

"Electro-Modulation Spectroscopy
of GaAs/AlGaAs Low Dimensional Structures"

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Abstract

Electroreflectance spectra are taken on GaAs/Al_xGa_{1-x}As single quantum wells (SQWs), multiple quantum wells (MQWs), and superlattices, in conjunction with other optical measurements, and compared to models of the lineshape.

Electric field-dependent electroreflectance and photocurrent measurements on four SQWs, of differing well width, demonstrate that the electroreflectance signal is modulated by the quantum confined Stark effect, and the evolution of the transition energies, linewidths and strengths are compared to detailed envelope function calculations. Optical interference results in the electroreflectance lineshape of a SQW being critically sensitive to the optical phase delay between the rays reflected from the well and the sample surface, which is verified by angle-dependent measurements on two SQW structures.

The sensitivity of the electroreflectance lineshape to layer depth makes it a particularly useful technique for characterising MQWs. By comparison of field-dependent electroreflectance, electrotransmission, transmission and photocurrent measurements, two MQWs are demonstrated to have abrupt monolayer changes in well width at certain stack depths. These monolayer splittings would be unresolvable under flatband conditions, but are enhanced here by the linear field change across the stack, caused by the background ionised impurity charge. Hence monolayer splittings are observed for wells as wide as 163 Å. Bias-dependent photocurrent and electroreflectance measurements on a third MQW, contained inside a n⁺-i-n⁺ doping arrangement, are used to determine the field profile, and are related to the carrier transport through the structure.

Electric field modulation of the superlattice dielectric function is also considered and compared to measurements on three type-II GaAs/AlAs structures. Electroreflectance is particularly sensitive to the weak transitions between Γ -point holes and X-point electrons, due to their large field-induced energy shift. Both $m=+\frac{1}{2}$ and $-\frac{1}{2}$ Stark-ladder transitions are observed for a (GaAs)₁₀/(AlAs)₈ structure, evolving linearly with field, at a rate in excellent agreement with the superlattice half period. Transitions to the X-point are also resolved for a (GaAs)₆/(AlAs)₆ superlattice.

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Related Publications

- (1) "The electroreflectance lineshape for a single quantum well: the dependence on angle of incidence and temperature",
A.P.Thorn, A.J.Shields, P.C.Klipstein, N.Apsley and T.M.Kerr,
J. Phys. C20,4229 (1987)
- (2) "Lineshape broadening and polarisation dependence of electroreflectance spectra of single quantum wells",
A.J.Shields, P.C.Klipstein and N.Apsley,
Semicond. Sci. Technol. 4,476 (1989)
- (3) "A variable pressure, low-temperature, diamond anvil cell",
A.J.Shields, R.A.Woolley, P.C.Klipstein, J.Simmons and R.H.Friend,
Semicond. Sci. Technol. 4,301 (1989)
- (4) "Electromodulation spectroscopy of metalorganic-vapor-phase-epitaxy-grown GaAs/AlGaAs multiple quantum wells",
A.J.Shields, P.C.Klipstein, J.S.Roberts and C.Button,
to be published in Phys. Rev. B42 (15 Aug. 1990)

"Using modulation spectroscopy measurements of the Stark effect in MOVPE grown GaAs/AlGaAs multiple quantum wells to observe monolayer thickness changes in wells of up to 158 Å in width",
A.J.Shields, P.C.Klipstein, J.S.Roberts and C.Button, 5th Int. Conf. Physics of Electro-optic Microstructures and Microdevices, Crete (1990), to appear in Superlattices and Microstructures
- (5) "Stark-ladder behaviour of the X-levels in a type-II GaAs/AlAs superlattice measured using electroreflectance spectroscopy"
A.J.Shields, P.C.Klipstein, M.Skolnick, C.Whitehouse and G.W.Smith,
to be published in Phys. Rev. B42 (15 Sept. 1990)
- (6) "Lineshape model for modulated reflectivity of multiple quantum wells",
A.J.Shields and P.C.Klipstein, submitted for publication (1990)

"Modulated reflectance lineshapes for quantum well systems",
A.J.Shields and P.C.Klipstein, 5th Int. Conf. Physics of Electro-optic Microstructures and Microdevices, Crete (1990),
to appear in Superlattices and Microstructures
- (7) "Carrier transport effects on the photocurrent spectra of an intrinsic GaAs/AlGaAs multiple quantum well with highly doped AlGaAs contact layers",
A.J.Shields, P.C.Klipstein, G.W.S.Smith, D.R.P.Guy and N.Apsley,
to be submitted (1990)

also

A.J.Shields, P.C.Klipstein, G.W.S.Smith, D.R.P.Guy and N.Apsley,
5th Int. Conf. Physics of Electro-optic Microstructures and Microdevices, Crete (1990),
to appear in Superlattices and Microstructures

1 Introduction

1.1 Background to this Research

Sophisticated methods of depositing thin films of semiconductor crystal on a flat substrate have been developed over the past two decades, where the growth can be controlled to proceed in monolayer planes [1.1]. Furthermore it is possible to switch the growth, allowing the preparation of single crystals comprised of thin layers of different materials. Heterostructures comprised of different alloys of $(\text{Al}_x\text{Ga}_{1-x})\text{As}$ have received the most attention and consequently produce the best quality layers at present. However, many other combinations of semiconductors have been produced, including some where the materials have a different lattice constant, so introducing strain into the layers [1.2].

The two or more components in the heterostructure will in general have different bandstructures, so allowing a wide range of conduction and valence band profiles to be engineered. In particular it is possible to trap carriers in a narrow layer, for example of GaAs, bounded by a material with a larger bandgap, eg. $\text{Al}_x\text{Ga}_{1-x}\text{As}$. Since the interfaces are abrupt in good quality material, this problem can be regarded, under the effective mass approximation, as a particle in a rectangular well. Quantum confinement effects are seen when the layer width is comparable to the de Broglie wavelength of the carrier, and the system is then referred to as a quantum well (QW). The bandstructure for this quasi-two-dimensional system is qualitatively different from that of its bulk semiconductor constituents. The energy associated with motion perpendicular to the plane of the QW is restricted to a series of "confinement levels", that form the thresholds of subbands, due to the free motion parallel to the plane of the well. Systems where the motion is further restricted to one or even zero dimensions, have also recently been studied [1.3]. Heterostructures where the carrier motion is restricted in one or more dimensions have been given the collective name of "low dimensional semiconductor (LDS) systems".

If QWs are formed in the narrow bandgap layer, for both the conduction and valence bands then the system is referred to as "type-I". Meanwhile in "type-II" systems, the minima of the conduction and valence bands lie in different layers. For a type-I system, optical transitions are allowed between electron and hole states with the same subband number, so that the absorption spectrum will resemble a series of steps, whose thresholds correspond to the bulk bandgap of the well material,

plus the sum of the electron and hole confinement energies. Large peaks are seen at energies just below the threshold of each step, which are due to the bound excitonic state formed by the Coulombic attraction of the electron and hole. Excitons can also be observed in bulk semiconductors at low temperature, but their strength is greatly enhanced in QWs, due to the electron and hole being squeezed together in the narrow well layer. Consequently excitons can be resolved in room temperature spectra of QWs.

As its name suggests a "single quantum well" (SQW) is a structure containing just one QW layer. A second class of heterostructure that has received intensive research is a regular stack of alternating layers of two materials, called a "superlattice" (SL). If the barrier layers are sufficiently thick then this structure can be regarded as a regular array of electronically isolated QWs, and is often referred to as a "multiple quantum well" (MQW). MQWs have often been studied in preference to SQWs as they produce a larger signal in many spectroscopies, while the layer uniformity is expected to be good. Measurements have been made on both SQWs and MQWs in this thesis and are reported in Chps. 5 and 6 respectively. It is shown that electro-reflectance produces quite different lineshapes for SQWs and MQWs, which is due to the technique being sensitive to the depth of the contributing QWs in the sample structure.

If the barriers of a SL are sufficiently thin, then the electronic states of adjacent wells begin to couple. The interaction of the envelope function in each well with the confining potential of its neighbours, leads to a hybridisation of the isolated well envelope functions into states extending across the entire SL. Meanwhile the degeneracy of the confined level of each well is lifted, producing a miniband of closely spaced energy levels. There has been a growing interest in such SLs over the past two to three years, fuelled by improvements in the quality of the layer interfaces, and miniband effects have been observed in a number of different heterostructures, as discussed in Chp. 7. Particular interest has centred on GaAs/AlAs SLs, because if the GaAs layers are sufficiently narrow, then the states associated with the X-point of the Brillouin zone can become the lowest in the conduction band. As these are confined in the AlAs layers, these SLs are then type-II. Measurements on three type-II GaAs/AlAs SLs are reported in Chp. 7.

The advances in interface quality have been accompanied by simultaneous improvements in the electrical behaviour of the material. There are two common methods for applying a uniform electric field to a

region of semiconductor, both of which are employed in this thesis. In the first the region of interest is placed in the intrinsic region of a p^+-i-n^+ doping profile, to which a reverse bias is applied. For GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ systems it is now possible to obtain fields close to the breakdown field using this arrangement. The other method is to grow the undoped region of interest on a highly doped substrate and then evaporate a suitable semitransparent Schottky barrier on the front surface.

Electric fields applied perpendicular to the plane of a QW have been found to produce very interesting effects upon their bandstructure, which has been the subject of intensive research over the past decade. In bulk semiconductors, the bound excitonic states are readily lifetime broadened by a small electric field, due to ionisation. This effect is qualitatively similar to the Stark effect observed for the Hydrogen atom. However, when a field is applied perpendicular to the plane of a QW, the separation of the carriers is restricted by the confining barriers. Hence the excitonic resonances can still be observed at fields that are many times the bulk ionisation threshold. Meanwhile the field can also produce large shifts of the exciton transition energies, due to changes in the electron and hole confinement energies. The confining barriers limit the field-induced reduction of electron-hole overlap and hence the simultaneous decline in the Coulombic interaction, while also reducing the size of the energy shift. The net effect is to produce large shifts of the exciton energy, with negligible increase in the linewidth and only a small reduction in the oscillator strength. This phenomenon has been called the "quantum confined Stark effect" (QCSE).

Entirely different effects are observed when an electric field is applied perpendicular to the layer planes of a SL, although again the changes in the bandstructure can be very dramatic, as discussed in Sec. 7.1. If the well width is narrow then the QCSE will be negligible. However the electric field can produce major changes in the zero-field miniband structure of SLs with narrow barriers. The field creates a potential drop between adjacent wells, so reducing their coupling. As the field is increased, the SL envelope functions become isolated to an increasingly small spatial region. Eventually the envelope functions become confined in individual QWs and the bandstructure is similar to a repeated version of that for an isolated QW. At moderate fields, a series of transitions can be seen due to electron and hole states centered around different wells. The separation of these transitions is

equal to the potential drop across one period, and so increases linearly with field.

Optical spectroscopy has proved a valuable tool for studying the bandstructure of LDS systems. The spectra can be analysed to yield transition energies, linewidths and strengths, which can be related to the electronic structure. The most popular spectroscopies have been photoluminescence (PL) and PL excitation (PLE), as these require minimal sample preparation. Transmission measurements can be made by etching away an absorbing substrate. Photocurrent (PC) spectroscopy has also become increasingly popular, especially for field-dependent studies, aided by the improvement in the electrical behaviour of samples. Transmission and PC spectroscopies have the advantage that they are readily performed at room temperature.

There have also been a number of studies on LDS systems using modulation spectroscopy, many of which are reviewed in Chp. 3. This involves measuring the change in the reflection or transmission spectrum produced by a modulated perturbation, using phase sensitive detection. It is more convenient to monitor the reflectivity as this does not require the prior removal of an absorbing substrate, as often necessary for transmission measurements. Most of the previous studies have therefore measured either electroreflectance (ER) or photorelectance (PR). While this thesis concentrates on ER, some electrotransmission (ET) measurements are also presented. In ER, the electric field is modulated by an applied alternating voltage, requiring contacts to be made to the sample, although this also facilitates a quantitative study of the electric field dependence. The need for contacts is avoided in PR, where the modulation is provided by a mechanically chopped laser beam. It has been suggested that the laser light has the effect of changing the electric field at the surface of the sample, so ER and PR could be expected to produce similar spectra.

ER is a particularly powerful method of measuring QW and SL systems, because their bandstructures are very sensitive to the electric field. It is also especially suited for measuring transitions above the bandedge, involving either higher subbands or bands, as it is only sensitive to critical points and so will not contain background contributions due to lower energy states. However modulation spectroscopies have the disadvantage that their lineshapes are often not as easily interpreted as those in PC, PL, PLE etc. A lineshape model is therefore needed to accurately extract the transition energies, linewidth and strengths. Such a model is presented in this thesis for electromodulation spectroscopy of SQWs, MQWs and short period SLs.

The past decade has seen a great deal of research into optical spectroscopy of LDS systems, and especially electric field effects. Much of this has been stimulated by the potential application of these structures in opto-electronic devices. The desire to transmit ever increasing volumes of information, at faster speeds, has led to digital communication using light transmitted along fibre optic cables. Optical communication also has the advantage over traditional electric signals in copper cables, that it is not prone to electrical "pick-up".

LDS systems have been exploited in several designs for optical lasers, modulators and detectors. QW lasers have been used in several applications, as they require a small threshold current, and the operating wavelength can be chosen using the well width [1.4]. Detectors using MQWs have been found to be efficient, while they are also wavelength selective to a certain extent [1.5,6]. Much of the current interest is focused on optical modulators incorporating MQWs [1.7]. These exploit the QCSE, relying on the large field-induced energy shift of the bandedge exciton, with negligible broadening, to produce a large change in the transmission. There have also been several designs based upon reflection geometries [1.8,9]. More recently optical modulators have been proposed, which rely on the field-induced change in the absorption of a short period SL [1.10].

Research into electric field effects in LDS systems is therefore essential for maximising the efficiency of such opto-electronic devices. For instance, part of this thesis examines the increase in linewidth of SQWs and MQWs with electric field, which will have a crucial bearing on device performance. The shape of the exciton tail below the bandedge, which is also studied here, again will have a significant effect on the efficiency.

QW systems have also been used to produce optical logic elements [1.11]. If the PC signal from a modulator is used to provide positive feedback, then an optically bistable switch can be constructed. (Meanwhile, negative feedback with a current source can be used to design a modulator with a controlled amount of absorption.) Optical logic could greatly speed up the processing between the detection and transmission of optical information.

There is also interest in using optical signals to interconnect chips inside a computer, again in order to improve their speed. GaAs/AlGaAs lasers, modulators and detectors for this purpose could be grown directly onto wafers doing electronic processing, which may be made of GaAs or even Si. Eventually though optical logic elements may

be integrated on the same chip as the interconnects in order to perform the processing using light signals.

1.2 Topics Studied in this Thesis

This thesis describes electric field-dependent optical measurements on GaAs/Al_xGa_{1-x}As SQWs, MQWs and SLs. The studies concentrate on ER and PC spectroscopies, although transmission, ET and PL are also collected on some of the samples for comparison. There are two closely related overall objectives for this work. Firstly to study the electric field dependence of the bandstructure of different LDS systems and secondly to examine ER lineshapes for the various structures.

Chp. 2 describes the bandstructure of isolated QWs, including valence band mixing and excitonic effects and how this is related to their optical transitions. This is followed by a discussion of the electric field dependence of the bandstructure of both bulk semiconductors and QWs. Exciton transition energies and oscillator strengths are calculated for QWs, with different well widths, as a function of the applied field.

Chp. 3 considers lineshape models for electromodulation spectra of SQWs, MQWs and bulk semiconductors. Throughout this thesis the electromodulation spectra are taken using a small, fixed square wave, offset by a variable d.c. bias. The modulated dielectric function is calculated by taking the difference in the QW dielectric function between the two extremes of the field modulation. An applied modulation can produce a change in the energy, linewidth and overlap integral associated with an excitonic transition, and the lineshapes produced by small changes in each of these are plotted in Sec. 3.2. The strengths of the energy and overlap modulated lineshapes are calculated for transitions between the different electron and hole confined states, in wells of various width, as a function of the applied field, using the model of the QCSE described in Chp. 2. The field-induced change in the energy is shown to be much more important than that in the overlap, for transitions between the lowest electron and hole confined states, for nearly all well widths and fields. The importance of the field-induced change in the linewidth is also assessed. This is the first time that QW electromodulation mechanisms have been treated in such detail.

Having calculated the field modulated dielectric function for the QW, the next step in deriving lineshape formulae for SQWs and MQWs, is

to relate this to the ER and ET. The overall reflectivity of a structure is calculated by considering optical interference of the rays reflected from each interface. It is shown to be a good approximation to consider only the ray reflected from the front surface of the sample, along with those from the QW(s), since the ray from the front surface is much more intense than the others. The reflectivity of each well can be approximately written as a linear function of its dielectric function. Consequently an explicit lineshape formula is derived for SQWs in Sec. 3.3, which shows the ER to be a linear sum of the real and imaginary parts of the QW modulated dielectric function ($\Delta\epsilon_r$ and $\Delta\epsilon_i$ respectively). The mixture is determined by a phase term, called the "lineshape phase" here, which is the optical phase delay between the interfering rays from the front surface and the QW. The various simplifications used in the determination of the ER lineshape formula are considered carefully, by comparison with a more exact transfer matrix calculation. The calculated spectra are also compared to experimental data taken on a SQW sample. This was the first simple analysis of optical interference effects upon the reflectivity lineshapes of a SQW, when originally published as a joint work in Ref. 1.12.

Sec. 3.5 is devoted to ER lineshapes of bulk semiconductors. The transitions are modulated by the Franz-Keldysh effect, rather than the QCSE, and so the spectra are qualitatively different to those for SQWs. There are several reasons for including an analysis of ER of bulk material in this thesis. Firstly, contrast is drawn with the model for QWs, there being a number of crucial differences between the two analyses. This highlights the errors of several recent studies that have modelled PR spectra of LDS systems using formulae relevant only to bulk semiconductors. Secondly the classical analysis of bulk ER is extended here to consider the case where the lower field of the modulation is non-zero. This produces a "beating" of the Franz-Keldysh oscillations, which is also observed experimentally. The analysis is also extended to consider the situation where the bulk region subject to the field is buried below the sample surface, as is the case for samples with a p^+-i-n^+ doping profile, which are frequently studied nowadays. This again results in the ER being a mixture of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, as for QW structures, except $\Delta\epsilon_r$ and $\Delta\epsilon_i$ are then determined by the Franz-Keldysh effect. The modified theory is used to model ER features from bulk regions contained inside the intrinsic region of several LDS samples. From this it is possible to extract the field in the intrinsic

region, and in each case it is found to be in good agreement with that expected due to the applied bias.

The model of the ER lineshape derived for a SQW is compared to experimental measurements in Chp. 5. The ideas on optical interference can be tested by examining the angle of incidence dependence of the lineshape. This varies the optical phase delay between the rays reflected from the front surface and the QW, and so the ER is expected to evolve between lineshapes resembling $\Delta\epsilon_r$ and $\Delta\epsilon_i$. Angle-dependent measurements are reported for two SQW structures in Sec. 5.2.1, and the ER lineshape of both is seen to evolve in the manner predicted by the model. The rate of the lineshape evolution is used to estimate the overlayer thickness between the QW and the front surface of the sample. Measurements on one of these SQWs were also published in Refs. 1.12 and 1.13. The dependence of the lineshape on the polarisation of the light (Sec. 5.2.2) and on topological variations across the sample surface (Sec. 5.2.3) are also examined.

The low temperature ER and PC spectra of four SQWs, with well widths of 59, 72, 88 and 106 Å, are studied as a function of the applied offset field. Least squares fits are made to the ER spectra, using the lineshape model of Sec. 3.3, in order to extract the exciton energies, linewidths, lineshape phases and transition strengths. The Stark shifts of the exciton transition energies measured in the ER and PC spectra, for each well, are found to be in very good agreement with those calculated. The field dependence of the strengths of the PC and ER features are also in good agreement with that predicted by the QCSE. The increase in the linewidth with field is examined. Unexpected behaviour is observed in the ER spectra taken with small applied biases, which is not predicted by the QCSE. As this appears to indicate a sudden change in the modulation mechanism, it is suggested that the observed phenomenon may be related to the depletion edge sweeping through the QW.

A minimum electric field must be applied to these four SQWs, at low temperature, in order to remove the photo-excited carriers from the well. The nature of this threshold is studied by measuring the PC signal as a function of the applied bias. PL spectra are also taken on the SQWs, over a complementary voltage range. It is shown that the PL intensity and lineshape, as well as the PC voltage threshold, is related to resonant tunnelling of carriers from the QW. The effect of the tunnelling on the ER spectra is also measured.

The model of optical interference is extended in Sec. 3.4 (and also Ref. 1.14) to consider ER lineshapes of MQWs, for the first time.

Each QW in the stack will make a different contribution to the reflectivity, depending upon its depth in the structure. The ER lineshape of a perfect MQW will therefore not be just a scaled up version of that for a SQW, as it is for spectroscopies depending solely on the imaginary part of the dielectric function, such as PLE and PC. The mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ will vary across the ER spectrum, due to changes in the optical penetration depth, and this is shown to result in complicated lineshapes. The model presented in Sec. 3.4 assumes a regular array of identical QWs, and hence another explicit ER lineshape formula is derived. Calculations are made for a single exciton and typical sample parameters. The lineshape is found to become increasingly more complicated than that for a SQW as the number of wells in the stack is increased. The analytical approach adopted here, allows a clear understanding of the origin of the different features in the calculated lineshape, and how they are affected by the various sample parameters.

Two experimental studies of MQW samples are reported in Chp. 6. In both of these it was not possible to make a direct comparison between the ER spectra and the lineshape model, as the QWs do not all have the same width or are not subject to the same field. However, despite this, optical interference is demonstrated to have the crucial effect on the lineshape. Due to the systematic changes in the field and/or well width across the stack, features are seen in the ER spectra deriving from different regions of the stack. It is shown that the depth of the regions producing these features, can be determined by measuring the angular dependence of the ER lineshape. It is also shown how the strength of the ER features from the back of the stack is greatly affected by absorption in the shallower layers, as again predicted by the model. The ER features have an associated lineshape phase, unlike most other spectroscopies, and this can be analysed to determine depths inside the sample structure. The ER measurements, complemented by other spectroscopies, thus allow a detailed elucidation of the well width and field profiles across the stack, in both of the experimental studies.

In the first study, reported in Sec. 6.2 (and also in Ref. 1.15), 17 K electric field-dependent ER, ET, PC and transmission measurements are made on two MOVPE grown GaAs/Al_{0.3}Ga_{0.7}As MQWs, which have either 88 or 163 Å wide wells, inside a p⁺-i-n⁺ doping profile. The E1H1 and E1L1 excitons are seen to move to lower energy with field in good agreement with the calculated Stark shifts. However the exciton peaks appear as a barely resolved doublet in the PC and transmission spectra, while this splitting is more clearly resolved in the ER and ET measurements. It is

argued that these splittings derive from sudden changes in the well width of one monolayer (ML) at certain depths in the stack. Under flatband conditions, these monolayer splittings would be smaller than the optical linewidth and hence unresolvable. However, the unintentional background doping of the MQW results in the field being larger at the back of the stack than at the front, which enhances the splitting. Hence monolayer splittings are observed in much wider wells ($163 \text{ \AA}$) than previously reported. The angle dependence of the ER lineshape is used to determine that the wider wells lie at the back of the stack. The bias dependences of the size of the splitting and the relative heights of two features in the different spectroscopies are examined, and also found to be in good agreement with the above microscopic model of the structure of the stack. The possible consequences for MOVPE grown opto-electronic devices incorporating MQWs are discussed.

The second experimental study of MQWs, in Sec. 6.3 (and Ref. 1.16), reports 17 K, bias-dependent PC and ER spectra taken on an MBE grown MQW, which was nominally $\text{GaAs}(100 \text{ \AA})/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}(90 \text{ \AA})$, placed inside a $n^+ - i - n^+$ profile. In this case the optical spectra are dominated by the highly non-uniform field profile set up by the dark current tunnelling through the structure. The ER and PC spectra are used to determine the field profile across the stack at different applied bias. The excitonic features are found to have a very unusual shape in the PC spectra, appearing as two peaks. However, a closer inspection shows that this unusual lineshape is actually due to the PC being collected only from the back of the stack, so that the centre of a single peak is removed by absorption in the shallower layers. This explanation of the PC lineshape is substantiated by the bias dependence of the PC spectra and also the angular dependence of the ER.

Chp. 7 considers SLs where the barriers are sufficiently narrow to produce significant minibandwidths. The electric field dependence of these SLs is qualitatively different from that of isolated QWs, and hence the ER transitions are modulated in a completely different manner. An existing envelope functions model of the SL bandstructure is modified in Sec. 7.1, to take account of the finite broadening, and thereby calculate the field modulated dielectric function. The analysis is kept as simple as possible, in order to highlight the phenomena due purely to interaction of adjacent wells.

The remainder of Chp. 7 is devoted to field-dependent ER and PC measurements on three GaAs/AlAs SLs. In each of these the GaAs layer width is very narrow, so that the lowest conduction band state is

confined in the AlAs layers and is associated with the X-point of the Brillouin zone. The lowest transition is therefore type-II, since it involves electrons and holes confined in the AlAs and GaAs layers respectively. The transition to the X-point conduction band is clearly resolved in the ER spectra taken on a GaAs(10 ML)/AlAs(8 ML) SL. The potential drop between adjacent layers, results in the field dependence of type-II transitions being quite different than that for type-I. Under an electric field the X-point transition is observed to split into a doublet, due to transitions between holes confined in a GaAs layer, and X-point electrons confined in the adjacent AlAs layers to both lower and higher potential. With increasing field these two transitions are seen to increase and decrease linearly in energy, at a rate in excellent agreement with the SL half period. This is despite the X-point transitions being too weak to be resolved in PC spectra taken under similar conditions. ER is particularly sensitive to transitions that are indirect in real space, as they have a large field-induced energy shift. The field-dependent behaviour of the X-point ER transitions in this structure are also reported in Ref. 1.17. The ER associated with the transitions to the Γ -point of the conduction band are also discussed, both for this SL and for a similar structure where the GaAs layers are 1 ML wider.

ER measurement on a GaAs(6 ML)/AlAs(6 ML) SL also resolved structure which is due to transitions to the X-point of the conduction band. In this case transitions from both the heavy and light (Γ -point) holes were seen. The bias dependence of both the transitions to the Γ - and X-point of the SL conduction band are examined.

2 Electronic Structure of Quantum Wells in an Electric Field

2.1 Single Particle States In A Rectangular Well

When a thin layer of semiconductor is sandwiched inside a wider bandgap material, potential discontinuities are created at the interfaces, resulting in confining wells for the carriers. Modern growth methods can ensure the change from one material to the other occurs abruptly within a few atomic layers, so that the sides of the well are steep. Meanwhile the background doping levels in the well and barriers can be kept small enough to make band bending effects minimal. Consequently it is a good approximation to consider these wells as rectangular. The following considers the electronic structure of such a quantum well, and is equally applicable to the conduction or valence band.

The electrons in the conduction band and holes in the valence band can be considered as behaving like electrons in free space, with the effect of the lattice approximated by giving the carriers an effective mass. This is called the effective mass approximation, which is an extremely powerful simplification widely used in semiconductor physics. It has proved very successful in predicting the electronic structure of QWs, despite the very abrupt changes in potential at the interfaces. Its validity is perhaps most questionable when dealing with structures containing layer widths of just a few lattice unit cells.

Carriers can therefore be considered to propagate through the lattice as plane-waves with a de Broglie wavelength calculated from the appropriate effective mass of the band. When the plane waves encounter a potential barrier they are partially reflected. If the barrier is infinitely high, the reflection is perfect and the transmission through the barrier zero. When two potential barriers are brought close enough to have a separation comparable to the de Broglie wavelength, quantum interference effects are produced. The carriers propagate back and forth in the well, being reflected off the two barriers and hence only certain values of the de Broglie wavelength will produce constructive interference, forming a time-independent state. If the barriers are infinitely high, there is no phase change on reflection and the k -vector in the z -direction (taken here as the normal to the QW plane) must satisfy $k_z = N\pi/L_w$, where L_w is the well width and N is a positive integer ($N = 1, 2, 3, \dots$).

Meanwhile, the propagation in the xy -plane is unrestricted, allowing all values of k_x and k_y . The quantisation of the k_z momentum,

therefore leads to the bandstructure being rearranged into a number of subbands, whose energy thresholds (E_N) and density of states per unit area (D_{2D}) are given by

$$E_N = \hbar^2 \pi^2 N^2 / (2 m_z L_w) \quad \dots 2.1$$

$$D_{2D} = m_{xy} / \pi \hbar^2 \quad \dots 2.2$$

where m_z and m_{xy} are the effective masses describing the z- and xy-motions respectively. Notice that the density of states of each subband is independent of energy and the subband number (N), so the total density of states of the band resembles a series of equally sized steps, as shown in Fig. 2.1. The value of the density of states at the subband edges equals that for a three-dimensional band, shown as the dashed line in Fig. 2.1 [2.1]. Dingle et al [2.1] were the first to observe a stepped absorption spectrum for QWs, suggestive of the formation of subbands, as discussed in Sec. 2.3. They found the threshold energies of the steps to be in approximate agreement with a calculation of the confinement levels (E_N) for the QWs.

In reality the confining barriers will have only a finite height (V_0) and the wavefunctions penetrate a small distance into the barrier. This increases the de Broglie wavelength and so reduces the energy of the confined levels. There is then a certain phase change on reflection off the barrier, which is given by the second term on the left-hand-side of Eq. 2.3. Hence the condition for constructive interference becomes

$$(2 m_w E)^{1/2} / \hbar L_w + 2 \tan^{-1} \{ (m_b E / m_w (V_0 - E))^{1/2} \} = N \pi \quad \dots 2.3$$

where m_w and m_b are the z-direction effective masses in the well and barrier. Masselink et al [2.2] have written down an expression which is valid for barriers of a finite thickness. In this case, in addition to a phase change upon reflection, the transmission is finite and the modulus of the reflectivity less than unity.

The energies (E) and wavefunctions (Φ) of the system can also be found (equivalently) by solution of Schrödinger's equation

$$-(\hbar^2/2m) \nabla^2 \Phi + V(z) \Phi = E \Phi \quad \dots 2.4$$

where $V(z)$ is the confining potential. Under the effective mass envelope function approximation, the wavefunction of a single particle can be written as the product

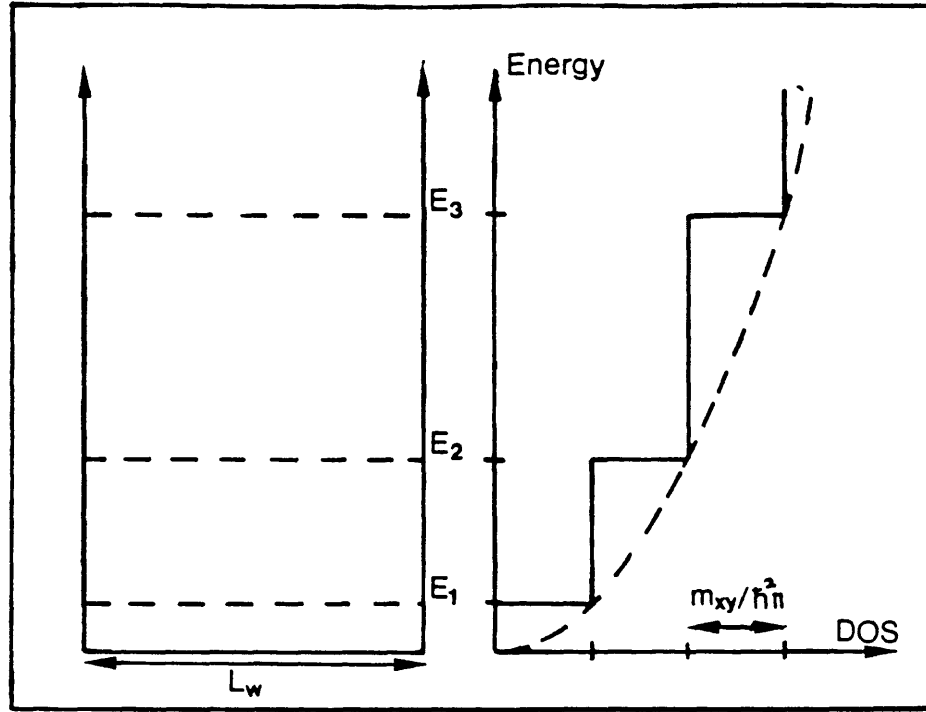


Fig. 2.1 : Schematic drawing of subband structure of a QW. The discrete energy solutions for the z -motion, form the thresholds of two-dimensional subbands, each with an energy independent density of states (DOS). The dashed line shows the density of states of bulk material.

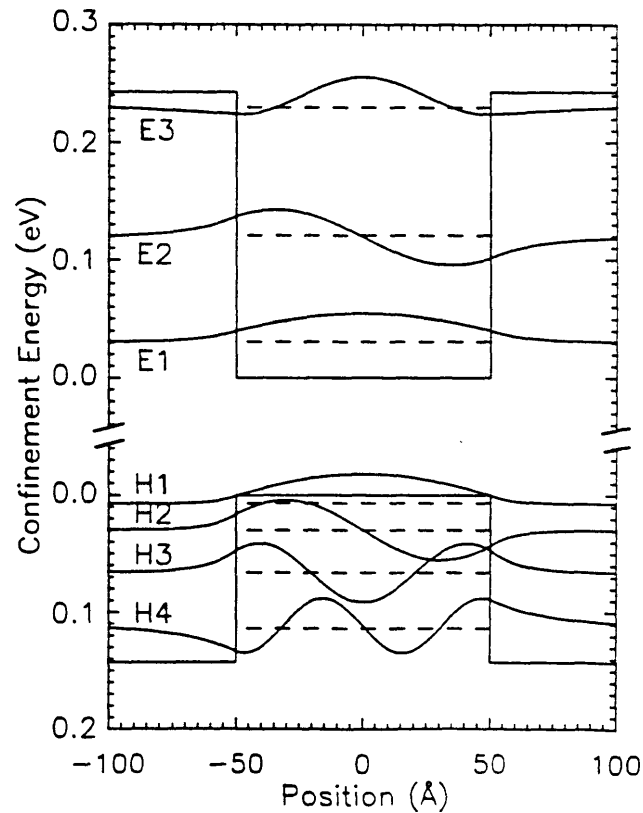


Fig. 2.2 : The zero-field envelope functions of the electron and heavy hole states confined in a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, calculated using the method described in text and the parameters listed in table 4.1.

$$\Phi = U_0(\underline{r}) \Psi(z) \exp \{ i (k_x \cdot x + k_y \cdot y) \} \quad \dots 2.5$$

where $\Psi(z)$ is the envelope function describing the z-motion; the exponential describes the free motion in the xy-plane and $U_0(\underline{r})$ is the zone centre Bloch function of the bulk semiconductor band forming the well. Schrödinger's equation (Eq. 2.4) can then be reduced to be one-dimensional

$$- (\hbar^2/2m) \cdot \delta^2 \Psi / \delta z^2 + V(z) \Psi = E_N \Psi \quad \dots 2.6$$

where E_N is the energy associated with the z-motion, which forms the threshold of a subband due to the free-electron-like motion in the xy-plane.

For the rectangular confining potential, $V(z)$, shown in Fig. 2.2, solution of Eq. 2.6 yields

$$\Psi(z) = a \exp (+ i k z) + b \exp (- i k z) \quad \dots 2.7$$

$$\text{where } k = (2 m \{ E - V(z) \})^{1/2} / \hbar \quad \dots 2.8$$

In the well, $V(z) = 0$, so that k has a real value (written $k = k_w$). The envelope function will therefore be oscillatory inside the well, corresponding to the sum of the forward and backward travelling plane waves being reflected off the barriers. In the barriers, $V(z) = V_0$, so that k is purely imaginary ($k = i k_b$) for energies $E < V_0$, and the envelope function is determined by the sum of an increasing and decreasing exponential. In order for the envelope function to be finite in the limit $z \rightarrow +\infty$, the b coefficient in Eq. 2.7 must be zero in the positive- z barrier. Similarly, $a = 0$ in the negative- z barrier, so as to keep the envelope function finite as $z \rightarrow -\infty$. Hence the envelope region in the well and two barrier regions can be written as

$$2z \leq -L_w, \quad \Psi = c \quad b_b \exp (k_b z) \quad \dots 2.9$$

$$-L_w \leq 2z \leq L_w, \quad \Psi = c \{ a_w \cos (k_w z) + b_w \sin (k_w z) \} \quad \dots 2.10$$

$$+L_w \leq 2z, \quad \Psi = c \quad \exp (- k_b z) \quad \dots 2.11$$

where b_b , a_w , b_w and c are constants.

The coefficient c in Eqs. 2.9-11 can be determined explicitly by applying the normalisation condition $|\langle \Psi | \Psi \rangle|^2 = 1$. The remaining three unknown coefficients (b_b , a_w , b_w) and the energy (E) are determined by the conditions that the envelope function (Ψ) and the

particle flux $m^{-1} \cdot (\delta\psi/\delta z)$ be continuous across the two well interfaces. The continuity conditions form four simultaneous equations, which yield discrete solutions for the four unknowns (E, b_p, a_w, b_w) , corresponding to the confined states of the QW. These can be found using the standard matrix method of solving an eigenvalue problem.

The symmetry of the QW potential about $z = 0$, allows a further simplification of the problem. The equations formed from the continuity conditions at $z = +\frac{1}{2} L_w$, are identical to those at $z = -\frac{1}{2} L_w$. Consequently, $b_p = +1$ and $b_w = 0$ for the solutions with a symmetric envelope function (ie. $N = 1, 3, \dots$), while $b_p = -1$ and $a_w = 0$ for the solutions with an antisymmetric envelope functions ($N = 2, 4, \dots$). However, the analysis has not been simplified in this manner, so as to allow a comparison with the situation under a z -direction electric field, where the potential is no longer symmetric.

The band alignment of the GaAs/ Al_xGa_{1-x} As hetero-system is such that the minimum of both the valence and conduction bands, at the Brillouin zone centre, lies in the GaAs layer, so that it is "type-I". Consequently a confining QW is formed for both the electrons and holes, as shown in Fig. 2.2. Several experimental measurements [2.3-6] have shown the conduction band to assume about 0.65 of the difference in the GaAs and Al_xGa_{1-x} As bandgaps. Fig. 2.2 plots the confined electron and heavy hole envelope functions calculated for a 100 Å GaAs/ $Al_{0.3}Ga_{0.7}$ As QW, using the parameters listed in table 4.1. It can be seen that there are three electron (labelled E1, E2, E3) and four heavy hole (H1, H2, H3, H4) confined states for this QW. As expected, the envelope functions can be seen to be oscillatory in the well, with an approximate number of half wavelengths equal to the subband number, while decaying exponentially in the barriers.

Optical transitions can occur between the various subbands in the valence and conduction band. Fig. 2.3 plots the variation in the sum of electron and hole confinement energies for the Mth electron and Nth hole subband, as a function of the well width, for a GaAs/ $Al_{0.3}Ga_{0.7}$ As QW, again using the parameters listed in table 4.1. The valence band of bulk GaAs is actually doubly degenerate at its minimum, as discussed in Sec. 2.2, and so transitions are possible between the electron and heavy hole (denoted by ENHM) and light hole (ENLM) subbands, as shown by Figs. 2.3.1 and 2.3.2 respectively.

When the well width is reduced there is an increase in the electron and hole confinement energies, as can be seen in Fig. 2.3. This results in fewer states being confined by the wells, although there is always at least one for both the valence and conduction band. This

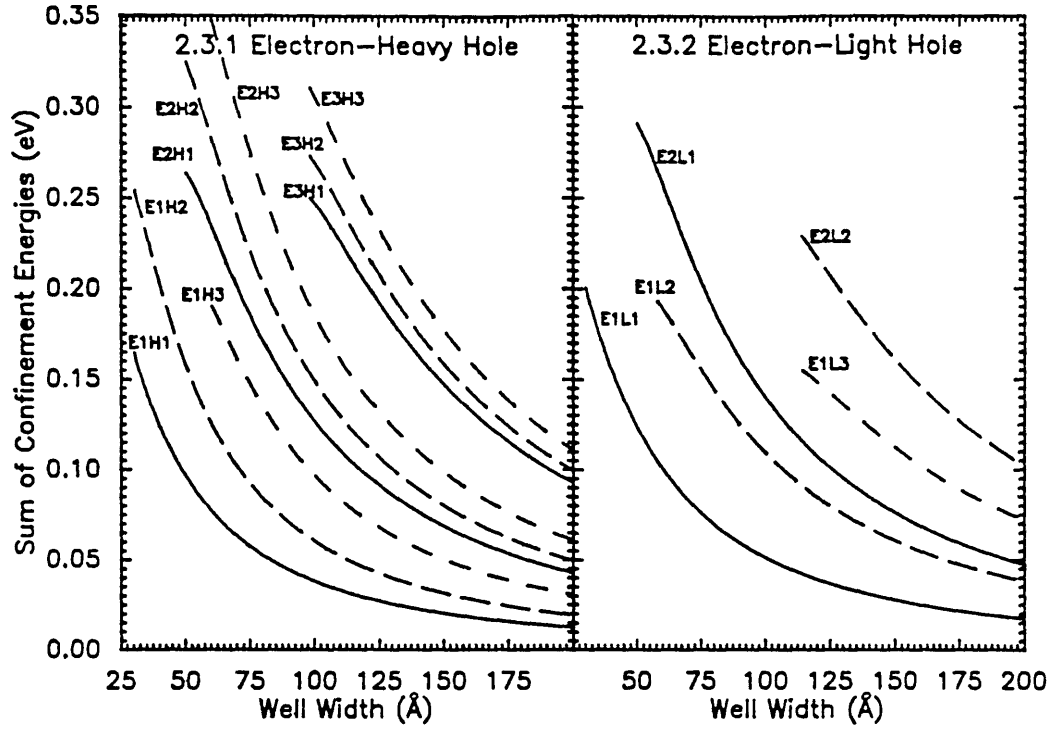


Fig. 2.3 : The sum of the zero-field electron and hole confinement energies of a GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW as a function of well width, for the heavy (2.3.1) and light (2.3.2) hole transitions, calculated using parameters of table 4.1.

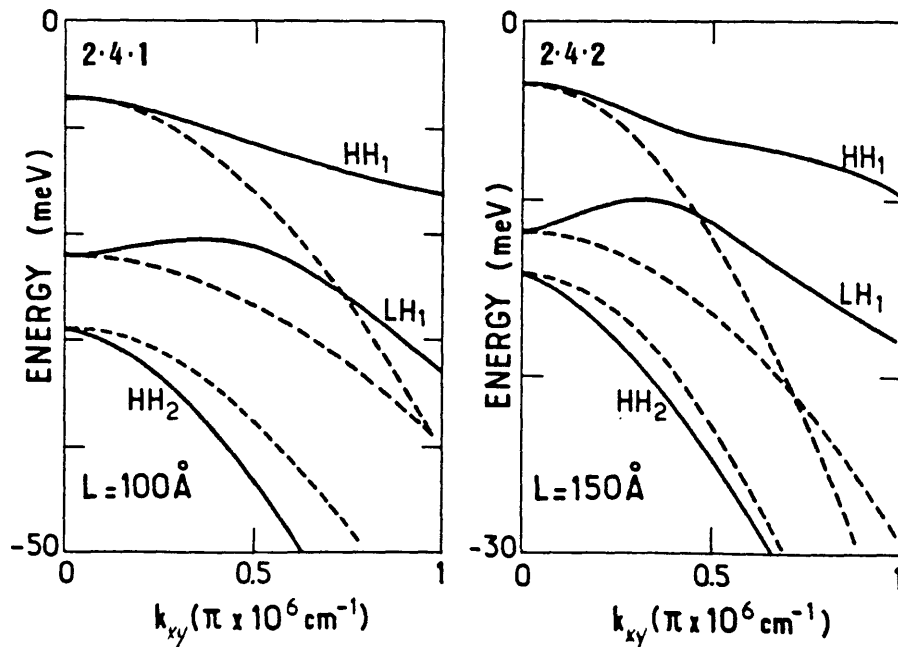


Fig. 2.4 : In-plane hole dispersion for a 100 (2.4.1) and 150 Å (2.4.2) GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, reported by Bastard and Brum [2.7]. The parabolic dispersions in the absence of the subband mixing are shown dashed.

is because the increased penetration of the envelope function into the barriers, counteracts the reduction in the well width, when the wells are very narrow. Although the finite barrier height significantly reduces the energy of the lowest subband (E_1), the excited states are found to be still approximately given by $E_N = N^2 \cdot E_1$, provided that the level is not too close to the top of the well.

For energies $E > V_0$, the value of k in Eq. 2.8 is imaginary in both the well and barriers. The barrier envelope function can therefore also be written as a real sum of sine and cosine terms, as is Eq. 2.10. The continuity conditions then provide solutions at every energy value, so giving rise to a continuum of levels. However, the envelope function is concentrated inside the QW region at certain energies, which are known as the "unconfined states" of the well.

2.2 Valence Band Structure

2.2.1 Bulk Semiconductor

The "tight binding" approach for calculating the bandstructure of semiconductors considers the crystal energy states as deriving from the orbitals of its constituent atoms or molecules. In bulk III-V semiconductors the conduction band is comprised of s atomic orbitals, which have an orbital angular momentum quantum number of $l = 0$, while the valence band is built from p atomic orbitals, with $l = 1$. Therefore the valence band is constructed from six atomic p orbitals, corresponding to the three states with different z-component of angular momentum ($m = -1, 0, 1$), each having a double degeneracy due to the electron spin $s = +\frac{1}{2}$ or $-\frac{1}{2}$. The total angular momentum (J) and its z-component (J_z), the resultant of addition of the orbital momentum and the spin, are good quantum numbers for the six state system. That is to say, the values of J and J_z ($=J, J-1 \dots -J$) define the time-independent eigenstates.

The interaction of the orbital angular momentum and the spin results in two possible values for the total angular momentum of $J = 3/2$ or $1/2$. The doubly degenerate $J = 1/2$ state is shifted to lower energy and is therefore called the "spin-orbit split-off" (SOSO) band, while the $J = 3/2$, $J_z = \pm 3/2$ and the $J = 3/2$, $J_z = \pm 1/2$ bands remain degenerate at the centre of the Brillouin zone.

It is informative to express the eigenfunctions $|J, J_z\rangle$ in terms of the time-dependent basis set $|P_j, s\rangle$, corresponding to the orbital

momentum states $|P_j\rangle$ with electron spin state $|s\rangle$. It can be shown [2.7] that

$$|3/2, \pm 3/2\rangle = 1/\sqrt{2} (|p_x, \pm 1/2\rangle \pm i |p_y, \pm 1/2\rangle) \quad \dots 2.12$$

$$|3/2, \pm 1/2\rangle = 1/\sqrt{6} (\pm |p_x, \bar{1}/2\rangle + i |p_y, \bar{1}/2\rangle - 2 |p_z, \pm 1/2\rangle) \quad \dots 2.13$$

$$|1/2, \pm 1/2\rangle = 1/\sqrt{3} (\pm |p_x, \bar{1}/2\rangle + i |p_y, \bar{1}/2\rangle + |p_z, \pm 1/2\rangle) \quad \dots 2.14$$

Eq. 2.12 shows that the $|3/2, \pm 3/2\rangle$ state is comprised only of the atomic orbitals aligned in the x- and y-directions. For these states there will be a small overlap between adjacent atomic sites in the z-direction, and hence the holes will have a large effective mass for transport in this direction. Consequently this band is referred to as the "heavy" hole (HH). Conversely from Eq. 2.13, the $|3/2, \pm 1/2\rangle$ is comprised mostly of p_z orbitals and will therefore have a larger mobility in the z-direction, giving a "light" hole (LH) mass. Notice that similar arguments lead to the conclusion that the $|3/2, \pm 3/2\rangle$ band is the lighter for transport along a direction in the xy-plane, showing that the bands are highly anisotropic. Meanwhile the SOSO band is expected to have a mass which is intermediate between that of the HH and LH band, for both the z- and xy-directions. The conduction band is comprised of s orbitals and so has an isotropic effective mass, in this simple picture.

In III-V semiconductors the bands are approximately parabolic near the Brillouin zone centre. Hence the dispersion is similar to that of free electrons, with an effective mass describing the band curvature. The maximum of the valence band is comprised of a HH and LH band which are degenerate at $\underline{k} = 0$. The preceding discussion has argued that the HH band has a larger effective mass for motion in the z-direction and so has a shallower dispersion than the LH band. However the dispersions of the two bands are highly anisotropic, so that this ordering of the effective masses and curvatures reverses for motion in the xy-plane.

The dispersion of the LH and HH, as well as all the other energy bands, can be calculated using a detailed tight binding model, see for instance Ref. 2.8. This confirms the anisotropy of the valence band that has been argued qualitatively above. However, the conduction and valence bands can also be described quantitatively, around the Brillouin zone centre, using the simpler $\underline{k} \cdot \underline{p}$ theory of Luttinger and Kohn [2.9], which lends a more intuitive insight into their nature. This calculates the interaction of the conduction, HH, LH and SOSO bands under the effective mass approximation. For semiconductors with a large bandgap,

eg. GaAs, it is a good approximation to ignore the interaction of the conduction and valence bands. Furthermore at energies near the maximum of the valence band, it is a good approximation to also ignore the influence of the SOSO band. The hybridisation of the HH and LH bands, due to their mutual interaction, can then be calculated using the matrix equation [2.7]

$$\begin{pmatrix} H_{hh} & c & b & 0 \\ c^* & H_{lh} & 0 & -b \\ b^* & 0 & H_{lh} & c \\ 0 & -b^* & c^* & H_{hh} \end{pmatrix} \begin{pmatrix} |3/2, +3/2\rangle \\ |3/2, -1/2\rangle \\ |3/2, +1/2\rangle \\ |3/2, -3/2\rangle \end{pmatrix} = E \begin{pmatrix} |3/2, +3/2\rangle \\ |3/2, -1/2\rangle \\ |3/2, +1/2\rangle \\ |3/2, -3/2\rangle \end{pmatrix} \quad \dots 2.15$$

where

$$H_{hh} = -(\hbar^2/2m_0) \{ (k_x^2 + k_y^2) (\gamma_1 + \gamma_2) + k_z^2 (\gamma_1 - 2\gamma_2) \} \quad \dots 2.16$$

$$H_{lh} = -(\hbar^2/2m_0) \{ (k_x^2 + k_y^2) (\gamma_1 - \gamma_2) + k_z^2 (\gamma_1 + 2\gamma_2) \} \quad \dots 2.17$$

$$c = (3^{1/2}\hbar^2/2m_0) \{ (k_x^2 - k_y^2) \gamma_2 - 2i k_x k_y \gamma_3 \} \quad \dots 2.18$$

$$b = (3^{1/2}\hbar^2/m_0) (k_x - ik_y) k_z \gamma_3 \quad \dots 2.19$$

and $\gamma_1, \gamma_2, \gamma_3$ are positive constants known as the Luttinger parameters.

At the Brillouin zone centre ($\underline{k} = 0$) the off diagonal terms in Eq. 2.15 are zero. Hence Eq. 2.15 reduces to four uncoupled equations, giving an $E(\underline{k})$ dispersion which is "free-electron-like", with effective masses for the $|3/2, \pm 3/2\rangle$ (HH), $|3/2, \pm 1/2\rangle$ (LH) bands, for motion in the z- and xy-directions, of

$$m_z \text{ (LH)} = (\gamma_1 + 2\gamma_2)^{-1} \quad \dots 2.20$$

$$m_{xy} \text{ (LH)} = (\gamma_1 - \gamma_2)^{-1} \quad \dots 2.21$$

$$m_z \text{ (HH)} = (\gamma_1 - 2\gamma_2)^{-1} \quad \dots 2.22$$

$$m_{xy} \text{ (HH)} = (\gamma_1 + \gamma_2)^{-1} \quad \dots 2.23$$

Notice that since the Luttinger parameters (γ_1, γ_2 and γ_3) are positive, this gives an ordering of the masses which is consistent with that argued earlier in a qualitative manner. The Luttinger parameters of GaAs have been estimated in a number of theoretical and also experimental studies, which rely on fitting values from the measured effective masses. Skolnick et al [2.10] performed cyclotron resonance measurements on GaAs and thereby estimated the valence band effective masses for different crystallographic axes. More recently there have

been estimates based upon measurements on GaAs/Al_xGa_{1-x}As QW structures, several of which are tabulated by Shanabrook et al [2.11]. This thesis uses the widely accepted values of $\gamma_1 = 6.8$ and $\gamma_2 = 1.9$, which yield $m_z(\text{HH}) = 0.34$, $m_z(\text{LH}) = 0.094$, $m_{xy}(\text{HH}) = 0.115$ and $m_{xy}(\text{LH}) = 0.204$ [2.12].

Moving away from the zone centre the diagonal terms are no longer zero and produce a mixing of the four hole bands. In practice this mixing is small, except when two bands are close in energy. In a QW the degeneracy of the HH and LH at $\underline{k} = 0$ is broken, which results in a greater mixing of the two bands as now discussed.

2.2.2 Quantum Wells

The degeneracy of the HH and LH bands at the Brillouin zone centre is lifted in a QW. For the coordinate system introduced earlier, the z-direction is perpendicular to the QW plane. The confinement results in only certain values of k_z being possible, corresponding to different subbands ($N = 1 \rightarrow \infty$), as discussed in Sec. 2.1. The HH, by virtue of its larger z-direction effective mass, has a smaller confinement energy than the LH, as can be inferred from Eq. 2.1. The motion of the holes in the plane of the QW is unrestricted, and hence the dispersions in the xy-plane can be expected to be parabolic, if valence band mixing effects are ignored. Since the LH band has a shallower curvature than the HH for motion in the xy-plane, there will be crossings of the various HH and LH subbands at points away from $\underline{k}_{xy} = 0$. There will be a strong hybridisation of the two subbands at these points in the dispersion, which will produce an "anti-crossing" and remove the degeneracy. Clearly then the bands are radically distorted from the simple parabolic shape, so that the effective mass will be a strong function of the momentum in the xy-plane.

The valence band structure of QWs can be calculated using an empirical tight binding model [2.13], or by using the $\underline{k} \cdot \underline{p}$ theory [2.14-17, 2.7] previously outlined for bulk semiconductors. The $\underline{k} \cdot \underline{p}$ analysis is extended to QWs by adding terms describing the confining potential $\{V(z)\}$ to H_{hh} (Eq. 2.16) and H_{lh} (Eq. 2.17), and replacing k_z by the operator $-i\partial/\partial z$. This is because the hole is no longer "free-electron-like" in the z-direction, ie the z-dependence of the envelope function is no longer given by $\exp(ik_z z)$, but must now be replaced by an envelope function, as in Eq. 2.5.

Fig. 2.4 shows the dispersion calculated by Bastard and Brum [2.7] for a 100 Å and a 150 Å GaAs/Al_{0.3}Ga_{0.7}As QW. The parabolic dispersions

produced in the absence of valence band effects are drawn as dashed lines. Clearly the mixing of the subbands causes a significant distortion of the free-electron-like dispersions, especially at k -vectors close to the anti-crossings. Notice that the curvature near the zone centre, for each of the three subbands shown, is also affected by the mixing. Most dramatic is the effect on the first light hole subband, which has an electron-like dispersion at $k_{xy} = 0$, resulting in a negative effective mass. Negative in-plane effective masses for the L1, and other subbands, have also been predicted by several other workers [2.17,2.13-14]. The detail in the valence band structure is very sensitive to the parameters used and the well width. However, the interaction with the other subbands will always result in a shallower in-plane dispersion for the lowest HH subband (ie. heavier m_{xy}) [2.7].

The off diagonal terms are zero at the Brillouin zone centre, as for bulk material, so the problem then reduces to the one-dimensional Schrödinger's equation for the single carrier confined states (Eq. 2.6, using the HH and LH masses for the z -direction of bulk material). Consequently the mixing will not alter the energies of the subband thresholds. However, the modification of the in-plane hole effective masses will alter their Coulombic binding with the electrons, as discussed in Sec. 2.4. The most dramatic effect of the valence subband mixing is to redistribute the oscillator strength of the various optical transitions from the valence subbands to those in the conduction band, as described in Sec. 2.3.

The Luttinger Hamiltonian can also be used to determine the effects of strain on the QW, which could be applied externally [2.18-22] or introduced internally by growth of lattice mismatched systems [2.17, 2.23,2.7,1.2]. The effect the strain is to shift the Brillouin zone centre energies of the HH subbands relative to those of the LH, and so it perturbs the valence bandstructure in a similar manner to the quantum confinement, again radically altering the in-plane dispersions.

2.3 Optical Transitions of a Quantum Well

Light can excite electrons to a higher energy state. The oscillator strength for a transition from an initial state with energy E_i and normalised wavefunction Φ_i , to a final state E_f , Φ_f , is given by Fermi's Golden Rule

$$f_{if} = (2/m_0 E) | \langle \Phi_f | \underline{e} \cdot \underline{p} | \Phi_i \rangle |^2 \quad \dots 2.24$$

where E is the photon energy ($E = E_f - E_i$), $\hat{p}$ is the momentum operator and $\underline{e}$ is the direction of polarisation of the electric field vector of the light.

In the envelope function approximation, the wavefunctions can be written as a product of the unit cell Bloch function (U) and an envelope function ($\equiv$), so that

$$f_{if} = (2/m_0 E) \left\{ \langle \equiv_f | \equiv_i \rangle \cdot \langle U_f | \underline{e} \cdot \hat{p} | U_i \rangle + \langle \equiv_f | \underline{e} \cdot \hat{p} | \equiv_i \rangle \cdot \langle U_f | U_i \rangle \right\}^2 \quad \dots 2.25$$

Eq. 2.25 can be used to determine the relative strengths of transitions within the same band (intraband) or between different bands (interband). It was discussed in Sec. 2.2 how the conduction and valence bands in III-V semiconductors are comprised of s and p atomic orbitals respectively. These atomic orbitals have opposite parity, so the second term in Eq. 2.25 is zero for transitions between the valence and conduction bands. Hence the oscillator strength for interband transitions will be proportional to the overlap integral of the electron and hole envelope functions. In contrast, the first term in Eq. 2.25 is zero for the intraband transitions, so that optical transitions are observed between the confined states which have opposite parity. Such intraband transitions have been observed in modulation doped samples, which have a highly populated $N = 1$ electron state [2.24].

The total envelope function ($\equiv$) in Eq. 2.25, can be separated into a product of a term ($\Psi(z)$) describing the confinement by the QW in the z-direction and a term due to the xy-motion. If the Coulombic attraction of the electron and hole is ignored, the motion in the xy-plane is free-electron-like, and is represented by a plane wave. The oscillator strength for transitions between the Mth valence and the Nth conduction subband will then be given by

$$f_{NM} = (2/m_0 E) P_{CV}^2 M_Z^2(N, M) \quad \dots 2.26$$

where the optical matrix element for transitions between the valence (v) and conduction (c) band, $P_{CV} = \left| \langle U_C | \underline{e} \cdot \hat{p} | U_V \rangle \right|$ and the overlap of the z-direction envelope functions, $M_Z(N, M) = \left| \langle \Psi_{e,N} | \Psi_{h,M} \rangle \right|$.

The value of P_{CV} depends upon the polarisation direction of the light ($\underline{e}$). For light polarised with the electric field vector perpendicular to the plane of the well, $\underline{e} \cdot \hat{p}$ reduces to $\hat{p}_z$. The hole Bloch functions can be written in terms of their constituent atomic orbitals, $|p_x\rangle$, $|p_y\rangle$ and $|p_z\rangle$, as in Eqs. 2.12-14, while the electron Bloch function is comprised of an $|s\rangle$ atomic orbital. Using the

orthogonality relationship, $\langle s | \hat{p}_i | p_j \rangle = \delta_{ij}$, the heavy and light hole transitions can be shown to have relative strengths of 0 and 1 respectively. Similarly for light polarised in the plane of the QW, the relative strengths are $P_{cv}(HH)/P_{cv}(LH) = 0.75/0.25$. Weiner et al [2.25] made absorption measurements on QWs using a waveguide geometry, and measured the relative strengths of the E1H1 and E1L1 transitions, to be in rough agreement with that predicted, for both of these two light polarisations.

If the wells are assumed to have infinite confining potential barriers, the electron and hole envelope functions have exactly the same form, and since the envelope functions of either carrier make an orthonormal set, there is a finite overlap only for transitions between the same subband number, ie $M_Z^2(N,M) = \delta(N,M)$. Hence the transitions between the same electron and hole subband number ($N = M$) are referred to as "allowed", while the others are called "forbidden". In real QWs, with a finite barrier height, the electrons and holes penetrate a different amount into barriers, creating a small overlap for forbidden transitions with the same parity, ie $(N-M) = \text{even}$. However, this overlap is small in comparison to that of an allowed transitions, for example, for the 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW discussed in Sec. 2.1, $M_Z^2(E1H1) = 0.988$ and $M_Z^2(E1H3) = 0.007$.

The optical spectra can be calculated by summing the oscillator strength over all the possible initial and final states, with the restraint that the energy and momentum be conserved. This yields the transition strength, at a photon energy of E , in the absence of the Coulombic interaction, to be given by

$$\varepsilon_i(E) = S_O \sum_{N,M} \int_0^\infty dE_{xy} D_{2D} M_Z^2(N,M) \delta(E_g + E_N + E_M + E_{xy} - E) \quad \dots 2.27$$

where the proportionality constant is given by [2.26,82]

$$S_O = \frac{4 \pi^2 e^2 \hbar^2}{m_O E L_w} \cdot \frac{2}{m_O E} \cdot P_{cv}^2 \quad \dots 2.28$$

The energy difference between the initial and final states has been written as a sum of the bulk bandgap (E_g); the electron (E_N) and hole (E_M) confinement energies; and the energy associated with the relative motion in the xy-plane (E_{xy}). The relative motion in the xy-plane has a constant density of states per unit area, given by $D_{2D} = m_O \cdot \mu_{xy} / \pi \hbar^2$, where μ_{xy} is the electron-hole reduced mass for motion in the

xy-direction. This assumes the in-plane dispersions are parabolic and so neglects the mixing of the valence subbands.

Other workers often prefer to calculate the absorption coefficient (α), which is related to the imaginary part of the dielectric function by the relation, $\alpha = (E/n_r \cdot \hbar \cdot c) \cdot \epsilon_i$, where n_r is the real part of the refractive index. The dielectric function and the absorption coefficient have been defined per unit volume of the QW in these relations, as is the common practice.

In the absence of valence band mixing effects, the integration over E_{xy} in Eq. 2.27 is trivial, yielding

$$\epsilon_i(E) = S_0 D_{2D} \sum_{N,M} M_Z^2(N,M) H(E_g + E_N + E_M - E) \quad \dots 2.29$$

where $H(x)$ is the unit step function defined by: $H(x < 0) = 0$, $H(x = 0) = \frac{1}{2}$, $H(x > 0) = 1$. Hence each pair of electron and hole subbands, will contribute a step to $\epsilon_i(E)$, with a threshold at $(E_g + E_N + E_M)$, and whose height is proportional to $M_Z^2(N,M)$. Since $M_Z^2 \approx \delta(N,M)$ at zero field, only steps due to the allowed transitions will be observed. Notice also that each step will have the same height, since D_{2D} is independent of N or M . Since P_{cv} is approximately independent of L_w [2.13] and $M_Z^2(N,N) \approx 1$, the height of each step can be expected to vary as L_w^{-1} . Consequently the total absorption of each subband in the QW, $(1 - \exp[-\alpha L_w])$, will be independent of its width.

Eq. 2.27 assumes each subband transition to have an infinitely narrow energy resolution, while in fact each will have a finite linewidth, due to a variety of broadening mechanisms. For instance, a fluctuation in the well width across the surface of the QW, would produce a variation in the electron and hole confinement energies. The finite linewidth can be included by replacing the Dirac-delta function in Eq. 2.27 by a lineshape function with an integrated strength of unity. This lineshape function must be included in the integral over E_{xy} and so will result in a broadening of the sharp edges at the threshold of each subband transition.

The stepped appearance of the QW absorption spectrum was first observed by Dingle et al [2.1], who made measurements on several GaAs/Al_{0.2}Ga_{0.8}As MQWs with different well widths, which are shown in Fig. 2.5. The spectra in Fig. 2.5 are complicated by the presence of large peaks just below the threshold of each subband transition. These are caused by the bound states produced by the Coulombic attraction of the electron and hole, which are discussed in detail in Sec. 2.4.

It was discussed in Sec. 2.2 how mixing of the valence subbands

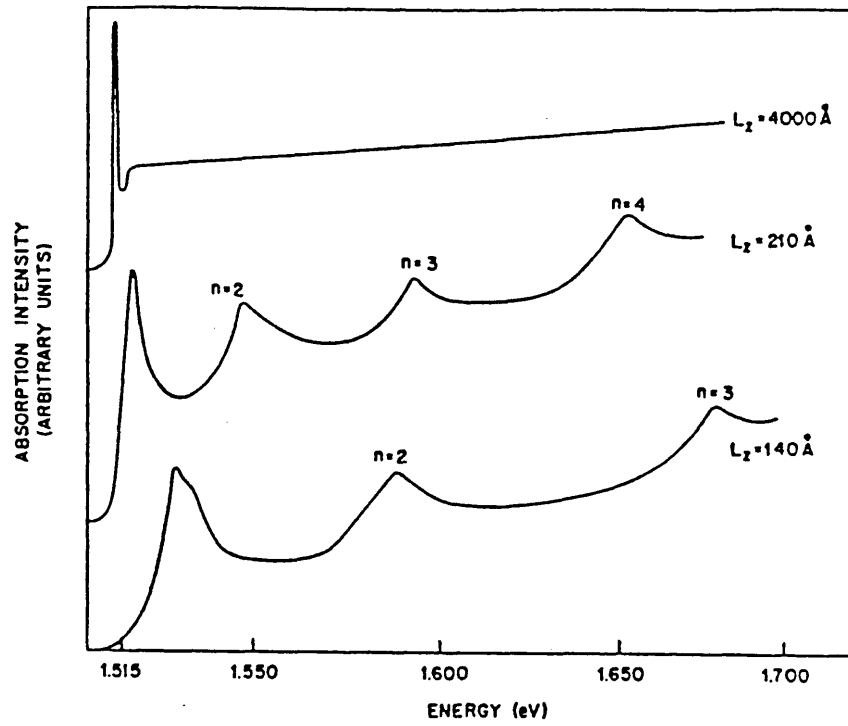


Fig. 2.5 : Absorption spectra (2K) of 4000, 210 and 140 Å GaAs/
Al_{0.2}Ga_{0.8}As QWs, measured by Dingle et al [2.1].

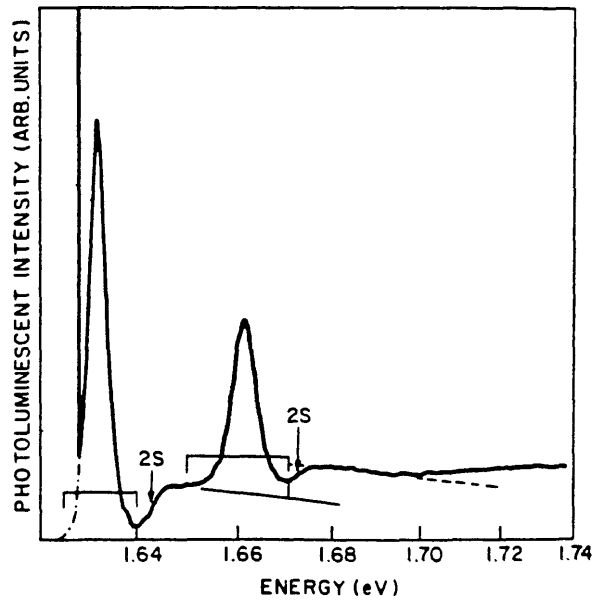


Fig. 2.6 : PLE spectrum of a 42 Å GaAs/Al_{0.37}Ga_{0.63}As SQW, measured by
R. Miller et al [2.31]. Temp = 6 K, PL detection = 1.6288 eV; Laser
power = 0.3 W/cm².

can result in the dispersion in the plane of the well being highly non-parabolic. At the zone centre, where the off-diagonal terms of the Luttinger Hamiltonian are zero, the hole subbands are purely heavy or light in nature, so that the energies of the continuum thresholds will not be altered by the mixing. However, the joint optical density of states (D_{2D}) for a pair of electron and hole subbands is proportional to its reduced mass, when parabolic bands are assumed. Consequently, the mixing can be expected to alter the density of states and distort the stepped appearance of the optical spectrum. The mixing will also affect the large peaks due to the bound electron-hole states that can be seen in Fig. 2.5. It is shown in Sec. 2.4 that these states have a finite k_{xy} , and so their binding energy and oscillator strength is significantly altered by the valence subband mixing away from zone centre.

2.4 Excitonic Interaction in a Quantum Well

Low temperature emission or absorption spectra of bulk semiconductors often show a sharp peak just below the bandgap. Wannier [2.27] was the first to propose that this is due to the bound state of the photoexcited electron and hole produced by their Coulombic attraction, called an exciton. The exciton binding energy and radius in bulk semiconductors can be estimated by an analogy with the Hydrogen atom, to be

$$R = (R_{Hyd} / n^2) \cdot (\mu / \epsilon_{rel}^2) \quad \dots 2.30$$

$$a = (a_{Hyd} \cdot n^2) \cdot (\epsilon_{rel} / \mu) \quad \dots 2.31$$

where R_{Hyd} and a_{Hyd} are the Hydrogen Rydberg constant and Bohr radius; $R_{Hyd} = m_0 \cdot e^3 / 32 \cdot \pi^2 \cdot \hbar^2 \cdot \epsilon_0^2 = 13.61 \text{ eV}$ and $a_{Hyd} = 4 \cdot \pi \cdot \hbar^2 \cdot \epsilon_0 / m_0 \cdot e^2 = 0.5292 \text{ \AA}$, with the principle quantum number, $n = 1, 2, 3, \dots$. The values of the binding energy and radius of the Hydrogen atom are modified for the semiconductor by its relative dielectric constant (ϵ_{rel}) and the reduced effective mass of the electron-hole pair (μ). For the ground state exciton ($n = 1$) in GaAs, $R = 3.7 \text{ meV}$ and $a = 150 \text{ \AA}$.

Elliott [2.28] showed that the oscillator strength of the excitonic states in bulk semiconductor vary as $f = F/n^3$, where F is a constant. Ralph [2.29] and Shinada and Sugano [2.30] repeated Elliott's analysis for excitons that are perfectly confined to two dimensions.

Schrödinger's equation for the planar Hydrogen atom has a similar solution to the three-dimensional case, except that the principle quantum number n is replaced with $(n-\frac{1}{2})$. Clearly then the reduced dimensionality produces an enhancement of the exciton binding energy and oscillator strength, of the ground state exciton, by factors of four and eight respectively.

Fig. 2.6 shows a low temperature PLE spectrum measured on a 42 Å GaAs/Al_{0.37}Ga_{0.63}As MQW by R. Miller et al [2.31]. This shows two large peaks due to the ground state exciton ($n = 1$) of the E1H1 and E1L1 transitions. Each of these peaks is accompanied by a shoulder to slightly higher energy, due to the merged contribution of the excited bound ($n \geq 2$) and unbound electron-hole states. The spectrum is clearly dominated by the contribution of the $n = 1$ excitonic state. Detailed calculations of the binding energy and oscillator strength of the $n = 1$ excitonic state are made in Sec. 2.4.1, while the weaker excited excitonic states and the continuum are discussed in Sec. 2.4.2.

2.4.1 Variational Calculations of Excitonic Ground State

The binding energy and oscillator strength of bound excitons in real QWs lies between the two- and three-dimensional limits, discussed above. This section describes calculations of the excitonic ground state in QWs not subject to an electric field. This is achieved by a solution of Schrödinger's equation and the most common approach has been to use a variational technique, such as that described by Bastard et al [2.32] or R. Miller et al [2.31]. The variational method described here will be extended later in Sec. 2.6.2 to calculate the excitonic binding energy and oscillator strength for a QW subject to a uniform electric field.

The exciton Hamiltonian can be written as a sum of terms describing the z -direction motion of the electrons ($\hat{H}_e$) and holes ($\hat{H}_h$), as in Eq. 2.6, and a third due to the motion in the xy -plane ($\hat{H}_{xy}$)

$$\hat{H} = \hat{H}_e(z_e) + \hat{H}_h(z_h) + \hat{H}_{xy}(z_e, z_h, \underline{r}_{xy}) \quad \dots 2.32$$

$$\hat{H}_{xy} = -\frac{1}{r_{xy}} \frac{\delta}{\delta r_{xy}} r_{xy} \frac{\delta}{\delta r_{xy}} - \frac{1}{r_{xy}^2} \frac{\delta^2}{\delta \theta_{xy}^2} - \frac{2}{\{(z_e - z_h)^2 + r_{xy}^2\}^{\frac{1}{2}}} \quad \dots 2.33$$

The motion in the xy -plane has been resolved into that of the centre of mass ($\underline{R}_{xy}$) and the relative coordinate ($\underline{r}_{xy}$). The first two terms in

Eq. 2.33 are due to the kinetic energy of the relative xy-motion, expressed in polar coordinates $\underline{r}_{xy} = (r_{xy}, \theta_{xy})$, while the third term derives from the Coulombic attraction of the electron and hole. The energies and distances in Eq. 2.33 have been expressed in units of R_∞ and a_∞ respectively, the binding energy and radius of bulk ground state excitons, calculated using Eqs. 2.30-31 and the reduced mass for motion in the xy-direction (μ_{xy}).

Eq. 2.33 has neglected the terms due to the kinetic energy of the centre of mass. The centre of mass is unaffected by the Coulombic interaction and therefore has a free-electron-like motion, which can be treated separately. The centre of mass energy is restricted by the requirement that the total momentum be conserved in the optical process. Thus it can be shown that the centre of mass energy is negligibly small compared to the binding energy of the relative motion.

The variational method involves assuming the form of the exciton envelope function (Φ) with certain unknown parameters, which are chosen so as to minimise the energy expectation value $\langle \Phi | \hat{H} | \Phi \rangle$. This has a value approximating to the exciton energy, provided a good guess was made for the form of Φ . The binding energy (E_{be}) is then the difference in the energy expectation value evaluated with and without the term describing the Coulombic attraction ($\hat{H}_{ex}$). Dropping $\hat{H}_{ex}$ in Eq. 2.32 yields the carrier confinement energies discussed in Sec. 2.1.

Bastard et al [2.32] considered an exciton envelope function of the form

$$\Phi = \Psi_e(z_e) \Psi_h(z_h) \phi(\underline{r}_{xy}, z_e, z_h) \quad \dots 2.34$$

where $\Psi_e(z_e)$ and $\Psi_h(z_h)$ are the electron and hole z-direction envelope functions respectively, found by solution of the single carrier Hamiltonians $\hat{H}_e$ and $\hat{H}_h$, and $\phi(\underline{r}_{xy}, z_e, z_h)$ describes the Hydrogenic part of the electron-hole state. The plane wave term due to the free-electron-like centre of mass motion in the xy-plane has again been ignored. Bastard et al assumed perfect confinement in the wells, hence $\Psi_e(z_e)$ and $\Psi_h(z_h)$ are given by $\Psi(z) = L_w^{-1} \cos(\pi.z/L_w)$. This assumption leads to an overestimate of the binding energy, as discussed below, especially for narrow QWs, where the envelope functions penetrate a significant depth into the barrier. Meanwhile, for the Hydrogenic part of the envelope function, Bastard et al employed two trial functions

$$\phi_1(r_{xy}) = (2/\pi)^{\frac{1}{2}} (1/\sigma_1) \exp(-r_{xy} / \sigma_1) \quad \dots 2.35$$

$$\phi_2(r_{xy}, z_e, z_h) = A_2 \exp(- \{ r_{xy}^2 + (z_e - z_h)^2 \}^{\frac{1}{2}} / \sigma_2) \quad \dots 2.36$$

where σ_1 and σ_2 are the variational parameters describing the separation of the hole and electron, and A_2 is a normalisation constant given in Ref. 2.31. ϕ_1 will only be accurate when the z-direction separation of the carriers ($z_e - z_h$) is small compared to that in the xy-direction (r_{xy}), ie for narrow QWs. However, ϕ_1 has the advantage of allowing a much simpler solution than ϕ_2 , as the motion in the z- and xy-directions can then be treated separately, as shown below. Some workers have also made calculations using a more general form than ϕ_2 , which includes two different variational parameters to describe the separation in the z- and xy-directions [2.33].

The variation with well width of the binding energy and the root mean square electron-hole separation in the xy-plane (defined later in Eq. 2.43), calculated by Bastard et al, are shown in Fig. 2.7. Decreasing the well width reduces the average separation of the electron and hole in the z-direction, and so increases their Coulombic attraction. This produces an increase in the binding energy and decrease in the average separation in the xy-direction. The curves for both trial envelope functions show that as $L_z \rightarrow 0$, $E_{ex} \rightarrow 4R_\infty$, which is the value for the perfect two-dimensional limit. However, the two-dimensional behaviour disappears with increasing well width, such that at $L_w = a_\infty$, $E_{be} = 2R_\infty$. As expected, calculations with the separable exciton envelope function (ie using ϕ_1) are not accurate for wide wells and only ϕ_2 yields the three-dimensional behaviour in the limit $L_w \rightarrow \infty$.

R. Miller et al [2.31] also made variational calculations of the exciton binding energies as a function of well width, using the non-separable trial envelope function of Eq. 2.36, and again assuming perfect confinement in the well. They compared the calculated 1s-2s separation for the E1H1 and E1L1 excitons, with that they measured in the PLE of a number of GaAs/Al_{0.37}Ga_{0.63}As MQW samples [2.36]. Fig. 2.6 shows an example of one of these PLE spectra for a 42 Å QW. The 2s position was taken as the midpoint of the rising edge in the spectra, due to the unresolved excited excitonic states and the continuum, as shown in Fig. 2.6. It can be seen in Fig. 2.8 that they found very good agreement with the calculated binding energies over a wide range of well widths. The agreement is perhaps surprising considering that they assumed infinite confining barriers, which tends to overestimate the binding energy, as discussed below. The anomaly may be explained by the decrease in the binding energy due to the finite barrier height being counteracted by an increase due to valence subband mixing, as discussed

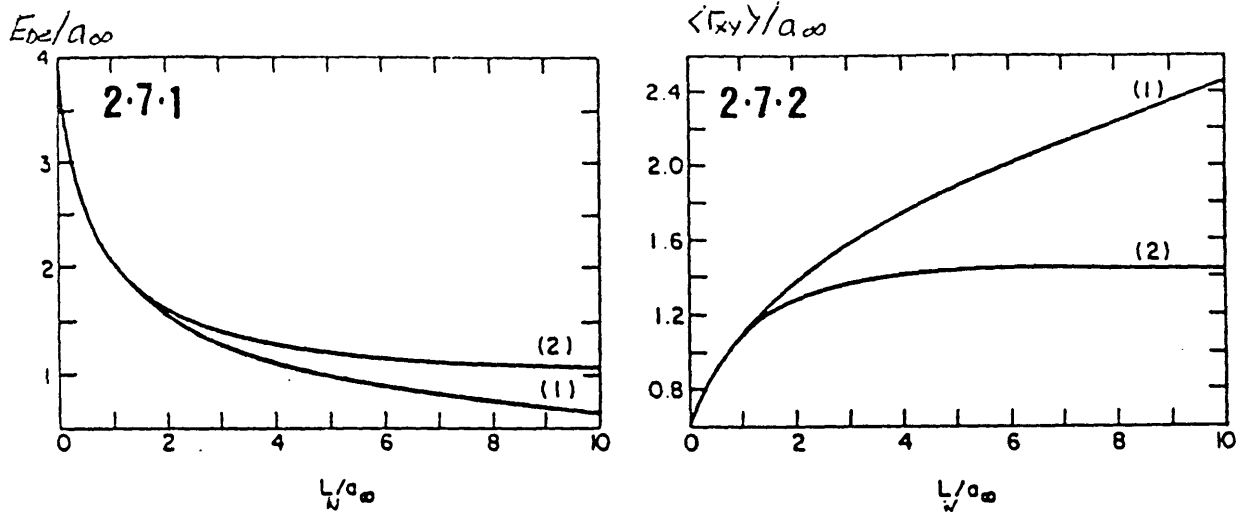


Fig. 2.7 : Variational calculation of Bastard et al [2.32] of the well width dependence of exciton binding energy (2.7.1) and its extension in xy-plane (2.7.2), for a GaAs QW with infinite barriers. Two variational envelope functions are considered (1) ϕ_1 and (2) ϕ_2 , as explained in text.

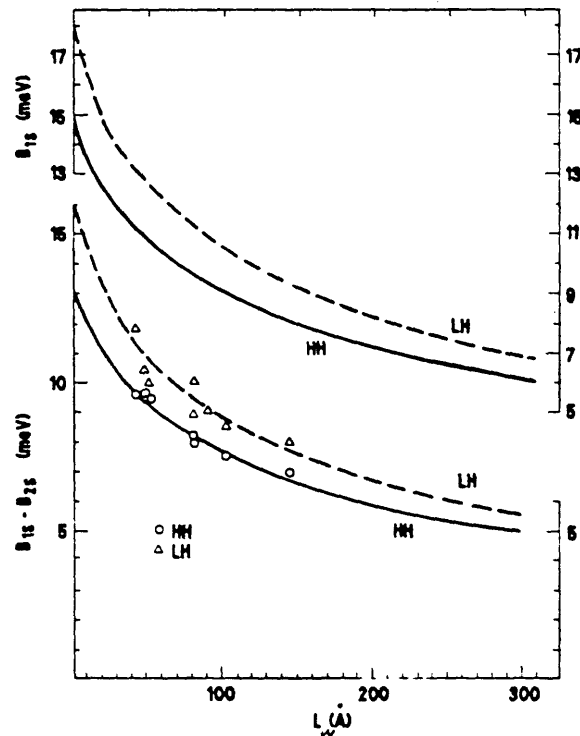


Fig. 2.8 : The separation of the 1s and 2s excitons, for E1H1 ($\circ$) and E1L1 ($\triangle$), in PLE spectra of GaAs/ $\text{Al}_{0.37}\text{Ga}_{0.63}\text{As}$ QWs (eg. Fig. 2.6). The solid lines show a variational calculation of the 1s-2s separation for E1H1 and E1L1. Also shown (dashed lines) are the calculated 1s binding energies. Taken from R. Miller et al [2.31].

below. An accurate comparison between theory and experiment is further complicated by uncertainty in the value of some of the parameters used in the calculation, as discussed later in Sec. 2.6.2.

The trial envelope function of Eq. 2.35 allows the motion in the z- and xy-directions to be considered separately. The exciton binding energy is then given by

$$E_{be} = \langle \Psi_e \Psi_h \phi_1 | \hat{H}_{xy} | \Psi_e \Psi_h \phi_1 \rangle \quad \dots 2.37$$

If ϕ has a dependence on z_e or z_h , as in Eq. 2.36, this separation is not possible as ϕ cannot then be cancelled from the $\langle \Phi | \hat{H}_{e(h)} | \Phi \rangle$ terms. The integral over r_{xy} and θ_{xy} in Eq. 2.37 can be solved explicitly, to yield [2.34]

$$E_{be}(\sigma_1) = \int_{-\infty}^{+\infty} dz_e |\Psi_e(z_e)|^2 \int_{-\infty}^{+\infty} dz_h |\Psi_h(z_h)|^2 E_O(z_e, z_h, \sigma_1) \quad \dots 2.38$$

$$E_O(z_e, z_h, \sigma_1) = \sigma_1^{-2} - 4 \sigma_1^{-1} P(2 |z_e - z_h| / \sigma_1) \quad \dots 2.39$$

The energies and distances in Eq. 2.39 are again written in units of R_∞ and a_∞ . The first term in Eq. 2.39 derives from the first term in $\hat{H}_{xy}$ and is due to the kinetic energy, while the second term is the potential energy, deriving from the Coulombic attraction. E_O , as defined by Eq. 2.39, will be negative after finding its minimum value. The function $P(x)$ is given by

$$P(x) = \int_0^\infty y (y^2 + x^2)^{-\frac{1}{2}} e^{-y} dy = \frac{1}{2} \pi x \{ H_1(x) - Y_1(x) \} - x \quad \dots 2.40$$

where $H_1(x)$ is the first-order Struve function and $Y_1(x)$ is the first-order Bessel function of the second-kind, both of which are defined in Ref. 2.35. $P(x)$ was calculated from a series expansion for $H_1(x)$ and $Y_1(x)$ at small x [2.35], and from an asymptotic expansion for $\{H_1(x) - Y_1(x)\}$ at large x [2.35]. $P(x)$ is unity at $x = 0$ and tends to zero as $x \rightarrow \infty$. Substituting $P(0) = 1$ into Eq. 2.39, recovers the result in the perfect two-dimensional limit, $\sigma/a_\infty = \frac{1}{2}$ and $E_{be}/R_\infty = -4$.

In the calculations presented here, Ψ_e and Ψ_h are determined using the exact analysis described in Sec. 2.1, which takes account of the finite barrier height. Again the parameters for this calculation are listed in table 4.1. This solution for Ψ_e and Ψ_h has the advantage of being applicable to the higher subband transitions, while also being readily adaptable to make field-dependent calculations of the binding energy, as described in Sec. 2.6.2. After solution for Ψ_e and Ψ_h , Eq.

2.38 is solved at a particular value of σ_1 , by a numerical integration over z_e and z_h . σ_1 is then varied to find the value that minimises $E(\sigma_1)$, using the Newton-Rhapson method [2.35].

The Coulombic interaction modifies the expression previously given for the oscillator strength, Eq. 2.26, as there is then no longer free-electron-like motion in the xy-direction. A more general expression than Eq. 2.26 includes a term describing the overlap of the electron and hole envelope functions in the xy-plane. The xy-motion is described by an envelope function for the relative coordinate in Eq. 2.34, $\phi(\underline{r}_{xy})$, in which case the square of the xy-overlap, per unit area of the xy-plane, is given by $|\phi(\underline{r}=0)|^2$. Hence the imaginary part of the dielectric function, at a photon energy of E , is given by

$$\epsilon_i(E) = S_0 \sum_{N,M,E_{xy}} |\phi(0)|^2 M_z^2(N,M) \delta(E_g + E_N + E_M + E_{xy} - E) \quad \dots 2.41$$

where the symbols have the same meaning as in Sec. 2.3. This expression is general to both the bound and unbound states of the relative motion in the xy-plane.

For the transitions between unbound electron-hole states for the xy-motion, the summation over E_{xy} in Eq. 2.41 must be replaced by an integration over a continuum, ie $\int_0^\infty A_w \cdot D_{2D} dE_{xy}$, where A_w is the area of the QW interface and D_{2D} is again the density of states per unit area. In the absence of the Coulombic interaction, the relative motion in the xy-plane is free-electron-like and can be represented by a plane wave, so that $|\phi(0)|^2 \rightarrow A_w^{-1}$. Eq. 2.41 will therefore reduce to the previous expression, Eq. 2.27, in the absence of the Coulombic interaction. However, the Coulombic interaction modifies the form of ϕ by reducing the mean electron-hole separation and thereby enhancing the value of $|\phi(0)|^2$. The enhancement declines with the kinetic energy of the relative motion (E_{xy}), so that the Coulombic interaction is negligible at large E_{xy} and $|\phi(0)|^2 \rightarrow A_w^{-1}$, as for free carriers. The transition strength is therefore significantly enhanced at the continuum threshold and decreases to the constant value for free-carriers with increasing photon energy. Shinada and Sugano [2.30] showed the transition strength of the unbound states to be enhanced by a factor of two at the continuum edge, in the perfect two-dimensional limit, while Chan et al [2.36] calculated the enhancement to be smaller for real QWs.

For the bound excitonic transitions, E_{xy} represents the binding energy and has discrete, negative values. Hence the summation over E_{xy} is replaced by a summation over the discrete bound excitonic states,

$n = 1, 2, 3, \dots, \infty$. For the $n = 1$ bound excitonic transition, represented by the variational envelope function of Eq. 2.35,

$$|\phi(0)|^2 = (2/\pi)^{\frac{1}{2}} \sigma_1^{-2} \quad \dots 2.42$$

As the well width is reduced, there will be a reduction in the variational parameter, due to the increased electron-hole attraction and this will produce an increase in the excitonic oscillator strength. Hence the absorption coefficient, defined per unit length of QW, will increase more rapidly than L_w^{-1} . Masomoto et al [2.33] measured the absorption strength of E1H1(1s) excitons in GaAs/AlAs MQWs with different well widths and observed good agreement with their variational calculation.

Another interesting parameter of the excitonic state, is its root mean square extent in the xy-plane, which can be determined explicitly, for the separable exciton envelope function, from the expectation value of r_{xy}^2 [2.32]. Similarly, the mean in-plane k-vector can also be determined from its expectation value, yielding

$$\langle \Phi | r_{xy}^2 | \Phi \rangle^{\frac{1}{2}} = (3/2)^{\frac{1}{2}} \sigma_1 \quad \dots 2.43$$

$$\langle \Phi | k_{xy} | \Phi \rangle = 1 / \sigma_1 \quad \dots 2.44$$

Fig 2.9 plots the well width dependence of the binding energy of the excitons formed between the various electron and heavy hole subbands in a GaAs/Al_{0.3}Ga_{0.7}As QW. The exciton envelope function was calculated using $\epsilon_{rel} = 12.2$ [2.31] and $\mu_{xy} = 0.042$, deduced from the Luttinger parameters given in Sec. 2.2. These yield $R_\infty = 3.84$ meV and $a_\infty = 154$ Å.

