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Optimal operation of industrial compressor stations in systems with large energy consumption

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Declaration of originality

I herewith certify that all material in this dissertation which is not my own work has been properly acknowledged.

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Abstract

The aim of the thesis is to study the optimal operation of compressor stations in systems with large energy consumption such as process systems and natural gas networks. Compressor stations include several compressors in parallel which usually account for the major part of the total energy consumed in the system. Therefore, the efficient operation and maintenance of the compressors could save energy, and reduce operational costs.

The development of optimisation frameworks and optimisation models can determine the decisions which lead to the minimisation of costs operating compressor stations. The modelling of the behaviour of the compressors is also an important topic of the current PhD study. Different types of models should be used according to the level of optimisation level (online and real time, and offline), considering the available resources such as compressor maps or process data.

The thesis developed a comprehensive real time optimisation framework which can reduce the power consumption of compressor stations compared to the case of operation with the existing industrial practices. The thesis also developed a mixed integer linear programming multi-period optimisation model to minimise total costs of the operation, for example electricity costs, and start up and shut down costs. Another contribution of the thesis is the integration of operation and maintenance of the compressors considering different types of maintenance activities such as major overhauls and the washing of compressors. For example, the proposed integrated framework can be used to generate the schedules of the online and offline washing of the compressors compared to existing approaches described by fixed periodical washing or washing when the degradation of the condition of the compressors has reached unacceptable limits.

The optimisation frameworks was applied to two industrial case studies, namely one air separation plant involving a network of air compressors in BASF, Ludwigshafen, Germany and one export natural gas compressor station operated by Statoil in Norway. The final chapters of the thesis discuss the contributions and assumptions of each method, and present potential new research areas deriving from the PhD study.

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"In today's rush we all think too much, seek too much, want too much and forget about the joy of just Being"
Eckhart Tolle

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Nomenclature

Indices/Sets

$a \in A$	process plants for compressed air utility
$ch1 \in CH1$	observations of variables of convex hull problem one
$ch2 \in CH2$	observations of variables of convex hull problem two
$ch3 \in CH3$	observations of variables of convex hull problem three
$e \in E$	air separation products (e.g., N_2 and O_2)
$i \in I$	compressors
$j \in J$	headers
$k \in K$	discrete areas of compressor maps
$u \in U$	air separation columns
$v \in V$	process variables in the steady-state detection (RTO)
$z \in Z$	storage tanks
$t \in T$	time periods
tr	time moment of the beginning of RTO calculations
$\delta \in \Delta$	external sources providing natural gas or product

Subsets

$\Delta_{(j)}$	set of external sources providing gas/product to header j
$J_{(i)}$	set of headers connected with compressor i
$J_{(a)}$	set of headers that are connected to process plant a
$J_{(u)}$	set of headers that are connected to air separation column u
T^{sf}	time window of scheduled maintenance
$U_{(z)}$	set of air separation columns connected to storage tank z
V'	set of monitored process variables in steady-state detection
$Z_{(e)}$	set of storage tanks that can store product e
$\Delta_{(j)}$	set of sources providing gas to header j
I^{dm}	set of compressors that are subject to fixed maintenance
$\tilde{I}^{dm}$	set of compressors under maintenance before optimisation
I^{b}	set of large compressors
I^{s}	set of small compressors
I^{NOFF}	compressors no under offline maintenance before optimisation
I^{fm}	set of compressors subject to flexible maintenance

I^{M}	set of compressors subject to maintenance
I^{M1}	set of compressors subject to light maintenance
I^{M2}	set of compressors subject to major overhaul
I^{NM}	set of compressors not subject to maintenance

Subscripts

act	actual conditions
act,s	actual conditions at the inlet (suction) of a compressor
aux	auxiliary variables
ch	choke (stonewall)
cor	corrected
d/out	discharge/outlet (outlet position of compressor)
dem	demand
el	electrical
g	gas
in/s	inlet/suction (inlet position of compressor)
mech	mechanical (refers to mechanical power)
meas	measured variables
ref	reference conditions
r.d	regression domain
sur	surge
std	standard conditions
STP	standard temperature and pressure
tot	tot

Superscripts

*	scaled parameters and variables in RTO optimisation model
fxd	fixed maintenance
min	minimum
max	maximum
o	initial conditions
out	outlet (refers to outlet of tanks)
util	utilities

Parameters

$b/(\mathbf{b})$	estimated coefficients of regression model of Eq. (4.4) [-]
b_{ch}	coefficients of expression for choke line [-]
b_h	coefficients of expression for head of a compressor [-]
b_L	coefficients of expression for minimum speed line [-]

b_s	coefficients of expression for surge line [-]
b_U	coefficients of expression for maximum speed line [-]
b_w	coefficients of expression for shaft power of a compressor [-]
b_{el}	coefficients of expression for electrical power consumption [-]
C_{el}	electricity cost [EUR/kWh]
C_{ext}	purchase cost of product/gas from external sources [EUR/kg]
C_D	cost of header change [EUR/change]
C_f	shutdown cost [EUR/shutdown]
$c_{f,MolW}$	ratio of actual and reference molecular weight [-]
$c_{f,P}$	ratio of actual and reference pressure [-]
$c_{f,T}$	ratio of actual and reference temperature [-]
C_{st}	start up cost [EUR/startup]
C_{of}	offline washing cost [EUR/wash]
C_{on}	online washing cost [EUR/wash]
c_{std}	conversion factor from MMsm^3/d to m^3/h [-]
d^o_{of}	duration of offline washing before optimisation [d]
d^o_{on}	duration of online washing before optimisation [d]
d_c	conversion factor, equal to 24$\cdot$ 3600 [-]
h	parameter of steady-state detection of RTO [-]
M^*	numbers for Big-M formulation [-]
M^{es}	Big-M numbers of equal split, Eqs. (5.71) – (5.72) [-]
M^{esm}	Big-M numbers of equal surge margin, Eqs. (5.75) – (5.76) [-]
M^{ext}	mass flow rate of gas/product from external sources [kg/s]
M_{dem}	demand in mass flow rate [kg/s]
$M_{in,x}$	extreme points of inlet mass flow rate data-set [kg/s]
M_{prod}	min/max boundaries of separation columns flows [kg/s]
$M_{r.d.}$	min/max boundaries of mass flow rate of compressor [kg/s]
M^{max}_{rec}	maximum recycle flow rate [kg/s]
M_w	mass flow rate of intercoolers[kg/s]
$MolW$	molecular weight [mol/kg]
N	min/max boundaries of rotational speed [RPM]
n_s	data included in a time window of a steady-state detection [-]
P_{dem}	demand in pressure [bar A]
P_{in}	pressure at the inlet of a compressor [bar A]
P_{op}	operational pressure of downstream process [bar A]
P_{out}	pressure at the outlet of a compressor [bar A]
$P_{d,x}$	extreme points of discharge pressure data-set [bar A]
$Q_{b/c}$	volumetric flow rate boundaries of compressor maps [m^3/h]
Q^{max}_{ch}	maximum volumetric flow rate at choke line [m^3/h]

Q_{dem}	demand in volumetric flow rate [m^3/h]
Q_{sur}^{min}	minimum volumetric flow rate at surge line [m^3/h]
Q_{std}	demand in volumetric flow rate [MMsm^3/d]
R	ideal gas constant, equal to 8,314 [kJ/(mol·K)]
R_F	recovery factor of online washing model [-]
R_g	gas constant [kJ/(kg·K)]
S	number of data per step of the steady-state detection [-]
T_d	temperature of the gas at the delivery point [K]
T_{in}	temperature at the inlet of a compressor [K]
$T_{in,x}$	extreme points of inlet temperature data-set [K]
t_S	number of days before the end of the time horizon [d]
$\tilde{v}_{tnk}^{stor}$	initial inventory of products in storage tanks [kg]
v_y	variance used in steady-state detection algorithm [-]
V_{dem}	demand of products for each period [kg]
V_{tnk}	min/max storage capacity for products in storage tanks [kg]
W^{max}	maximum shaft power [kW]
x	input measured variables of data-driven model (Eq. (4.4)) [-]
$\bar{x}$	mean value of input measured variables x
y	output predicted variable of data-driven model (Eq. (4.4)) [-]
Y_{ss}	defines steady-state status of a process variable [-]
$Y_{ss,system}$	defines steady-state status of a system [-]
Z	compressibility [-]
$\mathbf{z}$	input parameter vector of the RTO optimisation model
α_0, α_1	coefficients of load curve of headers [-]
γ^*	min. time period of two consecutive overhauls [d]
γ	min. time between two consequent online washing events [d]
ϵ	degradation rate of compressor [kW/d]
δS^o	initial cumulative hours of operation before optimisation [d]
η/Λ_C^{max}	max. number of compressors to be maintained in a period [-]
$\tilde{\eta}$	defines if a compressor is subject to fixed maintenance [-]
κ	conversion factor of mass flow rate [-]
$\lambda_{off/on}$	max. number of offline/online washings per time period [-]
λ_k	min. number of offline washing episodes [-]
Λ_{El}	number of electrical maintenance actions [-]
Λ_{Me}	number of mechanical maintenance actions [-]
μ	mean of a set of values [depends on parameter]
ν	duration of maintenance (general case) [d]
ν_{OV}	duration of major overhaul [d]
ν^{fxd}	duration of fixed maintenance of compressors [d]

$\tilde{\nu}^{fxd}$	total time that compressors have been under maintenance [d]
o	maximum online time after the startup of compressor i [d]
π	min/max boundaries of pressure ratio P_d/P_{in} [-]
ρ	density [kg/m^3]
σ	vol. % of components of air [-]/standard deviation [-]
τ	position of moving time window of steady-state detection [-]
τ^{fxd}	starting time of fixed maintenance model for compressor i [d]
τ^{ls}	end of maintenance time window T_{sf} [d]
τ^{es}	beginning of maintenance time window T_{sf} [d]
ϕ	polytropic exponent of compressors [-]
$\tilde{\phi}$	initial connection between compressor and header j [-]
$\tilde{\chi}$	initial operational status of compressors [-]
ψ	minimum shut down time [d]
$\tilde{\psi}$	continuous days of non-operation before optimisation [d]
ω	minimum running time [d]
$\tilde{\omega}$	continuous days of operation before optimisation [d]
Ω	maximum extra power because of degradation [kW]

Continuous variables

H	head of a compressor [kJ/kg]
M_{in}	mass flow rate at the inlet of a compressor [kg/s]
M_{rec}	recycling mass flow rate [kg/s]
$M_{tot}/\bar{M}$	total mass flow rate at the inlet of a header [kg/s]
M_{out}	mass flow rate at the outlet of a compressor [kg/s]
M_{prod}	mass flow rate of product e for air separation column u [kg/s]
N	shaft rotational speed [RPM]
$P_{d/out}$	outlet pressure of compressor (known also as discharge) [bar A]
Q	volumetric flow rate [m^3/h]
Q_{esm}	volumetric flow rate of surge line [m^3/h]
Q_{ym}	volumetric flow rate which refers to the surge margin [m^3/h]
V^{ext}	amount of products or gas from external source $\delta \in \Delta_{(j)}$ [kg]
V_{prod}	amount of separation columns products to buffer tanks [kg]
V_{tnk}^{out}	amount of products from storage tanks at time period [kg]
V_{tnk}^{stor}	inventory level of products in storage tanks at time period t [kg]
W	power [kW]
W_{cor}	corrected shaft power of a compressor [kW]
W_{el}^{c}	power consumed when compressor is not fouled [kW]
ΔS	cumulative time of the operation after the last maintenance [d]
ΔS_{aux}	auxiliary variable to linearise the online washing model [-]

$\Delta S_{on,aux}$	aux. variable to linearise online-offline washing model [-]
ΔW	extra power consumed from the motor of compressor i at t [kW]
λ_1	parametric variable of the convex hull problem one [-]
λ_2	parametric variable of the convex hull problem two [-]
λ_3	parametric variable of the convex hull problem three [-]
π	pressure ratio [-]
ω	mass flow rates in RTO optimisation model [kg/s]

Binary variables

D	defines a header change of a compressor
F	defines a shut down of a compressor
R_c	defines status of the recycle valve
S	defines a startup of a compressor
U_{El}	defines an electrical maintenance
U_{fm}	defines the start of maintenance (flexible maintenance case)
U_{Me}	defines a mechanical maintenance
U_{ov}	defines beginning of a major overhaul
W_{of}	defines the beginning of an offline washing
U_{of}	defines if a compressor is under offline washing
K_{of}	defines if the offline washing finished
X	defines operational status of a compressor
Y	defines connections between compressors and headers
λ^*	defines operating discrete area of compressor map
$\xi_{(i,i',j,t)}$	defines if compressors are connected to the same header

Abbreviations

CV	Coefficient Variation
DCS	Distributed Control System
EOH	Equivalent Operating Hours
EOT	Equivalent Operating Time
IGV	Inlet Guide Vane
LP	Linear Programming
MINLP	Mixed Integer Non-Linear Programming
MILP	Mixed Integer Linear Programming
MIP	Mixed Integer Programming
NLP	Non-Linear Programming
ONWE	Online Washing Episode
OFWE	Offline Washing Episode
RMSE	Root Mean Square Error

RPM	Rotations Per Minute
RSQ	R Squared
RTO	Real Time Optimisation
VSD	Variable Speed Drive
m.u.	Monetary units
n.u.	Normalised units
r.d	Regression domain

1 Introduction

1.1 Description of the chapter

The thesis elaborates on the optimal operation of compressor stations in process systems and natural gas networks. Chapter 1 provides the overview of the thesis. First, the description of the project Energy SmartOps is presented in Section 1.2, as the PhD study on the optimal operation of compressors is part of the project. Interactions with researchers of the project from different academic or technical institutes, and visits in industrial sites are few of the highlights of this initial training networking European project.

Section 1.3 introduces the research topic of the optimal operation of industrial compressor stations with multiple parallel centrifugal compressors in large energy systems. This section states the research problem and presents the key parameters to be considered so as to define the scope of the thesis. Moreover, Section 1.3 answers the question why this problem is important to be studied.

Two industrial case studies are presented in Section 1.4. The process plants of these two case studies include multi-stage centrifugal compressors which are connected in a parallel configuration. The first case study involves an air separation plant in BASF, Germany. This plant encompasses an air compressor station. The second plant operated by Statoil, Norway, involves a natural gas export compressor station which provides natural gas from Norway to Europe. Both compressor stations include compressors with high power consumption. The developed methodologies will be applied to these case studies.

Section 1.5 describes the aim and objectives of the thesis. This section explicitly states the aims of the current study and presents the outline of the thesis by describing the context and the interrelations among the chapters. Section 1.8 gives the structure of the thesis.

1.2 An introduction to the project: Energy SmartOps

The current study of the optimal operation of compressors is part of a European project, namely Energy SmartOps. Energy SmartOps aims for (a) reducing energy consumption in industrial applications and (b) training researchers through PhD

studies, networking and other activities. The project is funded by the European Community via Marie Currie ITN People Actions under the FP7 program.

The project involves several partners such as universities, research organisations of companies that supply technology, and end-user companies. The universities are Imperial College London, which is the project coordinator, Cranfield University, ETH Zürich, Politechnica Krakowska and Carnegie Mellon. Commercial organisations such as ABB R&D in Norway, Poland and Germany, and international technical training companies such as ESD Ltd are some of the partners. The end-user companies participants of the project are BASF (Germany), ThyssenKrupp (Italy) and Statoil (Norway).

There are fifteen European Early Stage Researchers, including the author of the thesis, whose aim is to generate and test methods for energy savings in large industrial sites. The developed methods will be applied to case studies provided by the project. In addition, the project focuses on the technical training, e.g. PhD studies, short technical courses, and personal development, e.g. soft skills courses, industrial experience, of the researchers. The researchers based in universities followed short-term placements in the industry (approximately three months) or in the research centres of commercial organisations such as ABB, and vice versa. The project is described by five main *Work Packages (WP)*:

- **WP1 - Electromachinery:** deals with the modelling of electro-mechanical systems and with the development of diagnostic algorithms for assessing performance and condition of industrial electric drives.
- **WP2 -Turbomachinery:** examines the development of models of compressors and algorithms to diagnose and monitor the performance of the compressors. Moreover the work package aims to optimally operate the compressors with the use of optimisation methods such as mixed integer linear programming.
- **WP3 - Maintenance and diagnosis:** studies the monitoring of a process using multivariate statistical analysis and examines the reactive performance-based maintenance planning for process plants.
- **WP4 - Energy optimisation:** deals with the energy management for mechanical and electrical/power electronics systems.
- **WP5 - Electricity optimisation:** investigates the electricity demand management in process plants, and the integration of energy management and production.

Figure 1.1 shows the different research areas of the work packages. The blue highlighted topics of the list on the right side of the figure show the key topics

with which the thesis is concerned. The topic of the thesis is related to *WP2 - Turbomachinery*. The WP2 involves the modelling of the steady-state behaviour of the compressors using two different approaches: data-driven models from process data and polynomial models derived from compressor maps. The modelling of the steady-state behaviour of a compressor describes its feasible window of operation and its power consumption as a function of key process variables such as temperature, mass flow rates and pressures.

The developed methods are applied to two different case studies. In the first case study, when compressor maps are not available (these maps are explained in Section 2.2.2), then process data have to be used to model power and the feasible window of operation. In the second case compressor maps are used to model the behaviour of the compressors. Optimisation formulations employ these models to optimise the compressors. The role of the optimisation is to increase the total efficiency of the compressor station, to reduce the total costs and decrease the wear of the compressors while at the same time all the operational and other constraints are satisfied. The diagnosis and management of faults is studied by other researchers in the project.

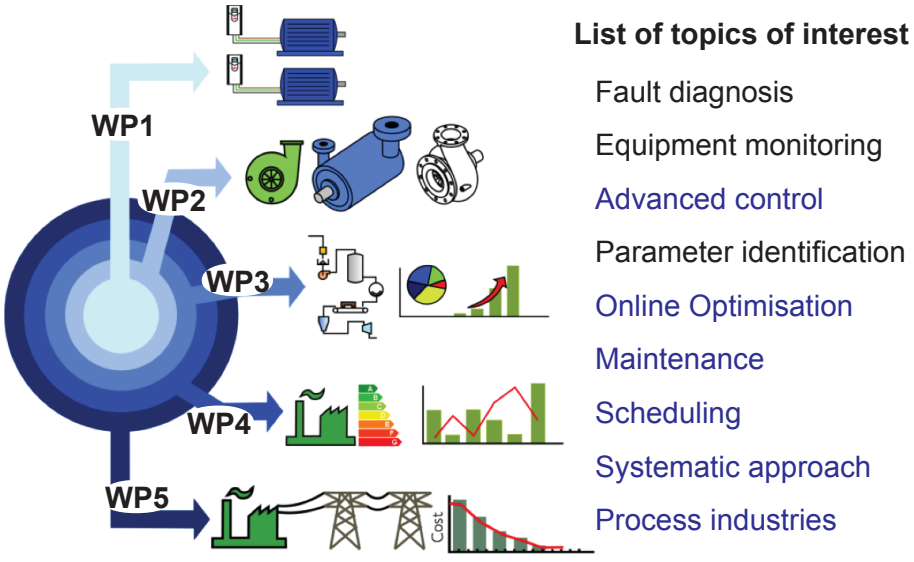


Figure 1.1: Graphical description of the five Work Packages of the Energy SmartOps (courtesy of Energy SmartOps consortium).

1.3 An introduction to the optimal operation of compressors

Nowadays, the process, and oil and gas industries work on the improvement of their operations. Government regulations, intense competition, changes in the markets and requirements for more environmental-friendly applications are some reasons which motivate the industries to improve their current practices. Each equipment unit of a plant is highly integrated with the overall process, therefore the consideration of the interactions among units, sub-systems and large systems is essential to achieve increased total efficiency, reduced total costs and better management of assets of a plant.

Many processes in these industries use compressors to achieve several objectives for their operations. Compressors are mechanical machines which can provide air for utilities (e.g. combustion or pneumatic tools), can recirculate fluids and can convey gas through a pipe. In some cases, processes consume large amounts of energy and the major part of this consumption comes from the operation of their compressors. In addition, the maintenance costs of compressors are relatively high. For these reasons, the efficient operation of compressors and their integrated management could save energy and reduce operational costs. Energy-intense applications of compressors can be found in the chemical and natural gas industry.

1.3.1 Compressed air

U.S. Department of Energy (2003) has outlined a broad range of applications of compressed air in industry. Table 1.1 shows examples of these applications. Compressed air is also used in oxidation, cryogenics, refrigeration, dehydration and aeration. According to Boyce (2003) typical applications in the process industry are: (a) air blower for Fluid Catalytic Cracking (FCC) unit, (b) gas recovery unit for FCC, (c) reformer recycle compressors, (d) ammonia plant nitric acid train, (e) cryogenic expander, and (d) re-compression processing and refrigeration systems.

Compressed air is considered as the fourth utility, after electricity, natural gas and water, in facilitating production activities according to Yuan et al. (2006). It is one of the most expensive utilities in a facility. It accounts for more than 10% of total industrial energy use for few selected countries as found in literature (Saidur et al., 2010). In the review on compressed energy by Saidur et al. (2010), one of the main remarks is that 70% to 90% of the total electricity bill of an air compressor system comes from the annual operating usage of compressors and other related components, such as air dryers and supporting equipment. Another remark is that the operating cost of an air compressor system accounts for more than three times than its maintenance and capital cost in the lifecycle of a compressor. Indeed,

Table 1.1: Examples of uses of compressed air in industrial sector (U.S. Department of Energy, 2003).

Industrial Sector	Examples
Apparel	Conveying, tool powering, control and actuators
Automotive	Stamping, tool powering, controls and actuators
Chemicals	Conveying, control and actuators
Food	Dehydration, bottling, cleaning, vacuum packing
Furniture	Tool powering, clamping, spraying
General manufacturing	Clamping, stamping, tool powering and cleaning
Lumber and wood	Sawing, hoisting, clamping, pressure treatment
Metals fabrication	Assembly station powering, injection molding
Petroleum	Process gas compressing, controls and actuators
Primary metals	Vacuum melting, control and actuators, hoisting
Pulp and paper	Conveying, controls and actuators
Rubber and plastics	Tool powering, forming, mold press
Stone, clay and galls	Conveying, blending, glass blowing, cooling
Textiles	Agitating liquids, clamping, conveying

the energy cost corresponding to the operation accounts for 78%, the capital cost for 16% and the maintenance for 6%. The review mentions that a non-optimal sizing of the compressors and inefficient control strategies of the operation result in increased operating costs and high energy consumption. It is also mentioned that the use of sophisticated control schemes can result in 12% energy savings.

In many industrial applications which require gas at large quantities of mass flows and high operational pressures there is a need for the use of several compressor operating in parallel. Compressors which are connected in parallel are capable for providing increased total mass flow in the application. An example is an air- and gas-supply network of a terepthalic acid manufacturing plant presented in the study by Han et al. (2004). The authors reported that this kind of network consumes the major part of the electrical power of the whole plant. The energy consumed from the network is estimated approximately 75-85% of the total consumption. Moreover, Mitra et al. (2012) mentioned that the air compressors of an air separation plant consumes the major proportion of the overall process, this statement is explained in Section 1.4. The first case study of the thesis, which is presented in Section 1.4, is a cyrogenic air separation plant which includes air compressors. The air compressors, which provide compressed air to the separation process, consume the major proportion of the power of the overall plant and this power accounts for more than 70%.

1.3.2 Natural gas compression

Natural gas is one of the primary sources of energy which is used for heating, cooking and generation of electricity in residential sector. Moreover industrial plants use natural gas as a basic source of energy for heating and power generation.

A large natural gas network can provide several tens of billion std m^3 of gas annually (Nørstebø et al., 2008). Kurz and Brun (2012a) explained the operations in the industrial oil and gas sector from the source, i.e. oil and gas fields, to the end-users. Rich natural gas containing condensate is provided from wells to gas processing plants which produce dry gas. The dry gas has to be transported to industrial or domestic final customers.

Compression applications of natural gas take place in different stages of the path of the gas from the wells until its delivery to the end-users, i.e. residential areas or industries. Figure 1.2 shows the main compression applications and their classification to upstream, midstream and downstream. According to Kurz and Brun (2012a) the upstream applications deal with the transportation of the natural gas from the wells to the processing-gas plants. The midstream applications transport the sales gas from the processing plant to the distribution points. The downstream applications involve the distribution of the gas to the market areas which are the end-users.

The *gas gathering* includes the flash gas compression which separates the natural from the crude oil both found mixed in the well (reservoir). Moreover, compressors are used to boost the flows of wells which have natural low pressures. *Gas lift* and *gas injection* use the following principle: gas is injected into the well to enhance the flow of the crude. *Export gas compressors* are used to compress gas from an offshore platform to shore through a sub-sea pipeline. The gas processing plants use compressors to increase the pressure of the gas coming from the gathering system to the operational pressure of the plant. These types of compressors are known as *boost compressors* according to Kurz and Brun (2012a). There are also compressors which increase the pressure of the gas at the exit of the plant to the pressure of the inlet of downstream pipelines.

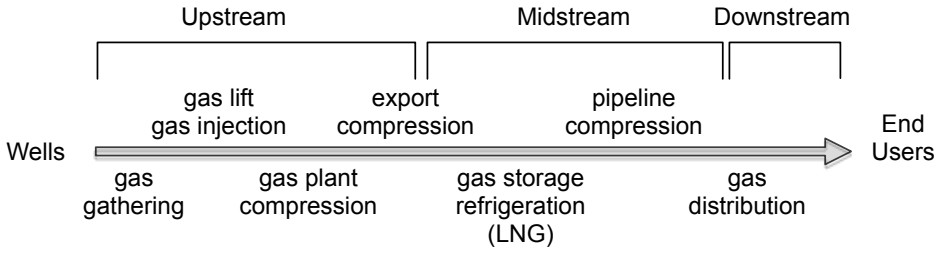


Figure 1.2: Applications of natural gas compression.

The produced dry gas which is ready for use by the end-users is called *sales gas*. Compressor stations are used to compress the sales gas from the exit of the plant to the delivery points, in case of gas transportation through pipelines. Schmidt et al. (2014) reports that this method of transportation is used when the distances of transportation are up to 4,000 km over land and 2,000 km off-shore. If the distances are greater than these, then the natural gas should be liquefied (LNG process) and transported in ships. Therefore, the liquefaction of gas involves a refrigeration process which has requirements in compression. Compressors are also used to store gas in reservoirs to deal with the uncertainty in the demand of the gas.

The pipeline transportation involves compressor stations in series to increase the pressure of the gas in order to overcome the friction losses in pipes. A compressor station can involve several compressor units which operate in parallel. Typically, a large number of compressor stations can be found in a gas network in different structures, for example a large network in US comprises several hundreds of pipelines and tens of compressor stations distributed in a strategic way (Ríos-Mercado and Borraz-Sánchez, 2015). However, there are gas networks which employ a small number of compressor stations, but with great power and flow capacity, to transport the gas through the long pipes of the gas network. One example is the second case study of the thesis which considers an export gas compressor station of Statoil which operates in the Norwegian gas network (Ministry of Petroleum and Energy, 2014).

The electrical motor of a single compressor in a export gas compressor station can consume up to 40 MW (Nørstebø et al., 2008). In the case where the drivers of the compressors are gas turbines, the gas needed to provide the energy to operate a natural gas network may reach up to 5% of the total volume of the transported gas (Wu et al., 2000; DeMarco and Elias, 2011). The compressors driven by the gas turbines are reported to consume 93% of this energy (DeMarco and Elias, 2011). In the long term, the operational costs outweigh the capital cost of the purchase of a compressor. This fact enables researchers and industries to focus on a more efficient operation considering the health condition of the compressors (for instance, effective maintenance policies), and environmental and government restrictions.

1.4 Introduction to the case studies

The project EnegySmartOps provided two case studies, an air separation plant involving a network of air compressors in BASF, Germany, and one export natural gas compressor station operated by Statoil in Norway.

1.4.1 BASF case study



Figure 1.3: Chemical complex of BASF in Ludwigshafen, Germany (Bertha Benz Realschule Wiesloch, 2008).

BASF is one of the world's leading chemical companies. Its end-products range from chemicals, plastics, performance products and crop protection to oil and gas. The headquarters of BASF is in Ludwigshafen in Germany. The industrial site of BASF in Ludwigshafen can be seen in Fig. 1.3. This site with two thousand buildings in a ten square kilometre (10 km^2) area is considered as the largest integrated chemical complex in the world (BASF, Corporate Website, 2015). The company's global headquarters and research centre employed approximately 33,000 employes in 2013.

BASF, Germany provided the first case study which is an air separation plant with a network of air compressors. This network comprises eleven multi-stage centrifugal compressors which provide compressed air to the air separation columns of the air separation plant. The air separation plant separates the compressed air into its basic components: oxygen, nitrogen and argon. Additionally, the compressors have to provide compressed air for utilities to other processes in the BASF complex. The electrical power needed to operate the plant is about 75MW, equivalent to the needs of a town with 50,000 residents. The major energy consumption comes from the compression of ambient air from the air compressors.

The performances of the compressors are not known and are not expected to be the same due to uneven operational patterns such as different continuous operational online time periods and different history of maintenance events. The users of the plant have to take decisions which involve discrete events, for example when and which compressors to start up or shut down. The considerations of the discrete events result in complex decision-making problems with combinatorial aspects. The objective of the decisions is the plant to meet the demand requirements at minimum operational cost.

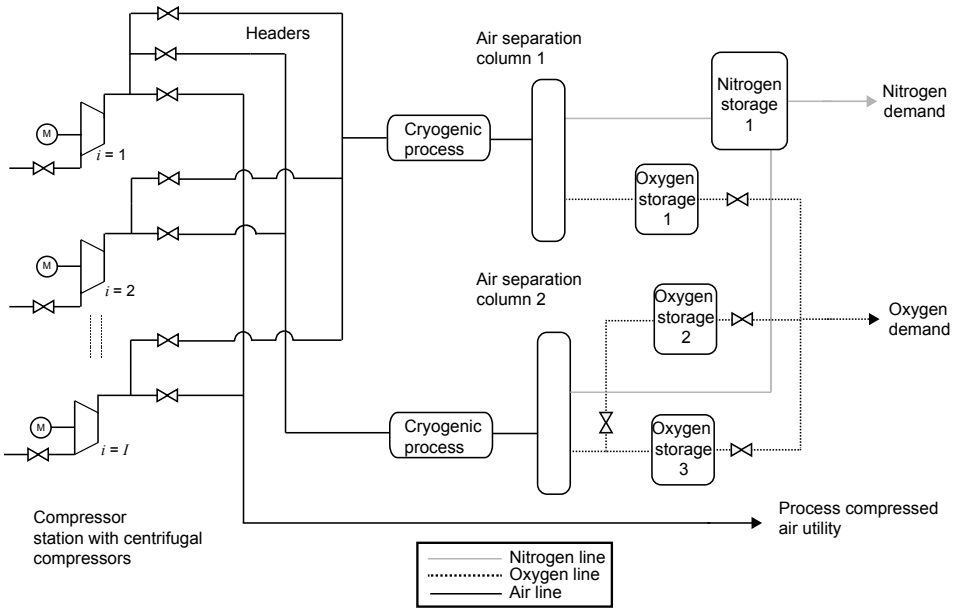
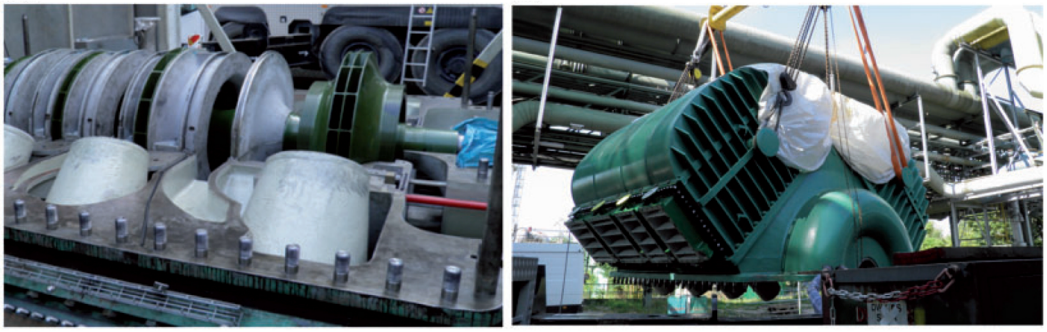


Figure 1.4: Topology of the operational units and gas streams in the air separation plant similar to the plant of BASF, Germany.

An air separation plant includes air separation units which are cryogenic separation columns integrated with heat exchangers, compressors, and expanders as can be seen in Fig. 1.4. Storage tanks are used to store oxygen and nitrogen from the cryogenic separation. The figure shows the cryogenic process after the compressor station and before the cryogenic air separation columns. The cryogenic process includes the purification of the compressed air, the main heat exchangers and the expanders. A more detailed description of the air separation process can be seen in Fig. 3.2.

The basic processes taking place in an air separation plant can be summarised in six steps. In the first step, ambient air is drawn through filters at the inlet of the plant. The filters remove dust and particles from the air. The filtered air is compressed in the second step. Air compressors consume energy to increase the pressure of the ambient air at a necessary pressure required from downstream processes. The pressure is approximately from five to ten bar. The compression of the air provides all the energy required for the refrigeration process which cools the air to the operational cryogenic temperatures (cryogenic temperatures are considered below -150°C). Moreover, modern air separation columns use expander turbines of which main role is the cooling of the processed compressed air and the turbines provide work to the air compressors, hence the overall efficiency is increasing.

The fourth step involves the removal of substances such as water and CO_2 from the air to prevent their freezing in downstream processes with low temperatures.



(a) (b)

Figure 1.5: BASF centrifugal multi-stage compressor with open body (a) and top casing (b) (Cicciotti et al., 2015).

According to Xu et al. (2011), filters such as molecular sieve adsorbers are used to achieve this objective. Then in step five, the highly integrated expanders and heat exchangers cool the processed compressed air to low temperature, approximately to -180°C to liquefy the air. The principle of the operation of an air separation plant is based on the refrigeration cycle and the throttling effect. The throttling effect (or Joule-Thompson) is the change in the temperature of a gas (or liquid) when it is expanded through a valve while at the same time heat is not exchanged in the environment. Additionally, the main heat exchanger (MHE) cools the purified compressed air to cryogenic temperatures with the use of streams of cold products and waste cold streams.

The final step includes the cryogenic separation columns which vaporise the liquid so as to selectively separate the air to its basic components at their different boiling points. According to Messer Group (2015) oxygen becomes liquid at temperature -183 °C and nitrogen at -196 °C. The exchange of mass and heat between the rising vapour and descending liquid causes continuous evaporation and condensation. The result of this process is the production of oxygen at the bottom and nitrogen at the top of the column. Argon can be separated with additional process steps.

In the case of the air separation plant of BASF the products are exclusively gases and they are stored in buffer tanks. The gaseous products are provided to internal and external users through pipelines. An internal customer may be a process plant on the same site which uses oxygen for its own processes.

As previously mentioned, the air compressors consume the majority of the total energy of the plant. The air compressors of BASF air separation plant constitute a network of compressors which are connected in parallel. There are eleven available compressors comprising of groups with similar nominal specifications. However, the groups of compressors differ in design, power rate, operating range, efficiency,

structure in terms of stages of compression and control method.

These air compressors are multi-stage centrifugal compressors with heat exchangers between each two sequential stages and an aftercooler after the exit of the last stage of the compressor. They are driven by electrical motors at constant speed. Figure 1.5 shows the open body of a multi-stage compressor of BASF during maintenance operations.

The case study focuses on the optimal operation of the existing compressor units without considering the purchase of new more efficient compressors. The plant managers and operators aim to coordinate the individual operations of the compressors to achieve the minimisation of the operational cost which mainly includes the electricity consumption of the electrical motors which power the compressors. The users of the plant deal with a decision-making problem which requires them to identify the best set points and configuration of the compressors which result in optimal operation. These decisions are related to the distribution of the load, and the scheduling and maintenance of the compressors.

The compressors have been operating for more than fifty years and their performance maps are not available, there is also a reduced number of available measurements installed in most of the compressors, and therefore the development of rigorous models of all the compressors which describe their performance is not feasible.

1.4.2 Statoil case study

The second case study examines the operation of an export natural gas compressor station in Norway. The compressor station is part of the Norwegian gas transport network which comprises 7,800 km of pipelines and there are pipelines which can reach up to 1,200 km length (Nørstebø, 2008). The operational pressure in the pipes can reach up to 210 bar. The Norwegian gas transport network is presented in Fig. 1.6. The blue nodes represent the main production wells of the gas network which are related to Kollsnes gas processing plant, namely Visund, Kvitebjørn and Troll fields. The red node represents the location of the export gas compressor station of interest, which is part of the Kollsnes plant. The Kollsnes plant supplies two offshore platforms Sleipner R and Draupener S/E represented by the black nodes. These two offshore platforms distribute the sales gas to the final terminals represented by the green nodes in the rest of Europe. Compared to other conventional gas networks, which involve several compressor stations distributed in series along the pipes, the Norwegian gas network uses only a small number of compressor stations which provide the necessary energy to the gas in order to reach its final destination.

The Kollsnes plant is a gas processing plant which involves a compressor sta-

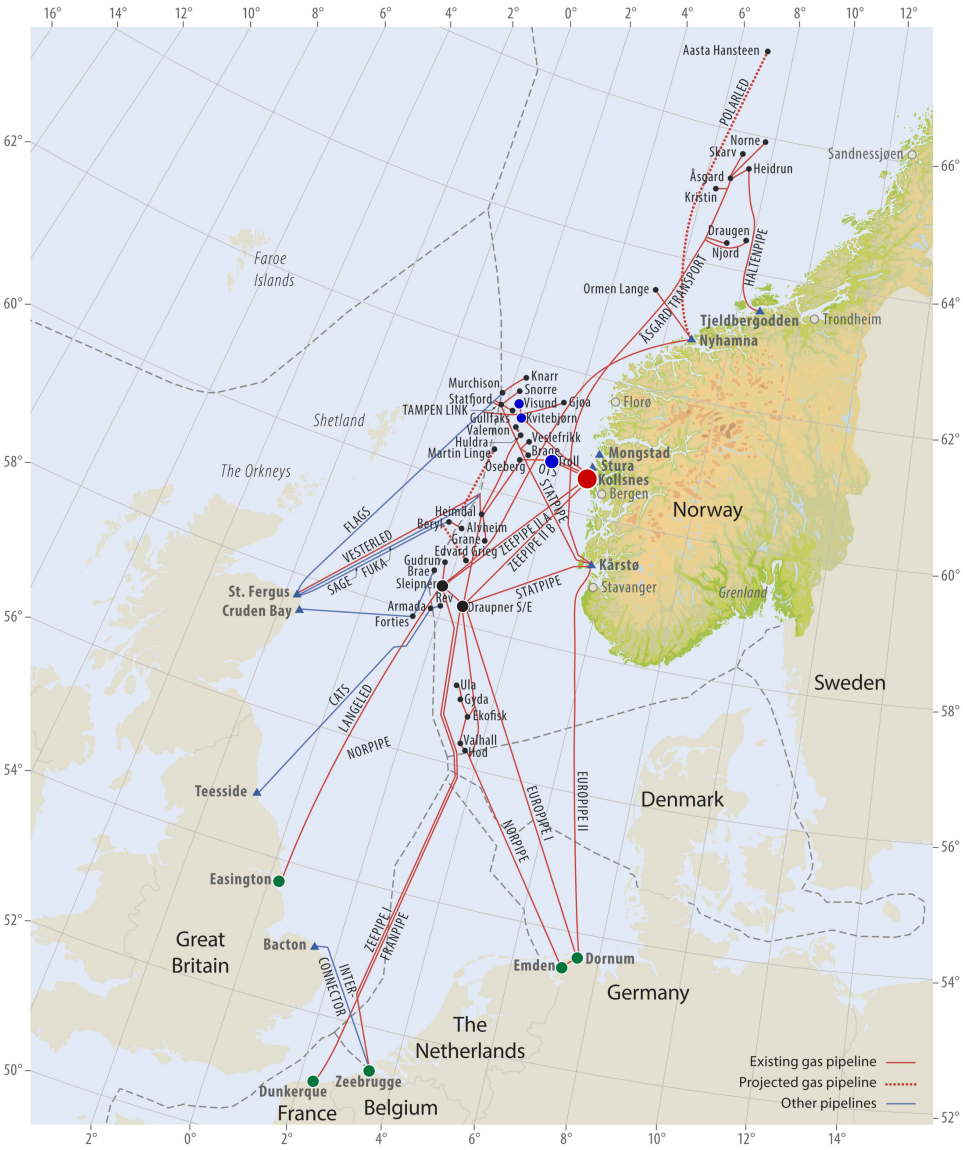


Figure 1.6: Norwegian gas network (Ministry of Petroleum and Energy, 2014).

tion with six large multi-stage centrifugal compressors operating in parallel. The compressor station can handle 143 million standard cubic metres per day according to the official website of Statoil (Statoil, 2015). A simplified schematic which shows the topology of the plant can be seen in Fig. 1.7. The Troll, and Kvitebjørn and Visund fields provide rich gas to the Dew Point Control (DPC) facilities and Natural Gas Liquids plant which process the gas. Produced dry gas is delivered through a common suction manifold to the inlets of the six multi-stage centrifugal compressors. The exits of the compressors can be provide gas either to header ZIIA or to header ZIIB towards to platforms Sleipner R (P-SLR) and Draupner S/E (P-DPR). The first five compressors are nominally identical and the sixth compressor has different characteristics compared to the other five compressors.

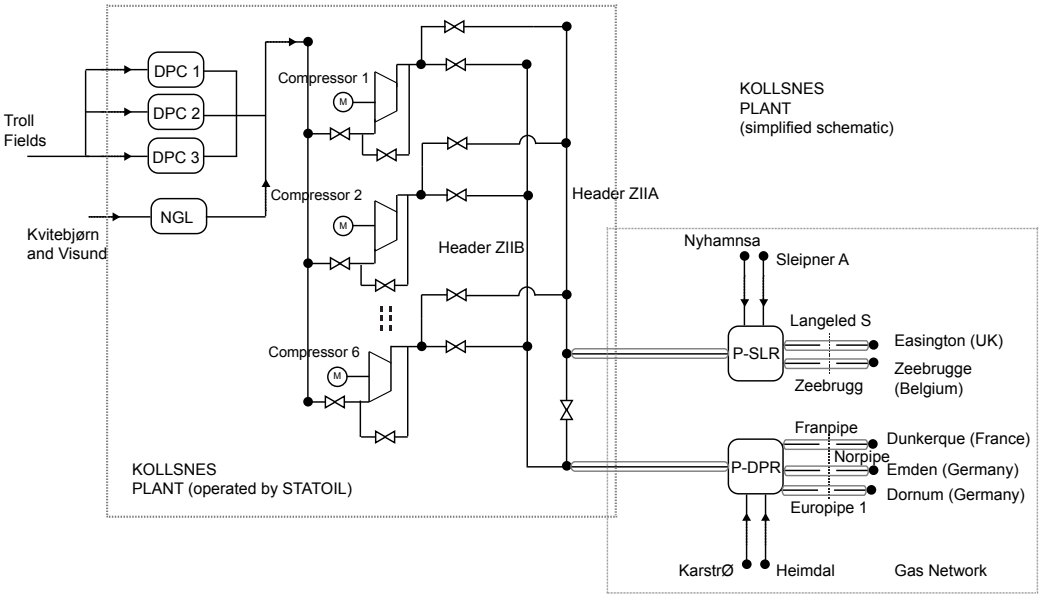


Figure 1.7: A schematic of the topology of the Kollsnes plant connected with a downstream pipeline network.

The compressors are controlled with variable speed drive systems which adjust the rotational speed of their shaft. The compressors have air coolers after the last stage. The diagram of Fig. 1.7 does not show them for simplicity.

Statoil company operates the Kollsnes plant. GASSCO company operates the downstream pipeline gas network. The compressors export natural gas at high pressures with inlet pressure more than 65 bar and outlet pressures approximately at 200 bar. The compressors have to meet the demand which is requested from the final terminals.

1.5 Scientific aim and objectives of the thesis

The aim of the current work is to achieve optimal operation of existing installed compressors in a compressor station of a chemical plant or an export natural gas station without considering the purchase of new more efficient compressors. The development of optimisation frameworks and optimisation models can determine the decisions which lead to the minimisation of operational costs of compressors which operate in parallel, as the focus of this thesis is on compressor stations which include compressors running in a parallel configuration. The modelling of the compressors is also an important topic of the current study. Different types of models of compressors should be used according to the level of the optimisation application (online and real time, and offline) and considering the availability of resources (provided compressor maps or process data).

Given the description of the aim, there are three major scientific objectives to be achieved:

Objective One: *is to develop a Real Time Optimisation (RTO) framework which optimally shares the load among parallel compressors in real time to deal with short-term changes in the operation.* In practice and as found in the literature in Chapter 3, and as proved with the use of real industrial data in Chapter 4, a compressor station usually involves dissimilar compressors. The problem of the optimal distribution of load aims to determine the loading of each compressor which results in the minimum total power consumption while all the constraints are respected including the satisfaction of the demand. The demand fluctuates over time and a real time optimisation approach can deal with the uncertainty in parameters, such as ambient conditions, model parameters, and demand profiles.

Objective Two: *is to develop a multi-period optimisation framework to optimally schedule the operations of compressors for long time periods.* The users of a compressor station have to take decisions which relate to discrete events such as starting up a compressor. These decisions can be determined from a multi-period optimisation framework which can optimise the operation for the total time periods under consideration. Therefore the solution, for example, of the first period has considered the requirements of the following periods. The same applies for the solutions of all the other time periods. The output of the framework is the schedule of the compressors which provides the best decisions such as the selection of the online compressors and the selection of stand-by compressors for each period.

Objective Three: *is to implement maintenance activities into the operational optimisation framework and achieve simultaneous optimal operation and maintenance.* The maintenance activities of a compressor station are, for example, the repair of mechanical and electrical parts, and the washing of the inside of the compressors with cleaning solutions. These activities improve the efficiency of the compressors, however they have to be scheduled in accordance with the operation, as maintenance activities require the shut down of compressors which influences the total capacity and the total efficiency of the compressor station.

Figure 1.8 describes the three objectives and their interconnections to achieve optimal operation of compressor stations. The Real Time Optimisation (RTO) provides the set points of the loads of the compressors which minimise the power. The selected compressors which should operate online are given from the output of the scheduling. The scheduling also considers the load sharing problem but at

an aggregated level. The real time optimisation provides set points considering models which take into account detailed information such as ambient conditions. If the scheduling considers the maintenance of the compressors, then the resulted integrated optimisation framework constitutes a more comprehensive systematic approach, but with more decisions to be taken.

On the right side of Fig. 1.8, there is the scale of the different optimisation problems in model complexity and level of decision. This means that the Real Time Optimisation framework employs detailed models which can be solved online and the scheduling problems use models with reduced complexity to deal with the computational burden as multi-period problems include a large number of variables depending on the duration of the total period under consideration.

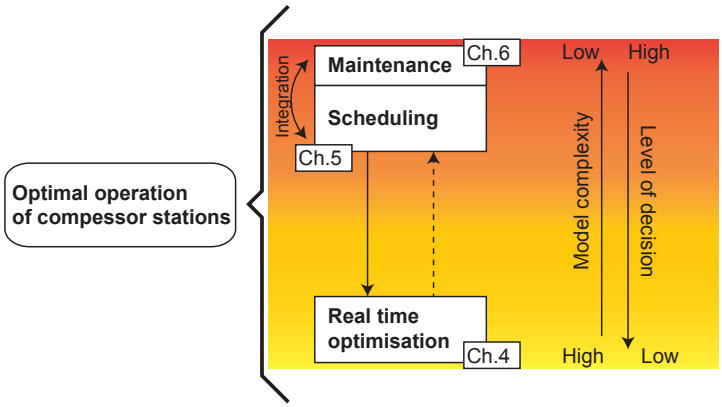


Figure 1.8: A schematic which illustrates the objectives of this thesis for achieving optimal operation of compressor stations.

1.6 Contributions of the thesis

1.6.1 Contribution to the real time optimisation of compressors

The literature review in Chapter 3 of the thesis will explain that the current state-of-the-art of the optimisation of compressors lacks a comprehensive real time optimisation framework to optimally share the mass flow of the compressors. The thesis developed this framework which is applied to a real case study using industrial data. The thesis argues that this framework can potentially reduce the operational costs of industrial plants with parallel compressors compared to the existing industrial practices. A key contribution is the consideration of data-driven models which express the power consumption of the compressor without the need to estimate their efficiency. These models are computationally efficient and suitable for an industrial application with limited availability of process measurements. Another contribution of the thesis is a multivariate steady-state detection method

applied to a system of parallel compressors for the needs of the real time optimisation. Three papers have been published from this work, two in conferences (Xenos et al., 2014a,b) and one in a journal (Xenos et al., 2015a).

1.6.2 Contribution to the scheduling of compressors for long periods

Compressor stations in many industrial applications include several compressors in parallel, however previous modelling approaches considered compressor stations as a single entity neglecting the operational aspects of the individual compressors. A breakthrough of the thesis is the development of mixed integer linear programming optimisation model of compressor stations with individual models for each compressor which can be used to minimise total costs of the stations, for example electricity cost and shutdown costs. This generic optimisation model expands the current state-of-the-art as:

- It can be used for different industrial applications which include compressors. The model will be applied to two different cases, namely an air compressor station of an air separation plant and an export natural gas station.
- It can employ data-driven models or models based on compressors maps to describe the behaviour of the compressors. The reason for using different modelling approaches is because a plant may not have sufficient instrumentation to provide enough data for the building of the data-driven models. Conversely, the compressor maps may not be known. Therefore, it is useful to have both methods available.
- It can be used to compare the existing operation of compressors based on typical industrial practices with the suggested optimal approach based on the scheduling of compressors. The thesis will carry out this comparison in the Statoil case study.

The work on the multi-period optimisation to optimally operate the compressors for long periods is in preparation for publication in a journal article. This work will present the results of the scheduling of compressors of the Statoil case study. These results will be presented in Chapter 5.

1.6.3 Contribution to the integration of operation and maintenance

The integration of operation and maintenance of compressors expands further the state-of-the-art of the scheduling and maintenance of compressors. The literature review will support this statement as previous research focused on the

maintenance of a single compressor without considering the interconnections between the maintenance and operational tasks of more than one compressor. The basic contributions are summarised below:

- The thesis provides a comprehensive framework for scheduling of compressors considering different types of maintenance activities of compressors. The maintenance activities refer to corrective mechanical and electrical repairs, major overhauls to replace major parts of the compressors, and washing of the compressors. The framework enables planning of maintenance tasks according to the scheduling of the compressors.

- A breakthrough is that the suggested integrated framework can be used to generate the schedules of the online and offline washing of the compressors. Indeed, the existing approaches for washing a compressor are fixed periodical washing or washing when the fouling has reached unacceptable levels.

- The thesis also presents further developments on the integration of maintenance and operation. The thesis proposes a rolling horizon approach which carries out the optimisation of the compressors as it would happen in reality. The rolling horizon approach deals with uncertainty to some extent and this is the first step for a future study on the optimisation of compressors under uncertainty.

The work on the integration of operation and maintenance has been published in two conference papers (Xenos et al., 2014c, 2015b) and in two journal articles (Xenos et al., 2015c, 2016).

1.7 Sponsors and acknowledgements

The PhD studies started in September 2011 and included industrial secondments to BASF Ludwigshafen, Germany and Statoil in Norway. The group of Automation Technology, Advanced Process Control in BASF hosted the author for several months and supported the project with technical expertise related to the BASF air separation case study. Dr Erling Lunde of Process Technology of Upstream Rotating Equipment, Efficient Facilities, Research, Development and Innovation (RDI) in Statoil, Norway hosted the author in an secondment in the premises of Statoil. Dr Lunde provided industrial knowledge and support for the natural gas case study of this thesis.

The PhD studies were fully funded from the Marie Curie FP7-ITN project "Energy savings from smart operation of electrical, process and mechanical equiment - ENERGY SMARTOPS", Contract No: PITN-GA-2010-264940.

1.8 Structure of the thesis

This chapter has introduced the project and the aims of this thesis. The next chapter, Chapter 2, explains the principles of the operation of a single industrial multi-stage centrifugal compressor. It also explains the compressor map, feasible window of operation, performance and power consumption, and control methods of an industrial compressor. The interactions of multiple compressors attached to an upstream and downstream system will be described. Moreover, the tasks of the operational management of compressors will be classified and explained. The current study deals with the supervisory control or real time optimisation, and scheduling of compressors which will be explained within the context of the operational management of industrial air and gas compressor stations.

Chapter 2 also defines the aspects of an optimisation problem using mathematical programming to solve it. The classification of different types of optimisation problems will be presented. Techniques for solving linear programming, nonlinear programming, mixed integer programming optimisation problems will be explained. These techniques are available in the software for optimisation GAMS (McCarl et al., 2008).

The literature review in Chapter 3 presents the studies on the optimisation of compressor stations from previous authors and determines the relevant state-of-the-art. The study of the literature reveals the limitations of the current state-of-the-art on the real time optimisation, scheduling and maintenance of compressors. Chapter 3 mentions explicitly the gap of knowledge and the open questions which this current study will try to answer.

Chapter 4 provides a methodology of real time optimisation for the optimal load sharing of compressors online. The methodology explains how to use industrial process data to develop data-driven models for the needs of a real time optimisation application. Chapter 4 also demonstrates the application of the RTO to the BASF case study, and after this the results are presented.

The scheduling of compressors can be examined with the formulation of a multi-period optimisation problem as it is explained in Chapter 5. An optimisation framework employing a Mixed Integer Linear Programming (MILP) model will be presented using continuous and binary variables to represent continuous and discrete operational aspects of the compressors respectively. Then, Chapter 5 will present the application of the MILP model to compressor stations when compressors maps are either known or not. The model will be applied to both BASF and Statoil case studies.

Chapter 6 presents the integrated framework for optimal operation and maintenance of compressor stations. First it begins with the description of a basic maintenance policy implemented into the operational model which is demonstrated in

Chapter 5. Then, the basic integrated model is implemented in a reactive scheduling approach which updates the parameters of the model in an iterative way, and in which the time horizon of the optimisation is moved by a step in each iteration. The next topic is the extension of the basic integrated framework for optimal operation and maintenance with the consideration of major overhauls of compressors. This framework employs a longer time horizon compared to the basic case. Then, Chapter 6 will present the optimal operation of compressors considering a condition-based maintenance policy. The maintenance policy, here, involves different types of washings such as online, offline and their combination.

Chapter 7 evaluates the outcome from the research and justifies that the outputs meet the requirements of the objectives presented in Chapter 1. Chapter 8 gives the conclusions of this thesis and the references are given in the end.

2 Background of operation of compressor stations

2.1 Description of the chapter

Chapter 2 provides the background knowledge which is necessary to understand the developed methodologies for the optimal operation of compressor stations in this current study. Therefore, the description of the principles of industrial centrifugal compressors and their operational aspects are introduced in this chapter. The focus of the study is on centrifugal compressors as this type of compressor is usually used in process systems and natural gas applications. This chapter also explains the major operational aspects of a compressor, namely characteristics, performance, control methods and maintenance, of a single centrifugal compressor. The systems known as compressor stations which include several compressors in series and in parallel are also mentioned. Then, Chapter 2 elaborates on the management of compressors and gives the definition of an automation pyramid of a compressor station which illustrates the hierarchy of different decisions of the operation of compressors. The chapter also explains the principles of mathematical programming, and computational tools are explained. Finally, optimisation techniques and solvers of commercial platforms of optimisation such as GAMS are reported.

2.2 Industrial compressors

2.2.1 Description of industrial centrifugal compressors

Classification of compressors

Industrial compressors can be classified into two main categories: positive displacement and dynamic. According to Fig. 2.1, the first category involves rotary compressors and reciprocating. Axial and centrifugal compressors constitute the dynamic compressors. Dynamic compressors provide continuous flow and positive displacement provide intermittent flow. The decision for selecting a suitable compressor for an application depends on factors such as operational range, pressure ratio, geometry, weight, power, efficiency, capital cost and availability in market.

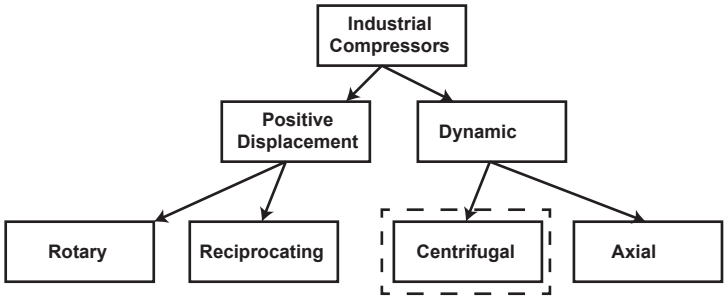


Figure 2.1: Basic classification of compressors based on U.S. Department of Energy (2003).

Table 2.1: Operational and performance characteristics of positive displacement, and axial and centrifugal compressors according to Boyce (2003).

Compressor	Pressure ratio			Efficiency	Operating range
	Industrial	Aerospace	Research		
Positive displacement	Up to 30	-	-	75% – 82%	-
Centrifugal	1.2 - 1.9	2.0 – 7.0	13	75% – 87%	Large
Axial	1.05	1.1 – 1.45	2.1	80% – 91%	Narrow

According to Boyce (2003), positive displacement compressors were widely used in operations in both process industry and transportation of natural gas up to and through 1960s when centrifugal compressors became more attractive.

Centrifugal compressors have higher thermodynamic efficiency and lower maintenance cost than the reciprocating compressors. The thermodynamic efficiency of a compressors is explained in Section 2.2.2. Moreover, centrifugal compressors are used in applications which request high flow rates and low pressure ratios. Pressure ratio is defined as the the outlet pressure over the inlet pressure of a compressor. The operating range defines the feasible operation in means of pressure and flow rate which is restricted by physical causes. The efficiency, operating range and feasible range of a compressor will be explained in detail in Section 2.2.2. Table 2.1 presents the operational characteristics of most common types of compressors used in the industries.

Reciprocating compressors are easy to maintain but they have to be maintained frequently. Centrifugal compressors are more reliable with a greater maintenance cycle than the reciprocating. However, due to the complexity of the centrifugal compressors, their repairs take longer. Additionally, the cost of the repair of a centrifugal compressor is lower than the repair cost of a reciprocating compressor. Centrifugal compressors are lightweight and more compact compared to reciprocating ones, hence they can be used for off-shore applications.

Components of an industrial multi-stage compressor

A *single-stage centrifugal compressor* is a machine wherein a gas is compressed by radial acceleration in an impeller with radial vanes and a surrounding metallic casing. A single-stage centrifugal compressor includes: (a) an impeller (rotor blades) which increases the kinetic energy of the inlet gas, (b) diffuser vanes (stator blades) which convert a part of the kinetic energy to an increase in pressure and (c) the vaneless area of the diffuser which collects the flow. To achieve high ratios of compression, several single-stage centrifugal compressors are arranged in series. These single stages in series of the resulting structure are attached to a rotating shaft. This structure is called *multi-stage centrifugal compressor* and it is known as a *compressor train* in the industrial field. Multiple (multi-stage) compressor trains are usually connected in series and their compressors do not have necessarily the same specifications, for example number of stages.

Figure 2.2 illustrates a sketch of an industrial multi-stage centrifugal compressor. The sketch focuses on the main components of the first stage of the multi-stage compressor which are: (a) inlet nozzle, (b) Inlet Guide Vanes (IGVs), (c) impeller, (d) diffuser, (e) return channel, (f) collector volute (or scroll) and (g) discharge nozzle. The configuration can be different in other machines. For instance, the inlet guide vanes may not exist. Another example of different configuration is that diffuser vanes can be used in the (vaneless) diffuser area after the impeller.

An industrial multi-stage compressor similar to this in Fig. 2.2 is usually powered either by a gas turbine or an electrical motor. Moreover, a multi-stage compressor typically is coupled with heat exchangers (intercoolers) between two sequential stages. The reason for this is to improve the thermodynamic efficiency of the compression as explained by Dixon and Hall (2010).

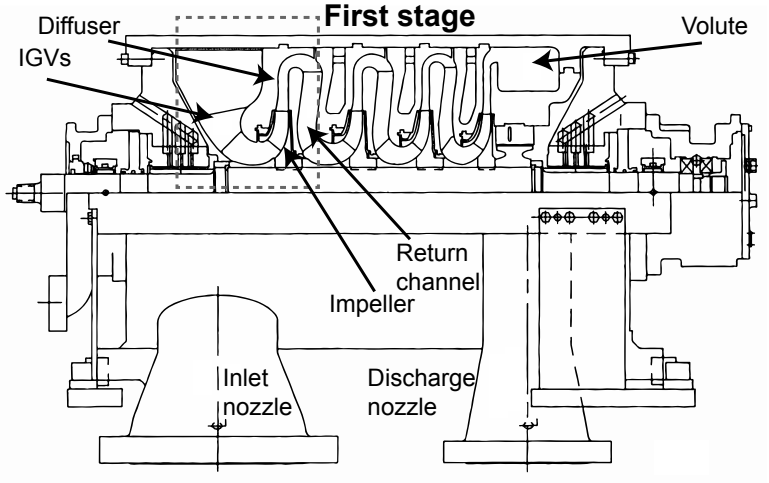


Figure 2.2: An industrial multi-stage compressor (Gresh, 2001).

2.2.2 Operation of a compressor

Definition of basic parameters of the operation

Figure 2.3 shows a compressor with two centrifugal compressor stages and an intercooler placed in between them. The compressor is connected with a downstream and an upstream process. The compressor conveys a gas from the upstream process with pressure, temperature and mass flow rate, P_{in}, T_{in}, M_{in}, to the downstream process with gas specifications P_{out}, T_{out}, M_{out}. There is a driver which provides power, W_{mech}, to the shaft of the compressor and the driver can be either a gas turbine or an electrical motor. A steam turbine is also used in the same way as the gas turbine to power a compressor, therefore when the text refers to gas turbines as drivers of compressors, it is implied that steam turbines can also be used instead of gas turbines.

The driver rotates the shaft with rotational speed N. This speed can be either constant or variable in the case of a Variable Speed Drive (VSD). A *variable speed compressor* is a compressor which employs this type of drive. Compressors can also have adjustable blades, known as Inlet Guide Vanes (IGVs), at their inlet, i.e. at the eye of the impeller. These blades are adjusted by an angle θ to change the operating point of the compressor. The use of an intercooler can reduce the inlet temperature, $T_{in,c}$, of the stream of gas which is coming from the exit of the first stage, and therefore the reduction in inlet temperature can thermodynamically improve the efficiency of the following stage (Dixon and Hall, 2010). The result is the increase of the total efficiency of the multi-stage compressor.

The operation of a compressor can be identified from the studying of compressor maps. Compressor maps provide information about the characteristics and performance of compressors. These maps are used to identify the operating point of a compressor and its efficiency at steady-state conditions, i.e. ambient temperature and pressure. The consideration of both the operating point, which describes

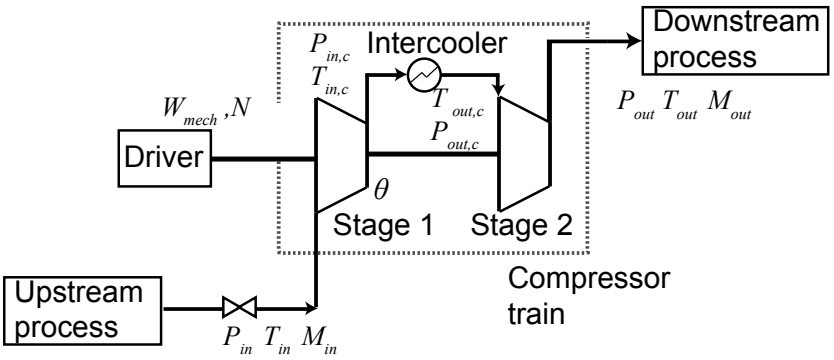


Figure 2.3: Operation of a two-stage (multi-stage) centrifugal compressor connected with an upstream and a downstream system.

pressure and flow rate, and the efficiency of the compressor can be used to estimate the power consumed by the gas at specific conditions.

Compressor map: characteristics and thermodynamic efficiency

A standard graphical representation used to describe the operation of a compressor is a compressor map which gives the characteristic curves of a variable speed compressor or a compressor with inlet guide vanes. In the case of single speed compressor without IGVs, there is only a single characteristic curve.

A typical corrected compressor map can be seen in Fig. 2.4. A general corrected compressor map takes into account inlet conditions, pressure P_{in} and temperature T_{in} and molecular weight $MolW$. For the sake of simplicity the $MolW$ parameter is omitted in this description, for example the compressor map of an air compressor normally does not consider the $MolW$. The reason of considering the inlet conditions is that a compressor map changes with respect to changes in inlet temperature or pressure.

A single characteristic curve of a compressor describes the relationship between pressure ratio $\pi = P_{out}/P_{in}$ and the corrected mass flow rate $M_d = M_{in}\sqrt{T_{in}}/P_{in}$ for a constant rotational speed N (Dixon and Hall, 2010). Figure 2.4 shows that a compressor with characteristics with different angles of IGVs. According to the manufacturer and the application, there are other descriptions of compressors maps which may consider for example discharge pressure, or isentropic or polytropic head on the vertical axis (Gresh, 2001). The horizontal axis in some compressor maps describe volumetric flow rate.

