Problems in the Optimum Planning of Multi-Enterprise Supply Chains

by

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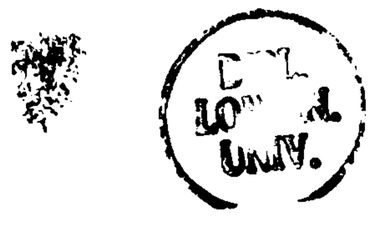

Abstract

In this thesis, optimisation of supply chains involving multiple enterprises is considered. In particular, methods to determine policies that optimise the performance of the supply chain as a whole are characterised. A key issue is to ensure adequate rewards for each supply chain participant. A mathematical programming modelling framework is presented for *fair*, optimised profit distribution between members of multi-enterprise supply chains utilising inter-echelon transfer prices. The proposed formulation is based on a novel approach applying game theoretical Nash-type models to find the optimal profit level for each enterprise subject to given minimum profit requirements. The supply chain planning problem is formulated as a mixed integer nonlinear programming model. A separable programming approach is then applied utilising logarithmic differentiation and approximations of the variables of the objective function.

Furthermore, methods for finding optimal policy parameter choices for dynamic supply chains are described. In particular, detailed procedures for the modelling of the physical and decision-making business process aspects of a supply chain are presented. Pragmatic, non-invasive policy and parameter modifications that improve commonly used performance measures are identified. These models are also utilised to assess and quantify the risks and benefits of supply chain operations modifications, introduction of new products into the supply chain, and financial strategy change. To optimise the dynamic simulation models, a number of approaches are developed and evaluated, including a mixed-integer linear programming, a gradient-based, and a simulated annealing approach. More specifically, the gradient-based method is based around a higher level optimiser that collects the objective function value results and gradients in terms of policy parameters from simulations. The applicability of all approaches are demonstrated through industrial case studies relevant to process systems engineering.

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Part I

Introducing Optimal Supply Chain Planning

Chapter 1

Introduction

1.1 The Supply Chain Problem

Data management complexity, communication complications, piecemeal management approaches, and a lack of general shared performance measures are some of the characteristics of supply chain planning. Supply chain modelling has been a specific research topic since the beginning of the 1990s. Previously as well as today, supply chain problems appeared under a variety of labels, *e.g.* material administration, manufacturing, logistics, and operations management. The areas of supply chain modelling, theory and practice have experienced a significant boom in the research community during the last few years. To emphasise this point, a search on "supply chain" for the year 2000 generates 174 articles in the Science Citation Index.

In short, supply chain activities involve procurement, processing, and distribution of materials across the extended enterprise. Often marketing, design, and business strategies are also incorporated in the terminology. Extended multi-enterprise supply chains usually have both a significant spatial reach (often globally) and long lead times. From the long-term perspective, supply chain design tries to meet business objectives and future customer needs by managing investment and divestment decisions. These comprise decisions about plant locations, plant capacities, out-sourcing, plant resources, distribution resources, transportation resources, adequate performance measures, supplier selection and maintenance. The medium-term supply chain planning decisions incorporate how much should be produced where and with what resources. Supply chain planning includes how best to utilise the manufacturing resources, transportation resources and

business processes available. On the short-term operational level, production scheduling and vehicle routing take place to ensure that production plans are executed so as to minimise costs, waste, and change-overs. Supply chain research comprises a wide span of areas such as inventory theory, forecasting, production planning, scheduling, optimisation, communications, management theory, coordination and control theory, systems theory, marketing, logistics, and purchasing theory.

Supply chain concepts are used to achieve enterprise efficiency and to be effective in delivering increasingly customised products at the time of product demand. Supply chain analysis, simulation and optimisation aim to achieve reduced costs and to create transparent systems able to support decision-making throughout the multi-enterprise network and thereby increasing corporate competitiveness. While the mathematical models that are used to represent physical processes and business systems gradually become more complex and demanding, the computational capability of common computers, that are used to solve the problems, is rapidly increasing. This makes it more feasible to handle complex modelling, simulation and optimisation of large systems. Evolving information interfaces between companies and a strong focus on capturing industrial information internally via large scale business systems further enable this progress.

In multi-enterprise supply chains, there exist many decision makers, several layers or echelons, and an extensive amount of data which is mostly not available to all members of the system. Consideration of the huge amount of data in the system creates extreme computational complexity. It is of great importance to develop methods to simplify and aggregate data in a way that does not remove any vital information about core physical processes and business processes from the model. At the same time, the aggregation should make it possible to solve the problem without a prohibitive computational cost.

An inherent problem in highly integrated supply chains is how to divide the payoff of collaboration while ensuring that solutions are optimal in terms of sourcing, production, inventory, and distribution policies. The solutions that are obtained from optimisation usually exhibit payoff distributions that are not sustainable from the long-term perspective because some of the supply chain players often deem their own share of profit disproportionately small compared to the others'. This perception of unfair solutions can result in adversarial attitudes and deteriorating integration of otherwise beneficial supply chain partnerships.

Mechanisms that apportion payoffs between supply chain companies must be transparent and clear to enable each company to apprehend the foundation of the profit distribution system. Transfer pricing of the products shipped between the supply chain companies provides a method of distributing the profit received from customers at the downstream nodes of the supply chain. The "mark-up" of the products at upstream nodes supplies sufficient profit for the respective companies to make these companies sustainable in terms of *e.g.* research investments, shareholder return, and satisfactory quarterly reports. Furthermore, transfer pricing fits well into current company practices of cost accounting and are therefore easy to integrate in common business frameworks.

This thesis aims at finding methods to model and optimise multi-enterprise supply chains. The objective is to obtain representative models that correctly and transparently explain, simulate, and optimise these complex systems. The methods developed utilise optimisation, heuristics, simulation, and game theory techniques, often biased towards algorithmic approaches. A general aim is to measure the relative value of the techniques developed by means of reliable performance indicators. More specifically, the objectives are to create models that can guarantee long-term profitability and sustainability for multi-enterprise supply chain networks and to find optimal methods of setting policy parameters so as to minimise costs subject to required business objective values in the overall supply chain network.

1.2 Thesis Outline

This thesis is divided into six further chapters. In chapter 2, a review of supply chain research is presented. The scope of this chapter is to cover general supply chain issues, previous developed models, and optimisation of supply chain problems. First, general supply chain concepts are introduced. The information sharing capability in contemporary industry is reviewed. Inter-company cooperation issues are further discussed, especially focusing on the concept of supply chain partnerships. Supply chain transfer pricing, with emphasis on profit sharing, is reviewed. Secondly, deterministic, stochastic, and simulation-based models for supply chain modelling are presented as well as appropriate methods for handling inventory modelling and forecasting. Finally, methods for optimisation of complex supply chain systems are reviewed. These methods include

linked simulation/optimisation procedures, multi-objective optimisation and heuristic approaches.

Chapter 3 presents a solution model for fair profit distribution between two supply chain participants. Basic game theory concepts are explained, together with a model based on the Nash bargaining solution. The concepts of mixed integer nonlinear programming (MINLP), branch-and-bound, global optimisation and spatial optimisation algorithms are briefly introduced. A novel spatial branch and bound Nash algorithm is also presented.

In chapter 4, a generalisation of the Nash concepts to the general multi-enterprise supply chain is developed. A general MINLP approach to determine the most appropriate transfer price level for products transferred between production sites as well as from production sites to distribution centres is first presented. The model also considers typical production, inventory, and distribution constraints. To solve the MINLP model, a separable programming approach is applied. To obtain a model of mixed-integer linear programming (MILP) form which has the desired properties of guaranteeing optimality, logarithmic differentiations and approximations of the non-linear variables of the objective function are performed. The approach is then evaluated for a number of test cases, which have up to six cooperating companies.

Chapter 5 describes a methodology for the physical and decision-making business process aspects of supply chain modelling. Pragmatic, non-invasive policy and parameter modifications that improve performance measures are investigated. The aim of the model is to assess and quantify the risks and benefits of changing supply chain operations. To illustrate typical applications of the model, examples in terms of new product introduction and long-term manufacturing shut-down case studies are presented.

In chapter 6, methods for finding optimal policy parameter choices for dynamic supply chain simulation are described. Several approaches, including an MILP approach, a gradient-based approach, and a simulated annealing approach, are developed. Evaluation of the approaches in terms of computational performance and solution quality is then performed for a simple dynamic inventory management problem and an industrial scale supply chain problem.

Chapter 7 concludes the findings of this thesis. Finally, an agenda for future work in the areas of multi-enterprise supply chain planning is presented.

Chapter 2

Multi-Enterprise Supply Chain Problems and Characteristics

The supply chain ideology has in recent years become one of the main approaches to achieve corporate effectiveness and efficiency. The terminology implies that a systems view is taken on the inter-linked network of cooperating companies rather than a functional or hierarchical one. Enterprises have generally experienced an increased degree of competitiveness by regarding their activities from a supply chain point of view. This is partially due to the evolving higher specialisation in a more differentiated market, with companies being less commonly vertically integrated. But first and foremost, competition drives companies to reduced cost structures with lower inventories, more effective transportation systems, and transparent systems able to support information throughout the supply chain. A single company rarely controls the production of a commodity as well as sourcing, distribution and retail activities. Supply chain networks comprise all these activities as well as all the links between them. Today, and even more so in the future, it is widely believed that supply chains rather than single companies will compete in the market (Monczka and Morgan, 1997). The Supply Chain Council defines the supply chain as:

> *The supply chain - a term increasingly used by logistics professionals - encompasses every effort involved in producing and delivering a final product, from the supplier's supplier to the customer's customer. Four basic processes - plan, source, make, deliver - broadly define these efforts, which include man-*

> *aging supply and demand, sourcing raw materials and parts, manufacturing and assembly, warehousing and inventory tracking, order entry and order management, distribution through all channels, and delivery to the customer.*

One of the most challenging issues is to create ways to integrate the supply chain utilising standardised concepts, which is why the Supply Chain Council have been developing the Supply Chain Operations Reference model (SCOR) since 1997. The model is used to measure supply chain performance, track weak areas, and develop methods for efficient solutions. The Supply Chain Council has several hundred members including some of the largest manufacturers in the world (*e.g.* Bayer, Compaq Computer Co., Eastman Kodak Co., Motorola Inc., Procter & Gamble, and Texas Instruments) (Lummus and Vokurka, 1999*a*).

Gable (1997) describes the history of the modern consumer goods industry as a series of successive steps. The first period after World War II is characterised by market driven trends where enterprises produced a relatively small number of products in bulk, though at prices accessible to the general public and generally of good quality. Distributors and retailers bought in large volumes. The distribution channels were slow and erratic. As more products were introduced the plants and warehouses started to become clogged with inventory, and starting from the mid 70s, an explosion in stock keeping units (SKUs) becomes evident. High supply chain costs made retailers receive lower margins on investment. A hostile environment evolved between manufacturers and retailers, because of trade promotions invented by manufacturers to induce larger demand for the new SKUs. Since the 80s, retailers have generally gained more power in the supply chains because of market dynamics. Because of inherent power relationships there is clearly a need for a rigorous systems approach to the analysis of supply chain performance and optimisation, in order for all supply chain players to reap the benefits associated with the individual consumer needs. In the early 21st century, a tremendous interest for corporate supply chain improvement is being manifested in terms of both the number of academic papers published and the software applications developed.

In the past, academic achievements in supply chain applications have been either relatively small scale and theoretical focusing on inventory replenishment dynamics or else on the network design of a new supply chain implementation. Shaw and Fulkerson (2000) state that models concentrated on analytical modelling of production systems in order

to obtain optimal solutions. These latter type of papers include strategic optimisation models which normally determine the location of factories and warehouses, the resource provision and capacity of production, and the distribution and warehouse resources. Production management is generally regarded in an aggregated fashion with simplified capacity constraints. Furthermore, operational problems regarding supply linked to production and production linked to distribution have been studied widely but with focus on individual components in the overall supply chain. The most interesting problems in the supply chain research today include how to tackle the inherent uncertainty in the systems and how to integrate all components in a single model. Uncertainty occurs due to operational discrepancies from planned activities, fluctuation of demand from forecasts and from new products, and from new operational practices being introduced. Hence, in the light of system complexity and the variability of constraint fulfillment due to the inherent uncertainty, one is sometimes forced to find "satisfactory" solutions rather than optimum solutions. Capturing the innovation and product introduction phases as well as forecast deviations and influence of business processes are also still relatively un-probed areas of supply chain research.

From a corporate perspective, Brady (2000) suggests that the largest savings in terms of supply chain activities are to be found in the customer/supplier relationship costs, inventory management and the degree of efficient utilisation of fixed assets. In this context, supply chain applications such as the first generation e-commerce software focused mainly on the buy and sell relations of each individual company and the recording of a vast number of financial measurements through enterprise resource planning (ERP) systems. Whereas this type of software definitely filled a gap for a number of multinational enterprises in terms of information collection, it did not create the huge savings many supply chain managers had hoped for. These applications focussed primarily on unplanned buying/selling activities whereas the planned buying and selling and the operational planning were still conducted by conventional methods. Newer applications incorporate to some extent the planning of supply chain activities to create smooth production patterns to fulfil customer demand. The buy/sell relationships are still very much emphasised albeit with more application geared towards coordination and synchronisation of production and distribution activities. Areas which are currently under development in newer applications are forward visibility of demand and the efficient incorporation of new products in

demand management and planning execution. Most products today display a high degree of demand variability and it is of vital importance to be able to cope with these dynamics, either by enhanced forecasting methods and market intelligence, or preferably via short production lead times and flexible manufacturing systems. Of course the degree to which demands for new products can be forecast accurately will depend on the business. The manufacturing tools will enable faster decision-making by bringing more information to the decision makers instantaneously. The vision within industry is that strategic sourcing of raw materials and promotional sales planning will soon be carried out continuously. Order management and fulfillment of orders requested are thought to be executed within seconds whereas the external supply/demand synchronisation should be handled within a few hours. Supply chains are soon thought to be able to fully introduce a new product within a number of weeks, at least in some leading business sectors.

The suggested improvements clearly demand strong internal supply chain understanding and supply chain partners who are also willing to develop continuously. Successful supply chain participants are likely to have well-developed customer and supplier relationships, planning systems that can handle a flexible demand environment, and systems that will capture the current internal and external pressures and opportunities facing the company and processes that determine where to buy, what and how to produce, what to promote, and what demands to satisfy.

2.1 General Multi-Enterprise Supply Chain Issues

In the context of the general evolution of industry, Wang (2000) regards the enablers for present day emerging industries such as micro-nano technology, biotechnology and advanced material technology are information technology, supply chain management, modelling and simulation, as well as human development and knowledge management factors.

There has been a substantial amount of industrial practice to achieve improved supply chain performance in terms of manufacturing high quality products at competitive prices with short lead times. Typical supply chain activities include: source, manufacture, buy, sell, move, merchandise, measure customer satisfaction in a multi-enterprise environment where the supply chain is constantly attacked by other competitive supply chains (Gable,

1997). Information support from ERP systems (comprising financial management, order processing, inventory management, purchasing, and bill-of-material management) and electronic data interchange (EDI) are complementary to the development of supply chain systems models.

Beamon (1998) provides a review of literature in multi-stage supply chain modelling together with an agenda for future research. She interprets the interest for supply chain performance measures, design and analysis as a result of an industry where costs of manufacturing are rising, resources are relatively becoming more scarce, product life cycles are shortened and the market place is globalised. Previous literature is divided into deterministic analytical models, stochastic analytical models, economic models and simulation models. Performance measures can either be qualitative or quantitative. The qualitative performance measures indicate to what degree a customer, internal or external, is satisfied or how flexible the supply chain is in terms of absorbing or responding to fluctuations in demand. The quantitative performance measures are either based on cost (cost minimisation, sales maximisation, profit maximisation, inventory investment minimisation or return on investment maximisation) or on customer responsiveness. In the research agenda Beamon highlights the demand for combined performance measures, analysis of critical supply chain variables, modelling of postponement of production (*i.e.* moving final assembly downstream in the supply chain), global supply chain modelling and demand distortion.

Bhatnangar *et al.* (1993) examine the areas of supply-production coordination, production-distribution and inventory-distribution, *i.e.* exploring integrated planning across pairs of company processes. They conclude that there is a need for coordinated efforts including supply planning, production and inventory planning, and distribution planning. They claim that determining reorder points, reorder quantities (lotsizing and safety stock issues), and the bull-whip effect (the destabilising effect of increasing volatility of demand as it passes upstream through the supply chain) are crucial factors that influence supply chain performance.

Backx *et al.* (1998) analyse the supply chain from a systems perspective. They emphasise the dynamics of the supply chain and the demand for adaptability to an increasingly changeable environment. Especially, in the chemical industry it seems as though performance measures are focussed on manufacturing rather than on dynamic operation.

Some early conceptual work were presented by the following authors: Bonney *et al.* (1996) present a hierarchical top-level model of a company in which approximate assessment can be provided before moving down a hierarchy. They investigate the consequences in terms of inventory levels, delivery performance and cash flows. O'Brien and Head (1995) work on developing a full business system environment to support just-in-time logistics. They investigate electronic system information sharing and its effects - costs and benefits - on partners, and the supply chain as a whole. Thomas and Griffin (1996) concentrate their efforts on coordinated planning between two or more stages in the supply chain, placing particular emphasis on models that would lend themselves to a total supply chain model. Slats *et al.* (1995) investigate the possibilities of improving the total logistics chain by building an extensive experimental environment, where suitable typical operations research models are integrated.

2.1.1 Supply Chain Management and Decision-Making

Supply chain management is defined by Mentzer (1999) as "systemic, strategic coordination of the traditional business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole". The supply chain concept is regarded to have its greatest merits from trying to integrate interrelated cross-firm business activities.

Management of supply chain activities include decisions about production (including decoupling of processes to increase flexibility), supply, demand fulfillment, distribution, inventory, financial issues, price, risk and continuous change. On all levels of the supply chain, decision-making procedures will determine the success of ideas implemented. The hierarchy of who is in charge of which decisions taken will be based on the time frame, local authority and the level of aggregation. Decision-making is generically divided into four levels based on the level of aggregation and involvement of senior management: scheduling, operational, tactical, and strategic levels. van Hoek (1998) states that the more integrated the supply chain network becomes the higher the focus on strategic issues.

While centralised management is appropriate for long-term strategic planning and tactical planning on a time-scale of years and months, local authority should generally

in part be administered to control short-term planning, operations, and scheduling. In a typical enterprise computer system, centralised decision-making functions on a top-down approach basis to control aggregated information. Decentralised systems work more directly with equipment and manpower on-line utilising real-time decision support to help solve disruptions and failures in plants and distribution centres.

Lee (2000) states that by building organisational linkages companies are enabled to share risk, costs and gains. But before this can be achieved companies must build an environment of trust by information and collaboration. One of the most important aspects of organisational linkages is that the reward systems give incentives for all members of the supply chain to work to improve common, agreed and clear measurable objectives. By letting the right people be responsible for the right matters (*i.e.* let the ones who have the strongest influences on the processes also be responsible for them), such as vendor management inventory and continuous replenishment planning implementations, detrimental effects of demand distortion can be minimised. Overall, shared performance measures are needed to ensure that all parties are accountable for their part of the united pursuits. Obviously, all supply chain partners should be able to access these measures continuously via shared information systems. In an earlier work, Lee and Billington (1992) report that it is common in many supply chains that one often discriminates against internal customers because external customers bring about real revenue. Explicit measures are stated for external customers but not for internal customers. This leads to less than satisfactory visibility of internal delivery performance.

Christopher (2000) reports that within industries that have implemented efficient production policies the benefits are often restricted to the factories and even though throughput times for car industries can be less than twelve hours, finished goods inventories can be up to two months worth of sales. The author argues that a more agile and market sensitive supply chain will be able to respond to demands both in terms of volume and variety of products. The conventional logistics rules and formulae that quantify inventory along the supply chain will be made invalid once appropriate information systems and integrated processes are in place within the supply chain partnership network. The author furthermore recommends the implementation of joint strategies, buyer-supplier teams, information transparency, and open-book accounting. He reasons that whereas the so called "lean" or "Japanese" production policies work well for high volume, low

variability, high predictability environments they do not perform well for environments where requirements for variety is high and for those where demand is volatile. The suggestion is to use the decoupling point as a watershed: up to the decoupling point lean policies should be used (which well may be based on forecasts) and beyond agile, flexible systems should be in place that work off real demand. By keeping generic inventories close to customers one can postpone final production and thereby increase the flexibility in meeting highly volatile consumer demands. The author concludes that the material decoupling point should lie as far downstream as possible thus enabling responsiveness to end customer demand whereas the information decoupling point to which the actual demand penetrates should lie as far upstream as possible.

Applequist *et al.* (2000) develop a deterministic mathematical programming model which accounts for demand and inventory uncertainties thereby providing a model that can balance supply chain risk in chemical manufacturing industries. The authors reason that the chemical industry supply chain complexity arise from a number of "dimensions", namely, physical scale, temporal scale and degree of uncertainty. Their model consists of statistical elements to account for uncertain parameter distributions, design and operation elements to determine investments as well as production plans and stochastic elements to determine revenue flows. A risk premium definition is introduced which obtains a trade-off of increasing return for increasing risk in terms of the net present value.

2.1.2 Echelon Interaction and Cooperation

Many authors have recently claimed that new supply chain paradigms are evolving. Bartezzaghi (1999) argues for more strategically flexible production models which can handle high degrees of complexity and incorporate novel product development with reduced delivery times as the primary objective. He suggests a more integrated and collaborative business approach with a delegation of core processes, thus combining local and global information to obtain multi-focussed, flexible processes.

Lummus and Vokurka (1999*b*) notice that many companies already have implemented systems for EDI or information exchange via the Internet. A strong move towards customised products and flexible customer service will further boost this development. Hence, supply chains evolve to have processes more open to visibility and information

sharing. Moad (1997) argues that the interfaces between companies are becoming more like "permeable membranes" in which information flows continuously rather than being fixed, discrete links between suppliers and customers. The direction is towards a single, extended enterprise that includes suppliers and customers, in which electronic commerce plays a very important rôle. Moad states that the cost of planning of production levels, sourcing of raw materials, handling inventory and delivering products to market is 14% of total costs in an average appliance manufacturer industry. Many manufacturers are giving direct access to information to suppliers and distribution channel partners for tracking inventory levels, shipment status, demand forecasts and other relevant data. This external sharing of information is lowering inventories and improving performance. The key requirements of sharing information are: common communication infrastructures; common data representation; fault tolerant communication systems; secure message systems and guaranteed user authentication.

Shaw and Fulkerson (2000) maintain that information systems are required in order for companies to integrate shop-floor activities with business processes like order fulfillment and product development. They emphasise that inter-organisational and intra-organisational operations coordination is vital to capture an adequate process view of the enterprise. This coordination is seen to occur in a form more resembling marketplace conditions with decentralised pricing and exchanges. The authors call for an increased capability of coordination of the business and the manufacturing parts of the supply chain. They conclude that the supply chain has received its popularity because of its rôle of executing business strategy in increasingly global, multi-enterprise business systems.

Partnerships are extremely popular as a concept among operations researchers and management consultants but they are difficult to implement due to opportunism and latent power relations between departments and separate enterprise business units (Maloni and Benton, 1997). Negotiations and profit-focus are still the most common practices in business. It is apparently not easy to achieve a shift from conventional adversarial attitudes towards openness, trust, and information sharing. However, this shift in attitudes is likely to occur in the near future according to many researchers as the benefits of cooperation becomes clear.

Mentzer *et al.* (2000) examine partnerships not only from a strategic viewpoint but also from an operational viewpoint and study how and to what extent partnerships are

formed. They consider operational partnerships (which requires less time, effort and investment than strategic partnerships) as a far more common and appropriate type of partnership in most cases in industry. Suggested surrounding pressures that give rise to partnerships are typically environmental uncertainty, global competition, and time and quality based competition. Some requirements for formation of partnerships are interdependence, trust, commitment, organisational compatibility, and top management vision. The degree of fulfillment of these prerequisites will determine the partnership formed, *i.e.* strategic or operational. In the following implementation phase, the partners will have to decide on issues such as information sharing, which technologies to use, utilisation of strategic interface teams, how to resolve organisational differences, ownership of invested assets, and establishment of joint performance measures. Finally, the authors view the major business performance outcomes that have to be evaluated as the improvement in economic performance, customer satisfaction and loyalty, and the further effectiveness of the relationship.

Bowersox *et al.* (1999) state that intra-firm coordination of supply chain activities such as sourcing, production and logistics make it possible to respond to the actual market volatility by the introduction of a seamless flow of demand information and reduced supply chain response times. The authors study so called lean launch strategies based on response based logistics in which inventory deployment methods for new product introduction are prescribed. They conclude that supply chain collaboration can reduce the risk when launching a new product.

Spekman *et al.* (1998) regard partnership activities as involving risk-sharing, collaborative design development, and achieving highly efficient processes. They point to the success stories of General Motor's Saturn project and the Toyota philosophy distinguishing between the superficial cooperation of many other car manufacturers and GM and Toyota's more closely integrated collaborative production processes. They argue that most companies today are well motivated to maintain long-term relationships, continuous information sharing and fair play in order to obtain supply chain integration, joint planning and technology sharing which is perceived to result in more competitive supply chain performance.

Lehtinen (1999) quantifies partnership collaboration from a field study perspective. In her study, a doubled cooperation rate within the manufacturing industry has been

proved in the last ten years and 20% of the sub-contractors now handle 80% of the orders. She maintains that delivery performance is the best order-winning criterion, rather than *e.g.* product quality. The elements constituting partnership are seen to be ideas and information exchange; joint problem solving; and long-term trust and faith. The study also shows that a large amount of commitment to "partnering" is required by the contenders and trust only builds up over time.

It seems that partnerships are most appropriate for single sourcing supply chain or supply chains with dual sourcing, *i.e.* just a few buyers and/or suppliers, with high initial investment, and research and development (R&D) costs. The switching cost for both suppliers and buyers to other partners in this type of industry can often automatically make partnership contracts binding and deter companies from finding temporarily cheaper alternatives. Dowlatshahi (1999) emphasises that a fully integrated partnership supply chain network tends to focus on strategic planning rather than on the price based issues in the interface of buyers and suppliers in the network and the prices of products are therefore determined on contracts instead of free market prices. Flexibility, service, and total costs of the overall network are measured as major performance objectives. Partnerships convey the benefits of vertical integration without the inherent risks of a single enterprise ownership. There has been little research performed to study actual performance implications of partnerships. An ability to measure and predict the performance of the supply chain is believed to strengthen a stable relationship between partners.

However, Gavirneni (2001) tries to estimate the benefits of cooperation in a modelled supply chain environment. The single supplier, n-retailer model considers a sequential order-shipment cycle for each time period in which (a) retailers place orders to the supplier after an inventory review, (b) supplier use information available to satisfy as much demand as possible, (c) retailers receive shipments from the supplier and tries to satisfy end customer demands, (d) holding costs are paid for excess inventories and penalty costs are paid for unsatisfied demand. The cooperation considered includes demand and inventory level sharing between supplier and each retailer and also inventory shipments between retailers. Furthermore, optimal ordering policies are considered for various levels of cooperation. The author concludes that benefits of cooperation increase with increased customer demand variability from forecasts, increased penalty costs for unsatisfied de-

mand, decreased supplier capacity, and decreased number of retailers.

Maloni and Benton (1997) have made a large review of what they call "Supply chain partnerships: Opportunities for operations research". They regard the supply chain partnership as equivalent to a strategic alliance, both of which they define as a relationship formed by two independent members of a supply chain to achieve specific benefits and objectives. The members aim to reduce total costs, inventories and increase the level of information sharing and thereby increase the operational and financial performance of the supply chain as a whole. Maloni and Benton emphasise the intention of the manufacturers not only to be concerned with price, but also to look for suppliers who will be able to work cooperatively in providing improved service, technological innovation and product design.

They notice that recently there has been a steady stream of conceptual and qualitative literature on supply chain partnership, which claims that mutually beneficial, win-win partnerships will be created through synergistic effects. But they also emphasise the difficulties of overcoming traditional social attitudes with corporate change as well as managerial, procedural and structural obstacles. In general, Maloni and Benton recognise that very little research has been performed in the quantitative field of supply chain partnerships and of supply chains as a whole. The authors conclude that there are large opportunities indeed for research within design, implementation and control of the extended supply chain.

Maloni and Benton further suggest supply chain partnerships as a way of bridging the barrier between suppliers and buyers, enabling manufacturers to ally with a reduced supplier base. The bargaining power of the manufacturer and the sheltering against interruptions in the supply chain in the competitive environment have always been regarded as sufficient reasons for implementing strategic alliances. The manufacturer in the conventional system could also try to negotiate lower costs, higher quality, reasonable delivery times and special needs. On the other hand, the partnership perspective leads to less adversarial attitudes, lower switching costs and decreased shipping errors. They also suggest that partnerships can lead to quantity and relationship based discounts as well as a decreased cost of quality. As customer satisfaction is becoming the most important factor to maximise in the objective function of the enterprise, partnerships comprising vertical relationships that involve long-term, interdependent planning, and operations

are regarded as being able to transform the way industries are run.

According to Graham *et al.* (1994), the advantages of partnerships are not visible until after at least three years of alliance. Reliance on a supplier leads to disastrous effects if the supplier does not meet the expectations. Companies also risk decreased competitiveness because of loss of partnership control and complacency with partners due to specialisation. Thus, the effects of supply chain partnerships have to be measured by adequate performance indicators. This is a demanding task indeed due to the inherent complexity of the systems to be analysed.

A recent study by Frohlich and Westbrook (2001) attests that a strong supply chain integration policy has great impact on enterprise performance. In fact, they state that the greater the degree of integration both upstream and downstream in the supply chain the better the performance. Their paper shows that integration and information sharing with only customers or only suppliers will not have a large impact on their performance measures. It is interesting to note the number and variety of measures improved by having an integrated supply chain policy. They include marketplace measures such as company market share, profitability and return on investments; productivity measures such as unit manufacturing costs, materials and overhead costs, procurement lead time, production lead time, delivery lead time, equipment changeover time, inventory turnover (sales/inventory), and labour productivity; non-productivity measures such as customer service and satisfaction, conformance quality, product variety, speed of product development, number of new products developed, on-time delivery, and supplier quality. All these performance indicators were strongly linked to the degree of integration in the 322 international companies that had been surveyed. In the light of this study, it is obvious that information sharing and supply chain integration is highly beneficial for business.

The issue of cooperation between companies in a supply chain which would otherwise be compete with one another has been called *coopetition* within literature. Bengtsson and Kock (2000) suggest that this type of relationship is complex since it has paradoxical logic due to the simultaneous conflicting interests and common interests. From their empirical evidence, it seems that companies more frequently cooperate in echelons upstream (*i.e.* away from the consumers) due to heterogeneity of resources in each supply chain company. They also point out that a focal firm often has to be involved in several separate relationships to maintain its competitive advantage.

To conclude, the benefits of supply chain partnerships are joint problem solving with an ability to plan and be able to forecast long-term profitability, to gain from multi-disciplinary expertise, to share future risks, to be able to share complex information, and to achieve transparency with resulting mutual business strategies. The drawbacks comprise a heavy reliance on another party implying a substantial amount of risk taking, loss of internal control, risks of adversarial attitudes leading to failed partnerships, potential implementation problems, a general lack of inter-company trust, difficulties in switching suppliers and distributors. However, the benefits of partnerships have been demonstrated clearly in areas of automotive and computer industries and visionary researchers do believe that many enterprises eventually will become virtual networks since the technological capabilities are present to create on-line systems spanning the entire supply chain (Port, 1999).

2.1.3 Transfer Pricing

Transfer pricing is a term that most commonly is utilised to denote an intra-company purchase cost, primarily used as a performance measure of specific sub-divisions of large enterprises. Abdallah (1989) defines transfer price as the price that a selling department, division or subsidiary of a company charges for a product or service supplied to a buying department, division or subsidiary of the same firm. The transfer pricing decisions affect marketing, production, location, transportation, and finance. It is useful to extend the terminology as to include the whole supply chain and thereby cover the inter-company price mechanisms if the supply chain is considered as one profit generating entity. O'Connor (1997) argues that transfer prices are utilised because there exists tension between general goals of a global company and internal goals of its subsidiaries. Furthermore, constraints from the legal environment involving taxes and tariffs further accentuate this tension. Many firms seem to design transfer pricing policies so as to minimise tax by taking advantage of tax structures in different countries.

Howe and Cox (1994) claim that the internal transfer price of an organisation does not affect the overall profitability or the financial measurements in general on the enterprise level. The transfer price applies to the local organisational level, thus enforcing market pressure into the organisation. In the supply chain of a single enterprise, a supplier is encouraged to sell to external customers if they would pay a higher price than the

downstream echelon. Similarly, the downstream unit tries to minimise the price of a delivered product. Howe and Cox point out that the internal collaborating echelons might be more reliable in terms of quality and schedule (a matter of dedication), a factor which has to be taken into consideration when determining the transfer price. Their results furthermore emphasise that local decision-making within the supply chain can be quite harmful if it is not linked to global evaluation of the performance.

Pfeiffer (1999) describes transfer pricing in a supply chain consisting of procurement, manufacturing and selling units of one single company. His theoretical model handles one commodity at each node and does not include any capacity constraints. He proposes a transfer price system governed by the headquarters, which fixes a specific transfer price level. Each node optimises its own decisions independently to maximise a given profit function, according to the price level fixed by the headquarters. After the decentralised optimisation, headquarters evaluates and collects the overall results obtained and chooses a new transfer price which leads to a higher overall profitability. A Lagrangean dual decomposition approach is utilised where each problem is a linear program (LP). The algorithm stops when the overall profit function is maximised. In common with the approaches proposed later in this thesis, Pfeiffer uses the transfer price as a value measure of the products. As would be expected, his conclusion is that the stock level and the production intensity (output/time) in the process system should be chosen to be minimal, thus minimising inventory and production costs subject to production requirements.

Alles and Datar (1998) claim that companies choose their cost-based transfer prices based on their competitive environment. The enterprise may cross-subsidise products in order to increase their ability to increase prices. Often, transfer prices for relatively lower cost products are increased while higher cost products are decreased. The authors give evidence that transfer prices are determined based on strategic decisions rather than by internal cost systems.

Jose and Ungar (2000) propose an approach to decentralised pricing optimisation of interprocess streams in chemical industry companies. Their iterative auction method determines the prices of process streams so as to maximise an objective for a single chemical company, while each division within the company is constrained by its available resources. The approach is interesting in that each division conceals its private information from the other parties within the so called "micro supply chain". It normally takes

several iterations for a model to converge and the user has to define the limited amount of slack resources utilised. One of the main conceptual differences between their approach and the one presented later in this thesis is that they regard the channel members as adversarial and competitive for resources rather than cooperative.

Vidal and Goetschalckx (2001) find that many global supply chain models assume that transfer prices are fixed and given. They state that it is widely accepted to regard utilisation of transfer prices by "arm's length principle", *i.e.* the involved parties are not considered to be part of the same company when transfer prices are set. The authors see the objectives of transfer pricing as (a) satisfy tax and other legal requirements, (b) production of profit maximisation by minimising global taxes and duties, (c) production of a fair framework for performance evaluation and motivation of all involved parties, (d) moving funds globally, (e) minimisation of exchange rate risk, (f) avoidance of exchange quotas and controls, (g) increase of profit share from joint ventures. A global distribution and transportation optimisation model is proposed which includes transportation, capacity constraints, inventory balances, and cost definitions. The resulting problem is non-convex with a linear objective function and linear and bilinear constraints. A heuristic successive LP solution is implemented, due to the large problem size, in which the nonlinear constraints are linearised by introducing new variables. By fixing iteratively either the flow variables or the transfer prices, the authors report that convergence can be obtained at least to a local optimum. They further introduce some heuristic rules to find an adequate initial transfer price starting point to improve the solution procedure.

Lindsey *et al.* (1996) show that as the degree of private information decreases in a buyer-seller relation, *i.e.* when more information is available to both parties, the amount of mutually acceptable price levels decreases also. They point out that in the bargaining process the parties show their beliefs and knowledge through their bidding behaviour. The surplus payoff that one party can attain depends on its level of private information. Lindsey *et al.* suggest that a split-the-difference procedure, where both seller and buyer submit sealed bids on a product, is advantageous compared to the classical seller-offer procedure, where the seller offers a price on a take it or leave it basis, since it makes use of both parties' private information. The procedure can thereby lead to successful sales more often. They show that given a fair amount of common values, the seller will also benefit from the split-the-difference procedure.

Li and Balachandran (1997) assert that if a system is characterised by asymmetric information, where one player has a different information supply source than the other player, then the optimal production levels are different from those of full information. This is irrespective of when the decision maker acquire his information. This implies that consideration has to be taken so as to somehow incorporate the way information is retrieved within the production planning system and the supply chain network as a whole.

2.1.4 Performance Assessment

A performance indicator can be defined as a variable indicating the effectiveness and/or the efficiency of a part or the whole of a system compared to a given target (Fortuin, 1988). Means to measure the success of supply chain implementations are not well defined in the literature. Since supply chain systems are complex systems with a multitude of facilities at each echelon, there are numbers of indicators that could be taken into consideration.

Measured performance related factors in supply chain management include various operational and long-term financial performance measures. Reduction of cost is the most common objective in many optimisation projects. Typical supply chain costs comprise operating costs such as inventory cost, handling cost and other administrative costs, distribution costs, manufacturing costs, raw material costs, maintenance costs, and transaction costs as well as fixed costs (location overhead costs, labour costs, manufacturing equipment), taxes, duties, and transfer costs. Responsiveness indicators include lead time, fill rate, on-time deliveries, and stock out probabilities. Flexibility measures include issues such as product changeover capability, product capacity changes, an ability to handle periods of poor manufacturing, supplier, and delivery performance, and an ability to accommodate new markets and competitors.

For marketing departments, maximisation of sales revenue is often the common objective. In industry, one often has to prioritise the customer satisfaction level arbitrarily against the cost measures. The asset utilisation metric is often primarily considered in the overall corporate risk management strategy but needs to be continuously evaluated in the ongoing supply chain improvement efforts. Manufacturing and development flexibility measures will become more important as the agent of change is built into the processes themselves. The variety of performance objectives generally makes it difficult

to achieve consistent and effective operations at the separate echelons in the supply chain since different departments strive for individual targets.

Lee and Billington (1992) also state that one of the main problems of supply chain management is autonomous management teams working in divergent directions driven by their own objectives. They emphasise that these operational performance measures may have little relation to the operational and/or strategic performance of the total supply chain and may even conflict with them. The authors also report that some of the overall performance metrics that can be used are total inventory turns, response time to orders, service fill rates, lateness relative to customer order due dates, and average back-order levels. Information systems should be linked to the production planning activities and include order forecasts, current backlogs, inventory levels, shipment due dates, and production capacities. They reason that failure to integrate these factors incur long planning cycles that increase forecast errors and reduce the ability to respond to updated customer orders. Hence, the manufacturing sites end up manufacturing the wrong products which leads to high stock levels and high back-order levels. Uncertainties that typically cause concern are reported to be supplier lead times, supplier delivery performance, machine downtimes, process yields, maintenance, lead times, and variability of demand. The magnitude of these uncertainties is often not clear to a typical supply chain manager and there are often no systems in place to track and document sources of uncertainty, claim the authors. Even though the paper by Lee and Billington is one of first in the explicit supply chain management literature, the conclusions about the need for design of processes for supply chain management, integration of databases throughout the supply chain, integration of control and planning systems, redesign of organisational incentives, and the inception of overall supply chain performance measures are still valid.

Neely *et al.* (1995) characterise types of performance indicators in systems analysis. These comprise quality, flexibility, cost, and time measures. A model may aim at enhancing one single performance indicator type, *e.g.* cost. The analysis on the highest strategic level should assess whether a performance measure reinforces the company's strategies, whether it matches the organisational culture, whether it is consistent with existing reward structures, whether some indicator focuses on customer satisfaction, and whether some measure focuses on how well a company fixes attention on its core activities. Concurrent performance measures are said to fail because they: encourage short-term

thinking; lack strategic focus and do not enhance performance since they do not provide data on quality, responsiveness, and flexibility; encourage local optimisation; encourage minimisation of deviation from a given standard rather than emphasising on continuous improvement; and fail to provide information on customer requirements and competitor performance.

Enterprise performance is often captured in terms of return on assets. This measure is quite appropriate for measuring supply chain success since it captures different aspects of performance. Revenue is to be maximised whereas the expenditure and the assets utilised to produce this revenue is to be minimised. In the supply chain context, revenue is maintained by forecasting, fulfilling and creating new customer demand. Managing the revenue stream also includes new product development and how to integrate new products in the supply chain activities. The reduction of costs is obviously closely linked to asset management. High cost efficiency and adequate asset management means high utilisation of available resources, be it production line utilisation, warehouse utilisation, inventory management or labour usage. In spite of this, asset management is often considered a more strategic issue than primarily reducing costs in the supply chain.

Christopher and Ryals (1999) state that an appropriate supply chain strategy can positively influence the main drivers for shareholder value which are revenue growth, operating cost reduction, fixed capital efficiency, and working capital efficiency. Increased reliability, responsiveness, and customer delivery service levels are suggested to boost the revenue flow. Apart from obvious reductions in transportation, inventory, handling and order processing costs that can be achieved by supply chain policies, there is also room for cost reductions through time compression and upstream logistics. Rationalisation of distribution and production networks through third-part logistics and manufacturing has furthermore lead to proportion of investments in fixed assets in terms of warehouses, distribution centres and local depots. By pipeline time compression and reduced order-to-cash cycle times, the working capital efficiency is seen by the authors to be significantly increased. They report that most companies do not know the true supply chain lead times of products and that unrealistically low costs of capital are employed. Supply chain time compression leads to higher responsiveness and lower inventory levels. The authors claim that this can be obtained via partnership sourcing and synchronised deliveries on the supply side, fighting of non-value added time and complexity and by

postponement of manufacturing internally, and by vendor managed inventory and continuous replenishment on the customer side.

Most supply chain models have used cost as the only performance indicator or combinations of cost and customer responsiveness measures, *e.g.* Cohen and Lee (1989), Cohen and Moon (1990), and Pyke and Cohen (1994) focus on cost; Towill *et al.* (1992), Lee and Sasser (1995), and Pyke and Cohen (1993) focus on cost and customer responsiveness. Arntzen *et al.* (1995) has many different costs, duties, and taxes as well as activity time as performance measures. Flexibility for future potential performance behaviour is exemplified by Voudouris (1996), who defines flexibility as slack in resource constraints (capacity less planned utilisation). The slacks are given for activity constraints (*e.g.* manpower and packaging lines) and are qualitatively differentiated by weights.

What has not often been considered when measuring supply chain performance is the way that supply chain partnerships and external relationships affect the key indicators. Somehow these factors are often taken for granted as fixed, thereby a lack in the supply chain understanding is manifested. Moreover, the potential overall financial impact of supply chain decisions is currently quite vague and difficult to implement ahead of implementation. Timme and Williams-Timme (2000) state that the impact that supply chain improvements have on company growth and capital utilisation is often not quantified and many aspects of profitability are excluded since they are not regarded as the field of the supply chain manager. Thereby, systems fail to provide information of the vital importance of supply chain metrics and of the impact that supply chain solutions can have on performance. Here, growth, profitability and capital utilisation are suggested as the financial key drivers affected by supply chain modifications. The key non-financial driver in a supply chain is typically some type of customer service level measure.

Beamon (1999) notices that numerical values do not capture the complex system performance of a supply chain. It is vague and difficult to use one single performance indicator to measure overall supply chain success. A large number of indicators have been developed for production, distribution, and inventory systems, but not even here is an agreed standard to be found. Beamon asserts that characteristics of supply chain performance measurement systems should include an ability to measure all relevant aspects of the performance, allow for comparison under different operating conditions, that the data should be measurable, and finally that the performance indicators should aim

in the same direction as the enterprise strategic goals. She finds three typical performance measurement types: (1) Resource focussed - aiming at a high level of efficiency; (2) Output focussed - aiming at a high level of customer service; (3) Flexibility focussed - measuring an ability to change to a variable environment quickly.

Gable (1997) concludes that while many companies may have a good level of short-term performance today, their interaction pattern with suppliers, buyers, customers, and other functions within the company significantly impairs long-term profits and competitiveness. The enterprise supply chain strategies aim to increase sales, distribute legitimate rôles to each channel member, and reinforce the brand image. Much of the large costs that consume enterprise profits are incurred between the separate nodes. Costs must be analysed at every level of the supply chain, including the final customer and the primary source. The most advanced supply chain markets are said to be clothing, automotive, and grocery industries where a program called "efficient customer response" is said to have saved the industry an estimated 11% of total systems cost.

Sabri and Beamon (2000) describe a model that incorporates traditional cost measures, customer service levels in terms of fill rates and volume and delivery flexibility measures for use in simultaneous optimisation of strategic and operational supply chain planning. The authors propose an iterative method which switches between a strategic sub-model and an operational sub-model, both of which are MILP models, until convergence is obtained. A multi-objective performance vector is used comprising the four required performance indicators. In both the strategic (volume flexibility) and the operational (delivery flexibility and customer service performance) sub-models, one has to pre-specify the desired lowest flexibility indices and customer service levels which act as model constraints.

Quantitatively, some researchers provide an idea of the scope of supply chain change. Krajewski and Ritzman (1996) state that 60% of total sales for a typical manufacturer is spent on purchased material. Thus, it is obviously desirable that the supplier is reliable and will deliver right quantities at the right time with short lead times. Metters (1998) quantifies the bull-whip effect, claiming that elimination of this effect would increase product profitability by 10-30%.

Bowersox *et al.* (2000) propose ten "mega-trends" that are going to change the execution of supply chain management in the 21st century and assess how far the average

Now	Later
Customer service	Relationship management
Adversarial	Collaborative
Forecast	Endcast
Experience	Transition strategy
Absolute value	Relative value
Functional	Process integration
Vertical integration	Virtual integration
Information hoarding	Information sharing
Training	Knowledge-based learning
Managerial accounting	Value-based management

Table 2.1: Ten supply chain mega-trends. (Bowersox *et al.*, 2000)

US company has reached in terms of change. These trends are displayed in table 2.1. The most prominent changes, and hence where the current performance is lowest, is in the areas of change management (transition strategy), relative value (measure profit margin rather than gross profit as ultimate objective), endcasting (primarily sharing of demand information to production planning), shifting towards collaborative attitudes, and knowledge-based learning (skill development in the context of overall process view of objectives, dynamics and measurements).

2.2 Modelling

As with any kind of modelling, supply chain modelling trades off computational efficiency against model detail. By choosing an appropriate level of abstraction and using clever computational techniques, efficient supply chain models that fulfil the criteria with respect to resemblance of the modelled environment can be obtained. The core focus of the model must be clear prior to the model development. For example, a supply chain planning model comprises objects for sales/pricing, ordering, production management, distribution management, inventory management and raw material sourcing/purchasing. Apart from these physical processes, the model should also include ways of describing the business processes under which the company operates. The most promising models

in this respect seem to be models that combine simulation of stochastic processes with optimisation. Supply chains of multiple enterprises are strongly influenced by external changes in the environment *e.g.* laws, consumer trends, and current research. External interfaces to account for these changes could thus also be incorporated for the supply chain to be competitive in a long-term perspective.

Recent Forrester surveys (Radjou, 2000) show that the most typical problems that global manufacturing companies face are poor visibility into plant operations, inaccurate demand forecasting and poor supply chain communication. When the companies face capacity constraints the most common actions are to work overtime, outsource, re-allocate production or to lose sales. Many companies have no means of actual analysis of what actions are preferable from a supply chain perspective. Thus, the models developed must provide a detailed understanding of the manufacturing and logistical functionality as well as incorporating links to upstream and downstream supply chain echelons. Hopefully, they should also be able to predict process limitations thus describing the best way of modifying operations so as to utilise existing assets in the supply chain in the best possible way and propose asset modifications if necessary. Benchmarks can be obtained by reviewing current processes and parameter or object modifications in the model can be performed that lead to achievement of better objectives.

Models that achieve the criteria of integrating enough supply chain detail to be practical often have the drawback of only providing a small number of feasible solutions without any suggestion as to which is the better solution and even less providing any knowledge as to whether any of the proposed solutions are close to the supply chain optimum. Without the information about what is achievable, *i.e.* the supply chain optimum, there are just various fragmented signs to base the decision-making on.

2.2.1 Deterministic Models

Though effectiveness of operations has improved lately due to the introduction of ERP systems and so called advanced planning systems, there is still wide interest in rigorous optimisation of supply chains delivering solutions of large-scale problems. Optimisation approaches present the opportunity of guaranteeing optimal or near optimal solutions within a specified margin.

Vidal and Goetschalckx (1997) extensively review primarily mixed integer programming (MIP) optimisation models. They focus on the relevant factors in the formulations, on specific characteristics of the solutions methods, and computational experiences.

Cohen and Lee (1989) deliver a model for making resource deployment decisions in a global supply chain network and solve different scenarios using an extensive MINLP model. The use of "structural solutions", *i.e.* discrete alternative solutions reduces the model to linear form; an MILP model. They also discuss several "policy options" for plant utilisation, supply and distribution strategies. The objective function and network constraints are quite detailed and contain terms such as taxes paid in different countries and requirement levels to establish manufacturing facilities in a country.

Cohen and Moon (1990) develop a model in which cost structure and supply chain strategies can be studied. They specifically conclude that transportation costs have a crucial effect in bringing supply chain costs down.

Kim and Lee (1998) explore fixed and variable capacity problems of jointly determining a unit's price and lot size for profit-maximisation under constant, price-dependent demand. They use Kuhn-Tucker conditions, marginal and sensitivity analyses and apply geometric programming to the constrained nonlinear maximisation problems that occur. Furthermore, they provide managerial implications on the optimal capacity decisions.

Arntzen *et al.* (1995) conclude after developing a very large MILP model, called the Global Supply Chain Model: "We also wonder how anyone can rely on heuristic solution methods in this arena". The program minimises cost or distribution times, meeting estimated demand, local restrictions, offset trade and maximum capacity. The model was implemented in restructuring the Digital Equipment Corporation and the authors claim it saved the company over $100 million.

D'Amours *et al.* (1999) are among the few authors who discuss multi-enterprise collaboration from a modelling perspective. They propose a network approach in which companies are selected and scheduled with the objective of fulfilling demand of one product on time at minimum cost for a supply chain including manufacture, storage, distribution and customer brokers. The manufacture, storage and customer are nodes in the network while the transportation costs are incurred at the arcs. The model constraints are quite simple and comprise mass balances between the nodes, capacity limits and integral flow quantities. Different bidding protocols are utilised for specific degrees

of information exchange between the separate enterprises. They distinguish between today's most common adversarial approach in which the cost is fixed for a specific delivery no matter when it is delivered as long as it is on time and in which no information is shared and the fully collaborative approach of sharing all supply chain information with a time-price dependency, with some intermediate steps in between the two. Their result shows that the collaborative approach is cheapest, but they maintain that the impact of information sharing in a networked organisation is yet to be fully understood. Based on their results, they also advise that inter-organisational systems should be implemented to provide for electronic data interchange.

Timpe and Kallrath (2000) describe an MILP model for application in chemical process industries, food industries and consumer goods industries. Different time scales are applied to production and distribution, apart from the normal considerations in this type of model such as raw materials usage, production, inventories and demand fulfillment. An example of this is to consider a production time scale in which the first twelve months of planning is split into 12 segments of 30 days each and there are four more segments of aggregated planning of 120 days each and on the commercial scale there would be equal length of each interval in the horizon. The authors further view one conceptual novelty in their paper to be the definition of capacity in a multi-site, multi-product production network in which they differentiate between capacity limits of tanks which have capacity limits and containers which are limited by the space to keep them. Binary variables are defined to ensure so that each tank can only hold one product at every time step.

Towill *et al.* (1992) discuss the actual implementation of dynamic models in industrial supply chains. They emphasise the importance of close collaboration between all participants in the supply chain, especially concerning free exchange of information about true market demand. The control system of the chain can then respond to real rather than distorted data.

2.2.2 Simulation-based Models

Perea *et al.* (2000) suggest an approach based on classical control theory for managing the supply chain operations. They compare three different control policies (fixed inventory control, inventory at orders' level and use of original demand signal) and their effect on costs, customer service levels and inventory oscillation. The supply performance is

highly dependent on the control laws and the authors conclude that trade-offs in key performance indicators must optimised in an objective function by means of control policy design. The assumptions used are the following: (a) the supply chain consists of independent nodes, (b) all decisions are made locally, (c) every node only has two inputs and outputs: material and information flows, (d) only nodes at the retail level have original customer demands visibility, (e) all information received at a node is from the nodes to which it is connected, and (f) each node operates as a pull system. These assumptions are based on the understanding of the supply chain as a set of completely separate entities with no information sharing a of core processes, thus the system is a private supply chain. This type of hybrid disconnected supply chain dynamic model is a key to understand the behaviour of operations in established supply chains and thorough analysis of the decision-making processes.

Berry and Naim (1996) study a real world supply chain which is replicated in simulation models and specifically try to find dynamic performance improvements by consecutive redesign strategies. They find it useful for supply chain designers to utilise simulation to improve on personal education and gathering of information. The main modelling components include control theory techniques, computer simulation techniques and statistical sampling techniques. Qualitatively, their methodology is based around pre-model systems analysis and post-model dynamic analysis. Typical supply chain quantities such as distributor order rates and actual stock levels are modelled as feedback flow of information. Dynamic quantities can be studied subject to design parameters such as desired stock levels, speed of stock recovery and lead times. The simulation models require the solution of difference equations which is handled by a proprietary spreadsheet software package. They conclude that a majority of industrial supply chains have not been designed explicitly and the inevitable result is that the dynamic performance is poor. Through dynamic supply chain modelling, comparisons can be performed as to study the relative benefits of various proposed redesign strategies.

van der Vorst *et al.* (2000) model a discrete event supply chain simulation system. They emphasise that mathematical approaches require too many simplifications to model realistic supply chains. Their model is a multi-echelon, multi-product system with periodic ordering. The authors recognise as supply chain components business processes, performance indicators, design variables, and business entities. Typical business processes

are order generation, production, order picking, and stock holding. Design variables are split at two levels, configuration variables such as IT and physical infrastructure and operational variables such as production quantities and timing of orders. Costs and various service levels measures are used as performance indicators in the model. Business entities are used to represent information flows and material flows or capacities and resources. In the context of model simulation, van der Vorst *et al.* suggest that the model should be able to calculate the state, time and place of each business entity after each transition and the level of the performance indicators.

Alfieri and Brandimarte (1997) describe an object-oriented methodology for supply chain simulation. They point out the complexity of finding the right trade-off between inventory and transportation costs and suggest a software environment to help decision-makers in evaluation of performance. The model considers network design issues, inventory management and transportation management. A simple example demonstrates how a supply chain can be modelled as a set of nodes including factories, stock-points, and demand points in which generic tasks are modelled through class inheritance.

Gjerdrum *et al.* (2000*a*) present a multi-agent simulation approach to optimise distinct parts of the supply chain network problem. Negotiations which determine order frequencies and timings occur between the members of the supply chain in order to find solutions that satisfy all parties involved.

2.2.3 Inventory Modelling

Hautaniemi and Pirttilä (1999) describe the choice of inventory policies in an assemble-to-order production environment. In industry, it is a common practice to choose inventory policies according to so called ABC-analysis normally based on unit value of products, criticality of production and maintenance. Higher value products (A-items) are managed around finding a balance between high customer service levels and low inventory. To maintain low costs, more advanced techniques such as MRP/DRP (Materials Requirements Planning and Distribution Resource Planning) are more frequently used than for lower value products. For lower value products (C-items), a simple ROP (Re-Order Point) system, which triggers orders as inventory reaches a pre-specified replenishment order point, is often sufficient. The main concern for C-items is to avoid stock outs and reduce the frequency of procurement and it is argued that the value of obsolete prod-

ucts is low compared to the cost of maintaining complex inventory policy systems. The authors examine the ABC-analysis classification based on value of production, lead time compared to the scheduling time, and demand distribution pattern and find that simpler policies often perform just as well as more advanced policies for many scenarios in the production environment described.

van Donselaar *et al.* (2000) examine how inventory planning policies influence the stability of production. The MRP logic is described as a system in which the logic takes into account the inventory, the scheduled receipts, the safety stock, the order quantity and the lead time for each component. The authors demonstrate a simulation experiment in which performance is evaluated in terms of service levels, inventory levels, and planning nervousness (the number of times an order is being re-scheduled). The stochastic element of the demand is modelled as a uniform distribution around the mean. The results suggest that input parameters such as the demand variability and demand quantities have large impact on the performance.

Gupta *et al.* (2000) investigate the strategic potential of managing inventory risk subject to customer demand satisfaction constraints. They suggest that guaranteed service levels can be achieved at relatively low additional cost. The two-stage supply chain modelled considers a medium term horizon (1-2 years) with multi-sites and demand uncertainty. Production decisions are made before demand realisation due to significant production lead times, whereas other supply chain decisions (such as inventory management) are delayed. Various demand regimes are recognised where different supply chain policies apply. A more detailed description of the stochastic programming model concepts utilised is given in Gupta and Maranas (2000). The solution is obtained through a chance constraint mathematical programming approach in which deterministic equivalents to stochastic elements are obtained through use of cumulative distribution functions. Inventory distributions are also calculated within the analytical framework by using normal distributions, expectation of supply occurring and calculation of the standard deviation of inventory at each production site.

Parlar (1997) investigates a continuous-review inventory problem with random supply interruptions. The supplier availability is modelled as a semi-Markov process. The inventory policy changes between the two cases: supplier available and supplier unavailable. He constructs an average cost per time objective function, in order to optimise the

inventory policy.

2.2.4 Forecasting

Product life cycles are shortening and fluctuations of customer demand are increasing. Furthermore, forecasts are more frequently used in production planning and management. Hence, the importance of forecasting techniques are vital to supply chain functionality - at least, until the supply chain cycle time has been reduced to next to nothing.

Lee and Billington (1992) state that inventory policies must be based on the demand uncertainty of every individual SKU. These uncertainties are dynamic in nature and there should therefore be systems in place to track these uncertainties. The authors emphasise that a system based on transaction volumes is simply not sufficient since it does not reflect the uncertainty of supply and demand at the specific supply chain node considered. Hence, forecasting methods used for production planning and inventory planning must incorporate dynamic forecast errors in both supply and demand so that the amount of inventory stocked is actually determined by the uncertainties that make it necessary to stock products in the first place.

Lee *et al.* (1997) identify the main causes of the bull-whip effect that eradicates the advantages of forecasting in many supply chains. These are continual updating signal processing of forecast demands, large order batches occurring infrequently with linked MRP and DRP planning processes that overlap, forward planning against price fluctuations, and shortage gaming. The authors contend that the bull-whip effect is the result of rational decision-making based on information available at the various supply chain nodes, but the bull-whip effects can be counter-acted by thorough supply chain understanding. Some strategies can be integration of information systems, better organisational relationships, and new incentive and measurement systems.

2.2.5 Linked Simulation and Optimisation

Subramanian *et al.* (2000) propose a computing architecture for linked mathematical programming and stochastic simulation which is utilised to assess risk in a pharmaceutical R&D pipeline. The authors attest that a single stochastic integer mathematical program including probability functions would result in high numerical complexity and the size of the considered problem would be too large to be efficiently solved. Instead, they propose

a linked simulation and optimisation model which combines combinatorial optimisation and discrete event simulation in order to evaluate the pipeline uncertainties and to control the risks.

Bose and Pekny (2000) model a combined forecasting, simulation, and optimisation architecture. Model predictive planning and scheduling is proposed in which the controlled variables relate to the production plan and output variables typically are related to inventory levels. The forecasting module first calculates the future target inventory so as to ensure required customer service levels at the same time as minimising inventory. The scheduling module then tries to achieve these inventory levels. The process is repeated until the end of the time horizon. Plant uncertainties and true demand levels comprise the disturbances going into the model. The authors discuss the overall efficiency of the business processes in terms of how much of promotional demand is known in advance and their relation to centralisation of control. They conclude they different policies of supply chain centralisation have to be implemented for various levels of business process efficiency.

2.2.6 Other Models

Androulakis and Reklaitis (1999) describe the supply chain planning problem as a coordinated operational system in which the plants, suppliers, shippers, and customers have their own decision variables which are also interrelated through production and demand constraints. Each of the entities in the chain have their own objectives implying that a meaningful, overall supply chain objective function cannot be constructed. The authors state that each of the operational units must be able to make its own distributed decisions on a suitable time scale in order for the supply chain to work efficiently. Hence, they regard the supply chain as "an *asynchronous distributed decision-making problem* which involves *asymptotic agreement with computation*". An important point is that the various entities in the supply chain have only a part of their decision domain shared with other entities. Therefore, all other decisions can be made locally. The resulting model thus tries to make global changes to the overall system by local adjustments. Each distributed unit minimises a local objective at the same time as maximising a common Lagrangian function by the generation of independent approximation of an ascent direction. The consensus problem has thus moved from the primal space to the dual space.

In a supply chain environment, argue the authors, asynchrony of operation results from optimal policies that have to be implemented continuously due to the dynamic nature of the problem. Although only simple problems are addressed in this paper, the reasoning behind the proposed algorithms hold a high degree of promise to solve distributed supply chain problems.

Homburg (1998) proposes a mathematical formulation for solving decision problems with multiple objectives. His idea is to use a two-level approach, where the top-level only provides general preference information. The base-level takes this information into account and determines a compromise solution. In the resulting multi-objective linear program formulation, a vector containing all decision makers' objective functions is to be maximised. Each decision-maker has to determine its weight functions on which there might also be restrictions (upper and lower bounds). By utilising pessimistic and ideal goal values the top-level provides general preference information (in terms of linear weight restrictions) from which the base level can then determine a compromise solution to the multi-objective linear program.

