagnation (Figure 3.14), so the hotspot has the same volume as a sphere of the same radius. Therefore the hotspot volume reduction is a consequence of the smaller scale of the hotspot, not a geometric feature of the aspherical hotspot shape. In the case of the fusion rate, the hotspot thermal energy is the dominant variable, because the fusion rate depends more strongly on the hotspot thermal energy:

$$\gamma_\alpha \propto \frac{E^{4.08}}{VM^{3.08}}. \quad (3.4.4)$$

The behaviour of γ_α as a function of initial perturbation amplitude is broken into its factors in Figure 3.17(b). W_α/W_r is larger for the perturbed stagnations at peak temperature ($t \approx 300$ ps) because the thermal energy has increased relative to the unperturbed case by a larger factor than the mass. After the temperature peak, the declining work rate and the increase in the thermal conduction rate cause the hotspot to lose energy faster than the unperturbed case. As material cools and leaves the hotspot, the rate of mass loss is greater than the rate of energy loss because the thermal energy density (thermal pressure) is approximately uniform, whereas the mass density is higher on the hotspot edge. The decreasing E/M results in a decreasing W_α/W_r , which positively feeds back into the decrease of E/M .

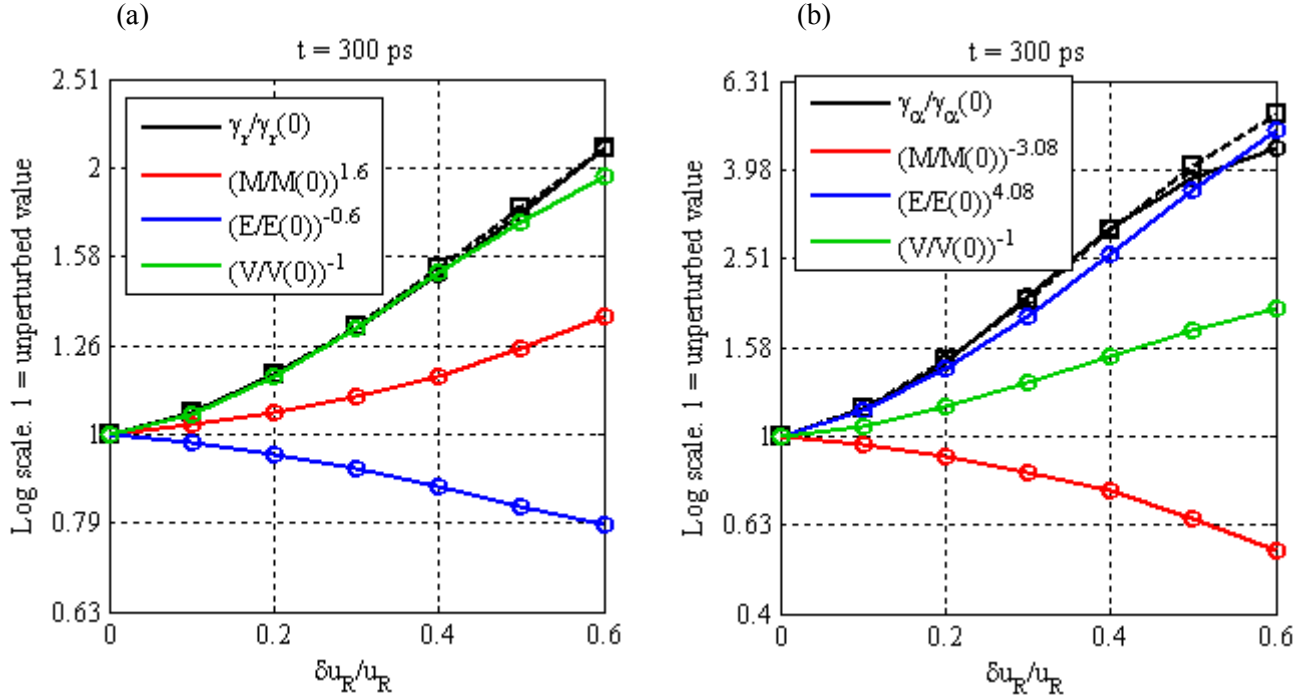


Figure 3.17: (a) Perturbation amplitude dependence of the work rate (black) broken into factors of the hotspot thermal energy (blue), mass (red) and volume (green). (b) The same for the fusion rate.

The compression of the temperature gradient at the hotspot boundary observed in the unperturbed stagnation is enhanced in the perturbed stagnation around $t = 200$ ps. The spike regions compress the fluid in their path and increase the temperature gradient, the bubble regions having the opposite effect (Figure 3.18). Overall, the thermal conduction is increased by this compression during the time that the $\langle u_R \rangle$ of the perturbed

target is higher (Figure 3.12). At 200 ps, $dT/dr|_{r=R}$ is 1.27 times higher in the spike regions than in the unperturbed simulation, compared with a reduction by 0.91 in the bubble regions. The volume-integrated source term (3.3.3) gives a value of W_{TC} 80% smaller than the value calculated directly from the code for this perturbation amplitude at this time, which appears to result from the additional thermal gradient compression by the perturbation. The thermal gradient begins to relax as the shell stagnates and $\langle u_R \rangle$ falls below the unperturbed value, and γ_{TC} falls back to a similar value to the unperturbed case at ~ 300 ps. A contributing factor to this is also the smaller hotspot surface area in the perturbed case at this time. The absolute power loss W_{TC} does increase with perturbation amplitude due to the higher central temperature in perturbed targets at this time, but since E is also proportional to the central temperature, γ_{TC} is approximately constant.

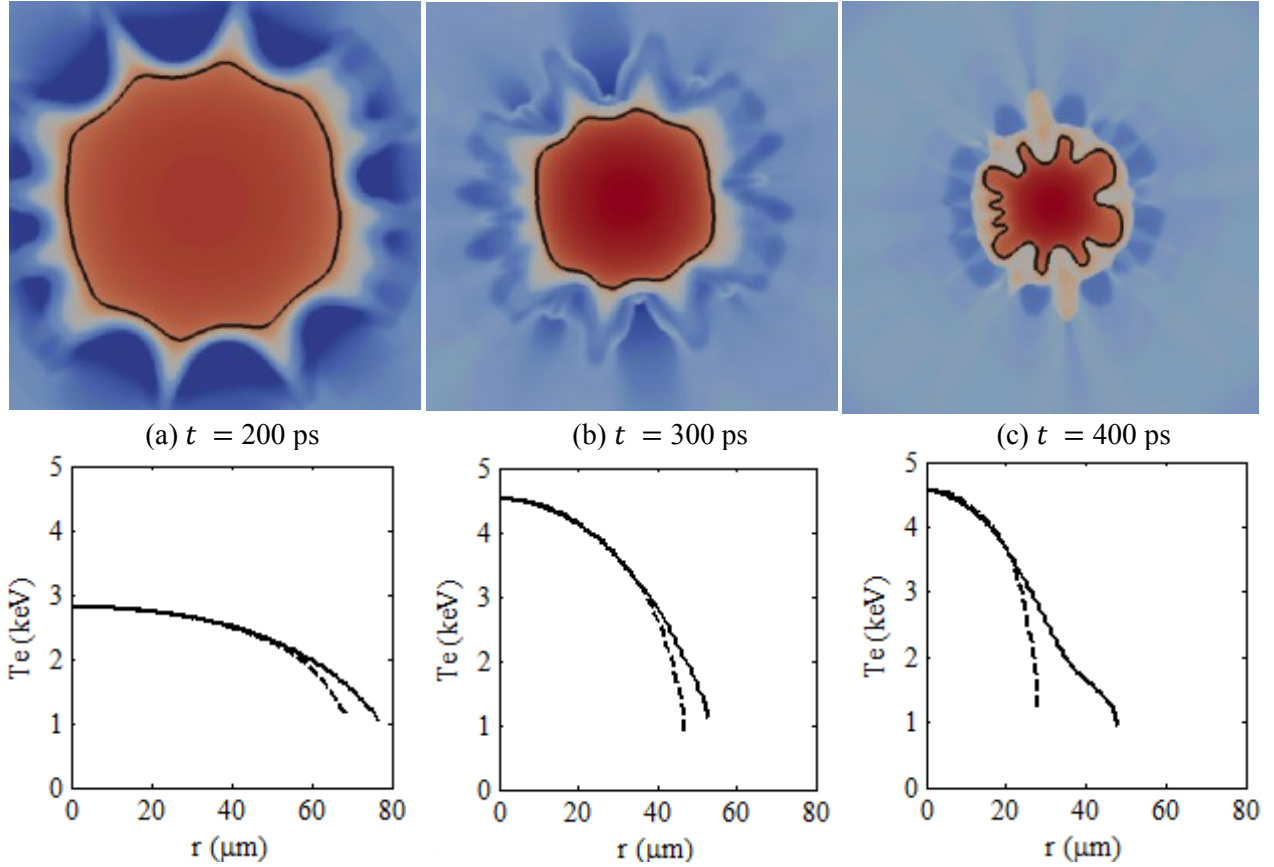


Figure 3.18: Hotspot electron temperature slices and lineouts through bubble (solid) and spike (dashed) regions at (a) $t = 200$ ps, (b) 300 ps and (c) 400 ps. The hotspot edge is shown as a black line in the temperature slices.

The second period of increased thermal conduction loss rate in the perturbed hotspot, beginning at $t \sim 350$ ps, occurs at the time that the hotspot's surface area begins to increase above $4\pi\langle R \rangle^2$ (Figure 3.19(b)).

The model given by (3.3.3) does not account for the effect of the perturbation on W_{TC} at this stage, but if (3.3.4) is used taking the surface area from the code, the variation of W_{TC} with perturbation amplitude is reproduced to within 13% (Figure 3.19(a)). The beginning of this surface-area dominated phase occurs sooner for larger perturbation amplitudes, as illustrated in Figure 3.19(a). In the $\delta u_R/u_R = 0.5$ simulation, the surface area is the largest factor in the γ_{TC} increase at $t = 400$ ps, but the $\delta u_R/u_R = 0.2$ simulation is still in the gradient-compression dominated phase at this time, and L^{-1} is the most significant factor. The onset of the surface-area dominated phase also depends on wavelength; in the shorter-wavelength $N = 162$ simulation it begins approximately 100 ps earlier, which agrees with the ~ 100 ps earlier deviation of the $N = 162$ simulation's surface area from $4\pi\langle R \rangle^2$ (Figure 3.14).

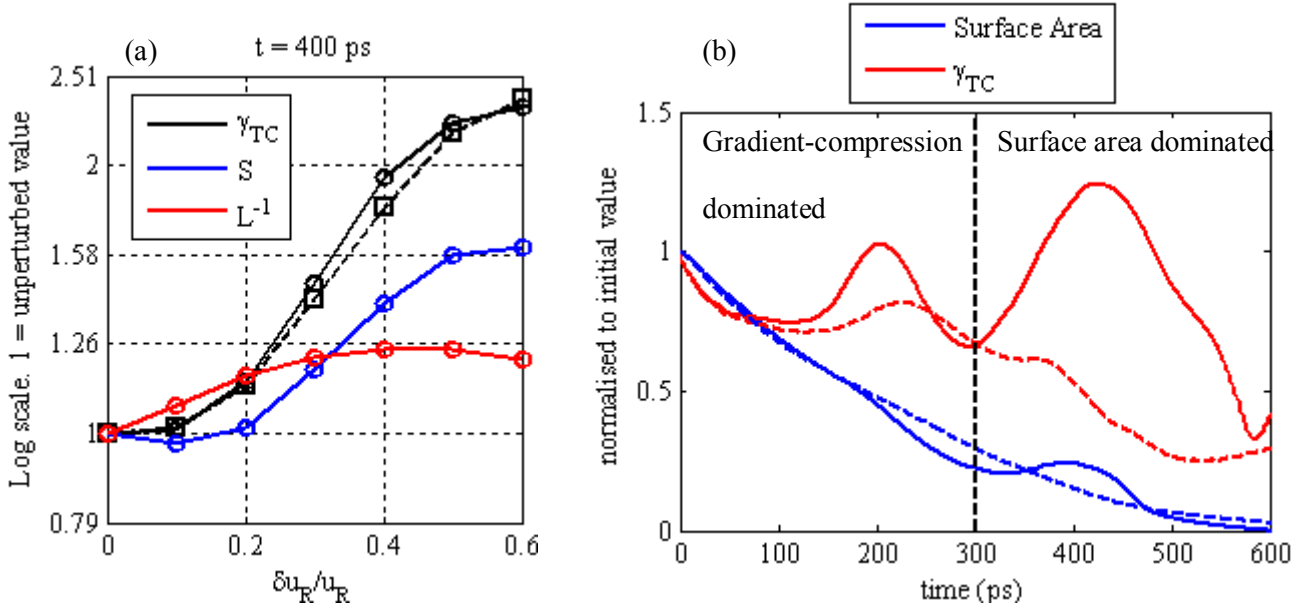


Figure 3.19: (a) Perturbation amplitude dependence of the thermal conduction rate (black) broken into factors of the hotspot surface area (blue), and thermal gradient (red). (b) Comparison of the thermal conduction loss rate in the $N = 42$ perturbed simulation (solid) and the unperturbed simulation (dashed).

In addition to the earlier onset of the surface area dominated thermal conduction, the $N = 162$ simulation has a slightly higher work rate around $t = 350$ ps, which can also be attributed to its higher S/V . Apart from these two effects, the shorter wavelength simulation differs little from the $N = 42$ simulation. The differences in work, radiation emission and fusion rates observed in the shape-deformed target were ultimately caused by the higher $\langle u_R \rangle$, which increased the rate of compression of the fuel in the

case of work, increased the density in the case of radiation, and increased the thermal energy via the work rate in the case of fusion. The shorter-wavelength perturbation has near-identical rates of these processes; the distribution of mass between the bubbles and spikes is the same, giving rise to a near-identical behaviour of $\langle u_R \rangle$ with time (Figure 3.20).

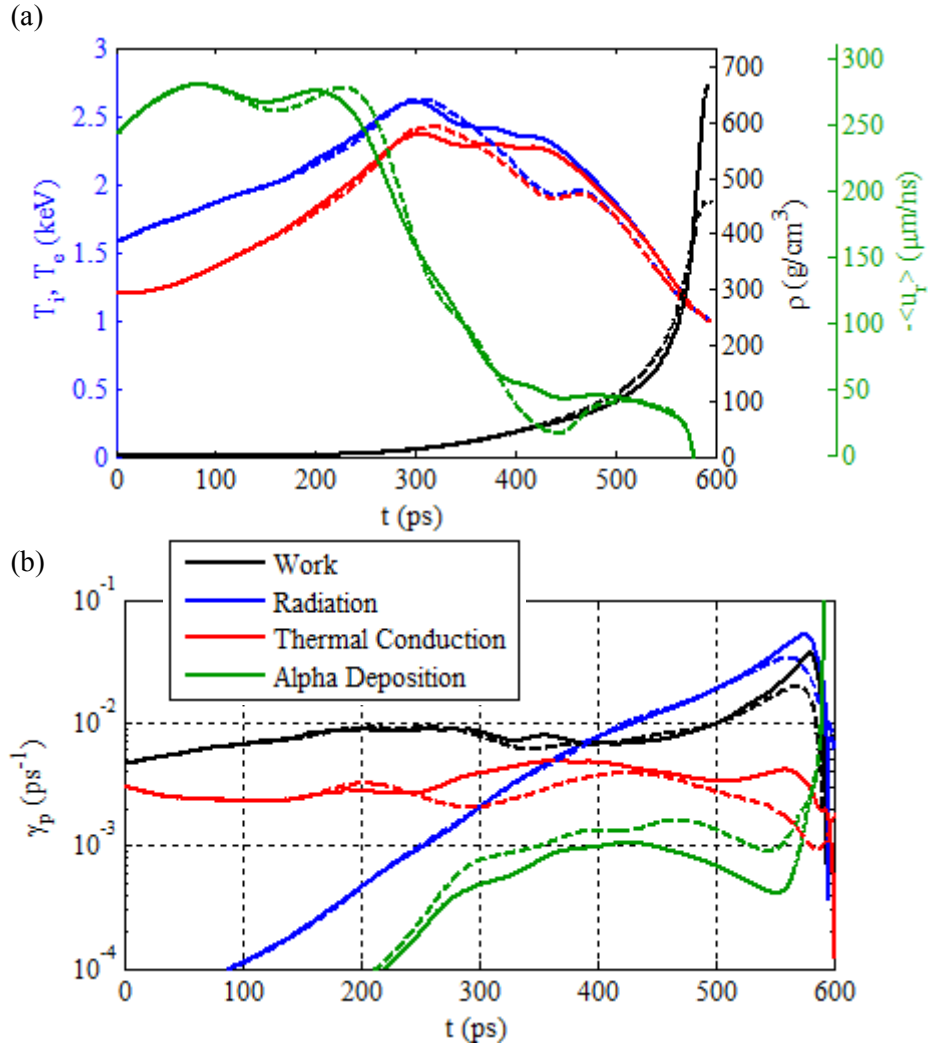


Figure 3.20: Comparison between (a) hotspot parameters and (b) energy loss/gain rates of $N = 162$ (solid) and $N = 42$ (dashed) stagnations.

If a hotspot bubble is thin enough, the local increase of the thermal cooling rate causes it to rapidly cool below the hotspot threshold temperature and collapse. Regions with a cooling timescale longer than the stagnation timescale will remain hot, and simply result in an increased global energy loss from the hotspot. Regions with a cooling timescale below the stagnation timescale will thermally equilibrate with the shell, and contract to its density due to the uniform pressure. This is the distinction between the two wavelength regimes

of perturbation, ‘shape deformation’ and ‘cold fuel mix’ (alternatively ‘thermal mix’). For simplicity, we will assume the small region of the hotspot edge we are interested in is spherical in shape. The characteristic cooling timescale for an enclosed sphere of hotspot material surrounded by shell material may be estimated by $\tau = 1/\gamma_{TC}$. Hence the cooling rate for a sphere of radius $\mathcal{R}$ is

$$\tau = \frac{E}{\kappa(\mathcal{R})S\nabla T|_{r=\mathcal{R}}} \approx \frac{e}{m_i} \frac{\mathcal{R}^2}{3q} \frac{\rho_c}{\kappa(\mathcal{R})(1 - T_{\mathcal{R}}/T_c)}. \quad (3.4.5)$$

Inserting typical values for a small, isolated region of hotspot material such as can be seen in Figure 3.22 ($\rho_c \approx 5\text{e}4 \text{ kg m}^{-3}$, $\kappa(\mathcal{R}) \approx 2\text{e}11 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$, $(1 - T_{\mathcal{R}}/T_c) \approx 1$, $q \approx 3$) gives a ‘mix threshold scale’ of

$$\mathcal{R}(\mu\text{m})^2 \approx \tau(\text{ps}). \quad (3.4.6)$$

So an enclosed sphere must be smaller than around $10 \mu\text{m}$ to thermally collapse within the 100 ps timescale over which the stagnation takes place. This sets an approximate scale boundary between the two perturbation wavelength regimes. The influence of perturbation wavelength on thermal mix is illustrated by Figure 3.11. Figure 3.11(a) shows an $N = 42$ perturbation in which the bubble ion temperature is around 2 keV . The hotspot bubbles in the $N = 162$ simulation (Figure 3.11 (b)) have an ion temperature of around 1 keV at the most radially-inward end, falling smoothly to a few hundred eV at their tips. Therefore the material in the $N = 162$ bubbles does not contribute significantly to the yield and can be considered to have left the hotspot, whereas the $N = 42$ bubbles are still part of the hotspot, although they are increasing its overall energy loss. While the mix threshold scale is sometimes quoted as being as high as Legendre mode 50 [48], this suggests modes as low as 12 ($\sim\sqrt{162}$) may be on the boundary.

Finally, it is interesting to note that beyond a particular initial velocity perturbation amplitude, the yield of $N = 42$ stagnations begins to increase (Figure 3.21). For large values of $\delta u_R/u_R$, the extra work done by the spikes in the heating phase of the stagnation raises the thermal energy sufficiently high to produce more neutrons in total, despite the earlier onset of the cooling phase. The relationship of the yield to the perturbation amplitude in an asymmetric, higher-bandwidth perturbation, and other effects not seen in the single-mode case, are discussed in the next section.

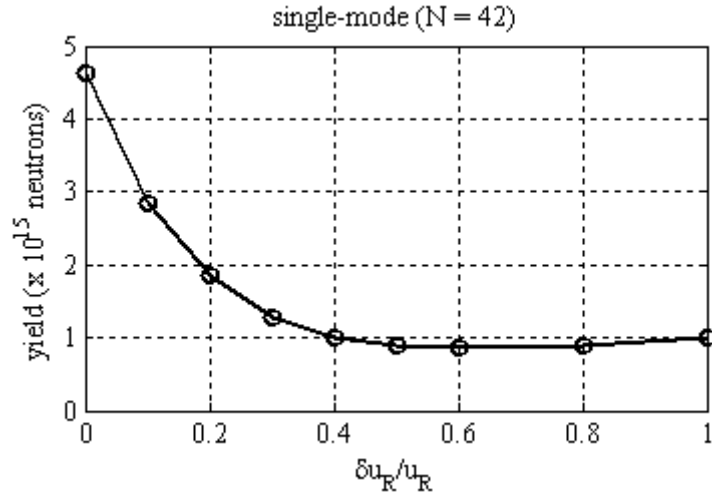


Figure 3.21: Yield vs. perturbation amplitude for $N = 42$ single-mode stagnations.

3.5) Additional effects on hotspot energy balance in multi-mode simulations

The asymmetric and multi-wavelength nature of a multi-mode perturbation alters the energy balance of the hotspot somewhat from the single-mode case. In this section, the differences in the energy balance caused by a particular multi-mode simulation are discussed. The initial amplitude of each node in this simulation is given by

$$\frac{\delta u_R}{u_R} \left(\frac{12}{N} \right)^{1/4} \xi \quad (3.5.1)$$

where ξ is a random variable distributed uniformly between 0 and 2. $\delta u_R / u_R$ is a total perturbation amplitude, and the simulation focused on in this section has a $\delta u_R / u_R$ of 0.5. The $N^{-1/4}$ scaling of the amplitudes is roughly equivalent to the $\sqrt{\lambda}$ dependance of the Rayleigh-Taylor spike velocity in the nonlinear phase. Modes between $N = 12$ and $N = 642$ are included, although $N = 642$ is on the threshold of resolvability. The $N = 12$ mode has its amplitude reduced by a factor of 2 because otherwise it grows much larger than the other modes, probably due to its large saturation amplitude. Figure 3.22 shows the shape of the multi-mode hotspot discussed in this section at $t = 300$ ps and $t = 400$ ps.

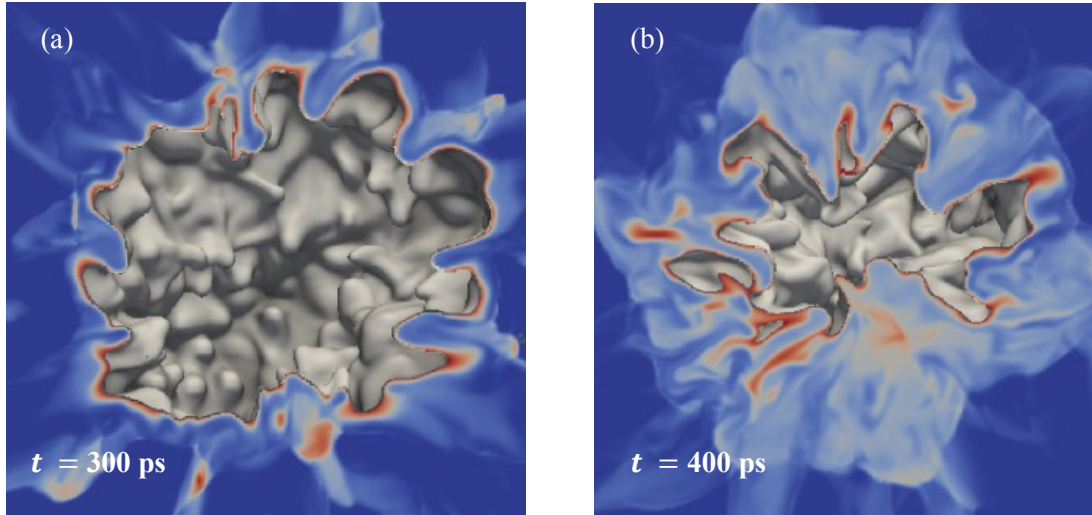


Figure 3.22: The grey contour shows the hotspot boundary in the multi-mode simulation discussed. The colourmap shows a slice of the ion temperature in the non-hotspot regions.

The total perturbation amplitude is scaled to match the experimental yield of $\sim 6 \times 10^{14}$ neutrons; the yield in the simulation is 5.99×10^{14} . Figure 3.23 shows the behaviour of the yield as the initial velocity-perturbation amplitude is varied. The yield curve smoothly intercepts the y-axis at the clean yield as expected, and does not tend to 0 with high perturbation amplitude. This is consistent with our observations of the effect of perturbations on the stagnation, where perturbation growth causes more work to be done early in time and allows some fusion to occur before the burn is quenched. In fact, the yield increases slightly after $\delta u_R/u_R = 0.65$, as observed in the single-mode case.

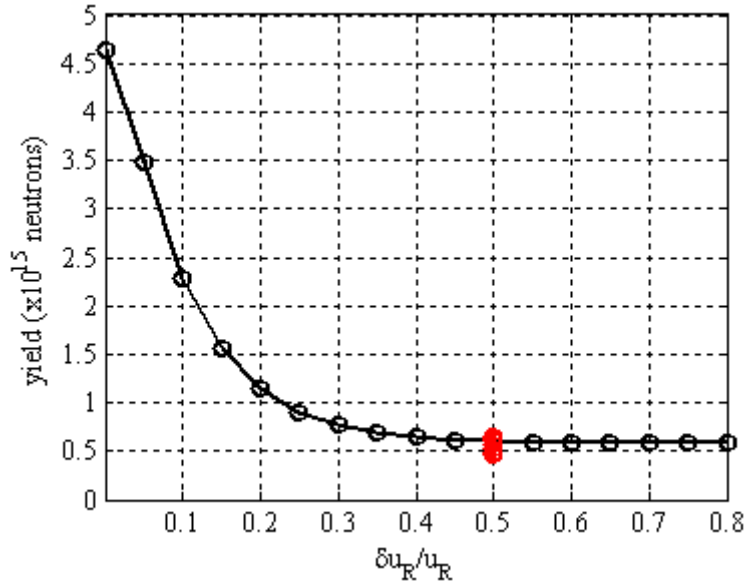


Figure 3.23: Neutron yield vs. perturbation amplitude at bang time. The red points give the values for 10 simulations with the same initial amplitude as the one described in the text but different random number seeds. The range of their yield values is 0.19×10^{15} neutrons.

Figure 3.24 shows the time-evolution of the state variables (a) and rates (b) in the multi-mode perturbed simulation (solid), compared with a single mode case (dashed). The single-mode velocity perturbation amplitude is chosen such that its initial $\langle u_R \rangle$ is the same as that of the multi-mode, which results in an amplitude of 0.8, larger than the simulation in Section 3.4. The neutron yield of the single-mode perturbation is 9.01×10^{14} , $3/2$ times larger than the multi-mode yield. The cause of the lower yield in the multi-mode simulation is the lower peak temperature. Two factors are responsible for this: the lower work rate before the temperature peak, and the higher thermal conduction rate at the time of the temperature peak.

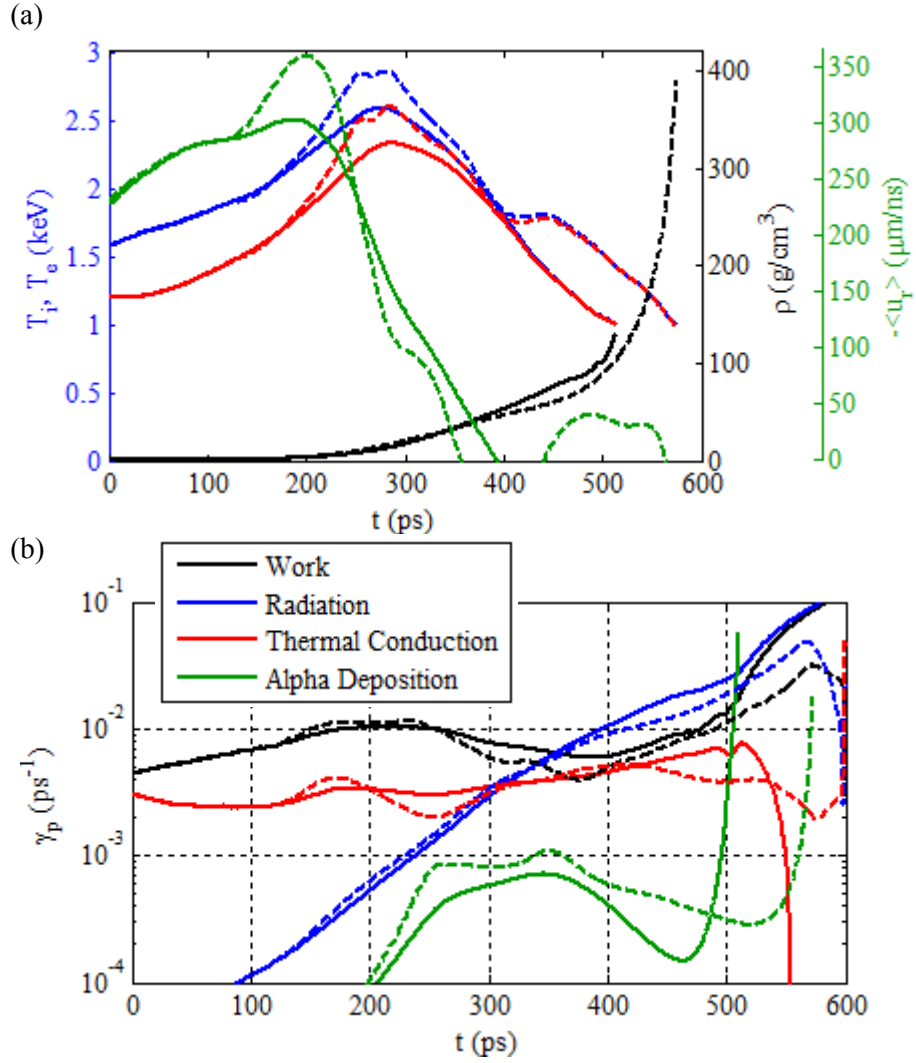


Figure 3.24: Comparison of (a) Hotspot parameters and (b) energy loss/gain rates of the multi-mode (solid) stagnation against an $N = 42$ stagnation (dashed) of the same initial average hotspot surface velocity $\langle u_R \rangle$ and total kinetic energy. The dotted line shows the rates for the unperturbed simulation.

The lower work rate before the temperature peak is related to the lower $\langle u_R \rangle$ present in the multi-mode simulation at around $t = 200$ ps. By the divergence theorem, the work rate is proportional to the divergence of the velocity field integrated over the hotspot, and the less divergent velocity field in the multi-mode simulation is probably the cause of the lower $\langle u_R \rangle$. The interaction between modes results in spike trajectories being directed off-centre; for example in the case that a small spike is located on the side of a larger spike, the transverse velocity field of the larger spike directs the trajectory of the smaller spike sideways. This can be seen occurring in Figure 3.22. In order to assess the impact of this on the work rate, we ran a single-mode simulation with each spike's trajectory aimed at a different random point within $40 \mu\text{m}$ of the hotspot centre. The total work done on the hotspot in the simulation with offset trajectories was 18% lower

than in the on-centre simulation. The offset simulation's work rate drops monotonically below that of the on-centre simulation as the spikes grow; at no point is there extra work done by the offset spikes. When the same procedure is carried out for a multi-mode simulation, an off-centre simulation does not show appreciably different behaviour to the simulation with centrally-aimed spikes, demonstrating that transverse trajectories of the spikes are generated spontaneously by mode-mode interaction in the multi-mode simulation. With a larger proportion of the velocity field directed off-centre, the multimode simulation has a less divergent velocity field and hence a lower work rate before the bang time.

As the perturbation grows larger, the multi-mode work rate rises above that of the single-mode. This is partly because there is more kinetic energy remaining in the multi-mode system because less has been thermalised so far – this can be seen in Figure 3.24(a) as a lower $\langle u_R \rangle$ in the single-mode case after $t = 260$ ps. In addition to this delay in thermalisation, it appears the multi-mode nature of the perturbation actually allows more thermal energy to be delivered to the hotspot in total. The total work done on the multi-mode hotspot by the end of the stagnation is 9% higher in the multi-mode case compared with the single-mode equivalent; it rises above the total work done in the single-mode case at $t = 318$ ps. $t = 300$ ps is around the time that neighbouring, transversely-moving spikes begin to collide, and the divergence of the velocity field in the surrounding hotspot material increases (with a negative sign) as they meet. $-\nabla \cdot \mathbf{u}$ is around $3 \times 10^{11} \text{s}^{-1}$ at the edge of a typical multi-mode bubble at $t = 400$ ps, compared with $4 \times 10^{10} \text{s}^{-1}$ in the single-mode case. This results in heating of hotspot bubbles; the ion temperature of a typical 20 μm -scale structure in the multi-mode simulation at $t = 400$ ps is 2 keV, compared with 1.3 keV in the equivalent single-mode simulation. In addition to locally increasing the work rate, collisions between neighbouring spikes may enhance the perturbation growth by combining the mass and kinetic energy of two or more spikes into a jet with a lower cross-sectional area. Such a jet interacts with a smaller amount of hotspot material per second and decelerates more slowly.

As with the single-mode case, the multi-mode thermal conduction rate is influenced by compression of the temperature gradient by the motion of the spikes, and the effect of the perturbation on the hotspot surface area. In the temperature gradient dominated phase, the single-mode hotspot has a higher thermal conduction rate than the multi-mode. During this time (around $t = 170$ ps), a higher Mach number in the

single-mode case is causing greater temperature gradient compression; the single-mode stagnation has a higher $\langle u_R \rangle$ at this time. As observed in the previous section, the single-mode thermal conduction rate falls briefly between this thermal-gradient dominated phase and the surface-area related thermal conduction increase phase, due to the falling Mach number and the smaller hotspot radius in the single-mode simulation compared with the unperturbed. However, the multi-mode thermal conduction rate does not fall off as sharply. The difference in thermal conduction rate between the single- and multi-mode targets at $t = 250$ ps is equal to the difference in their surface areas. As shown by Figure 3.25, both simulations' surface areas differ from $4\pi\langle R \rangle^2$ by approximately the same small amount at this time, so the greater surface area in the multi-mode simulation is primarily due to its larger radius.