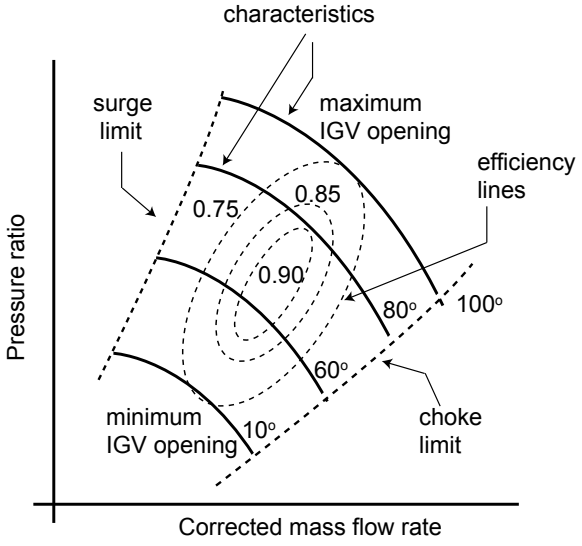


Figure 2.4: A typical compressor map of a single-stage centrifugal compressor with different angles of IGVs.

The operation between surge and choke for a constant speed curve (or a fixed angle of Inlet Guide Vanes) defines the operational range of this characteristic curve. The group of all the characteristics between minimum and maximum rotational speed (or Inlet Guide Vanes) constitutes the operational domain or *feasible window of the operation* of a compressor. The compressor cannot operate beyond the limits of this window due to physical, safety, power and mechanical constraints.

Surge is a phenomenon of reverse flow in the compressor. This phenomenon occurs when the machine compresses gas to high pressures at low flow rates. Helvoirt (2007) reports that surge not only reduces both performance and efficiency, but it can also cause damage to the compressor or other auxiliaries. Additionally, surge causes vibrations due to the reversal of the flow which results in unacceptable noise levels. Because of the undesired consequences of the effect of surge, compressors have to operate at a reduced operational window. This operational window is restricted by a left boundary, known as control surge line. This line is located to the right of the surge line and both lines are separated by a safety margin.

Choke or *stonewall* region is located at the right region of a performance map. A choke point is described as the operational point of a compressor which reaches at the maximum flow rate and the minimum head for a fixed speed curve. Mach number becomes equal to one, gas velocity has increased to acoustic velocity, and the flow cannot increase further. Bloch (2006) reports that choke can cause serious damage to the rotors and blades of multi-stage centrifugal or axial compressors. However, the choke phenomenon does not damage single-stage compressors to a great extent. Dixon and Hall (2010) explain the physical constraints of surge and choke with the use of fundamental aerodynamic and thermodynamic equations.

The upper and lower bounds of the speed (or bounds of IGVs) of a compressor map are defined by the specifications of the prime mover of the compressor, i.e. electric motor or gas turbine.

Figure 2.4 illustrates that a compressor map also provides the *efficiencies lines* which provide the efficiency of the compressor at every operating point on the map. These lines give the thermodynamic isentropic or polytropic efficiency (η_{is} or η_p) of a compressor. Isentropic and polytropic efficiencies are the most commonly used in the literature of turbomachinery. The *isentropic efficiency* is defined as the ratio of the useful energy input to fluid in unit time and the power input to the impeller and this expression is given by Eq. (2.1). Figure 2.5 defines the actual work, i.e power input to the impeller of one stage, and the isentropic work which is the useful energy input to the fluid through this stage without considering losses. The point A indicates the inlet of the stage of the compressor. The point B' denotes the outlet of the compressor with isentropic compression A-B' and point B refers to the actual compression A-B including losses.

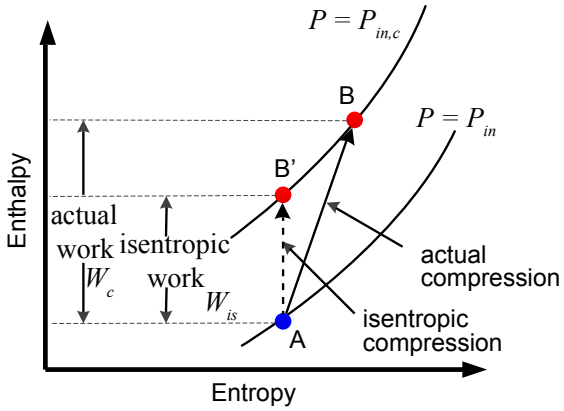


Figure 2.5: Enthalpy-entropy diagram which shows the isentropic and actual compression of the gas through a compressor stage.

The *polytropic efficiency* is estimated with the assumption that a compressor is composed of a large number of small stages. The benefits of the use of the polytropic efficiency is that this definition considers the pressure ratio of the machine in contrast to the isentropic efficiency. The polytropic efficiency is usually used to compare performances of different compressors. The polytropic efficiency η_p of one compressor stage with inlet A and exit B is given by the following expression using the notation of the first stage of the compressor in Fig. 2.3:

$$\eta_p = \frac{log((\frac{P_{in,c}}{P_{in}})^{\frac{\kappa-1}{\kappa}})}{log(\frac{T_{in,c}}{T_{in}})}$$

The κ in the above expression is the exponent of the isentropic change. The polytropic exponent ϕ is connected with κ through the following equation:

$$\frac{\phi-1}{\phi} = \frac{1}{\eta_p}\frac{\kappa-1}{\kappa}$$

Total efficiency of a compressor

A compressor is coupled with a drive through a shaft and other components such as a gearbox (GB). The power flow from the power source to the gas can be seen in Fig. 2.6. Electricity from the grid provides energy to electrical motors or natural gas is burnt by a gas turbine to rotate the shaft of the compressor. In either case, power flows in the power drivers (i.e. electrical motor or gas turbines) and this power can be defined as W_{in}. The input power W_{in} is converted into mechanical power W_{mech} which is transferred to the mechanical parts, i.e. gearbox, shaft and the impeller. The efficiency of the driver is η_{dr} and the connection between input and mechanical power is given by $W_{mech} = W_{in} \cdot \eta_{dr}$.

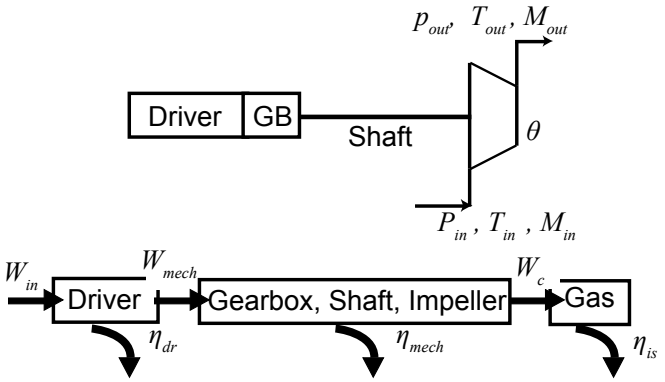


Figure 2.6: Power flow in the different components of a compressor.

The power W_c used to compress the gas is calculated by $W_c = W_{mech} \cdot \eta_{mech}$ and due to the isentropic efficiency:

$$\eta_{is} = \frac{W_{is}}{W_c} \tag{2.1}$$

and

$$W_{in} = \frac{W_{mech}}{\eta_{dr}} = \frac{W_c}{\eta_{dr} \cdot \eta_{mech}} = \frac{W_{is}}{\eta_{dr} \cdot \eta_{mech} \cdot \eta_{is}} = \frac{W_{is}}{\eta_{cmpr}} \tag{2.2}$$

where $\eta_{cmpr} = \eta_{dr} \cdot \eta_{mech} \cdot \eta_{is}$ is the total efficiency of the system which encompasses the driver, GB, shaft and impeller. The W_c is the actual power absorbed from the gas of the compressor and the W_{is} is the isentropic power equal to the minimum ideal power which assumes no thermodynamic losses. In the case of an electrical motor, the subscript dr is replaced by el, thus the power of the driver is W_{el}. The actual power W_c of a single compressor stage is given by:

$$W_c = M_{out}H_{out} - M_{in}H_{in} = M_{out}C_pT_{out} - M_{in}C_pT_{in} = M_{in}C_p(T_{out} - T_{in}) \tag{2.3}$$

where H_{in} and H_{out} are the enthalpies at the inlet and outlet of the compressor, C_p is the heat capacity at constant pressure. It is assumed that there are not leakages, thus $M_{in} = M_{out}$ from the mass balance of a compressor stage.

The isentropic power can be calculated from the following equations considering the relationship of pressures and temperatures from the isentropic change A-B', $(T_{B'}/T_A) = (P_{B'}/P_A)^{(k-1)/k}$:

$$W_{is} = M_{in}C_p(T_{out} - T_{in}) = M_{in}C_pT_{in}(\frac{T_{out}}{T_{in}} - 1) = M_{in}C_pT_{in}[(\frac{p_{out}}{p_{in}})^{(k-1)/k} - 1] \tag{2.4}$$

2.2.3 Maintenance of compressors

The maintenance of compressors is an essential factor of the general management of a compressor station which will be explained in Section 2.4. The management of the tasks of maintenance influences the availability and the reliability of the production of the plants which include the compressors. The downtime of the operation due to failures and unexpected breakdowns of compressors can lead to unsatisfied demands and increased economic losses. Moreover, the unsatisfied targets can cause problems between the commercial relationships between plant and its customers. This is because the plant would not be considered as reliable when the agreed contracts between plant and customers are violated. The choice of the best maintenance policy and the best maintenance plan of compressors has been studied from many researchers as maintenance tasks may give rise to increased maintenance and operational expenses if these tasks are not optimally scheduled.

Focusing on compressors and especially in industrial centrifugal compressors, there are several maintenance policies of compressors to decide, namely corrective, preventive and predictive. The *corrective maintenance* is related to the policy for fixing a compressor after it has been broken. According to Thorp et al. (2014) the mechanical components of a compressor with the most frequent failures are bearings, seals, compressor blades, compressor discs and inlet air filters.

The restoration or the maintenance activity of a compressor which has not failed at preselected intervals constitutes the *preventive maintenance* (Rao and Naikan, 2008). A typical preventive maintenance approach includes the use of a fixed schedule, optimised in advance for minimum cost according to Bohlin and Wärja (2010). However, unexpected failures may occur, and thus the users have to re-plan the maintenance of the compressors. A preventive maintenance approach involves inspections of the compressors which are performed to assess the condition of the compressors.

The *predictive maintenance* uses real time analysis of data to assess the condition of the compressors, therefore to decide the optimal moment to maintain a compressor based on this acquired information and its analysis. The development of advanced performance and condition monitoring tools based on the condition of the oil and vibrations of mechanical and electrical components are essential to achieve the application of predictive maintenance. The state-of-the-art in performance monitoring of compressors which can estimate performance and characteristics of a compressors is mentioned in the study by Cicciotti et al. (2015). This study presents model-based approaches which exploit real time process measurements, such as flow rates and pressures, to calculate the efficiency and characteristics of the compressors online.

Bertolini and Bevilacqua (2006) reported that the development of advanced

techniques of maintenance requires managerial and technical skills of the maintenance staff. The oldest and simplest method of maintenance, which is the corrective, is to change to a more advanced maintenance but also more complex and integrated policies such as a sophisticated preventive and predictive maintenance. The management of advanced maintenance policies requires the considerations of multiple components and multiple units which result in more complex methods compared to those used in the corrective approach. These complex methods may involve optimisation models which carry out an economic analysis to answer questions for example when the best moment to maintain a process unit is and what maintenance policy should be applied to a particular system.

The maintenance of the compressors can also be classified into two categories: (a) non-recoverable and (b) recoverable maintenance. The *non-recoverable* maintenance involves the repair (e.g. welding of material) or replacement of mechanical or electrical parts of the compressors. The replacement of spare parts is defined as a restoration action when a compressor has returned to perfect condition after this restoration action. Replacements of spare parts take place in major overhauls of the compressors which take longer compared to short shut downs to repair the compressor. The cost of the repair per overhaul has been estimated between \$472K – \$714K (Thorp et al., 2014).

The *recoverable* maintenance involves the washing of compressors to deal with the problem of fouling. The fouling of compressor is caused from the deposition of particles mainly on the blades of the compressors and annulus surfaces mainly due to dust mixed with air. Fouling causes over-consumption of the power and drop of full mass flow rate according to Martín-Aragón and Valdés (2014). The washing of the compressors can occur online, i.e. a solution is injected while the compressor operate online, or can occur offline, i.e. the compressor is washed with a solution with additional treatments when it is in offline. Several washing schedules have been suggested from Sánchez et al. (2009), Aretakis et al. (2012), Rao and Naikan (2008) and Hovland and Antoine (2004). Chapter 6 will suggest a condition-based maintenance approach which fall into the category of the predictive maintenance studies. Martín-Aragón and Valdés (2014) mention that the cost of fouling of the compressors has been estimated approximately 6.25 million per year and that an improved maintenance schedule could save 200,000 EUR/year per gas turbine.

2.3 Compressors integrated with other systems

2.3.1 Interactions between compressors and a downstream system

An *operating point* of a compressor is the intersection of its characteristic curve at a fixed rotational speed (or fixed angle of IGVs) and the characteristic curve of

the downstream system to which the compressor is attached. The characteristic of the downstream system is known as a *load curve* or demand curve. The reason for introducing these concepts is to explain that the downstream process influences the performance and power consumption of the compressor with which is connected.

Figure 2.7a illustrates three load curves of different downstream processes. It is assumed that the upstream process does not change the inlet conditions of the compressor, thus the focus is on the interactions of the downstream process and compressor. Kurz et al. (2012) described three main categories of downstream processes with which a compressor is usually connected. Load curve A describes a pipeline system in which the pressure increases when the mass flow through the pipes increases. Load curve B is used to represent systems in which the pressure does not change significantly with the change in mass flow. Refrigeration systems and process systems into which gas is fed at a specific discharge pressure are typical examples of this category (Bloch, 2006). Load curve C describes gas storage applications (Kurz and Brun, 2010) which are not included in the scope of this thesis.

Figure 2.7a also shows an Operating Point (OP) of the compressor described from a compressor map. The compressor is connected with a downstream system described by load curve A. The Operating Point on the compressor map gives information about the mass flow rate (M_{op}), pressure (P_{op}) of the gas supplied, opening of the IGV (θ_{op}) and isentropic efficiency ($\eta_{is,op}$) at steady-state conditions.

There are several methods to control a compressor as explained in the next Section 2.3.2. Figure 2.7b shows an example of changing operating point from OP1 to OP2. It shows that by increasing the opening of the IGVs by 20^o the

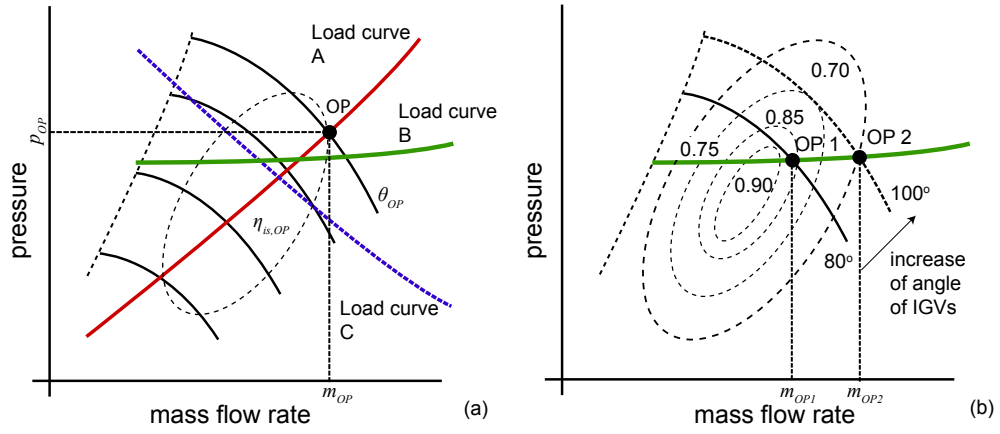


Figure 2.7: Various types of load curves and an Operating Point (OP) of a compressor connected with load curve A (a) and operating point change OP1 to OP2 by increasing the position of the IGVs (b).

operating point changes. The compressor at OP2 provides higher mass flow rate at a higher pressure than in the case of OP1, and moreover the efficiency of the compressor drops to 0.7. This information, pressure, mass flow rate and efficiency can be used to estimate the power consumed from the compressor (power in the shaft). By using the efficiency of the shaft and gearbox, the power provided by the driver can be computed.

2.3.2 Control methods

There are several control strategies to adjust the operating point of a compressor, namely use of variable speed drives, suction and discharge throttling, inlet guide vanes and recycling valve. More than one control strategy from the aforementioned can be applied to a compressor to change its operating point. A brief description of each control method is given below according to Lipták (2006) and Kurz et al. (2012):

- Compressors can employ *variable speed drives* which can modify the values of the rotational speed of the shaft. The drivers can be variable speed electrical motors, steam or gas turbines. This control method is considered as one of the most efficient.

- *Suction throttling* is mainly used for fixed speed compressors. The basic principle of this method is that a throttling device, which is usually a butterfly valve, varies the pressure of the inlet stream of gas at the inlet of the compressor, and hence this results in changing the mass flow rate. A drawback of this technique is that efficiency losses occur when the flow passes through the valve.

- *Discharge throttling* works in the same way as in the case of the suction throttling. In this case, throttle valve is located at the discharge side of the compressor. The suction and discharge throttling are considered as the least efficient methods to control a compressor.

- *Inlet Guide Vanes* (IGVs) are the actuators which modify the operating range of a compressor. Indeed, IGVs consist of several blades placed at the inlet of the compressor. In the instance of a multi-stage compressor, IGVs can be found at the inlet of the first stage exclusively.

- The flow can be recycled to modify the operating point of a compressor. However this method is used in a situation of emergency as this method of control is inefficient. Indeed, compressed gas is recycled back to the inlet of the compressor, therefore the energy to compress the gas is wasted.

- Multiple compressor units in parallel can be used to increase and decrease the total capacity of the compressor station by switching on or off individual compressors.

2.4 Management of compressor stations

2.4.1 Operational tasks of compressors

Figure 2.8a shows a pyramid of the typical decisions or control tasks of plant-wide automation of a process plant according to the ANSI/ISA-95 standard (Harjunkoski et al., 2009). The pyramid includes the *Process Control System* (PCS) which involves real time set point control and real time monitoring of the process in Level 0, 1 and 2. The time scale of Process Control System is seconds or fractions of seconds. The *Manufacturing Execution System* (MES) of Level 3 deals with the manufacturing operations and considers decisions for example the scheduling of the units of the operations for the next days or hours. The Manufacturing Execution System also considers the topic of the optimisation of the process by improving its performance in real time. Indeed, the optimisation suggests the best set points of the control system. On the top of the pyramid, in Level 4, is the *Enterprise Resource Planning* (ERP) which considers business planning and logistics. The high level decisions of the Enterprise Resource and Planning deals with markets, production targets and sales.

Figure 2.8b presents a pyramid of decisions of the operations of a compressor station. At the top of this pyramid, in Level 4, there are the targets of demand, given from the Central Dispatch Department in the case of compressors in the natural gas industries (Paparella et al., 2013). The targets of demand are the input of Level 3 of the pyramid. Level 3 includes the scheduling and maintenance

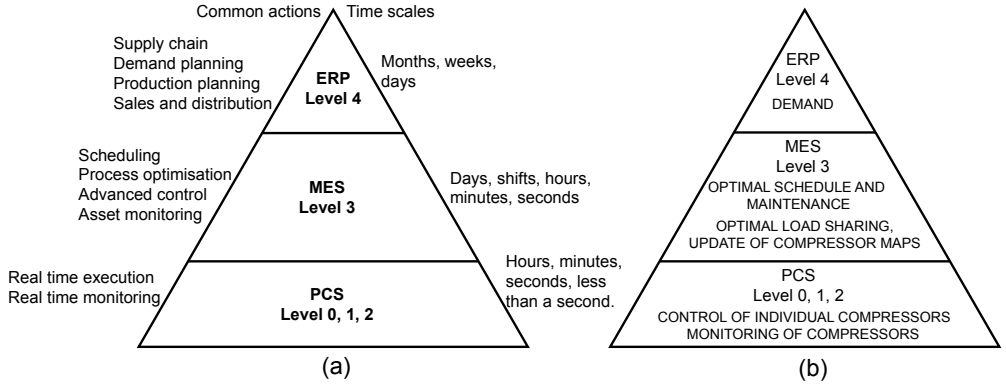


Figure 2.8: Decision pyramid of a plant according to the ANSI/ISA-95 (Harjunkoski et al., 2009) (a) and the corresponding decision pyramid of a compressor station (b).

of compressors. The same level also deals with the optimal distribution of the load among the compressors which provides set points, for example the set points of the speeds of the motors of the compressors, to the controllers of the control system. The lowest levels, Level 0, 1, 2, involve the control of the individual units and the monitoring of the process. The role of the controllers is to deal with disturbances with fast dynamics, thus to keep the given set points from the upper level constant. The *Real Time Optimisation* and the *scheduling* of compressors are considered as control tasks which belong in Level 3 of the decision pyramid in Fig. 2.8b.

2.4.2 Supervisory control: Real Time Optimisation (RTO)

According to Fig. 2.8a, Level 3 of the decision pyramid of a plant is concerned, among other tasks, with the online calculation of the optimal set points of the process operations of the plant. The calculation of the optimal set points can be also carried out for combined units of a plant. This procedure is known as *Real Time Optimisation* (RTO). According to Edgar et al. (2001), the application of RTO in chemical industry started in the 1960s. Initially, a large mainframe computer computed the optimal process set points which were subsequently sent to analogue controllers to adjust the operating points of the operation. In the 1970s, real time optimisation was implemented into computer control systems with a Distributed Control System (DCS). In this case the RTO was named supervisory control.

The computation of the optimal set points of the process variables can be determined from an optimisation problem. This optimisation problem, for which general description will be explained in Section 2.5, aims to minimise (or maximise) a performance criterion while at the same time considers constraints of the examined system including constraints derived from the steady-state modelling of the processes. These process models can be derived from fundamental knowledge (physics), experimental data or a combination of them. The set points, which are determined every day, hour or minute, can be temperatures, pressures or flow rates.

Furthermore, the performance criteria can be the increase of production rates, the minimisation of energy consumption or the minimisation of the operational cost. Seborg et al. (2004) defines the performance criterion as the operating profit. The constraints of the operation can be design, safety or environmental boundaries. For instance, the material of the blades of the compressor defines a maximum temperature for the gas stream flowing into the compressor. Another example is the flow rates boundaries which are dictated by geometrical restrictions of the compressor.

The implementation of the RTO in a process operation using a traditional reg-

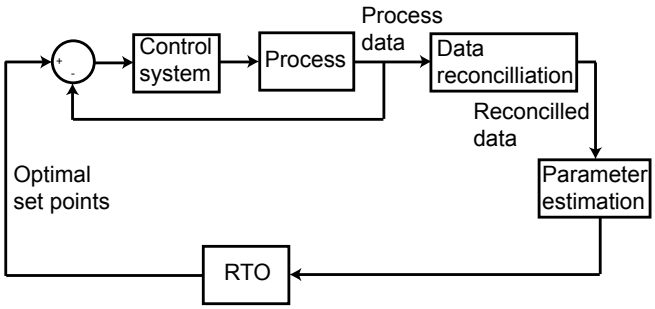


Figure 2.9: A block diagram describing the implementation of RTO with regulatory feedback control in a process operation (Seborg et al., 2004).

ulatory (feedback) control can be seen in Fig. 2.9. Real time optimisation handles the steady-state operation and the feedback control deals with the dynamics of the system. Hence, the outer RTO loop runs slower than the inner feedback control loop. Seborg et al. (2004) states that if the RTO model and the dynamic model of the regulatory control have substantial difference, this results in poor performance of the overall system.

According to the Fig. 2.9, real time optimisation includes the following steps:

- Determination of the steady-state operational conditions of the plant by monitoring process data,
- Reconciliation of the data,
- Estimation and update of the parameters of the process model,
- Calculation of new optimal set points,
- Implementation of the new set points through the control system.

The sequence of the tasks of the RTO starts when the process reaches a steady-state. Seborg et al. (2004) mentions that the existence of a steady-state is determined by monitoring a number of measurements, such as compositions and production rates, within allowable tolerances. A RTO run completes all the procedures (i.e. data transfer, reconciliation, update of the parameters of the process models, calculation and application of new set points of the controllers) during the steady-state before the conditions of the process change and a new calculation has to commence.

Edgar et al. (2001) states that the implementation of RTO and feedback control is not the normal approach in industrial practice. Advanced process control strategies such as model predictive control are used. Darby et al. (2011) reports

that the basic structure of Real Time Optimisation in cascade with Model Predictive Control has become the standard approach for implementing steady-state optimisation to improve the process which operates around nominal steady-states. Edgar et al. (2001) and Darby et al. (2011) present the integration of RTO and model predictive control in detail.

2.4.3 Operational planning and scheduling

Operational (or production) planning and *scheduling* are considered as separated levels according to Fig. 2.8a. However, an overlap exists between these two decision tasks in different levels in the pyramid in Fig. 2.8a. Edgar et al. (2001) reports that planning focuses on broad classes of products and provision of adequate manufacturing capacity at time horizons of years, months and days. Moreover, scheduling focuses on details of material flow, manufacturing and production at time horizons of weeks, days or hours. The activities of planning and scheduling according to Edgar et al. (2001) are summarised in Table 2.2.

Table 2.2: Planning and scheduling activities according to Edgar et al. (2001).

Corporate operations planning
Allocate production requirements to plants.
Balance facilitys capacity.
Optimise materials and product movements (supply chain).
Plant operations planning
Determine production plans.
Plan inventory strategy.
Determine raw materials requirements.
Plant scheduling
Determine run lengths.
Determine sequence of operations.
Provide inventory for production runs.

2.5 Optimisation (mathematical programming)

A major current focus in lowering production costs has enabled engineers to employ rigorous methods and tools for decision-making, such as *optimisation*, which is known as *mathematical programming*, to design and produce products both efficiently and economically. Optimisation techniques are being used in a large range of industries such as aerospace, chemical, electrical, construction and manufacturing industries. Typical examples of optimisation problems are: (a) identification

of the minimum weight of an aircraft, (b) decision of optimal operating conditions of a boiler in a process plant, (c) optimum design of electrical networks, (d) selection of the best site to construct a plant, (e) plan of maintenance and replacement of equipment to reduce operating costs in a manufacturing industry and (f) evaluation of plant data to develop a model of a process.

A wide variety of problems in design, construction and operation of chemical plants can be solved by optimisation. Focusing on operations of a plant, optimisation provides several benefits such as improvement of the performance of the processes, improved yields of the valuable products, reduced contaminants and undesired emissions, and reduced energy consumptions. Additionally, the optimisation of the operations results in reduced maintenance costs, longer times between shutdowns of expensive equipment and less wear of the machines. The purpose of Section 2.5 is to present the principles of optimisation theory and its application for solving industrial problems.

2.5.1 Definition of an optimisation problem

According to Edgar et al. (2001), an *engineering optimisation problem* can be described as follows: a process can be represented by some equations or experimental data, and the goal of the optimisation is the identification of the values of variables describing the process when the value of a single performance criterion becomes maximum or minimum under given circumstances. Both the given restrictions and the model of the process constitute the constraints of the mathematical problem.

General description

The statement of an optimisation problem is usually written in the following form (Rao, 2009):

Find $\mathbf{X} = [x_1 \dots x_n]^T$ which minimises $f(\mathbf{X})$

subject to the constraints:

$g_i(\mathbf{X}) \leq 0,\ i = 1, \dots, m$
$h_j(\mathbf{X}) = 0,\ j = 1, \dots, p$

Where:

- The $\mathbf{X}$ is a n-dimensional vector which describes the continuous variables of the problem, these variables are known as *design*, *decision*, *optimisation variables* or just *variables* of the optimisation problem.

- The $f(\mathbf{X})$ is the *objective function* of the problem which represents the performance criterion previously mentioned. In this case the objective function is single equation but it is possible to have more than one objective function to solve, i.e. multi-objective optimisation. The objective function can be linear or non-linear.
- The g_i are the *inequality constraints* which can be linear or non-linear.
- The h_j are the *equality constraints* which can be linear or non-linear.
- The n denotes the number of the variables, the m denotes the number of inequalities and the p is the number of the equalities of the optimisation problem.

An optimisation problem is defined by an objective function and constraints. Both the equalities and inequalities are the constraints. The constraints represent the restrictions of the optimisation problem. The quantities which are found inside the equations of both the objective function and the constraints can be classified into the decision variables, which represent the variables of the optimisation problem, and the *preassigned parameters*. The preassigned parameters are fixed values which cannot change during the procedure of optimisation and they are given as input. The procedure to identify the objective function and constraints of the problem is called *modelling*.

Constraints

All the values of the decision variables of the optimisation problem which satisfy the constraints define a feasible solution. The set of all feasible solutions constitutes the feasible region of the optimisation problem. Indeed, the combination of the equalities and inequalities determine the feasible region of a constrained optimisation problem.

Objective function

When there is more than one feasible solution of the optimisation problem, a performance criterion is employed to choose the best feasible solution among all of them. This performance criterion is called objective function. The choice of the objective function depends on the nature of the problem. For example, in a chemical plant the maximisation of the final product is the criterion preferred. In an air separation plant, the minimisation of the energy consumption can be a performance criterion.

Optimal points of a function

The optimality of the optimisation problem characterises the solution of the problem which is given by a n-dimensional vector $\mathbf{X}$. For the sake of graphical illustration, a decision variable x with $n = 1$-dimension is considered without losing generality. The optimality of each solution is characterised according to the following criteria:

- A solution x^* is considered *strong local minimum* of the optimisation problem if $f(x^*) < f(x),\ \forall\ x \in N(x^*, \delta)$, where δ-neighbourhood $N(x^*, \delta)$ is a set of feasible solutions within an arbitrary small distance δ from x^*.
- A *weak local minimum* is when $f(x^*) \leq f(x),\ \forall\ x \in N(x^*, \delta)$.
- A *strong local maximum* is when $f(x^*) > f(x),\ \forall\ x \in N(x^*, \delta)$.
- A *weak local maximum* is when $f(x^*) \geq f(x),\ \forall\ x \in N(x^*, \delta)$.
- A *global minimum* is when $f(x^*) < f(x),\ \forall\ x \in V$, where V is the feasible domain space of function f. The *global minimum* is also called *global optimum* of the function. For *global maximum*, the symbol $<$ is replaced with the $>$ in the later statement.

Convexity

Another useful definition for the characterisation of optimisation problems is convexity. The convexity criterion helps to characterise a set as *convex* or *non-convex*. Thus a set C is convex if:

$$C \text{ is convex } \iff \forall\ (x, y) \in C\ \wedge \lambda \in [0, 1] : ((1 - \lambda)x + \lambda y) \in C$$

A function $f : C \to \mathbf{R}$ defined on a nonempty subset of C of $\mathbf{R}^n$ and taking real values is called convex, if:

- the domain C of the function f is convex
- for any $x, y \in C$ and $\lambda \in [0, 1]$: $f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$.

If the above inequality is strict whenever $x \neq y$ and $0 \leq \lambda \leq 1$, then f is called *strictly convex*. A function f such that $-f$ is convex is called *concave*. An important characteristic of convexity is that if an optimisation problem (i.e. objective function and constraints) is convex then any local minimum is global minimum Edgar et al. (2001).

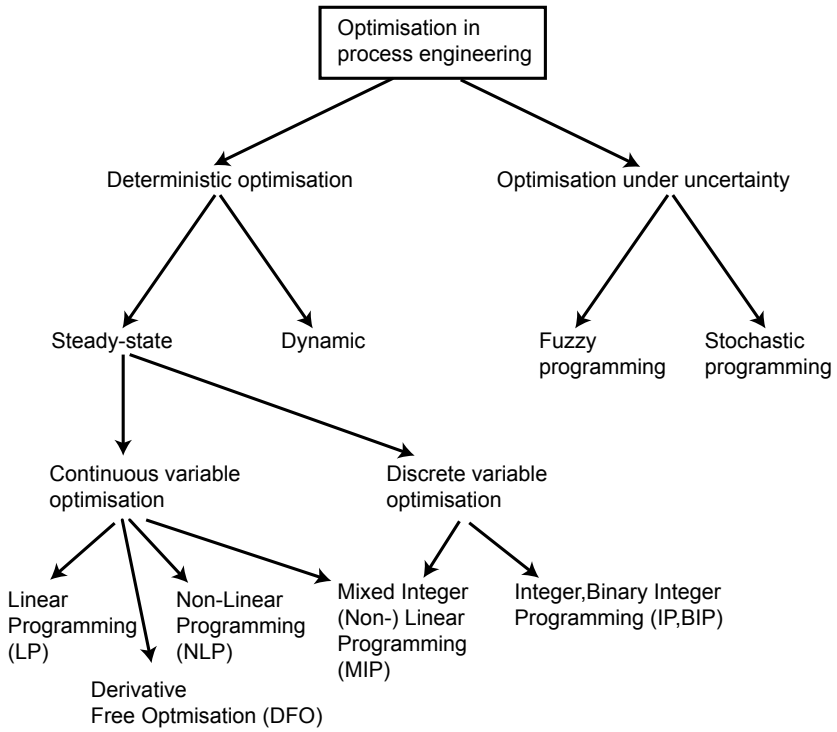


Figure 2.10: A general classification of optimisation problems in process engineering.

2.5.2 Classification of optimisation problems

Optimisation can be classified according to different criteria: for example the types of the variables (continuous, integer or mixed) and the consideration of uncertainty or not. Figure 2.10 illustrates a general overview of different types of optimisation problems based on the studies of Edgar et al. (2001), Biegler and Grossmann (2004), Sahinidis (2004) and Rao (2009).

Stochastic and deterministic optimisation

Depending on the nature of both variables and parameters of an optimisation problem, the problem can be classified into deterministic or stochastic. *Stochastic optimisation* deals with problems which involve uncertainty in the values of their variables or their parameters. Prices of electricity, heat transfer coefficients, demand of products, and ambient temperature and pressure are a few examples of parameters which can involve uncertainty in optimisation problems of process systems. *Deterministic optimisation* does not consider uncertainty of variables and parameters of the optimisation problems.

This PhD thesis will focus on the deterministic approach for modelling and optimising operations of process systems and systems with large energy consump-

tion such as export gas compressor stations. Figure 2.10 shows that deterministic optimisation problems are further classified into steady-state and dynamic optimisation. Dynamic optimisation employs time-dependent models to determine the optimal trajectory of a value of a process variable. Here, process operations are modelled in dynamic conditions using Differential-Algebraic Equations (DAEs). Typical examples of dynamic optimisation are optimisation of the scheduling of batch processes (b) optimisation of processes with unstable conditions (c) evaluation of control schemes (d) evaluation of safety studies and (d) identification of the optimal behaviour of the system during start up, upset, shut down according to Cervantes and Biegler (2009).

Deterministic steady-state optimisation solves algebraic equations and it does not involve differential equations. As shown in Fig. 2.10, *steady-state optimisation* can be further categorised into continuous or discrete optimisation depending on the type of the variables of the problem, i.e continuous, integer or a combination of them. In the case of the use of both continuous and discrete variables to describe an objective function or/and constraints, then the optimisation problem is considered as *Mixed Integer Programming* (MIP) problem.

Linear and Non-Linear Programming methods

Continuous variable optimisation involving only continuous variables can be divided into Linear Programming (LP) and Non-Linear Programming (NLP). The term of LP was given by George Dantzig in 1947 (Edgar et al., 2001). A *Linear Programming* problem is the optimisation problem which uses both linear objective function and linear constraints. On the other hand, a non-linear programming problem involves non-linear objective function or/and non-linear constraints. Linear programming problems have the generic form:

$$\text{maximise } \{f(\mathbf{X}) = c^T\mathbf{X} : \ A\mathbf{X} \leq b, \ \mathbf{X} \geq 0\}$$

where f is the linear function to be maximised (or minimised in a minimisation problem) and $\mathbf{X}$ is the vector of decision variables. The vectors of parameters c, b and matrix of coefficients A are given. The objective is to estimate a feasible solution (if there is any) which maximises (or minimises) the objective described by the affine function $f(\mathbf{X})$ which is equal to $c^T\mathbf{X}$ in the general description. The constraints given by a set of inequalities $A\mathbf{X}$ define a convex polytope on which the optimisation problem has to search the optimal solution for the decision variables $\mathbf{X}$.

The main algorithms which solve LP problems are the Simplex and the Interior-point algorithms. The *Simplex* algorithm is based on the search of the optimal

solution in the vertices of the polytope which defines the feasible region of the optimisation problem. The algorithm moves from one vertex to the following one until predefined criteria to be met. In contrast with the Simplex algorithm, an *Interior-point* algorithm finds improved search directions strictly in the interior of the feasible region.

The *Non-Linear Programming* method involves the optimisation of a problem if one or more of the equations of the the objective function f, inequalities g and equalities h is non-linear. The main challenge when dealing with NLP methods is that when there are several optima and the aim is to estimate the global optimum and not any local minimum. The main techniques to determine local solutions are gradient-based search methods such as Newton's method and steepest descent approach (Edgar et al., 2001). Rao (2009) explains and presents in detail methods for solving non-linear programming problems.

Mixed Integer Programming (MIP)

When an optimisation problem involves continuous, and integer and/or binary variables the problem is called Mixed Integer Programming (MIP) which is further classified to Mixed Integer Linear Non-Linear Programming (MILP and MINLP) problems depending on the type of the objective function and the constraints. The general mathematical formulation of the optimisation problem becomes:

Find $\mathbf{X},\ \mathbf{Y}$ which minimises $f(\mathbf{X}, \mathbf{Y})$
subject to the constraints:

$g_i(\mathbf{X}, \mathbf{Y}) \leq 0,\ i = 1, \ldots, m$
$h_j(\mathbf{X}, \mathbf{Y}) = 0,\ j = 1, \ldots, p$

where vector $\mathbf{X}$ takes continuous and vector $\mathbf{Y}$ takes integer and/or binary values. More specifically, the description of a MILP optimisation problem is given by the following formulation:

maximise $c^T\mathbf{X} + h\mathbf{Y}$
subject to:
$A\mathbf{X} + G\mathbf{Y} \leq b$
$\mathbf{X} \geq 0, \mathbf{Y} \geq 0$ and integer or binary

where $\mathbf{X}$ is the vector of positive continuous variables and $\mathbf{Y}$ is the vector of integer and/or binary variables. The vector coefficient $c,\ b,\ h$ and matrices of coefficients $A,\ G$ are given in the optimisation problem.

The values of the binary variables $\mathbf{Y}$ represent decisions which refer to events which should or should not occur. Therefore a variable Y_λ takes value one if an event λ should occur or value zero if this event should not occur. The event can be an on/off mode of an operation, a logical condition, fixed cost or a piecewise linear function decision.

The main algorithms used to solve Mixed Integer Programming Problems are the cutting plane algorithm of Gomory (1960) and the branch-and-bound algorithm of Land and Doig (1960). In addition, the algorithm of Balas Balas (1965) can efficiently deal with binary problems according to Rao (2009). Kopanos (2010) summarises other methods to solve mixed integer programming problems such as the branch-and-cut method, which is a combination of branch-and-bound and cutting plane algorithm, and disjunctive programming.

Other optimisation methods

Biegler and Grossmann (2004) reported optimisation techniques without derivatives, namely genetic algorithms, simulated annealing and classical direct search methods. These techniques have the advantage of easy implementation into a wide spectrum of problems. They are used before applying any sophisticated optimisation methods. Moreover, most of these techniques derived from heuristics. This type of optimisation is known as Derivative Free Optimisation (DFO) which is presented in the study by Biegler and Grossmann (2004).

2.5.3 Software and solvers

Software for optimisation

The software platform for optimisation problems mainly to be used in this study is GAMS (General Algebraic Modeling System). Software GAMS allows the user to model and solve linear, non-linear, and mixed integer programming optimisation problems. Since GAMS includes an integrated development environment, this adheres the use of third-party developed optimisation solvers. Examples of the most used solvers are CPLEX, CONOPT, DICOPT and BARON. The advantage of the GAMS software is that it presents and models the equations in a concise, human-readable algebraic description. Initially, GAMS was developed for applications related to economics and management science, however the researchers in the process system engineering community use GAMS to solve relevant engineering optimisation problems.

Solvers of GAMS

The most commonly used solver in GAMS is CPLEX or IBM ILOG CPLEX Optimisation Studio (IBM, 2015). CPLEX took its name from the Simplex method which is implemented in the C programming language at its original development stage. The CPLEX solver can deal with integer programming problems, large scale linear programming problems, and convex and non-convex quadratic programming problems.

2.6 Summary of the chapter

This chapter classified industrial compressors according to their design and principle of operation. The most common types of compressors used in the process and natural gas industries are centrifugal compressors. The basic principles of multi-stage centrifugal compressors were explained. Moreover, a compressor map defines the operation of the compressors and provides the feasible region of operation and efficiency of the compressor. The maintenance of a compressor plays a key role in its operation and lifecycle. The different types of strategies of maintenance, namely corrective, preventive and predictive were explained. Furthermore, the recoverable and non-recoverable maintenance of a compressor were mentioned.

This chapter also presented the management of the compressors with the use of an automation pyramid of different decision tasks of a process plants tailored to the specific tasks of a compressor station. The current study focuses on the Real Time Optimisation (RTO), and Scheduling and maintenance of compressors. These topics are related to the Level 3, Manufacturing Execution System, of the pyramid of the decision tasks. Finally, the principles of mathematical programming and the basic software tools to solve optimisation problems have been presented.

3 Literature review on the optimal operation of compressor stations

3.1 Description of the chapter

Chapter 3 presents the literature review on the topic of the optimal operation of compressor stations. Section 3.2 will classify the types of problems which deal with the improvement of the operation of the compressors considering different applications, and considering steady-state analysis or multi-period approach. Moreover, the optimisation problems which deal with optimal operation of compressors will be classified based on the types of variables that are used in the mathematical formulations. Binary variables are used to model decisions which deal with discrete events, for example starting up or shutting down a machine. Continuous variables represent flow rates and pressures.

Moreover, Chapter 3 focuses on studies from previous researchers on: (a) optimal load sharing in Section 3.3, (b) scheduling in Section 3.4 and (c) maintenance of compressors in Section 3.5. The presentation of the state-of-the-art in these topics will show the need for developing optimisation frameworks to deal with the optimal distribution of the load and with the scheduling of the compressors considering maintenance tasks of the individual compressors. Therefore, Section 3.6 explicitly reports the gaps of knowledge considering the literature review and justifies the contributions of the thesis.

3.2 Classification of problems of optimal operation of compressors

3.2.1 General overview

The optimisation of compressor stations is a topic which many researchers have studied in the recent years. Compressors are used in various applications where the nature of each application influences the objectives and the constraints of the optimisation problem. For instance, problems examining natural gas systems which transport gas through long pipes can consider the phenomenon of the linepack (Kurz et al., 2012). The linepack is a time-dependent phenomenon which de-

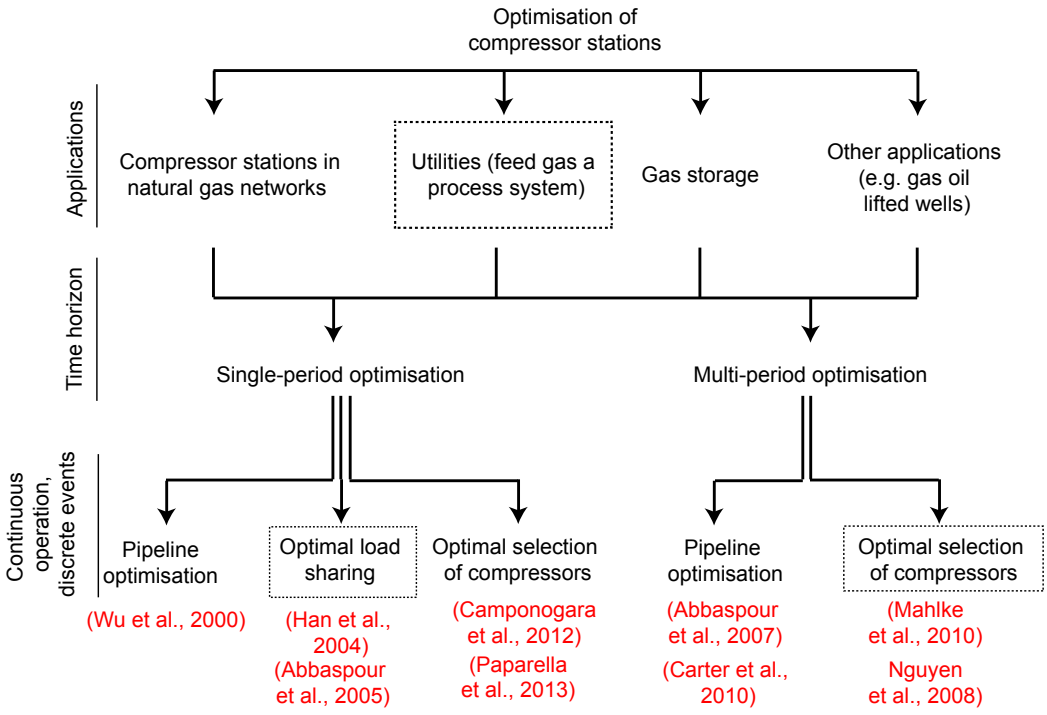


Figure 3.1: Classification of optimisation of compressor stations.

scribes the inherent capability of a pipe to store gas. The stored gas can be used for example when the operation cannot satisfy the demand due to an unexpected failure of a compressor (Kurz and Brun, 2012a). On the other hand, a supply compressor station of a chemical process provides gas at a constant pressure.

The optimal operation of gas compressors in different process systems has been studied by many researchers, for example Han et al. (2004), Hasan et al. (2009) and Sun and Ding (2014). These researchers employed a steady-state optimisation analysis (solution for one set of operational conditions) compared to the scheduling approach from van den Heever and Grossmann (2003) which used a multi-period approach. The latter study focused on the pipeline network and not on the operational aspects of the compressors, such as shut down and start up actions. Other studies focused on the optimal control of the compressors employed Model Predictive Control strategies, recent examples are the papers by Gopalakrishnan and Biegler (2013), and Zavala (2014).

Figure 3.1 shows the main groups of optimisation problems regarding: (a) the application, (b) the time horizon and (c) the consideration of continuous operation or/and discrete events. The set points of the compressors refer to continuous quantities. The selection of the compressors to operate online is modelled with the use of binary variables which describe discrete events such as startups and shutdowns.

3.2.2 Optimisation of compressors regarding the application

The top section of Figure 3.1 presents the classification of the optimisation of compressor stations considering four main categories of applications, namely natural gas networks, utilities, gas storage and other applications. Many researchers, for example Shaw (1994), Wu et al. (2000), Cobos-Zaleta and Ríos-Mercado (2002) and Borraz-Sánchez (2010), have studied the optimisation of natural gas networks. These studies focus on the solution of problems which involve complex networks of pipelines and compressors. Due to the complexity and the large-scale of these problems, a compressor station is modelled as a single compressor unit, even if a compressor station consists of multiple compressors. The optimisation of this kind of problem usually includes the basic storage modelling of the pipes, i.e. linepack. The linepack of the pipe provides flexibility to the system, nevertheless the compressors have to operate at higher pressures resulting in higher power consumption. This trade-off between flexibility and increased power consumption is investigated in these problems.

The second type of application refers to process systems which include utilities in which compressors spend most of the energy consumption of these systems. The compressors of utilities usually do not store the gas in the pipes, and thus the analysis of these systems do not focus on detailed modelling of the pipes. The reason is that the friction losses are not important compared to the previous category of the natural gas networks where the length of the pipes are hundreds of kilometres. A representative example of this type of application is the study by Han et al. (2004) which examines the case of an air- and gas-supply compressor station of a terephtalic acid manufacturing plant. The authors mentioned that such compressor stations consume more than 75% of the total electric power consumed in the plant. Widell and Eikevik (2010) studied the optimal operation of parallel screw compressors in refrigeration systems. Compressors in refrigeration systems consume large amounts of energy and can be used in oil and gas industry in a number of cases as Kurz and Brun (2012a) reports cooling a compressed gas, Liquefaction of Natural Gas (LNG) and removing liquids in a gas plant.

A typical example of the application of compressors for utilities is the first case study of this thesis which deals with an air compressor station of an air separation plant. This case study of the air compressors of an air separation plant has been introduced in Section 1.4.1. The role of a process system which includes a compressor station can be considered either as a utility or as a production plant. The description of the end-user of the process system defines the process system itself. For instance, if an air separation plant produces oxygen to sell to external customers, then the plant is considered as production plant. On the other hand if the downstream system of the plant is a major process which uses the oxygen

for producing the final products which are not oxygen or nitrogen, then the air separation plant is considered as a utility. Agha et al. (2010) mention that a utility is defined as any quantity containing energy and can be useful to an industrial process in manufacturing the final product. Moreover, Section 3.4.1 will present examples of the optimisation of utilities for steam production, articles in Chemical Engineering refer to these kinds of problems as "utilities".

The third type of application of compressors deals with the optimisation of compressors in gas storage and it has been studied by Kurz and Brun (2010). Figure 3.1 shows that the fourth category considers applications such as lifting gas from gas fields. Moreover, Kurz and Brun (2012a) reported that gas storage and gas lifting from gas fields are considered as midstream applications of gas compression in the oil and gas industry. Studies from Camponogara et al. (2012) and Silva and Camponogara (2014) formulated a MINLP model that supports the real-time decision-making problem in oil fields considering wells, reservoirs and compressors. The current work does not study the optimal operation of compressors in these types of applications.

3.2.3 Optimisation of compressors regarding time horizon

Many authors examined the optimisation of gas compressors in different applications considering different time horizons. According to Fig. 3.1, there is the classification of single- and multi-period optimisation.

Single-period optimisation

The authors who have dealt with problems from the former category usually do not explicitly name the optimisation problems as *single-period*. However, these studies deal with steady-state analysis and consider information of one time period. An example is when an optimisation problem uses a fixed set of values of parameters which are related to the operation such as inlet ambient conditions and demand.

Single-period optimisation can be further classified as (a) steady-state pipeline optimisation (Wong and Larson, 1968; Wu et al., 2000; Borraz-Sánchez, 2010; Carter et al., 2010), (b) optimal load sharing (Han et al., 2004; Abbaspour et al., 2005; Paparella et al., 2013) and (c) optimal selection of compressors (Wright et al., 1998; Cobos-Zaleta and Ríos-Mercado, 2002; Widell and Eikevik, 2010; Camponogara et al., 2012; Paparella et al., 2013).

Steady-state pipeline optimisation examines the optimal operation of the fuel cost minimisation problem (Borraz-Sánchez, 2010) considering information of one period (Carter et al., 2010). Section 3.3 presents the optimal load sharing topic in detail. Optimal selection of compressors is the third sub-category of single-period

optimisation. It involves discrete events in the formulation of the optimisation problem, for example variables which represent which compressor stations should operate (decision for online or offline mode) (Cobos-Zaleta and Ríos-Mercado, 2002) or binary variables for deciding assignments of gas-lift compressors to injection wells (Camponogara et al., 2012).

The binary variables represent decisions which are related to events which result in great changes in the system, for example switching on a compressor takes considerable time compared to the change of its set point. In addition, the total operational range of the system changes significantly after this change. Hence, without knowledge of the following trends in demand, a decision which would be optimal for the current moment, which is estimated from a steady-state analysis, may be sub-optimal in the long term in the case of considering a longer time period to optimise. Therefore a multi-period approach should consider binary events, and therefore formulations such as MILP should be included in multi-period approaches.

Multi-period optimisation

Multi-period optimisation employs information about the future based on forecasting methods. The forecast of the demand and parameters of the constraints are used in the optimisation model. The optimisation suggests decisions for the current operation and these decisions are based on the forecasted information. An example of a forecasted parameter is a scheduled maintenance of a compressor, which might influence the decisions for the current operation.

Multi-period optimal operation of compressors, which considers long time horizons, includes (a) pipeline optimisation with a fixed number of operating compressors (Marques and Morari, 1988; Abbaspour et al., 2007; Carter et al., 2010) and (b) optimal selection or scheduling of compressors (van den Heever and Grossmann, 2003; Nguyen et al., 2008; Mahlke et al., 2009). The current work will examine the scientific area on the scheduling of compressors, thus Section 3.4 provides a detailed literature review on the topic.

3.3 Optimal load sharing

Kurz et al. (2012) and Garcia-Hernandez and Brun (2012) reported that the installation of spare stand-by compressors in a station increases its flexibility. Spare compressors are used when the capacity of a station is not enough to satisfy the demand which is requested due to changes in the demand side of the plant. These changes are mainly caused because of changes in the markets (prices of products and electricity) and changes in the internal production strategies of the company

such as products specifications and amount of production.

Many authors and practitioners reported that it is difficult or impossible for different compressors in a compressor station to have identical characteristics and efficiencies (Abbaspour et al., 2005; Lipták, 2006; Milum, 2012; Rolls Royce, 2014). Moreover, these characteristics and efficiencies change over time due to fouling and erosion (Kurz and Brun, 2012b), and non-uniform maintenance plans which result in dissimilar compressor maps for the same compressor at different time periods (Forsthoffer, 2011; Paparella et al., 2013; Cicciotti et al., 2014a). The use of surrogate models and process data can predict the performance and characteristics of the compressors according to the paper by Tirnovan et al. (2008).

Many strategies have been used to share the load between compressors. Kurz et al. (2012) commented that if compressors have identical compressor maps, then the load can be equally split or they can operate at the same surge margin. Surge margin is the distance between operating point and surge. In the same work, it was reported that if two compressors have different sizes or different efficiencies the more efficient compressor should provide the base load and the second compressor, the less efficient one, has to deal with the fluctuations of the load. On the other hand Twohig (2011) reported that parallel compressors in a pharmaceutical fermentation process were decided to operate so as the most aged (and less efficient) compressor worked at its maximum flow rate (where its optimal point is) and the more efficient compressors should vary their operation to cover the fluctuations of the demand. Moreover, Lipták (2006) suggested a method in which the first step is to estimate the performance of the compressors in units of flow per units of power and the second is to load the units in order of their performance.

The approach of this thesis is to formulate an optimisation problem to distribute the load among the compressors. A few researchers (Jenícek and Kralík, 1995; Han et al., 2004; Abbaspour et al., 2005) studied the optimal load sharing of compressors operating in parallel in order to minimise the fuel consumed by the gas turbines drivers. However, these works did not present an online application which considers the practical aspects of the implementation of the actual optimisation, for example update of the maps and steady-state detection. Paparella et al. (2013) presented an online optimisation framework which updates the parameters of the models of the compressors online. The optimisation framework applied to a gas boosting station. The authors showed that surrogate models can be used to predict the performance of the compressors for online applications.

3.4 Scheduling and optimal selection of compressors

The optimisation of natural gas networks involving compressor stations received considerable attention in the last five decades (Sood et al., 1971; Marques and Morari, 1988; Mahlke et al., 2009; Carter et al., 2010). Many studies have focused on the optimisation of the overall network without zooming into the operational complexity of the compressor stations of the gas network, examples of these works are mentioned by Borraz-Sánchez (2010) and recently by MohamadiBaghmolaei et al. (2014).

Most of the studies on natural gas networks emphasise the use of optimal control, usually by employing dynamic programming. However, the operational aspects such as startup and shutdown, timing constraints and maintenance of the compressors are typically neither modelled explicitly nor optimised. The contributions from Uraikul et al. (2004) and Nguyen and Chan (2006) are some representative works that include a few operational decisions for the compressors. However, both studies assumed that compressors operate at a single operating point (i.e., non-partial load operation), and therefore the mass flow of individual compressors was not considered into the optimisation.

Compressor stations with parallel compressors are typically used to distribute gas from an upstream process to other downstream processes. A few researchers have addressed the optimisation of these systems. For example, van den Heever and Grossmann (2003) proposed a mathematical programming approach for the production planning and reactive scheduling problem of a hydrogen supply network. Emphasis was placed on the modelling of the pipeline network and not on the operation of the compressors. In particular, minimum run and shutdown times and costs for compressors were ignored. Jiao et al. (2012) presented a similar multi-period optimisation model for the scheduling of the operations of a hydrogen system involving multiple compressors. The authors considered the start up of the compressors, however they used a simplified approach to represent the feasible window of the operation of the compressors.

The work of Mahlke et al. (2009) used a mixed integer continuous variables approach to include discrete events into an optimisation transient problem to deal with optimal operation. However, the authors did not consider multiple compressors in parallel within each compressor station and a short time horizon was employed (24 hours). In a recent study by Paparella et al. (2013), the authors studied the optimisation of a compressor station with parallel compressors. Although this study used integer variables to represent the start up and shut down of a compressor, a steady-state analysis accounted only for one time point. None of the aforementioned studies included maintenance constraints.

A few studies considered discrete variables in their optimisation models, however the authors of these studies focused on the optimisation of a single time point: Wu et al. (2000) presented a steady-state pipeline analysis, Abbaspour et al. (2005) and Paparella et al. (2013) demonstrated the optimal load sharing of multiple compressors, and Camponogara et al. (2012) studied the optimal selection of compressors for a single-period of the optimisation problem as mentioned in Section 3.2.2.

3.4.1 Methodologies with similar formulation for other applications (e.g. utilities for process systems)

There is significant research contribution in the operational optimisation of utility systems for steam production or cogeneration (Iyer and Grossmann, 1997; Kim and Han, 2001; Varbanov et al., 2004; Velasco-Garcia et al., 2011; Chen et al., 2012; Luo et al., 2011, 2012). When a plant generates electricity and heat, this is called cogeneration. The formulation of the optimisation problem in these works involved the choice of the turbines to work online and the the estimation of the optimal distribution of their load to minimise operational costs. Other similar formulations can be seen in the publications by Kim et al. (2002), who studied the optimisation of the operation after a failure of a unit, and by Agha et al. (2010), who demonstrated the optimisation of the integration a production plant and a utility system for minimising the operational cost of both systems simultaneously.

Iyer and Grossmann (1997) presented a multi-period approach to minimise the operational costs of a utility system while at the same time the system satisfies the demands for water, steam and power. A MILP optimisation model considered discrete variables to represent the status of the units: on or off status. The output of the optimisation model is the choice of the operation of the units for each period. The formulation of the optimisation considered constraints for example for the start up and shut down of the units, called changeovers. The objective function included a term representing the purchase of utilities from external providers in the case the plant could not meet the demand.

The main differences between the problems of the optimisation of utilities for cogeneration of power and steam, and the optimisation of the gas compressors are: (a) different physical processes, and (b) different functions which describe the behaviour of their operating units, turbines in the case of utilities and compressors in the other case. In the case of utilities, the steam is at high pressure at the inlet of the turbines and at lower pressure at their exit. The compressors work in the reverse way, they increase the pressure of the gas. The models of turbines, which are well established as can be seen in Luo et al. (2011), are different from the models of compressors in terms of the description of the feasible window of

operation and power consumption. Therefore, the optimisation models include different constraints because turbines and compressors are described by different physical processes.

However, these two categories of problem have similarities. For example, in the cases which involve turbines users have to find the optimal trade-off between expanding gas to different headers and generating electricity. The same principle can be seen in the case with compressors where users have to decide which compressor to provide compressed gas to which header. In both cases, headers connected with compressors (a) and output expansion levels of turbines (b) have different pressures. Another common point is that the total load in both cases can be controlled by starting up stand-by units or switching off the online ones. The corresponding costs are included in both optimisation problems which employ a multi-period approach.

Another application which demonstrates similar mathematical formulation to the optimisation of compressors is the unit commitment problem (Raglend and Padhy, 2008; Padhy, 2004). Operators of the electrical grid should optimally schedule the operation of generators to satisfy the demand in electricity. The decision variables are the choice of the generator units and their power outputs. The objective is to minimise the total operational costs and a multi-period approach is used.

There are other contributions in the literature describing process applications with features similar to those found in the optimal operation of compressors. Among them, Rong and Lahdelma (2005) presented a linear programming planning model and optimisation algorithm for trigeneration. Thorin et al. (2005) proposed a mathematical programming model for the long-term planning of cogeneration systems in a competitive market environment. Kopanos et al. (2013) presented an optimisation framework for the energy production planning of a network of combined heat and power generators. Zhuan and Xia (2013) proposed a dynamic programming algorithm for the operational scheduling of a pumping station with multiple pumps. Recently, Kopanos and Pistikopoulos (2014) introduced a reactive scheduling rolling horizon framework based on a state-space representation and multiparametric programming. The proposed approach was applied in a network of combined heat and power units.

3.4.2 Operational planning of air separation plants

The process of the cryogenic air separation starts with the intake of atmospheric air that is usually filtered to remove dust. This air is compressed and re-cooled with the use of an aftercooler. Then, the compressed air is purified through molecular sieves (molecular filters) so as to remove carbon dioxide, gaseous hydrocarbons and

water vapour. The compressed air is cooled to cryogenic temperatures through the main heat exchanger and the expander. According to Zhu et al. (2011), the compressed air is split into two streams, the main stream and a low pressure stream after the expansion of the main stream through the expander. The two streams enter an air separation column which separates the air to its basic components of oxygen, nitrogen and argon. Figure 3.2 presents a simplified structure of a cryogenic air separation process having oxygen and nitrogen as products. Gaseous nitrogen collects in the top of the air separation column, and liquid oxygen collects at the bottom of the air separation column. The oxygen at the bottom is vaporised while nitrogen in liquid form is introduced at the top of the column. Flow rates, temperatures and heat flows are designed so that the required purity is met.

The final products can be transferred via pipelines to inter-connected local industrial customers. For pipeline transfer, products should be vaporised into their gaseous form. For other longer-distance customers, it is most practical and economical to transfer them in a liquid form. In this case, the separated gaseous products are passed through a liquefier and then placed into dedicated storage tanks before their shipment by marine or land transportation modes. A liquefier is the equipment which provides additional refrigeration (separately from this occurring in the cryogenic air separation) for liquefaction. Liquefiers use the same principle as the air separation process to cool a gas through compression, expansion and heat exchange.

The cryogenic separation of air is a very energy-intensive process that requires significant heat integration of heat exchangers and air separation columns, in order to improve the energy efficiency of the overall process. Indeed, most previous research has been dedicated to the heat integration of air separation plants. In

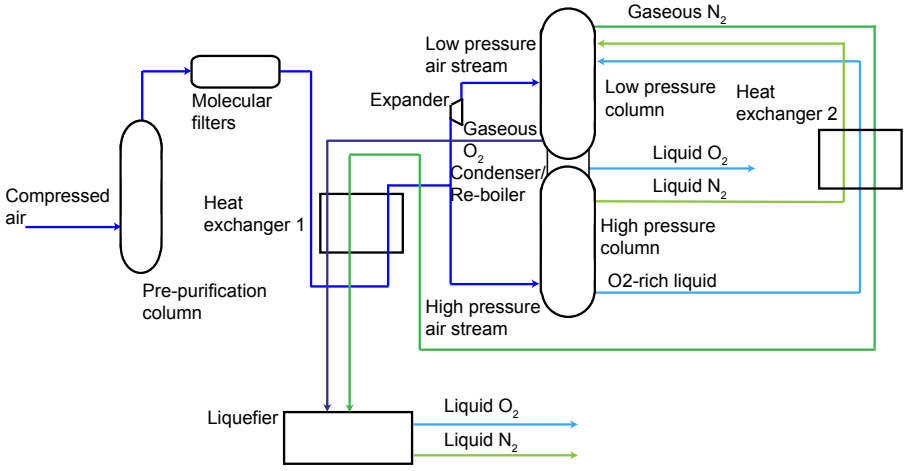


Figure 3.2: Simplified structure of a cryogenic air separation process with oxygen and nitrogen products.

cryogenic air separation, the network of compressors and the liquefiers are the main sources of power consumption.

There is a limited number of works concerning with the operational management of air separation systems. Ierapetritou et al. (2002) presented a MIP optimisation model which employed a black box model of the air separation plant. The objective was to minimise the cost of the operation of the air separation units, the cost of the liquefier and utility costs for lights and instruments. The model took into account three different operational modes of the plant, regular, assisted operation and shutdown. The resulting mixed integer linear programming model used binary variables to model these different modes. This deterministic model was included in a stochastic optimisation framework to consider uncertainty in the electricity prices. The optimisation of Ierapetritou et al. (2002) can generate the schedule of process operation modes and production rates.

Karwan and Keblis (2007) used a similar modeling approach to represent the operation of an air separation plant with different operational modes. In this study, the authors considered different electricity pricing contracts such as hourly based pricing (called as real time pricing) and time of use. Karwan and Keblis (2007) additionally considered product losses during configuration changes. Recently, Mitra et al. (2012) presented a MIP optimisation framework for the minimization of production, inventory and transition costs. The transition costs refer to the changes from one operational mode of the plant to another. This study also considered volatile electricity prices.

Following a different approach, Zhu et al. (2011) presented a NLP formulation that has as constraints a process model for the air separation process that accounts for mass and energy balances for the air separation columns, heat exchangers, and the throttle valves. The above articles have focused on the planning of the operational modes of the overall air separation plant, but did not address the detailed scheduling of the components of the air separation plant such as the compressors and the air separation columns.

3.5 Maintenance of compressors

Chapter 1 explained that the maintenance of a compressor may involve several actions such as bearing and rotor repairs, coupling and gear box maintenance, and inspections of the sealings and the motors. Moreover, the compressors which are placed in process gas applications mainly suffer from fouling. Fouling is the depositions of particles in the fluid to the airfoil. The depositions increase the roughness of the surfaces of the internal mechanical components of the compressors (e.g. impeller and diffuser area) and restrict the passage of the gas. Thus, the result

of the fouling is a decrease in performance and increase in power consumption for the same load compared to a non-fouled compressor.

