

Plasma cooling by thermal conduction in the magnetically closed solar corona with large spatial variations in the magnetic field strength

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ABSTRACT

Plasma cooling by thermal conduction is considered for solar coronal magnetic loops that have large variations in their magnetic field strength and hence cross-sectional area. In terms of a ‘cooling time’, for all cases considered an increase in the cross sectional area from base to apex leads to longer cooling times than for a loop with uniform area. Although this result is expected, this paper reveals, for the first time, a vast range of different cooling outcomes that depend on the details of the prescribed loop geometry. While a simple approach suggests that the cooling time could scale with the average area, $\langle A \rangle$, this only holds when the region of the most significant area variation and the hottest plasma coincide. Instead, analytic and quasi-analytic solutions show that this scaling can greatly exaggerate the effect of $\langle A \rangle$ on the cooling time. For example, there are cases in which the cooling time scales as $\ln(\langle A \rangle)$. Thus, conductive cooling needs to be considered on a case-by-case basis. The results can be understood in terms of an artificial ‘heating’ associated with a non-uniform loop area.

Key words: methods: numerical – Sun: activity – Sun: corona – Sun: Magnetic fields – MHD .

1 INTRODUCTION

Plasma cooling by thermal conduction in the magnetically closed solar corona (hereafter coronal loops) has been the subject of many studies. In such a geometry, conduction parallel to the magnetic field is the dominant process with conduction perpendicular to the field being generally negligible. For example, quasi-steady coronal loop models involve a balance between an imposed heating function and cooling by field-aligned thermal conduction and optically thin radiation (e.g. I. J. D. Craig, A. N. McClymont & J. H. Underwood 1978; P. C. H. Martens 2010). On the other hand when the heating varies rapidly with time, the initial temporal dependence of the plasma temperature is determined largely by thermal conduction which in turn determines the observed emission, in particular at X-ray and EUV wavelengths (e.g. P. J. Cargill 1994; F. Reale 2014; P. J. Cargill, H. P. Warren & S. J. Bradshaw 2015). Further the predicted hot emission from nanoflare heating (the ‘smoking gun’ of coronal heating) is very sensitive to the conductive cooling models used (e.g. W. T. Barnes, P. J. Cargill & S. J. Bradshaw 2016a, b).

While early work (e.g. J. L. Culhane, J. F. Vesecky & K. J. H. Phillips 1970) looked at conductive cooling in a loop with constant cross-sectional area, it was realized 50 yr ago (S. K. Antiochos & P. A. Sturrock 1976, hereafter AS76), that loops with a

cross-sectional area (magnetic field strength) that increases (decreases) from the footpoints to the apex cooled more slowly than loops with a uniform area. [In a one dimensional loop model, magnetic flux conservation implies BA is a constant, where $B(s)$ is the magnetic field strength and $A(s)$ defines the area profile.] Subsequently cooling in such geometries was investigated using simple models (e.g. S. K. Antiochos & P. A. Sturrock 1978; P. J. Cargill et al. 2022) and advanced magnetohydrodynamic (MHD) simulations (e.g. Z. Mikić et al. 2013; C. Froment et al. 2018; J. W. Reep et al. 2022).

This paper focuses on the simpler class of models, namely purely conductive cooling in a coronal loop with a non-uniform area. In earlier work (P. J. Cargill et al. 2022; P. J. Cargill, A. W. Hood & D. Johnson 2025), we argued that there was a simple scaling of the conductive cooling time-scale proportional to the average loop area between apex and chromosphere, as is discussed further in Section 2. P. J. Cargill et al. (2025), in a study of conductive cooling at a magnetic null, compared the results of such a model with full non-linear MHD results and found excellent agreement. However, in the process of the null point study, it became apparent that the simple scaling did not always work well. The aim of this paper is to address that topic further and, in particular, to establish some general results concerning conductive cooling in a loop geometry by studying a range of area profiles. We follow the approach of P. J. Cargill et al. (2025) and solve the conductive cooling equation by the method of separation of variables as first proposed by AS76. We find that there is a

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large range of previously unknown conductive cooling solutions that depend on the specific area profile.

Section 2 outlines the energy equation used, summarizes the simple scaling derived in our earlier work and describes briefly the method of solution of the time-dependent conduction equation. Section 3 presents both analytical and numerical solutions of the conduction equation for a range of area profiles, and demonstrates the existence of a wide range of cooling time-scale scalings with the functional form of the cross-sectional area. Section 4 summarizes the work, comments on the various approximations made and discusses applications to observed coronal cooling. The appendix presents solutions for a generalized (i.e. non-*Spitzer*) conduction coefficient.

2 EQUATIONS, ASSUMPTIONS, AND METHODS

The one-dimensional energy equation along a field line with a coordinate, s , and a prescribed cross-sectional area, $A(s)$, is in the usual notation:

$$\frac{\partial E}{\partial t} = -\frac{1}{A(s)} \frac{\partial}{\partial s} \left(A(s)v[E+p] + A(s)F_c \right) + Q - n^2 \Lambda(T) \quad (1)$$

with $E = p/(\gamma - 1) + \rho v^2/2$, $F_c = -\kappa_0 T^{5/2} dT/ds$ is the heat flux with $\kappa_0 = 8.12 \times 10^{-7}$ in c.g.s. units, $\Lambda(T)$ an optically thin radiative loss function (e.g. J. A. Klimchuk, S. Patsourakos & P. J. Cargill 2008) and $Q(s, t)$ an imposed heating function. There is also the equation of state: we assume a fully ionized electron-proton plasma: $p = 2nk_B T$. [Alternative conduction models are discussed in the Conclusions.]