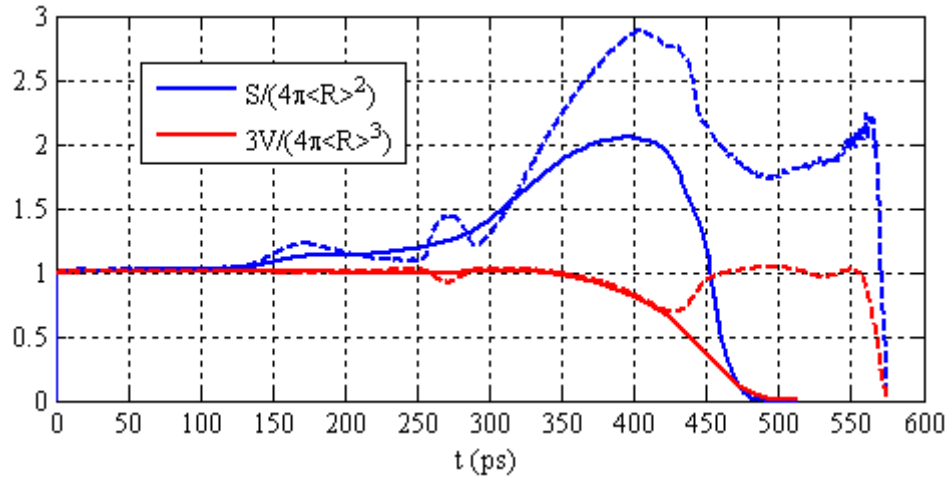


Figure 3.25: Deviation from the values of a sphere of equal radius of: surface area (blue) and volume (red) of the multi-mode perturbed hotspot (solid) and the equivalent $N = 42$ single-mode hotspot (dashed).

As collisions between neighbouring spikes begin to occur in the multi-mode hotspot, temperature gradient compression begins to have more of an effect on the thermal conduction rate. At $t = 400$ ps, the temperature gradient at the hotspot edge in the spike regions of the multi-mode simulation is typically around $0.3 \text{ eV}/\mu\text{m}$, compared to $0.2 \text{ eV}/\mu\text{m}$ in the single-mode hotspot and $0.16 \text{ eV}/\mu\text{m}$ in the unperturbed hotspot. Compared with the single-mode simulation, the multi-mode does not have a significantly lower thermal conduction rate at $t = 400$ ps despite having a 40% smaller surface area. Compared with the unperturbed simulation, the multi-mode has an 85% higher thermal conduction rate, but only a 35% higher surface area. The greater thermal gradient compression in the multi-mode hotspot is responsible for the rest of the increase. The effect of transverse spike trajectories closing regions off, as seen in Figure 3.22, can also create regions

with a thermal cooling timescale shorter than the hydrodynamic timescale of the stagnation, contributing to the thermal conduction loss rate.

The fusion rate is significantly lower in the multi-mode case, as expected from the lower temperature of the multi-mode hotspot. However, this is even true at $t = 400$ ps when both simulations have the same mass-averaged temperature, and the multi-mode has a higher average density. The analytic solution for the fusion rate (3.4.4) gives a 5% higher γ_α at $t = 400$ ps in the multi-mode relative to the single-mode, whereas the value from the simulation is 30% lower. This is probably because the distribution of temperatures in the multimode hotspot is biased towards the edge relative to the model 1D profile. Both the single-mode and the multi-mode hotspots have the same E/M (same mass-averaged temperature), but in the multi-mode case more of the thermal energy is located in the denser, cooler outer regions, whereas the single-mode hotspot has more thermal energy concentrated in the centre. The former configuration produces a lower fusion rate because of the fusion rate's stronger dependence on temperature than density.

Until $t \approx 400$ ps the multi-mode radiation emission rate does not differ much from the single-mode case, due to its weak temperature dependence. At about $t = 400$ ps, the higher density of the multi-mode simulation causes the radiation emission rate to increase above the single-mode case. This is caused primarily by a higher hotspot mass rather than a smaller volume. According to (3.4.3), the radiation emission rate is expected to be proportional to

$$\gamma_r \propto \frac{M}{V} T_{ma}^{-0.6}. \quad (3.5.2)$$

since the mass-averaged temperature $T_{ma} \propto E/M$ (as per equation (3.6.1)). So for the same mass-averaged temperature T_{ma} , as is the case at $t = 400$ ps (Figure 3.24(a)), one would expect $\gamma_r \propto M/V$. The hotspot volumes are almost exactly the same at this time, but the mass of the multi-mode hotspot is 9% higher, 8.17 μg compared with 7.46 μg . The higher hotspot mass is a consequence of the greater thermal conduction in the multi-mode hotspot. The radiation loss rate is 16% higher in the multi-mode simulation at this time. The volume factor contributes another 3%. The extra 4% may be caused by the higher radiation emission rates which can be seen on the tips of spikes and in regions where transversely-moving spikes are colliding (Figure 3.26). These are the regions possessing higher $-\nabla \cdot \mathbf{u}$, which as a result have a higher density and temperature than similar regions in the single-mode hotspot. This influences the radiation emission more than the fusion

rate because the regions on the edge of the hotspot only contribute a small amount to the total fusion rate due to its strong temperature dependence, but they have a more significant effect on the total radiation emission. If this effect is the source of the small discrepancy in the difference between the multi-mode and single-mode γ_r and that calculated from the analytic model, it is not a highly significant effect.

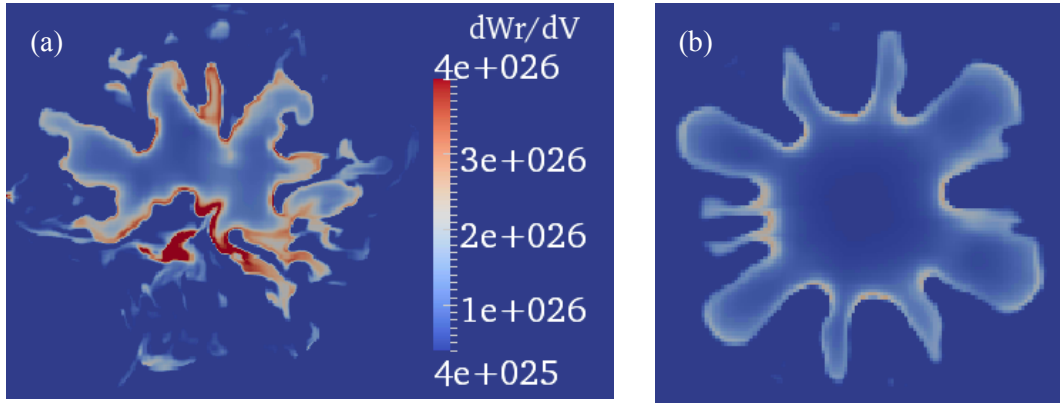


Figure 3.26: Emissivity slices of the multi-mode (a) and the equivalent single-mode $N = 42$ (b) stagnations at $t = 400$ ps. The multi-mode shows much greater emissivity in regions where the convergence of perturbation structures is compressing the fluid.

The generality of the preceding discussion to the random seed of the perturbation can be tested by running several multi-mode simulations with the random number generator seeded from the system clock. This has been performed for 10 additional simulations. The range of the yields is 1.9×10^{14} neutrons, a variation of $\pm 18\%$ around the mean yield of 5.4×10^{14} neutrons. There is some spread in the energy source rates, but the phenomenology is the same (Figure 3.27). In particular, the lower work rate and higher thermal conduction relative to the single-mode stagnation at the time of peak temperature occur independently of the random seed of the perturbation.

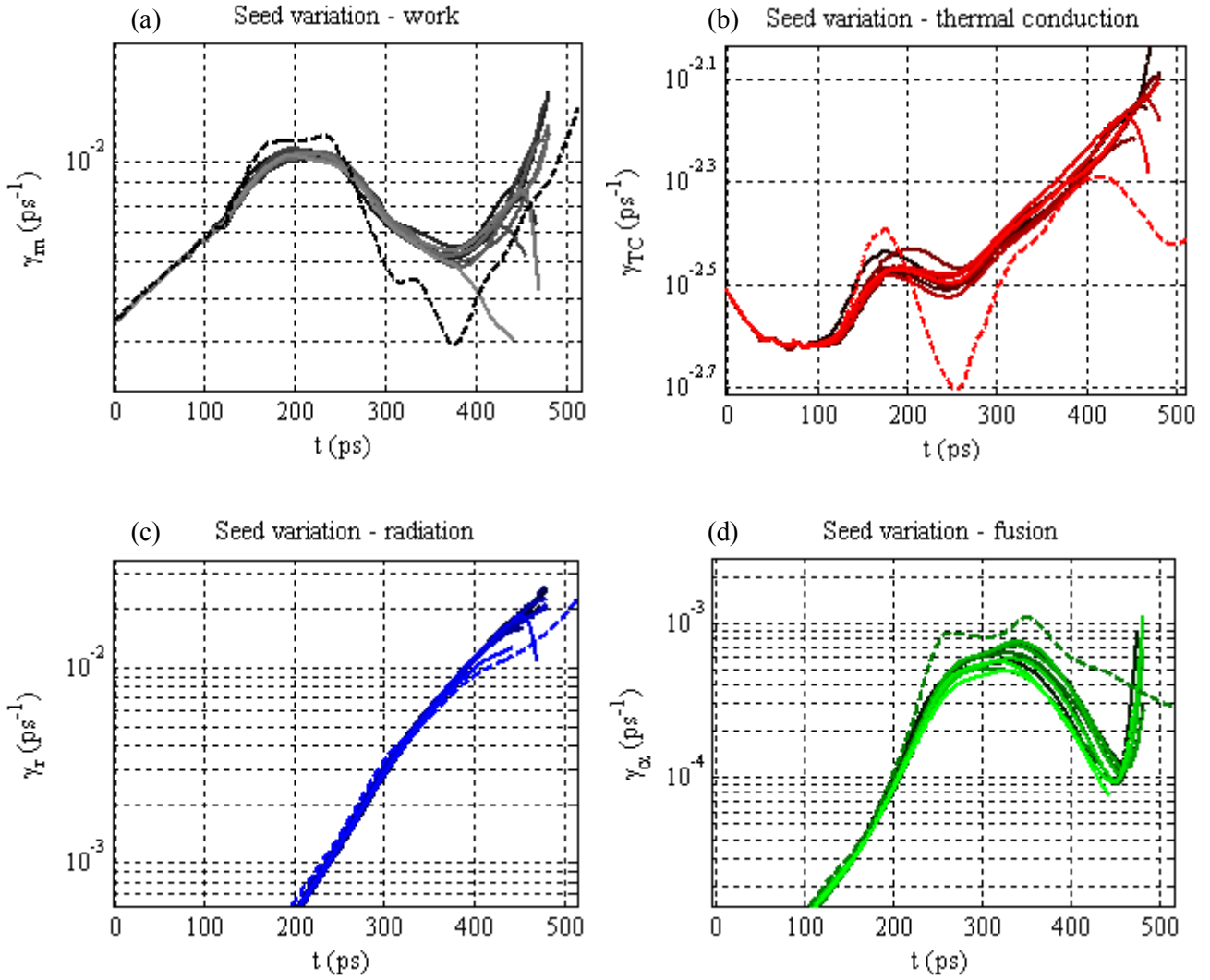


Figure 3.27: Variation of time-dependant work rates with the seed of the random number generator used to generate the multi-mode perturbation. Each differently-shaded solid line is generated with a different random seed. The dashed line shows the equivalent $N = 42$ single-mode perturbation for comparison.

3.6) Summary and discussion

The simple radial profiles upon which the hotspot converges during the stagnation phase allow exact calculation of the energy source terms acting on the hotspot. Of particular interest is the exact definition of the thermal scale length this enables, and the fact that the radiation loss and fusion generation rates take the same form as the volumetric rates, multiplied by the hotspot volume and a profile factor. It is also simple to write the energy source terms as functions of the hotspot mass, energy and volume. The 1D analysis is an improvement on 0D analysis such as that of Widner [1], Lindl [4] and Atzeni [8], which implicitly assumes the hotspot properties are uniform. While ignition conditions are usually generated by computational studies,

such as those carried out in 1D by Hermann et al. [12] and the 2D analysis used to develop the ITF parameter [28], an analytic study of the dependence of the separate hotspot energy source terms on the hotspot properties is useful in understanding the physics of the hotspot energy balance, and in particular in investigating with precision the effects of a perturbation on the energy balance of the hotspot at stagnation.

A concise way to assess the difference made by a 1D analysis of the hotspot energy balance is to produce a heating map akin to Figure 1.4 from the 1D volume-integrated source terms. Obviously, using T_c and $\rho_c R$ as the axes does not provide a meaningful comparison, since the temperature throughout most of the hotspot is lower than T_c and the hotspot ρR is higher than $\rho_c R$. A more meaningful measure of the hotspot temperature is the ‘mass averaged’ temperature

$$T_{ma} = \frac{m_i E}{3e M}. \quad (3.6.1)$$

which would be the same for a hotspot of any profile having the same thermal energy E and mass M . T_{ma} can be related to the hotspot central temperature T_c by

$$T_{ma} = \frac{\int T \rho dV}{\int \rho dV} = T_c / H_1(T_c, q). \quad (3.6.2)$$

The areal density of the 1D hotspot is given by

$$\langle \rho R \rangle = \int_0^R \rho dr = H_1(T_c, q/3) \rho_c R. \quad (3.6.3)$$

Each of the volume-integrated source terms may be written in terms of $\langle \rho R \rangle$, T_{ma} and R by substituting $\rho_c = \langle \rho R \rangle / H_1(T_c, q/3) R$ and $T_c = H_1(T_c, q) T_{ma}$ as follows:

$$W_m = \frac{8\pi}{3} \frac{e}{m_i} \langle \rho R \rangle T_{ma} R \langle u_R \rangle \times H_1(T_c, q) / H_1(T_c, q/3) \quad (3.6.4)$$

$$W_r = \frac{4\pi}{3} A_r \langle \rho R \rangle^2 T_{ma}^{1/2} R \times H_{3/2}(T_c, q) H_1(T_c, q)^{1/2} / H_1(T_c, q/3)^2 \quad (3.6.5)$$

$$W_\alpha = \frac{4\pi}{3} A_\alpha \langle \rho R \rangle^2 T_{ma}^4 R \times H_{-2}(T_c, q) H_1(T_c, q)^4 / H_1(T_c, q/3)^2 \quad (3.6.6)$$

$$W_{TC} = 4\pi A_{TC} q R T_R^{5/2} (T_{ma} H_1(T_c, q) - T_R) / \ln \Lambda \quad (3.6.7)$$

The heating map produced from these volume-integrated source terms is shown in Figure 3.28. The self-heating region (for which $\langle u_R \rangle > 0$) is much larger when the 1D profiles are taken into account (compare Figure 1.4). This is caused primarily by the lower thermal conductivity in the 1D model. In the 1D model, the thermal conductivity at the hotspot edge is used, whereas 0D Widner-type models use the thermal conductivity in the hotspot centre. Since the temperature at the hotspot edge is typically around 1/4 of the central temperature, this reduces the thermal conductivity by a factor of approximately $(1/4)^{5/2} \sim 0.03$. Consequently, the hotspot surface area to volume ratio $3/R$ can be significantly larger (ie. $\langle \rho R \rangle$ can be smaller) before thermal losses exceed the rate of fusion energy deposition. When alpha particle transport is included in the model (see Chapter 5), the reduced thermal conduction is compensated by alpha particle loss in this high T_{ma} and low $\langle \rho R \rangle$ region, and the self-heating region approximates that of the Widner-type model.

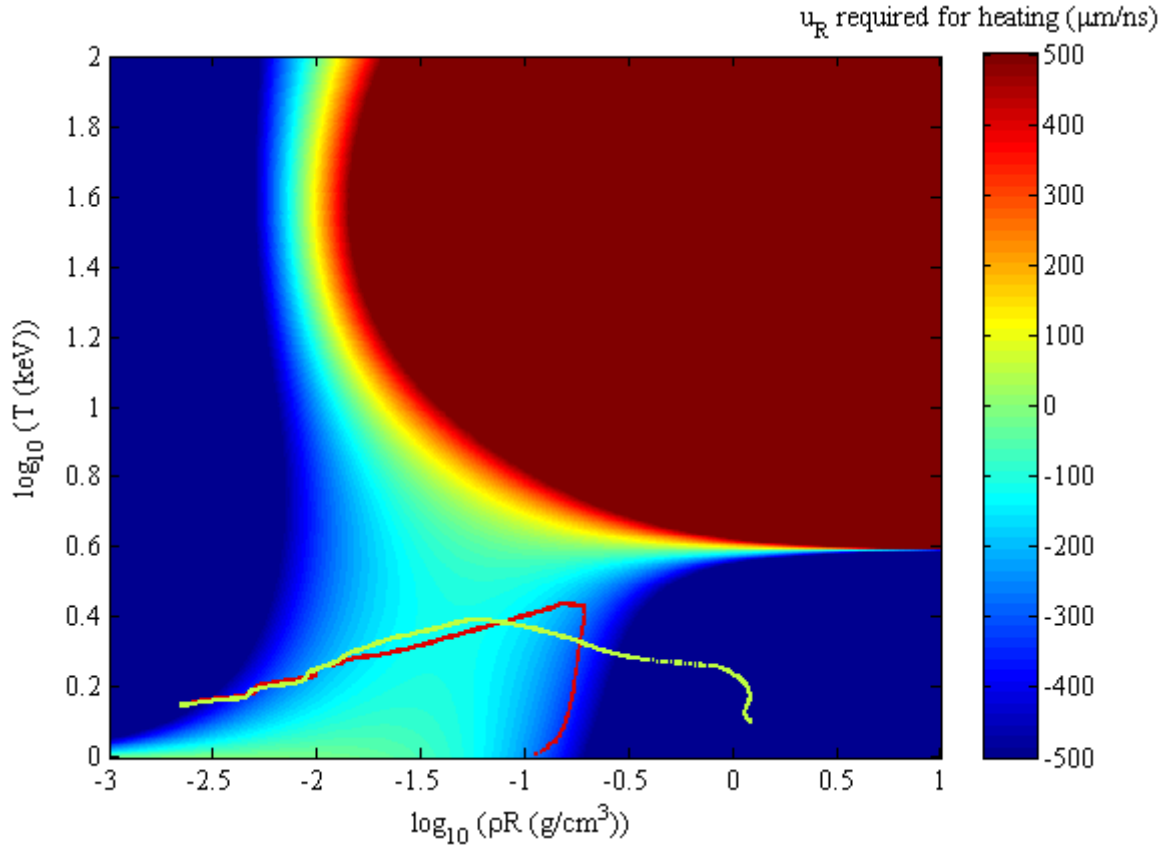


Figure 3.28: The $\langle u_R \rangle$ required for self-heating as a function of the areal density and mass-averaged temperature of the self-similar hotspot profiles, as calculated by the model developed in Section 3.3. Compare to Figure 1.4. The trajectories of the unperturbed stagnation (red) and $N = 42$ single-mode perturbed stagnation from Section 3.4 are shown.

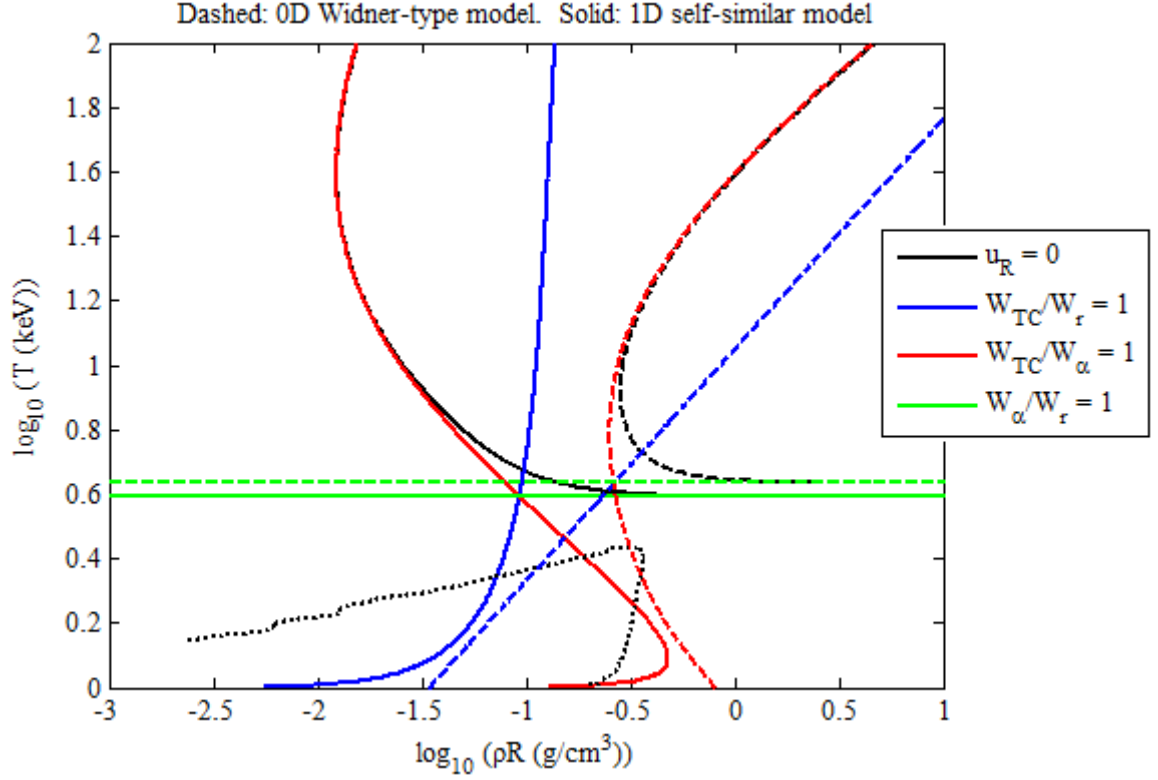


Figure 3.29: The self-heating or ‘ignition’ line (black) and lines separating the dominance of thermal conduction and radiation loss (blue), thermal conduction and alpha heating (red) and alpha heating and radiation loss (green). The solid line shows these lines as calculated by the 1D model using the self-similar profiles converged upon by the unperturbed hotspot. The dashed line shows these lines as calculated by the Widner-type 0D model outlined in Section 1.2. The unperturbed stagnation’s trajectory is shown as a dotted black line.

Various transition lines of interest are shown in Figure 3.29. The lower temperature bound of the self-heating region is set by the temperature at which $W_\alpha/W_r > 1$ (termed the ‘ideal ignition temperature’ by Lindl [4]). When $q_i = 3$, the 1D model calculates this to be 3.72 keV, slightly below the 0D model’s value of 4.32 keV. However, if $q_i = 1.5$ is used, the 1D model gets 4.31 keV, remarkably close to the 0D value. If the $q_i = 3$ result is appropriate as appears to be the case at peak burn (Figure 3.5(a)), then the (mass-averaged) temperature required for ignition at $\langle \rho R \rangle \sim 0.3 \text{ g/cm}^2$ is slightly lower than one might conclude from the 0D model. This line is also significant because it marks the temperature at which increasing the hotspot density causes an increase in the heating rate rather than a decrease. Similarly, the line separating the thermal-conduction dominated region from the radiative-loss dominated region ($W_{TC}/W_r = 1$) marks a point at which the hotspot density becomes a significant factor in the energy loss. The thermal conduction loss rate is

independent of the hotspot density, but once the hotspot crosses into the radiation-dominated regime, any compression increases the loss rate. In terms of $\langle \rho R \rangle \propto M/R^2$, an increase of $\langle \rho R \rangle$ at constant M decreases the loss rate in the thermal conduction regime, but increases it in the radiation-dominated regime. The transition into the radiative-loss regime ($W_{TC}/W_r < 1$) occurs at lower $\langle \rho R \rangle$ or higher T_{ma} values in the 1D model, due to the weaker thermal conduction loss of the 1D hotspot. The radiation loss rate is reproduced very well by the 0D Widner-type model along the trajectory of the unperturbed stagnation if the mass-averaged temperature is used, differing by only 7% at peak emission. This indicates that the shift of the $W_{TC}/W_r = 1$ line is primarily due to the change in thermal conduction loss, and the lower $W_\alpha/W_r > 1$ transition line is primarily due to a higher fusion rate for the 1D profile at a given T_{ma} and $\langle \rho R \rangle$. The larger ignition region is a combination of both of these effects.

The difference between the Widner-type model and the 1D model can be assessed by fitting the hypergeometric functions in (3.6.5) - (3.6.4) to power laws of T_c , and converting from T_c to T_{ma} according to the approximation $H_1(T_c, q) \approx T_c(\text{keV})^{0.5}$ shown in Figure 3.10. This procedure yields approximate formulae for the source terms entirely in terms of T_{ma} , $\langle \rho R \rangle$, $\langle u_R \rangle$ and R as per the Widner-type model:

$$W_m = 0.78 T_{ma}^{0.28} W_m^{wid} \quad (3.6.8)$$

$$W_r = (0.064 T_{ma}^2 + 0.7) W_r^{wid} \quad (3.6.9)$$

$$W_\alpha = (0.26 T_{ma}^2 + 0.31) W_\alpha^{wid} \quad (3.6.10)$$

with T_{ma} in keV. W_x^{wid} refers to the Widner-type formulae (1.2.1) - (1.2.3) and the fits were calculated for $q_e = q_i = 3$ and $2.5 \text{ keV} < T_c < 7 \text{ keV}$. The agreement between the Widner-type W_r and the 1D W_r along the trajectory of the unperturbed target is clear; the prefactor is 0.96 for $T_{ma} = 2 \text{ keV}$. However, at $T_{ma} = 3 \text{ keV}$ the prefactor is 1.28. The work rate prefactor is 0.95 for $T_{ma} = 2 \text{ keV}$ and 1.06 for $T_{ma} = 3 \text{ keV}$, so the Widner-type work rate agrees well for sub-ignition temperatures. The W_α prefactor is 1.35 for $T_{ma} = 2 \text{ keV}$ and 2.65 for $T_{ma} = 3 \text{ keV}$, which agrees with the lower $W_\alpha/W_r = 1$ temperature in the 1D model. The conversion of T_c to T_{ma} in the thermal conduction term is obvious and has been omitted since the large difference between this term and the Widner-type term has already been discussed.

Perturbations to an all-DT stagnating ICF target reduce the yield produced, an effect variously called ‘shape deformation’ or ‘cold fuel mix’/‘thermal mix’ in the literature depending on wavelength. The phenomenology of this process has been worked out computationally in this chapter. The perturbation was found to cause a number of interrelated effects, primarily by altering OD hotspot properties such as the total thermal energy, volume, and surface area, but also by causing the hotspot radial profiles to deviate from the profiles of the unperturbed hotspot.

Perhaps the most important effect of the hotspot perturbation is a higher radial velocity at the hotspot surface $\langle u_R \rangle$ before the temperature peak. The higher $\langle u_R \rangle$ can be explained by mass transfer from the bubbles to the spikes, which produces a higher average velocity over the hotspot surface for a given kinetic energy. The higher work rate this causes results in an earlier temperature peak compared to an unperturbed target. As a result of the extra thermal energy in the hotspot before the temperature peak, the rate of fusion is increased at this time relative to the unperturbed stagnation, although the fusion rate is still significantly lower than the other rates. Another result of the higher $\langle u_R \rangle$ before the temperature peak is that the hotspot volume is reduced, resulting in a higher density, which increases the radiation loss rate. The increased radiation loss rate, along with a falling-off of the work rate at peak temperature, prevents the temperature increasing any further, and the perturbed hotspot radiatively collapses around 200 ps sooner than the unperturbed.

The effect of the perturbation on the thermal conduction rate may be split into two regimes. Initially, the thermal gradient at the hotspot edge is compressed, which is another consequence of the higher $\langle u_R \rangle$ and occurs more strongly in the spikes than in the bubbles. This effect is probably not a significant factor in the reduced yield of the perturbed hotspot, since it is localised to the edge of the hotspot and only cools this region, having little effect on the total fusion rate in the hotspot. The gradient compression becomes less significant as the kinetic energy of the shell thermalises and $\langle u_R \rangle$ drops away. As the perturbation develops, the increase of the hotspot’s surface area above $4\pi R^2$ becomes the dominant factor in increasing the thermal conduction loss rate. The multi-mode simulation’s surface area reaches $\sim 8\pi R^2$ at peak, doubling the thermal conduction loss rate. The onset of this effect is wavelength dependant, with an $N = 162$ perturbation entering the surface-area dominated regime approximately 100 ps before the $N = 42$ perturbation. The bubbles of the $N = 162$ hotspot are small enough to thermally collapse within the timescale of the stagnation, and eventually

leave the hotspot. As they do so, the thermal conduction rate drops to that of the $N = 42$ hotspot and the fusion rate falls slightly due to the lost mass.

An inefficiency of thermalisation of kinetic energy by perturbed targets results from the anisotropy of ρu_r . The hotspot pressure is lower from the point of view of the spikes than it would be if the entire shell had the ρu_r of the spike regions, so they are not decelerated completely by the hotspot and are able collide with each other, heating the dense-fuel part of the spikes rather than the hotspot. In a related effect, the hotspot pressure is not adequately confined by the bubble regions, since the pressure is higher from the point of view of the bubbles than it would be if the entire shell had the ρu_r of the bubble regions. This causes the bubble regions to expand more quickly. It is commonly assumed that the main effect of a perturbation on the energy balance of a stagnating hotspot is the inefficient thermalisation of kinetic energy. However, time-integrating the extra thermal conduction loss during the surface-area dominated phase and the extra radiation loss due to the smaller radius, these effects are responsible for 0.38 kJ and 2.18 kJ of energy loss respectively, as opposed to the 1.7 kJ lost by inefficient thermalisation. This suggests that the radiation loss increase is at least as important as the inefficient thermalisation. Also, the time-dependence of these effects must be taken into account, and the fact that the thermal conduction loss rate and radiation loss rates are significantly increased around the temperature peak (and these rates are of similar orders of magnitude at this time) demonstrates that thermalisation inefficiency alone is not the single cause of the yield difference.

The effects mentioned above are common to single-mode and multi-mode hotspots. The multi-mode nature of a real perturbation causes several additional effects which result in a lower yield relative to an equivalent $N = 42$ single-mode perturbation. This lower yield is caused primarily by a lower peak temperature. The lower peak temperature results from a lower work rate caused by a less divergent velocity field, and a higher thermal conduction rate. The less divergent velocity field results from the incoherent superposition of perturbation modes generating vorticity. In a single-mode perturbation with spike trajectories directed off-centre, less work is done in total than in an on-centre perturbation. However, the multi-mode perturbation actually does more work in total than the equivalent single-mode due to the collision of neighbouring transversely moving spikes. This probably does not have a large effect on yield, since it heats the cooler regions at the hotspot edge. The multi-mode hotspot also has a higher thermal conduction rate than

the equivalent single-mode, which at first is due to the larger radius in the multi-mode hotspot. The greater thermal conduction results in a higher hotspot mass. As the stagnation progresses and the hotspot radius is reduced, the higher mass of the multi-mode hotspot results in a higher density, and therefore its radiation loss rate is slightly higher than the equivalent single-mode, which by this time has stagnated and has a similar volume to the multi-mode. This multi-mode radiation loss effect occurs after the temperature peak and is relatively minor. The main cause of the yield difference between single- and multi-mode perturbations were the work rate and radius-related thermal conduction rate differences, occurring before and at peak temperature respectively.

The phenomenology of the effect of the perturbation on the hotspot energy balance was shown to be independent of the random seed of the perturbation. It must be noted that the relative amplitudes of the modes remained the same in the analysis. One would expect that if the amplitudes of the modes were closer to one another, the differences between the multi-mode and single-mode perturbations would be enhanced, and the temperature peak would be lower still. Similarly, if the energy in the perturbation were concentrated more into a single mode and the spike amplitudes less random, the temperature peak would be higher, as seen in the equivalent single-mode simulation. The difference between the $N = 42$ and $N = 162$ single mode perturbations sheds light on what would happen if the perturbation spectrum were shifted to shorter wavelengths; the general phenomenology was much the same, but the surface-area related thermal conduction increase occurred sooner.

The yield of (multi- and single-mode) perturbed hotspots was found to tend toward a finite value, or even increase slightly, as the perturbation amplitude increases. This is due to the increased work done before the temperature peak, which raises the reactivity of the hotspot high enough to generate a significant number of fusion reactions before the thermal collapse occurs. However, to perturb the hotspot by an extreme amount in the expectation of achieving ignition-scale temperatures is not a viable strategy, as the hotspot $\langle \rho R \rangle$ is too low at the time the extra work is done and the hotspot is in a region where the ignition temperature is significantly higher than the ‘ideal ignition temperature’ of 3.92 keV.

This work draws particular attention to the importance of the average fluid velocity at the hotspot edge $\langle u_R \rangle$ in causing early thermalisation of kinetic energy and increased radiative loss. One may speculate that the yield reduction by a perturbation could be limited if the time-dependence of $\langle u_R \rangle$ were controlled. It is possible to imagine that the time dependence of $\langle u_R \rangle$ could be tuned by altering the drive pulse shape, perhaps to set up a fluid velocity profile in the shell that increases with radius. This way, the influence of the perturbation on the $\langle u_R \rangle$ could be compensated for, and the yield increased.

Chapter 4

Diagnostics

Producing and interpreting experimental data is a major challenge in ICF. The complexity, high energy-density, short timescale, and small spatial size of the system means there is a limited amount of information available. Target parameters and structure which are crucial to improving yield and achieving ignition can often only be detected indirectly. The majority of this chapter consists of production of synthetic diagnostics from NIF-relevant stagnation simulations, and investigation of how they are affected by the 1D and 3D properties of the hotspot. This allows assessment of the capabilities and limitations of the available diagnostics and comparison between our multi-mode simulation and the experimental data. First, the stagnation-phase target diagnostics fielded on the NIF are discussed.

4.1) Diagnostics fielded on the NIF

The key diagnostics relevant to the stagnation phase are laid out in Table 1, along with the target properties they are designed to detect. We will discuss each of them in turn.

Table 1: NIF Diagnostics relevant to the stagnation phase and the target parameters each measures.

