

Enhancing the Degrees of Freedom in Array Signal Processing

by

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The logo features a dark blue background with a white wave-like shape at the bottom. The text "Imperial College" is in a bold, dark blue font, and "London" is in a lighter blue font, both positioned within the white wave shape.

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Abstract

Future antenna array systems are expected to deliver extremely high data rates whilst combating challenges of exponential increase in user density, high interference levels and multipath dispersion effects. Towards this end, this thesis presents novel algorithms to enhance the “degrees of freedom” (DoF) of a multiple-input multiple-output (MIMO) antenna array system. Furthermore, arrays that possess time-varying array geometry, termed as “flexible” arrays, are investigated for the purpose of defence and internet of things (IoT) applications, such as unmanned aerial vehicle (UAV) clusters and multi-platform arrays.

The parametric channel model, which provides a tractable mathematical model for wireless channels, lays the foundation for all the algorithms in this thesis. Based on this model, three systems are proposed, that extend the DoF progressively. Firstly, the spatial-only system is explored, in terms of the relationships between the array geometry and various array performance criteria, such as the thresholds of detection and resolution and the capacity loss. Secondly, the spatial-only system is extended to a spatiotemporal array system, to extend the DoF beyond the spatial domain, by employing a novel transmitter, a multi-parameter channel estimator and a spatiotemporal beamformer that suppresses interference in space and time jointly. Thirdly, the spatiotemporal system is extended to a spatiotemporal-virtual system, by extracting and exploiting the Tx array geometry.

Finally, the concept of “flexible” array signal processing is introduced for sensor arrays with time-varying sensor positions, such as UAV clusters and towed arrays. Based on three underpinning models, namely, a parametric channel model, a physical mobility model of the flexible array and a mobility model of the targets, an arrayed-state space model is constructed that is utilised in conjunction with an extended Kalman filter, to continually track the sensor positions and target parameters. All the algorithms presented in this thesis are evaluated using computer simulation studies.

Declaration of Originality

I hereby declare that the intellectual content of this thesis is the product of my own research work. Some parts of this thesis have appeared earlier in reports submitted as part of my PhD and in journal and conference papers published during the course of this PhD. Where information and assistance was received from other sources, they have been acknowledged appropriately in the text and in the form of references provided in the bibliography.

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Table of Contents

Abstract	1
Declaration of Originality	2
Copyright Declaration	3
Acknowledgments	4
Table of Contents	5
List of Figures	8
List of Tables	11
Publications Related to this Thesis	12
Notation	13
Abbreviations	14
Chapter 1: Introduction	17
1.1 Fundamentals of Array Signal Processing	18
1.2 Challenges for Array Signal Processing	21
1.3 Thesis Scope and Organisation	24
Chapter 2: Parametric Channel Model and the Role of Array Geometry	27
2.1 Overview	28
2.2 Parametric Channel Model	31
2.3 Parameters of the Array Manifold	33
2.4 Relationships between Array Geometry and Array Performance Cri- teria	36

2.4.1	Detection and Resolution Thresholds	37
2.4.2	Capacity Loss	40
2.5	Summary	45
	Appendix 2.A: Proof of Eq. 2.23	46
Chapter 3:	Spatiotemporal Array Signal Processing	48
3.1	Overview	49
3.2	State of the Art: Space-time Array Signal Processing	51
3.3	Design of the Tx and Rx	54
3.3.1	Tx System Design	54
3.3.2	Rx System Design	56
3.3.3	Comparison of Array Bounds	61
3.4	Spatiotemporal Beamformer Design	62
3.4.1	Joint Delay–Doppler Estimation	63
3.4.2	DOA Estimation	64
3.4.3	Combined Fading Coefficient Estimation	64
3.4.4	Design of the Rx Spatiotemporal Beamformer Weights	66
3.5	Computer Simulation Studies	68
3.5.1	Evaluation of Channel Estimation	69
3.5.2	Evaluation of Interference Mitigation	71
3.6	Summary	73
Chapter 4:	Spatiotemporal-Virtual Array Signal Processing	74
4.1	Overview	75
4.2	State of the Art: Virtual Arrays	76
4.3	Tx and Rx System Design	78
4.3.1	Tx System Model	78
4.3.2	Rx System Model	80
4.4	Spatiotemporal-Virtual Beamformer Design	85
4.4.1	Joint Delay–Doppler Estimation	86

4.4.2	Joint DOA–DOD Estimation	86
4.4.3	Design of the Rx Spatiotemporal-Virtual Beamformer Weights	87
4.5	Computer Simulation Studies	88
4.5.1	Evaluation of the Channel Estimator Block	89
4.5.2	Evaluation of the ST-virtual Reception Block	90
4.6	Summary	94
Chapter 5:	Flexible Array Signal Processing	96
5.1	Overview	97
5.2	State of the Art: Flexible Arrays	100
5.2.1	Target/Source Localisation	100
5.2.2	Array Shape Tracking	103
5.3	Flexible Array Signal Processing Model	105
5.3.1	Received Signal Model	106
5.3.2	Flexible Array Mobility Model	107
5.3.3	Target Constant Velocity Mobility Model	112
5.4	Arrayed-State Space Model and Extended Kalman Framework for Flexible Arrays	113
5.5	Simulation Studies	117
5.5.1	Cluster of UAVs	117
5.5.2	Towed Arrays	120
5.6	Summary	122
Chapter 6:	Conclusions and Further Work	123
6.1	List of Contributions	126
6.2	Suggestions for Further Work	127
References		128

List of Figures

1.1	Illustration of the plane wave propagation model.	19
1.2	Illustration of thesis structure.	25
2.1	Illustration of the MIMO parametric channel model for the i -th user.	32
2.2	Illustration of the environment to study detection and resolution thresholds.	34
2.3	Illustration of the environment to study capacity loss.	35
2.4	Illustration of the 2D grid array geometries employed for evaluating the array performance criteria.	38
2.5	Comparison of the detection and resolution thresholds for the grid array geometries under evaluation.	39
2.6	Comparison of the capacity loss for the grid array geometries under evaluation.	44
2.7	Comparison of the capacity loss for the grid array geometries under evaluation when the desired user is located at the center of the interferer cluster.	45
3.1	Classification of spatiotemporal array receivers.	53
3.2	Baseband representation of the transmitter and channel of the i -th source in the proposed spatiotemporal system.	55

3.3	The 3D data cube and its vectorisation to form the spatiotemporal snapshot $\underline{x}[n]$	57
3.4	Block diagram of the spatiotemporal channel estimator and receiver.	58
3.5	Sample results of the delay–Doppler and DOA estimation of the spatiotemporal channel estimator.	70
3.6	Sample results of the estimation of the phase of the combined path fading coefficient of the spatiotemporal channel estimator.	70
3.7	Performance evaluation of the spatiotemporal channel estimator.	71
3.8	Comparison of the interference mitigation capabilities of the proposed spatiotemporal subspace receiver against other state of the art receivers.	72
4.1	Baseband representation of the transmitter and MIMO channel of the i -th source in the proposed spatiotemporal-virtual system.	79
4.2	Illustration of the format of the transmitted message across the Tx array.	80
4.3	Illustration of the proposed spatiotemporal-virtual channel estimator and receiver.	81
4.4	Illustration of the 3D data cube, projection and vectorisation for the spatiotemporal-virtual system.	82
4.5	Sample result at the output of the spatiotemporal-virtual channel estimator.	90
4.6	Performance evaluation of the spatiotemporal-virtual channel estimator.	91
4.7	Comparison of the dependence of output SNIR on the Tx array geometry (ULA) for various receivers.	93
4.8	Comparison of the dependence of output SNIR on the Tx array geometry (UCA) for various receivers.	93

4.9	Comparison of the degrees of freedom offered by the proposed spatiotemporal-virtual system as compared to other systems.	94
5.1	Classification of the current state of the art in flexible array signal processing.	101
5.2	Illustration of the proposed framework for flexible array signal processing that tracks the target parameters and array positions snapshot by snapshot.	106
5.3	Illustration of the UAV cluster with the unknown targets in the xy -plane.	111
5.4	True and tracked azimuth trajectories of the targets by the UAV cluster.	118
5.5	True and tracked azimuthal angular velocity trajectories of the targets by the UAV cluster.	119
5.6	True and tracked locations of the UAVs in the UAV cluster.	119
5.7	True and tracked azimuth trajectories of the targets by the towed array.	121
5.8	True and tracked azimuthal angular velocity trajectories of the targets by the towed array.	121

List of Tables

2.1	Ranking of the five array geometries based on the capacity loss. . .	46
3.1	Simulation parameters employed to evaluate the spatiotemporal beamformer	68
4.1	Simulation parameters employed to evaluate the spatiotemporal- virtual beamformer	89
5.1	Summary of steps to estimate target parameters and the flexible array positions using the extended Kalman filter	116
5.2	Difference in MSE of the predictions from the noiseless mobility model and the proposed algorithm	120

Publications Related to this Thesis

The following papers were accepted for publication during the Ph.D.

1. T. Gabillard, V. Sridhar and A. Manikas, "Capacity loss and antenna array geometry", in IEEE International Conference on Communications (ICC), 21-25 May 2017.
2. V. Sridhar and A. Manikas, "Target tracking with a flexible UAV cluster array", in IEEE Global Communications Conference (GLOBECOM), 4-8 Dec 2016.
3. V. Sridhar, T. Gabillard and A. Manikas, "Spatiotemporal-MIMO channel estimator and beamformer for 5G", IEEE Transactions on Wireless Communications, Sep 2016.
4. A. Manikas, V. Sridhar, and Y. Kamil, "Array of Sensors: A Spatiotemporal-State-Space Model for Target Trajectory Tracking", Proceedings of IEEE SAM 2016 (invited)
5. T. Gabillard, V. Sridhar, A. Akindoyin and A. Manikas, "Comparative Study of 2D Grid Antenna Array Geometries for Massive Array Systems," IEEE Globecom Workshops, San Diego, CA, 2015, pp. 1-6.
6. V. Sridhar, M. Willerton and A. Manikas, "Towed Arrays: Channel Estimation, Tracking and Beamforming", in Beamforming: Sensor Signal Processing for Defence Applications, Vol. 5, London, UK: Imperial College Press, 2015.

Notation

a, A	Scalar
$\underline{a}, \underline{A}$	Column Vector
$\mathbf{a}, \mathbb{A}$	Matrix
$(\cdot)^T, (\cdot)^*, (\cdot)^H, (\cdot)^{-1}$	Transpose, Conjugate, Hermitian transpose, Inverse
$\widetilde{(\cdot)}$	Uncertainties/Errors
$\exp(\underline{A})$	Element by element exponential of vector $\underline{A}$
$\underline{A}^b$	Element by element power
$\text{col}_{m:k}(\mathbb{A})$	Columns m to k of the matrix $\mathbb{A}$
$\det(\mathbb{A})$	Determinant of matrix $\mathbb{A}$
$\text{Re}\{\cdot\}$	Real component
$\angle a$	Phase of a
$\text{eig}_m(\mathbb{A})$	m -th eigenvalue of $\mathbb{A}$
$\ \underline{A}\ $	Euclidean norm of vector $\underline{A}$
$\text{trace}(\mathbb{A})$	Sum of the diagonal elements of matrix $\mathbb{A}$
$\mathcal{E}\{\cdot\}$	Expectation operator
$\approx$	Approximately equal
$\odot, \otimes$	Hadamard product, Kronecker product
$\forall$	For all
$\underline{0}_N, \underline{1}_N$	Column vector of N zeros and N ones
$\mathbb{I}_N$	$N \times N$ Identity matrix
$\mathbb{O}_{N \times M}$	$N \times M$ matrix of zeros
$\mathcal{R}, \mathcal{C}$	Set of real & complex numbers respectively

Abbreviations

A/D	Analogue-to-Digital
AIC	Akaike Information Criteria
AOSN	Autonomous Ocean Sampling Network
AUV	Autonomous Underwater Vehicle
CDMA	Code Division Multiple Access
CRB	Cramer Rao Bound
D/A	Digital-to-Analogue
DF	Direction Finding
DOA	Direction of Arrival
DOD	Direction of Departure
DoF	Degrees of Freedom
EKF	Extended Kalman Filter
ESPRIT	Estimation of Signal Parameters via Rotation Invariance Techniques
GLS	Generalised Least Square
IoT	Internet of Things
ISI	Inter Symbol Interference
KR	Khatri-Rao
LMS	Least Mean Squares

LOS	Line-of-Sight
MAI	Multiple Access Interference
MDL	Minimum Descriptor Length
MIMO	Multiple-Input Multiple-Output
ML	Maximum Likelihood
MSE	Mean Square Error
MUSIC	Multiple Signal Classification
NFR	Near Far Ratio
NLOS	Non-Line-of-Sight
PN	Pseudo Noise
RADAR	Radio Detection and Ranging
RF	Radio Frequency
RMSE	Root Mean Square Error
SAR	Synthetic Aperture Radar
SIMO	Single-Input Multiple-Output
SNIR	Signal to Noise plus Interference Ratio
SNR	Signal-to-Noise Ratio
SONAR	Sound Navigation and Ranging
SPARTA	Spatially Referenced Towed Array
STAR	Spatio Temporal Array
TDL	Tapped Delay Line
TDMA	Time Division Multiple Access
TOA	Time of Arrival
Tx, Rx	Transmitter, Receiver
UAV	Unmanned Aerial Vehicle
UCA	Uniform Circular Array

ULA	Uniform Linear Array
VANET	Vehicular Adhoc Network

Chapter 1

Introduction

Array signal processing plays a key role in a wide variety of domains, such as wireless communication technologies, target localisation and tracking using airborne and ground radar, remote sensing, clusters of autonomous vehicles on land, air and underwater and internet of things (IoT) applications such as wearables and smart homes, to name a few. The history of array signal processing may be traced back to the mid-19th century when widely-separated telescopes were proposed for high-resolution astronomical imaging. However, a majority of the technical advances in array signal processing have occurred in the last 30 years and may be attributed to the development of high speed inexpensive processors, advancement of integrated chip design and the availability of high rate analogue-to-digital (A/D) and digital-to-analogue (D/A) convertors. This was accompanied by the development of various algorithms and mathematical tools that enabled sensor array signal processing. Defence applications such as radar and sonar were the driving forces behind predominant advances in array signal processing. Defence and security segments, all over the world, have a keen interest in antenna array solutions such as multi-sensor surveillance systems to locate and track multiple targets with high resolution and jamming and anti-jamming systems. However, in the past decade, with shrinking sizes of antenna elements and low power and inexpensive electron-

ics, antenna arrays have started to play a major role in multi-input, multi-output (MIMO) wireless communications. Current wireless communication technologies, namely 3G and 4G, employ antenna arrays that give them the potential ability to enhance directivity, gain, signal-to-noise ratio (SNR), coverage, redundancy, interference cancellation, clutter suppression and provide heightened capabilities of detection, resolution and capacity. With the possibility of exploiting millimetre (mm-wave) bands, where the antenna element shrinks drastically, larger antenna arrays become plausible on the mobile handset. Thus, looking to the future, array signal processing is expected to be in the vanguard of future communication and defence technologies.

1.1 Fundamentals of Array Signal Processing

An antenna/sensor array may be defined as a collection of antennas/sensors distributed in 3-dimensional space operating together as a unit. Antenna array signal processing refers to the exploitation of the data obtained across all these sensors to perform three crucial tasks, namely source/target detection, channel estimation and reception or beamforming. Towards modelling and addressing these problems, the concept of the array manifold vector (array impulse response vector) is critical. Consider an N -element antenna array with sensor locations $\mathbf{r}$ given by

$$\mathbf{r} \triangleq [r_1, r_2, \dots, r_N]^T = [r_x, r_y, r_z] \in \mathcal{R}^{N \times 3}, \quad (1.1)$$

operating in the presence of one source lying in the far field of the array. The assumption of far field implies that the radius of propagation is large enough to assume that the propagating wavefronts are plane wave rather than spherical waveforms. Figure 1.1 illustrates the plane wave propagation model of the source. The far field distance ρ may be defined in terms of the Fraunhofer distance as

$$\rho \gg \frac{2D^2}{\lambda}, \quad (1.2)$$

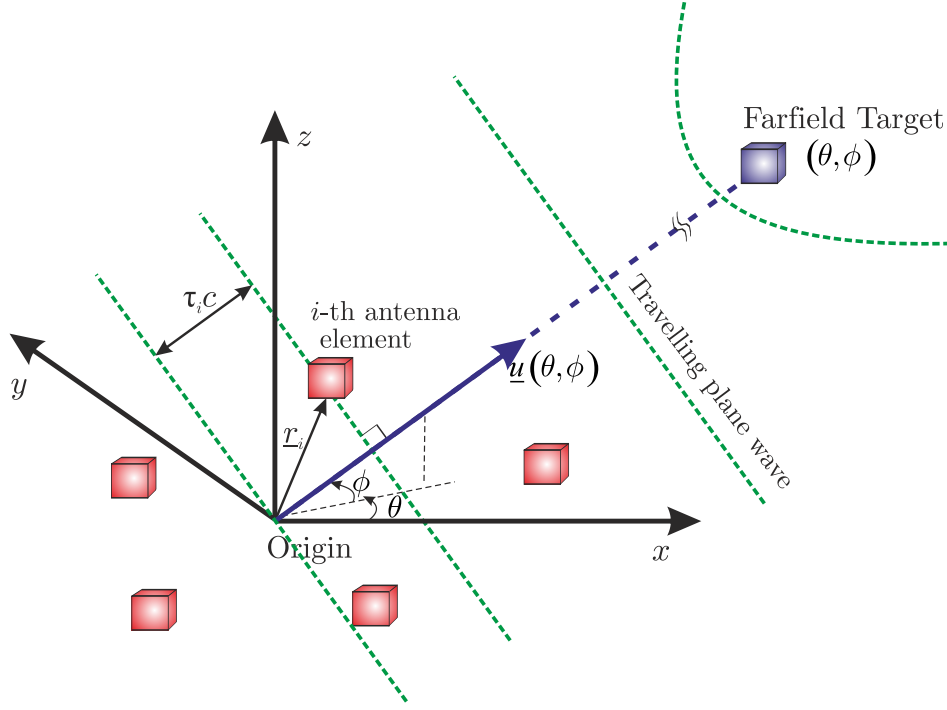


Figure 1.1: Illustration of the plane wave propagation model.

where D denotes the antenna array aperture given by

$$D = \max_{\forall i,j} \| \underline{r}_i - \underline{r}_j \|, \quad (1.3)$$

and λ denotes the wavelength of the signal utilised for propagation. With this assumption, the time delay τ_i between a wavefront impinging on antenna i and the origin is a simple function of the direction of arrival (DOA) (θ, ϕ) of the source and the location of the i -th antenna as follows

$$\tau_i = \frac{\underline{r}_i^T \underline{u}(\theta, \phi)}{c}, \quad (1.4)$$

where c is the speed of propagation in the medium and $\underline{u}(\theta, \phi)$ denotes the unit norm propagation vector given by

$$\underline{u}(\theta, \phi) = [\cos \theta \cos \phi, \sin \theta \cos \phi, \sin \phi]^T. \quad (1.5)$$

Thus, as the plane wave signal from a source propagates across various elements of the antenna array, the signal undergoes a phase shift corresponding to the time

delay given by Eq. 1.4. Extending this concept to all the elements in the antenna array, post down converting the signal to baseband, we may write the array impulse response vector $\underline{S}(\theta, \phi)$, termed as the array manifold vector, as follows

$$\begin{aligned}\underline{S}(\theta, \phi) &= \left[\exp\left(-j2\pi F_c \frac{\underline{r}_1^T \underline{u}}{c}\right), \exp\left(-j2\pi F_c \frac{\underline{r}_2^T \underline{u}}{c}\right), \dots, \exp\left(-j2\pi F_c \frac{\underline{r}_N^T \underline{u}}{c}\right) \right]^T \\ &= \exp\left(-j2\pi \frac{F_c}{c} [\underline{r}_x, \underline{r}_y, \underline{r}_z] \underline{u}(\theta, \phi)\right),\end{aligned}\quad (1.6)$$

where F_c denotes the carrier frequency. Note that, in the far field, the array manifold vector is not a function of the source range, i.e. the absolute distance of the source from the origin. Thus, the array manifold vector provides crucial information about the targets and the role of the array geometry in the system.

Based on the array manifold vector and the properties of the system environment, array signal processing techniques attempt to address the following tasks

- The detection problem: This problem is concerned with determining the number of incident signals on the array or number of targets/users in the environment of the array under consideration. There exist many approaches towards addressing this problem such as the Akaike Information Criterion (AIC) in [1] and Minimum Description Length (MDL) in [2].
- The estimation problem: This problem is concerned with the estimation of the parameters associated with the source or target such as time delay, DOA and Doppler frequency. A majority of the current literature focuses on direction finding algorithms such as the Multiple Signal Classification (MUSIC) algorithm [3] and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [4].
- The beamforming problem: This problem revolves around design of appropriate weights at the Rx and/or Tx to deliver high gains towards the desired user and thus acquire the desired signal whilst suppressing the interference in all possible domains [5]. This enables the Rx or Tx array to steer the beam adaptively based on the outcome of the estimation problem.

The three aforementioned problems form the crux of the research in array signal processing. Some applications may focus solely on the detection and estimation problems such as radar applications. The third step is more relevant to wireless communication receivers where interference cancellation and combating path loss are critical to establish a reliable radio link. The challenge of array signal processing is to perform these aforementioned tasks in challenging and hostile environments that pose severe fading, high pathloss, high interference and high clutter.

1.2 Challenges for Array Signal Processing

Array signal processing has been and is expected to be crucial for the advancement of a wide variety of applications. However, techniques in array signal processing are required to continuously evolve to meet the proliferating demands placed on array applications such as snowballing user density, swelling capacity requirements and heightened interference suppression. Moreover, with the possibility of integrating sensors on a wide variety of platforms, with data links to collect the sensor data, many unconventional array systems start to arise, such as arrays with time-varying array geometries considered in this thesis. Thus, array signal processing remains a “hot-bed” of research spanning across various disciplines. In the present work, the focus is on two broad areas of antenna array applications, namely, antenna arrays for mobile wireless communications and “flexible” arrays such as arrayed vehicular clusters. Solutions to address some crucial problems with respect to these applications are provided. An overview of the challenges being addressed is as follows

1. Requirement of the parametric channel model

Modelling the wireless channel where the array systems operate is key to the design of suitable array signal processing algorithms. In the context of

wireless communication systems, there has been extensive literature on the subject, with channel matrices that model the frequency selective wireless channel. However, a majority of these approaches are “non-parametric” since they characterise the channel either independent of channel parameters or utilise the statistical models of these channel parameters in place of their instantaneous values and often assume that independent parallel identical channels between different Tx and Rx array elements exist [6]. Subsequently, for future 5G wireless communications and beyond, these models are rendered unsuitable, since the channel estimation is extremely intractable with a huge number of unknowns and the underlying assumption of uncorrelated channels between Tx and Rx antennas starts to break down. Hence, there is a requirement of devising new channel models that are representative of the channel, as well as simplify the estimation procedure with more tractable parameters. This class of models known as “parametric models” is presented in this thesis and forms the backbone of the design of the various systems proposed throughout this thesis.

2. Importance of antenna array geometry

In the literature, a majority of the channel models for Tx and/or Rx array systems either ignore the array geometry or fall short of fully exploiting it towards the objective of the system. The array geometry starts to play a crucial role in scenarios where targets or users occur in clusters or where targets/users are spatially very close to one another. Such a scenario is extremely relevant to future urban communication scenarios where a number of users are expected to operate within a small region. Therefore, one of the crucial contributions in this thesis is to study the importance of array geometry towards the signal processing abilities of arrays for spatially close users and determine performance metrics that can be employed to select an appropriate array geometry as per the application at hand.

3. Increasing degrees of freedom

With spiralling data rates, user density and quality of service requirements, current array signal processing systems are required to possess a large number of dimensions, or alternatively deliver very high degrees of freedom. To such a degree, solutions that extend the degrees of freedom, especially for wireless communications, form one of the key contributions of this thesis. A straightforward and popular approach to increase the degrees of freedom is to employ “massive” systems that possess a large number of antennas. However, in this thesis, alternative means to enhance the degrees of freedom without increasing the number of antennas are explored. Firstly, techniques to extend the spatial-only MIMO system to a spatiotemporal MIMO system, that dictates a unique temporal structure at the Tx and exploits space and time jointly at the Rx to suppress interference, are presented. The degrees of freedom are then further extended by employing a spatiotemporal-virtual system that exploits the Tx array geometry additionally. Hence, increasing the degrees of freedom constitutes one of the core objectives addressed in this thesis.

4. Flexible array signal processing

Apart from arrays that possess time-invariant array geometry, certain applications utilise arrays composed of sensors with time-variant positions, termed as “flexible” arrays. Some examples of such applications are towed arrays, clusters of unmanned aerial vehicles (UAVs), clusters of satellites and clusters of underwater vehicles. With the growth of internet of things, flexible arrays are also expected to play a key role in wireless communications, where a number of mobile sensor arrays will be expected to operate alongside mobile targets or users. Flexible array signal processing refers to the detection, estimation and reception of signals received on flexible arrays. Such an array may experience both arbitrary and known motion (due to uncertainties

in the independent propulsion of each sensor) and arbitrary and unknown motion (due to external perturbations). In this scenario, typical array signal processing algorithms fail since they are built on the premise of a constant and known array geometry. Consequently, array signal processing algorithms need to be adapted to incorporate the time-varying array geometry and provide an ability to track this variation with time. This forms another major contribution of this thesis.

1.3 Thesis Scope and Organisation

The primary focus of this thesis is to study techniques to extend the degrees of freedom in antenna arrays, that possess time-invariant and time-varying array geometry (also known as “flexible” arrays). Algorithms and receiver designs for antenna arrays are proposed in the context of future wireless communications to address various challenges such as heightened user density, higher spatial discrimination amongst users/targets, higher interference suppression and higher data rate requirements. “Flexible” arrays are studied for applications such as UAV clusters and towed arrays, where an arrayed-state space model tracks the array geometry whilst estimating the target or source channel parameters. The performance of the algorithms presented in each chapter are investigated using computer simulation studies.

The rest of this thesis is organised into five chapters as shown in Fig. 1.2. As illustrated in Fig. 1.2, Chapters 2 to 4 are related to time-invariant arrays, with a focus on future communication systems, while Chapter 5 is related to time-varying arrays employed in applications such as towed arrays and UAV clusters. In Chapter 2, the concept of the array manifold for antenna arrays is introduced. Using the array manifold, a parametric channel model is introduced for a MIMO system that encompasses the antenna array geometries at the Tx and Rx and

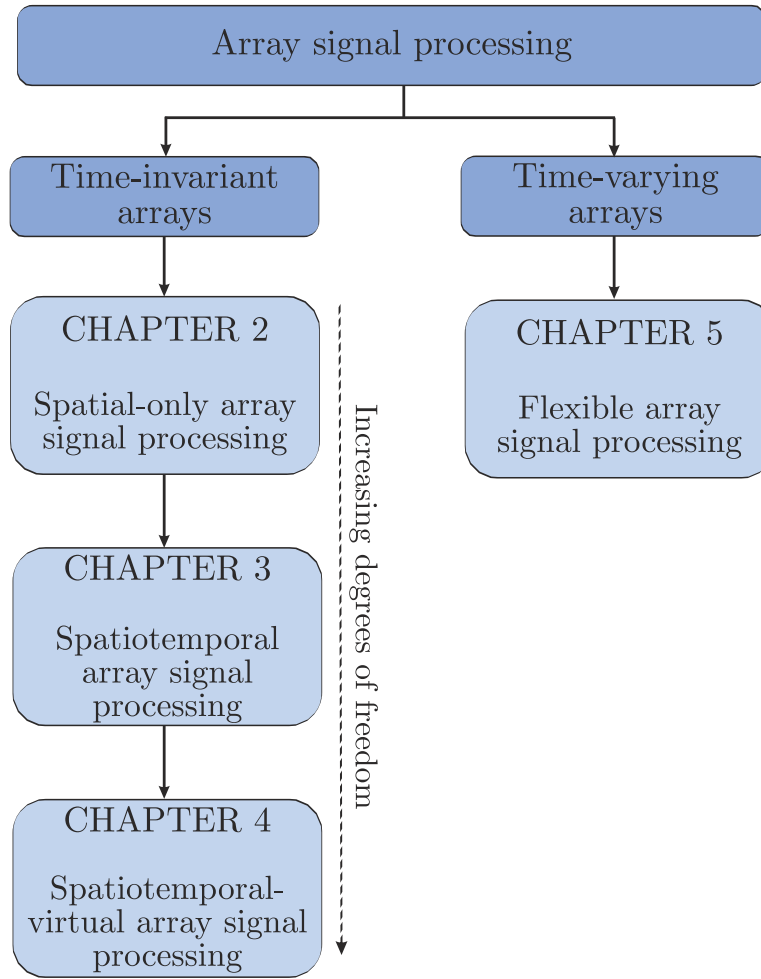


Figure 1.2: Illustration of thesis structure.

the source/target parameters. Equipped with this channel model and tools from differential geometry, the array manifold is utilised to study the impact of array geometry on the abilities of detection, estimation and capacity of the Rx array for two users spatially very close to each other.