Ertogral and Wu (2000) describe a production planning model for supply chain application based on auction theory in which agents compete based on local utilities. Specifically, the auction algorithm proposed assumes that each facility or agent first tries to find its locally best solution to the material flow (the shared variable) to be determined. Conflict prices or penalties for inconsistencies between the parties are then established. Weighting parameters are utilised to measure how much each facility "suffers" from an average of the disparate settings of the variables. The authors state that the applicability is large in distributed business environments and they report on more fair solutions with only small solution quality degradation from current so called monolithic models.

Petrovic *et al.* (1998) model the supply chain as a fuzzy set in an uncertain environment, thereby exploiting the opportunity to build a decision support system for the entire supply chain. They try specifically to determine stock levels and order quantities during a finite time horizon to obtain acceptable delivery performance. The model uncertainty comprises customer demand and raw material supply.

Theys (1998) structures the challenges of management in a "wired" world. He claims that old decision support systems are not adequate to model evolution of concurrent organisation systems, because of the high level of interactions and change. He believes

that complexity theory, particularly developed at the Santa Fe Institute, will lead to more appropriate models of these nonlinear dynamic systems.

In this chapter, recent literature from several fields concerning supply chain applications has been reviewed. In particular, model based and conceptual papers have been reviewed that are utilised to develop methods in the further chapters in this thesis. The next chapter specifically makes use of MIP methods and considers some simple and case study-based examples of optimisation for multi-enterprise supply chain planning.

Part II

Deterministic Supply Chain Modelling with Transfer Pricing

Chapter 3

Supply Chain Cooperation and Transfer Pricing Issues

Conflicting interests in general extended multi-enterprise supply chains frequently lead to problems in how to distribute overall value to each member of the supply chain. A simple approach to enhance the performance of a multi-enterprise supply chain is to maximise the summed enterprise profits of the entire supply chain subject to various network constraints. When the overall system is optimised in this fashion there is no automatic mechanism to allow profits to be *fairly* apportioned among participants. Solutions to this class of problems usually exhibit quite uneven profit distributions and are therefore impractical. They do however give an indication of the best possible total profit attainable in the supply chain as well as an indication of the best activities to carry out. The aspiration of this chapter is to extend the above approach so as to achieve solutions of similar quality overall but with equitable profit distributions.

Power relations exist in the supply chain and every member seeks to leverage as much value to themselves as possible. For example, manufacturing companies in a dominant rôle relative to other supply chain members may gain a structural advantage rendering opportunities for attaining higher profits than their suppliers and/or distributors. Possession of power is demonstrated by a relative capacity by owners of particular resources to safeguard value when participating in the supply chain. The distribution of physical products in the supply chain exists in an exchange relationship with the value chain, which distributes the revenue stream from customers to each member of the supply chain.

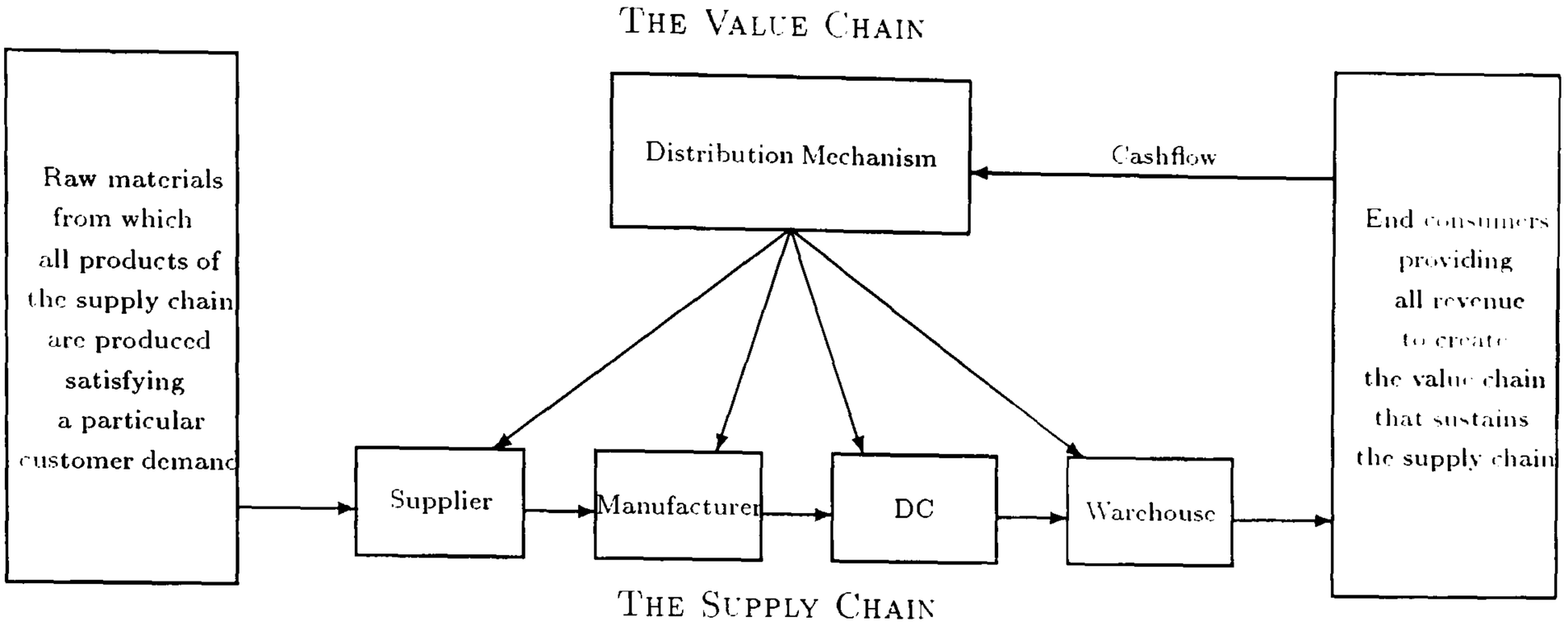


Figure 3.1: Relation between the supply chain and the value chain

The problem of distributing profits to each member of the supply chain is illustrated in figure 3.1. The concept is adapted from Cox (1999).

van Hoek (1998) raises the question of how to divide supply chain revenues among the players in the system, when there is no leading player to determine how distribution of benefits should be handled. This problem is of crucial importance to the problem considered here. He states that supply chain control is no longer based on direct ownership but rather on integration over interfaces of functions and enterprises. Traditional performance indicators limit the possibilities of optimising the supply chain network, since the measures do not correctly address the wide opportunities for improvement. As supply chain integration is no longer dependent on large vertical integration costs, but rather on interfaces, the supply networks become temporary products that will need to be able to cope with various enterprises being contained in it, since exit and entry barriers can be shared among the supply chain players. The supply chain measurement system should be able to give each member a clue - maybe even before entering - of *e.g.* how much return on investment can be expected from being a part of a particular supply chain. van Hoek proposes a framework where, as the supply chain players move from a cost saving approach to one of market extension and market creation, the focus of the performance measures move from efficiency towards customer service measures and integration measures. In the light of these problems, this chapter aims at obtaining quantitative values of a *fair* profit objective for each company in the supply chain based on the available supply chain resources.

In this chapter, the scope is to find solutions to this class of problems given a quantita-

tive prior power structure of the supply chain. The power relationship conveys a mapping from the value chain to the physical supply chain. It should be emphasised that the companies involved in the supply chain are in business only to create value for themselves. For example, sharing of information between partners will only take place if the supply chain players believe their profits will be leveraged. The underlying assumption is that the surrounding business environment is suitable for long-term partnerships and that the power relations between the companies can somehow be quantified in terms of minimum acceptable profit levels. These levels are clearly dependent on the customer market for the products manufactured in the supply chain as well as the vertical inter-company power relations in the supply chain. It should be added that these power relations may be affected by the existence of external competitors.

Optimal solutions to the supply chain planning problem are interesting because of the intrinsic need of efficiency in today's business environment. One of the most robust techniques to solve supply chain problems to optimality or a guaranteed margin of optimality is that of deterministic global optimisation. In this field, the most general form of mathematical programming problems are handled, namely non-convex mixed integer nonlinear programs. In this work, spatial branch-and-bound techniques that relax non-convexities are applied. As the solution space investigated is contracted, the relaxed solutions to the exact problem improve. After a finite number of subproblems are solved, a solution that can be guaranteed to be within a pre-specified margin is obtained.

The combination of game theory-based fair transfer prices and optimal mathematical programming provides solutions that can give rise to sustainable payoff levels for cooperating multi-enterprise supply chain companies at the same time as optimising sourcing, production, inventory, and transportations policies of all participating companies. As for all mathematical programming approaches, there is an upper limit for the problem sizes that can be tackled and aggregation strategies have to be implemented in order to model large scale supply chain network problems.

3.1 Game Theory for Inter-Enterprise Collaboration

Maloni and Benton (1997) propose four different approaches to modelling multi-entity supply chains: simulation, optimisation, game theory, and heuristics. In this chapter,

game theory is briefly introduced providing supportive ideas to optimisation of a multi-enterprise supply chain. The approach developed utilises spatial branch-and-bound global optimisation to obtain fair solutions that could provide long-term benefits to all players (partners) in the multi-enterprise supply chain. Game theory provides a means of achieving multi-entity solutions in the bargaining, collaboration and conflict situations that arise when many decision makers, especially belonging to separate business entities, have to agree on a common outcome. The presentation below is indebted to the excellent game theory overview by Straffin (1993).

Game theory is a logical analysis of conflict and cooperation and a game comprises at least two players. Examples of typical players might be companies, nations, politicians or normal human beings as in the game of chess. The strategies that any player can pursue are the possible courses of action available to the player. The strategies chosen by all players determine the outcome of the game. Every outcome is connected with numerical payoffs that each player will receive for that particular outcome.

A common, and quite obvious principle when optimising multi-player systems is that the outcome should be Pareto optimal. Pareto optimality can in game theory terminology be formulated:

> *An outcome of any two-player game is non-Pareto optimal if there exists another outcome of the game which would give both players higher payoffs, or it would give one player a higher payoff and the other player the same payoff. An outcome is Pareto optimal if there exists no such outcome.*

When approaching the problem of sharing supply chain multi-enterprise profits between partners in an optimal manner one naive idea would be to share profits evenly. From a game theory perspective this reasoning is flawed by two different arguments. The first is that utilities can neither be compared or added (five pounds means less to a millionaire than to a pauper). The second is that equal profit sharing ignores the asymmetries of the game structure, which implies that one player has a an advantage in a supposed negotiation or in the inter-player (*e.g.* inter-company) power relations.

The fathers of modern game theory, von Neumann and Morgenstern (1944), state that no player should accept cooperation if the solution results in less payoff than a non-cooperative solution, which in the multi-enterprise supply chain context means that no company should enter a partnership if it does not think it will benefit from it.

3.2 Nash Optimality

A very simple, *naive* single level approach to optimisation of the supply chain system presented later in this chapter can be to maximise the summed profits of A and B, *i.e.* $\max \Pi = \Pi_A + \Pi_B$. This approach is easy to handle computationally, but as will be seen it typically leads to an unfair profit distribution between A and B, since the transfer prices constitute a cost base for the inventory kept in the system and the optimisation minimises overall costs. Therefore, optimisation of the system minimises the value of products at A by establishing transfer prices as low as possible subject to lower profit requirements.

It is sometimes argued that this simple approach can be used by dividing the surplus of the optimisation via some fair side payment scheme but there will always be arguments to what the fair levels of side payments will be and the issue of distribution of value is not avoided. Therefore, the aim is to build in the profit distribution mechanism from the start into the actual production, inventory and transportation planning.

The approach developed here uses a game theory approach called the Nash model (Nash, 1950). This is built on four axioms that a rational bargaining solution should obey. These are characterised by *Pareto optimality*, *symmetry*, *scale invariance*, and *independence of irrelevant alternatives* (Conley and Wilkie, 1996). The solution delivers an optimal fair split of payoff to each of the rational players in a game, and the Nash approach is therefore a suitable method to solve the particular type of problems under consideration. If the status quo point (also referred to as the conflict point) of a game is (x_0, y_0), then the Nash solution is (x, y), $x \geq x_0$ and $y \geq y_0$, which maximises $(x - x_0)(y - y_0)$. Based on the assumption of a symmetric game where neither player has a competitive bargaining advantage (*i.e. a posteriori* to the determination of the status quo point), neither of the supply chain partners will have any reason to grant the other partner better terms than the latter is prepared to grant him. The symmetric game will thus have a symmetric solution. Both players will agree on a solution that allocates payoffs equitably between them subject to the status quo point.

Figure 3.2 shows the bargaining positions of the different players based on their original agreements, which in this case is the minimum acceptable profit levels, (Π_A^L, Π_B^L). The figure also shows the outcome of the game, the Nash solution. The solution according to Nash is the point (Π_A, Π_B) with $\Pi_A \geq \Pi_A^L$ and $\Pi_B \geq \Pi_B^L$ which maximises the product

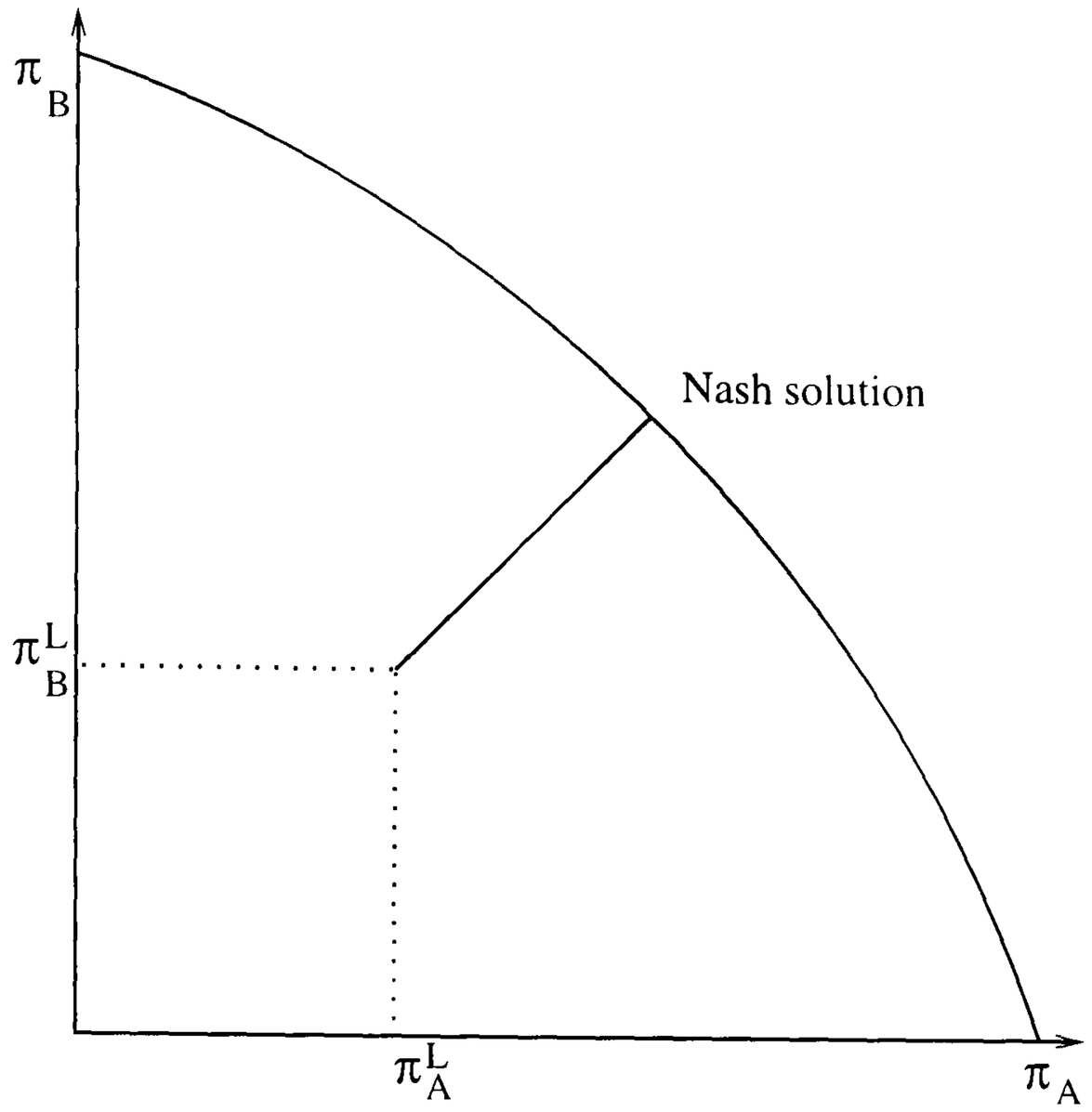


Figure 3.2: Nash solution

$(\Pi_A - \Pi_A^L)(\Pi_B - \Pi_B^L)$. The Nash solution can be regarded as a negotiation process where the status quo is a point where the bargaining will end up if no cooperation is to be had. The threat strategies leading to the choice of a status quo point imply that there is a separate game of choosing the optimal status quo point for each player when regarding power relations and negotiation capabilities. The Nash solution has been criticised for disadvantaging players with a low value status quo point, but the status quo point is representing the structural advantage in the game of one player player in favour of the other player.

Rubenstein (1982) discusses the bargaining solution from a practical standpoint as well as a normative standpoint. The problem of the players' "bargaining ability" is vital. In our proposed approach, we try to avoid this by stating that the head offices of both firms have agreed to utilise the Nash solution subject to various production constraints and initial lower bounds on utility. An interesting aspect of this work is not only the transfer pricing process but also how the production model can be solved and how it affects the production, inventory and transportation plans.

In this thesis, the term "transfer pricing" is used even though more than one company

is concerned to underline the nature of the supply chain as *one* profit generating entity. Vaysman (1998) recognises the way that managerial decisions are influenced by transfer pricing, but the production planning implications of this influence is not modelled. One could argue that the way the proposed model uses administered transfer pricing; top management in both firms agree on utilising the Nash bargaining model as a set of rules to both improve from the currently best solution obtainable. After that there is no room for negotiation. Thus, the combined effect of coordinated planning results in improved profits.

Vaysman states that one reason for applying transfer pricing in the first place is that "some divisional information is known only to divisional managers". Therefore, it is argued that a full-information approach cannot incorporate transfer pricing. Our proposal is that whereas this argument certainly has it merits, this work does not necessarily require that information is shared between the parties but rather that the production information can be handed to an external (*e.g.* academic) agent.

Baldenius *et al.* (1999) discuss the managerial aspects of a transfer pricing system and compare cost-based transfer pricing with negotiated transfer pricing. An important issue here is that if each division bears the full cost of its investment the future trend will be to adjust further investments downwards since only a share of the added surplus from the investment will be received by the division. They specify a model in which management is centralised and where "head office" specify generic rules of rights and obligations of the divisions. The divisions have symmetric but imperfect knowledge of the state variables. With respect to the view of the enterprise, they have a similar reasoning to what the relationships between the divisions are. The firms have information about their production processes but they do not know what production patterns will be utilised in the future horizon. Contrary to Baldenius *et al.*, the model proposed here does not consider future investments but is an operational planning model on the medium scale horizon.

3.3 Mixed Integer Non-Linear Programming

The objective function of the Nash model is non-convex and non-linear. Therefore, some common mixed integer non-linear program approaches and problems are reviewed in this

section. To solve the supply chain problem, obtaining guaranteed optimal solutions (to reinforce the quality of solutions) high-performance (due to problem size) optimisation techniques are required. In sections 3.4 to 3.6, useful optimisation techniques that will ensure optimality for this class of problems are reviewed.

Mixed integer non-linear programs (MINLP) provide a straight-forward modelling route to optimising almost any physical or engineering system. Problems associated with the solution of MINLPs include:

1. High computational burden (NP-completeness), requiring exponential solution times in the worst case (van den Heever and Grossmann, 1999)

2. Non-convexity leads to local solutions

A general MINLP can be formulated as follows:

$$\max_{x,y} \Phi(y, x)$$

s.t.

$$h(y, x) = 0$$

$$g(y, x) \leq 0$$

$$y \in \{0, 1\}^m, x \in \mathcal{R}^n$$

where Φ is the objective function to be maximised, y are the binary variables, x are the continuous variables, and g and h are the inequality and equality constraints, respectively.

The most common techniques for solving MINLPs are the Outer Approximation (OA) method (Duran and Grossmann, 1986) and the Generalised Benders Decomposition (GBD) method (Geoffrion, 1972). OA iterates between a master mixed-integer linear program (MILP) where the model is linearised around the non-linear program (NLP) solution points and the NLP subproblem where the binary variables are held fixed. GBD utilises a similar approach, but the solution is projected onto the space of binary variables only from the full mixed integer space of OA (Quesada and Grossmann, 1992).

In the area of long-term multi-period planning, Sahinidis *et al.* (1989) present solutions for this class of problems utilising branch-and-bound, cutting planes, GBD, and heuristics approaches. Shah (1998) recently presented a thorough overview of the liter-

ature in the area of optimisation techniques used in single- and multi-site planning and scheduling.

3.4 Global Optimisation

Non-convex problems such as the Nash problem cannot normally be guaranteed to obtain optimality within a certain margin, which is what we want to be able to achieve. Therefore, we need to explore the theory of global optimisation, so as to be able to guarantee optimality for the algorithm presented in section 3.8.

In the last decade, there has been a rapid growth of interest on global optimisation in chemical engineering, computational chemistry and molecular biology applications (Floudas, 1999). Early work in chemical engineering applications can be seen in Stephanopoulous and Westerberg (1975), who explain a method of a Lagrangian based method of successive linear approximations of non-separable terms, and Westerberg and Shah (1978), describing how upper bounds can be found for the dual bounds with applications to heat exchanger networks. A renewed interest in the field was initiated by the studies by Floudas *et al.* (1989).

Non-convex problems can be optimised through successive relaxations. The relaxation is constructed by either enlarging the feasible region or by underestimating the objective function. The relaxations normally convert the problem such that they are solvable with conventional methods. For example, concave functions in an interval can be underestimated by a straight line (Ryoo and Sahinidis, 1995). From the underestimation, a lower bound is obtained. By optimisation in the local region for the lower bound, an upper bound can be found. The global optimum is to be found in between the lower and upper bounds, see figure 3.3. For a typical algorithm, if the lower and upper bounds are sufficiently close the search stops. As long as some search criteria are not fulfilled the search region is partitioned into new subregions. The problem is then again solved for the new bounds.

Quesada and Grossmann (1993), (1995) present convex underestimators for linear fractional and bilinear problems within a branch-and-bound framework which originates in the works by Falk and Soland (1969), McCormick (1976) and Al-Khayyal and Falk (1983). Furthermore, Maranas and Floudas (1995) present trilinear convex underestima-

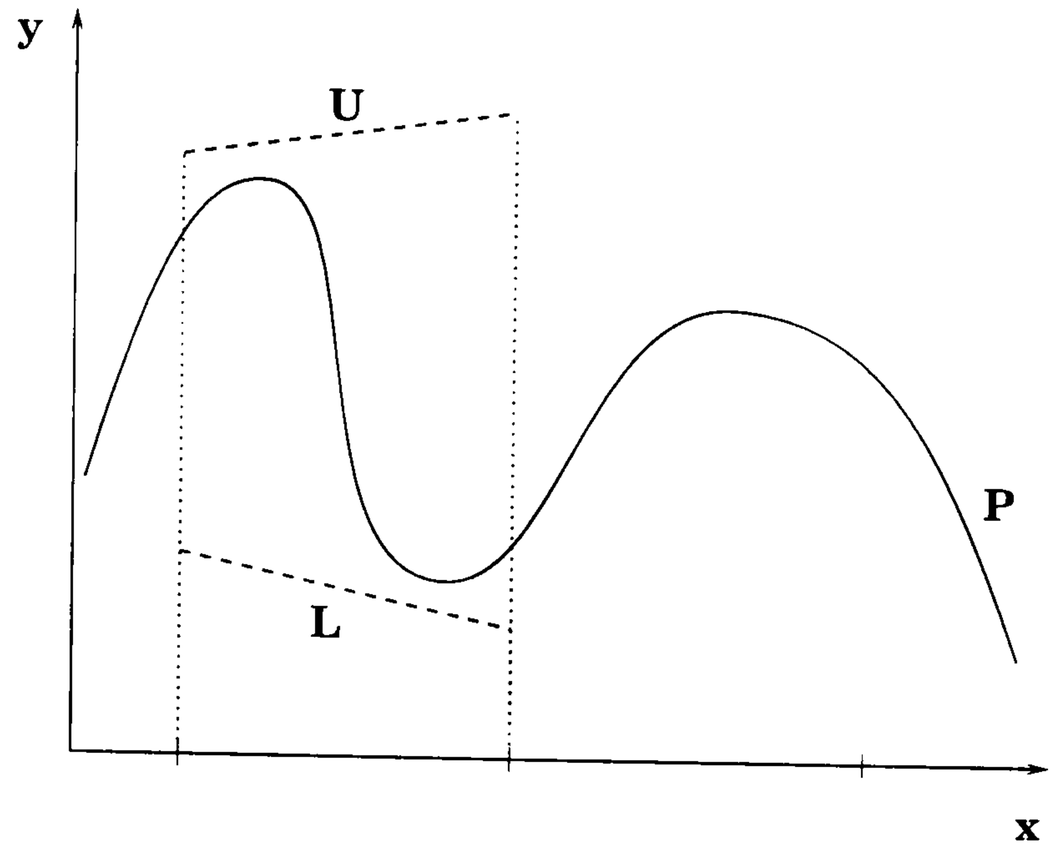


Figure 3.3: Lower and upper bounds in a local region

tors for general non-convex MINLPs. Benson (1996) presents an extensive overview of deterministic global optimisation algorithms.

3.5 Branch-and-Bound

The branch-and-bound approach is one of the most common approaches to global optimisation (Ryoo and Sahinidis, 1996). A general non-concave maximisation problem is considered, which is the same as a non-convex minimisation problem. Any maximisation problem can be stated as a minimisation problem with an objective function with reversed signs.

Lower and upper bounds are obtained over subregions of an original search space. To improve the value of the upper bound a branch-and-bound algorithm partitions a feasible region of the relaxed problem into a finite number of subregions. The subregions are tightened or removed from the algorithmic search based on feasibility or optimality. The bounds over the search space successively become tighter as the search progresses. The overall lower bound is updated when a feasible solution with a value higher than any previous solution is obtained. In a maximisation problem, any subregion with an upper bound no better than the current global lower bound, can be deleted from the search. If the concave (convex in minimisation terms) overestimation of the function is infeasible, subsequent problems on this branch can be deleted (pruned) as the infeasibility is also valid for the exact problem. The relative difference between the lower and upper

bounds at the end of the search indicates with which precision the global solution can be guaranteed to be the global optimum. A typical algorithm stops when the list of problems is exhausted or the relative difference between the lower and upper bounds is sufficiently small, *i.e.* within a pre-specified tolerance.

There are many different variations on how to select nodes for evaluation and when and how to branch. The common issue is that tight formulations are needed to exploit the special mathematical structure of the problem. The performance of the algorithm depends heavily on the method of selecting the concave relaxation of the maximisation problem in the search space. The smaller the difference between the exact and the approximate solution, the better the relaxation (Zamora and Grossmann, 1998). A typical minimisation algorithm for global optimisation (Ryoo and Sahinidis, 1996) is as follows:

1. Branching: Split the search region into a number of subregions, S_l, $l = 1, \cdots, s$.
2. Bounding: Select a subregion, S_l, and determine lower and upper bounds
   of the objective function, $\Phi_l^L(x,y) \leq \Phi(x,y) \leq \Phi_l^U(x,y), \forall\, x \in S_l$.
3. Global bounding: Set $\Phi_l^{L,k}(x,y) = \min \Phi_l^L(x,y) : l = 1, \cdots, s$
   and $\Phi_l^{U,k}(x,y) = \min \Phi_l^U(x,y) : l = 1, \cdots, s$
4. Termination criteria and new subproblem selection:

   If $|\Phi_l^L(x,y) - \Phi_l^U(x,y)| \leq \epsilon$,

   Terminate. A solution sufficiently close to the optimum has been found.

   Else,

   Select a subregion.

   Set $k = k + 1$.

   Start over from step 1.

3.6 Spatial Branch-and-Bound Optimisation

The advantage of deterministic, constrained global optimisation schemes is that they can be guaranteed to reach the global optimum within a finite (although sometimes large) amount of iterations. There are several approaches of deterministic global optimisation, including branch-and-bound, cutting plane and decomposition methods. In a spatial branch-and-bound algorithm lower and upper bounds of the objective function are first obtained to achieve an *a priori* idea of the size of the search space. Through successive

refinement of the subregion, *i.e.* tightened bounds and pruning (based on optimality convergence and feasibility criteria), progression is made towards a solution where the gap between the convex relaxation and the non-convex original function is sufficiently small. The tightness of the bounding process is of crucial importance for the rate of convergence.

In this context, it is assumed that the initial lower bounds on the profit objectives are obtained from the lowest acceptable profit levels of cooperating enterprises in a supply chain partnership. The initial upper bounds are obtained by maximisation of each individual enterprise profit level subject to model constraints including the lower profit requirement of other companies in the supply chain. If the supply chain can be improved in terms of efficiency, the global optimum can then be found between the lower and upper bounds. The optimisation problem is solved via a convex relaxation of the non-convex bilinear objective function, *i.e.* an MILP problem is solved in each node of the branch-and-bound procedure. Linear optimisation solvers, which are known for their speed and robustness, can thus be utilised. Partitioning is performed until the upper and lower bounds are sufficiently close to provide a relaxed solution close to the non-convex objective function value, to eventually render a guaranteed global optimal solution, given the convergence criteria. For similar spatial branch-and-bound approaches see Falk and Soland (1969), McCormick (1976), Ryoo and Sahinidis (1995), Quesada and Grossmann (1995), and Horst and Pardalos (1995).

3.7 A Two-Enterprise Supply Chain Model

One key issue in supply chain optimisation involving multiple enterprises is the determination of policies that optimise the performance of the supply chain as a whole while ensuring adequate rewards for each participant. To test approaches of solving this problem on a realistic supply chain, a first step is to develop a basic supply chain model.

The supply chain problem formulation developed in this chapter is a mixed integer non-linear programming (MINLP) model. Decision variables include inter-firm transfer prices, production and inventory levels, and flows of products between echelons, subject to a deterministic sales profile and lower profit requirements for each enterprise.

The first model presented considers two supply chain echelons, which belong to two

separate enterprises forming a partnership: a manufacture type company, A, and a distribution and sales type company, B. A delivers manufactured products to B, which sells the products to external customers with a short-term deterministic demand profile. The supply chain is illustrated in figure 3.4. In the model, which implicitly comprises four echelons - raw material suppliers, A, B, and the final customers, A and B are explicitly modelled. In A, I different products can be produced utilising R production resources. The prices for products shipped between A and B are called transfer prices. The decision makers in A and B have K discrete transfer price levels to choose from. Inventory can be held in either A or B. Costs include raw material, manufacturing, administrative (transaction), inventory, and transportation cost in A, and purchase, administrative, inventory, and transportation cost in B. No fixed costs are considered in this model. The revenue of A is equivalent to the material purchase cost of B and B's revenue is determined by the market prices of the products. The revenues less the corresponding costs constitute the objective functions ("profits") of A and B, respectively. The prior, lower profit requirements of the enterprises will be utilised by the optimisation as lower bounds on the objective profit variables. The overall system is optimised with two disparate objectives. First, overall profit optimisation of the system is the objective. Secondly, the aim is to find *fair* solutions of similar quality as the overall system optimisation but with an equitable profit distribution between the supply chain partners.

In order to model the system, the following supply chain assumptions are made:

- All information is available for all the overall supply chain optimisation.
- External demand and external sales prices are deterministic over a short to medium

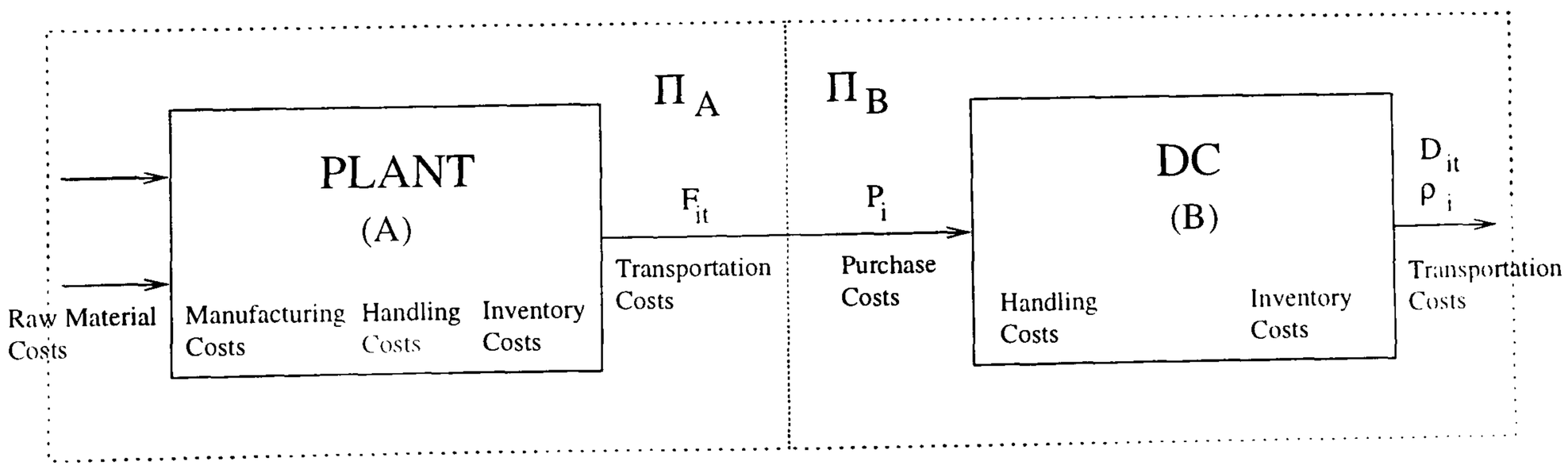


Figure 3.4: A two-enterprise supply chain

term time horizon.

- Alternative transfer price levels are available to choose from.
- A long-term supply chain partnership is established indicating that decision-makers can accept transfer prices which may or may not be the current market prices of the transferred products.

The overall problem can formally be stated as:

Given:

- time horizon,
- external demand,
- external sales prices,
- raw material costs,
- production data,
- inventory data,
- transportation data,
- minimum acceptable profit requirements,
- range of available transfer prices,

determine:

- production plan,
- inventory policies,
- transportation plan,
- transfer price levels,

so as to:

- find multi-enterprise strategies which result in optimal, *fair*, profitability of the enterprises;

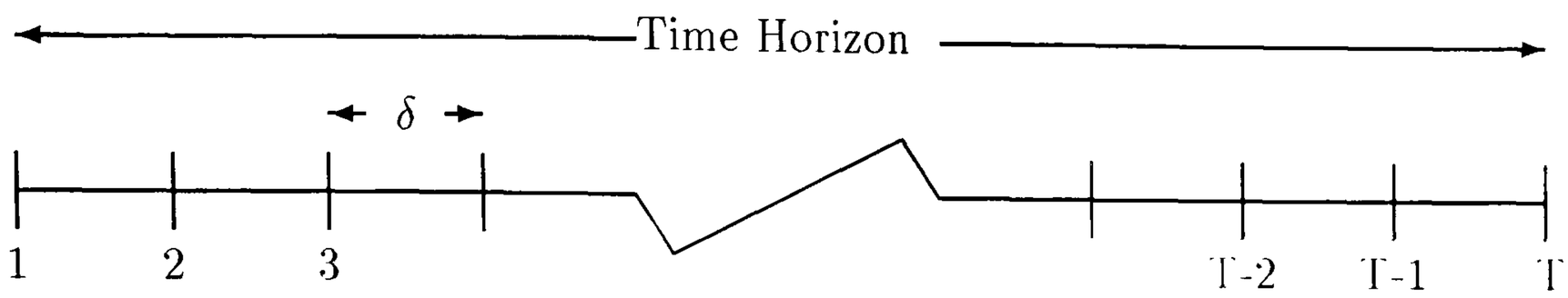


Figure 3.5: Time horizon discretisation

- fulfil demand.

In order to facilitate the mathematical model, a time discretisation approach is applied over a planning horizon, H, of T discrete time intervals of equal duration, δ, as illustrated in figure 3.5. The key variables in the formulation include production levels in A, inventory levels in A and B, flow levels of products between A and B, and transfer price levels. These are determined so as to maximise the profits of A and B, subject to production constraints, transportation constraints, inventory constraints, transfer price level constraints, mass balances, stock-out policies, and several cost constraints.

Mathematical Model

In the following sections, all constraints that appear in the model are presented. The model nomenclature is as follows:

Nomenclature

Sets

$i =$	Products	$(i = 1, \ldots, I)$
$k =$	Transfer price levels	$(k = 1, \ldots, K)$
$r =$	Production resources	$(r = 1, \ldots, R)$
$t =$	Time intervals	$(t = 1, \ldots, T)$

Parameters

UR_{ir} = Demand for resource r for production of product i

R_{rt} = Capacity of resource r for production of product i

RMC_i = Raw material costs of product i

PC_i = Production costs of product i

TC_i^A = A's transportation costs of product i

TC_i^B = B's transportation costs of product i

H_i^A = A's administrative cost parameter of product i

H_i^B = B's administrative cost parameter of product i

r = Inventory holding cost constant

$\hat{P}_{ik}$ = k available transfer price levels of product i

D_{it} = Customer demand quantity of product i at time t

δ = Number of days in a time interval

σ_i^A = A's minimum inventory level of product i

σ_i^B = B's minimum inventory level of product i

ρ_i = Market sales price of product i

Binary Variables

X_{ik} = 1 if kth price level is chosen for product i, 0 otherwise

Continuous Variables

$\Pi_A =$ A's profit

$\Pi_B =$ B's profit

$C^A =$ A's cost of operations

$C^B =$ B's cost of operations

$IC^A_{it} =$ A's inventory cost of product i at time t

$IC^B_{it} =$ B's inventory cost of product i at time t

$HC^A_{it} =$ A's administrative cost of product i at time t

$HC^B_{it} =$ B's administrative cost of product i at time t

$M_{it} =$ Production level of product i at time t

$I^A_{it} =$ A's inventory level of product i at time t

$I^B_{it} =$ B's inventory level of product i at time t

$P_i =$ Transfer price of product i

$\overline{XI}_{ikt} =$ Linearised inventory level of product i at time t, k available levels

$F_{it} =$ Amount of product i shipped from A to B of product i at time t

$\overline{FX}_{ikt} =$ Linearised internal shipment quantity of product i at time t, k available levels

$S_{it} =$ Amount of product i shipped to customer at time t

Production Constraints

The quantity of production resources utilised in A cannot exceed the resource availability level. In equation 3.1, UR_{ir} is the unit consumption of resource r required for production of one unit of product i. M_{it} is the production quantity of product i at time t. The quantity used of a production resource r at time t for a product i is $UR_{ir}\ M_{it}$. The amount of resource r available at time t is R_{rt}. The quantity used of a production resource summed over all products i must be less than the total resource availability level R_{rt} for each resource at each time t. Mathematically, we have:

$$\sum_i UR_{ir}\ M_{it} \leq R_{rt} \qquad \forall\ r, t \qquad (3.1)$$

Mass Balance Constraints

The inventory level in A of product i at time t, I_{it}^A, is equal to the amount at time $t-1$, $I_{i,t-1}^A$, added to the amount produced, M_{it}, less the amount shipped to B, F_{it}:

$$I_{it}^A = I_{i,t-1}^A + M_{it} - F_{it} \qquad \forall\, i,t \tag{3.2}$$

Similarly, for B, the inventory level of product i at time t, I_{it}^B, is equal to the amount at time period $t-1$, $I_{i,t-1}^B$, added to the amount purchased from A, F_{it}, subtracted by the amount shipped to customers, S_{it}:

$$I_{it}^B = I_{i,t-1}^B + F_{it} - S_{it} \qquad \forall\, i,t \tag{3.3}$$

Inventory Policies

To avoid stock-outs at A and B, pre-specified minimum inventory levels for each product i (σ_i^A, σ_i^B) are safeguarded by:

$$I_{it}^A \geq \sigma_i^A \qquad \forall\, i,t \tag{3.4}$$

$$I_{it}^B \geq \sigma_i^B \qquad \forall\, i,t \tag{3.5}$$

Customer Demand Constraints

A "hard" customer demand constraint which is fixed to fulfil variable demands at all time intervals t is used for all products i. S_{it} is the quantity of product i shipped at time t. D_{it} is the corresponding customer demand of product i at time t.

$$S_{it} = D_{it} \qquad \forall\, i,t \tag{3.6}$$

Profit Constraints

The profit constraints simply state that the profit of each of the supply chain companies is the revenue less the cost incurred at that supply chain node. The quantity of product i shipped to B, F_{it}, multiplied by the transfer price, P_{it}, summed over all time intervals t and products i constitutes the revenue of A. For B, the revenue is the quantity of product i shipped to customers, S_{it}, multiplied by the market sales price of product i, ρ_i, summed over all time intervals t and products i.

$$\Pi_A = \sum_i \sum_t F_{it}\ P_i - C^A \tag{3.7}$$

$$\Pi_B = \sum_i \sum_t S_{it}\ \rho_i - C^B \tag{3.8}$$

Transfer Price Levels

In the model, K transfer price levels for each product i transferred between A and B are available via the constant matrix $\hat{P}_{ik}$. X_{ik} is a binary variable which takes the value 1 if the kth price level is chosen for product i, and 0 otherwise. To determine the price level P_i, X_{ik} is multiplied by $\hat{P}_{ik}$ and summed over all transfer price levels available for that product. The constraint $\sum_k X_{ik} = 1$ ensures that only one of the k transfer price levels available is chosen for product i.

$$P_i = \sum_k \hat{P}_{ik}\ X_{ik} \qquad \forall\ i \tag{3.9}$$

$$\sum_k X_{ik} = 1 \qquad \forall\ i \tag{3.10}$$

$$X_{ik} \in \{0, 1\} \qquad \forall\ i, k \tag{3.11}$$

Inventory Cost Constraints

In constraints 3.12 and 3.13, the $\frac{r}{365}$ term is the interest lost per day for keeping inventory over a year's time, while δ is the number of days in each time interval t. The inventory cost is dependent on the outgoing prices associated with the given echelon. The inventory costs of A and B are therefore functions of the transfer price levels and of the sales prices to customers, respectively.

Inventory cost Value of products × interest lost value × inventory held

$$IC^A_{it} = \frac{P_i\ r\ \delta\ I^A_{it}}{365} \qquad \forall\ i, t \tag{3.12}$$

$$IC^B_{it} = \frac{\rho_i\ r\ \delta\ I^B_{it}}{365} \qquad \forall\ i, t \tag{3.13}$$

Administrative Cost Constraints

In constraints 3.14 and 3.15, the cost of handling goods is a constant, H^A_i and H^B_i, multiplied by the total amounts handled in A and B, respectively. In A, the amount is the flow of products shipped to B, F_{it}, added to the production amount, M_{it}. Similarly, B's total handled amount is the quantity shipped to customers, S_{it}, added to the purchased amount, F_{it}.

Administrative cost Administrative cost constant × (amount received + amount shipped)

$$HC_{it}^{A} = H_i^{A}\ (M_{it} + F_{it}) \qquad \forall\ i,t \tag{3.14}$$

$$HC_{it}^{B} = H_i^{B}\ (F_{it} + S_{it}) \qquad \forall\ i,t \tag{3.15}$$

A Cost Constraint

In constraint 3.16, the manufacturing cost, $(RMC_i + PC_i)\ M_{it}$, is summed up with the inventory cost, IC_{it}^{A}, the administrative cost, HC_{it}^{A}, and the transportation cost of products shipped to B, $TC_i^{A}\ F_{it}$, for all products i and time intervals t. This constraint defines the total cost incurred in A.

Manufacturing cost (Raw material cost constant + production cost constant) × amount produced

Transportation cost Transportation cost constant × amount shipped

$$C^{A} = \sum_i \sum_t (RMC_i\ M_{it} + PC_i\ M_{it} + IC_{it}^{A} + HC_{it}^{A} + TC_i^{A}\ F_{it}) \tag{3.16}$$

B Cost Constraint

The costs incurred in B are equivalent to those incurred in A, except that the raw material cost is substituted by the purchase cost, $F_{it}\ P_i$. Furthermore, there is no production in B thus no production cost.

Purchase cost Transfer price × amount purchased

$$C^{B} = \sum_i \sum_t (F_{it}\ P_i + IC_{it}^{B} + HC_{it}^{B} + TC_i^{B}\ S_{it}) \tag{3.17}$$

Non-negativity Constraints

For many of the variables there are natural non-negativity constraints. Production and flow variables, M_{it}, F_{it}, S_{it}, must be positive for all times and products, as must variable costs, HC_{it}^{A}, HC_{it}^{B}, IC_{it}^{A}, IC_{it}^{B}, C^{A}, C^{B}. Non-negative inventory levels in A and B are safeguarded by constraints 3.4 and 3.5. The transfer price levels chosen, P_i, cannot be negative in this model even though in principle it would be feasible to let A pay for goods delivered to B.

$$F_{it}, M_{it}, S_{it}, HC_{it}^{A}, HC_{it}^{B}, IC_{it}^{A}, IC_{it}^{B} \geq 0 \qquad \forall\ i,t \tag{3.18}$$

$$P_i \geq 0 \qquad \forall\ i \tag{3.19}$$

$$C^{A}, C^{B} \geq 0 \tag{3.20}$$

Linearisation Constraints

The model has several non-linear elements. These elements are chosen to be linearised to result in a mixed integer linear programming (MILP) formulation (but the problem is still non-linear) so as to obtain a problem formulation in which optimality for the overall optimisation approach can be guaranteed within a finite time horizon. In constraints 3.7 and 3.17 and in constraint 3.12, there are non-linearities in the terms $F_{it}\ P_i$ and $P_i\ I^A_{it}$, respectively. The following linearisations (based on Glover (1975)) are applied:

$$\overline{XI}_{ikt} \leq X_{ik}\ I^U_i \qquad \forall\ i,k,t \qquad (3.21)$$

$$I^A_{it} = \sum_k \overline{XI}_{ikt} \qquad \forall\ i,t \qquad (3.22)$$

$$\overline{FX}_{ikt} \leq X_{ik}\ F^U_i \qquad \forall\ i,k,t \qquad (3.23)$$

$$F_{it} = \sum_k \overline{FX}_{ikt} \qquad \forall\ i,t \qquad (3.24)$$

We define $\overline{XI}_{ikt} \equiv X_{ik}\ I_{it}$ and $\overline{FX}_{ikt} \equiv F_{it}\ X_{ik}$. These linearisations will introduce $2 \cdot i \cdot k \cdot t + 2 \cdot i \cdot t$ new constraints. The term $\sum_k \hat{P}_{ik}\ \overline{FX}_{ikt}$ is used instead of the nonlinear term $F_{it}\ P_i$ and the nonlinear term $P_i\ I^A_{it}$ is replaced by $\sum_k \hat{P}_{ik}\ \overline{XI}_{ikt}$. Upper limits for the flow of each product i between A and B are introduced, F^U_i in constraint 3.23, and for the inventory level, I^U_i in constraint 3.21. Furthermore, the linearised terms cannot take negative values.

$$\overline{XI}_{ikt}, \overline{FX}_{ikt} \geq 0 \qquad \forall\ i,k,t \qquad (3.25)$$

Summary of Formulation

In conclusion, the entire formulation is outlined as follows:

$$\max\ \Pi_A + \Pi_B$$

subject to

$$\Pi_A = \sum_i \sum_t \sum_k \hat{P}_{ik}\ \overline{FX}_{ikt} - C^A$$

$$\Pi_B = \sum_i \sum_t S_{it}\ \rho_i - C^B$$

$$\Pi_A \geq \Pi^L_A$$

$$\Pi_B \geq \Pi^L_B$$

$$\sum_i UR_{ir}\ M_{it} \leq R_{rt} \qquad \forall\ r,t$$

$$I^A_{it} = I^A_{i,t-1} + M_{it} - F_{it} \qquad \forall\ i,t$$

$$I^A_{it} \geq \sigma^A_i \qquad \forall\ i.t$$

$$IC^A_{it} = \frac{r\ \delta}{365} \sum_k \hat{P}_{ik}\ \overline{XI}_{ikt} \qquad \forall\ i,t$$

$$\overline{XI}_{ikt} \leq X_{ik}\ I^U_i \qquad \forall\ i,k,t$$

$$I^A_{it} = \sum_k \overline{XI}_{ikt} \qquad \forall\ i,t$$

$$HC^A_{it} = H^A_i\ (M_{it} + F_{it}) \qquad \forall\ i,t$$

$$C^A = \sum_i \sum_t (RMC_i\ M_{it} + PC_i\ M_{it} + IC^A_{it} + HC^A_{it} + TC^A_i\ F_{it})$$

$$P_i = \sum_k \hat{P}_{ik}\ X_{ik} \qquad \forall\ i$$

$$\sum_k X_{ik} = 1 \qquad \forall\ i$$

$$I^B_{it} = I^B_{i,t-1} + F_{it} - S_{it} \qquad \forall\ i,t$$

$$I^B_{it} \geq \sigma^B_i \qquad \forall\ i,t$$

$$IC^B_{it} = \frac{\rho_i\ r\ \delta\ I^B_{it}}{365} \qquad \forall\ i,t$$

$$HC^B_{it} = H^B_i\ (S_{it} + F_{it}) \qquad \forall\ i,t$$

$$C^B = \sum_i \sum_t ((\sum_k \hat{P}_{ik}\ \overline{FX}_{ikt}) + IC^B_{it} + HC^B_{it} + TC^B_i\ S_{it})$$

$$S_{it} = D_{it} \qquad \forall\ i,t$$

$$\overline{FX}_{ikt} \leq X_{ik}\ F^U_i \qquad \forall\ i,k,t$$

$$F_{it} = \sum_k \overline{FX}_{ikt} \qquad \forall\ i,t$$

$$\overline{XI}_{ikt}, \overline{FX}_{ikt} \geq 0 \qquad \forall\ i,k,t$$

$$F_{it}, M_{it}, S_{it}, HC^A_{it}, HC^B_{it}, IC^A_{it}, IC^B_{it} \geq 0 \qquad \forall\ i,t$$

$$P_i \geq 0 \qquad \forall\ i$$

$$C^A, C^B \geq 0$$

$$X_{ik} \in\ 0,1 \qquad \forall\ i,k$$

3.8 Nash-Based Algorithm

The proposed Nash problem is inherently non-linear and non-convex. A specialised algorithm is therefore developed based on McCormick (1976) relaxations and a successive refinement procedure to linearise the problem, *i.e.* the maximisation of the Nash objective function. The McCormick methodology is often difficult to apply since it is hard to find appropriate lower and upper bounds for the reformulated (approximate) problem. In this case, the lower bounds are given by each company's lower profit requirements. If the lower profit requirement point is suboptimal then the Nash algorithm will find the solution of the linearised problem that most fairly (subject to the Nash axioms) distributes the surplus amount obtained by the problem optimisation (within a pre-specified margin). The solution algorithm applied to the problem is outlined next and is also illustrated in figure 3.6.

1. **Convex Relaxation.** Specify minimum echelon profit levels: Π_A^L and Π_B^L.

2. Objective: Max $\Phi = (\Pi_A - \Pi_A^L)(\Pi_B - \Pi_B^L)$

3. Expand: $(\Pi_A - \Pi_A^L)(\Pi_B - \Pi_B^L) \simeq \hat{\zeta} - \Pi_A \Pi_B^L - \Pi_A^L \Pi_B + \Pi_A^L \Pi_B^L \equiv \hat{\Phi}$

4. Find upper bounds on Π_A and Π_B:

   - `Max` Π_B `s.t.` $\Pi_A \geq \Pi_A^L$ `and model constraints` $\Rightarrow \Pi_B^U$
   - `Max` Π_A `s.t.` $\Pi_B \geq \Pi_B^L$ `and model constraints` $\Rightarrow \Pi_A^U$

5. Apply McCormick relaxation constraints on original non-linearity, $\hat{\zeta}$ in $\hat{\Phi}$.

$$\hat{\zeta} \geq \Pi_A^U \Pi_B + \Pi_A \Pi_B^U - \Pi_A^U \Pi_B^U$$
$$\hat{\zeta} \leq \Pi_A^U \Pi_B + \Pi_A \Pi_B^L - \Pi_A^U \Pi_B^L$$
$$\hat{\zeta} \leq \Pi_A \Pi_B^U + \Pi_A^L \Pi_B - \Pi_A^L \Pi_B^U$$
$$\hat{\zeta} \geq \Pi_A^L \Pi_B + \Pi_A \Pi_B^L - \Pi_A^L \Pi_B^L$$

6. **Spatial branch-and-bound algorithm.** Put the relaxed problem with the bounds of step 4 in the candidate list, $\mathcal{L}$, given the lower bounds Π_n^L, $(n = A, B)$, in order to obtain the highest upper bounds, Π_n^U. The region investigated below is thus: $[\Pi_n^L, \Pi_n^U]$ on each objective variable. The best solution so far, Φ^{BSF} is set to $-\infty$.

   `Set` $\mathcal{L}$ `= { Fully relaxed problem }`, $\Phi^{BSF} = -\infty$.

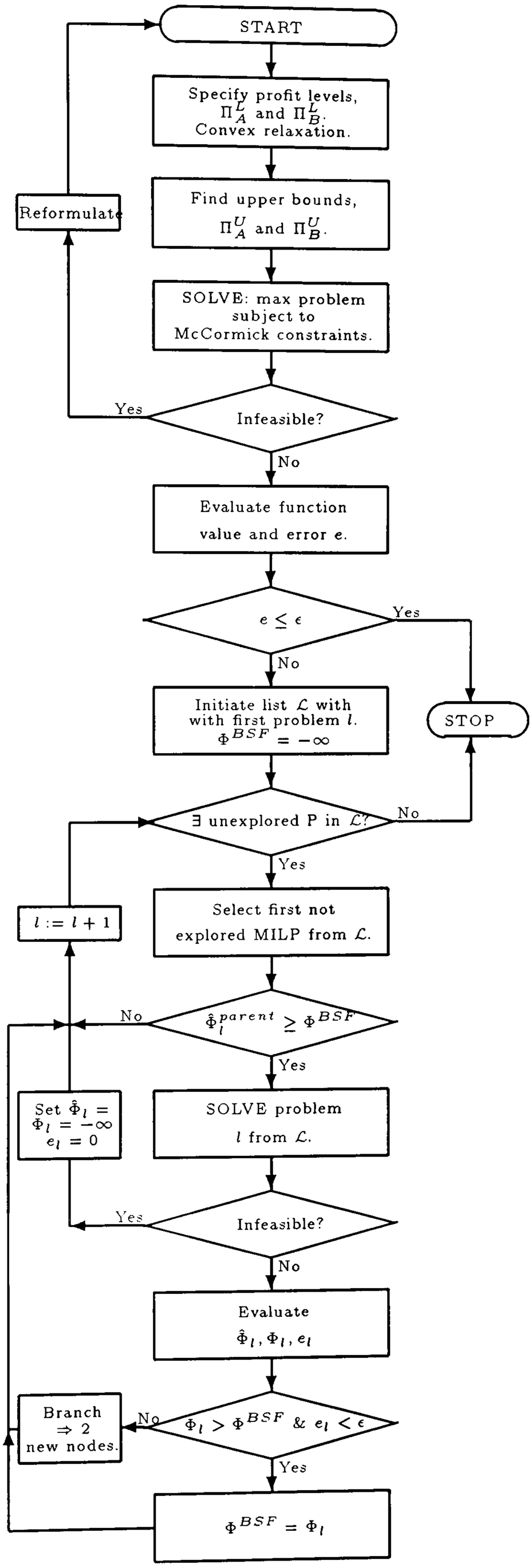


Figure 3.6: Spatial branch-and-bound algorithm

7. Choose the first problem from the list $\mathcal{L}$ if there is any node that has a prospect of yielding a better solution than the best feasible solution so far. If no problem in the list fulfills this criterion or all nodes have been explored, stop.

   Select the first, not explored MILP problem l from $\mathcal{L}$, if fully relaxed or $\hat{\Phi}_l^{parent} > \Phi^{BSF}$. If no $l \in \mathcal{L}$ satisfy $\hat{\Phi}_l^{parent} > \Phi^{BSF}$ or all l exhausted, STOP.

8. Solve the MILP chosen using the convex McCormick envelope described above. If the solution is feasible, evaluate the actual objective function and the error function (difference between actual and relaxed objectives). If there is no feasible solution set the objective values to a low value, *e.g.* $-\infty$, and the error to zero, so as not to further explore this branch of the search tree.

   Solve MILP problem for l from the candidate list $\mathcal{L}$.
   If infeasible set $\hat{\Phi}_l = -\infty$, $\Phi_l = -\infty$ and $e_l = 0$, where e_l the error measure ($e_l \equiv \frac{|\Phi - \hat{\Phi}|}{\Pi_A \Pi_B} = \frac{|\Pi_A \Pi_B - \hat{\zeta}|}{\Pi_A \Pi_B}$). Otherwise, evaluate $\hat{\Phi}_l$, Φ_l and e_l.

9. If the current value of the objective function is better than the best solution so far set current value to Φ^{BSF}. It is required that the current error is less than the relative tolerance, ϵ.

   If $(\Phi_l > \Phi^{BSF}$ and $e_l < \epsilon)$ set $\Phi^{BSF} = \Phi_l$.

10. If the current error is larger than the relative tolerance, split the current node search space range and create two children nodes with new ranges on the profit function that has the largest span. This partitioning can be performed in different fashions (Smith and Pantelides, 1999). For simplicity, the mid-point of the parent node search space is used as a divider for the children nodes (bisection). Finally, let the parent node of the children nodes be the current node.

    If $e_l \geq \epsilon$, then

    (a) Select n with maximum $[\Pi_n^{l,U} - \Pi_n^{l,L}]$, $n^\star$.

    (b) Create $l+1$ and $l+2$ and add to $\mathcal{L}$, where $\Pi_{n^\star}^{l+1,L} = \Pi_{n^\star}^{l,L} + \frac{1}{2}(\Pi_{n^\star}^{l,U} - \Pi_{n^\star}^{l,L})$ and $\Pi_{n^\star}^{l+1,U} = \Pi_{n^\star}^{l,U}$, while $\Pi_{n^\star}^{l+2,U} = \Pi_{n^\star}^{l,U} - \frac{1}{2}(\Pi_{n^\star}^{l,U} - \Pi_{n^\star}^{l,L})$ and $\Pi_{n^\star}^{l+2,L} = \Pi_{n^\star}^{l,L}$.

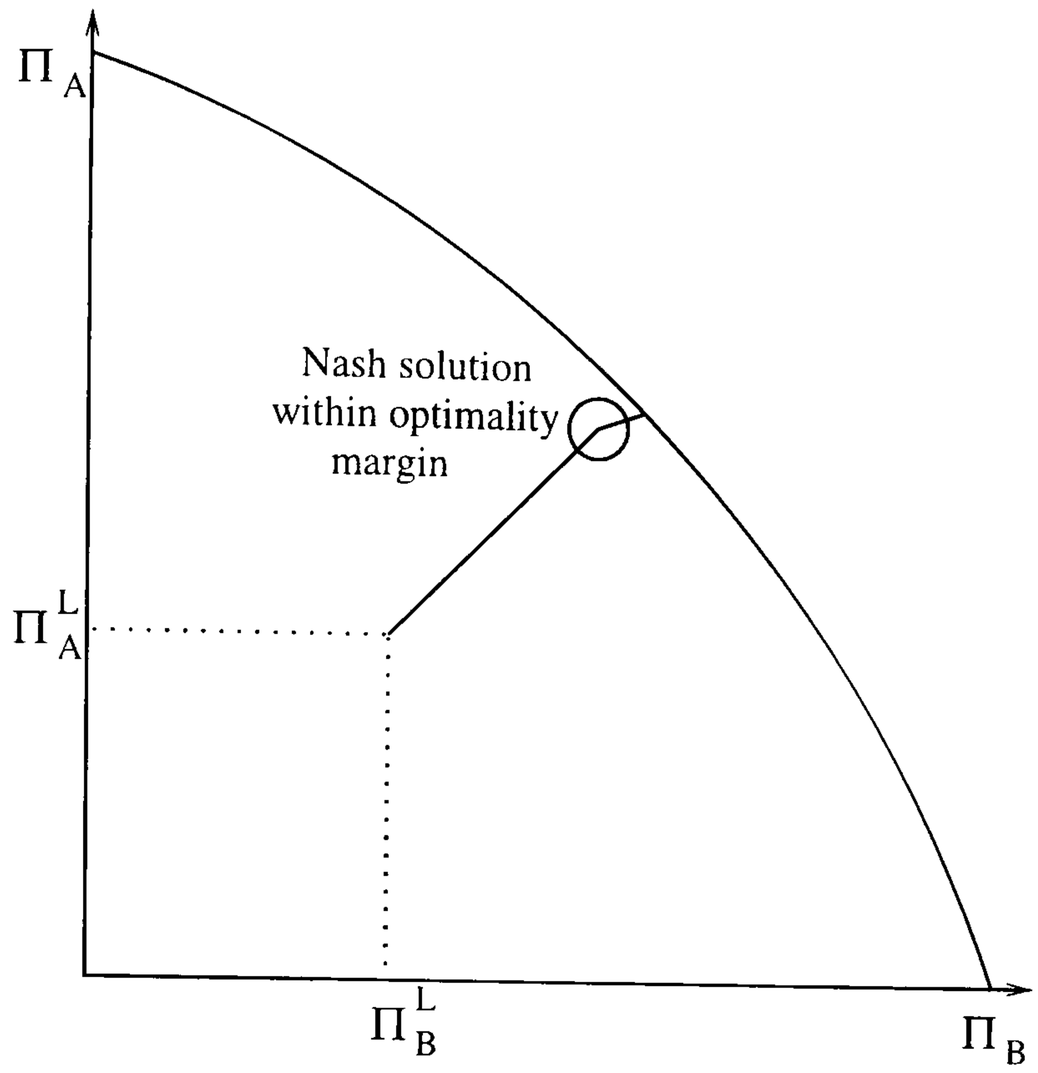


Figure 3.7: Improved Nash solution: The hybrid approach

(c) Set $\hat{\Phi}_{l+1}^{parent} = \hat{\Phi}_l$, $\hat{\Phi}_{l+2}^{parent} = \hat{\Phi}_l$.