Diagnostic	Measured parameters
Neutron Activation Diagnostics	Primary yield, angular variation in Down Scatter Ratio
Magnetic Recoil Spectrometer	Neutron Spectrum (Primary yield, ion temperature, Down Scatter Ratio)
Neutron Time of Flight detectors	Neutron Spectrum (ion temperature, Down Scatter Ratio)
Gamma Reaction History	Fusion burn history
South Pole Bang Time	X-ray burn history
Neutron Imaging System	2D Neutron images, Down Scatter Ratio
Gated X-ray Detectors	2D X-ray self-emission images, X-ray burn history

Neutron Activation Diagnostics (NADs) measure the absolute yield of primary (14.1 MeV) neutrons. NADs consist of samples of materials that undergo nuclear reactions in the presence of 14.1 MeV neutrons, in particular the $^{90}\text{Zr}(n,2n)$, $^{63}\text{Cu}(n,2n)$, and $^{65}\text{Cu}(n,2n)$ reactions. Reactions are chosen whose products decay to produce measurable radiation, and the activity of the samples after the shot provides the primary neutron yield measurement. NADs are positioned in at least 17 locations around the target chamber. An anisotropy is observed in primary neutron yield as measured by NADs at different locations. There are two possible reasons for this. Firstly, it may be an anisotropy in the proportion of neutrons which have been downscattered to lower energies due to an anisotropy in the target's ρR [49]. The proportion of neutrons that scatter once in the target (the 'Down-Scatter Ratio' or DSR) is linearly related to the areal density by [50]

$$\rho R(\text{g/cm}^2) = (20.4 \pm 6) \times \text{DSR}_{10-12 \text{ MeV}}, \quad (4.1.1)$$

so any deviation from the average in primary yield measured by a NAD is expected to be proportional to the deviation from the average of the ρR at that location. Secondly, it may be due to bulk fluid motion shifting the neutron spectrum to higher energy. The cross-section of the reactions used increases roughly linearly with energy at 14 MeV [49], and if the distribution of centre-of-mass velocities of the reacting D and T ions is shifted towards the detector, the neutrons produced will be shifted to higher energies, producing more reactions in the NADs and giving a false reading of higher yield. Energy shifts of the primary neutron spectra

of exploding pusher targets have corroborated the hypothesis that these targets have a net fluid velocity of up to 210 km/s, which is sufficient to explain the NAD anisotropy. However, it is not clear whether this is also the case for cryogenic layered DT implosions, as the neutron spectrum shift is not observed [51].

The Magnetic Recoil Spectrometer (MRS) provides a fully energy-resolved measurement of the neutrons produced by the target [3] [4]. The MRS uses a deuterated plastic (CD) foil to convert neutrons into deuterons whose energy is correlated to the neutron energy. Deuterons scattered in the forward direction are selected by an aperture, and then pass through a magnetic coil. The coil disperses the deuterons through an angle related to their energy, onto an array of detectors from which the spectrum is read. Non-deuterated plastic (CH) can also be used; deuterated plastic covers an energy range of 3-18 MeV and non-deuterated plastic covers 6-30 MeV. One of the main design considerations of the MRS is a tradeoff between neutron conversion efficiency and energy resolution; a thicker plastic foil results in a larger scattering probability, but a wider range of scattering angles pass through the aperture. As the neutron yield of the target increases, thinner foils can be used because a lower conversion efficiency is required for sufficient numbers of scattered deuterons to be produced. The error on the ion temperature as measured by the MRS on the NIF is calculated as $\pm 2.9\%$ with the thinnest foils, appropriate for yields of 10^{15} - 10^{19} neutrons, and $\pm 6.4\%$ for yields of between 10^{14} and 10^{15} neutrons [53].

Neutron time-of-flight (nTOF) detectors measure the neutron spectrum by recording the flux of neutrons as a function of time at the detector. For detectors sufficiently far from the target, the time of arrival of a given neutron is dominated by its kinetic energy, which allows the time-dependant signal to be converted into an energy spectrum. The neutron flux is measured by a scintillator coupled to a photodiode or a photomultiplier tube. The error on the ion temperature measurement from the Spec-E and Spec-A nTOF detectors used for sub-ignition-yield experiments is quoted as 12% [50]. Spec-E and Spec-A are located approximately 20 m from the target chamber centre. nTOF detectors are also placed at 4.5 m to measure the neutron bang time, with a time resolution of 100 ps [54].

The Gamma Reaction History (GRH) detector [55] provides an independent measurement of the fusion bang time, along with the fusion burn width. 16.75 MeV gamma rays are produced by DT fusion reactions with a branching ratio of 10^{-5} . The gammas are converted to optical-frequency photons by Gas Cherenkov Detectors, which are amplified by photomultiplier tubes before being recorded by streak cameras.

The time resolution of the GRH is estimated to be 20-100 ps [54]; the error on both the fusion bang time and burn width of shot N120205 is stated to be 30 ps. The GRH detector can also be used to measure the ablator areal density [56]. Scattering events between neutrons and ablator ions such as ^{12}C ions in plastic ablators produce gammas of a lower energy than the fusion gammas, which can be detected by one of the GRH's four channels. This provides a measure of the ablator areal density only, without the effect of the fuel included.

The South Pole Bang Time (SPBT) detector measures the X-ray bang time and burn width, respectively the peak and FWHM of the X-ray pulse from the hotspot. It is located on the south pole to avoid background from laser-plasma instabilities within the holhraum. It also uses graphite monochromator crystals with a bandpass at 11keV to filter out background radiation. The filtered X-ray pulse is converted to an electronic signal by a photoconductive detector. The error on the X-ray bang time and burn width measurements from the SPBT is estimated to be around 30 ps [57]. The X-ray pulse is also measured by a recently developed streak camera system, SPIDER [58], with a quoted temporal resolution of 10 ps and an absolute timing resolution (relevant to bang-time measurement) of 30 ps.

The neutron imaging system (NIS) produces spatially-resolved images of both primary and downscattered neutrons [59]. The neutrons pass through an array of 'pinholes' consisting of 20 cm-long tungsten blocks with cavities of triangular cross-section that taper in the middle. Behind the pinhole array is an array of scintillator fibres, the output of which is passed to a CCD via a microchannel plate and a fibre optic bundle. The microchannel plate is gated to select particular neutron energy ranges based on their time of arrival, which enables the production of downscattered images in various energy bands. The spatial resolution of the NIS is 20 μm and the images are time-integrated. The NIS also acts as a time-of-flight detector, with an absolute error on the ion temperature measurement of 0.5 keV [50].

Gated X-ray Diagnostics (GXD) produce time-resolved 2D images of the X-rays emitted by the hotspot for approximately 500 ps around the X-ray bang time [60][61]. The basic GXD layout consists of a series of pinholes incident on a gated MCP, which is imaged by a CCD or photographic film. CCDs are used for GXDs on OMEGA and Nova, as well as for DD shots on NIF in 2009. The CCD is sensitive to a high-neutron environment, displaying a degradation of the image characterised by bright points referred to as "stars". These artefacts make CCDs unusable on NIF above yields of 10^{13} , as produced by the DT layered implosion targets. Photographic film is used instead [60], as it responds to neutrons with a more uniform

background noise, with an amplitude of around 4% of the peak signal for a yield of 10^{15} neutrons. The images are postprocessed to reduce neutron-induced noise using by a “local mean filter” method in which anomalously bright individual pixels are set to the mean intensity of their neighbours. Two film-based GXDs, referred to as Hardened Gated X-ray Imagers (hGXI), are fielded on the NIF. One points through the North polar LEH and the other is positioned on the equator, looking through a 500 μm -diameter diagnostic window in the hohlraum [61]. The narrow field of view of the LEH and diagnostic window contributes to a reduction in background X-rays from the hohlraum. A series of Kapton (Polyamide) filters with a total thickness of 2.275 mm is also present to filter out the hohlraum X-ray background below 5.5 keV and provide shielding against debris and neutrons. The spatial resolution of the hGXI cameras is 10 μm and the temporal resolution is between 50 and 100 ps. The cameras also provide information about the X-ray bang time and burn width through the relative intensity of the time-gated images. For ignition-level yields of $>10^{15}$ neutrons, the neutron-related noise present on the hGXI cameras is expected to wash out the signal and greater neutron protection is required. The Active Readout In A Neutron Environment (ARIANE) camera is designed to deal with yields of this magnitude. The detector package of ARIANE is located 7 m from the target chamber centre in a neutron-shielded enclosure [62]. A CCD is used in ARIANE in the place of film. At this distance, the signal from the slower-moving neutrons arrives later than the X-ray signal, and by gating the CCD the neutron signal is removed.

4.2) Fusion and X-ray burn histories

By summing the radiation emission and DT fusion rate over the entire simulation domain at each timestep, synthetic fusion and X-ray burn histories are produced, comparable to those produced by the GRH, SPBT, and other diagnostics. This allows comparison of how the shape of the fusion and X-ray histories is altered by the perturbation, and in particular the effect of the perturbation on the bang times and burn widths. As well as providing a possible indicator of perturbation amplitude, the behaviour of the bang times and burn widths with perturbation amplitude has implications for other diagnostics, many of which integrate the neutron or X-ray emission over the burn history.

The burn histories of a multi-mode and an equivalent single-mode $N = 42$ simulation are shown in Figure 4.1. For sufficiently high perturbation amplitude, the fusion burn history of $N = 42$ simulations show a distinct ‘shoulder’ which is not present in the unperturbed simulation - the fusion pulse in the unperturbed simulation has a single distinct peak. The shoulder in $\rho^2 T^4$ is caused by the temperature peak, which corresponds approximately to the stagnation time. The global maximum of $\rho^2 T^4$ occurs later because the density is still increasing after peak temperature, which is why the $\rho^2 T^4$ shoulder is separate from the bang-time peak. In the unperturbed case, the temperature peak occurs close to the point of maximum density, so the time of peak temperature is close enough to the time of peak $\rho^2 T^4$ for there to be a single distinct peak. However, there is still a 50 ps difference between the temperature peak and the bang time in the unperturbed case. The peak-temperature shoulder is not as significant in the multi-mode simulation as in its equivalent $N = 42$ simulation, due to its lower peak temperature (Figure 4.1). Therefore the ‘double-peaked’ neutron burn history observed in the $N = 42$ simulations is a potential signature of a large-amplitude, single-wavelength perturbation present in the experiment.

The different behaviour of the X-ray burn history (Figure 4.1) is a consequence of its weaker temperature dependence. The $N = 42$ X-ray stagnation-time shoulder develops less strongly than in the fusion burn history. The X-ray emission rate also falls off less rapidly after the peak than the fusion rate in the $N = 42$ simulation because the density is still increasing after the bang time and density is a more dominant factor in the X-ray emission rate. As with the shoulder in the neutron burn history, this effect is less prominent in the multi-mode case. While the $N = 42$ hotspot still consists of separate bubbles after the stagnation, the multi-mode hotspot collapses completely at around $t = 450$ ps, so the multi-mode X-ray pulse is not extended in the same way. Therefore, a tail on the X-ray burn history is another potential signature of a large-amplitude, single-wavelength perturbation present in the experiment.

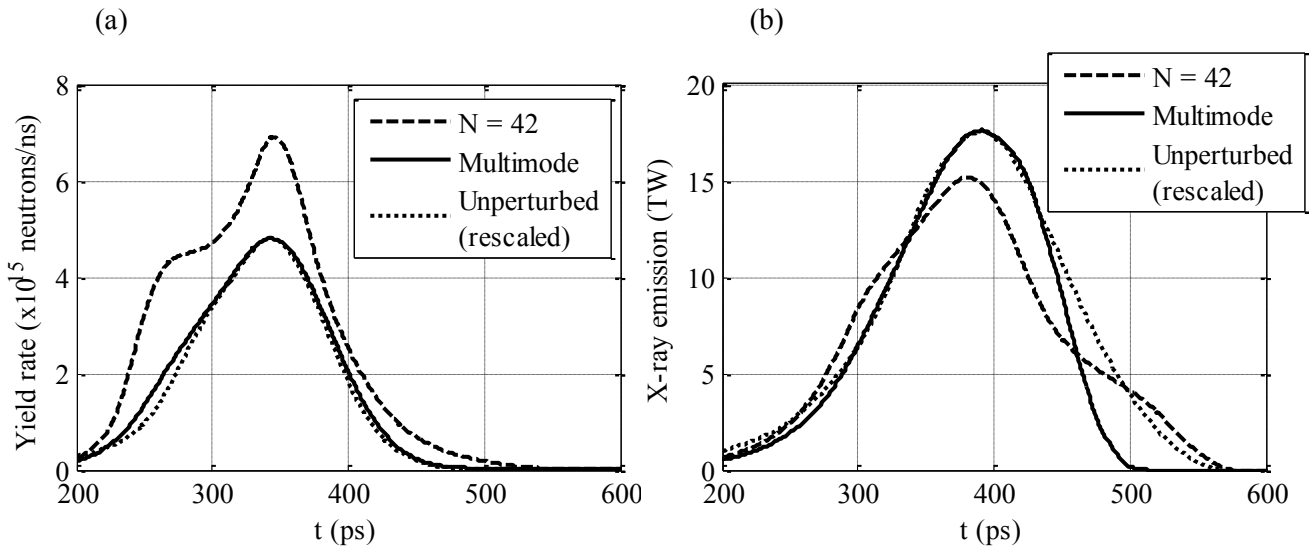


Figure 4.1: (a) Neutron and (b) X-ray burn histories for $N = 42$ single-mode (dashed) and multi-mode (solid) stagnations. The burn histories of the unperturbed simulation are shown with a dotted line, scaled to have the same peak amplitude and peak time as the multi-mode simulation.

The behaviour of the bang times and burn widths for multi-mode simulations is shown in Figure 4.2. Both X-ray and neutron bang times decrease monotonically with perturbation amplitude. The X-ray bang time occurs later because of the weaker temperature dependence. As shown in Figure 4.3, the trend of the difference in X-ray and neutron bang times is a general increase with perturbation amplitude, although the variation for perturbation amplitudes above $\delta u_R/u_R = 0.5$ is of the order of picoseconds, an order of magnitude below the resolution of the GRH and SPBT diagnostics.

The behaviour of the burn width with perturbation amplitude (Figure 4.2(b)) is more complicated. After an initial decrease, the X-ray burn width monotonically increases with perturbation amplitude for $\delta u_R/u_R > 0.15$. The magnitude of the increase is approximately 5 ps, whereas the yield varies by approximately 50% for this range of perturbation amplitudes. The neutron burn width displays the same initial decrease, then sharply increases for $0.15 < \delta u_R/u_R < 0.35$, before gradually monotonically decreasing. In the ‘sharp increase’ region, the time difference between the temperature peak and the fusion burn peak gets larger with perturbation amplitude, which causes a decreased peak burn rate and a larger FWHM of the fusion burn history. In the ‘monotonic decrease’ region, the peak burn rate does not decrease significantly, due to the early work delivered to the hotspot by the spikes. However, the higher perturbation amplitude means the energy loss from the hotspot is larger after the temperature peak, so the burn rate falls off sooner. This two-regime

behaviour suggests interpreting experimental burn width data is not simple. It may be possible to ascertain which regime of burn width behaviour experimental targets are in by their yields.

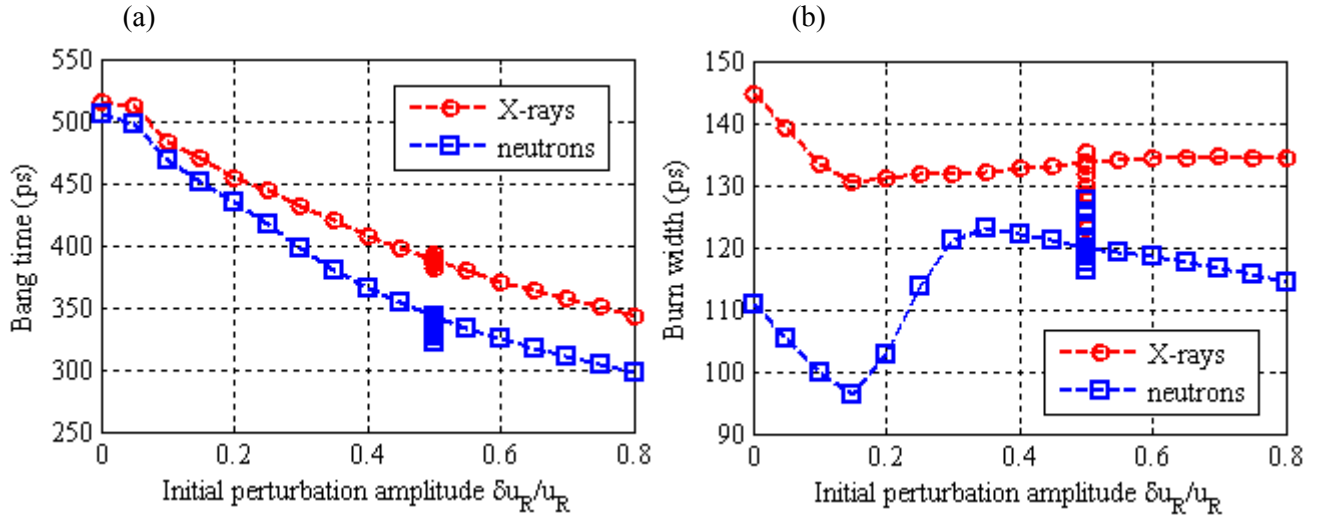


Figure 4.2: (a) Bang times (relative to peak velocity, the start of the simulation) for neutrons (blue square) and X-rays (red circles) as a function of initial perturbation amplitude. (b) Burn widths (FWHM of the burn history) for neutrons and X-rays as a function of initial perturbation amplitude. In both plots, the variation with the random seed of the perturbation is shown for $\delta u_R/u_R = 0.5$.

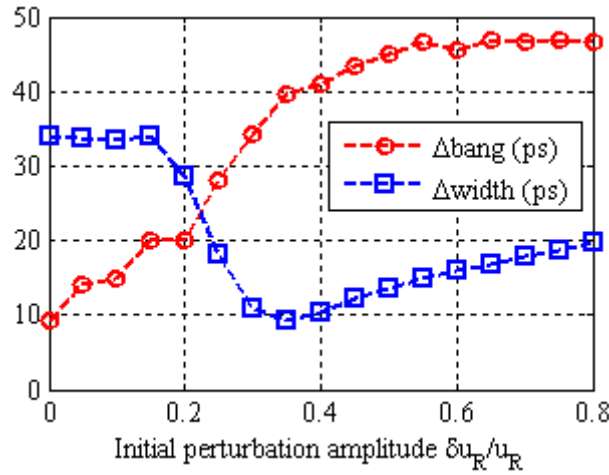


Figure 4.3: Difference in X-ray and neutron burn width (blue squares) and bang time relative to peak velocity (red circles) as a function of initial perturbation amplitude.

The experimental bang time difference for shot N120205 is 81 ps, and the errors on the bang times for the GRH and SPBT are 30 ps and 50 ps respectively, so the simulation is within the errors of the experiment, although there is a wide margin of error. The fusion burn width of shot N120205 was measured by the GRH

as 155 ps, with an error of 30 ps. The simulated burn width falls outside this error bar, but there is a relatively large variation in the burn width with the random seed of the perturbation, and the upper range of this variation falls within the experimental error.

4.3) Primary neutron spectra

Simulated neutron spectra are produced from Gorgon data by integrating over the production spectrum of each cell. The effect of neutron scattering is not taken into account. For a Maxwellian DT plasma with ion density n_i and temperature T , moving at an angle of $\cos \theta$ to the detector at a speed of v_f , the probability density of a neutron being detected with an energy E_n is given by Appelbe and Chittenden [63] as

$$R_{12}(E_n, v_f, T, \cos \theta) = n_i^2 \int_0^\infty f(E_r, v_f, T, \cos \theta) dE_r. \quad (4.3.1)$$

The function $f(E_r, v_f, T, \cos \theta)$ represents the dependance of the production spectrum on the distribution function of the reactants' energies E_r and is derived explicitly in [63]. To simulate the detected neutron spectrum from Gorgon data, the production spectrum must be integrated over the entire three-dimensional plasma:

$$R_{12}(E_n) = \iiint n_i(x, y, z)^2 \int_0^\infty f(E_r, v_f, T, \cos \theta) dE_r dx dy dz. \quad (4.3.2)$$

Since effects of the order of 10s of keV may be expected to occur [63] it is desirable to evaluate the spectrum $R_{12}(E_n)$ at ~ 400 points between 13 and 15 MeV. Calculating $R_{12}(E_n)$ at each of these points can be time-consuming because (4.3.2), a quadruple integral over the spatial grid and the distribution of relative reactant energies E_r , must be evaluated numerically for each E_n point. To speed the process, the n_i^2 values of the computational cells are binned into bins of v_f, T and $\cos \theta$, since the spectra of cells with similar values of v_f, T and $\cos \theta$ are approximately the same, reducing the number of times (4.3.1) must be evaluated. In other words, instead of the volume-integral in real space (4.3.2), we evaluate an integral in $(v_f, T, \cos \theta)$ -space:

$$R_{12}(E_n) = \iiint n_i^2(v_f, T, \cos \theta) \int_0^\infty f(E_r, v_f, T, \cos \theta) dE_r dv_f dT d\cos \theta, \quad (4.3.3)$$

and construct an array $n_i^2(v_f, T, \cos \theta)$ which has fewer points than the spatial grid $n_i(x, y, z)$. We find that 150 points in both the v_f and T directions and 20 in the $\cos \theta$ direction are sufficient for convergence of the calculated spectrum for the non-igniting stagnation simulations.

Figure 4.4(a) shows the neutron spectra produced by hotspots of various central ion temperatures T_c with the self-similar profile (3.2.2). The extra broadening of the spectrum for the higher- T_c hotspots is due to their higher temperature. By evaluating the production spectrum (4.3.1) for a uniform plasma, a relationship between FWHM of the neutron production spectrum and ion temperature can be established (Figure 4.4(b)). A good rule of thumb is that a 50 keV FWHM change results in a ~ 1 keV ion temperature change.

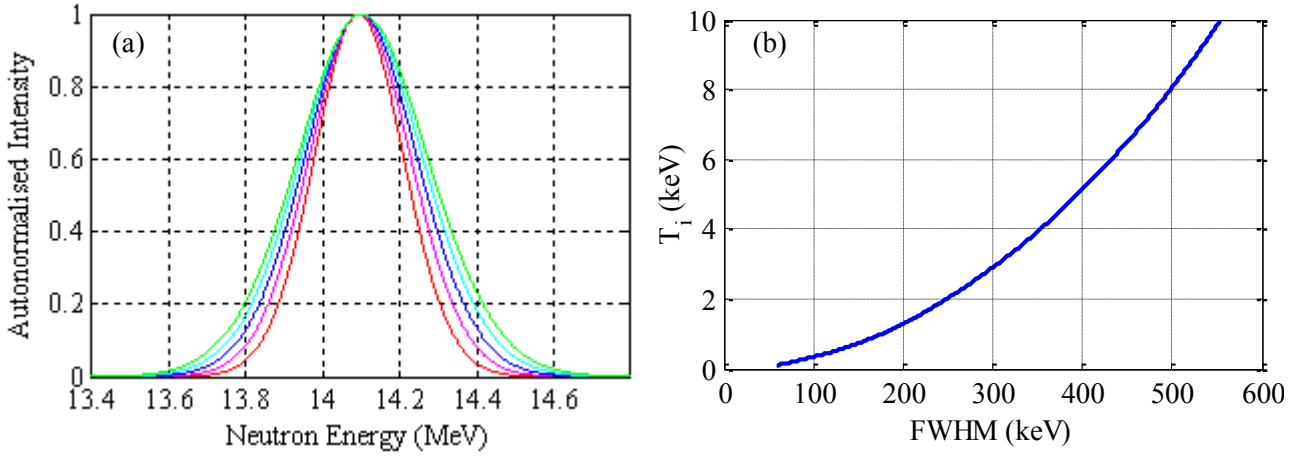


Figure 4.4: (a) Neutron spectra for unperturbed hotspots with central temperatures between 3 keV (red) and 7 keV (green), with motion Doppler broadening excluded. (b) FWHM of the neutron spectrum produced by a uniform plasma vs. its ion temperature.

All regions with a significant reaction rate contribute to the width of the production spectrum, so the ‘neutron temperature’ (temperature as diagnosed by the neutron spectrum) represents some sort of weighted average of the ion temperature throughout the hotspot. It is useful to assess how the neutron temperature compares with the actual ion temperature profile present in the hotspot. Figure 4.5 plots the neutron temperature of self-similar (3.2.2) hotspots against T_c ; there is a linear relationship between neutron temperature T_n and central temperature given by $T_n(\text{keV}) = 0.67T_c(\text{keV}) + 0.43$ keV. The neutron temperature agrees closely with the ‘burn-averaged’ ion temperature calculated by:

$$T_{ba} = \frac{\int_0^R \langle \sigma v \rangle \rho(r)^2 T(r) dr}{\int_0^R \langle \sigma v \rangle \rho(r)^2 dr} \quad (4.3.4)$$

the difference between the two is within 3% for $3 \text{ keV} < T_c < 7 \text{ keV}$. Using the $\langle \sigma v \rangle \propto T^4$ approximation to the reactivity (Figure 3.1), the burn-averaged ion temperature can be related to the central ion temperature by

$$T_{ba} = \frac{H(-3, T_c)}{H(-2, T_c)} T_c. \quad (4.3.5)$$

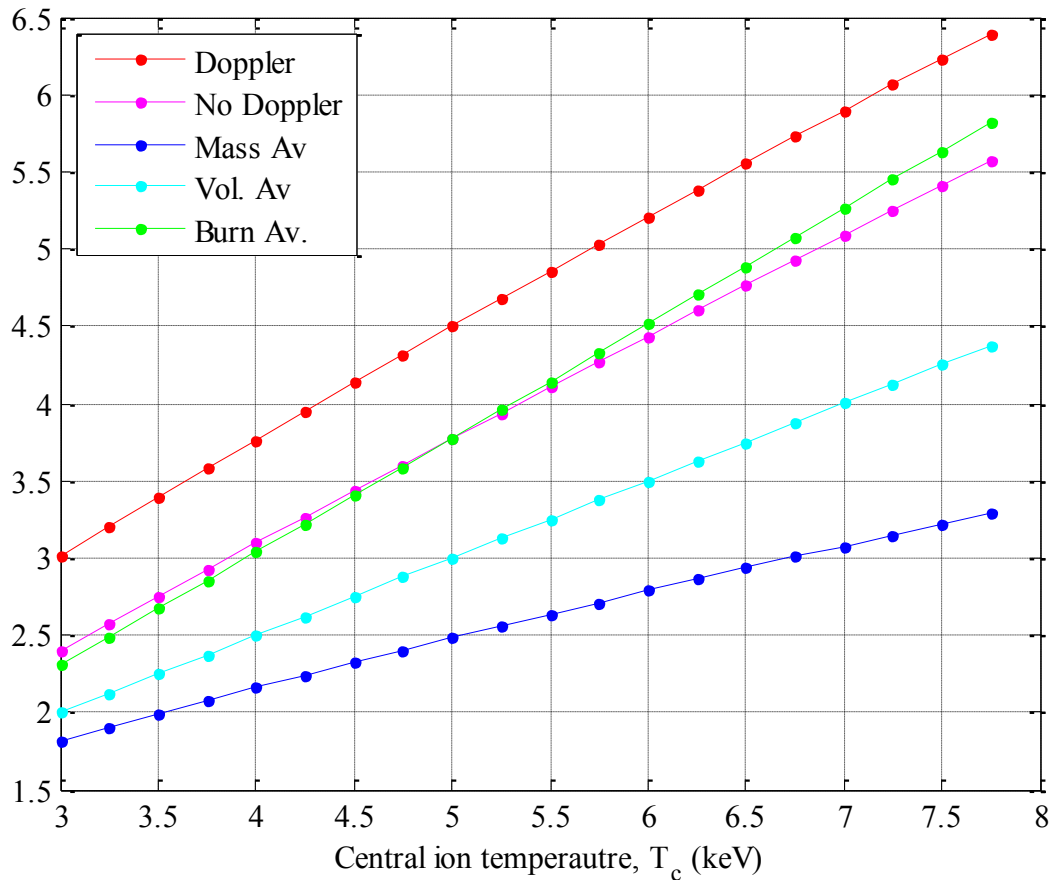


Figure 4.5: Temperatures as calculated from neutron production spectrum width for self-similar (3.2.2) hotspots as a function of central ion temperature, with (red) and without (pink) the inclusion of motion Doppler broadening. The velocity profile is held constant and the velocity at the hotspot edge is $300 \mu\text{m/ns}$. For comparison, the mass-averaged (dark blue), volume-averaged (light blue) and burn-averaged (green) hotspot temperatures are shown as a function of central ion temperature.

One must take care when using the neutron temperature as a measure of hotspot pressure. For example, [64] calculates the pressure of an experimental hotspot at maximum compression by assuming that

$P = 2n_e e T_n (\text{eV})$ and concludes that the Lawson-type confinement parameter $P\tau$ is a factor of three lower than that required for ignition. We may assess the error introduced by this assumption by writing the mass-averaged ion temperature as $T_{ma} = T_c \rho_c V / M$. Assuming that $n_e = M / (m_i V)$, as has implicitly been done in [64] by assuming that n_e is uniform in the hotspot, and using $P = 2e / m_i \times \rho_c T_c$, it follows that

$$P = 2n_e e T_{ma}. \quad (4.3.6)$$

Hence in order to calculate the hotspot pressure it is necessary to use the mass-averaged ion temperature instead of the neutron temperature. The mass-averaged ion temperature differs significantly from the neutron temperature, by a linear factor of

$$T_{ma}(\text{keV}) = 0.46 T_n(\text{keV}) + 0.73 \text{ keV}. \quad (4.3.7)$$

Hotter regions of the hotspot contribute more strongly to the neutron temperature than colder regions, due to the strong temperature dependence of the fusion cross-section, so the neutron temperature is significantly higher than the mass-averaged ion temperature. In conclusion, using the neutron temperature directly to diagnose hotspot pressure results in an overestimation of hotspot pressure by a factor of 1.42 for a neutron temperature of 3 keV.

Figure 4.5 also shows how the neutron temperature changes when the effect of motion Doppler broadening is included. The measured neutron temperature including the effect of motion Doppler broadening is an approximately constant 700 eV higher in these simulations, for which the velocity at the hotspot edge was 300 km/s. The motion broadening is lower than it would be for a uniform velocity profile, because the Kidder-like linear profile causes the velocity of the hottest material in the centre to be lower. As the target stagnates, the motion broadening is reduced; the mass-averaged radial velocity of the unperturbed target at bang time ($t \sim 500$ ps) is only $75 \mu\text{m/ns}$ (the velocity at the hotspot edge was $90 \mu\text{m/ns}$) and its neutron temperature is only increased by 200 eV due to motion Doppler broadening, at $T_{neutron} = 4.71$ keV. Since this is the time that contributes most to the time-integrated neutron spectrum, the spectrum seen by the detector is also not strongly affected by motion Doppler broadening. The time-integrated neutron spectrum

has a neutron temperature of 4.44 keV, which is 204 eV lower when the effects of motion broadening are not included.

A stagnation with a single spike exhibits a small shift of the neutron spectrum. For a single $N = 42$ -scale spike located on the North pole ($\theta = 0$), with the detector also located on the North pole, the neutron spectrum is shifted down in energy by approximately 10 keV relative to a detector located on the equator ($\theta = \pi/2$) – the higher velocity of the spike in the direction away from polar detector causes the neutron velocity in the detector frame to be reduced. As the target stagnates, the neutron spectrum shift becomes positive. This is not due to a reversal of the spike's direction by the stagnation; having greater momentum than the material on the opposite side of the target, the spike's direction of travel is not reversed. The cause is a jet launched in the opposite direction to the spike's motion. There is a low density cavity behind the imploding spike, and as the accretion associated with the stagnation of the target passes from the dense fuel region into the cavity region, it forms a release wave that converges on the axis of the spike (Figure 4.6). The release has a high velocity in the outward radial direction, and when it converges its ion temperature reaches the keV scale. In this particular simulation, the jet is formed before the bang time ($t \sim 500$ ps), and so the time-integrated spectrum in the polar detector exhibits a shift in the positive energy direction, caused by the jet. The time-integrated spectrum is also broader in the polar direction relative to the equatorial direction, because of the changing energy shift of the time-dependant spectrum, which gets manifested as a broadening when integrated over time. The polar spectrum has a 1.5% (65 eV) higher neutron temperature than the equatorial at bang time, which rises to 3.5% (129 eV) in the time-integrated spectrum.

Error! Reference source not found.(b) shows the motion broadening of the neutron spectrum as a function of time for an $N = 42$ simulation. The initial magnitude of the broadening is around 25 keV larger than that of the unperturbed simulation, because of the greater range of velocities present in the perturbed simulation. The broadening drops rapidly at 300 ps as the target stagnates. This occurs sooner than for the unperturbed simulation, simply because the stagnation occurs sooner. The fact that the spectrum broadening drops sooner for the $N = 42$ simulation does not reduce the time-integrated broadening, because the bang time also occurs earlier, so the weighting of the broader pre-stagnation spectra in the time-integration is similar to the weighting of the pre-stagnation spectra in the unperturbed case. In both cases, the post-

stagnation spectra contribute most strongly to the time-integrated spectra, and the $N = 42$ simulation still has more motion broadening post-stagnation than the unperturbed simulation. These two factors mean the $N = 42$ time-integrated spectrum has a greater amount of motion-broadening than the unperturbed time-integrated spectrum. The unperturbed time-integrated neutron temperature is increased by 4% ($\Delta T_n = 205$ eV, $T_n = 4.313$ keV) due to motion-doppler broadening, whereas the $N = 42$ time-integrated neutron temperature is increased by 7.1% ($\Delta T_n = 282$ eV, $T_n = 3.973$ keV).

There is no significant difference in the neutron spectrum for different lines-of-sight (LOS) in the $N = 42$ simulation due to the symmetry of the perturbation. The red squares in **Error! Reference source not found.**(b) show the broadening for a detector pointing directly at a spike, and the black triangles show the broadening for a detector pointing at a bubble region; these LOS are as different as is possible in this symmetrically-perturbed simulation. Due to the symmetry of the perturbation, the spike LOS passes through spike regions on both sides of the capsule, and the bubble LOS passes through bubbles on both sides. The two LOS differ in angle by 55° , which is not enough to produce a significant relative shift in the spectrum; two LOS differing by 45° in the single-spike simulation showed a relative shift no greater than 2 keV, which is the energy resolution used for the spectrum simulation process. The maximum difference in broadening between the two $N = 42$ LOS is 2 keV, and at no time are their peaks shifted relative to one another by more than 2 keV.