We are concerned with purely conductive cooling in a static atmosphere with spatially constant pressure. The aim is to isolate the interaction between conduction and cross-sectional area variations. The details of the following methodology have been presented in P. J. Cargill et al. (2022, 2025), so the discussion here is brief. Neglecting heating, radiation and the dynamic terms (but we discuss their likely effect in the conclusions), equation (1) becomes:

$$3nk_B A \frac{\partial T}{\partial t} = \frac{\partial}{\partial s} \left(A \kappa_0 T^{5/2} \frac{\partial T}{\partial s} \right) = \frac{\partial}{\partial s} \left(\frac{2\kappa_0 A}{7} \frac{\partial T^{7/2}}{\partial s} \right) \quad (2)$$

where now $p = p(t)$, $n = n(s)$ and $T = T(s, t)$. We work in a geometry typical of magnetically closed coronal loops with a length $2L$ between their boundaries and symmetry assumed about the mid-point ($s = L$). The loop has a summit temperature of T_1 and a footpoint temperature of T_0 where $T_1 > T_0$. Since the pressure is spatially constant, the summit density is given by $n_1 = n_0 T_0/T_1$. The areas at the apex and base are defined as A_1 and A_0 with $A_1 > A_0$ and, without loss of generality, we set $A_0 = 1$. Fig. 1 summarizes the geometry.

2.1 An approximate solution for non-uniform area

A simple estimate of the role of the area factor comes from an approach based on the Enthalpy Based Thermal Evolution of Loops (EBTEL) method (J. A. Klimchuk et al. 2008; P. J. Cargill, S. J. Bradshaw & J. A. Klimchuk 2012; P. J. Cargill et al. 2022). If we integrate equation (2) once over the length L between the base of the loop and its apex and assume that $dT/ds = 0$ at the latter, it becomes:

$$\frac{\langle A \rangle L}{\gamma - 1} \frac{dp}{dt} = F_0 = -(2/7)\kappa_0 (dT^{7/2}/ds)_{s=0}, \quad (3)$$

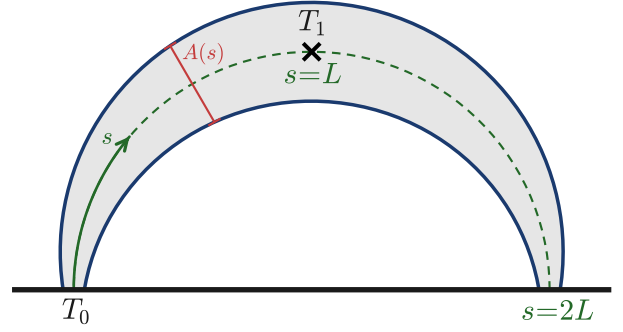


Figure 1. A diagram linking the key variables with the assumed, symmetric, loop model. The green dashed curve portrays a magnetic field line, with s varying along it. The two blue curves illustrate the variation in cross-sectional area, $A(s)$, along the (green) field line.

Table 1. Definition of various cooling times used in the text.

Cooling time	Definition	Equation
τ_c	Calculated value	(9)
τ_{c0}	Uniform area	(6)
τ_*	Approximate value	(6)
τ_{cool}	Time to cool by a prescribed factor	(25)

where the average area of the loop is $\langle A \rangle = (1/L) \int_0^L A ds$. This is an exact result. The right hand side is then approximated as (J. A. Klimchuk et al. 2008)

$$(2/7)(dT^{7/2}/ds)_{s=0} \approx \xi T_a^{7/2}/L, \quad (4)$$

where T_a is the apex temperature and ξ a numerical parameter that determines the magnitude of the heat flux at $s = 0$. equation (3) can then be integrated to give (P. J. Cargill et al. 2025):

$$T_a/T_{a0} = 1/(1 + t/\tau_*)^{2/5}, \quad (5)$$

with T_{a0} the initial apex temperature. We can then define a cooling timescale τ_*

$$\tau_* = \frac{21k_B n_a L^2}{10\kappa_0 T_{a0}^{5/2}} \langle A \rangle = \frac{21p_0 L^2}{20\kappa_0 T_{a0}^{7/2}} \langle A \rangle = \tau_{c0} \langle A \rangle, \quad (6)$$

where τ_{c0} is the cooling time for a uniform cross-sectional area and n_a is the constant apex density. τ_* then scales with $\langle A \rangle$ and the values p_0 , n_a , and T_{a0} are imposed at the start of the cooling phase. We have also set $\xi = 4/7$ here since this value of ξ is the same as that obtained from the AS76 model with a constant area and was also shown to be a good approximation for cooling times in a strictly two-dimensional null point geometry (P. J. Cargill et al. 2025). Values of ξ assumed in earlier literature for constant area are discussed in the Conclusions.

Equation (6) will be taken as a benchmark approximation for conductive cooling times and one purpose of the rest of this paper is to investigate the viability of equation (6) for a range of area profiles. To assist the reader, the various cooling times that we use are defined in Table 1.

2.2 Solution method

We solve equation (2) using the semi-analytic model first proposed for static conductive cooling by AS76. The detailed approach has been discussed in P. J. Cargill et al. (2025), although

in a null point geometry rather than a loop. Defining $G(s) = (2/7)[1/(2k_B n(s))]^{7/2}$, one obtains (on writing s in units of s/L):

$$\frac{3L^2}{2\kappa_0 p^{7/2}} \frac{dp}{dt} = \frac{1}{A} \frac{d}{ds} \left(A \frac{dG}{ds} \right) \quad (7)$$

As discussed by AS76 and P. J. Cargill et al. (2025), equation (7) can be solved by the method of separation of variables with each side of equation (7) equal to a constant, $-k^2$. The left hand side integrates immediately to give:

$$p(t)/p(t=0) = 1/(1+t/\tau_c)^{2/5} \quad (8)$$

where we can define a characteristic cooling time-scale

$$\tau_c = 3L^2/(5\kappa_0 p_0^{5/2} k^2). \quad (9)$$

The dependence of τ_c on the area now appears through the separation constant, k^2 , and therefore on the spatial temperature profile, $T(s)$. The solution of the spatial dependence requires three boundary conditions: two to determine the constants of integration and one to determine k^2 . If the hot end of the loop is at the apex ($s=1$), symmetry requires $dG/ds=0$ there. At the cold end ($s=0$), $G=G_0$ is imposed, so the two integration constants are then determined. Finally, imposition of the apex temperature gives $G(s=1)=G_1$, and this completes the solution. From this solution, the base heat flux and k^2 are determined and hence τ_c . Note though that in reality the base heat flux will be constrained by the importance of either optically thin radiation at the footpoint or an evaporative upflow there (S. K. Antiochos & P. A. Sturrock 1978). This is discussed further in the Conclusions. The present model assumes that below T_0 the downward flux is radiated away. See S. K. Antiochos & P. A. Sturrock (1976).