A rise in the number of startups of a compressor increases its Equivalent Operating Hours (EOHs) (Bohlin and Wärja, 2010). The increase in EOHs results in decreasing the time between two consequent technical inspections and bringing the compressor closer to a major overhaul (Aretakis et al., 2012). The fact that a compressor is under a major overhaul results in reduced availability and flexibility of the overall system. An example for taking the decision to switch off a compressor is a case where the demand is low, thus it might be beneficial the compressor to be shut down. However, there is a trade-off between the scenario of keeping the compressor online working at lower capacity (but in less efficient operational conditions) and the scenario of switching it off.

There is research on determining the optimal washing schedule of compressors. However, most of the authors have focused on the examination of a single compressor which is usually part of a gas turbine system, i.e. a compressor coupled with a turbine. Moreover, most of the studies are focusing on advanced models describing the performance of axial compressors. Rao and Naikan (2008) studied the optimal washing schedule of a single compressor to determine the best intervals for periodic online and offline washings. Indeed, the authors considered online and offline washing costs, costs of inspections, cost of downtime and cost of random failures. An optimisation algorithm with iterative steps was developed to minimise the total cost.

Martín-Aragón and Valdés (2014) studied the optimal scheduling of offline washing of gas turbine compressors with thermodynamic methods which connect performance and operational aspects of compressors (e.g. efficiency and mass flow) with the increase of the heat rate and the drop of the full load power of the compressor. Similarly to the latter study, Sánchez et al. (2009) assessed the economic influence of different offline washing programs of the compressor on the lost revenue of a gas turbine. Fabbri et al. (2011) carried out a thermo-economic analysis in order to compare online and offline washings of a gas turbine.

Aretakis et al. (2012) formulated a cost function which included costs for capital cost, fuel cost, offline washing cost, other maintenance cost (excluding washing cost) and labour cost of a single compressor for one-year analysis. An optimisation algorithm considered this cost function in conjunction with a generic performance model of the compressor. The optimisation minimises the cost in an iterative way according to minimisation criteria. The output of this optimisation framework is the optimal washing intervals. The optimisation solver was the Simplex Down-hill Method in Multi-dimensions which does not use derivatives. Another study from Bohlin and Wärja (2010) developed a mixed integer programming (MIP) optimi-

sation problem to deal with non-critical routine inspections and service tasks.

Boyce and Gonzalez (2007) carried out experimental trials for long time periods to estimate the optimal scheduling of online and offline washing schedules. Basic thermodynamic quantities such as isentropic efficiency and overall heat rate were used to compare the performance of different schedules and different solutions for injecting inside the compressor.

All the previous studies mainly focused on the maintenance of a single compressor and most of them examined exclusively the maintenance without considering how the partial operation of the compressors influences the maintenance and vice versa. Although Boyce (2003) referred to a concept of integration between operation and maintenance, the author called this integration Performance Based Total Productive Maintenance, however this reference did not present a method to achieve this integration.

There are also papers which studied the maintenance scheduling of single or parallel units in other applications such as biopharmaceutical manufacturing (Liu et al., 2014), wind turbine farms (Gustavsson et al., 2014), process plants (Sequeira et al., 2001; Hazaras et al., 2012) and power plants with gas engines (Castro et al., 2014). The article by Castro et al. (2014) deals with the maintenance scheduling of parallel gas engines and the work demonstrates many similarities with the topic of the current thesis. Castro et al. (2014) employed a mixed integer linear programming model which considered shut downs and startups of the engines, and constraints which involved industrial restrictions related to maintenance, for example where there were limited maintenance resources. However, the model considered an industrial case with identical engines and their power consumption was independent of time and operational conditions.

Therefore, there is an open question to answer when examining a compressor station which includes several compressors in operation, which is to optimally maintain them and at the same time to consider an operation at high performance. This framework should consider the state-of-the-art which is described by Li and Nilkitsaranont (2009) who demonstrated a prognostic approach to estimate the remaining time of a gas turbine until its next maintenance event. Moreover, constraints of the maintenance personnel should be implemented in this framework. These constraints have been reported in the study by Castro et al. (2014). An example is the consideration of limited personnel for the maintenance. Other constraints can be a maximum number of units than can be maintained at a given time and there is a number of units which can be maintained simultaneously by the same maintenance group.

3.6 Gaps of knowledge and contributions of the thesis

Gap of knowledge in optimal load sharing

Study of the literature revealed a lack of a systematic way to optimally share the load of compressors online considering varying operational conditions, such as atmospheric temperature and pressure, and demand requested from downstream processes. One of the assumptions mainly used is that individual compressors have the same characteristics and the same performance behaviour. Regarding this assumption, the conventional practice is to distribute the load evenly among the compressors or to apply other similar strategies described in Section 3.3. In addition, a few works presented optimisation strategies to share the load, however to the best of the author's knowledge there is no approach which considers a comprehensive online application with aspects such as update of maps and steady-state detection. Therefore, there is a need *for developing a Real Time Optimisation (RTO) framework which optimally shares the load among parallel compressors in real time to deal with short-term changes in operation.* This is Objective One of the thesis, as introduced in Section 1.5.

The current thesis will present a methodology for generating data-driven models of multi-stage centrifugal compressors with a water cooling system and IGVs, which is not the most commonly studied case. Compressors with variable speed drives and without IGVs is the most commonly studies case in the academic literature. Real industrial data are used to generate these data-driven models. The data-driven models are implemented into an optimisation model which computes the distribution of the load among the compressors in order to achieve reduced operational costs. The proposed optimisation is formulated in a generic way and can be applied to a compressor station with non-identical parallel single-or-multi-stage compressors in utilities or in process systems when operational data are available.

Gap of knowledge in scheduling of compressors

The thesis suggests a comprehensive framework which deals with the optimisation of the compressors in two different levels, long- and short-term optimisation. One of the contributions of the current study is to consider the schedule of the compressors and the optimisation of the process in real-time. Previous authors studied the scheduling of the compressors without taking into account the constraints of the individual compressors of the station and most of the works modelled the station as a single compressor, especially in studies for natural gas networks optimisation. Therefore, the Objective Two focuses on *the development of a multi-period optimisation framework to optimally schedule the operations of compressors for long time periods.*

Gap of knowledge in simultaneous maintenance and operation

The articles presented in Section 3.4 offer a starting point for the work proposed in the thesis on the scheduling of compressor stations. However, these articles do not address the simultaneous optimisation of maintenance and operational tasks of the compressors. The literature from one side examines the maintenance of a single compressor considering its performance and on the other side it examines the optimisation of the operation of network of compressors neglecting their maintenance tasks. Hence, the optimisation of a compressor station with multiple (not necessarily similar) compressors in parallel considering the maintenance of the individual units is still an open question. Objective Three will study the topic of the *implementation of maintenance activities into the operational optimisation framework*.

The current thesis will present a mathematical framework for the optimal operation considering condition-based maintenance which would be the state-of-the-art in the optimal operation and maintenance of compressors. The framework includes the basic operational constraints of the compressors considering operational aspects, for example the prediction of power consumption depending on operational conditions, the extra power consumption due to degradation, and minimum running and minimum shut down times of the compressors. The model also includes the constraints which describe the application of offline and online washing.

Moreover the account of the state-of-the-art suggests the multi-period optimisation of integrated operation and maintenance of natural gas compressor stations with multiple compressors in parallel. The maintenance model is related to major overhauls for long time periods of several months. This kind of optimisation is still an open question, therefore the thesis aims to develop and present an optimisation framework which deals with this integration.

The thesis also demonstrates a method to linearise an industrial gas compressor map using a Mixed Integer Linear Programming (MILP) formulation. The optimisation framework includes this formulation into a MILP model which considers operational aspects of the compressors (e.g. shut downs) and a maintenance model. The latter model includes major overhauls and minor maintenance activities, such as corrective maintenance, inspections and electrical tests of the drives. The final objective of the thesis is to demonstrate that the optimisation of the framework is able to provide the best decisions in order to reduce operational costs and increased flexibility and reliability.

3.7 Summary of the chapter

Chapter 3 presented the literature review on the topic of the optimal operation of compressor stations. The study of the state-of-the-art concludes that there is a need to develop systematic approaches to optimise multiple dissimilar industrial compressors in process systems and in natural gas networks. An online optimisation framework for the optimal distribution of the load of compressors is a solution for real-time applications. This method will be presented in the next chapter. Moreover, there is not a systematic approach to schedule the compressors for long time periods. Thus a multi-period optimisation approach should be developed to fill this gap of knowledge in Chapter 5. The previous state-of-the-art still has not presented a solution of the problem of the simultaneous maintenance and operation of compressors. This integration is presented in Chapter 6.

4 Real Time Optimisation (RTO) for online application

4.1 Description of the chapter

Chapter 4 presents a method to deal with the research topic of the thesis which is the optimal distribution of the load among parallel compressors in real-time. The proposed method is a Real Time Optimisation (RTO) framework. A compressor station usually involves dissimilar compressors which are typically operated in a way that the total load is evenly split or other load sharing strategies are applied. These strategies have been explained in Chapter 3.

The current chapter explains a framework which estimates the load of each individual compressor of the compressor station and the aim of the framework is the minimisation of operational costs including the power used by the electrical motors. The development of the models of compressors has to meet the requirements of an online application which runs calculations in real-time. Moreover, the models should consider the industrial information which is available from the existing hardware and software, for example the available measurements of process variables.

Section 4.3 presents the components of the RTO framework, namely monitoring and collection of process variables, validation of data, development and parameter estimation of data-driven models, optimisation and application of the optimal set points via the control system. The industrial case of the air separation plant in BASF which was introduced in Chapter 1 is explained in detail in Section 4.4. Section 4.5 gives the results and the conclusions are given in Section 4.6. Finally, the summary of the chapter is provided in the end.

The next Section 4.2 presents and explains a general integrated framework which connects RTO and scheduling. After this description, every computational step of the real time optimisation framework is explained. Chapter 4 involves research which has been published in the following conferences and journals:

- The paper titled 'Modeling and optimisation of industrial centrifugal compressor stations employing data-driven methods' (Xenos et al., 2014a) was presented in the conference *ASME Turbo 2014* and it introduced the method-

ology for modelling industrial multi-stage compressors with data-driven models. The principle of this method is the basis of the models used for modeling the compressors in Section 4.3.3.

- The application of the real time optimisation with improved models has been published in the research article with title 'Optimization of a network of compressors in parallel: Real Time Optimization (RTO) of compressors in chemical plants - An industrial case study' (Xenos et al., 2015a) in the journal of *Applied Energy*.

- The steady-state detection in Section 4.3.2 has been presented in the conference '*10th International Conference on Control, Control 2014* and the title of the conference paper was 'Preprocessing of raw data for developing steady-state data-driven models for optimising compressor stations'.

4.2 General integrated optimisation framework for the optimisation of compressor stations

The optimisation of compressors can be studied by two different approaches which consider different dynamics and deal with different objectives. The first approach is a real time optimisation problem which is related to optimal distribution of the load among compressors with a fixed configuration. The time scale of the problem here is minutes or hours. The second approach is the scheduling of the operation and maintenance of the compressors, this approach considers information predicting future trends of demands and it applies to time scales of days or weeks. The objective of scheduling is mainly to determine the choice of the compressors and the actual configuration within the system (e.g. connections between compressors and headers).

The RTO approach solves the problem of the optimal load sharing. This problem deals with the estimation of the set points of the compressors which optimise the process. The problem of optimal load sharing is also known as *optimal distribution of load* and *multi-compressor capacity optimisation*. The objective of this optimisation problem could be any interest of the users of the plant, however in the case of the optimal load sharing problem the objective involves all the terms which describe operational costs. Examples are the cost coming from the electricity consumed by the motors of the compressors or the purchase cost of the cooling water which is used by the intercoolers of multi-stage compressors.

The scheduling optimisation problem, by contrast, takes into account decisions which involve discrete events, for example switching on or shutting down a compressor. The scheduling can consider also decisions which involve the maintenance

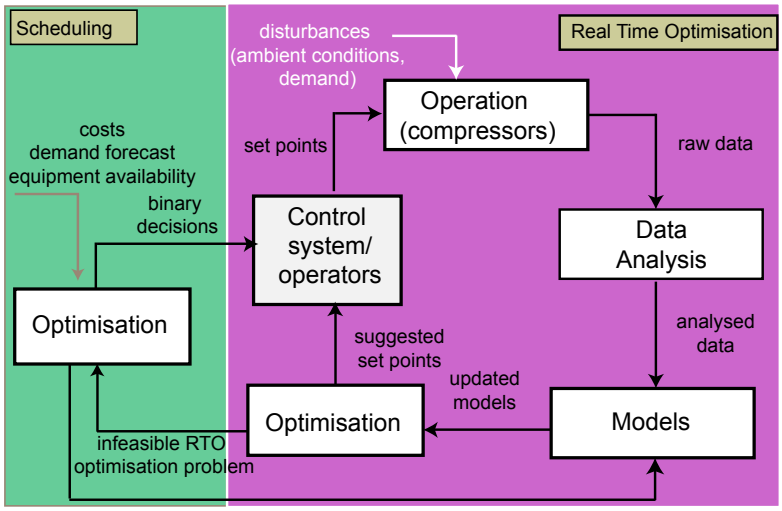


Figure 4.1: Integrated framework for the optimisation of compressor stations.

of the compressors. The objective of the scheduling includes cost terms which are related to operational, maintenance, start up, and shut down costs. The operational costs in the scheduling are estimated in an aggregated way, however RTO determines the operational costs in real time in a more refined way.

Although the two different approaches have different objectives, they are highly integrated. Figure 4.1 shows a simplified graphical representation of the two frameworks, of RTO and scheduling. The RTO uses the raw data of the measurements from the operation to develop models. The data analysis examines the quality of the data and cleans them from outliers. The optimisation uses the models to suggest optimal set points to the control system. The control system under the supervision of operators apply the set points to the compressors. This loop occurs in a frequent basis in order to deal with changes in parameters. The parameters change due to disturbances such as ambient conditions and demand.

When the actual demand given in the RTO is significantly different from that which is predicted in the scheduling problem, then the RTO problem may become infeasible. This is because the online compressors may not be able to meet the requirements of the demand side. In this case, the scheduling problem updates its models and a new schedule of the compressors is estimated. The new schedule gives the new configuration of the system which can satisfy the demand, for instance by bringing another compressor online. Another example is when compressors which are online can satisfy the demand, but the demand could be satisfied by fewer compressors, then, again, the scheduling problem has to be solved to estimate a new set of compressors for optimal operation (it is also possible that the overall analysis suggests keeping the same configuration). This integration will not be studied in depth as it is not part of the scope of the thesis. However, discussions

on this topic will be presented in Chapter 7.

The framework in Fig. 4.1 connects several decisions tasks from Level 0, 1, 2 and 3 of the automation pyramid in Fig. 2.8b presented in Chapter 2, for example optimal scheduling and maintenance, optimal load sharing and update of the maps, and control and monitoring of the operation of compressors. The optimal load sharing considers the updates of compressor maps which corresponds to the asset monitoring in Level 3 in the manufacturing execution system of pyramid in Fig. 2.8a.

4.3 A Real Time Optimisation (RTO) framework using data-driven models

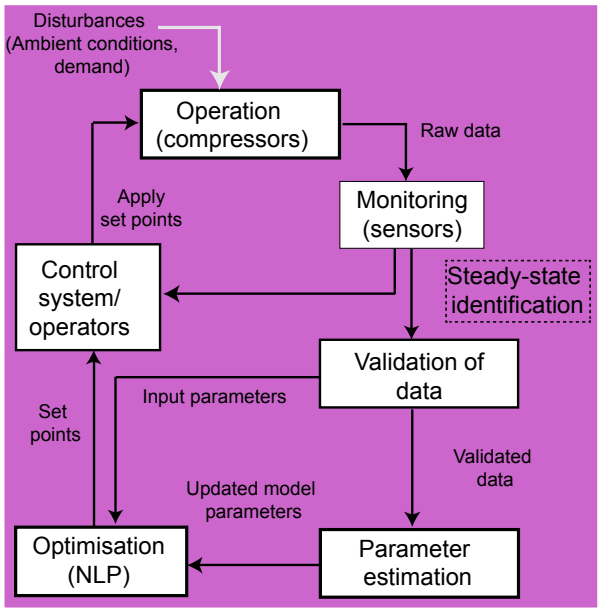


Figure 4.2: Detailed description of the structure of the components of the RTO.

A basic RTO scheme (Mansour and Ellis, 2008) can be tailored to the operation of parallel compressors. Figure 4.2 presents the components of the RTO proposed for compressors, and their relationships.

The sensors of the monitoring system collect process data of the operation such as mass flow rates, pressures and temperatures. A steady-state identification algorithm examines key process variables and identifies when the operation is in steady-state. Thus, if the operation is in steady-state, then the collected data should be validated (Prata et al., 2010; Mansour and Ellis, 2008). The next step is to use the validated data to update the models of the online compressors.

A NLP optimisation model employs these data-driven models and estimates

the set points of the controlled variables for optimal load sharing. These variables are the mass flow rates of the compressors. The set points are given to the control system which role is to apply and keep these points to the process until the next run of the RTO. The control system is also responsible to estimate the set points of the actuators which results in the values of these mass flow rates. The actuators of the compressors can be variable speed drives or inlet guide vanes.

The following sections will explain the basic components of the Real Time Optimisation of compressor stations: (a) analysis of process data and measurements, (b) steady-state detection, (c) development of the models, (d) assessment of the accuracy of the prediction of the models and (e) the definition of the optimisation model. The details of how the control system achieves the requested set point are not discussed as this topic does not concern the objectives of this study.

4.3.1 Process data and measurements

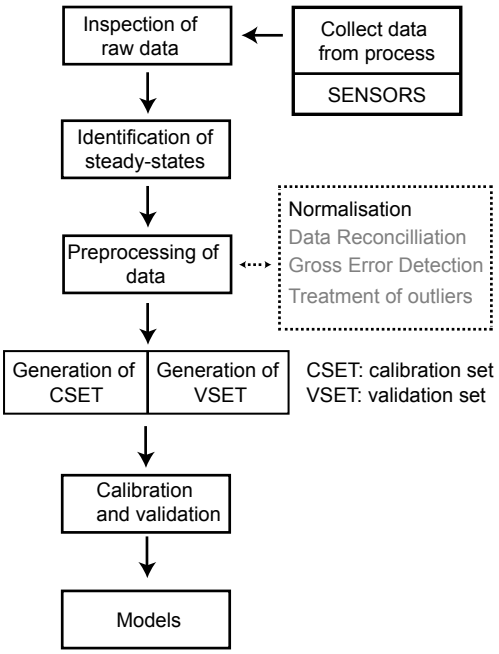


Figure 4.3: General methodology of offline development of data-driven models.

The methodology to develop models of compressors is based on the offline analysis of process data according to the procedure presented in Fig. 4.3. The compressors are centrifugal multi-stage, with a cooling system, driven by electrical motors with constant rotational speed. This methodology is referring to the less studied type of compressor, i.e. multi-stage centrifugal compressor with IGVs, compared to the case of a single-stage axial compressor. The steps of the development of data-driven models consist of several procedures as illustrated in Fig. 4.3. The

main steps in the procedure are the following:

Inspection of raw data: After the collection of the data from the operation, the first step is to examine the values of the data set through observation and to gain a general overview of its structure. Kadlec et al. (2009) mention the importance and reasons of this step for developing regression models of data-driven *soft sensors*.

Identification of steady-state analysis and preprocessing of stationary points: Steady-state regression models do not perform well when the collected data are collected in transition. There are several methods in the literature to estimate a steady-state of a process (Cao and Rhinehart, 1995; Jiang et al., 2003; Kelly and Hedengren, 2013). Kim et al. (2008) suggested an algorithm, which is simple to implement, to estimate the steady-states of a process. Section 4.3.2 will present a multi-variate steady-state detection for compressor stations based on the method proposed by Kim et al. (2008).

Preprocessing of data: Data have been normalised because different variables have different units, for example power is measured in kW and pressure in bar. In the current study, the maximum data point of the set is used to divide the set. The normalised data, then, have to be preprocessed. Data reconciliation, gross error detection and handling outliers constitute the procedure of the data preprocessing (Prata et al., 2010). Indeed, it is common to find outliers in raw data. The outliers can be ignored or to be replaced with predictions of the best value fit of the outlier replaced.

Generation of the calibration and validation set of data: This step involves the division of the total set of data into two smaller sets. The first set, the calibration set, is used to estimate the parameters of the regression model while the validation set examines if the generated model predicts well in a different set than in the calibration one. The data set used to calibrate the model defines the domain of validity of the model, which is called *regression domain*. Extrapolation of the model outside its regression domain may give values which are not true. Brooks et al. (1988) presented a study which characterises the domain of regression models.

Calibration and validation of data: The next step is to decide the structure of a model which describes the process. The process here is the compression of ambient air from a multi-stage centrifugal compressor, presented in Fig. 4.4. The

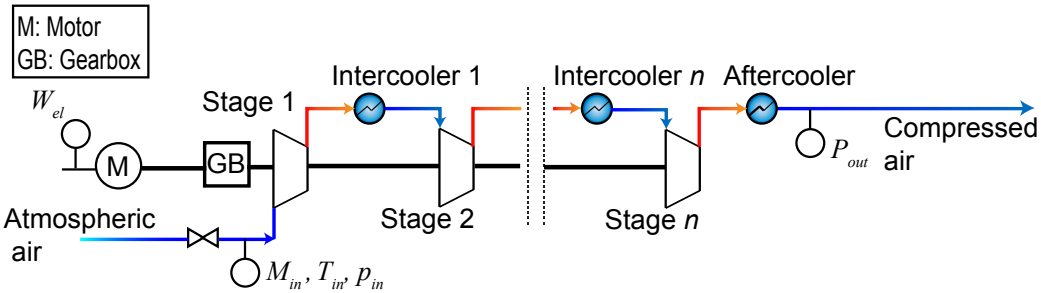


Figure 4.4: Multi-stage centrifugal compressor with inlet and outlet measurements.

model is calibrated and validated with the calibration and validation data sets respectively.

The models are developed from historical data. The use of these data-driven models allows the modelling of the multi-stage compressors with available measurements only at the inlet of the first and at the exit of the last stage. A rigorous model requires measurements between the stages of compression and analytical models of the intercoolers. Moreover, the use of a data-driven model in the optimisation reduces the computational burden compared to the use of a rigorous model which considers aerodynamics and thermodynamics of the fluid. The computational time of the optimisation plays an important role in an online application such as the RTO.

Figure 4.4 shows that the available process measurements can be found at the inlet of the first stage and the exit of the outlet of the last stage. The measurements at the inlet are the mass flow rate m_a at standard conditions, $T_{\text{STP}} = 273$ K (0 ^{o}C) and $P_{\text{STP}} = 100$ kPa (1 bar) (Wikipedia, 2015c). The specific gas constant of an ideal gas, which is based on a mean molar mass for dry air of 28.97 g/mol, is $R_g = R/MolW = 287.1$ J kg^{-1} K^{-1} (Wikipedia, 2015b). The electrical power of the motor W_{el} is also given from measurements.

4.3.2 Steady-state detection

The purpose of the steady-state detection algorithm is to identify the steady-states of the operation of the compressors to develop reliable data-driven models of industrial compressors similar to this in Fig. 4.4. Data-driven models are black boxes which hold a relationship between input and output variables. According to Cao and Rhinehart (1995) and Kim et al. (2008), these input and output variables should be close to steady-state to hold the validity of the mass and energy balances implied in the black box

A steady-state detection algorithm based on a moving window was developed. Figure 4.5 shows the application of the moving window to a data-set of a single

process variable. A moving data-set window is defined from a fixed number of data points of the process variable, n_s. The data included in the window are updated at each step τ, recent data are added and old data are discarded. The window moves every S number of data points. A process variable v has value $x_{(i,v,t)}$, where $i \in I$ (I is the set of compressors), $v \in V$ (V is the set of variables) and $t \in T^{tot}$ (T^{tot} is the set of data points which corresponds to the total sample time). Variable τ corresponds to a window with data $(t - n_s + 1, t)$. The sample rate of the data set (referring to the discretisation $t \in T^{tot}$), the n_s and the S are parameters which have to be tuned for a desired function of the steady-state detection algorithm relevant to the application.

At each step τ the data-set window moves to $t' = t + S$ and the standard deviation of the included data points in the window, $\sigma_{(i,v,\tau)}$, of compressor i and variable v is calculated from:

$$\sigma_{(i,v,\tau)} = \sqrt{\frac{1}{n_s} \sum_{t' \in (t-n_s+1,t)} (x_{(i,v,t')} - \mu_{(i,v,\tau)})^2} \tag{4.1}$$

where $\bar{x}_{(i,v,\tau)}$ is the mean of the data in the window:

$$\bar{x}_{(i,v,\tau)} = \frac{1}{n_s} \sum_{t' \in (t-n_s+1,t)} x_{(i,u,t')} \tag{4.2}$$

The steady-state algorithm detects a steady-state episode of a process variable v when a particular condition holds true, for example if three times the standard deviation $3\sigma_{(i,v,\tau)}$ is less than a predefined value $h_{(i,v)}$. This bound is chosen by

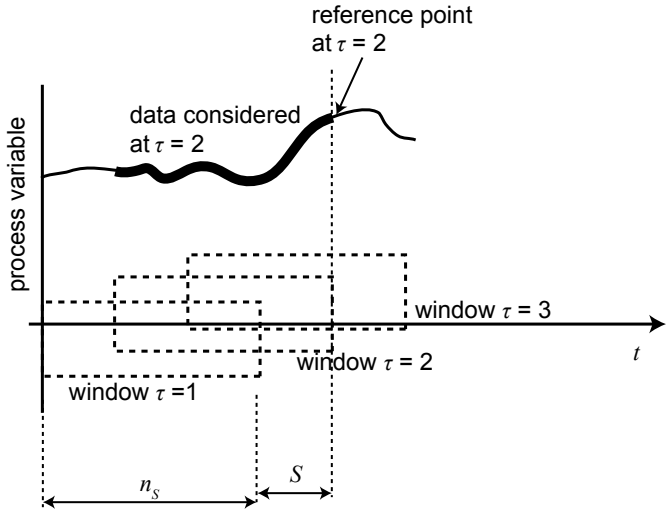


Figure 4.5: Moving time data set window.

the users according to their engineering judgement as there are not standardised values for this type of application.

The developed steady-state algorithm is multivariate and involves the examination of more than one variables to assess when the system is in steady-state. According to Mansour and Ellis (2008) a system is in steady-state when all the considered variables are in steady-state. The system in this study is the compressors which operate in parallel.

Hence, if a process variable v of compressor i is in steady-state at t then a binary variable $Y_{ss(i,v,t)}$ is equal to one, otherwise the variable takes the value zero. Therefore, the steady-state of the system of $i \in I$ compressors considering $v \in V' \subseteq V$ variables is estimated from the value of the variable $Y_{ss,system}$:

$$Y_{ss,system} = \prod_{i \in I} \prod_{v \in V'} Y_{ss(i,v,t)} \tag{4.3}$$

The output of the steady-state detection algorithm is a matrix of data with J' rows (variables) and T^* columns (number of final steady-states of the system) where $T^* \subseteq T^{tot}$.

4.3.3 Development of models

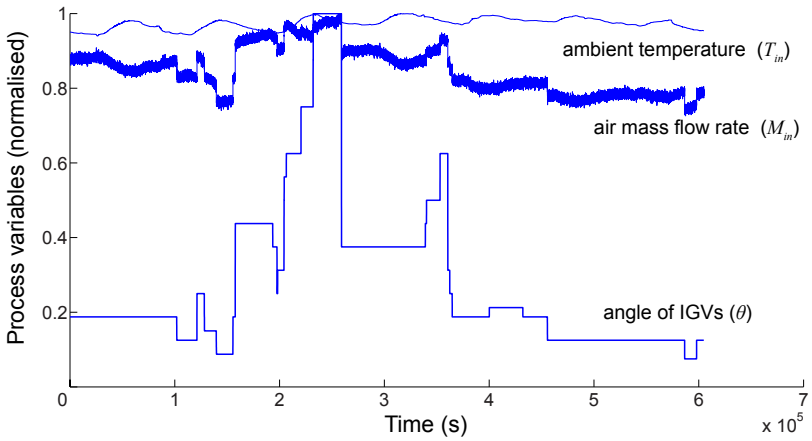


Figure 4.6: Several process variables (normalised) of the operation of an industrial centrifugal compressor over time.

The development of a black box model of the compressor system of Fig. 4.4 includes: the multi-stage compressor, the cooling system, the rotating shaft, gearbox and motor. According to the step tests in an industrial centrifugal compressor of the air compressors in the air separation plant in BASF, it was shown that the



Figure 4.7: Black box model which associates input with output variables.

power consumption of the motor mainly depends on the mass flow entering the compressor, M_{in}, the ambient conditions, T_{in}, P_{in}, and the pressure at the exit of the compressor (discharge pressure), P_{out}.

In this test the compressor is disconnected from the downstream process and receives different constant set points of the position of the IGVs for defined time periods. The outlet conditions of the compressor are kept constant. These step tests took place for one week and data with 10s sample interval, i.e 0.1Hz sample rate, were collected. The investigation of the behaviour of the process variables of the operation of this compressor helps to inspect which data should be taken into account for the development of data-driven models and helps to identify upper bounds for ideal steady-states.

Several measurements can be acquired from the Distributed Control System (DCS), for example the mass flow at the exit of the compressor, the temperature and pressure of the environment and the power consumed by the motors. Figure 4.6 shows three process variables, angle of the IGV (θ), mass flow rate (M_{in}) and ambient temperature (T_{in}), over time during the step tests. The values on the vertical axis are normalised. All of these variables influence the power consumption.

Figure 4.6 shows that the mass flow rate is considerably influenced by the θ. Indeed, the mass flow rate changes in a non-linear way with the change of the θ. When the θ changes from 0.19 to 0.43 between $1 \cdot 10^5$ and $2 \cdot 10^5$ s, the mass flow rate changes significantly compared to smaller changes in the mass flow rate from larger changes of the θ between $2 \cdot 10^5$ and $3 \cdot 10^5$ s. Moreover, when θ is fixed, the mass flow changes at a slow rate following inversely the change in the ambient temperature.

Figure 4.7 shows the procedure for the development of the model of the operation of a multi-stage compressor. A black box is used to predict the power which is consumed from the electrical driver W_{el} and this power is the output of the model. The black box model represents the power as function of process variables of the operation, $M_{in(i,t)}, T_{in}, P_{in}, P_{out}$. These variables represent the input of the model. A polynomial regression model is used to develop the black box model of each compressor i, by defining $x_{(i,k,t)} = [M_{in(i,t)}, T_{(in,t)}, P_{(t)}, P_{out(t)}]$ with $k = 4$ number of input variables and $y_{(i,t)} = W_{el(i,t)}$, then the following polynomial gives

the black box model of compressor i:

$$\begin{aligned} y^*_{(i,t)} = b_{(i,0)} + b_{(i,1)} \cdot x^*_{(i,1,t)} + b_{(i,2)} \cdot x^*_{(i,2,t)} + b_{(i,3)} \cdot x^*_{(i,3,t)} + b_{(i,4)} \cdot x^*_{(i,4,t)} \\ + b_{(i,5)} \cdot x^{*2}_{(i,1,t)} + b_{(i,6)} \cdot x^{*2}_{(i,2,t)} + b_{(i,7)} \cdot x^{*2}_{(i,3,t)} + b_{(i,8)} \cdot x^{*2}_{(i,4,t)} + \\ b_{(i,9)} \cdot x^*_{(i,1,t)} \cdot x^*_{(i,2,t)} + b_{(i,10)} \cdot x^*_{(i,1,t)} \cdot x^*_{(i,3,t)} + b_{(i,11)} \cdot x^*_{(i,1,t)} \cdot x^*_{(i,4,t)} \end{aligned} \quad (4.4)$$

where $y^*_{(i,t)} = y_{(i,t)}/y^{max}_{(i)}, x^*_{(i,k,t)} = x_{(i,k,t)}/x^{max}_{(i,k)}$ are the scaled variables of the regression models of compressors $i \in I$. The $x^{max}_{(i,k)}$, $y^{max}_{(i)}$ are the maximum variables of their respective calibration and validation sets. The parameters of the models, $b_{(i,m)}$ with $m = 0, 2, \ldots, 11$ are calculated with regression methods (Rosipal and Kramer, 2006).

The black box model described by Eq. (4.4) has been derived based on the analysis of the available input data and the output data (power). The key variables x_k have been chosen based on the observation of data (plots of power against the x_k variables), examining the correlation between power and x_k variables. Knowledge also on the physical connections between power and other variables such as temperatures and pressures has influenced the choice of the variables.

The choice of the form of the black box model and the terms of the polynomial of the right hand side of the equation is based on several iterations until to reach an acceptable level of accuracy of prediction, in other words reduced prediction error for a validation set of data. The Matlab function *stepwisefit* (Matlab, 2014) was employed to assess different regression models.

4.3.4 Assessment of the accuracy of the prediction of the models

To evaluate the accuracy of the prediction of the models the Coefficient of Variation of the Root Mean Square Error, CV(RMSE) is used (Wikipedia, 2014):

$$\text{CV(RMSE)} = \frac{\text{RMSE}}{\bar{y}} = \frac{\sqrt{(\sum_{t \in T^*} (y_{meas(i,t)} - y_{(i,t)})^2)/T^*}}{\bar{y}}$$

and the coefficient of determination, known as R-squared (RSQ) expresses how well the data fit the model (Matlab, 2014):

$$S_{res} = 1 - \frac{(\sum_{t \in T^*} (y_{meas(i,t)} - y_{(i,t)})^2)}{(T^* - 1) \cdot v_y}$$

where $\bar{y}$ is the mean and v_y is the variance of the predicted values of y, and y_{meas} are the measured variables.

4.3.5 Optimisation model of the RTO

The offline steady-state identification algorithm, presented in Section 4.3.2 is modified for online applications. The online steady-state identification examines if the $Y_{ss,system}$ is one at the current moment, t_r, considering V' variables of each compressor i at the window $(t_r - n_s - 1, t_r)$.

According to Fig. 4.3, raw data are collected from the operation after the steady-state detection and they are validated through data reconciliation. The validated data are used (a) to update the parameters of the models which are used from an optimisation model and (b) to provide the input parameters of this optimisation model. The set points of the mass flow rates, which are the output of the optimisation model, are the input to the control system. The controller can be a feedback controller. The controller deals with the application of the set points and adjusts the position of the actuators (IGVs) to achieve the desired flows.

There is usually a mismatch between models and reality due to fitting errors and performance changes due to fouling and erosion, as was also reported in (Paparella et al., 2013). This mismatch influences the shape of the objective function of the optimisation problem and consequently the estimation of the minimum of the total power consumed. To reduce this influence, the mass flow rates of the compressors were chosen as degrees of freedom of the optimisation problem. The position of the IGVs can be then adjusted from the controllers. An analysis of the models showed that the accuracy in prediction of the models does not improve significantly when the position of the IGVs is considered as an input variable in the models.

Defining the mass flow rates $\omega^*_{(i,t_r)}$ as optimisation variables (degrees of freedom or decision variables of the optimisation) and $\mathbf{z}^* = [x^*_{(2,t_r)} x^*_{(3,t_r)} x^*_{(4,t_r)}]$ as the input parameter vector of the optimisation model (Fig. 4.2) , then the optimal load distribution at a steady-state episode which starts at point $t = t_r$ is computed by the following optimisation formulation:

$$\min_{\omega^*} \quad \sum_{i \in I} W_{el(i,t_r)} \tag{4.5}$$

subject to:

$$W^*_{el(i,t_r)} = f_i(\mathbf{z}^*, \omega^*_{(i,t_r)}, \mathbf{b_{(i)}}), \qquad \forall i \in I \tag{4.6}$$

$$W_{el(i,t_r)} = W^*_{el(i,t_r)} \cdot y^{max}_{(i)}, \qquad \forall i \in I \tag{4.7}$$

$$M_{in(i,t_r)} = \omega^*_{(i,t_r)} \cdot x^{max}_{(i,1)}, \qquad \forall i \in I \tag{4.8}$$

$$\sum_{i \in I} M_{in(i,t_r)} = M_{dem(t_r)} \tag{4.9}$$

$$M_{(i)}^{min} \leq M_{in(i,t_r)}, \qquad \forall i \in I \tag{4.10}$$

$$M_{in(i,t_r)} \leq x_{(i,1)}^{max}, \qquad \forall i \in I \tag{4.11}$$

$$W_{el(i)}^{min} \leq W_{el(i,t_r)}, \qquad \forall i \in I \tag{4.12}$$

$$W_{el(i,t_r)} \leq y_{(i)}^{max}, \qquad \forall i \in I \tag{4.13}$$

Equation (4.5) describes the objective function which is the minimisation of the power consumption at steady-state which starts at $t = t_r$, and Eq. (4.6) describes the constraints which give the power of each compressor i as function of the $\mathbf{z}^*$ and normalised mass flow rates $\omega_{(i,t_r)}^*$. Equations (4.7) and (4.8) refer to the normalisation of the powers and mass flow rates. Equation (4.9) provides the mass balance between summation of the flows of the compressors and the demand, $M_{dem(t_r)}$, requested. Finally, Eqs. (4.10) – (4.13) define the regression domain.

The above optimisation formulation was implemented in Matlab$^{\text{TM}}$ using the optimisation function *fmincon* (Matlab, 2014b). The results from the optimisation on a real industrial case study with compressors of a process plant will be presented in Section 4.5.3. The description of the case study is given in the next section.

4.4 Description of the industrial case study

This section presents the application of the methodology of RTO to the air compressor station of multi-stage compressors in parallel in BASF, Germany, which distributes compressed air to different end-users. The end-users are air separation columns and plant-site utilities for compressed air. The air compressor station consumes the major part of the total energy in the air separation plant. Moreover, the power rating of the plant is several tens of MW.

The case study to be examined has the following characteristics:

- The compressor station involves three air multi-stage centrifugal compressors similar to the compressor depicted in Fig. 4.4.
- The compressors operate in parallel to supply an air separation column with compressed air. The air separation column requests compressed air of mass flow rate, M_{dem}, at a constant pressure, P_{op}.
- The three compressors are assumed to have the same specifications, which are power rate, minimum and maximum capacity and efficiencies, at the point of the commissioning of the plant. However, the compressors are not in the same condition during the time period of the study as will be shown in Section 4.5.

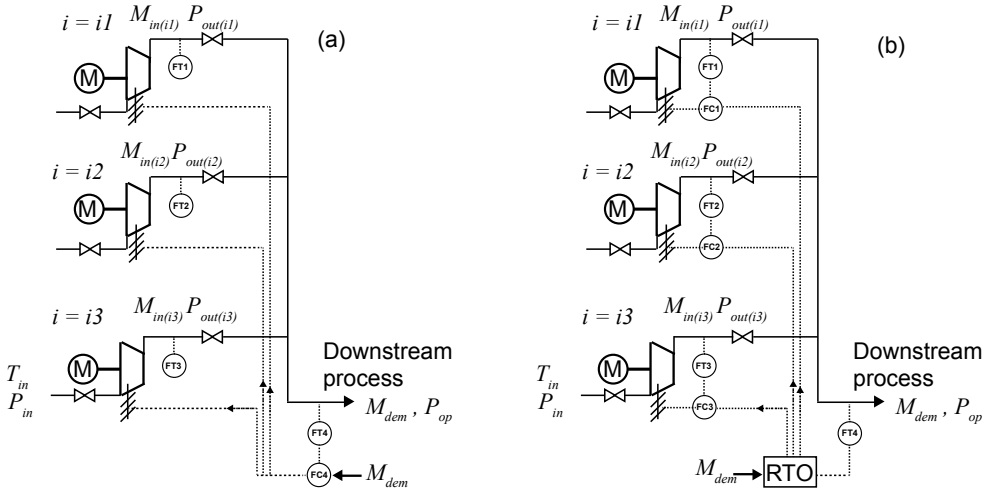


Figure 4.8: The current implemented (a) and suggested (b) control scheme of three parallel air multi-stage centrifugal compressors.

A description of the implemented control structure in the industrial plant can be seen in Fig. 4.8a. The set point of the demand M_{dem} is given as an input of the system to controller FC4. The flow transmitter FT4 measures the total mass flow rate of the compressed air provided by the compressors. The controller FC4 gives the same angle of the IGVs to each compressor in order to control their mass flow rates and meet the demand. By applying this control strategy the load should be shared equally among the compressors. The summation of the flows, total flow, measured from FT4 has to match with the M_{dem}. If there is a mismatch then the IGVs of all the compressors are adjusted uniformly (increase or decrease the same angle for all the compressors) to reach the desired M_{dem}. Referring to Fig. 4.8a, this control structure does not include the RTO block and instead sends the same point signal to each compressor.

The control strategy that the thesis suggests for optimal load sharing can be seen in Fig. 4.8b. The three flow transmitters FT1, FT2 and FT3 are used to transmit the individual flows at the exit of the compressors. The RTO receives the desired M_{dem} as input and the FT4 provides the total mass flow rate of the compressors. In this case controllers FC1, FC2 and FC3 work independently and they receive the values of the flows of the compressors according to the RTO computations. The RTO estimates the best set points for FC1, FC2 and FC3 based also on measurements of T_{in}, P_{in} and P_{op}. The individual feedback controllers FC1 – FC3 have to independently adjust the position of the IGVs of the compressors to reach the set points of the mass flows given from the RTO.

4.4.1 Practical challenges of the case study

The case study of the industrial air separation process has revealed several practical challenges which are summarised below. These challenges should be considered in order to achieve a realistic approach of the proposed methodology.

The BASF operators did not operate the compressors over their full range in the collected data set. Therefore, only a partial compressor map is captured in the regression models. Hence, the feasible window of operation of the model used for the optimisation is not defined from the actual physical limits of the compressor, i.e. surge, choke, and minimum and maximum angles of IGVs. Instead, this feasible operational window of the compressor is defined from the domain of the regression model (Brooks et al., 1988). The regression domain is a part of the actual operational window.

Figure 4.9a demonstrates that an operating point of the compressors comes from the intersection of the Compressor System (CS) curve and the load curve of the downstream process. The Compressor System curve is defined as the merged individual compressor characteristics of the parallel compressors assuming that the inlet conditions are the same of all compressors. The set of all the operating points of the corresponding data set defines the regression domain of the model of a compressor as can be seen in Fig. 4.9b. It is known from the plant that the operators operated well within the physical limits (surge and choke) of the compressors during the past operation.

A model is expected to be more accurate when the data set is collected over a shorter time period, for example one week than a model derived from a data set of several months because for longer period the compressor might have been in various states during this period. For example, a compressor is efficient immediately after

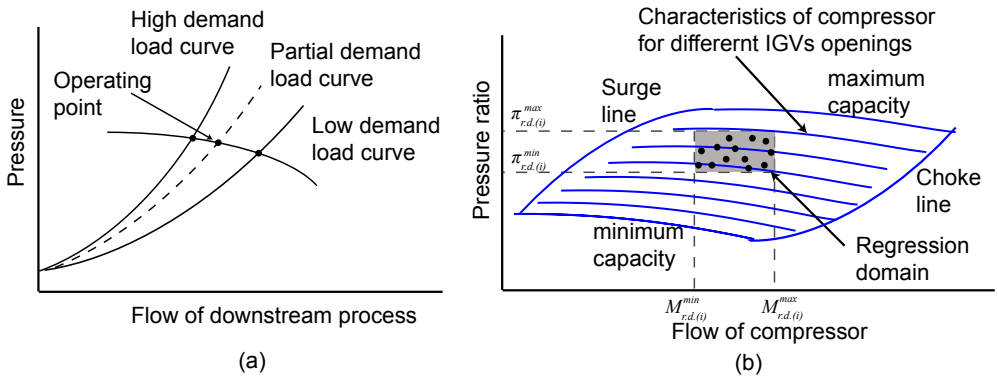


Figure 4.9: An operating point of the system defined by the intersection between load curve and characteristic of the system (CS curve) (a) and the feasible window of operation (i.e regression domain) of a compressor with IGVs (b).

maintenance and less efficient after many hours of operation. However, the range of the regression domain of the model is smaller in the case of a more accurate model. Moreover, for the purposes of the development of the models, the operators should ideally run the compressors at different operating points through the day so as to increase the range of the regression domain. However, such experimentation is not practically feasible when the plant is operating for commercial production.

In the case that the compressor was operated close to the surge or choke line, the regression domain cannot be assumed rectangular (see Fig. 4.9b) due to the physical restrictions. There are two options to deal with this issue: (a) use a convex hull for describing the regression domain, or (b) add an extra constraint in the optimisation model described by Eqs. (4.6) – (4.13). The use of a convex hull can be seen in Brooks et al. (1988) and in Mitra et al. (2012). The convex hull ensures a more tight regression domain which increases the validity of the model in its boundaries (Kahrs and Marquardt, 2007). The extra constraint can be a regression black box model of the outlet pressure which relates the mass flow rate and other parameters such as ambient temperature and pressure, $P_{out(i)} = g_i(\mathbf{z}, M_{in(i)}, \mathbf{c_{(i)}}),\ i \in I$ where $\mathbf{c_i}$ is the vector of the fitted parameters of the new regression model. The application of the convex hull is not used in the RTO application, however it is explained in detail in the next Chapter 5.

The data reconciliation step requires redundant measurements, for example extra measurements of the flow apart from these at the exit of the compressors. Unfortunately the industrial case study did not have these measurements, and hence the data reconciliation step could not be implemented.

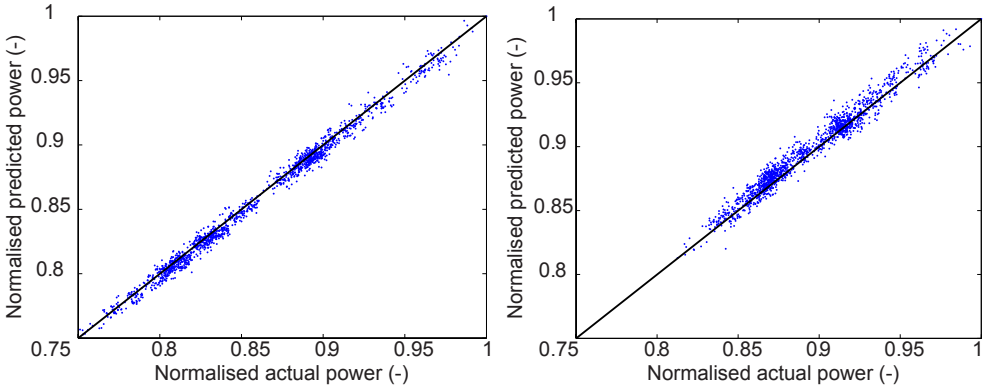
4.5 Results

4.5.1 Models of compressors

Table 4.1: Statistics of the fitting and validation of the regression models.

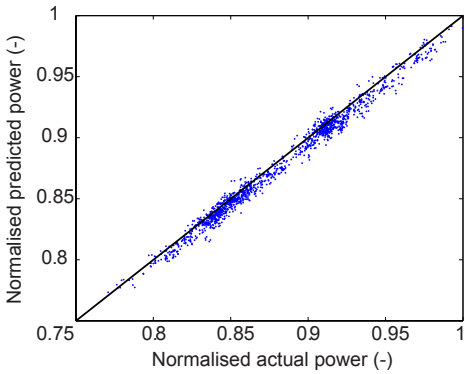
	Fitting	**Validation**	**Validation**
Compressors	**RSQ**	**RSQ**	**CV(RMSE)**
$i1$	0.992	0.992	0.54
$i2$	0.968	0.967	0.67
$i3$	0.987	0.988	0.57

Data from fifteen days of operation were used to develop the models of the compressors. The case study assumed that the performance of the models deteriorates gradually and that there was no event which has caused a relatively high discontinuous decrease in performance. Data of 129600 continuous operating points with



(a) Compressor $i1$

(b) Compressor $i2$



(c) Compressor $i3$

Figure 4.10: Prediction versus actual values of power of compressors $i1$, $i2$ and $i3$ in the validation set.

10s sample interval (0.1 Hz sample rate) were collected. The steady-state algorithm detected 7430 steady-state episodes in the examined data set for the system of the compressors. Of these episodes, 80% were used for the fitting of the model and the remaining 20% were used to validate the developed model.

Table 4.1 presents the statistics of the fitting, RSQ, and the validation, RSQ and CV(RMSE), of the models of the three compressors. The standard function *LinearModel.fit* of Matlab^{TM} (Matlab, 2014) was used to fit the data into the polynomial of Eq. (4.4). The computational time of the estimation of the coefficients is a few seconds. Figure 4.10 shows the prediction of the normalised power consumption of each compressor versus the measurements of the power. The axes present normalised values of the power due to confidentiality restrictions. The normalisation of the power consumption of each compressor is calculated by dividing the power with the y_i^{max}.

Compressor $i1$ shows the most accurate match between prediction and actual measurements of the power. The fitting of the model has also higher RSQ value

than in the other cases. In the case of compressor $i2$ the accuracy of the prediction is relatively less than of the other two compressors. The reason for this is that the measurements of the mass flow and power of compressor $i2$ are relatively more noisy than in the case the other two compressors. The CV(RMSE) of the models ranges between 0.54% and 0.67%. By comparing the mean RMSE results with other results from similar case studies (Han et al., 2004; Xenos et al., 2014b) these models can be considered of high accuracy for predicting power consumption of compressors.

Table 4.2 presents the minimum and maximum bounds of the mass flow rates and powers of the three compressors. The values shown in Table 4.2 represent the boundaries of mass flow rates and powers where they are divided by two scaling factors to keep the confidentiality agreement with the provider of the case study. In other words, the units are dimensionless and scaled. The table shows that compressors have different regression domains with compressor $i1$ having a larger feasible operational window than the other two. The values of the coefficients of the regression models of the three compressors can be seen in Tables 4.3 and 4.4.

Table 4.2: Boundaries of mass flow rates and power consumptions.

Compressor	$M_{in(i)}^{min}$	$x_{(i,1)}^{max}$	$W_{el(i)}^{min}$	$y_{(i)}^{max}$
$i1$	0.670	0.924	0.304	0.406
$i2$	0.676	0.836	0.295	0.365
$i3$	0.624	0.842	0.286	0.374

Table 4.3: First six coefficients of the regression models.

Compressor	b_0	b_1	b_2	b_3	b_4	b_5
$i1$	-1.24	-0.24	-2.00	3.13	2.26	0.96
$i2$	0.76	-20.61	5.98	23.84	-12.49	0.09
$i3$	9.22	-2.58	-16.97	-0.69	1.73	0.97

Table 4.4: Last six coefficients of the regression models.

Compressor	b_6	b_7	b_8	b_9	b_{10}	b_{11}
$i1$	1.27	-1.61	-0.99	-0.60	0.24	-0.16
$i2$	-9.08	-8.86	-0.37	14.00	-7.22	15.00
$i3$	9.20	-0.25	-1.52	-1.24	1.59	1.56

4.5.2 Illustrative example with industrial compressors

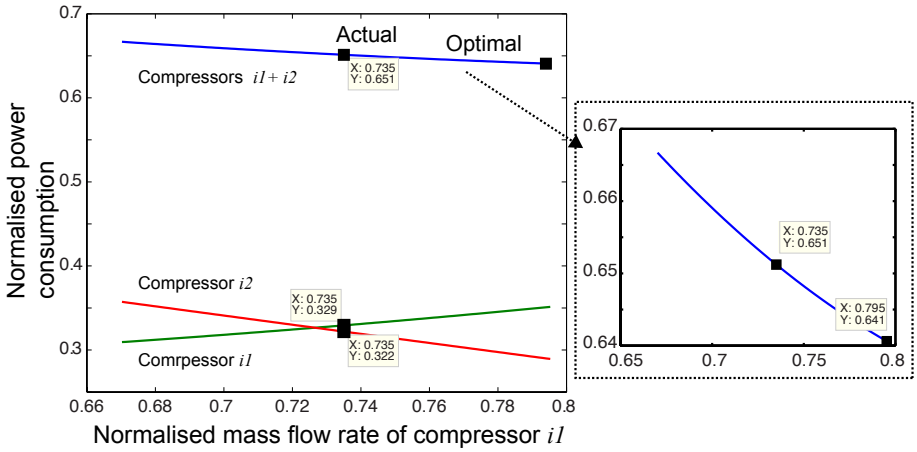


Figure 4.11: Example of the optimisation of two compressors in parallel.

An illustrative example of the two compressors $i1$ and $i2$ explains the optimal load sharing using the optimisation presented in Section 4.3.5. In this example, only two compressors are taken into account into the optimisation model. The rate of total mass flow which has to be delivered from the two compressors is M^*_{dem}. The summation of the mass flow rates of the two compressors $M_{in(i1)}$ and $M_{in(i2)}$ has to be equal to the mass flow rate of the demand.

Figure 4.11 presents the normalised power consumed from the two compressors individually and the combined normalised power consumption on vertical axis as a function of the normalised mass flow rate of the first compressor, $M_{in(i1)}$ on horizontal axis. Therefore, given the $M_{in(i1)}$, the $M_{in(i2)}$ equals $M^*_{dem} - M_{in(i1)}$. By increasing the mass flow rate of the first compressor, the compressor $i1$ consumes more power while compressor $i2$ consumes less power, assuming that all the other parameters are kept fixed, for example inlet temperature and pressure of the downstream process. Moreover, the compressors are restricted to operate from a minimum mass flow rate boundary, corresponding to compressor $i1$ equal to $\max\{M^{min}_{in(i1)}, M^*_{dem} - M^{max}_{in(i2)}\} = 0.670$ normalised units (n.u) and a maximum equal to $\min\{M^{max}_{in(i1)}, M^*_{dem} - M^{min}_{in(i2)}\} = 0.795$ n.u. when the value of the mass flow rate of the demand is 1.471 n.u.

The actual operation (point described in Fig. 4.11) is defined as the operation which took place in reality and the power consumed from the compressors is a result of the mass flow rates from the existing control scheme. Compressors $i1$ and $i2$ operated at $M_{in(i1)} = 0.735$ n.u. and $M_{in(i2)} = 0.736$ n.u. which can be assumed that the load was split evenly. As can be seen from Fig. 4.11, compressor $i1$ consumes more power than $i2$ by 2.1% under these conditions. This means that the compressors are in a different condition.

From the combined curve (Compressors $i1 + i2$), it can be observed that in the actual operation the mass flow rate of compressor $i1$ is $M_{in(i1)} = 0.735$ n.u. In this case, the total consumption is higher than operating at the point which compressor $i1$ has mass flow rate $M_{in(i1)} = 0.795$ n.u. The reduction in power is 1.54% in this case. In other words, this observation demonstrates that compressor $i1$ is more efficient than compressor $i2$ at these conditions. The more the compressor $i1$ operates against compressor $i2$ the higher is the total power reduction in the available search space. The upper boundary results due to the minimum mass flow rate boundary of compressor $i1$.

The above analysis and graphical representation is feasible for two compressors but if more than two compressors are involved in a compressor station, then optimisation deals with the estimation of the minimum value of the objective function while ensuring that the constraints hold valid.

4.5.3 Demonstration of Real Time Optimisation (RTO) application in parallel with real operation

Table 4.5: Three different cases of operation.

Case	Mass flow rates to estimate power using Eq. (4.4)
Actual operation	The mass flow rates are given from the real data
Equal split operation	The demand is split to equal mass flow rates
Optimal operation	The mass flow rates are given from the optimisation

Section 4.5.3 examines the application of the developed RTO methodology on a simulation of real time operation. The historical data are simulated as if given in real time and the RTO runs in parallel with the operation of the system which is depicted in Fig. 4.8a. The RTO determines the conditions for the optimal load sharing and the results coming from the computations of the RTO are compared with the operation that took place in reality. The RTO methodology was applied to the compressor system of the three parallel compressors for more than 12h.

The steady-state detection is configured to examine when the three compressors are in steady-state simultaneously for 40s. The inputs of the optimisation are collected during this period. After the RTO calculations, the system is examined if it is still in steady-state and if this holds true the RTO results are given to the proposed control system (see Fig. 4.8). The compressors would move to a new steady state that is different from the one in the historical data. The current approach examines each steady state of the historical data individually. The online steady-state detection algorithm identified 50 steady-state episodes where the first 16 can be seen in Fig. 4.12.

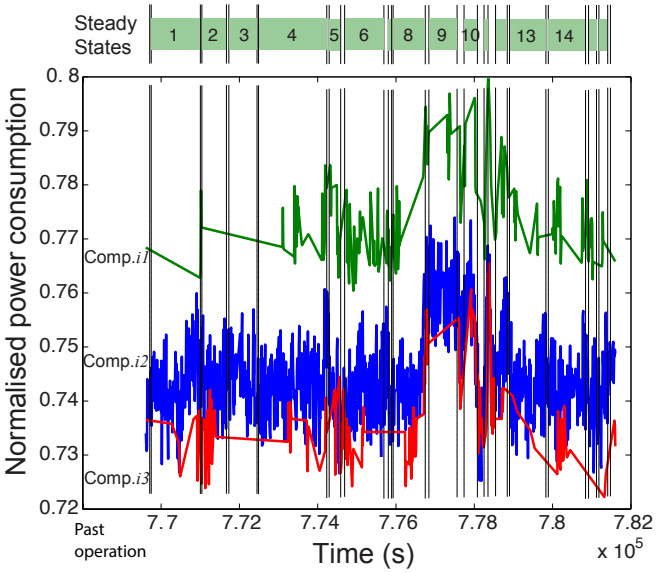


Figure 4.12: The first sixteen steady-state episodes of the system of the compressors, compressor $i1$ (green), compressor $i2$ (blue) and compressor $i3$ (red).

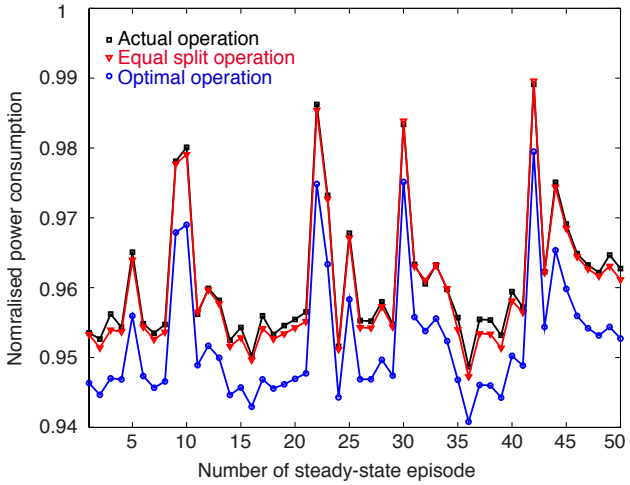


Figure 4.13: Normalised total power consumption during the periods of steady-state operation of the three compressors from actual, equal split and optimal operation.

It was assumed that the compressor conditions do not change significantly during the 15-day period of the collected data in Section 4.5.1 and the updated compressor maps from the 15-day time window were used in the optimisation. Figure 4.13 shows the normalised power consumption of the three compressors in three different cases for the examined 12 h: (a) actual operation, (b) equal split opera-

tion and (c) optimal operation. Table 4.5 shows these three case studies with the assumptions considered to estimate the power consumption.

The power consumption in Fig. 4.13 is a calculated quantity whose errors depend on errors in the quantities used in the calculation. The difference between the actual operation and equal split operation might be attributed to random variability in the data. However the power consumption for optimal operation is systematically lower and the difference cannot be accounted for by random statistical variation. The measurement error in the mass flow rate is less than +/-0.005 on the normalised mass flow rate scale. This applies to the results in Figs. 4.13 – 4.19.

Figure 4.13 shows that the actual operation and the equal split strategy do not show an important difference in the total power consumed from the compressors. Moreover, Figs. 4.14 – 4.16 show that the compressors are not equally split in the actual operation. Especially in the case of compressor $i3$, it can be seen that there is a difference between the mass flow rate from equal split operation and actual operation. Although the values of the mass flow rates in the case of the equal split are different from the mass flow rates from the actual operation, there is not much difference in the total power consumption.

On the other hand, Fig. 4.13 shows that the optimisation achieves reduction in the total power consumption in all 50 steady-state periods. Figure 4.14 shows that compressor $i1$ should work at higher mass flows than in the case of the actual and equal split operation and Fig. 4.15 shows that compressor $i2$ should work at lower mass flows. Figure 4.16 demonstrates that the optimal operation suggests that compressor $i3$ has to work at mass flow rates very close to the mass flows the actual operation which took place. In other words, the optimisation estimated

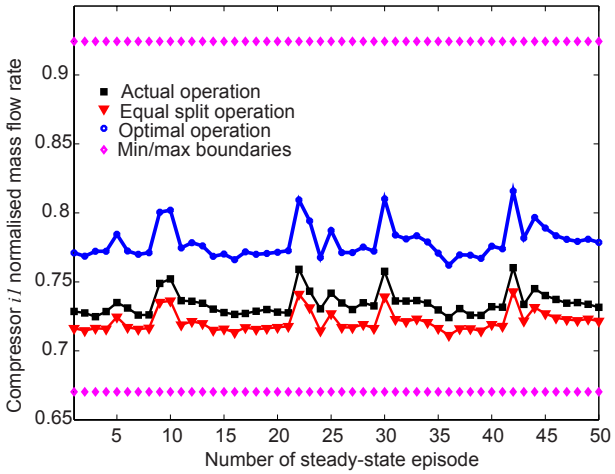


Figure 4.14: Compressor $i1$ normalised mass flow rate from three different cases.

that compressor $i3$ was operated well during the actual operation but compressor $i1$ as more efficient should be loaded more than compressor $i2$ which is less efficient under the current operational conditions.

It was observed that in actual operation compressor $i1$ and $i2$ were operated with equal load and compressor $i3$ dealt with the remaining load. The RTO method was applied exclusively to the two compressors $i1$ and $i2$ in the case of the 50 steady-state episodes. This is because the previous results from the optimisation showed that compressor $i3$ should not change operating point from this of the actual operation, therefore the mass flow of compressor $i3$ does not have any degree of freedom in the optimisation.

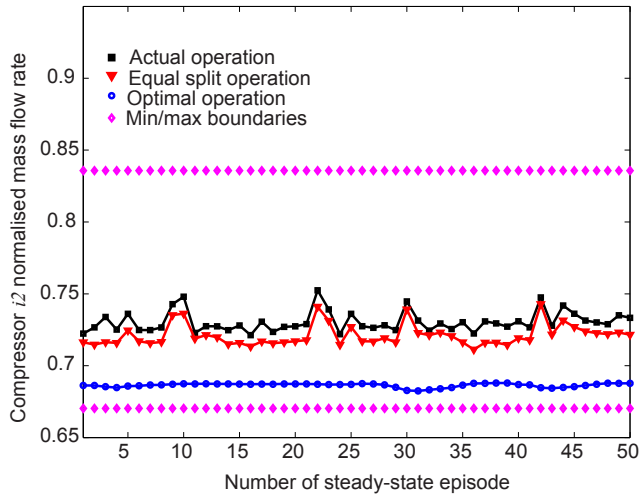


Figure 4.15: Compressor $i2$ normalised mass flow rate from three different cases.

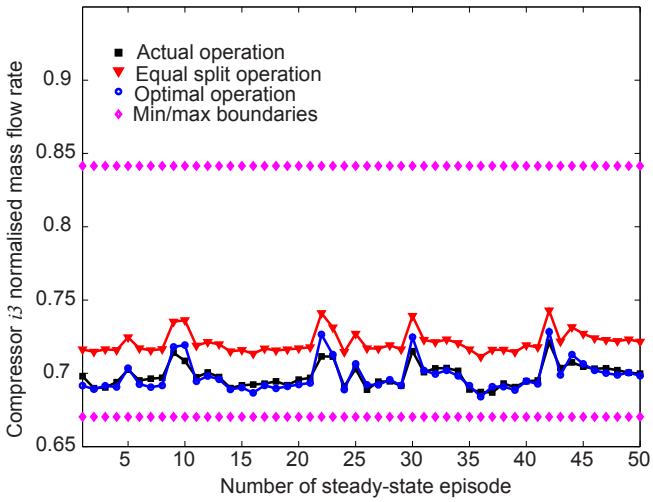


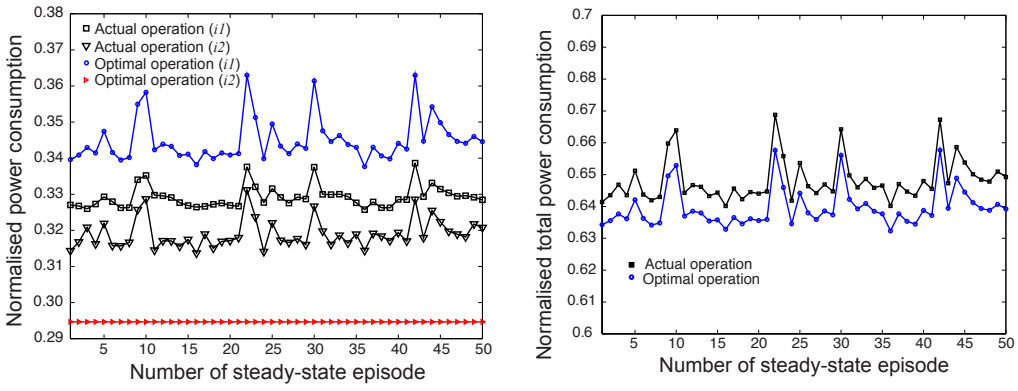
Figure 4.16: Compressor $i3$ normalised mass flow rate from three different cases.

In this case study, Fig. 4.17a shows the power consumption of the two compressors from the actual and optimal operation. It can be seen that compressor $i1$ consumes more power compared to compressor $i2$ in the actual operation in all 50 cases where the equal split strategy was applied. This shows again that compressors are not in the same condition and have different performance. Moreover, compressor $i1$ is less efficient than compressor $i2$ when the load is equally shared.

