Uncertainty Quantification and Race Car Aerodynamics

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ABSTRACT

Car aerodynamics is subjected to a number of random variables which introduce uncertainty into the down-force performance. These can include, but are not limited to pitch and ride height variations. Studying the effect of the random variations of these parameters is important to predict accurately the car performance during the race. Despite their importance the assessment of these variations is difficult and it cannot be performed with a deterministic approach.

In the open literature there are no studies dealing with this uncertainty in car racing aerodynamics modelling the complete car and assessing the probability of a competitive advantage introduced by a new geometry. A stochastic method is used in this work in order to predict the car down-force under stochastic variations and the probability to obtain better performance with a new diffuser geometry. A Probabilistic Collocation Method (PCM) has been applied to an innovative diffuser design to prove its performance with stochastic geometrical variations.

The analysis is conducted using a complete three dimensional CFD simulation with a k-ω turbulence closure, allowing the performance of the physical diffuser to be more accurately represented in a stochastic real environment. The random variables included in the analysis were pitch and ride height

variations at different speed conditions. The mean value and the standard deviation of the car down-force have been evaluated.

INTRODUCTION

The study of aerodynamic components for race cars demands that the performance is of a significant magnitude before implementation is deemed justified. Typical design of aerodynamic parts, such as diffusers, is often achieved using deterministic CFD simulations. In the case of Jensen [1] multiple deterministic cases such as in sideslip and roll were presented for an under-tray design. In this case [1] the CFD result was at least 50% greater than the actual down-force achieved. Unlike the reality, deterministic simulations do not take into account the probabilistic likelihood of the off-design geometrical variations. During the development of a part or product which is dependent upon complex fluid flows such as, in this case, the design of a diffuser for a racing car, there are a number of sources of error in the analytical stages of design. When concerning numerical simulations, errors arise from geometrical variations [2]. These errors are random and only stochastic methodologies, known as Uncertainty Quantification methods [3], can be used to account their effects. These methods have been developed in the aerospace sector to evaluate the impact of manufacturing errors on the life of the components [4-5].

A number of stochastic methods maybe currently used, the first of which includes applying a random input upon a number of simulations, as in the Monte-Carlo Method (MCM). This approach is accurate and robust directly simulating the stochastic nature of the varying inputs [6]. This approach is however very computationally demanding and time consuming. In order to obtain the probability distribution of CFD simulations with a reasonable computational time several alternative methods have been proposed in literature.

In order to study the stochastic nature of a turbine blade, D'Ammaro and Montomoli [7] have shown that by adapting the standard MCM to a Monte Carlo Method Lattice Sampling (MCMLS) the computation was two orders of magnitude less demanding whilst still producing accurate results. Furthermore using a Probabilistic Collocation Method (PCM) the result showed negligible error over the MCM approach, and was achieved with three orders of magnitude less computational time. The suitability of PCM is further acknowledged as suitable for flow uncertainty by Shi et al. [8].

Whilst reviewing alternative uncertainty quantification methods such interval analysis, sensitivity derivative, moment methods, Walters and Huyse [9] confirm that polynomial chaos approaches such as PCM provide results which comply with those of MCM despite the reduced computational effort. As a result of this finding, the PCM approach will be employed during this work.

Unlike typical uncertainty, or error quantification, this work aims not to bound the error based upon error in the input (numerical or simulation errors), but aims to characterise the error as a result of the stochastic nature of the system. That is, the variations in the scenario which are likely to occur and in this case the car set up [10].

In the open literature there are only two works related to car racing and aerodynamics, [11] and [12]. However in [11] the author analysed the impact of uncertainty on the aerodynamics of a single tire in isolation.

The present work is the first one to use UQ to study a complete car. This paper is based on the MSc thesis of the Bradford [12], under the supervision of Montomoli [12], for the design of the University of Surrey Formula Student car. In this paper a complete car has been analysed, considering as random variables, which are to be subjected to a stochastic variation, the pitch and ride height of the car. These variations will be used to obtain a probabilistic distribution for the down-force of the car. In the authors knowledge this is the first time a stochastic method has been applied to a race car diffuser design in this manner and to the overall aerodynamic of the vehicle. At the same time it is shown that UQ techniques maybe used to obtain the probability that new configuration, the new diffuser design, has better performance than the original one.

CFD METHODOLOGY

Meshing was carried out using ICEM CFD using a tetrahedral octree approach. Using a density feature the tetrahedral cells near the car were refined and expanded at a ratio of 1.2. Y+ values were checked for the mesh and were found to average at 40 which were sufficient for use with enhanced wall treatment. The surface mesh for the car is shown in figure 1.

Significant in the flow around a car, and particularly in the study of a diffuser, are the rotating wheels and the moving floor. These features were included in the simulation. The floor was set as a moving boundary at the same velocity as the free stream inlet, as shown in figure 2. Figure 2 shows the axes of rotating wheels, the street movement and the definition of the car height and pitch. The lateral surfaces and the top of the domain are defined as free stream conditions. The layout to simulate the moving street and the rotating wheel has been obtained from the work of Ponzi [13] that analysed the performance of a rear diffuser of a sport car.

The domain of the simulation was made sufficiently large as to ensure that there were no significant interactions between the car and the walls of the simulation. Dimensions for the domain are shown in figure 2, in terms of car lengths. The inlet condition is a velocity inlet where the flow is directly axially. Air at standard atmospheric pressure of 101325Pa is used as reference pressure.

To ensure the tires do not contact the floor tangentially and to reproduce the wheel deformation with contact to the ground, a small amount of cut-off was made to ensure a larger contact area, as common practice in car CFD [13]. The CFD solver used was CFX with a standard k-ω sst model and wall function.

Uncertainty Quantification: Probabilistic Collocation Method

In this work two variables are assumed affected by random variations: the pitch angle and the ride height, as shown in figure 2. The pitch of the car has been defined as rotation through the centre of mass of the vehicle. These parameters are subjects to random variations and are known only with a certain degree of accuracy. The question is how to obtain the downforce is these parameters are affected by uncertainty?

In literature a simple method to answer this question is to evaluate the downforce using a Monte Carlo method. Thousands of simulations must be carried out in order to obtain an accurate output with different variations of pitch and ride height. By increasing the number of the simulations, the stochastic representation of the output improves. This method is accurate but computationally prohibitive.