Fig. 2.9 also shows the binding energy of the E1H1 exciton calculated assuming perfect confinement in the well (dotted line), and is therefore similar to the curve calculated by Bastard et al, shown in Fig. 2.7. It can be seen that the E1H1 calculation using finite barriers gives a significantly smaller binding energy at all well widths. Taking a 100 Å wide QW, for example, the finite barrier calculation gives a binding energy that is 0.7 meV smaller than that for the infinite barrier case. The discrepancy is largest for very narrow well widths, where the envelope functions penetrate a large distance into the barriers, so producing a dramatic decrease in the binding energy. These conclusions are in agreement with those of Greene and Bajaj [2.37], who first reported the effect of the finite barrier height. Priester et al [2.38] later improved Greene and Bajaj's

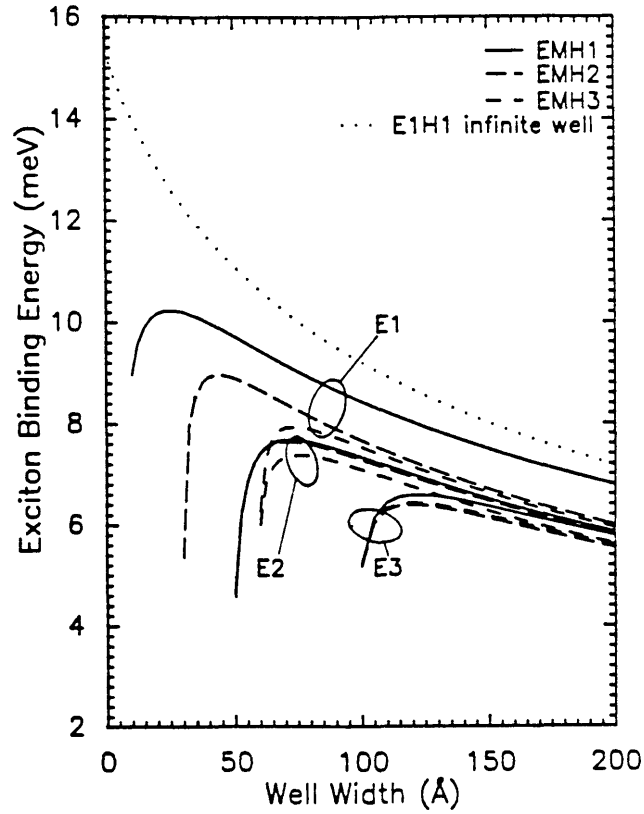


Fig. 2.9 : Well width dependence of the 1s excitonic binding energy (E_{be}) for the Nth electron and Mth heavy hole confined states in GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QWs, calculated using the variational method and parameters described in Sec. 2.4.1. The dotted line shows E_{be} for E1H1 assuming infinite barriers.

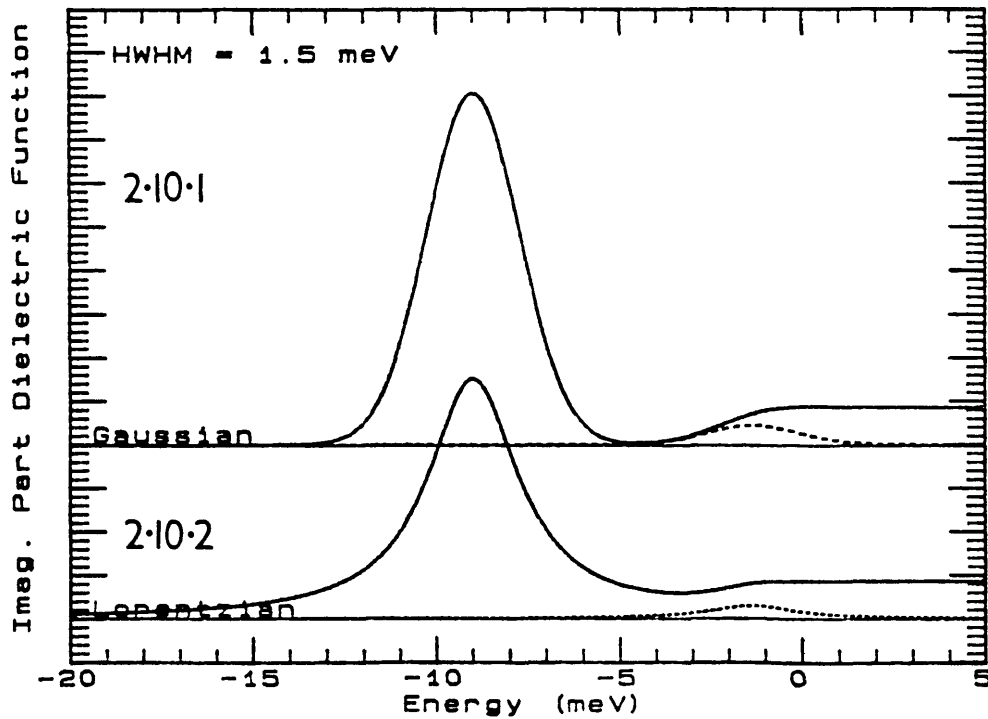


Fig. 2.10 : Calculated imaginary part of the dielectric function near the threshold of a subband transition, including excitonic and continuum contributions, using the method described in Sec. 2.4.2, taking $\Gamma = 1.5$ meV (HWHM), $v = 0.385$ ($E_{be} = 9$ meV) and either a Gaussian (2.10.1) or Lorentzian (2.10.2) broadening profile. The contributions of the excited excitonic states ($n = 2, 3, \dots$) are shown as dashed lines.

calculations by taking a different z-direction effective mass in the well and barrier, as in the calculations presented here. Lin et al [2.39] have reported experimental evidence for the E1H1 binding energy decreasing at very small well widths, from their thermally modulated PL measurements on GaInAs/InP SQWs.

It can be seen in Fig. 2.9 that the higher electron-heavy hole subband transitions have a smaller binding energy than E1H1. As also pointed out by Bastard and Brum [2.7], the binding energy of ENHM decreases with increasing hole (or electron) subband number, M (or N). This is because ψ_h (or ψ_e) is more uniformly distributed across the well width for states with a larger subband number, so reducing the Coulombic attraction to the other carrier. Notice that at each well width, the binding energy tends towards a minimum value with increasing subband number. This is because the z-direction extent of $\psi_{e(h)}$ is roughly limited to within the well width, provided that the confined level does not lie too close to the top of the well. When the confined level does approach near the top of the well, its envelope function extends far beyond the barrier interfaces, producing a dramatic decline in the binding energy.

Sanders and Chang [2.40] considered the effect of valence band mixing on the exciton binding energy as a function of the well width. They found that including the valence band mixing increased the binding energy of both the E1H1 and E1L1 excitons. For example, for a 100 Å GaAs/Al_{0.25}Ga_{0.75}As QW they calculated an enhancement of 1.1 and 2.7 meV respectively. Chan [2.41] calculated similar increases. Several experimental studies have measured a larger than expected exciton binding energy [2.42-44] or transition strength [2.33], and suggested this may be due to valence subband mixing effects.

The average value of the in-plane k-vector (k_{xy}) for the exciton state can be determined using Eq. 2.44. For the $n = 1$ E1H1 exciton of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW this yields, $\langle k_{xy} \rangle = 0.28 \pi \times 10^6 \text{ cm}^{-1}$. It can be seen from Fig. 2.4.1 (taken from Ref. 2.7), that the H1 and L1 in-plane dispersions of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, are shallower in the presence of the mixing, at this value of k_{xy} . Hence the subband mixing will tend to increase the H1 and L1 effective masses experienced by the exciton. This increase in the effective mass reduces the average separation of the electron and hole and thereby enhances their binding. Since the H1 dispersion is always shallower in the presence of valence band mixing [2.7], it will always result in an increase in the E1H1 binding energy. However, the higher subband transitions could have either an enhanced or diminished binding energy, depending upon whether

the in-plane hole mass is increased or reduced. This can be expected to greatly distort Fig. 2.9 for the higher subband transitions.

The most significant effect of the valence subband mixing is the quite dramatic redistribution of the exciton oscillator strength. The excitons will experience the strong hybridisation of the subbands away from $k_{xy} = 0$. The mixing alters the symmetry of the hole envelope functions, so that the "forbidden" exciton transitions gain oscillator strength at the expense of those that are "allowed". For instance, in a GaAs/Al_xGa_{1-x}As QW at a well width of about 100 Å, the threshold of the L1 subband lies close to that for H2, so that there is a strong mixing of their dispersions in the xy-plane. Consequently there is an increase in the transition strength of the E2L1 exciton, due to the H2 component of its hole envelope function [2.45,2.17]. This is balanced by a corresponding decrease in the strength of the E2H2 exciton. This explains the double peak observed around the E2H2 spectral region of the PLE spectrum measured by R. Miller et al [2.45]. More recently, there have been reports of mixing between higher excitonic states, as discussed in Sec. 2.4.2.

2.4.2 Excitonic Spectrum

Variational calculations of the excited excitonic states ($n \geq 2$) require a more complicated form for the excitonic envelope function. Klipstein et al [2.46] suggested a simpler method of calculating the excitonic spectrum, regarding the QW as lying between the two- and three-dimensional limits, which can be solved explicitly. Elliott [2.28] showed that the binding energy and oscillator strength of excited excitonic states vary as F/n^3 and R/n^2 respectively. Ralph [2.29] and Shinada and Sugano [2.30] showed that these relations also hold for perfect two-dimensional excitons, but with n replaced by $(n-\frac{1}{2})$. In the spirit of this analysis, Klipstein et al suggested that excitons in real QWs can be described by replacing n with $(n-v)$, where the dimensionality is parameterised by the quantity (v) , and lies between the two- and three-dimensional limits, $0 \leq v \leq 0.5$. Hence the exciton transition energy (E_n^{ex}) can be related to the continuum threshold (E^C) by

$$E_n^{\text{ex}} = E^C - R / (n-v)^2 \quad \dots 2.45$$

$$f_n^{\text{ex}} = F / (n-v)^3 \quad \dots 2.46$$

where R and F are constants, evaluated using the reduced mass for the

xy-motion.

Eqs. 2.45-46 can be used to model the contribution of the various excitonic states to the dielectric function (ϵ). Klipstein et al represented the ground state by a Lorentzian oscillator, with an energy E_1^{ex} , linewidth Γ_1^{ex} and oscillator strength f_1^{ex} . They modelled the continuum by taking an integral of oscillators, with a linewidth Γ^{C} and strength f^{C} , over a constant density of states D_{2D} . This can easily be extended to also include the excited excitonic states ($n = 2, 3, \dots, \infty$). The contribution of the bound and unbound excitonic states to the QW dielectric function can be written as

$$\epsilon = A \cdot \sum_{n=1}^{\infty} f^{\text{ex}} L(E, E_n^{\text{ex}}, \Gamma_n^{\text{ex}}) + A \cdot D_{2D} \cdot f^{\text{C}} \cdot \int_{E^{\text{C}}}^{\infty} L(E, \theta, \Gamma^{\text{C}}) d\theta \quad \dots 2.47$$

where A is a constant. The lineshape function (L) in Eq. 2.47 is taken to have a Lorentzian imaginary part, with unit integrated strength

$$L(E_0, \Gamma, E) = (2 \cdot E_0 / \pi) \cdot \{ (E_0^2 - E^2) + i 2 \Gamma E \}^{-1} \quad \dots 2.48$$

The relative strengths of the two terms in Eq. 2.47, due to the bound and unbound electron-hole states, can be related using the fact that the excitonic contribution in the limit of $n \rightarrow \infty$, must merge smoothly with the continuum. Differentiation of Eq. 2.45 with respect to n , yields the density of excitonic states per unit energy interval. This can be used to deduce the product of the excitonic oscillator strength and the density of excitonic states, which is constant and must be equal to the corresponding value for the continuum. It follows that the ratio of the ground state exciton and continuum oscillator strengths is given by [2.46]

$$r = f_1^{\text{ex}} / D_{2D} f^{\text{C}} = 2 R / (n-v)^3 \quad \dots 2.49$$

Klipstein et al took the relative strengths of the excitons and continua in the absorption spectra measured by D. Miller et al [2.47], to evaluate the dimensionality factor v , and hence estimate the exciton binding energy.

This simple analysis ignores the slow variation of the transition strength with energy in the continuum due to the Coulombic interaction, that was discussed in Sec. 2.4.1. The present analysis matches the continuum strength to that of the bound excitons at the band edge, and therefore ignores the reduction in the continuum strength with increasing energy. It also ignores the splitting of the different

angular momentum states associated with each principle quantum number.

Fig. 2.10.2 plots the imaginary part of the dielectric function calculated using Eqs. 2.45-49 and taking $\Gamma^C = \Gamma_n^{\text{ex}} = 1.5 \text{ meV}$, $R = 3.4 \text{ meV}$, $v = 0.385$. The linewidth is a typical low temperature value (half width at half maximum, HWHM) for a good quality GaAs/Al_xGa_{1-x}As QW with a well width of about 70-100 Å, as discussed later in Sec. 5.3. The broadening of the excited excitonic states and the continuum were taken to equal that of the ground state, although they may actually be larger, due to the greater spatial extent of the electron-hole system. The dimensionality factor (v) was chosen so as to give a $n = 1$ exciton binding energy of 9.0 meV, close to the value measured and calculated by R. Miller et al [2.31] for a 100 Å QW. The energy scale is expressed relative to the continuum threshold (E^C) in Fig. 2.10.2, while the vertical scale is adjusted so that the area under the $n = 1$ exciton peak is unity.

The curve in Fig. 2.10.2 is dominated by the large peak due to the ground state exciton at -9 meV. None of the higher excitonic states are resolved from the shoulder due to the continuum, for the linewidth used here. The contribution of the excited states is clearly much smaller than that of the ground state. The ratio of the oscillator strengths of the $n = 1$ state to that of the $n = 2$, must lie between the three- and two-dimensional limits of 8 and 27 respectively, and is actually about 18 for the value of the dimensionality parameter (v) chosen here.

Fig. 2.10.1 again shows the imaginary part of the QW dielectric function, calculated with the same parameters as in Fig. 2.10.2, but with the Lorentzian lineshape function replaced by a Gaussian. The Gaussian lineshape profile is chosen to have the same HWHM and area under the curve as the Lorentzian. The Gaussian is smaller in the wings of the lineshape, and so the spectral region containing the continuum edge and excited excitonic states is more clearly resolved. The Gaussian curve in Fig. 2.10.2 has a very slight peak just above the band edge energy. Notice that this does not correspond to the $n = 2$ exciton, but rather the overlapping contributions of all the excited excitonic states and the continuum.

The $n = 2$ excitonic state only becomes resolved when the HWHM linewidth is reduced below 1.5 meV, as demonstrated by Fig. 2.11, which plots the imaginary part of the dielectric function, using a Gaussian profile with various linewidths. The other parameters are taken to have the same values as in Fig. 2.10. It can be seen that a peak is not resolved at the position of the $n = 2$ exciton, until the linewidth is reduced to 0.5 meV. As the linewidth is further reduced the $n = 3$

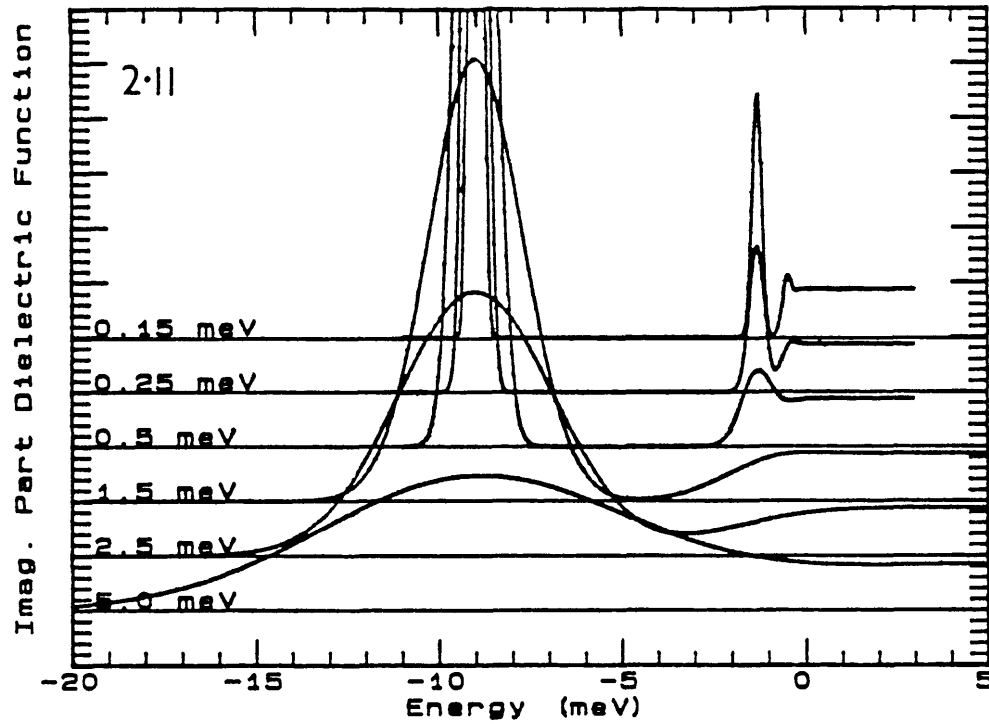


Fig. 2.11 : Calculated imaginary part of the dielectric function, using a Gaussian broadening profile, with different HWHM linewidths of between 5 and 0.15 meV.

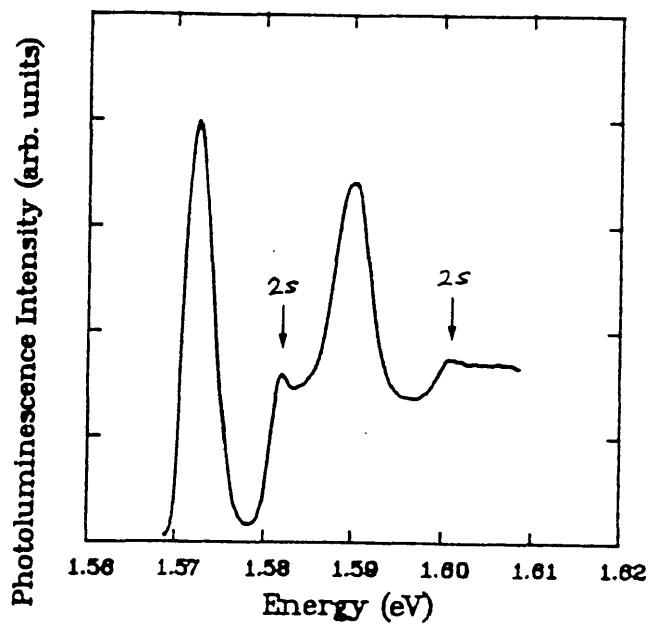


Fig. 2.12 : PLE spectrum of a 75Å GaAs/Al_{0.4}Ga_{0.6}As MQW, measured by Dawson et al [2.42]. Temperature = 8 K, PL detected at 1.569 eV, Laser Power = 5 W/cm².

exciton becomes resolved. In general, an excited excitonic state will not be clearly resolved until the linewidth is comparable to its separation from the next highest state.

The calculated excitonic spectrum of Fig. 2.10.1 is similar to the PLE spectrum measured by R. Miller et al [2.31] on a 42 Å GaAs/Al_{0.37}Ga_{0.53}As MQW, shown in Fig. 2.7. Two large peaks are seen due to the E1H1 and E1L1 ground state excitons, each accompanied by a shoulder to higher energy, due to the merged contributions of the excited excitons and the continuum. Separate peaks due to the $n = 2$ excitons are not resolved, which is consistent with the HWHM E1H1 linewidth being 2.3 meV here. Meynadier et al [2.6] have reported similar PLE spectra.

Fig. 2.12 shows the bandedge PLE spectrum measured by Dawson et al [2.42] on a 75 Å GaAs/Al_{0.35}Ga_{0.65}As MQW. In contrast to the spectrum of R. Miller et al, a peak is resolved at an energy between the ground state exciton and the continuum threshold for both the E1H1 and E1L1 transitions, which they attributed to the $n = 2$ exciton (2s). The sharpness of the E1H1(2s) peak is surprising, considering that the E1H1(1s) linewidth is 2.0 meV (HWHM). Comparison with Fig. 2.10, shows that the spectrum calculated with a smaller linewidth, does not show such a clearly resolved peak. The 2s excitons may be enhanced in the experimental spectrum by valence band mixing effects, not considered in this simple model, and which are discussed below. It is also peculiar that the size of the E1H1(1s) peak relative to the continuum level in the experimental spectrum (Fig. 2.12) is much smaller than that calculated in Fig. 2.10. Perhaps this is due to a fall in the efficiency of the dye laser or non-linearity of the detector used in the experiment. The 2s E1H1 exciton was also observed in the PL spectra of this and other samples [2.49].

There have been several more recent observations [2.49-51] of distinct peaks due to the 2s exciton in samples having narrower linewidths than those measured by R. Miller et al or Dawson et al. The narrower linewidth is achieved by using samples with a wider well width, thereby reducing the effect of interface roughness. Reynolds et al [2.50] measured the PLE spectrum of a 225 Å GaAs/Al_{0.35}Ga_{0.65}As MQW, which displayed linewidths of about 0.25 meV (HWHM), and resolved peaks which they attributed to the 2s and 3s excitonic state. The luminescence of this sample was complicated showing seven different peaks, which they attributed to the E1H1(1s) exciton from well regions having a sub-monolayer difference in the width. This suggests caution must be used in labelling the PLE peaks, however, they justified the 2s and 3s assignment by measuring the size of the diamagnetic shift. Their

et al [2.51] measured the PR spectrum of this sample and were able to observe both the 1s and 2s E1H1 excitons, although not the 3s. Collins et al [2.49] also measured distinct peaks due to the 2s E1H1 and E1L1 excitons in the PLE spectrum of a $160 \text{ \AA}$ GaAs/AlGaAs MQW, which again had very narrow linewidths. These peaks became unresolved when a small electric field was applied.

Recently there have been a number of experimental [2.49,2.52-54] and theoretical [2.54-56] studies concerning mixing of excitonic states, which is ignored by the simple theory presented here. The mixing of the LH and HH subbands was discussed in Sec. 2.2.2. For excitonic transitions it is necessary to include the Hydrogenic part of the envelope functions in the hybridisation. When an excitonic state of one subband transition, for instance E1H1(2p), becomes nearly degenerate with an excitonic state of a different transition, eg E1L1(1s), the mixing of their envelope functions causes a redistribution of their oscillator strengths.

Vina et al [2.52] have observed such mixing in the PLE of the sample studied by Collins et al [2.49], with applied electric and magnetic fields. With increasing electric field, the intensity of the E1L1(2s) peak is seen to decrease and a number of other peaks are seen to appear, which are attributed to the higher angular momentum excited states of the E1H1 exciton; namely 2p,3p,3d,4d. For this well width ($160 \text{ \AA}$), the E1H1 excited states lie just higher in energy than E1L1(1s). Consequently when the field is increased the E1H1 excited states move closer to the E1L1(1s), due to the larger Stark shift of H1 compared to L1. This increases the mixing of the exciton envelope function and enhances the excited states of the E1H1 at the expense of E1L1(1s) [2.54-55].

Other reports have observed mixing of the E1H2(1s) exciton and the excited states of E1L1. Collins et al [2.49] observed an unexpected peak in the vicinity of the E1H2(1s) exciton in the same $160 \text{ \AA}$ MQW. They assigned this peak to an excited excitonic state of E1L1, which has enhanced oscillator strength due to mixing of the L1 and H2 subbands. This excited state is not the 2s, as a separate E1L1(2s) peak is resolved at the lower fields. Note that the observation of the E1L1 excited state requires an electric field, as only then does the E1H2 transition have significant oscillator strength. They also observed this transition in field-dependent PC measurements of a $130 \text{ \AA}$ wide well, which had a larger linewidth. Yu et al [2.53] reported measuring a similar peak in the PC spectrum of a $85 \text{ \AA}$ GaAs/Al_{0.3}Ga_{0.7}As MQW in an electric field and again suggested it to be an excited excitonic state

of E1L1, which has enhanced oscillator strength due to the mixing of the L1 and H2 subbands. This transition can be also seen in the field-dependent PC spectra of the 88 Å and 106 Å GaAs/Al_{0.3}Ga_{0.7}As SQWs presented in Chp. 5.

2.5 Electric Field Effects On Free Carriers

2.5.1 Free Carrier Envelope Function in a Field : Franz-Keldysh Effect

Under the effective mass approximation, the Hamiltonian for a free carrier, of energy E, subject to a uniform electric field, F, applied along the z-direction becomes

$$\hat{H} = - (\hbar^2/2m) \delta^2 / \delta z^2 + e F z \quad \dots 2.50$$

and consequently Schrödinger's equation for the envelope function describing the motion in the z-direction can be written as

$$\delta^2 \psi(z) / \delta z^2 = Z \psi(z) \quad \dots 2.51$$

$$\text{where} \quad Z(z) = - (E - e F z) / \Theta \quad \dots 2.52$$

$$\text{and} \quad \Theta = (e^2 \hbar^2 F^2 / 2 m)^{1/3} \quad \dots 2.53$$

This differential equation has the general solution

$$\psi(Z) = a \text{Ai}(Z) + b \text{Bi}(Z) \quad \dots 2.54$$

where a and b are constants and Ai(Z) and Bi(Z) are the "Airy" functions, plotted in Fig. 2.13, which can be calculated using a series expansion [2.35].

The shape of these functions can be qualitatively deduced by analogy with the zero-field free carrier envelope function, with a linearly varying energy. The Ai(Z) function represents a carrier travelling in the positive z-direction. As the carrier travels from left to right in Fig. 2.13, it decelerates, and hence the period of the oscillations decreases along with its local k-vector. The increase in the amplitude of the oscillations can be thought of as arising from a larger probability density per unit length as the speed of the carrier slows. Eventually the kinetic energy of the carrier falls to zero at the point where, (E - eFz) = 0. When E < eFz, the carrier envelope

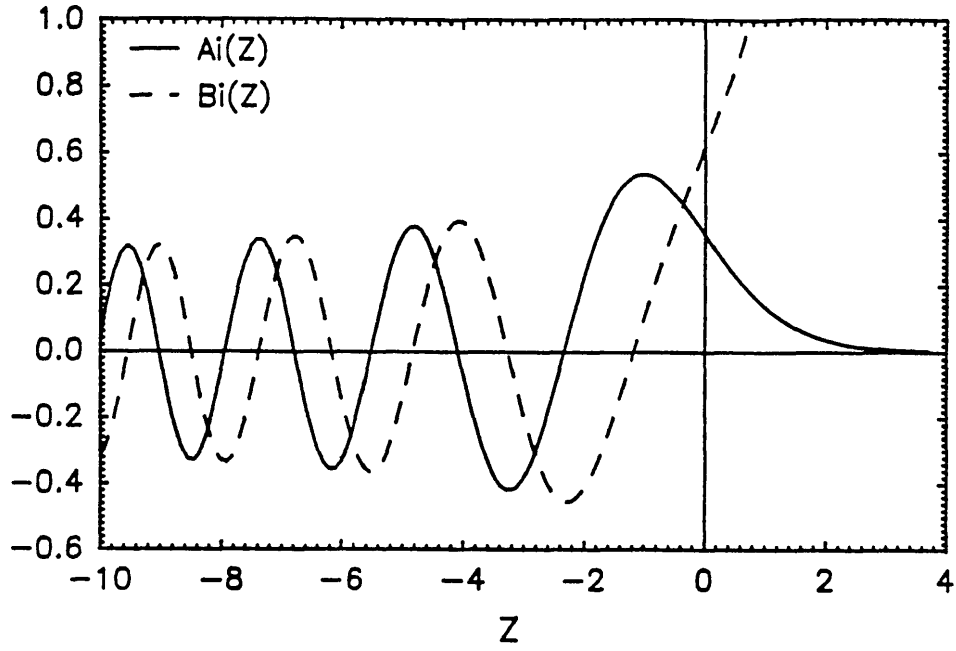


Fig. 2.13 : The Airy functions, $Ai(Z)$ and $Bi(Z)$; the general solutions for a free-carrier under uniform electric field.

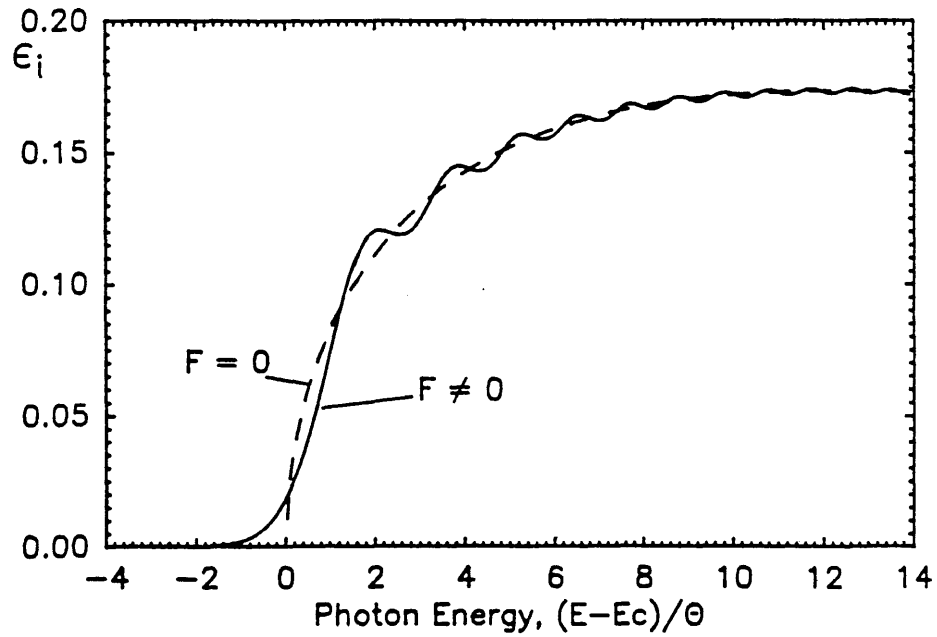


Fig. 2.14 : The imaginary part of the dielectric function of a bulk semiconductor near an Mo critical point, under zero (dashed line) and a finite (solid) applied electric field.

function decays rapidly, as it tunnels into the gap. The decay is more rapid than an exponential with linear argument, since the barrier height increases with distance. Similarly, the Bi(Z) function represents a carrier travelling in the negative z-direction, which tunnels out of the gap and accelerates in the negative z-direction. For the envelope function to remain finite as $z \rightarrow +\infty$, $b = 0$ in Eq. 2.54.

Optical spectra for transitions between the valence and conduction band can be calculated by evaluating the electron-hole overlap integral, and then integrating over the energy associated with the xy-motion, as in Eq. 2.27 [2.57]. Alternatively, the transition strength can be found by transforming the motion into centre of mass coordinates and then integrating over the energy associated with the xy-motion [2.58-61]. For an M_0 critical point, this yields the real and imaginary parts of the dielectric function to be given by

$$\epsilon_i(E) = (A \theta / E^2) F_1(\{E^C - E\} / \theta) \quad \dots 2.55$$

$$\epsilon_r(E) = (A \theta / E^2) G_1(\{E^C - E\} / \theta) \quad \dots 2.56$$

$$\text{where } F_1(x) = \pi \int_x^{+\infty} |Ai(y)|^2 dy = \pi [\{Ai'(x)\}^2 - x \{Ai(x)\}^2] \quad \dots 2.57$$

$$G_1(x) = \pi \int_x^{+\infty} Ai(y).Bi(y) dy = \pi [Ai'(x).Bi'(x) - x. Ai(x).Bi(x)] \dots 2.58$$

$Ai'(x)$ denotes $\delta Ai(x)/\delta x$, A is a constant and θ is evaluated using the reduced electron-hole effective mass, ie $m \rightarrow \mu$ in Eq. 2.53. The imaginary part of the dielectric function given by Eqs. 2.55 and 2.57 is plotted in Fig. 2.14. Also shown is the curve in the limit of the field tending to zero, when $F_1(x) \rightarrow (-x)^{\frac{1}{2}}.H(-x)$, where H is the unit step function. Hence F_1 is proportional to $\{E - E^C\}^{\frac{1}{2}}$, for $E > E^C$, at zero field, which can also be deduced from the three-dimensional free-carrier density of states. Similarly, the function $G_1(x) \rightarrow x^{\frac{1}{2}}.H(x)$, in the limit of the field tending to zero.

It can be seen that for a finite field, the imaginary part of the dielectric function has a tail below the energy of the zero-field bandgap (E^C), which is due to the tunnelling of the carriers into the gap. Meanwhile, for the photon energies above the bandedge there are oscillations in ϵ_i , about the zero-field curve. These Franz-Keldysh oscillations, as they are known, derive from the oscillations in the Airy function at energies $E > e F z$, and their period increases with the electric field. The absorption tail below the zero-field bandedge was first predicted in two independent reports by Franz [2.62] and Keldysh [2.63].

2.5.2 Bulk Semiconductor Excitons In An Electric Field

In bulk semiconductors, the bound excitons are rapidly ionised by an electric field. A classical estimate of the ionisation field can be made from the ratio of the exciton binding energy (E_{be}) and its radius (a). For GaAs these values are roughly $E_{be} = 3.7$ meV and $a = 150$ Å, yielding an ionisation field of only 2.5 kV/cm. Duke and Aleriff [2.64] have made a more detailed analysis, which confirms that the excitonic lines are rapidly broadened by small electric fields.

2.5.3 Quantum Well Under an Electric Field Parallel to Well

For electric fields applied along a direction in the plane of the QW, qualitatively similar effects are expected to those in bulk semiconductor. Again the field should rapidly dissociate the electron and hole, although the required ionisation field can be expected to be larger than for bulk, due to the enhanced exciton binding energy and reduced radius.

The continuum term in the dielectric function will also have a similar field dependence to bulk semiconductor. The dielectric function of two-dimensional free-carriers under an in-plane electric field, can be calculated in a similar manner as for three-dimensional free-carriers [2.61]. Again there will be an exponential tail at energies below the subband threshold, with broad oscillations about the zero-field dielectric function to higher energy. However, the two-dimensional subband will have a different zero-field dielectric function, having a stepped energy dependence, rather than the square root dependence shown in Fig. 2.14.

D. Miller et al [2.34] made absorption measurements on a GaAs/Al_{0.32}Ga_{0.68}As MQW with 95 Å wide QWs, with the field applied parallel to a direction in the plane of the wells, and these are shown in Fig. 2.15. It can be seen that the major effect of the in-plane electric field is a broadening of the exciton peaks, due to their field ionisation. The observed broadening agreed qualitatively with the theoretical predictions of Lederman and Dow [2.65] for excitons in the perfect two-dimensional limit. D. Miller et al also measured an electrotransmission spectrum, taken with a pulsed field applied along the plane of the layers. They were able to model the spectrum by assuming it to be produced by a field-induced change in the linewidth of the excitons.

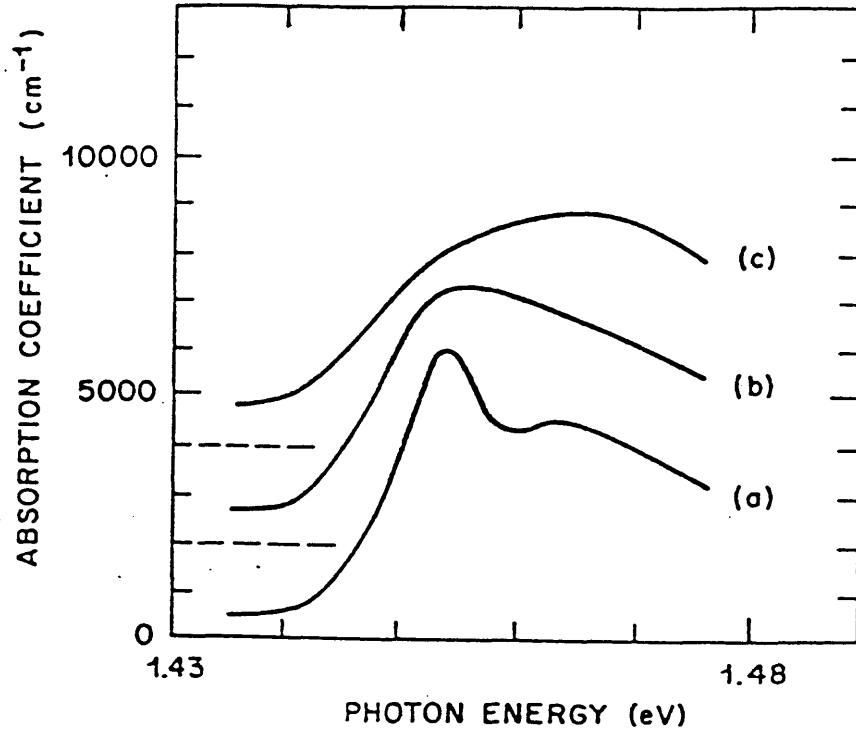


Fig. 2.15 : Absorption spectra (295 K) of 95 Å GaAs/Al_{0.32}Ga_{0.68}As MQW with different electric fields applied parallel to the QW plane of (a) 0, (b) 16 and (c) 48 kV/cm, measured by Miller et al [2.34].

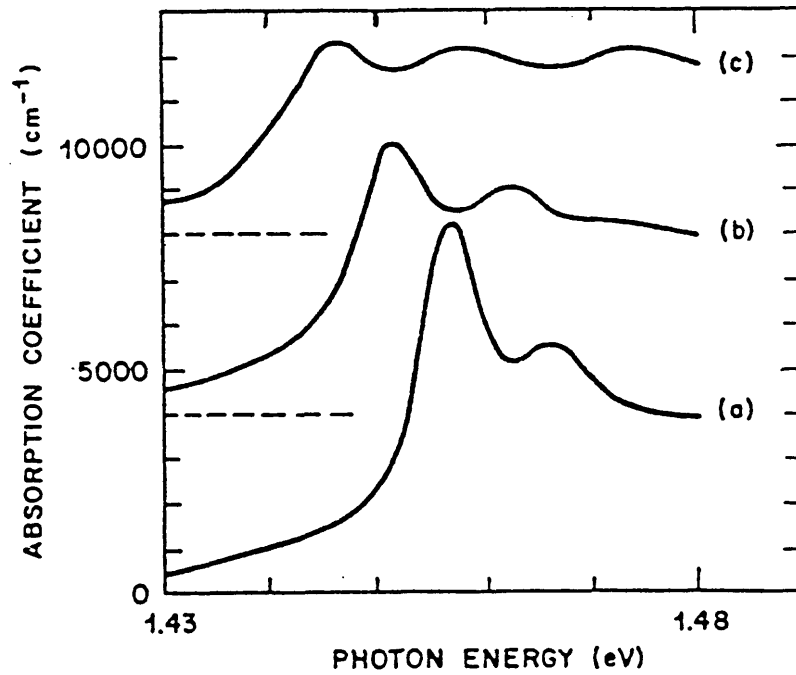


Fig. 2.16 : Absorption spectra (295 K) of 95 Å GaAs/Al_{0.32}Ga_{0.68}As MQW with different electric fields applied perpendicular to the QW plane of (a) 10, (b) 47 and (c) 73 kV/cm, measured by Miller et al [2.34].

2.6 Electric Field Effects on Quantum Wells :Quantum Confined Stark Effect

Electric fields applied to a QW in the direction perpendicular to the plane of the well, ie the z-direction, produce qualitatively different effects to those seen in bulk semiconductor. D.Miller et al [2.34,2.66] also made absorption measurements on their 95 Å GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ MQW sample, with a uniform electric field applied perpendicular to the plane of the QW, which are shown in Fig. 2.16. They saw a large downward energy shift of the $\text{E}_{1\text{H}1}$ and $\text{E}_{1\text{L}1}$ excitons, of up to 2.5 times the zero-field binding energy, while the exciton peaks remained resolvable to fields of about 100 kV/cm. This is in sharp contrast to the effect of a field applied parallel to the QW, shown in Fig. 2.15, where the exciton resonances are rapidly broadened and there is no discernible energy shift. They called the behaviour observed with the field applied perpendicular to the QW plane, the "quantum confined Stark effect" (QCSE).

Electric field-dependent photocurrent spectra taken on a number of SQW samples, are presented in Chp. 5. These show similar behaviour to that observed by D.Miller et al. Fields of up to 300 kV/cm are applied to a 59 Å GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, and the $\text{E}_{1\text{H}1}$ exciton peak is seen to shift to lower energy by more than 50 meV. Measurements are made on four SQW samples in Chp. 5, which are identical except for their well width, and their field dependencies are compared.

The energy shifts of the excitonic transition with electric field can be resolved into two components; the change in the electron and hole confinement energies and the change in the Coulombic binding energy. Sec. 2.6.1 calculates the evolution of the z-direction motion with the electric field, while the field-induced modification of the electron-hole Hydrogenic state is considered separately in Sec. 2.6.2.

2.6.1 Electric Field Dependence of the Quasi-Confined States

In the presence of an electric field, the carriers are able to tunnel from the QW, so that there are no truly confined states. However, the tunnelling is extremely weak for the lowest confined levels under low to moderate fields, due to the presence of the potential barriers. The QW can therefore be considered to have a number of "quasi-confined" states.

The energies and envelope functions of a QW under a z-direction electric field are again found by solution of Schrödinger's equation.

The confining potential $V(z)$ of the QW is added to the Hamiltonian of Eq. 2.50 to give

$$\hat{H}_z = - (\hbar^2/2m) \delta^2/\delta z^2 + V(z) + e F z \quad \dots 2.59$$

Again Schrödinger's equation can be written as Eq. 2.51, which has the general solution of Eq. 2.54, but now Eq. 2.52 is modified to include the QW potential

$$Z(z) = - (E - e F z - V(z)) / \theta \quad \dots 2.60$$

The argument of the Airy functions will be different on either side of the QW interfaces, due to the discontinuity in $V(z)$ and m (and therefore θ). As in the zero field case, the solutions are found by matching the envelope function (Ψ) and the particle flux $m^{-1}(\delta\Psi/\delta z)$ across the QW boundaries.

Sec. 2.6.1.2 considers the formal solution for the z -motion, using a similar analysis to that presented by Ikonik et al [2.57]. This shows that there is a continuous distribution of energy states at finite fields, rather than the discrete values (ie the confined levels) found at zero field. However, at certain energies, the envelope function becomes concentrated inside the well, and these can be regarded as "quasi-confined" states, with a certain linewidth and envelope function. The linewidth increases with field, so that each quasi-confined states eventually becomes unresolvable. At low fields, an energy, linewidth and strength can be defined for the optical transitions between the quasi-confined electron and hole levels, in an analogous way to the zero-field transitions between the perfectly confined levels. Calculations are made of the field evolution of these optical transitions between the quasi-confined states.

Before making this formal analysis, it is worthwhile considering the behaviour for wells with infinitely high confining potential barriers, as presented in Sec. 2.6.1.1. This ignores the field-induced tunnelling from the well, and so is not quantitatively or qualitatively accurate at high fields. However, it allows much insight into the behaviour at low fields, where the tunnelling of the quasi-confined states is negligible. The interband optical transitions between the low-field quasi-confined states are shown to have a remarkable field evolution, with dramatic changes in their energy and strength.

There have been many other approaches to the solution of Schrödinger's equation for a QW under a uniform electric field. These

have included: exact solutions taking account of the field-induced tunnelling from the well [2.57,67-68]; a Monte-Carlo calculation [2.69]; a resonant tunnelling method, which assumes an approximate form for the potential [2.34,70-71] and a variational method [2.72]. Other workers have assumed infinite confining barriers to allow: an exact explicit analysis [2.73-74]; a variational calculation [2.72]; and a perturbational method for low fields [2.72].

2.6.1.1 Explicit Analysis Assuming Infinite Barriers

The field dependence of the quasi-confined levels can be solved explicitly if the barriers are taken to be infinitely high. The envelope function in the well region is then given by Eq. 2.54, with the requirement that it must be zero at both of the interfaces. Hence

$$\frac{\text{Bi}[-E/\theta]}{\text{Ai}[-E/\theta]} = \frac{\text{Bi}[-(E - eFL_w)/\theta]}{\text{Ai}[-(E - eFL_w)/\theta]} \quad \dots 2.61$$

In the high field limit ($eFL_w \gg E^*$), the carrier is strongly polarised to the side of the well, so the right-hand barrier in Fig. 2.17 can be ignored, allowing the confining potential to be regarded as triangular. (E^* is the energy relative to the bottom corner of the triangular well. Both e and F are positive.) For the envelope function to remain finite as $z \rightarrow \infty$, the coefficient b in Eq. 2.54 must be zero. Applying the other boundary condition $\Psi(z=0) = 0$, yields the energy of the confined states (E_N^*) relative to the lower corner of the well, in terms of the zeroes of the Airy function $Z_0(N)$, which are tabulated in Ref. 2.35; $E_N^* \approx -\theta Z_0(N)$. It is more convenient to express the confinement energies relative to the potential at the midpoint of the well, as the electron and hole values can then be added directly, hence

$$E_N \approx -\frac{1}{2} e F L_w + \theta Z_0(N) \quad \dots 2.62$$

The first term on the right-hand-side of Eq. 2.62 is typically larger than the second for the $N = 1$ subband, and hence the energy shift is almost linear, in the high field limit. The shift is due to the potential drop between the midpoint and side of the well, where the carrier is tightly confined at high fields. The second term in Eq. 2.62 reduces the linear dependence slightly and represents the increase in the carrier kinetic energy as the carrier is squeezed into the corner of the well.

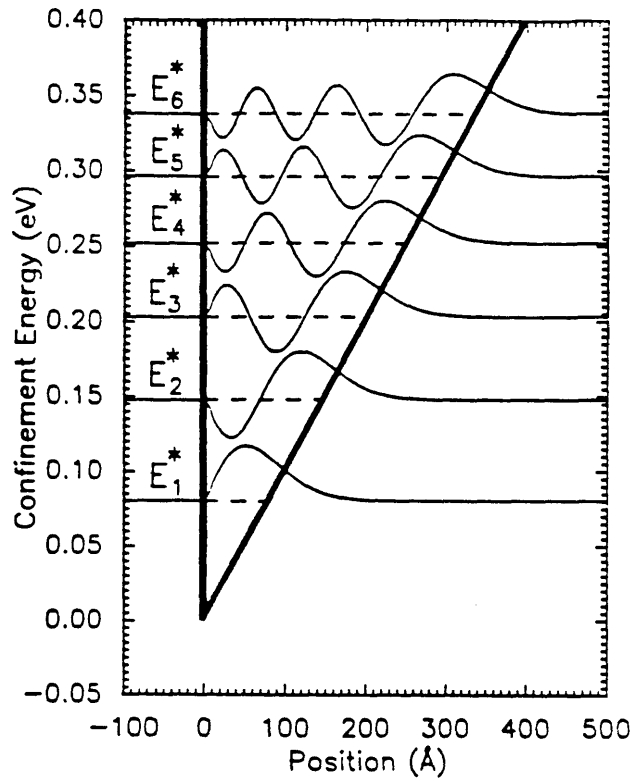


Fig. 2.17 : The first six confined states in an "infinite" triangular potential. The calculation was made for electrons ($m = 0.067$), under an electric field of 100 kV/cm.

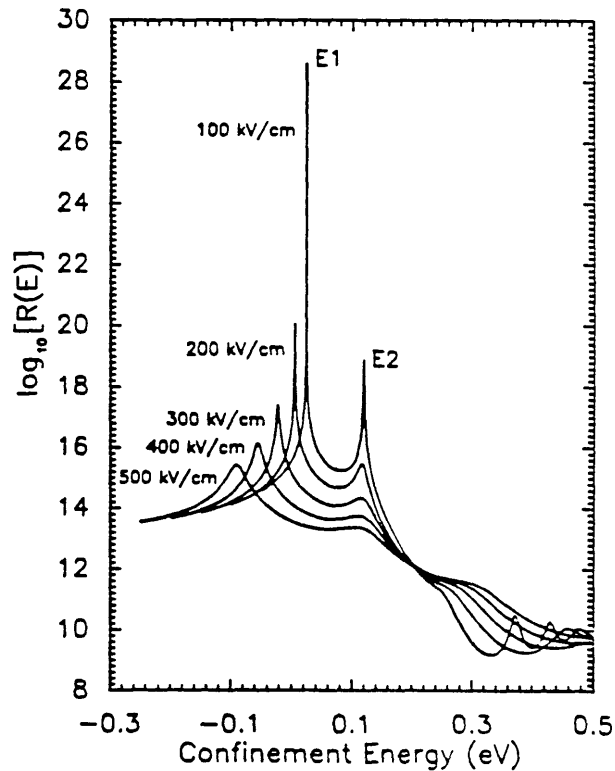


Fig. 2.18 : The function $R(E)$, defined by Eq. 2.72, for the conduction band of a 100 Å GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW under different uniform electric fields. The large resonances in $R(E)$ correspond to the quasi-confined states, E1 and E2. Calculated using the method described in Sec. 2.6.1 and the parameters listed in table 4.1.

The field dependence of the energy shift can be deduced in the low field limit using perturbation theory. The field-dependent term in the Hamiltonian (Eq. 2.59) is then regarded as a small perturbation to the zero-field eigenfunctions. It can be shown that the first non-zero contribution to the energy shift of the lowest subband (relative to the potential at the midpoint of the well) is of second-order in the field and given by [2.72]

$$E_1 \approx - 2.195 \ m e^2 F^2 L^4 / \hbar^2 \quad \dots 2.63$$

The perturbation expansion is only valid when the field-induced potential drop across the well is much smaller than the confinement energy ($eFL_w \ll E_1$). Consequently Eq. 2.63 is most accurate for narrow wells and for small fields. The quadratic field dependence of the energy shift in the low field limit can also be demonstrated using a variational calculation [2.65].

Both Eqs. 2.62 and 2.63 confirm that the Stark shift will be larger in wider wells, which is to be expected, since they allow a larger field-induced spatial separation of the electron and hole. It is also apparent that the energy shift increases with the effective mass of the carrier. This is explained by heavier carriers being more tightly confined to the side of the well by the field.

In contrast to the $N = 1$ subband, the higher subbands ($N \geq 2$) can shift to higher energy in the low-field limit. Again this can be argued using second-order perturbation theory. Eventually the field will become large enough to polarise the higher subband state towards the side of the well, when it will also have a negative energy shift. It requires a larger electric field before the well can be regarded as triangular for the higher subbands, due to their larger confinement energy. However, at fields sufficiently large to make Eq. 2.62 valid, it is clear that the Stark shift will decrease with subband number (N), since the coefficient of the second term ($Z_0(N)$) increases. The subband levels will be more equally spaced in the high field limit than at zero field, since the zeroes of the Airy function ($Z_0(N)$) have a slower than quadratic dependence on N .

2.6.1.2 Exact Analysis For Finite Barriers

When the finite potential of the barriers is taken into account, the envelope function can no longer be assumed to be zero at the QW interfaces. The envelope functions are again given by Eqs. 2.54 and

2.60, in the presence of a field, with $V(z) = V_0$ in the barriers and $V(z) = 0$ in the well. The coefficients of the Airy functions in Eq. 2.54 will be different in the two barrier and well regions. In order for the envelope function to be finite as $z \rightarrow +\infty$, the coefficient of $Bi(Z)$ in the positive z -direction barrier must be zero. Consequently the envelope function can be written in the form [2.57]

$$\text{For } z \leq -\frac{1}{2}L_w, \quad \Psi = c (a_b Ai(Z_b) + b_b Bi(Z_b)) \quad \dots 2.64$$

$$\text{For } -\frac{1}{2}L_w \leq z \leq +\frac{1}{2}L_w, \quad \Psi = c (a_w Ai(Z_w) + b_w Bi(Z_w)) \quad \dots 2.65$$

$$\text{For } z \geq +\frac{1}{2}L_w, \quad \Psi = c Ai(Z_b) \quad \dots 2.66$$

where $Z_w(b) = \{E - eFz - V(z)\}/\theta_w(b)$ and is different in the well and barrier material, due to the discontinuities in m and $V(z)$.

The envelope function given by Eq. 2.64-66 is normalised between the positions $z = -L$ and $z = +L$, in the limit of $L \rightarrow \infty$, so that

$$\int_{-L}^{+L} |\Psi(z)|^2 dz = 1 \quad \dots 2.67$$

$$\text{and} \quad \Psi(\pm L) = 0 \quad \dots 2.68$$

Eq. 2.67 can be solved explicitly in terms of the functions given by Eqs. 2.57-58. As $L \rightarrow \infty$, the integration over the negative- z barrier in Eq. 2.67 diverges, while those over the well and positive- z barrier remain finite and can consequently be ignored. The divergence of the integral over the negative- z barrier, derives essentially from the fact that the amplitude of $Ai(x)$ and $Bi(x)$ decreases slowly, as $(-x)^{-\frac{1}{4}}$, in the limit $x \rightarrow -\infty$. Thus it can be shown that

$$c^2 = (2 m_b e F / \hbar^2)^{1/6} \cdot \pi L^{-\frac{1}{2}} (a_b^2 + b_b^2)^{-1} \quad \dots 2.69$$

Notice that $c \rightarrow 0$, as $L \rightarrow \infty$, so that the envelope function is zero. However, the density of states increases as $L^{\frac{1}{2}}$, as shown below, which results in the transition strength remaining finite. The continuity requirement for Ψ and $m^{-1}(\delta\Psi/\delta z)$ at the two QW interfaces, forms four simultaneous equations for the four remaining coefficients (a_b, b_b, a_w, b_w) , which can be solved numerically using a standard matrix method. Before considering the solution for (a_b, b_b, a_w, b_w) , it is worthwhile considering its zero-field limit.

As the field tends to zero, the $Ai(Z_b)$ and $Bi(Z_b)$ functions in the barrier regions tend to exponential terms, $\exp(-k_b z)$ and $\exp(+k_b z)$ respectively, omitting the proportionality constant. For the zero-field

envelope function to be finite as $z \rightarrow -\infty$, requires $a_b = 0$. Meanwhile, the $Ai(Z_w)$ and $Bi(Z_w)$ functions in the well region tend to a sinusoidal dependence. Therefore, Eqs. 2.64-66 reduce, in the limit $F \rightarrow 0$, to those previously discussed for the zero-field case (Eqs. 2.9-11).

At zero field there are consequently three unknown coefficients (b_b, a_w, b_w), after normalisation using Eq. 2.67, in addition to the energy (E), for which a solution is sought. The requirement that Ψ and $m^{-1}(\delta\Psi/\delta z)$ be continuous at the two well interfaces, forms four equations for these four unknowns. This forms an eigenvalue problem which has discrete solutions, corresponding to the confined states of the QW. However, in the presence of an electric field, the condition that $a_b = 0$ is relaxed. There are then five unknowns and four equations, which means that a solution can be found for every energy. Consequently there is a continuous energy spectrum of solutions in the presence of an electric field.

At most energies, the solution reveals the coefficient a_b to be large, so that the envelope function is dominated by the $Ai(Z_b)$ term in the negative- z barrier. This decays exponentially with increasing z , as it approaches the QW, so that the amplitude of the normalised envelope inside the well is small. These solutions are therefore similar to those for the bulk barrier material, with the presence of the QW having little influence. However, at some energies, a_b is small and the normalised envelope function is concentrated inside the QW. These solutions are clearly analogous to those in the zero-field limit, when $a_b = 0$, and so are referred to as the "quasi-confined" states of the QW.

Indeed several workers have approximated the solution at finite fields by determining the energies where $a_b = 0$, a condition known as the Breit-Wigner parameterisation [2.67]. This results in a solution consisting of a number of discrete energies, rather than the continuous spectrum. While this is an excellent approximation at low field, it becomes increasingly poor as the field increases, as discussed later. Making the approximation of infinite confining barriers again reduces the analysis to an eigenvalue problem with discrete solutions, which again is invalid at high fields.

It has been discussed how the zero-field discrete solutions for the carrier motion in the z -direction, are replaced by a continuum at finite fields. The density of states for the z -motion can be calculated from Eq. 2.68, in an analogous way as for the well known treatment for free carriers, using the sinusoidal expansions for $Ai(Z)$ and $Bi(Z)$ as $Z \rightarrow -\infty$ [2.35]. Hence Eq. 2.68 yields, in the limit of $L \rightarrow \infty$,

$$(\delta N / \delta E) = (2m_b)^{\frac{1}{2}} L^{\frac{1}{2}} / \pi \hbar (eF)^{\frac{1}{2}} \quad \dots 2.70$$

The total optical spectrum for transitions between the valence and conduction band must be calculated by summing over all the possible initial and final states. At zero field this summation extends over the discrete electron and hole confined states $(\Sigma_{N,M})$, as in Eq. 2.41. However, for a finite field, the summation over N and M must be replaced by integrations over the continuous distribution of electron (E_e) and hole (E_h) energies. The imaginary part of the dielectric function, at a photon energy E , is then given by

$$\epsilon_i(E) = S_0 \sum_{E_{xy}} |\phi(0)|^2 \{ M_Z^u(E_e, E_h) \}^2 \times \int dE_e R_e(E_e) \int dE_h R_h(E_h) \delta(E_g + E_e + E_h + E_{xy} - E) \quad \dots 2.71$$

where the function $R_{e(h)}(E_{e(h)})$ is defined, for both the conduction and valence band, by

$$R(E) = c^2 \cdot (\delta N / \delta E) = \{ 4 m_b^2 / \hbar^4 e F \}^{1/3} (a_b^2 + b_b^2)^{-1} \quad \dots 2.72$$

and the overlap integral in Eq. 2.71, has been defined for the unnormalised electron ($\Psi_e^u = \Psi_e / c_e$) and hole ($\Psi_h^u = \Psi_h / c_h$) envelope functions, so that it has a finite value,

$$M_Z^u(E_e, E_h) = \int_{-L}^{+L} \Psi_e^u(z) \cdot \Psi_h^u(z) dz \quad \dots 2.73$$

Eq. 2.71 can again be used to determine the contribution of both the bound and unbound electron-hole states to ϵ_i , as described in Sec. 2.4.1. Each bound state, has a discrete negative value for E_{xy} , equal to its binding energy. For the unbound electron-hole states, the summation over E_{xy} in Eq. 2.71, must be replaced by an integration over the continuum extending from $E_{xy} = 0 \rightarrow \infty$. Notice that the calculation of either the bound or unbound contributions will be much more difficult for finite fields, than at zero-field, due to the integrations over E_e and E_h .

2.6.1.3 Single Carrier Quasi-Confined States

Fig. 2.18 plots the function $R(E)$, calculated for the conduction band of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW under different electric fields. The barrier height and effective masses used in this calculation are

listed in table 4.1, and are identical to those used to calculate the zero-field confined envelope functions, shown in Fig. 2.2. The largest fields shown in Fig. 2.18 are very extreme, being larger than the dielectric breakdown field of GaAs. It can be seen that the curve for 100 kV/cm shows two large resonances at 0.0236 and 0.1189 eV, with a profile resembling a Lorentzian function. These correspond to the quasi-confined states of the well, E1 and E2 respectively, formed when the coefficient a_b is small ($R \propto \{a_b^2 + b_b^2\}^{-1}$) and the normalised envelope function is concentrated inside the well. As the field is increased the E1 resonance broadens and shifts to lower energy. Meanwhile, the E2 resonance undergoes a smaller Stark shift and is more rapidly broadened by the field.

The broadening of the resonances can be explained in terms of the time dependence of the quasi-confined states. The probability density for carriers in any quasi-confined state of the QW under a finite field, will decay exponentially with time, due to tunnelling through the negative-z barrier. The lifetime of carriers in the well (τ) will decrease with the electric field, due to the reduction in the effective thickness of the barrier material for the confined carriers to tunnel through. The energy dependence of the confined state can then be found by taking the Fourier transform of this exponential decay with time. This produces a Lorentzian energy dependence, with a linewidth given by $\Gamma \approx \hbar/\tau$. This is simply a statement of the semi-classical Uncertainty Principle [2.75]. The lifetime of the carriers in the well can also be calculated using the WKB approximation [2.75-76].

The carriers are unable to tunnel from the well at zero field, so that the linewidth due to field-induced tunnelling will be zero. The resonances in $R(E)$ will therefore tend to Dirac-delta functions, situated at the energies of the zero-field confined states at 0.0304 and 0.1194 eV. There is also a third confined state very close to the top of the well at zero field, as can be seen in Fig. 2.2, which is not resolved even at the lowest field in Fig. 2.18.

Fig. 2.19 plots the field dependence of the linewidth (HWHM) of the electron resonances shown in Fig. 2.18, along with those of the light and heavy holes in a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, again calculated using the parameters in table 4.1. It can be seen that the linewidth of each quasi-confined level increases very rapidly above some field. This corresponds to the field-induced ionisation of the confined state. The linewidths of the E1 and H1 resonances increase at roughly the same rate with field and neither becomes significant until fields in excess of 300 kV/cm. For the value of the discontinuity ratio used here, 0.65,

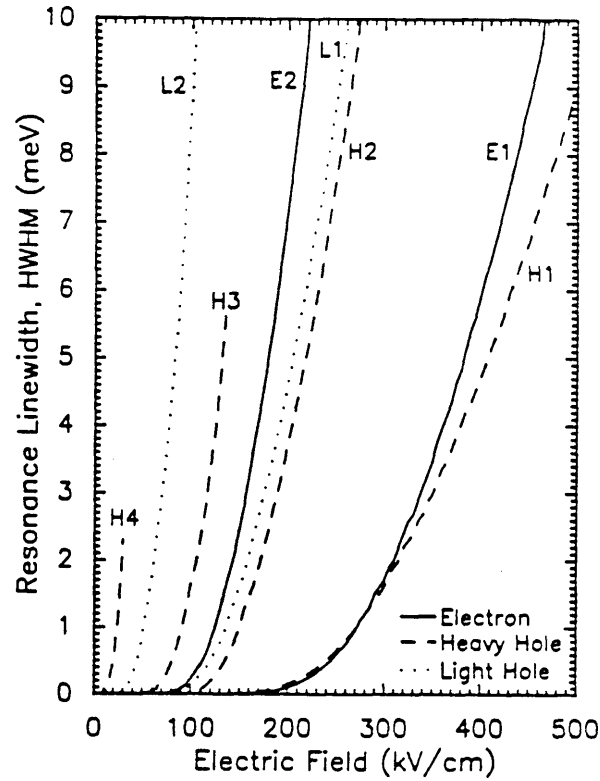


Fig. 2.19 : The linewidth (HWHM) of the resonances of a 100 Å GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, as a function of the applied field. The different curves show the variation for the electron resonances (solid lines), shown in Fig. 2.18, in addition to those for the heavy (dashed) and light (dotted) holes. The parameters are again taken from table 4.1.

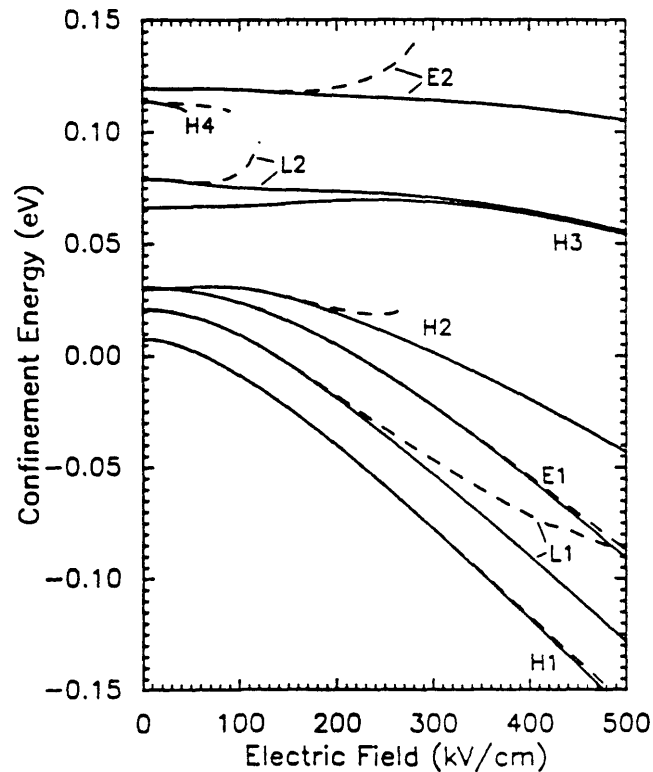


Fig. 2.20 : The solid lines show the energies of the maxima (E_{res}) of the electron, heavy hole and light hole resonances for a 100 Å GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, as a function of the applied electric field. The dashed lines show the energies determined using the Breit-Wigner parameterisation (see Sec. 2.6.1). Parameters from table 4.1.

the effect of the smaller potential barrier for the heavy holes, compared to that for the electrons, is approximately compensated by their heavier effective mass in the barrier. Meanwhile the linewidth of the L1 resonance increases much more rapidly than H1, being ≈ 4 meV (HWHM) at 200 kV/cm, due to its lighter barrier mass than the heavy hole. Notice also that the linewidths of the higher quasi-confined levels increase much more rapidly than those of the $N = 1$. This is because the higher confined states have a larger confinement energy and so experience a smaller effective barrier height to tunnel through. The linewidths of the $N = 1$ resonances show very little dependence upon the well width, except for very narrow QW, where the large confinement energies produce significant reductions in the effective barrier heights.

Fig. 2.20 plots the field dependence of the resonances energies (solid lines) for the 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW. These curves show the same general features as the infinite barrier calculations of Sec. 2.6.1.1. The Stark shift of the $N = 1$ states can be seen to be approximately quadratic at low fields, before becoming linear at high fields, when the carriers are tightly confined against the side of the well. The higher levels show a smaller Stark shift, as they are less readily polarised to the side of the well by the field, due to their larger confinement energy.

Fig. 2.20 also shows the quasi-confinement energies, calculated using the Breit-Wigner (B-W) parameterisation (dashed lines), ie the energies where $a_b = 0$, used by Austin and Jaros [2.67]. It can be seen that the B-W energies agree closely with the resonance maxima at low fields, but increasingly overestimate their values at high fields. The discrepancy happens at a lower field for the higher states. The B-W energy of each state can eventually no longer be found above a certain field, when the value of a_b no longer falls to zero. Comparison of Figs. 2.19 and 2.20 shows that the discrepancy between the resonance maxima and the B-W energies is only significant when the resonances are broad. The discrepancy is due to the B-W parameterisation ignoring the variation in b_b across the resonance, which is only valid if the resonance is narrow. Under this approximation, the maximum of the function R , which is proportional to $(a_b^2 + b_b^2)$, is indeed given by the energy where $a_b = 0$. Consequently, the B-W energies are only accurate at low fields and the high-field positive energy shifts have no physical significance.

2.6.1.4 Optical Transitions in the Narrow Resonances Approximation

The optical spectrum of a QW under an electric field must, in general, be calculated by integrating the overlap integral over the continuum of electron (E_e) and hole (E_h) energies associated with the motion in the z-direction as in Eq. 2.71. Notice that even after neglecting the Coulombic interaction, this will require considerable computation, due to the dependence of the unnormalised overlap integral (M_Z^u) on E_e and E_h . The resonances in $R_e(E_e)$ and $R_h(E_h)$, at E_e^{res} and E_h^{res} respectively, can be expected to produce broadened optical features. However, if the resonances are broad, these features will not occur at $E = E_g + E_e^{\text{res}} + E_h^{\text{res}} + E_{xy}$, as in the zero-field case, due to the variation in $M_Z^u(E_e, E_h)$.

In the limit of the resonances being much narrower than the confinement energy, the coefficients of the unnormalised electron and hole envelope functions in the well, ie a_w and b_w , can be regarded as constant over the energy range of the resonances. Hence the overlap integral in Eq. 2.71 can be replaced with its value at the resonance energies, ie. $M_Z^u(E_e, E_h) \rightarrow M_Z^u(E_e^{\text{res}}, E_h^{\text{res}})$, and be removed from the integration over E_e and E_h . Similarly, the energy (E_{xy}) and envelope function (ϕ) of the xy-motion can also be regarded as independent of E_e and E_h , and be determined using ψ_e^{res} and ψ_h^{res} , the envelope functions at the energies of the resonance maxima.

With this approximation, the computation of the optical spectra is greatly simplified, because the z-motion of the electron and hole can be considered separately, to obtain the energy (E^{res}), linewidth (Γ^{res}) and unnormalised envelope function ($\psi^u(E^{\text{res}})$) for each resonance. As discussed earlier, the lineshape of each electron and hole resonance is Lorentzian, provided that the resonance is narrow. Consequently the convolution of the hole (of width Γ_h^{res}) and electron (width Γ_e^{res}) resonance in Eq. 2.71, produces another Lorentzian function with a linewidth of $(\Gamma_h^{\text{res}} + \Gamma_e^{\text{res}})$. Hence Eq. 2.71 becomes

$$\epsilon(E) = S_0 \sum_{E_e^{\text{res}}, E_h^{\text{res}}, E_{xy}} |\phi(0)|^2 \frac{c_e^2(E_e^{\text{res}}) c_h^2(E_h^{\text{res}})}{\{M_Z^u(E_e^{\text{res}}, E_h^{\text{res}})\}^2} L(E, E_t, \Gamma_t) \quad \dots 2.74$$

where $L(E, E_t, \Gamma_t)$ is the Lorentzian broadening profile, defined by Eq. 2.48. The imaginary part of L is a Lorentzian function of unit integrated strength, centred at an energy $E_t = E_g + E_e^{\text{res}} + E_h^{\text{res}} + E_{xy}$, and with a linewidth of $\Gamma_t = \Gamma_e^{\text{res}} + \Gamma_h^{\text{res}}$. Notice that Eq. 2.74

also describes the real part of the dielectric function, since the real and imaginary parts of L are related by the Kramers-Kronig relations [2.61].

Eq. 2.74 is similar to the corresponding zero-field expression (Eq. 2.41), since (by definition) $M_Z^u = M_Z / \{|c_e|^2 \cdot |c_h|^2\}^{\frac{1}{2}}$. The Lorentzian function in Eq. 2.74, which replaces the Dirac-delta function of Eq. 2.41, describes the broadening due to the field-induced tunnelling from the well. Since the Lorentzian and Dirac-delta functions both have an integrated strength of unity, it is clear that the integrated transition strength will be independent of the linewidth, and hence the degree of field-induced broadening. There will be a redistribution of the oscillator strength between the different quasi-confined transitions with increasing field, as demonstrated in Sec. 2.6.1.5, due to the variation in M_Z^2 . However, the total integrated transition strength of all the quasi-confined transitions will remain constant [2.77-78,71]. Indeed, the integrated transition strength will still be unaltered at very high fields, when the resonances are broad and the spectrum can no longer be regarded as consisting of a number of quasi-confined transitions.

As discussed in Sec. 2.4.1, the contribution of the states with an unbound electron-hole motion in the xy-plane, can be determined by replacing the summation over E_{xy} with an integration over the continuum of states with $E_{xy} \geq 0$. The Lorentzian function in Eq. 2.74, will result in the sharp edge seen at the threshold of the continuum at zero-field, being smeared out at finite fields.

The contribution of the bound excitons to the optical spectrum can also be calculated using Eq. 2.74. The bound states have discrete values of E_{xy} . The xy-motion envelope function (ϕ) and binding energy ($E_{xy} = E_{be}$) can be determined using Φ_e^{res} and Φ_h^{res} . The Lorentzian function in Eq. 2.74, which has a width $(\Gamma_e^{res} + \Gamma_h^{res})$, will describe the homogeneous broadening due to the field-induced ionisation of the electron and hole. The Coulombic attraction will tend to reduce the field-induced dissociation of the electron and hole, so that the homogeneous exciton linewidth will be less than $(\Gamma_e^{res} + \Gamma_h^{res})$. However, the Coulombic attraction can be expected to have only a small effect upon the exciton linewidth, since it requires only a very small field to ionise the exciton in the absence of the confining barriers, as discussed in Sec. 2.5.2.

The coefficients c_e and c_h are found using the normalisation condition, Eq. 2.67. It was noted earlier that c_e and c_h tend to zero, as the limit of the normalisation integral, $L \rightarrow \infty$. However, for a

narrow resonance, the normalisation integral can be evaluated using a finite lower limit,

$$\Delta I_N(L) = \{ c(E_{\text{res}}) \}^{-2} = \int_{-L}^{\infty} | \Psi^u(z, E=E^{\text{res}}) |^2 dz \quad \dots 2.75$$

At the energy of a narrow resonance, $I_N(L)$ does not diverge until L is very large. This is essentially because the envelope function coefficients in the barrier (a_b and b_b) are very small, if the resonance is sharp. As L is increased from zero, $I_N(L)$ initially increases to a constant value (I_{N0}) when the normalisation integral has spanned all the quasi-confined envelope function. As L is increased further, the value of $I_N(L)$ does not increase above I_{N0} appreciably, until L gets very large, when $I_N(L) \rightarrow \infty$. Hence the envelope function can be approximately normalised by taking $c^{-2} = I_{N0}$.

It is stressed that this is only possible at the energy of a resonance maximum, and furthermore the resonance must be narrow. As the resonances become broader, $I_N(L)$ diverges at a smaller value of L . Therefore, under increasing field, the plateau where I_N is constant, becomes increasingly poorly defined, as the envelope function in the negative- z barrier becomes more significant. Consequently, at large fields, it becomes no longer possible to normalise the envelope function in this manner. This is consistent with the fields where the resonance becomes broad and Eq. 2.74 is no longer valid. The optical spectrum must then be calculated using the much more laborious Eq. 2.71.

The limit of the normalisation integral (L) can be taken as the position where the envelope function in the negative- z barrier becomes oscillatory, ie $L = (V_0 - E^{\text{res}})/eF$. To check the validity of the normalisation, the fractional increase in the normalisation integral after doubling its lower limit, ie $\Delta I_N = \{I_N(2L) - I_N(L)\}/I_N(L)$, is calculated. When ΔI_N is large, the resonance will be broad and Eq. 2.74 will no longer be valid. The optical spectrum will then have to be calculated using Eq. 2.71. At these fields it will no longer be meaningful to consider the spectrum as consisting of a number of quasi-confined transitions. In the present calculations, the oscillator strength was not calculated at fields where $\Delta I_N > 0.1$.

2.6.1.5 Calculations of Transitions Between Quasi-Confined States

Fig. 2.21 plots the electron and heavy hole quasi-confined envelope functions for a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW under an applied electric field of 100 kV/cm. As discussed in Sec. 2.6.1.4, these are the

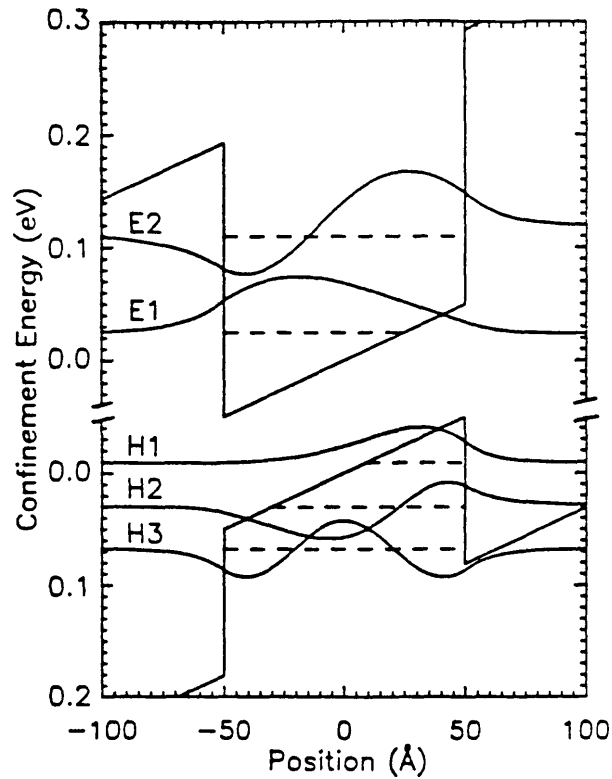


Fig. 2.21 : Electron and hole quasi-confined envelope functions for a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW under a uniform electric field of 100 kV/cm, calculated at the resonance energies, as described in Sec. 2.6.1, with parameters listed in table 4.1.

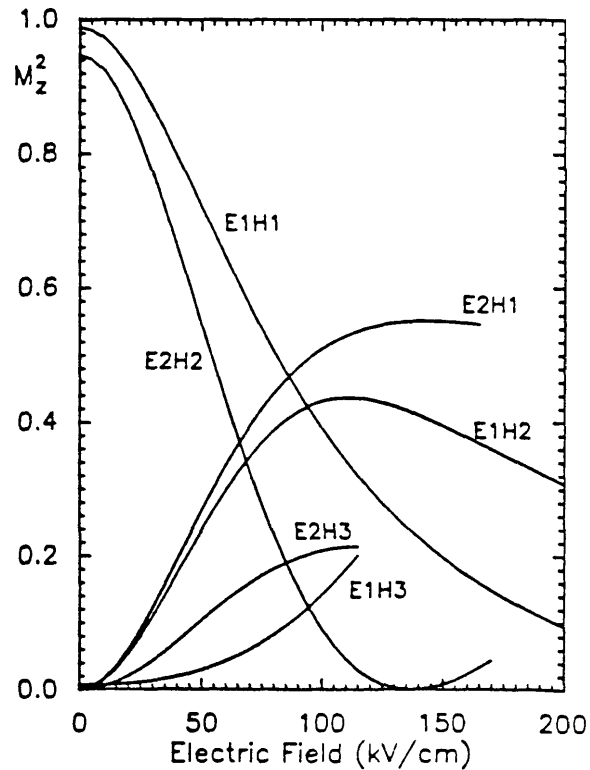


Fig. 2.22 : The field dependence of the square of electron - heavy hole overlap (M_z^2) for quasi-confined subband transitions (ENHM) of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW. Calculated using the parameters in table 4.1.

envelope functions determined at the energies of the maxima in $R(E)$, and were normalised over a finite z -range. The same parameters are used for this calculation, as for the corresponding zero-field envelope functions shown in Fig. 2.2, and these are listed in table 4.1. It can be seen that the electric field distorts the envelope functions, by polarising the electron and hole towards opposite sides of the well. This results in a reduction of the overlap of the allowed transitions ($N = M$) accompanied by an increase for the forbidden transitions. For instance, it can be seen in Fig 2.21, that the field causes local maxima in the E1 and H2 envelope functions to coincide, which will enhance the E1H2 overlap.

Fig. 2.22 plots the variation in the square of the normalised overlap integral ($M_z^2 = |\langle \psi_e^{\text{res}} | \psi_h^{\text{res}} \rangle|^2$) for the different electron-heavy hole (ENHM) quasi-confined transitions of the 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, as a function of electric field. It can be seen that the curve for each of the higher subband transitions is not shown above a certain field, corresponding to the field where the narrow resonances approximation is no longer valid, by the criterion of Sec. 2.6.1.4. Fig. 2.22 again demonstrates the field-induced decrease in the overlap of the allowed and the increase of the forbidden transitions. Notice that the E1H2 transition has a larger overlap than E1H1 for fields in excess of about 90 kV/cm. It can be seen that the overlap of the forbidden transitions reaches a peak at some finite field, when the local maxima of the electron and hole envelope functions coinciding.

Fig. 2.23 plots the field dependence of the sum of the electron and hole z -motion energy ($E_e^{\text{res}} + E_h^{\text{res}}$) for the E1H1 (solid lines) and E1L1 (dashed) transitions in GaAs/Al_{0.3}Ga_{0.7}As QWs of differing well width. These calculations were again made using the parameters listed in table 4.1. As expected the Stark shifts increase with the well width, due to the greater degree of polarisation of the electron and hole possible in the wider wells. The shift of the E1H1 transition in each well can be seen to be larger than that for E1L1, due to its larger hole mass.

The field dependence of the square of overlap integral (M_z^2) for E1H1 (solid lines) and E1L1 (dashed), in these QWs, is plotted in Fig. 2.24. The E1L1 overlap is not plotted at fields in excess of 200 kV/cm, where the L1 resonance is rapidly broadened by the field. It can be seen that the rate of decline in M_z^2 increases with the well width, which again is due to the larger field-induced polarisation of the electron and hole envelope functions in the wider wells. Therefore the larger Stark shift of wider wells, is necessarily accompanied by a stronger

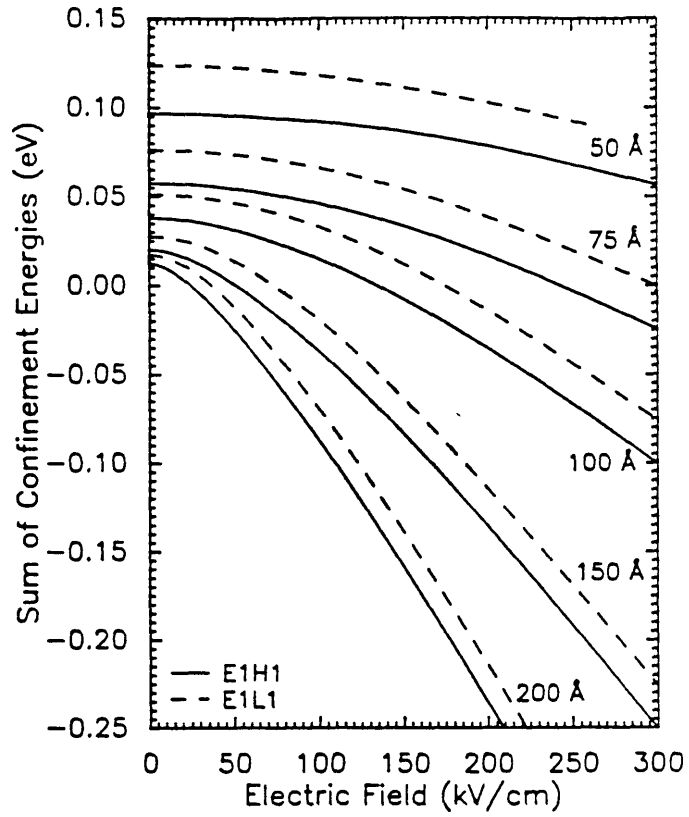


Fig. 2.23 : Calculated field dependence of the sum of the electron and hole quasi-confinement energies ($E_e^{\text{res}} + E_h^{\text{res}}$) for E1H1 (solid lines) and E1L1 (dashed) in GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QWs, with well widths of 50, 75, 100, 150 and 200 Å, using parameters in table 4.1.

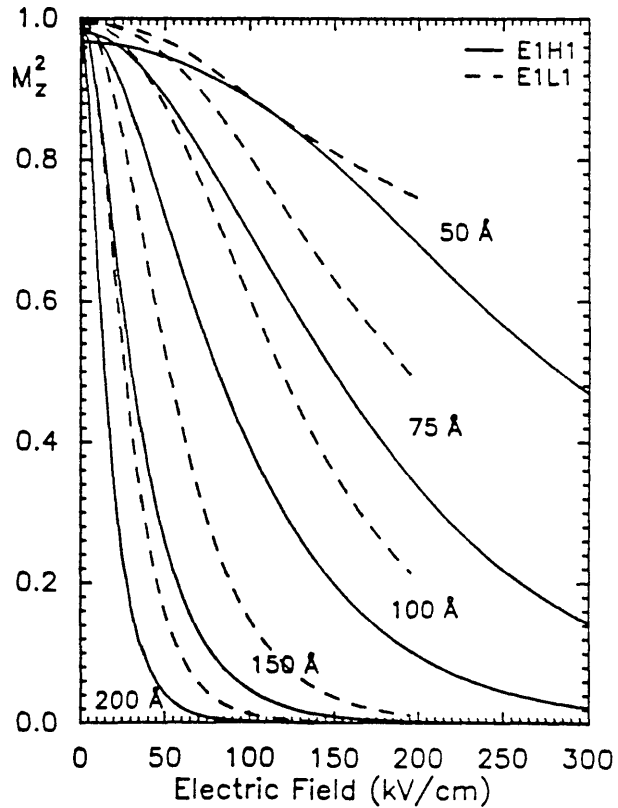


Fig. 2.24 : Calculated field dependence of the square of electron-hole overlap (M_z^2) for E1H1 (solid lines) and E1L1 (dashed) transitions in GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QWs, with well widths of 50, 75, 100, 150 and 200 Å, using parameters in table 4.1.

decline in their transition strength. The overlap integral of the E1L1 transition decreases less rapidly with field than E1H1, because the L1 envelope function is not so tightly polarised to the side of the well, as a result of its smaller mass.

2.6.2 Quantum Well Excitons in an Electric Field

There have been several reports of variational calculations of the change in the exciton binding energy with an electric field [2.34,79-81]. Each of these studies have used the separable envelope function, ie using ϕ_1 given by Eq. 2.35, although they differed in their choice for the z-direction envelope functions (ψ_e, ψ_h). D.Miller et al [2.34] took variational forms for ψ_e and ψ_h , which assumed infinite confining barriers. To recompense for the envelope function penetrating into the barrier, they used an effective well width, which gave the same zero field transition energy as a more exact calculation. Nojima [2.80] later corrected an error in an equation used by D.Miller et al, which caused them to overestimate the decline in the binding energy with field. Brum and Bastard [2.79] also took a variational form for ψ_e and ψ_h , except their's allowed for the finite barrier height. Hong and Singh [2.81] made variational calculations with the separable exciton envelope function, but used an exact numerical solution for ψ_e, ψ_h .

Nojima compared each of these calculations and showed that, although there is a general agreement in the trends with well width and field, there is considerable variation in the absolute values obtained. This derives from the different parameters used by the workers, particularly the dielectric constant and the xy-direction reduced effective mass, which each assumed to have the same value in the well and barrier material. Some workers took a value of the relative dielectric constant (ϵ_{rel}), which averaged over the values for bulk layers of the well (ϵ_w) and barrier (ϵ_b) material, $\epsilon_{rel} = (\epsilon_w \cdot \epsilon_b)^{\frac{1}{2}}$ [2.31,34,80-81]. Meanwhile, other workers took the value for bulk layers of well material, $\epsilon_{rel} = \epsilon_w$ [2.32,79]. The latter choice gives a significantly larger value for ϵ_{rel} and hence a reduced Coulombic attraction and binding energy. It is interesting to note that the value of ϵ_{rel} will have quite a large contribution due to the exciton itself, and this will depend on the photon energy. A more complete analysis would therefore have to be performed self-consistently.

I have made calculations of the electric field dependence of the E1H1 and E1L1 exciton binding energy in GaAs/Al_{0.3}Ga_{0.7}As QWs. These were made using the variational method described in Sec. 2.4.1, which

employs the separable form for ϕ and an exact solution for ψ_e and ψ_h , taking account of the finite barrier height. By using an exact solution for the z-motion, I have been able to calculate the binding energies to much larger fields than in the previous studies of Refs. 2.34,79-81. The z-direction effective masses, taken as different in the well and barrier, and the potential discontinuities are given by table 4.1. The calculations were made in the narrow resonances approximation, discussed in Sec. 2.6.1.4, so that ψ_e and ψ_h were evaluated at the resonant energies. The reduced masses for motion in the xy-direction (μ_{xy}) were taken as 0.042 and 0.050, for the heavy and light hole exciton respectively, deduced from the GaAs Luttinger parameters discussed in Sec. 2.2.1 and assumed to be the same in the well and barrier material. The dielectric constant was taken as $\epsilon_{rel} = (\epsilon_w \cdot \epsilon_b)^{\frac{1}{2}} = 12.2$.

Using the same parameters as Hong and Singh [2.81], I have been able to reproduce their curve for the E1H1 exciton in a 100 Å to within 0.1 meV. I found similar agreement with the calculations of Brum and Bastard [2.79] for a 200 Å QW. I also obtained good agreement with the curves calculated by Nojima [2.80] for several well widths, although I find the binding energy to be ≈ 0.3 meV smaller at 100 kV/cm, which is likely to be due to the inadequacy of their form for ψ_e and ψ_h at these fields.

Fig. 2.25 plots the field dependence of the E1H1 (solid lines) and E1L1 (dashed) exciton binding energy for GaAs/Al_{0.3}Al_{0.7}As QWs, with well widths of between 50 and 200 Å. It can be seen that the E1L1 excitons have a larger binding energy than E1H1, which is due to their heavier in-plane mass. The E1L1 binding energy is not shown at fields above 200 kV/cm in Fig. 2.25, where the L1 state is ionised by the field, and the narrow resonances approximation is no longer valid.

The E1H1 and E1L1 excitons show a similar field evolution. The decrease in the binding energy with increasing field is due to the reduction in the Coulombic attraction of the electron and hole as they are polarised to opposite sides of the well. This explains the more rapid decline in the binding energy for the wider wells. The binding energy for the wider wells tends to a constant value at high fields, as the increase in the z-separation of the electrons and holes is limited by the potential barriers. Notice that the binding energies of the 50 Å QW are only slightly reduced at the largest fields shown in Fig. 2.25, which would correspond to very large experimental fields.

Comparison of Figs. 2.23 and 2.25 shows that the field-induced change in the sum of the electron and hole quasi-confinement energies is typically much larger than the change in the binding energy. It can be

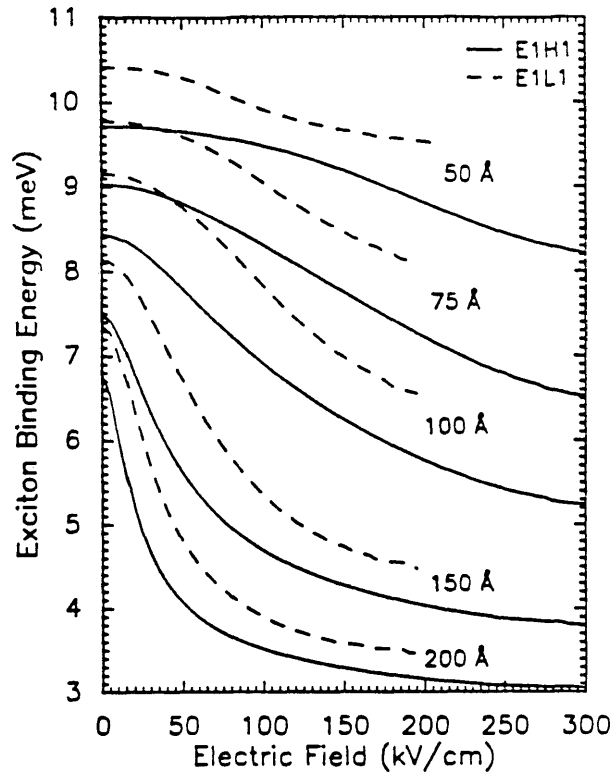


Fig. 2.25 : The field dependence of the E1H1 (solid lines) and E1L1 (dashed) exciton binding energies in GaAs/Al_{0.3}Ga_{0.7}As QWs, with well widths of 50, 75, 100, 150 and 200 Å, calculated using the variational method and parameters discussed in Sec. 2.6.2.