This is followed by Chapter 3, where the antenna array system is extended to a spatiotemporal antenna array system. Based on the parametric channel model introduced in Chapter 2 and a novel design of the transmitted signal, a spatiotemporal channel estimator and beamformer suitable for future 5G wireless communications is presented. It will be shown that the proposed spatiotemporal receiver utilises a fewer number of antennas to obtain a gain identical to that of a massive

array. A representative example where the proposed spatiotemporal beamformer consisting of only nine Rx antenna elements is illustrated to outperform a spatial-only massive MIMO receiver consisting of 80 antenna elements.

In Chapter 4, the spatiotemporal system presented in Chapter 3, is enhanced to a spatiotemporal-virtual array system that extends the degrees of freedom further by extracting the information about the Tx array geometry. It will be illustrated that the proposed receiver outperforms the spatiotemporal receiver in terms of the degrees of freedom and eliminates the performance dependence on the Tx array geometry. Thus, as indicated in Fig. 1.2, the systems presented in Chapters 2 to 4 increase the degrees of freedom of the communication system progressively.

Following this, in Chapter 5, a special class of applications that utilises “flexible” arrays with time-varying array geometry is studied. Two specific applications, namely, UAV clusters and towed arrays are considered. An arrayed-state space model that incorporates the flexible arrayed-parametric model, mobility model of the flexible array and mobility model of the targets, is utilised in conjunction with an extended arrayed-Kalman filter to track the target parameters and time-varying array sensor positions.

Finally, in Chapter 6, a brief summary and a discussion of the conclusions drawn from the simulations are presented. The chapter is concluded with an outline of the possible directions for future further work.

Chapter 2

Parametric Channel Model and the Role of Array Geometry

In this chapter, a “parametric” channel model that models the channel in terms of parameters such as the DOA, fading coefficients, propagation delays, Doppler frequencies of different users or targets and the array geometry is presented. This model is more tractable than “non-parametric” channel models, making it more suitable for future massive array communication systems. Based on this parametric channel model, three array performance criteria to measure the antenna array’s ability to distinguish spatially close users, namely, the detection threshold, resolution threshold and a novel concept termed as “capacity loss” are presented. A set of five array geometries are compared on the basis of these array performance criteria to illustrate the important role of selection of array geometry as per the objective of the system at hand. It is important to point out that, inline with the stringent requirements of future communication systems, a worst-case environment is considered for comparison rather than the more popular approach of average statistical performance.

2.1 Overview

Antenna or sensor arrays play a crucial role in a variety of applications ranging from communications, radar to geolocation systems. In order to design appropriate transmitter and receiver algorithms for antenna arrays, appropriately modelling the MIMO channel is crucial. There exists abundant literature that provide mathematical models for a MIMO channel, given by the matrix $\mathbb{H} \in \mathcal{C}^{N \times \bar{N}}$, between an $\bar{N}$ element Tx array and an N element Rx array. However, there are two major drawbacks with a majority of the approaches in the literature. The first shortcoming is that most of these approaches rely on “non-parametric” channel models such as the Kronecker-based model, Weichselbeger’s model, Sayeed’s model and Muller’s models [7]. For instance, in [8, 9], using non-parametric models, the channel capacity in various environments, such as channels with high interference, indoor line-of-sight (LOS) and channels with high antenna correlation, are analysed. In [10], using a non-parametric channel model for an indoor environment, the effect of multipath propagation on the channel capacity is studied. An analysis of the channel capacity and analytical expressions of the spectral efficiency for an arbitrary input covariance matrix, in the presence of antenna correlation and interference, is presented in [11]. In [12], based on the Kronecker channel model, approximate spatial correlation models for specific array configurations for a clustered MIMO channel are proposed. Such “non-parametric” models characterise the channel either independent of the target/user channel parameters or utilise the statistical models of these channel parameters in place of their instantaneous values. This implies that channel estimation would require to estimate $N\bar{N}$ “independent” unknowns of the channel matrix and the system would possess $\min(N, \bar{N})$ degrees of freedom. The primary assumption under all these models is the existence of independent parallel identical channels between different Tx and Rx array elements. However, for large antenna array systems that are expected to play a crucial role

in future communication systems, measurement and modelling campaigns show that the underlying non-parametric models become quite unsuitable and erroneous since the channel does not present rich scattering [13]. Moreover, with the trend of employing massive arrays, the matrix $\mathbb{H}$ would grow in size resulting in an extremely high number of unknowns. Hence, it is more meaningful to model the channel accurately in terms of tractable parameters such as the DOA, direction of departure (DOD), fading coefficients, propagation delays, Doppler frequencies of different users or targets and the array geometry. With such models, the number of unknowns required to reconstruct the channel matrix $\mathbb{H}$ is independent of the number of antennas. Such models are known as “parametric models” and will constitute the backbone of this thesis.

In the literature, there have also been some approaches that utilised parametric models to study the impact of interference on the antenna array system. In [14], using 2D ray models, the channel is parametrically modelled and the effect of antenna spacing on the channel capacity is studied, albeit for a simple 2×2 MIMO system. In [15], the effect of spatial correlation on the channel capacity is studied. However, the influence of the array geometry has been ignored in the aforementioned research and this constitutes the second major shortcoming. That is, a majority of the models either ignore the array geometry and do not factor this into the channel model, or fail to fully exploit it towards the objective of the system. The impact of this is especially pertinent in scenarios where targets or users occur in clusters or where targets/users are spatially very close to one another. There have been some works that attempt to optimise the array geometry to achieve a desired performance criterion such as maintaining a uniform parameter estimation performance across the whole field-of-view [16], resolving ambiguities [17], providing increased coverage [18], reducing sidelobe level [19], improving direction finding capabilities [20] and designing linear arrays (non-uniform or uniform) and planar arrays with predefined detection and resolution thresholds [21]. Another

crucial criterion is delivering maximum channel capacity and the impact of array geometry on this end goal has garnered attention in the recent years. For instance, in [22], the impact of four uniform array geometries on the ergodic capacity in a microcellular urban environment is studied, albeit in the absence of interference. In [23, 24], an analysis of the achievable capacity for two different sets of array geometries is estimated assuming the users are located uniformly within a sphere and a circle of known radius, respectively. However, this scenario is representative of the average capacity, rather than the worst-case capacity that is obtained when the users are spatially very close to each other, as expected in future dense urban environments. Thus, in this chapter, the crucial role of array geometry, in conjunction with a parametric channel model, is studied in the context of the ability of an array system to perform crucial array signal processing tasks in the presence of “spatially close users/targets”. Specifically with respect to capacity, instead of the common notion of average capacity, a novel concept of “capacity loss” is introduced to study the impact of the array geometry on the channel capacity with the worst-case scenario of spatially close users.

In summary, in this chapter, the relationship between the abilities of a multiple antenna array system and the associated array geometry is studied, using a family of geometries known as “grid” arrays as the baseline for comparison and analysis. In specific, the following crucial insights are presented:

- The detection and resolution bounds are expressed as functions of the arc length of a manifold curve for 2D “grid” antenna arrays.
- The concept of “capacity loss” is introduced and expressed in terms of the parameters of the array manifold.
- Typical studies assume multiple users to be randomly distributed in space. However, here a worst-case scenario of spatially close users or targets is considered. This is more relevant to applications such as communications for dense networks.

Note that these studies are relevant to a wide range of applications utilising multiple antennas.

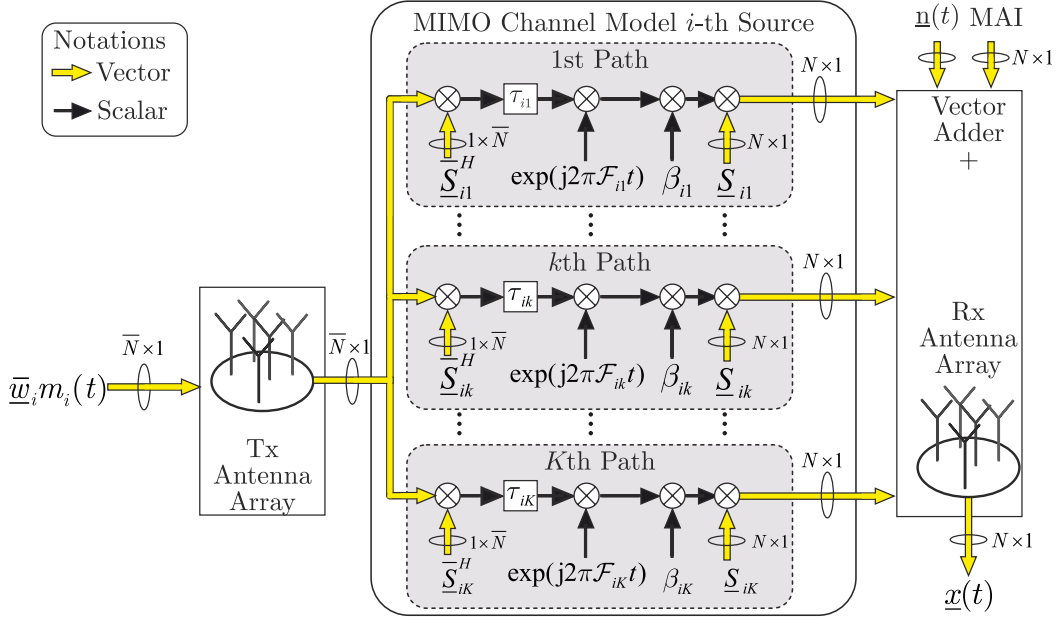
The remainder of this chapter is organised as follows. In Section 2.2, the parametric channel model that will be a crucial part of this thesis, is presented. This is followed by Section 2.3 where the basic parameters of an array manifold curve embedded in an N -dimensional complex space are presented. Section 2.4 is concerned with a comparative study of a number of antenna array geometries that belong to the family of “grid” arrays. In this analysis, detection and resolution thresholds and “capacity loss” are employed as figures-of-merit to compare array geometries for different applications. In Section 2.5, the chapter is summarised.

2.2 Parametric Channel Model

In this section, a parametric model describing a MIMO channel that is time-varying, multipath dispersive and frequency selective over the operating frequency band is presented. Figure 2.1 illustrates the MIMO channel model for the i -th source associated with K multipaths. This model is crucial for all the algorithms presented in the rest of this thesis. Consider a Tx antenna array of $\bar{N}$ antennas and a Rx antenna array of N antennas whose Cartesian coordinates are given by $[\bar{r}_x, \bar{r}_y, \bar{r}_z] \in \mathcal{R}^{\bar{N} \times 3}$ and $[r_x, r_y, r_z] \in \mathcal{R}^{N \times 3}$, respectively, in metres. The environment is assumed to consist of M users and every user is associated with K multipaths. The parameters i and k are used to represent the index of the source and multipath, respectively. The planewave spatial receiver array manifold vector for the k -th multipath of the i -th user is denoted by $\underline{S}_{ik} \triangleq \underline{S}(\theta_{ik}, \phi_{ik})$ and denotes the receiver array response for a plane wave arriving from a DOA¹ (θ_{ik}, ϕ_{ik}) and is given by

$$\underline{S}(\theta_{ik}, \phi_{ik}) = \exp \left(-j \left[r_x, r_y, r_z \right] \underline{k}(\theta_{ik}, \phi_{ik}) \right), \quad (2.1)$$

¹ θ is the azimuth angle measured anticlockwise with respect to the x -axis and ϕ is the elevation.

Figure 2.1: Illustration of the MIMO parametric channel model for the i -th user.

where $\underline{k}(\theta_{ik}, \phi_{ik})$ denotes the wavenumber vector defined as

$$\underline{k}(\theta_{ik}, \phi_{ik}) \triangleq \frac{2\pi}{\lambda} [\cos \theta_{ik} \cos \phi_{ik}, \sin \theta_{ik} \cos \phi_{ik}, \sin \phi_{ik}]^T, \quad (2.2)$$

with λ representing the wavelength of the carrier. Similarly, using $(\bar{\cdot})$ to denote all equivalent parameters associated with the transmit array $[\bar{r}_x, \bar{r}_y, \bar{r}_z]$, the transmit array manifold vector is denoted by $\underline{S}_{ik} \triangleq \underline{S}(\bar{\theta}_{ik}, \bar{\phi}_{ik})$, where $(\bar{\theta}_{ik}, \bar{\phi}_{ik})$ represents the DOD from the transmit antenna array. Using these definitions, the $N \times 1$ baseband signal obtained at the receive antenna array at the time instant t can be modelled as follows

$$\underline{x}(t) = \sum_{i=1}^M \sum_{k=1}^K \beta_{ik} \exp(j2\pi\mathcal{F}_{ik}t) \underline{S}_{ik} \underline{S}_{ik}^H \underline{w}_i m_i(t - \tau_{ik}) + \underline{n}(t), \quad (2.3)$$

where β_{ik} , $\mathcal{F}_{ik}$ and τ_{ik} represent the complex path fading coefficient, the Doppler shift and multipath delay, respectively. The transmit weight and message associated with the i -th user are denoted by $\underline{w}_i$ and $m_i(t)$, respectively. The Doppler shift is given by

$$\mathcal{F}_{ik} = -\frac{F_c v_{ik}}{c}, \quad (2.4)$$

where v_{ik} stands for the radial velocity of the i -th user's k -th path and F_c denotes the carrier frequency. (The positive value of the radial velocity represents the same direction as $\underline{k}$). Moreover, $\underline{n}(t)$ is the spatially and temporally white, zero mean complex additive Gaussian noise with an unknown variance σ_n^2 . The model illustrated by Fig. 2.1 and given by Eq. 2.3 embodies the basis of all the algorithms and systems proposed in this thesis and will be continually referred to as required. However, in this chapter, a simple case of M static users, each associated with a single path, operating in a single-input multiple-output (SIMO) system is considered and thus Eq. 2.3 simplifies to the following

$$\underline{x}(t) = \sum_{i=1}^M \beta_i \underline{S}_i m_i(t) + \underline{n}(t), \quad (2.5)$$

where β_i denotes the path fading coefficient associated with the i -th user (The subscript k has been dropped for simplicity). (Note that the scalar term $\overline{S}_{ik}^H \overline{w}_i$ has been absorbed within the path fading coefficient.)

2.3 Parameters of the Array Manifold

In this chapter, it is assumed that all the users lie on the xy -plane, i.e. all the users lie on a θ -curve [25]. Thus, the array manifold vector is a function of a single parameter namely the azimuth angle θ . The locus of all the array manifold vectors $\underline{S}(\theta)$, $\forall(\theta)$ is a curve $\mathcal{A}$ embedded in an N -dimensional complex space as illustrated in Figs. 2.2 and 2.3 and is defined as

$$\mathcal{A} \triangleq \{\underline{S}(\theta), \forall \theta \in [0^\circ, 360^\circ]\}. \quad (2.6)$$

It is clear from Eq. 2.6 that the array manifold is conventionally parameterised in terms of the azimuth angle θ . However, parameterisation in terms of the arc length s , which is the most basic feature of a curve and a natural parameter representing the actual physical length of a segment of the manifold curve, is more suitable.

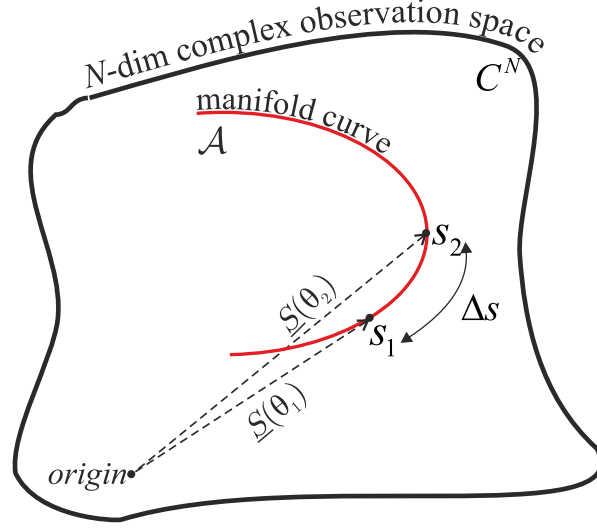


Figure 2.2: Illustration of the environment to study detection and resolution thresholds comprising of two closely spaced users on the array manifold $\mathcal{A}$ associated with manifold vectors $\underline{S}_1$ and $\underline{S}_2$ with an arc length Δs between the two users.

The arc length $s(\theta)$ along the manifold curve $\mathcal{A}$ is formally defined [25] as

$$s(\theta) \triangleq \int_0^\theta \left\| \frac{d\underline{S}(\theta)}{d\theta} \right\| d\theta. \quad (2.7)$$

The arc length between two points on the manifold curve at θ_1 and θ_2 is denoted by $\Delta s = s(\theta_2) - s(\theta_1)$ and is crucial in characterising the performance of the system [26]. In order to facilitate the analysis of the relationship between the various “figures-of-merit” described in this chapter and the array geometry, for the rest of this chapter, we represent the manifold vector of the i -th user as $\underline{a}_i$, in terms of the arc length, rather than the azimuthal representation ($\underline{S}(\theta_i)$) as follows

$$\underline{a}_i \triangleq \underline{a}(s_i). \quad (2.8)$$

Thus, using this, we may define the principal curvature $\kappa_1(s)$ at any point s [25] on the array manifold as

$$\begin{aligned} \kappa_1(s) &= \left\| \frac{d^2 \underline{a}(s)}{ds^2} \right\| \\ &= \frac{1}{\|\dot{\underline{r}}(\theta)\|^2} \left\| \dot{\underline{r}}^2(\theta) + \frac{j}{\pi} \mathbb{P}_{\dot{\underline{r}}(\theta)}^\perp \underline{r}(\theta) \right\|, \end{aligned} \quad (2.9)$$

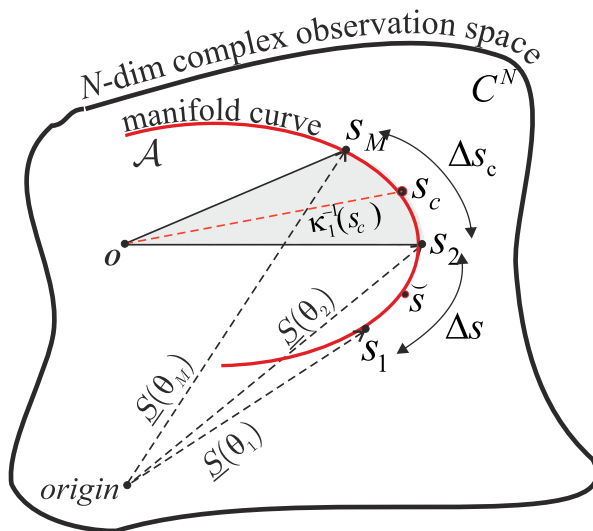


Figure 2.3: Illustration of the environment utilised to study capacity loss comprising of a desired user $\underline{s}_1$ and a cluster of interferers $\underline{s}_2, \dots, \underline{s}_M$. The cluster size is assumed to be Δs_c . The point O denotes the centre of the curvature of the arc on which the interferer cluster lies with radius equal to $\kappa_1^{-1}(s_c)$.

where

$$\underline{r}(\theta) = \underline{r}_x \cos \theta + \underline{r}_y \sin \theta, \quad (2.10)$$

$$\dot{\underline{r}}(\theta) \triangleq \frac{\partial \underline{r}(\theta)}{\partial \theta} = -\underline{r}_x \sin \theta + \underline{r}_y \cos \theta, \quad (2.11)$$

and $\mathbb{P}_{\dot{\underline{r}}(\theta)}^\perp$ represents the complement projection operator onto the subspace spanned by $\dot{\underline{r}}(\theta)$. That is,

$$\mathbb{P}_{\dot{\underline{r}}(\theta)}^\perp = \mathbb{I}_N - \dot{\underline{r}}(\theta) (\dot{\underline{r}}^H(\theta) \dot{\underline{r}}(\theta))^{-1} \dot{\underline{r}}^H(\theta). \quad (2.12)$$

In this chapter, two different user environments are considered. For the study of the detection and resolution thresholds, two users closely spaced on the array manifold curve are considered as shown in Fig. 2.2. For the study of capacity loss, a desired user and a cluster of interferers on the array manifold curve are considered as shown in Fig. 2.3. For the environment described in Fig. 2.3, using Eq. 2.9, we may find the principal curvature $\kappa_1(s_c)$ at the centre of the cluster of the interfering users, i.e. at point s_c given by

$$s_c \triangleq s(\theta_c) = \frac{(s_2 + s_M)}{2}. \quad (2.13)$$

The parameter s_c is shown in Fig. 2.3 along with $\Delta s_c = s_M - s_2$ that denotes the arc length of the cluster and corresponds to an angular separation $\Delta\theta_c = \theta_M - \theta_2$. Assuming a circular approximation² of the manifold curve in the region of the cluster, the radius of this segment is the inverse of the principal curvature $\kappa_1^{-1}(s_c)$ as also shown in Fig. 2.3. In this case, we may express the inner product between two array manifold vectors $\underline{a}(s_i)$ and $\underline{a}(s_j)$ in the cluster as

$$\underline{a}^H(s_i) \underline{a}(s_j) \approx N - 2\kappa_1^{-2}(s_c) \sin^2 \left(\frac{(j-i)\Delta s_c}{(M-2)2\kappa_1^{-1}(s_c)} \right), \quad (2.14)$$

with $2 \leq i, j \leq M$.

The above definitions and concepts will be utilised in the next section to analyse the relationships between the detection and resolution thresholds, capacity loss and the array geometry.

2.4 Relationships between Array Geometry and Array Performance Criteria

Consider a calibrated antenna array system consisting of N antennas operating in the presence of a desired static source and multiple static co-channel interferers. In this study, the sources are assumed to operate with a single antenna for simplicity. The desired source's worst-case performance scenario is dictated by the interfering source spatially closest to the desired source and is hence used as the baseline for comparison. The geometry plays an important role in the system's performance and the parameters introduced in Sections 2.2 and 2.3 will be employed in this section to provide a theoretical framework for studying and analysing different array geometries. More specifically, five antenna array geometries belonging to the class of "grid" arrays are considered for comparison. Figure 2.4 illustrates some representative examples of 2D grid arrays that are under consideration in this study.

²For larger Δs_c , corresponding to larger $\Delta\theta_c$, the circular approximation may not be valid and the second curvature $\kappa_2(s_c)$ of the array manifold curve may need to be calculated and used.

These geometries are 7×7 “filled”-grid (49 antennas), square (24 antennas), circular (18 antennas), concentric circular (28 antennas) and X-shaped (17 antennas) as shown in Fig. 2.4. The “filled”-grid array geometry is commonly known as the “grid” array. However, in this chapter the term “grid” arrays refers to the family of geometries obeying the following

$$\underline{r}_x^T \underline{r}_y = 0 \text{ and } \|\underline{r}_x\| = \|\underline{r}_y\|. \quad (2.15)$$

It is important to point out that all the geometries shown in Fig. 2.4 have been derived from the underlying “filled”-grid and chosen to have the maximum array aperture for the specified geometry. Due to the balanced symmetrical format of grid arrays, the associated array manifolds present unique properties that enables easier analysis as illustrated in the following sections. As figures-of-merit, the detection and resolution thresholds as well as a novel concept termed as “capacity loss” are employed and these are presented below.

2.4.1 Detection and Resolution Thresholds

In this subsection, the impact of the array geometry on the detection and resolution abilities of the array for two spatially closely spaced sources is investigated [27]. The environment is assumed to consist of one desired user and one interferer with an arc length separation of Δs along the array manifold curve as shown in Fig. 2.2.

Based on Chapter 8 of [26] and with reference to Fig. 2.2, it can be shown that the minimum $(\text{SNR} \times L)_{\text{det}}$ for the system to detect these two sources located close to each other in space (i.e. $\Delta\theta$ is small) is given by

$$(\text{SNR} \times L)_{\text{det}} = \frac{2}{\Delta s^2}. \quad (2.16)$$

Due to the special structure of grid arrays, the arc length Δs between user 1 ($\underline{S}(\theta_1)$) and user 2 ($\underline{S}(\theta_2)$) can be written as

$$\Delta s = \pi \|\underline{r}_x\| \Delta\theta_{1,2}, \quad (2.17)$$

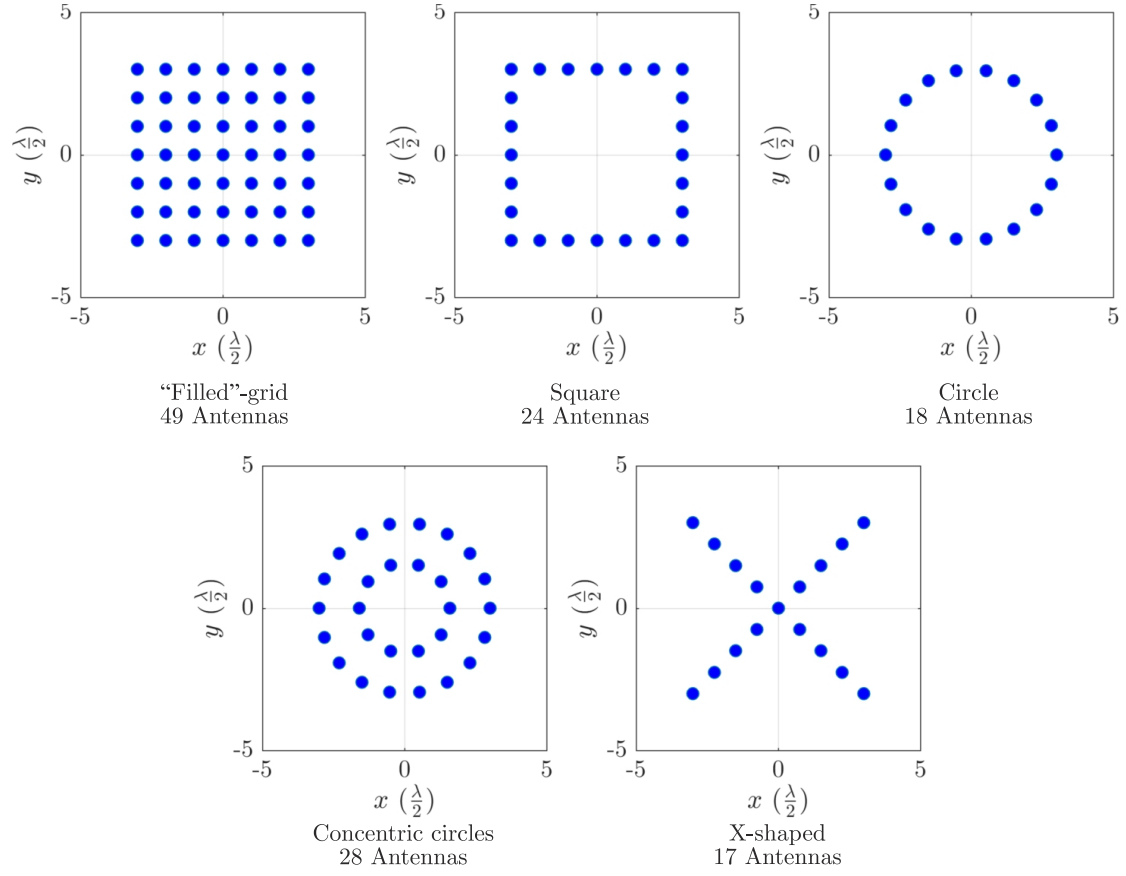


Figure 2.4: Illustration of the five 2D grid array geometries employed for evaluation namely (a) “filled”-grid ($N = 49$), (b) square ($N = 24$), (c) circle ($N = 18$), (d) concentric circle ($N = 28$) and (e) X-shaped ($N = 17$).

where $\Delta\theta_{1,2} = \theta_2 - \theta_1$. Thus, Eq. 2.16 simplifies to

$$(\text{SNR} \times L)_{\text{det}} \propto \frac{1}{\|r_x\|^2 \Delta\theta_{1,2}^2}. \quad (2.18)$$

In a similar manner, the resolution threshold with respect to the two sources in Fig. 2.2 provides a minimum $(\text{SNR} \times L)_{\text{res}}$ for the system to resolve the two sources [26],

$$(\text{SNR} \times L)_{\text{res}} = \frac{32}{\Delta s^4 \left(\kappa_1^2(\theta_m) - \left| \mathbf{1}_N^T \left(\frac{\underline{r}(\theta_m)}{\|\underline{r}(\theta_m)\|} \right)^3 \right|^2 - \frac{1}{N} \right)}, \quad (2.19)$$

where $\theta_m = (\theta_2 + \theta_1)/2$ and $\underline{r}(\theta_m)$ is evaluated using Eq. 2.10. Equation 2.19 indicates that, unlike detection, $(\text{SNR} \times L)_{\text{res}}$ depends on the first curvature $\kappa_1(\theta_m)$ and is not independent of the azimuth angles of the two sources.

