

A methodology for improved predictions of surface ground movements around shafts

David M. G. Taborda PhD DIC

(corresponding author)

Reader in Geomechanical Modelling

Department of Civil and Environmental Engineering, Imperial College London, London, UK

d.taborda@imperial.ac.uk

ORCID: 0000-0001-5391-2087

Antonio M. G. Pedro PhD DIC

Auxiliar Professor

Department of Civil Engineering, ISISE, ARISE, University of Coimbra, Coimbra, Portugal

amgpedro@dec.uc.pt

ORCID: 0000-0003-0271-7243

Helen Xia MEng ACGI

Geotechnical Engineer

Arup, London, UK

helen.xia@arup.com

Stuart Hardy MEng ACGI DIC PhD CEng FICE

Associate Director

Arup, London, UK

stuart.hardy@arup.com

Abstract

Shafts are typically employed in urban environments to provide access or ventilation to underground structures such as stations, railways or highways. The choice of design is determined, among other things, by the need to control settlements at the surface, often estimated during early design stages using empirical expressions. These have been shown to have limited accuracy, failing to account appropriately for the effect of shaft diameter on the ground movements associated with shaft excavation. This paper reviews empirical expressions available in the literature in the context of a large database of settlements induced by shaft excavation in London. A comprehensive set of detailed numerical analyses is performed to enable the development of a new set of expressions capable of predicting accurately the computed vertical and horizontal ground movements at the surface. The new expressions are shown to provide better predictions of the observed field data than predictive expressions available in the literature, establishing a new benchmark against which future proposals can be assessed.

Keywords

Excavations; shafts; ground movements; numerical modelling; UN SDG 11

List of notations

α – maximum normalised settlement

β – maximum normalised horizontal displacement

β_c – time-dependent normalised shotcrete stiffness

$\boldsymbol{\varepsilon}$ – strain tensor

ε_{vol} – volumetric strain

ϕ – angle of shearing resistance

ϕ_{int} – angle of shearing resistance of interface

ψ – angle of dilatancy

ψ_{int} – angle of dilatancy of interface

ν – Poisson's ratio

a – parameter of expression for determining horizontal displacements

b – parameter of expression for determining horizontal displacements

c – parameter of expression for determining horizontal displacements; cohesion

c_{int} – cohesion of interface

d – parameter of expression for determining horizontal displacements
 D – shaft diameter
 $D_{ext,PCS}$ – external shaft diameter for the zone supported by pre-cast concrete segments
 $D_{ext,SCL}$ – external shaft diameter for the zone supported by sprayed concrete lining
 $\overline{D_{ext}}$ – average external shaft diameter
 D_{ref} – reference diameter
 E – Young's modulus
 E_d – generalised deviatoric strain
 E_{28} – Young's modulus of shotcrete at 28 days
 G_{tan} – tangent shear modulus
 h – shaft wall thickness
 H – shaft depth
 H_{PCS} – extent of shaft supported by pre-cast concrete segments
 H_{SCL} – extent of shaft supported by sprayed concrete lining
 K_0 – at-rest coefficient of earth pressure
 k_h – horizontal permeability
 k_v – vertical permeability
 K_N – normal stiffness of interface elements
 K_T – tangential stiffness of interface elements
 K_{tan} – tangent bulk modulus
 n – shape factor defining extent of settlement trough
 N_{obs} – number of observations
 s – parameter controlling the effect of time on the stiffness of shotcrete
 S_h – horizontal displacement
 S_v – settlement
 t_{conc} – age of shotcrete
 t_u – unit time
 x – radial distance behind shaft wall
 y_{LC} – elevation of the top of London Clay
 EBS – Excavation before support
 PCS – Pre-cast concrete segments
 $RMSE$ – Root mean squared error
 SBE – Support before excavation
 SCL – Sprayed concrete lining

1. Introduction

With increasing urbanisation and associated need for infrastructure, underground structures, particularly those related with transportation, have become increasingly prevalent. In this context, the construction of vertical shafts, either temporary or permanent, are critical for the successful completion of most tunnelling projects (Xia, 2019). In recent years, several circular shafts with different sizes, diameter and depth, have been constructed in London and many more are expected in the near future (Schwamb & Soga, 2015; Faustin et al., 2018; Quigg, 2019).

As shown by Faustin et al. (2018) the construction method employed in the shaft construction is of paramount importance in the control and mitigation of the movements associated with their construction. Two main techniques are usually employed in the shaft excavation depending on the relative timing between excavation and the placement of the support: *Support Before Excavation (SBE)*, and *Excavation Before Support (EBS)*. Naturally, as shown by extensive monitoring data (Faustin et al., 2018), the movements associated with the latter technique are considerably higher and therefore need to be predicted with greater accuracy as there is greater potential to cause damage to nearby structures or services. However, until recently, the estimation of those movements, particularly at early design stages, was mostly based on an analytical expression proposed by New & Bowers (1994) based on the settlements measured at the Heathrow Express trial shaft. As a result, it is often suggested (New, 2017; Faustin et al., 2018) that the applicability of the New & Bowers (1994) expression remains largely confined to shafts of similar geometric characteristics (excavation depth, $H = 26\text{ m}$ and outer diameter, $D = 11\text{ m}$). In effect, despite several empirical expressions having been proposed in the literature for estimating both settlements (GCG, 2007; Mott MacDonald, 2013; Gaba et al., 2017; Newhouse, 2017) and subsurface ground movements (Le et al, 2019), they all rely, to some extent, on the prediction of that single case. An alternative approach has been suggested by Zheng et al. (2003), where a stack of spherical cavity contractions under constant volume conditions is adapted to the prediction of ground movements induced by the excavation of shafts. Similar to the traditional approach used for tunnelling works, this proposal employs the concept of volume loss to determine the distribution of horizontal and vertical displacements, both at the ground surface and at intermediate depths. In Zheng et al. (2003), values of volume loss of 0.01% for an *SBE* shaft, 0.075% for an *EBS* shaft and 0.15% for a combined *SBE* and *EBS* shaft were reported. Clearly, additional data from heavily instrumented shafts are required to allow a better understanding of how to estimate this parameter.

With the development of three major tunnelling projects in London, particularly the London Power Tunnels (LPT) project, it became possible to gather a large database of settlements induced by shaft

excavation. The results presented by Faustin et al. (2018) and later analysed in detail by Jones (2022) show that, although the original empirical proposal of New & Bowers (1994) has some merits, it cannot be fully generalised for other geometries, particularly for those consisting of larger shaft diameters and depths. By combining the extended settlement database presented in Faustin et al. (2018) with the results of detailed numerical modelling for a wide range of geometries, this paper proposes a new set of analytical expressions for estimating both settlements and horizontal ground movements at surface induced by excavation of shafts in London. A comprehensive comparison with other methods available in the literature is also undertaken, demonstrating that the proposed expressions lead to an increase in accuracy, while remaining sufficiently simple for use in the early stages of shaft design. Given the availability of data, the focus of this paper is on shafts in London Clay, although the followed approach can be generalised for any other well-characterised material.

2. Monitored shafts in London

2.1 Case studies

This study focuses on the prediction of ground movements associated with the excavation of shafts where the excavation progresses ahead of the placement of the support (*EBS*). Typically, excavation is carried out in 1.0 m to 1.2 m high sections, being comparable to the procedure adopted in tunnel excavation whereby the placement of the lining follows the excavation front (Faustin et al., 2018). Once the excavation is complete, the lining, either in the form of pre-cast concrete segments (*PCS*) or sprayed concrete (*SCL*), is constructed. Excavation can then restart for the subsequent ring, with this sequence being repeated until formation level is reached. As observed by Faustin et al. (2018), this methodology is adequate for construction in competent materials where substantial ingress of water is unlikely. In London, pre-cast concrete segments are used to support the superficial deposits that typically can be found overlying London Clay, with the support switching to sprayed concrete lining once excavation progresses into the latter material. In some cases, pre-cast concrete segments are jacked into the ground, which means that the support is already in place when excavation takes place and, therefore, this methodology is closer to *SBE* than to *EBS*, leading to smaller excavation-induced settlements. As expected, pre-installation of shaft linings (e.g. diaphragm walls, bored piles or steel sheet piles) – clearly the definition of a *SBE* process – reduces further the resulting settlements, as observed in several shafts built for the Crossrail project (Faustin et al., 2018). A final possible configuration exists combining *SBE* and *EBS* methodologies in a single shaft, with the former being employed in the top section of the shaft and the latter being used in the lower section. Examples of this approach include the Northern Line Extension shafts (Kennington Green, Kennington Park) as well as several of those in the Crossrail project (Fisher Street, Limmo Peninsula).

Table 1: Summary of the key characteristics of the EBS shafts reported by Faustin et al. (2018).

