

RADIATION HEAT TRANSFER
IN CYLINDRICAL FURNACES

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BY

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ABSTRACT

Experimental measurements were made on two small cylindrical furnaces, one being oil-fired with an end sink, the other being gas-fired with a wall sink. Heat balances were performed and the heat transferred to the sinks was found.

Various methods were used to predict the heat transfer to the sink and the predicted values were compared with those measured.

The method proposed by Hottel and Cohen was found to be accurate if sufficiently detailed analyses were undertaken. The reception factors necessary to the analyses of small cylindrical furnaces were calculated based on data of Hoffman. The relative effects of certain parameters on the accuracy of prediction were investigated. These were number of gas zones employed, representation of the furnace gas and variation of absorption coefficient between gas zones.

The Lobo-Evans and Hottel methods were analysed and a modified Lobo-Evans method is proposed which is thought to be a simple and effective design method.

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Chapter 1. Introduction.

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Chapter 1.Introduction.1.1. General.

Furnaces have been used for many centuries as a means of transferring large quantities of heat to localised sinks. In the widest sense the word furnace implies the attainment within an enclosure of temperatures above ambient. Usually these temperatures are obtained by burning a fuel and are sufficiently high to necessitate refractory walls.

Thus the first problem of furnace design is one of containing and using high temperatures. Once this problem has been surmounted attention is directed to the efficient conversion of the fuel into energy, and the absorption of this energy by the sink. During the last forty years much work has been directed towards explaining and predicting the performance of furnaces, and gradually sophisticated methods are being developed.

The heat transfer in a furnace takes place by radiation and convection. In most cases radiation is by far the larger contributor and also the most difficult to predict. Before considering radiation brief consideration will be given to convection.

1.2. Convection in furnaces.

Many furnaces contain a tubular heat sink and convection to these tubes is by no means insignificant. Such furnaces often have a second section in which the cooler flue gas yields heat, mainly by convection, to banks of tubes.

Many correlations exist which predict with reasonable accuracy the convection heat transfer coefficients between gas and tubes. McAdams ¹ presents a summary of such data.

The performance of convection sections of furnaces is correlated by Monrad ² who has added an allowance for the radiation which also takes place. Thus when convection predominates performance prediction may be quite accurate.

1.3. Radiation in furnaces.

The radiation in a furnace fired by burning fuel may originate either from the transient flame constituents or from the products of combustion. Separate consideration will be given to these below.

1.3 (a) Radiation from transient constituents.

The present state of knowledge of the behaviour of flames is far from complete. Much work is being done at Ijmuiden by the Flame Radiation Research Joint Committee ³.

Reviews of the present state of knowledge are available^{3a,3b,4,5}

It is not yet possible to calculate the effective emissivity of a flame from external considerations (fuel, burner etc.) only, and the standard experimental methods can lead to widely differing estimates^{6,7}.

In order to explain the transfer in a furnace the existence of a flame should be acknowledged, but because of the lack of data it has been customary to regard the furnace as being filled with products of combustion only. An attempt has been made by Thorneycroft and Thring⁶ to allow for the flame.

The flame was represented by an idealised zone and for emission purposes the zone was considered to be a solid with an overall effective emissivity. The zone was assumed to absorb as though it were products of combustion only. This representation appears reasonable providing an adequate estimation of the emissivity can be made. Thorneycroft⁷ used two methods for measuring emissivity, a modification of Sherman's⁸ total radiation method and the optical pyrometer method of Hottel and Broughton⁹. While these two methods agreed within themselves they produced emissivities varying from 0.5 to 0.89. Large variations in the estimated temperature of the flame zone resulted.

The heat transfer predicted in a furnace is very dependent on the temperature and emissivity of the flame zone. Thorneycroft states ⁷ that almost any desired fraction of the total theoretical heat transfer can be obtained by altering the assumed flame emissivity.

It would thus seem that until flame emissivities can be determined more accurately considerable errors may be introduced by allowing for the flame. A more accurate prediction of total heat transfer will probably be obtained assuming products of combustion only. If, however, the object is to predict local heat transfer rates in the furnace an allowance for the flame would seem necessary.

1.3(b) Radiation from products of combustion.

Most analyses of furnace performance have assumed that the fuel is instantaneously transformed into hot products of combustion without passing through a flame stage. Some of the many different approaches using this assumption are considered below.

1.4. Furnace correlations.

There have been proposed correlations of two different types, purely empirical correlations which make no assumptions about the internal behaviour of the gas and semi-theoretical correlations based on Stephan's Law.

1.4(a) Empirical correlations.

The earliest correlation was presented by Hudson ¹⁰ as a result of his work on steam boiler furnaces. The fraction of the heat input above the steam temperature which is absorbed by the ultimate receiver μ is given by

$$\mu = \frac{1}{1 + \frac{G \cdot C}{45}} \quad 1.1$$

This correlation was not very successful and Orrok ¹¹, with additional boiler data, modified it to allow for type of fuel,

$$\mu = \frac{1}{1 + \frac{G \sqrt{C}_0}{27}} \quad 1.2$$

This equation fitted the data but is patently invalid as firing rate tends to zero.

Many other considerations were proposed ^{12,- 18} for different types of furnace. Possibly the most advanced is that due to Wilson, Lobo and Hottel ¹⁹ who attempt to allow for different types of furnaces. It is :

$$\mu = \frac{1}{1 + \frac{\left[\frac{G \sqrt{Q/A'}}{1000} \right]}{4.4} + \frac{\left[\frac{G \sqrt{Q/A'}}{1000} \right]^2}{70}} \quad 1.3$$

where $A' = \alpha A_{cp} + \beta A_R$

and $\beta = \frac{1}{\frac{p_f}{1-p_f} + \frac{1}{F_R}}$

* for meaning of symbols see Nomenclature section.

This correlation is found to suffer from the same limitation as other empirical correlations. It can only be used for furnaces similar to those on which it is based, and cannot be used to predict the performance of a furnace shewing significant differences from the basic type or types.

Consideration of the large number of variables involved in a furnace shews that it is to be expected that an empirical correlation cannot be used outside a narrow range of variables.

1.4(b) Correlations with theoretical basis.

De Baufre²⁰ proposed a correlation based on Stephan's Law

$$q = A_c E \left[\left(\frac{T}{1000} \right)^4 - \left(\frac{T_s}{1000} \right)^4 \right] \quad 1.4$$

The term E is an effectiveness factor and was evaluated on a basis of heat liberation per unit furnace volume.

Unfortunately this basis yielded poor correlation of data.

Lobo and Evans²¹ suggested that Stephan's Law should be used as a basis and that individual factors should be evaluated either theoretically or empirically to the greatest possible accuracy. The proposed method involves the trial and error solution of the rate equation

$$q = .173 \phi \left[\left(\frac{T_g}{100} \right)^4 - \left(\frac{T_s}{100} \right)^4 \right] A A_{cp} + h_c A (T_g - T_s) \quad 1.5$$

with the gas side heat balance

$$q = H_{in} - M \cdot C_p \cdot T_g \quad 1.6$$

The term effective cold plane area αA_{cp} is evaluated from charts presented by Hottel²² which, while not being completely accurate²³, are the best available method.

The factor ϕ is based on Christiansen's formula²⁴ and is given by

$$\phi = \frac{1}{\frac{1}{F_s} + \frac{1}{F_c} - 1} \quad 1.7$$

and

$$F_s = p_f \left(1 + \left(\frac{A_R}{\alpha A_{cp}} \right) \frac{1}{1 + \left(\frac{p_f}{1 - p_f} \right) \frac{1}{F_{Rc}}} \right) \quad 1.8$$

The factor F_{Rc} was found empirically by Lobo and Evans but Hottel²⁵ gives equations for its evaluation.

The inherent limitation of this method is that the same temperature must serve for both gas exit temperature and mean radiating temperature in the enclosure. Despite this limitation quite good correlation can be obtained, especially for large furnaces.

Hottel²⁶ has extended the Lobo and Evans treatment to remove the necessity for gas exit temperature to equal mean radiating temperature. His correlation is essentially the Lobo and Evans equation plus an estimate of the difference between gas exit and mean radiating temperatures.

The following three equations must be solved by trial and error.

$$q = \overline{GS} \sigma (T_G^4 - T_1^4) + h_1 A_1 (T_G - T_1) \quad 1.9$$

$$\frac{H_{in} - q}{H_{in}} = \frac{T_G - B - T_o}{T_{AF} - T_o} \quad 1.10$$

$$B = \frac{q}{H_{in}} \cdot \frac{T_{AF} - T_o}{4} \quad 1.11$$

$\overline{GS}$ is the Lobo-Evans $\phi \alpha A_{cp}$ multiplied by a term $\frac{1-a^3}{1-a^4}$, where $a = T_1/T_G$, to render this interchange factor non-directional.

This method predicts furnace performance quite accurately. The one major empirical step is the selection of the divisor, taken as 4 by Hottel, in the equation for B. If this divisor were correlated effectively with type of furnace the method would be very accurate indeed.

1.5 Gas emission.

It has been mentioned that the emission in a furnace is taken as being from products of combustion only. The correlations of Lobo-Evans and Hottel presuppose a knowledge of the effective emission from a mass of hot gas.

Much work has been done, both by calculation and experiment, to try to establish the emission and absorption characteristics of gases.

1.5(a) Schack radiation charts.

Schack ²⁷ started with the Planck distribution law and, by integrating over all wave lengths and substituting values for absorption coefficient arrived at the total radiant energy emitted from a gas mass.

The Planck distribution law is a plot of intensity against wavelength with temperature as parameter. The values of constants taken gave the monochromatic intensity I_{λ} . The effective limits of absorption bands were determined and the total radiation found by graphical integration of the equation

$$E_T = \int_{\lambda_a}^{\lambda_b} I_{\lambda} d\lambda. \quad 1.12$$

This value of E_T is the total radiation for an absorption band $\Delta\lambda$ for an infinitely thick gas layer.

The total radiation for a thickness h of gas can be found from

$$E = E_T e^{-kh} \quad 1.13$$

However absorption coefficient k varies with wavelength. Schack plotted k as a function of λ , divided the area into strips and rearranged the strips to give a linear variation, or series of linear variations, with wavelength. By incorporating various rather unwieldy shape factors he was able to evaluate the total radiation.

In order to achieve accurate results very small increments must be used and even then, as the variation of intensity and absorption coefficient with temperature and wavelength are not known completely, may not be very accurate. Also Beer's Law (i.e. that the exponential term in equation 1.13 can be represented in terms of concentration) was assumed and this can lead to significant errors^{28,29,30} in the case of water vapour.

1.5(b) Mean beam length.

In Schack's work various factors were introduced to allow for the gas shape. Hottel has shown^{31,32} that by a suitable choice of variable, called mean beam length, these factors can be made to approach unity.

The energy absorbed in time t by a thickness x of gas, containing P atmospheres of absorbing constituent, from a monochromatic beam of radiation intensity I , originating from a surface dA , of direction such that θ is the angle between the beam and the normal to the surface dA is :

$$dE'_a = I t dA \cos \theta d\omega (1 - e^{-k'Px}) \quad 1.14$$

However k' varies with wavelength. If this variation can be represented by a straight line relation from zero to a value k , and if the equation is integrated over the hemispherical solid angle the total radiation from dA is

$$E_a = I_t dA \int_0^{2\pi} d\omega \cos \theta \left[1 - \frac{1 - e^{-kPx}}{kPx} \right] \quad 1.15$$

If the path length were constant and equal to L the expression becomes :

$$E_a = I_t dA \pi \left[1 - \frac{1 - e^{-kPL}}{kPL} \right] \quad 1.16$$

The ratio of the true absorption to that obtained assuming constancy of path length is called the shape factor :

$$\psi = \frac{\frac{1}{\pi} \int_0^{2\pi} d\omega \cos \theta \left[1 - \frac{1 - e^{-kPx}}{kPx} \right]}{\left[1 - \frac{1 - e^{-kPL}}{kPL} \right]} \quad 1.17$$

By choosing a suitable value of L it is possible to cause the value of ψ to remain close to unity and it may then be neglected. Hottel presents tabulated values of suitable mean beam lengths L.

This approach is similar but algebraically simpler than that of Schack. Hottel used the mean beam length to prepare total radiation charts^{31,32} which are subject to similar limitations as those of Schack, and estimated the total probable error at 30%.

1.5(c) Total emissivity determinations.

Much experimental work^{28,33,34,35,36} has been done on gas emission and the most comprehensive presentation is that of Hottel and Egbert²⁹. The net interchange of radiation between one radiating component of a gas and some

other body can be represented as

$$\begin{aligned} q/A]_{\text{net}} &= f_1 (L, P_G, P_T, C, T_G) \\ &- f_2 (L, P_G, P_T, C, T_G, T_S) \end{aligned} \quad 1.18$$

The effect of the composition C of the rest of the gas is not completely known. Hertz³⁷ found that with CO_2 absorption O_2 , N_2 and air were interchangeable but H_2 as diluent could increase the absorption.

A number of investigations^{28,29,30} have revealed that Beer's Law ($P_G L$ taken as single variable) is valid for CO_2 but not for NH_3 , HCl and H_2O .

Hottel and Egbert prepared their basic charts for emission from H_2O and CO_2 on the basis of

$$q/A]_{\text{net}} = f_1 (P_G L, T_G) - f_1 (P_G L, T_S) \quad 1.19$$

The water chart was for an idealised state of zero partial pressure of water vapour and a correction chart was prepared to allow for partial pressures greater than zero. Further correction charts were drawn up based on data of Fishenden³⁸ for water and Hertz³⁷ and von Bahr³⁰ for CO_2 to allow for variation in total pressure P_T . A correction chart based on the calculations of Eckert³⁶ was established to allow for superimposed radiation from CO_2 and H_2O .

Empirical equations were suggested, incorporating T_s , to allow absorptivities to be calculated from the emissivity charts.

The various correction charts are not thought to be very accurate but as their magnitudes are small the overall error should not be great. Hottel and Egbert claim that the radiation charts are accurate to 5%, while the resulting emissivity after corrections have been applied should be accurate to 10%. The absorptivities shew greater errors.

The total gas characteristics calculated in this manner assumed grey conditions. The real situation is approximated to by various empirical allowances.

1.5(d) Absorption bands.

Penner³⁹ has calculated the band absorption of diatomic gases, and Plass⁴⁰ has calculated the absorption of the 4.3 micron band of CO_2 . Penner has computed the total emissivity of CO_2 by considering individual bands and has confirmed the experimental measurements by Hottel. The calculation of band absorption for polyatomic molecules at high temperatures is difficult owing to the many energy levels which a molecule can occupy.

When full data for the band absorption of CO_2 and H_2O is available it will be possible to calculate the radiant transfer between gas and walls whose spectral absorptivity varies. At present the work of Hottel and Egbert²⁹ remains the best engineering approach.

1.6. Radiation in empty enclosures.

The interchange of radiation in an enclosure is dependent on the geometry of the enclosure. The basic concepts underlying these geometric factors can be seen by considering an enclosure containing non-absorbing gas. Certain electric furnaces can be described in this way.

For all surfaces occurring in practice emission and reflection are assumed to be diffuse, i.e. Lambert's Cosine Law is assumed valid. Although the actual emission at oblique angles is less than that predicted by Lambert's Law the errors for normal furnace surfaces are small.

The basic function is the view factor F_{12} , which is the fraction of all radiation leaving a particular surface S_1 which is intercepted by another surface S_2 . It is a purely geometric function which is obtained by integration. Two basic rules are used in the manipulation of these factors viz :

$$S_1 F_{12} = S_2 F_{21} \quad 1.20$$

$$\text{and} \quad \sum_{\text{all } i} F_{1i} = 1 \quad 1.21$$

View factors can be obtained analytically by light analogue methods and values are given in the literature ^{16,22,41-44}.

In an enclosure in which all the surfaces are not black, the intercepted radiation will be partly absorbed and partly

reflected. This reflected flux must be calculated if all interchange in the system is to be accounted for.

Consider an enclosure composed of grey source-sink surfaces having absorptivity equal emissivity and of no-flux surfaces which are partially absorbing and partially diffuse reflecting, but not necessarily grey. The refractory zone may be thought of as having a partial emissive power owing to the presence of each source-sink zone and a total emissive power equal to the sum of the partial emissive powers.

The total interchange between two source-sink zones, S_1 and S_2 , is the total of the direct interchange between these zones and the contributions due to multiple reflections and re-radiations from other surfaces as are consequent on the existence of S_1 and S_2 as net radiators in the system. By considering one zone only to be a radiator it is possible to evaluate these reflected fluxes using heat balances on all zones. This has been done for certain cases by Hottel ⁴⁵.

1.7. Radiation in gas-filled enclosures.

Two basic methods of treatment have been proposed, and are considered below.

1.7(a) Method due to Hottel and Cohen ⁴⁶.

The enclosure walls and volume are divided into zones small enough such that the zone may be considered

isothermal. The interchange of radiant energy between these zones is then evaluated. As this model can require the furnace to be divided into many zones the actual evaluation of the interchange can become very complicated.

Hottel and Cohen initially considered a grey gas and then presented a method of approximating to a real gas. The direct one-way radiation flux from one surface to another surface is represented by :

$$q_{s_i s_j} = \overline{s_i s_j} \sigma T_{s_i}^4 \quad 1.22$$

or from a surface to a gas zone,

$$q_{s_i g_j} = \overline{s_i s_j} \sigma T_{s_i}^4 \quad 1.23$$

The terms $\overline{s_i s_j}$ etc. were called direct interchange areas. They may be thought of as the product of the actual area of a surface zone or $4 \times \text{volume} \times \text{absorption coefficient}$ for a gas zone, and a reception factor f .

Hottel and Cohen present charts of the factors f for cubes of gas and squares of walls. The charts shew f as a function of the three co-ordinates, each divided by the cube edge B ; and kB where k is the absorption coefficient.

The enclosure walls are assumed to be diffuse reflectors and allowance is made for multiple reflections within the enclosure. Allowing for these reflections the

net interchange per unit difference in emissive powers between two zones is defined as $\overline{S_1 S_2}$, $\overline{G_1 S_2}$ etc. These terms all have the dimensions of area and are called total interchange areas.

The total interchange area is a function of geometry only providing emissivities and absorptivities are constant. By considering one zone only to be emitting and all other zones to be at absolute zero temperature and then drawing up heat balances over all zones Hottel and Cohen established the forms of the total interchange areas.

A real gas exhibits a variation of absorption coefficient with temperature and it is necessary to remove the grey gas assumption. The emissivity of a real gas is represented by the weighted sum of the contribution of a number of grey gases. Hottel and Cohen propose that a sufficiently accurate representation can be achieved using one grey and one clear gas. This involves evaluating some of the total interchange areas for two values of absorption coefficient. They also propose a method for estimating the mean temperature suitable for basing the gas properties on

Equations are presented for allowing for enthalpy flux and one way convection in the furnace. A heat balance is then built up for all heat transfer for every zone in the system. The heat balance equations are forcibly

linearised with T^4 as variable and solved by trial and error.

Thorneycroft and Thring^{6,47} applied the Hottel-Cohen treatment to a petroleum heater. They did not use the representation of a real gas but assumed the gas to be grey. An approximation was made in the mathematics which saved some computation without great loss in accuracy. They found good agreement between measured and calculated values of heat transfer.

The exchange factors f have been evaluated for a cylindrical system by Hoffman^{48,49,50}. He used the method for predicting heat transfer rates in a tall narrow chamber. Unlike the cube-square system the cylindrical f factors cannot be evaluated once for all. The factors are evaluated for a particular value of the height to diameter ratio. Hoffman evaluated these factors for a height to diameter ratio of unity. It is possible to calculate f factors for height to diameter ratios greater than unity from his results, but not for ratios less than unity.

Further work at M.I.T. is in progress²⁶ on the cylindrical system, the flow pattern in furnaces, the representation of absorptivity by a number of grey gases and the performance of end-fed plug flow furnaces.

1.7(b) Network analysis method.

Oppenheim⁵¹ has proposed a method, using the analogy of electric circuits, which allows for real gas and real walls by considering the energy emitted and absorbed in a spectral band and the equivalent band transmissivity. Application of this method requires data on the absorption bands of CO_2 and H_2O which is not yet available.

An approximation proposed by Bevans and Dunkle⁵² requires the knowledge of the total absorption of a gas in a narrow region of the spectrum in which the emissive power of the wall is essentially constant. Edwards⁵³ has shown that this approximation could be used in the case of non-grey walls with spectral absorptivity varying only slowly. As no band absorption correlations exist as yet for water it is not possible to use this method on normal furnace gases.

A method has recently been proposed by Edwards and Nelson⁵⁴ to estimate roughly the coarse spectral distribution of radiant energy in an enclosure containing isothermal CO_2 and H_2O . An approximate correlation for the band absorption based on the total emissivity of Hottel⁴⁵ is given.

Oppenheim and Bevans⁵⁵ say that the application of the network method is presently restricted by lack of knowledge of the properties of radiant systems. When

this knowledge becomes available the network analysis method will be a powerful tool in describing the radiant transfer between real gas and real walls.

1.8. Present work.

Very little attention has been given to the problem of radiation heat transfer in small furnaces. It is true that a complete theoretical analysis of all mechanisms of transfer will apply to both large and small furnaces, but such an analysis is not yet feasible and approximations or simplifications introduced to render a solution more easily obtainable do not necessarily apply equally to furnaces of significantly different configurations.

Most of the work on furnace correlations has been aimed at predicting the heat transfer in large tube stills. The only published application of a Hottel and Cohen ⁴⁶ type analysis of an industrial furnace is that of Thorneycroft and Thring ⁶ which is also for a large tube still.

Many small cylindrical furnaces are used industrially and it is desirable that the performance of these furnaces can be predicted. The present work is an attempt to facilitate this prediction, and was undertaken in the following manner.

The heat transfer in two small cylindrical furnaces was investigated experimentally. The furnaces were quite dissimilar, one being oil-fired with an end sink, the other being gas-fired with a wall sink.

Various correlations were used to attempt to predict the heat transfer measured.

The method proposed by Hottel and Cohen was applied to the furnaces. This involved establishing the reception factors between narrow sections of a gas filled enclosure. The effects of certain parameters, gas representation, number of zones, variation of absorption coefficient, on the predicted heat transfer were investigated.

Chapter 2. Experimental Investigations.

2.1. Introduction.

2.2. Pot furnace.

- 2.2(a) Plant.
- 2.2(b) Routine plant measurements.
- 2.2(c) Suction pyrometer.
- 2.2(d) Wall temperatures.
- 2.2(e) Surface temperatures.
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2.3. Superheater.

- 2.3(a) Plant.
- 2.3(b) Measurements.
- 2.3(c) Procedure.
- 2.3(d) Results.

Chapter 2. Experimental Investigations.

2.1. Introduction.

As a test of the methods of calculation which follow, a practical investigation was made of heat transfer in two cylindrical furnaces. The first, located at I.C.I. Dyestuffs Division, Huddersfield, was an oil-fired sulphuric acid pot concentrator, which essentially comprised a cylinder of aspect ratio unity radiating to an end sink. The second, at the Greenfield Works of Messrs. Courtaulds, was a gas-fired steam superheater which approximated to a cylinder of aspect ratio two radiating to a wall sink.

In both cases heat balances were carried out over the whole plane and the heat transfer estimated by the gas side enthalpy drop.

2.2. Pot furnaces.

2.2(a) Plant.

The plant layout is shewn diagrammatically in Fig. 2.1 and the pot is shewn in Fig. 2.2.

The pot furnace is a cylinder with a cast iron pot suspended in, and almost filling, the upper half of the volume.

The steel casing is lined with brick to a total

Fig 2.1.

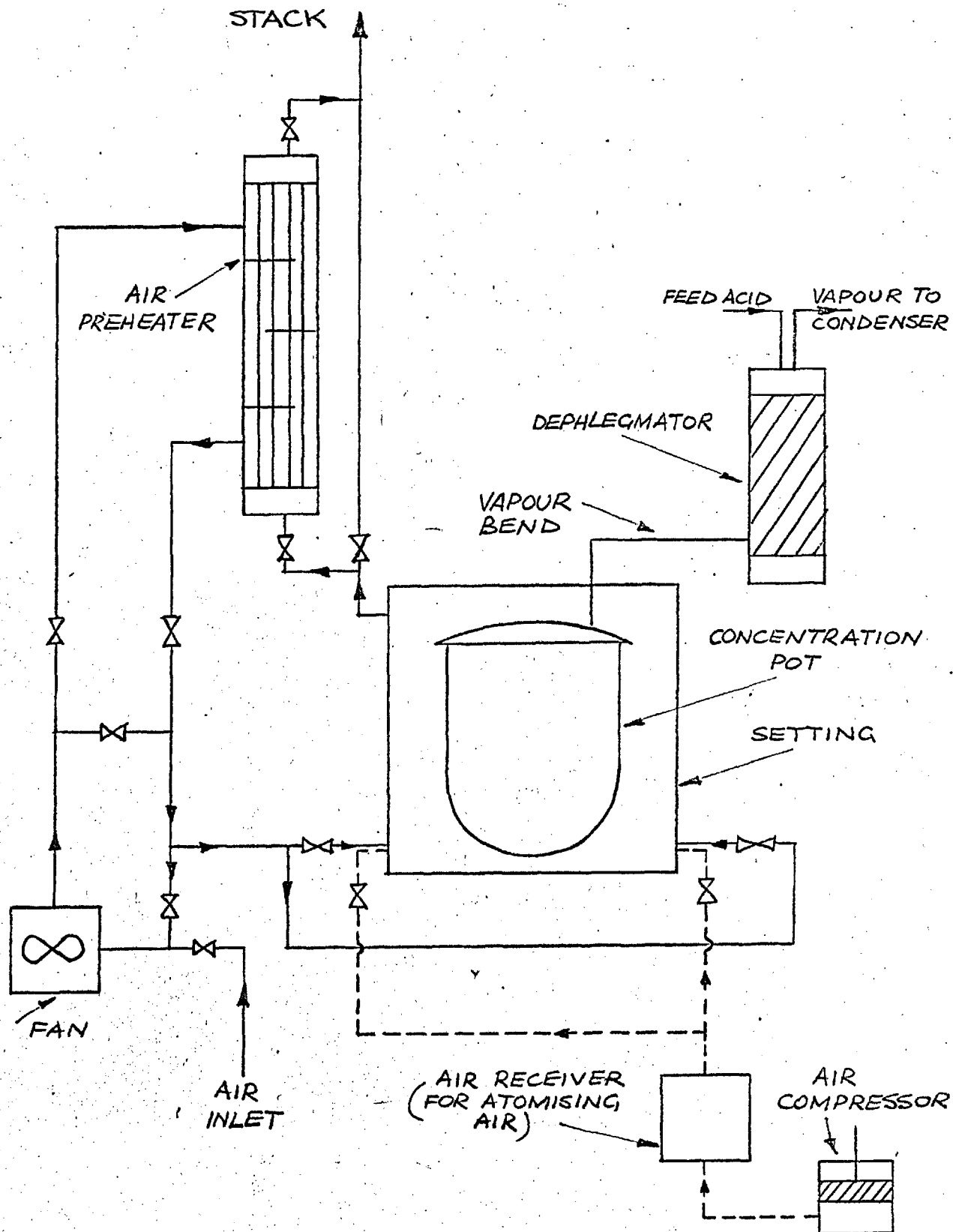
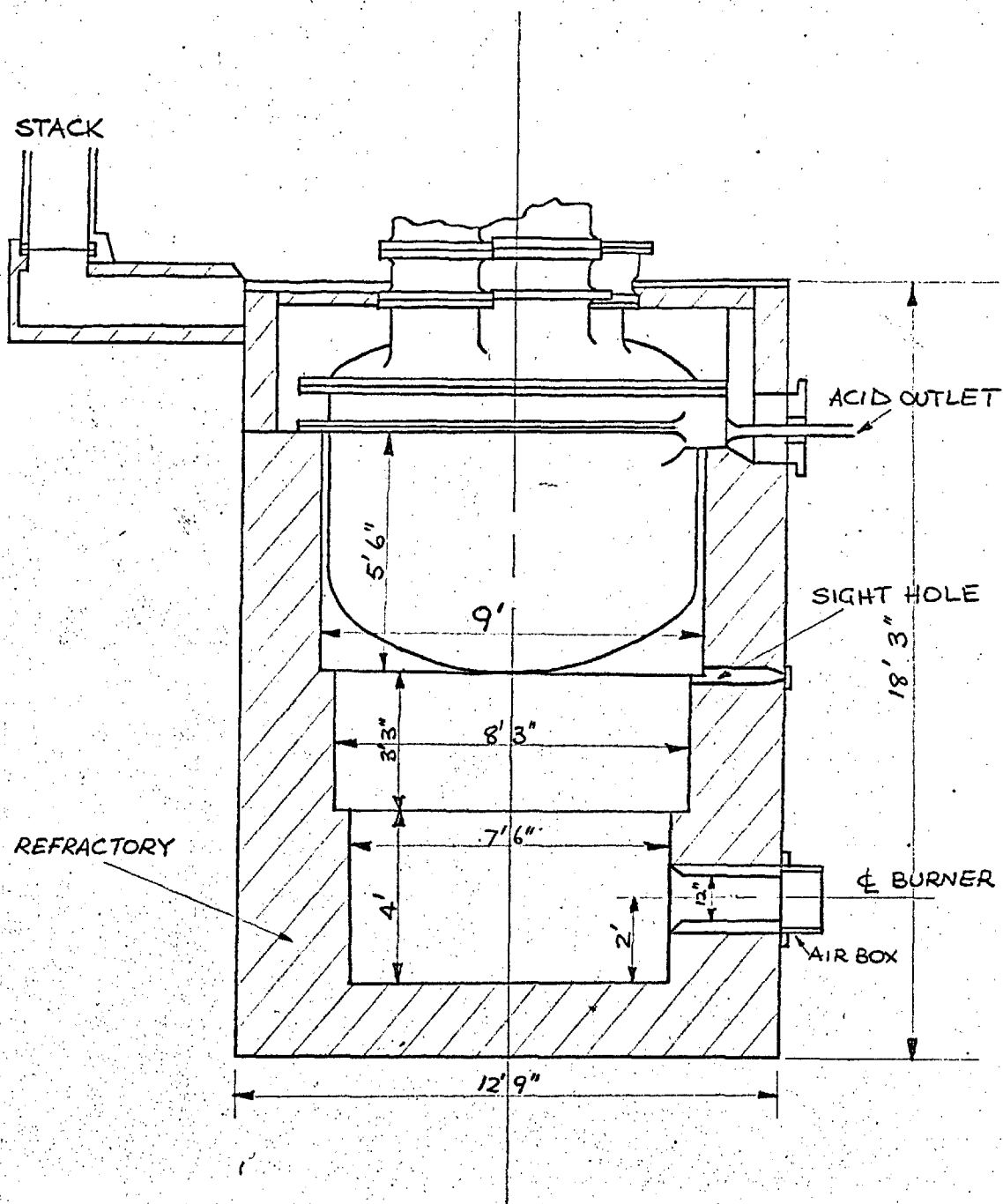
POT FURNACE PLANT LAYOUT

Fig 2.2.POT FURNACE

thickness of between $22\frac{1}{2}$ and 30 inches. The base of the setting is uncemented refractory and acid resistant brick. The pot approximates to a cylinder of diameter 8 ft. 6 ins. The annular passage between the pot and the setting allows hot gas to flow up and over the top of the pot. The radiation section below the pot is approximately a cylinder of height and diameter 8 ft.

Two heavy fuel burners are mounted tangentially 2 ft. from the setting base. The air used to atomise the fuel is supplied at a pressure of 25 psig. from a ring main and air receiver, pumped by a reciprocating compressor. The greater part of the combustion air is taken in by a centrifugal forced-draught fan, passed through the rectangular shell of a tubular air pre-heater and delivered to the two burners.

After passing over the top of the pot the products of combustion pass through a brick lined flue box and thence either through the tubes of the air pre-heater or directly to stack.