The results of the optimisation give a lower total power consumption of the compressors compared to the power consumption coming from the actual operation as can be seen in Fig. 4.17b. Compressor $i1$ is loaded more and compressor $i2$ less. This strategy results in a lower total power consumption. This means that compressors $i1$ and $i2$ work more efficiently when they operate at the mass flow rates that the RTO suggests. These mass flow rates can be seen in Figs. 4.18 and 4.19.

The above results show that an RTO framework can be applied to multiple compressors in parallel. The RTO can provide the automation of the optimisation of compressors in real-time. The online steady-state detection algorithm identifies when the operation is constant and the RTO scheme computes the optimal distribution of the load among the online compressors considering their updated characteristics and performances. The work has assumed that it is reasonable to neglect the optimisation of the transients of the system. This is a reasonable assumption because the duration of the transient operation between two steady-states is much smaller compared to the length of time the compressors stay at their new operating point.

As discussed earlier in the chapter, the RTO makes use of a fixed configuration of compressors. However, the selection of the online compressors is given from the



(a) Compressor power consumptions

(b) Total power consumption

Figure 4.17: Power consumption of compressors $i1$ and $i2$ from optimisation and actual operation (equal split).

second part of the framework in Fig. 4.1 which is presented in Section 4.2. Therefore, the scheduling of the compressors is also needed to provide the best selection of compressors. The following Chapter 5 deals with the optimal scheduling of the compressors.

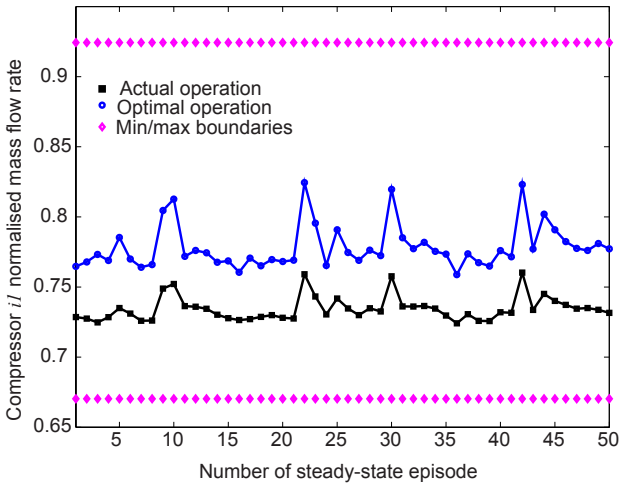


Figure 4.18: Compressor $i1$ normalised mass flow rate from actual and optimal operation.

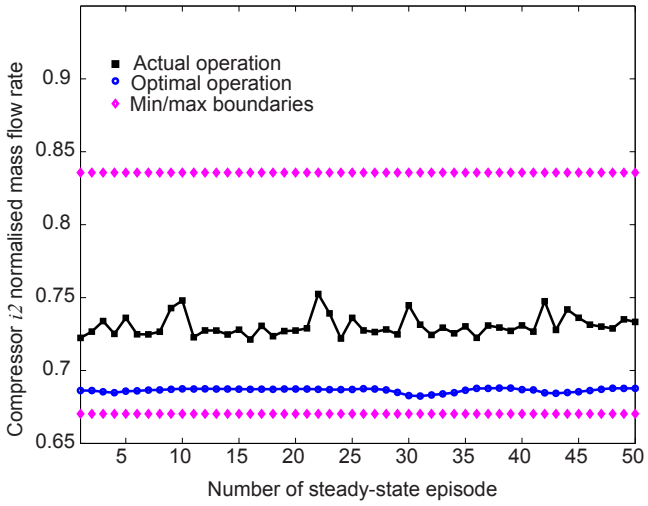


Figure 4.19: Compressor $i2$ normalised mass flow rate from actual and optimal operation.

4.6 Summary of the chapter

Chapter 4 proposed the integrated framework of real time optimisation and scheduling and demonstrated the former component of the framework: the online Real Time Optimisation (RTO). The RTO method collects raw data from the process. The data are used to update the models of the compressors which are used in an optimisation framework which reduces the power consumption in a period in which the operation is at steady-state. The optimisation deals with different operational conditions such as inlet temperature and pressure.

A real industrial case study of an air compressor station, part of an energy-intensive chemical process in BASF, Germany consuming several tens of MW was optimised using the developed RTO methodology. The comparison between the RTO application and the actual operation taken place in reality showed that the RTO method has the potential to reduce the total power consumption of the compressors.

Finally, the analysis of the real process data showed that the compressors which are assumed similar in means of specifications, in reality they are not according to the results. This is illustrated in Fig. 4.17a. This observation justifies the approach taken in the thesis, which is to assume that compressors in a compressor station are not identical.

5 Multi-period optimisation of the operation of compressor stations

5.1 Description of the chapter

Chapter 5 presents the methodology for optimal selection of compressors for long periods using a multi-period approach. The methodology is an optimisation framework which uses continuous and binary variables to estimate the minimum total costs of the operation of a compressor station while ensuring that the operational constraints of the compressors and the plant are satisfied. This framework will also implement the maintenance model which is presented in the following Chapter 6.

The problem of scheduling of compressors involves two different cases of modelling. The first case assumes that the models of the compressors are data-driven. The procedure of the building of these models is similar to that of the air compressors of the case study of the BASF air compressors (presented in Chapter 4). In the second case, the models of the compressors are represented by compressor maps which provide both power and efficiency as functions of head, rotational speed and volumetric flow rate. The compressor maps have been generated from experiments under various operational conditions. Statoil provided the maps of the gas natural compressors of the case study introduced in Section 1.4.2.

Mixed integer programming equations are used to describe the discrete operational functions of compressors and other process units of a plant. Examples of discrete operations include the start up and shut down of a compressor, and the assignment of online compressors to different downstream processes. A generic multi-period optimisation model for the optimisation of a plant which includes the compressor station involves these mixed integer programming equations and the models of the power consumption. These models are either the data-driven polynomial expressions or mixed integer programming formulations generated from compressor maps. The resulting multi-period optimisation model is solved using mathematical programming techniques, such as MILP.

A study was conducted to optimise the schedule of the BASF air separation plant in a collaboration with the postdoctoral researcher Georgios Kopanos (now lecturer at Cranfield University). The work with title 'Operational planning of a

network of compressors: The air separation plant case' (Kopanos et al., 2015) was published in the journal *Applied Energy* and is reported in Section 5.17. Subsequently, these methods were further extended in this thesis to a larger and more comprehensive study in a export gas plant operated by Statoil, which is presented in Section 5.17. A second publication is in progress to present the results from the Statoil case study.

5.2 Description of the general methodology

5.2.1 Description of a compressor station within a system

The scheduling of the compressors is used to coordinate the operational tasks of the compressors to achieve improved operation for long time periods. Examples of these operational tasks are distribution of load, start up events and maintenance activities.

The compressors are part of a broader system and they can compress different types of gases according to the application. Examples of systems which include compressors which compress air and natural gas have been presented in Section 1.3. The thesis examines one case of the air compressors of an air separation plant which compress ambient air (presented in Section 1.4.1) and a second case with compressors of an export gas plant which compress natural gas at high inlet pressures (presented in Section 1.4.2).

The models of the power consumption of the motors of compressors can be generated either with the use of process data or with the use of compressor maps. The availability of field measurements allows the use of the former type of modelling. When compressor maps are provided by the user, then the latter type of modelling can be used. In reality, a compressor map is generated from performance tests, which require field measurements. However, a compressor map varies over

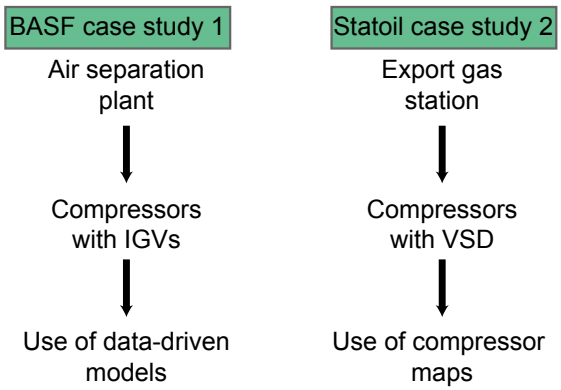


Figure 5.1: Classification of type of application, actuator of compressors and method of modelling of each case study.

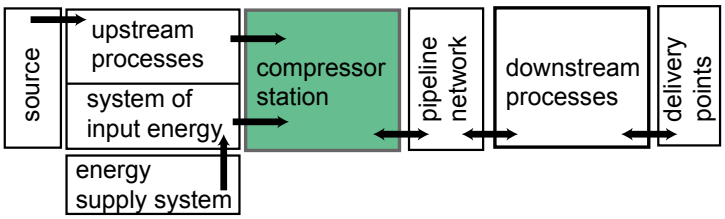


Figure 5.2: Components of a system which includes a compressor station.

time and after many hours of operation it does not reflect the actual condition of the compressor. On the contrary, the use of process data to generate data-driven models has the option to update the parameters of the models of the compressors on a frequent basis. However, these data are coming from normal operation without considering tests, and thus the field data are covering a limited domain of the total feasible operation compared to the full operating window provided by a compressor map. Both types of methods can be used to model either a compressor with adjustable inlet guide vanes and fixed rotational speed (i.e. compressors with IGVs), or a compressor with variable speed and fixed rotational speed (i.e. compressors with VSD). Figure 5.1 summarises the type of application, actuator of the compressors and method of modelling of each case study of the thesis.

Section 5.6 gives the description of the modelling of the behaviour of compressors using process data, and Section 5.7 presents the modelling with the use of compressor maps. The reason for using data-driven models in the air compressor station in BASF is that there are not available compressor maps, but there are available process measurements. On the contrary, the modelling of the operation of gas compressors in the natural gas case study can employ compressor maps as Statoil periodically carries out performance tests to generate compressor maps. Historical compressors maps were provided for the purposes of the current study.

A system with compressors includes several components and other sub-systems. Figure 5.2 illustrates a generic description of the main components and sub-systems, and the interactions among them. The scheduling problem considers the models of these sub-systems and their intercorrelated interactions. This generic description includes the cases of the air compressor station and the export gas compressors of the natural gas station.

- *Source:* The source provides the raw material to upstream processes. Example of a source is a well which provides rich natural gas containing condensate in the case of an export gas station. In the case of an air separation plant, the ambient air is the source.

- *Upstream processes:* The upstream processes pretreat the fluid (air or natural

gas) coming from the source in order to purify it from undesired components, for example dust from the ambient air in the case of air compressors. Gas processing plants process rich gas and they produce dry natural gas.

- *Input energy system:* The compressors require energy to work. Gas turbines or electrical motors power the compressors.
- *Energy supply:* The system of energy supply provides the energy to the drivers of the compressors, for example the electrical grid supplies electricity to the motors. Another example is a natural gas provider which provides natural gas as a fuel to the gas turbines drives.
- *Compressor station:* The compressor station is the main focus of the scheduling as most of the degrees of freedom are related to the operation of these machines.
- *Pipeline network:* It includes the short pipes which connect the discharge point of the compressors and the inlet of downstream processes. It also includes the valves and other pipes which connect the main pipes among them.
- *Downstream processes:* The downstream processes can be chemical processes which post-treat the compressed air coming from the compressor station, in the case of an air separation plant. In the case of the natural gas systems the downstream processes are the long pipes which convey the dry gas to other destinations. There is not treatment of the compressed gas, the "process" considered here is the transportation of the gas. Moreover, the storage of gas can be considered as downstream process.
- *Delivery point:* This sub-system includes the distribution of the final product to the final users, these who define the amount and properties of the final product. The demand of the compressor station is connected with the demand of the final users of the overall system.

According to Nørstebø (2008) the optimisation of compressors should be examined as a system including all the components which affect the decisions related to the operation of compressors. The consideration of a large-scale description of the problem and the inclusion of more degrees of freedom can lead to a greater margin of the improvement of the operation. However, the more components are included into the optimisation model, the more difficult and more non-linear the problem becomes. Moreover, the length of the time horizon influences the solution and as a larger time horizon considers more information this leads to more accurate decisions. On the other hand, the information coming from a long time horizon

embeds higher uncertainty compared to the case of a shorter time horizon. Therefore, there is a trade-off between short and long time horizon which influences the accuracy of the decisions. Nevertheless, this trade-off will not be examined in the current case study but it has been identified as topic for future work.

5.2.2 Examples of decisions

Figure 5.3 shows a generic natural gas compressor station which comprises of three compressor units which can provide compressed gas to two headers. This description assumes that the compressors have the same inlet conditions, however this can be modified without the loss of generality. The headers provide compressed gas to further downstream processes which are not described in the figure. The destination points correspond to systems which deliver gas or product to end-users. The same principles apply to the case of an air compressor station.

The suggested optimisation model considers a compressor station with multiple parallel compressors $i \in I$ which can be connected with different headers $j \in J$. Figure 5.3 illustrates that compressors can be in operational mode (online), for example compressor $i1$ and $i2$, or are in offline mode, for example compressor $i3$. The valves with black colour are closed and the valves with white colour are open. Moreover, compressors can supply gas to different headers. Figure 5.3 shows

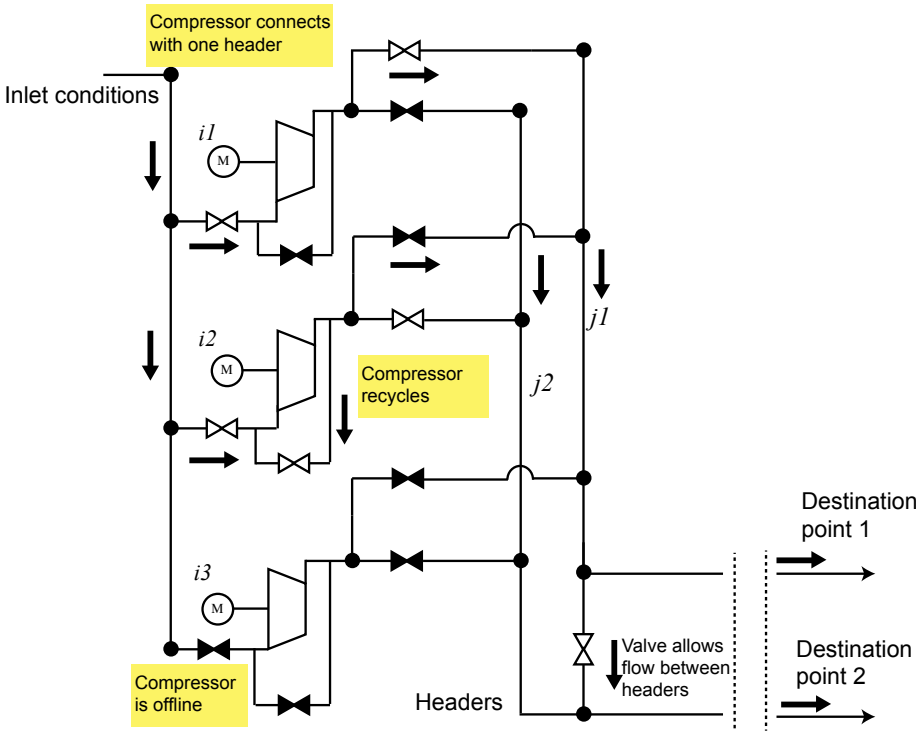


Figure 5.3: A typical compressor station.

that compressor $i1$ is supplying gas to header $j1$. The figure also shows that the headers are connected with a valve. If the valve opens, then gas flows from one header to another. All these are decisions related to discrete events, for example a compressor is on or off, a compressor is connected only with header $j1$ or not, and the valve between headers is open or closed.

These discrete events influence the quantities of the continuous operation, for example for the compressors which are online the users have to decide how much gas to provide to the headers. Moreover, the load of a compressor can be distributed among the headers with which are connected. The recycling flow rate can also take different continuous values. The decision of discrete actions and continuous quantities are related to each other. For example, if few compressors are chosen to operate online, then they have to supply more gas compared to the case of a greater number of online compressors. This trade-off between number of online compressors and mass flow rates is considered in the optimisation model in order to achieve the minimisation of the total costs.

5.2.3 Statement of the problem

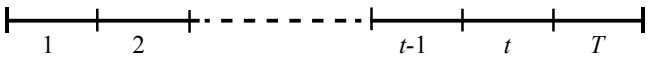


Figure 5.4: A uniform discretisation of the scheduling time horizon.

The multi-period optimisation model employs a finite time horizon T discretised in uniform time periods $t \in T$, as shown in Fig. 5.4. This approached is based on the method used by Iyer and Grossmann (1997) for operational planning of utilities. The demands are given and are assumed constant for each period. The purpose of the scheduling problem is to take the best decisions for each period and the objective is to achieve the minimisation of the total costs for the total time which equals to the summation of all the time periods. The statement of the scheduling problem should consider the following operational aspects of the compressors:

- The operation of a compressor is restricted to a feasible window of operation, because of physical, material and safety reasons which have been explained in Chapter 2. Dissimilar compressors have different boundaries of this feasible operational window. This window can be determined by compressor maps which are given by vendors or generated from experimental field tests. When maps are not available, then process data from the plant can be exploited to determine the feasible operational window. The modelling with the use of

process data is presented in Section 5.6 and The modelling with the use of compressor maps is explained in 5.7

- In reality, dissimilar compressors have different efficiencies. The efficiency of a compressor is a function of head (or pressure) and mass flow rate. The efficiency is known when a compressor map is given or when enough process data are available to calculate the efficiency through thermodynamic and aerodynamic equations (Dixon and Hall, 2010). The inclusion of the efficiency in the optimisation problem adds non-linearities, thus instead of the use of efficiency, the power consumption of the compressors is used in the optimisation models. The use of the power leads to linear formulations which is expected to significantly reduce the computational burden.

- A compressor can start up and shut down in a time period. It is assumed that these events can happen only one time in the beginning of the period.

- A compressor can be connected with different header after each time period. The change of the header incurs an operational cost, as will be explained and modelled in Section 5.5.

The optimisation formulation includes the models of the compressors which describe their power consumption and their feasible operational window. It also includes the operational aspects of the compressors namely start up, shut down, connection with and change of headers, and safety operational models of the compressors. The safety operational models are the recycling gas and blow-off air models. The BASF case study 1 considers a blow-off air model, and the natural gas compressors of Statoil case study 2 consider a recycling gas model. The mass balances in the headers, in other pipes and other process units are part of the constraints of the optimisation formulation. The mass balances in the destination points are also presented. The consideration of the initial state of the compressor station, and the boundary constraints (or terminal) complete the mathematical formulation of the framework.

The methodology employs binary and continuous variables in order to formulate a Mixed Integer Linear Programming (MILP) model. The reason to use these two types of variables is that the solution of the MILP model considers both continuous operational aspects, such as flow distribution, and discrete events, for example the selection of online compressors. As previously explained, both continuous and discrete aspects influence each other, therefore it is essential to solve the problem simultaneously.

The notation and explanation of the major binary and representative continuous positive variables can be seen below:

Major binary variables

$$X_{(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I \text{ is in operation during time period } t \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{(i,j,t)} = \begin{cases} 1, & \text{if compressor } i \in I \text{ serves header } j \in J_{(i)} \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}$$

$$S_{(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I \text{ starts up at the beginning of time period } t \\ 0, & \text{otherwise} \end{cases}$$

$$F_{(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I \text{ shuts down at the beginning of time period } t \\ 0, & \text{otherwise} \end{cases}$$

Representative positive continuous variables

$Q_{cor(i,j,t)}$ is the inlet corrected volumetric flow rate in m^3/h
$H_{cor(i,j,t)}$ is the corrected head in kJ/kg
$N_{cor(i,j,t)}$ is the corrected speed in RPM
$W_{cor(i,j,t)}$ is the corrected shaft power in kW
$M_{in(i,j,t)}$ is the inlet mass flow rate in kg/s
$P_{d(i,j,t)}$ is the outlet pressure of the compressor in bar

of compressor $i \in I$ connected with header $j \in J_{(i)}$ in period $t \in T$. The subset of headers $J_{(i)}$ is related to the feasible connections between compressors $i \in I$ and headers $j \in J$. The inlet and outlet variables are referred to the inlet and outlet of a compressor.

The inlet corrected volumetric flow rate Q_{cor} and corrected head H_{cor} are used for the description of the operation of natural gas compressors as can be seen in Section 5.7. The reason is that conventional compressor maps for natural gas are usually expressed in these quantities. Ambient conditions, compressibility and molecular weight are the main factors which influence the feasible area and performance of a natural gas compressors. On the other hand, inlet mass flow rate M_{in} and outlet pressure P_d are used for the description of air compressors. The molecular weight and compressibility varies insignificantly in the case of air compressors and the ambient conditions, such as temperature and pressure, are considered in the data-driven models of these types of compressors, as can be seen in Section 5.6.

The notation used describes variables, sets and parameters with the use of

subscripts and superscripts. The superscripts are used to specify the quantity, for example to define an upper boundary of a variable X, then X^{max} is used. The subscript set-indices of variables, sets and parameters are in brackets. The remaining letters are used to specify the quantities. In some cases through the text, the subscript set-indices may be omitted for better readability. Lower case letters (greek or latin) are used for parameters.

5.3 Minimum run and shutdown time

Constraints (5.1) and (5.2) define $S_{(i,t)}$ and $F_{(i,t)}$ binary variables through variables $X_{(i,t)}$. Parameters $\tilde{\chi}_{(i)}$ denote the operating status, on or off, of compressor i just before the beginning of the current scheduling horizon. Moreover, parameters $\tilde{\chi}_{(i)}$ can be calculated through parameters $\tilde{\varphi}_{(i,j)}$, which represent the active connection of compressor i to header j before the beginning of the current scheduling horizon. Both parameters $\tilde{\chi}_{(i)}$ and $\tilde{\varphi}_{(i,j)}$ are connected through expression $\tilde{\chi}_{(i)} = \sum_{j \in J_{(i)}} \tilde{\varphi}_{(i,j)}$, since a compressor can connect with at most one header at a time.

$$\begin{aligned} S_{(i,t)} - F_{(i,t)} &= X_{(i,t)} - \tilde{\chi}_{(i)}, && \forall i \in I, t = 1 \\ S_{(i,t)} - F_{(i,t)} &= X_{(i,t)} - X_{(i,t-1)}, && \forall i \in I, t > 1 \end{aligned} \tag{5.1}$$

$$S_{(i,t)} + F_{(i,t)} \leq 1, \qquad \forall i \in I, t \in T \tag{5.2}$$

Constraints (5.2) are omitted, if the costs of startup and shutdown are part of the objective function. For every compressor i, constraints (5.3) and (5.4) model the minimum run and shutdown time, respectively.

$$X_{(i,t)} \geq \sum_{t'=max\{1,t-\omega_{(i)}+1\}}^{t} S_{(i,t')}, \qquad \forall i \in I, t \in T, \omega_{(i)} > 1 \tag{5.3}$$

$$1 - X_{(i,t)} \geq \sum_{t'=max\{1,t-\psi_{(i)}+1\}}^{t} F_{(i,t')}, \qquad \forall i \in I, t \in T, \psi_{(i)} > 1 \tag{5.4}$$

5.4 Assignment of compressors to headers

A compressor i can supply fluid to at most one header $j \in J_{(i)}$ at a time and this is modelled through constraints (5.5). The reason for the exclusive operation between one compressor and a specific header is that headers have different pressures. If a compressor was connected with more than one header, then back

flows would occur from headers with higher pressure towards to headers with lower pressure.

$$\sum_{j \in J_{(i)}} Y_{(i,j,t)} = X_{(i,t)}, \qquad \forall i \in I, t \in T \tag{5.5}$$

5.5 Compressor-to-header assignment changes

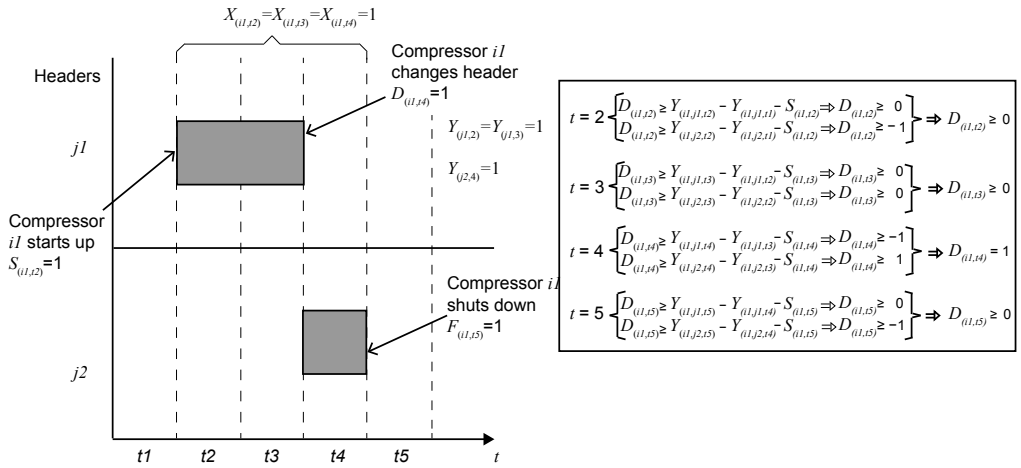


Figure 5.5: Modelling of header-changes for a compressor through constraints (5.6).

Any compressor i during its operation can change headers from one time period to another. The industrial practice concerns compressor-to-header re-assignments to be typically achieved manually through valve opening or closing by the technical personnel of the plant. For this reason, a small number of header-changes is desirable so as to: (a) reduce the use of personnel, (b) decrease the probability for potential human errors, (c) favour a more smooth operation (i.e. easier to implement, control, and revise) of the overall network of compressors and (d) avoid energy losses during the header-change due to venting of the fluid to the atmosphere (in the case of air compressors). Hence, the total number of these compressor-to-header changes is considered in the optimisation formulation which incorporates an associated penalty cost term, C_D, in the objective function as showed in Section 5.12.

In order to model compressor-to-header changes, the following set of binary decisions variables has been introduced:

$$D_{(i,t)} = \begin{cases} 1, & \text{if compressor } i \text{ changes header from time period } t-1 \text{ to } t \\ 0, & \text{otherwise} \end{cases}$$

The compressor-to-header changes are modelled by the following constraints:

$$\begin{aligned} D_{(i,t)} &\geq Y_{(i,j,t)} - \tilde{\varphi}_{(i,j)} - S_{(i,t)}, \qquad \forall i \in I, j \in J_{(i)}, t = 1 \\ D_{(i,t)} &\geq Y_{(i,j,t)} - Y_{(i,j,t-1)} - S_{(i,t)}, \quad \forall i \in I, j \in J_{(i)}, t > 1 \end{aligned} \tag{5.6}$$

Figure 5.6 depicts an illustrative example of how constraints (5.6) work. Because of the introduction of a penalty cost for header-changes in the objective function, the optimisation algorithm should try to force the binary variables $D_{(i,t)}$ to become zero, in other words the optimisation will try to decrease the header-changes to the minimum.

5.6 Modelling of compressors with the use of process data

The methodology which is used to determine the power consumption and feasible operational window of a multi-stage centrifugal compressor for the scheduling problem was described in Chapter 4. A black box data-driven model is used to model this behaviour. This modelling approach will be used only for the formulation of the optimisation model of the operation of air compressors of the BASF case study, as illustrated in Fig. 5.1. The use of compressor maps to model the behaviour of the compressors is used for the second case study and this modelling approach is presented in the following Section 5.7.

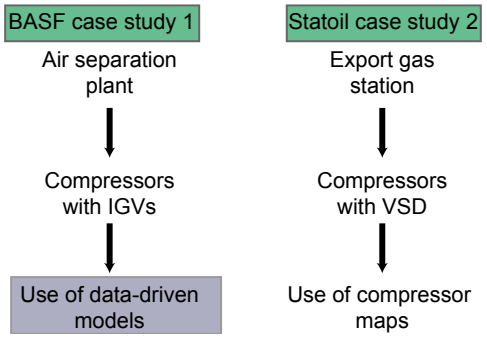


Figure 5.1 describing the applications of each case study. Section 5.6 examines the use of data-driven models.

5.6.1 Power consumption

The modelling of the power consumption of the motor of a multi-stage compressor can be derived from process data. Chapter 4 presented a black box method to represent the power consumption of the motor (W_{el}) as a function of: (a) process variables such as mass flow rates (M_{in}) and outlet pressures (P_d), and (b) parameters such as the inlet pressure (P_{in}). For the purposes of the scheduling problem the output power of the electrical motor of a compressor can be calculated by the following expression:

$$W_{el(i,j,t)} = b_{el(i,0)} Y_{(i,j,t)} + b_{el(i,1)} M_{in(i,j,t)} + b_{el(i,2)} (P_{d(i,j,t)} / P_{in(i,t)}) + b_{el(i,3)} M_{w(i,t)} \tag{5.7}$$

where $M_{w(i,t)}$ is the measured flow rate of the cooling water through the intercoolers of a compressor i. The b_{el} coefficients of this expression are calculated using data fitting methods in Matlab$^{\text{TM}}$. This expression is usually represented well by an expression with quadratic and bilinear terms, however a more simple linear form could be derived in some cases. The advantage of a linear objective function is the significant reduced computational effort to reach optimality, however a higher error in the results is usually expected.

This expression can describe compressors with Inlet Guide Vanes (IGVs) or inlet throttling valves. With IGV, the operating point of the compressor changes through the manipulation of the angle of the inlet guide vane that controls the outlet pressure. In the latter case, the operating point of the compressor changes by increasing the losses at the inlet of the compressor and in that way the inlet pressure can be modified.

The industrial case of BASF includes air compressors which use either IGVs or inlet throttling valves as actuators. However, the case study does not provide the inlet pressure at the eye of the compressor. The measurement of the inlet pressure, which is provided from the industrial partner, is at the filter of the inlet duct of the compressors which is approximately one bar for all the compressors. This work does not focus on the estimation of the manipulated variables, i.e. angles of IGVs or angles of throttling valves. Therefore, the expression of Eq. (5.7) describes well both cases of compressors for the purpose of scheduling, since the variable that is taken into account in the optimisation is the outlet pressure P_d. The estimation of the manipulated variables of the IGVs or angles of throttling valves should be part of the analysis of lower automation levels such as control.

Chapter 4 demonstrated that two similar compressors consume different amounts of energy at the same operating conditions. The efficiencies of the compressors and motors are embedded in the power consumption expression of Eq. (5.7). The reason to use this method is that the efficiencies cannot be explicitly modelled in this case study because many measurements are not available. The measurements are essential to carry out thermodynamic calculations to estimate the efficiency of a compressor. As previously mentioned, the use of the efficiency adds non-linearities in the models of the optimisation. Thus, the use of the power consumption of Eq. (5.7) in the objective function of Eq. (5.61) helps to avoid non-linear formulations which are hard to solve.

5.6.2 Feasible window of operation

The feasible window of operation of the compressor is defined from the domain of the regression model (Brooks et al., 1988). The group of all the operating points of a collected data set determines the regression domain of the model as can be seen in Figure 4.9b which was described in the previous chapter. It is also known from the plant that the operators of the compressors may not have operated the compressors over their full operational range in the collected data set. Therefore, a partial compressor map is captured in the regression models. This leads to a solution in a confined space. The extrapolation of the models leads to inaccurate results, as has been observed in a numerical study during the development of the regression models. Nevertheless, the operation coming from the optimisation using this operational space is expected to be more efficient than operation without optimisation.

For any compressor i, constraints described by Eqs. (5.8) and (5.9) denote the lower and upper bounds on the inlet compressed air mass flow rate and pressure ratio that the compressor could operate. These bounds define the feasible window of operation. The extreme parameters $M^{min}_{r.d.(i)}, M^{max}_{r.d.(i)}, pi^{min}_{r.d.(i)}$ and $\pi^{max}_{r.d.(i)}$ are estimated by the collected data set.

$$M^{min}_{r.d.(i)}\, Y_{(i,j,t)} \leq M_{(i,j,t)} \leq M^{max}_{r.d.(i)}\, Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \quad (5.8)$$

$$\pi^{min}_{r.d.(i)}\, Y_{(i,j,t)} \leq P_{d(i,j,t)}/P_{in(i,t)} \leq \pi^{max}_{r.d.(i)}\, Y_{(i,j,t)}, \quad \forall i \in I, j \in J_{(i)}, t \in T \quad (5.9)$$

From constraints (5.9), if $Y_{(i,j,t)} = 0$, then $P_{d(i,j,t)} = 0$. This zero pressure has no physical meaning, and is used for the calculation of the term of power consumption cost in the objective function (Eq. (5.61)), which is related to the power consumption given by Eq. (5.7).

5.6.3 Feasible window of operation with tight constraints: Convex hull application

The operational behaviour of the electrical power consumption of the motors of compressors can be formulated in a more detailed way that considers ambient temperature. The inlet ambient pressure in the case of the air separation plant does not change significantly, therefore it is not taken into account. This assumption also simplifies the demonstration of the method described in this section. Hence, the electrical power consumption of the motor of a compressor $W_{el(i,j,t)}$ is given by the following equation:

$$W_{el(i,j,t)} = b_{el(i,0)} Y_{(i,j,t)} + b_{el(i,1)} M_{in(i,j,t)} + b_{el(i,2)} P_{d(i,j,t)} + b_{el(i,3)} T_{in(t)} \quad (5.10)$$

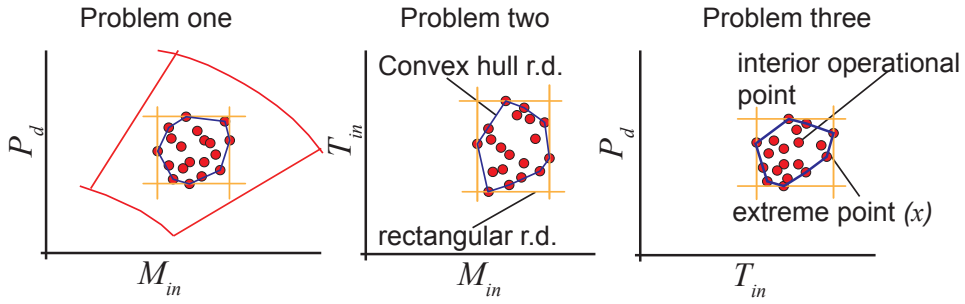


Figure 5.6: Convex hull problems with three process variables involved.

Equations (5.8) and (5.9) which describe the regression domain with a rectangular domain can be replaced by a domain based on the theory of the convex hull presented in the study of Vecchietti et al. (2003). The use of a convex hull provides more tight constraints of the regression domain (r.d.) compared to the case of a rectangular domain. Indeed, the convex hull can ensure higher confidence in the prediction of the power consumption compared to the case of a rectangular regression domain. The reason is that the the extrapolation in the prediction is minimised in the former case. The rectangular regression domain is defined by minimum and maximum values of the observed operating points of the provided data set, and thus this results in allowing Eq. (5.10) to take values which are between the tight domain defined by the convex hull and the rectangular domain.

Figure 5.6 shows a graphical illustration of the constraints which define the operational range of a compressor using rectangular and convex hull regression domains. The convex hull regression domain is defined from the operating points of the sets of the two variables M_{in} and P_d and the parameter T_{in}. The development of the constraints of each problem described by Eqs. (5.11) – (5.22) is presented in the article by Danzer et al. (1963). Kahrs and Marquardt (2007) discussed the validity of the model using the convex hull approach for data-driven models, and Mitra et al. (2012) applied a convex hull approach to data-driven models with mixed integer programming formulations.

Convex hull problem one involves variables M_{in} and P_d:

$$\sum_{ch1 \in CH1} \lambda_{1(i,j,ch1,t)} M_{in,x(i,ch1)} = M_{in(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.11}$$

$$\sum_{ch1 \in CH1} \lambda_{1(i,j,ch1,t)} P_{d,x(i,ch1)} = P_{d(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.12}$$

$$\sum_{ch1 \in CH1} \lambda_{1(i,j,ch1,t)} = Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.13}$$

$$\lambda_{1(i,j,ch1,t)} \leq 1, \qquad \forall i \in I, ch1 \in CH1, j \in J_{(i)}, t \in T \tag{5.14}$$

Convex hull problem two involves variables M_{in} and T_{in}:

$$\sum_{ch2 \in CH2} \lambda_{2(i,j,ch2,t)} M_{in,x(i,ch2)} = M_{in(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.15}$$

$$\sum_{ch2 \in CH2} \lambda_{2(i,j,ch2,t)} T_{in,x(i,ch2)} = T_{in(t)} Y_{(i,j,t)}, \quad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.16}$$

$$\sum_{ch2 \in CH2} \lambda_{2(i,j,ch2,t)} = Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.17}$$

$$\lambda_{2(i,j,ch2,t)} \leq 1, \qquad \forall i \in I, ch2 \in CH2, j \in J_{(i)}, t \in T \tag{5.18}$$

Convex hull problem three involves variables P_d and T_{in}:

$$\sum_{ch3 \in CH3} \lambda_{3(i,j,ch3,t)} P_{d,x(i,ch3)} = P_{d(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.19}$$

$$\sum_{ch3 \in CH3} \lambda_{3(i,j,ch3,t)} T_{in,x(i,ch3)} = T_{in(t)} Y_{(i,j,t)}, \quad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.20}$$

$$\sum_{ch3 \in CH3} \lambda_{3(i,j,ch3,t)} = Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.21}$$

$$\lambda_{3(i,j,ch3,t)} \leq 1, \qquad \forall i \in I, ch3 \in CH3, j \in J_{(i)}, t \in T \tag{5.22}$$

The parameters $M_{in,x(i,ch1)}$, $P_{d,x(i,ch1)}$ of convex hull problem one are the extreme points of the sets of the observed operational points of mass flow rates and pressures of compressors $i \in I$ with observations $ch1 \in CH1$. The number of observations of the variables of each compressor i can be different among them, therefore the number of observations in the set $CH1$ represents the maximum number of observations among all of the compressors $i \in I$. The extreme points are estimated by the Matlab function *convhull* (Matlab, 2015). The parameter λ_1 takes values between zero and one and it is used for the formulation of the convex hull problem. Parameters $\lambda_1, \lambda_2, \lambda_3$ do not have any physical meaning. The sets of observations of pressures and mass flow rates of each compressor i for the convex hull problem two and three are given by $ch2 \in CH2$ and $ch3 \in CH3$ respectively.

The equations of convex hull problem one, Eqs. (5.11) and (5.12), are satisfied if the values of the variables $M_{in(i,j,t)}$, $P_{d(i,j,t)}$ which represent an operating point belong to the regression domain of the corresponding convex hull of problem one in Fig. 5.6. Equations (5.11) and (5.12) are valid if there exists $\lambda_{1(i,j,ch1,t)}$ for $ch1 \in CH1$ such that Eqs. (5.13) and (5.14) are satisfied. The summation of the positive parameters $\lambda_{1(i,j,ch1,t)}$ for all $ch1 \in CH1$ has to be equal to one if the compressor i is connected to header j at period t, thus $Y_{(i,j,t)}$ is equal to one as it is described by Eq. (5.13). The other two convex hull problems are formulated in the same way. Therefore Eqs. (5.11) – (5.22) define the regression domain of the decision variables $M_{in(i,j,t)}$ and $P_{d(i,j,t)}$ which consider the key parameter $T_{in(t)}$.

5.7 Modelling with the use of compressor maps

Section 5.7 presents the methodology for modelling the behaviour of compressors using compressor maps. The compressor maps in this section refer to variable speed compressors working with natural gas. The second industrial case study examines these types of compressors as can be seen in Fig. 5.1.

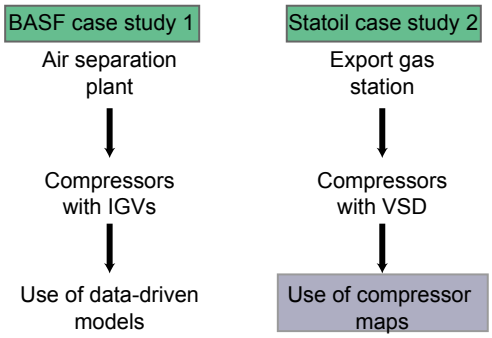


Figure 5.1 describing the applications of each case study. Section 5.7 examines the use of compressor maps.

The compressors maps can be given from the industry and they describe the shaft power consumption, head, discharge pressure, volumetric flow rate and efficiency. The maps consider constant reference conditions, i.e. reference temperature, $T_{ref(i)}$, reference pressure, $P_{ref(i)}$, and reference molecular weight, $MolW_{ref(i)}$. The volumetric flow rate, speed, head, pressure and power are defined as *corrected* at these conditions as can be seen in Eqs. (5.40) - (5.44).

5.7.1 Power consumption

Figure 5.7 shows a characteristic map which illustrates the feasible operational window of an industrial compressor restricted by surge, choke, minimum and maximum speed line. Moreover, the power consumed from the shaft is given as a function of the speed and volumetric flow rate as can be seen in Fig. 5.8. Both sets of curves are provided by the vendor, or by the user after conducting performance tests. In the natural gas case study, the optimisation minimises the power of the

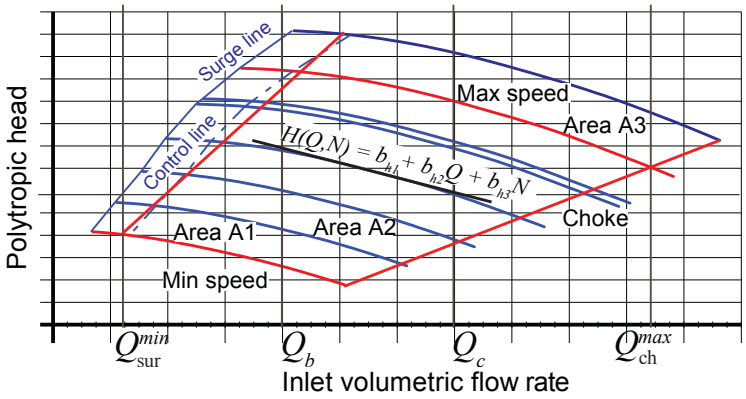


Figure 5.7: Characteristics of a gas compressor with VSD control.

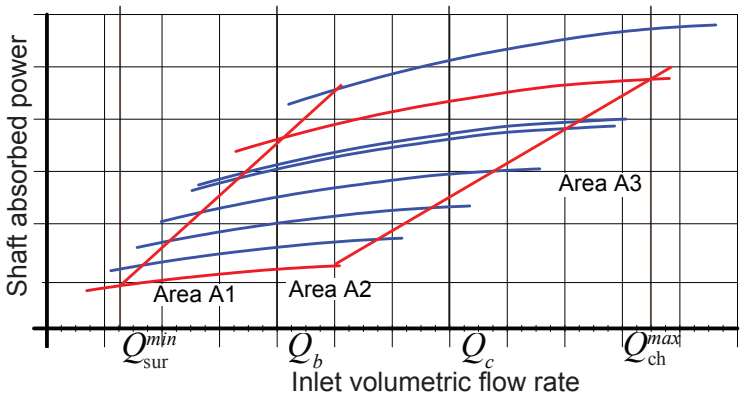


Figure 5.8: Power curves of a compressor with VSD control.

shaft of the compressors. Analysis of the losses of the gearbox of the industrial compressors suggest that they are less than 1%. These losses have been ignored for the purposes of the scheduling problem. For the same reason, the electrical losses in the electrical motors for various operating points are not considered.

The power and head of the natural gas compressors are represented by the MILP Eqs. (5.23) and (5.24) which include the auxiliary variables W_{aux}, H_{aux}, N_{aux} and Q_{aux}, and a binary variable λ^*. The linearisation of the power and head of the compressor involves the division of the maps into K areas. Figures 5.7 and 5.8 illustrate an example of the division of the polytropic head-volumetric flow rate map and power curves into three areas with non-uniform segments. The model becomes more accurate as the number of segments increases. However, increasing the number of areas requires greater computational burden. The reason for choosing three areas in this trade-off is that the modelling errors were of the same order as the effects that have already been neglected (e.g. gearbox losses).

$$\begin{aligned} W_{aux(i,j,k,t)} &= b_{w1(k,i)}\lambda^*_{(i,j,k,t)} + b_{w2(k,i)}Q_{aux(i,j,k,t)} \\ &\quad + b_{w3(k,i)}N_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, k \in K, t \in T \end{aligned} \tag{5.23}$$

$$\begin{aligned} H_{aux(i,j,k,t)} &= b_{h1(k,i)}\lambda^*_{(i,j,k,t)} + b_{h2(k,i)}Q_{aux(i,j,k,t)} \\ &\quad + b_{h3(k,i)}N_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, k \in K, t \in T \end{aligned} \tag{5.24}$$

The following equations describe the constraints which include the binary variable $\lambda^*_{(i,j,k,t)}$ used to linearise the characteristics (head-speed-volumetric flow rate) and the power (work-speed-volumetric flow rate) curves:

$$\sum_{k\in K} \lambda^*_{(i,j,k,t)} = Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.25}$$

$$W_{cor(i,j,t)} = \sum_{k\in K} W_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.26}$$

$$H_{cor(i,j,t)} = \sum_{k\in K} H_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.27}$$

$$N_{cor(i,j,t)} = \sum_{k\in K} N_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.28}$$

$$Q_{cor(i,j,t)} = \sum_{k\in K} Q_{aux(i,j,k,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.29}$$

$$N^{min}_{(i)} \lambda^*_{(i,j,k,t)} \leq N_{aux(i,j,k,t)} \leq N^{max}_{(i)} \lambda^*_{(i,j,k,t)},$$
$$\forall i \in I, j \in J_{(i)}, k \in K, t \in T \tag{5.30}$$

$$Q^{min}_{sur(i)} \lambda^*_{(i,j,k,t)} \leq Q_{aux(i,j,k,t)} \leq Q_{b(i)} \lambda^*_{(i,j,k,t)}, \quad k = 1, \tag{5.31}$$

$$Q_{b(i)} \lambda^*_{(i,j,k,t)} \leq Q_{aux(i,j,k,t)} \leq Q_{c(i)} \lambda^*_{(i,j,k,t)}, \quad k = 2, \tag{5.32}$$

$$Q_{c(i)} \lambda^*_{(i,j,k,t)} \leq Q_{aux(i,j,k,t)} \leq Q^{max}_{ch(i)} \lambda^*_{(i,j,k,t)}, \quad k = 3, \tag{5.33}$$
$$\forall i \in I, j \in J_{(i)}, t \in T$$

where the $N^{min}_{(i)}$ and $N^{max}_{(i)}$ are the minimum and maximum rotational speeds of each compressor i. The volumetric flow rates Q^{min}_{sur}, Q_b, Q_c and Q^{max}_{ch} are chosen in such a way to divide the compressor map in three areas. The number of the segments and their lengths are subject to users' judgement.

For each area $k \in K$ the power and head are estimated by various linear functions of speed and volumetric flow rate. The auxiliary variables are associated with each k area at which the compressor operates. The binary variable $\lambda^*_{(i,j,k,t)}$ defines on which area a compressor i connected with header j operates in period t. If a compressor i is not connected with any header, i.e. $Y_{(i,j,t)} = 0$ and $X_{(i,t)} = 0$, then the $\lambda^*_{(i,j,k,t)}$ equals to zero. This means that the auxiliary variables take the value zero and the corrected variables become zero, referring to Eqs. (5.23) - (5.33).

The linear expressions of power and head, functions of speed and volumetric flow rate, are developed using data fitting methods in Matlab$^{\mathrm{TM}}$. The procedure of the fitting of the data is similar to the methodology presented in Chapter 4. The provided compressor maps from Statoil were digitalised and stored as a table as has been explained by Abbaspour et al. (2005). The table provided the necessary data for the fitting. Figure 5.7 shows an example of the linearisation of the head curve, in area A2, using a linear function $H(Q, N) = H_{aux(i,j,k,t)}|_{k=2}$. The function is a first order polynomial with input data the volumetric flow rate and rotational speed, and output data the head for various operating points. Therefore, the

coefficients of Eqs. (5.23) and (5.24) are estimated from the fitting methods used similarly in Chapter 4. The mathematical modelling of the power curves $W(Q, N)$ follows the same procedure as that of the head.

5.7.2 Feasible window of operation

The operative points are restricted to the area defined from the four red curves defining the boundaries as can be seen in Fig. 5.7. Figure 5.7 illustrates that the head has to be smaller than the head defined by the control surge line (known as just control line) and the maximum speed line (Wu et al., 2000), these constraints are modelled with Eqs. (5.34) and (5.35) respectively. Equations (5.36) and (5.37) model the constraint which describes that the head has to be greater than the choke and the head of the minimum speed line (Wu et al., 2000). The parameters of the Eqs. (5.34) - (5.37), $b_{s1(i)}, b_{s2(i)}, b_{U1(i)}, b_{U2(i)}, b_{ch1(i)}, b_{ch2(i)}, b_{L1(i)}, b_{L2(i)}$, are estimated with similar fitting methods as explained previously. The head of each area $k \in K$ has to respect these constraints.

$$H_{cor(i,j,t)} \leq b_{s1(i)} Q_{cor(i,j,t)} + b_{s2(i)} Y_{(i,j,t)}, \tag{5.34}$$

$$H_{cor(i,j,t)} \leq b_{U1(i)} Y_{(i,j,t)} + b_{U2(i)} Q_{cor(i,j,t)}, \tag{5.35}$$

$$H_{cor(i,j,t)} \geq b_{ch1(i)} Q_{cor(i,j,t)} + b_{ch2(i)} Y_{(i,j,t)}, \tag{5.36}$$

$$H_{cor(i,j,t)} \geq b_{L1(i)} Y_{(i,j,t)} + b_{L2(i)} Q_{cor(i,j,t)}, \tag{5.37}$$

$$\forall i \in I, j \in J_{(i)}, t \in T$$

The mass flow rate M_{in} provided at the inlet of the compressor is given by the following equation:

$$Q_{act(i,j,t)} \rho_{(i,t)} = 3,600 M_{in(i,j,t)}, \quad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.38}$$

with the density estimated by:

$$\rho_{(i,t)} = \frac{10^4 \ P_{act,s(t)}}{R_{g(t)} \ 10^3 \ T_{act,s(i,t)} \ Z_{act,s(t)}} \tag{5.39}$$

The operating conditions (temperature $T_{act,s}$, pressure $P_{act,s}$, molecular weight $MolW_{act,s}$ and compressibility $Z_{act,s}$) are usually different from the reference conditions. The different inlet conditions are called *actual conditions*. The calculations to estimate the actual power consumption of the shaft of the compressor are done using the actual conditions, because this variable is used in the objective function to be minimised. The conversion between actual and corrected values is given from equations Eqs. (5.40) - (5.44) (Boyce, 2003).

$$Q_{cor(i,j,t)} = Q_{act(i,j,t)} \sqrt{c_{f,MolW(i,t)}} / \sqrt{c_{f,T(i,t)}}, \tag{5.40}$$

$$H_{cor(i,j,t)} = H_{act(i,j,t)} c_{f,MolW(i,t)} / c_{f,T(i,t)}, \tag{5.41}$$

$$N_{cor(i,j,t)} = N_{act(i,j,t)} \sqrt{c_{f,MolW(i,t)}} / \sqrt{c_{f,T(i,t)}}, \tag{5.42}$$

$$P_{cor(i,j,t)} = P_{act(i,j,t)} / c_{f,T(i,t)}, \tag{5.43}$$

$$W_{cor(i,j,t)} = W_{act(i,j,t)} c_{f,MolW(i,t)} / (c_{f,P(i,t)} \sqrt{c_{f,T(i,t)}}\ ), \tag{5.44}$$

$$\forall i \in I, j \in J_{(i)}, t \in T$$

where the coefficients of the constraints are estimated by $c_{f,T(i,t)} = T_{act,s(i,t)}/T_{ref(i)}$, $c_{f,MolW(i,t)} = MolW_{act,s(i,t)}/MolW_{ref(i)}$ and $c_{f,P(i,t)} = P_{act,s(i,t)}/P_{ref(i)}$. The composition of the gas is assumed constant at the inlet of each compressor which is provided from the main suction header of the compressor station.

The motor which drives a compressor i restricts the maximum limit of the shaft power W_{cor} of the compressor. This is modelled through constraints (5.45).

$$W_{cor(i,j,t)} \leq W_{(i)}^{max}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.45}$$

The following constraint connects head H_{act} and discharge pressure P_d of a compressor $i \in I$ connected with header $J_{(i)}$ for each period $t \in T$ (Gresh, 2001). The $P_{d(j,t)}$ is equal to $P_{act(i,j,t)}$, $j \in J_{(i)}$ of Eq. (5.43).

$$H_{act(i,j,t)} = Y_{(i,j,t)} \frac{Z_{act,s(t)} R_{g(t)} T_{act,s(i,t)}}{\phi_{(i)}} \left(\left(\frac{P_{d(j,t)}}{P_{act,s(t)}} \right)^{\phi_{(i)}} - 1 \right) \tag{5.46}$$

5.8 Other control methods

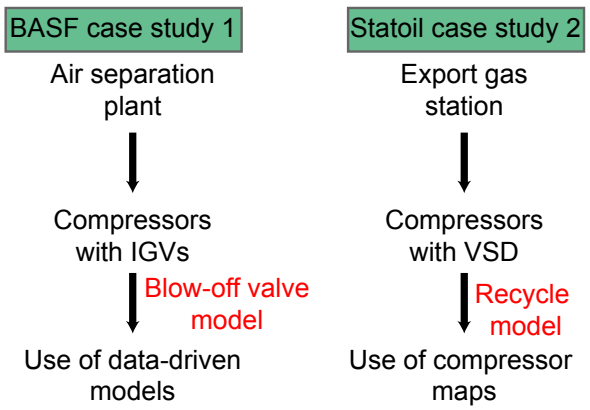


Figure 5.1 describing the applications of each case study and different control methods.

The previous methods presented the methodology of the operation of compressors with the use of variable inlet guide vanes, inlet throttling valves and variables speed drives. The first two cases are related to the air separation case study and the case with the variable speed drives refers to the natural gas case study. Section 5.8 will present the modelling of the operation of natural gas compressors when they are controlled with a recycle valve, and when compressed air is vent through a blow-off valve in the case of air compressors.

5.8.1 Recycle model

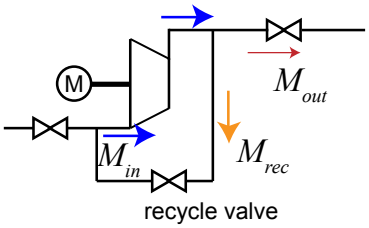


Figure 5.9: The gas flows of a compressor using recycling flow.

The mass balance of a single compressor which can use recycling of the gas is shown in Fig. 5.9. This method to control the mass flow rate is usually used in natural gas compressors, thus the following equations are related to the second case study of the export gas station. The compressor maps consider the M_{in} for the head and power calculations. The R_c binary variable models the on-off status of the recycle valve. The recycle flow control model is represented by the following equations (van den Heever and Grossmann, 2003):

$$M_{in(i,j,t)} = M_{out(i,j,t)} + M_{rec(i,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.47}$$

$$M_{rec(i,t)} \leq M^{max}_{rec(i)} R_{c(i,t)}, \qquad \forall i \in I, t \in T \tag{5.48}$$

$$R_{c(i,t)} \leq X_{(i,t)}, \qquad \forall i \in I, t \in T \tag{5.49}$$

5.8.2 Blow-off valve model

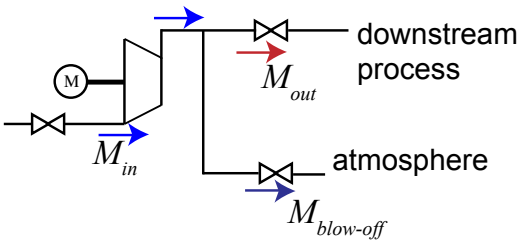


Figure 5.10: Gas flows of an air compressor with the use of a blow-off valve.

When the fluid of a process can satisfy environmental requirements, then a compressor can vent it into the atmosphere when it is necessary. In the case of air compressors, Fig. 5.10 shows the mass balance when there is a blow-off valve to vent the air. This mass balance $M_{in} = M_{out} + M_{blow-off}$ will be used in the optimisation model of the BASF case study (Fig. 5.1). The excess of compressed air produced is vented in the atmosphere. This may happen in the scenario of keeping compressors online, and their total minimum flow rate due to control surge line is higher than the demand.

5.9 Outlet pressure of compressors

The outlet pressure of a compressor is equal to the pressure of the header that the compressor serves, and is given by the corresponding load curve:

$$P_{d(i,j,t)} = \alpha_{0(j)}\, \bar{M}_{(i,j,t)} + \alpha_{1(j)}\, Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t \in T \tag{5.50}$$

Parameters $\alpha_{0(j)}$ and $\alpha_{1(j)}$ represent the coefficients of the load curve of header j. In order to avoid non-linearities in the load curve constraints (5.50), auxiliary variables $\bar{M}_{(i,j,t)}$ have been introduced. These variables denote the total compressed mass flow rate in the header j which is served by compressor i during time period t. Non-negative variables $\bar{M}_{(i,j,t)}$ are modelled through the following set of big-M constraints (Vecchietti et al., 2003):

$$\begin{aligned}
&\bar{M}_{(i,j,t)} \geq \sum_{i' \in I_j} M_{(i',j,t)} - M^*_{(j)}\,(1 - Y_{(i,j,t)}), \quad \forall i \in I, j \in J_{(i)}, t \in T \\
&\bar{M}_{(i,j,t)} \leq \sum_{i' \in I_j} M_{(i',j,t)} + M^*_{(j)}\,(1 - Y_{(i,j,t)}), \quad \forall i \in I, j \in J_{(i)}, t \in T \\
&\bar{M}_{(i,j,t)} \leq M^*_{(j)}\, Y_{(i,j,t)}, \qquad\qquad\qquad\qquad\quad \forall i \in I, j \in J_{(i)}, t \in T
\end{aligned} \tag{5.51}$$

Parameters $M^*_{(j)}$ are problem-specific large numbers that can be usually calculated by considering the maximum capacity of header j. They do not have any physical meaning, and they are used for the formulation of the big-M constraints.

5.10 Mass balances of air separation plant units

This section describes the constraints of the downstream processes of an air separation plant which includes air compressors, as shown Fig. 5.11. The major units are the air separation columns, the storage tanks of the products and the system which includes the utilities for compressed air.

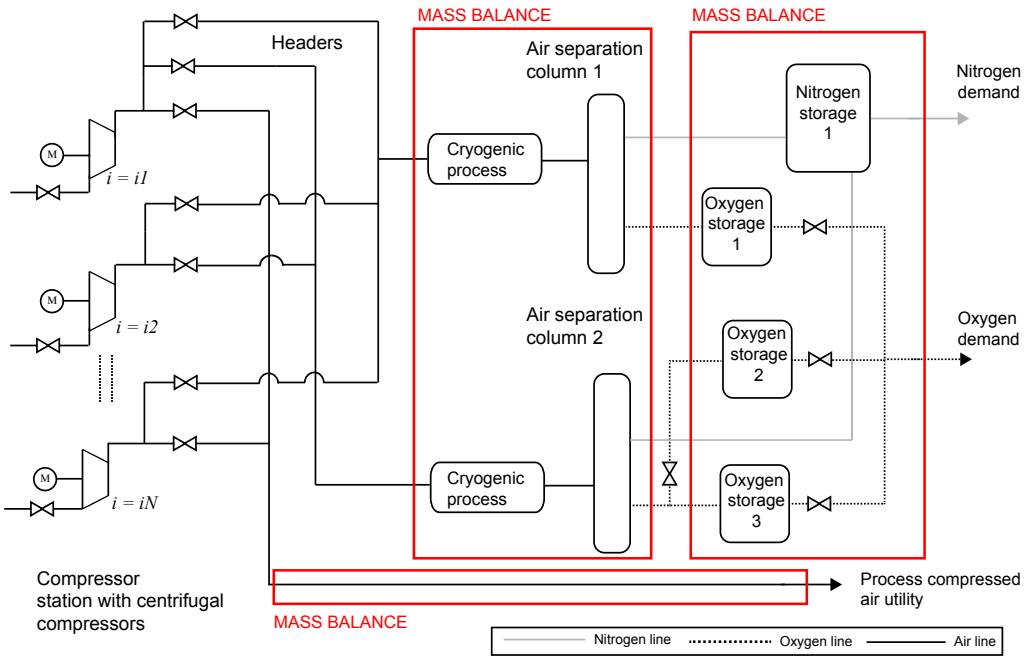


Figure 5.11: Mass balances of the units of an air separation plant.

5.10.1 Demand for compressed air utility

The compressors should satisfy the demand for compressed air utility $M^{util}_{dem(a,t)}$ for other process plants $a \in A$ in the site in each time period t, according to Eq. (5.52). The subset $J_{(a)}$ denotes the headers which are connected with process plant a. Figure 5.11 shows the air line (compressed air) at the bottom which leads to other process plants for utilities.

$$\sum_{i \in I} \sum_{j \in (J_{(a)} \cap J_{(i)})} M_{(i,j,t)} \geq M^{util}_{dem(a,t)}, \qquad \forall a \in A, t \in T \tag{5.52}$$

5.10.2 Air separation columns: mass flow rates for products

The outlet mass flow rate of product e from each air separation column u during each time period t is given by the following equation. The subset $J_{(u)}$ denotes the headers which are connected with air separation column u. The parameter $\sigma_{(e)}$ is the volumetric percentage of primary component of air e.

$$M_{prod(e,u,t)} = \sigma_{(e)} \sum_{i \in I} \sum_{j \in (J_{(u)} \cap J_{(i)})} M_{(i,j,t)}, \qquad \forall e \in E, u \in U, t \in T \tag{5.53}$$

Constraints (5.54) provide lower and upper bounds of the outlet mass flow rate of product e from each air separation column.

$$M^{min}_{prod(e,u)} \leq M_{prod(e,u,t)} \leq M^{max}_{prod(e,u)}, \qquad \forall e \in E, u \in U, t \in T \tag{5.54}$$

5.10.3 Storage tanks mass balances

The total amount of products e from the air separation column u that is provided to each storage tank $z \in Z_{(e)}$ during time period t is represented by variables $V_{prod(e,u,z,t)}$, and is given by Eq. (5.55). The parameter $\kappa_{(t)}$ is a conversion factor of mass flow rate to aggregated amount of mass flow.

$$\sum_{z \in Z_{(e)}} V_{prod(e,u,z,t)} = \kappa_{(t)}\, M_{prod(e,u,t)}, \qquad \forall e \in E, u \in U, t \in T \tag{5.55}$$

Hence, the demand for any product e in each time period t is met by extracting the necessary amount of product, $V^{out}_{tnk(e,z,t)}$, from the storage tanks $z \in Z_{(e)}$. The products could be also acquired from external sources $\delta \in \Delta_{(j)}$ with amount of provided flow $V^{ext}_{(e,t)}$, if the demand $V_{dem(e,t)}$ cannot be fully met by the internal production network.

$$\sum_{z \in Z_{(e)}} V^{out}_{tnk(e,z,t)} + \sum_{\delta \in \Delta_{(j)}} V^{ext}_{(\delta,e,t)} = V_{dem(e,t)}, \qquad \forall e \in E, t \in T \tag{5.56}$$

Constraints (5.57) correspond to the mass balance in the product storage tanks under the complete satisfaction of the demand for products. Variables $V^{stor}_{tnk(e,z,t)}$ denote the amount of product e that is stored in storage tank $z \in Z_{(e)}$ at the end of time period t. Parameter $\tilde{v}^{stor}_{tnk(e,z)}$ represents the initial inventory of product e in storage tank $z \in Z_{(e)}$.

$$\begin{aligned} V^{stor}_{tnk(e,z,t)} &= \tilde{v}^{stor}_{tnk(e,z)} + \sum_{u \in U_{(z)}} V_{prod(e,u,z,t)} - V^{out}_{tnk(e,z,t)}, \quad \forall e \in E, z \in Z_{(e)}, t = 1 \\ V^{stor}_{tnk(e,z,t)} &= V^{stor}_{tnk(e,z,t-1)} + \sum_{u \in U_{(z)}} V_{prod(e,u,z,t)} - V^{out}_{tnk(e,z,t)}, \quad \forall e \in E, z \in Z_{(e)}, t > 1 \end{aligned} \tag{5.57}$$

Constraints (5.58) define the lower and the upper inventory bounds. The upper inventory bound represents the storage capacity of the storage tank. Safety stocks are represented by the lower inventory bound.

$$V^{min}_{tnk(e,z)} \leq V^{stor}_{tnk(e,z,t)} \leq V^{max}_{tnk(e,z)}, \qquad \forall e \in E, z \in Z_{(e)}, t \in T \tag{5.58}$$

5.11 Export gas operational constraints

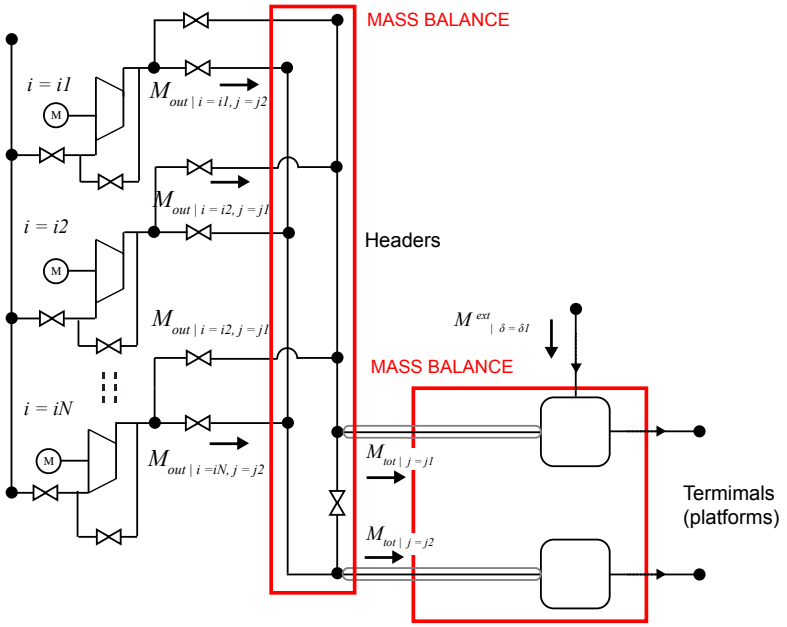


Figure 5.12: Export gas station diagram.

