

THEORY
HYDRODYNAMIC MODELLING OF CATASTROPHIC
FAILURE OF CYLINDRICAL FUEL STORAGE TANKS

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by

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A B S T R A C T

Most specifications for containment walls or bunds surrounding large cylindrical liquid fuel storage facilities are not adequate to cope satisfactorily with containment of spills in the case of a catastrophic failure of the storage tank. In such instances, the thin tank wall may effectively be considered to disappear instantaneously. This leaves a free standing column of liquid which proceeds to collapse under its own weight, spreading out radially along the ground in a mode known as "base surge". When the liquid collides with the surrounding containment wall, the build-up of liquid leads to the formation of a hydraulic jump which is reflected back towards the original site of the tank. For bunds of limited height, liquid may also simultaneously spill over the top of the containment wall.

This study presents a mathematical description of the collapse and spread of such a liquid column and the resulting impact against the containment boundaries. It aims to evaluate bund height and strength requirements for complete containment. The dispersive motion of a collapsing cylindrical liquid column is idealized as a problem of incompressible potential flow. The mathematical model is derived assuming inviscid flow, axial symmetry and negligible vertical accelerations. Under these assumptions the governing equations are partial differential equations of hyperbolic type in one radial space variable and time, and are conveniently solved by the method of characteristics. The motion of the liquid in the various space-time regions of collapse and consequent interactions is determined using an implicit second

order numerical method of characteristics.

Impact against bunds inclined both towards and away from the original site of the tank has been studied. Overflow in the case of bunds of limited height and forces imparted upon the containment wall have also been considered.

The results are presented in a form suitable for both comparison with data from small scale experiments and are used to make recommendations for specifications of industrial bund layouts.

TO MUM AND DAD

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In the bottom right hand corner of pages (167) to (211) a cartoon is shown of the collapse of a cylindrical liquid column within a circular banded area as predicted by the axisymmetrical model.

Nomenclature

A	-	Tank radius
A_n	-	Coefficient function
B_n	-	Coefficient function
c	-	Wave speed, height transformation variable ($= (gh)^{\frac{1}{2}}$)
c_n	-	Value of c at r_n
c_p	-	Value of c at r_p
c_1	-	Value of c in front of shock (region III)
c_2	-	Value of c behind shock (region V)
C_b	-	Value of c corresponding to H_b
C_o	-	Value of c corresponding to H_o
F_w	-	Load on containment wall
g	-	Acceleration due to gravity
h	-	Liquid surface height
h_b	-	Height of inclined bund as a function of r and
h_1	-	Liquid surface height in front of shock (region III)
h_2	-	Liquid surface height behind shock (region V)
H_b	-	Bund height
H_d	-	'Equal volume' specification bund height
H_o	-	Tank/column height
H_2	-	Maximum liquid surface height at containment wall
p	-	Pressure
p_o	-	Ambient air pressure
P	-	Pressure on containment wall
Q_s	-	Overflow as a fraction of total tank volume
r	-	Radial displacement
r_i	-	Radial displacement mesh point i

- r_n - Value of r at intersection point of negative characteristic with time level
- r_p - Value of r at intersection point of positive characteristic with time level
- R_b - Position of backward moving depletion front
- R_f - Position of forward moving surging front
- R_i - Boundary of "triangular section" in inclined containment wall approximation for $R_s > R_w$
- R_p - Distance to which overflow jet is projected
- $(R_p)_{\max}$ Maximum value of R_p
- R_p^* - Dimensionless value of R_p
- R_r - Position of reflection front from centreline
- R_s - Shock position
- R_s - Shock position for inclined containment walls
- R_t - Boundary of "triangular section" in inclined containment wall approximation for $> 90^\circ$
- R_w - Position of containment wall
- S_n - Slope of negative characteristic
- t - Time
- t_j - Time level j
- T_b - Time at which depletion front reaches centreline
- T_c - Time at which shock front reaches centreline
- T_i - Time at which shock and reflected depletion front meet
- T_o - Time column collapse is initiated
- T_p - Time at which projected overflow jet reaches ground level
- T_w - Time at which surging front reaches containment wall
- u - Horizontal liquid velocity
- u_a - Average liquid velocity in "triangular section" for inclined containment wall approximation

u_n	-	Value of u at r_n
u_p	-	Value of u at r_p
u_w	-	Wave speed of liquid over the bund
u_1	-	Value of u in front of shock (region III)
u_2	-	Value of u behind shock (region V)
U	-	Shock velocity
v	-	Vertical liquid velocity
V	-	Volume of liquid
V_w	-	Rate of change of liquid level at containment wall
x	-	Linear displacement (plane model)
x_b	-	Position of depletion front (plane model)
x_f	-	Position of surging front (plane model)
x_w	-	Position of containment wall (plane model)
y	-	Vertical displacement (plane model)
z	-	Vertical displacement (axisymmetric model)
Z	-	Similarity variable
α	-	Angle of deviation from the vertical (degrees)
γ	-	Ratio of specific heats
ϵ	-	Similarity variable
ι	-	Dimensionless displacement variable
η	-	Similarity variable
θ	-	Angle of containment wall inclination (degrees)
ρ	-	Liquid density
τ	-	Dimensionless time variable
ϕ	-	Riemann invariant
ψ	-	Riemann invariant

INTRODUCTION

Large liquid storage tanks are slumbering giants. As long as they continue to fulfil their function, they appear perfectly safe and present day society has become accustomed to them, even when seen in considerable numbers as on tank farms.

Of course, the danger lies inside them and if the contents escape, the effect on the environment can be considerable. Even ten million gallons of the most inert liquid, when slowly released, will eventually cause extensive flooding unless the spill can be satisfactorily contained. If, as is more likely to be the case, the liquid is toxic (ammonia), particularly volatile (LNG) or corrosive (sulphuric acid) the detrimental consequences of a spill are multiplied. If the liquid is combustible or explosive the potential danger will be hard to imagine.

Immediate containment of the spill is thus vitally important. It limits the danger to an enclosed area. Furthermore, if the liquid is highly volatile the small surface reduces the rate of vapour cloud formation; this in turn limits the danger to the environment from poisonous clouds or explosion.

The usual way to provide for containment of a possible spill is by surrounding the immediate area around with a bund, built to an "equal volume" specification : the volume enclosed by the containment wall being at least equal to the volume of the full tank within its perimeter. A safety factor up to 1.5 is sometimes applied. However, what often appears to have been overlooked is that the rate of release of the liquid can have a far more deciding influence on the consequences of tank failure. This can, for

instance, be illustrated by the following paragraph taken from the coroner's report on the East Ohio Gas Company disaster, published in 1945 (9).

"At 2.40 p.m. on October 20th, 1944, a 4200 m³ storage tank containing liquified natural gas (LNG) at a facility in Cleveland, Ohio, USA, suddenly gave way. Although the tank, and three others next to it, were surrounded by dykes and also had individual dykes leading to a pit, some liquid escaped the site and spread into streets, sewers, and basements. The vapours quickly ignited, causing fires and explosions. About 20 minutes later, the legs holding a second tank collapsed from the heat, releasing another 2100 m³ of LNG. The subsequent explosions shot flames more than half a mile into the air, and the temperature in some areas reached 3000°F. The fires and explosions killed 130 people, injured 225 others, and resulted in property damage estimated at seven million dollars. Had the spill occurred at a different time of day, casualties would have been much higher. At the time of the fire most children were at school, and most men were at work."

The sudden release of the entire contents of a large storage tank in a "Catastrophic Tank Failure" (CTF) results in a very strong radial dispersive base surge of liquid which is capable of destroying all but the strongest of containment structures. To put

this statement into perspective : it can be shown (Appendix 1) from estimates based on calculation of energy released, that the potential damage caused by the liquid surge is of the same order as that which may result from the explosive energy available from the ~~Vapour~~ contents of the tank. In the Cleveland incident, debris was carried up to 200 metres by the surging liquid which illustrates the magnitude of the transferred forces.

Furthermore, even in the presence of a sufficiently strong bund, the liquid may flow up and over the containment wall. In such cases a significant proportion will spread outside the enclosure, thus increasing the chances of causing further damage through the more readily appreciated dangers inherent in the nature of the liquid itself.

Therefore, on both counts, design of effective containment walls capable of dealing with even the most catastrophic spills, is of overriding importance, regardless of the physical or chemical properties of the stored liquid. It is with this specific aspect of bulk liquid storage that the Imperial College project on Catastrophic Tank Failure is concerned.

Much of the world's bulk liquid storage is held in large vertical cylinder tanks. Of particular interest is the storage of cryogenic liquids such as LNG and LPG due to their hazardous properties and abundance. The liquids are stored at atmospheric pressure in very large quantities, 50,000 m³ being not uncommon. In certain cases storage tanks hold up to 120,000 m³. The accident in Cleveland involved tanks which were very small compared with many around today. Despite improved tank design and stricter

safety regulations, a mechanical failure, a natural disaster or, now increasingly, terrorist and military sabotage, can lead to CTF. Although this type of failure is rare (reported incidents in Appendix (2)), when it occurs it does so with disastrous consequences. In view of the obvious importance of these consequences it is surprising that to date there is no comprehensive analysis available of the problem in its many aspects. However, since the issue is of considerable environmental, financial and political significance much of the work carried out or authorized is of a confidential nature and findings are therefore understandably not immediately made readily available.

Taking as the main objective the need to understand the flow from a CTF to such an extent that accurate requirements for containment of the resulting spill could be specified, the present research programme was set up along three lines. The first approach, which is the subject of this thesis, was to analyse the problem theoretically, by considering an idealised mathematical model of a collapsing cylindrical liquid column, the resulting flow and the interactions of the spreading liquid at the containment wall. From this elementary requirements of bunds for complete containment of a spill as well as the consequences of inadequate provisions can be evaluated. To complement the theoretical predictions, another part of the programme [T.S. Sharifi], was concerned with experimental modelling. In this study the collapse of small scale, vertical columns of water on a horizontal surface (water-table) are examined by high-speed photographic and optico-electrical techniques. Results consist of data, suitable for direct comparison with mathematical predictions, as well as of

values and information of general behaviour in more complex (e.g. asymmetric) situations, not readily evaluated by theoretical means.

A large proportion of liquids stored in bulk (e.g. LNG, LPG) have a vapour pressure above 1 bar at NTP. These are therefore stored in refrigerated tanks; when released in a spill their flow behaviour will be complicated by nucleate (LPG) or film (LNG) boiling. The third part of the research programme [A.J. Thew] is therefore concerned with the description and observation of these aspects of cold spills and with the evaluation of their influence on the findings from the other two studies.

Catastrophic Tank Failure can occur in a variety of ways. On one instance a clear vertical rupture of the tank wall has been reported, the shell subsequently carried over 100 meters sideways by the liquid propane contents. In another the tank failed by a circular rupture around its base, after which the shell lifted 25 meters upwards. These observations as well as calculations justify the starting point for CTF modelling, that, once failed, the tank shell is essentially irrelevant to the subsequent flow. Therefore, all research into the consequences of a CTF may begin with a model of a free standing cylindrical column of liquid, which proceeds to collapse under gravity.

As mentioned previously mathematical modelling gives useful predictions for practical situations and for comparison with small scale experiments, provided of course that the theoretical basis for the model is not scale related. The limitation in this type of research lies in the nature of modelling itself - idealization. Certain assumptions can and need to be made in order to simplify

the numerical solution of the overall governing laws of motion.

The first and most obvious assumption that can be made is the negligible contribution of viscous and surface tension effects. These forces will be insignificant compared with the inertial and gravitational contributions in the time span over which the liquid column collapses, spreads and collides with the containment wall. Viscous and surface tension forces will only have a significant effect if the liquid can spread over such a large area so that it is reduced to a slow moving thin layered spill. This situation will only arise far beyond the distance to the bund which is irrelevant to this work. Assuming furthermore no density variations in the motion, the problem becomes one of incompressible potential flow and the general equations of conservation of momentum reduce to Euler's equations in three space variables and time.

A further reduction in complexity of the solution is achieved by assuming axisymmetric column collapse. The main reasons for not attempting to solve Euler's equations for asymmetrical flow problems are the extensive mathematical effort and the prohibitive computational cost and storage requirements for an appropriate numerical method. Modelling of asymmetric collapses and spread is much better evaluated by physical experiments.

Having opted for an axisymmetrical model in a cylindrical polar coordinate system, a choice arises of which numerical method to employ. It is possible to apply a finite difference scheme to Euler's equations and to continuity in two dimensions. G.A. Hill et al [1980, (16)] solved these equations for both cartesian and cylindrical polar coordinate systems when looking at a dam break

problem with a sudden release of water onto a quiescent plane of liquid. Using a similar method for our problem would provide an accurate model of a collapsing liquid column. However, a further simplifying assumption can be made (when modelling a CTF) without loss of accuracy in determining the resultant motion at the bund, which is the main concern of this project. If it is assumed that horizontal acceleration terms dominate the motion as compared with the vertical ones (which is true for all but the first instances of the collapsing process), then the three governing equations (Euler's 2-D and continuity) reduce to the so called "shallow water equations". For columns initially squat this is reasonable to postulate. A squat column is defined as and limited to one whose height to diameter ratio is not more than unity, this being the case for most industrial storage tanks. This assumption does lead to an unrealistic representation of the initial collapsing motion, but over the entire duration of the motion - spill and the interaction of the liquid at the containment wall - it results in a practical model. Since our ^econcern lies more with an accurate representation of the motion at the bund than with a precise description of the initial collapse, this assumption is permitted. It is furthermore useful in that it reduces the complexity and cost of a numerical solution. As this research study is primarily one of offering quantitative answers to the genuine industrial problem of CTF and less a mathematical exercise for precise modelling of collapsing liquid columns, this is acceptable.

The shallow water equations can either be solved by a finite difference scheme or, very conveniently as they are partial differential equations of hyperbolic type, by a numerical method of

characteristics. Leaf et al [1979, (19)] compare several explicit numerical solutions for the water hammer problem, where the equations governing the system are similar to the shallow water equations. The method of characteristics (MOC) is found to be superior in both accuracy and computational cost to the Lax difference scheme and the Galerlium quadratic finite element method. A two-step Lax-Wendroff finite difference method is required to achieve the same accuracy as obtained by the MOC. Even here the MOC is needed to implement the boundary conditions, which in general are difficult to satisfy in finite difference schemes applied to this type of equations. We have used a second order implicit numerical MOC. This ensures stability and flexibility in the use of time step sizes as will be shown later in Chapter (2).

When the front edge of the surging liquid, resulting from the column collapse, collides with and builds up at the containment wall, a hydraulic jump or shock forms which is reflected back towards the original site of the tank. This study examines in particular the modelling of the perpendicular impact and the resulting shock formation and propagation for circular, vertical containment walls, to find values for the necessary wall height to prevent overflow. Approximations only, are offered in the case of inclined containment walls. Quantification of overflow in the event of bunds of limited height is studied along with the distance to which the overflow (jet) of liquid can be projected beyond the bund. The impact pressure and loading on the wetted area of the containment wall is also evaluated.

It should be noted that most storage tanks are enclosed within a rectangular banded area. This model cannot however cope

with this type of boundary since axial symmetry of flow is lost after impact.

The above aspects are treated theoretically and numerically in the first two chapters. We begin in Chapter (1) with the theoretical formulation of the problem. First a review of previous mathematical modelling in this area of research is given and utilized to introduce the basic concepts. The presentation of our model then follows. In Chapter (2) the numerical solution of this theoretical model is presented. The numerical MOC employed is discussed along with other techniques appropriate to the various areas of collapse, spread and consequent interactions at boundaries.

Chapter (3) describes the computer program. In Chapter (4) the solutions are offered and results are presented for a range of values of the various parameters involved. The discussion in Chapter (5) includes recommendations for complete liquid containment in the event of CTF, along with suggestions for further research.

CHAPTER 1

Mathematical Modelling

1.1 Introduction

The theoretical description of the three-dimensional flow, resulting from the axisymmetrical collapse of a cylindrical storage tank situated within a bounded area, is essentially a dynamic, two-dimensional mathematical problem in the vertical and radial plane. This shall be referred to as the axisymmetrical model. The description is based on the conservation equations of mass and momentum in a cylindrical-polar coordinate system and their solution is possible, subject to certain justifiable simplifications.

Although it is believed that this work represents the first comprehensive theoretical treatment of the problem of CTF, a number of authors have reported previously on work of a more limited or associated nature. As such these are either lacking in accurate description of what could possibly happen in the event of an industrial storage tank suffering CTF, or the work concentrates to a greater extent on the mathematical analysis itself and does not continue with attempts to model possible containment layouts. As far as possible all this work has recently been reviewed by Sharifi, [1980, (25)]. Here, only those studies which are of direct relevance to the work and methods of this research will be mentioned again both for the purpose of a review of past work and also as an introduction to the mathematical modelling of this problem.

1.2 Previous Work

Greenspan and Young [1978, (14)] provided the most comprehensive background work for the research undertaken in this thesis. They examined the sudden release of a large amount of liquid from a reservoir onto a horizontal plane, by considering both experimental and theoretical models. The experiments consisted of water contained within a cuboid reservoir where the front wall could be instantaneously lifted so as to allow the liquid to spill into a channel and collide with a containment barrier at the end of this channel. The corresponding theoretical study modelled the two-dimensional situation of the instantaneous release of liquid from the front of a rectangular reservoir onto a plane (Fig. (1.1)); i.e. a similar process to that of the classical dam break problem. As this comprises the two-dimensional mathematical modelling of a two-dimensional situation of linear (as opposed to radial) flow in a channel or plane, this model shall be referred to as the Plane model.

Greenspan and Young also analysed the impact of the surging liquid with the containment barrier theoretically so as to model the way in which the liquid is reflected from a vertical wall and to predict the amount which could spill over. Their physical experiments were mainly performed to complement the theoretical predictions in these two areas.

For an analogy with CTF this reservoir model effectively assumes a rectangular column behind a dam wall which suddenly gives way. This simple two-dimensional model describing the two-dimensional problem is illustrated by Figs. (1.1) and (1.2), which consider the height profiles of the liquid before and after

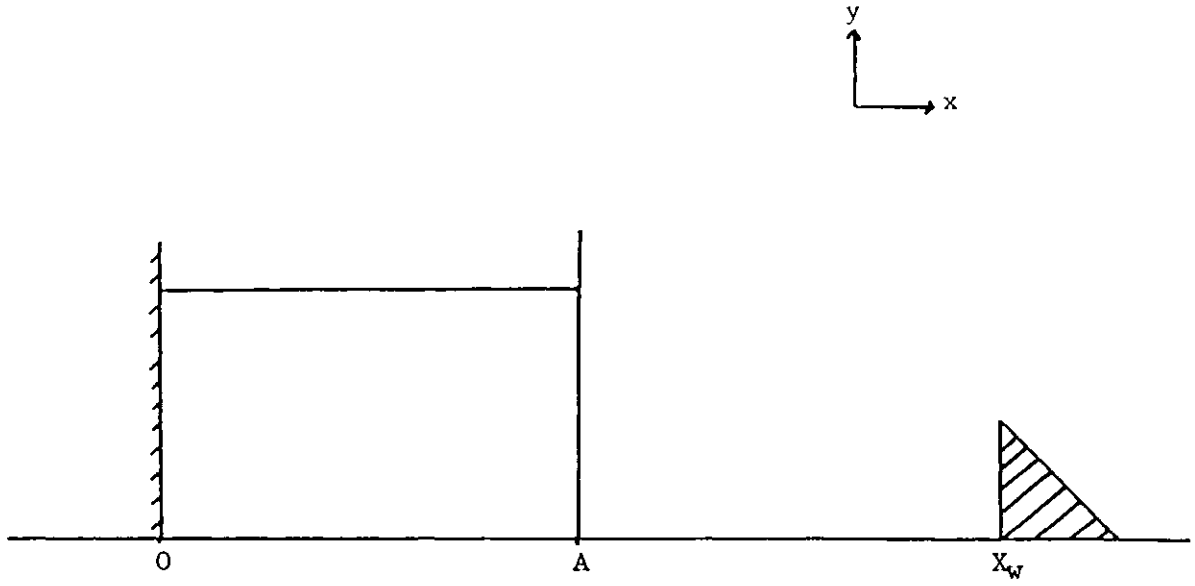


Figure 1.1 Intact reservoir before failure. $t = T_0$

Dam wall at $x = A$

Reservoir back wall at $x = 0$

Barrier at $x = X_w$

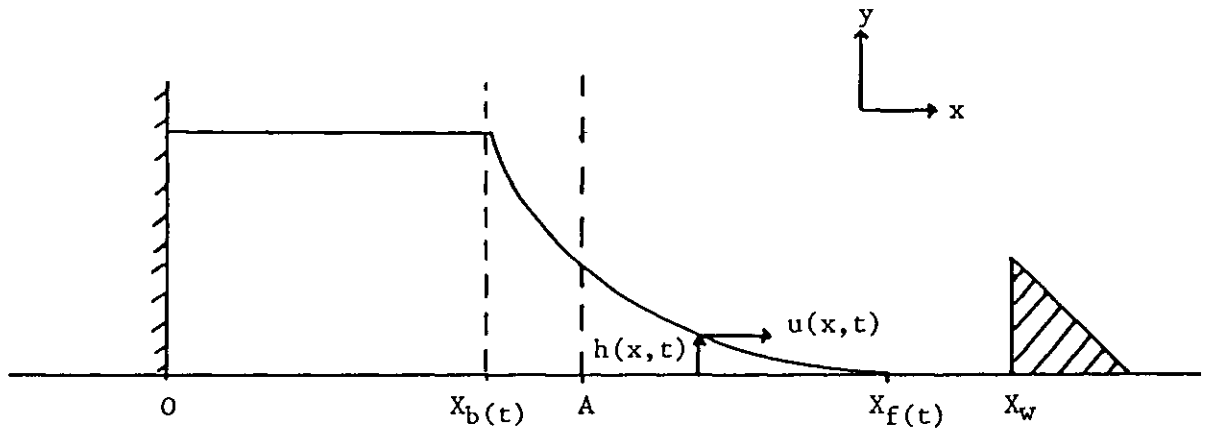


Figure 1.2 Liquid height profile after reservoir failure.

reservoir failure. The right hand side of the reservoir wall, the dam, is instantaneously removed at time, $t = T_0$, when the liquid collapses starting at $x = A$ and surges towards the barrier at $x = X_w$. Simultaneously, the reservoir is depleted starting at $x = A$ and progressing towards the intact back wall at $x = 0$.

To describe this process mathematically Greenspan and Young use the well known Shallow Water Equations (SWE's) given by Eqs. (1.2.1); these are the mass and momentum balances simplified by certain assumptions allowable in the circumstances of events of this type. These assumptions will be discussed in more detail in section (1.3.1).

$$\begin{aligned}\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} &= 0 \\ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + h \frac{\partial u}{\partial x} &= 0\end{aligned}\tag{1.2.1}$$

The SWE's in cartesian coordinates (Eqs. (1.2.1)) are partial differential equations of hyperbolic type in two dependent variables: horizontal liquid velocity, $u(x,t)$, and liquid depth or surface liquid height, $h(x,t)$. Here u and h have been assumed to be functions of the horizontal displacement, x and time, t only. As there is no dependence of the liquid velocity on vertical displacement in the shallow water theory, u is a height averaged velocity. The derivation of the SWE's will be considered later in section (1.3.1).

In this cartesian configuration which applies to the plane model, the above SWE's can be solved analytically ^{from} the time of collapse and spread until a physical boundary is encountered as shown in Fig. (1.2).

$$u(x,t) = \frac{2}{3} (\sqrt{gH_0} + \frac{x - A}{t})$$

(1.2.2)

$$h(x,t) = \frac{1}{9g} (2\sqrt{gH_0} - \frac{x - A}{t})^2$$

H_0 is the original reservoir height.

The derivation of this analytical solution is given in Appendix (3).

After impact of the 'forward' surging liquid front against the vertical barrier there is a liquid build up at the wall which forms a hydraulic jump or 'shock'. This liquid shock is reflected from the barrier and propagates back towards the site of the original reservoir. If the containment barrier is of limited height some of the liquid also spills over. An analytical solution of the shock formation and propagation was found to be practically impossible. Greenspan and Young use a series solution to describe the motion valid for a short time after impact. The method of characteristics is then applied to model the shock numerically.

Eqs. (1.2.1) can be written in their characteristic form:

$$u + 2(gh)^{\frac{1}{2}} = K_1$$

and

(1.2.3)

$$u - 2(gh)^{\frac{1}{2}} = K_2$$

on

$$\frac{dx}{dt} = u + (gh)^{\frac{1}{2}}$$

and

(1.2.4)

$$\frac{dx}{dt} = u - (gh)^{\frac{1}{2}}$$

respectively. K_1 and K_2 are constants.

The derivation is given in section (1.3.2).

Greenspan and Young implement a numerical routine based on the MOC to evaluate the motion in all space-time regions of the reservoir failure process: initial collapse and unhindered spread, reflection of the forward surging liquid at the barrier, reflection of the depletion wave in the reservoir at the back wall and reflection of the hydraulic jump at the back wall. The computer program created was then used to predict the amount of overflow for various values of parameters such as the original reservoir dimensions, the distance of the barrier from the dam and the barrier height. Details of this theoretical evaluation of overflow is discussed in section (1.3.7).

This thesis will confirm that this plane model cannot be used to describe accurately the collapse of a 3-D cylindrical column and the resulting flow. Nevertheless, much of the work by Greenspan and Young has been valuable ground work for the application of the MOC to problems of this type.

Although much of the earlier research by Penney and Thornhill [1952, (21)] relevant to this study was superseded by the work of Greenspan and Young, it is superior in that the former authors also consider the 2-D mathematical solution for the axisymmetric collapse of a liquid column. Penney and Thornhill's interest in this problem was inspired by the 'base surge' effect observed at the atomic weapon trials at Bikini. In the blast a tall column of water droplets mixed with air was formed. This relatively dense fluid then proceeded to collapse under its own weight within the less dense surrounding fluid (air). It was noted that the mode of

collapse resulted in a radial spreading of the fluid column at its base.

This problem is idealised as one of incompressible flow. The velocity potentials of the two fluids are taken as functions of two space variables and time and set to satisfy Laplace's equation. A general solution is developed in terms of an infinite series. They describe the first instances of collapse of columns either symmetrical about the vertical plane or vertical axis initially hemispherical in shape, in order to evaluate accelerations at various points along the interface of the two fluids.

More relevant however is their approximate solution for squat columns which is applicable to the process in the later stages of collapse. In this case vertical acceleration terms in the motion were neglected reducing the system of equations to the SWE's. These were solved by a numerical MOC. However, unlike Greenspan and Young, they had no reason to investigate interactions of the spreading liquid at boundaries and no description of numerical methods employed was mentioned.

A related area of relevant previous research is that of the physics of gas dynamics. Notably, the isentropic motion of gases expanding into a vacuum is governed by a set of equations very similar to those used in this research, namely by partial differential equations of hyperbolic type. Work by Stanyukovich [1960, (26)] has in this respect been very useful, particularly in terms of evaluating the motion at the centreline of cylindrical columns.

Another aspect of similarity of gas dynamics in the context of the axisymmetrical collapse of a liquid column, relates to the convergence of the propagating hydraulic jump (formed at the surrounding containment wall) at the axis of symmetry of the original tank. Butler looked at the disturbance of an almost stationary polytropic gas by a converging spherical or cylindrical shock wave. As the shock reaches the axis of symmetry it becomes very strong and singular. The resulting reflection was also studied.

Together Greenspan and Butler [1962, (13)] investigated further the isentropic expansion of a sphere or cylinder of gas into a surrounding vacuum. They derived a solution for the radial gas velocity and for the local speed of sound in terms of an infinite series:

$$u(r,t) + \frac{2}{\gamma-1} c(r,t) = \sum_{n=0}^{\infty} A_n(\eta) t^n$$

$$u(r,t) - \frac{2}{\gamma-1} c(r,t) = \sum_{n=0}^{\infty} B_n(\eta) t^n$$

where u is the gas velocity as a function of radial displacement and time

c the local speed of sound

η a similarity variable combining r and t

γ the ratio of specific heats

A_n, B_n coefficient functions

Although the derivation of terms in these series is far more appropriate for gases with particular values of γ , the first few terms in the expressions were found to be relevant and sufficiently accurate to describe the motion of a collapsing cylinder at very small times after the initiation of the motion.

All relevant problems in gas dynamics relate to the governing conservation equations in cylindrical polar coordinates for compressible flow:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} &= 0 \\ \frac{\partial}{\partial t} (\ln \rho) + u \frac{\partial}{\partial r} (\ln \rho) + \frac{\partial u}{\partial r} + \frac{u}{r} &= 0 \end{aligned} \tag{1.2.5}$$

where $p = p(\rho)$

$\rho = \rho(r, t)$, the density

$u = u(r, t)$, the radial gas velocity

For isentropic flow Eqs. (1.2.5) can be written in the form:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{2}{\gamma-1} \frac{c}{r} \frac{\partial c}{\partial r} = 0 \tag{1.2.6}$$

$$\frac{2}{\gamma-1} \frac{\partial c}{\partial t} + \frac{2}{\gamma-1} u \frac{\partial c}{\partial r} + \frac{c}{r} \frac{\partial u}{\partial r} + \frac{uc}{r} = 0$$

where $c = (p/\rho)^{\frac{1}{\gamma}}$

and $p = k \rho^\gamma$, k being a constant.

Eqs. (1.2.6) with $\gamma=2$ are identical to the SWE's (Eqs. (1.2.1)) with the substitution of variables $gh=c^2$. The gas density, here, essentially plays the role of the liquid height in the case of modelling a collapsing liquid column.

1.3 Theoretical Analysis and Derivations

The axisymmetrical 3-D problem defined above (section (1.1)) can be solved by applying a numerical MOC to the governing SWE's, derived in section (1.3.1). This method of solution is applied to the initial column collapse and unhindered spread as well as to the reflection of the depletion wave at the axis of symmetry of the tank. The treatment of the shock formation and propagation after impact of the surging liquid against the containment wall will be illustrated for both vertical and inclined barriers. In the case of vertical walls the solution can be considerably simplified whereas inclined walls require the application of a numerical MOC. The instantaneous pressure at impact and the force on all these types of walls will be considered. Quantification of overflow in the case of containment walls of limited height is also investigated, along with the possible distance the overflowing jet of liquid (see Fig. (1.3)) can travel beyond the banded enclosure.

1.3.1 Derivation of the SWE's in a Cylindrical-Polar Coordinate System

The derivation of the SWE's for a collapsing cylinder is

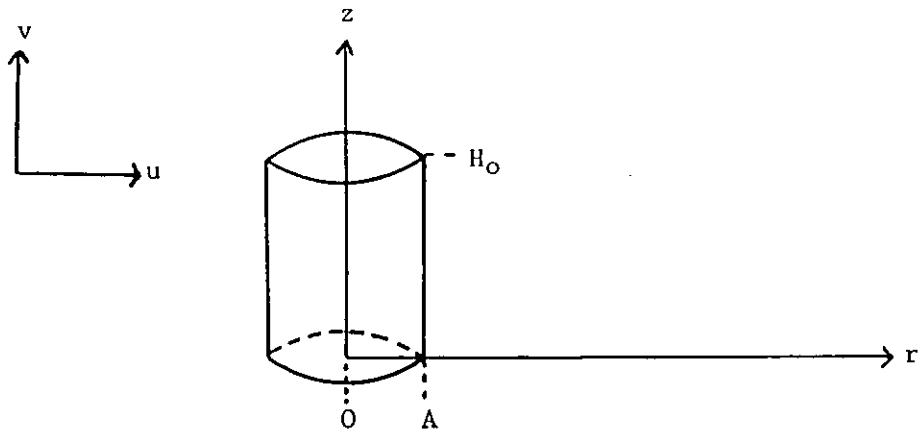
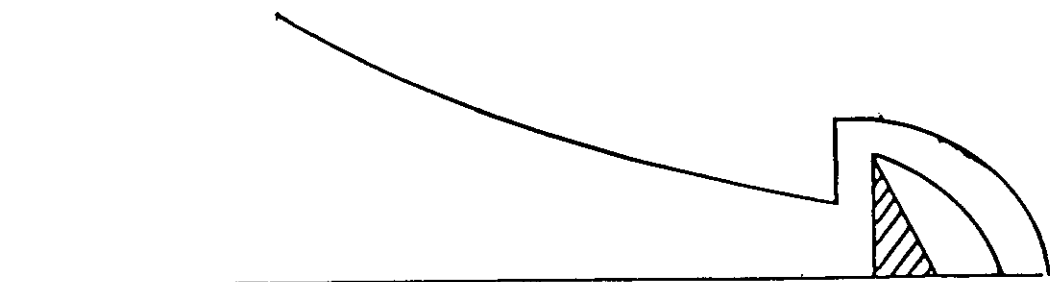


Figure 1.4 $\underline{U} = u(r,z,t)\underline{i}_r + v(r,z,t)\underline{i}_z$



Very first instances of impact - Liquid flowing up the vertical face of the containment dyke wall.



After liquid build up - Liquid is projected in the form of a jet beyond the dyke.

Figure 1.3

similar to that of the plane model first described by Ritter [1882, (23)]. The conservation of mass equation ('continuity') and Euler's [1755, (10)] equation for a cylindrical polar coordinate system (Fig. (1.4)) are, assuming axial symmetry:

$$\text{Continuity: } \frac{1}{r} \left[r \frac{\partial u}{\partial r} + u \right] + \frac{\partial v}{\partial r} = 0 \quad (1.3.2)$$

$$\text{Euler: } \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \quad (1.3.3)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g \quad (1.3.4)$$

Euler's equations are arrived at by neglecting, viscous terms in the equations of conservation of momentum and surface tension in the appropriate boundary conditions. This is allowed as, for example, a 30 m high tank containing LNG, has Reynolds and Weber numbers of 1.2×10^9 and 2.5×10^8 respectively. For a 10cm high experimental column of water, Re and We are respectively, 9.9×10^4 and 1.3×10^3 . It can thus be assumed that gravitational and inertial forces predominate in the motion. Even when comparing theoretical and small scale experimental data, neglecting all viscous and surface tension forces in the liquid, will be permitted for the time span over which the important motion takes place. Only when the liquid spill is reduced to a slow-moving film must

the above neglected forces come into consideration. In the problem at hand this phenomenon is not of importance. The density of the surrounding air is very small compared to that of the liquid and thus does not affect the motion.

The ground over which the liquid flows is assumed to be horizontal. Ground resistance to flow is neglected here. In the case of LNG and perhaps LPG spills, a vapour film may be formed between the ground and the flowing bulk of liquid due to the high rate of heat transfer. It is believed that the film may remain stable over the time span in which complete collapse and spread to the bund occurs. In the case of non-boiling spills, ground resistance contributions may be non-negligible. Dressler [1952, (8)] and Whitham [1954, (29)] describe modifications to the plane model SWE's for the effects of ground resistance. Dressler's work predicts that for Chézy ground resistance coefficients, $K < 10^{-6}$, liquid flow will be unaffected. Furthermore, Whitham derives an approximation for the distance behind the forward surging front where ground resistance effects are significant for a given value of K . The errors incurred by neglecting ground resistance effects were found to be acceptable for most situations (14) and consequently these factors have not been considered in this treatise.

Consider the vertical section of an axisymmetric liquid column collapse (Fig. (1.5)) and the initial conditions:

$$t \leq 0 : r \leq A, \quad h = H_0, \quad u = 0$$

$$r > A \quad h = 0$$

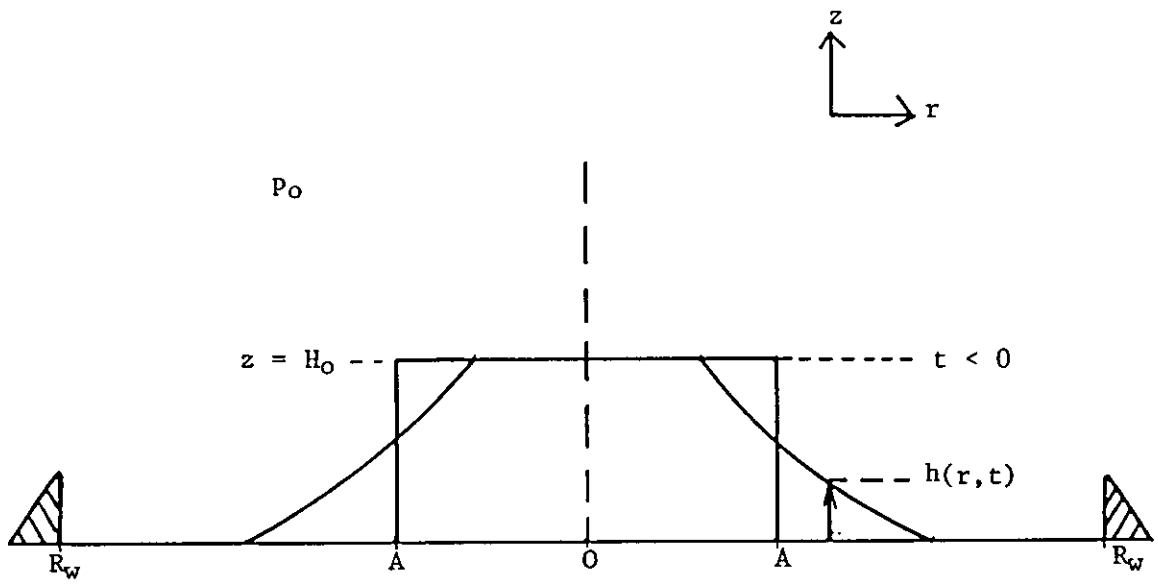


Figure 1.5 Vertical cross section of a collapsing liquid column.

It follows from Eq. (1.3.2) that

$$\int_0^h \frac{dv}{dz} dz = v \Big|_{z=h} - v \Big|_{z=0} = \int_0^h \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) dz \quad (1.3.5)$$

The vertical velocity at the ground is clearly zero. By definition a particle at the free surface of the liquid must have a velocity wholly tangential. Let the free surface in this situation be given by:

$$z = h(r, t) \quad (1.3.6)$$

It follows that the total derivative,

$$\frac{D}{Dt} (z - h(r, t)) = 0 \quad (1.3.7)$$

and hence Eq. (1.3.5) yields:

$$v \Big|_{z=h} = \frac{\partial h}{\partial t} + \frac{u \partial h}{\partial r} = - \int_0^h \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) dz \quad (1.3.8)$$

It is now furthermore assumed that vertical acceleration terms are negligible in this situation compared with horizontal acceleration terms. This is certainly inaccurate in the first instances of column collapse where the acceleration is nearly entirely in the vertical direction within and close to the tank, as is obvious from experimental work; see Martin and Moyce [1952, (20)]

and this work (Appendix (4)). In the first instances of motion this is also clear from Euler's equations. However, for modelling the collapse and its consequences as a whole this assumption is convenient and justifiable. The area and consequently the time span over which the entire process takes place is great enough to justify neglecting the effect of the errors incurred in the first moments of the collapse. It was calculated from Penney's work as well as from empirical data from experiments conducted in this research that the vertical accelerations in the motion are an order of magnitude less than the horizontal ones beyond a third of a cylinder radius from the edge of the tank at all times. The vertical acceleration is at least equal to the horizontal acceleration within a quarter of a tank radius from $r = A$ at all times. The vertical velocity in the motion was found to be negligible by the time a hydraulic jump has propagated back towards the tank edge for the closest of containment walls considered in this study. In other words, the ensuing evaluation of the motion will be inaccurate at the beginning of the collapsing process within and near to the tank but not however away from the tank nor at times after impact against the containment has occurred, where the practical interest lies.

So by neglecting vertical accelerations, horizontal velocity terms will increase faster initially and hence throughout the motion. Thus $|v| \ll |u|$ and Eq. (1.3.4) reduces to:

$$0 = -\frac{1}{\rho} \frac{dp}{dz} - g \quad (1.3.9)$$

or

$$p = p_0 + \rho g(h-z) \quad (1.3.10)$$

since $p = p_0$, the ambient air pressure at $z = h(r,t)$.

Hence,

$$\frac{\partial p}{\partial r} = \rho \cdot g \frac{\partial h}{\partial r} \quad (1.3.11)$$

and therefore Eq. (1.3.3) becomes:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = -g \frac{\partial h}{\partial r} \quad (1.3.12)$$

The right hand side (RHS) of Eq. (1.3.12) is independent of z , and therefore so must be the LHS, $\frac{Du}{Dt}$. When the liquid is initially at rest ($t \leq 0$) u is independent of z ; it remains so and is therefore independent of z at all times.

Hence the shallow water equations follow:

$$\text{Eq. (1.3.12)} \rightarrow \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + g \frac{\partial h}{\partial r} = 0 \quad (1.3.13)$$

$$\text{Eq. (1.3.8)} \rightarrow \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + \frac{uh}{r} = 0 \quad (1.3.14)$$

If, for the sake of convenience, the substitution, $c^2 = gh$, is introduced the two dependent variables assume the dimensions of velocity. c also has a significance in long wave theory. It can

be shown (17) that a disturbance or wave travelling along the surface of the liquid, for which the amplitude is much less than the liquid depth and governed by the above Eqns. (1.3.13) and (1.3.14), satisfies the wave equation. This is of practical significance as is illustrated later in section (1.3.2). Thus,

$$\text{Eq. (1.3.13)} \rightarrow \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + 2c \frac{\partial c}{\partial r} = 0 \quad (1.3.15)$$

$$\text{Eq. (1.3.14)} \rightarrow 2 \frac{\partial c}{\partial t} + 2u \frac{\partial c}{\partial r} + c \frac{\partial u}{\partial r} + \frac{uc}{r} = 0 \quad (1.3.16)$$

the initial conditions now being:

$$t \leq 0 : \quad r \leq A, \quad c = C_0 = (gH_0)^{\frac{1}{2}}, \quad u = 0$$

$$r > A, \quad c = 0.$$

At the leading edge of the collapsing column $c = 0$ as is shown later. At the front of the depletion wave running into the column u still remains equal to zero. Hence the term $\frac{uc}{r}$ in Eq. (1.3.16) vanishes at these two positions for all times of unhindered collapse, thus reducing Eqs. (1.3.15) and (1.3.16) to the form taken by the SWE's for the plane model. The analytical solution of the SWE's for this case, in the region of unhindered collapse, is shown in Appendix (3). Hence from this solution for

$u(r,t)$ and $h(r,t)$ given by Eqs. (1.2.2) the outward moving liquid front can be described by:

$$R_f = A + 2C_0 t \quad (1.3.17)$$

here $u = 2C_0$ and $c = 0$ and the inward moving front can be given by:

$$R_b = A - C_0 t \quad (1.3.18)$$

here $u = 0$, $c = C_0$.

1.3.2 Solution of the SWE's by the Method of Characteristics

By adding and subtracting Eqs. (1.3.15) and (1.3.16):

$$\frac{\partial}{\partial t} (u \pm 2c) + (u \pm c) \frac{\partial}{\partial r} (u \pm c) \pm \frac{uc}{r} = 0 \quad (1.3.19)$$

From the comparison of this expression with the total derivative of $u \pm 2c$,

$$d(u \pm 2c) = \frac{\partial}{\partial t} (u \pm 2c) dt + \frac{\partial}{\partial r} (u \pm 2c) dr = 0 \quad (1.3.20)$$

it can be readily seen from Eq. (1.3.19), that

$$\frac{d}{dt} (u \pm 2c) = \mp \frac{uc}{r} \quad (1.3.21)$$

if

$$u \pm c = \frac{dr}{dt} \quad (1.3.22)$$

or

$$u \pm 2c \pm \int \frac{uc}{r} dt = \text{constant} \quad (1.3.23)$$

when

$$\frac{dr}{dt} = u \pm c \quad (1.3.24)$$

respectively.

In other words the values of the LHS of Eqs. (1.3.23), sometimes referred to as the Riemann invariants, are constant for points moving with velocities $u \pm c$ respectively, i.e. they are constant along paths or 'characteristics' defined by Eqs. (1.3.24).

Characteristics are paths or trajectories along which any entities, such as physical disturbances are propagated. An infinitesimal disturbance gives rise to a propagation of this disturbance along a characteristic path. There is a discontinuity from rest to motion. In this case, the 'disturbance', i.e. the disappearance of the tank wall, generates two distinct 'families' of characteristics in the model: one 'forward' family whose leading characteristic (Eq. (1.3.17)) defines the position of the front surging towards the containment wall, and one 'backward' family whose leading characteristic (Eq. (1.3.18)) defines the position of the depletion front moving into the cylinder. These two propagating disturbances arising from the single initial disturbance will each

create a different disturbance on contact with any obstacle or boundary. Hence, from then on a new distinct family of characteristics will describe the system in the next space-time region.

The transformation of the SWE's into their characteristic form is very convenient. The relation between the variables of motion in the form of a trajectory in space-time allows for a solution progressing in time. In other words: the state of the motion at a given time can be deduced solely from the knowledge of the conditions at a previous time along with the appropriate boundary conditions. At first sight it may appear that the SWE's are easier to deal with in the form given by Eqs. (1.3.15) and (1.3.16); particularly since the RHS of Eqs. (1.3.24) is not known in advance and hence leaves the characteristic curves undefined. They must in fact be determined in the course of evaluating the unknown functions u and c from the simultaneous Eqs. (1.3.23), and this is not always straightforward as will be explained in more detail in the next chapter. Nevertheless, the formulation of this problem in characteristic terms is very useful to study the properties of the solution. The form of the solution evaluated in this way enables a representation of the way in which disturbances and their interactions move within the boundaries of the bounded enclosure.

Eqs. (1.3.23) and (1.3.24) define the system in all regions of the process: initial collapse, spread towards the containment wall, reflection from the containment wall, reflection of the depletion wave at the central axis and reflection of the converging hydraulic jump at the central axis. In the next sections the solution of the characteristic equations in these specific areas is

examined. A more detailed description of characteristics in general is given by Abbott [1966, (1)] and in relation to a collapsing column in Appendix (5). The characteristic diagram illustrating the entire process of collapse and resulting flow is given in Fig. (1.6) for an axisymmetric cylinder. The leading characteristics which bound the distinct space-time regions of motion are shown. Within each region lie the forward and backward families governing a specific part of the collapse process. The regions can be summarized as follows:

Region I

$A < r < R_w$, $0 < t < T_w = (R_w - A)/2C_0$. This defines the space time region outside the motion. The liquid is never present here.

Region II

$0 < r < A$, $0 < t < T_b = A/C_0$. Part of the liquid in the column is here as yet undisturbed.

Region III

$0 < r < R_w$, $0 < t < T_i$. Region of unhindered column collapse and liquid flow. Wave front surges towards the containment wall at $r = R_w$ and a depletion wave moves into the undisturbed liquid of the tank towards $r = 0$.

