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confinement**

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Analysis of simple shear tests with cell pressure confinement

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Analysis of simple shear tests with cell pressure confinement

The high cost of offshore infrastructure provides continuous encouragement for optimisation of design practices. Development of a more rational method to interpret results from simple shear tests with cell pressure confinement can reduce costs and improve reliability of offshore infrastructure. This paper addresses a commonly overlooked issue affecting design parameter selection: specimen shape varies from right cylinder to oblique cylinder after loading along a single shearing direction. Thus, normal horizontal stresses are not always equal to the cell pressure and their magnitude varies throughout the specimen lateral surface. An analysis is proposed that accounts for changing specimen geometry and lateral surface area during shearing and for the actual effect of cell pressure during testing. The analysis also describes how the intermediate principal stress can be assessed. Test results for medium dense silica sand are interpreted following de Josselin de Jong's alternative shearing mechanism hypothesis. Conventional interpretation methods yield conservative design parameters for this soil. Failure states develop when the intermediate principal effective stress is halfway between major and minor principal effective stresses. Typical results for the soil tested show equipment performance meets standard direct simple shear requirements for shear strain rate, vertical stress and specimen height control.

Keywords: Laboratory test; plane strain; intermediate principal stress; principal stress rotation; offshore sediments.

Introduction

Laboratory tests that allow direct application (or measurement) of shear stresses on the top and bottom horizontal boundaries of a soil specimen are among the earliest tests used in geotechnical engineering. These tests were originally envisioned as an improvement to direct shear tests (Kjellmann 1951) and attempt to emulate a special case of plane strain mode of deformation known as simple shear (Bjerrum & Landva 1966). Early versions of these tests have been referred to as direct simple shear or just simple shear tests, depending on the boundary conditions imposed by the apparatus used. No simple shear test (direct or otherwise) commonly used in practice can apply

complementary shear stresses to vertical specimen boundaries. As a result, analysis of simple shear test results invariably relies on interpretation. Despite this limitation, simple shear tests (direct or otherwise) are widely used worldwide. At present, the vast majority of these tests are conducted on cylindrical specimens. While the choice of cylindrical specimen geometry adds additional complexity to the rigorous interpretation of test results for specimens loaded along a single shearing direction, this aspect is often overlooked in most analyses, which typically fall back to idealised plane strain limit equilibrium assumptions. This compromise is because cylindrical soil samples are relatively easy to obtain using standard site investigation procedures and the resulting testing method is relatively straightforward.

The two most common types of simple shear tests on cylindrical specimens used in practice are the direct simple shear test (e.g. Bjerrum and Landva 1966, ASTM D6528-07) and the simple shear test with cell pressure confinement. Direct simple shear tests impose a zero lateral strain boundary condition through physical constraint (e.g. reinforced rubber membrane or stack of hollow discs). On the other hand, cell pressure-confined simple shear specimens are tested using standard unreinforced rubber membrane and subjected to a controlled lateral-stress boundary condition. In this paper, an alternative analysis of results from simple shear tests with cell pressure confinement is presented. The paper describes typical boundary conditions imposed in such tests and outlines an alternative procedure to properly account for the effect of evolving cylindrical specimen geometry and intermediate principal stress on the analysis of test results.

Simple shear test with cell pressure confinement

In order to make a clear distinction from direct simple shear tests, simple shear tests carried out under controlled cell pressure confinement will be simply referred to as

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3 simple shear tests in this paper (i.e., without the ‘direct’ qualifier). Earlier versions of
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5 this type of simple shear test were used for research in the United States (Peacock et al.
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7 1968), Canada (Finn et al. 1971) and Germany (Franke et al. 1979). A commercial
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9 version was even produced and became available in California in the 1980s. Testing
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11 methods and apparatus discussed in this paper developed from the original simple shear
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13 testing equipment and procedures used at The University of Western Australia (UWA)
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15 since the late 1990s (Mao and Fahey 2003). A detailed description of the new software
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17 and hardware capabilities of the new generation of UWA simple shear devices that has
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19 been developed over the last couple of years at COFS is covered in a companion paper.
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21 However, the main features of this new generation of UWA simple shear devices are
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23 schematically shown in Figure 1. Testing methods and apparatus characteristics that are
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25 relevant for the present discussion are described next.
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30 In a typical UWA simple shear test, a cylindrical specimen is subjected to a
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32 vertical stress (σ_{zz}) while the lateral stress induced by cell pressure (σ_c) is controlled
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34 separately (Figure 2a). Backpressure (u_{bp}) is applied to fully saturate the specimen
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36 following saturation procedures similar to those used in conventional triaxial tests
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38 (Head 1986). The pore pressure (u) is measured throughout the test so the effective
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40 vertical and confining stresses (σ'_{zz} and σ'_c , respectively) are known at anytime during
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42 the test. Once specimen saturation is achieved, the specimen is typically subjected to a
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44 drained consolidation stage (i.e., pore pressure change $\Delta u = u - u_{bp} \approx 0$) under a specified
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46 stress ratio $K (= \sigma'_c / \sigma'_{zz})$ that may or may not be equal to K_0 (Figure 2a). K_0
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48 consolidation can be achieved incrementally by monitoring strains and adjusting
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50 boundary stresses as needed, or using an automated control loop to yield negligible
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52 radial strains during consolidation. Drained anisotropic consolidation under controlled
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54 principal stress rotation (i.e., ‘inclined’ consolidation) can also be carried out
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(Zdravkovic and Jardine 2001), if required. As long as the shear stress on the horizontal plane (τ_{zx}) is zero and no shear deformation (γ_{zx}) is applied to the specimen (Figure 2), σ'_c is equal to the horizontal effective stresses ($\sigma'_{xx}=\sigma'_{yy}$). This convenience ends as soon as shearing deformation is imposed to the specimen ($\gamma_{zx}>0$). Horizontal effective stresses are not controlled during shearing. The only lateral stress component that is truly controlled throughout the test is σ_c (Fig. 2).

Shearing stages can be conducted under drained or undrained conditions. In this paper, only undrained shearing is described. This typically involves controlling either the horizontal displacement (U_{zx}) applied to the horizontal platen in strain-controlled tests, or the horizontal shearing load (S_{zx}) in stress-controlled tests (Figures 2b and 2c). If the specimen height (h), diameter (d), and horizontal cross sectional area (A_H) are evaluated at the end of consolidation and the typical constant-height boundary condition is imposed during undrained shearing (i.e., $U_{zz}=0$), τ_{zx} and γ_{zx} are defined as, respectively,

$$\tau_{zx} = \frac{S_{zx}}{A_H} = \frac{4S_{zx}}{\pi d^2} \quad (1)$$

$$\gamma_{zx} = \arctan\left(\frac{U_{zx}}{h}\right) \quad (2)$$

Equations 1 and 2 rely on the assumption that volume changes are not allowed during undrained shearing of a saturated specimen so the specimen diameter remains constant (i.e., $\Delta d=0$) despite the fact a flexible membrane is used. The theorems of Pappus-Guldinus can then be used to assess, respectively, the volume (V) and lateral surface area (A_L) of the oblique cylinder that defines specimen geometry during shearing as

$$V = h \frac{d}{2} \pi \frac{d}{2} = \frac{\pi d^2 h}{4} \quad (3)$$

$$A_L = \frac{\pi dh}{\cos \gamma_{zx}} = \int_0^{2\pi} \frac{h}{\cos \gamma_{zx}} \frac{d}{2} d\alpha \quad (4)$$

where $d\alpha$ is an infinitesimal arc along the perimeter of the specimen top boundary (Fig. 2b).

