

Modelling laboratory horizontal stress and coal permeability data using S&D permeability model

Ji-Quan Shi and Sevket Durucan*

Department of Earth Science and Engineering, Royal School of Mines, Imperial College London, London SW7 2BP, UK

*Corresponding author. Tel.: +44 (0) 20 7594 7374; E-mail address:
j.q.shi@imperial.ac.uk

Abstract

A modelling study was carried out on a set of recently published horizontal (radial) stress and permeability data, obtained under uniaxial strain conditions, using Shi and Durucan permeability model. During the permeability experiment, the no-displacement lateral boundary condition was maintained through adjusting the applied confining pressure as the pore pressure was reduced to simulate coalbed methane production. The reported variations in the effective horizontal stress for a San Juan Basin core sample were first matched by tuning the elastic properties (Young's Modulus and Poisson's ratio) for the coal sample, which were unknown. The Young's Modulus was found to fall in a rather narrow range of 2.35 to 2.55 GPa, with the corresponding Poisson's Ratio in the range between 0.25 and 0.35. The availability of both permeability and horizontal stress data allows for the determination of the cleat volume compressibility. It was found that the semi-log plot of permeability increase vs. reduction in the effective horizontal stress displayed a bi-linear trend. The results of this study have demonstrated that the published laboratory horizontal stress and permeability data can be adequately described by S&D permeability model.

Keywords: laboratory permeability experiment, uniaxial strain conditions, horizontal stress, model match, S&D permeability model.

Introduction

Unlike conventional gas reservoirs, coalbed reservoirs experience dynamic changes in (absolute) permeability during coalbed methane production through reservoir pressure depletion (primary recovery). It is recognised that, as the reservoir is being drawdown, two processes would occur which have opposing effects on coalbed permeability. On the one hand, reduction in pore pressure would generally lead to an increase in the reservoir effective stresses. On the other hand, coal matrix shrinkage induced by desorption of coal gas (predominately methane) tends to result in a reduction in the effective stresses. Changes in cleat permeability/porosity of coalbeds are controlled by these two processes, and in particular the overall trend is determined by their net impact on the effective stresses during methane production.

Analytical models have been developed to describe this dynamic change in coalbed permeability under uniaxial strain conditions considered to prevail in large-scale

reservoirs (e.g. Palmer and Mansoori, 1998; Shi and Durucan, 2004). Because laboratory permeability experiments are usually conducted under fixed confining pressure conditions, validation of the permeability models has so far relied almost exclusively on the limited field permeability data available (e.g. Mavor and Vaughn, 1998; Palmer et al., 2007; Clarkson et al., 2010; Shi and Durucan, 2010).

As the first of a two-part series, Mitra et al. (2012) recently reported an experimental study of coal permeability behaviour under uniaxial strain conditions. As well as the permeability response to a declining pore pressure, the corresponding variation in the applied horizontal (radial) stress, which was adjusted throughout the experiment to maintain the laterally constrained boundary condition, was also measured (Figure 1).

A modelling study of the permeability data for a San Juan Basin coal sample employing two commonly used permeability models was presented in the second paper (Liu et al., 2012). Liu et al. reported that one of the models used, Shi and Durucan (S&D) model (2004), underestimated the laboratory permeability data at low pore pressures (< 2 MPa) and proposed a modification to the S&D model. However, mistakes have been found in the published work. This study set out to rectify these errors in the application of S&D model to the laboratory data.