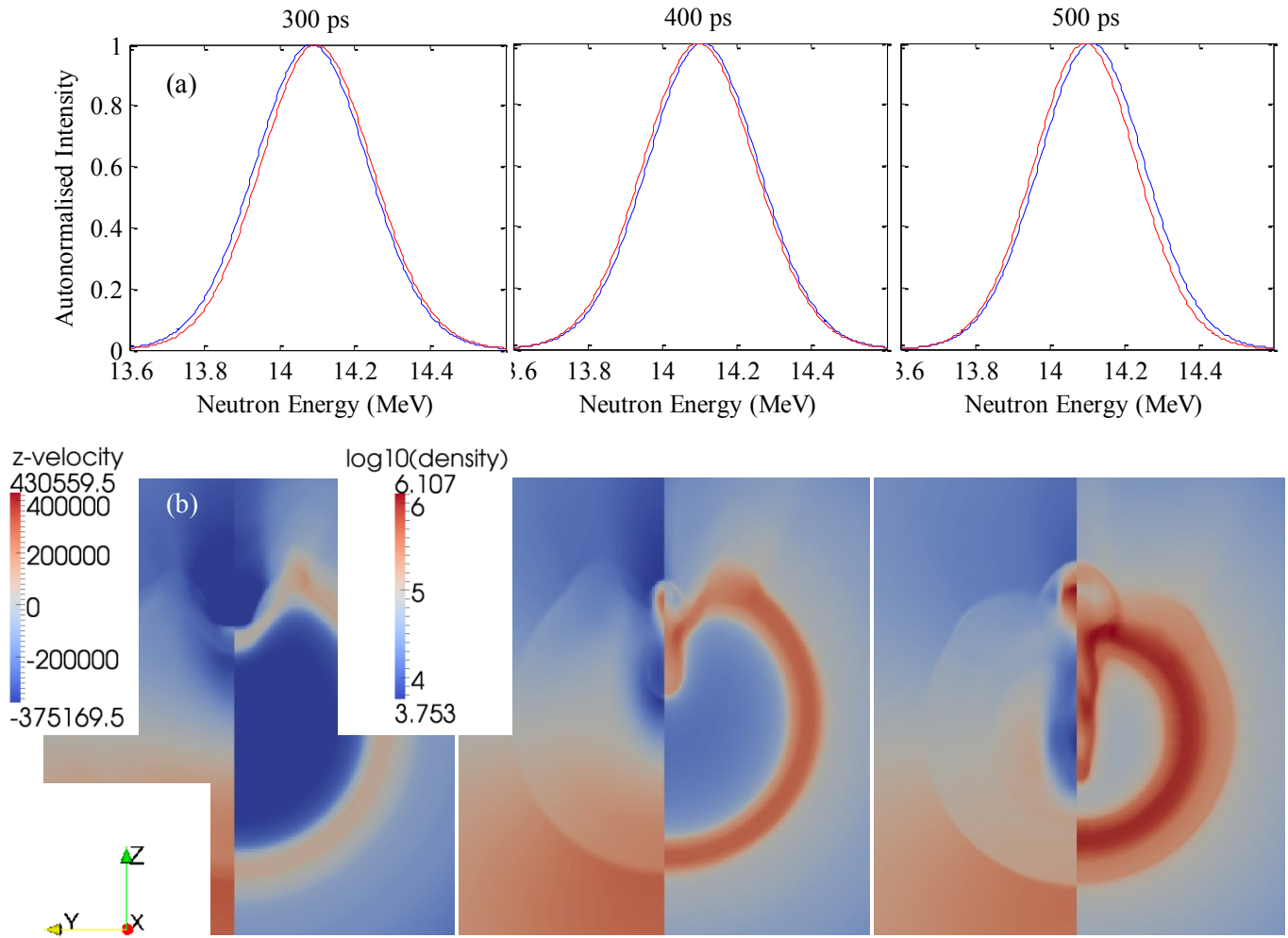


Figure 4.6: (a) Relative shift in energy spectrum peaks between detectors located parallel (blue) and perpendicular (red) to the motion of a spike at 200, 300 and 400 ps. (b) Component of velocity in the z -direction (left) and density (right) maps of a stagnating target with a single spike at 200, 300 and 400 ps.

Unlike the single-mode simulation, there is an LOS dependence in the neutron spectra of the multimode targets due to the anisotropy of the perturbation. In particular, the detector pointing down the z -axis ($\theta = 0$, blue circles in **Error! Reference source not found.**) shows an 8 keV broader spectrum at $t = 200$ ps and a 20 keV broader spectrum at $t = 300$ ps compared with the other two detectors. This is due to the large structure at $\theta = 90^\circ$ moving with a high velocity in the positive z -direction. The LOS dependence is greater post-stagnation than pre-stagnation because the fluid motion after the stagnation is dominated by the momentum anisotropy caused by the perturbation, whereas a large component of the fluid motion at $t = 200$ ps is related to the symmetric radial motion of the implosion. This LOS dependence is manifested in the time-

integrated spectra; the time-integrated spectrum at the $\theta = 0$ detector is broadened by 12% ($\Delta T_n = 540$ eV, $T_n = 4.332$ keV) by fluid motion, whereas the others are broadened by 9.8% ($\Delta T_n = 408$ eV, $T_n = 4.194$ keV) and 9.0% ($\Delta T_n = 375$ eV, $T_n = 4.161$ keV).

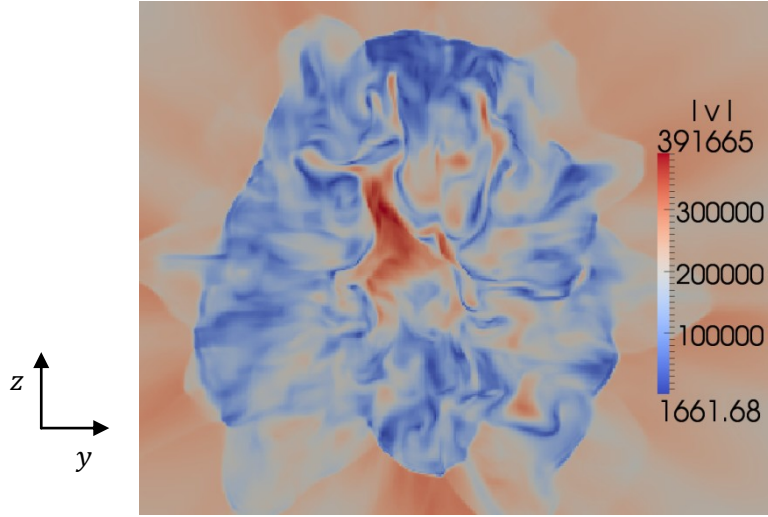


Figure 4.7: The magnitude of the velocity field of the multi-mode simulation at 450 ps. There is significant residual motion in what remains of the hotspot, causing broadening of the neutron spectrum and a line-of-sight dependence.

As with the unperturbed targets, there is a significant discrepancy between the neutron temperature as measured by neutron spectroscopy and the mass-averaged ion temperature. The $N = 42$ simulation has a T_{ma} of 2.50 keV at bang time, and its time-integrated neutron spectrum temperature is 3.69 keV without motion Doppler broadening, and 3.97 keV with motion Doppler included. The multi-mode simulation has a T_{ma} of 2.64 keV and an unbroadened time-integrated T_n of 3.79 keV, rising to 4.19 keV with motion Doppler included.

4.4) X-ray and neutron images

We post-process Gorgon simulations using a raytracing technique to produce synthetic X-ray images of the kind generated experimentally by the hGXI cameras, in order to compare between the hotspot shape in stagnation simulations and the X-ray images produced. The raytrace includes the effect of free-free

absorption assuming both the ice and the ablator are fully-ionised DT. The absorption is frequency resolved – the spectrum of each ray changes along its path.

If there were no absorption, the hotspot would appear brighter at the edge because the volumetric Bremsstrahlung rate is proportional to $P^2 T_e^{-3/2}$ and the hotspot pressure is uniform. Figure 4.8(a) shows the intensity map generated by a spherical hotspot with uniform pressure and the self-similar temperature profile (3.2.2) with $T_c = 4$ keV and $T_R = 1$ keV and a radius of $40\text{ }\mu\text{m}$, without the effect of self-absorption taken into account. Using the Planck opacity, the free-free absorption by a fully-ionised DT object of density ρ , electron temperature T_e and thickness L of thermal radiation emitted by a body of temperature T_r may be approximated by

$$\kappa_P = \exp(-9.35 \times 10^5 \times L \rho^2 T_e^{1/2} / T_r^4). \quad (4.4.1)$$

Modelling the shell as fully-ionised DT with $\rho = 10^5\text{ kg/m}^3$, $T_e = 200\text{ eV}$ and $L = 50\text{ }\mu\text{m}$, the exponent is -6.6 for a radiation temperature $T_r = 1\text{ keV}$, -0.41 for $T_r = 2\text{ keV}$, and -0.025 for $T_r = 4\text{ keV}$. Almost all the radiation from the edge of the hotspot is absorbed because the edge is cooler than the centre, resulting in an image which is brightest in the middle (Figure 4.8(b)). The quasispherical shape of the hotspot also contributes to the brightening of the image in the centre, since the image is a line integral in the direction perpendicular to the camera and the hotspot is wider in the centre. These effects influence the relationship between P0 as measured from X-ray self-emission images and the physical hotspot radius. The P0 is measured as the fundamental mode of the 17% intensity contour [61]. In Figure 4.9(a), the P0 of images of self-similar hotspots (3.2.2) produced using the Planck-opacity approximation (4.4.1) is plotted against the hotspot radius R ; the P0 is consistently smaller than R for the range of R shown. When a $10\text{ }\mu\text{m}$ spatial blur is included, the P0 is larger than R for $R \lesssim 23\text{ }\mu\text{m}$.

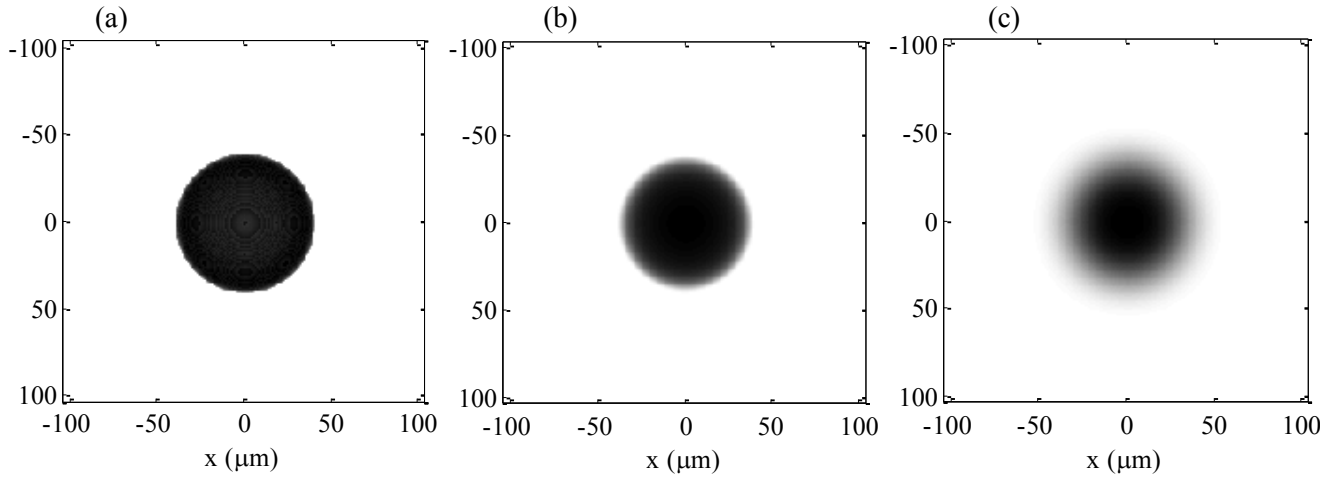


Figure 4.8: (a) Synthetic X-ray image of an unperturbed hotspot without the inclusion of absorption or blur. (b) The hotspot from (a) imaged with the effects of absorption by a uniform DT shell included. (c) The image from (b) blurred with a 10 μm Gaussian point-spread function.

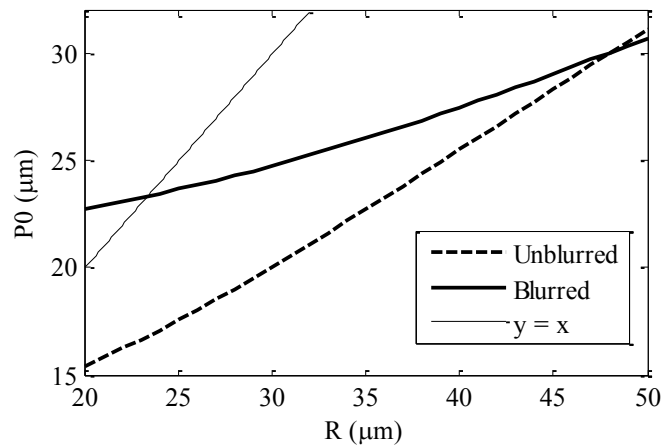


Figure 4.9: Dependence of P_0 on target radius as measured from the 17% intensity contour of unblurred (dashed) and blurred (solid) unperturbed X-ray images.

The GXD cameras fielded on the NIF use a Kapton filter which acts as a high-pass filter at 10 keV on the frequency spectrum of the self-emission, having qualitatively the same effect as the carbon in the ablator. When this effect is included in the synthetic X-ray images produced from our simulation (Figure 4.10(b)), the resulting image has a lower P_0 - 39 μm as opposed to 45 μm in the unfiltered image at 300 ps and 23 μm as opposed to 27 μm at 400ps. The image also appears less perturbed, since the lower-frequency emission from the colder bubble regions is filtered out. Figure 4.10(c) shows the same hotspot imaged using a filter with a Gaussian transmission spectrum centred on 500 eV with a width of 200 eV. This produces an image with a

more pronounced perturbation structure and a P0 of 48 μm at 300 ps and 38 μm at 400 ps, around 10 μm larger than the unfiltered image.

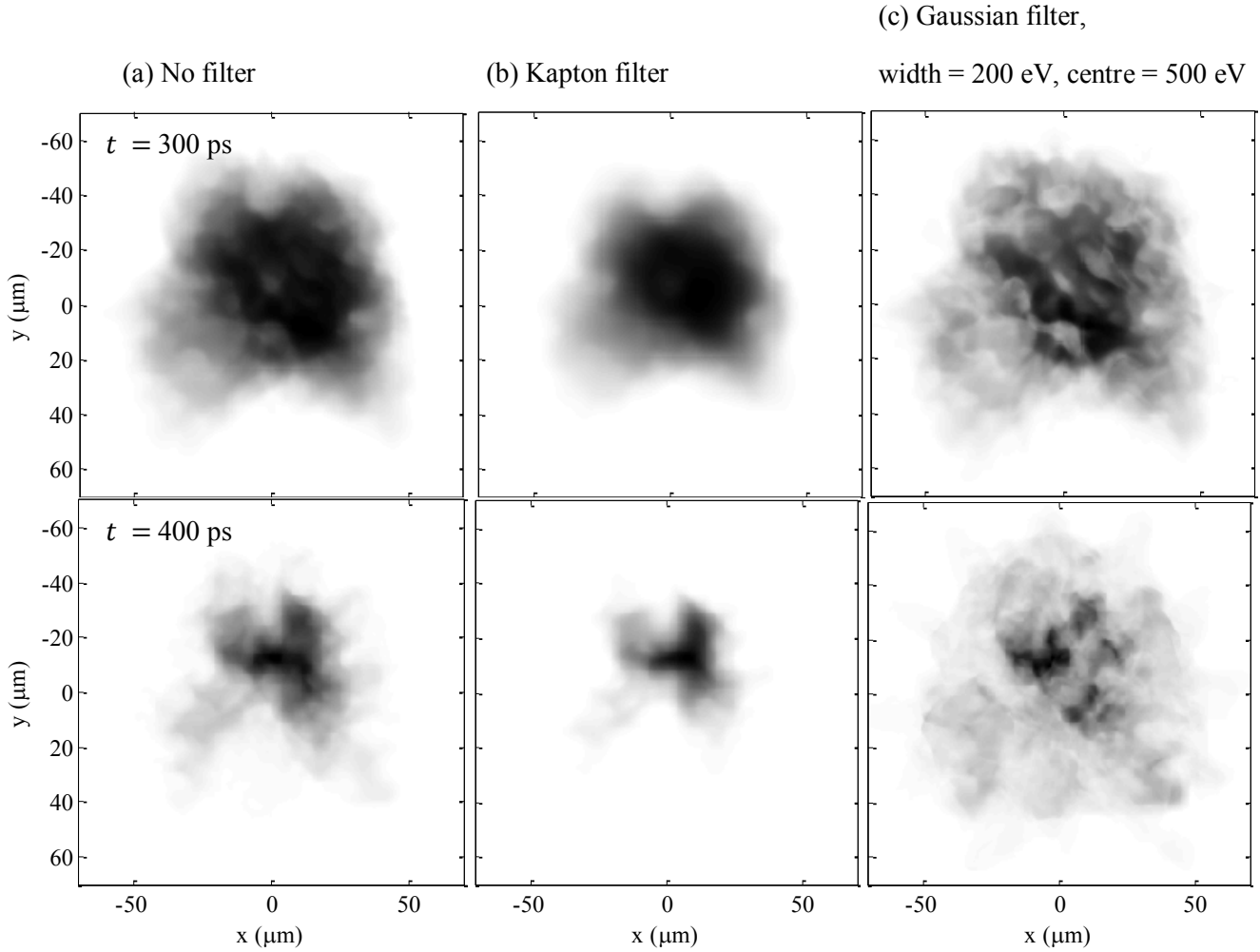


Figure 4.10: Synthetic X-ray images of multi-mode perturbed targets (a) with no filter, (b) with a 2.275 mm Kapton filter such as the one fielded on NIF and (c) with a Gaussian transmission spectrum centred on 500 eV with a width of 200 eV. The top images are produced at $t = 300$ ps and the bottom images at $t = 400$ ps.

X-ray images show a higher P0 and lower perturbation amplitude the earlier in time they are taken. Since perturbations move the X-ray bang time earlier in time, more perturbed simulations produce a larger P0 in their bang-time X-ray images despite the fact that their $\langle R \rangle$ drops monotonically below that of the unperturbed simulation. In other words, perturbed targets have a smaller $\langle R \rangle(t)$, but a larger $\langle R \rangle(t_{bang})$. The X-ray bang time of the unperturbed simulation is 515 ps, at which time its hotspot radius is 25 μm , with an X-ray image P0 of 23 μm . The multi-mode perturbed simulation's X-ray bang time is 390 ps, at which time its $\langle R \rangle$ is 38 μm , with an X-ray image P0 of 30 μm . The unperturbed simulation's $\langle R \rangle$ at this time is 44 μm . The

25/23 ratio of P_0 to $\langle R \rangle$ in the unperturbed case agrees with Figure 4.9, however, the perturbed target's ratio of 38/30 does not agree. This may be due to the heating of the outer regions of the hotspot by colliding spikes, as mentioned in Section 3.5, causing more and higher-frequency emission from these regions.

Figure 4.11 shows the Kapton-filtered synthetic self-emission images as above, with a $10\ \mu\text{m}$ Gaussian point spread function applied to imitate the spatial resolution of the GXD cameras on the NIF. The blurred images have a larger P_0 than the unblurred images – $30\ \mu\text{m}$ as opposed to $23\ \mu\text{m}$ at $t = 400\ \text{ps}$, and $42\ \mu\text{m}$ as opposed to $39\ \mu\text{m}$ at $t = 300\ \text{ps}$. The presence of $N = 42$ scale and smaller perturbation structures is not apparent from the images. Instead, the varying amplitude of the spikes gives the appearance of a larger-wavelength perturbation. This suggests that jets of varying amplitudes caused by artefacts such as dust, ablator defects or ice grooves could be among the causes of apparent low mode-number perturbations observed in experimental GXD images [19].

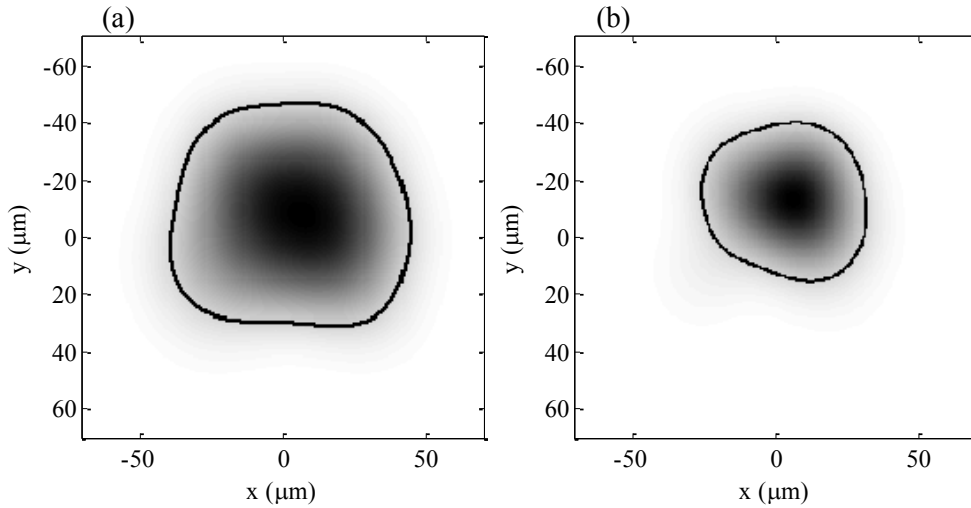


Figure 4.11: Kapton-filtered synthetic X-ray images at (a) $t = 300\ \text{ps}$ and (b) $t = 400\ \text{ps}$ with a $10\ \mu\text{m}$ spatial blur applied. The black line shows the 17 % intensity contour from which the P_0 , P_2 and P_4 are measured.

We produce synthetic primary neutron images by integrating the DT fusion rate along the direction normal to the detector. We do not include the effect of scattering. Unlike the X-ray self-emission images, neutron images are inherently brighter in the centre because the DT fusion rate depends on $P^2 T^2$ in the regime where $\langle \sigma v \rangle \propto T^4$ is valid, and the hotspot pressure is uniform. Figure 4.12 shows a neutron image generated by a spherical hotspot with uniform pressure and the self-similar temperature profile (3.2.2) with $T_c = 4\ \text{keV}$,

$T_R = 1$ keV and a radius of $40\text{ }\mu\text{m}$. The P_0 of the unblurred neutron images is almost identical to those of the X-ray self-emission images (Figure 4.13) – it is a constant $1.5\text{ }\mu\text{m}$ larger than the X-ray images across the whole range of hotspot radii shown. However, the larger ($20\text{ }\mu\text{m}$) spatial blur causes the blurred neutron images to have a P_0 approximately $15\text{ }\mu\text{m}$ greater than the P_0 of the blurred X-ray images. The P_0 of these images is also less sensitive to hotspot radius than for the X-ray images; the P_0 of the blurred images varies by only $5\text{ }\mu\text{m}$ for radii between $20\text{ }\mu\text{m}$ and $50\text{ }\mu\text{m}$ because the image is dominated by the blur (Figure 4.13).

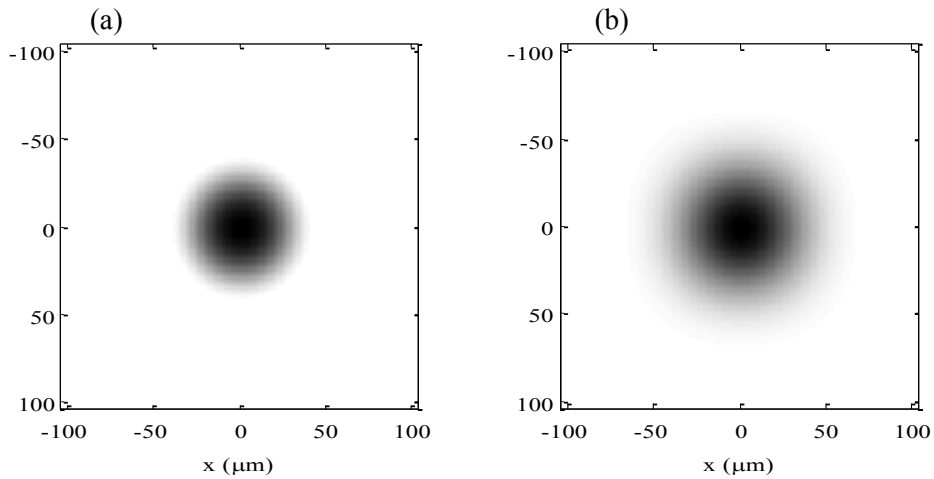


Figure 4.12: (a) Synthetic neutron image created by integrating the fusion reaction rate over the direction normal to the camera. (b) The image in (a) with a $20\text{ }\mu\text{m}$ spatial blur applied.

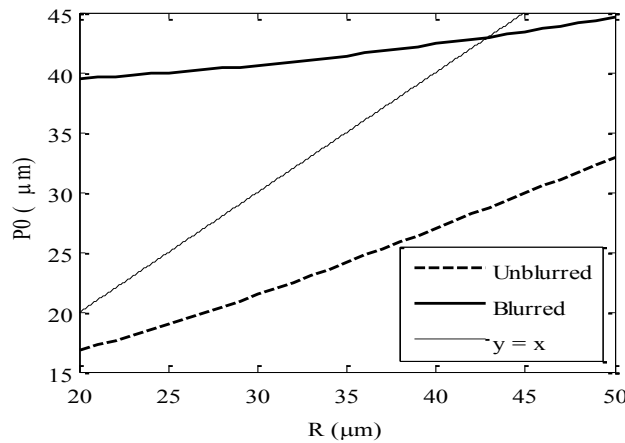


Figure 4.13: Dependence of P_0 on target radius as measured from the 17 % intensity contour of unblurred (dashed) and blurred (solid) unperturbed neutron images.

The experimental neutron images are time-integrated, however they are often taken to represent the state of the target at neutron bang time. The burn history of the target essentially applies a temporal blur to the image. The images produced at bang time appear similar to the time-integrated images (Figure 4.14). However, there is a difference in the properties of the hotspot measured from the images. The time-integrated image is slightly smaller; the P0 of the time-integrated image is $47.6\text{ }\mu\text{m}$, compared with the bang-time image's P0 of $50.1\text{ }\mu\text{m}$. The perturbation is also different in the time-integrated images – $P4/P0$ is 0.48% in the time-integrated image compared with 0.41% in the bang-time image. $P2/P0$ is 1.1% in the time-integrated image compared with 1.4% in the bang-time image. The perturbation in these images is very small, because the spatial blur washes out most of the perturbation structure. Neutron images are usually only sensitive to very large P2 and P4 perturbations generated by significant beam imbalance.

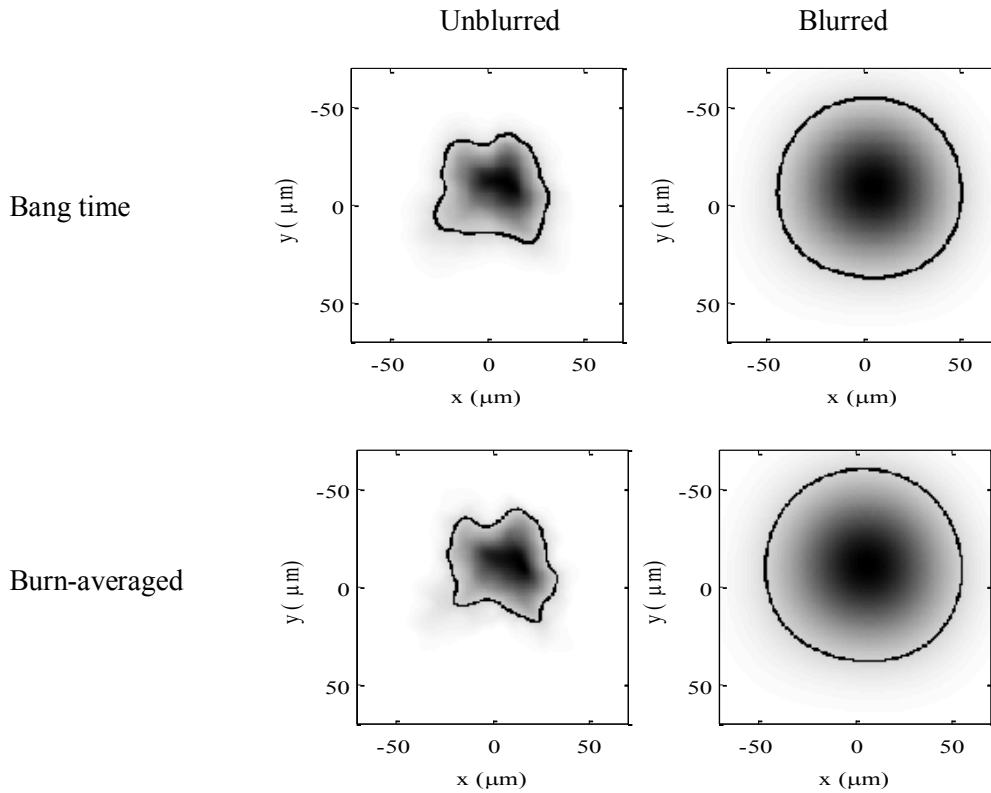


Figure 4.14: Neutron images of multi-mode perturbed targets. The images on the left are unblurred, and the images on the right are blurred with a $20\text{ }\mu\text{m}$ Gaussian point-spread function. The top images are taken at bang time, and the bottom images are burn-averaged to simulate the time-integrated images produced by the NIS. The black line shows the 17 % intensity contour from which the P0, P2 and P4 are measured.

4.5) Spatially-resolved ρR diagnostics

Neutron Activation Diagnostics and neutron spectrometers are able to provide an angularly-resolved measurement of the target's ρR by measuring the DSR at various positions around the target chamber. It is possible to compare our simulations to these data by integrating ρ over the radial coordinate r at random (θ, φ) -positions distributed uniformly over the surface of a sphere. These line integrals are used to produce maps of $\rho R(\theta, \varphi)$ (Figure 4.15). Note that in this section ρR refers to the total target areal density, not just the hotspot areal density. Since the experimental $\rho R(\theta, \varphi)$ measurements are time-integrated, it is necessary to produce burn-averaged $\rho R(\theta, \varphi)$ maps to make a meaningful comparison.

The $\rho R(\theta, \varphi)$ map of the $N = 42$ simulation at $t = 250$ ps shows a larger ρR in the bubble regions than the spike regions, in agreement with the density map shown in Figure 3.13(a). At this time, mass is flowing from the spike regions into the bubble regions according to the velocity field set up by the acceleration-phase Rayleigh-Taylor instability (RTI). As the deceleration-phase RTI develops, the direction of the transverse flow is reversed and mass begins to flow from the spike regions into the bubble regions. This explains why the spikes have a larger ρR at $t = 400$ ps. The burn-averaged $\rho R(\theta, \varphi)$ shows a larger ρR in the spikes than the bubbles because the bang time occurs at around $t = 350$ ps, although the amplitude of the variation of ρR is smaller in the burn-averaged map than at 400 ps.

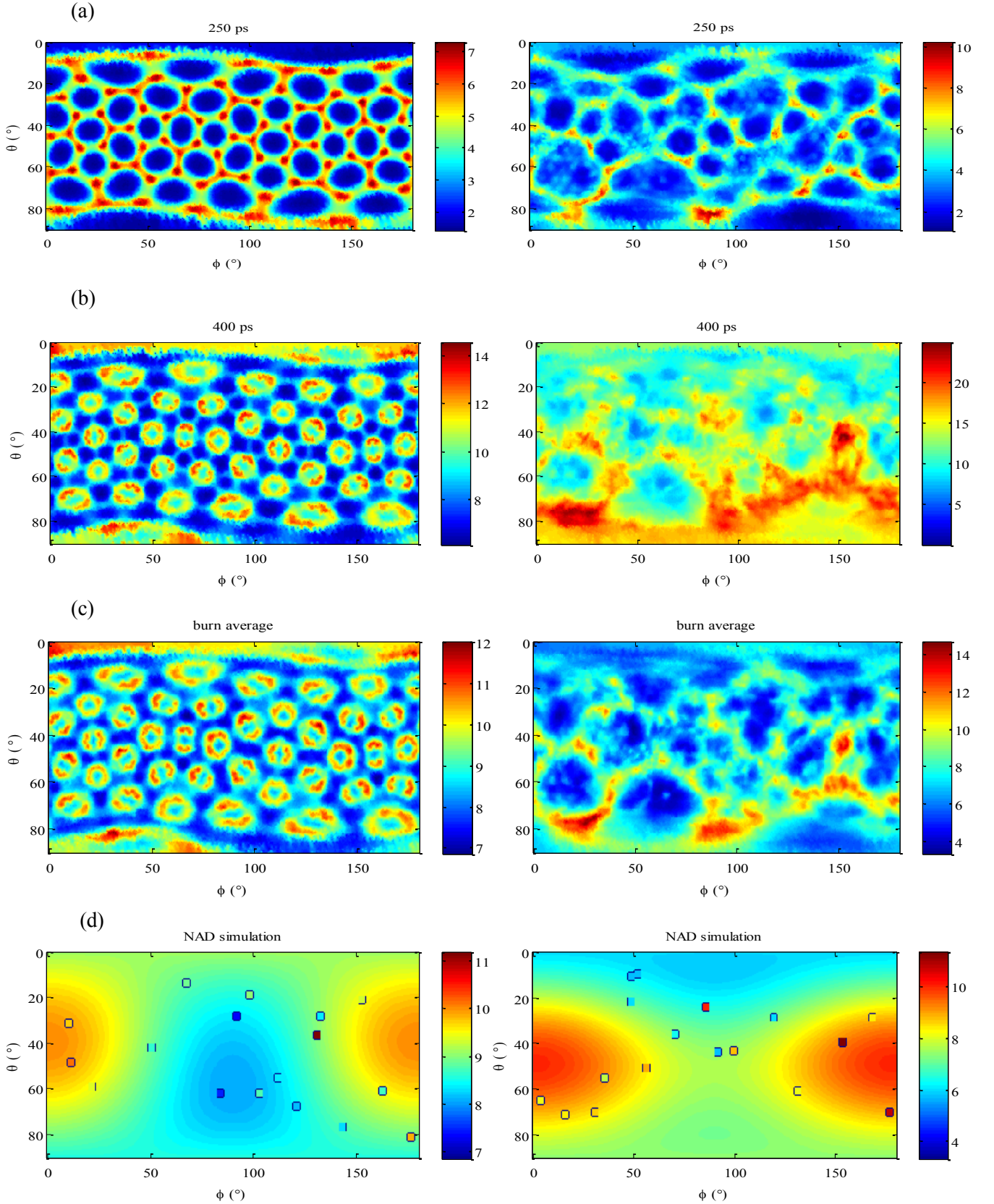


Figure 4.15: Projections of ρR onto the (θ, ϕ) -plane for $N = 42$ (left) and multi-mode (right) simulations. (a) is before stagnation, at $t = 250$ ps, (b) is after stagnation, at $t = 400$ ps, and (c) shows the burn-averaged

projections relevant to the NADs. In (d), 17 points from the burn-averaged maps have been selected at random and low-order spherical harmonics fit to them to produce the colourmap.

In the multi-mode simulation, the burn-averaged $\rho R(\theta, \varphi)$ maps have more long-wavelength structure than the initial perturbation. This is due to merging of neighbouring spikes occurring before the bang time. Spherical harmonic transforms of the $\rho R(\theta, \varphi)$ maps at 250 ps and 400 ps reveal significantly higher amplitudes of the low-order components at 400 ps than 250 ps (Figure 4.16). Since the bang time occurs at 350 ps, the spectrum of the burn-averaged $\rho R(\theta, \varphi)$ contains a combination of the low-order structure of the perturbation present at 400 ps and the higher-order structure of the perturbation present at 250 ps (Figure 4.16). The average ρR for shot N120205 is 8.98 kg/m², as calculated from the DSR values from the SPEC-E, SPEC-A and MRS spectrometers. The burn-averaged mean ρR of the $N = 42$ simulation is 8.88 kg/m², in close agreement with the measured value. The burn-averaged mean ρR of the multi-mode simulation is 82% of the measured value, at 7.35 kg/m², which is not within the 6% error bar of the ρR measurement from the spectrometers. The ρR at bang time is significantly larger than the burn-averaged ρR , at 13.14 kg/m².

