

Constructing deployment scenarios for reserve deliverability via adaptive robust optimization [★]

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ABSTRACT

Network congestion often hinders the deployment of reserves needed to balance forecast errors during real-time operations. A pertinent idea to tackle this challenge involves adding deployment scenarios of spatial distributions of forecast errors as contingencies to the day-ahead problem. However, current approaches disregard the effect of grid characteristics and the day-ahead schedule on the induced congestion and, consequently, reserve deliverability. In this work, we formulate a two-stage adaptive robust optimization problem to jointly consider interactions between day-ahead and real-time operations and forecast errors. Using a column-and-constraint algorithm, we iteratively construct deployment scenarios by finding the worst-case forecast error for reserve deliverability. Simulations on the RTS-GMLC system show that adding these scenarios to the day-ahead problem significantly reduces the frequency of congestion-driven reserve undeliverability. Notably, the choice and number of scenarios dynamically adapts to the day-ahead schedule.

1. Introduction

a) Background and motivation. To manage the growing imbalances between the day-ahead (DA) and real-time (RT) scheduling problems, partly caused by increasing variable renewable energy (VRE) forecast errors, system operators (SOs) reserve spare generation capacity in DA, which can be deployed in RT [1]. To ensure that the frequency of reserve shortages remains below a prescribed reliability target, SOs are increasingly using probabilistic forecasts to determine the amount of required reserves dynamically [2,3]. Typically, reserve requirements are estimated for the system as a whole or a limited number of large zones [4]. However, network congestion often renders reserves undeliverable in RT. For instance, a significant portion of ramping reserve was undeliverable in 2018 in California [5].

b) Related work. Several approaches of varying complexity could address the problem of undeliverable reserves. At one end of the spectrum, computationally intensive formulations explicitly model multiple scenarios for the spatial distribution of forecast errors to obtain DA schedules that ensure the desired RT reliability [6]. For instance, two-stage stochastic optimization problems endogenously determine reserve capacity at various locations by minimizing probability-weighted costs

over multiple scenarios with distinct spatial distributions of forecast errors. However, such methods may be impractical due to computational issues and lack of properties, desirable in markets [7]. As an alternative, robust optimization problems [8] ensure deliverability for all error realizations within an *uncertainty set* and closely resemble existing scheduling processes [9]. However, robust problems are sensitive to the design of the uncertainty set and often lead to overly conservative solutions. This conservativeness is to some extent reduced when robust problems are adaptive, considering multiple stages and allowing recourse actions at a later stage (e.g., RT redispatch) [10]. Solution approaches for adaptive robust problems usually assume a reserve activation policy [11] or iteratively find the worst-case realizations for a series of DA schedules [12]. Data-driven methods can be used to determine the parameters of reserve activation policies [13,14].

At the other end, less computationally intensive methods rely on heuristics and approximations for generating DA schedules. While they aim for higher RT reliability than practices with network-unaware reserve procurement, they usually lack theoretical guarantees. For instance, reserving capacity at each node to meet nodal reserve needs is an inner approximation of the adaptive robust problem's feasible region that does not require any network capacity for reserves. In practice, imposing reserve requirements at every node is often costly and sometimes

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impossible due to the lack of local reserve suppliers. When reserve requirements are nodal, there are provisions for limited contribution of out-of-node resources. The limits on the contributions are determined through conservative approaches [15] or heuristics [16,17].

Building upon the current practice of approximations with zonal reserve requirements, several articles propose a shift from zones with jurisdictional boundaries [18] to zones identified based on active and reactive power flow analysis [19] and grid characteristics [20,21]. To account for dynamic congestion patterns, Viafora et al. [22] presented a method for dynamically partitioning the grid into zones using probabilistic forecasts.

More recently, SOs started incorporating forecast error scenarios as contingencies in their DA schedule to support reserve deliverability in RT [23,24]; hereafter, we refer to them as *deployment scenarios*. By explicitly modeling RT redispatch actions under deployment scenarios, this approach advances the traditional deterministic modeling practice [25] and constitutes a second-best alternative to the optimal, but impractical, multi-stage stochastic and robust formulations. However, generating these deployment scenarios from probabilistic forecasts is an open question. Industry best practices typically account for a few *extreme scenarios* with heuristically selected spatial distributions of forecast errors [26], potentially overlooking other possible distributions that could lead to reserve undeliverability due to network topology and transmission constraints. Importantly, these deployment scenarios are chosen without considering how reserve deliverability depends on the DA schedule and network congestion.

c) Aim and contribution. In this work, we develop a method for constructing deployment scenarios from a set of probabilistic forecasts, which are then included as contingencies in the DA scheduling problem. We formulate a two-stage adaptive robust optimization (ARO) problem that jointly considers the DA schedule, worst-case forecast error realizations, and the full-recourse RT schedule. When the uncertainty set is polyhedral, the ARO problem is equivalent to the standard DA scheduling problem with deployment scenarios as additional contingencies. Using the column-and-constraint (CCG) [27] algorithm, we construct deployment scenarios corresponding to a series of iteratively updated DA schedules. We apply this method to the RTS-GMLC 2019 [28] system and provide original insights into the differences between CCG-constructed deployment scenarios and *extreme* deployment scenarios constructed following industry practice. We find that CCG-constructed deployment scenarios often include errors in both directions (i.e., actuals may be greater or lower than the DA point forecast) at different grid nodes, which contrasts extreme scenarios that include errors in a single direction. To account for the aggregation benefits of forecast errors, extreme scenarios reduce the reserve requirement at each node based on a uniform across-nodes policy. In contrast, the CCG-constructed deployment scenarios allow the DA problem to endogenously calculate what level of aggregation benefit (if any) can be leveraged, considering transmission constraints. Overall, our results show that incorporating deployment scenarios in the DA schedule improves reserve deliverability compared to network-unaware reserve procurement and that CCG-constructed deployment scenarios closely approximate prescribed reliability targets, while outperforming extreme scenario approaches.

The rest of the paper is structured as follows. Section 2 presents the preliminaries on the reserve deliverability problem. Section 3 describes the methodology proposed to construct deployment scenarios. Section 4 presents the experimental design and results. Finally, Section 5 summarizes conclusions and provides directions for further research.