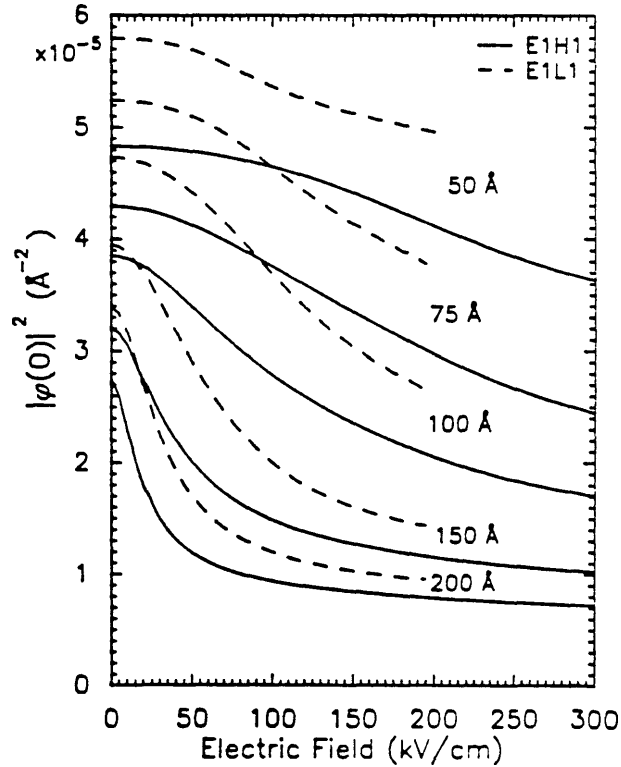


Fig. 2.26 : The field dependence of the square of the electron-hole overlap, per unit area, for motion in the xy-plane, $|\phi(0)|^2$, for the E1H1 (solid lines) and E1L1 (dashed) excitons in GaAs/Al_{0.3}Ga_{0.7}As QWs, with well widths of 50, 75, 100, 150 and 200 Å, calculated using parameters given in Sec. 2.6.2.

seen that the change in the binding energy will make the largest contribution to the total Stark shift, for wide wells at low fields.

Fig. 2.26 plots the evolution of $|\phi(0)|^2$ for the E1H1 (solid lines) and E1L1 (dashed) excitons with electric field, for the different wells widths. This quantity corresponds to the square of the overlap for the xy-motion, per unit area of the xy-plane, and is proportional to the exciton oscillator strength, as can be seen from Eqs. 2.41 and 2.74. The zero-field oscillator strength is strongest for the narrower wells, due to their stronger Coulombic interaction. The reduction in $|\phi(0)|^2$ with increasing field is again due to the field-induced increase in the z-separation of the electron and hole, which reduces their attraction. Comparison with Fig. 2.24 shows that the field dependence of the excitonic oscillator strength will be dominated by the variation in M_z^2 , rather than $|\phi(0)|^2$. Notice that the term due to the xy-motion will have most influence for wide wells at low field, as was also the case for the exciton energy.

The separable form of the exciton envelope function, used here, is only accurate when the z-separation of the electron and hole is much smaller than that in the xy-direction. The calculations of Bastard et al [2.32], shown in Fig. 2.7, demonstrated the separable form to be accurate for $L_w < 1.5 a_\infty$, at zero field. Since this corresponds to well widths of 231 and 194 Å for the E1H1 and E1L1 excitons respectively, the present calculations can be expected to be accurate at zero field. However, the average z-separation of the electron and hole increases with the electric field, and furthermore, this increase is more rapid for the wider wells. As a result, the binding energy and oscillator strength of the wider wells in Figs. 2.25 and 2.26 may be slightly overestimated at high fields. However, the field-induced increase in the z-separation of the carriers is limited by the confining barriers, which will consequently limit the error in the binding energy and the oscillator strength.

It was discussed in Sec. 2.4, how the valence subband mixing can quite radically alter the exciton binding energy and oscillator strength at zero field. Several workers have considered the effect of valence subband mixing on the field dependence of the exciton binding energy [2.82-83,17]. The mixing will modify the field dependence of the binding energy, due to the lateral extent of the exciton increasing with field, thereby reducing the average in-plane k-vector. The exciton will consequently experience a different mixture of hole subbands, so altering its binding energy and oscillator strength. However, this is a

second-order effect compared to the zero-field change in binding energy and oscillator strength produced by the mixing.

The electric field removes the spatial inversion symmetry of the z-direction and consequently the Kramers' degeneracy of the subbands away from $k_{xy} = 0$. This splitting in the dispersion of each subband depends on how strongly it is mixed with the other subbands. Consequently the two spin states of each subband have a different binding energy, and their energy separation will increase with the amount of subband mixing and the electric field. For a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, Broido [2.83] calculated a splitting in the E1L1 exciton of 0.6 meV at 50 kV/cm. This splitting may be too small to be resolvable, in present quality material, but will cause an additional amount of broadening.

3 Lineshape Models for Modulation Spectroscopy

3.1 Introduction

Modulation spectroscopy measures the change in the optical spectrum of a material produced by an applied perturbation. This is achieved by periodically varying the perturbation and monitoring the resultant optical signal at that frequency using phase sensitive detection. Since the mid-1960's there have been a large number of studies of bulk semiconductors using modulation spectroscopy, many of which have been reviewed by Cardona [2.61], Seraphin [3.1] and Aspnes [3.2]. The technique found wide popularity, because it is sensitive only to the critical points of the bandstructure, so that the signal due to structure above the fundamental bandgap is not obscured by a background deriving from lower energy bands. Its differential nature results in sharp spectral features, which are therefore well resolved. As a phase sensitive technique it can potentially produce a good signal-to-noise ratio.

In electromodulation studies the alternating perturbation is provided by an electric field. Either the reflectivity or transmission can be monitored at the frequency of the field, so yielding the electroreflectance (ER) or electrotransmission (ET). ER has received more attention than ET, as it does not require the prior removal of an absorbing substrate. The electric field can be applied using several methods. A semitransparent metallic layer can be evaporated onto the front surface to act as a Schottky barrier, which is then reverse biased with an applied voltage [3.3]. A field can also be created at the sample surface using a biased electrolyte [3.4]. More recently, it has become increasingly popular to selectively dope the sample, so as to place the region of interest inside the intrinsic region of a p^+-i-n^+ diode, the properties of which will be discussed in Chp. 4.

The 1980's have seen an increasing interest in the study of LDS systems using ER spectroscopy [3.5-10,1.12-17,2.68]. Mendez et al [3.5] measured GaAs/AlGaAs MQWs over the 1.3 - 5.5 eV spectral range, and observed structure due to a number of interband transitions. The transition energies were observed to be larger than in bulk GaAs, due to the confinement in the different bands. Erman et al [3.6] also observed a number of interband transitions in GaAs/Al_{0.54}Ga_{0.46}As SQWs using ER, and made a comparison with ellipsometry measurements. Alibert et al [3.7] measured the field dependence of the ER spectra of these SQWs and found a qualitative agreement with the QCSE. Klipstein et al [2.68]

compared the PLE and ER spectra of a GaAs/AlGaAs SQW and MQW, and observed several subband transitions with energies in rough agreement with a calculation.

There have also been a large number of experiments on LDS samples which study photorefectance (PR), where the modulation is created by a mechanically chopped laser beam incident on the sample [3.11-33,2.41]. Since PR does not require electrical contacts, it has the advantages of requiring little sample preparation, while also being non-destructive. Like ER, PR can be readily performed at room temperature, making it an attractive method for characterising LDS devices. It can also be used on samples with a semi-insulating substrate, which are more difficult to study with ER.

It is believed that the photoexcited carriers, produced by the laser beam in a PR experiment, neutralise part of the surface charge of the sample, so reducing its internal electric field [3.12,15,16]. Consequently PR spectra are also thought to be modulated by changes in electric field, suggesting that the ER and PR lineshapes should be similar. However, a recent report has suggested that photo-induced formation of free excitons can also give a significant contribution to the PR at low temperatures [3.24]. ER is perhaps preferable to PR for studying the bandstructure, as there is no ambiguity in the modulation mechanism, while the electric field can be carefully controlled and used as a continuous variable.

Many of these studies of ER and PR spectra of QWs have concentrated on the form of the lineshape for transitions between the lowest electron and hole subbands [1.13-15,2.33,3.14-16,18,22-23], which is discussed in detail in this chapter. The lineshape for transitions involving higher subbands confined in the QW, and also unconfined states with energies above the barrier potential, have also been examined [3.20,21,26,28]. Other studies have been less concerned with the lineshape itself, but have used ER or PR to measure transition energies as a function of some applied variable. Alibert et al [3.7] studied the ER spectra of SQW samples as a function of the applied field, while Voisin et al [3.9] have made field-dependent ER measurements on SLs. PR has been used to measure transition energies as a function of hydrostatic pressure [3.30-32], and also to study topological variations [3.19].

Although it is often more sensitive than conventional techniques, modulation spectroscopy has the disadvantage of being more difficult to interpret. Identifying the transition energies may not be as straight forward as in non-modulation spectroscopies, such as transmission, PC,

PL or PLE, where often the position of a peak is measured. A quick glance at Fig. 3.1, which plots a PC and ER spectrum of a SQW taken under similar conditions, illustrates this point. It can be seen that the energies of the PC peaks do not correspond to any particular points in the ER. Measurements of a transition energy as a function of some applied variable, may be complicated by the simultaneous variation of the lineshape of this modulated spectrum. As an additional complication, different transitions in the spectrum will not in general have the same lineshape. Consequently an appropriate lineshape model is needed to analyse the spectra accurately.

Modulation spectra can be more complicated than non-modulation techniques for several reasons. Spectroscopies, such as PC, PLE, or absorption, are essentially determined by the imaginary part of the dielectric function, and can be quite easily related to the bandstructure. However, in modulation spectroscopy the lineshape also depends upon how the dielectric function is perturbed. Conversely though, this property also allows the effect of the modulating parameter to be studied. As an additional complication, modulated reflectance measurements depend on both the real and imaginary parts of the dielectric function, as in fact is also the case for the non-modulated reflectance. The mixing coefficients for the real and imaginary parts can vary across the spectrum and also with an applied variable.

The objective of this chapter is to derive formulae for electro-modulated lineshapes of SQWs and MQWs. These not only allow the transition energies to be accurately determined from the spectra, but also the linewidths and the transition strengths. It is shown that ER spectra can also contain information on the depths of the QWs inside the sample structure. The derivation of the lineshape formulae is considered in two stages; firstly the change in the dielectric function produced by the field modulation is calculated and then this is related to the ER and ET. The field modulated dielectric function due to the 1s bound excitonic state is determined using the method described in Sec. 2.6. The contribution of the 1s state was shown to dominate the optical spectra in Chp. 2, so that the excited excitonic states and the continuum can be ignored. The relative importance of the field-induced change in the exciton energy, linewidth and oscillator strength is calculated for different subband transitions, in wells of various width, as a function of the d.c. offset field. Field modulation of the dielectric function of short period SLs occurs by an entirely different mechanism than the QCSE, and is discussed separately in Chp. 7.

The modulated dielectric function is then related to the ER and ET

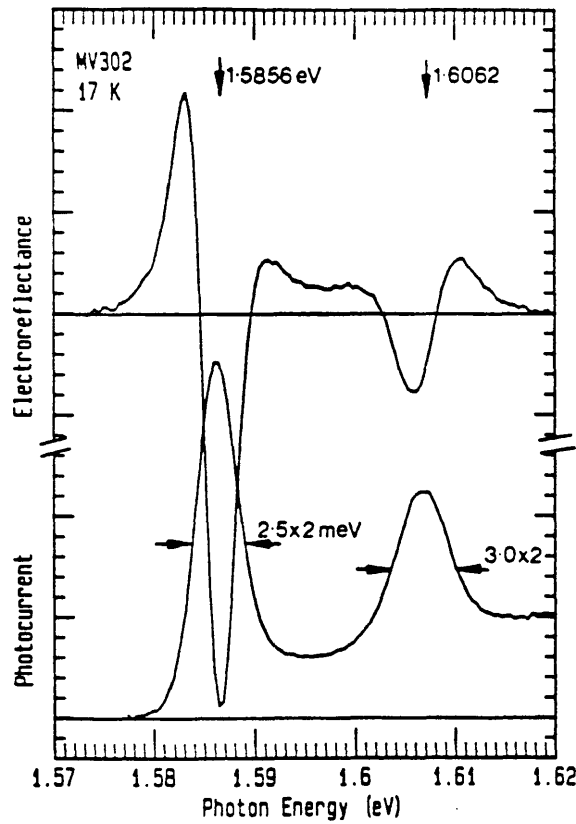


Fig. 3.1 : PC and ER spectra taken on MV302 (a 59 Å GaAs/Al_{0.3}Ga_{0.7}As SQW), at 17 K, with the light incident at 20° and under an applied bias of -2 V (47 kV/cm). The ER was modulated by 1.0 V (12 kV/cm) peak-to-peak. Further experimental details are given in Sec. 5.1.

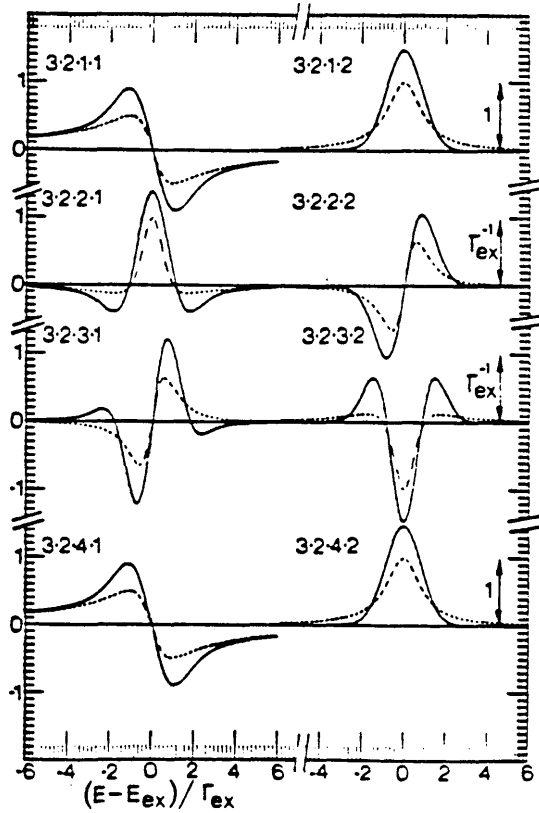


Fig. 3.2 : The contribution of a single exciton to the real (3.2.1.1) and imaginary (3.2.1.2) parts of the QW dielectric function, using either a Lorentzian (dashed lines) or Gaussian (solid) broadening profile, with HWHM of Γ . Also shown are the partial differentials with respect to the exciton energy (3.2.2.1-2), linewidth (3.2.3.1-2) and overlap (3.2.4.1-2).

of SQWs in Sec. 3.3. This involves considering the optical interference between rays arising from the different interfaces of the sample structure. By simplifying the analysis as far as is valid, a simple explicit formula is derived for the lineshape. This allows a clear understanding of the role of each parameter in determining the lineshape. The validity of the approximations used in the derivation of this formula are tested by a comparison with a more exact numerical calculation. The formulae derived in Sec. 3.3 do not depend on the form of the modulated dielectric function and so are general to any modulation spectroscopy. The nature of the ER lineshapes for the E1H1 and E1L1 transitions are then examined in detail. Calculations are made using the parameters of a GaAs/Al_{0.3}Ga_{0.7}As SQW and compared to experimental ER spectra taken on this sample. The model will be applied to further measurements on a number of SQWs in Chp. 5.

Modulated reflectance lineshapes for MQWs are considered in Sec. 3.4. In these structures the analysis is more complicated than for SQWs, due to the increased number of interfaces and hence interfering rays. Again the various rays are summed to yield another explicit lineshape formula, relating the modulated dielectric function to the ER and ET. Calculations are made using typical parameters and it is shown that the MQW lineshape becomes increasingly more complicated, as the number of QWs in the stack increases. Again the analytical approach adopted here, shows clearly the effect of each sample parameter, allowing the origin of the complicated structure in the lineshape to be discussed. The model is compared to ER and ET measurements on MQWs in Chp. 6.

The ER of bulk semiconductors is discussed in Sec. 3.5. There are several motivations for considering bulk semiconductors in a thesis primarily concerned with the study of LDS structures. Firstly, contrasts are drawn with the analysis for SQWs, in order to highlight the deficiencies in the analysis of some recently reported PR measurements on LDS samples. Secondly, the standard analysis is extended to consider the effect of (i) the field modulation being small, (ii) the region of bulk semiconductor being buried in the sample structure. Such buried layers are found in samples with a p⁺-i-n⁺ doping profile, which are now increasingly common, thanks to improvements in the technology of electric field-dependent devices. This analysis is used to determine the electric fields in various LDS samples which also contain regions of bulk semiconductor in their intrinsic region.

3.2 Electric Field Modulation of the Quantum Well Dielectric Function

3.2.1 Dielectric Function of a Quantum Well

It was discussed in Sec. 2.4.2 how the zero-field optical spectra of QWs are dominated by the contributions due to the excitonic ground state. This is readily apparent in the PLE spectra shown in Fig. 2.6 and 2.12, which are dominated by the large peaks due to the 1s state of the E1H1 and E1L1 excitons. Calculations of the excitonic spectrum, shown in Fig. 2.10, also showed the contribution of the lowest electron-hole bound state to be much larger than that of the excited bound states or the unbound continuum. The 1s excitonic oscillator strength is larger in QWs than in bulk semiconductor due to the enhanced Coulombic attraction resulting from the confinement of the electron and hole in the same narrow layer. As a consequence the excitonic peaks are even seen to dominate the room temperature spectra of QWs [2.26].

In bulk semiconductors, the bound excitonic features are rapidly broadened by an electric field, due to the dissociation of the electron and hole. However, for QWs under a field perpendicular to the plane of the well, the potential barriers prevent the ionisation, so that the excitonic features are preserved, as demonstrated by the spectra shown in Fig. 2.16.

The ER spectra of QWs are also dominated by the 1s excitonic contribution, as clearly demonstrated by Fig. 3.1. This plots the PC and ER spectra of a 59 Å GaAs/Al_{0.3}Ga_{0.7}As SQW, taken at 17 K and under a field of 47 kV/cm, on the same energy scale. The nature of the ER lineshape is discussed in the following sections, but for now notice that large features can be seen corresponding to the familiar E1H1 and E1L1 1s excitonic peaks of the PC spectrum. A similar agreement was seen between the energies of the ER features and the PC 1s excitonic peaks at room temperature and at other applied electric fields. Klipstein et al [2.68] also found the low temperature E1H1 and E1L1 ER features of a GaAs/Al_{0.2}Ga_{0.8}As SQW occurred at the same energy as the PLE excitonic peaks. Shanabrook et al [3.15] compared the PR and PLE excitonic energies of a MQW, at temperatures between 6 and 250 K, and found agreement to better than 1 meV.

The present analysis therefore neglects the contribution of the excited excitonic states and the continuum to the dielectric function of the well, and considers only the excitonic ground state. The dielectric function of the QW (ϵ_w) is regarded as consisting of contributions due to the optical transitions of the QW (ϵ_{ex}), on top of a large real

background, with a value equal the dielectric constant of the barrier material (ϵ_b), ie. $\epsilon_w = \epsilon_b + \Sigma \epsilon_{ex}$.

The contribution of the excitonic transitions to the dielectric function of a QW under a uniform electric field, in a direction perpendicular to the plane of the well (z-direction), was considered in Sec. 2.6. The carriers are able to tunnel from the QW, so that each discrete zero-field confined electron (hole) energy level for the z-motion, is replaced by a resonance, with confinement energy $E_{e(h)}^{res}$ and linewidth $\Gamma_{e(h)}^{res}$. If these resonances are narrow, the contribution of each excitonic transition associated with a pair of electron and hole quasi-confined states, can be written as

$$\epsilon_{ex}(E) = S_0 V_{ex} L(E_{ex}, \Gamma_{ex}, E) \quad \dots 3.1$$

$$\text{where} \quad E_{ex} = E_g + E_e^{res} + E_h^{res} + E_{be} \quad \dots 3.2$$

$$\Gamma_{ex} = \Gamma_e^{res} + \Gamma_h^{res} \quad \dots 3.3$$

$$V_{ex} = |\phi(0)|^2 \cdot |\langle \psi_e^{res} | \psi_h^{res} \rangle|^2 \quad \dots 3.4$$

$L(E_{ex}, \Gamma_{ex}, E)$ is the Lorentzian lineshape function, given by Eq. 2.48, and S_0 is a constant given by Eq. 2.28, regarded as independent of field. In the present analysis, S_0 is determined from the zero-field integrated absorption strength (A_{ex}), reported by Masumoto et al [2.33], using the relation $A_{ex} = S_0 V_{ex}(F=0) E_{ex} / (n_r \cdot c \cdot \hbar)$. The xy-motion envelope function ($\phi(r_{xy})$) and binding energy (E_{be}) are again evaluated using the z-motion envelope functions at the energies of the resonances, ie ψ_e^{res} and ψ_h^{res} , as in Sec. 2.6. Notice that L is complex, so that Eq. 3.1 describes both the contribution to the real and imaginary part. The imaginary part of L is maximum at a photon energy of E_{ex} , has a linewidth (HWHM) of Γ_{ex} , and an integrated strength of unity.

Notice that V_{ex} determines the field dependence of the integrated transition strength, since S_0 is taken to be constant. The factor in Eq. 3.4 of $|\langle \psi_e^{res} | \psi_h^{res} \rangle|^2$ is the square of the overlap integral for the electron and hole motions in the z-direction. Meanwhile, the other factor in Eq. 3.4, $|\phi(0)|^2$, is the square of the overlap integral for the electron and hole motion in the xy-plane, per unit area of the xy-plane, expressed in relative coordinates. Consequently, V_{ex} is the square of the exciton overlap in all three spatial dimensions, per unit area, which from now on will be simply referred to as the "exciton overlap".

In the presence of inhomogeneous broadening mechanisms, there will be a statistical distribution of excitons with different energies,

linewidths and integrated transition strength. The lineshape function in Eq. 3.1 will then be determined by the convolution of this statistical distribution and the Lorentzian homogeneous profile. The statistical distribution could have any form, in general, depending upon the particular broadening mechanism. Considering the case of a Gaussian distribution of exciton energies, with constant linewidth and strength, the lineshape function is then approximately given by

$$L(E_{ex}, \Gamma, \gamma, E) = -i (\pi \pi^{1/2} / 4 \gamma) W\{ (E_{ex} - E) / \gamma + i \Gamma \} \quad \dots 3.5$$

where $W(x+iy)$ is the Error function tabulated in Ref. 2.35. The Lorentzian homogeneous profile is taken to have a HWHM linewidth of Γ , which is convoluted with a Gaussian distribution with a HWHM of $\gamma / (\ln 2)^{1/2}$ and an integrated area of unity. The negative energy pole of the Lorentzian function, at $E = -E_{ex}$, is ignored in the derivation of Eq. 3.5, as it negligibly small near $E = E_{ex}$.

For the case where the Gaussian distribution is much broader than the Lorentzian profile, ie. $\gamma \gg \Gamma$, the imaginary part of the Lorentzian profile tends to the Dirac-delta function, $\delta(E_{ex} - E)$, and hence the imaginary part of Eq. 3.5 reduces to the Gaussian distribution function itself. Meanwhile the real part is given by the convolution of the Gaussian distribution and $1/(E_{ex}^2 - E^2)$, which is determined by the Dawson Integral $D(x)$, also tabulated in Ref. 2.35. This convolution of the Gaussian distribution with the real part of an unbroadened Lorentzian lineshape, is exactly equivalent to taking the Kramers-Kronig transform [2.61] of the Gaussian function. This yields the profile, when determined purely by Gaussian broadening, to be

$$L(E_{ex}, \gamma, E) = (1/\pi) D\{(E_{ex} - E) / \gamma\} + i (1/\pi^{1/2} \gamma) \exp\{-\{(E_{ex} - E) / \gamma\}^2\} \quad \dots 3.6$$

3.2.2 Electric Field Modulated Dielectric Function

The modulated dielectric function is defined as the change in the dielectric function produced by an alternating perturbation. In the case of electromodulation, this perturbation is an alternating electric field. The modulated dielectric function can be separated into terms due to the changes in the exciton energy ($\Delta \epsilon_E$), linewidth ($\Delta \epsilon_\Gamma$) and overlap ($\Delta \epsilon_V$)

$$\Delta \epsilon = \Delta \epsilon_E + \Delta \epsilon_\Gamma + \Delta \epsilon_V \quad \dots 3.7$$

Since small voltage modulations are used in the experiments performed in this thesis, the field-induced change can be approximated by first-order-differentials of Eq. 3.1 and hence

$$\Delta \epsilon_E \approx S_O (\delta L / \delta E_{ex}) V_{ex} (\delta E_{ex} / \delta F) \Delta F \quad \dots 3.8$$

$$\Delta \epsilon_T \approx S_O (\delta L / \delta \Gamma_{ex}) V_{ex} (\delta \Gamma_{ex} / \delta F) \Delta F \quad \dots 3.9$$

$$\Delta \epsilon_V \approx S_O L (\delta V_{ex} / \delta F) \Delta F \quad \dots 3.10$$

where ΔF is the size of the electric field modulation. Note, that although the case of electromodulation has been considered here, Eqs. 3.8-10 are in fact general to any small applied perturbation.

Fig. 3.2 shows the real (Fig. 3.2.1.1) and imaginary (3.2.1.2) parts of the lineshape function of a single exciton, for both Lorentzian and Gaussian broadening. Also shown are their partial differentials with respect to E_{ex} (3.2.2), Γ_{ex} (3.2.3) and V_{ex} (3.2.4). The imaginary parts of the Gaussian and Lorentzian profiles have been chosen to have the same HWHM linewidth (written now as Γ_{ex} for both) and integrated area. The energy scale is expressed relative to the exciton energy and in units of Γ_{ex} , and hence the vertical scale for the energy- and linewidth-differential lineshapes are in units of $1/\Gamma_{ex}$. The curves in Fig. 3.2 are therefore independent of the linewidth, provided that it is much smaller than the exciton energy ($\Gamma_{ex} \ll E_{ex}$).

It can be seen that the Gaussian and Lorentzian broadened curves in Fig. 3.2 are similar in shape. Notice, however, that the Gaussian profile results in larger subsidiary maxima for $\text{Real}[\delta L / \delta E_{ex}]$ and $\text{Imag}[\delta L / \delta \Gamma_{ex}]$. Notice also that $(\delta L / \delta \Gamma_{ex})$ has similar lineshape to $(\delta L / \delta E_{ex})$, especially for the Lorentzian profile, but with their real and imaginary parts interchanged. Both $(\delta L / \delta E_{ex})$ and $(\delta L / \delta \Gamma_{ex})$ resemble the first-energy-differential of L , while $(\delta L / \delta V_{ex})$ is given by the unmodulated lineshape itself.

The strengths of the different lineshapes shown in Fig. 3.2 are determined by $(\delta E_{ex} / \delta F)$, $(\delta \Gamma_{ex} / \delta F)$ and $(\delta V_{ex} / \delta F)$, as expressed by Eqs. 3.8-3.10. These field-differentials depend upon the parameters of the QW system, especially the well width, and which quasi-confined states are involved. Sec. 3.2.3 assesses the strengths of $\Delta \epsilon_E$ and $\Delta \epsilon_V$ for the E1H1 and E1L1 transitions in wells of different width, using the QCSE calculation detailed in Sec. 2.6. In real QWs, the low-field linewidth will be determined by inhomogeneous mechanisms, with only a small component due to field-induced tunnelling. The increase in the linewidth with field $(\delta \Gamma_{ex} / \delta F)$ can therefore not be determined using the

QCSE calculations of Sec. 2.6 and the strength of $\Delta\epsilon_F$ is therefore discussed separately in Sec. 3.2.5. The modulation of the higher subband transitions is quite different to that for E1H1 and E1L1 and is discussed in Sec. 3.2.4.

3.2.3 Energy- and Overlap-Modulation of Lowest Subband Transitions

The field dependence of the exciton energy (E_{ex} in Eq. 3.2) and overlap (V_{ex} in Eq. 3.4) were calculated in Sec. 2.6. This was done by using an appropriate variational form for the envelope function of the xy-motion, so that the z- and xy-motions could be considered separately, in Secs. 2.6.1 and 2.6.2 respectively. Fig. 2.23 shows the variation in the sum of the electron and hole z-motion energies ($E_e^{res} + E_h^{res}$) with electric field, for E1H1 (solid) and E1L1 (dashed) in GaAs/Al_{0.3}Ga_{0.7}As QWs with well widths of 50, 75, 100, 150 and 200 Å. Meanwhile Fig. 2.25 shows the field dependence of the exciton binding energy (E_{be}) in the same QWs. It can be seen that the field-induced change in ($E_e^{res} + E_h^{res}$) is typically much larger than that in E_{be} . The exciton overlap (V_{ex}) is given by the product of the square of the overlap for the z-motion ($|\langle \phi_e^{res} | \phi_h^{res} \rangle|^2$), shown in Fig. 2.24, and the square of the overlap for the xy-motion per unit area ($|\phi(0)|^2$), shown in Fig. 2.26. Notice that the field dependence of V_{ex} will also be dominated by the variation in the z-motion term.

The differentials of the exciton energy ($\delta E_{ex}/\delta F$) and overlap ($\delta V_{ex}/\delta F$) with respect to the electric field have been calculated, in order to determine the strengths of $\Delta\epsilon_E$ (see Eq. 3.8) and $\Delta\epsilon_V$ (Eq. 3.10). Figs. 3.3 and 3.4 plot $V_{ex} \cdot (\delta E_{ex}/\delta F)$ and $(\delta V_{ex}/\delta F)$ respectively, which are the coefficients determining the strength of the energy- and overlap-modulated lineshapes in Fig. 3.2, ie $(\delta L/\delta E_{ex})$ and $(\delta L/\delta V_{ex})$. The curves in Figs. 3.3 and 3.4 will therefore show the dependence of the strengths of $\Delta\epsilon_E$ and $\Delta\epsilon_V$ on a variable offset field (F), with a small and fixed modulation (ΔF).

Consider firstly the field dependence of $V_{ex} \cdot (\delta E_{ex}/\delta F)$, shown in Fig. 3.3. The solid lines show the variation for E1H1, while the dashed are for E1L1. The Stark shift is quadratic at low fields, before becoming approximately linear at larger fields, as was discussed in Sec. 2.6.1 and is evident in Fig. 2.23. Therefore, the absolute value of $(\delta E_{ex}/\delta F)$ varies linearly at small fields and tends to a constant value, equal to the well width, in the high-field limit. Hence $|V_{ex} \cdot (\delta E_{ex}/\delta F)|$ (Note : absolute value !) will increase linearly at low fields, due to $(\delta E_{ex}/\delta F)$, and then decreases again at high field, as $(\delta E_{ex}/\delta F)$ levels

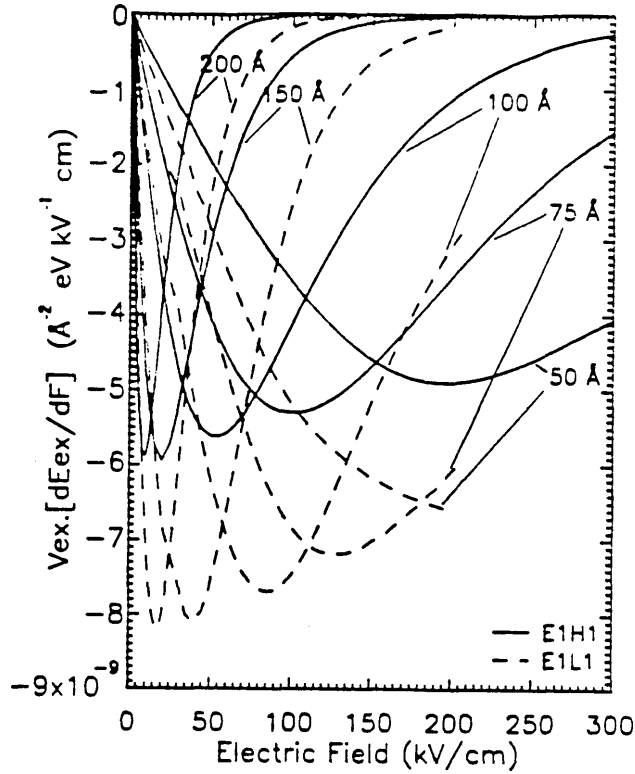


Fig. 3.3 : The vertical axis plots the product of the exciton overlap (V_{ex}) and the partial differential of the exciton transition energy with respect to field ($\delta E_{ex}/\delta F$), ie $V_{ex} \cdot (\delta E_{ex}/\delta F)$. This quantity determines the strength of $\Delta \epsilon_E$, see Eq. 3.8. The curves show the field dependences determined for the E1H1 (solid lines) and E1L1 (dashed) transition in a 50, 75, 100, 150 and 200 Å GaAs/ $Al_{0.3}Ga_{0.7}As$ QW.

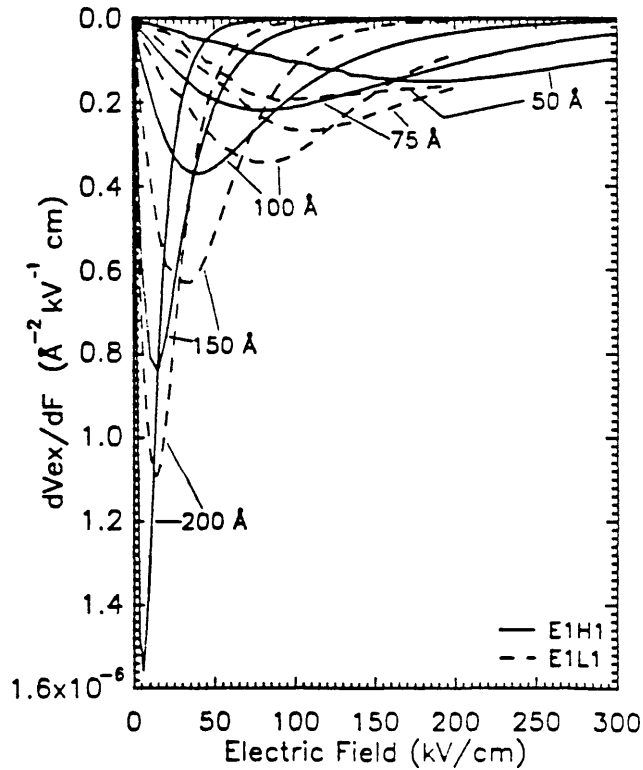


Fig. 3.4 : The calculated field dependence of $(\delta V_{ex}/\delta F)$ for the E1H1 (solid lines) and E1L1 (dashed) transitions of a 50, 75, 100, 150 and 200 Å wide GaAs/ $Al_{0.3}Ga_{0.7}As$ QW. This quantity determines the strength of $\Delta \epsilon_V$, as demonstrated by Eq. 3.10.

off and the decrease in V_{ex} becomes more important. The value of $|V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)|$ reaches a maximum at a smaller field in wider wells, reflecting the sharper decline in V_{ex} and also that $(\delta E_{\text{ex}}/\delta F)$ levels off at a lower field. It can be seen in Fig. 3.4 that $|(\delta V_{\text{ex}}/\delta F)|$ behaves in a similar manner, increasing linearly at low fields, before decreasing again above some value of the field.

The curves in Fig. 3.3 and 3.4 can be compared to judge the relative strengths of $\Delta\epsilon_E$ and $\Delta\epsilon_V$. The energy- and overlap-modulated lineshapes in Fig. 3.2, $(\delta L/\delta E_{\text{ex}})$ and $(\delta L/\delta V_{\text{ex}})$, have relative strengths of $1/\Gamma_{\text{ex}}$ and 1 respectively. Hence the ratio of the amplitudes of $\Delta\epsilon_V$ and $\Delta\epsilon_E$ can be found by dividing Eq. 3.10 by 3.8, to give

$$(\Delta\epsilon_V / \Delta\epsilon_E) = S_{V/E} \Gamma_{\text{ex}} \quad \dots 3.11$$

$$\text{where } S_{V/E} = (\delta V_{\text{ex}}/\delta F) / \{ V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F) \} \quad \dots 3.12$$

The quantity $S_{V/E}$, defined by Eq. 3.11, is plotted in Fig. 3.5 for the E1H1 transition of the 50, 75, 100, 150 and 200 Å GaAs/Al_{0.3}Ga_{0.7}As QW. It can be seen that for each of the well widths, the ratio $S_{V/E}$ changes from some zero-field value to approximately the same high-field value. Notice that the strength of $\Delta\epsilon_V$ will be largest in wide wells at low fields. The ratio $S_{V/E}$ is not shown for the E1L1 transitions in Fig. 3.5 for the sake of clarity, but it shows a similar well width and field dependence to E1H1, with slightly smaller absolute values.

The strength of $\Delta\epsilon_V$ relative to $\Delta\epsilon_E$ will increase with the linewidth, according to Eq. 3.11. However, even when the HWHM linewidth is as large as 10 meV, $\Delta\epsilon_E$ will be several times larger than $\Delta\epsilon_V$, for nearly all the well widths and fields considered in Fig. 3.5. The low temperature PC spectrum shown in Fig. 5.1, for a 59 Å GaAs/Al_{0.3}Ga_{0.7}As QW, has a E1H1 linewidth (HWHM) of 2.5 meV. For this case, $\Delta\epsilon_E$ will be many times larger than $\Delta\epsilon_V$.

The dominance of $\Delta\epsilon_E$ over $\Delta\epsilon_V$ derives from the large energy shifts produced by the QCSE, which can be many times the exciton linewidth. This can be argued qualitatively from the observation that a change in the exciton energy of $\Delta E_{\text{ex}} = -\Gamma_{\text{ex}}$, will produce a contribution to $\Delta\epsilon$ of similar strength as the complete quenching of the exciton, ie $\Delta V_{\text{ex}} = -V_{\text{ex}}$.

3.2.4 Higher Subband Transitions

The Stark shifts for the higher quasi-confined states are

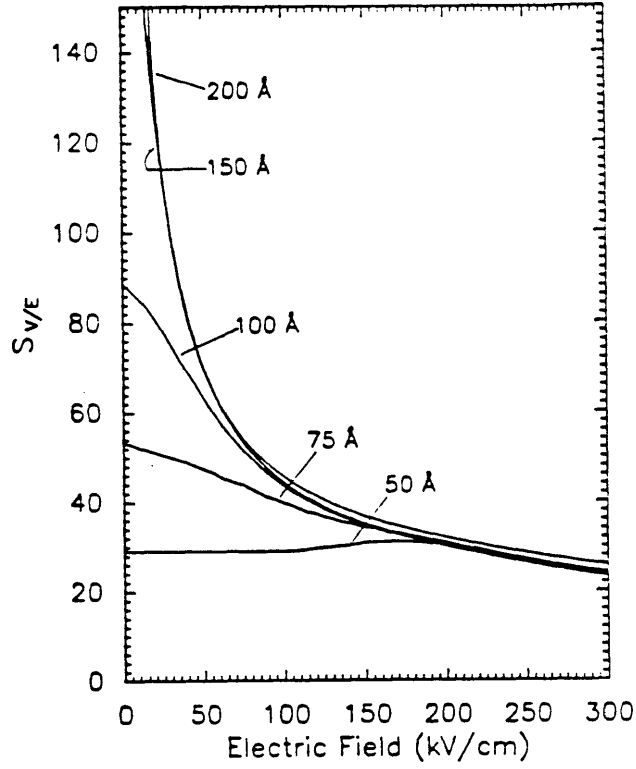


Fig. 3.5 : Calculated field dependence of $S_V/E = (\delta E_{ex}/\delta F) / \{V_{ex} \cdot (\delta E_{ex}/\delta F)\}$ for the E1H1 transition of a 50, 75, 100, 150 and 200 Å GaAs/ $Al_{0.3}Ga_{0.7}As$ QW. This quantity determines the relative strength of $\Delta\epsilon_V$ compared to $\Delta\epsilon_E$, see Eqs. 3.11-12.

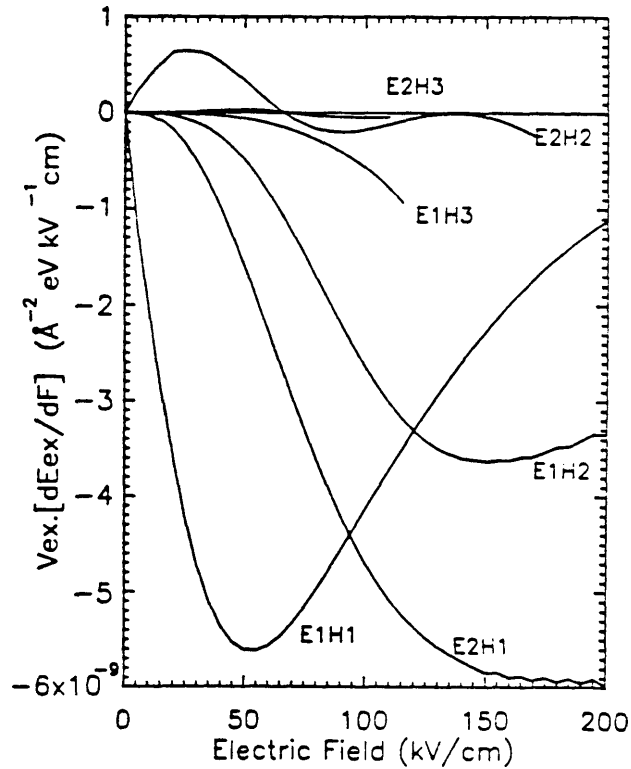


Fig. 3.6 : The field dependence of $V_{ex} \cdot (\delta E_{ex}/\delta F)$ for transitions between the electron and heavy hole subbands of a 100 Å GaAs/ $Al_{0.3}Ga_{0.7}As$ QW. This quantity determines the strength of $\Delta\epsilon_E$, see Eq. 3.8.

typically smaller than those for the $N = 1$ state, see Fig. 2.20, calculated for a $100 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW. This will diminish the strength of $\Delta\epsilon_E$ for the higher subbands transitions compared to E1H1. Meanwhile the z-motion overlap integral has a more complicated variation with field for the higher subband transitions, as shown in Fig. 2.22. Figs. 3.6 and 3.7 plot $V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)$ and $(\delta V_{\text{ex}}/\delta F)$ respectively, for the exciton transitions between the different electron and heavy hole quasi-confined states of a $100 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW. Hence the curves in Figs. 3.6 and 3.7 will be proportional to the strengths of $\Delta\epsilon_E$ and $\Delta\epsilon_V$, given by Eqs. 5.8 and 5.10, for the higher subband transitions. The curves are not plotted above the field where the narrow resonance approximation is no longer valid, by the criterion of Sec. 2.6.1.4. It should be remembered that V_{ex} will be significantly altered by the valence subband mixing for the higher states, as discussed in Sec. 2.4.1, which is ignored in these calculations.

Firstly consider the modulation of the parity allowed E2H2 transition. Fig. 3.6 shows that $V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)$ is initially positive for the E2H2 transition, but then turns negative at higher fields. Notice that the absolute value of $V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)$ is typically much smaller for E2H2 than that for E1H1, which is due to its smaller Stark shift.

Considering now the variation of $(\delta V_{\text{ex}}/\delta F)$ in Fig. 3.7 for E2H2, it can be seen that this is initially negative at low field, with its absolute value increasing to a maximum at some field, before decreasing again with further field. This is similar to the variation for the other allowed transition; E1H1. However, notice that $(\delta V_{\text{ex}}/\delta F)$ for E2H2 becomes positive at some field (about 140 kV/cm), due to the field-induced coincidence of local maxima in the E2 and H2 envelope functions. If the E2H2 exciton was not rapidly broadened at fields of around 160 kV/cm, making the narrow resonances approximation invalid, $(\delta V_{\text{ex}}/\delta F)$ would be seen to turn negative again with further field and eventually tend towards zero.

Since $M_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)$ is considerably weaker for E2H2 than E1H1, the contribution of $\Delta\epsilon_V$ to the modulated dielectric function is likely to be much more significant for the E2H2 transition. The contribution of $\Delta\epsilon_V$ will be further enhanced by the linewidth of the E2H2 transition being larger than that for E1H1 [1.12]. Consequently, the E2H2 electromodulated lineshape can be expected to be determined by the overlap-modulated lineshape in Fig. 3.2.4.1-2 $(\delta L/\delta V_{\text{ex}})$ and resemble the unmodulated dielectric function. However, there may be a complicated evolution of the lineshape with field, since $(\delta V_{\text{ex}}/\delta F)$ will be zero at some fields, so that the lineshape may then resemble $(\delta L/\delta E_{\text{ex}})$. When

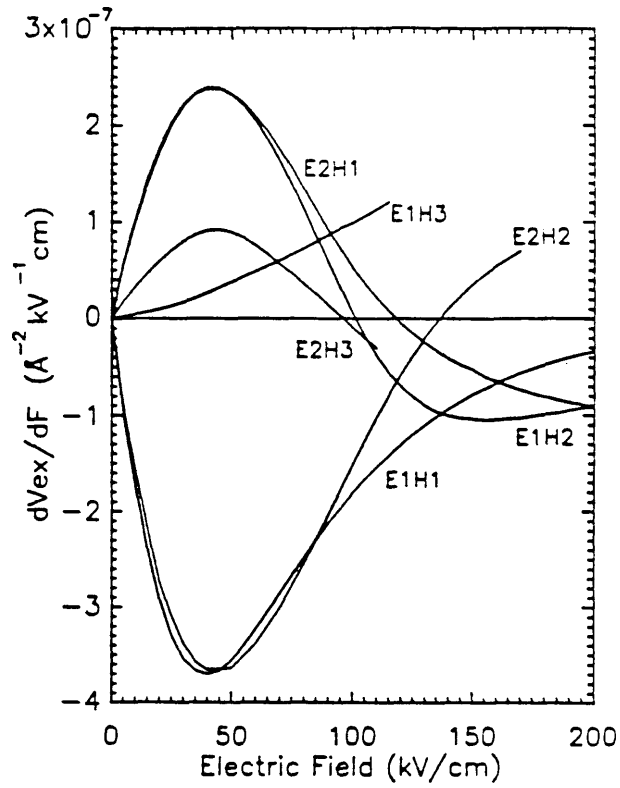


Fig. 3.7 : The calculated field dependence of $(\delta V_{ex}/\delta F)$ for the transitions between the electron and heavy hole subbands of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW. This quantity determines the strength of $\Delta\epsilon_V$, see Eq. 3.10.

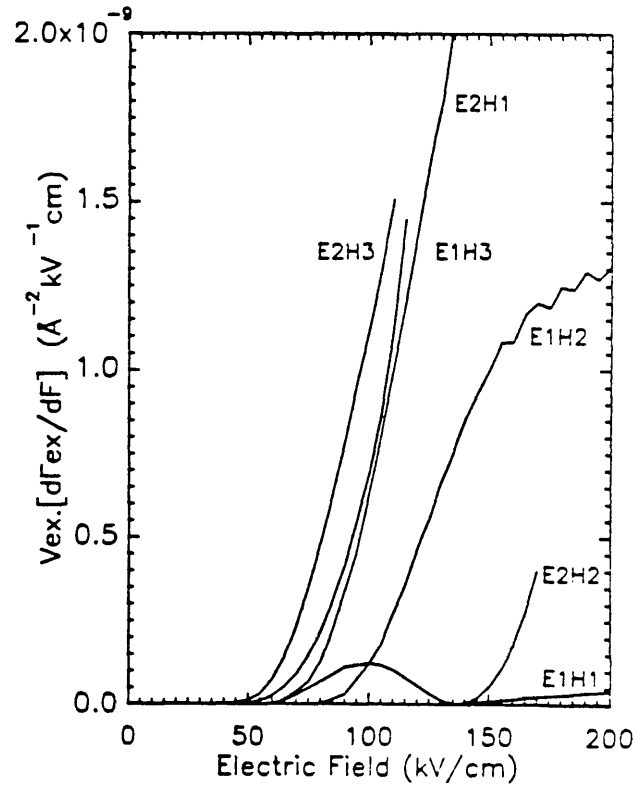


Fig. 3.8 : The field dependence of $V_{ex} \cdot (\delta \Gamma_{ex}/\delta F)$ for transitions between the electron and heavy hole subbands of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW. This quantity determines the strength of $\Delta\epsilon_T$, see Eq. 3.9.

the field is increased further, the lineshape will again resemble $(\delta L/\delta V_{ex})$, but will have an opposite sign to that at lower fields. It is possible that a large increase in the E2H2 linewidth with field might cause $(\delta L/\delta \Gamma_{ex})$ to dominate over both $(\delta L/\delta E_{ex})$ and $(\delta L/\delta V_{ex})$, as discussed below in Sec. 3.2.5.

For the forbidden E1H2 transition of this QW, the Stark shift is negative at all fields, and again the value of $|V_{ex} \cdot (\delta E_{ex}/\delta F)|$, in Fig. 3.6, increases to a maximum, although at a higher field than for E1H1, before falling back towards zero. Meanwhile, $\delta V_{ex}/\delta F$, in Fig. 3.7, is positive at the lowest fields, for this forbidden transition, before becoming negative at higher fields, when the H2 envelope function is confined near the side of the well. For the E1H2 transition, $\Delta\epsilon_V$ will be larger than $\Delta\epsilon_E$ at low fields, where the overlap is small and increases rapidly with field. It can be inferred from Figs. 3.6 and 3.7 that $\Delta\epsilon_E$ becomes stronger at moderate fields (around $F = 100$ kV/cm), while $\Delta\epsilon_V$ becomes weaker.

3.2.5 Linewidth-Modulation

The increase in the transition linewidth with field, due to the tunnelling of carriers from the well, will result in a linewidth-modulated contribution to $\Delta\epsilon$. Fig. 3.8 plots $V_{ex} \cdot (\delta \Gamma_{ex}/\delta F)$ for the ENHM transitions of a 100 Å GaAs/Al_{0.3}Ga_{0.7}As QW, which is the coefficient determining the strength of $\Delta\epsilon_T$ in Eq. 3.9. The relative strengths of $\Delta\epsilon_T$ and $\Delta\epsilon_E$ can be deduced by dividing Eq. 3.9 by Eq. 3.8, to yield

$$(\Delta\epsilon_T / \Delta\epsilon_E) = S_{T/E} = (\delta \Gamma_{ex}/\delta F) / (\delta E_{ex}/\delta F) \quad \dots 3.13$$

Comparison of Fig. 3.8 with Figs. 3.6 and 3.7 shows that $\Delta\epsilon_T$ will become significant at the fields, where either the electron or hole resonance is rapidly broadened. However, note that the rapid increase in Γ_{ex} will also result in the transition becoming unresolvable.

For the transitions between the lowest electron and hole subbands, the linewidth due to the field-induced tunnelling is typically negligibly small compared to that due to inhomogeneous mechanisms. The field dependence of the low temperature E1H1 linewidth of four MOVPE SQWs is measured in Chp. 5 and plotted in Fig. 5.15. It can be seen that the linewidth increases most rapidly with fields for the widest wells. The relative size of $\Delta\epsilon_T$ and $\Delta\epsilon_E$ for these SQWs can be deduced from comparisons of the increases in the linewidths (Fig. 5.15) and

decreases of the transition energies (Fig. 5.12) with electric field. Clearly the Stark shifts are much larger than the changes in linewidth for each of the four SQWs, so that the amplitude of $\Delta\epsilon_E$ will be at least five times larger than that of $\Delta\epsilon_I$.

It was pointed out earlier that the curves for $(\delta L/\delta E_{ex})$ and $(\delta L/\delta \Gamma_{ex})$ in Fig. 3.2 are similar in shape, but with their real and imaginary parts interchanged. In fact this is generally true for any lineshape that can be expressed as $L(E_{ex}-E+i\Gamma_{ex})$. The Lorentzian in Eq. 2.48 can be written in this form by ignoring the pole at $E = -E_{ex}$. Differentiation of this general form for the lineshape produces the equalities

$$(\delta L/\delta E_{ex}) = i (\delta L/\delta \Gamma_{ex}) = - (\delta L/\delta E) \quad \dots 3.14$$

Hence $(\delta L/\delta E_{ex})$ is the negative of the differential of the lineshape with respect to the photon energy, $(\delta L/\delta E)$. Meanwhile $(\delta L/\delta \Gamma_{ex})$ is also given by the first-energy-differential, but with a $\pi/2$ phase factor in the argand diagram. Hence the combined contribution of the energy- and linewidth-modulated terms can be expressed as

$$\Delta\epsilon_E + \Delta\epsilon_I = - S_0 V_{ex} (\delta L/\delta E) (\delta E_{ex}/\delta F) \Delta F (1 - i S_I/E) \quad \dots 3.15$$

Therefore the effect of the linewidth-modulation is to mix the contribution of the real and imaginary parts of $(\delta L/\delta E)$ to $\Delta\epsilon_r$ and $\Delta\epsilon_i$. It will be shown in Sec. 3.3, that the ER is a linear mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$. Hence the effect of the field-induced change in the linewidth, will be merely to alter the coefficients of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ in the ER lineshape. However the ER will still be determined by $\text{Real}[(\delta L/\delta E)]$ and $\text{Imag}[(\delta L/\delta E)]$.

Sec. 3.2 has demonstrated that $\Delta\epsilon$ can be assumed to be proportional to $(\delta L/\delta E)$ for the E1H1 and E1L1 transitions in typical QWs. The contribution of $\Delta\epsilon_V$ has been shown to be much smaller than $\Delta\epsilon_E$, being most significant in wide wells at low fields and when the linewidth is large. Meanwhile, $\Delta\epsilon_I$ is also typically smaller than $\Delta\epsilon_E$, although, since $\Delta\epsilon_I$ and $\Delta\epsilon_E$ have a similar form, except with their real and imaginary parts interchanged, the effect of the change in linewidth is just to mix $\text{Real}[(\delta L/\delta E)]$ and $\text{Imag}[(\delta L/\delta E)]$ in $\text{Real}[\Delta\epsilon]$ and $\text{Imag}[\Delta\epsilon]$.

3.3 Modulation Spectroscopy of Single Quantum Wells :

Optical Interference Effects

Having calculated the modulated QW dielectric function, this section now seeks to relate this to the modulated reflectance and transmission, ie the ER and ET. A QW sample necessarily contains several interfaces, and the overall reflectance is determined by optical interference of the rays from each. The reflectance of layered systems can be modelled using an optical transfer matrix method [3.34], which can be used to calculate ER spectra of QW structures [2.46]. However this is a numerical method, which allows little insight into the importance of each parameter.

A clearer understanding of the problem can be gained by analytically considering only the most important interfaces in the structure. The reflection at the semiconductor/air interface is much stronger than any other in the sample, and consequently only this ray need be considered along with those from the QW interfaces. Sec. 3.2.1 considers the interference of the rays reflected from the two interfaces of the QW, in order to calculate its reflectance in terms of its dielectric function. Then this QW reflectance is considered along with that of the semiconductor/air interface, in order to deduce the overall sample reflectance in Sec. 3.3.2. This can then be differentiated to deduce a ER lineshape formula (Sec. 3.3.3), whose nature is discussed for the transitions between the lowest electron and hole subbands (Sec. 3.3.4).

Both parts of this derivation use the expressions for the reflectivity (r) and transmission (t) of the etalon formed between two parallel interfaces with individual amplitude reflectivities r_1 and r_2 and transmissions t_1 and t_2

$$r = (r_1 + r_2 e^{ig}) / (1 + r_1 r_2 e^{ig}) \quad \dots 3.16$$

$$\text{and} \quad t = t_1 t_2 e^{i\frac{1}{2}g} / (1 + r_1 r_2 e^{ig}) \quad \dots 3.17$$

$$\text{where} \quad g = 2 k L n \cos(\theta) \quad \dots 3.18$$

The quantity g represents the optical phase delay between the rays reflected from the two interfaces, which are separated by a medium of refractive index n and thickness L . The angle between the direction of light propagation in this medium and the normal to the interfaces is θ , and k is the wavevector of the light in vacuum.

3.3.1 Reflectance and Transmission of an Isolated Quantum Well

The two interfaces of a QW with its barrier material, form an optical etalon, as shown in Fig. 3.9. If the two barrier interfaces are assumed to be identical, $r_2 = -r_1$ and $t_1 \cdot t_2 = (1 - r_1^2)$. Substituting these relations into the Eqs. 3.16 and 3.17 yields the amplitude reflectance and transmission of the QW to be given by

$$r_w = r_1 \{ 1 - \exp(ig_w) \} / \{ 1 - r_1^2 \exp(ig_w) \} \quad \dots 3.19$$

$$t_w = \{ 1 - r_1^2 \} \exp(i \frac{1}{2} g_w) / \{ 1 - r_1^2 \exp(ig_w) \} \quad \dots 3.20$$

$$\text{where } g_w = 2 k L_w n_w \cos \theta \quad \dots 3.21$$

If the refractive index of the well and barrier regions are written as n_w and n_b respectively, then the reflectivity of the QW interface is given by Fresnel's relation $r_1 = (n_b - n_w)/(n_b + n_w)$. In this model the refractive index of the well is regarded as being equal to that of the barriers plus a contribution due to the optical transitions of the QW. If it is assumed that $|(n_w - n_b)/2n_b| \ll 1$, or equivalently $|r_1| \ll 1$, Eqs. 3.19-20 can be further simplified by ignoring the second-order terms in r_1 . Hence,

$$r_w \approx i (\epsilon_w - \epsilon_b) \sin(\frac{1}{2} g_w) \exp(i \frac{1}{2} g_w) / 2 n_b^2 \quad \dots 3.22$$

$$t_w \approx \exp(i \frac{1}{2} g_w) \quad \dots 3.23$$

where ϵ_w and ϵ_b are the dielectric functions of the well and barrier respectively ($\epsilon = n^2$). The approximation, $|r_1| \ll 1$, is not crucial to the derivation of the lineshape formulae, however it allows the QW reflectance and transmission to be written as having a linear dependence on the QW dielectric function, so lending a more immediate understanding of the form of the lineshape.

The validity of this approximation can be judged by considering the ER of MV302, a 59 Å GaAs/Al_{0.3}Ga_{0.7}As SQW sample (MV302), whose layer details are listed in Chp. 5. Fig. 3.1 shows ER and PC spectra taken on this sample at a temperature of 17 K, under an offset field of 47 kV/cm. Masumoto et al [2.33] reported the zero-field integrated absorption strength (A_{ex}) of a 59 Å QW to be about 430 cm⁻¹eV. If the exciton absorption peak is assumed to have a Gaussian profile, with a HWHM of Γ_{ex} , then the value of the peak absorption will be $(\ln 2/\pi)^{1/2} \cdot (A_{ex}/\Gamma_{ex})$. Taking $\Gamma_{ex} = 2.5$ meV, the HWHM of the E1H1 peak in Fig. 3.1, yields a peak absorption of 81000 cm⁻¹. This is equivalent to

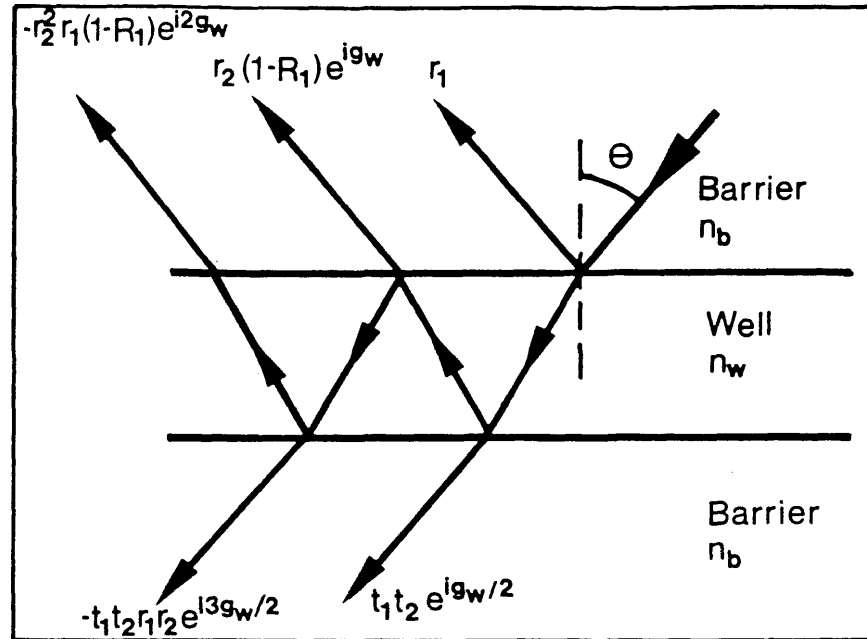


Fig. 3.9 : Schematic drawing of the interfering rays reflected from the two interfaces of an isolated quantum well.

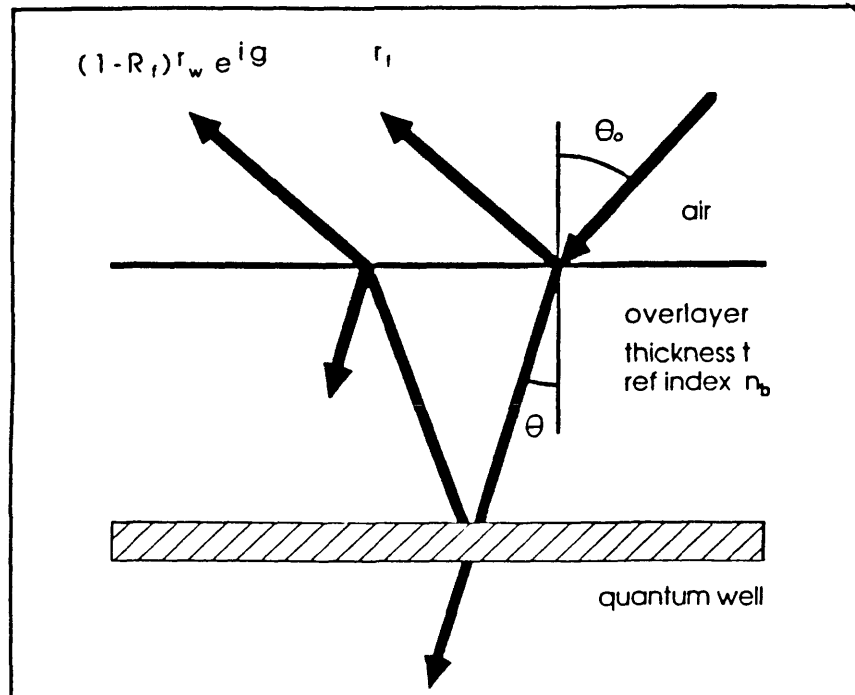


Fig. 3.11 : The most important rays to be considered in calculating the modulated reflectance of a SQW structure; those from the QW and the semiconductor/air interface at the front surface. The other interfaces of the sample can be ignored. The second (and higher) reflections of the QW can also be neglected. The layers between the QW and front surface are replaced by an effective overlayer, as discussed in text.

$\text{Imag}[n_w] = 0.5$ at the exciton peak, and hence $|r_1| = 0.07$. Therefore it is reasonable to assume $|r_1| \ll 1$, and ignore the second-order terms in $|r_1|$, even at the peak energy where r_1 will be largest. Furthermore, since the integrated absorption strength decreases with the well width, this approximation is likely to be even better for the other SQWs studied in Chp. 5, which have wider wells. The peak absorption will also be smaller at room temperature, due to the larger linewidths. Taking a Lorentzian profile, with the same values of Γ_{ex} and A_{ex} , would further reduce the peak height. Therefore the situation considered here is the worst case, and even then the approximation is very good.

Fig. 3.10.1 plots the calculated real and imaginary parts of the amplitude reflectance of a 59 Å QW. The QW dielectric function, shown in Fig. 3.10.2, is taken to be comprised of two Gaussian broadened 1s excitons, due to E1H1 and E1L1, which make a contribution to the QW dielectric function given by Eqs. 3.1-4. The exciton energies and linewidths were taken from the -2 V PC spectrum of MV302 in Fig. 3.1. The exciton overlaps (V_{ex}) were taken to have the values calculated for a 59 Å GaAs/Al_{0.3}Ga_{0.7}As QW at this field. The constant S_0 in Eq. 3.1, for E1H1, was deduced from the integrated zero-field absorption strength of 430 cm⁻¹eV, reported by Masumoto et al [2.33] for this well width. The value of S_0 for E1L1 was taken to be a third that for E1H1, due to the ratio of the squares of the optical matrix elements (P_{CV}^2), as discussed in Sec. 2.3.

The QW reflectance curves labelled "a" were calculated using Eq. 3.22. Also shown is an exact solution (labelled "b"), considering the multiple reflections across the QW, calculated using Eq. 3.19 and Fresnel's reflectivity formula for r_1 . It can be seen that the discrepancies between curves "a" and "b" are small, so further justifying the neglect of the terms in r_1^2 . Notice that the overall QW reflectance is small, $|r_w| < 0.02$, and less than the reflectance of either of the two interfaces, $|r_1|$. This is because the reflection from the front interface of the well partially cancels with that from the back, if the well is much thinner than the wavelength of the light in the semiconductor (approximately 2400 Å for GaAs/AlGaAs QW bandgap). Notice also that $\text{Real}[r_w]$ resembles $-\text{Imag}[\epsilon_w]$, while $\text{Imag}[r_w]$ resembles $\text{Real}[\epsilon_w]$, which is in accordance with Eq. 3.22, since $\exp(i \frac{1}{2} g_w)$ will be close to unity for narrow wells.

By making the additional assumption that the QW is much thinner than the wavelength of light in the material, third-order terms in g_w and higher can be ignored and hence Eq. 3.22 further simplified to

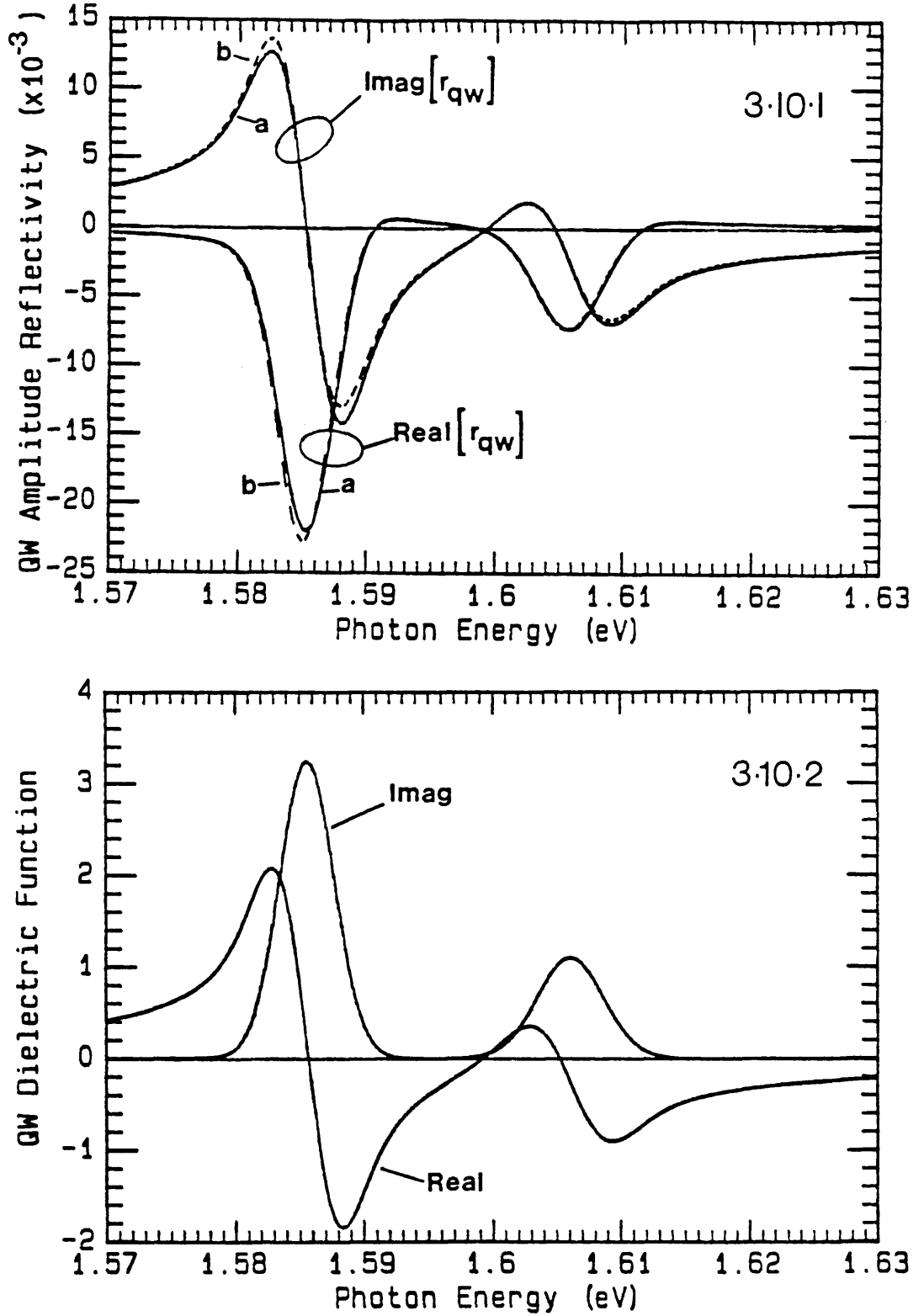


Fig. 3.10 : The real and imaginary parts of the amplitude reflectance (3.10.1) of an isolated QW (59 Å), calculated using either Eq. 3.22 (curve "a") or else the more exactly with Eq. 3.19 (curve "b"). Also shown are the real and imaginary parts of the QW dielectric function used for the calculation (3.10.2). This takes two Gaussian broadened excitons with the parameters relevant to the E1H1 and E1L1 excitons in the PC spectra of Fig. 3.1; for E1H1 $E_{\text{ex}} = 1.5856$ eV, $\Gamma_{\text{ex}} = 2.5$ meV, $V_{\text{ex}} = 4.294 \times 10^{-5}$ Å⁻², $S_0 = 420.3$ Å²eV; while for E1L1 $E_{\text{ex}} = 1.6062$ eV, $\Gamma_{\text{ex}} = 3.0$ meV, $V_{\text{ex}} = 5.346 \times 10^{-5}$ Å⁻², $S_0 = 140.1$ Å²eV.

$$r_w \approx i (k L_w \cos \theta / 2 n_b) (\epsilon_w - \epsilon_b) \exp(i \frac{1}{2} g_w) \quad \dots 3.24$$

Eq. 3.24 can also be derived by regarding the QW as an infinitely thin polarisable medium and applying Maxwell's boundary conditions, as done by Klipstein and Apsley [2.46]. In fact allowing a finite thickness for the QW has little effect upon the lineshape, due to the variation in g_w being very slow over the range of energies of the transition.

3.3.2 Reflectance of a Single Quantum Well Structure

This section derives an expression for the reflectance of a SQW structure, and it is again illustrated by calculations made using the parameters of sample MV302. Table 3.1 lists the thickness and refractive indices of the layers in MV302, whose growth specification is listed in Chp. 5. The GaAs(15 Å)/AlGaAs(15 Å) SLs included in the layer specification are regarded as 195 Å wide $\text{Ga}_{0.15}\text{Al}_{0.85}\text{As}$ layers. The indices were taken as the values measured by Aspnes et al [3.35] for $\text{Al}_x\text{Ga}_{1-x}\text{As}$, at the energy of the E1H1 transition of MV302 at room temperature.

x	thickness (μm)	index
0.35	0.96	3.379
0.3	0.38	3.407
0.15	0.0195	3.506
0.3	0.0050	3.407
0.0	0.0059	-----
0.3	0.0050	3.407
0.15	0.0195	3.506
0.3	0.38	3.407
0.35	0.96	3.379
0.0	0.3	3.657
0.0	substrate	3.657

Table 3.1 : Thicknesses and refractive indices of the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers in MV302. The indices are those measured in Ref. 3.35. The layers between the QW and front surface can be replaced by an effective overlayer, which is 1.3645 μm thick of average index 3.389.

The reflectivity of the front surface will be much larger than any other interface in the sample, due to the large discontinuity in the refractive index. The ER of SQWs can therefore be calculated, to a very good approximation, by considering only the ray reflected from the front surface along with those from the QW. In this analysis, the combined reflectance of the two interfaces of the QW, is taken as that deduced in

Sec. 3.3.1. Hence the SQW sample can be analysed as an etalon formed between the QW and the front surface, as depicted in Fig. 3.11. An effective overlayer is taken between the QW and front surface, which has the same optical thickness as the individual layers that it replaces. For the layer details listed in Table 3.1, the total overlayer thickness is given by $\Sigma L = 1.3645 \mu\text{m}$, which is taken to have the average refractive index $(\Sigma n.L)/(\Sigma L) = 3.389$.

It was demonstrated in Sec. 3.3.1 that the QW reflectivity is typically very small, $|r_w| < 0.02$ for the QW considered here, and consequently only the first ray reflected from the QW need be considered, yielding

$$R \approx R_f + 2 (1-R_f) \text{Real} [r_f r_w \exp(ig_o)] \quad \dots 3.25$$

where g_o is the phase delay for the round-trip across the overlayer between the top surface and the well. The front surface amplitude and intensity reflectivity are written as r_f and R_f respectively. Note that r_f will be negative. This approximation is equivalent to taking only the first-order terms in r_w .

The three curves plotted in Fig. 3.12 show the intensity reflectance calculated for MV302, calculated with different lineshape models. Curve "a" shows the reflectance calculated using Eq. 3.25. This approximates the layers between the front surface and the QW by an effective overlayer, and takes only the first reflected ray from the QW. The calculation takes r_w in Eq. 3.25 as that shown by curve "b" in Fig. 3.11. Fig. 3.12 also shows a calculation of the SQW reflectance (curve "b"), which again assumes an effective overlayer, but this time takes all the multiple reflections inside the etalon formed by the QW and the front surface, using Eq. 3.16. Comparison of curves "a" and "b" shows that ignoring the double (and higher) reflections from the QW introduces only a small error in the overall SQW reflectance.

The reflectance of the SQW structure can also be calculated using an optical transfer matrix method [2.46,3.34]. Using this method to calculate the SQW reflectance, with a homogeneous effective overlayer between the QW and front surface, exactly reproduced curve "b". This is to be expected, of course, since the two calculations merely use a different method to solve the same problem. However, it demonstrates the calculating routines to be accurate.

The SQW reflectance was also calculated using the optical transfer method and using the complete layer details listed in Table 3.1, and this is plotted as curve "c" in Fig. 3.12. It can be seen that curve

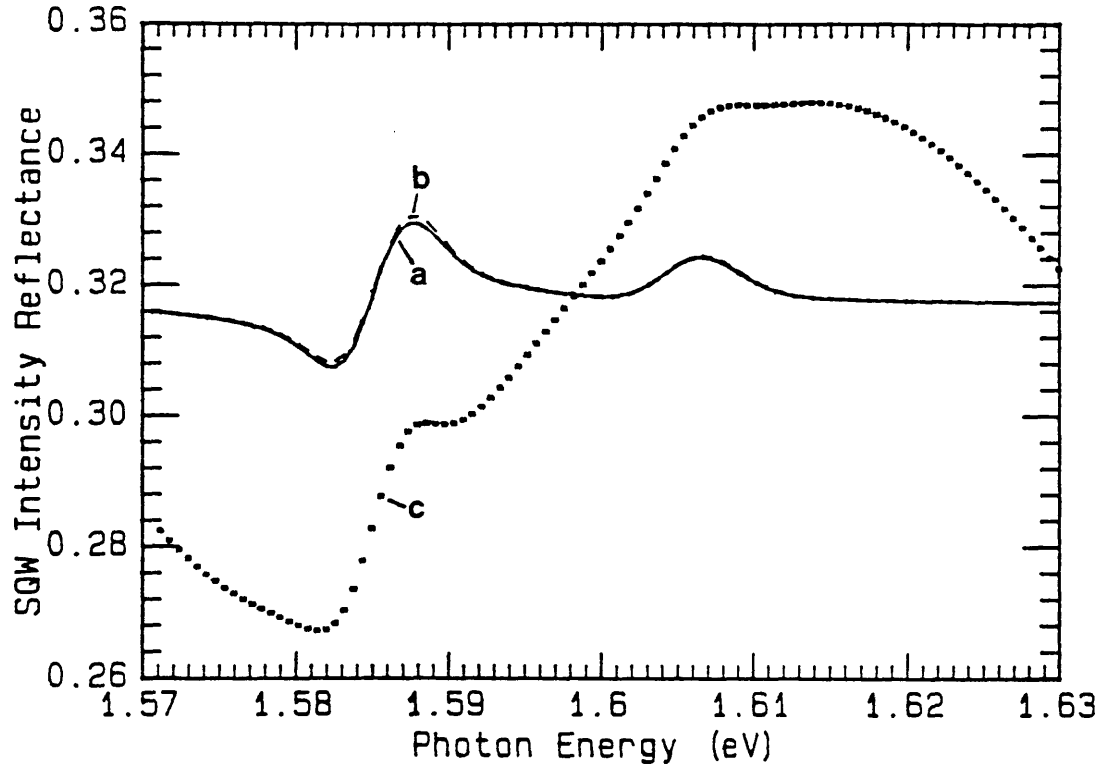


Fig. 3.12 : The intensity reflectance of the MV302 (59 Å) SQW structure, calculated using different approximations. The analysis for curve "a" considered an effective overlayer ($L = 1.3645 \mu\text{m}$, $n = 3.389$) and took just the first ray from the QW, ie Eq. 3.25. Curve "b" again considered an effective overlayer, but this time took all the rays reflected between the QW and the front surface, using Eq. 3.16. Curve "c" considered the rays from all the interfaces of the sample structure, details of which are listed in table 3.1, and uses a transfer matrix method. The QW dielectric function was regarded as consisting of a E1H1 and E1L1 exciton term, identical to those in Fig. 3.10.

"c" shows similar shaped features to curves "a" and "b" at the exciton energies, but these are superimposed on broad oscillations. These oscillations derive from the etalons formed between the front surface and the interfaces of the sample other than those at the QW. The large refractive index mismatch ($\Delta n = 0.28$) between the GaAs buffer and $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As } n^+$ layer is especially important. The reflectivities of some of the buried interfaces are comparable to, or larger than, that of the QW. They therefore producing oscillatory structure in the overall reflectance as large as the features due to the QW, so that Eq. 3.25 does not accurately determine the reflectance. However, the reflectivities of these other interfaces are not modulated by the electric field, and so the differential of Eq. 3.25 will accurately determine the ER, as demonstrated below.

3.3.3 Electroreflectance of a Single Quantum Well Structure

The modulated intensity reflectance (ie ER) can be found by substitution of Eq. 3.22 for r_w into Eq. 3.25 and then differentiating, to yield

$$\Delta R \approx B \{ \Delta \epsilon_r \sin(g) + \Delta \epsilon_i \cos(g) \} \quad \dots 3.26$$

$$\text{where} \quad B = -2 (1-R_f) r_f \sin(\tfrac{1}{2} g_w) / 2 n_b^2 \quad \dots 3.27$$

$$\text{and} \quad g = g_o + \tfrac{1}{2} g_w \quad \dots 3.28$$

$\Delta \epsilon_{r(i)}$ is the change in the real (imaginary) part of the QW dielectric function produced by the perturbation, that was discussed in Sec. 3.2. The sign of the ER, and hence $\Delta \epsilon$, is defined by the high field value minus that at the lower field of the modulation, ie $\Delta R = R(F+\Delta F)-R(F)$, $\Delta F > 0$. The phase factor g , which will be referred to as the "lineshape phase", is the round-trip phase delay for propagation between the front surface and the midpoint of the QW. Further discussion of this derivation is given in Refs. 1.12 and 1.13.

Fig. 3.13 shows the calculated ER spectrum, using the same parameters as in the reflectance of Fig. 3.12, which are those relevant to the PC spectrum of MV302 shown in Fig. 3.1. The ER in Fig. 3.13 has again been calculated using different approximations (labelled "a", "b" and "c"), corresponding to those used in generating the reflectance in Fig. 3.12. The three curves plotted correspond to calculations considering : ("a") an effective overlayer and only the first ray reflected from the QW, as in Eqs. 3.26-28; ("b") an effective overlayer

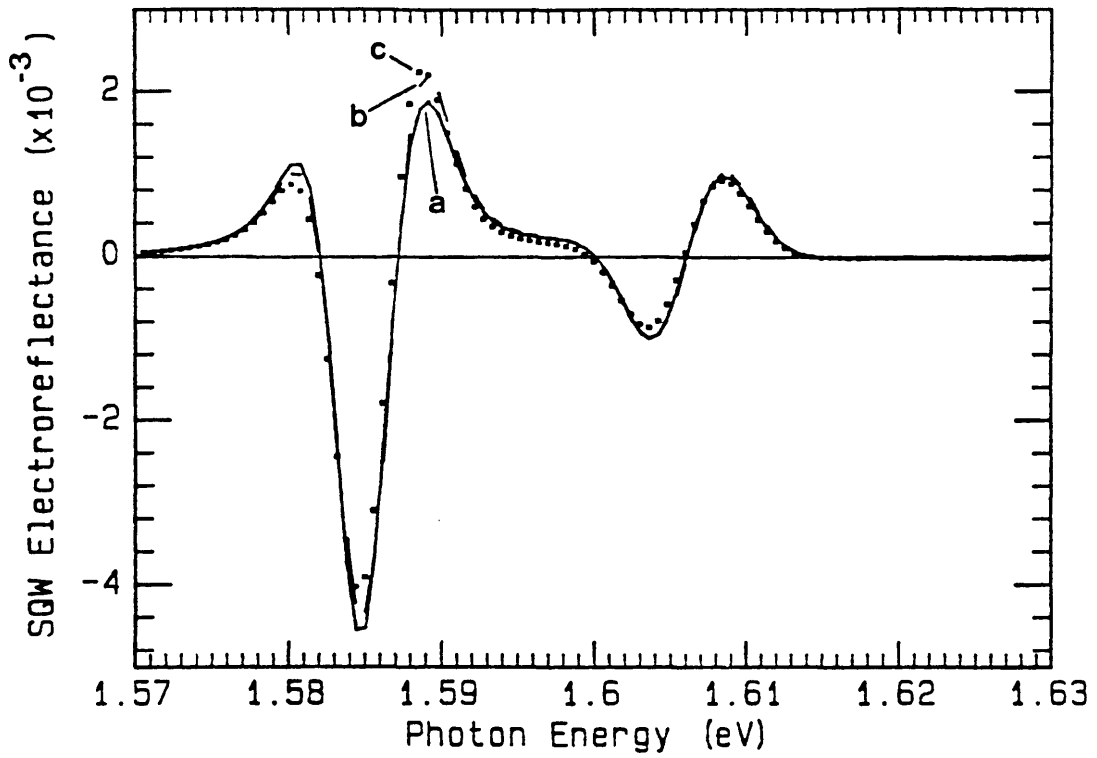


Fig. 3.13 : Calculated ER spectra for sample MV302. The curves "a", "b", and "c" employed the different approximations described in caption of Fig. 3.12. The curves were generated using the modulations :
 E1H1 $\Delta E_{\text{ex}} = 0.70$ meV, $\Delta \Gamma_{\text{ex}} = 0.12$ meV, $\Delta V_{\text{ex}} = 1.07 \times 10^{-6} \text{ \AA}^{-2}$;
 E1L1 $\Delta E_{\text{ex}} = 0.69$ meV, $\Delta \Gamma_{\text{ex}} = 0.12$ meV, $\Delta V_{\text{ex}} = 1.27 \times 10^{-6} \text{ \AA}^{-2}$.

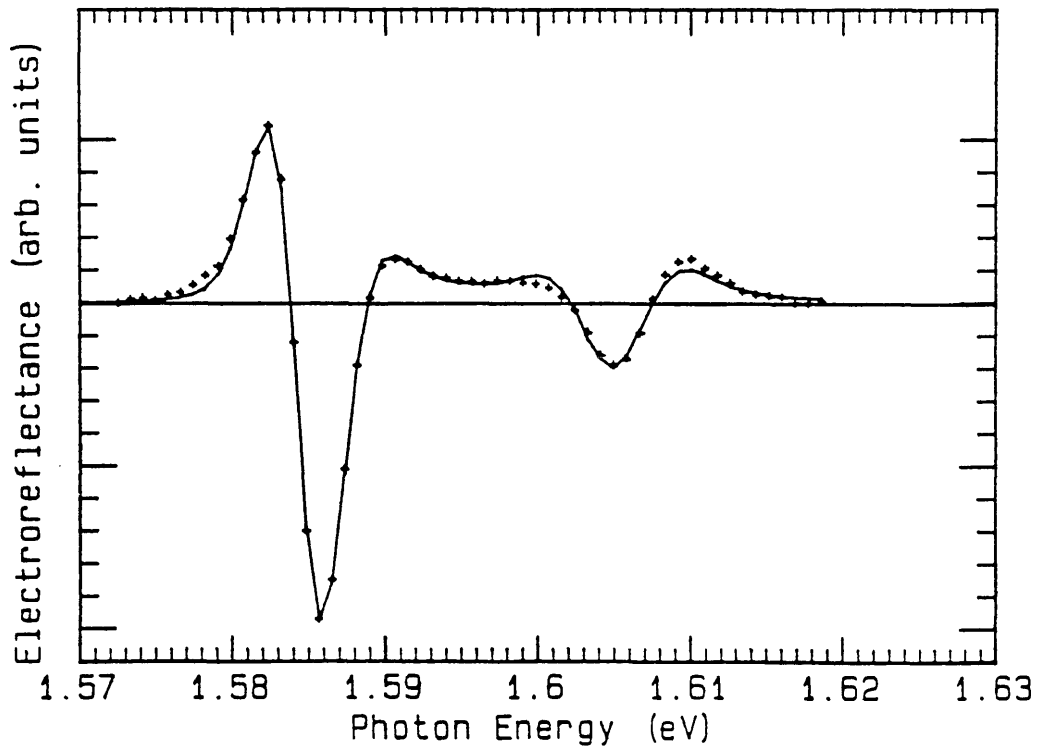


Fig. 3.14 : The points (+) reproduce the experimental ER spectrum of MV302 plotted in Fig. 3.1. The solid line is a least squares fit of the lineshape model producing curve "a" in Fig. 3.13, with fitted parameters
 $E_{\text{ex}}(\text{E1H1}) = 1.5855$ eV, $\Gamma_{\text{ex}}(\text{E1H1}) = 2.8$ meV, $E_{\text{ex}}(\text{E1L1}) = 1.6056$ eV,
 $\Gamma_{\text{ex}}(\text{E1L1}) = 2.9$ meV, $S_0(\text{E1L1})/S_0(\text{E1H1}) = 0.30$, $L = 1.3448 \text{ \mu m}$.

and all the rays reflected from the QW; and ("c") all the sample interfaces using an optical matrix method. The discrepancy between curves "a" and "b" is small, as was also the case for the non-modulated reflectance in Fig. 3.12. This time though there is also very good agreement with the optical matrix calculation "c". The broad oscillatory structure observed in curve "c" of Fig. 3.12 is not modulated by the electric field and so does not appear in the ER of Fig. 3.13. The sample interfaces that are ignored in Fig. 3.11 can be considered as slightly altering r_f and R_f . Hence they produce a variation in the first term of Eq. 3.25 (R_f) comparable to R_w , but only very slightly changing the coefficients of the second term, which determine ΔR .

Notice that the calculated ER in Fig. 3.13 is similar in shape to the experimental spectrum of Fig. 3.1. The theoretical ER was produced assuming a Gaussian lineshape and using the values of ΔE_{ex} and ΔV_{ex} , calculated for the E1H1 and E1L1 transitions in a 59 Å GaAs/ $Al_{0.3}Ga_{0.7}As$, under the experimental d.c offset field of 47 kV/cm with a peak-to-peak modulation of 12 kV/cm. The value of $\Delta \Gamma_{ex}$ was taken as that measured in the PC measurements, which is small and has little effect on the ER lineshape. The discrepancy between the calculated and measured curves derives largely from an error in the value of the optical thickness of the overlayer. The lineshape is very sensitive to the overlayer thickness, which will be discussed in Sec. 3.3.4, so this parameter must be varied in order to obtain a better agreement.

Fig. 3.14 shows a least squares fit of the lineshape model to the ER spectrum, where the overlayer thickness, the E1H1 and E1L1 exciton energies, the E1H1 and E1L1 linewidths, and the ratio of the strength of the E1L1 contribution to that of E1H1, were varied in order to find the best fit, as described in Sec. 4.4. The experimental ER spectrum was measured in arbitrary units, so a scaling factor was also allowed to vary, making a total of seven parameters. It can be seen that the lineshape model produces a good fit to the data. The small discrepancy probably arises from the exciton profile not being a true Gaussian, although the fit with a Lorentzian profile was considerably worse. The fitted energies and linewidths were close to those measured in PC spectroscopy in Fig. 3.1. Meanwhile the effective overlayer thickness is fitted as being roughly 120 Å narrower than the sample specification, which may be due to a small error in either the values of the average refractive index employed or in the growth rate, from which the layer thickness was determined.

Note that Eqs. 3.26-3.28, giving the ER lineshape in terms of the modulated dielectric function of the QW, are equally valid for any form of $\Delta\epsilon$. The legitimacy of this lineshape formula does not depend on whether the dielectric function is dominated by excitonic or continuum contributions, or how those contributions are modulated by the electric field, or indeed any other perturbation. Eqs. 3.26-3.28 will even be valid for bulk semiconductor where the region under a field is buried inside the structure, as discussed in Sec. 3.5, where $\Delta\epsilon_r$ and $\Delta\epsilon_i$ are determined by the Franz-Keldysh effect.

3.3.4 Electroreflectance Lineshape for Lowest Subband Transitions

The ER lineshape can be calculated from Eqs. 3.26-3.28, using the modulated dielectric function from Eqs. 3.8-3.10. It was deduced in Sec. 3.2, that $\Delta\epsilon_v$ is usually much weaker than $\Delta\epsilon_E$, for the E1H1 and E1L1 transitions. Meanwhile $\Delta\epsilon_r$ is also usually smaller than $\Delta\epsilon_E$, but in any case it can be easily combined with $\Delta\epsilon_E$ using Eq. 3.15. Hence

$$\Delta R \approx C \{ (\delta L_r / \delta E) \sin(g) + (\delta L_i / \delta E) \cos(g) \} \quad \dots 3.29$$

$$\text{where } C = -B S_O V_{ex} (\delta E_{ex} / \delta F) (1 + S_{\Gamma/E}^2)^{\frac{1}{2}} \Delta F \quad \dots 3.30$$

$$\text{and } g = g_O + \frac{1}{2} g_W - \tan^{-1} \{ S_{\Gamma/E} \} \quad \dots 3.31$$

	g	ΔR in Eq. 3.29	lineshape description
a	$2m \pi$	$\Delta R \propto +(\delta L_i / \delta E)$	negative-zero-crossing
b	$(2m+1/2) \pi$	$\Delta R \propto +(\delta L_r / \delta E)$	negative-peak
c	$(2m+1) \pi$	$\Delta R \propto -(\delta L_r / \delta E)$	positive-zero-crossing
d	$(2m+3/2) \pi$	$\Delta R \propto -(\delta L_i / \delta E)$	positive-peak

Table 3.2 : SQW lineshapes for different values of the lineshape phase.
(m is an integer)

Eq. 3.29 shows that the ER is a linear sum of the real and imaginary parts of the first-differential of the lineshape function with respect to the photon energy, the mixture determined by the lineshape phase g . Considering a single exciton in the dielectric function, the lineshape would be expected to follow the evolution indicated in table 3.2 and shown in Fig. 3.15, with increasing g , as the ER "rotates" between the real and imaginary parts of $(\delta L / \delta E)$. The lineshape labelled "a" in Fig. 3.15, will be referred to as a "negative-zero-crossing", since it is zero at the exciton energy with a negative gradient. Similarly lineshape "c" will be called a "positive-zero-crossing".