11. Return to step 7.

Goto 7.

Single Level and Hybrid Approach

The single level approach regards the supply chain as being one profit-generating centre, and is used as a bench-mark supply chain performance indicator to evaluate the performance of the Nash algorithm. The overall objective is to maximise the profit of the entire supply chain which is equal to the summed profits of A and B, *i.e.* $\Pi = \Pi_A + \Pi_B$. The obtained value will be the highest summed supply chain profit attainable (within the optimality margin).

Furthermore, a hybrid improvement phase is considered in which the single level approach can be utilised on the Nash solution so as to attain a truly Pareto optimal solution. The hybrid approach is illustrated in figure 3.7. For instance, on the best achievable node of the Nash solution scheme we may have a marginal gap of 4%, with a given margin of optimality of 5 %. This means that the obtained solution can be guaranteed to be within this pre-specified margin of overall possible profit for the stated

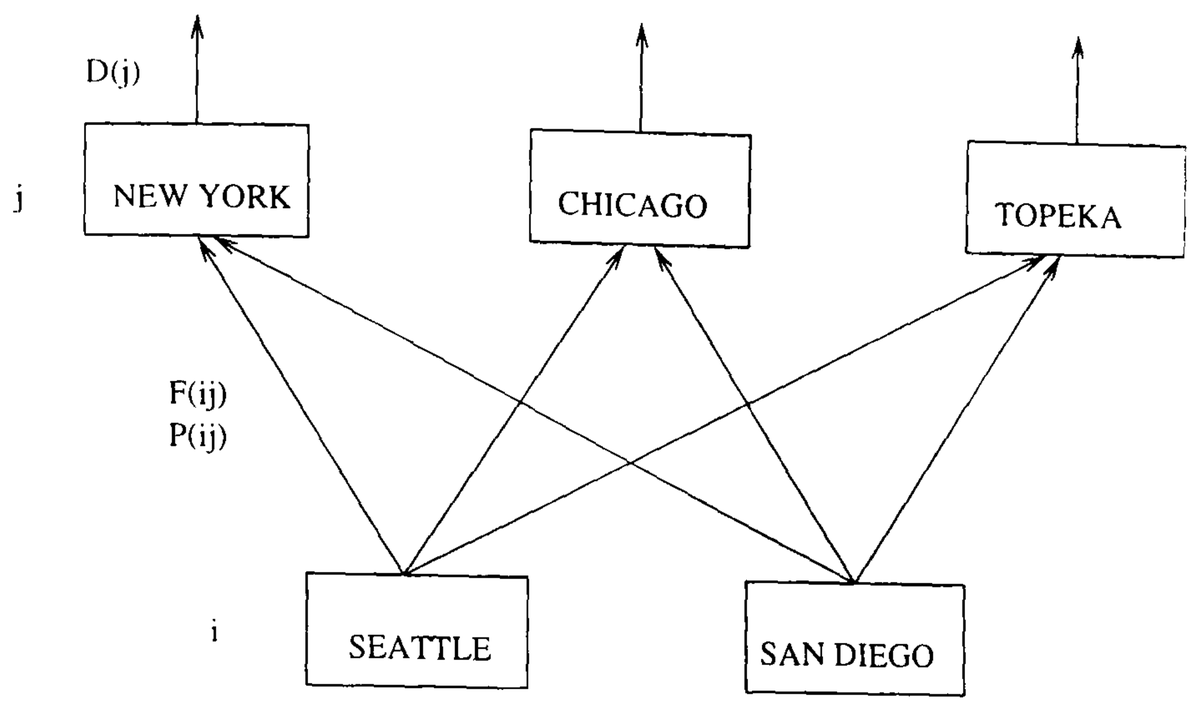


Figure 3.8: The transportation problem

problem. By setting the profits obtained by the branch-and-bound approach as lower bounds for the single level optimisation, a slightly better solution may be achievable. In other words, $\Pi_A + \Pi_B$ is maximised, subject to $\Pi_A \geq \Pi_A^\star$, $\Pi_B \geq \Pi_B^\star$ (where $\Pi_A^\star$ and $\Pi_B^\star$ are the solution profits of the Nash approach), and other constraints. This approach is mainly considered for large-scale problem when the spatial branch-and-bound algorithm requires large CPU times.

3.9 The Transportation Problem

To demonstrate the basic features of the approach developed, we focus our attention on a classical transport problem as proposed by Dantzig (1963). In this problem, the maximum available supply of products at two canning plants in Seattle and San Diego (company A) and customer demand at three demand points New York, Chicago, and Topeka (company B) are given. Given also is the unit cost of transportation between all the five nodes in the supply chain network. The solution of the problem determines optimal transportation quantities and links so as to minimise the transportation cost.

The basic transportation problem is modified by addition of a handling cost at the plants based on the shipped quantities, pre-specified unit revenues incurred at the demand points and a number of possible transfer price levels for each product transported between the nodes in the supply chain network. The problem has consequently been converted into maximisation of profit of the plant and distribution centre level, see figure 3.8. In other words, the levels of F_{ij} and P_{ij} are determined so as to achieve a fair distribution of profit between the level i and j. The computational results of the transportation problem

can be studied in table 3.1. In the table we have in turn: the node number, the relaxed solution to the Nash problem, $\hat{\Phi}_l$, the exact value of the Nash function, Φ_l, the relative value of the error (which for an acceptable solution should be less than the tolerance), ϵ_l, and finally the values of the objectives, Π_A and Π_B. The pre-specified tolerance in this example is 1%. The three first nodes have quite high errors, and they are branched. Node number 4 has a value of 2583.5, and a node error that is satisfactory, 0.4%, and we do not further branch on this node. Nodes 5 and 7 are within the tolerance, but with a lower objective function value than node 4. As can be seen, branching is carried out on nodes 6, 8, and 11. The overall solution on the subsequent nodes does not improve from the solution of node 4. Node 4 is thus stated as the best solution of the algorithm.

3.10 A Three-Enterprise Supply Chain Model

In this section, a generalisation of the two-enterprise model to a three-enterprise supply chain partnership is presented. The model considers three nodes in a supply chain, which are owned by three different enterprises: two manufacturing type companies, A and B, and a distribution and sales type company, C. These three enterprises constitute a single sourcing supply chain partnership with three echelons. A delivers intermediate products to B, which in turn delivers final products to C, which sells the final products to external customers.

I different products are produced in A using R production resources and are supplied downstream. In B, J products are produced using P production resources and the I intermediate products purchased from A and then the products are supplied downstream to C. The time horizon consists of T discrete time intervals. The decision makers in A, B, and C have K transfer price levels to choose from, with disparate price ranges for intermediate products i and final products j. Inventory can be held in either A, B, or C. Costs include raw material, manufacturing, administrative (transaction), inventory, and transportation cost at A. The costs of B are the same except for the added purchase cost incurred by purchase of products from A. C's costs are purchase, administrative, inventory, and transportation cost. The revenue of A is the purchase cost of B and the revenue of B is the purchase cost of C and revenue of C is determined by the market sales prices of the final products j. The revenues less the corresponding costs constitute

Node	A^L	A^U	B^L	B^U	$\hat{\Phi}_l^{parent}$	Parent node	$\hat{\Phi}_l$	Φ_l	ϵ_l	Π_A	Π_B
1	50	150	50	150			5071.988	2572.506	0.246	100.720	100.720
2	50	150	100	150	5071.988	1	3447.578	2318.079	0.114	84.476	117.238
3	50	150	50	100	5071.988	1	3409.465	2324.890	0.109	118.189	84.09
4	100	150	100	150	3447.578	2	2624.233	2583.508	0.004	100.828	100.828
5	50	100	100	150	3447.578	2	2581.325	2581.325	4.48E-17	100.000	101.627
6	100	150	50	100	3409.465	3	2616.250	2287.725	0.033	119.325	83.000
7	50	100	50	100	3409.465	3	2500.000	2500.000	9.09E-17	100.000	100.000
8	100	150	75	100	2616.250	6	2614.198	2324.890	0.029	118.189	84.095
9	100	150	50	75	2616.250	6	1842.569	1842.569	1.47E-16	123.703	75.000
10	125	150	75	100	2614.198	8	2033.504	2033.504	1.41E-16	125.000	77.113
11	100	125	75	100	2614.198	8	2586.200	2430.693	0.016	113.362	88.362
12	100	125	87.5	100	2586.200	11	2579.563	2506.261	0.007	109.394	92.197
13	100	125	75	87.5	2586.200	11	2409.519	2409.519		114.251	87.500

Table 3.1: Spatial branching: The transportation problem

the objective functions of A, B, and C, respectively. The objective function utilises the extended Nash-Harsanyi model (Harsanyi, 1977*a*; Shields *et al.*, 1999). The generalised Nash-Harsanyi extension to the Nash solution proposes, similarly to the Nash model discussed earlier, that if a status quo point for n players is x_n^L, then the fair solution point is x_n, $x_n \geq x_n^L$, which maximises $\prod_n (x_n - x_n^L)$. Thus, the objective function for the three-enterprise supply chain problem is max $\Phi = (\Pi_A - \Pi_A^L)(\Pi_B - \Pi_B^L)(\Pi_C - \Pi_C^L)$.

The McCormick constraints from the Nash algorithm needs to be extended for the three-echelon supply chain to include trilinear terms (to substitute the profit term $\Pi_A \Pi_B \Pi_C$), see below. Bilinear terms have been linearised utilising the following McCormick overestimation approximations for the bilinear terms ($\Pi_A \Pi_B, \Pi_A \Pi_C, \Pi_B \Pi_C$) of the three-echelon example:

$$\widehat{\Pi_A \, \Pi_B} \leq \Pi_A^U \, \Pi_B + \Pi_A \, \Pi_B^L - \Pi_A^U \, \Pi_B^L$$
$$\widehat{\Pi_A \, \Pi_B} \leq \Pi_A \, \Pi_B^U + \Pi_A^L \, \Pi_B - \Pi_A^L \, \Pi_B^U$$
$$\widehat{\Pi_B \, \Pi_C} \leq \Pi_B^U \, \Pi_C + \Pi_B \, \Pi_C^L - \Pi_B^U \, \Pi_C^L$$
$$\widehat{\Pi_B \, \Pi_C} \leq \Pi_B \, \Pi_C^U + \Pi_B^L \, \Pi_C - \Pi_B^L \, \Pi_C^U$$
$$\widehat{\Pi_A \, \Pi_C} \leq \Pi_A^U \, \Pi_C + \Pi_A \, \Pi_C^L - \Pi_A^U \, \Pi_C^L$$
$$\widehat{\Pi_A \, \Pi_C} \leq \Pi_A \, \Pi_C^U + \Pi_A^L \, \Pi_C - \Pi_A^L \, \Pi_C^U$$

Maranas and Floudas (1995) proposed a set of convex underestimators with tighter bounds than the corresponding bilinear products, *e.g.* $\widehat{\Pi_A \, \Pi_C} \times \Pi_B$. An overestimation of the trilinear term with six linear functions is introduced. The derivation of these bounds is given in Appendix A.

$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^L \Pi_C^L + \Pi_A^U \Pi_B \Pi_C^L + \Pi_A^U \Pi_B^U \Pi_C - \Pi_A^U \Pi_B^U \Pi_C^L - \Pi_A^U \Pi_B^L \Pi_C^L$$
$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^L \Pi_C^L + \Pi_A^U \Pi_B \Pi_C^U + \Pi_A^U \Pi_B^L \Pi_C - \Pi_A^U \Pi_B^L \Pi_C^U - \Pi_A^U \Pi_B^L \Pi_C^L$$
$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^U \Pi_C^U + \Pi_A^L \Pi_B \Pi_C^L + \Pi_A^L \Pi_B^U \Pi_C - \Pi_A^L \Pi_B^U \Pi_C^L - \Pi_A^L \Pi_B^U \Pi_C^U$$
$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^U \Pi_C^U + \Pi_A^L \Pi_B \Pi_C^U + \Pi_A^L \Pi_B^L \Pi_C - \Pi_A^L \Pi_B^L \Pi_C^U - \Pi_A^L \Pi_B^U \Pi_C^U$$
$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^U \Pi_C^L + \Pi_A^L \Pi_B \Pi_C^L + \Pi_A^U \Pi_B^U \Pi_C - \Pi_A^U \Pi_B^U \Pi_C^L - \Pi_A^L \Pi_B^U \Pi_C^L$$
$$\Pi_A \widehat{\Pi_B \, \Pi_C} \leq \Pi_A \Pi_B^L \Pi_C^U + \Pi_A^U \Pi_B \Pi_C^U + \Pi_A^L \Pi_B^L \Pi_C - \Pi_A^L \Pi_B^L \Pi_C^U - \Pi_A^U \Pi_B^L \Pi_C^U$$

$i =$	$(\text{Prod}_1, \text{Prod}_2)$
$k =$	$(0.4. 0.8, \ldots, 4)$
$r =$	(Heat exchanger, Reactor)
$t =$	$(1, \ldots, 12)$

Table 3.2: Sets for example 1

3.11 Computational Experiments

Three approaches - the single level optimisation approach, the Nash algorithm, and the "hybrid" combination - of solving the two-enterprise supply chain transfer price problem given sets of base case parameters are discussed. Two examples have been implemented to demonstrate the techniques developed. For the three-enterprise supply chain model one test problem has been developed and is described in section 3.11.4. The models have been implemented in GAMS (Brooke *et al.*, 1997) using the CPLEX (Lustig, 1998) MILP solver. The computational performance given is for a Sun SPARC Ultra 60 workstation using a margin of optimality of 5% for all MILP solutions, except for example 1 where the marginal gap is set very small so as to obtain truly optimal solutions. The tolerance, ϵ, is 0.1% for example 1 and 5% for example 2.

3.11.1 Example 1

The first example considers a very simple manufacturing process in the production plant, A. Two products are produced sharing one heat-exchanger and one reactor and are shipped to the downstream distribution centre, B. The available transfer price levels represent a discretisation of a continuous span of prices. The overall model time horizon of interest is one year. In this example, a time discretisation interval, δ, of one month is used. The lower profit requirements of A and B are $\Pi_A^L = \Pi_B^L = 2500$. Table 3.5 provides profit levels and computational results for the single level approach and the Nash algorithm. The data for this example are given in tables 3.2, 3.3, and 3.4.

Single Level Approach

In the single level approach the whole supply chain is optimised as a single profit-centre. A low transfer price level is desirable when optimising the whole supply chain as this

$$UR_{ir} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$RMC_i = \begin{pmatrix} 1.2 \\ 0.9 \end{pmatrix}$$

$$PC_i = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$$

$$TC_i^A = \begin{pmatrix} 0.6 \\ 0.75 \end{pmatrix}$$

$$\sigma^A = \sigma^B = 0$$

$$r = 0.15$$

$$\delta = 30.42$$

$$R_i = \begin{pmatrix} 24.3 \\ 21.9 \end{pmatrix}$$

$$I_i^U = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$$

$$P_i^U = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

$$F_i^U = \begin{pmatrix} 60 \\ 60 \end{pmatrix}$$

$$H^A = \begin{pmatrix} 8.8 \\ 8.7 \end{pmatrix}$$

$$H^B = \begin{pmatrix} 1.0 \\ 1.2 \end{pmatrix}$$

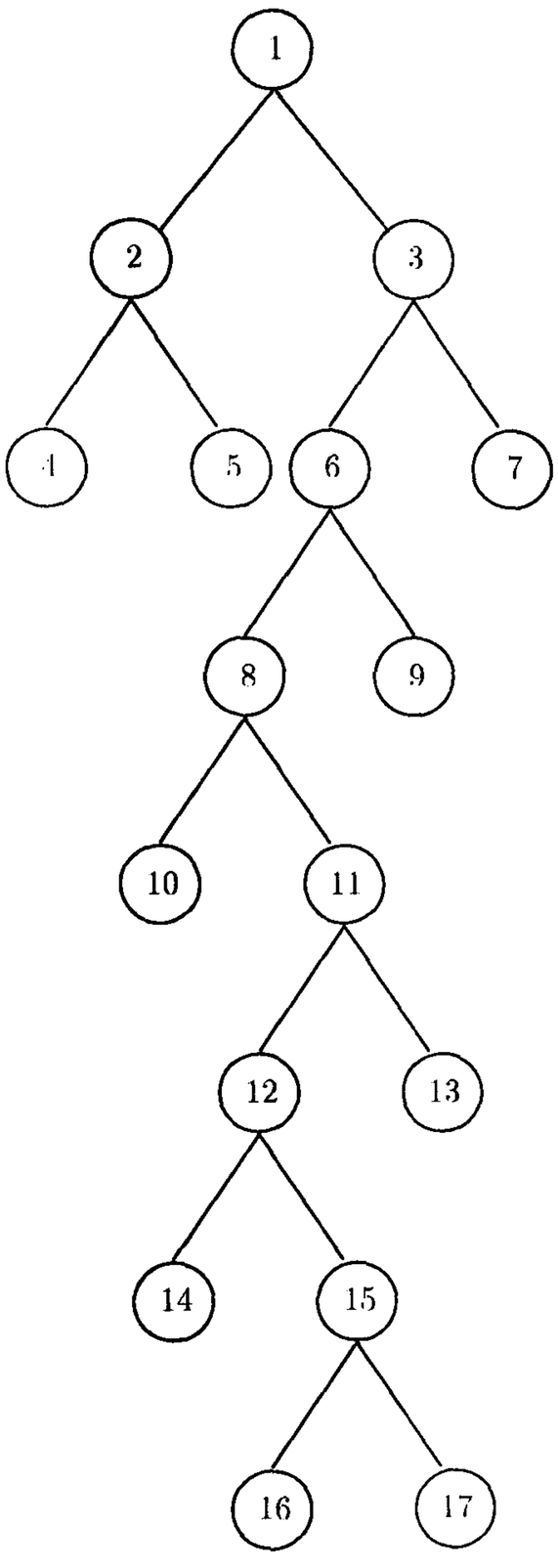
$$TC_i^B = \begin{pmatrix} 0.9 \\ 1.1 \end{pmatrix}$$

Table 3.3: Time independent parameters for example 1

$$R_{rt} = \begin{pmatrix} 70 & 65 & 58 & 50 & 85 & 85 & 85 & 85 & 85 & 85 & 78 & 72 \\ 80 & 70 & 70 & 56 & 40 & 85 & 85 & 85 & 85 & 85 & 82 & 75 \end{pmatrix}$$

$$D_{it} = \begin{pmatrix} 45 & 10 & 15 & 20 & 0 & 150 & 10 & 10 & 10 & 10 & 10 & 190 \\ 10 & 15 & 15 & 15 & 0 & 185 & 5 & 5 & 5 & 5 & 5 & 195 \end{pmatrix}$$

Table 3.4: Time dependent parameters for example 1



$\Pi_A^L = 2500, \Pi_B^L = 2500$

Relative tol = 0.001

1. $e = 0.24, \Phi = 5.89 * 10^6, \hat{\Phi} = 11.8 * 10^6$
2. $e = 0.11, \Phi = 5.24 * 10^6, \hat{\Phi} = 7.85 * 10^6$
3. $e = 0.11, \Phi = 5.23 * 10^6, \hat{\Phi} = 7.84 * 10^6$
4. $e = 1.8 * 10^{-4}, \Phi = 5.89 * 10^6, \hat{\Phi} = 5.90 * 10^6$
5. $e = 6.7 * 10^{-4}, \Phi = 6.06 * 10^6, \hat{\Phi} = 6.08 * 10^6$
6. $e = 7.0 * 10^{-3}, \Phi = 5.92 * 10^6, \hat{\Phi} = 6.10 * 10^6$
7. $e = 3.8 * 10^{-17}, \Phi = 5.89 * 10^6, \hat{\Phi} = 5.89 * 10^6$
8. $e = 7.0 * 10^{-3}, \Phi = 5.92 * 10^6, \hat{\Phi} = 6.10 * 10^6$
9. $e = 1.6 * 10^{-16}, \Phi = 4.47 * 10^6, \hat{\Phi} = 4.47 * 10^6$
10. $e = 1.6 * 10^{-16}, \Phi = 4.53 * 10^6, \hat{\Phi} = 4.53 * 10^6$
11. $e = 7.0 * 10^{-3}, \Phi = 5.92 * 10^6, \hat{\Phi} = 6.10 * 10^6$
12. $e = 7.0 * 10^{-3}, \Phi = 5.93 * 10^6, \hat{\Phi} = 6.10 * 10^6$
13. $e = 3.8 * 10^{-17}, \Phi = 5.56 * 10^6, \hat{\Phi} = 5.56 * 10^6$
14. $e = 3.8 * 10^{-17}, \Phi = 5.66 * 10^6, \hat{\Phi} = 5.66 * 10^6$
15. $e = 4.0 * 10^{-3}, \Phi = 5.93 * 10^6, \hat{\Phi} = 6.02 * 10^6$
16. $\hat{\Phi}^{\text{parent}} < \Phi^{\text{BSF}}$
17. $\hat{\Phi}^{\text{parent}} < \Phi^{\text{BSF}}$

Figure 3.9: Nash algorithm, spatial branch-and-bounding: Example 1

implies low inventory cost in A. The problem is that this policy induces low profits for A. Profits of A and B are 3045 and 6898, respectively.

Nash Approach

Figure 3.9 illustrates in detail the procedure of the spatial branch-and-bound algorithm presented in section 3.8. The error (the relative difference between the convex relaxation

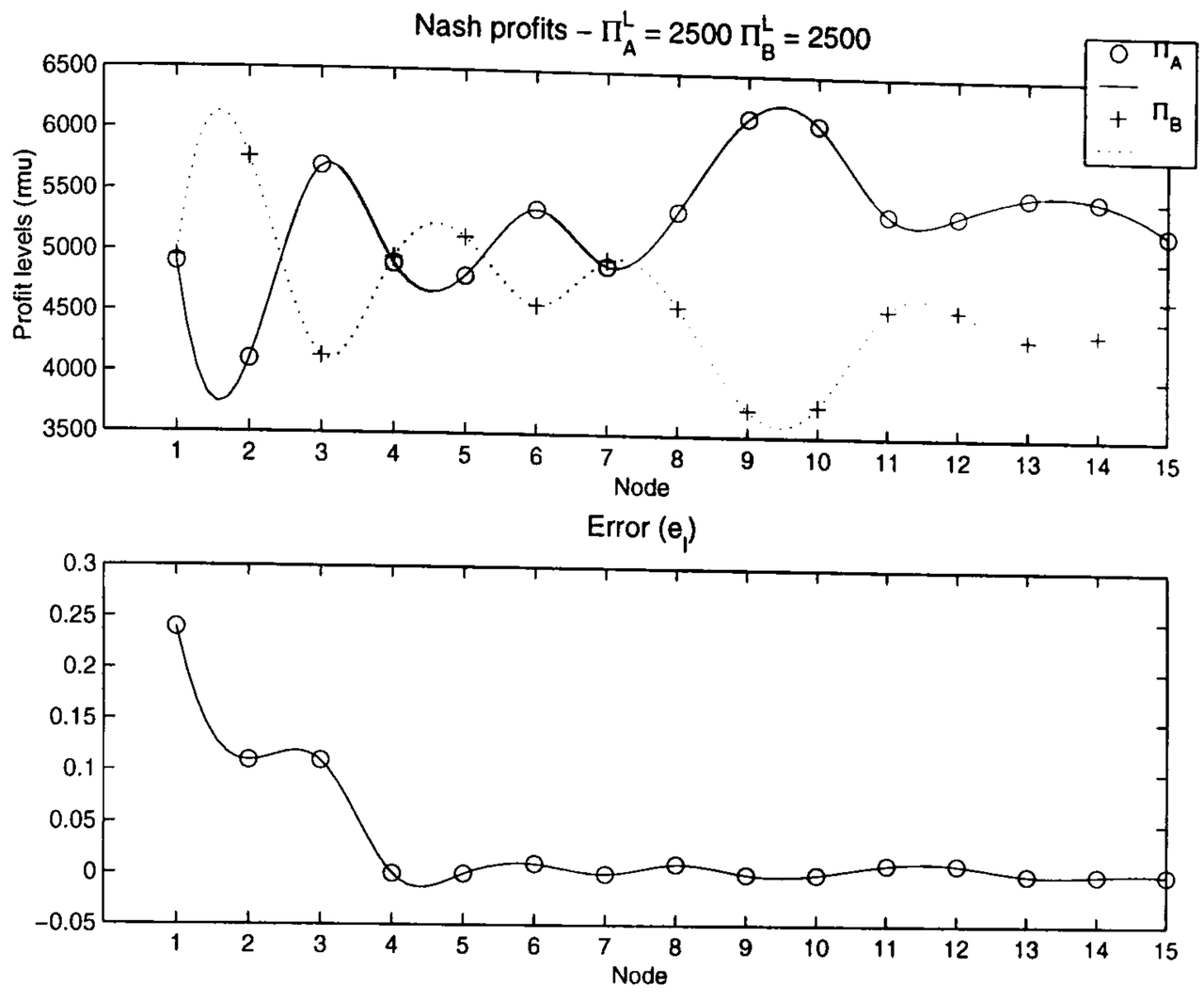


Figure 3.10: Nash algorithm, profit levels and error progression: Example 1

of the bilinear objective and the objective itself) is denoted by e. The value of the exact and the relaxed objective are denoted by Φ and $\hat{\Phi}$, respectively. In total, 15 nodes have been explored. With a tolerance of 5%, instead of 0.1%, 7 MILPs instead of 15 would be solved. Node 5 is the best node with an objective value on the exact function of 6.06×10^6. Figure 3.10 illustrates the echelon profits as well as the error during the solution procedure. It is clear that the algorithm examines four nodes to find the first acceptable solution (*i.e.* within the tolerance). As can be seen in table 3.5, the Nash algorithm generates profit levels of A and B of 4808 and 5127, respectively. A fair split of profits (*cf.* the single level approach where profits were 3045 and 6898, respectively) subject to the Nash axioms is obtained.

3.11.2 Example 2

A two-enterprise example[1] is considered with one plant manufacturing final products and one distribution centre with a more complex manufacturing process than in example 1. In the production plant, the material in the first step of the process is a liquid feed which is dried in a tower to obtain four different powders which in turn are packed in various sizes to make 18 final products of which 15 are currently in demand. They belong to four different families, based on the intermediate used for production. The plant production

[1]This example is adapted from Yee, K. L., Shah, N. (1997). Scheduling of multistage fast-moving consumer goods plants, *Journal of the Operational Research Society*, **48**, 1201-1214.

Table 3.5: Computational results for examples 1 and 2

Computation	Gap (%)	Constraints	Variables	Binary	Nodes	CPU (s)	Π_A	Π_B	$\Pi_A + \Pi_B$
Example 1 - single	0	781	729	20	1	3.5	3045	6898	9943
Example 1 - Nash	0	785	730	20	15	66.2	4808	5127	9935
Example 1 - hybrid	0	781	729	20	1	2.4 + Nash	4808	5127	9935
Example 2 - single	0.08	26889	24560	150	1	232	5065	52480	57545
Example 2 - Nash	1.7	26893	24561	150	7	10k	18110	37499	55609
Example 2 - hybrid	0.24	26889	24560	150	1	464 + Nash	18110	39344	57454

process is illustrated in figure 3.11. The packing of the goods uses six different packaging lines and a limited amount of manpower and material handling resources. Base feed ("Feed") is converted into four intermediate products ("Int_1" to "Int_4") by four separate processes ("Proc_1" to "Proc_4"). The intermediate products are then sent to packaging ("Packline_1" to "Packline_6"). Final products ("P_{11}" to "P_{43}" (products P_{13}, P_{23}, and P_{33} are currently not required by customers)) are obtained by using different packaging lines in order to make products of different shape and size. The distribution centre output restrictions are characterised by an irregular deterministic demand profile for a time horizon of eight weeks with a time discretisation interval of one day and fixed sales prices throughout this period. As is quite typical within industrial systems today, products are delivered downstream at the end of each week in the planning period, *i.e.* days $7, 14, 21, \ldots$.

The following quantities of the production and distribution problem are determined while maximising the profit levels of the separate enterprises:

- Production levels in A
- Inventory levels in A
- Flow of products between A and B
- Transfer prices of the products
- Inventory levels in B

Single Level Approach

In figures 3.12 and 3.13, the flow rates between A and B, the production level in A, and the inventory levels in A and B for product 3-4 are presented. In figure 3.14, the transfer price levels for some of the products, *i.e.* products 3-1 to 3-5, are shown. The transfer price established is the middle bar for each product in the figure and the two other bars show the available transfer price range. The production plan described by the figures satisfies demand with minimised inventory at transfer prices that meet lower profit requirements ($\Pi_A^L = 5000$ and $\Pi_B^L = 25000$) for both enterprises. The profits of A and B are 5065 and 52480, respectively. The profit of A has barely improved from the status quo level, and the result is considered strictly uneven based on the Nash axioms. A will thus

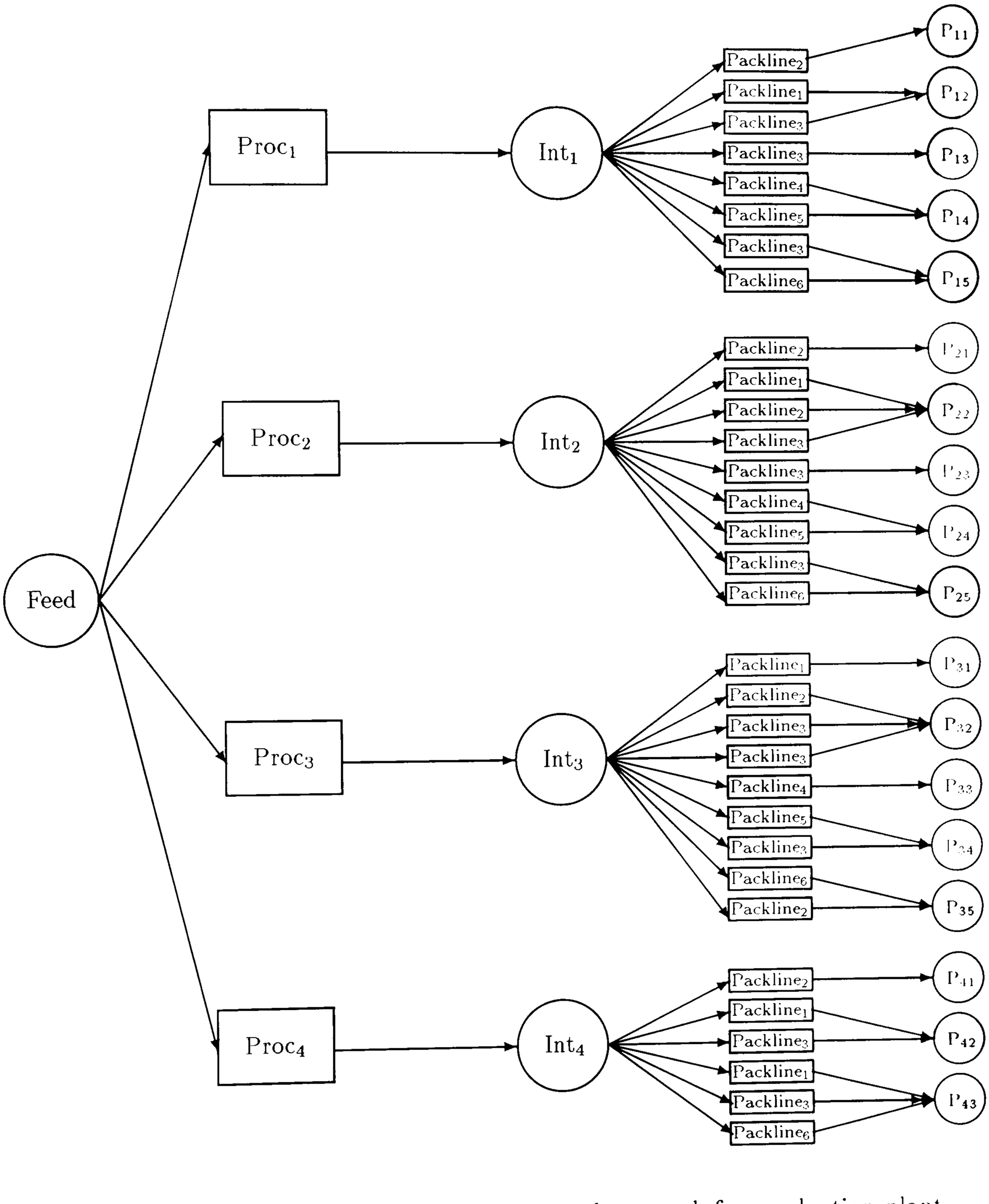


Figure 3.11: Two-enterprise example: State-task network for production plant

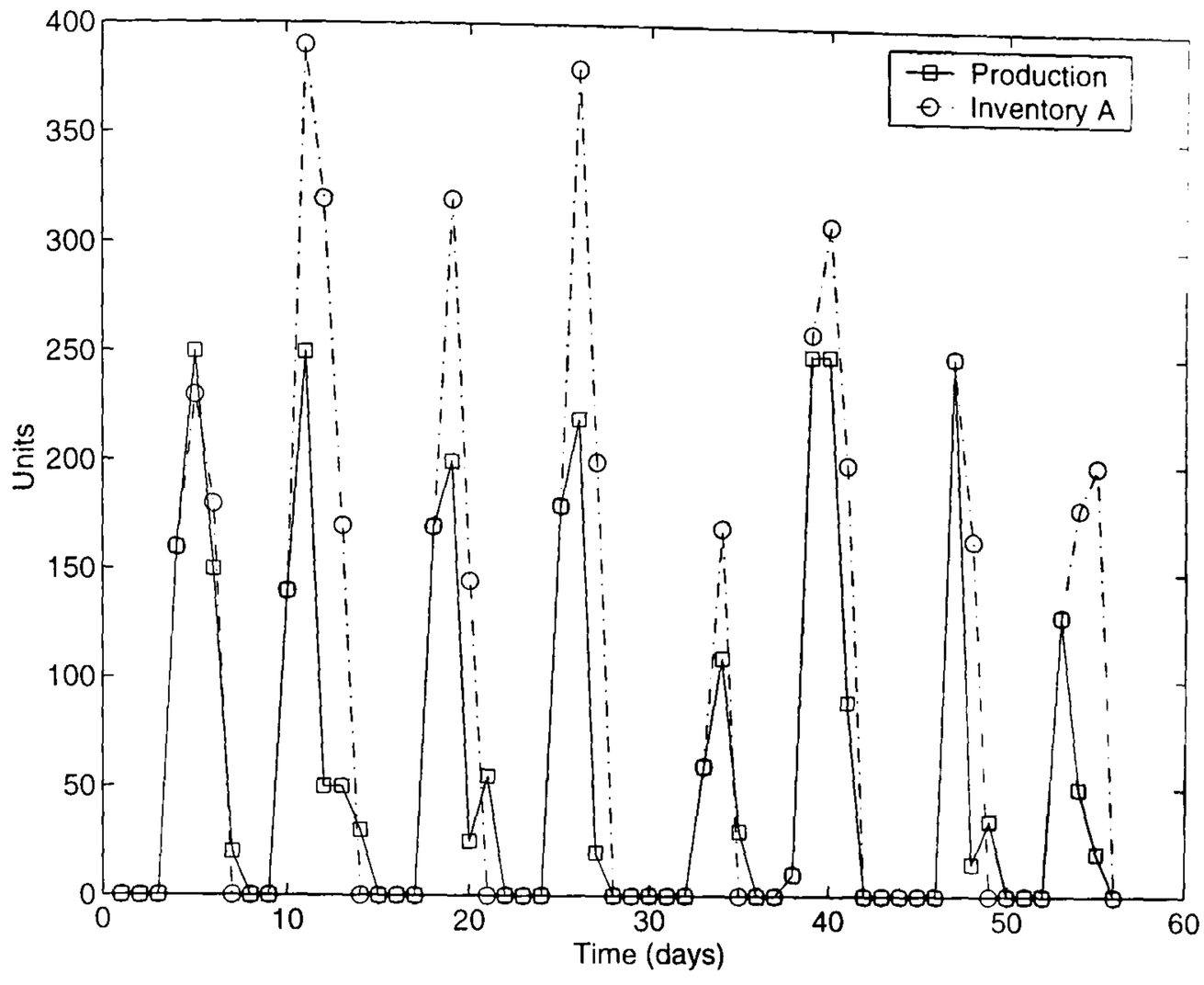


Figure 3.12: A associated variables - production quantity and inventory: Single level approach

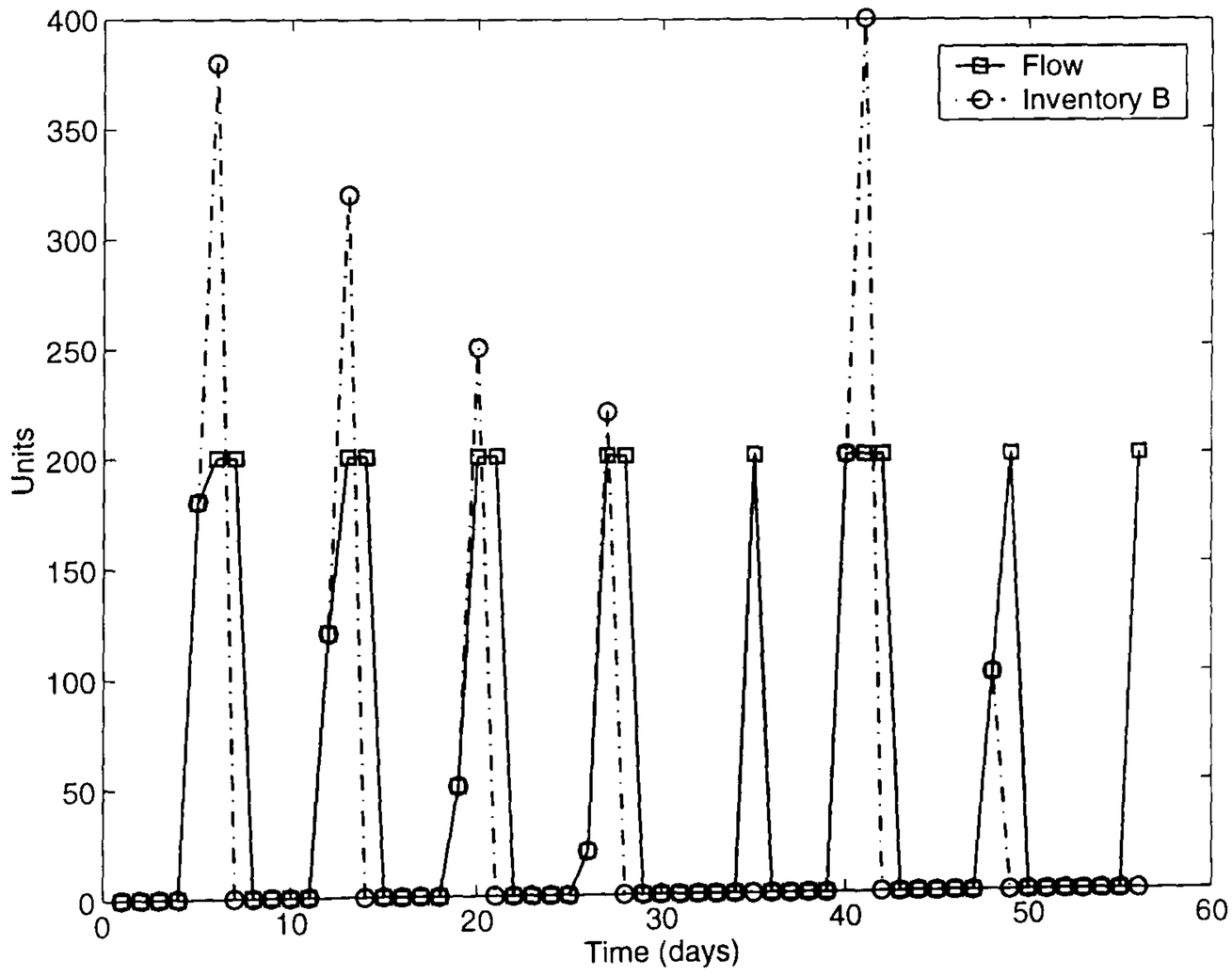


Figure 3.13: B associated variables - flow and inventory: Single level approach

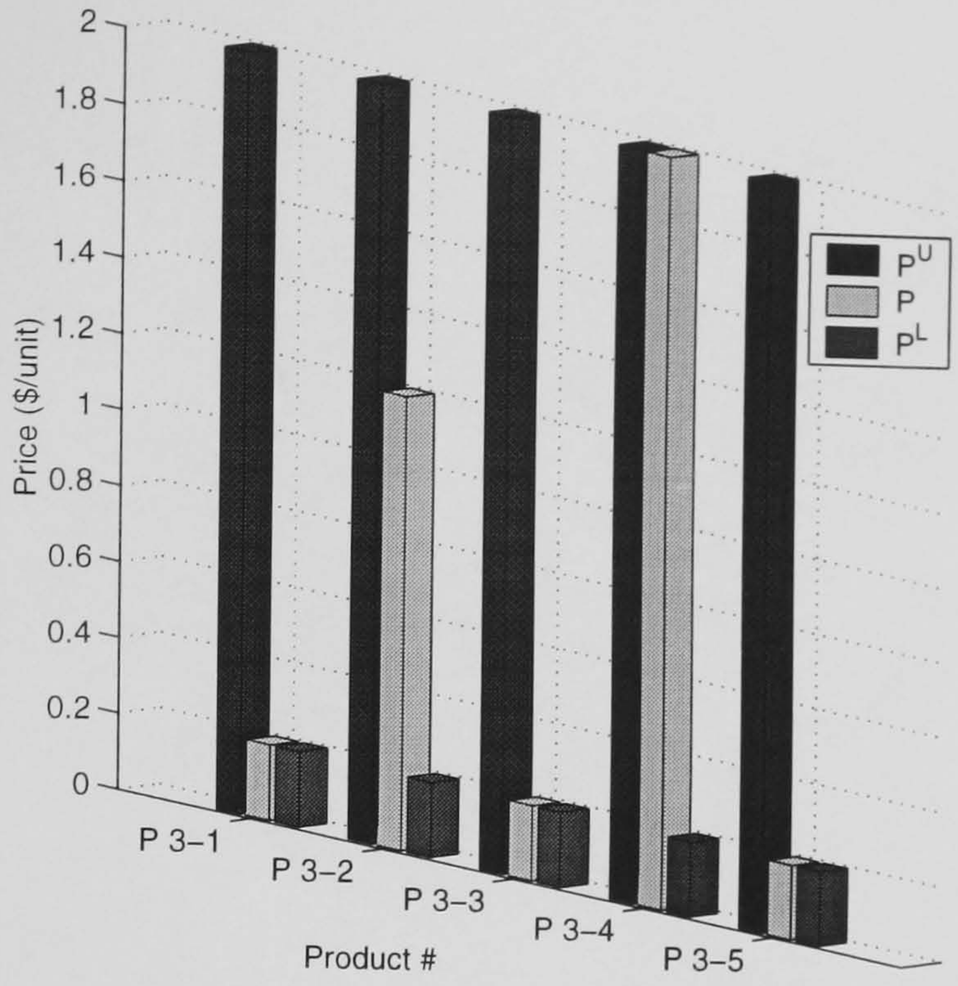


Figure 3.14: Transfer prices: Single level approach

challenge the result with reference to the current value of the Nash product (Harsanyi, 1977*a*) and claim the single level approach to be unfair, and a more fair solution has to be found via the Nash approach to secure long-term profitability of both nodes of the supply chain system.

Nash Approach

In figures 3.15 and 3.16, the Nash production plan can be compared to that of the single level approach. It is clear that the production plans are similar, albeit not identical. Both approaches lead to fulfilled customer demand, but with a different profit distribution based on the differences in transfer prices, see figure 3.17 and compare to figure 3.14. Figure 3.18 displays the enterprise profits for the different alternatives. The first set of bars shows the lower profit requirements of the two enterprises and the total supply chain profit level, Π_{tot}, prior to the optimisation of the system. The second set of bars shows the profits using the single level optimisation approach. It is clear that the single level optimisation results in an uneven solution, where only B benefits from the supply chain optimisation. The third set of bars shows the profit distribution after the Nash optimisation. The Nash approach results in a equitable split of payoffs, subject to the prior lower profit requirements and to the Nash axioms. In figure 3.19, it can

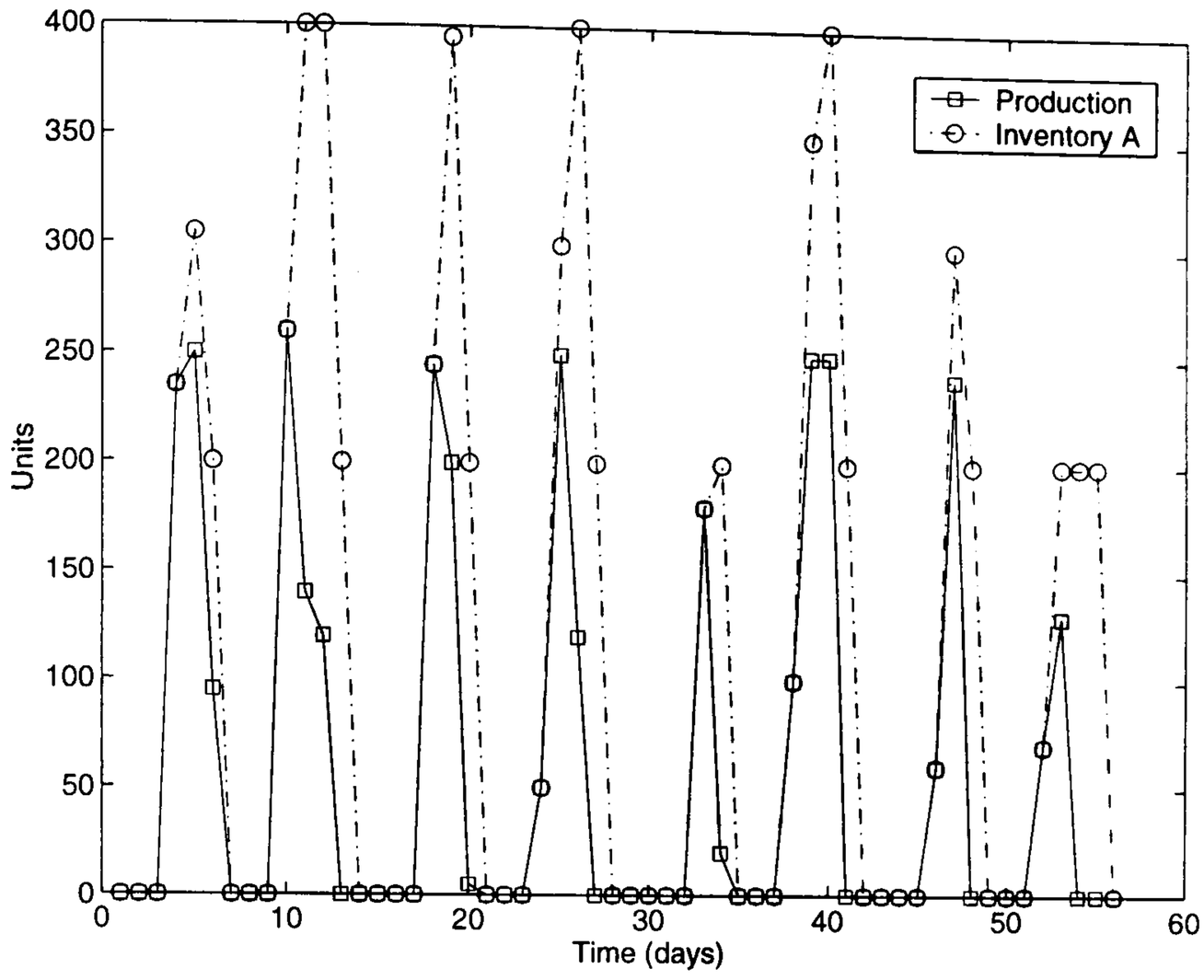


Figure 3.15: A associated variables - production quantity and inventory: Nash approach

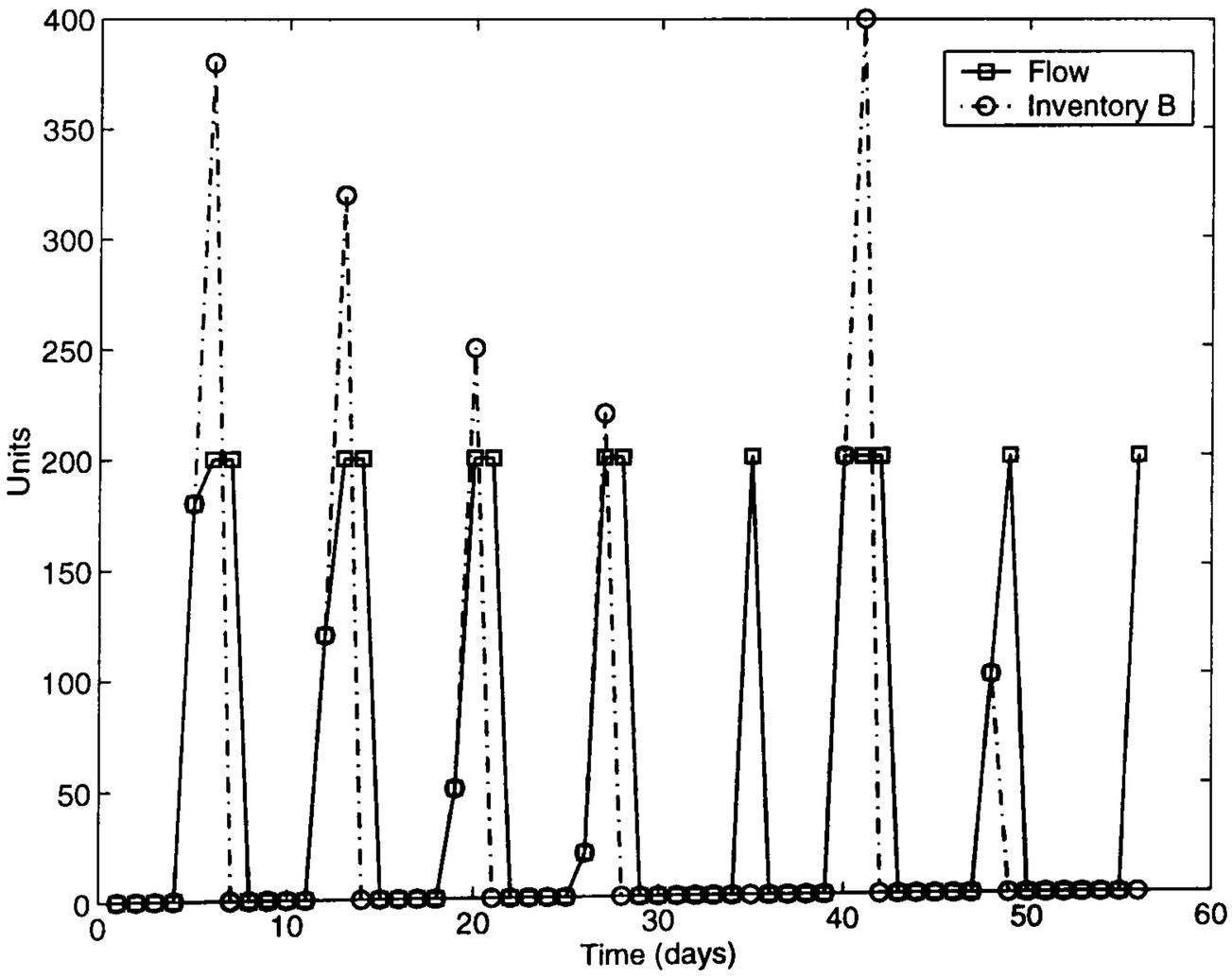


Figure 3.16: B associated variables - flow and inventory: Nash approach

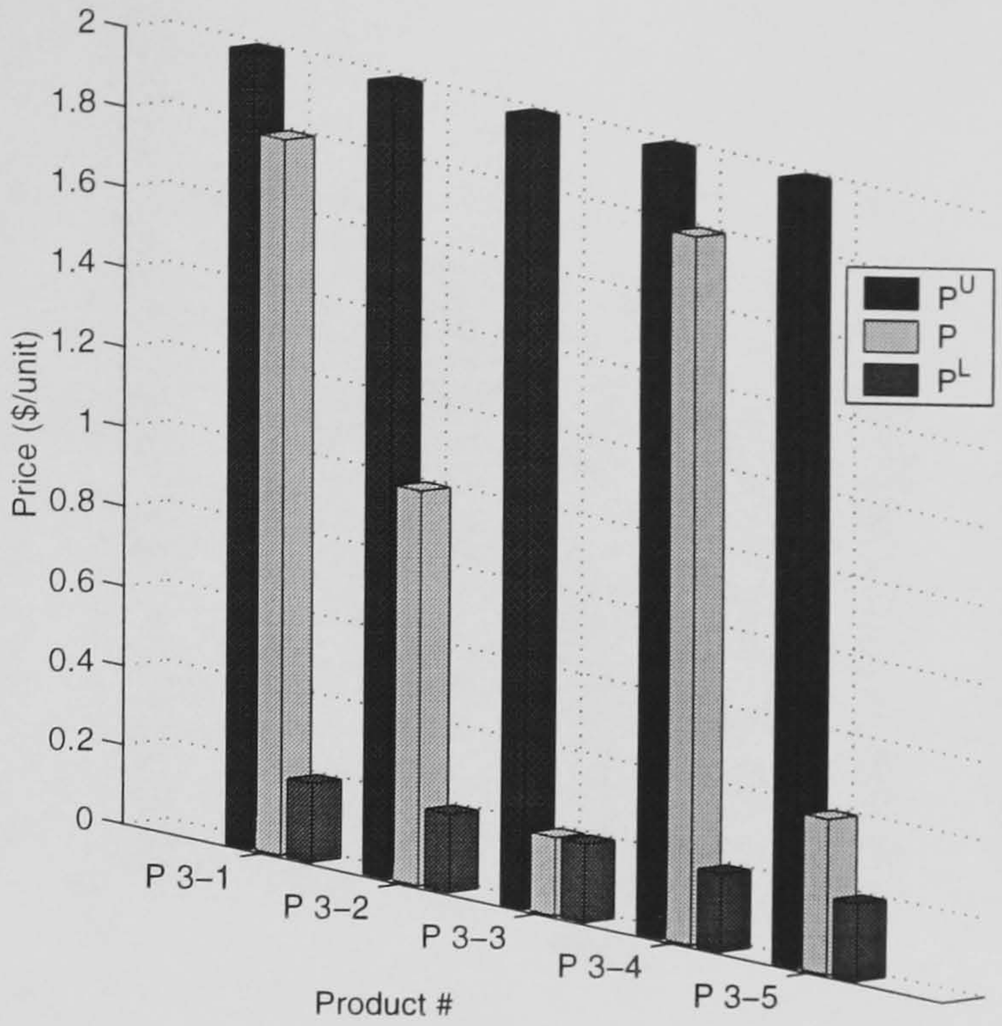


Figure 3.17: Transfer prices: Nash approach

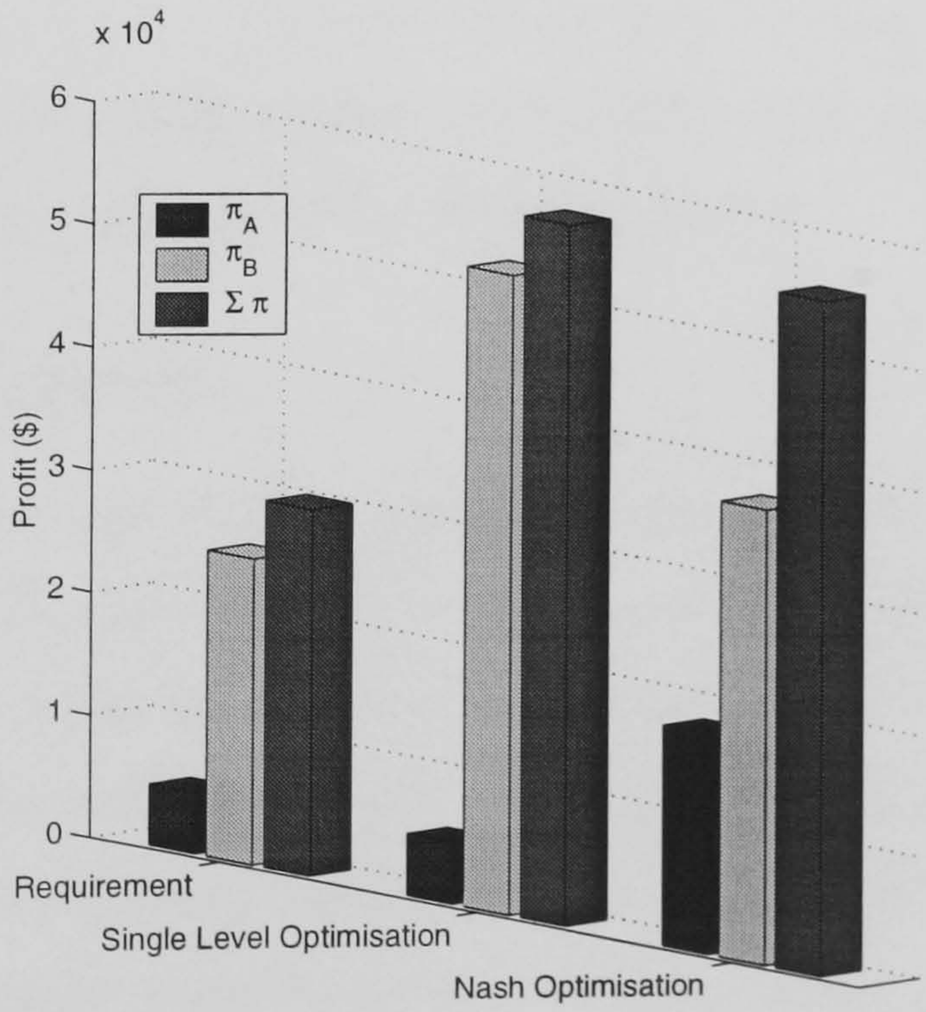


Figure 3.18: Two-enterprise profits before and after supply chain optimisation

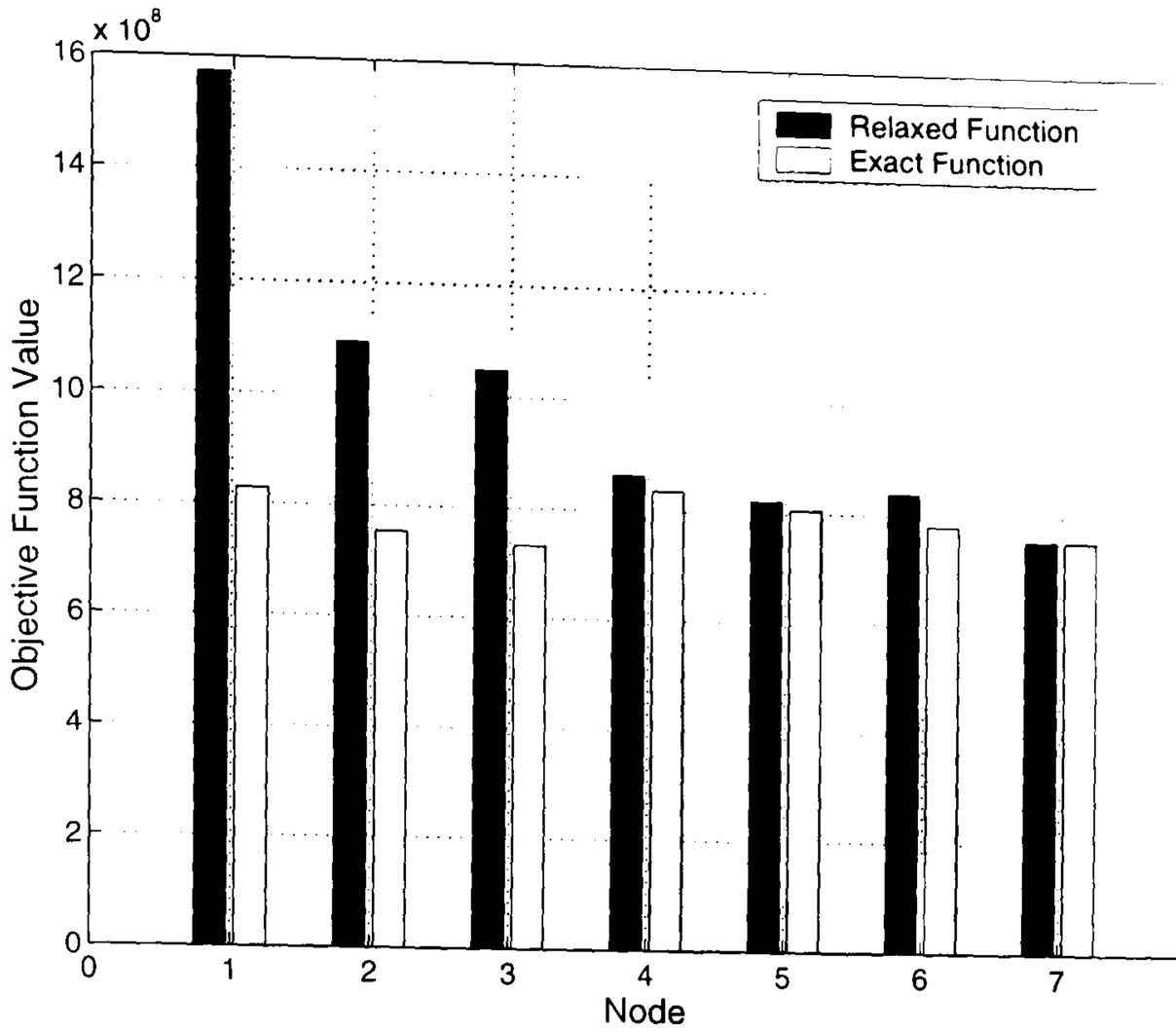


Figure 3.19: Relaxed and exact values of objective function for each node: Nash approach

be seen that the algorithm successively fulfills the error criterion, *i.e.* the relaxed and the exact functions converge. The computational results are displayed in table 3.5. The effect of having a lower profit requirement point with an inequitable profit distribution is tested. B has higher prior profit expectations than A, which leads to a solution with a profit distribution of *ca.* 70/30 in favour of B. Profits of A and B are 18110 and 37499, respectively (*cf.* single level approach - 5065 and 52480).