For this reason a collocation method has been chosen because it offers the possibility to use a small number of simulations to represent the stochastic output, in this case the Probabilistic Collocation Method (PCM). The basic idea of collocation methods is to use a finite number of simulations to represent the stochastic output (that we will call y(x,ξ), where ξ is the standardised input random variable and x the position in the domain). In order to use a finite number of data, we must make some assumptions on the shape of the probability output. In this way with ad hoc input values, we can use a finite number of simulations to evaluate the output statistics. It is the same problem of fitting a polynomial with a specified number of points. The only difference is that the input points follow a probability distribution. Using the PCM as described in [3 and 14], the stochastic output, $y(\boldsymbol{x},\boldsymbol{\xi})$, is a combination of the coefficients $a(\boldsymbol{x})$ and the multi-dimensional polynomials $\Psi_j(\boldsymbol{\xi})$, which are function of the standardized input random variable $\boldsymbol{\xi}$.

$$y(\boldsymbol{x},\xi) = \sum_{i=0}^{P} a_i(\boldsymbol{x})\psi_i(\xi)$$

Where

$$P = \frac{(n+d)!}{n!\,d!}$$

Where n is the number of independent random variables and d is the degree of polynomial expansion. In this case there are two independent random variables, pitch and ride height and a second order polynomial is chosen.

When determining mean down-force value, by treating a number of parameters as random variables, such as changes in pitch angle and ride height, the data points can be used to create an field fit by a Gaussian distribution [15]. Hermite polynomials are used to represent $\psi_i(\xi)$, as they are orthogonal with zero mean.

This can be written, to second order:

$$y(\boldsymbol{x}, \xi_1, \xi_2) = a_0 + a_1(\boldsymbol{x})\xi_1 + a_2(\boldsymbol{x})\xi_2 + a_3(\boldsymbol{x})\xi_1\xi_2 + a_4(\boldsymbol{x})\left({\xi_1}^2 - 1\right) + a_5(\boldsymbol{x})\left({\xi_2}^2 - 1\right)$$

$$\psi_1(\xi) = \xi_1$$

$$\psi_2(\xi) = \xi_2$$

$$\psi_3(\xi) = \xi_1\xi_2$$

$$\psi_4(\xi) = {\xi_1}^2 - 1$$

$$\psi_5(\xi) = {\xi_2}^2 - 1$$

Where y is the output, in this example downforce value, x is the random variable and ξ is the stochastic part of the input variable x. After the samples, or collocation points are determined by simulation results, the next stage is to evaluate the deterministic coefficients a_i. The deterministic coefficients however are independent of the problem in question and can therefore be used for a number of similar scenarios [16].

The deterministic coefficients can be found by solving the following [17]:

$$\begin{bmatrix} \psi_0(\xi_{i,0}) & \cdots & \psi_P(\xi_{i,0}) \\ \vdots & \ddots & \vdots \\ \psi_0(\xi_{i,P}) & \cdots & \psi_P(\xi_{i,P}) \end{bmatrix} \begin{pmatrix} a_{i,0} \\ \vdots \\ a_{i,P} \end{pmatrix} = \begin{pmatrix} y_{i,0} \\ \vdots \\ y_{i,P} \end{pmatrix}$$

These were applied to the sampled results and to determine a mean value the following formula was used to find the mean downforce value:

$$\mu = \frac{\sum Wy}{\sum W}$$

As the minimum number of collocation points required is 6 for this problem the sum was normalised by the sum of the weighting for the appropriate points.

The model used in this paper has been fully validated in [7, 5,14]. The code has been validated against Monte Carlo Simulations in several applications, from multi-physics problems [5] to turbulence closures uncertainty [14] and geometrical uncertainty [4].

Table 1 shows the variations which were considered in this work and the number of simulations required to build the stochastic space:

Table 1 Car ride variation test cases

Test [-]	Pitch angle variation [deg]	Ride height variation [mm]
1	0	0
2	0	20
3	0	-20
4	1	0
5	-1	0
6	1	20
7	-1	20

This allows a two dimensional stochastic space as input when considering pitch angle and ride height as random variables. The seven simulations are used by the PCM method to obtain the mean value and the standard deviation of the real car under random variations of the pitch angle and ride height. The basic idea of the methodology explained above is to obtain the performance of the car under stochastic variations as weighted average of the solution in the collocation points, the seven CFD simulations.

The impact of car ride variations has been evaluated at different car velocities, v=10-30 m/s. The velocity has not been considered as random parameter even if this can be done. The present work focuses only on the asset variation of the car. In terms of Reynolds number based on car length, the car is in the range Reynolds=$1.51x10^6$, $3.03 \, x10^6$, $4.54 \, x10^6$

RESULTS

The overall goal of this work is to show the benefits of the new diffuser design in comparison with the clear configuration. This has been done with a standard “deterministic comparison” and considering the random variations, Uncertainty Quantification comparison. As random variations, Gauss distributions for the pitch angle and the car height have been used. There are no experimental data available to define the input PDF and the choice of the Gauss PDF is arbitrary. However similar methods have been used by the same authors [5] to use as input experimental measured distributions.

The Uncertainty Quantification comparison gives to the designer the probability to achieve higher downforce under random variations of the car position. Moreover a scenario analysis has been carried out to evaluate the probability to obtain a better performance in acceleration, skid pad test, sprint and endurance.

The clean car configuration is the original car without under-tray and diffuser, as shown in figure 2.

Down-Force comparison: Deterministic Comparison vs Uncertainty Quantification

1. **Deterministic comparison**

The primary objective of the diffuser design was the implementation of down-force on the car, or the negation of lift. In the deterministic case the baseline pressure distributions along the centreline of the car, for a velocity of v=10m/s, are shown in figure 3. The bottom graph shows the pressure coefficient distribution on the midplane of the car. The bottom silhouette is the section of the car with the same cutting plane. The arrows indicates the region where the force on the car is positive or negative. Starting from the front, the pressure on the bottom part is higher than the top one, generating lift. Despite a small acceleration that generates downforce all the front part of the car is producing lift. At y=0.5 the flow start accelerating and the rear part of the car, despite the small acceleration induced by the pilot helmet, is producing a small downforce. In the baseline configuration, shown in figure 3, it is possible to observe that most of the downforce is generated by the safety structure behind the pilot helmet.

Starting with this distribution, the car has been modified with a wide undertray area and two lateral diffuser. The pressure distribution of this new configuration and the local pressure are shown respectively in figure 4 and 5. In both cases the velocity is 10 m/s.

Comparing figure 3 and 4, it is evident the low pressure region as a result of the diffuser section between 0.76 and 1.9 metres in longitudinal position. During this position the lower surface exhibits a lower pressure than the corresponding upper surface. This difference is responsible for a negative lift.