The pot concentrator acts as a reboiler for concentrating sulphuric acid. Weak acid about 71% H_2SO_4 is taken from storage, concentrated to about 94.5% H_2SO_4 and sent to receiving tanks. Typical loading is 22 tons 100% H_2SO_4 per day.

2.2(b) Routine plant measurements.

The temperature of pot body, column head, pre-heated air inlet and outlet, flue gas inlet and outlet to and from the air pre-heater were continuously recorded on a Kent recorder. All these thermocouples, except that in the pot body, were checked with either mercury-in-glass thermometers or other thermocouples. Since the thermocouple measuring flue gas inlet temperature gave readings varying with its position in the thermopocket, it was replaced by a mercury-in-glass thermometer inserted directly into the duct.

Ambient and cold acid temperatures were measured using mercury-in-glass thermometers. Wet and dry bulb temperatures were recorded at the air inlet.

The automatic CO₂ meter was found to be erratic so a Monoc CO₂ meter was used at intervals during a trial. One reading per trial was checked using an Orsat apparatus. The Monoc CO₂ meter was standardised in the laboratory against an Orsat apparatus.

The strength of the acid was determined analytically from bulked samples taken during trials. The consistency of the acid meters was checked by a mass balance on the H₂SO₄.

The oil Tyler meter was checked by collecting and weighing timed samples.

2.2(c) Suction pyrometer.

The hot gas temperature at the exit to the radiation section was measured using a BISRA water cooled suction pyrometer. A traverse was made from the centre of the furnace to the wall in 5 inch steps. A water in glass manometer was used to indicate suction. Sufficient air flow was maintained through the ejector to ensure more than the recommended minimum hot gas flow ⁵⁶ at all times.

2.2(d) Wall temperature.

Eight holes were drilled through the casing and refractory of the radiation section at various heights and orientations. Chromel-alumel thermocouples were inserted. These were protected by refractory and supported in mild steel tubes. The hot junctions were arranged flush with the inside walls so that a record of the temperature distribution could be taken. Readings were checked by interchanging thermocouples during trials.

2.2(e) Surface temperatures.

Several surface pyrometers were tested for consistency and a Cambridge pyrometer was found to give repeatable readings. The instrument was standardised in the laboratory using beakers of liquid at known temperatures. The various surfaces of the plant were divided into zones

over which the temperature did not vary by more than 10°C . The average temperatures of these zones were recorded during trials.

2.2(f) Procedure.

Four trials were carried out at loads between 16 and 28 tons/day 100% H_2SO_4 .

During the trials attempts were made to equalise pot pressure with atmospheric to reduce air leaks.

The load was changed in the evening and the plant allowed to achieve steady conditions overnight. The attainment of steady conditions was checked through the morning and measurements were made in the afternoon.

2.2(g) Results.

The readings and intermediate results are presented as Appendix 1.1. The overall heat balances are summarised in the following table.

Table 2.1. Pot furnace heat balance.

	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
Load tons H_2SO_4 /day	25.0	19.5	27.6	16.8
Net heat in oil 10^6 Btu/hr	2.4140	1.8655	2.6190	1.6210
Heat in air "	<u>.0570</u>	<u>.0670</u>	<u>.0685</u>	<u>.0880</u>
Total heat input "	2.4710	1.4825	2.6875	1.6590
Heat to acid "	1.8050	1.3780	1.9660	1.1870
% of total	73.0	71.3	73.2	71.6
Stack heat loss "	.2980	.3230	.4050	.2440
% heat loss	12.1	16.7	15.1	14.7
Surface heat loss "	.3050	.2750	.3880	.2080
% of total	14.4	14.2	14.4	12.5
% total unaccounted for	0.50	-2.2	-2.7	1.2

The results of the suction pyrometer traverses are given in Table 2.2

Table 2.2. Suction pyrometer readings.

Distance from furnace centre mm .	Temperature °C.			
	Trial 1	2	3	4
16	798	763	834	836
21	799	808	855	857
26	817	828	873	881
31	839	849	892	894
36	859	862	904	909
41	873	871	921	919
46	857	849	911	908
48	826	769	888	879
49	739	716	861	826
51	631	-	-	-

The wall thermocouple readings are given in Table 2.3.

Table 2.3. Wall thermocouple readings.

Thermocouple No.	Temperature °C.			
	Trial 1	2	3	4
1	458	465	547	561
2	716	699	773	720
3	809	764	835	841
4	863	818	894	886
5	638	629	677	643
6	879	873	907	890
7	1019	984	1033	1039
8	768	751	775	771

The thermocouple placings are shewn with reference to the burners in the following table.

Table 2.4. Wall thermocouple placings.

Item	Height above casing bottom ft-ins.	Angular position degrees
Openside burner	3-9	0
Close side burner	3-9	180
Thermocouple No.1	13-3	103
2	9-7	74
3	9-0	230
4	8-3	146
5	6-4	347
6	5-10	74
7	5-2	146
8	5-0	230

2.3. Superheater.

2.3(a) Plant.

The plant layout is shewn diagrammatically in Fig.2.3 and the superheater is shewn in Fig. 2.4.

The superheater is a cylinder with a tube bundle in the shape of a truncated cone almost as large as the enclosure.

The furnace enclosure is 6 ft. 3 ins. high and has a diameter of 3 ft. 10 ins. The bundle has an overall height of 6 ft. $1\frac{1}{2}$ ins. The top header has an overall diameter of 4 ft. 2 ins. and is recessed into the wall, while the bottom header is 3 ft. 3 ins. in diameter. The tubes are approximately $1\frac{1}{2}$ ins. in diameter.

Two burners, taking towns gas, are mounted on top of the furnace in a tunnel of 1 ft. 6 ins. diameter which introduces the hot gas into the chamber. The bottom take-off is a 1 ft. 6 ins. diameter duct leading to an out of use scroll mixer-cooler. The hot gas is then led to a box steam pre-heater. The flue gas is finally cooled and run down to storage before being used as an inert atmosphere.

The preheater and superheater heat, in series, low pressure steam which is used for catalyst regeneration.

Fig 2.3.

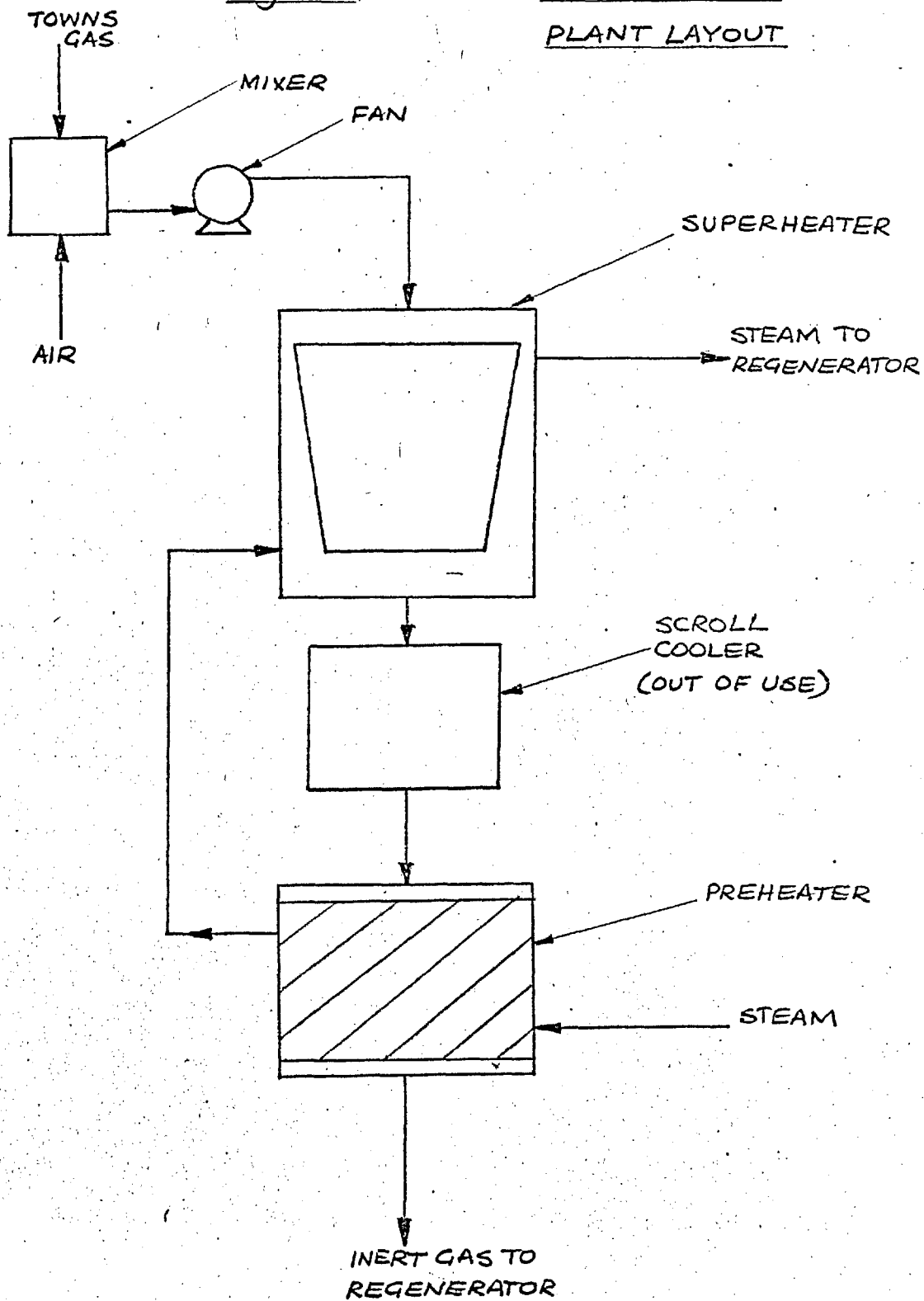
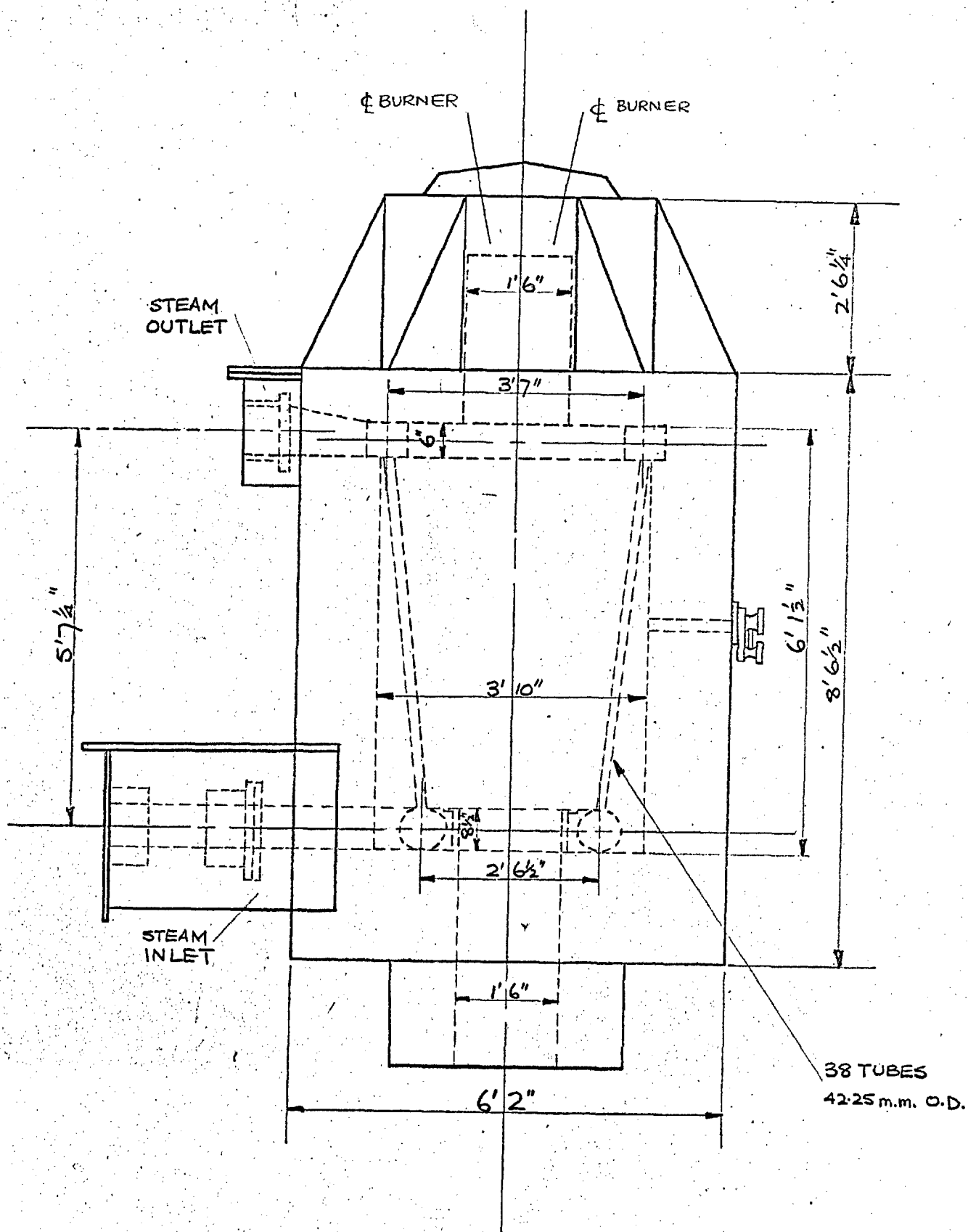
SUPERHEATER
PLANT LAYOUT

Fig 2.4. SUPERHEATER



2.3(b) Measurements.

Steam pressures were measured using existing pressure gauges. The thermocouples on the steam line were checked and two new thermocouples inserted. The steam flow was automatically recorded by a proportional indicator. The indicator was stripped, cleaned and reassembled. The flue gas temperatures leaving the superheater and preheater were measured using a single shield metal suction pyrometer, developed by the Gas Research Council. Sufficient air flow was maintained to ensure the recommended ⁵⁷ hot gas flow.

Ambient and wet and dry bulb temperatures were recorded.

The towns gas flow was recorded using the Gas Board meter. Both air and gas flows were indicated on orifice flowmeters. The flue gas was analysed using an Orsat apparatus. The Gas Board provided special analyses and calorific values measured during the trials.

The surface of the plant was divided into sections over which the temperature did not vary significantly. The temperature of each zone was recorded using a Cambridge surface pyrometer.

2.3(c) Procedure.

As the superheater was integrated with a complex plant network it was not possible to vary loads. A trial run was made to familiarise the investigators with the equipment. The next day, after checking for steady conditions, the results used in the calculations were taken.

2.3(d) Results.

The readings and calculations are presented in Appendix 1.2. The overall heat balance is shown in the following tables.

<u>Table 2.5.</u>	<u>Overall heat balance.</u>	
	10^6 Btu/hr.	%
Enthalpy in	1.7787	100
Stack loss	.2983	16.8
Enthalpy to steam in superheater.	.7114	40.0
Enthalpy to steam in preheater	<u>.4781</u>	<u>26.9</u>
Total to steam	1.1895	66.9
Heat losses	.1974	11.1
unaccounted for	.0935	5.2

Table 2.6.Superheater heat balance.

	10^6 Btu/hr.	%
Heat to superheater	1.7787	
Heat ex superheater in gas enthalpy drop	<u>.9152</u>	
Enthalpy drop	.8635	100
Heat to steam	.7114	82.4
Heat losses	<u>.1176</u>	<u>13.6</u>
	.8290	96.0
unaccounted for	.0345	4.0

Chapter 3. Evaluation of Reception Factors f .

3.1. Theory.

- 3.1(a) Surface to surface reception.
- 3.1(b) Gas to gas reception.
- 3.1(c) Elemental reception factors F .

3.2. Geometric representation of the furnaces.

- 3.2(a) Configuration of pot furnace.
- 3.2(b) Configuration of superheater.
- 3.2(c) Choice of zone size.

3.3. Derivation of f from F .

- 3.3(a) Starting data.
- 3.3(b) Integrations to establish F_{ee} and f_{ee} .
- 3.3(c) Integration of F to yield f .
- 3.3(d) Addition of reception factors.
- 3.3(e) Tabulated reception factors f .
- 3.3(f) Check on accuracy of reception factors.

Chapter 3. Evaluation of Reception Factors f.

3.1. Theory.

Consider a cylindrical enclosure containing an absorbing gas of constant absorption coefficient. The treatment suggested by Hottel and Cohen⁴⁶ requires the surfaces and gas to be divided into zones such that each zone may be considered isothermal. In a cylindrical system the zones are taken to be cylindrical to avoid great mathematical complexity. This system has been investigated in detail by Hoffman.⁵⁰

The fraction of the total radiation that leaves any zone which is intercepted by another zone, taking into account the absorption of radiation by the intervening gas, is called the reception factor [or (direct) view factor or (direct) interchange factor]. These reception factors must be evaluated.

3.1(a). Surface to surface reception.

From the above definition:

$$q_{s_1 \rightarrow s_2} = A_{s_1} f_{s_1 s_2} \sigma T_{s_1}^4 \quad 3.1.$$

This one way exchange is also known to be directly proportional to the area of each zone as viewed from the

other and inversely proportional to the square of the separating distance R (Fig. 3.1). The absorption by the gas over the distance R will reduce the intensity at dA_2 .

Thus:

$$\frac{dq_{s_1 \rightarrow s_2}}{\pi} = \frac{1}{\pi} (dA_1 \cos \theta_1 - dA_2 \cos \theta_2) / R^2 \cdot e^{-kR} \cdot \sigma T^4 \quad 3.2.$$

In the above equation the effects of refraction⁶⁰ are neglected as the refractive index of a gas is close to unity.⁴⁶

When equation 3.2. is integrated over finite areas equations 3.1. and 3.2. are equal and f_{ss} can be found. For the general case of zone i and $j = i + \alpha$ (α being a whole number describing the separation of i from j) and when all length terms are normalised with respect to cylinder diameter δ ,

$$f_{ss\alpha} = \frac{1}{16\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^1 d\left(\frac{z_2}{\delta}\right) \int_{\alpha}^{\alpha+1} d\left(\frac{z_1}{\delta}\right) \left[1 - \cos(\phi_2 - \phi_1)\right]^2 \frac{e^{-k\delta\left(\frac{R}{\delta}\right)}}{\left(\frac{R}{\delta}\right)^4} \quad 3.3.$$

where

$$\left(\frac{R}{\delta}\right)^2 = \left(\frac{r_1}{\delta}\right)^2 + \left(\frac{r_2}{\delta}\right)^2 - \left[2 \frac{r_1 r_2}{\delta^2} \cdot \cos(\phi_2 - \phi_1)\right] + \left(\frac{z_2 - z_1}{\delta}\right)^2$$

and

$$\frac{r_1}{\delta} = \frac{r_2}{\delta} = \frac{1}{2}$$

The reception factors f_{es} and f_{ee} may be found similarly and are defined in Table 3.1.

3.1(b). Gas to gas reception.

The emissivity or absorptivity of an element of gas, on a volume basis, is $4k$.⁶¹ The radiation from volume V_1 of gas intercepted by volume V_2 is,

$$q_{V_1 \rightarrow V_2} = 4kV_1 f_{V_1 V_2} \sigma T_G^4 \quad 3.4.$$

Some of the radiation emitted by volume V_1 is absorbed by volume V_1 . This fraction is f_{gg_0} .

Consider the differential volume dV_1 (Fig. 3.1) radiating over 4π steradians. Of the total radiation leaving dV_1 the fraction reaching an area dA normal to the incident radiation is $\frac{dA \cdot e^{-kR}}{4\pi R^2}$. The amount absorbed in volume dV_2 is,

$$dq_{dV_1 \rightarrow dV_2} = 4kV_1 \frac{dA}{4\pi R^2} \left[1 - e^{-k \frac{dV_2}{dA}} \right] \sigma T_{G_1}^4 \cdot e^{-kR} \quad 3.5.$$

As the volumes are elemental, only the first two terms of the exponential are considered,

$$dq_{dV_1 \rightarrow dV_2} = \frac{4kV_1 k dV_2}{4\pi R^2} \cdot e^{-kR} \sigma T_{G_1}^4 \quad 3.6.$$

After integrating equation 3.6. and equating with equation 3.4. the reception factor is,

$$f_{gs} = \frac{k\delta}{\pi^2} \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^1 d\left(\frac{z_1}{\delta}\right) \int_{\alpha}^{\alpha+1} d\left(\frac{z_2}{\delta}\right) e^{\frac{-k\delta(\frac{R}{\delta})}{(\frac{R}{\delta})^2}} \quad 3.7.$$

The reception factors f_{sg} and f_{eg} may be found similarly and are defined in Table 3.1.

3.1(c). Elemental reception factors F.

Direct analytical integration of these integrals was not possible. A convenient way of facilitating the evaluation is to neglect initially all integrations with respect to $\frac{z_1}{\delta}$ and $\frac{z_2}{\delta}$, the vertical variables. Since these variables also occur in the combination $\frac{z_2 - z_1}{\delta} = h$, it is convenient to define the functions F_{ss} , F_{es} etc. shown in Table 3.2. The quantities F_{ss} , F_{es} etc. are termed elemental reception factors and give the fractional exchange between zones of thickness or width $d\left(\frac{z_1}{\delta}\right)$ and/or $d\left(\frac{z_2}{\delta}\right)$ at a separation h . The substitution $h = \frac{z_2 - z_1}{\delta}$ and integration with respect to $\frac{z_1}{\delta}$ and/or $\frac{z_2}{\delta}$ leads to the reception factors f_{ss} f_{es} etc.

Hoffman⁵⁰ manipulated the expressions for these elemental reception factors and partially integrated them analytically to yield the expressions given in Table 3.3. These integrals he numerically integrated on a digital computer. The final integrations to yield f from F were done by

TABLE 3.1. Reception Factors

$$f_{es} = \frac{2}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \int_0^1 d\left(\frac{z_1}{\delta}\right) \left\{ \frac{1}{2} - \frac{r_2}{\delta} \cos(\phi_2 - \phi_1) \right\} \left(\frac{z_2 - z_1}{\delta} \right) \frac{e^{-k\delta \left(\frac{R}{\delta}\right)}}{\left(\frac{R}{\delta}\right)^4}$$

3.8

$$\frac{r_1}{\delta} = \frac{1}{2}$$

$$f_{ee} = \frac{4}{\pi^2} \left(\frac{z_2 - z_1}{\delta} \right)^2 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \frac{e^{-k\delta \left(\frac{R}{\delta}\right)}}{\left(\frac{R}{\delta}\right)^4}$$

3.9

$$f_{sg_\alpha} = \frac{k\delta}{2\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^1 d\left(\frac{z_1}{\delta}\right) \int_\alpha^{\alpha+1} d\left(\frac{z_2}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \left[\frac{1}{2} - \left(\frac{r_2}{\delta}\right) \cos(\phi_2 - \phi_1) \right] \frac{e^{-k\delta \left(\frac{R}{\delta}\right)}}{\left(\frac{R}{\delta}\right)^3}$$

3.10

$$\frac{r_1}{\delta} = \frac{1}{2}$$

$$f_{eg_\alpha} = \frac{4k\delta}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_\alpha^{\alpha+1} d\left(\frac{z_1}{\delta}\right) \left(\frac{z_2 - z_1}{\delta} \right) \frac{e^{-k\delta \left(\frac{R}{\delta}\right)}}{\left(\frac{R}{\delta}\right)^3}$$

3.11

$$\text{where } \left(\frac{R}{\delta}\right)^2 = \left(\frac{r_1}{\delta}\right)^2 + \left(\frac{r_2}{\delta}\right)^2 - \left[\frac{2r_1 r_2}{\delta^2} \cos(\phi_2 - \phi_1) \right] + \left(\frac{z_2 - z_1}{\delta} \right)^2$$

3.12

TABLE 3.2 Elemental Reception Factors

$$F_{ss}(h, k\delta) = \frac{1}{16\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \left[1 - \cos(\phi_2 - \phi_1) \right]^2 \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^4} \quad 3.13$$

$$F_{es}(h, k\delta) = \frac{2h}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \left[\frac{1}{2} - \frac{r_2}{\delta} \cos(\phi_2 - \phi_1) \right] \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^4} \quad 3.14$$

$$F_{ee}(h, k\delta) = \frac{4h^2}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^4} \quad 3.15$$

$$F_{sg}(h, k\delta) = \frac{k\delta}{2\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \left[\frac{1}{2} - \frac{r_2}{\delta} \cos(\phi_2 - \phi_1) \right] \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^3} \quad 3.16$$

$$F_{eg}(h, k\delta) = \frac{4k\delta h}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^3} \quad 3.17$$

$$F_{gg}(h, k\delta) = \frac{k\delta}{\pi^2} \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{1}{2}} \frac{r_1}{\delta} d\left(\frac{r_1}{\delta}\right) \int_0^{\frac{1}{2}} \frac{r_2}{\delta} d\left(\frac{r_2}{\delta}\right) \frac{e^{-k\delta \left(\frac{R}{\delta} \right)}}{\left(\frac{R}{\delta} \right)^2} \quad 3.18$$

TABLE 3.3 Manipulated Elemental Reception Factors
(due to Hoffman⁵⁰)

$$F_{ss}(h, k\delta) = \frac{2}{\pi} \int_0^1 \frac{x^4 e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{\sqrt{1-x^2} (x^2+h^2)^2} dx \quad 3.19$$

$$F_{es}(h, k\delta) = \frac{8h}{\pi} \int_0^1 \frac{x^2 \sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{(x^2+h^2)^2} dx \quad 3.20$$

$$F_{ee}(h, k\delta) = e^{-k\delta h} - hk\delta e^{-k\delta h} - \frac{4h^2}{\pi} \int_0^1 \frac{\sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{x^2+h^2} dx \\ + \frac{4hk\delta}{\pi} \int_0^1 \frac{\sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{(x^2+h^2)^{\frac{1}{2}}} dx + 2hk\delta [F_{gg}(h, k\delta)] \quad 3.21$$

$$F_{sg}(h, k\delta) = \frac{2k\delta}{\pi} \int_0^1 \frac{x^2 \sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{(x^2+h^2)^{\frac{3}{2}}} dx \quad 3.22$$

$$F_{eg}(h, k\delta) = 2k\delta e^{-k\delta h} - \frac{8k\delta h}{\pi} \int_0^1 \frac{\sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{(x^2+h^2)^{\frac{1}{2}}} dx \\ - 4k\delta h [F_{gg}(h, k\delta)] \quad 3.23$$

$$F_{gg}(h, k\delta) = \frac{k\delta}{2} [-E_i(-k\delta h)] - \frac{2k\delta}{\pi} \int_0^1 [-E_i(-k\delta(x^2+h^2)^{\frac{1}{2}})] (1-x^2)^{\frac{1}{2}} dx \quad 3.24$$

where $-E_i[-k\delta(x^2+h^2)^{\frac{1}{2}}] = \int_{k\delta(x^2+h^2)^{\frac{1}{2}}}^{\infty} \frac{e^{-t}}{t} dt.$

plotting and using Simpson's Rule.

It is important to notice that the reception factors f cannot be applied universally to all cylindrical zone configurations. A series of f factors is evaluated for a particular value of the ratio height of zone to cylinder diameter. Thus if a height to diameter ratio of unity is taken, only furnaces which can be broken into zones of height equal diameter can be investigated.

The elemental reception factors F are of universal application. By substituting the correct value for $h = \frac{z_2 - z_1}{\delta}$ and integrating with respect to $\frac{z_1}{\delta}$ and $\frac{z_2}{\delta}$ any zone configuration can be evaluated.

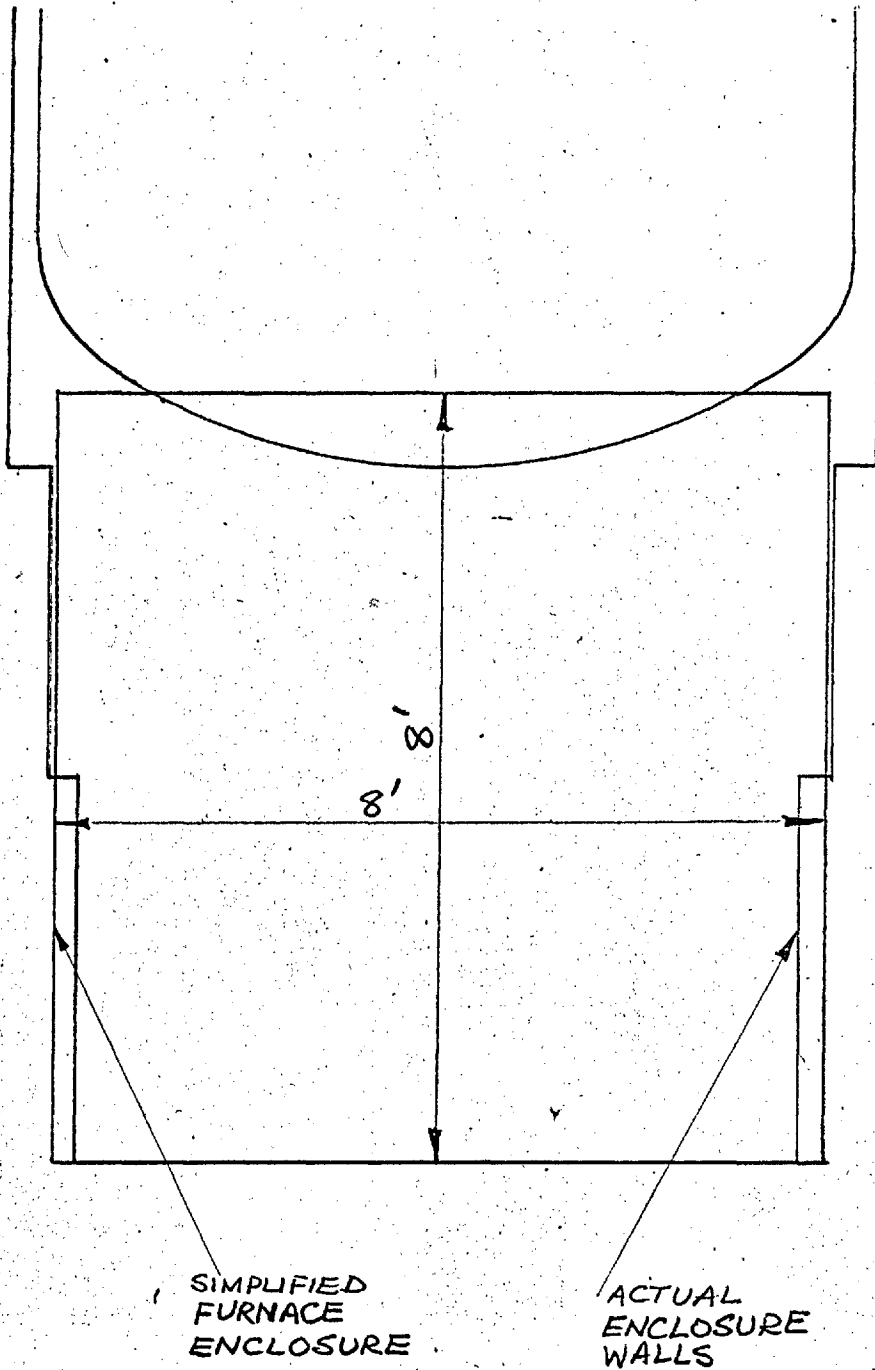
3.2. Geometric representation of the furnaces.

The reception factors for a gas-filled cylindrical enclosure divided into cylindrical zones can be evaluated. The furnaces under consideration are not perfect cylindrical enclosures but must be represented as such.

It is necessary to select zone sizes suitable for the furnaces so that the elemental reception factors F can be integrated to yield useful reception factors f .

3.2(a). Configuration of pot furnace.

A cross-sectional view of the pot furnace enclosure is shown in Fig. 3.2.