2. Preliminaries

In this section, we describe the operating framework (Section 2.1), and formulate the DA scheduling problem with deployment scenarios for the deliverability of reserves as a two-stage ARO problem (Section 2.2).

Notation. We use bold lowercase (uppercase) font for vectors (matrices) and calligraphic font for sets. Let $|\cdot|$ be the set cardinality and $\mathbf{1}$ be a vector of ones with appropriate size. Forecasts are denoted by $\hat{\cdot}$; the value of decision variables at optimality is denoted by $(\cdot)$. For a positive integer Q , we define $[Q] = \{1, \dots, Q\}$.

2.1. Operating framework

Consider a power system where $\mathcal{N}$ is the set of nodes, $\mathcal{L}$ is the set of lines, and $\mathcal{G}$ is the set of generators. For the i th period, the point forecast for the net demand (load minus VRE production) is $\hat{\mathbf{d}} \in \mathbb{R}^{|\mathcal{N}|}$, and $\xi \in \mathbb{R}^{|\mathcal{N}|}$ is a random variable that describes the forecast error, defined as actual minus DA point forecast. The SO has access to a set $\mathcal{E}$ of K scenarios with forecast errors that account for spatial correlations, where $\mathcal{E} = \{\xi_k\}_{k \in [K]}$. In practice, this information can be provided by methods that generate multivariate scenarios from marginal probabilistic forecasts using a Copula function — see, e.g., Carmona and Yang [29]. We note the aggregate net demand error at the system level with ξ^{agg} and compute its scenario-specific value, ξ_k^{agg} , as $\mathbf{1}^\top \xi_k$. For brevity, the period index i is omitted.

Consider a prescribed reliability level $\alpha \in (0, 1)$, where α is typically high, e.g., 0.90, 0.95. The system-wide reserve requirements for the i th period are estimated by

$$\hat{p}^+ = q_{\frac{1+\alpha}{2}}(\{\xi_k^{\text{agg}}\}_{k \in [K]}), \quad \hat{p}^- = q_{\frac{1-\alpha}{2}}(\{\xi_k^{\text{agg}}\}_{k \in [K]}), \quad (1)$$

where $q_u(\cdot)$ is a function that takes as input K scenarios and returns the u th empirical quantile. In words, the likelihood of ξ^{agg} being in the interval $[\hat{p}^-, \hat{p}^+]$ is $100 \cdot \alpha\%$. Typically, it holds that $\hat{p}^- \leq 0$ and $\hat{p}^+ \geq 0$, which implies that downward and upward reserves are procured to balance negative and positive errors of net demand, respectively.

The SO solves two sequential optimization problems. In DA, the SO schedules energy and reserves. In RT, the SO adjusts the schedules to ensure cost-effective and reliable operations. As we are concerned with reserve undeliverability, the second stage objective only considers reliability.¹

a) DA problem. The DA (or first-stage) scheduling problem is given by

$$\min_{\mathbf{p}, \mathbf{r}^+, \mathbf{r}^-} (\mathbf{c}^e)^\top \mathbf{p} + (\mathbf{c}^+)^\top \mathbf{r}^+ + (\mathbf{c}^-)^\top \mathbf{r}^-, \quad (2a)$$

$$\text{s.t. } \mathbf{1}^\top \mathbf{p} = \mathbf{1}^\top \hat{\mathbf{d}}, \quad (2b)$$

$$\mathbf{1}^\top \mathbf{r}^+ \geq \hat{p}^+, \quad \mathbf{1}^\top \mathbf{r}^- \geq -\hat{p}^-, \quad (2c)$$

$$\mathbf{M}(\mathbf{A}\mathbf{p} - \hat{\mathbf{d}}) \leq \mathbf{f}^{\text{max}}, \quad (2d)$$

$$-\mathbf{M}(\mathbf{A}\mathbf{p} - \hat{\mathbf{d}}) \leq \mathbf{f}^{\text{max}}, \quad (2e)$$

$$\mathbf{p} + \mathbf{r}^+ \leq \mathbf{p}^{\text{max}}, \quad (2f)$$

$$\mathbf{p} - \mathbf{r}^- \geq \mathbf{p}^{\text{min}}, \quad (2g)$$

$$\mathbf{p}, \mathbf{r}^+, \mathbf{r}^- \geq \mathbf{0}. \quad (2h)$$

Problem (2) computes the least-cost energy ($\mathbf{p} \in \mathbb{R}^{|\mathcal{G}|}$) and reserve schedule ($\mathbf{r}^+, \mathbf{r}^- \in \mathbb{R}^{|\mathcal{G}|}$) (2a) to satisfy the net demand forecast (2b) and dynamic reserve requirements (2c), subject to transmission feasibility constraints (2d)–(2e) and constraints for minimum and maximum generating power (2f)–(2h). Here, $(\mathbf{c}^e, \mathbf{c}^+, \mathbf{c}^-)$ represent linear costs, $(\mathbf{p}^{\text{max}}, \mathbf{p}^{\text{min}}, \mathbf{f}^{\text{max}})$ represent technical limits for energy and line flows, $\mathbf{M} \in \mathbb{R}^{|\mathcal{L}| \times |\mathcal{N}|}$ is the Power Transfer Distribution Factors (PTDF) matrix, and $\mathbf{A} \in \mathbb{R}^{|\mathcal{N}| \times |\mathcal{G}|}$ is the node incidence matrix that maps generators to nodes.