This section refers to the description of the operation of an export natural gas compressor station which can be seen in Fig. 5.12. The summation of the individual flows to each header is equal to the total mass flow rate at the inlet of the header:

$$M_{tot(j,t)} = \sum_{i \in I} M_{out(i,j,t)}, \qquad \forall j \in J_{(i)}, t \in T \tag{5.59}$$

The outlet pressure of each compressor has to be equal to the inlet pressure of the header is connected with. The headers are connected with long pipes which convey the compressed gas to platforms (Fig. 5.12).

The formulation of the optimisation problem considers the inlet of the pipes as the delivery point of the export gas station. It assumed that the linepack (stored amount of gas in the pipes) and the downstream pressures are given. Additionally, the demand mass flow rate $M_{dem(j,t)}$ of each pipe connected with each header $j \in J$ and pressure $P_{dem(j,t)}$ are used in actual conditions as they are measured from the plant. The mass balance at the delivery point, i.e. after the aftercooler of the outlet of the compressors, can be seen in the following equation:

$$M_{tot(j,t)} + \sum_{\delta \in \Delta_{(j)}} M^{ext}_{(\delta,t)} \geq M_{dem(j,t)}, \qquad \forall j \in J_{(i)}, t \in T \tag{5.60}$$

where $M_{dem,g(j,t)}$ is estimated from actual conditions using the following equations for every header $j \in J$ in every time period $t \in T$ (Nørstebø, 2008):

$$Q_{dem,act(j,t)} = \frac{P_{std}}{P_{dem(j,t)}} \cdot \frac{T_{d(t)}}{T_{std}} \cdot \frac{Z_{d(t)}}{Z_{std}} \cdot Q_{std} \cdot c_{std}$$

$$M_{dem(j,t)} = \frac{Q_{dem,act(j,t)} \cdot P_{d(j,t)} \cdot 10^4}{R_{g(t)} \cdot Z_{act,s(t)} \cdot 3.6 \cdot 10^4}$$

where the standard conditions are given, $P_{std} = 1.01325$ bar, $T_{std} = 288.15$ K, and $Z_{std} = 1$. The c_{std} stands for a conversion factor ($=10^5/24$) in order to convert Q_{std} (given in MMsm^3/d) into $Q_{dem,act(j,t)}$ (in m^3/h).

The $T_{d(t)}$ is the temperature of the gas at the delivery point assumed constant in this work and the $Z_d(t)$ equals to $Z_{act,s(t)}$ as compressibility (and composition) of the gas are assumed constant. The gas constant is given by $R_{g(t)} = R/MolW$ where $R = 8{,}314$ kJ/(mol·K) is the ideal gas constant.

The variable $M^{ext}_{(\delta,t)}$ represents the mass flow rate of natural gas (in kg/s) provided from external sources $\delta \in \Delta$. In this work other external sources are not considered, however in order to give a comprehensive description of this variable is included in the presentation of the model.

5.12 Objective function

Considering both cases of air and natural gas compressors, the objective is to minimise the electricity cost of the motors of the compressors, the start up and shut down costs, the cost when a compressors changes header and the procurement costs of gas/products from external sources. At the same time the constraints described in Sections 5.3 - 5.11 related to each case study have to be respected. The general form of the objective function is given by Eq. (5.61).

$$min \overbrace{\sum_{t\in T} C_{el(t)} \sum_{i\in I} \sum_{j\in J_{(i)}} d_c W_{el(i,j,t)}}^{\text{electricity cost}} + \overbrace{\sum_{t\in T} \sum_{i\in I} (C_{st(i)} S_{(i,t)} + C_{f(i)} F_{(i,t)})}^{\text{start up and shut down cost}} + \overbrace{\sum_{t\in T} \sum_{i\in I} C_D D_{(i,t)}}^{\text{change header cost}} + \overbrace{\sum_{t\in T} \sum_{e\in E} \sum_{\delta\in\Delta_{(j)}} C_{ext(\delta,e,t)} d_c M^{ext}_{(\delta,e,t)}}^{\text{purchase products/gas cost}} \tag{5.61}$$

The first term of the objective function represents the total electricity cost of the motors where the $C_{el(t)}$ is the electricity cost in EUR/kWh and the d_c is the factor to convert electrical power consumption to energy used over the time period. The electrical power consumption of the compressors $W_{(el)}$ in the objective function is estimated from either Eq. (5.7) or (5.10) in the case of the air compressors. In

the case of the natural gas compressors, the power consumption is given by Eq. (5.44). The binary variables $S_{(i,t)}$ and $F_{(i,t)}$ are equal to one if the compressor starts up or shuts down respectively. The start up cost is $C_{st(i)}$ and the shutdown cost is $C_{f(i)}$. The parameter C_D corresponds to the cost which occurs when a compressor changes header between period $t-1$ and t. The cost $C_{ext(\delta,e,t)}$ refers to the purchase price (in EUR/kg) of gas or product e from external sources $\delta \in \Delta$.

The formulated optimisation problem is a MILP model which minimises the objective function of Eq. (5.61) subject to constraints described in Sections 5.3 - 5.11 considering the specific case study, for example the case of an air compressor station in an air separation plant or an export gas compressor station. The optimisation problem has been implemented in the optimisation platform GAMS (McCarl et al., 2008). The optimisation determines the values of the optimisation variables which minimise the objective function. These values describe the best decisions the user of the plant should take to achieve optimal operation, the major decisions can be seen below:

- Which compressors to operate in period t: $X_{(i,t)}$
- Which header a compressor should connect with in period t: $Y_{(i,j,t)}$
- When to start up a compressor: $S_{(i,t)}$
- When to shut down a compressor: $F_{(i,t)}$
- What is the optimal distribution of load: $M_{in(i,j,t)}$
- What is the optimal purchase from external sources: $M^{ext}_{(\delta,e,t)}$

5.13 Maintenance constraints - Given maintenance schedule

The maintenance tasks of compressors are typically predefined before the optimisation of the production. The duration $\nu^{fxd}_{(i)}$ of a maintenance task is given. Two types of preventive maintenance tasks are considered in the basic operational model: (a) *maximum runtime-based*, (b) and *fixed*. The first type of maintenance is related to the maximum run time of the compressors. Fixed maintenance tasks takes place in a predefined time horizon, such that the starting and completion times of the maintenance tasks are known.

Maximum runtime-based maintenance tasks The performance of a compressor depends on its current condition. Since the performance of the compressor

deteriorates during its utilisation, there is often a time limit on its continuous operation. There is a maximum run time, $o_{(i)}$, after which the compressor should be switched off, and maintenance takes place to prevent mechanical damage and the energy-inefficient use of the compressor. The runtime-based maintenance policy can be considered as a simplified case of the condition-based maintenance policy (Li and Nilkitsaranont, 2009). For the sake of simplicity here, the duration of these maintenance tasks is considered to be equal to the minimum shutdown time. For every compressor, the maximum run time constraints are given by:

$$\sum_{t'=max\{1,t-o_{(i)}\}}^{t} X_{(i,t')} \leq o_{(i)}, \qquad \forall i \in I, t \in T \tag{5.62}$$

If there is a maximum time that compressors could remain idle (i.e., maximum shutdown time), similar types of constraints are derived.

Fixed maintenance tasks The starting times $\tau_{(i)}^{fxd}$ and the duration $\nu_{(i)}^{fxd}$ of the fixed maintenance tasks are known. For this reason, fixed maintenance tasks are modelled by setting to zero (from the starting to the completion of the maintenance task) the operating binary variables $X_{(i,t)}$ of compressors $i \in I^{dm} \subseteq I$ that are subject to fixed maintenance:

$$X_{(i,t)} = 0, \qquad \forall i \in I^{dm}, t = \tau_{(i)}^{fxd}, ..., (\tau_{(i)}^{fxd} + \nu_{(i)}^{fxd} - 1) \tag{5.63}$$

These constraints generally hold, since the maintenance duration is typically at least equal to the minimum shutdown time.

5.14 Initial state of the network

The initial state of the network is partially described by the initial inventory level of every storage tank $\tilde{v}_{tnk(e,z)}^{stor}$ and the initial operating status of every compressor $\tilde{\varphi}_{(i,j)}$ ($\rightarrow \tilde{\chi}_{(i)}$). Variables $\tilde{v}_{tnk(e,z)}^{stor}$ are found in constraints (5.57), and binary variables $\tilde{\varphi}_{(i,j)}$ are found in (5.1). Also needed is information on the startup and shutdown from the previous scheduling horizon. The following set of constraints allows past information regarding startup and shutdown to be carried over at the beginning of the current scheduling horizon:

$$X_{(i,t)} = 1, \qquad \forall\ i \in I, t = 1, ..., (\omega_{(i)} - \tilde{\omega}_{(i)}) : 0 < \tilde{\omega}_{(i)} < \omega_{(i)} \tag{5.64}$$

$$X_{(i,t)} = 0, \qquad \forall\ i \in I, t = 1, ..., (\psi_{(i)} - \tilde{\psi}_{(i)}) : 0 < \tilde{\psi}_{(i)} < \psi_{(i)} \tag{5.65}$$

Parameter $\tilde{\omega}_{(i)}$ denotes the total number of time periods at the end of the past scheduling horizon that compressor i has been operating since its last startup. Parameter $\tilde{\psi}_{(i)}$ denotes the total number of time periods at the end of the past scheduling horizon that compressor i has not been operating since its last shutdown. Figure 5.13 shows how the past startup information is captured from the above constraints.

In the case of a compressor i which has been in operation at the end of the previous scheduling horizon (i.e., $\tilde{\omega}_{(i)} > 1$), then the total run time from its last startup of the past horizon should be carried over to the current scheduling horizon so as to model successfully the maximum run time limits. Constraints (5.66) describe the initial state of the network in respect of the maximum run time.

$$\sum_{t'=max\{1,t-(o_{(i)}-\tilde{\omega}_{(i)})\}}^{t} X_{(i,t')} \leq (o_{(i)}-\tilde{\omega}_{(i)}), \qquad \forall i \in I, t \leq (o_{(i)}-\tilde{\omega}_{(i)}+1) : \tilde{\omega}_{(i)} > 1 \tag{5.66}$$

Regarding fixed maintenance tasks, the initial maintenance state (i.e., under maintenance or not) of every compressor should be carried over from the previous scheduling horizon. To consider this properly, for the compressors $i \in \tilde{I}^{dm}$ that are under maintenance at the beginning of the current scheduling horizon and their maintenance tasks had already started in the previous scheduling horizon, constraints (5.63) are modified as follows.

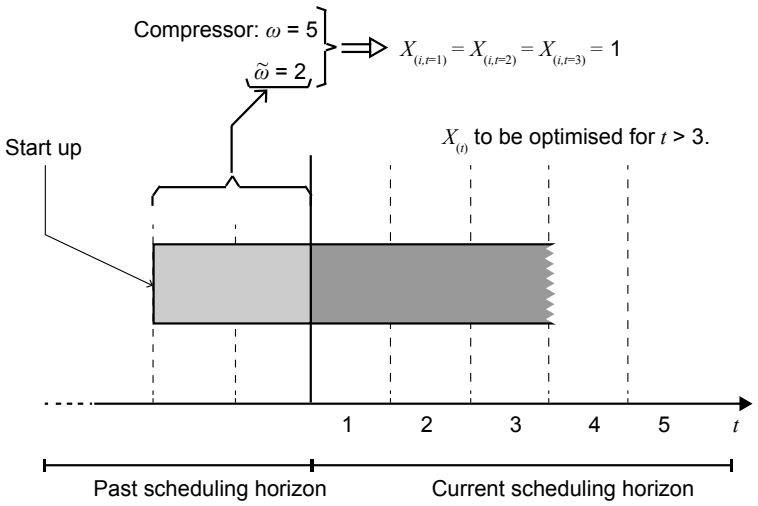


Figure 5.13: Carryover of past startup information to model minimum run time.

$$
\begin{aligned}
&\tilde{\eta}_{(i,t)} = 1, && \forall i \in (I^{dm} \setminus \tilde{I}^{dm}), t = \tau^{fxd}_{(i)}, ..., (\tau^{fxd}_{(i)} + \nu^{fxd}_{(i)} - 1) \\
&X_{(i,t)} = 1 - \tilde{\eta}_{(i,t)}, && \forall i \in (I^{dm} \setminus \tilde{I}^{dm}), t = \tau^{fxd}_{(i)}, ..., (\tau^{fxd}_{(i)} + \nu^{fxd}_{(i)} - 1) \\
&\tilde{\eta}_{(i,t)} = 1, && \forall i \in \tilde{I}^{dm}, t = 1, ..., (\nu^{fxd}_{(i)} - \tilde{\nu}^{fxd}_{(i)}) \\
&X_{(i,t)} = 1 - \tilde{\eta}_{(i,t)}, && \forall i \in \tilde{I}^{dm}, t = 1, ..., (\nu^{fxd}_{(i)} - \tilde{\nu}^{fxd}_{(i)})
\end{aligned}
\tag{5.63$'$}
$$

Parameter $\tilde{\nu}^{fxd}_{(i)}$ denotes the total time that compressor i has been under maintenance (since the start of the maintenance task) at the end of the previous scheduling horizon. In other words, $(\nu^{fxd}_{(i)} - \tilde{\nu}^{fxd}_{(i)})$ represents the remaining time that compressor i has to be under maintenance in the current scheduling horizon. Parameter $\tilde{\eta}_{(i,t)}$ denotes if compressor i is under maintenance in time period t. For $\tilde{\eta}_{(i,t)} = 1$, the corresponding $X_{(i,t)}$ becomes zero. Equation (5.63$'$) refers to the set of compressors $i \in (I^{dm} \setminus \tilde{I}^{dm})$ which defines these compressors which have to be under maintenance but their maintenance have not started before the beginning of the optimisation.

5.15 Terminal constraints

The optimisation determines a solution which minimises the total cost of the operation of the plant and the solution applies only within the boundaries of the optimisation problem. For instance, the solution for the optimal operation for one month may suggest to work the compressors as long as possible without maintaining them during the month to avoid the costs of shut down. However, the current optimisation does not consider the requirements for a following scheduling problem and it is possible to have an infeasible solution or a non-optimal solution compared to the case where the two consequent scheduling problems have been solved simultaneously.

Another example is the use of the storage tanks to reduce the production, therefore the power consumption. However, the tanks would be empty at the end of the scheduling horizon and the following optimisation problem would not have the flexibility to utilise the tanks at the beginning of the new time horizon.

A practical issue for using a relatively short time horizon of one month is that the uncertainty in the forecast of parameters, such as the production targets, increases as the the length of the horizon becomes greater.

For these reasons, the optimisation has to consider boundary conditions at the end of the time horizon. These boundary conditions can be called terminal constraints. These boundary conditions take into account the requirements of the following scheduling problem. Equation (5.67) ensures that the contents of

the tanks are full at the end of the time horizon, $t = T$, in the case of the air separation plant.

$$V^{stor}_{tnk(e,z,t)} = V^{max}_{tnk(e,z)}, \qquad \forall e \in E, z \in Z, t = T \tag{5.67}$$

The terminal constraints have to be investigated using heuristics from historical operation and literature. The decisions of these constraints are strongly related with the particular configuration of the compressors of the plant.

5.16 Equal split and equal surge margin operation

The most common industrial strategies to share the loads of compressors are the equal split and equal surge margin methods. The *equal split* method assumes that the load of the compressors is split evenly among the compressors, and the *equal surge margin* operation considers an equal distance between the operating points of all the compressors and their respective control surge lines. The control surge line or just control line is a limit on the right of the surge of a compressor and it is known.

These strategies are less sophisticated than the optimal distribution of the load this thesis is suggesting. The optimal distribution of the load is related to different operating points of the individual compressors. In addition, the equal split and equal surge margin strategies insert extra constraints into the optimisation model and this increases the computational burden as shown in Section 5.17.4.

The following equations will be implemented in the optimisation formulation of the natural gas compressors case study (the equations describing the equal split strategy of Section 5.16.1 could be used also for the formulation of the optimisation of the networks of compressors of the BASF case study). Section 5.16.2 presents a formulation of equal surge margin strategy among parallel compressors which require the use of compressor maps. The given compressor maps provide the descriptions of the control surge lines.

5.16.1 Constraints of the equal split strategy

A binary variable $\xi_{(i,i',j,t)}$ is defined for the equal split cases as follows:

$$\xi_{(i,i',j,t)} = \begin{cases} 1, & \text{if compressors } i \text{ and } i' \text{ are connected with header } j \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}$$

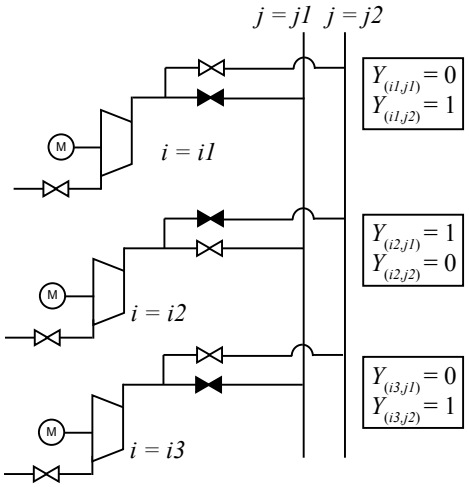


Figure 5.14: An example of three compressors and two headers.

The following inequalities, which represent logical operations, identify which compressors work at the same header:

$$\xi_{(i,i',j,t)} \geq Y_{(i,j,t)} + Y_{(i',j,t)} - 1,$$
$$\forall i \in I, j \in J_{(i)}, i' \in I : i+1 \leq i' \leq I, t \in T \tag{5.68}$$
$$\xi_{(i,i',j,t)} \leq Y_{(i,j,t)},$$
$$\forall i \in I, j \in J_{(i)}, i' \in I : i+1 \leq i' \leq I, t \in T \tag{5.69}$$
$$\xi_{(i,i',j,t)} \leq Y_{(i',j,t)},$$
$$\forall i \in I, j \in J_{(i)}, i' \in I : i+1 \leq i' \leq I, t \in T \tag{5.70}$$

The equal distribution of the load of the compressors which operate at the same header is given by the following constraints:

$$M_{out(i,j,t)} \leq M_{out(i',j,t)} + M^{es}(1 - \xi_{(i,i',j,t)}),$$
$$\forall\ i \in I,\ j \in J_{(i)},\ i' \in I : i+1 \leq i' \leq I, t \in T \tag{5.71}$$
$$M_{out(i,j,t)} \geq M_{out(i',j,t)} - M^{es}(1 - \xi_{(i,i',j,t)}),$$
$$\forall\ i \in I,\ j \in J_{(i)},\ i' \in I : i+1 \leq i' \leq I, t \in T \tag{5.72}$$

An example of the function of the constraints can be seen in Fig. 5.14 which shows the operation of three compressors connected with two headers. It is assumed that compressor $i1$ is working at header $j2$, compressor $i2$ at header $j1$ and compressor $i3$ is operating at header $j2$. The Eqs. (5.68) – (5.70) examine the pairs of compressors $i1$ and $i2$, compressors $i1$ and $i3$ and compressors $i2$ and $i3$ if they are attached to each header $j1$ and $j2$.

Therefore, for the first scenario of pair of compressors $i1$ and $i2$, and header $j1$, Eq. (5.68) can be written as $\xi_{(i1,i2,j1,t)} \geq 0+1-1=0 \rightarrow \xi_{(i1,i2,j1,t)} \geq 0$. Equations (5.69) and (5.70) give $\xi_{(i1,i2,j1,t)} \leq 0$ and $\xi_{(i1,i2,j1,t)} \leq 1$ respectively, thus the final value of $\xi_{(i1,i2,j1,t)}$ is zero. This is translated that compressors $i1$ and $i2$ are not operating at header $j1$ in period t. If the same procedure applies for the pair of compressors $i1$ and $i3$ and header $j2$ then the $\xi_{(i1,i3,j2,t)}$ is equal to one and this explains that compressors $i1$ and $i3$ are operating to the same header $j2$.

The values of the binary variables ξ are used by Eqs. (5.71) and (5.72). The latter equations estimate the outlet mass flow rate of each compressor. In the equal split strategy, if the compressors are attached to the same header then the mass flow rates have to be the same. The parameter M^{es} is estimated considering the values of individual mass flow rates of the compressors. It is important to note that the outlet mass flow rate of each compressor has a minimum due to the restriction of the minimum speed and control surge line.

5.16.2 Constraints of the equal surge margin

The constraints of the equal surge margin strategy are given by the following equations:

$$Q_{esm(i,j,t)} = H_{cor(i,j,t)}/b_{s1(i)} - b_{s2(i)},$$
$$\forall i \in I, j \in J_{(i)}, t \in T \tag{5.73}$$
$$Q_{cor(i,j,t)} \leq M^{esm} Y_{(i,j,t)},$$
$$\forall i \in I, j \in J_{(i)}, t \in T \tag{5.74}$$
$$Q_{cor(i,j,t)} \leq (Q_{esm(i,j,t)} + Q_{ym(j,t)}) + M^{esm}(1 - Y_{(i,j,t)}),$$
$$\forall i \in I, j \in J_{(i)}, t \in T \tag{5.75}$$
$$Q_{cor(i,j,t)} \leq (Q_{esm(i,j,t)} + Q_{ym(j,t)}) - M^{esm}(1 - Y_{(i,j,t)}),$$
$$\forall i \in I, j \in J_{(i)}, t \in T \tag{5.76}$$

Equation (5.73) is the equality form of Eq. (5.34) which now estimates the volumetric flow rate Q_{esm} which corresponds to the control line intersecting with H_{cor}. Figure 5.15 describes the operation of two dissimilar compressors, A and B, operating to the same header. The head H_{cor} is the same for both compressors and the surge margin Q_{ym} is estimated equal for both compressors in order to satisfy the demand of this header. However, the Q_{esm} and Q_{cor} of the two compressors are different. The optimisation model which examines the optimal operation of compressors with Equal Surge Margin estimates which compressors $i \in I$ to operate to each header j in every period t and the surge margin is estimated for each header j in each period t, i.e. $Q_{ym(j,t)}$, from Eqs. (5.74) – (5.76).

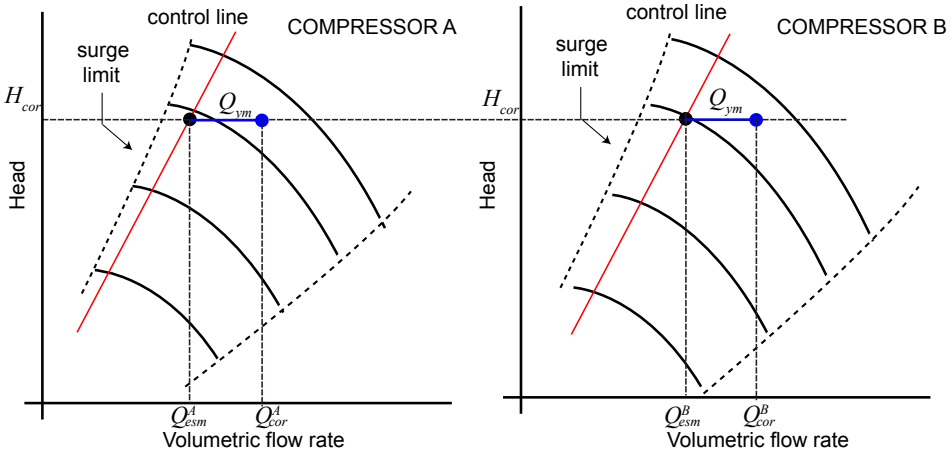


Figure 5.15: An example of two compressors working with equal surge margin.

5.17 Numerical application of the methodology

5.17.1 Description of the case study 1 (air separation plant)

Section 5.17 considers the industrial air separation plant of BASF in Ludwigshafen, Germany (Fig. 1.4). There are five small compressors with throttling valves, $i \in I^s = \{i1, i2, i3, i4, i5\}$, and six large compressors with IGVs, $i \in I^b = \{i6, i7, i8, i9, i10, i11\}$. There are three headers. The first header $j1$ collects the compressed air for utilities in the industrial complex of BASF. The other two headers $j2$ and $j3$ are connected with two air separation columns $u1$ and $u2$, respectively. Oxygen and compressed air are the products of the industrial plant. Table 5.1 gives the main operating data for the eleven compressors. These data have been derived from the analysis of historical data of the industrial air separation plant. The costs for startup and shutdown of the compressors are estimated from Nguyen et al. (2008). These costs of the large compressors $i6 - i11$ are approximately 60% higher than those of the small compressors $i1 - i5$.

Section 5.17 examines two examples, which are defined in Table 5.2, to demonstrate the optimisation framework for optimal operation of compressors. The first

Table 5.1: Normalised compressor operating bounds of outlet mass flow rates and pressure (%).

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$	$i11$
$M^{min}_{r.d.(i)}$	41.6	34.0	38.0	35.5	34.9	58.2	47.5	48.0	55.0	48.8	53.6
$M^{max}_{r.d.(i)}$	58.5	55.7	55.2	55.8	56.5	88.4	87.6	83.7	83.4	87.0	87.7
$\pi^{min}_{r.d.(i)}$	52.6	44.2	49.5	50.3	48.4	53.7	46.3	50.1	52.1	45.7	47.4
$\pi^{max}_{r.d.(i)}$	68.9	64.8	70.0	59.8	62.6	64.9	69.3	69.3	66.5	69.2	69.8

Table 5.2: Description of the examples of the air separation case study.

Example	Approach	Brief description
Example 1-A	Deterministic	Simplified example of the industrial case
Example 1-B	Deterministic	Industrial case with deterministic demand

example is a modified version of the industrial problem to highlight the main features of the proposed general optimisation framework. In comparison with the original industrial problem, this illustrative example, Example 1-A, involves: (a) a smaller number of compressors; (b) nitrogen as an additional product; (c) a storage tank for nitrogen; (d) the possibility for purchasing the air separation products from external sources; and (e) demand profiles with more fluctuations in comparison to the industrial data of BASF. The second example, Example 1-B, considers the industrial problem of BASF under deterministic demand profiles for products.

A total scheduling horizon of 30 days, divided into one-day periods, is studied in all examples. The optimisation model of the operation of the air separation plant involves the following constraints:

- Constraints of the minimum run and shutdown time, described by Eqs. (5.1) – (5.4).
- Constraints of the assignment of compressors to headers, described by Eq. (5.5).
- Constraints of the changes of compressors to headers, described by Eq. (5.6) in Section 5.5.
- Constraints which model the feasible window of operation of the compressors with the use of a rectangular regression domain, described by Eqs. (5.8) and (5.9).
- Constraints of the blow-off venting of the air, described in Section 5.8.2.
- Constraints which model the pressure of the headers with which the compressors are connected. Equation (5.50) provides the relationship of pressure and mass flow rate through a header and Eq. (5.51) is used to estimate the total mass flow rate in the headers.
- Constraints of the mass balances of the air separation units, given by Eqs. (5.52) – (5.58).
- Constraints of a given maintenance schedule including runtime-based maintenance constraints of Eq. (5.62), and given starting times and duration of maintenance tasks described by Eq. (5.63) or (5.63′)

- Constraints of the initial state of the compressors, described by Eqs. (5.64), (5.65) and (5.66).
- Constraints of the equal split strategy, described by Eqs. (5.68) – (5.72).
- Constraints of the equal surge margin strategy, described by Eqs. (5.73) – (5.76).

The objective function for the case study of the air separation plant is given by Eq. (5.61) in Section 5.12 with the power consumption of the motors of the compressors to be given by Eq. (5.7). Equation (5.7) is used in conjunction with the rectangular regression domain.

All given data and reported results are normalised and made dimensionless due to confidentiality reasons. All optimisation problems have been solved in GAMS/CPLEX 11.1, under default configurations, in an Intel(R) Core(TM) i7-2600 CPU @3.4 GHz with 8 GB RAM. A zero optimality gap (OPTCR) has been imposed in all problems instances.

The optimality gap represents the distance between the best integer solution, which satisfies all the integer requirements so far, and the best estimate, which is the bound of the best integer solution (McCarl et al., 2008). When CPLEX finds a solution where the OPTCR is below, for this instance, 0.5%, then the solver terminates the computational procedure and provides the solution to the user. Problem instances have been solved to zero optimality gaps, if not otherwise stated.

5.17.2 Example 1-A: Illustrative example of the air separation plant

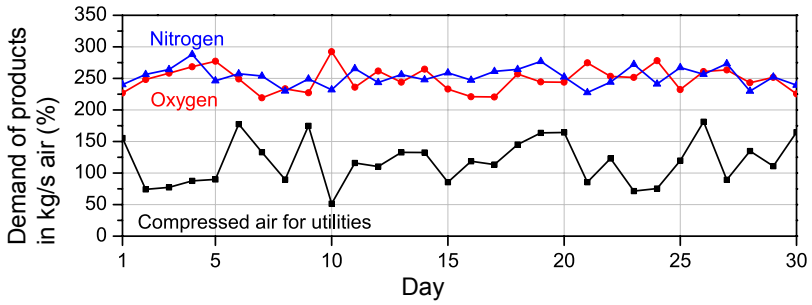


Figure 5.16: Example 1-A: Normalised demands for products.

This illustrative example considers a modified version of the industrial air separation plant described by Fig. 1.4. In this example, the air separation plant consists of eight compressors $I := \{i1 - i4, i8 - i11\}$. There is no initial configuration of the compressors as the history of the previous operation is not considered.

Table 5.3: Example 1-A: Main parameters.

Symbol	Value	Unit	Comment
t	1	d	Duration of each time period
T	30	d	Duration of time horizon
$\omega_{(i)}$	6	d	Minimum run time
$\psi_{(i)}$	3	d	Minimum shutdown time
$o_{(i)}$	15	d	Maximum run time for small compressors
$\nu_{(i)}$	4	d	Duration of maintenance
$\sigma_{(O_2)}$	0.21	-	Volumetric fraction of O_2 in air
$\sigma_{(N_2)}$	0.78	-	Volumetric fraction of N_2 in air
$C_{el(t)}$	0.0984	m.u./kWh	Electricity cost per kWh
$C_{D(i)}$	1,000	m.u./change	Cost for header-change
$C_{ext(O_2,t)}$	0.05	m.u./kg	Purchase price for kg of O_2
$C_{ext(N_2,t)}$	0.01	m.u./kg	Purchase price for kg of N_2

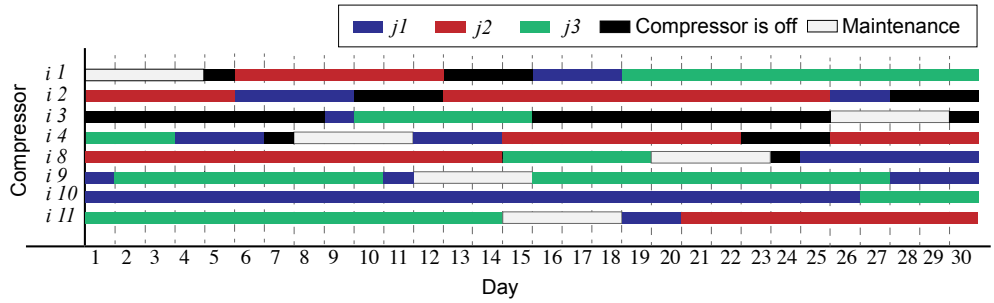
Table 5.3 provides the values of the main parameters of this example. A maximum run time $o_{(i)} = 15$ days for each small compressor is considered, while large compressors are not restricted regarding maximum run times. It is assumed that the demands for products are known at the beginning of the planning horizon, and they are deterministic. Figure 5.16 shows the demands per time period. The minimum storage level for each tank is 20% of its maximum capacity.

The outputs of the air separation columns are oxygen and nitrogen. It is assumed 100% conversion of the provided compressed air at the inlet of the air separation columns. Oxygen and nitrogen products can be either stored into their corresponding storage tanks or can be used directly to satisfy the demand. In addition, oxygen and nitrogen could be acquired from external sources under certain purchase prices. Purchase prices of products and electricity tariffs are assumed constant over time.

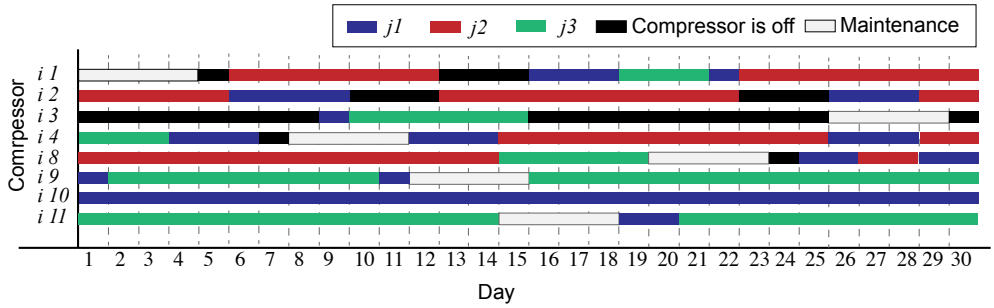
The illustrative example examines two different instances. The first instance is considered as baseline and is identified as Instance 1-A.1. It considers a fixed maintenance plan and a relatively high change header cost ($C_{D(i)} = 1,000$ m.u./change). The second instance (Instance 1-A.2) is a slightly modified version of Instance 1-A.1 with a lower change header cost ($C_{D(i)} = 250$ m.u./change).

Instance 1-A.1 In this problem instance, the maintenance tasks are predefined. According to this fixed maintenance plan, the maintenance tasks for compressors $i1$, $i3$, $i4$, $i8$, $i9$ and $i11$ have been pre-scheduled to start in Days 1, 26, 8, 20, 12 and 15, respectively.

Figure 5.17a displays the optimal compressors schedule (found by solving the proposed model) for the thirty-day time horizon considered. The Gantt chart of Figure 5.17a provides information regarding the active connections between



(a) Fixed maintenance and $C_{D(i)} = 1,000$ m.u./header-change for Instance 1-A.1.



(b) Fixed maintenance and $C_{D(i)} = 250$ m.u./header-change for Problem 1-A.2.

Figure 5.17: Problem 1-A: schedules for all problem instances.

compressors and headers as well as the operating status of the compressors. The vertical axis shows the status of each compressor i and the horizontal axis shows the time periods in Days (d). The blue, red and green colour of a bar per period indicates that an online compressor is connected with headers $j1$, $j2$ and $j3$ in this period respectively. The bars with black colour indicate that compressors are offline and the white bars indicate that compressors are under maintenance in the respective periods.

According to Figure 5.17a, two compressors are connected to each header at most times. However, three compressors work in parallel on days where the demand for products is high. For example, when the demand for oxygen is high at Day 10, one small and two large compressors ($i3$, $i9$ and $i11$) are connected to header $j3$ with compressed air. Moreover, three compressors work at header $j1$ when there is a peak in demand in compressed air for utilities at Days 6, 9 and 26. The results show that two large compressors ($i9$ and $i10$) work at header $j1$ at day 1, when there is also a peak in the demand. When the demand in compressed air for utilities is low (such as in Days 2, 3, 10, 23 and 24) only one large compressor ($i10$) is connected to header $j1$.

In addition, Figure 5.18 shows the load distribution for each compressor in each day. Small compressors are energy-efficient when they operate at maximum capacity, while large compressors are energy-efficient over a broader operational

area. For these reasons, the large compressors accommodate the demand fluctuations by adjusting their mass flow rates while small compressors tend to operate at maximum capacity.

Figure 5.19a presents the total compressed air supplied to each header for each time period. The mass flow rate of compressed air in header $j1$ is equal to the compressed air for utilities as shown in Figure 5.16. Figure 5.19b shows the compressed air load distribution proportion between the two air separation columns $u1$ and $u2$. As expected, column $u2$ produces more than column $u1$, due to the fact that column $u2$ has a higher production capacity. Finally, purchase of nitrogen is reported on Day 15. This is due to the fact that the demand for nitrogen cannot be satisfied because two large compressors are under maintenance on this day.

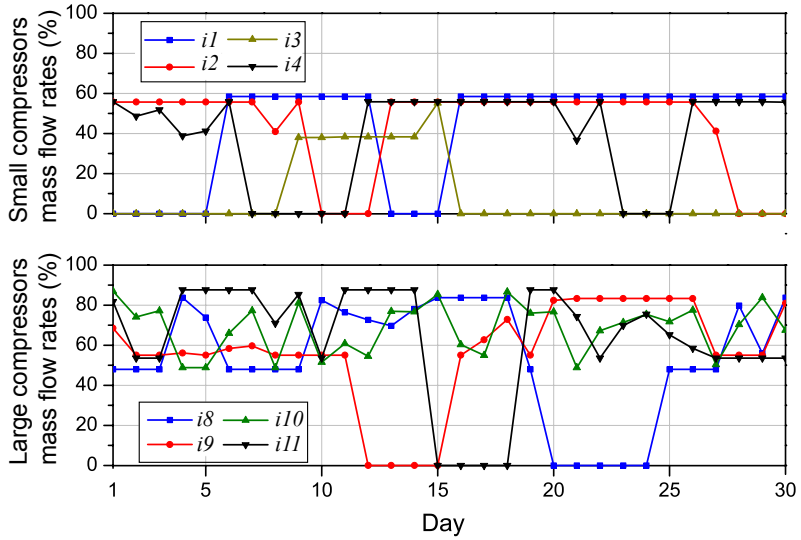


Figure 5.18: Optimal distribution of (normalised) load of compressors in Instance 1-A.1

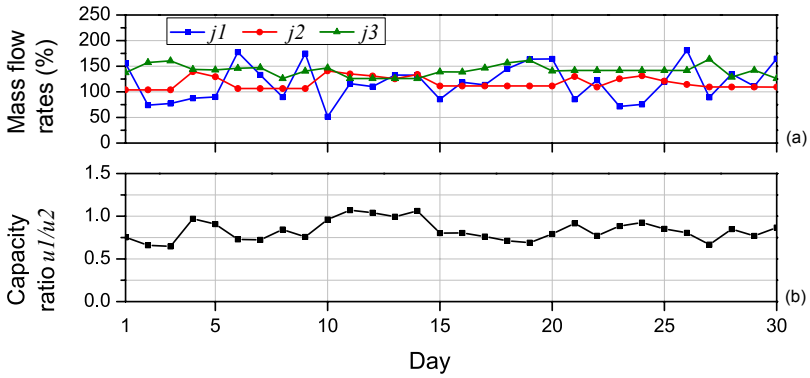


Figure 5.19: Instance 1-A.1: (a) total compressed air supplied to each header, and (b) production capacity ratio of column $u1$ and $u2$.

Instance 1-A.2 A slightly modified version of Instance 1-A.1 is considered with the aim to show how a variation in the cost for changing header affects the optimal schedule. Here, the cost for changing header is lower than that of Instance 1-A.1, $C_{D(i)} = 250$ m.u./change. All remaining data are the same.

Figure 5.17b shows the optimal schedule for the compressors. As expected, in the optimal schedule of this problem instance there are more header changes than in that of Instance 1-A.1. More specifically, there are 16 header changes in Instance 1-A.2 and 12 header changes in Instance 1-A.1. Due to the different associated costs for changing header, the total changing header cost in Instance 1-A.2 (4,000 m.u.) is lower that that of Instance 1-A.1 (12,000 m.u.). Nevertheless, according to Table 5.4, the total cost without considering the change header cost (shown in the Obj* column) did not change significantly between the two problem instances.

Moreover, the results show that the computational time decreases significantly, if the header change cost increases. This can be seen in Table 5.4 which shows the number of nodes of the branch-and-bound algorithm which is related to the CPU time. This trend is reasonable since high header change costs, could restrict the operational flexibility and as a consequence potentially decrease the solution search space. Similarly to Instance 1-A.1, there are purchases of nitrogen in Day 15, because the air separation plant cannot meet the demand for nitrogen since two large compressors are under maintenance on this day.

Table 5.4: Computational results for all problem instances of the illustrative Example 1-A.

Instances	Eqs.	Binary variables	Continuous variables	Nodes	CPU (s)	Obj (m.u.)	Obj* (m.u.)
1-A.1	8,574	1,770	2,580	9,742	293	90.98	86.64
1-A.2	8,574	1,770	2,580	321,355	4,903	87.67	86.51

5.17.3 Example 1-B: Industrial example

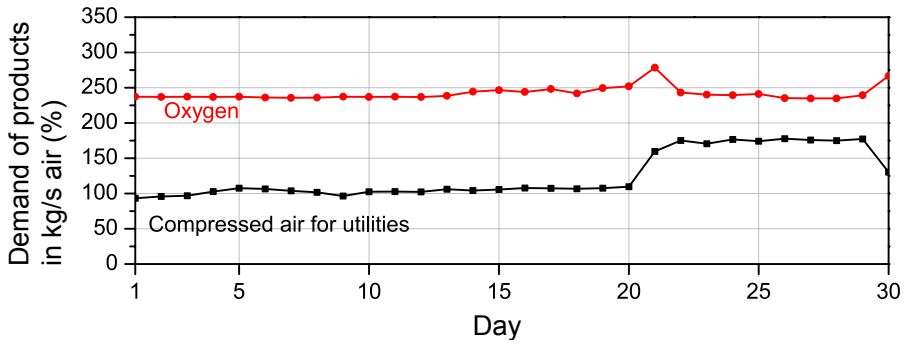


Figure 5.20: Normalised demand for products for Example 1-B.

Table 5.5: Example 1-B: initial condition (i.e. $t = 0$) for all compressors.

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$	$i11$
Header	-	$j1$	-	-	$j2$	$j2$	-	$j1$	-	$j3$	$j3$
$\tilde{\omega}_{(i)}$	0	6	0	0	25	22	0	10	0	36	40
$\tilde{\psi}_{(i)}$	6	0	18	2	0	0	30	0	29	0	0

Table 5.6: Information of maintenance tasks of the compressors for Example 1-B.

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$	$i11$
$\tau_{(i)}^{fxd}$	26	-	-	7	15	2	-	-	22	-	-
$\nu_{(i)}^{fxd}$	5	-	-	3	5	3	-	-	3	-	-

Here, the proposed optimisation framework is applied to a deterministic industrial case study of the air separation plant of BASF in Ludwigshafen, Germany. Oxygen and compressed air for utilities are the products, and their normalised demand profiles are given in Figure 5.20. The plant also produces nitrogen but the production targets of the plant are given based on the demand for oxygen. The current research study will show how to determine the schedule of the compressors.

The cost of changing header is equal to 250 m.u./change and purchases of oxygen are not allowed. Small and large compressors could operate for maximum 20 and 60 consecutive days respectively, apart from compressor $i5$ which can operate for a maximum of 50 consecutive days. In addition, large compressors have a minimum run time of six days while compressors $i1$, $i2$, $i3$, $i4$, $i5$ have minimum run times equal to five, six, seven, six and five days respectively.

All the related past data that fully describe the condition of the compressors at the beginning of the scheduling horizon (i.e. $t = 0$) can be found in Table 5.5 which provides information regarding: (a) the active compressor-to-header connection ($\tilde{\phi}_{(i,j)}$), (b) the total duration that each compressor has been operating from its last startup ($\tilde{\omega}_{(i)}$), and (c) the total duration that each compressor has not been operating since its last shutdown ($\tilde{\psi}_{(i)}$). For example, at the beginning of the scheduling horizon, compressor $i2$ is connected to header $j1$ and it has been operating for six time periods (not necessarily serving $j1$ in all of these periods) since its last startup. Also, at the beginning of the scheduling horizon, compressor $i7$ has not been operating for 30 time periods since its last shutdown.

Example 1-B examines the optimal schedule of the compressors considering fixed maintenance. Table 5.6 gives the start and duration of the constraints of the maintenance actions. The results from the optimisation are summarised as follows: 11,545 equations with 2,310 binary and 3,150 continuous variables. The problem solved after 139 s and the nodes were 1,704. The value of the minimised objective function achieved was 55.61 m.u.

Figure 5.21 displays the optimal schedule for Example 1-B. The figure shows that nine compressors are used to cover the demand over the 30 days, and compressors $i1$ and $i3$ do not operate in any time period. Moreover, there are no header changes in this example. It should be noted that the initial compressor conditions influence the obtained schedule. For instance, compressor $i10$, which has been operating continuously for 36 days before the beginning of the scheduling horizon, shuts down at the beginning of time period 25 because it reaches its maximum run time of 60 days.

Figure 5.22 shows the normalised load distribution for each compressor for every time period. Similarly to Example 1-A, small compressors usually operate close to (or exactly at) their maximum flow rate, while the large compressors change their flow rate over a larger range to cover the demand fluctuations. This result was expected because small compressors are actuated with inlet throttling valves while large compressors have inlet guide vanes. In practice, the throttling of the small compressors makes them inefficient at lower mass flows. This phenomenon was observed from the past operation of the plant which shows a similar operational strategy. Therefore, despite the fact that the regression models used do not explicitly assess the performance of each compressor, they captured implicitly the efficiency of the compressors. Indeed, the small compressors operate close to their maximum in the industrial air separation plant studied. In addition, a number of large compressors, such as compressors $i6$ and $i10$, tend to operate at a fixed mass flow rate while other large compressors with better efficiency in a larger range cover the demand variations.

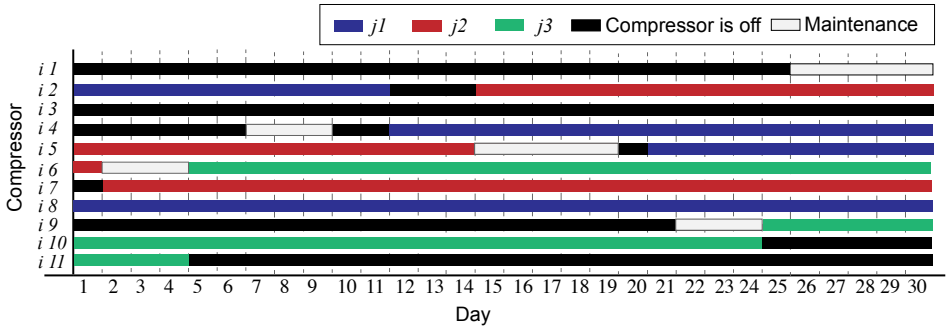


Figure 5.21: Optimal schedule for industrial Example 1-B.

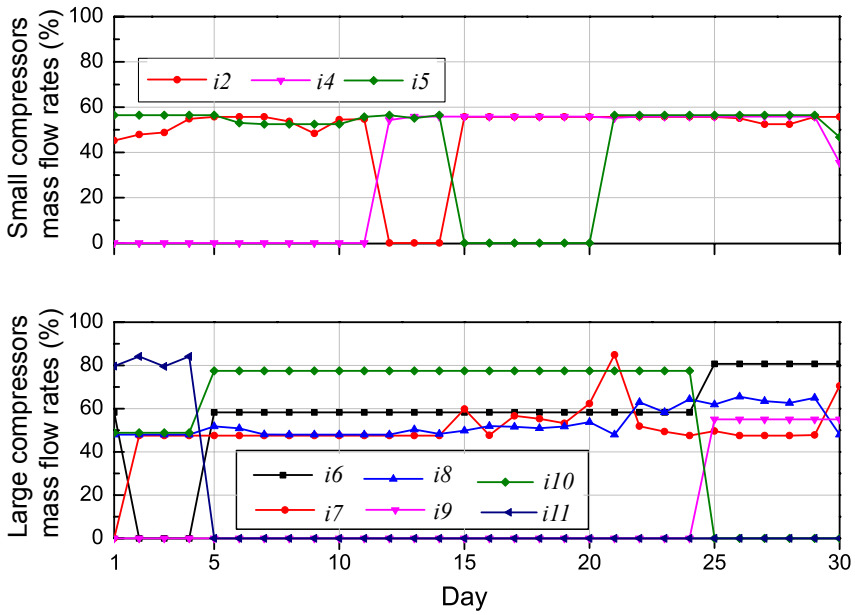


Figure 5.22: Normalised load of compressors for industrial Example 1-B.

5.17.4 Example 2: Industrial example of an export gas compressor station

Description of case study 2 (Statoil export gas compressor station)

The methodology of the multi-period operational model of parallel compressors is applied to an industrial example using data from Statoil. This example considers a deterministic optimisation problem with a time horizon of thirty days with time periods of one day each.

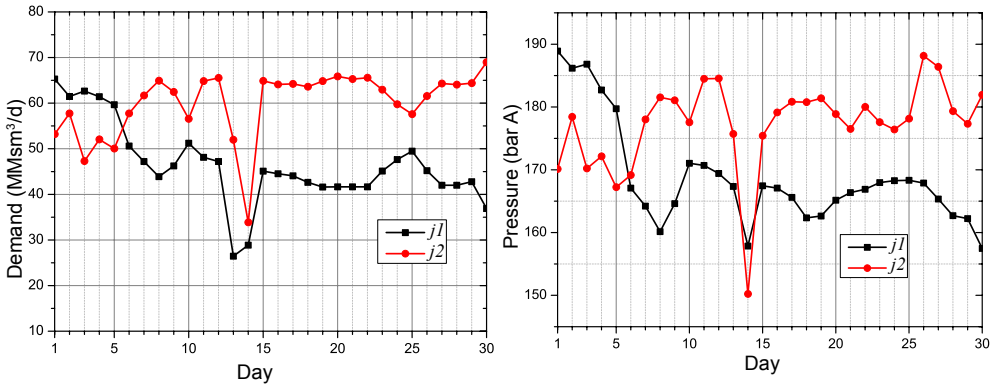
The industrial Example 2 considers Kollsnes export gas compressor station, shown in Fig. 5.12, with six multi-stage centrifugal compressors. The range of their nominal power is from 34 to 37 MW. The original case encompasses five small compressors $I^s = \{i1, i2, i3, i4, i5\}$ which have same characteristics and one large compressor $i6 \in I^b$ which has larger operating range and higher efficiency than the other five small compressors. It is noted that the efficiency of a compressor varies

Table 5.7: Initial state of the compressors.

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$
$\tilde{\phi}(i, j1)$	1	1	0	0	0	0
$\tilde{\phi}(i, j2)$	0	0	1	1	0	1
$\tilde{\omega_{(i)}}$	50	20	55	14	0	20
$\psi_{(i)}$	0	0	0	0	25	0
$\tau_{(i)}^{fxd}$	15	18	-	-	25	20
$\nu_{(i)}^{fxd}$	2	2	-	-	2	2

Table 5.8: Parameters of compressors.

	T_{ref}	P_{ref}	$MolW_{ref}$	$\omega_{(i)}$	$\psi_{(i)}$	$o_{(i)}$
I^s	273.15	75	17.3	5	3	65
I^b	299.65	76.7	18.1	9	3	75



(a) Given demand flow rate.

(b) Given pressure.

Figure 5.23: Input demand of flow rate and pressure of each delivery point for headers $j1$ and $j2$.

according to the volumetric flow rate and polytropic head for fixed conditions (i.e. inlet pressure and temperature, compressibility and molecular weight).

Table 5.7 gives the initial connection of a compressor i with headers $j1$ and $j2$, described by the binary parameter $\tilde{\phi}_{(i,j)}$. The parameter $\tilde{\omega}_{(i)}$ represents the continuous days of operation of the online compressors before the beginning of the optimisation. Parameter $\tilde{\psi}_{(i)}$ gives the continuous days of non-operation of the compressors which have been shut down before the beginning of the optimisation. Moreover, Table 5.7 provides the beginning $\tau_{(i)}$ and duration $\nu_{(i)}$ of the fixed maintenance of the compressors. Table 5.8 shows the values of the reference conditions of each set of compressors. It also provides the minimum running time, $\omega_{(i)}$, the minimum shut down time, $\psi_{(i)}$, and the maximum running time, $o_{(i)}$.

It is assumed that the temperature of the delivery point is constant, $T_{d(t)} = 315.15$ K. The composition and compressibility of the gas are assumed and given constant over time, with molecular weight $MolW_{act,s(t)} = 17.3$ and compressibility $Z_{act,s(t)} = 0.809$. The polytropic exponent in Eq. (5.46) is estimated from the compressor maps $\phi_{(i)} = 0.35$ and the electricity price is assumed fixed for the purposes of the scheduling problem with $C_{el(t)} = 0.098$ EUR/kWh (European Commission, Eurostat, 2015).

Moreover, the start up cost is estimated equal to the energy spent to operate a compressor for ten hours at its nominal rate is based on the experience of the industry and analysis of historical data. The shut down cost was set equal to the

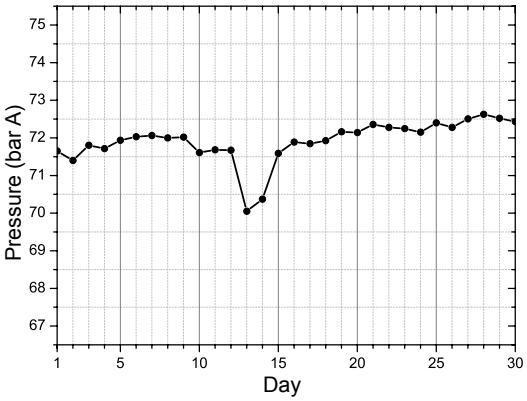


Figure 5.24: Upstream inlet pressure.

half of the start-up cost. The inlet temperature of the gas is assumed constant over time for different compressors. The temperature for the compressors $i \in I^s$ is 280 K and the temperature for compressors $i \in I^b$ is 294 K.

Historical industrial data from past years are collected for formulating a case study. Data from one summer month provided the demand in mass flow rate (Fig. 5.23a) and in pressure (Fig. 5.23b). The inlet pressure of the gas compressor station is also given (Fig. 5.24) for this month.

The developed mathematical models which are applied to the Statoil export gas compressor station involve the following constraints:

- Constraints of the minimum run and shutdown time, described by Eqs. (5.1) – (5.4).
- Constraints of the assignment of compressors to headers, described by Eq. (5.5).
- Constraints of the changes of compressors to headers, described by Eq. (5.6) in Section 5.5.
- Constraints which model the power consumption and feasible window of operation with the use of compressor maps. These constraints are described by Eqs. (5.23) – (5.46).
- Constraints of the model of the recycle of the flow, described by Eqs. (5.47) – (5.49).
- General constraints of the operation of an export gas compressor station given by Eqs. (5.59), (5.60).
- Constraints of a given maintenance schedule including runtime-based maintenance constraints of Eq. (5.62), and given starting times and duration of maintenance tasks described by Eq. (5.63).

- Constraints of the initial state of the compressors, described by Eqs. (5.64), (5.65) and (5.66).
- Constraints of the equal split strategy, described by Eqs. (5.68) – (5.72).
- Constraints of the equal surge margin strategy, described by Eqs. (5.73) – (5.76).

The objective function for the case study of the natural gas export station is given by Eq. (5.61) in Section 5.12. Compressor maps are used to estimate the power consumption through Eqs. (5.23), (5.26) and (5.44). As previously mentioned, the motor and gearbox losses are not considered, thus $W_{el} = W_{act}$. The constraints, objective function and the given input data were implemented into the optimisation platform GAMS using the MILP solver CPLEX 1.1 in the same system as in Example 1 with Intel(R) Core(TM) i7-2600 CPU @3.4GHz with 8GB RAM.

Twelve different optimisation problems have been developed to illustrate the use of the mathematical model for optimal operation of export gas compressors. Table 5.9 presents the description of the scheduling problems of which each considers (a) different method for sharing the load, and (b) different number of large compressors. The real configuration of the plant includes only one large compressor installed. Problems 6 – 8 and 10 – 12 consider the hypothetical scenario of the use of three large compressors. The reason to examine this case is that Statoil considers to purchase large compressors in the next years. Finally, the twelve problems examine different predefined values of the OPTCR criterion.

Therefore, the *Optimal* method for optimally sharing the load among the compressors considers the operating set points of the compressors as decision variables of the optimisation problem. The set points of the compressors are not correlated through any constraint and they are free to take any value within their feasible windows of operation. This method is proposed in this current research study and it is compared with typical industrial policies such as equal split and equal surge margin. The optimal method is part of the suggested optimisation framework of this PhD thesis. The *Equal Split* (ES) method refers to a typical industrial method which shares the total load evenly among the compressors and it was mathematically described in Section 5.16.1. An other common industrial policy to distribute the total load is the one which refers to the operation with equal distance (margin) between operation and surge of every compressor connecting to the same header, namely *Equal Surge Margin* (ESM) method.

Table 5.9: Description of twelve problems for optimal operation of compressors with different specifications, for example different methods for sharing load, i.e. Optimal, Equal Split (ES), Equal Surge Margin (ESM), and use of various number of large compressors.

Problem	Method for sharing the load	# of large compressors	OPTCR (%)
1	Optimal	One	0.5
2	Optimal	One	0.0
3	ES	One	0.5
4	ESM	One	0.5
5	Optimal	Two	0.5
6	Optimal	Three	0.5
7	ES	Three	0.5 **(1.16)**
8	ESM	Three	0.5 **(1.06)**
9	Optimal	Two	0.0
10	Optimal	Three	0.0 **(0.14)**
11	ES (given Y)	Three	0.0
12	ESM (given Y)	Three	0.0

Results and discussions of Example 2

Industrial example of Statoil with different methods for sharing the load
The Problems 1, 3 and 4 examine the optimal scheduling of compressors employing Optimal, Equal Split and Equal Surge Margin methods with one large compressor (compressor $i6$) and the optimality gap OPTCR is set to 0.5%.

Figure 5.25 shows that the optimisation problem with Optimal method (Problem 1) is an easier computationally problem to solve compared to the other two problems of Equal Split (Problem 3) and Equal Surge Margin (Problem 4). Indeed, the Equal Split problem includes 900 more binary variables than the Optimal and Equal Surge Margin problems because of the consideration of Eqs. (5.68) – (5.72). The Equal Surge Margin problem involves 420 more continuous variables than both the Optimal and Equal Split problems as it considers Eqs. (5.73) – (5.76).

The results in Fig. 5.26 show that the solution of the scheduling Problem 1, which considers the Optimal method to distribute the total load, achieves decreased total costs by 0.36% and 0.38% compared to total costs from the scheduling Problem 3 with Equal Split method and scheduling Problem 4 with Equal Surge Margin method respectively. The differences between the Optimal and the latter methods are not significantly great because the real industrial case includes only one dissimilar compressor compared to the other five similar compressors of the compressor station. The current case study considered the same compressor map for modelling the five similar compressors. In reality, it is expected all the compressors to be dissimilar to some extent as has been shown in Chapter 4.

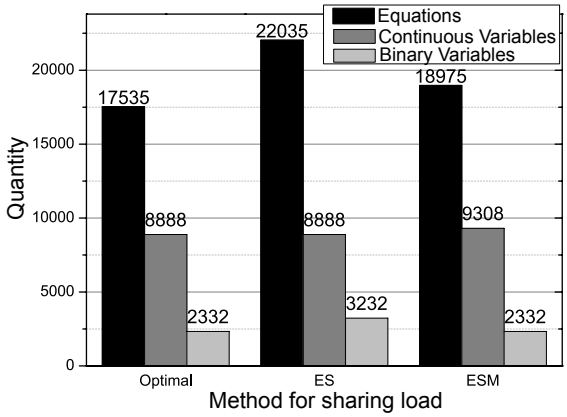


Figure 5.25: Specifications of the model of the scheduling with three different methods for sharing the total load: Optimal, Equal Split (ES) and Equal Surge Margin (ESM).

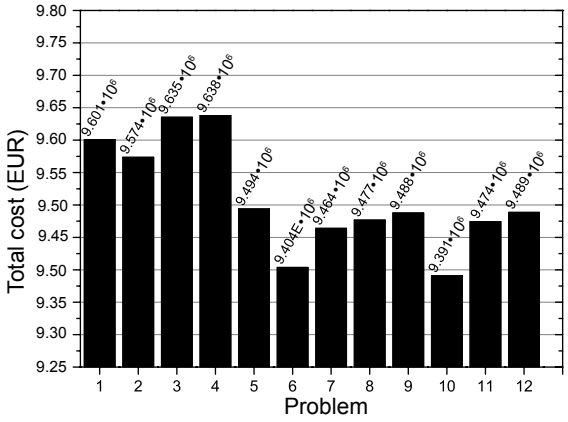
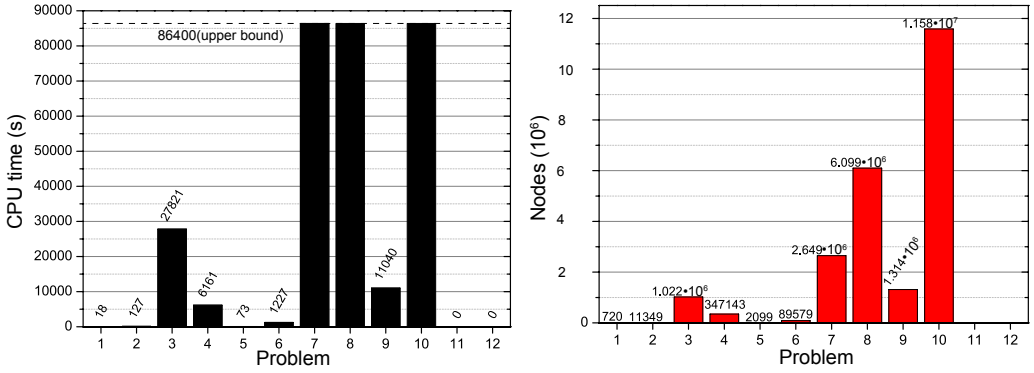


Figure 5.26: Total costs in EUR for thirty days optimisation for the twelve optimisation problems described in Table 5.9.

The computational time for solving the scheduling problem with Equal Split method is approximately 1534 times greater than the time needed to solve the Optimal scheduling problem which computational time is only approximately 18 s as it is shown in Fig. 5.27a. The scheduling problem with the Equal Split method has a large number of binary variables and this increases significantly the computational time compared to the scheduling problem with the Equal Surge Margin method with an increased number of continuous variables. Figure 5.27a shows these computational times. The number of nodes of the branch-and-bound tree (Section 2.5.2) of each optimisation problem can be seen in Fig. 5.27b.

The Problem 2 of Example 2 examines the scheduling with Optimal method with predefined value of OPTCR equal to 0.00%. The solution of this problem provides a reduced total cost by 0.28% compared to the total costs of Problem 1



(a) CPU time

(b) Nodes

Figure 5.27: Example 2: computational time and nodes of the GAMS solutions.

with OPTCR value equal to 0.5%. However, the computational time of Problem 2 is 127 s and relatively larger than the computational time of Problem 1 which is 18s.

A conclusion from the comparison between Problem 1, and Problems 3 and 4 is that the scheduling problem with the Optimal method provides a better solution compared to the typical industrial policies which can lead to significant reductions in operational costs. Most importantly and in addition to the previous conclusion, the scheduling problem which considers Optimal method is computationally less expensive than the scheduling with Equal Surge Margin and Equal Split methods which consider a greater number of binary and continuous variables.

Moreover, the scheduling versions with the equal split and equal surge methods are the better versions of the current applied industrial policies of operation for two reasons. First, the operation with Equal Split (ES) method is influenced by the simultaneous optimisation of the selection of the compressors, and equal distribution of total load among the dissimilar compressors. On the contrary, in the industry this approach which considers simultaneously selection of compressors and distribution of load is not established yet, thus the equal distribution of load is applied to a selection of compressors based on the experience of the operators. Second, in the case of the Equal Surge Margin scheduling, the surge margin is optimised for every time period t. However, the current industrial strategy is to estimate the surge margin according to a given selection of compressors. Hence, it is expected the scheduling with Optimal solution to significantly improve the real current operation under the consideration of the assumption taken into account.

Different scenarios with more large compressors An extension of the industrial case study is the examination of the economic benefits from the use of more large compressors in the current configuration. In practice, the benefits from

the optimal distribution of load against typical load sharing strategies stem from the fact that the compressors which operate in parallel are dissimilar. In an ideal case in which all the compressors have the same specifications and conditions (i.e. performance), then there would not be any significant improvement if the optimal method is applied instead of the typical load sharing strategies. In practice a compressor station involves dissimilar compressors due to uneven operation and uneven maintenance of the compressors.

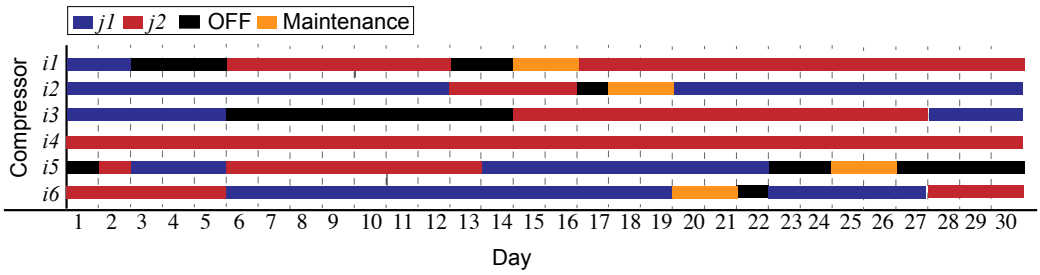
According to Table 5.9, Problems 5 and 6 examine the scheduling of compressors with the following characteristics: (a) use of Optimal load sharing method, (b) use of two and three large compressors with the same specifications as compressors $i6$ and (c) the value of the OPTCR is predefined to 0.5%. Problems 9 and 10 examine similar cases to Problems 5 and 6 with two and three compressors respectively, and the only difference is that the value of OPTCR is set to 0.0%. Therefore, the sets of compressors with two large compressors for Problems 5 and 9 would be $I^s = \{i1, i2, i3, i4\}$ and $I^b = \{i5', i6\}$, and the sets of compressors for Problems 6 and 10 are $I^s = \{i1, i2, i3\}$ and $I^b = \{i4', i5', i6\}$. Compressors $i4'$, $i5'$ are assumed exactly the same as compressor $i6$.