At constant height, Eq. 3 indicates the volume of the oblique cylinder at any stage of undrained shearing is identical to the specimen volume at the end of consolidation. Inspection of Eq. 4 (and Fig. 2) shows that, during undrained shearing, σ_c acts on an inclined lateral surface area that is larger than the lateral surface area of the specimen at the end of consolidation by a factor inversely proportional to $\cos(\gamma_{zx})$. This corresponds to lateral surface areas about 1, 2 and 5% larger than the lateral surface area of the specimen at the end of consolidation for nominal shear strains equal to 10, 20 and 30%, respectively. Equation 4 can also be obtained through direct integration, as illustrated by its rightmost term.

Simple shear tests under cell pressure confinement have been typically carried out at UWA on specimens with height to diameter ratio (h/d) after consolidation equal to about 0.4 (most values between 0.38 and 0.42). This value is close to the upper bound (0.32) used by Vucetic and Lacasse (1982) for direct simple shear tests, for which no significant influence of this parameter on soil response was observed. Deformation of a cylindrical specimen along a single radial direction (e.g. the x-direction) such as that containing plane ABCD in Fig. 2b inevitably leads to radial non-uniformities (Doherty and Fahey 2011). For soil elements on the dry side of the critical state line and/or with relatively high initial shear stiffness, this pattern of deformation and specimen geometry may induce a noticeable zone of strain localisation in both direct simple shear (Budhu

1984, Dabeet et al. 2015) and simple shear specimens (Joer et al. 2011), such as that schematically illustrated by the plane that intersects points B, F and D in Fig. 2b and 2c, which will be referred to as the β plane. But regardless of whether or not direction BFD defines a clear failure plane in the specimen (Joer et al. 2011), the stress components on this inclined β plane can always be determined at any stage of the test. However, such an analysis requires proper assessment of the actual shape, boundary conditions and kinematics associated with the deformed specimen geometry during shearing. Thus, changes in lateral surface area prescribed by Eq. 2 and the non-trivial influence of σ_c resulting from the actual mode of deformation of the specimen (Fig. 2b and 2c) must be properly taken into account. Only after such assessments are carried out, stress path analyses similar to that described by Wroth (1984) for the horizontal plane (i.e., plane BGC in Fig. 2) can be conducted for other planes such as the β plane. A framework outlining additional checks required to carry out such analysis is discussed next.

Truncated oblique cylinder equilibrium

If the oblique cylinder defining the deformed configuration of simple shear specimens (Fig. 2b) were to be intersected by a hypothetical plane containing points BFD (i.e., the β plane), a truncated oblique cylinder would result. The intersection surface containing points BFD defines an ellipse with area

$$A_{\beta} = \frac{\pi d}{4} \sqrt{h^2 + (d - h \tan \gamma_x)^2} = \frac{\pi d h}{4 \sin \beta} \quad (5)$$

where β is the angle between the horizontal direction and the β plane direction (Fig. 2c).

For three-dimensional force equilibrium, the truncated oblique cylinder shown in Fig. 2 is re-plotted in Figure 3 in slightly different form. Forces N_{fdcg} and N_{fgh}

represent resultant forces due to the application of cell pressure on the inclined curved surfaces FDCG and FGB (Fig. 3a), respectively. Both N_{fdcg} and N_{fgb} are perpendicular to their point of application on those surfaces. Identical forces N_{fgb} and N_{fdcg} act on the back side of the lateral surface of the truncated oblique cylinder (Fig. 3b), but those are not shown in Fig. 3a, for clarity. A few comments can be made at this point about these forces. First, the point of application of such forces has been schematically shown in Fig. 3 to correspond to the actual point of application that results from proper integration of $\sigma_c dA$ over the corresponding region of interest. Secondly, the components of forces N_{fgb} projected along the x-axis are consistent with the S_{zx} direction (i.e., cell pressure applied to regions FGB facilitates shearing), whereas the opposite is true for N_{fdcg} (i.e., cell pressure acting on regions FDCG resists shearing). Along line FG (or line EFG, in Fig. 2), σ_c has no effect on specimen equilibrium along the x- and z-directions. Finally, the vertical projections of forces N_{fgb} and N_{fdcg} along the z-axis are downwards and upwards, respectively, thus having opposite effects on vertical equilibrium of the truncated oblique cylinder.

Equilibrium analysis along the y-direction yields no useful results since motion is not allowed along the y-direction. The remaining translational equilibrium equations along the x and z directions produce, respectively

$$S_{zx} + 2N_{fgb} \cos\left(\frac{\pi}{3}\right) \cos\left(\frac{\gamma_{zx}}{3}\right) + N_{\beta} \sin(\beta) = 2N_{fdcg} \cos\left(\frac{2\pi}{9}\right) \cos\left(\frac{5\gamma_{zx}}{9}\right) + S_{\beta} \cos(\beta) \quad (6)$$

$$2N_{fdcg} \cos\left(\frac{2\pi}{9}\right) \sin\left(\frac{5\gamma_{zx}}{9}\right) + N_{\beta} \cos(\beta) + S_{\beta} \sin(\beta) = 2N_{fgb} \cos\left(\frac{\pi}{3}\right) \sin\left(\frac{\gamma_{zx}}{3}\right) + N_{zz} \quad (7)$$

where

$$N_{f_{gb}} = \sigma_c A_{f_{gb}} = \sigma_c \frac{\pi dh}{16 \cos(\gamma_{zx})} \tag{8}$$

$$N_{f_{dcg}} = \sigma_c A_{f_{dcg}} = \sigma_c \frac{3\pi dh}{16 \cos(\gamma_{zx})} \tag{9}$$

$$N_{\beta} = \sigma_{\beta} A_{\beta} = \sigma_{\beta} \frac{\pi dh}{4 \sin(\beta)} \tag{10}$$

$$S_{\beta} = \tau_{\beta} A_{\beta} = \tau_{\beta} \frac{\pi dh}{4 \sin(\beta)} \tag{11}$$

$$N_{zz} = \sigma_{zz} A_H = \sigma_{zz} \frac{\pi d^2}{4} \tag{12}$$

$$S_{zx} = \tau_{zx} A_H = \tau_{zx} \frac{\pi d^2}{4} \tag{13}$$

For $\gamma_{zx}=0$, equations 6-9 can be simplified to yield the particular case of equilibrium representing the stress state of a right cylinder element at the end of consolidation (Joer et al. 2011).