For a loop with a uniform area, $k^2 = 2(G_1 - G_0)$ and so $\tau_c = 3L^2/(10\kappa_0 p_0^{5/2} G_1(1 - G_0/G_1))$. For $G_0 \simeq 0$, i.e. a cool and high density footpoint, $\tau_c = \tau_{c0} = 21p_0 L^2/(20\kappa_0 T_{a0}^{7/2})$, as noted earlier.

While we present our results in terms of two cooling time ratios: τ_c/τ_{c0} and τ_c/τ_* , it is useful to consider some physical numbers. As a benchmark, we take $L = 25$ Mm, the apex and footpoint temperatures as 10 and 1 MK, respectively, the apex density is 10^9 cm^{-3} , so that p_0 is $0.276 \text{ dyne cm}^{-2} \text{ s}^{-1}$. This gives $\tau_{c0} = 5.73$ s. Different values of, for example, L involve a simple rescaling.

3 RESULTS

The aim is to present a range of area profiles for which the change in area ranges from happening predominately near $s=1$ (for example a null point: see P. J. Cargill et al. 2025) to slowly throughout the loop (e.g. a dipole field), to near the footpoints (an abrupt constriction at the base of the corona). The general solution is presented in Section 3.5. However, this revealed a broad range of scalings of τ_* with $\langle A \rangle$. This encouraged us to generate a number of analytic solutions to pin down the physics behind these results. These are presented in the following three sub-sections.

In investigating non-uniform areas, our general approach is to treat the ratio of base to apex cross-section as being independent from the intermediate area profile. Thus, for a fixed apex area A_1 , we can vary the location and sharpness of the transition from small to large area.

3.1 Analytic solutions: step-function and related area profiles

Consider first:

$$A(s) = 1 + \frac{A_1}{2} \left(1 + \tanh \left[\frac{s - s_m}{s_1} \right] \right). \quad (10)$$

Analytic solutions are possible in the limit $s_1 \rightarrow 0$ so that the area jumps abruptly at $s = s_m$ from $A = 1$ for $s < s_m$ to $A = A_1$ for $s \geq s_m$. This might correspond to the base of a loop for small s_m or the vicinity of a current sheet or null for large s_m . The right hand side of equation (7) then becomes a simple second order equation in both parts of the loop, and a solution can be obtained by matching AdG/ds (the heat flux) and G (the temperature) at $s = s_m$. This gives a solution:

$$G = G_0 - k^2 s^2/2 + k^2 s \left[s_m + (1 - s_m)A_1 \right], \quad s < s_m, \quad (11)$$

$$G = G_1 + k^2 \left[s - (s^2 + 1)/2 \right], \quad s > s_m, \quad (12)$$

$$k^2 = \frac{2(G_1 - G_0)}{1 + 2s_m(1 - s_m)(A_1 - 1)}, \quad (13)$$

and so τ_c follows from equation (9). For a prescribed value of A_1 , the minimum of k^2 , and so the longest cooling time, has $s_m = 0.5$ (midway between the apex and base) and this corresponds exactly to the scaling in equation (6): $k^2 = 2G_1/\langle A \rangle$ for $G_0 \ll G_1$, where $\langle A \rangle = s_m + (1 - s_m)A_1$. Further, the values of τ_c are symmetric about $s_m = 0.5$.

The upper left panel of Fig. 2 shows τ_c/τ_{c0} for small $s_1 = 0.01$, with the ratio of the apex to base area increasing from 1 to 50^{12} and s_m varies between 0.1 to 0.5 with s_m increasing from lower to upper curves. (Only five cases of s_m are shown due to the symmetry around $s_m = 0.5$: thus $s_m = 0.9$ has the same values of τ_c as $s_m = 0.1$, etc.) For $s_m = 0.5$, τ_c increases by a factor of 25 over τ_{c0} for the largest area variation. However, as s_m increases (decreases) above (below) 0.5, the cooling times decrease for a given value of A_1 , typically by a factor between 2 and 3 when $s_m = 0.1$. The constant area result is recovered when $s_m = 0$ and 1, though care is needed as s_m approaches but does not equal 0 or 1.³

The results can be understood in terms of two related effects that are evident in the limit of large A_1 . In this case:

$$k^2 = \frac{(G_1 - G_0)}{s_m \langle A \rangle} \quad (14)$$

and:

$$\tau_c = 2\tau_{c0} \langle A \rangle (s_m/L), \quad (15)$$

where s_m is now written in physical units. This demonstrates that the effective cooling length in the approximation of the heat flux equation (4) is decreased by a factor s_m/L which increases as s_m

¹For all figures in Section 3.1–3.4 we show numerical solutions of equation (7), and have confirmed that the numerical and analytic solutions are in agreement

²Throughout this paper, we consider large values of A_1 , partly because this range makes the underlying scalings clearer.

³If we set $s_m = \delta_1$ as s_m approaches zero, and $s_m = 1 - \delta_2$ as s_m approaches one, we find, for large $\langle A \rangle$ and G_1 also large that $k^2 = 2G_1/(1 + 2\delta_1 A_1)$ and $k^2 = 2G_1/(1 + 2\delta_2 A_1)$. However, when s_m equals 0 or 1, $A_1 = A_0$, and one recovers the uniform area value. Given this problem, we limit s_m to the range $0.1 \leq s_m \leq 0.9$.