Fig 3.2.POT FURNACE ENCLOSURE WITH
SIMPLIFIED ENCLOSURE SUPERIMPOSED

This enclosure must be represented by a cylinder which can be divided into a whole number of equal zones. So that the effect of different zone sizes can be investigated, these zones should be of such dimensions that they can be combined together in different ways and still represent the total enclosure by a whole number of zones. The most flexible representation is a cylinder of height equal to diameter.

The dished bottom of the pot is represented as a disc heat sink filling the top of the enclosure. Because of the wall recesses this is a reasonable approximation. The enclosure is taken to have height and diameter equal to 8 ft. This causes the bottom of the pot to encroach slightly into the top gas zone. With the smallest zones used this encroachment was only 20% of the volume of the top zone and can be off set against the excess volume at the edges.

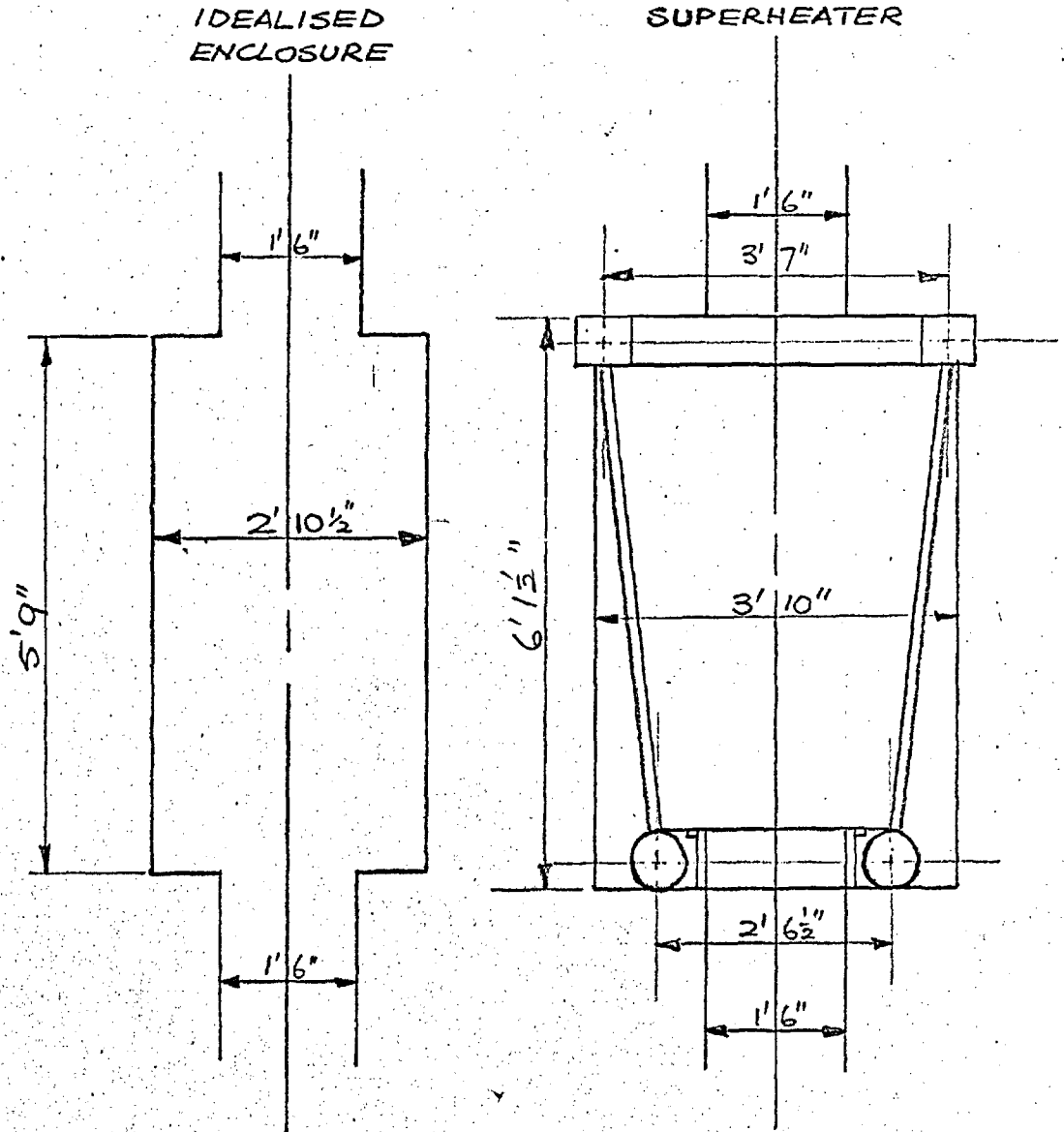
The assumed configuration of the furnace is shown superimposed on Fig. 3.2.

3.2(b). Configuration of superheater.

The cross-section of the superheater is shown in Fig. 3.3.

Again the furnace must be represented by a cylinder of simple aspect ratio. In this case the walls of the enclosure must represent the heat sink.

Fig 3.3. SIMPLIFIED SUPERHEATER BODY



The dead space behind the tubes and bottom header has been neglected and the enclosure representation, a cylinder height equal twice diameter, is shown in Fig. 3.3. For the purpose of radiation the tubes have been assigned an effective plane area and emissivity. They are still considered to be tubes when convection is calculated.

3.2(c). Choice of zone size.

In a cylindrical enclosure the zones extend across the furnace and the one variable is zone height. Throughout the derivation of reception factors dimensions have been normalized with respect to cylinder diameter. The variable defining zone size is therefore taken to be aspect ratio, i.e. $\frac{\text{height}}{\text{diameter}}$.

For the pot furnace which is a cylinder of height and diameter equal 8 ft, the smallest aspect ratio considered is $\frac{1}{10}$. It would seem likely that there will be little increase in accuracy of representation by considering a breakdown of more than ten gas zones. With this configuration it is possible to combine the zones in groups of 2, 5 and 10. The enclosure can thus be treated as a single zone or divided into 2, 5 or 10 zones.

As the geometric configuration assumed for the superheater approximates but loosely to its actual shape, the

maximum number of zones considered is four. The enclosure height is twice its diameter thus the basic zones have an aspect ratio $\frac{1}{2}$.

3.3. Derivation of f from F.

The basic breakdown of the pot furnace is a cylinder of height equal diameter split into ten equal zones. The reception factors f must be established for zones of aspect ratio $\frac{1}{10}$. To allow for all reception in the enclosure factors f_{ss_0} to f_{ss_9} , f_{es_0} to f_{es_9} etc. must be evaluated.

To establish the reception factors for the superheater the integrations must be extended to a cylinder of height equal twice diameter. Zones of aspect ratio $\frac{1}{2}$ are considered and the factors f_{ss_0} to f_{ss_3} etc. must be evaluated.

3.3(a). Starting data.

The elemental reception factors F are of universal application to the cylindrical system. For them to apply to a particular system the substitution $h = \frac{z_1 - z_2}{\delta}$ must be made and integrations with respect to $\frac{z_1}{\delta}$ and/or $\frac{z_2}{\delta}$ must be carried out.

The elemental reception data were obtained from Hoffman⁶² for the cases F_{ss} , F_{es} , F_{gg} , F_{sg} and F_{eg} . These were evaluated by Hoffman by numerically integrating the

expressions in Table 3.3. The values of the F-factors for varying h at values of $k\delta$ of 0, 0.5, 1, 2 and 3 were available.

Sufficiently detailed data for the case F_{ee} were not available so the expression for F_{ee} (Table 3.3) was integrated graphically.

3.3(b). Integrations to establish F_{ee} and f_{ee} .

The manipulated expression for the elemental reception factor F_{ee} is given in Table 3.3, equation 3.21:-

$$F_{ee}(h, k\delta) = e^{-k\delta h} - hk\delta e^{-k\delta h} - \frac{4h^2}{\pi} \int_0^1 \frac{\sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{x^2 + h^2} dx$$

$$+ \frac{4hk\delta}{\pi} \int_0^1 \frac{\sqrt{1-x^2} e^{-k\delta(x^2+h^2)^{\frac{1}{2}}}}{(x^2 + h^2)^{\frac{1}{2}}} dx + 2hk\delta [F_{gg}(h, k\delta)]$$

The two integrals, I_1 and I_2 respectively, were integrated graphically.

For given values of $k\delta$ and h the expression under each integral sign was evaluated for x being 0, 0.02, 0.06, 0.1, 0.2, 0.3, 0.5, 0.7, 0.9 and 1.0. These values were plotted on squared paper against x and the values of both integrals established by square counting.

Such integration was carried out for h equal to 0.1, 0.2, 0.3, 0.5, 0.7, 0.9, and 1.0, and for $k\delta$ equal to 0.5, 1.0, 2.0 and 3.0. F_{ee} followed by substitution in equation 3.21.

For the case $k\delta = 0$, a simpler method was possible. On substitution of $k\delta = 0$, equation 3.21 reduces to,

$$F_{ee}(h, k\delta = 0) = 1 - \frac{4h^2}{\pi} \int_0^1 \frac{\sqrt{1-x^2}}{x^2 + h^2} dx \quad 3.25$$

which can be separated to,

$$F_{ee}(h, k\delta = 0) = 1 - \frac{4h^2}{\pi} \int_0^1 \frac{dx}{\sqrt{1-x^2}(x^2+h^2)} - \frac{x^2 dx}{\sqrt{1-x^2}(x^2+h^2)} \quad 3.26$$

$$= 1 - \frac{4h^2}{\pi} [I_3 - I_4] \quad 3.27$$

It is known that

$$\int_0^1 \frac{x^2}{1+qx^2} \cdot \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2q} \left[1 - \frac{1}{\sqrt{1+q}} \right] \quad 3.28$$

so by substituting $q = \frac{1}{h^2}$

$$I_4 = \int_0^1 \frac{x^2 dx}{\sqrt{1-x^2}(x^2+h^2)} = \frac{\pi}{2} \left[1 - \frac{h}{\sqrt{1+h^2}} \right] \quad 3.29$$

It is known that

$$\int_0^1 \frac{1}{1+qx^2} \cdot \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2} \frac{1}{\sqrt{1+q}} \quad 3.30$$

so by substituting $q = \frac{1}{h^2}$

$$I_3 = \int_0^1 \frac{1}{x^2 + h^2} \cdot \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2} \frac{1}{h\sqrt{1+h^2}} \quad 3.31$$

thus

$$\begin{aligned} \frac{4h^2}{\pi} [I_3 - I_4] &= \frac{4h^2}{\pi} \cdot \frac{\pi}{2} \left[\frac{1}{h\sqrt{1+h^2}} - 1 + \frac{h}{\sqrt{1+h^2}} \right] \\ &= 2h\sqrt{1+h^2} - 2h^2 \end{aligned} \quad 3.32$$

whence

$$F_{ee}(h, k\delta = 0) = 1 - 2h\sqrt{1+h^2} + 2h^2 \quad 3.33$$

Values of $F_{ee}(h, k\delta = 0)$ for various values of h can be calculated from equation 3.33.

Consideration of equations 3.9 and 3.15 shews that

$$f_{ee}(k\delta, h) = F_{ee}(k\delta, h) \quad 3.34$$

Thus the computed values of F_{ee} are the required reception factors f_{ee} . The tabulated values are given in Table 3.6, those values not calculated being obtained by interpolation.

3.3(c). Integration of F to yield f.

The elemental reception factors F_{ss} , F_{gg} , and F_{sg} must be integrated with respect to $\frac{z_1}{\delta}$ and $\frac{z_2}{\delta}$ and the factors F_{es} and F_{eg} integrated with respect to $\frac{z_1}{\delta}$.

Consider the surface to surface factors F_{ss} . The tabulated values of F_{ss} were plotted against $\frac{z_2}{\delta}$ on sheets of 22 x 17 ins. squared paper. The integration with respect to $\frac{z_1}{\delta}$ was performed by choosing values of $\frac{z_2}{\delta}$ and integrating over the range $\frac{\Delta z}{\delta} = \frac{1}{100}$, starting from the origin. These values were summed at intervals $\frac{\Delta z}{\delta} = \frac{1}{10}$ for each $\frac{z_2}{\delta}$. The integrals over the range $\frac{\Delta z}{\delta} = \frac{1}{10}$ were plotted against the corresponding $\frac{z_2}{\delta}$ and integrated with respect to $\frac{z_2}{\delta}$, again at intervals of $\frac{\Delta z}{\delta} = \frac{1}{10}$, to give f_{ss} .

This process of integration was used to evaluate f_{ss} , f_{sg} , f_{es} and f_{eg} for cylinders of aspect ratio $\frac{1}{10}$ up to a separation $\frac{z_2 - z_1}{\delta} = 1$ for values of $k\delta$ of 0, 0.5, 1, 2 and 3. These values are given in Tables 3.4 to 3.9.

The superheater enclosure is of aspect ratio 2. The reception factors f were extended for cylinders having aspect ratio 0.5 up to a separation $\frac{z_2 - z_1}{\delta} = 2$. This was done in the same manner as above and the values of f are given in Tables 3.4 to 3.9.

3.3(d). Addition of reception factors.

The reception factors f have now been established for cylindrical zones of aspect ratio $\frac{1}{10}$. It is possible to build reception factors for larger zones.

Consider a cylinder of aspect ratio $\frac{4}{10}$. The value of f_{ss_0} for this cylinder, i.e. the fractional reception by the curved surface of radiation originating from that surface, can be calculated knowing the values of f_{ss_0} , f_{ss_1} , f_{ss_2} and f_{ss_3} for cylinders of aspect ratio $\frac{1}{10}$.

$$\left[f_{ss_0} \right]_{\text{aspect ratio } 0.4} = \left[\frac{4f_{ss_0} + 6f_{ss_1} + 4f_{ss_2} + 2f_{ss_3}}{4} \right]_{\text{aspect ratio } 0.1}$$

3.35

In this manner values of f_{ss} , f_{gg} and f_{sg} for zones of larger aspect ratio can be established.

To establish f_{es} and f_{eg} for a zone of larger aspect ratio it is necessary only to sum the required number of smaller reception factors, e.g.:-

$$\left[f_{es_0} \right]_{\text{aspect ratio } 0.4} = \left[f_{es_0} + f_{es_1} + f_{es_2} + f_{es_3} \right]_{\text{aspect ratio } 0.1}$$

3.36

In the case of end to end reception no arithmetic is needed as

$$\left[f_{ee_0} \right]_{\text{aspect ratio } 0.4} = \left[f_{ee_3} \right]_{\text{aspect ratio } 0.1}$$

3.37

In the above manner the reception factors f were evaluated for aspect ratios of 0.2, 0.5 and 1.0. The calculated values are shewn in Tables 3.4 to 3.9.

3.3(e). Tabulated reception factors f .

The various reception factors, f_{ss} , f_{es} , f_{ee} , f_{gg} , f_{sg} and f_{eg} , for the different values of zone aspect ratio and $k\delta$, are given in Tables 3.4 to 3.9 respectively.

All the reception factors were plotted against $k\delta$ so that factors for any $k\delta$ could be obtained.

3.3(f). Check on accuracy of reception factors.

It has been shewn how the basic factors for zones of aspect ratio $\frac{1}{10}$ can be built up to give factors for a zone of aspect ratio unity. Hoffman^{so} calculated factors for zones of aspect ratio unity. The two sets of f -factors are shewn in Table 3.10.

The agreement between the two sets of calculated factors is close. The average discrepancy is 0.17% and the standard deviation of the discrepancy is .53%.

A further method of checking is possible. Considering one zone, the equation $f_{ss_o} + f_{se_o} + f_{se_o} + f_{sg_o} = 1$ must be true. This method of checking was used when

Table 3.4. Surface-surface reception factors f_{ss} .

Aspect ratio of zone	$k\delta$ No.	0	0.5	1.0	2.0	3.0
0.1	f_{ss} 0	.09492	.06908	.05125	.03034	.01972
	1	.08525	.06043	.04328	.02389	.01405
	2	.07114	.04846	.03343	.01641	.00876
	3	.05829	.03849	.02570	.01160	.00556
	4	.04689	.03013	.01932	.00814	.00358
	5	.03739	.02324	.01426	.00563	.00228
	6	.02937	.01774	.01067	.00411	.00142
	7	.02270	.01345	.00787	.00319	.00095
	8	.01819	.01019	.00580	.00253	.00089
0.2	9	.01440	.00776	.00428	.00199	.00033
	f_{ss} 0	.18017	.12951	.09453	.05423	.03377
	1	.14291	.09792	.06792	.03416	.01857
	2	.09473	.06099	.03930	.01676	.00749
	3	.05942	.03609	.02174	.00852	.00303
0.5	4	.03674	.02080	.01188	.00512	.00122
	f_{ss} 0	.38208	.26676	.18890	.10079	.05859
	1	.20266	.12980	.08361	.03706	.01678
	2	.06431	.03224	.01877	.00685	.00171
1.0	3	.02180	.00950	.00416	.00081	.00017
	f_{ss} 0	.58473	.39656	.27250	.13786	.07538

Table 3.5. End-surface reception factors f_{es}

Aspect ratio of zone	k_0 No	0	0.5	1.0	2.0	3.0
0.1	f_{es} 0	.18015	.16742	.15656	.13447	.11980
	1	.14696	.12452	.10687	.07989	.05989
	2	.11878	.09407	.07529	.04928	.03270
	3	.09561	.07155	.05390	.03152	.01884
	4	.07662	.05454	.0391	.02076	.01111
	5	.06156	.04200	.02874	.01359	.00667
	6	.04974	.03234	.02111	.00915	.00404
	7	.04025	.02508	.01565	.00635	.00248
	8	.03263	.01962	.01165	.00445	.00155
0.2	f_{es} 9	.02694	.01540	.00874	.00301	.00098
	0	.32711	.29194	.26343	.21436	.17969
	1	.21439	.16562	.12919	.08080	.05154
	2	.13815	.09654	.06793	.03435	.01778
	3	.08999	.05742	.03676	.01550	.00652
0.5	f_{es} 4	.05957	.03502	.02039	.00746	.00253
	0	.61812	.51210	.43181	.31592	.24234
	1	.21112	.13444	.08589	.03655	.01572
	2	.07960	.03920	.02000	.00533	.00143
1.0	f_{es} 3	.03560	.01410	.00575	.00093	.00016
	0	.82924	.64654	.51770	.35247	.25806

Table 3.6. End-end reception factors f_{ee} .

Aspect ratio of zone	$k\phi$ No.	0	0.5	1.0	2.0	3.0
0.1	f_{ee} 0	.81900	.56742	.48875	.58080	.80060
	1	.67208	.48168	.42743	.50470	.64335
	2	.55360	.39610	.34434	.38185	.44391
	3	.45480	.32000	.27400	.27050	.28500
	4	.38200	.26029	.21626	.18408	.16935
	5	.32056	.21300	.16450	.12000	.09700
	6	.27102	.17475	.12157	.07375	.04960
	7	.23104	.14400	.09250	.04450	.02500
	8	.19828	.11950	.07200	.02718	.01200
	9	.17160	.09952	.05786	.01963	.00668
0.2	f_{ee} 0	.67208	.48168	.42743	.50470	.64335
	1	.45840	.32000	.27400	.27050	.28500
	2	.32056	.21300	.16450	.12000	.09700
	3	.23104	.14400	.09250	.04450	.02500
	4	.17160	.09952	.05786	.01963	.00668
0.5	f_{ee} 0	.38200	.26029	.21626	.18408	.16935
	1	.17160	.09952	.05786	.01963	.00668
	2	.09160	.04900	.02100	.00450	.00100
	3	.05573	.01994	.00713	.00092	.00012
1.0	f_{ee} 0	.17160	.09952	.05786	.01963	.00668

Table 3.7.

Gas-gas reception factors, f_{gg} .

Aspect ratio of zone	$k\delta$ No.	0.5	1.0	2.0	3.0
0.1	f_{gg} 0	.06670	.11912	.21140	.28068
	1	.03312	.05642	.09150	.11443
	2	.01679	.02800	.04045	.04448
	3	.01083	.01728	.02266	.02129
	4	.00739	.01150	.01348	.01206
	5	.00540	.00791	.00844	.00685
	6	.00406	.00554	.00551	.00405
	7	.00307	.00401	.00370	.00247
	8	.00237	.00300	.00235	.00167
0.2	f_{gg} 0	.09982	.17554	.30290	.39511
	1	.03876	.06485	.09753	.11234
	2	.01550	.02409	.02903	.02613
	3	.00829	.01150	.01158	.00871
	4	.00481	.00616	.00506	.00352
0.5	f_{gg} 0	.15145	.26142	.42986	.53900
	1	.03754	.05846	.07521	.07559
	2	.00832	.01034	.00766	.00489
	3	.00310	.00297	.00131	.00048
1.0	f_{gg} 0	.18899	.31988	.50507	.61459

Table 3.8. Surface-gas reception factors f_{sg}

Aspect ratio of zone	k No.	0.5	1.0	2.0	3.0
0.1	f_{sg} 0	.09208	.16488	.27500	.35984
	1	.05019	.08406	.12714	.15096
	2	.02773	.04394	.05740	.05848
	3	.01765	.02704	.03206	.02860
	4	.01209	.01754	.01868	.01532
	5	.00881	.01188	.01122	.00848
	6	.00630	.00824	.00718	.00484
	7	.00456	.00570	.00458	.00268
	8	.00336	.00404	.00302	.00164
	9	.00243	.00296	.00204	.00088
0.2	f_{sg} 0	.14227	.24894	.40214	.51080
	1	.06165	.09949	.13700	.14826
	2	.02532	.03700	.04032	.01042
	3	.01298	.01703	.01508	.01042
	4	.00686	.00837	.00633	.00342
0.5	f_{sg} 0	.22462	.38075	.58043	.70056
	1	.05981	.08874	.10390	.09779
	2	.01163	.01348	.00941	.00495
	3	.00346	.00325	.00142	.00049
1.0	f_{sg} 0	.28442	.46950	.68432	.79835

Table 3.9. End-gas reception factors f_{eg} .

Aspect ratio of zone	$k\delta$ No	0.5	1.0	2.0	3.0
0.1	f_{eg} 0	.07457	.14184	.25913	.35476
	1	.04994	.08846	.14064	.16980
	2	.03526	.05912	.08292	.08940
	3	.02565	.04124	.05240	.05080
	4	.01930	.02966	.03408	.02956
	5	.01488	.02172	.02224	.01780
	6	.01171	.01604	.01508	.01096
	7	.00932	.01182	.00992	.00628
	8	.00736	.00934	.00664	.00364
	9	.00594	.00734	.00468	.00240
0.2	f_{eg} 0	.12451	.23030	.39976	.52456
	1	.06091	.10036	.13532	.14020
	2	.03418	.05138	.05632	.04736
	3	.02103	.02786	.02500	.01724
	4	.01330	.01668	.01132	.00604
0.5	f_{eg} 0	.20472	.36032	.56916	.69432
	1	.04921	.06626	.05856	.04108
	2	.01740	.01840	.01035	.00445
	3	.00740	.00630	.00215	.00056
1.0	f_{eg} 0	.25393	.42658	.62772	.73540

Table 3.10. Values of f for aspect ratio unity.

$k\delta$	0	0.5	1.0	2.0	3.0
Calculated					
f_{ss_0}	.58473	.39656	.27250	.13786	.07538
f_{es_0}	.82924	.64654	.51770	.35247	.25806
f_{ee_0}	.17160	.09952	.05786	.01963	.00668
f_{gg_0}		.18899	.31988	.50507	.61459
f_{sg_0}		.28442	.46950	.68432	.79835
f_{eg_0}		.25393	.42658	.62772	.73540
Hoffman					
f_{ss_0}	.58520	.39518	.27206	.13721	.07529
f_{es_0}	.84181	.64324	.51078	.35137	.26099
f_{ee_0}	.17157	.09956	.05786	.01962	.00669
f_{gg_0}		.18838	.31924	.50268	.61616
f_{sg_0}		.28047	.46742	.68210	.78683
f_{eg_0}		.25068	.42658	.62901	.74570

reading off values of f from the plots at particular $k\delta$ before calculating direct interchange areas (see section 4.2(f)).

Chapter 4. Interchange Areas.

4.1. Introduction.

4.2. Direct interchange areas.

- 4.2(a) Wall temperatures and emissivities.
- 4.2(b) Mean beam length.
- 4.2(c) Emissivity of the gas.
- 4.2(d) Representation of a real gas.
- 4.2(e) Calculation of absorption strength.
- 4.2(f) Calculation of direct interchange areas.

4.3. Total interchange areas.

- 4.3(a) Surface-surface total interchange areas.
- 4.3(b) Gas-surface total interchange areas.
- 4.3(c) Gas-gas total interchange areas.
- 4.3(d) Solution of the simultaneous equations.
- 4.3(e) Real gas allowance.
- 4.3(f) Tabulated total interchange areas.

Chapter 4 Interchange Areas.

4.1 Introduction.

The various reception factors for exchange of radiation between cylindrical zones have been established. By applying these factors to a specific enclosure the transfer of radiant energy in that enclosure can be accounted for.

Two factors⁴⁶ are introduced to facilitate the explanation of the radiation interchange. The direct interchange area is used when dealing with direct exchange of radiation between zones. The total interchange area accounts for all radiant exchange including wall reflections.

4.2 Direct interchange areas.

Consider a furnace enclosure divided into cylindrical zones such that each zone may be taken as isothermal. The absorptivity of the gas is assumed independent of wave length and temperature. This restriction, that the gas is grey, is removed later.

The direct one-way exchange of radiant energy between two zones is described in the following manner for surface to gas, gas to gas, etc:-

$$q_{s_i g_j} = \overline{s_i g_j} \sigma T_{s_i}^4 \quad 4.1$$

$$q_{g_i g_j} = \overline{g_i g_j} \sigma T_{g_i}^4 \quad 4.2$$

The surface zone s_i may be the curved wall or the flat end bounding a cylindrical gas zone. The term q is the one-way radiant heat transfer from the source zone, indicated by the

first subscript, to the receiver zone, indicated by the second subscript. The terms $\overline{s_i g_j}$ etc. are called direct interchange areas and are the products of two kinds of terms. The first has the dimensions of area and is the actual area of a surface zone or (4) (volume) (absorption coefficient) for a gas zone. The second is the reception factor f .

When two zones are at the same temperature the net flux between them must be zero. As the net flux is the difference of two one-way flux terms it follows that:-

$$\overline{g_i s_j} = \overline{s_j g_i} \quad 4.3$$

To establish the direct interchange areas for a system it is necessary to find the relevant reception factors f . The reception factors have been established as functions of $k\delta$. It is thus necessary to evaluate a representative absorption coefficient k for the enclosure. This coefficient is dependent upon wall and gas temperatures and emissivities. As the value of k chosen is used in calculations to find zone temperatures and yet is dependent on those temperatures a trial and error method must be used. A value of k is assumed and the calculations laid out in Chapters 4 and 5 are made to arrive at approximate zone temperatures. These zone temperatures are used to improve the estimated absorption coefficient. The absorption strengths $k\delta$ for all the cases considered are calculated and the respective reception factors and direct interchange areas evaluated.

4.2 (a) Wall temperatures and emissivities.

The positions and temperatures of the thermocouples in the pot furnace are given in Tables 2.3 and 2.4. The temperatures are plotted against height in furnace in Fig.4.1.

Thermocouples 5 and 8 were placed behind burners and might be expected to record lower temperatures than the average for their level. This tendency is most marked near the level of the burners. Thermocouple 7 is directly apposite a burner and would probably indicate temperatures higher than the average for that level. The lines drawn on Fig. 4.1 are considered the most likely variations of wall temperature allowing for the placing of the thermocouples. From these curves mean wall temperatures, used in the calculation of absorption coefficients, were found.

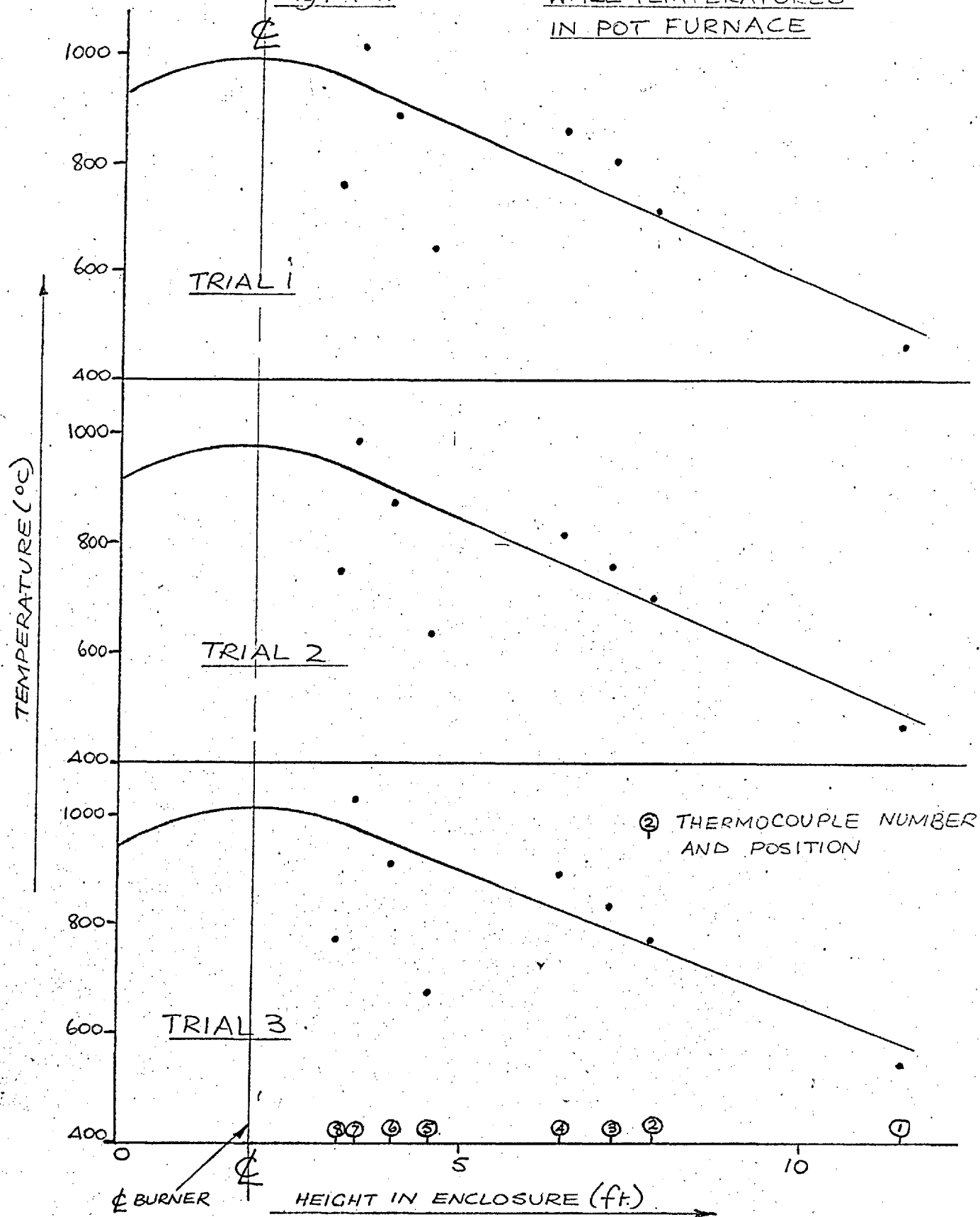
The walls of the superheater were taken to be the steam tubes and an effective temperature was calculated from the measurements of steam temperature recorded in Appendix 1.2.

Determinations of the emissivity of firebrick shew a large range of values. The results of a number of workers ^{63,64,65,66,67} were averaged for the temperatures found and an overall mean emissivity of 0.75 was taken. The results of a number of workers ^{25,66,67} indicate that the cast iron pot can be assumed to have an emissivity of 0.9.

The ends of the super heater are firebrick and contain inlet and exit ports. The ports were assigned an emissivity of unity and, after allowing for relative area, the effective emissivity of the enclosure ends was taken to be 0.8.