Test methods and typical results

During a typical displacement-controlled undrained shearing stage of a simple shear test carried out on a saturated specimen, U_{zx} is varied (and so is γ_{zx} , since $U_{zz}\approx 0$) following a specified shear strain rate while S_{zx} is measured (and so is τ_{zx} , since the specimen's horizontal cross-sectional area does not change). The opposite takes place during load-controlled tests as S_{zx} is varied while U_{zx} is measured. The vertical stress σ_{zz} is kept constant to emulate field stress paths (Lambe 1967) deemed to be relevant to typical offshore geotechnics applications (Andersen 2004, Randolph and Wroth 1981). In

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3 simple shear tests with cell pressure confinement, this can be done by varying the cell
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5 pressure σ_c by whatever amount may be required to ensure $\Delta N_{zz}=0$, as schematically
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7 shown in Figure 4a. Thus, variations in vertical effective stress are a mirror image of
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9 pore pressure changes induced in the soil during undrained shearing. This also forms the
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11 theoretical basis for the interpretation of direct simple shear test results (Dyvik et al.
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13 1987). In direct simple shear tests, however, pore pressures are not measured and
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15 changes in σ'_{zz} are assumed to be directly equivalent to measured σ_{zz} changes required
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17 to keep constant specimen height (thus volume).
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21 Figure 4b shows the probable variation of the same parameters so as to yield the
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23 same vertical effective stress response during undrained shearing if the cell pressure
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25 were to be kept constant. This expected response is supported by numerical modelling
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27 results conducted by Doherty and Fahey (2011) on normally consolidated soil. It should
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29 be noted that the stress boundary condition shown in Fig. 4b is not used to carry out
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31 simple shear tests with cell pressure confinement at UWA. However, the idealised
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33 response shown in Fig. 4b implies pore pressures would have to adjust further to
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35 compensate for changes in total vertical stress as well. Thus, pore pressure changes for
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37 this boundary condition can no longer be a mirror image of σ'_{zz} changes, as shown in
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39 Fig. 4a. While the boundary conditions shown in Fig. 4b are not representative of
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41 testing methods used in UWA simple shear tests, the latest generation of simple shear
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43 devices recently developed at UWA does have the capability now to impose this
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45 boundary condition as well, if required. A possible disadvantage of this constant cell
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47 pressure boundary condition is that none of the boundary normal stresses (σ_{xx} , σ_{yy} and
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49 σ_{zz}) will remain constant during undrained shearing, as discussed previously.
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55 The typical undrained response of an anisotropically-consolidated medium-
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57 dense clean silica sand specimen in a strain-controlled simple shear test with cell
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pressure confinement is shown in Figure 5. The specific gravity (ASTM D854-10), minimum void ratio (ASTM D4253-06 Method 1A) and maximum void ratio (ASTM D4254-06 Method B) of the sand tested are 2.65, 0.494 and 0.639, respectively. The specimen was reconstituted using dry deposition (Ishihara 1996) and its diameter, net height (h), and relative density after consolidation were equal to 69.8 mm, 30.9 mm, and 59%, respectively. The change in shear stress on the horizontal plane along with the pore pressure response of the soil are shown in Fig. 5a. These are response parameters that are actually measured during the test. As described earlier, σ_c is the test parameter that is controlled by the system (Fig. 5b) so as to yield the desired $\Delta\sigma_{zz}=0$ condition, while the specimen height is maintained constant via active or passive height control. The latter was used to obtain the results shown in Fig. 5. Passive height control performance is illustrated in Figure 6. The shear strain used in the test (0.05%/s) is well within the limits required in ASTM D6528-07 for similar (but not identical) direct simple shear tests, including testing stages between the shear strain at 50% of the peak shear stress (γ_{50}) and the end of the test (Fig. 6a). Vertical strains during shearing are within 0.05%, which corresponds to the level of tolerance required by ASTM D6528-07 for direct simple shear tests (Fig. 6b). For the most part of the test (shear strains less than about 20%), the specimen height is maintained within about 0.015 mm (15 μ m). The slight jump at around 20% shear strain is due to effective stress reversal ($\sigma'_{xx}=\sigma'_{zz}$) on the vertical ABCD plane as the vertical load transitions from compression to extension and a mechanical compliance of about 0.010 mm (10 μ m) becomes apparent. In simple shear tests with cell pressure confinement, specimens are back pressure saturated and sudden changes in vertical actuator motion due to system compliance do not necessarily correspond to changes in specimen height. This is supported by the relatively smooth response in all other test parameters shown in Fig. 5.

The cell pressure becomes equal to the vertical stress at a shear strain of around 18% (Fig. 5b). Thus, it may be expected that effective stress reversal on the ABCD plane (i.e., $\sigma'_{xx}=\sigma'_{zz}$) may occur at a shear strain slightly larger than 18% for this test, as discussed previously. However, along the EFG line (Fig. 2b), which is parallel to the ABCD plane, effective stress reversal ($\sigma'_{yy}=\sigma'_{zz}$) is already taking place at a shear strain of 18% since the normal effective stress acting on the EFG line (σ'_{yy}) is equal to the intermediate effective principal stress (σ'_2) along that line at anytime during the test. This variation in effective horizontal stress throughout the lateral surface of test specimens may be partially related to the non-uniform stress contours that develop within cylindrical simple shear specimens subjected to single directional shear loads at around failure conditions (Doherty and Fahey 2011). However, comparisons between real experimental results and output from numerical simulations must always be carried out with caution to ensure that the boundary conditions (Fig. 4) and testing details (e.g. h/d ratio, cap roughness, etc.) assumed in numerical simulations are truly representative of real testing conditions. Actual boundary conditions used in typical simple shear tests with cell pressure confinement are those schematically shown in Fig. 4a, not Fig. 4b. Nevertheless, since effective stress paths during undrained shearing may be primarily influenced by the soil state after consolidation, such a general comparison may still be useful.

Stress states for the truncated oblique cylinder

Inspection of Equations 6 through 13 shows that the only variables left to discover are the normal and shear stress components on the inclined β plane, σ_β and τ_β , respectively. All other variables can be either measured or deduced from proper inspection of the geometry of deformation of the specimen at any stage of the test.

The β plane does not need to be necessarily associated with an observed failure plane (Joer et al. 2011) – it is simply used here to allow assessment of the average stress state on an additional plane intersecting the truncated oblique cylinder, other than the horizontal plane (de Josselin de Jong 1971). In turn, this allows a second, independent point to be defined to represent the stress Mohr circles for the soil element being tested in spite of the fact no complimentary shear stresses are applied to the lateral surface of the specimen, which has always limited the complete assessment of the stress state at a point for simple shear tests on cylindrical specimens. This alternative framework is still a simplified approach to analyse results deriving from simple shear tests on cylindrical specimens since the average stress states derived for both the horizontal plane BGC and the inclined β plane solely relate to the plane strain state associated with vertical plane ABCD. However, this analysis does take into account the real shape of the deformed specimen during shearing as well as the actual contribution of the cell pressure on soil response, test control, and in the three dimensional force equilibrium analysis underpinning the proposed framework.