Region IV

$0 < r < R_i$, $T_b < t < T_c$. A weak reflection from the centre of the tank results from the the depletion wave converging at $r = 0$. (R_i, T_i) is the position where this reflection meets the hydraulic jump.

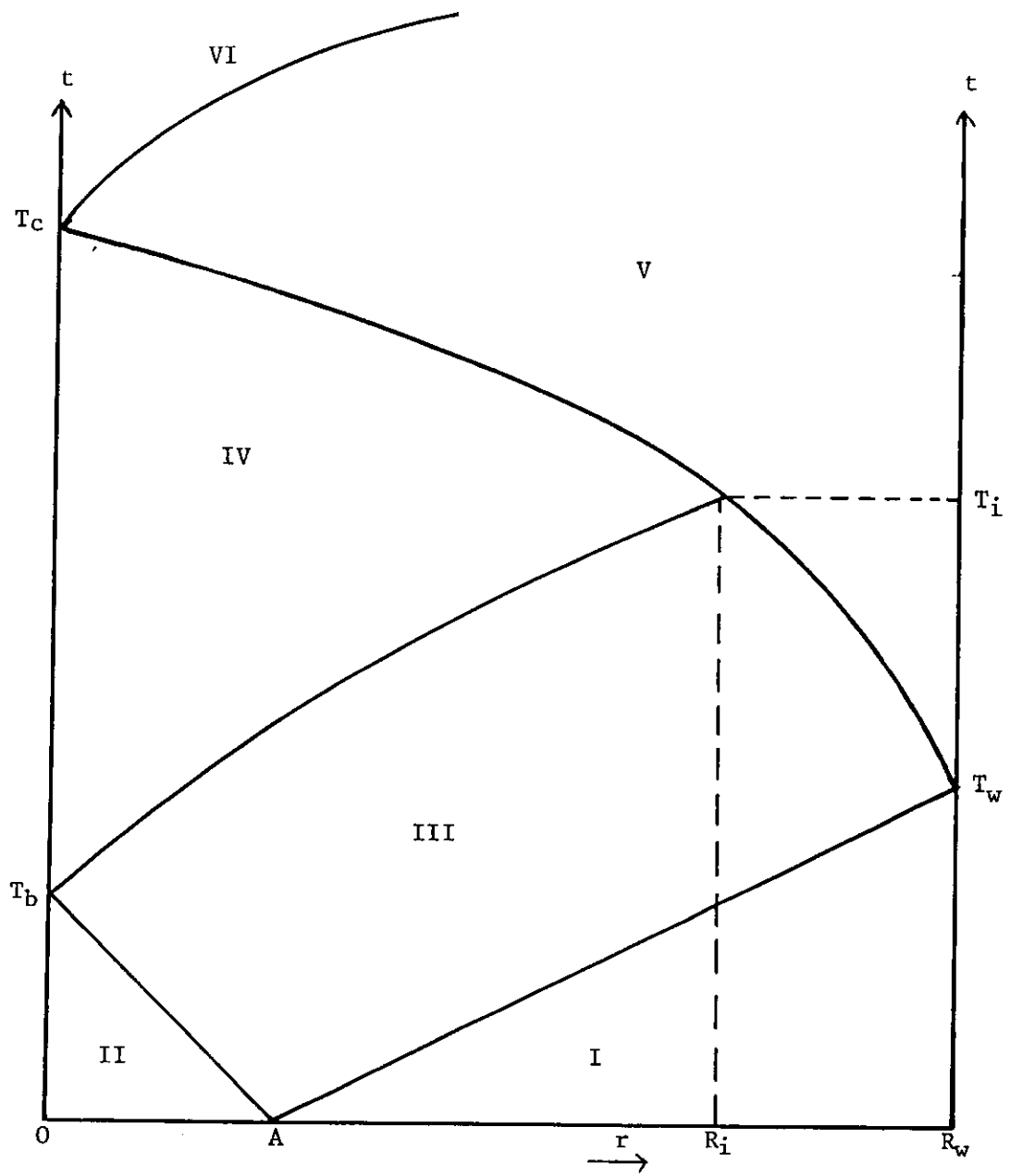


Figure 1.6

Region V

$0 < r < R_w$, $T_w < 0 < T_c$. The hydraulic jump reflected from the containment wall propagates back towards $r = 0$, swallowing the weak reflection from region IV at (R_i, T_i) .

Region VI

$0 < r < R_w$, $t > T_c$. The shock formed in region V travels up and down between the centre line and the containment wall indefinitely as no energy is assumed to dissipate according to the model. This phenomenon will be referred to as 'sloshing'.

Reference to these numbered regions will be made throughout the text.

1.3.3 Unhindered Cylinder Collapse - Region III

In the shallow water theory describing the collapse of a liquid column, there is a discontinuity in the motion the instant the tank wall disappears. Therefore, to implement the numerical MOC, the values of u and c need to be obtained at a time just after the collapse has started. Hence a solution for the SWE's is required at an early stage of the collapsing process.

Due to the similarity with the type of equations employed in gas dynamics, use can be made of a series solution, for a set of hyperbolic differential equations, derived by Greenspan and Butler (13). In this method the partial differential equations are reduced to a set of ordinary differential equations by postulating a solution in the form of two infinite series:

$$\begin{aligned}\phi &= \sum_{n=0}^{\infty} A_n(\eta) t^n \\ \psi &= \sum_{n=0}^{\infty} B_n(\eta) t^n\end{aligned}\tag{1.3.25}$$

where for our case: $\eta = \frac{r-A}{C_0 t}$

ϕ, ψ are the Riemann invariants $u \pm 2c$ respectively.

A_n, B_n are coefficient functions evaluated by equating terms after substitution of Eqs. (1.3.25) in the SWE's, Eqs. (1.3.15) and (1.3.16). Solving for these coefficients is more suitable for gases with particular values of γ , the ratio of specific heats. In the case of the SWE's it becomes extremely difficult to solve analytically for coefficients where $n > 1$.

If only the first term in each series is considered the solution for the SWE's obtained from $\phi = \phi(A_0, t)$ and $\psi = \psi(B_0, t)$ is,

$$\begin{aligned}u(r, t) &= \frac{2}{3} \left(C_0 + \frac{r-A}{t} \right) \\ c(r, t) &= \frac{1}{3} \left(2C_0 - \frac{r-A}{t} \right)\end{aligned}\tag{1.3.26}$$

which is in fact the analytical solution of the SWE's for the plane model in region III. Hence for the axisymmetric model there must be a compensation for the dispersive radial flow, as opposed to the 'plane' channel type flow, in the following terms of the series.

By deriving the (n=1)-coefficients and solving the resulting simultaneous equations,

$$\phi = \phi (A_0, A_1, t) \quad (1.3.27)$$

$$\psi = \psi (B_0, B_1, t)$$

the following expressions for u and c are obtained:

$$u = 2C_0 - 2C_0Z + ZC_0(5Z^{\frac{1}{2}} - 3Z - 2) \quad (1.3.28)$$

$$c = ZC_0 + \frac{ZC_0t}{2} (7Z^{\frac{1}{2}} - 3Z - 4)$$

where, $Z = \frac{1}{3} (2 - \eta)$

Comparison of these values of u and c evaluated at a small time after the collapse begins, $t = T_0$, reveals a small but significant difference. The exact solution for coefficients with $n > 1$ is extremely difficult and tedious. The contributions made by A_1 and B_1 to the value of u and c are already small compared with those of the A_0 and B_0 . The contributions that A_2 and B_2 will make to Eqs. (1.3.28) at a small time, $t = T_0 + \Delta t$, will be negligible. All that is required in practice to initiate a reliable numerical solution is an accurate value of u and c at some time, however small, after the start of the collapse. For this Eqs. (1.3.28) will be sufficiently accurate at a very small time after collapse and this is therefore all that is required.

This initialisation for the numerical routine enables a

description of the forward flow until the spreading liquid reaches the containment wall and of the backward flow until the depletion wave arrives at the centre of the tank, i.e. the motion in region III.

1.3.4 Impact against a Vertical Containment Wall - Region V

This section deals with the perpendicular interaction of the spreading liquid column with the rigid containment wall. In other words the case of a circular wall surrounding a cylindrical storage tank is considered. The face of the containment wall is taken to be vertical. Inclined walls are examined in the next section.

On collision of the spill against the face of the wall at $t = T_w$, liquid rushes upwards and accumulates rapidly to a considerable height. Due to the continual presence of liquid flowing towards the wall, a hydraulic jump or bore forms and moves with increasing speed back towards the original site of the tank. It should be noted that although there is no explicit vertical velocity in the shallow water theory, the law of conservation of mass enables the model to account for the rise in liquid level after impact.

According to the solution of the SWE's in region III, the approaching liquid front has zero depth and zero slope and therefore a discontinuity arises at (R_w, T_w) requiring a solution for the bore formation at a small time after impact, $T_w + \Delta t$, before the jump conditions can be implemented. To obtain this, series solutions for the liquid velocity and height as well as for the bore position and velocity, as formulated by Greenspan and

Young (14) are used for a short time after impact. It is justified to use this plane model solution, as mathematically the problem of impact and hydraulic jump formation can be treated linearly over a small radial displacement at all points other than at the axis of symmetry at the centre of the tank, where curvature effects are of crucial importance. The series solutions are reported in Appendix (6).

For modelling purposes the developed bore is assumed to have a vertical jump as shown in Fig. (1.7),

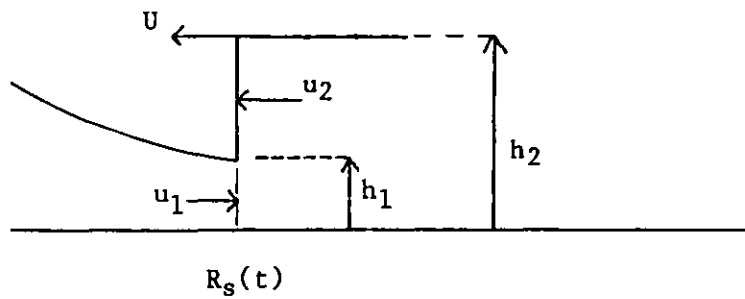


Figure 1.7

where subscripts 1 and 2 represent the motion in front of and behind the jump respectively. U is the bore velocity and R_s represents the bore position. The jump conditions were found to be most convenient in the following form:

$$U = u_1 - c_2 \left(\frac{M^2 + 1}{2} \right)^{\frac{1}{2}} \quad (1.3.29)$$

$$u_2 = u_1 - c_1 (M^2 - 1) \left[\frac{1}{2} \left(1 + \frac{1}{M^2} \right) \right]^{\frac{1}{2}} \quad (1.3.30)$$

where : $M = \frac{c_2}{c_1}$

The derivation of the above equations is shown in Appendix (6). The solution of the jump conditions requires the evaluation of the bore height, h_2 . For this purpose it is assumed that h_2 is independent of r and a function of t only, i.e. the liquid between the front of the bore (shock front) and the containment wall is always level. This is consistent with work by Stanyukovich (26) who used a similar system of equations applied to problems in gas dynamics, and experimentally substantiated for liquid spills by Denham and Luker [1979, (7)]. A more advanced numerical solution which does not use this assumption was developed and will be described in section (2.4). However, as its results were found to be very similar to those obtained using the above approximation and were far more costly to obtain in terms of computer time, the adequate simplified method is used for vertical containment walls.

Thus using the level bore assumption, the situation at $t > T_w$ is as shown in Fig. (1.8).

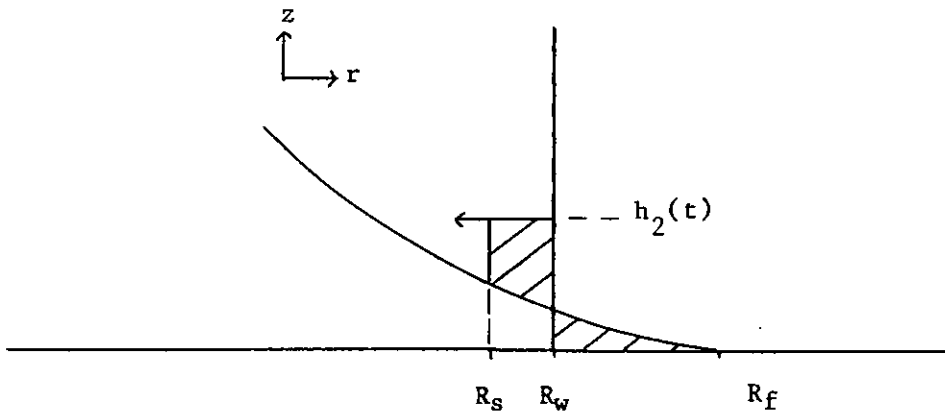


Figure 1.8

$$h_2(t) = \frac{V}{\pi(R_w^2 - R_s^2)} \quad (1.3.31)$$

where V is the volume under the height profile from R_s to R_f (the theoretical position of the liquid front in the absence of a containment wall).

$$V = \int_{R_s}^{R_f} 2\pi r h_1(r, t) \quad (1.3.32)$$

with $h_1(r, t)$, the liquid height in region III, given numerically for the axisymmetric model. For the 2-D plane case the function $h_1(r, t)$ is given analytically. Hence the bore height in this case can be given continuously, until the motion in front of the jump is defined by region IV:

$$h_2 = \frac{4C_o^2(R_f - R_s)^3}{27gR_f^2(R_w - R_s)} \quad (1.3.33)$$

The shock velocity is clearly defined by

$$v = \frac{dR_s}{dt} \quad (1.3.34)$$

This completes the formulation of the problem in region V.

1.3.5 Reflection of the Depletion Wave at the Centreline - Region IV

The depletion wave starts at $r=A$, $t=0$ and converges at the centre of the cylinder at $r=0$, $t=T_b=A/C_0$ where a weak reflection is then created which moves forwards to the containment wall. In the plane model, the depletion wave is reflected at the back wall of the reservoir. An analytical solution for the position of the reflection front, applicable to the plane case, was first derived by Landau [1944, (18)], for a similar system of equations in gas dynamics:

$$R_r = 2C_0t - 3A\left(\frac{C_0t}{A}\right)^{1/3} - A \quad (1.3.35)$$

At this point,

$$u(R_r, t) = 2C_0\left[1 - \left(\frac{A}{C_0t}\right)^{2/3}\right] \quad (1.3.36)$$

$$c(R_r, t) = C_0\left(\frac{A}{C_0t}\right)^{2/3}$$

For the axisymmetric model, however, there is no analytical solution for the reflected front. Use is made of an approximation developed by Stanyukovich (26), valid for a short time $t_1 < T_b + \Delta t$. The reflected front is given by,

$$R_r(t_1) = C_0t - A \quad (1.3.37)$$

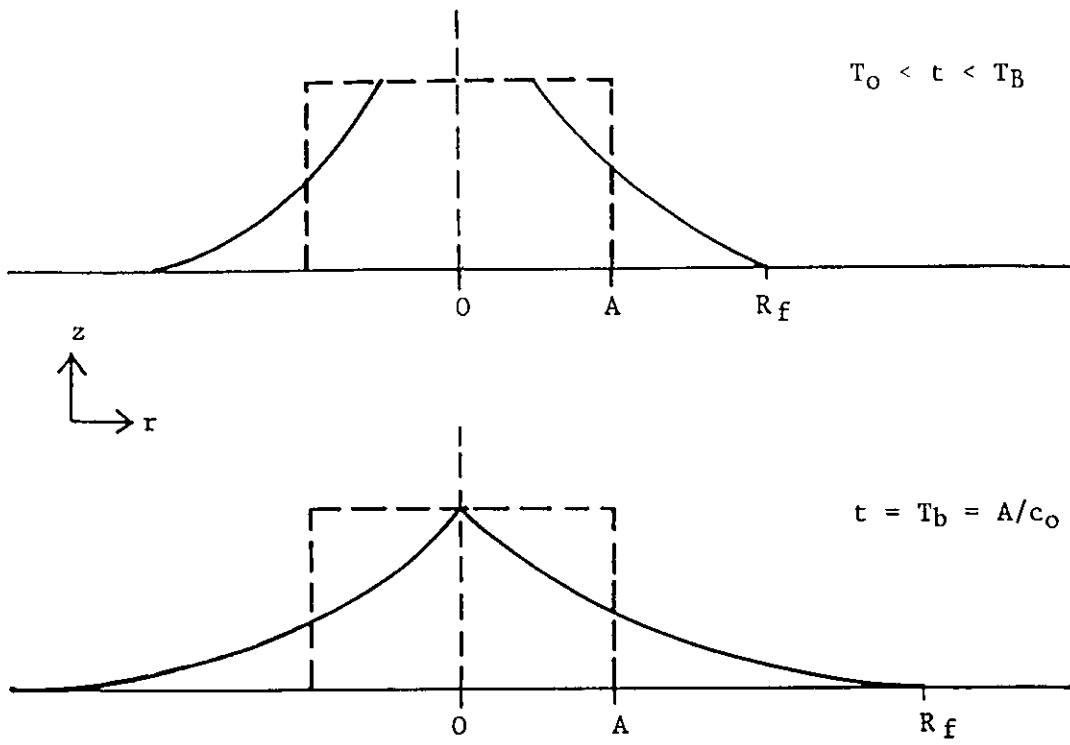


Figure 1.9

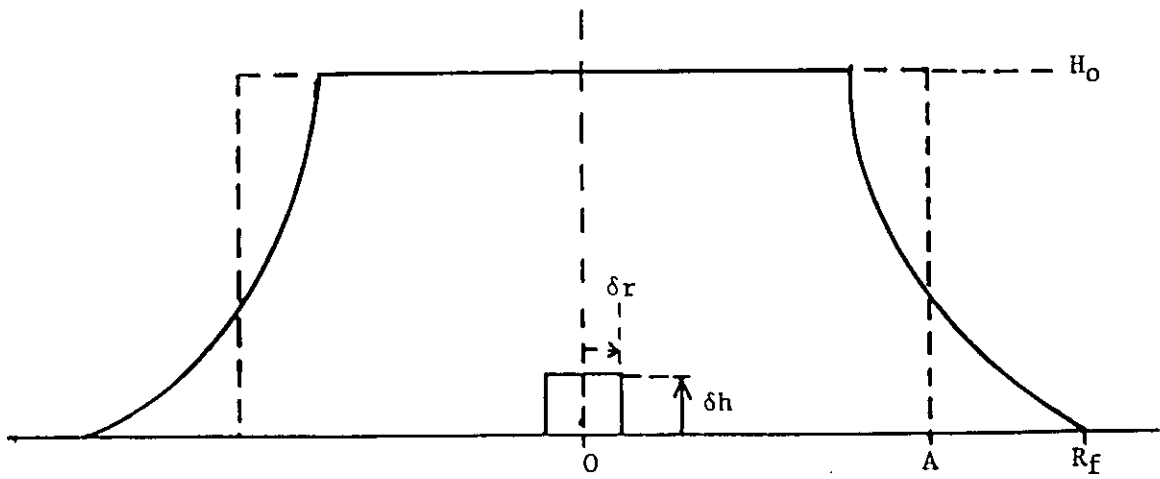


Figure 1.10

and the liquid velocity and wave speed, in the region $r < R_r(t_1)$, by:

$$c(r, t_1) = \frac{3}{\pi} C_0 \left(\frac{A}{C_0 t} \right)^{3/4} \left[1 - \left(\frac{r}{2C_0 t} \right)^2 \right]^{\frac{1}{2}}$$

$$u(r, t_1) = 2 \left[C_0 - c(r, t_1) \right] \left[1 - \left(\frac{r}{2C_0 t} \right)^2 \right]^{-1} \quad (1.3.38)$$

An iterative routine within the numerical MOC is used to evaluate the motion in region IV at times, $t > t_1$. This will be described in more details in section (2.3). Recalling the SWE's in their characteristic form,

$$u \pm 2c \pm \int \frac{uc}{r} dt = \text{constant} \quad (1.3.39)$$

on

$$\frac{dr}{dt} = u \pm c \quad (1.3.40)$$

it is obvious that a mathematical singularity arises at $r=0$ where the liquid velocity, u , is obviously also zero. Hence the integral term in Eqs. (1.3.39) will be undefined here. To facilitate the evaluation of this term at $r=0$, it may be written in the form,

$$\int \frac{uc}{r} \frac{dr}{u \pm c}$$

using Eqs. (1.3.40).

Now consider a control volume (Fig. (1.10)), $\delta h \times \delta r$, which

is elongated in height and remains at constant volume, the total volume being

$$V = \pi r^2 h = \pi r^2 \frac{c^2}{g}$$

As the volume is conserved,

$$\frac{dV}{dt} = \frac{\pi}{g} \left[r^2 2c \frac{dc}{dt} + c^2 2r \frac{dr}{dt} \right] = 0 \quad (1.3.41)$$

The radial velocity at which the control volume is shrinking is

$$u = \frac{dr}{dt} \quad (1.3.42)$$

Hence, combining Eqs. (1.3.41) and (1.3.42):

$$\frac{uc}{r} = -\frac{dc}{dt} \quad (1.3.43)$$

Therefore the integral term in the characteristic equations,

$$\int \frac{uc}{r} dt = - \int \frac{dc}{dt} \frac{dr}{u \pm c}$$

So in the limit as r approaches zero,

$$\lim_{r \rightarrow 0} \left(\frac{uc}{r(u \pm c)} \right) = \pm \left(\frac{1}{c} \frac{dc}{dt} \right)_{r=0} \quad (1.3.44)$$

This completes the formulation of the problem in region IV for solution by the numerical method of characteristics.

1.3.6 Inclined Containment Walls

In the previous sections modelling of containment has dealt exclusively with straight vertical walls. A bund surrounding a storage facility can differ from this in two essential ways: firstly it may be inclined and not vertical and secondly it may be of limited height, i.e. the liquid level, $h_2(R_w)$, is not always less than the bund height, H_b . In this section we will consider in particular consequences of the former. There are two good reasons for this. One is that for the purpose of better containment there are, at least in principle, obvious advantages for a wall sloping towards the tank as shown in Fig. (1.11). Secondly, most existing bunds are of the compacted dyke type, the face sloping away from the tank as shown in Fig. (1.12).

Evaluation of the motion in region V for these two cases is not as straightforward as for vertical containment walls. The assumption of a level bore is less readily justified and there is as yet no theoretical or experimental evidence to this effect. To resolve this situation the more complicated numerical MOC could be implemented to model the flow in region V. However, this method of solution would be hampered by 'vertical' boundary conditions at the containment wall due to its inclination. In the case of vertical containment walls there is nowhere, throughout the motion, any obstacle to vertical flow above ground level. When considering inclined walls both horizontal and vertical boundary conditions are

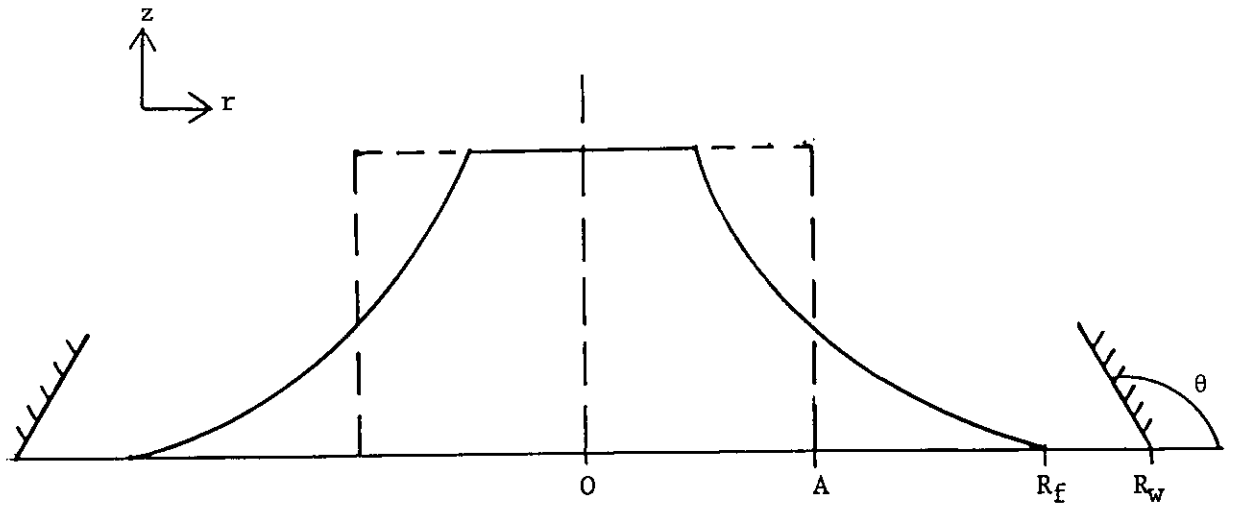


Figure 1.11 Angle of inclination $\theta > 90^\circ$

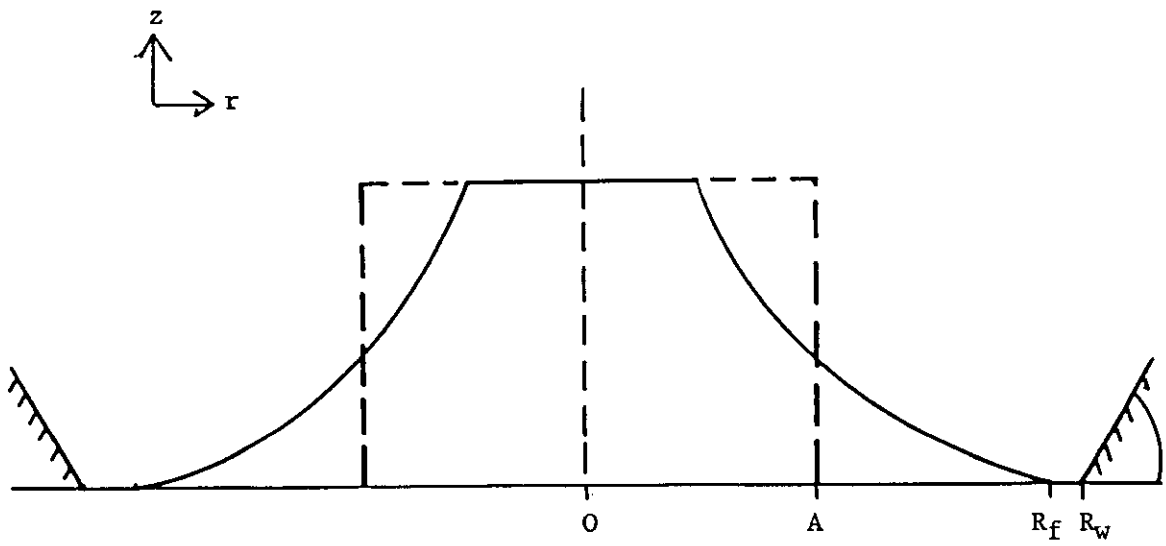


Figure 1.12 Angle of inclination $\theta < 90^\circ$

imposed on the motion and it is these vertical restrictions that add a new dimension to the problem which cannot be taken into account by the shallow water theory.

This problem can be clarified by the following simplified example (Fig. (1.13)). In the case of a bund inclined at an angle, θ , greater than the perpendicular, the separation of the shock position from the face of the containment wall is a function of z , unlike in the case of a vertical wall where this is constant. Consequently, when dealing with inclined walls, the liquid velocity behind the shock can no longer be independent of z . This conflicts with the shallow water theory, or rather, the shallow water theory is, strictly speaking not adequate for solving the motion in region V when considering inclined walls. An accurate solution to this problem will therefore require an entirely new theoretical approach.

However, within the scope of the shallow water theory the following approximate solution is offered. With minor modifications to the geometrical considerations and by assuming an average velocity near to the wall (see Fig. (1.14)), the numerical method applied to vertical walls for level bores, was used for cases where the deviation, of the angle of inclination, from the vertical is small. Under these circumstances results are unlikely to differ widely from values obtained for vertical containment walls. It was noted that for small angles α ($= |90 - \theta|$), the corresponding values of θ yielded identical results for the bore height as a function of time. This would suggest that the angle of inclination does not have a first order effect on the bore height,

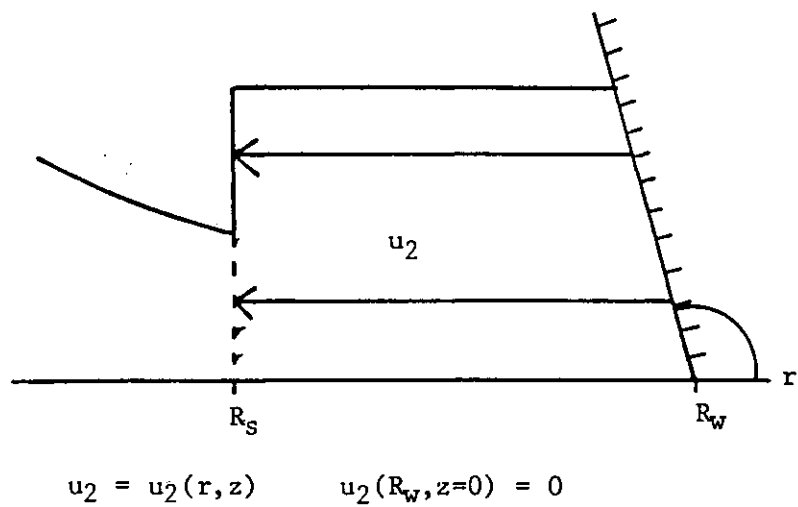
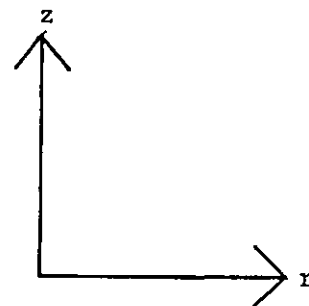
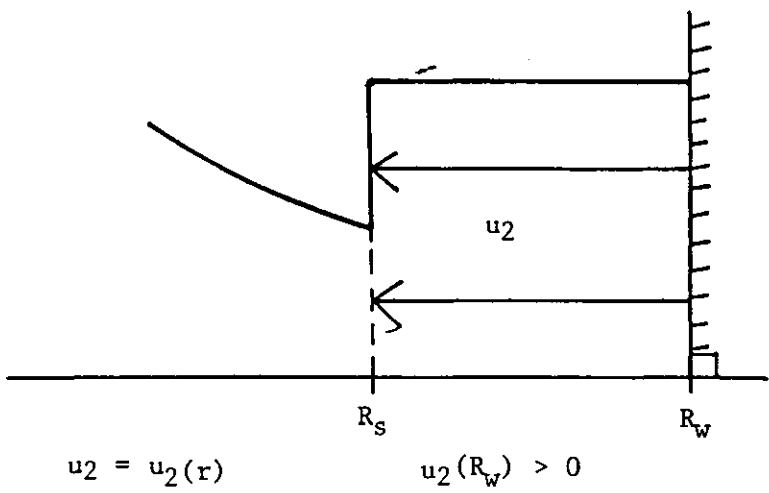


Figure 1.13

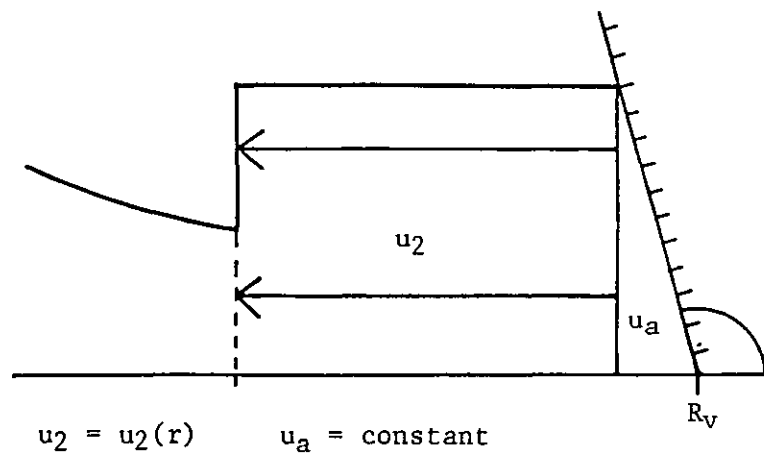


Figure 1.14

or mathematically speaking the relationship between the bore height and α , in terms of a Taylor series, is:

$$h_2(t, \alpha) \sim h_2(t) \Big|_{\alpha=0} + \alpha^2 \frac{\partial^2 h_2}{\partial \alpha^2} \Big|_{\alpha=0} + \dots \quad (1.3.45)$$

The average difference in $h_2(t, \alpha)$ for angles of $\theta = 98^\circ, 82^\circ$ is 2% over the time range $T_w < t < T_c$, the error reaching 5% for $\alpha = 14^\circ$ and increasing more rapidly as α becomes larger. For all values of α in the range $40^\circ < \theta < 150^\circ$ the numerical routine is well behaved. The results for all angles within this range are not necessarily valid, but they do offer some indication of the degree of error involved by assuming a level bore. For a sufficiently wide range of angles that would be considered in practice, the angle of inclination of a containment wall, only has a second order effect on the liquid level behind the shock. Hence the shallow water theory can, under these conditions, be used as a reasonable approximation despite the vertical variation in the motion. Illustrating this second order effect by means of the numerical routine derived for vertical walls does not however justify assuming a level bore in the case of inclined bunds. To obtain a more accurate solution for this case and also to show that the second order effect is valid for a wider range of angles than predicted by the vertical wall method, the numerical MOC is here used in region V. This allows for a variation in the liquid level between the shock position and the containment wall.

As may be recalled from the solution for the vertical wall problem, a discontinuity arises in modelling the collision of the

surging liquid against the containment wall; a solution at a short time after impact is thus required. The initial conditions predicted by Greenspan and Young's series solutions (see Appendix (6)) strictly only apply to vertical walls, supplying values for the liquid height and velocity behind the jump, the shock velocity and shock position at a time $t = T_w + \Delta t$. However, it was noted from numerical experimentation that only the evaluation of the liquid height and velocity was of crucial importance for the accurate computation of the shock motion at later times. When inaccurate values of the shock velocity, U , and the initial displacement of the shock from the wall, ΔR_s , were used to initialise the numerical routine, the effect on the evaluation of the motion further ahead in time was negligible. This is due to the fact that initially the rate of change in liquid height is far greater than the rate of change of shock position. The effect of a change in ΔR_s of up to 200% is unnoticeable beyond times 5% greater than T_w .

Greenspan and Young's initial solutions for the liquid height and velocity behind the shock are used with minor adjustments in certain cases due to the geometric conditions imposed by the inclination of the wall. The shock velocity and position are calculated accordingly. This involves much tedious algebra and therefore all changes to these initial conditions are presented in Appendix (7). Having established the basis for the routine, the motion is henceforth solved using the numerical MOC. Details of this method applied to region V are described in the next chapter.

For inclined walls ($\theta < 90^\circ$) which deviate significantly from the vertical, the bore will take longer to form and to start moving back towards the centre of the enclosure. In extreme cases no

hydraulic jump will be established. When the computer program assumes the presence of walls of unlimited height (in order to predict the maximum eventual liquid level), the liquid will in these cases flow up the slope to a maximum distance and flow back again. The angle at which a shock no longer forms can be predicted.

For the general case of $\theta < 90^\circ$, an important modification to the theory must be noted. Beyond R_w (see Fig. (1.15)), the elevation of the wall face above the horizontal must be taken into account when considering the wave speed, c . For this the following is assumed:

$$\begin{aligned} c &= \left[gh_2(r,t) \right]^{\frac{1}{2}} & ; r < R_w \\ c &= \left[g(h_2(r,t) - h_b(r)) \right]^{\frac{1}{2}} & ; r > R_w \end{aligned} \tag{1.3.46}$$

where $h_b(r)$ is the height of the containment wall face above ground level.

Walls inclined with angles $\theta > 90^\circ$ are only considered up to $\theta = 135^\circ$. Not only would walls of this degree of inclination be impractical to build but the resultant height for a given length of wall would be much lower for greater angles and thus defeat the purpose of better containment.

1.3.7 Overflow

Of the two assumptions concerning the containment wall in the

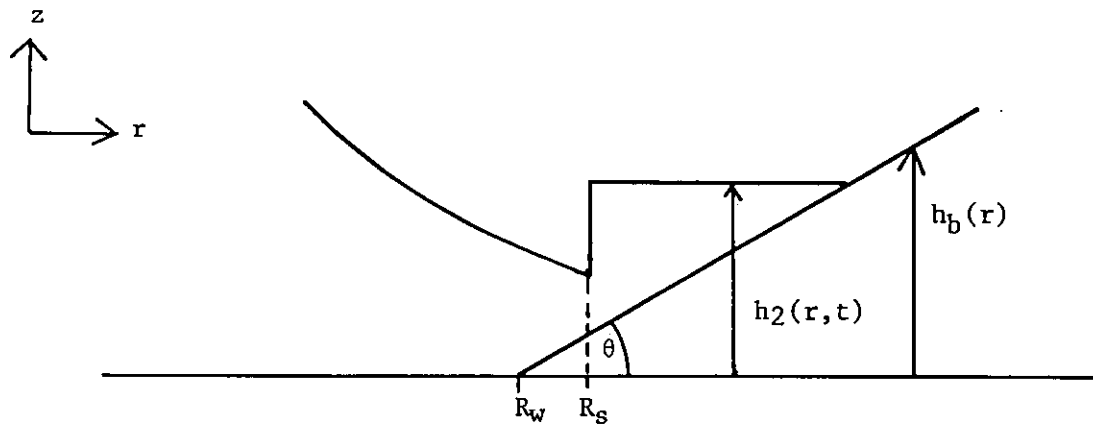


Figure 1.15

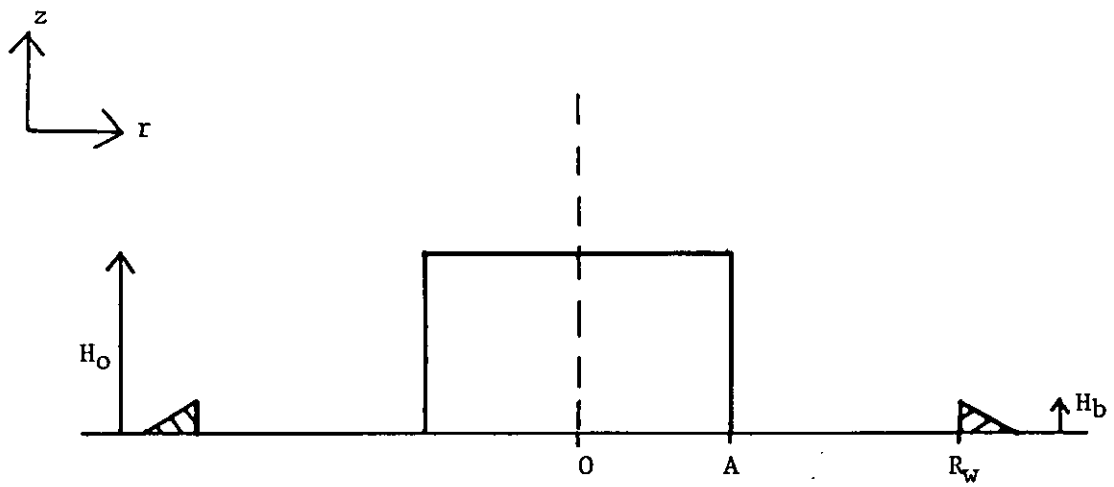


Figure 1.16 To scale

initial mathematical model, mentioned at the beginning of the previous section, the most important is that of unlimited height. In this context a wall of unlimited height is a wall sufficiently high to prevent overflow. In nearly all industrial situations bunds are of inadequate height for containment purposes in the event of a CTF. Most are built to such a height that the volume of the storage tank equals the volume within the perimeter of the containment walls. A circular bund surrounding a cylindrical tank of height, H_o , would under these conditions have a height,

$$H_d = \frac{A^2 H_o}{R_w^2} \quad (1.3.47)$$

See Fig. (1.16).

In the event of a CTF the liquid will readily build up, after impact, above the bund height. When this happens it is assumed, for the purpose of the model, that no further disturbance travels backwards into the enclosure as the result of overflow. From application of long wave theory it can be shown that the wave speed equals the liquid velocity over the bund. Therefore,

$$U_w = c(R_w, t) - C_b$$

where $c(R_w, t) > C_b = (gH_b)^{\frac{1}{2}}$ and

H_b is the bund height.

From continuity,

$$u(R_w, t)h(R_w, t) = U_w(h(R_w, t) - H_b)$$

the liquid velocity at the bund can be given by:

$$u(R_w, t) = 0$$

when $c(R_w, t) \leq C_b$ and

$$u(R_w, t) = \frac{[c(R_w, t)^2 - C_b^2]^{3/2}}{c(R_w, t)^2} \quad (1.3.48)$$

when $c(R_w, t) > C_b$.

To compute the total volume which flows over the bund integration is required over the time range for which $c(R_w, t) > C_b$. For a bund with the specification $H_b = H_d$, overflow must by definition cease before the time, $t = T_c$, when the returning shock reaches the centre of the enclosure. Thus the total volume spilled, excluding that due to the sloshing effects in region VI which were found to be negligible - and therefore not included here - will be given by:

$$Q_s = 2\pi R_w \int_{T_w}^{T_c} u(R_w, t) h(R_w, t) dt \quad (1.3.49)$$

or using Eq. (1.3.48).

$$Q_s = \frac{2\pi R_w}{g} \int_{T_w}^{T_c} [c(R_w, t)^2 - C_b^2]^{3/2} dt \quad (1.3.50)$$

The second assumption about containment used in the model was that of a vertical wall. For inclined bunds, with angles $\theta < 90^\circ$,

the overflow will, with falling values of θ , increase progressively due to a positive horizontal component of liquid velocity at the wall associated with the surge. A similar, small but non-negligible and additional negative component is present when dealing with inclined walls for which $\theta > 90^\circ$. These velocity components can easily be included in Eq. (1.3.49) and are calculated by considering the rate at which the liquid builds up along the face of the inclined bund.

When overflow occurs it takes the form of a jet which is projected outside the bunded enclosure. The distance which the jet travels can be computed from horizontal and vertical velocity components of the liquid at the top of the bund. The horizontal velocity, $u(R_w, t)$, is given by Eq. (1.3.48). As there is no explicit vertical velocity in the shallow water theory the rate of change of liquid height at the containment wall is used. This shall be denoted by, V_w , where

$$V_w = \frac{d}{dt} (h(R_w, t))$$

Assuming that the bund is of zero width at the top and that the ground level beyond is equal to that inside the enclosure, the movement of the jet can be treated as a simple projectile problem. Thus the time of flight of the jet is given by,

$$T_p = \left[\left(\frac{V_w}{g} \right)^2 + \frac{2H_b}{g} \right]^{\frac{1}{2}} - \frac{V_w}{g} \quad (1.3.51)$$

and the distance of projection by:

$$R_p(t) = R_w + u(R_w, t)T_p \quad (1.3.52)$$

Calculations of this type are, for instance, of importance when estimating the pressure impact of the jet on adjacent installations if the distance from the wall is less than $R_p - R_w$.

1.3.8 Hydrostatic Loading and Impact Pressure on the Containment

Wall

At the moment of impact the advancing liquid strikes the base of the containment wall with a velocity, $u = 2C_0$. Since the front of the inviscid liquid is theoretically of zero thickness, there is a mathematical discontinuity at, $t = T_w$. After impact the velocity of the liquid, rising up the wall in the form of a hydraulic jump, is modelled as entirely horizontal, independent of height and zero at the face of the wall. It is therefore only possible to predict the pressure (or stress) due to the incoming liquid accurately, at the moment of impact, $t = T_w$. Thus

$$P(T_w) = \rho(2C_0)^2 = 4\rho gH_0 \quad (1.3.53)$$

where ρ is the liquid density.

According to the model, for $t > T_w$, the pressure at the base of the wall is only due to the hydrostatic head at $r = R_w$, given by

$$P(t) = \rho gh_2(t) \quad (1.3.54)$$

Thus, because of the idealized description of the flow behind the

shock, Eq. (1.3.54) is insufficient to predict correctly the total pressure at the base of the containment wall due to both impact and hydrostatic pressures. This omission to make proper allowance for the contribution to the force on the wall due to the incoming surging liquid beyond, $t = T_w$, also affects the calculations for the total load on the containment wall. It would however be reasonable to expect that this neglected term becomes small by the time the hydraulic jump has propagated a certain distance from the containment wall. The incoming surging liquid would then in practice be 'fully absorbed' by the returning bore so that no effect is felt at the wall, the force then being entirely hydrostatic.

The force or load on the containment wall can be taken as the hydrostatic head of the liquid at $r = R_w$, integrated over the wetted area. Thus,

$$F_w(t) = \pi R_w \rho g h^2(R_w, t) \quad (1.3.55)$$

Ignoring impact forces, the load on the wall must under the above assumptions, reach a maximum when the liquid height is at its peak. Whether the shock has moved sufficiently far at this point, so that force of impact from the surging liquid can be neglected or not will be discussed in Chapter (5).

CHAPTER 2

Numerical Formulation

2.1 Introduction

In this chapter the numerical implementation of the two models, outlined in the previous chapter, is described. An account is presented of the principles involved in the numerical method of characteristics and of various algorithms suggested in the literature. The method developed in this study is discussed in detail and compared with the one used by Greenspan and Young (14).

The adaption, of the selected numerical MOC, to the various phases of the CTF model is given together with further numerical techniques of specific relevance to the different space-time regions.

To summarize the descriptions given in Chapter (1), the general shallow water equations which govern the motion in all space-time regions of collapse are:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + 2c \frac{\partial c}{\partial r} = 0 \quad (2.1)$$

$$\frac{2\partial c}{\partial t} + 2u \frac{\partial u}{\partial r} + c \frac{\partial u}{\partial r} + \frac{Kuc}{r} = 0$$

where $K = 0$ represents the plane case and

$K = 1$ the axisymmetric case.

In their characteristic form the Riemann invariants are:

$$u \pm 2c \pm K \int \frac{uc}{r} dt = k \quad (2.2)$$

on the respective characteristic slopes,

$$\frac{dr}{dt} = u \pm c \quad (2.3)$$

where k is a constant which will clearly be different on different curves including those within the same family.

To describe the various numerical methods available, the above equations will be used as an example. K is taken as zero, since the additional integral term will only complicate the illustrations. The evaluation of the integral term for $K = 1$ is explained later in section (2.3). The plane case is described by:

$$u + 2c = k_1 \quad (2.4)$$

$$u - 2c = k_2$$

on the sets of curves, defined by:

$$\frac{dr}{dt} = u + c \quad (2.5)$$

$$\frac{dr}{dt} = u - c$$

respectively.