Designation	Shaft location	Construction method	Outer diameter, D [m]	Excavation depth, H [m]	Support thickness [mm]	Ground conditions
SJW2	St John's Wood no. 2	PCS	6.5	34.5	225	Made Ground (4.6m)
		SCL	10.6	12.5 (47.0)	300	Superficial Deposits (1.2m) London Clay
N&B	Heathrow	PCS	11.0	16.0	Not reported	Made Ground (2.0m)
		SCL		10.0 (26.0)		Superficial Deposits (4.0m) London Clay
ISL	Islington	PCS	11.1	27.7	300	Made Ground (3.2m)
		SCL	11.9	9.9 (37.6)	240	London Clay (28.3m) Lambeth Group
ECRT	Earls Court Road	PCS	11.1	29.9	300	Made Ground (1.9m)
		SCL	11.9	9.6 (39.5)	250	Superficial Deposits (0.8m) London Clay
SPC	St Pancras	PCS	11.1	35.7	300	Made Ground (2.0m)
		SCL	12.0	9.1 (44.8)	305	London Clay (43.6m) Lambeth Group
CGR	Channel Gate Road	PCS	13.1	22.2	325	Made Ground (2.2m)
		SCL	13.1	10.7 (32.9)	312	London Clay
WIL	Willesden	PCS	13.2	20.3	350	Made Ground (1.7m)
		SCL	13.9	8.7 (29.0)	225	London Clay
KG2	Kensal Green no. 2	PCS	13.2	30.2	350	Made Ground (1.9m)
		SCL	14.0	12.5 (42.7)	275	London Clay
EDR	Eade Road	PCS	13.2	26.7	325	Made Ground (1.4m)
		SCL	14.2	12.6 (39.3)	338	London Clay (37m) Lambeth Group
SJW1	St John's Wood no. 1	PCS	13.2	33.1	350	Made Ground (2.9m)
		SCL	14.4	12.5 (45.6)	315	Superficial Deposits (1.2m) London Clay
KG1	Kensal Green no. 1	PCS	15.7	17.7	350	Made Ground (1.8m)
		SCL	16.4	8.8 (26.5)	240	London Clay
HBY	Highbury	PCS	15.7	25.4	350	Made Ground (2.5m)
		SCL	16.7	9.8 (35.2)	320	London Clay (25.8m) Lambeth Group

Given the above, from the full database described in detail in Faustin et al. (2018) containing 27 different shafts in London, 11 *EBS* shafts from the London Power Tunnels project are selected in this study to be added to the Heathrow Express trial shaft described in New & Bowers (1994). These shafts are summarised in Table 1 in terms of geometrical characteristics (outer diameter D and excavation depth H) and lining type (*PCS* or *SCL*), together with a summary of the ground conditions. Clearly, while a good range of shaft depths is represented in this database (26.0 m – 47.0 m), the variety of

diameters is slightly more limited (10.6 m – 16.7 m, with the PCS part of St John's Wood no.2 shaft reducing to 6.5 m). Finally, it is important to note that, according to Faustin et al. (2018), the settlements recorded at the two shafts at Kensal Green (KG1 and KG2) were deemed anomalous as very small movements were observed. These were justified by the slow rate of construction employed due to the presence of contaminated ground. In the next sections, the displacements measured during construction of each of these 11 shafts are described and compared to the empirical methodologies available in the literature.

2.2 Empirical methodologies

Five methodologies are considered herein for estimating shaft displacements induced by their excavation. A detailed review and assessment of these methods is available in Newhouse (2017) and Jones (2022) and are only briefly outlined in this section. The earliest attempt at producing an analytical approach to estimate settlements due to shaft excavation was proposed by New & Bowers (1994) based on the field observations from the Heathrow Express trial shaft, which was characterised by a diameter $D = 11.0\text{ m}$ and a depth of $H = 26\text{ m}$:

$$S_v [m] = \alpha \cdot H \cdot \left(1 - \frac{x}{H}\right)^2 \quad (1)$$

where S_v is the settlement at a distance $x [m]$ behind the wall and $\alpha[\%]$ is a parameter which was suggested to take the value of 0.06%. Clearly, based on this expression, the settlement is assumed to be zero for distances greater than the shaft depth (i.e. $x > H$), while the maximum settlement, which is estimated to occur immediately behind the wall (i.e. $x = 0\text{ m}$), is 0.06% of the shaft depth. It is important to note that, as discussed in New (2017), the value suggested for α was determined based solely on the field monitoring installed at the Heathrow shaft, indicating that its use for other geometries may not necessarily yield reasonable results. Therefore, New (2017) proposed that it is taken as a fitting parameter, together with a 'shape factor' $n[-]$ that controls the distance from the shaft wall at which the settlement reduces to zero ($x = n \cdot H$):

$$S_v [m] = \alpha \cdot H \cdot \left(1 - \frac{x}{n \cdot H}\right)^2 \quad (2)$$

The settlements observed in the shafts listed in Table 1 were analysed by New (2017), who reported a consistent trend whereby larger shafts induce larger settlements, though considerable scatter in the data was present. The latter is unsurprising considering the difficulties posed by monitoring large structures in dense urban areas, particularly in the vicinity of the excavation, where construction activities and the movement of heavy plant may render accurate readings difficult. Based on the measured data, New (2017) proposed that an upper bound (i.e. conservative) value for the settlement could be obtained using Equation (2) with $\alpha = 0.06\%$ and $n = 1.5$. These values correspond to the

same maximum normalised settlement as proposed in New & Bowers (1994), with the effects of the shaft excavation extending to a larger radial distance, meaning that potential distortions due to differential settlements are smaller.

An alternative proposal for estimating the settlements based on the shaft geometry was established for the Crossrail project in GCG (2007), where a Gaussian distribution is adopted to ensure compatibility with the procedures adopted for tunnels and boxes:

$$S_v [m] = \alpha \cdot H \cdot e^{1/2} e^{-\frac{\left(x + \frac{2}{3}n \cdot H\right)^2}{2\left(\frac{2}{3}n \cdot H\right)^2}} \quad (3)$$

where α [%] (maximum normalised settlement) and n [–] (normalised radius of influence) are given as linear functions of the shaft diameter (see Table 2), up to $D = 25$ m, above which point the geometry ceases to impact these parameters. It is reported in GCG (2007) that this curve is designed to be reasonably conservative and hence is expected to provide an upper limit in terms of settlements. Additionally, in GCG (2007) a methodology to establish conservative estimates for horizontal ground movements due to shaft excavation is also provided, which is given in Equation (4), where positive displacements correspond to movements towards the excavation. Mott MacDonald (2013) proposed using the same general shape later also employed by New (2017) (Equation (2)) but using the parameters for maximum settlement and radius of influence suggested in GCG (2007) (Table 2). Following an extensive analysis of the settlements recorded in the shafts discussed in Faustin et al. (2018), Jones (2022) obtained values of α and n for Equation (2) that depend on the construction method (Table 3). Furthermore, Jones (2022) suggests that the effect of the diameter on the predicted settlements can be considered by introducing this geometric characteristic in the calculation of the extent of the settlement trough using $n = 0.1 \cdot D$.

$$S_h [m] = \left(1 + \frac{3}{2} \frac{x}{nH}\right) \cdot S_v \quad (4)$$

Table 2: parameters α and n to use in the expressions proposed by GCG (2007) and Mott MacDonald (2013).

Diameter	α (%)	n
$D < 10$ m	$0.006 \cdot D$	1
10 m $< D < 25$ m	$0.006 \cdot D$	$1 + (D - 10)/15$
$D > 25$ m	0.15	2

Table 3: parameters α and n to use in Equation (2) according to Jones (2022).

Shaft type	α (%)	n
<i>SBE</i> (jacked <i>PCS</i>)	0.035	1.7
<i>EBS</i>	0.035	1.1
Combined <i>SBE</i> & <i>EBS</i> (sheet pile/secant pile wall followed by shotcrete)	0.045	2.8
Combined <i>SBE</i> & <i>EBS</i> (jacked <i>PCS</i> followed by shotcrete)	0.045	1.4

2.3 Assessment of empirical methodologies

To understand the key shortcomings of the currently available empirical expressions, the root mean squared error (*RMSE*) calculated using Equation (5) was determined for each of the 12 shafts listed in Section 2.1:

$$RMSE [m] = \sqrt{\frac{\sum_{i=1}^{N_{obs}} (S_{v,predicted,i} - S_{v,observed,i})^2}{N_{obs}}} \quad (5)$$

where N_{obs} is the number of datapoints reported for each shaft, while $S_{v,predicted,i} [m]$ and $S_{v,observed,i} [m]$ are, respectively, the settlements yielded by the empirical expressions (Equations (1), (2) and (3)) and those observed in the field. Clearly, a smaller value of *RMSE*, shown throughout this work in $[mm]$, corresponds to a closer fit between the prediction and the observed data, with the opposite being true for larger differences between these two datasets. Moreover, it is important to note that, for the case of the New (2017) expression (Equation (2)), the values of α and n were calculated using a least-squares approach, meaning that each of the shafts is characterised by a different combination of these parameters, as reported in Table 4. Note that, in Table 4, the value of diameter given $\overline{D_{ext}}$ is the average external diameter whenever data in Table 1 allows its calculation using:

$$\overline{D_{ext}} [m] = \frac{D_{ext,PCS} \cdot H_{PCS} + D_{ext,SCL} \cdot H_{SCL}}{H_{PCS} + H_{SCL}} \quad (6)$$

where $D_{ext,PCS} [m]$ and $H_{PCS} [m]$ are, respectively, the external diameter and depth of excavation for the section of the shaft that uses precast concrete segments, while $D_{ext,SCL} [m]$ and $H_{SCL} [m]$ correspond to the same quantities but for sprayed concrete lining.

Table 4: errors associated with the use of empirical expressions to predict observed settlements due to shaft excavation.

Shaft	$\overline{D_{ext}}$ [m]	H [m]	New & Bowers (1994) RMSE [mm]	GCG (2007) RMSE [mm]	Mott Macdonald (2013) RMSE [mm]	Jones (2022) RMSE [mm]	α (%)	n	RMSE [mm]
SJW2	7.6	47.0	94.21	52.51	35.27	4.59	0.02689	0.7771	1.24
N&B	11.0	26.0	2.24	4.55	1.90	11.43	0.06313	1.0671	1.80
ISL	11.3	37.6	7.94	27.27	17.58	5.28	0.04318	1.2637	1.59
ECRT	11.3	39.5	6.92	36.52	18.14	4.89	0.05199	0.8625	2.31
SPC	11.3	44.8	3.56	36.48	13.64	8.01	0.05077	1.0449	1.78
CGR	13.1	32.9	8.48	70.34	37.18	15.19	0.07361	0.6728	3.05
WIL	13.4	29.0	10.00	63.02	41.92	2.88	0.03536	0.8891	1.45
KG2 (□)	13.4	42.7	72.93	311.18	223.76	32.53	0.02816	0.6067	2.47
EDR	13.5	39.3	14.59	128.78	85.89	12.09	0.05044	0.8213	5.91
SJW1	13.5	45.6	34.36	241.34	168.90	7.92	0.04247	0.7681	1.69
KG1 (□)	15.9	26.5	30.65	204.98	164.56	9.54	0.01834	1.4328	0.15
HBV	16.0	35.2	4.20	85.65	39.30	1.62	0.04374	1.4393	1.20
RMSE (◊)			18.65	74.65	45.97	7.39			2.20

Notes: (□) the measurements for these shafts were affected by slow rate of construction, see Section 2.1; (◊) average value of RMSE for all the shafts with the exception of KG1 and KG2.