Perhaps the main advantage of this analysis is that the average stress state for the β plane can be inferred for any level of shearing deformation imposed to the specimen during the test. This includes (but is not limited to) the initial specimen geometry (right cylinder) at the start of shearing (Fig. 2a), which should be associated with a value of S_{zx} equal to zero, strictly speaking, as well as the more complex geometry associated with the truncated oblique cylinder (Fig. 2b), as described previously.

If the stress coordinates for the horizontal and β planes are defined as $(\sigma'_{zz}, \tau_{zx})$ and $(\sigma'_{\beta}, \tau_{\beta})$, respectively, the location of these two points on each stress Mohr's circle is schematically shown in Figure 7 for six arbitrary stages of the test previously shown in Fig. 5: at the end of anisotropic consolidation (i.e., corresponding to the first and

smallest Mohr circle shown in Fig. 7), and at five additional shear strain levels during undrained shearing (5, 10, 15, 20 and 25%). For convenience, positive shear stresses leading to clockwise rotation around the y-axis (Fig. 3a) are plotted downwards in Fig. 7 to allow the β plane direction to be defined directly from the plot.

In addition to the two stress paths related to the evolution of stress states for the horizontal and β planes, Figure 7 also depicts the Mohr circle for each strain level mentioned above along with the estimated pole location for each circle. Mohr circles were generated using the simple procedure described in Appendix A so as to fit and include the two stress states defining each circle. The angle β was calculated along with coordinates $(\sigma'_\beta, \tau_\beta)$ for each nominal value of γ_{zx} used in the analysis. Table 1 summarises the five input parameters used in the analysis (obtained from test results shown in Fig. 5) as well as three key output results. The β plane inclination (simply defined following a clockwise rotation from the horizontal direction in Fig. 7) increases slightly from 23.9° at the end of consolidation to 26.5° at a shear strain of 25%. In spite of the typical variability associated with experimental results and the simple approach used to produce Mohr circles and pole locations, it is possible to fit two lines intersecting the pole of each Mohr circle with inclinations equal to either zero or β for the horizontal or β planes, respectively. This is schematically shown for the first and last stress Mohr circles presented in Fig. 7 and suggests the analysis yields realistic values for σ'_β and τ_β .

Stress and strength simple shear parameters

Simple shear strength for vertical plane ABCD

The undrained monotonic response of medium dense sand cannot be properly described using a simple elasto-plastic model outside its initial elastic range. Such response may

include up to three additional relevant states, namely phase transformation, peak and critical states (Murthy et al. 2007). For simplicity, three straightforward methods were used to assess the secant friction angle (ϕ) of the soil (Bolton 1986) associated with the plane strain boundary condition presumed to apply to the vertical plane ABCD. In the first method, ϕ was derived from the average stress state on the horizontal plane (σ'_{zz} , τ_{xz}). In the second method, the traditional plane strain interpretation for a right cylinder specimen was used, which implies the average stress components on horizontal and vertical planes are (σ'_c , τ_{zx}) and (σ'_{zz} , τ_{xz}), respectively (i.e., horizontal stress is equal to the cell pressure and complimentary shear stresses are applied to the vertical sides AB and CD of the ABCD element shown in Fig. 2c). Method 3 is based on the truncated oblique cylinder analysis presented in this paper and displayed in Fig. 7. Table 2 summarises the results obtained with these three methods.

Results shown in Fig. 7 and Table 2 indicate ϕ values derived from the average stress state associated with the horizontal plane (Method 1) are conservatively estimated in the case of the medium-dense silica sand tested. In this method, the maximum τ_{zx}/σ'_{zz} stress ratio yielding the maximum value for ϕ equal to about 34° can only be defined at large strains for stress components associated with the horizontal plane. Since the horizontal plane does not represent the stress state at failure (Fig. 7), this method yields conservative values for the peak friction angle (ϕ_p). The critical state friction angle (ϕ_c) may be reasonably estimated using this approach, as the stress state related to the horizontal plane moves towards the failure envelope of the soil at large strains (Ladd and Edgers 1972).

Stress paths for the plane strain stress invariants s' and t for methods 2 and 3 are superimposed in Figure 8 to the same stress Mohr circles (determined using Method 3) shown earlier in Fig. 7. For shear strains between 5 and 15%, the traditional approach of

assuming complimentary shear stresses apply to vertical sides AB and CD and the horizontal stress is equal to the cell pressure (Method 2) underestimates ϕ_p for the medium dense sand tested (see top insert in Fig. 8). Within this strain range, underestimated values of ϕ from Method 2 are primarily due to the fact this method overestimates σ'_{xx} (by assuming it to be equal to σ'_c) rather than errors in the assessment of the lateral surface area of the specimen.

Method 3 yields the highest values of ϕ that can be obtained for each Mohr's circle of stress (see top insert in Fig. 8) as the method accounts for the entire stress state defined by each Mohr's circle. Maximum stress obliquity (thus ϕ_p , for this soil) takes place at a shear strain of around 10% (Fig. 8), which corresponds to the third Mohr's circle shown in Figs 7 and 8. Naturally, Method 1 is oblivious to this feature of soil behaviour as it relies solely on measurements made on the horizontal plane.

The undrained shear strength (s_u) of the soil is equally predicted regardless of the method used (see lower insert plot in Fig. 8). This observation must be taken with caution. This does not mean s_u estimated from Method 1 (based on the shear stress acting on the horizontal plane τ_{zx}) is a good predictor of failure conditions in the soil. It simply means that τ_{zx} is a good predictor for the deviatoric invariant of the stress tensor in plane strain (t). Soil failure is controlled by pore pressures and thus by the effective stresses in the soil, which are in fact poorly predicted by either s_u or τ_{zx} , naturally. For normally consolidated soil with easily predictable elasto-plastic response, s_u may be directly related to failure conditions. However, this assumption does not hold for more complex soil responses such as those including phase transformation and/or strain softening, as discussed previously.

Intermediate principal stress parameters in simple shear

As described earlier, line EFG represents a particular location on the lateral surface of the specimen where the cell pressure is always equal to the normal horizontal stress component perpendicular to plane ABCD. Thus, by definition, the value of the intermediate effective stress σ'_2 ($=\sigma'_{yy}=\sigma'_c$) can be assessed at any stage of a simple shear test with cell pressure confinement along line EFG. However, σ'_2 values obtained before localisation develops (i.e., $\gamma_{zx}<20\%$) may be more reliable than those at larger strains (Wood et al. 1979). Values of σ'_2 determined by this approach are shown in Table 2 along with the intermediate principal stress coefficient (b) (Bishop 1966) and Lode's angle (θ), which are defined, respectively, as

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{\sigma'_2 - \sigma'_3}{\sigma'_1 - \sigma'_3} \quad (14)$$

$$\theta = \arctan \left[\frac{1}{\sqrt{3}} (2b - 1) \right] \quad (15)$$

where θ is in radians.