2.2.1 The Numerical Method of Characteristics

The numerical method of characteristics is a solution which progresses in time from an initial time, $t = T_0$, where the values of the dependant variables are known at a range of points or nodes of the discretized independant variable r . Using the above Eqs. (2.4) and (2.5) the method can now be illustrated in its most basic form - an explicit method.

Let u and c be defined at time, T_0 , at each point of the set of evenly spaced values of $r : r_1, r_2, \dots, r_n$. r_1 and r_n are the boundary points. Through all of these points pass two characteristics defined by Eqs. (2.5). From the values of u and c at these points, the slope of the two characteristics through each point can be evaluated. Taking a small enough spacing between points, δr , the position $P(r,t)$ in space-time, at which opposite (positive and negative) characteristics, passing through adjacent points, intersect, may be approximated. The characteristics are not necessarily straight. $P(r,t)$ will therefore only be exactly predicted when $\delta r \rightarrow 0$.

At any point $P(r,t)$ where, $r_1 < r < r_n$ and $t > T_0$, the values of u and c and will be related to u and c at the points from which the two characteristics, forming $P(r,t)$, originate. This relation is defined by Eqs. (2.4) and shown diagrammatically in Fig. (2.1).

Hence if at the point $A(r,t)$ the velocity and wave speed are denoted by u_a and c_a respectively, then from Eqs. (2.4) two simultaneous equations in u_a and c_a are obtained:

$$u(r_2, T_0) + 2c(r_2, T_0) = u_a + 2c_a$$

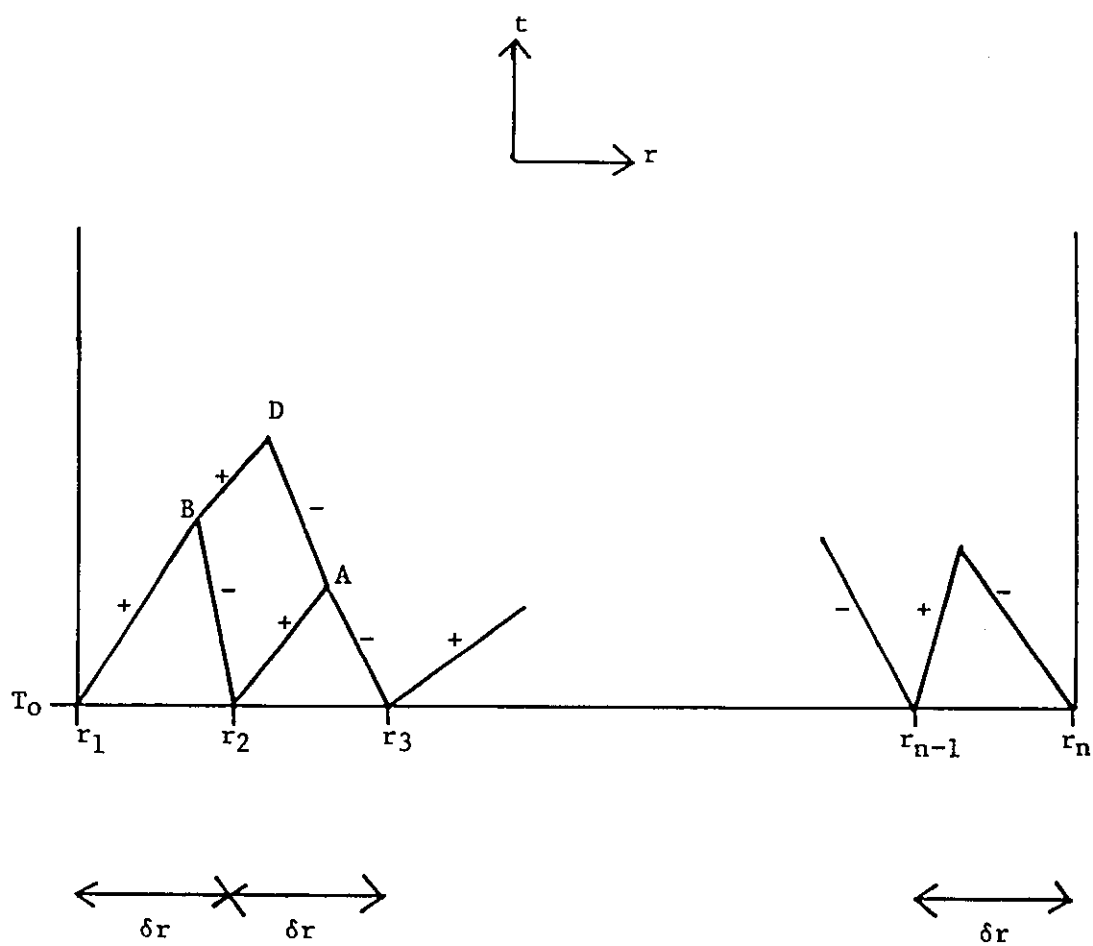


Figure 2.1

$$u(r_3, T_0) - 2c(r_3, T_0) = u_a - 2c_a$$

or,

$$u_a = \frac{1}{2} \left[u(r_2, T_0) + 2c(r_2, T_0) + u(r_3, T_0) - 2c(r_3, T_0) \right]$$

$$c_a = \frac{1}{4} \left[u(r_2, T_0) + 2c(r_2, T_0) - u(r_3, T_0) + 2c(r_3, T_0) \right]$$

Similarly, u_b and c_b could be evaluated at $B(r, t)$ and hence using the above process starting from points A and B, the motion at $D(r, t)$ can also be computed. Therefore, using the boundary conditions where necessary, the dependant variable can be calculated in this manner throughout the entire space-time region concerned.

The above algorithm is a very basic and simple numerical method known as a 'three-point' method. This explicit numerical solution is very easy to implement but requires a relatively small value of δr to arrive at a solution of satisfactory accuracy. It is further limited by instabilities as errors are cumulative in time. For Eqs. (2.1) this method is only stable in region III (see Fig. (1.6)) when $K = 1$ and in regions III and IV when $K = 0$.

In Fig. (2.1) the nodes where the motion is defined are scattered irregularly within the boundaries. Should the value of u or c be required at a specific point in space-time, further errors would be introduced by interpolation. In a more sophisticated, explicit three point method discussed by Abbott [1979, (2)], the motion is evaluated at points on a regular (r, t) mesh. See Fig. (2.2). As in the previous example let the motion be defined at a series of evenly spaced points : $r_1, r_2, \dots, r_n$,

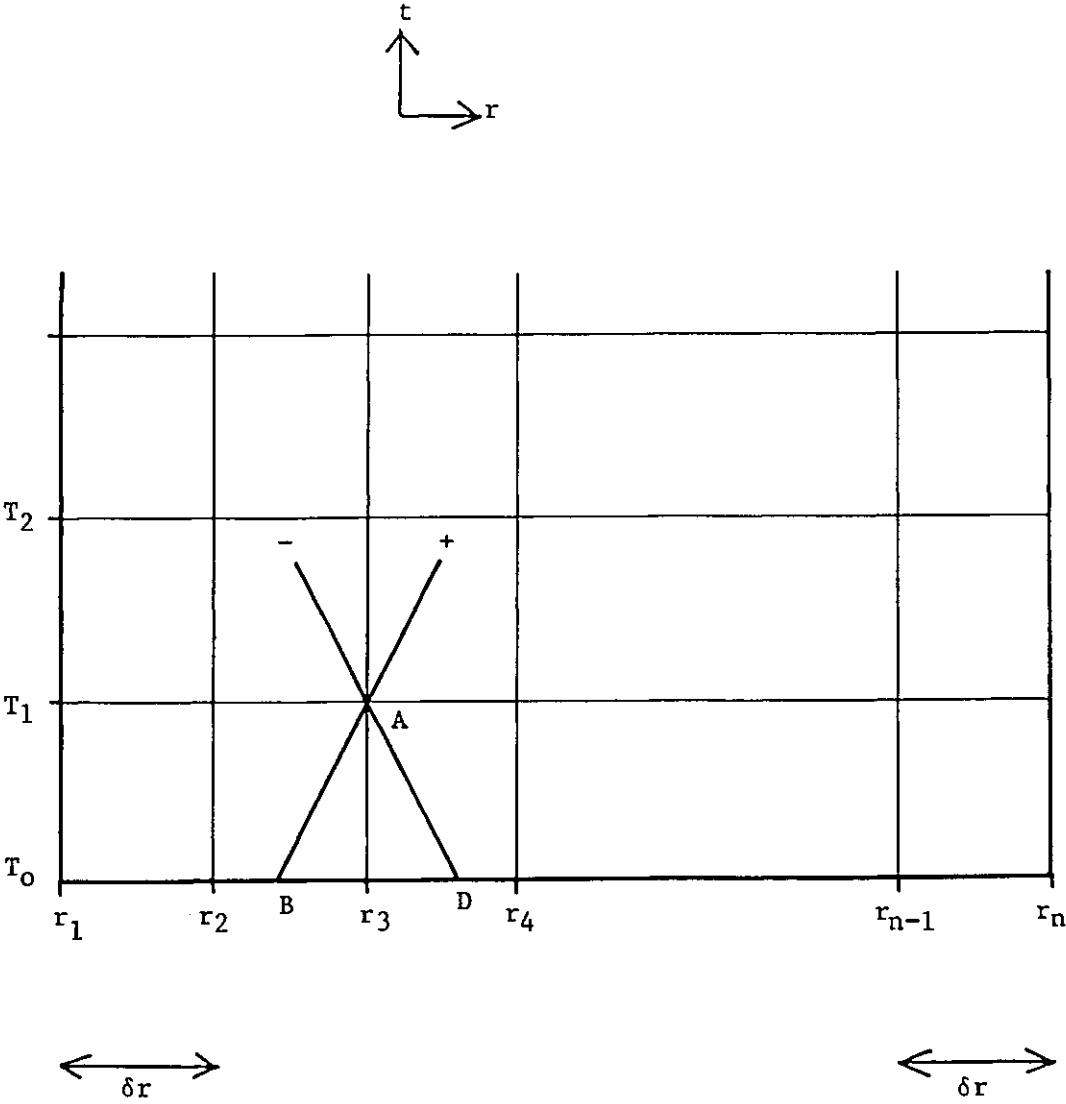


Figure 2.2

at time $t = T_0$. At node $A(r_3, T_1)$, u_a and c_a , would be evaluated by considering the positive and the negative characteristics passing through this point. These would respectively intersect the $(t=T_0)$ -line at points B and D. Since the positions of B and D and the values u_a and c_a are unknown, the slopes of these two characteristics are approximated by evaluating them at (r_3, T_0) , the error being a function of δt . Using these values of the slopes the position of B and D can be located. From interpolation along the line, $t = T_0$, where u and c are given at a finite number of nodes, u_b , c_b and u_d and c_d can be computed.

Greenspan and Young (14) have refined this method further by subsequently re-evaluating the slopes of AB and AC at point A using the values of u and c obtained there. The entire process is reiterated until u_a and c_a have converged sufficiently. This method hereby becomes implicit leading to a stable solution in all space-time regions of the plane model ($K=0$). A detailed description of their method is presented in Appendix (8).

All the methods mentioned above are considered to be 'first order' methods since, the slopes of the characteristic curves at any node, are calculated at or iterated about, only one point.

Another consideration necessary when looking at more accurate methods of solution is the type of interpolation used. It would obviously be inconsistent to use a linear interpolation scheme with a numerical method which is superior to the first order ones described above. The converse is also true. Linear interpolation results in first order accuracy in problems of this type.

As a way of improving accuracy, a 'four-point' method can be applied to the SWE's. This was first used by Hansen [1965, (15)] in

problems of plasticity. Here an element banded by two positive and two negative characteristics is considered as shown in Fig. (2.3). The points of intersection are denoted by A, B, C, D, in such a way that AB spans a positive characteristic. Although, in a system so defined, known conditions at any two opposite points are sufficient to determine the conditions at a third, the conditions at the remaining point will also be defined. The system is however, not over determined so long as exact relations between the dependant variables at A, B, C, D are used. Here in lies the reason why this second order method converges faster to an accurate solution than first order three-point methods. The solution at the third point does not only depend on the conditions at the two adjacent, opposite points but also on its relation to the conditions at point four. When comparing three and four-point methods it is obvious that the number of characteristic curves considered differs by a factor of two and the number of points by one. Thus the superiority in terms of speed of solution or accuracy of the four-point method over the three-point ones, is achieved at the expense of mathematical complexity and computational storage.

This can be taken further by considering higher order symmetries as illustrated, for instance, by Fig. (2.4). Higher order methods are in general only advantageous for use in linearized schemes (2). For non-linear problems such as the SWE's, these methods are very complicated to implement. As furthermore, the situation in region V already requires extensive storage, for reasons outlined in section (2.4), the principles of the three-point method had to be adopted for this work. For further details on higher order methods see, for instance, Abbott (2).

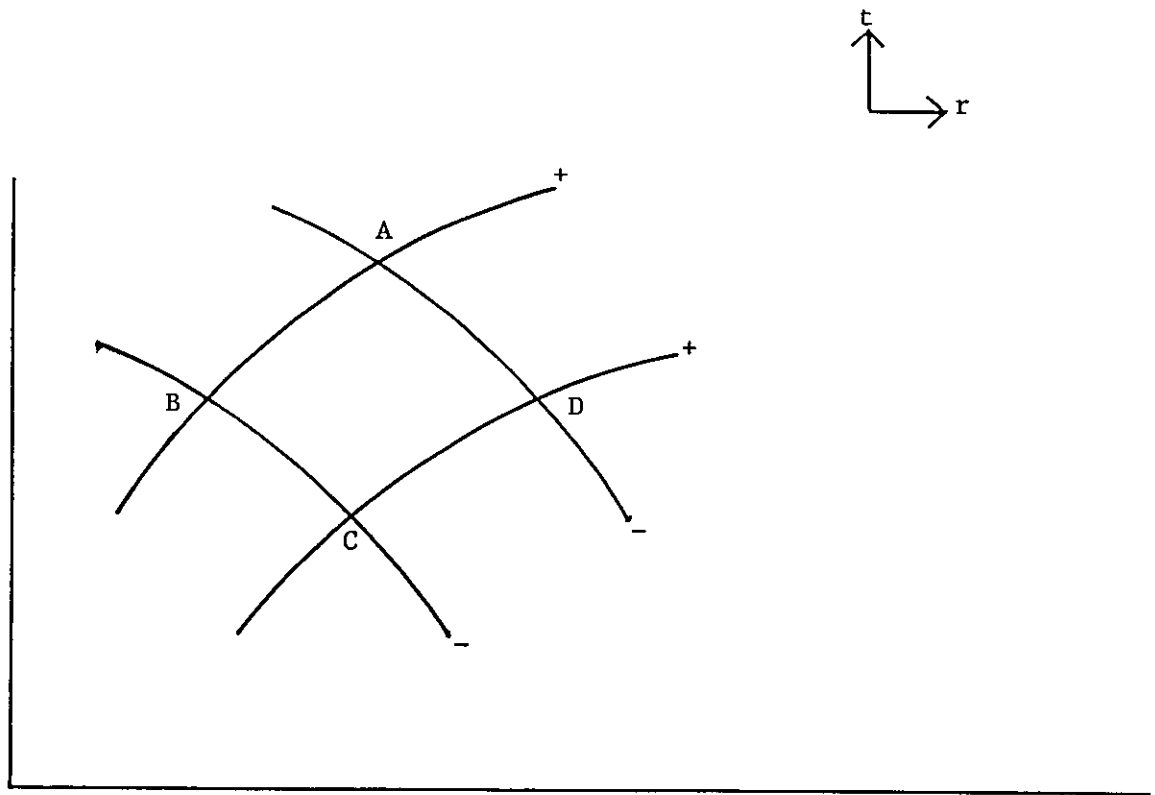


Figure 2.3

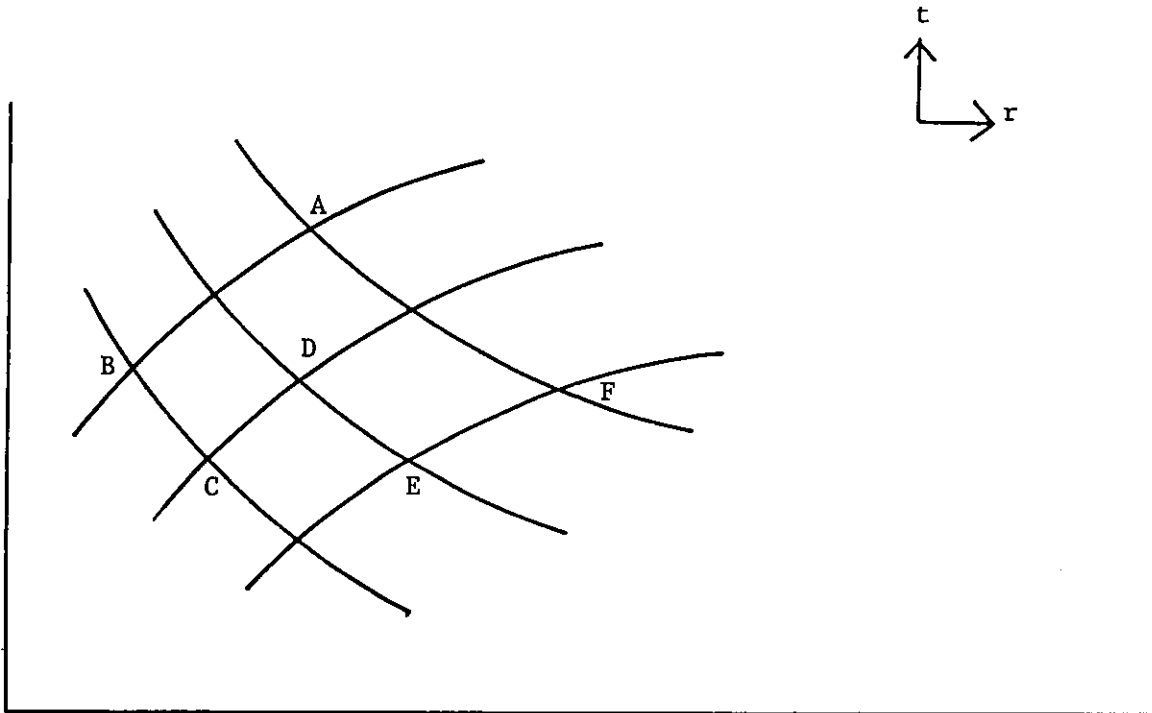


Figure 2.4

2.2.2 A Second Order Three-Point Method

The numerical MOC used for this modelling work is an implicit second order three-point method, which combines stability with relative simplicity of implementation. It is an improvement of the Greenspan method, which in turn was a development of the method outlined by Abbott (2).

To begin the explanation of this method, consider the positive and negative characteristics passing through a point $A(r_i, t_{j+1})$ at which u and c are to be evaluated. The point A lies on an (r, t) mesh, which, for the illustration of the numerical method, is assumed to be evenly spaced in both directions and where the values of u and c are known at time level j . The following, obvious but important points must now be made:

- 1) The slopes of the positive and negative characteristics are not necessarily of opposite sign, nor positive or negative in value, as is for instance shown in Fig. (2.5).
- 2) The characteristics are approximated to straight lines between two time levels.
- 3) If r_p is the point at which the positive characteristic which passes through A intersects the $(t = t_j)$ -line and r_n the negative counterpart, then in this implicit scheme, r_p and r_n do not necessarily lie within the nodes, r_{i+1} and r_{i-1} , adjacent to r_i , as is shown in Fig. (2.5). To ensure stability in explicit schemes, r_p and r_n must lie between r_{i+1}

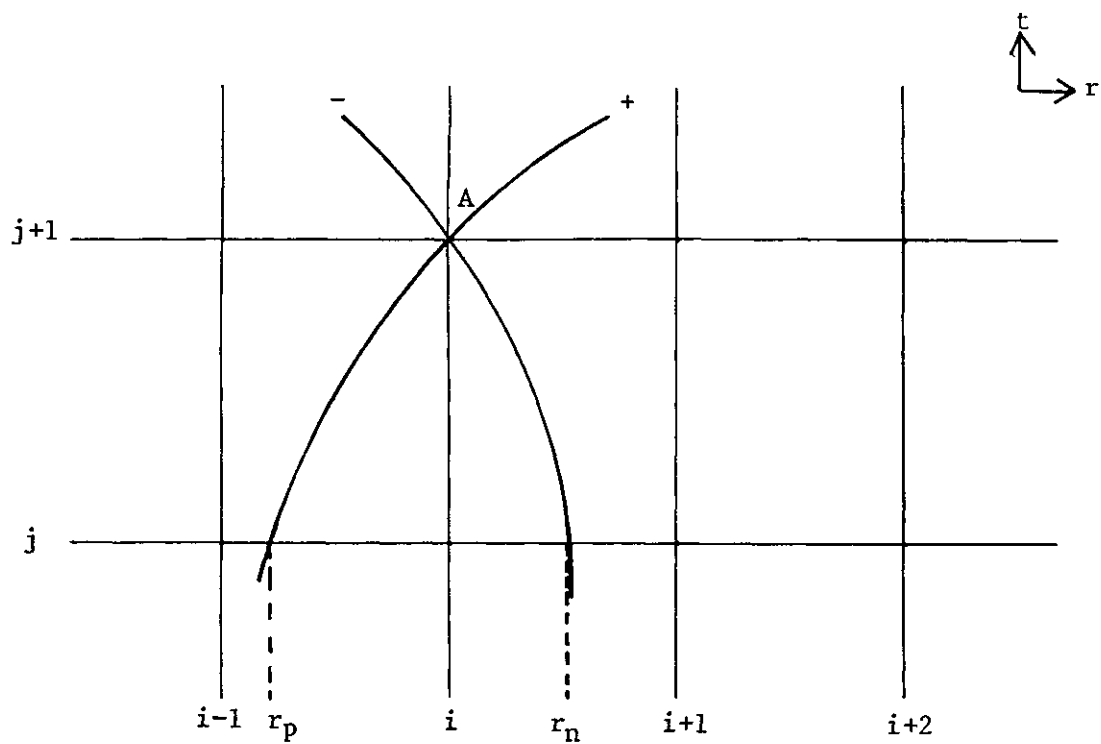


Figure 2.5

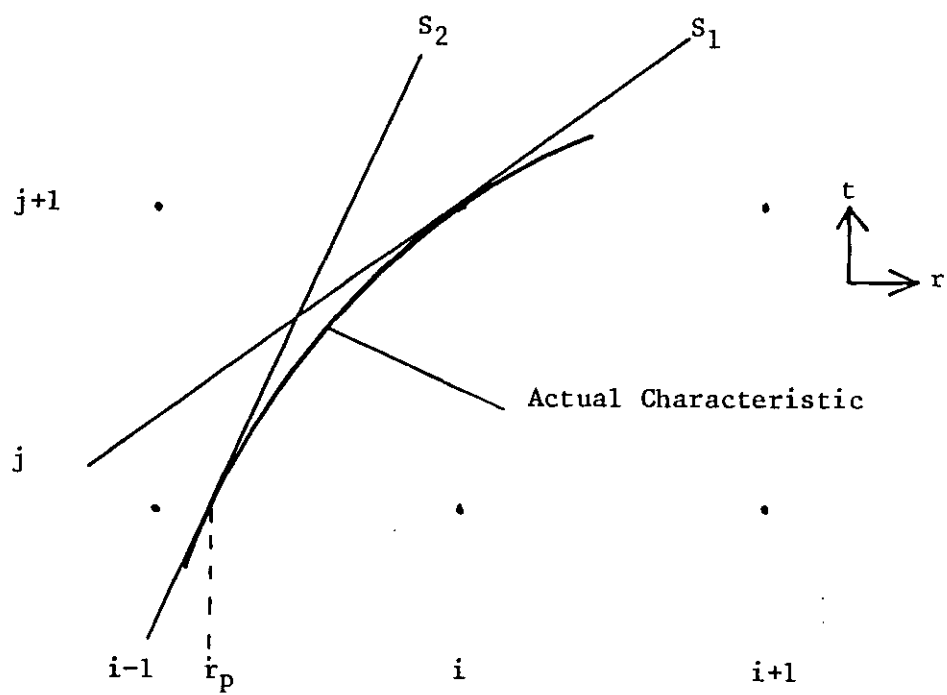


Figure 2.6

and r_{i-1} . In such cases the limit for the time step is given by the Courant-Friedrich-Levy criterion:

$$\delta t \leq \frac{\delta r}{|\bar{u}_j + \bar{c}_j|}$$

where $\bar{u}_j$ and $\bar{c}_j$ represent the maximum values of u and c at time step j . This represents a severe restriction on computation when dealing with the SWE's. With implicit schemes the use of far greater time steps is possible.

The points (r_p, t_j) and (r_i, t_{j+1}) , by definition, lie on the same positive characteristic and (r_n, t_j) and (r_i, t_{j+1}) on the same negative characteristic. The equations of these two paths are given by Eqs. (2.5). For a small value of δt the following approximations can be made:

$$\left(\frac{dr}{dt}\right)_+ \approx \frac{r_i - r_p}{\delta t} \tag{2.6}$$

$$\left(\frac{dr}{dt}\right)_- \approx \frac{r_i - r_n}{\delta t}$$

Thus, to calculate r_p and r_n , the slopes, $\frac{dr}{dt}$, need to be evaluated using $u \pm c$ respectively. The point at which this calculation is performed is of crucial importance as is explained next.

Consider the positive characteristic (Fig. (2.6)) between

(r_p, t_j) and (r_i, t_{j+1}) ; the description is similar for the corresponding negative characteristic. As was described previously for Abbott's three-point method, the slope is evaluated at (r_i, t_j) as an approximation, which would in effect be the value of the slope of the characteristic passing through (r_i, t_j) , not of the characteristic under consideration. This is however a reasonable approximation for regular families of characteristics, the accuracy limited of course by δt . In region V (Fig. (1.6)), however, the characteristics within the same family have a rapid change in slope with respect to time from characteristic to characteristic. Therefore this approximation serves only as a good initial value, allowing for the evaluation of u and c at r_p (and r_n) and hence at (r_i, t_{j+1}) by the process used by Abbott (described previously).

Even when iterating further to achieve a more accurate result, care must be taken when calculating the slope of the characteristic in question. In Abbott's three-point method this slope is evaluated at (r_i, t_j) and no further iteration is performed. Greenspan begins his method in the same manner and then iterates by re-evaluating this slope at (r_i, t_{j+1}) for improved accuracy (see Appendix (8)). As is shown on an exaggerated scale in Fig. (2.6), the slope S_1 , evaluated at (r_i, t_{j+1}) , will not necessarily lead to the optimum value of r_p .

There is a choice of three points at which this slope may be evaluated: (r_p, t_j) , (r_i, t_j) and (r_i, t_{j+1}) . The second point offers no possible re-evaluation of the slope. By experimenting numerically it was found that choosing the average value of the slopes at the other two points mentioned, will achieve the greatest accuracy and maximum rate of convergence. This method is

consistently superior to any other method of combination of the slopes at the three points.

The algorithm employed for this implicit second order three-point method can now be presented.

- 1) The characteristic gradients are approximated by:

$$\frac{dr}{dt} \approx u(r_i, t_j) \pm c(r_i, t_j) \quad (2.7)$$

- 2) r_p and r_n are evaluated from the finite difference Eqs. (2.6).

- 3) u and c are computed at these two points with the aid of Chebyshev (4) polynomial interpolation. The two node points on either side of r_p or r_n are used for this purpose. Four points are thus available for this interpolation unless there is a boundary immediately adjacent to r_p or r_n , in which case only three points are used.

- 4) The simultaneous Eqs. (2.4) give the values of u and c at the mesh point under consideration:

$$u(r_i, t_{j+1}) + 2c(r_i, t_{j+1}) = u_p + 2c_p \quad (2.8)$$

$$u(r_i, t_{j+1}) - 2c(r_i, t_{j+1}) = u_n - 2c_n$$

or

$$u(r_i, t_{j+1}) = \frac{1}{2} [u_p + 2c_p + u_n - 2c_n] \quad (2.9)$$

$$c(r_i, t_{j+1}) = \frac{1}{4} [u_p + 2c_p - u_n + 2c_n]$$

5. The slopes of the two characteristics in question are thus re-evaluated in the manner described above, using the values of the motion at points (r_i, t_{j+1}) and (r_p, t_j) or (r_n, t_j) :

$$\left(\frac{dr}{dt}\right)_+ = \frac{1}{2}[u(r_i, t_{j+1}) + c(r_i, t_{j+1}) + u_p + c_p] \quad (2.10)$$

$$\left(\frac{dr}{dt}\right)_- = \frac{1}{2}[u(r_i, t_{j+1}) - c(r_i, t_{j+1}) + u_n - c_n] \quad (2.11)$$

6. r_p and r_n are re-evaluated as in step (2), using Eqs. (2.6).

7. As for step (3).

8. $u(r_i, t_{j+1})$ and $c(r_i, t_{j+1})$ are re-computed iteratively by averaging the result of Eqs. (2.9) with the previous values of $u(r_i, t_{j+1})$ and $c(r_i, t_{j+1})$.

$$u^{k+1}(r_i, t_{j+1}) = \frac{1}{4}[u_p^{k+1} + 2c_p^{k+1} + 2u^k(r_i, t_{j+1}) + u_n^{k+1} - 2c_n^{k+1}] \quad (2.12)$$

$$c^{k+1}(r_i, t_{j+1}) = \frac{1}{8}[u_p^{k+1} + 2c_p^{k+1} + 4c^k(r_i, t_{j+1}) - u_n^{k+1} + 2c_n^{k+1}]$$

where k denotes the iteration number; $k > 1$.

The iterative procedure is continued until, $|u^{k+1}(r_i, t_{j+1}) - u^k(r_i, t_{j+1})|$ and $|c^{k+1}(r_i, t_{j+1}) - c^k(r_i, t_{j+1})|$ are sufficiently small.

This more accurate three-point method for evaluating the motion offers a stable solution and allows for the use of significantly larger time steps.

To solve the SWE's in any region of the collapse and ensuing flow process, an initial solution of u and c is required for a set of points spanning the mesh at a given time. Appropriate boundary conditions in each space-time region must also be supplied. These two conditions are given in Chapter (1).

Although the motion in region III and the leading characteristic in region IV (Fig. (1.6)) can be determined analytically for the plane model, it was thought useful to solve the entire problem for this case, using the above numerical MOC, for the following two important reasons.

- 1) The analytical solutions for this plane model would serve as a test for the accuracy of the numerical method employed.
- 2) Rather than re-writing a completely new and separate numerical routine for the more complicated axisymmetric model a reliable computer program written first for the relatively simple plane model, could be modified to allow for, amongst other differences, the integral term in the characteristic equations governing the cylindrical case.

The numerical methods presented above were illustrated using the plane model Eqs. (2.4) and (2.5). The evaluation of the additional, integral term in (Eqs. (2.2) governing the axisymmetric model will be outlined in the next section.

2.3 Additional Numerical Methods for the Axisymmetric Model

A radial-integral term is present in the axisymmetric characteristic equations. These are defined by:

$$u + 2c + \int \frac{uc}{r} dt = k_1 \quad (2.13)$$

$$u - 2c - \int \frac{uc}{r} dt = k_2$$

where k_1 and k_2 are constant for each individual characteristic curve. Eqs. (2.13) are used in steps (4) and (8) of the algorithm described in the previous section to model the axisymmetric case. Hence by applying Eqs. (2.13) to step (4), the equations for the axisymmetric model, corresponding to Eqs. (2.8) of the plane model, are:

$$u(r_i, t_{j+1}) + 2c(r_i, t_{j+1}) + \int_{(i,j+1)}^r \frac{uc}{r} dt = u_p + 2c_p + \int_{(p,j)}^r \frac{uc}{r} dt \quad (2.14)$$

$$u(r_i, t_{j+1}) - 2c(r_i, t_{j+1}) - \int_{(i,j+1)}^r \frac{uc}{r} dt = u_n - 2c_n - \int_{(n,j)}^r \frac{uc}{r} dt$$

or (cf. Eqs. (2.9)), by solving the above simultaneous equations:

$$u(r_i, t_{j+1}) = \frac{1}{2}[u_p + 2c_p + u_n - 2c_n + PI + NI] \quad (2.15)$$

$$c(r_i, t_{j+1}) = \frac{1}{4}[u_p + 2c_p - u_n + 2c_n + PI + NI]$$

where,

$$PI = \int_{(p,j)} \frac{uc}{r} dt - \int_{(i,j+1)} \frac{uc}{r} dt \quad (2.16)$$

$$NI = \int_{(i,j+1)} \frac{uc}{r} dt - \int_{(n,j)} \frac{uc}{r} dt \quad (2.17)$$

Using the general equations defining the characteristic curves, Eqs. (2.5), the two above equations become:

$$PI = \int_p \frac{uc}{u+c} \frac{dr}{r} - \int_i \frac{uc}{u+c} \frac{dr}{r} = \int_{r_p}^{r_i} \frac{uc}{u+c} \frac{dr}{r} \quad (2.18)$$

$$NI = - \int_{r_n}^{r_i} \frac{uc}{u-c} \frac{dr}{r} \quad (2.19)$$

The solution of these integrals is obtained numerically. The integral is evaluated at four points in the range r_p to r_i (or r_n to r_i) using the Chebyshev polynomials (which are already defined to perform the necessary interpolations in the numerical MOC). These four values are then integrated numerically to solve for PI and NI. The solution of these definite integrals in region IV, at $r = 0$, where a singularity arises, is shown in section (1.3.5).

A further problem which arises in region IV, when considering the axisymmetric model, concerns the implementation of the boundary conditions. For the plane model the leading characteristic in region IV, which also defines the boundary between regions III and

IV, is defined analytically. For the axisymmetric model, however, an iterative solution is required starting with the approximation for the leading characteristic offered by Stanyukovich (26) and presented in section (1.3.5). The position of the reflected front in region IV, R_r , is defined by Eq. (1.3.37) for a short period between, $t = T_b$ and $t = T_b + \Delta t$, where T_b represents the commencement of the motion in region IV. The values of u and c between $r = 0$ and $r = R_r$ at $t = T_b + \Delta t$ are given by Eqs. (1.3.38). The time step, $(t_{j+1} - t_j)$, used in the numerical MOC throughout regions III and IV, is greater than Δt . To evaluate the motion at the first full time step into region IV ($t_1 = T_b + t_{j+1} - t_j$), given the values of u , c and R_r at an initial and smaller time $t_0 = T_b + \Delta t$ the following algorithm is used:

- 1) $R_r(t_1)$ is computed by linearly projecting the positive characteristic which passes through $(R_r(t_0), t_0)$ to time level t_1 .
- 2) $u(r, t_1)$ and $c(r, t_1)$ are evaluated at all mesh points between $r = 0$ and $r = R_r(t_1)$ by the numerical MOC in the usual manner (section (2.2.2)), taking the projected characteristic mentioned in step (1) as the boundary between regions III and IV. The value of $c(r=0, t_1)$ is unknown and therefore an iterative solution is required assuming that the values of c at the centreline and at the mesh point adjacent to the centreline are equal.
3. A more accurate value of the slope of the leading

characteristic is computed by averaging the new value of the slope calculated at $(R_r(t_1), t_1)$ with the old value calculated at $(R_r(t_0), t_0)$.

4. A new value of $R_r(t_1)$ can thus be obtained allowing for further iterations in the manner described above until successive values of $R_r(t_1)$ differ little.

The calculation procedure is identical for all further time steps in region ^{IV} except that the computations take place over the larger time step, $(t_{j+1} - t_j)$.

2.4 Implementation of the Numerical Method of Characteristics in Region V

For a vertical containment wall the height of the bore is assumed to vary with time only. This allows for a simplification of solution so that the numerical MOC need not be applied to model the flow in region V. Instead a different numerical method is implemented, which is described in section (2.5).

The MOC is however applied to region V,

1. as a solution for the motion in the case of inclined containment walls, where the level bore assumption does not necessarily hold;
2. as a verification for the assumption of a level bore in the case of vertical walls.

The description of the numerical MOC in region V, given

below, uses the case of a vertical wall as an example to simplify the explanation. This will serve as a basis for understanding the application to inclined containment walls, which follows in section (2.6).

Consider a developed shock for which the height and velocity are known at a time t_j over the entire span of the bore, from $R_s(t_j)$ to R_w . This is illustrated in Fig. (2.7) as a shock of simple, non-linear, but otherwise arbitrary, profile, $h_2(r, t_j)$. This type of outline does of course include the level bore profile assumed for vertical walls. In Fig. (2.7), the profile of the liquid height in front of the shock, $h_1(r, t)$ is defined in region III. The liquid height profile behind the shock a time, t_{j+1} , (where the motion is to be evaluated) is also illustrated by an equally arbitrary profile. For the purpose of example the velocity profiles behind the shock would have served equally well.

In Fig. (2.8) the characteristic diagram corresponding to Fig. (2.7) has been drawn. The time step, $(t_{j+1} - t_j)$, needs to be small enough, so that all mesh points at which $u(t_{j+1})$ and $c(t_{j+1})$ are to be evaluated, lie within the range Δr . As is shown in Fig. (2.8), Δr is defined by the positive characteristic through $(R_s(t_j), t_j)$ and the negative characteristic through (R_w, t_j) . By restricting the evaluation of $u(t_{j+1})$ and $c(t_{j+1})$ in this range, problems of interpolation along the wall and the shock are eliminated. The slopes of these two characteristics are given by:

$$Ch_+ : u_2(R_s(t_j), t_j) + c_2(R_s(t_j), t_j)$$

$$Ch_- : -c_2(R_w, t_j)$$

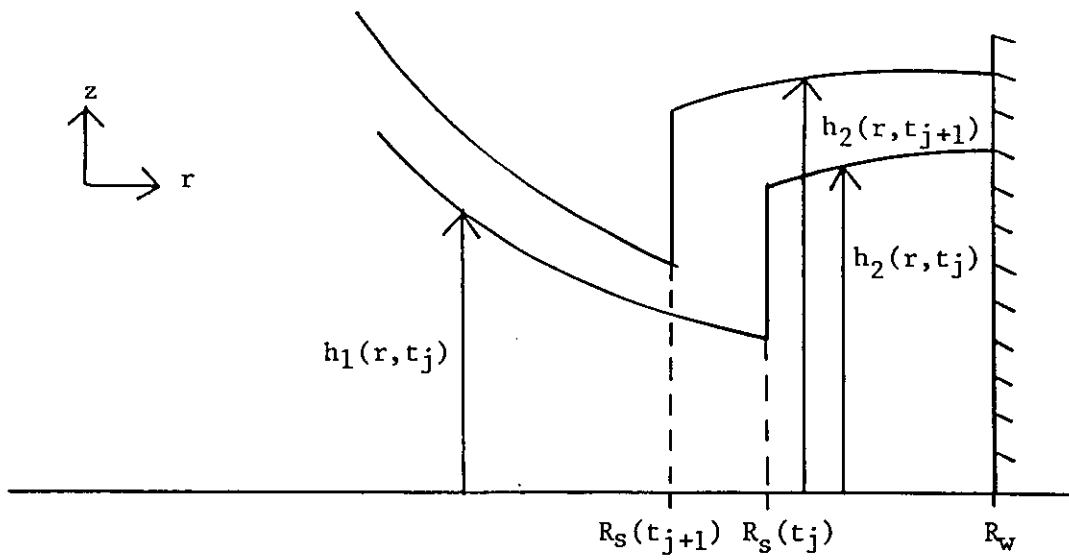


Figure 2.7

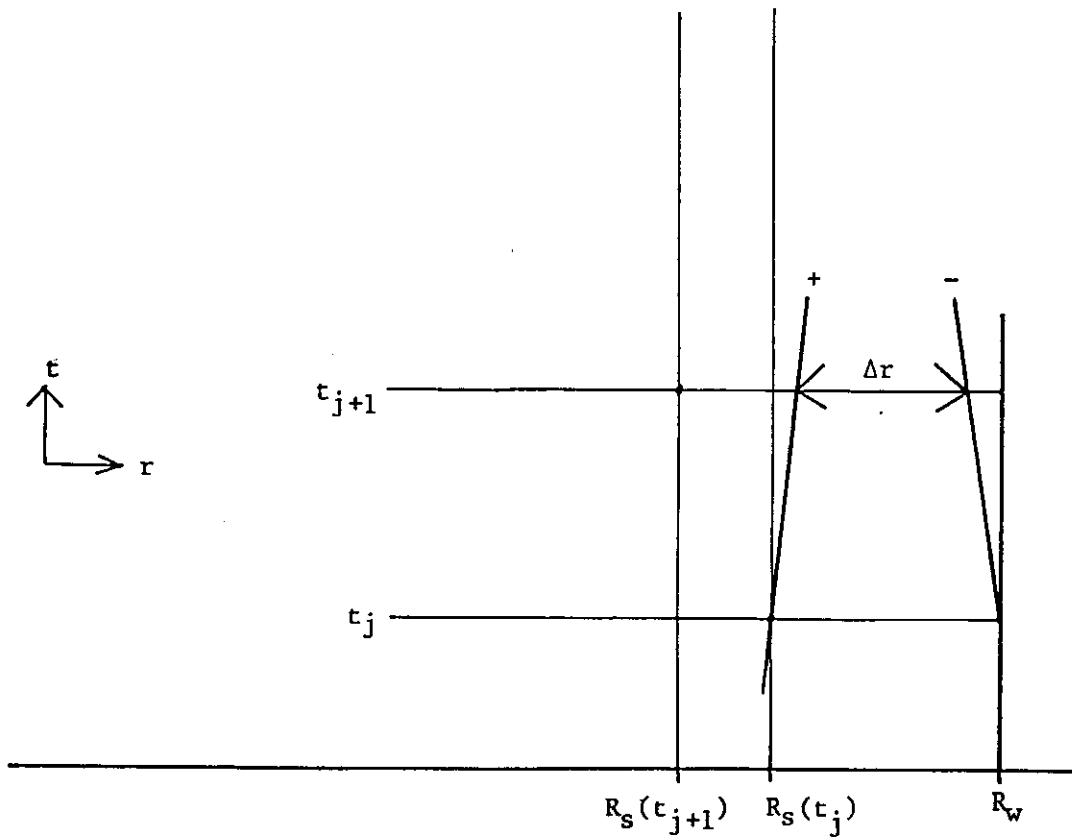


Figure 2.8

After evaluation of the motion at all points within the range Δr , by the three-point numerical MOC (section (2.2.2)), c can be evaluated at $(R_s(t_{j+1}), t_{j+1})$ and (R_w, t_{j+1}) by extrapolation. The liquid velocity, u_2 , is already defined at these two points by the shock condition, Eq. (1.1.30) and by the boundary condition at the wall respectively.

The time step, $(t_{j+1}-t_j)$, is selected so that the distance for extrapolation is less than the radial mesh spacing in the range, Δr . This criterion results in very small time steps at the start of the motion in region V, which gradually increase as the shock propagates away from the containment wall. The crucial calculation in this numerical method is the evaluation of the shock position, R_s , at $t = t_{j+1}$. The following nested iterative algorithm is used:

- 1) A suitable time step is computed according to the criterion discussed above.
- 2) the shock velocity, U , is evaluated using shock condition, Eq. (1.3.29) with u_1 , c_1 and c_2 evaluated at $(R_s(t_j), t_j)$.
- 3) The new shock position, $R_s(t_{j+1})$ is computed using forward differencing:

$$R_s(t_{j+1}) = R_s(t_j) + (t_{j+1}-t_j)U$$

- 4) The motion is evaluated at time level, t_{j+1} , at all mesh points in the range Δr , using the numerical MOC (section 2.2.2)).

5) $c_2(R_s(t_{j+1}), t_{j+1})$ is calculated by extrapolating the Chebyshev polynomial defining $c_2(r, t_{j+1})$ in the range Δr which was evaluated in step (4). $u_2(R_s(t_{j+1}), t_{j+1})$ is given by Eq. (1.3.30).

6) The slope, S_n , of the negative characteristic passing through $(R_s(t_{j+1}), t_{j+1})$ is computed at this point using the values of c_2 and u_2 derived in step (5):

$$S_n(t_{j+1}) = u_2(R_s(t_{j+1}), t_{j+1}) - c_2(R_s(t_{j+1}), t_{j+1})$$

7) The point, r_n , at which the above mentioned negative characteristic intersects with time level, t_j , is calculated from the value of S_n :

$$r_n = R_s(t_{j+1}) - (t_{j+1} - t_j) S_n(t_{j+1})$$

8) The Chebyshev polynomials defining $u(t_j)$ and $c(t_j)$ in the range Δr are used to evaluate u_n and c_n by interpolation.

9) The slope of the negative characteristic passing through the point r_n will be defined at (r_n, t_j) , by

$$S_n(t_j) = u_n - c_n$$

10) The average of $S_n(t_{j+1})$ and $S_n(t_j)$ provides a new value of the slope $S_n(t_{j+1})$ of the characteristic passing through the point, $(R_s(t_{j+1}), t_{j+1})$.

- 11) Steps (7) to (10) are repeated until successive values of r_n are sufficiently close.
- 12) The shock velocity, U , is re-evaluated, using u_1 , c_1 , c_2 at $(R_s(t_{j+1}), t_{j+1})$.
- 13) Steps (3) to (12) are repeated, reselecting the time step if necessary, until the shock position, $R_s(t_{j+1})$ has converged.

This method's accuracy can be verified by comparing the values of $u_2(R_s(t_{j+1}), t_{j+1})$ determined by the shock condition and by extrapolation. It is found to be very accurate, admittedly at the expense of extremely high computer costs.

2.5 The Solution in Region V for Vertical Containment Walls

As discussed in section (1.3.4) it may be assumed that the bore propagating away from the wall in region V is of constant height; i.e. the liquid surface height, h_2 , varies with time only. The equations defining the motion in region V for the level bore assumption, derived in Appendix (6) are:

$$\frac{dR_s}{dt} = U = u_1 - c_2 \left[\frac{M^2 + 1}{2} \right]^{\frac{1}{2}} \quad (2.20)$$

$$u_2 = u_1 - c_1 (M^2 - 1) \left[\frac{1}{2} \left(1 + \frac{1}{M^2} \right) \right]^{\frac{1}{2}} \quad (2.21)$$

$$c_2 = c_2(R_s, t, c_1) \quad (2.22)$$

where $M = \frac{c_2}{c_1}$

Suffices 1 and 2 denote the motion immediately in front of and behind the shock respectively; u_1 , u_2 , c_1 and c_2 are functions of R_s and t . u_1 and c_1 are defined numerically in region III.

In essence the problem is to determine the progression of the shock position in time by solving the non-linear ordinary differential Eq. (2.20). Once this is established the liquid velocity and height can be evaluated at R_s from Eqs. (2.21) and (2.22). Since the liquid height is assumed constant between R_s and R_w , the liquid velocity will vary linearly with r from zero at the containment wall to u_2 at the position of the shock.