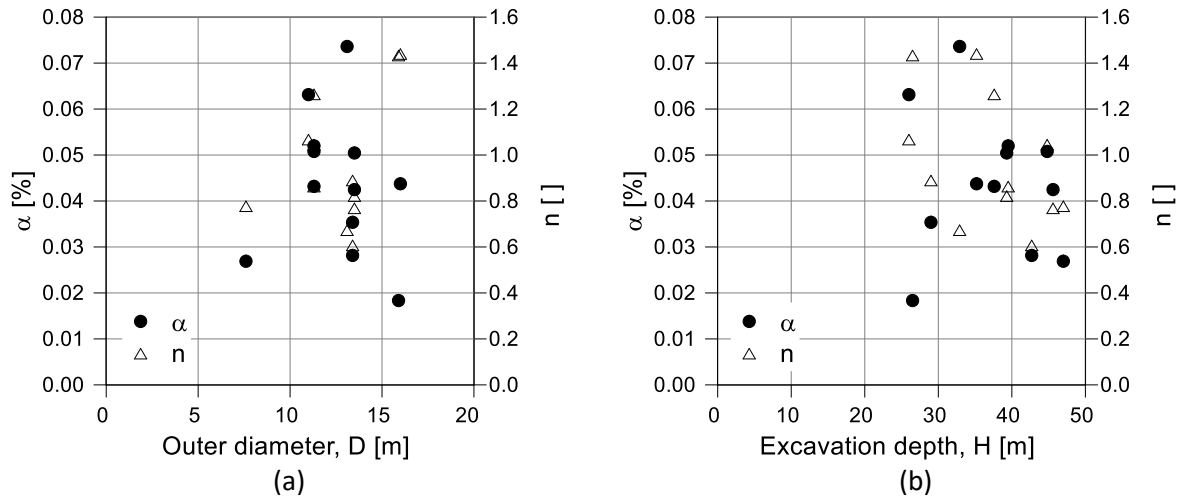


Figure 1: variation of parameters α and n for the New (2017) expression with (a) shaft diameter and (b) excavation depth.

Based on the obtained results, it is interesting to note that the original New & Bowers (1994) proposal performs generally well, and in effect better than the alternative expressions with the exception of that by Jones (2022). While this may seem surprising, it is important to consider the range of geometries present in this list of shafts: apart from SJW2, all shafts have a diameter between 11.3 m and 16.0 m, which are not too dissimilar from that at the origin of the New & Bowers (1994) expression

(11.0 m). In effect, for this expression, apart from the HBY shaft, a clear trend emerges where errors are larger for smaller (SJW2) and larger diameters (EDR, SJW1), matching the conclusions outlined by New (2017). Conversely, the *RMSE* values associated with the GCG (2007) and Mott MacDonald (2013), while larger on average, seem to be slightly more uniform across the different geometries, validating, to a certain extent, the introduction of empirical relationships between the geometric characteristics of the shaft and the expected values of α and n . This is further verified by the significantly better performance of the expression introduced by Jones (2022) which, in addition to having been calibrated based on this database, improves the expression by New & Bowers (1994) by including explicitly the effect of diameter on the extent of settlement (n). Also shown in Table 4 is the result of the calibration of the New (2017) expression for each of the considered shafts. As expected, this process results in uniformly lower values of *RMSE* for all the considered geometries. However, this is only accomplished through a substantial variation in values of both α (from 0.01834% to 0.07361%) and n (from 0.6025 to 1.4393). Furthermore, these values do not appear to be easily predictable: in Figure 1, the obtained values of α and n are plotted against the diameter and depth of the various shafts, with no clear trend discernible for either of the two geometric characteristics.

Figure 2 shows the monitoring data compared to each of the various proposals, generally corroborating the above observations and demonstrating that, as previously discussed, the expressions suggested in GCG (2007) and Mott MacDonald (2013) are indeed conservative. This is clear in the case of the HBY shaft, where both expressions provide an upper limit to the observed data while New & Bowers (1994) and, in particular, Jones (2022) predicted slightly lower settlements. Note that the curves obtained using New (2017) combined with the parameters in Table 4 and Figure 1 are not presented in Figure 2 as they are individually calibrated based on the respective data, rather than being predictive. The curves labelled “Proposal” in Figure 2 correspond to the new expressions established in this work and will be discussed in subsequent sections.

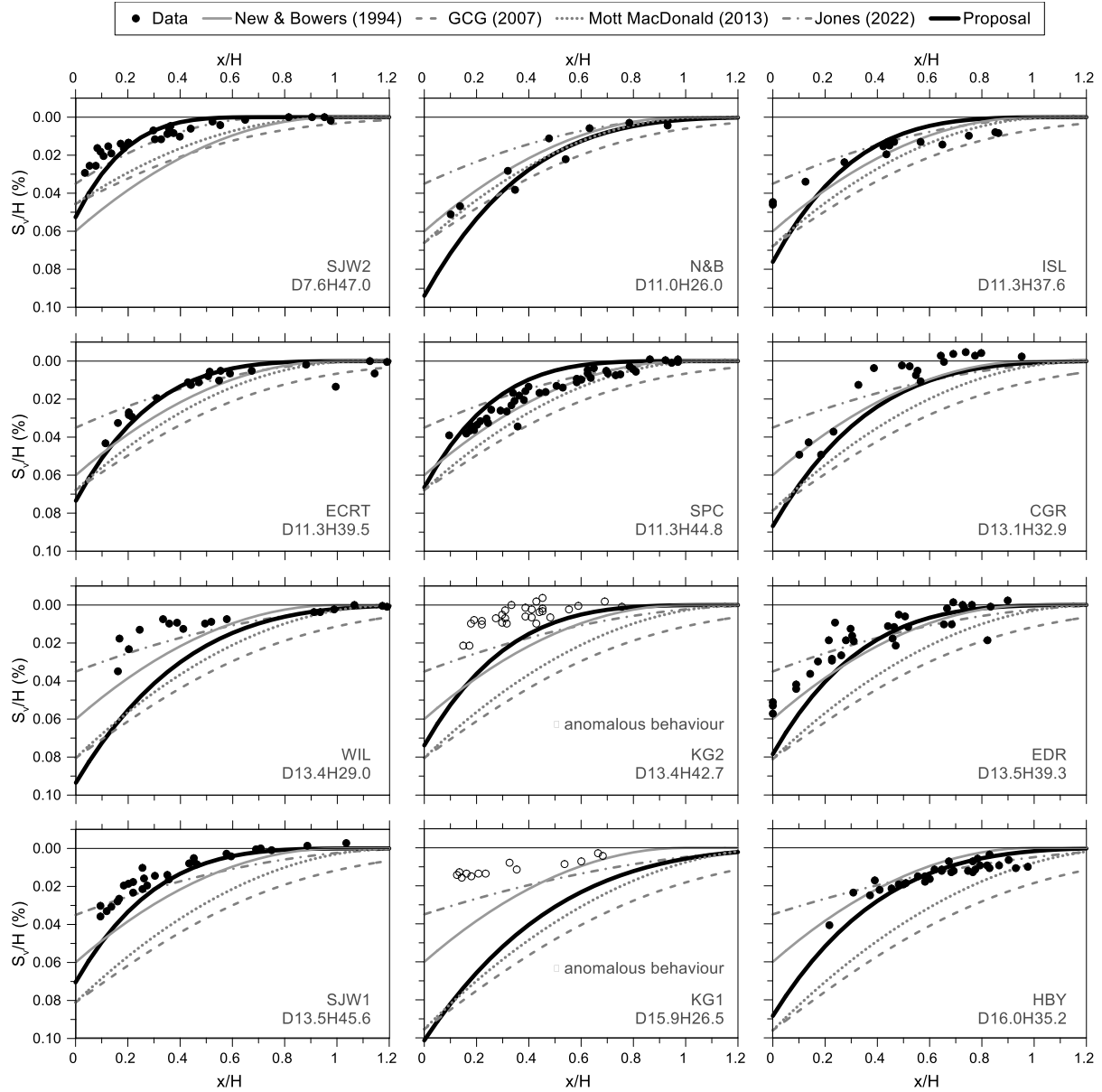


Figure 2: comparison between monitored data, expressions published in the literature and the proposed expression for the shafts reported in Faustin et al. (2018).

3. Development of a new expression

3.1 Description of the problem

The performance of the available empirical expressions characterised in Section 2.3 suggests that, while the assembled database of monitored shafts provides an invaluable source of data for verification and validation of design methodologies, it is clearly insufficient to generate new approaches for estimating the ground movements caused by the excavation of shafts. Indeed, the proposal of a new expression requires substantially more data, ideally covering a wider range of geometries than the limited set currently available in the literature. To address this gap, axisymmetric numerical analyses are performed using PLAXIS 2D (Bentley Systems, 2023), with details of the

modelling approach provided in the next section. Importantly, to enable the study of a variety of geometries, a parametric representation of a shaft with diameter D , depth H and wall thickness h was created, as shown in Figure 3. A simplified version of a typical ground profile in London was adopted, consisting of 2 m of Made Ground overlaying London Clay, with the water table located at the top of this layer. It was assumed that the shaft was excavated in 1 m steps, the top half of the shaft was supported using underpinned pre-cast concrete segments, while sprayed concrete was used for its lower half, thus replicating the typical EBS construction methodology adopted in London, as described for the parametric model in Table 5. Once the full excavation depth is reached, the vertical and horizontal ground movements at the surface between the edge of the shaft wall and a distance of $2H$ from this point are extracted.