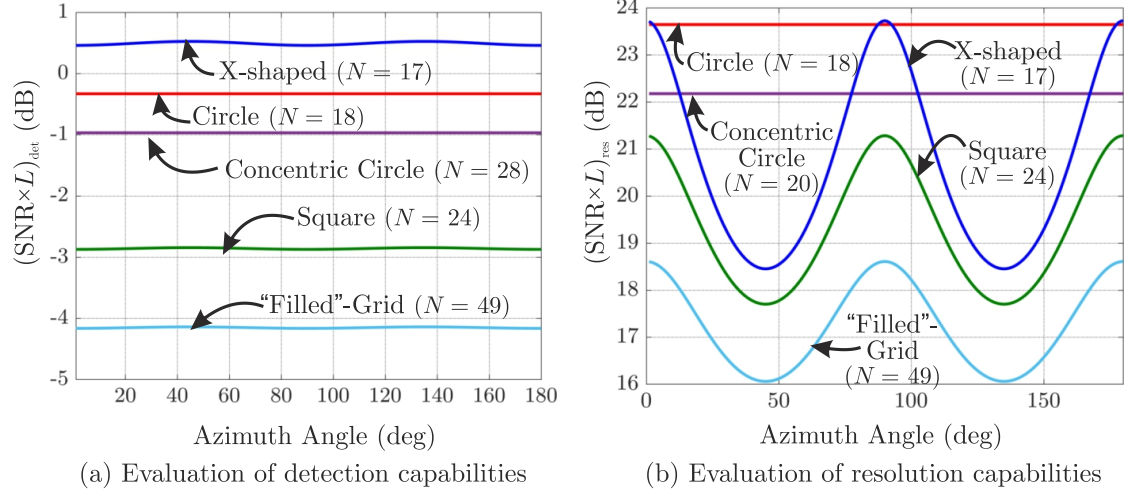


Figure 2.5: (a) Detection threshold $(\text{SNR} \times L)_{\text{det}}$ versus varying azimuth angle (b) Resolution threshold $(\text{SNR} \times L)_{\text{res}}$ versus varying azimuth angle. The angular separation $\Delta\theta_{1,2}$ is maintained at 3° .

Figures 2.5a and 2.5b illustrate the thresholds of detection and resolution presented in Eqs. 2.18 and 2.19 for the five grid array geometries under consideration. The elevation angles are maintained at 0° while θ_m is varied. The angular separation between the two users $\Delta\theta_{1,2}$ is maintained at 3° . Since the resolution threshold is a function of the first curvature, it varies with the azimuth angle as expected while the detection threshold remains independent of the azimuth angle. For both the detection and resolution thresholds, the "filled"-grid array followed by the square array provides the best performance while the circular and the X-shaped arrays perform the worst. One can observe that the number of antennas is the crucial factor that determines these thresholds. However, the result shows that the square array yields acceptable detection and resolution thresholds for a much lower number of antennas than the "filled"-grid array. This is due to the fact that the determining factor is $\|\underline{r}_x\|$ (remember $\|\underline{r}_x\| = \|\underline{r}_y\|$ for all grid arrays) rather than the number of antennas N itself. This implies that by optimising the placement and inter antenna spacing, for the same number of antennas N , lower detection and resolution thresholds may be obtained.

2.4.2 Capacity Loss

In this subsection, a novel concept of “capacity loss” will be defined and the impact of array geometry on the same will be investigated. Let us recall Eq. 2.5 that models the signal $\underline{x}(t)$ received at an N element Rx antenna array, operating in an environment consisting of M static users with one multipath each, where the i -th user is associated with the Rx manifold vector $\underline{S}_i$, message $m_i(t)$ and path fading coefficient β_i , as follows

$$\underline{x}(t) = \sum_{i=1}^M \beta_i \underline{S}_i m_i(t) + \underline{n}(t). \quad (2.20)$$

Using Eq. 2.20, the covariance matrix $\mathbb{R}_{xx} \in \mathcal{C}^{N \times N}$ of the signal $\underline{x}(t)$ can be written as

$$\mathbb{R}_{xx} = \mathcal{E} \{ \underline{x}(t) \underline{x}^H(t) \} = \sum_{i=1}^M P_i \underline{S}_i \underline{S}_i^H + \sigma_n^2 \mathbb{I}_N, \quad (2.21)$$

where P_i denotes the received power of the i -th user. In this case, the channel capacity C is formally defined as the maximum of the mutual information $\mathcal{I}$ between the transmitted desired signal $m_1(t)$ and the signal received at the output of the antenna array $\underline{x}(t)$, assuming perfect knowledge of the desired array manifold vector $\underline{S}_1$. That is,

$$C = \max_{m_1} \mathcal{I} \{ m_1(t); \underline{x}(t) | \underline{S}_1 \}. \quad (2.22)$$

It can be shown that the channel capacity given by Eq. 2.22 can be written as follows (refer to Appendix 2.5)

$$C = B \log_2 \left(\frac{\det(\mathbb{R}_{xx})}{\det(\mathbb{R}_{xx} - P_1 \underline{S}_1 \underline{S}_1^H)} \right) \text{ bits/sec}, \quad (2.23)$$

where B is the channel bandwidth. By defining the matrix $\mathbb{A}$ as follows

$$\mathbb{A} \triangleq [\underline{S}_2, \dots, \underline{S}_M] \in \mathcal{C}^{N \times (M-1)}, \quad (2.24)$$

Eq. 2.23 can be rewritten as

$$C = B \log_2 \left(\frac{\det \begin{bmatrix} N + \sigma_n^2, & \sqrt{P_1} \underline{S}_1^H \mathbb{A} \mathbb{P} \\ \sqrt{P_1} \mathbb{P} \mathbb{A}^H \underline{S}_1, & \mathbb{P} \mathbb{A}^H \mathbb{A} \mathbb{P} + \sigma_n^2 \mathbb{I}_{M-1} \end{bmatrix}}{\sigma_n^2 \det (\mathbb{P} \mathbb{A}^H \mathbb{A} \mathbb{P} + \sigma_n^2 \mathbb{I}_{M-1})} \right), \quad (2.25)$$

where $\mathbb{P} \in \mathcal{C}^{(M-1) \times (M-1)}$ is the matrix of signal power given by

$$\mathbb{P} = \begin{bmatrix} \sqrt{P_2}, & 0, & \cdots & 0 \\ 0, & \sqrt{P_3}, & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0, & 0, & \cdots & \sqrt{P_M} \end{bmatrix}. \quad (2.26)$$

The difference between the capacity in the presence of co-channel interference, given by Eq. 2.25, and the capacity in the absence of interference is defined as the “capacity loss”. This capacity loss denoted by C_{loss} is the negative contribution of the $M - 1$ interferers in diminishing the capacity and is given as follows

$$C_{\text{loss}} = -\log_2 \left(1 - \frac{\underline{b}^H \mathbb{D}^{-1} \underline{b}}{N + \sigma_n^2} \right) \text{ bits/sec/Hz}, \quad (2.27)$$

where

$$\underline{b} = \sqrt{P_1} \mathbb{P} \mathbb{A}^H \underline{S}_1, \quad (2.28)$$

$$\mathbb{D} = \mathbb{P} \mathbb{A}^H \mathbb{A} \mathbb{P} + \sigma_n^2 \mathbb{I}_{M-1}. \quad (2.29)$$

It can be noted that the absence of interference, i.e. $\mathbb{P} = \mathbb{O}_{M-1}$, implies that the capacity loss is equal to 0 bits/sec/Hz. More importantly, this term is negligible when the interferers are spatially far away from the desired user. However, if the interfering sources are spatially close to the desired user, (as this is expected with high probability in future high density communications) the capacity loss is expected to be significant. Hence, a study of the variation in C_{loss} in terms of the array geometry can present key insights that would be crucial for the design of future systems.

In order to establish the relationship between the capacity loss and the antenna array geometry, the capacity loss defined by Eq. 2.27 is expressed as functions of arc lengths and the first curvature of the array manifold defined in Section 2.3. The environment being studied is illustrated by Fig. 2.3 and consists of the desired user and $M - 1$ remaining interferers that are in the form of a interferer cluster. Let us denote them by their corresponding array manifold vectors parameterised by the arc length rather than the azimuth, i.e. $\underline{a}_1$ and $\underline{a}_2, \dots, \underline{a}_M$, respectively. The impinging angle of the desired user is θ_1 while the $M - 1$ interferers have angles of $\theta_2, \dots, \theta_M$, respectively. We assume that the angular separation between the desired user and the cluster of interferers is small enough to result in a capacity loss. For the sake of simplicity, we assume that the desired user and all interferers are received with unity power. That is, $P_1 = P_2 = \dots = P_M = 1$. Recalling Eq. 2.27, the matrix $\mathbb{D} \in \mathcal{C}^{(M-1) \times (M-1)}$ may be expressed as follows

$$\mathbb{D} = \begin{bmatrix} \underline{a}_2^H \underline{a}_2 & \underline{a}_2^H \underline{a}_3 & \dots & \underline{a}_2^H \underline{a}_M \\ \underline{a}_3^H \underline{a}_2 & \underline{a}_3^H \underline{a}_3 & \dots & \underline{a}_3^H \underline{a}_M \\ \vdots & \vdots & \ddots & \vdots \\ \underline{a}_M^H \underline{a}_2 & \underline{a}_M^H \underline{a}_3 & \dots & \underline{a}_M^H \underline{a}_M \end{bmatrix} + \sigma_n^2 \mathbb{I}_{M-1}. \quad (2.30)$$

Consequently, $\mathbb{D}$ can be expressed in terms of the array manifold parameters introduced in Section 2.3 as follows

$$\mathbb{D} \approx -2\kappa_1^{-2}(s_c) \sin^2 \left(\mathbb{M} \frac{\Delta s_c}{2\kappa_1^{-1}(s_c)(M-2)} \right) + N \underline{1}_{M-1} \underline{1}_{M-1}^T + \sigma_n^2 \mathbb{I}_{M-1}, \quad (2.31)$$

where $\mathbb{M}$ is the following Toeplitz matrix

$$\mathbb{M} \triangleq \begin{bmatrix} 0, & 1, & \dots & M-2, \\ 1, & 0, & \dots & M-1, \\ \vdots & \vdots & \ddots & \vdots \\ M-2, & M-1, & \dots & 0 \end{bmatrix}. \quad (2.32)$$

Remember that, s_c denotes the centre of the interferer cluster (see Fig. 2.3) and

$\kappa_1(s_c)$ is given by Eq. 2.9. In a similar fashion, $\underline{b}$ in Eq. 2.27 may be written as

$$\underline{b} = \begin{bmatrix} \underline{a}_1^H \underline{a}_2 \\ \underline{a}_1^H \underline{a}_3 \\ \vdots \\ \underline{a}_1^H \underline{a}_M \end{bmatrix} \in \mathcal{C}^{(M-1) \times 1}, \quad (2.33)$$

which may be, in turn, expressed in terms of the array manifold parameters as follows

$$\underline{b} \approx N \underline{1}_{M-1} - 2\kappa_1^{-2}(\check{s}) \sin^2 \left(\left(\Delta s \underline{1}_{M-1} + \frac{\Delta s_c}{(M-2)} \mathbb{M} \begin{bmatrix} 1 \\ \underline{0}_{M-2} \end{bmatrix} \right) \frac{1}{2\kappa_1^{-1}(\check{s})} \right), \quad (2.34)$$

where $\check{s}$ is the centre of $\Delta s = s_2 - s_1$, i.e.

$$\check{s} = \frac{s_1 + s_2}{2}. \quad (2.35)$$

Equation 2.27, in conjunction with Eqs. 2.31 and 2.34, establishes an important relationship between the capacity loss and the most important parameters of the array manifold, namely the arc length and the principal curvature which are, in turn, dictated by the array geometry and the propagation environment. Let us investigate this crucial design equation for the array geometries in Fig. 2.4.

The propagation environment was assumed to be composed of one desired user and three interferers. The Δs_c was set so that the angular separation between the cluster of interfering users is set to 3° , i.e. $\Delta\theta_c = 3^\circ$. In addition, the Δs was set such that the angular separation $\Delta\theta_{1,2}$ was varied between 0° and 7° . For this environment, the resulting capacity loss for the five geometries in Fig. 2.4 is illustrated in Fig. 2.6 with variable $\Delta\theta_{1,2}$ and variable θ_1 . Table 2.1 gives the performance ranking of the geometries shown in Fig. 2.4. An interesting result is that the ‘‘Square’’ array and X-shaped array yield a result very similar to that of the ‘‘Filled’’-grid array even though the ‘‘Filled’’-grid has a higher number of antennas. This can be explained by applying the result obtained in Eq. 2.27 for this set of

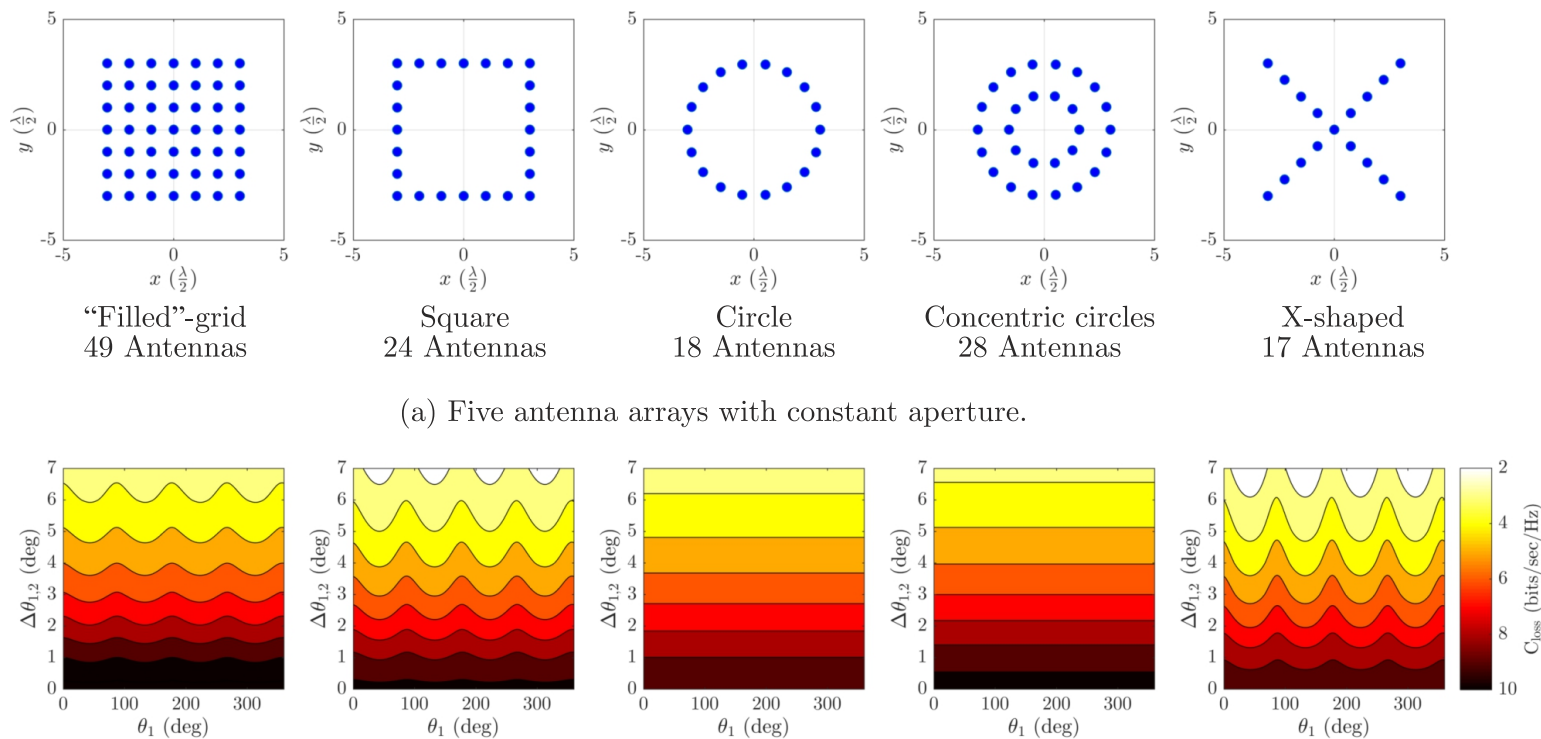


Figure 2.6: Performance of grid arrays in Fig. 2.4 in terms of the capacity loss with varying θ_1 and $\Delta\theta_{1,2}$ (The performance plot is presented below the illustration of the corresponding array geometry).

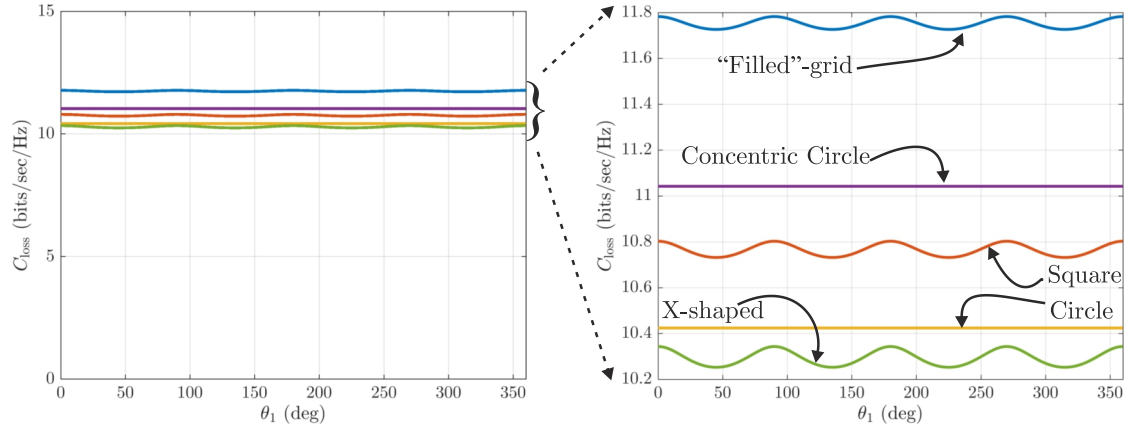


Figure 2.7: Capacity loss for the case that the desired user is located at the center of the interferer cluster, i.e. ($s = s_c$), plotted for $\forall s_1$ (i.e. $\forall \theta_1$). Cluster Δs_c is set such as to fix $\Delta \theta_c = 3^\circ$.

geometries that illustrates an interplay between three different parameters namely $\|\underline{r}_x\|$, κ_1 and N that results in the ranking given in Table 2.1. The relationship between these parameters dictates the performance ranking.

Now, consider the scenario where the desired user is located at the center of a cluster of 4 interferers (i.e. $M = 5$) with $\Delta \theta_c = 3^\circ$. The DOA of the desired user θ_1 is variable and corresponds to the horizontal axis in Fig. 2.7 which illustrates the capacity loss for the geometries shown in Fig. 2.4. It can be observed that the capacity loss is approximately the same for all geometries although the small difference in capacity loss is dictated by the number of antennas N . Overall, the results in this subsection, highlight the importance of the “capacity loss” derived in Eq. 2.27. This expression can be utilised as an optimisation cost function for future array design algorithms, especially for propagation environments where high user density needs to be tackled.

2.5 Summary

In this chapter, the parametric channel model was introduced and presented. Based on this model and equipped with analytical metrics on the array mani-

Table 2.1: Ranking of the five array geometries based on the capacity loss.

Array geometry	$\ \underline{r}_x\ $	Manifold length ℓ_m	Ranking
“Filled”-grid	14	276.35	4th
Square	12.08	238.51	2nd
Circle	9	177.65	3rd
Concentric circles	9.68	191.04	last
X-shaped	8.22	162.17	best

fold curve, various grid array geometries were studied and compared by employing relevant performance criteria of an array system as figures-of-merit. The performance criteria employed were (i) detection and resolution thresholds and (ii) the novel concept of “capacity loss” that clearly measures the degradation in capacity due to interferers spatially very close to the desired user. Five geometries were examined by picking subsets of antennas from a 7×7 “filled”-grid array and their performance was evaluated in the context of a multi-source system where users are located close together in space as expected in future dense communication networks. The results illustrated that the performance of antenna arrays may be improved and managed efficiently by an intelligent choice of array geometry as per the task at hand, when utilised in conjunction with a parametric channel model. The results also presented key relationships that can be utilised as cost functions in future works to perform dynamic selection of array geometry based on the goals of the array system.

Appendix 2.A: Proof of Eq. 2.23

Let us recall the expression for capacity for a desired user given by Eq. 2.22

$$\frac{C}{B} = \max_{m_1} \mathcal{I} \{m_1(t); \underline{x}(t) | \underline{S}_1\}. \quad (2.36)$$

For a deterministic channel, the mutual information $\mathcal{I}$ may be written as

$$\mathcal{I}(\underline{x}(t); m_1(t)) = \mathcal{H}(\underline{x}(t)) - \mathcal{H}(\underline{x}(t) | m_1(t)), \quad (2.37)$$

where $\mathcal{H}(\cdot)$ denotes the entropy. Since the environmental noise is considered Gaussian and the maximum information is obtained when the transmitted signals are Gaussian, the array output $\underline{x}(t)$ is a Gaussian process and its entropy may be written as

$$\mathcal{H}(\underline{x}(t)) = \log_2 \det(\pi e \mathbb{R}_{xx}), \quad (2.38)$$

where $\mathbb{R}_{xx}$ denotes the covariance matrix associated with $\underline{x}(t)$. Similarly, the conditional entropy may be written as

$$\mathcal{H}(\underline{x}(t) \mid m_1(t)) = \log_2 \det(\pi e (\mathbb{R}_{xx} - P_1 \underline{S}_1 \underline{S}_1^H)), \quad (2.39)$$

where $\underline{S}_1$ is the manifold vector associated with the desired user. Thus, Eq. 2.36 may be written as

$$C = B \log_2 \left(\frac{\det(\mathbb{R}_{xx})}{\det(\mathbb{R}_{xx} - P_1 \underline{S}_1 \underline{S}_1^H)} \right). \quad (2.40)$$

Chapter 3

Spatiotemporal Array Signal Processing

In this chapter, a novel approach to extend the degrees of freedom of an antenna array system operating in a wireless communication environment is proposed. The approach proposed is in contrast to “massive” systems, where the number of antenna elements are increased dramatically, with the downside of severe constraints on digital beamforming due to power and cost restrictions arising from a multitude of radio frequency (RF) chains. The spatiotemporal beamformer proposed in this chapter extends the degrees of freedom by exploiting space and time domains jointly, with the aid of intelligent Tx signal design and an array manifold “extender” at the Rx. The proposed approach outperforms existing approaches in terms of achieving near-far resistance capabilities, heightened interference mitigation and consequently resulting in increased degrees of freedom. A representative example illustrates that the proposed approach, utilised in conjunction with a small array of 9 antennas, outperforms a traditional massive array of 80 antennas.

3.1 Overview

In Chapter 2, the important role of the Rx array geometry encompassed within a parametric channel model was highlighted towards achieving improved detection, resolution and capacity in an array system. However, current array signal processing systems are becoming increasingly inadequate to cater to future applications that are required to deliver high degrees of freedom (DoF). More specifically, in the domain of wireless communications, with the advent of internet of things and spiralling data and connectivity requirements, there is an imminent requirement to explore means to expand the degrees of freedom provided by the communication system being employed. For instance, consider the upcoming mm-wave communications technologies, that proposes the utilisation of the millimetre wave band to augment the already saturated lower frequency bands to address ever-growing demands on data rates and traffic volumes. At these frequencies, due to smaller antenna sizes, many more antennas may be integrated at the user-end devices, bringing advanced antenna array signal processing algorithms to the fore. However, a major limitation at these frequencies is the increased propagation losses [28]. In addition, the mm-wave channel is known to be frequency selective [29, 30, 31], while foliage losses and rain attenuation at these frequencies are significant and can limit the coverage severely. Furthermore, attenuation factors due to obstacles in non-line-of-sight (NLOS) communications are much higher as compared to their counterparts in the 1-3 GHz communications band. Another critical problem would be the interference management arising from high user density in small cell deployments. In such environments, typical power control mechanisms will be much harder to operate. All the aforementioned issues can be addressed by enhancing the degrees of freedom delivered by the communication system. An increase in the DoF maps to a number of interrelated objectives namely

1. ability to resolve and deliver highly pointed beams towards closely spaced

sources leading to increased interference cancellation and thus heightened capacity,

2. increased accuracy of channel estimation and
3. ability to cater to an increased number of sources.

The degrees of freedom directly scales with the number of dimensions of the system as per the definition in [32, 33]. Therefore, extending the dimensions, and hence the DoF, beyond the Rx spatial array dimensions, forms the motivation of this chapter to address the challenges in future communication systems.

Consider a system that is characterized by N_{dim} dimensions. This implies that every distinguishable signal can be characterised by a unique manifold vector $\underline{S}(\underline{p}) \in \mathcal{C}^{N_{\text{dim}} \times 1}$ where $\underline{p} \in \mathcal{C}^{Q \times 1}$ is the list of Q parameters that characterises the manifold vector. For example, the received signal at an Rx array as given by Chapter 2, when utilised with no post-processing, constitutes an N -dimensional system. As stated earlier, in order to expand the DoF of the array system, the parameter N_{dim} needs to be increased. This may be achieved by extending the manifold vector in a number of different ways such as incorporating additional system or channel parameters, or by simply increasing the number of antennas in the system. One of the most popular and straightforward approaches is to utilise hundreds or thousands of antenna array elements known as massive arrays or massive MIMO [34, 35]. An increase of N antennas increases the degrees of freedom by N . However, utilising an independent RF chain for each antenna element of a large array is not sustainable with the given constraints of power, manufacturing cost and available area. Some methods in literature propose a rollback to the strategy of hybrid beamforming, i.e. combining analogue and digital beamforming. This is with the objective of reduced design complexity and power burden arising from the numerous analogue front-end chains to support the increased number of antennas. In [36], a hybrid beamformer consisting of analogue subarrays and a digital beamformer is proposed. However, it exploits the spatial dimension alone

and employs an expensive iterative DOA estimation procedure requiring more iterations at a lower SNR. A hybrid beamformer maximising the receiver output SNR is proposed in [37]. However, the downside with such approaches is the inferior beamforming performance and accuracy. The research carried out in this thesis does not focus on such approaches. On the other hand, in this research, other means to expand the DoF that do not require as many antennas, while retaining the benefits of digital beamforming, are explored. Towards this, in this chapter, the spatial array system of Chapter 2 is extended to a spatiotemporal array system, by exploiting space and time jointly. The two main enablers of such a system are the intelligent design of the Tx signal and the extension of the spatial manifold vector to that of a spatiotemporal manifold vector.