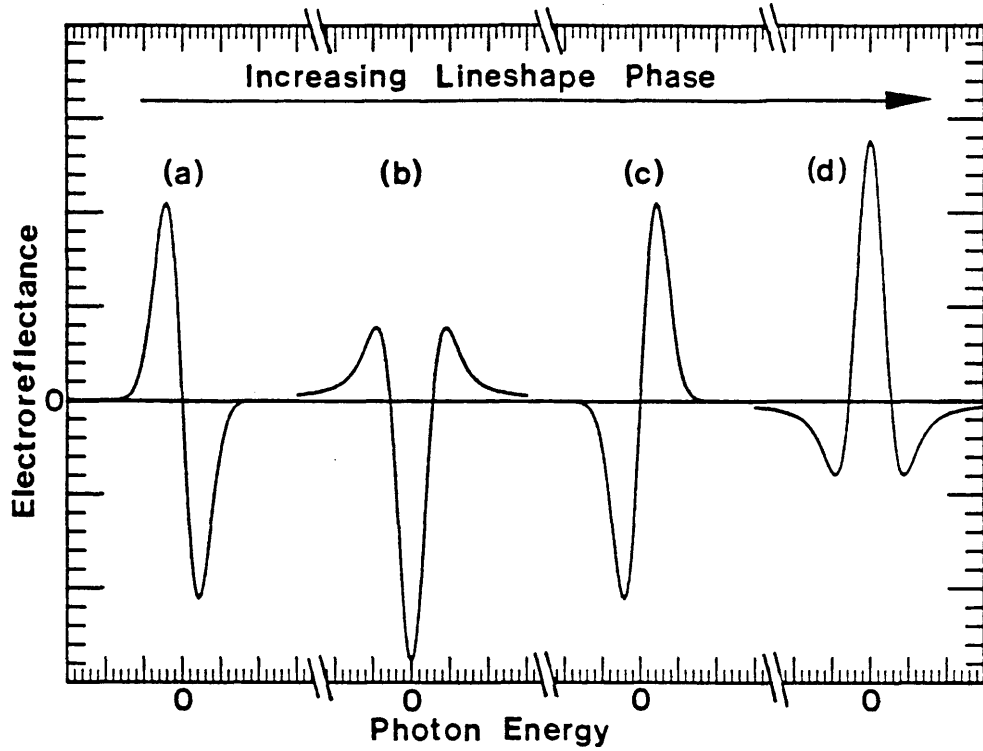


Fig. 3.15 : The evolution of the ER lineshape of a single exciton in a SQW, produced by a variation in the lineshape phase.

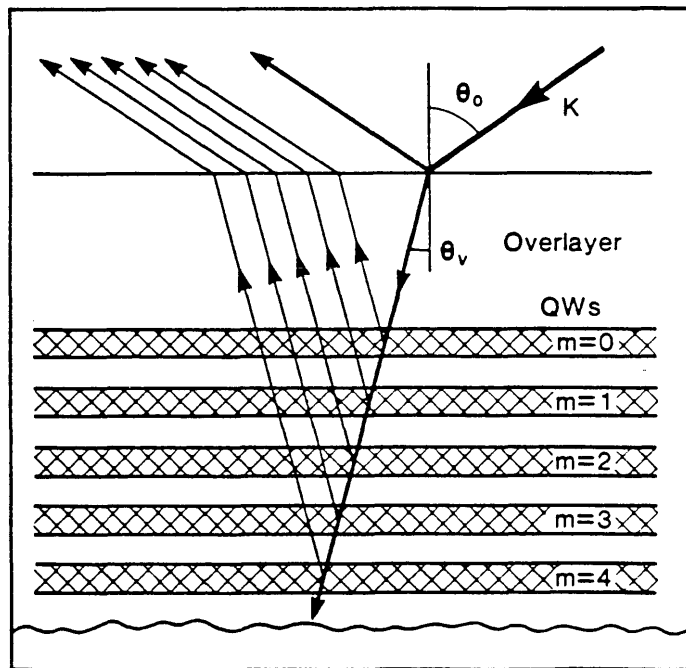


Fig. 3.16 : Schematic of the important rays determining the ER of a MQW structure. The double (and higher) reflections from the QWs can be ignored.

Lineshapes "b" and "d" are also named according to their nature at the exciton energy.

The ER lineshape of Fig. 3.1 can now be understood. For the E1H1 exciton the value of g in Eq. 3.29 is such that the ER lineshape is intermediate between the curves labelled "a" and "b" in Fig. 3.15. Meanwhile, the value of g is slightly different for the E1L1 exciton, because it lies to higher energy, and so it is intermediate between curves "b" and "c".

The first two terms determining the lineshape phase in Eq. 3.31, due to the optical phase change for light making the round-trip across the overlayer and the QW are determined by Eq. 3.18. The first term is much larger than the second, since typically the overlayer is much thicker than the QW. It is a good approximation to take the refractive index of the QW as equal to that in the overlayer in Eq. 3.31, so that the first two terms can be combined, by taking an overlayer thickness which extends to the midpoint of the well. The third term determining the lineshape phase in Eq. 3.31 is also small, since typically the shift of the transition energy is much larger than the change in the linewidth with field, as discussed in Sec. 3.2.5.

The lineshape phase is therefore dominated by the phase change in making the round-trip across the overlayer, which is given by Eq. 3.18, using the total thickness of material from the front surface to the middle of the QW (L) and an average refractive index (n), which was defined earlier. For typical overlayer thicknesses, especially in samples incorporating a p^+-i-n^+ doping profile, the lineshape phase will be many radians. For instance, Chp. 5 describes experiments on a sample with a $1.9\text{ }\mu\text{m}$ overlayer, which has an average refractive index of 3.44. The lineshape phase can be calculated to be 96.9 radians, for the energy of the room temperature QW bandgap and an angle of incidence of 0° . Consequently the precise lineshape, ie which mixture of curves in Fig. 3.15, will be difficult to predict, as a mere 1.6% error in the overlayer optical thickness ($n.L.\cos\theta$) will change the lineshape phase by $\pi/2$, drastically altering the lineshape.

The lineshape phase could be varied by changing the overlayer thickness, either by the growth of several samples or by etching a single sample. However, the large value of the lineshape phase, allows more cunning ways of varying g . Inspection of Eq. 3.18 shows that g can also be varied through its dependence on either the angle of propagation in the overlayer (θ) or the photon energy. Changing the angle of incidence (θ_0) of the light on the structure, produces only a small change in the angle in the overlayer, due to the large discontinuity in

the refractive index at the air/semiconductor interface, but this is sufficient to produce a significant change in g . Meanwhile the transition energy can be changed through the dependence of the bulk bandgap on temperature [3.36,1.12].

Notice that the third term determining the lineshape phase in Eq. 3.31, is a constant for a fixed bias. This term will therefore not effect the evolution of the lineshape with angle of incidence or temperature. The only effect of the field-induced change in the linewidth will be to rotate all the ER spectra by the same amount. Since the precise lineshape phase of any particular spectrum is very sensitive to the exact value of the overlayer optical thickness, as discussed above, this change in g due to the linewidth-modulation, may be difficult to detect.

To test the derived lineshape formula, ER spectra have been taken as a function of the angle of incidence on two SQW samples, which have a different overlayer thickness. These measurements are presented in Chp. 5 and show a lineshape evolution in excellent agreement with Eq. 3.29-31. The angle-dependent measurements on one of these samples is also reported in Ref. 1.12. Ref. 1.12 also describes measurements made on this sample as a function of the temperature, performed by a co-worker, Peter Thorn, for which a similar lineshape evolution was observed.

This analysis of optical interference effects has also recently been confirmed by Zheng et al [3.37], who measured the reflectance of four $100 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ SQW samples, which had different overlayer thicknesses of between 100 and $1000 \text{ \AA}$. These showed an evolution in the lineshape, which is in agreement with Eq. 3.20. Theis et al [3.24] have also observed interference effects in the reflectance and PR spectra of a $175 \text{ \AA}$ GaAs/AlGaAs SQW. They etched $400 \text{ \AA}$ from the overlayer of their sample and saw a lineshape change in both spectra, explainable by a change in lineshape phase of roughly $\pi/2$.

3.4 Modulation Spectroscopy Lineshapes of MQWs

The analysis of optical interference effects in SQWs is now extended to MQW systems, where the wells are assumed to be identical and equally spaced. The barriers are assumed to be sufficiently thick to ensure that the electronic structure of each well can be considered as being the same as that of an isolated QW. In such a structure, each of the QWs will contribute a different lineshape to the overall

reflectance, depending upon its depth below the sample surface. Again an analytical approach is adopted here, which allows the essential elements influencing the lineshape to be determined. The approximations used in deriving the explicit lineshape formula are assessed by also making more exact calculations using a transfer matrix method.

The ER lineshape formula for MQWs is derived in Sec. 3.4.1. This is then used to calculate the lineshape in Sec. 3.4.2, taking typical parameters and an increasing number of QWs in the stack. The nature of the lineshape is discussed along with the effect of the various parameters. The ET lineshape of MQWs is discussed in Sec. 3.4.3 and contrasted with that for ER.

3.4.1 Electroreflectance Lineshape Formula

The QWs are assumed to be identical and regularly spaced inside the sample. The combined reflectivity of the two interfaces of each well is again taken as r_w , derived in Sec. 3.3.1, and all other reflections inside the structure are ignored, except that from the front surface, as shown schematically in Fig. 3.16. Summing the reflected rays and again ignoring the small double (and higher-order) reflections from the QWs, which are negligibly small, yields the overall amplitude reflectance of the structure to be

$$r_{mqw} \approx r_f + (1-R_f) \exp(ig_o) \sum_{m=0}^N r_w \exp(img_{per}) \quad \dots 3.32$$

where g_{per} is the phase delay for the round-trip between adjacent wells and the stack contains a total of $(N+1)$ wells. Optical absorption in the stack is also included in Eq. 3.32 through the imaginary part of g_{per} (ie $\text{Imag}[n_w]$).

The summation is trivial if the reflectivity (and hence dielectric function) is the same for each of the wells. In this case, solving for the intensity reflectance yields

$$R_{mqw} \approx R_f + 2 (1-R_f) \text{Real} \left[r_f r_w \exp(ig_o) \frac{(1-\exp(i\{N+1\}g_{per}))}{(1-\exp(ig_{per}))} \right] \dots 3.33$$

Notice that Eq. 3.33 is similar to the result derived for a SQW (Eq. 3.25), with an extra factor in the second term on the right-hand-side. This extra factor is similar to the amplitude distribution produced in the image plane of a diffraction grating [3.38]. The analogy is not

surprising, as the diffraction pattern of a grating is the Fourier transform of the slit pattern in the object plane, while the ER is the transform of the distribution with depth of the QWs (see Eq. 3.32). Substituting for the well reflectance from Eq. 3.22 yields

$$\Delta R_{\text{mqw}} \approx B \text{ Real } [i \sin(\frac{1}{2} \{N+1\} g_{\text{per}}) \Delta \epsilon_w \exp(ig)] \quad \dots 3.34$$

$$\text{where } B = - ((1-R_f) r_f / n_b^2) \cdot \sin(\frac{1}{2} g_w) / \sin(\frac{1}{2} g_{\text{per}}) \quad \dots 3.35$$

$$\text{and } g = g_o + \frac{1}{2} N g_{\text{per}} + \frac{1}{2} g_w \quad \dots 3.36$$

Notice that the phase factor (g) in Eq. 3.36 is the lineshape phase determined at the midpoint of the MQW stack. The two terms, $\sin(\frac{1}{2} g_w)$ and $\sin(\frac{1}{2} g_{\text{per}})$ in Eq. 3.35, derive from small thicknesses inside the structure, and so have only a slow variation with energy, which will not have much effect on the lineshape.

3.4.2 Discussion of Calculated Lineshapes

The solid lines in Fig. 3.17 plot the ER lineshape of a single exciton, for MQWs with between 1 and 200 QWs in the stack, calculated using Eq. 3.33. The thickness of the well and barrier layers were taken as 88 Å and 60 Å respectively. The exciton energy was taken as 1.55 eV, with an integrated absorption strength of 240 cm⁻¹eV, and the broadening profile was assumed to be Gaussian with a 1.5 meV HWHM. These parameters are close to those relevant to the E1H1 exciton of the GaAs/Al_{0.3}Ga_{0.7}As MQW (MV370) reported in Sec. 6.2, at low temperature. The linewidth of the MQW sample studied in Sec. 6.2 is found to be larger than that of the 88 Å SQW, grown under similar conditions, reported in Chp. 5. It is shown in Sec. 6.2 that this larger linewidth is due to a systematic variation of the well width and electric field across the MQW stack. Since the analysis presented here assumes the wells in the MQW stack to be identical, the linewidth was taken to be that of the SQW, which is about 1.5 meV (HWHM). The exciton profile was assumed to be Gaussian, as this was found to give a good fit to the ER spectrum of this SQW at low temperature, as shown in Chp. 5. The modulated QW dielectric function was calculated using a modulating Stark shift of 0.5 meV, corresponding to a peak-to-peak modulation of 0.5 V around a d.c. offset of 0 V applied to the MQW sample of Chp. 6. The overlayer thickness between the front of the stack and the top surface of the sample was taken as zero in these calculations, as its effect just rotates the entire lineshape, in a similar manner as for SQWs. The

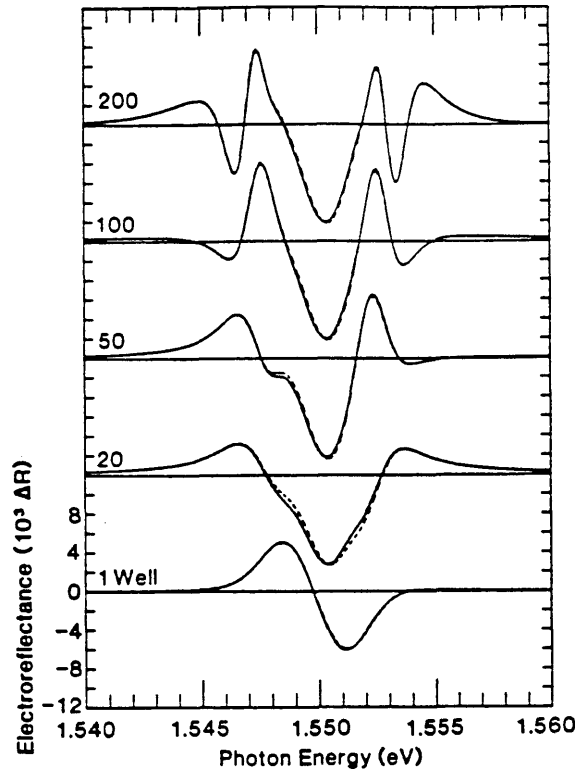


Fig. 3.17 : The calculated ER lineshape of an exciton in a MQW with different numbers of wells in the stack. The solid lines were obtained with the explicit analysis detailed in Sec. 3.4, while the dashed are obtained using a transfer matrix method. The layer widths were taken as : wells = 88 Å, barriers = 60 Å, overlayer = 0 Å. The exciton profile was taken as Gaussian with $E_{\text{ex}} = 1.55$ eV, $\Gamma_{\text{ex}} = 1.5$ meV, $A_{\text{ex}} = 240 \text{ cm}^{-1}\text{eV}$ (integrated absorption strength), while the modulating Stark shift was taken to be $\Delta E_{\text{ex}} = 0.5$ meV.

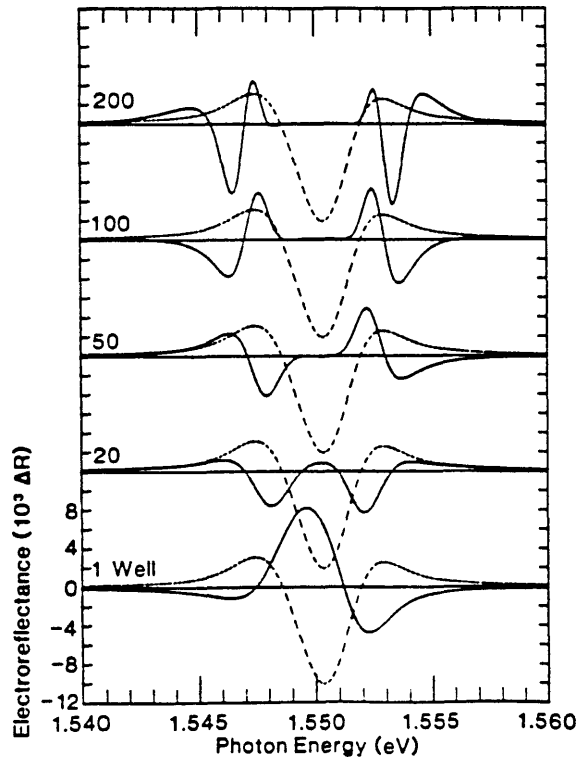


Fig. 3.18 : The ER lineshape of a MQW with different numbers of wells, calculated with identical parameters to Fig. 3.17, decomposed into the terms deriving from the front (dashed lines) and back (solid) of the stack.

angle of incidence was also taken to be zero.

The dashed lines in Fig. 3.17 plot another calculation of the ER lineshape, which uses a transfer matrix method [3.34,2.46] and takes the same parameters as discussed above. The transfer matrix method is a numerical technique that takes account of all the reflected rays, and so is more exact than the explicit analysis detailed in Sec. 3.4.1. It can be seen that the curves calculated using the exact and explicit analyses are almost indistinguishable, so justifying the neglect of the second-order terms in r_w . The explicit analysis has the considerable advantage that it allows a clearer understanding of the role of each of the sample parameters, as discussed below.

Fig. 3.17 demonstrates that the strength of the ER signal does not increase with the number of wells in the stack, as is the case for most other spectroscopies, such as PLE, PC, or absorption. This is explained by the reflected ray from each of the wells having a different lineshape, depending upon its depth inside the sample structure (and hence its lineshape phase). The rays reflected from the different wells therefore tend to destructively interfere and the strength of the ER signal does not increase when the total stack thickness is increased above an eighth of the wavelength of light in the material, which is typically about $300 \text{ \AA}$ for the GaAs/AlGaAs QW bandgap. Adding more QWs to the stack changes the phase of the overall reflectance, but does not increase its strength.

Notice that the calculated lineshape for $N = 0$ in Fig. 3.17, ie the case of one well in the stack, is equivalent to that for a SQW with zero overlayer thickness, which is proportional to $\delta\epsilon_1/\delta E$ (see Eq. 3.29). This is consistent with the lineshape formulae for MQWs (Eqs. 3.34-36) reducing to those applicable to SQWs (Eqs. 3.26-28) when $N = 0$, as would be expected. As the number of wells increases, the MQW lineshapes in Fig. 3.17 become increasingly more complicated than those for a SQW, with extra features appearing in the wings of the lineshape. Essentially this is because the absorption depth changes as a function of energy across the exciton profile, thereby affecting the lineshape phase. For the absorption peak height used in these calculations, the penetration depth is only about $0.25 \text{ \mu m}$ at the central energy of the exciton peak, which is less than the stack thicknesses typically employed in MQWs. Note that if the stack was non-absorbing (a hypothetical situation), then the lineshape would be similar to that for a SQW, with a lineshape phase given by the midpoint of the stack (see Eqs. 3.34-3.36).

The ER in the central region of the lineshape, where the

absorption is large, will originate from the front of the stack, while in the wings the absorption coefficient falls towards zero and so the lineshape phase will be that for the middle of the stack. Concentrating firstly on the central region of the lineshape, As the number of the wells is increased, the lineshape phase initially changes very rapidly and the lineshape is therefore seen to rotate. Then when the stack thickness starts to exceed the light penetration depth, there will only be a gradual change in the lineshape, as more wells are added. Hence the ER lineshape around the centre of the exciton feature will be sensitive to the overlayer thickness and the optical penetration depth.

The origin of the structure in the wings of the lineshape in Fig. 3.17 can be deduced by rewriting Eq. 3.34 as the sum of two terms

$$\Delta R_{mqw} \approx \frac{1}{2} B \text{ Real } [\Delta \epsilon_w \exp(i g_o - i \frac{1}{2} g_{per} + i \frac{1}{2} g_w) - \Delta \epsilon_w \exp(i g_o + i (N+\frac{1}{2}) g_{per} + i \frac{1}{2} g_w)] \dots 3.37$$

The first term in Eq. 3.37 is the ER from the front of the QW stack and the second term is that from the deepest part of the stack. This then is equivalent to regarding the MQW stack as a thick homogeneous layer of material, which extends from the depths of half a period shallower than the middle of the first QW to half a period deeper than the middle of the final QW. It is therefore the position of the front and back of the stack which have most influence on the lineshape. The finite thickness of the well and barrier introduce correction terms (the sinusoidal terms in Eq. 3.35), but these have a slow energy dependence, and therefore do not greatly affect the lineshape. This can be thought as due to the reflections from adjacent wells in the body of the stack tending to cancel with one another, leaving only contributions from the edges of the stack.

The two terms in Eq. 3.37 are plotted separately in Fig. 3.18. The contribution from the front of the stack (dashed lines), does not change as the number of wells increases, and this term dominates the centre of the ER profile. Clearly the second term in Eq. 3.37 (solid lines), from the back of the stack, is responsible for the features in the wings of the lineshape in Fig. 3.17. With increasing number of wells, the lineshape phase of the second term varies very rapidly, as the depth of the back of the stack increases. Adding just 300 Å to the stack thickness will rotate the features in the wings from a contribution resembling $(\delta \epsilon_r / \delta E)$ to one like $(\delta \epsilon_i / \delta E)$, or vice-versa. Notice also that the contribution of the back falls to zero at the

centre of the lineshape, as it is limited by absorption in the stack.

3.4.3 Electrotransmission of a Structure with an Etched Substrate

The transmission of a MQW structure can be deduced by again considering optical interference of the different transmitted rays. For GaAs/AlGaAs structures, the absorbing substrate must be etched away to allow transmission measurements. Hence there is another semiconductor/air interface at the back of the etched cavity, from which there will be a strong reflection. For samples that have not been etched, as were considered previously, the reflection from the back of the structure can be ignored, as it is strongly attenuated by absorption in the substrate.

In this analysis the wells are again assumed to be identical and regularly spaced. The overall transmission and reflectance of the stack of wells are written as r_{stk} and t_{stk} respectively. Ignoring the double and higher-order reflections from the wells, which were demonstrated to be very small in Sec. 5.3.1, yields

$$r_{\text{stk}} = r_w \sum_{m=0}^N \exp(i m g_{\text{per}}) \quad \dots 3.38$$

$$t_{\text{stk}} = (t_w t_b)^N t_w \quad \dots 3.39$$

where the stack contains $(N+1)$ wells and N barriers and t_b is the phase change on travelling across a barrier layer.

The reflectivity of the stack was previously considered in Secs. 3.4.1-2, where it was noted that each QW will contribute a different lineshape to the overall stack reflectivity, depending upon the value of the phase factor, $\exp(i m g_{\text{per}})$. Hence the reflections from the different wells will destructively interfere, resulting in the overall stack reflectivity being of comparable size to that for an isolated well.

The stack amplitude transmission will simply equal the product of the transmission of each well and barrier layer, as given by Eq. 3.39, after again ignoring the second-order and higher reflections from the QW. The amplitude transmission of an isolated well (t_w) is given by Eq. 3.23, ie $t_w \approx \exp(i k n_w L_w \cos \theta)$, to first-order in $|(n_w - n_b)/2n_b|$. It was demonstrated in Sec. 5.3.1 that $|(n_w - n_b)/2n_b|$ is typically small, so making this expression for t_w a good approximation. Notice that $\text{Imag}[n_w]$ will describe the absorption in the well. Similarly the propagation across a barrier will cause a phase change, so that

$t_b = \exp(i k n_b L_b \cos \theta)$, where L_b is barrier thickness. The angle of propagation relative to the normal of the plane of the well, θ , is taken to be the same in the well and barrier layer here, which is a justifiable approximation if $|(n_w - n_b)/n_b| \ll 1$. Hence Eq. 3.23 can be rewritten as

$$t_{stk} \approx \exp(i \frac{1}{2} g_{stk}) \exp(i \frac{1}{2} \text{Real}[g_{trn}]) \exp(-\frac{1}{2} \text{Imag}[g_{trn}]) \quad \dots 3.40$$

$$\text{where } g_{stk} = 2 k n_b \{ (N+1) L_w + N L_b \} \cos \theta \quad \dots 3.41$$

$$g_{trn} = 2 k (n_w - n_b) (N+1) L_w \cos \theta \quad \dots 3.42$$

The first factor in Eq. 3.40, $\exp(i \frac{1}{2} g_{stk})$, is the phase change on propagating across the stack, if the wells were replaced by layers with a refractive index equal to that of the barriers. The second factor in Eq. 3.40, $\exp(i \frac{1}{2} \text{Real}[g_{trn}])$, is the modification to this phase delay due to the optical transitions of the QW, while the third factor describes the absorption due to the QW transitions.

Notice from Eq. 3.40 that $|t_{stk}|$ will get progressively smaller as the number of wells $(N+1)$ increases. The increase in the strength of $(1 - |t_{stk}|)$ with N , is in contrast to the behaviour of $|r_{stk}|$, which does not increase with N , due to the destructive interference of the rays from each well, as discussed earlier.

The overall transmission of the structure, consisting of a MQW stack inside an etched semiconductor cavity, is again calculated by considering the rays with a less than second-order dependence on r_w . These rays can be divided into two categories, which are shown schematically in Figs. 3.19.1 and 3.19.2. Fig. 3.19.1 shows the rays that are transmitted through the structure without reflection from the QW. These rays are the most intense to be transmitted and will dominate the unmodulated transmission spectrum. The two air/semiconductor interfaces have a large reflectivity and so multiple reflections inside the cavity, as shown in Fig. 3.19.1, cannot be ignored. The intensity of the rays shown in Fig. 3.19.1 will be reduced by absorption in the stack. Fig. 3.19.2 shows the transmitted rays which experience a single reflection from a QW. The intensity of these rays will be much weaker than those in Fig. 3.19.1, but their contribution to the ET signal will be of a comparable strength.

The analysis of the interfering rays is simplified if the two interfaces of the cavity are assumed to be smooth and parallel, with the same reflectivity. The amplitude (incident from the air side of the

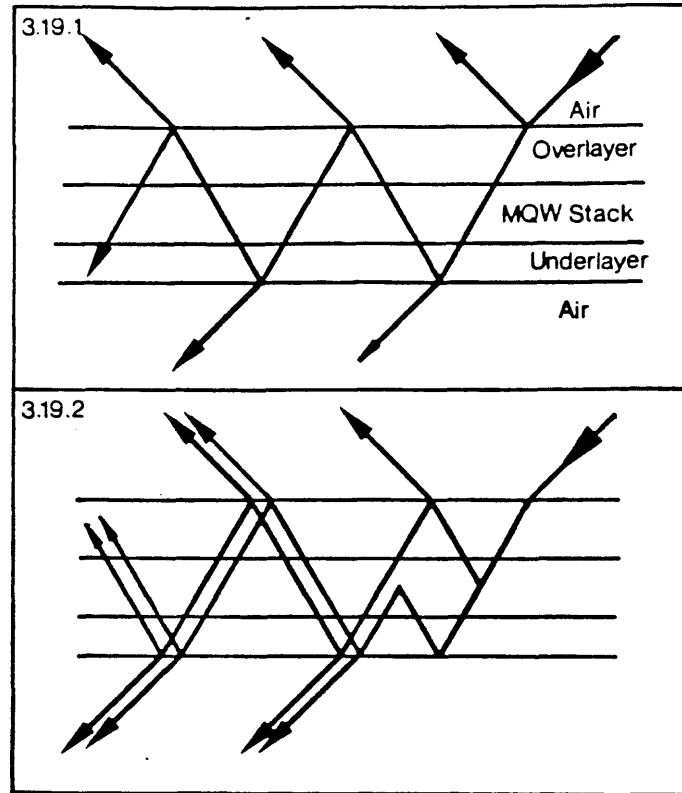


Fig. 3.19 : Schematic of the most important rays transmitted through a structure containing a MQW stack imbedded inside an etched cavity. The stack has been replaced by an effective interface with reflectivity and transmission given by Eqs 3.38 and 3.39. The rays shown in 3.19.1 are not reflected at any of the QWs, but are slightly reduced by the absorption of the stack, while the rays in 3.19.2 undergo a single reflection from a QW.

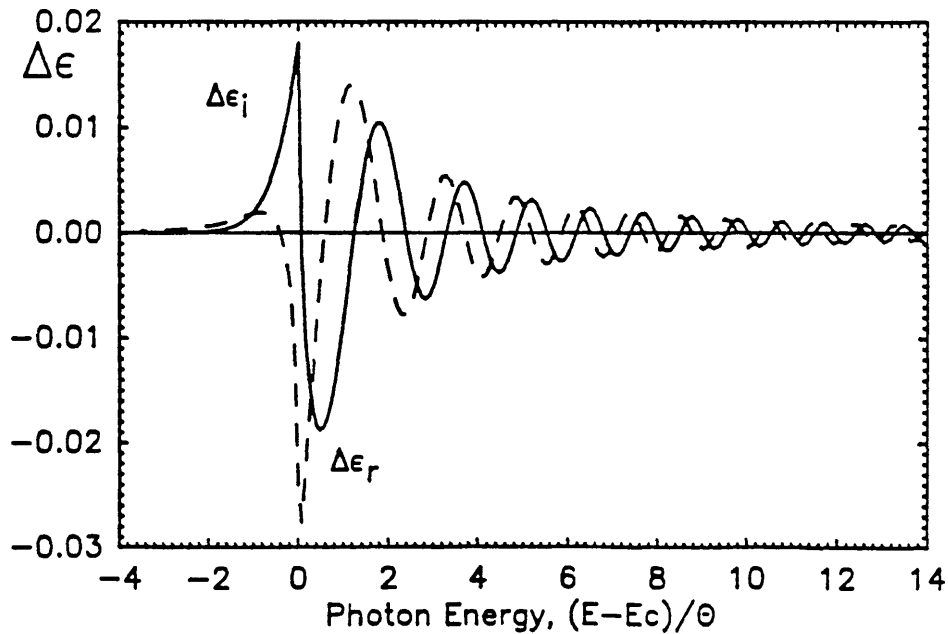


Fig. 3.20 : The real (dashed line) and imaginary (solid) parts of the field modulated dielectric function of bulk semiconductors at the fundamental bandgap, due to the Franz-Keldysh effect. The lower field of the modulation is zero. Note that $\Delta\epsilon_i$ is the difference between the two curves shown in Fig. 2.14.

interface) and intensity reflectivity are written as r_f and R_f respectively. The optical phase delay for a round-trip across the overlayer between the top of the MQW stack and the front surface of the cavity is written as g_o , while g_u is the phase change for the round-trip across the underlayer between the bottom of the MQW stack and the back surface of the cavity. Either g_o or g_u can have a non-zero imaginary part, if the overlayer or underlayer contain absorbing layers. Summing all the transmitted rays shown in Fig. 3.19.1 and 3.19.2 yields amplitude transmissions of t_0 and t_1 respectively, given by

$$t_0 = \frac{(1-R_f) t_{stk} \exp(i \frac{1}{2} \{g_o + g_u\})}{1 - R_f t_{stk}^2 \exp(i \{g_o + g_u\})} \quad \dots 3.43$$

$$t_1 = -t_0 r_{stk} r_f \{\exp(ig_o) + \exp(ig_u)\} \quad \dots 3.44$$

The denominator of Eqs. 3.43 describes the multiple reflections between the front and back of the cavity. The total amplitude transmission, is then approximated by, $t \approx t_0 + t_1$.

When there are only a few QWs in the stack, the value of t_{stk} is close to unity and r_{stk} is small. The rays shown in Fig. 3.19.1 will be much more intense than those in Fig. 3.19.2 and will dominate the unmodulated transmission spectrum. However, both the rays shown in Fig. 3.19.1 and 3.19.2 contain a first-order dependence on $|(n_w - n_b)/n_b|$, and consequently both must be considered in calculating the first-order signal due to the QW. Clearly then the explicit relations for the intensity transmission and ET will be quite complicated, when there are only a few wells in the stack.

As the number of well in the stack increases, the strength of the contribution to the ET of the rays shown in Fig. 3.19.1 increases, while that of the rays in Fig. 3.19.2 does not. This can be deduced by inspecting Eqs. 3.43 and 3.45, and noting that the strength of $|r_{stk}|$ does not increase appreciably with N , while $(1 - |t_{stk}|)$ does. Consequently the signal due to the QW will be determined by the rays shown in Fig. 3.19.1 and those in Fig. 3.19.2 can be ignored. Taking the complex conjugate of Eq. 3.43 yields the total intensity transmission to be given by

$$T \approx \frac{(1-R_f)^2 T_{stk}^2 \alpha_{cav}}{1 + R_f^2 T_{stk}^2 \alpha_{cav}^2 - 2 R_f T_{stk} \alpha_{cav} \cos(g_{cav} + \text{Real}[g_{trn}])} \quad \dots 3.45$$

where $g_{cav} = g_o + g_u + g_{stk}$, is real and represents the total phase

change for the round-trip across the etched cavity, if the well layers are replaced by barrier material, and $\alpha_{\text{cav}} = \exp(-\text{Imag}[g_o + g_u])$ describes the absorption by the layers in the cavity other than the QWs. The intensity transmission of the stack, is from Eq. 3.40, given by

$$T_{\text{stk}} \approx \exp \left(- (N+1) k L_w \text{Imag}[\epsilon_w] \cos \theta / n_b \right) \quad \dots 3.46$$

At energies below the MQW bandedge, $\text{Imag}[\epsilon_w] = 0$ and hence $T_{\text{stk}} = 1$. If the cavity does not contain any other absorbing layers, ie $\alpha_{\text{cav}} = 1$, then the transmission spectrum will show large oscillations due to the variation of the denominator of Eq. 3.45. These Fabry-Perot oscillations, as they are known, are caused by the interference of the light reflected from the front and back of the cavity, and their period will be determined by the optical thickness of the cavity. The oscillations can be suppressed by placing absorbing layers in the cavity; by inclining the back surface of the cavity relative to the front; or coating one face with an anti-reflection layer stack.

For stacks with a large number of QWs, the absorption of the stack at energies above the bandedge will be strong, so that $T_{\text{stk}} \ll 1$. Consequently the denominator of Eq. 3.45 will be close to unity and hence the overall intensity transmission will be simply given by the relation, $T \approx (1-R_f)^2 T_{\text{stk}} \alpha_{\text{cav}}$. Thus the imaginary part of the dielectric function can be determined from the logarithm of the transmission spectrum, at energies where the stack absorption is strong.

Notice that T_{stk} is determined solely by the imaginary part of the QW dielectric function. Consequently the overall intensity transmission of the structure will have only a weak dependence on $\text{Real}[\epsilon_w - \epsilon_b]$, due to the term $\text{Real}[g_{\text{trn}}]$ in the denominator of Eq. 3.45. If this term is ignored, differentiation of Eq. 3.45 yields the ET to be given by

$$\Delta T \approx - G(E) (1-R_f)^2 (N+1) k L_w n_b^{-1} \cos \theta \text{Imag}[\Delta \epsilon_w] \quad \dots 3.47$$

$$\text{where } G(E) = \frac{(1 - R_f^2 \alpha_{\text{cav}}^2 T_{\text{stk}}^2) T_{\text{stk}} \alpha_{\text{cav}}}{\{1 + R_f^2 \alpha_{\text{cav}}^2 T_{\text{stk}}^2 - 2 R_f \alpha_{\text{cav}} T_{\text{stk}} \cos(g_{\text{cav}})\}^2} \quad \dots 3.48$$

Hence the ET will be approximately determined by $\text{Imag}[\Delta \epsilon_w]$ alone, with the amplitude being determined by an oscillatory function, $G(E)$, that has the same period as the Fabry-Perot oscillations of the unmodulated transmission spectrum. The period of the Fabry-Perot oscillations is typically much larger than the linewidth of a QW transition, so that

$G(E)$ should not significantly distort the ET lineshape.

Notice that the strength of the ET signal will increase with the number of wells in the stack, while the ER of MQWs with an unetched substrate, considered in Sec. 3.4.1-2, does not. Notice also that the ET will be insensitive to the arrangement of the QW layers inside the stack. This again is in contrast to the ER of MQWs with an unetched substrate, where the signal is determined by a mixture of $\text{Real}[\Delta\epsilon_w]$ and $\text{Imag}[\Delta\epsilon_w]$, depending upon the depth of the front and back of the stack. However, the ET will also have some dependence on $\text{Real}[\Delta\epsilon_w]$ for MQWs with only a few wells, due to the rays shown in Fig. 5.19.2.

3.5 Electroreflectance of Bulk Semiconductors

The previous sections have considered ER lineshapes for QWs, where the field is applied perpendicular to the plane of the well. For this field geometry the dielectric function of the QW is modulated by the QCSE, as discussed in Sec. 3.2. An applied electric field produces qualitatively different behaviour on bulk semiconductors; the Franz-Keldysh effect, which was discussed in Sec. 2.5. This results in very different ER lineshapes for bulk material. The analysis presented in this section is also relevant to QW samples, where the electric field is applied in a direction in the well plane.

The analysis of bulk ER in this section, adopts the same approach as employed for QWs of (1) deciding which optical transitions have most influence on the ER; (2) examining the effect of electric field and thereby deducing the form of modulated dielectric function; and finally (3) relating the modulated dielectric function to the ER. At each stage comparison is made with the QW analysis.

The classical analysis of bulk semiconductor ER, as for instance found in Refs. 2.61, 3.1 or 3.2, is extended here to consider the case of small field modulations. The model is also extended to consider the situation where the semiconductor region producing the ER is located below the surface of the sample. This is applicable to p^+i-n^+ diodes, for instance, where the region subject to the field is buried under some highly doped contact layers. In this situation optical interference effects, similar to those discussed for QW samples, will then be important. The formulae derived for bulk semiconductors, in this section, are used to determine the electric fields in some QW samples which also contain regions of bulk material.

3.5.1 Dielectric Function

Excitons are readily ionised by an electric field in bulk semiconductors, so their contribution to the ER spectra will be negligible for fields larger than a few kV/cm. Consequently the continuum terms can be expected to dominate bulk ER.

The real and imaginary parts of the dielectric function of a bulk semiconductor under a uniform field, near the fundamental bandgap (E^C), are given by Eqs. 2.56 and 2.55 respectively. Fig. 2.14 plots the form of ϵ_i at zero and a finite field. In the zero-field limit, ϵ_i increases as $(E-E^C)^{\frac{1}{2}}$ above the zero-field bandedge, as discussed in Sec. 2.5.1. Meanwhile at a finite field, there is a tail below the zero-field bandedge, and oscillations about the zero-field curve above E^C . This phenomenon is known as the Franz-Keldysh effect.

3.5.2 Electric Field Modulated Dielectric Function

The modulated dielectric function is again defined as the difference in the dielectric function between the two extreme applied fields. Since the form of ϵ_i at a finite field shows oscillations about the zero-field curve, it is convenient to take the lower field as zero, and the real and imaginary parts of the field modulated dielectric function are then given by

$$\Delta\epsilon_r = (A \theta^{\frac{1}{2}} / E^2) G(\{E^C-E\}/\theta) \quad \dots 3.49$$

$$\Delta\epsilon_i = (A \theta^{\frac{1}{2}} / E^2) F(\{E^C-E\}/\theta) \quad \dots 3.50$$

where $\theta = e^2 \hbar^2 F^2 / 2 \mu$. The functions in Eqs. 3.49 and 3.50 are known as the electro-optic F- and G-functions, and are given by the difference in the finite and zero-field curves for F_1 and G_1 in Eqs. 2.57 and 2.58, so that

$$G(x) = \pi [\{Ai'(x).Bi'(x)\} - x \{Ai(x).Bi(x)\}] - \sqrt{(x)}.H(x) \quad \dots 3.51$$

$$F(x) = \pi [\{Ai'(x)\}^2 - x \{Ai(x)\}^2] - \sqrt{(-x)}.H(-x) \quad \dots 3.52$$

where $H(x)$ is the unit step function. Fig. 3.20 plots the electro-optic F- and G-functions, defined by Eqs. 3.52 and 3.51. The first term in Eqs. 3.51 and 3.52, involving the Airy functions and their derivatives, describes the dielectric function in a finite field, while the second term, involving the unit step function, is the zero-field limit. It can

be seen that the F-function is simply the difference in the finite- and zero-field curves in Fig. 2.15.

The Airy functions can be written as analytical formulae in the limits $x \rightarrow +\infty$. For large negative x , $Ai(x)$ and $Bi(x)$ are sinusoidal with a decreasing amplitude [2.35], which can be used to show

$$\Delta\epsilon_r(-x) \approx - (A (E-E^C)^{\frac{1}{2}} / \pi E^2) \cos(4 x^{3/2} / 3) \quad \dots 3.53$$

$$\Delta\epsilon_i(-x) \approx - (A (E-E^C)^{\frac{1}{2}} / \pi E^2) \sin(4 x^{3/2} / 3) \quad \dots 3.54$$

Eqs. 3.53-54 therefore approximate the decaying oscillations seen at energies well above the bandedge in Fig. 3.20.

The above analysis considered the situation where the field is modulated between some finite value and zero. However, the experimental studies of this thesis use small modulations, around a variable d.c. bias, as the QW spectra can then be regarded as differential in nature. Fig. 3.21.1 plots the imaginary part of the dielectric function at two finite values of the field, F_1 and F_2 . Notice that the period of the oscillations is different for the two curves. The imaginary part of the modulated dielectric function, for a field varying between F_1 and F_2 , will then be given by the difference in the two curves in Fig. 3.21.1, so producing the curve in Fig. 3.21.2. Notice that this curve for $\Delta\epsilon_i$ again shows Franz-Keldysh oscillations above the bandedge, as in Fig. 3.21, but now these have an oscillatory envelope of larger energy period. This is the well known phenomenon, called "beating", that arises from the subtraction of two sinusoidal functions of slightly different period.

When the lower field of the modulation is finite, the modulated dielectric function can be written as,

$$\Delta\epsilon_r = (A \theta^{\frac{1}{2}} / E^2) \{ G_1(\{E^C-E\}/\theta_1) - G_1(\{E^C-E\}/\theta_2) \} \quad \dots 3.55$$

$$\Delta\epsilon_i = (A \theta^{\frac{1}{2}} / E^2) \{ F_1(\{E^C-E\}/\theta_1) - F_1(\{E^C-E\}/\theta_2) \} \quad \dots 3.56$$

where $\theta_{1(2)} = (e^2 F_{1(2)}^2 \hbar^2 / 2 \mu)^{1/3}$ and F_1 and G_1 were defined in Eqs. 2.57 and 2.58. If the F- and G-functions are again approximated as being sinusoidal at energies well above the bandedge, ie $x \rightarrow -\infty$, it can be shown that

$$\Delta\epsilon_r \approx +(2A(E-E^C)^{\frac{1}{2}}/\pi E^2) \cdot \cos(2\{x_1^{3/2}+x_2^{3/2}\}/3) \cdot \sin(2\{x_1^{3/2}-x_2^{3/2}\}/3) \quad \dots 3.57$$

$$\Delta\epsilon_i \approx -(2A(E-E^C)^{\frac{1}{2}}/\pi E^2) \cdot \sin(2\{x_1^{3/2}+x_2^{3/2}\}/3) \cdot \sin(2\{x_1^{3/2}-x_2^{3/2}\}/3) \quad \dots 3.58$$

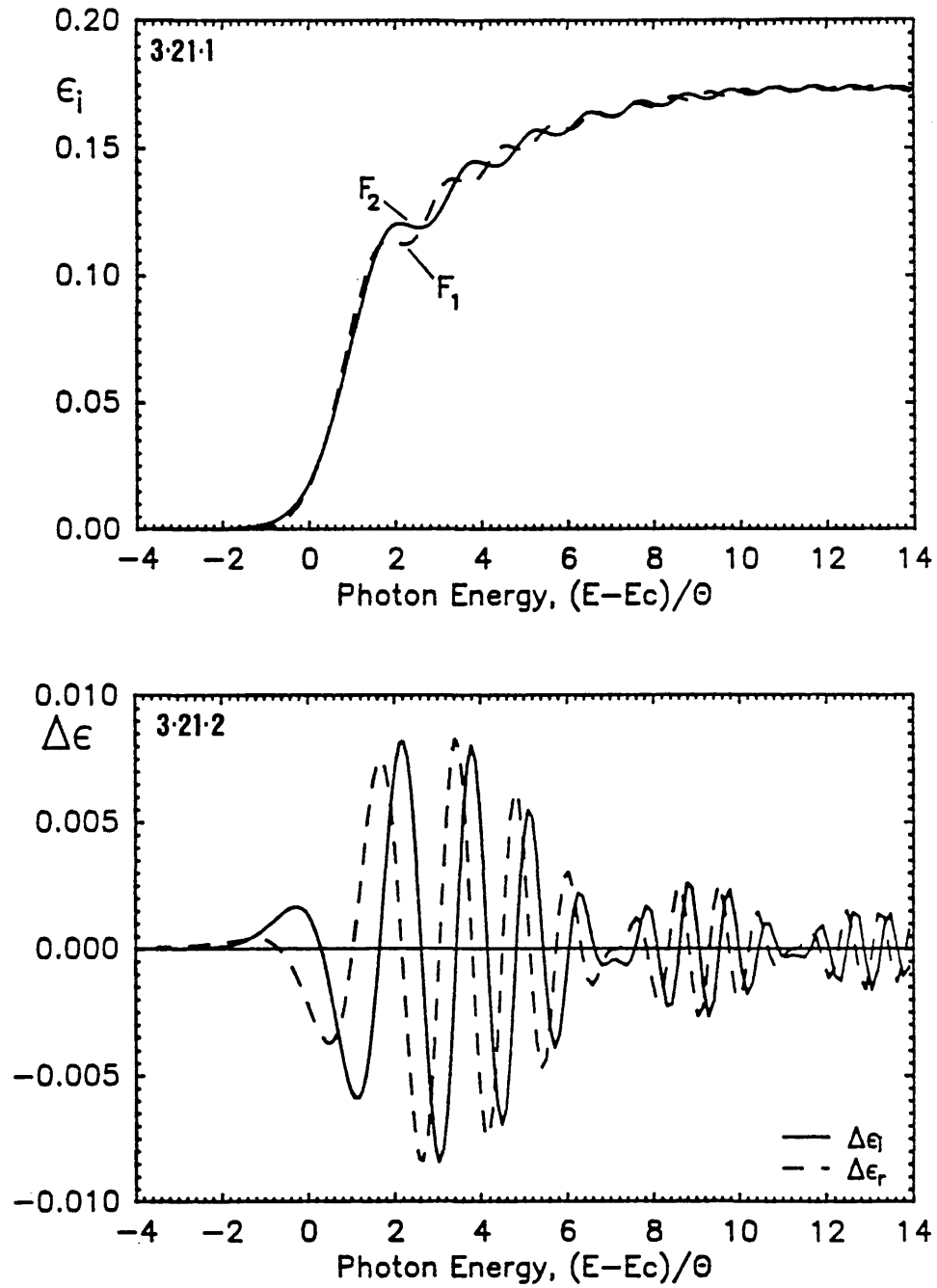


Fig. 3.21 : The imaginary part of bulk semiconductor dielectric function at two different applied fields is shown in 3.21.1. The difference in these curves gives the imaginary part of the field modulated dielectric function shown as the solid line in 3.21.2. The real part of modulated dielectric function is also shown in 3.21.2 as the dashed line.

where $x_{1(2)} = (E^C - E)/\theta_{1(2)}$ and θ_1 and θ_2 are determined using the fields F_1 and F_2 respectively.

Comparison of Eq. 3.57 (3.58) with Eq. 3.53 (3.54) shows that the Franz-Keldysh oscillations in Fig. 3.21.2 have a period determined by the "average" field, $F = 2/(1/F_1 + 1/F_2)$. The second sinusoidal term in Eqs. 3.57 and 3.58 will have a larger period, determined by the field $F = 2/(1/F_1 - 1/F_2)$, and this describes the envelope of the Franz-Keldysh oscillations. Eqs. 3.57 and 3.58 reduce to Eqs. 3.53 and 3.54 for the case of $F_1 = 0$ or $F_2 = 0$, as can be expected.

Fig. 3.22 shows a clear example of this beating of the Franz-Keldysh oscillations. The experimental ER spectrum in Fig. 3.22 was taken on a sample (G512) containing a region of bulk GaAs semiconductor. The layer specification of this sample and the other experimental details are given in Chp. 7. Fig. 3.22 also shows the ER lineshape calculated using the modulated dielectric function given by Eqs. 3.55-56. (The ER was related to $\Delta\epsilon_r$ and $\Delta\epsilon_i$ using Eq. 3.26, since the region of bulk semiconductor contributing to the ER is buried below the surface, as discussed later in Sec. 3.5.3.) The field extremes were taken to be 21 and 28 kV/cm for the calculated lineshape, which are close to the expected ideal values of 26 and 31 kV/cm, deduced from the experimentally applied bias.

It can be seen that the model gives a good description of the periods of both the Franz-Keldysh oscillations and their envelope in the experimental spectrum. However, the oscillations were observed to decay more rapidly with energy than predicted by the model. This is likely to be due to the neglect of the finite broadening parameter and the Coulombic attraction in this calculation.

The effect of a finite broadening can be included by convoluting the unbroadened curves of Fig. 3.20 with an appropriate profile [2.61], or by allowing the argument of the F- and G-functions to be complex [2.60]. The effect of a finite linewidth (Γ) is to smooth the sharp features at the bandedge in Fig. 3.19 and also to cause a faster decay of the oscillations with energy [2.61]. This will give a better agreement between the measured and calculated spectra in Fig. 3.22. For the case when the field is small, such that $\theta \ll \Gamma$, it has been shown by Aspnes and co-workers [3.39-40] that the modulated dielectric function is determined by the third-differential of the non-modulated dielectric function with respect to the photon energy.

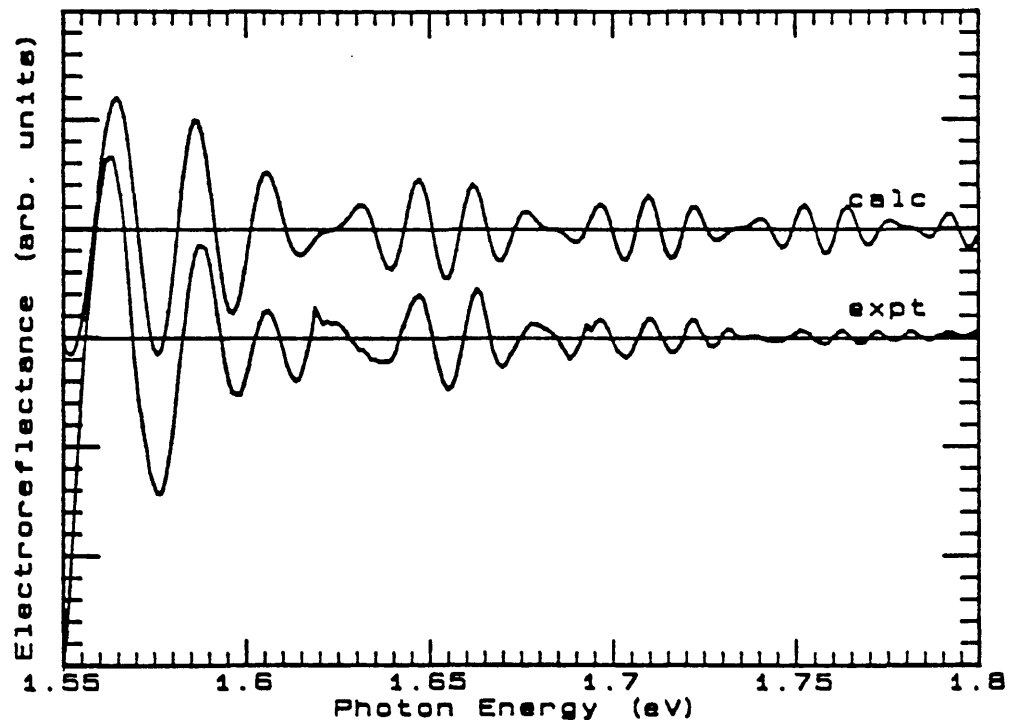


Fig. 3.22 : An ER spectrum measured on sample G512, showing features deriving from a region of bulk GaAs inside the sample. Also shown is a theoretical spectrum calculated using the method described in Sec. 3.5. Notice the "beating" of the Franz-Keldysh oscillations. G512 had identical layer details to G485, described in Sec. 7.3.2, except the (6,6) GaAs/AlAs SL is replaced by 0.777 μm of (9,36) SL. The applied bias was -5 V, with a modulation of 1 V_{ptp}. $T = 17$ K. $\theta_0 = 45^\circ$.

3.5.3 Electroreflectance Lineshape

To calculate the reflectance, and hence the ER, of a SQW sample, it was necessary to consider interference of the ray from the QW with that from the front surface. For a homogeneous bulk semiconductor, subject to a uniform electric field, there is only one interface, at the front surface of the sample, and its reflectivity is determined by Fresnel's relation. Fresnel's relation can be differentiated to yield

$$\Delta R = \alpha(\epsilon_r, \epsilon_i) \Delta \epsilon_r + \beta(\epsilon_r, \epsilon_i) \Delta \epsilon_i \quad \dots 3.59$$

where α and β are called the Seraphin coefficients [3.41]. It can be shown that $\beta = 0$, at the fundamental bandedge, so the ER will be determined solely by $\Delta \epsilon_r$. This is in complete contrast to the SQW case, where both the real and imaginary parts are equally important.

Glembocki and Shanabrook [3.14] and Theis et al [3.23] have attempted to model PR lineshapes of MQW samples using $\Delta \epsilon_r$ alone, justifying this approach by arguing that the β Seraphin coefficient is zero near the fundamental bandedge. However their argument is erroneous. The Seraphin coefficients, derive from differentiation of the Fresnel relation at the semiconductor/air interface of a semi-infinite slab of bulk material. This is completely different from the situation in a QW sample, where the QW(s) are buried inside a material of similar dielectric function. Hence the Seraphin coefficients are not valid for SQW or MQW samples. In bulk semiconductors, the inequality $\alpha \gg \beta$, originates from the large discontinuity in ϵ_r at the front surface, at energies near the fundamental bandedge. For a QW, however, both ϵ_r and ϵ_i are only slightly enhanced above that of the barrier, so that $\Delta \epsilon_r$ and $\Delta \epsilon_i$ are of similar strength.

Both these studies [3.14,23] use least squares fitting to deduce the shape of the exciton broadening profile. However it seems very dubious to extract subtle information on the profile, after ignoring the contribution of $\Delta \epsilon_i$. In particular, Theis et al found they could improve the fit by either allowing the β Seraphin coefficient to be non-zero, or by allowing a change in the linewidth to modulate the dielectric function in addition to the Stark shift. Both of these steps mimic the effect of adding a contribution from $(\delta \epsilon_i / \delta E_{ex})$ which they have ignored. This could explain the unreasonably large values fitted by Theis et al for the field-induced change in the exciton linewidths.

Returning again to the ER of bulk semiconductors, it is possible to have a contribution from $\Delta \epsilon_i$ when the field is non-uniform, as then

the light is reflected from a range of depths inside the sample [3.42-43]. Similarly optical interference can also be expected to be important in bulk ER, if the region subject to the field is buried inside the sample structure, such as in a p^+-i-n^+ diode. Doping the sample to produce a p^+-i-n^+ profile has become an increasingly popular method of applying an electric field. In previous ER studies of bulk semiconductors it has been more common to evaporate a semitransparent Schottky barrier on the surface of the sample [3.3] or use a suitable electrolyte [3.4].

When the bulk semiconductor region is buried below the surface, the ER will again be a linear mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, as in Eq. 3.26, that was derived for a SQW. However, $\Delta\epsilon_r$ and $\Delta\epsilon_i$ will then be determined by the Franz-Keldysh effect, ie Eqs. 3.55-56. The lineshape phase, g in Eq. 3.26, will be determined by the optical penetration depth into the bulk region, added to the depth of the front of the bulk region below the sample surface. At energies high above the bandedge, $E \gg E^C$ (and hence $x \rightarrow -\infty$), Eqs. 3.57-58 can be substituted into Eq. 3.26 to yield

$$\Delta R \approx -(2 A B (E-E^C)^{\frac{1}{2}} / \pi E^2) \cdot \sin(2\{x_1^{3/2} - x_2^{3/2}\}/3) \\ \times \sin(2\{x_1^{3/2} + x_2^{3/2}\}/3 - g) \quad \dots 3.60$$

3.5.4 Aspnes' Third Derivative Functional Form

Aspnes and co-workers [3.3,39,44] proposed a lineshape formula for bulk semiconductors, which included a general phase term (θ) mixing the real and imaginary parts of the dielectric function

$$\Delta R = A \Gamma^{-n} e^{i\theta} (E^C - E + i \Gamma)^n \quad \dots 3.61$$

The energy dependence of the zero-field dielectric function of bulk semiconductors is given by $\epsilon \propto (E^C - E + i\Gamma)^{\frac{1}{2}}$, as in Eq. 3.61 with $n = \frac{1}{2}$. Aspnes and co-workers [3.39-40] showed that for low fields, such that $\Gamma \gg \theta$, the modulated dielectric function is determined by the third-differential with respect to energy, $\Delta\epsilon \propto \delta^3(E^2 \cdot \epsilon) / \delta E^3$. This is the low-field limit of the Franz-Keldysh effect. Hence $n = -5/2$ is taken in Eq. 3.61 in order to describe the ER of bulk semiconductors in the low-field limit. The phase factor, θ in Eq. 3.61, which causes a mixing $\Delta\epsilon_r$ and $\Delta\epsilon_i$ in the ER, is usually treated as a phenomenological factor to be determined from the spectrum. Eq. 3.61, using $n = -5/2$, is known as the "third-derivative functional form" (TDFFF).

Eq. 3.61 bears some similarity to the lineshape formula for SQWs that was derived in Sec 3.3 (Eq. 3.29), provided that the exciton broadening profile in Eq. 3.29 is taken to be Lorentzian. The contribution of a Lorentzian broadened exciton to the dielectric function has an energy dependence given by $\epsilon \propto (E^C - E + i\Gamma)^{-1}$ (ie $n = -1$), ignoring the small negative energy pole. It was discussed in Sec. 3.2 that for the E1H1 and E1L1 QW transitions, the modulation of the dielectric function is approximately determined by the first-differential with respect to the photon energy, $\Delta\epsilon \propto \delta\epsilon/\delta E$. Consequently, Eq. 3.61 along with $n = -2$, will approximately describe the ER lineshape of Lorentzian broadened E1H1 and E1L1 excitons in a QW.

Notice that Eq. 3.61 also contains a phase (θ) mixing $\Delta\epsilon_r$ and $\Delta\epsilon_i$, which is equivalent to the lineshape phase (g) in Eq. 3.29. However, Eq. 3.61 does not predict the energy dependence of the phase, given by 3.18, which can have a significant effect on the lineshape. This is apparent in the experimental ER spectrum of Fig. 3.1, where, it was noted earlier, the lineshape phases of the E1H1 and E1L1 exciton are different. Likewise Eq. 3.61 does not predict the remarkable sensitivity of the ER lineshape to the angle of incidence, demonstrated in Chp. 5. Note that if an applied perturbation, such as temperature, electric field, or hydrostatic pressure, changes the transition energy of a QW, then it can also be expected to alter the lineshape, due to the energy dependence of g (Eq. 3.18). This change in the lineshape is not predicted by the TDFF. The energy dependence of the phase given by Eq. 3.18, described the evolution of the ER lineshape with sample temperature observed by Thorn et al [1.12]. It may also account for the change in the PR lineshape with temperature measured by Glembocki et al [3.13,3.16] (see Fig. 1 of Ref. 3.13). Similarly, it could also explain the comment of Kangarlu et al [3.30], who used the TDFF to model the PR spectra of a GaAs/AlGaAs MQW at different applied hydrostatic pressures, and noted a variation in the value of the fitted phase (θ).

Several authors have used Eq. 3.61, taking $n = -5/2$, to make lineshape fits to the PR of MQW samples [3.18,19,26,30-32]. Their analysis is difficult to justify physically, as it corresponds to regarding the ER as deriving from the Franz-Keldysh effect of three-dimensional continuum terms. A proper analysis must take $n = -2$, corresponding to the QCSE of a bound exciton. Despite their error, these workers have been able to obtain reasonable lineshape fits. Glembocki and Shanabrook [3.14] suggested that taking $n = -5/2$ in Eq. 3.61, mimics the lineshape produced by the first-energy-differential of a Gaussian broadened exciton profile. If the excitons had a Gaussian

profile, as their own measurements suggested, this would explain the fortuitous agreement found by the workers using the TDFF formula.

The result for bulk semiconductors that the ER is determined by the third-energy-differential is a consequence of the electrons and holes being accelerated in opposite directions by the field (the Franz-Keldysh effect) [3.41,42]. The acceleration of the electrons and holes is prevented by the confining barriers in a QW under a field in the direction perpendicular to the plane of the well. The large field-induced energy shift of the lowest subbands, results in the ER of the E1H1 and E1L1 transition of a QW being determined by the first-energy-differential. This is true for both the exciton and continuum terms of the xy-motion of the QW. However, if the field was applied along a direction in the plane of the QW, the electron and hole could then freely accelerate, apart from a small correction due to the Coulombic attraction, and the ER would again be given by the third-differential, in the low-field limit.

The first-differential result for a QW must tend toward the third-differential of bulk semiconductors as the well width increases. This will happen at fields where the carriers can tunnel freely from the well, as discussed in Sec. 2.6.1. However, note that at these fields the resonance width of the quasi-confined state will be very broad, so that it will no longer be meaningful to divide the optical spectrum into a number of discrete quasi-confined transitions.

The behaviour for a SL must also tend towards that of bulk when the barrier width is reduced. As the barrier width is reduced, there is an increase in the minibandwidth of the subband states, and the envelope functions become delocalised through the SL. (The bandstructure of SLs is discussed in Sec. 7.1). If the minibandwidths are larger than the potential drop across the MQW produced by the field, the carriers will be able to accelerate through the SL, perpendicular to the layers. This will produce lineshapes similar to those predicted by the Franz-Keldysh effect, and they can be expected to be third-differential in the low-field limit. Shen et al [3.20,21] made lineshape fits to the PR spectra of GaAs/AlGaAs MQWs, and found the lowest subband transitions to be well described by a first-differential lineshape, while the higher transitions were third-differential. They suggested that this was because the electrons in the higher subbands could accelerate across the MQW.

3.6 Summary of Lineshape Models and Their Limitations

Lineshape models for the modulated reflectance and transmission spectra of SQWs, MQWs and bulk semiconductors were presented in this chapter. The approach adopted was to firstly calculate the change in the dielectric function produced by the modulation, and then relate this to the modulated reflectance and transmission. In each case explicit formulae were derived which clearly elucidated the most important parameters determining the lineshape. The emphasis was placed on electric field modulation, although many of the equations are general to the modulated dielectric function taking any form.

The modulated dielectric function due to a single ground state bound exciton was considered here, and resolved into components due to small changes in the exciton energy, linewidth and overlap. The strengths of the energy- and overlap-modulated contributions were calculated for the specific case of a small and fixed electric field modulation. This was done for the different subband transitions in QWs of various widths, as a function of the applied d.c. offset field. It was found that, for the E1H1 and E1L1 transitions, the energy-modulated lineshape was dominant for most well widths and d.c. fields, with the overlap-modulated lineshape being most important in wide wells at low fields. Hence $\Delta\epsilon_r$ and $\Delta\epsilon_i$ were found to be approximately given by $(\delta\epsilon_r/\delta E_{ex})$ and $(\delta\epsilon_i/\delta E_{ex})$ respectively. For a single bound exciton these gave lineshapes resembling a "negative-peak" and a "zero-crossing". The change in overlap was found to be more important for the higher subband transitions, where the Stark shifts are smaller. It was shown that modulation of the exciton linewidth and energy produces similar lineshapes, both being proportional to first-energy-differential, but with their real and imaginary parts interchanged. As a consequence a significant degree of linewidth-modulation was shown to just change the phase of the ER lineshape.

Having calculated the field modulated dielectric function of a QW, the next step was to relate this to the ER and ET of SQWs and MQWs. The reflectance of a sample structure was determined by considering optical interference between the rays reflected from its various interfaces. The air/semiconductor boundary at the front surface has a much larger reflectivity than any other interface, so it was shown a good approximation to consider only this ray along with those from the QW(s). The reflectivity of an isolated QW was determined by considering interference of the rays from its two barrier interfaces. It was shown that these tend to cancel, resulting in the overall QW reflectivity

being small, for typical parameters.

For SQW structures it was therefore appropriate to consider only the rays from the front surface and the first-order reflection of the QW. The validity of this approximation was verified by comparison with a more exact transfer matrix calculation considering the reflections from all the sample interfaces. The analysis yielded the result that the ER is given by a linear sum of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, with the mix being determined by the lineshape phase, which corresponds to the optical phase delay between the rays from the front surface and the QW. The lineshape will therefore be sensitive to the overlayer thickness between the QW and front surface and also the angle of the light in the overlayer. The calculated lineshape was found to give good agreement with the experimental spectrum of a SQW.

The SQW ER lineshape formula will be tested much more rigorously in the next chapter, where ER measurements on five different SQWs are reported. In particular the dependence of the lineshape on the angle of the incident light will be examined. Varying the angle of incidence changes the lineshape phase, and hence the ER for the E1H1 and E1L1 excitons can be expected to cycle between lineshapes resembling $(\delta L_r/\delta E_{ex})$ and $(\delta L_i/\delta E_{ex})$. This very dramatic evolution of the lineshape will be a clear test of the validity of the lineshape formula. Least squares fits will also be made to experimental spectra to obtain a closer comparison with the model. This will allow the transition energies, linewidths and strengths to be accurately extracted from the spectra.

The model of optical interference was then extended to consider ER of MQW structures. As for SQWs, only the first-order reflections from the QWs were considered, along with the strong reflection from the front surface. The QWs in the stack were assumed to be identical and regularly spaced, in order to derive another explicit lineshape formula. This was again verified by more exact transfer matrix calculations. The lineshape was calculated for different numbers of QWs in the stack, and found to become increasingly more complicated with the stack thickness. In particular, increasingly strong structure in the wings of the lineshape was seen as the number of wells was incremented. The strength of the MQW ER signal did not increase significantly with the number of wells, in contrast to other spectroscopies, such as PC, PLE, absorption, or ET.

The analytical approach adopted here allowed the origin of these very complicated lineshapes to be understood. Unlike spectroscopies that depend solely on the imaginary part of the QW dielectric function,

the reflectance of each QW in the stack has a different lineshape, depending upon its depth below the sample surface. The rays reflected from each QW will tend to interfere destructively, so that the overall reflectance of the MQW shows only contributions from the edges of the stack. This destructive interference results in the overall reflectivity of the MQW being not much stronger than that of a SQW.

The MQW ER lineshape was decomposed into these two contributions; one from the front of the stack and the other from the back. The contribution from the front dominated the centre of the ER exciton lineshape. Meanwhile the contribution of the back of the stack was limited by absorption at energies near the centre of the exciton profile, but produced large features in the wings of the lineshape. Hence the phase of the features at the centre and in the wings of the lineshape was determined by the depths of the front and the back of the stack respectively. The structure in the wings of the lineshape, strengthens with the number of QWs in the stack, due to the increase in the total absorption.

ET measurements on GaAs/AlGaAs MQWs can be made after etching away the absorbing substrate. The front and the back of the cavity have a strong reflectivity, which results in large Fabry-Perot oscillations in the transmission, when the absorption in the cavity is small. It was shown that for a MQW with a large number of wells in the stack, the ET will be approximately determined by $\Delta\epsilon_i$ alone. However, for a SQW, the ET lineshape will also have some dependence on $\Delta\epsilon_r$, due to rays that are transmitted after a single reflection from the QW and one of the interfaces of the cavity. The ET spectra of MQWs will not be affected by the position of the QWs, and the strength of the signal will increase with the number of QWs, in contrast to ER of MQWs. This will make the ET spectra of MQWs easier to interpret than ER spectra, which can show extra structure due purely to interference effects, as discussed above.

The calculations for both SQWs and MQWs considered only the lineshape produced by the ground state exciton. There will also be a smaller contribution due to the continuum and excited excitonic states of each subband transition. A more complete dielectric function was not considered here, as this would introduce a large number of parameters, obscuring the effects produced purely by optical interference. Absorption by the E1H1 continuum may have an especially significant effect on the MQW ER lineshape especially, as it will tend to reduce the higher energy side of the ground state excitonic feature, deriving from the back of the stack.

The QWs were assumed to have an identical dielectric function for

the derivation of the MQW ER lineshape formula. Measurements on two GaAs/Al_{0.3}Ga_{0.7}As MQWs are presented in Chp. 6, which show that there is a systematic variation in the transition energy across the stack. In this case the summation over the different wells in the stack can no longer be performed easily. Analytical analysis is then more difficult, although a numerical solution using an optical transfer matrix approach is possible. A variation in the transition energy will severely alter the contribution from the back of the stack, as it will then experience a different absorption by the shallower layers. The model for the MQW ER lineshape is therefore inappropriate for exact lineshape fitting of the experimental spectra measured in Chp. 6.

Despite the limiting assumption of a homogeneous stack, the basic principles discussed here will be still valid for analysis of the MQW ER lineshapes in Chp. 6. Features are seen in the the MQW ER spectra due to the front and back of the stack, the lineshapes of which are found to depend upon their depths in the sample and the absorption in shallower layers. It is shown in both Chps. 6 and 7, that the angular dependence of the features observed in the ER spectra is particularly useful in determining the depth of the layers producing them. This information on layer depths can not be extracted from spectra depending solely on the imaginary part of the QW dielectric function, such as PLE, PC or ET. Hence the use of ER in conjunction with other spectroscopies can be a powerful tool in determining the microscopic structure of a stack of QWs.

Excitons are rapidly ionised by an applied electric field in bulk semiconductors and so the ER spectrum will be dominated by contributions from the continuum, in contrast to QWs. The Franz-Keldysh effect results in the modulated dielectric function decaying exponentially below the bandedge, while showing oscillations that extend over a wide energy range above the gap. It was shown that there will be a "beating" of the Franz-Keldysh oscillations, for field modulations where the lower extreme is non-zero. For a small modulation, the period of the Franz-Keldysh oscillations is determined by the d.c. offset field, while the beat period is dependent on the size of the modulation. This beating phenomenon was observed experimentally, and the measured spectra were in qualitative agreement with those calculated. The most major limitation of this calculation of the bulk modulated dielectric function is the neglect the finite broadening parameter and also the Coulombic interaction.

The classical analysis of bulk semiconductor ER considers a semi-infinite slab of material, subject to an uniform electric field. Hence

the only interface is that situated at the front surface, and its large discontinuity in ϵ_r at energies near the fundamental bandedge, results in the ER being approximately proportional to $\Delta\epsilon_r$ alone. This is in complete contrast to LDS systems, where the QWs are necessarily buried beneath a barrier layer. For LDS systems the interference of the rays from the front surface of the sample and the QW(s), results in the ER being determined by a mixture of $\Delta\epsilon_r$ and $\Delta\epsilon_i$. However, optical interference will also be important in bulk semiconductors, if the region subject to the field is buried inside the structure, such as in a sample with a $p^+ - i - n^+$ doping profile. In such a case the ER is again expected to be determined by a linear mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ as for QW systems, but with $\Delta\epsilon$ being determined by the Franz-Keldysh effect.

4 Methods of Experimentation and Analysis

4.1 Experimental Methods

Fig. 4.1 shows a schematic diagram of the apparatus used in the ER experiments performed here. This versatile arrangement can be easily modified to also perform ET, PC, transmission or reflectivity measurements. In electromodulation experiments an alternating bias is supplied to the sample and the reflected or transmitted beam detected in the Silicon photodiode. The lock-in amplifier receives its reference waveform from the signal generator, so detecting the a.c. component of the signal in the photodiode, which constitutes the ER or ET signal. For unmodulated measurements of reflection or transmission, the beam incident on the sample is mechanically chopped and the lock-in reference taken from the chopping unit. Likewise PC signals can be measured by attaching the sample to the current input of the lock-in, so that the sample itself acts as the detector. The various aspects of the system are now considered individually.

4.1.1 Incident Light

A monochromatic beam was produced using a broad-band 80 W Tungsten or 450 W Xenon lamp, filtered by a single grating monochromator of 0.3 m focal length. The second- and higher-order diffracted beams from the grating were removed by a low pass optical filter. The spectral width, which is set by the spectrometer slit size, was varied depending on the linewidth of the sample, and it was estimated by measuring the width of the system response produced by a narrow line source.

There is always a small discrepancy between the spectrometer scale and the true wavelength, due to a slight misalignment of the grating position. This discrepancy was found to vary with the scale setting in a non-linear fashion, changing from 8 nm when the scale read 600 nm, to 13 nm at 900 nm, for one of the gratings. The variation could be closely approximated by a second-order polynomial, the coefficients of which were determined by fitting the measured wavelengths of a large number of known spectral lines, produced by standard Helium and Mercury sources, to their tabulated values. When the grating was removed, and then replaced again, the error in the wavelength scale was found to change by less than 0.2 nm. The kinematic mounting of the grating ensures the reproducibility of its calibration. However, the scale was recalibrated each time the grating was moved, in order to maximise its

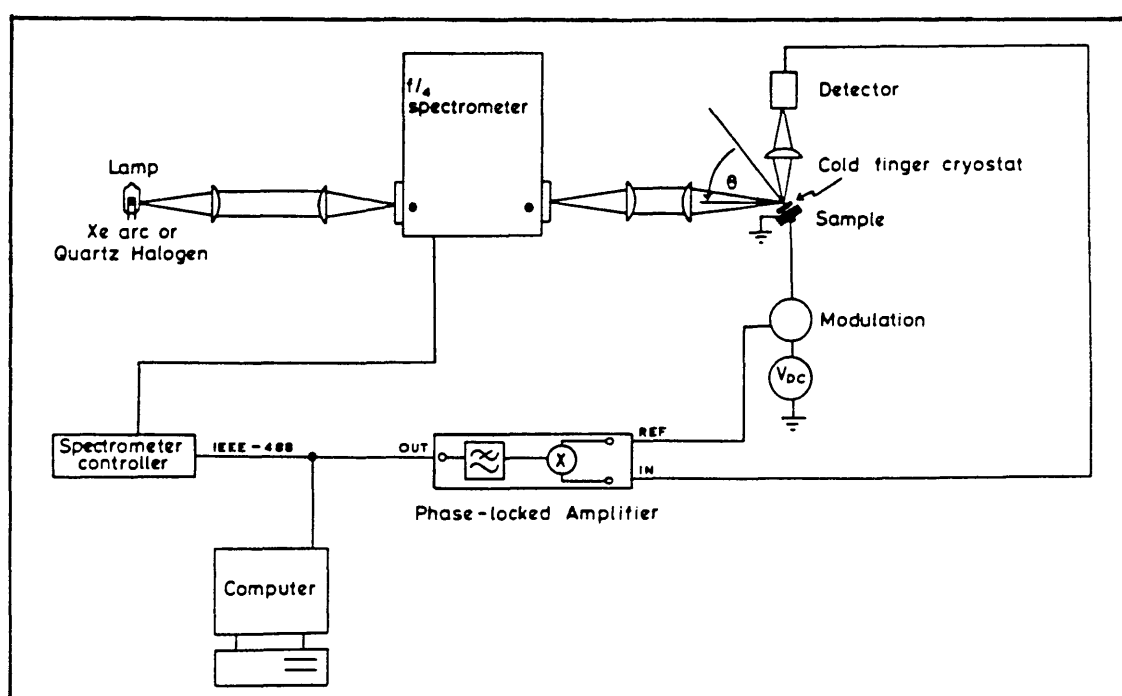


Fig. 4.1 : Schematic diagram of the apparatus used for ER experiments.

a hole in the cold-finger immediately behind the sample.

Spectra could be taken as a function of angle of incidence in the continuous-flow cryostat, by mounting the sample on a rotating insert, which had a plane mirror placed at 90° to the sample plane. This steered the reflected beam out through the same cryostat window as it entered, after which it was deflected by a 50% reflecting beamsplitter, into the Silicon photodetector. In the closed-cycle cryostat, the angle of incidence was restricted by the position of the cryostat windows to be either near 0° or 45° for reflection measurements.

4.1.3 Detection

The reflected or transmitted beams were measured in a Silicon photodiode connected to the current input of the lock-in, except for PC measurements where the sample itself was used at the detector. The small electromodulated signal was detected by setting the reference frequency of the lock-in with the output of the signal generator. Meanwhile the d.c. signals were measured by mechanically chopping the incident beam. The frequency of the bias modulation or mechanical light chopper was chosen at a value where the noise was minimal, usually between 70 and 420 Hz and away from the harmonics of the mains supply. Typically the pre- and post-integration times of the lock-in were set to be 0.3 and 0.1 sec respectively.

4.1.4 Sample Biasing

Electric fields were generated across the intrinsic region of the sample studied here by applying a bias voltage. All the samples measured in this thesis had one of two different doping arrangements; the intrinsic region was either sandwiched between highly doped p^+ and n^+ regions or else lay at the top of the structure above highly doped layers. In the first case, ohmic contacts were made to the p^+ and n^+ regions to form a p^+-i-n^+ diode, while in the second, a thin semitransparent layer of Gold was evaporated on top, to act as a Schottky barrier. The doping levels of the various regions were determined by capacitance-voltage (C-V) measurements, as described in Sec. 4.2. Most of the samples were processed into mesas in order to obtain larger breakdown voltages. This processing was done either by Graham Monnington at the Royal Signals and Radar Establishment, Malvern (RSRE), Tony Rivers at University College London (UCL), or by myself at Imperial College.

accuracy.

All the spectra have been normalised to the system response. This was measured separately by placing the Silicon photodetector directly in the chopped incident beam. Alternatively, a small proportion of the incident beam could be removed using a weakly reflecting beamsplitter and measured in a second detector and lock-in. The former method proved more convenient and was therefore used more frequently. The Xenon lamp produced a number of very large spikes, where the light intensity could increase by an order of magnitude in less than 1 nm. Unfortunately these coincided with the spectral region of the room temperature bandgap of a GaAs/AlGaAs QW. The presence of the spikes in the spectra could not be removed using either of the above methods of normalisation, presumably because of non-linearity of either the sample or the Silicon detector. Consequently the Tungsten lamp, which had a slowly varying response, was used over this spectral range. The use of the Xenon lamp was restricted to low temperature measurements, or to room temperature measurements on very narrow QWs, where the spectral features lay to higher energy than the spikes.

4.1.2 Sample Mounting and Cryogenics

For room temperature measurements the samples were mounted on an xyz-translation pin on top of a rotation stage. In this way the sample could be accurately positioned on the rotational axis. The angle between the incident beam and the normal to the sample surface could be varied with an accuracy of about 0.5° , using the scale on the rotation stage.

Low temperature measurements were made in either a closed-cycle or a continuous-flow variable temperature Helium cryostat. The temperature was estimated from the resistance of a calibrated Silicon diode in thermal contact with the sample. The closed-cycle and continuous-flow cryostats could produce minimum temperatures of about 17 and 2 K respectively. Care was taken to avoid straining the sample in low temperature experiments, due to the different thermal expansion coefficients of the sample and its holder. The strain was minimised by attaching the sample to its mounting by just a single corner. Most of the measurements in this thesis were made in the closed-cycle cryostat, where the sample is cooled by attaching it to a cold-finger, suspended in a vacuum. Hence care had to be taken to ensure good thermal contact between the sample holder and the cold-finger. This was particularly difficult in the transmission geometry, where there must necessarily be

For the electromodulation measurements a square wave bias voltage was applied to the sample. The peak-to-peak (ptp) height of this square wave, was kept as small as possible, so that the spectra could be treated as arising from a small change in the transition energy, linewidth or strength and therefore be differential in nature. Increasing the size of the modulation was found to broaden many spectra, resulting in some features becoming unresolved. For each sample the modulation was reduced until the lineshape was found to stop changing. Spectra were taken with different offset voltages and a fixed size modulation. The size of the voltage offset and the modulation were measured with a digital voltage meter and/or an oscilloscope. Care was taken not to exceed the reverse breakdown voltage or the forward bias current threshold of the diode.

Unmodulated spectroscopies were performed as a function of field by applying a variable bias to the sample, using a battery and potentiometer. The photocurrent could also be scanned as a function of the applied bias, at a fixed wavelength, by connecting the sample to the analog output of the lock-in. Care was taken in the PC measurements, especially when using a cryostat, that neither of the sample leads were connected to earth, which would have introduced noise into the circuit.

4.1.5 Microcomputer Control

The experiment was controlled by a microcomputer, whose basic function was to step the monochromator through a range of wavelengths, and read the signal detected by the lock-in. The software allowed the limits and point separation of the wavelength range to be chosen. The signal could be averaged in a combination of three ways; by increasing the lock-in time constants, the number of reads at each wavelength, or the number of scans. This allowed small signals to be averaged over many hours to obtain clean spectra. At the end of the scan the data was stored on a diskette, along with the details of the experiment.

Several advantages were gained by digitising the spectra. They could then be numerically divided by the system response and corrected for the error in the wavelength scale. The spectra could be plotted, with chosen scales for the axes, in a number of different formats, for example as a function of the photon energy or superimposed on several other spectra taken under slightly different conditions. The spectra could also be readily processed in various ways, for instance, taking the differential or integral or finding all the maxima and minima. It also allowed a numerical comparison, using least squares fitting, of

experimental ER spectra and lineshape models, as described in Chp. 3. It was especially time conserving that this processing or lineshape fitting could be performed repeatedly on different spectra, taken at, for instance, various electric fields or angles of incidence.

The software gave the option of reading a second lock-in at each wavelength point, allowing the system response to be simultaneously measured, or two signals, for example the PC and the reflectivity, to be recorded in the same scan. There was also the option of reading the voltage across the thermal diode, during the experiment and converting this to a temperature.

In addition to scanning the wavelength, the routine could program the lock-in to provide a variable d.c. bias, in order to study signals as a function of voltage. The variable analog voltage port of the lock-in was also used in a separate section of the software that measured the current-voltage (I-V) characteristic of the sample. The current was measured in a ammeter also connected to the bus of the microcomputer. The lock-in could also be programmed to measure the noise as a function of frequency, in order to chose the best setting for the signal generator or chopper.

4.1.6 Photoluminescence

PL measurements were made on a separate optical table. The PL was produced by a 4W Argon ion laser, with the sample mounted in the continuous-flow Helium cryostat. The light was filtered by a double grating monochromator, with a focal length of 0.85 m, and detected in a extended S20 photomultiplier tube, employing photon counting. Again the experiment was computer controlled by a routine similar to that for the ER arrangement. PLE experiments could be performed using a Xenon arc lamp and single grating spectrometer source, similar to that described in section 4.1.

4.2 Electric Fields in p^+i-n^+ Diodes

A uniform electric field can be conveniently applied across a region of semiconductor by arranging it to lie inside the intrinsic region of a p^+i-n^+ diode. This technique is frequently employed in devices exploiting the QCSE. In a perfect diode the doping of the n- and p-type regions is very high, while that of the i-region is negligible, so that the diode acts like a parallel plate capacitor, with

all the applied voltage dropped across the i-region. The field (F) across the i-region is then uniform and given by the total voltage drop, which is the sum of the applied bias (V_{app}) and the built-in voltage (V_{bi}), divided by the intrinsic region thickness (L_i); $F = (V_{bi} - V_{app})/L_i$. Since V_{bi} is positive here, a negative value of V_{app} corresponds to the diode being placed under reverse bias.

In a real device the doping of the n^+ - and p^+ -regions will not be infinite, resulting in part of the voltage also being dropped in these layers, consequently producing a smaller field in the i-region. The i-region may also have a significant background doping level, resulting in a change in the electric field across the region, as predicted by Gauss' law. This has important consequences for devices containing a MQW within a p^+ -i- n^+ diode, as the field and hence the Stark shift is different for each QW in the stack, resulting in a smearing of the optical spectra. Chp. 6 describes experiments on MQWs where this spatial variation of the electric field is important. In samples with only one QW (ie a SQW), this broadening will not arise, of course, and furthermore, the field at the midpoint of the i-region, the position of the QW in all of the SQW structures studied here, is fortunately not affected at moderate or large biases. However, it is shown below, that the field at the midpoint will be reduced at the lowest biases, when the i-region is not fully depleted.

The bias-dependent field profile of a real p^+ -i- n^+ structure can be deduced by application of Gauss' law and invoking charge conservation. If the doping concentrations are assumed uniform in each region, they each have a linear field variation, as shown in Fig. 4.2. The analysis involves straight forward algebra, yielding the field at the midpoint of the i-region to be approximated by

$$F \approx \{ (V_{bi} - V_{app}) / L_i \} \cdot \{ 1 / (1 + \pi/n) \} \quad \dots 4.1$$

where p , π and n are the doping concentrations of the p^+ -, i- and n^+ -regions respectively and the i-region is assumed to be lightly p-type, which is often the case for undoped MOVPE AlGaAs, where carbon acceptors are the most abundant impurity [4.1-2]. Eq. 4.1 includes the first-order correction to ideal behaviour, due to the doping level of the n^+ -region being comparable to that of the i-region. At very large applied biases, the second-order (and higher) terms in (π/n) become increasingly important and Eq. 4.1 is then no longer accurate. However, for the SQW samples studied in Chp. 5, Eq. 4.1 is a good approximation for $(V_{bi} - V_{app}) \ll 500$ V.

The other important deviation from ideal behaviour occurs at lower biases and derives from the background impurity charge of the i-region not being fully depleted. The above analysis for a p^+-n-n^+ is then no longer valid. By considering a $n-n^+$ junction, it can be shown that the i-region becomes completely depleted at an applied biases V_{dep} , given by

$$(V_{bi} - V_{dep}) \approx \left\{ \frac{e \pi L_i^2}{2 \epsilon_{rel} \epsilon_0} \right\} \cdot (1 + \pi/n) \quad \dots 4.2$$

As expected $(V_{bi} - V_{dep})$ increases with the background doping level and width of the i-region. At voltages such that $|V_{bi} - V_{app}| < |V_{bi} - V_{dep}|$, the field is dependent on the square root of the total voltage drop, instead of the linear relationship of Eq. 4.1. The field at the midpoint of the i-region is then :

$$F \approx \left\{ \frac{2 e \pi (V_{bi} - V_{app})}{\epsilon_{rel} \epsilon_0 (1 + \pi/n)} \right\}^{\frac{1}{2}} - \frac{e \pi L_i}{\epsilon_{rel} \epsilon_0} \quad \dots 4.3$$

As the applied bias is reduced, the depletion edge eventually reaches the midpoint and the field falls to zero, at an applied bias given by :

$$(V_{bi} - V_{app}) = (V_{bi} - V_{dep}) / 4 \quad \dots 4.4$$

It is therefore important to determine the doping levels of the various regions before estimating the field. These can be deduced from capacitance measurements performed as a function of bias. Orthodox C-V measurements on the diodes yield the "junction" carrier concentration $(1/\pi + 1/n)^{-1}$, at biases where the i-region is not fully depleted [3.36]. Since usually $n \gg \pi$, this approximates to the level in the i-region. Alternatively, a doping profile for the entire structure can be produced by etching the sample and making C-V measurements between the exposed surface and a Schottky contact [4.3]. The latter method accurately determines the concentration of the highly doped n^+ - and p^+ -regions, but is unreliable for the i-region, due to the etch progressing at different rates across the sample surface.

Such C-V measurements were used to determine the doping levels in the samples studied in this thesis. Etched C-V measurements were made at the University of Sheffield on the four MOVPE grown p^+-i-n^+ SQW samples studied in Chp. 5, allowing the estimates of $p = 1 \times 10^{18} \text{ cm}^{-3}$ and $n = 3 \times 10^{17} \text{ cm}^{-3}$. There was some fluctuation between the samples; specifically the doping of the n^+ -region was determined to be about

$4\text{-}6 \times 10^{16} \text{ cm}^{-3}$ in two of the mesas. Inspection of Eq. 4.1 shows that this causes a reduction in the field of less than 2%, which is not significant. In any case, the low estimate for n could derive from an uneven etch during the C-V measurement, since the n-type layers lie at the back of the structure. For simplicity all the SQW samples were taken to have the same p and n doping levels, the values quoted above, since moderate fluctuations about these values will not significantly alter the field.

The doping of the intrinsic region in these samples was determined using orthodox C-V measurements. These produced a doping profile which varied over the intrinsic region, reaching a maximum at the QW. As this was probably due to charge trapping effects in the well, the minimum value in the doping profile was taken, which was about $2 \times 10^{15} \text{ cm}^{-3}$. A larger value of n does not greatly alter the electric field at biases where the i-region is fully depleted; for instance a value of $1 \times 10^{16} \text{ cm}^{-3}$ produces only a 4% reduction. However, it does have a dramatic effect on the fields at the edges of the i-region, as shown in Fig. 4.2. Furthermore, a larger bias is then required to deplete the i-region, before the linear relationship (Eq. 4.1) becomes valid. Therefore the background doping of the i-region will significantly reduce the field at its midpoint at low biases.