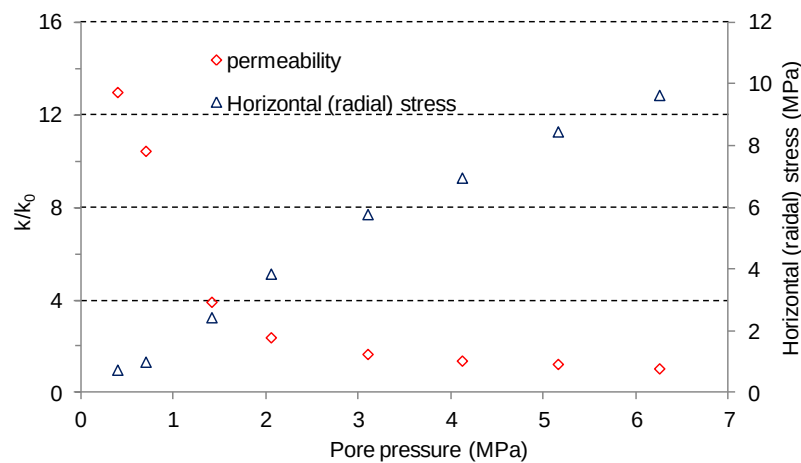


Figure 1 Variations in coal permeability and the horizontal stress measured for a San Juan Basin sample under uniaxial strain conditions (re-plotted using data digitised from Mitra et al., 2012). During the experiment, the pore pressure was reduced from an initial value (p_0) of 6.2 MPa down to about 0.4 MPa. The permeability is expressed as the ratio to its initial value (k_0) at p_0 .

Match of experimental data using S&D model

In S&D model (2004), change in cleat permeability of coal varies exponentially with changes in the effective horizontal stress $\Delta\sigma$

$$\frac{k}{k_0} = e^{-3c_f\Delta\sigma} \quad (1)$$

where c_f is the cleat volume compressibility (MPa^{-1}), and changes in the effective horizontal stress, under uniaxial strain conditions, is given by

$$\Delta\sigma = \sigma - \sigma_0 = -\frac{\nu}{1-\nu}\Delta p + \frac{E\varepsilon_{smax}}{3(1-\nu)}\left(\frac{p}{p+P_\varepsilon} - \frac{p_0}{p_0+P_\varepsilon}\right) \quad (2)$$

where E and ν are Young's Modulus and Poisson's ratio of coal, ε_{smax} is the maximum volumetric swelling strain and P_ε is the pressure at which the swelling strain $\varepsilon_s = 0.5\varepsilon_{smax}$.

To highlight the influence of the elastic properties on the two terms, Eq. (2) maybe written as

$$\Delta\sigma = \frac{\nu}{1-\nu}[-\Delta p + \frac{E\varepsilon_{smax}}{3\nu}\left(\frac{p}{p+P_\varepsilon} - \frac{p_0}{p_0+P_\varepsilon}\right)] \quad (3)$$

It can be seen that the strength of the shrinkage term relative to the compaction term, which determines the overall trend of $\Delta\sigma$, is affected only by the ratio E/ν , while the magnitude of $\Delta\sigma$ is also impacted by ν .

Eq. (1) may be written as

$$\ln \frac{k}{k_0} = -3c_f\Delta\sigma \quad (4)$$

Thus, the plot of $\ln(k/k_0)$ vs. $(-\Delta\sigma = \sigma_0 - \sigma)$ can be used to estimate the cleat compressibility.

Variable, as well as constant, cleat volume compressibility has been used to fit both experimental and field permeability data (e.g. McKee et al., 1998; Robertson and Christiansen, 2007), with the compressibility generally decreasing with increasing net stress applied on the coal. Palmer (2009) noted that the (water) permeability data for several San Juan Basin cores reported by Seidle et al. (1992) displayed bi-linear behaviour (Figure 2 in Palmer, 2009). Zheng et al. (2012) showed that the cleat compressibility was strongly dependent on both the effective stress and the pore pressure for two dry Chinese bituminous coal samples from the Qinshui Basin and Junggar Basin. Shi and Durucan (2010) used a stress-dependent cleat compressibility to match the permeability data from the San Juan Basin coalbed reservoirs.

Previous work

The input parameters to S&D model include the cleat volume compressibility c_f , the elastic properties (E and ν), and the free swelling strain properties ($\varepsilon_{\max}$ and P_ε) of coal. The input parameters used by Liu et al. (2012) are presented in Table (column 2). Note that the elastic properties (E and ν) of the San Juan Basin coal sample were unavailable and their values ($E = 3.52$ GPa, $\nu = 0.32$) were selected with reference to their respective range reported in the literature (Liu et al., 2012). The cleat volume compressibility was obtained from the experimental data using Eq. (4) (Mitra et al., 2012). Unfortunately, instead of using the slope of $\ln(k/k_0)$ - $(-\Delta\sigma)$ plot to determine the cleat volume compressibility, the $\log(k/k_0)$ - $(-\Delta\sigma)$ plot was actually used (Figure 2). The slope in the figure gives a cleat compressibility of $0.276/3 = 0.092$ MPa⁻¹. Given that the slopes of the two plots differ by a factor of $\ln(10)$ or approximately 2.3, the correct cleat volume compressibility should be 2.3 times the value used in the modelling study by Liu et al. (2012), i.e. 0.212 MPa⁻¹.