3.11.3 Hybrid Approach

In table 3.5, it can be seen that the algorithm uses significantly more CPU time than does the single level approach. This is because a number of MILP optimisations have to be performed in the spatial branch-and-bound search tree. Since a margin of optimality of 5% is used, it can be useful to make use of the results given for the final Nash solution and use the single level approach on the profit results obtained by the Nash approach so as to obtain a truly Pareto optimal solution, mentioned earlier as the "hybrid" approach. When the hybrid concept is utilised it can be seen that with the Nash approach fair profit levels are established with a summed overall profit of just less than 0.2% than the single level approach for the second example. Profits of A and B are, after applying the hybrid approach, 18110 and 39344, respectively. For example 1, it does not make sense to use

the hybrid approach since the Nash solution is strictly optimal.

3.11.4 Three-Enterprise Test Problem

This problem[2] is similar to example 2 described in section 3.11.2. A base chemical manufacturing plant (A) is linked to a downstream specialty chemical plant (B) which in turn is linked to a distribution centre (C). A horizon of six weeks at a one-day discretisation is modelled. The manufacturing problem in plant A is the following. The material in the first step of the process is a feed which is dried in a tower to four different powders which in turn are packed to make 5 products. The packing of the products uses six pack-lines and a limited amount of manpower and material handling resources. In plant B, the intermediate products from plant A are mixed with specific amounts of additives and shipped onwards to C. The distribution centre faces a deterministic demand and fixed sales prices for the time horizon of six weeks. The following quantities are established for the production and distribution problem, which maximise the profit levels of the separate enterprises (i are the intermediate products and j are the final products):

- Production level of i in A
- Inventory level of i in A
- Flow of products between A and B
- Transfer prices of the products between A and B
- Inventory level of i and j in B
- Production level of j in B
- Flow of products between A and B
- Transfer prices of the products between B and C
- Inventory level of j in C

In figures 3.20, 3.21, and 3.22, the variables for one of the intermediate products and one of the final products of the three-enterprise test problem are shown. Figure 3.20 shows the inventory level of intermediate product in B, I_{it}^{B}, the amount produced of intermediate product, M_{it}^{A}, and the distribution flow of the intermediate product, F_{it}^{1}

[2]This example is also based on Yee, K. L., Shah, N. (1997). Scheduling of multistage fast-moving consumer goods plants, *Journal of the Operational Research Society*, 48, 1201-1214. However, some parts have been removed to make the problem size smaller.

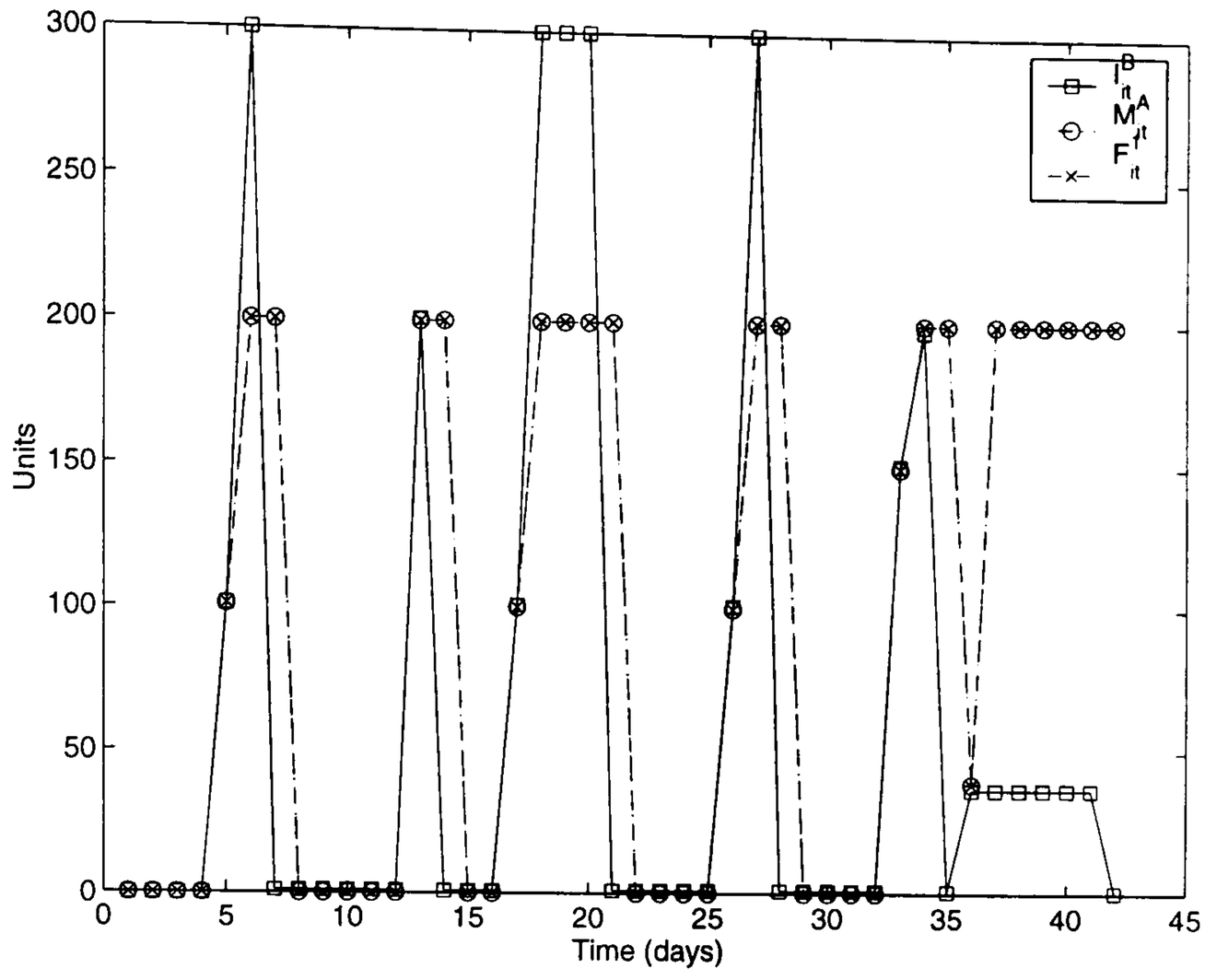


Figure 3.20: Variables associated with intermediate product i

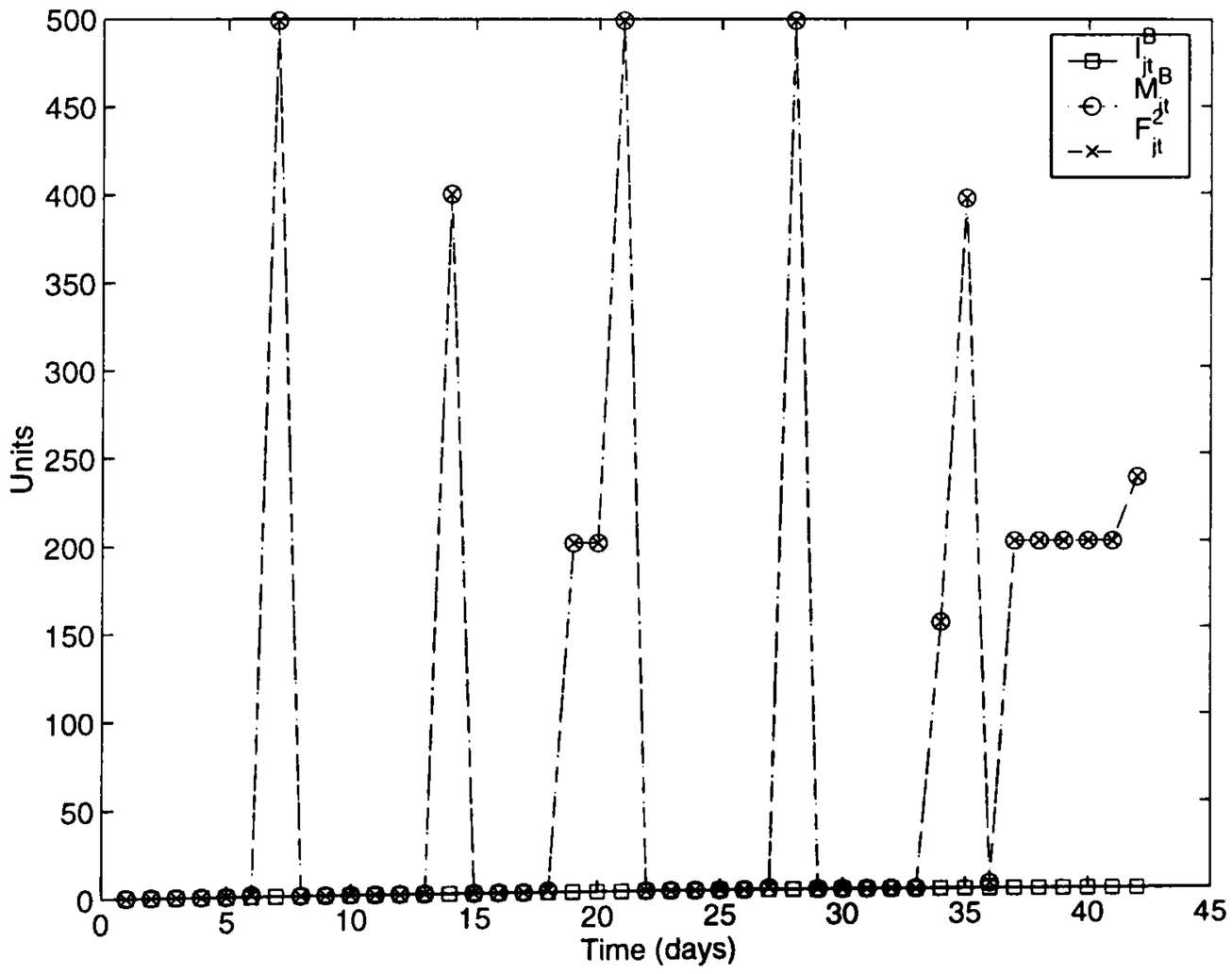


Figure 3.21: Variables associated with final product j in B

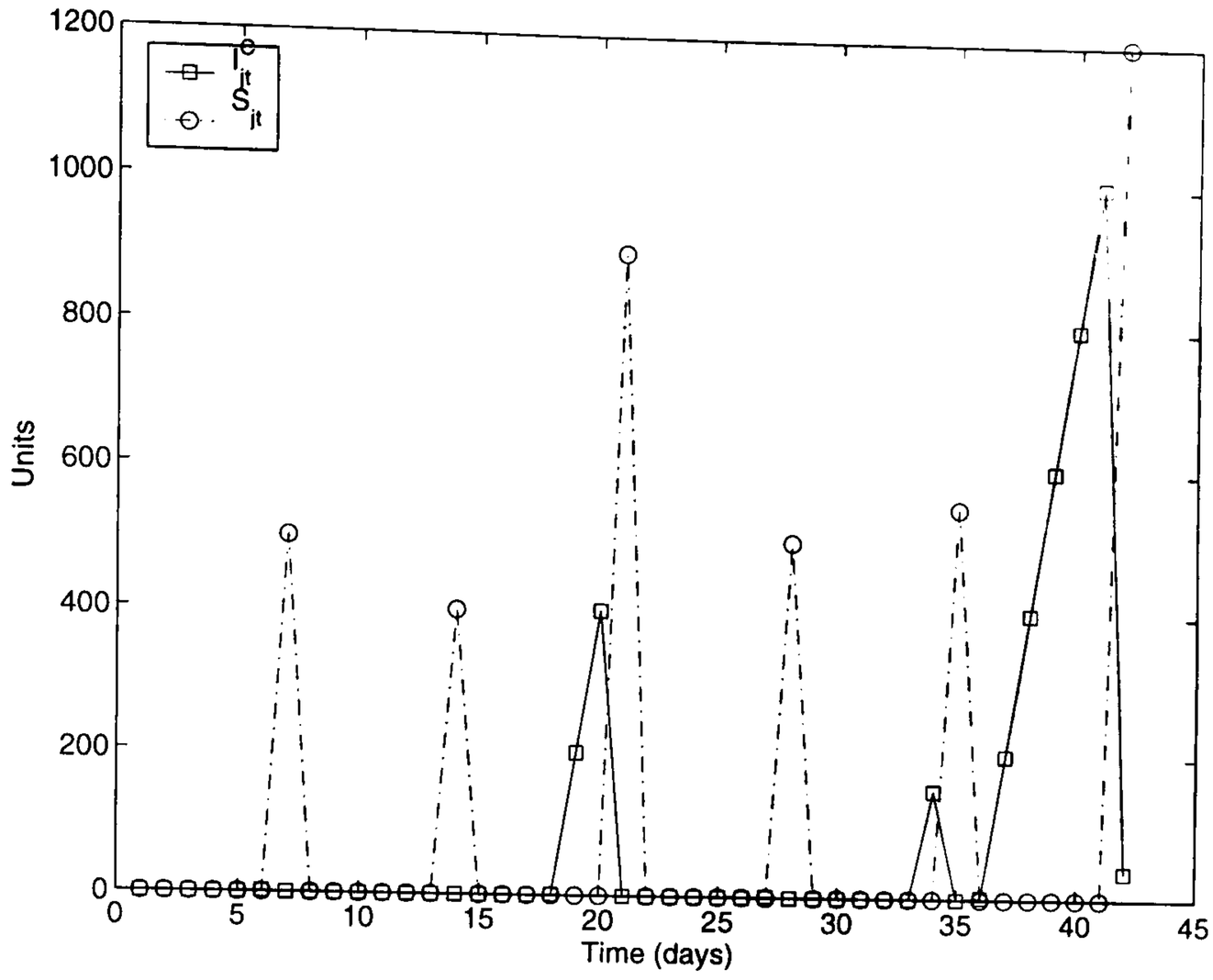


Figure 3.22: Variables associated with final product j in C

over the six week horizon. Figure 3.21 displays the inventory level of final product in B, I^B_{jt}, the amount produced of final product, M^B_{jt}, and the distribution flow of final product to C, F^2_{jt}. Finally, figure 3.22 shows the inventory level in C, I^C_{jt}, and the amount shipped to external customers, S_{jt}.

The transfer price levels established by the optimisation are displayed in figures 3.23 and 3.24. Figure 3.23 shows the transfer prices of intermediate products between A and B. In figure 3.24, the prices of final products between B and C are displayed. The transfer price levels determined result in the enterprise profits shown in figure 3.25. The first set of bars shows the lower profit requirements of the three enterprises and the total supply chain profit, Π_{tot}, prior to the optimisation of the system. The second set of bars shows the profits after the Nash optimisation approach. It is clear that a relatively fair solution has been found.

The computational results of this test problem are as follows. 111 spatial branch-and-bound nodes are evaluated. Each node requires *ca.* 10 min CPU time, *i.e.* the overall CPU requirement is about 18 hours. There are 12209 constraints, 10366 continuous variables and 120 binary variables in the model. The relative tolerance, ϵ, is set to 10% and the solution node obtained has a relative error of 8%.

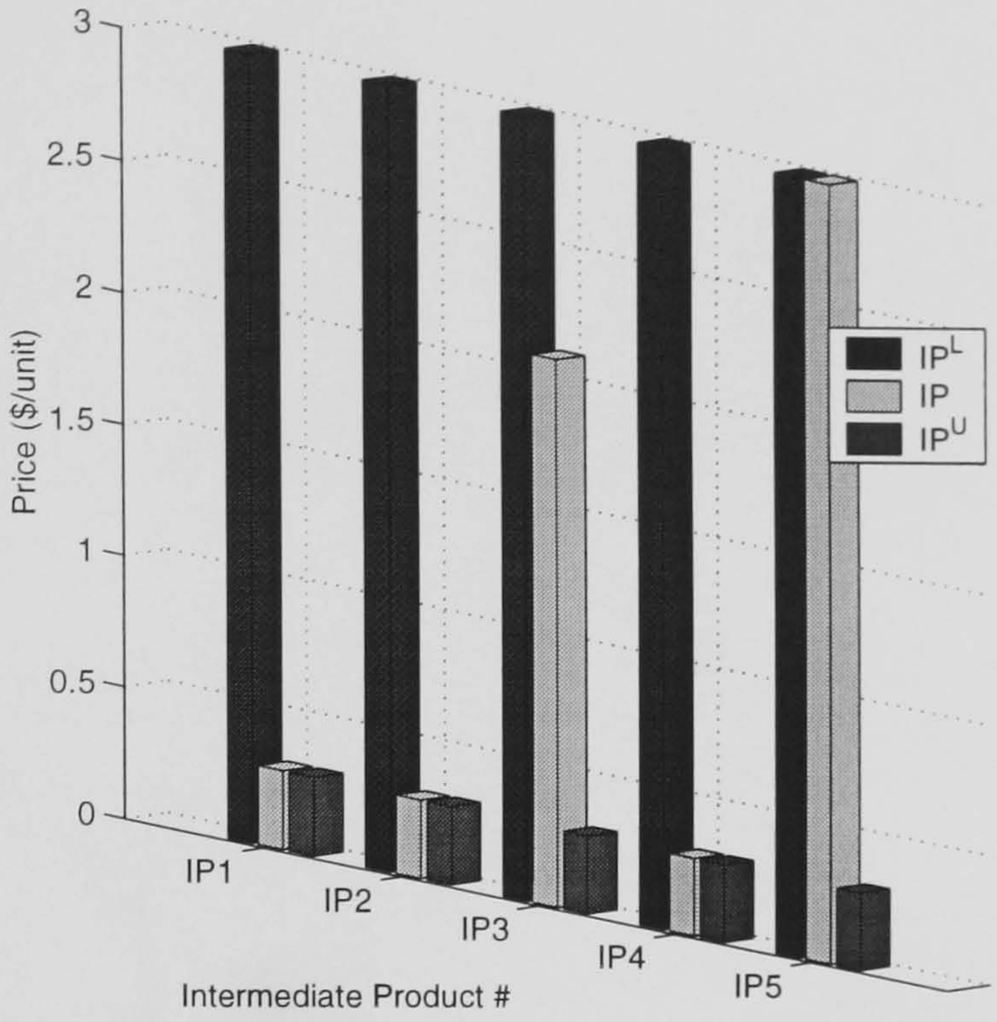


Figure 3.23: Transfer prices: Between A and B

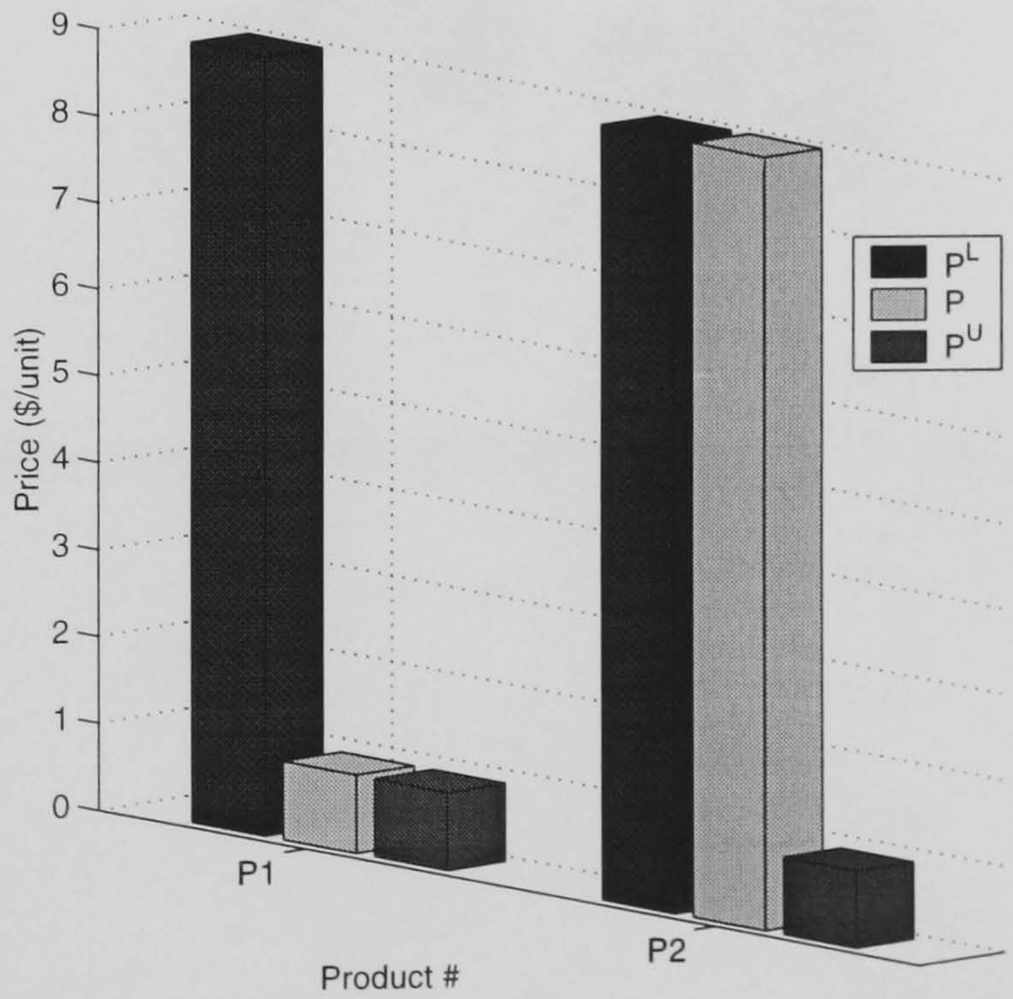


Figure 3.24: Transfer prices: Between B and C

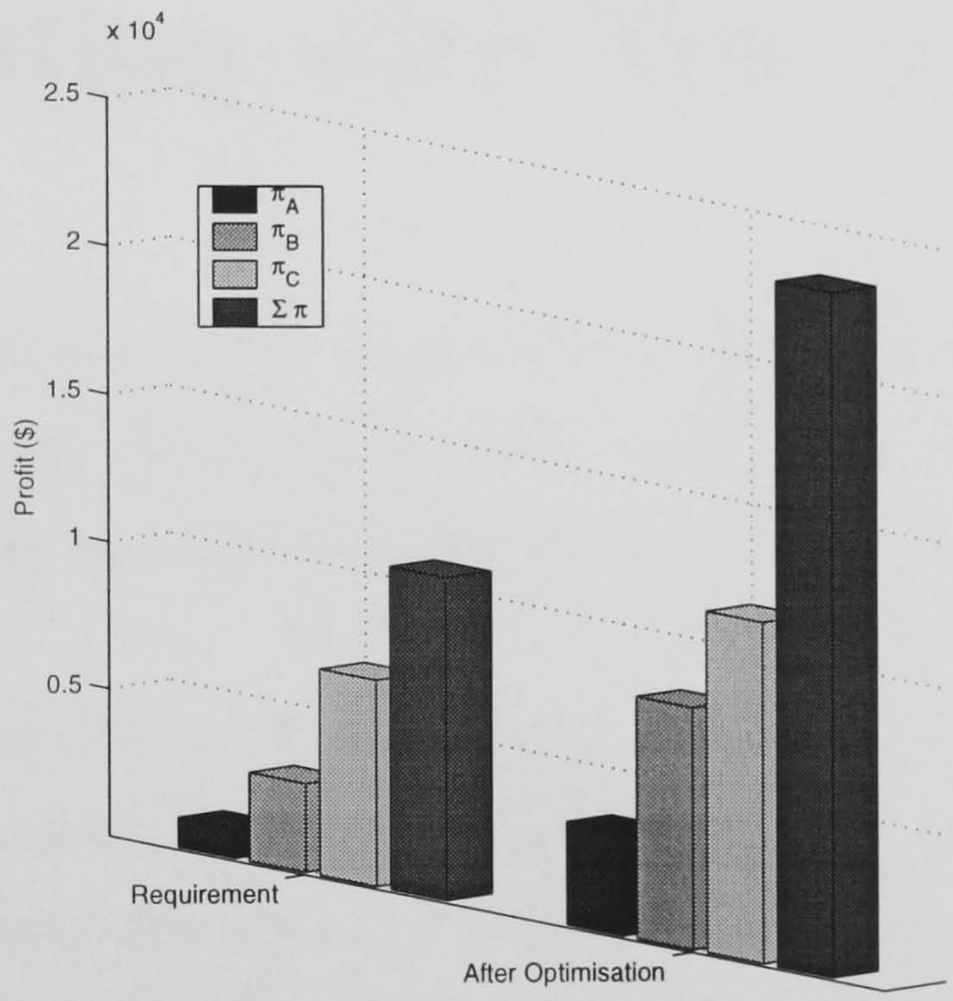


Figure 3.25: Three-enterprise profits before and after supply chain optimisation

Chapter 4

General Supply Chain Cooperation with Transfer Pricing

The aim in this chapter is to find solutions to the general class of problems presented in chapter 3 given a quantitative prior power structure of the supply chain. A model capturing any number of cooperating companies in a multi-enterprise supply chain is presented.

Ballou *et al.* (2000) stress the importance of common objectives in the entire supply chain. Unattainable improvements for single companies in terms of cost savings and customer service enhancements can be obtained by cooperative companies. The authors point out that problems arise if some of the firms benefit at the expense of the others. The conflict resolution between supply chain partners must thus be of focal interest and to keep the coalitions intact the rewards of cooperation must be redistributed. They identify three means to achieve this: (1) *Metrics* could be developed to capture the nature of inter-organisational cooperation to simplify benefit analysis. (2) *Information sharing* mechanisms could transfer information about the benefits of cooperation among the members in the supply chain. (3) *Allocation methods* could be developed that *fairly* distribute the rewards of cooperation between the members.

We attempt to address these three issues quantitatively in this chapter. An approach for simultaneously optimising the profits in multi-enterprise supply chain partnerships is proposed, in which the separate firms try to optimise their profits *fairly* given their lower profit requirement levels as well as their technology standard. We first present a mixed-

integer nonlinear programming approach to determine the most appropriate transfer price level for products transferred between production sites as well as from production sites to distribution centres. The MINLP model is tackled in a separable programming approach in which the utilisation of logarithmic differentiation and approximations of the variables of the objective function result in a model of MILP form. The solution obtained includes the transfer prices, production levels, timing and quantities of products delivered and resources utilised. The inventory cost at the production plant depends on the transfer price level and is therefore not arbitrary as regarding the overall supply chain optimisation.

4.1 A Nash Approach

As mentioned in the previous chapter, a single level approach of optimising the supply chain system can be to maximise the summed profits of all members, m, *i.e.* $\max \Phi = \sum_m \pi_m$. As we have seen, this approach typically leads to an *unfair* profit distribution. Hence, an approach will be required that can produce a fair profit distribution subject to similar overall performance.

The generalised Nash-Harsanyi (Harsanyi, 1977*b*) extension to the Nash solution described in the previous chapter proposes that if each member has a lower profit requirement point, x_m^L, then the fair solution point is x_m, $x_m \geq x_m^L$, which maximises $\Phi = \prod_m (x_m - x_m^L)$. If the minimum acceptable profit levels for a three-player game, say, are π_m^L $(m \in \{A, B, C\})$ then the Nash solution is the point π_m with $\pi_A \geq \pi_A^L$, $\pi_B \geq \pi_B^L$ and $\pi_C \geq \pi_C^L$ which maximises the product $(\pi_A - \pi_A^L)(\pi_B - \pi_B^L)(\pi_C - \pi_C^L)$.

The proposed Nash problem for any number of supply chain members is inherently nonlinear and nonconvex. The problem statement subject to constraints is to maximise the objective function $\Phi = \prod_m \pi_m$, with π_m defined below in constraint 4.10, subject to production resource constraints (4.1), inventory constraints (4.2), demand constraints (4.3), inventory cost constraints (4.4), administrative cost constraints (4.5), manufacturing cost constraints (4.6), purchase cost constraints (4.7), and transportation cost constraints (4.8). The transfer prices are defined via discrete decision variables according to constraints (4.11) and (4.12). The resulting nonlinearities in constraints (4.4), (4.7), and (4.10) are replaced by the linearisation variables defined in constraints (4.15), (4.16),

(4.17), and (4.18).

The nonlinearities in the objective function are tackled in this work via a separable programming approach to linearise the problem, *i.e.* the maximisation of the Nash objective function in which for each company (member m) the lowest acceptable profit level is given. If the lower profit requirement point is suboptimal then the Nash algorithm finds the solution of the problem that most fairly (subject to the Nash axioms) distributes the surplus amount obtained by the problem optimisation. The solution approach applied is based on a separable technique that is described in Appendix B.

Generally, separable programming approaches express nonlinear objective functions and constraints as a sum of functions involving only one variable. For the details of how the separable approach has been implemented in the proposed model, see Appendix B.

4.2 A General Supply Chain Modelling Framework

A general supply chain is considered, which is composed of N different enterprises. There are three different types of nodes in the supply chain. A "primary" company produces intermediate products which are delivered to a "secondary" company, where the intermediates are converted to final products. Finally, a "tertiary" company, is a warehouse or distribution centre (where no production takes place) from which the final products are delivered to customers.

In any company of the supply chain, product i can be produced utilising R production resources. In secondary companies, products are produced using production resources and intermediate products purchased from primary companies and then the products are supplied downstream to tertiary companies. The prices for products shipped between primary companies and secondary companies, and secondary companies and tertiary companies are called *transfer prices*. The decision makers in the primary, secondary, and tertiary companies have K discrete transfer price levels to choose from. Inventory can be held in all companies. Costs include raw material, manufacturing, administrative, inventory, purchase, and transportation cost. The revenue is determined by sales prices or transfer prices multiplied by the amount shipped to downstream nodes. The revenues less the corresponding costs constitute the objective function ("profit") for each company. The assumptions of the model and the problem formulation are the same as presented in

the previous chapter.

4.3 Mathematical Formulation

In order to facilitate the mathematical model, a time discretisation approach is applied over a planning horizon, H, of T discrete time intervals of equal duration, δ, as illustrated in figure 3.5.

The key variables in the formulation include production levels in primary companies and secondary companies, inventory levels in all companies, flow levels of products between primary companies and secondary companies and secondary companies and tertiary companies, and transfer price levels for the product flows. These are determined so as to maximise the profits of all companies, subject to production resource constraints, transportation constraints, inventory constraints, transfer price level constraints, mass balances, and cost constraints. The model is described in detail below. First, the following notation is introduced.

Indices

m: member

i, j: products

r: resource

k: transfer price available

t: time

Sets

R_m: Resources of m

P_r: Set of products using r

SM_i: Set of members selling i

$\overline{T}_{im}$: Set of members receiving i from m

T_{im}: Set of members providing i to m

O_m: Set of products produced by m

Parameters (m denotes member in all definitions below)

α_{imr}: Demand for resource r for production of product i

A_{mrt}: Capacity for resource r at time t

ρ_{ijmr}: Unit consumption of resource r and intermediate product i for the production of product j

D_{imt}: Customer demand quantity of product i at time t

r: Inventory holding cost constant

δ: Time period duration (in days)

η_{im}: Market sales price of product i

HC^0_{im}: Handling cost constant of product i

MIC^0_{imr}: Manufacturing cost constant, using resource r for production of i

PC^0_{im}: Purchase cost constant of product i

TC^0_{im}: Transportation cost constant of product i

$\hat{P}_{imm'k}$: k available transfer price levels for product i

Binary Variables

$X_{imm'k}$: 1 if kth price level is chosen between members m and m' for product i; 0 otherwise

Continuous Variables

B_{imt}: Batch size of product i at time t

I_{imt}: Inventory level of product i at time t

$F_{imm't}$: Amount shipped of product i between members m and m' at time t

S_{imt}: Amount shipped of product i to customers at time t

IC_m: Inventory cost

$P_{imm'}$: Transfer price of product i between members m and m'

HC_m: Handling cost

MIC_m: Manufacturing cost

PC_m: Purchase cost

TC_m: Transportation cost

C_m: Overall cost

π_m: Profit

4.3.1 Production Resource Constraints

The amount of each production resource utilised over any period t at the production site cannot exceed the plant availability level, A_{mrt}, in that period. Thus, for each member m the utilisation of production resource r for each product i is $\alpha_{imr}B_{imt}$, where α_{imr} is the unit consumption of resource r for production of one unit of product i and B_{imt} is the amount produced of product i at time t. The utilisation of production resources for each member m, resource r, and time interval t should be less than the resource availability level, A_{mrt}.

$$\sum_{i \in P_r} \alpha_{imr} B_{imt} \leq A_{mrt} \qquad \forall\ m, r \in R_m, t \qquad (4.1)$$

4.3.2 Inventory Constraints

The inventory level of product i at time t for each member m, I_{imt}, is equal to the amount at time $t-1$, $I_{im,t-1}$, added to the amount produced by member m, B_{imt}, added to the amount received, $\sum_{m' \in T_{im}} F_{im'mt}$, less the amount used to produce other products by consuming product i, $\sum_{j \in O_m} \sum_{r \in R_m} \rho_{ijmr} B_{jmt}$ (ρ_{ijmr} is the unit consumption of resource r and intermediate i to produce a downstream product j), and the amount shipped to other members, $\sum_{m' \in \overline{T}_{im}} F_{imm't}$, and the amount shipped to customers, S_{imt}. Hence:

$$I_{imt} = I_{im,t-1} + B_{imt} + \sum_{m' \in T_{im}} F_{im'mt} - \sum_{j \in O_m} \sum_{r \in R_m} \rho_{ijmr} B_{jmt} - \sum_{m' \in \overline{T}_{im}} F_{imm't} - S_{imt} \qquad \forall\ i, m, t \qquad (4.2)$$

4.3.3 Demand Constraints

A "hard" external demand constraint is used for all products i which is fixed to fulfil variable demands at all time intervals t. S_{imt} is the shipped quantity of product i at time t. D_{imt} is the corresponding customer demand of product i at time t.

$$S_{imt} = D_{imt} \qquad \forall\ i, m \in SM_i, t \qquad (4.3)$$

4.3.4 Cost Constraints

Inventory Cost

The inventory cost depends on the outgoing prices associated with the member m:
$\sum_i \sum_t \sum_{m' \in \overline{T}_{im}} \frac{r\ \delta}{365} I_{imt} P_{imm'}$ if other members m' consume products i or $\sum_i \sum_t \frac{r\ \delta}{365} I_{imt} \eta_{im}$

(where η_{im} is the constant selling price to customers over the time horizon) if products are sold to customers. Consequently[1]:

$$IC_m = \sum_i \sum_t \frac{r\ \delta}{365} I_{imt} \Big(\sum_{m' \in \overline{T}_{im}} P_{imm'} + \eta_{im} \Big) \qquad \forall\ m \tag{4.4}$$

Administrative Cost

The cost of handling product i for member m at time t is a constant, HC^0_{im}, multiplied by the total amounts handled, be it amount produced, B_{imt}, amount sold, S_{imt}, incoming amount, $\sum_{m' \in T_{im}} F_{im'mt}$, or outgoing amount, $\sum_{m' \in \overline{T}_{im}} F_{imm't}$. For each member, two of the terms will normally be non-zero, *i.e.* one incoming amount and one outgoing amount.

$$HC_m = \sum_i \sum_t HC^0_{im} \Big(B_{imt} + S_{imt} + \sum_{m' \in \overline{T}_{im}} F_{imm't} + \sum_{m' \in T_{im}} F_{im'mt} \Big) \qquad \forall\ m \tag{4.5}$$

Manufacturing Cost

The manufacturing cost for producing i is a constant, MC^0_{imr} (depending on the resources utilised), multiplied by the amount manufactured, B_{imt}:

$$MC_m = \sum_i \sum_t \sum_{r \in R_m} MC^0_{imr} B_{imt} \qquad \forall\ m \tag{4.6}$$

Purchase Cost

The purchase cost for member m consists of the costs of raw materials purchased from external producers (given by a constant cost for each raw material i, PC^0_{im}, multiplied by the usage, B_{imt}). Furthermore, intermediate products can also be purchased from other members m'. In this case, the purchase cost is obviously the price of the intermediate product, $P_{im'm}$, multiplied by the amount purchased, $F_{im'mt}$, summed over all members providing the intermediate product.

$$PC_m = \sum_i \sum_t \Big(PC^0_{im} B_{imt} + \sum_{m' \in T_{im}} P_{im'm} F_{im'mt} \Big) \qquad \forall\ m \tag{4.7}$$

[1] It should be added that constraints (4.4) are mathematically correct when only one member receives material i from member m. However, for the general case of multiple receiving members, these constraints do provide an adequate approximation to capture various supply chain trade-offs.

Transportation Cost

The transportation cost is determined by the amount shipped from member m either to customers, S_{imt}. or to other members, $\sum_{m' \in \overline{T}_{im}} F_{imm't}$. The sum is multiplied by a product transportation constant, TC^0_{im}.

$$TC_m = \sum_i \sum_t TC^0_{im}(S_{imt} + \sum_{m' \in \overline{T}_{im}} F_{imm't}) \qquad \forall\ m \qquad (4.8)$$

Overall Cost Constraints

The overall cost is simply the sum of the inventory cost, the administrative cost, the manufacturing cost, the purchase cost, and the transportation cost. This amount constitutes the total cost base for member m.

$$C_m = IC_m + HC_m + MC_m + PC_m + TC_m \qquad \forall\ m \qquad (4.9)$$

4.3.5 Profit Definition Constraints

The profit definition constraints simply state that the profit of each member of the supply chain is the revenue less the cost incurred at that supply chain node. The revenue is either the selling price to customers, η_{im}, multiplied by the amount sold, S_{imt}, or the transfer price to other consuming members m', $P_{imm'}$, multiplied by the amount shipped, $F_{imm't}$. The profit, π_m, of each member , m, is then defined by:

$$\pi_m = \sum_i \sum_t (\eta_{im} S_{imt} + \sum_{m' \in \overline{T}_{im}} P_{imm'} F_{imm't}) - C_m \qquad \forall\ m \qquad (4.10)$$

4.3.6 Transfer Price Levels

It is assumed that K discrete transfer price levels for each product i transferred between members are defined via the parameter $\hat{P}_{imm'k}$. To determine the price level $P_{imm'}$, the binary decision variable $X_{imm'k}$ is multiplied by $\hat{P}_{imm'k}$ and summed over all transfer price levels for that product. For each product i, only one transfer price level is chosen since $\sum_k X_{imm'k} = 1$.

$$P_{imm'} = \sum_{k \in TP_i} \hat{P}_{imm'k}\ X_{imm'k} \qquad \forall\ i, m, m' \in \overline{T}_{im} \qquad (4.11)$$

$$\sum_k X_{imm'k} = 1 \qquad \forall\ i, m, m' \in \overline{T}_{im} \qquad (4.12)$$

4.3.7 Linearisation Constraints

There are non-linearities in constraints (4.4), (4.7), and (4.10), respectively. This is due to the product of the transfer price, $P_{imm'}$, and inventory level, I_{imt}, and the product of the transfer price and the amount shipped, $F_{imm't}$. Therefore, we introduce the following variables to convert the nonlinear model above to a linear equivalent:

$$\overline{XI}_{imm'kt} \equiv X_{imm'k}\ I_{imt} \tag{4.13}$$

$$\overline{XF}_{imm'kt} \equiv X_{imm'k}\ F_{imm't} \tag{4.14}$$

An appropriate upper bound for the amount shipped of each product i from member m to $m' \in \overline{T}_{im}$ is introduced, $F^U_{imm'}$, and for the inventory level of product i for each member m, I^U_{im}.

I^U_{im}: Upper bound on inventory for product i

$F^U_{imm'}$: Upper bound on flow of product i between members m and m'

The following linearisation constraints are introduced.

$$\overline{XI}_{imm'kt} \leq X_{imm'k}\ I^U_{im} \qquad \forall\ i, m, m' \in \overline{T}_{im}, k, t \tag{4.15}$$

$$I_{imt} = \sum_{m' \in \overline{T}_{im}} \sum_{k \in TP_i} \overline{XI}_{imm'kt} \qquad \forall\ i, m, t \tag{4.16}$$

$$\overline{XF}_{imm'kt} \leq X_{imm'k}\ F^U_{imm'} \qquad \forall\ i, m, m' \in \overline{T}_{im}, k \in TP_i, t \tag{4.17}$$

$$F_{imm't} = \sum_{k \in TP_i} \overline{XF}_{imm'kt} \qquad \forall\ m, m' \in \overline{T}_{im}, i, t \tag{4.18}$$

Consequently, the term $\sum_k \hat{P}_{imm'k}\ \overline{XF}_{imm'kt}$ is used instead of the nonlinear term $F_{imm't}\ P_{imm'}$. Also, the nonlinear term $P_{imm'}\ I_{imt}$ is replaced by $\sum_{m' \in \overline{T}_{im}} \sum_k \hat{P}_{imm'k}\ \overline{XI}_{imm'kt}$.

4.4 Computational Experiments

For all solutions, the relative error is defined as the relative difference between the actual and the approximated value: rel error $= \frac{\Phi - \hat{\Phi}}{\Phi}$, where $\Phi = \prod_m (\pi^\star_m - \pi^L_m)$ at the optimal solution point $\pi^\star_m$. In figure 4.1, the error depicted is the difference between the actual and the approximated value *i.e.* $f(\lambda^\star)$ less $\hat{f}(\lambda^\star)$.

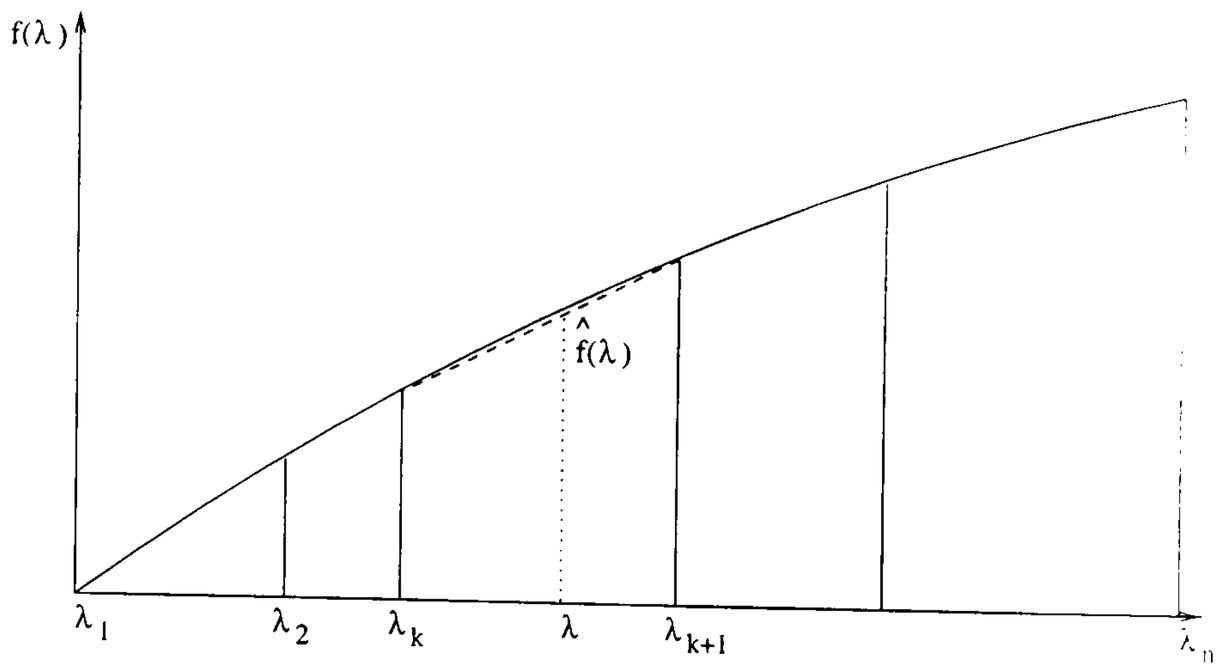


Figure 4.1: Piecewise linear approximation of a function f

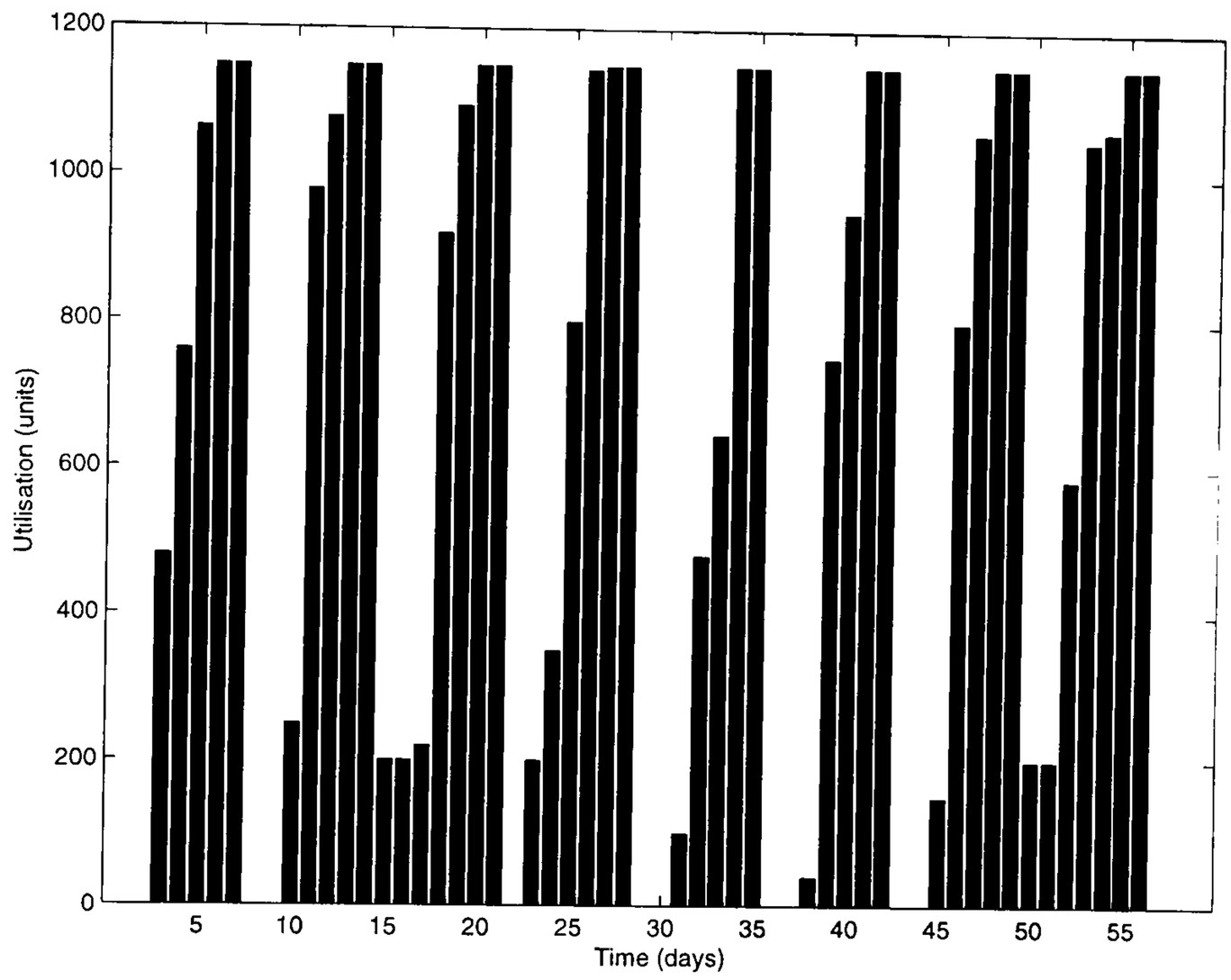


Figure 4.2: Two-enterprise example: Usage of resource tower at plant

4.4.1 A Two-Enterprise Example

The two-enterprise example[2] introduced in the previous chapter is considered for which the production process is shown in figure 3.11. The following quantities of the production and distribution problem are determined while maximising the profit levels of the separate enterprises: production resource utilisation, production levels, inventory levels, flows, and transfer prices of the products.

The production resource utilisation for the tower resource over the production horizon is displayed in figure 4.2. The maximum availability each day is 1150 units processed. It can be seen that the resource is at full capacity (at the upper bound) before each

[2] All the examples presented in this chapter are adapted from Yee, K. L., Shah, N. (1997). Scheduling of multistage fast-moving consumer goods plants, *Journal of the Operational Research Society*, 48, 1201-1214.

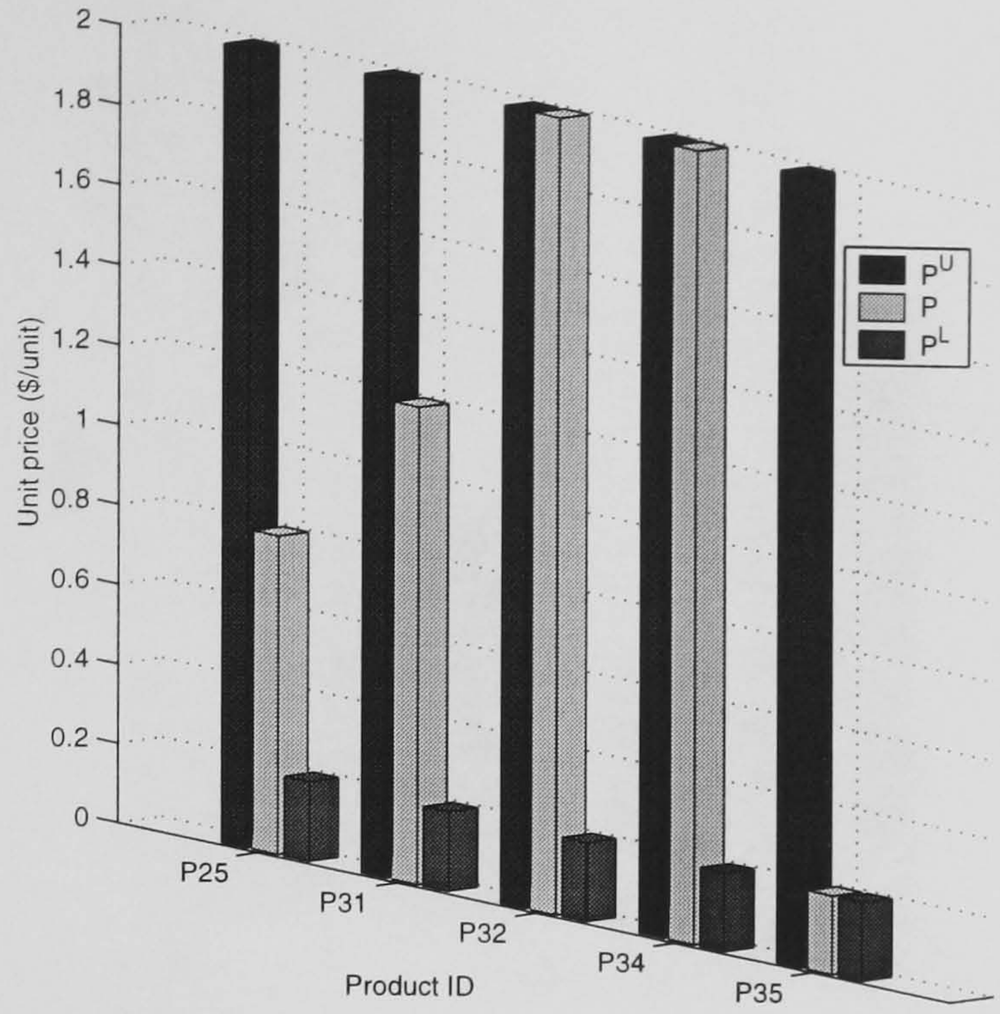


Figure 4.3: Two-enterprise example: Transfer prices for five products

customer delivery (*e.g.* $t = 6, 7, 13, 14$) and that the tower is not utilised at all during 10 time intervals (*e.g.* $t = 1, 2, 8, 9, 21$). This policy leads to minimised inventory cost and is therefore favoured to a policy in which production is evenly spread over the horizon. The transfer price levels for five of the 15 products (products P_{25}, P_{31}, P_{32}, P_{34}, and P_{35}) established by the optimisation are displayed in figure 4.3. As can be seen, some of the product prices take the minimum and maximum values in the available range while other product prices take on intermediate values. The optimal transfer price levels determined result in the enterprise profits shown in figure 4.4. The first set of bars shows the lower profit requirements of the two enterprises and the total supply chain profit, $\Sigma\,\pi$, prior to the optimisation of the system. The second set of bars shows the profits obtained after the Nash optimisation approach is used.

The profits for the secondary and tertiary companies are 17786 and 37890, respectively, given lower profit requirement levels of (5000, 25000). The λ-multipliers (see Appendix B for details) take non-zero values for $\lambda_{31} = 0.282$, $\lambda_{41} = 0.718$; $\lambda_{12} = 0.001$, and $\lambda_{22} = 0.999$. As can be seen, two adjacent λs take non-zero values as is guaranteed by the separable approach. The error at the solution point is 0.013%. In table 4.1, the results of this and the experiments presented below are shown. The CPU requirement for this example is roughly half an hour. The naive approach would have given profit

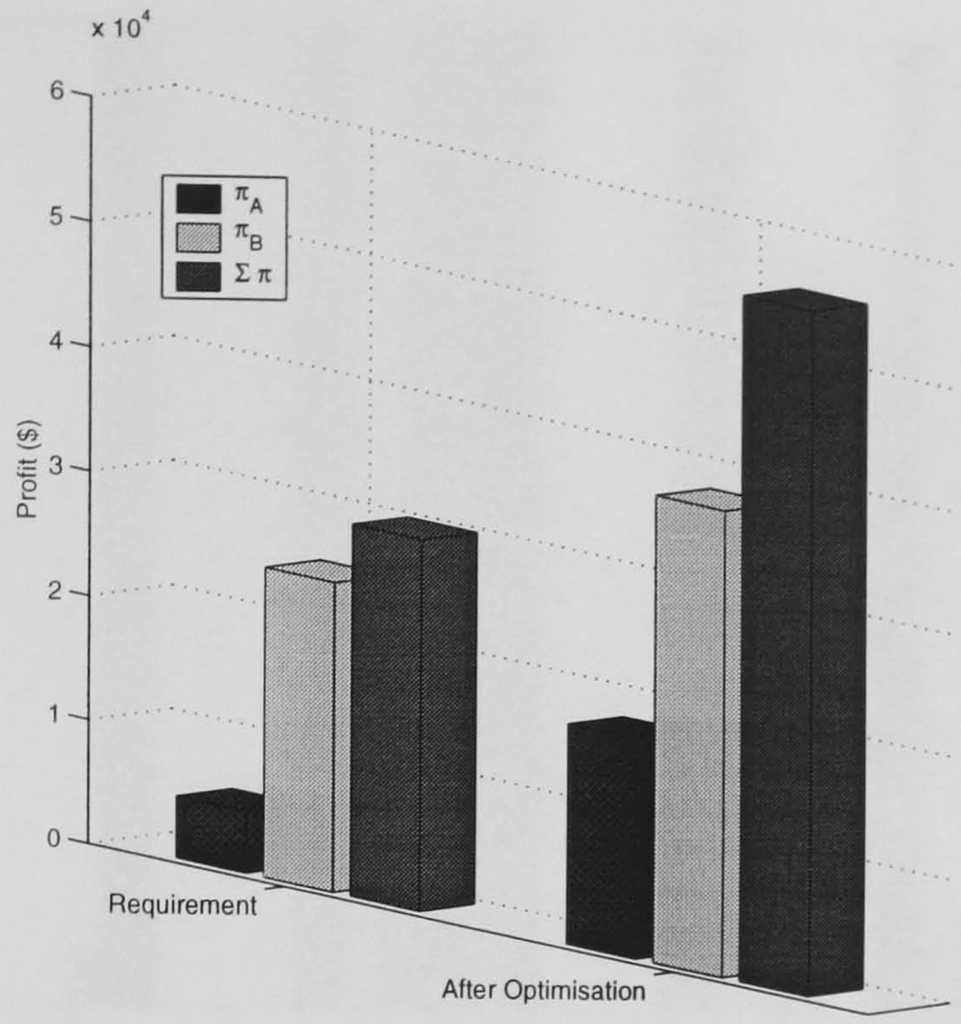


Figure 4.4: Two-enterprise profits before and after supply chain optimisation

Example	Binaries	Continuous	Constraints	Gap	CPU (s)	Relative Error (%)
2 Enterprises	150	25354	27692	5%	2115	0.013
3 Enterprises	120	14217	9777	5%	542	0.60
5 Enterprises	170	26945	22157	5%	19	0.20
6 Enterprises	340	51039	38241	5%	646	0.023

Table 4.1: Computational performance for four examples

levels of 5065 and 52480, respectively. The result of the proposed approach implies an equitable profit split and is more sustainable for the supply chain as a whole.

Table 4.1 gives the problem size for the different examples, the marginal gaps used when solving the problems, the CPU time required on a Sun SPARC Ultra 60, and finally the relative error of the solutions obtained. The solution error is under 1% for all examples which is to be considered a favourable result for the overall approach.

4.4.2 A Three-Enterprise Single Sourcing Example

In the three-enterprise example, a base chemical manufacturing plant (primary company A) is linked to a downstream specialty chemical plant (secondary company B) which in turn is linked to a distribution centre (tertiary company C). A horizon of six weeks using

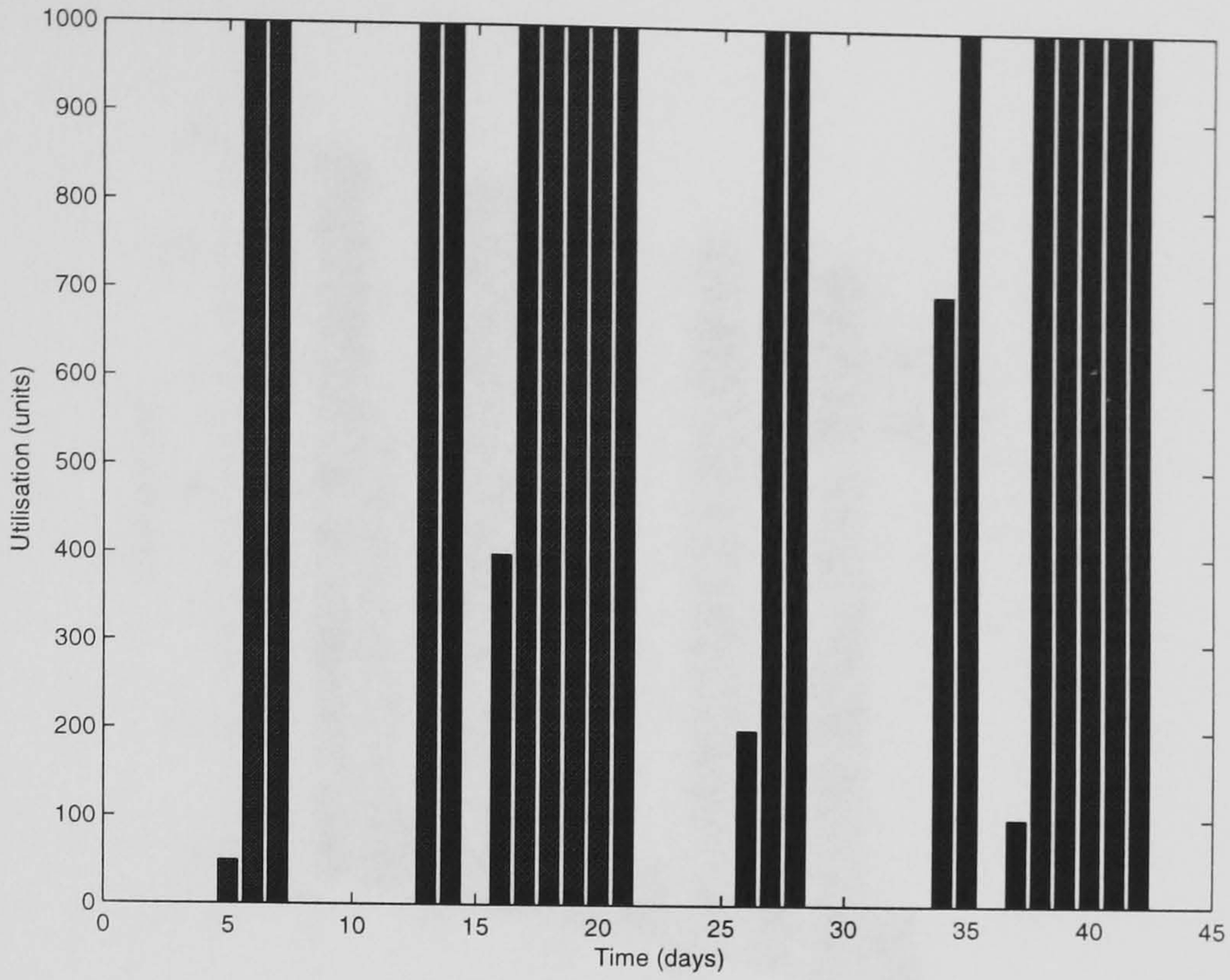


Figure 4.5: Three-enterprise example: Usage of resource tower at A

one-day time discretisation is modelled in this example. The manufacturing problem in the primary company is the following. Material in the first step of the process is a feed which is dried in a tower to four different powders which in turn are packed to make 5 intermediate products. In the secondary plant, the intermediate products are mixed with specific amounts of additives to produce five final, unpacked products which are then packaged. The packaging of the products in the secondary company uses three packaging lines and a limited amount of manpower and material handling resources. The two final packed products (made up of the five unpacked products) are shipped downstream to the tertiary company. The tertiary company faces a deterministic demand and fixed sales prices for the time horizon of six weeks.