From Eq. (2.20):

$$R_s(t_{j+1}) - R_s(t_j) - \int_{t_j}^{t_{j+1}} U(R_s, t) dt = 0 \quad (2.24)$$

The initial value of $R_s(t_j)$ and $c_2(t_j)$ for $j = 1$ is defined at a short time after impact, $t_1 = T_w + \delta t$, by Eqs. (A6.14) and (A6.13), given in Appendix (6). The zero of Eq. (2.24) is found using a reverse communication bisection method (6). The integrand is calculated iteratively at four points over the range t_j to t_{j+1} at $R_s = (R_s(t_j) + R_s(t_{j+1}))/2$. The time step used in region V is much smaller than in region III. Therefore, the values of u_1 and c_1 required at the four points where the integrand is computed, must be evaluated by interpolation both in displacement and in time. The Chebyshev polynomials provide an accurate means for interpolating between displacement mesh points at the two adjacent time levels in region III. Similar Chebyshev polynomials as a function of time, are created for $c_1(r)$ and $u_1(r)$. These are required to uphold the consistency of accuracy of interpolation

between the larger time levels of region III. Therefore, when evaluating the integrand, u_1 and c_1 are also computed by interpolating along the vertical lines through the adjacent displacement mesh points. This is summarized in Fig. (2.9).

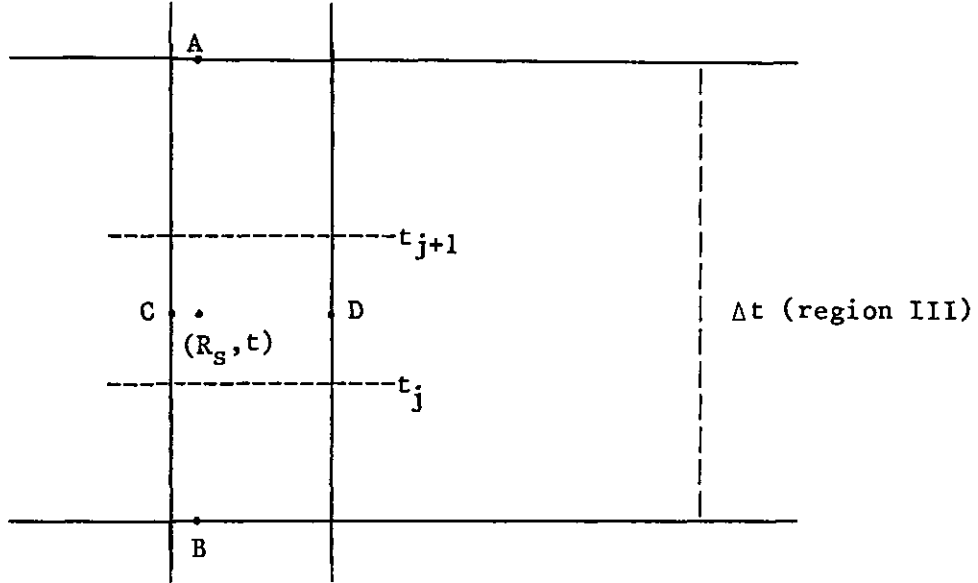


Figure (2.9) Not to scale. $\Delta t \gg (t_{j+1} - t_j)$

$u_1(R_s, t)$ and $c_1(R_s, t)$ are obtained by interpolating to find u and c at A, B, C and D and then again to find u and c at (R_s, t) .

Within the bisection routine four values of the integrand are then numerically integrated from t_j to t_{j+1} , thus arriving at a value for $R_s(t_{j+1})$. This process is repeated in time until the propagating bore reaches $r = 0$.

In the early stages of the motion the height of the bore changes very rapidly, while the shock position varies relatively slowly in time; hence the need for a much smaller time step in region V. From numerical experimentation it was found that for the accuracy in region V to be consistent with that in region III, the

time step in region V is required to be 100 times smaller than for computation in region III. The mesh spacing in the radial direction is also reduced considerably near the wall to overcome the above mentioned problem. As the bore propagates further away from the wall the restriction on the mesh spacing and time step size can be relaxed.

This numerical method, assuming a level shock, requires only a small fraction of the computer time used when applying the numerical MOC in region V.

2.6 Inclined Containment Walls

As was mentioned in section (1.3.6), the presence of an inclined barrier imposes a vertical boundary condition on the flow, leading to a variation in the horizontal velocity as a function of the shock height in the liquid. An accurate solution for the motion in this case would require a completely new approach to the theoretical model. Having established the second order effect of the angle of inclination on the liquid bore height, it was decided to use shallow water theory as a reasonable approximation for modelling the flow behind the shock. The necessary modifications to the initial conditions, required to start the routine in region V are given in Appendix (7). Henceforth the motion can be computed by the numerical MOC as described in section (2.4). The only further approximation necessary for the evaluation of the shock motion in the case of inclined containment walls, concerns the liquid velocity near the wall where, by virtue of the vertical boundary conditions, this cannot be independent of height. The

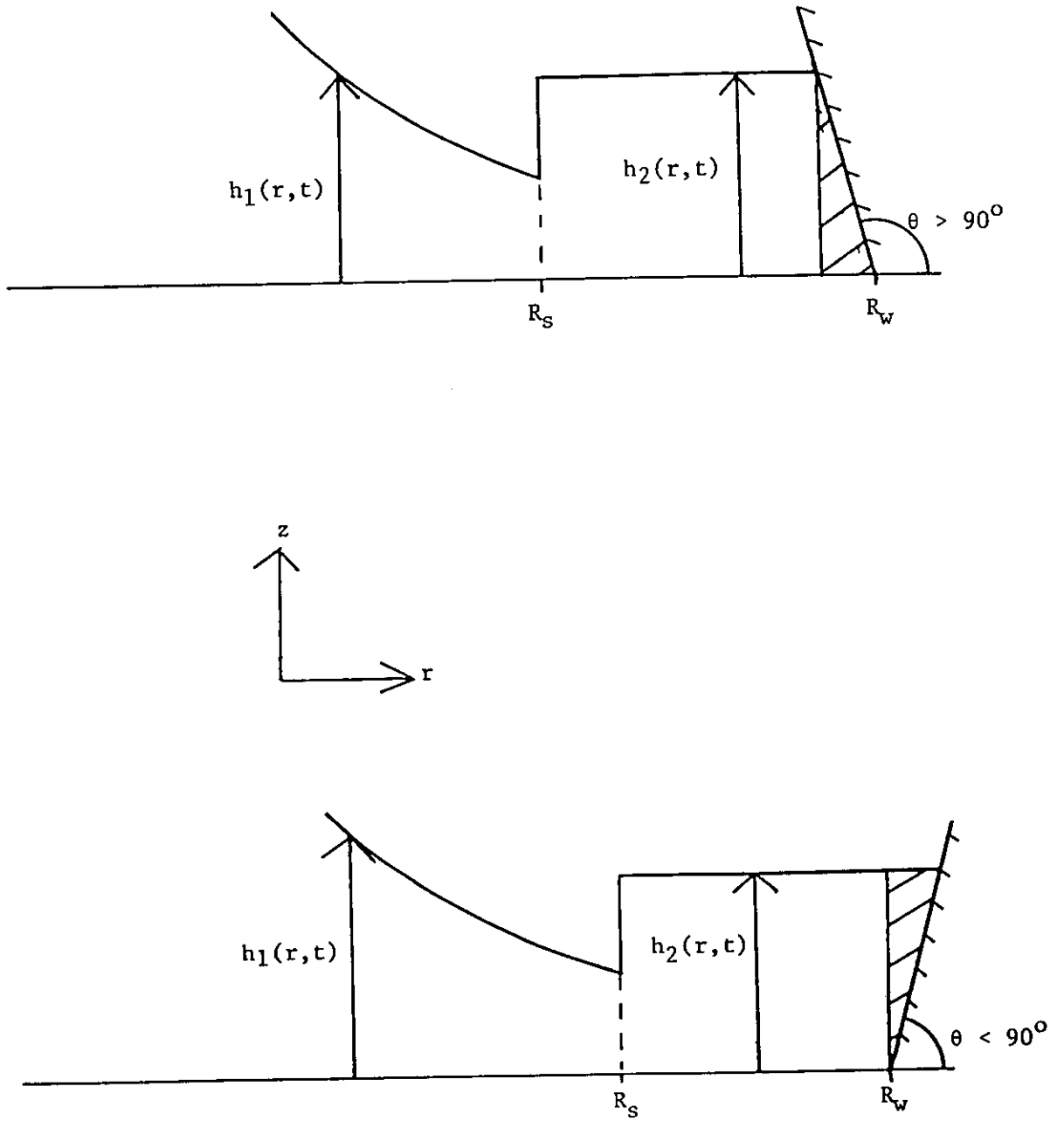


Figure 2.10 The liquid velocity in the shaded 'triangular section' is assumed to be constant and equal to an average value there-in.

velocity near the wall (see Fig. (2.10) is averaged within the 'triangular section' behind the shock, according to Eq. (A7.4) derived in Appendix (7). The liquid velocity in the 'rectangular section' of the hydraulic jump is calculated in the usual manner from the numerical MOC. Where a characteristic projects to a point within the 'triangular section', the averaged velocity is assumed. The value of the liquid velocity at any point within the 'triangular section' is small in relation to velocities elsewhere behind the shock, irrespective of whether the above velocity averaging-approximation is made or not, since those points are near to the wall. Hence, as the calculations for the liquid height and velocity are additive in the numerical MOC (Eqs. (2.13) the error on these values will be small.

2.7 Overflow and Continuity

When considering a bund of limited height and consequently, overflow, the conditions determining the nature of the returning shock in region V will differ. Mass and momentum must be conserved after impact and overflow. When using the numerical MOC in region V for inclined bunds, the boundary conditions Eqs. (1.3.48) will ensure continuity. However, the numerical routine used in the case of vertical walls, where a level of shock is assumed, requires modification to satisfy conservation of mass. Even with overflow occurring it can be assumed that a level shock persists (7). Eqs. (1.3.50) and (2.22) are combined to define $c_2(R_g, t)$ in the numerical routine presented in section (2.5) for the case of overflow:

$$c_2(R_s, t) = gh_w(R_s, t) - \frac{1}{\pi(R_w^2 - R_s^2)} \int_{t_j}^{t_{j+1}} [c_w^2(t) - c_b^2]^{3/2} dt \quad (2.25)$$

where $h_w(=c_w^2/g)$ represents the liquid height in the event of no overflow and c_b is the transformed bund height.

For an accurate evaluation of the liquid height at the bund in the event of overflow, a further iterative process is involved using Eq. (2.25).

To test for continuity and thus for the above iterative calculation of $h_2(R_w, t)$, the total quantity of liquid that has overflowed at any time can be compared with the liquid remaining within the banded enclosure to ensure that mass is conserved in the numerical routine.

2.8 Conservation of Momentum

A useful criterion for estimating the accuracy of the computation, of the motion in region V, is obtained by considering the conservation of momentum of the liquid in transition from region III to region V. For the vertical wall algorithm this supplies a verification of the validity of the level shock assumption. When dealing with inclined bunds, the error incurred by assuming a constant, average velocity near to the containment wall, can also be estimated.

According to the shallow water theory, momentum is conserved after impact of the surging liquid (region III) against the containment wall, where a hydraulic jump is formed and reflected back (region V) towards $r = 0$. Consider Fig. (2.11) where the liquid height profile is illustrated for the two cases of the

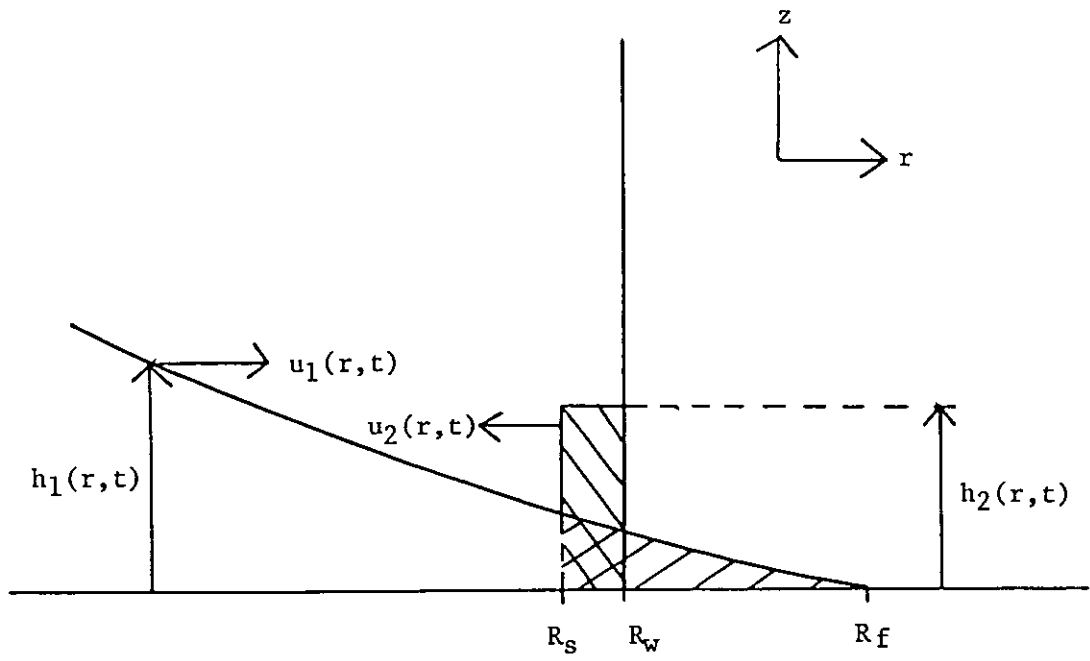


Figure 2.11

presence and absence of a containment wall. The total momentum of the unhindered liquid between R_s and R_f should equal the total momentum of the liquid between R_s and R_w in the case of a barrier of unlimited height restricting the forward flow. Hence,

$$\int_{R_s}^{R_f} 2\pi r \rho u_1(r,t) h_1(r,t) dr = \int_{R_s}^{R_w} 2\pi r \rho u_2(r,t) h_2(r,t) dr \quad (2.26)$$

Both sides of the equations are calculated numerically using the Chebyshev polynomials describing the motion in each region of flow. Eq. (2.40) can be simplified for the plane model where $u_1(r,t)$ and $h_1(r,t)$ have analytical solutions. For the case of a vertical wall the above equality reduces to:

$$\frac{t}{27g} \left[2C_0 - \frac{(R_s-A)^3}{t} \right] \left[C_0 + \frac{(R_s-A)}{t} \right] = \frac{1}{2} u_2(R_s)(R_w-R_s)h_2 \quad (2.27)$$

2.9 Numerical Solution in Region VI

A singularity occurs at the centreline when modelling the converging hydraulic jump in transition from region V to region VI. Butler's (3) numerical method mentioned previously is used to overcome the mathematical singularity which occurs at $r = 0$. The liquid height and velocity distribution behind the converging shock along with the shock velocity need to be supplied to this routine for a given small distance from the point of singularity ($r = 0$). These input data are readily available from the results obtained in region V.

The flow in region VI is relatively unimportant within the context of this project and was computed to complete the evaluation of the motion resulting from a CTF. Details of the numerical method are thus given in Appendix (9).

CHAPTER 3

Computational Techniques

3.1 Introduction

Of particular interest in the main computer program developed in this study are the numerical methods and computational techniques used. The program was first written for the plane model to solve the flow in regions III, IV and V and then adapted to evaluate the flow in these regions for the axisymmetric model. It incorporates options for inclined containment walls and overflow. Two different sub-programs are used to evaluate the motion in region V: one is written for the level shock assumption and the other uses the numerical MOC. Data generated by the main program are used in separate programs to evaluate the pressure and load on a containment wall, and also as input to Butler's numerical method for evaluating the motion in region VI. For computational details of this method see reference (3).

Several graphics-programs were developed to present the results in a more convenient form but these are not discussed here as they are purely functional. The overview of the main program is shown in Fig. (3.1). The program listings with comment statements are given in Appendix (10).

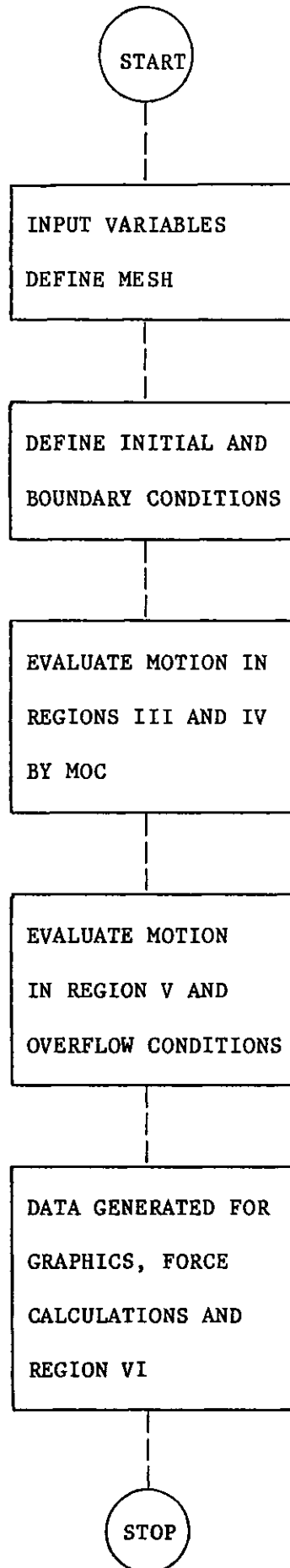


Figure 3.1

3.2 Input Variables and Definition of Mesh

The following parameters are required as input variables for the program:

- A - Tank radius
- BHR - Radius to height ratio of the tank (for scaling
 output data)
- DRI - largest displacement step size
- DT - time step
- RW - Distance from centre of tank to containment
 wall
- THETA - Angle of inclination of containment wall
- RFR, DFR - Parameters determining the variable mesh layout

The above input variables are sufficient for producing a complete set of results for any situation - experimental or industrial - since the height of the liquid at any point in space-time is always linearly proportional to the original column height. Similarly the velocity of the liquid is always linearly proportional to $(gH_0)^{\frac{1}{2}}$. This can be readily concluded from the characteristic equations. (Eqs. (2.2)).

A variable mesh is introduced, to limit round off errors, when calculating both the reflection of the depletion wave from the centreline (region IV) and the shock formation and propagation (region V). In region IV it was found that to ensure the accuracy of the iterative procedure described in section (2.3), the mesh had to be refined by a factor 10 near the centreline. Due to the wide

discrepancy in the rates of change of shock position and liquid bore height (especially in the moments immediately subsequent to impact) a more refined mesh is required to reduce interpolation errors. These errors arise when computing the values of u_1 and c_1 in front of the shock which are defined in region III. Since the initial shock propagation away from the wall is slow, the accurate determination of u_1 and c_1 is crucial to the evaluation of the shock motion. This can only be achieved with a mesh far less coarse than is sufficient for the computation of the motion in region III. Refining the mesh over the entire domain of calculation in order to keep this constant would result in much superfluous and costly computation. It was thus decided to incorporate a variable mesh, up to 100 times finer near the containment wall than in the greater part of the computational domain.

The mesh covers half a radial cross section starting at the centre of the tank ($r = 0$) and ending at the containment wall, ($r = R_w$). Calculations proceed firstly from left to right ($r = 0$ to $r = R_w$) and then in time. The mesh extends in constant time steps (except in region V as is explained later) until the hydraulic jump converges on itself at $r = 0$.

3.3 Implementation of the Numerical Method of Characteristics

The values of u and c at the first time step in region III are provided by the initial conditions, Eqs. (1.3.28). Evaluation of the motion at the second time step differs from the calculation at all subsequent time steps. No interpolation is necessary as u

and c are defined in terms of continuous functions at the previous time step (Eqs. (1.3.28)), rather than at a finite number of mesh points. Hence direct evaluation rather than interpolation can be used to save computational time. Since the function initialising the motion at the first time step is non-linear, a secant-type iteration method is used to find the points r_p and r_n (see section (2.2.2.)) at which, respectively, the positive and negative characteristics intersect the previous time level. A NAG FORTRAN library routine (28) is used which locates the zero of a continuous non-linear function by means of a continuation method based on a secant iteration.

At all following time steps, calculations involve interpolation with discretized functions of u and c . When a complete 'sweep' at one time level has been performed and the values of u and c have been established at all mesh points at that time level, a Chebyshev (4) polynomial fit is made to both discretized functions. This facilitates interpolation and presents u and c in a convenient form for integration purposes. Chebyshev polynomials were found to give a good fit to u and c over the range of mesh points concerned, thus resulting in low errors of interpolation.

The integrand in the characteristic equations,

$$\frac{uc}{u+c} \frac{1}{r}$$

is evaluated at four points over the interval concerned (see section (2.3)) using the established Chebyshev functions for u and

c. A finite difference method developed by Gill and Miller [1972, (12)] is used to integrate over the interval; it is particularly accurate when dealing with an unequally spaced mesh.

The optimization of the characteristic slope evaluation is performed using another NAG library routine which locates the root of a function by a combination of linear interpolation and extrapolation with a bisection method (6). It uses reverse communication to evaluate the function, the function being the summed average of the two expressions defining the slope of the characteristic at each time level (see Eqs. (2.10) and (2.11)). This method was found to guarantee convergence and is very efficient with respect to computational cost. It is central to the numerical MOC and because of its repetitive use at every mesh point it can meet the requirements of low computational time without loss in accuracy.

The motion in region III is evaluated far beyond the time of impact against the containment wall since u_1 and c_1 (the conditions in front of the shock) need to be defined for the computation of the hydraulic jump propagation until the shock reaches the centreline, at $t = T_c$. Evaluating u_1 and c_1 within the boundaries $r = 0$ and $r = R_w$ up to, $t = T_c$ ensures a wide array of values of the motion in front of the jump. This is necessary as the shock calculations are iterative and require a sufficiently wide range of u_1 and c_1 about $R_s(t)$.

The flowchart for the evaluation of the motion in region III by the numerical MOC is shown in Fig. (3.2).

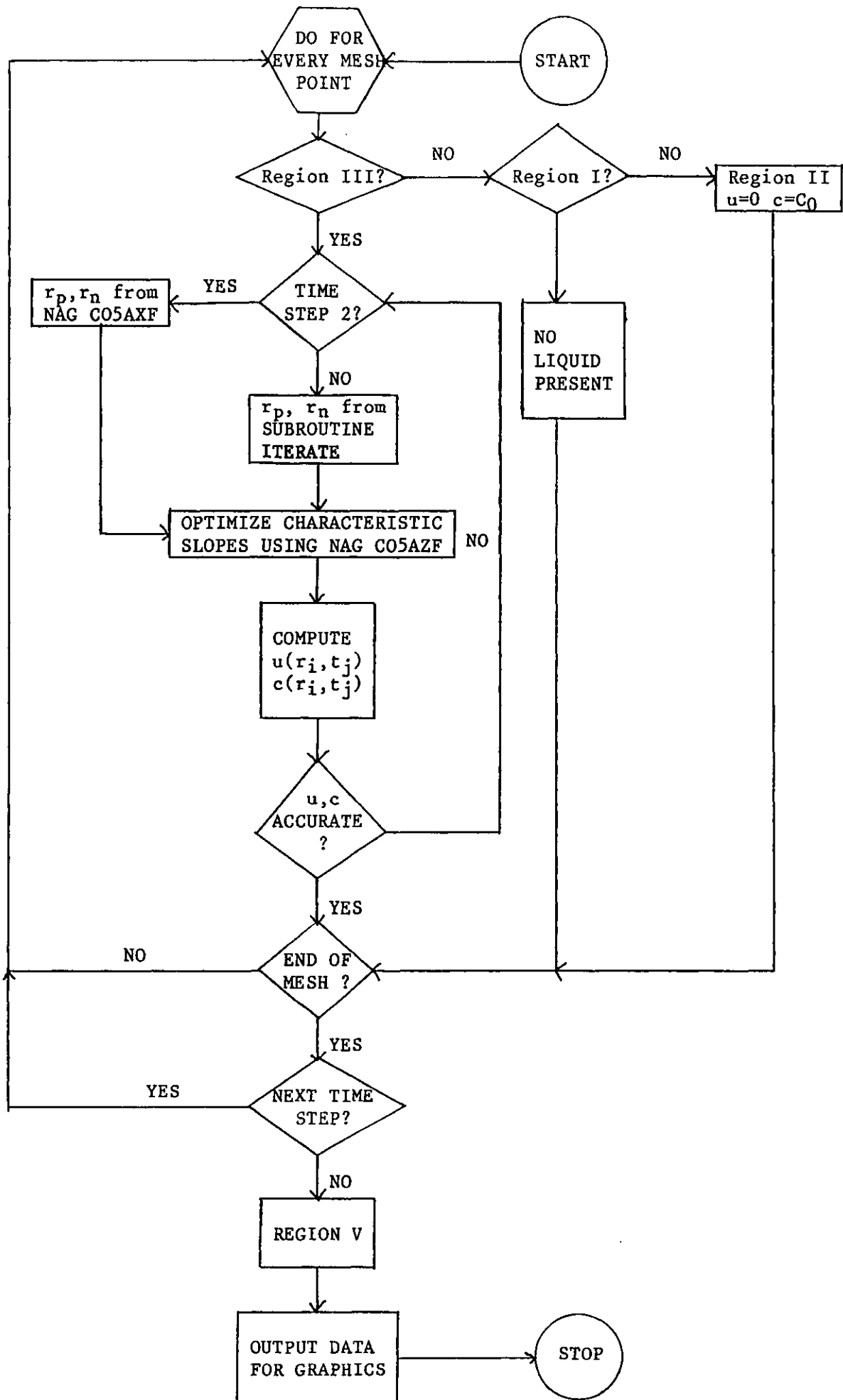


Figure 3.2

3.4 Region V

When considering flow at vertical containment walls, a level shock is assumed. The solution takes the form of an integration in time of the non-linear shock equation presented previously (Eq. 2.24). Because of the large difference between the rates of change of the shock height and the shock position, immediately after impact, it was found necessary to use a much smaller time step than that used for calculations in region III and IV. Eventually a time step size of up to one hundredth of the original one had to be used to ensure stability of the routine and a consistent accuracy of solution.

Recalling Eq. (2.24),

$$R_s(t_{j+1}) - R_s(t_j) - \int_{t_j}^{t_{j+1}} U(R_s, t) dt = 0 \quad (3.1)$$

it is obvious that the shock velocity, U , is a function of the shock height, the shock position and time and hence a function of the velocity and height of the incoming liquid. The shock height can be computed, as a function of shock position and time, purely on the basis of the geometry of the type of containment wall assumed, since the bore is taken as level. After evaluation of the shock velocity, U , at four points over the range t_j to t_{j+1} the integral of Eq. (3.1) is determined by a NAG quadrature routine (12). The root of Eq. (3.1) is then found by using the same bisection method (6) which is used to find the optimum value of the slope in the numerical MOC.

The values of the motion flowing into the shock are

calculated by interpolation between mesh points and between time levels since u and c in region III are only defined at every larger time step used in the computations of that region. To maintain a consistent accuracy of shock calculation it was found necessary to incorporate Chebyshev polynomials in time along lines of constant radial displacement in the mesh (see section (2.5)).

Per unit area of mesh covered the evaluation of the motion in region V is much more expensive, in terms of computer time than in regions III or IV. This is almost entirely due to the finer mesh that is necessary, especially at early times, in region V. The flowchart for the level shock routine is shown in Fig. (3.3).

When dealing with inclined containment walls for which the assumption of a level shock can lead to greater errors, the numerical MOC is applied to solve the motion in region V. The algorithm and the numerical techniques used are basically the same as those outlined for the solution in region III (see section (3.3)) with two major exceptions. Firstly, to avoid interpolations across the wall or the shock front, the time step is limited so that the relevant characteristics are bounded by the containment wall and the position of the hydraulic jump. This leads initially to a very small time step (about three orders of magnitude smaller than for the level shock routine described above). The criterion for the size of this time step depends on the required number of mesh points within the bounded region, R_s to R_w , which in turn depends on the distance the shock has propagated away from the wall. In Fig. (3.4) C_- represents the limiting negative characteristic passing through $(R_s(t_j), t_j)$. The motion is

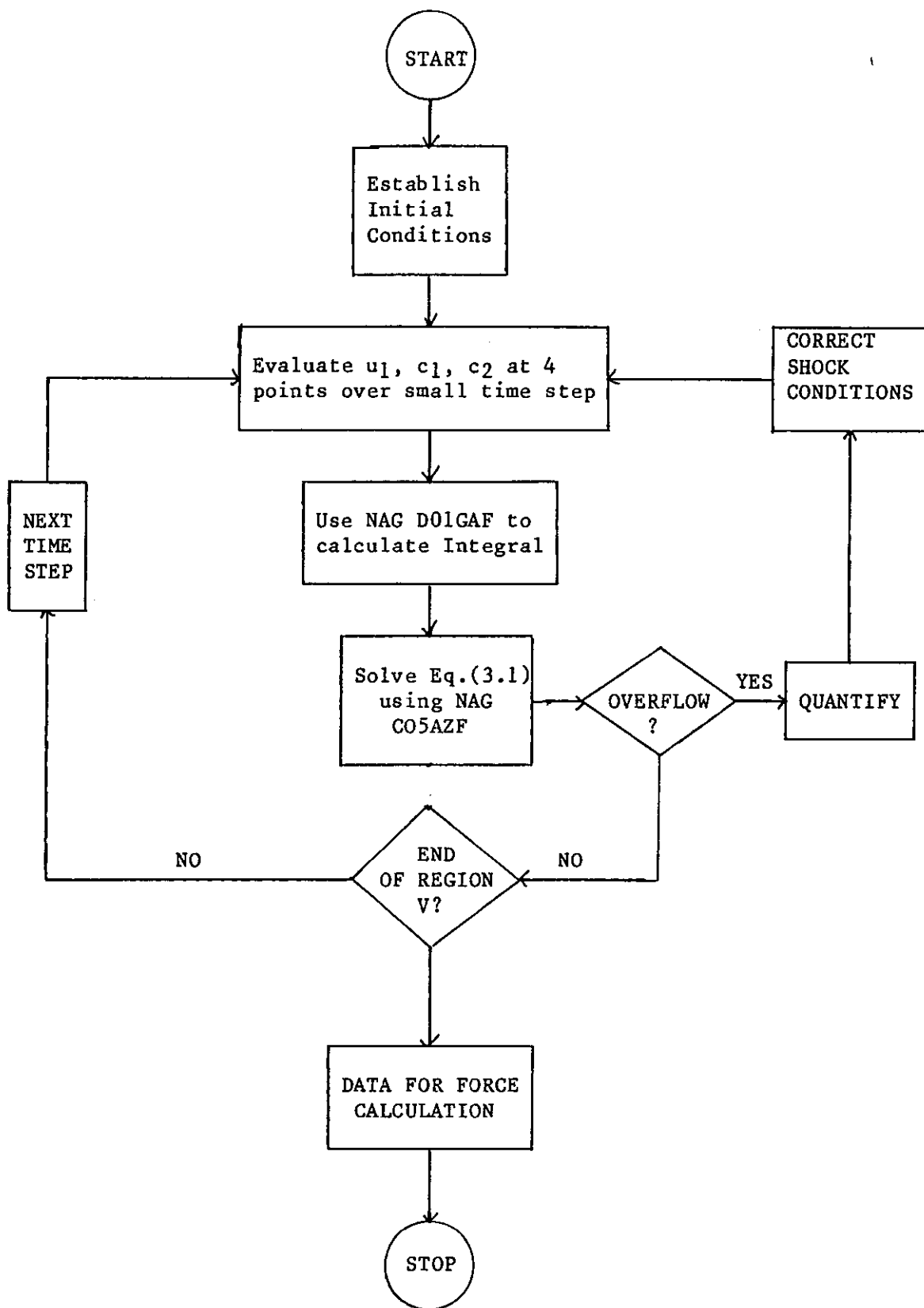


Figure 3.3

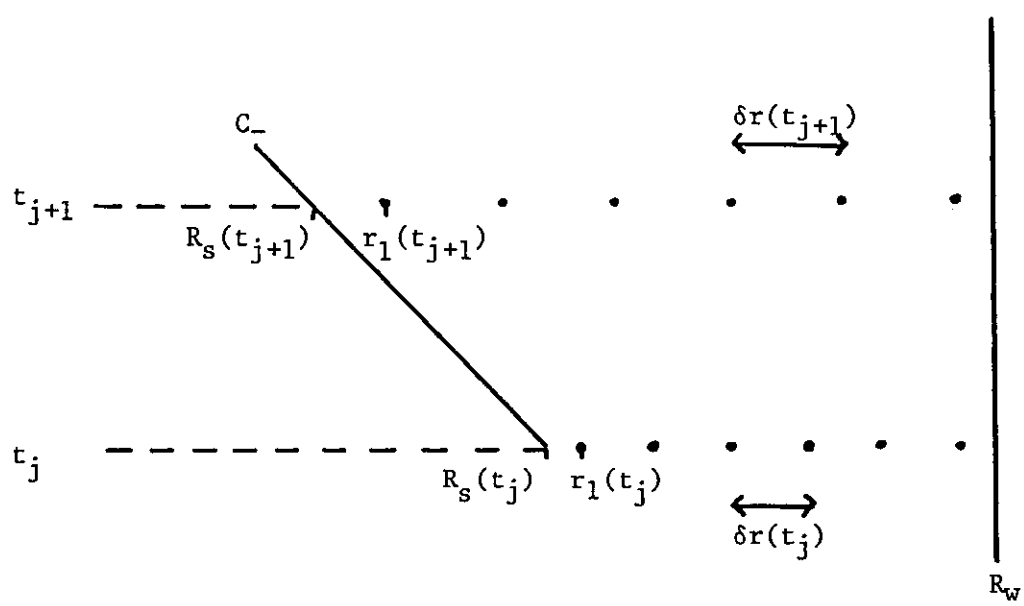


Figure (3.4)

completely defined at time level, t_j . Having chosen the number of mesh points required at time level, t_{j+1} , between $r_l(t_{j+1})$ and $r_n(t_{j+1})$, the step size is evaluated so that:

$$r_l(t_{j+1}) - R_s(t_{j+1}) < \frac{\delta r(t_{j+1})}{2} \quad (3.2)$$

In other words the distance over which extrapolations are performed, $r_l(t_{j+1}) - R_s(t_{j+1})$, is maintained to at least half the mesh spacing $\delta r(t_{j+1})$. This reduces extrapolation errors. The above evaluation is found within an iterative procedure in the computer program which optimizes the slope of C_- .

This leads on to the second major difference between the numerical MOC algorithms in regions III and V. Since the shock position is a variable boundary, a variable mesh spacing is required. The mesh is redefined at each time level according to the criterion given in Eq. (3.2) and according to the length of the field of calculations, i.e. the distance $(R_w - R_s)$.

The conditions described above, necessary for the accuracy of the method, lead to a very high computational cost which is about an order of magnitude greater than that for the routine evaluating the motion in region V when assuming a level shock.

3.5 Overflow

An option for evaluating the overflow conditions is incorporated in the computer program. The bund height can be specified or an "equal volume" criterion can be applied.

The numerical integration of Eq. (1.3.49) is performed by the same NAG library routine (12) which evaluates the integral in the characteristic equations.

In the case of the level shock assumption this integration is executed within an iterative routine which is necessary to satisfy mass conservation with respect to overflow. When using the numerical MOC in region V this requirement is satisfied by the boundary conditions. The iterative method re-evaluates the shock condition after four time steps on the basis of the amount of liquid which has flowed over during this period. Overflow is then re-computed and the process is continued until satisfactory convergence is achieved with respect to the shock height and shock position. When the motion in region V is terminated the entire overflow integration is re-assessed to compute an accurate quantity.

3.6 Accuracy and Stability

In the case of explicit numerical methods the stability criterion is analytically defined (see section (2.2.2)). Theoretically a stability analysis for the numerical MOC applied to the SWE's is very difficult and was not undertaken here. However, no divergence or oscillation was noted in the computation within a wide range of possible tank/containment wall configurations, even for excessively large values of the mesh spacing and the time step. This does of course not imply that the solutions under these conditions are accurate or satisfactory, only that the numerical procedure is stable. The iterative, implicit second order method

is therefore, in practice, only restricted by the required accuracy of solution. It was decided that all solutions would be iterated until successive values differed by less than $\frac{1}{2}\%$.

Whether the solution is correct or not can only be justified by a) comparison with analytical results, or in the absence of these, by b) the arguments supporting the use of the numerical techniques applied to the modelling equations. The numerical solution for the plane model in regions III and IV compared favourably with the corresponding analytical solutions. The " $\frac{1}{2}\%$ - accuracy" criterion mentioned above leads, in this case, to numerical values which differ by up to 0.04% from the analytically derived solutions.

For the " $\frac{1}{2}\%$ -accuracy" criterion a step length of one tenth of the tank radius is used when applying the numerical MOC in region III and IV. This step length is reduced further by a factor of ten in the vicinity of the centreline of the tank to minimize errors of interpolation in the iterative scheme developed for region IV (see section (2.3)). The mesh spacing in region V for the numerical MOC is initially set at up to one hundredth of that used in region III. This criterion is gradually relaxed as the shock moves away from the containment wall.

After performing a sensitivity analysis, a time step of $0.1(A^2/gH_0)^{\frac{1}{2}}$ was chosen for the computation throughout regions III and IV, to comply with the " $\frac{1}{2}\%$ -accuracy" criterion. As mentioned in section (3.4) the time step is considerably reduced for the numerical MOC in region V, initially by up to a factor of 10,000.

The numerical solution of the non-linear ordinary differential equation which determines the shock conditions in

region V, assuming a constant liquid height behind the jump, is not unconditionally stable. It was decided to reduce the mesh spacing in region V close to the containment wall progressively by a factor of up to 100 times that used in region III, i.e. up to a thousandth of the tank radius. Given this criterion, the time step for which the numerical method applied to this region is stable, is about $1/25^{\text{th}}$ of that used in regions III and IV. To maintain the "accuracy" of $\frac{1}{2}\%$ as mentioned previously, the time step is reduced by a further factor of 4.

CHAPTER 4

Results

4.1 Introduction

The first and most important result from the two mathematical models described previously is the maximum liquid height at the surrounding wall, which is the main concern regarding containment purposes. These and other additional areas of interest, such as the quantification of liquid spilling over the bund (when this is of limited height) and the pressure exerted on the wetted area of the containment walls, as a function of parameters such as the original tank height, tank diameter, distance between the bund and the tank and bund height in case of overflow, will be described. The significant difference of the results predicted by the two models is illustrated, demonstrating the expected greater relevance of the axisymmetric model in evaluating the consequences of a catastrophic failure of a cylindrical storage facility. The results for overflow and for momentum balances after impact against the containment wall are used to estimate the overall accuracy of the numerical routines by considering the degree of conservation. (See sections (2.7) and (2.8)).

As previously outlined the collapse of a column proceeds unhindered until the surging liquid front collides with the surrounding containment wall. The shallow water theory predicts a

constant wave front speed of $2(gH_0)^{\frac{1}{2}}$ from the instant the column begins to collapse. For a tank 30 m high and 60 m in diameter this signifies a speed of 34 ms^{-1} or about 75 mph. The time it takes for the liquid to reach the containment wall is therefore predicted to be very small: about $\frac{1}{2}$ and $2\frac{1}{2}$ seconds for a wall situated respectively at $1\frac{1}{2}$ and 4 tank radii from the centre of the tank. The duration of the motion in region V, when the liquid is reflected from the containment wall back to the centre of the banded enclosure in the form of a hydraulic jump, is about ten times as long. The motion in region VI, where the liquid 'sloshes' up and down between $r = 0$ and $r = R_w$, is relatively unimportant with respect to the maximum attainable liquid height at the containment wall and consequently with respect to overflow and pressure considerations. It is therefore evident that the most significant part of the motion resulting from a CTF is over very quickly, lasting no more than about 20 seconds for the vast majority of industrial scale cases.

Results are presented for bunds at distances, R_w , from the centre of the tank (or from the back wall in the plane model), ranging from 1.5 to 4 tank radii, A . To consider containment walls closer than $1.5A$ is not justified since, as has been shown, vertical acceleration effects do not become negligible over such short ranges of flow. Bunds further away than $4A$ are not common in industrial situations.

The liquid depth throughout the motion is always linearly proportional to the original column height, H_0 . R_w and A are thus the only parameters which need to be varied in order to produce a complete set of results which are independent of scale. The

displacement is thus presented in a dimensionless form as a function of A . All liquid heights are given as a fraction of the original column height and the liquid velocities and wave speeds are given as multiples of $(gH_0)^{\frac{1}{2}}$. In order to relate to real industrial situations the time is given in absolute terms, with reference to specific values of the appropriate parameters concerned, unless otherwise stated.

Two dimensional, superimposed, cross-section, liquid height profiles and pseudo three dimensional graphics of collapsing columns are used throughout this chapter to clarify the results and to present an overall picture of the motion which results from a CTF within a banded enclosure. Additional data, with several values for each critical parameter, are given in Appendix (11).

4.2 Unhindered Column Collapse

It is of interest to show the profiles of a collapsing column in the early stages of the motion (region III) to illustrate the difference achieved by using an axisymmetrical model instead of a plane model. Figs. (4.2) and (4.3) are superimposed cross-sections of a collapsing column at various times before the front edge reaches the containment wall. These figures respectively show the free surface elevation of the liquid for a cylindrical tank with an initial height to radius ratio of unity and a plane rectangular reservoir of corresponding dimensions. The axis of symmetry of the tank in the axisymmetric model corresponds to the back wall of the reservoir in the plane model. Fig. (4.1) is a pseudo 3-D representation of a column shortly after collapse is initiated.

Collapsing Column - Region III

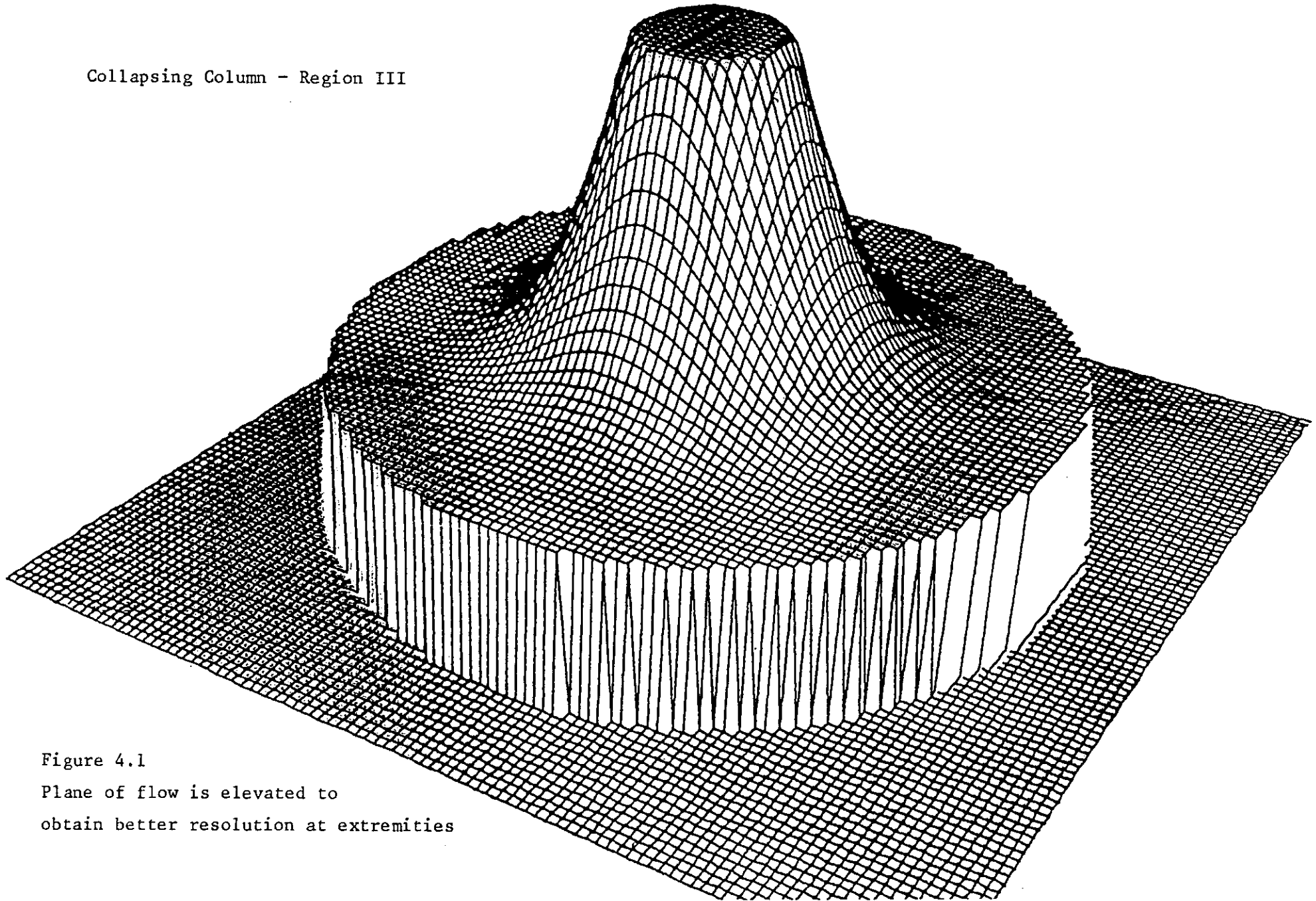


Figure 4.1

Plane of flow is elevated to
obtain better resolution at extremities

Free Surface Height - Region III

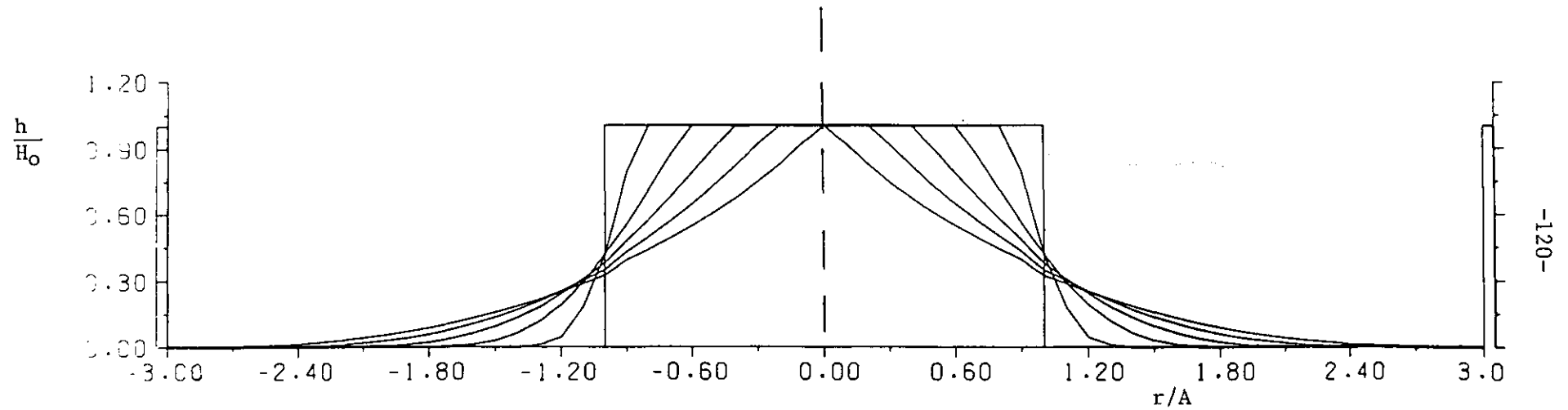


Figure 4.2 Axisymmetric Model, $t < T_w$, $R_w/A = 3.0$

Time step = $0.2(A^2/gH_0)^{\frac{1}{2}}$

Free Surface Height - Region III

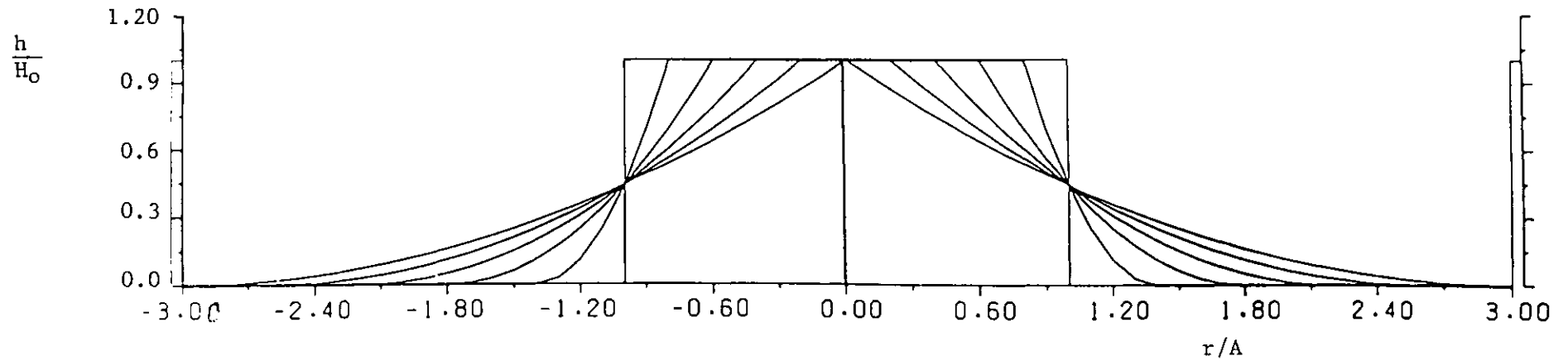


Figure 4.3 Plane Model, $t < T_w$, $R_w/A = 3.0$

Back Wall at $r/A = 0.0$

Figure is doubled up on both sides of bund wall for comparison
with axisymmetric model.