It would appear that Liu et al. (2012) also failed to constrain the elastic properties used for the core sample by fitting Eq. (2) to the experimental stress data. As shown in Figure 3, the stress reductions ($\sigma_0 - \sigma$) computed using the elastic properties listed in Table 1 (column 2) are much higher than those measured in the laboratory.

Table 1 Input parameters to S&D model used in Liu et al. (2012) and this study.

S&D model input parameters	Liu et al.	This study
Cleat volume compressibility c_f , MPa ⁻¹	0.092	<i>0.194</i> ($p \geq 1.4$ MPa) <i>0.485</i> ($p < 1.4$ MPa)
Young's Modulus E , GPa	3.52	<i>2.35 – 2.55</i> ($p \geq 1.4$ MPa) <i>Progressively reduced by up to 60%</i> ($p < 1.4$ MPa)
Poisson's Ratio ν , fraction	0.32	<i>0.25 – 0.35</i>
$\varepsilon_{\max}$	0.01075	0.01075
P_ε , MPa	4.16	4.16

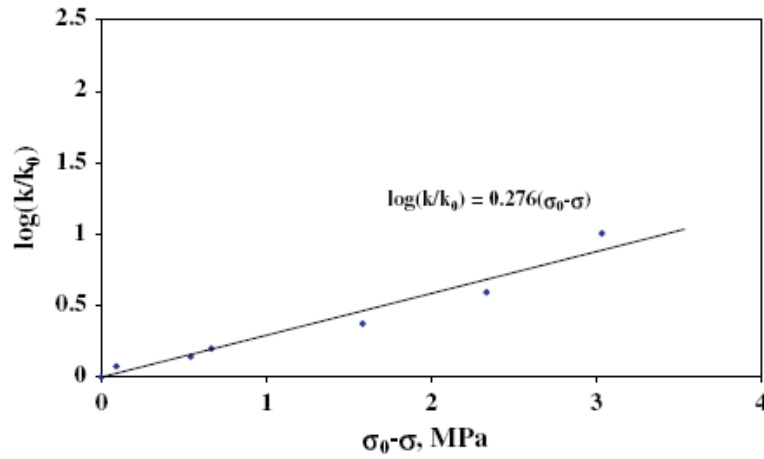


Figure 2 The slope of $\log(k/k_0)$ - $(-\Delta\sigma)$ plot used by Liu et al. (2012) to estimate the cleat volume compressibility (after Mitra et al., 2012).

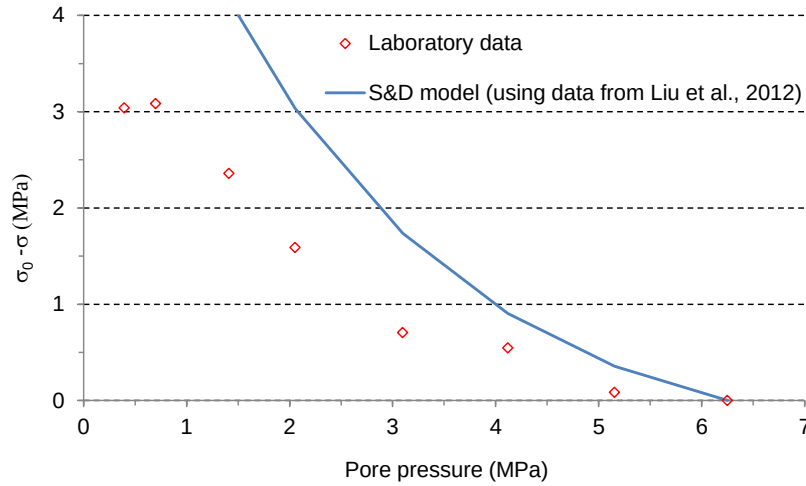


Figure 3 Over-prediction of the horizontal stress data of Mitra et al. (2012) by S&D model (Eq. (2)) using the elastic properties from Liu et al. (2012).