The production resource utilisation for the tower in primary company A is displayed in figure 4.5. The maximum resource availability is 1000 units. The tower is mainly utilised before a customer delivery as in the previous example. Figure 4.6 shows the transfer prices of intermediate products (products P_1 to P_5) between A and B. The transfer prices span the whole available range of possible values. The transfer price levels determined result in the enterprise profits shown in figure 4.7. The solution implies that all companies increase their profits by a fair amount. The profits for A, B, and C are 4757, 9218, and 12365, respectively, given a lower profit requirement levels of (1000, 3000,

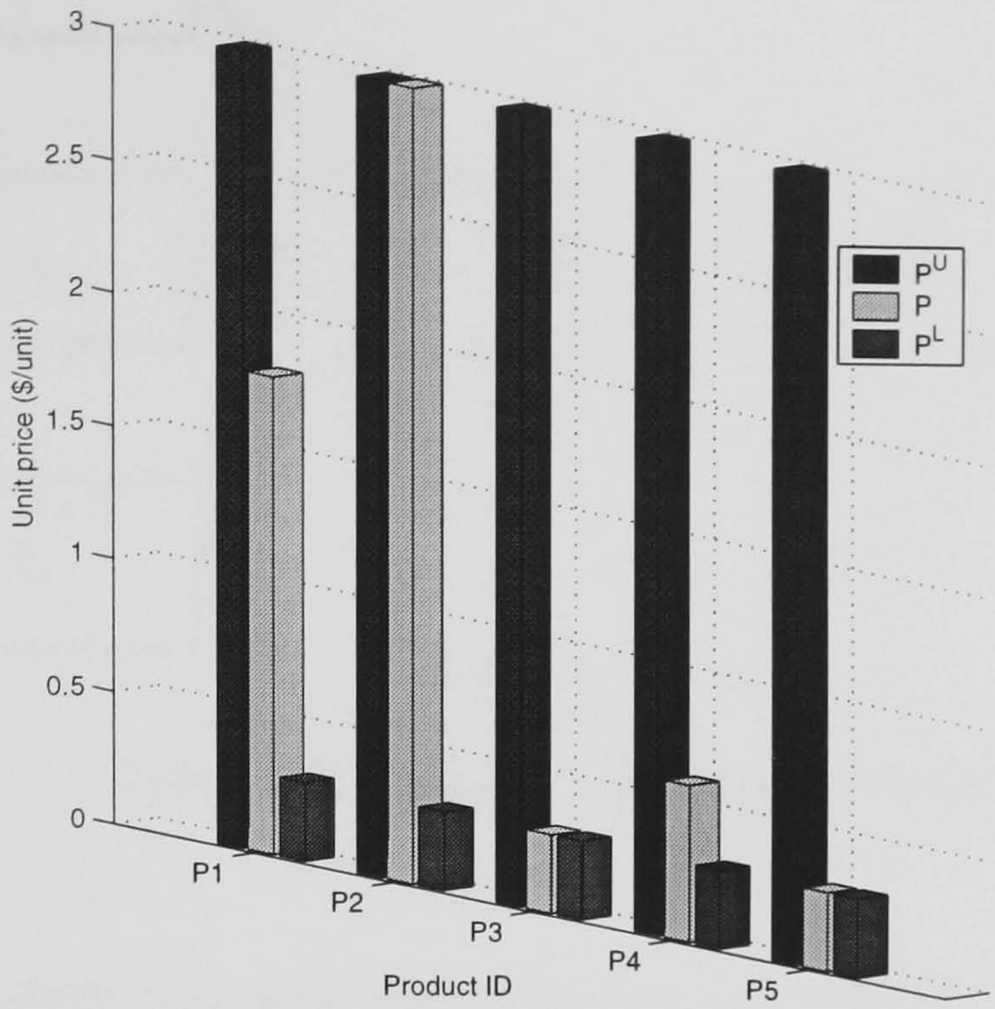


Figure 4.6: Three-enterprise example: Transfer prices between A and B

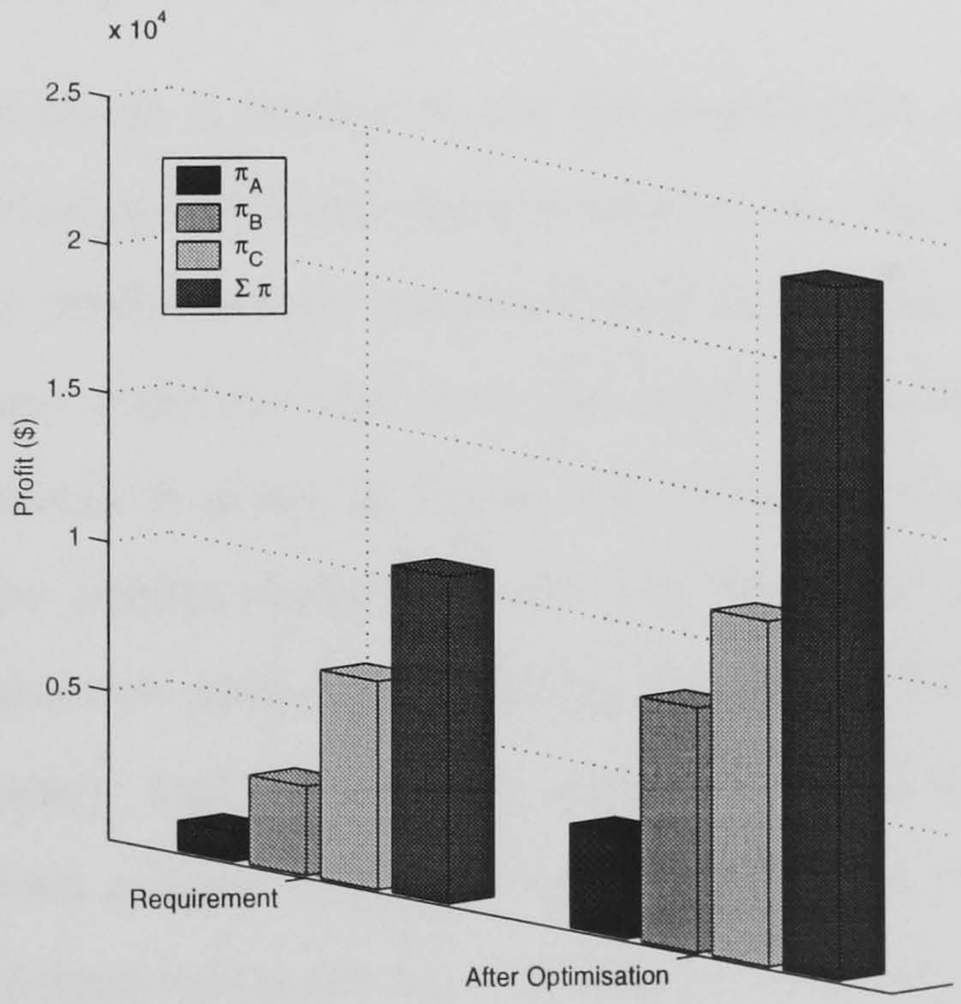


Figure 4.7: Three-enterprise profits before and after supply chain optimisation

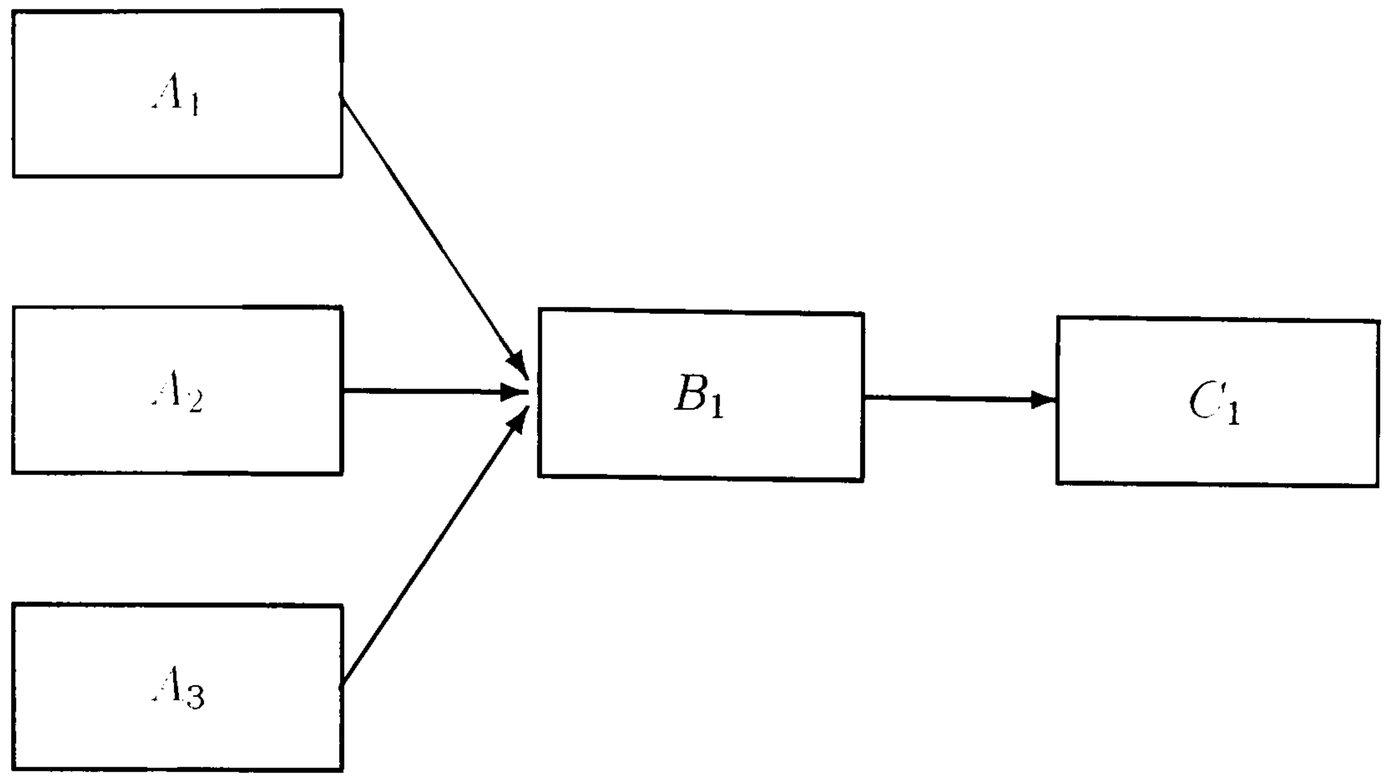


Figure 4.8: Five-enterprise connectivity

7000). The λ-multipliers take non-zero values for $\lambda_{11} = 0.792$, $\lambda_{21} = 0.208$; $\lambda_{32} = 0.782$, $\lambda_{42} = 0.218$; $\lambda_{43} = 0.954$, and $\lambda_{53} = 0.046$. The CPU requirement is slightly less than 10 minutes and the error at the solution point (which again is the difference between the piecewise linear approximation of the objective function to maximise in the separable approach and the actual function value) is 0.60%.

4.4.3 A Five-Enterprise Example

The five-enterprise example is similar to the three-enterprise one except that there are three specialised producers of intermediate products, A_1, A_2, and A_3 (primary companies). For some of the products they compete based on different production costs and for others they are the only suppliers that have the required production resources available. The connectivity network is given in figure 4.8. The determined transfer price levels result in the enterprise profits shown in figure 4.9. Company A_1 takes advantage of its slightly superior production costs and available resources. The profits for A_1, A_2, A_3, B_1 (secondary company), and C_1 (tertiary company) are 3590, 2467, 3000, 5000, and 9681, respectively, given a lower profit requirement levels of (0, 0, 0, 2000, 4000). The λ-multipliers take non-zero values for $\lambda_{41} = 0.803$, $\lambda_{51} = 0.197$, $\lambda_{32} = 0.533$, $\lambda_{42} = 0.467$, $\lambda_{43} = 1$, $\lambda_{44} = 1$, $\lambda_{45} = 0.080$, and $\lambda_{55} = 0.920$. The CPU requirement is 19 seconds and the error at the solution point is 0.20%. It may seem unusual to have a lower computational time for this example which has a larger problem size than the previous one, but the CPU requirements have been found to be sensitive to the choice of the lower

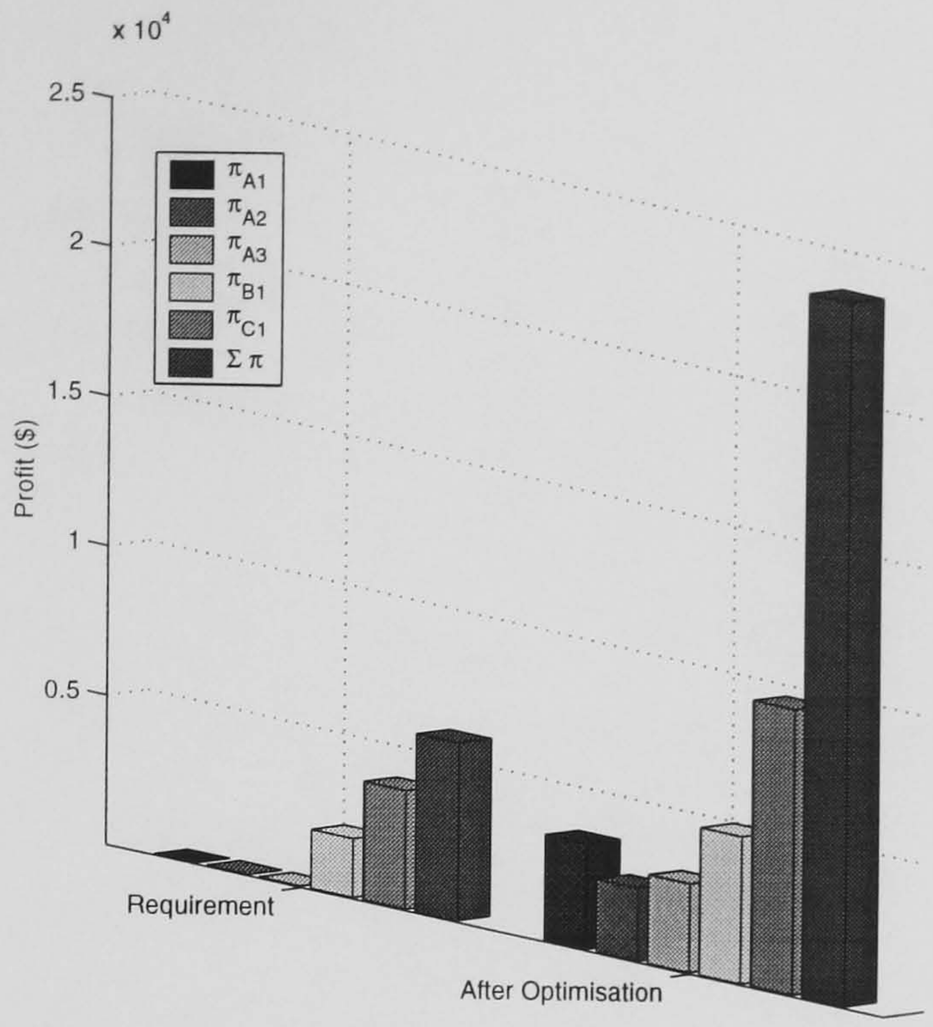


Figure 4.9: Five-enterprise profits before and after supply chain optimisation

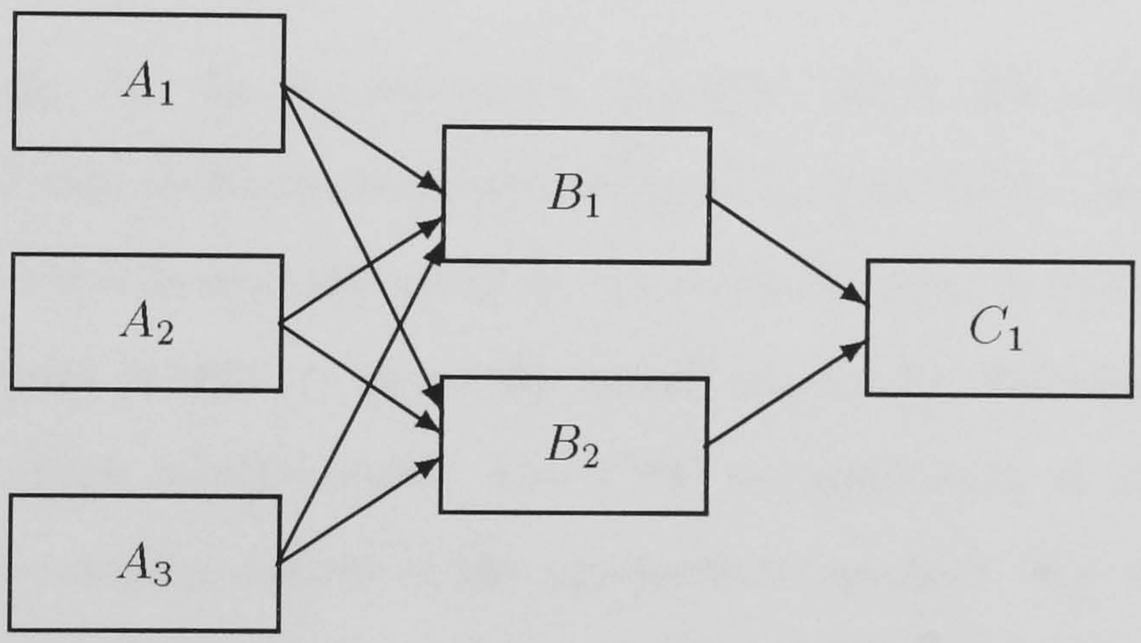


Figure 4.10: Six-enterprise connectivity

requirement point. It should also be noted that in this example there are more production resources available to fulfil the same demand. Therefore, it is less demanding to find feasible solutions in this example.

4.4.4 A Six-Enterprise Example

The six-enterprise example has the same properties as the five enterprise example and an extra final producer, B_2. The connectivity network is given in figure 4.10. The enterprise profits are shown in figure 4.11.

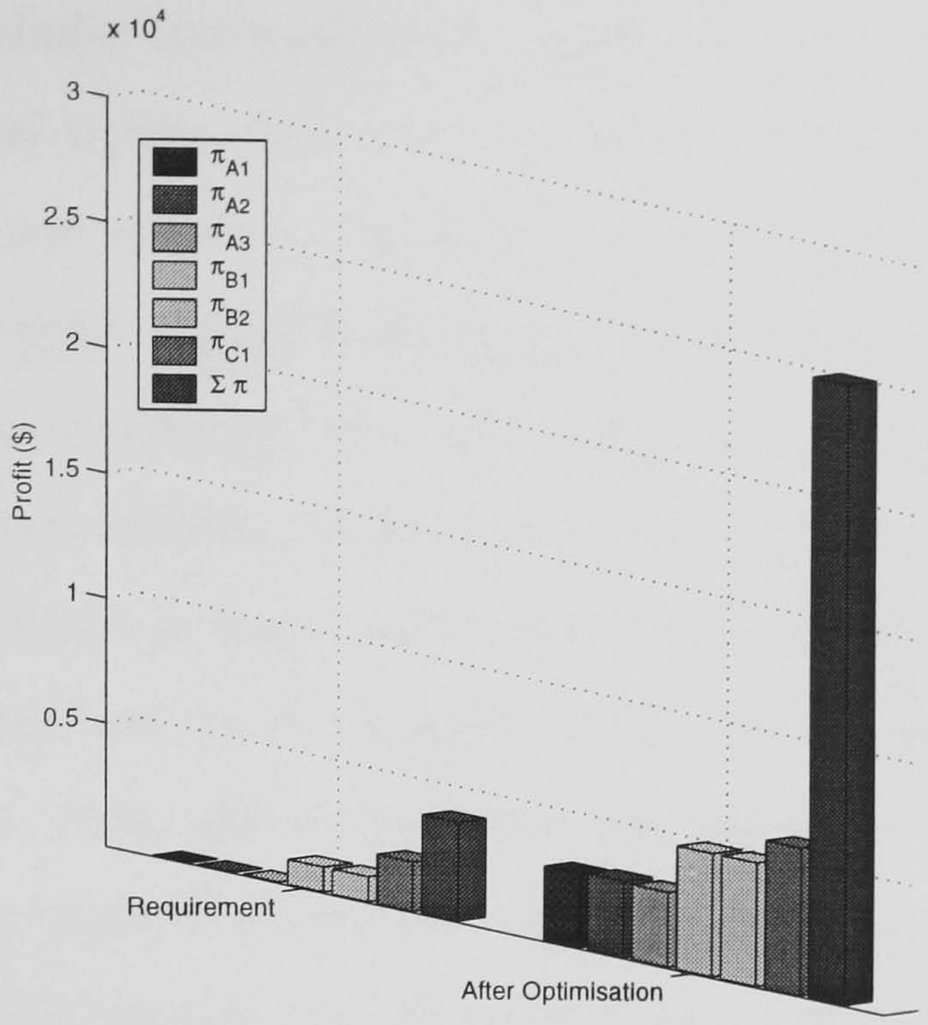


Figure 4.11: Six-enterprise profits before and after supply chain optimisation

The profits for A_1, A_2, A_3, B_1, B_2, and C are 3000, 3085, 3031, 5000, 5000, and 6000, respectively, given lower profit requirement levels of (0, 0, 0, 1000, 1000, 2000). The CPU requirement is 646 seconds and the error at the solution point is 0.023%.

The computational errors are generally small, especially considering that these are large-scale supply chain applications. The CPU requirements of the first example is higher due to more complex nature of the production problem. For the other examples, the CPU requirements vary because of the time to find an integer solution within the marginal gap (5%).

4.5 Transfer Pricing Summary

In this chapter, an approach to sustainable profit sharing in an n-enterprise supply chain has been considered. A separable programming approach, which utilises the game theoretical, fair bargaining concepts developed by Nash and Harsanyi, has been presented, in which semi-continuous transfer prices have been used in order to distribute profits between supply chain partners. The proposed model is a mixed integer non-linear programming problem aiming to determine production resource utilisation, production levels, inventory levels, flows, and transfer prices of the products in the supply chain

network so as to maximise the profit levels of the separate enterprises fairly.

The computational results show that the proposed method produces equitably distributed profits with low errors on the solutions. We believe there is a strong future for optimised multi-enterprise supply chain systems which utilise top level enterprise performance measures as objectives, through which the production process activities are planned directly. In this chapter, we have described such a system.

Although the approach is based on deterministic demand data, the key result to be retained for operational use are the transfer prices. Hence, representative demands can be used to determine these, and shorter-term operational scheduling tools can be used to manage the supply chain in practical situations of dynamic demands.

The work presented assumes that all relevant data can be shared by all supply chain members, thus allowing the supply chain network to have a single optimisation model. However, it is still quite common in practice that members are reluctant to make all information available to their partners. The development of suitable optimisation systems characterised by private information is presented in the next part of this thesis. However, first a framework for the modelling of large-scale industrial supply chains is presented.

Part III

Methods for Optimal Dynamic Supply Chain Planning

Chapter 5

Simulation of Supply Chain Systems

Stochastic, dynamic simulation is one way of modelling and evaluating performance of large-scale supply chain systems. The system can be treated at an appropriate level of detail and stochastic parameters can be integrated in a straightforward manner. The supply chain simulation model proposed in this chapter incorporates production, inventory, distribution and retail functions as well as mimicking the business processes utilised to determine policy parameters.

When scenarios are evaluated in terms of performance requirements on variables such as customer service levels and costs, confidence bounds on these variables can be calculated through simulation repetitions. Furthermore, reliable future estimates can be obtained and utilised to determine appropriate values on the controllable input parameters. On the medium-scale horizon, these parameters are often the only available controls, especially in investment intensive process industries. Most mathematical programming models consider network design problems and these decisions are taken infrequently on strategic grounds whereas the simulation models proposed are thought to be used to make sure that the operational policies satisfy the current performance requirements.

In this chapter, we describe a procedure for modelling of the physical and decision-making business process aspects of a supply chain. A specialty chemical process with various international markets, several secondary manufacturing plants and a few primary manufacturing plants illustrates the model. Pragmatic, non-invasive policy and param-

eter modifications that improve performance measures such as average inventory levels, probability of stock outs and customer service levels are identified. The model is also utilised to assess and quantify the risks and benefits of changing supply chain operations. The demand management models are based around fixed order cycle processes in which the distribution resource planning (DRP) process with dynamic safety stock (SS) utilises customer forecasts and available inventory planning policies to determine order patterns from the secondary manufacturing in a pre-specified time window. The manufacturing process (based on finite capacity scheduling) then determines order release dates.

Supply chain manufacturing in this sector is furthermore divided into primary and secondary production. Primary production considers bulk manufacturing of products that has not been differentiated to customer needs. This part of the production mainly aims at providing the rest of the supply chain with raw material for further conversion. In many chemical industries, primary production is performed in campaigns over several weeks. Secondary manufacturing aims at providing markets with required goods at the required times. Activities typically include customisation and packaging.

The combined effect of secondary manufacturing delivery performance and forecast accuracy is studied in scenario simulations with demand variability sampled from random distributions. Secondary manufacturing manages its raw material inventory and sends delivery orders to the campaign-based primary manufacturing. In the primary stage, the specialty chemicals are manufactured through a multi-stage process. One of the key features of the model is that it captures the physical supply chain as well as the business and decision processes that drive it. The general structure of the proposed supply chain model is illustrated in figure 5.1.

The scope of this chapter is to stress the importance of continuous supply chain performance assessment by improved forecasting and demand management procedures, and to illustrate how stochastic simulation modelling of the entire supply chain can be achieved to aid this assessment.

5.1 Introduction

Many typical supply chains have production and distribution activities that span several countries, and product markets that are to be found globally. The opportunities for

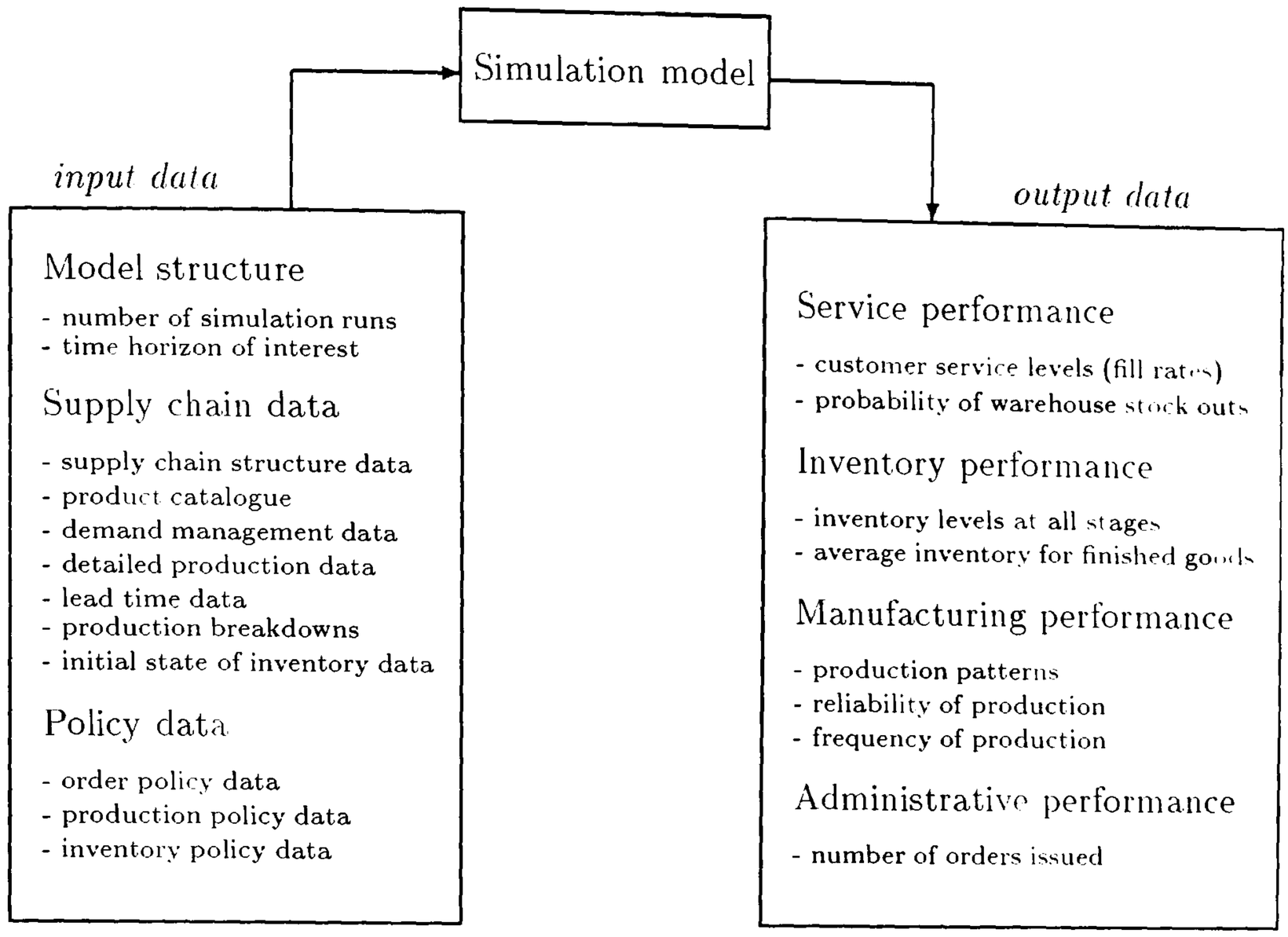


Figure 5.1: Simulation model structure

supply chain improvements are large *e.g.* in terms of cutting costs of keeping inventory throughout the supply chain while maintaining high customer service levels. Furthermore, the manufacturing processes can be improved so as to employ current working capital and labour more efficiently.

McGuffog and Wadsley (1999) regard active supply chain cooperation between key players as a means to obtain reduced uncertainties and thereby reduced costs and increased service. To satisfy orders, prior planning needs to be performed among the supply chain partners especially with respect to the authority to take action, the process of decision-making and the communication systems implemented. Typical demand uncertainties arise from the degree of order timing predictability, order size predictability, influence of institutional and random factors rather than true demand, and data accuracy in terms of true demand, prices, delivery points and timings. The authors claim that whereas management analyses new product development, acquisitions, and financial control, there is a lack in terms of achieving simplification and standardisation protocols across the supply chain. They propose several items, which are displayed in table 5.1, to simplify the supply chain management.

In the last few years, there has been a multitude of efforts focused on providing im-

Key processes
Supporting flows of information
Definitions of each data element
Definitions of shared, collaborative master data files
Linked data codes to accompany the master data files

Table 5.1: Standardisation to simplify supply chain management (McGuffog and Wadsley, 1999)

provements of supply chain management and optimisation. These efforts span a wide range of models: from commercial enterprise resource planning systems and so called advanced planning systems to academic achievements in mixed-integer linear programming, fuzzy set programming and multi-agent systems.

In this chapter, we are interested in the *simulation* of key business processes, the physical assets, and the decision-making processes of supply chains. By being able to predict the behaviour of these processes, the aim of the model is to enhance the overall performance in terms of key measures.

The specialty chemical supply chain under investigation in this chapter is broadly divided into three main echelon types: (1) Primary manufacturing, (2) secondary manufacturing (formulation and packaging) and (3) marketing and distribution (demand management). In the external downstream supply chain there are also several distributor warehouses and consumer retailers. In this system, the forecast demand must be matched against resources such as equipment capacity, material, and manpower. The supply chain market trends are moderately dynamic and the response time (supply chain lead-time) is quite long.

As Perea *et al.* (2000) claim, many supply chains operate very much as a linked "pull system" which is driven by orders between the echelons. Products available to be delivered are removed from the backlog of orders that is created when an end customer requests a product. The succession of events is carried upstream by replenishment orders. Plants collect the orders placed and create production plans to satisfy them. The production plan and the order system are the main controlling entities of such a pull type supply chain.

To comply with set standards (such as customer service levels) at reduced cost: all

stages in the supply chain should be considered in the improvement process. Reduced costs can be achieved through lower inventory levels throughout the chain and a thorough understanding of how working capital and manpower capacity is utilised.

The specialty chemical supply chain is characterised by pull manufacturing driven by customer demand forecasts and sales. The marketing and distribution function pulls finished products from the secondary manufacturing according to forecasts and firm orders from retailers and wholesalers. The secondary manufacturing then pulls material from primary manufacturing to maintain its raw material availability.

5.1.1 Opportunities and Scope

The scope of this study is to achieve visibility of the key business processes, decision-making processes and product flows throughout the supply chain. Through repeated simulations of these activities, the benefits and risks of current operations and possible modifications to these operations will become clear.

The system processes are benchmarked and possible changes are quantified in terms of typical, standard performance measures. The model provides means of reducing risks and provides business and operational decision-making support.

Cost reductions can be achieved through efficiency of production and optimal customer service levels in most supply chains. From an operational view-point, the system may obtain the desired goals through tighter control of stock keeping unit (SKU) inventories and reduced lead times through decoupling points, thus resulting in increased responsiveness and flexibility. Identification of capacity bottlenecks and other long-term strategic supply chain issues are also typical desired results.

5.1.2 Objectives of Study

The first step of a general supply chain study is to quantify and benchmark the operational policies and business processes in accordance with the performance indicators. Thus, the operational policies become clear and inconsistencies in the model and/or policy implementation become apparent. A qualitative study should be performed to illustrate how the performance measures are utilised at different stages of the supply chain.

Particularly, one aim of the simulation procedures is to see how key performance

indicators are influenced by the main supply chain parameters that are, more or less, easy to modify by managerial policy adjustments. In the next chapter, methods to optimise these parameters are proposed and evaluated. The supply chain parameters of interest in this chapter are the following:

Primary manufacturing:

- Production capacities (hr/wk)
- Production rates for continuous processes and campaigns (kg/hr)
- Safety stocks at each production stage (kg)
- Raw material safety stock (kg)
- Initial stock levels at each stage (kg)

Secondary manufacturing:

- Production capacities (hr/wk)
- Production rates (kg/hr or units/hr)
- Production flexibility (% of nominal production rates/capacities)
- Inbound bulk intermediate product safety stock (kg)
- Initial stock levels (units or kg)
- Finished tablets intermediate bulk products (units)
- Minimum order quantity (units or kg)
- Order increments (units or kg)

Demand management:

- Finished packaged product safety stock (units)
- Minimum order quantity (units)
- Order increments (units)
- Forecast accuracy (%)
- Initial stock levels (units)

5.1.3 Non-Invasive Policies and Parameters

One of the main differences between this study and similar supply chain studies is that an entirely pragmatic approach is followed. The aim is to use the standards and the *as-is* policies and parameters. This enables easy-to-change, low risk policies to improve performance measures (no need for new systems installations), with maintained familiarity with business processes for all decision-makers in the supply chain. To summarise:

- The approach is non-invasive in that it will not suggest modifications to the physical infrastructure or software systems, but rather work within the existing framework to improve it as far as possible.

- Detailed dynamic modelling of the supply chain functionality is carried out, including:

  - The dynamic physical system;
  - The associated business processes;
  - The decision support tools for the business processes.

Hence, the model replicates and benchmarks the existing system and then suggests alternative parameters/policies with confidence.

5.2 A Supply Chain Approach

The simulation of the system covers a standard forecast time horizon. In the simulation model, demands and production disturbances are sampled from random distributions. These demands and production disturbances samples are taken at each time step throughout the time horizon. The system then simulates multiple simulation series (*i.e.* the entire time horizon is repeated with new samples) so that various scenarios can be evaluated in terms of expected supply chain behaviour and potential risks. Market dynamics are studied in terms of forecast instability. Secondary manufacturing is given special attention since it is liable for a large portion of the responsiveness in meeting varying demand. The influence of product interactions, maintenance and capacity constraints on downstream market performance measures are also considered. The primary manufacturing processes are modelled by means of campaign plans and a type of simple

control law that manages production timings etc. The models are linked together and the overall network behaviour is analysed.

5.2.1 Data Collection and Definitions

The data collected give some representative examples of the requirements to undertake a supply chain investigation of this kind. Below, the required data in terms of general, demand management and production data are given. Some of these data are required in the simulation model but mainly form the basis for a comparison between the model and the real world supply chain that is replicated in the model.

General data:

- target service levels;
- key performance indicator requirements;
- relative product sizes and importance (typically used to determine safety stock requirements);
- actual system inventory and orders;
- Supply chain structure
  - distribution centre, warehouse, market and factory locations;
  - system lead times;
  - supply chain connectivity.

Demand management data:

- historical sales;
- sales forecast with forecast accuracy for future time horizons;
- safety stock policies (forward cover);
- supply plans agreed by manufacturing;
- annual volumes and values of products;
- order quantities and order increments.

Production data:

- campaign frequencies;
- batch sizes and production items (resources) available;
- production mechanisms (recipes):
- perceived capacity constraints;
- product interactions;
- maintenance;
- perceived bottlenecks;
- production cycle times.

Furthermore in detail, the production data is divided into primary data comprising stage production capacities, initial stage stock, and target stage level stocks and secondary data regarding handling of excess orders, line production rates, number of production hours, the availability of production flexibility, initial stocks, and safety stock policies at the various stages of secondary manufacturing.

Actual sales data for a control period and forecast data for future periods are also collected. These data can be used to obtain an order plan adhering to company inventory policies. Furthermore, the historical forecast accuracy should be measured so that the forecast data can then be modified with the expected variability of demand. Moreover, some orders determined will occasionally be delayed or expedited due to various factors (such as factory capacity limitations and demand planner intelligence). These variances can be either based on historical information, thus the delays are sampled from random distributions, or may be a result of the manufacturing process due to conflicting products and/or machine breakdowns. The system model could thus simulate both the physical process and the decision-making process in the supply chain.

5.2.2 Model Building and Validation

A staged approach is pursued, in which the focus moves upstream in the supply chain from the market end. This approach enables the orders to be tracked from the customer

all the way up to the primary production. The study includes how well information is handled in the network, the forecast accuracies, main interactions between nodes in the supply chain, how well production resources are utilised, what the production bottlenecks are, etc. Each node of the supply chain is modelled and tested separately, thus validating the model behaviour without too much external parameter noise.

5.2.3 Process Benchmarking

As the model results are matched with actual values for a historical reference period, we are able to benchmark the current processes in terms of the measured performance indicators. Thereby, we can study what effect various policy parameters would have on the indicators and thus explore how well tuned the model input parameters (the policy parameters such as minimum order quantity (MOQ), safety stocks) are.

5.2.4 Exploration of Strategy Alternatives

After the benchmarking process is complete, the model can then be used to recommend what pragmatic modifications are appropriate and what the likely effects (benefits) will be in terms of the performance measures. The risks of the modified operational policies will also be quantified.

The following sections (5.3–5.5) describe a model incorporating the most vital characteristics of supply chain activity. The model provides the basic means of measuring supply chain activities and how they affect the most prominent performance indicators. The structure of the model is shown in figure 5.2.

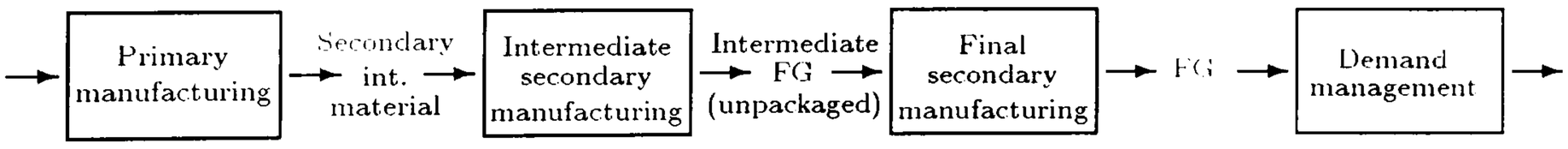


Figure 5.2: A supply chain model

5.3 Demand Management Model

In demand management, it is important to balance inventories of finished goods (FG) so as to achieve both good customer service levels and low inventory and handling costs.

The size of inventories is usually determined by the importance of the product and the volatility of its demand. It is also important to create reasonably smooth order patterns for manufacturing and to convey information about market trends upstream in the supply chain so that the future can be anticipated and activities can be planned accordingly.

Typically, demand management involves handling information about the market (actual demand and future forecasts), determining the demand management buffer stocks of finished products and linking these stocks to manufacturing orders. By linking the output to the input, the key input control parameters and the uncontrollable noise influence on the performance indicators can be measured and evaluated.

By choosing the right value for the input parameters, the required demand management performance measures can be achieved, thereby lowering costs at sufficiently high customer service levels. The model can also estimate the inherent risk of the policy adjustment.

The expected inventory levels of the system under current average order deferral times (the time between when an order is supposed to arrive and when it actually arrives - normally estimated based on historical measures) can be studied as well as the potential effects of changes in safety stock on customer service levels, probability of stock outs, and inventory costs.

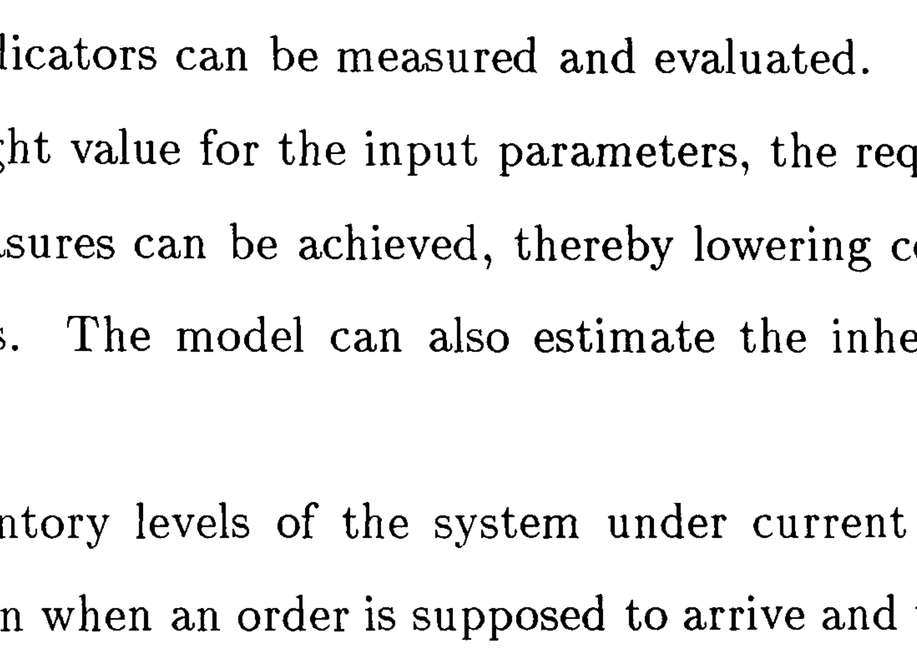


Figure 5.3: Demand management model order process

A simplified sketch of the demand management procedure at each time step is shown in figure 5.3. In this figure, the DRP process registers the received finished goods from secondary manufacturing, checks the current FG inventory levels and the future forecasts in order to plan the orders in the order list, issue orders with first requested dates (FRD) and occasionally removes excessive or unnecessary future orders. Then, the upstream

secondary manufacturing materials requirements planning (MRP) process is consulted in order to update the order list with estimated release dates (ERD). The order is then complete and other activities take place. Afterwards, the list is updated and the current time is incremented.

The following data compose the demand management model input:

- Number of weeks' safety stock
- Minimum order quantity
- Order increment
- Initial inventory level
- Forecasts for time horizon demand
- Forecast accuracy

We obtain the following output information:

- Order patterns and inventory levels, number of orders issued
- Average inventory levels with upper and lower confidence intervals over repeated simulations
- Probability of stock outs
- Customer service levels

By studying the relationship of the output and the input, we can study how the key input control parameters (especially the weeks' safety stock and the MOQ) and the uncontrollable noise (especially the demand variability and the secondary manufacturing order deferral patterns) affect the performance measures (especially the customer service level and the inventory cost).

Typically, the secondary manufacturing process comprises the specialised production and packaging of each finished good for each specific market. The main aim of primary manufacturing is to produce sufficient amounts of product at all times to provide secondary manufacturing with the raw material that is required to meet market demands of FG. The model simulates tools like DRP for warehouse inventory planning and MRP-II

for manufacturing where these are used for decision-making and highlights how best to use the assets and systems in place. One of the aims of the model is to be used as a tool for decision support. Figure 5.4 displays the main elements of the simulation model and the relative occurrence of each step in the process. The loop procedure is repeated for time periods 1 to H (the time horizon).

5.3.1 Fixed Order Cycle Process

This procedure determines order patterns within a pre-specified time window via DRP. Typically, the model is based on actual sales data for a control period and on forecast data for future periods. These data are utilised to obtain an order plan (with order arrival dates and order quantities) adhering to a fixed order cycle (FOC) policy. The properties comprising an order are displayed in figure 5.5. For comparison, McGuffog and Wadsley (1999) state that an order consists of: (a) order type, (b) supply chain participants (customer and supplier), (c) date of supply, (d) product name, (e) product quantity. The forecast data are then modified with demand variability (sampled from a normal distribution). The FOC policy, see figure 5.6, determines the size of receipts (ordered products) for each week during the planned time horizon. The receipts are obtained through a simple mass balance for each individual week. The mass balance is shown in equation 5.1, where I_t is the inventory at time t, I_{t-1} is the inventory at time $t-1$, R_t is the scheduled receipt to arrive at time t and F_t is the demand forecast at time t.

$$I_t = I_{t-1} + R_t - F_t \qquad \forall\, t \tag{5.1}$$

The inventory at any time should not be lower than the expected demand of a pre-specified number of weeks (the *number of weeks' cover* or the *safety stock*). This can be expressed as $SS_t = \sum_{t=t^\star}^{t^\star+k} F_t$, where k is the number of weeks' cover. The model determines PR_t, the currently planned receipts, based on the MOQ and order increments, such that $I_t \geq SS_t$ and therefore $PR_t \geq SS_t + F_t - I_{t-1} - R_t$. Then, if t is within the scheduling horizon, an order of the size PR_t is issued and becomes a scheduled receipt, R_t.

However, in the dynamic environment simulated there are primarily two reasons why this mass balance will not reflect the actual situation in the market. The actual demand,

Initialisation

Initialise program with initial inventories, safety stocks and forecasts etc.

Create initial demand management orders based on previous ordering pattern.

Begin Loop ($t = 1, \ldots, H$)

Planning cycle

Do DRP for demand management for each product (SKU).

Do master requirements planning for final secondary manufacturing.

Do master requirements planning for intermediate secondary manufacturing including adding produced material to final secondary manufacturing raw material stocks (intermediate FG).

Simulation cycle

Remove intermediate FG inventory utilised in final secondary manufacturing.

Remove intermediate FG inventory for external markets.

Remove secondary intermediate material used in secondary batch processes.

Time step demand management model, secondary manufacturing model including adding FG stock to demand management.

Send requirements for secondary intermediate material of secondary manufacturing plants to primary manufacturing.

Do staged production for primary manufacturing plants.

Add secondary intermediate material from primary manufacturing to secondary manufacturing model.

Time step primary manufacturing model.

Increase current time.

End Loop

Figure 5.4: Master supply chain procedure

- Order ID
- Producer
- Customer
- Product ID
- Order quantity
- Purity
- Pack size
- First requested date
- Estimated release date

Figure 5.5: Properties of a demand management order

Schedule determines the orders requested from factory at each future time step.

(a) Check the size of safety stock.

(b) Do mass balance including previously agreed receipts.

(c) If inventory less than safety stock, determine

the new orders so that inventory is higher than safety stock.

If the time period planned is within the scheduling time horizon part, *e.g.* between 2-16 weeks into the future, insert the order in the appropriate place in the order queue based on the date required.

Figure 5.6: FOC scheduling functionality

D_t, at time t (which is now the actual simulated time period rather than the planned time period) will differ from the forecast, F_t. This is what is known as demand variability. Also, the planned receipts R_t are subject to change: some orders may be deferred and others may be expedited due to secondary manufacturing planning and execution issues. This will incur a different schedule of actual orders received, R'_t. The resulting mass balance is stated in equation 5.2.

$$I_t = I_{t-1} + R'_t - D_t \qquad \forall\, t \qquad (5.2)$$

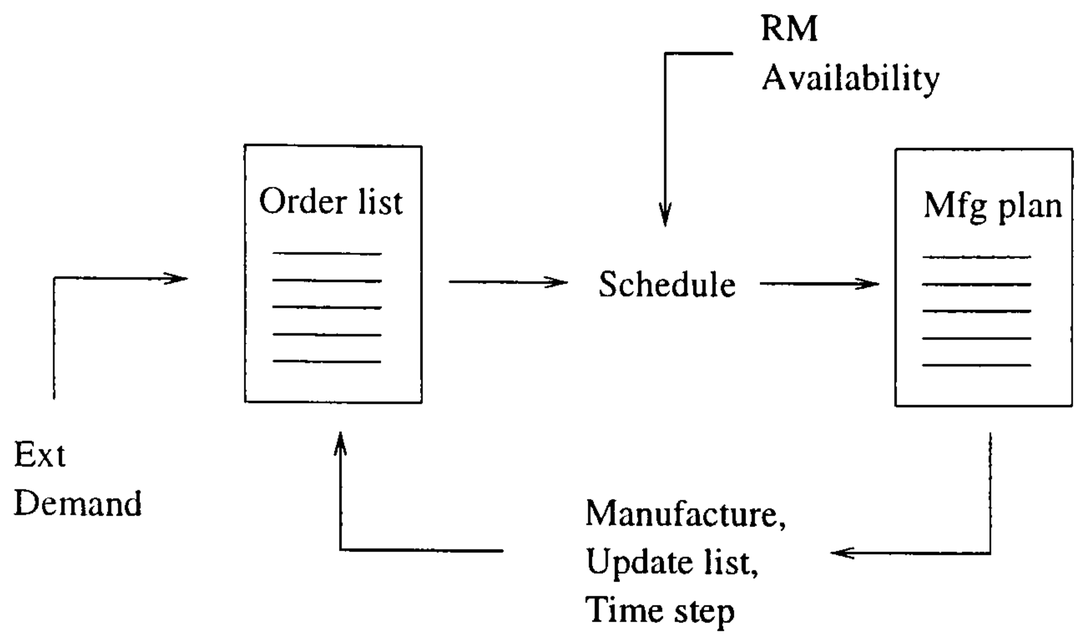


Figure 5.7: Secondary production order process

5.4 Secondary Manufacturing Model

The basic functions of the secondary manufacturing (packaging) model are to:

- read orders from the downstream demand management model;
- manage finished product inventory and work-in-progress;
- create order queue to scheduling based on orders from demand management;
- create orders for upstream production;
- schedule fixed orders (sorting of confirmed orders);
- create task schedule;
- create variability in task length based on samples from random distribution;
- maintain secondary manufacturing unpackaged material availability.

The secondary manufacturing process involves the specialised production (formulation) and packaging of each product SKU for each specific market. Due to product interactions and limited capacity, it is important to keep buffer stocks of each SKU in the system in order to safeguard against deviations between product sales forecasts and actual sales. The size of orders received from demand management, normally the product MOQ, is determined by an agreement between the factory and the demand management. The order size does not generally correspond one to one with the factory batch size of any given product. The factory schedules production of finished products to maintain a safety stock so as to provide the demand management with the orders required. A viable

production task schedule is created after the product orders have been sorted in a first in first out (FIFO) manner. The secondary production procedure at each time step is displayed in figure 5.7.

The order process between demand management and secondary manufacturing is often characterised by a changeable period where orders can be added or deleted and a firm period where orders cannot be modified. This implies that if for instance an unpredicted rise in market demand occurs in the firm period then the system will be unable to respond to this change and the result is a lowered service level in the marketplace. Furthermore, machine availability at secondary manufacturing is occasionally reduced due to maintenance. The down-time is modelled as a reduction of capacity in the MRP process occurring if a random number $\in (0, 1)$ is less than a pre-specified limit, l^U (based on historical estimates). For instance, machine unavailability might occur on average every 10^{th} time period, thus $l^U = 0.1$. A description of the procedure is given in figure 5.8.

MRP procedure determines the estimated release date (ERD) for external orders.

(a) Read the elements of the order list one by one.

(b) Determine if over- or under-time is required. This is controlled by the deviation between first estimate of ERD and requested date.

(c) Determine, for the order, production time $= \frac{\text{order quantity}}{\text{production rate}}$

(d) Check if machine breakdown present within this time period.

(e) Do main production determining step: Check that current raw material availability is sufficient. Else increment the ERD and increment week.

If production time + the utilisation of equipment for the present week is less than the maximum utility available, then the ERD will be the present week. Else update the future weekly plan.

Figure 5.8: MRP scheduling functionality - final secondary manufacturing (packaging)

5.4.1 Intermediate Manufacturing

The formulation of tablets can normally be considered a four-step process (Gilmore and Smith, 1996) involving dispensing, granulation, compression and packaging. Some manufactured products require a fifth step of coating (between compression and packaging)

in which colours and flavours are added. In the first step, raw materials are weighed in specified quantities. Then granules of a specified particle size are manufactured (granulation). The tablets are then formed in the compression step; four steps are involved in this process, namely: filling pre-compression, compression and ejection. Finally, appropriate coating is performed and the tablets are packaged as required.

The intermediate secondary production (formulation) is handled in a similar fashion as the final secondary manufacturing. The main difference is that the raw material amount is determined by the SKU due to different purity requirements in the final product. Furthermore, due to dedicated production facilities the equipment is utilised fully for each time period. Therefore, there is also no under- or overproduction available. The procedure is shown in figure 5.9. The order properties that constitute an order between

Intermediate MRP schedule determines, for the orders requested from secondary final manufacturing, the estimated release date (ERD)

(a) Read the elements of the secondary order list one by one.

(b) Determine raw material amounts for product based on recipe.

(c) Determine, for the order, production time $= \frac{\text{order quantity} \times \text{production time for batch}}{\text{batch size}}$

(d) Do main production determining step: Check that current raw material availability is sufficient. Else move the ERD and update weekly plan.

If production time + the utilisation of equipment for the present week is less than the maximum utility available, then the ERD will be the present week. Also, check remainder time units available in the week so as to use up all production time available.

Else update the future weekly plan.

Figure 5.9: MRP scheduling functionality - intermediate secondary manufacturing

final secondary manufacturing and intermediate secondary manufacturing are displayed in figure 5.10. As mentioned in figure 5.4, the order list procedure includes adding new inventory as part of the production process. This is performed by checking current orders in the order list with ERD = 0. These orders are removed and the order quantity in this order receipt is added to the appropriate inventory variable downstream in the supply chain.

- Product ID
- Order Quantity
- Estimated Release Date

Figure 5.10: Properties of a secondary manufacturing order (internal order)

5.4.2 Product Interactions

Frequently, product interactions at manufacturing sites lead to production schedule conflicts, which in turn result in delays to the orders to be delivered. In particular, peaking seasonal demands for some products often coincide with those of other products. These peaks can often be tracked in demand management forecasts available at the market level. It is common in many supply chains that this information is not made fully available to the manufacturing sites until it is very late.

5.4.3 Capacity Constraints

In practice, capacity constraints are often a bit more slack than described in many common models (especially optimisation models). Overtime work during periods of high demand is more a rule than an exception in industry. In terms of this model, the capacity is therefore "semi-fixed"; *i.e.* we allow some deviation from planned capacity at the production lines during busy and unoccupied periods.

5.5 Primary Manufacturing Model

Primary manufacturing is of a different nature than the secondary manufacturing. Larger batches of bulk product (serving several downstream secondary sites) can be produced than the equivalent finished product amounts manufactured in the FG specific secondary manufacturing, due to the lower cost of inventory upstream in the supply chain. The bulk product inventory planning, *i.e.* for material going into the final manufacturing steps at the secondary sites therefore becomes somewhat more effortless than the SKU planning at the demand management level. The production of raw material is quite steady, although the integrated scheduling of tasks and task-lengths is a quite complex assignment, *cf.* the example supply chain study later in this chapter and Shah and Pantelides (1992).

As mentioned before, primary manufacturing is about producing sufficient amounts of bulk product to provide secondary manufacturing with the raw material that is required to meet market demands. Therefore, at some periods of the year and especially when product demand is ramping, it is quite acceptable to keep some inventory of bulk product at primary sites to buffer for future periods. The choice of the "right amount stocked" must be carefully made. In this model, we are able to track bulk product inventory and make decisions about primary manufacturing policies based on information about the rest of the supply chain and the product market demands.

Some processes, as suggested by Minegishi and Thiel (2000), require a linked push and pull mode in which the supply side to an inventory node or "point of synchronisation" works in a push fashion and the demand side acts in a pull fashion. The demand side then works from definite orders and the supply side works from forecasts.

One way of determining production profiles and utilisation of manufacturing requirements is to use a type of linked proportional controller to be used in the considered serial processing structure with dedicated production equipment. Recently, Perea-López *et al.* (2001) presented a frame-work for such control laws for the entire supply chain regarding the system as decentralised without any global coordinating mechanisms. Their approach suggests that at each node decision are made locally based on material pulled from downstream nodes. The proportional control laws compared include fixed inventory control (similar to the approach suggested below), control based on the incoming orders, control with respect to the current backlog, and control based on actual demand signals from the retailer level. The authors do not give any conclusive evidence as to what control law gives the best performance but suggest that the policies have different advantages in terms of cost reduction and customer service levels fulfillment.

The idea used here is to determine the production rate at stage s by comparing its inventory at time $t-1$ with a set point inventory at time t. If the inventory, $I_{s,t-1}$, is higher than the set-point target stock, $I^{\star}_{st}$, then no production is necessary; otherwise, the production is proportional (k) to the deficit, subject to constraints on the maximum production rate, $P^{\max}_{st}$, and the available inventory at stage $s-1$ ($I_{s-1,t-1}$). $I^{\star}_{st}$ and k have to be determined according to model characteristics. The procedure is expressed mathematically in equation 5.3.

$$P_{st} = \min(\min(P^{\max}_{st}, I_{s-1,t-1}), \max(k(I^{\star}_{st} - I_{s,t-1}), 0)) \quad \forall\ s,t \tag{5.3}$$

The updated inventory is then determined through a normal mass balance by the previous inventory, $I_{s,t-1}$, the production rate, P_{st}, the consumption rate for downstream production, $P_{s+1,t}$, and the demand (if present) of finished bulk products by secondary manufacturing sites, D_{st}.

$$I_{st} = I_{s,t-1} + P_{st} - P_{s+1,t} - D_{st} \qquad \forall\ s,t \qquad (5.4)$$

For stages (processes) which have shared equipment, special procedures have to be observed. Normally these procedures include a fixed campaign length, $t^{\star}_{cl}$ and a change-over period, $t^{\star}_{co}$. The algorithm for two stages, P_{s-1} and P_s, sharing the same equipment can be implemented as displayed in figure 5.11, assuming that the second stage is under investigation and that $t^{\star}_{co} = 1$. The algorithm determines the production pattern taking into account previous inventory levels and the set-point target stock levels. If the inventory at stage s is less than at $s-1$ and both are below their target stock levels, production occurs at stage s. The production is set by default to the previous level of production in a campaign since it is assumed that the level of utilisation is not going to increase during a campaign. However, if the stock used of a previous stage at a previous time, say $I_{s-1,t-1}$, is not sufficient to meet production then the production is set so that it can be supported by the inventory level rather than the maximum capacity. Finally, if it is time for a clean-out, then the production levels at the considered stages has to be 0.

5.6 Stochastic Simulation Procedure

Supply chain simulation is a complex field of study since it comprises such a large number of concurrent operations. Stochastic supply chain simulation is based upon assumptions about the causes of supply chain uncertainty as well as the degree of uncertainty present at the various sources. In the system, discrete events occur at the various echelons and drive the supply chain decision-making processes to respond. One approach to simulate the stochastic element is to let the uncertain parameters follow a statistical distribution and then to take samples from this distribution to obtain input data throughout the simulation time horizon. One of the key features of supply chain simulation models is that it is straightforward to incorporate uncertainties.

Description and analysis of effects of the following activities could typically be included in a dynamic, stochastic supply chain model: raw material orders; campaign

If $P_{s,t-t^{\star}_{co}} == 0$ AND $P_{s-1,t-t^{\star}_{co}} == 0$

If $I_{s,t-1} < I^{\star}_{st}$ AND $I_{s-1,t-1} < I^{\star}_{s-1,t}$

If $I_{s,t-1} < I_{s-1,t-1}$

$P_{st} = \min(\min(P^{\max}_{st}, I_{s-1,t-1}), \max(k(I^{\star}_{st} - I_{s,t-1}).0))$

$P_{s-1,t} = 0$

Else

$P_{s,t} = 0$

$P_{s-1,t} = \min(\min(P^{\max}_{s-1,t}, I_{s-2,t-1}), \max(k(I^{\star}_{s-1,t} - I_{s-1,t-1}), 0))$

Else

$P_{st} = \min(\min(P^{\max}_{st}, I_{s-1,t-1}), \max(k(I^{\star}_{st} - I_{s,t-1}), 0))$

$P_{s-1,t} = \min(\min(P^{\max}_{s-1,t}, I_{s-2,t-1}), \max(k(I^{\star}_{s-1,t} - I_{s-1,t-1}).0))$

Else if $P_{s,t-1} \neq 0$

$P_{st} = \min(P_{s,t-1}, I_{s-1,t-1})$

$P_{s-1,t} = 0$

Else if $P_{s-1,t-1} \neq 0$

$P_{st} = 0$

$P_{s-1,t} = \min(P_{s-1,t-1}, I_{s-2,t-1})$

If $P_{s,t-1} \neq 0 \ldots P_{s,t-t^{\star}_{cl}} \neq 0$ **Or** $P_{s-1,t-1} \neq 0 \ldots P_{s-1,t-t^{\star}_{cl}} \neq 0$

$P_{st} = 0$

$P_{s-1,t} = 0$

Figure 5.11: Pseudo-code for shared production equipment between stages

production; line production; production orders; distribution; ordering; demand management; forecasting; sales orders; inventory management, including inventory associated with various stages of manufacture and how they interact; raw material sourcing, including multi-enterprise issues and interaction; product development and introduction; flexibility etc. In our approach, the complete supply chain is dynamically simulated over a pre-specified time horizon (*e.g.* two years). First, initialisation of the model is performed with respect to fixed policy parameters, business processes and initial stock levels. At each time step (*e.g.* each week) of the time horizon input parameters and model data from previous time steps are collected. Actual sales and machine breakdown

```
Loop over each time period in the time horizon
    within each simulation, collect info on current stock levels,
    demands, sales, stock outs, minimum and maximum stocks etc.
Determine average max and min stocks and demands
Determine statistics such as variances and standard deviations and use
    these to find confidence intervals on stocks and average stocks
Determine service levels
Write performance results to a file (to be done at repetitions for
    different safety stocks or other input parameters)
```

Figure 5.12: Performance collection in simulation runs

are generated from stochastic distributions. Stock positions, current supply chain orders and forecasts are then updated and evaluated. This procedure is illustrated in figure 5.12. When any model variables violate the current procedures or policies (such as the safety stock) the model issues new directions of action (such as issuing new orders).