At y=0.76 there is a small spike induced by the under-tray blockage. The effect is small if compared to the beneficial effect induced by the new configuration. For about 50% of the car length there is clear region of downforce.

Figure 5 shows more in detail the pressure distribution under the car at a velocity of 10 m/s. As expected from figure 3 and 4, the axial extention of the low pressure region is bigger with the new geometry, left side. At the same time it is possible to observe that this region is relatively wide and uniform, highlighted by the dashed isocontour line.

Similarly the results for the car with and without the diffuser at 20 and 30 m/s are shown in the same manner in Figure 6 and Figure 7. This shows a significantly decreased pressure distribution along the lower surface of the car maintaining the upper part almost unchanged. Increasing the velocity the overall downforce increases but the pressure coefficient distribution is almost the same.

This is not surprising because the Reynolds number range is 1.51×10^{6}-4.54×10^{6} and the flow is fully turbulent. For this Reynolds number range it is not expected any variation of the Cp.

In dimensional form, the variation in lift force as a function of velocity is shown in figure 8 which allows comparison to the clean car configuration. Figure 8 shows three lines, the green line indicates the clean case lift for different velocities, the blue line the case with the diffuser and the red line the increment of downforce changing the Reynolds number. The standard deterministic comparison confirms that the new design is able to improve the downforce for the range of velocity considered, v=10-30m/s.

Figure 9 shows the deterministic effect of the variation of the pitch on the pressure distribution. On the left side there is the car with the diffuser with pitch=0deg, on the right half of the car with pitch=-1deg and half with pitch=1deg. By changing the pitch the overall pressure changes and the effect is evident. However it is difficult to know exactly the car pitch and/or the car height. For this reason in the following paragraph these data will be assumed as random distribution.

2. Uncertainty Quantification Comparison

By using PCM is possible to evaluate the mean value of car downforce under the influence of random variations for the pitch and the height. As a result of this, performance parameters (downforce, centre of pressure position) can have the PCM uncertainty quantification applied to give more representative mean values which encompass the variations of random variables.
Table 2 shows the comparison of the downforce (negative lift) for various velocities between a deterministic and a stochastic CFD simulation (mean value) for the case of the car with the diffuser (as reference there is also the baseline configuration).

At the same time the probability to have a downforce higher with the new configuration than with the old one has been evaluated. The probability to have higher downforce with the new configuration is between 90 to 94%.

For the stochastic case is also possible to define the confidence region with 68.2% of confidence (1 σ). The same region is shown in figure 10

	Velocity [m/s]		
	10	**20**	**30**
	Reynolds number		
	1.51×10^6	**3.03×10^6**	**4.54×10^6**
L Clean car output [N]	-12.90	-65.90	-195.00
L Deterministic [N]	-14.77	-62.97	-192.06
L Stochastic, UQ, mean [N]	-3.83	-21.86	-43.19
L Stochastic, UQ, σ [N]	8.57	29.63	96.50
% Stochastic - Deterministic (mean)	14.51%	4.44%	1.51%
% Probability to have higher down-force with the new design	90%	92%	94%

Table 2 Difference in the lift between deterministic and stochastic outputs

Performance increase and competition scenario analysis

To determine the viability of the diffuser a scenario based test was simulated relative to the original baseline car. There are a number of events over which the performance can be compared. This section aims to give an overview of these events and subsequently determine a value for the relative performance increase. Four different conditions are analysed: skid pad, acceleration, sprint and endurance.

1. Skid pad: The skid pan test involves two concentric circular track sections. Two consecutive laps are carried out on each circle before transition to the other circle. The layout is shown in Figure 11
2. Acceleration: The acceleration course is a 75 metre straight section of track. This is known prior to the event.
3. Sprint and endurance: Simulation of the sprint and endurance event poses an issue as the details of the track layout change with each event. Furthermore knowledge of the layout prior to the event is unknown. Based on previous events and cars running with GPS loggers (FSAE Sim) there are however courses which can be used. The course layout from 2011 Formula SAE West competition is shown in Figure 12.

Using a simulator from Formula SAE [19] the results provide an approximation to the percentage increase expected from a diffuser. The values outputted from the simulator feature a varied aerodynamic down-force input, with all else held constant. The results are shown in Table 3.

Time, seconds	Acceleration	Skid Pad	Sprint	Endurance
μ	4.98	5.206	78.684	1550.888
μ+σ	4.98	5.238	79.069	1558.579
μ−σ	4.97	5.162	78.280	1542.801
Clean Car	4.97	5.281	79.567	1568.534
Probability less time with New Design	15%	97%	96%	98%

Time change from clean car, seconds	Acceleration	Skid Pad	Sprint	Endurance
μ	0.01	-0.075	-0.883	-17.646
μ+σ	0.01	-0.043	-0.498	-9.955
μ−σ	0.0	-0.119	-1.287	-25.733

% Time change from clean car	Acceleration	Skid Pad	Sprint	Endurance
μ	0.20%	-1.42%	-1.11%	-1.12%
μ+σ	0.20%	-0.81%	-0.63%	-0.63%
μ−σ	0.00%	-2.25%	-1.62%	-1.64%

Time, seconds	Acceleration	Skid Pad	Sprint	Endurance
μ	4.98	5.206	78.684	1550.888
Deterministic	4.97	5.281	79.567	1568.534
% Time Change	0.20%	-1.42%	-1.11%	-1.12%

Table 3 Event predictions

This shows a performance increase in each event albeit the acceleration event. The greatest percentage time improvement was the skid pad, followed by endurance and sprint.

At the same time the probability to obtain better performance for each specific scenario with the new geometry have been calculated. Excluding the acceleration test, where the greater weigth of the diffuser affects the results, the probability in all the other tests to have a better time is between 96 to 98%.

Taking the mean of the percentage time change across all the events this yields the mean percentage time difference as a direct result of applying the diffuser to the car. The resulting value is shown in

Table 3. Over a representative endurance circuit the proposed diffuser improves the car time by between 0.6 and 1.4%, with a mean of 1.12%. Across all the events this equates to a mean time change of -0.86%. Within one standard deviation this puts the result between -1.33 and -0.52%.

The mean result from the stochastic approach can be compared to the result which would otherwise be obtained using deterministic methods. This can be compared in terms of event times.