¹ To highlight the article's novel contributions, we solve a single-interval optimization problem. The core methodology can be extended for scheduling applications with commitment decisions and inter-temporal constraints.

b) RT problem. Given fixed first-stage decisions $\mathbf{x} = (\mathbf{p}, \mathbf{r}^+, \mathbf{r}^-)$ and a realized net demand forecast error ξ_0 , the RT (or second-stage) redispatch problem that computes recourse actions $\mathbf{p}^{\text{rec}} \in \mathbb{R}^{|\mathcal{G}|}$ is given by

$$\min_{\mathbf{p}^{\text{rec}}, \mathbf{g}^+, \mathbf{g}^-, \mathbf{e}^+, \mathbf{e}^-} (\mathbf{c}^{\text{viol}})^T (\mathbf{g}^+ + \mathbf{g}^- + \mathbf{e}^+ + \mathbf{e}^-), \quad (3a)$$

$$\text{s.t. } \mathbf{1}^T \mathbf{p}^{\text{rec}} = \mathbf{1}^T \xi_0, \quad (3b)$$

$$\mathbf{M}(\mathbf{A}\mathbf{p}^{\text{rec}} - \xi_0) \leq \mathbf{f}^{\text{max}} - \mathbf{f}^e + \mathbf{e}^+, \quad (3c)$$

$$-\mathbf{M}(\mathbf{A}\mathbf{p}^{\text{rec}} - \xi_0) \leq \mathbf{f}^{\text{max}} + \mathbf{f}^e + \mathbf{e}^-, \quad (3d)$$

$$\mathbf{p}^{\text{rec}} \leq \mathbf{r}^+ + \mathbf{g}^+, \quad (3e)$$

$$-\mathbf{p}^{\text{rec}} \leq \mathbf{r}^- + \mathbf{g}^-, \quad (3f)$$

$$\mathbf{g}^+, \mathbf{g}^-, \mathbf{e}^+, \mathbf{e}^- \geq \mathbf{0}, \quad (3g)$$

where $\mathbf{f}^e = \mathbf{M}(\mathbf{A}\mathbf{p} - \mathbf{d})$ is shorthand for the DA scheduled energy flow. Note that the available transmission capacity for recourse actions is $\mathbf{f}^{\text{max}} - \mathbf{f}^e$ and $\mathbf{f}^{\text{max}} + \mathbf{f}^e$. Constraint (3b) ensures the RT balance of supply and demand. Constraints (3c)-(3d) and (3e)-(3f) ensure that recourse actions are within the available transmission capacity and generation reserved in DA, respectively. If the reserves are insufficient, e.g., RT schedules exceed capacities reserved in DA or cannot be delivered due to grid congestion, then the slack variables $\mathbf{g}^-, \mathbf{g}^+ \in \mathbb{R}^{|\mathcal{G}|}$ and $\mathbf{e}^-, \mathbf{e}^+ \in \mathbb{R}^{|\mathcal{L}|}$ become positive [30]. The objective function (3a) imposes a large penalty $\mathbf{c}^{\text{viol}}$ on non-zero slack values.

2.2. Adaptive robust optimization formulation

We now formulate a two-stage ARO problem that enables reserve deliverability and show that it can be equivalent to (2) augmented with deployment scenarios. To streamline notation, we rewrite problem (2) in a compact form as

$$\min_{\mathbf{x}} \mathbf{c}^T \mathbf{x}, \text{ s.t. } \mathbf{B}\mathbf{x} \leq \mathbf{b},$$

where $\mathbf{x} = (\mathbf{p}, \mathbf{r}^+, \mathbf{r}^-)$, $\mathbf{c} = (\mathbf{c}^e, \mathbf{c}^+, \mathbf{c}^-)$, and $(\mathbf{B}, \mathbf{b})$ parameterize the set of linear inequalities² that represent the feasible set of (2). Next, we rewrite problem (3) as

$$\min_{\mathbf{p}^{\text{rec}}, \mathbf{s} \geq \mathbf{0}} (\mathbf{c}^{\text{viol}})^T \mathbf{s}, \quad \text{s.t. } \mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{p}^{\text{rec}} - \mathbf{s} \leq \mathbf{E}\xi_0 + \mathbf{h},$$

where $\mathbf{x}$ are the fixed first-stage decisions, $\mathbf{s} = (\mathbf{0}, \mathbf{g}^+, \mathbf{g}^-, \mathbf{e}^+, \mathbf{e}^-)$, and $(\mathbf{H}, \mathbf{D}, \mathbf{E}, \mathbf{h})$ are constructed appropriately to represent the feasible set of (3).³

The two-stage ARO problem with full recourse, considering only feasibility penalties, is given by

$$\min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} + \max_{\xi \in \mathcal{U}} \min_{\mathbf{p}^{\text{rec}}(\xi), \mathbf{s}(\xi)} (\mathbf{c}^{\text{viol}})^T \mathbf{s}(\xi), \quad (4a)$$

$$\text{s.t. } \mathbf{B}\mathbf{x} \leq \mathbf{b}, \quad (4b)$$

$$\mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{p}^{\text{rec}}(\xi) - \mathbf{s}(\xi) \leq \mathbf{E}\xi + \mathbf{h}, \quad \forall \xi \in \mathcal{U}, \quad (4c)$$

$$\mathbf{s}(\xi) \geq \mathbf{0}, \quad \forall \xi \in \mathcal{U}, \quad (4d)$$

where second-stage decisions $\mathbf{p}^{\text{rec}}(\xi), \mathbf{s}(\xi)$ are a function of ξ and $\mathcal{U}$ is an uncertainty set that covers potential realizations of net demand error ξ the system has to be reliable against. For example, $\mathcal{U}$ could coincide with $\mathcal{E}$, or be a subset or an approximation of it.

The choice of $\mathcal{U}$ is critical for both the performance guarantees and the computational cost associated with solving (or approximating) (4). When $\mathcal{U}$ is a polyhedral set, then the worst-case cost occurs at one of its extreme points (vertices). Let $\mathcal{V}$ be a discrete set that contains all the vertices of $\mathcal{U}$. Then, (4) is equivalent to

$$\min_{\mathbf{x}, \{\mathbf{p}^{\text{rec}}, \mathbf{s}\}_{\xi} \in \mathcal{V}} \mathbf{c}^T \mathbf{x} + \eta, \quad (5a)$$

² Each equality constraint is replaced with two opposite inequalities.