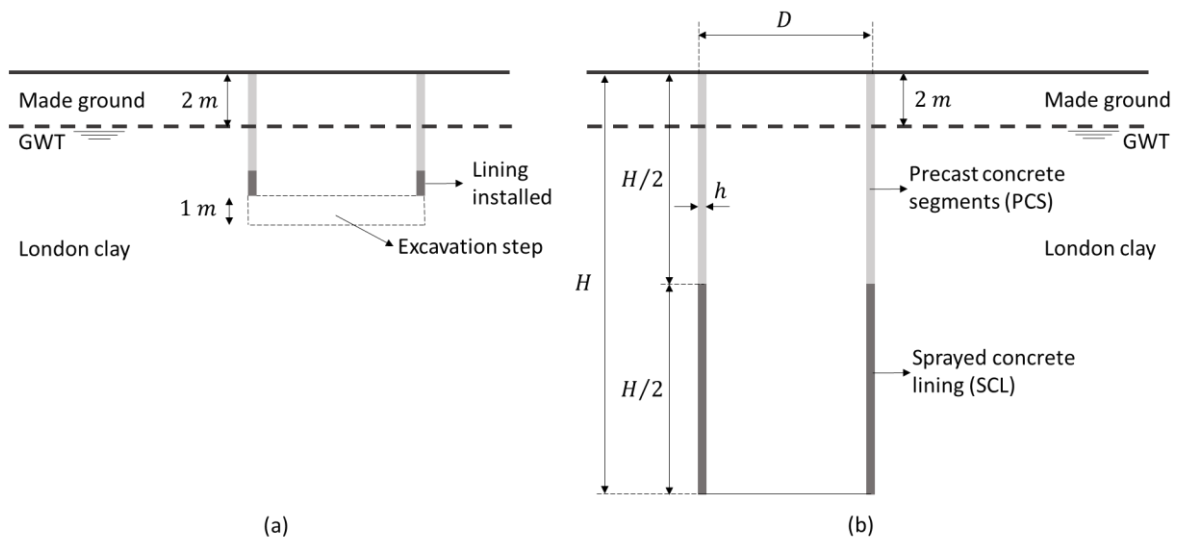


Figure 3: parametric definition of the analysed shaft problem (a) during construction and (b) in the final excavation stage.

Table 5: outline of the parametrised construction sequence of the shaft.

Phase	Duration	Activity
1	t_u	Construction of concrete segments and excavation of the top 2 m
2	t_u	Excavation to 3 m
3	t_u	Excavation to 4 m, construction of concrete segment to 3 m
...		
$H/2 - 1$	t_u	Excavation to $H/2$ m, construction of concrete segment to $(H/2 - 1)$ m
$H/2$	t_u	Excavation to $(H/2 + 1)$ m, application of shotcrete to $H/2$ m
...		
$H - 1$	t_u	Excavation to H m, application of shotcrete to $(H - 1)$ m
H	t_u	Application of shotcrete to H m

Using the Python API available in PLAXIS2D (Bentley Systems, 2023), a script was created such that the geometry and construction sequence of each shaft are procedurally generated given a set of parameters $\{D, H, h, t_u\}$, reducing substantially the time associated with setting up, running and

postprocessing the numerical models, enabling a large database to be created for the development and calibration of new expressions. A total of 20 cases were selected, covering diameters from 5 m to 30 m and excavated depths ranging from 20 m to 60 m. As shown in Figure 4, the analysed geometries clearly encompass those of the shafts reported in the literature, extending considerably the available data towards larger diameters. Moreover, in terms of aspect ratio, H/D , the monitored shafts were characterised by values of this quantity between ≈ 1.7 (KG1) and ≈ 6.2 (SJW2), while the analysed geometries extend this to a significantly wider interval: 0.67 ($H = 20$ m, $D = 30$ m) $\leq H/D \leq 12.0$ ($H = 60$ m, $D = 5$ m). Lastly, a constant value of wall thickness h of 0.35 m was assumed, together with an advance rate of $t_u = D/10$ days. It is important to note that the definition of the latter is significant in the context of these analyses since, as discussed in Section 3.2, a time-dependent mechanical response was prescribed for shotcrete. The impact of t_u is otherwise limited since the permeability of the soil deposit is very low.

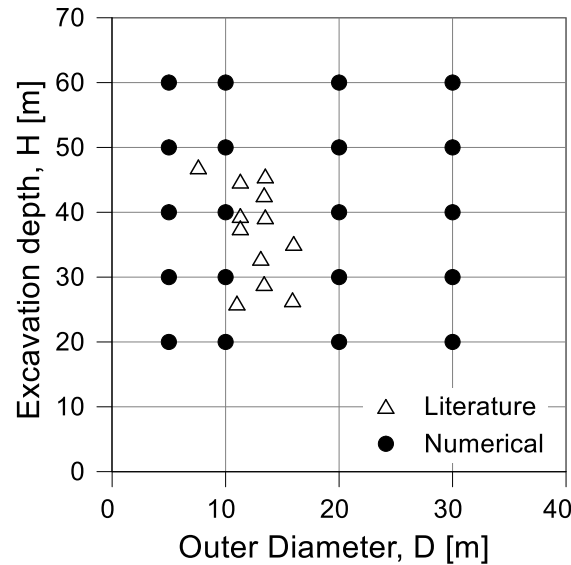


Figure 4: geometrical characteristics of the shafts reported in the literature and those of the modelled shafts.

3.2 Details of the analysis

The excavation of each shaft, as outlined in the previous section, was simulated using coupled consolidation analyses, taking advantage of the axisymmetric nature of the problem. For both lateral boundaries, representing the axis of symmetry on the left-hand side of the geometry and the far field on the right-hand side, displacements and water flows in the horizontal direction were restricted. To avoid boundary effects, the boundary on the right-hand side, after a careful analysis of all the geometries considered in this study, was placed at a distance from the shaft wall of $3 \cdot H$, with a minimum of $2 \cdot D$. Following a similar approach, the bottom boundary, where no displacements were allowed, was placed at a depth below the excavation level of $2 \cdot H$, with a maximum of 100 m. The soil domain was discretised using 15-noded triangular elements, with quartic shape functions and 12

Gauss points (Bentley Systems, 2023), with the mesh creation process ensuring that the average element size in the zone around the shaft wall, where smaller elements were consistently employed, was kept constant between the different geometries. As a result of this process, the number of elements varied between 7123 ($D = 5\text{ m}, H = 20\text{ m}$) and 9438 ($D = 30\text{ m}, H = 60\text{ m}$). The initial conditions were generated assuming a hydrostatic pore water pressure profile in equilibrium with the ground water table located at the top of the London Clay (i.e. 2 m depth), values of bulk unit weight of 18 kN/m^3 and 20 kN/m^3 for the Made Ground and the London Clay, respectively, and the variation of the at-rest coefficient of earth pressure K_0 presented in Figure 5(a), which is based on that used in Gawecka et al. (2017) and Sailer et al. (2019). The Made Ground was assumed to be dry, while London Clay behaved in an undrained manner with anisotropic permeability given by:

$$\log_{10} k_v = 0.0215 \cdot (y - y_{LC}) - 9.427 \geq 1 \times 10^{-12} \text{ [m/s]} \quad (7)$$

$$k_h = 2.0 \cdot k_v \quad (8)$$

where k_v and k_h denote vertical and horizontal permeability [m/s], respectively, y is the vertical coordinate where a value of zero corresponds to the ground surface and y_{LC} is the elevation of the top of the London Clay (-2 m). This variation replicates that adopted in Avgerinos et al. (2016) – shown in Figure 5(b) together with the field data gathered by Hight et al. (2007) – and was introduced in the analysis through a User-Defined Flow Model (Bui et al., 2019), as detailed in Taborda et al. (2023a). Clearly, the use of a hydrostatic pore water pressure distribution is a simplification of the underdrained profile typically encountered in the London basin (Simpson et al., 1989). However, given that hydrostatic conditions lead to smaller mean effective stress levels and hence softer ground conditions, such an assumption contributes to conservative estimates of ground movements. Moreover, the extent of underdrainage at a given site is a function of several aspects of local ground conditions (e.g. thickness of London Clay, characteristics of soil layers composing the Lambeth Group, depth of the lower aquifer), all of which are difficult to parametrise.

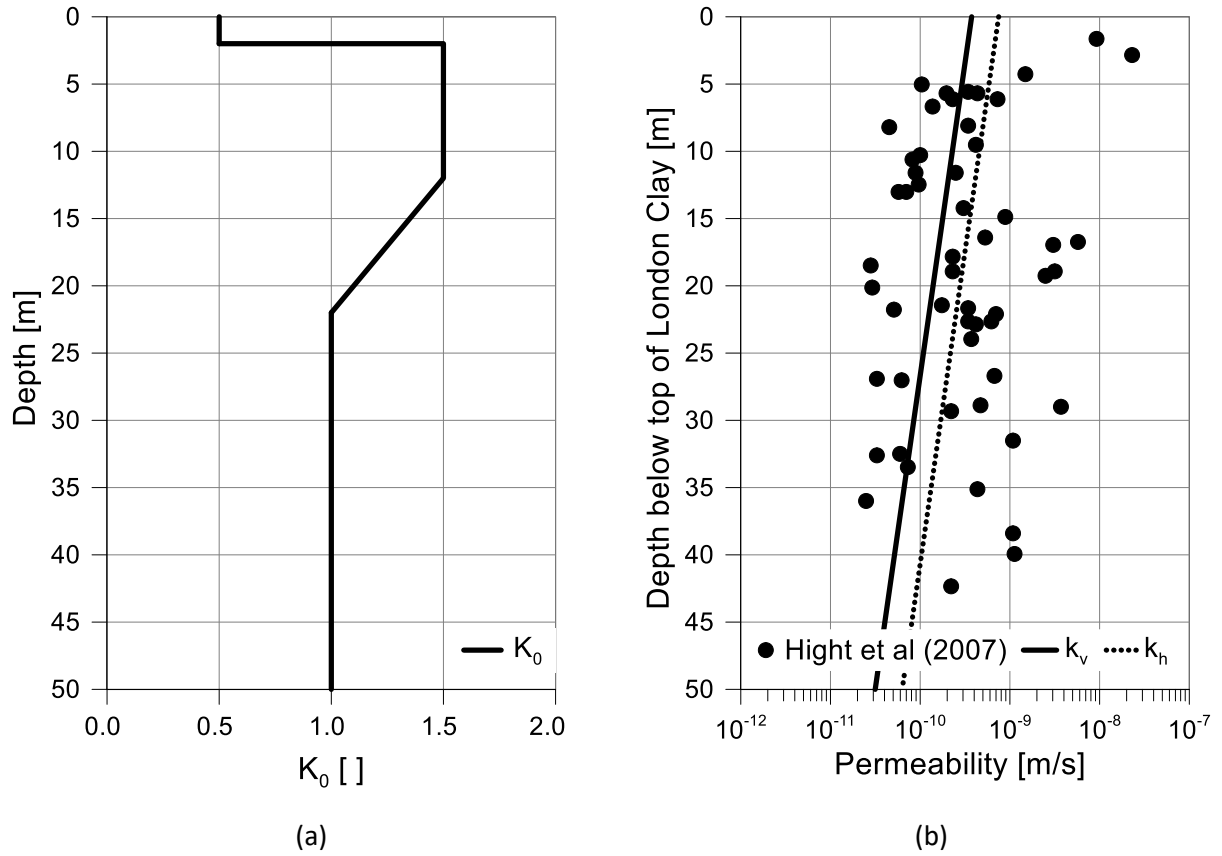


Figure 5: (a) modelled profile of at-rest coefficient of earth pressure (K_0) and (b) assumed permeability variation with depth compared with field data gathered by Hight et al. (2007).