CONCLUSIONS

In authors' knowledge this is the first application of stochastic methods to race car aerodynamics.
A new diffuser has been designed for a sport car and the improved performance on the car has been proved for different velocities. By using an Uncertainty Quantification method, the mean downforce value has been predicted considering the uncertainty in the car set-up. By using a probabilistic collocation method the confidence level of the simulation can be evaluated if the set-up is affected by uncertainty. The probability that the new design would give better performance than the original one has been also evaluated.
Using this representative mean downforce value, considering random variations in pitch and ride height, the track performance can be evaluated both relative to the clean car configuration, and the result which would arise if the stochastic approach had not been used.
The results have been used to evaluate the time change in different race scenarios. A probabilistic CFD simulation has been used to predict the loop time and the accuracy of this result due to the uncertainty in the car configuration. Even considering only the mean value, the time difference between stochastic and deterministic values is about 1% for most of the scenarios analysed with a level of confidence between 96 to 98%. This difference, small in absolute terms, is enough to win or lose a car racing competition.

NOMENCLATURE

d	Degree of the polynomial
nv	Number of variables
L	Down-force

PCM	Probabilistic Collocation Methods
UQ	Uncertainty Quantification
x	Random variable
μ	Mean value
σ	Standard Deviation
$\tilde{Y}$	Polynomial approximation of a generic f
H_i	Orthogonal base of Stochastic space
a_i	Generic terms of Hi
ξ	Vector defining random inputs
$aj(\boldsymbol{x})$	Deterministic coefficients PCM
$\Psi_j(\boldsymbol{\xi})$	Multi-dimensional orthogonal polynomials
$w_j(\boldsymbol{\xi})$	Quadrature weights

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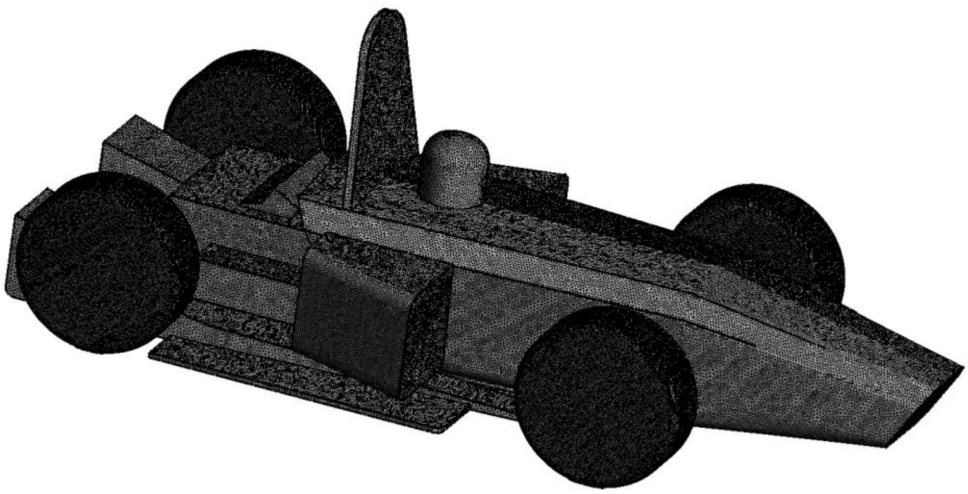

Figure 1 Surface mesh image over car with the proposed diffuser and under-tray

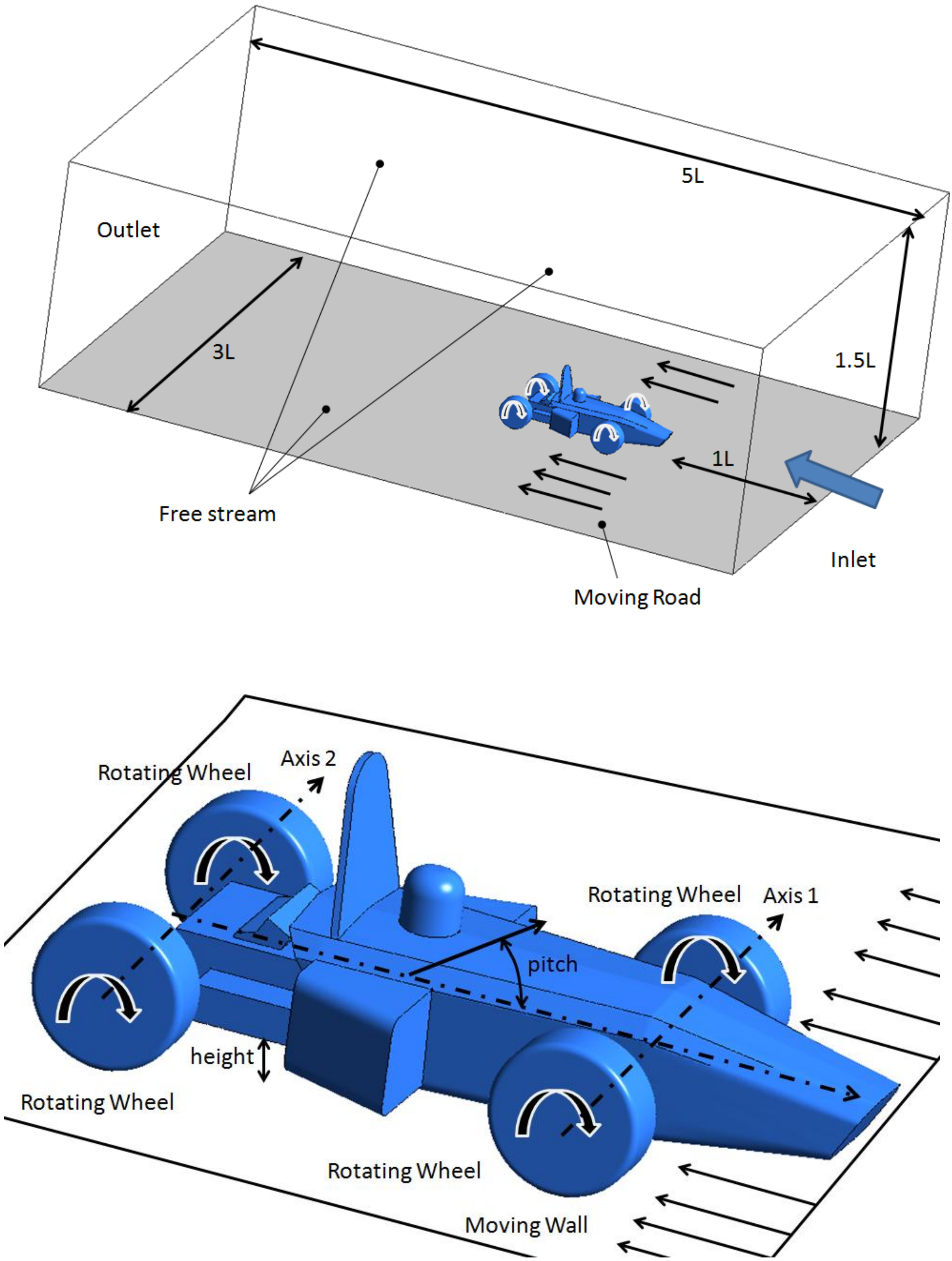

Figure 2 Schematic view of the computational domain with dimensions in terms of car lengths for the clean geometry. Layout for the rotating wheel similar to Ponzi [13]