Figure 5.26 shows that Problem 5 with two compressors achieves decreased total operational costs compared to the Problem 1 with one large compressor by 1.11%. Problem 6 with three large compressors achieves decreased total operational costs compared to the Problem 1 by 2.05%. These calculations considered OPTCR equal to 0.5%.

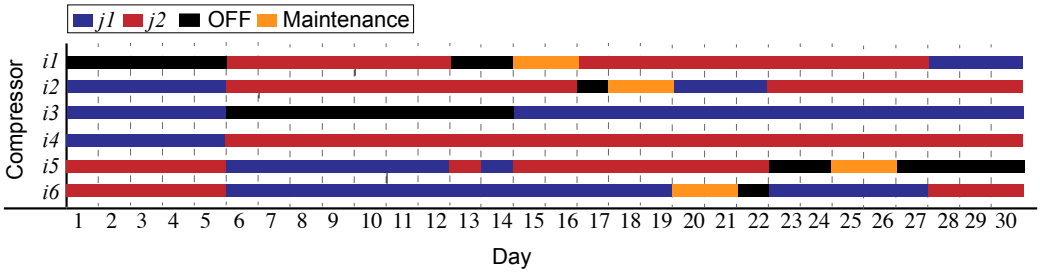
The comparison between Problems with one (i.e. Problem 2) , two (i.e. Problem 9) and three large compressors (Problem 10) with OPTCR = 0.0% shows similar results. The Problem 9 achieves decreased total operational costs compared to Problem 2 by 0.90% and the difference between Problem 10 with three large compressors and Problem 2 is 1.91%. Indeed, in this representative industrial example, the reduced operational costs for one month amounting to 88,857 EUR if two large compressors are used, and to 182,707 EUR if three large compressors are employed instead of only one large compressor in both cases. It is important to notice that again the set with large compressors considers compressors which have the same specifications.

From a computational point of view, the more large compressors are considered in the optimisation problem, the longer it takes to find the optimal solution. Problem 10 reached the predefined time criterion of 24h (equal to 86,400 s) as can be seen in Fig. 5.27a and the number of nodes of the computational tree are significantly greater than those of Problem 9. Furthermore, the solution of Problem 9 has a larger number of nodes than the solution of Problem 2. Problems 2 (with one large compressor) and Problem 9 (with two large compressors) reached a solution

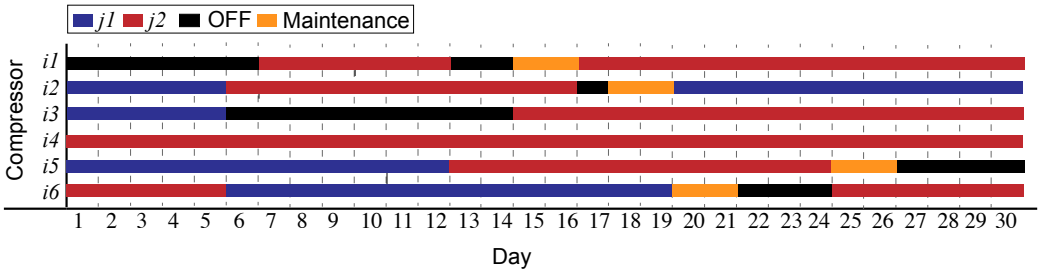
with OPTCR = 0.0%, however the Problem 10 reached a solution with OPTCR = 0.14%. The values of the OPTCR of the actual solutions of the optimisation problems can be seen in bold font and in brackets next to the predefined values of the OPTCR (in plain font) in Table 5.9. If the final OPTCR values in bold font are omitted, this means that the optimisation problems have been solved at the predefined OPTCR values in plain font.



(a) Problem 2: Scheduling with one large compressor $i6$.



(b) Problem 9: Scheduling with two large compressors $i5'$ and $i6$.



(c) Problem 10: Scheduling with three large compressors $i4'$, $i5'$ and $i6$.

Figure 5.28: Example 2: schedules for Problems 2, 9 and 10.

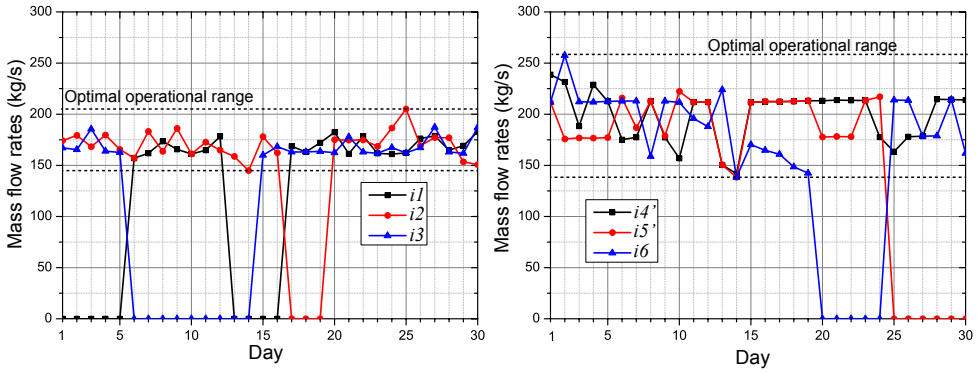
Table 5.10: Total costs and distribution of costs in EUR for Problem 2 (one large compressor), Problem 9 (two large compressors) and Problem 10 (three large compressors).

Problem	Total	Change header	Start up	Shut down	Electricity
2	9,573,620	8,000	203,688	101,844	9,260,088
9	9,487,763	12,000	206,640	103,320	9,165,803
10	9,390,916	5,000	206,640	103,320	9,075,956

Figure 5.28 provides the Gantt Chart of the scheduling of each problem with one, two and three large compressors respectively. The first observation, is that the number of the large compressors under consideration influences the number of changes, and the number of startups and shutdowns of the compressors in each problem according to Fig. 5.28 and Table 5.10. The problem with the greatest number of large compressors, in Fig. 5.28c, shows that the operation involves fewer changes and each compressor operates to one header for longer periods than in the other two cases. It can be observed that in the first two problems there are instances in which compressors operated to a head only for one time period, and after the end of this period they changed connection, for instance compressor $i5$ on Day 2 for Problem 2. Compressor $i5$ on Days 13 and 14 for Problem 9 is another example. The initial condition influences the scheduling for all instances. For instance, compressor $i3$ shuts down on Day 6 as it is close to reach its maximum allowed continuous operational days and compressor $i4$ remains online mostly for all time periods in all cases as it is the one which has been operated less than the other compressors before the start of the optimisation.

In order to illustrate the benefits of the scheduling with the Optimal load sharing approach compared to the Equal Split and Equal Surge Margin approaches when three large compressors are installed, Problem 7 and Problem 8 dealt with the Equal Split and Equal Surge Margin load sharing methods respectively. However, both problems were computationally difficult to solve and the optimal solutions found were these with OPTCR = 1.16% and 1.06% respectively after reaching the maximum computational time limit. Figure 5.26 shows that the total costs from Problems 7 (Equal Split) and 8 (Equal Surge Margin) are greater than the costs of Problem 6 (Optimal), however, there is uncertainty in their relative respective differences because Problems 7 and 8 have been solved for different OPTCR values.

The impact of different sharing load methods on the total costs of scheduling The previous paragraph showed that the consideration of Equal Split and Equal Surge Margin methods increases the complexity of the optimisation problems and the optimal solution of the corresponding scheduling problems with three large compressors cannot be estimated at 0% OPTCR. For this reason, this paragraph will examine the differences between scheduling problems with Optimal and the other load typical sharing (i.e. Equal Split and Equal Surge Margin) methods when the selection of the compressors is given in the case of the scheduling with typical load sharing methods. The consideration of a given selection of compressors results in the simplification of the optimisation models as they will not involve binary variables. The current industrial policy does not provide a simultaneous solution of best selection of compressors and optimal distribution



(a) Small compressors (b) Large compressors

Figure 5.29: Example 2: mass flow rates for Problem 10.

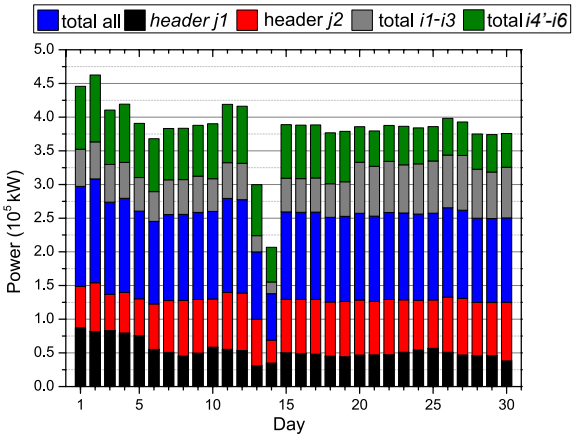
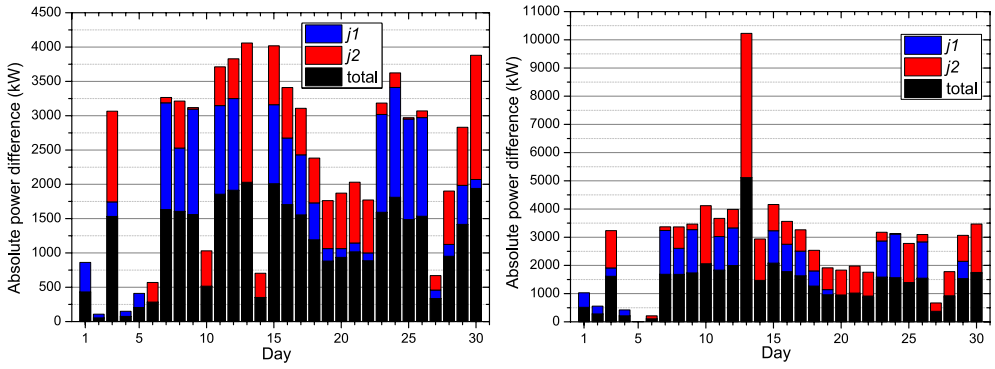


Figure 5.30: Power consumption of the station (total all), of small compressors (total $i1-i3$), of large compressors (total $i4'-i6$) and power consumption per header $j1$ and header $j2$ per each time period.

of load. Hence the analysis in this paragraph will help to identify the benefits of the scheduling of compressors with Optimal method compared to the typical industrial methods.

Problems 11 and 12 consider the solution of the scheduling with Equal Split and Equal Surge Margin method for sharing the load among the compressors with a given selection of compressors for every day of the thirty day optimisation problem described in the beginning in Section 5.17.4. Because there not any binary variables included in the optimisation, then the computational time to solve both problems is approximately 0.28s and therefore there are no nodes shown in Fig. 5.27b for Problems 11 and 12. The optimal solution of Problem 10 with Optimal method for sharing load achieves 0.88% and 1.03% reduced operational costs compared to Equal Split and Equal Surge Margin methods.

Figure 5.29 illustrates the mass flow rates of small and large compressors per



(a) ESM against Optimal (b) ES against Optimal

Figure 5.31: Power consumption absolute differences (in kW) between scheduling with Optimal and other sharing methods (ESM/ES) per each day.

time period for 30 days. As can be observed, there is not any pattern of operation related to the flows of the compressors. Nevertheless, the operational range of the small compressors is more narrow than that of the large compressors. Moreover, Fig. 5.30 displays the power consumption of the compressor station, of each header and the groups of small and large compressors. From Day 20 and onwards the power consumption of small compressors become greater than that of the large compressors. This is because two out of three large compressors have to be maintained according to the given maintenance plan.

Figure 5.31 illustrates the absolute difference in power in kW between the scheduling with Optimal method and the scheduling with Equal Surge Margin (Fig. 5.31a) or Equal Split (Fig. 5.31a). The two latter methods are given the selection of the compressors. In the first comparison the difference in power can range from 54 kW up to more than 2 MW. Similarly in the second comparison with the scheduling with Optimal and Equal Split method the difference can reach the value of 5 MW. However, the total power difference for 30 days is greater in the case of Optimal against Equal Surge Margin as mentioned previously. Nevertheless, this example shows the importance to consider the selection and distribution of the compressors simultaneously.

5.18 Summary of the chapter

A general mathematical framework for the optimisation of the operational tasks of multiple parallel compressors has been presented. The outputs of the framework are the best decisions of the operation which result in reduced total costs. The framework can be used for the optimisation of the operation of compressor stations of systems with large energy consumption. The air compressors of an air separation

plant and the export natural gas compressor station of a gas network are two examples of these systems.

The mathematical formulation of the suggested framework considered a MILP multi-period approach with uniform discretised time periods. This mathematical framework includes the modelling of compressors with the use of process data or with the use of compressors maps. The mathematical framework with the former modelling approach was applied to the industrial case of the air separation plant and the latter modelling approach was applied to the export gas compressor station.

The operative set points, startup, shutdown and change-header costs of the compressors were determined for achieving optimal operation. Both continuous and binary variables were used to model the operational constraints of the compressors and the overall plant. Initial state of the system and terminal constraints were defined and maintenance actions were given as input along with demands and other parameters such as values of the costs. The resulting MILP model was implemented in GAMS and the CPLEX solver was employed to find the optimal solution.

The application of the framework to the case study of the air separation plant demonstrated the outputs of the MILP model (e.g Gantt chart, set points of mass flow rates) and validated the model by comparing the results of the scheduling with industrial practices. An industrial example based on historical data showed that the optimal schedule can provide insights and generic trends for improving the operation. For instance, the small compressors should work at maximum capacity and the large compressors to satisfy the remaining demand. Another example is that compressors should operate with the minimum number of changes among headers for the considered time period of optimisation.

The application of the framework to the case study of the export gas station studied the following cases with the respective remarks:

- *Comparison of scheduling cases with different load sharing methods for the current industrial Statoil case (one large compressor under consideration)*: The computational results showed that the optimisation problem with scheduling with Equal Split and Equal Surge Margin methods are computationally more demanding to solve compared to the scheduling with Optimal method. Nevertheless, the scheduling with Optimal method achieves slightly better results for the total costs of the operation. It was also observed that small changes in the values of OPTCR affects the solution significantly.

- *Investigation and quantification of the operational and economic impact of different scenarios with more large compressors under consideration*: The

more large compressors are used, the greater is the improvement in the operation. The use of two and three large compressors can reduce the operational costs by approximately 1% and 2% respectively. However, the computational time to solve the optimisation problem with more than one large compressor increases with the number of the large compressors under consideration. The optimisation problems with Equal Split and Equal Surge Margin sharing methods reached solutions with OPTCR = 1.16% and 1.06% respectively after 24 h computational time with the predefined OPTCR criterion was set to 0.0% and the predefined time termination criterion was set to 24h.

- *Examination of the impact of different sharing load methods on the total costs of scheduling*: the configuration of the compressors and connections of compressors with headers are determined from the solution of the scheduling problem with Optimal load sharing method. An example showed that the latter scheduling problem achieves 0.88% and 1.03% reduced operational costs compared to the cases with Equal Split and Equal Surge Margin methods respectively. The two latter cases used fixed configuration of compressors and connections of compressors and headers from the solution of the former scheduling problem.

6 Integration of optimal operation and maintenance

6.1 Description of the chapter

The maintenance of centrifugal compressors is an important topic for the industry as the maintenance cost accounts for 6% of the total cost of the life cycle of a compressor. Chapter 2 reported that the specifications of the maintenance of centrifugal compressors, for example period of a cycle and cost of maintenance service, differ compared to these of other types of compressors. Centrifugal compressors have longer maintenance cycles but a maintenance repair lasts longer than other types of compressors. Moreover, centrifugal compressors deal with the problem of fouling which causes erosion and corrosion. These two phenomena cause a decrease in performance and therefore an increase in power consumption.

The aim of the maintenance of compressors is to reduce the number of unexpected breakdowns and failures. A failure of a compressor may affect other process units which are connected with the compressor, and thus this failure can result in downtime of the plant and lost production. The maintenance includes tasks such as replacement of components (for instance bearings and blades of impeller), inspection of mechanical and electrical parts, and cleaning of the surfaces of the inside of the compressor.

Maintenance and operation seem to be highly dependent on each other. The purpose of Chapter 6 is to develop methods for integrating maintenance and operation, and prove this dependance and quantify it. Therefore, the integration of operation and maintenance will be presented by mathematical models which deal with different cases of maintenance of compressors, for example minor corrective maintenance actions, major overhauls and different washing methods.

Section 6.2 presents a basic maintenance model which considers the start times of the maintenance events as degrees of freedom in the optimisation model. Indeed, the estimation of the start times of the maintenance events is restricted by a given time window defined by the earliest and latest start time of the maintenance. In addition, the compressors have maximum running time constraints, as a maximum runtime-based maintenance model is also taken into account (this model is presented in Section 5.13).

The basic model can be adapted in a rolling time horizon framework which is based on the principles of a model predictive control tool, but for scheduling purposes. The time horizon of the multi-period optimisation problem is shifting by a number of periods (one period will be used) and the parameters are updated in every shift. Section 6.3 presents the algorithm of this rolling horizon approach and gives the results from its application to the case of the air separation plant.

Section 6.4 presents the extension of the basic integration model considering longer time periods and a larger set of maintenance constraints. This extended model deals with the simultaneous optimisation of operation and maintenance including major overhauls. This model also considers aspects of the maintenance of compressors, for example different groups for mechanical and electrical repairs and inspections, and restrictions for the maximum number of compressors that can be repaired at the same period. These restrictions were mentioned in the study by Castro et al. (2014).

Section 6.5 presents a model for condition-based maintenance for the optimisation of the washing schedules of industrial centrifugal compressors. The assumptions of the model are presented in detail and the model is applied to the air compressors of the air separation plant of case study one. The comparison between online, offline washing and their combination is presented. The framework also examines scenarios of compressors with different degradation rates and scenarios with different numbers of available compressors to operate.

Chapter 6 involves research which has been published in the following conferences and journals:

- Results from the basic model of the integrated operation and maintenance of Sections 6.2 and 6.3 have been published in the paper 'Operational planning of a network of compressors: The air separation plant case' (Kopanos et al., 2015) in the journal *Applied Energy*.

- The research in Section 6.4 initially was presented in the conference *Turbo Expo 2015* with title 'Optimal operation and maintenance of gas compressor stations: An integrated framework applied to a large-scale industrial case' (Xenos et al., 2015b). The conference paper was recommended for journal by the reviewers of the conference and therefore the original work with improvements was published in the *Journal of Engineering for Gas Turbines and Power* (Xenos et al., 2015c).

- The work on the condition-based maintenance of compressor is published in the journal *Computers & Chemical Engineering* with the title 'Operational optimization of networks of compressors considering condition-based maintenance' (Xenos et al., 2016) and is presented in Section 6.5.

- A similar work titled as 'Operational optimization of compressors in parallel considering condition-based maintenance' (Xenos et al., 2014c) was published in the conference *European Symposium on Computer Aided Process Engineering (ESCAPE 24)*.

The application of the basic maintenance model (Section 6.2) and the rolling time horizon framework (Section 6.3) considers the BASF case study 1 of the air separation plant. The Section 6.4 consider the Statoil case study 2 of the export natural gas compressor station. The Section 6.5 examines a condition-based maintenance approach and is related to the case study 1. Chapter 6 will present numerical examples related to the case studies for each optimisation model after the end of the section which describes each model.

6.2 Basic model of integrated operation and maintenance

6.2.1 Basic maintenance constraints

Section 6.2 focuses on activities of preventive maintenance such as repairs, replacements of mechanical and electrical parts, and inspections. These maintenance tasks should take place within a predefined time window, and their exact starting or completion times are additional decisions to be made in the optimal framework which was presented in the previous Chapter 5. This approach is defined as *flexible maintenance*. In order to model these types of maintenance tasks, the following set of binary decisions variables is introduced:

$$U_{fm(i,t)} = \begin{cases} 1, & \text{if the maintenance task of compressor } i \text{ begins in time period } t \\ 0, & \text{otherwise} \end{cases}$$

For compressors $i \in I^{fm} \subseteq I$ that are subject to flexible maintenance, the corresponding maintenance tasks should start within a given time window $t \in [\tau^{es}_{(i)}, \tau^{ls}_{(i)}] \subseteq T$:

$$\sum_{t=\tau^{es}_{(i)}}^{\tau^{ls}_{(i)}} U_{fm(i,t)} = 1, \qquad \forall i \in I^{fm} \tag{6.1}$$

The earliest and latest starting times of each compressor i are given by parameters $\tau^{es}_{(i)}$ and $\tau^{ls}_{(i)}$ respectively. The latest starting time plus the duration of the maintenance task has to be lower or equal to the last time period of the scheduling

horizon of interest: $(\tau^{ls}_{(i)} + \nu_{(i)}) \leq T$. The reason is to ensure that the maintenance task will be completed within the given scheduling horizon. Constraints (6.2) ensure the unavailability of compressor i throughout the duration of the flexible maintenance task $\nu_{(i)}$.

$$X_{(i,t)} + \sum_{t'=max\{\tau^{es}_{(i)}, t-\nu_{(i)}+1\}}^{min\{\tau^{ls}_{(i)}, t\}} U_{fm(i,t')} \leq 1, \qquad \forall i \in I^{fm}, t = \tau^{es}_{(i)}, \ldots, (\tau^{ls}_{(i)} + \nu_{(i)} - 1) \tag{6.2}$$

The fixed maintenance tasks introduced in the previous chapter can be modelled as flexible maintenance tasks, if they are included in the set I^{fm}, and the $\tau^{es}_{(i)} = \tau^{fxd}_{(i)}$ and $\tau^{ls}_{(i)} = (\tau^{fxd}_{(i)} + \nu_{(i)} - 1)$ are defined. An incomplete flexible maintenance task that started in the previous scheduling horizon, in the current scheduling horizon is a fixed maintenance task that can be modelled by constraints (5.63′) found in Chapter 5.

6.2.2 Maintenance tasks restrictions

Simultaneous maintenance of many compressors is undesirable. From a management point of view, the maintenance of several compressors during the same time limits the operational flexibility in the possibility of unexpected events. Moreover, the maintenance manpower may be limited, and therefore a maximum number of compressors $(\eta_{(t)})$ could be maintained simultaneously in every time period. This management aspect regarding maintenance tasks can be modelled by the following constraints:

$$\sum_{i \in (I^{dm} \cup \tilde{I}^{dm})} \tilde{\eta}_{(i,t)} + \sum_{\substack{i \in I^{fm} \\ \tau^{es}_{(i)} \leq t \leq (\tau^{ls}_{(i)} + \nu_{(i)} - 1)}} \sum_{t'=max\{\tau^{es}_{(i)}, t-\nu_{(i)}+1\}}^{min\{\tau^{ls}_{(i)}, t\}} U_{fm(i,t')} \leq \eta_{(t)}, \qquad \forall t \in T \tag{6.3}$$

6.2.3 Integrated framework

The integrated model for operation and maintenance is given by the following constraints:

- minimum run and shutdown time in Section 5.3,
- assignment of compressors to headers in Section 5.4,
- compressor-to-header assignment changes in Section 5.5,

- feasible operational window described by a rectangular regression domain in Section 5.6,
- blow-off valve model in Section 5.8.2,
- outlet pressure of compressors in Section 5.9,
- mass balances of the units of air separation plant in Section 5.10,
- given maintenance tasks in Section 5.13,
- initial state of the network in Section 5.14,
- terminal constraints in Section 5.15,
- basic maintenance constraints in Section 6.2.1,
- maintenance task restrictions in Section 6.2.2.

The objective function is this of Eq. (5.61) using the data-driven model expression of power consumption of Eq. (5.7). An illustrative and industrial example will be presented in the following sections to demonstrate the use of the integrated approach compared to the optimal operation with pre-scheduled maintenance.

6.2.4 Example 3: Illustrative example

Example 3 is a development of Example 1-A that was explained in Section 5.17.2 on page 145. The description of the illustrative Example 3 of the air compressors of an air separation plant has been explained in the description of Example 1-A. Example 3 assumes that the header cost is 1,000 m.u/change and that the maintenance tasks have not been pre-scheduled. Table 6.1 gives the values of the earliest and latest starting times of the flexible window of maintenance of the compressors. The aim of this example is to demonstrate that a better schedule in terms of total costs can be obtained if maintenance and operational tasks for compressors are simultaneously considered in the optimisation.

The number of equations are 8,695 with 1,894 binary and 2,580 continuous variables. The nodes of the optimal solution are 139,173 and the CPU time solution is 10,695 s. The objective value is 81.92 m.u. The value of the objective function without considering the cost for changing header is 78.25 m.u.

Due to the inclusion of the degrees of freedom $U_{fm(i,t)}$ of each compressor i, there are 124 more binary variables in this example compared to the example with fixed maintenance presented in the previous chapter in Section 5.17.2. This results in an increase in the computational time. Figure 6.1 displays the optimal schedule of the compressors. The maintenance plan is an output of the optimisation.

Table 6.1: Example 3: Earliest and latest starting times for flexible maintenance tasks (in Days).

	$i1$	$i2$	$i3$	$i4$	$i8$	$i9$	$i10$	$i11$
$\tau^{es}_{(i)}$	1	-	1	5	10	10	-	1
$\tau^{ls}_{(i)}$	15	-	26	25	25	25	-	26

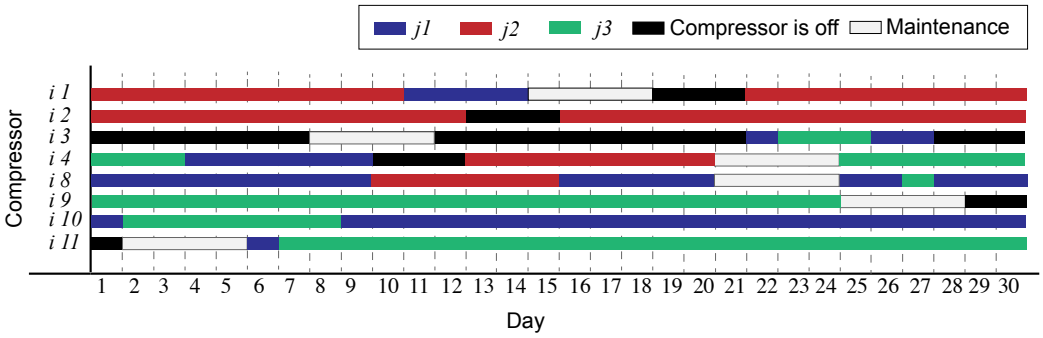


Figure 6.1: Example 3: Optimal schedule considering flexible maintenance policy.

The results show that the total cost is reduced by 10% compared to the previous Example 1-A (page 149) with fixed and given maintenance, showing that consideration that a flexible maintenance policy provides better results than in the case of fixed maintenance given as input. Figure 6.2a shows the aggregated total cost of the case with fixed maintenance ($C_{D(i)} = 250$ m.u.) and flexible maintenance policy with the same change header cost as the former case per each day.

One reason that flexible maintenance leads to cost savings is that the maintenance tasks are scheduled in such a way that the number of startups and shutdowns are decreased in comparison with the schedules of the previous problem instances. Figure 6.2b shows that the power consumption, startup and shutdown costs are reduced when flexible maintenance policy is considered. The case with the fixed maintenance plan can be considered as the upper bound for the flexible maintenance plan case. The total change header cost in the integrated approach with eleven header changes is approximately the same as in the case with fixed maintenance with the lower change header cost and twelve header changes.

Also, as previously mentioned, purchase of nitrogen was reported at Day 15 in both cases with fixed maintenance, due to the fact that the plant could not satisfy the demand. The reason is that two large compressors were under pre-scheduled maintenance on this day. However, in the integrated approach the maintenance tasks are scheduled optimally in such a way that the demand for products could be met without the need of any product purchases from external sources. Therefore, this example has demonstrated clearly the significant benefits of the simultaneous optimisation of operational and maintenance tasks.

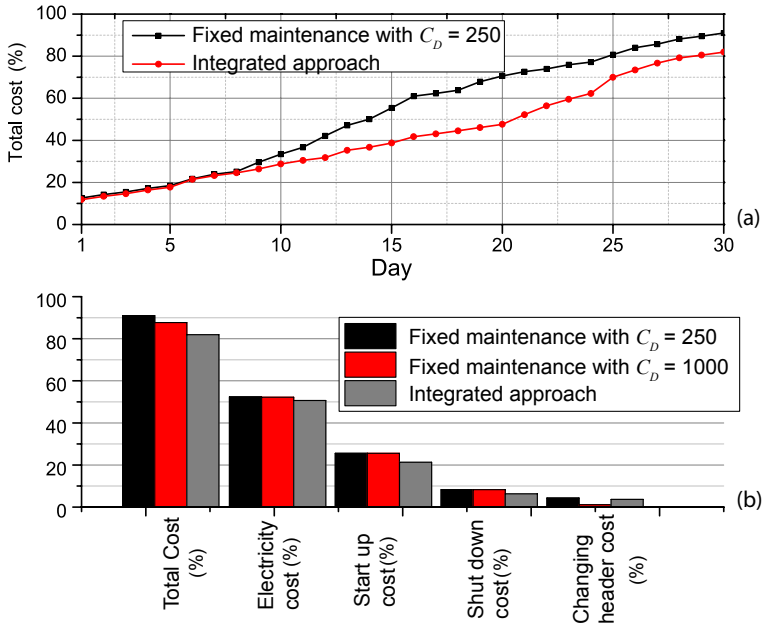


Figure 6.2: Example 3: Aggregated normalised objective value for fixed and integrated approach cases (a), and normalised total cost breakdown of fixed maintenance (two different C_D) and integrated approach (b).

6.2.5 Example 4: Industrial example

Example 4 builds on the Example 1-B that was presented in Section 5.17.3 on page 149. Section 6.2.5 presents two problem instances for Example 4 which consider the integration of operation and maintenance. Instance 4.1 will present the concept without including maintenance restrictions of manpower (explained in Section 6.2.2) and Instance 4.2 will include these restrictions. Table 6.2 gives the earliest and latest times of the maintenance tasks. The compressors which have

Table 6.2: Information of maintenance tasks of the compressors.

Instance	Parameter	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$	$i11$
Instance 4.1	$\tau^{es}_{(i)}$	1	-	-	1	1	1	-	-	1	-	-
Instance 4.2	$\tau^{ls}_{(i)}$	26	-	-	28	25	28	-	-	28	-	-

Table 6.3: Computational results for all instances of the industrial examples. The Fixed Maint. case refers to the Example 1-B in Section 5.17.3.

	Eqs.	Binary variables	Continuous variables	Nodes	CPU (s)	Obj (m.u.)
Fixed Maint.	11,545	2,310	3,150	1,704	139	55.61
Instance 4.1	11,683	2,444	3,150	58,626	7,721	54.65
Instance 4.2	11,817	2,594	3,150	82,517	14,000	54.65

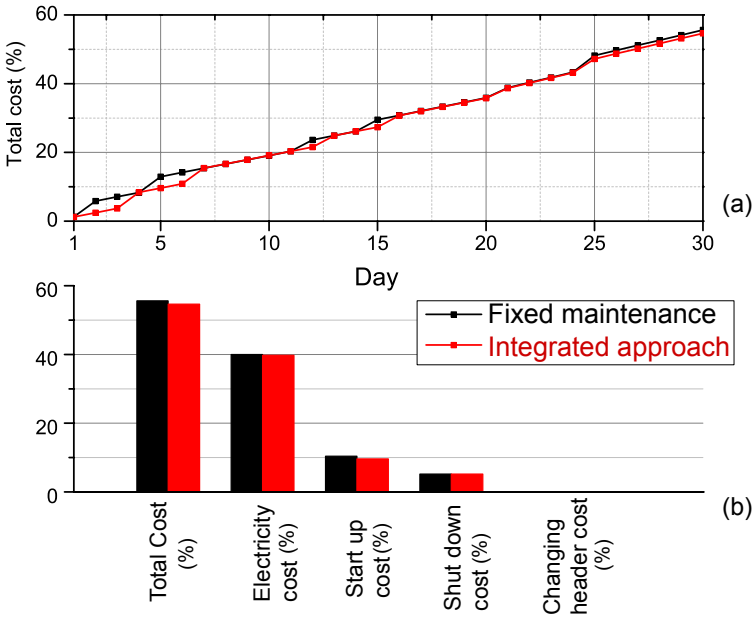
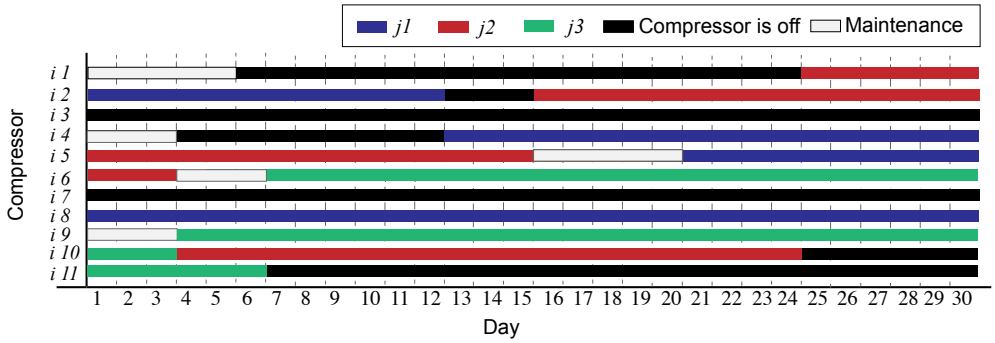


Figure 6.3: Fixed maintenance (Example 1-B) and integrated approach (Example 4: Instance 4.2): aggregated normalised objective value (a), and normalised total cost breakdown (b).

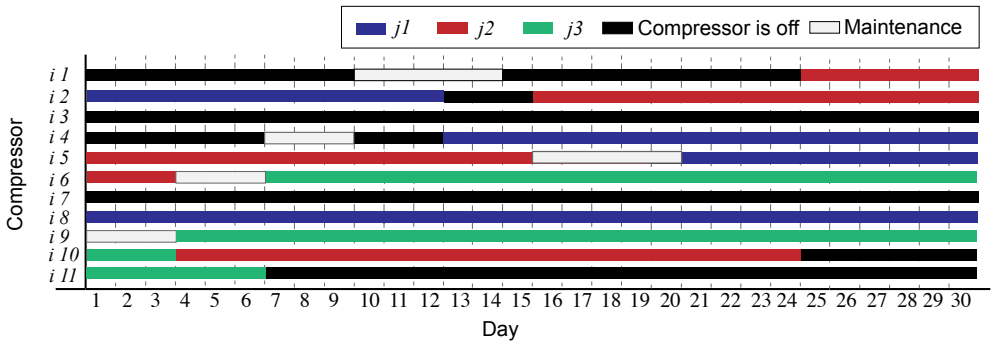
to carry out maintenance are the same as in the case in Section 5.17.3 and the durations of the maintenance tasks are again the same for each compressor. Table 6.3 show the computational results of all the problems solved.

Instance 4.1 Figure 6.4a gives the schedule for the unconstrained (i.e. no restrictions of manpower) flexible maintenance case. Due to the simultaneous optimisation of maintenance and operational tasks, the maintenance tasks are scheduled differently from the case with fixed maintenance in such a way that the total cost is decreased by 1.8%. Moreover, all maintenance tasks are completed before Day 21, where demand for products increases significantly. In Instance 4.1, there are several time periods where maintenance tasks take place simultaneously. For example, from Day 1 to 3, three maintenance tasks are performed simultaneously, and in Day 4 and 5, two maintenance tasks take place at the same time. However, simultaneous maintenance has manpower implications. For this reason, a flexible maintenance policy with restrictions to avoid simultaneous maintenance tasks is presented in Instance 4.2.

Instance 4.2 In this constrained flexible maintenance case, it is assumed that one maintenance task at most can take place in every time period, hence $\eta_{(t)} = 1$. Figure 6.4b displays the resulting optimal schedule of this problem instance. As



(a) Instance 4.1



(b) Instance 4.2 with restrictions and $\eta_{(t)} = 1$.

Figure 6.4: Optimal schedules of compressors.

it can be seen from Table 6.3, the total cost of Instance 4.2 is the same with that of Instance 4.1. The solution of Instance 4.1 is exactly the same with Instance 4.2 apart of the start times of the maintenance tasks. Indeed, in Instance 4.2 the maintenance tasks of compressors $i1$ and $i4$ are shifted to the right so they do not occur in the same time periods. Hence, these two changes do not affect the total cost of Instance 4.1 since they do not add any cost in the objective function, and the rest of the schedule is the same as that in Instance 4.1. Although Instance 4.1 and Instance 4.2 have the same total cost, Instance 4.2 gives a better schedule (in terms of flexibility and management perspective), since maintenance tasks are distributed more uniformly over the scheduling horizon of interest.

Figure 6.3a provides a comparison for the aggregated normalised total cost between the fixed maintenance case of Example 1-B in Section 5.17.3 and the integrated approach of Instance 4.2 (or Instance 4.1 as both have the same cost). Figure 6.3b shows the normalised total cost breakdown for all problem instances. Similarly to the results from the illustrative Example 3 of Section 6.2.4, it is observed that the costs of startup and power consumption decrease under a flexible maintenance policy (i.e., Instance 4.1 and Instance 4.2). In these cases, the cost of power consumption has been reduced by 0.62%, and startup cost declines by

7.70% in comparison with those in the case with fixed maintenance.

Overall, it has been demonstrated that the simultaneous optimisation of maintenance and operational tasks could provide better solutions and decrease the total cost compared to the case of optimal operation with given maintenance. As was shown, the better coordination of the maintenance tasks helps to operate the compressors more efficiently (a decrease in power consumption by 0.62% was observed) and to reduce the startups of the compressors (reduction by 7.70%). The operation with reduced number of startups increases the lifetime of the compressors.

6.3 Basic maintenance model in a rolling time horizon framework (reactive scheduling)

6.3.1 Model

The basic model of integrated operation and maintenance of Section 6.2 is formulated in such a way that it can be used within a rolling horizon framework known as reactive scheduling. According to van den Heever and Grossmann (2003) reactive scheduling is the problem which updates a schedule dynamically as the constraints and parameters change over time. Focusing on the scheduling of compressors, the initial state of the compressors (cumulative operating time or shut down time) and the flows of demands are examples of parameters which change over time. Moreover, an unexpected breakdown of a compressor is also considered in the update of the constraints.

The rolling horizon concept is based on the principles of the model predictive controller. According to Kopanos and Pistikopoulos (2014) the rolling horizon framework solves an optimisation model for a scheduling horizon of certain length, known as *prediction horizon*. A second time horizon, which is a subset of the prediction horizon and known as control horizon, is used to apply the decisions of the prediction horizon. This second time horizon usually has duration of one time period. The optimisation problems are solved iteratively by moving forward the prediction time horizon (consequently the control horizon moves along with the prediction horizon) after each iteration. In each iteration, the solution of the control horizon is applied. After each iteration, the optimisation problem receives feedback, for instance updated demands and state of the system.

Figure 6.5 illustrates an example of the rolling horizon concept with a fixed prediction horizon of three time periods and a control horizon of a single period. The problem needs to be solved for five times periods, thus there are five iterations (i.e. $iter_{tot} = 5$). In the first iteration, the initial state of the problem is known. The optimisation problem is solved and the results of the optimisation are applied only for the first time period. The output of the control horizon of the optimisation

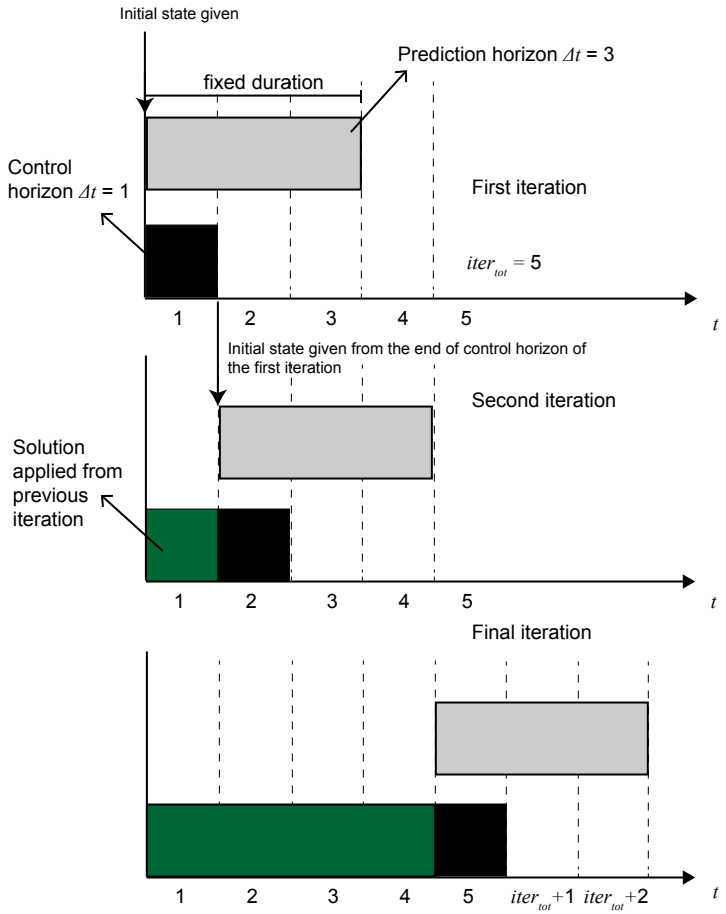


Figure 6.5: Example of rolling horizon approach.

problem of the first iteration are given as input to the second optimisation problem for the second iteration. In the second iteration, the control horizon along with the prediction horizon moved by one time period. After solving the problem iteratively for five time periods, the solutions for the five periods have been applied.

Figure 6.6 displays a representative algorithm for rolling horizon optimisation of the compressor station of the air separation plant. The algorithm is used to demonstrate the implemented solutions after a predefined number of iterations $iter_{tot}$. At the initialisation step for $iter = 1$, the prediction and control horizon, and the initial state of the system are given. The prediction horizon is moving by one time period and the parameters of the optimisation model are updated in each iteration. For example, the initial connections of the compressors with the headers for each optimisation problem in each iteration are given by Eqs. (6.4) – (6.6). Equation (6.4) defines the parameter of the initial connections $\tilde{\varphi}$ for the first

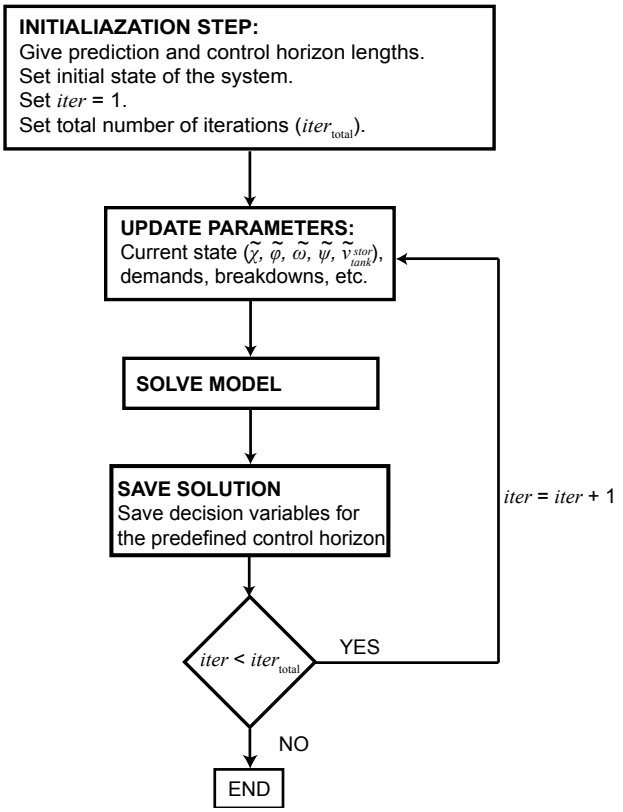


Figure 6.6: A representative rolling horizon algorithm.

iteration, i.e. $iter = 1$, where the value of $\tilde{\varphi}^0$ is given. Equations (6.5) and (6.6) calculate the parameter of the initial connections $\tilde{\varphi}$ of the optimisation problem of each iteration $iter$ given from the output of the problem of every previous iteration $iter - 1$. The auxiliary parameter $\tilde{\varphi}^{aux}_{(i,j,it)}$ is used to temporarily save the output of each $Y_{(i,j,t)}$ in each iteration. Another example is the update of the demand of the product e of the air separation plant given by Eq. (6.7). The parameter $V^{aux}_{dem(e,t,it)}$ represents the given forecasted demand of product e for the prediction horizon $t \in T$ in each iteration $it = iter,\ iter = 1, \ldots, iter_{tot}$. The remaining parameters $\tilde{\chi}, \tilde{\omega}, \tilde{\psi}, \tilde{v}^{stor}, \tilde{\nu}^{fxd}$ are updated in a similar way. The prediction horizon has the general description $T = (iter, iter + 1, \ldots, iter + \Delta H - 1)$, where the ΔH is the duration of the fixed prediction horizon.

The updated parameters are given to the optimisation model. GAMS solves the optimisation problem and provides the solution (i.e. decision variables) which would be applied for the predefined control horizon. As the purpose of the algorithm is to demonstrate the implemented solution after $iter_{tot}$ iterations, then the solution is saved. This solution is also used to update the parameters of the next iteration. The algorithm terminates when the iterations reach the predefined number of total iterations $iter_{tot}$.

$$\tilde{\varphi}_{(i,j)} = \tilde{\varphi}^{0}_{(i,j)}, \qquad \forall i \in I, j \in J_{(i)}, t = 1, iter = 1 \tag{6.4}$$

$$\tilde{\varphi}_{(i,j)} = \tilde{\varphi}^{aux}_{(i,j,it)}, \qquad \forall i \in I, j \in J_{(i)}, t > 1, it = iter - 1, iter = 2, \ldots, iter_{tot} \tag{6.5}$$

$$\tilde{\varphi}^{aux}_{(i,j,it)} = Y_{(i,j,t)}, \qquad \forall i \in I, j \in J_{(i)}, t = iter, it = iter, iter = 2, \ldots, iter_{tot} \tag{6.6}$$

$$V_{dem(e,t)} = V^{aux}_{dem(e,t,it)} \qquad e \in E, t \in T, it = iter, iter = 1, \ldots, iter_{tot} \tag{6.7}$$

6.3.2 Numerical example

Description of the example

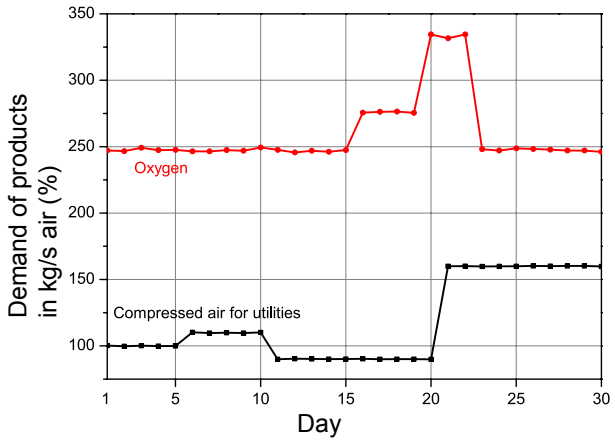


Figure 6.7: Normalised demand for products (deterministic values).

The rolling time horizon framework is applied to the case study of the air separation plant in BASF. The aim is to show how the proposed integrated optimisation framework can cope with a real-life case under the presence of uncertainty in demand. The specifications of the optimisation problem are the following. Oxygen and compressed air are the products of the plant, change header cost is equal to 250 m.u./change, and purchases of oxygen are not allowed.

The reactive scheduling problem will been solved via the rolling horizon approach displayed in Figure 6.6 for 30 days (i.e. there are 30 iterations). A time period is equal to one day. A prediction horizon of 21 time periods, and a single-period control horizon have been employed. In each iteration, a scheduling problem for the next 21 time periods (i.e., the prediction horizon) is solved with updated information regarding the current state of the overall system and the demand for

products. The solution of the first time period of the current prediction horizon is only applied in the corresponding iteration, i.e. $t = iter$.

In this illustrative problem, demands for products are not deterministic but instead may fluctuate over time. For this reason, the demand profiles are updated after each iteration for a given prediction horizon. As the length of the control horizon is one time period, then an iteration is carried out after one time period. Figure 6.7 shows the actual normalised demand profile for oxygen and compressed air for utilities for thirty days. Hence, the optimisation problem has to be updated and solved for thirty iterations.

The adjective "actual" indicates that this normalised demand profile represents the case in which the demand is known and is not expected to change over these days (i.e. deterministic values of the demand). This is the case in which the demand for products for all 30 days is known with certainty at the beginning of the time horizon of each iteration and is called *perfect information case*. One would solve a deterministic scheduling problem for the scheduling horizon of 30 days. The perfect information solution is the best solution that one could obtain. However, in practice, it is impossible to find this solution due to the uncertainty in the demand forecasting. In the real-life case the demand involves uncertainty. Therefore in the rolling horizon approach a forecasted demand for the prediction horizon is provided at each day. As the duration of the prediction horizon is 21 days, then there is a new demand prediction for the following 21 days after every day. The forecasted demand for each prediction horizon at each day is not presented in the Figure 6.7 for better readability.

The initial condition of the compressors is the same as in Table 5.5. It is assumed that some compressors follow a fixed maintenance policy and others a flexible maintenance policy to demonstrate the use of the framework for a general scenario. Compressors $i9$ and $i11$ have been pre-scheduled for maintenance in Days 20 and 15 respectively. Compressors $i2$ and $i6$ are under flexible maintenance, and their maintenance tasks must start in time window defined by earliest time start $\tau^{es}_{(i)} = 1$ (i.e. Day 1) and latest time start $\tau^{ls}_{(i)} = 15$ (i.e. Day 15). The duration of each maintenance task is three days. Maximum run times for compressors are the same as in Example 4. Moreover, an unexpected breakdown of compressor $i5$ on the second day is considered.

Results

A total number of 30 iterations has been solved. An iteration represents a scheduling problem which employs a prediction horizon of 21 days and a time limit of 1,800 s of the CPU time was set for each iteration. The computational time for every iteration can be seen in Figure 6.8. On average, solutions are obtained in low

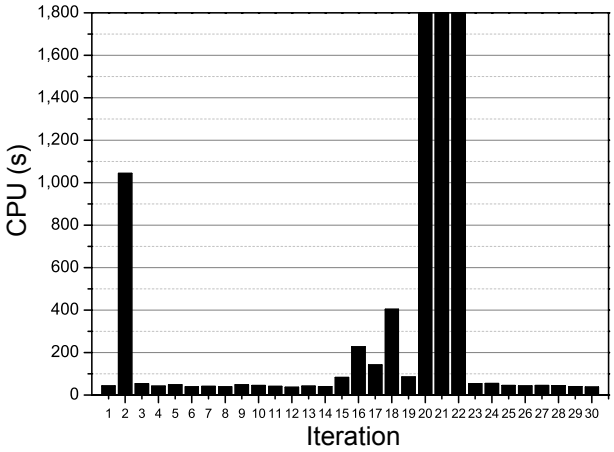


Figure 6.8: Computational CPU time in s for each iteration.

computational times. Negligible optimality gaps have been reported for iterations 20 to 22, where the imposed time limit was reached. As discussed by Kopanos and Pistikopoulos (2014), longer prediction horizons result in bigger mathematical model sizes, and as a consequence the necessary computational time will probably increase too.

Figure 6.9 shows how the overall schedule was built through the solutions obtained in each iteration. The last Gantt chart gives the implemented schedule. The compressors that do not operate in any iteration are not included in Figure 6.9. In the third iteration, the schedule of the compressors for the first and second day have been implemented, and the schedule for the third day is set fixed. The schedule for Days 4,. . . ,23 will be re-evaluated in the next iterations considering updated information (e.g. demands and state of compressors). As can be seen in the Gantt chart of the implemented schedule, the schedule for Days 4 – 23 in the third iteration is different from the final solution on these days.

Another example is that the schedule in the second iteration determined that compressor $i5$ should operate at header $j2$ on Day 2. However, in the second iteration, the updated information gives that compressor $i5$ must stay offline on Day 2 due to an unexpected breakdown. Thus, the practical value of the reactive scheduling approach can be seen here, as a new schedule is generated for Day 2 considering the updated information of the state of the compressors.

The normalised load distribution of each compressor is given in Figure 6.10. The figure shows that compressors $i3$, $i4$, and $i9$ remain idle during the thirty-day schedule. It should be noted here that it is known that compressor $i3$ is the least energy-efficient compressor, and the results of all case studies give clear evidence of it since they show that compressor $i3$ should stay offline or should operate at limited capacity.

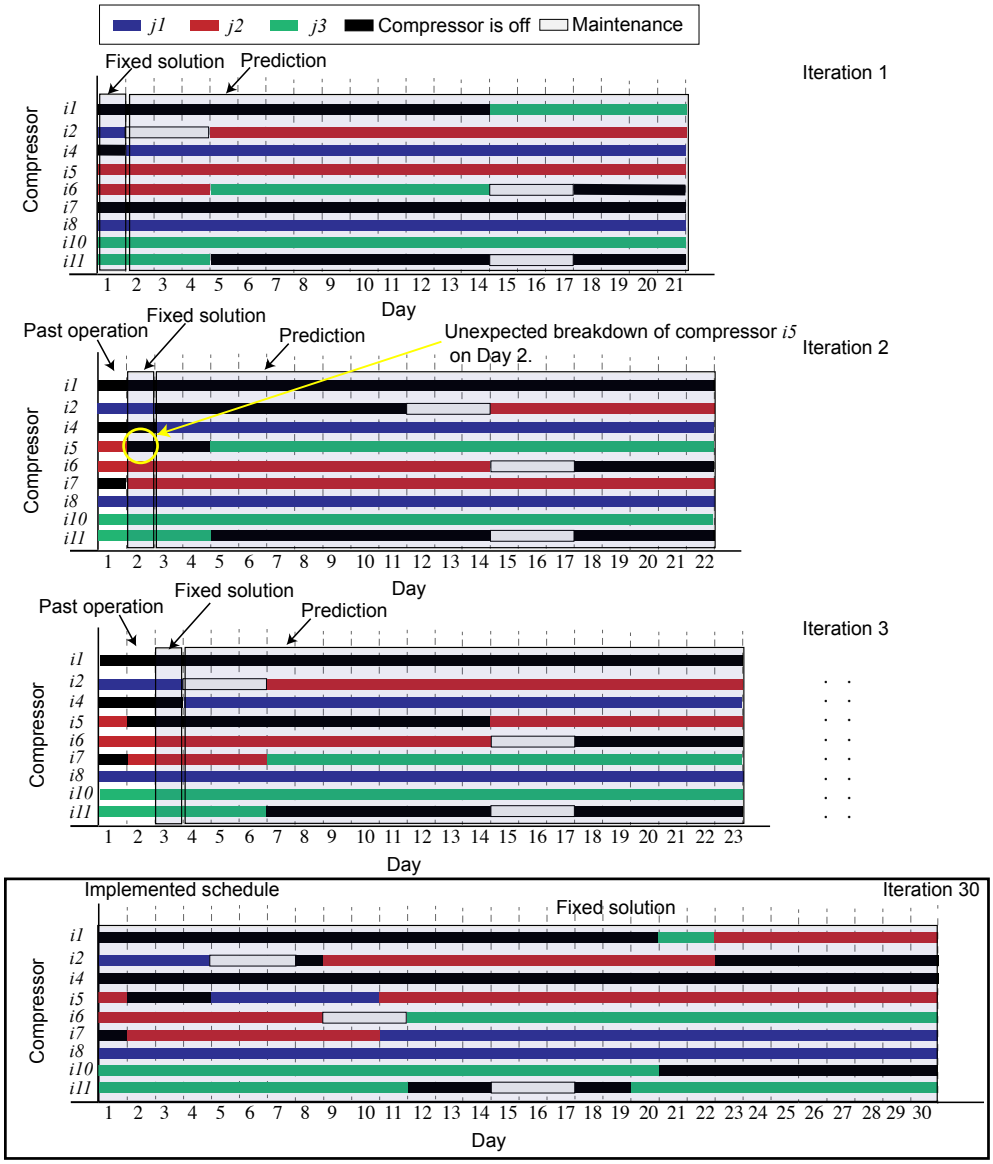


Figure 6.9: Schedule generation via rolling horizon.

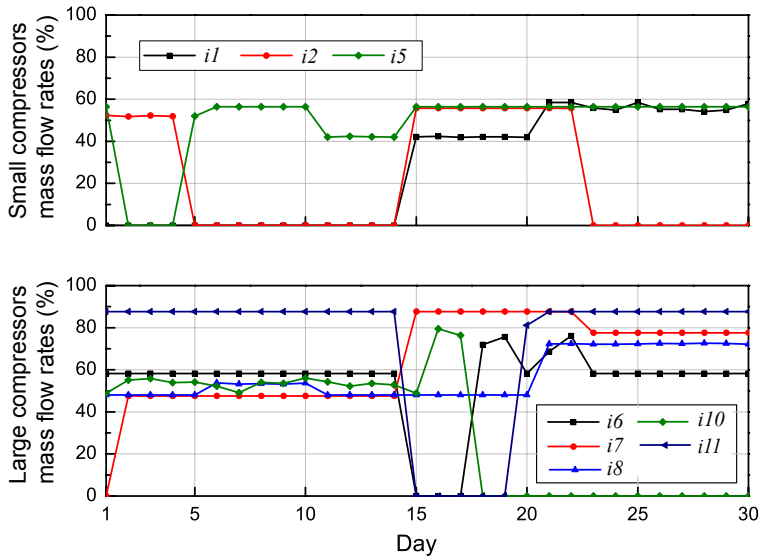


Figure 6.10: Normalised mass flow rates of small and large compressors.

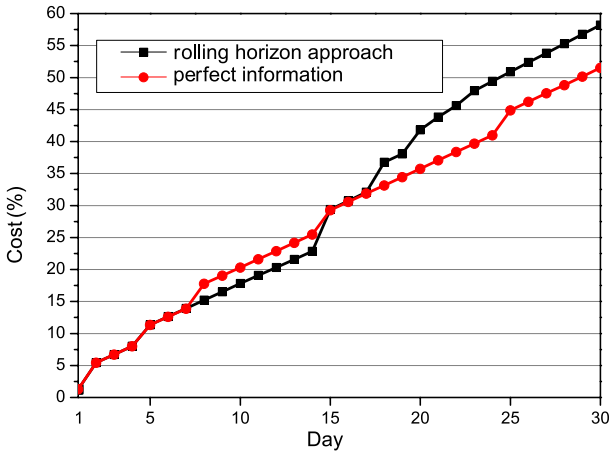


Figure 6.11: Aggregated normalised objective value for the rolling horizon and perfect information solution.

The aggregated costs of the perfect information case and the solution derived by the rolling horizon approach are shown in Figure 6.11. The results show that the rolling horizon solution is 11% worse than that of the perfect information case. The obtained solution could be improved, if the accuracy in the forecast of the demand is improved and the length of the prediction horizon increases. However, in practice as the length of the prediction horizon increases, the forecasting accuracy naturally decreases. Overall, through this example it has been shown the applicability of the basic integrated framework of operation and maintenance of compressors to deal with relevant scheduling problems in dynamic production environments.

6.4 Maintenance model including major overhauls

6.4.1 Maintenance model

Table 6.4: Descriptions of maintenance tasks.

Maintenance	Type	Example	Compressor set	Shutdown duration
Overhaul	Major	Rotor replacement	I^{M2}	ν_{OV}
Mechanical	Light	Vibrational analysis	I^{M1}	1 d
Electrical	Light	VSD inspection	I^{M1}	1 d

This section refers to an integrated model of operation and maintenance of natural gas compressors and the Statoil export gas station will be considered for the numerical applications. Section 6.4.1 demonstrates a comprehensive maintenance model which considers different types of maintenance tasks: (a) major overhauls, (b) mechanical maintenance and (c) electrical maintenance corrective tasks. A major overhaul is a long term maintenance with duration $\nu_{OV(i)}$ days and the compressor has to be off during this period.

A mechanical corrective maintenance involves minor mechanical repairs, leak tests and vibrational analysis and it is assumed that it lasts for one day. Thus, the compressor has to be off in this day. The electrical corrective maintenance involves minor electrical repairs, inspection of the VSD drives of the motors of the compressors and other types of inspections. It is assumed that the electrical corrective maintenance also lasts for one day. The compressor also has to be off in this day. Table 6.4 summarises the description of each maintenance. The following assumptions of the maintenance model are taken into consideration:

- The cost of every type of maintenance is not involved in the objective function as the cost of maintenance is assumed independent from the main variables of operation.

- Two sets of compressors are considered for maintenance activities: compressors $i \in I^M$ which are subject to maintenance and compressors $i \in I^{NM}$ which are not scheduled to be maintained.
- The industrial policy is that two different types of maintenance activities cannot happen in the same time period. Only one maintenance group can occupy the space for repairing a compressor in each day.
- There is a minimum time between two consequent major overhauls of two different compressors. The reason is that a major overhaul includes replacements of major spare parts of the compressor, for example the replacement of the rotor. It takes time for the rotor to be repaired. When the rotor is repaired, then a second compressor can have a major overhaul by replacing its used rotor with the revamped one.

There are two different sets of compressors which are related respectively to a major overhaul $i \in I^{M2} \subseteq I^M$ and to light maintenance $i \in I^{M1} \subseteq I^M$. The light maintenance includes mechanical, electrical repairs and inspections of the equipment. The maintenance tasks have to be scheduled within a given time window $t \in T^{sf}_{(i)} = (\tau^{es}_{(i)}, \tau^{ls}_{(i)}) \subseteq T$. The maintenance model considers three sets of binary variables:

$$U_{ov(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I^{M2} \text{ starts an overhaul maintenance in period } t \\ 0, & \text{otherwise} \end{cases}$$

$$U_{Me(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I^{M1} \text{ shuts down for mechanical maintenance} \\ & \text{actions in period } t \\ 0, & \text{otherwise} \end{cases}$$

$$U_{El(i,t)} = \begin{cases} 1, & \text{if compressor } i \in I^{M1} \text{ shuts down for electrical maintenance} \\ & \text{actions in period } t \\ 0, & \text{otherwise} \end{cases}$$

When a mechanical or electrical maintenance episode takes place a compressor should shut down and these constraints are modelled by Eqs. (6.8) and (6.9):

$$X_{(i,t)} \leq (1 - U_{Me(i,t)}), \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.8}$$

$$X_{(i,t)} \leq (1 - U_{El(i,t)}), \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.9}$$

Equations (6.10) and (6.11) describe the constraints where a number of planned mechanical or electrical maintenance episodes (Λ_{Me} or Λ_{El}) have to take place within the time period of the given time window $T^{sf}_{(i)}$.

$$\sum_{t \in T} U_{Me(i,t)} = \Lambda_{Me}, \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.10}$$

$$\sum_{t \in T} U_{El(i,t)} = \Lambda_{El}, \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.11}$$

Equation (6.12) models the assumption that electrical and mechanical maintenance tasks cannot happen for each compressor in the same period:

$$U_{Me(i,t)} + U_{El(i,t)} \leq 1, \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.12}$$

Equations (6.13) and (6.14) consider a maximum number of mechanical (Λ^{max}_{Me}) and electrical (Λ^{max}_{El}) maintenance tasks which can take place in a time period, by considering limited maintenance manpower.

$$\sum_{i \in I^{M1}} U_{Me(i,t)} \leq \Lambda^{max}_{Me}, \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.13}$$

$$\sum_{i \in I^{M1}} U_{El(i,t)} \leq \Lambda^{max}_{El}, \qquad \forall i \in I^{M1}, t \in T^{sf}_{(i)} \tag{6.14}$$

In the case of a major overhaul, the binary variable $U_{ov(i,t)}$ takes the value one when a compressor $i \in I^{M2}$ starts the maintenance. The maintenance lasts for $\nu_{OV(i)}$ days. Compressors $i \in I^{M2}$ which are subject to major overhaul have to start maintenance within a given time window $T^{sf}_{(i)}$ according to Eq. (6.15). Equation (6.16) models the unavailability of the compressors due to maintenance for $\nu_{OV(i)}$ days similar to Eq. (6.2).

$$\sum_{t \in T^{sf}_{(i)}} U_{OV(i,t)} = 1, \qquad \forall i \in I^{M2} \tag{6.15}$$

$$X_{(i,t)} + \sum_{t'=max\{\tau^{es}_{(i)}, t-\nu_{OV(i)}+1\}}^{min\{\tau^{ls}_{(i)}, t\}} U_{OV(i,t)} \leq 1, \qquad \forall i \in I^{M2}, t = \tau^{es}_{(i)}, ..., (\tau^{ls}_{(i)} + \nu_{OV(i)} - 1) \tag{6.16}$$

Equation (6.17) assumes a maximum number of compressors Λ^{max}_{C} which can be repaired at the same time period for each time period $t \in T$ and Eq. (6.18) guarantees a minimum time γ^* between two consequent beginnings of major overhauls of different compressors due to the availability of a single spare rotor.

$$\sum_{\substack{i\in I^{M2},\\ \tau^{es}_{(i)}\le t\le(\tau^{ls}_{(i)}+\nu_{OV(i)}-1)}} \sum_{t'\in max\{\tau^{es}_{(i)},t-\nu_{OV(i)}+1\}}^{min\{\tau^{ls}_{(i)},t\}} U_{OV(i,t')} \le \Lambda^{max}_{C} \tag{6.17}$$

$$\sum_{i\in I^{M2}} \sum_{t'=max\{1,t-\gamma^{*}+1\}}^{t} U_{OV(i,t')} \le 1, \qquad \forall t \in T \tag{6.18}$$

6.4.2 Integrated framework with focus on major overhauls

The objective is to minimise the electricity cost of the motors of the compressors, the start up and shut down costs, and the change cost when a compressor changes header while at the same time all the constraints of the integrated model have to be respected. The objective function is Eq. (5.61) which is duplicated below.

$$min \overbrace{\sum_{t\in T} C_{el(t)} \sum_{i\in I} \sum_{j\in J_{(i)}} d_c W_{el(i,j,t)}}^{\text{electricity cost}} + \overbrace{\sum_{t\in T} \sum_{i\in I} (C_{st(i)} S_{(i,t)} + C_{f(i)} F_{(i,t)})}^{\text{start up and shut down cost}} + \overbrace{\sum_{t\in T} \sum_{i\in I} C_D D_{(i,t)}}^{\text{change header cost}} + \overbrace{\sum_{t\in T} \sum_{e\in E} \sum_{\delta\in\Delta_{(j)}} C_{ext(\delta,e,t)} d_c M^{ext}_{(\delta,e,t)}}^{\text{purchase products/gas cost}} \tag{5.61}$$

The integrated operational and maintenance model is focused on major overhauls and maintenance tasks which are subject to limited maintenance manpower and other constraints such as the constraint on the number of maintenance tasks that can take place in the same time period. The integrated framework couples the constraints of the operational model given by constraints of:

- minimum run and shutdown time in Section 5.3,
- assignment of compressors to headers in Section 5.4,
- compressor-to-header assignment changes in Section 5.5,
- power consumption and feasible operational window with the use of compressor maps in Section 5.7,
- recycle model in Section 5.8.1,
- general operational constraints of the export gas station in Section 5.11,
- maximum run time constraints in 5.13,
- initial state of the network in Section 5.14, and
- the maintenance model described in Section 6.4.1.