Time step = $0.2(A^2/gH_0)^{\frac{1}{2}}$

Note the simultaneous effects of a forward 'surging wave' moving outwards and a depletion wave moving inwards from the position of the original tank wall at $r = A$. The speed of the depletion wave is $(gH_0)^{\frac{1}{2}}$, the forward wave moving at twice this speed as previously mentioned. Even for the widest of tanks, which can measure up to 80 m in diameter, the depletion wave will terminate after about 2 seconds if the liquid in the tank was originally 30 m high.

It must be noted from Fig. (4.3) that all the height profiles derived from the plane model pass through the point ($r/A = 1$, $h_1/H_0 = 4/9$). This occurs until the relection of the depletion wave from the back wall moves outside the boundary of the original reservoir ($r < A$). For the axisymmetric model this is not the case. In Fig. (4.2) the profiles can be seen to pass through a lower point at $r/A = 1$ at each successive time step. This effect arises from the radial dispersive motion of a collapsing cylindrical column: the area covered by the spreading liquid increases with r^2 , not with r . Thus the cross-sectional area under the profiles shown in Fig. (4.2) is not constant. Furthermore the liquid present immediately behind the advancing front is therefore of lower depth and consequently moves faster there, than in the plane model. In other words, the axisymmetric model predicts an advancing front behind which the rates of change of liquid height and velocity with respect to r , are smaller than for the plane model. This justifies more accurately the use of the word, 'front', and illustrates a stronger base surge effect, as noted in the physical experiments of both Martin and Moyce (20) and of this study.

The values of the liquid velocity corresponding to Figs.

Horizontal Liquid Velocity - Region III

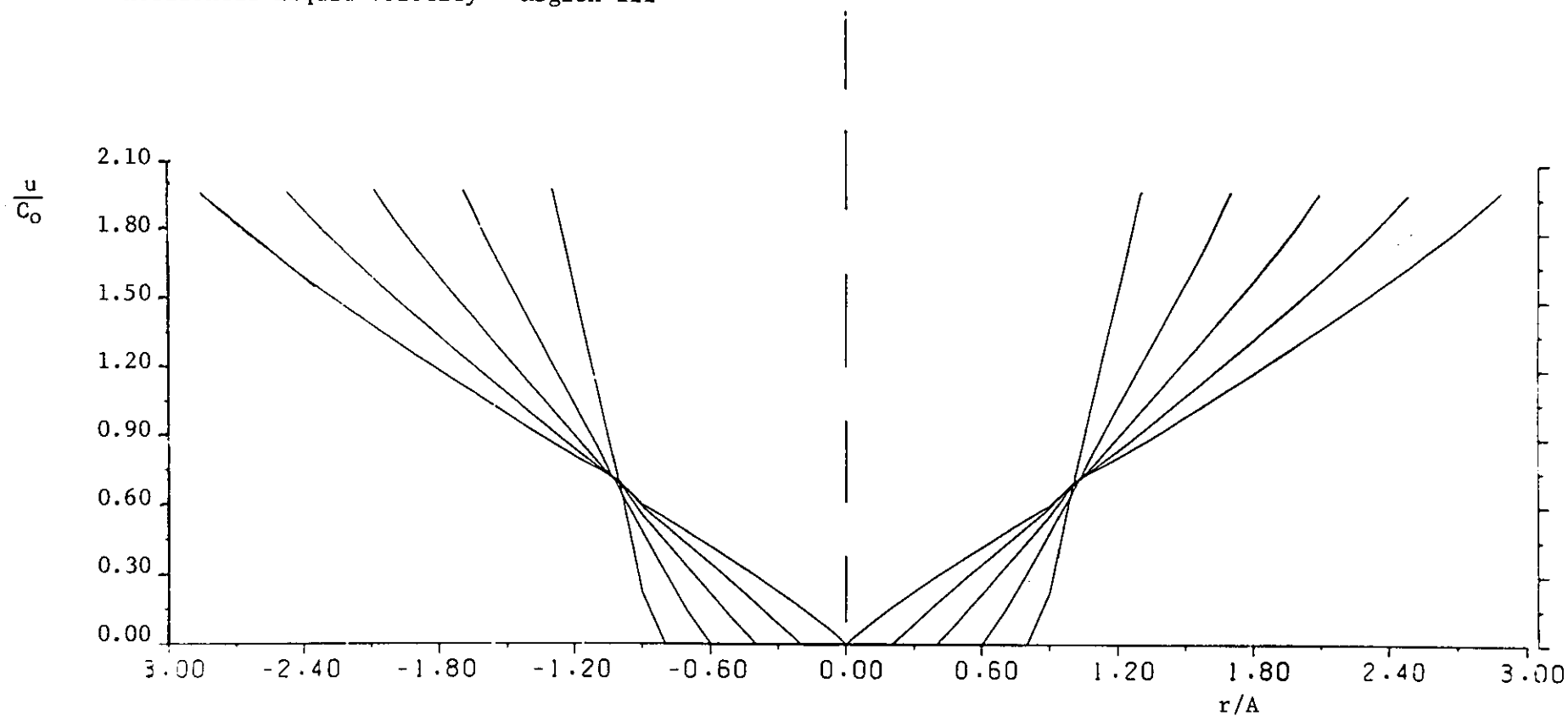


Figure 4.4 Axisymmetric Model, $t < T_w$, $R_w/A = 3.0$

(Resolution quality is poor due to coarse mesh refinement in computer graphics program).

$$\text{Time step} = 0.2(A^2/gH_0)^{\frac{1}{2}}$$

Horizontal Liquid Velocity - Region III

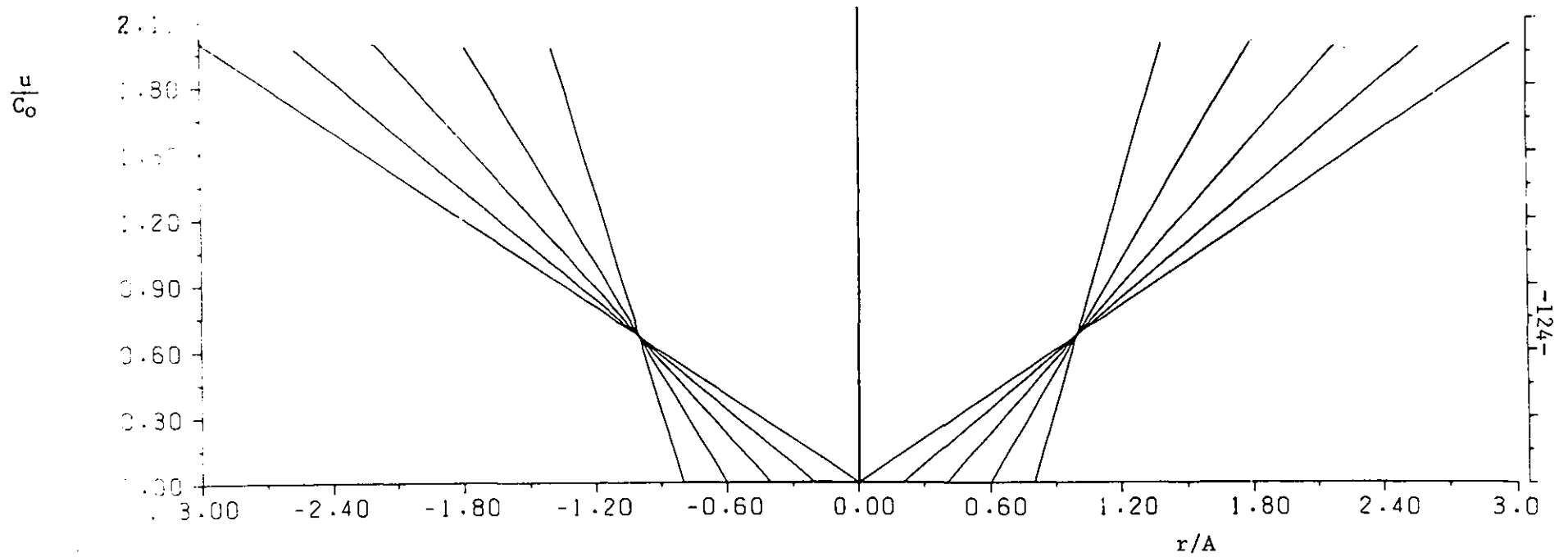


Figure 4.5 Plane Model, $t < T_w$, $R_w/A = 3.0$

Back Wall at $r/A = 0.0$

Figure is doubled up on both sides of back wall for comparison
with axisymmetric model.

Time step = $0.2(A^2/gH_0)^{1/2}$

(4.2) and (4.3) are given in Figs. (4.4) and (4.5).

4.3 Vertical Containment Walls of Unlimited Height

Both models were used with containment walls of sufficient height to avoid overflow. This allows for the prediction of the maximum height to which the liquid can possibly rise at the containment wall. The minimum bund height to prevent overflow is thereby obtained. Results from the plane model are in very good agreement with Greenspan's, who used a model of this type. At this stage it is important to mention that for both models the liquid level as a function of displacement and time is linearly proportional to the original column height throughout the duration of the motion; i.e. anywhere along the liquid surface including at the containment wall. This is to be expected since, when viscous and ground resistance effects are neglected in the models, the process of column collapse, subsequent spread and reflection from a bund is determined by the continuous interchange of hydrostatic and kinetic energy.

Figs. (4.6) (axisymmetric model) and (4.7) (plane model) show liquid height profiles at regular time intervals superimposed on each other. The axisymmetric model predicts a significantly lower height behind the shock than the plane model. Thus the maximum height, H_2 , which the liquid reaches at the containment wall also differs. Perhaps less obvious is the marked difference between the two models of the variation of the maximum liquid level as a function of the bund separation from the tank. The plane model predicts values of H_2 from 93% to 77% of the original tank height

Free Surface Height - Regions III, IV, V

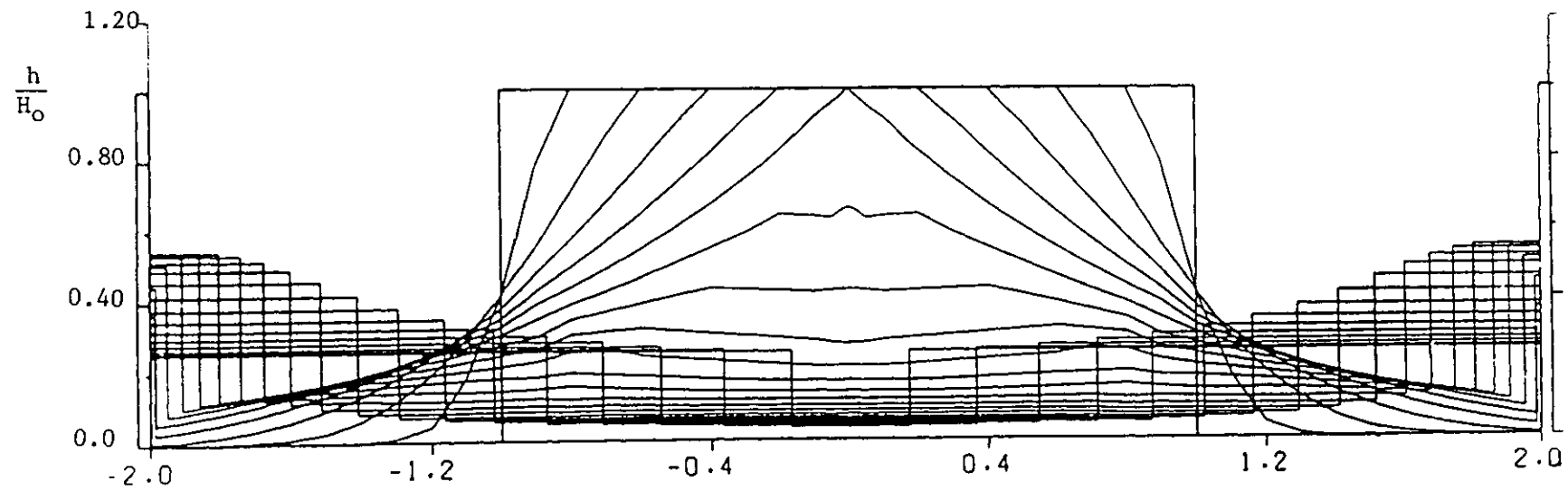


Figure 4.6 Axisymmetric Model

$$R_w/A = 2.0$$

$$\theta = 90^\circ$$

$$\text{Time step} = 0.2(A^2/gH_0)^{1/2}$$

Free Surface Height - Regions III, IV, V

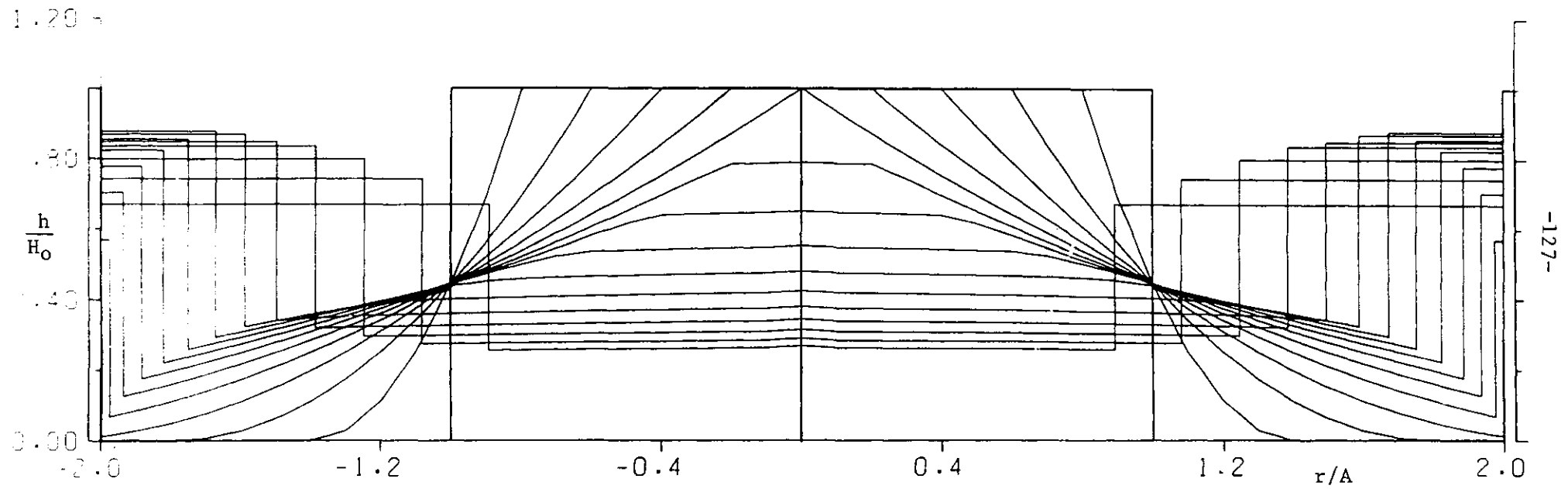


Figure 4.7 Plane Model, $R_w/A = 2.0$, $\theta = 90^\circ$

Back Wall at $r/A = 0.00$

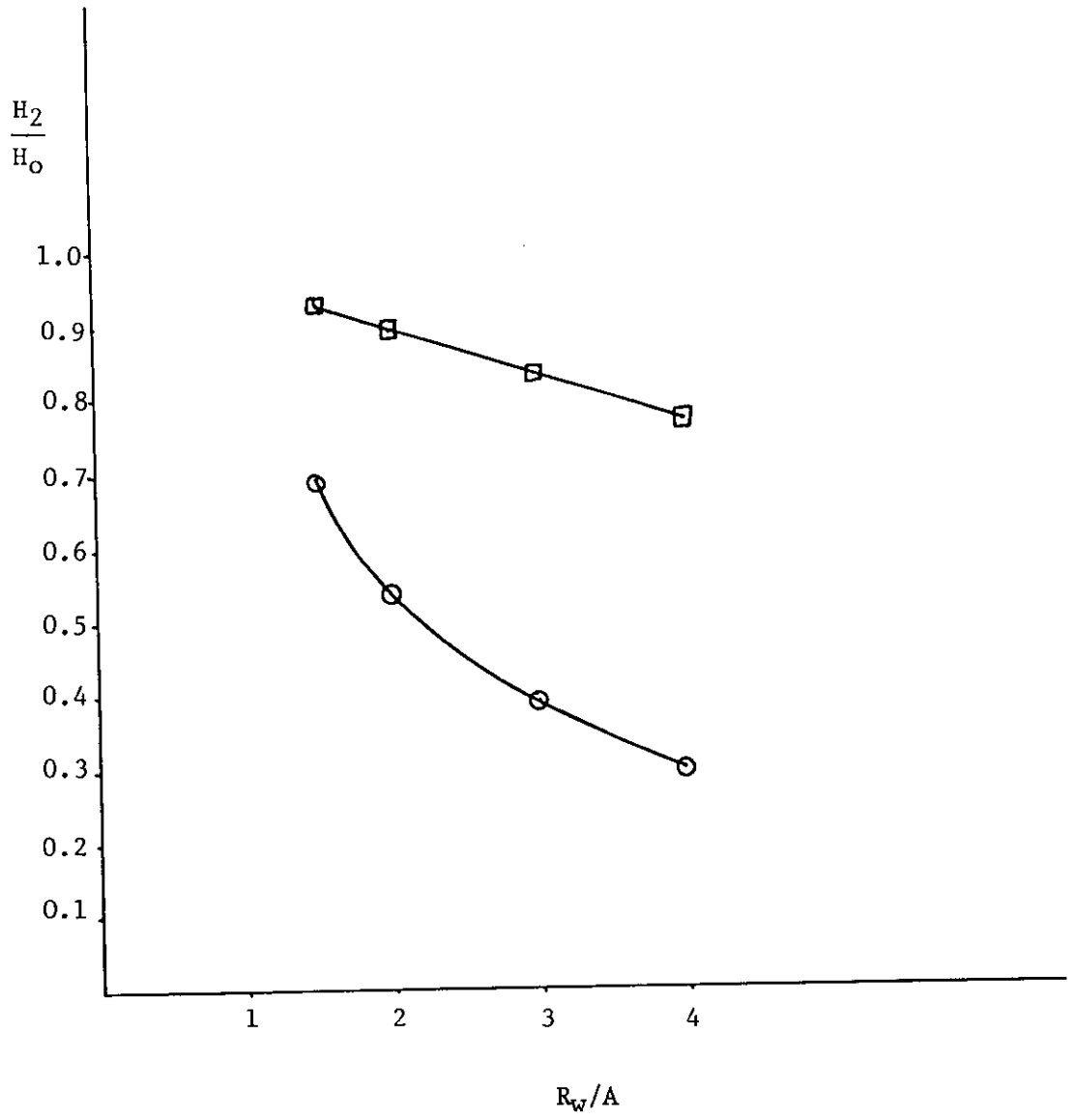
Time step = $0.2 (A^2/gH_0)^{\frac{1}{2}}$

in the range $1.5 < R_w/A < 4$. The axisymmetric model evaluates H_2 as 69% to 30% of H_0 in this range of bund distances, as is shown in Fig. (4.8). Fig. (4.9) shows the liquid level at the containment wall as a function of the dimensionless time, $(t - T_w)/T_w$, for both models and for various values of R_w . The variation of u and c in front of the discontinuity is linear in time and distance for the plane model, whereas the radial dispersive term in the axisymmetric model results in non-linear variations for the liquid velocity and wave speed. This affects the hydraulic jump formation and propagation and accounts for the differences predicted by the two models. The importance of the greater variation of H_2 with R_w predicted by the axisymmetric model will become more apparent in the section describing overflow, but it is useful to point out at this stage that as distinct from the indications of the plane model, the results from the axisymmetric model suggest that building a bund further away from the tank leads to a significant reduction in the height required for complete containment in the event of a CTF.

Having established the significant difference between the hydraulic jump heights predicted by the two models, the remainder of the results to be presented in this chapter, will mostly be given for the axisymmetric model only, which is more relevant to real situations. In the following sections on overflow, inclined containment walls and forces imparted after impact, results for the plane model will differ according to the results presented in this section, the liquid height at the containment wall being the critical parameter.

Superimposed profiles for $R_w/A = 1\frac{1}{2}, 3, 4$, similar to Fig.

Shock Height vs. Containment Wall Distance



⊙ Axisymmetric Model

□ Plane Model

Figure 4.8 $\theta = 90^\circ$

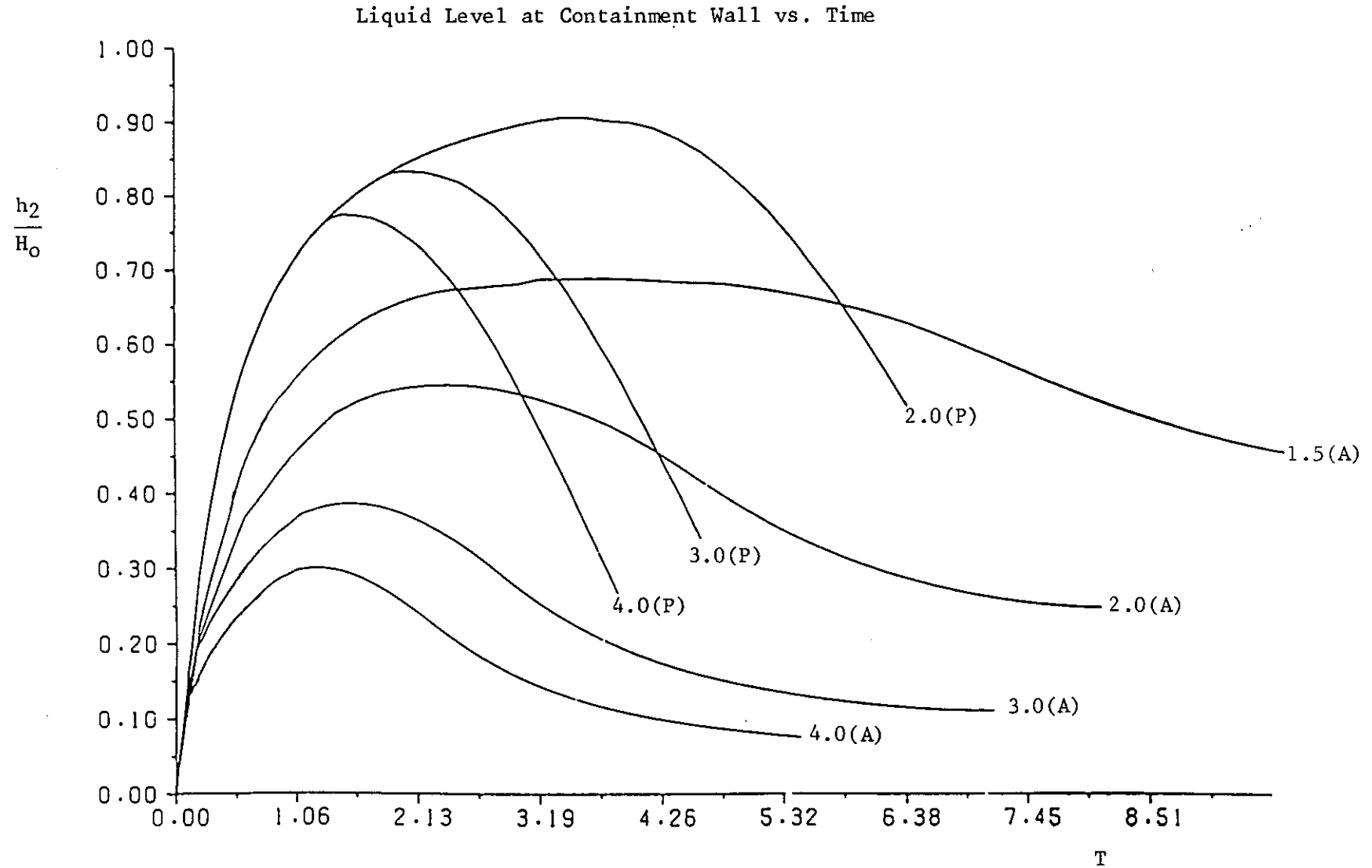


Figure 4.9 $T = (t - T_w)/T_w$

P = Plane Model: $R_w/A = 2.0, 3.0, 4.0$

A = Axisymmetric Model: $R_w/A = 1.5, 2.0, 3.0, 4.0$

$\theta = 90^\circ$

(4.6) are given in Appendix (11).

4.4 Vertical Bunds of Limited Height

Overflow calculations were performed for models with containment walls at various distances from the tank centre in the range mentioned previously and also for wall heights between 10% and 50% of the original tank height. The bund height, H_d , for which the enclosure volume equals the tank volume is also considered.

Fig. (4.10) shows the fractions of the total tank content which spill over the containment wall, as a function of bund height. For a fixed bund height overflow decreases with R_w/A (Fig. 4.11). The significant reduction in overflow achieved by increasing the distance of the containment wall from the tank is only predicted by the axisymmetric model. (See section (4.3)).

H_d will be smaller the further the bund is situated from the tank; H_d is $\propto \frac{1}{R_w}$ for the plane case and $\propto \frac{1}{R_w^2}$ in the axisymmetric case. The plane model predicts little variation of the liquid level at the bund with R_w . It follows that the amount of overflow increases when considering containment walls of height, H_d , further away from the tank. The more appropriate axisymmetric model does however indicate a significant reduction in liquid level at bunds further away from the tank. In this model increasing R_w has two opposite effects on the amount of overflow: 1) it will increase since $H_d(R_w)$ decreases with R_w and, 2) it will decrease since $H_2(R_w)$ decreases with R_w . The overall effect is still one of more overflow as the bund is placed further away (Fig. 4.12) but the

Percentage Overflow vs. Bund Height

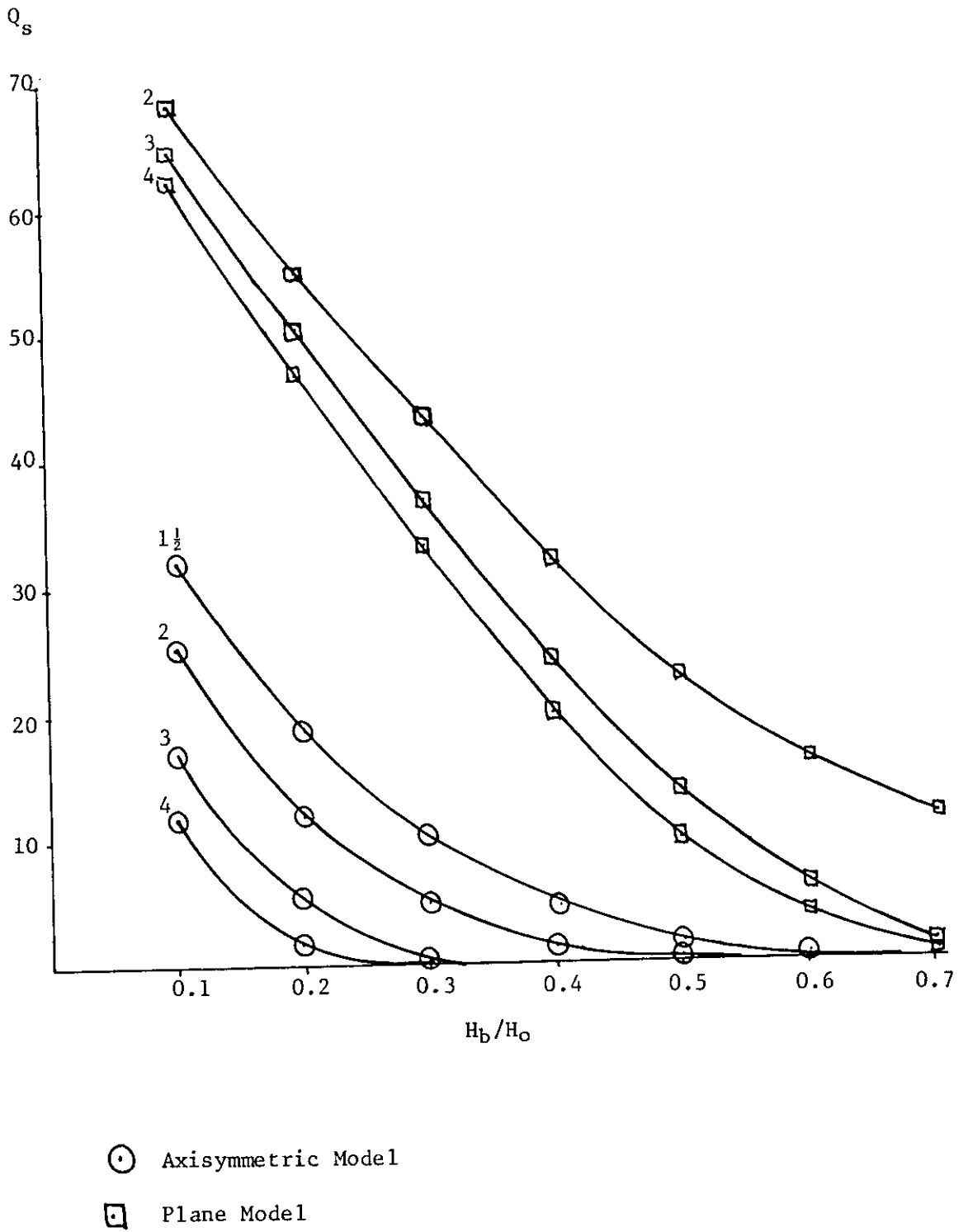


Figure 4.10 $R_w/A = 1.5, 2.0, 3.0, 4.0$

Percentage Overflow vs. Bund Distance

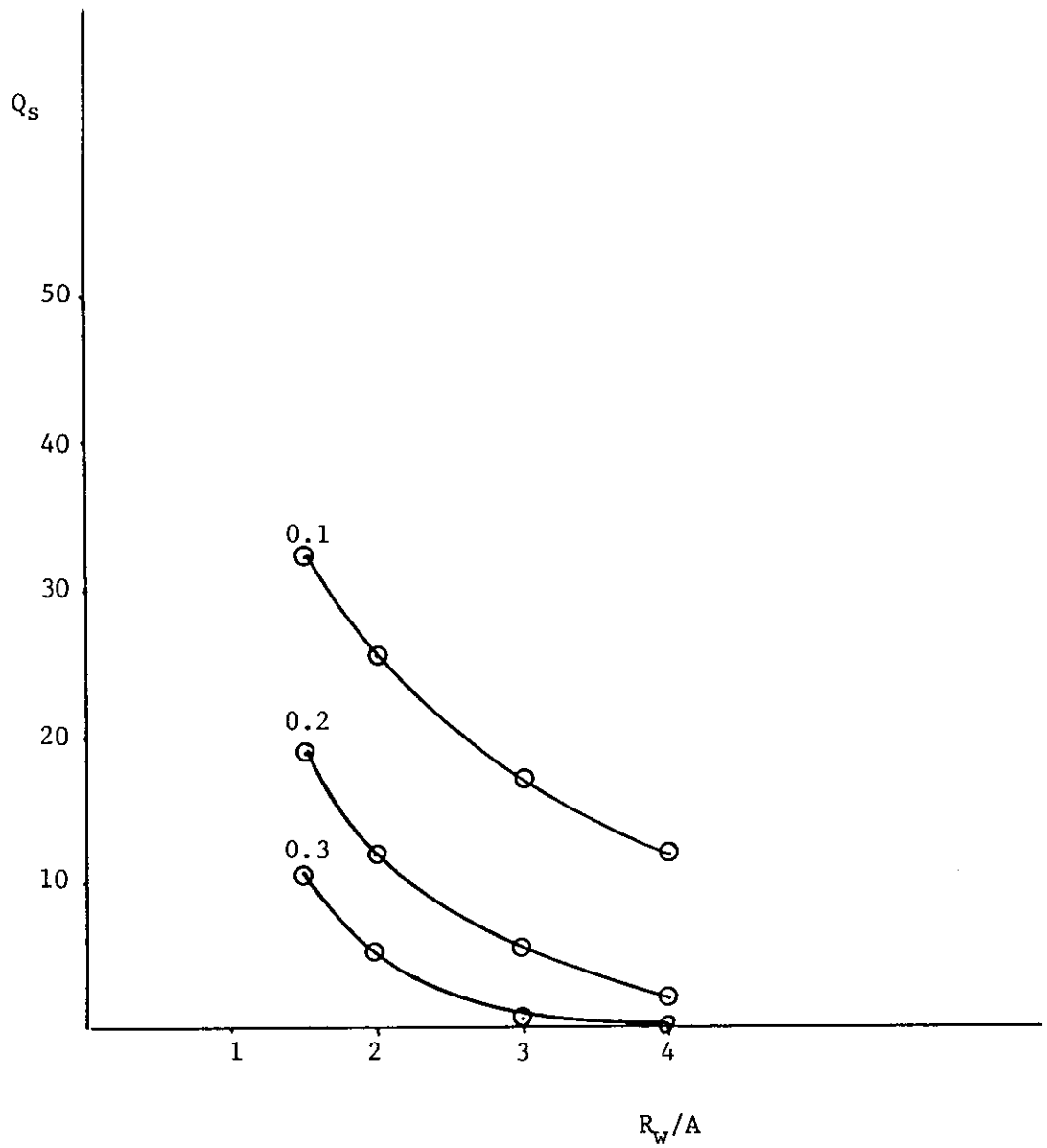
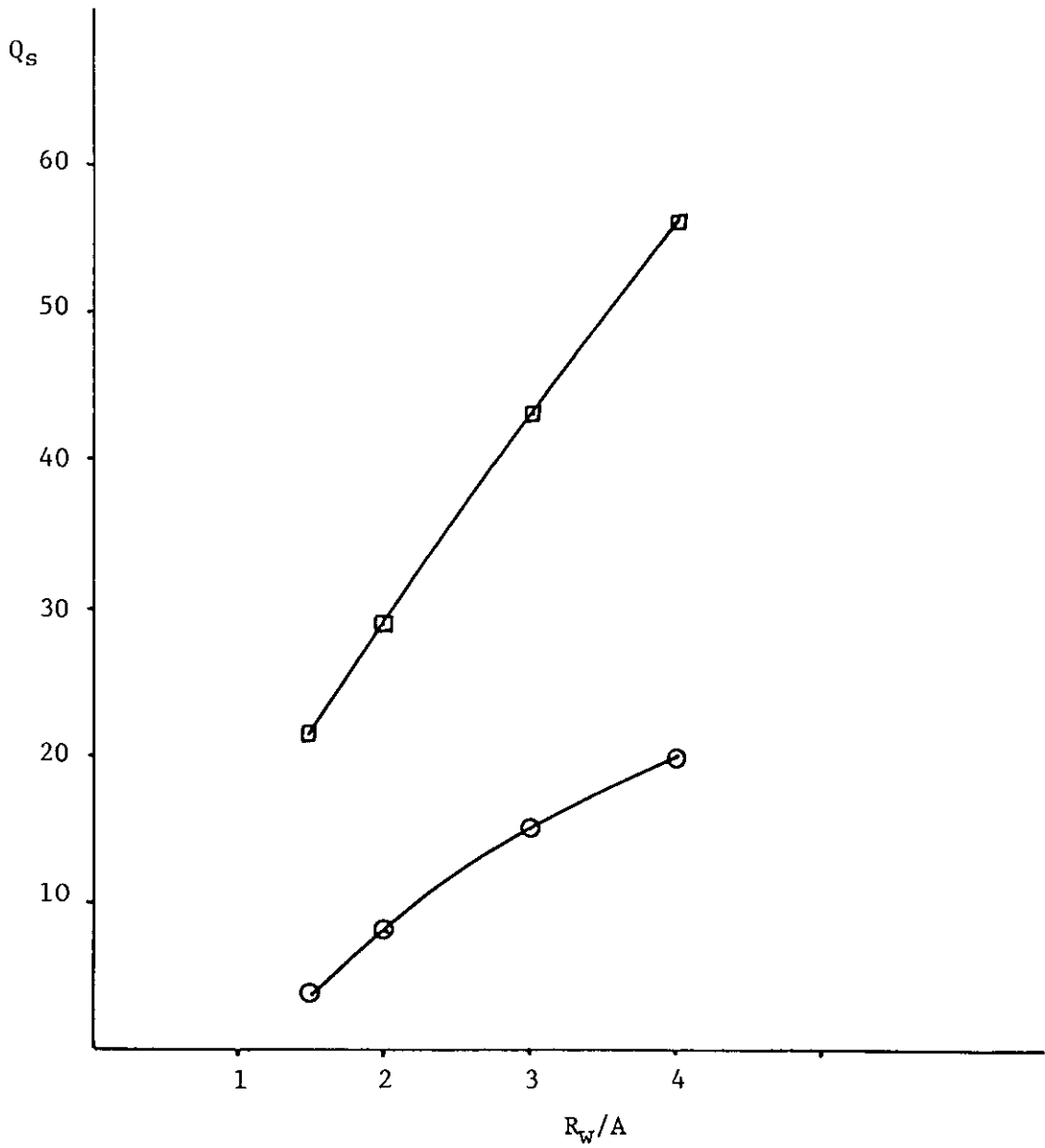


Figure 4.11 Bund heights, $H_b = 0.1, 0.2, 0.3$

$\theta = 90^\circ$

Percentage Overflow vs. Bund Distance



⊙ Axisymmetric Model

□ Plane Model

Figure 4.12 Bund height, $H_b = H_d$

$\theta = 90^\circ$

trend is not as accentuated as predicted by the plane model.

Overflow calculations also allow for a convenient check of the overall mass conservation of the numerical routine as is explained in section (2.7). For vertical containment walls variation in the volume of liquid accounted for never exceeds $\frac{1}{2}\%$ of the original tank volume which points to the accuracy of the numerical method.

As was observed in previous experimental work (7), overflow takes the form of a jet which is projected outside the enclosure. From equations derived in section (1.3.7) graphs were plotted showing the maximum distance of projection beyond the bund as a function of the bund height (Fig. (4.13)) and also as a function of the bund distance (Fig. (4.14)). From the latter plot it can be seen that if bunds of 'equal volume' specification are used, where $H_b = H_d$, then a maximum distance of jet projection occurs when the bund is placed at approximately two tank radii from the centre of the tank. Fig. (4.15) shows, as a function of time, the distance of projection and the liquid level at the bund (which in the case of overflow is always lower than in the case of a containment wall of unlimited height). This illustrates that the peak distance reached by the jet is attained just before time, $t = T_{\max}$ when the liquid comes to its maximum height at the bund. This is due to the fall in the vertical velocity component, which is highest at, $t = T_w$ and decreases to zero at $t = T_{\max}$.