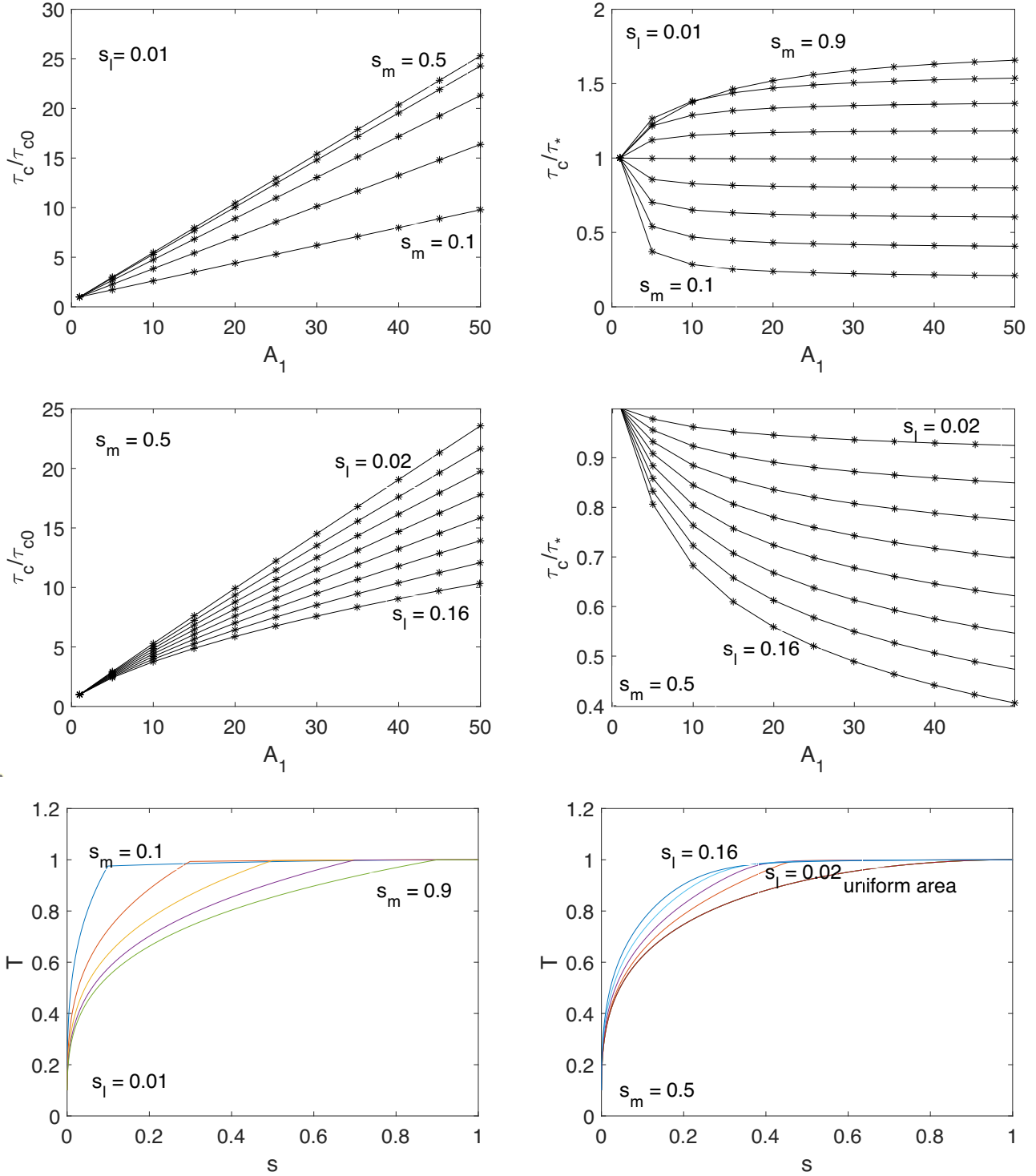


Figure 2. Upper left: The ratio of the cooling time, as defined in the text (τ_c), to the cooling time for a loop of uniform area (τ_{c0}) as a function of apex area for a step-function area profile. s_m increases upwards for each curve from 0.1 to 0.5. Upper right shows the ratio τ_c / τ_* as a function of A_1 with s_m now increasing from bottom to top. The middle two panels show the corresponding results for a tanh profile with $s_m = 0.5$ and s_l increasing downwards from 0.02 to 0.16. The lower panels show the temperature profile normalized to its apex value for various values of s_m and s_l for $A_1 = 50$.

increases. Conversely $\langle A \rangle$ decreases as s_m increases. This gives good agreement with the full solution shown in Fig. 2.

Further understanding arises when we recall that the conduction term for non-uniform area can be written as:

$$\frac{1}{A} \frac{d}{ds} \left(A \frac{dG}{ds} \right) = \frac{d^2 G}{ds^2} + \frac{1}{A} \frac{dA}{ds} \frac{dG}{ds}, \quad (16)$$

with the first term on the right hand side just the conductive term for a uniform area (e.g. J. F. Vesecky, S. K. Antiochos & J. H. Underwood 1979; P. J. Cargill et al. 2022). It has been pointed out that when dA/ds and dG/ds are both positive, the second term is an effective heating (same sign as Q in equation 1). This effect is present in all solutions to the conduction equation having a variable area, but we will show that its precise profile is very significant. For the present area profiles, where A jumps from small to large at $s = s_m$, there is a localized ‘heating’ pulse, which has the effect of producing a plasma close to isothermal to the right of s_m , as is shown in the lower left panel of Fig. 2. The effective cooling distance of the loop is then not L but s_m , which accounts for the scaling in equation (15).

We are also interested in how the approximate cooling time, τ_* given in equation (6), compares with these solutions. The upper right panel of Fig. 2 shows the ratio τ_c/τ_* for the same values of s_m , as before, with s_m decreasing from the upper curve to the lower curves. If this scaling holds, then all values of s_m should be straight lines with $\tau_c = \tau_*$. Clearly, this is not the case. As s_m decreases, the loop cools more rapidly than the simple expression implies.