Fig. 4.1.

WALL TEMPERATURES IN POT FURNACE



4.2 (b) Mean beam length.

When a cylindrical enclosure contains an absorbing or emitting medium the walls of the enclosure experience a particular radiant flux density. The mean beam length of this enclosure is the radius of a hemisphere of the same medium which would cause this same radiant flux density at the centre of the hemisphere.

Consider a cylindrical enclosure made up of n basic zones each of volume V . The flux leaving the gas mass is the total emission minus the self-absorption and is,

$$\text{flux} = 4k \ nV \ (1 - f_{gg}) \ \sigma \ T^4 \quad 4.4$$

where f_{gg} refers to the total enclosure volume and is given in terms of zone reception factors by,

$$f_{gg} = n f_{gg_0} + 2 \ (n-1) \ f_{gg_1} + 2 \ (n-2) \ f_{gg_2} + \dots \quad 4.5$$

To find the flux density the flux at the walls must be divided by the total wall area A .

$$\text{Flux density} = \frac{4knV \ (1-f_{gg}) \ \sigma \ T^4}{A} \quad 4.6$$

The flux density at the centre of a hemisphere of radius L of the same gas is,

$$\text{Flux density} = 1 - e^{-kL} \ \sigma \ T^4 \quad 4.7$$

Equating equations 4.6 and 4.7 gives,

$$1 - e^{-kL} = \frac{4knV \ (1-f_{gg})}{A} \quad 4.8$$

If the cylinder diameter δ is taken to be the characteristic dimension of the system then $L = \psi \delta$, where ψ is the factor by which the diameter must be multiplied to give the mean beam length.

Thus,

$$1 - e^{-k\psi\delta} = \frac{4knV(1-f_{gg})}{A} \quad 4.9$$

The equation 4.9 must be solved by trial and error as f_{gg} is a function of $k\delta$ which is a function of mean beam length.

The values of ψ for different $k\delta$ calculated from the above equation are given in Table 4.1.

Table 4.1.

$k\delta$	ψ
0	2/3
0.5	.630
1.0	.603
2.0	.539
3.0	.490

Mean beam length for cylinder with height equal diameter radiating to whole surface as a function of absorption strength.

These values were plotted so that mean beam lengths for any value of $k\delta$ could readily be obtained.

4.2 (c) Emissivity of the gas.

If a gas is grey its emissivity is independent of temperature and equal to its absorptivity. In fact absorptivity is known to be strongly dependent on the temperature of the radiating surface as well as the gas temperature, whereas emissivity depends mainly on the gas temperature. To circumvent this difficulty Hottel⁴⁵ has suggested that both the emission and absorption phenomena can be satisfactorily approximated by

$$\alpha_{\text{ave}} = \epsilon_{\text{ave}} = \epsilon_{G_{T_{\text{ave}}}} \cdot K' \quad 4.10$$

where
$$K' = \frac{4 + a + b - c}{4}$$

and $\epsilon_{G_{T_{\text{ave}}}}$ is the gas emissivity evaluated at the arithmetic mean of the gas and surface temperatures. The factors a and b are the slopes of the $\ln \epsilon_G - v - \ln PL$ and the $\ln \epsilon_G - v - \ln T$ curves and have been tabulated by Hottel. The empirical constant C has a value 0.45 for H_2O and 0.65 for CO_2 . This method of representing gas emission and absorption is recommended providing the wall and gas temperature do not differ by a factor greater than two.

4.2 (d) Representation of a real gas.

A real gas, in contrast to a grey gas, shows variation in emissivity and absorptivity with wavelength and temperature. The characteristic absorption in certain spectral regions exhibited by a real gas causes the absorbable wavelengths to be filtered out and the transmittance of the gas for the remainder of the radiation approaches unity. The grey gas assumption thus leads to prediction of too large an interchange

between gas and sink zones, and too small an interchange between source-sink surfaces.

If a grey gas is assumed the absorption coefficient may be calculated from,

$$\epsilon_L = 1 - e^{-kL} \quad 4.11$$

where L is the mean beam length.

Hottel has shewn^{26,45} that it is possible to represent the variation of emissivity of a real gas by the weighted sum of the contributions from a number of grey and clear gases.

$$\epsilon_x = \sum_i a_i (1 - e^{-k_i x}) \quad 4.12$$

with $\sum a_i = 1$.

The simplest representation is to replace the real gas by a grey gas of absorption coefficient k and weighting factor a and a clear gas of absorption coefficient zero and weighting factor $(1 - a)$. The total emissivity in this case is related to path length x by,

$$\epsilon_x = a (1 - e^{-kx}) \quad 4.13$$

This exponential can be fitted to the actual emissivity - x - path length curve at three path lengths. As the wall emissivity is high (0.75) the first surface reflection is of major importance²⁹ and subsequent reflections are much less important owing to rapid alternation. The three key points for curve-fitting are chosen as zero path length, a path length equal to the mean beam length L for the system and a path length twice this value $2L$.

The values of a and k may be found from the following equations,

$$a = \frac{(\epsilon_L)^2}{2\epsilon_L - \epsilon_{2L}} \quad 4.14$$

$$kL = \ln \frac{\epsilon_L}{\epsilon_{2L} - \epsilon_L} \quad 4.15$$

The derivations of these equations are given in Appendix 2.

A better approximation to a real gas can be obtained using a representation of two grey and one clear gas. In this case the emissivity is given by,

$$\epsilon_x = \sum_{i=1}^3 a_i (1 - e^{-k_i x}) \quad 4.16$$

with $k_3 = 0$ and $\sum a_i = 1$.

The emissivity - v - path length curve can now be fitted at two further points and these are taken to be $3L$ and $4L$. The values of a_1 , a_2 , k , and k_2 can be found by solving the four simultaneous equations (see Appendix 2):-

$$\epsilon_L = a_1 (1 - e^{-k_1 L}) + a_2 (1 - e^{-k_2 L}) \quad 4.17$$

$$\epsilon_{2L} = a_1 (1 - e^{-2k_1 L}) + a_2 (1 - e^{-2k_2 L}) \quad 4.18$$

$$\epsilon_{3L} = a_1 (1 - e^{-3k_1 L}) + a_2 (1 - e^{-3k_2 L}) \quad 4.19$$

$$\epsilon_{4L} = a_1 (1 - e^{-4k_1 L}) + a_2 (1 - e^{-4k_2 L}) \quad 4.20$$

In the representation of the gas so far considered the properties a and k are average values for the whole enclosure. It has been assumed that the gas properties can be represented effectively by such average values. However it is possible to go one step further towards a rigorous solution.

If the interchange between a given zone g_1 and each other zone in the system is evaluated with a k based on the temperature of that zone the errors should be small. The interchange with adjacent zones should be accurately accounted for as their temperatures, and hence their k values, will not be very different from those of zone g_1 . The interchange with zones distant from g_1 will be in error but as these terms are small some error may be tolerated. Thus if the total interchange areas (see section 4.3) for the system are evaluated, allowing each gas zone in turn to be the original emitter and basing calculations on the k for that zone, a more accurate pattern of transfer can be found.

To undertake this procedure the temperature of each gas zone must be estimated and the various k calculated and then, after calculating all interchange in the system, if the estimated and calculated temperatures are significantly different the zone temperatures must be re-estimated. If such a method is used the total interchange areas lose their symmetry and $\overline{G_1 G_2}$ does not equal $\overline{G_2 G_1}$.

4.2 (e) Calculation of absorption strength $k\delta$

The gas composition was calculated from the experimental results (Appendix 1) and the partial pressures of H_2O and CO_2 found. A mean beam length L was assumed and the functions PL calculated. The mean enclosure wall temperatures were estimated from the experimental data (section 4.2 (a)). A mean gas temperature for the enclosure was assumed. Using the various charts given by Hottel⁴⁵ the values of the effective emissivities ϵ_L , ϵ_{2L} etc. were calculated from equation 4.10. By substitution in the equations given in section 4.2 (d) the required value of $k\delta$ was established.

The assumed value of mean beam length was checked for consistency with the calculated $k\delta$ (section 4.2 (c)) and if necessary a new mean beam length was estimated.

With the value of $k\delta$ established the various reception factors f were found and the total interchange areas for a one gas zone system were evaluated. The total heat balance calculations (see chapter 5) were performed and the mean gas zone temperature found. If this temperature was significantly different from the assumed gas temperature a new value was estimated and the whole procedure repeated.

In this manner the values of $k\delta$ and a for the different trials and gas representations were evaluated. These values of $k\delta$ and a are listed overleaf in Table 4.2 (Values of absorption strength $k\delta$ and weighting factor a)

FURNACE	TRIAL	GAS REPRESENTATION	$k\delta$	a
Pot	1	grey	0.3545	---
Pot	1	1 grey 1 clear	1.79	0.285
Pot	1	2 grey 1 clear	0.4928	0.35912
Pot	1	2 grey 1 clear	9.632	0.09548
Pot	1	grey - variable k	zone 1 0.4069	---
Pot	1	grey - variable k	zone 2 0.3189	---
Pot	1	1 grey 1 clear and	zone 1 1.927	0.311
Pot	1	variable k	zone 2 1.696	0.268
Pot	2	grey	0.335	---
Pot	2	1 grey 1 clear	1.958	0.259
Pot	3	grey	0.338	---
Pot	3	1 grey 1 clear	1.747	0.279
S/heater	-	grey	.311	---
S/heater	-	1 grey 1 clear	1.89	0.256

4.2 (f) Calculation of direct interchange areas.

The reception factors f are found at the correct $k\delta$ from the charts of $f - v - k\delta$ established in chapter 3. The direct interchange area is the relevant reception factor multiplied by the area of the source zone or by $4kV$ if a gas zone is the source. In this manner all direct interchange areas for the system, $\overline{es}_0, \overline{es}_1, \dots, \overline{gg}_0, \overline{gg}_1, \dots$ etc., were established. The first letter designates source zone, the second letter receiver zone and the subscript the separation of the two zones.

A check on the accuracy of the reception factors obtained from the plots was possible. Consider the curved wall

of a zone. The reception factors must satisfy the equation,

$$f_{ss_0} + 2f_{se_0} + f_{sg_0} = 1 \quad 4.21$$

Many similar equations can be formed enabling all reception factors, and hence direct interchange areas, to be checked.

In this way any errors in curve plotting and reading were found and corrected.

4.3 Total interchange areas.

Real furnace walls are partial reflectors of radiation and their reflection is much more nearly diffuse than specular. This reflected radiation is distributed among all gas and surface zones and, although absorption quickly reduces the strength of the radiation, the reflection process continues indefinitely. The total interchange area between two zones is defined as the net interchange of radiation per unit difference in emissive powers, and includes the contributions from multiple reflections. They are designated $\overline{S_i S_j}$, $\overline{G_i S_j}$, $\overline{G_i G_j}$ where the first letter refers to source and the second to receiver. To establish a total interchange area knowledge of all zones in the system is required.

If the emissivities and absorptivities of the system are assumed to be independent of temperature then the total interchange areas are also independent of temperature and may be evaluated at any convenient temperature. It is convenient to assign a temperature of absolute zero to every zone save one. The temperature of this one zone is taken such that its black emissive power is unity. It is now possible in a convenient manner to follow the radiation from this one zone throughout the system.

There will be a radiant flux to and from every surface in the system owing solely to original emission from the one zone and the multiple reflections within the enclosure. The flux density leaving a surface is termed R with a pre-subscript to denote the original emitter and a final subscript to denote the reflecting surface. Thus if surface s_1 were the only original emitter the flux density leaving surfaces $s_2, s_3, \dots$ would be $s_1 R_{s_2}, s_1 R_{s_3}, \dots$, while the flux density leaving s_1 would be $s_1 R_{s_1} + \epsilon_{s_1}$ to allow for the original emission.

The total reflected flux away from any surface s_j is equal to the reflected flux times the zone area A_{s_j} . If this reflected flux is multiplied by the ratio of absorptivity (equal to emissivity ϵ_{s_j}) of the surface to reflectivity ρ_{s_j} of the surface the rate of energy absorption is obtained. Since the emissive power of the original emitter was made equal to unity this rate of energy absorption must be equal to the desired total interchange area,

$$\overline{S_i S_j} = \overline{S_j S_i} = \frac{A_{s_j} \epsilon_{s_j}}{\rho_{s_j}} s_i R_{s_j} \quad 4.22$$

$$\overline{G_i S_j} = \overline{S_j G_i} = \frac{A_{s_j} \epsilon_{s_j}}{\rho_{s_j}} g_i R_{s_j} \quad 4.23$$

The problem of finding total interchange areas is now reduced to one of determining the various reflected fluxes. This can be done by solving the system of simultaneous equations obtained from writing a radiant heat balance on each

surface. The methods are shown in sections 4.3 (a) and 4.3 (b).

In the nomenclature used for total interchange areas end zones and surface zones are given the letter S. These surface zones are numbered in order from the top of the furnace. Thus if there are n gas zones the two end zones are termed S_1 and S_{n+2} . The gas zones are also numbered from top to bottom of furnace.

4.3 (a) Surface-surface total interchange areas.

The surface zone s_1 is allowed to be the sole original emitter in the system. The total rate of energy impingement on a surface is equal to the contributions from all surfaces in the system. If the sum of all these terms is multiplied by the surface reflectivity the result must equal the reflected flux,

$$\overline{s_1 s_1} (s_1 R_{s_1} + \epsilon_{s_1}) + \overline{s_2 s_1} s_1 R_{s_2} + \overline{s_3 s_1} s_1 R_{s_3} + \dots = \frac{A_{s_1} s_1 R_{s_1}}{\rho_{s_1}}$$

$$\overline{s_1 s_2} (s_1 R_{s_1} + \epsilon_{s_1}) + \overline{s_2 s_2} s_1 R_{s_2} + \overline{s_3 s_2} s_1 R_{s_3} + \dots = \frac{A_{s_2} s_1 R_{s_2}}{\epsilon_{s_2}}$$

etc.

4.24

The terms $\overline{s_1 s_1}$, $\overline{s_1 s_2}$ are the relevant direct interchange areas.

Equation 4.24 can be transposed to, (see overleaf)

$$\frac{(\overline{s_1 s_1} - A_{s_1})}{\overline{e_{s_1}}} \overline{s_1} R_{s_1} + \overline{s_2 s_1} \overline{s_1} R_{s_2} + \overline{s_3 s_1} \overline{s_1} R_{s_3} + \dots = -\epsilon_{s_1} \overline{s_1 s_1}$$

$$\overline{s_1 s_2} \overline{s_1} R_{s_1} + \frac{(\overline{s_2 s_2} - A_{s_2})}{\overline{e_{s_2}}} \overline{s_1} R_{s_2} + \overline{s_3 s_2} \overline{s_1} R_{s_3} + \dots = -\epsilon_{s_1} \overline{s_1 s_2}$$

etc.

4.25

This yields a set of n simultaneous equations where n is the number of surface zones present and therefore the number of required reflected fluxes. The coefficients on the reflected fluxes can be set up as a matrix and the simultaneous equations solved by inverting the matrix and multiplying by the final column of right hand side terms.

If each other surface zone in the system is allowed in turn to become the original emitter further sets of simultaneous equations can be established. The matrix is identical in each case but a different final column is obtained. The solution is thus the inversion of the matrix and multiplication by a series of final columns. After substitution of the direct interchange areas previously obtained the matrix and final columns for the case of three surface zones is given by,

$$\begin{vmatrix} \frac{-A_e}{\overline{e_e}}, \overline{se_o}, \overline{ee_o} \\ \overline{es_o}, \overline{ss_o} - \frac{A_s}{\overline{e_s}}, \overline{es_o} \\ \overline{ee_o}, \overline{se_o}, -\frac{A_e}{\overline{e_e}} \end{vmatrix} \begin{vmatrix} 0, -\epsilon_s \cdot \overline{se_o}, -\epsilon_e \cdot \overline{ee_o} \\ -\epsilon_e \cdot \overline{es_o}, -\epsilon_s \cdot \overline{ss_o}, -\epsilon_e \cdot \overline{es_o} \\ -\epsilon_e \cdot \overline{ee_o}, -\epsilon_s \cdot \overline{se_o}, 0 \end{vmatrix}$$

4.26

4.3 (b) Gas-surface total interchange areas.

If a gas zone is allowed to be the original emitter a similar series of heat balances can be made.

$$\overline{s_1 s_1} g_i R_{s_1} + \overline{s_1 s_2} g_i R_{s_2} + \overline{s_1 s_3} g_i R_{s_3} + \dots \overline{g_i s_1}) = \frac{A_{s_1} g_i R_{s_1}}{e_{s_1}}$$

$$\overline{s_1 s_2} g_i R_{s_1} + \overline{s_2 s_2} g_i R_{s_2} + \overline{s_2 s_3} g_i R_{s_3} + \dots \overline{g_i s_2}) = \frac{A_{s_2} g_i R_{s_2}}{e_{s_2}}$$

4.27

If these equations are transposed it is found that the matrix of coefficients on the reflected fluxes is the same as for the surface to surface case. The final column is

$$-\overline{g_i s_1}, -\overline{g_i s_2}, -\overline{g_i s_3} \text{ etc.}$$

Thus if each gas zone in turn is allowed to be the original emitter a further series of final columns is obtained which on multiplication by the inverted matrix yield the required reflected fluxes.

4.3 (c) Gas-gas total interchange areas.

If gas zone g_m is the original emitter then the total reception by any other gas zone g_n is equal to the direct radiation $\overline{g_m g_n}$ plus the sum of the products of the reflected flux at each surface $g_m R_{s_i}$ ^{times} the fraction of that flux which reaches and is absorbed by the gas zone g_n , $\overline{s_i g_n}$. As the black emissive power of zone g_m is unity the total interchange area is given by,

$$\overline{G_m G_n} = \overline{g_m g_n} + \sum_i (g_m R_{s_i}) (\overline{s_i g_n}) \quad 4.28$$

By allowing each gas zone in turn to be the original emitter all the $\overline{GG}$ may be obtained.

4.3 (d) Solution of the simultaneous equations.

When 5 or 10 gas zones were investigated the London University Mercury computer was used. A method of matrix inversion and post multiplication by series of final columns⁶⁸ was used to obtain the reflected fluxes $g_i R_{s_j}$ and $s_i R_{s_j}$. When smaller numbers of gas zones were investigated the method developed by Crout⁶⁹ was used with an electric desk calculator. All reflected fluxes were evaluated to six significant figures.

This method was used in preference to the determinant method proposed by Hottel and Cohen⁴⁶ as it very greatly reduced the amount of computation.

The total interchange areas $\overline{SS}$, $\overline{SQ}$, $\overline{GG}$ were derived from the reflected fluxes using the desk calculator. Again six significant figures were carried.

The total interchange areas were checked for errors in computation by substitution in the following equations.

$$\sum_i \overline{s_j s_i} + \sum_i \overline{s_j g_i} = \epsilon_{s_j} A_{s_j} \quad 4.29$$

$$\sum_j \overline{g_i g_j} + \sum_j \overline{g_i s_j} = 4kVg_i \quad 4.30$$

4.3 (e) Real gas allowance.

It has already been mentioned (section 4.2 (d)) that a real gas can be represented by a number of grey gases. If one grey and one clear gas are used the total interchange areas must be evaluated at $k\delta = 0$ and $k\delta = k\delta$ and the total interchange areas to be used become,

$$\overline{SS} = a (\overline{SS})_k + (1-a) (\overline{SS})_{k=0} \quad 4.31$$

$$\overline{GS} = a (\overline{GS})_k \quad 4.32$$

$$\overline{GG} = a (\overline{GG})_k \quad 4.33$$

If two grey and one clear gas are used the total interchange areas become,

$$\overline{SS} = a_1 (\overline{SS})_{k_1} + a_2 (\overline{SS})_{k_2} + (1-a_1-a_2) (\overline{SS})_{k=0} \quad 4.34$$

$$\overline{GS} = a_1 (\overline{GS})_{k_1} + a_2 (\overline{GS})_{k_2} \quad 4.35$$

$$\overline{GG} = a_1 (\overline{GG})_{k_1} + a_2 (\overline{GG})_{k_2} \quad 4.36$$

When k is allowed to have a different value for each gas zone all total interchange areas are evaluated for each value of k . The values taken were those considered likely to yield the most accurate representation. Thus $\overline{G_1 S_1}$ is based on the k for gas zone G_1 . For the total interchange area between the two ends of the furnace the value of k for the whole furnace (as calculated above where k is not allowed to vary) was used.

4.3 (f) Tabulated total interchange areas.

The total interchange areas for all furnace trials and gas representations investigated are listed in Tables 4.3 to 4.20.

Table 4.3. Total interchange areas for pot furnace
trial 1: 10 gas zones: 1 grey-1clear: $k_6 = 1.79$

GG		4 6	0.479624	1 4	0.270269
1 1	2.19646	4 7	0.288312	1 5	0.158365
1 2	1.03596	4 8	0.182955	1 6	0.0977459
1 3	0.498665	4 9	0.124910	1 7	0.0612932
1 4	0.299364	4 10	0.0923198	1 8	0.0411591
1 5	0.187997	5 5	2.14100	1 9	0.0281702
1 6	0.123518	5 6	1.00402	1 10	0.0204874
1 7	0.0829721	5 7	0.481462	1 11	0.0159036
1 8	0.0584572	5 8	0.291954	1 12	0.0779048
1 9	0.0408288	5 9	0.189844	2 1	1.87187
1 10	0.0328733	5 10	0.137618	2 2	0.554291
2 2	2.15919	6 6	2.14295	2 3	1.12352
2 3	1.01347	6 7	1.00763	2 4	0.542517
2 4	0.485235	6 8	0.487609	2 5	0.262976
2 5	0.290694	6 9	0.302830	2 6	0.153924
2 6	0.182134	6 10	0.209454	2 7	0.0949124
2 7	0.120183	7 7	2.14865	2 8	0.0596784
2 8	0.0813598	7 8	1.01681	2 9	0.0405447
2 9	0.0589428	7 9	0.503461	2 10	0.0284419
2 10	0.0437706	7 10	0.331133	2 11	0.0217197
3 3	2.14598	8 8	2.16310	2 12	0.104094
3 4	1.00580	8 9	1.04138	3 1	1.16216
3 5	0.480496	8 10	0.546986	3 2	0.270092
3 6	0.287793	9 9	2.20440	3 3	0.542478
3 7	0.180935	9 10	1.11400	3 4	1.11673
3 8	0.120393	10 10	2.33098	3 5	0.538433
3 9	0.0835908	GS		3 6	0.260578
3 10	0.0641244	1 1	3.22269	3 7	0.152529

Table 4.3 contd.

4 4	2.1480	1 2	1.14427	3 8	0.0943299
4 5	1.00354	1 3	0.554487	3 9	0.0598734
GS		6 3	0.0950940	8 9	1.12472
3 10	0.0415878	6 4	0.152643	8 10	0.555038
3 11	0.0306680	6 5	0.260160	8 11	0.290654
3 12	0.141461	6 6	0.537496	8 12	0.970850
4 1	0.768477	6 7	1.11495	9 1	0.132415
4 2	0.158534	6 8	0.539309	9 2	0.0206047
4 3	0.263123	6 9	0.264019	9 3	0.0286020
4 4	0.538547	6 10	0.159554	9 4	0.0416573
4 5	1.11448	6 11	0.107110	9 5	0.0618781
4 6	0.537276	6 12	0.437797	9 6	0.0985496
4 7	0.260148	7 1	0.249153	9 7	0.159587
4 8	0.152768	7 2	0.0414547	9 8	0.271497
4 9	0.0953610	7 3	0.0598885	9 9	0.555556
4 10	0.0619530	7 4	0.0944459	9 10	1.14388
4 11	0.0453783	7 5	0.152763	9 11	0.587510
4 12	0.205563	7 6	0.261022	9 12	1.56425
5 1	0.525229	7 7	0.539180	10 1	0.106776
5 2	0.097995	7 8	1.11768	10 2	0.0161403
5 3	0.154114	7 9	0.543686	10 3	0.0220885
5 4	0.260707	7 10	0.271313	10 4	0.0309390
5 5	0.537293	7 11	0.172042	10 5	0.0455105
5 6	1.14405	7 12	0.641501	10 6	0.0679403
5 7	0.537487	8 1	0.174126	10 7	0.107759
5 8	0.261022	8 2	0.0283403	10 8	0.173216
5 9	0.154567	8 3	0.0406820	10 9	0.292852
5 10	0.0985228	8 4	0.0599167	10 10	0.589639
5 11	0.0675017	8 5	0.0952675	10 11	1.20154

Table 4.3 contd.

5 12	0.294738	8 6	0.154494	10 12	2.69350
6 1	0.354811	8 7	0.263895	SS	
6 2	0.0615366	8 8	0.543586	1 1	2.49424
SS		3 6	0.656463	6 8	0.800314
1 2	6.18988	3 7	0.544492	6 9	0.677416
1 3	4.89606	3 8	0.450098	6 10	0.572624
1 4	3.93165	3 9	0.371750	6 11	0.486509
1 5	3.18976	3 10	0.308318	6 12	1.75036
1 6	2.60862	3 11	0.264861	7 7	1.04793
1 7	2.15399	3 12	1.00127	7 8	0.952099
1 8	1.80361	4 4	1.03341	7 9	0.814950
1 9	1.53032	4 5	0.931017	7 10	0.695936
1 10	1.32113	4 6	0.786630	7 11	0.595968
1 11	1.16716	4 7	0.659227	7 12	2.14354
1 12	5.43900	4 8	0.549207	8 8	1.06285
2 2	1.04985	4 9	0.456650	8 9	0.970700
2 3	0.939577	4 10	0.380588	8 10	0.838415
2 4	0.789795	4 11	0.319631	8 11	0.725619
2 5	0.658032	4 12	1.19646	8 12	2.64093
2 6	0.543916	5 5	1.03443	9 9	1.08610
2 7	0.447548	5 6	0.933813	9 10	1.00017
2 8	0.367482	5 7	0.791381	9 11	0.876094
2 9	0.301887	5 8	0.666046	9 12	3.27174
2 10	0.256303	5 9	0.558105	10 10	1.12374
2 11	0.219418	5 10	0.468195	10 11	1.04903
2 12	0.849095	5 11	0.395225	10 12	4.08791
3 3	1.03716	5 12	1.44234	11 11	1.18869
3 4	0.932171	6 6	1.03897	11 12	5.17875
3 5	0.785830	6 7	0.940438	12 12	1.61093

Table 4.4 Total interchange areas for pot furnace.

trial 1: 10 gas zones: grey gas: $k\delta = .3545$

GG		3 9	0.0426577	8 10	0.144629
1 1	0.406382	3 10	0.0356158	9 9	0.420924
1 2	0.220851	4 4	0.405686	9 10	0.239704
1 3	0.130715	4 5	0.219390	10 10	0.432029
1 4	0.0901182	4 6	0.130053	GS	
1 5	0.0647388	4 7	0.0905851	1 1	2.65445
1 6	0.0507683	4 8	0.0664164	1 2	1.111106
1 7	0.0406267	4 9	0.0537926	1 3	0.642862
1 8	0.0327164	4 10	0.0448944	1 4	0.379888
1 9	0.0266752	5 5	0.405784	1 5	0.258578
1 10	0.0222838	5 6	0.219936	1 6	0.192181
2 2	0.407162	5 7	0.131075	1 7	0.145376
2 3	0.220285	5 8	0.0920399	1 8	0.113328
2 4	0.130198	5 9	0.0685226	1 9	0.0886920
2 5	0.0898264	5 10	0.0564679	1 10	0.0715601
2 6	0.0648078	6 6	0.406859	1 11	0.0581385
2 7	0.0512930	6 7	0.221585	1 12	0.268264
2 8	0.0413789	6 8	0.133325	2 1	1.97430
2 9	0.0339167	6 9	0.0951740	2 2	0.645228
2 10	0.0282468	6 10	0.0724885	2 3	1.10580
3 3	0.406122	7 7	0.409196	2 4	0.639848
3 4	0.219542	7 8	0.224704	2 5	0.378214
3 5	0.129765	7 9	0.137585	2 6	0.257924
3 6	0.0898237	7 10	0.100522	2 7	0.192376
3 7	0.0652773	8 8	0.413343	2 8	0.146339
3 8	0.0520703	8 9	0.230335	2 9	0.115174

Table 4.4 contd.

GS		5 3	0.257529	7 9	0.651016
2 10	0.0914531	5 4	0.376888	7 10	0.396147
2 11	0.0757966	5 5	0.637423	7 11	0.284159
2 12	0.333454	5 6	1.10200	7 12	0.946583
3 1	1.48254	5 7	0.639174	8 1	0.503656
3 2	0.381973	5 8	0.380446	8 2	0.0898461
3 3	0.639927	5 9	0.263108	8 3	0.115203
3 4	1.10252	5 10	0.200835	8 4	0.148386
3 5	0.637853	5 11	0.158970	8 5	0.196395
3 6	0.377280	5 12	0.575176	8 6	0.264223
3 7	0.257908	6 1	0.703288	8 7	0.387051
3 8	0.148161	6 3	0.191898	8 9	1.22114
3 10	0.118035	6 4	0.257506	8 10	0.664500
3 11	0.0958814	6 5	0.377764	8 11	0.414199
3 12	0.396900	6 6	0.639289	8 12	1.23659
4 1	1.13924	6 7	1.10488	9 1	0.438491
4 2	0.260171	6 8	0.643265	9 2	0.0731029
4 3	0.378134	6 9	0.385997	9 3	0.0918635
4 4	0.637744	6 10	0.270513	9 4	0.118785
4 5	0.10142	6 11	0.210945	9 5	0.153187
4 6	0.637726	6 12	0.731474	9 6	0.202897
4 7	0.378042	7 1	0.593929	9 7	0.272642
4 8	0.259589	7 2	0.114362	9 8	0.398008
4 9	0.195960	7 3	0.146194	9 9	0.665800
4 10	0.152095	7 4	0.193184	9 10	1.13950
4 11	0.123760	7 5	0.259669	9 11	0.689012
4 12	0.478813	7 6	0.381015	9 12	1.65088
5 1	0.885495	7 7	0.643715	10 1	0.375216
5 2	0.193205	7 8	1.11080	10 2	0.0594486

Table 4.4 contd.