³ In addition to slack variables, $\mathbf{s}$ includes $\mathbf{0}$ for the two opposite inequalities, replacing (3b).

$$\text{s.t. } \mathbf{B}\mathbf{x} \leq \mathbf{b}, \quad (5b)$$

$$\eta \geq (\mathbf{c}^{\text{viol}})^T \mathbf{s}_{\xi}, \quad \forall \xi \in \mathcal{V}, \quad (5c)$$

$$(\mathbf{p}_{\xi}^{\text{rec}}, \mathbf{s}_{\xi}) \in \Omega(\mathbf{x}, \xi), \quad \forall \xi \in \mathcal{V}, \quad (5d)$$

where η represents the worst-case violation cost, $(\mathbf{p}_{\xi}^{\text{rec}}, \mathbf{s}_{\xi})$ are the wait-and-see second-stage decisions for each vertex in $\mathcal{V}$, and $\Omega(\mathbf{x}, \xi) = \{(\mathbf{p}^{\text{rec}}, \mathbf{s}) : \mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{p}^{\text{rec}}(\xi) - \mathbf{s}(\xi) \leq \mathbf{E}\xi + \mathbf{h}, \mathbf{s}(\xi) \geq \mathbf{0}\}$ is the feasible set of the second-stage problem, given $\mathbf{x}, \xi$. The objective function (5a) consists of the DA scheduling cost and the worst-case violation cost. Note that (5d) includes the RT constraints for particular realizations of ξ , which could be interpreted as reserve deployment scenarios. Hence, the ARO formulation formally justifies the industry intuition of adding deployment scenarios to the DA problem. When (5d) is satisfied without slack activation for all vertices, η equals zero. If any slack is non-zero, then η is positive, and supply-demand balance is not guaranteed within $\mathcal{U}$. In those cases, it is worth examining whether the system lacks reserves at the system level or at specific nodes and how accurate model inputs, such as $\mathcal{U}$ and $\mathbf{c}^{\text{viol}}$, are.

The ARO reformulation shows that the success of the deployment scenarios hinges on the careful construction of these scenarios. The number of vertices in $\mathcal{U}$ may be exponential to the dimension of ξ , making vertex enumeration impractical for solving (5) to optimality.

3. Constructing deployment scenarios

In this section, we describe two methods for constructing deployment scenarios, assuming a polyhedral uncertainty set presented in Section 3.1. First, we describe a method that only uses DA forecasts of net demand as inputs and resembles industry practices (Section 3.2). Second, we develop a method that additionally leverages grid characteristics and DA decisions, which comprise our key contribution (Section 3.3).

3.1. Uncertainty set

In this work, we assume that SOs aim to ensure reserve deliverability for RT scenarios with ξ^{agg} lying within an interval bounded by the system-wide reserve requirements $\hat{\rho}^+, \hat{\rho}^-$, and nodal net demand errors lying within a “box”, given by

$$\mathcal{U}_a^{\text{agg}} = \{\xi : \hat{\rho}^- \leq \mathbf{1}^T \xi \leq \hat{\rho}^+\}, \quad (6)$$

$$\mathcal{U}^{\text{box}} = \{\xi : \min_{k \in [K]} \{\hat{\xi}_k\} \leq \xi \leq \max_{k \in [K]} \{\hat{\xi}_k\}\}, \quad (7)$$

where the box is bounded by the lowest and highest forecast error scenario (element-wise operation).

The uncertainty set for problem (4) is given by the intersection $\mathcal{U} = \mathcal{U}_a^{\text{agg}} \cap \mathcal{U}^{\text{box}}$, which implies that $\mathcal{U} \subseteq \mathcal{U}_a^{\text{agg}}$ and $\mathcal{U} \subseteq \mathcal{U}^{\text{box}}$. In practice, SOs can also use other polyhedral uncertainty sets that may better reflect the distribution of forecast errors. The quality of uncertainty sets can be assessed in terms of coverage, density, or interpretability.

3.2. Extreme deployment scenarios from forecasts

This approach resembles industry practice [26] and constructs a set of two extreme deployment scenarios $\mathcal{S}^{\text{depl}}$: one for positive and one for negative forecast errors. This approach assumes that the most challenging case for reserve deliverability is when all errors are in the same direction (positive or negative), which implies a high correlation among them. Here, we follow the same logic and distribute $\hat{\rho}^+, \hat{\rho}^-$ to the nodes such that all errors are in the same direction. First, for the j th node, we calculate allocation factor e_j^+ , which accounts for the $\frac{1+\alpha}{2}$ -level marginal quantile forecasts as follows:

$$e_j^+ = \frac{q_{\frac{1+\alpha}{2}}(\{\hat{\xi}_{j,k}\}_{k \in [K]})}{\sum_{n \in \mathcal{N}} q_{\frac{1+\alpha}{2}}(\{\hat{\xi}_{n,k}\}_{k \in [K]})}. \quad (8)$$

In words, the allocation factors are proportional to the quantiles of nodal forecast errors. We construct the upward deployment scenario that includes in each node n an upward error equal to $\xi_n^+ = \hat{\rho}^+ \cdot e_n^+$. An additional Euclidean projection step onto $\mathcal{U}$ is applied. We follow the same approach for the downward direction.

3.3. Constructing deployment scenarios via CCG

In this section, we construct deployment scenarios considering their dependency on (i) problem parameters (e.g., grid topology) and (ii) first-stage decisions. We apply a CCG algorithm [27] to iteratively add vertices, such that $\mathcal{S}^{\text{depl}} \subseteq \mathcal{V}$.

To streamline notation, let

$$Q(\bar{\mathbf{x}}) = \max_{\xi \in \mathcal{U}} \min_{\mathbf{p}^{\text{rec}}(\xi), \mathbf{s}(\xi) \in \Omega(\bar{\mathbf{x}}, \xi)} (\mathbf{c}^{\text{viol}})^T \mathbf{s}(\xi) \quad (9)$$

be the worst-case objective value of the second-stage problem given fixed first-stage decisions $\bar{\mathbf{x}}$. The CCG algorithm is summarized in [Algorithm 1](#). First, we consider an empty set of deployment scenarios, $\mathcal{S}^{\text{depl}}$, solve (5), and estimate a lower bound LB on the RT violation cost (lines 1, 3). Next, we fix the first-stage decisions and approximate the worst-case cost of the second stage $Q(\bar{\mathbf{x}})$ (line 4), which returns an approximately worst-case scenario $\bar{\xi}$ and an upper bound UB on the RT violation cost (line 5). If the worst-case scenario $\bar{\xi}$ has a non-zero violation cost, it is added to the set of deployment scenarios (lines 6–9). The algorithm terminates when $\text{UB} = \text{LB}$ or when the maximum number of iterations M is reached. M can be chosen based on the number of deployment scenarios the SO can add to the DA schedule.