To obtain statistically significant results of the stochastic process, the simulation is repeated a number of times. At each run, informative data are collected. Finally, the results of all the simulations are combined to determine probabilistic measures of supply chain performance.

5.7 A Supply Chain Example

A simple supply chain example[1] is shown in figure 5.13. The company has primary production facilities located in Europe, secondary production facilities in Asia and America which serve several downstream customers, among others Japan and the USA, respectively. On the demand management side, the markets considered in this example are Japan and the USA. There are two main SKUs in the Japanese market: products A and B. Also, in the US market there are two principal products, C and D. There are also several other product SKUs that share secondary manufacturing resources (in Asia and America) which are handled by other warehouses not explicitly modelled *e.g.* in Europe. The customer service level (CSL) deemed sufficient for this supply chain by decision-makers is typically 97% (called the target CSL). The low figure is due to large

[1] This example is based on Gjerdrum *et al.* (2000*b*).

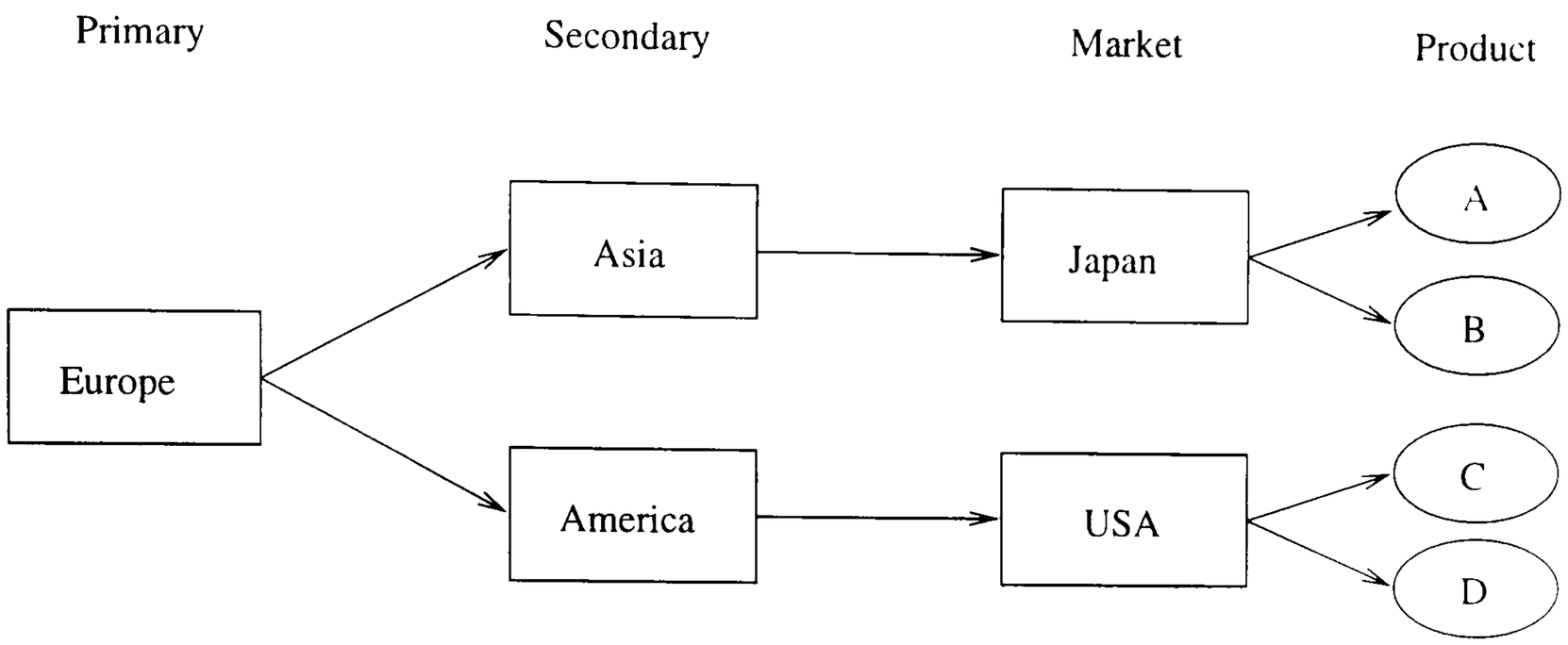


Figure 5.13: A simple supply chain

inventories held at external wholesaler warehouses downstream in the supply chain, which will buffer against any temporary stock outs. Occasional SKU stock outs are therefore not considered to be harmful since the end consumer is not affected. Therefore, the aim of this case study is to manage inventory effectively while maintaining the target CSL. The demand management SKU data are presented in table 5.2, whereas the production data are given in table 5.3. Note that in the secondary stage, material is converted

	Product A	Product B	Product C	Product D
Safety stock (weeks)	4	4	4	4
MOQ (units)	3,000	2,000	5,000	20,000
Deviation from forecast (%)	25	20	25	25
Package size (tablets)	12	30	30	30
Initial stock (units)	15,000	15,000	60,000	60,000

Table 5.2: Demand management SKU data

from intermediate product measured in *kg* to final product measured in units (packs of tablets). Finally, the primary manufacturing data are displayed in table 5.4. As can be seen, only the most vital process data are incorporated in the model yet there is enough information to make it possible to study effects at all stages of the supply chain and capturing typical supply chain dynamics. The deviation from forecast denotes the expected variability of demand sampled from a normal distribution centred around the forecast value (*i.e.* the standard deviation). The demand forecast data for one of the products (A) are displayed in table 5.5.

	Asia	US
Safety stock (kg)	50	150
Intermediate product order quantity (kg)	30	130
Production capacity (hrs/wk)	40	60
Production rate (units/hr)	300	1,200
Flexibility of production (%)	50	25
Initial intermediate product stock (kg)	200	180

Table 5.3: Secondary manufacturing site base case data

Production capacities		Stage safety stocks		Initial stock levels	
Stage 1	150	Stage 1	1200	Stage 1	1000
Stage 2	150	Stage 2	1200	Stage 2	1000
Stage 3	250	Stage 3	2000	Stage 3	2000
Stage 4	250	Stage 4	2000	Stage 4	2000
Stage 5	100	Stage 5	1600	Stage 5	1500

Table 5.4: Primary manufacturing base case data

5.8 Supply Chain Performance

One of the most important supply chain performance indicators is the amount of unutilised working capital in the chain. By closely tracking simulated inventory one can take substantial costs out of the supply chain. This must be done while maintaining required customer service levels, low probabilities of stock outs and supply chain efficiency in general. Therefore, we simulate the inventory levels as well as the more traditional customer focused aspects of the supply chain performance. Each Monte Carlo simulation is repeated a large number of times to produce statistically trustworthy results.

5.8.1 Supply Chain Example Preliminaries

The experiments presented in table 5.6 below demonstrate the response of the supply chain to changes in internal and external parameters. These experiments are useful in that they provide information on whether current policies and external factors can be modified while maintaining strong supply chain performance.

Time	Demand								
1	4500	22	6229	43	7312	64	6906	85	6840
2	3735	23	5280	44	7270	65	7319	86	7781
3	3459	24	5844	45	6821	66	5524	87	5946
4	4227	25	5472	46	6903	67	5881	88	6059
5	4844	26	7103	47	5708	68	5822	89	5639
6	4970	27	5704	48	10256	69	6932	90	6015
7	3835	28	5870	49	10860	70	6059	91	6340
8	5071	29	5639	50	10128	71	6468	92	6434
9	6070	30	6808	51	9186	72	6168	93	7076
10	5532	31	5534	52	9077	73	6637	94	7061
11	5315	32	6614	53	5337	74	6084	95	7114
12	6056	33	6708	54	4826	75	8355	96	6667
13	6265	34	6515	55	4284	76	7811	97	7331
14	5199	35	6040	56	4092	77	7650	98	8364
15	6208	36	5319	57	6885	78	7277	99	6669
16	6014	37	5958	58	5964	79	6428	100	12137
17	6126	38	7210	59	6760	80	7651	101	12717
18	6033	39	6137	60	6005	81	6762	102	10360
19	7019	40	5393	61	8161	82	6437	103	10160
20	5121	41	7492	62	6595	83	7629	104	9007
21	6130	42	7179	63	8012	84	7164		

Table 5.5: Demand forecast points for base case, product A

The customer service level (CSL) is defined as "part-fill on time" or sum of sales over sum of demands (see equation 5.5). When there is sufficient inventory, the sales equal demand. When inventory runs out, it is assumed that the available inventory can be sold while the rest of the demand is left unfulfilled. Consequently, sales are defined as in equation 5.6. The average inventory, $\overline{I}$, is defined as the inventory I_t summed over all time steps, divided by the number of time steps, $n = 104$, see equation 5.7.

$$\text{CSL} = \frac{\sum_t S_t}{\sum_t D_t} \tag{5.5}$$

$$S_t = \min(I_{t-1}, D_t) \qquad \forall\, t \tag{5.6}$$

$$\overline{I} = \frac{\sum_t I_t}{n} \tag{5.7}$$

The probability of stock out (PSO) is simply the number of times when there is no inventory divided by the total number of data points, *i.e.* the time horizon length multiplied by the number of simulation runs. $\overline{I}$ is the time horizon average finished product (SKU) inventory.

The scenarios studied are (except for the base case scenario):

1. High demand variability represents a complex market to forecast - in this case the forecast variability has been doubled for products A-D compared to the values given in table 5.2.

2. High order quantity shows what happens if a typical policy parameter is modified (MOQ is trebled).

3. The demand increase in the soaring demand case for product A is 50 units each time period into the future, *i.e.* $x_t = f_t + 50 \times t$, where x_t contains the average new demands and f_t contains the average base case demands.

4. Similarly, the collapsing demands conform to the following rule: $x_t = f_t - 25 \times t$. Similar rules have been applied to the other SKUs.

Although these parameters obviously affect the performance levels (see table 5.6), it seems that the demand management performance levels are not severely affected by the demand variability or the order quantity. The ramping and decreasing demands give conservative measures, since it is assumed that the forecast will not be updated during the time horizon. The US market is able to handle a soaring demand better due to fewer

conflicting products. Predictably, both the customer service level and probability of stock out are reduced when demand decreases for the future time horizon (*i.e.* actual demand is lower than the forecast). Recalling that the requirement for the service level is 97%,

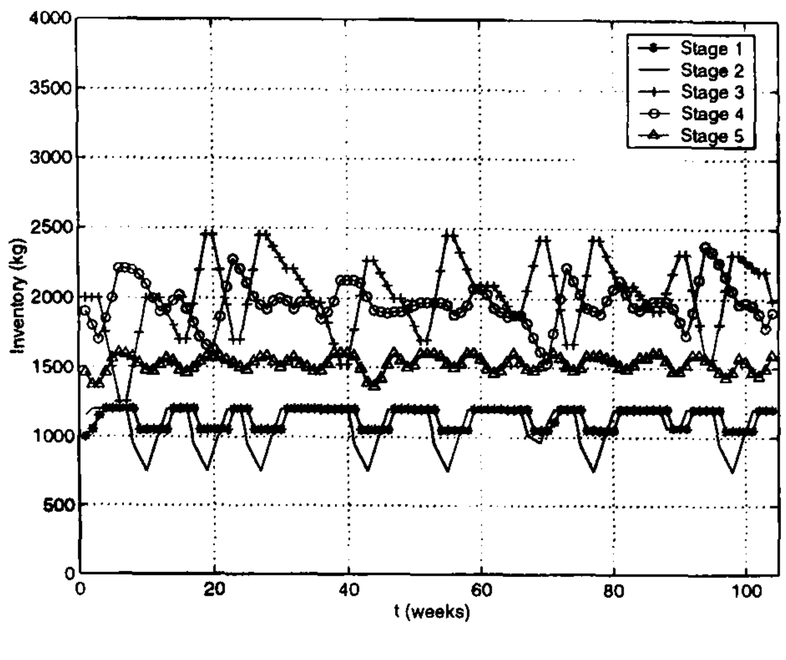
Experiment	Product A (Asia)			Product C (US)		
	CSL (%)	PSO (%)	$\bar{I}$	CSL (%)	PSO (%)	$\bar{I}$
Base case	99.03	1.01	28094	99.91	0.24	83779
Scenario 1	96.30	2.69	28308	98.35	1.88	86390
Scenario 2	99.13	0.96	28586	99.97	0.10	86365
Scenario 3	70.78	23.1	18806	96.45	3.41	90261
Scenario 4	98.60	1.44	24300	99.50	0.72	78251

Table 5.6: Supply chain experiments

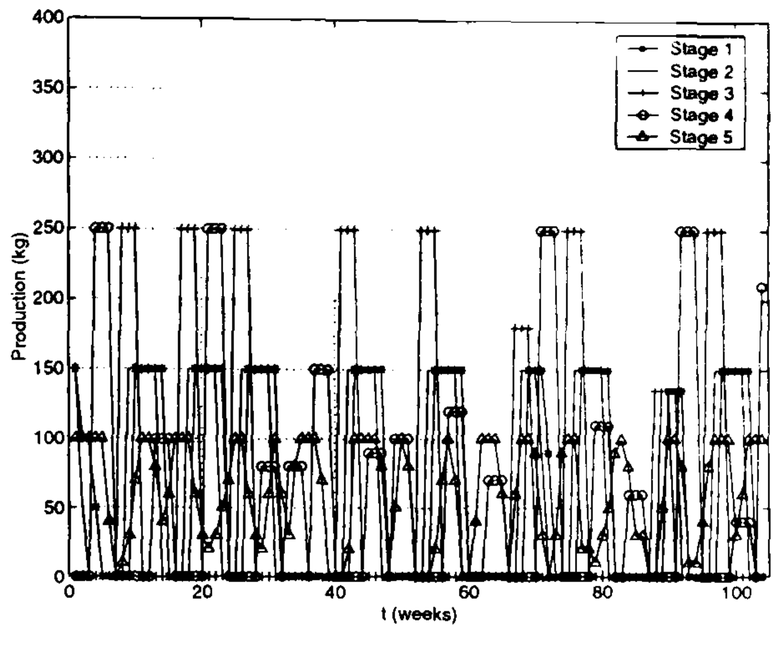
there is some scope for reduced safety stocks, particularly if the variability of demand with regards to forecast is thought to be right and soaring demands are not likely to occur. In the next chapter, some methods to provide a means to setting the appropriate levels for policy parameters are examined.

5.8.2 Effects on Performance Indicators and Production Echelons

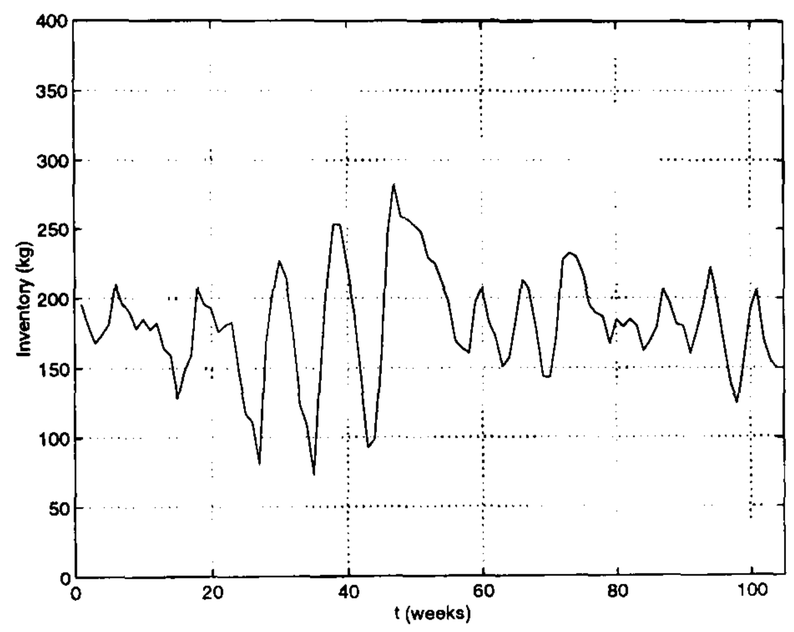
Some of the main supply chain variables are displayed in figure 5.14 to illustrate the dynamics of the linked supply chain system. When considering alterations to current procedures and policies in one part of the supply chain, it is meaningful to study the effects the new policies would have on the rest of the system. Figure 5.14(a) shows the inventory of the five production stages in the primary manufacturing unit in Europe for a demand management policy with base case forward stock cover. It can be seen that the primary production policy plans for the future time horizon seems adequate if the aim is to keep a steady amount of bulk product safety stock at the primary production site. The corresponding production planned for the time horizon are displayed in figure 5.14(b). In figure 5.14(c), the finished primary product inventory at one secondary site (in Asia) is presented throughout the time horizon. Figure 5.14(d) demonstrates the fluctuation of finished unpackaged secondary products at the same site. Here, some products included in manufacturing exported as bulk product (unpackaged) to the European ("EurExp")



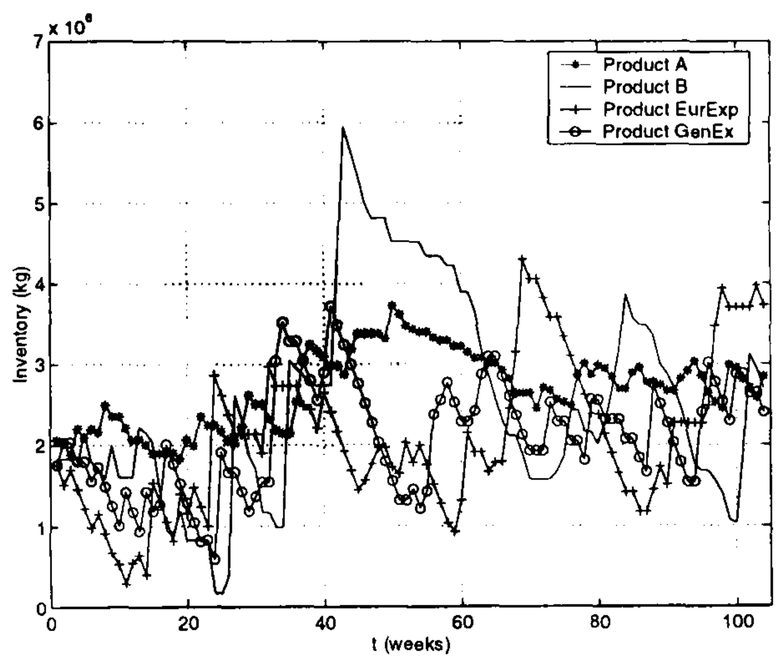
(a) Primary inventory (Europe)



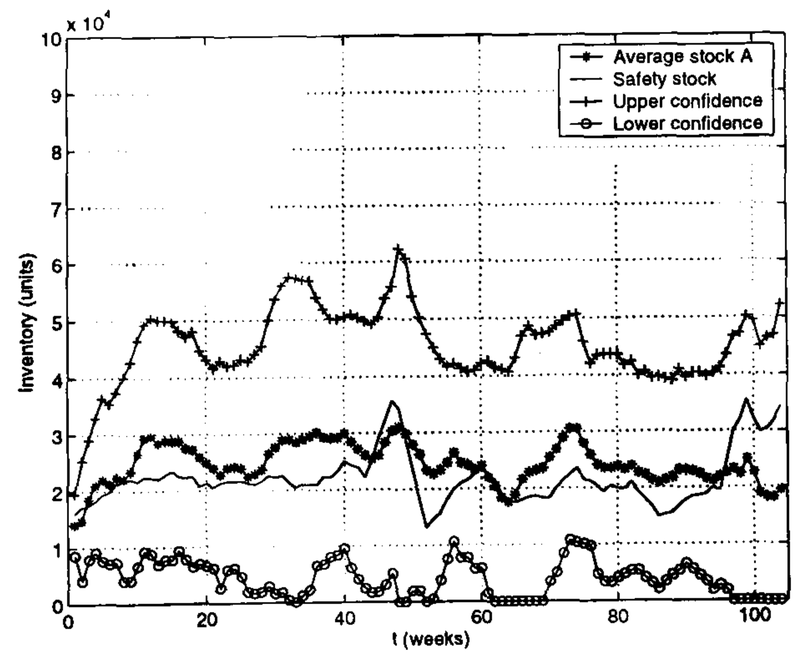
(b) Primary production profile (Europe)



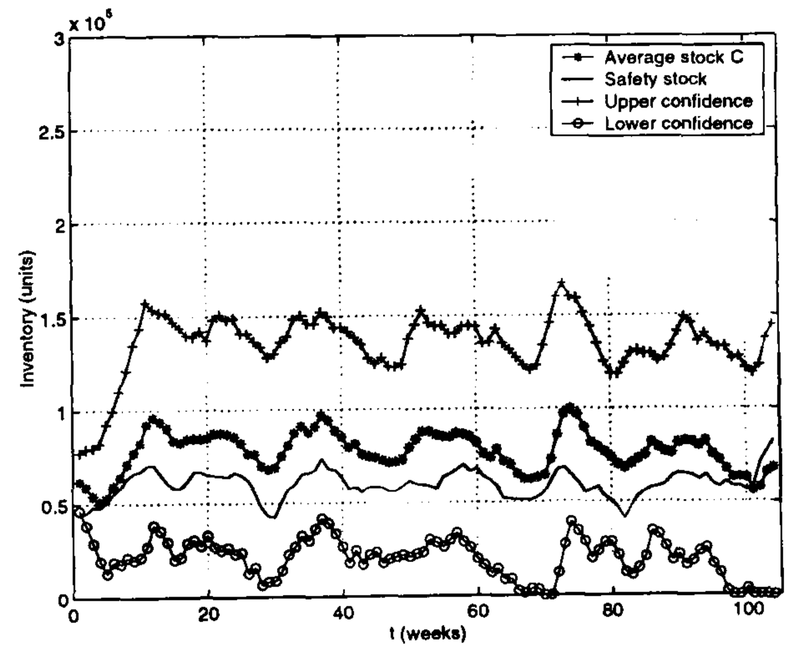
(c) Primary product inventory (Asia)



(d) Finished unpackaged products (Asia)



(e) Final product A inventory (Japan)



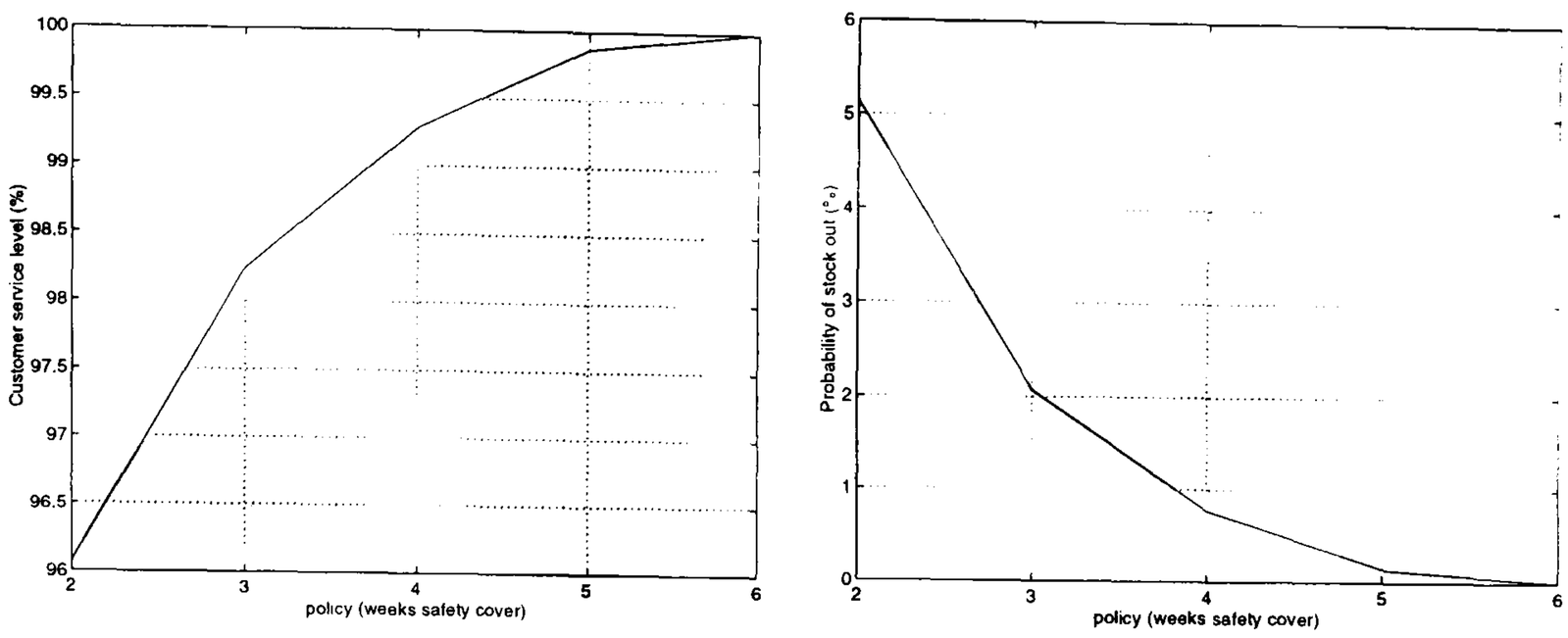
(f) Final product C inventory (US)

Figure 5.14: Major supply chain variables

and international markets ("GenEx") are also shown. Considering both primary and secondary manufacturing, it can be concluded that the policies proposed do not cause any grave detrimental effects on the system. Thus, we analyse the continuous system behaviour for changes made in downstream node policies before they have been implemented and thereby enabling minimisation of any risks these policy modifications might entail.

Finally, figures 5.14(e) and 5.14(f) show the finished packaged product warehouse inventories for two products (A and C) in the markets considered, Japan and US. The typical inventory for a forward cover of 4 weeks is shown for products A and C. It can be seen that stock outs are extremely unlikely to occur in the near future. The dips in inventory levels occur due to demand increases in conflicting products. In these figures, "Average stock" is the average inventory at time t for a number of samples, "Upper confidence" and "Lower confidence" are the upper and lower 95% confidence limits of the inventory levels, and "Safety stock" is the dynamic safety stock.

In figure 5.15, the effect of policy parameter modifications on key performance indicators in the supply chain are displayed. First, in figure 5.15(a), the customer service level defined as "part fill on time" is shown for different policies of weekly forward cover stock for product C. It can be seen that a policy of 3 weeks of forward cover will be sufficient to satisfy the requirement of a target CSL of 97%. Secondly, figure 5.15(b) shows the expected number of stock outs for various safety stock policies for product C in the US market. The value represents the risk of a stock out occurring during the simulated period. In figure 5.15(c), the resulting average inventory of product C for various stock policies are shown. This value can then be used to calculate the market inventory cost. Finally, in figure 5.15(d), the numbers of orders issued as a function of the MOQ policy parameter is shown. As can be seen, the orders decrease with an increase in size of orders thus decreasing the administrative cost.



(a) Customer service level (b) Probability of stock out

(c) Average inventory (d) Orders issued

Figure 5.15: Effect of policy parameter changes on key performance indicators

5.9 New Product Introduction

One application of the developed model is to evaluate current and suggested policies in terms of supply chain management in particular with respect to the new product introduction phase. To improve supply chain processes, modelling of the information sharing between echelons must be carried out in order to obtain effective collaboration and assess the risks and benefits of adjusted policies.

Appropriate measurements of the expected influence on key performance indicators have to be considered when launching a new product into the supply chain and then onto the market place. Currently, in many chemical industries, the product decisions are based on market intelligence rather than on thorough supply chain risk management. Modelling and simulation are important tools to support this evaluation.

5.9.1 Illustrative Example

An illustrative example is presented in order to demonstrate the effects of introduction of a new product into an existing supply chain. The issues studied include:

- whether acceptable customer service levels can be maintained for existing products;
- what service level is expected for the new product;
- whether primary production can cope with the increase in demand or whether new resources must be made available;
- what safety stocks are appropriate.

The scenario is that a new product E is to be introduced in the US market in the supply chain described previously. The supply chain scenario is illustrated in figure 5.16. Management is keen to know whether production resources are sufficient to satisfy fore-

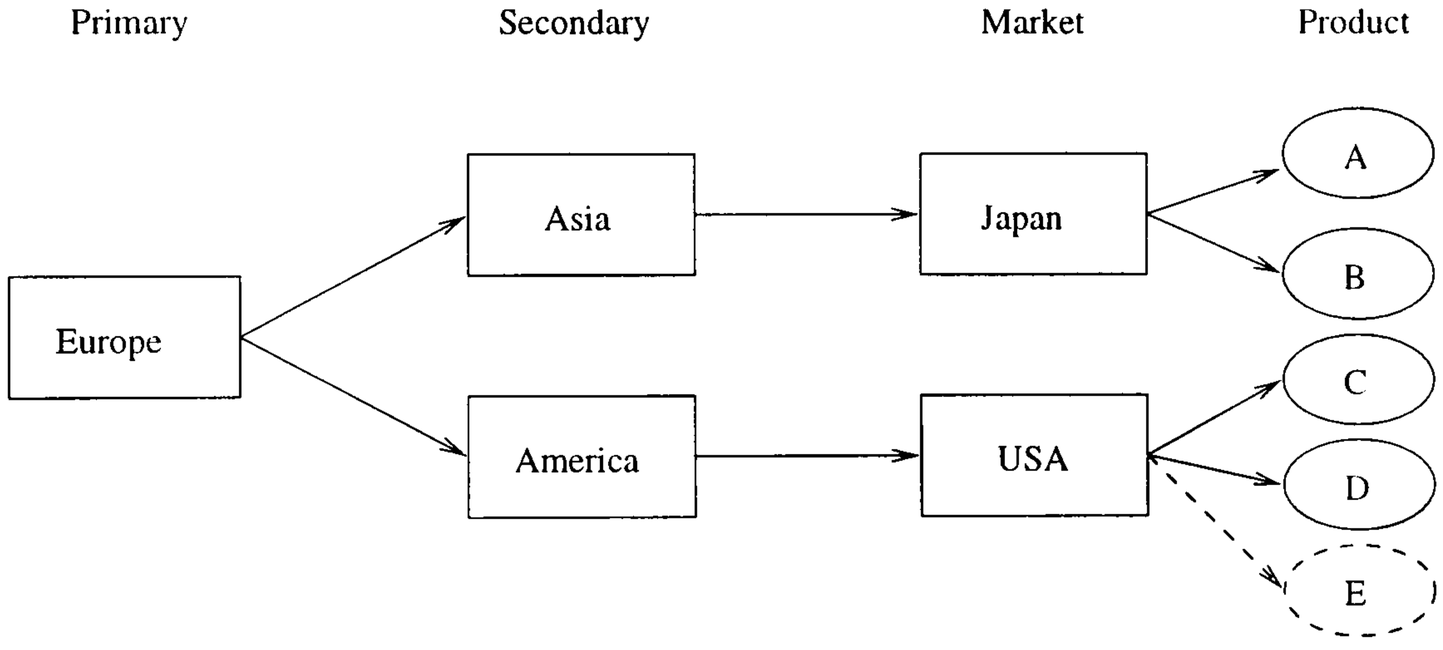


Figure 5.16: A new product introduction in an existing supply chain

cast demand for product E (see table 5.7) while simultaneously maintaining reasonable inventory levels and customer service for existing mature products in the entire supply chain. In table 5.8, the original input data for demand management together with the suggested policy for the new product E are displayed. Primary and secondary production data are as previously displayed in tables 5.3 and 5.4. Figure 5.17 displays the behaviour of the system with the new product introduced. The customer service level is low for product E due to high demand after week 50, see figure 5.17(f). The effect of the ramping demand for product E can be examined. It is obvious from the figure that the system cannot cope with the increased demand at the end of the period. However, because of

Time	Demand								
1	400	22	4600	43	8800	64	13000	85	17200
2	600	23	4800	44	9000	65	13200	86	17400
3	800	24	5000	45	9200	66	13400	87	17600
4	1000	25	5200	46	9400	67	13600	88	17800
5	1200	26	5400	47	9600	68	13800	89	18000
6	1400	27	5600	48	9800	69	14000	90	18200
7	1600	28	5800	49	10000	70	14200	91	18400
8	1800	29	6000	50	10200	71	14400	92	18600
9	2000	30	6200	51	10400	72	14600	93	18800
10	2200	31	6400	52	10600	73	14800	94	19000
11	2400	32	6600	53	10800	74	15000	95	19200
12	2600	33	6800	54	11000	75	15200	96	19400
13	2800	34	7000	55	11200	76	15400	97	19600
14	3000	35	7200	56	11400	77	15600	98	19800
15	3200	36	7400	57	11600	78	15800	99	20000
16	3400	37	7600	58	11800	79	16000	100	20200
17	3600	38	7800	59	12000	80	16200	101	20400
18	3800	39	8000	60	12200	81	16400	102	20600
19	4000	40	8200	61	12400	82	16600	103	20800
20	4200	41	8400	62	12600	83	16800	104	21000
21	4400	42	8600	63	12800	84	17000		

Table 5.7: Demand forecast for product E

	Product A	Product B	Product C	Product D	*Product E*
Safety stock (weeks)	4	4	4	4	*4*
MOQ (units)	10,000	5,000	5,000	20,000	*1.000*
Deviation from forecast (%)	25	20	10	10	*50*
Pack size (units)	12	30	30	30	*40*
Initial stock (units)	15,000	15,000	60,000	60,000	*10.000*

Table 5.8: Demand management data for new product introduction example

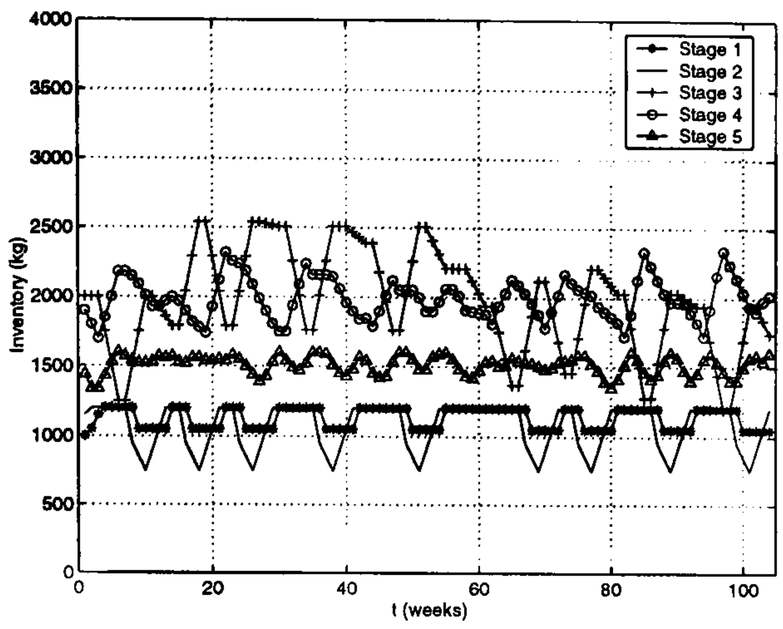
the uncertainty of the forecast it is probably wise to postpone other decisions until a later date, when more information about the success of product E has been gathered. With this scenario, the new product E will have an average inventory level of 23656 and a customer service level of 70.7%. The impact of the new product as the demand for the product increases is substantial in terms of primary material demand. As can be seen from figures 5.17(a) and 5.17(b), the primary production is more occupied than in the previous base case, albeit not as much as to affect the secondary stocks in the US. see figure 5.17(c), or to cause detrimental effects on the other parts of the supply chain. see figures 5.17(d) and 5.17(e). The current situation can be alleviated temporarily by increasing the numbers of shifts at the packaging lines at the secondary site in the US. However, if the trend of increased demand continues in the long run (if the forecasts are correct) more primary manufacturing resources are likely to be required to cope with the increase in overall product demand.

It is clear that many different scenarios can be studied with this method. The policy parameters are adjusted so that the desired result in terms of output parameters, such as inventory costs and customer service levels are balanced.

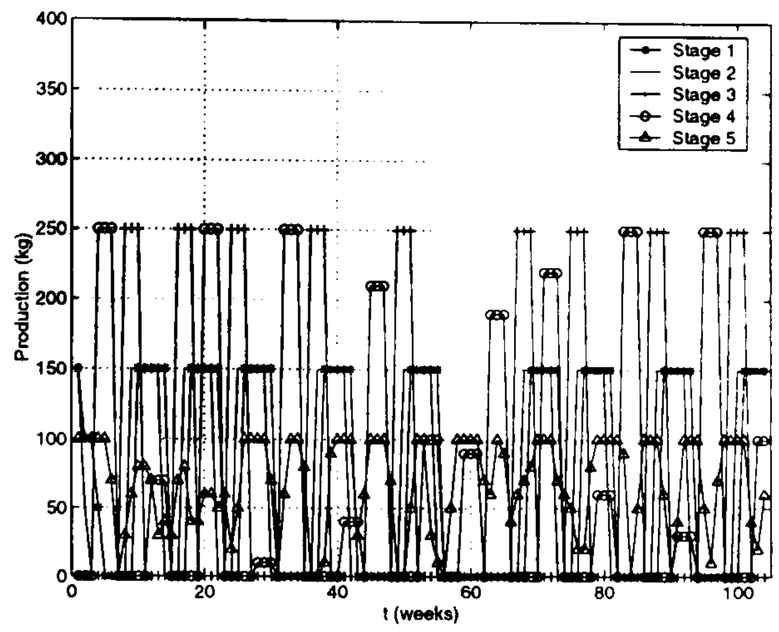
5.10 Implications of Manufacturing Shut-Downs

In primary manufacturing, there is a need to maintain efficiency at the same time as providing raw material to the downstream supply chain. Occasionally, the production paths have to be altered, major maintenance has to be performed, or new production equipment needs to be installed. However, these changes cannot always take place while maintaining full service to the downstream stages and under some circumstances a shut-down of one or several primary production stages is necessary. Because of the significant effects such a shut-down may have on the rest of the supply chain, it is sensible to plan the shut-down activities well in advance. In this section, we demonstrate the effects of shut-down activities and consider how major disruptions that could affect end customers can be avoided.

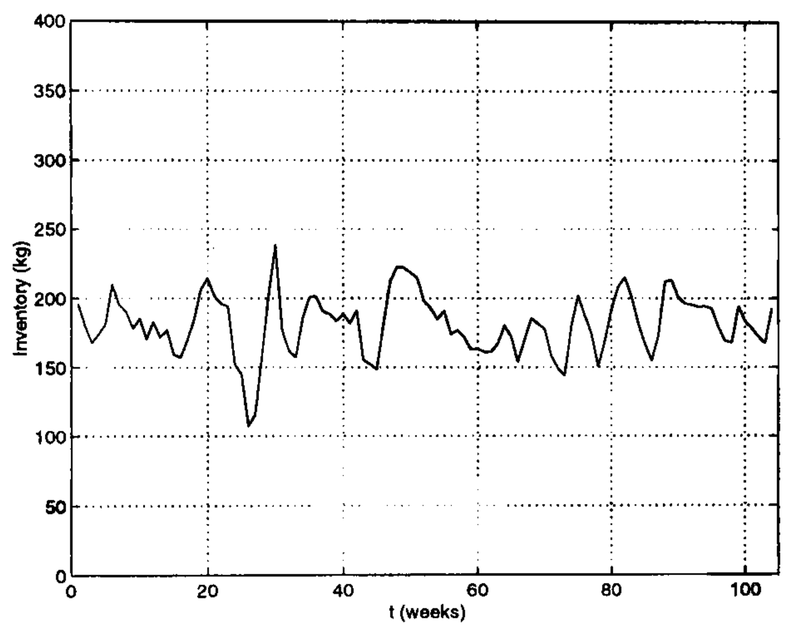
The effects of a short total primary factory shut-down are shown in figure 5.18. The shut-down is scheduled for first quarter of the second year (weeks 53-66). The inventory is approaching low values at the end of the shut-down (shown in figure 5.18(a)). However,



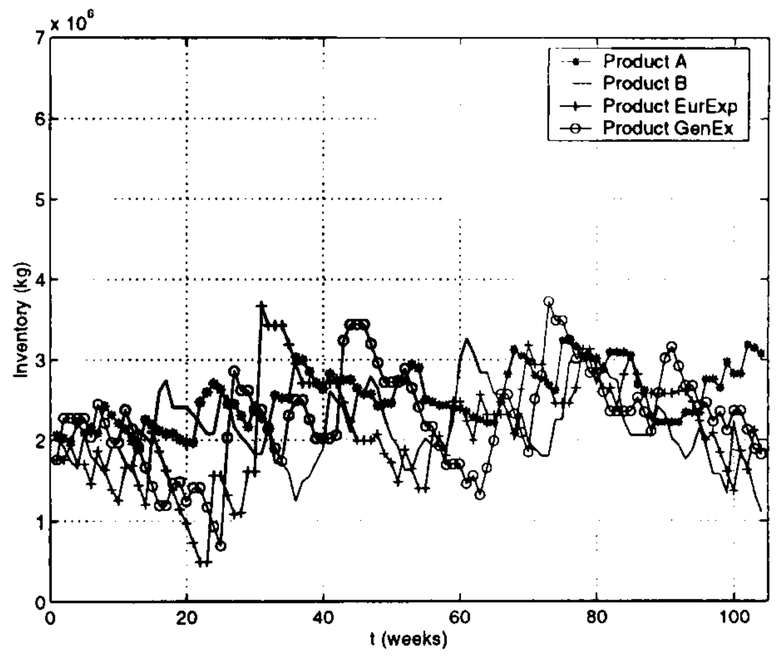
(a) Primary inventory (Europe)



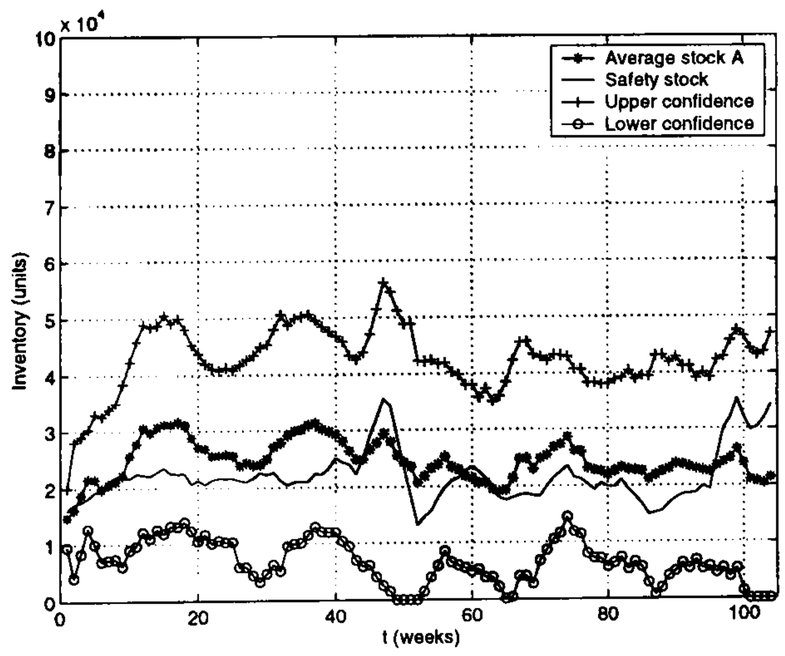
(b) Primary production profile (Europe)



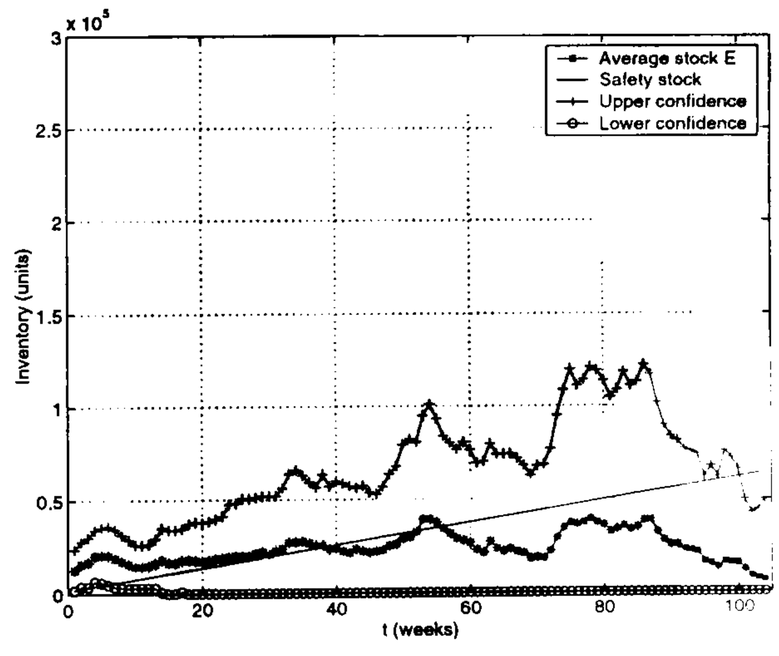
(c) Primary product inventory (US)



(d) Finished unpackaged products (Asia)



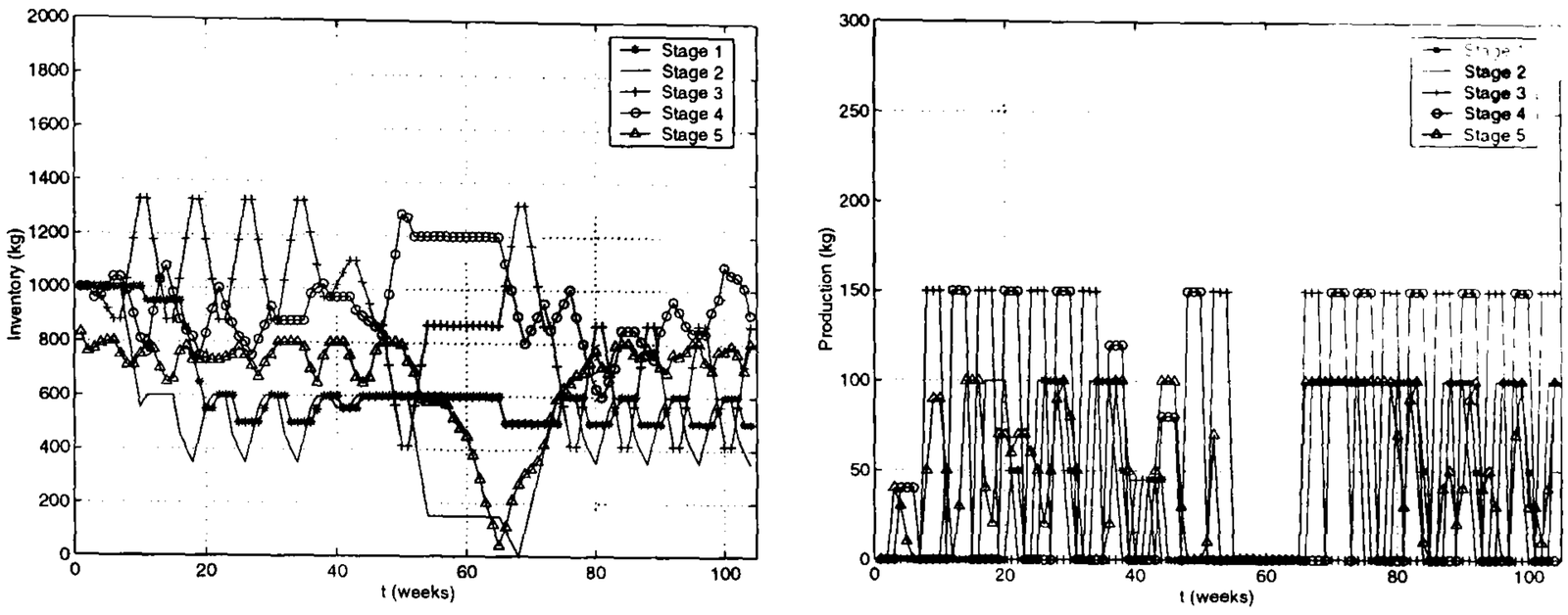
(e) Final product A inventory (Japan)



(f) Final product E inventory (US)

Figure 5.17: Effect of new product introduction on supply chain variables

it can be seen that stage 5 stocks do not reach zero and therefore the rest of the chain should not be affected.



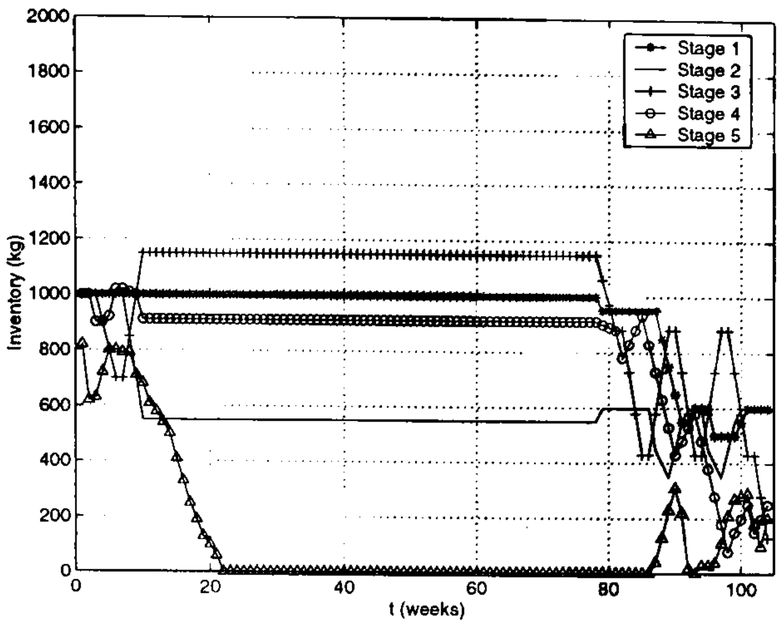
(a) Primary inventory (Europe)

(b) Primary production profile (Europe)

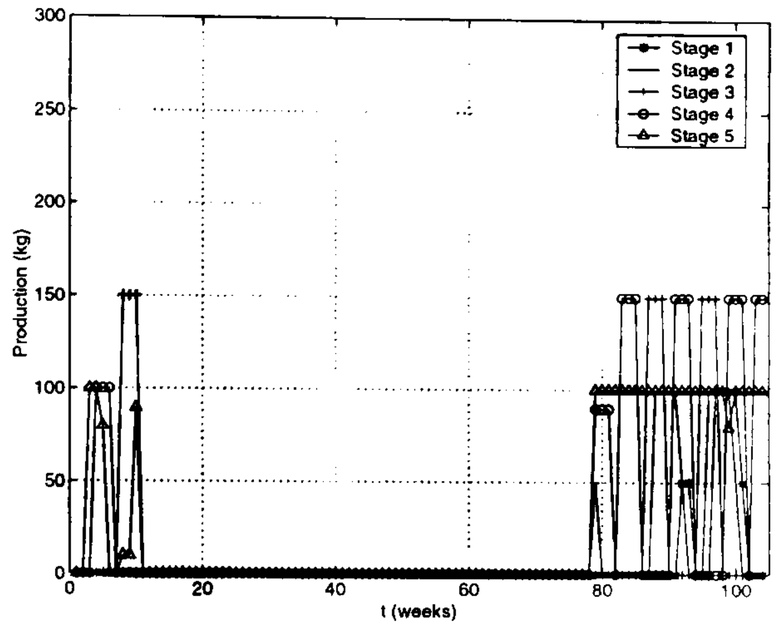
Figure 5.18: Effects of short shutdown

In figure 5.19, the effects of a long shut-down are shown in terms of overall supply chain performance. The primary manufacturing effects are shown in figures 5.19(a) and 5.19(b). It can be seen that the supply of stage 5 material runs out around week 20, ten weeks after production stopped in Europe. It takes a few additional weeks for the effects to show at the Asian plant because of pipeline stocks (due to transportation lead times). Thus, as shown in figure 5.19(c), inventory starts dropping in week 23. However, it will not reach stock out until week 40. Then, when the supply of primary product has ceased, the finished unpackaged stocks successively diminish to reach stock out between weeks 50 and 70 (in figure 5.19(d)). Finally, in figures 5.19(e) and 5.19(f), the effects of the shut-down can be seen in the finished product warehouses. In figure 5.19(e), the effects are delayed due to the postponement strategy implemented in Asia secondary production, whereas figure 5.19(f) shows that the effects of secondary stock outs (not shown) immediately transfer to the warehouse in week 40.

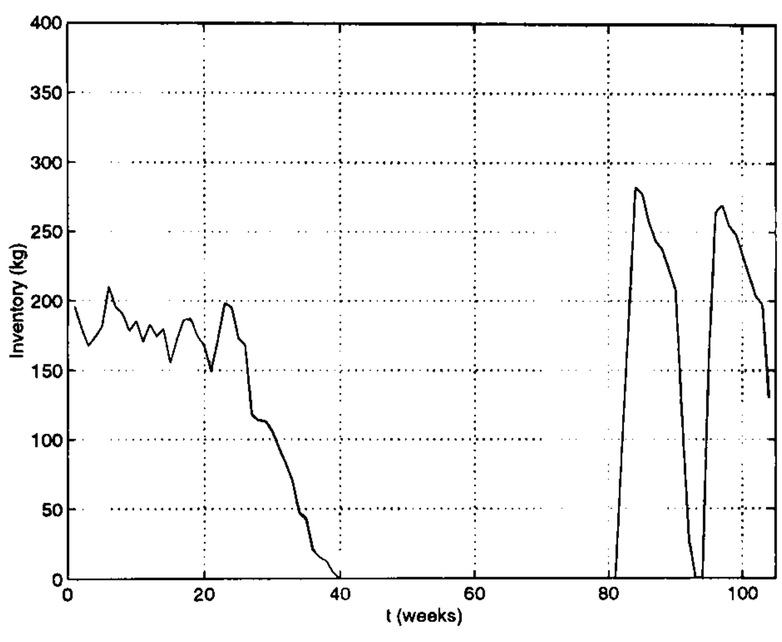
Figure 5.20 shows how the effects on down-stream supply chain echelons may be relieved by ramping up primary stocks in advance of the shut-down. By setting target stock levels for stage 5 to build up a higher level of safety stock at the time when the shut-down occurs, it is possible not to diminish the inventory entirely until production starts again (as can be seen in figure 5.20(a)). However, this obviously requires some



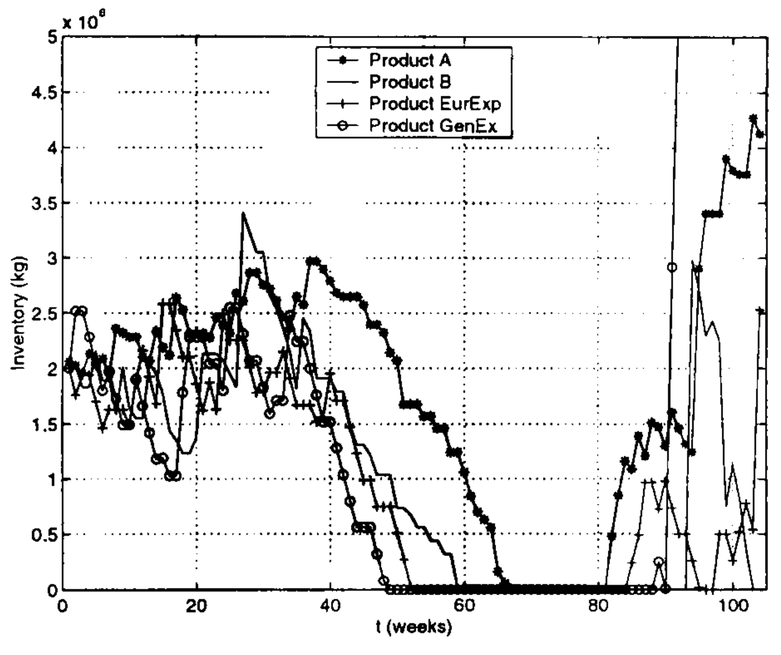
(a) Primary inventory (Europe)



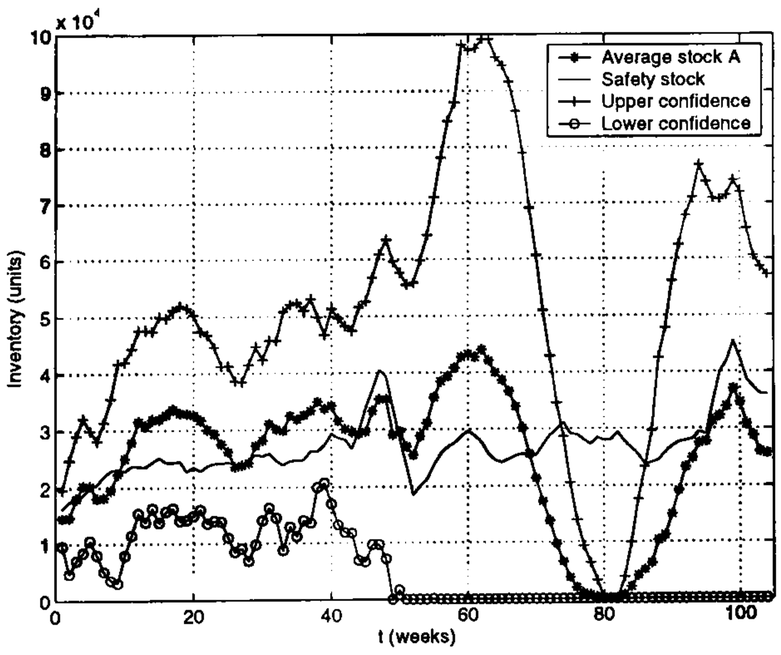
(b) Primary production profile (Europe)



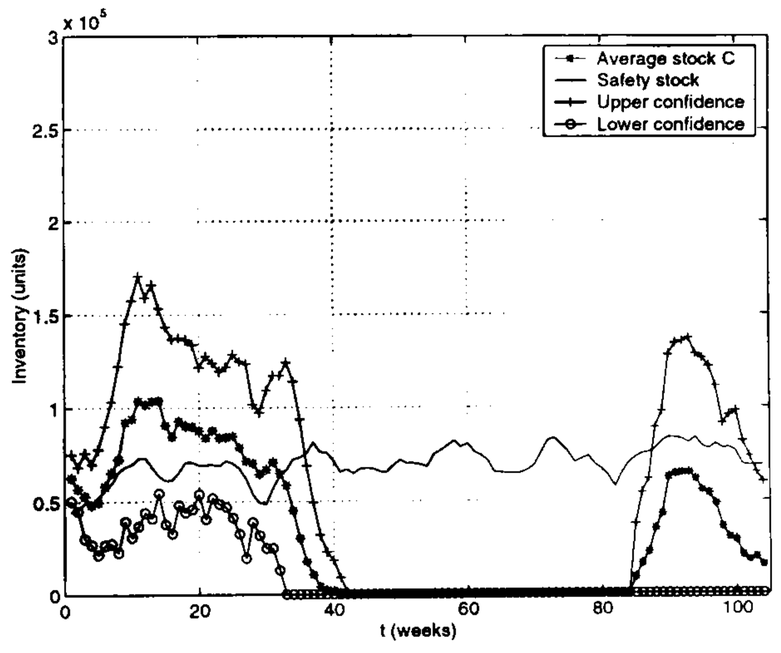
(c) Primary product inventory (Asia)



(d) Finished unpackaged products (Asia)

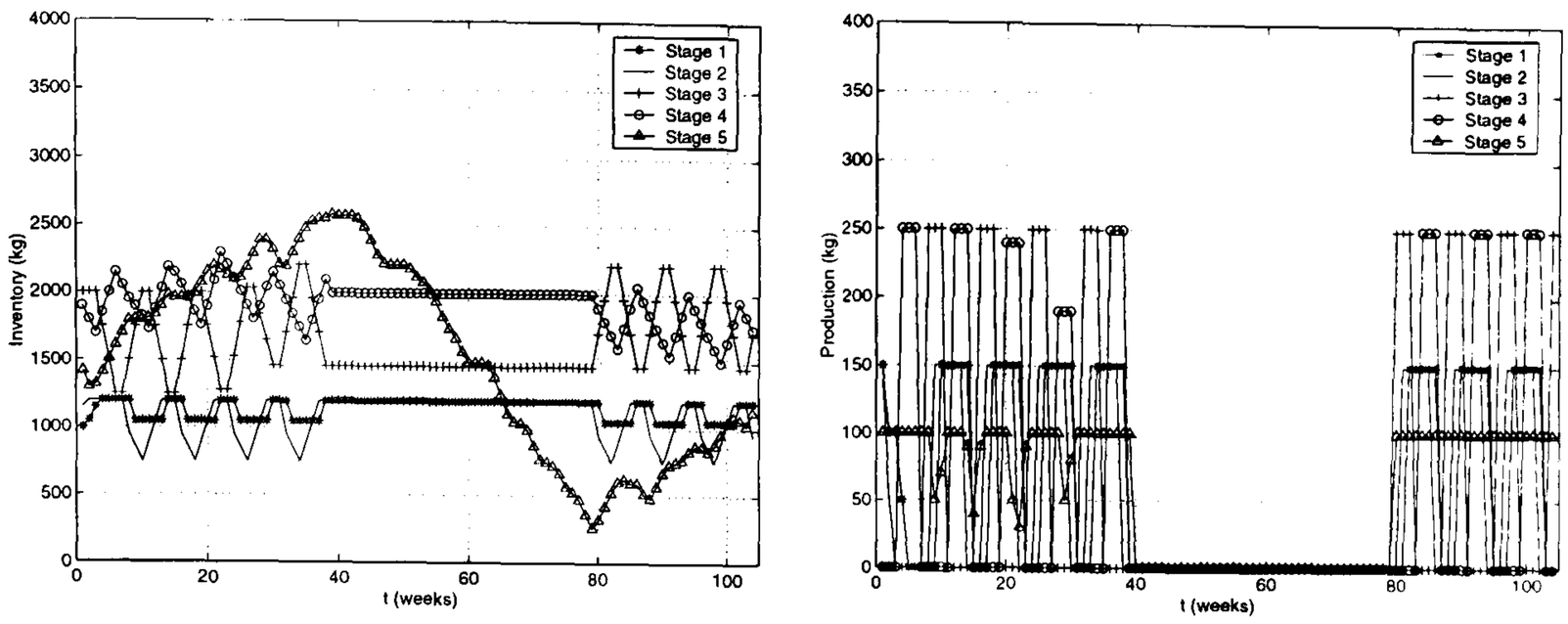


(e) Final product A inventory (Japan)



(f) Final product C inventory (US)

Figure 5.19: Major primary shut-down effect on supply chain variables



(a) Primary inventory (Europe)

(b) Primary production profile (Europe)

Figure 5.20: Ramping primary stocks before major shut-down

spare capacity at the primary site where the production will be more constrained than otherwise (see figure 5.20(b)).