E as a tuning parameter to match horizontal stress data

In this study, E is used as the main tuning parameter to match the measured horizontal stress data. It was found that, with the exception of the last two pressure points, a close match to the experimental data could be achieved by using a Young's Modulus in a rather narrow range between 2.35 and 2.55 GPa, with the corresponding Poisson's ratio varying from 0.25 ($E = 2.35$ GPa) to 0.35 ($E = 2.55$ GPa). An example of the model match is illustrated in Figure 4.

The model over-estimates the reduction in the effective stress for the last two pressure points. During the experiment, the effective horizontal stress decreased steadily, from its initial value of ~ 3.4 MPa, as the pore pressure was reduced from 6.2 MPa (Figure 5). It is noted there was a slight rebound at the last pressure point. One plausible

explanation is that there was a reduction in the Young's Modulus of the core sample as the effective horizontal stress was reduced to below 1 MPa ($p = 1.4$ MPa).

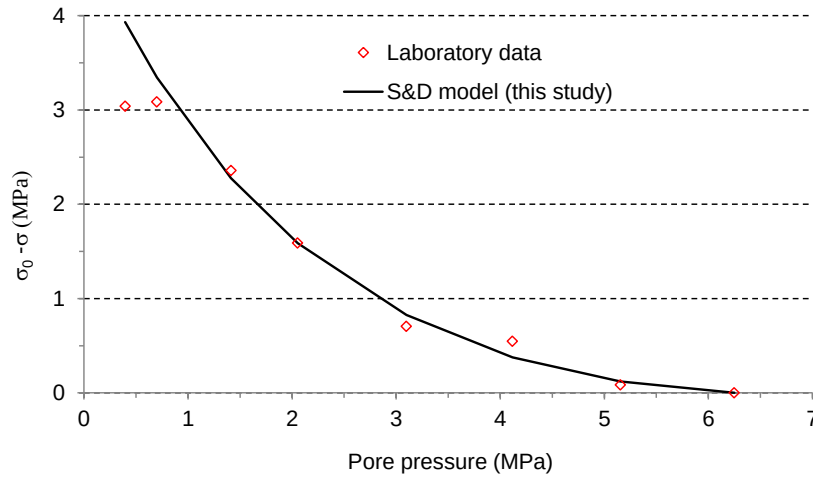


Figure 4 Match of the effective horizontal stress using Eq. (2) with $E = 2.45$ GPa and $\nu = 0.3$.

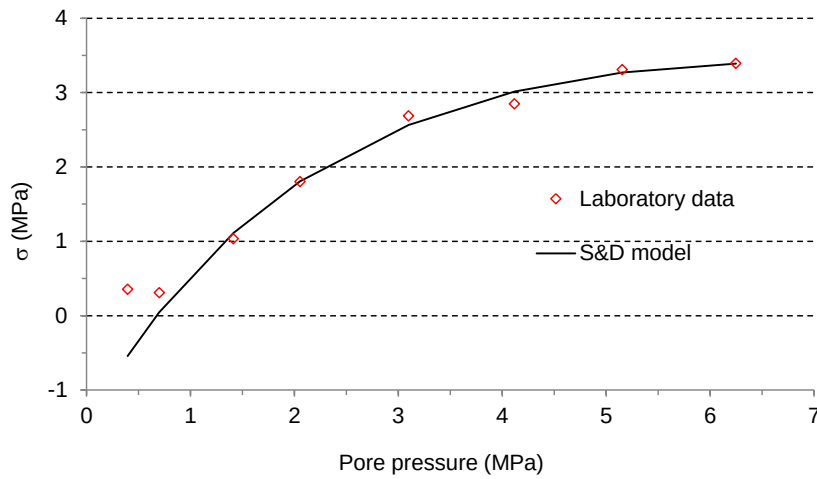


Figure 5 Variation in the effective horizontal stress and model match. Note that the effective horizontal stress varies non-linearly with the pore pressure.