Fig. 4.3 shows the variation of the field at the centre of the intrinsic region with the applied bias for the parameters relevant to the MOVPE SQW mesas studied in Chp. 5; $L_i = 0.819 \text{ } \mu\text{m}$, $V_{bi} = +1.9 \text{ V}$, $n = 3 \times 10^{17} \text{ cm}^{-3}$, $p = 1 \times 10^{18} \text{ cm}^{-3}$, $n = 2 \times 10^{15} \text{ cm}^{-3}$ and $\epsilon_{rel} = 11.77$. The field was also calculated with a larger value of the i-region doping, $n = 1 \times 10^{16} \text{ cm}^{-3}$. The value of the relative dielectric constant of the i-region, ϵ_{rel} , was taken as the value measured by Aspnes et al [3.35] for $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ at the energy of the QW bandgap. The curves are an exact calculation, not employing the approximations of Eqs. 4.1 to 4.3, although the deviations between these equations and the exact relations are negligibly small. It can be seen that for the larger intrinsic region doping, it takes a significant voltage ($V_{app} \approx +0.7 \text{ V}$) to produce a field at the midpoint, and this field is reduced below the ideal value until the i-region becomes fully depleted (at $V_{app} \approx -3 \text{ V}$). As predicted earlier the field is close to that expected for a perfect p^+-i-n^+ diode at larger reverse biases.

The width of the intrinsic region and the built-in voltage have a more critical effect on the value of the field than the doping levels. The intrinsic region thickness was taken in the above calculation to have the value specified in the growth procedure. The Fermi level

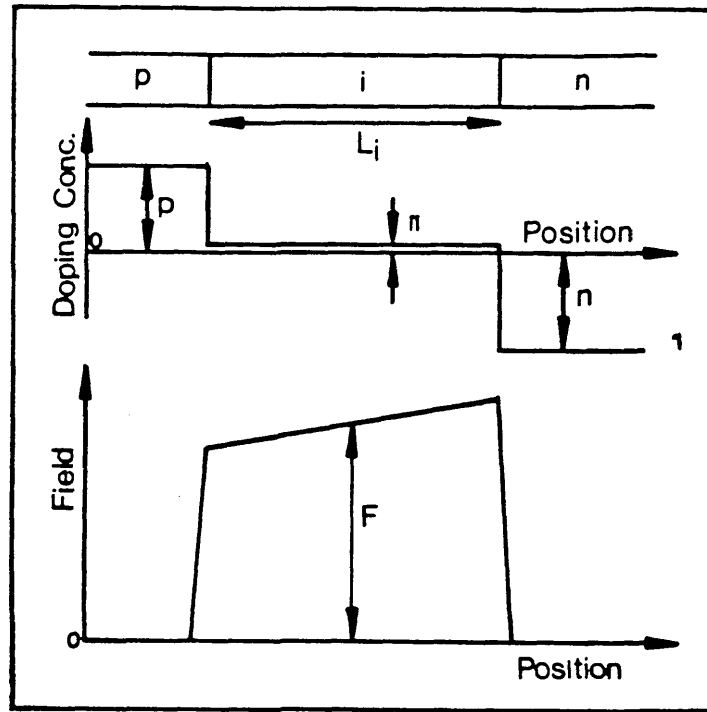


Fig. 4.2 : Schematic of the electric field profile in a p^+-i-n^+ diode, in which each region is uniformly doped and the i-region is lightly p-type.

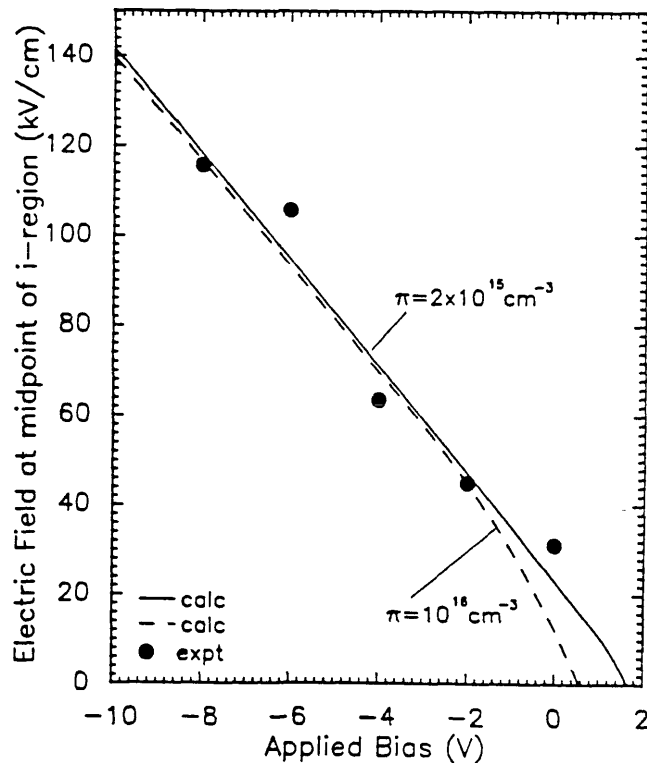


Fig. 4.3 : The calculated bias dependence of the electric field at the centre of the i-region of a p^+-i-n^+ diode, taking the i-region impurity concentration to be $\pi = 2 \times 10^{15}$ (solid line) or $1 \times 10^{16} \text{ cm}^{-3}$ (dashed). The data points show the experimental electric field, deduced from the ER spectra of a region of bulk $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ semiconductor contained inside a p^+-i-n^+ .

should lie close to the acceptor (donor) level in the highly doped $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$ p^+ (n^+) layers at low temperature. The built-in voltage of the diode should therefore be approximately given by the bandgap minus the donor and acceptor binding energies, which yields a value of about 1.9 V. Further evidence for this choice was given by the low temperature current-voltage characteristic, where the current was seen to increase sharply near +1.9 V.

Verification of the electric fields calculated in this manner was sought from the ER signal from the bulk $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ layers included in the structure of the MOVPE grown SQWs studied in Chp. 5. These bulk layers are also contained in the i-region of the p^+ -i- n^+ doping profile of the samples. It was shown in Sec. 3.5 that the ER spectra of bulk semiconductors show oscillations above the bandedge, whose period increases with the applied electric field, as determined by Eq. 3.60. A value of the electric field was fitted to the measured positions of the zero-crossings in the ER spectra, taken at several applied biases on one of the SQWs (MV307) studied in Chp. 5.

The values of the electric field estimated from the Franz-Keldysh oscillations at different applied biases are plotted as the data points in Fig. 4.3. It can be seen that these agree with the expected calculated field (for $n = 2 \times 10^{15} \text{ cm}^{-3}$ curve) to within 10 kV/cm at all applied biases. There is no perceivable systematic difference in the expected and fitted field that suggests an error in the value of any of the parameters used. Although the Franz-Keldysh oscillations of the barrier material do not allow an accurate determination of the field at any particular applied bias, they at least confirm that the developed field is close to that expected. There may be some error in the field determined from the ER of the barrier layers, due to non-uniformity of the electric field across the intrinsic region, caused by the background impurity level. Also the oscillations may be obscured by features from the highly doped $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$ regions, where the electric field, although falling to zero within a small distance (corresponding to the depletion depth), is also modulated.

4.3 Calculation of the Quantum Confined Stark Effect

Measurements of the QCSE in $\text{GaAs}/\text{Al}_x\text{Ga}_{1-x}\text{As}$ SQWs and MQWs presented in Chps. 5-7 are modelled using the effective mass, envelope function analysis detailed in Sec. 2.6. This section discusses the parameters used in the calculations. The electron and hole resonances

are assumed to be narrow (see Sec. 2.6.1.4), so that the electron and hole z -motions can be solved separately. The $n = 1$ bound exciton transition energy, overlap and linewidth component, due to the field-induced tunnelling, are then given by Eqs. 3.2-4. The z -motion of the electron and hole are solved using the exact analysis of Sec. 2.6.1. Meanwhile, the relative xy -motion is solved variationally by assuming a separable form for the exciton envelope function, as described in Sec. 2.6.2. This model neglects the mixing of the valence subbands, discussed in Sec. 2.2.

The parameters used for calculations of the field dependence of the z -motion of the electron and hole in GaAs/Al_xGa_{1-x}As QWs are listed in table 4.1. The confinement energies and the electron-hole overlap integral are most sensitive to the parameters of the well itself; its width and effective masses. Meanwhile, the linewidth of the quasi-confined states is most sensitive to the barrier height and mass. The barrier parameters also have a significant effect on the energies of the states near the top of the well, as then a significant part of the its envelope function lies in the barrier.

Parameter	Value	Ref.
E_g (eV)	1.247.x	4.4
m_{el}	0.067 + 0.0835.x	2.4
m_{hh}	0.34 + 0.412.x	2.4
m_{lh}	0.094 + 0.066.x	2.4
η	0.65	2.3

Table 4.1 : Parameters for Al_xGa_{1-x}As used in calculation of z -motion.

The well effective masses are taken as the values now widely employed, which have been found to give the best agreement with measured energies of allowed and forbidden transitions [2.3-4]. The barrier masses are more uncertain, but have only a small effect on the energies of the lowest confined levels. The fraction of the bandgap discontinuity taken up by the conduction band has been estimated by several recent studies to lie in the range 0.57 to 0.68 [2.3-2.6], with the value $\eta = 0.65$ being used here.

For the xy -motion, the relative dielectric function and electron-hole reduced mass are taken to have the same value in the well and barrier. The value of ϵ_{rel} was taken as an average of that for the well and barrier material, $\epsilon_{rel} = (\epsilon_w \cdot \epsilon_b)^{\frac{1}{2}} = 12.2$, using the values quoted for bulk Al_xGa_{1-x}As in Ref. 3.35. The effective masses for the xy -motion were chosen to be consistent with the Luttinger parameters

determining the z-direction masses of the well, as discussed in Sec. 2.2. Thus the electron-heavy hole and electron-light hole reduced masses for the xy-motion were taken as 0.042 and 0.050 respectively.

The thickness of an epitaxially grown layer is often quite different from that intended, so the well width was fitted from a measured transition energy. This was done for each of the SQW and MQW samples studied in Chps. 5 and 6, using the E1H1 PC peak at 0 V and room temperature. The room temperature energy was chosen, because this eliminates errors in the value of the low temperature bandgap, due to uncertainty in the sample temperature and small unavoidable strain shifts. Also the transition can be measured at a lower bias for SQWs at room temperature, as discussed in Sec. 5.4, reducing the uncertainty in the Stark shift. The room temperature GaAs bandgap was taken to be 1.4247 eV [4.4].

4.4 Electroreflectance Lineshape Fitting

In the least squares method a set of N data points, (x_1, y_1) , $(x_2, y_2) \dots (x_N, y_N)$, such as an experimental ER spectrum, are compared to a model relationship $y = f(x)$. Certain specified parameters in the model relationship are then varied in order to minimise Q^2 , where

$$Q^2 = \frac{\sum_{i=1}^N \{ y_i - f(x_i) \}^2}{\sum_{i=1}^N \{ y_i \}^2} \quad \dots 4.5$$

This can be done numerically by considering small variations in the model parameters [4.5]. The quantity Q in Eq. 4.5, which is referred to here as the "fit quality", can be used as a measure of the agreement between the fitted and experimental set of points. Notice that Q has been normalised to the absolute strength of the ER spectrum, so that the values obtained for different spectra can be compared.

Chps. 3 and 5 report least squares fits made to ER spectra taken on GaAs/Al_xGa_{1-x}As SQWs, using the model detailed in Sec. 3.3. This provides a detailed comparison of the data and lineshape model, as well as allowing the parameters of the lineshape model to be extracted.

For the lineshape fit shown in Fig. 3.14, the theoretical lineshape was calculated in the following manner. The QW dielectric function was taken to be comprised of two contributions due to the E1H1 and E1L1 ground state bound excitons, as given by Eq. 3.1, with a

Gaussian broadening profile (Eq. 3.6). The modulated dielectric function was calculated by allowing a field-induced change in the exciton energies and overlaps, calculated using the method described in Sec. 4.3, and a field-induced change in the linewidths, determined from that measured in the PC spectra. The best fit was then found by varying seven parameters; the overlayer thickness, the E1H1 and E1L1 exciton energies, the E1H1 and E1L1 linewidths, the strength of the E1L1 feature relative to E1H1 and a scaling factor.

This analysis makes several approximations, discussed in Sec. 3.3. The layers between the QW and the front surface are replaced by an effective overlayer, the QW reflectivity is written as a linear function of its dielectric function and the double reflections from the QW are ignored. Each of these were shown to be good approximations in Sec. 3.3. Hence, the lineshape model used here corresponds to curve "a" in Fig. 3.13. Lineshape fits made with the more exact calculations, curves "b" and "c" of Fig. 3.13, were not found to be significantly better or worse, and yielded similar fitted parameters.

Further lineshape fittings are made in Sec. 5.3, using a similar procedure as that described above, except that the field-induced change in the exciton overlap is ignored. As discussed in Sec. 5.3, this is a valid approximation if the linewidth is small. The ER lineshape formula can then be simplified to Eq. 3.29. The lineshape function in Eq. 3.29, $L(E_{\text{ex}}, \Gamma_{\text{ex}}, E) = L_r + i L_i$, is taken to be either Gaussian (Eq. 3.6) or Lorentzian (Eq. 2.48).

Eight parameters were varied in the lineshape fittings made in Sec. 5.3; the energy (E_{ex}), linewidth (Γ_{ex}), phase (g in Eq. 3.29) and strength (C in Eq. 3.29) of each exciton. The small field-induced change in overlap has been ignored here, so as to allow the field dependence of the fitted value for the ER transition strength (C), to be directly compared to that given by Eq. 3.30. Lineshape fittings made using the more general lineshape model, Eqs. 2.26-28, were not found to be significantly better or worse, and yielded similar fitted parameters.

5 Measurements on Single Quantum Wells

This chapter describes measurements on five different GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ SQWs. PC, ER and PL spectra were taken on four MOVPE grown GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ SQWs, which had identical structures except for the width of the well layer, being either 59, 72, 88 or 106 Å. In addition to having very narrow optical linewidths, these samples had excellent electrical properties, allowing the application of large electric fields. Another SQW structure was also studied, this time grown by MBE and containing a 78 Å GaAs/ $\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$ QW. This sample had larger optical linewidths and poorer electrical properties than the MOVPE grown SQWs, but also had a thicker overlayer between the QW and the front surface, making it better suited to study the variation of the ER lineshape with the angle of the incident light.

The experiments described in Sec. 5.2 are mainly concerned with optical interference effects upon the SQW ER lineshape, that were modelled in Sec. 3.3. Eq. 3.26 predicts the ER lineshape of a SQW to be given by a linear mix of the real and imaginary parts of the modulated QW dielectric function, $\Delta\epsilon_r$ and $\Delta\epsilon_i$ respectively. The mix is determined by the "lineshape phase", the optical phase delay between the rays reflected from the QW and the front surface of the sample, g in Eq. 3.26. Sec. 3.2 demonstrated the modulation of E1H1 and E1L1 transitions to be dominated by their field-induced energy shifts. Hence the modulated dielectric function will be determined by the first-energy-differential of the exciton broadening profile, ie $\Delta\epsilon \propto \delta L/\delta E$. The ER lineshape of the E1H1 and E1L1 features will consequently be approximated by a linear mix of $(\delta L_r/\delta E)$ and $(\delta L_i/\delta E)$, depending upon the value of the lineshape phase, as expressed by Eq. 3.29, tabulated in table 3.2 and shown diagrammatically by Fig. 3.15.

Sec. 5.2.1 reports the dependence of the ER spectra of both the MOVPE and MBE grown SQWs on the angle of the incident light. For both sample structures the lineshape is seen to evolve with angle of incidence, due to the change in the lineshape phase, in the manner predicted by the model of Sec. 3.3. Sec. 5.2.2 discusses the subtle differences observed between spectra taken with TE and TM polarised light, which are not predicted by the lineshape model, but are shown to also be related to optical interference effects. Sec. 5.2.3 examines topological variations in the ER lineshape across the wafer.

Sec. 5.3 examines the QCSE in the four MOVPE grown SQWs. Field-dependent PC spectra are taken on each of the four samples, and the evolution of the exciton energies and oscillator strengths are compared

to calculations made using the method detailed in Sec. 2.6. ER spectra are also taken on each of the four SQW samples with different applied offset biases, and lineshape fits are made to these in order to extract the exciton energies, linewidths, lineshape phases and transition strengths. The evolution of the energies and transition strengths are again compared to that predicted by the QCSE. The increase in the exciton linewidth with field, for the four wells, is also discussed and compared to that predicted by the field-induced tunnelling. It is shown that the broadening is determined by inhomogeneous mechanisms.

A minimum field is required to produce a low temperature PC signal and the nature of this threshold, for the MOVPE grown SQWs, is described in Sec. 5.4.1. Field-dependent PL was also measured on some of the MOVPE SQWs (Sec. 5.4.2), in order to study the complementary range of fields not covered by PC spectroscopy. The field dependence of the PL, and also the PC threshold, are dominated by carrier transport effects, and a qualitative explanation of the observed behaviour is presented in Sec. 5.4.3. Sec. 5.4.4 describes low-field ER measurements made on the four SQWs, which showed unexpected behaviour.

5.1 Experimental Details

Five different SQW samples are studied in this chapter. The first is a nominally 100 Å wide GaAs/Al_{0.2}Ga_{0.8}As SQW (GEC256) grown by MBE at GEC Hirst Research Centre by Dr T.M.Kerr. The structure was grown on a n⁺ GaAs substrate and consists of 1.5 µm n⁺ Al_{0.4}Ga_{0.6}As; 0.2 µm undoped Al_{0.2}Ga_{0.8}As graded from Al_{0.4}Ga_{0.6}As; a 100 Å undoped GaAs QW; 0.2 µm undoped Al_{0.2}Ga_{0.8}As graded to Al_{0.4}Ga_{0.6}As; 1.5 µm p⁺ Al_{0.4}Ga_{0.6}As; and finally a 0.2 µm p⁺ GaAs cap. The room temperature E1H1 transition energy measured in this and a previous study [2.68] suggests that the actual well width is 78 Å, which is much narrower than the intended value. Ohmic contacts were made to the n⁺ and p⁺ regions in order to allow the application of an electric field.

Four MOVPE grown SQW samples are also studied, which were grown by Dr. J.Roberts at the University of Sheffield. The samples were grown within a short time span and so should have similar interface quality and doping levels. The layers were grown on a n⁺ GaAs substrate orientated 2° off the [001] crystal axis and consisted of 0.3 µm n⁺ GaAs buffer; 0.96 µm n⁺ Al_{0.35}Ga_{0.65}As; 0.38 µm undoped Al_{0.3}Ga_{0.7}As; an undoped SL; 50 Å undoped Al_{0.3}Ga_{0.7}As barrier; the undoped QW; 50 Å undoped Al_{0.3}Ga_{0.7}As barrier; an undoped SL; 0.38 µm undoped

$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$; $0.96\text{ }\mu\text{m p}^+\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$; and finally $0.24\text{ }\mu\text{m p}^+\text{GaAs}$ cap. The samples were grown to have the nominal well widths of $60\text{ }\text{\AA}$ (MV302), $80\text{ }\text{\AA}$ (MV307), $100\text{ }\text{\AA}$ (MV301), $120\text{ }\text{\AA}$ (MV303). However the room temperature E1H1 exciton transition energy, suggested significantly smaller values of $59\text{ }\text{\AA}$ (MV302), $72\text{ }\text{\AA}$ (MV307), $88\text{ }\text{\AA}$ (MV301) and $106\text{ }\text{\AA}$ (MV303). Notice that the structure includes two SLs, consisting of 6 periods of $\text{GaAs}(15\text{ }\text{\AA})/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}(15\text{ }\text{\AA})$, the purpose of which were to trap carbon impurities in the system, before the growth of the well region. In this way it was hoped to limit the impurity concentration near the well and so produce a narrower linewidth. The bandgap of this SL lies to much higher energy than that of the QW, and so does not obscure the optical transitions between the lowest electron and hole subbands, although it does limit the study of the higher confined levels.

The various layers in the MOVPE grown SQWs were doped so that the QW lies at the midpoint of the intrinsic region of a $\text{p}^+\text{-i-n}^+$ diode, which allows the application of large variable electric fields, as discussed in Sec. 4.2. The samples were prepared into $600\text{ }\mu\text{m}$ square mesas and ohmic contacts made to the front and back regions. This processing removed the GaAs capping layer. The electrical behaviour of the mesas was found to be excellent, allowing reverse biases in excess of -25 V ($\approx 320\text{ kV/cm}$) in some cases. The breakdown voltage was more modest in other mesas, although still very good, and a field in excess of 140 kV/cm could be applied to each SQW. The mesas showed a sharp increase in current, at 17 K , at forward biases of between $+1.7$ and $+2.0\text{ V}$, justifying the value of $+1.9\text{ V}$ taken for the built-in voltage in the calculation of the fields in Sec. 4.2. The impurity concentrations of the various regions, determined by C-V measurements, were also discussed in Sec. 4.2.

Either a 450 W Xenon or a 80 W Tungsten light source, filtered by a single grating monochromator, were used for the measurements. The spectral resolution was chosen to be much smaller than the linewidth of the sample being measured. All the spectra were normalised to the spectrum of the incident beam. The room temperature measurements were made with the sample mounted on a calibrated rotation stage, so that the angle between the light and the normal to the sample surface could be varied, to an accuracy of better than $\frac{1}{2}^\circ$. The axis of rotation was aligned to be perpendicular to the plane of incidence to better than 5° . For the low temperature measurements, the samples were mounted on the cold-finger of a closed-cycle He cryostat, and a calibrated thermal diode mounted on the finger indicated a temperature of about 17 K .

The increase in the E1H1 transition energy was found to be a few meV smaller than expected after cooling from room temperature, suggesting the sample temperature to be higher than that of the cold-finger. Care was taken to avoid introducing strains upon cooling the sample, as discussed in Sec. 4.1. A good measure of the degree of strain in the sample is the splitting of the E1H1 and E1L1 transitions, due to its sensitivity to uniaxial stress [2.22]. Upon cooling the separation of the E1H1 and E1L1 exciton peaks was found not to change significantly.

5.2 Optical Interference Effects on the Electroreflectance Lineshape

5.2.1 Angle of Incidence Dependence

5.1.1.1 Sample GEC256

Fig. 5.1 shows ER spectra taken on sample GEC256 at room temperature, with the light polarised in TE configuration, and incident at different angles relative to the normal of the sample surface. The applied voltage was modulated by 1 V_{ptp} around a d.c. offset of 0 V. The spectra show strong features related to the E1H1 and E1L1 exciton transitions at 1.463 and 1.475 eV respectively. Notice how the lineshape of the E1H1 feature evolves from a "negative-zero-crossing" at 20°, to a "positive-peak" at 40° and then a "positive-zero-crossing" at 66.5°. This sequence of lineshape evolution is in agreement with that predicted by Eq. 3.29, table 3.2 and Fig. 3.15, remembering that the lineshape phase decreases with increasing angle of incidence.

The ratio of the principal peaks (R_{pp}) in the ER spectra, defined as the height of the second largest peak in the lineshape divided by the height of the largest (see inset of Fig. 5.2), are plotted against the cosine of the angle in the overlayer in Fig. 5.2. (For the moment concentrate on the TE data, corresponding to the spectra in Fig. 5.1. The TM data will be discussed later). The value of R_{pp} has a maximum value of unity for the zero-crossings and a minimum of about 0.4 for the positive-peak. For a QW dielectric function comprised of a single exciton this plot would be expected to be symmetric about the minimum, provided that the broadening profile is also symmetric. The asymmetry of Fig. 5.2 is due to the overlap of the E1L1 feature with the E1H1 lineshape. The E1L1 contribution reduces the size of the higher energy subsidiary peak of the E1H1 feature at angles of incidence near 40°, resulting in the two subsidiary peaks of E1H1 becoming equal in height

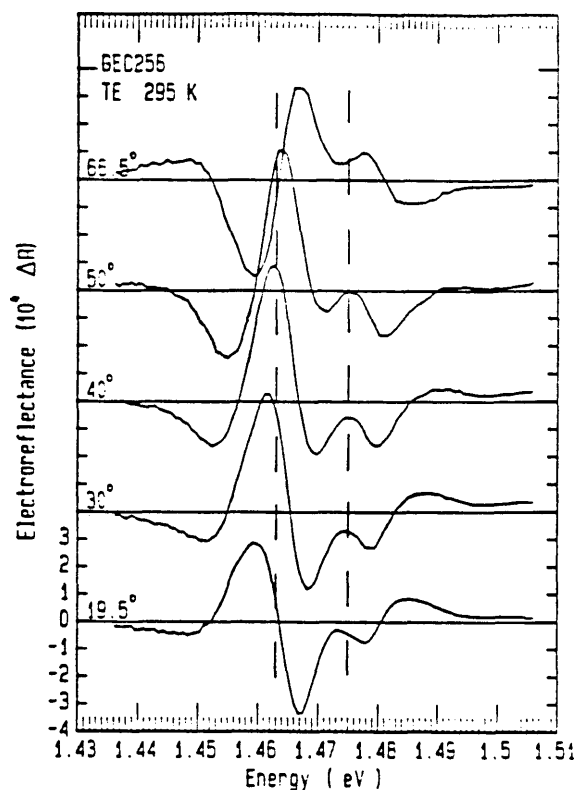


Fig. 5.1 : ER spectra taken on sample GEC256 at room temperature, with the light incident at different angles to the normal of the sample surface (θ_0) and in TE polarisation.

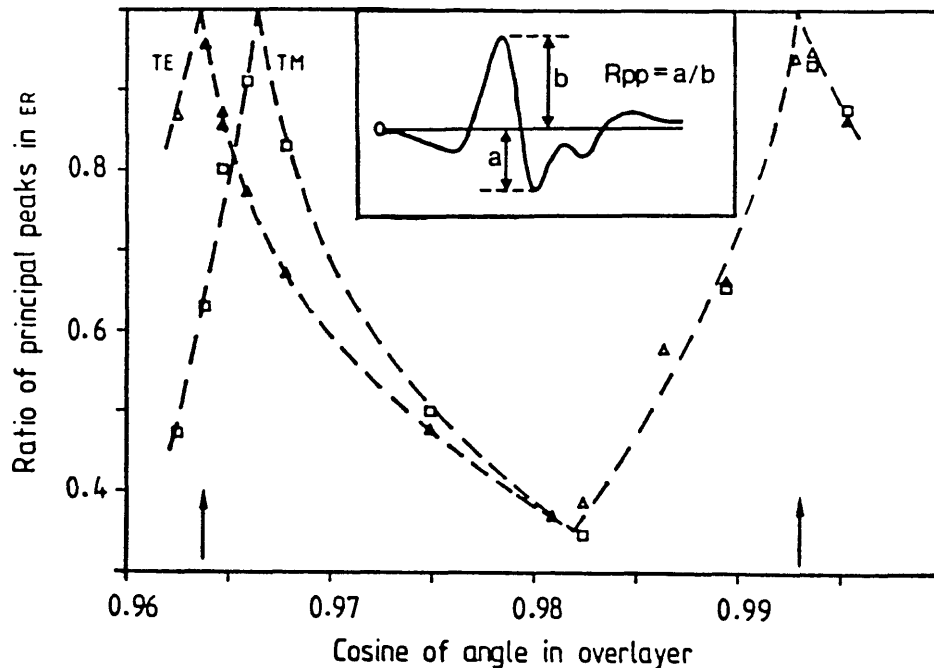


Fig. 5.2 : The evolution of the room temperature ER spectra taken on GEC256 (Fig. 5.1) with angle of incidence, for either TE (Δ) or TM ($\square$) polarised light. The vertical axis plots the ratio of the principal peaks (R_{pp}) of the spectra, defined in the inset, while the horizontal axis plots the cosine of the angle in the overlayer. The dashed lines are guides to the eye.

at a smaller angle of incidence.

An inversion of the lineshape corresponds to a change in the lineshape phase of $\Delta g = \pi$. The distance between the two zero-crossings in Fig. 5.2, $\Delta(\cos\theta) = 0.0296 \pm 0.001$, can be used, along with Eq. 3.18, to estimate the overlayer thickness as $(2.1 \pm 0.1) \mu\text{m}$. This is close to the value of $1.9 \mu\text{m}$ given in the layer specification, which was estimated from the growth rate. The discrepancy in the deduced value of the overlayer thickness may derive from either the uncertainty in the average refractive index of the overlayer, an error in the growth rate, or the presence of a thin film on the surface of the sample as discussed in the Sec. 5.2.2. Note that although the presence of the overlapping E1L1 transition distorts the position of the minimum Rpp in Fig. 5.2, it will not affect the separation of the two points where $R_{pp} = 1$, as the entire lineshape (including E1L1) is inverted between these two angles.

I have also measured the ER lineshape of the E2H2 transition in this sample, as a function of the angle of incidence, and this is reported in Ref. 1.12. Fig. 1 of Ref. 1.12 shows that the E2H2 feature undergoes a similar evolution and is seen to approximately invert upon changing the angle from 21° to 64° . This confirms that the mixing of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ produced by the optical interference effects, does not depend upon how the dielectric function is modulated, as was pointed out in Sec. 3.3. The angular dependence of the E1H1 lineshape at 50 K, which is reported in Ref. 1.13, showed a similar evolution as that seen in Fig. 5.1, from which the overlayer thickness was estimated to be $(2.0 \pm 0.1) \mu\text{m}$.

A co-worker, Peter Thorn, has measured the temperature dependence of ER spectra from this sample [1.12]. This again shows an evolution of the E1H1, E1L1 and E2H2 lineshapes, except that in this case the linewidth also changes with temperature. The lineshape is seen to evolve from a negative-zero-crossing, to a negative-peak, to a positive-zero-crossing, to a positive-peak and finally towards a negative-zero-crossing again as the sample was cooled, consistent with the increase in the lineshape phase produced by the transition energy shifting to higher energy. The overlayer thickness was estimated to be $(2.0 \pm 0.4) \mu\text{m}$ from the Rpp plot for the spectra taken at different temperatures [1.12].

5.2.1.2 Sample MV307

I have also made room temperature ER measurements on the four MOVPE grown SQWs as a function of the angle of incidence. This section concentrates on the data for MV307, which had a $72 \text{ \AA}$ wide GaAs/

$\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, although the others showed very similar behaviour, as discussed below. The modulating voltage was chosen to be 2 V_{ptp} about an offset of -0.5 V.

Fig. 5.3 shows spectra collected on MV307 at angles of incidence between 15° and 75° , using TE polarised light. The spectra show features due to the E₁H₁ and E₁L₁ excitons at 1.477 eV and 1.495 eV respectively. These are more clearly resolved than in Fig. 5.1, due to the smaller optical linewidths and larger H₁-L₁ separation in this sample. Again the lineshape is seen to evolve in the manner predicted by Eq. 3.29, summarised in table 3.2 and displayed in Fig. 3.15. However, this time the overlayer thickness is not large enough to produce a complete inversion of the lineshape upon changing the angle of incidence.

The ratio of principal peaks for the spectra in Fig. 5.3 are plotted in Fig. 5.4 as a function of the cosine of the angle in the overlayer. Fig. 5.4 shows similar behaviour to Fig. 5.2, except that the separation of the points where $R_{pp} = 1$ is increased, due to the smaller overlayer thickness. Notice also that the minimum value of R_{pp} is about 0.31, which is smaller than measured for the MBE grown SQW, probably due to the high energy side of the E₁H₁ lineshape being less obscured by the E₁L₁ feature. The horizontal separation of the two points where R_{pp} is minimum and maximum in Fig. 5.4 should correspond to $\Delta g = \pi/2$, which facilitates an estimate of the overlayer thickness of 1.1 μm , close to the intended growth thickness of 1.3645 μm .

The ER spectra of the other MOVPE SQW samples, which differ from MV307 only in their well width, were also measured as a function of the angle of incidence. These were similar to those in Fig. 5.3, with two large features at the energy of the E₁H₁ and E₁L₁ excitons. However the lineshape phase was found to be different for each of these samples at any one angle, suggesting that they have a different overlayer thickness. (This is also apparent in the low temperature ER spectrum taken on each SQW, as can be seen in Fig. 5.18.) Note that a change in the overlayer thickness of just 310 Å is sufficient to alter the lineshape phase by $\pi/2$. The rate of the evolution of the lineshape with angle of incidence, as measured by a R_{pp} plot, was found to be similar for all the MOVPE SQWs. This is as expected, since the rate of the evolution depends on the absolute thickness of the overlayer. Meanwhile the lineshape at any one angle will depend on the residual of the lineshape phase exceeding an integer number of 2π , and will therefore be very sensitive to the exact overlayer thickness.

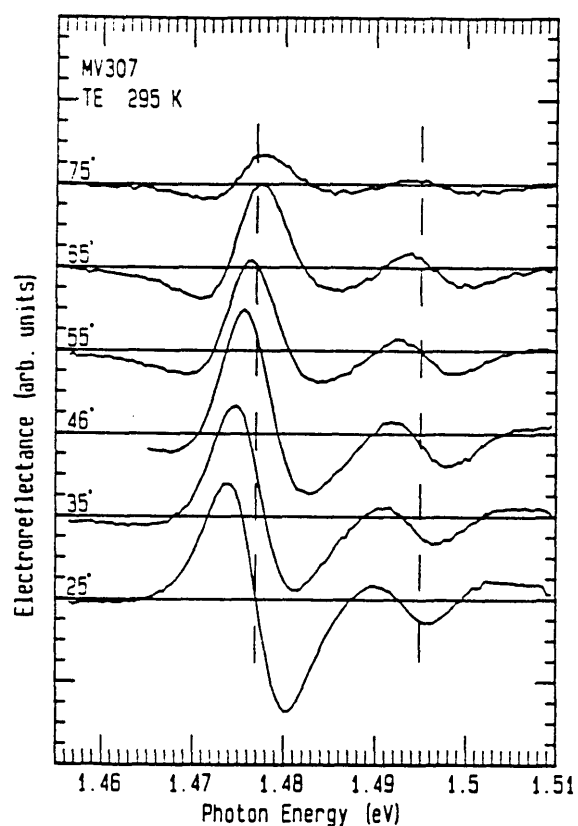


Fig. 5.3 : Room temperature ER spectra taken on MV307 with different angles of incidence and TE polarisation.

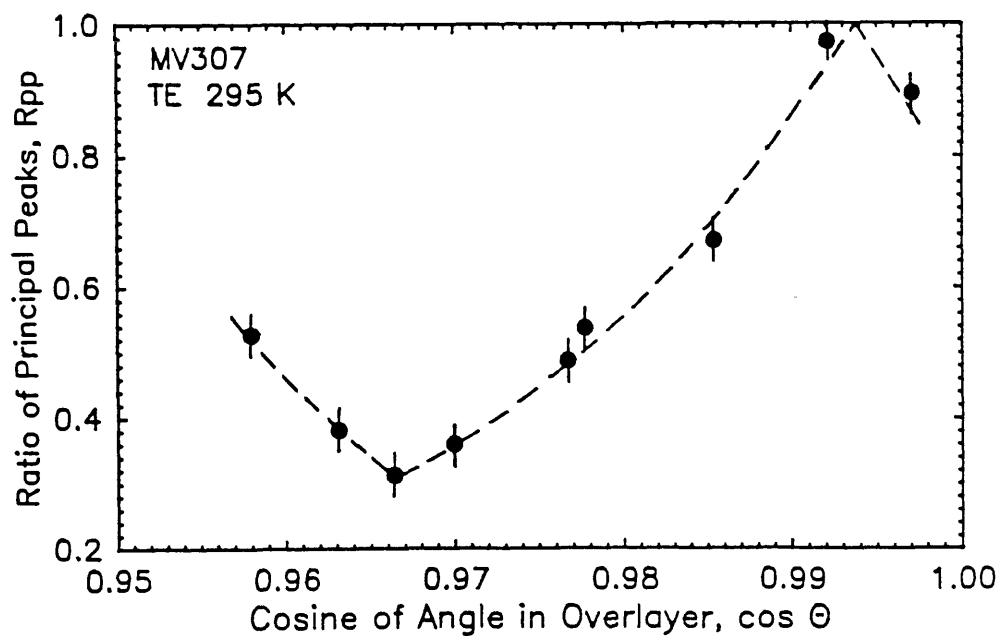


Fig. 5.4 : Ratio of principal peaks in the room temperature ER spectra of MV307 (Fig. 5.3), using TE polarised light. The line is a guide to the eye.

5.2.2 Polarisation Dependence

All the spectra presented in Sec. 5.2.1 were taken using TE polarised light, where the electric field vector is polarised in the plane of the QW. The ER lineshape measured with the magnetic field vector of the light polarised in the plane of the QW (TM), were found to be subtly different from the TE spectra. Fig. 5.5 shows two spectra taken on GEC256 under the same conditions as in Sec. 5.2.1.1, with the light incident at 65° , and polarised in either TE or TM configuration. The two spectra can be seen to be similar, but notice that there is a small difference in their lineshape phases. For the TE spectrum, $R_{pp} = (0.87 \pm 0.01)$, with the positive lobe of the lineshape being largest, while for TM, $R_{pp} = (0.80 \pm 0.01)$, but with the negative lobe now being largest.

Fig. 5.2 also shows the angle of incidence dependence of R_{pp} for the spectra taken with TM polarisation, in addition to the previously discussed TE data on GEC256. Notice that the TM spectra evolve more rapidly with the angle of incidence, than TE. This divergence between the angular dependence with TE and TM polarisations was also observed for spectra taken at 50 K on this sample, as reported in Ref. 1.13.

It is possible that this effect is due to the different response curves for the light source, monochromator grating etc. with TE and TM polarisations. However, as discussed in Sec. 4.1, care has been taken to eliminate the system response. As an additional check, measurements were repeated with the axis of rotation of the sample itself rotated through 90° . This interchanged the system response curves for the two polarisations. A plot of R_{pp} against $\cos(\theta)$, for the spectra collected with this new geometry, was almost identical to that in Fig. 5.2, with again the positive-zero-crossing occurring at a smaller angle of incidence (ie larger $\cos(\theta)$) using TM polarisation. This demonstrates conclusively that the effect is not due to the spectral response of the apparatus.

An explanation for the observed difference between the ER lineshapes produced by TE and TM polarised light is not immediately obvious from the lineshape model of Sec. 3.3. Assuming the dielectric functions of the QW and the overlayer are isotropic, the only effect of changing the polarisation, will be to change the reflectivity of the front surface, and hence the scaling coefficient (B) in Eq. 3.26. Consequently, the TE and TM lineshapes would be expected to have the same lineshape, although having a different strength, at most angles of incidence. The TE and TM lineshapes would only be different at those

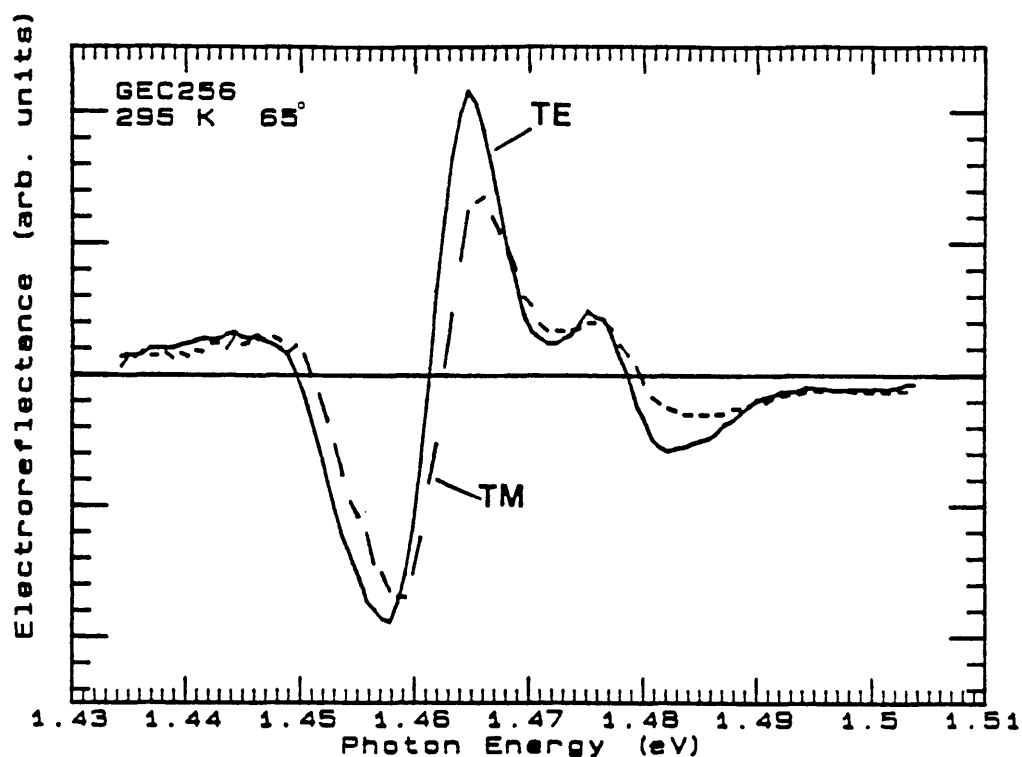


Fig. 5.5 : Room temperature ER spectra taken on sample GEC256, with the light incident at an angle of 65° , and polarised in either TE (solid lines) or TM (dashed) polarisation.

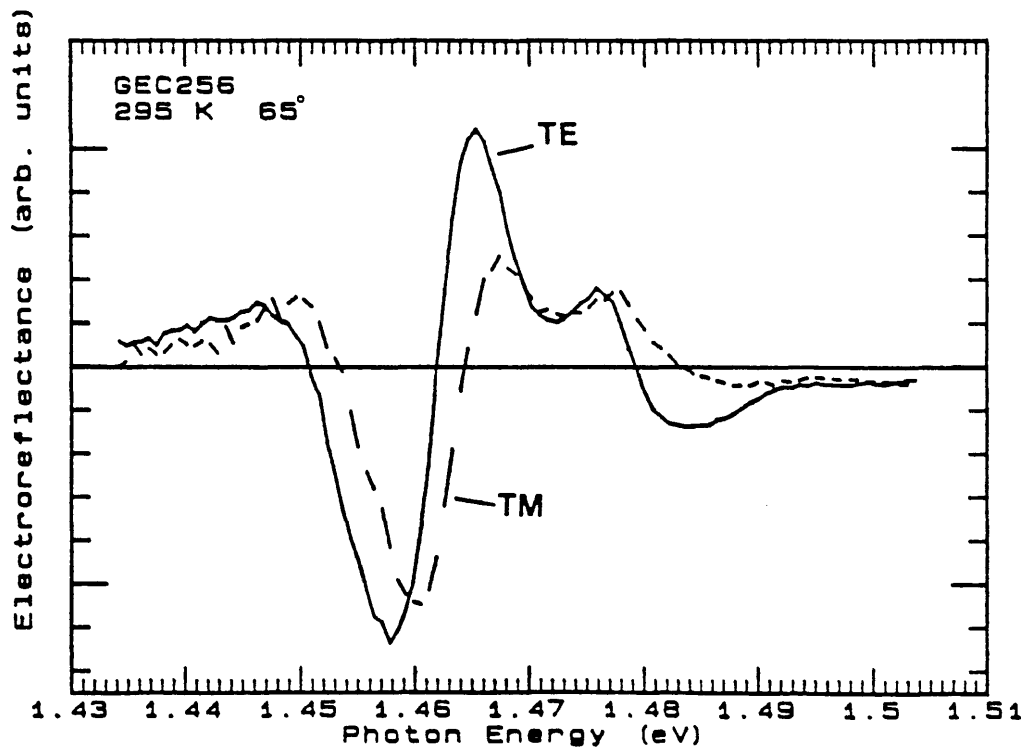


Fig. 5.7 : ER spectra taken on sample GEC256 under identical conditions to those in Fig. 5.5, except that the light is now incident on a different region of the sample surface.

narrow range of angles near the Brewster condition, given by $\tan^{-1}(3.44) = 74^\circ$, when the reflectivity of the front surface with TM polarisation will be as small as that of the QW. However the difference between the TE and TM spectra observed here occurs at angles that are too small to be explained by Brewster effects. In any case it may not be possible to observe a change in the TM lineshape due to Brewster effects, due to the finite spread of angles incident on the sample.

Several explanations for the difference between the TE and TM ER lineshapes were considered in Ref. 1.13. It was shown that the effect of the anisotropy in the QW dielectric function, for light with its electric field vector polarised in different directions, as discussed in Sec. 3.3, was two orders of magnitude too small to explain the observed phenomenon. Similarly, the anisotropy in refractive index of the overlayer, caused by lattice mismatch strain between the various layers, was also much too small to explain the behaviour.

A more plausible explanation for the difference in the lineshape between the two polarisations is the presence of an unwanted thin film on top of the sample. This thin film would form an optical etalon, so resulting in a phase change for light reflecting from the front surface of the sample. The phase change for the ray reflected from the front surface will alter its optical phase delay from the ray reflected from the QW, so producing a change in the lineshape phase of the ER. This phase change due to the thin film will be sensitive to the angle of incidence and the polarisation, if the film has a refractive index intermediate between that of the semiconductor and air. It is possible that the film could be related to oxidation of the AlGaAs through the 0.2 μm GaAs capping layer. This seems likely, since the ER lineshapes measured with TE and TM polarisation were still found to be different after chemically cleaning the sample using the standard procedures for GaAs.

ER from a SQW structure with a thin film on the surface can be modelled using Eq. 3.25, after allowing the amplitude reflectivity of the front surface (r_f) to be complex. r_f can be determined using the reflectivity formula for an optical etalon, Eq. 3.16. Alternatively the optical transfer matrix method discussed in Sec. 3.3 could also be used. Fig. 5.6 shows the calculated variation of R_{pp} with the cosine of the angle in the overlayer, for a structure consisting of a 1.9285 μm thick overlayer of $n = 3.44$, covered by a 700 $\text{\AA}$ film with a refractive index of 2.9. The theoretical spectra were calculated using Eq. 3.25 and a QW dielectric function comprised of a E1H1 and E1L1 Gaussian broadened exciton, with the parameters fitted to the room temperature spectra in

Sec. 5.2.1.1 (see Ref. 1.13 for further details). It can be seen in Fig. 5.6 that a splitting is predicted between the two polarisations similar to that observed in experiment (Fig. 5.2). A more detailed comparison of the experimental and calculated curves is difficult due to the large number of unknowns, such as the exact shape of the thin film and the real and imaginary parts of its refractive index.

This thin film on the surface could also explain the discrepancy between the overlayer thickness, deduced from the evolution of the TE lineshape with the angle of incidence, and that specified in the growth routine. The extra phase shift on reflection from the thin film at the top surface is angle-dependent, so that the two angles where $R_{pp} = 1$ on Fig. 5.2 will be displaced horizontally by different amounts. The effect will be largest on spectra taken with TM polarised light, but will also affect the TE data to a lesser degree, thereby upsetting the estimate of the overlayer thickness.

5.2.3 Topological Dependence

5.2.3.1 Sample GEC256

Recall that Fig. 5.5 shows two ER spectra taken on sample GEC256 at room temperature, using either TE or TM polarised light, incident at an angle of 65° . The spectra shown in Fig. 5.7 are taken under identical conditions, except that the light spot has been positioned on a different area of the sample surface, a few millimeters distant. (The spectra reported in Sec. 5.2.1.1 are all taken with the light spot in the same position as in Fig. 5.5).

Concentrating first of all on the TE spectra in Figs. 5.5 and 5.7, it can be seen that they have a very similar lineshape, but with a slightly different lineshape phase. The TE excited spectrum in Fig. 5.5 has $R_{pp} = (0.87 \pm 0.01)$, with the positive lobe being largest; while the TE curve in Fig. 5.7 has $R_{pp} = (0.86 \pm 0.01)$, but now with the negative lobe being largest. (The similarity of the value of R_{pp} for the TM spectra in Fig. 5.5 and the TE spectra in Fig. 5.7 is purely coincidental.) Least squares fits to these two TE spectra, using Gaussian broadened exciton profiles and the method described in Sec. 4.4 and Ref. 1.13, suggests the average overlayer thickness to be different by $(85 \pm 20) \text{ \AA}$ in the two areas of the sample probed. Meanwhile the lineshape fits suggested the E1H1 transition energies to be different by $(0.4 \pm 0.2) \text{ meV}$, which could be explained by a change in the average

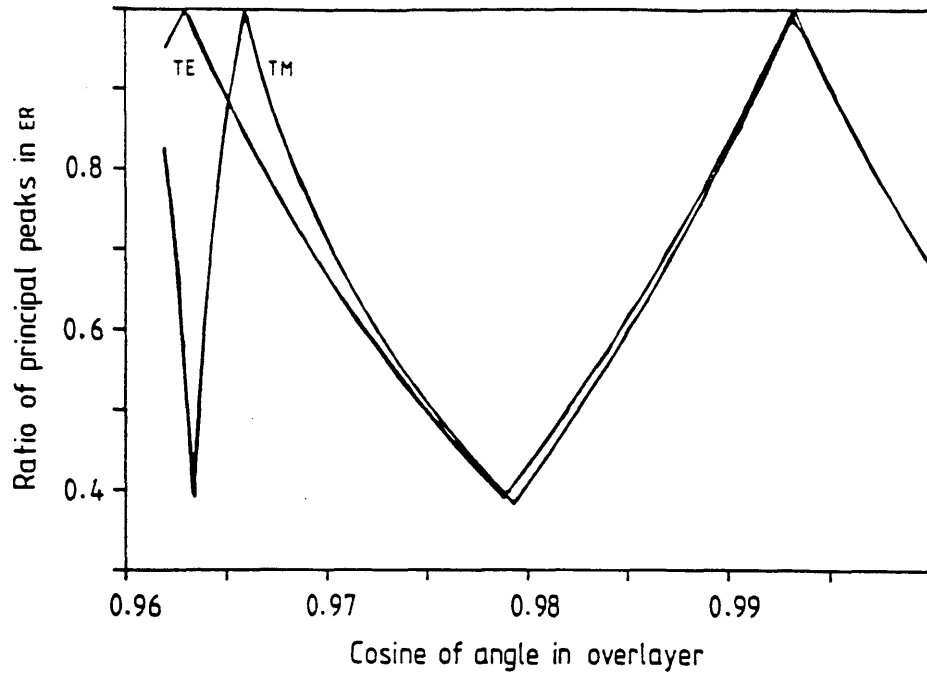


Fig. 5.6 : The calculated variation in the ratio of principal peaks against the cosine of angle in overlayer, using parameters relevant to GEC256, for comparison with the experimentally measured Fig. 5.2. The theoretical ER spectra were calculated for a structure with an overlayer thickness of $t = 1.9286 \mu\text{m}$ and refractive index $n = 3.44$, capped by a thin film of $t' = 0.07 \mu\text{m}$ and $n' = 2.9$. The QW dielectric function was calculated using Gaussian broadened E1H1 and E1L1 1s excitons, with the parameters fitted to the ER spectra shown in Fig. 5.1.

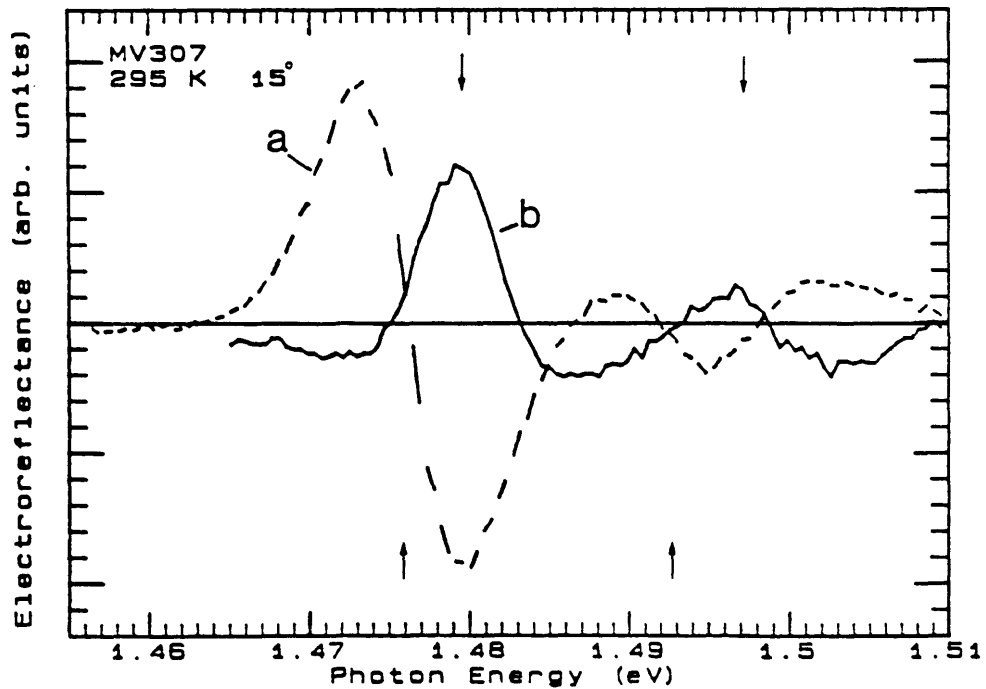


Fig. 5.8 : Room temperature ER spectra taken on two different mesas of sample MV307, using TE polarised light, incident at an angle of 15° .

well width of $(0.7 \pm 0.4) \text{ \AA}$ between the two regions. This change in the transition energy alone, produces negligible change in the lineshape phase.

A larger difference is observed between the spectra taken at the two sample positions using TM polarised light. In this case, for the TM spectrum in Fig. 5.5, $R_{pp} = (0.80 \pm 0.01)$ with the positive lobe being larger; while for Fig. 5.7 the lineshape is closer to being a negative-peak with $R_{pp} = (0.46 \pm 0.01)$. As the difference in the TE and TM spectra is believed to be due to the presence of an unwanted, although unavoidable, thin film on the sample surface, as discussed in Sec. 5.2.2, it seems that there is also a topological variation in the film across the sample surface. The topological variation of the film may also account for at least part of the smaller difference seen between the phases of the two TE lineshapes.

5.2.3.2 Sample MV307

Fig. 5.8 shows ER spectra taken on two different mesas of MV307, the $72 \text{ \AA}$ MOVPE grown SQW. The layers of MV307 were grown simultaneously on two separate pieces of substrate, and the two mesas studied here come from either wafer. Both spectra were taken at room temperature, with TE polarised light, incident at 15° . The spectra taken with TM polarised light had a very similar lineshape to the TE spectra, as expected at this small angle of incidence.

It can be seen that there is a dramatic difference in the lineshape of the two mesas, with the curve labelled "a" resembling a positive-peak lineshape, while curve "b" is close to a negative-zero-crossing lineshape. This approximate $\pi/2$ change in the lineshape phase suggests curve "b" to be caused by an overlayer which is about one eighth of the wavelength of the light in the material thicker, and in fact least squares fits made to the two spectra indicate the difference to be $(316 \pm 18) \text{ \AA}$. It should be noted that since the ER lineshape is unaltered by an increase in the lineshape phase of 2π , corresponding to an overlayer thickness of roughly $1240 \text{ \AA}$, the difference may actually be much larger. The least squares fits also indicate that the E1H1 transition energy of spectrum "b" is $(3.3 \pm 0.3) \text{ meV}$ lower than that of curve "a", which suggests the well width of this mesa to be wider by $(2.7 \pm 0.3) \text{ \AA}$. This change in the transition energy is responsible for only a tiny fraction of the change in the lineshape phase, which must therefore be due to variation in the overlayer thickness. Large topological variations in layer thicknesses are

typical of MOVPE grown material and derive from the substrate being slightly tilted with respect to the direction of the flow of the reactants. This explains both the observed topological variations in the QW and overlayer thicknesses.

5.3 Measurements of the Quantum Confined Stark Effect

This section describes PC and ER measurements performed on the four MOVPE grown SQWs as a function of the applied bias. The samples were mounted on the cold-finger of the closed-cycle Helium cryostat, maintained at about 17 K.

5.3.1 Field-Dependent Photocurrent Spectra

Fig. 5.9 shows 17 K PC spectra taken on each of the four SQWs with an applied bias of -2 V (47 kV/cm). This is the smallest reverse voltage at which a PC signal could be measured at low temperature, as a minimum threshold field was needed to remove the photoexcited carriers from the well, as discussed in Sec. 5.4.1. These spectra resemble the familiar bandedge absorption profile of a QW discussed in Sec. 2.3, with two sharp peaks due to the E1H1 and E1L1 excitonic ground state. The narrow linewidths displayed here reflect the good sample quality, as discussed later. It can be seen that the transitions lie to higher energy in the narrower wells, due to the increase in the carrier confinement energies.

Spectra taken on MV302 (59 Å) with different applied biases, corresponding to fields of between 47 and 322 kV/cm, are shown in Fig. 5.10. As expected, both the E1H1 and E1L1 peaks are seen to move to lower energy. The separation of E1H1 and E1L1 increases slightly with field, as its heavier mass results in a larger Stark shift for H1 than L1. The shifts are accompanied by an increase in the linewidth with field, as the broadening mechanisms become more efficient. The heights of the exciton peaks can also be seen to decrease, due in part to the increase in the linewidth and also the decrease in the electron-hole overlap. Notice that the peaks remain resolved until very large fields, showing that the barriers do indeed limit the field ionisation of the exciton, as expected. However, the exciton peaks are no longer resolved at the highest fields in Fig. 5.10, and two broad steps are then seen due to the E1H1 and E1L1 continua.

Fig. 5.11 shows the field evolution for a wider well; that of

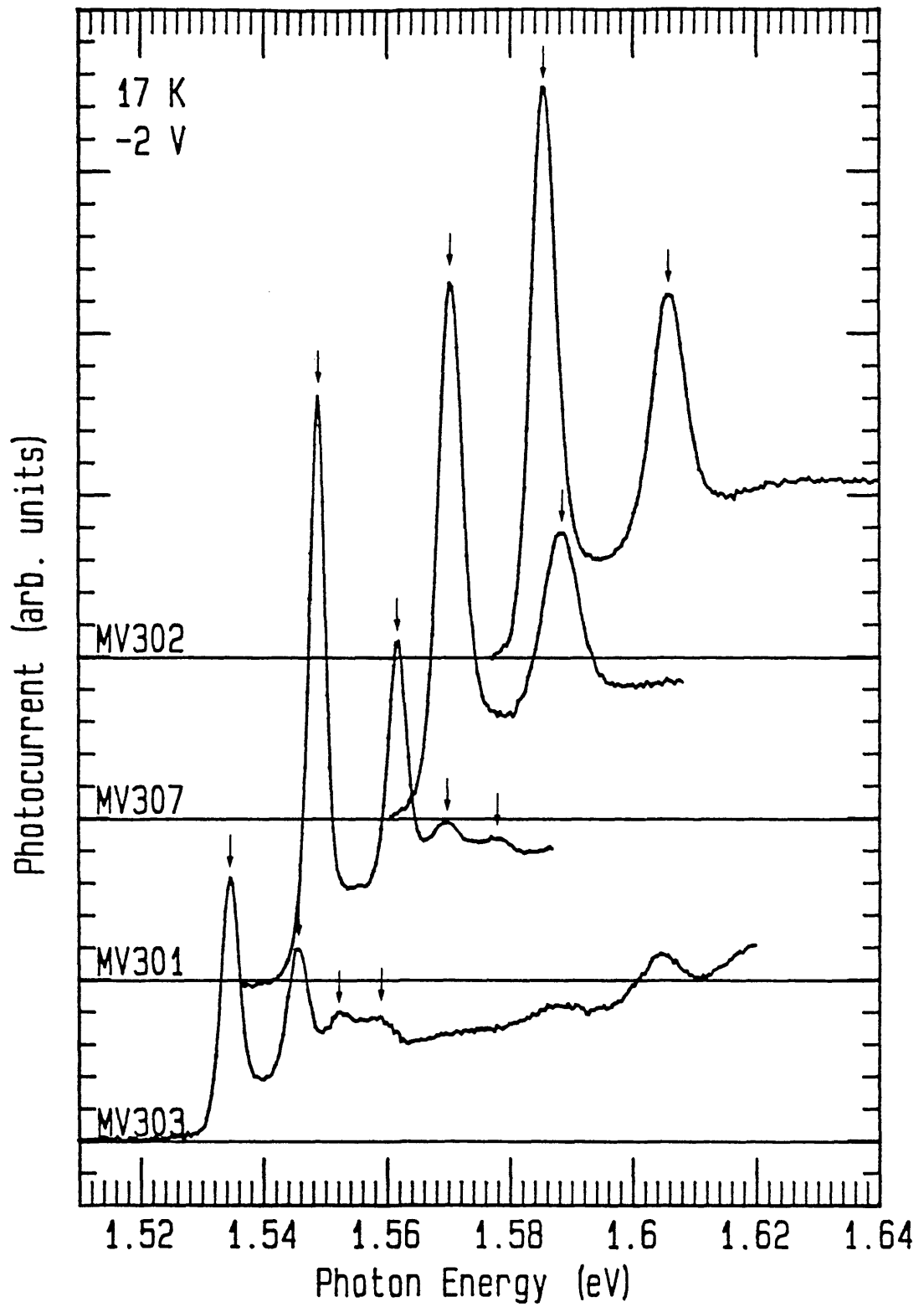


Fig. 5.9 : PC spectra taken on the four MOVPE grown GaAs/Al_{0.3}Ga_{0.7}As SQW samples, taken at a temperature of $T = 17$ K and with an applied bias of -2 V.

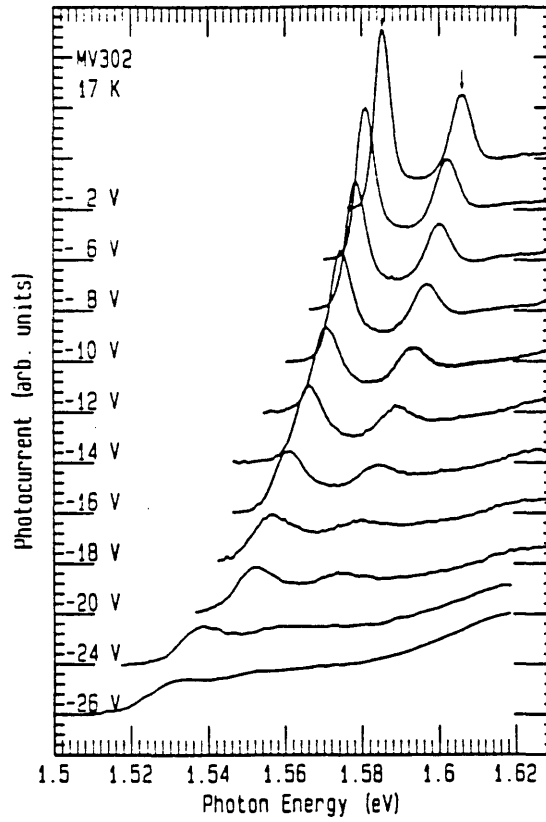


Fig. 5.10 : PC spectra taken at $T = 17$ K on sample MV302 ($L_w = 59$ Å), with different applied biases.

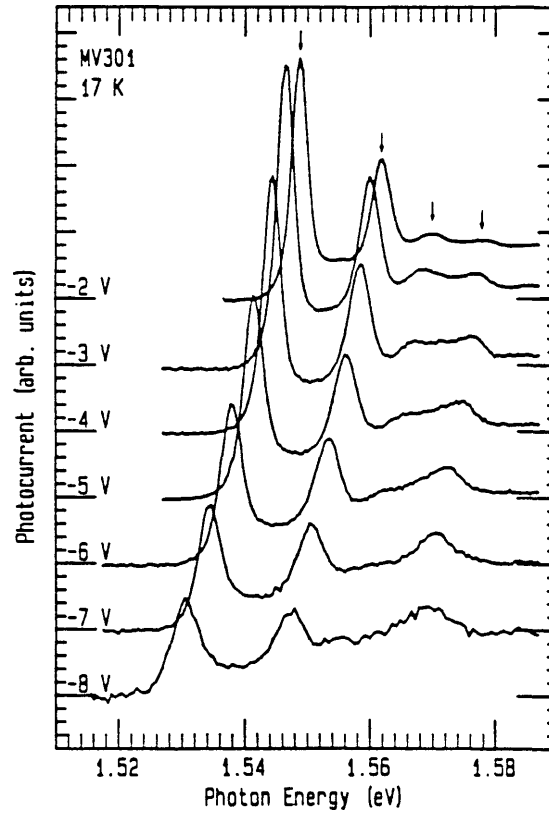


Fig. 5.11 : PC spectra taken at $T = 17$ K on sample MV301 ($L_w = 88$ Å), with different applied biases.

MV301, where the well width is $88 \text{ \AA}$. This again shows the shift to lower energy of the E1H1 and E1L1 excitonic peaks, with a simultaneous increase in the linewidth and decrease in the peak height. Notice that the energy shift and reduction of the transition strengths occurs more rapidly with field than in MV302, due to the greater degree of electron-hole polarisation possible in this wider well. This sample also shows some extra peaks that were not observed for MV302; in particular, the two small peaks at energies above the E1L1 exciton seen in the -2 V spectrum.

The higher energy feature of these two, which is at 1.5778 eV in the -2 V spectrum of Fig. 5.11, is the E1H2 exciton (1s) transition. This assignment explains the strengthening of this peak with field, caused by the increase in the overlap of E1 and H2 envelope functions. It also moves to lower energy more slowly than the E1H1 exciton, reflecting the smaller Stark shift of H2 compared to H1. The E1H2 (1s) exciton can also be observed in MV303 ($106 \text{ \AA}$), corresponding to the peak seen at 1.5586 eV in the -2 V spectrum of Fig. 5.9. The bias dependence of the PC spectra of MV303 (not shown), showed a similar increase in the strength of the E1H2(1s) peak with field. The ER features from the other higher subband transitions in these SLs were obscured by features from the SLs, also contained inside the sample structure.

The other unexplained peak (at 1.5697 eV) in the -2 V spectrum of MV301 in Figs. 5.11 and 5.9, does not strengthen with increasing reverse bias. This peak is also seen (at 1.5522 eV) in the -2 V spectrum of MV303 ($106 \text{ \AA}$) in Fig. 5.9, although it was not observed for the two narrowest wells ($59 \text{ \AA}$, $72 \text{ \AA}$). Yu et al [2.53] and Collins et al [2.49] have also observed this transition and suggested it to be an excited excitonic state of E1L1, as discussed in Sec. 2.4.1. Indeed the PC spectrum published by Yu et al for their $85 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ MQW at 80 kV/cm, is very similar to that of my $88 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ SQW at 72 kV/cm (the -4 V spectrum in Fig. 5.11). The separation of the peaks due to the observed excited and ground state of the E1L1 exciton is 7.8 meV and 6.6 meV for MV301 ($88 \text{ \AA}$) and MV303 ($106 \text{ \AA}$) respectively at -2 V (47 kV/cm). This compares to a separation of 8.1 meV reported by Yu et al for their $85 \text{ \AA}$ well at 46 kV/cm.

The excited E1L1 excitonic state observed for MV301 and MV303 is much stronger than predicted by a simple analysis that ignores mixing of the valence subbands. Fig. 2.10 shows the dielectric function of an exciton transition, calculated using a HWHM linewidth of 1.5 meV, similar to the 1.55 meV measured for the -2 V E1H1(1s) PC peak of MV301 (1.55 meV). Note that a distinct peak due to any of the excited exciton

states is not resolved in Fig. 2.10. The enhanced strength in the experimental spectra is produced by a mixing of the L1 and H2 subbands, as discussed in Sec. 2.4.1. The excited excitonic state of E1L1 is not resolved at the highest fields in Fig. 5.11 as its energy separation from the E1H2(1s) is then larger, due to its larger Stark shift, so resulting in less mixing. Likewise the feature is strongest in the two wider wells in Fig. 5.9, since then the energy of the excited E1L1 exciton will be closest to E1H2(1s). When the well width is reduced, the separation of H2 and L1 increases, so reducing their mixing. The spectra of the two narrower SQWs in Fig. 5.9, around the spectral region of the E1L1 continuum edge, is similar to that in the PLE spectrum of Fig. 2.6, measured by Miller et al [2.31] on a 42 Å QW, and also the calculated spectra of Fig. 2.10, which ignore the valence subband mixing.

None of the four wells in this study unambiguously resolved the E1H1 continuum, which can be seen as a rising edge in the spectra of Miller et al [2.31], shown in Fig. 2.6. This rising edge is also resolved in the spectrum of Yu et al [2.53], despite the linewidth being slightly larger in their sample than MV301, which has a similar well width. Meanwhile, others have resolved peaks due to the E1H1(2s) exciton with samples displaying a comparable linewidth [2.42]. The E1H1 continuum and excited excitonic states are not resolved in the present study, probably because of the lineshape of the excitonic profile. The 1s excitons show an asymmetric lineshape, which is broader to higher energy, thus obscuring the structure in the expected region of the excited E1H1 excitonic transitions. This point is demonstrated by the calculated curves in Figs. 2.10.1 and 2.10.2, which were both generated using a linewidth of 1.5 meV (HWHM), similar to MV301 at -2 V. Fig. 2.10.2 was generated with a Lorentzian excitonic profile and is broader in the wings, than the Gaussian broadened profile in Fig. 2.10.1. The high energy side of the E1H1(1s) excitons in Fig. 5.9 is broader in the wings, like that in Fig. 2.10.2, and this obscures the E1H1 bandedge region. Perhaps this difference in the high energy side of the lineshape is due to a different broadening mechanism in these MOVPE grown samples, than in the MBE samples used in the previous studies.

5.3.2 Comparison to Calculations of the Quantum Confined Stark Effect

The variation of the E1H1 and E1L1 exciton transition energies with electric field, for each of the SQWs, are plotted as the open

triangles in Figs. 5.12 and 5.13 respectively. (The open squares are the energies measured in ER spectroscopy, discussed later.) The field is calculated using the exact expression for a p^+-i-n^+ diode and the parameters discussed in Sec. 4.2. As expected the Stark shifts increase with the well width, due to the greater possible spatial separation of the electron and hole. The Stark shift of the E1H1 transition is larger than that of the E1L1, due to the heavy hole being more tightly confined to the side of the well by the field. The measured PC energies are compared to those calculated (solid lines) in the manner described in Sec. 4.3 using the fitted well widths. This calculation includes the field-induced change in the exciton binding energy. A good agreement can be seen between the measured and calculated Stark shifts for all four SQWs.

The symbols in Fig. 5.14 plot the field dependence of the E1H1 excitonic transition strength measured in the PC spectra of the four SQWs. The transition strength was determined by numerically integrating the E1H1 PC peak, with the upper limit of this integration taken as the energy where the exciton peak joined the higher energy continuum. (The error in integrated PC transition strength was determined by the uncertainty in the position of this upper limit.) Only the biases where all the photoexcited carrier were collected (see Sec. 5.4.1) are plotted in Fig. 5.14. It can be seen that the transition strength decreases more rapidly for the wider wells, as expected, due to the greater field-induced polarisation of the electron and hole. The solid lines in Fig. 5.14 show the field dependence of the square of the exciton overlap (V_{ex}), given by Eq. 3.4, for the four well widths, again calculated using the method and parameters discussed in Sec. 4.3. The PC spectra were measured in relative units and the set of experimental integrated intensities for each SQW have consequently been scaled so as to coincide with the calculated variations at -6 V (95 kV/cm). In general, a good agreement can be seen between the measured and calculated transition strengths. However, the strength of the E1H1 PC peak in the 59 and 106 Å QW decline slightly less rapidly than expected. This discrepancy may be caused by an increasing contribution with field, from the broadened continuum, to the high energy side of the E1H1 exciton peak.

5.3.3 Low Temperature Photocurrent Linewidth

Fig. 5.15 shows the field dependence of the linewidth of the E1H1 PC peak measured for the four SQWs, at 17 K. The linewidth is defined as the half-width between the energies of the peak maximum and the

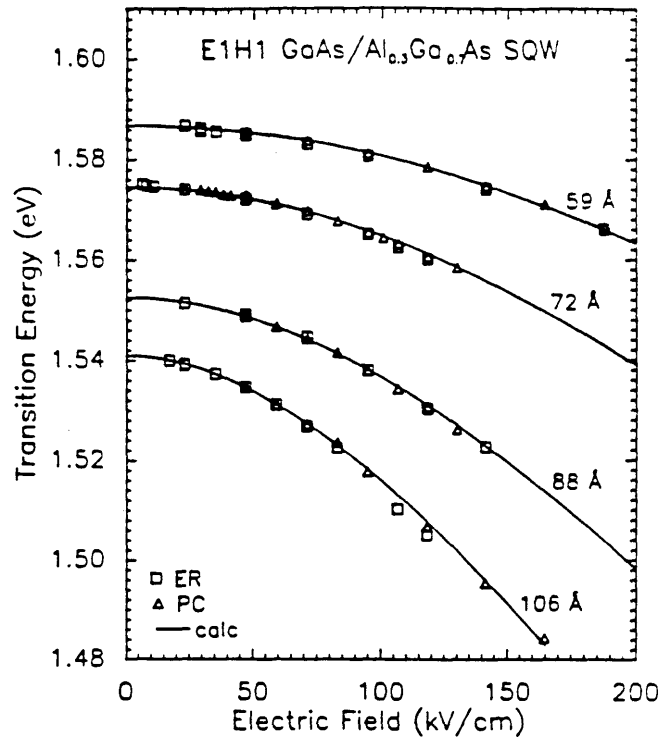


Fig. 5.12 : The energies of the E1H1 exciton PC peak (Δ), along with the E1H1 energy fitted to the ER spectra ($\square$), for each of the four MOVPE grown SQW samples, as a function of the applied field. The solid lines show the Stark shifts of each well, calculated in the manner described in Sec. 4.3.

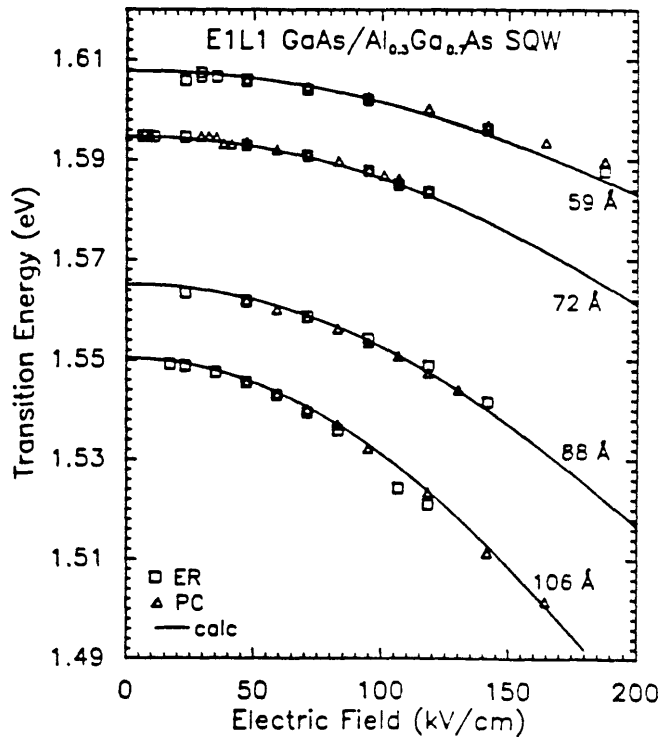


Fig. 5.13 : The field dependence of the E1L1 exciton energy in each of the four SQWs, as measured by the PC peaks (Δ), fitted from the ER spectra ($\square$) and calculated (—).

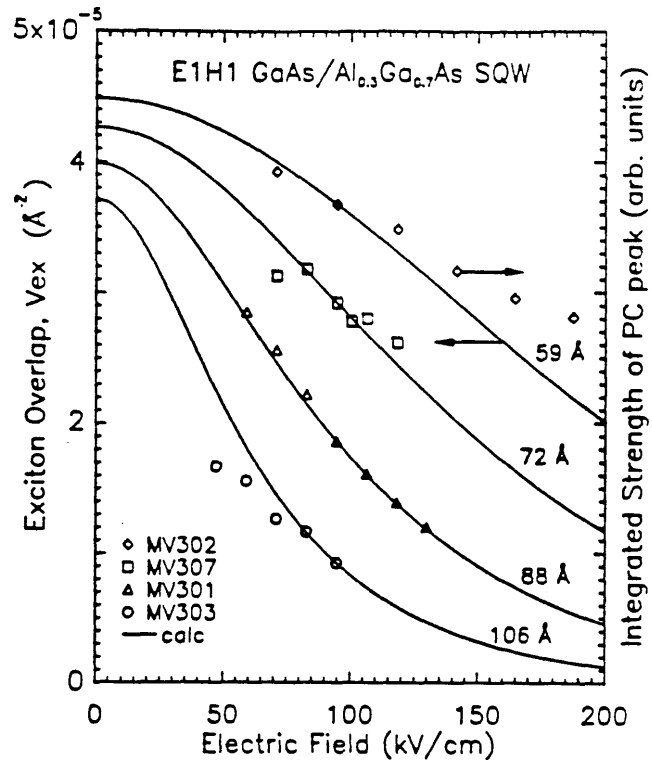


Fig. 5.14 : The symbols show the field dependence of the integrated strength of the E1H1 PC peak for the four MOVPE grown SQWs. The solid lines are the calculated E1H1 exciton overlap, V_{ex} , defined by Eq. 3.4. The experimental points for each QW have been scaled to match the theoretical curves at 94.7 kV/cm (-6 V).

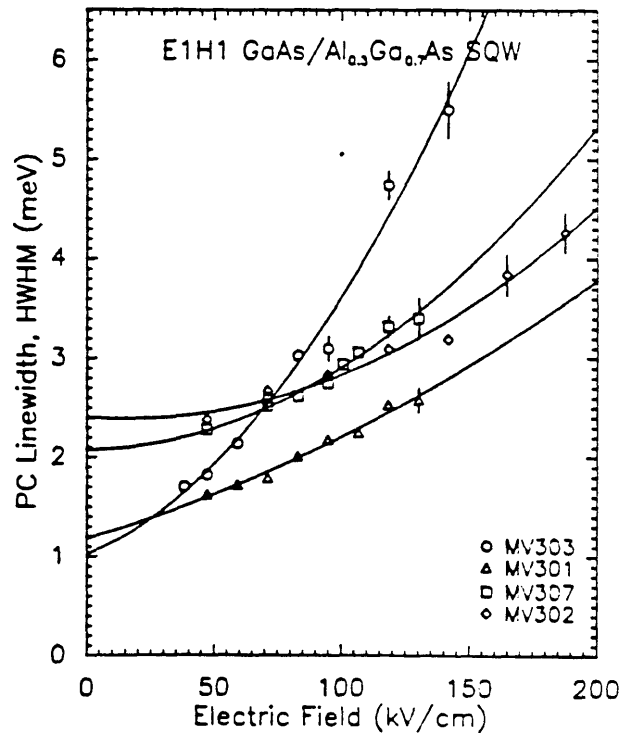


Fig. 5.15 : The field dependence of the 17 K linewidth (HWHM) of the E1H1 PC peaks of the four MOVPE SQWs. The lines are guides to the eye.

half-maximum to lower energy. This allows the linewidth to still be measured when the half-maximum to higher energy becomes obscured by the broadened continuum. The lines are drawn through the experimental points as guides to the eye. It can be seen that the projected linewidth at zero field increases with the well width. However, the linewidth of the wider wells increases more rapidly with field, so that the widest well has the largest linewidth at fields greater than about 80 kV/cm.

It was discussed in Sec. 2.6 that the linewidth will increase with electric field, due to the tunnelling of the carriers from the well. However, the sum of the linewidths of the E1 and H1 resonances was calculated to be less than 0.5 meV, for each well, at these electric fields. Clearly the measured increase in the linewidth with field is much larger than this, and must therefore be determined by inhomogeneous mechanisms.

Roughness of the QW interfaces can be expected to cause a broadening of the optical transitions, as this produces a spatial variation in the well width and hence the confinement energies. In the analysis of Singh and Bajaj [5.1], the roughness is considered as producing islands at the interface, where the position of the boundary changes in monolayer (ML) steps ($1 \text{ ML} = 2.83 \text{ \AA}$). The exciton will experience a number of different well widths, if its diameter in the plane of the QW is much larger than the lateral extent of these islands, so producing an inhomogeneous broadening of the transitions. Meanwhile, if the islands extend further in the plane of the QW than the exciton diameter, a number of transitions will be resolved, originating from excitons located in regions where the well width differs by exact integer multiples of a ML.

The importance of interface roughness on the transition lineshape has been demonstrated by several experimental studies. Splittings have been resolved in spectra, due to exact monolayer variations in the well width [5.2-7]. Other studies have examined the dependence of the linewidth on the well width, in samples grown under identical conditions [5.8-10, 5.2, 5.4-5], as equal amounts of roughness in each QW, will produce an increase in the linewidth with decreasing mean well width.

Fig. 5.16 plots the E1H1 linewidths (FWHM) measured in several of these studies (labelled a-e), details of which are listed in table 5.1. Linewidths measured using PL spectroscopy have also been included in Fig. 5.16, although it should be noted, that these need not necessarily be the same as the excitation linewidth, especially if the recombination occurs by an extrinsic process, which is argued in Sec. 5.4.2 to be the

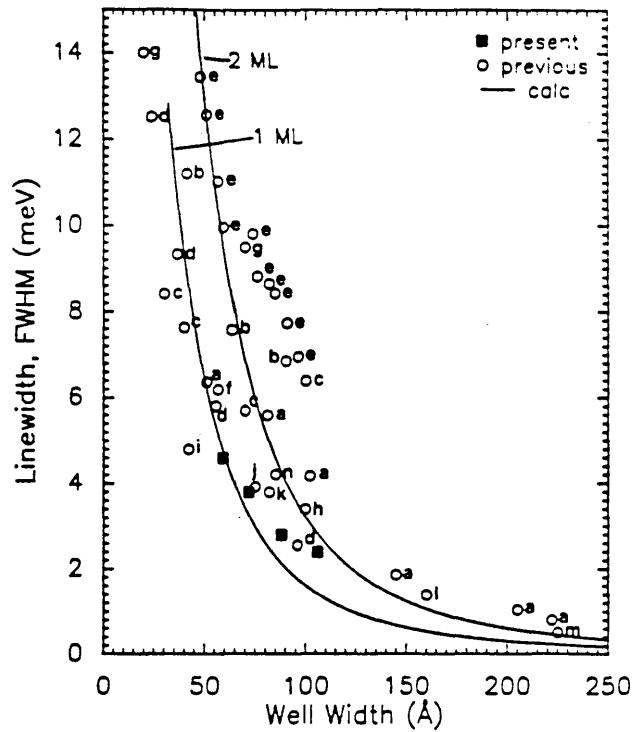


Fig. 5.16 : The well width dependence of the E1H1 exciton linewidth (FWHM) of GaAs/Al_xGa_{1-x}As QWs, measured in a number of different optical studies. This compares the interpolated zero-field linewidths of the four MOVPE grown SQWs studied here (■), with those reported in previous optical studies (○), the details of which are listed in table 5.1, along with a key for the letters on the diagram. The solid lines are the calculated FWHM linewidth produced by a 1 or 2 monolayer variation in the well width.

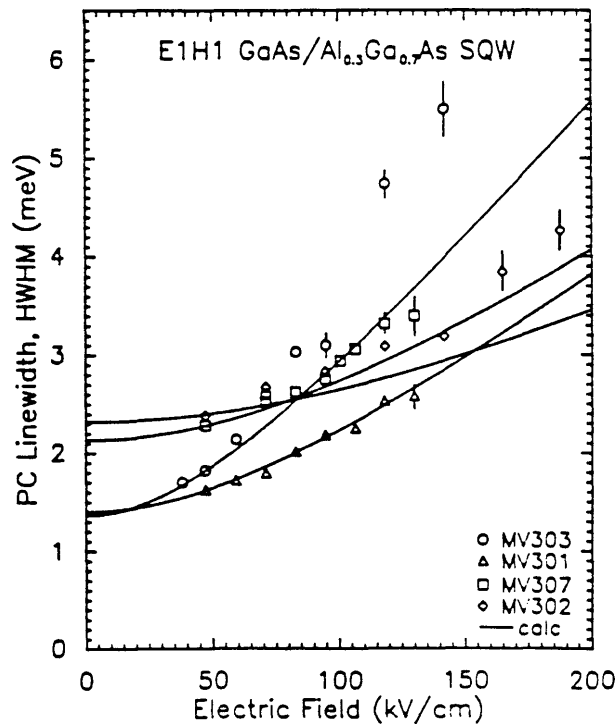


Fig. 5.17 : The calculated field dependence of the E1H1 linewidth (HWHM) in the four MOVPE grown SQWs, assuming the linewidth to be determined by well width variations. The symbols are the measured PC HWHM, as in Fig. 5.15. The degree of well width variation in each well has been chosen, so as to agree with the measured linewidth at -2 V (47.1 kV/cm).

case for the SQWs studied here. Also plotted, as individual points (labelled f-n), are the linewidths measured in some other spectra discussed in this thesis. The solid lines in Fig. 5.16 show the change in the E1H1 confinement energy (E) of a GaAs/Al_{0.3}Ga_{0.7}As QW, produced by a change in the well width (L_w) of 1 or 2 MLs;

ie $E(L_w - \frac{1}{2} \Delta L_w) - E(L_w + \frac{1}{2} \Delta L_w)$, where $\Delta L_w = 1$ or 2 MLs.

For each study of a series of QWs with different widths in Fig. 5.16 (labelled a-e), the linewidth can be seen to increase with decreasing well width, in a similar manner to the calculated curves. This suggests that fluctuations in the well width are the dominant broadening mechanism, and that there is roughly an equal amount of roughness in each well. However, notice that the decrease in the measured linewidths with well width, is not quite as rapid as that predicted by the solid lines, so that the linewidth is larger than expected for wide wells ($L_w > 120 \text{ \AA}$). This suggests that either, there is a greater amount of roughness in wells with a larger mean width, or that other broadening mechanisms make a significant contribution to the linewidth of wide wells.

Id.	Author	Ref.	Expt	T(K)	x	Sam	Growth	T(°C)
-	This study	-	PC	17	0.3	SQW	MOVPE	670
a	Weisbuch et al	5.9	PLE	?	0.3	MQW	MBE	690
b	Frijlink et al	5.10	PL	?	?	?	MOCVD	?
c	Kawai et al	5.11	PL	4	0.54	SQW	MOCVD	780
d	Fukunaga et al	5.6	PL	4	0.5	SQW	MBE	600
e	Sakaki et al	5.5	PL	77	1.0	MQW	MBE	560
f	Voilliot et al	5.8	PL	5	0.33	SQW	MBE	635
g	Kobayashi et al	5.15	PL	2	0.3	SQW	MOCVD	600
h	Miller et al	5.13	PLE	5	0.26	SQW	MOCVD	750
i	Miller et al	2.36	PLE	6	0.37	SQW	MBE	?
j	Dawson et al	2.37	PLE	8	0.35	MQW	MBE	630
k	Dawson et al	2.37	PLE	8	1.0	MQW	MBE	630
l	Collins et al	2.39	PC	10	?	SQW	MBE	?
m	Reynolds et al	2.40	PLE	2	0.35	MQW	MBE	600
n	Yu et al	2.43	PC	2	0.3	MQW	MBE	?

Table 5.1 : Some low temperature measurements of the E1H1 exciton linewidth (FWHM) in GaAs/Al_xGa_{1-x}As QWs, which are plotted in Fig. 5.16. The list includes the spectroscopy employed, the experimental temperature, the Al concentration (x) of the barrier, the type of QW structure, the growth method and the substrate temperature during growth. All these samples were growth without interruptions at the interfaces.

The solid squares in Fig. 5.16 plot the zero-field E1H1 PC

linewidths measured for the four SQWs in this study, which were grown under identical conditions. It can be seen that for the narrowest wells, the value of the linewidth is in rough agreement with a 1 ML variation in the well width. It can be seen that the linewidths of these four SQWs compare favourably with those previously reported. Fig. 5.16 demonstrates the linewidth of the SQWs studied here to be as narrow as can be attained using conventional growth methods and are determined by roughly a 1 ML fluctuation in well width.

Notice that the linewidth of the four SQWs (solid squares in Fig. 5.16) does not decrease quite as rapidly with the well width, as predicted by the calculated curves, as discussed above for the previous studies. This again suggests that another broadening mechanism may be important in the wider wells. Another possible source of broadening, suggested by Bar-Joseph et al [5.13], derives from the background impurity concentration in the region of the QW. These impurities are ionised by the electric field, so producing a randomly fluctuating electric field inside the intrinsic region. The component of this random field in the plane of the QW will tend to ionise the bound excitons, so reducing their lifetime, and increasing the linewidth. In contrast to well width fluctuations, the linewidth due to this broadening mechanism will increase with the well width, due to excitons in wider wells being more readily ionised. This mechanism may be especially important in these MOVPE grown samples, where the background impurity concentration is typically larger than in MBE material, due to Carbon contamination [4.1-2].

Smaller linewidths than observed in Fig. 5.16 can be achieved by interrupting the growth at the interfaces [5.4-5,5.7]. This allows time for the atoms to migrate along the growth surface and fill the vacant sites, so producing smoother interfaces. The islands at the interfaces are then typically larger than the exciton diameter, producing a series of spectral features, due to regions where the well widths differ by integer multiples of a ML [5.4-5,5.7]. The linewidth of each feature is typically smaller than those in Fig. 5.16 [5.4-5]. However, this interruption in the growth may also result in a large number of impurities being trapped inside the material. This can be expected to have a detrimental effect on the linewidth at finite electric fields, when these impurities will be ionised. Consequently, samples grown with interruptions may be of limited advantage for field-dependent studies of QWs. Layer quality can also be improved, without interrupting the growth, by alternating the supply of the Group III (Al or Ga) and Group V (As) atoms. This speeds the migration of the atoms along the surface

to fill the vacant sites, thereby again producing smoother interfaces. This has been achieved by a modification of both the MBE [5.14] and also MOCVD [5.15] methods.

Broadening due to variations in the well width will produce an increase in the linewidth with electric field, as was observed for the four SQWs in Fig. 5.15. This is because the Stark curves for two different well widths diverge with increasing field. Furthermore the linewidth will be expected to increase more rapidly with field for wider wells, in qualitative agreement with the behaviour displayed in Fig. 5.15.

Fig. 5.17 plots the field dependence of the calculated change in the E₁H₁ exciton energy, of each of the wells, produced by a fixed change in well width. The change in the well width, for each of the SQWs, was chosen so as to match the measured HWHM of the PC peak at -2 V (47.1 kV/cm), corresponding to $\Delta L_w = 1.4 \text{ \AA}$ for MV302; $1.9 \text{ \AA}$ (MV307); $1.9 \text{ \AA}$ (MV301) and $2.8 \text{ \AA}$ (MV303). Notice again that it takes a larger degree of interface roughness to produce the linewidth observed in the wider QWs, suggesting other mechanisms may also be important.