Values of b and θ calculated using Method 3 are illustrated graphically by the stress Mohr circles for three-dimensional stresses shown in Figure 9, which correspond to a single, arbitrarily selected level of deformation ($\gamma_{zx}=15\%$). At this strain level, the stress state of the soil must lie within the hatched area. Variations in both b and θ during undrained shearing are shown in the insert plot of Fig. 9. At a shear strain of around 18%, which may be associated with the stage of the test around which effective stress reversals start to take place and the β plane may turn into a failure plane (Fig. 7), Lode's

angle takes a value of around 0° while σ'_2 is located midway between the major and minor effective principal stresses with $b \sim 0.5$ (Randolph and Wroth 1981).

For simple shear tests with cell pressure confinement on medium dense sand specimen with geometry and dimensions similar to those discussed in this paper, the stress state on the horizontal plane does not represent the worst failure condition for the soil element of interest. Simple shear failure conditions may be more representatively prescribed by the stress state associated with a hypothetical inclined plane (Budhu 1984, Joer et al. 2011). For the soil tested, this condition starts to develop at shear strains between 15 and 20%, when the effective intermediate principal stress is midway between the major and minor principal effective stresses ($b \sim 0.5$ and $\theta \sim 0^\circ$). This failure condition is clearly illustrated in Fig. 7 when the beta plane stress path starts to level off for the Mohr circle with $\gamma_{zx} = 15\%$, while the horizontal plane stress path remains unchanged and continues to move at constant slope with increasing shear strains. If the maximum stress obliquity (thus ϕ_p) may be of interest for design, for example, the designer must realise ϕ_p mobilisation takes place at earlier strains ($\sim 10\%$) with an intermediate principal stress ratio of around 0.45. Thus, it is important for design engineers relying on simple shear test results to clearly identify what failure means (along with its corresponding design parameters) in the context of their analysis rather than simply trying to derive simple shear design parameters using procedures originally designed for direct simple shear tests. Interpretation of simple shear tests following analyses that neglect the true effect of σ'_2 on soil response may be in error and should be avoided.

Conclusions

In simple shear tests with cell pressure confinement subjected to shear deformation

along a single horizontal direction, the shape of the test specimen varies from right cylinder to that of an oblique cylinder. As such, the horizontal stress is not equal to the cell pressure at all times during the test. The horizontal stress varies throughout the lateral surface of the specimen.

Typical testing methods and results are discussed for a medium dense clean silica sand specimen. Equipment performance and soil response is shown to be consistent with testing requirements expected for similar direct simple shear tests in terms of shear strain rate, vertical stress, and specimen height control.

An analysis of equilibrium was presented that accounts for the actual, deformed specimen shape, the changing lateral surface area of the specimen during shearing, as well as the actual effect of cell pressure during the test. An alternative framework was described and used to interpret test results, which shows that conventional methods of interpretation of simple shear test results yield conservative shear strength parameters for the medium dense silica sand tested.

The alternative framework outlined in this paper also illustrates how the intermediate principal effective stress can be assessed in these tests. For the medium dense silica sand tested, failure states develop when the intermediate principal effective stress is midway between the major and minor principal effective stresses.

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Appendix A – Procedure to draw a Mohr circle of stress based on two previously-defined stress states from simple shear tests:

1. Identify two representative stress states, for example H (σ'_{zz} , τ_{zx}) and B (σ'_{β} , τ_{β}).
2. Draw a line (chord) intersecting points H and B (Fig. A1). Alternatively, define the equation of the line (chord) intersecting these two points analytically or numerically (using a computer spreadsheet).
3. Draw the perpendicular bisector of the chord defined in (2). Alternatively, define the equation of a line (bisector) that is perpendicular to the chord defined in (2) and that passes through the chord centre (i.e., point X along the chord, which is equally distant from points H and B).
4. Extend the chord bisector until it intersects the horizontal (normal effective stress) axis at point C. As the perpendicular bisector of a chord always passes through the centre of a circle, this allows locating the centre (and radius) of the stress Mohr circle of interest, which also includes the two original stress states defined in (1). Alternatively, calculate the effective normal stress defining the centre of the Mohr circle by assuming a value of zero for the shear stress in the equation of the bisector line defined in (3).
5. Draw the complete Mohr circle of stress. Alternatively, define the equation of the circle based on the coordinates of its centre defined in (4) as well as the coordinates of the two points defined in (1).

Analytical steps described above can be easily programmed into a computer spreadsheet for increased accuracy of numerical calculations (as opposed to a purely graphical solution). This analytical/numerical approach was used to generate the results discussed in this paper.

Table 1. Input parameters used in the analysis and output results

γ_{zx}	σ'_c	σ'_{zz}	τ_{zx}	U_{zx}	σ'_β	τ_β	β
(rad)	(kPa)	(kPa)	(kPa)	(mm)	(kPa)	(kPa)	(deg)
0.00	42	102	0	0.00	91	24	23.9
0.05	53	71	43	1.55	50	45	24.4
0.10	110	152	94	3.10	103	98	24.9
0.15	213	251	159	4.67	170	154	25.4
0.20	328	303	196	6.26	207	164	25.9
0.25	408	329	219	7.89	222	164	26.5

Table 2. Calculated parameters and soil friction angles for the three methods used

	Method 1			Method 2					Method 3								
γ_{zx}	τ_{zx}/σ'_{zz}	ϕ		σ'_1	σ'_3	s'	t	ϕ	σ'_1	σ'_2	σ'_3	b	θ	s'	t	ϕ	
(rad)	(deg)			(kPa)	(kPa)	(kPa)	(kPa)	(deg)	(kPa)	(kPa)	(kPa)	(deg)			(kPa)	(kPa)	(deg)
0.00	0.00	0		102	42	72	30	25	102	42	42	0.00	-30	72	30	25	
0.05	0.61	31		106	18	62	44	46	105	53	14	0.43	-5	60	46	50	
0.10	0.62	32		228	34	131	97	48	219	110	22	0.45	-3	121	99	55	
0.15	0.63	32		392	72	232	160	44	381	213	59	0.48	-1	220	161	47	
0.20	0.65	33		512	119	315	196	38	511	328	118	0.53	2	315	197	39	
0.25	0.66	34		591	147	369	222	37	593	408	149	0.58	6	371	222	37	

Fig. 1. New generation of UWA simple shear apparatus: (a) schematic diagram and (b) revised force and displacement transducer strategy

Fig. 2. Soil element in simple shear with cell pressure confinement: (a) end of consolidation, (b) during shearing, and (c) 2D representation of vertical plane ABCD intersecting the middle of the specimen

Fig. 3. (a) 3D and (b) top views of boundary forces acting on truncated oblique cylinder (Note: normal forces due to cell pressure application on the back side of the lateral surface area are not shown in part (a) for clarity)

Fig. 4. Idealised variations in pore pressure and effective vertical stress with increasing shear strain during undrained simple shear test with cell pressure confinement on normally consolidated soil for (a) constant vertical stress and (b) constant cell pressure boundary conditions