The following piecewise variations in the Young's Modulus as the pressure is reduced from p_0 have been considered with $p_1 = 1.4$ MPa, $p_2 = 0.7$ MPa and $p_3 = 0.4$ MPa,

$$E = E_0, \quad p_0 \geq p \geq p_1$$

$$E = E_1, \quad p_1 > p \geq p_2$$

$$E = E_2, \quad p_2 > p \geq p_3$$

For piecewise varying Young's Modulus, Eq. (2) maybe re-casted as

$$\Delta\sigma = \Delta\sigma_1, \quad p_0 \geq p \geq p_1 \quad (5a)$$

$$\Delta\sigma = \Delta\sigma_1(p_1) + \Delta\sigma_2, \quad p_1 > p \geq p_2 \quad (5b)$$

$$\Delta\sigma = \Delta\sigma_1(p_1) + \Delta\sigma_2(p_2) + \Delta\sigma_3, \quad p_2 > p \geq p_3 \quad (5c)$$

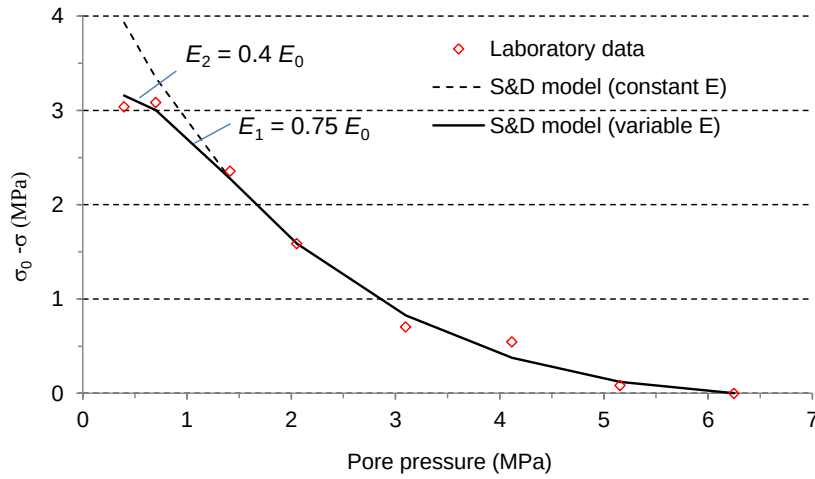
with

$$\Delta\sigma_1 = -\frac{\nu}{1-\nu}(p - p_0) + \frac{E_0 \varepsilon_{smax}}{3(1-\nu)} \left(\frac{p}{p+p_\varepsilon} - \frac{p_0}{p_0+p_\varepsilon} \right) \quad p_0 \geq p \geq p_1 \quad (6a)$$

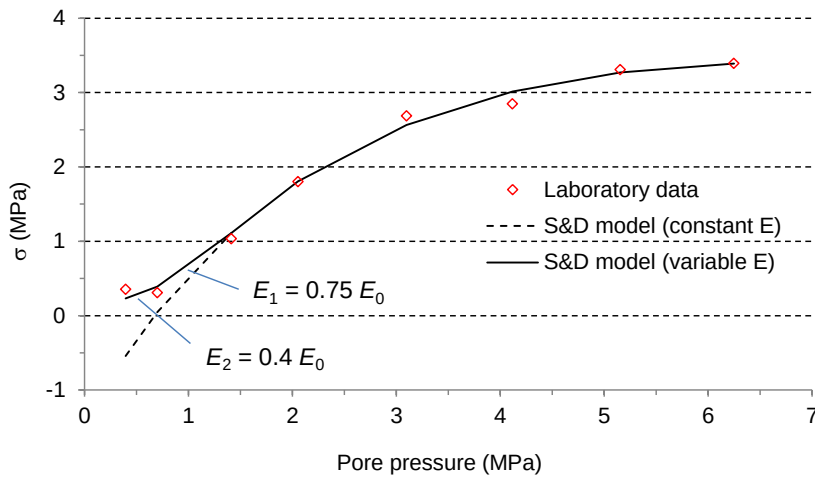
$$\Delta\sigma_2 = -\frac{\nu}{1-\nu}(p - p_1) + \frac{E_1 \varepsilon_{smax}}{3(1-\nu)} \left(\frac{p}{p+p_\varepsilon} - \frac{p_1}{p_1+p_\varepsilon} \right) \quad p_1 > p \geq p_2 \quad (6b)$$

$$\Delta\sigma_3 = -\frac{\nu}{1-\nu}(p - p_2) + \frac{E_2 \varepsilon_{smax}}{3(1-\nu)} \left(\frac{p}{p+p_\varepsilon} - \frac{p_2}{p_2+p_\varepsilon} \right) \quad p_2 > p \geq p_3 \quad (6c)$$