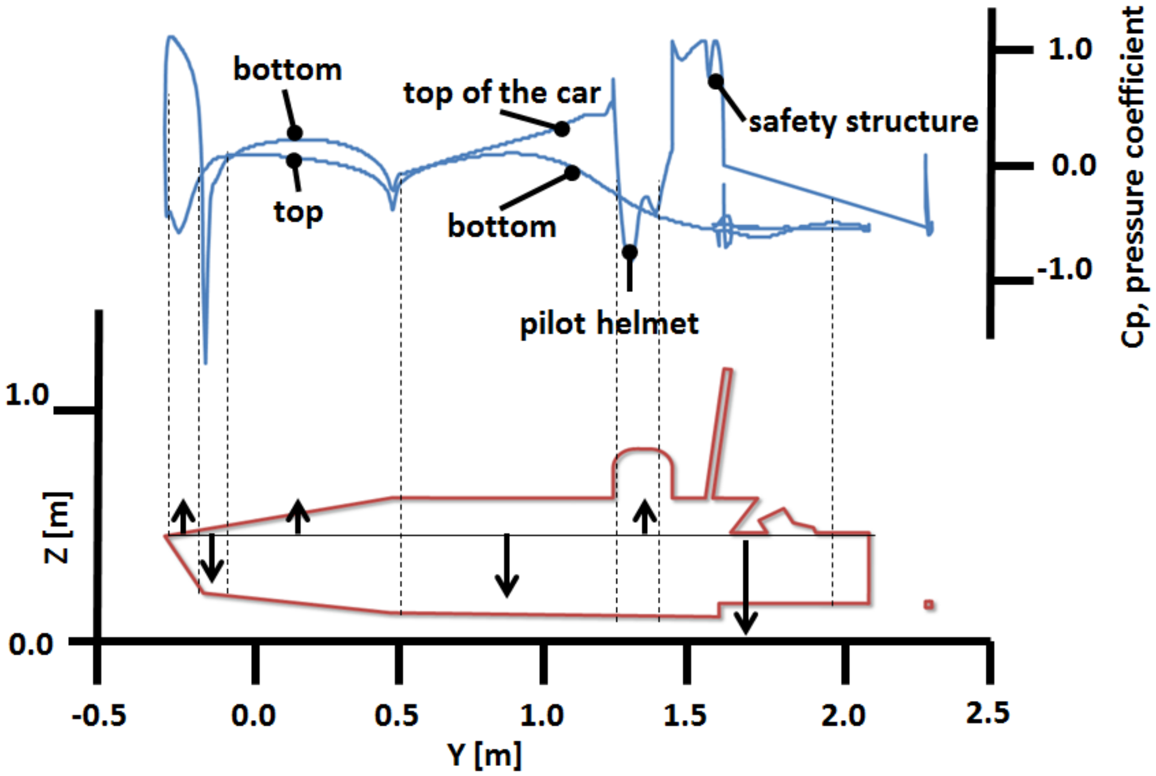

Figure 3 Baseline car pressure distribution, v=10m/s, Reynolds=1.51x10^6 , along mid-plane.

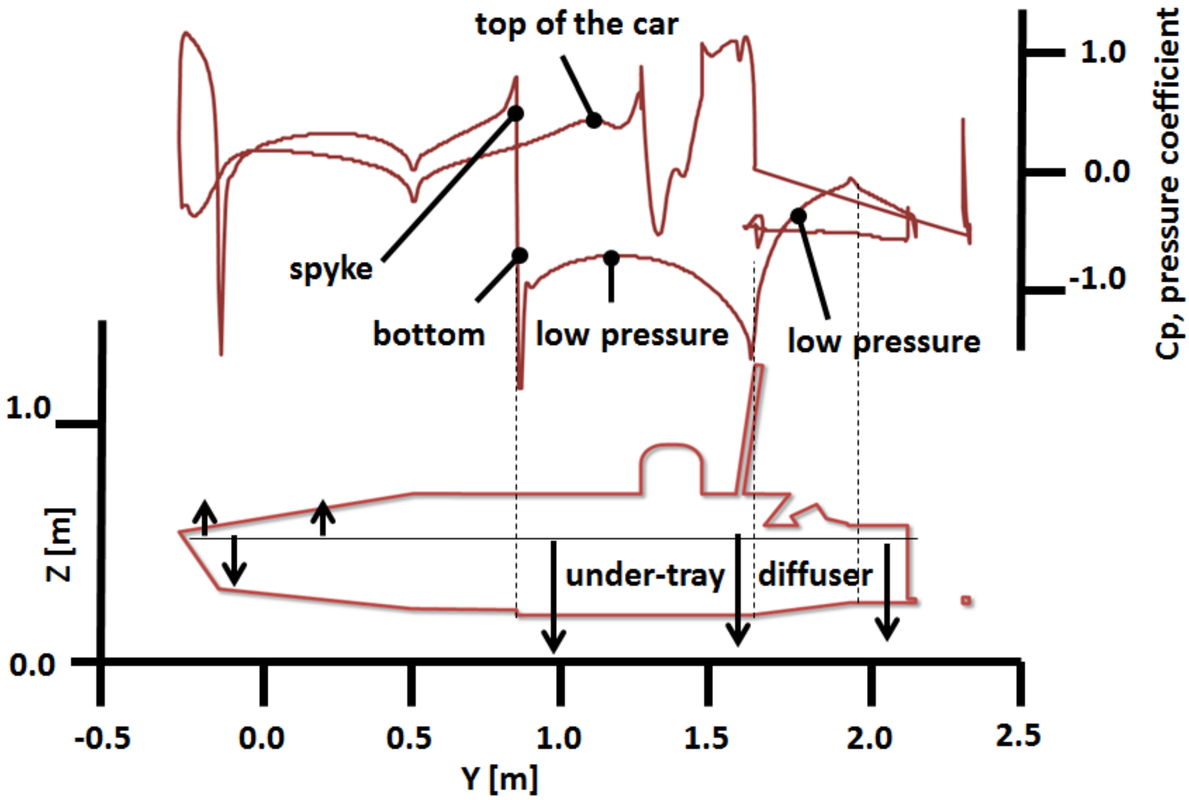

Figure 4 Diffuser car distribution, v=10m/s, Reynolds=1.51x10^{6}along mid-plane.