Maximum Jet Projection vs. Bund Height

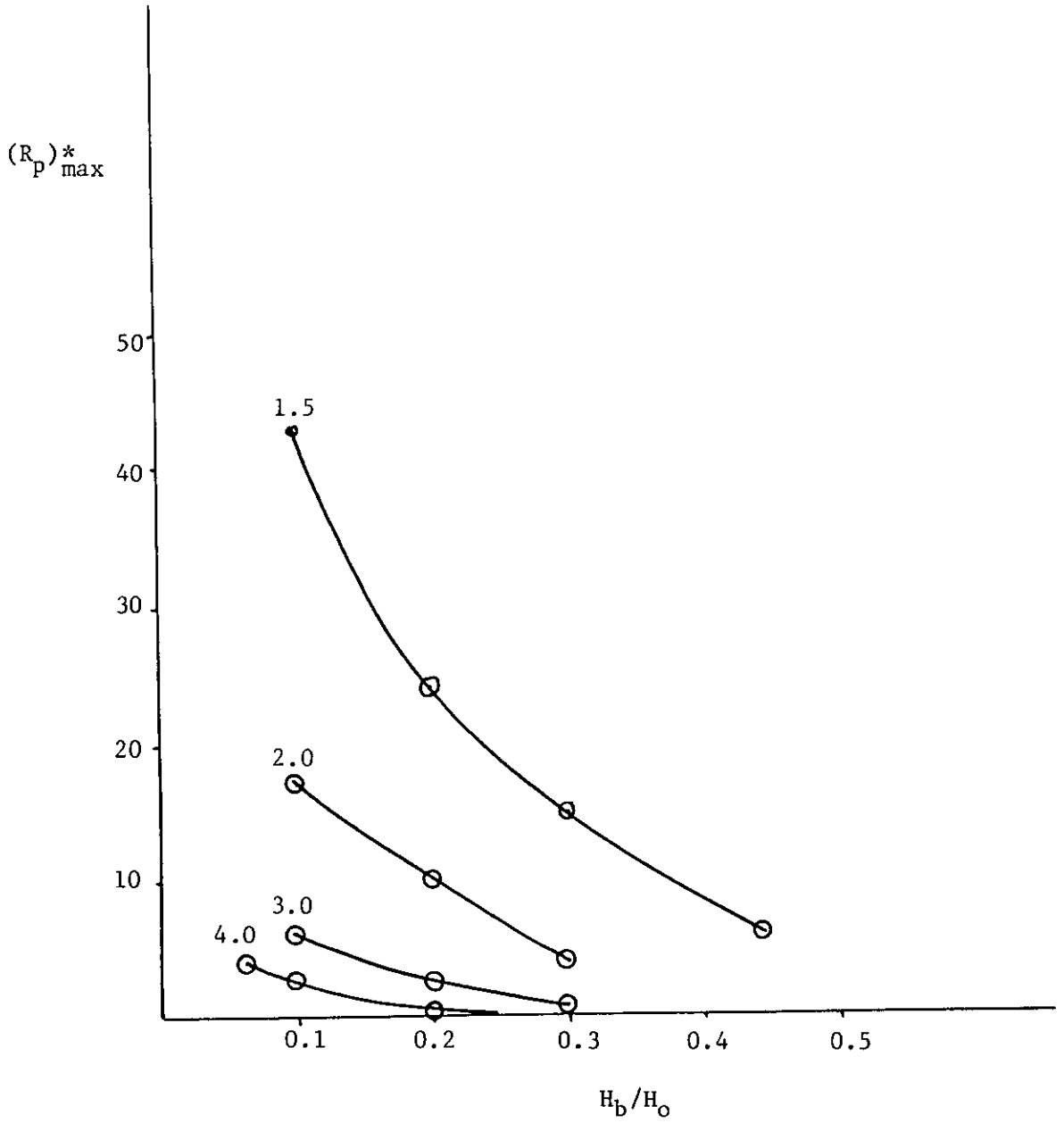


Figure 4.13 Bund Distance $R_w/A = 1.5, 2.0, 3.0, 4.0$

$$(R_p)^*_{\max} = \frac{(R_p)_{\max} - R_w}{R_w} \times 100\%$$

$$\theta = 90^\circ$$

Maximum Jet Projection vs. Bund Distance

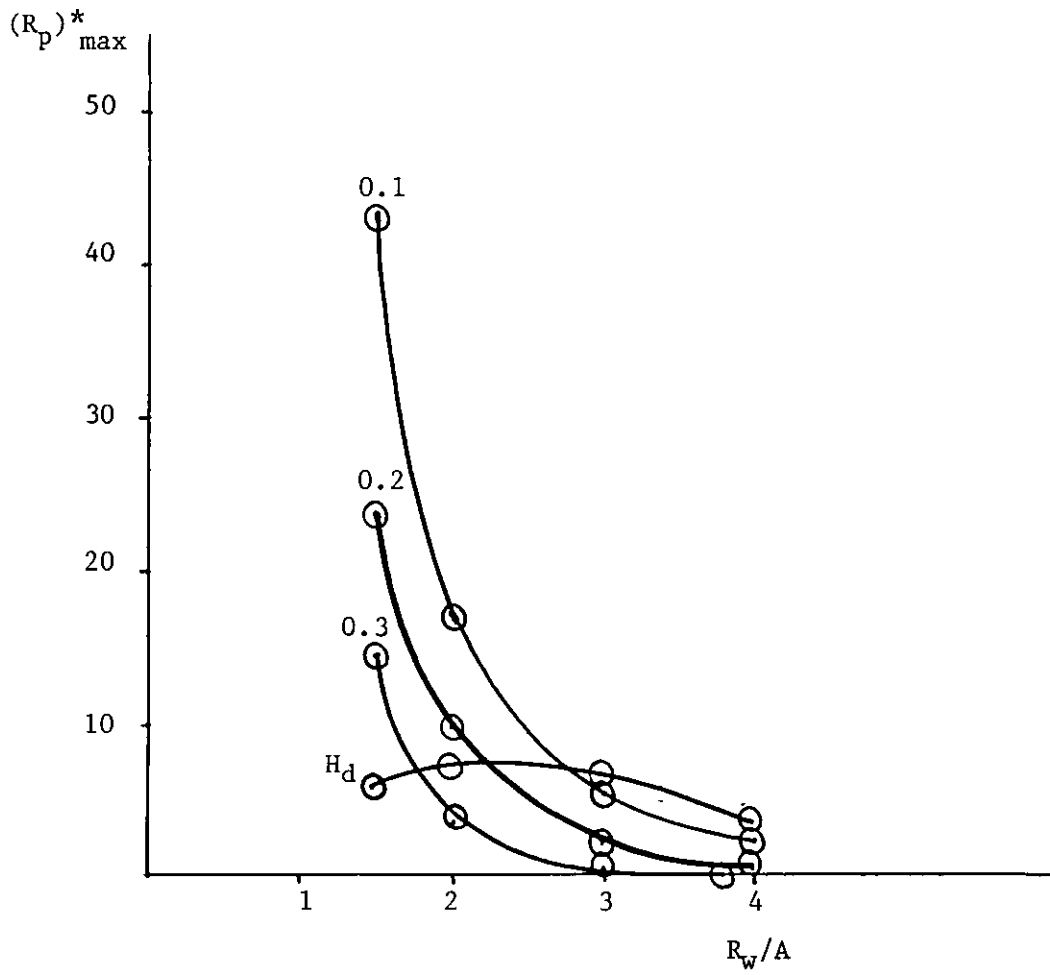
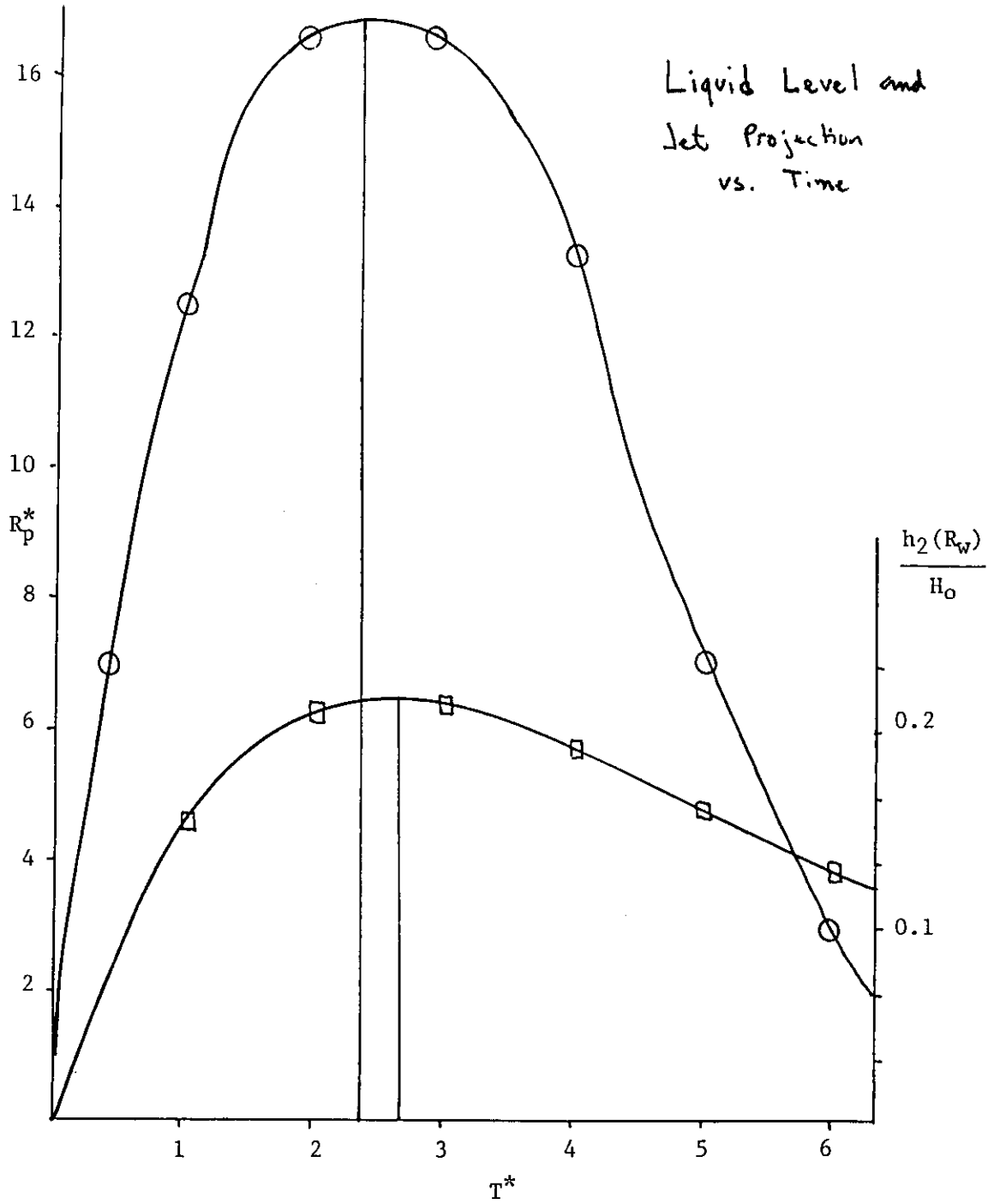


Figure 4.14 $(R_p)^*_{\max} = \frac{(R_p)_{\max} - R_w}{R_w} \times 100\%$

Bund height $H_b/H_o = 0.1, 0.2, 0.3, H_d$

$\theta = 90^\circ$



○ Distance of jet projection, R_p^* , vs. time

□ Liquid level at bund, h_2 , vs. time

$$R_p^* = \frac{R_p - R_w}{R_w} \times 100\%$$

$$T^* = \frac{t - T_w}{T_w}$$

Figure 4.15 $R_w/A = 2.0$, $\theta = 90^\circ$

4.5 Inclined Containment Walls of Unlimited Height

It did appear in section (1.3.6) that the effect of the angle of inclination of the containment wall only has a second order effect on the liquid height behind the shock. This was concluded from numerical experimentation which showed that for two angles, $\theta > 90^\circ$ and $\theta < 90^\circ$, where the deviation from the vertical is the same, the difference in shock height at all times was small in the range $|90^\circ - \theta| < 50^\circ$. Fig. (4.16) shows the liquid level at the containment wall, $h_2(R_w)$, for angles in this range, at three different times:

- a) T_1 - maximum liquid height attained
- b) T_2 - maximum deviation in liquid height for corresponding angles ($T_2 < T_1$)
- c) T_3 - liquid height at time, $t > T_1$

Graph T_2 in Fig. (4.16) shows the maximum deviation from symmetry, which reaches about 10% for the corresponding angles, $\theta = 60^\circ$ and $\theta = 120^\circ$. The deviation is about 20% for angles, $\theta = 45^\circ, 135^\circ$. From this it is clear that the use of the SWE's as approximate modelling equations for inclined containment is only justifiable for a small range of angles, which depends on the error that is considered to be acceptable. The motion at the wall can thus be predicted from this model and Fig. (4.16) then serves as an indication of the accuracy of these results.

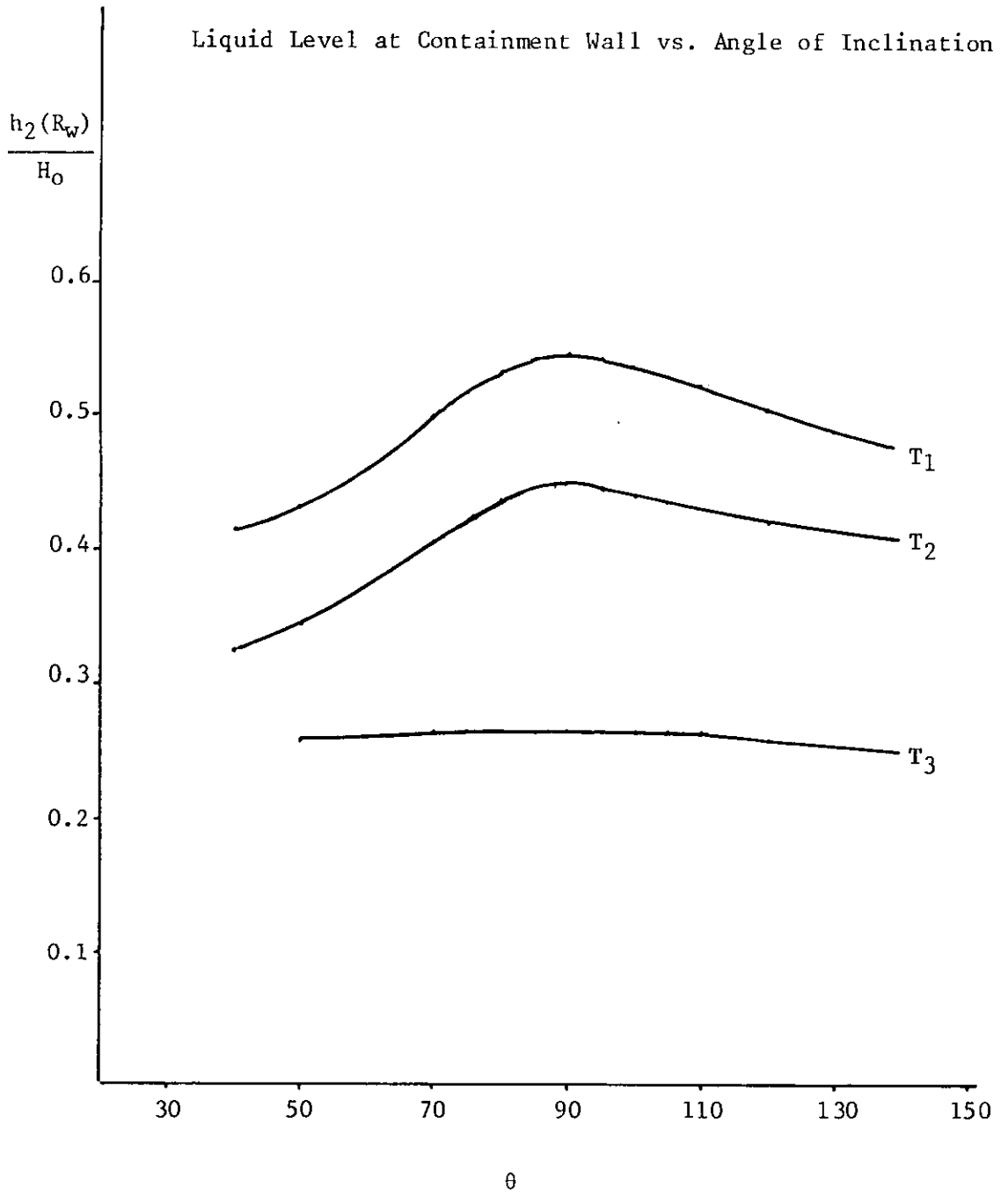


Figure 4.16 $T_1 = T_{\max}$

$$T_2 < T_{\max}$$

$$T_3 > T_{\max}$$

The results for inclined containment walls, presented in this section are evaluated using the numerical MOC as described in section (2.4). This method allows for a variation in liquid surface height between the shock position, R_s and the containment wall defined by, $R_w(r)$, in the inclined case. The discrepancy between the hydraulic jump profiles evaluated by the numerical MOC and by the level shock assumption (used for vertical walls) is shown in Fig. (4.17) for the angles $\theta = 60^\circ, 80^\circ, 90^\circ, 100^\circ, 120^\circ$. The height profiles are shown for the time step corresponding to, $t = T_1$, in Fig. (4.16)). As mentioned above the results for angles outside this range, derived from the SWE's suffer from errors larger than 10% in the approximation used to model the motion and these are therefore not presented. The comparison of the two numerical methods used to model the motion in region V is also given for the case $\theta = 90^\circ$, which serves as a reference and illustrates the validity of the level shock assumption for the case of vertical walls. From Fig. (4.17) it can clearly be seen that for $\theta < 100^\circ$ the hydraulic jump is almost level. Only for angle $\theta > 100^\circ$ does the liquid profile behind the shock start to deviate significantly from the horizontal. The extent to which it does depart from the horizontal loses its significance for angles where the evaluation of the liquid height by the SWE's is itself already subject to large errors. Similarly for angle of inclination significantly less than the vertical, the prediction by this method of a nearly level bore is equally unjustifiable.

For the range of angles where the SWE's can be assumed to apply, it can be seen from Fig. (4.17) that the hydraulic jump is reflected from the wall more rapidly the greater the angle of inclination.

Height Profiles at $t = T_1$

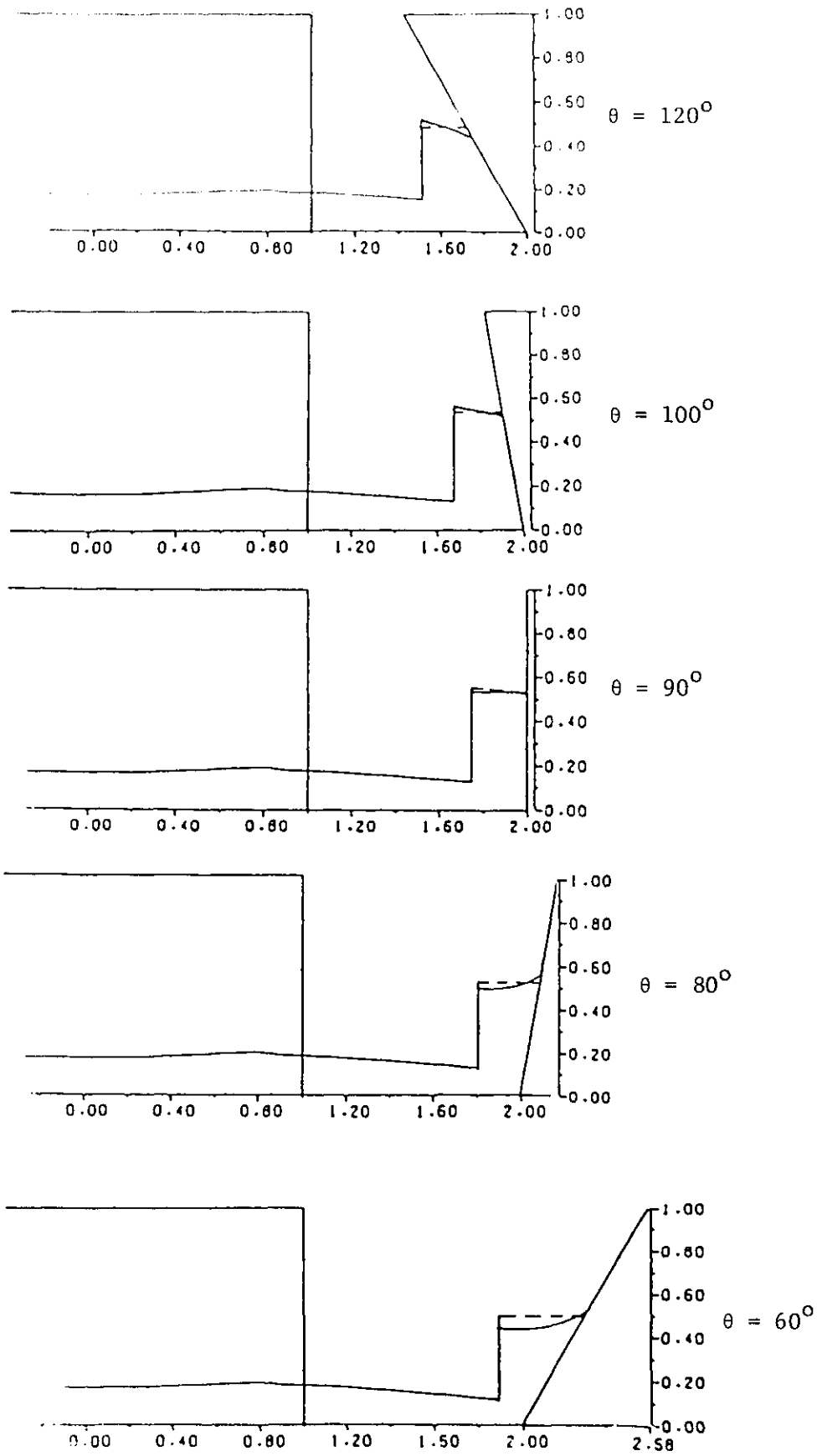


Figure 4.17 Comparison of the 'numerical MOC' and the 'level shock assumption' methods applied to region V.

----- level shock assumption

———— numerical MOC

except for $\theta = 90^\circ$ where this is the reverse.

Profiles correspond to time $t = T_1$ (Fig. (4.16))

4.6 Inclined Bunds of Limited Height

As can be expected overflow, in the case of bunds inclined at angles, $\theta < 90^\circ$, will be much greater than for vertical bunds, due to the horizontal liquid surge up the face of the inclined wall. The flow as a function of bund inclination is shown in Fig. (4.18). Containment is not found to be greatly improved for bunds with angles, $\theta > 90^\circ$. This is particularly true for low bunds: comparison of vertical bunds with bunds inclined at 120° shows a decrease in overflow of 6% for bunds at height, $H_b = H_o/10$, and a decrease of 25% for bunds of height, $H_b = H_o/4$.

For bunds inclined at more than 30° from the vertical, results may be subjected to large errors corresponding to the trend shown in Fig. (4.16); hence the dotted lines in Fig. (4.18).

A better way of deducing the extent to which the SWE's are inappropriate for modelling the flow in region V for inclined bunds can be determined by the errors incurred in the conservation of mass with overflow and in the conservation of momentum after impact when considering a wall of unlimited height. Table (4.1) shows these values for several angles, θ .

The distance, R_p , to which the overflowing jet is projected beyond the bund is also much greater if the bund is inclined at angles, $\theta < 90^\circ$ (Fig. (4.19)). $(R_p)_{\max}$ for $\theta > 90^\circ$ follows a similar tendency to the overflow curves of Fig. (4.18), i.e. a slight decrease is observed when θ increases.

Percentage Overflow vs. Bund Inclination

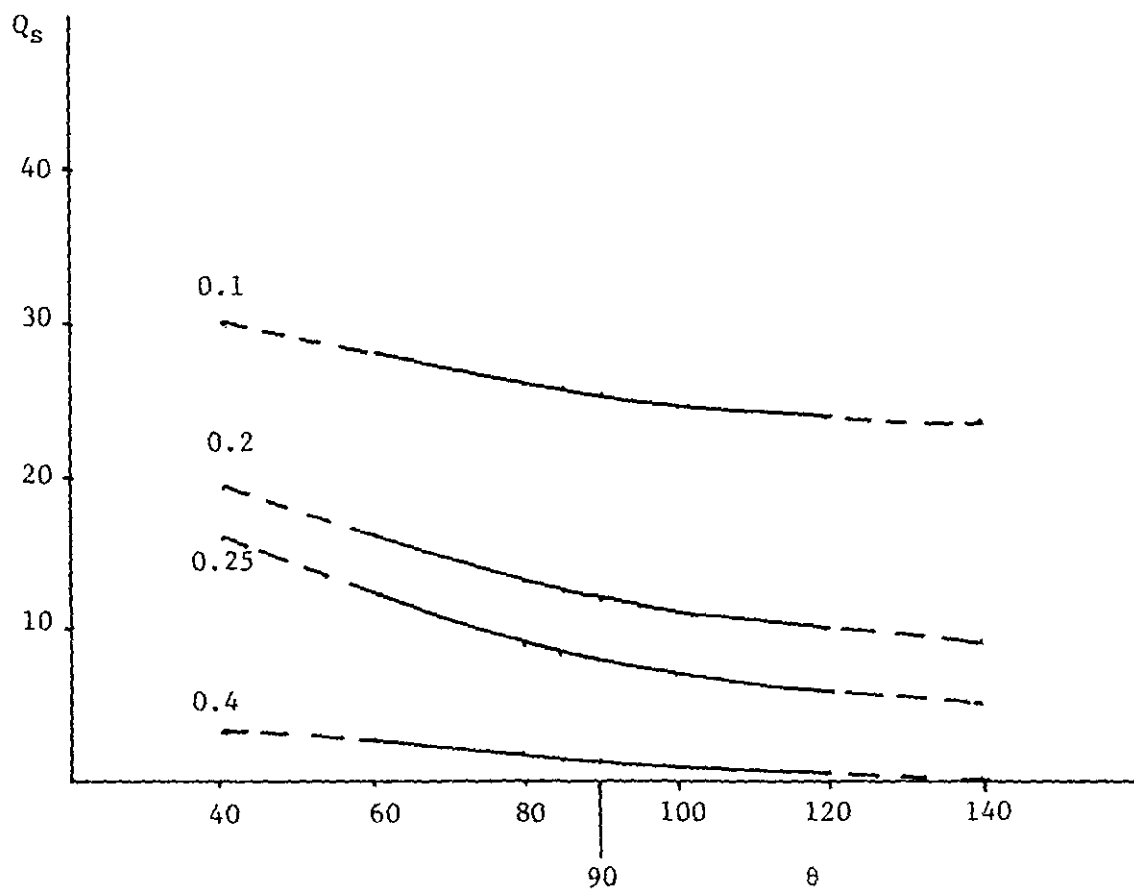


Figure 4.18 Bund heights, $H_b/H_o = 0.1, 0.2, 0.25, 0.4$

$$R_w/A = 2.0$$

Error Estimates

Angle of Inclination	Mass Conservation % Error		Momentum Conservation % Error	
	Max.	Mean	Max.	Mean
40	12.39	9.53	27.35	20.26
60	8.68	6.89	18.26	13.96
80	4.26	3.51	8.75	6.83
85	1.92	1.63	3.64	2.92
90*	0.54(0.59)	0.46(0.57)	1.03(1.37)	0.86(1.29)
95	1.33	1.13	2.83	2.11
100	3.78	3.06	7.37	5.68
120	8.10	6.28	16.04	11.86
140	10.91	8.08	25.90	18.77

Table 4.1 All above percentage errors are calculated for the numerical MOC routine and show a loss of mass or momentum. *The figures in brackets for a vertical wall are the values for the level shock assumption routine.

Maximum Jet Projection vs. Bund Inclination

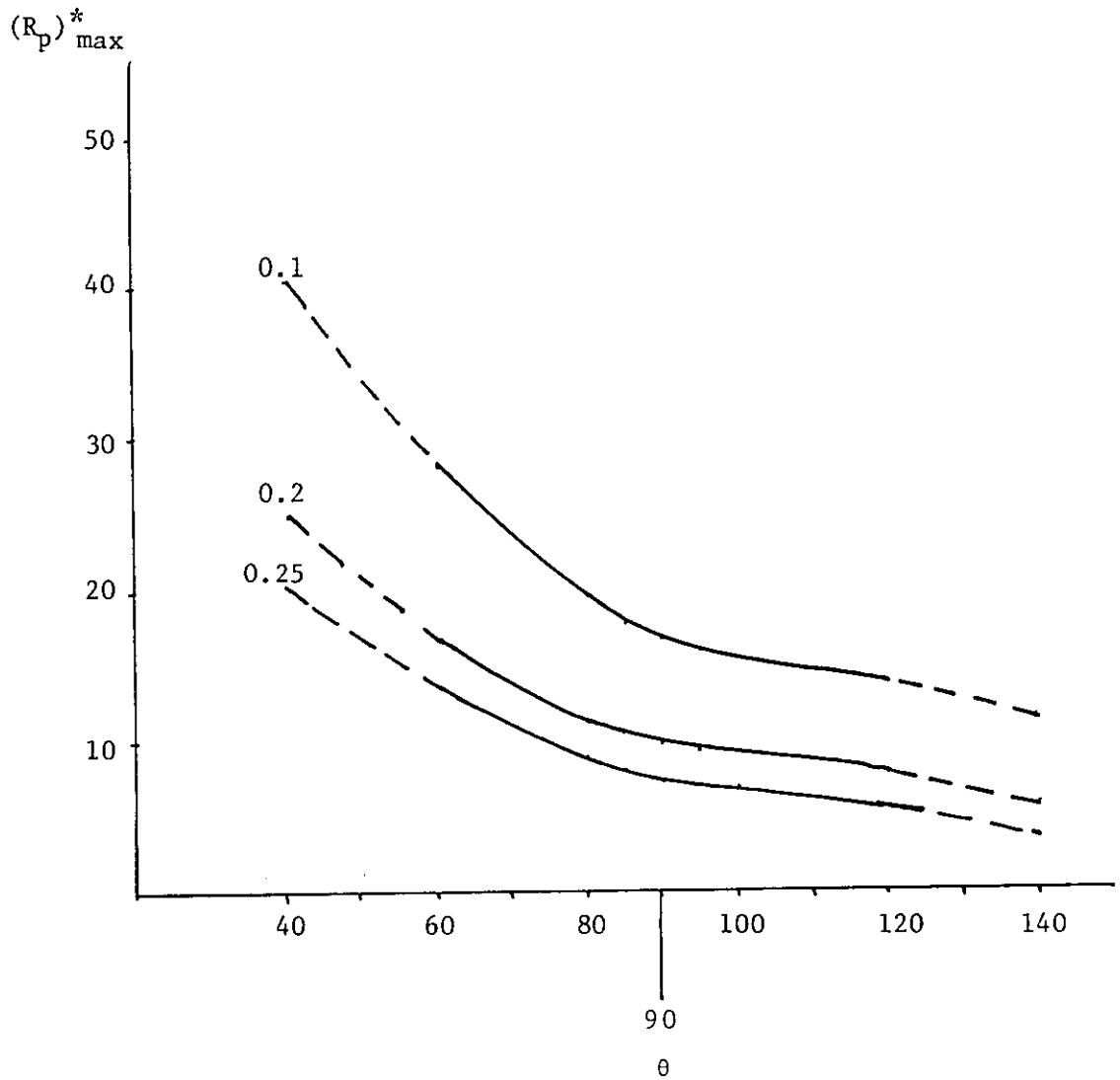


Figure 4.19 Bund height, $H_b = 0.1, 0.2, 0.25$

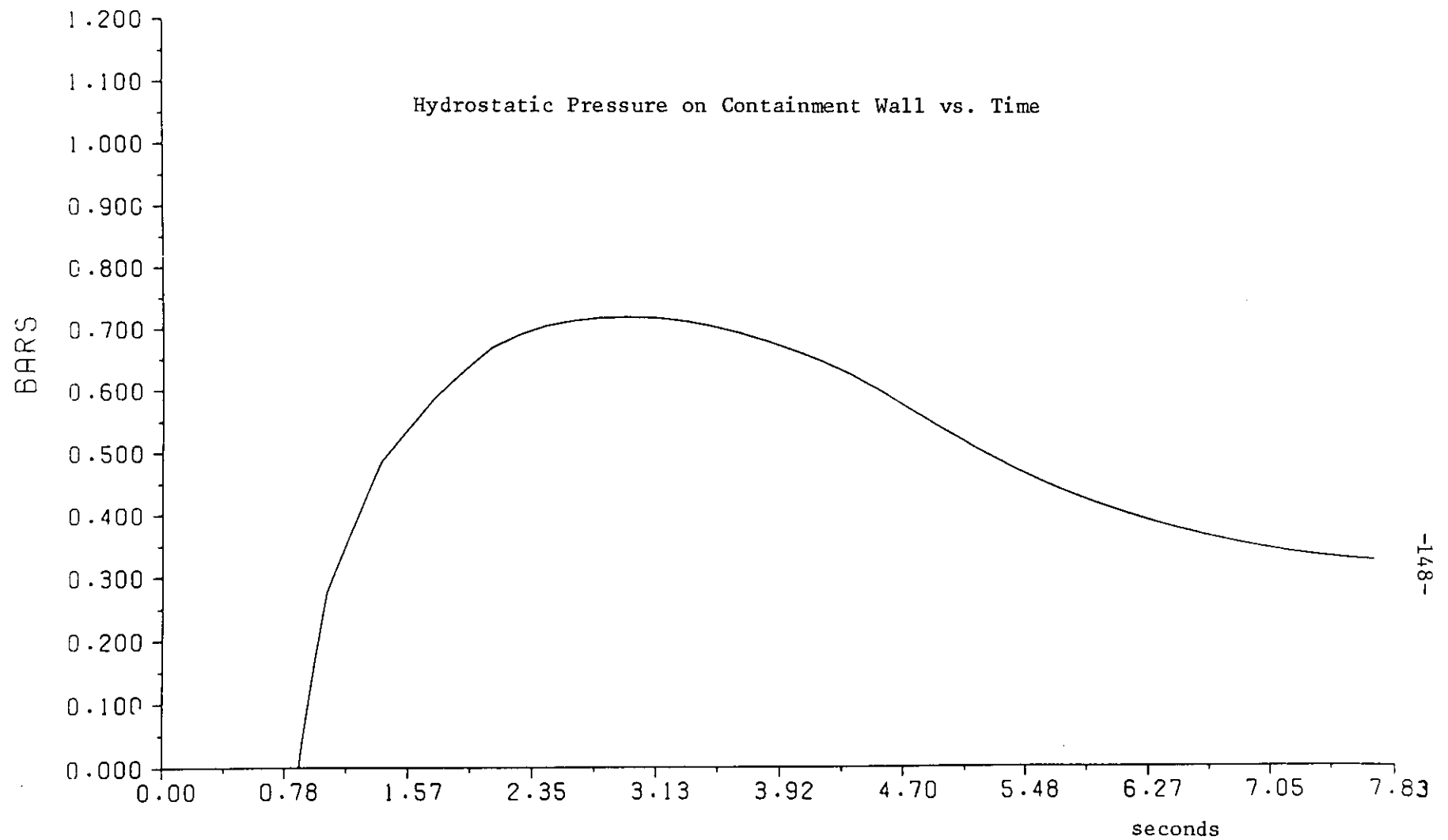
$$(R_p)^*_{\max} = \frac{(R_p)_{\max} - R_w}{R_w} \times 100\%$$

$$R_w/A = 2.0$$

4.7 Load and Impact Pressure on Containment Walls

The hydrostatic load or force on a vertical containment wall due to the liquid build-up after impact is shown in Fig. (4.21) as a function of time for an LNG spill. The load is proportional to the square of the liquid height at the wall. The corresponding hydrostatic pressure at the base of the wall, which is directly proportional to the liquid height at the containment wall, is shown in Fig. (4.20). This is predicted to be zero at the moment of impact as there is, at that time, no hydrostatic head. However, the pressure due to impact of the surging liquid at $t = T_w$, can be evaluated (section (1.3.8)). For the example in Fig. (4.20) where $H_0 = 30$, the impact pressure at $t = T_w$, assuming a liquid LNG density of 450 kg m^{-3} , is 5.3 bars which is far greater than the maximum hydrostatic pressure. Although the model cannot predict the impact pressure at the base of the wall for times, $t > T_w$, it is obvious that the hydraulic jump which forms and propagates away from the wall will absorb the impact pressure of the incoming surging liquid. The impact pressure can be expected to fall very rapidly after $t = T_w$. From previous experiments (7) it was noted that by the time the liquid height at the containment wall is at its peak, the shock has propagated sufficiently far away from the wall for the effect of the incoming liquid on the pressure at the base of the wall to be negligible. Thus, the maximum total pressure is likely to occur at the moment of impact and then fall sharply to rise again to the maximum hydrostatic value as defined by Fig. (4.20).

Table (4.2) shows, for different liquids, the initial impact



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Figure 4.20 $A = 30.00 \text{ m}$

$R_w = 60.00 \text{ m}$

$H_o = 30.00 \text{ m}$

$\theta = 90.00$

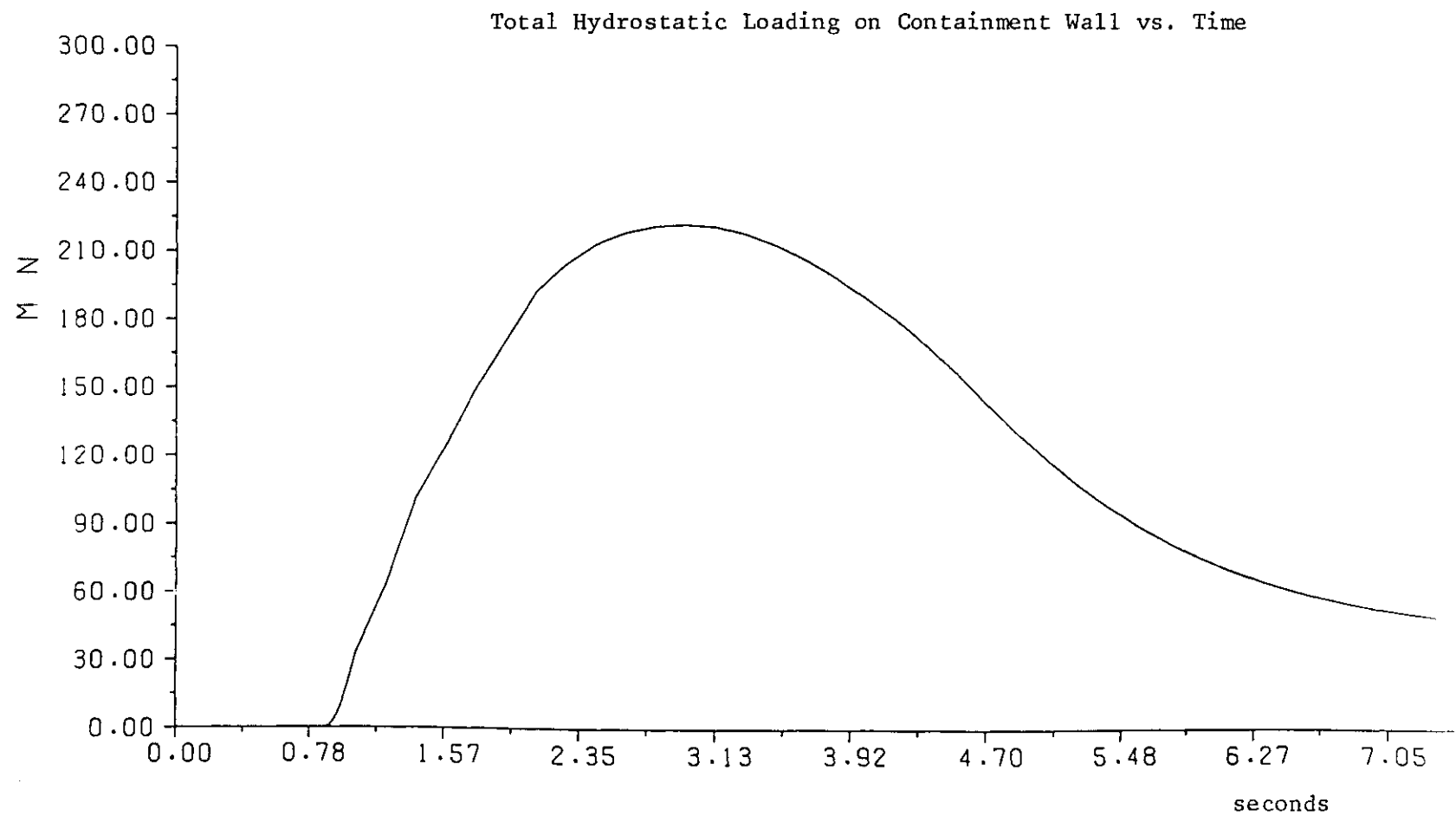


Figure 4.21 $A = 30.00$ m
 $R_w = 60.00$ m
 $H_o = 30.00$ m
 $\theta = 90.00$

Tank Contents	Density	Impact Pressure at T_w /bars	Maximum Hydrostatic Pressure in bars	Maximum Hydrostatic Loading in MN
LNG	450	5.3	0.7	220
LPG	600	7.1	0.9	294
Ammonia	690	8.1	1.1	337
Petroleum	800	9.4	1.3	391
Methanol				
Ethanol				
Crude Oil				
Naptha	1000	11.8	1.6	489
Water				

Table 4.2 Impact pressure, maximum hydrostatic pressure and maximum hydrostatic loading on the containment wall for a storage tank initially 30 m high and 40 m in diameter with the bund 20 m from the edge of the tank.

pressure, maximum hydrostatic pressure and load for a tank initially 30 m high and 40 m in diameter with a containment wall 40 m from its centre.

CHAPTER 5

Conclusions

5.1 Appraisal of the Model

The suitability of the mathematical model developed in this study and its applicability to the description of a catastrophic failure of a cylindrical storage tank within a bunded area will depend mainly on the validity of the following four assumptions discussed previously:

- 1) negligible vertical acceleration in the flow,
- 2) negligible effect of ground resistance on the flow,
- 3) inviscid flow and
- 4) axisymmetric collapse.

Short of considering exceptional (and most unlikely) circumstances for which the Reynolds number is very low, the third assumption is certain to hold for all industrial and experimental cases, as was explained in Chapter (1). The fourth assumption is almost a prerequisite to the viability of this study. The problem of an asymmetric column collapse is very complex and difficult to solve mathematically. Given that no complete 3-dimensional model is as yet generally available, modelling the consequences of a CTF as an axisymmetric column collapse was therefore considered to be the most worthwhile next phase for investigating spills of this type. Furthermore, considering only the initial spread, not

overflow, the highest overall rate of escape of liquid will occur if the column collapse is completely unhindered, i.e. axisymmetric.

The first assumption, and to a lesser extent the second, are however less trivial and this determines foremost the limitation of the model as a way of describing the consequences of a CTF. As was argued in the first chapter, assuming the vertical accelerations to be negligible leads to a satisfactory description of the motion away from the original location of the tank. In particular the accurate prediction of the liquid level at the containment wall is of crucial importance. This factor will determine the required bund height, shape and strength if complete containment, in the event of a CTF, is required. It will also determine the amount and nature of overflow for a prescribed bund height.

In addition, as was argued in section (1.3.6), vertical acceleration contributions can be neglected for a small range of inclined containment walls close to the perpendicular.

Given the appropriate conditions, the effect of ground resistance may not have significant influence on the motion when considering cryogenic liquids such as LNG where film boiling may occur. For other liquids, where no vapour layer is present between the ground and the flowing liquid, over the short duration when the important part of the motion takes place, ground resistance may have a non-negligible effect. The implications of including vertical acceleration and ground resistance effects in a mathematical model will be discussed in more detail in the last section.

It is believed that this axisymmetric model now provides the most satisfactory results available to date for predicting the

consequences of a CTF. The limits of applicability of this idealized model, for any experimental or industrial situation, may be summarized by recapitulating the following requirements, which must be taken into consideration when the results presented in this work are to be used:

- (1) The column must be squat ($H_0/2A < 1$) so that vertical accelerations may be neglected.
- (2) The containment wall must be sufficiently far from the tank ($R_w/A < 1\frac{1}{2}$) so that the duration of flow is long enough for the dissipation of initial vertical motion.
- (3) At containment walls, the deviation of the angle of inclination from the vertical must be small ($|90^\circ - \theta| < 20^\circ$). Otherwise vertical acceleration contributions become significant at the wall due to the additional vertical boundary restriction imposed on the flow.
- (4) The Reynolds number must be large ($Re \gg 1$) for viscous effects to be ignored.
- (5) The Weber number must be large ($We \gg 1$) for surface tension effects to be neglected.
- (6) The Chézy resistance factor must be small ($K < 10^{-6}$) for ground resistance effects to be negligible (8).
- (7) Account must furthermore be taken of the fact that the model assumes an axisymmetric flow pattern.

Having established the limits of applicability of the plane and axisymmetric theoretical models, the accuracy of the numerical solution must be considered. The results obtained by applying the

numerical MOC, developed in this work, to the plane model agree exactly with the analytical solutions available for regions III and IV. Furthermore, this numerical method is also in good agreement with Greenspan's results for the plane model in all regions of the flow. The application of the numerical MOC in region V also validates the use of the less rigorous (but also less costly) level shock assumption.

The accuracy of the results predicted by the axisymmetric model can only be argued for on the basis that the identical numerical techniques used in the plane model lead to a correct solution of the similar modelling equations. The additional numerical methods necessary for the solution of the more complicated axisymmetric model can only be justified by the arguments for their application, which have been discussed at length in the first three chapters.

Comparison with the theoretical work of Hill et al (16), who modelled both the plane and axisymmetric cases and included vertical variation in the liquid velocity, is not possible primarily because they considered the collapse of a liquid column onto a quiescent bed of liquid (identical to that in the tank) rather than onto a dry surface. This results in a very different flow pattern. They also presented results for the plane model only.

Comparison with experimental results, such as undertaken by Greenspan and Young (14) for the plane model, can at this stage not be made for the axisymmetric model since no comprehensive results are as yet available. Even though experimental data cannot absolutely guarantee the accuracy of the theoretical results, they

do give an indication of the validity of the trends established by the model. The results obtained from the axisymmetric model are as would be expected for a model of this type, i.e. a radially dispersive flux of mass and momentum is predicted, in contrast with the channel or plane type flow indicated by the model considered by Greenspan and Young.

5.2 Discussion of Results

It is thus believed, given the greater similarity between the model and the event of a cylindrical tank collapsing, that the axisymmetric model is superior to the plane model for describing a CTF. The use of the plane model to describe the consequences of flow resulting from a CTF can be misleading, as is shown by Greenspan's extrapolated conclusion, about the application of the plane model to cylindrical storage tank collapses (Liquefied Gases Safety Report to the Congress of the United States, [1978, (22)]). Greenspan expected (correctly) that the liquid height at the bund would be lower when considering radial flow from a collapsing cylindrical tank than when considering linear channel-type flow from a collapsing rectangular reservoir. This effect is clearly demonstrated in Chapter (4) : the axisymmetric model predicts a significantly lower liquid level at the bund than does the plane model. However, Greenspan goes on to conclude that in the case of a cylindrical storage tank surrounded by an "equal volume" specification bund of height, $H_b = H_d$, the overflow can be predicted by the plane model. This is clearly erroneous : Fig. (4.12) shows to what extent the overflow predictions from the two

models differ for this case ($H_b = H_d$). The reasoning behind Greenspan's conclusion lies with the fact that an axisymmetric model would entail a lower value of H_d than the plane model (see section (4.4)). He assumes that the increased overflow effect of having a lower bund height, H_d , in the axisymmetric would be cancelled out by the decreased overflow effect due to the lower liquid level at the bund predicted by the axisymmetric model. In other words Greenspan expected that these opposing effects in an axisymmetric model would counteract each other sufficiently to allow the use of a plane model for predicting overflow with bunds of this type. It has however been shown in this study that the axisymmetric model predicts very different results from the plane model and that the effect of the lower liquid level at the bund is the overriding factor.

Since the liquid height at the containment wall is predicted as significantly lower by the axisymmetric model than by the plane model, related quantities, such as the required bund height for complete containment, load on the wall and overflow (in the case of a bund of limited height), will also be lower. Other parameters however, are predicted to be the same by both models, such as the initial impact pressure which is a function of the velocity of the forward moving front.

Furthermore, it is apparent from the results of the axisymmetric model that the maximum liquid level at the bund decreases significantly with the increased distance between the bund and the tank. Thus because of the decrease in head of liquid above the top of bund, overflow decreases markedly as a function of

bund distance. Following on from this the distance to which the overflowing jet of liquid is projected beyond the containment wall can readily be appreciated and shown to be far greater when a bund of given height stands nearer to the tank. For bunds of "equal volume" specification, the maximum jet projection occurs when the bund is placed at about one tank diameter from the centre of the tank as is illustrated in Fig. (4.14). Thus if complete containment of a CTF spill is not essential and a bund of a certain (insufficient) height is to be built, whether to an "equal volume" specification or otherwise, it would be better, with the above considerations in mind, to place it well beyond one tank diameter from the tank centre.

As is to be expected flow over inclined bunds is found to increase sharply for most bunds with angles of inclination, $\theta < 90^\circ$. However, this effect is less pronounced for low bunds in which case the liquid head above the top of the bund rapidly becomes the main driving force as opposed to the increased horizontal surge effect due to the bund's inclination. In other words, low bunds soon become swamped after impact so that the effect of the inclination is relatively small. The increase in distance of overflow jet projection is found to follow a similar trend to that of the overflow itself when comparing vertical and inclined bunds of this type.

Greenspan and Young also studied the spill over inclined barriers ($\theta < 90^\circ$) and predicted a significantly lower increase in overflow with respect to vertical containment walls than predicted by this study. However since they do not mention their method of

calculation nor provide any explanation about the deficiency of the shallow water theory for modelling flow over inclined bunds, a qualified comment on the difference between their study and this work is not possible.

For bunds inclined in the opposite direction ($\theta > 90^\circ$), the model developed in this study predicts no significant reduction in overflow. It is found that the head of liquid above the top of the bund is the far greater determining factor. Whether this finding of only slightly improved containment is true, for the small range of angles (90° to 110°) for which the model is considered to be valid, is arguable and cannot be confirmed without comparison with experiment. However it is reasonable to expect that if a containment wall ($\theta > 90^\circ$) is steeply inclined and sufficiently high so that it is not swamped by the level of the liquid at the wall, the liquid is most likely to be 'thrown back' due to the shape of the containment wall. Consequently overflow should be markedly reduced. Therefore, as the model only predicts a slight decrease in overflow for all bunds inclined at more than 90° , it is important to include vertical acceleration terms when attempting to describe mathematically the impact against inclined bunds of this type.

In tank farms only one bund is often built to surround several tanks. On the assumption that only one tank within its perimeter is likely to fail at any one time, the bund is built to a height so that the volume it encloses equals or is slightly greater than the largest tank within its perimeter. In the event of a CTF however, given the impact pressure predicted by the model and with

added support from reports of actual CTF's (Qatar) an adjacent tank may readily fail under liquid impact, especially if the tank is only partially full. The height of the bund would under these circumstances of combined spills be totally insufficient. It is thus recommended that each tank has its own separate enclosure and that either a) the bund is built to a specification for complete containment in the event of a CTF or b) if overflow is acceptable, that the bund and layout are made to such requirements that the overflowing jet of liquid will not reach the tank in the adjacent enclosures. How each of these conditions can be satisfied has been detailed in this study.

5.3 Suggestions for Further Work

The shallow water theory as applied to the case of an axisymmetric column collapse and resulting flow within a circular bunded area can be assumed to have been taken to its limit in this study. The axisymmetric model developed here allows for the evaluation of the required bund height for complete containment, quantification of overflow in the case of bunds of limited height and impact and loading calculations on the containment wall. These values can be predicted for containment walls sufficiently far from the tank and for a small range of inclined bunds.