The ratio τ_c/τ_* in turn determines the parameter ξ discussed earlier. ξ represents how F_0 , the heat flux at the lower boundary, is influenced by the complete temperature structure, $T(s)$, and in our units we can write $F_0 = dG/ds(s=0)$ which in turn leads to an expression

$$\xi = F_0/G_1 = (2/7)(dG/ds)_0/G_1 \quad (17)$$

using equation (4). ξ can also be expressed in terms of the cooling times ratio:

$$\xi = 4/7(\tau_c/\tau_*) \quad (18)$$

Thus larger heat fluxes at the base increase ξ and decrease the cooling time. For the step function area profile, we find

$$\left(\frac{dG}{ds}\right)_{s=0} = k^2(s_m + (1 - s_m)A_1) \quad (19)$$

and so

$$\xi = \frac{4}{7}[s_m + (1 - s_m)A_1]/[1 - 2s_m(1 - s_m)(1 - A_1)] \quad (20)$$

Recall that when $A_1 = 1$, $\xi = 4/7$, which is also the value for $s_m = 0.5$. The cases where $\tau_c < \tau_*$ imply that $\xi > 4/7$ and hence there is a larger heat flux through the base of the loop than is assumed in equation (6). This can also be seen in the lower left panel of Fig. 2 where the smaller values of s_m have a steeper temperature gradient at $s = 0$.

Next, we increase the distance over which the area changes from $s_l = 0.02$ to $s_l = 0.16$ with $s_m = 0.5$. This does not permit analytic solutions and the middle panels of Fig. 2 shows the numerical results in the same format as the upper ones. The cooling time now changes by a significant factor for the same $\langle A \rangle$: the broadening of the area transition decreases the cooling time by about a factor of 3 for the largest s_l and A_1 . Also, while the scaling in equation (6) was satisfied exactly for the discontinuous area profile with $s_m = 0.5$, this is no longer the case. Once again, the changing area has a tendency to make the upper part of the loop more isothermal (lower right), thus decreasing the effective cooling length. By examining the ‘effective heating’, we find that the maximum of this heating shifts towards the footpoint from the midpoint as s_l increases.

We also find that for a given A_1 , as s_l increases from 0.02 to 0.16, the parameter ξ also increases. The right lower panel of Fig. 2 shows that for an abrupt transition at $s_m = 0.5$, the temperature to the left declines more smoothly than for the broader area variation. For small ratios of A_1 the increase in ξ is relatively small: 20 per cent for an area factor of 3, while for the larger area factors it increases by a factor of 2.5.

3.2 Analytic solutions: linear area profiles

A second analytic solution of equation (7) arises for a linear area profile of the form $A(s) = 1 + \alpha s$. This, and the next two examples, come closest to representing cooling in a dipole field. Note that symmetry around $s = L$ implies a discontinuous value of dA/ds there, but equation (16) shows that dA/ds is multiplied by dG/ds which is zero at the apex, so this discontinuity has no effect on the solution. Integrating equation (7) gives:

$$\frac{G(s) - G_0}{k^2} = \frac{(1 + \alpha)^2}{2\alpha^2} \ln(1 + \alpha s) - \frac{s^2}{4} - \frac{s}{2\alpha}, \quad (21)$$

so that

$$k^2 = (G_1 - G_0) / \left[\frac{(1 + \alpha)^2}{2\alpha^2} \ln(1 + \alpha) - \frac{1}{4} - \frac{1}{2\alpha} \right] \quad (22)$$

On expanding the logarithm term, this gives $\tau_c = \tau_{c0}$ for $\alpha = 0$. For large α we find:

$$k^2 = 2(G_1 - G_0) / [\ln(\alpha) - 1/2], \quad (23)$$

so that τ_c now scales with $\ln(\alpha) = \ln(2\langle A \rangle)$. The left panel of Fig. 3 shows τ_c/τ_{c0} (stars) and τ_c/τ_* (circles). The right hand panel shows some temperature profiles as a function of α . We see that the increase in the cooling time as A_1 increases is smaller than for the previous step function examples, a factor of 3.5 between $A_1 = 1$ and $A_1 = 50$, and this should be compared with an expected factor of 25 from equation (6). Again one can understand the results in terms of the ‘effective heating’ argument above. Here, this effective heating is localized toward the footpoint, leading to a more isothermal corona (see right panel of Fig. 3), again decreasing the effective cooling length. Using equation (17), for large α , ξ scales as $\alpha/\ln(\alpha)$, and increases from 4/7 for uniform area to 4.1 for the largest area factor.

3.3 Analytic solutions: hyperbolic function

The next example has $A(s) = \cosh^2 \lambda / \cosh^2[\lambda(1 - s)]$ so that in terms of A_1 , $\lambda = \cosh^{-1}(\sqrt{A_1})$. This has a solution

$$G - G_0 = \frac{k^2}{4\lambda^2} (\cosh(2\lambda) - \cosh[2\lambda(1 - s)]) \quad (24)$$

so that

$$k^2 = 4\lambda^2(G_1 - G_0) / [\cosh(2\lambda) - 1] \quad (25)$$

On setting $\cosh(2\lambda) \approx 1 + (2\lambda)^2/2!$ and letting $\lambda \rightarrow 0$, we obtain the required result for a uniform area.

Varying A_1 over the same range as before, we find that the ratio τ_c/τ_{c0} is larger than for the linear profile, taking on a value of 7 for $A_1 = 50$ while the ratio τ_c/τ_* is also larger, 0.4 for $A_1 = 50$. We can understand this in terms of the effective heating from equation (16). Compared to the linear area profile, the hyperbolic function has this effective heating moved toward the loop apex.

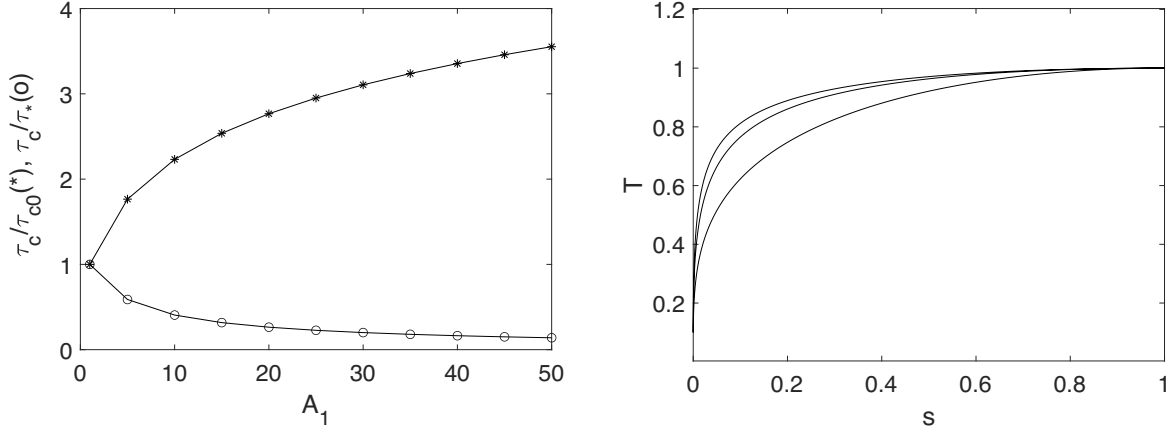


Figure 3. The cooling time ratios (left) as a function of apex area for a linear area profile. The right hand panel shows the temperature profiles normalized to its apex value for three values of α , 0, 4, and 8 going from right to left.