6.4.3 Example 5: Industrial example

Description of case study 2 (Statoil export gas station) for 90 days

The methodology of the integrated model with the maintenance constraints including major overhauls events is applied to Example 5 using industrial data from Statoil. This example builds on Example 2 on page 152. Example 5 examines problem Instance 5.1 which is a deterministic optimisation problem and considers a time horizon of 90 days with one day periods. This problem instance is denoted as the baseline case which will be compared to problem Instance 5.2 which considers given fixed maintenance tasks, .

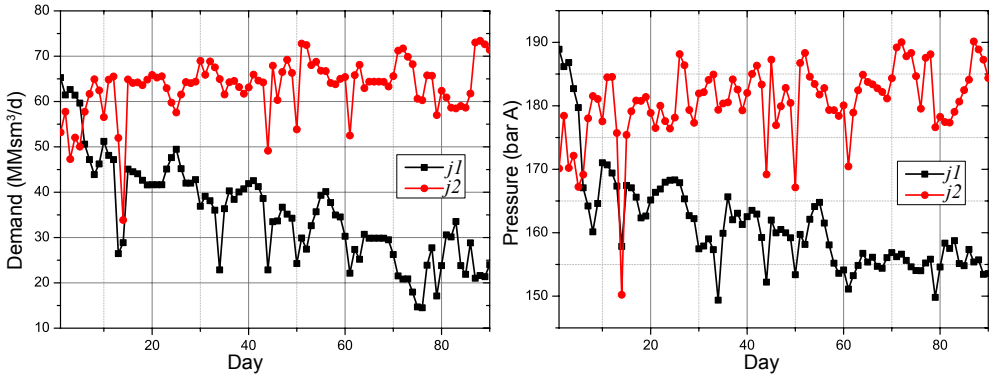
Example 5 considers Kollsnes export gas compressor station with six multistage centrifugal compressors as can be seen in Fig. 5.12. The compressor station is the same one which was described in Section 5.17.4 in the previous chapter. Table 5.7 gives the initial connection of compressors and headers and duration and the parameters $\tilde{\omega}_{(i)}$ and $\tilde{\psi}_{(i)}$ of each compressor. Table 5.8 gives the reference conditions and parameters of the minimum running time ($\omega_{(i)}$), the minimum shut down time ($\psi_{(i)}$) and the maximum running time ($o_{(i)}$). Instance 5.1 considers flexible maintenance and Table 6.5 provides the start and finish of the windows $t \in T^{sf}_{(i)}$ of each compressor i.

It is assumed that there is no maintenance activity before or close to the start of the time horizon $t = 1$ for simplicity. Two compressors are due for major overhaul, $I^{M2} = \{i2, i5\}$ and the remaining compressors $I^{M1} = \{i1, i3, i4, i6\}$ are in need of minor maintenance activities. In addition, it is assumed that each compressor $i \in I^{M1}$ requires one mechanical and one electrical maintenance task. In addition, no more than two mechanical or electrical maintenance events can take place in one time period.

The remaining parameters stay the same. In Instance 5.1 and Instance 5.2 historical industrial data from three summer months are collected. These data provide the demand in mass flow rate (Fig. 6.12a) and in pressure (Fig.6.12b). The inlet pressure of the gas compressor station is also given (Fig. 6.13) for these months. The integrated optimisation models of Instance 5.1 and 5.2, and the given input data were implemented into the optimisation platform GAMS using the MILP solver CPLEX 1.1 with the system with Intel(R) Core(TM) i7-2600 CPU @3.4GHz with 8GB RAM. The optimality gap OPTCR was set 1%.

Table 6.5: Values of start and finish of flexible windows.

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$
$\tau^{es}_{(i)}$	1	1	1	1	1	1
$\tau^{ls}_{(i)}$	90	81	90	90	81	90



(a) Given demand flow rate. (b) Given pressure.

Figure 6.12: Input demand of flow rate and pressure of each delivery point for headers $j1$ and $j2$ for 90d.

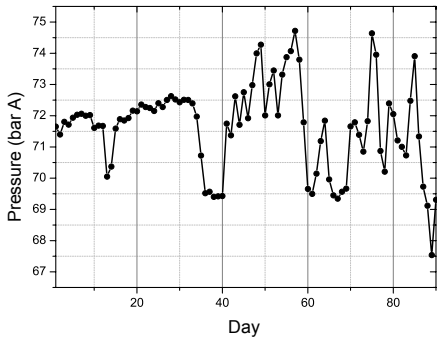


Figure 6.13: Upstream inlet pressure.

Instance 5.1 (baseline case) The optimisation model of Instance 5.1 consists of 54,046 equations, 7,920 binary and 26,640 continuous variables, and 234,705 nodes. The CPLEX provided a solution when the optimality gap of 1% was reached after 43,201 s of CPU time.

The results of the optimisation suggest how many compressors should operate at each header, as can be seen in Fig. 6.14. The optimal solution suggests keeping a constant number of compressors operating to each header as much as possible as the optimiser minimises the number of the unnecessary startups and shutdowns. Especially in the case of the second header $j2$, three compressors are used between Day 15 and Day 88. Although the number of the compressors is constant over long time periods, the electricity cost fluctuates. For example, the electricity used (i.e the power used because the cost of kWh is fixed) fluctuates significantly between Days 35 and 49 (the black line variates, as shown in Fig. 6.14), but the number of the online compressors is fixed for both headers (blue line stays constant, as shown in Fig. 6.14). On Day 44 there is a large drop in electricity cost due to lower demand requested and there is not any switch off. This means that the

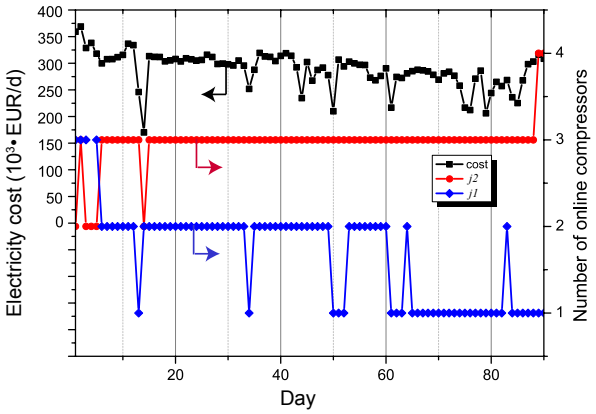


Figure 6.14: Electricity cost and number of online compressors at each header.

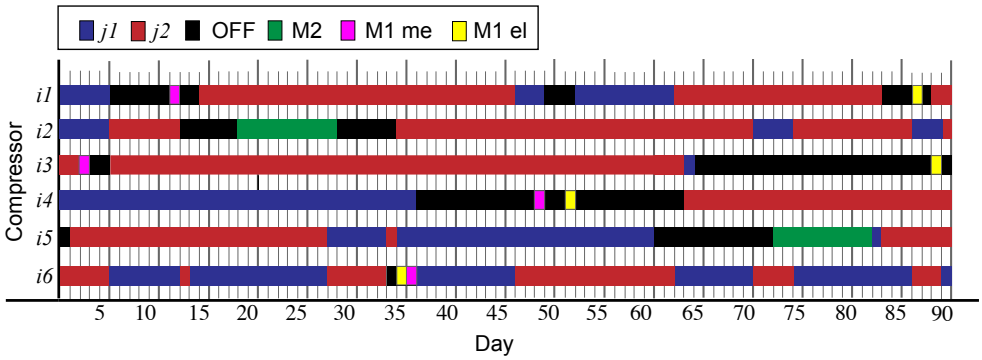


Figure 6.15: Schedule from optimisation of Instance 5.1.

optimiser finds that it is more beneficial to reduce the capacity of a compressor rather than switching it off.

The optimal scheduling resulting from the optimisation can be seen in Fig. 6.15. The schedule shows the status of the compressors, e.g. online and offline status, header connection, maintenance activity, and the start and duration of each operational task. According to Fig. 6.15, compressor $i6$ operates 97% of the total examined time period. The other compressors operate less than this. The reason for this is that compressor $i6$ is the most efficient. Therefore, keeping efficient compressor $i6$ online for more time improves the overall efficiency of the system. Moreover, compressor $i6$ changes headers more frequently than the other compressors. The result is that the efficiency of the overall system increases against the change-header costs of compressor $i6$. Another remark is that compressor $i6$ shuts down at Day 34 when the demand is low. Moreover, the power consumption of the other compressors is relatively low on this day as can be seen in Fig. 6.14.

The maintenance tasks suggested from the optimisation have the flexibility to be shifted within the period the compressors are off. This, mathematically, means that there are multiple solutions as the shift of the timings of the maintenance

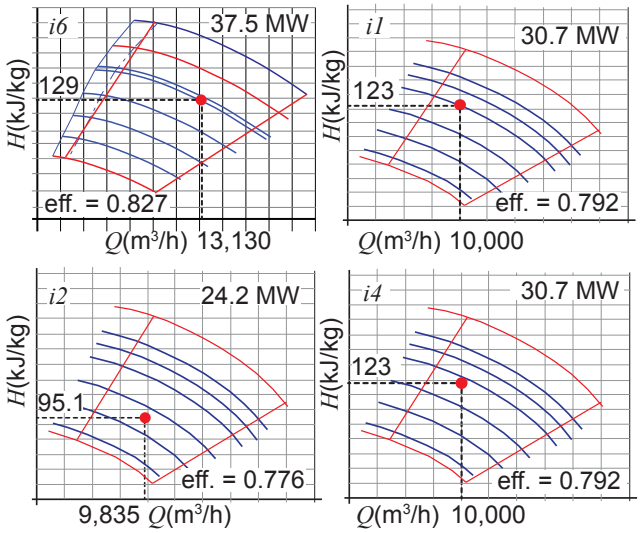


Figure 6.16: Optimal load distribution for Day 71.

activities will not change the value of the objective function. The constraints are not violated and there is no additional cost in the objective function. Therefore, the site operators would have some flexibility to manage the optimum maintenance taking account of other factors such as delivery times of spare parts.

Figure 6.15 also shows that compressors with same characteristics, compressors $i1$ - $i5$, have to apply different decisions for their startups and shutdowns in terms of number and duration. For example, compressor $i1$ should start up three times and operate with an approximate utilisation factor of 81%. On the other hand, compressor $i5$ should start up two times with utilisation factor of 74%. The reason for these differences are the different initial connections between compressors and headers, different operational hours before the optimisation and different duties for maintenance.

Figure 6.16 illustrates the optimal distribution of four compressors online at Day 71, in which compressors $i6, i1$ and $i4$ are connected with header $j2$ and compressor $i2$ with header $j1$. The quantities on the figures are presented in actual conditions [1]. The optimal distribution of load shows that compressor $i6$ handles more capacity than either of compressor $i1$ and $i4$. Compressor $i6$ operates at higher head, nevertheless it has higher efficiency (eff.) than the other compressors operating at the same header with lower flow rate and lower head. The remaining load is split to compressors $i1$ and $i4$ as they have the same characteristics (equal split load). Compressor $i2$ works at significantly lower efficiency than the other compressors due to the lower pressure and mass flow rate being requested (Figs.

[1]The operating conditions (e.g., temperature $T_{act,s}$ and pressure $P_{act,s}$) are usually different from the reference conditions. The different inlet conditions are called actual conditions.

5.23a and 5.23b).

Table 6.6 gives the details of the operational conditions suggested of the optimisation for five days (Days 71 - 75). The $M_{(header)}$ is the total mass flow rate in the header in kg/s. As can be seen, there is a difference between actual and corrected values. This is due to a change of inlet pressure. Compressor $i6$ handles from 36.7% up to 39.1% of the mass flow provided from three compressors in total to header $j2$. In Day 75, it operates alone providing 100% of the gas requested. Figure 5.23a shows that the demand is very low in Day 75 where the solver suggests that the best combination is compressor $i6$ to operate alone at the header with the lower demand and the other three to the header with the higher requested demand. Another solution could be to switch off a compressor, however after two days after Day 75 the demand was forecast to become higher, and this is the reason the optimiser did not choose this option.

Table 6.6: Suggested operating conditions of $i6$ for five days.

Day	71	72	73	74	75
Actual					
M (kg/s)	232.1	238.6	225.6	212.2	137.1
$M/M_{(header)}$(%)	38.5	39.3	38.1	36.7	100
Q (m^3/h)	13130	13574	12934	12000	7459
H (kJ/kg)	128.8	130.3	129.7	128.0	92.0
W (MW)	37.2	39.1	36.8	33.2	14.6
Corrected					
Q (m^3/h)	13303	13753	13105	12158	7557
H (kJ/kg)	132.2	133.8	131.1	131.4	94.4
W (MW)	36.1	37.7	35.2	32.2	14.7

Instance 5.2 (fixed maintenance case) In order to examine the importance of the selection of the starting time points of the maintenance tasks into the optimisation model, another optimisation problem with fixed maintenance is formulated. This problem Instance 5.2 assumes that all the maintenance activities are placed between days 59 and 90, and all the constraints of the maintenance model are respected. Therefore, Eqs. (6.8) - (6.18) are not considered in the optimisation model and the objective function is the same as in problem Instance 5.1.

Figure 6.17 shows the scheduling of the compressors for Instance 5.2. The maintenance tasks have been arranged at the end of the time horizon. Comparing the two schedules from Figs. 6.15 and 6.17, problem Instance 5.2 shows that optimal operation is influenced greatly by the non-inclusion of the maintenance into the optimisation. The decisions for the operation of compressors have significantly

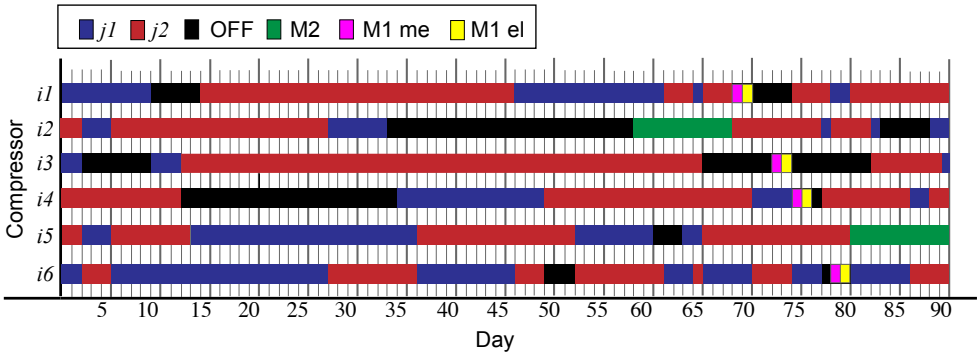


Figure 6.17: Schedule from optimisation of case B.

changed even at the beginning of the time horizon as can been seen for the case of compressor $i5$. Another remark is that the fixed maintenance schedule reduces the flexibility of the operation: for example compressor $i6$ is utilised approximately 93% over the total time of the optimisation in Instance 5.2. Thus, the overall system is expected to be less profitable.

Table 6.7 compares the results from problem Instance 5.1 and Instance 5.2. There is a significant difference in the start up, shut down and change header costs, however the electricity cost does not increase significantly from Instance 5.1 to Instance 5.2. If this difference related to the electricity cost is translated to absolute numbers, 54,000 EUR are saved in three months by considering the selection of the times of the maintenance as a degree of freedom. The overall savings are approximately 0.85% or 224,000 EUR in three months. The last column of Table 6.7 gives the contribution of each cost to the total cost of the base case. The electricity cost is the dominant one. Therefore, a small improvement in the schedule of the compressors results in significant reduction in the overall savings.

Section 6.4.3 presented the scheduling of compressors for two cases, the baseline with integrated operation and maintenance (Instance 5.1) and Instance 5.2 without considering maintenance decisions in the optimisation. The proposed optimisation deals with a difficult problem which includes many decisions to take which include discrete events and continuous quantities. The scheduling approach deals with a

Table 6.7: Comparison between problem Instance 5.1 (baseline case) and Instance 5.2 (fixed maintenance case).

Cost (10^3 EUR)	Instance 5.1	Instance 5.2	% diff.	% of total cost
Start Up	304.1	407.4	25.4	1.15
Shut Down	152.0	203.7	25.4	0.56
Change	23.0	38.0	39.5	0.09
Electricity	25,974	26,028	0.21	98.20
Total	26,453	26,677	0.84	100

deterministic approach of the problem. The extension to the optimisation into a reactive scheduling approach which updates the information of the forecast parameters can cope with uncertainty to some extent. A reactive approach of optimal operation of compressors parts of process systems has been presented in Section 6.3. The reactive approach can also deal with unplanned events such as failures of compressors which have not been scheduled.

6.5 Condition based-based maintenance: washing of compressors

6.5.1 Introduction

Section 6.5 studies the optimal operation and maintenance of the compressors of the case study of the air separation plant. It presents the optimisation of compressors with a condition-based maintenance model which considers the degradation of the condition of the compressors. This maintenance model focuses on online and offline washing, two different cleaning procedures which reduce the extra power used by the compressors due to fouling. Different case scenarios examine the influence of different types of washing methods on the total costs of operation and maintenance. Finally, Section 6.5 demonstrates the benefits of the optimisation and proves that washing schedules and operation have to be examined simultaneously and not separately, in contrast to common industrial practice presented in the literature in Chapter 3.

Chapter 2 explained that *fouling* is the depositions of particles in the fluid onto the airfoil. The deposits increase the roughness of the surfaces of the internal mechanical components (for example, impeller and diffuser area) and restrict the passages areas of the fluid. An example of a fouled impeller can be seen in Fig. 6.18. Thus, the result of fouling is a decrease in performance and increase in power consumption for the same load compared to a non-fouled compressor.

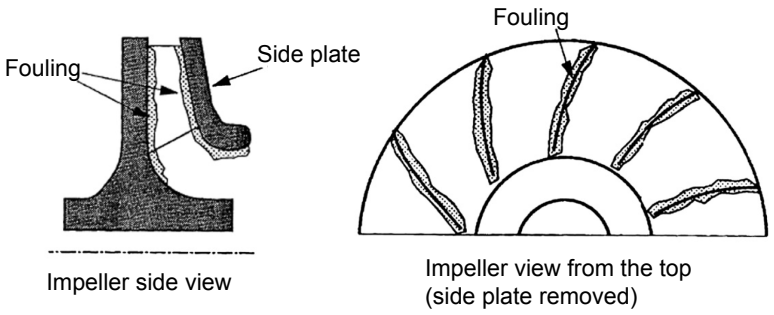


Figure 6.18: Side and top view of a fouled impeller (Forsthoffer, 2011)

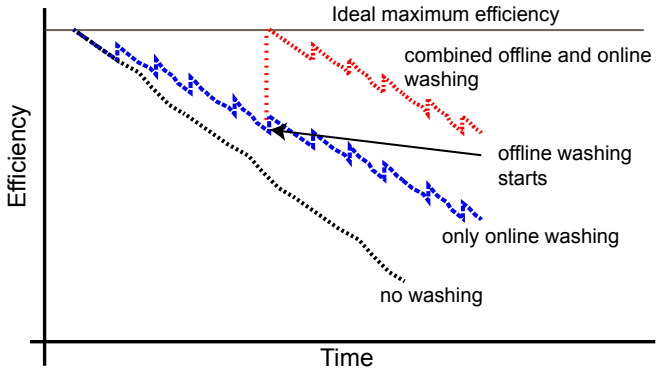


Figure 6.19: The qualitative trend of the efficiency over time considering different types of washing methods.

There are two common strategies to deal with the problem of fouling, namely offline and online washing. The offline washing takes place when a compressor is not operating and the online washing cleans the compressor online without interrupting its operation. An *Offline Washing Episode* (OFWE) is complete after several cleaning steps when the compressor is in offline mode. The purpose of the washing of these cleaning steps is to recover the efficiency of the compressor which results in decreasing the extra power consumed. Fabbri et al. (2011) report that supplementary maintenance tasks, such as mechanical inspections, can be included during a typical OFWE, hence the total duration of this OFWE can be a few days.

Another method for improving the performance of a compressor is to inject a cleaning solution inside the compressor while it is operating online, this procedure is known as *Online Washing Episode* (ONWE). The advantage of this method is that the efficiency improves without shutting down the compressor to wash. However, the recovery of the performance is smaller than in the case of an offline washing episode. Figure 6.19 shows the qualitative trends of the efficiencies when a compressor (a) is not washed, (b) is washed online exclusively and (c) is cleaned with a combination of offline and online washing. According to Fabbri et al. (2011), an ONWE cannot take place if the ambient temperature is less than 14^oC because ice is created which can damage the blades of the compressor.

The following mathematical framework for the optimal operation of a compressor station with multiple parallel compressors considers a condition-based maintenance approach focusing on preventing the effects from fouling. The framework includes the basic operational model of the air separation plant which was presented in Section 6.2.3. The modelling includes the prediction of power consumption depending on operational conditions, the extra power consumption due to degradation, and offline and online washing maintenance which will be explained in the following Sections 6.5.2 and 6.5.3.

6.5.2 Model of offline washing maintenance

The offline washing maintenance model considers the following set of binary variables:

$$W_{of(i,t)} = \begin{cases} 1, & \text{if an offline washing event of a compressor } i \text{ starts in period } t \\ 0, & \text{otherwise} \end{cases}$$

$$U_{of(i,t)} = \begin{cases} 1, & \text{if a compressor } i \text{ is under offline washing in period } t \\ 0, & \text{otherwise} \end{cases}$$

$$K_{of(i,t)} = \begin{cases} 1, & \text{if the offline washing of a compressor } i \text{ has finished in period } t \\ 0, & \text{otherwise} \end{cases}$$

Figure 6.20 illustrates an example of an episode of offline washing. The offline washing starts at time period $t3$ and it lasts until $t5$. Therefore, the compressor is available from period $t6$. In this example the compressor remains off for periods $t6$ and $t7$ and starts up in period $t8$.

Equations (6.19) – (6.22) connect the basic variables of the description of an Offline Washing Episode (OFWE). The duration of an OFWE is described by parameter $\nu_{(i)}$ and it varies according to the type of the compressor and the supplementary maintenance actions that are related to the washing maintenance such as mechanical inspections. When an offline washing episode starts, i.e. $W_{of(i,t)} = 1$, then the compressor must stay offline during this period t, i.e. $X_{(i,t)} = 0$. This is modelled with Eq. (6.23) which also satisfies the case when a compressor has to be switched off, i.e. $X_{(i,t)} = 0$, for reasons other than washing, therefore binary variable $W_{of(i,t)}$ can take the value zero.

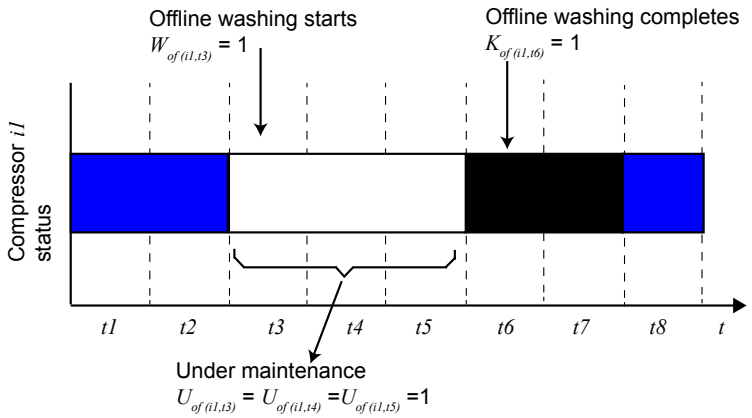


Figure 6.20: Offline washing episode and binary variables explanation.

$$W_{of(i,t)} - K_{of(i,t)} = U_{of(i,t)} - U_{of(i,t-1)},$$
$$\forall i \in I,\ (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t > 1) \tag{6.19}$$

$$W_{of(i,t)} - K_{of(i,t)} = U_{of(i,t)},$$
$$\forall i \in I,\ d^o_{of(i)} = 0,\ t = 1 \tag{6.20}$$

$$W_{of(i,t)} + K_{of(i,t)} \leq 1,$$
$$\forall i \in I, (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t \in T) \tag{6.21}$$

$$1 - U_{of(i,t)} + \sum_{t'=max\{1,t-\nu_{(i)}+1\}}^{t} W_{of(i,t')} = 1,$$
$$\forall i \in I, (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t \in T) \tag{6.22}$$

$$X_{(i,t)} \leq 1 - U_{of(i,t)},$$
$$\forall i \in I, t \in T \tag{6.23}$$

$$U_{of(i,t)} = 1,$$
$$\forall i \in I,\ d^0_{of(i)} > 0,\ 1 \leq t \leq (\nu_{(i)} - d^o_{of(i)}) \tag{6.24}$$

The optimisation model also considers OFWEs that occurred before the start of the time window of the optimisation. This information is part of the input of the model and describes the initial state of the system. The initial state of the system influences significantly the optimisation results as has been demonstrated in the work by Kopanos and Pistikopoulos (2014). The parameter $d^o_{of(i)}$ gives the duration of a compressor which has already been in maintenance before the beginning of the time horizon of the optimisation. If $d^o_{of(i)} > 0$, then the compressor has to continue being maintained up to the time period $(\nu_{(i)} - d^o_{of(i)})$. Equation (6.24) describes this constraint.

Equation (6.25) models the maximum number (λ_{off}) of maintenance events which can take place in one time period. The simultaneous maintenance episodes in one time period (e.g. in one day) may be restricted due to industrial policies.

$$\sum_{i \in I} U_{of(i,t)} \leq \lambda_{off}, \qquad \forall t \in T \tag{6.25}$$

The constraints described by Eqs. (6.26) – (6.30) provide the models of the degradation rate and recovery of the extra power consumption after an offline washing episode. The main assumptions of the degradation and power recovery model are summarised below:

- The degradation rate of a compressor depends on the type of the machine

and on the cumulative time of operation after the last maintenance. A linear function between extra power consumption due to degradation, ΔW, and cumulative operational time ΔS is employed. The expression is $\Delta W_{(i,t)} = \epsilon_{(i)} \Delta S_{(i,t)}$, where $\epsilon_{(i)}$ is the degradation rate of each compressor.

- It is assumed that the effects from the surroundings do not change abruptly during a short time period. For instance, the construction of a new plant close to the compressor station might increase the dust intake in the compressors and the fouling would increase. The linear correlation between extra power consumption and cumulative time period of operation would then not hold true. An advanced performance monitoring diagnostic tool could deal with this problem in a rolling horizon scheduling approach similar to this presented in Section 6.3.

- When a compressor is switched off and remains offline without maintenance, then the condition of the compressor stays as it was before the shutdown. In reality, corrosion can continue between the fouling deposits and the blades, also during stand-by of the machine according to Meher-Homji et al. (2001).

- The additional power consumed due to degradation is assumed constant during the period t, and it does not depend on the operating point.

- When a compressor is washed offline it is assumed that the efficiency is fully recovered, in other words the extra power consumption due to degradation becomes zero immediately after the washing, see Fig. 6.19. According to Diakunchak (1992) a residual decrement of 1% of the degradation might still be observed after an offline washing, however for the purposes of the demonstration of the framework this residual is not modelled.

The assumption of a linear correlation between extra power consumption due to the accumulation of fouling and time holds true up to a particular amount of fouling (Li and Nilkitsaranont, 2009). The current method employs values of the degradation rate ϵ derived from the analysis of Cicciotti et al. (2014a) on the compressors of the same air separation case study presented in Chapter 1.

Although there is not a well defined model at the moment, it is possible that parameters apart from the cumulative time increase the accumulation of fouling. Sánchez et al. (2009) presented a list of factors which influence the build up of fouling, however they expressed the fouling accumulation as a function of time. The rate at which the performance deteriorates is influenced by a large number of factors, this fact makes the prediction of the degradation rate impossible without infield observations of past behaviour of the compressor.

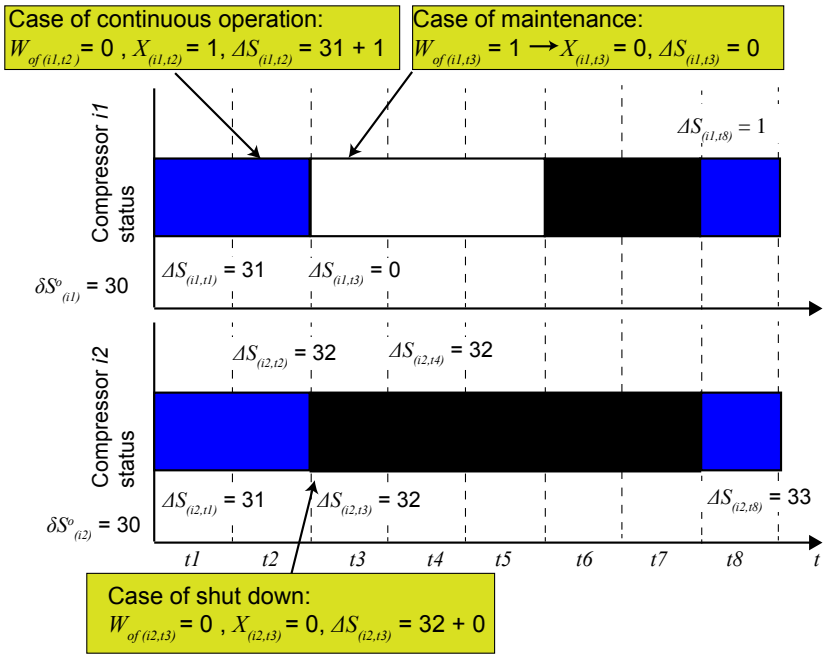


Figure 6.21: Three different case scenarios of the status of a compressor: continuous operation, maintenance and shut down.

The lack of detailed understanding or of a definitive model for fouling justify the assumption that the extra power consumption due to degradation depends on the cumulative time of operation is sufficient for the current study.

As the time is discretised uniformly in time periods $t \in T$, a continuous variable $\Delta S_{(i,t)}$ is used to enumerate the cumulative time of operation at each time period. Figure 6.21 explains graphically the function of this variable. Equations (6.26) and (6.27) give the mathematical description of the $\Delta S_{(i,t)}$. When a compressor is already under maintenance, i.e. $d^{o}_{of(i)} > 0$, then Eq. (6.28) sets $\Delta S_{(i,t)}$ equal to zero for the remaining time period $(\nu_{(i)} - d^{o}_{of(i)})$. The $\delta S^{o}_{(i)}$ gives the duration of a compressor which has been operated before the start of the optimisation. This parameter is part of the input of the initial state of the system.

Equation (6.29) gives the expression of the extra power a compressor consumes because of the degradation. The degradation rate $\epsilon_{(i)}$ depends on the size of the compressor i (Cicciotti et al., 2014a). The parameter $\Omega_{(i)}$, in Eq. (6.30), is a maximum boundary restricting the extra power consumption due to degradation. This parameter is derived from the relationship of the extra power and cumulative days, $\Omega_{(i)} = \epsilon_{(i)} o_{(i)}$, where $o_{(i)}$ is the maximum running time.

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - W_{of(i,t)}),$$
$$\forall i \in I,\ (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t > 1) \tag{6.26}$$

$$\Delta S_{(i,t)} = (\delta S^o_i + X_{(i,t)})(1 - W_{of(i,t)}),$$
$$\forall i \in I, d^o_{of(i)} = 0,\ t = 1 \tag{6.27}$$

$$\Delta S_{(i,t)} = 0,$$
$$\forall i \in I,\ d^o_{of(i)} > 0\ ,\ 1 < t \leq (\nu_{(i)} - d^o_{of(i)}) \tag{6.28}$$

$$\Delta W_{(i,t)} = \epsilon_{(i)} \Delta S_{(i,t)} X_{(i,t)},$$
$$\forall i \in I, t \in T \tag{6.29}$$

$$\Delta W_{(i,t)} \leq \Omega_{(i)},$$
$$\forall i \in I, t \in T \tag{6.30}$$

Equations (6.26) and (6.27) involve bilinear terms some comprising multiplication of binary with binary variables and others the multiplication of binary with continuous variables. Equations (6.31) – (6.36), which use the principles of the Big-M formulation (Vecchietti et al., 2003), convert these non-linear constraints into linear. The reason is to relax the optimisation problem and avoid to solve a hard mixed integer non-linear programming optimisation model.

$$\Delta S_{(i,t)} \leq o_{(i)}(1 - W_{of(i,t)}),$$
$$\forall i \in I,\ (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t \in T) \tag{6.31}$$

$$\Delta S_{(i,t)} = 0,$$
$$\forall i \in I,\ d^o_{of(i)} > 0\ ,\ 1 \leq t \leq (\nu_{(i)} - d^o_{of(i)}) \tag{6.32}$$

$$\Delta S_{(i,t)} \leq \delta S^o_i + X_{(i,t)} + o_{(i)} W_{of(i,t)}, \quad \forall i \in I, d^o_{of(i)} = 0,\ t = 1 \tag{6.33}$$

$$\delta S^o_i + X_{(i,t)} - o_{(i)} W_{of(i,t)} \leq \Delta S_{(i,t)}, \quad \forall i \in I, d^o_{of(i)} = 0,\ t = 1 \tag{6.34}$$

$$\Delta S_{(i,t)} \leq \Delta S_{(i,t-1)} + X_{(i,t)} + o_{(i)} W_{of(i,t)},$$
$$\forall i \in I,\ (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t > 1) \tag{6.35}$$

$$\Delta S_{(i,t-1)} + X_{(i,t)} - o_{(i)} W_{of(i,t)} \leq \Delta S_{(i,t)},$$
$$\forall i \in I,\ (d^o_{of(i)} > 0,\ (\nu_{(i)} - d^o_{of(i)}) < t \leq T)\ \vee (d^o_{of(i)} = 0,\ t > 1) \tag{6.36}$$

Moreover, Eq. (6.29) involves the multiplication of the continuous variable $\Delta S_{(i,t)}$ with the binary variable $X_{(i,t)}$. The reason for this multiplication is that

the extra power due to degradation, $\Delta W_{(i,t)}$, at a period t should not be added into the objective function when the compressor i is offline but not washed. On the other hand, the previous time periods in which the compressor has been operating must be registered through $\Delta S_{(i,t)}$ for each period t. This leads to a non-linear equation, where the following formulation can convert the constraints into linear constraints:

$$\Delta W_{(i,t)} \leq \Omega_{(i)} X_{(i,t)}, \qquad \forall i \in I, t \in T \tag{6.37}$$

$$\Delta W_{(i,t)} \leq \epsilon_{(i)} \Delta S_{(i,t)} + \Omega_{(i)}(1 - X_{(i,t)}), \qquad \forall i \in I, t \in T \tag{6.38}$$

$$\epsilon_{(i)} \Delta S_{(i,t)} - \Omega_{(i)}(1 - X_{(i,t)}) \leq \Delta W_{(i,t)}, \qquad \forall i \in I, t \in T \tag{6.39}$$

As discussed in the assumptions, if the model considered the assumption of Meher-Homji et al. (2001) that a compressor continues being fouled while it is offline, then Eqs. (6.26) and (6.27) should exclude the $X_{(i,t)}$ in the first parenthesis. This would lead to an easier mathematical problem to solve. The current optimisation problem considers the case that a compressor does not become fouled during offline mode.

6.5.3 Model of online washing

The online washing maintenance model considers the following set of binary variables:

$$U_{on(i,t)} = \begin{cases} 1, & \text{if online washing of a compressor } i \text{ occurs in time period } t \\ 0, & \text{otherwise} \end{cases}$$

The injection of a cleaning solution into the compressor can have unwanted side effects such as corrosion of the blades if done too frequently. For this reason, it is assumed a minimum time period $\gamma_{(i)}$ of each compressor i between two consequent Online Washing Episodes (ONWEs):

$$U_{on(i,t)} = 0,$$
$$\forall i \in I, d^o_{on(i)} < \gamma_{(i)} \ , \ 1 \leq t \leq (\gamma_{(i)} - d^o_{on(i)}) \tag{6.40}$$
$$\sum_{t'=max\{t-\gamma_{(i)}+1,1\}}^{t} U_{on(i,t')} \leq 1,$$

$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t \in T) \vee (d^o_{on(i)} < \gamma_{(i)}, (\gamma_{(i)} - d^o_{on(i)}) < t \leq T) \tag{6.41}$$

Equation (6.40) is valid when a compressor has been washed online for time period $d^o_{on(i)}$ before the start of the optimisation and the $d^o_{on(i)}$ is smaller than the $\gamma_{(i)}$. For any other case, Eq. (6.41) holds true.

Equation (6.42) describes the fact that there is also a maximum λ_{on} ONWEs of different compressors that can take place in each time period of the time horizon. The reason for considering this constraint is that it is possible the online washing infrastructure could not support simultaneous online washing as described by Boyce and Gonzalez (2007). In any other case, this constraint can be omitted.

$$\sum_{i \in I} U_{on(i,t)} \leq \lambda_{on}, \qquad \forall t \in T \tag{6.42}$$

An online washing event cannot take place when the compressor is offline. Equation (6.43) describes that if a compressor i is online at period t, i.e. $X_{(i,t)} = 1$, then it has the option to be washed, i.e. $U_{on(i,t)} = 1$ or it can be decided to operate without being washed, i.e. $U_{on(i,t)} = 0$. However, if the compressor is offline, i.e. $X_{(i,t)} = 0$, variable $U_{on(i,t)}$ has to be equal to zero in this time period.

$$U_{on(i,t)} \leq X_{(i,t)}, \qquad \forall i \in I, t \in T \tag{6.43}$$

The recovery model for online washing, which is described in Eqs. (6.44) and (6.45), assumes that $\Delta S_{(i,t)}$ is reduced by a recovery factor R_F. For instance, if the recovery factor is $R_F = 0.2$ and the operating period is $\Delta S_{(i,t)} = 100$ days, then the final operating period after an online washing would be $\Delta S_{(i,t)} \cdot (1 - R_F) = 80$ days. Then the $\Delta W_{(i,t)}$ is estimated based on the duration of this period. The variable $\Delta S_{(i,t)}$ represents the Equivalent Operating Time (EOT) presented by de Backer (2000) and Bohrenkamper et al. (2000). The online washing model is completed with the consideration of the degradation model, which is the same one considered in the case of the offline washing in the previous Section 6.5.2 and it is defined by Eqs. (6.29) and (6.30).

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - R_F \cdot U_{on(i,t)}), \qquad \forall i \in I, t > 1 \tag{6.44}$$

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)})(1 - R_F \cdot U_{on(i,t)}), \qquad \forall i \in I, t = 1 \tag{6.45}$$

The formulation of the constraints in Eqs. (6.44) and (6.45) shows that the multiplication of binary and continuous variables is more complicated compared to the corresponding case of the offline washing. To overcome this complexity, the linearisation of these constraints employs an auxiliary variable $\Delta S_{aux(i,t)}$. The linearised constraints of Eqs. (6.44) and (6.45) can be seen below:

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)}) - \Delta S_{aux(i,t)}, \qquad \forall i \in I,\ t = 1 \tag{6.46}$$

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)}) - \Delta S_{aux(i,t)}, \quad \forall i \in I,\ t > 1 \tag{6.47}$$

$$\Delta S_{aux(i,t)} = 0,$$
$$\forall i \in I, d^o_{on(i)} < \gamma_{(i)},\ 1 \leq t \leq \gamma_{(i)} - d^o_{on(i)} \tag{6.48}$$

$$\Delta S_{aux(i,t)} \leq o_{(i)} U_{on(i,t)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t \in T) \vee (d^o_{on(i)} < \gamma_{(i)}, t > (\gamma_{(i)} - d^o_{on(i)})) \tag{6.49}$$

$$\Delta S_{aux(i,t)} \leq (\Delta S_{(i,t-1)} + X_{(i,t)}) R_F + (1 - U_{on(i,t)}) o_{(i)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t > 1) \vee (d^o_{on(i)} < \gamma_{(i)}, t > \gamma_{(i)} - d^o_{on(i)}) \tag{6.50}$$

$$(\Delta S_{(i,t-1)} + X_{(i,t)}) R_F - (1 - U_{on(i,t)}) o_{(i)} \leq \Delta S_{aux(i,t)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t > 1) \vee (d^o_{on(i)} < \gamma_{(i)}, t > \gamma_{(i)} - d^o_{on(i)}) \tag{6.51}$$

$$\Delta S_{aux(i,t)} \leq (\delta S_i^o + X_{(i,t)}) R_F + (1 - U_{on(i,t)}) o_{(i)},$$
$$\forall i \in I, d^o_{on(i)} \geq \gamma_{(i)}, t = 1 \tag{6.52}$$

$$(\delta S_i^o + X_{(i,t)}) R_F - (1 - U_{on(i,t)}) o_{(i)} \leq \Delta S_{aux(i,t)},$$
$$\forall i \in I, d^o_{on(i)} \geq \gamma_{(i)}, t = 1 \tag{6.53}$$

The degradation model used in the online washing considers the linearisation of the constraint of Eq. (6.29) which was described earlier when discussing Eqs. (6.37) – (6.39) exactly as in the case of the offline washing degradation model.

6.5.4 Model of combined online and offline washing

Section 6.5.4 describes the scenarios when both offline and online washings are available to clean the compressors. In this case, the optimisation model employs all the binary variables $W_{of(i,t)}$, $U_{of(i,t)}$, $K_{of(i,t)}$ and $U_{on(i,t)}$. The optimisation model includes the constraints in Eqs. (6.19) – (6.25) from the model of offline washing in Section 6.5.2, and the constraints in Eqs. (6.40) – (6.43) from the model of online washing in Section 6.5.3.

The degradation and recovery model is a combination of the two different washing models. This combined model is described by the constraints in the general form in Eqs. (6.29) – (6.30) and the linearised form in Eqs. (6.37) – (6.39). The recovery model in the combined online and offline washing scenario is given by Eqs. (6.54) and (6.55).

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - W_{of(i,t)})(1 - R_F \cdot U_{on(i,t)}),$$
$$\forall i \in I,\ t > 1 \tag{6.54}$$

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)})(1 - W_{of(i,t)})(1 - R_F \cdot U_{on(i,t)}),$$
$$\forall i \in I,\ t = 1 \tag{6.55}$$

Equations (6.54) and (6.55) can be linearised with the use of an auxiliary variable $\Delta S_{on,aux(i,t)}$ as follows:

$$\Delta S_{(i,t)} \leq o_{(i)}(1 - W_{of(i,t)}),$$
$$\forall i \in I,\ (d^o_{of(i)} = 0, t \in T) \vee (d^o_{of(i)} > 0, t > (\nu_{(i)} - d^o_{of(i)})) \tag{6.56}$$

$$\Delta S_{(i,t)} = 0,$$
$$\forall i \in I,\ d^o_{of(i)} > 0\ ,\ 1 \leq t \leq (\nu_{(i)} - d^o_{of(i)}) \tag{6.57}$$

$$\Delta S_{(i,t)} \leq \Delta S_{on,aux(i,t)} + o_{(i)} W_{of(i,t)},$$
$$\forall i \in I,\ (d^o_{of(i)} = 0,\ t > 1) \vee (d^o_{of(i)} > 0,\ t > (\nu_{(i)} - d^o_{of(i)})) \tag{6.58}$$

$$\Delta S_{on,aux(i,t)} - o_{(i)} W_{of(i,t)} \leq \Delta S_{(i,t)},$$
$$\forall i \in I,\ (d^o_{of(i)} = 0,\ t \in T) \vee (d^o_{of(i)} > 0,\ t > (\nu_{(i)} - d^o_{of(i)})) \tag{6.59}$$

$$\Delta S_{on,aux(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)}) - \Delta S_{aux(i,t)},$$
$$\forall i \in I, (d^o_{of(i)} = 0,\ t > 1) \vee (d^o_{of(i)} > 0,\ t > (\nu_{(i)} - d^o_{of(i)})) \tag{6.60}$$

$$\Delta S_{on,aux(i,t)} = (\delta S_i^o + X_{(i,t)}) - \Delta S_{aux(i,t)},$$
$$\forall i \in I, d^o_{of(i)} = 0,\ t = 1 \tag{6.61}$$

$$\Delta S_{aux(i,t)} \leq o_{(i)} U_{on(i,t)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t \in T) \vee (d^o_{on(i)} < \gamma_{(i)}, t > \gamma_{(i)} - d^o_{on(i)}) \tag{6.62}$$

$$\Delta S_{aux(i,t)} = 0,$$
$$\forall i \in I, d^o_{on(i)} < \gamma_{(i)},\ 1 \leq t \leq \gamma_{(i)} - d^o_{on(i)} \tag{6.63}$$

$$\Delta S_{aux(i,t)} \leq (\Delta S_{(i,t-1)} + X_{(i,t)}) R_F + (1 - U_{on(i,t)}) o_{(i)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t > 1) \vee (d^o_{on(i)} < \gamma_{(i)}, t > \gamma_{(i)} - d^o_{on(i)}) \tag{6.64}$$

$$(\Delta S_{(i,t-1)} + X_{(i,t)})R_F - (1 - U_{on(i,t)})o_{(i)} \leq \Delta S_{aux(i,t)},$$
$$\forall i \in I, (d^o_{on(i)} \geq \gamma_{(i)}, t > 1) \vee (d^o_{on(i)} < \gamma_{(i)}, t > \gamma_{(i)} - d^o_{on(i)}) \tag{6.65}$$

$$\Delta S_{aux(i,t)} \leq (\Delta S_{(i,t-1)} + X_{(i,t)})R_F + (1 - U_{on(i,t)})o_{(i)},$$
$$\forall i \in I, d^o_{on(i)} \geq \gamma_{(i)}, t = 1 \tag{6.66}$$

$$(\delta S^o_i + X_{(i,t)})R_F - (1 - U_{on(i,t)})o_{(i)} \leq \Delta S_{aux(i,t)},$$
$$\forall i \in I, d^o_{on(i)} \geq \gamma_{(i)}, t = 1 \tag{6.67}$$

6.5.5 Objective function

The objective function is given by Eq. (6.68) and it includes the operating and maintenance costs, and start up and shut down costs. The first term represents the total electricity cost. The $W_{el(i,t)}$ gives the power used by compressor i in time period t. This power is equal to the summation of: (a) the power consumed when the compressor is clean, $W^c_{el(i,t)}$, which depends on the operating conditions, for example mass flow rate and pressure, and (b) the extra power consumption $\Delta W_{(i,t)}$ due to degradation. The $\Delta W_{(i,t)}$ depends on the cumulative time periods of operation. The parameter d_c is the duration of the period of the finite time horizon and $C_{el(t)}$ is the electricity price in [m.u./kWh], where m.u. is the monetary units. The power consumption of the clean compressor $W^c_{el(i,t)}$ is given by Eq. (5.7).

The $C_{st(i)}$, $C_{f(i)}$ are the costs related to startup and shutdown of compressor i respectively. The units of the costs are in [m.u./event], where the event corresponds either to a startup or a shutdown. The $C_{of(i)}$ and $C_{on(i)}$ are the costs of the offline and online washings, and their units are in [m.u./wash]. The C_D is the cost when a compressor changes header, in units [m.u./change], and the $C_{ext(e,t)}$ is the cost to purchase product $e = \{O_2, N_2\}$ at period t, [m.u./kg of product e] related to external sources in the case that the demand cannot be satisfied from the air separation plant.

$$min \quad \overbrace{\sum_{t\in T} C_{el(t)} \sum_{i\in I} \sum_{j\in J_{(i)}} d_c \cdot W_{el(i,t)}}^{\text{electricity cost}} + \sum_{t\in T}\sum_{i\in I} \overbrace{(C_{st(i)}S_{(i,t)} + C_{f(i)}F_{(i,t)}}^{\text{start up and shut down cost}}$$
$$+ \overbrace{C_{of(i)}W_{of(i,t)} + C_{on(i)}U_{on(i,t)})}^{\text{washing costs}} + \overbrace{\sum_{t\in T}\sum_{i\in I} C_D D_{(i,t)}}^{\text{change header cost}} + \overbrace{\sum_{t\in T}\sum_{e\in E} C_{ext(e,t)} d_c M^{ext}_{(e,t)}}^{\text{purchase products}} \tag{6.68}$$

Equation (6.68) presents the complete objective function when online and offline washings are available. When the analysis of a compressor station involves only

one type of washing then the objective function considers the respective terms of costs. This objective function is an extension of the objective function used in the Section 5.12.

6.5.6 Terminal constraints of maintenance model

Equations (6.69) and (6.70) give the constraint regarding the offline washing of compressors. They describe that a minimum number of λ_k Offline Washing Episodes (OFWEs) have to take place within the time period $[T - t_S^*,\ T]$, where $t_S^* = t_S + max\{\nu_{(i)}\}$ and t_S is a number of days before the end of the time horizon.

$$\sum_{t'=T-t_S^*}^{T} \sum_{i \in I} K_{of(i,t')} = \lambda_k \tag{6.69}$$

$$\sum_{t'=T-t_S^*}^{T} K_{of(i,t')} \leq 1, \qquad \forall i \in I^{NOFF} \tag{6.70}$$

The set $I^{NOFF} \subseteq I$ involves all the compressors which have not been under offline maintenance before the start of the optimisation. Equation (6.70) guarantees that a compressor will be maintained only one time in the time period $[T - t_s^*,\ T]$. Moreover, the constraint described by Eq. (5.67) which ensures that the contents of the storage tanks are full at the end of the time horizon is taken into account in the optimisation formulation.

The terminal constraints have to be investigated using heuristics from historical operation and literature. The decisions of these constraints are strongly related with the particular configuration of the plant of the compressors in the plant.

6.5.7 Description of numerical application

The condition-based maintenance optimization is tested in the industrial air separation plant of BASF in Ludwigshafen, Germany. This section considers the eleven multi-stage centrifugal compressors of the plant. Again, the compressors supply three headers with compressed air and there are five small compressors, $i \in I^s = \{i1, i2, i3, i4, i5\}$, driven by electrical motors with nominal power rating of some MW, $W_{el,s}^{No}$. Moreover, there are six large compressors, $i \in I^b = \{i6, i7, i8, i9, i10, i11\}$ with power rate of more than ten MW each, $W_{el,b}^{No}$, and $W_{el,b}^{No} = 2W_{el,s}^{No}$. Table 5.1 gives the feasible region, i.e. boundaries of mass flow rates and pressures, of the compressors derived from data analysis of past operation.

The description of the plant has been described in Section 5.10 in which Fig. 5.11 gives the schematic of the plant. The first header, $j1$, collects the compressed

air for utilities in the industrial complex of BASF. The other two headers $j2$ and $j3$ are connected with two air separation columns $u1$ and $u2$ respectively.

The optimisation model uses a finite time horizon of thirty days and uniform time period of one day. Table 6.8 provides the main parameters of the optimisation model. This example considers minimum run time $\omega_{(i)}$ equal to nine days for the large compressors and for the small compressors equal to five, six, seven, six and five days for compressors $i1, i2, i3, i4$ and $i5$ respectively. The values of the maximum running times and the rest of the parameters such as $\nu_{(i)}$, $\gamma_{(i)}$ and λ_{on} are selected to reflect typical industrial practices. The value of the R has been chosen based on a reasonable average recovery factor according to observations in literature.

A typical duration for offline washing with other minor maintenance actions (such as inspections and corrective repairing) is three days. This value has been used for all the compressors apart from compressors $i1$ and $i5$ for which the time periods of the maintenance actions account for two and five days respectively.

Table 6.8: Main parameters of operation and maintenance.

Symbol	Units	Values	Comment
t	d	1	Time period
T	d	30	Total number of periods of the horizon
$\omega_{(i)}$	d	5 - 7	Minimum run time of small compressors
$\omega_{(i)}$	d	9	Minimum run time of large compressors
$o_{(i)}$	d	65	Maximum run time of small compressors
$o_{(i)}$	d	75	Maximum run time of large compressors
$\nu_{(i)}$	d	2 - 5	Duration of OFWEs
$\gamma_{(i)}$	d	10	Minimum duration between two ONWEs
R	-	0.2	Recovery factor after an ONWE
λ_{on}	-	1	Maximum ONWEs in a time period
λ_{off}	-	3	Maximum OFWEs in a time period
λ_k	-	2	Parameter of Eq. (6.69)
t_s	d	10	Parameter of Eqs. (6.69) and (6.70)

Table 6.9: Cost parameters of operation and maintenance.

Costs	Units	Values	Comment
$C_{st(i)}$	m.u./start up	44.55	start up cost of small compressor
$C_{st(i)}$	m.u./start up	89.10	start up cost of large compressor
$C_{of(i)}$	m.u./wash	4.32	offline wash. cost of small compressor
$C_{of(i)}$	m.u./wash	8.60	offline wash. cost of large compressor
$C_{on(i)}$	m.u./wash	0.86	online wash. cost of small compressor
$C_{on(i)}$	m.u./wash	1.72	online wash. cost of large compressor
C_D	m.u./change	2.27	cost for changing header

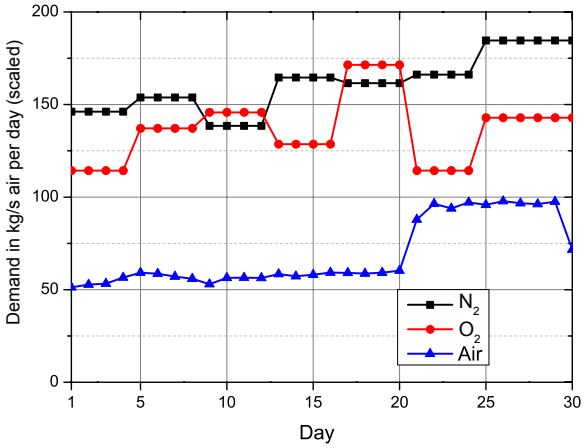


Figure 6.22: Production targets of the air separation plant for thirty days.

The ratio between extra power consumption and nominal power for one month, $\Delta W/W_{el}^{No}$ is 4.5% and 3.3% of the small and large compressors respectively. These values are based on the work from Cicciotti et al. (2014a) which used a detailed first principles model of an industrial compressor to estimate the profile of the degradation over time. The degradation rates are calculated based on monitoring parameters (for example performance) of the compressor using a linear approximation.

Table 6.9 provides the values of all the major parameters of the costs of the objective function. The start up costs for small and large compressors were assumed equal to the energy consumed from the motors of the compressors for half a day. In this example it is assumed that there is not a major energy consumption for switching off a compressor, therefore this cost is zero. The washing costs are estimated using the assumptions from Aretakis et al. (2012) and Fabbri et al. (2011). Therefore, Table 6.9 gives the values of the costs of a washing episode in this particular case study. The cost for changing header has been explained in Chapter 5.

A baseline study is formulated to examine the optimal operation and maintenance plan of compressors with different types of washings. This case uses the information from Tables 5.8, 6.8 and 6.9. Moreover the information of the initial state of the system is provided by Table 6.10. The base case employs modified production targets for O_2, N_2 and compressed air for utilities from real industrial data. Figure 6.22 gives the production targets for O_2 and N_2 in kg of compressed air (scaled units).

Table 6.10: Initial state of the system.

Compressor	*i1*	*i2*	*i3*	*i4*	*i5*	*i6*	*i7*	*i8*	*i9*	*i10*	*i11*
Header	-	$j1$	-	-	$j2$	$j2$	-	$j1$	-	$j3$	$j3$
$\tilde{\omega}_{(i)}$	0	6	0	0	40	22	0	20	0	40	55
$\tilde{\psi}_{(i)}$	30	0	18	20	0	0	30	0	30	0	0
$\delta S^{o}_{(i)}$	20	6	50	0	40	22	20	20	0	40	55
$d^{o}_{of(i)}$	1	0	0	3	0	0	0	0	1	0	0
$d^{o}_{on(i)}$	-	2	-	-	-	-	-	-	-	1	5

6.5.8 Results and discusions

All given data and reported results are normalised and made dimensionless due to confidentiality reasons. All optimisation problems have been solved in GAMS/CPLEX 11.1, under default configurations, in an Intel(R) Core(TM) i7-2600 CPU @3.4 GHz with 8 GB RAM. A zero optimality gap has been imposed in all problems instances.

The baseline study considers three different optimisation problems with respect to each available washing method: only online washing (ON), only offline (OFF) and both washings (ON+OFF). The specifications of each optimisation problem and the values of the objective functions of each solution can be seen in Table 6.11.

The number of variables of the OFF case is larger than the ON case and the number of variables in the ON+OFF case is considerably larger than in the OFF case due to the consideration of the extended mathematical model described in Section 6.5.4. Furthermore, Table 6.11 shows that solving the problem in the OFF case takes relatively more time than the other two cases. The reason for this is that, in the OFF case, a compressor can be washed only if it is offline. Therefore, the offline washing cost is implicitly connected with the cost of a start up. Indeed, if a compressor which has been washed after a shut down has to to operate again, then it has to start up with a start up penalty. On the other hand in the ON case the online washing can prolong the operational time of the compressor without shutting it down.

Table 6.11: Problem specifications and values of the objective function for each scenario.

Cases	Eqs.	Binary variables	Continuous variables	Nodes	CPU (s)	Obj (m.u.)
ON	14,276	2,640	4,380	514	51	983
OFF	14,638	3,297	4,050	749	137	1,022
ON+OFF	16,601	3,627	4,707	522	85	996

Offline washing results

Figure 6.23a gives the Gantt chart of the scheduling of the compressors of the OFF case. The figure gives the operational status of each compressor (online or offline mode), the connections between compressors and headers and the washing status over a time horizon for thirty days.

The general trend of the solution of the optimisation is that the compressors should operate as much as possible providing compressed air to the headers with which the compressors are connected in the first time period. A remark is that the connections compressors-headers in the first period is not necessarily the same with those of the initial state as can be seen in Table 6.10. For example, compressor $i8$ can satisfy the load of header $j1$ without the contribution of another compressor as the load of this header is relatively low compared to the load of the air separation units connected with headers $j2$ and $j3$ (see Fig. 6.22). In the latter cases, more than one compressor is needed to meet the demand at each header.

As can be observed from the results, the initial state influences the optimal solution. For instance, Fig. 6.23a shows that in the beginning of the optimisation compressor $i11$ is switched off and it is not used at all. In addition, in the beginning of optimization compressor $i2$ has changed from header $j1$ to $j3$. The reason for this is that compressor $i11$ has higher minimum limit of mass flow rate than compressor $i2$, therefore compressor $i11$ switches off and compressor $i2$ satisfies the

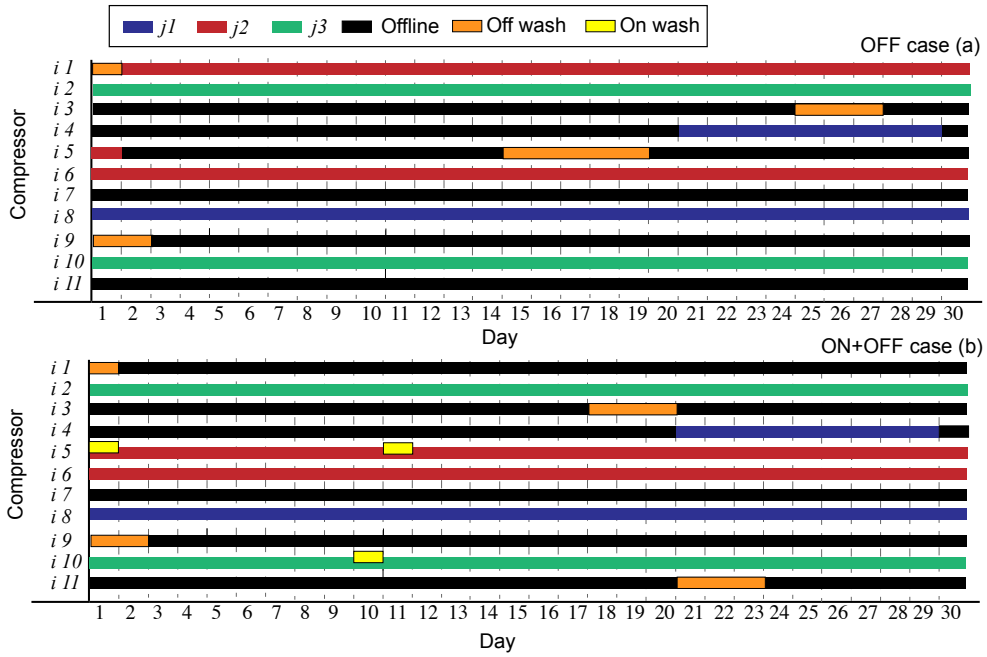


Figure 6.23: Gantt charts of the scheduling of the compressors for baseline case with only offline washing (a) and both online and offline washing (b).

load of header $j3$ along with compressor $i10$. These changes support the further decisions which switch off compressor $i5$ at Day 2 and start up compressor $i1$ connecting to header $j2$.

The optimization suggests maintaining compressors $i3$ and $i5$ which have been operated more than the others as can be seen from the values of $\delta S^{o}_{(i)}$ in Table 6.10. Hence, this decision leads to lower maintenance costs compared to the scenario which could have decided the washing of a larger compressor such as compressor $i11$. Table 6.9 shows that the offline washing cost of a small compressor is lower than the cost of large compressor. In the section of the time horizon between Day 15 and Day 30, the two washing episodes of compressors $i3$ and $i5$ are derived from the terminal constraints in Eqs. (6.69) and (6.70) with the consideration of $\lambda_k = 2$, and $t^*_S = 15$ d.

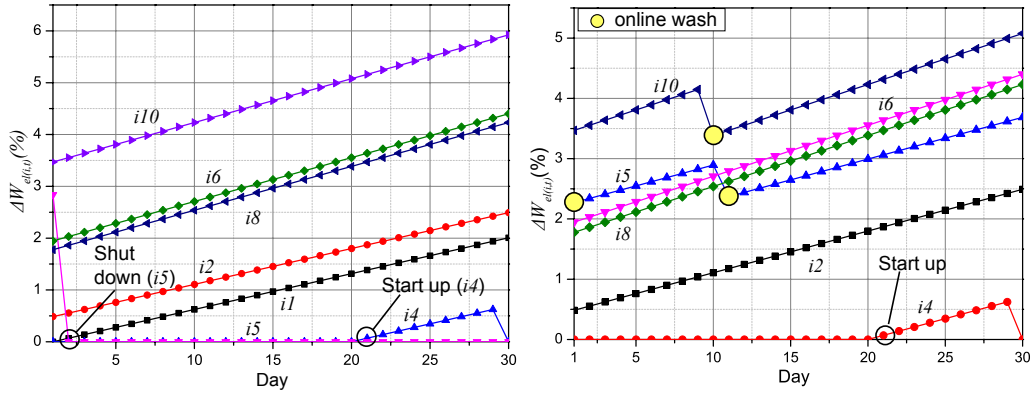
The previous statements show that there are many factors which influence the best decisions for optimal operation of the compressors and these factors depend on the knowledge of the future information such as forecast of the demand and the decisions for the succeeding time periods. Moreover, the values of the parameters of the problem influence these decisions and examples of these parameters are the initial operating and shut down times, and the values of the penalty costs.

Thus, even the most well-trained and experienced team of managers and operators could find it difficult to take the best decisions without having a sophisticated optimisation model based on the systematic use of the previous mentioned information. The nature of the problem is combinatorial involving a large number of scenarios, and therefore it is impractical and probably impossible to identify the optimal patterns for the best decisions when there is not the use of optimisation.

Results from the combination of offline and online washing

Figure 6.23b illustrates the Gantt chart of the scheduling of the compressors of the base case with available both online and offline washings, ON+OFF case. The solution of the ON case results in the same Gantt chart as in the ON+OFF case apart from the existence of offline washing episodes, therefore the ON Gantt chart is not presented. Additionally, the numbers and times of the online washing episodes (ONWEs) are the same in both cases.

The results from the schedule of the ON+OFF case show that online washing is used along with offline. In this case compared to the OFF case, compressor $i5$ is not switched off but it is washed online. Hence, both schedules included cleaning of compressor $i5$, the OFF case with offline washing, but the ON+OFF with online and keeping the compressor on. The reason is that this small compressor is the second most fouled one after compressor $i11$. As the schedule of the OFF case switched off compressor $i11$ the same can be observed in the schedule of the



(a) OFF base case

(b) ON+OFF base case

Figure 6.24: Extra power consumption (scaled) over time.