If improvements are to be made in the area of mathematical modelling of CTF's, the following three items are considered to be the most important:

- 1) Most storage tanks are surrounded by non-circular bunded areas.

This present model fails to describe the motion as soon as axial flow is lost, i.e. when the liquid hits the non-circular bund. From experiments (7) dealing with square containment areas it can be seen that liquid converges into the corners after initial impact at the midpoints of the four walls. It then proceeds to build up to a greater height and causes far greater overflow in the corner, than it would in the case of a circular banded area. It should be possible to model this situation using the SWE's in their asymmetric form. This type of model would furthermore allow for the description of a loss of liquid through a limited vertical section of the tank wall.

It should be noted at this point that because of the vastly increased overflow effect which occurs at the corners within a rectangular banded area, industrial practice is beginning to discourage their use. The rounding off of corners is recommended. A square banded area could be modified, for instance, by inserting walls at 45° across the corners so as to create an octagonal enclosure. The axisymmetric model developed in this study could, under these and other similar circumstances, approximate the resultant flow in the event of a CTF.

- 2) Including vertical variation in the motion would allow for the following improvements:
 - a) modelling of a CTF within a circular containment area situated close to the tank,

- b) accurate description of the flow over a wider range of inclined bunds than considered in this study and,
- c) modelling of non-squat storage tank collapses.

Case (a) may be of particular importance since the move in industry today is towards installation of relatively tall circular bunds close to the storage tank. The study of point (b) would shed some light on the expected contribution towards better containment that could be made by inclining a bund at angles, $\theta > 90^\circ$. Including vertical accelerations would also allow for the prediction of the impact pressure at times after the initial time of collision of the surging liquid against the containment wall. A mathematical model which incorporates vertical acceleration terms would require the solution of Euler's equations (Eqs. (1.3.3) and (1.3.4)) and continuity (Eq. (1.3.2)), perhaps by means of a finite difference scheme.

- 3) Neglecting ground resistance contributions in this model should of course have the effect, if any, of predicting more conservative results. However, it would be useful to calculate the extent to which ground resistance affects the flow, particularly in circumstances for which the boundary layer is significant as determined by Whitham (29). The incorporation of the Chézy resistance term as defined by Dressler (8) would be relatively straightforward in a finite difference scheme such as mentioned above. It would however be more difficult to perform a solution similar to Dressler's or Whitham's (which they applied to the plane model) to the axisymmetric model (or

for that matter to an asymmetric model) on the basis of the shallow water theory. Incorporation of ground resistance effects will furthermore open the way for prescribing quantitatively, practical measures for slowing down the surging liquid front and thus reduce impact and overflow at the containment wall.

Two mathematical options for further modelling thus become apparent : the first is to solve the 3-D SWE's for the asymmetric problem of spill and containment for dealing with point (1), and the second is to solve Euler's 2-D equations and continuity, thereby including vertical acceleration. The latter option has the additional advantage of including, with relative ease, the problem of ground resistance. The potential improved modelling achievable through either of these two options can, from the above comments, be seen to be mutually exclusive. The former would allow for the advantages outlined in point (1) above and the second option for advantages given in points (2) and (3). In addition, with the experience gained from the implementation of the numerical methods used in this study, it is considered that the second option is easier to carry out and more effective with regards to cost and effort although both options would require greater mathematical and computational endeavour than was necessary for this study. It is therefore recommended that the above alternatives be carefully considered before undertaking the solution of an improved model.

Further work using the results developed in this study lies in the comparison with experimental data. Given that the results

presented in this thesis are normalized with respect to the tank and containment wall dimensions and that the liquid height and velocity at any point in space-time are proportional to the original column height, direct comparison with experimental values is straightforward.

Behaviour of the flow, more easily modelled by experimental means, such as ground resistance effects, asymmetric flow within rectangular banded areas and impact against or flow over inclined barriers will give an indication as to the suitability of the approximations used in this theoretical study and consequently the necessity for further mathematical modelling can be qualified.

As has been mentioned previously LNG and LPG form a large proportion of the world's bulk storage. In the event of a CTF the cryogenic nature of these liquids may have an important effect on the flow and therefore on the requirements for containment. To accurately model this situation would require the incorporation of effects resulting from boiling phenomena and thus a complete re-assessment of the theory is necessary.

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APPENDIX 1

Potential Liquid Flow and Explosive Vapour Energy Estimations
for an LPG Storage Tank

The potential destructive capacity of liquid held within a storage tank can be compared to the potential explosive energy of a combustible vapour mixture inside the tank by considering for example these alternatives for an LPG storage tank 30 m high and 60 m in diameter.

Such a tank full of liquid can readily be shown to have a total potential hydrostatic energy of about 1.5×10^{10} J. The same tank filled with a stoichiometric propane/air mixture at the boiling point of propane would have a potential energy of combustion of 3.8×10^{11} J.

However a far more realistic situation would be that of a tank, for instance, 90% full of liquid within a vapour mixture in the top 3 m of the tank. The above calculations for this case leads to values of 1.2×10^{10} J and 3.8×10^{10} J respectively.

The likelihood of a stoichiometric propane/air mixture forming in the vapour space of a tank can be considered to be very small indeed. The richest mixture in which an explosion could propagate would contain 9.5% fuel by volume. This mixture would release a maximum energy of combustion of 2.2×10^8 J from the initial vapour phase in the top of the tank. A more realistic value, however, would be about 10% of this value since the tank roof would fail almost immediately after the explosion is initiated due to the pressures produced under such conditions.



The potential energy of a liquid released in the event of a CTF can be more realistically estimated from practical evidence. In one reported incident the liquid surge swept part of the tank shell and roof, weighing in excess of 350 tonnes, to a distance of about 100 m. It can be shown that the energy required for this is about 3.2×10^8 J.

Furthermore, since the tank shell was swept away by a limited radial section of the liquid tank contents and since the effect of a vapour cloud explosion within the tank is expected to dissipate more rapidly with distance, the potential damage over the whole range of a bunded area (and beyond) is likely to be greater by liquid forces than by explosive forces for most conceivable situations.

APPENDIX 2

A History of Well Documented Catastrophic Tank Failures

The catastrophic failure of a storage tank is a rare event and has not always been well documented. This is mainly due to the shocked state of witnesses at a scene of devastation and to sensitive nature of the consequences. A few of the better known and publicized occurrences are summarized below.

i) In Cleveland Ohio, in 1944 a 4200 m³ LNG storage tank failed due to improper design. The spreading liquid pushed debris to a distance of 200 yards from the original site of the tank. The liquid took to the drainage canals and ignited. It reached neighbouring inhabited areas and the resulting fires and explosions caused the death of 130 people. An adjacent tank collapsed due to the heat, releasing a further 2100 m³ of LNG. The estimated financial loss was put at \$7 million in 1945 prices. This accident virtually stopped the use of LNG in the U.S.A. for 20 years. One recommendation made by the Bureau of Mines report (9) on the disaster was: "Storage containers for liquified gases should be isolated from other parts of the plant and should be provided with dikes (bunds) large enough to confine the entire contents of the tank in case of failure."

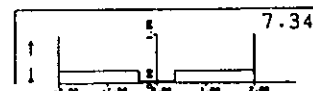
ii) In Addyston Ohio, in 1976 a 2000 m³ methanol tank was struck by lightning, igniting the vapour mixture inside causing the tank wall to yield at the bottom flange. The entire tank shell and roof



lifted 25 m upwards and landed on the surrounding bund. The liquid overflowed on all sides over the containment wall, caught alight and set fire to two other tanks in adjacent enclosures, one of which was fortunately an empty (methanol) tank. The other contained 2500 m³ of acrylonitrile which eventually burst. Its spilled contents ignited immediately. Only the presence of the empty methanol tank prevented the blaze from spreading to the rest of the tank farm.

iii) In 1977 a 45000 m³ storage tank 90% full with liquefied propane gas failed catastrophically in Qatar in the Middle East. The tank shell and part of the roof were carried over the surrounding bund 100 m away. The propane was ignited destroying the entire liquefaction storage and cracking plant. Seven people were killed and villages one mile away were scorched by the fire.

A survey of storage tank failures published in 1954 by the American Petroleum Institute (24) reported on 33 such incidents. 11 of these were complete failures, 4 of which occurred during water testing. The other tanks contained either crude oil or gas oil at the time of collapse. All these tanks had capacities in the range of 8000 to 25000 m³. In 7 out of the 11 incidents of particular interest, the description of the tank failure and subsequent effect on the tank shell was reported. In only one of these 7 detailed cases was the tank not swept away by the surging liquid. In the other 6 cases it was reported that the tank either failed at the bottom and ripped around the circumference of its base, or that the tank shell ripped up the side. In both types of



failure the shell was folded open by the collapsing liquid column and washed away, mostly halting on contact with the surrounding bund.

APPENDIX 3

Analytical Solution of the Shallow Water Equations for Unhindered

Collapse in the Plane Model

The unhindered collapse of a plane reservoir or the classical dam break problem have been previously described by Stoker [1957, (27)]. The analytical solution presented here is valid until the liquid reaches one of the boundaries such as the back wall of the tank or the containment wall. This is illustrated in Fig. (1.2). As shown by Ritter (23), the SWE's for this case in cartesian coordinates are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + 2c \frac{\partial c}{\partial x} = 0$$

(A1.1)

$$\frac{2\partial c}{\partial t} + 2u \frac{\partial c}{\partial x} + c \frac{\partial u}{\partial x} = 0$$

The initial conditions for $t \leq 0$ are:

$$c = C_0 \text{ if } 0 \leq x \leq A$$

(A1.2)

$$c = 0 \text{ if } x > A$$

$$u = 0$$

where $C_0 = (gH_0)^{\frac{1}{2}}$



On dimensional grounds a similarity solution of the form,

$$\eta = \frac{x-A}{t} \quad (A1.3)$$

is sought. Eqs. (A1.1) then yield:

$$(u - \eta) \frac{\partial u}{\partial \eta} + 2c \frac{\partial c}{\partial \eta} = 0 \quad (A1.4)$$

$$2(u - \eta) \frac{\partial c}{\partial \eta} + c \frac{\partial u}{\partial \eta} = 0$$

And hence by resubstitution

$$u = K + \frac{2}{3} \eta \quad (A1.5)$$

$$c = K - \frac{1}{3} \eta$$

where K is a constant.

Using Eqs. (A1.1) in their characteristic form:

$$u \pm 2c = \text{constant} \quad \text{on} \quad \frac{dx}{dt} = u \pm c \quad (A1.6)$$

it can be seen, taking the positive or 'forward' characteristic equation (Eq. (A1.6)) that

$$3K = \text{constant} \quad \text{on} \quad \frac{dx}{dt} = 2K + \frac{1}{3} \eta \quad (A1.7)$$



As $c = C_0$, $u = 0$ at $x = A$, $t = 0$, so

$$K = \frac{2}{3} C_0 \quad (A1.8)$$

Hence:

$$u = \frac{2}{3} (C_0 + \frac{x-A}{t}) \quad (A1.9)$$

$$c = \frac{1}{3} (2C_0 - \frac{x-A}{t}) \quad (A1.10)$$

Also from Eq. (A1.7)

$$\frac{dx}{dt} = \frac{4}{3} C_0 + \frac{1}{3} \frac{x-A}{t} \quad (A1.11)$$

or

$$x = B t^{1/3} + 2C_0 t + A \quad (A1.12)$$

where B is a constant.

It can be seen from Eqs. (A1.9) and (A1.10) that B must be zero, otherwise u, c become unbounded as $t \rightarrow 0$. Hence the leading edge of the forward surging liquid is described by:

$$x = X_f = 2C_0 t + A \quad (A1.13)$$

where from Eqs. (A1.9) and (A1.10),

$$u(x = X_f) = 2C_o \quad (A1.14)$$

$$c(x = X_f) = 0$$

This defines the forward front until it hits the bund at X_w .
 Arising from a similar procedure, a description of the depletion
 wave into the tank or the backward moving front is obtained:

$$X_b = A - C_o t \quad (A1.15)$$

where

$$u(x = X_b) = 0 \quad (A1.16)$$

$$c(x = X_b) = C_o$$

The depletion front hits the back wall at $x = 0$ where these
 equations cease to be valid.

To summarize this solution for the plane model in region
 III, the equations describing the motion of unhindered reservoir
 collapse (Eqs. (A1.9) and (A1.10)) can be mapped onto a
 characteristic diagram. See Fig. (A5.1).

APPENDIX 4

The Extent of Vertical Accelerations in the Initial Stages of the
Collapse of a Cylindrical Liquid Column

In the first instances of column collapse it is evident from experimental work that the motion in and near to the tank is dominated almost entirely by vertical velocity terms and not by horizontal ones as assumed in the shallow water theory. This is shown very clearly by the high speed film records of a small scale collapsing column experiment as shown in Figure A4.1. Frames (A) to (H) show the axisymmetric collapse of a virtually intact vertical cylindrical column of water. The motion is nearly entirely vertical within and also (but to a lesser degree) in the immediate vicinity of the column. The motion at the surging front is nearly totally horizontal. From such records as presented in Fig. (A4.1) it was determined that vertical accelerations are at least an order of magnitude less than horizontal accelerations in all areas outside the circular perimeter - one third of the original column radius away from the edge of the cylinder. The vertical acceleration of the liquid is at least equal to the horizontal acceleration within a quarter of the cylinder radius away.

From frames (A) to (D) it will be noticed that the collapsing liquid column has a rippled profile. The cause of this might be the vibrations of the lifting mechanism transmitted to the rising containment cylinder which originally holds the liquid at rest before collapse is initiated.

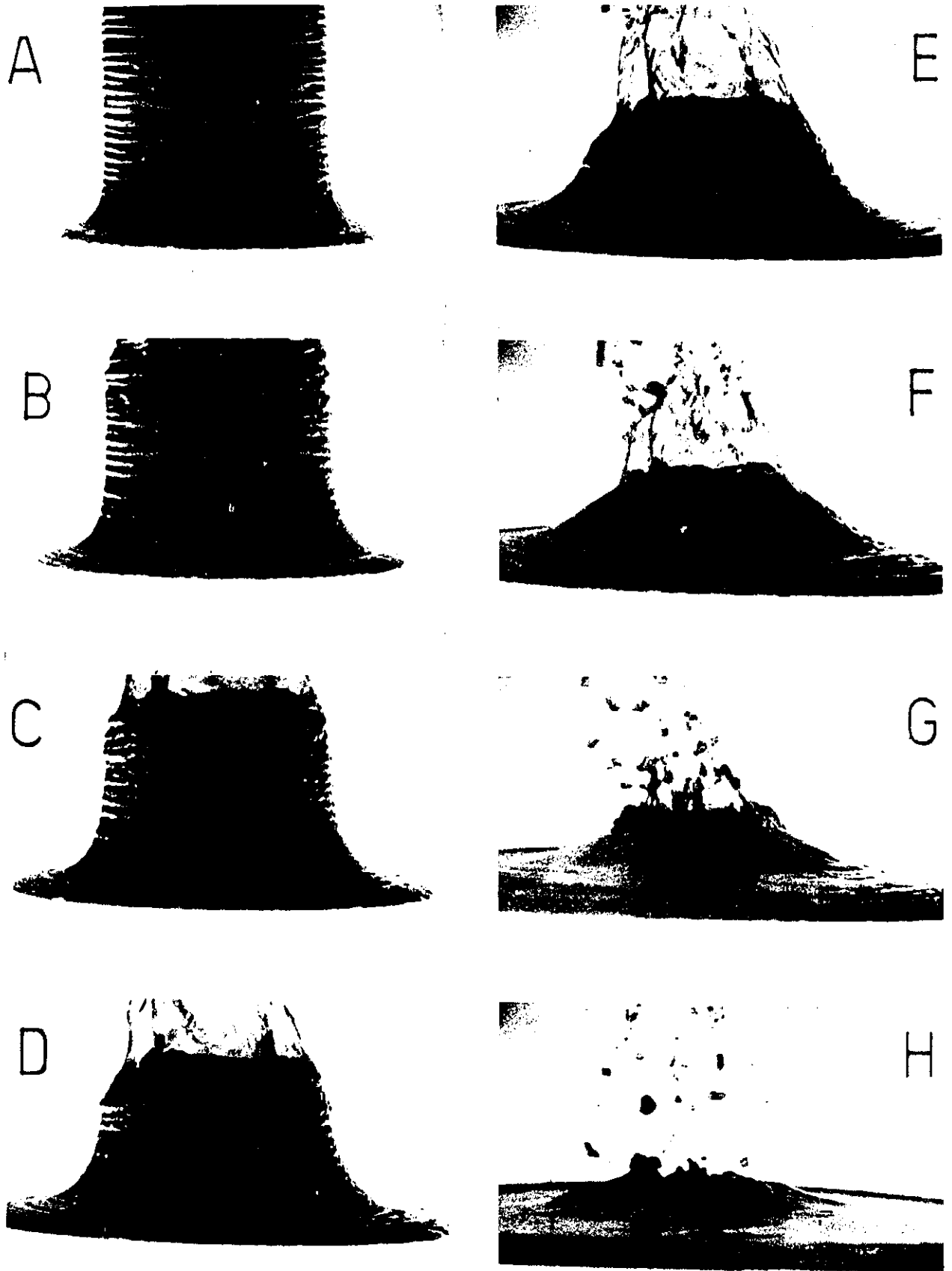


Figure A4.1



APPENDIX 5

Characteristics

The concept of characteristics can suitably be explained by their application in region III, where the liquid spread is unhindered. For reference see Stoker (3). For the sake of convenience the plane model will be used for illustrative purposes allowing for an analytical solution of all characteristics within each family. Use of the axisymmetrical model would require the discrete evaluation of all characteristic curves other than the leading ones.

In the problem under examination the general solution for the characteristic curves is:

$$\frac{dx}{dt} = u \pm c \quad (A5.1)$$

depending on whether the motion is considered to be forward or backward respectively. Fig. (A5.1) shows the translation of the characteristic curves into a height profile curve of a collapsing reservoir. The leading forward and backward characteristic shown here define the position, in time, of the forward surging liquid front and the depletion wave front into the reservoir. The actual height profile between these two points, X_b and X_f is defined by the family of characteristics which span the radial section bounded by the two leading characteristics. This so called 'fan' of characteristics of both families is shown in Fig. (A5.2). The

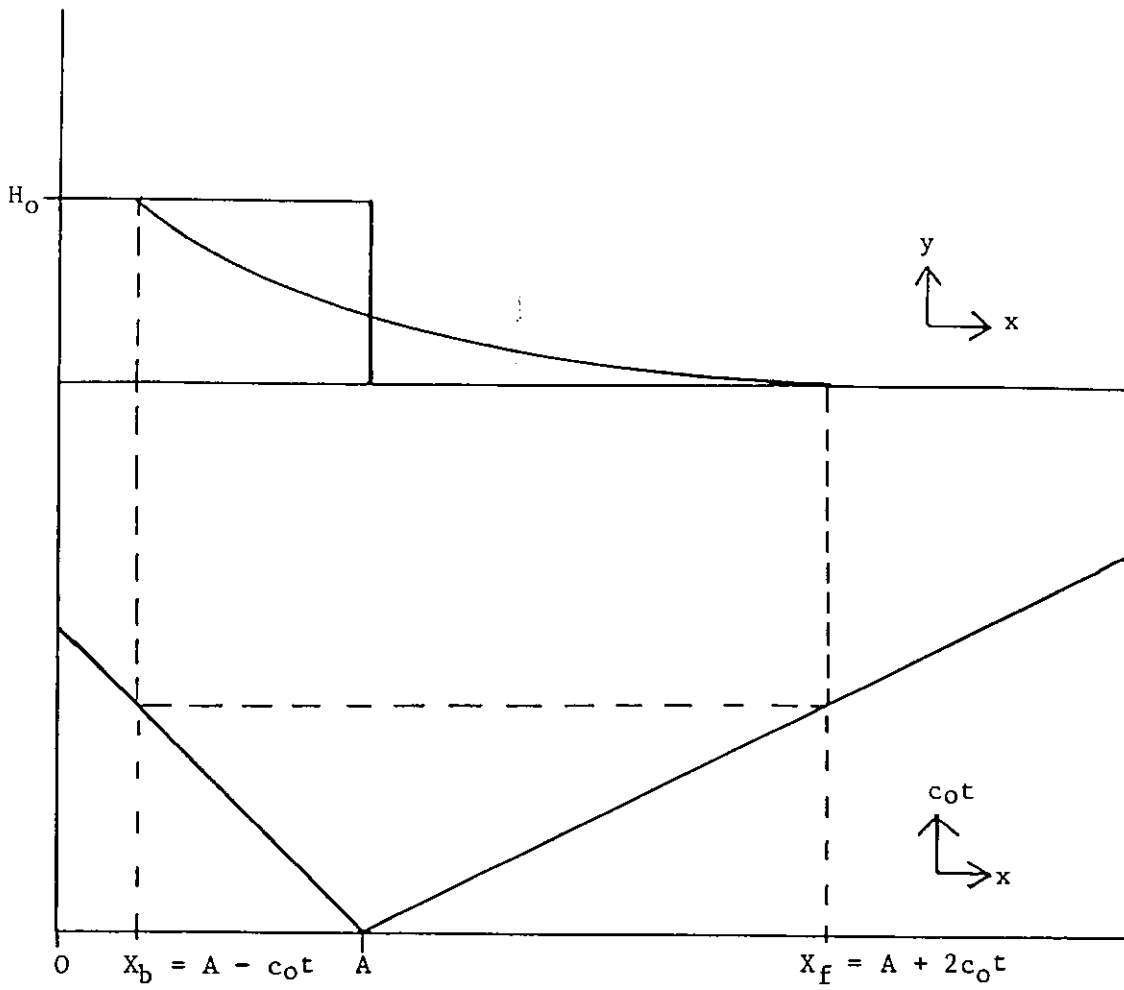


Figure A5.1

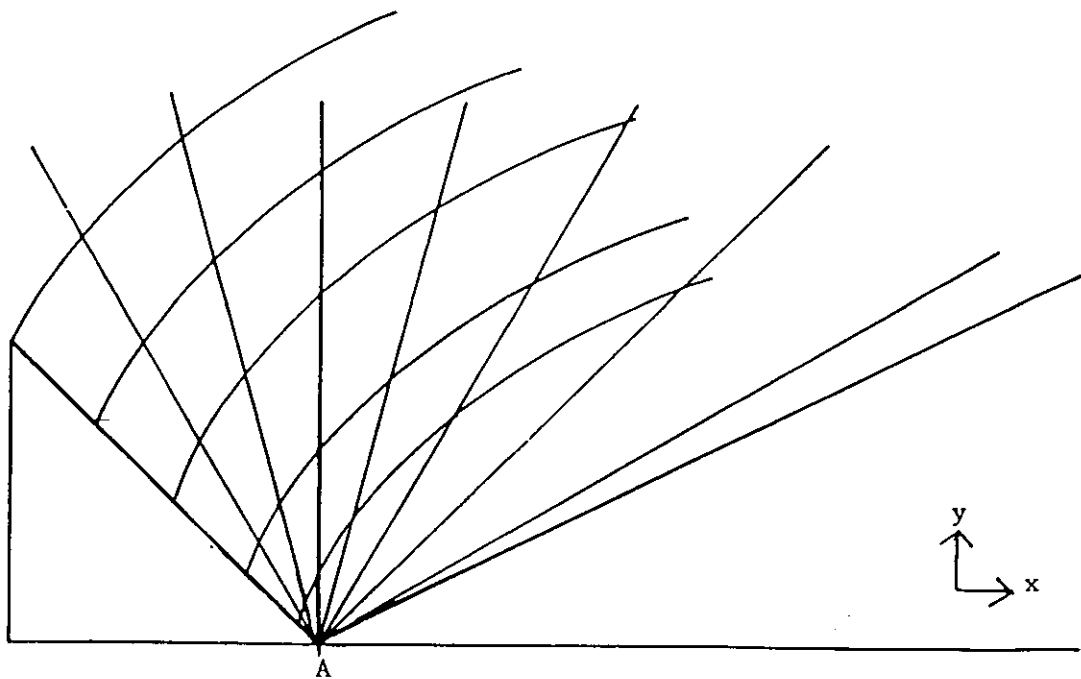


Figure A5.2

backward or negative characteristics radiate out from $x = A$, $t = 0$ and are defined by

$$X_n = A + B_n t \quad (A5.2)$$

The positive characteristics start from the leading negative characteristic and asymptotically attain the slope of the leading positive characteristic. These are defined by:

$$X_p = A + B_p t^{1/3} (2C_0 \frac{t}{x} - 1)^{-1} \quad (A5.3)$$

B_n and B_p are parameters and A and C_0 are given by the dimensions of the original reservoir. Eqs. (A5.2) and (A5.3) are the solutions of Eqs. (A5.1) in the space time region of initial reservoir collapse and unhindered spread.

Each region, as was described in Fig. (1.6)), has its two families of crossing characteristics, along which constant values, $u \pm 2c$ are carried. This provides a solution by means of two simultaneous equations at each point of intersection in space time, formed by the two families of characteristics. In each region, other than the region of initial collapse and spread for the plane model, the characteristic curves need to be defined numerically as analytical descriptions such as in Eqs.(A5.2) and (A5.3) are practically impossible. In the axisymmetric model this is true throughout the entire process.

APPENDIX 6

Shock Equations

The shock conditions appropriate to the reflection of a hydraulic jump from a rigid vertical wall are outlined below. Use is made of the treatment by Courant and Friedrichs [1948, (5)], for shock propagation in gases. Greenspan and Young's (14) series solution valid for short times immediately after impact against the containment wall is also presented.

Consider the following fully developed hydraulic jump or shock travelling back from the containment wall in Fig. (A6.1).

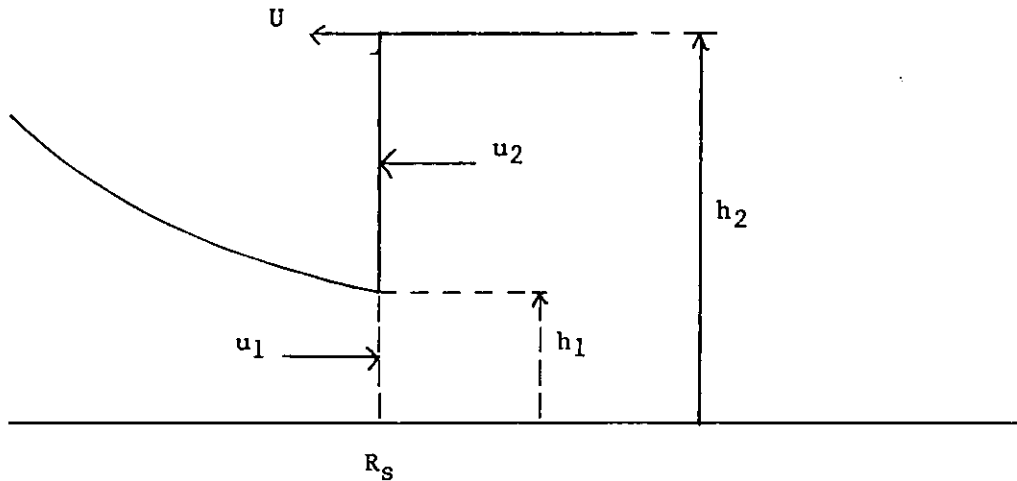


Figure A6.1

Subscripts 1 and 2 denote the motion in front of and behind the shock. U is the shock velocity and R_s the shock position in time. It should be noted that the values of the incoming motion (u_1 and h_1) are independent of the motion behind the shock. u_1 and

h_1 are given analytically in the plane model by Eqs. (1.2.2) and numerically in the axisymmetric model.

a) General Shock Conditions

Conservation of mass across the shock yields:

$$\rho_1(u_1 - U)h_1 = \rho_2(u_2 - U)h_2$$

or

$$(u_1 - U)h_1 = (u_2 - U)h_2 \quad (A6.1)$$

since incompressible flow is assumed.

Conservation of momentum gives:

$$\rho_1 U_1(u_1 - U)h_1 - \rho_2 u_2(u_2 - U)h_2 = \frac{1}{2}\rho_2 g h_2^2 - \frac{1}{2}\rho_1 g h_1^2 \quad (A6.2)$$

or by rearrangement

$$u_1 v_1 h_1 - u_2 v_2 h_2 = \frac{1}{2}g(h_2^2 - h_1^2) \quad (A6.3)$$

where $v = u - U$

From Eq. (A6.1) the mass flux is defined as:

$$m = v_1 h_2 = v_2 h_2 \quad (A6.4)$$

Therefore, Eq. (A6.3) becomes:



$$m(v_1 - v_2) = \frac{1}{2}g(h_2^2 - h_1^2) \quad (A6.5)$$

or using Eq. (A6.4)

$$v_1 h_1 (v_1 - \frac{v_1 h_1}{h_2}) = \frac{1}{2}g(h_2^2 - h_1^2) \quad (A6.6)$$

Rearranging yields:

$$U = u_1 - \left[\frac{gh_2}{h_1} \frac{h_2 + h_1}{2} \right]^{\frac{1}{2}} \quad (A6.7)$$

or using $h = c^2/g$

$$U = u_1 - \left[\frac{c_2^2}{2} \frac{c_2^2 + c_1^2}{c_1^2} \right]^{\frac{1}{2}} \quad (A6.8)$$

Hence

$$U = u_1 - c_2 [(M^2 + 1)/2]^{\frac{1}{2}} \quad (A6.9)$$

where $M = c_2/c_1$

Eqs. (A6.4) and (A6.7) combine to yield:

$$u_2 - U = \frac{h_1}{h_2} \left[\frac{gh_2}{h_1} \frac{h_1 + h_2}{2} \right]^{\frac{1}{2}} \quad (A6.10)$$

Then using Eq. (A6.9),

$$u_2 = u_1 + \left[\frac{h_1}{h_2} - 1 \right] \left[\frac{gh_2}{h_1} \frac{h_1 + h_2}{2} \right]^{\frac{1}{2}} \quad (\text{A6.11})$$

or using $h = c^2/g$

$$u_2 = u_1 - c_1 (M^2 - 1) \left[\frac{1}{2} \left(1 + \frac{1}{M^2} \right) \right]^{\frac{1}{2}} \quad (\text{A6.12})$$

Eqs. (A6.9) and (A6.12) thus define the shock conditions for a propagating hydraulic jump in region V.

b) Series Solution for Short Times After Impact

Greenspan and Young have provided an exact explicit series solution for the shock formation and propagation which is valid for a short time after impact. The liquid velocity, u , and transformed liquid height, c , are represented by:

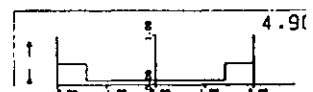
$$u = \tau f_0(\tau) + \tau^2 f_1(\tau) + \dots \quad (\text{A6.13})$$

$$c = \tau^{\frac{1}{2}} g_0(\tau) + \tau^{3/2} g_1(\tau) + \dots$$

where

$$\tau = \frac{t - T_w}{T_w}$$

and



$$\eta = \frac{X_w - x}{X_w \tau^2}$$

The position of the shock front is given by:

$$\eta_s = z_0 + z_1 \tau + \dots \quad (\text{A6.14})$$

The shock velocity is therefore defined by:

$$U = \frac{d}{d\tau} [\tau^2 \eta_s] = -2z_0 \tau - 3z_1 \tau^2 - \dots \quad (\text{A6.15})$$

Substituting Eqs. (6.13) and (6.14) into the shock equations, Eqs. (A6.9) and (A6.12) and using the solution of the SWE'S in region III, Greenspan and Young arrived at values for the above coefficients as follows:

$$f_0(\eta) = \eta \qquad f_1(\eta) = \frac{5}{3} \frac{\eta^3}{k_0^2} + \frac{2k_1^3}{k_0}$$

$$g_0(\eta) = k_0 \qquad g_1(\eta) = \frac{-\eta^2}{2k_0} + k_1$$

$$z_0 = \frac{8}{27 k_0^2} \qquad z_1 = -\frac{1}{4} \left[f_1(z_0) + \frac{6z_0}{k_0} g_1(z_0) + 8z_0 - 6z_0^2 \right]$$

$$\text{where } k_0 = \frac{2^{5/4}}{3^{1/2}}$$

$$\text{and } k_1 = \frac{-5(3)^{1/2}}{9(2)^{1/4}} (2^{1/2} - 1/9)$$

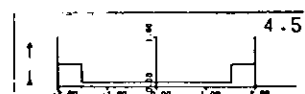


APPENDIX 7

Modification of the Initial Solution for the Shock Formation
(in Region V) for Inclined Containment Walls

As mentioned in section (1.3.6) modifications are necessary for the initial values, supplied by Greenspan and Young's series solutions, when considering inclined containment walls. The liquid velocity near to the wall and for a short time after impact is strongly dependent on the vertical displacement. In order to be able to apply the shallow water theory an average velocity near the wall is assumed as an approximation. The nature of the modification particular to both types of inclined bunds are now considered separately.

In principle it would be expected that, for a containment wall inclined at an angle, $\theta > 90^\circ$, the liquid will be reflected faster with increasing values of θ . The liquid height build-up would be limited by the overhanging wall, thus forcing it to propagate back more rapidly. Consider the initial conditions $R_s(T_w + \delta t)$ and $h_2(r, T_w + \delta t)$ for a vertical wall, as given by Greenspan and Young's series solution. See Fig. (A7.1). These initial values predict a very small variation in $h_2(r, T_w + \delta t)$ over the range R_s to R_w and therefore the hydraulic jump is considered to be level for this starting time. As was explained in section (1.3.6) the rate of change of the jump height is initially far greater than the rate of change of the bore position and therefore accurate initiation of the latter variable was found to be



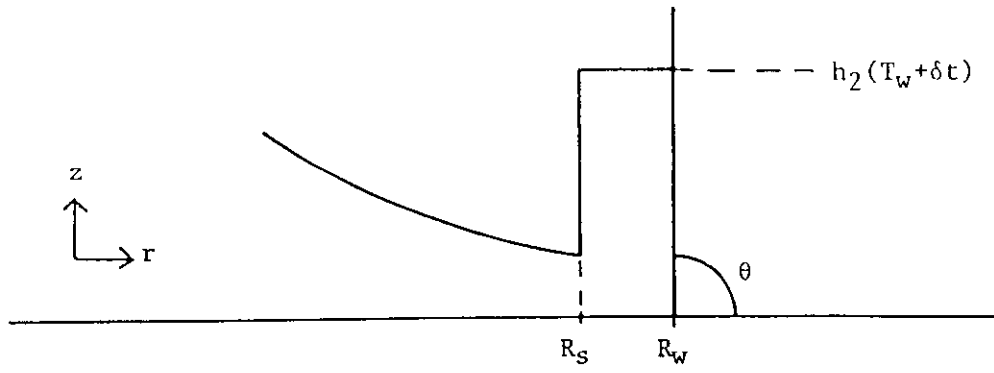


Figure A7.1 $\theta = 90^\circ$

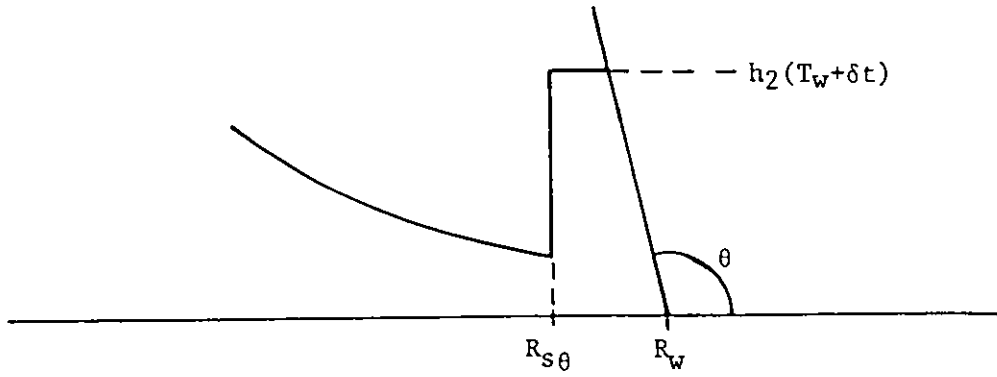


Figure A7.2 $\theta > 90^\circ$ $R_{s\theta} = R_s$

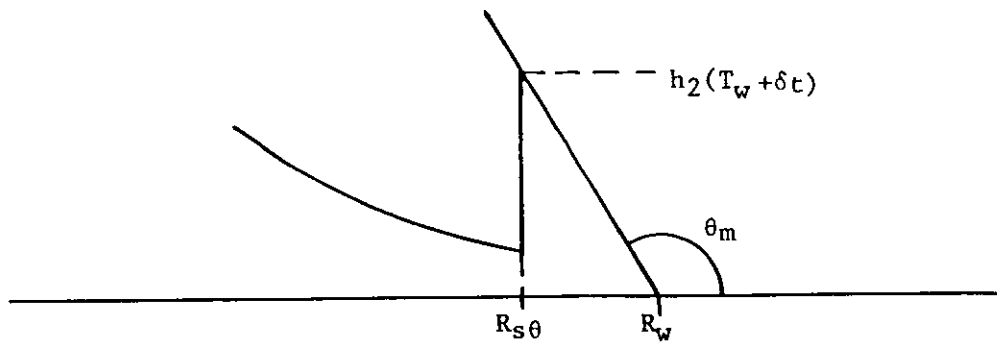


Figure A7.3 $\theta > 90^\circ$ $R_{s\theta} = R_w + h_2/\tan \theta$

unimportant. Thus for inclined containment walls $h_2(R_w, T_w + \delta t)$ is taken as for vertical walls and the bore position $R_s(T_w + \delta t)$ is amended assuming that the volume of liquid behind the shock is equal in both cases. The shock for containment walls where $\theta > 90^\circ$ can, under the above assumption, take two forms shown in Figs. (A7.2) and (A7.3). The value of the modified shock position, $R_{s\theta}$, is given by

$$R_{s\theta} = R_w^2 + \frac{-h^3 - 3R_w \tan^2 \theta h_2^2 + 3V \tan^2 \theta / \pi}{3 \tan^2 \theta h_2} \quad (A7.1)$$

where V is the volume of liquid behind the shock.

$$V = \pi A^2 H_0 - \int_0^{R_{s\theta}} h(r, T_w + \delta t) 2\pi r dr \quad (A7.2)$$

The integral is evaluated numerically using the values of the liquid height computed in region III. If the angle of inclination is so great that a solution for Eq. (A7.1) is impossible, then the initial bore height is reduced to a level where the length of the shock becomes zero, as shown in Fig. (A7.3), and the value of $R_{s\theta}$ is calculated accordingly.

The initial shock velocity, $U(T_w + \delta t)$ can obviously not be taken from Greenspan's series solution. It can, however, be estimated using backward differencing and by considering the



average distance the shock front travels between the face of the inclined wall and $R_{s\theta}$. Over the time span T_w to $T_w + \delta t$,

$$U(T_w + \delta t) = \frac{dR_s}{dt} \approx \frac{(R_{s\theta} - R_w - h_2/2 \tan \theta)}{2\delta t} \quad (A7.3)$$

As was mentioned in section (1.3.6) the liquid velocity behind the shock must, for $\theta \neq 90^\circ$, be a function of vertical displacement. To accommodate this inconsistency into the shallow water theory it is assumed that the liquid bounded within the 'triangular' section of the shock (see Fig. (A7.4)) has a uniform average velocity, u_a . The liquid velocity in the 'rectangular' section will vary linearly (since a level bore is assumed in the initial conditions) from u_a to u_2 with respect to r . u_2 is given by the shock condition, Eq. (1.3.30). Furthermore, the liquid velocity in the 'triangular' section is assumed to be uniform throughout the calculations in region V. This seemed the only justifiable approximation for use within the bounds of the shallow water theory. Only in the first instance of the motion in region V is the quantity of liquid in the 'triangular' section non negligible compared with that in the rectangular section. The overall error throughout the motion incurred by this approximation is, therefore, small. The value of u_a is computed by considering the conservation of mass and momentum over the 'triangular' section. Thus:

$$u_a(t) = 3P_f(t)\tan^2\theta/\pi h_2(h_2^2 + ah_2 + b) \quad (A7.4)$$



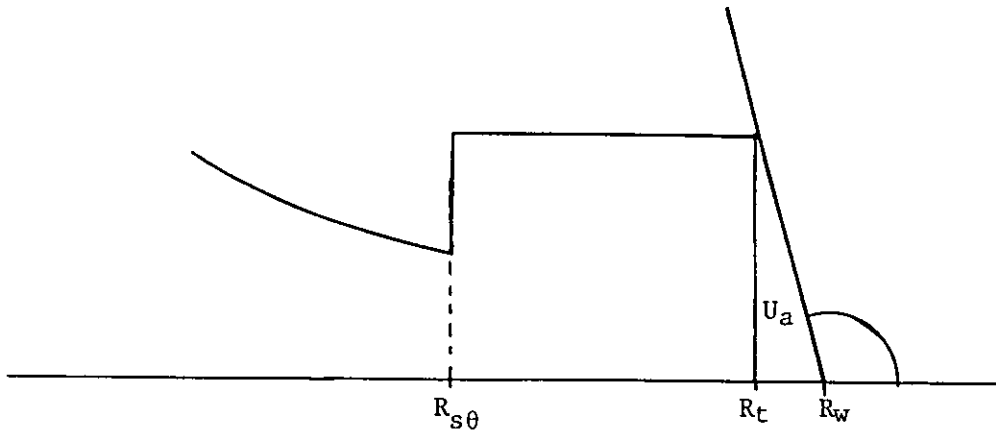


Figure A7.4

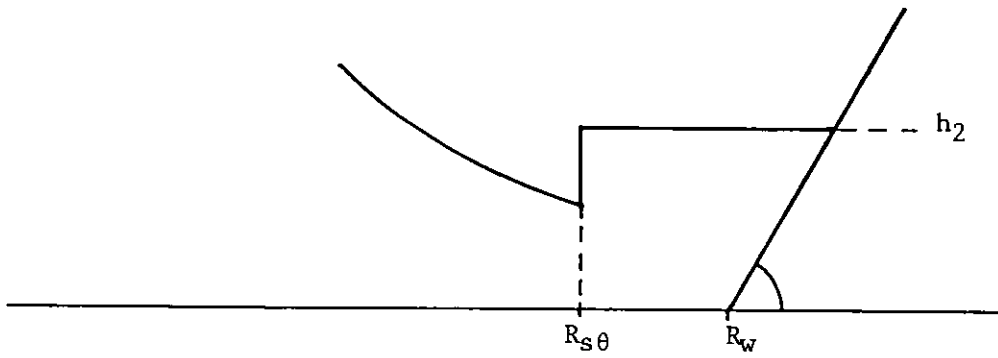


Figure A7.5 $R_{s\theta} < R_w$

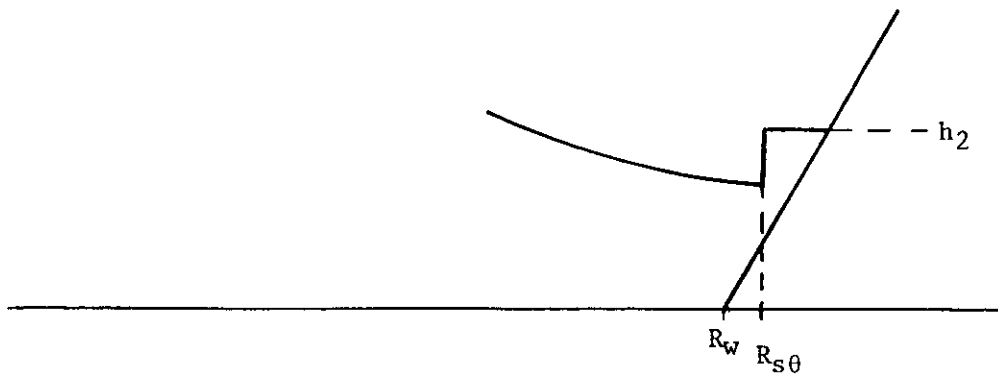


Figure A7.6 $R_{s\theta} > R_w$

where $a = 3 R_w \tan \theta$

$$b = 3 \tan^2 \theta (R_w^2 - R_t^2)$$

R_t is the value of r bounding the triangle. (See Fig. (A7.4)).

$P_f(t)$ is the momentum of the liquid bounded by the triangle. $P_f(t)$ is evaluated by considering the liquid flow in the absence of a containment wall and is computed numerically.

For the case $\theta < 90^\circ$ the initial shock position can either be situated before or behind $r = R_w$, as shown in Figs. (A7.5) and (A7.6). The modifications to the values of R_s , U , u_2 are performed in a similar manner to those presented for $\theta > 90^\circ$. For $\theta < 90^\circ$, $R_{s\theta}$ is given by Eq. (A7.1) when $R_{s\theta} < R_w$. If $R_{s\theta} > R_w$, $R_{s\theta}$ is calculated by solving the cubic equation:

$$R_{s\theta}^3 + cR_{s\theta}^2 + d = 0 \quad (A7.5)$$

where $c = -3h_2/4 \tan \theta$

$$d = - (R_w + \frac{h_2}{\tan \theta})^3 - \frac{3V}{\pi \tan \theta}$$

and V is given by Eq. (A7.2).