5.11 Simulation Model Summary

A supply chain simulation model comprising both decision-making processes and physical processes has been developed. Specific issues relevant to each type of echelon have been modelled. The system is typically utilised to evaluate proposed modifications to current policies at all levels in the supply chain.

The aim is to take a non-invasive approach to both the physical and the organisational system. Hence the model "mimics" tools like DRP and MRP-II where these are used for decision-making and highlights how best to use the assets and systems in place.

In the next chapter, the objective is to demonstrate methods to optimise the parameters that are used to control supply chain operations. The supply chain system described is used as a case study to illustrate how this optimisation can be performed.

Chapter 6

Supply Chain Parameter Optimisation

Policy parameters are essentially the control settings that management can utilise to affect supply chain operation. Parameters should be easy to control without investments or other major infrastructure redesigns. Thus, typical parameters are safety stock levels and reorder policies and to a lesser degree production capacities and production hours. Parameter optimisation aims at finding values for the supply chain policy parameters that improve supply chain efficiency and effectiveness.

However, it is challenging to obtain simultaneous optimised solutions for complex supply chains. Conventional techniques cannot simply handle such large problem sizes that optimisation of dynamic supply chain model comprises. Hence, a decomposition approach is presented in this chapter as a means to tackle the scale and confidentiality properties of supply chain problems. The decomposition approach divides the problem into one aggregate top-level model that collects information from the local models and then optimises the local model parameters according to some given criteria. In the applications presented, the local models are typically represented by a type of simulation model similar to that presented in the previous chapter.

6.1 Decomposition and Hidden Models

In some cases, a single supply chain model cannot be built due to confidential data. The complication of information not being made public brings about a need for models that

represent private information on the single enterprise level, and where the information made public is a selected subset of the private information. The supply chain model needs features to support selection procedures for which information that can be made public and which needs to be retained by the enterprise. In this context, each entity of the

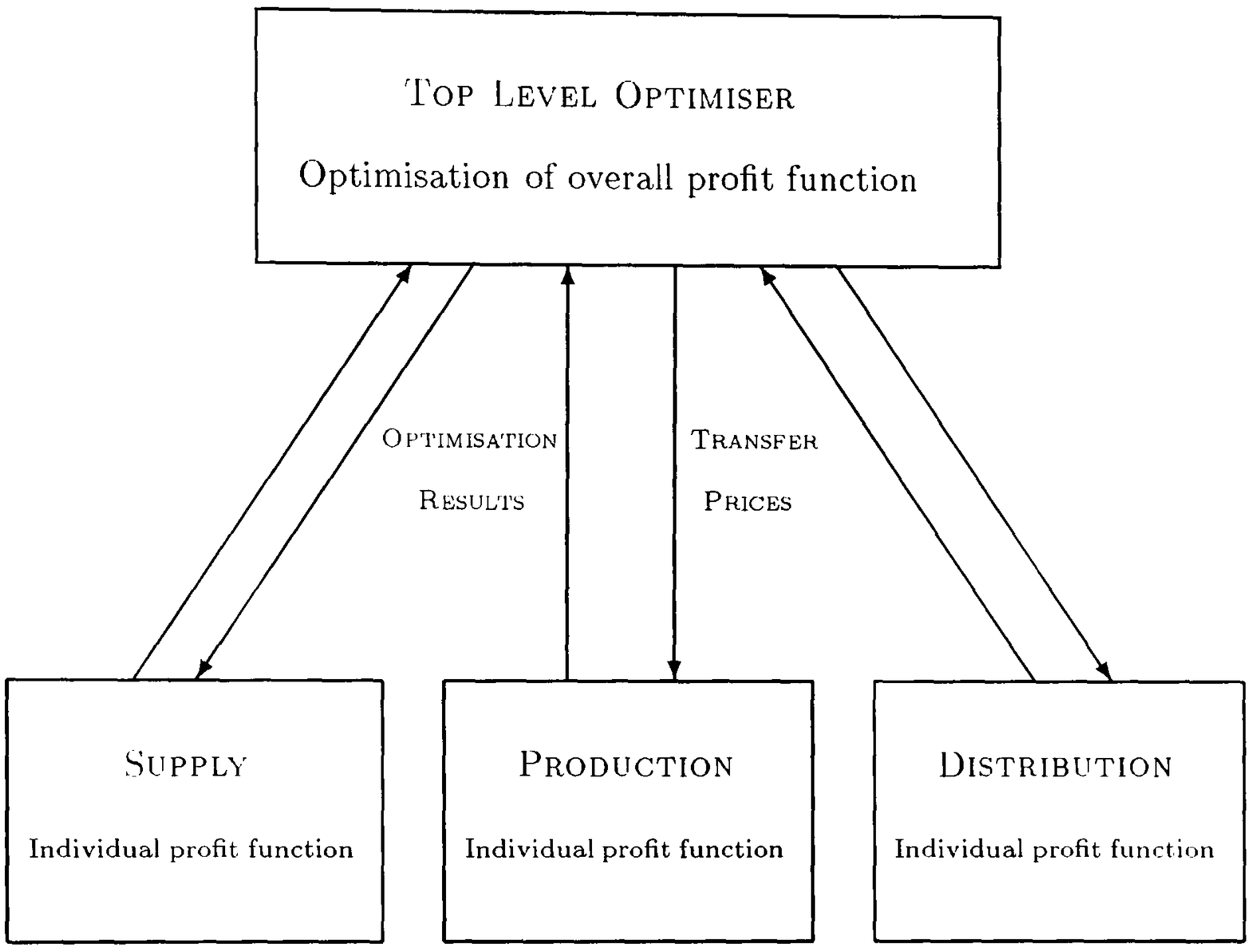


Figure 6.1: Attaining supply chain optimisation with hidden information

supply chain network is represented privately or publicly. For instance, each individual node within the supply chain network might optimise an individual profit function or other performance measure determined by management. A centralised optimiser collects and evaluates the results obtained. The top level optimiser can, based on the results determine some fixed quantities for the overall supply chain, such as the transfer prices proposed by Pfeiffer (1999), see figure 6.1. Algorithmic approaches that successively attain better values of an overall performance measure seem promising both in the light of the computational effort involved in solving the entire supply chain network problem and the requirement for private information models. The models can be based around the idea of supporting distributed knowledge in the sense that the models should be able to capture behaviour of different components in the supply chain network.

With respect to this work, the general idea is to use optimisation tools on the top

level. The optimisation procedure aims at maximising some performance measure of the overall supply chain. Parameters from the optimisation level (*e.g.* parameters such as the safety stock or minimum order quantity considered in the previous chapter) are transferred to a distributed simulation/local optimisation level, where object models of the supply chain components are utilised to simulate the dynamic, stochastic behaviour of the complex system. Thus, the simulation could provide information to the high level optimisation process in terms of constraint violation, gradients on the performance measures, and constraints in terms of the parameters provided, see figure 6.2. On the optimisation level these gradients are used to re-optimise the overall system so as to further improve the top level performance measures. The overall algorithm proceeds until no further improvement is possible.

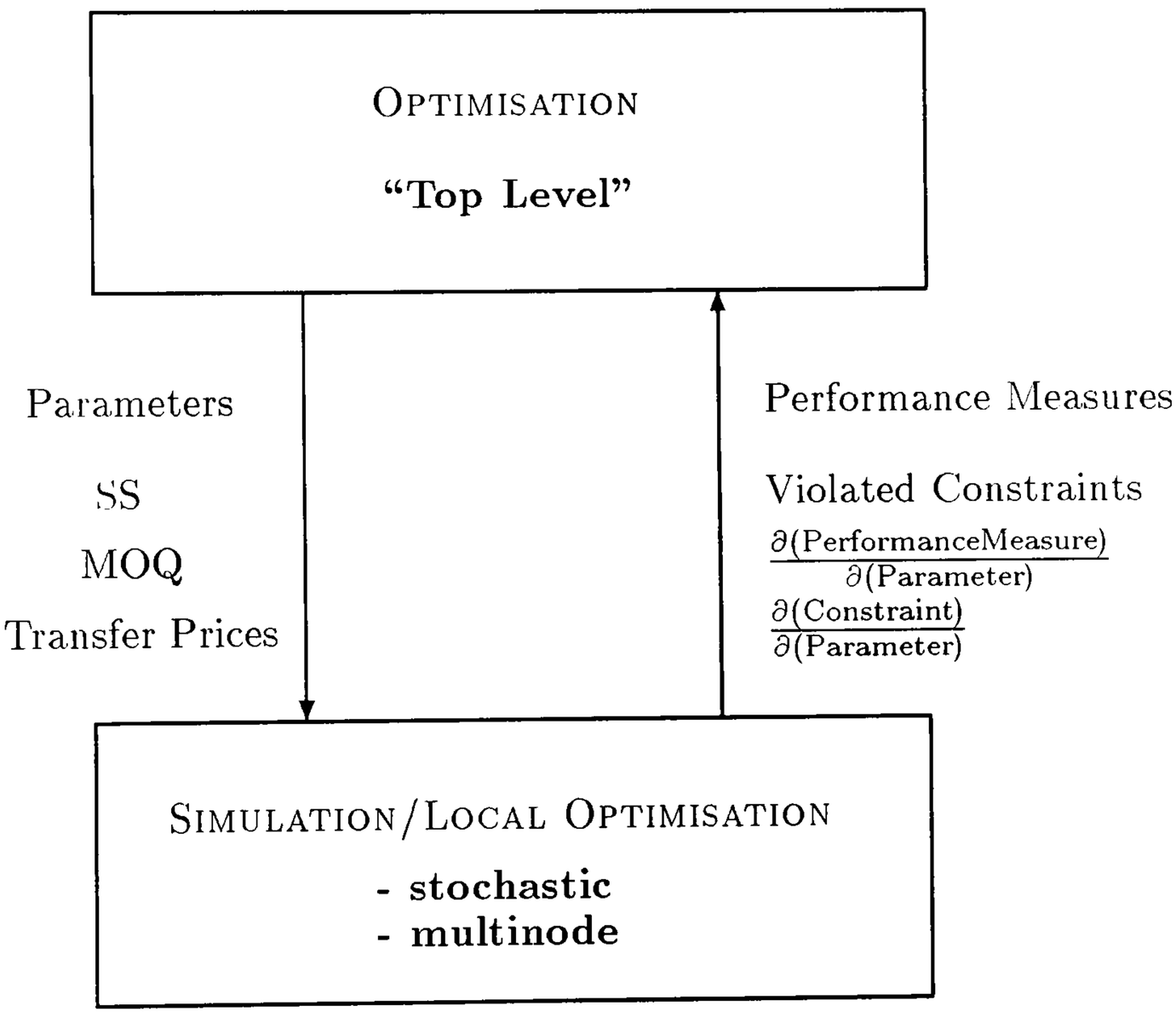


Figure 6.2: Combined optimisation and simulation supply chain approach

6.2 Application to Parameter Optimisation

In the next sections, approaches are proposed that suggest a way of tackling the richness of information in a typical industrial specialty chemical supply chain by considering some of the major control parameters and their effect on key performance indicators.

The scope is the medium to long term planning of an existing supply chain. Issues that arise in a supply chain network on this scale are forecasting and demand management decisions, production decisions and distribution decisions. Typically, demand management has to forecast the likely sales quantities for each SKU and based on the numbers obtained determine the size of finished goods safety stock. Production management then has to make decisions about the location of production, production resource availability and what SKUs should be prioritised. These actions include how to exploit regional differences to maximise returns and identify improvements that will improve supply chain performance. The distribution of material has to be considered concurrently with the production planning so as to find out how best to fulfil the requirements of each echelon.

Not only is this problem rich in information and the numbers of intra-echelon and inter-echelon actions that have to be performed to produce a consumer chemical in large companies, but also on the multi-enterprise macro scale there is evidence that the sharing of information about certain cost elements is not straightforward from a managerial perspective. Therefore, an approach that will not reveal these elements but rather focus on some optimisation properties would be an important incentive for further advances in large-scale supply chain optimisation.

Obviously, the technique must be computationally viable as well as represent and/or mimic both the physical and the business processes in the actual supply chain in question. The aim is that the dynamic, stochastic simulation model presented in the previous chapter should consider the appropriate level of abstraction as well as capturing the essential activities at each echelon in the chain and the interaction between the different levels. To represent the stochastic nature of the system, one can either utilise random distributions for each uncertain parameter or one could find a pertinent sampling sequence technique that effectively stratifies the whole parameter space. Normally, the second approach is more computationally efficient.

6.3 Supply Chain Simulation Modelling

As mentioned in the previous chapter, the key methodology followed is to select a few key products that are of vital importance to the supply chain. Then the overall supply chain model is developed at an appropriate level of abstraction with regards to the

physical system and the decision processes. The system is validated and benchmarked and sensitivity analysis of performance indicators to the policy parameters is performed. Finally, a linked simulation and optimisation procedure can be performed.

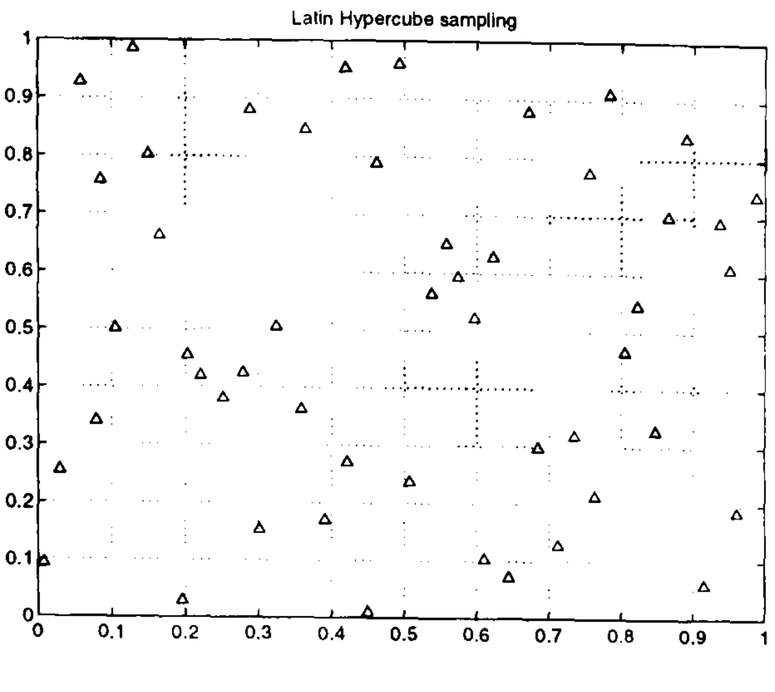
Stochastic simulation of a supply chain requires a relevant way of describing the operation conditions under which the supply chain is likely to work. Therefore, an efficient sampling technique that covers the whole input parameter space is to be chosen. The first step is to choose which uncertainties β are the main causes of supply chain disruptions. These could for example be the future forecast demands, lead times and production down-time. The values of β could be sampled via *e.g.* Monte Carlo sampling, Latin Hypercube Sampling or shifted Hammersley sequence sampling. The most common approach is Monte Carlo simulation. In this work, Latin Hypercube Sampling (LHS) is chosen because of its faster convergence properties.

In LHS, the idea is to divide the input space of each uncertain parameter β into n non-overlapping intervals of equal probability. However, each interval occurs only once in the sampling sequence. The size of each interval is determined by the distribution function for the parameter in question. The actual value of β in an interval is determined through an appropriate random function distribution (Diwekar and Kalagnanam, 1997).

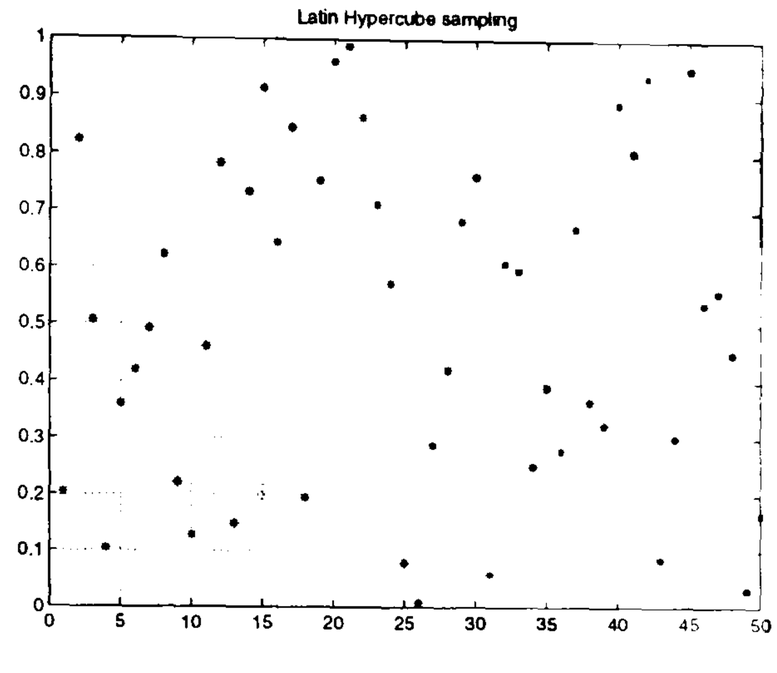
In figure 6.3, the LHS process is illustrated for both a typical uniform distribution in the interval (0, 1) and the equivalent Gauss distribution with a standard deviation of 0.1 and mean of 0.5. Especially, in figure 6.3(b), it can be seen that each interval of 0.1 (*e.g.* (0.2, 0.3)) contains 5 values throughout the sampling horizon. These values are randomised in the particular interval of 0.02.

The idea proposed in this work is to use an extensive simulation approach to account for the large uncertainties present in a supply chain and then to optimise some key parameters with respect to an overall objective and requirements of desired supply chain performance in terms of service fulfillment. However, to obtain statistically significant results of the stochastic simulation model, the simulation is repeated over a number of runs with samples obtained from the LHS process. In general terms, the objective is to choose values for a number of optimisation parameters, θ, subject to some uncertain parameters, β, so as to optimise a number of desired state variables, x.

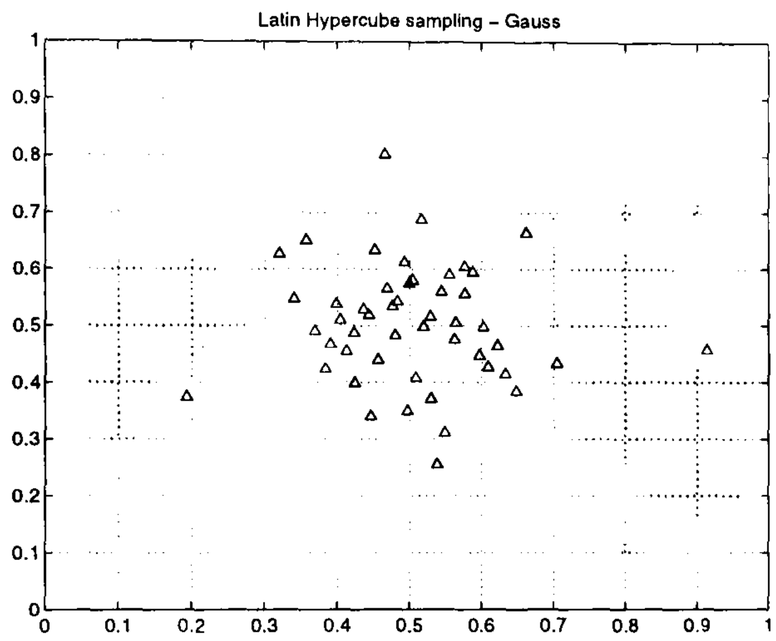
The overall problem can formally be stated as follows:



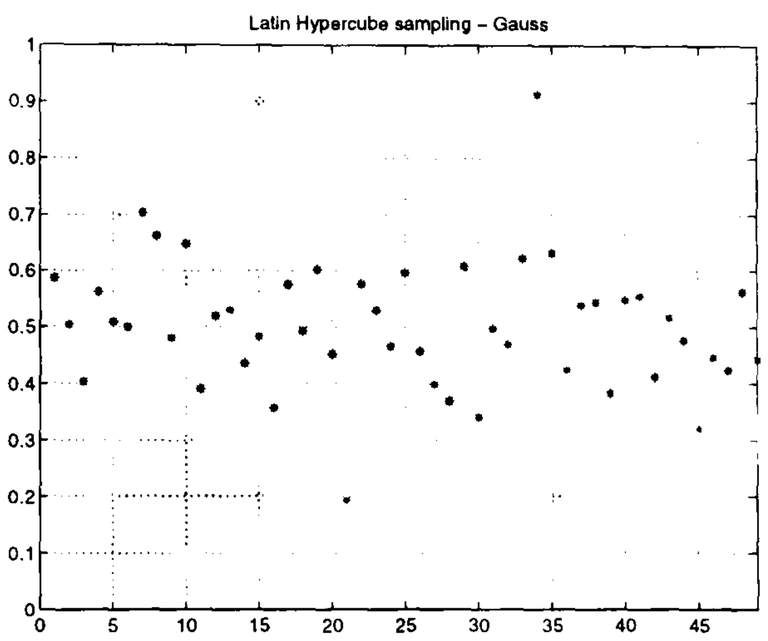
(a) Uniform two-dimensional LHS



(b) Uniform LHS as successively chosen



(c) Gauss two-dimensional LHS



(d) Gauss LHS as successively chosen

Figure 6.3: Example of Latin Hypercube sampling (50 points $\in$ (0, 1))

Given:

- time horizon,
- volumes and value of products,
- forecast demand,
- historical forecast accuracy,
- order quantities and order increments,
- primary production data (such as campaign lengths, batch sizes, dynamic target stock levels),

- secondary production data (cycle times, product interactions, batch sizes, bottlenecks, maintenance etc.)
- initial inventories,
- safety stock policies,
- transportation data,
- minimum acceptable requirements in terms of key performance indicators.

determine:

- policies parameters such as order sizes and safety stock levels

so as to:

- optimise a supply chain performance criterion considering various trade-offs of supply chain wide low average inventory levels and high customer service levels.

In this chapter, three alternative approaches for finding optimal supply chain policy parameters are described. The aim is to provide policy parameter modifications to decision-makers so as to minimise cost while maintaining required levels of service etc. A simple reorder point warehouse model and an industrial scale model are considered to evaluate the performance of these approaches. In general industrial scale examples, the choice of such parameters can be quite arbitrary. Here, the parameters investigated are dynamic safety stock levels and minimum order quantities. The dynamic repetitive three-stage supply chain simulation model is fed with samples from a random distribution utilising Latin Hypercube Sampling to cover the uncertain parameter space in terms of forecast demand.

The first approach presented is an MILP model in which the global optimal solution for the policy parameters is found. However, such a model cannot efficiently be applied to full-scale supply chain applications due to excessive computational requirements.

Secondly, a simulated annealing approach is described. For each stochastic modification of the policy parameters, a simulation level function evaluation is performed, thus enabling an assessment of how well the new parameters settings perform in terms of the local objective. The simulated annealing approach provides new modifications until a stopping criterion is enforced.

The final approach presented is a gradient-based method. Here, a higher level optimiser collects the results in terms of objective values and gradients from the simulation level. The gradients are obtained by introducing small perturbations to the current levels of the policy parameters in the simulation model. Hereby, gradients can be evaluated and passed on to the optimisation level. The policy parameters are divided into a constraining set and a cost set, where the cost parameters are optimised. The values of policy parameters after optimisation are then taken as the starting point of a new simulation. The values obtained are added to the optimiser data and new optimised parameter values are determined.

The aim of the motivating example presented next is to determine which approach could be used most efficiently on problems of a larger scale in terms of computational effort, and to evaluate the probability of finding good results.

6.4 Motivating Example

A small single-product inventory management example[1] is studied based on a reorder point (ROP) procedure, *i.e.* decision parameters P and Q (P is the reorder point and Q is the reorder quantity) and output minimisation variable I (the cumulative inventory) and constraining variable SL (the service level). When the inventory of the warehouse drops to a level lower than P an order of size Q is triggered. An order can only be scheduled if there is no order already due to arrive within the lead time. Given are the (randomly generated) daily demands, D_t, the time horizon length, initial inventory position, I_0, the reorder lead time, L, and an initial guess for the decision parameters ((P_0, Q_0): used in the simulated annealing and gradient-based approaches). The aim then is to determine the decision parameters so as to minimise the cumulative inventory over the time horizon subject to the service level constraint.

Because the motivating example assumes pre-specified demands throughout the time horizon, there is one optimal solution of the policy parameters ($P^\star$, $Q^\star$). However, due to the nature of reorder point procedure, the optimal solution is quite sensitive to small changes in the actual demands. Hence, for practical purposes it is more important to find reasonable values for the parameters rather than the one optimal solution. The aim of the simulated annealing and the gradient-based approach presented is to provide

[1] This example is a typical simple inventory problem. For more details see Silver, E. A. (1998). *Inventory management and production planning and scheduling*. Wiley. New York.

solutions close to the optimal solution in a short amount of computational time.

6.4.1 Simple Simulation Inventory Model

In the stochastic simulation model utilised in the simulated annealing and gradient-based approaches below, the following equations are used to determine the performance. Equation 6.1 defines the inventory levels, equation 6.2 defines the sales at t, equation 6.3 defines the overall service level and equation 6.4 defines the cumulative inventory. In the mass balance, F_t is the planned size of orders to arrive at time t. Also, the following logic is used: If an order has not already been triggered and if $I_{t^\star} < P$, then $F_{t=t^\star+L} = Q$, which is to say that if the inventory at time, $t^\star$, is less than the reorder point, P, then an order of the size of the reorder quantity, Q, is scheduled to arrive after the lead time, L, has expired.

$$I_t = I_{t-1} - S_t + F_t \qquad \forall\, t \tag{6.1}$$

$$S_t = \min(I_{t-1}, D_t) \qquad \forall\, t \tag{6.2}$$

$$SL = \frac{\sum_t S_t}{\sum_t D_t} \tag{6.3}$$

$$I = \sum_t I_t \tag{6.4}$$

The service level (SL), which is the total amount of product sold divided by the total amount of product demanded (equation 6.3), has to be fulfilled according to pre-specified requirements. While the main objective of the model is to minimise the cumulative inventory, the SL is required to be 100%. For each time period, the material balance of the inventory is evaluated considering that occasionally the inventory is not sufficient to fulfil the product demand.

6.4.2 Test Cases

Ten different demand scenario cases are created which comprise 100 time periods (*e.g.* days) and which are thoroughly evaluated to test the efficiency of the approaches. The test cases comprising the computational experiments are created to test a range of problems and to identify the number of iterations required to find acceptable solutions. The demands are created by considering a probability of whether the demand does or does not exist for every time period. Furthermore, the demands are set to be within pre-specified lower and upper bounds. In table 6.1, the input data used to create the test cases are

shown. The demand sequences are generated from uniform distributions in which D_u and D_l are the upper and lower bounds. γ is the uniform probability of an order occurring at any given time period, *i.e.*, if $\gamma = 0.7$, there is a 70% chance of an order occurring in any given time period. Finally, the initial inventory at time 0, I_0, is also shown. The replenishment lead time, L, is specified to be 4 time periods in all test cases. To illustrate

	γ	D_u	D_l	I_0
Case 1	0.7	100	50	400
Case 2	0.8	80	40	400
Case 3	0.5	500	100	800
Case 4	0.6	1200	700	6000
Case 5	0.4	2300	400	4500
Case 6	0.75	10000	2500	50000
Case 7	0.3	15000	7000	27000
Case 8	0.35	700	500	2000
Case 9	0.6	800	100	2100
Case 10	0.9	1200	400	5000

Table 6.1: Test case input data

a typical demand pattern, the generated demands for case 1 for time periods 1–100 are shown in table 6.2.

6.5 Approaches

6.5.1 MILP Inventory Management Model

The model presented in this section is an MILP formulation which replicates the considered inventory management problem. Whereas MILP approaches ensure optimality, they tend to be relatively slow and make use of extensive computational resources for certain classes of problems. However, it makes sense to utilise the MILP results as a reference and benchmark both in terms of solution quality and computational performance. Comparisons and trade-offs can then be made between the proposed approaches. The following terminology is used:

Time	Demand								
1	99.8	21	94.28	41	94.18	61	0	81	0
2	0	22	0	42	0	62	63.51	82	78.23
3	0	23	0	43	69.34	63	60.43	83	96.63
4	93.41	24	57.79	44	71.38	64	75.21	84	0
5	0	25	52.6	45	0	65	59.36	85	80.06
6	82.9	26	85.38	46	0	66	75.45	86	58.52
7	0	27	50.47	47	78.62	67	82.79	87	0
8	88.84	28	70.33	48	0	68	0	88	59.78
9	67.04	29	53.66	49	99.22	69	63.24	89	92.13
10	57.85	30	0	50	0	70	0	90	91.06
11	0	31	78.44	51	92.38	71	72.46	91	66.99
12	76.36	32	98.43	52	90.15	72	75.46	92	59.13
13	73.6	33	0	53	0	73	0	93	72.69
14	0	34	50.68	54	72.97	74	80.68	94	90.75
15	0	35	0	55	77	75	95.87	95	0
16	0	36	71.89	56	61.74	76	0	96	60.24
17	0	37	0	57	89.51	77	85.48	97	94.95
18	62.81	38	65.47	58	98.55	78	65.49	98	0
19	59.11	39	0	59	63.8	79	97.35	99	85
20	74.5	40	86.03	60	81.43	80	91.73	100	56.02

Table 6.2: Demand points for case 1

Index

t: Time periods

Parameters

P^U: Maximum value of P

Q^U: Maximum value of Q

I_0: Initial stock level

L: Lead time duration in time periods

I^U: Maximum stock level

D_t: Demand quantity of product at time period t

Binary Variables

X_t: $= 1$ if order raised at time period t; 0 otherwise

W_t: $= 1$ if stock is below P at time period t; 0 otherwise

Continuous Variables

P: Reorder point

Q: Reorder quantity

F_t: Amount received at time period t

Objective function

The objective is to minimise the cumulative inventory over the time horizon subject the constraints presented below.

$$\min \sum_t I_t \tag{6.5}$$

Material balance

The material balance is stated in equation 6.6. Before the lead time has expired, no amount of product can be expected to be received, see equation 6.7.

$$I_t = I_{t-1} - D_t + F_t \qquad \forall\, t \tag{6.6}$$

$$F_t = 0 \qquad t = 1, \ldots, L \tag{6.7}$$

Logic constraints

Constraint 6.8 states that an order is raised at t if stocks are below P ($W_t = 1$) and no orders in the period $t-1, t-2, \ldots, t-L+1$. Constraints 6.9 and 6.10 ensure that an order is not raised if an order is already raised during the lead time or the stock is above P. W is determined through constraints 6.11 and 6.12. If I_t is less than P, W is 1.

$$X_t \geq W_t - \sum_{\theta=1}^{L-1} X_{t-\theta} \qquad \forall\, t \qquad (6.8)$$

$$X_t \leq W_t \qquad \forall\, t \qquad (6.9)$$

$$X_t \leq 1 - X_{t-\theta} \qquad \forall\, t, \theta = 1, \ldots, L-1 \qquad (6.10)$$

$$W_t \geq \frac{P - I_t}{I^U} \qquad \forall\, t \qquad (6.11)$$

$$1 - W_t \geq \frac{I_t - P}{I^U} \qquad \forall\, t \qquad (6.12)$$

Order quantity definitions

In constraints 6.13 to 6.15, the size and timing of orders are determined. If $X_{t-L} = 1$ (*i.e.* an order is raised), then F_t is set to the reorder quantity, Q. If on the other hand, $X_{t-L} = 0$, then F_t is set to 0.

$$F_t \geq Q - Q^U(1 - X_{t-L}) \qquad \forall\, t \qquad (6.13)$$

$$F_t \leq Q \qquad \forall\, t \qquad (6.14)$$

$$F_t \leq Q^U X_{t-L} \qquad \forall\, t \qquad (6.15)$$

6.5.2 Simulated Annealing Approach

As can be seen below in the test results section, the MILP approach provides global minima but it requires a long time to reach the optimal solutions (see table 6.3) for the motivating example problems. This is clearly unacceptable since industrial supply chain models are much larger than the motivating example. Therefore, approaches are applied to the motivating example to try to reduce the computational cost. First, a simulated annealing approach (Dowsland, 1995) is presented in figure 6.4.

1. **Initialisation.** In the initialisation step of the simulated annealing approach, the objective function value, f_0, is set to a high value and the initial values for the parameters, s_0, are chosen. Furthermore, the initial temperature, T_{ita} [2], and the

[2] T, N, and α are parameters used in the simulated annealing approach.

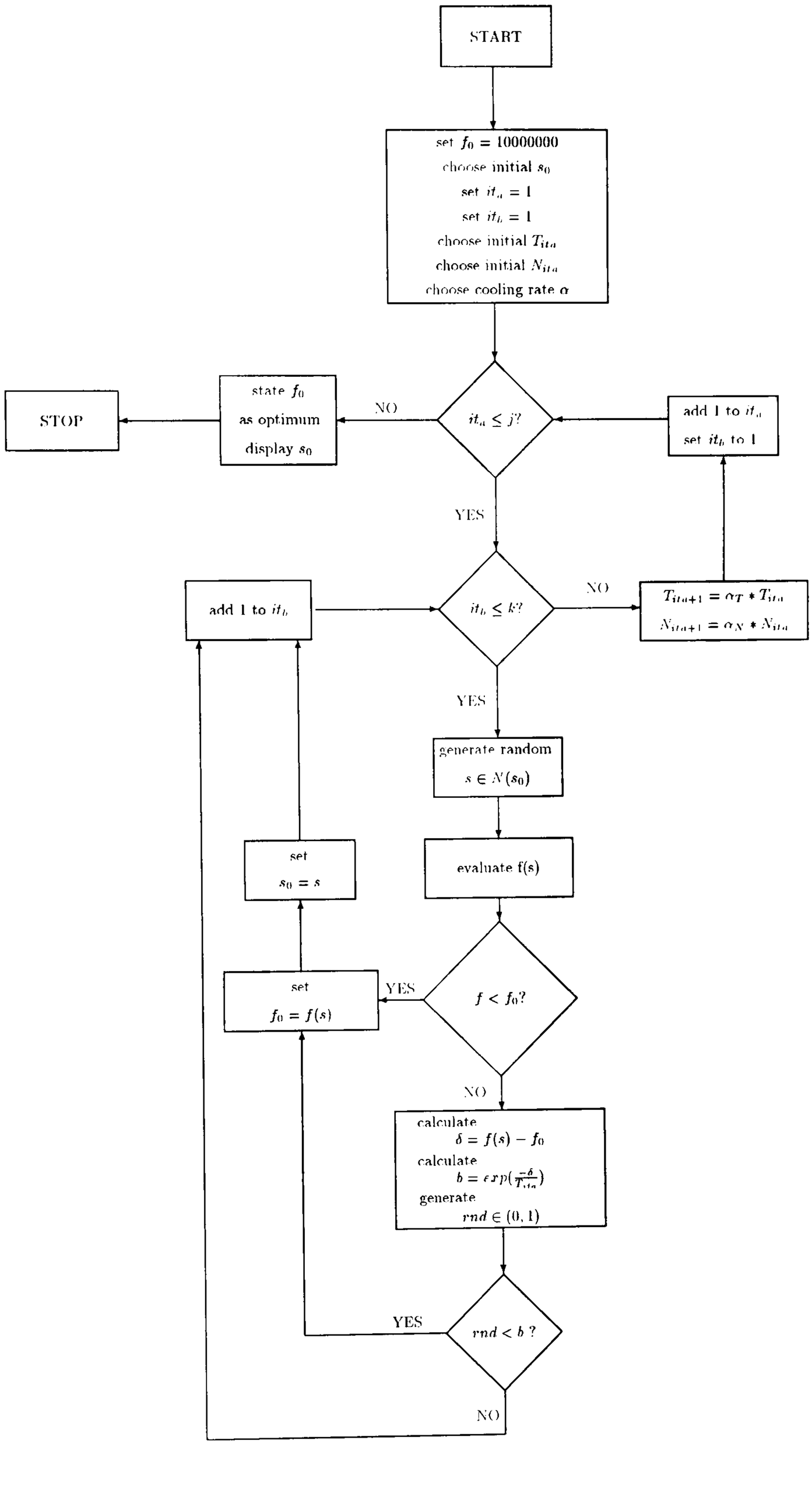


Figure 6.4: Simulated annealing algorithm

initial neighbourhood, N_{ita}, as well as the cooling rate are specified. The iteration counters, it_a $(1,\ldots,j)$ and it_b $(1,\ldots,k)$, are set to 1.

2. **Outer loop.** In the outer loop, temperature and neighbourhood are updated according to the chosen cooling mechanism and the outer iteration counter, it_a, is incremented and it_b is set to 1.

3. **Inner loop.** In the inner loop, a random value of the control parameters, s, is obtained from a random number distribution in the neighbourhood of s_0. The objective function value, f, for this s is evaluated in the function call to the simulation model. Then, the value f is compared to the previously accepted value f_0. If lower, then s and f are set to become new s_0 and f_0 and counter it_b is incremented. If higher, the difference, $\delta = f - f_0$, is calculated and used to evaluate the stochastic acceptance criterion. The criterion depends on the current value of the "temperature" parameter (T) and the magnitude of the increase of the objective obtained in the local simulation function evaluation.

The acceptance criterion is $rnd < exp(\frac{-\delta}{T})$, where rnd is a random number between 0 and 1. The exponential function expresses the probability of the acceptance of an uphill move. The function could be written as $f = exp(-a)$ with $a = \frac{\delta}{T}$. As a goes from 0 to positive infinity the acceptance criterion function goes from 1 to 0. At the very start of the process the temperature is high, hence a is low and uphill moves are likely to be accepted. As the process progresses, the temperature is lowered and acceptance of uphill moves decreases, and higher values of the objective function become successively more unlikely until they are essentially forbidden. Once a new point has been accepted (either by having a smaller objective function value or if the uphill move is accepted), the new point is stored as new values of s_0 and f_0. When the desired number of iterations has been performed $(j \times k)$, the algorithm stops.

The simulated annealing approach evaluates the objective function at an arbitrary initial point and then compares the objective function at a point within the neighbourhood of the last accepted point. The neighbourhood is shrunk by a factor α_N in the outer loop. This approach is similar to the random descent approach, in that the neighbourhood is explored randomly and only downhill moves are accepted. Simulated annealing differs in accepting uphill moves if the acceptance criterion is met. Compared to a random descent

approach, the advantage of simulated annealing is clearly that it provides the chance of escaping a local minimum through the occasional uphill moves and the chances of finding solutions close to the global minimum are increased.

In terms of implementation, a constraint stating a required service level, SL_r, of 100% is imposed by a penalty function that is added to the simulation model. The model imposes a penalty, P_i, for every iteration which are less than 100% in terms of service level, SL_i, by multiplying the difference in SL_i from the requirement by a very large number (big-M), thus making it "expensive" to have an SL other than the required level.

$$P_i = M \times (SL_r - SL_i) \qquad \forall\ i\text{: } SL_i \leq SL_r \tag{6.16}$$

The chosen cooling properties are linear:

$$T_i = \alpha_T\ T_{i-1} \tag{6.17}$$

where α_T = constant.

6.5.3 Gradient-Based Supply Chain Optimisation

In this section, a gradient-based approach is proposed as follows: The state variables x can belong to a constraining set, C, or a cost set, E, or both. The idea being that C contains properties that need to be controlled (*e.g.* customer service levels) and E comprises a number of variables to optimise (*e.g.* supply chain inventory levels - *i.e.* inventory costs, or the number of orders issued in the system - *i.e.* transaction costs).

The general problem formulation is to optimise an objective function $E(\theta, x, \beta)$ subject to $C(\theta, x, \beta) \geq 0$. θ_0, the initial guess of optimisation parameters, is chosen arbitrarily or according to current values in operation. Then a number of simulations are performed over the distributions of the uncertain parameters β. Thereby, estimates of E and C are obtained for likely future scenarios. By introducing a small perturbation to θ_k, $\delta\theta$, and re-performing the simulations, the derivatives $\frac{\partial E}{\partial \theta_k}$ and $\frac{\partial C}{\partial \theta_k}$ can be obtained. Thus all data needed to perform the optimisations are obtained. The embedded optimisation problem then takes the following form which essentially is a linearisation of the overall problem in the space of θ.

$$\min \; E \tag{6.18}$$

s.t.

$$E \geq E(\theta_k) + \frac{\partial E}{\partial \theta_k}(\theta - \theta_k) \qquad \forall\, k \tag{6.19}$$

$$C(\theta_k) + \frac{\partial C}{\partial \theta_k}(\theta - \theta_k) \geq 0 \qquad \forall\, k \tag{6.20}$$

θ_k contains information about previously values of the parameters and $\frac{\partial C}{\partial \theta_k}$ and $\frac{\partial E}{\partial \theta_k}$ contains information with regards to the previously obtained gradients. Re-performing the simulations with the new values of the optimisation parameters and introducing new perturbations (obtaining more gradients) results in new parameters for the optimisation. The process is illustrated in figure 6.5.

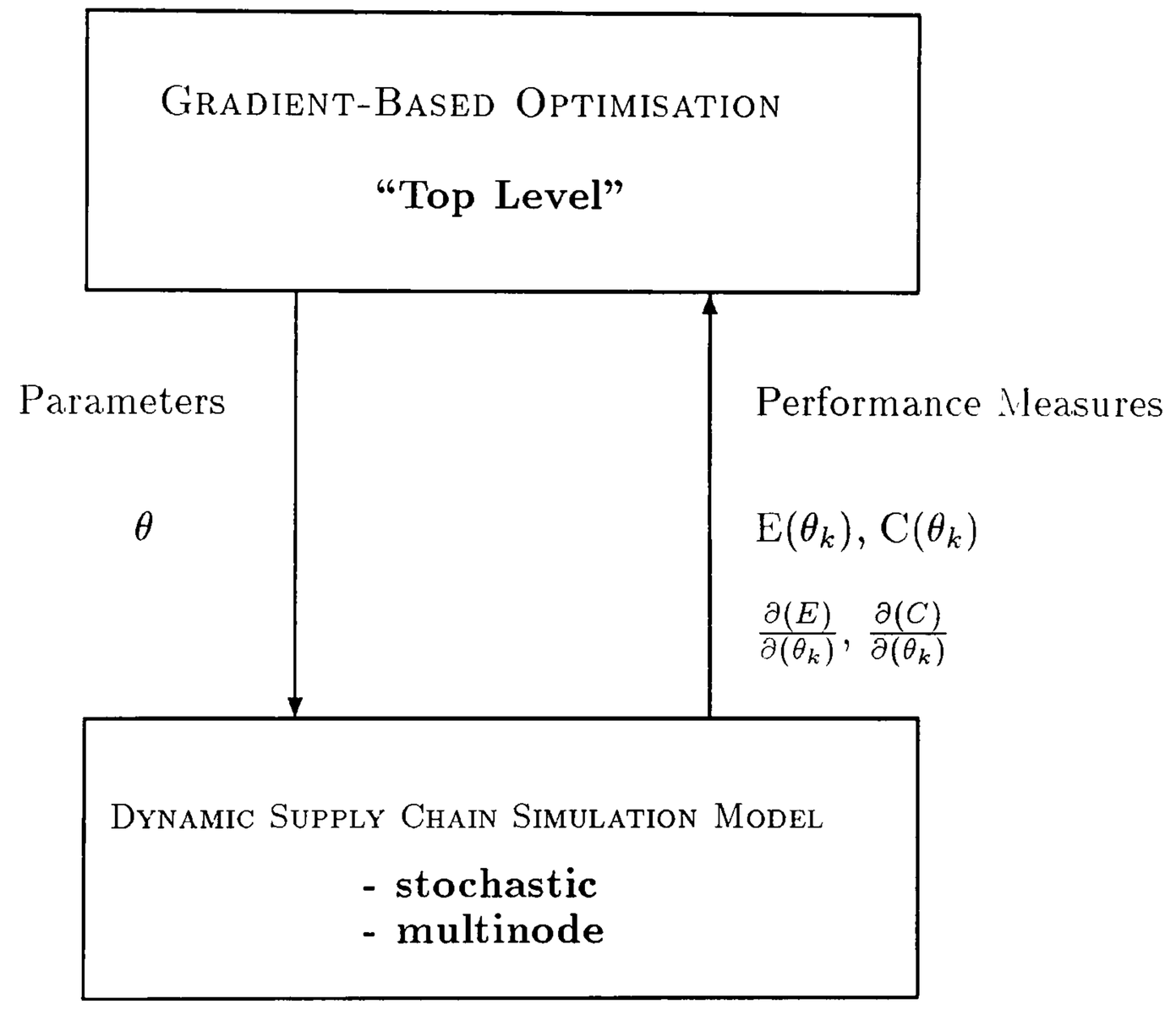


Figure 6.5: A gradient-based supply chain optimisation procedure

Simple Gradient-Based Model

The application of the gradient-based optimisation procedure described above is given next with respect to the motivating reorder point example. The cost set, E, is replaced by the inventory objective variable, I, and the constraining set, C, is replaced by the service

level, SL. The parameters, θ, comprise the array (P, Q). Furthermore, the parameters P and Q are perturbed by a small amount in the simulations so that the gradients in terms of the output variables, SL and I, can be obtained *e.g.* $\frac{\partial SL}{\partial P} = \frac{\delta SL - SL}{\delta P - P}$. The following notation is used:

Parameters

SL^L:	Minimum acceptable service level
$I(Q', P')$:	Value of I at point of simulation
$SL(Q', P')$:	Value of SL at point of simulation
Q':	Value of Q at point of simulation
P':	Value of P at point of simulation
$\frac{\partial I}{\partial Q'}$:	Value of partial derivative of I w.r.t. Q
$\frac{\partial I}{\partial P'}$:	Value of partial derivative of I w.r.t. P
$\frac{\partial SL}{\partial Q'}$:	Value of partial derivative of SL w.r.t. Q
$\frac{\partial SL}{\partial P'}$:	Value of partial derivative of SL w.r.t. P

Continuous Variables

I:	Cumulative horizon inventory
SL:	Service level
P:	Reorder point
Q:	Reorder quantity

The implementation of the embedded optimisation problem for the reorder point procedure is given in equations 6.21 to 6.23.

$$\min \; I \tag{6.21}$$

s.t.

$$I \geq \overline{I_k}(Q', P') + \frac{\partial I}{\partial Q'_k}(Q - Q'_k) + \frac{\partial I}{\partial P'_k}(P - P'_k) \qquad \forall\, k \tag{6.22}$$

$$SL_k(Q', P') + \frac{\partial SL}{\partial Q'_k}(Q - Q'_k) + \frac{\partial SL}{\partial P'_k}(P - P'_k) \geq SL^L \qquad \forall\, k \tag{6.23}$$

However, it is not necessary nor perhaps beneficial for the convergence properties to use all the previously generated gradients. Thus, rather than implementing constraints 6.22 and 6.23 for all k, we use only one previously generated set of gradients (*i.e.* $k = K - 1$). In order not to extrapolate too far from the local region explored in the linearisation,

the concept of a trust region is introduced. The trust region is a local domain that the optimisation is allowed to explore. A small μ is chosen for the values of the previous iteration, P'_{K-1} and Q'_{K-1}, which determines the range of the region explored. Hence:

$$Q'_{K-1}(1-\mu) \leq Q \leq Q'_{K-1}(1+\mu) \tag{6.24}$$

$$P'_{K-1}(1-\mu) \leq P \leq P'_{K-1}(1+\mu) \tag{6.25}$$

However, at the $(k+1)^{th}$ iteration, the inventory simulation model evaluation of the optimal parameters for the k^{th} iteration may make it infeasible to reach the required service level (SL^L) in the optimisation. This is due to a combination of the local linearisation (based on the derivatives obtained from the simulation model) and the trust region requirements (in the optimisation model). Therefore, a penalty function (constraint 6.26) is introduced with respect to low values of service level and conditions for when the penalty should be imposed (constraint 6.27). This allows for a reduction in the value in the imposed level of SL^L (constraint 6.23) which is instead imposed in the penalty function. The penalty function has the same characteristics as the one previously presented in the simulated annealing section. The introduced variable f is to be minimised and any values of SL that are lower than the required service level, SL_r, are penalised.

$$f = I + M \times (SL_r - SL)Z \tag{6.26}$$

$$SL \geq SL_r(1-Z) \tag{6.27}$$

where Z is a binary variable (1 if $SL \leq SL_r$; 0 otherwise).

$SL \cdot Z$ is a bilinear product. Therefore, the positive continuous variable $\overline{SLZ} \equiv SL \cdot Z$ is introduced. Two further constraints are required for $\overline{SLZ}$.

$$\overline{SLZ} \leq SL_r Z \tag{6.28}$$

$$\overline{SLZ} \leq SL \tag{6.29}$$

Since SL is not previously explicitly defined in the model, the introduction of a new constraint to determine the level of this variable is required.

$$SL = SL_k(Q', P') + \frac{\partial SL}{\partial Q'_k}(Q - Q'_k) + \frac{\partial SL}{\partial P'_k}(P - P'_k) \qquad k = K-1 \tag{6.30}$$

To summarise, the entire formulation is as follows:

$$\min f = I + M \cdot SL_r \cdot Z - M \cdot \overline{SLZ} \tag{6.31}$$

s.t.

$$I \geq \overline{I_k}(Q'.P') + \frac{\partial I}{\partial Q'_k}(Q - Q'_k) + \frac{\partial I}{\partial P'_k}(P - P'_k) \qquad k = K - 1 \tag{6.32}$$

$$SL_k(Q', P') + \frac{\partial SL}{\partial Q'_k}(Q - Q'_k) + \frac{\partial SL}{\partial P'_k}(P - P'_k) \geq SL^L \qquad k = K - 1 \tag{6.33}$$

$$SL = SL_k(Q', P') + \frac{\partial SL}{\partial Q'_k}(Q - Q'_k) + \frac{\partial SL}{\partial P'_k}(P - P'_k) \qquad k = K - 1 \tag{6.34}$$

$$SL \geq SL_r(1 - Z) \tag{6.35}$$

$$\overline{SLZ} \leq SL_rZ \tag{6.36}$$

$$\overline{SLZ} \leq SL \tag{6.37}$$

6.6 Test Results

6.6.1 MILP Approach Results

In table 6.3, the optimal results from the MILP approach are shown. The test cases are solved to global optimality so that the results of the other proposed approaches can be compared with the best solutions possible. As can be seen, the computational requirements vary significantly but are generally in the range of several hours. The results from the other approaches are shown in later sections as comparisons are made with these results.

	Obj. value	$P^\star$	$Q^\star$	CPU (s)
Case 1	20639	253.4	314.0	23969
Case 2	16286	213.4	276.3	22882
Case 3	70096	1083	781	38996
Case 4	222403	3485	2800	53829
Case 5	235875	3877	2007	4663
Case 6	1.9851M	27580	25388	23610
Case 7	1.92M	16418	22240	11164
Case 8	98126	1226	1044	9678
Case 9	122337	1565	1461	22456
Case 10	232659	3804	3896	16269

Table 6.3: MILP results

6.6.2 Simulated Annealing Approach Results

The input data for the simulated annealing approach are given in table 6.4. Nh is the

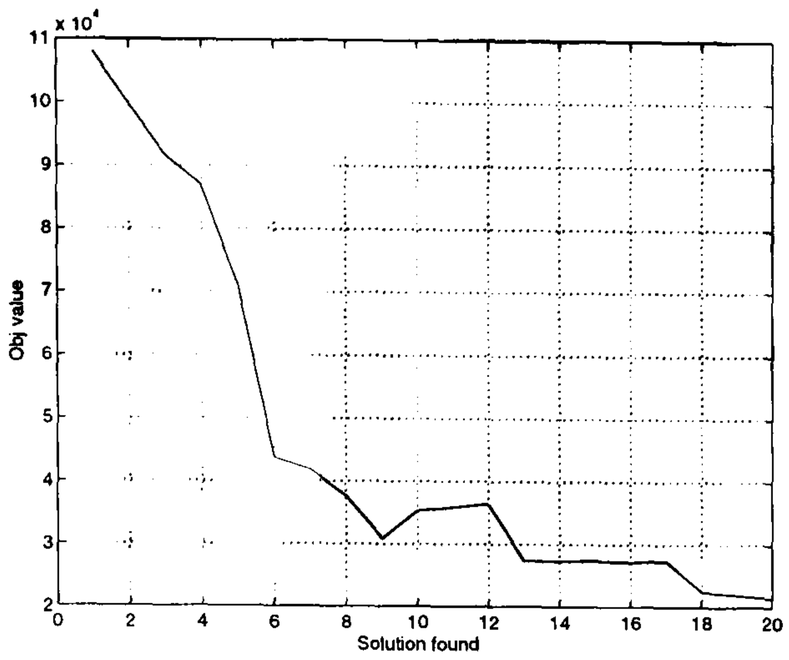
	P_0	Q_0	Nh
Case 1	800	800	200
Case 2	800	800	200
Case 3	3500	3500	700
Case 4	9000	9000	2000
Case 5	11000	11000	2500
Case 6	100000	100000	20000
Case 7	80000	80000	18000
Case 8	5000	5000	1500
Case 9	6000	6000	2000
Case 10	15000	15000	3000

Table 6.4: Input data for simulated annealing

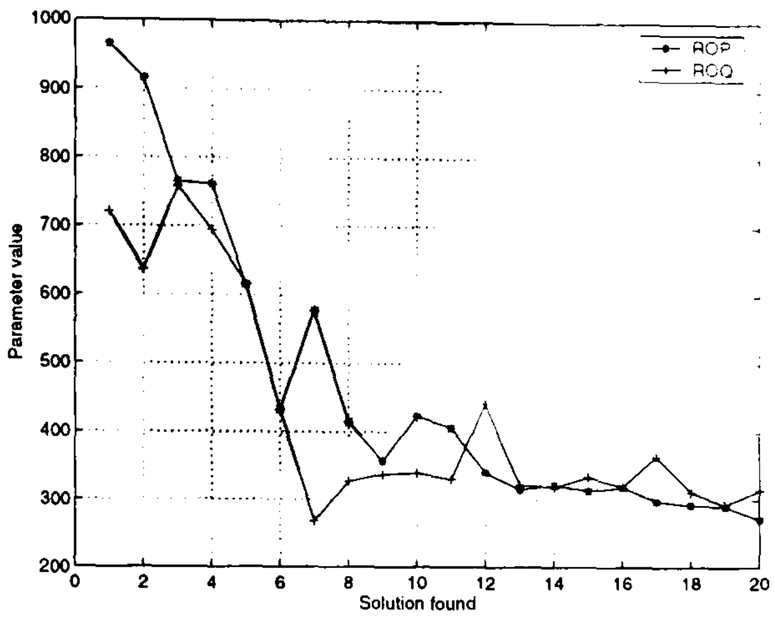
neighbourhood in which the search is taking place. The annealing parameters are set to $T_0 = 1000$, $\alpha_T = 0.8$ and $\alpha_N = 0.99$.[3] A typical progression of the algorithm for a test case is illustrated in figure 6.6. Specifically, figure 6.6(a) shows the progression of the accepted solution value which is to be minimised. Figure 6.6(b) displays the value of the input parameters that are to be determined and figure 6.6(c) shows the progression of the temperature. Finally, in figure 6.6(d), the function value with regards to the iteration number is displayed rather than with respect to the number of the accepted solutions obtained. For case 1, the progression of the objective function is shown in figure 6.7 (10 runs each of 10000 iterations). At each iteration, the maximum, average, and minimum values of the objective function are displayed. This figure illustrates how many iterations might be used on average to obtain a specified accuracy and the kind of distribution that is to be expected. It can be seen that when the number of iterations is reduced, the quality of the solution deteriorates. It can be concluded that at least 500 iterations is required to obtain a solution within 20% of the optimum. However, this value may vary significantly depending on the application and the annealing parameters.

Ten successive runs with different random seeds are performed in order to evaluate

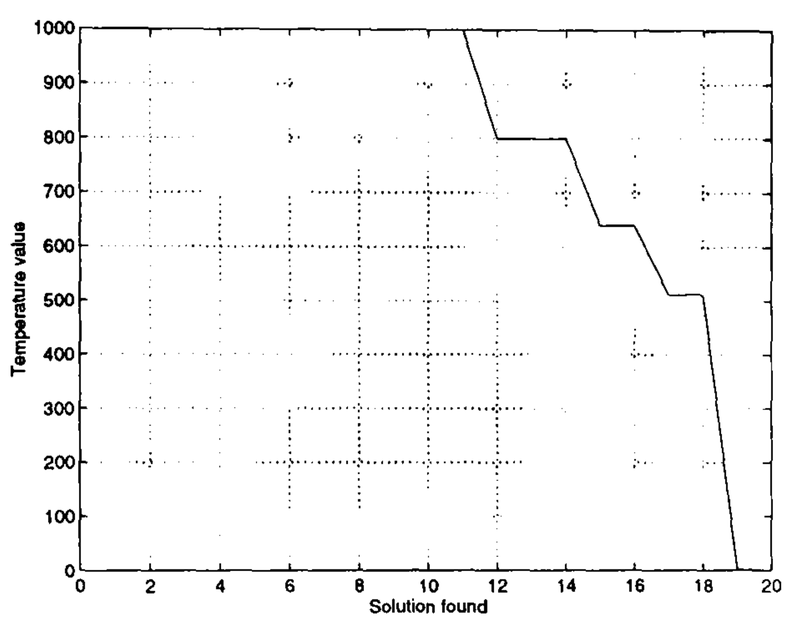
[3]The values of T_0, α_T, and α_N are in the range generally recommended in literature and have been obtained through testing. The values of P_0 and Q_0 are specified to be sufficiently far away from the optima to evaluate critically the simulated annealing properties. The values of Nh are a proportion of



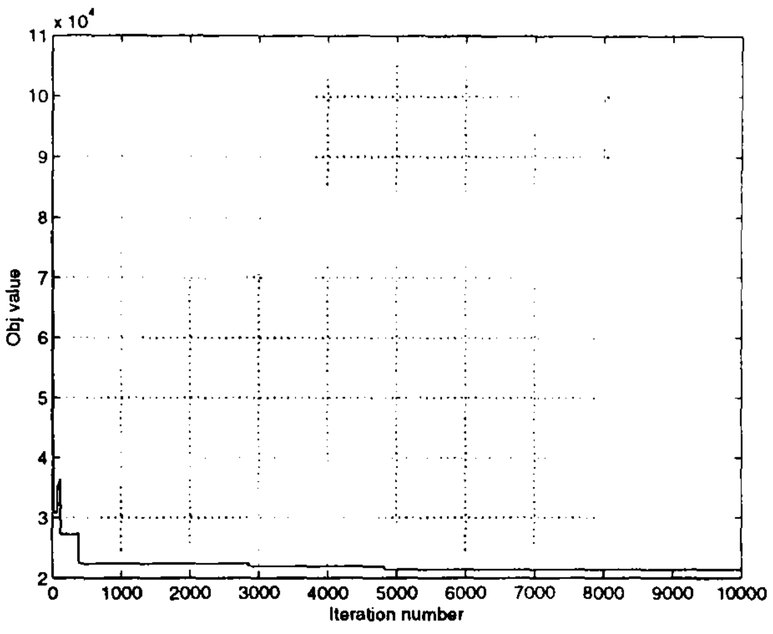
(a) Solution progression



(b) Parameter progression

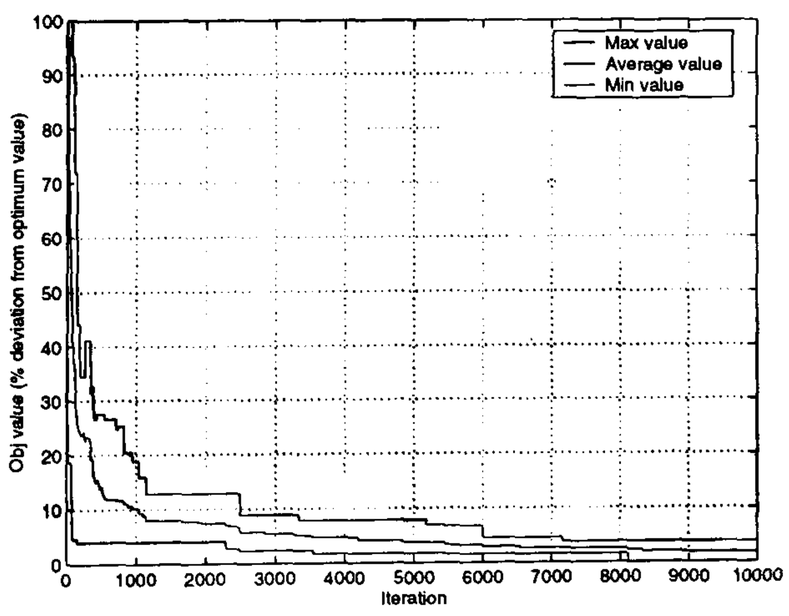


(c) Temperature progression

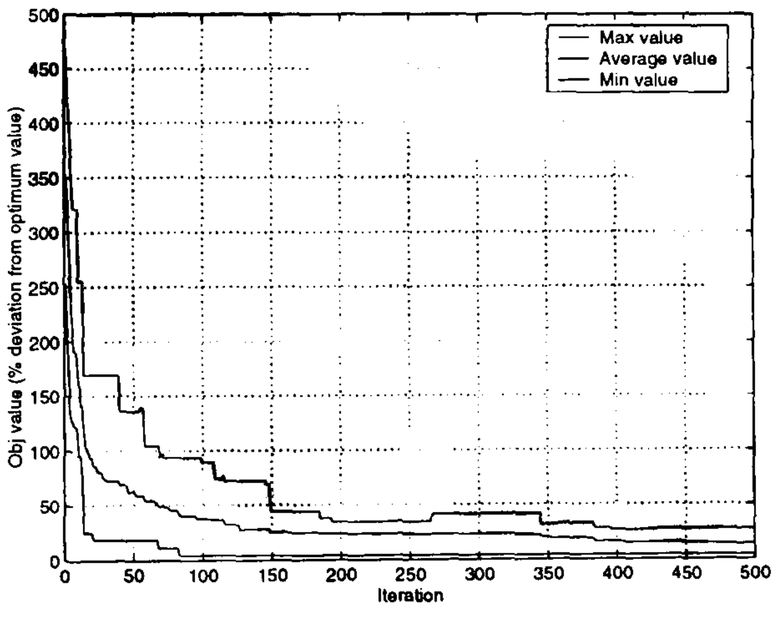


(d) Value during iterations

Figure 6.6: Solution evolution simulated annealing - test case 1



(a) 10000 iterations shown



(b) Zoom on the first 500 iterations

Figure 6.7: Solution progression simulated annealing - test case 1

the performance of the simulated annealing approach. In table 6.5, the best, average and worst function values in terms of the deviation from the optimal solutions are shown. To be able to compare the parameter settings (P and Q) with the MILP approach, the

	Best value	Worst value	Average value	P	Q
Case 1	0.01%	4.47%	1.75%	254.1	314.0
Case 2	4.65%	5.58%	5.20%	220.2	276.6
Case 3	1.36%	6.87%	3.45%	840.3	854.5
Case 4	4.31%	95.6%	15.1%	3492	2801
Case 5	0.05%	1.49%	0.63%	3559	2007
Case 6	1.29%	4.20%	2.81%	27620	25388
Case 7	4.91%	90.1%	28.3%	16356	22213
Case 8	8.97%	68.0%	21.5%	1370	1045
Case 9	0.03%	35.9%	5.04%	1889	1407
Case 10	0.01%	2.17%	0.98%	3586	3896

Table 6.5: Simulated annealing test results: Deviations from optima and parameter settings

settings for the best values for the parameters are also shown for the ten test cases. It can be seen that for most of the test cases the average values of the objective functions are within 10% of the optima. However, for some cases the distributions are quite wide.