This chapter is organised as follows. In Section 3.2, a brief overview of current spatiotemporal or space-time approaches is presented. This is followed by Section 3.3 where the mathematical models and designs of the transmitter and receiver blocks are presented. Equipped with these models, the designs of the spatiotemporal channel estimator and weight design blocks are presented in Section 3.4. The proposed spatiotemporal beamformer is subjected to detailed computer simulation studies which are presented in Section 3.5. The chapter is concluded in Section 3.6.

3.2 State of the Art: Space-time Array Signal Processing

There have been some approaches in the literature that attempt to integrate the spatial and temporal domains to extend the degrees of freedom. In this section, a brief overview of the current state of the art in spatiotemporal (also known as space-time in the literature) array signal processing is presented. In general terms,

spatiotemporal techniques may be defined as approaches that utilise the $(N \times 1)$ signal received across the antenna array in conjunction with N_t temporal snapshots to extend the degrees of freedom. The approaches in literature may be broadly classified as shown in Fig. 3.1. Although the approaches are discussed at the Rx in this section, all these approaches may be utilised either at the Tx or Rx or both. There are a few approaches that utilise pilots, i.e. are non-blind, and are not the focus of this section.

Decoupled space-time single-user approaches process the spatial and temporal information one after the other (in any order) rather than processing both domains jointly. The term “single-user” approaches implies that these approaches attempt to isolate the desired user’s signal treating the other users as interferers, whilst “multi-user” approaches are interested in decoding and demodulating all users. The spatial domain in such approaches may employ algorithms such as MUSIC [3], min-norm MUSIC [38, 39], smoothed MUSIC [40, 41], root-MUSIC [42] or ESPRIT [4] and the temporal domain may utilise various properties of the structure of the transmitted message to further isolate the desired signal. For instance, in [43], a decoupled 2D space-time RAKE receiver and a corresponding blind equalisation algorithm for a code division multiple access (CDMA) system is proposed. In [44], a decoupled space-time receiver is proposed where the first stage utilises the spreading sequence of the desired user to despread the data, whilst the second stage utilises the most significant eigenvector from the eigendecomposition of the spatio-frequency correlation matrices.

On the other hand, spatiotemporal approaches exploit spatial and temporal domains “jointly” by essentially extending the manifold from a N -dimensional space to NN_t dimensions, where N_t corresponds to the temporal dimensions. Such approaches may be further classified as single-user or multi-user receivers. There have been some approaches in literature, as discussed in the survey in [45], that propose blind multichannel identification or equalisation methods using joint

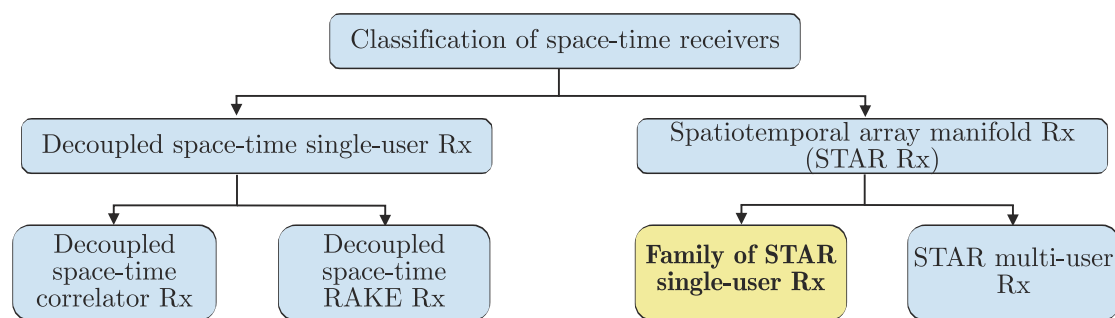


Figure 3.1: Classification of spatiotemporal array receivers. The spatiotemporal beamformer proposed in this chapter belongs to the family of single-user STAR receivers as highlighted by the yellow box.

space-time processing. However, these techniques are predominantly applicable for approaches employing time division multiple access (TDMA). In [46], a single-user STAR receiver is proposed based on a parametric model. However, an estimate of the channel matrix is assumed known and continuously tracked based on DOA and time of arrival (TOA) estimation in conjunction with the received data. This has been enhanced to a multi-user receiver in [47, 48], where an iterative channel identification approach is utilised leading to high complexity that increases with the number of interferers and number of antennas, making it unsuitable for the large antenna arrays of future 5G systems. The SpatioTemporal ARray (STAR) manifold beamformer presented in this chapter integrates space and time in a single manifold vector, called the spatiotemporal manifold vector and belongs to the branch of single-user beamformers highlighted in Fig. 3.1. This considerably empowers the receiver to provide narrow spatiotemporal multi-beams and combat the path loss by focusing the antenna array gain towards a specific path's directions of interest at the right time (delay) [49]. Unlike earlier systems, the proposed beamformer explicitly compensates multiple access interference (MAI) and estimates DOA, delay, radial velocity (Doppler frequency) and complex path gains of all paths of the desired user. Also, the channel estimation algorithm employed is blind, unlike systems with pilots such as in [50]. In the next sections, the design

of the proposed spatiotemporal beamformer, consisting of a channel estimator and weight design block, is mathematically modelled and evaluated for various channel environments.

3.3 Design of the Tx and Rx

Consider a MIMO environment where M sources operate in the same frequency band with the transmitter and receiver arrays employing $\bar{N}$ and N antennas respectively. For the i -th source, we may split the overall system model into three entities namely the transmitter model, channel model and receiver model. The parametric channel model, which is independent of the system being employed, has already been introduced in Chapter 2. Thus, in this section, the transmitter and receiver blocks characteristic to the spatiotemporal beamformer system are presented.

3.3.1 Tx System Design

Each of the M co-channel sources employ a unique pseudo noise (PN)-code of period $\mathcal{N}_c$. With reference to Fig. 3.2, consider at point A the input data symbol stream of the i -th source denoted by $\{a_i[n]\}, \forall n$. Without any loss of generality, the transmitted symbols are assumed to be of unity magnitude. Then, the data stream $\{a_i[n]\}$ of the i -th source, with a symbol period T_{cs} , is spread by a PN-code sequence of chip period T_c . The use of PN-code is just a representative example of how we can integrate a modulation scheme with antenna array geometry. Note that instead of a PN-code we can use a Hadamard/Walsh sequence, or a chirp-pulse signal. Let us define the vector $\underline{c}_i$ such that

$$\underline{c}_i = \underbrace{[\alpha_i[1], \alpha_i[2], \dots, \alpha_i[\mathcal{N}_c]]^T}_{\text{one PN-code period}} \in \{+1, -1\}. \quad (3.1)$$

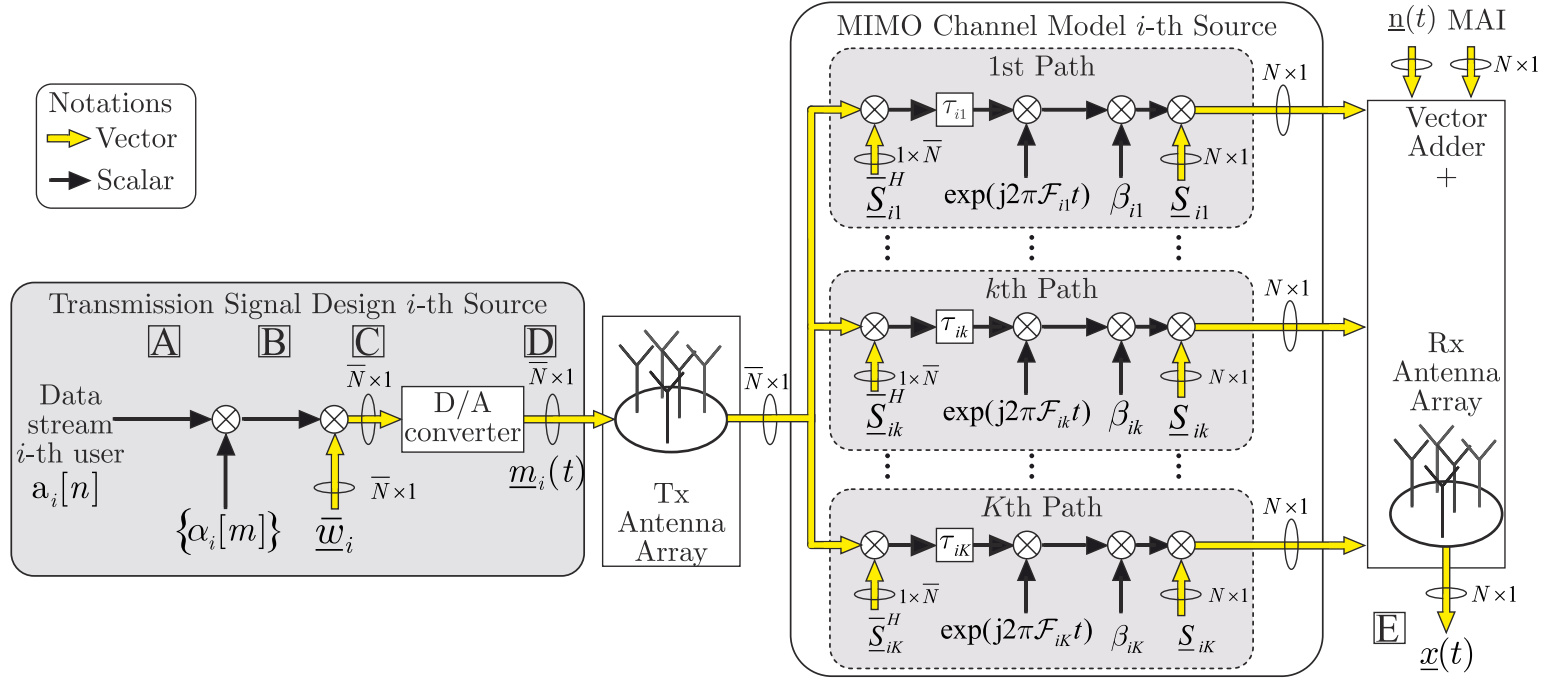


Figure 3.2: Baseband representation of the transmitter and channel of the i -th source in the proposed spatiotemporal system.

With $T_{cs} = \mathcal{N}_c T_c$, for the $(\mathcal{N}_c n + m)$ -th interval, the transmitted symbol at point B is given by $\mathbf{a}_i[n] \alpha_i[m]$, $\forall m \in \{1, 2, \dots, \mathcal{N}_c\}$. Following this, the transmitter weight of the i -th source $\underline{\mathbf{w}}_i \in \mathbb{C}^{\bar{N} \times 1}$ is applied on the modulated and spread data stream vector, at point C of Fig. 3.2, to steer the transmitter main lobe towards a desired direction during the instant $(\mathcal{N}_c n + m)$, yielding $\underline{\mathbf{w}}_i (\mathbf{a}_i[n] \alpha_i[m])$. Note that the transmitter weights $\underline{\mathbf{w}}_i$ may be designed based on the feedback from the receiver. The design of these transmitter weights is out of the scope of this chapter. The sequence of samples of the n -th symbol of the i -th source, obtained at point C, can be written in the form of a matrix $\mathbb{M}_i[n]$ given by

$$\mathbb{M}_i[n] = \underline{\mathbf{w}}_i (\mathbf{a}_i[n] \mathbf{c}_i^T) \in \mathbb{C}^{\bar{N} \times \mathcal{N}_c}, \quad (3.2)$$

Finally, the $(\bar{N} \times 1)$ message signal vector obtained at the output of the D/A convertor (see point D) is transmitted onto the Tx antenna array of $\bar{N}$ antennas.

3.3.2 Rx System Design

The received signal $\underline{\mathbf{x}}(t)$, which is shown at point E in Fig. 3.2 and Fig. 3.4, is fed to a preprocessor consisting of a discretiser of a sampling period T_c and a bank of N tapped delay lines (TDLs) of length $2\mathcal{N}_c$. Since the generic case of asynchronous systems is considered, the transmitted data symbols do not arrive at the antennas at the beginning of their interval but with a relative delay. Thus, during the nT_{cs} period, the TDLs contain the contribution of the n -th Tx symbol plus some information of the previous $(n - 1)$ and next $(n + 1)$ symbols. In this thesis, we use the term inter symbol interference (ISI) to represent this contribution arising from this preprocessor. Figure 3.3a shows the 3D data cube which represents the received discretised signal, where the n -th slice of the data cube matrix is of size $N \times 2\mathcal{N}_c$ corresponding to the data stored in the bank of TDLs at the nT_{cs} period. The vectorisation of the transposed n -th data slice provides the n -th spatiotemporal snapshot $\underline{\mathbf{x}}[n] \in \mathbb{C}^{2N\mathcal{N}_c \times 1}$ as shown in Fig. 3.3b. Hence, the output

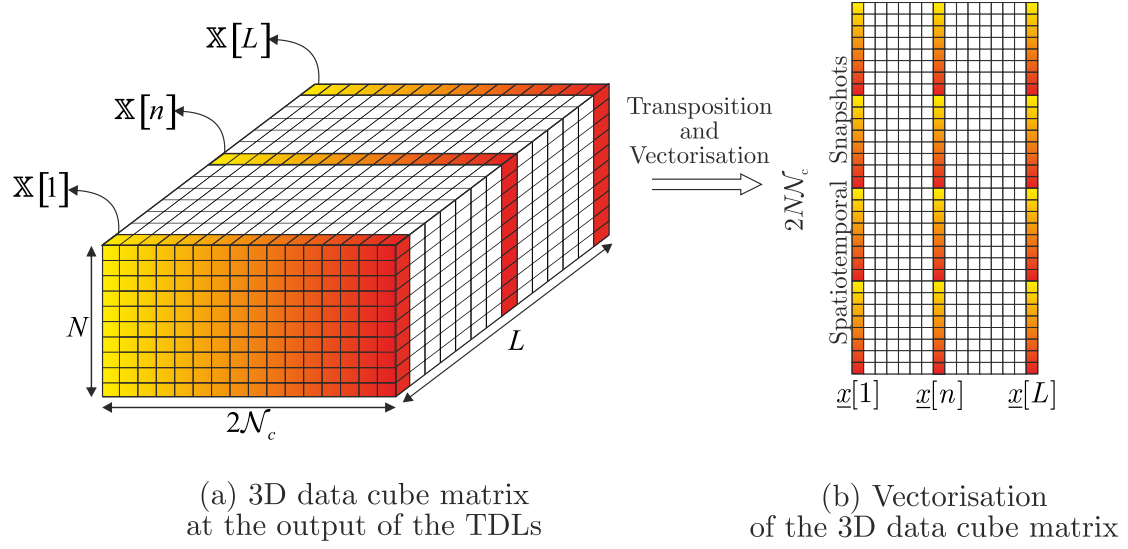


Figure 3.3: The 3D data cube and its vectorisation to form the spatiotemporal snapshot $\underline{x}[n]$.

$\underline{x}[n]$ at point F in Fig. 3.4 can be written as

$$\underline{x}[n] = \text{vec} \left(\mathbb{X}^T [n] \right), \quad (3.3)$$

and this section presents the mathematical model of the same. Thus, the vector $\underline{x}[n]$ contains the n -th symbol of the desired source along with ISI, MAI and noise effects.

In order to model these contributions, for the i -th source and k -th path, the concept of the array manifold vector is extended to the STAR manifold vector $\underline{\mathbf{h}}_{ik} \in \mathcal{C}^{2N_c \times 1}$ [51] given as follows

$$\underline{\mathbf{h}}_{ik} = \underline{\mathbf{S}}_{ik} \otimes \left(\mathbb{J}_{2N_c}^{l_{ik}} \begin{bmatrix} \underline{\mathbf{c}}_i \\ \underline{\mathbf{0}}_{N_c} \end{bmatrix} \odot \underline{\mathbf{F}}_{ik} \right) = \mathbb{A}_{ik} \underline{\mathbf{S}}_{ik}, \quad (3.4)$$

where $\mathbb{A}_{ik} \in \mathcal{C}^{2N_c \times N}$ is the linear mapping between the space-only manifold vector $\underline{\mathbf{S}}_{ik}$ and the STAR manifold vector $\underline{\mathbf{h}}_{ik}$ defined as

$$\mathbb{A}_{ik} = \mathbb{I}_N \otimes \left(\mathbb{J}_{2N_c}^{l_{ik}} \begin{bmatrix} \underline{\mathbf{c}}_i \\ \underline{\mathbf{0}}_{N_c} \end{bmatrix} \odot \underline{\mathbf{F}}_{ik} \right). \quad (3.5)$$

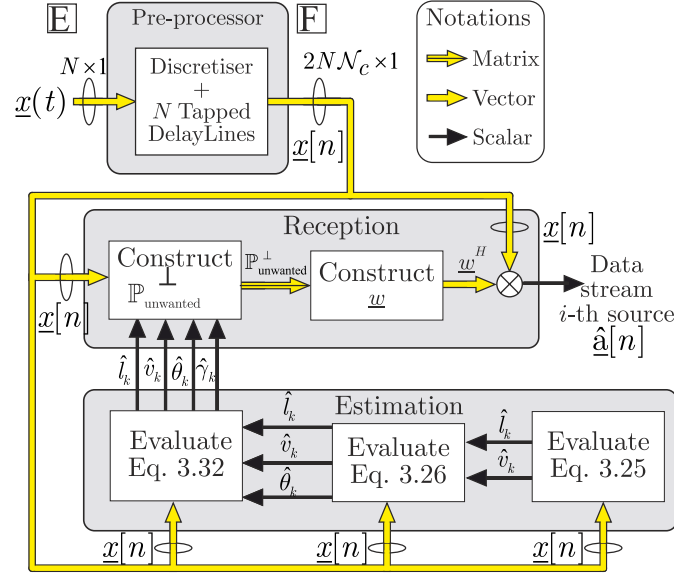


Figure 3.4: Block diagram of the preprocessor consisting of the discretiser, TDLs, channel estimator and the STAR subspace receiver.

The matrix $\mathbb{J}_P$ (or $\mathbb{J}_P^T$) is a $P \times P$ matrix and the power l of $\mathbb{J}_P$ (or $\mathbb{J}_P^T$) when applied to a vector $\underline{x} \in \mathcal{C}^{P \times 1}$, i.e. $\mathbb{J}_P^l \underline{x}$ (or $(\mathbb{J}_P^T)^l \underline{x}$), downshifts (or upshifts) the vector $\underline{x}$ by l elements and is given as follows

$$\mathbb{J}_P = \begin{bmatrix} \underline{0}_{P-1}^T & 0 \\ \mathbb{I}_{P-1} & \underline{0}_{P-1} \end{bmatrix}. \quad (3.6)$$

In Eq. 3.4, the parameter $l_{ik} = \left\lfloor \frac{\tau_{ik}}{T_c} \right\rfloor \bmod \mathcal{N}_c$ stands for the discrete relative path delay. The temporal variation within a symbol arising from the Doppler frequency shift is modelled as the vector $\underline{\mathcal{F}}_{ik} \in \mathcal{C}^{2\mathcal{N}_c \times 1}$ written as

$$\underline{\mathcal{F}}_{ik} = \exp \left(\mathbf{j} [0, 1, \dots, 2\mathcal{N}_c - 1]^T 2\pi \mathcal{F}_{ik} T_c \right), \quad (3.7)$$

where $\mathcal{F}_{ik}$ denotes the Doppler frequency given by

$$\mathcal{F}_{ik} = -\frac{F_c v_{ik}}{c}, \quad (3.8)$$

and v_{ik} stands for the radial velocity of the i -th source's k -th path with F_c denoting the carrier frequency.

Using the STAR manifold vector defined in Eq. 3.4, we can model the n -th spatiotemporal snapshot $\underline{x}[n]$ as follows

$$\underline{x}[n] = \sum_{i=1}^M [\mathbb{H}_i, \mathbb{H}_i^{\text{comb}}] \begin{bmatrix} \left(\underline{\gamma}_i \odot \underline{f}_i[n] \right) \mathbf{a}_i[n] \\ \left(\underline{\gamma}_i \odot \underline{f}_i[n-1] \right) \mathbf{a}_i[n-1] \\ \left(\underline{\gamma}_i \odot \underline{f}_i[n+1] \right) \mathbf{a}_i[n+1] \end{bmatrix} + \underline{\mathbf{n}}[n], \quad (3.9)$$

where $\mathbf{a}_i[n]$ represents the symbol transmitted by the i -th source and $\underline{\mathbf{n}}[n]$ represents the sampled noise. The matrix $\mathbb{H}_i$ contains the STAR manifold vectors of all the multipaths of the i -th source, constructed as follows

$$\mathbb{H}_i = [\underline{\mathbf{h}}_{i1}, \underline{\mathbf{h}}_{i2}, \dots, \underline{\mathbf{h}}_{iK}] \in \mathcal{C}^{2N\mathcal{N}_c \times K}. \quad (3.10)$$

In Eq. 3.9, the matrix $\mathbb{H}_i^{\text{comb}} \in \mathcal{C}^{2N\mathcal{N}_c \times 2K}$ contains delayed versions of the matrix $\mathbb{H}_i$ (ISI) and is defined as

$$\mathbb{H}_i^{\text{comb}} = [(\mathbb{I}_N \otimes (\mathbb{J}_{2\mathcal{N}_c}^T)^{\mathcal{N}_c}) \mathbb{H}_i, (\mathbb{I}_N \otimes \mathbb{J}_{2\mathcal{N}_c}^{\mathcal{N}_c}) \mathbb{H}_i]. \quad (3.11)$$

In addition, the vector $\underline{\gamma}_i \in \mathcal{C}^{K \times 1}$ contains the path fading coefficients and the gain provided by transmit beamforming for all the multipaths of the i -th source. That is,

$$\underline{\gamma}_i = \underline{\beta}_i \odot \left[\overline{\underline{S}}_{i1}^H \underline{\overline{w}}_i, \overline{\underline{S}}_{i2}^H \underline{\overline{w}}_i, \dots, \overline{\underline{S}}_{iK}^H \underline{\overline{w}}_i \right]^T, \quad (3.12)$$

where the vector $\underline{\beta}_i \in \mathcal{C}^{K \times 1}$ given by

$$\underline{\beta}_i = \left[\underline{\beta}_{i1}, \underline{\beta}_{i2}, \dots, \underline{\beta}_{iK} \right]^T, \quad (3.13)$$

contains the K multipath fading coefficients of the i -th source. Finally, the vector $\underline{f}_i[n] \in \mathcal{C}^{K \times 1}$ incorporates the temporal variation arising from the Doppler shift from one symbol to the other, defined as

$$\underline{f}_i[n] = \exp \left(j2n\pi [\mathcal{F}_{i1}, \mathcal{F}_{i2}, \dots, \mathcal{F}_{iK}]^T T_{cs} \right). \quad (3.14)$$

Without any loss of generality, the first source is assumed to be the desired, i.e. $i = 1$, and thus $\underline{x}[n]$ can be rewritten as follows to distinguish the desired signal,

ISI, MAI and noise component

$$\begin{aligned}
 \underline{x}[n] = & \overbrace{\mathbb{H}_1 \left(\underline{\gamma}_1 \odot \underline{f}_1[n] \right) \mathbf{a}_1[n]}^{\text{Desired term}} + \overbrace{\mathbb{H}_1^{\text{comb}} \begin{bmatrix} \left(\underline{\gamma}_1 \odot \underline{f}_1[n-1] \right) \mathbf{a}_1[n-1] \\ \left(\underline{\gamma}_1 \odot \underline{f}_1[n+1] \right) \mathbf{a}_1[n+1] \end{bmatrix}}^{\text{ISI term}} \\
 & + \overbrace{\sum_{i=2}^M \left[\mathbb{H}_i^{\text{comb}}, \mathbb{H}_i \right] \begin{bmatrix} \left(\underline{\gamma}_i \odot \underline{f}_i[n-1] \right) \mathbf{a}_i[n-1] \\ \left(\underline{\gamma}_i \odot \underline{f}_i[n+1] \right) \mathbf{a}_i[n+1] \\ \left(\underline{\gamma}_i \odot \underline{f}_i[n] \right) \mathbf{a}_i[n] \end{bmatrix}}^{\text{MAI term}} + \overbrace{\underline{\mathbf{n}}[n]}^{\text{noise}}. \quad (3.15)
 \end{aligned}$$

Based on Eq. 3.15, the theoretical covariance matrix of the received signal

$\mathbb{R}_{xx} \in \mathcal{C}^{2N\mathcal{N}_c \times 2N\mathcal{N}_c}$ can be written as

$$\begin{aligned}
 \mathbb{R}_{xx} &= \mathcal{E} \{ \underline{x}[n] \underline{x}^H[n] \} \\
 &\stackrel{\triangle}{=} \overbrace{\mathbb{H}_1 \mathbb{G}_1 \mathbb{H}_1^H}^{\mathbb{R}_{\text{des}}} + \overbrace{\mathbb{H}_1^{\text{comb}} (\mathbb{I}_2 \otimes \mathbb{G}_1) (\mathbb{H}_1^{\text{comb}})^H}^{\mathbb{R}_{\text{ISI}}} \\
 &\quad + \overbrace{\sum_{i=2}^M \left[\mathbb{H}_i^{\text{comb}}, \mathbb{H}_i \right] (\mathbb{I}_3 \otimes \mathbb{G}_i) \left[\mathbb{H}_i^{\text{comb}}, \mathbb{H}_i \right]^H}^{\mathbb{R}_{\text{MAI}}} + \overbrace{\sigma_n^2 \mathbb{I}_{2N\mathcal{N}_c}}^{\mathbb{R}_{\text{nn}}}, \quad (3.16)
 \end{aligned}$$

where $\mathbb{G}_i \in \mathcal{C}^{K \times K}$ represents the multipath power gain matrix given by

$$\begin{aligned}
 \mathbb{G}_i &= \mathcal{E} \left\{ \left(\underline{\gamma}_i \odot \underline{f}_i[n] \right) \mathbf{a}_i[n] \mathbf{a}_i^*[n] \left(\underline{\gamma}_i \odot \underline{f}_i[n] \right)^H \right\} \\
 &= \text{diag} \left\{ \underline{\gamma}_i \odot \underline{\gamma}_i^* \right\} \odot \mathcal{E} \left\{ \underline{f}_i[n] \underline{f}_i^H[n] \right\} \\
 &= \text{diag} \left\{ \underline{\gamma}_i \odot \underline{\gamma}_i^* \right\}, \quad (3.17)
 \end{aligned}$$

since $\mathcal{E} \{ \mathbf{a}_i[n] \mathbf{a}_i^*[n] \} = 1$ and $\mathcal{E} \left\{ \underline{f}_i[n] \underline{f}_i^H[n] \right\} = \mathbb{I}_K$. Note that with the L spatiotemporal snapshots shown in Fig. 3.3, the practical covariance matrix corresponding to the theoretical one given by Eq. 3.16, can be obtained as follows

$$\mathbb{R}_{xx} = \frac{1}{L} \left[\underline{x}[1], \underline{x}[2], \dots, \underline{x}[L] \right] \left[\underline{x}[1], \underline{x}[2], \dots, \underline{x}[L] \right]^H. \quad (3.18)$$

3.3.3 Comparison of Array Bounds

As stated in [25], the estimation performance of an arrayed system depends on the radius of the uncertainty sphere centered at the true value on the manifold. This uncertainty sphere arises due to the finite sampling effect characterised by the number of snapshots L and the SNR. The Cramer–Rao bound (CRB), a theoretical lower bound on the variance of the estimation error of the azimuth (θ) for a source, for space-only systems, has been obtained as [25]

$$\text{CRB}_{\text{space-only}}[\theta] = \frac{1}{2(\text{SNR} \times L) \dot{s}^2(\theta)}. \quad (3.19)$$

Here $\dot{s}(\theta) = \frac{\partial s}{\partial \theta}$ denotes the rate of change of the arc length s of the array manifold and is a function of the array geometry. For instance, for linear arrays of N sensors, $\dot{s}(\theta) = \pi \|\underline{r}_x\| \sin \theta$. However, to evaluate the CRB for the spatiotemporal system employed in this chapter, Eq. 3.19 needs to be extended to incorporate the STAR manifold vector. The rate of change of the arc length of the extended manifold $\dot{s}_{\mathcal{H}}(\theta)$ has been derived in [51] as

$$\dot{s}_{\mathcal{H}}(\theta) = \sqrt{\sigma_{\mathbb{A}}^2 \dot{s}^2(\theta) + \sigma_{\dot{\mathbb{A}}}^2 N}, \quad (3.20)$$

where $\mathbb{A}$ is a linear transformation, as introduced in Eq. 3.5, mapping the space-only manifold to the STAR manifold vector (Note that the elevation angle ϕ has been set to 0 for simplicity). In Eq. 3.20, $\sigma_{\mathbb{A}}^2$ and $\sigma_{\dot{\mathbb{A}}}^2$ are defined as follows

$$\sigma_{\mathbb{A}}^2 = \frac{\text{Tr}\{\mathbb{A}^H \mathbb{A}\}}{N}, \quad (3.21a)$$

$$\sigma_{\dot{\mathbb{A}}}^2 = \frac{\text{Tr}\{\dot{\mathbb{A}}^H \dot{\mathbb{A}}\}}{N}. \quad (3.21b)$$

Using the transformation defined in Eq. 3.5 along with Eqs. 3.21a and 3.21b, the following particular properties can be obtained

$$\sigma_{\mathbb{A}}^2 = \mathcal{N}_c, \quad (3.22a)$$

$$\sigma_{\dot{\mathbb{A}}}^2 = 0. \quad (3.22b)$$

Hence, Eq. 3.20 simplifies to

$$\dot{s}_{\mathcal{H}}(\theta) = \sqrt{\mathcal{N}_c} \dot{s}(\theta). \quad (3.23)$$

Using this in conjunction with Eq. 3.19, we can derive the CRB of the proposed spatiotemporal system as follows

$$\boxed{\text{CRB}_{\text{STAR}}[\theta] = \frac{1}{\sqrt{\mathcal{N}_c}} \text{CRB}_{\text{space-only}}[\theta]}. \quad (3.24)$$

This illustrates that, for the same number of antennas, the proposed spatiotemporal system outperforms the corresponding space-only system in terms of the estimation error.