GS		2 8	0.317986	5 11	0.343648
10 3	0.0759706	2 9	0.257064	5 12	0.30107
10 4	0.0966014	2 10	0.211117	6 6	0.977233
10 5	0.124970	2 11	0.179125	6 7	0.878555
10 6	0.161468	2 12	0.737729	6 8	0.737224
10 7	0.213544	3 3	0.986200	6 9	0.615008
10 8	0.286475	3 4	0.879059	6 10	0.514393
10 9	0.415851	3 5	0.729655	6 11	0.432452
10 10	0.689073	3 6	0.599072	6 12	1.61853
10 11	1.7074	3 7	0.489071	7 7	0.984931
10 12	2.22339	3 8	0.395110	7 8	0.889491
SS		3 9	0.319604	7 9	0.752059
1 1	2.18981	3 10	0.261213	7 10	0.634797
1 2	6.19517	3 11	0.219019	7 11	0.541448
1 3	4.84736	3 12	0.865727	7 12	2.02033
1 4	3.83784	4 4	0.977351	8 8	0.999786
1 5	3.05769	4 5	0.873602	8 9	0.909130
1 6	2.44648	4 6	0.727066	8 10	0.777919
1 7	1.97808	4 7	0.599031	8 11	0.669736
1 8	1.61307	4 8	0.491487	8 12	2.54026
1 9	1.34475	4 9	0.400242	9 9	1.02534
1 10	1.13256	4 10	0.327671	9 10	0.942497
1 11	1.00859	4 11	0.273630	9 11	0.822656
1 12	4.76318	4 12	1.06185	9 12	3.20043
2 2	1.00760	5 5	0.974726	10 10	1.06871
2 3	0.893056	5 6	0.873559	10 11	1.00035
2 4	0.739386	5 7	0.729417	10 12	4.05230
2 5	0.605422	5 8	0.603885	11 11	1.14547
2 6	0.492565	5 9	0.499208	11 12	5.18680
2 7	0.396102	5 10	0.411289	12 12	1.44880

Table 4.5. Total interchange areas for pot furnaces.

trial 1: 5 gas zones: 1 grey - 1 clear: k₀ 1.79

GS		4 6	1.28896	3 7	2.64237
1 1	5.09101	4 7	1.61743	4 4	3.96505
1 2	3.37577	5 1	0.240560	4 5	3.24310
1 3	1.23396	5 2	0.0877689	4 6	2.35125
1 4	0.408499	5 3	0.180488	4 7	3.89644
1 5	0.170053	5 4	0.434471	5 5	4.08775
1 6	0.0869053	5 5	1.29293	5 6	3.43888
1 7	0.183051	5 6	3.52160	5 7	5.91275
2 1	1.93705	5 7	4.25411	6 6	4.40812
2 2	1.23398	SS		6 7	9.26178
2 3	3.30688	1 1	2.46466	7 7	1.58955
2 4	1.21008	1 2	11.0815	GG	
2 5	0.402845	1 3	7.12253	1 1	6.42122
2 6	0.180097	1 4	4.76640	1 2	2.29545
2 7	0.348809	1 5	3.33883	1 3	0.789724
3 1	0.884096	1 6	2.49329	1 4	0.345699
3 2	0.409314	1 7	5.44871	1 5	0.177855
3 3	1.21035	2 2	3.96431	2 2	6.28778
3 4	3.30261	2 3	3.16465	2 3	2.24875
3 5	1.21840	2 4	2.89272	2 4	0.777571
3 6	0.433351	2 5	1.49251	2 5	0.367747
3 7	0.735782	2 6	1.05060	3 3	6.28006
4 1	0.425542	2 7	1.85407	3 4	2.26607
4 2	0.170834	3 3	3.92729	3 5	0.845321
4 3	0.402905	3 4	3.16941	4 4	6.33407
4 4	1.21818	3 5	2.23007	4 5	2.42198
4 5	3.32843	3 6	1.56492	5 5	6.75650

Table 4.6 Total interchange areas for pot furnace.

trial 1: 5 gas zones: grey gas: $k \delta = 3545$

GS		4 6	1.75806	3 7	2.36672
1 1	4.62455	4 7	2.18555	4 4	3.71691
1 2	3.50340	5 1	0.816621	4 5	2.99244
1 3	1.65605	5 2	0.301492	4 6	2.12347
1 4	0.788838	5 3	0.494709	4 7	3.64189
1 5	0.464740	5 4	0.851470	5 5	3.84117
1 6	0.298110	5 5	1.76552	5 6	3.21168
1 7	0.604009	5 6	3.68647	5 7	5.74132
2 1	2.62502	5 7	3.87023	6 6	4.21277
2 2	1.65923	SS		6 7	9.23427
2 3	3.47701	1 1	2.15967	7 7	1.42721
2 4	1.64976	1 2	11.0382	GG	
2 5	0.797480	1 3	6.89727	1 1	1.25344
2 6	0.490784	1 4	4.42894	1 2	0.570791
2 7	0.878289	1 5	2.96299	1 3	0.271499
3 1	1.59246	1 6	2.14605	1 4	0.166871
3 2	0.789323	1 7	4.77272	1 5	0.111796
3 3	1.64834	2 2	3.77739	2 2	1.24759
3 4	3.48269	2 3	2.95256	2 3	0.568086
3 5	1.67161	2 4	1.97721	2 4	0.275535
3 6	0.841910	2 5	1.29097	2 5	0.177818
3 7	1.30947	2 6	0.872038	3 3	1.24912
4 1	1.10095	2 7	1.60707	3 4	0.4577099
4 2	0.466574	3 3	3.69711	3 5	0.294019
4 3	0.798164	3 4	2.92773	4 4	1.26868
4 4	1.67483	3 5	1.99505	4 5	0.612535
4 5	3.53214	3 6	1.35746	5 5	1.33039

Table 4.7 Total interchange areas for pot furnace.

trial 1: 2 gas zones: 2 grey - 1 clear: $k\delta = 9.632$ ^{.4928}

GG		2 2	2.38690	1 3	8.00517
1 1	81.0241	3 1	2.35664	1 4	5.48331
1 2	4.59971	3 2	13.6678	2 2	20.8988
2 2	81.6470	4 1	1.15673	2 3	12.1474
SG		4 2	6.39603	2 4	6.26440
1 1	7.66657	SS		3 3	22.3199
1 2	1.46085	1 1	2.24739	3 4	16.9800
2 1	13.3572	1 2	20.4549	4 4	1.44171

Table 4.8 Total interchange areas for pot furnace.

trial 1: 2 gas zones: 1 grey - 1 clear: $k\delta = 1.79$

GG		2 2	2.53647	1 3	8.07830
1 1	23.3911	3 1	2.57049	1 4	5.50691
1 2	5.00050	3 2	12.6792	2 2	21.0610
2 2	23.9464	4 1	1.47937	2 3	12.2654
SG		4 2	5.66418	2 4	6.31561
1 1	6.47916	SS		3 3	22.4912
1 2	1.79950	1 1	2.29995	3 4	17.2819
2 1	12.3956	1 2	20.7878	4 4	1.47003

Table 4.9 Total interchange areas for pot furnace.

trial 1: 2 gas zones: grey gas: $k\delta = .3545$

GG		2 2	4.66619	1 3	7.1937
1 1	5.09957	3 1	4.56331	1 4	4.84075
1 2	1.58203	3 2	14.8720	2 2	19.7508
2 2	5.38931	4 1	1.36709	2 3	11.0378
SG		4 2	7.5408	2 4	5.67237
1 1	9.02157	SS		3 3	21.1049
1 2	1.82154	1 1	1.99106	3 4	17.0055
2 1	14.2372	1 2	20.4109	4 4	1.30592

Table 4.10. Total interchange areas for pot furnace.

trial 1: 2 gas zones: 1 grey-1 clear: variable k: $k\delta = 1.927$
 1.696

G_1G_1	28.6072	1 4	1.65997	1 3	8.07830
G_1G_2	5.88623	2 1	1.64855	1 4	5.50691
G_2G_1	4.41744	2 2	2.37886	2 2	20.3712
G_2G_2	20.6864	2 3	11.6169	2 3	12.2654
GS		2 4	5.22718	2 4	6.31561
1 1	7.65327	SS		3 3	22.9748
1 2	13.9821	1 1	2.2995	3 4	17.5358
1 3	2.80740	1 2	20.3028	4 4	1.47003

Table 4.11 Total interchange areas for pot furnaces.

trial 1: 2 gas zones: grey gas: variable k: $k\delta = 4.069$
 3.189

G_1G_1	6.56635	1 4	1.55049	1 3	7.19370
G_1G_2	2.01956	2 1	1.68312	1 4	4.84075
G_2G_1	1.31281	2 2	4.30961	2 2	18.9073
G_2G_2	4.43754	2 3	12.8793	2 3	11.0378
GS		2 4	6.89701	2 4	5.67237
1 1	10.0415	SS		3 3	21.8274
1 2	16.1556	1 1	1.99106	3 4	17.2949
1 3	5.02982	1 2	19.8909	4 4	1.30592

Table 4.12 Total interchange areas for pot furnace.

trial 2: 2 gas zones: 1 grey-1 clear: $k\delta = 1.958$

GG		2 2	2.28979	1 3	8.23869
1 1	24.3826	3 1	2.32414	1 4	5.63431
1 2	4.97907	3 2	11.9907	2 2	21.3892
2 2	24.9245	4 1	1.39389	2 3	12.5147
SG		4 2	5.33361	2 4	6.43744
1 1	6.39567	SS		3 3	22.8405
1 2	1.69229	1 1	2.34476	3 4	17.418
2 1	11.7365	1 2	20.9551	4 4	1.48508

Table 4.13 Total interchange areas for pot furnace.

Trial 2: 2 gas zones: grey gas: $k\delta = .335$

GG		2 2	4.52260	1 3	7.34482
1 1	4.63758	3 1	4.41937	1 4	4.93320
1 2	1.45365	3 2	14.2360	2 2	20.0579
2 2	4.90468	4 1	1.80116	2 3	11.2654
SG		4 2	7.23841	2 4	5.78987
1 1	8.65910	SS		3 3	21.3302
1 2	1.73857	1 1	2.02856	3 4	17.1468
2 1	13.6218	1 2	20.5826	4 4	1.32899

Table 4.14 Total interchange areas for pot furnace.

trial 3: 2 gas zones: 1 grey-1 clear: $k\delta = 1.747$

GG		2 2	2.47989	1 3	8.15567
1 1	21.9029	3 1	2.51161	1 4	5.55907
1 2	4.73555	3 2	12.2419	2 2	21.2412
2 2	22.4355	4 1	1.42801	2 3	12.3763
SG		4 2	5.48782	2 4	6.37621
1 1	6.57770	SS		3 3	22.6832
1 2	1.73845	1 1	2.31930	3 4	17.3844
2 1	11.9635	1 2	20.9112	4 4	1.48233

Table 4.15 Total interchange areas for pot furnace.

trial 3: 2 gas zones: grey gas: $k\delta = .338$

GG		2 2	4.52260	1 3	7.34482
1 1	4.63758	3 1	4.41937	1 4	4.93320
1 2	1.45365	3 2	14.3260	2 2	20.0579
2 2	4.90468	4 1	1.30116	2 3	11.2654
SG		4 2	7.23841	2 4	5.78987
1 1	8.65910	SS		3 3	21.4302
1 2	1.73857	1 1	2.02856	3 4	17.1468
2 1	13.6218	1 2	20.5826	4 4	1.32899

Table 4.16 Total interchange areas for superheater.

4 gas zones: 1 grey-1 clear: $k\delta = 1.89$

GS		1 3	0.875460	5 5	3.17751
1 1	0.784015	1 4	0.360474	5 6	2.48864
1 2	1.58118	1 5	0.186769	6 6	0.166913
1 3	0.302088	1 6	0.208619	GG	
1 4	0.0331599	2 2	3.17751	1 1	2.93865
1 5	0.00585024	2 3	1.70739	1 2	0.567470
1 6	0.00422815	2 4	0.616423	1 3	0.0697618
2 1	0.0967324	2 5	0.2522001	1 4	0.0116815
2 2	0.3011906	2 6	0.186769	2 2	2.83318
2 3	1.53220	3 3	2.97909	2 3	0.554742
2 4	0.296005	3 4	1.64555	2 4	0.0697618
2 5	0.0363400	3 5	0.616423	3 3	2.83318
2 6	0.0181699	3 6	0.360474	3 4	0.567470
SS		4 4	2.97909	4 4	2.93865
1 1	0.166913	4 5	1.70739		
1 2	2.48864	4 6	0.875460		

Table 4.17 Total interchange areas for superheater.

4 gas zones: grey gas: $k\delta = .311$.

GS		2 5	0.140234	2 4	0.478597
1 1	0.808994	2 6	0.0969651	2 5	0.183616
1 2	1.78491	SS		2 6	0.130414
1 3	0.544967	1 1	0.151676	3 3	2.84337
1 4	0.139681	1 2	2.48153	3 4	1.50415
1 5	0.0532092	1 3	0.797886	3 5	0.478597
1 6	0.0459052	1 4	0.286556	3 6	0.286556
2 1	0.242351	1 5	0.130414	4 4	2.85337
2 2	0.543735	1 6	0.143865	4 5	1.56561
2 3	1.71619	2 2	3.05588	4 6	0.797886
2 4	0.525204	2 3	1.56561	5 5	3.05588

Table 4.17 contd.

5 6	2.48153	1 3	0.0425365	3 3	0.482277
6 6	0.151676	1 4	0.0169849	3 4	0.152631
GG		2 2	0.482277	4 4	0.504459
1 1	0.504459	2 3	0.146656		
1 2	0.152681	2 4	0.0425365		

Table 4.18 Total interchange areas for pot furnace.

trial 1: 1 gas zone: 1 grey-clear: $k\delta = 1.79$

$G_1 G_1$	56.9481	1 3	5.82564	SG	
SS		2 2	68.0599	1 1	8.57559
1 1	1.95258	2 3	23.5594	2 1	30.0387
1 2	28.9040	3 3	1.21018	3 1	7.11919

Table 4.19 Total interchange areas for pot furnace.

trial 1: 1 gas zone: grey gas: $k\delta = .3545$

$G_1 G_1$	13.4474	1 3	5.20946	SG	
SS		2 2	62.3653	1 1	10.9024
1 1	1.61236	2 3	22.5338	2 1	38.3778
1 2	27.5136	3 3	1.02581	3 1	8.9291.

Table 4.20 Total interchange areas for superheater.

trial 1 gas zone: grey gas : $k\delta = .311$

$G_1 G_1$	3.05718	1 3	0.201291	SG	
SS		2 2	23.5408	1 1	1.19262
1 1	0.073285	2 3	3.70850	2 1	11.0191
1 2	3.70850	3 3	0.073285	3 1	1.18262

Chapter 5. Total Heat Balances.

5.1. Introduction.

5.2. Enthalpy terms.

5.3. Convection terms.

5.3(a) Convection transfer coefficient for pot furnace.

5.3(b) Effect of convection transfer coefficient on total heat transferred.

5.3(c) Convection transfer coefficient for superheater.

5.4. Setting up the heat balances.

5.5. Solution of the heat balances.

Chapter 5 Total Heat Balance.

5.1 Introduction.

The various terms describing the interchange of radiant energy have been calculated. Heat balances are now set up for each zone in the system. Convection, enthalpy flux and wall losses are included with radiation so that all energy exchange is accounted for.

The furnaces investigated were under steady conditions so there are no storage terms. The effects of gas to gas convection, or eddy diffusion between gas zones, have been neglected.

The various energy terms are evaluated and, as radiation is the largest contributor, all terms are linearised with σT^4 (hence forward termed W) as variable. The simultaneous heat balances over each zone are solved by trial and error.

5.2 Enthalpy terms.

Both furnaces considered are end-fed cylinders. The enthalpy terms are evaluated assuming plug flow.

The energy transferred across the boundary between two gas zones is $G.C_p.T$ where G is mass flow rate, C_p is mean specific heat and T the temperature of the zone from which the gas flows.

The flow rates and compositions of the furnace gases have been calculated (Appendix 1). The specific heat used in all enthalpy terms is the mean specific heat⁷⁰ between 0 and 900°C. As accurate mean specific heats between absolute zero and the desired temperature were not available the datum of 0°C

was used throughout for specific heat. The average mean specific heat used is accurate for most zones but slightly in error for the hottest zones. The maximum error on any single enthalpy term is not more than 3% in the 10 gas zone cases and less than 2% in the 2 gas zone cases. The accuracy of quoted mean specific heats does not seem to warrant the calculation of separate values for each zone.

The enthalpy terms are linearised in W and enthalpy flux coefficients η are defined as,

$$\eta_{g_i g_j} = \frac{G \cdot C_{pT_{g_i}}}{W_{g_i}} \quad 5.1$$

such that when multiplied by W they describe the one-way enthalpy transfer.

The input enthalpy, called MH_{in} , is the total calorific value plus sensible heat injected into the furnace. Mean specific heats for fuel and air are again evaluated from the datum 0°C .

5.3 Convection terms.

The one-way convective flux to or from a gas or surface zone is given by $h_c AT$, where T is the temperature of the zone from which the heat is transferred, A is the common area and h_c is the convective transfer coefficient, assumed constant over the area A . Again linearisation in W is employed and convective flux coefficients γ are defined as,

$$\gamma_{s_i g_j} = \frac{h_c A_{s_i} T_{s_i}}{W_{s_i}} \quad 5.2$$

5.3 (a) Convection transfer coefficient for pot furnace.

To evaluate the terms γ values of h_c for exchange between gases and plane or slightly curved surfaces must be found.

Colburn⁷¹ presented equations for heating and cooling of fluids at plane surfaces, one of them having been derived theoretically by Pohlhausen⁷². The limits of applicability are given by McAdams⁵⁸. The value of h_c derived from these equations is $h_c = 0.23$. Tribus⁷³ suggested a modification which gives a value $h_c = 0.31$. These equations appear to be applicable over short distances only.

An equation for fluid flow in vertical pipes in the viscous region, supposedly allowing for both natural and forced convection, was presented by Martinelli et al⁷⁴. This has been modified by McAdams¹ to allow for free and forced convection in opposite directions. The coefficient obtained was $h_c = 1.61$. When applying this value to the furnace being considered it must be remembered that Martinellis' data was for narrow tubes.

McAdams¹ suggested that when velocities are low the values of h_c for both natural and forced convection should be evaluated and the larger used. He presents equations for calculating h_c for natural convection. By the criteria given the flow in the present case is on the dividing line between laminar and turbulent. The values of h_c obtained assuming laminar and turbulent flow respectively are $h_c = 0.63$ and $h_c = 0.78$. Consideration of the swirls and the recirculation near the flame leads to the conclusion that the actual flow will be turbulent.

From the calculations a reasonably representative value of convective transfer coefficient would seem to be $h_c = 1$. In view of the widely differing estimates of h_c no attempt was made to evaluate it for varying furnace conditions.

As there was so much variation in the values of h_c obtained the effect of this variation on the calculated total heat transfer was investigated.

5.3 (b) Effect of convection transfer coefficient on total heat transferred.

The first estimate of mean gas temperatures for trial 1 on the pot furnace led to the absorption strength being taken as $k\delta = 1.64$. The interchange areas and total heat balances based on this $k\delta$ were evaluated for a 2 gas-zone system and the temperatures found resulted in an improved estimate of $k\delta = 1.79$ (see section 4.2 (e)) for general method). The calculations based on $k\delta = 1.64$ were used to investigate the effect of different values of h_c .

The total heat balances over all zones pot furnace trial 1 were evaluated with values of $h_c = 0.5, 1$ and 2 . The zone temperatures ($^{\circ}\text{C}$) found are given below.

Table 5.1

h_c	0.5	1.0	2.0
zone g_1	909	904	897
g_2	1107	1104	1102
s_2	791	792	792
s_3	896	899	901
s_4	885	887	890

The net heat transferred to the sink was calculated as 1.22, 1.26 and 1.30×10^6 B.t.u./hr. for values of h_c equal 0.5, 1 and 2 respectively.

The effects of considerable variations in h_c on the total heat transferred are quite small. Thus even if the selected value $h_c = 1$ is in error the calculated heat transferred should not be greatly in error.

5.3 (c) Convection transfer coefficient for superheater.

Colburn⁷¹ suggested equations for gases flowing over banks of tubes for both staggered and in-line conditions. These equations are for fully developed turbulence normally taken¹ as being achieved with banks 10 rows deep. Pierson⁷⁵ has provided factors whereby the coefficients for tube banks less than 10 deep can be obtained for Colburn's equations.

In the present case there is only one row of tubes but these are near the furnace wall which will increase the turbulence. To allow for this the factor for staggered tube banks 2 rows deep has been used.

In view of the imprecise nature of these calculations a value of convective transfer coefficient ~~was~~ based on average furnace gas properties was evaluated. This value, $h_c = 2$, was used for all zones.

5.4 Setting up the heat balances.

A heat balance for each zone in the system is established. The sign convention used requires heat transferred to the zone in question to be positive. All heat exchange between the zone is accounted for.

For a 2 gas zone, 2 wall zone and 2 end zone system for the pot furnace, numbering from the top, the heat balances over zones g_2 and s_2 are given below

$$\begin{aligned}
 & W_{g_1} (\overline{G_1 G_2}) + W_{g_2} (\overline{G_2 G_2} - 4kaVg_2 - \gamma_{g_2 s_3} - \gamma_{g_2 s_4} - \eta_{g_2 g_1}) \\
 & + W_{s_1} (\overline{S_1 G_2}) + W_{s_2} (\overline{S_2 G_2}) + W_{s_3} (\overline{S_3 G_2} + \gamma_{s_3 g_2}) \\
 & + W_{s_4} (\overline{S_4 G_2} + \gamma_{s_4 g_2}) + MH_{in} = 0
 \end{aligned} \tag{5.3}$$

$$\begin{aligned}
 & W_{g_1} (\overline{G_1 S_2} + \gamma_{g_1 s_2}) + W_{g_2} (\overline{G_2 S_2}) + W_{s_1} (\overline{S_1 S_2}) \\
 & + W_{s_2} (\overline{S_2 S_2} - \epsilon_{s_2} A_{s_2} - \gamma_{s_2 g_1}) + W_{s_3} (\overline{S_3 S_2}) + W_{s_4} (\overline{S_4 S_2}) - q_{s_2} = 0
 \end{aligned} \tag{5.4}$$

The term q_{s_2} in equation 5.4 represents losses through the furnace wall. These losses were determined and are given in Appendix 1.

5.5 Solution of the heat balances.

All the coefficients on $W (= \sigma T^4)$ in the heat balance equations 5.3, 5.4 etc. have been calculated save the η and γ terms. The calculation of these requires a knowledge of zone temperature T . The value of T for each zone is guessed and the corresponding values of η and γ are evaluated. The heat balances now form a system of simultaneous equations each having a number of W terms equal to the number of equations.

The value of W for the sink zones is known and can be substituted into the equations. This reduces the numbers of terms and equations by the number of sink zones. If there are N gas zones in the system the number of simultaneous

equations to be solved is equal to $(2N + 2 - \text{number of sink zones})$.

The equations are solved for W and the zone temperatures T found. If the calculated T differs from the estimated T used to evaluate η and γ then a new estimate of zone temperature must be made and the solution process repeated. This trial and error cycle was repeated until the discrepancy was not more than:-

- 1° for single gas zone system
- 2° for 2 gas zone system
- 5° for 5 and 10 gas zone systems.

Also the discrepancies must shew a reasonably balanced distribution in sign (i.e. all zone temperatures in error by +1° was not considered solved).

The pot furnace representations using 1 and 2 gas zones and all the superheater representations were evaluated on an electric desk calculator. The pot furnace 5 and 10 gas zone analyses were solved using the London University Mercury Computer.

The results of these solutions are given in Chapter 6.

Chapter 6. Calculated Heat Transfer.

- 6.1. Introduction.
- 6.2. Measured heat transfer.
 - 6.2(a) Pot furnace.
 - 6.2(b) Superheater.
- 6.3. Heat transfer predicted by theoretical analyses.
 - 6.3(a) Range of variables investigated.
 - 6.3(b) Calculated results.
- 6.4. Heat transfer predicted by correlations.
 - 6.4(a) Wilson-Lobo-Hottel correlation.
 - 6.4(b) Lobo-Evans correlation.
 - 6.4(c) Method proposed by Hottel.

Chapter 6 Calculated Heat Transfer.

6.1 Introduction.

The heat transferred to the sink from the pot furnace enclosure was calculated from the gas enthalpy drop. The heat transferred to the sink in the superheater was found from enthalpy changes for both gas and steam. These calculations established the actual heat transfer and served as standards with which the heat transfer predicted by various methods was compared.

The method of Hottel and Cohen⁴⁶ was used to predict the zone temperatures and heat transfer. Three pot furnace trials and the superheater trial were analysed. Furnace enclosures were considered as 1,2,4,5 or 10 gas zones with 3,4,6,7 or 12 surface zones. The gas was represented as a grey gas, a grey-clear mixture or a mixture of two grey gases and one clear gas. The effect of allowing gas absorption coefficient to vary was investigated with a 2 zone analysis using grey and grey-clear gas representations.

The heat transfer predicted by the methods of Wilson, Lobo, Hottel¹⁹ and Lobo, Evans²¹ were calculated. The Hottel extension²⁶ of the Lobo, Evans method and a modification of this extension were used to predict the heat transfer in the furnaces.

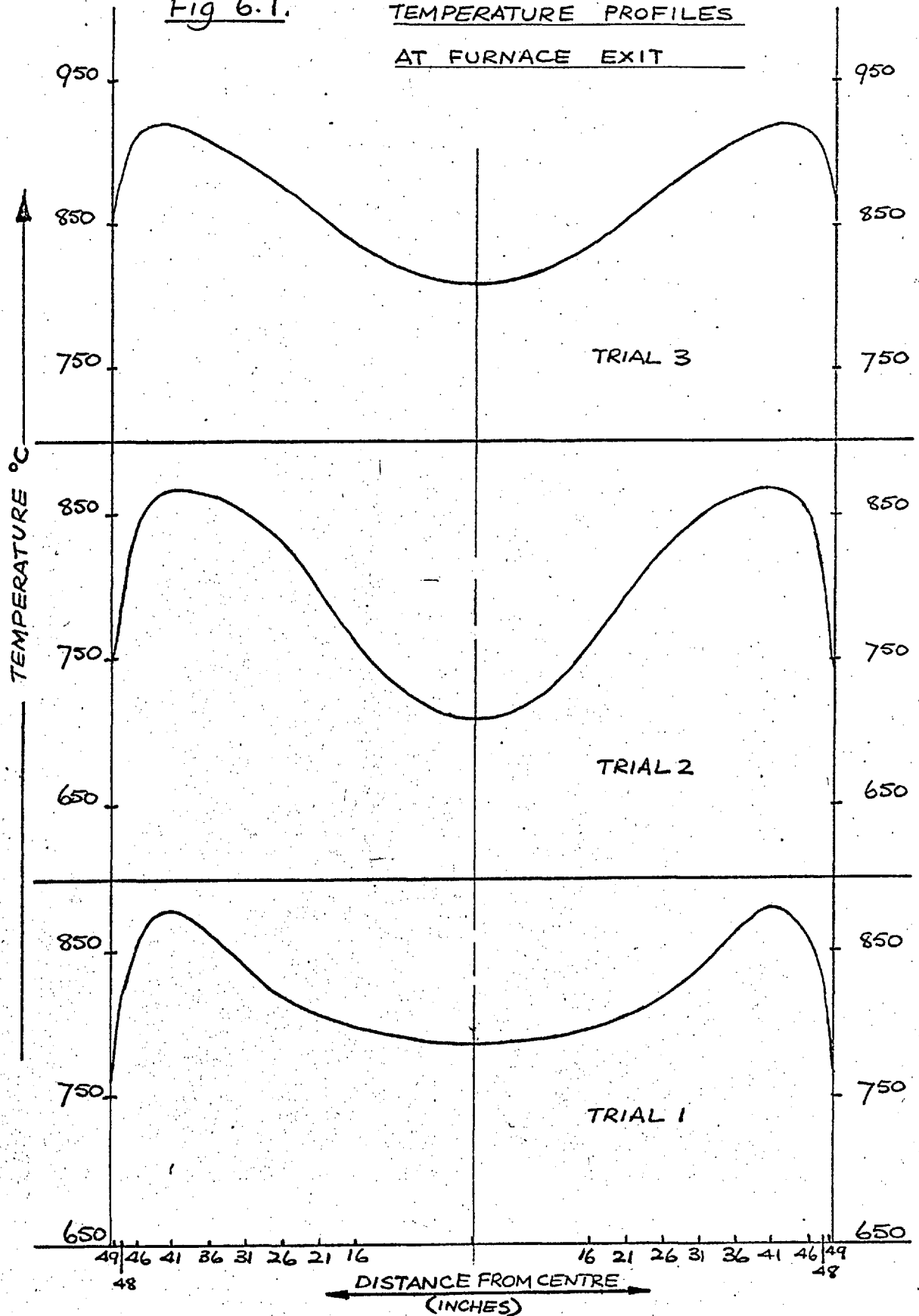
6.2 Measured heat transfer.

The experimental results given in Chapter 2 and Appendix 1 were used to evaluate the heat transferred from the furnace enclosures to the heat sinks.

6.2 (a) Pot furnace.

Three pot furnace trials (Nos.1, 2 and 3) were investigated to give a range of operating conditions. The heat transferred to the sink from the furnace enclosure was found from the difference between heat input and the enthalpy flow leaving the enclosure. The average gas exit temperature was found from the suction pyrometer traverses (Chapter 2) assuming a symmetrical temperature distribution. The temperature profiles are shown in Fig.6.1.

The various terms are calculated below with a datum of 0°F:-			
	Trial 1.	2.	3.
Cv of oil B.t.u/lb	17,850	17,850	17,850
oil rate lb/hr.	134	104	146
heat 10^6 B.t.u/hr.	2.3919	1.8564	2.6061
air temp.ex.preheater °F	416	518	491
ambient air temp. °F	86	88	79
mean air temp. °F (80% through preheater)	350	432	409
mean Cp B.t.u/lb °F	.247	.249	.248
air flow lb/hr.	2760	3127	3541
heat 10^6 B.t.u/hr.	0.2386	0.3364	0.3592
oil flow rate lb/hr.	134	104	146
specific heat B.t.u/lb	.465	.465	.465
temperature °F	200	200	200
heat 10^6 B.t.u/hr.	0.0125	0.0097	0.0136
total heat input 10^6 B.t.u/hr.	2.6430	2.2025	2.9789

Fig 6.1.TEMPERATURE PROFILES
AT FURNACE EXIT

	Trial 1.	2.	3.
Flue gas rate lb/hr.	2894	3231	3687
mean Cp B.t.u/lb	.2770	.2726	.2747
gas exit temp. °F	1512	1461	1589
heat 10^6 B.t.u/hr.	1.2120	1.2868	1.6093
wall heat losses 10^6 B.t.u/hr.	0.045	0.031	0.045
total heat ex.furnace 10^6 B.t.u/hr.	1.2570	1.3178	1.6543
total heat in 10^6 B.t.u/hr.	2.6430	2.2025	2.9789
total heat out 10^6 B.t.u/hr.	1.2570	1.3178	1.6543
heat to sink q_s 10^6 B.t.u/hr.	1.3860	0.8847	1.3246

The values of heat transferred to sink q_s are given below.

	q_s (10^6 B.t.u/hr.)
Trial 1	1.3860
Trial 2	0.8847
Trial 3	1.3246

6.2 (b) Superheater.

The enthalpy changes on the gas and steam sides have already been calculated (Chapter 2), and are summarised below:-

heat into furnace	10^6 B.t.u/hr.	1.7787
enthalpy leaving furnace	10^6 B.t.u/hr.	<u>0.9152</u>
enthalpy drop in furnace	10^6 B.t.u/hr.	0.8635
wall heat losses	10^6 B.t.u/hr.	<u>0.1176</u>
heat to sink	10^6 B.t.u/hr.	0.7459
enthalpy gain by steam	10^6 B.t.u/hr.	0.7114

It will be seen that there is a 5% difference between the estimates of the heat transferred to the sink given by gas and steam side enthalpy balances. The best estimate of the heat transferred to the sink was taken as being the average of the two calculated values.

Superheater q_s (10^6 B.t.u/hr) 0.7287

6.3 Heat transfer predicted by theoretical analyses.

The heat transferred to the sinks of the furnaces has been measured. The theoretical method described in Chapter 3, 4 and 5 was used to predict the heat transfer to the sinks and the accuracy of prediction was found by comparison with measured heat transfer.

The method used required the furnace to be split into isothermal zones. The temperatures of these zones were calculated by an iterative method, and the problem was considered solved when there was no significant difference (see section 5.5) between the estimated and calculated zone temperatures. When the zone temperatures were known the net heat transferred to the sink was found by substitution into the total heat balance over the sink zone.

6.3 (a) Range of variables investigated.

The pot furnace and the superheater are both small cylindrical furnaces but are of very different configuration. The pot furnace is oil-fired, has an aspect ratio of unity and a heat sink approximating to a disc at one end. The superheater is gas-fired, has an aspect ratio of two and a heat sink which is taken as the walls of the enclosure.

Three trials on the pot furnace were analysed, each having a different load, fuel rate and percentage excess air. One trial on the superheater was analysed.

The accuracy of the method of analysis is dependent on the number of zones considered. If the zones are to be truly isothermal an infinite number of zones would be necessary. Trial 1 on the pot furnace was analysed using 1,2,5 and 10 gas zones to follow the variation in predicted heat transfer with number of zones.