In table 6.6, the computational performance of the simulated annealing approach and the MILP approach are compared. The two columns show the CPU requirements in seconds for the MILP approach and the typical requirements for one simulation run of the simulated annealing algorithm for 10000 iterations. The MILP algorithm requires up to 15 hrs (case 4) to find the optimal value. In general, the simulated annealing approach reaches a reasonable solution within seconds.

Knowing the true optima, the simulated annealing runs (with 10000 iterations) have been interrupted as soon as the objective function reached a value within 10% of the optima. In this case, most runs require less than 1 s and 1000 iterations. Since the MILP approach is solved to global optimality, it can be considered unfair in terms of computational requirements to compare the optimal solutions obtained by the MILP

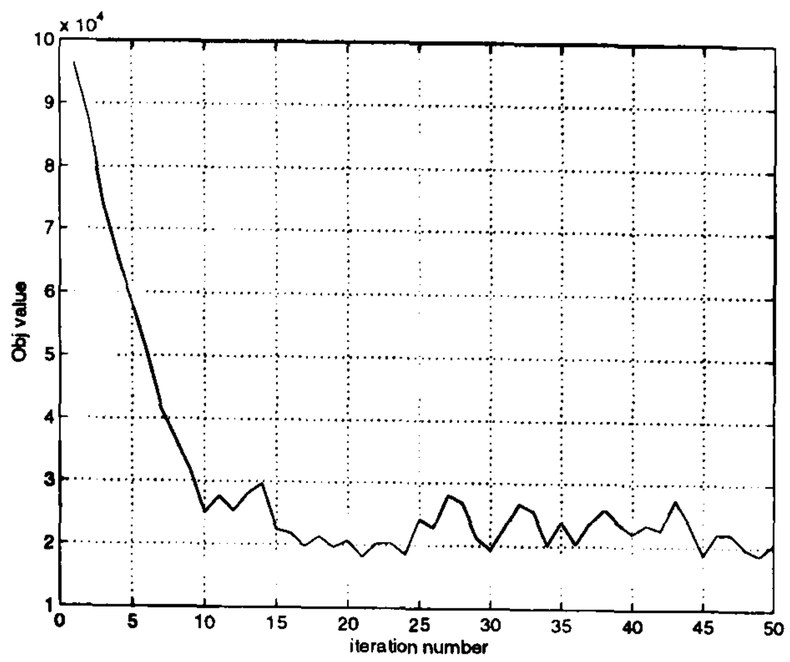
	CPU MILP (s)	CPU SA (s)
Case 1	23969	4.56
Case 2	22881	4.83
Case 3	13349	4.64
Case 4	53829	4.81
Case 5	3778	4.30
Case 6	23610	4.33
Case 7	11163	4.64
Case 8	1996	4.58
Case 9	17172	4.50
Case 10	16269	5.06

Table 6.6: CPU time for MILP and simulated annealing approaches

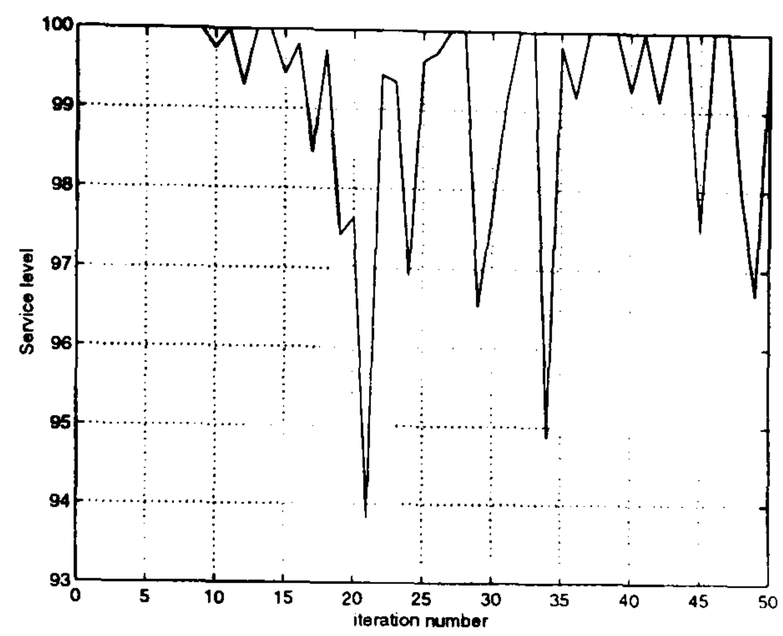
approach to the near-optimal solutions of the simulated annealing approach. However, if the optimality criterion is replaced by a 5% margin of optimality, the improvement in computational cost is only 20% and for 10% margin of optimality the requirement is 16051 s for test case 1, implying a saving of 33%.

6.6.3 Gradient-Based Approach Results

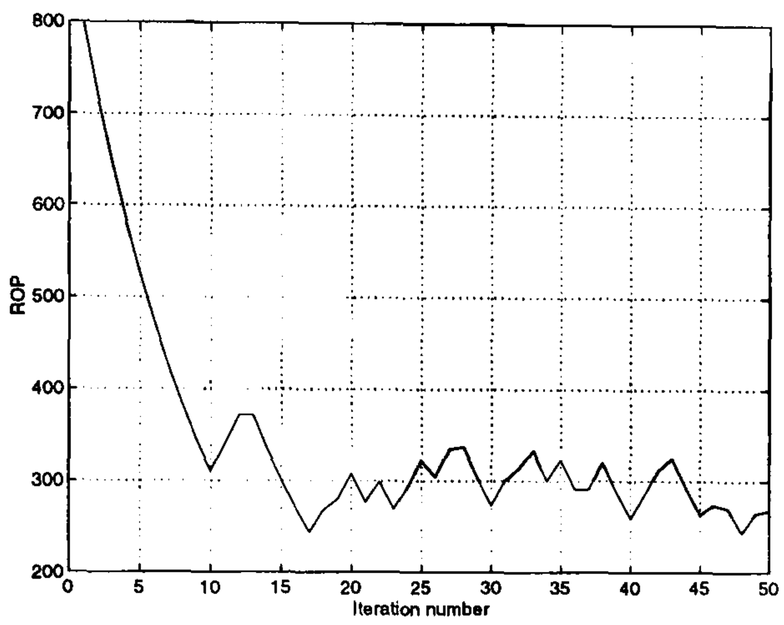
The input data (P_0, Q_0) for the gradient-based approach are the same as previously displayed in table 6.4. The solution evolution of the gradient-based approach (50 iterations) is illustrated in figures 6.8 and 6.9 for a trust region of 10% and 3%, respectively. The choice of trust region size clearly influences the performance of the algorithm. For a trust region of 10% case, the algorithm oscillates around the optimal solution (figure 6.8(d)), because of the large neighbourhood explored at each iteration. The procedure is able to find values close to the actual optimum ($P = 253.4$ and $Q = 314$) already after about 17 iterations (see figures 6.8(c) and 6.8(d)). In the 3% trust region case, the algorithm finds a local minimum after 30 iterations (see figures 6.9(c) and 6.9(d)). The service level is not permitted to deviate from the required 100% as much as for the 10% case (*cf.* figures 6.8(b) and 6.9(b)). Thus, the parameters cannot move away from that point and the values found are both slightly above 300. However, recall that with the simu-



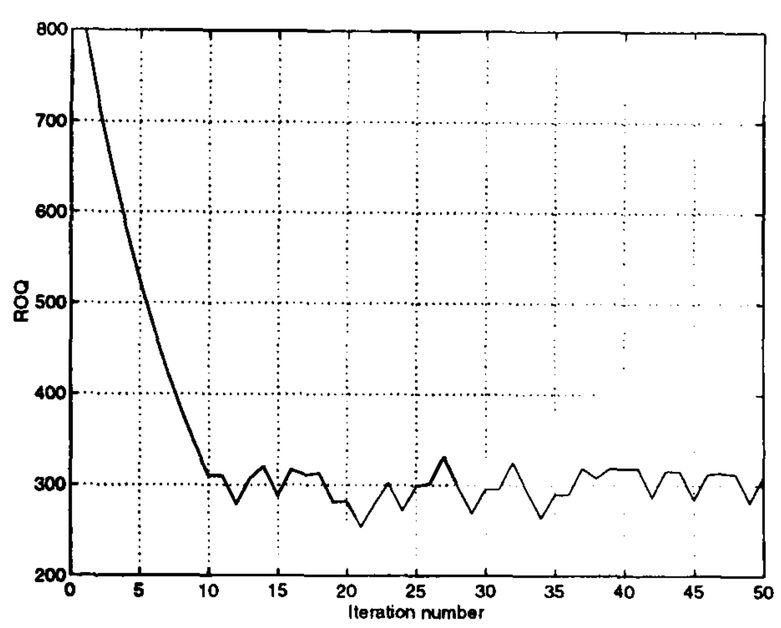
(a) Solution progression



(b) Service level progression



(c) P progression



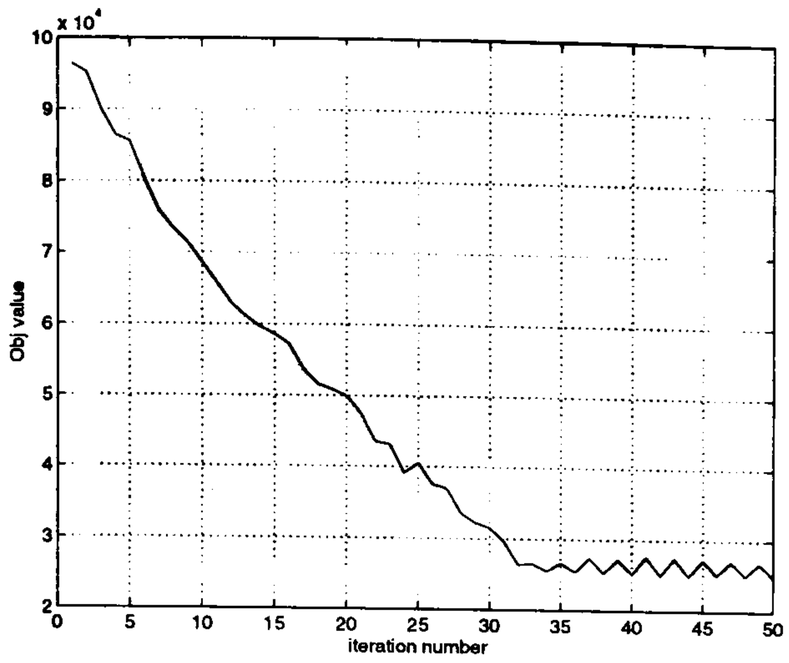
(d) Q progression

Figure 6.8: Solution evolution - gradient-based approach, large trust region

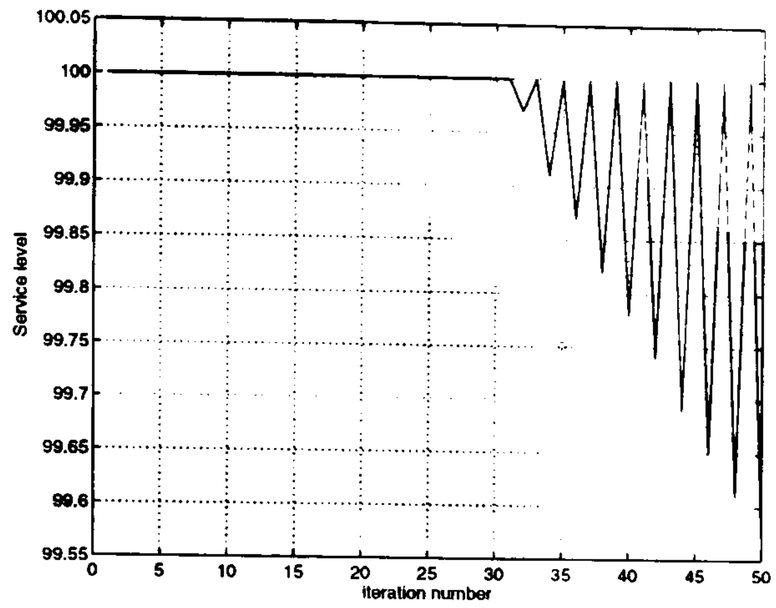
lated annealing approach, about 500 iterations were normally required to obtain a level of solution quality within 20% from the optimum. Comparing figures 6.8(a) and 6.9(a), it can be seen that with a large trust region, a solution close to the optimum (20639) is obtained faster.

In terms of the 10 test cases, the algorithm achieves acceptable results at 100% service levels for most of the cases (table 6.7). The table shows the number of iterations required to obtain the solution, the values of the parameters, (P, Q), along with the deviation from the optimal MILP solutions at a point of 100% service level. In terms of CPU time, all examples require less than 0.5 s and the values are therefore not shown in the table.

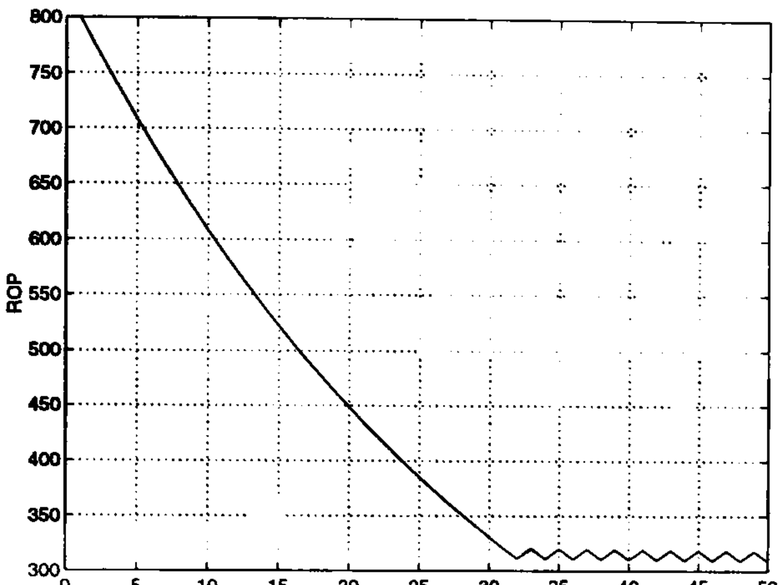
The essential advantage of this approach is the low number of iterations required to obtain these results. In addition, several solutions with a low objective function value and



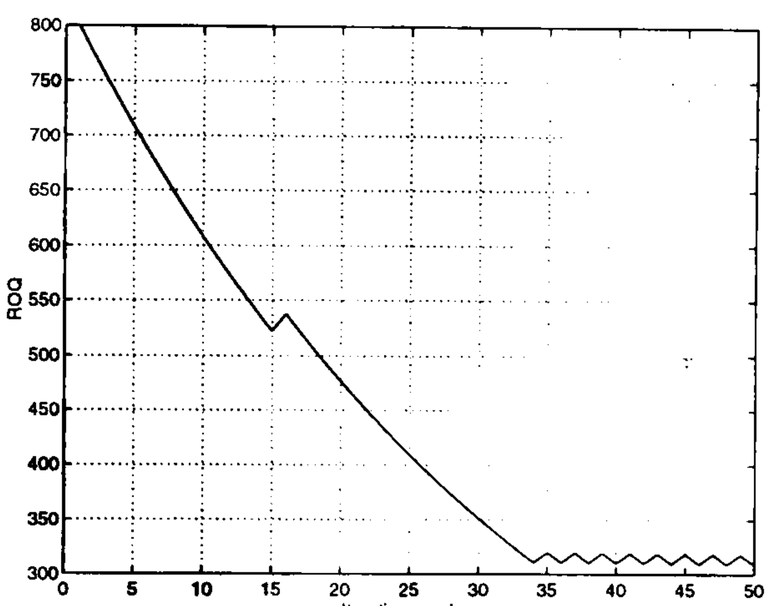
(a) Solution progression



(b) Service level progression



(c) P progression



(d) Q progression

Figure 6.9: Solution evolution - gradient-based approach, small trust region

fairly high service level provide promising starting points for simulated annealing runs that could be performed in sequence. For case 1, some of these starting points are shown in table 6.8. However, this may not be a major concern since dynamic demands present in real-world applications will be based on forecasts that are subject to change. The approaches presented can be expected to offer good solutions for inventory management problems in that they provide policy parameter levels that are much better than typical levels prior to optimisation. Thus, in practical applications, the policy parameters can be far away from the optimal levels and significant improvements can be provided by the approaches described.

To conclude the motivating example test case results, both the simulated annealing and the gradient-based approaches produce reasonable results with low computational

	It. req.	P	Q	Obj. value
Case 1	33	283	312	7.64%
Case 2	32	227	278	12.4%
Case 3	7	918	917	16.9%
Case 4	21	3505	3505	36.5%
Case 5	7	2403	4463	50.3%
Case 6	24	24173	36110	19.6%
Case 7	6	22500	22500	19.6%
Case 8	19	1539	1026	25.6%
Case 9	19	1644	1644	12.6%
Case 10	40	3346	4089	9.27%

Table 6.7: Gradient-based approach test results at 100% service level

	Obj. value	SL	P	Q
1	21873.80	98.36	273.47	325.68
2	21870.70	99.83	278.70	310.41
3	21819.70	99.81	278.59	310.28
4	21773.10	99.91	282.13	303.95
5	21549.10	99.66	270.73	312.43
6	20383.20	99.61	264.26	308.98
7	20907.40	99.22	290.27	299.69
8	20815.00	99.78	270.73	310.09
9	20768.30	99.76	270.63	309.97

Table 6.8: Suggested simulated annealing initial starting points for case 1

cost. However, for the applications in large-scale supply chain model the time consuming factor is how many times the objective function (*i.e.* the number of iterations) is evaluated. In the gradient-based approach, the objective function needs to be evaluated three times per iteration, since the approximation of the gradients (one for $\frac{\partial I}{\partial Q}$, $\frac{\partial SL}{\partial Q}$ and one for $\frac{\partial I}{\partial P}$, $\frac{\partial SL}{\partial P}$) requires two extra objective function evaluations. The simulated annealing requires only one function evaluation per iteration. Nevertheless, the gradient-based approach still performs better if only a small number of iterations is acceptable due to simulation overhead. However, a combination of both approaches might be beneficial so as to find solutions of higher quality faster. One would then first find the region of interest with the gradient-based approach and then proceed to search locally with a relatively small neighbourhood using the simulated annealing approach to find a near-optimal solution.

6.7 Illustrative Industrial Scale Example

In the industrial large-scale supply chain model[4], the aim is to optimise a number of control parameters while maintaining key performance indicators at desired levels. The parameters primarily regarded are the safety stock cover (SS) in number of weeks at the demand management warehouse and the minimum order quantity (MOQ) specified between the secondary manufacturing and the demand management. The evaluation measures observed are the service level, SL, and the average inventory, $\bar{I}$. By choosing these performance indicators we try to obtain a balance between the conflicting interests of low inventory levels and high service levels. However, the problem of how to trade-off these various performance measures in the resulting multi-objective function value is considered an issue for management to determine. Due to the vast problem size, it is not feasible to describe the problem in an MILP formulation. The simple simulation inventory model has been replaced by the industrial scale simulation model described in the previous chapter. The application to the simulated annealing approach is first presented. Then, the performance of the gradient-based approach is evaluated.

A linked simulation-optimisation approach of the type presented here generally requires many simulation runs in order to prove convergence to a global optimum. However, the computational bottleneck in these experiments is clearly the extensive time it takes

[4]The industrial example is the same as the example presented in the previous chapter and is based on Gjerdrum *et al.* (2000*b*).

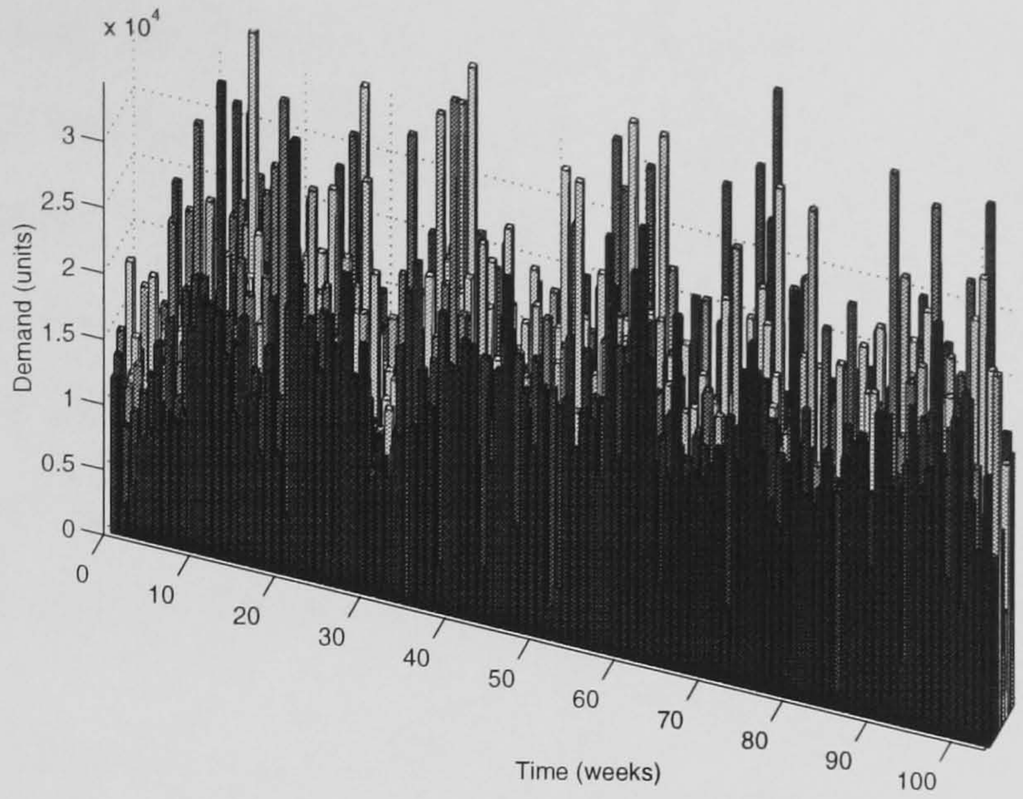


Figure 6.10: Latin Hypercube sampling for product C.

to perform the simulations whereas the computational requirements to perform optimisation is comparatively small. Therefore, the idea is to utilise an algorithm which is as greedy as possible so as to minimise the number of iterations required to obtain a feasible solution close to the optimum.

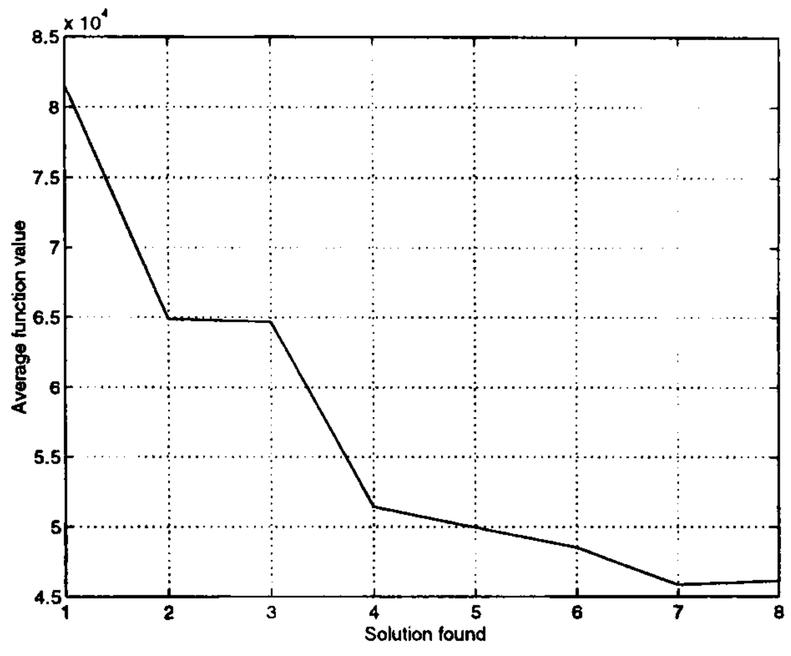
6.7.1 Simulated Annealing Approach

The simulation system presented in the previous chapter is based on Monte Carlo principles and the statistical properties of these trials are somewhat lacking because the number of iterations that has to be performed to obtain reliable results is quite large. Thus, to obtain more reliable results Latin Hypercube sampling is applied in the dynamic simulation to stratify the uncertain demand values throughout the time horizon.

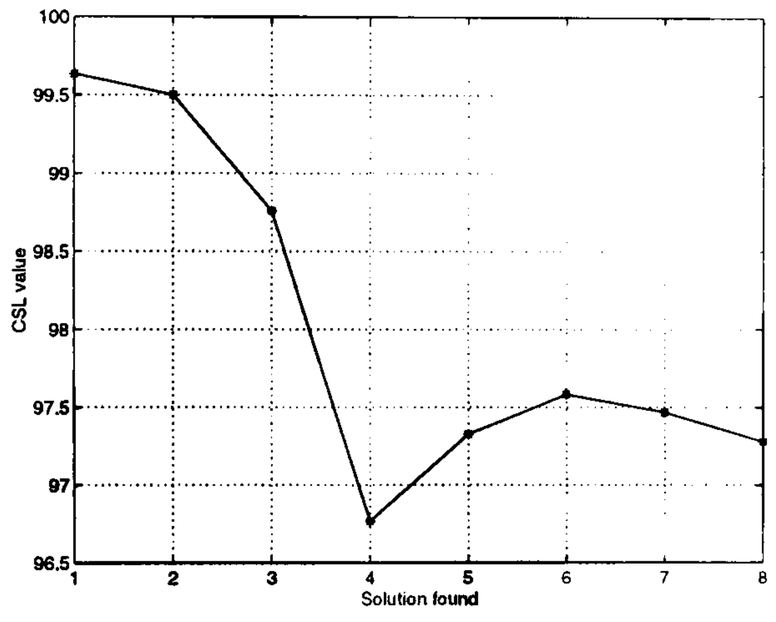
An example of the stratified uncertain space of the demand for product A is given in figure 6.10. For each time step of the time horizon (104 weeks) 10 values have been sampled so as to cover the likely range of demand values given the standard deviation of the variability of the forecast (Gaussian LHS has been utilised). To display the performance of the simulated annealing process, figure 6.11 displays some of the model variables. Specifically, figure 6.11(a) shows the progression of the solution value which is the average inventory value, $\bar{I}$, with a penalty imposed on customer customer service levels below the requirements (in this case $SL_r = 97\%$), see equation 6.38.

$$f = \bar{I} + M(SL_r - SL) \qquad \text{if } SL_i \leq SL_r \qquad (6.38)$$

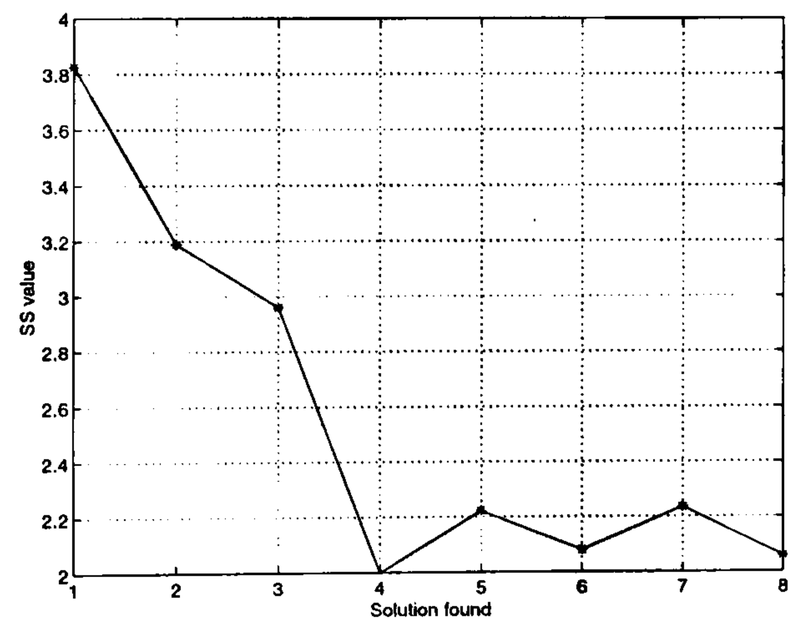
Thus, the value of the big-M constant is pre-determined so as to properly trade off the value of an excellent service level against the value of a low inventory level. In this case a level of big-M of 30000 has been chosen which means that a 1% decrease in service level corresponds to a decrease of 30000 in average inventory throughout the time horizon. In figure 6.11(a), it can be seen that the lowest value of the function value (average inventory + penalty constraint) attainable is slightly above 45000 units of product C. Figures 6.11(c) and 6.11(d) display the value of the policy parameters SS and MOQ and



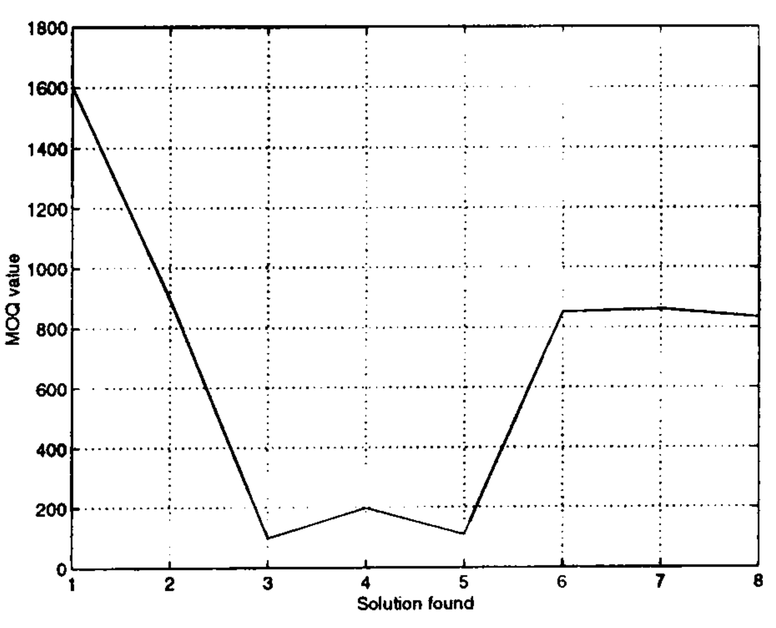
(a) Solution progression



(b) Service level progression



(c) Parameter SS progression



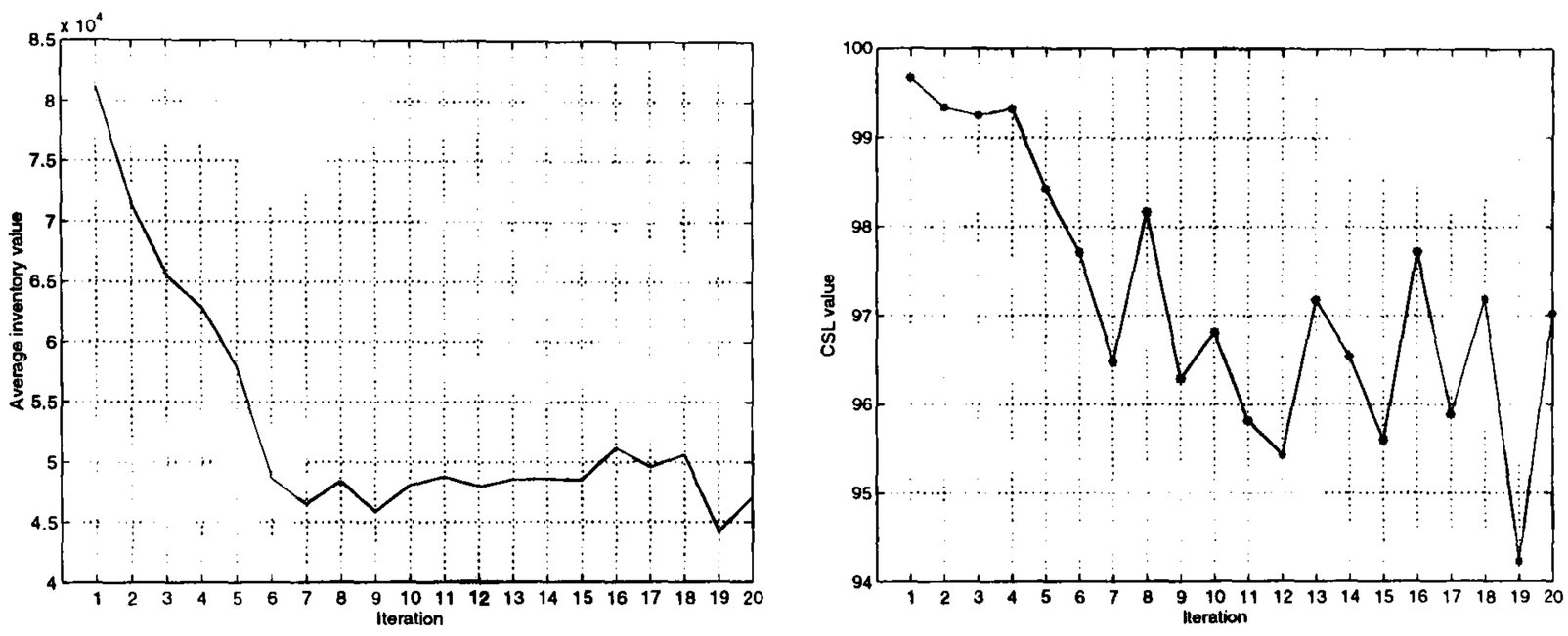
(d) Parameter MOQ progression

Figure 6.11: Solution evolution - simulated annealing approach

figure 6.11(b) shows the value of the service level. In fact, the last solution found indicates a safety stock of 2.06 weeks and an MOQ of 833 units resulting in an average stock value of 46117 units and a service level of 97.3%. The solutions obtained in figure 6.11 require 21564 s on a Sun SPARC Ultra 60 and comprise 200 simulated annealing iterations.

6.7.2 Gradient-Based Approach

When applying the gradient-based approach to the large-scale supply chain model, the same LHS sampling of the uncertainties as for the simulated annealing has been used. In figure 6.12 (the values in the figure are collected from the simulation level), it can be seen that a rapid progression towards the conditions providing low inventory costs are obtained. Specifically, figure 6.12(a) shows that the inventory level for the product investigated rapidly decreases with the number of iterations until the constraint of 97% service level is violated at iteration 7 (see figure 6.12(b)). Furthermore, in figures 6.12(c)



(a) Inventory level progression

(b) Service level progression

(c) Parameter SS progression

(d) Parameter MOQ progression

Figure 6.12: Solution evolution - gradient-based approach

and 6.12(d), the corresponding values of the policy parameters (the trust region is set to 10%) are shown. Considering the service level and the safety stock figures, it seems that a level of a little more than 2 weeks' cover is the most cost efficient given the

requirements of 97% service level. The best solution found is at iteration 20 providing an average inventory level of 47089 with a service level of 97.0%. The corresponding parameter settings are a safety stock of 2.23 weeks and an MOQ of 4085 units. From the gradients ($\frac{\partial \bar{I}}{\partial MOQ_k}$) obtained during the search it is clear that the MOQ parameter has little influence on the cost and the service level. However, a higher level is advantageous with respect to supply chain efficiency in terms of numbers of orders issued (although this is not considered an objective of this optimisation exercise). After obtaining the results from both the simulated annealing and the gradient-based approach, it can be concluded that a safety stock of a little more than two weeks would be appropriate to obtain the desired service level. The policy modifications result in an average inventory reduction of *ca.* 45% from the product C base case average inventory level of 83779 (see table 5.6).

In terms of computational requirements, 20 iterations (which is equivalent to 60 function calls since two gradients also has to be evaluated for each iteration) takes 3878 s using the gradient-based approach which should be compared to the 21564 s for 200 iterations of the simulated annealing approach resulting in comparable objective values and safety stock levels. It can be concluded that a good solution is found faster with the gradient-based approach. However, the quality of the solution is found to be slightly better using the simulated annealing approach.

6.8 Parameter Optimisation Summary

In this chapter, the scope has been to evaluate approaches that provide parameter settings that reduce cost while maintaining performance requirements for large-scale supply chain applications. The proposed algorithms have centred around the idea of a linked simulation and optimisation methodology in which the simulation can provide the detail of modelling required to appropriately replicate the supply chain functionality. The optimisation executes the simulation model as a simple function call to evaluate the current parameter settings.

The methodology proposed has the advantage of being applicable to multi-enterprise supply chain problems since the optimisation only requires a few values from the simulation which can be performed locally. Thus, there should be no need to disclose any proprietary company information to third parties.

Chapter 7

Conclusions and Further Work

In this thesis, the opportunity for certain aspects of multi-enterprise supply chain collaboration supported by modelling and optimisation has been demonstrated. From recent literature within the field, described in the first part of the thesis, it is clear that information sharing and supply chain partnership result in operational and financial benefits. The capability of information handling and accessibility within the supply chain network as a whole is crucial for the development of multi-enterprise supply chain applications. The aim of the optimisation procedures developed in the transfer price part of the thesis has been to maximise the chosen performance measures across the supply chain equitably, so as to ensure long-term profitability of all enterprises in the supply chain network.

In the dynamic supply chain modelling part of this thesis, a framework has been presented for the modelling of large-scale supply chains based on both the physical properties of the system and the inherent business and the decision-making processes that control the operation. Then, optimisation procedures to determine settings for policy parameters based on the inherent decision-making processes of the system have been developed.

7.1 Conclusions

In deterministic, multi-enterprise problem formulations as described in the second part of the thesis, it is generally difficult to apply optimisation when modelling more than one objective function at the same time, since it is not clear how utility (*e.g.* enterprise profits) should be distributed among the participating parties. In this work, supply chain

networks were modelled as mixed integer non-linear programming models with transfer prices, production levels, inventory policies, and distribution flows as decision variables. The objective has been to achieve solutions that *fairly* distribute profit between enterprises at the same time as optimising the supply chain objective variables and fulfilling customer demand.

For comparison, the summed enterprise profits of the whole supply chain were first simply maximised (*i.e.* treating the supply chain as a single level profit centre). When the overall system is optimised in this fashion there is no mechanism to allow profits to be apportioned fairly to all participants. Indeed, solutions to this class of problems usually exhibit very uneven profit distributions and are therefore impractical. They do however give an indication of the best possible total profit attainable in the supply chain. The ambition was then to extend the approach to achieve solutions of similar quality overall but with equitable profit distributions. In order to achieve a more fair split of the profits, a Nash approach was applied to the supply chain profit division problem. The proposed procedure maximises the product of the deviations of the achievable profit levels of each profit centre by the given *status quo* profit levels.

A spatial branch-and-bound algorithm was presented which utilises the game theory bargaining concepts developed by Nash. The results obtained were compared with those obtained by using a simple single level optimisation approach which focus on the total profit generation of the entire supply chain. Transfer prices were used in order to distribute profits in models with two and three supply chain partners. The computational results of several examples show that the proposed method produces profits very close to the single level supply chain optima but much more equitably distributed. Complications were seen to be excessive computational effort due to large spatial branch-and-bound search trees, the larger scale of the systems, and bilinear *and* trilinear terms in the objective function rendering more complex relaxations of the exact problem. However, results were generated and convex trilinear overestimations were developed.

To overcome the problems of finding a general methodology for profit sharing among any number of multi-enterprise supply chain partners an approach was developed based on generalised Nash-Harsanyi concepts. The resulting MINLP model was tackled using a separable programming approach in which the application of logarithmic differentiation and approximations of the variables of the objective function result in a model of

MILP form. The solution obtained includes the transfer prices, production levels, timing and quantities of products delivered and resources utilised. The inventory cost at the production plant depends on the transfer price level and is therefore not arbitrary as regarding the overall supply chain optimisation. The results were published in Gjerdrum *et al.* (2001*b*).

From the recent supply chain literature, it has been seen that companies work in an integrated manner and there exist information systems that make it possible to share production information between supply chain partners. There seems to be a strong future for optimised multi-enterprise supply chain systems which utilise top level enterprise performance measures as objectives for operational planning activities. In the transfer pricing part of the thesis, it has been assumed that all relevant data can be shared by all supply chain players, thus allowing the supply chain network to have one single optimisation model. However, it is still quite common in practice that players are reluctant to make information available to partners (especially that relating to cost structures).

The need for private models arises from the strictly confidential cost structures at the plant level, typical for many industries. If performance measures are developed in such a way that they are allowed to be made public, then the need of private models may be eradicated. Thus, the choice of performance measures defines how the supply chain information could be modelled. If, for instance the performance measure is in terms of customer service level, stock turnover, or other indicators which do not require internal cost data for their calculation it might be possible to use public models since the information then is non-confidential.

In order to model the efficiency of top level non-confidential policy parameters, a full scale simulation methodology was described in the third part of the thesis for modelling of the physical and decision-making business process aspects of supply chain application. Pragmatic, non-invasive policy and parameter modifications that improve performance measures such as average inventory levels, probability of stock outs and customer service levels were identified. The model was also utilised to assess and quantify the risks and benefits of changing supply chain operations policies. Applications of the simulation model have been presented in Gjerdrum *et al.* (2000*b*) and Gjerdrum *et al.* (2001*a*).

To optimise the dynamic simulation models described, a number of approaches were developed and evaluated. The methods are based on mixed-integer linear program-

ming, gradient-based, and simulated annealing concepts. Specifically, the gradient-based method is based around a top level optimiser that collects the results in terms of objective function values and gradients in terms of policy parameters from simulations. The methods provide parameter settings that reduce cost of operations subject to pre-specified performance requirements.

The ideas developed in the latter part of the thesis can be used to determine settings in a multi-enterprise supply chain environment since simulation of each node can be performed independently and the optimisation of policy parameters is based on the obtained values for some key performance indicators which can be chosen arbitrarily. If these performance measures are not confidential, there is no inherent need to share information between companies and optimisation can be performed based on private ownership of information.

7.2 Recommendations for Future Work

Next, a number of possible suggestions for future work that relate to the work presented in this thesis are presented. The aim is to provide the reader with ideas about how a framework for future supply chain modelling and analysis could be accomplished based on the understanding gathered through the work to date.

Production and Planning

Most commercial supply chains involve a large number of products that have to be closely monitored in terms of every processing step. These processing steps have limited capacities and planning has to be performed wisely so as to maximise resource utilisation and minimise non-value added time. However, the details of production plans, in particular when linked to upstream and downstream supply chain activities for integrated supply chain planning, can make the task at hand computationally unmanageable. One major issue for further research is how to aggregate products with respect to the details of production planning. In particular, evaluation of aggregation has to be performed both in terms of the cost value that accurate production plans entail and in terms of the increased ability to satisfy consumers. Moreover, how the aggregation procedure can enable more reliable future scenario planning for supply chain wide applications is a related topic of

study. With respect to supply chain uncertainty and the stochastic supply chain process, an interesting subject is how long into the future production plans can be maintained with acceptable validity.

Evaluation of how various linked production planning and forecasting methods actually affect the overall supply chain dynamics could be made. When manufacturing spans several supply chain echelons or is horizontally distributed, it would be valuable to evaluate how linked distributed optimised solutions compare with centralised supply chain production planning and furthermore how production plans can be modified when dynamic changes such as customer demand variations are imposed on the supply chain network.

Supply chains are subject to constant change, not only in terms of how the processes are run but also which members are present and what products are manufactured. As the processes are modified, it would be worth considering how to choose key performance indicators so as to capture the most relevant, critical aspects of business and production. Moreover, further research could be done as to evaluate net present values of future scenarios in terms of prospective supply chain participants, processes and products.

Inventory and Distribution Management

More thorough quantitative analyses of what inventory management approaches are effective to obtain desired goals of low cost, low maintainability and high service are required. It is also of interest to try to determine where the supply chain inventory should be positioned along the supply chain to achieve low cost and high responsiveness and which members are best suited to be responsible for the inventory at hand.

The distribution mechanisms are inherent parts of supply chain characteristics. However, in this thesis, these mechanisms have only been briefly mentioned. There is wide scope for further research in this area. Some matters to be explored are how integration of distribution flows and production plans can be maintained in the most efficient manner and what type of tracking mechanism should be used in supply chain models so as to represent the distribution flows of raw materials, unfinished products and finished products.

Supply Chain Performance

Key performance indicators have been used in this thesis to measure the rôle that policy parameters have in supply chain operations. However, a more detailed approach is required to capture all facets of supply chain improvement. For instance, the cost of manufacturing flexibility is one interesting path of study. If activities can be planned long in advance, then this would allow for cost reduction in terms of manpower and equipment utilisation. Defining metrics such as dynamic pricing policies based on committed delivery dates between upstream and downstream echelons would be one possible way of capturing these phenomena.

What is further required is to define performance measures and to be able to model them generically based on the degree of confidentiality. Furthermore, ways of expressing and calculating the performance measures quantitatively for multi-enterprise supply chains are needed. Supply chain collaboration on the multi-enterprise level involves two or more companies that interact in order to meet combined business objectives. In order to collaborate, the companies need a point of contact. One matter for further research is to what extent total integration benefits the supply chain performance and process functionality and whether it is possible to achieve desired aims of performance with only discrete information exchanges *e.g.* with respect to some parameters measuring current operations at the different supply chain nodes. There is also scope for further research into how valuable it is to preserve confidentiality and whether it is possible to perform collaboration and synchronisation of execution without any sharing of confidential data.

All evidence shows that service is of paramount importance to companies. In particular, service levels to key customers are normally perceived to be prioritised no matter how difficult their demands are to fulfil. It would be of interest to study the costs associated with such differentiated treatment especially in terms of production and distribution. Moreover, further research could be made into how other typical supply chain policy parameters affect end consumer service levels and thereby also analysing the impact that higher service levels entail in terms of operational and fixed costs.

In order to incorporate uncertainties in supply chain external and internal factors, further research is suggested in terms of the development of forecasting tools that can be used to predict the future behaviour. The quality of the forecast should be used to determine the validity of any plans that are created and give insight into how long into the

future it is meaningful to make plans for various processes. Feedback mechanisms that measure the deviation between forecast behaviour and actual behaviour of the system could be developed so as to improve forecast accuracy. It would also be interesting to assess what types of forecasting tools are effective for different circumstances.

Modelling and Optimisation

The fundamental question when trying to build a framework for normative transfer pricing policies is how to impose the pricing rules on the participating companies in a multi-enterprise supply chain. It would be of interest to pursue a study of how the inherent power relations between supply chain partners could be incorporated so as to capture the actual negotiation principles involved and find a pragmatic approach to agreement in pricing policies that could be useful for any type of power relations. Furthermore, how returns of excellent supply chain execution based on supply chain-wide investments are shared is very interesting field of study indeed. In particular, research should be performed into how profits in reality could be apportioned from the company receiving end consumer revenue which shares its profits with other supply chain members so as to make way for improved performance in the long run.

Based on the transfer pricing modelling and analysis performed in this thesis, it would be of interest to investigate the measurement of actual costs in supply chain models. Interesting subjects of study would be where to specify the mark-up of product value, how much value-added a particular process reflects, and how the cost base could be used to modify the operational policies. The analysis would hopefully provide increased transparency and introduce new paths to supply chain effectiveness.

It is not always appropriate to model all instances of supply chain applications for total public accessibility of information within partnerships. Thus, there is a call for models that further evaluate how "private" as well as "public" information can be captured. In this respect, it would be of interest to explore how other decomposition-type approaches for multi-stage planning models than the one proposed in this thesis would perform in a multi-enterprise supply chain environment.

The dynamic supply chain model demonstrated in this thesis is based on pragmatic principles and captures how specific companies operate both in terms of physical movements of material and the related business processes. Even though this type of tool is

valuable in the assessment of current performance of operations and adjustments of main policy parameters, there is a need for more flexible models to be developed. One main objective of such models is that they should be easy to modify for people involved in both operations and strategy on their level of expertise. This could be realised by having a modular structure of all key components of the model which replicates *e.g.* inbound and outbound logistics, material conversions, inventory management policies and the trade agreements between the supply chain members. A model built of modular blocks would allow for increased clarity, readability and flexibility.

Supply chains are, as mentioned, a set of participating members and processes that are subject to change. A development of a node structure used to represent the flexible nature of these networks would be most beneficial in terms of modelling and analysis. In particular, when participants are removed or added, it would be interesting to measure the costs and the benefits of these changes. Furthermore, the measurement and sharing of induced rewards coming from modifications such as new technology investments could be studied. Another crucial point of research would be how to maintain and ensure profitability during supply chain simultaneous change of demands, participants and processes. A changeable node-based network structure would perhaps enable such problems to be addressed.

Simulation of supply chain networks aims at capturing the true dynamics of the actual systems. There is scope for further investigations of the overall supply chain impacts of input data disturbances in terms of step changes, trend changes etc. These studies hopefully lead to a better understanding of how manufacturing bottlenecks can be avoided by introducing measures before external events occur.

For supply chain optimisation applied to planning, the opportunity to find one global optimal solution is fairly limited since the inherent dynamics modify the input data (which is often not very reliable in the first place). Therefore, it makes sense to use robust, fast, near-optimal techniques to determine how policies and plans could be set. The techniques developed need to be thoroughly evaluated in terms of ease of use, reliability and supply chain integration capabilities. Further stochastic methods such as tabu search and genetic algorithms could be explored as well as sensible methods to integrate local optimisation techniques with overall supply chain objectives.

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Appendix A

Linearisation of Trilinear Terms

Maranas and Floudas (1995) show that a convex lower bounding function of a trilinear term xyz is a generalisation of the McCormick constraints inside the rectangular region $[x^L, x^U] \times [y^L, y^U] \times [z^L, z^U]$ with x^L, y^L, $z^L \geq 0$. They show that there exist three different convex lower bounding schemes for trilinear terms. Similarly, we have for the overestimation of the McCormick constraints:

$$
\begin{aligned}
xyz \leq s_0 &= \min\{x^U s_1 + x s_1^L - x^U s_1^L,\ x s_1^U + x^L s_1 - x^L s_1^U\} \\
\text{where} \quad s_1 &= \min\{y^U z + y z^L - y^U z^L,\ y z^U + y^L z - y^L z^U\} \\
\text{and} \quad s_1^L &= y^L z^L,\ s_1^U = y^U z^U,
\end{aligned}
$$

$$
\begin{aligned}
xyz \leq s_0 &= \min\{y^U s_1 + y s_1^L - y^U s_1^L,\ y s_1^U + y^L s_1 - y^L s_1^U\} \\
\text{where} \quad s_1 &= \min\{x^U z + x z^L - x^U z^L,\ x z^U + x^L z - x^L z^U\} \\
\text{and} \quad s_1^L &= x^L z^L,\ s_1^U = x^U z^U,
\end{aligned}
$$

$$
\begin{aligned}
xyz \leq s_0 &= \min\{z^U s_1 + z s_1^L - z^U s_1^L,\ z s_1^U + z^L s_1 - z^L s_1^U\} \\
\text{where} \quad s_1 &= \min\{x^U y + x y^L - x^U y^L,\ x y^U + x^L y - x^L y^U\} \\
\text{and} \quad s_1^L &= x^L y^L,\ s_1^U = x^U y^U.
\end{aligned}
$$

After elimination of s_0 and s_1 and with the substitution of s_1^L and s_1^U we obtain the convex overestimation of the function trilinear function xyz:

$$
\begin{aligned}
xyz \leq \min\{ & x y^L z^L + x^U y z^L + x^U y^U z - x^U y^U z^L - x^U y^L z^L, \\
& x y^L z^L + x^U y z^U + x^U y^L z - x^U y^L z^U - x^U y^L z^L, \\
& x y^U z^U + x^L y z^L + x^L y^U z - x^L y^U z^L - x^L y^U z^U, \\
& x y^U z^U + x^L y z^U + x^L y^L z - x^L y^L z^U - x^L y^U z^U\}.
\end{aligned}
$$

$$\begin{aligned} xyz \leq \min\{\ & xy^U z^L + x^L yz^L + x^U y^U z - x^U y^U z^L - x^L y^U z^L, \\ & xy^U z^U + x^L yz^L + x^L y^U z - x^L y^U z^U - x^L y^U z^L, \\ & xy^L z^L + x^U yz^U + x^U y^L z - x^U y^L z^L - x^U y^L z^U, \\ & xy^L z^U + x^U yz^U + x^L y^L z - x^L y^L z^U - x^U y^L z^U\}, \end{aligned}$$

$$\begin{aligned} xyz \leq \min\{\ & xy^L z^U + x^U yz^U + x^L y^L z - x^U y^L z^U - x^L y^L z^U, \\ & xy^U z^U + x^L yz^U + x^L y^L z - x^L y^U z^U - x^L y^L z^U, \\ & xy^L z^L + x^U yz^L + x^U y^U z - x^U y^U z^L - x^U y^L z^L, \\ & xy^U z^L + x^L yz^L + x^U y^U z - x^L y^U z^L - x^U y^U z^L\}. \end{aligned}$$

When the duplicate forms have been eliminated six linear functions are obtained, which provide an upper bound on xyz:

$$\begin{aligned} xyz \leq \min\{\ & xy^L z^L + x^U yz^L + x^U y^U z - x^U y^U z^L - x^U y^L z^L, \\ & xy^L z^L + x^U yz^U + x^U y^L z - x^U y^L z^U - x^U y^L z^L, \\ & xy^U z^U + x^L yz^L + x^L y^U z - x^L y^U z^L - x^L y^U z^U, \\ & xy^U z^U + x^L yz^U + x^L y^L z - x^L y^L z^U - x^L y^U z^U, \\ & xy^U z^L + x^L yz^L + x^U y^U z - x^U y^U z^L - x^L y^U z^L, \\ & xy^L z^U + x^U yz^U + x^L y^L z - x^L y^L z^U - x^U y^L z^U\}. \end{aligned}$$

Appendix B

A Separable Programming Approach

Consider a continuous strictly convex function in one variable, $f(x)$, to be minimised. This function can be approximated over an interval as a piecewise linear function, see figure 4.1.

Using n grid points the function f can be linearly approximated as (Bazaraa and Shetty, 1979):

$$\bar{f}(x) = \sum_{k=1}^{n} \lambda_k \, f(x_k) \tag{B.1}$$

$$\sum_{k=1}^{n} \lambda_k = 1 \tag{B.2}$$

$$\lambda_k \geq 0 \qquad \forall \, k \tag{B.3}$$

The constraints $\sum_{k=1}^{n} \lambda_k = 1$ and $\lambda_k \geq 0$ and the convexity requirement guarantee that two adjacent nodes take non-zero values. A nonlinear problem P with j separable functions to be optimised

$$\max_{x} \sum_{j=1}^{m} f_j(x) \tag{B.4}$$

$$\text{subject to} \sum_{j=1}^{m} g_{ij}(x_j) \leq p_i \qquad \forall \, i \tag{B.5}$$

$$x_j \geq 0 \qquad \forall \, j \tag{B.6}$$

can be approximated over the n grid points (not necessarily equidistant). P have the properties that the f_j functions are strictly concave (remember that maximisation of a

concave function has the same properties as minimisation of a convex one) and g_{ij} are convex for $i = 1, \ldots, p$. In the following function, $\hat{f}_j$ are the separable portions of the objective function

$$\hat{f}_j = \sum_{k=1}^{n} \lambda_{kj}\, f(x_{kj}) \qquad \forall\, j \qquad \text{(B.7)}$$

$$\sum_{j=1}^{m} \sum_{k=1}^{n} \lambda_{kj}\, g_{ij}(x_{kj}) \leq p_i \qquad \forall\, i \qquad \text{(B.8)}$$

$$\sum_{k=1}^{n} \lambda_{kj} = 1 \qquad \forall\, j \qquad \text{(B.9)}$$

$$\lambda_{kj} \geq 0 \qquad \forall\, k, j \qquad \text{(B.10)}$$

The overall problem then becomes to maximise the following function:

$$\max \hat{F} = \sum_{j=1}^{m} \sum_{k=1}^{n} \lambda_{kj}\, f(x_{kj}) \qquad \text{(B.11)}$$

$$\sum_{j=1}^{m} \sum_{k=1}^{n} \lambda_{kj}\, g_{ij}(x_{kj}) \leq p_i \qquad \forall\, i \qquad \text{(B.12)}$$

$$\sum_{k=1}^{n} \lambda_{kj} = 1 \qquad \forall\, j \qquad \text{(B.13)}$$

$$\lambda_{kj} \geq 0 \qquad \forall\, k, j \qquad \text{(B.14)}$$

The generalised Nash objective is to maximise the product of profit deviations of the enterprises, *i.e.* $\Phi = \prod_{j=1}^{m} (\pi_j - \pi_j^L)$ where m is the number of participating companies, π_j is the profit of company j and π_j^L is the lower profit requirement point of company j. The objective function can be rewritten via logarithmic differentiation as $\ln \Phi = \sum_j \ln(\pi_j - \pi_j^L)$. If the separable approach is utilised for the Nash problem, with logarithmic differentiation, the reformulated problem becomes:

$$\max\, \hat{\Phi} = \sum_{j=1}^{m} \sum_{k=1}^{n} \lambda_{kj}\, \ln(\pi_{kj} - \pi_{kj}^L) \qquad \text{(B.15)}$$

$$\sum_{j=1}^{m} \sum_{k=1}^{n} \lambda_{kj}\, g_{ij}(x_{kj}) \leq p_i \qquad \forall\, i \qquad \text{(B.16)}$$

$$\sum_{k=1}^{n} \lambda_{kj} = 1 \qquad \forall\, j \qquad \text{(B.17)}$$

$$\lambda_{kj} \geq 0 \qquad \forall\, k, j \qquad \text{(B.18)}$$

To return to the original notation introduced in chapter 4, the explicit change in the formulation is in constraint 4.10. A new objective function is introduced, constraint

(B.20). Also, the constraints relating to the λ-values are introduced as constraints (B.21) and (B.22). It can be seen that the convexity properties hold, since $f_m = \ln(\pi_{km} - \pi^L_{km})$ is maximised and is strictly concave (equivalent to minimisation of a convex function) and π_{km} is linear and therefore convex.

$$\sum_{k=1}^{n} \lambda_{km}\, \pi_{km} = \sum_{i} \sum_{t} \sum_{m' \in \overline{T}_{mi}} F_{imm't}\, P_{imm'} - C_m \qquad \forall\, m \tag{B.19}$$

$$\max\, \hat{\Phi} = \sum_{m} \sum_{k=1}^{n} \lambda_{km}\, \ln(\pi_{km} - \pi^L_m) \tag{B.20}$$

$$\sum_{k=1}^{n} \lambda_{km} = 1 \qquad \forall\, m \tag{B.21}$$

$$\lambda_{km} \geq 0 \qquad \forall\, k, m \tag{B.22}$$

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