Using a progressively reduced E as the pressure falls below p_1 , a much improved match to the experimental stress data is obtained (Figure 6). It can be seen that the predicted horizontal stress now remains positive throughout the experiment (Figure 6b).



(a) $\sigma_0 - \sigma$



(b) σ

Figure 6 Improved model predictions using a progressively reduced E for the last two pressure points ($E_0 = 2.45$ GPa).

Bi-linear relationship for $\ln(k/k_0) - (-\Delta\sigma)$

Examination of the $\ln(k/k_0) - (-\Delta\sigma)$ reveals that the data points could perhaps better represented by a bi-linear relationship. As shown in Figure 7, the cleat compressibility increased from $c_{f1} = 0.194$ to $c_{f2} = 0.508 \text{ MPa}^{-1}$, about 2.6 times of its initial value when the reduction in the effective horizontal stress rose to above 2.36 MPa, with the corresponding effective horizontal stress (σ) reduced to approximately 1 MPa (Figure 6b) and the pore pressure lowered to 1.4 MPa. Note the potential Klinkenberg effect on the permeability is largely negligible here (see Appendix).

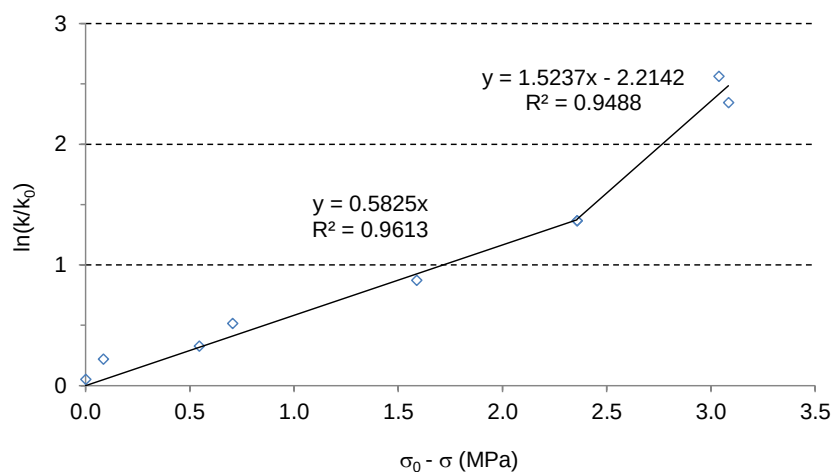


Figure 7 The $\ln(k/k_0) - (-\Delta\sigma)$ plot fitted to a bi-linear relationship.

Accordingly, Eq. (1) may be rewritten as

$$\frac{k}{k_0} = e^{-3c_{f1}\Delta\sigma_1} \quad p_0 \geq p \geq p_1 \quad (7a)$$

$$\frac{k}{k_0} = \frac{k(p_1)}{k_0} \frac{k}{k(p_1)} = \frac{k(p_1)}{k_0} e^{-3c_{f2}\Delta\sigma_2} \quad p_1 > p \geq p_2 \quad (7b)$$

$$\frac{k}{k_0} = \frac{k(p_2)}{k_0} \frac{k}{k(p_2)} = \frac{k(p_2)}{k_0} e^{-3c_{f2}\Delta\sigma_3} \quad p_2 > p \geq p_3 \quad (7c)$$

where $k(p_1)$ and $k(p_2)$ are the permeability at p_1 and p_2 respectively, and $\Delta\sigma_i$ ($i = 1, 2, 3$) are given by Eqs. (6).