The experimentally measured PC linewidths are again plotted on Fig. 5.17 for comparison with this calculation. Notice that although the calculation correctly predicts the general trends with field and well width, the measured linewidth of three of the SQWs increases more rapidly with field. The discrepancy is largest for the widest well, possibly suggesting there to be a significant contribution from other broadening mechanisms. The broadening due to the background impurity charge will also increase with the applied field. This is because the applied field results in an increase in the exciton diameter in the plane of the QW, so that the exciton is more readily ionised by the (internal) electric fields produced by the impurity charge.

5.3.4 Field-Dependent Electroreflectance Spectra

Fig. 5.18 shows 17K ER spectra collected on each of the four MOVPE grown SQW samples. The angle of incidence and the modulating bias used in each case are listed in table 5.2. The d.c. offset bias was set to be around -2 V in each case, so the ER can be compared to the corresponding PC spectra shown in Fig. 5.9. Comparing the two figures it can be seen that the energies of the ER features are in good agreement with the excitonic peaks of the PC spectra, as was even more clearly demonstrated by Fig. 3.1 for MV302. This again confirms that the ER lineshape is dominated by excitonic features. Similar agreement

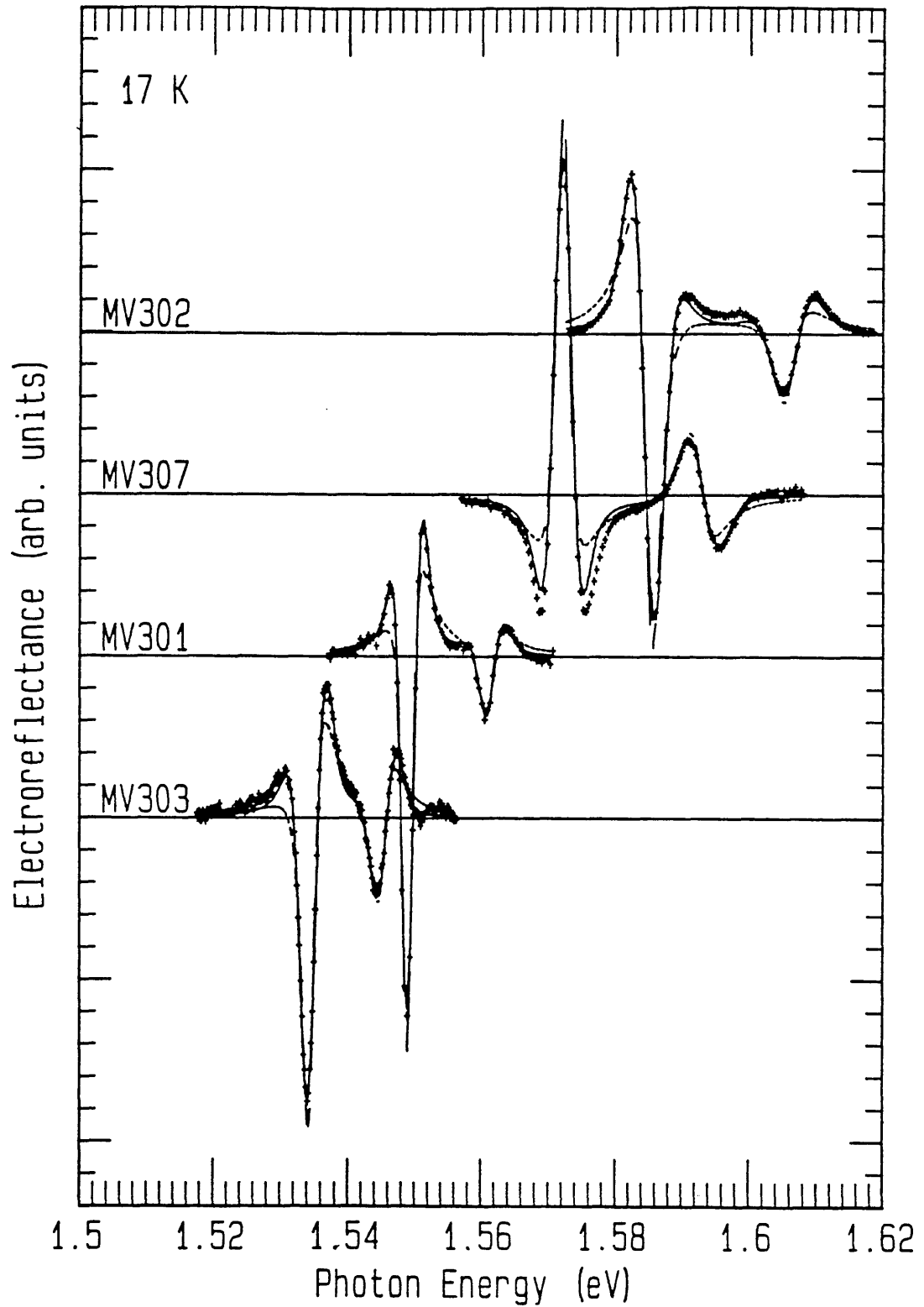


Fig. 5.18 : The data points show 17 K ER spectra taken on the four MOVPE grown SQWs, at biases near -2 V. The other parameters are listed in table 5.2. The solid lines show least squares fits made to the spectra, as described in the text, using either Gaussian (solid lines) or Lorentzian (dashed) exciton broadening profile. The fitted parameters are also listed in table 5.2.

is also observed for room temperature spectra and at other electric fields. Notice that the lineshape phase of the E1H1 and E1L1 ER features is different in each of the spectra, which is due to the angle of incidence, the QW transition energies and the optical thickness of the overlayer, being different in each sample.

Sample	MV302	MV307	MV301	MV303	Units
Bias	-2.0	-2.0	-1.8	-2.0	Volts
Mod	1.0	0.4	0.2	0.5	Vptp
θ_o	20	45	45	24	degrees
Gaussian Lineshape Fits					
Q	0.0770	0.1531	0.0599	0.1002	-
E(E1H1)	1.58508(3)	1.57207(4)	1.54918(2)	1.53461(2)	eV
E(E1L1)	1.6060 (2)	1.5928 (3)	1.5613 (1)	1.5455 (1)	eV
Γ (E1H1)	2.13 (1)	1.77 (2)	1.34 (1)	1.61 (1)	meV(HWHM)
Γ (E1L1)	2.62 (6)	2.68 (12)	1.61 (5)	1.71 (3)	meV(HWHM)
g(E1H1)	48 (1)	-89 (2)	109 (1)	122 (1)	degrees
g(E1L1)	129 (5)	-15 (8)	130 (5)	129 (4)	degrees
C(L/H)	0.30 (2)	0.49 (4)	0.25 (1)	0.37 (1)	-
Lorentzian Lineshape Fits					
Q	0.2205	0.3008	0.1842	0.2184	-
E(E1H1)	1.58507(6)	1.57207(7)	1.54920(4)	1.53465(4)	eV
E(E1L1)	1.6057 (3)	1.5925 (4)	1.5612 (2)	1.5452 (1)	eV
Γ (E1H1)	2.44 (6)	1.99 (6)	1.50 (4)	1.86 (4)	meV(HWHM)
Γ (E1L1)	2.9 (3)	3.0 (4)	2.0 (2)	2.1 (1)	meV(HWHM)
g(E1H1)	48 (3)	-88 (3)	109 (3)	124 (2)	degrees
g(E1L1)	119 (11)	-27 (13)	125 (13)	142 (16)	degrees
C(L/H)	0.33 (6)	0.49 (11)	0.23 (6)	0.41 (4)	-

Table 5.2 : Experimental details of 17 K ER spectra shown in Fig. 5.18, along with the parameters obtained from the least squares fits, using either Gaussian or Lorentzian broadened lineshapes. The errors in the final digit of the fitted values are shown in brackets. Eight parameters are varied in total. The ER is measured in arbitrary units, so only the ratio of the E1L1 to E1H1 ER transition strengths, C(L/H), is listed here.

The lines in Fig. 5.18 show least squares fits made to the experimental spectra, using the method described in Sec. 4.4, and taking either a Gaussian (solid line) or Lorentzian (dashed) exciton broadening profile, as indicated in Fig. 5.18. The transition energy, linewidth, strength and lineshape phase of the E1H1 and E1L1 ground state excitons were varied in order to find the best fit. The fitted parameters are listed in table 5.2, along with the "fit quality" (Q), defined in Sec.

4.4, which is the quantity minimised by the least squares procedure, and is therefore a measure of the agreement between the data and theory.

It can be seen in Fig. 5.18 that the fitted spectra generated using a Gaussian profile are in better agreement with the data, than those produced with the Lorentzian. This is substantiated by the values of Q listed in table 5.2. Notice also that there is no systematic variation in the exciton profile with well width.

In contrast, lineshape fits to low temperature ER spectra taken on GEC256, the MBE grown SQW studied earlier, were better described by the Lorentzian profile, as I previously reported in Ref. 1.13. It is not clear at present why the Gaussian profile gives the better agreement with low temperature spectra taken on the MOVPE SQWs, while the Lorentzian is better for the MBE grown SQW. However, the linewidths are significantly smaller in the MOVPE SQWs of the present study, so that the broadening may be limited by a different mechanism.

Fig. 5.19 shows spectra taken on sample MV302 (59 Å) at different applied biases between -2 and -24 V. As in the PC spectra of Fig. 5.10, it can be seen that the large features due to the E1H1 and E1L1 excitons move to lower energy with increasing field. The transition energies measured in ER are in close agreement with the position of the PC peaks in Fig. 5.10, as demonstrated later in Sec. 5.3.5. It can also be seen in Fig. 5.19 that the width of the ER features increases with field, as also observed in the PC spectra. The phase of the lineshape of both the excitonic features is seen to evolve slowly as the bias is varied, which is largely due to the change in the transition energy, as discussed in Sec. 5.3.5.

ER spectra were also taken on the other three SQW samples as a function of the applied electric field, and these showed qualitatively similar behaviour to that shown in Fig. 5.19 for MV302. Least squares fits were made to the field-dependent ER spectra in an identical fashion as in Fig. 5.18, using the Gaussian exciton profile. The fits to the ER spectra taken on MV302 are shown as the solid lines in Fig. 5.19. The quality of the fit was found to degrade slightly with increasing field, although the Gaussian profile was still found to give better agreement than the Lorentzian. The evolution of the fitted exciton transition energies, linewidths, strengths and lineshape phases are now discussed.

5.3.5 Evolution of Parameters Fitted to Electroreflectance Spectra

The open squares in Figs. 5.12 and 5.13 show the E1H1 and E1L1 exciton transition energies respectively, fitted to the field-dependent

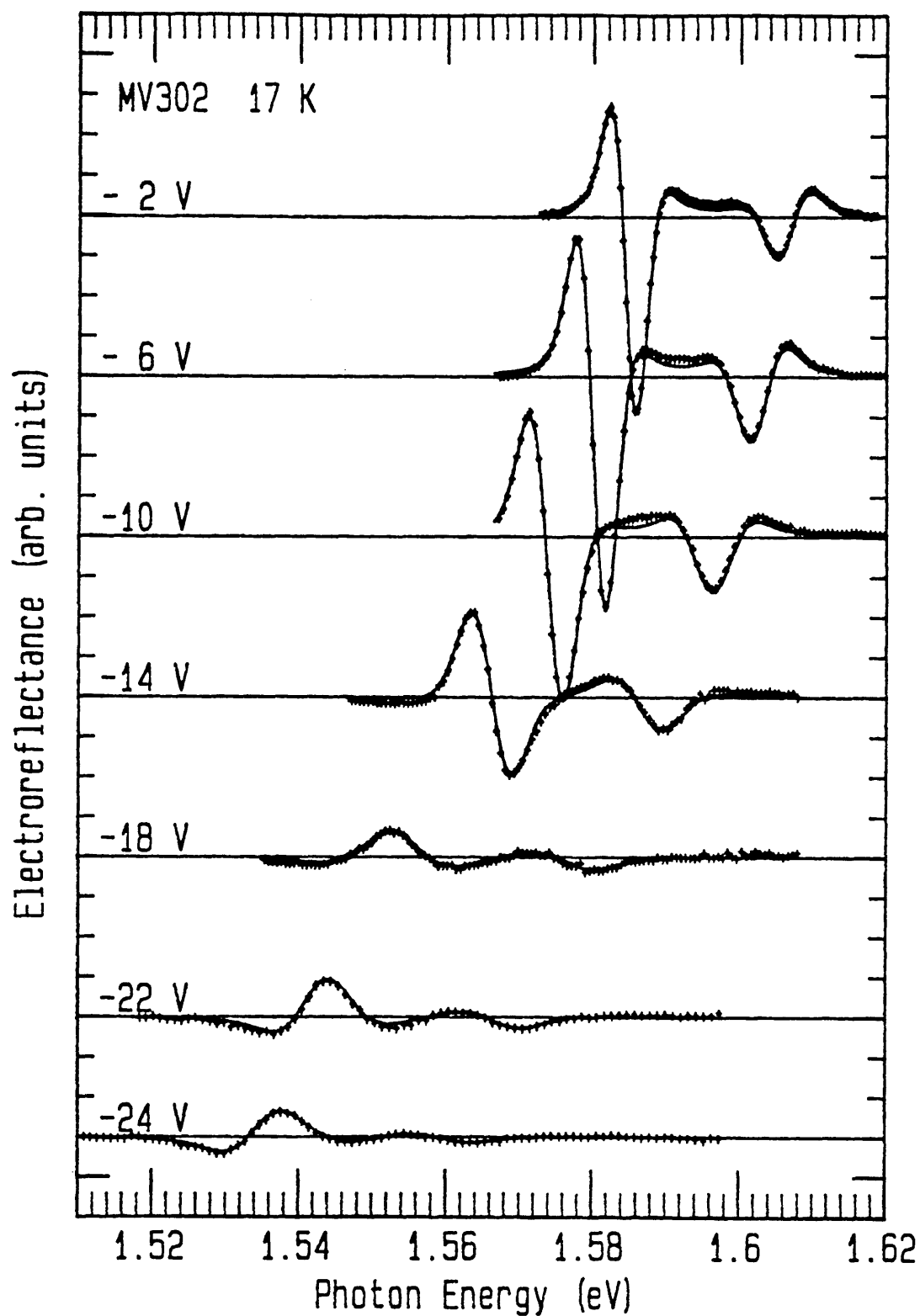


Fig. 5.19 : The data points show 17 K ER spectra taken on MV302 ($59 \text{ \AA}$) at different applied d.c. offset biases, with a fixed $1.0 \text{ V}_{\text{ptp}}$ modulation. The solid lines are least squares fits using a Gaussian exciton broadening profile.

ER spectra taken on the four MOVPE SQWs. It can be seen that these are in close agreement with the energies of the PC peaks (open squares), and also the calculated Stark shift (solid lines).

Fig. 5.20 plots the field dependence of the E1H1 (solid circles) and E1L1 (solid triangles) ER linewidths (HWHM) fitted to the ER spectra of MV302, assuming Gaussian broadened exciton profiles, shown in Fig. 5.19. Also plotted is the E1H1 linewidths (HWHM) measured in the PC spectra of Fig. 5.10 (open circles), and previously plotted in Fig. 5.15. It can be seen that the E1H1 linewidths deduced from the ER fits are smaller than those measured in the PC spectra, and that the discrepancy increases with the field. This was also found to be the case for the other SQWs. This may be due to the exciton profile not being a true Gaussian and therefore not be significant. Lineshape fits made to the spectra using the Lorentzian profile produced larger linewidths, although the quality of the fit was then poorer than for the Gaussian profile. The fitted ER linewidth in Fig. 5.20 may therefore be an underestimate, although they still demonstrate the trend in the linewidth with field.

The solid lines in Fig. 5.20 show the calculated E1H1 and E1L1 linewidths due to the field-induced tunnelling of the electron and hole from the well. This was calculated for an ideal 59 Å GaAs/Al_{0.3}Ga_{0.7}As QW, using the method detailed in Sec. 2.6.1. It therefore ignores the presence of the SLs in the barriers of the sample structure, which will tend to produce a more rapid increase in the linewidth with field. It can be seen that the calculated E1L1 resonance is very broad at fields in excess of 200 kV/cm, caused by the ionisation of the L1 state. However, notice that the fitted E1L1 ER linewidth does not increase as rapidly at fields above 200 kV/cm, as the calculations predict. A reasonably narrow E1L1 feature can also be seen in the PC (see Fig. 5.10) and ER (see Fig. 5.19) spectra, at reverse biases larger than -16 V (210 kV/cm). This suggests that either, the valence band barrier height, or the light hole barrier mass, is larger than the values given by table 4.1. The light hole barrier mass was taken as that at the Γ -point, which may be a poor approximation, if the light hole barrier wavefunction is also influenced by the other maxima in the valence band, as commented upon by Duggan et al [2.3].

It can be seen in Fig. 5.19 that the lineshape phase of the E1H1 and E1L1 excitons show a slow variation with field. A similar variation was also observed for the other SQWs. Since the lineshape phase has a linear dependence on the photon energy, $g = 2 \pi n \cos \theta E / \hbar c$, the phase can be expected to decrease due to the Stark shift. Fig. 5.21

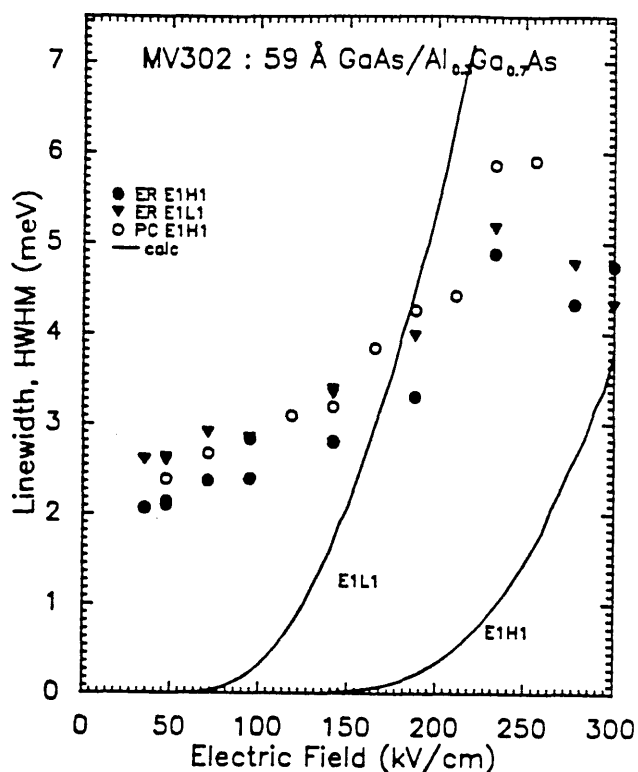


Fig. 5.20 : The field dependence of the HWHM linewidths fitted to 17 K ER spectra of MV302 (shown in Fig. 5.19) for E1H1 (●) and E1L1 (▼). Also plotted are the E1H1 linewidths measured in the PC spectra of Fig. 5.10 (○). The solid lines show the calculated linewidth due to field-induced tunnelling.

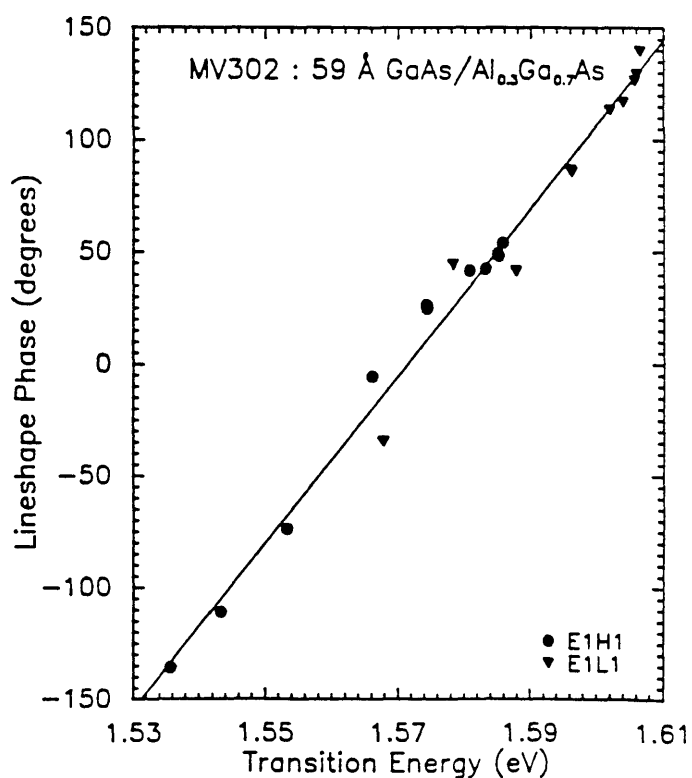


Fig. 5.21 : The variation of the E1H1 (●) and E1L1 (▼) lineshape phases fitted to the 17 K ER spectra of MV302 (Fig. 5.19), as a function of the fitted exciton transition energy.

plots the E1H1 (solid circles) and E1L1 (solid triangles) lineshape phases deduced from the lineshape fits to the ER spectra of MV302, against the fitted exciton energy. This shows the expected linear variation of the E1H1 and E1L1 phase with energy. Notice also that the larger lineshape phase of the E1L1 features compared to that for E1H1, in any one spectrum, is due to its larger transition energy, so that the E1H1 and E1L1 points in Fig. 5.21 lie on the same straight line.

The straight line drawn through the points in Fig. 5.21 has a gradient of 66 rad/eV. Meanwhile the gradient due to the change in the transition energy alone, is given by $(2t/n \cos\theta / \hbar c) = 47$ rad/eV. This was calculated using the average refractive index ($n = 3.389$) and overlayer thickness ($t = 1.3645 \mu\text{m}$) determined in table 3.1 and taking $\theta_0 = 20^\circ$. The more rapid evolution of the measured phase, than that predicted, can in part be explained by the change in the refractive index of the overlayer with the photon energy. Aspnes et al [3.35] measured the zero-field refractive index of $\text{Al}_{0.315}\text{Ga}_{0.685}\text{As}$ to decrease from 3.456 at 1.6 eV to be 3.404 at 1.5 eV. Allowing for this rate of change of n with E , increases the estimate of the gradient to 54 rad/eV, in better agreement with that measured. However this explains only part of the discrepancy, which suggests that the refractive index of the overlayer inside the intrinsic region may also decrease significantly with field.

Fig. 5.22 plots the field dependence of the fitted strength of the E1H1 (solid circles) and E1L1 (solid triangles) ER features, which correspond to the coefficient C in Eq. 3.29. The solid lines show the calculated variation of $V_{\text{ex}} \cdot (\delta E_{\text{ex}} / \delta F)$ for the E1H1 and E1L1 excitons of a $59 \text{ \AA}$ $\text{GaAs}/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW. Eq. 3.30 demonstrates that this quantity determines the field dependence of C , if the field-induced change in the exciton linewidth and overlap can be ignored. The curve in Fig. 5.22 calculated for the E1L1 exciton was multiplied by a factor of 1/3, equal to $P_{\text{CV}}^2(\text{E1L1})/P_{\text{CV}}^2(\text{E1H1})$ for the light polarisation used here, as discussed in Sec. 2.3. The experimental spectra were measured in relative units, so the fitted ER strengths have all been scaled by the same factor, to allow comparison with the calculation.

It can be seen that there is general agreement between the measured and calculated field dependences, confirming that the QW dielectric function is modulated by the QCSE and that the change in the exciton energy is dominant over that in the linewidth and overlap, as predicted in Sec. 3.2. The ER transition strength increases linearly at low fields, because the Stark curve is quadratic and $(\delta E_{\text{ex}} / \delta F)$ is therefore linear. At higher fields the Stark curve becomes linear, so

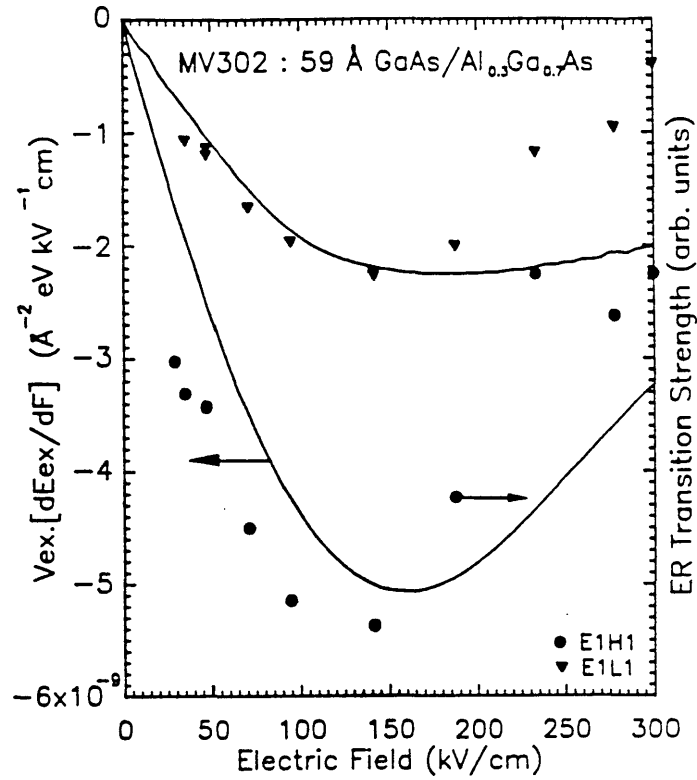


Fig. 5.22 : The field dependence of E1H1 (●) and E1L1 (▼) transition strengths, ie C in Eq. 3.29, fitted to 17 K ER spectra of MV302 (Fig. 5.19). The solid lines show the variation of the exciton overlap, V_{ex} , defined by Eq. 3.4, for E1H1 and E1L1 in a 59 Å GaAs/Al_{0.3}Ga_{0.7}As QW.

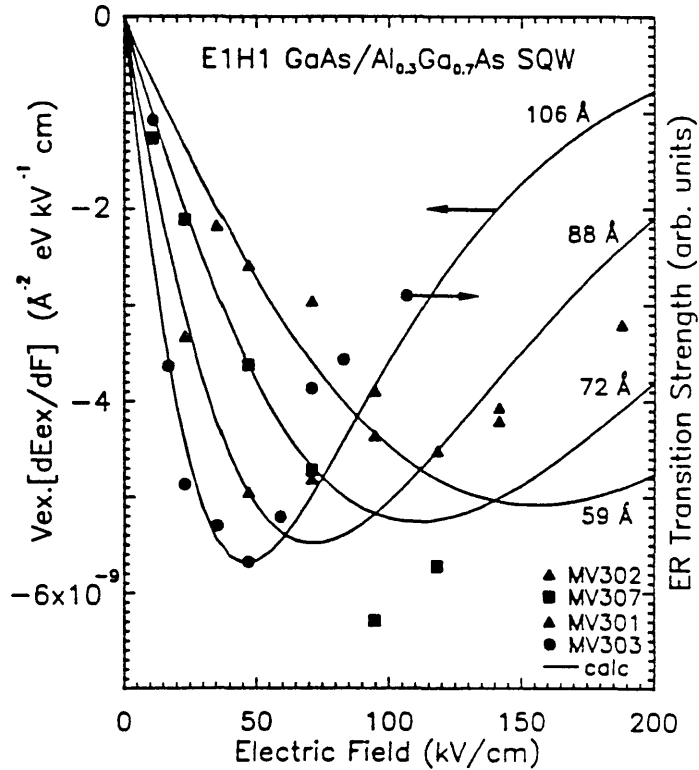


Fig. 5.23 : The field dependence of the E1H1 transition strengths in each of the four MOVPE grown SQWs. The solid lines show the calculated variation of V_{ex} for E1H1 in each well. The data points for each sample have been scaled to match the calculated curves at -2 V (47.1 kV/cm).

that $(\delta E_{\text{ex}}/\delta F)$ levels off, while the decrease in V_{ex} becomes more significant, causing the ER transition strength to decline again.

The decrease in the ER transition strength at high field, is more readily apparent in the wider wells. Fig. 5.23 plots the field dependence of the E1H1 transition strength measured in each of the SQWs. The solid lines again show the calculated variation in $V_{\text{ex}} \cdot (\delta E_{\text{ex}}/\delta F)$ for each well width. As the ER spectra were measured in relative units, the set of points for each SQW has been scaled to match the calculated curves at -2 V (47.1 kV/cm). Again a general agreement between the measured and calculated variation in the ER transition strength is observed. It can be seen that the transition strength increases most rapidly for the wider wells at low fields, as expected, due to their larger Stark shift. Notice also the decline in the ER strength of the widest well at fields in excess of about 50 kV/cm.

5.4 Low-Field Measurements on Single Quantum Wells

This section describes a number of measurements on the MOVPE SQWs at low biases. A threshold voltage is needed to remove the photo-excited carriers from the well at low temperature, and the nature of this is discussed in Sec. 5.4.1. The electrons and holes that cannot escape from the well recombine to produce a photoluminescence signal. The field-dependent PL spectra from these SQWs show unexpected structure, which is discussed in Sec. 5.4.2. The increase in the PC signal with field, and corresponding decrease in the PL, are argued in Sec. 5.4.3 to be determined by resonant tunnelling of the carriers from the well into the SLs, also contained inside the sample structure. Sec. 5.4.4 reports ER measurements made over the same range of biases, which again shows evidence for the resonant tunnelling, as well as further unexpected behaviour at lower fields.

5.4.1 Voltage Threshold for Low Temperature Photocurrent

A minimum reverse bias must be applied to the MOVPE grown SQW samples to produce a PC signal at low temperature. The nature of this threshold was studied by measuring the PC signal as a function of the applied bias, at a fixed photon wavelength. These measurements were made with the same experimental arrangement as that described in Sec. 5.1, with the sample temperature again maintained at 17 K.

Fig. 5.24 shows PC against applied bias scans collected on MV301

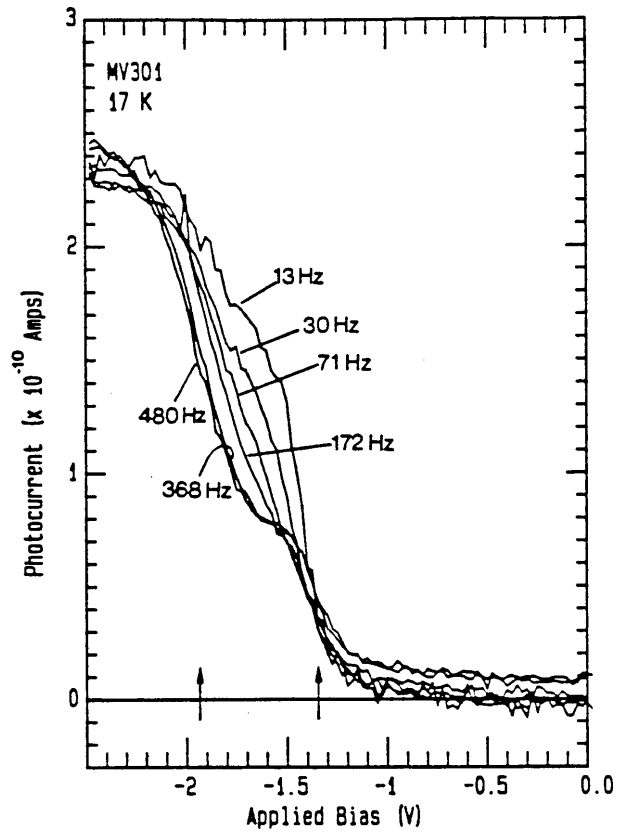


Fig. 5.24 : PC against applied bias for MV301 ($L_w = 88 \text{ \AA}$), using different chopping frequencies for the incident light. The PC is excited by light of energy, $E_{ph} = 1.5869 \text{ eV}$ and power, $P_{ph} = 36 \text{ nW/cm}^2$ and the sample temperature was 17 K.

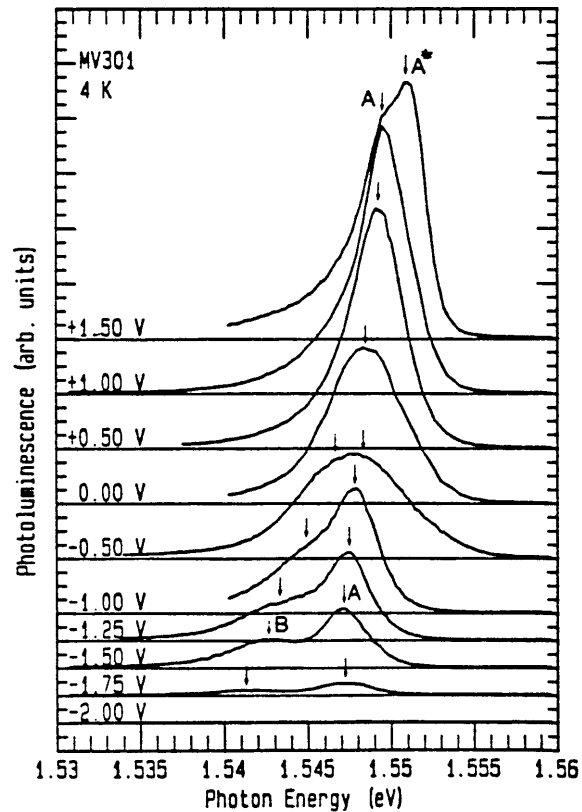


Fig. 5.25 : PL spectra taken on MV301 (at 4 K), under different applied biases, excited by laser light of $E_{ph} = 2.4091 \text{ eV}$ and $P_{ph} = 4.2 \text{ mW/cm}^2$

(88 Å), taken using several different chopping frequencies of the incident beam. The spectrometer energy (E_{ph}) was fixed at 1.5869 eV. This energy was chosen in a flat region of the PC spectrum, so that the curves reflect the variation in the quantum efficiency with field and do not show structure due to optical transitions being swept through the excitation energy. Photo-excited carriers are only excited into the well at this photon energy. The excitation power (P_{ph}) was estimated to be 36 nW/cm², from the size of the PC (I_{ph}) at reverse biases where its value was found to saturate, using $P_{ph} = (1-R) \cdot E_{ph} \cdot I_{ph} \cdot \eta / (2 \cdot e \cdot A)$, where A is the area of the mesa ($A = 0.0036$ cm²), R is the reflectivity of the front surface ($R = 0.30$) and the quantum efficiency (η) has been taken to be unity at the biases where the PC saturates.

It can be seen in Fig. 5.24 that the PC signal is zero until the reverse bias starts to exceed -1 V. Then, for the highest frequency curves, the PC rises in two quite steep steps, the midpoints of which lie near -1.3 and -1.9 V. The PC signal saturates near -2.2 V and does not increase appreciably with further reverse bias. Hence the quantum efficiency can be assumed to be unity in this region, allowing the value of the incident power to be estimated from the PC, as discussed above. PC against photon energy spectra, taken at biases on either of the two high-frequency plateaux in Fig. 5.24, had the same shape, differing only in the size of the signal and a small Stark shift, consistent with the change in field. It can be seen that reducing the chopping frequency causes an increase in the height of the -1.3 V plateau, so that the onset of the second rising edge becomes smeared.

The curves in Fig. 5.24 were found to be insensitive to the exciting wavelength, providing the photon energy was chosen to lie in a featureless region of the SQW PC spectrum. This is not surprising since the carriers will quickly relax to the lowest energy state. As the temperature was increased the threshold edge smeared out, so that at room temperature a PC signal could be measured at forward biases approaching the "turn-on" voltage. This can be understood as arising from thermal excitation of the carriers from the well.

Some variation in the behaviour of the PC threshold was found between the four SQW samples. For all the other SQWs, the PC was again zero at 0 V and then increased sharply with increasing reverse biases between -0.5 and -2 V, reaching a saturated value near -2 V. The PC again increased in two steps for MV303 (106 Å), whose midpoints occurred at voltages of -1.0 and -1.5 V. However, for MV302 (59 Å) there was only a single sharp threshold at -1.35 V. The reason for this variation between the samples is not clear, although it may be due to the

different well widths, or to a unintentional variation in some other property of the samples, such as their intrinsic region doping levels or their barrier thicknesses.

Tarucha and Ploog [5.16] have also made low temperature PC against bias scans, in their case upon GaAs/AlAs MQW samples. They observed maxima in the PC at the biases where the E1 confined level aligned with the E2, E3 or E4 level of its neighbour. It is argued in Sec. 5.4.3 that the PC threshold in this SQW sample is also influenced by resonant tunnelling of the carriers from the QW. However, before discussing the transport mechanisms, the closely related field dependence of the PL will be examined. The QW will only produce luminescence when the transport of the carriers from the field is slower than the recombination time. Hence the range of biases where the PL and PC signals can be detected are naturally complementary.

5.4.2 Field-Dependent Photoluminescence Spectra

The PL of some of these MOVPE SQWs was also studied at different applied biases. The data taken on MV301 (88 Å) is presented here, although the spectra recorded for MV303 (106 Å) are very similar, as discussed later. The PL was collected on the same mesas that were used to study the PC in Secs. 5.3.1 and 5.4.1 and ER in Sec. 5.3.4.

Luminescence was excited using the 514.5 nm line of an Argon ion laser. Most of the laser light will be absorbed in the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers at this photon energy, and indeed a large PL peak was observed at the energy of the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ bandgap. However, there was also a significant PL signal at the energy of the QW bandgap, showing that many of the photo-excited carriers relaxed into the well before recombination. The PL signal was collected by a double grating spectrometer of 0.85 m focal length, set to have a spectral resolution of about 0.1 nm. The sample was maintained at a temperature of 4 K in a continuous flow He cryostat. Experiments were performed with a variable low excitation power, which was estimated from the size of the saturated reverse bias PC, in the same manner as described in Sec. 5.4.1.

The PL spectra taken on MV301 at different applied biases, and with an excitation power of 4.2 mW/cm², are plotted in Fig. 5.25. A single peak at 1.5496 eV (labelled "A") can be seen in the +1.0 V spectrum, with a HWHM linewidth of 1.75 meV. The E1H1 transition energy at zero-field, interpolated from the PC measurements in Fig. 5.12, is 1.5525 eV. This yields a Stokes shift for the PL of 3.6 meV, after allowing for the temperature difference between the PC and PL

experiments. This value may be a few meV larger if the temperature in the PC experiments was actually larger than 17 K, as discussed earlier. Notice that the +1.0 V E1H1 PL linewidth is slightly larger than that measured in the -2 V PC spectrum (1.55 meV) and their lineshapes are rather different.

The presence of a Stokes shift suggests that the recombination is extrinsic, possibly occurring by a E1H1 exciton localised on an ionised impurity. At the larger forward bias shown in Fig. 5.25, of +1.5 V, the PL appears to consist of a doublet (labelled "A" and "A^{*}") separated by about 1.1 meV. The low energy peak of this pair is again attributed to recombination of the E1H1 exciton bound to an ionised impurity, as in the +1.0 V spectrum. The upper energy component of this doublet (A^{*}) may possibly be due to recombination of an E1H1 exciton bound to a neutral donor. The splitting could then be qualitatively explained by the smaller binding energy of an electron-hole pair bound to a neutral rather than ionised donor. This assignment could also explain why the intensity of peak A^{*} decreased rapidly when the bias was reduced from +1.5 to +1.0 V, since the depletion edge would be expected to reach the QW at such a bias, thereby ionising the impurities.

When the forward bias was reduced, peak A in the +1.0 V PL spectrum initially showed a modest shift to lower energy, due to the QCSE. Then, at biases between -1 and -2 V, the total PL intensity is seen to rapidly decrease. This quenching of the PL is in rough agreement with the range of biases where an increase in the PC signal was observed in Sec. 5.4.1. However, rather unexpectedly, the PL peak is also seen to broaden, so that at -0.5 V the linewidth is 3.7 meV (HWHM). Notice that two peaks are resolved in Fig. 5.25 at reverse biases larger than -0.5 V. The lower energy of these two peaks (labelled "B") shifts rapidly to lower energy, while also rapidly declining in intensity relative to the higher energy peak (A). The broad PL peaks at 0.0 and -0.5 V are therefore probably due to two similar sized peaks, whose energy separation is too small for them to be resolved. The HWHM linewidth of peak A has recovered to be 1.8 meV in the -1.5 V spectrum, close to its value in the +1.0 V spectrum.

The energies of the PL peaks are plotted in Fig. 5.26, as a function of the applied bias. The peak labelled A evolves slowly with bias, due to the Stark shift of the E1H1 transition, while peak B shows a much larger energy shift, with a linear dependence on the bias. The variation of the two PL peaks can be interpolated to intersect at approximately -0.14 V. This again suggests that the broad PL spectra seen at 0.0 and -0.5 V are due to two unresolved peaks.

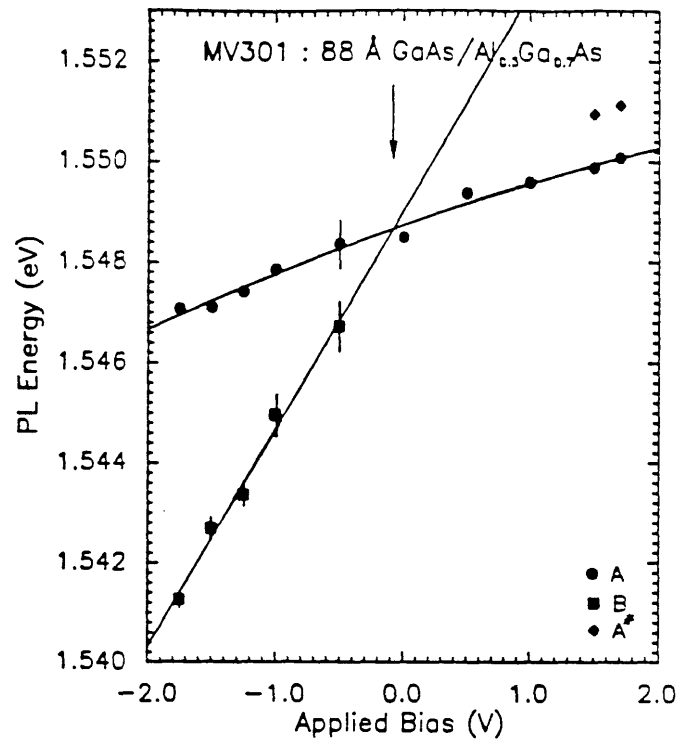


Fig. 5.26 : Field dependence of the energies of the PL peaks measured on MV301, shown in Fig. 5.25. The lines passing through the data are guides to the eye.

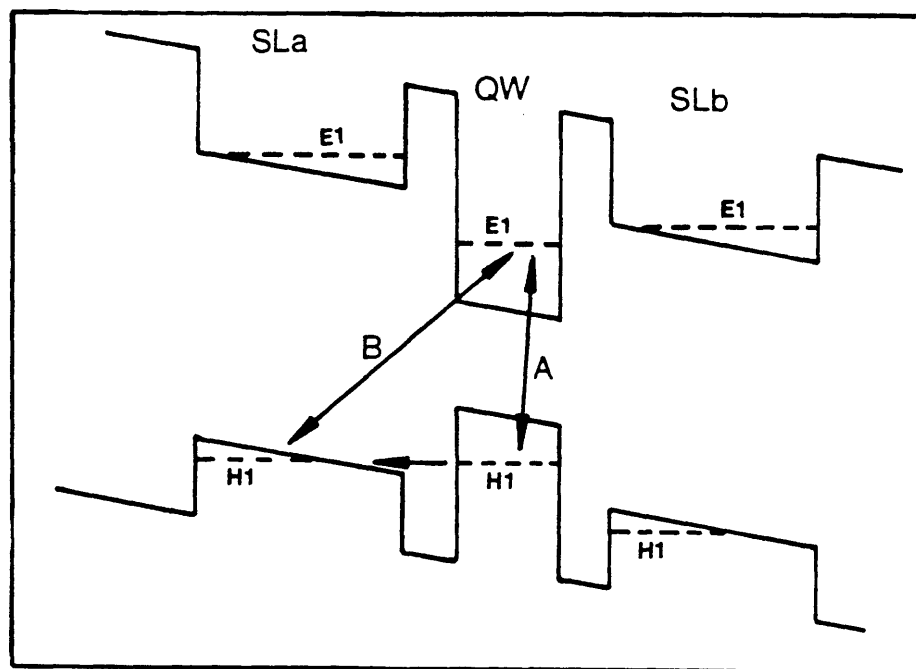


Fig. 5.27 : Schematic diagram of the valence and conduction band profile for the SQW samples. Notice that the two GaAs(15 Å)/Al_{0.3}Ga_{0.7}As(15 Å) SLs on either side of the QW, have been replaced with an Al_{0.15}Ga_{0.85}As layers, labelled "SLa" and "SLb".

When the excitation power was increased to 70 mW/cm², the spectra were again seen to broaden and then narrow with increasing field, although this time a splitting into two peaks was not resolved. This is presumably due to an increase in the linewidth of the individual peaks. However, when a much lower intensity of 0.35 mW/cm² was used, there was no apparent splitting or broadening of the spectra.

Measurements on MV303 (106 Å), using an excitation power of 8.4 mW/cm², produced PL spectra very similar to those for the 88 Å QW in Fig. 5.25. With increasing field the PL peak was again seen to broaden and then divide into two peaks, one of which decreased in relative intensity and shifted very rapidly to lower energy. A plot of the bias dependence of the PL peaks of MV303 was also very similar to that for MV301, Fig. 5.26. Peak B was seen to evolve linearly at a similar rate as in Fig. 5.26, while peak A had a slightly larger energy shift with bias than for MV301, due to the larger Stark shift of this wider well. Again the two PL peaks were seen cross at an applied bias of about 0 V.

5.4.3 Transport of the Photo-excited Carriers

At room temperature the carriers have sufficient thermal energy to be excited over the potential barriers of the QW, and so produce a PC signal in forward bias. However, at low temperature the carriers have much less thermal energy and their transport will be determined by tunnelling through the barriers. This tunnelling is enhanced by an applied field, so resulting in an increase in the PC signal, with corresponding decrease in the PL. This section seeks to explain the bias dependence of the low temperature PL and PC observed in Secs. 5.4.1 and 5.4.2.

The carriers must tunnel through the 50 Å Al_{0.3}Ga_{0.7}As barriers into the 6 period GaAs(15 Å)/ Al_{0.3}Ga_{0.7}As(15 Å) SLs, contained within the layer structure. Since these SLs have very narrow layers, their electronic structure will be approximated by that of a 195 Å wide Al_{0.15}Ga_{0.85}As/Al_{0.3}Ga_{0.7}As QW. The system can then be thought of behaving like three coupled QWs, as depicted in Fig. 5.27. The two SL regions are labelled "SLa" and "SLb" in the following discussion, as indicated in Fig. 5.27.

The field produces a potential drop across the structure which enhances the tunnelling of the electrons from the QW into SLb and the holes from the QW to SLa. However, the tunnelling rates can be expected to be negligible, until they are resonantly enhanced by the alignment of the confined levels in the QW and the SLs. Hence, at the lowest

electric fields, the electron and hole tunnelling times will be much longer than the recombination time. Consequently there will be a large PL intensity and negligible PC signal, as was observed in the experiments described in Sec. 5.4.1-2.

The tunnelling of the holes from the QW into SLa will be greatly enhanced at the field where their lowest confined levels align. It was calculated to require a uniform field of about 26 kV/cm to align the H1 levels of the 88 Å QW and SLa. This calculation took the potential discontinuities and effective masses in table 4.1, along with the layer widths specified in the growth procedure, except for the 88 Å well layer. Meanwhile the E1 levels of the QW and SLb were calculated to be degenerate at 43 kV/cm. The larger field required to align the electron levels, is a consequence of the greater discontinuity of the GaAs and $\text{Al}_{0.15}\text{Ga}_{0.85}\text{As}$ potentials in the conduction band, compared to that in the valence band.

A significant proportion of the holes will transfer from the QW to SLa when their H1 levels align. Meanwhile the electrons will still be confined to the QW, since the E1 level of the QW will still be lower in energy than that of SLb. Electrons in the QW could then recombine with holes confined in either the QW or SLa. These two recombination processes could explain the origin of the two peaks (A and B respectively) observed in the PL spectra of Fig. 5.25. The two PL peaks were found to be degenerate at -0.14 V, which corresponds to an electric field of 25 kV/cm. (The experimental field was estimated with the parameters previously discussed in Sec. 4.2.) This is in very good agreement with the field needed to align the H1 levels of the QW and SLa, calculated to be 26 kV/cm. This assignment for PL peaks A and B, could also explain the sudden appearance of peak B at $V_{\text{app}} \leq 0.0$ V, as only then does the H1 level of SLa become populated. (Remember that the broad PL peaks seen at 0.0 and -0.5 V were due to the unresolved presence of peak B). For increasing reverse bias, the H1 level of SLa continues to move to lower potential relative to H1 of the QW, which explains the rapid linear energy shift of peak B. The decrease in the intensity of peak B relative to that of peak A with reverse bias, could be understood as resulting from a reduction in the hole population of SLa, by the diminished tunnelling, when the H1 levels in the QW and SLa go out of alignment.

The E1 levels of the QW and SLb will become degenerate at a field of about 43 kV/cm. At this bias both the electrons and holes will be able to tunnel freely from the QW into the SLs. This calculated field is in good agreement with the quenching of the PL near -2 V (47 kV/cm),

and the corresponding saturation of the PC signal. This suggests that the electrons and holes can tunnel from the SLs to the contacts at these fields.

The calculations discussed above assume the electric field to be uniform across the structure. The separation of the positive charge in SLa and a net negative charge in the QW will create a dipole, producing a field opposing the hole tunnelling. The hole population in SLa and the field between the QW and SLa will adjust to find steady state conditions. The field profile across the structure will then be non-uniform and its calculation will require a self-consistent analysis, balancing the various tunnelling and recombination processes for the reservoirs of charge in the QW and SLs. However at lower biases, where all the charge is confined inside the well, the field can be expected to be uniform, so the calculation of the field required for alignment of the H1 levels in the QW and SLa will still be accurate. Likewise, the field for alignment of the E1 levels in the QW and SLb, will also not be affected, since then all the charge is swept from the region.

The simple arguments presented here do not predict the size of the PC signal at the moderate biases, where the holes can tunnel from the QW but the electrons cannot. At reverse biases larger than about -2 V, it becomes possible for both electrons and holes to tunnel freely from the well, which explains the rapid increase in the PC centred at -1.9 V in Fig. 5.24. However, the origin of the threshold centred near -1.3 V in Fig. 5.24 is not clear at present. The frequency dependence of this lower PC threshold suggests that it is produced by a carrier transport process that occurs over a millisecond time period. Meanwhile, the variations in the behaviour of the threshold between samples, suggests that it may be very sensitive to the exact values of the layer widths etc. The behaviour at these biases may therefore be difficult to explain without a self-consistent analysis, where all the layer widths and time constants for the different charge transfer processes in each region are known precisely.

5.4.4 Electroreflectance Lineshapes at Low Electric Fields

ER measurements were made at low biases to determine if the resonant tunnelling effects described in Sec. 5.4.3, which dominate the emission spectra, have any influence upon the excitation spectra. The data points in Fig. 5.28 show 17 K ER spectra taken on MV301 (88 Å) with a fixed modulation of 0.2 V_{ptp} and a variable d.c. offset of between -1.8 and +1.2 V. The modulation was kept as small as possible, so as to

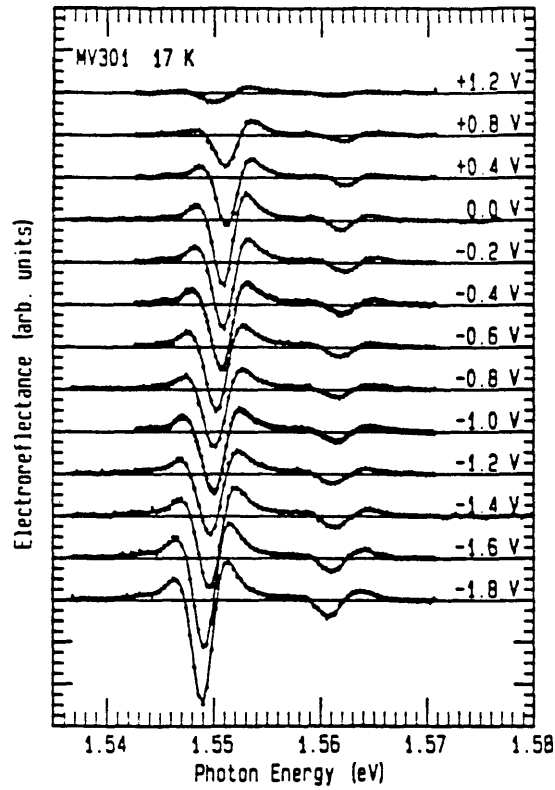


Fig. 5.28 : ER spectra (+) taken on sample MV301 ($L_w = 88 \text{ \AA}$) at different applied d.c. offset biases between -1.8 and +1.2 V, with a fixed 0.2 V_{ptp} modulation, $\theta_0 = 45^\circ$ and $T = 17 \text{ K}$. The solid lines show lineshape fits made to the spectra, as discussed in the text.

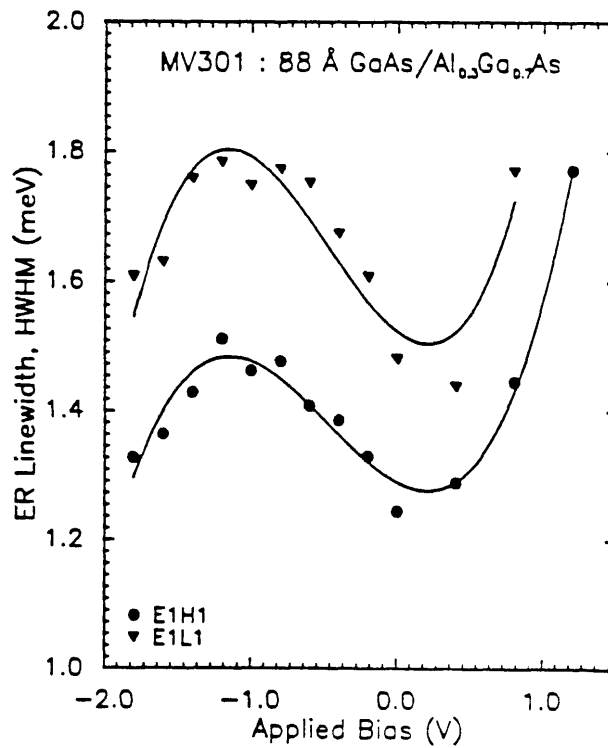


Fig. 5.29 : The bias dependence of the E1H1 (●) and E1L1 (▼) linewidths (HWHM) fitted to the ER spectra of MV301 shown in Fig. 5.28. The solid lines are guides to the eye.

allow a good voltage resolution. The angle of the incident light was 45° and the other experimental details were kept identical to those described in Sec. 5.3.4.

It can be seen in Fig. 5.28 that both the E1H1 and E1L1 features show a slow evolution to higher energy with decreasing field, with a simultaneous decline in their strength. This is in good agreement with the behaviour predicted by the QCSE, that was discussed in Sec. 5.3.5. The ER transition strengths decline due to the reduction in the modulating Stark shift.

Least squares fits were made to these spectra, shown as the solid lines in Fig. 5.28, using the method detailed in Sec. 4.4 and Gaussian exciton broadening profiles. The fitted parameters revealed a subtle field dependence, not readily apparent from the spectra shown in Fig. 5.28. Fig. 5.29 plots the variation in the fitted E1H1 and E1L1 linewidths with the applied d.c. offset bias. The fitted ER linewidths in Sec. 5.3.5 were found to gradually increase at reverse biases larger than -2 V. However, it can be seen that the variations of the E1H1 and E1L1 linewidths, at reverse biases smaller than -2 V, have unexpected local maxima near -1 V. The increase in the linewidths at biases between 0.0 and -2.0 V is likely to be related to the resonant tunnelling of holes into SLa, as discussed below, which was argued to occur over this voltage range in Sec. 5.4.3. The fitted ER transition energies were found to decrease gradually with field in the expected manner.

The PL linewidth was seen to increase dramatically at biases near 0 V in Fig. 5.25. It was argued in Sec. 5.4.3 that the H1 levels of the QW and SLa come into alignment near 0 V, so that the holes can tunnel into SLa. The fact that a similar dramatic increase in the ER linewidth was not observed at 0 V, suggests that the hybridisation of the H1 state of the QW and SLa is minimal. The small increase in the ER linewidths in Fig. 5.29 is unlikely to be due to a hybridisation of H1 and L1, as discussed below. The broad PL peak is therefore simply due to two unresolved peaks deriving from the transitions $E1(QW) \rightarrow H1(QW)$ and $E1(QW) \rightarrow H1(SLa)$. The $E1(QW) \rightarrow H1(SLa)$ transition is not observed in the ER spectra because the electron-hole overlap is extremely small.

The slight increase in the ER linewidths at biases between 0 and -2 V may be due to the internal electric field, produced by the spatial separation of net positive charge in SLa and negative charge in the QW, as discussed in Sec. 5.4.3. This separation of net charge occurs because the holes can tunnel into SLa while the electrons are trapped inside the well, at these biases. The internal electric field created

by the separation of net charge in the QW and SLa will tend to limit the hole tunnelling, so that an equilibrium is attained. If this internal field has a small random component in the plane of the well, due to a non-uniform charge distribution in the well and SLa, this will tend to ionise the exciton and cause an additional broadening. When the reverse bias exceeds -2 V, the electrons and holes can both tunnel freely from the well, so that there is no net charge separation. This turns off the internal electric field and the extra broadening.

Further unexpected behaviour was observed, at lower d.c. offset fields, when the size of the modulating bias was increased. Fig. 5.30 shows ER spectra taken on MV301, under identical conditions to those for Fig. 5.28, but with a larger modulation of 1.0 V_{ptp}. The 0 V spectrum in Fig. 5.30 is similar to that in Fig. 5.28, with the E₁H₁ feature resembling a "negative-peak" lineshape. However, notice that as the forward bias is increased, the lineshape phase of the ER feature changes, so that at +0.8 V the E₁H₁ feature is closer to a "negative-zero-crossing" lineshape. Note that this change in the ER lineshape phase in Fig. 5.30 is seen at a much smaller field than the broadening of the PL spectra, suggesting that the two phenomena are unrelated.

This rotation in the phase of the ER features at low biases was more clearly observed for spectra taken on MV307 and MV302. Fig. 5.31 displays 17 K ER spectra collected on MV302, using a modulating bias of 0.4 V_{ptp} and applied biases of between -1.0 and +0.4 V. The other experimental details are identical for the spectra taken on MV302, shown in Fig. 5.19.

The spectrum at -1.0 V in Fig. 5.31 is similar to those measured in Fig. 5.19, where it was shown previously that the QW dielectric function is modulated by the Stark shift. As the reverse bias is reduced the ER signal decreases, due to the decrease in the modulating Stark shift, and eventually, at -0.3 V, the signal is zero, suggesting the field at the QW is then very small. However, under further decreasing reverse bias and moving into forward bias, the signal is seen to unexpectedly increase again. Furthermore the signal is actually larger than it was at -0.5 V. With further increasing forward bias the ER signal was again seen to be very small at $V_{app} = +0.4$ V.

The unexpected ER spectra, at $-0.2 \text{ V} < V_{app} < +0.3 \text{ V}$, again show features due to the E₁H₁ and E₁L₁ excitons, with the transition energies being roughly the same as in the -0.5 V spectrum. The lineshapes are again reminiscent of the first-energy-differential of the dielectric function, but it is also apparent that there is a phase change in the

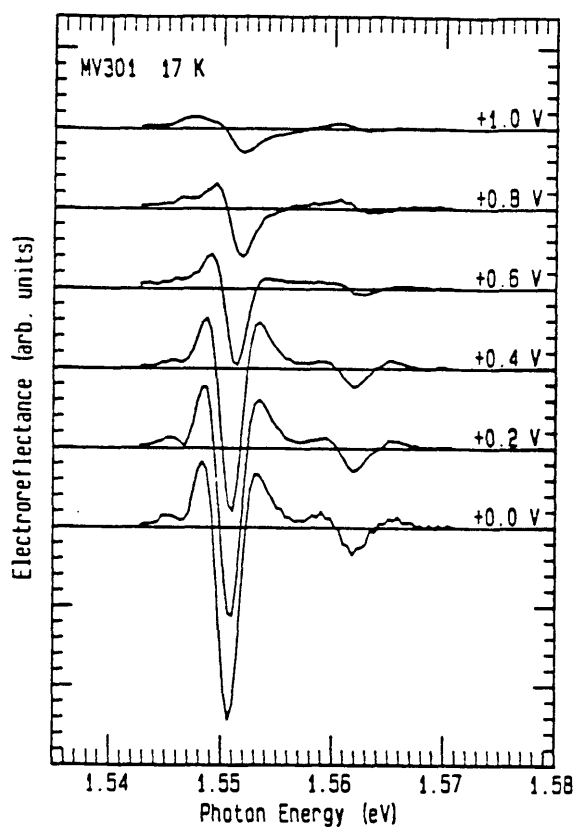


Fig. 5.30 : ER spectra (solid lines) taken on MV301 with different applied d.c. biases, under identical conditions to those in Fig. 5.28, except that the modulation was increased to 1.0 V_{ptp}.

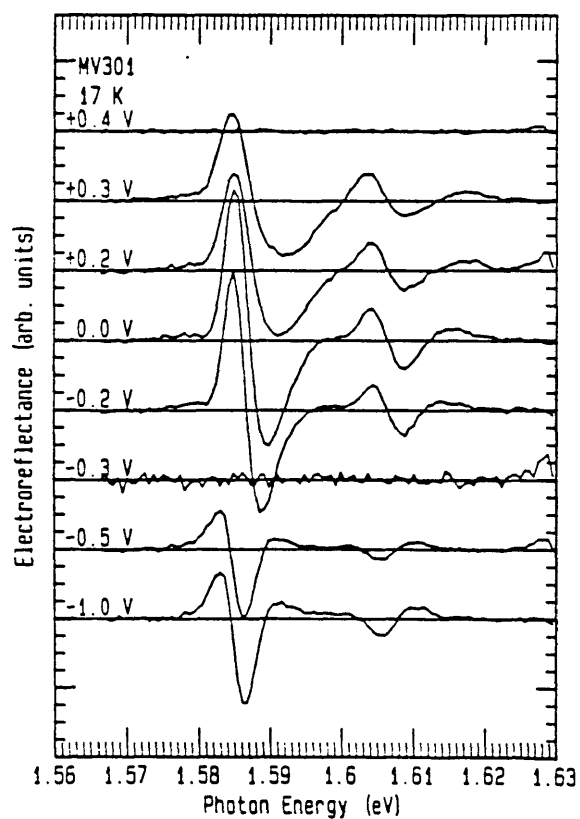


Fig. 5.31 : ER spectra taken on MV302 ($L_w = 59 \text{ \AA}$) at different d.c. biases between -1.0 and +0.4 V, with a 0.4 V_{ptp} modulation. $T = 17 \text{ K}$ and $\theta_0 = 20^\circ$.

lineshape of about 90° ; the -0.5 V spectrum has $R_{pp} = 0.56$ for the E1H1 feature, with the higher energy negative peak being larger, compared to $R_{pp} = 0.74$ for the -0.2 V spectrum with the lower energy positive peak being largest. This phase change could be due to the unexpected ER signal being modulated by a field-induced change in the linewidth, rather than the transition energy. Further evidence for this is given by the apparent increase in the width of E1H1 feature upon changing the applied bias from -0.2 to +0.3 V. The shape of the ER spectra suggests that the linewidth of the E1H1 exciton increases more rapidly on the high energy side.

The behaviour observed in Fig. 5.31 suggests that there is an increase in the exciton linewidth at biases between -0.2 and +0.3 V, which gives rise to a strong ER signal. Since the amplitude of the modulating bias is 0.4 V_{ptp}, this voltage range suggests that the linewidth increases suddenly when the d.c. bias is close to 0.0 V. The fact that the unexpected signal is detected over a narrow voltage range, which is close to the bias where the modulating Stark shift decreases to zero, suggests that it may be related to the depletion edge reaching the QW. For this to occur at a bias of +0.0 V, requires an average intrinsic region doping of $3.6 \times 10^{15} \text{ cm}^{-3}$, which is rough agreement with the value indicated by the C-V measurements, discussed in Sec. 4.2. When the depletion region reaches the well, it may become populated by a large carrier concentration, which could cause the observed increase in the linewidth.

This could also explain the phase change seen in the ER spectra of MV301 in Fig. 5.30. For MV301 the lineshape rotation starts to occur at a d.c. offset bias of about +0.6 V, with a 1.0 V_{ptp} modulation. This suggests that the depletion edge reaches the QW at +1.1 V, requiring an average background doping for the i-region of $1.6 \times 10^{15} \text{ cm}^{-3}$, again within the range suggested by the C-V measurements. It is not clear why a linewidth-modulated lineshape was not observed in the spectra of Fig. 5.28, taken on MV301 using a smaller modulation of 0.2 V_{ptp}. The ER signal remained very small when the forward bias was increased above the largest values shown in Fig. 5.28. The lineshape rotation in MV301 was observed even more clearly when the modulation was further increased to be 2.0 V_{ptp}. There was then a very clear change in the lineshape phase of roughly 90° when the offset bias was changed from 0.0 to +0.5 V.

A lineshape rotation was also observed in the ER spectra taken on MV307, at similar biases as for MV301 in Fig. 5.30. However, MV303 did not show a similar effect, which could be explained by it having a lower intrinsic region impurity concentration than the other three samples.

5.5 Summary

Angle-dependent ER measurements made on each of the five GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ SQWs studied in this chapter confirmed the importance of optical interference effects in determining the lineshape. It was predicted in Sec. 3.3, that optical interference between the rays reflected from the QW and the front surface of the sample, results in the ER lineshape being approximated by a linear mixture of the real and imaginary parts of the modulated QW dielectric function, depending upon the optical phase delay between the two rays. The phase delay was varied by changing the angle of the incident light on the sample and the lineshape was seen to evolve between lineshapes resembling $\Delta\epsilon_r$ and $\Delta\epsilon_i$, in close agreement with the predictions of Sec. 3.3. This very dramatic evolution of the ER lineshape with the angle of incidence is conclusive verification of the importance of optical interference effects in determining the lineshape.

The rate of the lineshape evolution with the angle of incidence, as measured by the ratio of principal peaks of the spectra, was used to estimate the overlayer thickness. It was also demonstrated how the ER lineshape can be used to measure topological variations in the well width and overlayer thickness. Hence ER could provide a useful technique for characterising QW structures, with the advantages that it does not require the prior removal of the substrate and can be performed at room temperature. Studying PR spectroscopy would also dispense with the need for electrical contacts.

ER could therefore be used to monitor the decrease of an overlayer thickness during an etching process. A suitable corrosive electrolyte could be used to both perform the etch and make the front electrical contact. Alternatively PR could be studied to avoid the need for electrical contacts. PR or electron-beam ER [5.17] could also potentially be used to measure the increase in the overlayer thickness during a growth process, thereby providing an alternative to the RHEED technique used in MBE [5.18]. The overlayer could consist of many different layers, provided that they do not absorb at the photon energy of the transition being monitored. A thin QW layer could be deposited deep in the layer stack, so as to provide a suitable transition to monitor the ER/PR lineshape. The energy of the ER/PR transition could also be used to determine the substrate temperature [5.19].

Sec. 5.3 described field-dependent PC and ER measurements on the four MOVPE grown GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ SQWs, with well widths of either 59, 72, 88 or 106 Å. The field dependence of the PC energies and the

integrated strength of the E1H1 peak was found to be in good agreement with a calculation of the QCSE for the QWs. In addition to proving the validity of the QCSE calculations, the PC measurements also demonstrated the estimates of the electric field in these p^+-i-n^+ diodes to be accurate. The E1H2 exciton was also observed in the PC spectra of the two wider wells and was found to have a smaller Stark shift than E1H1, while its strength increased with field, as again predicted by the QCSE. The two widest wells also showed another PC peak, due to an excited excitonic state of E1L1, which was given enhanced oscillator strength by the mixing of the H2 and L1 subbands.

Field-dependent ER measurements were also made on the four SQWs and the exciton energies, linewidths, transition strengths and lineshape phases were extracted using least squares fitting of the lineshape model. The ER transition energies were in excellent agreement with those measured in PC spectroscopy, with the Stark shifts being in close agreement with those calculated from the QCSE. The field dependence of the ER transition strength was also observed to be in good agreement with that predicted in Sec. 3.2, due to the QCSE. The lineshape phase was observed to change slowly with the applied field, due to the change in the transition energy and the refractive index of the overlayer.

Peculiar behaviour, not predicted by the QCSE, was observed in several measurements taken on these four MOVPE grown SQWs at low biases, and these were described in Sec. 5.4. The rise in the PC signal with field, and the corresponding quenching of the PL, was argued to be determined by the resonant tunnelling of the carriers from the well into the SLs also contained inside the sample structure. At applied biases between roughly 0 and -2 V, the holes are able to resonantly tunnel into SLa, while the electrons remain confined inside the well. This explained the peculiar splitting of the PL spectra at biases near 0 V, and the rapid shift of one of the split PL peaks to lower energy, as the reverse bias was increased towards -2 V. At reverse biases exceeding -2 V, both the electrons and holes can freely tunnel from the well, and the PC rapidly saturates, while the PL intensity falls to zero.

This explained many aspects of the observed behaviour, although the nature of the PC threshold, at biases between 0 and -2 V, is not entirely understood. A better understanding of the PC threshold would require further measurements on samples where the i-region doping and all the relevant layer widths have been carefully characterised. It would then be revealing to study the dependence of the PC threshold on the chopping frequency and power of the incident light and the sample temperature.

ER spectra were also taken over the range of biases where the resonant tunnelling effects in the PL and PC thresholds were observed. The linewidth was seen to increase slightly at those biases where the holes could resonantly tunnel into SL_a, while the electrons remained confined in the well. This was argued to be due to a random in-plane component of the internal field between the net charge confined in SL_a and the QW.

Further unexpected behaviour in the ER spectra was observed at lower fields. The ER features were seen to undergo a sudden phase change of 90°, that suggested a rapid broadening of the transitions. The sweeping of the depletion region through the QW was tentatively proposed as a mechanism for this increase in the linewidth. The behaviour was found to be different for the four SQW samples, with the effect being most dramatic for MV302. It was suggested that this was due to a different intrinsic region doping in each sample. Again some aspects of the behaviour were difficult to understand, for instance, the dependence on the size of the modulating bias. The effect therefore requires further investigation, which would be best performed on SQW samples with different levels of i-region doping, that have been carefully measured.

6 Measurements on Multiple Quantum Wells

6.1 Introduction

The MQW structures studied in this chapter have wide barriers, so that the carrier envelope functions are isolated in individual QWs. The MQW stacks can therefore be regarded as a collection of QWs having the same bandstructure. Previous experimental studies of quantum confinement effects have often measured MQW rather than SQW structures, since for most spectroscopies, such as PL, PLE, PC or absorption, these provide a larger signal. Interest in MQW structures has also been stimulated by their application in several devices, such as optical modulators [1.7-9].

A lineshape model for ER spectra of MQWs was presented in Sec. 3.4. It was discussed how the contribution of each QW to the ER spectrum had a different mixture of the real and imaginary parts of the modulated QW dielectric function, determined by its depth in the structure. Consequently the ER of a MQW was predicted not to be simply a scaled up version of that for a SQW, as is the case for spectroscopies depending only on the imaginary part of the dielectric function, such as PLE, PC, absorption. The optical penetration depth varies across the spectrum and hence so does the mixing of the real and imaginary parts of the modulated QW dielectric function. The QWs were assumed to be identical and evenly spaced in the analysis of Sec. 3.4, so that the contribution of each to the reflectivity could be added to yield an explicit lineshape formula. It was found that the lineshape of a single exciton consisted of a large central feature, flanked by two smaller features to lower and higher energies. The large central feature derives from the front of the stack, where the optical penetration depth will be small, while the subsidiary features are seen at energies where the absorption coefficient has decreased sufficiently to allow light to penetrate to the back of the MQW stack.

In the following studies the well width and electric field is found to vary across the stack. This prevents a direct comparison of the lineshape model of Sec. 3.4 and the experiment spectra. However, it is shown how optical interference ^{effects} are none the less very important in determining the ER spectra. The experimental studies show ER features associated with different regions of the stack, and the lineshape phases of these are consistent with their depths in the sample structure. The ER from the back of the stack is also crucially affected by absorption in the shallower layers, as was also highlighted by the model of Sec.

3.4.

Three MQW samples are studied in this chapter, each of which are GaAs/Al_{0.3}Ga_{0.7}As structures. Measurements on two of these samples are reported in Sec. 6.2. They had identical layer details, except for their well widths which were either 88 Å or 163 Å. Large fields were applied by arranging the MQWs in the intrinsic region of a p⁺-i-n⁺ diode. They were grown in the same MOVPE reactor, and under identical conditions, to the SQWs reported in Chp. 5. These samples are therefore ideal for studying the differences in the optical properties of MOVPE grown SQW and MQW structures. Bias-dependent, low temperature measurements are made using four different spectroscopic techniques; ER, ET, PC and transmission. It is emphasised that each of these provide complementary information, that allows a detailed elucidation of the microscopic structure of the MQW stacks. The bias and angular dependence of the ER lineshape is particularly useful in determining the depth in the sample of the layers producing the different spectral features.

The third sample, a GaAs(100 Å)/Al_{0.3}Ga_{0.7}As(90 Å) MQW structure, is reported in Sec. 6.3. Low temperature ER and PC measurements are made. This MQW stack is arranged in the intrinsic region of a n⁺-i-n⁺ doping profile. It is argued that the tunnelling of the dark current determines the electric field profile in the structure, which results in quite bizarre optical spectra. Again the angular dependence of the ER lineshape is found to be extremely useful in determining the depth of the layers producing the different features.

All the measurements on the MQW samples are summarised in Sec. 6.4. The deductions made about the MOVPE grown MQW stacks are discussed along with the implications for opto-electronic devices. The value of ER spectroscopy is compared to other techniques.

6.2 Measurements on Two MOVPE Grown GaAs/AlGaAs MQW Structures

Two MOVPE grown GaAs/Al_{0.3}Ga_{0.7}As MQW structures are studied in this section, which are identical except for their well width. They are arranged inside a p⁺-i-n⁺ doping profile, allowing the application of large electric fields. By comparing the various spectroscopies on the two samples and also the field dependence of these spectra, systematic variations across the stack are detected. This work is also reported in Ref. 1.15. Sec. 6.2.1 is devoted to a summary of the experimental details, while Sec. 6.2.2 describes the various spectra taken on the two

samples, which are then analysed and discussed in Sec. 6.2.3.

6.2.1 Experimental Methods

The two GaAs/Al_{0.3}Ga_{0.7}As MQW samples were grown in the same MOVPE reactor, and under identical conditions, as the MOVPE SQWs studied in Chp. 5, by Drs. J.Roberts and C.Button at the University of Sheffield. The two structures differ only in the thickness of the GaAs QW layers, which were 88 Å in MV370 and 163 Å in CB8, as determined from the room temperature, low-bias transition energy. As for the MOVPE grown SQWs, studied in Chp. 5, the well widths are significantly less than the intended values of 100 and 180 Å for MV370 and CB8 respectively. In both MQW structures the Al_{0.3}Ga_{0.7}As barrier layers were grown to be 60 Å and there were 50 wells in total. The structure was grown on a [100] GaAs n⁺ substrate, which was inclined 3° towards [110] and consists of 0.45 µm n⁺ GaAs buffer; 0.45 µm n⁺ Al_{0.6}Ga_{0.4}As; 0.6 µm n⁺ (2x10¹⁷ cm⁻³) Al_{0.3}Ga_{0.7}As; the undoped MQW; 400 Å undoped Al_{0.3}Ga_{0.7}As; and finally 0.3 µm of p⁺ (7x10¹⁷ cm⁻³) Al_{0.3}Ga_{0.7}As. The MQWs were arranged inside the intrinsic region of p⁺-i-n⁺ diodes to allow the application of a variable electric field. The detailed growth procedure is described in Ref. 4.2.

The two wafers were processed into 600 µm square mesas and ohmic contacts made to the top and substrate. The substrate was etched from some of the samples to allow transmission experiments through 200 µm square windows. The doping levels of the intrinsic regions were estimated by C-V measurements to be 1-2x10¹⁵ cm⁻³ for MV370 and 8x10¹⁴ cm⁻³ for CB8. The major impurity of the intrinsic MOVPE material is known to be Carbon acceptors in the AlGaAs layers [4.1-2], and hence this explains why the impurity concentration is higher in MV370, where a larger fraction of the thickness of the MQW is comprised of AlGaAs. The doping levels of the n⁺ and p⁺ regions were estimated using Polaron C-V data to have the values quoted in the above layer details, and are large enough to ensure that nearly all the applied voltage appears across the intrinsic region of the sample.

The I-V characteristic of the substrate-intact mesa resembled that of a conventional diode, with a breakdown voltage of more than -15 V (about 200 kV/cm) and a forward bias threshold of about +1.4 V at 17 K. Similar reverse voltages could be applied to the transmission sample, although the increase in current under forward bias was not so steep (+5 µA @ +3 V). The reason for this is unclear, but it suggests the

presence of a series resistance in the contact, which should not effect the field across the MQW, except at those positive biases where there is a significant current.

The optical arrangement consisted of a 80 W Tungsten or 450 W Xenon lamp, filtered by a single grating (f4) spectrometer, and detected with a silicon photodiode. The spectral resolution was set to much smaller than the observed linewidth and all the spectra were normalised to the spectral response of the system. The sample was mounted on the cold-finger of a closed-cycle Helium cryostat maintained at 17 K. ER measurements were performed at two angles of incidence; 16° and 45° .

PC and ER measurements were made on the substrate-intact mesa, while transmission and ET data were collected on substrate-etched mesas. Some PC spectra were also collected on the substrate-etched mesas to check that the optical features are identical in the two mesas. The modulating voltages for the ER and ET experiments were kept as small as possible, so that the spectra could be regarded as differential-like, and so that the lineshape did not narrow upon reduction of the a.c. component of the bias. For the ER measurements a fixed square wave alternating bias of 0.5 V_{ptp} was applied on top of a variable d.c. level, while the ET measurements were modulated by 0.1 V_{ptp}.

6.2.2 Results

6.2.2.1 MV370 : $88 \text{ \AA}$ wide wells

Fig. 6.1.1 shows PC and ER spectra taken on MV370 with an applied bias offset of 0 V. The ER spectra were taken at two different angles of incidence, for the reasons discussed later. The PC resembles the familiar absorption spectrum of a QW, with the E1H1 exciton peak at 1.554 eV and the E1L1 exciton at 1.567 eV. The E1H1 linewidth of 2.7 meV (HWHM) is larger than that measured on the SQW of similar well width studied in Sec. 5.3, which was grown in the same reactor under identical conditions. At a larger field (47 kV/cm), the SQW showed a HWHM linewidth of 1.5 meV. The wider linewidth of the MQW is clearly related to the kink that can be seen in the absorption tail.

Notice that the ER lineshape in Fig. 6.1.2 is quite different to that observed for SQWs in Chp. 5. It was discussed in Chp. 3 how the SQW lineshape is given by a linear combination of the first-energy-differential of the real and imaginary part of the excitonic contributions to the QW dielectric function, as shown in Fig. 3.15. Consequently the SQW lineshape has in general three lobes above and

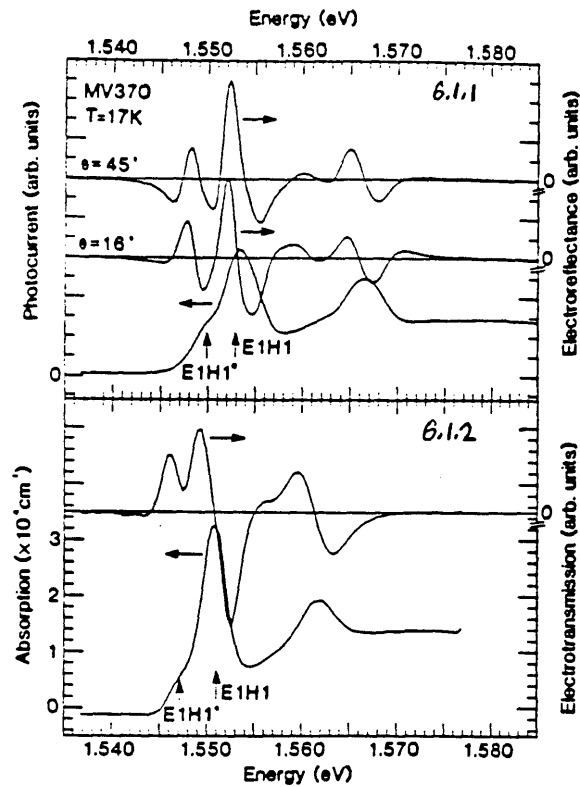


Fig. 6.1 : Four spectra taken on sample MV370 ($L_w = 88 \text{ \AA}$) at 17K and bias offset of 0 V. PC and ER measurements (6.1.1) were collected on a mesa with its substrate intact, while the transmission and ET data (6.1.2) are for another mesa with an etched substrate. The ER spectra were measured at two angles of incidence; 16° and 45° . The modulating bias was 0.5 V_{ptp} for ER and 0.1 V_{ptp} for ET.

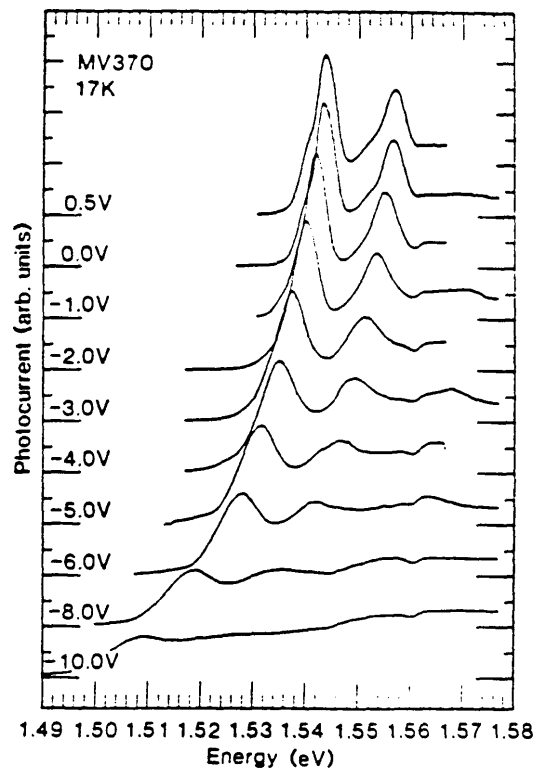


Fig. 6.2 : PC spectra taken on MV370 ($L_w = 88 \text{ \AA}$), with different applied biases between +0.5 V and -10 V, and a sample temperature of 17 K.

below the energy axis, while it is apparent in Fig. 6.1.1 that the ER feature of the E1H1 and E1L1 excitons in the MQW spectrum have five or six. It can be seen that the additional lobes are clearly related to the kink in the PC tail.

The low energy feature in the MQW ER lineshape, marked E1H1* in Fig. 6.1.1, which is associated with the kink in the PC tail, can also be seen in the ET and absorption spectra, shown in Fig. 6.1.2, which again were taken at an applied bias offset of 0 V. These measurements were taken on a different mesa of MV370, which had an etched substrate. The excitonic peaks lie a few meV lower in energy for the transmission measurements, with the small energy shift due to either the transmission mesa having wider wells or the sample temperature being slightly higher than 17 K. This is possible because it is more difficult to make good thermal contact between the sample and the cold-finger of the cryostat in the transmission geometry, where a heat sink cannot be placed immediately behind the sample. The energy shift between the two mesas would require a temperature of about 32 K for the substrate-etched mesa experiments. This is of little consequence since the linewidth does not start to increase significantly until much higher temperatures. PC spectra taken on the substrate-etched and the substrate-intact mesas had a very similar lineshape, but the spectra of the substrate-etched mesa was shifted a few meV lower, as in the transmission spectrum.

The absorption spectrum is often calculated by taking the logarithm of the transmission, as for instance in Ref. 2.33. This procedure is valid provided that the absorption of the cavity is strong, allowing the multiple reflections between the cavity interfaces to be ignored. However, the absorption spectrum in Fig. 6.1.2 was calculated using Eq. 3.45, the denominator of which describes the multiple reflections. This helps to suppress the Fabry-Perot oscillations seen at energies below the MQW bandedge. The optical thickness of the cavity was estimated from the spacing of the oscillations in the transmission below the MQW bandgap, and the reflectivity of the front and back of the cavity were taken as that of an air/bulk semiconductor interface. The contribution of the MQW transitions to the optical thickness of the cavity, ie $\text{Real}[g_{\text{trn}}]$ in the denominator of Eq. 3.45, was ignored.

The absorption coefficient in Fig. 6.1.2 is defined relative to the total MQW thickness, ie. including both the barriers and the wells. The integrated absorption strength of the E1H1 exciton is $240 \text{ cm}^{-1}\text{eV}$ (now expressed relative to the thickness of the GaAs layers alone), where the upper limit of the integration was taken to be 1.554 eV. This is close to the value reported by Masumoto et al [2.33] for this well

width of $250 \text{ cm}^{-1}\text{eV}$.

Fig. 6.2 shows PC spectra taken on MV370 with applied biases between +0.5 and -10 V. It can be seen that the E1H1 and E1L1 excitons move to lower energy with increasing electric field due to the QCSE, in a similar manner to the behaviour seen for the SQWs in Sec. 5.3.1. As the field increases, the linewidth broadens such that the excitons become difficult to resolve at the largest biases, while the kink in the E1H1 tail becomes difficult to resolve at lower reverse biases of about -2 V.

Figs. 6.3 and 6.4 display the evolution with bias of the ER and ET spectra taken on MV370 respectively. Notice that in the ET spectra the low energy feature remains discernible up until about -8 V, when it is finally obscured by the increase in the linewidth. This again demonstrates the enhanced sensitivity of modulation spectroscopy. The low energy feature can be seen to change more rapidly with bias in the ER spectra of Fig. 6.3. Notice that at the smallest biases in Fig. 6.3, the different lobes of the E1H1 ER feature are of similar size, while the relative size of the lower energy lobes decreases rapidly when the field is increased.

6.2.2.2 CB8 : $163 \text{ \AA}$ wide wells

PC spectra taken on CB8 as a function of the applied bias are shown in Fig. 6.5. These were taken on a mesa with an etched substrate. The lowest bias spectrum shows two peaks near the bandedge, due to the E1H1 (at 1.5248 eV) and E1L1 (at 1.5291 eV) excitons. The E1H1 peak has a HWHM of 2.4 meV. With increasing bias these transitions undergo large shifts to lower energy, along with a large decrease in their transition strength. This is more pronounced than in MV370 due to the wider well width in CB8. The "forbidden" E1H3 transition can also be seen at the lowest voltages at 1.5497 eV. As expected this transition shows a smaller Stark shift and increases in strength with field, due to the enhancement of the overlap of the E1 and H3 quasi-confined states. The E1H2 transition can be seen in the 0 V spectrum as a kink at 1.5325 eV, just above the E1L1 peak. This peak becomes more clearly resolved at higher fields.

Fig. 6.6 shows ET spectra taken on CB8 at different applied biases. This again demonstrates the large energy shifts and changes in the transition strengths due to the QCSE. Notice that there is again unexpected structure in the ET spectrum at energies below the E1H1 exciton transition, as was also the case for MV370. This structure can

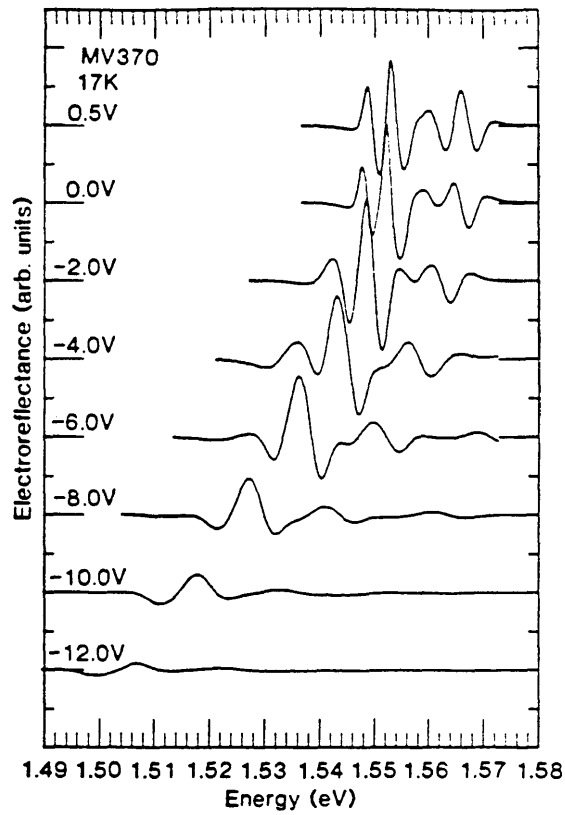


Fig. 6.3 : ER spectra taken on MV370 ($L_w = 88 \text{ \AA}$), with different offset biases. Taken with an angle of incidence of 16° , a modulating voltage of $0.5 \text{ V}_{\text{ptp}}$ and a temperature of 17 K .

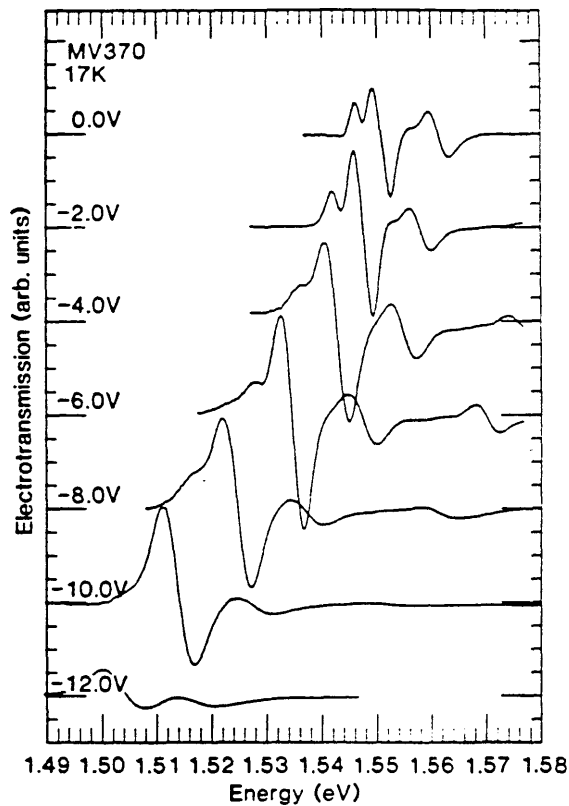


Fig. 6.4 : ET spectra taken on MV370 ($L_w = 88 \text{ \AA}$), with different offset biases. The light was incident normal to the sample surface, the modulating voltage was $0.1 \text{ V}_{\text{ptp}}$ and the sample temperature was 17 K .