3.4 Spatiotemporal Beamformer Design

In this section, based on the Tx and Rx models in Section 3.3, a spatiotemporal beamformer is presented. As shown in Fig. 3.4, this beamformer consists of a STAR-subspace receiver utilising the estimated parameters, denoted by $(\hat{\cdot})$, from a channel estimator. The design of these blocks is discussed in the next few sections. Without any loss of generality, the first source is considered to be the desired source.

Based on Eqs. 3.15 and 3.18, the parameters of interest of all the paths of the desired source (i.e. $i = 1$) to be estimated are as follows: relative multipath delay, channel fading coefficients, Doppler frequency and DOA. Here onwards, the subscript $i = 1$ is dropped for notational convenience. As shown in the estimation block of Fig. 3.4, the aforesaid complex estimation problem is broken down into multiple smaller units of lower complexity. The search procedures are carried out in the baseband. Firstly, the delay and Doppler frequency are jointly estimated, followed by the estimation of DOA. Then, the combined contribution of the channel fading coefficients and transmit beamforming weights is estimated.

These estimation procedures are blind and are based on the assumptions of plane wave propagation, known array geometry and fully calibrated arrays. (With no loss of generality, the elevation angle is set to zero i.e. $\phi_{ik} = \bar{\phi}_{ik} = 0 \forall i, k$). The proposed framework utilises

- the extended array manifold which is a mathematical object embedded in a multidimensional complex space that entirely characterizes an antenna array (of a given geometry) for the whole set of values in the parameter space
- the preprocessed received array data, and in particular its second order statistics, which can provide a linear space spanned by the signals (signal subspace),

where the solution of the estimation problem is the intersection of the non-linear subspace-(i) and the linear subspace-(ii).

3.4.1 Joint Delay–Doppler Estimation

In this first stage of joint estimation of delay and Doppler frequency, since the DOA is unknown, the spatial dimension provided by the N antennas cannot be exploited. Hence, the subspace is reduced from $2N\mathcal{N}_c$ to $2\mathcal{N}_c$ dimensions by a simple rearrangement of the slices $\mathbb{X}[n]$ of the 3D data cube in Fig. 3.3. Using the eigendecomposition of the covariance of the rearranged matrix $[\mathbb{X}^T[1], \dots, \mathbb{X}^T[L]]$ of size $\mathcal{C}^{2\mathcal{N}_c \times NL}$, the projection on the noise subspace $\mathbb{P}_n$ is obtained. Note that, although the number of dimensions is reduced, increased number of snapshots are obtained due to the format of the rearrangement. The parameters corresponding to the desired source will lie in the signal subspace spanned by the temporal part of the manifold vector in Eq. 3.4, given by $\mathbb{J}_{2\mathcal{N}_c}^{l_k} [\underline{c}^T, \underline{0}_{\mathcal{N}_c}^T]^T \odot \underline{\mathcal{F}}_k$, containing the PN-code of the desired source and a Doppler component. Using the fact that this subspace is orthogonal to the estimated noise subspace $\mathbb{P}_n$, the cost function $\xi(l_k, \mathcal{F}_k)$ estimates the delay–Doppler jointly for the k -th multipath of the 1st

source (i.e. desired source) and can be defined as follows

$$\xi(l_k, \mathcal{F}_k) = \frac{\left(\mathbb{J}_{2\mathcal{N}_c}^{l_k} \begin{bmatrix} \underline{c} \\ \underline{0}_{\mathcal{N}_c} \end{bmatrix} \odot \underline{\mathcal{F}}_k \right)^H \left(\mathbb{J}_{2\mathcal{N}_c}^{l_k} \begin{bmatrix} \underline{c} \\ \underline{0}_{\mathcal{N}_c} \end{bmatrix} \odot \underline{\mathcal{F}}_k \right)}{\left(\mathbb{J}_{2\mathcal{N}_c}^{l_k} \begin{bmatrix} \underline{c} \\ \underline{0}_{\mathcal{N}_c} \end{bmatrix} \odot \underline{\mathcal{F}}_k \right)^H \mathbb{P}_n \left(\mathbb{J}_{2\mathcal{N}_c}^{l_k} \begin{bmatrix} \underline{c} \\ \underline{0}_{\mathcal{N}_c} \end{bmatrix} \odot \underline{\mathcal{F}}_k \right)}. \quad (3.25)$$

The peaks obtained from the 2-dimensional search using this cost function over a 2D grid yield the delay and Doppler frequency values corresponding to the multipaths of the desired source. The number of evaluations is not a function of the number of antennas but depends purely on the chosen 2D grid. Furthermore, the evaluations are independent from each other and can be done in parallel.

3.4.2 DOA Estimation

The design of this cost function follows the logic stated previously that the intersection of the noise subspace spanned by $\mathbb{P}_n$ and the extended STAR manifold object will yield the set of all STAR manifold vectors $\underline{\mathbf{h}}_k$ (see Eq. 3.4) of the desired source's paths. Hence, for $k = 1, \dots, K$ pairs of estimated $(\hat{l}_k, \hat{\mathcal{F}}_k)$, a 1D search is carried out over the space-time cost function given by

$$\xi(\theta_{\hat{l}_k, \hat{\mathcal{F}}_k}) = \frac{\underline{\mathbf{h}}_k^H(\theta_{\hat{l}_k, \hat{\mathcal{F}}_k}) \underline{\mathbf{h}}_k(\theta_{\hat{l}_k, \hat{\mathcal{F}}_k})}{\underline{\mathbf{h}}_k^H(\theta_{\hat{l}_k, \hat{\mathcal{F}}_k}) \mathbb{P}_n \underline{\mathbf{h}}_k(\theta_{\hat{l}_k, \hat{\mathcal{F}}_k})}, \quad (3.26)$$

where $\mathbb{P}_n$ represents the projection on the noise subspace obtained from the eigen-decomposition of the covariance matrix $\mathbb{R}_{xx}$.

3.4.3 Combined Fading Coefficient Estimation

Post estimating the delay, Doppler frequency and DOA, the residual combined parameter γ_k to be estimated consists of the complex path fading coefficient and the transmit beamforming gain for the k -th path of the first source. In this section,

a near-far algorithm to estimate the amplitude and phase of γ_k is described. The steps in the algorithm are as follows

1. The subspace $\mathbb{R}_{k,\text{un}}$ consisting of MAI, noise and the other $K - 1$ paths of the desired source is constructed by subtracting the effects of $\hat{\mathbf{h}}_k$ (estimated in Sections 3.4.1 and 3.4.2) from the data covariance matrix

$$\mathbb{R}_{k,\text{un}} = \mathbb{R}_{xx} - \|\hat{\gamma}_k\|^2 \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H. \quad (3.27)$$

By partitioning the space spanned by the columns of $\mathbb{R}_{k,\text{un}}$ into “unwanted” signal subspace and “noise” subspace, the projection operator $\mathbb{P}_{k,\text{un}}$ is formed using the eigenvectors corresponding to the “unwanted signal”. This, in turn, can be utilised to construct $\mathbb{P}_{k,\text{un}}^\perp$ to isolate the k -th path of the desired source, that is

$$\mathbb{P}_{k,\text{un}}^\perp = \mathbb{I}_{2N\mathcal{N}_c} - \mathbb{P}_{k,\text{un}}. \quad (3.28)$$

In Eq. 3.27, the amplitude $\|\hat{\gamma}_k\|$ can be estimated using the following cost function inspired from [52]

$$\|\hat{\gamma}_k\| = \arg \min_{\|\gamma\|} \xi_k(\|\gamma\|), \quad (3.29)$$

where

$$\xi_k(\|\gamma\|) = \sum_{\substack{m=1 \\ \text{eig}_m < 0}}^{2N\mathcal{N}_c} (1 + \text{eig}_m(\mathbb{R}_k(\|\gamma\|))) + 10 \log_{10} \left(\sum_{\substack{m=1 \\ \text{eig}_m < 0}}^{2N\mathcal{N}_c} |\text{eig}_m(\mathbb{R}_k(\|\gamma\|))| \right), \quad (3.30)$$

with

$$\mathbb{R}_k(\|\gamma\|) = \mathbb{R}_{xx} - \text{eig}_{\min}(\mathbb{R}_{xx}) \mathbb{I}_{2N\mathcal{N}_c} - \|\gamma\|_k^2 \hat{\mathbf{h}}_k \hat{\mathbf{h}}_k^H. \quad (3.31)$$

The aforementioned 1D cost function of Eq. 3.30 is searched for the amplitude $\|\gamma\|$ that reduces the rank of the received signal covariance matrix $\mathbb{R}_{xx}$ by one.

2. By exploiting the noise tolerance of the modulation and utilising all the available symbols in the observation interval of L snapshots, the phase of $\hat{\gamma}_k$

can be estimated as follows

$$\angle \hat{\gamma}_k = \left\{ \frac{1}{L} \sum_{n=1}^L \left\{ \angle \left(f_k^*[n] \hat{\mathbf{h}}_k^H \mathbb{P}_{k,\text{un}}^\perp x[n] \right) - \hat{\gamma}_{k,\text{gu}} \right\} \bmod \psi - \psi_0 \right\} + \angle \hat{\gamma}_{k,\text{gu}}. \quad (3.32)$$

where $\angle \hat{\gamma}_{k,\text{gu}}$ is an initial guess of the phase of $\hat{\gamma}_k$ obtained with the help of $\mathbb{P}_{k,\text{un}}^\perp$ as below

$$\angle \hat{\gamma}_{k,\text{gu}} = \angle \mathbf{a}^*[1] \hat{\mathbf{h}}_k^H \left(\mathbb{P}_{k,\text{un}}^\perp x[1] \right). \quad (3.33)$$

Here $\mathbf{a}[1]$ is the first symbol of every observation interval which is assumed to be the header (i.e. known). There are two important points to be noted here. Firstly, the role of $\mathbb{P}_{k,\text{un}}^\perp$ is to make the procedure near–far resistant. Secondly, after suppressing the interference, two levels of noise suppression are utilised. First level of obtaining $\angle \hat{\gamma}_{k,\text{gu}}$ relies on the inherent noise tolerance of the modulation while the second level uses a maximum likelihood (ML) averaging approach to obtain an improved estimate of the phase. Note that this is under the assumption that the phase deviation introduced by the noise is within $\pm\psi$, where ψ is the minimum phase difference between two constellation points and ψ_0 is the initial modulation angle in the chosen modulation scheme.

3.4.4 Design of the Rx Spatiotemporal Beamformer Weights

In this subsection, the estimated channel parameters are employed to construct spatiotemporal beamforming weights to steer very high gain towards the directions of interest. The steps to build these weights are as follows

1. As a first step, the unwanted subspace of the desired source $\hat{\mathbb{R}}_{\text{unwanted}}$ is required to be estimated. Towards this, the contribution of the desired source to the received signal covariance matrix, namely $\hat{\mathbb{R}}_{\text{des}}$ in Eq. 3.16, has to be removed from the received signal covariance matrix $\mathbb{R}_{xx}$ as follows

$$\hat{\mathbb{R}}_{\text{unwanted}} = \mathbb{R}_{xx} - \hat{\mathbf{H}}_1 \hat{\mathbf{G}}_1 \hat{\mathbf{H}}_1^H. \quad (3.34)$$

2. By performing the eigendecomposition of the estimated $\hat{\mathbb{R}}_{\text{unwanted}}$, the matrix $\mathbb{E}_s$ whose columns contain the eigenvectors spanning the corresponding unwanted signal subspace can be obtained. Using this matrix $\mathbb{E}_s$, a projection operator orthogonal to the MAI and ISI may be built as follows

$$\mathbb{P}_{\text{unwanted}}^\perp = \mathbb{I}_{2N\mathcal{N}_c} - \mathbb{E}_s (\mathbb{E}_s^H \mathbb{E}_s)^{-1} \mathbb{E}_s^H. \quad (3.35)$$

3. The projection operator from step 2 is used to form the STAR-subspace weights as

$$\underline{w}_{\text{sub}} = \mathbb{P}_{\text{unwanted}}^\perp \hat{\mathbb{H}}_1 \left(\hat{\mathbb{H}}_1^H \mathbb{P}_{\text{unwanted}}^\perp \hat{\mathbb{H}}_1 \right)^{-1} \hat{\underline{\gamma}}. \quad (3.36)$$

An estimate of the transmitted symbol vector $\hat{\mathbf{a}}[n]$ (point A in Fig. 3.2) is formed as

$$\hat{\mathbf{a}}[n] = \underline{w}_{\text{sub}}^H \underline{x}[n]. \quad (3.37)$$

The performance of this receiver is compared to two other extensively used receivers

- single-user STAR-RAKE receiver
- multi-user STAR-decorrelating receiver.

The weights for the STAR-RAKE receiver are given as:

$$\underline{w}_{\text{RAKE}} = \hat{\mathbb{H}}_1 \hat{\underline{\gamma}}. \quad (3.38)$$

The multi-user STAR-decorrelating receiver (zero-forcing receiver) exploits the knowledge of the entire channel given by

$$\mathbb{H} = [\mathbb{H}_1^{\text{comb}}, \mathbb{H}_1, \dots, \mathbb{H}_M^{\text{comb}}, \mathbb{H}_M], \quad (3.39)$$

where $\mathbb{H}_i$ represents the contribution of the i -th user. The version of this receiver introduced in [53] as been employed here, expressed as

$$\underline{w}_{\text{dec}} = \text{col}_{2K+1:3K} \left\{ \left[\mathbb{H} (\mathbb{H}^H \mathbb{H})^{-1} \right] \right\} \hat{\underline{\gamma}}, \quad (3.40)$$

where the notation $\text{col}_{2K+1:3K}$ has been used to indicate that the $(2K+1)$ -th to $(3K)$ -th columns are extracted.

3.5 Computer Simulation Studies

In this section, simulation studies are presented to evaluate the performance of the proposed spatiotemporal beamformer and the associated parameter estimator. Without any loss of generality, both the Tx and Rx antenna arrays are considered to be uniform circular arrays (UCA) of 7 and 9 elements, respectively, lying in the xy -plane. The Cartesian coordinates of the elements of these two arrays are

Table 3.1: Simulation Parameters

Parameter	Value	Parameter	Value
$\bar{N}$	7	T_c	0.1 ms
$\mathcal{N}_c$	15 chips	N	9
M	4 sources	K	2 paths per source

indicated in $(\frac{\lambda}{2})$ units) in Eqs. 3.41 and 3.42, respectively.

$$[\bar{r}_x, \bar{r}_y, \bar{r}_z]^T = \begin{bmatrix} 1.1, & 0.7, & -0.3, & -1.0, & -1.0, & -0.3, & 0.7 \\ 0.0, & 0.9, & 1.1, & 0.5, & -0.5, & -1.1, & -0.9 \\ 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0 \end{bmatrix}, \quad (3.41)$$

$$[r_x, r_y, r_z]^T = \begin{bmatrix} 1.4, & 0.9, & -0.0, & -1.0, & -1.4, & -1.2, & -0.4, & 0.5, & 1.3 \\ 0.3, & 1.1, & 1.5, & 1.1, & 0.2, & -0.8, & -1.4, & -1.3, & -0.7 \\ 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0 \end{bmatrix}. \quad (3.42)$$

Table 3.1 provides the parameters employed in the simulations. The parameters follow a uniform distribution in their respective intervals i.e., the DOD and DOA between 0 and 360 degrees, the delays between 0 to $\mathcal{N}_c T_c - 1$, the Doppler between 0 and 2000 Hz and the complex path coefficients follow a uniform phase distribution between 0 and 360 degrees and the gain follows a Rayleigh distribution. Also, for the sake of simplicity, a steering vector towards a DOD of 90 degrees is utilised as the transmit weight per user.

3.5.1 Evaluation of Channel Estimation

In this section, the channel estimator block, illustrated in Fig. 3.4, is evaluated in terms of its performance in the presence of interference and noise. Consider one desired source operating in the presence of 3 co-channel interfering sources in a frequency selective channel with 2 paths per source. Let us assume an observation interval of 200 channel symbols ($L = 200$) during which the channel parameters of the desired user to be estimated are $(2T_c, 500 \text{ Hz}, 100^\circ)$ and $(10T_c, 1500 \text{ Hz}, 300^\circ)$. All users are assumed equipowered with an input SNR of 20 dB. Received signal data generated using random transmitted signal data for the above environment corresponding to $L = 200$ was utilised to obtain the results of the proposed approach as shown in Figs. 3.5 and 3.6.

An important parameter in subspace based approaches is the number of snapshots L used in the estimation procedure. As the value of L increases, the practical covariance matrix provides a better approximation of the theoretical covariance matrix. Figure 3.7a shows the root mean square error (RMSE) as a function of $\text{SNR} \times L$ averaged over 200 trials for all the subspace based approaches in the estimator for a fixed value of L at 200 and equal powers for all sources. The SNR was varied from 5 to 30 dB for the simulation. The error in the estimation of the delay is not shown since it is zero through the whole range of $\text{SNR} \times L$. The results indicating a decrease in RMSE with increasing values of $\text{SNR} \times L$ is in-line with the performance bounds presented in Section 3.3.3. A higher value of L may yield better means to combat lower SNR at the cost of the assumption that the channel remains constant over this longer interval of time. The CRB for the DOA parameter for the proposed spatiotemporal system was presented in Section 3.3.3. Figure 3.7b presents an evaluation of the proposed algorithm against this theoretical result in terms of the variance of the DOA error. It can be observed that the performance of the proposed system approaches the theoretical bound closely.

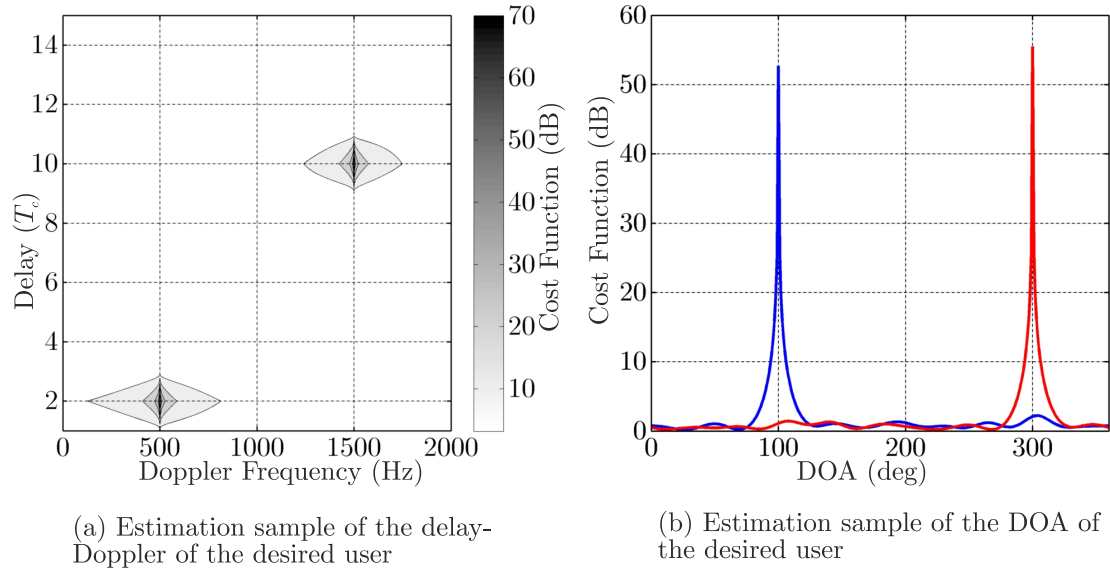


Figure 3.5: Sample result of the output of (a) the joint delay-Doppler estimation and (b) DOA estimation for the desired source operating with 3 other interfering sources. The SNR and NFR are set to 20 dB and 0 dB, respectively.

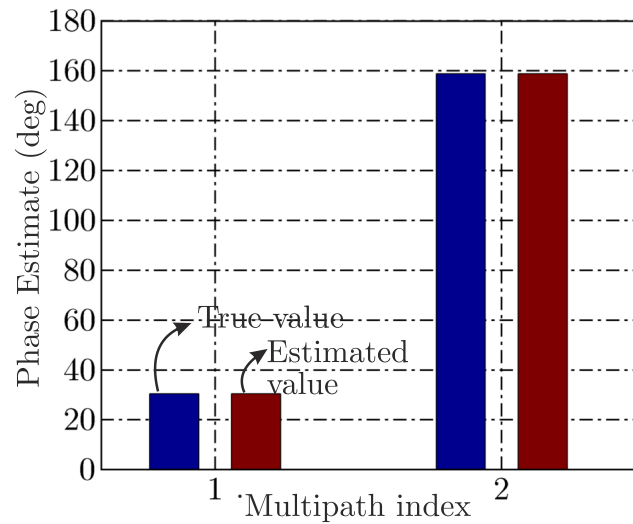


Figure 3.6: Sample result of the output of phase estimation of the combined path fading coefficient for the desired source operating with 3 other interfering sources.

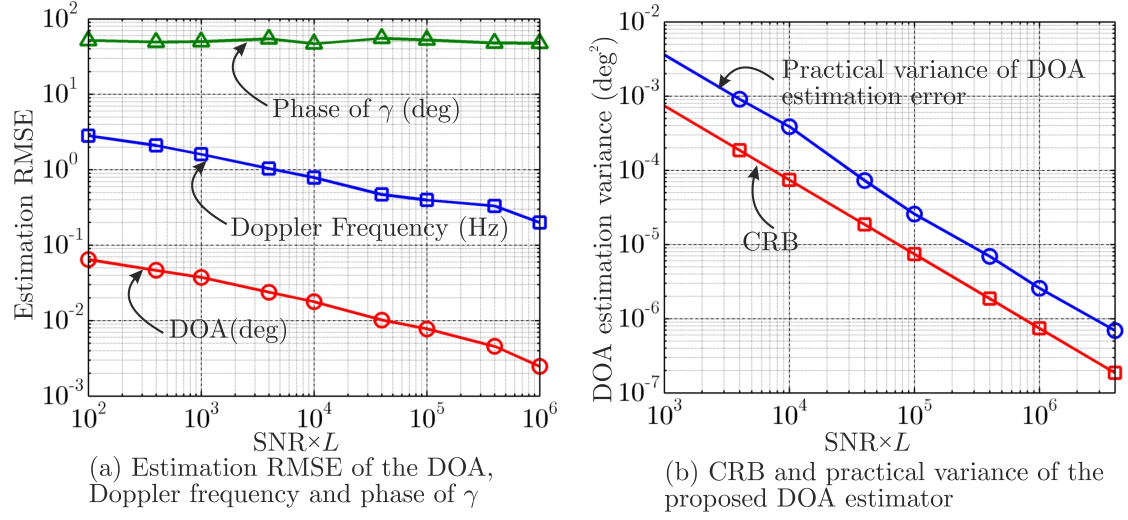


Figure 3.7: Performance evaluation of the spatiotemporal channel estimator.

3.5.2 Evaluation of Interference Mitigation

Figure 3.8 shows the performance of the proposed spatiotemporal-subspace receiver (whose weights are given by Eq. 3.36) against other spatiotemporal receivers such as the spatiotemporal-RAKE (Eq. 3.38), spatiotemporal-decorrelating (Eq. 3.40) and traditional spatial-only massive MIMO (80 antennas) beamformer (enhanced with subspace set of weights and RAKE weights) in the presence of high interference and noise in terms of the output signal to noise interference ratio (SNIR_{out}). All the systems being compared utilise identical channel resources. The SNIR_{out} parameter represents the level of interference present in the beamformed signal and evaluates the performance of the reception system to mitigate interferences. The estimation of this parameter is based on $\mathbb{R}_{\text{ISI}} + \mathbb{R}_{\text{MAI}} + \mathbb{R}_{\text{nn}}$, the covariance matrix of the undesired components of the received signal, and $\mathbb{R}_{\text{des}}$, the covariance matrix of the desired signal

$$\text{SNIR}_{\text{out}} = \frac{\underline{w}^H \mathbb{R}_{\text{des}} \underline{w}}{\underline{w}^H (\mathbb{R}_{\text{ISI}} + \mathbb{R}_{\text{MAI}} + \mathbb{R}_{\text{nn}}) \underline{w}}. \quad (3.43)$$

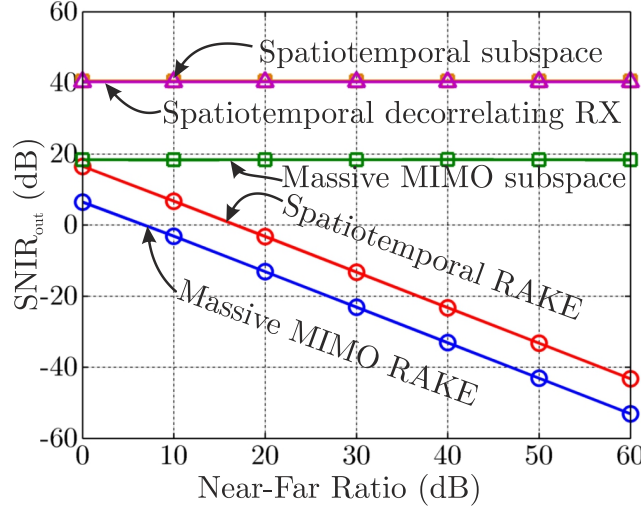


Figure 3.8: Comparison of the proposed spatiotemporal subspace receiver with other spatiotemporal and spatial-only receivers in terms of the near-far ratio (NFR) (200 trials)

By employing a spatiotemporal-RAKE receiver that does not perform interference cancellation, the expression simplifies to

$$\text{SNIR}_{\text{out}} = N\mathcal{N}_c \frac{P_s}{\sigma_n^2 + P_{\text{int}}}. \quad (3.44)$$

where P_{int} is the residual power contribution from MAI and ISI and P_s is the desired signal's power at the Rx. Clearly, as shown in Fig. 3.8, the SNIR_{out} of the RAKE receivers decreases dramatically, since they are unable to provide appropriate levels of performance in the presence of interference. However, the spatiotemporal RAKE receiver outperforms the spatial RAKE receiver, due to the gain arising from heightened dimensions. However, if appropriate spatiotemporal-subspace weights are utilised, this expression simplifies to

$$\text{SNIR}_{\text{out}} = N\mathcal{N}_c \text{SNR}_{\text{in}}, \quad (3.45)$$

as there is complete interference cancellation ($P_{\text{int}} \rightarrow 0$). Thus, the subspace receivers, provide steady performance over a huge range of near-far ratios (NFR) from 0 to 60 dB by incorporating terms to explicitly suppress interference. Such receivers are best suited for applications that are required to cater to high source

density, where tolerance to the near-far problem is critical. As illustrated in Fig. 3.8, this single-user subspace receiver achieves a performance very close to a multi-user receiver (spatiotemporal-decorrelating receiver) without the channel knowledge of the interferers. The performance is also an indicator that at varying NFR levels, the channel estimation errors, if any, lie within the input error tolerance boundary of the receiver.