The behaviour of Made Ground was assumed to be linear elastic-perfectly plastic, with its stiffness being described by $E = 10 \text{ MPa}$ and $\nu = 0.2$, its strength given by the Mohr-Coulomb failure criterion with $\phi = 30^\circ$ and $c = 0 \text{ kPa}$, with non-associated plasticity characterised by $\psi = 0^\circ$. Conversely, the mechanical response of London Clay was described using the non-linear elastic-perfectly plastic IC MAGE M01 model (Taborda et al., 2023b), implemented into PLAXIS 2D as a User-Defined Soil Model (UDSM). According to this model, the material stiffness is simulated using the non-linear elastic relationships for the tangent shear and bulk moduli – G_{tan} and K_{tan} , respectively – proposed in Taborda et al. (2016) and reproduced in Appendix B together with the adopted model parameters (Table B.1). The strength of London Clay is defined by the Mohr-Coulomb failure criterion with $\phi = 25^\circ$ and $c = 5 \text{ kPa}$, with the plastic response described by an angle of dilatancy of $\psi = 12.5^\circ$. The model is integrated using a modified Euler sub-stepping algorithm with automatic error control as proposed by Sloan (1987) and Sloan et al. (2001), with the specific implementation for PLAXIS UDSMs detailed in Taborda et al. (2023d).

In terms of excavation support, the concrete segments adopted in the top half of the shaft were modelled as linear elastic with their stiffness described by $E = 37.0 \text{ GPa}$ and $\nu = 0.2$, whereas for

the lower half of the shaft, where sprayed concrete was used, a non-linear time-dependent model was adopted. According to this simple model, implemented into PLAXIS as a UDSM (Taborda et al., 2023c), the stiffness of the shotcrete follows the expression given by the CEB-FIP Model Code 90 (CEB-FIP, 1993):

$$E(t_{conc}) [GPa] = \beta_c(t_{conc}) \cdot E_{28} \quad (9)$$

$$\beta_c(t_{conc}) [-] = \sqrt{\exp \left[s \cdot \left(1 - \left(\frac{28}{t_{conc}/1 \text{ day}} \right)^{1/2} \right) \right]} \quad (10)$$

where $E_{28} [GPa]$ is the stiffness of the material at 28 days, $t_{conc} [days]$ is the age of the concrete and s is a parameter which depends on the type of concrete. In the present case, $E_{28} = 30 \text{ GPa}$ and $s = 0.25$ were used to describe the evolution of the Young's modulus of shotcrete, together with $\nu = 0.2$.

Lastly, the interaction between the lining and the adjacent ground is modelled using interface elements with a strength reduced to two thirds of that of the soil, i.e. $c_{int}/c = \tan \phi_{int}/\tan \phi = 2/3$, with a dilatancy of $\psi_{int} = 0^\circ$. The stiffness of these elements was set to $K_N = K_T = 10^6 \text{ kN/m}^3$, where the subscripts N and T denote the normal and tangential directions, respectively. These values have been observed to be sufficiently large to avoid excessive displacements in either direction.

3.3 Proposed expression for settlements

The settlements extracted from each of the 20 finite element simulations listed in Table A.1 were first analysed independently to determine the combination of parameters α and n that reproduced the calculated settlement troughs with greatest accuracy when using the following expression:

$$S_v [m] = \alpha \cdot H \cdot \left(1 - \frac{x}{n \cdot H} \right)^4 \quad (11)$$

Note that this expression has a similar form to those proposed by New & Bowers (1994) and New (2017), with the only difference being the exponent, which has been increased from 2 to 4 as this led to substantial improvements in the reproduction of the calculated troughs. Two key relationships were observed: $\alpha [\%]$ is well predicted as a function of $H/\sqrt{D_{ref} \cdot D}$ (Figure 6(a)), while the value of n is seen to vary with H/D (Figure 6(b)). By using non-linear regression, the numerical results for the 20 analyses performed can be reproduced with a high degree of accuracy using Equation (11) combined with the following expressions for α and n :

$$\alpha [\%] = 0.154 \cdot \exp \left(-0.063 \cdot H / \sqrt{D_{ref} \cdot D} \right) \quad (12)$$

$$n [-] = 2.818 \cdot \left(\frac{H}{D}\right)^{-0.719} \quad (13)$$

where, in Equation (12), D_{ref} is a reference diameter (assumed here to take the value of 1 m), which is introduced to ensure dimensional consistency. It is interesting to note that, in Figure 6, the values of α and n established for the numerical results broadly agree with those obtained for the shafts reported in the literature. In particular, Figure 6 suggests that the proposed relationships bound the field data, thus providing conservative estimates of the maximum settlement (α) and settlement trough width (n), although this can only be verified for the relatively narrow range of aspect ratios for which field data are available.

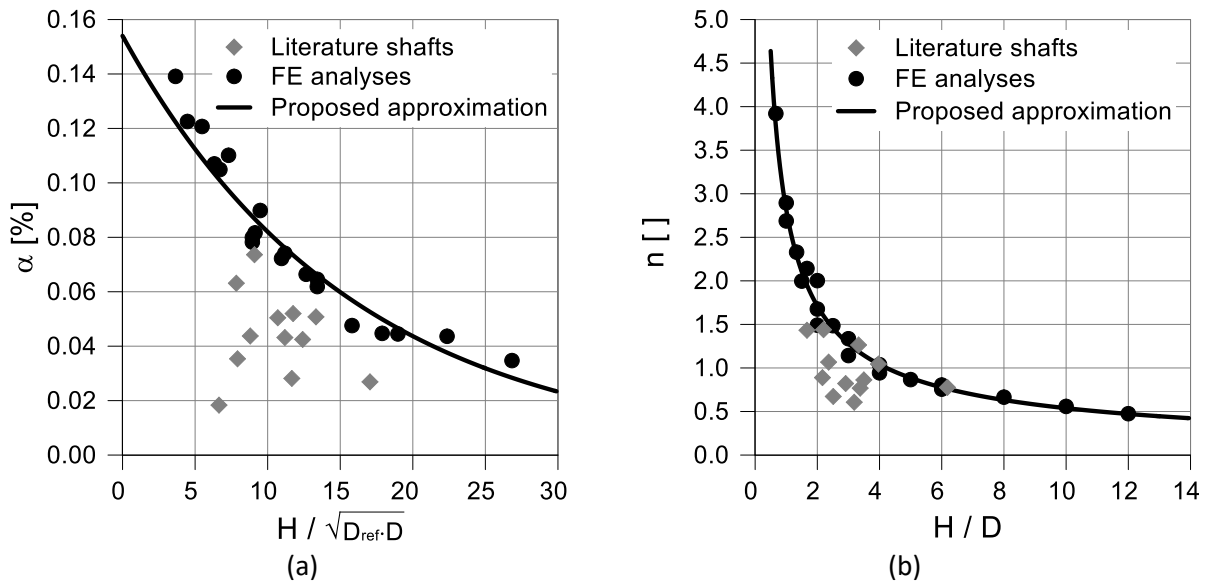


Figure 6: reproduction of the numerical results and comparison with literature shafts in terms of quantities (a) α and (b) n .

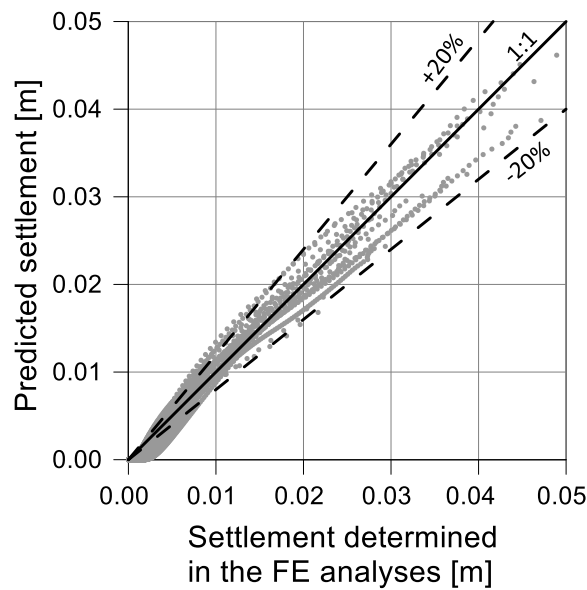


Figure 7: comparison between the settlement determined in the FE analyses and that given by the proposed expression.

The accuracy of the proposed expressions in reproducing the numerical database is demonstrated in Figure 7, where the numerical data is compared directly against that obtained using Equations (11-13). It can be seen that a large proportion of data points is clustered around the 1:1 line and within the $\pm 20\%$ bounds. Such high accuracy level is also visible in terms of settlement troughs for selected geometries in Figure 8. For a shaft depth of 20 m, the expressions by GCG (2007) and Mott MacDonald (2013) show reasonable accuracy, particularly in terms of scaling up the settlements with the shaft diameter. Conversely, as expected, the expressions by New & Bowers (1994) and Jones (2022) predict the same maximum normalised settlement for the three chosen diameters in Figure 8 (5 m, 10 m and 30 m), despite this value increasing from 0.08% to 0.15% in the numerical models. For the very deep shafts ($H = 60$ m) the same scaling with the diameter is observed for the expressions by GCG (2007) and Mott MacDonald (2013), though the associated increase in displacements appears to be substantially overestimated. As expected, given the adopted calibration procedure, Equations (11-13) are seen to reproduce the numerical data with excellent accuracy.