The furnace gas can be represented in many ways, the simplest being to assume a grey gas. Real gas characteristics can be approximated to by considering mixtures of grey and clear gases. Trial 1 of the pot furnace was analysed using two zones with the gas represented as grey, a grey gas-clear gas mixture and a mixture of two grey and one clear gas. All analyses carried out for all the furnace trials were done using both a grey gas and a single grey gas-clear gas mixture to represent the furnace gas. Thus the relative effect of these two gas representations over a wide range of variables can be found.

Throughout the above calculations an average absorption strength $k\delta$ for the whole enclosure was estimated. The two zone analysis of trial 1 on the pot furnace was also calculated with a different value of $k\delta$ for both gas zones. This was done for both grey and grey-clear mixture gas.

6.3 (b) Calculated results.

The heat balances over all zones of unknown temperature were solved simultaneously for zone temperature. The net heat transferred to the sink q_s was then found from the heat balance over the sink zone or zones. The final estimated and calculated zone temperatures ($^{\circ}\text{C}$) are listed below together with the values of q_s (10^6 B.t.u/hr), for all cases investigated, in Tables 6.1 to 6.10. The values of q_s are also expressed as percentages of the measured heat transferred to the sink.

Table 6.1 Pot furnace trial 1.

	grey gas $k\delta$.3545		grey-clear gas $k\delta$ 1.7	
	est ^d .	calc ^d .	est ^d .	calc ^d .
g_1	877	877	905	907
g_2	1103	1102	1101	1103
s_2	615	815	795	794
s_3	928	927	900	898
s_4	918	919	885	885
	$q_s = 1.32923 \quad 95.9\%$		$q_s = 1.26076 \quad 91.0\%$	

Table 6.2 Pot furnace trial 1.

grey gas $k \delta$.3545
 est^d. calc^d.

g ₁	790	794
g ₂	837	841
g ₃	893	890
g ₄	930	925
g ₅	963	958
g ₆	1005	1000
g ₇	1045	1042
g ₈	1100	1099
g ₉	1190	1190
g ₁₀	1345	1348
s ₂	755	757
s ₃	797	799
s ₄	838	837
s ₅	370	870
s ₆	897	899
s ₇	923	925
s ₈	952	952
s ₉	978	975
s ₁₀	998	998
s ₁₁	1010	1013
s ₁₂	980	980

$$q_s = 1.42246 \quad 102.6\%$$

10 gas zones.

grey-clear gas $k \delta$ 1.79
 est^d. calc^d.

830	828
870	874
935	932
970	967
1000	998
1040	1038
1075	1075
1125	1125
1200	1201
1340	1345
746	750
790	794
832	831
862	864
890	890
915	918
940	943
968	967
987	989
1010	1005
984	980

$$1.37525 \quad 99.2\%$$

Table 6.3 Pot furnace trial 1.

	grey gas $k\delta$.3545	
	est ^d .	calc ^d .
g_1	825	825
g_2	908	906
g_3	980	978
g_4	1075	1072
g_5	1250	1250
s_2	775	776
s_3	847	848
s_4	910	906
s_5	956	956
s_6	990	990
s_7	955	960

$$q_s = 1.39766 \quad 100.8\%$$

5 gas zones.

	grey-clear gas $k\delta$ 1.79	
	est ^d .	calc ^d .
	865	860
	945	945
	1015	1016
	1100	1101
	1252	1253
	768	768
	840	842
	900	898
	945	945
	980	980
	960	959

$$q_s = 1.34972 \quad 97.4\%$$

Table 6.4 Pot furnace trial 1.

	grey gas $k\delta$.3545	
	est ^d .	calc ^d .
g_1	946	945
s_2	807	806
s_3	813	813

$$q_s = 1.20432 \quad 86.9\%$$

1 gas zone.

	grey-clear gas $k\delta$ 1.79	
	est ^d .	calc ^d .
	966	967
	795	794
	799	800

$$q_s = 1.15023 \quad 83.0\%$$

Table 6.5. Superheater

	grey gas $k\delta$.311	
	est ^d .	calc ^d .
g_1	1355	1355
g_2	1125	1125
g_3	980	979
g_4	865	867
s_1	860	861
s_6	635	634

$$q_s = 0.79721 \quad 109.4\%$$

4 gas zones.

	grey-clear gas $k\delta$ 1.89	
	est ^d .	calc ^d .
	1380	1380
	1185	1186
	1050	1050
	940	941
	850	847
	615	614

$$q_s = 0.72548 \quad 99.6\%$$

Table 6.6. Pot furnace trial 1.

	grey gas $k\delta$, 0.4069	
	$k\delta_2$ 0.3189	
	est ^d .	calc ^d .
g_1	873	873
g_2	1128	1126
s_2	823	824
s_3	938	938
s_4	927	927

$$q_s = 1.37351 \quad 99.1\%$$

2 gas zones variable k.

	grey-clear gas $k\delta$, 1.927	
	$k\delta_2$ 1.696	
	est ^d .	calc ^d .
	900	898
	1129	1128
	800	801
	905	905
	894	892

$$q_s = 1.29014 \quad 93.1\%$$

Table 6.7 Superheater.

	grey gas $k\delta$.311	
	est ^d .	calc ^d .
g_1	1050	1049
s_1	690	690
s_3	710	710

$$q_s = 0.64415 \quad 88.4\%$$

1 gas zone.

	grey-clear gas $k\delta$ 1.89	
	est ^d .	calc ^d .
	1095	1096
	675	675
	695	695

$$q_s = 0.57611 \quad 79.1\%$$

Table 6.8 Pot furnace trial 2.

	grey gas $k\delta$	est ^d .	calc ^d .
g_1	790	789	
g_2	965	964	
s_2	715	715	
s_3	810	810	
s_4	800	800	

$$q_s = 0.88021 \quad 99.5\%$$

Table 6.9. Pot furnace trial 3.

	grey gas $k\delta$	est ^d .	calc ^d .
g_1	885	883	
g_2	1090	1088	
s_2	810	809	
s_3	915	913	
s_4	905	905	

$$q_s = 1.29422 \quad 97.7\%$$

Table 6.10. Pot furnace trial 1.

	est ^d .	calc ^d .
g_1	895	894
g_2	1100	1101
s_2	795	794
s_3	905	905
s_4	890	892

2 gas zones.

	grey-clear gas $k\delta$	est ^d .	calc ^d .
	1.958		
	815	815	
	970	969	
	697	698	
	783	783	
	772	772	

$$q_s = 0.83178 \quad 94.0\%$$

2 gas zones.

	grey-clear gas $k\delta$	est ^d .	calc ^d .
	1.747		
	905	907	
	1095	1094	
	785	785	
	885	886	
	875	873	

$$q_s = 1.21734 \quad 91.9\%$$

2 gas zones.

2 grey gas-1 clear gas mixture.

$$k\delta_1 = .04928$$

$$k\delta_2 = 9.632$$

$$q_s = 1.25508 \quad 90.6\%$$

The values of heat transferred to the sink q_s for all trials are listed in Table 6.11.

Table 6.11. Values of q_s (10^6 B.t.u/hr.)

Pot furnace trial 1.	q_s	% of q_{s_m}
1 gas zone: grey gas	1.20432	86.9
1 gas zone: grey/clear gas	1.15023	83.0
2 gas zones: grey gas	1.32923	95.9
2 gas zones: grey/clear gas	1.26076	91.0
2 gas zones: 2 grey/clear gas	1.25508	90.6
*2 gas zones: grey gas	1.37351	99.1
*2 gas zones: grey/clear gas	1.29014	93.1
5 gas zones: grey gas	1.39766	100.8
5 gas zones: grey/clear gas	1.34972	97.4
10 gas zones: grey gas	1.42246	102.6
10 gas zones: grey/clear gas	1.37525	99.2
Pot furnace trial 2.		
2 gas zones: grey gas	0.88021	99.5
2 gas zones: grey/clear gas	0.83178	94.0
Pot furnace trial 3.		
2 gas zones: grey gas	1.29422	97.7
2 gas zones: grey/clear gas	1.21734	91.9
Superheater.		
1 gas zones: grey gas	0.64415	88.4
1 gas zones: grey/clear gas	0.57611	79.1
4 gas zones: grey gas	0.79721	109.4
4 gas zones: grey/clear gas	0.72548	99.6

* The results marked with an asterisk were calculated allowing absorption coefficient k to vary between gas zones.

The effects of the different variables on the calculated value of q_s are considered in Chapter 7.

6.4 Heat transfer predicted by correlations.

The predicted heat transfer to the sink was calculated for the three trials on the pot furnace and the super-heater trial using various correlations.

6.4 (a) Wilson - Lobo - Hottel correlation.

Wilson, Lobo and Hottel¹⁹ have proposed an empirical equation which quite accurately described the heat transfer in a number of large tube stills. Allowances were suggested so that the correlation could be applied to different types of furnaces. For furnaces other than box type tube stills they proposed,

$$\mu = \frac{1}{1 + \left(\frac{G \sqrt{Q/A'}}{\frac{1000}{4.4}} \right)^2 + \left(\frac{G \sqrt{Q/A'}}{\frac{1000}{70}} \right)^2} \quad 6.1$$

where

$$\mu = \frac{\text{heat to sink}}{\text{input enthalpy}}$$

$$Q = \text{net heating value of fuel}$$

$$G = \text{air/fuel ratio}$$

$$A' = \alpha A_{cp} + \beta A_R \quad 6.2$$

where

$$\alpha A_{cp} = \text{equivalent cold plane area}$$

$$A_R = \text{refractory area}$$

$$\beta = \frac{1}{\frac{P_f}{1-P_f} \frac{1}{F_R} + 1} \quad 6.3$$

where

p_f = flame emissivity (from chart given by Lobo & Evans²¹)

F_R = view factor (given by Lobo & Evans²¹)

The calculated values of heat transferred to the sink q_s are listed in Table 6.12 and are also expressed as per centages of the measured heat transfer to the sink q_{s_m} .

Table 6.12. Wilson-Lobo-Hottel correlation.

	q_s (10^6 B.t.u/hr.)	% of q_{s_m}
Pot furnace trial 1	1.591	115
Pot furnace trial 2	1.181	133
Pot furnace trial 3	1.635	123
Superheater	1.366	187

6.4 (b) Lobo-Evans correlation.

Lobo and Evans²¹ proposed a correlation based on Stephan's Law. It comprises a trial and error solution of a rate of equation and a heat balance with the same value serving for both mean radiating temperature of the gas and gas exit temperature. The equations are,

$$q_s = .173 \phi \left[\left(\frac{T_g}{100} \right)^4 - \left(\frac{T_s}{100} \right)^4 \right] \alpha A_{cp} + h_c A (T_g - T_s) \quad 6.4$$

$$q_s = H_{in} - M \cdot C_p \cdot T_g \quad 6.5$$

where

T_g = mean gas temperature

T_s = sink temperature

αA_{cp} = effective cold plane area

h_c = convection transfer coefficient

- A = area for connective transfer
 H_{in} = input enthalpy
 M = gas flow rate
 C_p = mean gas specific heat
 ϕ = effectiveness factor obtained from charts or an equation due to Hottel²⁵.

The calculated values of heat transferred to the sink q_s are listed in Table 6.13.

Table 6.13.

Lobo-Evans correlation.

	q_s (10^6 B.t.u/hr)	% of q_{s_m}
Pot furnace trial 1	1.204	87
Pot furnace trial 2	0.800	90
Pot furnace trial 3	1.166	88
Superheater	0.614	84

6.4 (c) Method proposed by Hottel.

Hottel²⁶ has proposed an extension of the Lobo-Evans correlation which allows the mean radiating and gas exit temperatures to be different. This difference is called B . The method requires the trial and error solution of three equations, a rate equation, a heat balance and an equation defining B . These equations are,

$$q_s = \overline{GS} \sigma (T_g^4 - T_s^4) + h_c A (T_g - T_s) \quad 6.6$$

$$\text{where} \quad \overline{GS} = \frac{1-a^3}{1-a^4} \left[\frac{AT}{\frac{1}{e_g} + \frac{1}{ce_s} - 1} \right] \quad 6.7$$

$$\frac{H_{in} - q_s}{H_{in}} = \frac{T_g - B - T_o}{T_{AF} - T_o} \quad 6.8$$

$$B = \frac{q_s}{H_{in}} \cdot \frac{T_{AF} - T_o}{4} \quad 6.9$$

Where the symbols are as in the preceding section and,

σ = Stephan's Law constant

a = T_s/T_g

C = fraction cold

ϵ_g = gas emissivity

ϵ_s = sink emissivity

A_T = total furnace area

T_{AF} = pseudo adiabatic flame temperature
assuming no dissociation.

T_o = datum temperature

These equations were solved and the values of heat transferred to sink obtained are listed in Table 6.14.

Table 6.14 Hottel correlation.

	q_s (10^6 B.t.u/hr)	% of q_{s_m}
Pot furnace trial 1	1.438	104
Pot furnace trial 2	0.930	105
Pot furnace trial 3	1.372	104
Superheater	0.762	105

Equation 6.9 defining B was modified to become,

$$B = \frac{q_s}{H_{in}} \frac{T_{AF} - T_o}{5} \quad 6.10$$

Values of the heat transferred to the sink using equation 6.10 to define B were calculated and are listed in Table 6.15.

Table 6.15. Modified Hottel correlation.

	$q_s (10^6 \text{ (B.t.u/hr)})$	% of q_{s_m}
Pot furnace trial 1	1.374	99
Pot furnace trial 2	0.890	101
Pot furnace trial 3	1.313	99
Superheater	0.730	100

Chapter 7. Discussion.

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7.5. Furnace correlations.

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7.6. Conclusions.

CHAPTER 7. Discussion

7.1. Introduction.

Experimental investigations were carried out on two small cylindrical furnaces. Four trials were made on the pot furnace which has an oil fired enclosure of aspect ratio unity and an end sink. One trial was made on the superheater which has a gas fired enclosure of aspect ratio two and a tube bundle approximating to a wall sink. Heat balances were drawn up and the heat transferred to the sink q_{sm} was found from the measurements taken.

The values of elemental reception factors for interchange between differential elements of a gas-filled cylindrical enclosure were obtained from Hoffman ⁶². These elemental factors were integrated to give reception factors between cylindrical zones of aspect ratio $\frac{1}{10}$ up to a separation of one cylinder diameter for values of absorption strength $k\delta$ of 0, 0.5, 1, 2 and 3. From these factors further reception factors were built up for aspect ratios of $\frac{1}{5}$, $\frac{1}{2}$ and 1. The reception factors for zones of aspect ratio $\frac{1}{2}$ were extended to a separation of two cylinder diameters.

A Hottel-Cohen ⁴⁶ type of analysis was applied to three of the pot furnace trials using two gas zones and four surface zones and both grey and grey-clear mixture gas

representations. One trial on the pot furnace was analysed using 1, 2, 5 and 10 gas zones. This involves a total number of zones of 4, 6, 12 and 22 respectively. These analyses were performed using both grey and grey-clear mixture gas representations. The same trial was analysed using two gas zones and representing the gas as a two grey gas - one clear gas mixture. Also using this two gas zone framework analyses were performed with grey and grey-clear gas allowing the value of absorption coefficient k to vary between gas zones. The superheater trial was analysed using 1 and 4 gas zones with both grey and grey-clear mixture gas representations.

In all these analyses the predicted heat transfer to the sink q_s was compared with that found from the experimental measurements q_{s_m} .

The variation in predicted heat transfer with number of gas zones was examined. The effects of three different gas representations were investigated for one trial, and the relative effects of grey and grey-clear gas representations were found for a wide range of variables. The effects of allowing temperature and then temperature and absorption coefficient to vary were investigated.

The methods of Wilson-Lobo-Hottel ¹⁹, Lobo-Evans ²¹ and Hottel ²⁶ were applied to the furnaces. The

Methods of Lobo-Evans and Hottel were examined in detail.

7.2. Reception factors.

The reception factors for a cylindrical system have been evaluated for aspect ratio $\frac{1}{10}$ and values of absorption strength $k\delta$ of 0, 0.5, 1, 2 and 3. These factors have been used to derive reception factors for aspect ratios larger than $\frac{1}{10}$. By building the factors up to an aspect ratio unity the calculated results were compared with those of Hoffman⁵⁰. The average discrepancy found was 0.17% and the standard deviation of the discrepancy was 0.53%.

Further details of the calculations and results are given in Chapter 3.

7.3. Accuracy of experimental measurements.

The prime objective of the experimental work was to provide an accurate measure of the heat transferred to the sink q_{sm} in the furnaces. It has been mentioned in Chapter 2 that all instruments were checked and precautions were taken to ensure accurate measurements. Nevertheless some discrepancies have been found in the calculated heat balances.

In the trials on the pot furnace the heat unaccounted for varied between 0.5 and 2.7% of the heat input. The heat transferred to the pot was based on the gas-side enthalpy drop which was found by measuring the gas temperature at the enclosure exit by traversing with a suction

pyrometer. The temperatures recorded by the pyrometer should be accurate, and the assumption of a symmetrical temperature distribution is reasonable. In the absence of detailed knowledge of the flowpattern near the enclosure exit an average temperature was assumed and no weighting applied. This may introduce an error.

The values of q_{s_m} for the pot furnace should certainly be accurate to 5% and could well be accurate to 3%.

The heat balance on the superheater shewed a discrepancy of 4% and the overall heat balance a discrepancy of 5%. There was a 5% difference between the values of q_{s_m} based on gas and steam side enthalpy drops.

The value of heat transferred to the sink q_{s_m} is considered to be accurate to 5%.

7.4. Theoretical method for predicting heat transferred.

The theoretical analyses are very tedious to evaluate but do have the advantage that if the correct data are used, a large number of zones employed, and the enclosure and gas are correctly represented, the predicted heat transfer will be very close to the heat transfer occurring in practice.

In the present analyses certain small inaccuracies arise through the method of calculation. A single mean specific heat was used for all enthalpy terms to avoid

having to recalculate specific heats for each estimate of gas temperature. For most gas zones this mean specific heat is not in error and in the extreme case of the hottest zone in a 10 gas zone analysis the error is no more than 3%. The overall error introduced by this approximation will not be greater than 1%.

The value of the convective transfer coefficient h_c was difficult to establish for the pot furnace. The best value seemed to be $h_c = 1$. Calculations showed that values of $h_c = 0.5$ and 2.0 produced 3% variations in the estimate of the heat transferred. Certain errors may be introduced by assuming a value of unity for h_c but these should not be large.

Very little error will be introduced by the assumption that the heat sink can be considered as a disc covering the top of the furnace.

In total the errors inherent in the calculation method should not be greater than 3% for the pot furnace.

In order to represent the superheater as a cylindrical enclosure with a wall sink, rather sweeping simplifications were made. This may introduce a significant error but no estimate of its magnitude can be made. The error introduced by assuming a mean specific heat affects the superheater calculations in the same way as the pot furnace calculations

discussed above.

7.4(a) Method of solution of simultaneous equations.

Hottel and Cohen ⁴⁶ suggested a determinant form of solution to arrive at the total interchange areas $\overline{SS}$ and $\overline{SG}$. This method is grossly inefficient. For the solution of a 10 gas zone case 265 determinants would have to be solved.

The method employed was that of inverting the matrix formed from the coefficients in the simultaneous equations and then multiplying by a matrix formed from the columns of constant terms. The total interchange areas $\overline{SS}$ and $\overline{SG}$ were obtained for the 10 gas zone case by inverting one 12 x 12 matrix and then multiplying by a 12 x 12 matrix and also by a 12 x 10 matrix.

The total interchange areas $\overline{GG}$ are obtained from the equation:

$$\overline{G_m G_n} = \overline{G_m G_m} + \sum_i \overline{G_m}^R s_i s_i^n \quad 7.1$$

Thomeycroft suggests that a simplification be made to avoid these multiplications. The value of $\sum_n \overline{G_m G_m}$ is known and all $\overline{G_m G_m}$ are known. Thomeycroft suggests that $\sum_n \overline{G_m G_n}$ be subtracted from $\sum_n \overline{G_m G_m}$ and the residual be divided equally between each of the $\overline{G_m G_n}$ factors. While this will result in all $\overline{GG}$ being in error, the error on any $\overline{GG}$ should be small. Thomeycroft says that the absorption by gas zones is small

and the greater the number of wall reflections the nearer the above simplification is to accuracy.

In a very large furnace it might be reasonable to evaluate $\overline{GG}$ for adjacent zones accurately and use the above approximation only for interchange between distant zones. In a small furnace all $\overline{GG}$ are significant and should be evaluated accurately. In most refractory lined furnaces wall emissivities are high (0.7 or higher) and the first reflection is most important²⁹. As the simplification is most inaccurate when wall emissivity is high it would not seem applicable to the present case, with a wall emissivity of 0.75.

If the amount of computation involved in evaluating the $\overline{GG}$ seems too large to be done by hand a simple computer program can be written to perform the calculations.

7.4(b) Variation in predicted heat transfer with number of zones.

Trial 1 on the pot furnace was calculated using 1, 2, 5 and 10 gas zones (4, 6, 12 and 22 zones including surface zones) with the gas being taken as grey and a grey-clear mixture. The various values of predicted heat transfer to the sink q_s are given in Table 7.1. These values are plotted against number of gas zones in Fig. 7.1.

Fig 7.1.

VARIATION IN HEAT TRANSFERRED
TO THE SINK Q_s WITH NUMBER OF
GAS ZONES FOR POTFURNACE TRIAL I

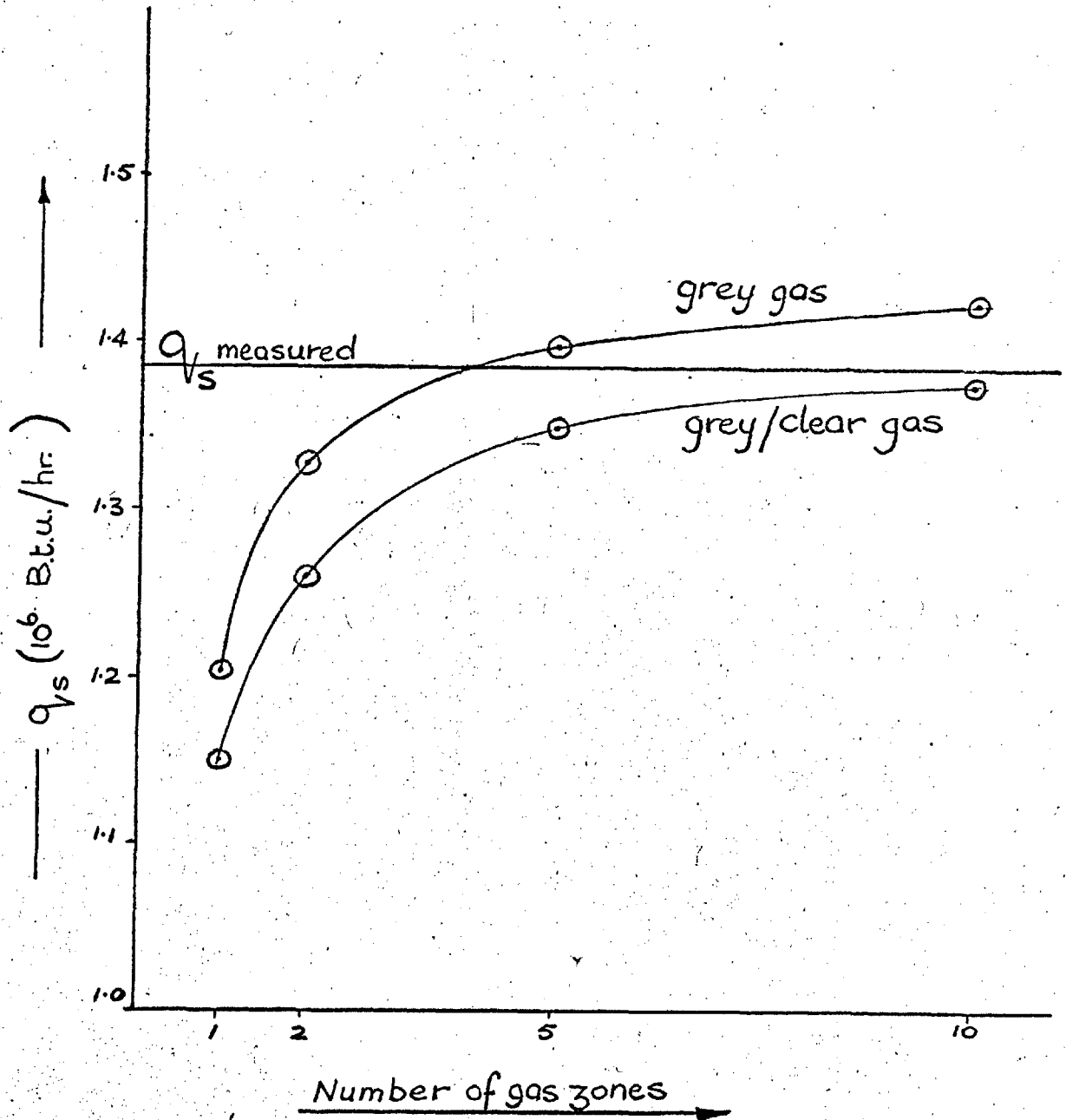


Table 7.1.

<u>Pot furnace trial 1.</u>		
	$q_s (10^6 \text{ Btu/hr})$	% of q_{s_m}
grey gas		
1 gas zone	1.20432	86.9
2 gas zones	1.32923	95.9
5 gas zones	1.39766	100.8
10 gas zones	1.42246	102.6
grey-clear gas		
1 gas zone	1.15023	83.0
2 gas zones	1.26076	91.0
5 gas zones	1.34972	97.4
10 gas zones	1.37525	99.2

The curves on Fig. 7.1. shew that as the number of gas zones is increased a more accurate estimate of q_s is obtained. As the number of gas zones is increased the relative effect of increasing the number of gas zones is decreased. From the shape of the curve it appears that increasing the number of gas zones above 10 would produce little increase in the value of q_s .

The value of q_s predicted using 10 gas zones and a grey-clear gas representation is very close to the value found experimentally.

The variation in q_s with number of gas zones is similar for both gas representations. With a large number of

zones, and therefore an accurate analysis, the assumption of a grey gas leads to an overestimate in the heat transferred to the sink.

7.4(c) Predicted heat transfer in different trials on the pot furnace

Two gas zone analyses were made of three trials on the pot furnace, using both grey and grey-clear mixture gas representations. The results are shown in Table 7.2.

Table 7.2.

Pot furnace 2 gas zones.

grey gas	q_s (10^6 Btu/hr)	% of q_{s_m}
trial 1	1.32923	95.9
trial 2	0.88021	99.5
trial 3	1.29422	97.7
grey-clear gas		
trial 1	1.26076	91.0
trial 2	0.83178	94.0
trial 3	1.21734	91.9

The percentages of the measured heat transfer predicted by the analyses show good agreement. For the grey gas case this percentage is $97.7 \pm 2\%$ and for the grey-clear gas case $92.3 \pm 2\%$. Remembering that 3% inaccuracies are quite possible in both q_s and q_{s_m} this agreement is good.

7.4(d) Predicted heat transfer in different furnaces.

It is not possible to compare the superheater and pot furnace results directly as different assumptions were made regarding configuration. The basic shape of the superheater is different and therefore an n gas zone analysis would not necessarily be comparable with an n gas zone analysis of the pot furnace.

Superheater analyses were made using 1 and 4 gas zones and grey and grey-clear gas representations. The results are listed in Table 7.3. The results for the 1 gas zone analyses of trial 1 on the pot furnace and the values for a 4 gas zone analysis obtained by interpolation on Fig.7.1 are included in Table 7.3. for comparison.

Table 7.3.

Superheater	$q_s (10^6 \text{ Btu/hr})$	% of q_{sm}
1 gas zone grey	0.64415	88.4
1 gas zone grey-clear	0.57611	79.1
4 gas zone grey	0.79721	109.4
4 gas zone grey-clear	0.72548	99.6
Pot furnace trial 1		
1 gas zone grey		86.9
1 gas zone grey-clear		83.0
4 gas zone grey		100.0
4 gas zone grey-clear		96.5

The variation of q_s with number of zones appears to be much more marked in the superheater. The 1 gas zone analysis gives a value of percentage of measured heat transfer for the grey-clear case lower than for the pot furnace while the 4 gas zone analysis gives a value higher. By comparison with Fig. 7.1. it would appear that a 10 gas zone analysis of the superheater would give a value of q_s considerably larger than the measured value. Rather sweeping simplifications were made in idealising the shape of the superheater and this is probably the cause of the error.

The cause of the greater variation in q_s with zone size for the superheater possibly lies in the configuration of the furnace. For the pot furnace with an end sink a typical ray from the hot end has to pass through the cooler gas to reach the sink. In general most rays reaching the sink will have passed through gas of varying temperature to reach the sink. In the superheater the radiation from a small part of the gas can reach the sink at the walls having passed only through gas at the same temperature. In general rays will pass through gas of relatively small range of temperature. From this it may be expected that a 1 gas zone analysis, involving the assumption of isothermal gas, will be less accurate for the superheater. Also increasing the number of gas zones will constitute a relatively larger step

towards reality in the superheater than in the pot furnace. From this it may be concluded that for a furnace with a wall sink analyses using small numbers of gas zones will not be accurate.

The inaccuracies which can arise from a wall sink furnace can be seen from the calculated temperatures of the two end zones of the superheater for the 1 and 4 gas zone analyses.

Superheater, 4 gas zones:

	grey	grey-clear
Temp. of top end °C	860	850
Temp. of bottom end °C	635	615

Superheater, 1 gas zone:

	grey	grey-clear
Temp. of top end °C	690	675
Temp. of bottom end °C	710	695

In the 4 gas zone analysis the top end is over 200°C hotter than the bottom end. Such a difference is reasonable as the hot gas input is at the top. For the 1 gas zone analysis, where the gas is assumed to be of uniform temperature, the method would predict the same temperatures for both ends. In fact, owing to the different values of heat loss from the ends found experimentally, the bottom end temperature is predicted to be slightly higher than the top end temperature.

7.4(e) Predicted heat transfer with different gas representations.

A 2 gas zone analysis was made of the results of trial 1 on the pot furnace using three different gas representations. These were grey gas, one grey gas-clear gas mixture and two grey gas-clear gas mixture. The predicted heat transfer is shown in Table 7.4.

Table 7.4.

Pot furnace trial 1 2 gas zones.

gas	$q_s 10^6 \text{ Btu/hr}$	% of q_{sm}	% difference
grey	1.32923	95.9	4.9
grey/clear	1.26076	91.0	0.4
2 grey/clear	1.25508	90.6	

The relative increase in accuracy of a grey-clear gas over a grey gas representation is much larger than the increase of a 2 grey-clear gas over a grey-clear gas representation. It would seem that a significant increase in accuracy is obtained by using a grey-clear gas to represent a real gas rather than assuming the gas to be grey. The increase in accuracy obtained by using a more complicated gas representation than grey-clear is small. Considering the possible errors in the values of q_s the effort involved in using a 2 grey-clear gas (all total interchange areas must be evaluated at two different values of $k\delta$) is not worth while.

The increase in computation caused by using grey-clear rather than grey gas is not so large; only the $\overline{SS}$ total interchange areas at $k\delta = 0$ are needed extra to the full set of grey values.

The values of q_s have been calculated using grey and grey-clear gas representations for a range of variables. The difference in predicted heat transfer using these gas representations has been found. The term difference is defined as:

$$\frac{q_s(\text{based on grey gas}) - q_s(\text{based on grey-clear gas}) \times 100}{q_s(\text{measured})}$$

Values of this difference are given in Table 7.5.

Table 7.5.

furnace	trial no.	no. of gas zones	difference %
pot	1	1	3.9
pot	1	2	4.9
* pot	1	2	6.0
pot	1	5	3.4
pot	1	10	3.4
pot	2	2	5.5
pot	3	2	5.8
Superheater	-	1	9.3
Superheater	-	4	9.8

*In this case the value of k was allowed to vary between gas zones.