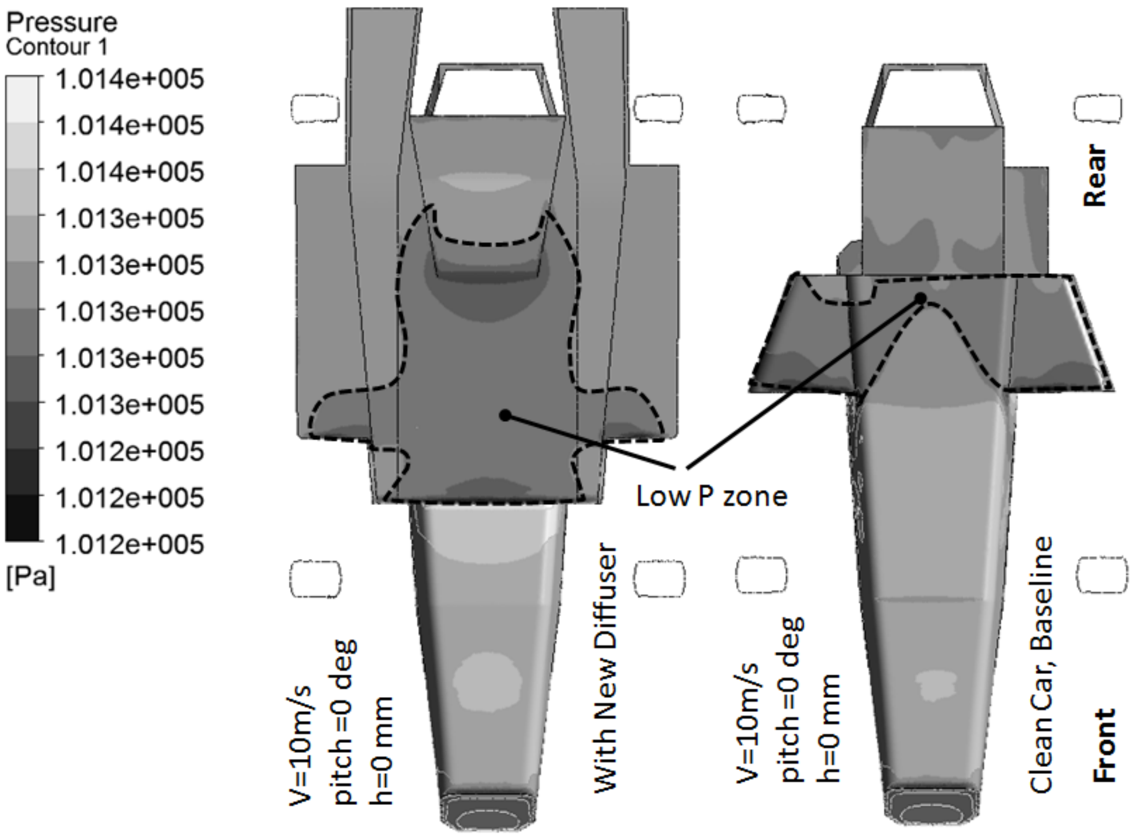

Figure5 Diffuser impact on car aerodynamics, v=10m/s

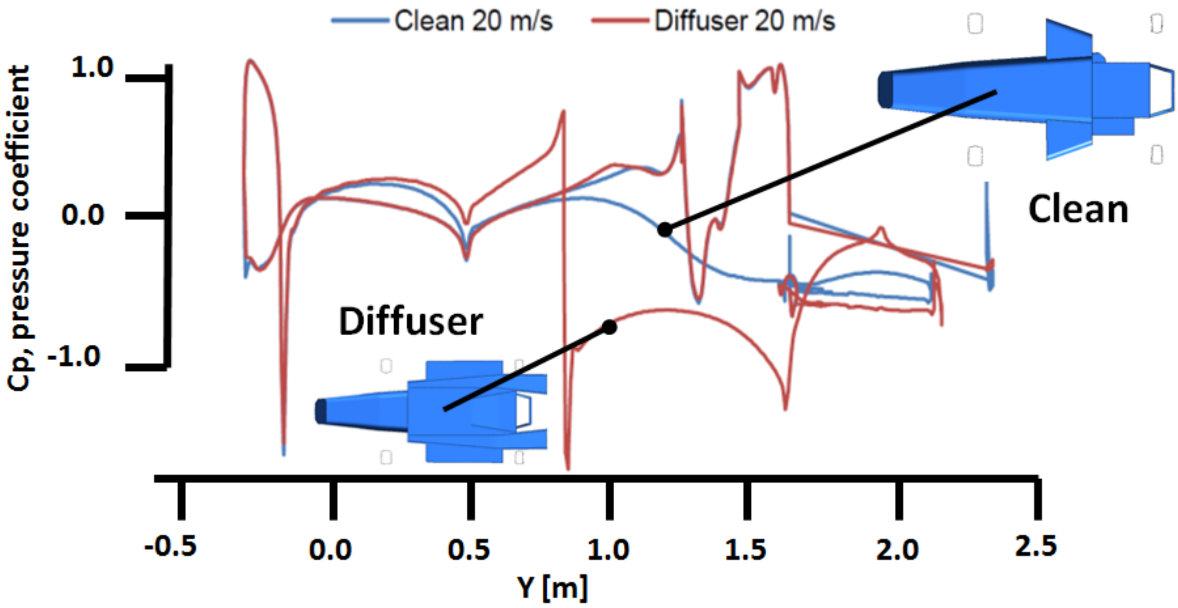

Figure 6 Pressure distribution along mid-plane at 20m/s, Reynolds=3.03x10^{6}

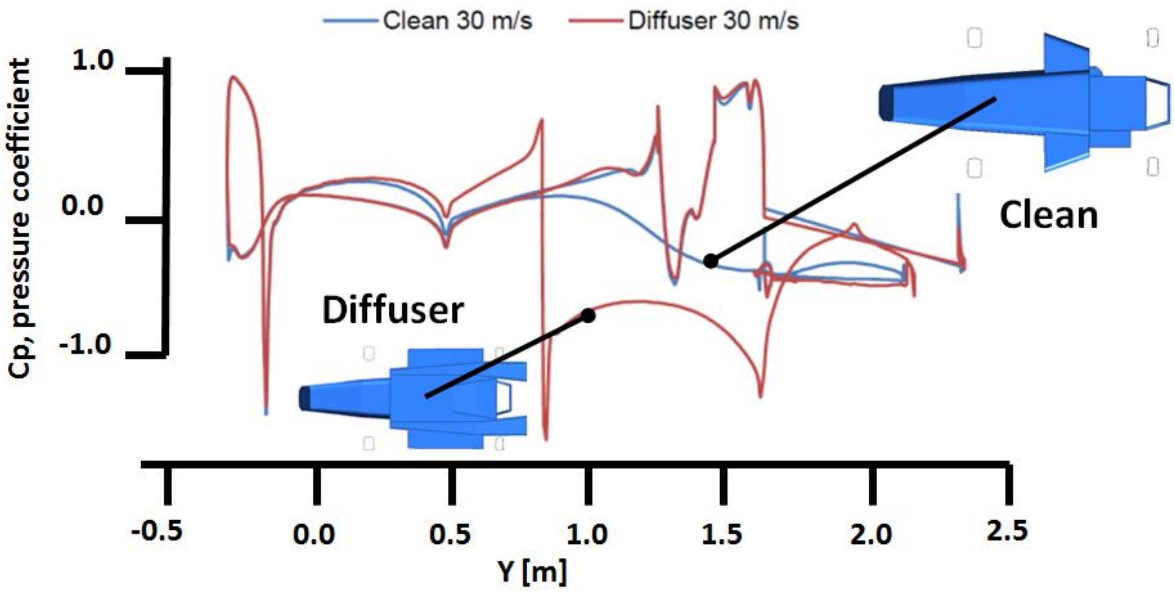