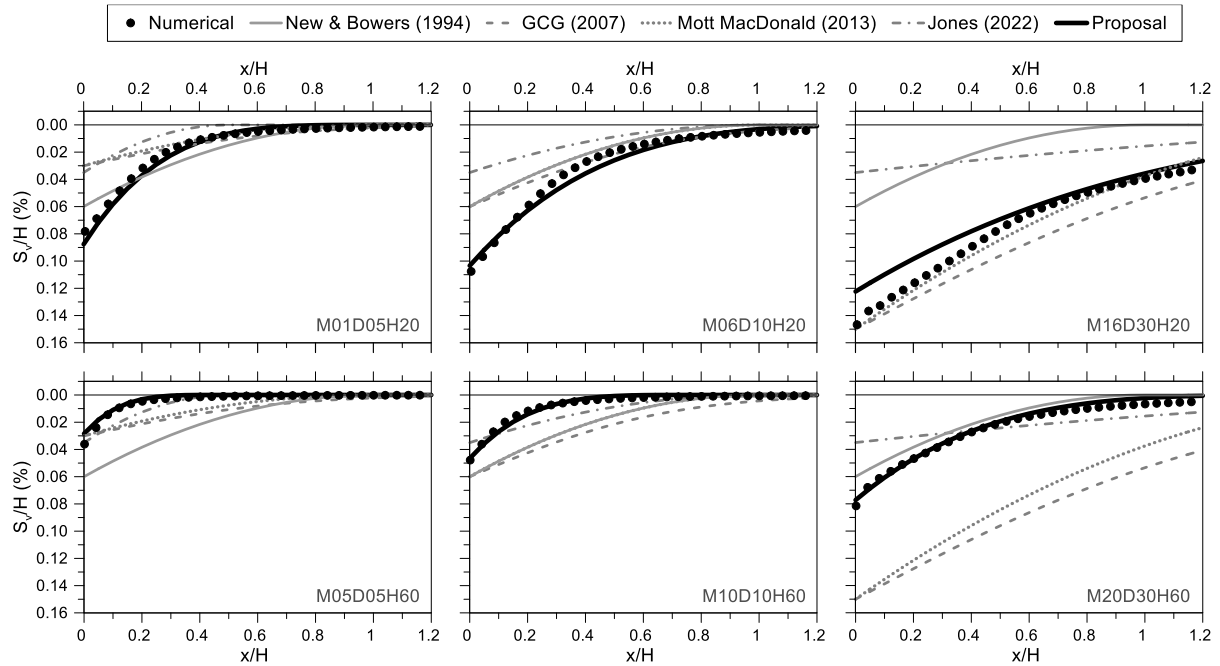


Figure 8: comparison between the settlements determined in selected numerical models and those obtained using the empirical expressions proposed in the literature and in this study.

A detailed assessment of the errors associated with the different expressions when reproducing the numerical database is shown in Figure 9, where a logarithmic scale has been adopted for the *RMSE* axis. It is interesting to note that, as discussed above, both GCG (2007) and Mott MacDonald (2013) have an excellent performance for the shallower shafts ($H = 20$ m), with a uniform error across all diameters, which would suggest that a correct scaling law for α and n had been achieved. However, as the depth of the shaft increases, the errors associated with these two sets of expressions increase both on average and in terms of variability. Indeed, for the deeper shafts ($H = 60$ m), there are two

orders of magnitude difference in error between the largest diameter ($D = 30\text{ m}$) and smallest diameter ($D = 5\text{ m}$) shafts. Conversely, the expression proposed by New & Bowers (1994) exhibits an increasing error with diameter solely for $H = 20\text{ m}$. For larger depths, the lowest error appears to be achieved for different values of diameter: $D = 10\text{ m}$ for $H = 30\text{ m}$, $D = 20\text{ m}$ for $H = 40\text{ m}$, 50 m and 60 m . The scaling of n based on the diameter introduced in Jones (2022) does alter significantly this trend, with the variation of $RMSE$ with diameter shifting towards a monotonically increasing pattern with values overall below those associated with New & Bowers (1994). Lastly, the expressions proposed by this study (Equations (11-13)) lead to smaller errors across all diameters and shaft depths, with only a slight trend of increasing error with diameter being observed, suggesting that a sharper variation of α and n with diameter could have been adopted.

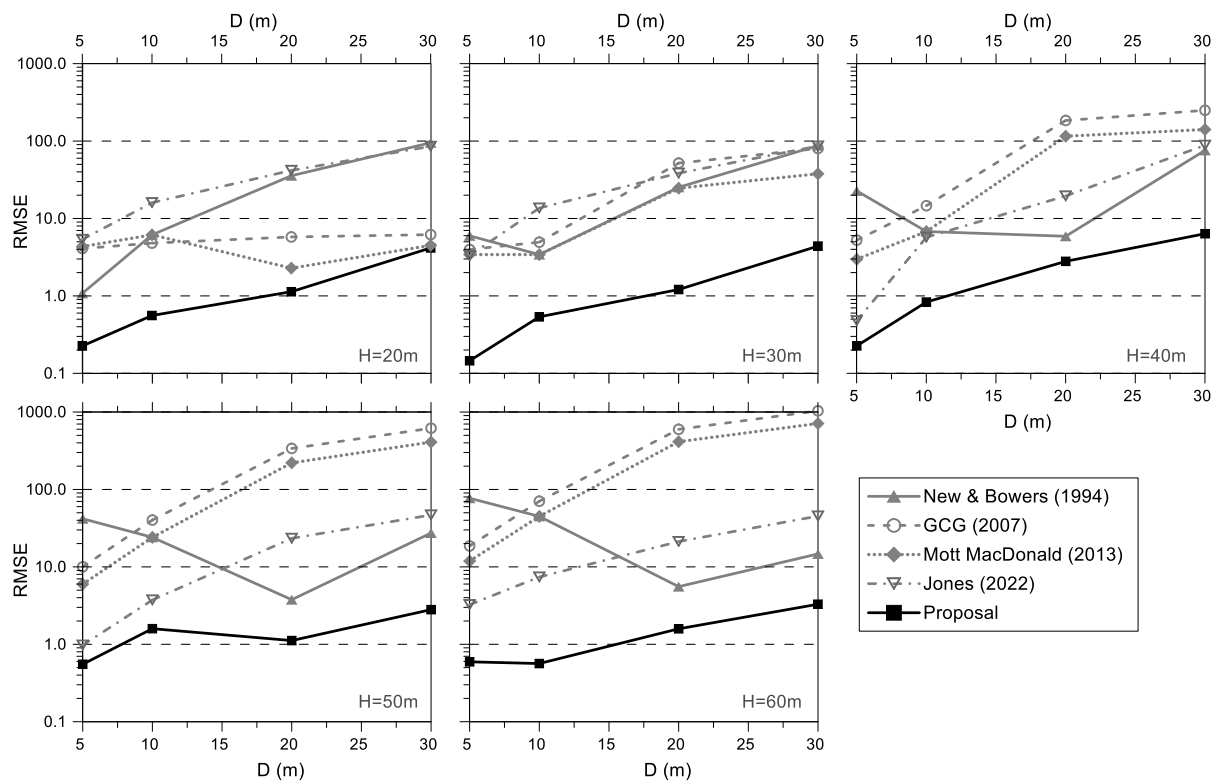


Figure 9: variations of $RMSE$ for the entire synthetic database associated with the empirical expressions in the literature and proposed in this study.

A more challenging test for the expressions arising from this study consists of attempting to reproduce the field data gathered by Faustin et al. (2018), given that the underlying numerical models represented an idealised shaft in London, rather than replicating the ground conditions and geometric characteristics specific to each of the shafts in the literature. The obtained curves are presented in Figure 2, where a reasonable agreement with the field data is observable, particularly in terms of maximum settlement and extent of the affected area. However, it should be noted that the obtained settlement troughs are generally more abrupt than those suggested by the monitoring data, although this does represent a more conservative estimate of the potential distortions associated with

differential settlements. Overall, it can be argued that the proposed expressions represent an improvement in terms of accuracy when replicating the available database. This is further supported by the values of *RMSE* for these shafts, which are listed in Table 6, with this metric being on average lower for the new expressions (Equations (11-13)), apart from those proposed by Jones (2022) which have been directly calibrated based on these field data. In effect, given that Equations (11-13) reproduce accurately the numerical data (Figure 7 and Figure 8), the low values of *RMSE* in Table 6, together with Figure 2, serve as a validation of the modelling approach adopted in the numerical analyses (Section 3.2), particularly when considering the introduced simplifications.

Table 6: errors associated with the use of proposed expression to predict observed settlements due to shaft excavation.

Shaft	$\overline{D}_{ext}$ [m]	H [m]	$H/\sqrt{\overline{D}_{ext} \cdot D_{ref}}$ [-]	$H/\overline{D}_{ext}$ [-]	Equation (11)		
					α (%)	n	<i>RMSE</i>
SJW2	7.6	47.0	17.05	6.18	0.05261	0.7603	10.55
N&B	11.0	26.0	7.84	2.36	0.09398	1.5182	6.70
ISL	11.3	37.6	11.19	3.33	0.07612	1.1873	22.62
ECRT	11.3	39.5	11.75	3.50	0.07345	1.1459	3.16
SPC	11.3	44.8	13.33	3.96	0.06651	1.0467	10.26
CGR	13.1	32.9	9.09	2.51	0.08686	1.4534	13.46
WIL	13.4	29.0	7.92	2.16	0.09349	1.6176	32.81
KG2 (□)	13.4	42.7	11.66	3.19	0.07385	1.2248	46.58
EDR	13.5	39.3	10.70	2.91	0.07850	1.3070	24.88
SJW1	13.5	45.6	12.41	3.38	0.07046	1.1745	16.71
KG1 (□)	15.9	26.5	6.65	1.67	0.10132	1.9518	138.90
HBV	16.0	35.2	8.80	2.20	0.08846	1.5986	3.36
<i>RMSE</i> (◇)							14.45

Notes: (□) the measurements for these shafts were affected by slow rate of construction, see Section 2.1; (◇) average value of *RMSE* for all the shafts with the exception of KG1 and KG2.