Fig. 5. Typical (a) stress-strain and pore pressure responses and (b) variations in vertical and confining stresses (both total and effective) with increasing shear strain for medium-dense silica sand specimen subjected to undrained shearing in simple shear test with cell pressure confinement

Fig. 6. System performance for simple shear test with cell pressure confinement shown in Fig. 5: (a) strain rate control and (b) specimen height change using passive height control

Fig. 7. Mohr's circles of stress at six stages of undrained shearing during a simple shear test with cell pressure confinement on medium-dense silica sand (first and last circles are shown with solid symbols along with directions of the horizontal and β planes intersecting the pole of those circles)

Fig. 8. Plane strain stress paths and strength parameters for methods 2 and 3

Fig. 9. Mohr's circles for three-dimensional stress at a point of the truncated oblique cylinder at a single stage of shearing deformation during undrained simple shear test with cell pressure confinement

Fig. A1. Mohr circle derived from two stress states from a simple shear tests

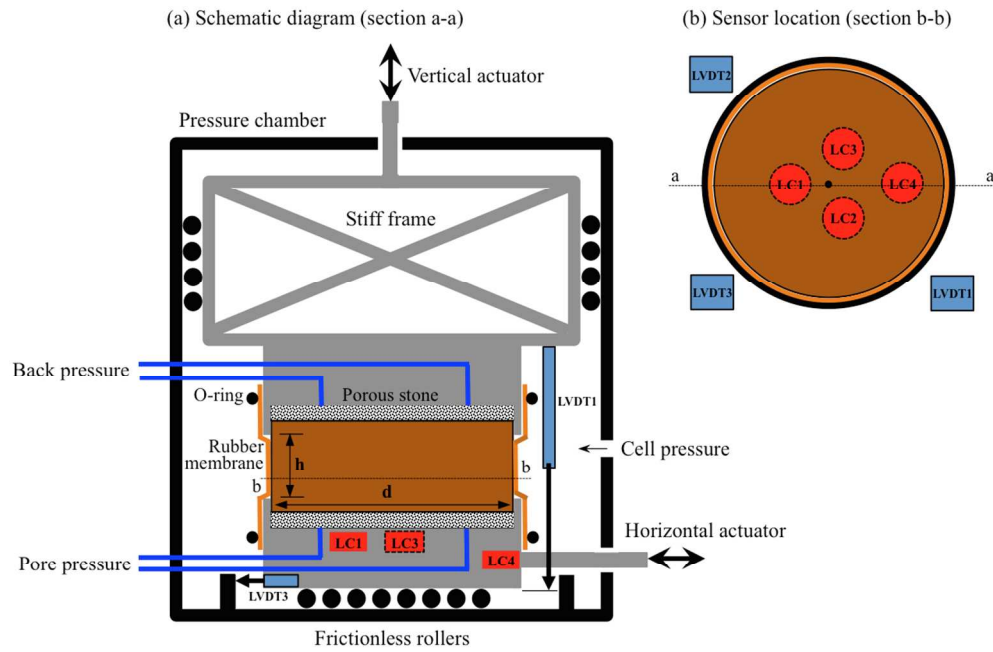


Figure 1
453x296mm (72 x 72 DPI)

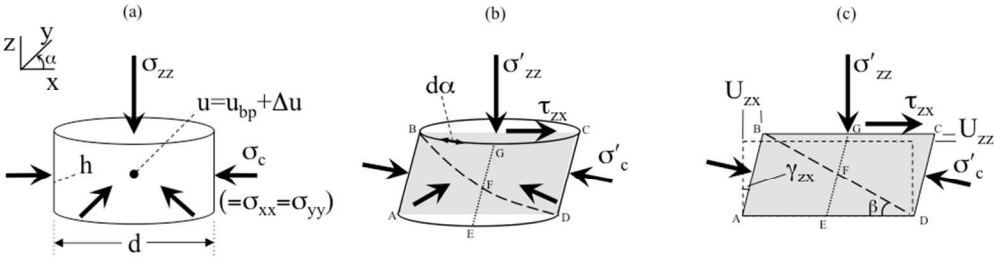


Figure 2
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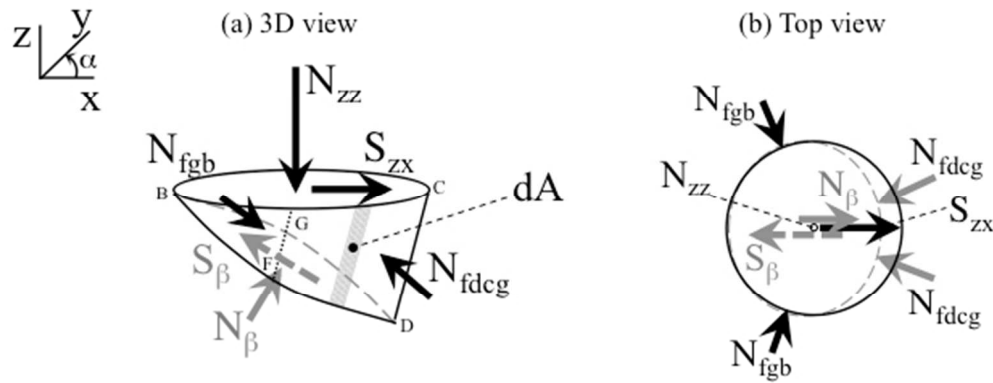


Figure 3
219x86mm (72 x 72 DPI)