The range of ρR in the multi-mode simulation agrees with the NAD data published for shot N120205 [49]. [49] calculates the ρR at a given location Ω_1 relative to a reference location Ω_2 in terms of the ratio of the unscattered yield measurements at these locations, $Y_A(\Omega_1)/Y_A(\Omega_2)$. This ratio is normalised to the value measured for the assumed-isotropic emission of an exploding-pusher capsule to eliminate systematic uncertainties (denoted $Y_B(\Omega_1)/Y_B(\Omega_2)$), resulting in a quantity referred to as the ‘specific activity ratio’, which essentially measures the unscattered-neutron yield asymmetry:

$$R_{SA}(\Omega_1) = \frac{Y_A(\Omega_1)/Y_A(\Omega_2)}{Y_B(\Omega_1)/Y_B(\Omega_2)}.$$

This ratio is related to the ρR at Ω_1 by

$$\rho R(\Omega_1) = \rho R(\Omega_2) - 47.5 \ln(R_{SA}).$$

The maximum and minimum measured R_{SA} values for shot N120205 are 1.07 and 0.85 respectively, giving a ρR range of 10.9 kg/m². The range of the burn-averaged ρR in the multi-mode simulation is 11.4 kg/m².

The error on the measured values of R_{SA} is approximately 3%, which results in an error on $47.5 \ln(R_{SA})$ of approximately 0.6 kg/m^2 ; therefore the ρR variation in the multi-mode simulation agrees with the available NAD data.

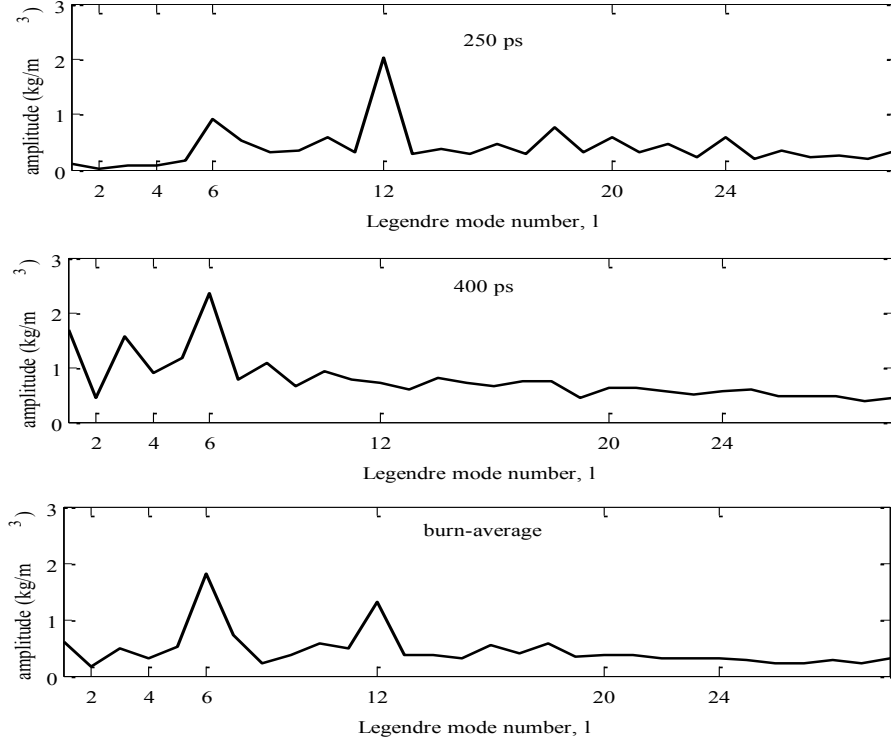


Figure 4.16: Spectra of the ρR projections of the multi-mode simulation.

[49] attempts to assess the spectrum of the ρR variation by fitting the low-order spherical harmonics given by $l = 1, m = 0, 1$ and $l = 2, m = 0$ to the $R_{SA}(\Omega_1)$ data. The negative $l = 2$ mode present in the fit is interpreted as an oblateness of the target shape, which is corroborated by the self-emission images. There is also a significant $l = 1$ mode causing a higher R_{SA} value at one side of the equator than the other. This is not considered to be a ρR anisotropy, but is explained as a result of anisotropic bulk fluid motion in the exploding pusher used for calibration which is not present in the layered implosion itself. An alternative explanation is observed in our simulations, in which anisotropic momentum associated with the perturbation is transferred to the hotspot at stagnation (Figure 4.7). Post-stagnation hotspot bulk fluid motions of a similar magnitude are also observed in 3D Hydra simulations of layered cryogenic capsules [40].

Given the small number of data points available, it is also possible that the NAD data is a result of a shorter-wavelength perturbation aliasing as an $l = 1$ and $l = 2$ perturbation. By randomly selecting 17 points from the burn-averaged $\rho R(\theta, \varphi)$ maps from our simulations we can simulate the effect of sampling the $\rho R(\theta, \varphi)$ distribution with a limited number of NADs. Figure 4.15(d) shows a fit of the $l = 1, m = 0, 1$ and $l = 2, m = 0$ spherical harmonics to 17 points taken from the $N = 42$ and multi-mode simulations. The RMS deviations of the data points from the fits are 0.74 kg/m^2 and 1.32 kg/m^2 respectively. The amplitudes of the $l = 1, m = 0, 1$ and $l = 2, m = 0$ modes of the fit are 1.45 kg/m^2 , 3.86 kg/m^2 and 1.93 kg/m^2 respectively. For comparison, the amplitudes of the $l = 1, m = 0, 1$ and $l = 2, m = 0$ modes of the spherical harmonic transform of the full $\rho R(\theta, \varphi)$ dataset are 0.52 kg/m^2 , 0.094 kg/m^2 and 0.092 kg/m^2 respectively. This demonstrates that it is possible to interpret a shorter-wavelength perturbation as a longer-wavelength one by taking a sufficiently low number of samples, and that the NAD observations of an $l = 1$ and $l = 2$ perturbation in shot N120205 are consistent with this scenario. However, it should be noted that different choices of the randomly sampled ρR measurements can produce different fits, so there is a < 1 probability of choosing ρR samples that give $l = 1$ and $l = 2$ mode perturbations at a particular location. If there is a systematic $l = 1$ or $l = 2$ perturbation in NIF shots, this could be differentiated from an aliased sampling of a short-wavelength perturbation because the latter would not be expected to produce the same result from shot to shot. However, if there were a random $l = 1$ or $l = 2$ perturbation, this could not be distinguished from a random shorter-wavelength perturbation. Similarly, if there were a systematic short-wavelength perturbation, this could not be distinguished from a systematic $l = 1$ or $l = 2$ perturbation by the NADs – although there is no *a priori* reason why a systematic short-wavelength perturbation should occur.

4.6) Summary and discussion

A series of synthetic diagnostics has been produced from NIF-relevant stagnation-phase ICF simulations. A comparison between the synthetic diagnostics produced from our multi-mode simulation and the experimental data for shot N120205 published by Clark et al. [22] is provided by Table 2. The perturbation amplitude of the multi-mode simulation was chosen as $\delta u_R / u_R = 0.5$ in order to match the simulated yield to the experimental yield.

Table 2: Comparison between diagnostics generated from the multi-mode simulation and experimental diagnostics from NIF shot N120205.

	Multi-mode simulation	Experiment
P0 (μm)	30 (23 unblurred)	23
P2/P0	0.06 (0.10 unblurred)	0.15
Neutron Temperature (keV)	3.79 (4.19 with Doppler)	3.39 ± 0.22 (MRS), ± 0.41 (nTOF)
DSR (%) (ρR (g/cm ²))	3.6 (0.74)	4.3 (0.88) ± 0.25 (0.05)
Yield (neutrons)	6.0×10^{14}	5.6×10^{14}

Overall, the simulations do not agree entirely with the experiment, suggesting that there are other causes of yield degradation on the NIF besides perturbations in the wavelength range present in our simulations. The discrepancy between simulated and experimental implosion trajectories was included in the initial conditions used, so this does not explain the rest of the difference. Since the perturbation amplitude was matched to the yield only, it is possible that another perturbation amplitude provides a better match to all the available data; according to Figure 3.23 the yield does not vary greatly above $\delta u_R/u_R \approx 0.4$. It is also possible that the thermal conductivity in the real target is higher than the conductivity used in our simulations, which would explain the lower neutron temperature. If the thermal conductivity were increased in the simulation, the perturbation amplitude could be made lower and the yield still be reduced to experimental levels. This would result in a later bang time, giving a lower P0 and possibly higher P2/P0 and higher ρR .

The P0 and P2/P0, which are measured from the X-ray images, agree more closely when the effects of image blur are neglected. Qualitatively comparing our synthetic X-ray images (Figure 4.10) with published experimental images suggests that the unblurred image is actually a more realistic representation of the experimental data. If the appropriate Gaussian blur width were somewhat smaller than 10 μm , the P0 of the simulations would agree more closely with the experiment.

The main focus of this chapter was an investigation of the capabilities and limitations of the NIF diagnostics. The effect of perturbation amplitude on bang time and burn width was investigated. The bang

time relative to peak velocity was found to reduce monotonically with multi-mode perturbation amplitude. However, absolute experimental measurement of the bang time relative to peak velocity is difficult. The difference between X-ray and neutron bang times is simpler to measure, but it is not sensitive to perturbation amplitude for perturbation amplitudes on the scale necessary to reduce the yield to NIF experimental levels. It is more sensitive for relatively low perturbation amplitudes and may serve as a measure of perturbation amplitude in this regime. Also, if a lack of sensitivity of the bang time difference to the perturbation amplitude were measured, it would suggest that the perturbation is in the high-amplitude regime – however, such a measurement is not likely to be possible if there is no other means of measuring or controlling the perturbation amplitude. In the high-perturbation-amplitude regime where bang time difference is not sensitive to perturbation amplitude, the difference in X-ray and neutron burn widths is approximately linear with perturbation amplitude. The range of the burn width difference is 10 ps in this regime, so the resolution of the GRH and SPBT (30 ps and 50 ps respectively) would have to be improved by factor of at least 10 in order for this to become useful. In conclusion, burn width difference and bang time difference are not viable measures of perturbation amplitude with the currently available diagnostics. The shape of the X-ray and neutron burn histories was found to be altered by the perturbation, showing an increased negative skew (wider in the negative time direction), so if the diagnostics were accurate enough to measure this, this may be used as a signature of perturbation amplitude. However, as with the other effects, this is probably too small to be measured with currently existing diagnostics.

The ion temperature inferred from neutron spectra was found to correspond closely to the burn-averaged ion temperature, if there are no significant motion Doppler effects. The ion temperature inferred from neutron spectra was found to be significantly higher than the mass-averaged and volume-averaged hotspot ion temperatures. It was shown that if the temperature as calculated from the neutron spectrum is used to diagnose the hotspot pressure, the hotspot pressure can be overestimated by a factor of around 1.5.

It was found that motion Doppler broadening broadens the primary neutron spectrum by an amount on the scale of the resolution of the MRS and nTOF detectors. However, the broadening is lower than would be expected for the instantaneous spectrum of a uniform imploding shell because the neutron emission peak is close to the stagnation time and the hottest part of the hotspot has the lowest velocity due to the Kidder

velocity profile. Significantly more motion Doppler broadening is observed in perturbed targets. In multi-mode targets, more broadening was observed than in single-mode targets, raising the measured temperature by an extra 120 eV. This agrees with the recent result of Clark et al. [22], who find that the residual kinetic energy remaining in the hotspot at stagnation brings the simulated temperature closer to experimental levels. A significant line-of-sight dependence of the neutron temperature was also observed in the multi-mode simulation, resulting from the anisotropy of the perturbation. Experimental line-of sight dependence of the neutron spectrum width has been observed by the SPECA and SPECE nTOF detectors [65], although the magnitude of the discrepancy is not available. Also, the line-of-sight variation of the neutron spectrum width in the simulation was approximately 100 eV, slightly below the detector resolution. Therefore it appears that perturbations of the wavelength range studied cannot cause an LOS discrepancy in the neutron spectrum large enough to be observed, and any such observation must be explained by other forms of perturbation such as a P1 caused by global asymmetry in the ice layer thickness.

It was found that the P0 of X-ray and neutron images is usually significantly smaller than the hotspot radius, which occurs due to the self-absorption of radiation from the relatively cool hotspot edge even when filtering effects are not included. Image blur increases the P0 and reduces the sensitivity of the P0 to the hotspot radius, particularly in neutron images. These observations agree with the conclusion of [66] that “the X-ray image is actually showing the inner part of a larger, more poorly compressed hotspot than is inferred from the simulations”. It is vital that this effect is accounted for in the interpretation of the hotspot radius from experimental images. According to Edwards [16], the hotspot density is inferred using measured values of the hotspot yield, volume, burn width, and temperature according to

$$Y \propto V t_{burn} \langle \sigma v \rangle (T) n^2 \quad (4.6.1)$$

where Y is the yield, and t_{burn} is the burn width. The hotspot volume V is measured by the hotspot P0 from X-ray and neutron images, and if the P0 is significantly smaller than the hotspot radius, this would lead to an overestimate of n . Referring to Figure 4.9, if the measured P0 from a blurred X-ray image is $25 \mu\text{m}$, the target radius is $30 \mu\text{m}$, resulting in a factor of $(30/25)^{3/2} = 1.31$ overestimation of n .

The Kapton filter used on the hGXI cameras reduces the P0 and creates images with less short-wavelength perturbation structure. Due to this and the effects of image blur, a series of short-wavelength perturbations of varying amplitude is able to alias as a long-wavelength perturbation in X-ray and neutron images. The Kapton-filtered X-ray images produced from the multi-mode simulation show little evidence of perturbations beyond mode 4, although no mode-4 perturbation was explicitly included in the initialisation and there is a strong presence of perturbations of higher mode number. An independent measure of perturbation amplitude and spectrum is also provided by Neutron Activation Diagnostics, which diagnose angular variations of ρR using relative activations of samples placed around the target chamber. However, the low number of NADs available means that this perturbation measure can also be subject to aliasing. Given that it is possible to reduce the target yield to experimental levels by applying an intermediate-mode perturbation such as the $N = 42$ perturbation (corresponding to approximately $l = 6.5$), a diagnostic capable of resolving these modes would be invaluable. Increasing the number of NADs seems a viable way of achieving this, since it is simply a case of fielding a larger number of a currently existing, technically simple diagnostic. 2D X-ray backlighting during the coast phase is another possibility. The resolution of the 2D X-ray backlighting capability on the NIF is not currently high enough to clearly observe intermediate-wavelength perturbations, but there are plans to improve the resolution in the near future.

Chapter 5

Alpha Transport and Burning Targets

This chapter examines igniting targets, in which alpha particle energy deposition is a significant energy source and the dynamics is dominated by the propagation of a burn wave. The basic hydrodynamics of burn propagation are reviewed with reference to 1D simulations. The influence of alpha particle transport on the burn propagation is investigated by comparing the burn dynamics in simulations using a variety of alpha particle transport models. The relationship between alpha particle confinement and the properties of the hotspot is then investigated in 1D and 3D.

5.1) Basic dynamics of the burn phase

An igniting ICF capsule enters a qualitatively different regime of behaviour from a non-igniting capsule. In the non-igniting regime, the main source of energy in the hotspot is the work done by the imploding fluid. The igniting regime is characterised by a positive feedback loop in which a high ion temperature causes fusion alphas to deposit energy into the plasma, which raises the ion temperature, causing more fusion. For ion temperatures below ~ 32 keV, the energy is deposited into the electron species according to equation (2.1.6) and transferred into the ions by electron-ion collisions. Above this temperature, the ion thermal velocity is sufficiently close to the alpha particle velocity that a significant proportion of the alpha particle energy is

deposited directly into the ions. The runaway heating process of ignition is finally quenched by the expansion of the target.

In first stage of the burn phase, some of the energy produced by fusion reactions is transported into the dense fuel, ablating the fuel into the hotspot. This propagating-burn design improves the gain of the target, since less mass must be raised to ignition temperature to burn a given amount of fuel. The original scheme for ICF involved simultaneous ignition of the entire fuel mass without the use of a hotspot, but it was soon realised that the use of a propagating burn is more efficient [6]. The ablation front or ‘deflagration’ propagating through the dense fuel layer is not qualitatively different from the ablation front at the hotspot edge in non-igniting targets, except that transport of energy by alpha particles can provide a significant contribution to the heating of the dense fuel. Electron thermal conduction still plays a role and may dominate over alpha particle transport depending on the properties of the hotspot [67].

A simple but conceptually useful model of the deflagration is to consider a two-component system consisting of the hotspot and the region of the dense fuel which has been compressed by the accretion shock, each having uniform fluid profiles [68]. In this model, energy transport from the hotspot raises the specific thermal energy of the fluid on the inner edge of the shocked fuel layer by $\Delta\varepsilon = \varepsilon_h - \varepsilon_s \approx \varepsilon_h$. The subscript h indicates the hotspot and s indicates the shocked fuel. The rate of mass ablation is therefore

$$\frac{dM_h}{dt} = (1 - \chi) \frac{W_\alpha}{\Delta\varepsilon} + \frac{W_{TC}}{\Delta\varepsilon} \approx M_h((1 - \chi)\gamma_\alpha + \gamma_{TC}) \quad (5.1.1)$$

where χ is the fraction of the energy produced as alpha particles deposited within the hotspot and M_h is the total hotspot mass. γ_α and γ_{TC} are the fusion and thermal conduction rates as defined in Chapter 3. The speed of the deflagration front relative to the fluid (its propagation speed) is

$$u_d = \frac{1}{4\pi R^2 \rho_s} \frac{dM_h}{dt} \quad (5.1.2)$$

which can be written in terms of the energy transport rates using (5.1.1) as

$$u_d = \frac{M_h}{4\pi R^2 \rho_s} ((1 - \chi)\gamma_\alpha + \gamma_{TC}). \quad (5.1.3)$$

The pressure increase of the hotspot caused by the thermal energy deposited by the alpha particles is transmitted from the deflagration front to the accretion shock subsonically through the dense fuel (for clarification of these terms, see Figure 1.2). This causes the accretion shock to accelerate, since its speed is proportional to $\sqrt{P}$. As the burn process progresses, a transition can occur from the subsonic deflagration process into a supersonic burn wave in the form of a high-temperature shock known as a detonation. In a detonation, the temperature of the shocked region becomes large enough to ignite the freefalling (unshocked) fuel directly, without the need for heating by alpha particles or thermal conduction. This can be caused by the hotspot pressure becoming high enough relative to the pressure in the dense fuel that the hotspot pressure propagates through the dense fuel supersonically, raising the dense fuel temperature to ignition temperatures. The deflagration-to-detonation transition is observed to occur by this compressive scenario in [68]. Alternatively, it is possible for the deflagration to catch up with the accretion if the fraction of alpha particle energy transported to the deflagration front is sufficiently high. This results in the entire region behind the accretion shock having the properties of the hotspot, and so the accretion shock becomes a detonation because all the material behind it is burning. Gauthier et al. [69] find that with the effect of neutron energy deposition included, this scenario occurs in an LMJ target because preheat of the dense fuel by neutrons increases the range of the alpha particles, whereas the compressive deflagration-to-detonation transition is hindered by the preheat.

5.2) A simulation of the burn-phase dynamics

In order to investigate some of the details of the burn phase, and in particular to study alpha particle transport in burning hotspots, it is necessary to generate burning hotspots in simulations. In this chapter published peak-velocity profiles for an LMJ direct-drive target design are used [70]. The LMJ peak-velocity profiles ignite in 1D simulations, allowing study of the burn phase. Med103 [71], an updated version of the 1D Lagrangian code Medusa [72], was used for the hydrodynamic simulations of the burn phase in order to ensure adequate resolution of the ablation front in burning targets. Med103 contains a variety of physics model options. The same models as those used in Gorgon were used for the burn-phase simulations, except for radiation transport,

which is dealt with by a multi-group diffusion model. In this section, alpha particle energy deposition is treated locally and instantaneously as in Gorgon. The next section investigates alternative alpha particle transport models and their influence on the burn dynamics.

The implosion velocity of the LMJ target is $450 \mu\text{m/ns}$, significantly higher than in the NIF target. The shell of the LMJ target is also further from the centre at the time of peak velocity, with a hotspot radius of $210 \mu\text{m}$. As before, the time of peak velocity is designated $t = 0$ for ease of discussion. The burn phase begins at approximately $t = 350 \text{ ps}$ in the LMJ target, when the alpha particle energy deposition rate exceeds the rates of the other processes in the system. The hotspot ion temperature profile can be fit by the model profile of Chapter 3 (3.2.2) at the beginning of the burn phase (Figure 5.2(a)). The central electron temperature is lower than the central ion temperature at all times, which may be due to the fact that both radiation and thermal conduction energy loss terms apply primarily to the electron species, whereas shock heating deposits energy primarily into the ion species. The alpha particles also deposit their energy directly into the ions at ion temperatures above 32 keV , but the ion temperature is still higher than the electron temperature before this point. In the region where energy is deposited by thermal conduction, the electron temperature is approximately 15% higher than the ion temperature during the burn phase, which is not true in non-burning targets due to the high equilibration rate. The electron temperature profile cannot be fit by the model profile (3.2.2) but resembles the parabolic, Marshak-like profile discussed in Appendix B. While this may be a consequence of the radiation transport model used in these simulations, radiation transport is not expected to be significant in burning hotspots and the form of the profile is more likely to be due to the higher thermal conductivity in a hotter hotspot.

The velocity profile in the burn phase is modified from linearity by the ablation front. The inward velocity of the ablating material causes a region in which the inward velocity is larger than it would be in the linear profile (Figure 5.2(c)). Energy deposited near the point of maximum density creates a pressure gradient away from the hotspot edge, causing the inward fluid motion (Figure 5.2(d)). This perturbation from the linear velocity profile can be seen in the non-igniting target at $t = 500 \text{ ps}$ in Figure 3.6, but it is much more prominent in the igniting target during the deflagration stage (Figure 5.2(d)).

The transition between deflagration and detonation occurs rapidly, between approximately $t = 500$ ps and $t = 560$ ps. The transition is of the compressive type. The pressure at the ablation front is increasing with time due to the ignition process, and this is transmitted subsonically to the accretion, setting up a pressure gradient between the ablation region and the accretion shock. Since the hotspot pressure is increasing faster than linearly with time, the pressure gradient driven into the dense fuel decreases with radius – the dense-fuel pressure gradient can be fit by a decaying exponential. A velocity profile in the dense fuel is set up of approximately the form

$$u_r = u_{max} - ar \quad (5.2.1)$$

(Figure 5.2(c)) and the material at lower r -values catches up with the material further out, compressing the dense fuel. Eventually the pressure gradient is large enough that a shock propagates through the dense fuel.

The propagation of this shock can be seen when the temperature profile exhibits a high-temperature burning region radially outward of the monotonically decreasing hotspot region (visible at $t = 550$ ps in Figure 5.2(a)). This is transmitted to the accretion shock in under 10 ps. Upon arrival at the accretion surface, it is partially reflected back into the hotspot, causing a high density and temperature at the centre, as can be observed in the (r, t) diagrams (Figure 5.1). The peak temperature occurs as the reflected shock hits the centre. As the shock reflects, the dense fuel layer expands such that its mass is distributed over the entire burning region; therefore, the density of the burning region is a factor of five higher than the hotspot density was during the deflagration stage. 87% of the 2.68×10^{18} fusion reactions occur during the detonation stage as a result of this high density and temperature in the burning region. The burn during the detonation stage lasts a relatively short time – the burn width of the LMJ target is 50 ps. The high expansion velocity of the detonated material – of the order of $1000 \mu\text{m/ns}$ – causes rapid expansion cooling and ultimately quenches the burn. A thermal wave propagates ahead of the detonation shock into the freefalling fuel, preheating the electron species (Figure 5.2(a) and Figure 5.1).

In the simulation in this section, the deflagration front is driven by electron thermal conduction due to the assumption that alpha particle energy deposition is instantaneous and local. The transport of fusion energy to the hotspot edge directly by alpha particles is investigated in the next section.

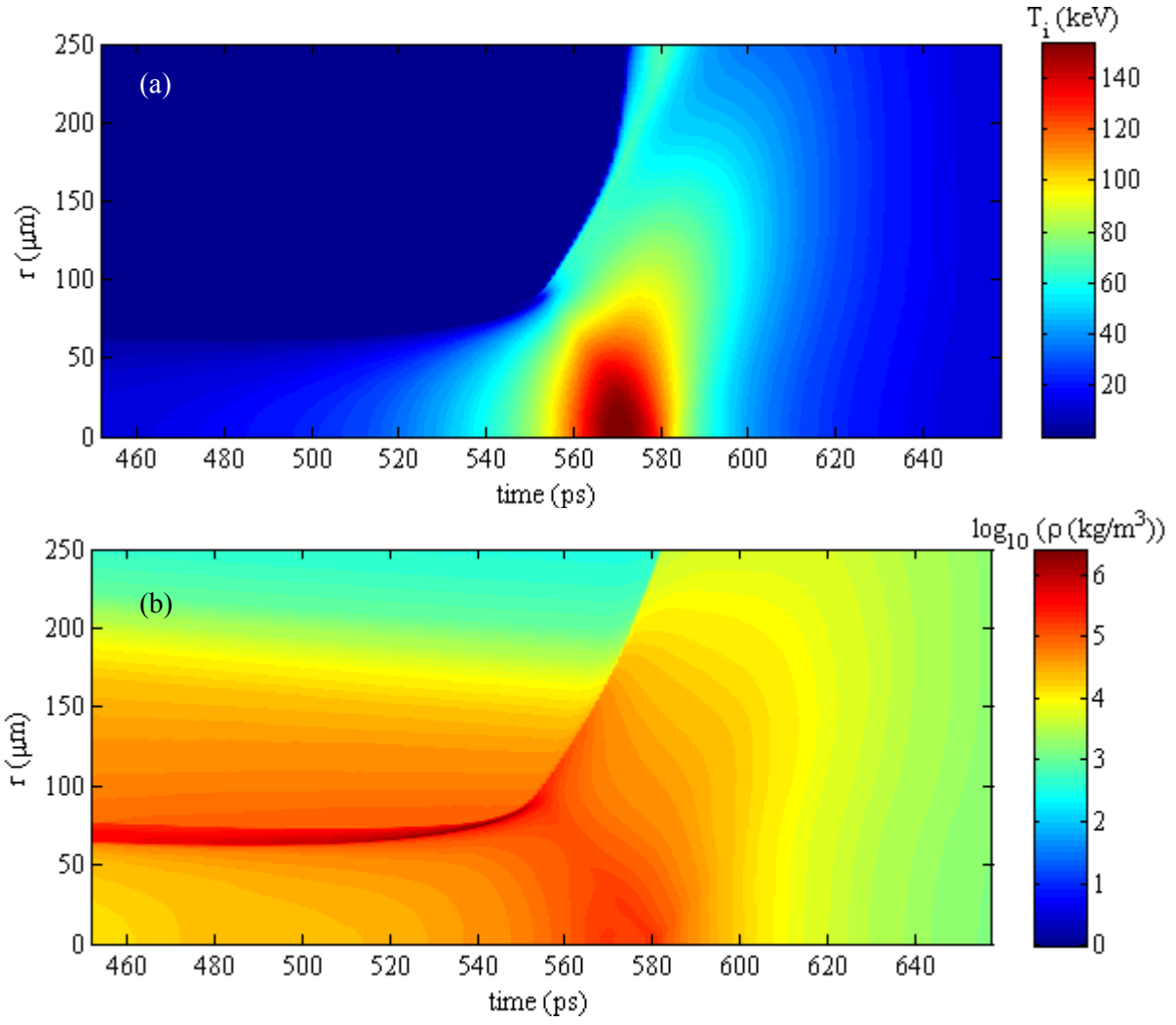


Figure 5.1: Time evolution of (a) ion temperature and (b) DT mass density profiles in the burning LMJ point design target with instantaneous local alpha particle energy deposition.

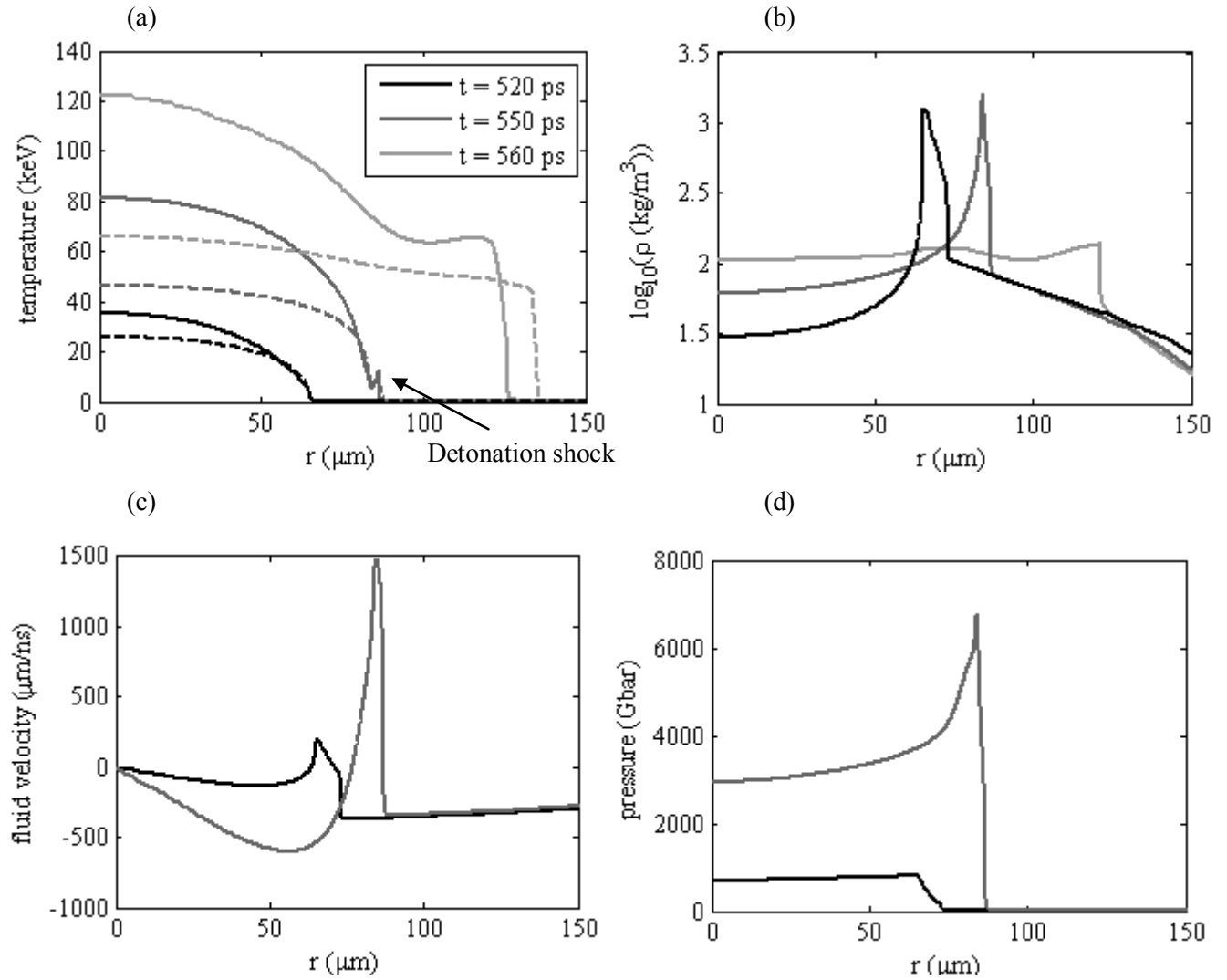


Figure 5.2: Radial profiles of (a) ion temperature (solid) and electron temperature (dashed), (b) DT mass density, (c) fluid velocity, and (d) pressure, for various times representing the deflagration stage, the deflagration-to-detonation transition, and the detonation stage of the burn propagation. The fluid velocity and pressure plots do not show the detonation stage, in order to preserve clarity.

5.3) The effect of alpha particle transport on the burn phase in unperturbed targets

The dynamics of the burn phase are highly dependant on the transport of energy by the alpha particles generated by fusion reactions. Energy deposited within the hotspot increases its temperature, contributing to the ignition feedback loop, and energy transported out of the hotspot heats the cold fuel, ablating it and propagating the deflagration front. The balance between these two effects can be parameterised by a variable,

χ , which is the fraction of the total energy created as fusion alphas that is deposited within the hotspot boundary. When the fusion rate is comparable to or greater than the other source terms, it is important to model alpha particle transport accurately.

The ‘kinetic’ model of alpha particle transport used in this work is based on [73]. A similar method is derived in [74] and applied for ICF plasmas in [75]. The fluid assumption is maintained for the DT and the electrons. This assumption is supported by Sherlock and Rose [76]. Sherlock and Rose [76] examine the distribution function of a 0D DT plasma with a large component of high-energy alpha particles, using a Monte Carlo model for the deuterium, tritium and alpha species. The electron species is treated as a Maxwellian fluid and the ions are initialised in a Maxwellian distribution. It is found that under these conditions the deuterium and tritium distribution functions stay very close to Maxwellian.

The kinetic model treats the alpha species as a set of discrete particles. The interaction between the computational particles and the background fluid is dealt with by applying slowing-down and scatter rates based on the alpha particle velocity and the background fluid properties. The full set of slowing-down and scatter rates is given in the next paragraph, but first we review a simple approximate derivation of the perpendicular scatter rate as given by Spitzer [77]. The slowing-down and scatter rates are derived by considering Coulomb collisions under the assumption of small-angle scattering, ie. that the impact parameter b of each collision is within a certain range, between the Debye length ($b = b_{max}$) and the impact parameter required for scattering through 90° ($b = b_0$). The change in the test particle’s velocity w perpendicular to its motion in a single small-angle collision is given by

$$\Delta w_\perp = 2wb_0/b. \quad (5.3.1)$$

It is assumed that collisions occur frequently enough that the rate of velocity deflection can be averaged over the impact parameter. As usual for a diffusive process, $\Delta w_\perp$ averages to 0 over a large number of collisions but the average of $\Delta w_\perp^2$ is finite. $\Delta w_\perp^2$ may be averaged over a cylinder defined by the test particle’s speed and the range of impact parameters to produce a continuous scattering rate:

$$\partial_t w_{\perp}^2 = \int_{b_0}^{b_{max}} 2\pi b^2 w n_f \Delta w_{\perp}^2 db = 8\pi w^3 n_f b_0^2 \ln \Lambda. \quad (5.3.2)$$

$2\pi b^2 w n_f$ is the probability density per unit time of colliding at an impact parameter of b , with n_f being the background fluid number density. Substituting in $b_0 = ZZ_f e^2 / 4\pi \epsilon_0 m w^2$ from Rutherford scattering, with m and Z being the mass and atomic number of the test particle, gives:

$$\partial_t w_{\perp}^2 = A_D / w, \quad (5.3.3)$$

where $A_D = Z^2 Z_f^2 e^4 n_f \ln \Lambda / 2\pi m^2 \epsilon_0^2$.