If an oracle that optimally solves $Q(\bar{\mathbf{x}}_j)$ for a fixed $\bar{\mathbf{x}}_j$ is available (step 4), then (5) is solved to optimality when [Algorithm 1](#) converges. In practice, however, computing $Q(\bar{\mathbf{x}}_j)$ is challenging because problem (9) is a nonconvex max – min problem. Solution approaches reformulate the problem as a mixed-integer problem or rely on heuristics [27]. Here, we use the *alternating direction method* (ADM) heuristic to approximate $Q(\bar{\mathbf{x}}_j)$ (9), which has been shown to perform well in similar problems [31].

First, strong duality is applied to the inner min problem for a fixed $\bar{\mathbf{x}}$, reformulating the max – min problem into a max problem given by

$$\max_{\pi, \xi \in \mathcal{U}} \pi^T (\mathbf{H}\bar{\mathbf{x}} - \mathbf{E}\xi - \mathbf{h}), \quad (10a)$$

$$\text{s.t. } -\mathbf{D}^T \pi = \mathbf{0}, \quad (10b)$$

$$\mathbf{0} \leq \pi \leq \mathbf{c}^{\text{viol}}, \quad (10c)$$

where π denotes dual variables of the constraints in $\Omega(\bar{\mathbf{x}}, \xi)$.⁴ Second, Problem (10), which has a bilinear objective, is solved via [Algorithm 2](#). The algorithm relies on iteratively optimizing a relaxed linear program, where part of the decision variables are treated as constants (either π or ξ). Given fixed first-stage decisions $\bar{\mathbf{x}}$ and an initial guess ξ^{init} , we maximize (10) over π (assuming $\xi = \xi^{\text{init}}$), which provides a lower bound LB^Q (line 3). Then, we maximize (10) over $\xi \in \mathcal{U}$, while π is fixed at the solution, which provides an upper bound UB^Q (line 4). The local bounds are iteratively updated until convergence ($\text{UB}^Q = \text{LB}^Q$), which is then guaranteed to be a local optimum that satisfies the Karush-Kuhn-Tucker conditions [31], or the algorithm terminates when the maximum number of iterations L is reached. We set $\bar{Q}(\bar{\mathbf{x}}_j)$ as the average between UB^Q, LB^Q and return the worst-case scenario $\xi^{\text{wc}} = \xi_l$ (line 7).

The initial guess ξ^{init} can be critical to the convergence of [Algorithm 2](#). Here, we develop a heuristic that chooses as initial guesses forecast error scenarios that could aggravate congestion. First, we select the set of lines that are congested in at least one period in DA or RT when the solution of (2) is followed. Then, for each line l , we find the forecast error ξ that results in the highest power flow increase in the

same direction as the DA flow f_l^e , obtained from (2), as follows

$$\max_{\xi \in \mathcal{U}^{\text{box}}} |f_l^e - \mathbf{M}_l \xi|. \quad (11)$$

(11) admits a closed-form solution (note that f_l^e is fixed), which is a vertex of $\mathcal{U}^{\text{box}}$. The vertex includes, for each node j , the maximum nodal forecast error, when term $(\text{sign}(f_l^e) \cdot \mathbf{M}_{l,j})$ is negative; and the minimum nodal forecast error when the same term is positive. This vertex is then projected onto $\mathcal{U}$ to obtain an initial guess. In addition, we use the two extreme deployment scenarios from [Section 3.2](#) as initializations ξ^{init} . That way, the use of the CCG-constructed scenarios will result in a level of reliability at least as good as the one achieved by using the extreme scenarios. Out of all initializations, we pick the scenario ξ^{wc} that leads to the highest $\bar{Q}(\bar{\mathbf{x}}_j)$.

The computational overhead of the CCG algorithm is anticipated to be similar to that of other iterative solutions currently implemented by system operators [32]. Note that Algorithm 2 can be trivially parallelized for multiple initializations.

Algorithm 1 Column-and-constraint generation.

Input: Problem (5), maximum number of scenarios M .

Output: $\mathcal{S}^{\text{depl}}$

- 1: Initialize $\mathcal{S}_0 = \emptyset$, $\text{UB} = \infty$, $\text{LB} = -\infty$, $j = 0$.
 - 2: **while** $\text{UB} - \text{LB} \geq 0$ and $j \leq M$ **do**
 - 3: Solve (5) with $\mathcal{V} = \mathcal{S}_j$ and set $\bar{\mathbf{x}}_j = \mathbf{x}^*$, $\text{LB} = \eta^*$.
 - 4: Approximate $Q(\bar{\mathbf{x}}_j)$ using $\bar{Q}(\bar{\mathbf{x}}_j)$.
 - 5: Set $\xi_j^{\text{wc}} = \bar{\xi}$, $\text{UB} = \bar{Q}(\bar{\mathbf{x}}_j)$.
 - 6: **if** $\bar{Q}(\bar{\mathbf{x}}_j) > 0$ **then**
 - 7: Update $\mathcal{S}_{j+1} = \mathcal{S}_j \cup \{\xi_j^{\text{wc}}\}$, $j \leftarrow j + 1$.
 - 8: **end if**
 - 9: **end while**
 - 10: Return $\mathcal{S}^{\text{depl}} = \mathcal{S}_j$.
-

Algorithm 2 Alternating direction method.

Input: First-stage decisions $\bar{\mathbf{x}}$, maximum number of iterations L , initialization ξ^{init} .

Output: $\bar{Q}(\bar{\mathbf{x}})$, ξ

- 1: Initialize by $\text{UB}^Q = \infty$, $\text{LB}^Q = -\infty$, $l = 0$, $\xi_l = \xi^{\text{init}}$.
 - 2: **while** $\text{UB}^Q - \text{LB}^Q \geq \epsilon$ and $l \leq L$ **do**
 - 3: Solve $\max_{\pi, \xi \in \mathcal{U}} \pi^T (\mathbf{H}\bar{\mathbf{x}} - \mathbf{E}\xi - \mathbf{h})$, s.t. $-\mathbf{D}^T \pi = \mathbf{0}$,
 set $\pi_l = \pi^*$, update LB^Q .
 - 4: Solve $\max_{\xi \in \mathcal{U}} \pi_l^T (\mathbf{H}\bar{\mathbf{x}} - \mathbf{E}\xi - \mathbf{h})$, update UB^Q .
 - 5: $l \rightarrow l + 1$
 - 6: **end while**
 - 7: Return $\bar{Q}(\bar{\mathbf{x}}) = \frac{\text{UB}^Q + \text{LB}^Q}{2}$, $\xi \leftarrow \xi_l$.
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4. Numerical experiments and results

In this section, we summarize the experimental design ([Section 4.1](#)), discuss an illustrative example ([Section 4.2](#)) and analyze results for annual simulations of the RTS-GMLC 2019 system ([Section 4.3](#)).