For relatively small values of θ a shock does not necessarily form immediately after impact and at least for some time, the liquid merely flows up the face of the containment wall. In this case (see Fig. (A7.7)) it is nonetheless assumed, for the purpose of numerical computation, that a shock is present at R_i , the



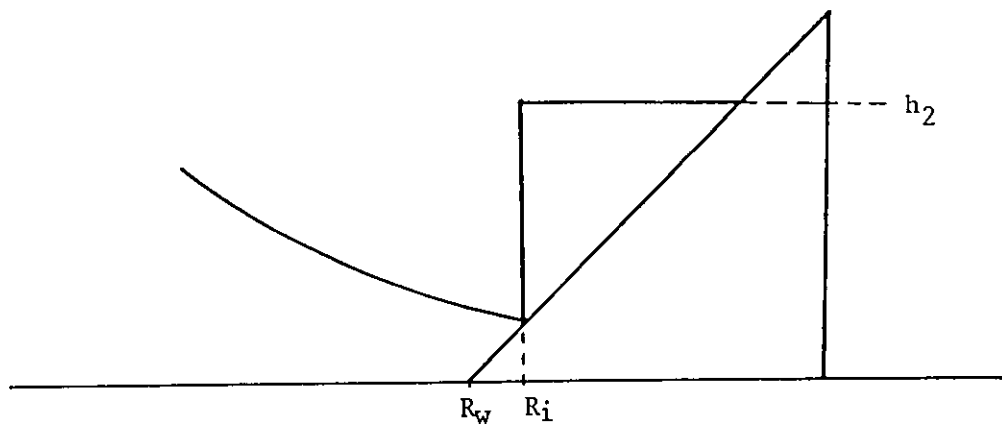


Figure A7.7

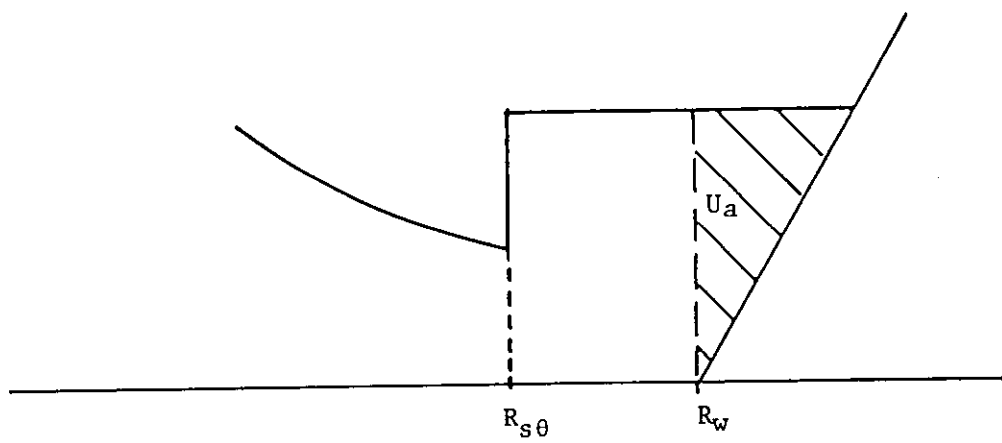


Figure A7.8

contact point of the surface of the incoming liquid and the containment wall. The surge up the face of the containment wall is therefore represented mathematically by the 'triangular' section behind R_i . The condition, $R_s = R_i$ is imposed, h_2 is modified according to Eq. (A7.5) and the calculations proceed in the usual manner. R_i is computed numerically in the axisymmetric model. In the plane case R_i has an analytical solution:

$$R_i = A + e - \frac{(e^2 - 4ft^2)^{\frac{1}{2}}}{2}$$

where $e = 2C_0t + 9gt^2 \tan \theta$

$$f = 9g \tan \theta (R_w - A) + 4C_0^2$$

g is the gravitational constant

If $R_s < R_i$, the modified value of h_2 is:

$$h_2 = \frac{j - (j^2 - 4k)^{\frac{1}{2}}}{2}$$

where $j = R_i - R_{s\theta} + h(R_i, T_w + \delta t) \tan \theta$

$$k = h(R_i, T_w + \delta t) (R_i - R_w) / \tan \theta$$

$h(r, t)$ is given in region III.

For the case $\theta < 90^\circ$ the initial shock velocity is also given by Eq. (A7.3).

The liquid velocity behind the shock beyond R_w assumed to be uniform is averaged within the triangular section (Fig. A7.8) according to Eq. (A7.4). As before the liquid velocity in the



'rectangular' section between R_w and $R_{s\theta}$ varies u_a to u_2 . In this case ($\theta < 90^\circ$) however, the size of the 'triangular' section is not negligible compared to the 'rectangular' section for a longer time span, especially if θ is relatively small. The error that arises from this approximation of average velocity within the triangular section for the case $\theta < 90^\circ$ is still small but greater than for the case $\theta > 90^\circ$ where the error is acceptable. This is nevertheless believed to be the most satisfactory approximation possible within the constraints of the shallow water theory.



APPENDIX 8

The Greenspan and Young Algorithm for the Numerical
Method of Characteristics

In the two dimensional plane model the SWE's are:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + 2c \frac{\partial c}{\partial x} = 0$$

$$2 \frac{\partial c}{\partial t} + 2u \frac{\partial c}{\partial x} + c \frac{\partial u}{\partial x} = 0$$

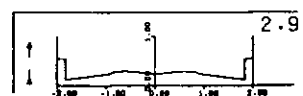
In their characteristic form the above equations can be given by,

$$u \pm 2c = \text{constant} \quad (\text{A8.1})$$

on the characteristic curves, respectively given by:

$$\frac{dr}{dt} = u \pm c$$

To evaluate the motion (u and c) at a series of mesh points, $x_i (i = 1, n)$, Greenspan and Young use a first order, implicit numerical MOC. With reference to Fig. (A8.1), consider the problem of determining the values of u and c at the mesh point (x_i, t_{j+1}) , where the values of u and c are known at all points of the previous time step, t_j and at the appropriate boundaries.



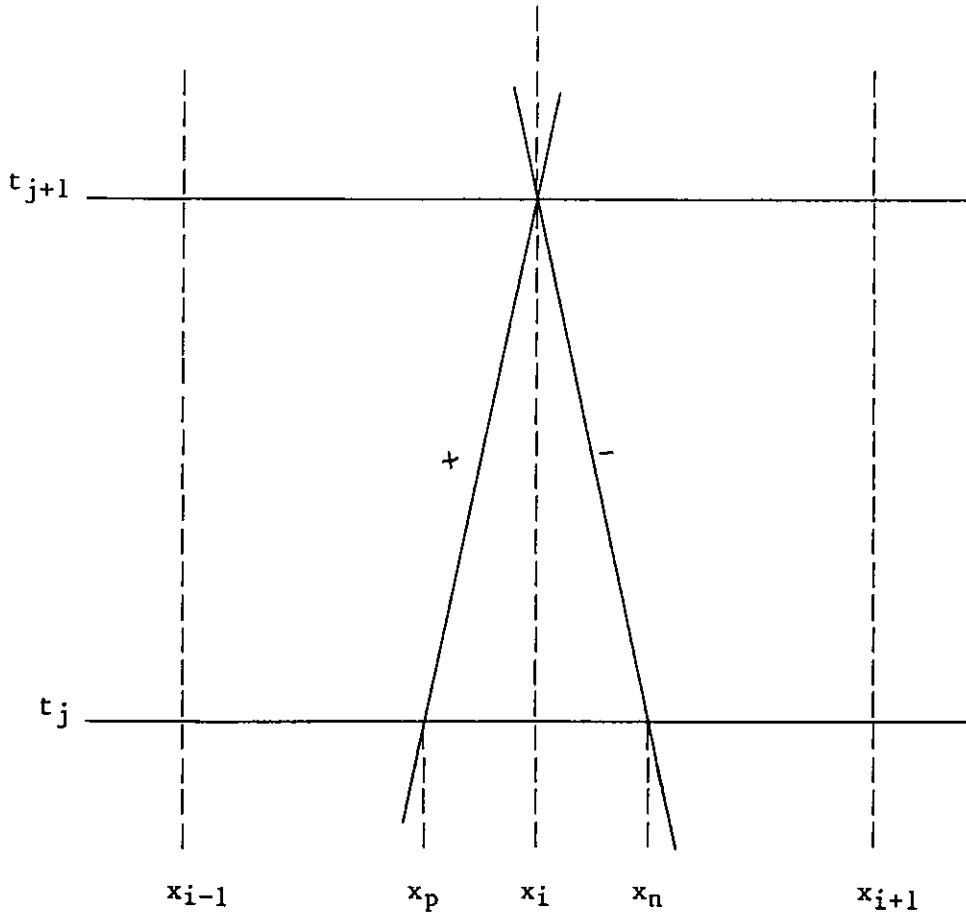


Figure A8.1

Fig. (A8.1) shows the positive and negative characteristics passing through the point, (x_i, t_{j+1}) , for which the liquid velocity, $u(x_i, t_{j+1})$ and the wave speed, $c(x_i, t_{j+1})$ are to be evaluated according to the following algorithm:

- 1) The values of u and c at (x_i, t_j) are used to calculate x_p and x_n by approximation of the slope of the two characteristics to:

$$\left(\frac{dx}{dt}\right)_+ \approx \frac{x_i - x_p}{t_{j+1} - t_j} \approx u(x_i, t_j) + c(x_i, t_j) \quad (\text{A8.2})$$

$$\left(\frac{dx}{dt}\right)_- \approx \frac{x_i - x_n}{t_{j+1} - t_j} \approx u(x_i, t_j) - c(x_i, t_j)$$

2) u_p , c_p and u_n , c_n , the respective values of u and c at x_p and x_n , are obtained using linear interpolation between the two mesh points either side of x_p and x_n .

3) $u(x_i, t_{j+1})$ and $c(x_i, t_{j+1})$ are then obtained from the characteristic equations, Eqs. (A8.1).

$$u(x_i, t_{j+1}) = \frac{1}{2}(u_p + 2c_p + u_n - 2c_n) \quad (\text{A8.3})$$

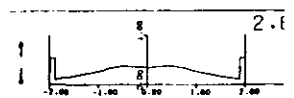
$$c(x_i, t_{j+1}) = \frac{1}{4}(u_p + 2c_p - u_n + 2c_n)$$

4) The slope of the characteristics is then re-evaluated using the above values of u and c at (x_i, t_{j+1}) instead of (x_i, t_j) , in Eqs. (A8.2).

5) u_p , c_p and u_n and c_n are then re-determined as in step (2).

6) The re-iterated values of $u(x_i, t_{j+1})$ and $c(x_i, t_{j+1})$ are obtained by averaging the new results from Eqs. (A8.3) with the previous values of $u(x_i, t_{j+1})$ and $c(x_i, t_{j+1})$.

The iterations follow in this way until successive values of x_p and x_n are sufficiently close.



APPENDIX 9

Numerical Solution of the Axisymmetric Shallow Water

Equations in Region VI

The convergence of the propagating hydraulic jump (region V) at $r = 0$, results in a reflected shock which proceeds to move back towards the surrounding containment wall. This resulting motion is defined by region VI. In an analogous problem in gas dynamics Butler (3) presents a solution for a strong shock converging to the centre of a sphere or cylinder of an almost stationary polytropic gas. This solution can be used to evaluate, the motion at the point of singularity, $r = 0$, and the resulting reflection.

On dimensional grounds a transformation is sought, of the type:

$$\epsilon = \frac{t}{\alpha r^{1/\alpha}} \quad (\text{A9.1})$$

where α is a constant and $r = 0$, $t = 0$ represents the singular point of the converging shock.

The SWE's for the axisymmetric model can thus be written in the form:

$$[1 - (u \pm c)\epsilon] \left[\frac{du}{d\epsilon} \pm 2 \frac{dc}{d\epsilon} \right] = (1 - \alpha)(u \pm c)(u \pm 2c) - \alpha u c - \frac{c^2(1 - \alpha)}{(1 - u\epsilon)} \quad (\text{A9.2})$$



where u , c , r and t are all in dimensionless form.

Using a similarity variable, $\eta = \epsilon^{-\sigma}$, the reflection of the converging shock from the centreline can thus be described by:

$$\sigma \eta \left[2(1-m) \frac{dn}{d\eta} - \frac{n dm}{d\eta} \right] = 2n(2\alpha m - 1) \quad (A9.3)$$

$$\sigma \eta \left[2(1-m) \frac{dm}{d\eta} - \frac{2n dn}{d\eta} \right] = m(1-\alpha m) - 2\alpha n^2 - \frac{(1-\alpha)n^2}{(1-m)}$$

where σ is positive and equal to the smallest index for which $\eta m(\eta)$ and $\eta n(\eta)$ are regular at $\eta = 0$ and where $m = u\epsilon$ and $n = c\epsilon$.

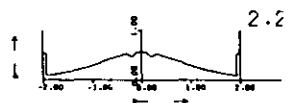
By expanding m and n in terms of η , substituting these expressions in Eqs. (A9.3) and equating coefficients a solution can be obtained for u and c behind the shock. The first terms in the expansions yields:

$$\alpha = \frac{m_0 - n_0}{m_0 - n_0 - m_0 n_0} \quad (A9.4)$$

where the suffix o represents the first term in the expansion of m and n in powers of η .

The following algorithm is used for the motion behind the reflected shock. Initial values of u and c defined by the incoming motion at the converging shock (region V) are taken at some small time before the singularity is reached at $t = 0$. Therefore, at this time, $t = -t_i$, $r = r_i$, $u = u_i$ and $c = c_i$.

An initial approximation for α is then obtained from Eq. (A9.4) with $m_0 = \epsilon_i u_i$ and $n_0 = \epsilon_i c_i$. Here $\epsilon_i = t_i / \epsilon r_i^{1/\alpha}$. With



this value of α and the constant values, u_i and c_i , $\frac{du}{d\epsilon}$ and $\frac{dc}{d\epsilon}$ are calculated from Eq. (A9.2) and then represented in the form of Chebyshev polynomials over the range $(-1, \epsilon_i)$.

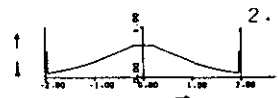
A second approximation to u and c is then obtained by integrating these expressions. These new values of u and c are then also expressed as Chebyshev polynomials and substituted in Eqs. (A9.4) and (A9.2) to re-evaluate the above mentioned derivatives. Iteration continues until successive values of α are sufficiently close.

The converged value of α is then used to evaluate the motion behind the reflected shock from Eqs. (A9.3). The reflected shock conditions are computed using the shock equations in the form:

$$c_2^2 = c_1^2 + \frac{1}{2} \left[\left(\frac{1}{\epsilon} - u_1 \right)^2 - \left(\frac{1}{\epsilon} - u_2 \right)^2 \right] \quad (\text{A9.5})$$

$$1 - u_2 = \frac{1}{2} \left(\frac{1}{\epsilon} - u_1 \right) + \frac{2}{3} \frac{c_1^2}{\left(\frac{1}{\epsilon} - u_1 \right)}$$

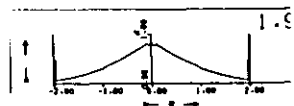
All that is required to compute the motion in region VI at all subsequent times is the values for u_1 and c_1 in Eqs. (A9.5). This is supplied by the computations performed in region V.



APPENDIX 10

Computer Program Listing

The main FORTRAN computer program, including the two different sub-programs for the computation in region V is presented on microfilm which is located on the inside of the back cover. A list defining the variables used is given at the beginning of each subroutine.



APPENDIX 11

Additional Results

The figures presented in this chapter show the superimposed surface height profiles and the variation of hydrostatic pressure and loading with time, for values of $R_w/A = 1.5, 3.0, 4.0$; i.e. those not shown in Chapter (4).

A microfiche is enclosed (back cover) which shows the entire space-time array for both the horizontal liquid velocity, u and the transformed height c . Values are presented in the following dimensionless forms:

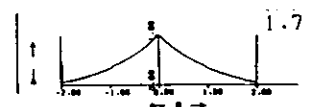
$$\text{Time} \quad : \quad T^* = t/(A^2/gH_0)^{\frac{1}{2}}$$

$$\text{Displacement} \quad : \quad R = r/A$$

$$\text{Velocity} \quad : \quad u^* = u/(gH_0)^{\frac{1}{2}}$$

$$\text{'Wave Speed'} \quad : \quad c^* = c/(gH_0)^{\frac{1}{2}}$$

Array blocks of c^* and u^* are presented alternately starting with c^* . Each array block ranges over the entire time range and over a limited range of R .



Free Surface Height - Region III, IV, V

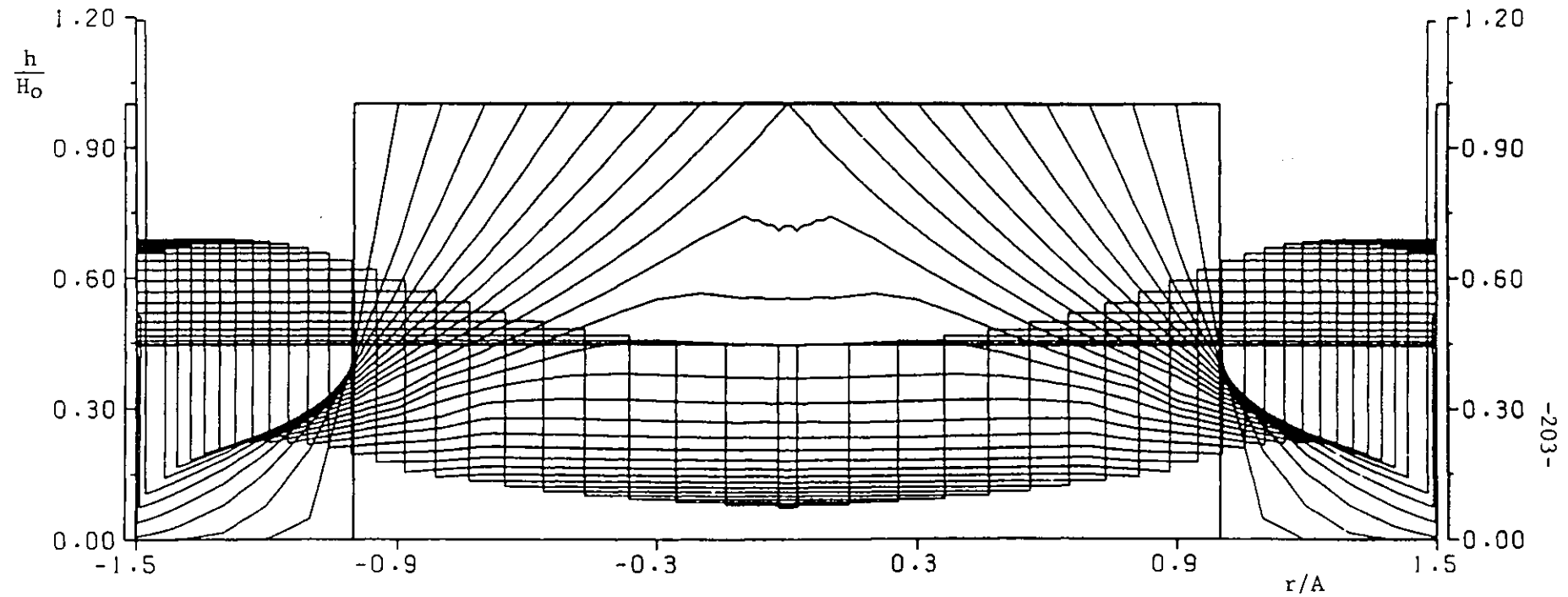


Figure All.1 Axisymmetric Model

$$R_w/A = 1.5$$

$$\theta = 90^\circ$$

$$\text{Time step} = 0.1 (A^2/gH_0)^{1/2}$$

Free Surface Height - Regions III, IV, V

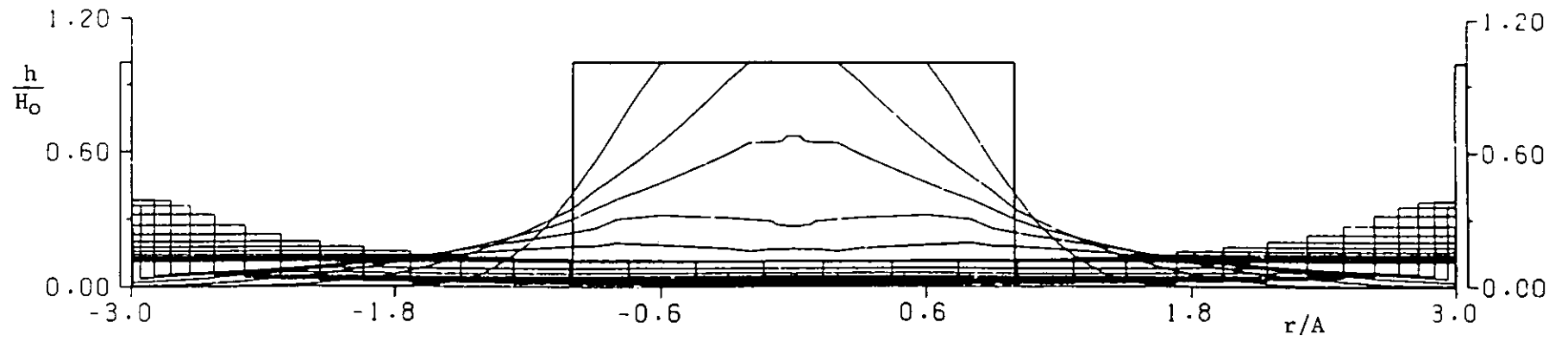
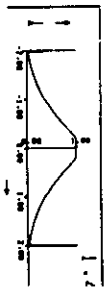


Figure All.2 Axisymmetric Model

$$R_w/A = 3.0$$

$$\theta = 90^\circ$$

$$\text{Time step} = 0.4 (A^2/gH_0)^{1/2}$$



Free Surface Height - Regions III, IV, V

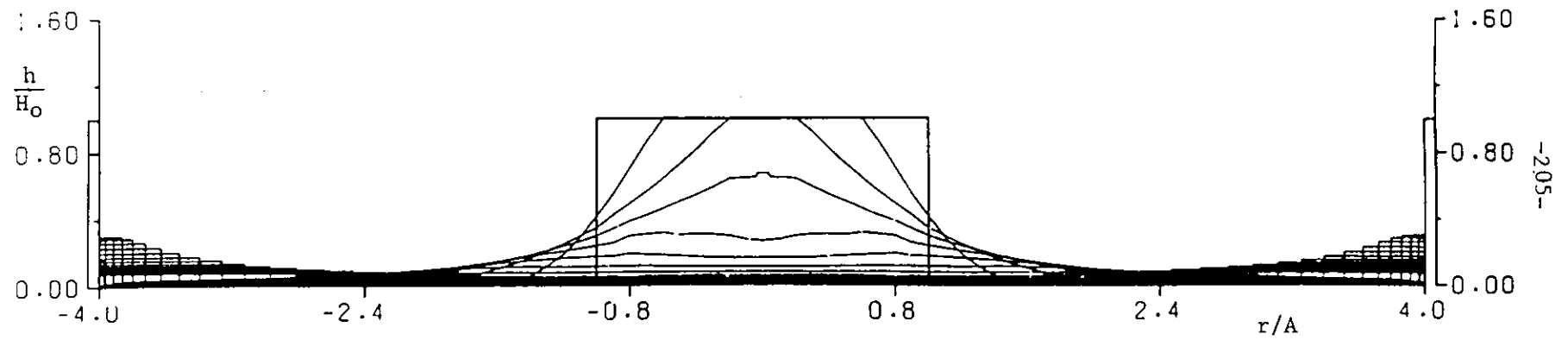
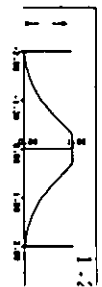


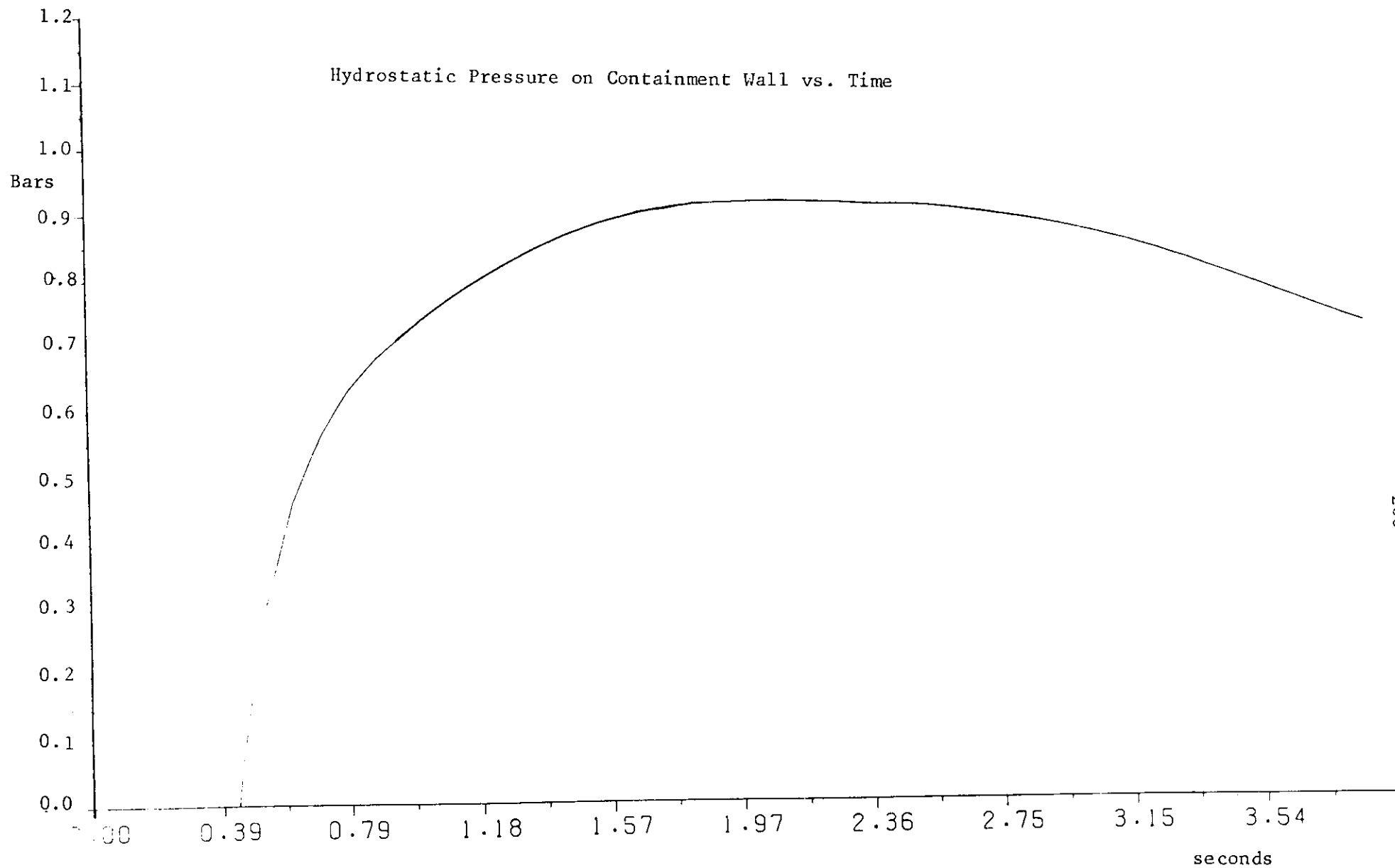
Figure All.3 Axisymmetric Model

$$R_w/A = 4.0$$

$$\theta = 90^\circ$$

$$\text{Time step} = 0.4 (A^2/gH_0)^{1/2}$$





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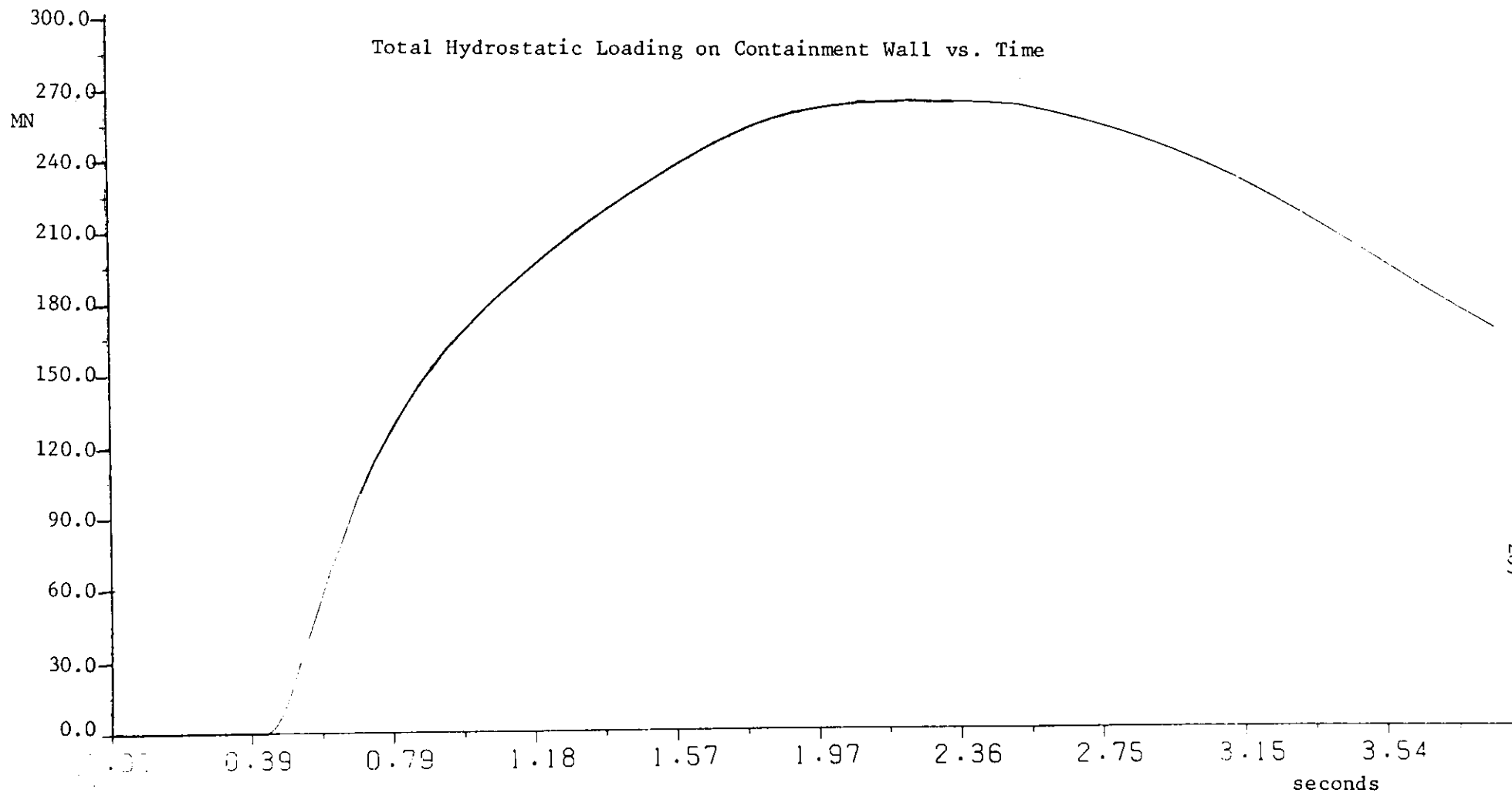
Figure A11.4 $A = 30.00 \text{ m}$

$R_w = 45.00 \text{ m}$

$H_o = 30.00 \text{ m}$

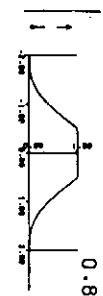
$\theta = 90^\circ$





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Figure A11.5 $A = 30.00 \text{ m}$
 $R_w = 45.00 \text{ m}$
 $H_o = 30.00 \text{ m}$
 $\theta = 90^\circ$



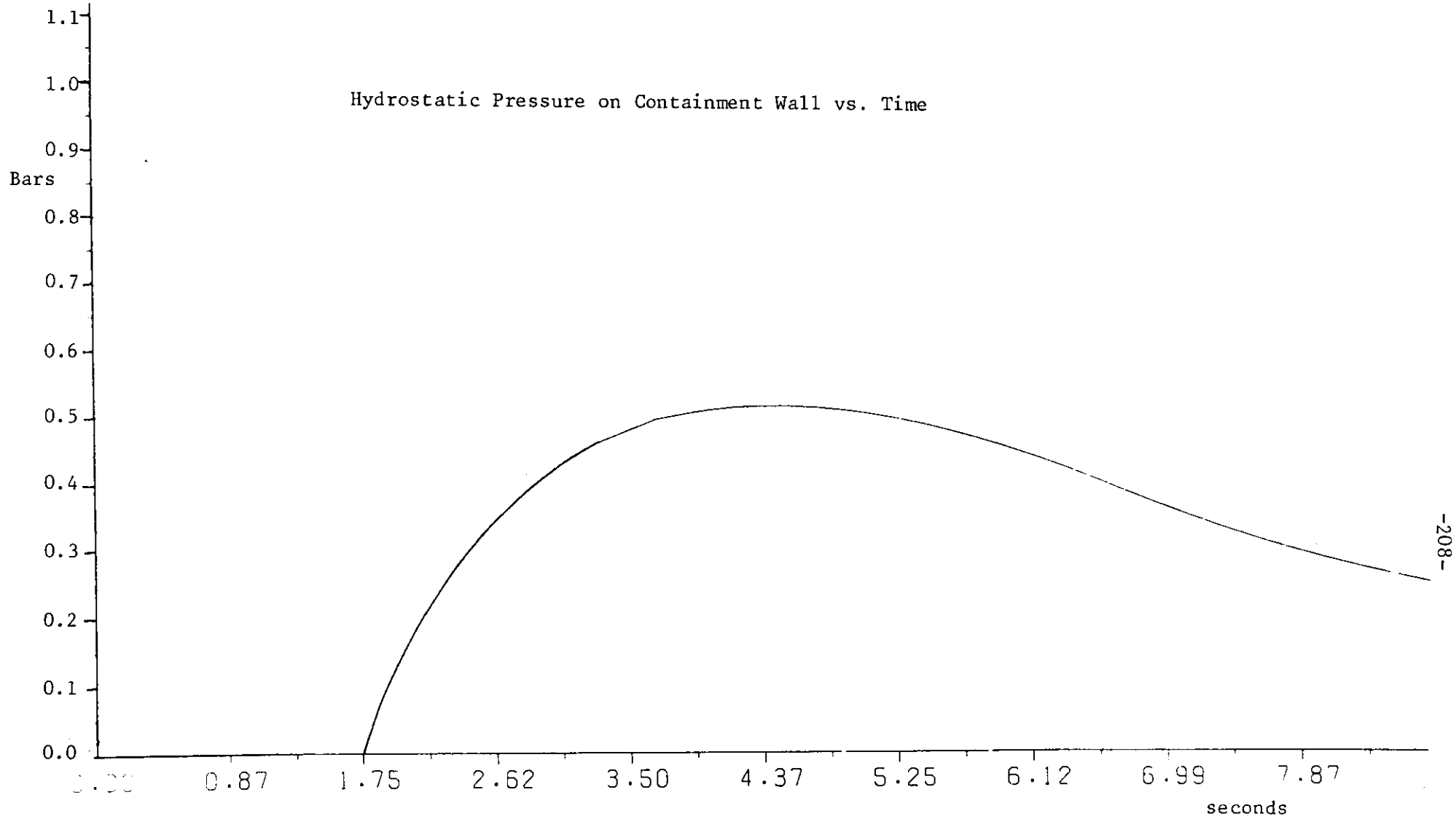
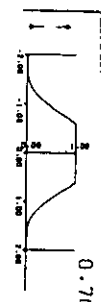


Figure A11.6 $A = 30.00 \text{ m}$
 $R_w = 90.00 \text{ m}$
 $H_o = 30.00 \text{ m}$
 $\theta = 90^\circ$



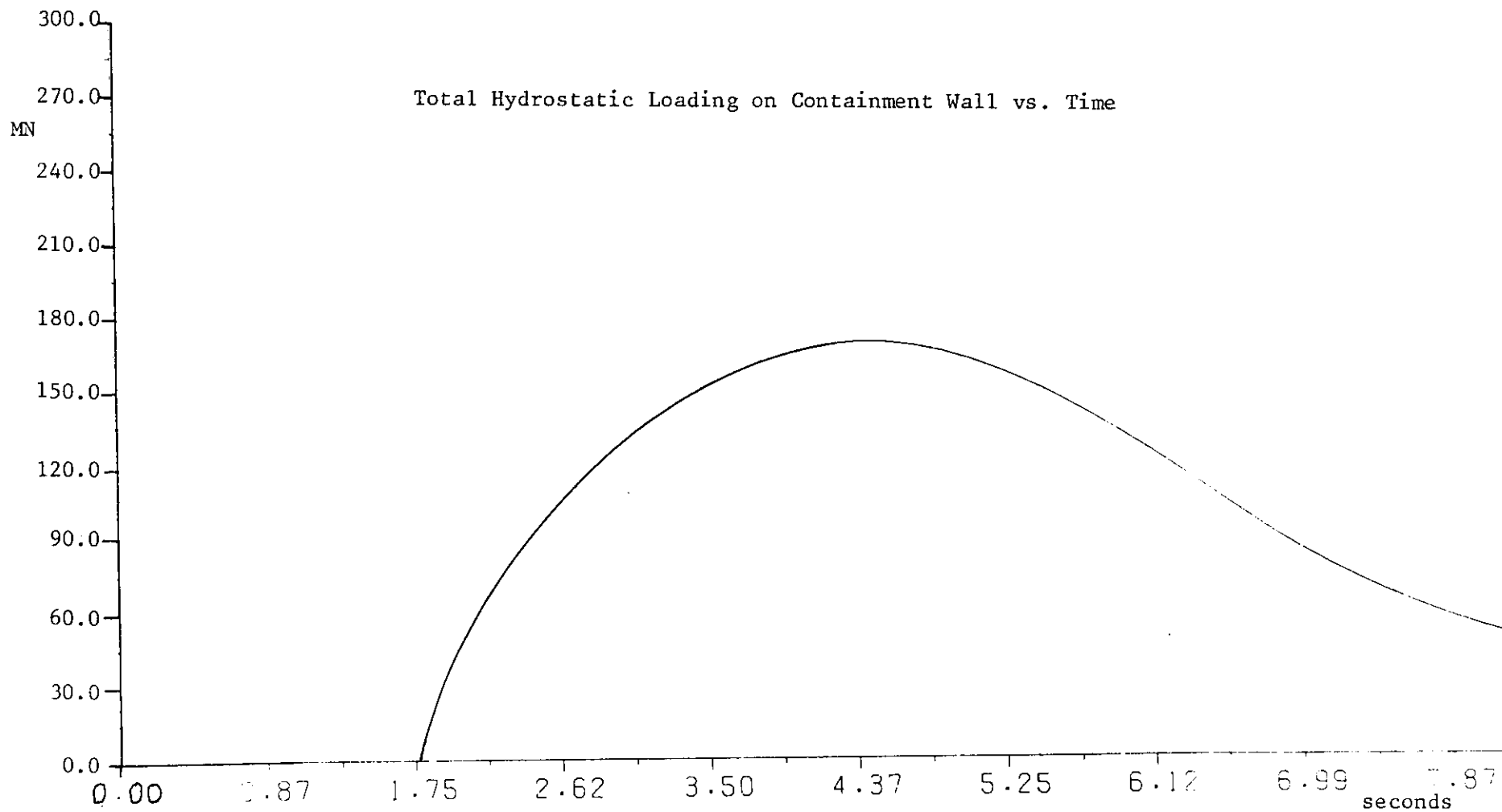
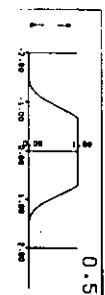


Figure A11.7 $A = 30.00 \text{ m}$
 $R_w = 90.00 \text{ m}$
 $H_o = 30.00 \text{ m}$
 $\theta = 90^\circ$



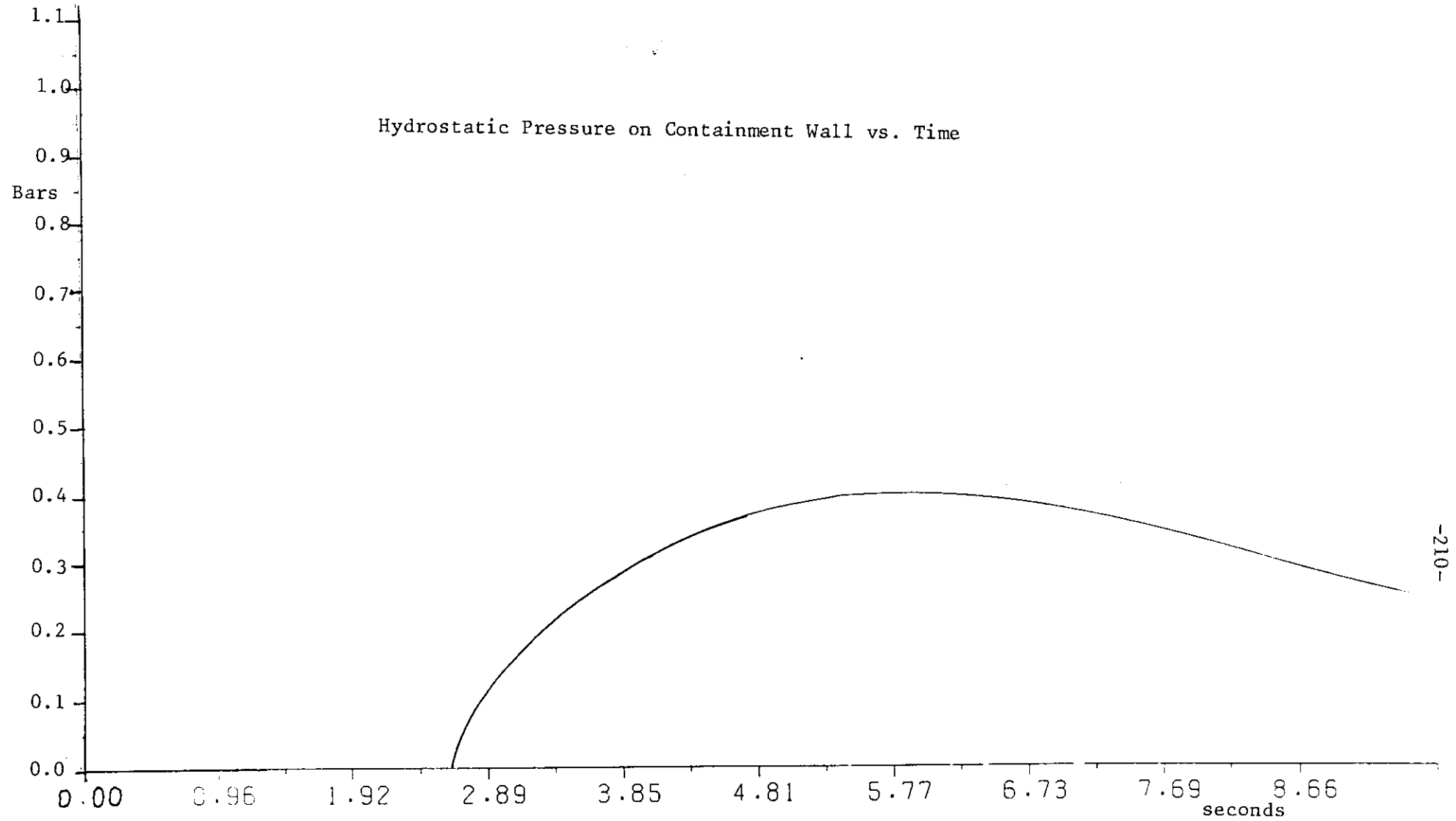
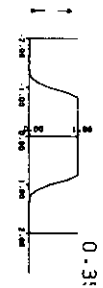


Figure All.8 $A = 30.00 \text{ m}$
 $R_w = 120.00 \text{ m}$
 $H_o = 30.00 \text{ m}$
 $\theta = 90^\circ$



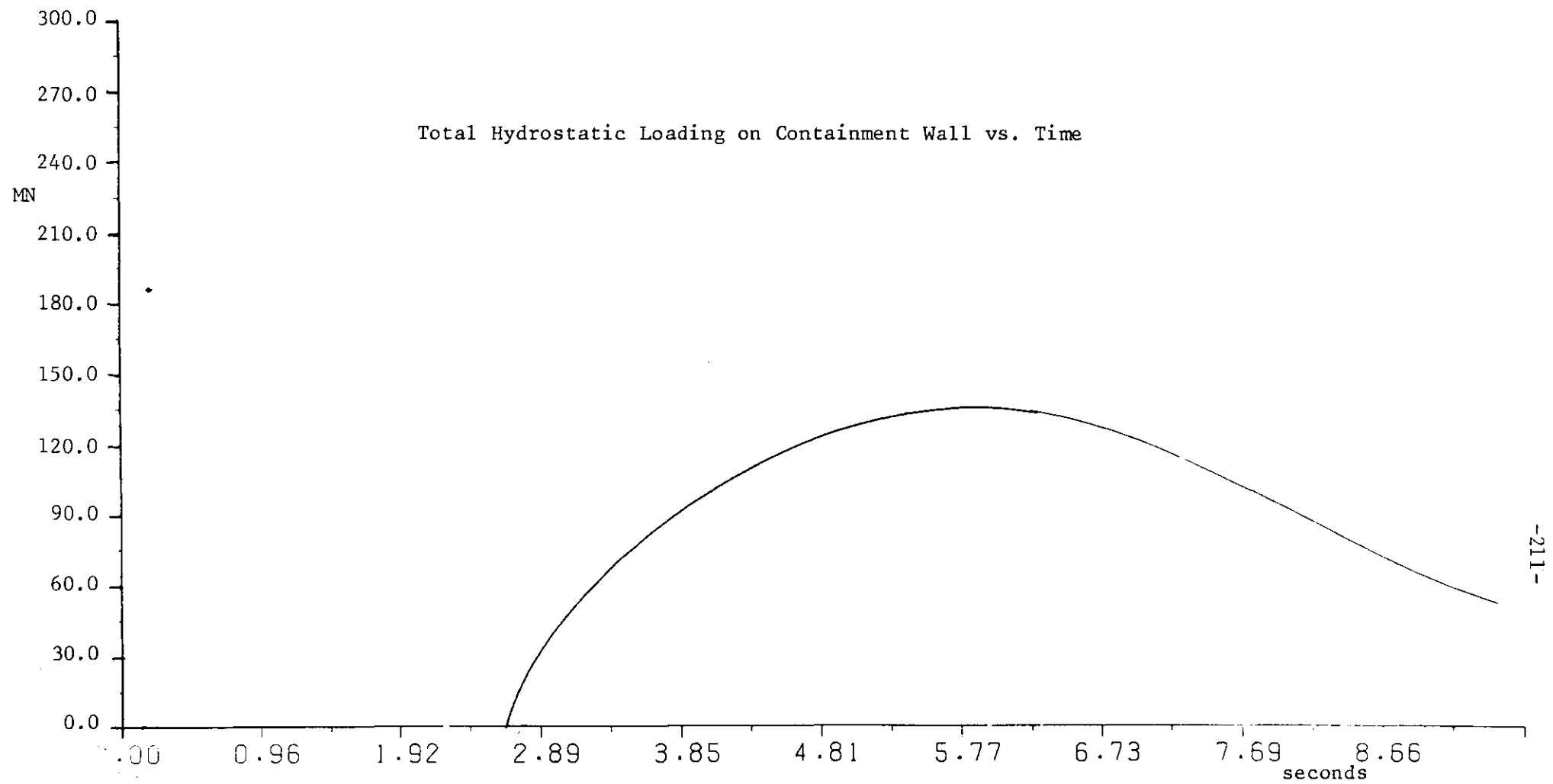


Figure A11.9 $A = 30.00 \text{ m}$
 $R_w = 120.00 \text{ m}$
 $H_o = 30.00 \text{ m}$
 $\theta = 90^\circ$