The preceding analysis assumes the mass of the background fluid particles $m_f \gg m$. The same analysis for general m_f , which integrates the collisions over a Maxwellian distribution of the background particles, was performed for stellar dynamics by Chandrasekhar [78] and expressed for a plasma by Spitzer [77]. These are the rates used in the kinetic model of Sherlock [73]. Along with the perpendicular scattering rate $\partial_t w_{\perp}^2$, a parallel scattering rate $\partial_t w_{\parallel}^2$ and a friction-like slowing down rate $\partial_t w_{\parallel}$ is derived, which taken together completely describe the evolution of the motion of the test particle:

$$\partial_t w_{\perp}^2 = (\varphi(x) - G(x)) A_D / w, \quad (5.3.4)$$

$$\partial_t w_{\parallel}^2 = G(x) A_D / w, \quad (5.3.5)$$

$$\partial_t w_{\parallel} = -G(x) A_D (1 + m/m_f) / v_{th}^2, \quad (5.3.6)$$

$$G(x) = \frac{\varphi(x) - \varphi'(x)}{2x^2}. \quad (5.3.7)$$

v_{th} is the thermal velocity of the background fluid and $x = w/v_{th}$. $\varphi(x)$ is the error function, which is equal to the fraction of the fluid particles which are moving slower than the test particle. The coefficients $G(x)$ and $(\varphi(x) - G(x))$ are plotted in Figure 5.3. Notice that $G(x)$ peaks when $w = v_{th}$, meaning that the maximum slowing-down rate $\partial_t w_{\parallel}$ occurs at this point. When $w \gg v_{th}$, $(\varphi(x) - G(x)) \sim 1$, so the high- m_f result is recovered; slow fluid particles behave like heavy fluid particles. At the creation velocity of alpha particles

from DT fusion, 1.29×10^7 m/s, most of the slowing-down is due to the electrons, but most of the scattering is due to the ions because scattering does not depend as strongly on the thermal velocity.

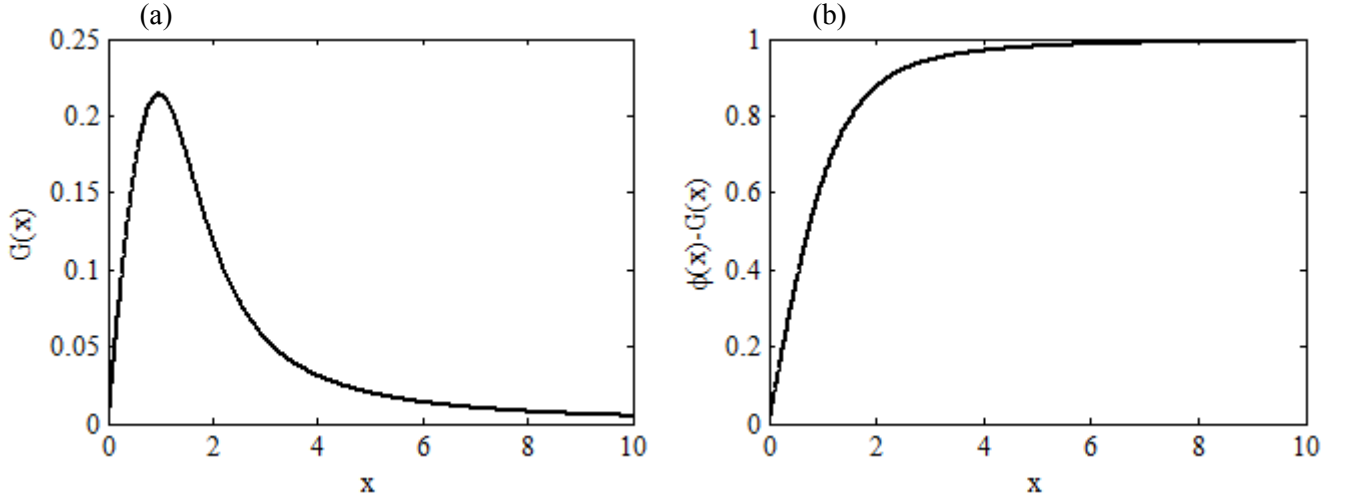


Figure 5.3: x -dependance of $G(x)$ and $\varphi(x) - G(x)$, which control the drag and scatter rates of a test particle in a Maxwellian background.

In order to apply this model to the transport of DT fusion alphas, computational particles are generated in each cell according to the fusion rate in that cell. Each computational particle represents many real particles. To minimise statistical fluctuations, the number of real particles per computational particle is chosen based on the average thermal energy of cells over the system:

$$N_{comp} = N_{\alpha} \frac{3.52 \text{ MeV}}{100 \langle E \rangle}, \quad (5.3.8)$$

where $\langle E \rangle$ is the mean thermal energy per cell across the system and N_{α} is the number of alpha particles generated in a cell in a particular timestep. A good alternative for $\langle E \rangle$ would be to use the minimum thermal energy of the cells in the hotspot. The initial velocity of the computational particles is 1.29×10^7 m/s and the velocity direction is chosen randomly, uniformly distributed over 4π radians. The number of computational particles generated in a cell is limited to $\max(N_{\alpha}/N_{comp}, 6)$ to reduce under-resolution of the emission direction. Each particle's initial position is randomly chosen within the cell. Cells with a fusion rate below a certain threshold do not produce any computational particles in order to reduce the computational load.

The motion of the computational particles is evolved according to the rates (5.3.4) - (5.3.6), using the method described by Sherlock [73]. These rates were derived for a fluid with zero bulk velocity, and it is necessary to transform into the fluid frame to apply them computationally. The velocity v of a given computational particle is transformed into the fluid frame by $c = v - u$ where u is the fluid velocity. It is then useful to rotate the coordinate system into a frame in which the particle velocity is aligned in the z -direction, giving $(w_x, w_y, w_z) = (0, 0, c)$. The drag rate can then be applied by simply incrementing w_z by

$$\Delta w_z = \partial_t w_{\parallel} \Delta t. \quad (5.3.9)$$

Parallel scattering is applied by incrementing w_z by a velocity increment chosen from a Gaussian distribution of standard deviation $\sigma_{\parallel} = \sqrt{\partial_t w_{\parallel}^2 \Delta t}$. For perpendicular scattering, the perpendicular scattering increment $\Delta w_{\perp}$ is chosen randomly from a Gaussian distribution of standard deviation $\sigma_{\perp} = \sqrt{\partial_t w_{\perp}^2 \Delta t}$ and the perpendicular scattering angle θ is chosen with uniform probability between 0 and 2π , leading to velocity increments of

$$\Delta w_x = \Delta w_{\perp} \cos(\theta), \quad (5.3.10)$$

$$\Delta w_y = \Delta w_{\perp} \sin(\theta). \quad (5.3.11)$$

Once the particle velocity has been updated in this reference frame, it is necessary to rotate back to the original coordinate system and add the fluid velocity to return to the lab frame. The timestep is limited according to

$$\Delta t = \min \left(\frac{\Delta s}{w}, \frac{w}{N \partial_t w_{\parallel}}, \frac{w}{N \partial_t w_{\parallel}^2}, \frac{w}{N \partial_t w_{\perp}^2} \right). \quad (5.3.12)$$

The first condition ensures the particle does not skip all or part of a fluid cell; Δs is the path length to the edge of the current cell. The other three conditions ensure the evolution of the particle's velocity is adequately resolved. This is typically only significant at the end of the particle's trajectory when the velocity is changing rapidly. At the start of the particle's trajectory, its high speed means the first condition dominates. N is a factor greater than 1, set to 40 in the simulations in this work, though $N \sim 10$ should suffice.

Energy and momentum transfer between the particles and the fluid is handled as follows [73]. The total momentum and energy lost by particles in a given cell is summed for each hydrodynamic timestep. The momentum lost by the particles is added to the fluid cell. The total kinetic energy gained by the fluid cell can be calculated from its momentum gain. The thermal energy gained by the fluid cell is

$$\Delta E = (\Delta E)_{particles} - (\Delta E)_{kinetic} , \quad (5.3.13)$$

where $(\Delta E)_{particles}$ is the total energy lost by particles in the cell and $(\Delta E)_{kinetic}$ is the kinetic energy gained by the cell. This procedure ensures conservation of energy and momentum within the system.

Alpha particle transport is also often modelled using a diffusion model. A multi-group diffusion model is derived by Cormann et al. [79]. Under the assumption that the velocity direction of the alpha particle species is isotropic everywhere, the Fokker-Planck equation reduces to a diffusive equation in terms of the number density of particles within an infinitesimal energy interval. In other words, the distribution function $n(\mathbf{x}, \mathbf{v})$ can be expressed simply in terms of the magnitude of the velocity, $n(\mathbf{x}, v^2)$. When the particle energy mv^2 is stored by a set of discrete grid points the diffusion equation becomes

$$\frac{dn_g}{dt} = \nabla \cdot D_g \nabla n_g - \frac{n_g}{\tau_g} + \frac{n_{g+1}}{\tau_{g+1}}, \quad (5.3.14)$$

where n_g is the number density of particles in energy group g , τ_g is the rate of transition of particles from group g to group $g - 1$, and D_g is a spatial diffusion coefficient, which is derived in a flux-limited form in [79]. The highest-energy group has its upper boundary at the alpha production energy (3.52 MeV) and has the fusion source term in the place of n_{g+1}/τ_{g+1} . Energy deposition into the fluid is handled by adding $E_g - E_{g-1}$ thermal energy into the fluid for each particle that transitions between energy group g and group $g - 1$, where E_g is the upper energy boundary of group g .

A single-group diffusion model can be written in terms of the energy density of the alpha species [80]:

$$\frac{dE_\alpha}{dt} = \nabla \cdot D_\alpha \nabla E_\alpha - \frac{E_\alpha}{F_e \tau_{\alpha e}} + \frac{dW_\alpha}{dV} - \frac{5}{2} \rho E_\alpha \frac{dV}{dt}, \quad (5.3.15)$$

where dW_α/dV is the volumetric rate of fusion energy generation, $\tau_{\alpha e}$ is the alpha-electron collision time, F_e is the fraction of energy deposited into electrons, and the final term accounts for the change in energy density due to expansion or contraction of the fluid (V here is the specific volume). $F_e\tau_{\alpha e}$ gives the total energy exchange rate between the alphas and both ions and electrons. This single-group model has been implemented in Medusa for comparison with the kinetic model.

The typical energy transport properties of each model are illustrated by Figure 5.4. Each model has been run on the same set of typical hotspot profiles (Figure 5.4(a)), with the fluid motion switched off. The local model is simply the reaction rate, and results in most of the alpha particle energy being deposited in the centre where the plasma is hottest. The kinetic model exhibits a ‘Bragg peak’ toward the hotspot edge, caused primarily by the $\sim 1/w^2$ dependence of the stopping power (5.3.6). Since most of the particles are generated in the centre, they are reaching the end of their trajectory at the hotspot edge, so they lose their energy most rapidly there. The higher density and lower temperature towards the hotspot edge also contribute to this effect. The diffusion model deposition curve does not reproduce the Bragg peak, producing an approximately uniform deposition profile which only falls off by 15% in the first 88% of the hotspot radius. However, the multi-group diffusion model is able [81] to qualitatively reproduce the Bragg peak in 1D with 10 energy groups, possibly fewer.

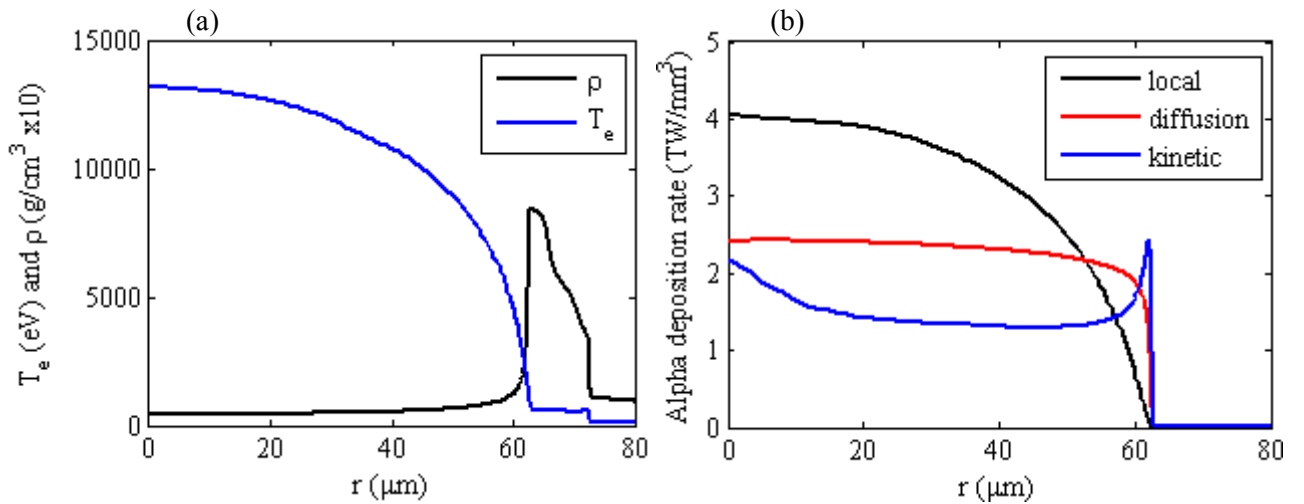


Figure 5.4: (a) Typical deflagration-phase fluid profiles used for comparison of alpha particle transport models. (b) Comparison of volumetric energy deposition rates for local, diffusion and kinetic models of alpha particle transport, as calculated for the profiles in (a).

The magnitude of the alpha particle transport produced by each model can be parameterised by the χ variable introduced in Section 5.1 to represent the fraction of alpha energy deposited within the burning region of the hotspot. The boundary between the ‘burning region’ and the ‘ablation region’ is defined in terms of the fluid density, as

$$R_a = r(\rho = e\rho_c). \quad (5.3.16)$$

ie. the point at which the density is a factor of e greater than the central density. χ is the fraction of alpha particle energy deposited within this radius. For the hotspot in Figure 5.4, the kinetic model calculates a χ value of 0.74, compared with 0.88 in the diffusion model and 0.98 in the local model.

The effect of alpha transport on the dynamics of the burn phase can be investigated by running the alpha transport models inline with Medusa and making comparisons between the dynamics produced under the various approximations. The yields, peak temperatures, and peak temperature times of the LMJ target run using the various alpha transport models are given in Table 3.

Table 3: Comparison of yields, peak temperatures, and peak temperature times of the burning LMJ target run using the local, delay, diffusion and kinetic models

Model	Neutron yield	Peak T_i (keV)	Peak T_e (keV)	T_i peak time t (ps)
Local	2.62×10^{18}	155	73	570
Delay	1.70×10^{18}	80	37	610
Diffusion	2.28×10^{18}	95	47	610
Kinetic	1.01×10^{18}	50	32	643

The effect of the finite collision time between the alpha species and the background fluid can be decoupled from the effect of the alpha particles’ motion by setting $D_g = 0$ in the diffusion model. This approximation is referred to as the ‘delay’ model. The alpha energy density in the delay model is advected with the fluid. The delay model produces a significantly lower and later temperature peak than the local model. The delay model temperature drops monotonically below the local model because of the thermal lag in

the alpha particle species (Figure 5.5). The thermal lag in the kinetic model is greater than that of the delay or diffusion models, which is part of the reason for its still later peak temperature time.

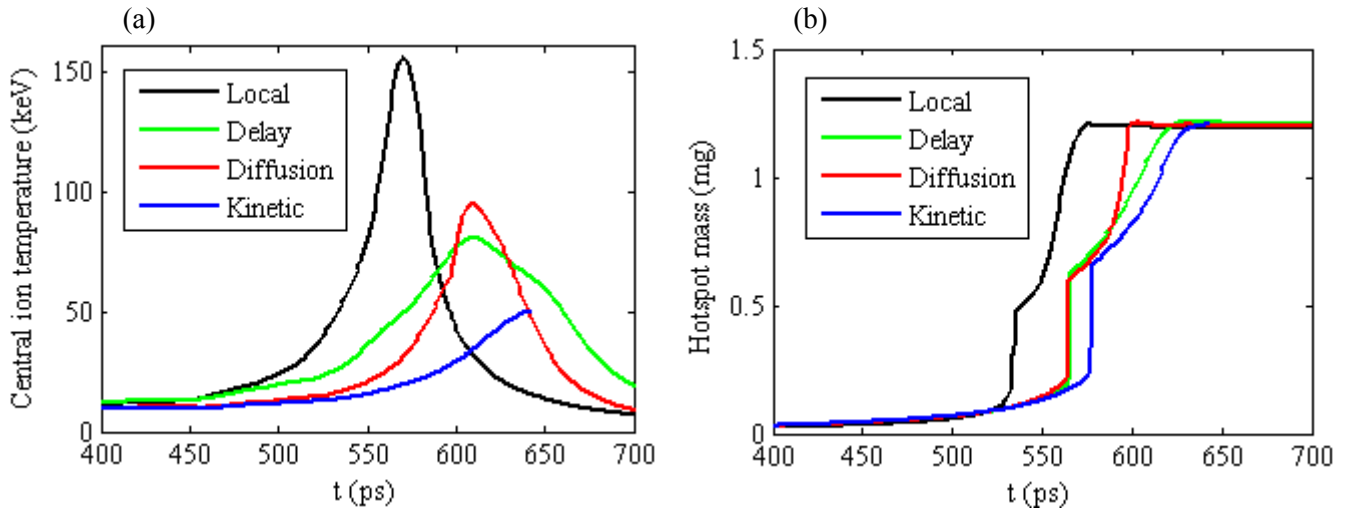


Figure 5.5: (a) Time dependence of central ion temperature as calculated by each of the alpha particle transport models. (b) Time dependence of hotspot mass as calculated by each of the alpha particle transport models. Hotspot mass is equivalent to the position of the hotspot radius in mass coordinates, and therefore gives the position of the burn wave in a Lagrangian frame. The sudden jump occurs at the deflagration-to-detonation transition.

The thermal lag in the delay model delays the transition to the detonation phase. The transition to the detonation phase can be observed as a large, sudden increase in hotspot mass (Figure 5.5(b)). The hotspot mass is equivalent to the position of the hotspot edge in mass coordinates, and therefore gives the Lagrangian position of the burn wave as a function of time. The transition occurs when the pressure gradient in the dense fuel is large enough to drive a shock through the dense fuel. The dense-fuel pressure gradient in the delay model is lower at a given time compared with the local model because the hotspot pressure is lower as a function of time due to its lower temperature, resulting in the later transition to the detonation phase. Despite the fact that the diffusion model has a lower temperature than the delay model at the time of the detonation transition and at all times before, the diffusion model detonates at approximately the same time as the delay model. This is due to the higher hotspot density in the diffusion model resulting from ablation of the dense fuel. This ablation does not significantly increase the hotspot volume due to the high mass density of the fuel, so there is the same amount of thermal energy in a similar volume compared with the delay model, resulting in

a similar pressure. The central pressure in the diffusion-model hotspot is only 5% lower at $t = 550$ ps; this small difference is compensated by a larger pressure peak at the ablation region resulting from alpha particle energy deposition.

Models with more energy transport by alpha particles have lower temperatures during the deflagration phase, the effect of the thermal energy loss on the fusion rate not being compensated by the resulting mass increase. This is why the diffusion model has a lower temperature than the delay model at all times except around the peak. The higher temperature at peak in the diffusion model occurs during the detonation stage. More of the detonated hotspot is reacting in the diffusion model because the thermal energy is distributed more evenly. In the detonation stage of the diffusion model, the temperature of the burning material is such that the temperature exponent of the fusion reactivity $m < 2$ in the hottest part of the hotspot. Since the volumetric fusion rate depends on temperature as $dW_\alpha/dV \propto P^2 T^{m-2}$ and the pressure is approximately uniform, the effect of the alpha particle transport in evening out the hotspot temperature actually results in a more efficient burn in the detonation stage for the diffusion model. This effect does not occur as strongly in the kinetic model since it reaches lower temperatures. The slightly higher hotspot mass in the kinetic model does not compensate for the energy loss in either the deflagration or detonation stage, since the transported energy is present in low-temperature material with a smaller fusion reactivity.

Figure 5.6 compares the temperature and pressure profiles of the LMJ target as calculated by the various alpha transport models, 50 ps before peak temperature (shortly before the detonation) and at peak temperature (shortly after the detonation). As expected, the temperature in the outer regions of the hotspot is closer to that of the centre in the models with more energy transport by alpha particles. However, the pressure is still approximately uniform except at the pressure peak occupying a few-micron region at the hotspot edge, where the majority of the electron conduction and alpha particle energy is deposited. The kinetic model has a significantly larger pressure peak in the deflagration stage (Figure 5.6), which corresponds to a higher inward velocity of the ablated fluid ($125 \mu\text{m/ns}$ as compared to $103 \mu\text{m/ns}$ in the local model at $t = 500$ ps). The post-detonation temperature profiles show more preheat of the electrons ahead of the detonation when alpha transport is included. Whatever the alpha particle confinement properties of the hotspot, there will always be

some transport of energy ahead of the detonation by alpha particles, since alpha particles are being generated in the plasma immediately behind the detonation front.

Since the temperature peak is caused by a shock hitting the axis, the effect of the finite alpha particle collision time reduces the peak temperature reached. In the local model, when the shock hits the centre it causes a large spike in the fusion rate which reinforces the high central pressure (Figure 5.6(b)). In the delay model, the energy feedback from fusion is distributed over time, so the central pressure spike is not as large – however, the central temperature spike persists for longer as seen in Figure 5.5(a).

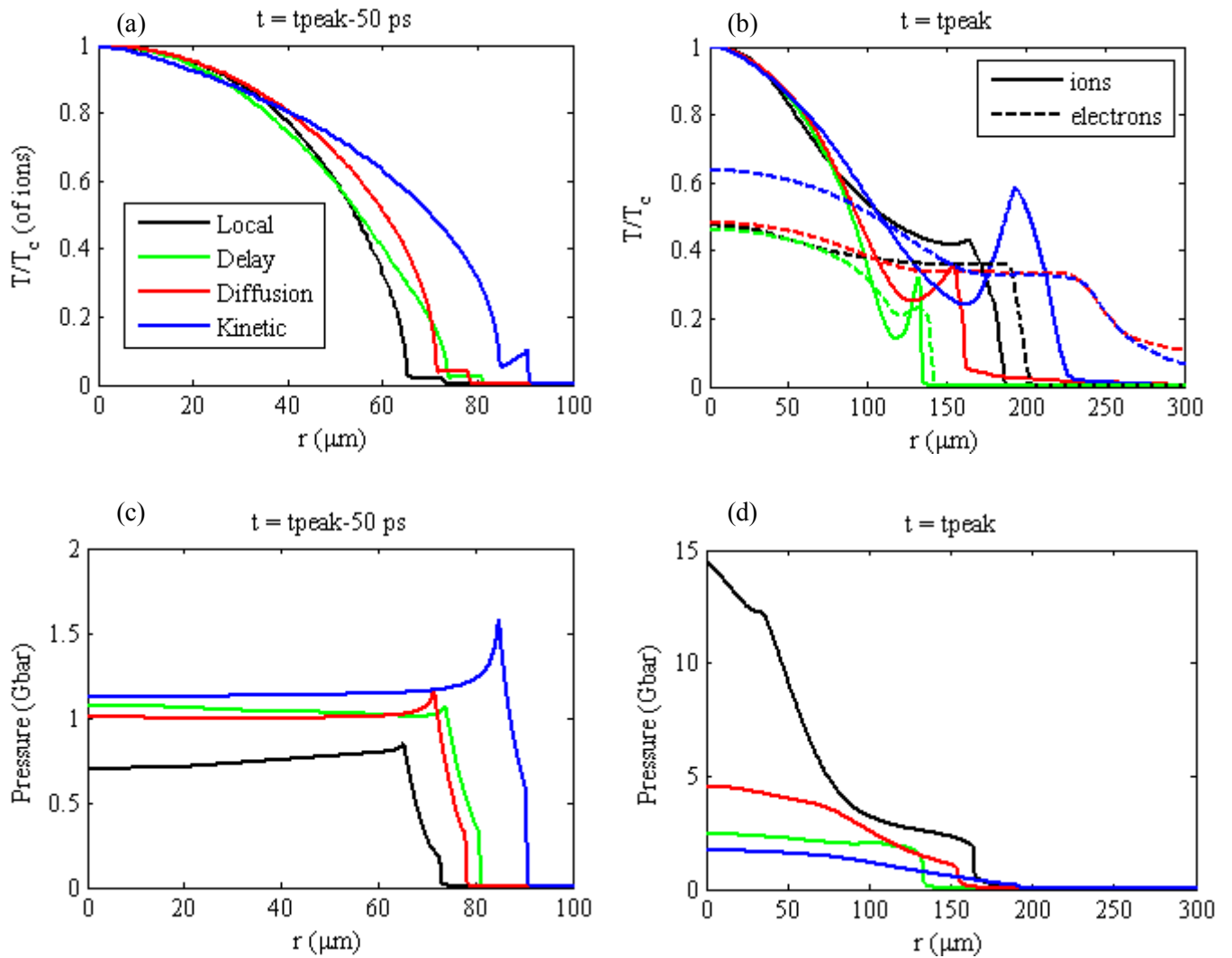


Figure 5.6: (a) Ion temperature profiles as calculated by each of the alpha transport models, 50 ps before the peak temperature occurs. Each profile is scaled to its central temperature. (b) Ion (solid) and electron (dashed) temperature profiles for each model at the time of peak temperature. (c) Pressure profiles 50 ps before the peak temperature occurs. (d) Pressure profiles at peak temperature.

The alpha particle confinement properties of the hotspot can be parameterised in terms of the burning region's 'transparency' to alpha particles $\tau = l_\alpha/R_\alpha$ (as defined in [68]) where l_α is the alpha particle range, given in [68] as

$$l_\alpha = 1.07 \frac{T_e(\text{keV})^{3/2}}{\rho \ln \Lambda} \text{ m.} \quad (5.3.17)$$

This alpha particle range is valid while the alpha particle's motion is dominated by slowing-down on the electrons, ie. when the ion temperature $T_i \lesssim 30 \text{ keV}$, which is true for most of the deflagration phase in the diffusion and kinetic model simulations. For the kinetic simulation of the LMJ target, τ begins around 3 at the start of the deflagration phase, and drops to 1.3 by the beginning of the detonation phase. In the diffusion-model simulation the minimum value of τ is slightly higher, at 1.7.

In order to ascertain empirically the relationship between τ and χ , the kinetic model was run for a set of unperturbed hotspots with various parameters. This was carried out using Gorgon, although since the hydrodynamic capabilities of the code were switched off (the background fluid was held static) the particular hydrocode used is of no consequence. Holding the background fluid static is necessary to relate a single value of χ with a single value of τ . A fluid state during the deflagration phase was used, calculated by the local model with uniform profiles in the hotspot and shell as the initial conditions. The deflagration-phase profiles generated were similar to those in the LMJ target. The static deflagration-phase background fluid configurations were rescaled by a variety of factors in either temperature, density or grid spacing. Rescaling the grid spacing has the effect of varying R_α . χ was measured for each rescaled configuration using the kinetic alpha transport model to provide χ calculations for a variety of τ values. The results are shown in Figure 5.7. A linear fit of τ to $\ln \chi$ yields

$$\chi = \exp(-0.110\tau - 0.0378). \quad (5.3.18)$$

This provides a means of calculating the expected χ of an unperturbed hotspot based on its temperature and areal density.

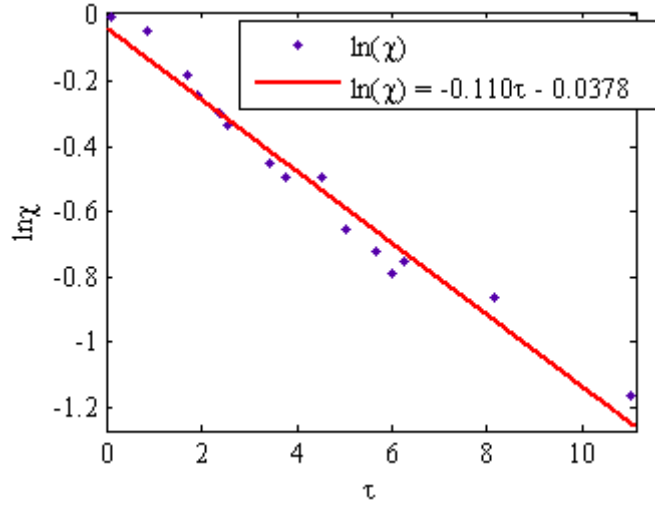


Figure 5.7: χ and τ values calculated from kinetic simulations on static unperturbed background fluids in Gorgon.

Figure 5.8(a) shows the variation of the confinement parameter χ with time for the various alpha transport models. The local and delay models have a χ of 1 during the deflagration phase as expected. During the deflagration-to-detonation transition, the χ of these models drops below 1 because alpha particles are being generated and deposited in the region of the dense fuel through which the detonation has passed, which is beyond the ablation region of the hotspot. The confinement parameter returns to 1 when the detonation has combined with the accretion shock. The single-group diffusion model calculates χ larger by approximately 0.1 relative to the kinetic model during the deflagration phase. The confinement parameter of both models increases with time during the early deflagration phase, which is matched by a decreasing hotspot transparency τ . The hotspot transparency decrease is a result of the decreasing hotspot radius, which increases ρR (Figure 5.8(b)). As the deflagration phase progresses and the hotspot temperature significantly increases, the decreasing ρR is compensated by the increase in $T_e^{3/2}$ and the hotspot transparency becomes constant with time for a period of approximately 100 ps before the detonation occurs. The hotspot transparency is lower in the kinetic model than the diffusion model during this time due to the kinetic-model hotspot's lower electron temperature.

Figure 5.8(a) shows the value of χ calculated from the kinetic-model hotspot transparency according to (5.3.18) alongside the value of χ calculated by the kinetic model. The value calculated from (5.3.18)

deviates from the simulated value of χ by less than 10% from around $t = 380$ ps until the end of the deflagration phase, the time at which the alpha particles are significant in the dynamics of the hotspot. It is less accurate before this time, but still calculates a value of χ differing by around 0.1 from the simulated value.

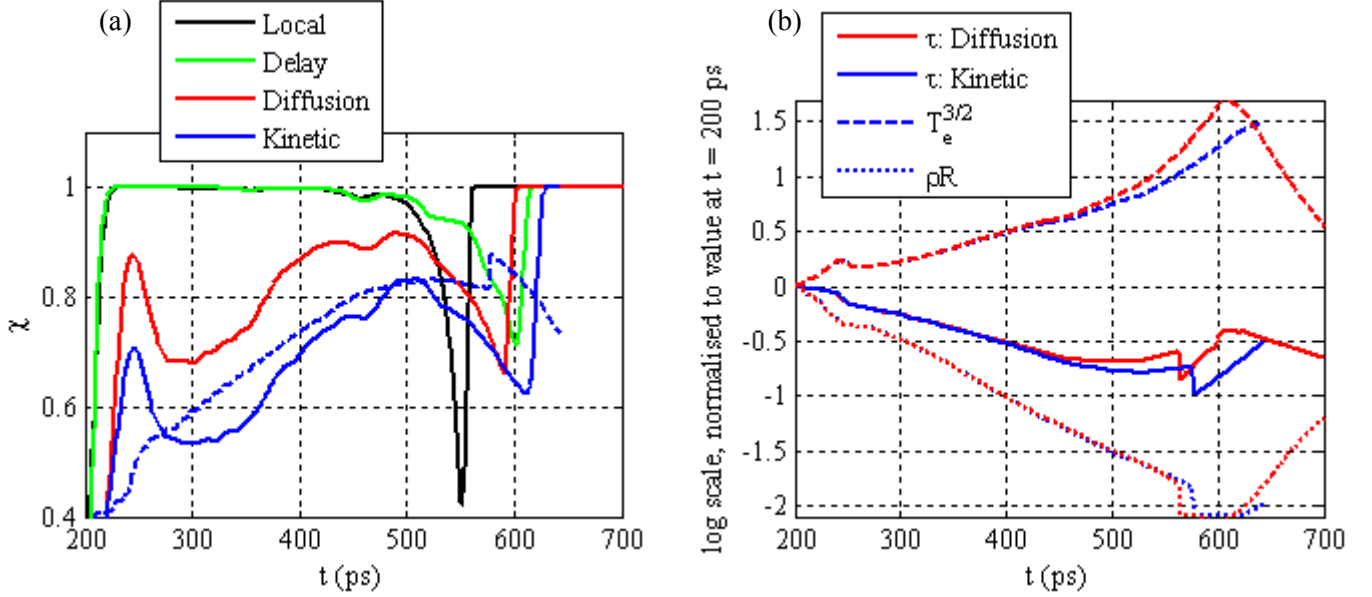


Figure 5.8: (a) Time variation of the alpha particle confinement in 1D Medusa simulations of the LMJ point design target for each of the alpha transport models. The dotted blue line shows the χ value calculated according to (5.3.18) for the kinetic simulation. (b) Change of the hotspot transparency to alpha particles τ with time (solid), broken into its factors of $T_e^{3/2}$ (dashed) and ρR (dotted).

5.4) Alpha transport in perturbed hotspots

In the previous section, the fraction of alpha particle energy deposited in unperturbed hotspots was investigated in relation to the hotspot properties. Perturbing the hotspot results in a change in the alpha confinement properties in relation to the unperturbed case. In order to investigate this, a set of perturbed burning hotspots was generated using Gorgon with various perturbation amplitudes. The kinetic model of alpha particle transport was run as a post-process on static background fluid configurations taken from the perturbed burning simulations, as in the unperturbed study used to generate Figure 5.7. Each perturbed background configuration was taken at the same time t . The perturbation amplitude is measured as the $\delta R/R$ of the perturbation at this time. The dependence of the confinement parameter χ on $\delta R/R$ is shown in Figure

5.9(a). χ is shown for two definitions of the ablation region of the hotspot: $\rho = e\rho_c$ as in the previous section, and $\rho = \rho_{\max}/\sqrt{e}$ where $\rho_{\max}$ is the peak density over the entire system, which defines a surface slightly further from the centre.