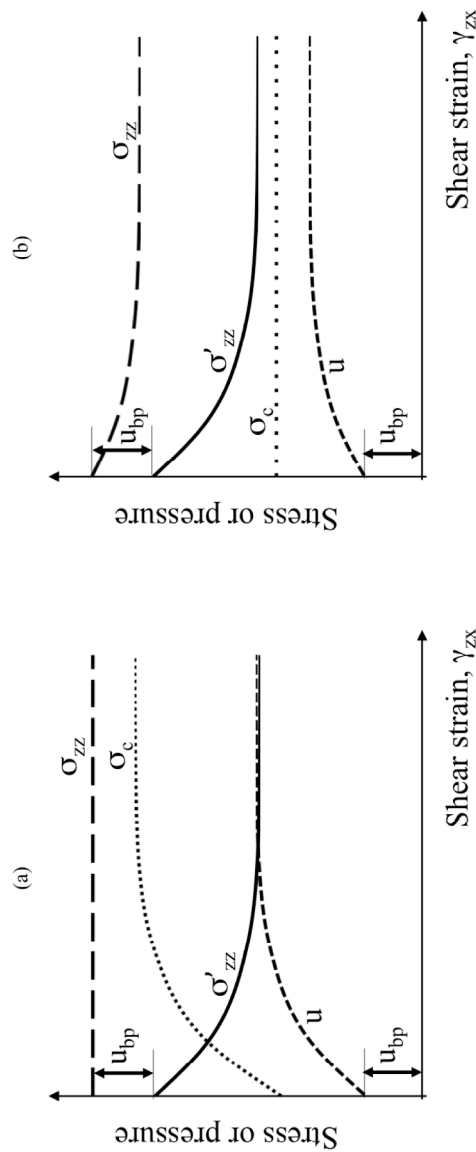


Figure 4
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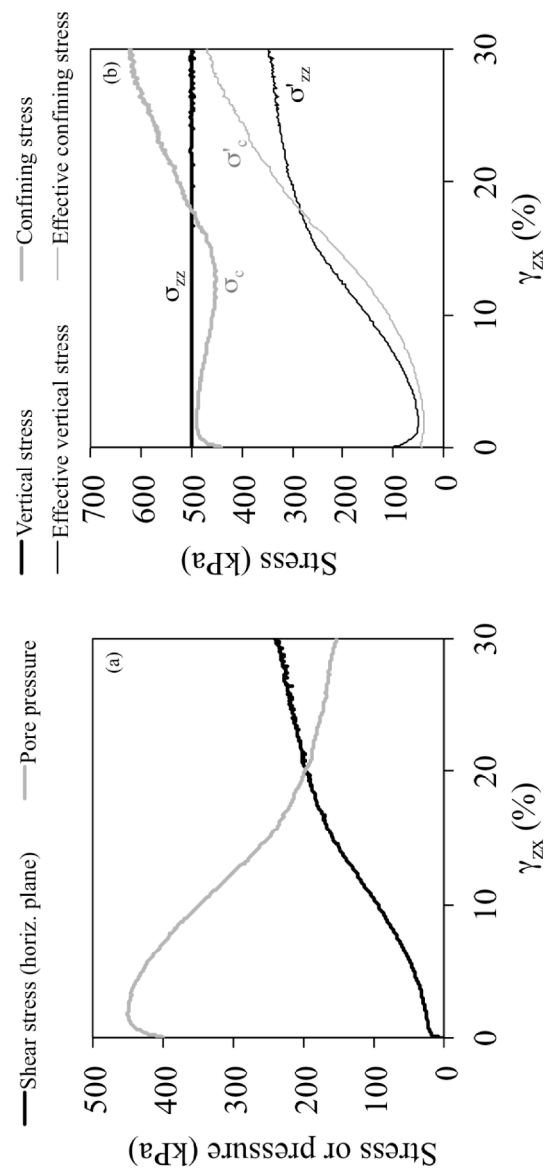


Figure 5
297x420mm (300 x 300 DPI)

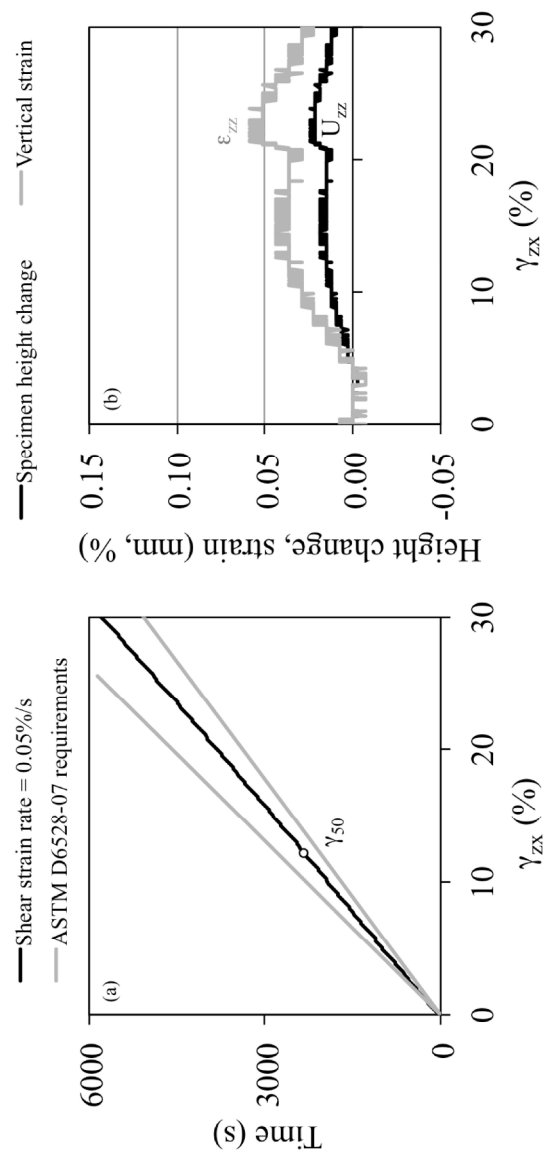


Figure 6
297x420mm (300 x 300 DPI)

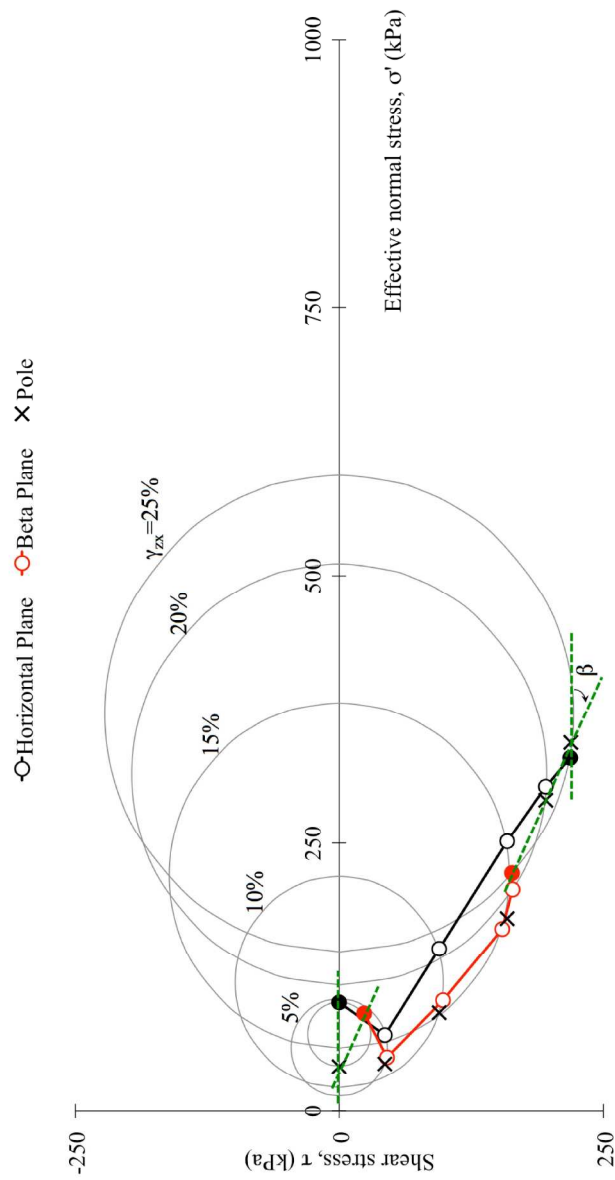


Figure 7
297x420mm (300 x 300 DPI)