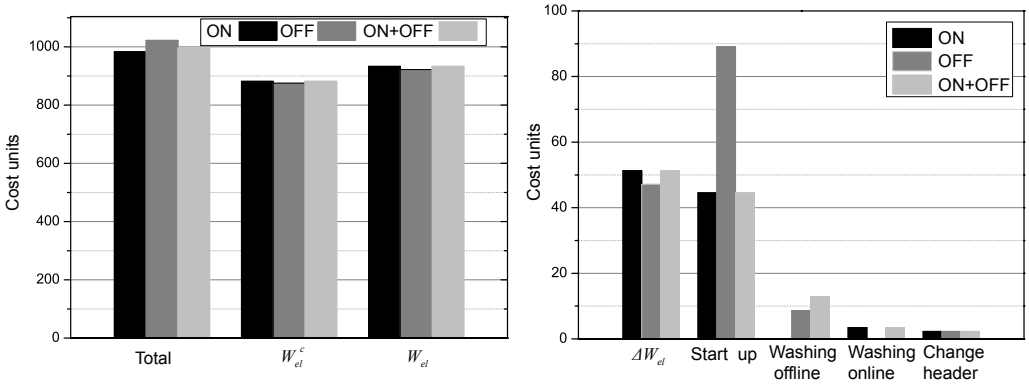
ON+OFF case. In the latter case, compressor $i5$ remains online and therefore only compressors $i7$ and $i11$ can be washed offline. The other smaller compressors cannot be washed offline as compressors $i1$, $i4$ have been washed before the beginning of the optimization and the other compressors $i2$, $i5$ remain online over the whole time horizon.

Figure 6.24 illustrates the extra power consumption $\Delta W_{el(i,t)}$ of each compressor i in scaled units, this means that if the real extra power is $\Delta W^*_{el(i,t)}$ in kW and a fixed parameter $\Delta W^s_{(i)}$ is given in kW, then the vertical axis shows the scaled variable $\Delta W_{el(i,t)} = \Delta W^*_{el(i,t)}/\Delta W^s_{(i)}$ with no units.

The ON case results in minimum total cost compared to the other two cases as can be seen in Fig. 6.25a. However, the ON case does not include terminal constraints as cases OFF and ON+OFF considering them. The total Equivalent Operating Time (EOT) in days of the compressors are 405, 321 and 280 in ON, OFF and ON+OFF cases respectively. Therefore, the ON+OFF case achieves 2.5% lower total cost than the one in the OFF case and the compressors have operated 41 days less. Therefore, the former case achieved to meet the demand with decreased operational costs and to wear less the compressors due to less equivalent operating time compared to the latter case.

Figure 6.25b displays that in the case of ON+OFF the total cost is lower, even if the costs of the washings are higher, than in the case of OFF. However, the total electricity W_{el} consumed in the ON+OFF is higher than in the case of the OFF. This demonstrates that the maintenance strategy considerably influences the operation. The main reason that the schedule in the ON+OFF case is less expensive than the OFF is that the online washing complements to the offline, thus this results in fewer start up events. The results which show that a compressor has to operate online continuously as much as possible is mostly in line with the industrial policy. This is justified as Fig. 6.23a shows that compressor $i5$ shuts

down after 41 days and Fig. 6.23b shows that the same compressor operates for 70 days equal to 53.2 equivalent operating days.



(a) Total cost and electricity costs.

(b) Distribution of costs.

Figure 6.25: Baseline case: total and electricity costs (a) and other costs (b) for thirty days.

6.5.9 Different degradation rates

Description of numerical examples

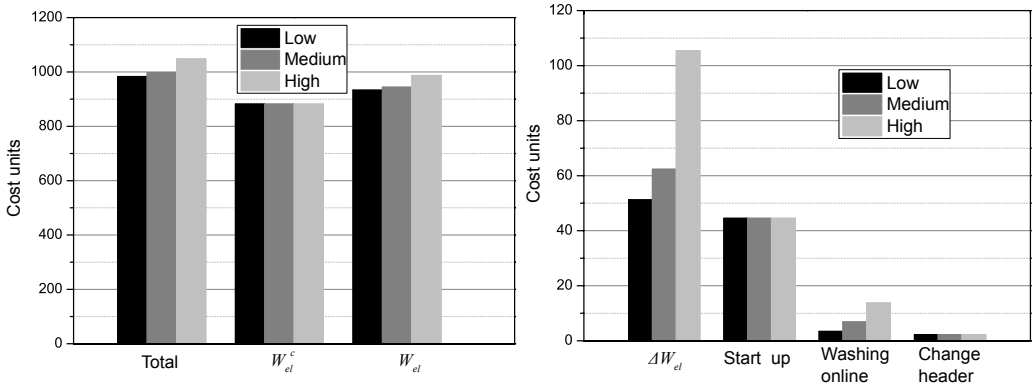
Section 6.5.9 examines the influence of different degradation rates on the scheduling of the compressors. In the baseline case the degradation rate is based on 4.5% and 3.3% extra power consumption per month of the small and the large compressors respectively in the air separation plant in BASF, Ludwigshafen, Germany. These degradation rates define the low degradation rate case (Low case). A Medium and a High case consider 6% and 9% extra power consumption per month for small and large compressors. Aretakis et al. (2012) stated that 10% extra power consumption per month is relatively significant but it can also be realistic.

The structure of the network of compressors and headers, the air separation units, storage tanks and customers are the same as in those of the baseline case. Moreover, the input from the Tables 5.8, 6.8, 6.9 and 6.10 remain the same.

Online washing

Figure 6.26 presents the total cost and distribution of costs per case with different degradation rates, Low, Medium and High, when there is only online washing. According to Fig. 6.26a, the higher the degradation rate is, the more the total cost increases. Figure 6.26b shows that the increase in total cost is a result of the increased cost of the total extra power consumption and the washing costs. The difference in total extra power, ΔW_{el}, between High and Medium is much higher than in the difference between Medium and Low cases. The number of online

washings are three, five and ten for Low, Medium and High cases respectively. The power of the cleaned compressors, W_{el}^{c}, is the same for all three cases and this means that different degradation rates do not influence the operation. This is explained from the fact that the compressors do not have differences in configuration in the three schedules according to optimisation output, and the start up and change-header costs are the same.



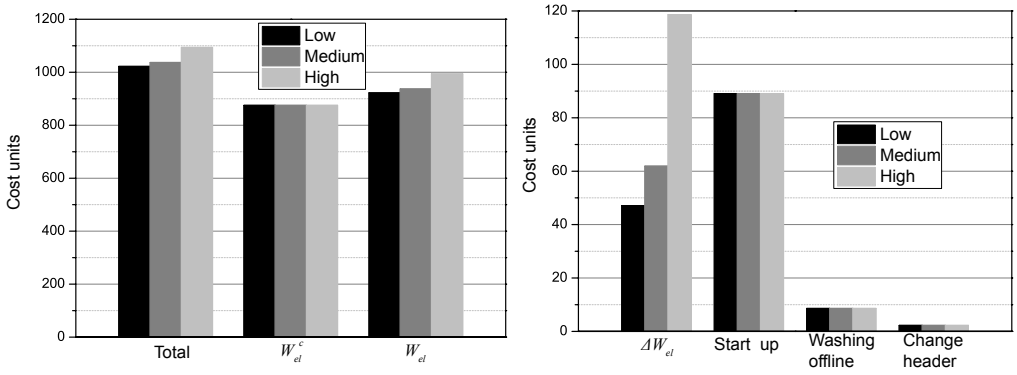
(a) Total cost and electricity costs.

(b) Distribution of costs.

Figure 6.26: Total cost and electricity costs (a) and other costs (b) for different degradation rates when there is only online washing for thirty days.

Offline washing

Figure 6.27 illustrates the total cost, electricity cost distribution and other costs when only offline washing is available. The decisions of operation and maintenance of the Medium and High cases are the same with those of the baseline case. For this reason the W_{el}^{c} and offline washing costs are the same for all three cases. The difference in total extra power between medium and low degradation rate is



(a) Total cost and electricity costs.

(b) Distribution of costs.

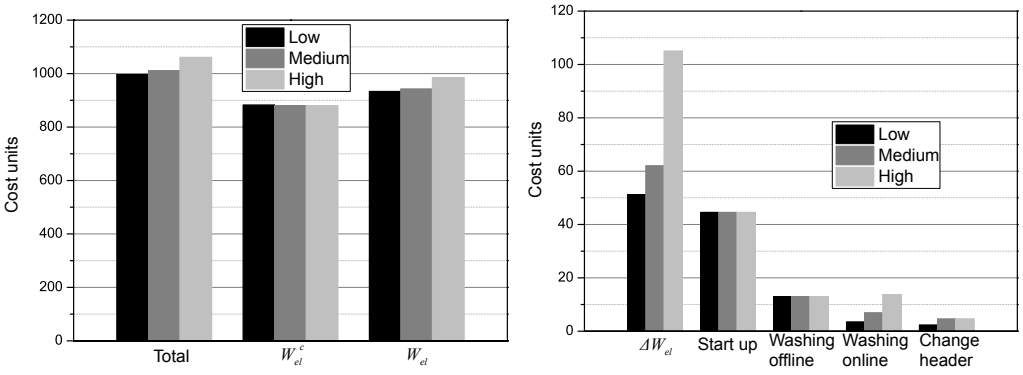
Figure 6.27: Total cost and electricity costs (a) and other costs (b) for different degradation rates when there is only offline washing for thirty days.

31.3%, and the difference between high and medium is 91.6%. This results in higher total cost in the High case. Moreover, the optimisation suggests keeping the online compressors operating as much as possible to avoid start up costs or possible change-header costs.

Both online and offline washing

Figure 6.28 shows that the combination of online and offline washings achieves reduced total costs by 2.5%, 2.5% and 3.0% in each degradation rate case respectively compared to the OFF case presented in the previous paragraph (Offline washing). However, only in the High case the extra power ΔW_{el} cost is lower in the case of the ON+OFF case than in the OFF case. The total cost of the OFF case includes increased start up costs, on the other hand the ON+OFF case is more flexible with lower total start up cost. The previous observations support the use of both online and offline washing, especially when the Equivalent Operating Time (EOT) of the ON+OFF case is reduced by 35% compared to that of the OFF case. This shows that the operation of compressors in the former case would be more flexible for a following scheduling problem as the compressors have been maintained in a more optimal way in the current scheduling problem.

Figure 6.29 shows the scheduling of the compressors and the online and offline washing episodes timings in the case of the high degradation rate (High case). The resulting schedule is different than the schedule when the degradation rate is lower, as shown in the schedule of the base case in Fig. 6.23b. The optimisation does not suggest washing a compressor after the minimum possible period, $\gamma_{(i)}$ days, as can be seen in the case of compressor $i2$. This demonstrates that a periodic fixed schedule, which is usually used in a typical industrial preventive maintenance



(a) Total cost and electricity costs.

(b) Distribution of costs.

Figure 6.28: Total cost and electricity costs (a) and other costs (b) for different degradation rates when both offline and online washing are considered for thirty days.

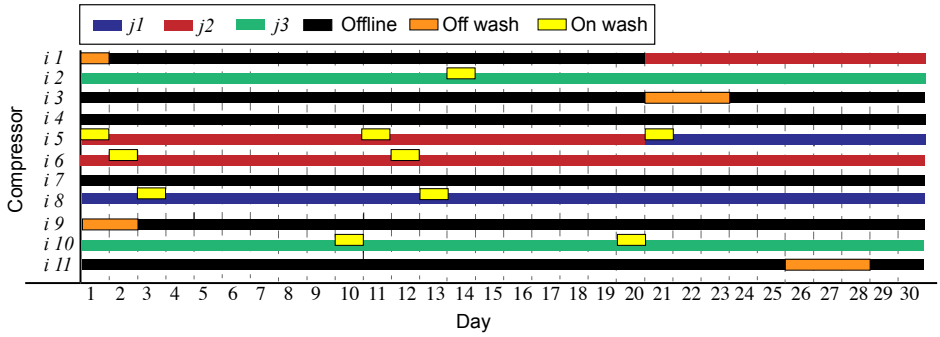


Figure 6.29: Scheduling of compressors with online and offline washing, and high degradation rates.

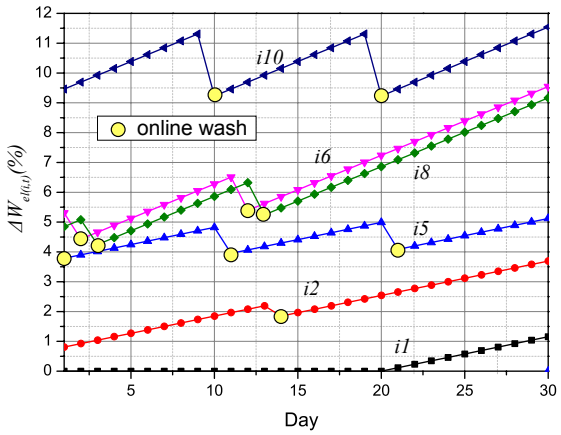


Figure 6.30: Extra power consumption of compressors with online and offline washing, and high degradation rates.

strategy, would increase the total costs if maintenance and operation had not been integrated in the optimisation model. Figure 6.29 shows that compressor $i5$ should be maintained in a periodic way. This does not apply to the maintenance frequency of the other compressors according to the figure.

Figure 6.30 gives the profile of the extra power ΔW_{el} of each compressor over the time horizon of the optimisation. The online washing model assumed that the application of online washing recovers a fixed percentage of the extra power of a compressor at the period the washing occurs. This means that for higher values of extra power consumption, the effect of the washing becomes greater. Moreover, Figure 6.29 shows that the optimisation suggests washing the more fouled compressors (shown in Table 6.10), $i5$, $i6$, $i8$ and $i10$, with a high frequency. Compressor $i5$ is the most fouled among the small compressors and it is washed three times, which is the maximum possible number of washing episodes in the total period of the optimisation. Compressor $i2$ is the least fouled, thus the optimisation suggests only one washing as the fouling accumulation is not high.

6.5.10 Case with a less flexible system

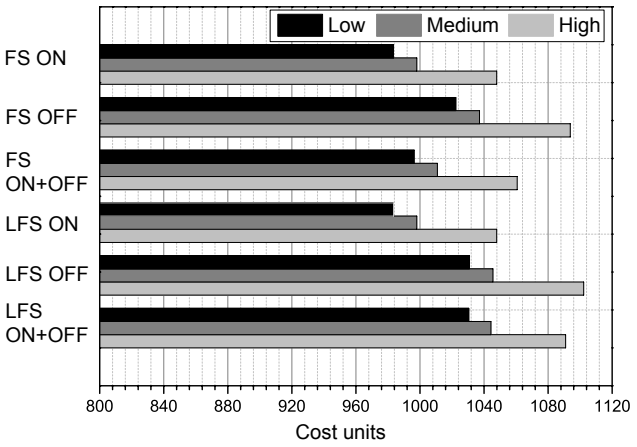


Figure 6.31: Comparison of total costs between a flexible system, Flexible System (FS), and a Less Flexible System (LFS)

Section 6.5.10 investigates the case with a reduced number of installed compressors compared to the baseline case. The motivation is that compressor stations in the industry may not involve a large number of spare compressors. This study examines the behaviour of the operation and maintenance of a less flexible system considering different methods to wash the compressors.

The system of the base case which includes all the compressors is called Flexible System (FS) and the system with the reduced number of compressors is called Less Flexible System (LFS). The LFS considers three small compressors $i \in I^s = \{i2,\ i4,\ i5\}$ and four large compressors $i \in I^b = \{i6,\ i8,\ i10,\ i11\}$.

Figure 6.31 shows the results from the Less Flexible System (LFS) and Flexible System (FS) study for each washing method used, i.e. ON, OFF and ON+OFF, and for each degradation rate case, i.e. Low, Medium and High. The results illustrate that the less flexible system is associated with higher total costs in all cases compared to the flexible system, apart from the cases with the only available online washing (case ON). Indeed, the costs of both systems in the ON case are the exactly the same.

The less flexible system has higher total cost in the OFF scenario and in all degradation rate cases by 0.8% and increased total cost in the ON+OFF case by approximately 3%. There is no difference in the ON case. This explains that the online washing along with the offline and with the use of spare compressors (i.e. flexible system) improves significantly the operation compared to the less flexible system case. This remark is important as compressors eventually have to be washed offline at some point.

The impact of the flexibility of a system, which is associated with the increased number of spare compressors, on the total cost can be seen in the case of the Flexible System (FS) where the ON+OFF washing reduces the total cost compared to the OFF washing case. However, this reduction is less important in the case of the Less Flexible System (LFS).

The capital costs in the case of a compressor station with a reduced number of parallel units are lower compared a compressor station with a number of spare units. According to literature (Saidur et al., 2010), the capital cost of a compressor is significantly lower than the operational and maintenance costs in its lifecycle which is usually more than 30 years. Therefore, a framework to determine optimal operation and maintenance of an existing installation of compressors is necessary to reduce operational costs independently if the selection of the number of the compressors and the decision of the proper sizing of the compressors was successful at the front end design of the plant. A sophisticated approach which examines the simultaneous optimal selection of the installed compressors and their sizing, and the operational planning of the plant can be a future topic of research.

6.6 Summary of the chapter

This chapter presented different optimisation models and frameworks to cope with the integrated operation and maintenance of compressors. First, a basic maintenance model was presented in Section 6.2. This model is coupled with the operational optimisation model which was presented in the previous Chapter 5. The basic maintenance model determines the times of maintenance of compressors restricted to a given time window. An illustrative example of the air separation plant (Example 3) compared the optimisation with given fixed maintenance and the optimisation with the use of the integrated framework. The comparison showed a reduction in total cost of the integrated approach compared to the fixed maintenance by 10%. An industrial example (Example 4) demonstrated a reduction in power cost and at the same time a schedule of the compressor with reduced number of start ups of compressors is provided compared to the case with fixed maintenance.

In Section 6.3, the integrated optimisation model of operation and maintenance is implemented into a rolling time horizon framework which is known as reactive scheduling. This framework solves optimisation problems which correspond to scheduling of the compressors in every iteration in which the time horizon is moving forward ("rolling"). The rolling time horizon framework can cope with real-life problems as it deals with uncertainty to some extend. The framework deals with uncertainty in parameters such as demands and availability of compressors by

updating the parameters in every iteration. The application of the rolling time horizon approach can be improved by estimating the best length of the time horizon. There is a trade-off between length of the time horizon and accuracy in the forecasting of the parameters. Moreover, the longer the time horizon the more benefit is expected assuming that the forecasting error of parameters is constant for different lengths of time horizons.

Section 6.4 presented a framework for integrated optimal operation and maintenance of large compressors of an export gas compressor station of Statoil in Norway for long periods. The framework receives input, for example the forecasted demand in pressure and in flow, the upstream pressure of the gas and the initial state of the configuration of the compressors. Industrial compressor maps were implemented into a MILP model. Moreover, the MILP model considers the operational and maintenance activities of the compressors. The framework uses the MILP model to suggest the best decisions which minimise the total costs. The results of the optimisation framework illustrated the best schedule of the operation and maintenance of the compressors which would be a basis for an industrial tool to help the users of the station to decrease costs, increase the lifecycle of the compressors and guarantee flexibility of the overall system. A comparison with a case with fixed maintenance schedule, showed that the integrated framework achieves savings of 224,000 EUR in three months (i.e. 0.84% improvement) from the optimal solution of the case which does not consider the maintenance as a degree of freedom.

Finally, Section 6.5 presented an integrated optimisation framework which can be used to optimise operation and maintenance of multiple compressors with different types of washings, namely offline and online, and their combination. This method was tested in a numerical example of the BASF air separation plant. The optimisation framework determines when and how often to wash the compressors using each washing method. The results showed that the best maintenance strategy to decrease the total costs and at the same time to minimise the wear of the compressors is a combination of offline and online washing. In the case which a compressor network involves spare compressors, the use of both washings decreases the total costs and increases the availability of the compressor significantly. The results have shown that the optimisation framework can provide enhanced decision support leading to optimal operation and maintenance. By the modification of the constraints of compressors and plant, the optimisation framework can be applied to other systems which include large power rate compressors working in parallel.

7 Critical evaluation and suggestions for future work

7.1 Description of the chapter

This chapter evaluates the outcomes of the research study on the optimisation of compressor stations in different industrial applications. Chapter 1 defined the objectives of the research study:

- Objective One was to develop a Real Time Optimisation (RTO) framework to optimally share the load among parallel compressors in real time to deal with short-term changes in the operation.
- Objective Two was to develop a multi-period optimisation framework to optimally schedule the operations of compressors for long time periods.
- Objective Three was to implement maintenance activities into the operational optimisation framework and achieve simultaneous optimal operation and maintenance.

Section 7.2 presents an evaluation of the achievements of the objectives. There is a discussion on the research work for each objective, and the relevant scientific contributions are identified. The strengths and assumptions of the methods used to achieve each objective are examined, and the future work related to each objective is reported. Finally, Section 7.3 highlights the recent outcomes and developments following on from the research study of the thesis.

7.2 Evaluation of the achievements of the objectives

7.2.1 Objective One

Description of research work for Objective One

Chapter 4 focused on the first objective which was the development of the Real Time Optimisation (RTO) of parallel compressors. The chapter presented a comprehensive optimisation framework which can use raw industrial data in real time to update process models used in an optimisation model. The operation of

the RTO can be see in Figure 4.2. The suggested optimisation model optimises the compressors in means of minimum power consumption and provides the best set points of the individual loads of the compressors to the control system. The framework uses an online data detection algorithm to identify when the system is in steady-state, so an RTO loop can initiate. This framework was applied to the air compressors of the BASF air separation plant with the use of industrial historical data provided by the industrial partner.

Contributions of the optimisation framework for Objective One

The major contributions related to the first objective are summarised below:

- The RTO framework employed data-driven models which update the models for estimating power consumption and the feasible window of operation, so as to represent the real-time conditions of the compressors at the point of the optimisation. The PhD study demonstrated that the operational costs are reduced with the use of the RTO compared to operation with industrial strategies (Xenos et al., 2015a).

- The RTO framework is the state-of-the-art for optimising a network of compressors, i.e. compressors connected in parallel configuration. This contribution is considered novel as the RTO framework is applied to the air compressor station in BASF which includes compressors which have been operating for many years. The original motivation of this study to optimise the operation of compressors was that compressors with similar nominal spec-

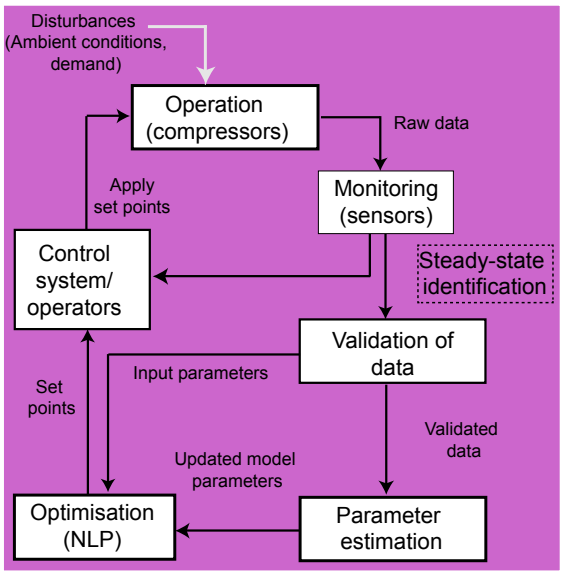


Figure 4.2: Detailed description of the structure of the components of the RTO.

ifications (at the point of their purchase) have different power consumptions and characteristics after many years of operation.

- The description of the integrated approach of the RTO (supervisory control) and the upper automation level of scheduling highlighted the need to examine the interactions among them. An example is that if the RTO problem is infeasible, this means that a new configuration of the compressors has to be chosen. The selection of the best compressors to operate online is one of the outputs of the scheduling.

- The application of the RTO to a system of compressors demonstrated the development of algorithms for offline and online steady-state detection using a multivariate approach. The thesis showed which process variables should be monitored by the algorithm. Moreover, the RTO algorithm optimises the operation considering slow changes in the ambient conditions of the operation. The thesis has presented a method suitable for detection of steady operation of all compressors in a network of parallel compressors (Xenos et al., 2014b).

- Another contribution of the suggested RTO approach is that the data-driven models of the compressors express the power consumption of the motors of the compressors as an explicit function of basic process variables. Therefore, there is no need to estimate efficiency. This is beneficial, because estimating the efficiency on-line while the compressor is operating is a difficult problem. For the needs of the RTO this is sufficient to estimate the optimal distribution of the total load as the optimisation minimises the summation of the power consumptions of the online compressors without the need to determine the efficiencies. The efficiency of the system is not calculated but for the needs of the optimisation its value is not necessary.

  The reason is it difficult to estimate the efficiency of the compressors is because of the lack of measurements of the multi-stage compressors. This problem is common in industrial applications which involve multi-stage compressors. For example, the compressors of Statoil did not have enough measurements to calculate the efficiency with the use of process data. The use of the efficiency in the objective function of the RTO adds non-linearities which results in a harder optimisation problem to solve. The computational time to solve the optimisation problem is an essential factor for online applications. Therefore, the thesis describes a new way of thinking to model the behaviour of compressors for the purposes of online optimisation and this method can help to develop a practical industrial RTO implementation. This method was presented in a conference paper (Xenos et al., 2014a), and the full work in the journal article (Xenos et al., 2015a).

Assumptions and future work related to Objective One

The assumptions of the developed RTO approach are the following:

- As previously mentioned, measurements from a multi-stage compressor may be limited, and therefore the development of a rigorous physical-based model is not possible. Then, the use of a data-driven model is an attractive way to represent the power consumption as a function of process variables. However, there are some limitations by using a data-driven model. The data used to generate the model define a regression domain and the model is valid within the regression domain. Indeed, the regression domain is developed from data of past operation. Therefore, if the operation is restricted to a small operational range, then the model is reliable within a small region. However, the optimal point may exist outside of this feasible region. The limitation is that the optimisation could not find this point and most importantly the user of the RTO application does not know if the operation can be improved further.

- Local areas of the regression domain may involve only a few observations or even no observations. The data-driven model is expected to predict with lower accuracy in these areas compared to areas with many observations. However, the boundaries of the regression domain define these areas as valid areas of the model. Kahrs and Marquardt (2007) have discussed this topic and the authors have suggested the use of a criterion which determines the confidence of the prediction of the model for this part of the region. For the objectives of the PhD and demonstration of the RTO framework, this thesis did not investigate this topic of the local areas with sparse observations of the regression domain. However, a future industrial implementation of the RTO needs to consider the suggestion from Kahrs and Marquardt (2007).

- According to Romagnoli and Sanchez (2000), basic methods to reconcile industrial data require redundancy in the system under consideration. The reconciliation of the data can increase the accuracy of the data-driven models. The redundant measurements will be used for mass and energy balances calculations. The system of the case study 1 is an air compressor station which involves several parallel air compressors. For example, for the mass balance of the system, a redundant measurement would be a measurement at the downstream header of the compressors which will provide the total mass flow. However, the installation in the air compressor station in BASF does not have extra measurements and the company cannot provide redundant measurements, thus the reconciliation of data is impossible for this case.

The installation of new instrumentations to capture the necessary redundant measurements requires a capital cost. An open question is if the benefit of the extra installation can repay this investment. The influence of the data reconciliation on the modelling of compressors and consequently on the load sharing optimisation could be a topic for future work. The outcome of this future work could suggest a method for data reconciliation of models of compressors for the purpose of industrial implementation.

- The online application of the RTO did not study the transitional periods from one steady-state before the application of the set points from the RTO and the subsequent steady-state after the application of these set points. The reason for this is that the dynamic behaviour of the compressor station is out of the scope of the thesis. However, observations from industrial visits and real tests showed that when the compressors change operating point, they reach a new steady state in a relative short period and so the impact of neglecting the transients is likely to be minimal. Further study needs to investigate the actual application of the RTO to the plant through real experiments or by developing a dynamic model. This has to be included in the list of actions for the investigation of the practical implementation of the RTO to the BASF air separation plant as presented in the next Section 7.3.1.

- A suggestion for future work is for the the data-driven models to use tighter constraints. The general description of the convex hull was described in Section 5.6.3, but MILP equations were used for the scheduling problem. The implementation of the convex hull approach in the RTO model has to use the algebraic form as it is presented in the paper by Kahrs and Marquardt (2007). This implementation is expected to increase the accuracy of the models of the RTO. The accuracy of the models is a critical topic for models generated from process measurements. The first reason is that predictions of the power consumption are not reliable outside of the domain defined by the observations which fit the model of the power consumption. The second reason is that the objective function of the model of RTO (shown on page 95) depends on the power consumptions of the online compressors. Therefore, the accuracy of the models influences the optimal solution related to the selected set points. If the models are inaccurate, then the selected points from the minimisation of the objective function cannot be trusted.

- The PhD thesis did not study the RTO with the use of first-principles models as the necessary measurements were not available for all the online compressors. One of the compressors in BASF had installed measurements and a

PhD student in the Department of Mechanical Engineering in Imperial College London in the same European project which funded the current PhD study developed a first-principles model of this compressor. This model is based on a large set of non-linear thermodynamic, aerodynamic and mass balance equations. This model can estimate the power consumption of the BASF multistage centrifugal compressor for large range of operation and for various operational conditions as the model is based on physical equations. Future work can investigate how to generate models for the other compressors based on the developed model of the one compressor. Then, a second investigation could use the developed models of the online parallel compressors into the RTO framework replacing the data-driven models. This investigation could examine if this approach is computationally efficient and compare the results with the results from the RTO using data-driven models of the same compressors.

- In the contribution of the first objective it was mentioned that the thesis identified a need to investigate the interactions between RTO (supervisory control) and the upper automation level of scheduling. This topic is not studied in the thesis due to the time restriction of the PhD studies. However, this topic is essential as the real operation would need to deal with these interactions in case both RTO and scheduling are implemented in a compressor station. A remark here is that the RTO topic is responsible of the process operation department of the compressor station, however the scheduling of the compressor is part of the decisions of departments at a higher level of plant hierarchy. The integration of RTO and scheduling will need to be a joint effort between these departments.

7.2.2 Objective Two

Description of research work for Objective Two

Chapter 5 dealt with the second objective of the thesis and presented a general mathematical framework which can estimate the schedule of parallel compressors for long time periods. The mathematical framework develop a multi-period approach with a Mixed Integer Linear Programming (MILP) model including binary and continuous decision variables. The continuous variables represent quantities such as mass flow rates and pressures of compressors at each time period. The binary variables model discrete events, for example a compressor starting up, shutting down and changing header in each time period.

The output of the framework is a Gantt chart which shows the operational mode of each compressor, i.e. online or offline mode, and the connection of each

compressor to each header per time period. Moreover, the aggregated mass flow rates of the compressors are estimated. These decisions minimise the total costs of the compressor station which include power consumption, shut down, start up and change-header cost.

For the purposes of scheduling, the thesis examined one case with air compressors of an air separation plant which compress ambient air (presented in Section 1.4.1) and a second case with compressors of an export gas plant which compress natural gas at high inlet pressures (presented in Section 1.4.2). Figure 5.1 shows that the scheduling problem of case 1 considered data-driven models for compressors with inlet guide vanes and the problem of case 2 employed models based on compressor maps for compressors with variable speed drives.

Contribution of the optimisation framework for Objective Two

The results of the scheduling of compressors on the air separation plant have been published in collaboration with Dr Georgios Kopanos (Kopanos et al., 2015). The scientific results of the scheduling of natural gas compressor station presented in Section 5.17.4 is in preparation for publication. The major contributions of the thesis related to the second objective are summarised below:

- The novelty of the research study on the scheduling of compressors stems from the fact that the developed optimisation approach in this thesis emphasises the operation of individual non-identical compressor units. Previous authors such as Ierapetritou et al. (2002) and Mitra et al. (2012) who worked on the optimisation of the operation of air separation plants considered the plant as a black box without considering the individual compressors as degrees of freedom. Hence, there was no method to optimally operate the individual compressors of the plant. Furthermore, in the case of the optimisa-

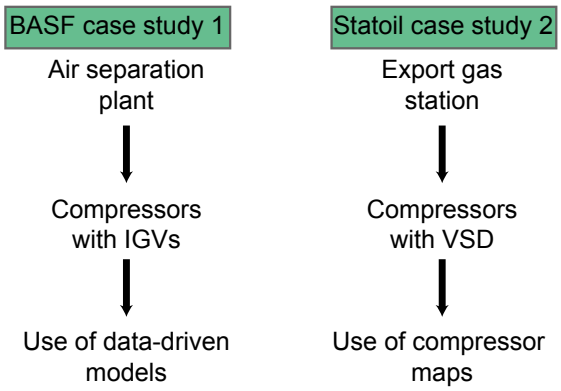


Figure 5.1: Classification of type of application, actuator of compressors and method of modelling of each case study.

tion of natural gas networks, a compressor station with multiple compressors in parallel was modelled as a single compressor unit assuming that all the compressors of the station have similar characteristics and performances over time.

In practice, the operators of a process plant or of a compressor station of a natural gas network have to decide which compressors to operate online and how much to load them. Indeed, these decisions influence the major power consumption of the whole plant (in the case of the process plant) or of the whole natural gas network. The PhD thesis developed a new method which allows to take the best decisions to minimise the total costs of the compressor station including the cost of the energy consumed. This new method is a multi-period optimisation problem involving operational aspects (such as startups and change-headers actions) and the power consumptions of the individual compressors of the compressor station of the plant.

- Chapter 5 presented a generic systematic approach for modelling and optimisation of compressor stations. This means that this approach can be used for different types of applications which involve compressors. The thesis presented the formulations of optimisation models for two cases, an air compressor station and a natural export gas station (shown in Fig. 5.1). The main operational aspects of compressors of various applications are the same, for example constraints for minimum and maximum run or shut down times.

  Another remark is that the basic model for optimal operation of compressors can be a extended for the use for other compressor applications. An example is the optimisation model for integrated operation and maintenance of compressors which was demonstrated in Chapter 6.

- The expression of the power consumption and the feasible window of operation of compressors can be modelled with different methods. Chapter 5 demonstrated a data-driven method using process data (based on the methodology in Chapter 4) and a method which uses compressors maps provided by performance tests. Chapter 5 demonstrated how first to digitise and store a compressor map as a table, and subsequently to use the table for developing mixed integer linear programming equations which describe the behaviour of the compressor.

- The use of data-driven models in the scheduling problem considered the power consumptions of the compressors as functions of their basic process variables. As explained in the contributions of the thesis in the RTO case

(page 221), the use of this type of data-driven model is beneficial because it does not need to estimate the efficiencies of the compressors. The inclusion of the efficiencies of the compressors would add non-linearities in the scheduling problem and this would result in a hard MINLP model that would be difficult to solve. It is important to notice that in the case of the natural gas case study the industrial partner provided compressor maps including the information of the efficiencies (in the case of the air separation plant the efficiencies of the compressors were not given), however the scheduling problem of the natural gas stations in Section 5.7.1 on page 125 considered only the power consumptions and not the available efficiencies to avoid non-linearities in the optimisation model.

- Chapter 5 also presented extended MILP models capturing typical industrial approaches to the optimisation of export natural gas compressors. Compared to the unconstrained distribution of load (Optimal method), these models optimise the operation with different methods for distributing the load including equal split and equal surge margin. The extended models can be seen in Section 5.16 on page 140. The scheduling versions with the equal split and equal surge methods are the better versions of the current applied industrial policies of operation. The equal split and the equal surge margin methods for sharing the load are influenced by the simultaneous optimisation of the selection of the compressors. On the contrary, in the industry this approach which considers simultaneously selection of compressors and distribution of load is not established yet, thus the equal distribution of load and the equal surge margin are applied to a selection of compressors based on the experience of the operators and this will result in increased total costs.

- The optimisation models which were developed for the cases of the air compressors and the natural gas compressors can be used as a tool by their users to examine "what if" scenarios. For example, the numerical Example 2 in Section 5.17.4 examined the scenarios of the operation of different installation of compressors assuming that the plant replaced the small compressors with larger ones.

Assumptions and future work related to Objective Two

The assumptions and future work suggestions related to Objective Two can be seen below:

- The parameters of the costs of the change-header C_D, shutdown C_f and startup C_{st} of the objective function described by Eq. (5.61) on page 135

have to be estimated through experimental data and further studies. The thesis used values for these coefficients based on the literature and on the industrial experience of the users of the plants of BASF and Statoil. The values of the cost coefficients influence the output of the optimisation (i.e. values of decision variables and objective value) as they weight the cost terms of the objective function. Thus, the accurate estimation for each case study (air separation or natural gas compressors) is essential for a future implementation of the technology on the plant.

- The electrical losses of the motors which depend on different operational conditions are not included in the models which used compressor maps in the natural gas case study. The implementation of this modelling aspect would improve the accuracy of the optimisation results and it is important for an industrial application. On the other hand, data-driven models consider these losses, as explained in Section 4.3.3 on page 93.

- The solutions in the scheduling problem may be restricted because of the restricted domain of the feasible window of operation of the the data-driven models. It has been observed that the regression domain of a compressor which has been exclusively operating at a specific header is influenced by the past operation to this header. For instance, the range of the operational pressure of this header is between 5.0 and 5.5 bar for a specific period. Therefore, the model of the compressor operated at this header is defined based on this pressure range. In reality, the compressor can run at higher pressures than 5.5 bar. For the optimisation calculations this compressor would not be able to connect to another header with pressure more than 5.5 bar. Therefore, an optimisation solution with a change of the compressor to this header would be impossible as there are no past data for this pressure range.

  The users of the plant operate the compressors in order to satisfy the requested demand based on their experience. One solution to increase the search space of the optimisation is to run online tests so as the compressors and headers to operate within a larger range. The design of these tests can be an interesting topic for research, as an online test implies direct costs of the actual production operation, but if these tests help to carry out a more effective scheduling then there would be an economic benefit for the plant. This analysis can be new topic for future work. Moreover, these online tests can be used for the improvement of the data-driven models in the RTO approach.

  In the case of the models which use compressor maps, the latter problem is not observed as the compressor map provides a broader operational window.

In practice the compressor maps have been generated through performance tests but not on a frequent basis.

- The optimisation framework for the air separation case study used a linear function for the data-driven model given by Eq. (5.7). The use of a more complex function with quadratic and bilinear terms is expected to increase the accuracy of the prediction of the power consumption, however the scheduling model is expected to become more difficult to solve. The trade-off between accuracy and computational time should be examined. Another interesting study is to examine if the schedules from each method have major differences. For the purposes of the demonstration of the optimisation framework this thesis considered the linear functions, however, the implementation of this approach in the industry needs to carry out an investigation for models with the best trade-off between accuracy and computational power.

- In the case of the optimisation of the compressor station of the natural gas case the outlet pressure of the compressors was given in the PhD study. This means that the linepack [1] of the downstream pipes of the compressors was not included in the optimisation as a degree of freedom. With the inclusion of this decision variable, the optimisation model will be non-linear according to basic flow equations (van den Heever and Grossmann, 2003) and the resulting MILP model will be a MINLP. In the investigation of the linepack optimisation, the optimisation should determine the values of the $P_{d(j,t)}$ in Eq. (5.46). In this case, the optimisation should take into account flow and mass balance equations of the downstream pipes. The new optimisation model will consider the trade-off between the stored amount of natural gas in the pipes which influences the pressure at the outlet of the compressors and the power consumption of the compressors which depends on this outlet pressure too. The stored natural gas in the pipes provides flexibility desired by the plant to deal with fluctuations in the demand during the day. Moreover, unexpected breakdowns of the compressors can be handled with the help of the linepack.

  The thesis did not consider the downstream pipes in the natural gas case study to simplify the formulated optimisation model of the compressor station for the needs of the current PhD study. However, an extension of the PhD work can be very attractive for studying the whole gas network or part of the gas network including compressor stations (there can be more than one compressor station in the network) and pipes considering their linepack

[1] The linepack is a time-dependent phenomenon which describes the inherent capability of a pipe to store gas.

effects. This large-scale approach can be a new research topic which can expand the category of the multi-period pipeline optimisation which was explained on page 72. One suggestion for a future step can be the consideration of the single export gas station Kollsnes (from case study 2) with the inclusion of its downstream pipes. The downstream pipes Zeepipe IIA and IIB have approximately 300 km length and 1m diameter each, which means that they can store great amount of gas per day.

- The formulation of the scheduling of the compressors using equal split and equal surge margin (presented in Section 5.16) should be revisited as these problems are difficult to solve even with the use of a MILP model. As was discussed in Example 2 on page 161, Problems 7 and 8 could not find a solution with 0% OPTCR after 86,400 s.

- In the natural gas case study, the compressibility of the delivery point of the export gas station was assumed constant. This means that the compressibility of the gas at the outlet of the compressor is assumed the same as that of the inlet. The fact that the temperature at the delivery point is assumed constant, and that also the pressure at the delivery point is also known, then the compressibility can be estimated with the use of generalised compressibility factor graphs (Wikipedia, 2015*a*). For academic purposes this assumption does not influence the validity of the model of the thesis, however, this assumption has to be investigated in the case of the industrial implementation.

- The current study used different durations of the time horizons of the examined scheduling problems. All the formulations in Chapter 5 employed four-week time horizons. However, the following Chapter 6 which studied the integrated framework for operation and maintenance considered durations of three weeks in the case of the rolling horizon approach in Section 6.3, and three months in the case study of the natural gas compressor station in Section 6.4.

  The duration of the time horizon influences the solution of the scheduling problem, as a larger time horizon which considers information for longer periods will result in more accurate decisions (in the ideal case of deterministic information). On the other hand, the information coming from a long time horizon embeds higher uncertainty compared to the case of a shorter time horizon. Therefore, there is a trade-off between short and long time horizon which influences the accuracy of the decisions. The investigation of the trade-off between length of horizon and uncertainty is part of the future work.

- The objective function for the case study of the air separation plant is given by Eq. (5.61) on page 135 with the power consumption of the motors of the compressors to be given by Eq. (5.7) on page 121. Equation (5.7) is used in conjunction with the rectangular regression domain defined by the contraints (5.8) and (5.9). The expression of power consumption of Eq. (5.10) and the convex hull given by Eqs. (5.11) – (5.22) could be used instead of the previous equations of power consumption and rectangular regression domain. The resulting optimisation model is expected to increase the accuracy of the predictions, however the computational problem would be harder to solve. The thesis did not present results from the optimisation model with implemented the convex hull due to time constraints of the PhD studies. However, this approach is considered in the work under development which is described in Section 7.3.2. This work is based on the optimisation model of this thesis.

- In practice, the benefits from the optimal distribution of load against typical load sharing strategies stem from the fact that the compressors which operate in parallel are dissimilar. In an ideal case in which all the compressors have the same specifications and conditions (i.e. performance), then there would not be significant improvement if the optimal method is applied instead of the typical load sharing strategies. In practice, a compressor station involves dissimilar compressors due to uneven operation and uneven maintenance of the compressors.

  However, in Section 5.17.4 two different compressor maps were used to model small and large compressors. Statoil provided the same map, which was generated at a specific time point, for the five small compressors and a second compressor map for the large compressor, which was generated at a different time point. A next step is to carry out an analysis for optimal operation of the natural gas compressor station after the industrial partner providing industrial maps for all six compressors generated at the same time moment. This can quantify the magnitude of the savings of the station with a better accuracy.

7.2.3 Objective Three

Description of research work for Objective Three

Chapter 6 dealt with the third objective of the thesis. It presented optimisation models and frameworks to cope with the integrated operation and maintenance of compressors. The developed methodologies can deal with maintenance namely minor corrective maintenance actions (for example repairs and replacements of

mechanical parts), major overhauls, and online and offline washing.

A basic maintenance model was presented in Section 6.2. This model is integrated with the operational optimisation model which was presented in Chapter 5. The basic maintenance model determines the start times of maintenance of compressors restricted to a given time window. In Section 6.3, the integrated optimisation model of operation and maintenance is implemented into a rolling horizon framework. The rolling horizon framework can cope with real-life problems as it deals with uncertainty to some extent. Section 6.4 presented a framework for integrated optimal operation and maintenance of compressors of an export gas compressor station for long periods. The integrated model includes different types of maintenance tasks such as major overhauls, and mechanical and electrical maintenance corrective tasks. Section 6.5 presented an integrated optimisation framework which can be used to optimise operation and maintenance of multiple compressors with different types of washings, namely offline and online, and their combination. Table 7.1 summarises the sections of Chapter 6.

Contributions of the optimisation framework for Objective Three

The contributions of the thesis related to the third objective are summarised below:

- The integrated approach of maintenance and operation can be a tool for the operators and managers of industrial compressor stations to reduce their energy consumption, to minimise the wear of the compressors with the minimisation of the number of startups and shutdowns, and to achieve a more practical operation by reducing the number of changes of the headers. The optimisation will find the best compromise among these costs.

- Chapter 6 demonstrated that maintenance and operation should be taken into account simultaneously in the optimisation. The results showed that the integration of maintenance and operation achieved reduced operational costs compared to the case of optimal operation with fixed maintenance. This was reported in Example 3 which studied the BASF air compressors in

Table 7.1: Description of sections of Chapter 6 with their respective optimisation models and types of maintenance.

Section	Optimisation model	Type of maintenance	Case study
6.2	Basic	Flexible maintenance	BASF
6.3	Basic in rolling horizon	Flexible maintenance	BASF
6.4	Extended basic	Major overhauls	Statoil
6.5	Extended basic	Online, offline washing	BASF

Section 6.2.4. By taking into account the integrated approach, the total cost of the operation reduced by 10% compared to the case which neglected the maintenance in the optimisation (Example 1-A). The industrial Example 4 also demonstrated a reduction in the total costs by 1.8% with the use of the integrated approach compared to the case of given maintenance (Example 1-B). Example 5 demonstrated that a natural gas compressor station can reduce its total costs with the use of the integrated operation and maintenance. The estimated cost reduction was 224,000 EUR for a period of three months.

- Chapter 6 presented that the integrated approach can deal with different types of maintenance activities. Every industrial case has different policies for maintenance and different resources, such as equipment and manpower. The optimisation framework is flexible to include maintenance constraints which are defined according to each specific industrial case.

- The demonstration of the basic integrated framework into the rolling horizon framework shows that the application of the integrated model can deal with uncertainty in parameters to an extent. In each iteration, parameters and constraints are updated and unexpected failures can be also taken into account.

- The methodology of the integrated approach which considered the washing of the compressors (e.g. online, offline) identified what are the model requirements for the prediction of fouling of multi-stage centrifugal compressors. When an accurate fouling model is developed, then the condition-based maintenance approach can become a practical predictive maintenance strategy in industry.

Assumptions and future work related to Objective Three

The integrated framework considers all the assumptions of the operational model of the compressors in Chapter 5. Moreover, there are a few more assumptions and suggestions for future work regarding the integrated approach:

- In Section 6.5, the condition-based maintenance approach is based on assumptions which are summarised in Section 6.5 on page 196.

- The non-linear equations of the model in Section 6.5 were linearised with the use of Big-M formulation, for example Eqs. (6.26) and (6.27). The model of this section has to be revisited as the resulting linear constraints are not computationally efficient. During the PhD studies, it was observed

that the real industrial problem could not be solved (unacceptable computational time), thus the numerical application on page 205 considered reduced demand which simplifies the optimisation problem.

- There is not a well defined model for predicting the accumulation of fouling at the moment. It is expected that parameters in addition to the cumulative time increase the accumulation of fouling. The rate at which the performance deteriorates is influenced by a large number of factors, this fact makes the prediction of the degradation rate impossible without infield observations of past behaviour of the compressor. The aim of Section 6.5 was to show the practical use of the optimisation framework and its potential use when advanced performance monitoring methods and explicit models of the accumulation of the fouling are available in the future. Although the assumption that the extra power consumption due to degradation depends on the cumulative time of operation and is sufficient for the current study, future work needs to examine the consideration of detailed models for fouling. Another PhD student in the Department of the Mechanical Engineering in Imperial College London in the same European project which funded the current PhD study is studying the topic of the fouling of multistage centrifugal compressors. Results from her work may be used to update the models of the condition-based maintenance optimisation.

- Section 6.5 compared a flexible system with a great number of compressors and a less flexible system with a smaller number of installed compressors. The capital costs in the case of a compressor station with a reduced number of parallel units are lower compared a compressor station with a number of spare units. The developed framework in this thesis determines optimal operation and maintenance of an existing installation of compressors. A sophisticated approach which examines the simultaneous optimal selection of the installed compressors and their sizing, and the operational planning of the plant can be a future topic of research.

- In Section 6.5, the models of the extra power consumption have to be improved to consider parameters such as mass flow rate and pressure. The development of advanced performance monitoring techniques may improve the prediction of the degradation with the update of the parameters of the models. The implementation of these techniques with a reactive scheduling approach similar to this used in Section 6.3 can lead to an effective real life industrial solution. This is part of future work. Furthermore, the identification of the terminal constraints of the integrated model (and the operational model) is still an open question which depends on the particular industrial

case. This issue could be resolved by solving a larger problem with a longer time horizon. The thesis focused on the academic aspects of this problem, thus the industrial implementation needs to study further the topic of the terminal constraints and their connections with the respective time horizon.

- If the integrated model is not implemented in a rolling horizon framework, then the problem assumes that the parameters are deterministic. The consideration of uncertainty in the parameters of the optimisation model, such as demand, have been presented in Section 6.3. The optimisation models in the other sections, for example the model considering the major overhauls for the natural gas case study, and the model considering online and offline washings for the air compressors can be implemented in a rolling horizon framework and this is part of future work. Furthermore, the update of the models in the rolling horizon framework has not been investigated. The thesis mainly focused on the deterministic approach of the scheduling of compressor and presented one case which can deal with uncertainty to some extend. More robust techniques to study the optimisation of compressors under uncertainty can be the next development of the work of this PhD study.

7.2.4 Summary of the contributions

Table 7.2 summarises all the major suggestions and classifies them as immediate improvements or as potential new research areas.

Table 7.2: Summary of the major topics for future work (IFOM stands for Integrated Framework of Operation and Maintenance).

Incremental improvement	Potential new research area
Examine the transitional periods in the RTO application	Work on methods to increase the reliability of the RTO data-driven models
Implement convex hull approach in the developed RTO	Develop a RTO approach with first-prinicples models
Examine influence of data-reconciliation on the RTO results	Investigate interactions between RTO and scheduling
Use different data-driven models in the scheduling objective function	Develop methods for online tests for improving RTO and scheduling models
Investigate trade-off between time horizon and information accuracy	Study linepack optimisation in the developed optimisation framework
Implement convex hull approach in the scheduling model	Expand field of pipeline optimisation with the inclusion of parallel compressors
Quantify savings when all compressor maps are available	Develop realistic fouling models to integrate with the IFOM
Examine connection between horizon duration and terminal constraints	Study integrated design and operation of compressors considering maintenance
Revisit optimisation models in Sections 5.16 and 6.5	Study optimisation of compressors under uncertainty

7.3 Latest outcomes of the thesis

7.3.1 Impact of the research

The research work in this thesis has great potential for impact on real industrial applications. In May 2015, the author of this thesis made a proposal and gained funding for working on the development of the proof-of-concept of the RTO for achieving energy savings in compressor stations in chemical plants. The funding is provided by the EPSRC Impact Acceleration Awards 2015 through Imperial College London. The author of the thesis is now employed at Imperial College London on that project leading a study for proof of the RTO concept in an industrial context and to provide quantified evidence for the savings that can be achieved. Real experiments on the air separation plant of BASF have been planned to quantify the savings. A comparison between operation with the application of the RTO and typical operation without the use of RTO will estimate these savings.

One of the aims of the proposed project is to develop a pre-commercialisation optimisation platform which can support the operators of a compressor station to distribute the load of multiple compressors. The project will test the methodologies developed and presented in this thesis and will identify the industrial considerations which represent the real-life situation and lead to a proof-of-concept that the optimisation platform can minimise the energy consumption in a real plant.

The project has to investigate that the energy savings are close to the values presented in the thesis under the consideration of practical constraints. These constraints can be the robustness of the optimization for various scenarios, the consideration of uncertainty in the data, and the availability of data to develop accurate models in every scenario. Moreover, the RTO will be tested online to guarantee solutions with a high level of confidence within an optimisation loop. The suggested platform has to also consider the available control structures and be integrated with the existing installed hardware and software structures of the plant.

BASF highlighted that the algorithms from the proposed project have the potential to be implemented commercially for optimization of compressor stations in BASF and in other plants. This means that the technology could be transferred to other industrial areas of BASF. Moreover, Statoil and any other company which operates compressor stations could potentially also use this state-of-the-art application. Another important development step is that the optimization platform should be fully automated. This development will follow after a period of its manual use on the plant which will start to deliver the energy savings and build trust in its decisions.

Finally, there may be potential for even broader interest. The work on real-

time optimization of compressors was presented at the launch of the Caterpillar Innovation and Research Centre on February 24th 2015, at which the Provost James Stirling signed the partnership between Imperial College and Caterpillar. The industrial visitors from Caterpillar identified uses for the topics in the proposal in a different sector, namely heavy automotive machinery.

7.3.2 Further work on the optimal operation of compressors considering energy management

Introduction

This section describes research work which is under development in a different but related area of the energy management of compressor stations. The reason this work is presented here is that it was done during the PhD thesis and uses the work on the optimisation of compressors as a foundation.

It is well established that industrial plants participate in electricity energy markets to take advantage of electricity price variations, for instance by increasing the production when electricity is cheap. However, there is another electricity market called reserve market, which could be of interest to the operators of industrial sites. Therefore, industrial sites with large energy consumption can participate in these markets through demand-side management whereby power producers and consumers (i.e. the industrial sites which participating in the reserve markets) communicate in a collaborative way to maintain stability in the electricity grid[2]. Industrial plants consume large amounts of energy and their operation is easier to control compared to the case of distributed residential users. The air separation plant can be considered as an example of a large industrial plant.

The electricity consumers (for example an air separation plant) can participate in different technologies of demand-side management which have been summarised by Palensky and Dietrich (2011). The technologies of interest are Demand Response (DR) programs which can be categorised into non-dispatchable programs

[2]The electricity grid delivers electricity from the generators to consumers via the transmission and distribution grids.

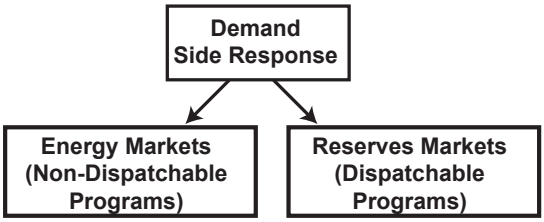


Figure 7.1: Classification of different demand-side response schemes.

and dispatchable programs as can be seen in Fig. 7.1. The Non-Dispatchable Programs which are related to energy markets allow the Transmission System Operator[3](TSO) to influence the power consumption in the demand side by giving predefined electricity prices. Dispatchable Programs, which are related to reserve markets enable the TSO to control the power consumption in the demand side by giving instructions in accordance to the requirements of electricity grid. In contrast to non-dispatchable programs, participants receive incentive payments in terms of being successful in satisfying the requirements of the electricity grid through orders given by the TSO.

An example of a dispatchable program is the demand offered by electrical consumers by reducing their load when the demand-supply balance is threatened. The reduced power which is offered by the consumer to the electrical grid is defined as *positive power reserve* according to the definition given in Fabozzi et al. (2013). Moreover, an electricity consumer can participate in a dispatchable program which can request from the consumer to increase power consumption in some situations when there is an excess of generation of electricity. In this case, the consumer has to increase its power consumption and the extra power the plant consumes is defined as *negative power reserve*.

However, the purpose of a plant is to produce products and not to provide services to the electricity grid through the reserve markets. This means that there is a need to assess the flexibility of a plant to provide services to the electricity grid while at the same time ensuring that the plant meets the requirements of its demand. This is the motivation to carry out an economic analysis of the operation of an air separation plant which participates in demand-side management schemes such as Demand Response (DR) programs. This analysis can quantify the incentives of the air separation plant which should receive in order to participate in these schemes.

Previous works and gap of knowledge

There are studies which focused on the scheduling of air separation plants with the considerations of energy constraints. Chapter 3 mentioned the studies by Ierapetritou et al. (2002), Karwan and Keblis (2007) and Mitra et al. (2012) which employed MIP models in an optimisation framework for the minimisation of production costs considering electricity prices. These studies used binary variables mainly to model the discrete operational modes of the operation of the plant, but neglecting the operation of individual compressors. Another study which considered variable electricity prices in the optimization of the operating conditions of

[3]The TSO is a company that is responsible for operating, maintaining and developing the transmission system for a control area and its interconnections.

air separation plants is from Zhu et al. (2010) without considering discrete actions such as startups and shutdowns of operational units.

The state-of-the-art of the participation of air separation plant in a dispatchable program has been recently presented by Zhang et al. (2015). The authors considered the simultaneous considerations of non-dispatchable and dispatchable programs. They also formulated a robust model for the uncertainty nature of the reserve markets. This study, however, focused on the concept of the air separation plant as an electricity producer using the stored liquid products to power an in-site cryogenic turbine providing electricity to the grid. Their model of the air separation plant was based on Mitra et al. (2012) using the same discrete operational modes. At present, practically no industrial loads are collaborating with the TSO of a grid through a dispatchable program, but the potential value of this collaboration for case studies such as an aluminium production have been examined by ALCOA (Todd et al., 2009). The limited number of participants in dispatchable programs can be explained due to the lack of a well defined framework for interaction between electricity grid and consumers. This is a question currently being addressed in Europe by ENTSO-E (The European Network of Transmission System Operators for Electricity)

Description of under development work

The work under development examines the participation of an air separation plant in a demand side response scheme without any extra installation. The work is based on the work presented in this thesis. The advantage of the suggested method stems from the detailed modelling of the compressors and air separation units which define the power consumption, capacity constraints and operational aspects with discrete nature of each unit (i.e. compressor, air separation units), for instance shutting down a compressor. An air separation column is rarely shut down and only for maintenance reasons. The modelling of the suggested method neglects the on or off modes of a column due to the short time horizon to be considered. The modelling considers the operation of compressors which influence the overall electricity costs of the air separation plant.

Performing a cost-benefit analysis is one method to ensure that the plant remains profitable. Process operators require decision making tools to find the optimal offer price so that the site can competitively participate in electricity markets, while maintaining profitability. Hence, the work under development proposes an optimisation framework to examine two case studies: (a) a non-dispatchable program considering various contracts of day-ahead markets (e.g. variable hourly electricity prices) with the simultaneous minimization of the operational costs of the plant and (b) a dispatchable program which requests flexibility from the com-

pressors by means of the modification of the baseline operation of the plant to offer power reserve to the TSO. Moreover, the work will propose how non-dispatchable and dispatchable programs can be managed together in the same plant.

Preliminary results were presented in the poster session of the conference *European Symposium on Computer Aided Process Engineering (ESCAPE 25)*. The full work is in preparation for publication in the journal of *Applied Energy*.

7.4 Summary of the chapter

Chapter 7 presented a critical evaluation of the achievements of the objectives of the thesis. It summarised the description of each objective and for each objective the chapter presented the respective contributions. Moreover, the chapter described and critically discussed the assumptions used for the mathematical frameworks presented in the thesis. Also, the topics for future work were classified in cases of immediate improvement and of potential new research area. Table 7.2 summarised the major topics for future work for both cases. Section 7.3 presented the impact of the research as the RTO will be studied further considering real life constraints. The RTO methodology is planned to be tested in the air separation plant of BASF and the results will be compared with the typical operation of the BASF users. There is also academic work in progress which is based on the PhD work on the optimisation of compressors. Section 7.3.2 introduces the work on the energy management of compressor stations of chemical plants.

8 Conclusions

8.1 Statement and justification of objectives

Chapter 1 stated that the aim of the PhD thesis was to achieve optimal operation of existing installed compressors of systems with large energy consumption. The thesis focused on two industrial systems, namely an air separation plant and an export natural gas compressor station. The introduction of the thesis described three objectives to achieve the above aim:

- Objective One was to develop a Real Time Optimisation (RTO) framework which optimally shares the load among parallel compressors in real time to deal with short-term changes in the operation.
- Objective Two was to develop a multi-period optimisation framework to optimally schedule the operations of compressors for long time periods.
- Objective Three was to implement maintenance activities into the operational optimisation framework and achieve simultaneous optimal operation and maintenance.

In Section 3, the literature review identified that there was a need to develop a systematic online approach to optimally distribute the load among the parallel compressors of the station. Chapter 4 dealt with the first objective of the thesis and presented the development and application of a real time optimisation framework for load sharing among compressors. The literature review identified that previous authors neglected the operation of parallel non-identical compressors of a compressor station. Thus, the second objective focused on this gap of knowledge to develop a methodology to schedule the compressors of a system for long time periods. Chapter 5 suggested a multi-period optimisation framework which was applied to the air separation and to the natural gas station case studies. The literature review also revealed the need to examine the simultaneous operation and maintenance of compressors. Chapter 6 dealt with this third objective of the thesis and presented the optimal operation compressors considering different types of maintenance activities such as corrective maintenance actions, major overhauls and washing of the compressors.

8.2 Conclusions of thesis for Objective One

The main evidence of achieving Objective One is the development of a comprehensive Real Time Optimisation framework which can optimally share the load among parallel compressors in real time and deal with short-term changes in operation. The description of the RTO framework was presented in Chapter 4. This work made it possible to demonstrate reduced operational costs on a simulation of real time operation of the industrial compressors of the air separation plant of BASF using historical data. Until now, there was not a systematic way to optimally share the load of compressors online considering varying operational conditions. Strategies in common use are to share the load using equal split among compressors, or running them with equal surge margin. The work of the PhD has shown a way forward to use updated models of the compressor from process data and estimate the best distribution of load among the compressors using optimisation. Moreover, the work has shown that the new method of RTO can be applied at a large scale relevant to the industry. Three papers have been published from this work, two in conferences (Xenos et al., 2014a,b) and one in a journal (Xenos et al., 2015a).

8.3 Conclusions of thesis for Objective Two

The main evidence of achieving Objective Two is the development of a multi-period optimisation framework which can optimally schedule the operations of compressors for long time periods. Chapter 5 presented the development of this framework using a mixed integer linear programming multi-period model. This work made it possible to generate the optimal schedule of compressors of the air separation plant of BASF and the export natural gas station of Statoil. Until now, compressor stations were treated as single units in natural gas optimisation problems. In previous work on the optimisation of air separation plants, the compressor station was modelled as a black box along with the air separation units. The work on the PhD study developed a method to schedule compressors which establishes the current state-of-the-art of the topic of the optimisation of compressor stations for long periods. The schedule can reduce the total costs of the operation, for example electricity cost, start up and shut down cost. The application of the method on the BASF industrial case study showed that the optimal schedule can improve the operation for a horizon of one month. The application of the method on the Statoil case study realised that the scheduling of compressors with the optimal load sharing method can achieve 0.88% and 1.03% reduced operational costs compared to the cases with equal split and equal surge margin methods respectively. Developments of this work have been published in one journal paper (Kopanos

et al., 2015) and another journal paper with Xenos as first author is in progress.

8.4 Conclusion of thesis for Objective Three

The main evidence of achieving Objective Three is the implementation of maintenance activities into the operation optimisation framework which achieves optimal operation and maintenance as was presented in Chapter 6. Chapter 6 presented the integrated framework of operation and maintenance with different types of maintenance activities such as major overhauls and washing of the compressors. The framework was applied to the case of the BASF air separation plant to show the benefits from the optimal operation with flexible maintenance compared to operation disregarding decisions for maintenance activities. The framework also made it possible to demonstrate the advantage of simultaneous optimisation and maintenance including major overhauls in the case of the Statoil natural gas case study for a three month time horizon. The literature examined the maintenance of a single compressor considering its performance and it also examined the optimisation of the operation of network of compressors neglecting their maintenance tasks. Therefore, the PhD study provides a new method for the optimisation of a compressor station with multiple compressors in parallel considering their maintenance. Four papers have been published from this work, two in conferences (Xenos et al., 2014c, 2015b) and two in journals (Xenos et al., 2015c, 2016).

8.5 Future work

The PhD study presented methodologies and frameworks for optimising the compressors of chemical plants and natural gas compressor stations. The previous sections of this chapter justified that the work in the thesis met the objectives given in the introduction. However during the PhD studies and after the achievement of the objectives several topics related to the optimisation of compressors were identified as future work:

- The RTO implementation in a plant needs to improve the robustness of the data-driven models considering the real life operation and the practical limitations related to the availability of process measurements.
- A new potential research area is to examine the use of first-principles models in the RTO framework, as this research area needs to work on complex physical models for online applications.
- The thesis identified the need to examine the interactions and possibly integration of automation levels of RTO and scheduling.

- The inclusion of the linepack effect and the modelling of the downstream pipes in the optimisation problem of natural gas compressor stations would result in a new research topic.
- The thesis studied optimal operation of an existing installation of compressors. However, it identified that the design of the compressors and their production planning of the compressors could be studied simultaneously.
- Furthermore, the current PhD study mainly focused on the deterministic approach of the scheduling of compressors. Therefore, the scheduling of compressors under uncertainty is another possible research area.

Immediate and other improvements for future work have been highlighted in Section 7 and a summary of them is in Table 7.2. The current PhD work has already launched two works which are under development, one work is the investigation of the RTO implementation in the BASF plant and the second work is the study of the participation of plants with compressor stations in electricity reserve markets. Both works are based on the scientific work developed in this thesis.

8.6 Final comment

Compressors are essential components of the chemical and petroleum industry and they consume large amounts of energy. The improvement of their operation will result in an impact for the industry, the environment and the society. The thesis has managed successfully to develop methods to potentially help the users of real plants to efficiently operate their compressors. Furthermore, the work of this thesis can be a foundation for further research on new areas such as the optimal energy management of systems with large energy consumption. The penetration of renewables in the conventional power generation system and the future increase in electricity demand will require to examine both energy efficiency and new methods to deal with these new circumstances and the optimal energy management of compressor stations can be one of this methods.

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