3.4 Proposed expression for horizontal displacements

Having performed the finite element analyses described in Section 3.2, the obtained results were further explored in terms of simulated ground movements at the surface in the horizontal direction, as only an approximate predictive relationship exists for shafts in London Clay, which is based on a procedure established for boxes (GCG, 2007). Based on the observations of the calculated displacements between the back of the shaft wall and up to radial distances of $2H$, the following expression is proposed:

$$S_h [m] = H \cdot \frac{\left(\frac{\beta + a \cdot \bar{x}^c}{1 + b \cdot \bar{x}^c} \right)}{1 + d \cdot \bar{x}^3} \quad (14)$$

where $\bar{x} [m/m]$ is the normalised radial distance behind the wall given as x/H , and β , a , b , c and d are quantities which are determined through nonlinear regression. Note that the displacements given by this expression are assumed to be positive in the direction towards the excavation. A quick observation of this expression suggests that, at the wall, the displacement is equal to $\beta \cdot H$, meaning that this quantity has the same role for horizontal displacements as that of α for settlements. The regression using the finite element data suggested that, similar to the expression for vertical settlement (Equation (11)), the expression above remains a function of two parameters only, H and D , since β , b and c are constants and thus independent of the geometry ($\beta = 0.00763 \%$, $b = 110.09$, $c = 1.340$), while a and d are described as functions of the aspect ratio H/D :

$$a [-] = 0.0609 \cdot \left(\frac{H}{D} \right)^{-0.625} \quad (15)$$

$$d [-] = 0.821 \cdot \left(\frac{H}{D} \right)^{1.807} \quad (16)$$

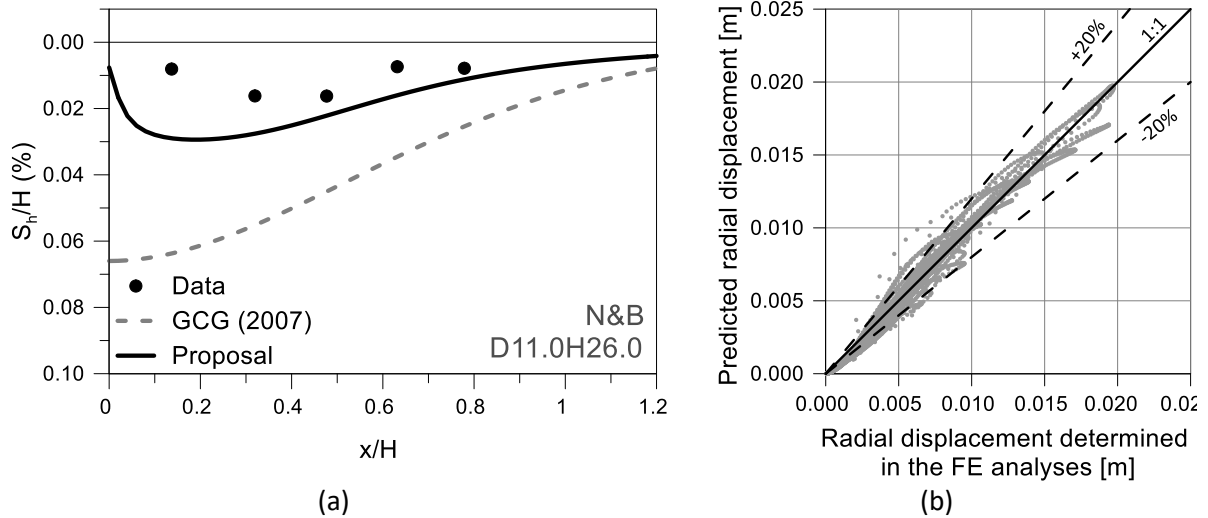


Figure 10: assessment of the proposed expression through comparison with (a) field data and (b) numerical data.

The accuracy of the proposed expression is shown in Figure 10(a) by comparing its predictions with the only field dataset available in terms of horizontal displacements (the S-line reported in New & Bowers (1994), which was thought to contain more reliable measurements than the T-line). Clearly, both the magnitude and the trend – an increase of horizontal displacement with the distance to the wall, followed by a gradual reduction – are adequately estimated by the proposed expression. Similarly, in terms of the numerical dataset (Figure 10(b)), there is a good agreement between the calculated and predicted displacements, as shown by the higher concentration of points around the

1:1 line, with the majority of points falling within the $\pm 20\%$ bounds. Lastly, Figure 11 depicts variations of normalised horizontal displacements with the distance to the shaft for selected geometries, comparing the finite element data, the proposed expression and the corresponding predictions using the methodology described in GCG (2007). Unsurprisingly, the calibrated expressions reproduce the numerical data with excellent accuracy, particularly for deeper shafts and independently of the shaft diameter. Conversely, the expression proposed in GCG (2007) appears to overestimate the effect of diameter and shaft depth, leading to considerably larger displacements for larger values of these geometric characteristics. Clearly, additional field evidence is required not only to validate the finite element predictions but also to assess the accuracy of the expressions proposed in this paper and in the literature.

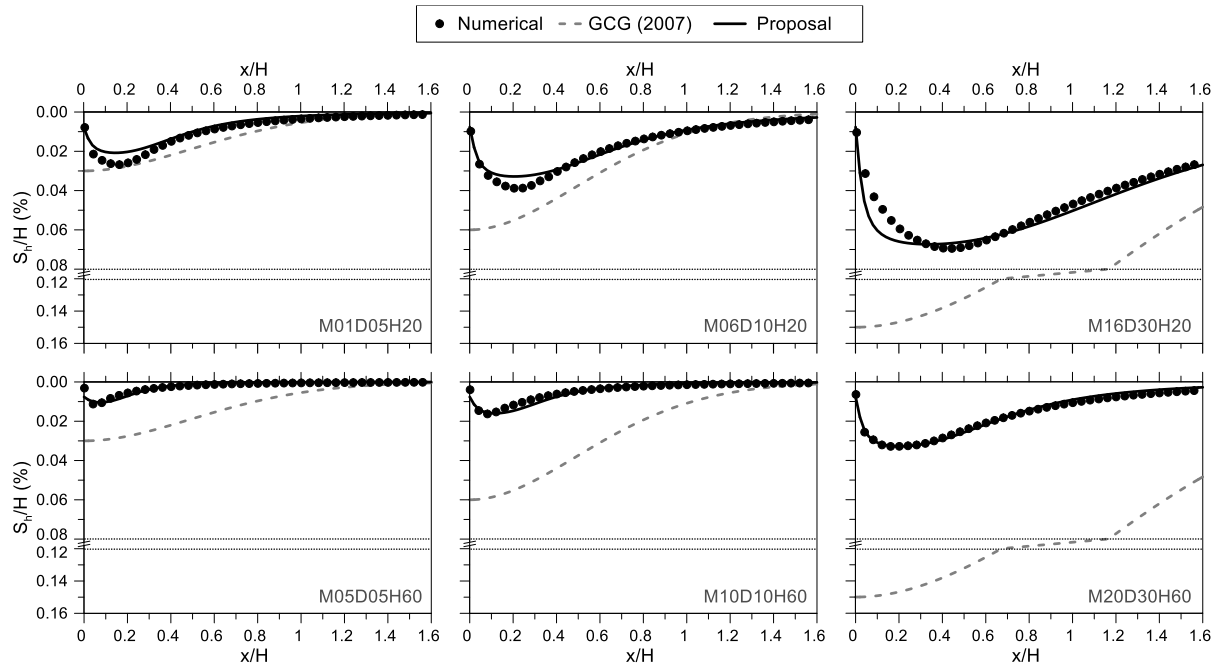


Figure 11: comparison between the horizontal displacements determined in selected numerical models and those obtained using the empirical expressions proposed in the literature and in this study.

4. Conclusions

A detailed study of ground movements induced by shaft excavation was undertaken in this paper, focusing on the measured data presented in New & Bowers (1994) and Faustin et al. (2018). The various expressions available in the literature (New & Bowers, 1994; GCG, 2007; Mott MacDonald, 2013; New, 2017; Jones, 2022) were reviewed and their predictions were compared against the field database. The calibration of a general expression based on that proposed in New (2017) against this dataset failed to reveal consistent trends for parameters α (normalised settlement at the wall) and n (normalised radius of influence of shaft excavation). However, by performing 20 coupled-

consolidation finite element analyses in PLAXIS 2D simulating the excavation of shafts of different geometries in London Clay, it was demonstrated that α and n could be reasonably predicted based on quantities related to the aspect ratio of the shaft ($H/\sqrt{D_{ref} \cdot D}$ for α and H/D for n). This allowed a new set of expressions to be developed that, as expected, were shown to replicate accurately the numerical data. More importantly, with the exception of the curves proposed by Jones (2022) which were calibrated directly based on the field monitoring gathered by Faustin et al. (2018), the new expressions were also able to out-perform the previously available empirical relationships when reproducing the available field data, even though the numerical models included idealised representations of the ground conditions in the London Basin, rather than simulating the specific stratigraphy and hydraulic conditions of each shaft.

A similar approach was followed for horizontal displacements at the surface induced by the excavation of shafts. A new expression was proposed which was shown to replicate the numerical results with excellent accuracy. The obtained results were also shown to be consistent with the limited data gathered for the Heathrow Express trial shaft by New & Bowers (1994), particularly in terms of predicting the maximum horizontal displacement at a considerable distance from the wall, rather than immediately behind the wall. However, it is clear that additional field data is required to conduct a detailed assessment of the accuracy of the expression, such that it can be used with confidence during early stages of shaft design. Given the lack of field validation, the developed expression should be seen as an indication of the horizontal displacements expected from a numerical analysis.

Two new expressions have been proposed for estimating surface ground movements induced by shaft excavation, which take into account both the diameter and the depth of the shaft. These were calibrated based on detailed finite element analyses and, where possible, were shown to yield predictions which are consistent with available field data. The expressions have been implemented in the form of a spreadsheet to encourage their use (Taborda et al., 2024).