As can be seen in Figure 8, the model prediction of permeability increase using $c_{f1} = 0.194 \text{ MPa}^{-1}$ is in excellent agreement with the experimental data for $p \geq 1.4 \text{ MPa}$ and a much improved match can be obtained by using an enlarged cleat volume compressibility ($c_{f2} = 2.5c_{f1}$, slightly lower than 2.6 times estimated from Figure 7 for a better match) for $p < 1.4 \text{ MPa}$. The input parameters used in this study are listed in Table 1 (third column).

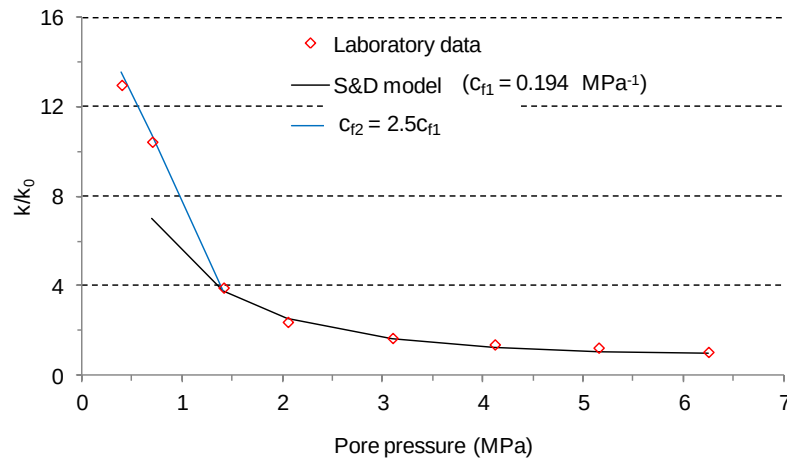


Figure 8 Model prediction of the permeability increase using a constant and a variable cleat volume compressibility.

Concluding remarks

The experimental work by Mitra et al. has provided valuable first-hand data on the response to pressure drawdown under uniaxial strain conditions of not only coal permeability, but also the horizontal stress for validating analytical permeability models. The results of this study demonstrate that the laboratory experimental data can be adequately described by S&D permeability model.

References

- Clarkson, C.R., Pan, Z., Palmer, I., Harpalani, S., 2010. Predicting sorption-induced strain and permeability with depletion for coalbed reservoirs. *SPE J* (March):152-9.
- Hu, G., Wang, H., Fan, X., Yuan, Z., Hong, S., 2009. Mathematical Model of Coalbed Gas Flow with Klinkenberg Effects in Multi-Physical Fields and its Analytic Solution, *Transp. Porous Med* 76:407–420.
- Jones, F.O., Owens, W.W., 1980. A laboratory study of low permeability gas sands. *J. Pet. Technol.* 32:1631–1640.
- Klinkenberg, L.J., 1941. The Permeability of Porous Media to Liquids and Gases. In: *Drilling and Production Practices*. American Petroleum Institute, New York.
- Liu, S., Harpalani, S., Mitra, A., 2012. Laboratory measurement and modelling of coal permeability with continued methane production: Part 2 – Modelling results. *Fuel* 94:117-124.
- Mavor, M.J., Vaughn, J.E., 1998. Increasing coal absolute permeability in the San Juan Basin Fruitland formation. *SPE Res Eval Eng* 201-6.