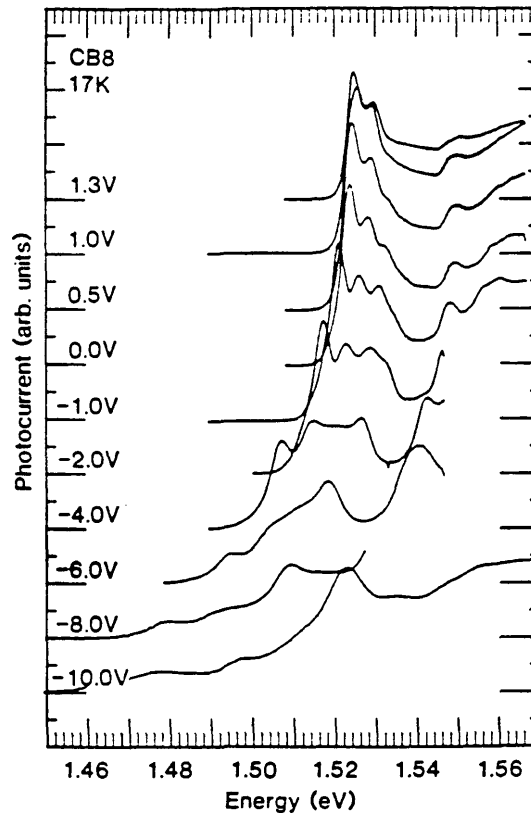


Fig. 6.5 : PC spectra taken on CB8 ($L_w = 163 \text{ \AA}$), with different applied biases and a sample temperature of 17 K.

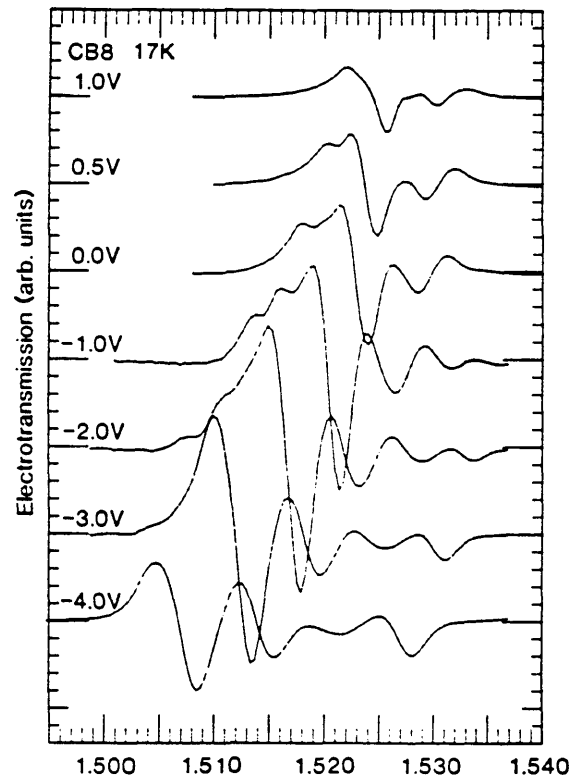


Fig. 6.6 : ET spectra taken on CB8 ($L_w = 163 \text{ \AA}$) with different offset biases. The modulating voltage was 0.1 V_{ptp} and the sample temperature was 17 K.

also be seen in the PC and transmission spectra, although it is much more difficult to discern, as was also the case for MV370. The unexpected structure in the ET spectra of CB8 are not so well resolved as for MV370 and becomes indistinguishable under quite a small electric field. Another difference between this phenomenon in the two MQWs, is that two peaks can be seen below the main E1H1 peak in some of the spectra of Fig. 6.6, for example at -1 V, while only one is ever observed for MV370. Also it is clear in Fig. 6.6 that the splitting increases more rapidly with field for CB8.

6.2.3 Analysis and Discussion

6.2.3.1 Quantum Confined Stark Effect

The exciton transition energies measured in the bias-dependent PC spectra taken on the substate-intact mesa of MV370 (Fig. 6.2) are plotted as the closed symbols in Fig. 6.7. The open symbols in Fig. 6.7 show the ET transition energies measured on the substrate-etched mesa (Fig. 6.4), which were taken to correspond to zero-crossings in the ET spectra, consistent with the transitions being modulated by their Stark shift. The energies of the zero-crossings in the ET spectra agree with the peaks of the absorption spectra, which lends further confidence to this assignment. It can be seen that the ET energies lie a few meV lower than those measured in PC, which is either due to a difference in the well widths or temperature between the two mesas studied, as discussed earlier. The ET transitions show a slightly larger Stark shift, which suggests that the substrate-etched mesa has wider wells.

The solid lines in Fig. 6.7 show the Stark shifts calculated for the E1H1 and E1L1 excitons in a $88 \text{ \AA}$ GaAs/Al_{0.3}Ga_{0.7}As QW, using the method and parameters detailed in Sec. 4.3. This includes the small change in the exciton binding energy with field. It can be seen that there is a reasonable agreement between the calculated and measured Stark shifts, although the measured energies decrease slightly more rapidly. This is probably due to the well width being larger than $88 \text{ \AA}$, the value fitted to the room temperature E1H1 transition energy. Alternatively the experimental field may have been underestimated due to an error in the intrinsic region thickness.

Fig. 6.8 plots the Stark shifts measured in the PC and ET spectra of CB8, which were both taken on the same substrate-etched mesa. The ET transition energies were again taken as zero-crossings in the spectra. This sample shows larger energy shifts than MV370, due to its wider well

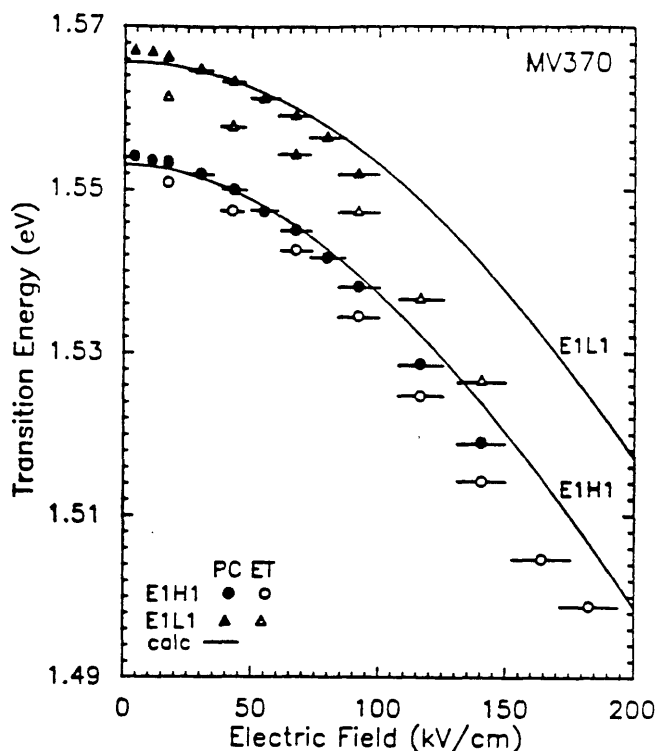


Fig. 6.7 : The solid symbols show the field dependence of the E1H1 (●) and E1L1 (▲) exciton PC peaks measured on MV370, while the open symbols plot the variation of the E1H1 (○) and E1L1 (△) energies measured in the ET spectra. The solid lines show the Stark shifts calculated for a $88 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, using the method and parameters detailed in Sec. 4.3.

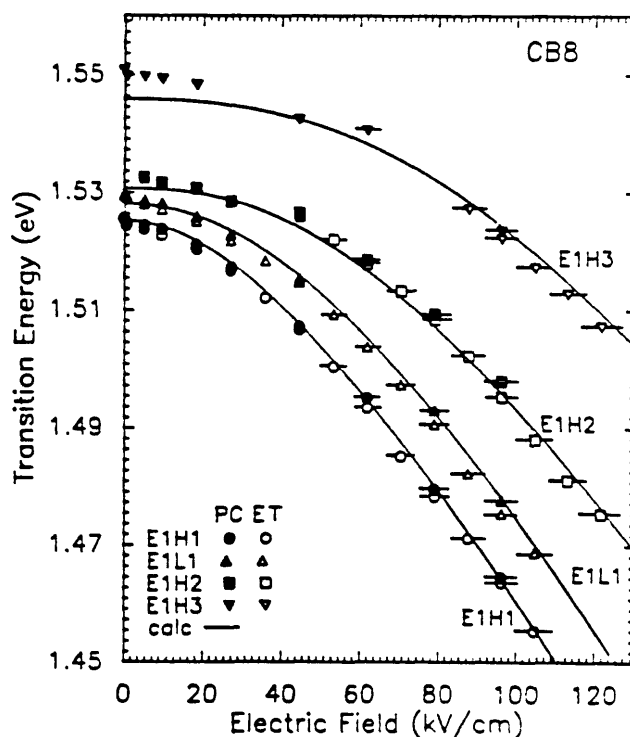


Fig. 6.8 : The solid symbols show the field dependence of the E1H1 (●), E1L1 (▲), E1H2 (■), and E1H3 (▼) exciton PC peaks measured on CB8, while the corresponding open symbols plot the energies measured in the ET spectra. The solid lines show the Stark shifts calculated for a $163 \text{ \AA}$ GaAs/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ QW, using the method and parameters of Sec. 4.3.

width. The Stark shifts were again calculated using the method detailed in Sec. 4.3, using a well width of $163 \text{ \AA}$. It can be seen that there is good agreement between the measured and calculated Stark shifts, for both the allowed and forbidden transitions.

6.2.3.2 Exciton Lineshape

It was discussed in Sec. 3.4, that the MQW ER lineshape is expected to be complicated by a change in the lineshape across the spectrum, due to the light penetration depth being a sensitive function of the photon energy. This results in the ER spectrum consisting of a main central feature, originating from the front of the stack, flanked by smaller subsidiary features at the energies where the absorption coefficient has decreased sufficiently to allow light to be reflected from the back surface of the MQW stack. However, these optical interference effects cannot explain the measured lineshapes alone.

The PC spectrum is determined by the imaginary part of the QW dielectric function. Similarly, the transmission and ET spectra will also be determined by the imaginary part of the QW dielectric function, with only a small dependence on the real part, for the thick MQW stacks used in these structures, as discussed in Sec. 5.4.3. Consequently the PC, transmission and ET spectra will not be affected by the depth of the layers in the structure. The observed splitting of the exciton features in the PC, transmission and ET spectra must therefore be caused by the presence of two different transition energies in the MQW dielectric function, which have been marked E_{1H1} and E_{1H1}^* in Figs. 6.1.1 and 6.1.2. Furthermore, since the total absorption coefficient is the average of that of the individual wells, this change in the transition energy must occur relatively abruptly, as otherwise only an increase in linewidth would be seen and the two transitions would be smeared into one. The kink in the E_{1H1} exciton peak of the PC and transmission spectra is due to the poorly resolved overlap of two peaks. A similar splitting can be seen in the E_{1L1} exciton feature using all four spectroscopies.

Fabry-Perot oscillations can be seen in the transmission spectrum at energies below the MQW bandgap, due to interference of light reflected from the front and back surface of the etched window. However the period of these oscillations is much larger than the MQW linewidth and will therefore not affect the lineshape. This cannot therefore be the cause of the low energy feature (E_{1H1}^*) seen in the ET spectra, which is confirmed by the presence of the same splitting in the ER and

PC experiments performed on mesas without an etched window. Extrinsic effects can also be discounted as an explanation for the lower peak, as its transition strength is too large.

Two explanations can be envisaged for different transition energies in the MQW stack; variation of the well width or variation of the electric field with depth. The two lowest energy peaks in the 0 V ET spectrum of MV370 are separated by approximately (3.2 ± 0.2) meV, which would require a change in well width of about $4.5 \text{ \AA}$ (1.6 monolayers). Meanwhile for CB8, where the splitting in the 0 V ET is about (3.5 ± 0.2) meV, a very large well width change of $22 \text{ \AA}$ is required.

The second possible cause of the splitting, the variation of the electric field across the MQW stack, necessitates the presence of charge in the intrinsic region. The background doping of the intrinsic region in MV370 of $1\text{-}2 \times 10^{15} \text{ cm}^{-3}$ produces a change in the electric field between the front and back of the stack of 12-25 kV/cm. Since the intrinsic region is lightly p-type, the field at the back of the stack is larger than that at the front. If the impurity concentration is assumed constant, then the field distribution across the stack will be linear. Meanwhile for CB8, the background doping is $8 \times 10^{14} \text{ cm}^{-3}$, resulting in the field being 14 kV/cm larger at the back of the stack. The total field change across the stack for the two samples should be fairly similar, since they both contain the same total thickness of $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ layers ($50 \times 60 \text{ \AA}$), and so should have the same Carbon contamination. This linear field increase cannot give the abrupt change in transition energy necessary to explain the splitting in the spectra, although its effect will make the transition energy at the back of the stack lower, due to the larger Stark shift.

The only source of charge in the MQW stack, besides the background doping, would have to derive from the carriers themselves. Recently Grahn et al [6.1-2] have reported photocurrent tunnelling experiments in MQWs which indicate the presence of different field domains within the MQW stack. For instance at the lowest fields, there are two field domains; in one the field is such as to align the first and second confinement levels of adjacent wells and so promote tunnelling, while in the other region the field is zero. This could also produce a splitting in the spectra due to the abrupt change in the field. However this cannot explain the results on this sample. Firstly the ELH1 feature of both samples shows a smoothly increasing Stark shift with bias, as is clearly evident in Figs. 6.7 and 6.8, while in the field domain situation the field in the two regions is independent of

bias (see Fig. 3 of Ref. 6.1). The magnitude of the shift is in agreement with that calculated for the expected field, produced by the applied bias across the $p^+ - i - n^+$ diode. The field across the stack is therefore determined by the ratio of the total voltage drop and the intrinsic region width, and not by the alignment of levels in adjacent wells, as in the field domain case. Also the variation of the separation and relative height of the two features with bias in both ER and ET can be well described by an abrupt well width change, as discussed below, but not by an abrupt electric field change. In any case a sudden change in the electric field would require a large carrier density at some point in the stack, which could not be supported by the very small excitation powers used in this experiment. In this experiment the excitation power is about 25 nW, compared with 10 mW used in the report of field domain behaviour in Ref 6.1. Clearly field domain formation of the type studied by Grahn et al cannot explain our results and is not discussed further.

It is unrealistic to separate the contribution of a well width change from the variation in the field across the intrinsic region, due to the impurity charge. If the wider wells were at the back of the stack the two effects would combine to increase the splitting. Then a smaller change in the well width would be needed. Firstly considering MV370, if, for instance, there was a monolayer change in the well width, this would require an additional change in the field of about 10 kV/cm, to produce the splitting observed in the 0 V spectrum. This increase in the average field between two regions of the stack having a different well width, is in rough agreement with the total field drop across the i-region of 12-25 kV/cm, deduced from the C-V data, as discussed later.

Meanwhile the splitting in the ET spectra of CB8 at 0 V is of a similar size; about 3.5 meV. In this wider well sample, a monolayer well width change produces a smaller change in the transition energy, while the electric field change has a greater effect, due to the Stark shifts being larger. A 3.5 meV splitting requires a change in field of about 6 kV/cm along with the monolayer increase in well width. Again this change in the average field of adjacent well width regions is in approximate agreement with the total field change across the stack of about 14 kV/cm, determined from the C-V measurements. The double splitting in the spectra of CB8 is evidence of two abrupt changes in the well width across the stack. Alternatively there could be a single change in the well width, with the middle transition being due to the $E1L1$ transition of the wider well. However, considering the variation of the splittings and the relative height of the features with bias, the

former explanation seems the more likely. Perhaps the fact that there are three well width regions in this sample, derives from the MQW stack being thicker than in MV370, due to it having wider wells.

Notice that under zero-field conditions the monolayer energy splitting would be very small and unresolvable. For a 88 Å well a monolayer increase in well width produces a reduction in the E1H1 exciton energy of 2.0 meV, while for a 163 Å well this value is just 0.45 meV. However the linear increase in field across the stack enhances the splitting, making it readily apparent, especially in the modulation spectroscopies. Observation of monolayer splittings for these relatively wide wells is only possible due to the background doping of the i-region.

The size of the splittings in both the samples is in good agreement with the region at the back of the stack having the widest wells and the largest fields. Also the field evolution of the spectra, discussed later, suggest the wider wells are at the back. It is possible to confirm whether the low energy feature comes from deeper in the stack by measuring the ER at different angles of incidence. The lineshape model in Chp. 3 predicts that the ER evolves more rapidly with angle of incidence for deeper layers, and this was confirmed by the measurements on SQWs in Chp. 5. Fig. 6.1.1 shows two ER spectra taken at 16° and 45°. It can be seen that the lowest energy lobes of the excitons (E1H1* and E1L1*) are radically altered by the change of the angle of incidence, while the main E1H1 and E1L1 features are much less so. A detailed analysis is difficult due to the overlap of the different features and also the lineshape from the back of the stack being greatly complicated by absorption in the front of the stack, as discussed in Sec. 3.4. However, it is clear that the lower energy lobes of the excitonic features are more affected by the change in the angle of incidence, confirming that they derive from a region deeper inside the sample.

Photoluminescence has also been measured at 4.2 K on MV370 after etching off the layers in front of the MQW stack [6.3]. With 632 nm excitation, where the penetration depth is comparable to the stack thickness, two peaks are just resolved at 1.5519 and 1.5545 eV, corresponding to the two E1H1 transitions energies seen by the other techniques of Fig. 6.1 at a slightly higher temperature. The height of the 1.5545 eV peak is about half that of the other, but the two peaks heights become almost equal when 514.5 nm excitation, with a shorter penetration depth, is used. This is consistent with the thinner wells lying toward the front of the stack.

6.2.3.3 "Real-Space Distribution of States"

Fig. 6.9 gives a pictorial explanation of the observed splittings for the case of two well width regions, as in MV370. Fig. 6.9.1 shows the variation in the well width across the stack, with an abrupt monolayer increase at some depth. Meanwhile the linear increase in the field across the stack is shown in Fig. 6.9.2. The transition energy of any particular QW can be found by projecting the field at its depth, onto the Stark curve for the appropriate well width (Fig. 6.9.3). Fig. 6.9.3 therefore represents the distribution of transition energies in the stack, which can be thought of as a "real-space distribution of states". The analogy with a one dimensional free carrier density of states in a bulk semiconductor is useful, because for the MQW the QWs are equally separated in depth, like the free carrier states are in k-space. Also the variation of energy with k-vector in a bulk semiconductor is parabolic, while the variation of the energy in real-space in Fig. 6.9.3 is also approximately parabolic. The total absorption of the stack (Fig. 6.9.4) can be found by placing a broadened exciton at each of the individual QW energies in Fig. 6.9.3. The individual QWs within one of the width regions are not resolved as their energy separation is smaller than the linewidth, and so only broad features can be seen due to each of the two regions. Fig. 6.9.3 shows that there is an energy range where there are no transitions in the stack (zero "real-space distribution of states"), deriving from the presence of the monolayer step, and this results in the splitting seen in the experimental spectra.

In these samples the wider wells are at the back of the stack. However, it is equally likely that a growth process could result in the wider wells being at the front of the stack. The effect of the linear field change would then oppose the well width change, and there would be a smaller overall spread of energies, particularly when the applied field was small. However it should still be possible to see features in the spectra due to the monolayer change, because this produces a range of energies where there is a double contribution in the stack, which could be thought of as an increase in the "real-space distribution of states".

6.2.3.4 Bias Dependence of Exciton Lineshape

When a larger reverse bias is applied, the field at each point in the intrinsic region increases, although the drop in field across the

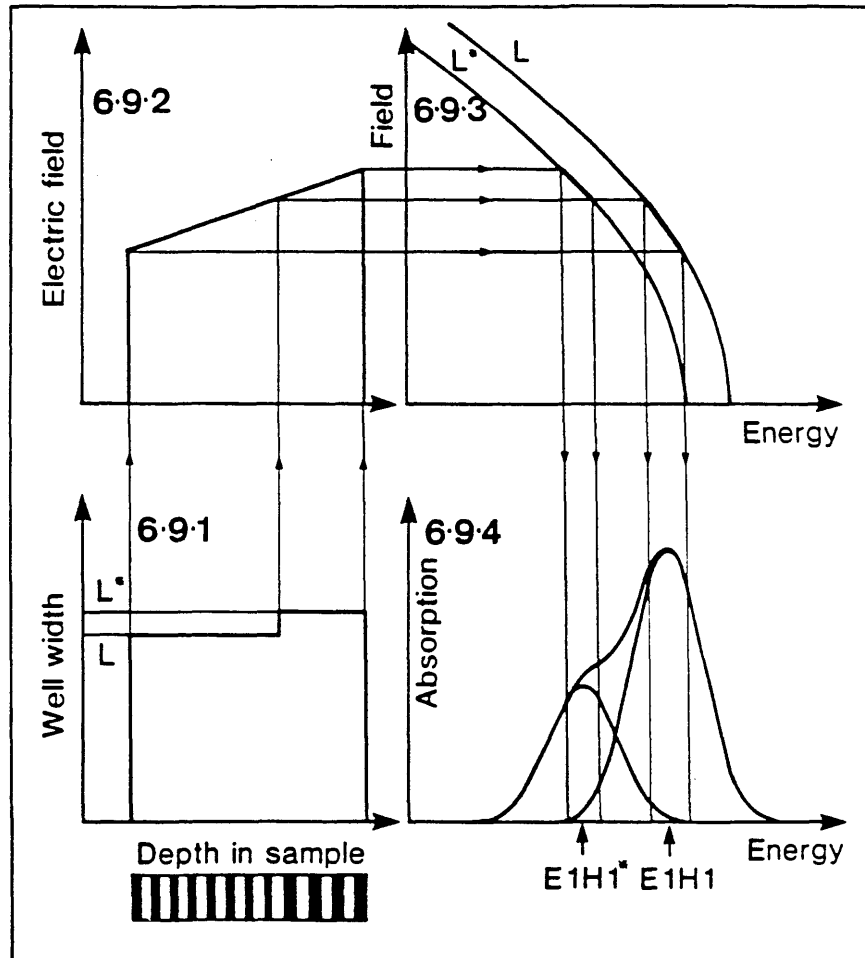


Fig. 6.9 : Pictorial representation of the origin of the splittings seen in the MQW excitonic features. The variation in the well width across the stack is shown in 6.9.1, while 6.9.2 displays the variation of field. The distribution of transition energies in the stack is represented in 6.9.3 and this gives an absorption spectrum like that in 6.9.4.

stack remains unchanged. The linear increase in field across the stack, depicted in Fig. 6.9.2, is then shifted up the field axis. Since the gradient of the Stark curve is steeper at larger fields, this results in an increase in the splitting of E_{1H1} and E_{1H1}^* .

Fig. 6.10 plots the splitting of the E_{1H1} feature against the electric field for MV370. The splitting is determined from the separation of the positive peaks in the ET spectra. The error associated with the estimate increases with field, as the features are poorly resolved. The solid line shows a calculation of the change in the energy, from those shown in Fig. 6.7, after allowing an increase in both the well width of 1 monolayer and the field of 10 kV/cm. It can be seen that there is good agreement between the measured and calculated splitting.

The wide well region of the stack can be very roughly taken to have half the thickness of the narrow well region, as suggested by the relative heights of the E_{1H1} and E_{1H1}^* features in the absorption spectra of Fig. 6.1.2. The centre of the narrow and wide well regions will therefore lie at approximately $1/3$ and $5/6$ the depths of the stack respectively. A field change between these two positions of 10 kV/cm, as suggested by the size of the observed splitting, corresponds to a total field change across the stack of 20 kV/cm, which is in agreement with the value determined by the C-V measurements of 12-25 kV/cm.

The experimental fields in Figs. 6.7 and 6.10 were calculated using the formula for a p^+-i-n^+ diode described in Sec. 4.3. The n^- and p^- region doping levels were taken to be those given in the layer specification, while the i -region doping was taken as $1.7 \times 10^{21} \text{ cm}^{-3}$, as suggested by a 20 kV/cm drop in field across the stack. The built-in voltage was taken as +1.6 V, as suggested by the low temperature current threshold. The total i -region thickness was determined using the fitted well width of 88 Å, the nominal barrier width of 60 Å and the nominal spacer width of 400 Å. The field was determined at the position of the centre of the narrow well region, which is $1/3$ of the total stack thickness from the front of the stack, although this is only a few kV/cm lower than the field at the midpoint of the stack. The horizontal error bars reflect the possible variation in the position of the middle of the narrow well region and also in the thickness of the i -region.

The measured and calculated E_{1H1} splittings for CB8 are shown in Fig. 6.11, and again show a reasonable agreement. The experimental splittings are determined from the peaks in the ET spectra in identical fashion as for MV370. The calculated curves assume (a) an increase in the well width of 1 monolayer and in the field of 6 kV/cm and (b) an

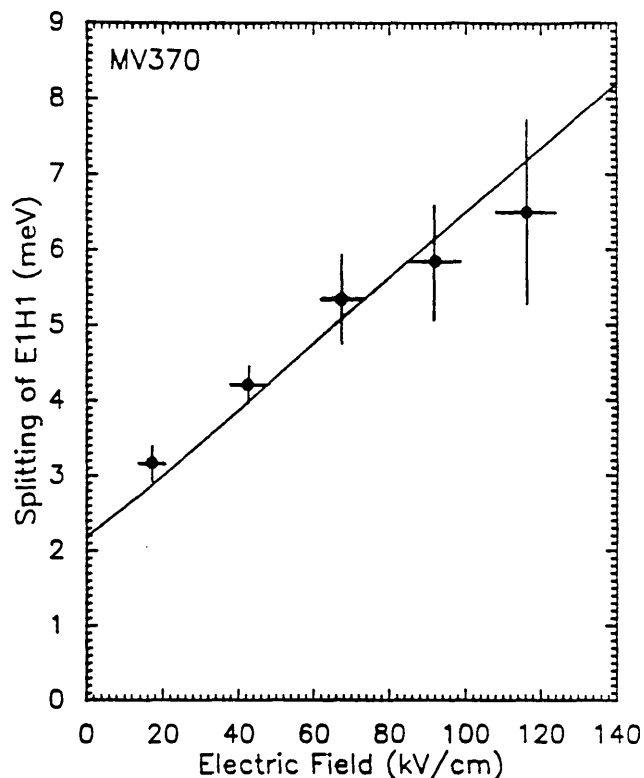


Fig. 6.10 : The data points plot the field dependence of the energy splitting of the E1H1 feature in the ET spectra of MV370. The solid lines show the calculated energy difference for a one monolayer increase in well width and an increase in the electric field of 10 kV/cm.

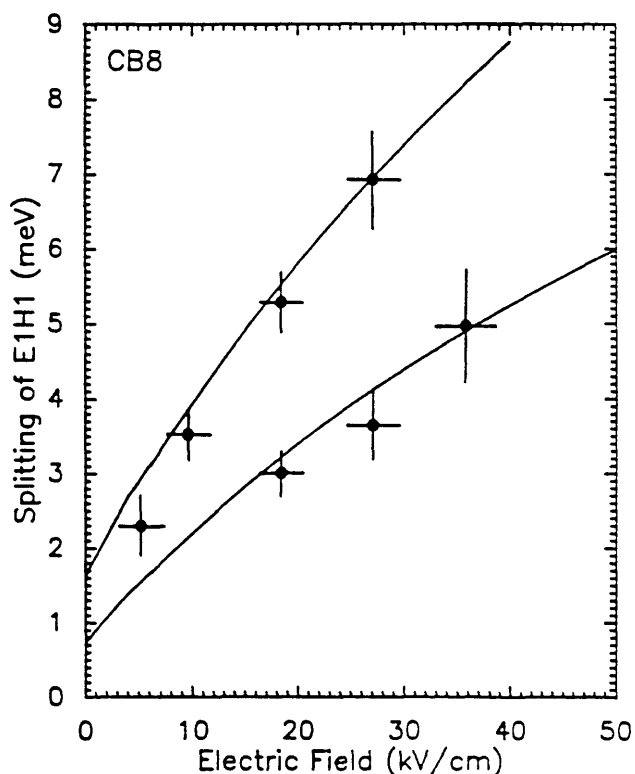


Fig. 6.11 : The points show the energy splittings of the peaks in the ET spectra of CB8 as a function of the applied field. The solid lines show the calculated change in the exciton energy after a one monolayer increase in well width and an increase in the electric field of 6 kV/cm (lower curve) and a two monolayer increase in well width and a field increase of 9 kV/cm (upper curve).

increase of 2 monolayers and 9 kV/cm. The relative thickness of the three well width regions in CB8 is more difficult to judge, due to the rapid decline in the oscillator strength with field, for this wide well. However taking each to have an equal width, would result in the widest and narrowest well width regions being separated by about $2/3$ of the total thickness of the stack. The size of the splitting therefore suggests the total field drop across the intrinsic region to be about 12 kV/cm, in good agreement with the value deduced from the C-V measurements of 14 kV/cm. The experimental fields in Fig. 6.8 and 6.11 were determined in a similar manner as for MV370, using the parameters appropriate to CB8.

It was noticed that at 0 V, the features of the wide well width region (E_{1H1}^*) of MV370 are more prominent in the ER spectrum than the ET. In ET the relative height of the different features is determined by their oscillator strength, the size of their modulating energy shift and the thickness of material producing the signal. This is because the detected light passes through all the material in the stack. However the size of the ER signal is insensitive to the thickness of the material, as increasing the number of wells only changes the phase of the lineshape, as discussed in Sec. 3.4. At low fields the ER features of the two regions should be, and are, fairly similar in size. However, the linewidth increases with the reverse bias and the absorption tail of the higher energy feature (E_{1H1}), due to the layers nearer the front of the stack, overlaps with the energy of the feature from the deeper layers (E_{1H1}^*). As the light reflected from the back layers must make two passes through the absorbing front layers, its contribution to the spectrum is reduced.

The individual linewidths of the features from the two regions depend upon the linear increase in field within each region. From Figs. 6.2-4 it is apparent that the linewidth of E_{1H1} and E_{1H1}^* increases along with their separation. Since the other broadening contributions, such as that due to the variation in well width within each region due to interface roughness, also increase with bias as suggested by the measurements on SQWs in Chp. 5, the splitting becomes unresolved with increasing reverse bias. This can be seen in the bias evolution of all four spectroscopies of MV370 (Figs. 6.2-4) and is also true for CB8 (Figs. 6.5-6).

6.2.3.5 Interface Quality and Growth Aspects

The optical linewidths of the samples studied here are quite

large, but it has been emphasised that this is due largely to the variation of the well width and electric field between the wells. The low temperature E1H1 linewidth of MV370 is about 3.4 meV (HWHM), when the average field in the stack is about 50 kV/cm, compared to 1.5 meV (HWHM) for the MOVPE SQW studied in Sec. 5.3.1, which has a similar well width and was grown under identical conditions. The narrow linewidth of the SQW sample suggests that the interface quality is very good for samples grown under these growth conditions. Furthermore, part of the SQW linewidth may derive from inhomogeneous broadening mechanisms other than the well width fluctuation, as discussed in Sec. 5.3.3.

The observed splittings, discussed above, require a relatively abrupt change in the transition energy at specific points in the stack, and it has been argued that this can only be produced by a change in the well width. The samples were grown on off-axis substrates, so might be expected to contain "pseudo-smooth" interfaces, in which the lateral separation of the monolayer steps is about $50 \text{ \AA}$, about one sixth of the exciton diameter. However, in the present case the MQW stacks were grown at $650 \text{ }^{\circ}\text{C}$, with a fast growth rate of between 3 and 4 monolayers per second. Calculations of the growth mechanisms in MBE, predict that step flow will be less important than island nucleation under these conditions, as for a perfectly oriented [001] substrate [6.4].

The MOVPE reactor used to prepare the samples appears to produce MQW structures in which the wells near the top of the stack are narrower than those near the bottom. It is not clear whether this is related to the mass flow of metal-organic precursors into the reactor, or to other related instabilities. The pressure of the reactor was monitored during the growth runs and found to be very steady, never varying more than ± 2 torr in about 980 torr. It is not possible at this time to provide a clear explanation for the growth mechanism which leads to the observed relatively abrupt changes of the well width at certain points in the stack. However this mechanism may be driven by some chemical process which favours growth of uniform layers when switching between $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and GaAs, where the Ga deposited is not an integer number of monolayers. The mechanism is unclear, but may be analogous to that observed by Tsuchiya et al [6.5] which drives the spontaneous growth of a coherent tilted superlattice. In their case, when (GaAl)As is deposited onto the planes between steps on an off-axis substrate, spontaneous ordering occurs, with the preferential formation of AlAs close to the step region. The tentative suggestion is made that in the present case, when the growth is switched from GaAs to $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$, perhaps a partially grown layer of GaAs may be more susceptible to

replacement by the aluminium bearing alloy, than when the GaAs layer is nearly complete. Such a mechanism could explain the abrupt change in well width observed in these MQW stacks, but at present it is only possible to speculate.

6.3 Measurements on a GaAs/AlGaAs MQW with a n^+ -i- n^+ Doping Profile

This section describes bias-dependent ER and PC measurements on another MQW structure, which has 100 Å GaAs wells and 90 Å $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ barriers. However this sample is unusual in that the undoped MQW is clad by two highly doped n^+ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ regions. Consequently the field profile across the intrinsic region of this sample is non-uniform, being determined by the tunnelling of the dark current through the structure. It is shown that this influences the shape of the optical spectra. These results are also reported in Ref. 1.16.

6.3.1 Experimental Methods

This sample was grown in the Varian Gen II MBE reactor at RSRE by Dr. G.W.Smith. It was grown on an n^+ GaAs substrate, cleaved for growth along the [001] crystallographic direction. The layer specification consisted of 1.0 μm n^+ GaAs buffer layer; 0.5 μm n^+ ($6 \times 10^{17} \text{ cm}^{-3}$) $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$; the undoped stack, consisting of 105 repeats GaAs(100 Å)/ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ (90 Å); another undoped 100 Å GaAs layer; 0.5 μm n^+ ($7 \times 10^{17} \text{ cm}^{-3}$) $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$; and finally 600 Å n^+ ($1 \times 10^{18} \text{ cm}^{-3}$) GaAs cap. The actual well width was determined from the E1H1 transition energy to be 93 Å. The wafer was etched down to the back n^+ layers to form 1 mm diameter cylindrical mesas. It was suspected that the samples grown in this reactor have a very narrow layer of p-type material unintentionally deposited between the substrate and the first epilayer. The back contact was made to the etched surface in order to electrically bypass this layer, while another ohmic contact was made to the top of the mesa.

The 17 K I-V characteristic is non-linear, being flatter at lower bias for both polarities. It is also asymmetric; the leakage currents being larger for negative biases applied to the front. (The convention used in all this section is that a positive bias corresponds to the front of the mesa being at a higher voltage than the back.) In positive bias, the current is +10 μA at +10 V, while for negative biases it takes only -6.9 V to produce this magnitude of current. It was hoped that the i-region would act as a high resistance, taking all the voltage dropped

across the sample. However it is clear from the I-V characteristic that the structure is not behaving like a linear resistor and it is likely that the current is determined by tunnelling of electrons through the MQW and $0.5 \mu\text{m Al}_x\text{Ga}_{1-x}\text{As}$ contact layers. This is also supported by the fact that the currents are much larger at room temperature, where thermionic processes would be expected to enhance their value.

ER and PC spectra have been taken on this sample as a function of the applied bias. The sample was mounted on the cold-finger of the closed-cycle Helium cryostat maintained at 17 K. The ER measurements were made with a fixed modulating voltage of 1.0 V_{ptp} and a variable d.c. offset. The light was incident on the sample at either 12° or 45° .

6.3.2 Electroreflectance and Photocurrent Spectra and Analysis

Fig. 6.12 shows 17 K PC spectra obtained with different applied positive biases. A threshold of approximately +1 V was needed to produce a PC signal. The E1H1 exciton feature in the +1 V spectrum of Fig. 6.12, centered at 1.5526 eV, is comprised of two peaks separated by approximately 4 meV. Similarly the light hole exciton feature near 1.565 eV displays a splitting of about 8.1 meV. With increasing bias, the exciton structure shows a Stark shift to lower energy, while the higher energy peak of each pair increases in size relative to the lower.

At first glance, it is compelling to explain this splitting of the E1H1 and E1L1 excitons by a variation in the well width, as for the MQW samples studied in Sec. 6.2. A 4.0 meV splitting would require two well widths which differ by $6.3 \text{ \AA}$ (2.6 monolayers). However, it seems most unlikely that these would be present without intermediate widths also. The background doping of the i-region in this sample is expected to be very low, as is typical of good quality MBE material, so that any splitting due to a well width variation, will not be significantly enhanced by a linear increase in the field across the stack, as for the samples in Sec. 6.2. Additionally the variation of the separation and the height of the peaks with bias is difficult to explain in terms of well width splittings.

A more likely explanation for the usual exciton lineshapes in Fig. 6.12, is that the PC signal derives only from the deepest layers in the MQW stack. The split excitonic features are then due to a single peak, where the signal in the central region of each transition has been reduced by absorption in the front of the MQW stack. The PC spectrum is like a transmission measurement of the front of the stack, with the back of the stack acting as the detector. The very large reduction near the

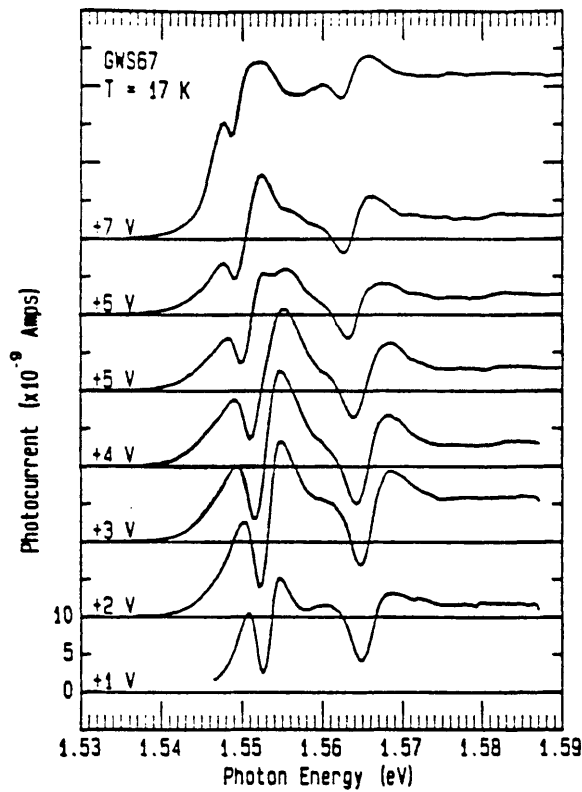


Fig. 6.12 : PC spectra taken on sample GWS67, with different applied positive biases of between +1 and +7 V. The sample temperature was 17 K.

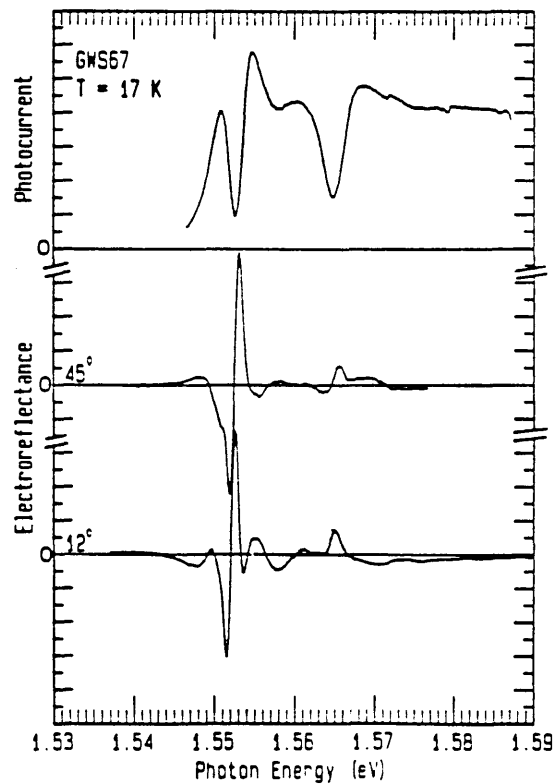


Fig. 6.13 : Two ER spectra taken on GWS67, with an applied offset bias of +1.0 V and modulation of 1 V_{tp}, and with the light incident at an angle of either 12° or 45° to the sample normal. Also shown is the PC spectrum taken with an applied bias of +1.0 V. The sample temperature was 17 K.

centre of the exciton features, suggests that the PC is collected from only a small region at the back of the stack. This can also explain why the E1H1 and E1L1 features in Fig. 6.12 are of roughly equal size, as the absorption of the heavy hole exciton peak will be stronger.

In the +1 V spectrum the higher energy PC maxima of the excitons are larger than the lower energy maxima. As the bias is increased, the higher energy peaks further increase relative to the lower ones. This can be explained by the field being larger at the front of the stack, shifting the exciton transition to lower energy and resulting in the dip in the PC signal being displaced from the centre. In the highest voltage spectrum the dip is completely removed from the maximum of the background. Meanwhile the background shows barely any Stark shift, suggesting that the field is smaller at deeper points in the stack. A dip at the centre of the PC peak can also be seen for the E2H2 exciton feature, where the apparent splitting is larger, due to the larger linewidth of this transition.

Fig. 6.13 again plots the +1 V PC spectrum, along with two ER spectra, also taken with an applied d.c. bias of +1 V and at angles of incidence of 12° and 45° . It can be seen that the ER spectra show large features corresponding to the energies of the dips in the E1H1 and E1L1 exciton PC. These are superimposed on smaller and broader features. The preceding arguments suggest that the large ER feature derives from the front of the stack, while the broader background is due to its deeper regions. The feature from the front of the stack is stronger, due to the electric field being larger in this region, although absorption will also reduce the contribution from the back.

It is possible to determine if the large features in the ER spectra, and hence the dips in the PC lineshape, are due to layers nearer to the front of the MQW stack, by studying the ER lineshape as a function of the angle of incidence. The ER lineshape model of Chp. 3, predicts that the signal from the deeper layers evolves more rapidly with the angle of incidence, and this was confirmed by the measurements on SQWs in Sec. 5.2. Comparing the two ER spectra in Fig. 6.13, taken at 12° and 45° , it can be seen that both large central features of the E1H1 and E1L1 transitions are only slightly altered by the change in the angle of incidence, while the background features undergo an approximate inversion. Similar evolution with angle is observed for spectra taken at other biases. This confirms that the central feature of the excitonic lineshape originates from nearer the surface of the sample, while the background feature comes from deeper inside the MQW stack. The expected change in the lineshape phase produced by the change in the

angle of the incident light, can be calculated using Eq. 3.18. For the depth of the front of the MQW stack, taken as that specified in the growth procedure, the change in the angle produces a change in the lineshape phase of $\Delta g = 34^\circ$, while for the back of the stack $\Delta g = 157^\circ$. These values are approximately consistent with the lineshape rotations observed in Fig. 6.13.

Fig. 6.14 shows the dependence of the ER spectrum upon an applied positive bias. With increasing bias, the large E1H1 and E1L1 features Stark shift to lower energy, continuing to track the dips in the PC spectra. The +9 V spectrum has Stark shifts of -5.4 and -3.8 meV for the E1H1 and E1L1 transitions respectively, requiring a field of 53 kV/cm applied to a 93 Å GaAs/Al_{0.3}Ga_{0.7}As QW. Smaller features to higher energy than the main strong transition appear as the bias is increased. These are probably caused by deeper regions in the MQW stack where the electric field is smaller.

Fig. 6.15 shows the PC spectra of GWS67 under different applied negative biases. The PC flows in the opposite direction than when a positive bias is applied, and the signal is also about five times larger. At small negative voltages the spectra have a similar shape to those taken with positive biases, again showing a dip at the centre of the transition. Again this behaviour indicates that only the carriers photo-excited at the back of the MQW stack are collected. However, in this case the size of the dips relative to their background is smaller, while the overall signal is larger. This suggests that a larger region of the back of the stack can contribute under negative biases, compared to the opposite voltage polarity. It can be seen that the dips in the E1H1 and E1L1 peaks get smaller as the negative bias is increased. At the largest negative bias (-6 V) the PC resembles a more familiar absorption spectrum of a QW, with a sharp peak at 1.5526 eV, due to the E1H1 exciton, with a HWHM of 1.45 meV. From this transition energy the well width was fitted to be 93 Å. A small bump can be seen 10.6 meV above the light hole exciton at 1.5649 eV. This bump may be due to an excited excitonic state of E1L1, which has enhanced oscillator strength due to the mixing of the L1 and H2 subbands, as was evident in the 88 and 106 Å GaAs/Al_{0.3}Ga_{0.7}As SQWs studied in Sec. 5.3.1. The bump may alternatively be due to the E1L1 continuum. The E1L1 (1s) binding energy was calculated to be 9.3 meV for a 93 Å GaAs/Al_{0.3}Ga_{0.7}As QW. However, this calculation ignores valence subband mixing effects, which are expected to enhance the E1L1 binding energy by a few meV [2.40].

The ER spectra for negative biases are complicated, suggesting that the electric field in the stack may be highly non-uniform. At

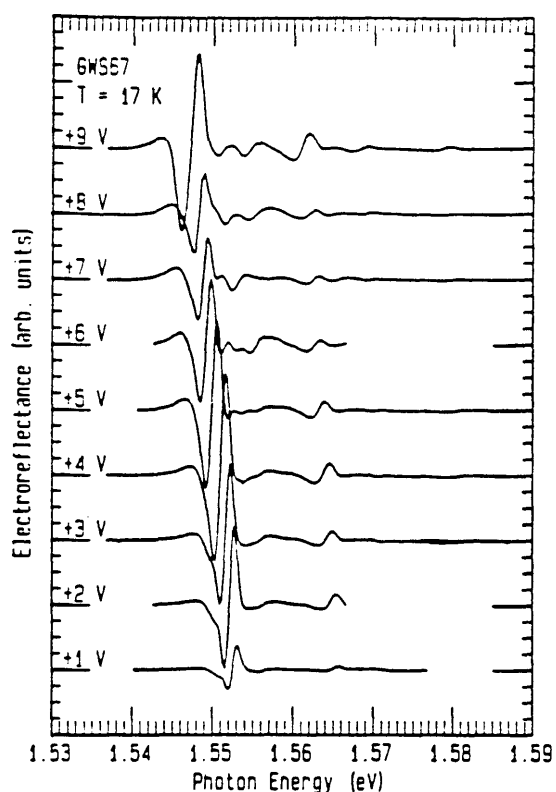


Fig. 6.14 : ER spectra taken on GWS67, with different positive applied offset biases of between +1.0 and +9.0 V. The modulation was 1.0 V_{ptp}, the sample was maintained at 17 K and the angle of incidence was 45°.

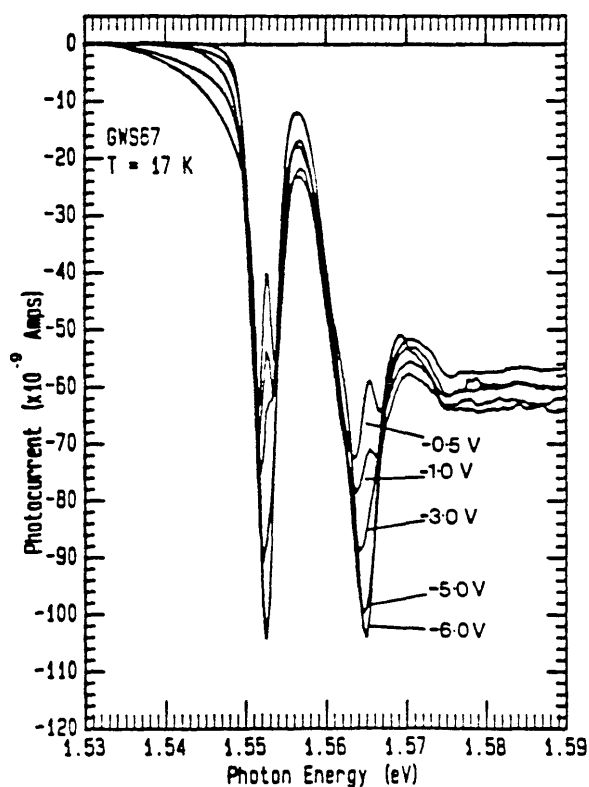


Fig. 6.15 : PC spectra taken on GWS67, with different applied biases of between -0.5 and -6 V. These were taken with the bias applied in opposite polarity to those in Figs. 6.12-14. The sample temperature was again 17 K.

small biases the spectra resemble those for positive bias. When the voltage is increased some features Stark shift to lower energy, although by a smaller amount than when an equal positive bias is applied. As in positive bias, other features, originating from lower field regions, can be seen lying to higher energy. However, under negative bias, these features are of similar size to those with the largest Stark shift, making the different contributions to the ER spectra difficult to resolve.

6.3.3 Bias Dependence of the Electric Field Profile

To summarise the deductions made from the optical spectra, it seems that the photo-excited carriers were not collected from the front of the MQW stack, when small biases of either polarity were applied. This was deduced from the unusual PC lineshape, which showed a dip at the centre of each excitonic peak. The dependence of the ER lineshape on the angle of incidence, confirmed that the dips in the PC peaks, derive from the front of the stack. As an increasing positive bias was applied, the dip in the exciton PC gradually became smaller, showing that the region at the front of the stack not contributing to PC signal was gradually being reduced. The dips seen in the PC peaks Stark shifted under an increasing positive bias, while the background features underwent a much smaller energy shift, showing that most of the voltage was dropped at the front of the stack. The strength of the ER features also suggested that the field was larger at the front of the stack.

A much larger PC signal was seen under negative biases, suggesting the carriers were being collected from a wider region at the back of the stack, than for positive biases. Also, it took a smaller negative voltage to remove the dips from the PC, suggesting that carriers from the whole depth of the stack were then collected. The Stark shifts were smaller under negative biases, than for positive biases, suggesting the field in the stack to be smaller.

The dark current through the structure is much larger than the photo-induced current, and so will determine the field profile in the structure. The applied voltage will therefore be dropped at the positions in the sample structure which show most resistance to the dark current. The photo-excited current is much smaller than the dark current, and will therefore have little effect upon the band bending inside the structure. For the positive bias spectra in Fig. 6.12, the PC signal is less than about $+0.02 \mu\text{A}$, while the dark current varies between $+1$ and $+10 \mu\text{A}$. Meanwhile for the negative bias spectra in Fig.

6.15, the maximum PC is about $-0.1 \mu\text{A}$, compared to dark currents of between -1 and $-10 \mu\text{A}$.

The field profile deduced from the PC and ER measurements, can be explained if the dark current in the structure is limited by tunnelling through the $0.5 \mu\text{m}$ n^+ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ barrier in front of the MQW stack. This may seem surprising as the layer at the back of the stack is $0.5 \mu\text{m}$ n^+ $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ and so should provide a larger barrier, even if tunnelling occurs via the X-point minimum in the $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ conduction band. Perhaps the contact at the back is made directly to the MQW, due to the etch not completely reaching the back layers, so bypassing the $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ layer. Indeed the $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ should become highly resistive at low temperature, as the carriers freeze out into the deep levels associated with X-point minimum of the conduction band. Since large currents can be passed by the sample, it must be the case that the carriers are bypassing the back $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ contact layer.

Fig. 6.16.1 shows the expected band bending at zero applied bias, which is produced by electrons diffusing from the n^+ regions into the undoped MQW. The dark current is of course zero when the applied bias is zero. It is proposed that the dark current sees a negligible barrier at the back of the stack, so that the bias dependence of the field profile is determined by the junction between the MQW and the front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact layer, shown in Fig. 6.16.1.

Under positive bias, the photo-excited electrons travel toward the front of the structure, while the holes travel towards the back. The photo-excited electrons are unable to contribute to the PC, because they are blocked by the front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact. Meanwhile the photo-excited holes see a negligible barrier at the back contact and can contribute to the PC. However, only those holes photo-excited near the back of the stack will survive long enough before recombination to contribute. Hence, the absorption in the front of the stack results in dips in the PC signal near the centre of each exciton peak, as seen in Fig. 6.12.

When the positive bias is increased, the junction between the MQW and the front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact behaves like a reverse biased diode, as shown in Fig. 6.16.2. The field at the front of the MQW stack increases so as to promote the tunnelling of the dark current through the $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact. This explains why the dips in the PC spectra of Fig. 6.12, and the corresponding large ER features in Fig. 6.14, undergo large Stark shifts to lower energy with applied positive bias.

For negative biases the PC signal is comprised of photo-excited electrons travelling towards the back of the stack. They find

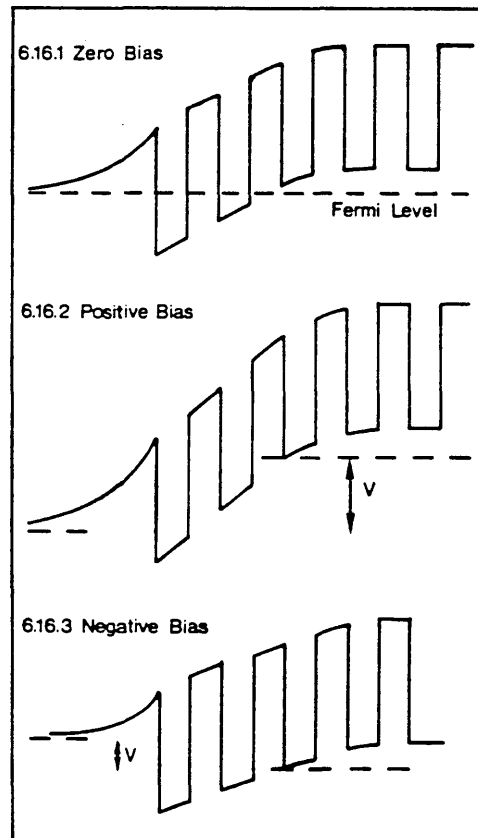


Fig. 6.16 : Schematic diagram of the conduction band profile of sample GWS67, at the junction between the undoped MQW stack and the front n^+ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact layer, under zero (6.16.1), a positive (6.16.2) and a small negative (6.16.3) applied bias.

travelling though the MQW much easier than the holes, due to their smaller effective mass, so a larger fraction of the stack can contribute, explaining the larger PC signals seen with this voltage polarity.

When the negative bias is increased, the junction between the MQW and front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact behaves like a forward biased diode, explaining the larger dark current measured with this voltage polarity. The depletion depth in the n^+ $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact is reduced, under a small negative bias, and so is the field at the front of the MQW stack, as shown in Fig. 6.16.3. When the negative biases is increased to some threshold value, the depletion width in the $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact will be zero, and the field at the front of the MQW will be approximately zero. Any further applied negative bias, above this threshold value, will be dropped across the entire stack thickness. The increase in field across the stack, results in the photo-excited electrons being collected from a larger depth, extending from the back of the stack. Eventually all the photo-excited electrons are collected and the negative contribution in the PC spectrum disappears, as is evident in Fig. 6.15. The fields in the stack will be much smaller under negative biases, because the applied voltage will be dropped across the entire stack. At the largest applied negative bias in Fig. 6.15 (-6 V), the field across the stack will be less than 30 kV/cm, which can be expected to produce a Stark shift for the E1H1 exciton of about 1.8 meV. The field will actually be less than 30 kV/cm, due to the threshold voltage for the junction between the MQW and the front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact.

6.4 Summary and Conclusions

Bias-dependent, low temperature ER, ET, PC and transmission measurements on two $\text{GaAs}/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ MQWs were reported in Sec. 6.2. The four spectroscopies provided complementary information which allowed a detailed elucidation of the structure of the MOVPE grown MQW stacks. In particular each optical technique showed splittings of the E1H1 and E1L1 excitonic transitions, that were explained by a relatively sudden change in the well width at certain depths in the stacks. The size of the splittings suggested changes in the well width of one monolayer. Under zero-field conditions, the energy splitting of the different well width regions would be smaller than the optical linewidth and they would not be resolved. However in this experiment, their energy separation was enhanced by the linear change in the electric field across the

stack, due to the unintentional background p-type doping of the intrinsic region, which is a common feature of MOVPE grown material. This has allowed the observation of monolayer splittings in wells as wide as 163 Å.

This non-uniformity of well width and field across the stack will have a detrimental effect upon the optical linewidth and could consequently degrade the performance of electro-optic devices. This is especially true for those which rely on the QCSE, as the linewidth increases rapidly with field. The experiments have shown that for the growth conditions used, the non-uniformity in well width across the stack is correlated; relatively sudden steps of one monolayer seem to occur at certain depths. MV370 had a 0.74 μm thick stack and showed two different well width regions, while CB8 had a 1.09 μm stack and three regions. Perhaps the MQW optical linewidth could be reduced by limiting the stack thickness to a smaller value, where there would be only one width. Meanwhile reducing the stack thickness would also decrease the broadening due to the linear variation of the field across the MQW. Alternatively, it would be of benefit to allow the monolayer steps and electric field broadening to act in opposition, by deliberately placing wider wells in the weaker field, which is contrary to the situation for the samples studied here.

It has been stressed that each of the spectroscopies used provides different information. The modulation techniques produce differential spectra that are much more sensitive to the monolayer splittings. The PC, transmission and ET spectra depend solely on the imaginary part of the dielectric function and so the observed splittings must be the consequence of different transition energies in the stack. The only possible explanation for this is a relatively sudden change in the well width of one monolayer at some depth. The transmission (or PC) can be used to deduce the absorption spectrum, and the relative heights of the different features in this are proportional to the thickness of the two well width regions, assuming they have the same transition strengths. For ET spectra the relative height of the features will also be affected by the size of Stark shift produced by the modulating field in the two regions.

In contrast ER depends on both the real and imaginary parts of the dielectric function. The mixture is determined by the phase delay between the interfering rays reflected from the contributing layers and the top surface. By studying the evolution of the ER lineshape with the angle of incidence, the lower energy part of the E1H1 and E1L1 exciton lineshapes were deduced to derive from a deeper region inside the stack.

Hence the wider wells are located at the back of the stack, where the field is largest. This agrees with the size of the observed splitting, in both of the samples, at 0 V and also the increase in the splittings with field. In addition the field dependence of the relative strength of the low energy part of the E1H1 lineshape can only be explained if it derives from the back of the stack. ER signals do not increase in strength as the number of wells in the stack increases, unlike the other spectroscopies. Therefore, for low biases, the low energy part of the ER lineshape has a similar strength to the higher energy part. However, the linewidth increases with the applied field, so that the strength of the ER feature deriving from the back of the stack is rapidly reduced by the absorption in the front of the stack.

The lineshape model presented in Sec. 3.4 calculated the ER lineshape of a single exciton for a perfect MQW, where there is a uniform field across the stack and all the wells have identical width. The MOVPE MQW samples studied here have quite sudden changes in the well width at certain depths and also a linear increase in the field across the stack. This limits the comparison of the measured and calculated spectra. The model in Sec. 3.4 ignores the systematic change in transition energy moving across the MQW, which has been represented in Sec. 6.2 by the idea of a "real-space distribution of states". However the concept of the phase change of the ER contribution from each well with depth inside the MQW stack is still valid. Exact modelling of the MQW ER lineshape for imperfect stacks has not been attempted, because it will be very sensitive to the exact form of the absorption profile of the front of the stack.

Similarly it was not possible to make a direct comparison of the lineshape model of Sec. 3.4 and the ER measurements on the GaAs(90 Å)/Al_{0.3}Ga_{0.7}As(100 Å) MQW reported in Sec. 6.3. In this case the comparison was prevented by the field profile across the stack being non-uniform, due to the n⁺-i-n⁺ doping profile in the sample. It was suggested that the band bending in the sample was determined by tunnelling of the dark current through the structure. Despite a direct comparison not being possible, interference effects were again shown to have a very important effect upon the MQW ER lineshape. As before the angular dependence of the ER lineshape was used to determine the depths of the layers producing the different structure in the spectra. This allowed the very strange PC lineshapes of the E1H1, E1L1 and E2H2 excitons to be interpreted.

The ER lineshape of both the E1H1 and E1L1 excitons showed a large feature, superimposed upon a smaller and broader background. The large

ER features corresponded to the unusual minima observed near the centre of the PC excitonic peaks. When the angle of the incident light was changed, the large features in the ER were only slightly altered, while the smaller background features underwent an approximate inversion. These rotations in the ER lineshapes are consistent with the large feature deriving from the front of the MQW stack and the smaller background feature coming from the back. Consequently it was deduced that the PC signal derives only from the back of the stack, with the dips being due to absorption by layers at a shallower depth.

It was proposed that the dark current is limited by tunnelling through the front $n^+ \text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact layer, while the back $n^+ \text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$ layer is bypassed. The non-linearity of the I-V characteristic, and its temperature dependence, also suggested that the current is determined by tunnelling processes. This front $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ contact also limits the collection of the photo-excited carriers travelling towards the front of the structure, so that the PC signal is comprised only of those carriers travelling to the back of the structure.

For positive applied biases, only the holes are collected, as they travel toward the back of the stack. Furthermore, only those holes for which the transit time across the MQW is less than the recombination time, can survive to produce a PC signal. Hence only the holes excited near the back of the stack are detected and dips are seen in the PC due to absorption in the shallower regions of the MQW. When an increasing positive bias is applied, the field at the front of the stack increases, so as to promote the tunnelling of the electrons forming the dark current into the front $n^+ \text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ region. Consequently the dips in the PC excitonic peaks, and the corresponding ER features, were seen to Stark shift to lower energy.

Under a negative bias, dips were again seen near the centre of the PC peaks. However, in this case it was clear that a smaller portion of the PC peaks had been removed by the absorption in the shallower layers. This is because for this voltage polarity, it is the electrons that move toward the back of the stack, which are able to tunnel more rapidly than the holes, so allowing a larger fraction of the back of the stack to contribute. When the negative bias was increased the dips disappear, showing that the photo-excited electrons were being collected from the entire stack.

7 Measurements on Short Period Superlattices

Improvements in the interface quality of epitaxial layers has allowed the growth of very narrow quantum wells with reasonable optical linewidths. Meanwhile the layer widths can be carefully controlled, allowing the preparation of homogeneous MQW stacks with narrow wells and barriers, which are sometimes referred to as "short period superlattices" (SLs). If the barrier width is sufficiently thin the carrier envelope function can penetrate into the adjacent wells. In such a situation the isolated well envelope functions hybridise, being replaced by envelope functions extending throughout the SL. The isolated well confinement energy level is then replaced by a miniband of closely lying states. SLs are interesting because their bandstructure is intermediate between that of an isolated QW and bulk alloy, which represent the extremes where the envelope functions are either confined or "free", in one of the crystal directions.

The electric field dependence of the SL bandstructure is particularly interesting, since the potential drop across one period caused by the field, reduces the interaction between adjacent QWs. Consequently, the electric field can be used to localise the low-field extended envelope functions into individual wells. This produces a radical change in the optical response of the SL. There have been several recent optical studies of SLs, each of which, with one exception, have relied on non-modulation techniques, such as PL, PLE, PC or absorption. It is shown that the strong electric field dependence of the bandstructure, makes field modulation spectroscopy a particular sensitive means of measuring SLs. This chapter discusses the field modulated dielectric function of SLs and describes ER measurements on three samples.

The three SL samples studied here are composed of alternating layers of GaAs and AlAs. In AlAs the conduction band minimum lies at the X-point of the Brillouin zone. Since the X-point of AlAs is lower in energy than that of GaAs, the electrons at the X-point are confined by the SL potential in the AlAs layers. If the GaAs layers are sufficiently narrow, the Γ -point electron levels, confined in the GaAs, can be raised higher in energy than the X-point levels, confined in the AlAs layers. The lowest bandgap is then "type-II", as the holes and electrons are spatially separated in the GaAs and AlAs layers respectively. Type-II SLs have also received considerable recent research. The transitions to the X-point conduction band states are much weaker than those to the Γ -point, and can consequently be difficult

to measure. It is shown that ER is particularly sensitive to the type-II transitions, allowing their measurement in some SLs, where similar PC measurements did not resolve structure. ER is used to measure the qualitatively different field evolution of both the type-I and type-II transitions in these SLs.

Sec. 7.1 discusses some theoretical aspects of the bandstructure of SLs and reviews related work in the literature. The simple and elegant theory of Bleuse et al [7.1], describing the formation of minibands in SLs with narrow barriers and their evolution with electric field, is described in Sec. 7.1.1. This is then adapted in Sec. 7.1.2 to consider the effect of a finite broadening parameter, allowing the field modulated dielectric function to be calculated. The nature of the type-II transitions and their electric field dependence is discussed in Sec. 7.1.3.

PC and ER measurements on three different GaAs/AlAs SL structures, all of which are type-II, are presented in this chapter. Two of these, had similar structures, which were GaAs(28 Å)/AlAs(22 Å) and GaAs(31 Å)/AlAs(22 Å), and are the subject of Sec. 7.2. Experiments on a SL with very narrow GaAs and AlAs layers of just 6 monolayers are presented in Sec. 7.3. Finally conclusions are drawn about the merits of using ER to measure the bandstructure of SLs.

7.1 Superlattice Bandstructure

7.1.1 Stark-Ladder Formation in an Electric Field

Consider a single energy band in an ideal SL composed of $(2N+1)$ equally spaced QWs. If the barrier width is sufficiently narrow, then there is a significant leakage of the carrier envelope function into the adjacent wells. This results in a hybridisation of the isolated well envelope functions into states that extend throughout the entire SL stack, while the degeneracy of the $(2N+1)$ confined levels in the $(2N+1)$ isolated wells is lifted, forming a miniband of $(2N+1)$ closely spaced energy levels, as described, for an ideal system, by the Kronig-Penney model [7.2]. The minibandwidth increases as the barrier width is reduced, due to the enhanced coupling between wells. In the limit of the barrier width being zero, the minibandwidth tends to infinity and the SL dispersion will tend to that of the bulk alloy of the SL constituent materials.

When a field (F) is applied, the degeneracy of the levels in the

isolated wells (in the absence of SL effects) is lifted. At large enough fields, the envelope functions in each well will be similar to those for isolated QWs. The SL bandstructure is then comprised of $(2N+1)$ levels that are evenly spaced by the potential drop across one period, equal to $e.F.d$, where d is the SL period. This is referred to as a "Stark-ladder" of states.

There is an intriguing evolution of the SL bandstructure between the miniband and isolated well extremes, described above. The boundary between the two regimes occurs at those fields where the potential drop between adjacent wells is comparable to the zero-field minibandwidth. At these fields the envelope function is localised over several periods, as shown schematically for the electron state in Fig. 7.1. For GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$ SLs, the electron envelope function can be distributed over several periods at a certain field, while that for the heavy holes is isolated in individual wells, as depicted in Fig. 7.1. This is a consequence of the heavy hole having a much larger barrier mass and so a smaller minibandwidth. The distributed electron envelope function will have a significant overlap with a number of the hole Stark-ladder states, resulting in the interband optical spectrum showing several features, separated by an energy $e.F.d$. The relative height of these features will depend on the distribution of the electron envelope function. These can be considered as transitions between electrons and holes centred around different wells and hence the assignment of $m = \dots -2, -1, 0, 1, 2 \dots$ in Fig. 7.1. With increasing field the electron envelope function in Fig. 7.1 will become increasingly isolated into the central well, resulting in the oscillator strength of the oblique ($m \neq 0$) Stark-ladder transitions decreasing to zero.

Stark-ladder transitions have been observed by Mendez et al [7.3] in field-dependent PL and PC spectra of a GaAs/ $\text{Al}_{0.35}\text{Ga}_{0.65}\text{As}$ SL. Five equally spaced transitions were observed for a SL with a $65 \text{ \AA}$ period, corresponding to the transitions shown in Fig. 7.1. These transitions showed a linear energy shift with field and the gradient was close to that expected from the period. Bleuse et al [1.10] observed similar behaviour in field-dependent transmission measurements on $\text{Al}_{0.24}\text{Ga}_{0.24}\text{In}_{0.52}\text{As}/\text{Ga}_{0.47}\text{In}_{0.53}\text{As}$ SLs. Stark-ladder transitions of up to sixth order have been observed by Barrau et al [7.4] in the PC spectra of a $\text{GaAs}(30 \text{ \AA})/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}(30 \text{ \AA})$ SL.

Voisin et al [7.5] have made ER measurements on a $\text{GaAs}(33 \text{ \AA})/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}(33 \text{ \AA})$ SL. The ER spectra showed rich structure, although no detailed analysis of this was attempted. Measurement of the ER signal as a function of the lower limit of the modulation bias, at

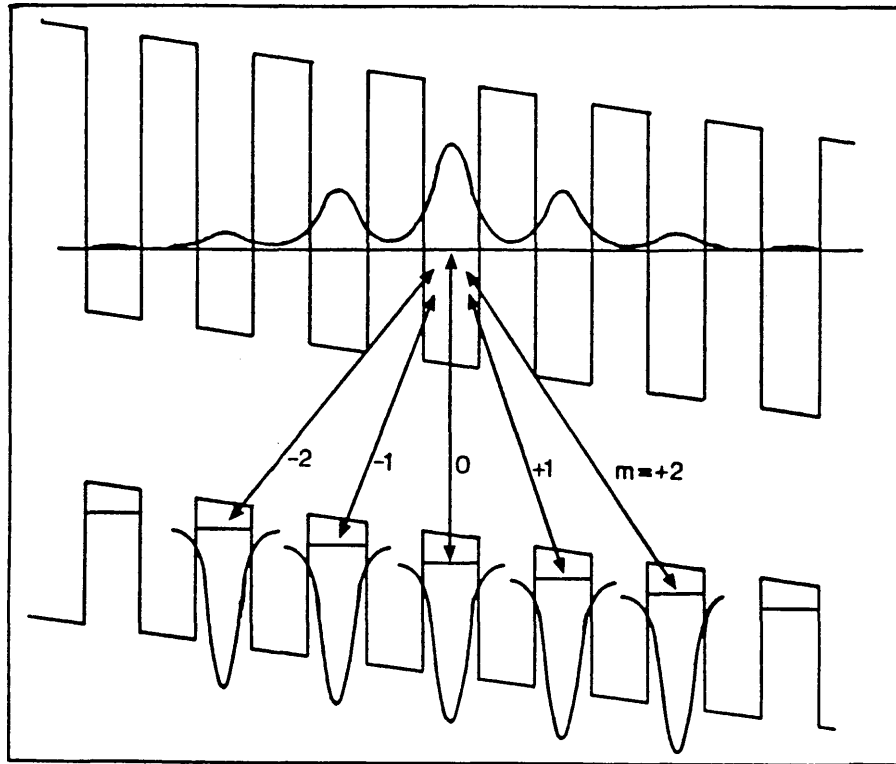


Fig. 7.1 : Schematic drawing of the electron and hole envelope functions in a type-I SL.

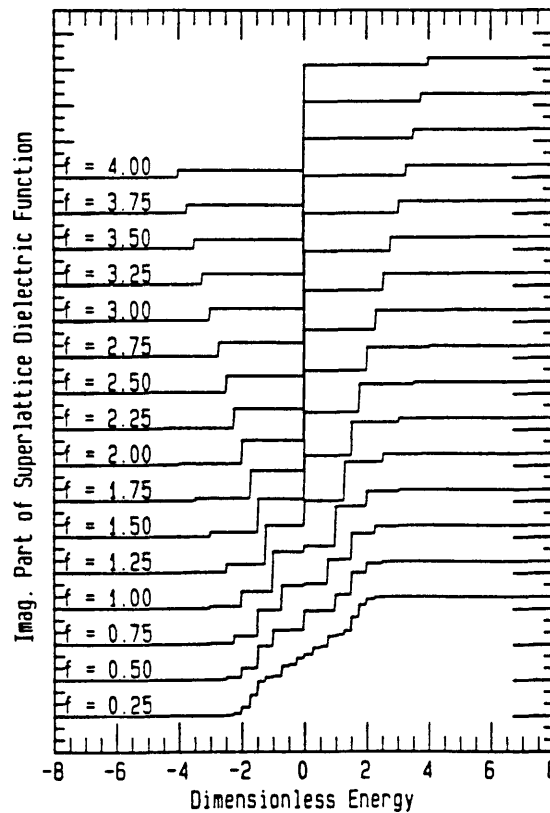


Fig. 7.2 : Imaginary part of the SL dielectric function, calculated using Eq. 7.4, under different electric fields. The energies and fields are expressed in the dimensionless units defined in the text.

energies below the SL bandedge, produced oscillatory curves in agreement with the expected Stark-ladder behaviour. In these experiments they fixed the maxima of the applied bias and varied the minimum voltage. In contrast, all the ER measurements in this thesis use small fixed voltage modulations about a variable d.c. bias, which produces differential spectra. Increasing the size of the modulation was found to broaden the ER spectra presented in Sec. 7.2-3, with the result that fewer features were resolved.

Stark-ladder phenomena in a bulk semiconductor, due to transitions between an electron and hole on different lattice sites, were in fact predicted some years ago [7.6]. However these have never been observed experimentally, as the small semiconductor lattice period, which is just 2.83 Å for GaAs, results in a Stark-ladder splitting too small to be resolvable in the best quality material. The recent growth of artificial semiconductor systems with a much larger period, ie a SL, has allowed the recent measurements of Stark-ladder behaviour.

Bleuse et al [7.1] calculated the field-dependent SL absorption spectra using a tight binding nearest neighbour calculation, which ignores band mixing and the Coulombic attraction of the electron and hole. The SL eigenstates, $\chi(z)$, are expanded in terms of the isolated QW z-motion envelope functions, $\psi(z)$

$$\chi(z) = \sum_{n=-N}^{+N} C_{nv} \psi(z - n d) \quad \dots 7.1$$

This assumes only one energy level per well. The Schrödinger equation is then solved by only considering the interaction of the isolated well envelope function with its own confining potential and that of its nearest neighbours. This leads to the miniband structure described by the Kronig-Penney model at zero field, with a bandwidth of 4σ , where σ is the transfer integral between nearest neighbours. The SL envelope functions are then found to be distributed over the entire SL stack.

In the presence of an electric field, such that $(2N+1) e F d \gg \sigma$, which is typically true even at very small fields, the miniband is replaced by a Stark-ladder spectrum

$$E = E_0 + m e F d, \quad -N \leq m \leq N \quad \dots 7.2$$

with the coefficients in Eq. 7.1 given by :

$$C_{nv} = J_{n-v} (2\sigma / e F d) \quad \dots 7.3$$

E_0 is the isolated well confinement energy and J_m is the Bessel function of integer order m . This is strictly only true in the limit $N \rightarrow \infty$, but is a good approximation for typical SLs [7.1].

The optical spectrum can be calculated from the overlap integral of the electron and hole envelope functions, yielding :

$$\epsilon_i(\Omega) = B \sum_{m=-N}^{+N} J_m^2(2/f) \cdot H(\Omega - mf) \quad \dots 7.4$$

where H is the unit step function and B is a constant given by $B = S_0 \cdot D_{2D}$, using the same nomenclature as Eq. 2.29. The electric fields and energies in Eq. 7.4 have been expressed in terms of the dimensionless parameters, $f = e F d / (\sigma_C + \sigma_V)$ and $\Omega = (E - E_{sqw}) / (\sigma_C + \sigma_V)$ respectively, where E_{sqw} is the transition energy of the isolated QW. Eq. 7.4 assumes each Stark-ladder transition to contribute a step to the dielectric function, corresponding to an unbroadened transition between two-dimensional subbands, in the absence of the Coulombic interaction.

Fig. 7.2 shows the spectra calculated using Eq. 7.4 for different values of the field. The Bessel functions were calculated using the series approximation quoted in Ref. 2.35. The spectra in Fig. 7.2 are identical to those reported by Bleuse et al [7.1]. It can be seen that the spectrum consists of a series of steps, whose energy separation increase linearly with field according to Eq. 7.2, while Eq. 7.4 quantifies the field dependence of the height of each step. It can be seen that the total integrated absorption of all the Stark-ladder transitions is constant, in accord with the "Sum Rule" [2.77]. Notice that even at quite moderate fields, where the potential drop across a period equals the minibandwidth ($f = 4$), the envelope functions are strongly localised inside one well. Under further increasing field, the strength of the central step increases, at the expense of the oblique transitions, as the envelope functions become further isolated into individual wells.

At small finite fields there is a complicated distribution of the oscillator strength between many different Stark-ladder transitions. At zero field, the absorption profile would tend towards the smooth curve predicted by the Kronig-Penney model. Eq. 7.4 shows that the size of the $\pm m$ th transition is determined by $J_m^2(2/f)$. Analysis of this Bessel function reveals that the height of each step initially oscillates with increasing field, before reaching a maximum when $(2/f) \approx m$. Under further increasing f , J_m falls to zero, except for $m = 0$ which rises to unity.

7.1.2 Electric Field Modulated Dielectric Function

In order to model experimental spectra it is necessary to take account of the finite broadening of the Stark-ladder transitions. The optical linewidth of a typical SL can be expected to be quite large, as a monolayer well width variation produces a large spread in the confinement energies for narrow wells, as demonstrated by Fig. 5.16.

The spectra measured on various SLs [7.3-5,1.10] showed the presence of distinct peaks that are indicative of bound excitonic states. Consequently the isolated well dielectric function is considered to be dominated by the 1s bound excitonic term here, as was also assumed in the analysis of isolated QWs considered in Sec. 3.2. Fig. 7.3 shows the imaginary part of the dielectric function calculated using Eq. 7.4, but with the unit step function replaced by a Gaussian lineshape function, with a FWHM of 1σ ,

Modulated spectra have been calculated by taking the difference in the SL dielectric function between two field extremes. Fig. 7.4 shows the modulated imaginary part of the dielectric function, generated using a small change in the field ($\Delta f = 0.01$), at different offset fields. The experimental ER spectra presented later were also taken with a small peak-to-peak modulation about a variable d.c. offset.

It can be seen in Figs. 7.3 and 7.4, that the transition strength is concentrated over the range of energies covered by the zero-field miniband ($-2 \leq \Omega \leq +2$). The features related to the different Stark-ladder transitions are clearly resolved in the high-field spectra of Figs. 7.3 and 7.4. The width of each individual transition is determined by the broadening parameter. Notice that the individual oblique ($m \neq 0$) Stark-ladder transitions contribute a zero-crossing to the spectra in Fig. 7.4, characteristic of the energy-differential of the the imaginary part of a single exciton dielectric function, ie $\text{Imag}[\delta L/\delta E]$ shown in Fig. 3.2.2.2. This is not surprising considering the very large field-induced energy shift of these transitions. Notice also that the zero-crossings are of opposite sense for the increasing ($m > 0$) and decreasing ($m < 0$) transitions.

Comparison of Figs. 7.3 and 7.4 shows that field modulation spectroscopy emphasises the oblique ($m \neq 0$), Stark-ladder transitions, over the direct ($m = 0$) contribution. This is due to the large field-induced energy shift of the oblique transitions. Likewise, the Stark-ladder transitions with a larger order number (m) will be emphasised. The extra sensitivity of ER to oblique Stark-ladder transitions is demonstrated by the measurements on GaAs/AlAs SLs

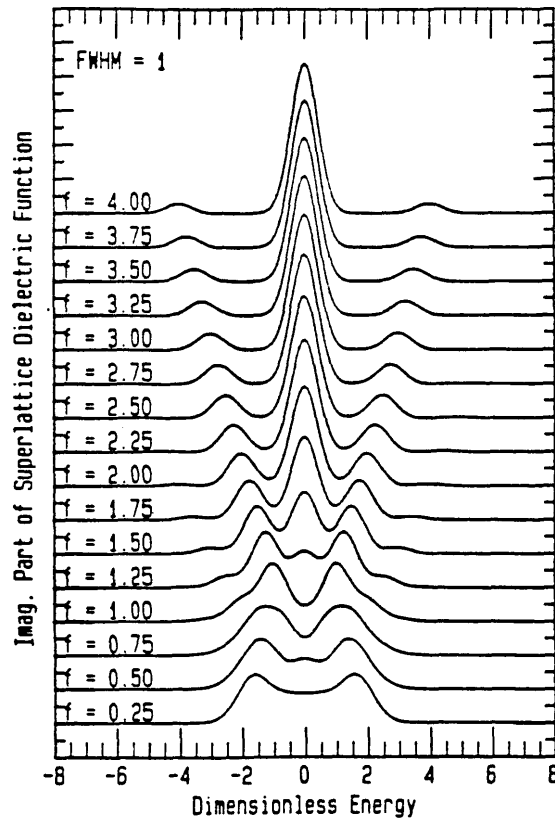


Fig. 7.3 : Field dependence of the imaginary part of the SL dielectric function, calculated with each **Stark**-ladder transition represented by a Gaussian broadened exciton peak, with a FWHM of σ .

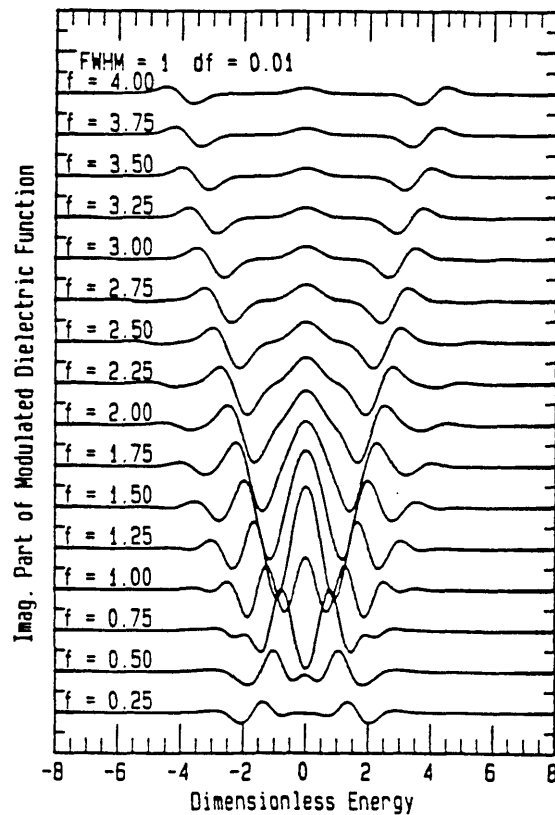


Fig. 7.4 : The imaginary part of the field modulated SL dielectric function, at different applied offset fields. The modulation was taken as $\Delta f = 0.1$. The FWHM of the exciton peaks was taken as 1σ .

presented in the Sec. 7.2.

As the field is reduced, the individual Stark-ladder transitions in Figs. 7.3 and 7.4 become unresolved. The peaks in Fig. 7.3 and the zero-crossings in Fig. 7.4 are then no longer found at the energies predicted by Eq. 7.2. For low fields ($f < 1$), the individual Stark-ladder transitions overlap and the average separation of the peaks in Fig. 7.3, and the zero-crossings in Fig. 7.4, is determined by the linewidth. For the smallest field, both the spectra in Figs. 7.3 and 7.4 are dominated by structure near the energy of the bandedges, where the oscillator strength is largest. Notice that as the field is increased from the lowest values, the features located at the edges of the miniband in Fig. 7.3 and 7.4, evolve towards the centre of the miniband.

Calculations with a smaller linewidth demonstrate the low-field behaviour more clearly. Fig. 7.5 shows the imaginary part of the dielectric function at different fields, calculated with a FWHM linewidth of 0.2σ . The gross behaviour in Fig. 7.5 again shows a redistribution of the zero-field oscillator strength from the edges of the miniband, towards its centre. However, it can be seen that the detailed evolution is very complicated. Similar behaviour can be observed in the modulated spectra calculated with this narrow linewidth.

The modulated real part of the SL dielectric function showed qualitatively similar behaviour to the imaginary part in Fig. 7.4. The lineshape of each individual oblique Stark-ladder transition, in the high-field limit, is then a positive- or negative-peak lineshape ($\pm \text{Real}[\delta L/\delta E]$ in Fig. 3.2.2.1), instead of the zero-crossings seen in Fig. 7.4. Similarly at low fields, where the individual ladder transitions cannot be resolved, the lineshape phase of the entire spectrum is just rotated by $\frac{1}{2}\pi$, with the zero-crossings being replaced by maxima or minima etc. Choosing broadened continuum contributions, instead of 1s excitons, also changes the lineshape of the individual ladder transitions, but not the general evolution of the spectra with field.

The MQW lineshape model of Sec. 3.4, and the experimental measurements on MQWs in Chp. 6, stressed that the ET depends essentially on the imaginary part of the modulated dielectric function alone, while ER also depends upon the real part. The mix of real and imaginary parts in the ER is determined by optical interference of the light reflected from the front surface of the sample and the QWs. Sec. 3.4 described how the mix of the real and imaginary parts can be complicated in multiple well systems, as the optical penetration depth varies across

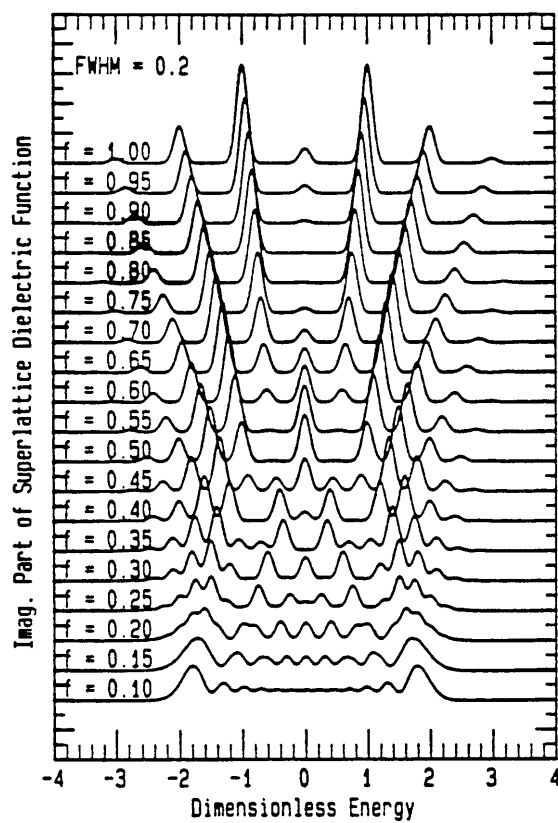


Fig. 7.5 : Field dependence of the imaginary part of the SL dielectric function, calculated using a small linewidth of 0.2σ .

the spectrum. These effects can also be expected to be important in the ER spectra of SL samples.

The model of the electric field modulated dielectric function of a SL, described in this section, has been greatly simplified. A number of approximations have been made, which are now discussed individually.

(1) The analysis has ignored the field-induced polarisation of the isolated well envelope functions, ie the QCSE. However, this will be negligible for the very narrow well widths typically found in short period SLs.

(2) Each well has been assumed to contribute only one level to the miniband in the valence and conduction band. However, this will be a poor approximation at the fields where the potential drop between adjacent wells is comparable to the separation of different confined levels in any one well. At these fields there will be a hybridisation of the two different confined states in adjacent wells, which will produce anti-crossing effects in the optical spectra. A more complete analysis would then require both confined states to be included in the summation of Eq. 7.1. This mixing of different confined states will be most important for the H1 and L1 states, especially in SLs with wide wells.

(3) Each Stark-ladder transition has been regarded as contributing a broadened 1s excitonic term to the overall SL dielectric function, and the contribution of the excited excitonic and the continuum states have been ignored. Furthermore, the binding energy and the square of the electron-hole overlap for the xy-direction per unit area, ie $|\phi(0)|^2$ in the nomenclature of Eq. 2.34, of the 1s exciton has been taken to be independent of field. A more complete analysis might also include a broadened continuum term, and calculate the 1s binding energy and xy-direction overlap using a variational method, as in Sec. 2.6.2. This has not been done here, so as to keep the contributions of the individual Stark-ladder transition as simple as possible, in order to identify the effects due to the interaction of adjacent wells. Some justification for the simplification used here is given by the success in modelling the SQW ER lineshape in Chps. 3 and 5 using only the 1s excitonic state.

Whittaker et al [7.7] have made a better analysis of the field dependence of the SL bandstructure, which includes the effect of the Coulombic attraction. Other workers have made zero-field calculations of excitonic states in SLs [7.8-9]. Each of these calculations demonstrated the zero-field SL bandstructure to be dominated by bound

excitons associated with the miniband edges, as in Fig. 7.3. However, the more detailed calculations [7.7-9] show the binding energy of the exciton associated with the top of the miniband to be larger than that associated with the bottom.

At low fields, Whittaker et al calculate a very complicated variation of the oscillator strength across the minibandwidth. Again the gross behaviour shows a redistribution of the oscillator strength, away from the edges of the miniband, towards its centre, as in Fig. 7.3. However, this is more complicated than in Figs. 7.3 and 7.5, due to mixing of the excited excitonic states in adjacent wells with 1s.

At high fields, Whittaker et al predict qualitatively similar behaviour to that seen in Fig. 7.3, with the Coulombic interaction only causing a small change in the energy and oscillator strength of each Stark-ladder transition. There will be a small change in the binding energy of each Stark-ladder transition with field, due to the change in the strength of the electron-hole interaction. For instance, the binding energy of the $m = 0$ transition will increase with field, as the electron and hole become confined inside the same well, and it will eventually tend towards the value for an isolated QW. Indeed this change in the binding energy has been observed in several experimental studies [7.3,7,10], as a small red shift of the $m = 0$ Stark-ladder transition. The field-induced change in the binding energy will have little effect on the ER lineshape of the oblique transitions, as it is negligible compared to the linear energy shift given by Eq. 7.2. However, it may have more effect on the $m = 0$ transition, which will otherwise be modulated solely by the change in the oscillator strength.

Whittaker et al also found the oscillator strength to be concentrated in the $m = 0$ ladder transition at high fields, as in Fig. 7.3. However, their calculations showed the $m = -1$ Stark-ladder transition to be more intense than the $m = +1$. This is not predicted by the simple calculations presented here; the curves in Fig. 7.3 are symmetric about their central energy ($\Omega = 0$). Whittaker et al pointed out that the energy separation of the $m = 0$ and $m = -1$ transitions will always be smaller than that of $m = 0$ and $m = +1$, due to the larger binding energy of $m = 0$ state. Hence simple perturbational arguments suggest that the $m = -1$ state will be more strongly mixed with $m = 0$, than $m = +1$, and will consequently have a larger oscillator strength.