Once again we can calculate ξ :

$$\xi = \frac{4}{7} \frac{\lambda \sinh \lambda}{\cosh 2\lambda - 1} \quad (26)$$

so that for large λ , we find $\xi = 4\lambda \tanh(2\lambda)/7$. We also obtain the correct value of ξ as λ approaches zero.

3.4 Analytic solutions: line and point dipoles: AS76

Next, we address the analytic solutions of AS76. Fig. 2 of that paper makes clear that the cooling time does not follow a simple scaling with $\langle A \rangle$. For $A_1/A_0 > 10$, the point dipole cools more slowly than the line dipole, and $\langle A \rangle$ of the line dipole is greater than for the point dipole, the opposite to the expected scaling. The heat flux for the line dipole is also greater than that of the point, indicative of faster cooling (AS76 Fig 4). We have regenerated the AS76 results for both cases. They are in fact very similar to the linear area profile discussed above and, for large area variations, they show a similar logarithmic scaling with the area factor. While the point dipole is more complicated to analyse, the cooling is slower than the line dipole but does not get close to the $\langle A \rangle$ scaling.

3.5 A more general cross-section

It is obvious from these results that the simple scaling $\tau_c \sim \tau_*$ fails so long as the parameter ξ is fixed at $4/7$. However, as we will next show, there are regimes where these approximations do work well, such as for null point cooling in our previous paper. We consider an area profile of the form:

$$\begin{aligned} A(s) &= 1 + f_1 \sin(\pi s/2s_0)^{a_f}, \quad s < s_0 \\ A(s) &= 1 + f_1, \quad s_0 < s < L. \end{aligned} \quad (27)$$

where s_0 is taken to be close to unity. We generate a similar range of solutions to those shown above. Twelve values of the parameter a_f are considered, ranging from 0.2 to 500⁴. Fig. 4 shows a series of area profiles. We are able to confine the area change to different parts of the loop with respect to the hotter end so that small a_f confines the area change to the footpoints (such as a transition region constriction), large a_f to the apex (similar to the magnetic

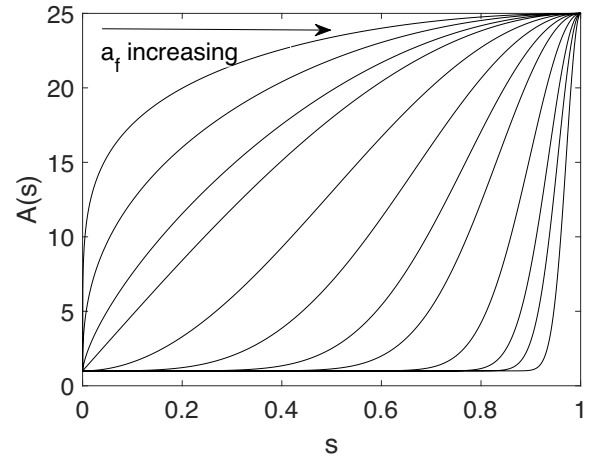


Figure 4. The area profiles for the 12 values of a_f in equation (27). a_f increases from left to right. $A_1 = 25$.

null) and an intermediate value such as $a_f = 1$ is similar to the linear profile discussed above and $a_f = 4$ to the hyperbolic one which we noted were similar to the dipole field.

The upper and lower panels of Fig. 5 show the ratios τ_c/τ_{c0} and τ_c/τ_* as a function of A_1 and $\langle A \rangle$, respectively. The left upper plot is colour coded according to two different ranges of a_f . In all other plots the direction of increasing a_f is shown either on the plot or in the caption.

Looking at the τ_c/τ_{c0} results first, we see that for all values of A_1 (and hence $\langle A \rangle$), this ratio is >1 so that an area change always slows conductive cooling. However, the effect is relatively small when the area change is confined to near the footpoints and, to a lesser degree, the apex. The lower left plot shows a more subtle situation. The maximum cooling time over all values of $\langle A \rangle$ and a_f occurs for a_f between 4 and 15 and A_1 between 10 and 15, with the cooling time falling off rapidly on either side.

Looking now at τ_c/τ_* , the upper right plot indicates that use of τ_* can underestimate the cooling time by up to 50 per cent when the area change is localized at the apex and over-estimate it by increasingly large values as the area change becomes first uniform and then concentrated at the footpoint. The discrepancy can be as large as a factor 10 for the largest A_1 . The ratio is near

⁴The 12 values are 0.2, 0.4, 0.7, 1, 2, 4, 8, 15, 40, 100, 200, 500.