- McKee, C.R., Bumb, A.C., Koenig, R.A., 1988. Stress-dependent permeability and porosity of coal and other geologic formations. *SPE Formation Evaluation* 3 (1), 81–91.
- Mitra, A., Harpalani, S., Liu, S., 2012. Laboratory measurement and modelling of coal permeability with continued methane production: Part 1 – Laboratory results. *Fuel* 94:110-116.
- Palmer, I., Mansoori, J., 1998. How permeability depends on stress and pore pressure in coalbeds, a new model. *SPE Reservoir Evaluation and Engineering* 1(6): 539–544 SPE-52607-PA.
- Palmer, I., Mavor, M., Gunter, B., 2007. Permeability changes in coal seams during production and injection. In: 2007 International Coalbed Methane Symposium.
- Palmer, I., 2009. Permeability changes in coal: analytical modeling. *International Journal of Coal Geology* 77 (1–2), 119–126.
- Robertson, E.P., Christiansen, R.L., 2007. Modeling laboratory permeability in coal using sorption-induced strain data. *SPE Reservoir Evaluation and Engineering* 10 (3), 260–269.
- Seidle, J.P., Jeanson, M.W., Erickson, D.J., 1992. Application of matchstick geometry to stress dependent permeability in coals. *SPE Rocky Mountain Regional Meeting*. Casper, Wyoming.
- Shi, J.Q., Durucan, S., 2004. Drawdown induced changes in permeability of coalbeds: a new interpretation of the reservoir response to primary recovery. *Transport in Porous Media* 56(1):1–16.
- Shi, J.Q., Durucan, S., 2010. Exponential growth in San Juan basin Fruitland coalbed permeability with reservoir drawdown: model match and new insights. *SPE Reservoir Evaluation and Engineering* 13(6):914–925.
- Wu, Y.S., Pruess, K., Persoff, P., 1998. Gas flow in porous media with Klinkenberg effects. *Transp. Porous Media* 32: 117–137. doi:10.1023/A:1006535211684.
- Zheng, G., Pan, Z., Chen, Z., Tang, S., Connell, L.D., Zhang, S., Wang, B., 2012. Laboratory study of gas permeability and cleat compressibility for CBM/ECBM in Chinese coals. *Energy Exploration & Exploitation* 30 (3):451-476.

Appendix: Klinkenberg effect on gas permeability

The Klinkenberg effect refers to the observations that the gas permeability of a porous medium tends to increase as the gas pressure is reduced, and the effect becomes more pronounced at low pressures. The Klinkenberg factor b (Klinkenberg, 1941) has been introduced to account for this effect

$$k = k_{\infty} \left(1 + \frac{b}{p}\right) \quad (\text{A-1})$$

where k_{∞} is the gas permeability at sufficiently high pressures. It has been shown (Jones and Owens, 1980; Wu et al. 1998) that b decreases with increasing permeability and can be fitted into the following relationship

$$b = \beta k_{\infty}^{-0.36} \quad (\text{A-2})$$

where $\beta = 0.251 \text{ Pa (m}^2)^{0.36}$

Thus, variations in the gas permeability due to the Klinkenberg effect as the gas pressure is reduced from p_0 (in unit of Pa) may be estimated

$$\frac{k_{KB}}{k_0} = \frac{1 + \frac{b}{p}}{1 + \frac{b}{p_0}} \quad (\text{A-3})$$

Figure A-1 presents the computed increase in the gas permeability for three different k_{∞} values spanning two-orders of magnitude. The permeability of the San Juan Basin core sample is believed to be greater than 1 mD. It is thus concluded that the impact of Klinkenberg effect on the dynamic behaviour of the gas permeability may be ignored in this study.

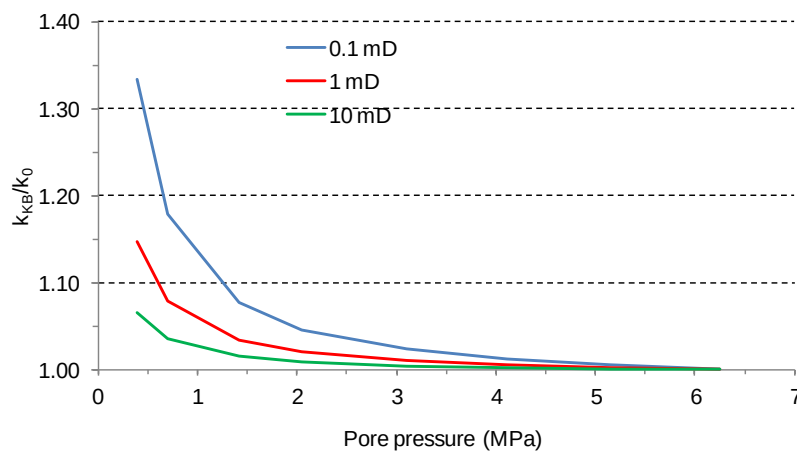


Figure A-1 Estimated influence of Klinkenberg effect on gas permeability for three different k_{∞} values spanning two-orders of magnitude.