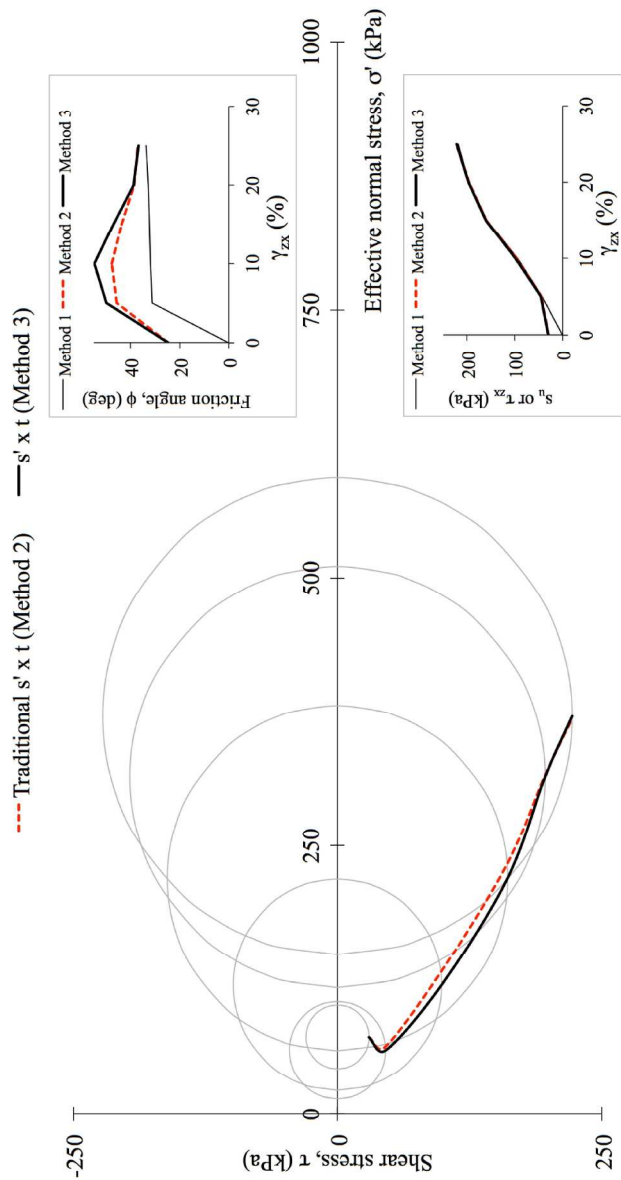


Figure 8
297x420mm (300 x 300 DPI)

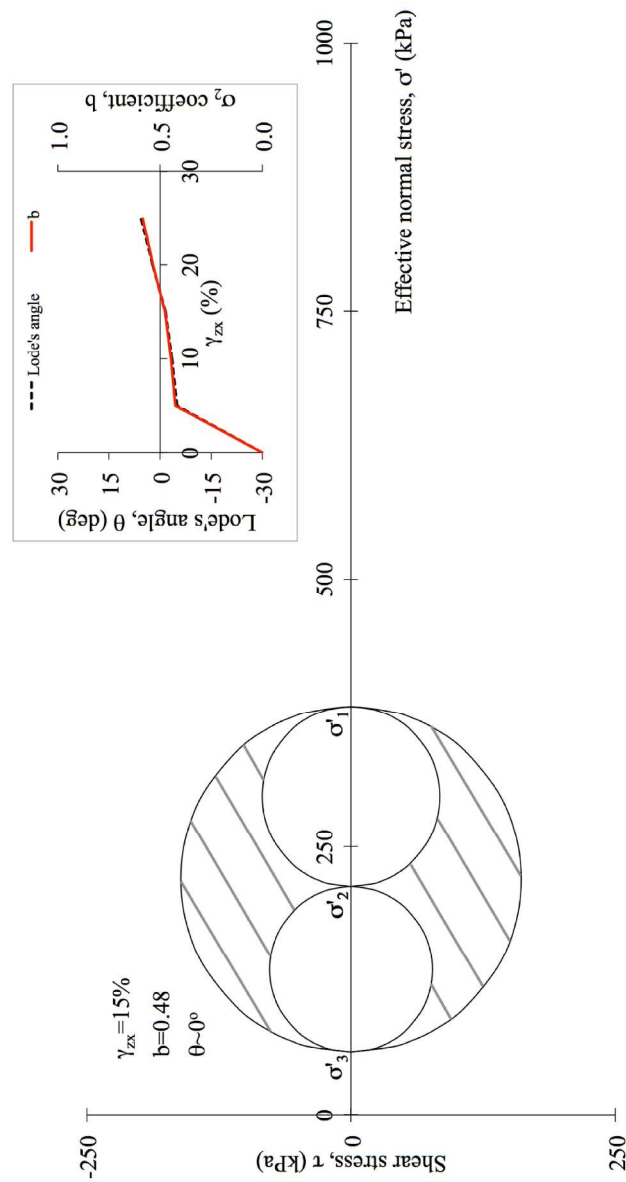


Figure 9
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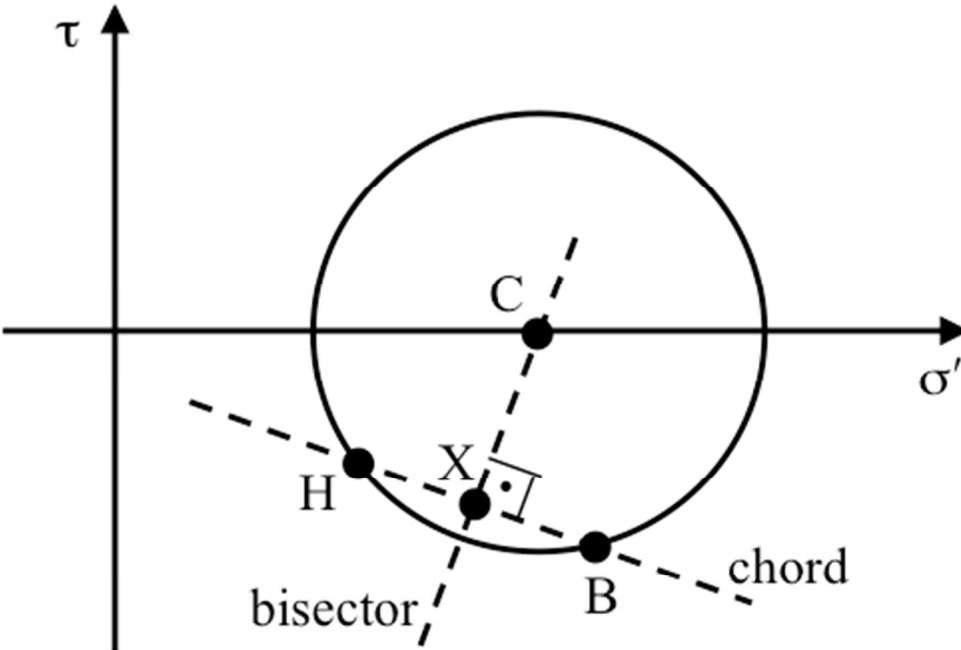


Figure A1
177x120mm (72 x 72 DPI)