Figure 7 Pressure distribution along mid-plane at 30m/s, Reynolds=4.54×10^6

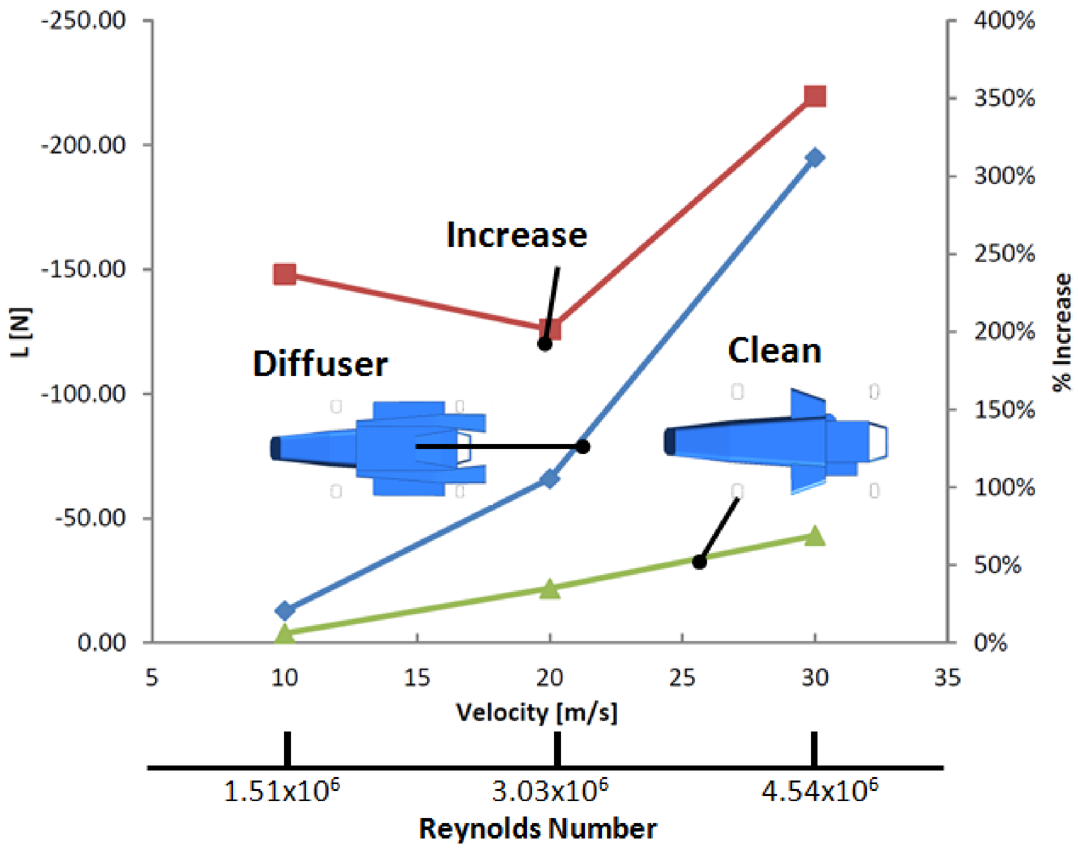

Figure 8 Lift coefficient values against velocity. Deterministic results comparison against clean car.