Acknowledgements

For the purpose of open access, the authors have applied a Creative Commons Attribution (CC BY) licence to any author-accepted manuscript version arising.

References

- Avgerinos, V., Potts, D. M. & Standing, J. R. (2016) The use of kinematic hardening models for predicting tunnelling-induced ground movements in London Clay. *Geotechnique*. 66 (2), 106-120.
- Bentley Systems (2023) PLAXIS2D 2023.1 - Reference Manual.

Bui, T. A., Casarella, A., Di Donna, A. & Brinkgreve, R. B. J. (2019) *Advanced Thermo-Hydro-Mechanical modelling features for practical applications in energy geotechnics, XVII European Conference on Soil Mechanics and Geotechnical Engineering, Reykjavik*

CEB-FIP (1993) CEB-FIP Model Code 90. Thomas Telford, London.

Faustin, N. E., Elshafie, M. Z. E. B. & Mair, R. J. (2018) Case studies of circular shaft construction in London. *Proceedings of the Institution of Civil Engineers - Geotechnical Engineering*. 171 (5), 391-404.

Gaba, A., Hardy, S., Doughty, L., Powrie, W. & Selemetas, D. (2017) *Guidance on embedded retaining wall design*. CIRIA. Report number: C760.

GCG (2007) *Settlement estimation procedure: box excavations and shafts*. London, Crossrail. Report number: 1D0101-G0G00-01004.

Gawecka, K. A., Taborda, D. M. G., Potts, D. M., Cui, W. J., Zdravkovic, L. & Kasri, M. S. H. (2017) Numerical modelling of thermo-active piles in London Clay. *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering*. 170 (3), 201-219.

Hight, D. W., Gasparre, A., Nishimura, S., Minh, N. A., Jardine, R. J. & Coop, M. R. (2007) Characteristics of the London Clay from the Terminal 5 site at Heathrow Airport. *Geotechnique*. 57 (1), 3-18.
<https://doi.org/10.1680/geot.2007.57.1.3>

Jones, B. (2022) *Soft Ground Tunnel Design*. Oxon, CRC Press.

Le, B. T., Goodey, R. J. & Divall, S. (2019) Subsurface ground movements due to circular shaft construction. *Soils and Foundations*. 59 (5), 1160-1171.

Mott MacDonald (2013) *Impacts of tunnels in the UK*. London, HS2.

New, B. (2017) Settlements due to shaft construction. In: *Tunnels and Tunnelling International*, September 2017 pp. 16-17.

New, B. M. & Bowers, K. H. (1994) Ground movement model validation at the Heathrow Express trial tunnel. *Tunnelling '94: Papers presented at the seventh international symposium, 'Tunnelling'94'*. Boston, MA, Springer US, pp. 301-329.

Newhouse, J. (2017) *Ground movement due to shaft construction*. MSc thesis. Imperial College London.

Quigg, G. (2019) Battersea cable tunnel: powering regeneration in central London, UK. *Proceedings of the Institution of Civil Engineers - Civil Engineering*. 172 (2), 77-82.

- Sailer, E., Taborda, D. M. G., Zdravkovic, L. & Potts, D. M. (2019) Fundamentals of the coupled thermo-hydro-mechanical behaviour of thermo-active retaining walls. *Computers and Geotechnics*. 109, 189-203.
- Schwamb, T. & Soga, K. (2015) Numerical modelling of a deep circular excavation at Abbey Mills in London. *Geotechnique*. 65 (7), 604-619.
- Simpson, B., Blower, T., Craig, R. N. & Wilkinson, W. B. (1989) *The engineering implications of rising groundwater levels in the deep aquifer beneath London*. CIRIA.
- Sloan, S. W. (1987) Substepping schemes for the numerical integration of elastoplastic stress-strain relations. *International Journal for Numerical Methods in Engineering*. 24 (5), 893-911.
- Sloan, S. W., Abbo, A. J. & Sheng, D. (2001) Refined explicit integration of elastoplastic models with automatic error control. *Engineering Computations*. 18 (1/2), 121-194.
- Taborda, D. M. G., Kontoe, S. & Tsiampousi, A. (2023a) *IC MAGE Flow Model 01 – Nonlinearly varying anisotropic permeability (Version 1.0)*. Zenodo. <https://doi.org/10.5281/zenodo.7796383>
- Taborda, D. M. G., Kontoe, S. & Tsiampousi, A. (2023b) *IC MAGE Model 01 – Strain-hardening/softening Mohr-Coulomb failure criterion with isotropic small strain stiffness (Version 2.1)*. Zenodo. <https://doi.org/10.5281/zenodo.8239422>
- Taborda, D. M. G., Kontoe, S. & Tsiampousi, A. (2023c) *IC MAGE Model 07 – Simple non-linear time-dependent stiffness model for concrete (Version 1.4)*. Zenodo. <https://doi.org/10.5281/zenodo.8234140>
- Taborda, D. M. G., Kontoe, S. & Tsiampousi, A. (2023d). *IC MAGE UMIP - Universal model interface for PLAXIS (Version 3.5)*. Zenodo. <https://doi.org/10.5281/zenodo.8226527>
- Taborda, D. M. G., Pedro, A. M. G. & Hardy, S. (2024) *Implementation of new analytical solutions for the prediction of surface ground movements due to shaft excavation in London Clay (Version 1.0)*. Zenodo. <https://doi.org/10.5281/zenodo.10657730>
- Taborda, D. M. G., Potts, D. M. & Zdravkovic, L. (2016) On the assessment of energy dissipated through hysteresis in finite element analysis. *Computers and Geotechnics*. 71, 180-194.
- Xia, H. (2019) Numerical analysis of shafts in London Clay. Final Year Project. Imperial College London.
- Zheng, C., Franza, A. & Jimenez, R. (2023) Analytical prediction of ground movements due to circular shaft construction in clayey soils. *Canadian Geotechnical Journal*. 60 (8), 1132-1139.

Appendix A

The key characteristics of the shafts selected for numerical analyses in order to produce a dataset for the calibration of the new expressions are listed below:

Table A.1: geometric characteristics of the shafts analysed numerically.

Designation	D [m]	H [m]	h [m]	t_u [day]
M01D05H20	5.0	20.0	0.35	0.5
M02D05H30		30.0		
M03D05H40		40.0		
M04D05H50		50.0		
M05D05H60		60.0		
M06D10H20	10.0	20.0	0.35	1.0
M07D10H30		30.0		
M08D10H40		40.0		
M09D10H50		50.0		
M10D10H60		60.0		
M11D20H20	20.0	20.0	0.35	2.0
M12D20H30		30.0		
M13D20H40		40.0		
M14D20H50		50.0		
M15D20H60		60.0		
M16D30H20	30.0	20.0	0.35	3.0
M17D30H30		30.0		
M18D30H40		40.0		
M19D30H50		50.0		
M20D30H60		60.0		

Appendix B

The behaviour of London Clay was modelled using the IC MAGE M01 model which combines a Mohr-Coulomb failure criterion with a non-linear hypoelastic formulation for reproducing the variation of stiffness over a wide range of strains (Taborda et al., 2023b). The strength of London Clay was modelled using a Mohr-Coulomb failure criterion with non-associated plasticity, characterised by parameters $\varphi = 25^\circ$, $c = 5 \text{ kPa}$ and $\psi = 12.5^\circ$. In terms of stiffness, a nonlinear formulation proposed by Taborda et al. (2016) was adopted, where the tangent shear (G_{tan}) and bulk (K_{tan}) moduli are defined by the expressions:

$$G_{tan} = G_{ref} \cdot \left(\frac{p'}{p'_{ref}} \right)^{m_G} \cdot \left[R_{G,min} + \frac{1 - R_{G,min}}{1 + \left(\frac{E_d}{a_G} \right)^{b_G}} \right] \geq G_{min} \quad (\text{B.1})$$

$$K_{tan} = K_{ref} \cdot \left(\frac{p'}{p'_{ref}} \right)^{m_K} \cdot \left[R_{K,min} + \frac{1 - R_{K,min}}{1 + \left(\frac{|\varepsilon_{vol}|}{a_K} \right)^{b_K}} \right] \geq K_{min} \quad (B.2)$$

where G_{ref} [kPa] and K_{ref} [kPa] describe, respectively, the maximum shear and bulk moduli at a reference mean effective stress of p'_{ref} [kPa], m_G and m_K define the non-linearity of the dependency of stiffness on the mean effective stress, a_G , b_G and $R_{G,min}$ are parameters determining the reduction of tangent shear modulus with deviatoric strain, E_d (the second invariant of the strain tensor, $\boldsymbol{\varepsilon}$, see Equation B.3), and a_K , b_K and $R_{K,min}$ are parameters describing the reduction of tangent bulk modulus with the volumetric strain, ε_{vol} (the first invariant of the strain tensor, see Equation B.4). Lastly, G_{min} [kPa] and K_{min} [kPa] are lower limits for the material stiffness which are neither stress- nor strain-dependent.

$$E_d(\boldsymbol{\varepsilon}) = \left(\frac{2}{3} \cdot [(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2] + \gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2 \right)^{\frac{1}{2}} \quad (B.3)$$

$$\varepsilon_{vol}(\boldsymbol{\varepsilon}) = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad (B.4)$$

The full set of parameters for IC MAGE M01 – adopted from previous studies presented in Gawecka et al. (2017) and Sailer et al. (2019), where analyses using these parameters were shown to reproduce the observed behaviour of pile foundations and retaining walls in London Clay, respectively – are listed in Table B.1.

Table B.1: material parameters adopted for the IC MAGE M01 model.

Shear modulus	G_{ref} [kPa]	m_G	a_G	b_G	$R_{G,min}$	G_{min} [kPa]
	51743.33	1.0	5.6×10^{-5}	0.9	0.06450	2667.0
Bulk modulus	K_{ref} [kPa]	m_K	a_K	b_K	$R_{K,min}$	K_{min} [kPa]
	26692.73	1.0	1.27×10^{-4}	1.8	0.13275	5000.0
Plastic behaviour	φ [°]	c [kPa]	ψ [°]			
	25	5	12.5			