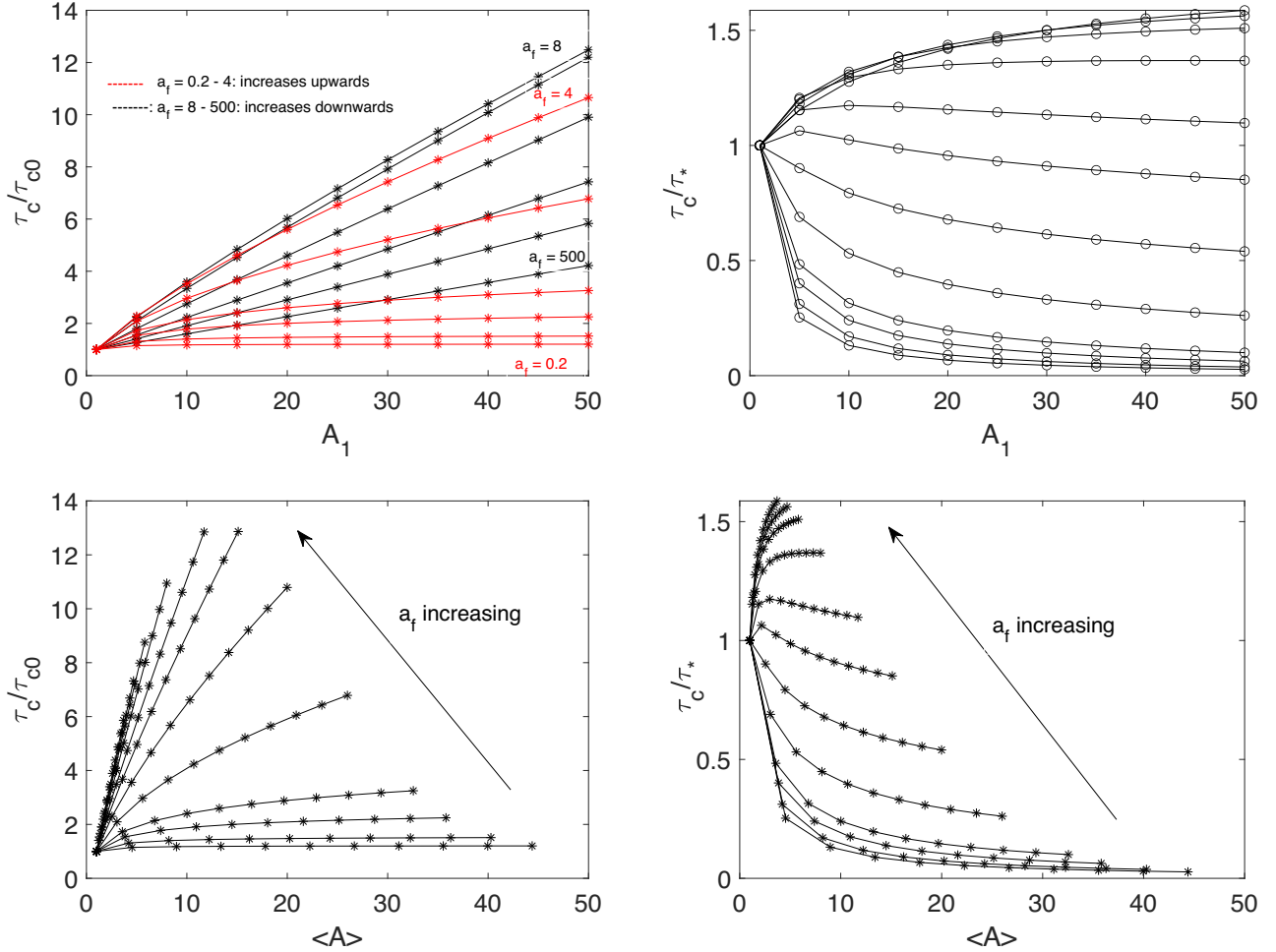


Figure 5. Upper left and right. The ratios τ_c/τ_{c0} and τ_c/τ_* as a function of A_1 . The red (black) curves in the upper left show a_f in the range 0.2–4 and 8–500, respectively. a_f increases in the direction shown on the plot. The upper right plot has a_f decreasing from upper to lower. The lower two curves show the same ratios as a function of $\langle A \rangle$. In all cases, the complete range of a_f are shown, increasing from 0.2 to 500 as indicated by the direction of the arrow.

unity for $a_f = 8$ which bears a resemblance to the null point geometry in our earlier paper. These results serve to demonstrate the vast range of possible solutions to what has been perceived as a well understood problem.

4 DISCUSSION

This paper has returned to a long standing problem in solar coronal physics, namely the nature of plasma cooling by thermal conduction in coronal loops with a varying cross-sectional area. While this problem has a long history, and has been understood in very general terms, we have revealed for the first time the large range of plasma cooling that can arise for different area profiles.

Simple considerations indicate that conductive cooling times can be expected to scale with an average loop cross-section ($\langle A \rangle$). However, our analytic solutions for one case indicate that a log-scaling occurs for a linearly changing area, and indeed a whole continuum of cooling time scalings arise from weaker than logarithmic to the average area. Thus, in very general terms, conductive cooling in such loops needs to be considered on a case-by-case basis, especially for large area variations. Perhaps serendipitously, our study of null point cooling came close to the simple $\langle A \rangle$

scaling of the cooling time. This arises because the $\langle A \rangle$ works best when the main area variation and hot plasma coincide.

Thus, the **AS76** methodology and the generalisations presented here prove to be very powerful tools. It can be readily extended to cases where $\kappa = \kappa_* T^m$, as discussed in the appendix, and we here denote the constant as κ_* to emphasize that it is not the same as κ_0 . Here, we find that, in addition to a power law decay of temperature with time, there are values of m where the cooling becomes catastrophic, such that the temperature reaches zero in a finite time. A particular class of such solutions corresponds to the turbulent cooling model of N. H. Bian, E. P. Kontar & A. G. Emslie (2016a); N. H. Bian et al. (2016b) applied to a loop in initial pressure equilibrium.

We have restricted ourselves to studying static thermal conduction. In another pioneering study S. K. Antiochos & P. A. Sturrock (1978) proposed evaporative conductive cooling. In this case, the loop pressure remains constant and an upflow driven by the downward conduction leads to an increase in the loop density, which in turn leads to slower conductive cooling. This solution also relies on the separation of variables approach which leads to a non-linear equation for G , but is amenable to our analysis, though analytic solutions appear unfeasible for non-uniform ar-

eas. The neglect of optically thin radiation will also be violated as the loop cools. In simple terms one can argue that a loop cools firstly by conduction then by radiation (e.g. P. J. Cargill, J. T. Mariska & S. K. Antiochos 1995), with the transition from one to the other occurring at 3–4 MK for a loop with an initial temperature of 10 MK (e.g. P. J. Cargill 1994). This requires a full numerical simulation (e.g. J. W. Reep et al. 2022).