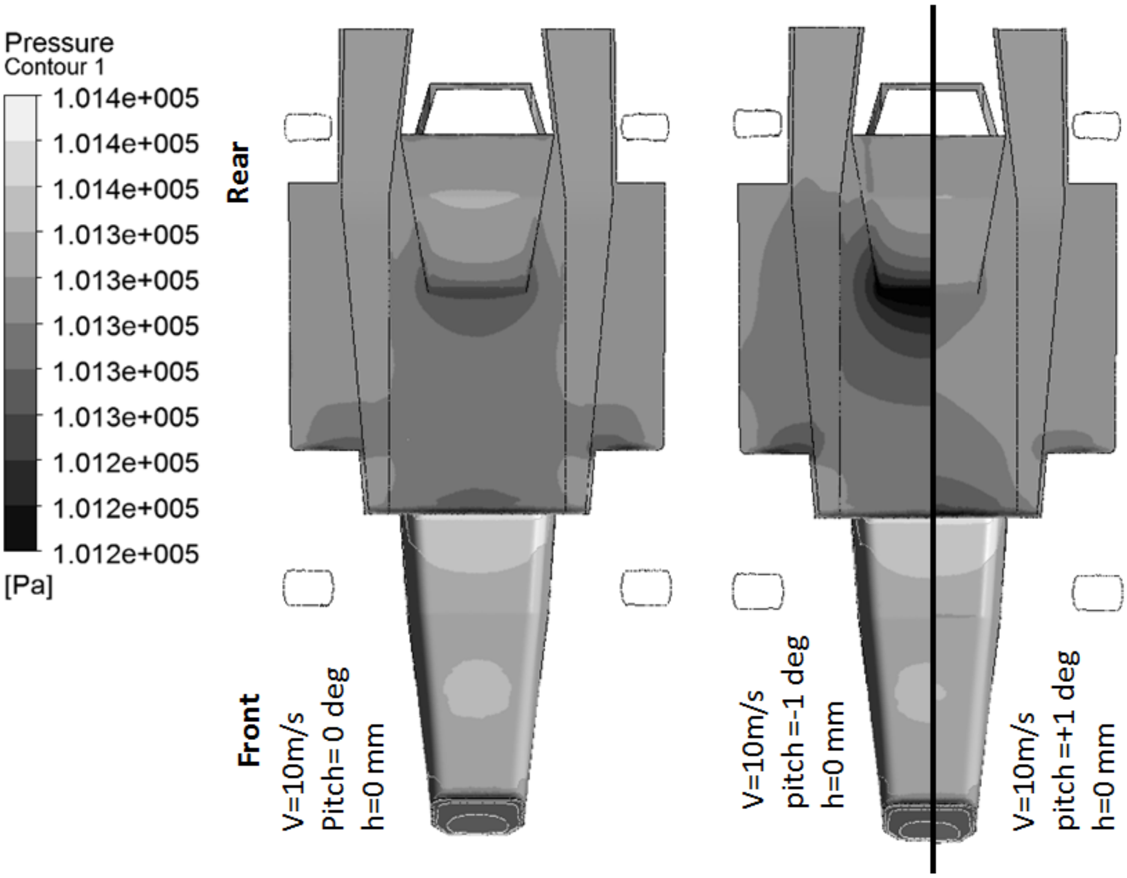

Figure 9 Effect of pitch angle on aerodynamics, pitch=1 / -1deg

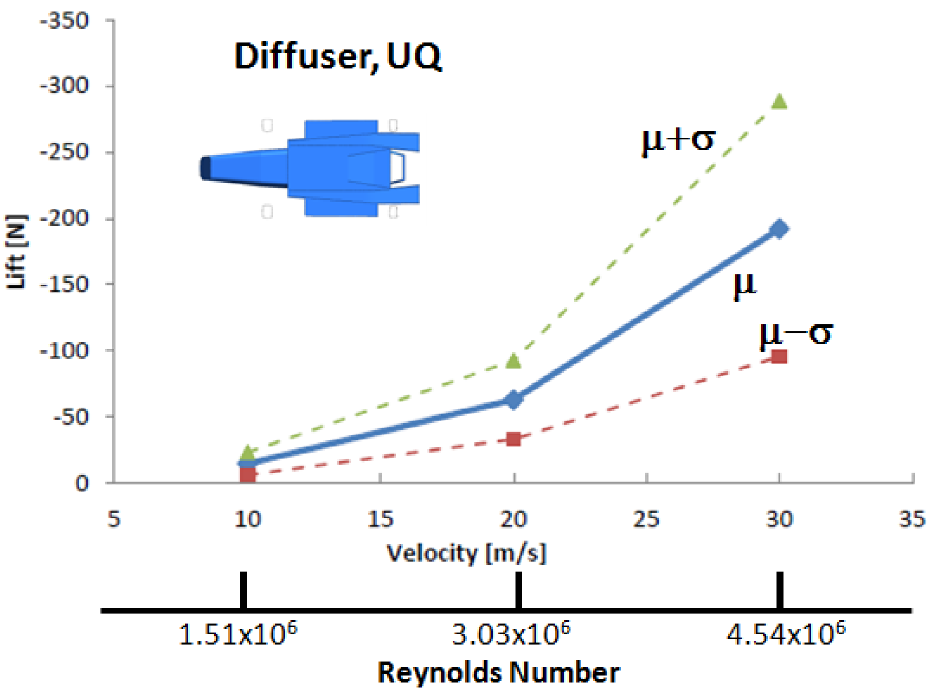

Figure 10: The expected downforce value, μ, and the lines of +/- 1 standard deviation

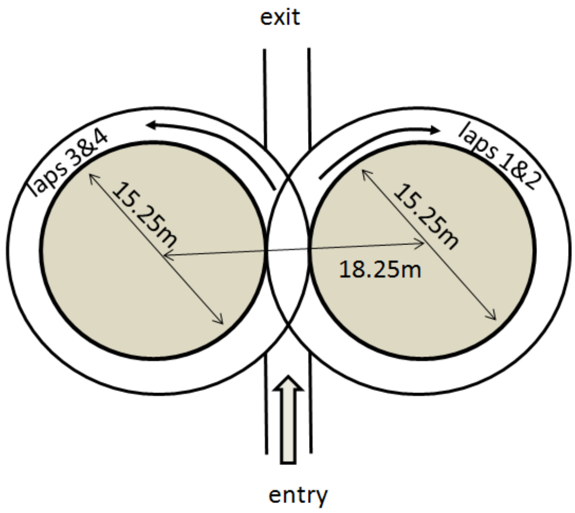

Figure 11: Skid pad circuit layout. Reproduced after [18]

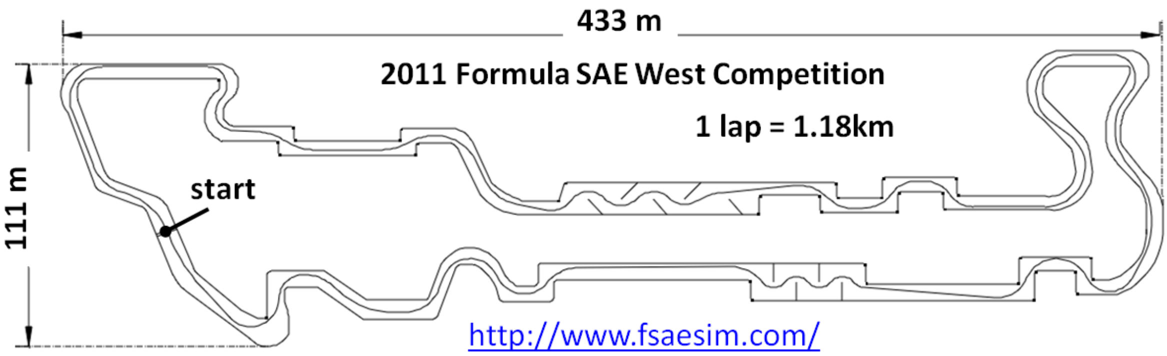

Figure 12: Map of the endurance course, from 2011 Formula SAE West competition [19]