The differences average about 5% for the pot furnace and about 10% for the superheater.

For a real gas, with its characteristic absorption of certain wavelengths, the absorbable wavelengths are gradually filtered out as a ray passes through the gas and the transmittance of the gas for the remainder of the radiation approaches unity. A grey gas has a single overall value of transmittance and absorbs radiation of all wavelengths.

A curve of $\log \epsilon - \log PL$ for a real gas is convex²⁶, whereas the relation for a grey gas is a straight line. The straight line relation for a grey gas is keyed to the real gas curve at one point. The selected point is the mean beam length of the enclosure, and only at this point does the value of grey gas emissivity equal the real gas value. For rays which travel further than the mean beam length the grey gas assumption overestimates emissivity, while for rays travelling shorter distances emissivities are underestimated. From the relative shapes of the real gas and grey gas emissivity relations, remembering that the point of intersection represents the emissivity of a ray travelling the average path length, it will be seen that the total effect of the overestimation of emissivity at high path lengths is greater than the underestimation at low path

lengths. The assumption of a grey gas will thus lead to an overestimate of the interchange between gas and sink zones.

The grey gas assumption will also lead to an underestimate of the exchange between wall and sink zones. In both furnaces considered the heat input is carried by the gas and the exchange from gas to sink is most important. The grey gas assumption thus produces an overestimate of the heat transferred to the sink.

The pot furnace has an end sink which represents $\frac{1}{6}$ of the total internal area. The remaining $\frac{5}{6}$ is refractory wall. The superheater has a wall sink which covers $\frac{4}{5}$ of the total internal area while the refractory ends cover only $\frac{1}{5}$ of the area. In the pot furnace there is relatively much more exchange of radiation from wall to sink than in the superheater. This exchange is underestimated by assuming a grey gas. Thus the underestimated term will be relatively much larger for the pot furnace. It is thus reasonable to expect the net overestimation in q_s found assuming a grey gas will be less than for the superheater. The assumption of a grey gas leads to greater error the greater the proportion of the the internal area represented by the sink.

From the above it might be expected that the use of a 2 grey-clear gas representation would effect a relatively larger improvement in the prediction of q_s in the super-

heater than it did in the pot furnace.

Thorneycroft and Thring⁶ assumed a grey gas and quoted work by Smith⁷⁶ which they said shewed that in certain cases a grey gas may give a better result than a grey-clear representation of a real gas for a theoretical speckled furnace. The calculations by Smith shew that a better result is obtained assuming a grey gas inside grey walls and sinks than by assuming a real gas, grey sink and no-flux or white walls. Smith found this to be true when actual wall emissivities had values greater than 0.4. Hottel⁴⁵ suggested that as a first approximation to a solution interpolation between the two methods investigated by Smith could be used.

The furnaces under investigation do not approximate to the speckled case, which requires intimate mingling of wall and sink zones. Also the no-flux or white wall assumption is not used. Providing a sufficiently large number of zones is considered the assumption of an approximately real gas inside grey walls must lead to a better result than a grey gas inside grey walls. The only situation in which this would not be true would be if the sink to wall proportion fortuitously caused the overestimation of exchange from gas to sink to cancel the underestimation of exchange from wall to sink. The results obtained in the present work suggest that for this to be true the sink must represent a

a very small proportion of the internal area.

7.4(f) Predicted heat transfer with varying absorption
coefficient

In a one gas zone analysis both gas temperature and absorption coefficient are represented by mean values for the whole enclosure. When 2 gas zones are considered the restriction of uniformity of gas temperature is lifted. The restriction of uniform absorption coefficient can be removed by evaluating k for both gas zones and calculating the interchange based on the appropriate properties. The effects of removing these restrictions are shown in Table 7.6. for trial 1 on the pot furnace with grey and grey-clear gas representations.

Table 7.6. Pot furnace trial 1

<u>grey-clear gas</u>	q_s	% of q_{sm}	% difference
1 gas zone	1.15023	83.0	8.0
2 gas zones	1.26076	91.0	2.1
2 gas zones variable	1.29014	93.1	
<u>grey gas</u>			
1 gas zone	1.20432	86.9	9.0
2 gas zones	1.32923	95.9	3.2
2 gas zones variable	1.37351	99.1	

The relative increase in accuracy obtained by

allowing k to vary through the enclosure is less than that obtained by allowing temperature to vary, but is still significant. If k is allowed to vary the complexity of calculation is greatly increased as all total interchange areas must be evaluated for each value of k and that value which describes the interchange most closely is then used. A greater increase in accuracy is obtained by allowing k to vary through the gas than is obtained by using more complex approximations to a real gas than a grey-clear mixture. If a really accurate prediction of heat transfer is required the value of absorption coefficient should be allowed to vary.

7.4(g) Calculated temperatures.

The wall zone temperatures calculated for trial 1 on the pot furnace using 10 gas zones are plotted on Fig 7.2. The estimated best curve representing the variation in actual wall temperature (see Fig. 4.1 and section 4.2(a)) is also shewn. The curves are of different shapes at the end of the furnace as the calculated curve assumes heat input to the first zone whereas in fact the burner centre lines are 2 ft. from the end. Apart from this there is good agreement between the two lines.

The gas zone temperatures calculated for trial 1 on the pot furnace using 10 gas zones are plotted on Fig. 7.3. The temperatures calculated assuming both grey and grey-clear

Fig 7.2.

VARIATION IN CALCULATED AND
EXPERIMENTAL WALL TEMPERATURES
FOR TRIAL 1 ON THE POT FURNACE
USING 10 GAS ZONES

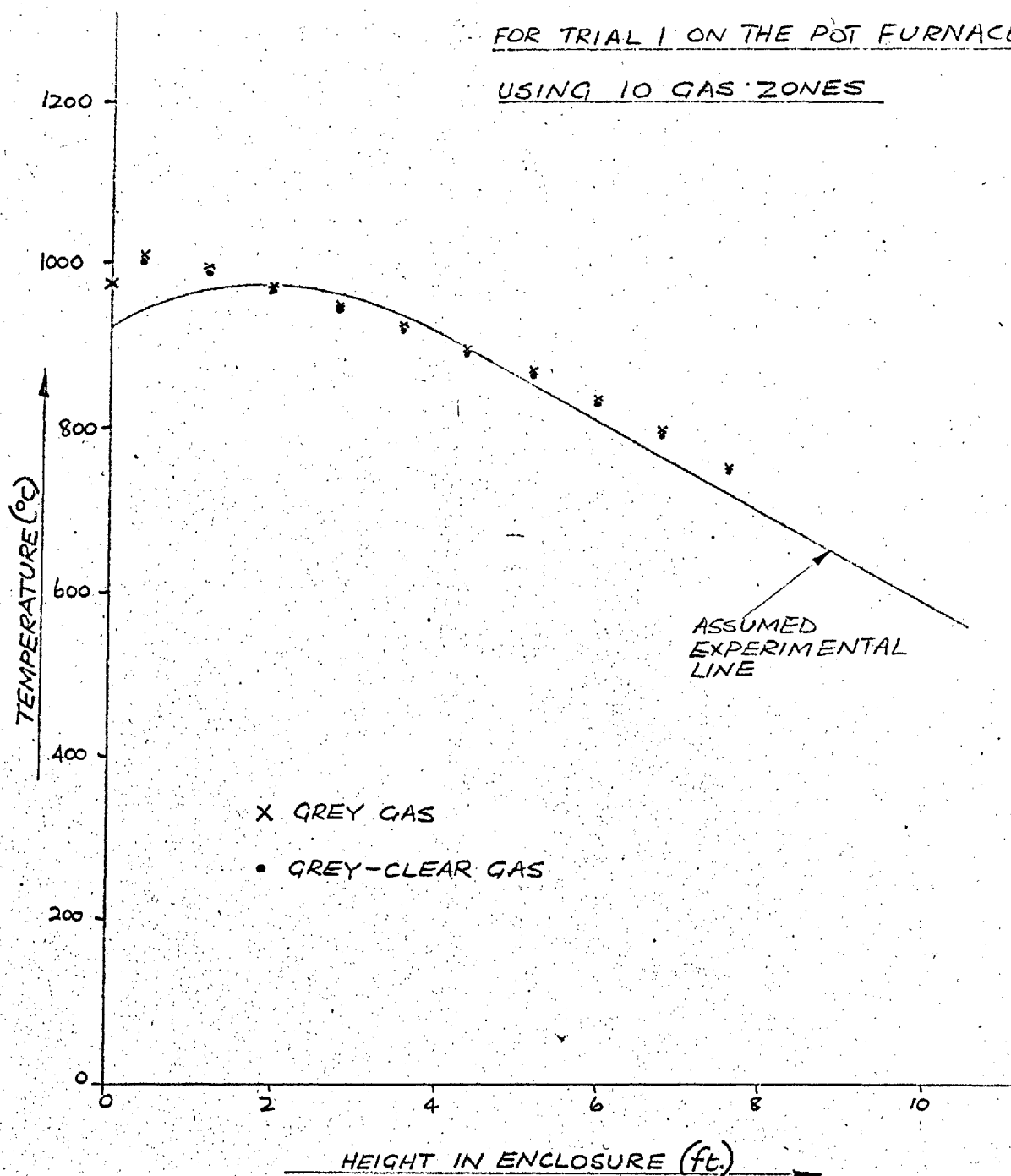
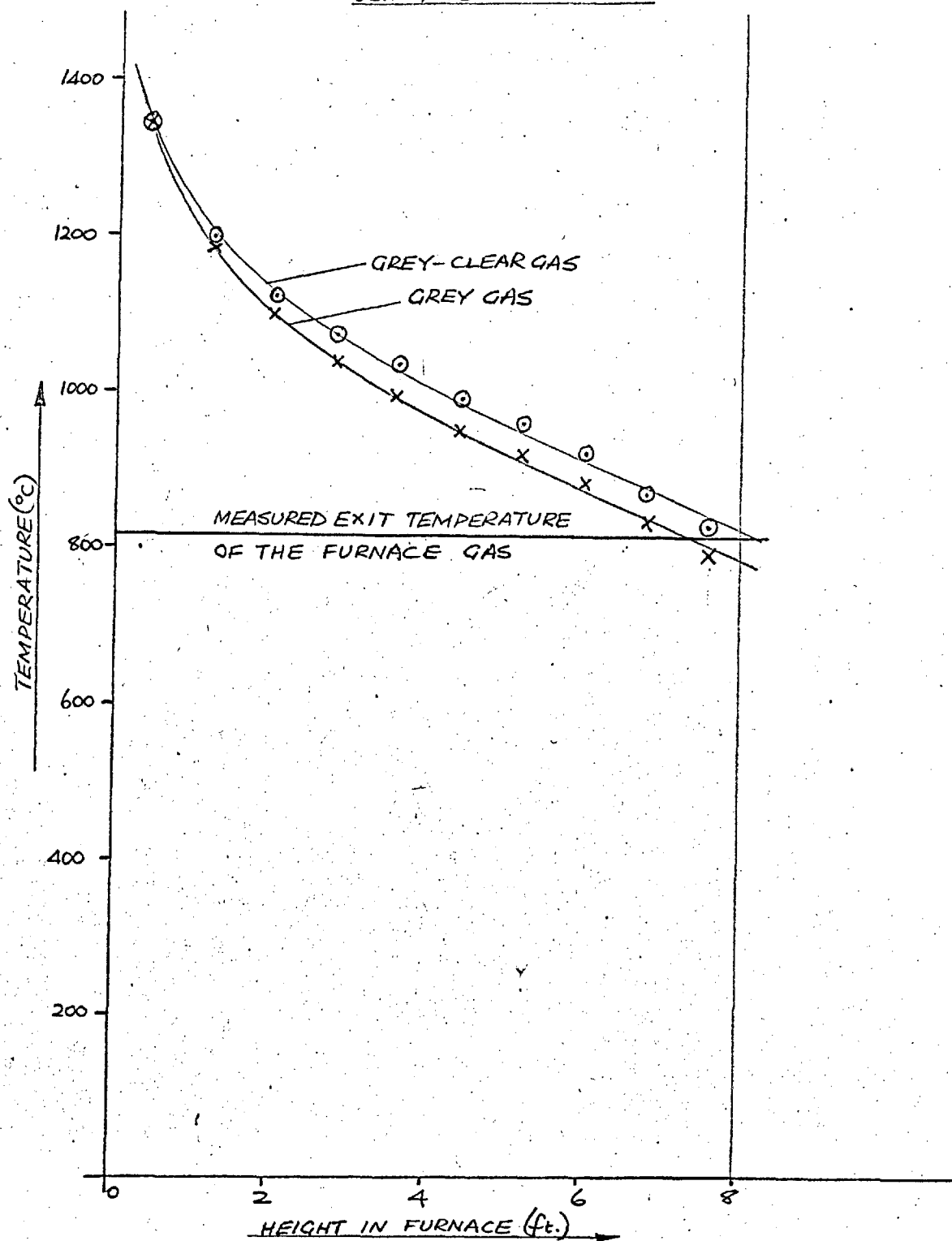


Fig. 7.3.

CALCULATED TEMPERATURE PROFILES
FOR TRIAL 1 ON POT FURNACE
USING 10 GAS ZONES



mixture gas representations are shewn. The temperature of the gas leaving the radiant enclosure, calculated from the measured temperature profile Fig. 6.1, is shewn. There is good agreement between measured and calculated exit temperatures.

The theoretical model appears to be a good representation of the pot furnace.

7.4(h) Flames

The pot furnace was fired by burning atomised fuel oil and two luminous flames existed in the furnace. The superheater was gas fired and most of the combustion took place in the tunnel leading into the furnace. In the analyses of both furnaces the existence of luminous flames has been neglected.

The presence of a luminous flame should be acknowledged if a complete description of the transfer processes within a furnace is required, but great problems arise when this is attempted. To fit in with the zone treatment the flame shape must be approximated to fill a basic zone. The mean overall emissivity and surface area of the flame must be found and a pseudo emissivity calculated based on the area of a unit zone. This was done by Thorneycroft and Thring⁶, who allowed the flame to emit as a solid body but absorb as products of combustion. The two methods used⁷ for measuring the

flame emissivity gave widely differing values and, as the flame zone was by far the largest single contributor to the exchange, the predicted heat transfer was subject to considerable uncertainty.

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In a recent paper Yardley and Patrick found that allowing for the two extremes of flame type, high velocity *and low velocity highly luminous* non-luminous/ made little difference in the calculated thermal efficiencies of the furnaces.

Although much work³ is being done on the study of flames it is not yet possible to predict flame emissivity from external considerations only.

Until such time as flame emissivities can be obtained with accuracy the best assumption for calculating overall heat transfer must be the instantaneous conversion of the fuel into products of combustion. The approximate allowance for the existence of a flame may however be most valuable when local rates of heat transfer are investigated.

7.5. Furnace correlations

7.5(a) Wilson-Lobo-Hottel¹⁹ correlation

The Wilson-Lobo-Hottel¹⁹ correlation was applied to three pot furnace trials and the superheater trial. Despite the allowances made in this correlation for varying type of furnace the values of heat transferred were not accurate. There are a large number of variables associated with furnace

performance and an empirical correlation cannot be expected to account for these outside the range of variables on which its derivation was based. This correlation, like many others, was based on large tube still data and does not correlate accurately the performance of the furnaces under consideration here.

7.5(b) Lobo-Evans correlation.

The method proposed by Lobo and Evans²¹ is essentially the simultaneous solution of a rate equation based on Stephan's Law and a heat balance. The same temperature is allowed to serve as mean radiating and gas exit temperature, subject to certain restrictions about the latter.

For the trials on which the correlation was based Lobo and Evans arbitrarily treated forward radiation to the convection sections of the furnaces separate from, and subsequent to, radiation to the sink in the radiant section. By doing this they calculated a fictitious outlet temperature from the radiation section which was above the actual outlet temperature. This calculated temperature turned out to be on average very close to the actual mean radiating temperature. If Lobo and Evans had considered all transfer originating within the radiant enclosure instead of just that fraction received in the radiant section their results would have been very different.

The two furnaces under consideration here do not have convection sections which receive a significant amount of forward radiation. It is not clear which temperature Lobo and Evans recommend should be taken as mean radiating temperature in this case. They say that the gas exit temperature should be used but in their applications of the method they used a fictitious gas exit temperature. It will be shown below that the use of true gas exit temperature for mean radiating temperature would result in large underestimations of the heat transfer. The most reasonable treatment would appear to be the simultaneous solution of the rate and heat balance equations for the total radiation in the enclosure. The same temperature is allowed to serve as mean radiating and gas exit temperature. To distinguish this method from that actually used by Lobo and Evans. This method will be termed modified Lobo-Evans.

The heat transferred to the sink q_s predicted by the modified Lobo-Evans method is shown in Table 7.7.

Table 7.7. Modified Lobo-Evans

	$q_s (10^6 \text{ Btu/hr})$	% q_{sm}
Pot furnace trial 1	1.204	87
Pot furnace trial 2	0.800	90
Pot furnace trial 3	1.166	88
Superheater	0.614	84

The modified Lobo-Evans method for trial 1 on the pot furnace predicts the same heat transfer as did a 1 gas zone analysis using the Hottel-Cohen treatment with a grey gas. This is to be expected as the assumptions are similar and the view factor for the pot furnace is correctly allowed for in the Lobo-Evans treatment. The 1 gas zone grey gas analysis of the superheater^{gwe} a value of heat transferred of 88% of that measured. This is close to the value given in Table 7.7; the difference probably lies in the approximate view factor given by Lobo and Evans.

So that the implications of the modified Lobo-Evans method can be investigated certain assumptions are made. Radiant transfer is dependent on emissivity and the fourth power of temperature. If the temperature dependence of emissivity is incorporated into the temperature terms the index on temperature is reduced from 4 to a value p . The rate equation becomes radiative heat transfer q_r equal to some factor F times the difference between gas temperature to the power p and the sink temperature T_s also to the power p . The convective transfer is neglected for simplicity.

$$q_r = F(T^p - T_s^p) \quad 7.1.$$

The heat balance can be written as enthalpy drop q_e equals input enthalpy H_i minus the outlet enthalpy, which is mass flow rate m times mean specific heat C_p times outlet

temperature T .

$$q_e = H_i - m C_p T \quad 7.2.$$

These two equations are drawn on figure 7.4.

The modified Lobo-Evans solution is at the intersection of the two lines. This intersection occurs at the value T_L which satisfies both equations 7.1 and 7.2 and the value q_L is the heat transferred to the sink estimated by the modified Lobo-Evans method.

The actual heat transferred to the sink q_a , the real mean radiating temperature of the gas T_g and the outlet temperature of the gas T_o are put on the diagram. It is obvious that T_o cannot exceed T_L and except in the case of a perfectly stirred furnace T_o must be lower than T_L . It is thus apparent that the modified Lobo-Evans method must underestimate the heat transfer in a furnace.

$$q_L < q_a \quad 7.3.$$

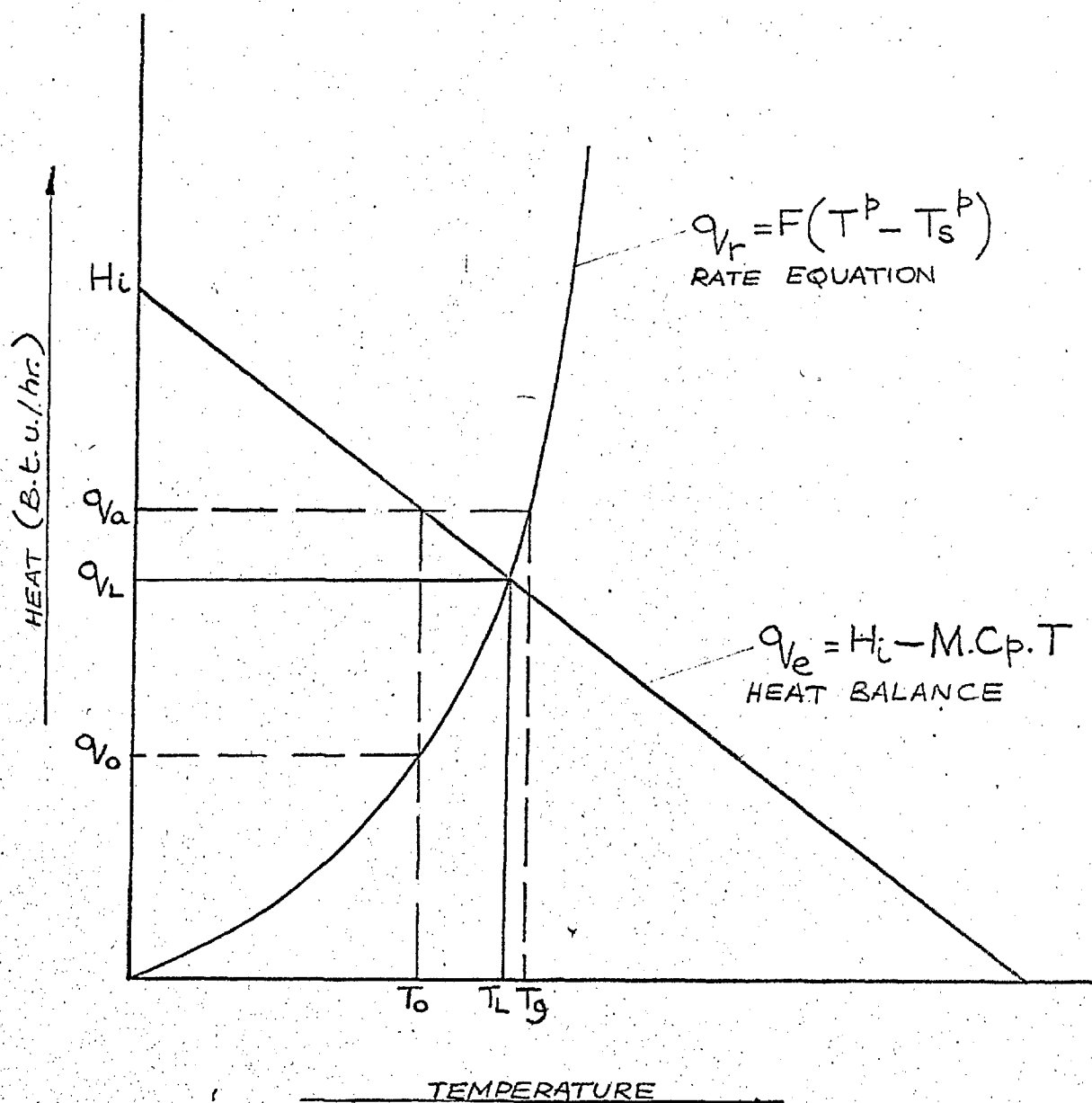
It must be emphasised that the modified Lobo-Evans method does not consist of using the actual gas exit temperature as the estimate of mean radiating temperature. Such a procedure results in the value for heat transferred of q_o . It is apparent that

$$q_a > q_L > q_o \quad 7.4.$$

It is also apparent, from the shape of the two curves, that q_L will lie nearer to q_a than to q_o . Thus

Fig 7.4.

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although the modified Lobo-Evans method must always underestimate the heat transfer the predicted q_L will be much nearer to the actual heat transfer q_a than to that obtained q_o by assuming that the actual outlet temperature T_o can serve as mean radiating temperature. If we let the difference between T_L and T_o be ΔT then,

$$q_a - q_L = m C_p \Delta T \quad 7.5$$

and,

$$q_L - q_o = F \left[(T_o + \Delta T)^p - T_o^p \right] \quad 7.6$$

or,

$$q_L - q_o = F T_o^p \left[1 + \frac{\Delta T}{T_o} \right]^p - F T_o^p \quad 7.7$$

The bracketed term can be expanded and by substitution of typical values it was found that terms higher than $\left[\frac{\Delta T}{T_o} \right]^2$ could be neglected. Thus,

$$\frac{q_L - q_o}{q_a - q_L} = \frac{F T_o^p \left[\frac{\Delta T}{T_o} + \frac{(p-1)}{2} \left(\frac{\Delta T}{T_o} \right)^2 \right]}{m C_p \Delta T} \quad 7.8.$$

or,

$$\frac{q_L - q_o}{q_a - q_L} = \frac{F p}{m C_p} \left[T_o^{p-1} \left(1 + \frac{p-1}{2} \frac{\Delta T}{T_o} \right) \right] \quad 7.9.$$

Substitution of the results into equation 7.9. (taking p to be 3.2)

gave values of $\frac{q_L - q_o}{q_a - q_L}$ of about 2.

A simpler but less accurate guide to the placing of q_L with respect to q_a and q_o can be obtained by taking the first term of the expansion only.

$$\frac{q_L - q_o}{q_a - q_L} = \frac{F_p T_o^{p-1}}{mCp} \quad 7.10.$$

This is in fact the ratio of the slopes of the radiation and enthalpy curves on Fig. 7.4. Thus,

$$\frac{q_L - q_o}{q_a - q_L} = \frac{\frac{d}{dT}(q_r)}{\frac{d}{dT}(q\Delta H)} \quad 7.11.$$

The value of the ratio obtained using equation 7.11 is typically about 80% of the more accurate value from equation 7.9.

The modified Lobo-Evans method underestimates the heat transferred to the sink. If the values of q_L calculated are multiplied by a factor 1.15 the results shown in Table 7.8 are obtained

Table 7.8.

	1.15 x q_L as % of q_{s_m}
Pot furnace trial 1	100
Pot furnace trial 2	103
Pot furnace trial 3	101
Superheater	97

The agreement to 3% is within the estimated accuracy of the measured heat transfer q_{s_m} .

The method proposed by Lobo and Evans applies to furnaces having convection sections. The forward radiation to the convection section is neglected. Only radiation which is received within the radiant section is considered. For these furnaces Lobo and Evans calculated a fictitious exit temperature which then served as a mean radiating temperature. The reception of radiation from the radiant enclosure by the convection section must be calculated separately. The Lobo-Evans method can either overestimate or underestimate the reception of heat in the radiant section.

As Lobo and Evans gave no clear method for calculating furnaces not having significant forward radiation from radiant to convective sections, the modified Lobo-Evans method was developed. This modified Lobo-Evans method involves the solution of a rate and a heat balance equation. The method will always underestimate the heat transfer. In the furnaces investigated a factor of 1.15 can be applied to correlate the performance to 3%.

This modified Lobo-Evans method can equally be applied to furnaces with a convection section. It will account for all the radiation originating in the radiant section. A simple factor, similar to the one calculated above, can be applied to improve the estimate of heat transfer to the sink. It is suggested that for normal design

purposes this modified Lobo-Evans method provides a simple and effective solution.

7.5(c) Hottel correlation.

Hottel²⁶ has suggested an extension of the Lobo-Evans method to remove the restriction of the same value applying for both mean radiating temperature and gas exit temperature.

The rate equation,

$$q_H = \overline{GS} \sigma (T_G^4 - T_S^4) + h_c A (T_G - T_S) \quad 7.12.$$

must be solved simultaneously with a heat balance, which is expressed as,

$$\frac{H_i - q_H}{H_i} = \frac{T_g - B}{T_{AF}} \quad 7.13.$$

where

$\overline{GS}$ is an exchange factor

T_g is the gas mean radiating temperature

T_{AF} is a pseudo adiabatic flame temperature assuming no dissociation

B is the difference between mean radiating temperature T_g and gas outlet Temperature T_o

$$B = T_g - T_o \quad 7.14$$

q_H is heat to sink estimated by Hottel method.

In order to solve equations 7.12 and 7.13 a value for B must be guessed. Hottel suggests that for many furnaces

a suitable guess for B can be obtained from the equation,

$$B = \frac{q_H}{H_{in}} \cdot \frac{T_{AF}}{4} \quad 7.15.$$

Thus equations 7.12, 7.13 and 7.15 are solved together to give a value for q_H .

The values of heat transferred to the sink predicted by the Hottel extension of the Lobo-Evans method are shown in Table 7.9.

Table 7.9.

	q_H (10^6 Btu/hr)	% of q_{s_m}
Pot furnace trial 1	1.438	104
Pot furnace trial 2	.930	105
Pot furnace trial 3	1.372	105
Superheater	.762	105

The equation for guessing B (equation 7.15) was proposed by Hottel based on experience. As with any such equation the estimate can usually be improved by basing the constants on data for the case under consideration. The values of heat transfer predicted by the Hottel method were calculated using the following modified equation for B.

$$B = \frac{q_H}{H_{in}} \cdot \frac{T_{AF}}{5} \quad 7.16$$

The results obtained are shown in Table 7.10 as a percentage of the measured heat transfer.

Table 7.10.

	% of q_{sm}
Pot furnace trial 1	99
Pot furnace trial 2	100
Pot furnace trial 3	101
Superheater	99

The agreement to 1% of the measured heat transfer is considerably better than the experimental accuracy, and must be considered to some extent fortuitous.

If the convective transfer in equation 7.12 is neglected the rate equation and the heat balance equation can be expressed diagrammatically; this is done in Fig. 7.5. The value of heat transfer estimated by this method q_H is shown and the values of gas outlet temperature T_o and mean radiating temperature $T_g = B + T_o$ are also shown.

The estimated heat transfer q_L using the modified Lobo-Evans method is shown by broken lines for comparison. The value of q_L must always be lower than q_H , providing B is not zero.

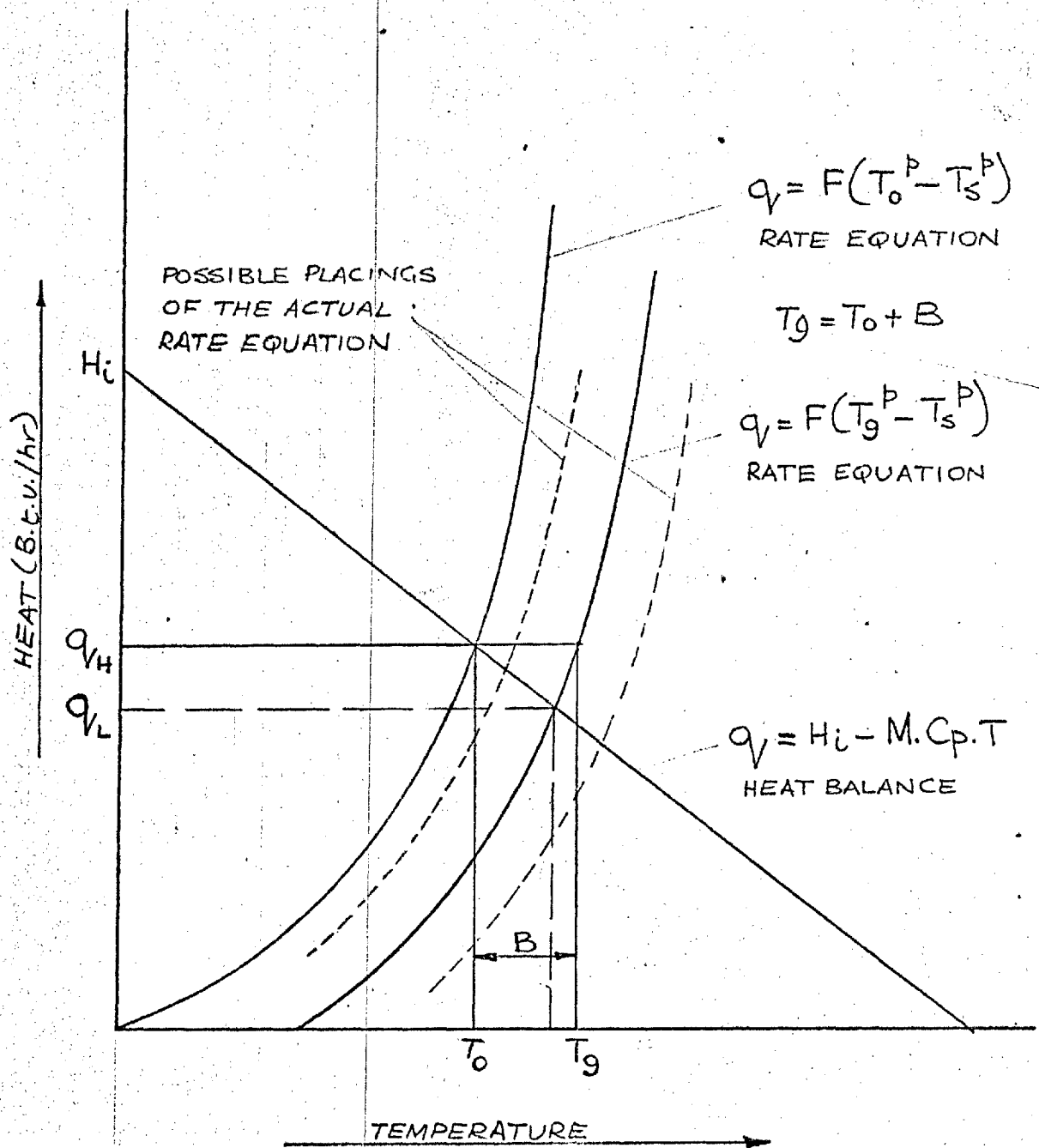
Consideration of equation 7.15 shows that Hottel's suggested estimate of B can be written much more simply as,

$$B = \frac{T_{AF} - T_G}{3} \quad 7.17.$$

Also equation 7.16 can be rewritten as

Fig 7.5.