3.6 Summary

In this chapter, a spatial-only system was extended to a spatiotemporal array system, to extend the DoF without increasing the number of antennas and resorting to hybrid beamforming. The spatiotemporal array system exploits space and time jointly by utilising an intelligent design of the Tx signal and extending the spatial manifold vector to a spatiotemporal manifold vector. The spatiotemporal beamformer design, that consists of a channel estimator block and a receiver weight design block, was presented. This beamformer was evaluated with varying levels of noise and interference. Simulation results indicated that the estimation and reception processes are highly tolerant to the near-far problem and would enable the system to function without power control management algorithms. Results illustrated that the proposed beamformer, in conjunction with a small array of nine antennas, yields a performance 20 dB better than a traditional MIMO system of 80 antennas.

Chapter 4

Spatiotemporal-Virtual Array Signal Processing

This chapter is concerned with extending the degrees of freedom above and beyond that of the spatiotemporal beamformer in Chapter 3, by exploiting the concept of virtual arrays. Virtual array signal processing has been investigated extensively in the domain of radar, where the transmitted message is often known completely and exploited at the Rx. However, communication systems do not enjoy this privilege. Thus, in this chapter, towards communication systems, by employing an intelligent Tx signal design and a corresponding manifold extender at the receiver, a spatiotemporal-virtual array system that exploits the Rx and Tx array geometries jointly with the temporal domain is presented. Results illustrate that the proposed system outperforms spatiotemporal systems and traditional spatial-only array systems, in terms of heightened degrees of freedom and an interference mitigation performance independent of the Tx array geometry.

4.1 Overview

In Chapter 3, the spatial array system of Chapter 2 was extended to a spatiotemporal array system in order to increase the degrees of freedom. The spatiotemporal array system exploits both spatial and temporal domains jointly, by an intelligent design of the Tx signal and subsequently exploiting this special structure at the receiver. However, the key question here is the following: Have all spatial domains yet been exploited in these systems? The answer is no, since the Tx array geometry is another key spatial domain that has been ignored.

In this chapter, the spatiotemporal array system is extended to a spatiotemporal-virtual array system, by additionally incorporating the Tx array geometry without placing any constraints on the array geometries at the Tx and Rx. Also, more importantly, the approach in this chapter requires no knowledge of the transmitted signal unlike the virtual arrays employed in radar [54], where the transmitted signal is completely known and is exploited to construct matched filters at the receiver. The problem statement that forms the crux of this chapter may be described as twofold

1. provide solutions that exploit all “spatial” (Tx and Rx) and temporal domains to provide heightened user resolution and interference cancellation and
2. improve the ability to accommodate more sources.

In the literature, both of these aforementioned objectives are often interchangeably termed as “degrees of freedom”. For instance, in [55, 56] the degrees of freedom is identical to channel multiplexing gain and a formal definition of this is provided in [33] as the coefficient of the logarithm of the SNR (when SNR is high). Note that this is the definition for degrees of freedom used in this chapter. However, in [57] this is interpreted as the number of sources that can be served by the array system. Please note that the objective (1) and (2), albeit related, are different.

While both of the objectives directly scale with the number of dimensions of the system, the proportionality ratio for objective (2) varies with the technique utilised at the transmitter.

This chapter is organised as follows. Section 4.2 provides a brief overview of the current state of the art with respect to virtual arrays in applications such as radar and communication systems. In Section 4.3, the Tx and Rx models of the spatiotemporal-virtual MIMO model are presented. Based on these models, the design of the spatiotemporal-virtual channel estimator and subspace beamformer is presented in Section 4.4. The performance of the proposed channel estimator and beamformer is presented in Section 4.5. Finally, the chapter is concluded in Section 4.6.

4.2 State of the Art: Virtual Arrays

Virtual arrays have been studied extensively in the context of MIMO radar systems. These approaches revolve around appropriate Tx signal design and utilisation of the complete knowledge of the Tx signal at the receiver. A majority of the radar literature focuses on methods to explicitly obtain and exploit the output of virtual arrays by exploiting transmit signal diversity with respect to code division [58], frequency division [59], time division [60] or space-time codes [61]. There have also been studies where the impact of different array geometries on the resultant virtual array geometry and radar system performance have been studied both empirically and analytically [62, 63, 64]. Some of the research focuses on extensions and enhancements of virtual radar such as preprocessing algorithms consisting of smearing filter banks [65] to deal with broadband waveforms, utilising Khatri-Rao (KR) in conjunction with MIMO radar [66] and novel DOA estimation algorithms for Doppler radar systems using spatiotemporal-virtual arrays [67]. However, radar systems possess the advantage of complete knowledge of the Tx

signal at the receiver that communication systems do not possess.

Virtual arrays for non-radar applications have also been explored to some extent in the literature. There have been some approaches in literature that attempt to expand the spatial degrees of freedom at the Rx. A class of approaches known as minimum redundancy arrays [68] attempt to design an optimum linear array with minimum redundant spacings, for a fixed number of antennas, that enables the array to maximise its resolution capabilities (i.e. increase the DoF). However, no closed form expressions, for the array geometry and the achievable degrees of freedom, exist. Certain special structures known as “coprime arrays” are exploited in conjunction with compressed sensing approaches to construct virtual arrays offering increased DoF [69]. Some approaches extend a single element Rx array to a virtual array by physically moving the antenna elements’ 3D coordinates or alternatively exploiting the motion of the antenna array sensors [70, 71, 72]. Another school of thought is to employ fourth order cumulants rather than just employing second order statistics, using the concept of “co-arrays” [73, 74], and is experimentally evaluated in [75] using a three element antenna array. However, the approach is restricted to non-Gaussian sources and the computation of fourth order cumulants is expensive in terms of computations and snapshots required. In [57], the concept of co-array is further extended to “nested-arrays” that proposes a systemic way to produce a virtual array from a uniform linear array (ULA), leading to an increase in the number of users that may be resolved by the system. This may be utilised in conjunction with an extension of the KR product based MUSIC algorithm in [76, 77] towards estimation. The KR product based approach is utilised in [78] for DOA estimation for wideband signals. However, a majority of the aforementioned techniques are limited to linear arrays. Moreover, predominantly all of them ignore the Tx array geometry that may be further exploited to provide a virtual array that is a function of the Tx and Rx array geometry, similar to radar systems. This forms the crux of this chapter. It is important to note that

the approach proposed in this chapter goes hand in hand with approaches such as the KR product based virtual arrays at the Rx to further extend the DoF.

4.3 Tx and Rx System Design

As in Chapters 2 and 3, consider a MIMO environment where M co-channel sources operate with Tx and Rx arrays consisting of $\overline{N}$ and N antennas respectively. The parametric channel model has been introduced and described in Chapter 2. In this section, the transmitter and receiver blocks of the proposed spatiotemporal-virtual approach are presented.

4.3.1 Tx System Model

One instance of the Tx waveform that enables the receiver to extract the spatial information provided by the Tx array is presented in this section. Consider the data stream of the i -th source denoted by $\mathbf{a}_i[n]$ (point A in Fig. 4.1) with a symbol period of T_{cs} . The data stream of the i -th source is spread by columns of the PN-code matrix $\mathbb{C}_i^T$ (point B in Fig. 4.1). The code matrix $\mathbb{C}_i \in \mathcal{R}^{\mathcal{N}_c \times \overline{N}}$ is given by

$$\begin{aligned} \mathbb{C}_i &= [\underline{c}_{i1}, \underline{c}_{i2}, \dots, \underline{c}_{im}, \dots, \underline{c}_{i\overline{N}}] \\ &= [\overline{c}_{i1}, \overline{c}_{i2}, \dots, \overline{c}_{ij}, \dots, \overline{c}_{i\mathcal{N}_c}]^T. \end{aligned} \quad (4.1)$$

In Eq. 4.1, $\underline{c}_{im} \forall m = 1, \dots, \overline{N}$ denotes the PN-code associated with the m -th antenna of the i -th source given by

$$\underline{c}_{im} = [\alpha_{im}[1], \alpha_{im}[2], \dots, \alpha_{im}[\mathcal{N}_c]]^T \in [+1, -1], \quad (4.2)$$

with a chip period T_c . Note that the symbol period and chip period are related as follows $T_{cs} = \mathcal{N}_c T_c$. Figure 4.2 is an illustration of the structure of the transmitted message known to the Tx and Rx. The symbol stream is then weighted by the

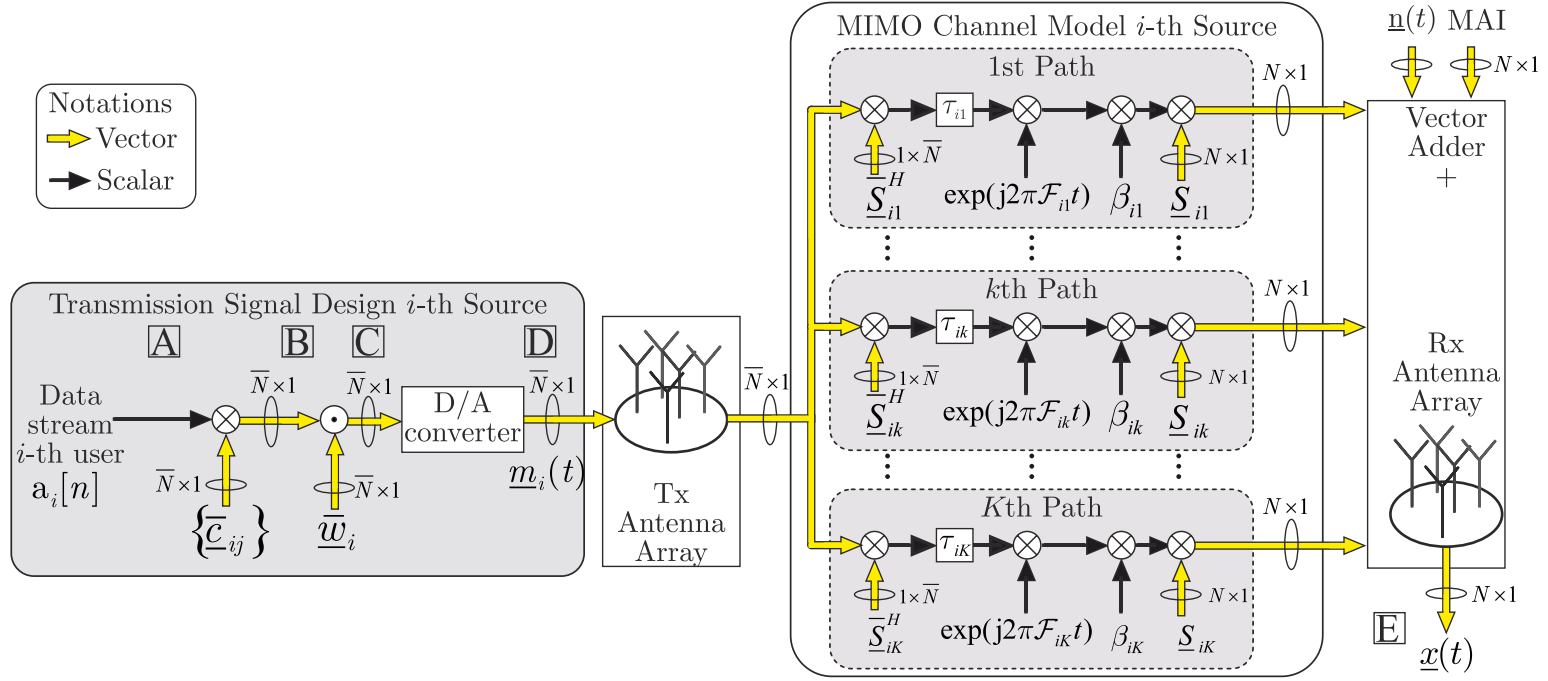


Figure 4.1: Baseband representation of the transmitter and MIMO channel of the i -th source in the proposed spatiotemporal-virtual system.

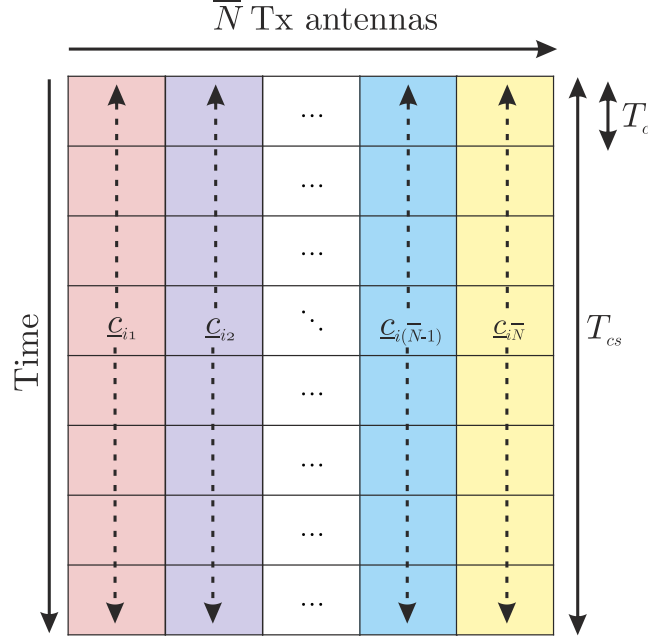


Figure 4.2: Illustration of the format of the transmitted message across the Tx array.

transmit weight $\underline{w}_i \in C^{\bar{N} \times 1}$ towards a desired angle (point C in Fig. 4.1). The transmitter weights may be designed based on the feedback of the receiver and is out of the scope of this chapter. Therefore, the chips corresponding to the n -th symbol of the i -th user transmitted across $\bar{N}$ antennas is denoted by the matrix $\mathbb{M}_i[n] \in C^{\bar{N} \times \mathcal{N}_c}$ (point C in Fig. 4.1) and can be written as

$$\mathbb{M}_i[n] = \mathbb{C}_i^T \odot \{(\underline{w}_i \mathbf{a}_i[n]) \underline{1}_{\mathcal{N}_c}^T\}. \quad (4.3)$$

Finally, the analogue message signal $\underline{m}_i(t) \in C^{\bar{N} \times 1}$ obtained at point D in Fig. 4.1 is transmitted over the Tx antenna array of $\bar{N}$ antennas.

4.3.2 Rx System Model

The $(N \times 1)$ received signal at point E in Fig. 4.3 (at the output of the channel as described in Chapter 2) is discretised and passed through a manifold extender. The first stage of this manifold extender consists of N TDLs of length $2\mathcal{N}_c$ that results in a 3D data cube. In the second stage, the slices of this 3D cube

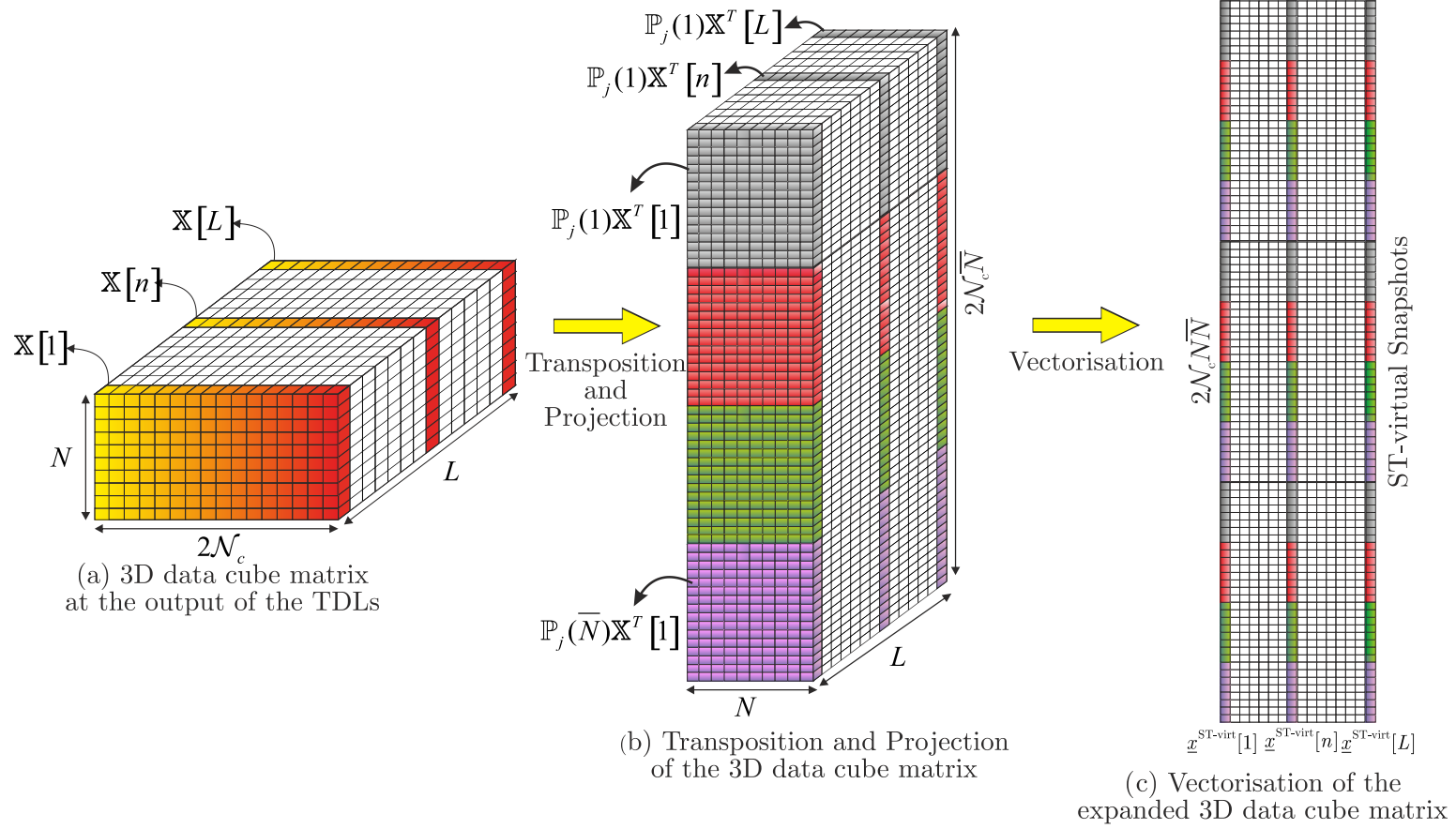


Figure 4.4: Illustration of the 3D data cube, projection and vectorisation for the spatiotemporal-virtual system.

Eq. 4.4, $\mathbb{N}[n]$ denotes the contribution of noise in the 3D data cube. Also, $\underline{S}_{ik}$ and $\overline{S}_{ik}$ denote the Rx and Tx array manifold vectors corresponding to the k -th path of the i -th user as described in Chapter 2. The matrix $\mathbb{J}_{2\mathcal{N}_c}$ is the shifting matrix as defined in Eq. 3.6 in Chapter 3. Also, $\underline{\mathcal{F}}_{ik} \in \mathcal{C}^{2\mathcal{N}_c \times 1}$ denotes the Doppler vector given by

$$\underline{\mathcal{F}}_{ik} = \exp \left(j2\pi \mathcal{F}_{ik} [0, \dots, 2\mathcal{N}_c - 1]^T T_c \right), \quad (4.6)$$

and l_{ik} denotes the discretised delay. With no loss of generality, the first user (i.e. $i = 1$) is considered to be the desired user. In order to isolate the signal corresponding to every Tx antenna of the desired user, let us define $\overline{N}$ projector operators $\mathbb{P}_{\mathbb{B}_{1m}}^\perp \in \mathcal{C}^{2\mathcal{N}_c \times 2\mathcal{N}_c}$, $\forall m = 1, 2, \dots, \overline{N}$ as follows

$$\mathbb{P}_{\mathbb{B}_{1m}}^\perp = \mathbb{I}_{2\mathcal{N}_c} - \mathbb{B}_{1m} (\mathbb{B}_{1m}^H \mathbb{B}_{1m})^{-1} \mathbb{B}_{1m}^H, \quad (4.7)$$

and $\mathbb{B}_{1m} \in \mathcal{C}^{2\mathcal{N}_c \times K(\overline{N}-1)}$ is obtained as follows

$$\mathbb{B}_{1m} = \left[\mathbb{J}_{2\mathcal{N}_c}^{l_{11}} \begin{bmatrix} \mathbb{C}_{1m} \\ \mathbb{O}_{\mathcal{N}_c \times (\overline{N}-1)} \end{bmatrix} \odot \underline{\mathcal{F}}_{11} \mathbf{1}_{\overline{N}-1}^T, \dots, \mathbb{J}_{2\mathcal{N}_c}^{l_{1K}} \begin{bmatrix} \mathbb{C}_{1m} \\ \mathbb{O}_{\mathcal{N}_c \times (\overline{N}-1)} \end{bmatrix} \odot \underline{\mathcal{F}}_{1K} \mathbf{1}_{\overline{N}-1}^T \right], \quad (4.8)$$

where $\mathbb{C}_{1m} \in \mathcal{R}^{\mathcal{N}_c \times (\overline{N}-1)}$ is given by

$$\mathbb{C}_{1m} = [\underline{c}_{11}, \dots, \underline{c}_{1(m-1)}, \underline{c}_{1(m+1)}, \dots, \underline{c}_{1\overline{N}}]. \quad (4.9)$$

The projection operator in Eq. 4.7 isolates the contribution of each Tx antenna of the desired user whilst preserving the second order characteristics of the noise. Thus, the spatiotemporal-virtual snapshot obtained at the single-user receiver (with desired user as first user) $\underline{x}_1[n] \in \mathcal{C}^{2N\overline{N}\mathcal{N}_c \times 1}$ may be obtained by vectorisation of the following concatenated matrices i.e.

$$\underline{x}_1[n] = \text{vec} \left(\begin{bmatrix} \mathbb{P}_{\mathbb{B}_{11}}^\perp \mathbb{X}^T[n] \\ \mathbb{P}_{\mathbb{B}_{12}}^\perp \mathbb{X}^T[n] \\ \vdots \\ \mathbb{P}_{\mathbb{B}_{1\overline{N}}}^\perp \mathbb{X}^T[n] \end{bmatrix} \right). \quad (4.10)$$

The spatiotemporal-virtual snapshot $\underline{x}_1[n]$ may be written as follows

$$\underline{x}_1[n] = \underline{x}_d[n] + \underline{x}_{\text{MAI}}[n] + \underline{n}[n], \quad (4.11)$$

where $\underline{x}_d[n]$ denotes the components of the desired user (1st user) given by

$$\begin{aligned} \underline{x}_d[n] = & \sum_{k=1}^K \beta_{1k} \left\{ \left(\underline{h}_{1k}^{\text{STv}} \odot \underline{\bar{w}}_1^{\text{ext}} \right) \mathbf{a}_1[n] \exp(j2\pi \mathcal{F}_{1k} n T_c) \right. \\ & + \left(\underline{h}_{1k, \text{prev}}^{\text{STv}} \odot \underline{\bar{w}}_1^{\text{ext}} \right) \mathbf{a}_1[n-1] \exp(j2\pi \mathcal{F}_{1k} (n-1) T_c) \\ & \left. + \left(\underline{h}_{1k, \text{next}}^{\text{STv}} \odot \underline{\bar{w}}_1^{\text{ext}} \right) \mathbf{a}_1[n+1] \exp(j2\pi \mathcal{F}_{1k} (n+1) T_c) \right\}, \end{aligned} \quad (4.12)$$

where $\underline{\bar{w}}_1^{\text{ext}} \in \mathcal{C}^{2\bar{N}N_c \times 1}$ denotes the extended Tx weight and is given by

$$\underline{\bar{w}}_1^{\text{ext}} = \underline{1}_N \otimes \underline{\bar{w}}_1 \otimes \underline{1}_{2N_c}. \quad (4.13)$$

Also, $\underline{x}_{\text{MAI}}[n]$ denotes the components corresponding to the multiple access interference given by

$$\begin{aligned} \underline{x}_{\text{MAI}}[n] = & \sum_{i=2}^M \sum_{k=1}^K \beta_{ik} \left\{ \underline{h}_{ik}^{\text{STv}} \mathbf{a}_i[n] \exp(j2\pi \mathcal{F}_{ik} n T_c) \right. \\ & + \underline{h}_{ik, \text{prev}}^{\text{STv}} \mathbf{a}_i[n-1] \exp(j2\pi \mathcal{F}_{ik} (n-1) T_c) \\ & \left. + \underline{h}_{ik, \text{next}}^{\text{STv}} \mathbf{a}_i[n+1] \exp(j2\pi \mathcal{F}_{ik} (n+1) T_c) \right\}. \end{aligned} \quad (4.14)$$

In Eq. 4.12, the spatiotemporal-virtual manifold vector has a unique format for the paths of the desired user that we attempt to isolate, i.e. $\underline{h}_{1k}^{\text{STv}}$ may be written as

$$\underline{h}_{1k}^{\text{STv}} = \underline{S}_{1k} \otimes \left\{ \left(\underline{\bar{S}}_{1k}^* \otimes \underline{1}_{2N_c} \right) \odot \mathbb{P}_{\mathbb{B}_1}^\perp \left(\mathbb{J}_{2\bar{N}N_c}^{l_{1k}} \underline{c}_1^{\text{ext}} \odot \underline{\mathcal{F}}_{1k}^{\text{ext}} \right) \right\}, \quad (4.15)$$

where $\underline{c}_1^{\text{ext}} \in \mathcal{C}^{2\bar{N}N_c \times 1}$ and $\underline{\mathcal{F}}_{1k}^{\text{ext}} \in \mathcal{C}^{2\bar{N}N_c \times 1}$ are given as

$$\underline{c}_1^{\text{ext}} = [\underline{c}_{11}^T, \underline{0}_{N_c}^T, \underline{c}_{12}^T, \underline{0}_{N_c}^T, \dots, \underline{c}_{1N}^T, \underline{0}_{N_c}^T]^T, \quad (4.16)$$

$$\underline{\mathcal{F}}_{1k}^{\text{ext}} = \underline{1}_{\bar{N}} \otimes \underline{\mathcal{F}}_{1k}. \quad (4.17)$$

and $\mathbb{P}_{\mathbb{B}_1}^\perp \in \mathcal{C}^{2\overline{N}\mathcal{N}_c \times 2\overline{N}\mathcal{N}_c}$ is given by

$$\mathbb{P}_{\mathbb{B}_1}^\perp = \begin{bmatrix} \mathbb{P}_{\mathbb{B}_{11}}^\perp & \mathbb{O}_{2\mathcal{N}_c} & \cdots & \mathbb{O}_{2\mathcal{N}_c} \\ \mathbb{O}_{2\mathcal{N}_c} & \mathbb{P}_{\mathbb{B}_{12}}^\perp & \cdots & \mathbb{O}_{2\mathcal{N}_c} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{O}_{2\mathcal{N}_c} & \mathbb{O}_{2\mathcal{N}_c} & \cdots & \mathbb{P}_{\mathbb{B}_{1\overline{N}}}^\perp \end{bmatrix}.$$

Equation 4.15 may also be rewritten as a function of the virtual array manifold as follows

$$\underline{h}_{1k}^{\text{STv}} = \left(\mathbb{I}_N \otimes \left[\mathbb{P}_{\mathbb{B}_{11}}^\perp, \mathbb{P}_{\mathbb{B}_{12}}^\perp, \dots, \mathbb{P}_{\mathbb{B}_{1\overline{N}}}^\perp \right]^T \mathbb{H}_{1k} \right) \left(\underline{S}_{1k} \otimes \overline{S}_{1k}^* \right). \quad (4.18)$$

Recall the spatiotemporal manifold vector introduced in Chapter 3 given as

$$\underline{h}_{1k}^{\text{ST}} = \underline{S}_{1k} \otimes \left\{ \mathbb{J}_{2\mathcal{N}_c}^{l_{1k}} \begin{bmatrix} \underline{c}_{1k} \\ \underline{0}_{\mathcal{N}_c} \end{bmatrix} \odot \underline{\mathcal{F}}_{1k} \right\}. \quad (4.19)$$

Comparing Eqs. 4.19 and 4.15, note that the second term in Eq. 4.19, i.e. the spatiotemporal manifold vector, is extended by employing the Tx array manifold vector. This, in effect, increases the number of dimensions of the spatiotemporal system from $2N\mathcal{N}_c$ to $2N\overline{N}\mathcal{N}_c$ in the proposed spatiotemporal-virtual system. Using the transmitter, channel and receiver models, a spatiotemporal-virtual beamformer is presented in the next section.