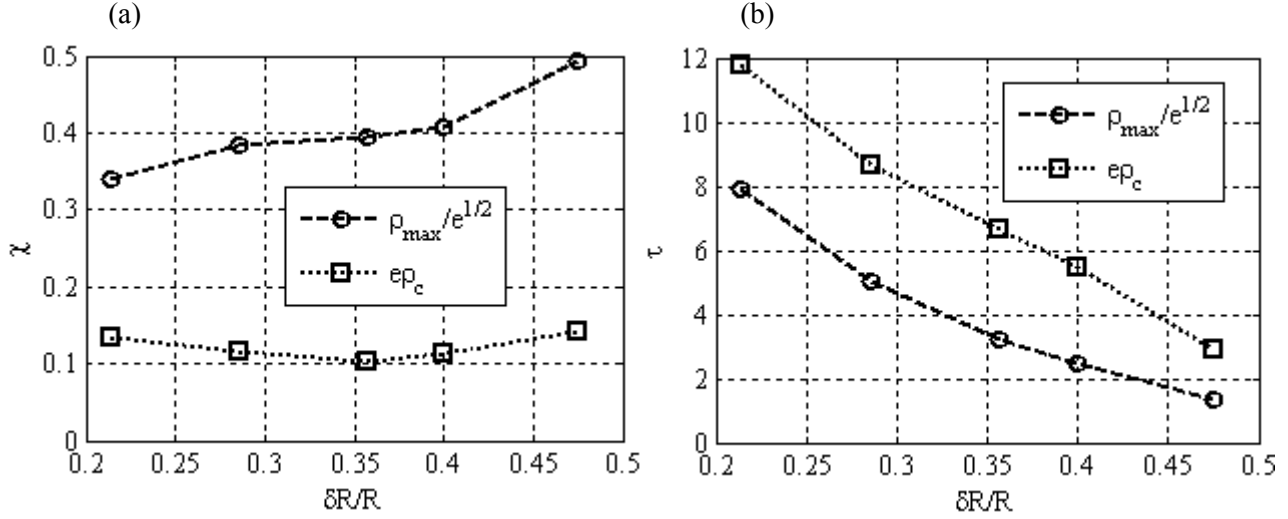


Figure 5.9: (a) Variation of the alpha particle confinement parameter χ with the perturbation amplitude $\delta R/R$ as calculated by the kinetic model in Gorgon for static perturbed background fluid configurations. (b) Variation of the burning region's transparency to alpha particles, τ , with perturbation amplitude for the same background fluid configurations. Two definitions of the boundary between the 'burning region' and the 'ablation region' are shown.

Intuitively, it might be expected that a more perturbed target reduces alpha particle confinement because a larger proportion of possible alpha particle trajectories reach the ablation region at higher energies; in other words, the hotspot surface-area is larger. However, in Figure 5.9(a) χ increases with perturbation amplitude in some cases. This is because, at this stage in the stagnation, the perturbation decreases T_e and increases ρR , so more-perturbed targets have a smaller alpha-particle transparency τ (Figure 5.9(b)). In order to calibrate for the effect of the decreasing hotspot transparency, the χ value can be normalised to the χ value of an equivalent unperturbed hotspot (5.3.18), $\chi_{1D} = \exp(-0.110\tau - 0.0378)$. χ/χ_{1D} can be interpreted as the 'shape-related' component of the alpha particle confinement. Between a perturbation amplitude of 0.21 and 0.48, the proportion of alpha particle energy deposited within the $e\rho_c$ density contour falls from 51% of the value in an equivalent unperturbed hotspot to 20% (Figure 5.10), demonstrating that the perturbation to the

hotspot's shape is responsible for greater fusion energy loss. This confirms the intuitive hypothesis that the increased surface area of perturbed hotspots should lead to inferior alpha confinement.

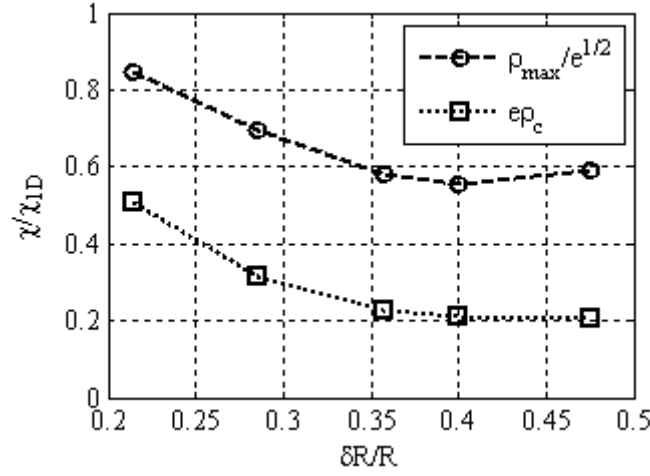


Figure 5.10: Perturbation amplitude dependance of the confinement parameter χ normalised to the value estimated for a 1D hotspot with the same alpha particle transparency τ , denoted χ_{1D} . The effect of a perturbation on alpha particle energy loss is to decrease the total energy deposited within the burning region by χ/χ_{1D} .

Slices of ion temperature and volumetric alpha energy deposition rate are shown in Figure 5.11, for $\delta R/R = 0.4$ and $\delta R/R = 0.48$. Those for perturbation amplitudes below $\delta R/R = 0.4$ are qualitatively the same as the $\delta R/R = 0.4$ slice. The volumetric alpha energy deposition rate is approximately a factor of 2 higher in the spike regions than the bubble regions in the $\delta R/R = 0.4$ hotspot, and a factor of 3 higher in the $\delta R/R = 0.48$ hotspot. The spike regions have a lower ρR at this time than the bubble regions, as observed in Figure 4.15, and so they are more transparent to alpha particles. The extra deposition on the spikes may lead to a ‘fire polishing’ effect in which the deflagration moves faster in the spike regions, reducing the perturbation growth rate. According to (5.1.3), the deflagration speed is proportional to the proportion of alpha energy transported into the ablation region ($1 - \chi$), and since the value of $(1 - \chi)$ is larger in the spike regions, one would expect a faster deflagration propagation speed there.

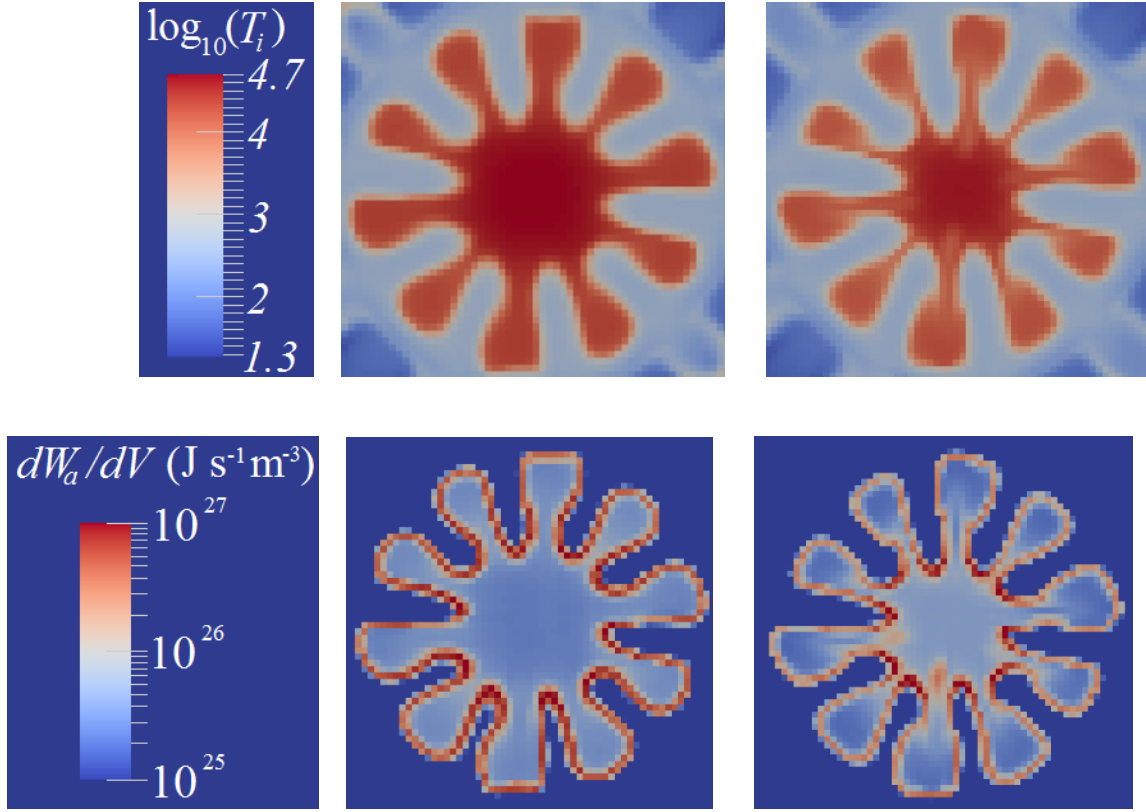


Figure 5.11: Top: $\log$ - ion temperature slices from the $\delta R/R = 0.4$ (left) and $\delta R/R = 0.48$ (right) hotspots. Bottom: volumetric alpha energy deposition rate slices from the $\delta R/R = 0.4$ and $\delta R/R = 0.48$ hotspots.

Contrary to the general trend, χ/χ_{1D} as measured by the $\rho = \rho_{max}/\sqrt{e}$ density contour increases in the $\delta R/R = 0.48$ hotspot relative to the $\delta R/R = 0.4$ hotspot (Figure 5.10). This appears to be due to the presence of the ‘Kelvin-Helmholtz rollups’ on the spikes of the $\delta R/R = 0.48$ perturbation, which are beginning to form in the $\delta R/R = 0.4$ hotspot but are more developed in the $\delta R/R = 0.48$ hotspot. These features develop in the nonlinear phase of the perturbation growth due to the shear flow on the tips of the spikes causing a Kelvin-Helmholtz instability to occur locally in the spike region, which results in a widening of the spike region. The reason for the χ increase may be that the Kelvin-Helmholtz structures compress the material in the paths leading to the bubble regions, increasing the slowing-down rate of alpha particles moving through these regions and therefore increasing the proportion of alpha particle energy deposited in a relatively low-density part of the hotspot. Since these regions have a density between $e\rho_c$ and $\rho_{max}/\sqrt{e}$, this does not affect the value of χ for the region within the $\rho = \rho_{max}/\sqrt{e}$ contour (Figure 5.10).

A P2-like perturbation can be generated by scaling the radial position of an unperturbed fluid configuration by the 2nd order spherical harmonic: $r' \propto Y_2^0 r$ with $\rho(r') = \rho(r)$ and $T(r') = T(r)$. Half of the density and volumetric alpha particle energy deposition rate slices for a prolate target generated in this manner are shown in Figure 5.12. The alpha particle heating rate is approximately 4/3 greater at the pole of this hotspot than on the equator. This may be due to the fact that alpha particles are generated with isotropically distributed velocity vectors, but there is less surface area per solid angle at the pole, so the alpha particle flux is higher. It must be noted that this does not necessarily accurately represent the conditions of a real P2-perturbed target, as the P2 perturbation of a real target can form a high-density column in the centre with separate cavities at the poles [66].

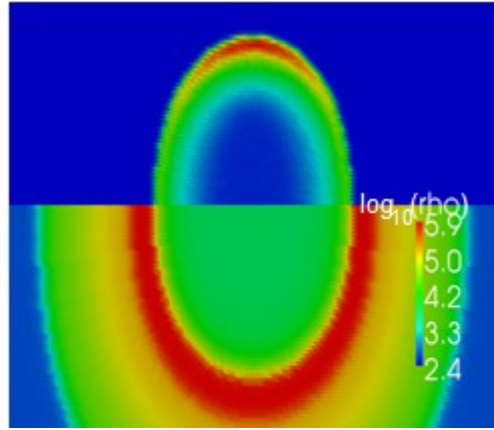


Figure 5.12: Top: log- volumetric alpha energy deposition rate slice showing half of a hotspot with a P2 perturbation. Bottom: log-density slice of the same P2-perturbed hotspot.

5.5) Summary and discussion

In this chapter, the dynamics of burning targets were discussed, with particular focus on the influence of alpha particle transport. The initial stage of the burn phase consists of a deflagration propagated by energy transport by alpha particles and electrons. In some circumstances, the burn wave transitions from a deflagration to a detonation, in which a shock propagates radially outward, igniting the fluid it compresses. Unless the alpha particle range is relatively long, the transition between these two stages is caused by the increasing hotspot pressure compressing and eventually shocking the dense fuel layer. The peak temperature occurs after this transition to detonation, when the shock that has passed from the hotspot through the fuel layer partially

reflects off the accretion and converges back into the centre, causing a high central temperature. This high central temperature is reinforced by a burst of fusion, an effect made significantly weaker by alpha particle transport due to the finite timescale and non-locality of the energy deposition. Since a large proportion of the yield is produced during the detonation phase (82% in our local-model simulation), this represents one way in which alpha particle transport can cause a significant reduction in yield. Since the hotspot pressure is lower as a function of time when alpha transport is included, the transition to detonation occurs later and the detonation progresses more slowly, which is another mechanism of yield reduction by alpha particle transport.

The fraction of fusion energy deposited within the burning region, χ , can be related to the hotspot fluid properties in terms of the ‘hotspot transparency to alpha particles’, $\tau = l_\alpha/R$, which is the ratio of the alpha particle range to the size of the burning region. χ was fit to τ for a series of static background fluid configurations to produce the relationship $\chi = \exp(-0.110\tau - 0.0378)$, which was validated against the dynamic 1D kinetic simulation. Since τ is a function of ρR and T , the energy-balance model developed in chapter 3 can be modified to include alpha particle transport by multiplying the W_α term by $\chi(\rho R, T)$. This alters the heating map as shown in Figure 5.13. The self-heating region is now much more co-incident with the one produced by the Widner-type 0D model, because hotspots in the high- T , low ρR region have a very high hotspot transparency to alpha particles, so the alpha particle loss compensates for the reduction in thermal conduction by the relatively low thermal conductivity at the hotspot edge. The trajectories of the local-model (red) and kinetic-model (green) igniting LMJ hotspots are shown in Figure 5.13. Both models ignite as they enter the self-heating region. The slope of the kinetic-model simulation is shallower during the deflagration phase since the rate of energy deposition is lower for a given ρR . The transition to the detonation phase can be seen as a sudden increase in ρR (and a temperature drop since the same energy is distributed over more mass) followed by a rise in temperature. It is interesting to note that the local-model hotspot follows the edge of the self-heating region after the detonation has occurred. This is because the self-heating region corresponds approximately to the self-heating region of the Widner-type 0D model, which becomes appropriate for the relatively uniform fluid in the detonation region. The detonated local-model hotspot follows the edge of the self-heating region because if it cools into the self-heating region, it heats again and leaves it; in this sense its trajectory is stable. The ρR remains approximately constant during the detonation;

the reason for this is not immediately obvious. The peak temperature of the local-model hotspot is therefore given by the temperature at the top edge of the self-heating region for the ρR at which it detonates. However, the kinetic-model hotspot is beginning to cool at a lower temperature than that of the self-heating region edge. The parameterisation of $\chi(\rho R, T)$ was calculated for the deflagration phase, and χ may be lower for the detonation phase. Also, the fluid velocity at the edge of the detonated region is higher than the $500 \mu\text{m/ns}$ upper limit of the colourmap in Figure 5.13, so the combination of the expansion cooling this causes and the possible extra alpha particle energy loss is causing the kinetic-model hotspot to cool despite being inside the calculated self-heating region. Since the local-model hotspot is expanding more quickly than the kinetic model yet the detonated local-model hotspot does not cool within the calculated self-heating region, one would expect that the alpha particle transport is the decisive factor.

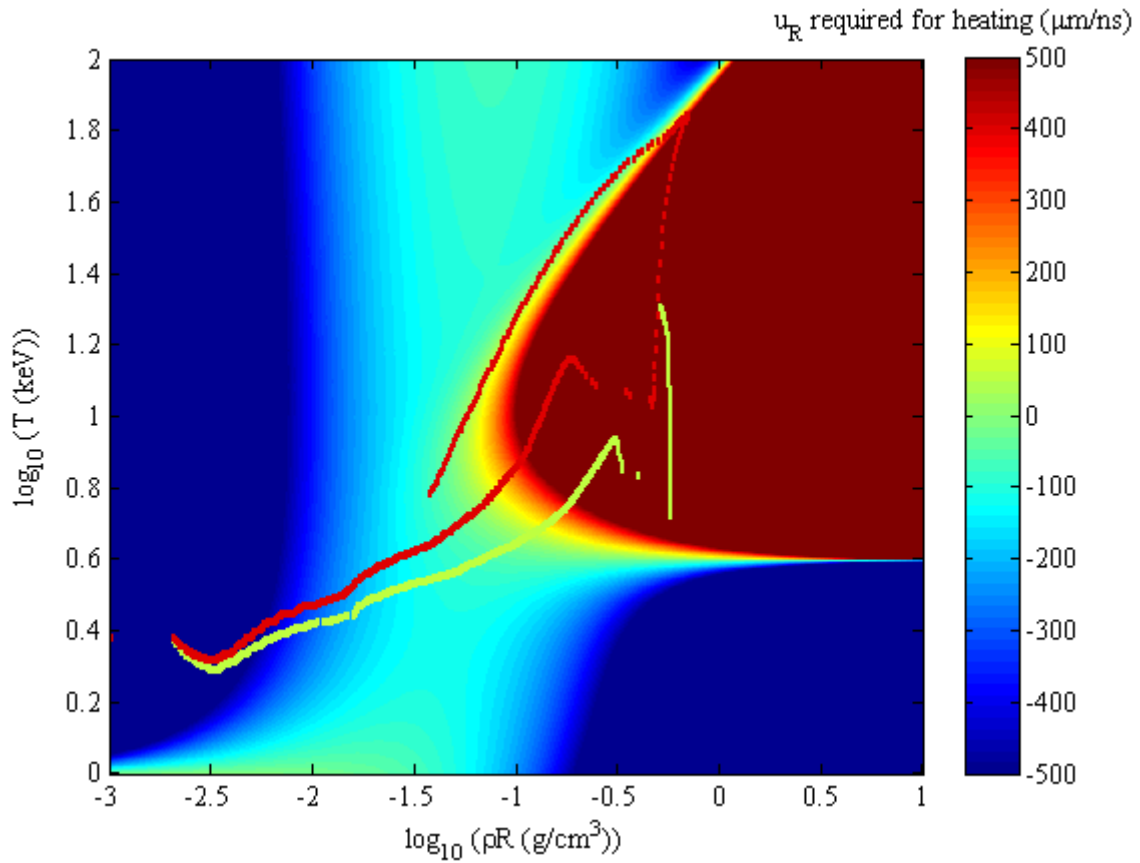


Figure 5.13: Hotspot heating map with the effects of alpha particle transport included. The trajectories of the LMJ hotspot are shown as calculated by the instantaneous local deposition model (red) and the kinetic alpha particle transport model (green).

The effect of a perturbation on alpha particle confinement was investigated. $N = 42$ perturbed targets were generated with a range of perturbation amplitudes, and a fluid configuration near bang time used for post-processing with the kinetic alpha particle transport model. It was found that the alpha particle confinement parameter χ increases as the perturbation amplitude increases due to the perturbation cooling the hotspot, causing a decrease in the hotspot transparency to alpha particles, τ . However, using the relationship found between τ and χ for unperturbed hotspots, it was shown that perturbed hotspots confine less alpha particle energy compared with unperturbed hotspots of the same τ - the perturbed hotspot geometry causes extra alpha particle energy loss. For a perturbation amplitude $\delta R/R = 0.4$, the alpha particle energy deposited within the $e\rho_c$ density contour was reduced by a factor of 0.2 relative to the value expected of an equivalent unperturbed hotspot. When the perturbation amplitude increased above this to $\delta R/R = 0.48$, there was less alpha particle deposition present at the hotspot edge in the bubble regions, which appears to be due to an effect of the shape of the Rayleigh-Taylor spikes in the nonlinear regime restricting the available paths of the alpha particles to the bubble regions, or compressing the fluid in those paths. The increased alpha particle energy loss for a perturbed hotspot shape is a further energy loss mechanism caused by perturbations in addition to those described in Chapter 3, and may be critical in marginally-igniting targets in which the maximum magnitude of the fusion heating rate is of the order of the other energy source rates.

Chapter 6

Conclusions and Future Work

A computational and theoretical study of the physics of ICF targets in the stagnation phase has been described. The majority of the simulation work was carried out using the multidimensional hydrocode Gorgon. Modifications made to Gorgon to study the stagnation phase of ICF include a stable and accurate thermal conduction solver, an imploding computational grid, and a scheme to generate suitable initial conditions. Each of these modifications is a step towards using Gorgon for full ICF implosion simulations, including the perturbation generator, which is used in a modified form to generate pre-shot capsule surface perturbations. Additional elements such as sufficiently accurate equation-of-state, opacity and radiation transport modelling are currently in development by others and it is intended that Gorgon will be capable of accurate full-capsule implosion simulations in the near future.

A 1D model of hotspot energetics

A study of the hotspot energy balance at stagnation was presented in Chapter 3. As well as being essential for ignition, the balance of the energy source terms in the hotspot at stagnation appears to be the dominant factor in determining the yield of non-igniting targets. The standard 0D ‘Widner-type’ theoretical development of the energy source terms assumes that the fluid properties are uniform within the hotspot. In a simulation of a pseudo-1D unperturbed stagnating hotspot, this was found to be adequate for the mechanical

work source term, but not the radiation, thermal conduction, or fusion terms. In particular, the thermal conduction term must be calculated using the conductivity at the hotspot edge, but 0D models use the conductivity in the hotspot centre, which is usually higher by a factor of more than 10. An improvement to the 0D approach was made by taking advantage of the self-similar temperature profile upon which the hotspot was observed to converge in simulations, and the uniform hotspot pressure caused by the low Mach number throughout most of the hotspot. When analytically integrated over these profiles and expressed as functions of the hotspot mass-averaged temperature T_{ma} , areal density $\langle \rho R \rangle$, radius R and edge fluid radial velocity $\langle u_R \rangle$, the source terms were found to be

$$W_r = \frac{4\pi}{3} A_r \langle \rho R \rangle^2 T_{ma}^{1/2} R \times H_{3/2}(T_c, q) H_1(T_c, q)^{1/2} / H_1(T_c, q/3)^2 \quad (5.5.1)$$

$$W_\alpha = \frac{4\pi}{3} A_\alpha \langle \rho R \rangle^2 T_{ma}^4 R \times H_{-2}(T_c, q) H_1(T_c, q)^4 / H_1(T_c, q/3)^2 \quad (5.5.2)$$

$$W_{TC} = 4\pi A_{TC} q R T_R^{5/2} (T_{ma} H_1(T_c, q) - T_R) / \ln \Lambda \quad (5.5.3)$$

$$W_m = \frac{8\pi}{3} \frac{q_e}{m_i} \langle \rho R \rangle T_{ma} R \langle u_R \rangle \times H_1(T_c, q) / H_1(T_c, q/3). \quad (5.5.4)$$

W_r , W_α and W_m differ from the 0D Widner-type versions by simple ‘profile factors’ given by ratios of hypergeometric functions, shown here on the right of the multiplication sign. The thermal conduction term differs somewhat more, as it takes account of the different thermal conductivity at the hotspot edge. Approximate forms of the 1D source terms in terms only of T_{ma} , $\langle \rho R \rangle$, $\langle u_R \rangle$ and R are given in Section 3.6 (equations (3.6.8) – (3.6.10)), which allow direct comparison with the Widner-type model. The 1D model calculates a considerably lower hotspot edge velocity $\langle u_R \rangle$ required to heat the hotspot in regions of low $\langle \rho R \rangle$, due to the reduced thermal conductivity at the hotspot edge. The ‘ideal ignition temperature’, for which $W_\alpha = W_r$, is slightly lower according to the 1D model in the case that the radial exponents of the ion and electron temperature profiles q_i and q_e are both approximately 3, which occurs at peak burn when the ion-electron equilibration rate is high.

The 1D analytic study of the hotspot energy source terms could be extended to 3D by making valid assumptions about the angular variation of the hotspot radius $R(\theta, \varphi)$ caused by the perturbation, then carrying out the volume integrals for this $R(\theta, \varphi)$. A preliminary attempt at this was carried out by the author

for a single-mode perturbation. $R(\theta, \varphi)$ was assumed to be a Bessel function centred on $\theta = 0$ to represent a single spike, which is then multiplied by the number of spikes to get the source term for the whole hotspot. The resulting volume-integrated source terms are the 1D source terms with an additional factor based on the perturbation amplitude $\delta R/R$. An additional complication arises because the work source term no longer has the radial fluid velocity parallel with the hotspot surface, although the temperature gradient is, due to the definition of the hotspot in terms of an isotherm. It may be necessary to make a more valid assumption for $R(\theta, \varphi)$, although the work was not carried far enough to be sure that a Bessel function is not appropriate. The current energy-balance model is time-independant. It may be worthwhile to generate a fully time-dependant model of the stagnation process by feeding the energy loss back into the hotspot pressure. The hotspot central density would be easily calculated as a function of time by taking advantage of the linear velocity profile, and the central temperature would follow from this and the pressure.

The influence of perturbations on hotspot energy balance

The 1D model developed facilitated a detailed investigation of the phenomenology of the influence of perturbations from spherical symmetry of an all-DT system on the hotspot energy balance during the stagnation phase. For single-mode perturbations, the change in the energy balance is produced by a change in $\langle u_R \rangle$, an increase of the hotspot surface area above $4\pi R^2$, and inefficient thermalisation of kinetic energy caused by the discrepancy between the hotspot pressure and the ram pressure ρu^2 of the spikes and bubbles. The temperature peak occurs almost 200 ps earlier for a single-mode perturbation with 42 spikes. This is due to the higher $\langle u_R \rangle$ early in time, which can be explained by a redistribution of mass from spikes to bubbles by the acceleration-phase Rayleigh-Taylor instability. The work rate falls off after the temperature peak, because $\langle u_R \rangle$ drops due to extra shell kinetic energy having been thermalised, and possibly by the shifting of mass back from the bubbles to the spikes in the deceleration-phase RTI. Around this time, the thermal conduction rate increases due to the larger hotspot surface area, which reinforces the cooling of the hotspot. The thermal conduction loss rate increase was found to occur sooner for shorter perturbation wavelengths, and the thermal collapse time of the hotspot bubbles was shorter for shorter wavelengths. The radiation loss rate also increases relative to an unperturbed target because the hotspot has been compressed to higher density by the higher

$\langle u_R \rangle$. Using the analytic thermal conduction source term, the thermal-collapse timescale of an isolated high temperature cavity at the hotspot edge can be related to the spatial scale of the cavity according to $\tau(\text{ps}) \approx R(\mu\text{m})^2$, which defines the scale boundary between ‘shape perturbations’ and ‘cold-fuel mix’ by requiring that τ be below the hydrodynamic timescale for thermal mix to occur. The $N = 162$ perturbation was found to thermally mix within the timescale of the stagnation, and since $N = 162$ corresponds approximately to a mode number $l = 13$, this result indicates that the transition to mix-like behaviour occurs for longer wavelengths than the $l \gtrsim 50$ often quoted.

When multiple perturbation modes are included in a single target, the main difference is that the temperature is lower around the peak. This can be explained primarily by the more divergent velocity field of the multi-mode perturbation, which is caused by the generation of off-centre spike trajectories by the superimposing of various perturbation modes, and results in a lower work rate. The thermal conduction rate at the time of peak temperature is also higher in the multi-mode simulation; since it has a lower $\langle u_R \rangle$ than the equivalent single-mode simulation before the temperature peak, it has a larger hotspot radius at this time, and so its surface area is larger. These factors combined result in a factor of 1.8 reduction in yield due to multi-mode effects for the simulations examined, leading to a total yield reduction by a factor of 9 compared with the unperturbed case.

The perturbation study raises the possibility of tuning the hotspot parameters at stagnation to optimise the stagnation process. In particular, the time dependence of the fluid velocity at the hotspot edge $\langle u_R \rangle(t)$ could be tuned to mitigate many of the detrimental effects of a perturbation. This could be investigated using the currently-existing stagnation-phase simulation capabilities of Gorgon by rescaling the initial conditions in a hydrodynamically-similar manner, with the scale factors depending on the radial coordinate. Hydrodynamically-similar scalings are those that scale from one set of solutions of the fluid equations to another [82], and a similar procedure was used to generate the ignition energy scaling law of Basko and Johner [11]. It may be possible using this method to find peak-velocity 1D target configurations which generate a $\langle u_R \rangle(t)$ pulse that optimises the hotspot energy balance at stagnation with the effects of a perturbation taken into account. Once a peak-velocity configuration were found that optimises the hotspot energy balance at stagnation, ablation/acceleration-phase simulations could be carried out to find the drive

pulse shape required to generate the necessary peak-velocity profiles. A pressure boundary condition could be used in the place of a thermal radiation drive if the fine details of the radiation drive were problematic, as in the ignition-energy study of Herrmann et al. [12].

Diagnosis of 0D and 3D hotspot properties at stagnation

The stagnation-phase simulations used for the energy-balance study were also used to generate synthetic diagnostic data, which allowed comparison between the physical parameters of the stagnating hotspot and the features of the diagnostics produced. This was presented in Chapter 4. The hotspot ion temperature as diagnosed by the width of neutron spectra was compared with the physical hotspot temperature. The ‘neutron temperature’ was found to correspond closely to the ‘burn-averaged’ hotspot temperature, given by an average of the ion temperature over the hotspot volume weighted by the reactivity. Since this produces a relatively high measure of the hotspot temperature, the hotspot pressure can be overestimated if it calculated using the neutron temperature; the correct temperature to use is the mass-averaged temperature, which is considerably lower.

In a 3D multi-mode perturbed hotspot, motion Doppler broadening of the spectrum was seen to cause the measured neutron temperature to rise by ~ 400 eV, compared to only ~ 200 eV for an $N = 42$ single-mode perturbation due to the spherical symmetry of the latter resulting in less residual kinetic energy at bang time. This agrees with the result of Clark [22] for a 3D simulation compared with a 2D simulation, in which the imposed symmetry of the 2D simulation has a similar negative effect on the residual kinetic energy at bang time. Since the residual momentum in the multi-mode simulation is directed in one particular direction, there is a line-of-sight dependence of the neutron spectrum width, reminiscent of a discrepancy observed experimentally between the spectral widths of the SPECE and SPECA nTOF detectors. However, the 100 eV discrepancy in the simulation was below the detector resolution.

The effects of self-absorption and the high-pass filters on the X-ray imagers reduce the P_0 measured from X-ray self-emission images below the hotspot radius. This effect also occurs for neutron images due to the temperature dependence of the fusion reactivity. If this is not accounted for, it results in an overestimation of the hotspot radius, which would add to the discrepancy in density between simulation and experiment

already seen. Even without the effects of image blur, the visibility of perturbations of the $N = 42$ scale and above is poor in X-ray images due to the self-absorption and filtering effects. When image blur is included, it is found that short-wavelength perturbations varying in amplitude around the capsule can alias as a long-wavelength P2 or P4 perturbation. This is also true for neutron images. The diagnosis of ρR perturbations by NADs is also subject to aliasing because of the small number of NADs available. The difference between the X-ray and neutron burn widths was found to vary with perturbation amplitude, but is not sensitive enough to perturbation amplitude to be used as a diagnostic using current equipment. Therefore, we conclude that intermediate-scale perturbations such as those in our simulations could be present on the NIF at stagnation without being detected. A 2D X-ray backlighting capability has recently been developed on the NIF, which is able to make images of the imploding target during the coast phase, at which time the spatial scale of the target is larger so it may be possible to resolve perturbations above mode 4. An efficient way to improve the capability to detect perturbations at stagnation might be to increase the number of NADs around the target chamber, since these diagnostics are simple in design.

The diagnostics produced from our simulations show larger values of P0 and the neutron temperature, and smaller values of P2/P0 and ρR than the simulations (Table 2). Since the yield generated by the perturbed simulations tends to the experimental yield, it may be that a simulation with a different perturbation amplitude matches the diagnostics and gives a similar yield. This has not yet been examined because it would be a highly time-consuming process. Each 640^3 -cell simulation takes around 12 hours to run and the dataset necessary for generating time-integrated diagnostics takes a large amount of storage space, so each dataset would need to be deleted before the next were generated. Also, a successful match to the data is not guaranteed. However, it may be valuable if it showed that the presence of an all-DT perturbation consisting of the ‘intermediate’ scale mode numbers not usually considered important for yield degradation is actually a scenario consistent with the NIF data. Another likely explanation of the discrepancy between the simulation and experiment seems to be that the thermal conductivity in the experiment is higher than the thermal conductivity used in our code. If the thermal conductivity were higher the hotspot temperature would be lower, which may bring the neutron temperature into agreement with the experiment. The experimental yield could therefore be achieved for a lower perturbation amplitude, which would bring the bang times later,

causing a lower P_0 , higher ρR as measured by the NADs, and potentially a higher P_2/P_0 since the perturbation would be at a later stage in its development.

Alpha particle transport in burning hotspots

The physics of the burn phase was investigated in Chapter 5. The role of alpha particle transport in the burn process was investigated by comparing a fully kinetic alpha particle transport model with models that omit elements of the kinetics. The main points of interest are the effect of energy loss from the burning region delaying the deflagration-to-detonation transition by reducing the time-dependant hotspot pressure, and the reduction in the peak temperature by the delay in the feedback of alpha particle energy to the temperature peak that is created in the centre shortly after the target detonates. The fraction of alpha particle energy deposited in the burning region, χ , was related to the parameters of the deflagration-phase hotspot empirically by fitting χ to the ratio of the alpha particle range to the hotspot radius, τ , which is a function of the hotspot electron temperature and areal density. This made it possible to include the effect of alpha particle transport as an additional loss term in the 1D model of energy balance developed in Chapter 3. When this is done, the self-heating region in $(\rho R, T)$ -space returns to approximately the position of the Widner-type self-heating region, although lower values of $\langle u_R \rangle$ are required to heat the hotspot at high T and low ρR than in the Widner-type model.

By postprocessing perturbed fluid configurations of stagnated targets with the kinetic alpha particle transport model in 3D, it was found that a perturbation significantly reduces χ relative to an unperturbed hotspot with the same τ . Significantly more alpha particle energy deposition was observed to occur in the spike regions, where the hotspot areal density is lowest, than the bubble regions. Since the speed of propagation of the deflagration is linearly related to $(1 - \chi)$, ie. to the proportion of alpha particle energy transported to the ablation region, it is likely that this would lead to a ‘fire polishing’ effect in which alpha particle transport reduces the perturbation growth rate. The kinetic model of alpha transport in Gorgon is in principle capable of running inline with the hydrodynamics, and fire polishing in burning or marginally-igniting ICF targets would be an interesting area to investigate using this capability.