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$$B = \frac{T_{AF} - T_G}{4} \quad 7.18.$$

The value guessed for B is empirical and can lead to various placings of the rate equation. The rate equation based on the actual mean radiating temperature may be above or below that obtained from the estimate of B. This is shewn on Fig. 7.5 by dotted lines. It is thus possible to overestimate or underestimate the heat transfer if the Hottel method is used for predicting furnace performance.

If furnace data are available, and the value for B can be estimated, the Hottel method affords a method for correlating performance. If no data are available a value for B must be guessed.

The modified Lobo-Evans method has the advantage that it must always underestimate the heat transfer. A factor, of the order 1.15, can be estimated to improve the accuracy of the prediction of the transfer. Guessing this factor is subject to the same inaccuracies as is guessing the factor B. The probable accuracies of the two methods are comparable and the modified Lobo-Evans method is simpler to use than the Hottel method.

7.6 Conclusions

The major conclusions arising out of this work are listed below.

1. Reception factors for the cylindrical system have been established for aspect ratio $1/10$ at various values of absorption strength up to a separation of one cylinder diameter. Reception factors for aspect ratio $1/2$ have been established up to a separation of two cylinder diameters. Reception factors can be built up from the above values for larger aspect ratios which are also multiples of $1/10$ or $1/2$.
2. Analyses of the Hottel-Cohen type can be very accurate if calculated in sufficient detail. A large number of zones is required, a real gas representation must be employed and the value of absorption coefficient should be allowed to vary between zones. The amount of computation is immense and if the method is to be used for design purposes the whole calculation procedure should be arranged as one computer program.
3. The prediction of heat transfer increases in accuracy as the number of gas zones considered increases. The relative increase in accuracy obtained by considering an extra gas zone decreases rapidly as the number of gas zones increases. For a narrow furnace with a wall sink, such as the superheater, the errors arising from considering a small number of gas zones are relatively larger than for a furnace with an end sink such as the pot furnace.
4. The assumption of a grey gas leads to an overestimation of the transfer between gas and sink and to an underestimation

of the transfer between wall and sink. These errors are inherent in the approximations involved in assuming a grey gas. If the heat input to the furnace is carried by the gas an overestimate of the heat transferred to the sink will result.

5. A significant improvement in accuracy is obtained by using a grey gas-clear gas mixture to represent the real furnace gas. Relatively little increase in accuracy is obtained by a representation of a mixture of two grey gases and one clear gas.
6. A significant improvement in accuracy is obtained by allowing the value of the absorption coefficient to vary between zones. This improvement is greater than that gained by using a gas representation of two grey - one clear mixture rather than one grey - one clear mixture.
7. The improvement in predicted heat transfer obtained by using a grey - clear gas mixture rather than a grey gas was about 5% for the pot furnace and 10% for the superheater. The greater the proportion of the internal area covered by sink zones the greater is the error introduced by assuming a grey gas.
8. Lobo and Evans proposed a method for calculating the heat received in the radiant section of furnaces which have significant forward radiation to a convection section. A modified Lobo-Evans method has been developed which will account for

all radiation originating in the radiant section. This method will always underestimate the heat transfer. A factor can be applied to this method which renders it subject to the same degree of uncertainty as the method proposed by Hottel, yet it is simpler to evaluate. This modified Lobo-Evans method provides a simple and effective design method.

9. Empirical correlations of the Wilson-Lobo-Hottel type which are based on large tube-still data cannot be expected to predict accurately the heat transfer in small furnaces. Methods of predicting furnace heat transfer which have a theoretical basis can be expected to apply equally to both large and small furnaces.

Appendix 1.Experimental Results and Calculations.A.1.1. Pot furnace.A.1.1(a) Readings.

The averaged results for each trial are given in Table A.1.1.

Table A.1.1

TRIAL		1	2	3	4
Pot body temp.	°C	317	325	317	318
Preheater air inlet temp.	°C	108	102	96	98
Column Lead temp.	°C	135	134	138	135
Flue gas inlet temp.	°C	348	343	380	385
Flue gas outlet temp.	°C	200	196	215	218
Preheater air outlet temp.	°C	213	270	255	263
Ambient temp. at base	°C	28	31	26	23
Ambient temp. at top	°C	30	32	27	24
Wet bulb temp.	°C	23.5	26.0	19.5	17.0
Spent acid temp.	°C	28	32	32	27
Oil delivery temp.	°F	200	185	185	180
Oil return temp.	°F	178	176	172	162
Feed acid strength	% H_2SO_4	70.7	71.1	70.9	71.3
Make acid strength	% H_2SO_4	94.4	94.6	94.5	94.4
Tyler meter oil	gall/hr.	15.01	11.65	16.35	10.12
Make acid	gall/hr.	136.0	105.7	150.0	91.3
CO ₂ content stack gas	%	10.5	7.25	9.0	9.4

A.1.1.(b) Oil meter calibration.

Final meter reading	g.	125988.6
Initial meter reading.	g.	<u>125983.0</u>
Volume collected	g.	5.6
Weight of drum + oil	lb.	61.5
Weight of drum	lb.	<u>11.5</u>
Weight of oil	lb.	50.0
Density lb/indicated gallon		8.93
Temp. at collection	°F	185

A.1.1.(c) Heat losses.

The heat losses were calculated from the equations⁵⁸

$$Q = A \sigma \epsilon \left[\left(\frac{T}{100} \right)^4 - \left(\frac{T_a}{100} \right)^4 \right]$$

$$Q = A c (\Delta \theta)^{5/4}$$

A.1.1.

and $C = 0.27$ for vertical wall

0.35 for horizontal plate facing upwards

$0.5/D^{0.25}$ for pipework.

The convection losses are for surfaces in still air. The measured temperatures and calculated heat losses for Trial 1 are given in Table A.1.2. The heat losses for all four trials are given in Table A.1.3.

Table A.1.2.

LOCATION	Area sq.ft.	Temp. °C.	Radi- ation loss B.t.u/ hr.	Convection loss B.t.u/hr.	Total loss B.t.u/ hr.
Setting casing.	12	220	9,884	4,766	
" "	120	65	9,342	5,758	
" "	235	70	20,819	13,340	
" "	88	68	7,536	4,681	
" "	64	72	6,079	3,853	
" "	197	75	20,244	12,925	
" "	10	120	2,527	1,561	
" "	11	150	4,249	2,457	
" "	7	130	2,049	2,031	
" "	<u>18</u>	90	<u>2,632</u>	<u>1,692</u>	
" "	762		85,361	53,064	138,425
Setting base	128				36,000
Setting cover	10	130	3,090	2,503	
" "	25	115	6,081	5,109	
" "	23	90	3,551	3,044	
" "	49	85	6,845	5,824	
" "	23	100	4,309	3,689	
" "	<u>130</u>		<u>23,876</u>	<u>20,169</u>	<u>44,045</u>
Vapour bend lagged horiz.	33	60	2,202	796	
" " vertical	20	110	4,470	2,690	
" " "	6	140	<u>2,130</u>	<u>1,203</u>	
			8,802	4,689	13,491
Agitator mounting	15	300	6,118	5,100	11,218
Gear stand	25	72	2,215	1,419	3,634
Flue box	90	100	12,572	10,697	23,269

continued.....

Table A.1.2. continued.

LOCATION	Area sq.ft.	Temp. °C.	Radi- ation Loss B.t.u/ hr.	Convection loss B.t.u/hr	Total loss B.t.u/ hr.
Conical inlet	31	125	5,808	4,972	10,780
Air preheater between thermocouples (see table A.1.5)			5,524	3,217	
Air preheater bare faces	3	325	2,372	1,114	
" " bottom sides	8	108	1,316	846	
Air preheater flanges	14	395	16,069	9,396	
" " top	19	180	5,871	4,756	
			31,152	19,329	50,481
Dephlegmator	130	18	2,564	1,295	3,859
Ducting	150	65	11,678	7,198	18,876
					<u>355,078</u>
					<u>355,000</u>

Table A.1.3. Heat losses in all trials.

TRIAL	TOTAL HEAT LOSS
1	355,000
2	292,000
3	389,000
4	208,000

A.1.1.(d) Air balance.

The percentage excess air was calculated knowing the CO_2 content of the stack gas, the oil composition and the air humidity.

Table A.1.4. Total air flow.

TRIAL	1	2	3	4
Oil rate lb/hr	134	104	146	90.4
Theo. air requirements lb/hr	14.12	14.12	14.12	14.17
" " " lb/hr	1892	1468	2062	1281
CO_2 content stack gas %	9.80	6.85	8.39	8.74
Excess air %	45.9	113.0	71.8	63.8
Total air flow lb/hr	2760	3127	3541	2098

As there are leaks to the setting the flow of combustion air through the preheater has been estimated from a heat balance on the preheater. The heat loss from the surface of the air preheater between the temperature measuring points was found from detailed measurements of surface temperature using a surface pyrometer.

Table A.1.5. Heat loss between temperature measuring points on air preheater for Trial 1.

Area	Temp. °C	Convection B.t.u/hr.	Radiation B.t.u/hr.	Total loss B.t.u/hr.
16	80	1096	1900	2996
1.5	150	335	630	965
3	90	282	410	692
5.5	55	172	260	432
36	45	402	800	1202
20	40	210	350	560
152	35	540	1354	1894
				8700

Table A.1.6. Heat balances over air preheater.

Trial	Gas flow lb/hr	T gas °F	T air °F	Gas heat loss B.t.u/hr.	Heat loss B.t.u/hr.	Heat to air B.t.u/hr.	air flow lb/hr
1	2894	266	369	209,700	8,700	201,000	2242
2	3231	264	356	224,700	8,600	216,100	2495
3	3687	297	410	293,500	12,300	281,200	2820
4	2188	301	430	177,200	11,000	166,200	1591

Table A.1.7. Total air balance.

TRIAL	1	2	3	4
Total air flow lb/hr	2760	3127	3541	2098
Flow thro' preheater lb/hr	2242	2495	2820	1591
%	81.2	79.8	79.6	75.8
Atomising air lb/hr	125	125	125	125
%	4.6	4.0	3.6	6.4
Air leaks lb/hr	393	507	596	382
%	14.2	16.2	16.8	18.2

A.1.1. (e) Acid concentration.Table A.1.8.

TRIAL	1	2	3	4
Spent acid strength %	70.7	71.1	70.9	71.3
Make acid strength %	94.4	94.6	94.5	94.4
Dephl. top temp. °C.	135	134	138	135
Spent acid temp. °C.	28	32	32	27
lb. spent acid/lb make acid.	1.335	1.331	1.333	1.324
Enthalpy spent acid B.t.u/lb.	-112	-106	-106	-111
Enthalpy make acid B.t.u/lb	184	184	184	184
	396	391	394.3	383
Heat out/lb make acid B.t.u	580	575	578.3	567
Heat in/lb make acid B.t.u	-149.5	-141.1	-141.3	-147
Heat reqd./lb make acid B.t.u	729.5	716.1	719.6	714
Heat reqd./lb H ₂ SO ₄ B.t.u.	772.8	757.0	761.5	756.4
Acid rate lb 100% H ₂ SO ₄ /hr.	2337	1820	2580	1569
Total heat reqd. 10 ⁶ B.t.u/hr	1.805	1.378	1.9660	1.187

A.1.1. (f) Heat to stack.Table A.1.9.

TRIAL	1	2	3	4
Preheater outlet temp. °F	392	385	419	424
Mean Cp. B.t.u/lb	0.263	0.260	0.262	0.263
Stack gas flow lb/hr	2894	3231	3687	2188
Heat in stack gas	298,000	323,000	405,000	244,000

A.1.1. (g) Total Heat input.Table A.1.10.

TRIAL	1	2	3	4
Oil rate lb/hr	134	104	146	90.4
Cv oil B.t.u/hr	17,850	17850	17,850	17,850
Heat as Cv 10 ⁶ B.t.u/hr	2.3919	1.8564	2.6061	1.6136
Oil temp. °F	200	185	185	180
Sensible heat in oil B.t.u/hr	12,460	8,950	12,560	7,570
Ambient air temp. °F	86	89.6	80.6	75.2
Air flow lb/hr	2760	3127	3541	2098
Air heat content B.t.u/hr	57,000	67,000	68,500	38,000

A.1.2. Superheater.A.1.2 (a) Readings.

The averaged results taken are given overleaf. State 1 refers to inlet gas and outlet steam, state 2 to intermediate conditions and state 3 to outlet gas and inlet steam.

Table A.1.11.

STATE	1	2	3
Steam pressure psig.	24	2.35	1.5
Millivolts	8.23	-	27.1
Steam temperature °C	202	390	652
Flue gas mV	-	41.15	14.35
Flue gas temp. °C	-	995	351

Ambient temp. 0°C

Steam flow 72% of 4000 lb/hr

Gas glow, meter 4101 cu.ft/hr.

Air flow, orifice 18,026 cu.ft/hr.

Analyses.

	Towns gas	Flue gas.
CO ₂	3.0	12.0
O ₂	0.4	-
C ₄ H ₈	1.6	-
CO	19.8	1.0
CH ₄	28.2	-
H ₂	32.0	0.8
N ₂	15.2	86.2

Measured Gross Cv 504 B.t.u/cu.ft.

During the course of the trial ambient temperature fluctuated around 0°C. As the temperature fell below 0°C mist formed. The air has therefore been assumed saturated at 0°C.

A.1.2. (b) Gas analyses and calorific value.

For stoichiometric combustion:-

	mole % +	O ₂	→	CO ₂	+	H ₂ O	+	N ₂
CO ₂	3.0			3.0				
O ₂	0.4							
C ₄ H ₈	1.6	9.6		6.4		6.4		
CO	19.8	9.9		19.8				
CH ₄	28.2	56.4		28.2		56.4		
H ₂	32.0	16.0				32.0		
N ₂	15.2						15.2 + 395.1	
		91.9		57.4		94.8	410.3	
91.9 × $\frac{32}{28}$ × $\frac{79}{21}$ = 395.1								

Thus flue gas comprises,

CO ₂	57.4	2525.6 lb
H ₂ O	94.8	1706.4
N ₂	410.3	11,488.4
		15,720.4 lb

$$\text{theo. \% CO}_2 \text{ (dry basis)} = \frac{57.4}{467.7} = 12.3 \%$$

The Orsat analysis gives:

CO ₂	12.0	12.0	The CO and H ₂ figures are taken to be equal.
CO	1.0	0.9	
O ₂	-	-	
H ₂	0.8	0.9	
N ₂	86.2	86.2	

$$n_{\text{Stoi CO}_2} = \frac{n_{\text{CO}_2} + n_{\text{CO}}}{1 + 3.76 \frac{n_{\text{CO}} + n_{\text{H}_2}}{2}} = 12.6 \%$$

The agreement between the values 12.3 and 12.6 indicates that the analyses are consistent.

The excess air percentage is found from,

$$x = \frac{n_{O_2} - \frac{n_{CO} + H_2}{2}}{.266 (r_N - r_N \frac{n_{CO_2} + H_2}{2}) - (n_{O_2} - \frac{n_{CO} + H_2}{2})}$$

Where r_N is the N_2/C ratio in the fuel. Thus % excess air
 $x = -4\%$

The oxygen deficient combustion may now be calculated,

		Theo O_2	actual O_2	CO_2	H_2O	N_2	CO	H_2
CO_2	3.0			3.0				
O_2	0.4		-0.4					
C_4H_8	1.6	9.6	9.6	6.4	6.4			
CO	19.8	9.9	7.9	15.8			4.0	
BH_4	28.2	56.4	56.4	28.2	56.4			
H_2	32.0	16.0	14.0		28.0	15.2		4.0
N_2	15.2					376.2		
		91.9	87.5	53.4	90.8	391.4	4.0	4.0

$$O_2 \text{ used} = 91.9 \times 0.96 = 87.9 = 87.5 + 0.4$$

After condensing out the water these products of combustion are compared with the Orsat analysis.

		Orsat
CO_2	11.8	12.0
N_2	86.4	86.2
CO	0.9	0.9
H_2	0.9	0.9

The agreement is satisfactory.

The actual inlet air can now be found,

	moles	lb
O ₂	87.5	2,800
N ₂	376.2	10,533.6
		13,333.6 lb air

Thus lb air/100 cu.ft. fuel gas = $\frac{13,333.6}{359} = 37.1$

Humidity of air = .0036 lb/lb dry air.

Thus 2.7 moles H₂O per 100 moles of fuel gas are brought in with the air.

The apparent gross calonfic value is found from the Cvs of the pure constituents ⁷⁰ for both stoichiometric and deficient combustion.

	Cv	stoi	deficient
C ₄ H ₈	3040	1.6	1.6
CO	318	19.8	15.8
CH ₄	995	28.2	28.2
H ₂	320	32.0	28.0
	Cv	495	469

Thus the deficient Cv is $\frac{469}{495} \times 100 = 94.7\%$ of the stoichiometric value. The gross Cv of the gas was actually measured as 504 B.t.u/cu.ft. Thus the Cv for deficient combustion is taken as $504 \times .947 = 477$ B.t.u/cu.ft.

The weight % water in the flue gas is calculated as,

$$\% \text{ H}_2\text{O} = \frac{1634.4}{15,111.8} \times 100 = 10.82.$$

Thus the correction to give net Cv is,

$$\text{Cv correction} = \frac{42.1 \times .1082 \times 1055}{100} = 48$$

where 42.1 is the weight of flue gas per 100 cu.ft. fuel.
Thus the net calorific value is,

$$\text{net Cv} = 477 - 48 = 429$$

The inlet air flow is 18,867 cu.ft./hr or 1514 lb/hr.

A.1.2 (c) Gas enthalpies.

Datum 0°C, 32°F.

The gas flow is 4101 cu.ft./hr or 213 lb/hr. The flue gas flow is 1727 lb/hr. Flue gas mean specific heats are,

$$0 - 350^{\circ}\text{C} \quad \text{Cp} = .273 \quad \text{B.t.u/lb } ^{\circ}\text{F}$$

$$0 - 1000^{\circ}\text{C} \quad \text{Cp} = .296 \quad "$$

$$\text{Cv inlet heat } 10^6 \text{ B.t.u/hr} = 1.7593$$

Sensible heat with

$$\text{inlet gas } 10^6 \text{ B.t.u/hr} = .0194$$

$$\text{Total inlet heat } 10^6 \text{ B.t.u/hr} = \underline{1.7787}$$

The enthalpy leaving the superheater is given by,

$$1727 \times .296 \times 1791 = 0.9152 \times 10^6 \text{ B.t.u/hr.}$$

The enthalpy leaving the preheater is given by,

$$1727 \times .273 \times 632 = 0.2983 \times 10^6 \text{ B.t.u/hr.}$$

A.1.2. (d) Steam enthalpies.

State	T°F	Psig.	H B.t.u/lb	flow lb/hr
in preheater	396	24	1234	2880
ex. preheater	734	2.35	1400	2880
ex. superheater.	1206	1.5	1647	2880

$$\text{Enthalpy gain in preheater} = 0.4781 \times 10^6 \text{ B.t.u/hr}$$

$$\text{Enthalpy gain in superheater} = 0.7114 \times 10^6 \text{ B.t.u/hr}$$

A.1.2 (e) Heat losses.

As there was a considerable wind blowing, estimated as 15 m.p.h, the method for calculating heat losses is that due to Nelson ⁵⁹. He presents charts based on an equation of the form:

$$h \begin{matrix} \text{(convection} \\ \text{+ radiation)} \end{matrix} = 1.5 + .00058 (\Delta T)$$

with different lines for different wind speeds. The still air line gives results which agree well with the standard equations.⁵⁸ On this basis the following table was drawn up.

Table A.1.12. Heat losses.

ITEM	A sq.ft.	T °F	h_{nelson} B.t.u./°F ft ²	heat loss B.t.u/hr
Preheater	93	36	4.6	15,400
"	4	45	4.7	850
"	20	108	5.1	11,000
Cooler	41	36	4.6	6,800
"	34	54	4.75	8,700
"	12	72	4.85	4,200
"	15	90	5.0	6,750
Ducting	30	54	4.75	5,100
"	40	72	4.85	20,950
Total preheater				79,800
Superheater bottom	30	90	5.0	13,500
" body	15	108	5.1	8,300
" "	20	72	4.85	7,000
" "	30	18	4.5	2,400

continued.....

Table A.1.12 continued.

ITEM	A sq.ft.	T °F	h _{nelson} B.t.u/hr °F	heat loss B.t.u/hr
Superheater body	100	54	4.75	25,650
Pipework	30	72	4.85	10,500
Superheater top	24	90	5.0	10,800
" "	20	45	4.7	4,200
" "	23	54	4.75	5,900
Burner mounting	6	234	6.0	8,400
" "	4	288	6.4	7,400
" "	4.5	414	7.3	13,600
Total superheater				117,600
Total heat loss				197,400

Appendix 2.

Equations to find absorption strength $k\delta$ and weighting factor a .

A.21 Grey gas-clear gas mixture.

If the emissivity - v - path length curve is fitted at L (mean beam length) and $2L$ then,

$$\epsilon_L = a(1 - e^{-kL}) \quad A.2.1$$

$$\epsilon_{2L} = a(1 - e^{-2kL}) \quad A.2.2$$

Dividing equation A.2.2 by A.2.1 gives,

$$\frac{\epsilon_{2L}}{\epsilon_L} = 1 + e^{-kL}$$

$$\text{thus } e^{-kL} = \frac{\epsilon_{2L}}{\epsilon_L} - 1 = \frac{\epsilon_{2L} - \epsilon_L}{\epsilon_L}$$

$$\text{whence } e^{kL} = \frac{\epsilon_L}{\epsilon_{2L} - \epsilon_L}$$

$$\text{thus } kL = \ln \frac{\epsilon_L}{\epsilon_{2L} - \epsilon_L}$$

$$\therefore k\delta = \frac{1}{\psi} \frac{\ln \epsilon_L}{\epsilon_{2L} - \epsilon_L} \quad A.2.3$$

From equation A.2.1

$$a = \frac{\epsilon_L}{1 - e^{-kL}} = \frac{\epsilon_L}{2 - (1 + e^{-kL})}$$

Multiplying top and bottom by ϵ_L ,

$$a = \frac{\epsilon_L^2}{2\epsilon_L - a(1 + e^{-kL})(1 - e^{-kL})}$$

$$\text{thus } a = \frac{\epsilon_L^2}{2\epsilon_L - a(1 - e^{-2kL})}$$

$$a = \frac{\epsilon_L^2}{2\epsilon_L - \epsilon_L^2} \quad \text{A.2.4}$$

A.2.2 Two grey gas - one clear gas mixture.

In this case the emissivity - v - path length curve can be fitted at L , $2L$, $3L$ and $4L$. The equations linking emissivity with absorption coefficient, are then,

$$\epsilon_L = a_1 (1 - e^{-k_1 L}) + a_2 (1 - e^{-k_2 L}) \quad \text{A.2.5}$$

$$\epsilon_{2L} = a_1 (1 - e^{-2k_1 L}) + a_2 (1 - e^{-2k_2 L}) \quad \text{A.2.6}$$

$$\epsilon_{3L} = a_1 (1 - e^{-3k_1 L}) + a_2 (1 - e^{-3k_2 L}) \quad \text{A.2.7}$$

$$\epsilon_{4L} = a_1 (1 - e^{-4k_1 L}) + a_2 (1 - e^{-4k_2 L}) \quad \text{A.2.8}$$

By elimination of the unknowns a_1 , a_2 and $e^{-k_2 L}$ these equations can be reduced to the quadratic:-

$$\begin{aligned} & e^{-2k_1 L} (\epsilon_{3L}^2 + \epsilon_L^2 - \epsilon_L \epsilon_{3L} - \epsilon_L \epsilon_{2L}) \\ & + e^{-k_1 L} (\epsilon_{2L}^2 + \epsilon_L \epsilon_{4L} - \epsilon_{2L} \epsilon_{3L} - \epsilon_L \epsilon_{2L}) \\ & + (\epsilon_{2L}^2 + \epsilon_{4L}^2 + \epsilon_L \epsilon_{4L} - \epsilon_L \epsilon_{3L} - \epsilon_{2L} \epsilon_{4L} - \epsilon_{2L} \epsilon_{3L}) = 0 \quad \text{A.2.9} \end{aligned}$$

The value of k_1 can be obtained from the standard solution of a quadratic equation. The other unknowns can then be found by substitution for k_1 and elimination.

NOMENCLATURE.

The nomenclature systems employed in original references have been adopted throughout. Where relevant the number of the equation in which a symbol first appears is given.

A	Area
A'	Effective area, eqn. 1.3.
A_{c_p}	Cold plane area, eqns. 1.3. & 1.5.
A_R	Refractory area, eqn. 1.3.
A_T	Total furnace area, eqn. 6.7.
a	Weighting factor, eqn. 4.12.
a	Slope of $\log E - v - \log PL$ curve, eqn. 4.10.
a	Temperature ratio, eqn. 1.9.
B	Difference between mean radiating and gas exit temperatures, eqn. 1.11.
b	Slope of $\log E - v - \log T$ curve, eqn. 4.10.
C	Fraction of furnace area cold, eqn. 6.7.
C	Fuel rate, lbs. fuel per hour per sq. ft. water cooled surface, eqn. 1.1.
C	Composition of gas, eqn. 1.18.
C_o	Fuel rate, lbs. equivalent good bituminous coal per hour per sq. ft. water cooled surface, eqn. 1.2.
C_p	Specific heat.
C_v	Calorific value.
c	Natural convection factor, eqn. A.1.1.

α	Exponent on temperature ratio in obtaining α from ϵ , eqn. 4.10.
D	Pipe diameter.
d	Diameter, eqn. A.1.1.
E	Effectiveness factor, eqn. 1.4.
E	Emission rate, energy per unit time.
E_T	Total emission, eqn. 1.12.
e	Subscript denoting end or end zone.
$\overline{\epsilon\epsilon}$	$\left. \begin{array}{l} \text{End-end} \\ \text{End-gas} \\ \text{End-surface} \end{array} \right\} \text{ direct interchange areas.}$
$\overline{\epsilon g}$	
$\overline{\epsilon s}$	
F	Factor, eqn. 7.1.
F	Elemental reception factors, Tables 3.2 & 3.3.
$F_{i,j}$	View factor, eqns. 1.20 & 1.21.
F_{RC}	View factor, eqn. 1.8.
F_S	Factor, eqn. 1.8.
f	Reception factor, Table 3.1.
f	Function, eqns. 1.18 & 1.19.
G	Air/fuel ratio, eqns. 1.1., 1.2. & 1.3.
G	Mass flow rate, eqn. 5.1.
$\overline{GG}$	$\left. \begin{array}{l} \text{Gas-gas} \\ \text{Gas-surface} \end{array} \right\} \text{ total interchange areas.}$
$\overline{GS}$	
g	Subscript denoting gas or gas zone.
$\overline{ge}$	$\left. \begin{array}{l} \text{Gas-end} \\ \text{Gas-gas} \\ \text{Gas-surface} \end{array} \right\} \text{ direct interchange areas.}$
$\overline{gg}$	
$\overline{gs}$	
H	Enthalpy.

H_i	Input enthalpy.
h	Combination of variables $\frac{z_2 - z_1}{\delta}$ giving normalised separation, Tables 3.2. & 3.3.
h_c	Convective transfer coefficient.
I	Intensity of radiation.
I_1, I_2	Integrals, eqns. 3.21. & 3.27.
i, j	Subscripts denoting general case.
K'	Factor defined by eqn. 4.10.
k	Absorption coefficient.
$k\delta$	Absorption strength.
L	Mean beam length.
M	Mass flow rate.
$MH_{i,m}$	Input enthalpy.
m, n	Subscripts denoting general case.
N	Number of gas zones.
n	Number of unit volumes, eqn. 4.4.
n_{O_2} etc.	Number moles O_2 .
P	Pressure.
P_G	Partial pressure of gas G.
P_T	Total pressure.
p_c	Cold plane emissivity, eqn. 1.8.
p_f	'Flame' emissivity, eqn. 1.8.
Q	Net heating value of fuel fired, eqn. 1.3.
Q	Heat transfer by natural convection, eqn. A.1.1.

q	Heat transferred.
q_c	Heat transfer by convection.
q_e	Heat transfer by enthalpy change.
q_r	Heat transfer by radiation.
q_s	Heat transfer to sink.
q_{sm}	Measured heat transfer to sink.
R	Separating distance between elements, Fig. 3.1.
$s_i^R s_j$	Reflected flux, eqn. 4.22.
r	Radius, Fig. 3.1.
r_n	N_2/O ratio in fuel.
$\overline{SG}$ $\overline{SS}$	Surface-gas Surface-surface } total interchange areas.
s	Subscript denoting surface or surface zone.
$\overline{se}$ $\overline{sg}$ $\overline{ss}$	Surface-end Surface-gas Surface-surface } direct interchange areas.
T	Temperature.
T_a	Ambient temperature.
T_{AF}	Pseudo-adiabatic flame temperature assuming no dissociation, eqn. 1.10.
T_c	Temperature of combustion products leaving chamber, eqn. 1.4.
T_o	Outlet temperature, eqn. 7.6.
T_o	Datum temperature, eqn. 1.10.
t	Time.
V	Volume.
W	Emissive power = σT^4

x	General variable, Table 3.3.
x	Percentage excess air in section A.1.2.
x	Path length, eqn. 1.14. and Appendix 2.
y	Path length in Appendix 2.
z	Height variable, Fig. 3.1.
z/δ	Normalised height variable.
α	Whole number describing separation between elements, eqn. 3.3.
α	Effective view factor, eqn. 1.5.
α	Absorptivity.
β	Effectiveness factor for refractory surface, eqn. 1.3.
γ	Convective flux coefficient defined by eqn. 5.2.
δ	Enclosure diameter.
Δ	Difference.
ϵ	Emissivity.
η	Enthalpy flux coefficient defined by eqn. 5.1.
θ	Angle, Fig. 3.1.
θ	Temperature, eqn. A.1.1.
λ	Wavelength.
μ	Refractive index.
μ	Fraction of heat input above steam temperature absorbed by sink, eqn. 1.1.
μ	Fraction of total heat above atmospheric temperature absorbed by sink, eqn. 1.2.

μ	Fraction of heat available above 60°F absorbed by sink, 1.3.
ρ	Reflectivity.
σ	Stephan-Boltzmann constant.
τ	Transmissivity.
ϕ	Angle, Fig. 3.1.
ϕ	Factor defined by eqn. 1.7.
ψ	Shape factor, eqn. 1.17.
ω	Solid angle.

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