4.4 Spatiotemporal-Virtual Beamformer Design

In this section, the design of the receiver, consisting of the channel estimator and beamformer weight design blocks are presented. The data cube shown in Fig. 4.4 obtained at the output of the “manifold extender” forms the input to the receiver. The channel estimator is required to estimate all channel parameters of the desired source. Thus, assuming the first source to be the desired source i.e. $i = 1$, the channel parameters to be estimated are $(\theta_{1k}, \bar{\theta}_{1k}, l_{1k}, \mathcal{F}_{1k}), \forall k = 1$ to K . (The subscript $i = 1$ is dropped for notational convenience). Thus, the estimation stage

is a four-dimensional search and can be split into two two-dimensional searches. Firstly, the delay and Doppler frequency shift may be estimated followed by the DOD and DOA. It is assumed that there exists an interval of L snapshots over which the channel parameters may be assumed to be constant. Note that the estimation of the path fading coefficient is not presented in this chapter since algorithms akin to those presented in [49] and in Chapter 3 may be employed.

4.4.1 Joint Delay–Doppler Estimation

Since information about the DOD and DOA is not available at this stage, the spatial domain cannot be exploited. Hence, the incoming data cubes corresponding to snapshots $n = 1$ to L are rearranged to form $\mathbb{Y} \in \mathcal{C}^{2N_c \times NL}$ given as

$$\mathbb{Y} = [\mathbb{X}^T[1], \mathbb{X}^T[2], \dots, \mathbb{X}^T[n], \dots, \mathbb{X}^T[L]], \quad (4.20)$$

where $\mathbb{X}[n]$ denotes the n -th slice of the 3D data cube shown in Fig. 4.4. By performing an eigendecomposition of the covariance matrix of $\mathbb{Y}$, the projector operator to the noise subspace $\mathbb{P}_n$ may be obtained. Using this, the delay and Doppler frequency can be jointly estimated with the cost function $\xi(l_k, \mathcal{F}_k)$ given by

$$\xi(l_k, \mathcal{F}_k) = \frac{\det(\mathbb{H}_k^H \mathbb{H}_k)}{\det(\mathbb{H}_k^H \mathbb{P}_n \mathbb{H}_k)}, \quad (4.21)$$

where $\mathbb{H}_k \triangleq \mathbb{H}(l_k, \mathcal{F}_k)$ is given by Eq. 4.5. The peaks of the cost function $\xi(l_k, \mathcal{F}_k)$ correspond to the desired user's delay and Doppler frequency shift respectively.

4.4.2 Joint DOA–DOD Estimation

For every pair of estimated delay and Doppler frequency shift $(\hat{l}_k, \hat{\mathcal{F}}_k)$ of the desired user, the DOA and DOD need to be estimated. Towards this, the covariance matrix $\mathbb{R}_{xx} \in \mathcal{C}^{2N\bar{N}N_c \times 2N\bar{N}N_c}$ is constructed as follows

$$\mathbb{R}_{xx} = \frac{1}{L} \sum_{n=1}^L \underline{x}[n] \underline{x}^H[n], \quad (4.22)$$

where $\underline{x}[n]$ is obtained as illustrated in Eq. 4.10. By performing an eigendecomposition of the matrix $\mathbb{R}_{xx}$, the eigenvectors spanning the signal subspace and the projection operator to the noise subspace $\mathbb{P}_n$ may be obtained. We may now employ the extended spatiotemporal-virtual manifold vector associated with the desired user as given by Eq. 4.15, in conjunction with the following cost function to yield the DOA and DOD corresponding to every pair of $(\hat{l}_k, \hat{\mathcal{F}}_k)$

$$\xi(\theta_k, \bar{\theta}_k) = \frac{(\underline{h}_k^{\text{STv}} \odot \underline{\bar{w}}^{\text{ext}})^H (\underline{h}_k^{\text{STv}} \odot \underline{\bar{w}}^{\text{ext}})}{(\underline{h}_k^{\text{STv}} \odot \underline{\bar{w}}^{\text{ext}})^H \mathbb{P}_n (\underline{h}_k^{\text{STv}} \odot \underline{\bar{w}}^{\text{ext}})}, \quad (4.23)$$

where $\underline{h}_k^{\text{STv}} \triangleq \underline{h}^{\text{STv}}(\theta_{l_k, \mathcal{F}_k}, \bar{\theta}_{l_k, \mathcal{F}_k})$ is given by Eq. 4.15. The peaks of the cost function $\xi(\theta_k, \bar{\theta}_k)$ correspond to the desired user's DOA and DOD, respectively.

4.4.3 Design of the Rx Spatiotemporal-Virtual Beamformer Weights

In this subsection, using the estimated channel parameters of the desired user in Sections 4.4.1 and 4.4.2, in conjunction with path fading coefficients, the design of the spatiotemporal-virtual beamformer weights are proposed to obtain the estimate of the transmitted message $\mathbf{a}[n]$. In order to receive $\mathbf{a}[n]$ the following steps are carried out

1. Firstly, the desired user's contribution is suppressed from the receiver covariance matrix $\mathbb{R}_{xx}$ (Eq. 4.22) to obtain the $\hat{\mathbb{R}}_{\text{unwanted}}$ as follows

$$\hat{\mathbb{R}}_{\text{unwanted}} = \mathbb{R}_{xx} - \mathbb{H}^{\text{STv}} \text{diag}(\underline{\beta}) (\mathbb{H}^{\text{STv}})^H, \quad (4.24)$$

where $\mathbb{H}^{\text{STv}} \in \mathcal{C}^{2N\bar{N}\mathcal{N}_c \times K}$ is given as

$$\mathbb{H}^{\text{STv}} = [\underline{h}_1^{\text{STv}}, \dots, \underline{h}_K^{\text{STv}}]. \quad (4.25)$$

Also, the vector $\underline{\beta} \in \mathcal{C}^{K \times 1}$ is written as

$$\underline{\beta} = [\beta_1, \dots, \beta_K]^T. \quad (4.26)$$

2. By performing an eigendecomposition of $\hat{\mathbb{R}}_{\text{unwanted}}$, the significant eigenvector matrix $\mathbb{E}_s$ can be obtained to construct a projection operator towards the space spanned by the desired user given by

$$\mathbb{P}_{\text{unwanted}}^\perp = \mathbb{I}_{2N\bar{N}\mathcal{N}_c} - \mathbb{E}_s (\mathbb{E}_s^H \mathbb{E}_s)^{-1} \mathbb{E}_s^H. \quad (4.27)$$

Thus, the subspace spatiotemporal-virtual beamformer weight $\underline{w}_{\text{sub}}^{\text{STv}}$ may be obtained as

$$\underline{w}_{\text{sub}}^{\text{STv}} = \mathbb{P}_{\text{unwanted}}^\perp \mathbb{H}^{\text{STv}} \left\{ (\mathbb{H}^{\text{STv}})^H \mathbb{P}_{\text{unwanted}}^\perp \mathbb{H}^{\text{STv}} \right\}^{-1} \underline{\beta} \in C^{2N\bar{N}\mathcal{N}_c \times 1}. \quad (4.28)$$

4.5 Computer Simulation Studies

To evaluate the performance of the proposed spatiotemporal-virtual systems, computer simulation studies have been performed. Without any loss of generality, the Tx and Rx arrays are assumed to be UCAs lying on the xy -plane with the Tx array consisting of 7 antenna elements and the Rx array consisting of 9 antenna elements whose geometries are given in half-wavelengths by Eqs. 4.29 and 4.30, respectively.

$$[\bar{r}_x, \bar{r}_y, \bar{r}_z]^T = \begin{bmatrix} 1.1, & 0.7, & -0.3, & -1.0, & -1.0, & -0.3, & 0.7 \\ 0.0, & 0.9, & 1.1, & 0.5, & -0.5, & -1.1, & -0.9 \\ 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0 \end{bmatrix}, \quad (4.29)$$

$$[r_x, r_y, r_z]^T = \begin{bmatrix} 1.4, & 0.9, & -0.0, & -1.0, & -1.4, & -1.2, & -0.4, & 0.5, & 1.3 \\ 0.3, & 1.1, & 1.5, & 1.1, & 0.2, & -0.8, & -1.4, & -1.3, & -0.7 \\ 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0, & 0.0 \end{bmatrix}. \quad (4.30)$$

However, in the reception studies, uniform linear arrays (ULAs) are also considered at the Tx and Rx to compare the dependence on geometry for different receivers. For the purpose of simulations, an observation interval of $L = 200$ snapshots is considered. Table 4.1 illustrates the simulations parameters employed for

the simulations. The parameters follow a uniform distribution in their respective intervals i.e., the DOD and DOA between 0 and 360 degrees, the delays between 0 to $\mathcal{N}_c T_c - 1$, the Doppler between 0 and 2000 Hz and the complex path coefficients follow a uniform phase distribution between 0 and 360 degrees and the gain follows a Rayleigh distribution.

Table 4.1: Simulation Parameters

Parameter	Value	Parameter	Value
M	4	T_s	0.1 ms
K	2	N	9
$\mathcal{N}_c$	31 chips	$\bar{N}$	7

4.5.1 Evaluation of the Channel Estimator Block

In this subsection, the performance of the proposed spatiotemporal-virtual estimator is evaluated. As illustrated in Table 4.1, the desired user is assumed to operate in the presence of three other interferers with two multipaths per user. Figure 4.5a indicates a sample result at the output of joint delay and Doppler frequency estimation yielding peaks at the parameters of the desired user, namely at $(4T_c, 700 \text{ Hz})$ and $(12T_c, 1500 \text{ Hz})$. Similarly, Fig. 4.5b indicates the corresponding result at the output of joint DOA and DOD estimation for the pairs of estimated delay and Doppler frequency yielding peaks at the parameters of the desired user, namely at $(40^\circ, 60^\circ)$ and $(220^\circ, 250^\circ)$.

As illustrated in Chapter 3, a key performance parameter for subspace based estimation approaches is the number of snapshots L required to achieve a certain RMSE at a specified SNR. A higher number of snapshots yields a better approximation of the theoretical covariance matrix. Figure 4.6 illustrates the performance of the estimation algorithms presented in Sections 4.4.1 and 4.4.2 in terms of the RMSE of the estimated parameters versus the $\text{SNR} \times L$. (The parameter L is fixed at 200 while the SNR is varied.) The error in estimation of the delay is not plotted

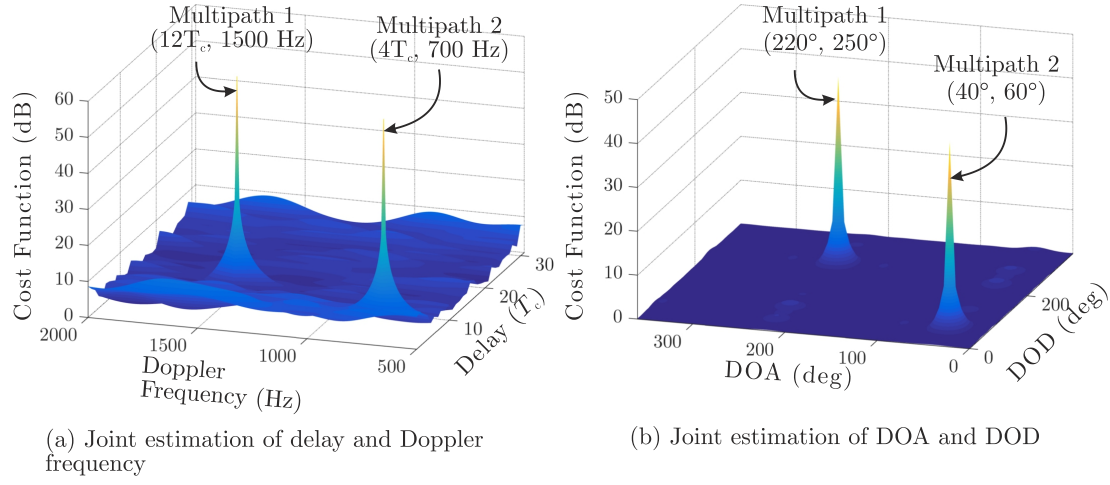


Figure 4.5: Sample result of the output of (a) the joint delay-Doppler estimation and (b) the joint DOA-DOD estimation for the desired source operating with 3 other interfering sources. The SNR and NFR are set to 20 dB and 0 dB respectively.

since this remains zero. As expected, the error in the estimation of Doppler frequency, DOA and DOD decreases with an increase in SNR, since the estimation of the noise subspace improves with an increase in number of snapshots for subspace based approaches.

4.5.2 Evaluation of the ST-virtual Reception Block

In this subsection, the performance of the proposed spatiotemporal-virtual MIMO receiver presented in this chapter is compared against spatial-only (i.e. massive MIMO) and the spatiotemporal receiver. Let us recall the definition of the SNIR_{out} defined in [49] and in Chapter 3 given as

$$\text{SNIR}_{\text{out}} = \frac{\underline{w}^H \mathbb{R}_{\text{des}} \underline{w}}{\underline{w}^H (\mathbb{R}_{\text{unwanted}} + \mathbb{R}_{\text{ISI}} + \mathbb{R}_{\text{nn}}) \underline{w}}, \quad (4.31)$$

where $\underline{w}$ denotes the beamforming weight employed at the receiver, $\mathbb{R}_{\text{des}}$ denotes the desired signal covariance matrix, $\mathbb{R}_{\text{unwanted}}$ denotes the unwanted signal covariance matrix, $\mathbb{R}_{\text{ISI}}$ denotes the covariance matrix of the inter symbol interference and $\mathbb{R}_{\text{nn}}$ represents the noise covariance matrix. Clearly, the value of the metric

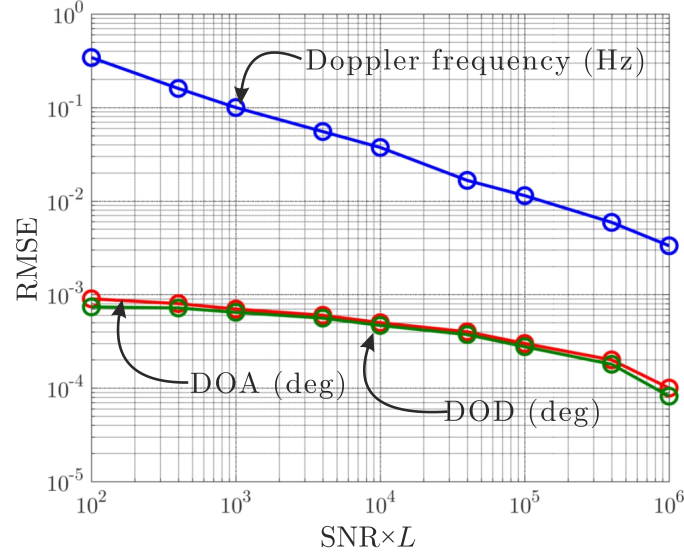


Figure 4.6: Estimation error of the parameters being estimated versus the $\text{SNR} \times L$ (The estimation error of the delay is not shown since it remains zero) (100 trials)

depends on the type of receiver weight design algorithm being employed. For a number of applications, tolerance to the near-far problem, i.e. constant performance to varying near-far power ratio levels, is a critical requirement. It has been illustrated in Chapter 3 that subspace beamformers exhibit nearly constant performance for NFR levels ranging from 0 to 60 dB. Thus, the subspace weights in Chapter 3 are extended to spatiotemporal-virtual subspace beamformer weights given by Eq. 4.28. Equipped with these weights, the interference may be completely suppressed and Eq. 4.31 simplifies to the following

$$\text{SNIR}_{\text{out}} = N_{\text{dim}} \frac{P_s}{\sigma_n^2}, \quad (4.32)$$

where P_s is the desired signal's power at the Rx. Assuming the desired Tx signal's power to be unity, Eq. 4.32 simplifies to the following for the spatial-only,

spatiotemporal and spatiotemporal-virtual receiver

$$\text{SNIR}_{\text{out}}^{\text{space-only}} = N \frac{G_{\text{Tx}}}{\sigma_n^2}, \quad (4.33a)$$

$$\text{SNIR}_{\text{out}}^{\text{ST}} = N \mathcal{N}_c \frac{G_{\text{Tx}}}{\sigma_n^2}, \quad (4.33b)$$

$$\text{SNIR}_{\text{out}}^{\text{ST-virt}} = N \overline{N} \mathcal{N}_c \frac{1}{\sigma_n^2}, \quad (4.33c)$$

where G_{Tx} represents the gain that arises from Tx beamforming. However, the gain G_{Tx} in the spatial-only and spatiotemporal systems, are variable and determined by the array geometry, DOD and the transmit weight. Also, note that the two aforementioned systems have no means to estimate the DOD and, as a result, are unable to control the variation in this parameter. The result of this on the output SNIR is illustrated in Figs. 4.7 and 4.8 where the proposed beamformer is compared with spatial-only and spatiotemporal beamformers for two sets of Tx array geometries, namely, a ULA and UCA consisting of 7 antennas respectively. The DOD of the desired user is varied (for simplicity, all multipaths of the desired user are assumed to have the same DOD) and the transmit weight is designed as a steering vector beamformer with the mainlobe at 90° . The spatiotemporal and spatial-only beamformers exhibit varying levels of output SNIR_{out} depending on the DOD and the Tx array geometry, whereas the proposed spatiotemporal-virtual beamformer provides a performance independent of the DOD and the Tx array geometry. This is because the spatiotemporal-virtual beamformer is capable of estimating the DOD and utilising the Tx array geometry to enhance the system dimensions, rather than ignoring this as an uncontrollable parameter.

Figure 4.9 illustrates a comparative study of the output SNIR for the beamformers under comparison, with an increasing number of “signals” in the system, for a UCA being employed at the Tx. These signals are equivalent to an increasing number of sources and/or multipaths. In this study, the number of multipaths per source is varied with the number of sources M fixed at 4. Figure 4.9 shows that the output SNIR of the spatial-only system is very low, when the number of signals is

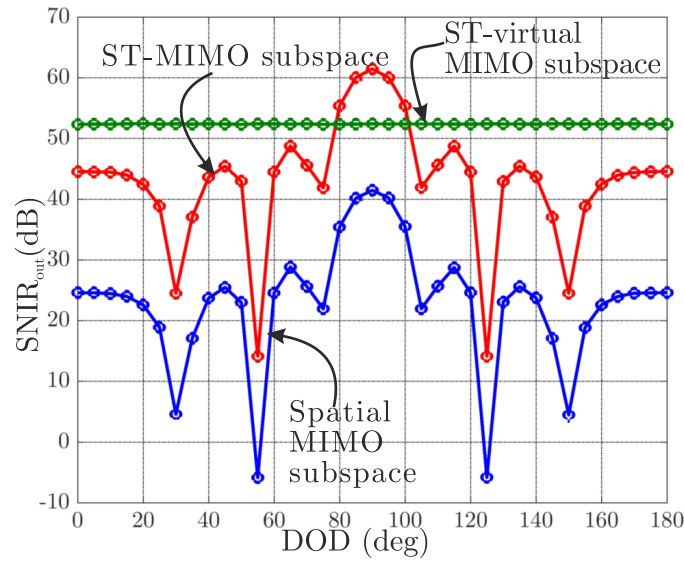


Figure 4.7: Comparison of output SNIR versus the DOD between the spatial-only, spatiotemporal, spatiotemporal-virtual subspace beamformers with a ULA being employed at the transmitter. The transmit weight is assumed to a spatial-only steering vector beamformer directed to 90 degrees. The NFR and SNR are both set to 10 dB. (100 trials)

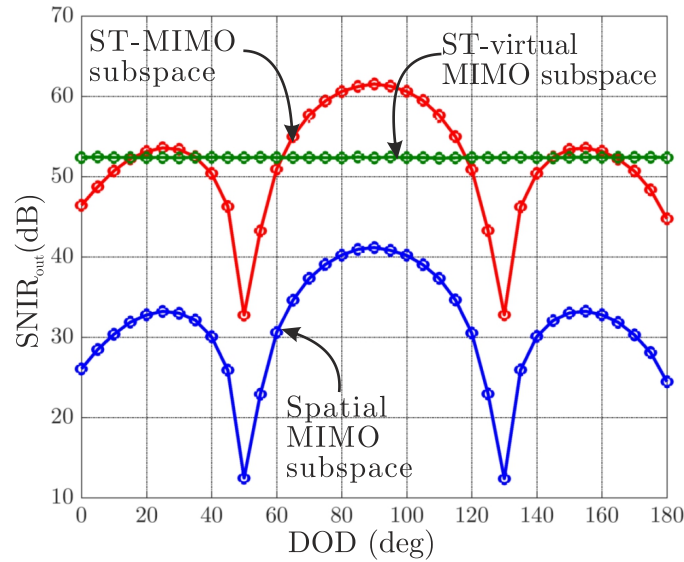


Figure 4.8: Comparison of output SNIR versus the DOD between the spatial-only, spatiotemporal, spatiotemporal-virtual subspace beamformers with a UCA being employed at the transmitter. The transmit weight is assumed to a spatial-only steering vector beamformer directed to 90 degrees. The NFR and SNR are both set to 10 dB. (100 trials)

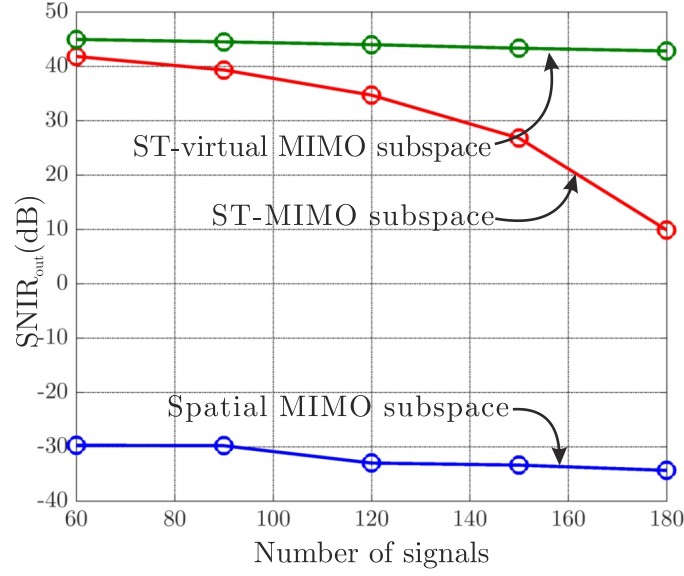


Figure 4.9: Comparison of output SNIR versus the number of signals for the spatial, spatiotemporal and spatiotemporal-virtual beamformer with subspace beamformer weights employed at the receiver. A UCA of 7 antennas is employed at the Tx. The DOD of the desired user's multipaths are set to 45 degrees (This corresponds to the trough region in Fig. 4.8)

high, since the covariance matrix has insufficient dimensions to construct the interference subspace. It can be observed that the spatiotemporal-virtual system supports a higher number of signals than the spatiotemporal receiver. To be precise, the spatiotemporal system provides an ability to accommodate $2N\mathcal{N}_c/3$ signals while the proposed spatiotemporal-virtual system accommodates $2N\bar{N}\mathcal{N}_c/3$.

4.6 Summary

In this chapter, the spatiotemporal array system (Chapter 3) was extended to a spatiotemporal-virtual array system, thus exploiting all spatial domains (Tx and Rx) jointly with the temporal domain, resulting in an increase in the degrees of freedom. A novel design of the transmitted signal and a corresponding receiver design that exploits the knowledge of the format of the Tx signal, to extract information about the Tx array geometry, in addition to the Rx spatial and temporal

domains, was presented. The solutions that were presented served the objectives of heightened interference cancellation whilst accommodating more sources/signals in the system. Also, the spatiotemporal-virtual subspace beamformer illustrated a performance independent of the Tx array geometry unlike the spatiotemporal subspace beamformer.

Chapter 5

Flexible Array Signal Processing

In this chapter, a unique class of arrays, termed as “flexible” arrays, that possess time-varying array geometry, is presented. Such arrays are relevant to a number of applications such as towed arrays and vehicular clusters, such as UAVs or satellites. For instance, a drone within a cluster of UAVs may experience perturbations from gusts of wind or imperfections arising from the controllers dictating the formation of the UAVs. Furthermore, with the advent of internet of things, “flexible” arrays are envisaged in the domain of wireless communications, in the form of sensors across static and mobile platforms that may move independently, whilst functioning together as an array. Thus, this chapter explores array signal processing techniques for such “flexible” arrays, since conventional techniques break down, due to the assumption of perfectly known and fixed array geometry. The approach presented in this chapter utilises the parametric model in [Chapter 2](#) (albeit with a time-varying array geometry), in conjunction with the mobility model of the array and the targets, to build a state space model that can be utilised to track both the target parameters and the array positions simultaneously. The proposed solution is evaluated using computer simulation studies, using towed arrays and UAV clusters as baseline applications.

5.1 Overview

In the earlier chapters, techniques to utilise all possible degrees of freedom in arrays with time-invariant array geometry were explored by jointly exploiting the Rx spatial domain, temporal domain and the Tx spatial domain. However, certain applications are required to utilise and exploit signals received on arrays whose sensors have time-variant positions. Such arrays with time-varying array geometries are termed as “flexible” arrays. Flexible array signal processing refers to the detection, estimation and reception of signals received on such flexible arrays. In this scenario, the array manifold becomes a function of time. Hence, typical array signal processing algorithms break down, since they are built on the premise of a constant and known array geometry. Flexible arrays find applications in a number of areas. Some of these applications are as follows

- **Towed arrays:** A towed array comprises of an array of hydrophones that are attached with equidistant spacing on a cable structure which may be several miles long and is towed by a submarine or ship. The acoustic signals received at these hydrophones are collected and routed back to the towing vessel for further analysis. Towed arrays are often used in naval defence applications to detect acoustic signatures of ships [79, 80]. They are also used for oil and gas exploration in marine waters. They are often employed for Sound Navigation And Ranging (SONAR) applications such as seismic exploration for oceanographic study. Another new area of research for SONAR systems is that of a SPATIally Referenced Towed Array (SPARTA). This is a multi-platform system consisting of a small fixed array and a small vehicle mounted array. Using arrays on different moving platforms makes it possible to achieve larger apertures and hence better reception. The challenge with towed arrays or multi-platform systems such as SPARTA, is that the motion of the towing vessel and external factors such as water drag, results in uncertainties in

the array sensor positions. There have been a number of approaches in the literature that have attempted to determine the deviation of the array's shape from linearity. However, the focus of the work in this chapter is on the case where the shape of the array changes significantly during the observation interval, rather than just translation or rotation of the array or very slow variations in array shape.