We also wish to comment on what is meant by the widely used term ‘cooling time’. This is often taken to assume some sort of exponential decay in temperature with a ‘cooling time’ taken to be the time for the temperature to fall by a factor e though that is only the case for a simple Newton’s law of cooling model where a small temperature perturbation is introduced in a plasma. In fact, going back to J. L. Culhane et al. (1970), for non-linear conduction, the cooling has followed power law decay as in equation (9). However, the term has real meaning only when considered over a certain range of temperatures. Defining the time taken for the temperature to cool by a certain factor as τ_{cool} , we find:

$$\tau_{\text{cool}} = \tau_c \left((T_{\text{start}}/T_{\text{end}})^{5/2} - 1 \right) \quad (28)$$

where T_{start} and T_{end} are the chosen start and finish temperatures over which one is interested in. Clearly $\tau_{\text{cool}} > \tau_c$ for many temperature ratios. For example, cooling by a factor of two gives $\tau_{\text{cool}} = 4.66\tau_c$. Once again τ_{cool} needs to be considered on a case-by-case basis when there are area variations.

Why does this matter? The nature of conductive cooling has been of considerable interest in determining the lifetimes of hot, impulsively heated coronal plasmas such as large flares. Especially important are long-lived post-flare loops where a perceived ‘cooling time’ is much shorter than the observed decay time of the emission, which in turn suggests long-term heating (e.g. R. A. Kopp & G. W. Pneuman 1976; P. J. Cargill & E. R. Priest 1982). In recent years, this interest has extended to non-flaring hot sources, as detected in active regions (e.g. F. Reale et al. 2009; P. Testa et al. 2011; J. T. Schmelz & S. Pathak 2012) and at magnetic null points (X. Cheng et al. 2023). Various effort have been made to extend cooling times, such as turbulent heat conduction (N. H. Bian et al. 2016a) and area variations (AS76). Both approaches have considerable uncertainties. Turbulent conduction involves several almost entirely unknown parameters and area-driven changes, as we have shown, depend rather sensitively on the area profile. The latter problem can, to a degree be mitigated by hydrodynamic simulations using extrapolated magnetic fields.

Finally, we return to the parameter ξ introduced earlier. This has taken on a variety of values in simple models: the analytic work of, for example, P. J. Cargill (1994) and P. J. Cargill & J. A. Klimchuk (2004) set $\xi = 1$, while the EBTEL model sets $\xi = 2/7$ (J. A. Klimchuk et al. 2008). But it now appears that, for uniform loops, $2/7$ is too small and 1 too big compared with the correct value of $4/7$. The former (latter) systematically suppresses (enhances) conduction. For more general area profiles, a large range of values are possible, and this paper has provided some guidance as to what values may be appropriate in a given situation.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: GENERALIZED CONDUCTION COEFFICIENTS AND TURBULENT COOLING MODELS

Consider a scaling $\kappa = \kappa_* T^m$. Then the conduction equation becomes:

$$3nk_B \frac{\partial T}{\partial t} = \frac{\partial}{\partial s} \left(\kappa_* T^m \frac{\partial T}{\partial s} \right) = \frac{\kappa_*}{m+1} \frac{\partial^2 T^{m+1}}{\partial s^2}. \quad (A1)$$

For $m > -1$ this gives physical solutions. Defining $G = 1/(m+1)(1/(2k_b n))^{m+1}$, we obtain a separable equation as before which gives for $m \neq 0$

$$p/p_0 = \left[1 + 2\kappa_* m p_0^m k^2 t / 3L^2 \right]^{-1/m}, \quad (A2)$$

and the solution for G is identical to that in the text. For constant area $k^2 = 2G_1$ so that we find:

$$p/p_0 = \left[1 + \text{sgn}(m) \frac{4\kappa_*}{3L^2} \frac{m}{m+1} \frac{T_0^{m+1} t}{p_0} \right]^{-1/m}, \quad m \neq 0, \quad (A3)$$

where $\text{sgn}(m)$ is the sign function and this reduces to the earlier value for $m = 5/2$.

Clearly the case $m = 0$ needs special treatment, and the result is exponential temperature decay. The introduction of the sgn function represents a fundamental change in the solution as m

becomes negative. For $m > 0$ there is asymptotic decay, i.e. the temperature approaches zero as time becomes large. When $m < 0$, there is catastrophic cooling, i.e. the temperature reaches zero in a finite time.

Of special interest in this context is the conduction mode of N. H. Bian et al. (2016a, 2016b). The model argues that small-scale magnetic field fluctuations can scatter electrons, leading to an inhibition of conductive cooling. The important aspect of the model is that the anomalous mean free path is independent of density, so that there is a density dependence in the conduction coefficient. The model has several parameters related to the turbulence, and the reader is referred to N. H. Bian et al. (2016a) for a discussion of these. We incorporate these into the constant κ_* since it is not our intention to analyse their results fully.

For $\kappa \propto nT^{1/2}$, on substitution for n , gives $m = -1/2$. This leads to a scaling $p/p_0 = 1/(1 - t/\tau_c)^2$, where τ_c is now a generic cooling time, so catastrophic cooling arises. On the other hand, N. H. Bian et al. (2016b) assume that the density⁵ is constant, leading to $m = 1/2$ and their equation (32). The difference lies in

our model starting from a pressure equilibrium, and N. H. Bian et al. (2016a) modelling very rapid heating which may not be in pressure equilibrium. Thus, the plasma density appears nowhere in their expressions for plasma cooling, unlike our AS76-based model.

Finally, it should be noted that the spatial temperature structure solution for G is independent of the factor m , though of course the temperature profile is dependent on m . In our analytic examples, we have shown that the separation constant k^2 can be written in the form $k^2 = 2G_1 F(\text{area})$ where $F(\text{area})$ is a general functional representation of the area models. One can then deduce that the influence of the area is largely independent of the value of m .

⁵Although there can be a density dependence in the parameter R

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