4.1. Experimental design

For a given reliability level α , we contrast three approaches: (i) DSW, where we solve (2) with system-wide reserve requirements estimated from (1); (ii) EXT, where we solve (5) with two extreme deployment scenarios constructed with method shown in [Section 3.2](#); and (iii) CCG, where we solve (5) with deployment scenarios constructed by applying CCG, as shown in [Section 3.3](#), for $\mathcal{U} = \mathcal{U}_\alpha^{\text{agg}} \cap \mathcal{U}^{\text{box}}$.

⁴ We slightly abuse notation as (10c) should be omitted for the dual corresponding to (3b) as it concerns an equality constraint with no feasibility slack.

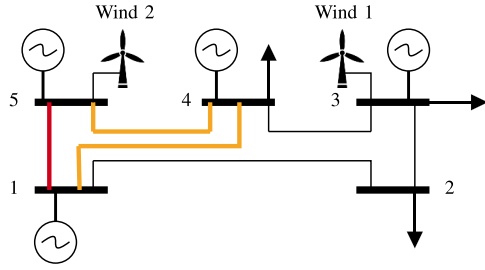


Fig. 1. Schematic of the 5-bus system. Red color indicates line congestion in DA DSW schedule. Orange color indicates RT line congestion given the DSW decisions, causing deliverability problems.

4.2. Illustrative example

We first discuss results for a modified IEEE 5-bus system [33]. The results can be easily verified by the reader and provide intuition for appreciating the results for the larger system in the next section. The system has two wind power plants, shown in Fig. 1. Wind plant 1 is located in node 3 alongside the most expensive generator, whereas wind plant 2 is located in node 5 alongside the cheapest generator. We consider wind forecast errors following a zero-mean multivariate normal distribution, where

$$\Sigma = \begin{bmatrix} 0.141 & 0.001 \\ 0.001 & 0.141 \end{bmatrix} (\text{pu})^2$$

is the covariance matrix. We sample $K = 1000$ DA scenario forecasts for ξ_k , calculate the system-wide requirements $\hat{p}^+, \hat{p}^-$ for $\alpha = 0.95$, and construct two sets of deployment scenarios using EXT and CCG. To assess out-of-sample performance, we sample an additional 1000 scenarios (representing realized errors), and solve the RT problem (3), for $c^{\text{viol}} = 1000$ \$/MWh. In addition to DSW, EXT, and CCG, we solve (5) to optimality via vertex enumeration (V-enum).

Fig. 2 plots the uncertainty set $\mathcal{U}$, which is enclosed by the red curve, and the respective deployment scenarios. CCG terminates after adding a single scenario, ξ_1^{wc} , located at the bottom left vertex, to the DA problem whereas EXT adds two deployment scenarios, and V-enum adds all six extreme points shown in Fig. 2. Table 1 presents the average results for scenarios with realized error that belongs to $\mathcal{U}$. Both EXT and CCG lead to higher DA costs and lower RT penalties compared to DSW. For EXT, 3.3% of out-of-sample observations within $\mathcal{U}$ violate RT constraints (i.e., have at least one RT slack variable with non-zero value). This is an order of magnitude lower than the 54.4% obtained under DSW. CCG performs best in terms of reliability, with 0% violations. In this case, CCG found the same solution as V-enum, while only using a single scenario.

Examining the DA decisions obtained by DSW, we observe that the generator at node 5 is the sole reserve supplier, line 1 – 5 is congested, and lines 1 – 4, 4 – 5 are close to becoming congested. When upward reserves are needed due to negative errors in the wind plant 1 (over-forecast of wind production), the orange lines shown in Fig. 1 become congested and the reserves procured at node 5 cannot be delivered. The EXT deployment scenario ξ^+ accounts for the impact of constraints on reserve deliverability, but does not consider the worst-case of forecast errors in wind plant 1 as the grid is ignored during scenario construction. In contrast, the CCG deployment scenario ξ_1^{wc} fully mitigates reserve undeliverability within the uncertainty set $\mathcal{U}$. In the DA solution derived by CCG, the more expensive generator at node 3 provides some reserves to compensate for the potential forecast errors in wind plant 1, while the energy schedule remains unchanged. While, in this example, the DA schedule changed only in terms of reserve schedules, we will see in the next section that inclusion of deployment scenarios in DA can lead to changes in the energy schedule and the available transmission capacity for recourse actions.

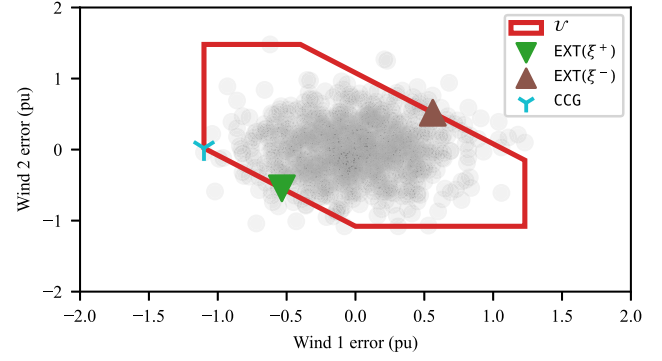


Fig. 2. Deployment scenarios found for the 5-bus system ($\alpha = 0.95$). Grey points indicate the sampled scenarios, used to construct the uncertainty set.

Table 1

Results for the 5-bus system ($\alpha = 0.95$).

	DA cost (10^3 \$/h)	Av. RT cost (10^3 \$/h)	Viol. Prob. (%)
DSW	131	134	54.4
EXT	140	1	3.3
CCG	142	0	0
V-enum	142	0	0

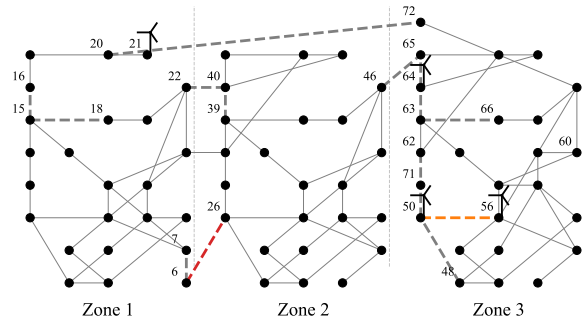


Fig. 3. Grid topology of RTS-GMLC 2019 System. Red color indicates line congestion after DSW is solved and orange color indicates RT line congestion given the DSW decisions, for an illustrative period. Dashed lines are used to initialize the ADM.