Despite its inadequacies, this analysis has elucidated the gross behaviour that can be compared with experimental spectra. To summarise some of the main points, different behaviour is seen in the various

field extremes :-

- (1) at high field ($f \gg 1$), provided the ladder separation also exceeds the linewidth, the various Stark-ladder transitions are resolved, evolving linearly with field and interpolating to the miniband centre, as given by Eq. 7.2;
- (2) at low fields ($f \ll 1$), miniband behaviour is observed with the structure being found close to the energies of the miniband edges;
- (3) at intermediate fields, structure is seen over the energy range of the miniband whose separation is determined by the linewidth. Features are observed to evolve from the miniband edges, towards the centre of the miniband.

7.1.3 X-Point Conduction Band States in GaAs/AlAs Superlattices

Fig. 7.6 shows the relative positions of the Γ - and X-points of the GaAs and AlAs conduction bands. In AlAs, the conduction band minimum lies at the X-point of the Brillouin zone and at a lower energy than the X-point of GaAs. Consequently the electrons at the Γ - and X-points in GaAs/AlAs SLs are confined in the GaAs and AlAs layers respectively. The height of the confining potential barriers can be deduced from the Γ - and X-point bandgaps, marked in Fig. 7.6, which are taken to have the values listed in table 7.1, and the value of the discontinuity ratio, taken here to be 0.65. The value of the discontinuity ratio has a critical effect upon the relative positions of the bottom of the X- and Γ -point potentials of the SL, $\Delta V(\text{CB}; \Gamma\text{-X})$, and consequently upon the relative energies of the Γ and X confined levels. Recent measurements have indicated the discontinuity ratio to lie in the range 0.6 to 0.7 [2.3-6], which gives a variation in $\Delta V(\text{CB}; \Gamma\text{-X})$ of 0.106 to 0.266 eV.

If the GaAs layers are sufficiently narrow, the confinement of the Γ -point electrons will raise their energy above that of the X-point electrons confined in the AlAs layers. The X-levels are then the lowest in the SL conduction band and the bandgap is indirect in both momentum- and real-space. Such a SL is referred to as "type-II". Recent measurements [7.12] suggest that GaAs/AlAs SLs are type-II when the GaAs layer is narrower than about 35 Å, in approximate agreement with the value of $\Delta V(\text{CB}; \Gamma\text{-X})$ produced by a discontinuity ratio of 0.65.

The oscillator strength for transitions between the valence and conduction bands is given by Eq. 2.26, under the effective mass approximation. The optical matrix element, P_{cv} in Eq. 2.26, will be smaller for transitions to the X-point of the conduction band, rather

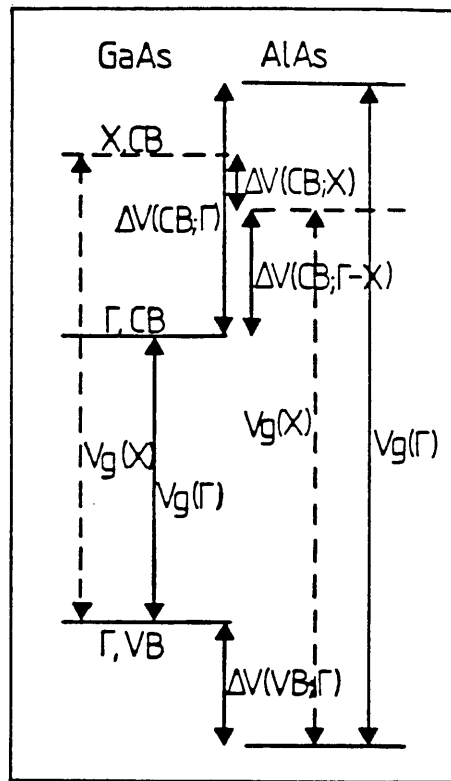


Fig. 7.6 : Schematic drawing of the relative positions of the Γ - and X-points of the conduction band and Γ -point of the valence band in GaAs and AlAs.

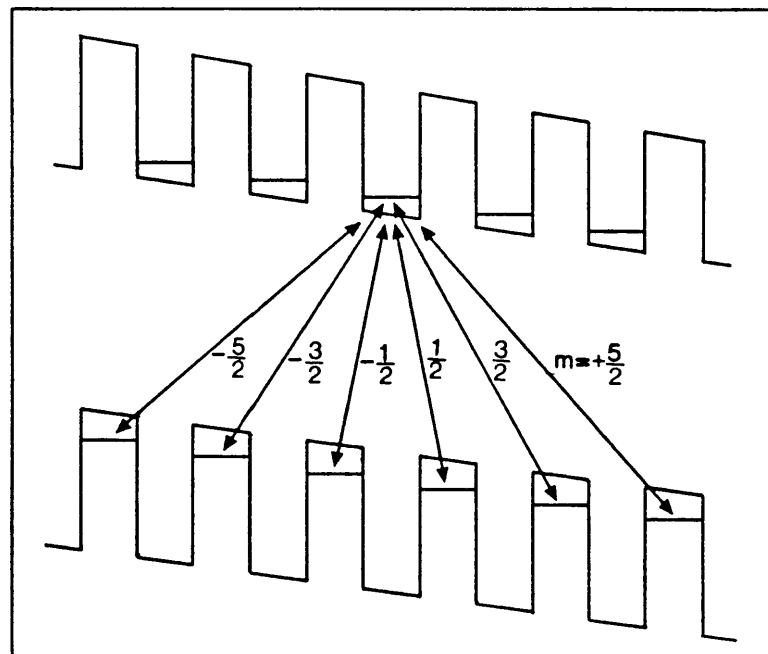


Fig. 7.7 : Schematic drawing of the Stark-ladder transitions in a type-II SL.

than the Γ -point, due to the symmetry of the X-point Bloch functions. In fact there must be a degree of mixing between the Γ - and X-point wavefunctions for P_{CV} to be non-zero, which can be estimated using first-order perturbation theory [7.13]. The origin of the potential creating the mixing is unclear, but it has been suggested that it could derive from either the SL potential or interface roughness [7.13-15].

parameter	GaAs	AlAs
$V_g(\Gamma)$	1.425 eV	3.020 eV
$V_g(X)$	1.900 eV	2.168 eV
$m(E1;\Gamma)$	0.067	0.150
$m(E1;X_z)$	0.23	0.19
$m(E1;X_{xy})$	1.3	1.1
$m(HH)$	0.34 [2.4]	0.76
$m(LH)$	0.094 [2.4]	0.15
$m(SOS0)$	0.15	0.24
discontinuity ratio		0.65 [2.3]
$\Delta V(CB;\Gamma)$		1.037 eV
$\Delta V(VB;\Gamma)$		0.558 eV
$\Delta V(CB;X)$		0.290 eV
$\Delta V(CB;\Gamma-X)$		0.185 eV

Table 7.1 : The potential discontinuities and effective masses used in calculations of Chp. 7. All the parameters are taken from Ref. 7.11 unless otherwise specified

The overlap of the electron and hole envelope functions, M_z in Eq. 2.26, will be also be small for the type-II transitions, since the holes and electrons are spatially separated in the GaAs and AlAs layers respectively. Hence the X-point transitions of the SL can be expected to be much weaker than those to the Γ -point of the conduction band. Note also that the interband selection rule for transitions between the Γ -point electrons and holes discussed in Sec. 2.3, of conservation of the subband number, will no longer be valid.

The optical transitions between the Γ -point of the valence band and X-point of the conduction band are restrained by the requirement that the optical momentum be conserved. The X-point has a six-fold degeneracy, with $\underline{k}$ values of $\pm(2\pi/a) \cdot [1,0,0]$, $\pm(2\pi/a) \cdot [0,1,0]$ and $\pm(2\pi/a) \cdot [0,0,1]$, where a is the lattice period. These are labelled X_x , X_y and X_z respectively and the convention used here takes the z-direction ($[0,0,1]$) as being perpendicular to the SL layer planes. Conservation of the momentum parallel to the SL planes forbids transitions from the Γ -point valence band to the X_x and X_y (written collectively as X_{xy}) conduction band. At first glance it appears that

conservation of the momentum in the z-direction, also forbids transitions to the X_z -point. However the SL periodicity reduces the crystal momentum in the z-direction, so that the X_z -point is "folded" back to the Brillouin zone centre, if the SL period contains an even number of unit cells. This transition then becomes allowed and is referred to as "pseudo-direct".

The preceding arguments predict that the only transitions between the Γ -point of the valence band and X-point conduction band allowed by momentum conservation, are those involving the X_z -minimum. Additionally these are only allowed when the SL has an even number of monolayers in the period. However the other transitions can also be observed in experiment, if the transition occurs via an intermediate state at the Brillouin zone centre. The electron can be scattered from the X-point to $\underline{k} = 0$, by either a phonon or an ionised impurity. In the case of phonon scattering this gives rise to peaks in the PL spectra spaced a phonon energy below the separation of the Γ -point hole and X-point electron, which are called "phonon side-bands" [7.13,15].

Optical transitions between the X-point electrons and Γ -point holes have been identified in a number of ways. Luminescence spectra are sensitive to the lowest energy bandgap, while excitation spectra are dominated by the transitions to the Γ -point electron states, which have a much larger transition strength. A large Stokes shift between the principal PL and PLE features, as measured by several authors [7.13-18], is thus indicative of type-II SLs. The X-point transitions can also be identified by their very long PL decay time, due to the small electron-hole overlap [7.13-15]. PL of both type-I and type-II SLs can show the presence of zero-phonon and phonon-assisted lines. However, their phonon satellites have been measured to exhibit quite different character [7.18,13]. For type-II SLs three phonon side-bands are observed, with energies in good agreement with X-point phonons of either GaAs or AlAs [7.13,14,18].

The X_z -valleys have a larger effective mass for motion in the z-direction than the X_{xy} -valleys. Hence for a SL grown along the [0,0,1] crystallographic direction, the X_z levels will have a lower confinement energy than X_{xy} . However the slightly larger lattice constant of AlAs compared to GaAs, results in the AlAs layers being subject to a compressive biaxial strain, which raises the X_z levels relative to X_{xy} . This opposes the effect of the smaller X_z confinement energy, resulting in the X_{xy} levels being lowest in SLs where the AlAs layer width exceeds a certain value. The measurements of van Kesteren et al [7.19] and Scalbert et al [7.20], suggested X_{xy} to be lower than

X_z when the AlAs layer width exceeds 55 - 70 Å.

The spatial separation of the X-point electrons in the AlAs layer and the Γ -point holes in the GaAs, leads to different electric field-dependent behaviour for type-II transitions, than for the type-I case discussed in Sec. 7.1.1-2. The type-II Stark-ladder transitions will therefore occur over odd half-integer multiples of the period, ie taking $m = \pm 1/2, \pm 3/2, \dots$ in Eq. 7.2, as shown schematically in Fig. 7.7. Note that the X-electrons have a much heavier effective mass in the GaAs layers, than the Γ -electrons do in AlAs, so they experience a much larger barrier, and are much more localised. Consequently the overlap of the X-point electrons and Γ -point holes will be small and only the $m = \pm \frac{1}{2}$ ladder transitions are likely to be observed.

Meynadier et al [7.12] studied the electric field-dependent PL spectra of a GaAs(35 Å)/AlAs(80 Å) SL. This SL is type-II and they observed recombination of the X-point electrons confined in the AlAs layers and the GaAs Γ -point holes. With increasing field this transition moved to higher energy at a rate in agreement with half the period of the SL, suggesting it to be the $m = +\frac{1}{2}$ Stark-ladder transition. An anti-crossing was observed between this and the Γ -point ($m = 0$) transition, at a sufficiently large field, and under further field the SL became type-I. This study did not reveal the presence of the downward moving $m = -\frac{1}{2}$ X-point Stark-ladder transition, as the electron-hole overlap is smaller for this state, due to the electric field-induced polarisation of the X-electron envelope function in the relatively wide AlAs layer. Experiments on a GaAs(28 Å)/AlAs(22 Å) SL, studying PL and PC as reported by Whittaker et al [7.7], in addition to ER measurements made by myself and co-workers [1.17], did show the $m = -\frac{1}{2}$ transition, as described later. Both the increasing and decreasing X-point Stark-ladder transitions were found to evolve linearly with field, at a rate in excellent agreement with the SL half period.

7.2 Measurements on a (10,8) and (11,8) GaAs/AlAs Superlattice

This section describes measurements on two similar MBE grown GaAs/AlAs SLs. Sample GWS97 is a (10,8) SL, while GWS85 is (11,8). The nomenclature used here for the SL layer dimensions is to write (n,m), where n and m are the number of monolayers (ML) in the GaAs and AlAs layers respectively. The results on GWS97 are also briefly reported in Ref. 1.17.

7.2.1 Experimental Details

The GaAs/AlAs SL samples studied here were grown in a Varian Gen-II MBE machine, at RSRE (Malvern), by Dr. G.W.Smith. Sample GWS85 contained of 100 repeats of undoped (11,8) GaAs/AlAs SL, while GWS97 had 100 repeats of (10,8). The entire sample structure consisted of 1 μm of n^+ buffer layers on a n^+ substrate; a 100 Å undoped GaAs spacer; the undoped SL; and finally a 240 Å undoped GaAs capping layer. A piece of each wafer was prepared into circular mesas with diameters ranging between 0.5 and 2.0 mm and semitransparent Gold Schottky barriers were evaporated on the top surfaces. Ohmic contacts were made to the Schottky barriers and the back surface. I am grateful to Dr. G.Monnington of RSRE for processing these samples.

The mesas had a forward bias threshold current of about +0.8-0.9 V at 17 K, corresponding to the Schottky barrier height and hence taken to equal the built-in voltage. The leakage currents were typically -10 μA at reverse biases of about -2.5 to -3.5 V, allowing a maximum applied field of 60-70 kV/cm. The increase in the leakage current with reverse bias was gradual and large values of -50 μA could be stably maintained without damaging the diode. This is in contrast to the electrical behaviour of the MOVPE p^+-i-n^+ diodes, grown at the University of Sheffield, that were studied in Chps. 5 and 6. In these the leakage currents were much smaller and the breakdown biases larger, although allowing a leakage current of more than a few μA to flow in the Sheffield samples usually impaired their performance.

The electric field (F) was calculated from the applied bias (V) using the formula $F = (V_{bi}-V)/t_i$. The built-in voltage (V_{bi}) was taken as the forward bias current threshold of +0.9 V, while the total intrinsic region width (t_i) was taken as the value specified in the growth procedure. C-V measurements showed the background doping of the intrinsic region to be small, so that the field across the SL can be regarded as uniform.

I have made low temperature PC and ER measurements on both SLs as a function of the applied electric field. The samples were mounted on the cold-finger of the closed-cycle Helium cryostat maintained at a temperature of 17 K. Two light sources were employed; a 80 W Tungsten lamp and a 450 W Xenon arc lamp, along with a single grating monochromator, to produce a beam of 0.25 nm HWHM.

Field-dependent PC and PL experiments have also been performed on GWS85 and GWS97, by Dr. M.S.Skolnick at RSRE, and these results are reported in Ref. 7.7. These PC measurements used a dye-laser light

source and so produced a better signal-to-noise ratio than the PC spectra I collected, as discussed later. The experiments described in Ref. 7.7 were performed on different pieces of the GWS85 and GWS97 wafers than the measurements presented here.

The two SL samples studied here, and also in Ref. 7.7, form part of a series of GaAs/AlAs SLs grown at RSRE, where the AlAs layer width was fixed at 8 monolayers. Experiments on previous samples in this series made by Dr. Skolnick at RSRE, which were grown under similar conditions, except that the substrate was semi-insulating, are reported in Refs. 7.13 and 7.18. These measurements showed a (12,8) SL to be type-I, while a (11,8) structure was found to be type-II [7.18]. This deduction was based upon the Stokes shift between the PL and PLE spectra; the relative intensities of the PL signal from the two SLs; and the nature of the PL phonon satellites observed for each SL [7.18]. By applying a hydrostatic pressure, the energy of the Γ -point in the SL conduction band was increased relative to that of the X-point, and the (12,8) was converted to type-II [7.13]. Consequently both the (11,8) and (10,8) SL studied here are expected to be type-II.

7.2.2 Photocurrent Measurements

Fig. 7.8 shows 17 K PC spectra measured on the (11,8) GaAs/AlAs SL (GWS85) with different applied biases. These were excited using the Tungsten lamp source and monochromator. The E1H1 and E1L1 transitions of the two SLs can be seen at energies near 1.82 and 1.88 eV respectively. Two distinct peaks are observed in the E1H1 region at low fields, which are due to the excitons associated with either minibandedge. Meanwhile only one peak is seen in the larger reverse bias spectra of Fig. 7.8, produced by the $m = 0$ Stark-ladder transition. Notice that shoulders can just be discerned on the E1H1 peak of some spectra, for example -0.5 V, which are due to the oblique ($m = \pm 1$) Stark-ladder transitions. These become unresolved at larger reverse biases due to the increase in the noise present in the spectra. There is also an increase in the background signal at energies below the E1H1 transition with reverse bias, which is caused by the carriers photo-excited in the GaAs layers being able to tunnel through the SL.

Fig. 7.9 shows a 17 K PC spectrum measured on the (10,8) SL with an applied bias of 0 V. This shows two peaks near 1.866 and 1.92 eV, due to the E1H1 and E1L1 transitions, both accompanied by a broad shoulder to high energy. The E1H1 and E1L1 energies are higher in this (10,8) SL, than the (11,8) SL, due to the smaller GaAs layer width.

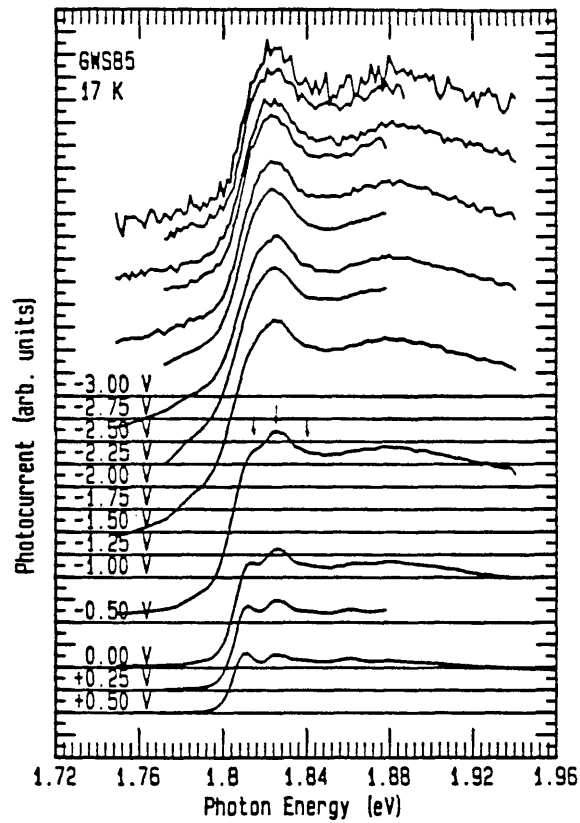


Fig. 7.8 : PC spectra measured on GWS85, a (11,8) GaAs/AlAs SL, at different applied biases, using a low-intensity light source and sample temperature of 17 K.

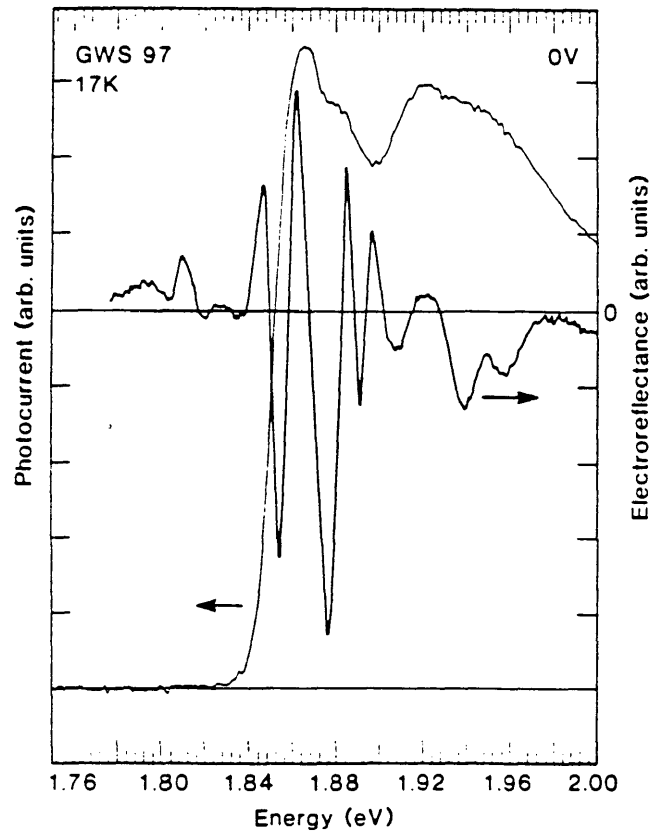


Fig. 7.9 : PC and ER spectra taken on GWS97, a (10,8) GaAs/AlAs SL, with an applied offset bias of 0 V, using a low-intensity light source. The sample temperature was 17 K and a 0.4 V_{ptp} modulation was used for ER.

The PC measurements on these SLs, using a low-intensity light source, were limited by the noise present in the spectra, caused by fluctuations in the leakage currents through the diodes. At -3 V the reverse leakage current was -6 μA for GWS85, while the photo-excited current was only about 4 nA. This noise obscures the features due to the oblique E1H1 Stark-ladder transitions at the higher biases in Fig. 7.8, where the leakage currents are large, while shoulders can only just be discerned at the moderate biases. The leakage currents were even larger in GWS97, thereby limiting the low-intensity PC measurements to a narrow range of reverse biases.

The signal-to-noise ratio can be greatly enhanced by using a more intense light source. PC measurements have also been made on these two SLs using a dye-laser light source and these are reported in Ref. 7.7. Fig. 7.10 show the dye-laser PC spectra taken on the (10,8) SL, with different applied biases. The PC excited by the dye-laser was roughly 1000 times larger than that produced in the low-intensity experiment described earlier. Notice that the E1H1($m = \pm 1$) transitions again appear as broad shoulders on the E1H1($m = 0$) peak, although these can be more clearly discerned than in the low-intensity PC spectra. The energies of the increasing ($m = +1$) and decreasing ($m = -1$) E1H1 Stark-ladder transitions were found to evolve linearly with field, at a rate in close agreement with Eq. 7.2, using the SL period (d) specified in the growth procedure, for both the (10,8) and (11,8) SL [7.7].

The dye-laser excited PC measurements on the (10,8) SL also revealed structure near 1.81 eV, due to transitions between electrons confined at the X-point and holes at the Γ -point, written here as X1H1. Under an electric field, the X1H1 feature split into a doublet due to the $m = +\frac{1}{2}$ and $m = -\frac{1}{2}$ Stark-ladder transitions, shown schematically in Fig. 7.7. It can be seen in Fig. 7.10 that the X1H1 features are much weaker than E1H1, as expected, appearing as very weak broad shoulders with slight excitonic character, on top of a rising edge due to the E1H1. The increasing ($m = +\frac{1}{2}$) and decreasing ($m = -\frac{1}{2}$) X1H1 Stark-ladder transitions were shown to evolve linearly with field in Ref. 7.7, again at a rate in good agreement with Eq. 7.2. Meanwhile the X1H1 transition was not resolved in the dye-laser PC study of the (11,8) SL, due to the proximity of the much larger E1H1 feature. However, the X1H1($m = \pm\frac{1}{2}$) transitions were observed in PL measurements made on both SLs, also reported in Ref. 7.7, and they were again found to evolve linearly with field at a rate in good agreement with Eq. 7.2.

Notice that the E1H1 PC feature has a very similar lineshape in the 0 V PC spectra shown in Figs. 7.9 and 7.11, taking using the low-

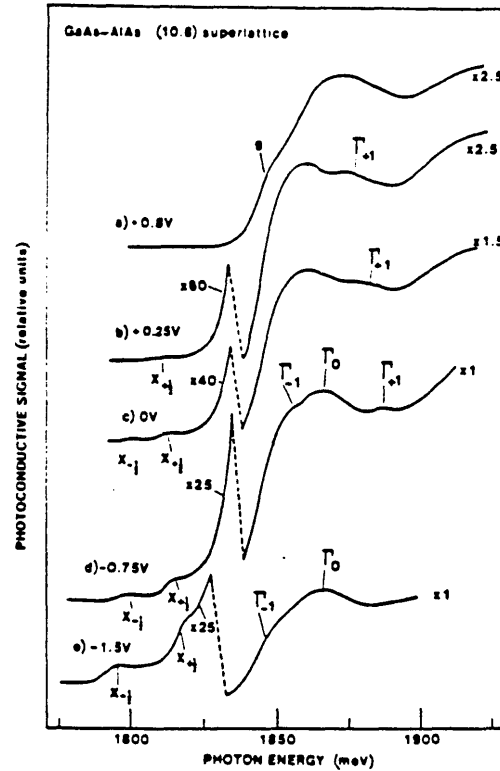


Fig. 7.10 : PC spectra (4 K) measured on GWS97, at different applied offset biases, using a dye-laser light source, reported by Whittaker et al [7.7].

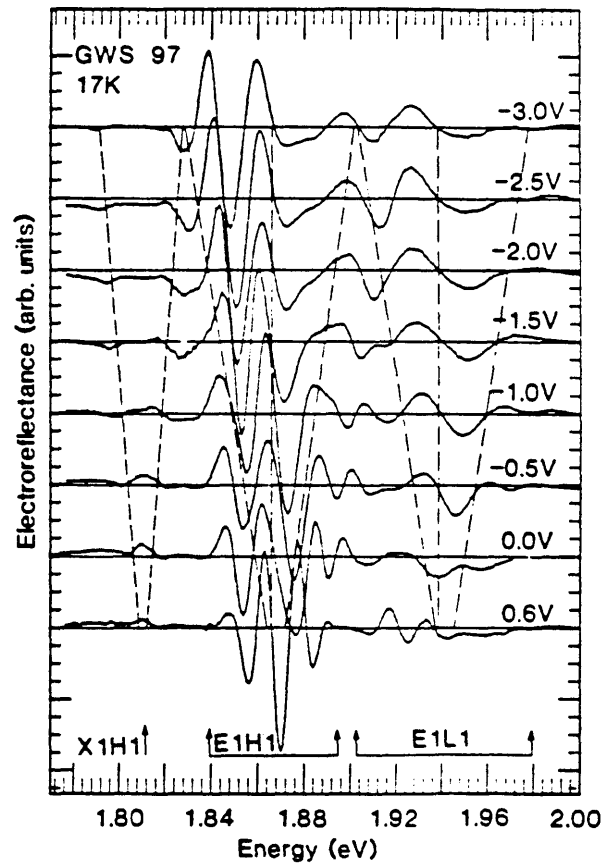


Fig. 7.11 : ER spectra taken on GWS97 at different applied offset biases, using a low-power light source. The light was incident at 45° , the temperature was 17 K and the modulating bias was fixed at 0.4 V_{ptp}. The dashed lines show the expected Stark-ladder behaviour for the E1H1, E1L1 and X1H1 transitions.

and high-intensity light sources respectively. (There is also a small energy shift between the two PC spectra, due to the average layer widths being slightly different in the two pieces of the wafer used for the experiments.) However, notice that the low-intensity PC spectrum does not resolve any structure at the energy of the X1H1 transition measured with the dye-laser light source. The X1H1 feature is at least two orders of magnitude weaker than E1H1 in the dye-laser PC measurements, making it too weak to be resolved above the noise present in the low-intensity PC experiment. The X1H1 features were also unresolved in the low-intensity PC spectra taken at other applied biases, even after several hours of signal averaging.

Fig. 7.9 also shows an ER spectrum taken on the (10,8) SL at a bias of 0 V, using the low-intensity light source. It can be seen that this ER spectrum does reveal structure due to the X1H1 transition, near 1.81 eV, despite no corresponding structure being resolved in the low-intensity PC spectrum. It was discussed in Sec. 7.1.2, that ER can be expected to have extra sensitivity to oblique ($m \neq 0$) Stark-ladder transitions, compared to those that are direct in real-space ($m = 0$), due to their large field-induced energy shift. Hence ER will enhance the strength of the X1H1($m = \pm \frac{1}{2}$) features allowing them to be observed using a low-intensity light source.

ER measurements are also unaffected by the noise associated with the leakage current in the sample, since the ER signal is detected in a separate Si photodiode. The ER spectra could therefore be collected over a wider voltage range than the low-intensity PC, with biases approaching the safe operating limit of the diode. The leakage current in these samples could be reversibly increased to as large as -50 μ A, when a significant electroluminescence was detected by the Si photodiode.

7.2.3 Electroreflectance Measurements

ER spectra measured on the (10,8) GaAs/AlAs SL (GWS97) are shown in Fig. 7.11. These were taken with different d.c. offset biases and a fixed voltage modulation of 0.4 V_{ptp}. The angle of incidence was 45°, and the temperature was again maintained at 17 K. Features can be seen around 1.87 and 1.94 eV, at low biases, which are due to the E1H1 and E1L1 minibands respectively. These energies are in good agreement with the centre of the corresponding features in the low-intensity PC spectrum of Fig. 7.9. A weaker feature can be seen around 1.81 eV in the ER spectra of Fig. 7.11, which is due to the X1H1 transition. The

X1H1 transition is now discussed in detail, before returning to the features associated with the E1H1 and E1L1 minibands.

Both the increasing X1H1($m = +\frac{1}{2}$) and decreasing X1H1($m = -\frac{1}{2}$) Stark-ladder transitions are clearly visible in Fig. 7.11, shifting in the expected linear manner, as indicated by the dashed lines. The inset of Fig. 7.12 shows an expanded view of the X1H1 transitions at a typical bias of -1 V. The X1H1($-\frac{1}{2}$) lineshape is taken to be a positive-zero-crossing, while the X1H1($+\frac{1}{2}$) is taken to be a negative-zero-crossing. This designation is based upon the observed symmetry of the lineshapes and is also consistent with the shape of the spectra at other biases. (Note that the two X1H1 lineshapes lie on a slowly varying background.) Notice that the two lineshapes are the inverse of one another, which is to be expected, since the electric field shifts the transitions in opposite energy directions. The strength of the $m = +\frac{1}{2}$ transition increases relative to the $m = -\frac{1}{2}$ with increasing field, due to its greater mixing with the Γ -electron state. Eventually, at the largest biases, the $m = +\frac{1}{2}$ transition becomes obscured by the downward moving E1H1 Stark-ladder transition.

Fig. 7.12 plots the energies of the zero-crossings, due to the X1H1 transitions in Fig. 7.11, as a function of the applied offset field. This demonstrates the expected linear Stark-ladder behaviour. The two solid lines are least squares fits to the experimental points, and their gradients, which are $-(25.1 \pm 0.7) \text{ \AA}$ and $+(24.3 \pm 0.7) \text{ \AA}$, are in excellent agreement with the half period of $25.5 \text{ \AA}$ specified in the growth procedure. The two lines also extrapolate to the same zero-field position, as expected, of $(1811.0 \pm 0.2) \text{ meV}$.

As discussed earlier, the X1H1($\pm\frac{1}{2}$) transitions in the (10,8) SL could only be resolved in PC spectroscopy when a more intense light source was used. The X1H1 features measured in the dye-laser excited PC resemble broad shoulders, with either no discernible or else only a very weak exciton peak. Meanwhile the ER spectra produce sharper structure, due to its differential nature, thus making the X1H1 transition energies easier to judge than in the PC. The PC spectra could be numerically differentiated to produce sharper structure, although this would introduce more noise than using the phase sensitive ER technique. ER is also fundamentally different from numerically differentiated non-modulation spectra, since the field-induced energy shift of each Stark-ladder transition is proportional to its order number, m .

Fig. 7.13 shows the ER spectra taken on the (11,8) GaAs/AlAs SL (GWS85) with different applied offset biases. These were collected using a modulating bias of 0.1 V_{ptp}. Structure due to the E1H1 and E1L1

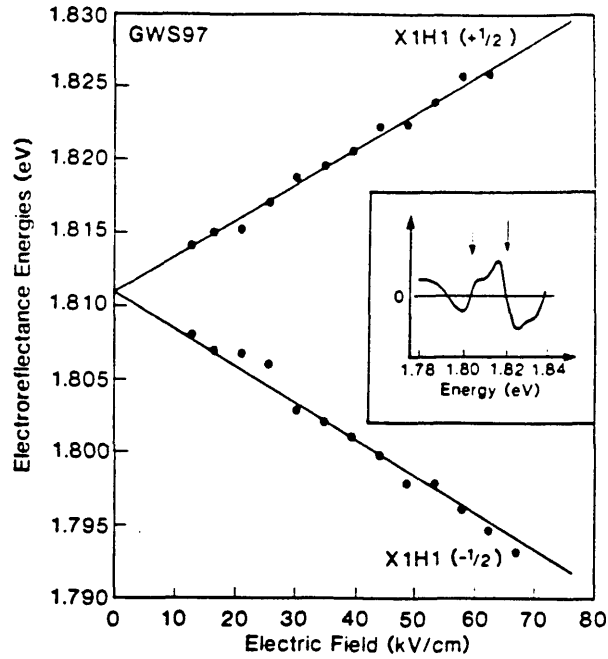


Fig. 7.12 : The points are the energies of the two X1H1 Stark-ladder transitions ($m = \pm \frac{1}{2}$) in Fig. 7.11. The transition energies were taken to correspond to zero crossings, as indicated in the -1 V spectrum shown as an inset. The solid lines are the best linear fit to the data.

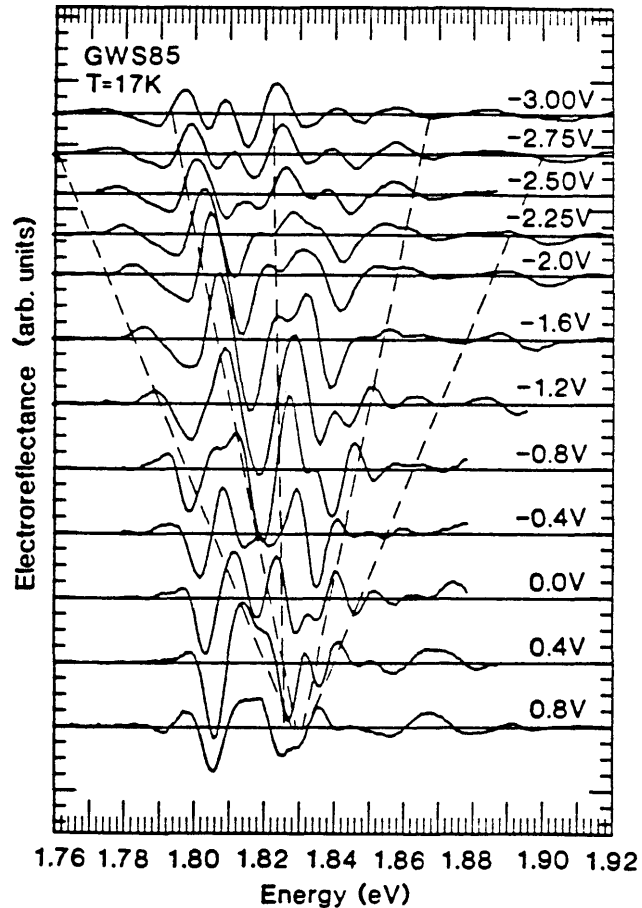


Fig. 7.13 : ER spectra taken on GWS85 under different applied offset biases, with a fixed modulation of 0.1 V_{ptp}, using a low-power light source. $\theta_0 = 45^\circ$. $T = 17$ K. The dashed lines show the expected Stark-ladder behaviour for the E1H1 transition.

transition can be resolved around 1.825 and 1.87 eV respectively. These energies are lower than for the (10,8) SL, due to the GaAs layer being one monolayer wider. The GaAs layer width has less effect upon the X1H1 transition energy and consequently the separation of the E1H1 and X1H1 is reduced in the (11,8) SL. The X1H1 feature is therefore obscured by the tail of the much stronger E1H1 feature in Fig. 7.13, but can just be resolved as weak structure in some of the lower bias spectra, for example near 1.791 eV in the 0 V. At larger biases the X1H1 feature becomes completely obscured by downward moving features from the E1H1 transition. The proximity of the E1H1 transition, also prevented the X1H1 transition being resolved in the dye-laser PC study reported in Ref. 7.7. However, as mentioned earlier, the X1H1 transition could be clearly followed in the PL spectra measured in Ref. 7.7, as this emphasises the transition across the lowest energy gap of the SL.

The ER spectra of both SLs show a great deal of structure over the regions of the E1H1 and E1L1 minibands, which is very sensitive to the applied bias. The ER spectra in Figs. 7.11 and 7.13 clearly show much more structure for the E1H1 and E1L1 transitions than the PC measurements of Figs. 7.8-10 and could therefore potentially yield more information on the SL bandstructure. However, this also makes the ER spectra more difficult to interpret. After a brief discussion of some theoretical considerations of ER of SLs, the ER features due to E1H1 and E1L1 transitions are discussed firstly for the (10,8) and then for the (11,8) SL.

The general features of the spectra can be compared to the simple model outlined in Sec. 7.1.2. However it should be remembered that this calculates only the real ($\Delta\epsilon_r$) and imaginary ($\Delta\epsilon_i$) parts of the field modulated SL dielectric function, while the ER will actually be a mixture of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ due to optical interference effects, as discussed in Sec. 3.3-4. Consequently it is not possible to predict if the Stark-ladder energies will correspond to peaks or zero crossings in the ER spectra. This uncertainty in the lineshape of each Stark-ladder transition makes ER spectra more difficult to interpret than spectroscopies such as PC, PL or PLE, where the position of a peak can simply be measured. This was less of a problem for the X1H1 transitions, where the two Stark-ladder transitions could be fully resolved, allowing their lineshapes to be judged from the symmetry of the spectra.

It was discussed in Sec. 3.4 that the mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ in the ER spectrum of a MQW changes across the spectrum, due to the change in

the optical penetration depth. This can also be expected to be important in the SL samples studied in this chapter. Therefore the mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ may be different for each Stark-ladder transitions and furthermore the mixture may change with bias. These effects however did not hamper the analysis of the X1H1 Stark-ladder transition that were taken to be zero crossings at all biases. In this region of the spectrum, the absorption coefficient is small and the light can be expected to penetrate to the back of the SL stack, so that the lineshape phase is determined by the midpoint of the stack. The lineshape phase would therefore not be expected to vary across the X1H1 region of the spectrum or with bias, making the spectra much easier to interpret.

The (10,8) SL was measured to have a minibandwidth of (40 ± 5) meV, for E1H1, in the dye-laser excited PC study [7.7]. This is in rough agreement with the extent of the E1H1 feature in the low-field ER spectra of Fig. 7.11, which can be expected to extend half a linewidth above and below the minibandedges, as can be seen in the calculated curves of Fig. 7.4. The quantity σ ($=\sigma_c+\sigma_v$) in the model of Sec. 7.1.2 can therefore be taken to have a value $\sigma = 10$ meV. The experimental spectra can be compared to the model calculation of Fig. 7.4, after converting the experimental field (F) to the dimensionless quantity $f = e.F.d / \sigma$. The largest applied reverse bias -3 V (72 kV/cm), corresponds to $f = 3.7$ for this SL, where the SL bandstructure should be in the high-field ("Stark-ladder") regime discussed in Sec. 7.1.2. The E1L1 feature covers a wider energy range in the +0.6 V spectrum, suggesting a larger minibandwidth. This is to be expected since the lowest light hole miniband will have a width comparable to that of the electron, while that of the heavy hole will be negligibly small.

To aid the interpretation of the ER spectra, the expected energies of the different Stark-ladder transitions have been marked by the lines running between the spectra in Fig. 7.11. The gradient of the oblique Stark-ladder lines was taken to equal an exact integer multiple of the period, as given by Eq. 7.2. The zero-field energies were determined from the low-intensity PC measurements.

Consider the -2.5 V ER spectrum of the (10,8) SL in Fig. 7.11. A bias of -2.5 V corresponds to a field of 63 kV/cm, or $f = 3.2$ expressed in dimensionless units. Comparison of the experimental spectrum with the calculated curves in Fig. 7.4 shows there to be a poor agreement. It can be seen that the E1H1 feature of the experimental ER signal is concentrated between the expected E1H1(0) and E1H1(-1) energies. The E1L1 feature also appears to be largest between the expected $m = 0$ and $m = -1$ energies. The reason for the discrepancy between the measured ER

and the curves in Fig. 7.4 is not clear, although it may be due to a change in the lineshape phase across the spectrum, as discussed earlier. Note that the ER signal is concentrated within the spectral region where the change in the absorption coefficient will be largest, ie between $m = 0$ and $m = -1$. The large lobes in the -2.5 V spectrum may therefore be related to a modulation of the light penetration depth.

The zero crossings of the ER spectra taken on the (10,8) SL, including those shown in Fig. 7.11, are plotted in Fig. 7.14, as a function of the applied field. It was discussed earlier that each individual Stark-ladder transition energy could correspond to a positive peak, a negative peak, or a zero crossing in the ER, and is most likely to be intermediate between two of these lineshapes. It is therefore an arbitrary decision to plot all the zero crossings, although a plot of the peak positions did not look qualitatively different.

It can be seen that some of the zero crossings plotted in Fig. 7.14 evolve with field at a rate close to that expected for the Stark-ladder transitions. For instance, a positive zero crossing evolves to lower energy close to the expected $E_{1H1}(-1)$ Stark-ladder line for fields greater than about 50 kV/cm. Similarly a line of positive zero crossings can be seen evolving close to the expected $E_{1L1}(+1)$ ladder line, at fields greater than about 25 kV/cm. Meanwhile the expected $m = 0$ energy appears close to a negative zero crossing for E_{1H1} , at all fields, and for E_{1L1} , at fields in excess of about 25 kV/cm. Notice also that there is a complicated evolution of the zero crossings in the spectral region between the expected $E_{1H1}(0)$ and $E_{1L1}(0)$ energies.

The ER spectra taken on the (11,8) SL, shown in Fig. 7.13, display even more detail for the Γ -point transitions. Again the expected energies of the Stark-ladder transitions, inferred from the PC measurements, are drawn on Fig. 7.13. Concentrating on the -2 V spectrum, it can again be seen that, the ER is strongest between the expected $E_{1H1}(-1)$ and $E_{1H1}(0)$ energies, as was also the case for the (10,8) SL. Notice that a negative peak appears close to the expected $E_{1H1}(0)$ energy, at reverse biases larger than -2 V. This is likely to be due to the $E_{1L1}(-1)$ transition, as supported by its evolution to lower energy with increasing field. The increase in the strength of this feature with field, and the decline in the $E_{1H1}(0)$ feature, suggests a hybridisation of the H1 and L1 envelope functions in adjacent wells at this field. This mixing was not observed for the (10,8) SL, because the L1 - H1 energy separation was larger in the narrow wells.

Fig. 7.15 plots the energies of all the zero crossings in the ER spectra taken on the (11,8) SL, along with the expected Stark-ladder

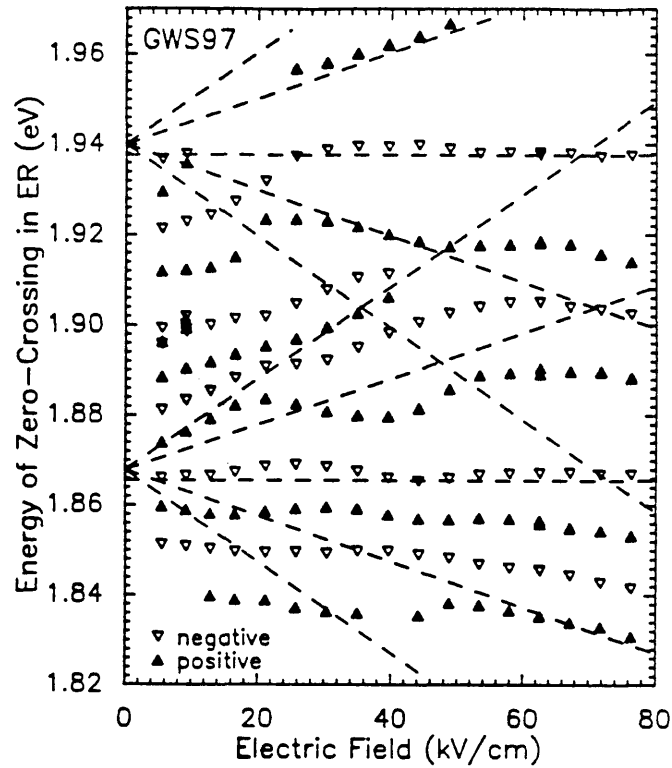


Fig. 7.14 : The positive- ($\blacktriangle$) and negative- (∇) zero-crossings measured in the field-dependent ER spectra of GWS97 (Fig. 7.11). The distinction "positive" or "negative" refers to the gradient of the ER against photon energy at the zero-crossing.

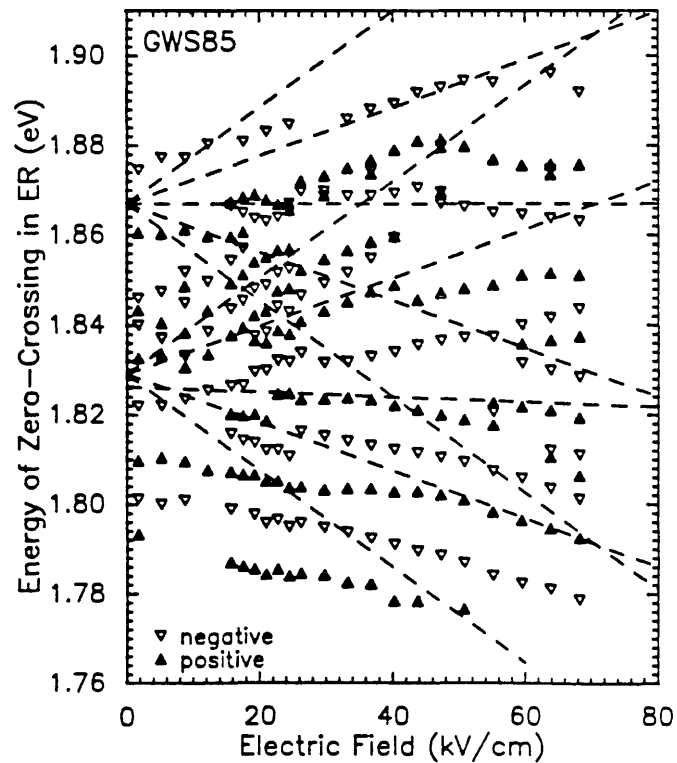


Fig. 7.15 : The zero-crossings measured in the field-dependent ER spectra of GWS85 (Fig. 7.13).

evolution. Again positive zero crossings can be seen evolving at a rate close to that expected for the $E1H1(-1)$ transition, for fields greater than about 50 kV/cm. Negative zero crossings can be seen to evolve close to the expected $E1L1(+1)$ energy at fields greater than roughly 30 kV/cm. Notice that there is again a very complicated evolution in the energy range between the $E1H1(0)$ and $E1L1(0)$ expected energies. At low fields, zero crossings can be seen (near 1.83-1.86 eV) to evolve at rates close to that expected for the $m = +1, +2$ and $+3$ Stark-ladder transitions. The very detailed structure of the spectra in this region may derive from a mixing of the $H1$ and $L1$ envelope functions. This will produce a more delocalised heavy hole envelope function and thereby enhance the oblique Stark-ladder transitions. Notice that at the highest biases the $E1L1(-1)$ transition is expected to be almost degenerate with $E1H1(0)$, which produces the previously mentioned structure in the ER spectra. This is also manifested by the negative zero crossings seen around 1.83 eV for the largest fields in Fig. 7.15, which evolves to a lower energy close to the $E1L1(-1)$ line.

7.3 Measurements on a (6,6) GaAs/AlAs Superlattice

The sample studied in this section is a GaAs/AlAs SL with very narrow wells and barriers that are just 6 monolayers wide. This shows qualitatively different behaviour to that observed for the (10,8) and (11,8) SL in Sec. 7.2.

7.3.1 Kronig-Penney Calculations

The zero-field transition energies of this SL were calculated with a simple effective mass Kronig-Penney model, using the parameters listed in table 7.1, which ignored excitonic effects and the strain shifts caused by the layer mismatch. Non-parabolicity of the Γ -point valleys of the conduction and light hole valence bands, due to their mutual interaction, was considered in the manner described by Nelson et al [7.21], but the other valleys were assumed to be parabolic. It is stressed that this is not expected to give an accurate description of the band non-parabolicity at energies considerably higher than the band minima. Hence the calculation can only be regarded as approximate for this SL, where the confinement energies will be large.

The Kronig-Penney model calculated the $E1H1$ miniband to extend from 2.011 to 2.100 eV, corresponding to a minibandwidth of 89 meV.

Meanwhile, the E1L1 miniband edges were calculated to lie at 2.111 to 2.308 eV respectively. At the largest reverse bias studied here (-13 V) the potential drop across one period is just 22 meV. Since this is much less than the calculated E1H1 and E1L1 minibandwidth, the (6,6) SL is expected to be in the low-field limit discussed in Sec. 7.1.2, where the spectral features are concentrated at the minibandedges.

The X_z 1H1 miniband was calculated to extend from 1.884 to 1.887 eV for the (6,6) structure, while X_{xy} 1H1 covers 1.947 to 2.026 eV. After allowing for the lattice mismatch strain, the X_z will still be lower than X_{xy} in this SL, which is in agreement with the measurements of van Kesteren et al [7.19] on this sample and by Scalbert et al [7.20] on other similar SLs.

7.3.2 Experimental Details

Sample G485 is a (6,6) GaAs/AlAs SL, grown by MBE at the Philips Research Laboratory (Redhill). The layer specification consisted of 1.01 μ m GaAs buffer layers; 206 repeats of (6,6) GaAs/AlAs SL; and finally 0.1 μ m GaAs cap. The substrate was n^+ GaAs, cleaved for growth along the [001] crystallographic direction and none of the epilayers were intentionally doped.

The sample was prepared into cylindrical mesas with diameters between 0.5 and 2.0 mm. Semitransparent Gold Schottky barriers were evaporated on the top surfaces of the mesas, while ohmic contacts were made to the substrates. The SL could therefore be subjected to a variable electric field (F) by application of a bias (V), whose value was calculated using the ideal relation, $F = (V_{bi} - V)/t_i$. The built-in voltage (V_{bi}) was taken to be +0.9 V, the value reported for a Gold Schottky barrier on GaAs [3.36], while the total intrinsic region thickness (t_i) was taken as that specified in the growth procedure. The mesas could withstand maximum reverse voltages of between -10 and -13 V, at 17 K, allowing a maximum field of about 60 to 70 kV/cm to be applied.

Electric field-dependent PC and ER spectra were collected on the SL using the arrangement described in Chp. 4, with the samples mounted on the cold-finger of a closed-cycle Helium cryostat maintained at 17 K. The ER was collected using a fixed voltage modulation of either 1.0 or 2.0 V_{ptp}, around a variable offset bias. The angle of incidence was set to be either 17° or 45°.

7.3.3 Photocurrent Measurements

Moore et al [7.17] have made PL and PLE measurements on a (6,6) GaAs/AlAs SL similar to G485. This sample was prepared in the same MBE reactor as G485 and had an identical structure, except that it was grown on a semi-insulating substrate. Their 5 K PL and PLE spectra are shown in Fig. 7.16. The PL shows a large peak at 1.899 eV, which they ascribe to recombination of Γ -point heavy holes and X_2 -point electrons. This transition is also seen in the excitation spectrum as the shoulder near 1.926 eV. Moore et al explained the apparent 27 meV Stokes shift between the PL and PLE, by localisation of the recombining carriers in wider regions of the potential wells, which was supported by the temperature dependence of the spectra. Scanning to higher energy reveals the E1H1 transition as a strong rising edge in the PLE, whose midpoint lies near 2.06 eV. A separate feature due to the E1L1 transition was not resolved, which they suggested was due to the large linewidths produced by well width fluctuations in such narrow QWs. A GaAs/AlAs (8,8) SL did show distinct E1H1 and E1L1 features.

Fig. 7.17 shows a 17 K PC spectrum taken by myself on G485, the (6,6) GaAs/AlAs SL, under an applied bias of 0 V. The sharp peak at 1.5165 eV and the background to higher energy derive from the bulk GaAs layer inside the intrinsic region. A second sharp rising edge can also be seen, whose midpoint lies at 2.013 eV, which is due to the E1H1 transition of the SL. This E1H1 PC edge is lower in energy than the corresponding feature measured in the PLE spectrum by Moore et al, shown in Fig. 7.16. The discrepancy can be explained by a variation in the average GaAs layer width, between the two samples studied, of less than a monolayer. The value measured in the present study is also in good agreement with the bottom of the E1H1 miniband, obtained from the simple Kronig-Penney calculation, at 2.011 eV. As in the PLE spectrum, a separate feature due to the E1L1 transition is not resolved in PC.

The Kronig-Penney calculation and the PLE measurements predict the transitions to the X-point electron states to lie to lower energy than E1H1. This region of PC spectrum shows very weak peaks at approximately 1.567, 1.725, and 1.866 eV. However this tiny ripple on the bulk PC signal is likely to be due to Fabry-Perot etalon effects between light reflected from the front and back surface of the GaAs buffer layers. The period of the ripple implies a Fabry-Perot cavity of 1.1 μm thickness, in good agreement with the width of the GaAs buffer layer. The X-point transitions of the SL are expected to be weak, which makes them difficult to resolve in PC spectroscopy, over the ripples from the

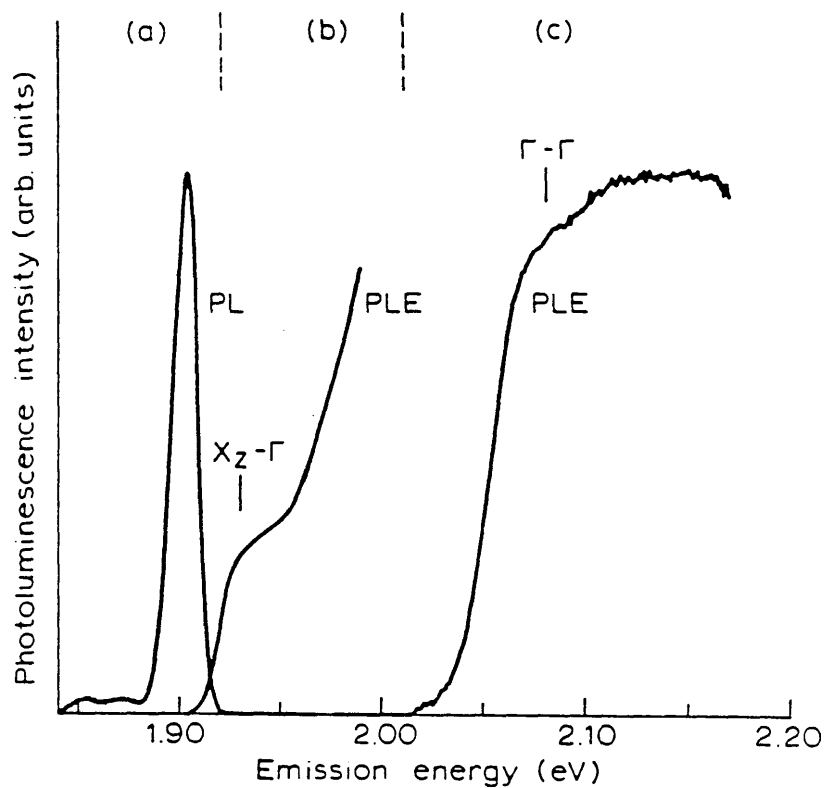


Fig. 7.16 : Zero-field PL and PLE spectra measured on a (6,6) GaAs/AlAs SL by Moore et al [7.17]. Note that the part of the PLE spectrum plotted in region (b) has been magnified. $T \approx 5$ K.

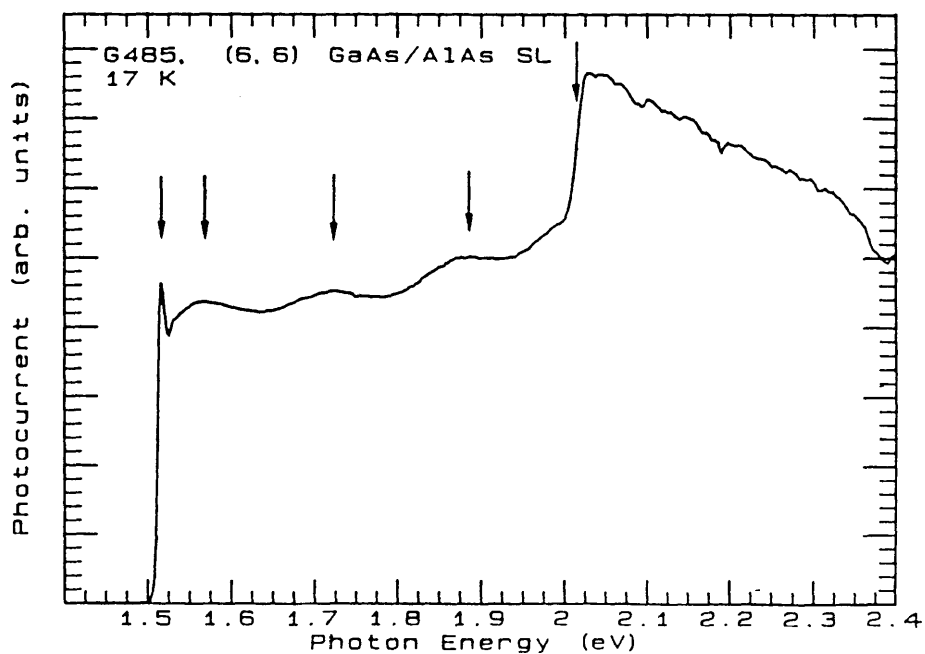


Fig. 7.17 : PC spectrum taken on sample G485, a (6,6) GaAs/AlAs SL. The offset bias was 0 V and the sample temperature was maintained at 17 K.

bulk GaAs layers.

When the reverse bias was increased the E1H1 feature of the SL in the PC spectra did not change significantly. However, the PC signal below the E1H1 bandedge increased quite abruptly around -1.5 V, presumably because the carriers photo-excited in the buffer layers could then tunnel through the SL. At low biases, there was also a change in the phase of the PC signal detected by the lock-in, of about 50° , for excitation energies above and below the E1H1 SL bandedge. Again this is probably related to tunnelling of the photo-excited carriers through the SL.

7.3.4 Electroreflectance Measurements

ER spectra have also been taken on the (6,6) SL at different applied biases and angles of incidence. The different spectral regions of the ER are now discussed separately.

Fig. 7.18 shows the ER spectra recorded between photon energies of 1.55 and 1.8 eV, at reverse offset biases of -5, -9 and -13 V, with a fixed modulating voltage of 1 V_{ptp}. The features in Fig. 7.18 are due to the Franz-Keldysh oscillations of the bulk GaAs buffer layer. The period of the oscillations can be seen to increase with reverse bias, as expected, due to the increase in the offset field. These spectra again show the beating of the Franz-Keldysh oscillations discussed in Sec. 3.5, due to the lower field of the modulation being non-zero.

The positions of the zero crossings in the spectra of Fig. 7.18 have been compared to those predicted by the lineshape model in Sec. 3.5. This model allows for the effect of the contributing layer being buried in the structure, which results in the ER being a mixture of the real and imaginary parts of the bulk semiconductor modulated dielectric function, as discussed in Sec. 3.5. The average depth of the contributing region was taken to be 1.6 μm , corresponding to the midpoint of the buffer region. The analysis yields offset fields of 26.6, 41.5 and 64.2 kV/cm for the -5, -9 and -13 V spectra respectively. These values are in good agreement with the fields deduced from the applied biases, using the ideal relation in Sec. 7.3.2, of 28.1, 47.2 and 66.3 kV/cm respectively. This shows that the field in the i-region is close to that expected from the applied bias.

The ER recorded in the spectral region of the Γ -point direct edge of the SL shows much more detail than the PC. Fig. 7.19 plots two 17 K ER spectra, taken with the light incident at either 17° and 45° . The

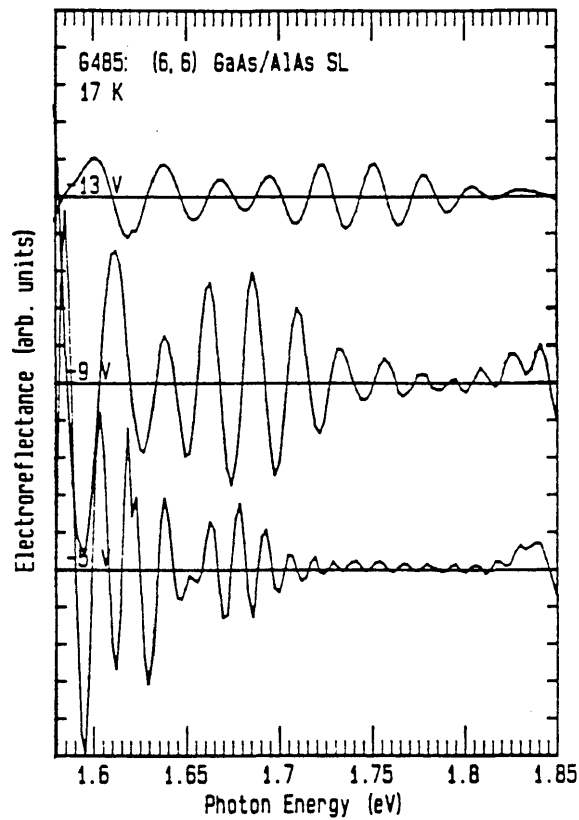


Fig. 7.18 : ER spectra taken on G485, with offset biases of -5, -9 and -13 V and a fixed modulation of 1 V_{ptp}. The sample temperature was 17 K and the angle of incidence was 17°. This spectral region shows the Franz-Keldysh oscillations deriving from the fundamental bandgap of the bulk GaAs buffer layer.

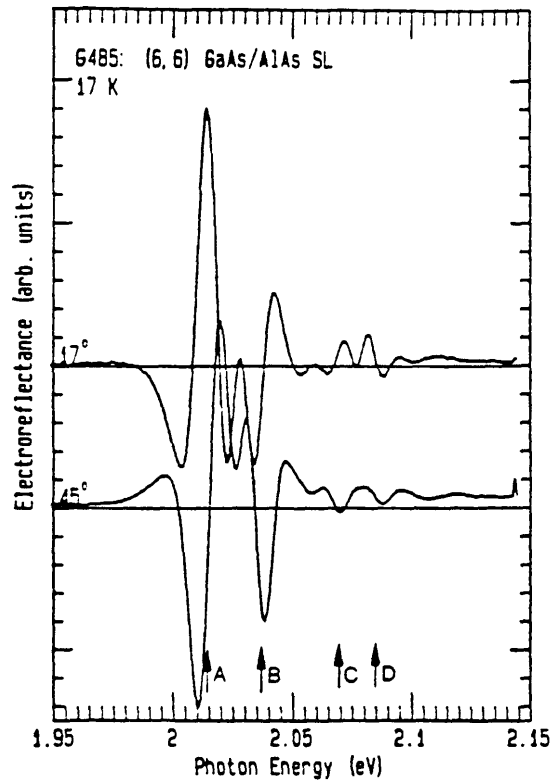


Fig. 7.19 : ER spectra taken on G485 at two different angles of incidence; 17 and 45°. This spectral range shows the Γ -point transitions of the (6,6) SL. The offset bias was 0.0 V, the modulation was 1.0 V_{ptp} and the sample temperature was 17 K.

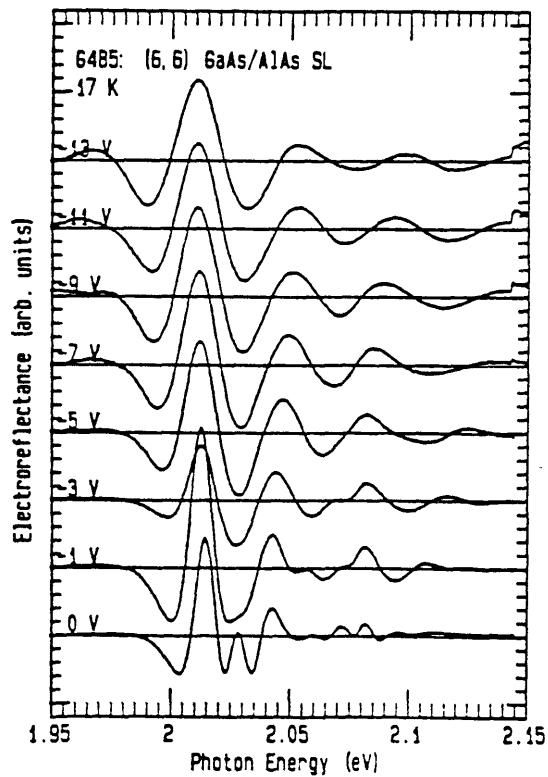


Fig. 7.20 : ER spectra taken on G485 at different offset biases between 0 and -13 V, showing the Γ -point transitions of the (6,6) SL. The modulating bias was fixed at 1.0 V_{ptp}, the angle of incidence was 17° and the temperature was 17 K.

applied offset bias is 0 V and the modulation is 1.0 V_{ptp} for both spectra. The ER spectra show two large features at 2.014 and 2.037 eV, labelled "A" and "B" respectively in Fig. 7.19, which are ascribed to the E1H1 transition of the SL. The energy of feature A is in close agreement with the midpoint of the rising edge measured in the 0 V PC spectra, and also the calculated energy of the bottom of the E1H1 miniband, obtained from the simple Kronig-Penney model. The Stark-ladder separation (e.F.d) is expected to be just 1.7 meV at this field, so that the ER spectrum can be expected to only show features at the minibandedges, due to the very large minibandwidth. The origin of the observed splitting of E1H1, into features A and B is unclear, although possible explanations are discussed later.

Two weaker features, labelled "C" and "D" in Fig. 7.19, can be seen to higher energy, near 2.069 and 2.085 eV. These features are tentatively ascribed to the E1L1 transition of the SL, which was not resolved in the PC or PLE spectra. The Kronig-Penney calculation places the bottom of the E1L1 miniband to higher energy, at 2.111 eV, although it should be remembered that this is not expected to be very accurate, due to the simplistic treatment of the effective mass. Features C and D could alternatively be due to the top of the E1H1 miniband, although the Kronig-Penney calculation also places this to a much higher energy.

The evolution of the ER lineshape with angle of incidence can be used to determine the depth of the layers producing each feature in the spectra of Fig. 7.19. In Sec. 5.2.1, the angular evolution of the ER lineshape of two SQWs, was used to determine the thickness of the overlayer between the well and front surface of the sample. In Sec. 6.2, the angular evolution was used to determine that the splitting observed in the ER spectra derived from wells of different width at the front and back of the stack.

Feature A in Fig. 7.19 has a symmetric lineshape, with a positive peak at the central energy, in the 17° spectra, while B resembles an anti-symmetric zero-crossing lineshape. When the angle of incidence was changed to be 45°, these lineshapes were seen to evolve, so that A resembled a zero-crossing, while B looked like a negative-peak, see Fig. 7.19. The lineshapes of features C and D are also seen to evolve with the angle of incidence, although it is difficult to be as precise about the form of the lineshapes, as they are not so well resolved, and also appear to lie on a broad background.

Notice that the lineshape phases of features A and B change by roughly the same amount, suggesting that they derive from the same depth inside the sample. Hence the splitting of the E1H1 feature can not be

due to a variation in either the well width or electric field across the stack, as was the case for the MQWs in Sec. 6.2. Likewise the splitting cannot be caused by optical interference effects, like those discussed in Sec. 3.4, as then the split features derive from the front and the back of the MQW stack. The change in the lineshape phase of roughly $\frac{1}{2}\pi$, suggests the ER to originate from an average depth of about $1.3\ \mu\text{m}$ inside the sample, which is rather larger than expected, since the midpoint of the SL lies only $0.6\ \mu\text{m}$ below the surface.

The origin of the splitting in the E1H1 and E1L1 features can therefore not be explained by optical interference effects or variations in the well width or electric field across the stack. Intra-well monolayer changes in the well width can also be discounted as these produce a change in the E1H1 energy of over 70 meV, much larger than that observed here. Features B and D are also unlikely to be due to the top of the E1H1 and E1L1 minibands, as this would suggest minibandwidths much smaller than those calculated.

Fig. 7.20 shows ER spectra taken at an angle of incidence of 17° , with a fixed modulation of 1.0 V_{ptp} and different d.c. offset biases of between 0 and -13 V. It can be seen that the width of the features increases with reverse bias, so that the splitting of features A and B becomes difficult to resolve at -1 V. The same is true for features C and D. At the larger reverse biases, oscillations are seen covering a wide spectral range in Fig. 7.20, whose amplitude decays with increasing energy. These high reverse bias spectra are reminiscent of the Franz-Keldysh oscillations observed for bulk semiconductors. It can be seen that the period of the oscillations in Fig. 7.20 increases with the applied reverse bias, as expected for Franz-Keldysh behaviour.

It is not surprising that the ER of this SL should resemble that of bulk material, considering the very large minibandwidth. The SL can be considered as having a similar bandstructure to bulk material, but with a modified effective mass for motion in the direction of the SL stratification. It should in principle be possible to estimate the effective mass along the direction of stratification of the SL from the period of these oscillations. However, this analysis has not been attempted here, as it must take account of the large broadening parameter of the SL transitions and also the Coulombic interaction at these energies near the bandedge, both of which were ignored in Sec. 3.5. Alternatively, interface roughness could also result in the (6,6) SL bandstructure resembling that of bulk $\text{Al}_{0.5}\text{Ga}_{0.5}\text{As}$ alloy.

For all the spectra in Fig. 7.20, changing the angle of incidence to be 45° , caused a rotation of the entire lineshape of approximately

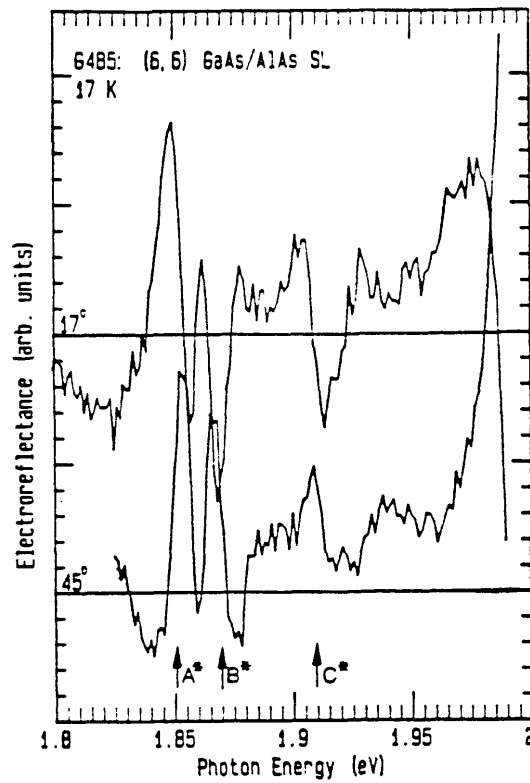


Fig. 7.21 : ER spectra taken on G485 with the light incident at either 17° or 45° , showing features due to transitions to the X-point of the SL conduction band. The offset bias was 0.0 V, the modulation was 2.0 V_{ptp}

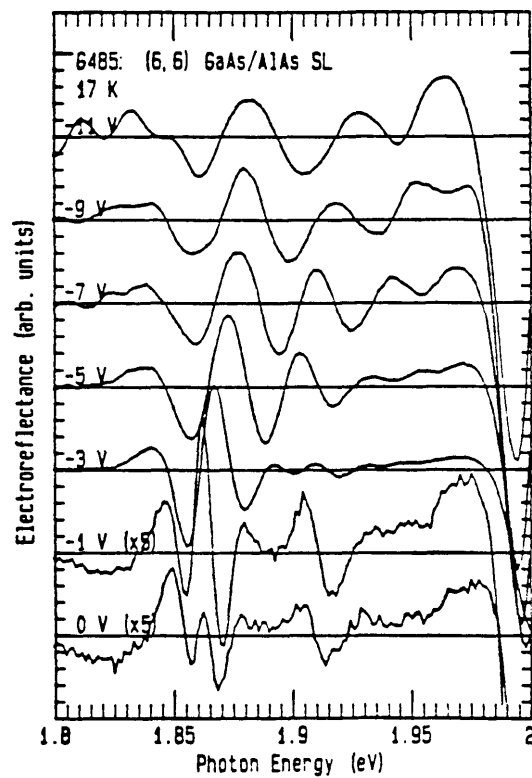


Fig. 7.22 : ER spectra taken on G485 at different offset biases between 0 and -11 V, in the region of the SL X-point transitions. Note that the 0 and -1 V spectra have been scaled by a factor of 5. The modulating bias was fixed at 2.0 V_{ptp}, the angle of incidence was 17° and the temperature was 17 K.

$\frac{1}{2}\pi$, similar to that shown in Fig. 7.19 for the 0 V spectrum. Hence the minima and maxima in the spectra taken at one angle of incidence, approximately corresponded to zero crossings at the other angle of incidence. This again shows that the SL stack is homogeneous in its optical response, and the observed features do not derive from different regions of the stack.

ER spectra were also taken over the region of the spectrum expected to contain the transitions to the X-point of the SL conduction band. Fig. 7.21 shows two 17 K ER spectra taken with the light incident at either 17° or 45° . The offset bias was 0 V and the modulation was 2.0 V_{ptp}. The spectra show three features at approximately 1.851, 1.869 and 1.909 eV, labelled "A*", "B*" and "C*" in Fig. 7.21, which lie on top of a broad oscillatory background due to the tail of the E1H1 feature to higher energy. These features are roughly 20 times weaker than those in Fig. 7.19 due to E1H1, despite the modulating bias being twice as large.

The two lower energy features (A* and B*) appear as a closely spaced doublet, as was also the case for the features labelled A and B in Fig. 7.19. The separation of A* and B* is (18 ± 4) meV, compared to 24 meV for A and B, although this is difficult to judge accurately without lineshape fitting. Again there is a change in the phase of the entire lineshape, of about $\frac{1}{2}\pi$, between the spectra taken at the two angles of incidence in Fig. 7.21, so that the peaks in one spectrum approximately coincide with the zero crossings of the other. (Note that the lineshapes are complicated by the broad background oscillations due to the E1H1 transition, which also rotate when the angle of incidence is changed.) The fact that the lineshape phases of A* and B* differ by a similar amount between the two spectra in Fig. 7.21, again suggests that they originate from the same average depth inside the sample structure.

Features A* and B* in Fig. 7.21 are tentatively ascribed to the X₂1H1 transition of the SL. The Kronig-Penney model calculates the X₂1H1 miniband around 1.885 eV, in approximate agreement with the position of features A* and B*. A more detailed calculation by Munoz et al [7.22] places this transition at 1.87 eV. However it should be remembered that there is an uncertainty in the X-point transition energies of as much as ± 0.070 , due to the uncertainty in the value of the discontinuity ratio. The X₂1H1 transition energy measured in the PLE spectra of a similar (6,6) SL, shown in Fig. 7.16, was significantly higher. As this was also the case for the E1H1 transition, this is likely to be due to the average layer width being slightly larger in the

sample studied here. The origin of the splitting into A^* and B^* , is again not clear, but is presumably related to the similar splitting of the E_{1H1} transition into A and B.

The feature at 1.909 eV in Fig. 7.21, labelled C^* , could be due to the $X_{xy}1H1$ transition. The Kronig-Penney calculation predicts this miniband to extend from 1.946 to 2.003 eV. Lattice mismatch strain will lower these energies by 16-23 meV [7.19-20]. However, feature C^* is perhaps more likely to be due to the X_z1L1 transition. This is supported by the separation of C^* and A^* in Fig. 7.21 being 58 meV, similar to the 55 meV separation of C and A in Fig. 7.19. A splitting similar to that observed for E_{1L1} (into C and D) was not resolved for feature C^* , although it does appear to be broader than A^* .

The bias dependence of the ER taken in this spectral region is shown in Fig. 7.22. These were taken with the light incident at 17° . The modulation was again fixed at 2.0 V_{ptp} and the d.c. offset bias was varied between 0 and -11 V. As for the Γ -point spectra in Fig. 7.20, the width of the features increases with bias, so that the splitting of features A^* and B^* soon becomes unresolved. Again broad oscillations are seen over a wide spectral range, at the higher biases, reminiscent of the Franz-Keldysh lineshapes of bulk semiconductors. The Franz-Keldysh oscillations originating from the buffer layers can also be seen at the lowest photon energies, and largest reverse biases, shown in Fig. 7.22. Changing the angle of incidence to be 45° , at any of the biases in Fig. 7.22, again rotated the entire lineshape by approximately $\frac{1}{2}\pi$, as demonstrated for the 0 V spectrum in Fig. 7.21.

It is surprising that the ER of the E_z1H1 transitions should resemble Franz-Keldysh oscillations at the higher biases, considering that X_z1H1 was calculated to have a minibandwidth of only 3 meV. This transition would therefore be expected to exhibit Stark-ladder behaviour like that observed for sample GWS97 in Sec. 7.2. The presence of oscillations over a wide energy range, suggests the minibandwidth to be larger than that calculated by the simple Kronig-Penney model.

It is also surprising that the period of the oscillations in Fig. 7.22 is similar to that for the E_{1H1} transitions in Fig. 7.20. For instance the -11 V spectrum has an average period of (44 ± 4) meV in Fig. 7.22, while it is (42 ± 3) meV in Fig. 7.20. (These values should be regarded as the average period at photon energies near the bandedge, since the period of Franz-Keldysh oscillations is dependent on the photon energy.) The Franz-Keldysh effect predicts that the period of the oscillations is rather insensitive to the electron-hole reduced mass (μ), depending on $\mu^{-1/3}$, however, the very large difference in the Γ -

and X-point conduction band effective mass should still produce quite different periods. Using the effective masses listed in table 7.1, predicts the period of the $X_{\text{z}}1\text{H}1$ oscillations to be 1.66 times smaller than that for $\text{E}1\text{H}1$. Meanwhile for the $X_{\text{xy}}1\text{H}1$ miniband, the period of the oscillations would still be expected to be 1.28 times smaller than $\text{E}1\text{H}1$.

These two points suggest that the oscillations seen in the high bias spectra of Fig. 7.22 do not derive from the $X_{\text{z}}1\text{H}1$ transition of the SL. It is also possible that these oscillations are caused by the transition between the spin-orbit split-off and conduction band of the bulk GaAs buffer layer, which is expected to occur near 1.858 eV. (ER features due to the corresponding transition of the SL were also observed, centred at 2.34 eV.) This would produce Franz-Keldysh oscillations over a wide energy range, whose period would be similar to that of the $\text{E}1\text{H}1$ transition of the SL. However, this transition is unlikely to be the cause of the split features (A^* , B^* , C^*) seen at low biases, which therefore probably derive from the X-point transitions of the SL, as discussed above.

7.4 Conclusions

Low temperature field-dependent PC and ER measurements have been reported on three GaAs/AlAs SLs in this chapter, which were (10,8), (11,8) and (6,6) structures. In each case the ER spectra were found to reveal more detailed structure than the PC. However, this structure was often so detailed as to make the ER spectra difficult to interpret. A simple model of the field modulated dielectric function ($\Delta\epsilon_r + i \Delta\epsilon_i$) of a SL was also presented and compared to the ER spectra where appropriate.

The simple model presented here is a modification of the nearest neighbour tight binding calculation of Bleuse et al [7.1]. The SL dielectric function was considered as consisting of contributions due to the ground excitonic state of each Stark-ladder transition. The energy and strength of each exciton was assumed to be determined purely by the solution of the z-motion, given by the nearest neighbour tight binding calculation. The contribution of each Stark-ladder transition considered here is very simplistic. This limitation, as well as the other approximations employed in the model, were discussed in Sec. 7.1.2. However, the simplistic approach has the advantage of clearly elucidating the SL behaviour alone, ie the interaction of neighbouring

wells.

The ER measurements on the (10,8) and (11,8) SLs revealed detailed structure for the Γ -point interband transitions (E1H1 and E1L1) that was very sensitive to the applied field. The zero crossings measured in these spectra were plotted as a function of the applied field, and some of these were found to evolve at a rate close to that expected for Stark-ladder behaviour. However, a detailed comparison between the measured lineshapes and the calculated modulated dielectric function showed the agreement to be poor. This was disappointing, considering that the Stark-ladder behaviour of the E1H1 transition had been carefully characterised, by a separate PC study of the SLs using a dye-laser light source.

It was suggested that the ER spectra are complicated by optical interference effects, which result in the ER signal being determined by a linear mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, depending upon the optical penetration depth below the front surface of the sample. The optical penetration depth will change dramatically with photon energy, around the Γ -point bandgap of the SLs. This will produce a change in the mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ across the ER spectrum, like that discussed for MQW samples in Sec. 3.4, and consequently produce complicated lineshapes. Furthermore, the absorption coefficient for any Stark-ladder transition, and hence the mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, will also change with bias. A detailed analysis of the optical interference effects has not been attempted here, as it will require detailed knowledge of the SL absorption profile.

The field modulated dielectric function of SLs would be better studied using ET spectroscopy. The ET of multiple well samples is determined essentially by $\Delta\epsilon_i$ alone, and so will not be affected by the interference effects complicating the ER. These ET measurements would retain the extra sensitivity to the oblique Stark-ladder transitions, discussed in Sec. 7.1.2.

It may also be the case that the GaAs/AlAs SLs samples studied here are not ideal for observing Stark-ladder behaviour, since the oblique transitions could only just be resolved in the dye-laser PC spectra of the parallel study in Ref. 7.7. Previously reported measurements on GaAs/Al_xGa_{1-x}As SLs, where $x = 0.35$ [7.3] or $x = 0.3$ [7.4], produced better resolved oblique Stark-ladder features. The larger linewidths of the GaAs/AlAs SLs studied here is probably due to the narrower well width, which enhances the broadening due to interface roughness. The leakage currents in these samples were also quite large, which limited the maximum field that could be applied. The complicated structure observed for E1H1 and E1L1 in ER, could perhaps be more

readily interpreted in samples with a narrower linewidth, that could withstand larger applied fields.

The GaAs/AlAs SLs of the present study, had the compensating attraction of allowing the transitions between the Γ -point of the valence and X-point of the conduction band to be observed, since each SL was type-II. The X-point lies to lower energy in AlAs than GaAs, with the result that the X-point transitions are indirect in real-space, as well as k -space. Hence the X1H1 transition evolve linearly with field, in a similar fashion as the oblique E1H1 Stark-ladder transitions, but with the order number now taking values $m = \pm 1/2, \pm 3/2, \pm 5/2$. The spatial separation of the electron and hole for the X1H1 transitions results in their transition strength being much weaker than that for E1H1.

Both the $m = -\frac{1}{2}$ and $m = +\frac{1}{2}$ Stark-ladder transitions were clearly resolved in the ER measurements on the (10,8) SL. As expected they were seen to evolve linearly with field, at a rate in excellent agreement with the SL half period. This was despite the transitions being too weak to be detected in the PC measurements using the same lamp and monochromator source. The extra sensitivity of the ER spectra derives from the very large energy shift of these oblique Stark-ladder transitions with field.

The X1H1 features of the (10,8) SL were much easier to interpret than those due to E1H1, partially because they could be clearly resolved from the other spectral features. Also the absorption of the X1H1 transitions is much smaller and so a change in the the mix of ϵ_r and ϵ_i , due to a variation in the optical penetration depth, will be less important across this region of the spectrum. The X1H1 transitions could also be resolved in the ER spectra of the (11,8) SL at low applied biases, although they became obscured by the nearby, and much stronger, E1H1 feature at larger biases.

ER and PC measurements were also made on a (6,6) SL as a function of the applied bias. This SL was expected to have very large minibandwidths for the Γ -point transitions and so only show structure at the miniband edges. Features were resolved in the ER which were tentatively ascribed to the lower edges of the E1H1 and E1L1 minibands. A corresponding E1L1 feature was not resolved in the PC spectra taken on this SL, or in the PLE spectra taken on a similar SL in another study. When the field was increased, the ER spectra resembled the Franz-Keldysh oscillations of a bulk semiconductor, which is not surprising considering the minibandwidth is expected to be very large.

ER features were also observed in the spectral region of the X1H1

transition of the (6,6) SL, which had a similar lineshape to the E1H1 feature. There was also another feature to higher energy, whose separation from X1H1 equalled the observed spacing of E1H1 and E1L1, suggesting it to be due to X1L1. However, when the field was increased, the ER in the spectral region of the X-point transitions also changed to resemble Franz-Keldysh oscillations, which is surprising considering the minibandwidth is expected to be small for X1H1. The oscillations seen at higher biases, may therefore be due to transitions between the Γ -point conduction and spin-orbit split-off bands of the bulk GaAs buffer layer inside the intrinsic region.

8 Conclusions

In this chapter I wish to comment upon a theme common to much of the theoretical and experimental studies of this thesis; the role of optical interference in determining reflectance lineshapes. The reader seeking a detailed summary covering all the aspects of this work should consult the final section of each chapter. A briefer precis can be found in the extended abstract of Sec. 1.2.

It was shown in Sec. 3.3, that the ER lineshape of a SQW can be determined by considering only the rays reflected from the front surface along with those from the well interfaces. This yields the ER to be approximated by $\Delta R \approx B \text{ Real}[i \Delta \epsilon \exp(i g)]$, where $\Delta \epsilon = \Delta \epsilon_r + i \Delta \epsilon_i$ is the field modulated dielectric function of the well; g is the "lineshape phase", the optical phase delay between the rays reflected from the front surface and the well; and B is a constant.

This model for the ER of a SQW was verified experimentally, in Sec. 5.2.1, by measuring the lineshape of two different structures as a function of the angle of incidence. Increasing the angle of incidence produced a decrease in the lineshape phase and hence the ER was seen to evolve between lineshapes resembling $\Delta \epsilon_r$ and $\Delta \epsilon_i$. Since $\Delta \epsilon_r$ and $\Delta \epsilon_i$ will have opposite symmetry about the exciton energy, as can be deduced from the Kramers-Kronig relations, this represents a very dramatic change in the ER lineshape. The rate at which the lineshape evolves with the angle of incidence was used to estimate the overlayer thickness between the well and the front surface, and this was found to be in good agreement with that specified in the growth procedure, for both the SQW structures.

The rays reflected from each well in a MQW stack, will contribute a different lineshape to the overall reflectance, depending upon their depth in the structure. Hence the rays will tend to destructively interfere, resulting in the overall reflectance of the MQW being little stronger than that of a SQW. It was shown in Sec. 3.4, that the absorption in the MQW stack results in very complicated MQW ER lineshapes for a single exciton. This is essentially because the optical penetration depth changes with the photon energy, so changing the mix of $\Delta \epsilon_r$ and $\Delta \epsilon_i$ in the ER. By making an explicit analysis, the complicated ER lineshape was shown to consist of two contributions, due to the front and back of the stack. This explicit analysis was demonstrated to be accurate by comparison with a more exact optical transfer matrix calculation.

Measurements were made on three MQW structures in Chp. 6. Each of

these MQWs showed a systematic variation in the exciton transition energy across the stack, which prevented a direct comparison with the MQW ER lineshape model of Sec. 3.4. Despite this, it was demonstrated how the absorption and phase change of the light propagating across the stack, has a critical effect upon the ER lineshape.

Sec. 6.2 reported low temperature field-dependent measurements on two MOVPE grown GaAs/Al_{0.3}Ga_{0.7}As MQWs, with p⁺-i-n⁺ doping profiles. It was demonstrated how ER measurements provide information not obtainable with other spectroscopies. In particular, the evolution of each ER feature with the angle of the incident light was used to determine the depth of the wells producing the feature. Furthermore, the strength of the ER feature also provided information on the position of the contributing wells in the stack, since it is sensitive to the absorption in shallower layers.

By comparing the ER spectra with ET, PC and transmission measurements also taken on the two MOVPE grown MQWs, at different applied biases, the well width was deduced to change abruptly by one monolayer at certain points in the stack. Under flatband conditions, these monolayer splittings would be smaller than the linewidth and therefore be unresolvable. However, there was also a linear change in the field across the stack in these samples, due to the background impurity charge of the i-region, which enhanced the splittings. This allowed a monolayer splitting to be resolved for a 163 Å wide well.

Low temperature bias-dependent ER measurements were also made on another GaAs/Al_{0.3}Ga_{0.7}As MQW in Sec. 6.3, this time arranged inside a n⁺-i-n⁺ doping profile. ER features were observed, which were shown to derive from low- and high-field regions inside the stack. The angular dependence of the ER lineshape was again used to determine the depths of these low- and high-field regions, and this indicated a field profile that could be explained by the tunnelling of the dark current through the structure. This allowed the bizarre PC spectra, also measured on the sample, to be interpreted. It was deduced that the PC was only collected from the back of the stack, and consequently the centre of each exciton peak in the PC spectrum was removed by the absorption in shallower layers.

Optical interference effects may also have complicated the ER lineshapes measured for the Γ -point transition in the GaAs/AlAs SLs studied in Sec. 7.2. The mixing of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ was undesirable in this case, as it complicated a detailed study of the Stark-ladder behaviour of the Γ -point transitions. The field dependence of the SL bandstructure would be better studied using ET, which is essentially

determined by $\Delta\epsilon_i$ alone.

It was stressed how electromodulation spectroscopy will be a sensitive method of measuring the SL bandstructure, due to its strong electric field dependence. In particular the oblique Stark-ladder transitions will have a large field-induced energy shift. This allowed the $m = \pm\frac{1}{2}$ Stark-ladder transitions of a type-II (10,8) GaAs/AlAs SL to be observed in ER, despite them being too weak to be resolved in PC spectra taken with the same light source. The $m = \pm\frac{1}{2}$ transitions were both found to evolve linearly with field, at a rate in very good agreement with the SL half period.

Optical transitions to the X-point of the conduction band were also observed in ER measurements on a (6,6) GaAs/AlAs SL, despite them being undetectable in PC measurements. ER also revealed structure due to transitions involving the light hole valence band, with both the X- and Γ -points of the conduction band, which were not observed in both PC and PLE measurements. Furthermore, it was shown how Franz-Keldysh oscillations, originating from regions of bulk semiconductor inside this sample, could be used to estimate the electric field, after optical interference effects had been handled correctly.

This thesis has aimed to demonstrate that electro-optical effects in QWs and SLs are not only of interest as building blocks for devices, but can also form the basis of a very sensitive spectroscopy. By comparison with complementary optical techniques, I have attempted to demonstrate the potential of this spectroscopy when applied to new multilayer materials. At the same time I have tried to contribute towards a better understanding of the underlying mechanisms producing the electromodulated spectra.

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