In summary, the main subject of the work presented here has been the effect of perturbations on the energetics of the stagnation, and the potential for intermediate-wavelength perturbations large enough to significantly reduce the yield to remain undetected by currently existing diagnostics. It is hoped that this work will form part of an effort for the presence and effects of three-dimensional DT perturbations of intermediate wavelength to become better understood. In particular, it is hoped that the modelling and phenomenological understanding of the effects of these perturbations on the hotspot energy balance might lead to the development of methods of mitigating those effects.

Appendix A:

The Ideal Ignition Temperature

The 1D model derived in Chapter 3 allows calculation of the ‘ideal ignition temperature’, at which alpha deposition exceeds radiation loss, ie.

$$\frac{W_\alpha}{W_r} = \frac{H_{-2}(T_c, q_i)}{H_{3/2}(T_c, q_e)} \frac{A_\alpha}{A_r} T_c^{7/2} > 1. \quad (5.5.5)$$

This is the minimum temperature required for ignition for any $\langle \rho R \rangle$. W_α/W_r is plotted as a function of T_c in Figure A.1(a). The central temperature for which $W_\alpha/W_r = 1$ is 9.29 keV for $q_i = 3$, significantly higher than the 4.53 keV threshold temperature in a uniform hotspot. However, when various hotspot temperature averages are considered, the ideal ignition temperature is lower. Three averages of the hotspot temperature may be defined as follows:

$$T_{va} = \frac{\int T dV}{\int dV} = H_{-1}(T_c, q) T_c, \quad (5.5.6)$$

$$T_{ma} = \frac{\int T \rho dV}{\int \rho dV} = T_c / H_1(T_c, q), \quad (5.5.7)$$

$$T_{ba} = \frac{\int \langle \sigma v \rangle \rho^2 T dV}{\int \langle \sigma v \rangle \rho^2 dV} = \frac{H_{-3}(T_c, q)}{H_{-2}(T_c, q)} T_c. \quad (5.5.8)$$

Appendix A

These are the volume-averaged, mass-averaged and burn-averaged temperatures respectively. The W_α/W_r curves for each of these temperature measures are shown in Figure A.1(b), and the volume-averaged, mass-averaged and burn-averaged temperatures for which $W_\alpha/W_r = 1$ are shown in Table 4.

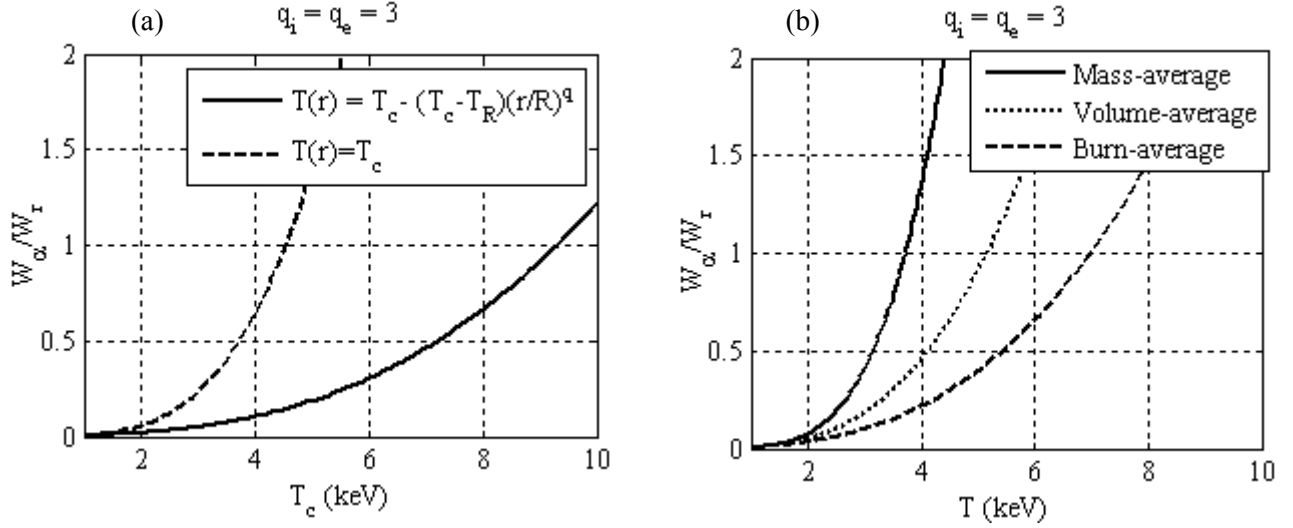


Figure A.1: (a) Ratio of fusion rate to radiation emission rate as a function of central temperature for the temperature profile of the unperturbed hotspot (3.2.2) with $q = 3$ (solid) and for a hotspot of uniform temperature (dashed). (b) Ratio of fusion rate to radiation emission rate for the unperturbed hotspot (3.2.2) as a function of three different measures of the global hotspot temperature.

Table 4: Central, volume-averaged, mass-averaged, and burn-averaged temperatures for which $W_\alpha/W_r = 1$

	$q_i = q_e = 3$	$q_i = 1.5, \quad q_e = 3$
T_c	9.29 keV	11.54 keV
T_{va}	5.14 keV	6.27 keV
T_{ma}	3.72 keV	4.31 keV
T_{ba}	6.97 keV	8.66 keV

Since the ratio of radiation emission to alpha particle production depends only on T_c , it is always proportional to $(E/M)^{2b}$. The ratio of (3.3.20) and (3.3.21) gives:

$$\frac{W_\alpha}{W_r} \propto \left(\frac{E}{M}\right)^{4.68} \propto T_{ma}^{4.68}. \quad (5.5.9)$$

Appendix B:

Source Terms for a Parabolic Hotspot Temperature Profile

With high levels of thermal conduction or with radiation transport included, the hotspot may adopt a parabolic temperature profile of the form

$$T = T_c \left(1 - \left(\frac{r}{r_f} \right)^2 \right)^{1/n} \quad (5.5.10)$$

resembling a Marshak wave, where r_f is the radius at which $T = 0$ and n is the thermal conductivity exponent, $\kappa \propto T^n$. For Spitzer conductivity, $n = 5/2$; for an optically-thick medium with diffusive radiation transport, $n = 4$ [83]. A similar analysis to that of Section 3.3 can be performed for such a hotspot. Maintaining the assumption that the hotspot boundary location R is at a particular temperature T_R , r_f can be expressed in terms of T_c , T_R and R as

$$r_f^2 = \frac{R^2}{1 - (T_R/T_c)^n}. \quad (5.5.11)$$

In the ideal parabolic profile $T = 0$ at $r = r_f$ so it is more physically realistic to cut this region out of the analysis, truncating the hotspot at T_R as before. Substituting (5.5.11) into (5.5.10) produces a temperature

Appendix B

profile resembling the model profile observed in the unperturbed simulations, with $q = 2$ and all temperatures raised to the power of n :

$$T^n = T_c^n - (T_c^n - T_R^n)(r/R)^2. \quad (5.5.12)$$

The general volume integral for the radiation emission and fusion power in the parabolic hotspot is

$$\int_0^R T(r)^{-m} r^2 dr = \mathcal{H}_{m,n}(T_c) \frac{R^3}{3} T_c^{-m} \quad (5.5.13)$$

where the profile factor $\mathcal{H}_{m,n}(T_c) = {}_2F_1(3/2, m/n; 5/2; (1 - (T_R/T_c)^n))$. This is similar to the hypergeometric function in Section 3.4, with $q = 2$, m replaced by m/n and $(1 - T_R/T_c)$ replaced by $(1 - (T_R/T_c)^n)$. The radiation emission power is therefore

$$W_r = \mathcal{H}_{3/2,n}(T_c) A_r \rho_c^2 T_c^{1/2} V \quad (5.5.14)$$

and the fusion power is

$$W_\alpha = \mathcal{H}_{-2,n}(T_c) A_\alpha \rho_c^2 T_c^4 V. \quad (5.5.15)$$

The profile factors $\mathcal{H}_{3/2,n}(T_c)$ and $\mathcal{H}_{-2,n}(T_c)$ are plotted in Figure B.1 for $n = 5/2$ and $n = 4$. Between $T_c = 3$ keV and $T_c = 7$ keV, these functions are fit to within 3% by

$$\mathcal{H}_{3/2,5/2}(T_c) \approx 1.57 T_c (\text{keV})^{0.24}, \quad (5.5.16)$$

$$\mathcal{H}_{3/2,4}(T_c) \approx 1.70, \quad (5.5.17)$$

$$\mathcal{H}_{-2,5/2}(T_c) \approx 0.54 T_c (\text{keV})^{-0.079}, \quad (5.5.18)$$

$$\mathcal{H}_{-2,4}(T_c) \approx 0.59. \quad (5.5.19)$$

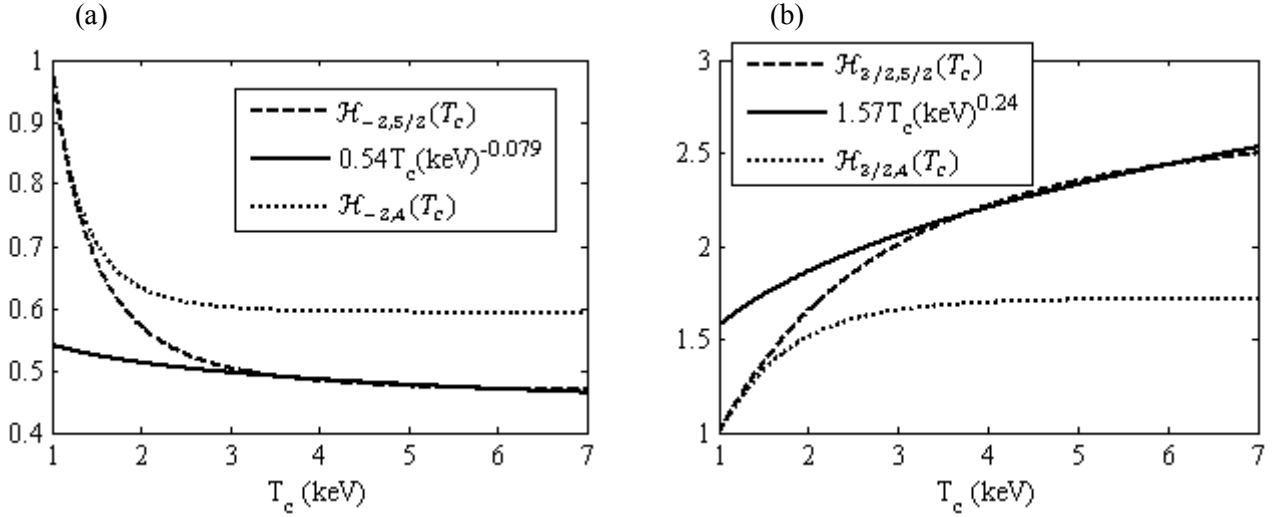


Figure B.1: (a) Temperature dependence of the radiation emission 'profile factor' for parabolic hotspots with conductivity exponent $n = 5/2$ (dashed) and $n = 4$ (dotted). The $n = 5/2$ profile factor is fit by $0.54T_c(\text{keV})^{-0.079}$. (b) Temperature dependence of the fusion 'profile factor' for parabolic hotspots. The $n = 5/2$ profile factor is fit by $1.57T_c(\text{keV})^{0.24}$.

The ratio of the fusion rate to the radiation emission rate is as before:

$$\frac{W_\alpha}{W_r} = \frac{\mathcal{H}_{-2,n}(T_c)}{\mathcal{H}_{3/2,n}(T_c)} \frac{A_\alpha}{A_r} T_c^{7/2}. \quad (5.5.20)$$

This is plotted for $n = 5/2$ and $n = 4$ in Figure B.2. For $n = 5/2$, $W_\alpha/W_r = 1$ at $T_c = 7.34$ keV, 1.95 keV less than for the profile in the unperturbed simulation. Since the temperature profile decays more gradually with radius in the parabolic hotspot, the temperature in the parabolic hotspot is higher at all points except the boundary and the centre for the same central temperature. For $n = 4$, $W_\alpha/W_r = 1$ at $T_c = 6.14$ keV. This is lower than for $n = 5/2$ because the higher conductivity exponent results in a more uniform temperature profile, so for the same central temperature the temperature is higher throughout the hotspot in the $n = 4$ case. However, it is not appropriate to compare hotspots on the basis of central temperature because a parabolic hotspot of a given central temperature contains more thermal energy than the hotspot of the unperturbed simulation with the same central temperature. It is more meaningful to compare the mass-averaged temperatures.

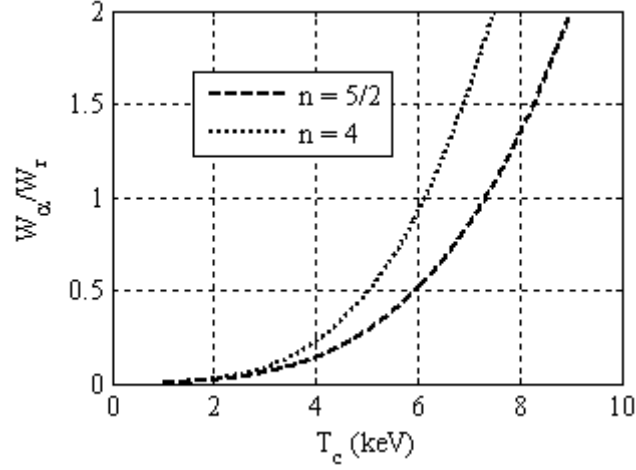


Figure B.2: Ratio of fusion rate to radiation emission rate as a function of central temperature for a parabolic hotspot with the conductivity exponent $n = 5/2$ (dashed) and $n = 4$ (solid).

The formulae for the volume-averaged, mass-averaged, and burn-averaged temperatures are also the same as for the simulated profiles in Section 3.3 with $H_m(T_c, q)$ replaced by $\mathcal{H}_{m,n}(T_c)$ (as is the mass formula). W_α/W_r is plotted against these temperature measures in Figure B.3, for $n = 5/2$ and $n = 4$. The spread of the temperature-average W_α/W_r curves is narrower for $n = 4$ than $n = 5/2$ because the $n = 4$ hotspot is more uniform. For $n = 5/2$, the mass-averaged temperature at which $W_\alpha/W_r = 1$ is 3.48 keV, lower than for the unperturbed simulation. Of course, except for at high $\langle \rho R \rangle$ ignition is more complicated than simply achieving $W_\alpha/W_r > 1$, so this does not necessarily mean that a parabolic hotspot is easier to ignite than one with less evenly distributed thermal energy.

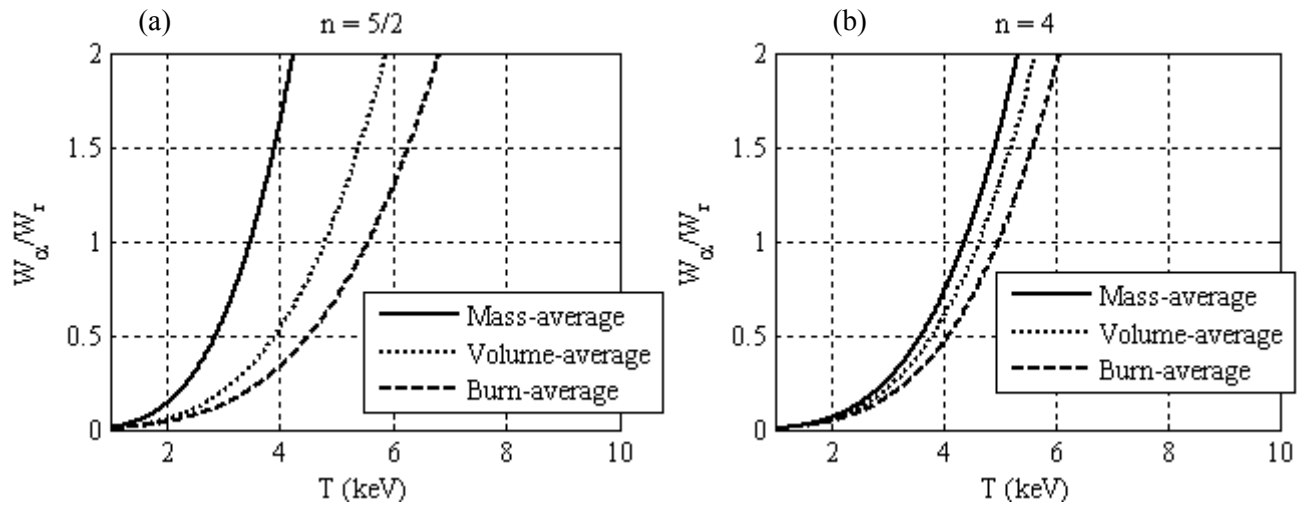


Figure B.3: Ratio of fusion rate to radiation emission rate for a parabolic hotspot as a function of three different measures of the global hotspot temperature, for conductivity exponents (a) $n = 5/2$ and (b) $n = 4$.

The thermal conduction rate in the parabolic hotspot is given by the surface integral (3.3.2). The thermal gradient at the hotspot boundary is given by

$$dT/dr|_{r=R} = -\frac{2(T_c^n - T_R^n)}{R} \frac{T_R^{1-n}}{n}. \quad (5.5.21)$$

As with the radiation emission and fusion terms, this is the same as for the simulated unperturbed profiles with $q = 2$ and the temperatures raised to the power of n . An additional term T_R^{1-n}/n also arises. The thermal gradient scale length at $r = R$ is

$$L = \frac{nR}{2T_R((T_c/T_R)^n - 1)}. \quad (5.5.22)$$

The work rate is unchanged for the parabolic hotspot because it derives directly from the isobaricity of the hotspot.

Bibliography

- [1] M. H. Widner, “Fuel energy balance studies of pellet ignition requirements,” 1979.
- [2] C. D. Zhou and R. Betti, “A measurable Lawson criterion and hydro-equivalent curves for inertial confinement fusion,” *Physics of Plasmas*, vol. 15, no. 10, p. 102707, 2008.
- [3] J. D. Lawson, “Some Criteria for a Power Producing Thermonuclear Reactor,” *Proc. Phys. Soc. B*, vol. 70, no. 6, pp. 6–10, 1957.
- [4] J. Lindl, “Development of the indirect-drive approach to inertial confinement fusion and the target physics basis for ignition and gain,” *Physics of Plasmas*, vol. 2, no. 11, pp. 3933–4024, 1995.
- [5] J. Nuckolls, L. Wood, A. Thiessen, and G. B. Zimmerman, “Laser Compression of Matter to Super-High Densities: Thermonuclear (CTR) Applications,” *Nature*, vol. 239, p. 139, 1972.
- [6] G. S. Fraley, E. J. Linebur, R. J. Mason, and R. L. Morse, “Thermonuclear burn characteristics of compressed deuterium-tritium microspheres,” *Physics of Fluids*, vol. 17, no. 2, p. 474, 1974.
- [7] J. Meyer-ter-Vehn, “On energy gain of fusion targets : the model of Kidder and Bodner improved,” *Nuclear Fusion*, vol. 22, no. 4, p. 561, 1982.
- [8] S. Atzeni and J. Meyer-ter-Vehn, *The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter*, 1st Edition. Oxford University Press, 2009, p. 480.
- [9] M. Murakami and Meyer-ter-Vehn, “Indirectly driven targets for inertial confinement fusion,” *Nuclear Fusion*, vol. 31, no. 7, pp. 1315–1331, 1991.
- [10] W. K. Levedahl and J. D. Lindl, “Energy scaling of inertial confinement fusion targets for ignition and high gain,” *Nuclear Fusion*, vol. 37, no. 2, pp. 165–173, 1997.
- [11] M. Basko and J. Johner, “Ignition energy scaling of inertial confinement fusion targets,” *Nuclear Fusion*, vol. 38, no. 12, pp. 1779–1788, 1998.
- [12] M. Herrmann, M. Tabak, and J. Lindl, “A generalized scaling law for the ignition energy of inertial confinement fusion capsules,” *Nuclear Fusion*, vol. 41, no. 1, pp. 99–111, Jan. 2001.
- [13] S. Atzeni and J. Meyer-ter-Vehn, “Comments on the article: A generalized scaling law for the ignition energy of inertial confinement fusion capsules,” *Nuclear Fusion*, vol. 41, no. 4, p. 456, 2001.
- [14] A. Kemp, J. Meyer-ter-Vehn, and S. Atzeni, “Stagnation Pressure of Imploding Shells and Ignition Energy Scaling of Inertial Confinement Fusion Targets,” *Physical Review Letters*, vol. 86, no. 15, pp. 3336–3339, Apr. 2001.
- [15] J. D. Lindl, P. Amendt, R. L. Berger, et al., “The physics basis for ignition using indirect-drive targets on the National Ignition Facility,” *Physics of Plasmas*, vol. 11, no. 2, p. 339, 2004.
- [16] J. Edwards, “Progress towards ignition on the NIF.” 2013. Available: lasers.llnl.gov/workshops/user_group_2013/docs/edwards_progress.pdf

- [17] D. S. Clark, S. W. Haan, A. W. Cook, et al., “Short-wavelength and three-dimensional instability evolution in National Ignition Facility ignition capsule designs,” *Physics of Plasmas*, vol. 18, no. 8, p. 082701, 2011.
- [18] G. R. Magelssen, J. a Cobble, I. L. Tregillis, et al., “Single-shell direct-drive capsule designs to study effects of perturbations on burn,” *Journal of Physics: Conference Series*, vol. 244, no. 2, p. 022006, Aug. 2010.
- [19] R. H. H. Scott, D. S. Clark, D. K. Bradley, et al., “Numerical Modeling of the Sensitivity of X-Ray Driven Implosions to Low-Mode Flux Asymmetries,” *Physical Review Letters*, vol. 110, no. 7, p. 075001, Feb. 2013.
- [20] J. D. Moody, P. Michel, L. Divol, et al., “Multistep redirection by cross-beam power transfer of ultrahigh-power lasers in a plasma,” *Nature Physics*, vol. 8, no. 4, pp. 344–349, Feb. 2012.
- [21] H. F. Robey, P. M. Celliers, J. L. Kline, et al., “Precision Shock Tuning on the National Ignition Facility,” *Physical Review Letters*, vol. 108, no. 21, p. 215004, May 2012.
- [22] D. S. Clark, D. E. Hinkel, D. C. Eder, et al., “Detailed implosion modeling of deuterium-tritium layered experiments on the National Ignition Facility,” *Physics of Plasmas*, vol. 20, no. 5, p. 056318, 2013.
- [23] A. Nikroo, K. C. Chen, M. L. Hoppe, et al., “Progress toward fabrication of graded doped beryllium and CH capsules for the National Ignition Facility,” *Physics of Plasmas*, vol. 13, no. 5, p. 056302, 2006.
- [24] J. A. Koch, B. J. Kozioziemski, J. Salmonson, et al., “Optical and X-ray Characterization of Groove Profiles in DT Ice Layers,” in *18th Target Fabrication Meeting*, 2008.
- [25] G. Bennett, M. Herrmann, M. Edwards, et al., “Fill-Tube-Induced Mass Perturbations on X-Ray-Driven, Ignition-Scale, Inertial-Confinement-Fusion Capsule Shells and the Implications for Ignition Experiments,” *Physical Review Letters*, vol. 99, no. 20, p. 205003, Nov. 2007.
- [26] B. A. Hammel, H. A. Scott, S. P. Regan, et al., “Diagnosing and controlling mix in National Ignition Facility implosion experiments,” *Physics of Plasmas*, vol. 18, no. 5, p. 056310, 2011.
- [27] R. Kishony and D. Shvarts, “Ignition condition and gain prediction for perturbed inertial confinement fusion targets,” *Physics of Plasmas*, vol. 8, no. 11, p. 4925, 2001.
- [28] S. W. Haan, J. D. Lindl, D. A. Callahan, et al., “Point design targets, specifications, and requirements for the 2010 ignition campaign on the National Ignition Facility,” *Physics of Plasmas*, vol. 18, no. 5, p. 051001, 2011.
- [29] O. L. Landen, J. Edwards, S. W. Haan, et al., “Capsule implosion optimization during the indirect-drive National Ignition Campaign,” *Physics of Plasmas*, vol. 18, no. 5, p. 051002, 2011.
- [30] M. J. Edwards, J. D. Lindl, B. K. Spears, et al., “The experimental plan for cryogenic layered target implosions on the National Ignition Facility—The inertial confinement approach to fusion,” *Physics of Plasmas*, vol. 18, no. 5, p. 051003, 2011.
- [31] W. H. Goldstein, R. Rosner, C. Barnes, et al., “Science of Fusion Ignition on NIF,” 2012.
- [32] “NIF Project Status.” [Online]. Available: lasers.llnl.gov/newsroom/project_status/index.php.

Bibliography

- [33] D. Clark, “Personal Communication.” .
- [34] H. S. Bosch and G. Hale, “Improved formulas for fusion cross-sections and thermal reactivities,” *Nuclear Fusion*, vol. 32, no. 4, pp. 611–631, Apr. 1992.
- [35] N. Niasse, “Personal Communication.” .
- [36] L. Lapidus and G. F. Pinder, *Numerical Solution of Differential Equations in Science and Engineering*, 1st Edition. Wiley-Blackwell, 1982.
- [37] P. L. T. Brian, “Finite-Difference Method of High-Order Accuracy for the Solution of Three-Dimensional Transient Heat Conduction Problems,” *A. I. Ch. E Journal*, vol. 7, no. 3, pp. 367–370, 1961.
- [38] J. Douglas, “Alternating direction methods for three space variables,” *Numerische Mathematik*, vol. 4, no. 1, pp. 41–63, Dec. 1962.
- [39] A. Wakatani, “A parallel and scalable algorithm for ADI method with pre-propagation and message vectorization,” *Parallel Computing*, vol. 30, no. 12, pp. 1345–1359, Dec. 2004.
- [40] D. Clark, “Detailed Implosion Modeling of DT-Layered Experiments on the National Ignition Facility,” in *Bull. Am. Phys. Soc.*, 2012.
- [41] S. Atzeni and M. Temporal, “Mechanism of growth reduction of the deceleration-phase ablative Rayleigh-Taylor instability,” *Physical Review E*, vol. 67, no. 5, p. 057401, May 2003.
- [42] D. Layzer, “On the Instability of Superposed Fluids in a Gravitational Field,” *The Astrophysical Journal*, vol. 122, p. 1, 1955.
- [43] H. Kenner, *Geodesic Math and How to Use It*, 2nd Edition. University of California Press, 2003, p. 172.
- [44] R. E. Kidder, “Laser-driven compression of hollow shells: power requirements and stability limitations,” *Nuclear Fusion*, vol. 16, no. 1, pp. 3–12, 1976.
- [45] R. Le Levier, G. J. Lasher, and F. Bjorklund, “Effect of a Density Gradient on Taylor Instability,” 1955.
- [46] D. S. Clark and M. Tabak, “Linear and nonlinear Rayleigh-Taylor growth at strongly convergent spherical interfaces,” *Physics of Fluids*, vol. 18, no. 6, p. 064106, 2006.
- [47] P. Reinicke and J. Meyer-ter-Vehn, “The point explosion with heat conduction,” *Physics of Fluids A: Fluid Dynamics*, vol. 3, no. 7, p. 1807, 1991.
- [48] S. P. Regan, R. Epstein, B. A. Hammel, et al., “Hot-spot mix in ignition-scale implosions on the NIF,” *Physics of Plasmas*, vol. 19, no. 5, p. 056307, 2012.
- [49] D. L. Bleuel, C. B. Yeamans, L. A. Bernstein, et al., “Neutron activation diagnostics at the National Ignition Facility (invited).,” *The Review of scientific instruments*, vol. 83, no. 10, p. 10D313, Oct. 2012.

- [50] M. Gatu Johnson, J. A. Frenje, D. T. Casey, et al., “Neutron spectrometry-an essential tool for diagnosing implosions at the National Ignition Facility (invited).,” *The Review of scientific instruments*, vol. 83, no. 10, p. 10D308, Oct. 2012.
- [51] M. Gatu Johnson, D. T. Casey, J. A. Frenje, et al., “Measurements of collective fuel velocities in deuterium-tritium exploding pusher and cryogenically layered deuterium-tritium implosions on the NIF,” *Physics of Plasmas*, vol. 20, no. 4, p. 042707, 2013.
- [52] D. T. Casey, J. A. Frenje, M. Gatu Johnson, et al., “The magnetic recoil spectrometer for measurements of the absolute neutron spectrum at OMEGA and the NIF,” *Review of Scientific Instruments*, vol. 84, no. 4, p. 043506, 2013.
- [53] J. A. Frenje, D. T. Casey, C. K. Li, et al., “Probing high areal-density cryogenic deuterium-tritium implosions using downscattered neutron spectra measured by the magnetic recoil spectrometer,” *Physics of Plasmas*, vol. 17, no. 5, p. 056311, 2010.
- [54] N. I. C. D. Team, “Diagnostics for the 2011 National Ignition Campaign,” January, 2011.
- [55] H. W. Herrmann, N. Hoffman, D. C. Wilson, et al., “Diagnosing inertial confinement fusion gamma ray physics (invited).,” *The Review of scientific instruments*, vol. 81, no. 10, p. 10D333, Oct. 2010.
- [56] N. M. Hoffman, D. C. Wilson, H. W. Herrmann, and C. S. Young, “Using gamma-ray emission to measure areal density of inertial confinement fusion capsules,” *The Review of scientific instruments*, vol. 81, no. 10, p. 10D332, Oct. 2010.
- [57] A. G. MacPhee, D. H. Edgell, E. J. Bond, et al., “A diamond detector for X-ray bang-time measurement at the National Ignition Facility,” *Journal of Instrumentation*, vol. 6, no. 02, pp. P02009–P02009, Feb. 2011.
- [58] S. F. Khan, P. M. Bell, D. K. Bradley, et al., “Measuring x-ray burn history with the Streaked Polar Instrumentation for Diagnosing Energetic Radiation (SPIDER) at the National Ignition Facility (NIF),” p. 850505, Oct. 2012.
- [59] M. D. Wilke, S. H. Batha, P. A. Bradley, et al., “The National Ignition Facility neutron imaging system,” *The Review of scientific instruments*, vol. 79, no. 10, p. 10E529, Oct. 2008.
- [60] S. Glenn, J. Koch, D. K. Bradley, et al., “A hardened gated x-ray imaging diagnostic for inertial confinement fusion experiments at the National Ignition Facility,” *The Review of scientific instruments*, vol. 81, no. 10, p. 10E539, Oct. 2010.
- [61] G. A. Kyrala, S. Dixit, S. Glenzer, et al., “Measuring symmetry of implosions in cryogenic Hohlraums at the NIF using gated x-ray detectors (invited).,” *The Review of scientific instruments*, vol. 81, no. 10, p. 10E316, Oct. 2010.
- [62] P. M. Bell, D. K. Bradley, J. D. Kilkenny, et al., “Radiation hardening of gated x-ray imagers for the National Ignition Facility (invited).,” *The Review of scientific instruments*, vol. 81, no. 10, p. 10E540, Oct. 2010.
- [63] B. Appelbe and J. Chittenden, “The production spectrum in fusion plasmas,” *Plasma Physics and Controlled Fusion*, vol. 53, no. 4, p. 045002, Apr. 2011.
- [64] S. H. Glenzer, D. A. Callahan, A. J. MacKinnon, et al., “Cryogenic thermonuclear fuel implosions on the National Ignition Facility,” *Physics of Plasmas*, vol. 19, no. 5, p. 056318, 2012.

- [65] L. Suter, “Personal Communication.” 2012.
- [66] O. S. Jones, C. J. Cerjan, M. M. Marinak, et al., “A high-resolution integrated model of the National Ignition Campaign cryogenic layered experiments,” *Physics of Plasmas*, vol. 19, no. 5, p. 056315, 2012.
- [67] S. Yu, O. N. Guskov, V. B. Krokhin, and P. N. Rozanov, “Similarity solution of thermonuclear burn wave with electron and alpha-conductivities,” *Nuclear Fusion*, vol. 16, no. 6, pp. 957–962, 1976.
- [68] S. Atzeni and A. Caruso, “Inertial Confinement Fusion: Ignition of Isobarically-Compressed D-T Targets,” *Nuovo Cimento*, vol. 80 B, no. 1, pp. 71–102, 1984.
- [69] P. Gauthier, F. Chaland, and L. Masse, “Deflagration-to-detonation transition in inertial-confinement-fusion baseline targets,” *Physical Review E*, vol. 70, no. 5, p. 055401, Nov. 2004.
- [70] J. Sanz, J. Garnier, C. Cherfils, et al., “Self-consistent analysis of the hot spot dynamics for inertial confinement fusion capsules,” *Physics of Plasmas*, vol. 12, no. 11, p. 112702, 2005.
- [71] A. Djaoui, “A User Guide for the Laser-Plasma Simulation Code : MED 103,” 1996.
- [72] J. P. Christiansen, D. E. T. F. Ashby, and K. V Roberts, “Medusa: A One-Dimensional Laser Fusion Code,” *Computer Physics Communications*, vol. 7, pp. 271–287, 1974.
- [73] M. Sherlock, “A Monte-Carlo method for coulomb collisions in hybrid plasma models,” *Journal of Computational Physics*, vol. 227, no. 4, pp. 2286–2292, Feb. 2008.
- [74] M. F. Ivanov and V. F. Shvets, “The Method of Stochastic Differential Equations for Computing the Kinetics of a Plasma With Collisions,” *USSR. Comput. Math. Phys.*, vol. 20, no. 3, pp. 146–155, 1981.
- [75] A. M. Oparin, S. I. Anisimov, and J. Meyer-ter-Vehn, “Kinetic simulation of DT ignition and burn in ICF targets,” *Nuclear Fusion*, vol. 36, no. 4, pp. 443–452, 1996.
- [76] M. Sherlock and S. J. Rose, “The persistence of Maxwellian D and T distributions during burn in inertial confinement fusion,” *High Energy Density Physics*, vol. 5, no. 1–2, pp. 27–30, Jun. 2009.
- [77] L. Spitzer, *Physics of Fully Ionized Gases*. Interscience, 1956.
- [78] S. Chandrasekhar, “Dynamical Friction I. General Considerations: The Coefficient of Dynamical Friction,” *Astrophysical Journal*, vol. 97, p. 255, 1943.
- [79] E. G. Cormann, W. E. Loewe, G. W. Cooper, and A. M. Winslow, “Multi-Group Diffusion of Energetic Charged Particles,” *Nuclear Fusion*, vol. 15, pp. 377–387, 1975.
- [80] S. Atzeni, “The physical basis for numerical fluid simulations in laser fusion,” *Plasma Physics and Controlled Fusion*, vol. 29, no. 11, pp. 1535–1604, 1987.
- [81] S. Vickers, “Personal Communication.” .
- [82] M. Murakami and S. Iida, “Scaling laws for hydrodynamically similar implosions with heat conduction,” *Physics of Plasmas*, vol. 9, no. 6, p. 2745, 2002.
- [83] Y. B. Zel’dovich and Y. P. Raizer, *Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena*, 2nd Edition. Dover, 2002.