4.3. RTS-GMLC 2019 system

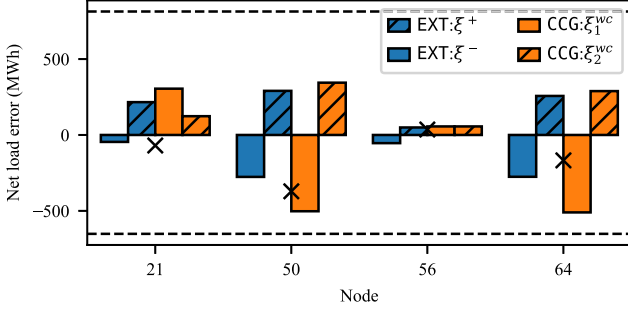
a) *System information.* The RTS-GMLC 2019 System [28] has 73 buses, 120 transmission lines, 73 conventional generators, 4 wind power plants, and 56 solar power plants, organized in 3 zones (Fig. 3). We assume perfect demand forecasts and consider imperfect forecasts for the 60 VRE plants. We use time series data provided by [34] and probabilistic forecasts provided by [29], comprising $K = 500$ scenarios. The data set covers a full year at hourly granularity. Exploratory data analysis indicated that the 4 wind plants at nodes 21, 50, 56, and 64, and the solar plant at node 60 account for approximately 75% of the total forecast errors in terms of absolute magnitude.

b) *Implementation hyper-parameters.* We compare DSW, EXT, and CCG for levels of $\alpha = \{0.90, 0.95, 0.99\}$, and use a high violation penalty of $c^{\text{viol}} = 1000$ \$/MWh. For the DA scheduling problem (2), we also include an additional variable that allows for VRE curtailment, bounded by the respective point forecast. For CCG, we run the alternating direction method with $L = 20$, which is never reached, and set the maximum number of scenarios for each period at $M = 10$. We initialize Algorithm 2 with the extreme scenarios derived for EXT and with scenarios found using (11) for 15 lines that are during at least one period congested in DA

Table 2

Results for a simulation year for the RTS-GMLC system.

		Av. DA cost (10 ³ \$/h)	Av. RT cost (10 ³ \$/h)	Viol. Prob. (%)
$\alpha = 0.90$	DSW	64.57	10.69	16.96
	EXT	65.65	1.00	2.92
	CCG	78.31	0.02	0.08
$\alpha = 0.95$	DSW	68.24	11.20	16.92
	EXT	69.66	0.94	2.40
	CCG	82.51	0.01	0.15
$\alpha = 0.99$	DSW	75.91	12.00	17.14
	EXT	78.53	0.51	2.56
	CCG	90.26	0.06	0.55

**Fig. 4.** Deployment scenarios for EXT and CCG for an illustrative period. Dashed lines indicate $\hat{\rho}^+$, $\hat{\rho}^-$. The $\times$ marker indicates realized errors.

or RT, according to the DSW-based results — see Fig. 3 for a visualization of the selected lines.

c) Annual performance. Table 2 presents annual averages for performance metrics under the different approaches, for $\alpha = \{0.90, 0.95, 0.99\}$. The metrics are reported for periods with realized RT error falling within the respective $\mathcal{U}$. The percentage of observations falling in $\mathcal{U}$ for each experiment is approximately 71% for $\alpha = 0.90$; 76% for $\alpha = 0.95$; and 82% for $\alpha = 0.99$. This undercoverage is attributed to the quality of the probabilistic forecasts, which are an exogenous input in this work. From Table 2, as α increases, the DA cost increases and RT cost decreases for all methods, which is expected as the reserve requirements increase with α . DSW, as expected, performs worst in terms of reliability, with an average violation frequency of approximately 17% for all α , but performs best in terms of DA cost. Considering that we report the frequency of violations when the realized error falls in $\mathcal{U}$, we would expect a fully reliable method to have a frequency of violations close to 0, i.e., assuming (4) is solved to optimality. Indeed, CCG achieves a violation frequency $\leq 1\%$ in all cases, whereas EXT is second-best in terms of reliability. To understand why these few RT violations within $\mathcal{U}$ persist, we examine whether the DA schedule is feasible without using any slack variables (i.e., $\eta^* = 0$). For instance, for $\alpha = 0.95$, 10 simulated periods have non-zero RT penalty cost and belong to $\mathcal{U}$. Among these 10 periods, 8 have $\eta^* = 0$ ⁵. The RT violations during these periods indicate that the ADM failed to find the worst-case error scenario.

Whereas CCG does not provide any guarantees about reliability outside $\mathcal{U}$, the results are similar to the ones inside $\mathcal{U}$, with CCG leading to a lower violation frequency and higher DA cost compared to EXT. For example, for $\alpha = 0.95$, the actual error is outside $\mathcal{U}$ for 24% of the periods. Over these periods, EXT and CCG have an average DA cost of 66.85 and 80.99 10³\$/h, respectively; while they record RT violations for 52.11% and 39.65% of the periods, respectively.

⁵ Two periods have $\eta^* > 0$, which either indicates that c^{viol} is low or that there is no feasible DA schedule that guarantees zero constraint violations in $\mathcal{U}$. In practice, for such cases, the operator can increase the value of c^{viol} .

The CCG achieves better performance in terms of reliability by adjusting the choice and number of vertices added as deployment scenarios, considering the dynamic congestion patterns. For instance, for $\alpha = 0.95$, CCG recovers 3 or more deployment scenarios in approximately 36% of the time. Lastly, it is worth noting that while EXT procures the same amount of upward and downward reserve as DSW, this is not the case for CCG which might choose to procure additional reserve due to congestion anticipated in deployment scenarios with spatial distribution of forecast errors different than EXT. For $\alpha = 0.95$, in respectively 10.9% and 20.8% of the periods CCG procures more up or down reserve than DSW, which implies that the full aggregation benefit cannot be leveraged due to transmission constraints. For these periods, on average 3.6 deployment scenarios are included in the DA schedule.

d) Impact of initialization. We examine the sensitivity of the CCG algorithm w.r.t. the starting points by running it with an alternative set of starting points, including only the extreme scenarios. This results for $\alpha = 0.90/0.95/0.99$ in a violation probability in $\mathcal{U}$ of 1.34/1.36/1.48%, which is better than EXT and worse than the results in Table 2, highlighting the benefit of selecting starting points using the initialization strategy in (11).

e) Illustrative period. We illustrate how CCG constructs better deployment scenarios by examining a particular period (with $\alpha = 0.90$) for which the actual error realization falls inside $\mathcal{U}$ and EXT leads to deliverability issues (i.e., non-zero RT penalty), while CCG does not. For the selected period, the only VRE generating electricity is wind; Fig. 4 plots the deployment scenarios of EXT and CCG alongside the system-wide requirements $\hat{\rho}^+$, $\hat{\rho}^-$ and the realized error, showing only nodes 21, 50, 56 and 64, corresponding to the four wind plants. For both EXT and CCG, the aggregate error in each deployment scenario is equal to either $\hat{\rho}^+$ or $\hat{\rho}^-$, but its distribution among nodes differs. For CCG, the first deployment scenario, ξ_1^{wc} , has an aggregate error equal to $\hat{\rho}^-$; compared to ξ_1^{wc} leads to a much higher allocation in nodes 50, 64 (RT wind production higher than forecasted) whereas nodes 21, 56 have a smaller error with opposite sign; thus, ξ_1^{wc} represents a less correlated setting. Concerning the second deployment scenario, ξ_2^{wc} , we observe that all nodal errors are positive (RT wind production lower than forecasted) with the aggregate error equal to $\hat{\rho}^+$; and the allocation is not proportional to the width of the prediction interval at each node, with relatively to EXT a higher allocation at nodes 50, 64. The line connecting nodes 6 – 26 is congested in the DSW solution of the DA problem.

In RT, a large negative net load error (RT wind production higher than forecasted) occurs in node 50 (see $\times$ marker in Fig. 4); in turn, under DSW, the re-dispatch causes congestion in line 50 – 56 and leads to activating the respective slack variable (approximately 96MW); the same slack is activated in RT for EXT but at a lower level than in DSW (47MW), meaning that EXT only partially resolves the deliverability issue. In contrast, CCG does not activate any slacks in RT. To understand this effect, we examine the aggregate energy and reserve schedule illustrated in Fig. 5. Firstly, CCG schedules more energy in zone 2 by curtailing a portion of wind production in node 50, thus creating available transmission capacity for recourse actions of 187MW in line 50 – 56, which is higher than the 97MW created by EXT. Secondly, CCG shifts upward reserve schedule from zone 1 and 2 to zone 3, whereas downward reserves shift from zone 3 to zone 1 and 2, which is due to the fact that the deployment scenarios consider higher forecast errors in nodes 50, 64. This can also be understood from the sign of the respective entries within the PTDF matrix, which are negative for nodes 50, 64 towards line 50 – 56 and thus the second CCG-scenario creates bigger available transmission capacity for recourse actions than EXT. The rest of the time periods offer similar insights. Namely, the CCG deployment scenarios allocate the uncertainty non-proportionally across nodes. As a result, CCG yields adapted energy-reserve schedules and available transmission capacity for recourse actions, fit for balancing different spatial distributions of forecast errors.

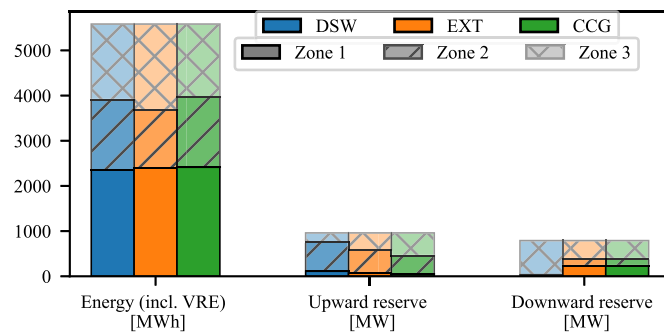


Fig. 5. DA schedule for an illustrative period, aggregated per zone.

5. Conclusion

Transmission congestion often renders reserves undeliverable during real-time operations, threatening system reliability. Emerging industry practices add a set of deployment scenarios as contingencies to the day-ahead scheduling problem to enhance reserve deliverability during real-time operations. However, industry practices for constructing deployment scenarios typically ignore the interdependency between deployment scenarios, grid characteristics, and day-ahead schedules. In this work, we leverage adaptive robust optimization to formulate a two-stage problem that jointly considers the day-ahead schedule and worst-case scenarios for reserve deliverability, and solve it via a column-and-constraint algorithm to generate reserve deployment scenarios. We conduct simulations for the RTS-GMLC 2019 system using two sets of scenarios: one set is constructed via the CCG algorithm and another based on a method that resembles current industry practice. Overall, the deployment scenarios constructed via the CCG algorithm significantly lower the number of periods with undeliverable reserves and are more appropriate to meet the system operator's reliability targets. Contrary to prevailing industry heuristics, these deployment scenarios often include forecast errors in opposite directions, which have the effect of aggravating the congestion of transmission lines. In addition, simulation results show that the number of deployment scenarios and the spatial distributions of errors dynamically adapt to the day-ahead conditions. Future work could quantify the benefits of the proposed approach for scheduling processes that include commitment decisions and inter-temporal constraints. Further work could expand the CCG algorithm to include provisions for cases where no feasible day-ahead schedule is found to be robust within the uncertainty set. Alternative algorithms for solving the adversarial problem could also be tested. Lastly, it is interesting to examine generating deployment scenarios directly from data, thus bypassing the need for iterative solution techniques.

CRedit authorship contribution statement

Guillaume Van Caelenberg: Writing - review & editing, Writing - original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization; **Akylas Stratigakos:** Writing - review & editing, Writing - original draft, Visualization, Validation, Supervision, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization; **Elina Spyrou:** Writing - review & editing, Writing - original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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