- **Clusters of vehicles in formation:** There has been an increased interest in utilising groups of autonomous vehicles in adaptable formations for a variety of applications. Such groups are able to interact autonomously with the environment and perform adverse tasks beyond the ability of individual vehicles [81]. Some of these application areas are listed below
 - **Clusters of UAVs:** UAVs find applications in a variety of areas such as remote sensing, commercial aerial surveillance, domestic policing, oil and gas exploration, post battle surveillance for casualties, scientific research and synthetic aperture radar. As UAV capabilities improve, a cluster of UAVs has been proven to perform better as compared to a single UAV in terms of time and efficiency [82]. More importantly, if the UAVs' formation is designed for a good array performance, array signal processing algorithms could be exploited to give better coverage, clarity and resolution. The major challenge with UAV clusters is twofold: tackling arbitrary and known movement since each UAV has its own propulsion system and arbitrary unknown movement due to turbulent sources e.g. wind. Hence, flexible signal array processing algorithms come into play here.
 - **Micro-satellite clusters:** Satellite formation flying refers to the idea of satellites working together in formation, to accomplish the objective of one larger and more expensive satellite. With improved means of

autonomous flying, this technology has become more viable. There have been simulation based investigations to illustrate the improvement in synthetic aperture radar (SAR) imagery from a cluster of satellites as compared to a single one [83, 84]. However, for the best results, one needs to account for the satellites' orbital deviations and drifts due to other effects such as earth oblateness, atmospheric drag and solar and lunar gravity.

- **Clusters of AUVs:** Another interesting area of research is autonomous oceanographic sampling networks (AOSN) [85]. Each node in the AOSN consists of a base buoy, an acoustic modem, point sensors, an energy source and a selectable number of autonomous underwater vehicles (AUVs). Such a system is expected to provide spatiotemporal sampling of the ocean surface as compared to 2D techniques such as thermistor chains. As in UAV clusters, the challenge in processing the signals from AUVs will be twofold: uncertainties in their independent propellers and external factors like water drag.

Apart from the advantages of using flexible arrays in the applications mentioned in this section, it has been analytically proven in [86] that a flexible array is more robust to ambiguity errors and to the presence of correlation between sources. However, the common requirement for all these applications, is a model which incorporates the time-varying array geometry and an ability to track this variation with time. Some techniques in literature have been proposed towards flexible array signal processing and an overview of the state of the art is presented in the next section. In this chapter, using a flexible arrayed parametric model of the signal received at a flexible array, mobility model of the flexible array and mobility model of the targets, a state space model is presented that can track the sources as well as the array positions. The model is evaluated with two flexible array applications,

namely towed arrays and UAV clusters.

This chapter is organised as follows. In Section 5.2, an overview of the current state of the art in flexible array signal processing is presented. This is followed by Section 5.3, where the underpinning models, namely, the flexible arrayed parametric model of the received signal, mobility model of the flexible array and the mobility model of the targets are presented. Equipped with these models, Section 5.4 describes the proposed state space model framework that may be utilised in conjunction with an extended Kalman filter to track the target parameters and sensor positions simultaneously. Section 5.5 presents the simulation studies carried out with the proposed solution for the applications of towed arrays and clusters of UAVs. Finally, the chapter is concluded in Section 5.6.

5.2 State of the Art: Flexible Arrays

A few techniques have been proposed in the literature to tackle the problems arising with detection, estimation and reception on flexible/time-variant arrays. Figure 5.1 suggests an overall classification of techniques for flexible array signal processing. In the following paragraphs, a brief insight into this classification is presented.

5.2.1 Target/Source Localisation

The first category of techniques attempts to perform the task of source localisation with time-varying arrays. The ML estimator is one of the most frequently used techniques and is usually designed by splitting the estimation problem to individual optimisation tasks over the DOAs and signal/noise parameters [87]. In [88], the ML estimator is specifically designed for a SPARTA system. However, such estimators require a multidimensional search and are often computationally expensive. To alleviate this, a generalized least squares (GLS) estimator is proposed

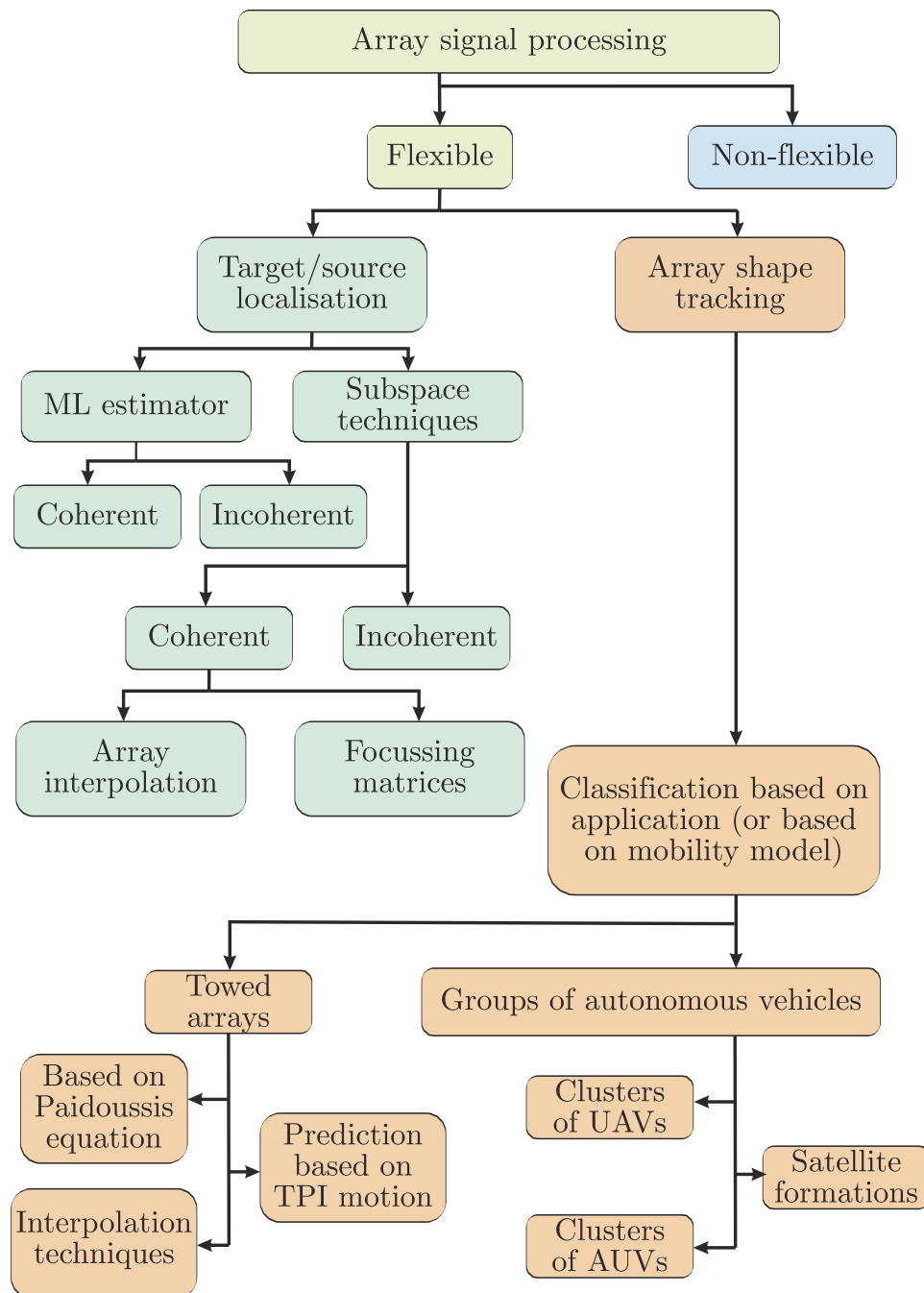


Figure 5.1: Classification of the current state of the art in flexible array signal processing.

in [89] which is computationally much simpler. A sub-optimal conditional ML estimator is described in [90] using a deterministic signal model. Alternatively, there are incoherent techniques in which the array is split into rigid subarrays where a simplified ML estimator could be utilised for each subarray [91]. In [91], this is applied for the case of a towed array, where the final bearing is obtained as the mean of the subarray bearings. However, these estimates are not very accurate and place a severe limitation on the number of sensors and types of perturbations (namely sinusoidal travelling disturbance).

There are also techniques which tweak the traditional subspace approach for time-varying arrays. In incoherent subspace techniques, the array is split into subarrays, such that the spatial covariance of the measurements of the subarray can be considered constant [92]. The MUSIC algorithm is then performed per subarray and the resulting spectra are combined in a coherent manner to yield the final source bearing. This technique starts to face problems in effective combining when the SNR is low. On the other hand, coherent subspace techniques attempt to draw concepts from wideband to narrowband transformations. With respect to flexible arrays, this would imply transforming the time-variant covariance matrix to a time-invariant covariance matrix. One approach is to use array interpolation, where a set of transformation matrices is utilised to map the time-variant manifold to a time-invariant manifold of a virtual inflexible array [87]. These transformation matrices are utilised to coherently combine the time-variant covariance matrices and the resultant matrix forms the input to the MUSIC algorithm. The major drawback of this technique is that the transformation holds true only for signals within an interpolation sector and hence limits the observation space to a small spatial region. An alternative technique known as focussing matrices firstly obtains a preliminary estimate of the DOAs using the deterministic modified or Gaussian modified beamformer [87]. Using this, a set of focussing angles are obtained by QR decomposition, after which the steps of the technique are similar to the array

interpolation method. The array interpolation technique handles much faster array shape changes than the focussing matrices technique. Though, both techniques require much lower computation effort than the ML estimator, the accuracy of the DOA estimates were found to be much lower than ML.

5.2.2 Array Shape Tracking

To be able to receive and decode the signals at the output of the flexible array, techniques to estimate the sensor positions accurately at any instant are required. In [93], a least mean squares (LMS) adaptive filtering algorithm is employed at the output of the sensors of the time-varying array to track the array shape using a pilot or known reference signal. However, most of the techniques proposed towards this objective depend on the type of application being employed. The mobility model associated with the application are used to design tracking models to continuously track the array shape, or design feedback controllers that adjust the array back to its original geometry. In this section, a few techniques towards this end goal are discussed.

For towed arrays, a class of techniques employ the Paidoussis equation that describes the dynamic behaviour of a thin flexible array towed through water. In [94], this equation is discretised to form a state space model with the traverse displacements of the array forming the states. Along with the measurements from the depth sensors and compass readings, this model can be used with a recursive Kalman filter to estimate the traverse displacements and hence the array shape. In [95], the discretised Paidoussis equation is used along with the compass sensor readings to predict the tow point induced motion. Then, the tail compass sensor data is used to do fine adjustments to the overall array shape. A complete analytic description of the motion of the towed array, when the towing vessel strays from the ideal path is provided in [96], also incorporating effects such as water drag. Another

class of techniques utilises interpolation, such as the quartic spline approximation to a space curve, to estimate the towed array shape using available compass sensor readings at certain locations of the array [97]. There are also techniques, such as those in [98, 99], where the shape of the towed array is expressed in terms of the towed vessel's path or from an accelerometer at the leading edge of the towed array.

For groups of UAVs, the challenge is to maintain the UAVs in a specific formation as commanded from the ground. In [100], UAV trajectories or formations to maximise overall estimation observability resulting in accurate target tracking is proposed. Often due to weight limitations, the UAVs are equipped only with rudimentary position sensors or navigation systems. Hence, to compensate errors due to these sensors and external turbulence such as wind, controller algorithms are implemented so that the UAVs can correct their positions iteratively. One class of techniques employs decentralised controllers i.e. each UAV has a local controller and exchanges no information with other vehicles. In [101], each UAV is assigned a linear kinematic model, under the constraint that each vehicle except the leading one has state information about the vehicle ahead of it. This implies a platoon like formation and is expected to be stable against structural perturbations and vehicle perturbations (wind gust and dynamic model uncertainties). A very simplified linear decentralised controller is described in [102], where a power output based objective function is optimised, when vehicles deviate from their positions. Another class of techniques employs distributed controllers rather than a centralised one to keep the vehicles in position [103]. Here, each UAV is equipped with a local controller, which receives information about its own state as well as the state of a subset of other vehicles. It does not consider absolute errors and rather looks at relative positioning with respect to other vehicles, ensuring directivity is maintained.

Satellite formations could be either in the form of multiple satellites in the

same chosen orbit trailing each other at specified distances or in the form of a densely packed cluster where the satellites occupy closely spaced orbits. In either of these cases, the shape of the formation could be perturbed due to independent propulsion errors or other factors such as gravitational effects and earth oblateness. Hence, controllers similar to that of UAVs, but based on a different mobility model are used to correct the positions of the satellites. In [104], a non-linear tracking control mechanism is proposed by modelling the follower satellite motion relative to the leader in terms of the “Hill’s Equation”. Other feedback control laws to incorporate unaccounted perturbations in the orbits of satellites, due to higher order gravitation effects, are discussed in [105]. A decentralised controller at every satellite is proposed in [106]. The controller uses Kepler’s differential central gravitational model to define how the states are propagated between measurements.

Similar to the case of UAV clusters, groups of AUVs require a control mechanism against water drifts and drag. A simple control strategy for stable orientation of autonomous vehicles is presented in [107], especially illustrating its applicability to a groups of AUVs.

5.3 Flexible Array Signal Processing Model

Consider a “flexible” antenna array consisting of N antennas with time-varying array geometry $\mathbf{r}(t) \in \mathcal{R}^{N \times 3}$ at a time instant t operating in the presence of M sources. Figure 5.2 illustrates the proposed framework for flexible array signal processing that tracks the target parameters as well as the flexible array geometry. As illustrated in Fig. 5.2, three mathematical models form the basis of the proposed solution, namely,

1. the mathematical model of the signal received at the flexible array incorporating all the channel parameters of interest and the time-varying array geometry,

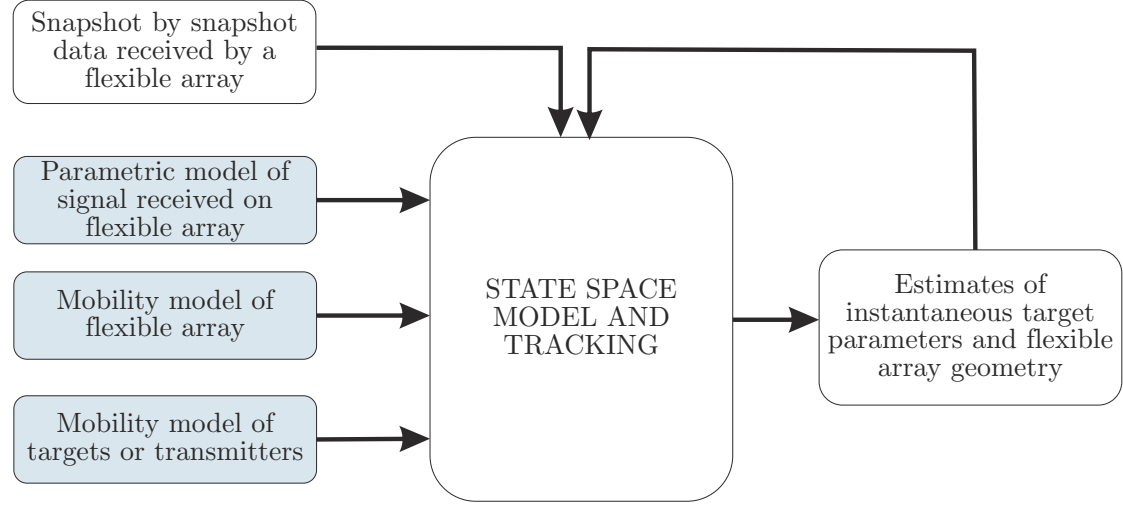


Figure 5.2: Illustration of the proposed framework for flexible array signal processing that tracks the target parameters and array positions snapshot by snapshot.

2. the mobility model of the flexible arrays under study incorporating the physical forces of the environment in which the flexible array has been deployed and
3. the mobility model of the targets or sources in the environment.

These mathematical models are discussed in more detail in the sections below.

5.3.1 Received Signal Model

Let us assume that we sample at a rate T_s and the time instant $t_l = lT_s$ denotes the l -th snapshot within an observation interval of L snapshots. Furthermore, let us assume that the flexible array, whose instantaneous locations are given by $\mathbf{r}(t_l) = [r_x(t_l), r_y(t_l), r_z(t_l)] \in \mathcal{R}^{N \times 3}$, operates in the presence of M narrowband far field, single antenna, sources. It is assumed that there exists one path that is of much higher power than the other multipaths of the source. Each source is assumed to be moving with an unknown constant radial velocity. The signal $\underline{x}(t_l) \in \mathcal{C}^{N \times 1}$ received at the flexible array at a time instant t_l can be modelled as

follows

$$\underline{x}(t_l) = \sum_{i=1}^M \beta_i \underline{S}_i(t_l) m_i(t_l) \exp(j2\pi l \mathcal{F}_i T_s) + \underline{n}(t_l), \quad (5.1)$$

where for the i -th user, $\underline{S}_i(t_l) \in \mathcal{C}^{N \times 1}$ denotes the time-varying array manifold vector, $\beta_i \in \mathcal{C}^{1 \times 1}$ represents the path fading coefficient, $m_i(t_l) \in \mathcal{C}^{1 \times 1}$ represents the delayed message received at the antenna array, $\mathcal{F}_i$ denotes the Doppler frequency and $\underline{n}(t_l) \in \mathcal{C}^{N \times 1}$ represents the observation noise. The time-varying array manifold vector is given by

$$\begin{aligned} \underline{S}_i(t_l) &\triangleq \underline{S}(\mathbf{r}(t_l), \theta_i(t_l), \phi_i(t_l)) \\ &= \exp\left(-j \frac{2\pi F_c}{c} \mathbf{r}(t_l) \underline{u}_i(t_l)\right), \end{aligned} \quad (5.2)$$

where F_c denotes the carrier frequency, c denotes the speed of the transmitted signal in the transmission medium and the wavenumber vector $\underline{u}_i(t_l)$ is given by

$$\begin{aligned} \underline{u}_i(t_l) &\triangleq \underline{u}(\theta_i(t_l), \phi_i(t_l)) \\ &= [\cos(\theta_i(t_l)) \cos(\phi_i(t_l)), \sin(\theta_i(t_l)) \cos(\phi_i(t_l)), \sin(\phi_i(t_l))]^T, \end{aligned} \quad (5.3)$$

where $(\theta_i(t_l), \phi_i(t_l))$ denotes the azimuth and elevation respectively at time instant t_l . In this chapter, the array elements are assumed to lie in the same plane i.e. $\underline{r}_z(t_l) = \underline{0}_N$.

5.3.2 Flexible Array Mobility Model

Depending on the application where the flexible array is being employed, a mobility model describing the evolution of the flexible array geometry with respect to time, in terms of all the physical forces present, may be derived. The choice of an appropriate mobility model is crucial for the design of appropriate algorithms and to reproduce the scenarios under study accurately. A generic mobility model may be expressed as follows

$$\begin{bmatrix} \underline{r}_x(t_l) \\ \underline{r}_y(t_l) \end{bmatrix} = \mathbb{F} \begin{bmatrix} \underline{r}_x(t_{l-1}) \\ \underline{r}_y(t_{l-1}) \end{bmatrix} + \begin{bmatrix} \underline{u}_x(t_l) \\ \underline{u}_y(t_l) \end{bmatrix} + \begin{bmatrix} \tilde{\underline{r}}_x(t_l) \\ \tilde{\underline{r}}_y(t_l) \end{bmatrix}, \quad (5.4)$$

where $\underline{r}_x(t_l), \underline{r}_y(t_l) \in \mathcal{R}^{N \times 1}$ denote the instantaneous x and y array positions respectively. The known transition matrix $\mathbb{F} \in \mathcal{R}^{2N \times 2N}$ and known driver vectors $\underline{u}_x(t_l), \underline{u}_y(t_l) \in \mathcal{R}^{N \times 1}$ are defined by the application under study and dictate the predictable or nominal array positions. This is determined by the known forces (such as propeller velocity/rotation angle) and the physics of the medium (such as fluid propagation or orbital theories). The errors in the array positions namely $\tilde{\underline{r}}_x(t_l), \tilde{\underline{r}}_y(t_l) \in \mathcal{R}^{N \times 1}$ arise from approximations in the mobility model and other external disturbances such as drifts/winds not modelled by the transition matrices and the driver terms. It is assumed that the first and second order statistics of these noise vectors is available. In this chapter, the two main flexible array applications under study are that of towed arrays and UAV clusters. Hence, mobility models relevant to these applications will be discussed in the next few paragraphs.

5.3.2.1 Towed Arrays

The conventional towed array consists of a neutrally buoyant cylinder with equi-spaced sensors placed on it. The towed array is attached to the towing vessel by a heavier-than-water cable and the other end of the towed array may either be attached to a rope drogue or left as a loose end. In typical scenarios, if the towing vessel maintains a constant velocity, the towed array remains linear. However, if the towing vessel changes its speed or heading or experiences turbulence, the towed array may be deformed. The most popular approach to model the motion of the towed array is the Paidoussis equation which describes the dynamic behaviour of a thin flexible array towed through water. As illustrated in [108], the motion in the horizontal and vertical dimensions may be analyzed independently. Hence, the Paidoussis equation is obtained in the form of two equations by (i) balancing the horizontal component of variable tension in the cylinder with the tangential viscous force (ii) balancing the transverse momentum with the vertical component of variable tension, normal viscous force, inviscid force due to acceleration of vir-

tual mass of cylinder and terms dependent on bending stiffness [96]. This equation is simplified greatly when the length of the towed array is much greater than its diameter. Using the simplified Paidoussis state evolution equations for the horizontal and vertical displacements, the following mobility model for a towed array may be derived

$$\begin{bmatrix} \underline{r}_x(t_l) \\ \underline{r}_y(t_l) \end{bmatrix} = \mathbb{F} \begin{bmatrix} \underline{r}_x(t_{l-1}) \\ \underline{r}_y(t_{l-1}) \end{bmatrix} + \begin{bmatrix} \underline{u}_x(t_l) \\ \underline{u}_y(t_l) \end{bmatrix} + \begin{bmatrix} \tilde{\underline{r}}_x(t_l) \\ \tilde{\underline{r}}_y(t_l) \end{bmatrix}, \quad (5.5)$$

where

$$\mathbb{F} = \begin{bmatrix} \alpha_x \mathbb{J}_N & \mathbb{O}_N \\ \mathbb{O}_N & \alpha_y \mathbb{J}_N \end{bmatrix}, \quad (5.6)$$

$$\underline{u}_x(t_l) = d\mathbb{L}(\mathbb{1}_{N-1} - \alpha_x \mathbb{J}_{N-1} \mathbb{1}_{N-1}) - \mathbb{L} \begin{bmatrix} \rho_x(t_l) \\ \underline{0}_{N-2} \end{bmatrix}, \quad (5.7)$$

$$\underline{u}_y(t_l) = \mathbb{L} \begin{bmatrix} \rho_y(t_l) \\ \underline{0}_{N-2} \end{bmatrix}, \quad (5.8)$$

$$\tilde{\underline{r}}_x(t_l) = -\mathbb{L}\underline{w}_x(t_l), \quad (5.9)$$

$$\tilde{\underline{r}}_y(t_l) = \mathbb{L}\underline{w}_y(t_l), \quad (5.10)$$

with

$$\mathbb{L} = \begin{bmatrix} 0, & 0, & \cdots & 0 \\ 1, & 0, & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1, & 1, & \cdots & 0 \\ 1, & 1, & \cdots & 1 \end{bmatrix} \in \mathcal{R}^{N \times N-1}, \quad (5.11)$$

$$\mathbb{J}_{N-1} = \begin{bmatrix} \underline{0}_{N-2}^T & 0 \\ \mathbb{I}_{N-2} & \underline{0}_{N-2} \end{bmatrix} \in \mathcal{R}^{N-1 \times N-1}, \quad (5.12)$$

where α_x and α_y represent the damping coefficients along the x and y axis, respectively, d denotes the fixed intersensor spacing, $\rho_x(t_l)$ and $\rho_y(t_l)$ represent the

known displacements of the first segment of the array at time instant t_l along the x -axis and y -axis, respectively, and $\underline{w}_x(t_l)$ and $\underline{w}_y(t_l)$ represent the uncertainty terms in the mobility model along the x -axis and y -axis, respectively. Therefore, Eq. 5.5 provides a mobility model for a towed array inline with the generic model presented in Eq. 5.4.

5.3.2.2 Cluster of UAVs

Choosing an appropriate mobility model is crucial to vehicular cluster applications. Models such as random way-point and Gauss-Markov model have been widely used towards modelling the distributions of vehicles. However, these models fail to capture all the constraints of aeronautical applications such as the inability of UAVs to make sharp turns and changes in velocity. Thus, taking these factors into account, mobility models for UAVs may be classified into three broad categories [109]. The first class of mobility models follows a semi-random distribution of UAVs, i.e. the variation in the UAV array geometry can be split into the sum of a known and an unknown perturbation in array geometry. An example of such a model is the semi-random circular mobility model proposed in [110] and such models are well suited for search and rescue missions. A second class of mobility models are based on a predetermined flight plan and are more suited for cargo and transportation applications [111]. The third class of mobility models that are employed for patrolling applications resemble swarms of vehicles and are closer in structure to the random way-point models in vehicular adhoc networks (VANETs). In this chapter, the first group of mobility models is considered. Semi-random mobility models may be further classified as group mobility models and entity mobility models. In this chapter, the group random mobility model will be considered, where the UAVs move as a group in formation around a fixed reference point, with each UAV permitted an element of random motion whose statistical properties are known. Therefore, with no loss of generality, the group motion is

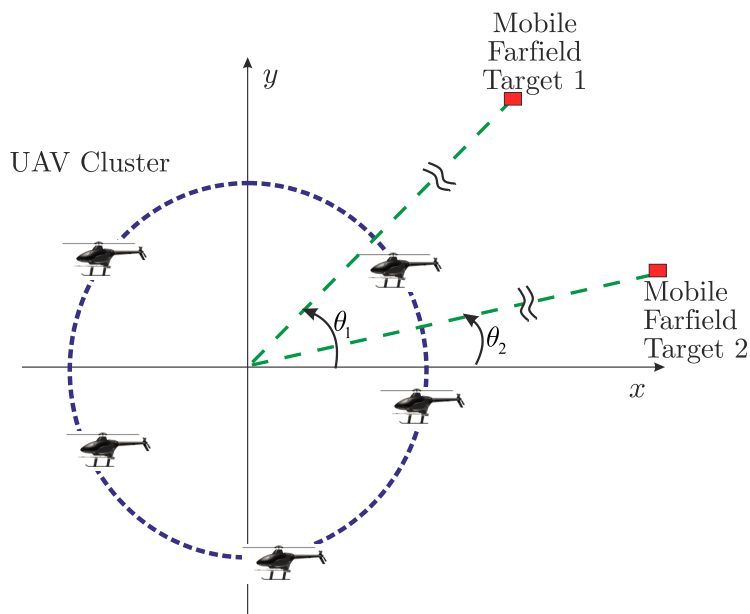


Figure 5.3: Illustration of the UAV cluster with the unknown targets in the xy -plane.

assumed to be split into two terms, namely, a known fixed rotation about the reference UAV and an unknown random component associated with each UAV, whose statistics are known. Figure 5.3 is an illustration of this mobility model. Hence, the mobility model for the UAV array may be written as

$$\begin{bmatrix} \underline{r}_x(t_l) \\ \underline{r}_y(t_l) \end{bmatrix} = \mathbb{F} \begin{bmatrix} \underline{r}_x(t_{l-1}) \\ \underline{r}_y(t_{l-1}) \end{bmatrix} + \begin{bmatrix} \tilde{\underline{r}}_x(t_l) \\ \tilde{\underline{r}}_y(t_l) \end{bmatrix}, \quad (5.13)$$

where

$$\mathbb{F} = \begin{bmatrix} \cos \omega T_s & -\sin \omega T_s \\ \sin \omega T_s & \cos \omega T_s \end{bmatrix} \otimes \mathbb{I}_N, \quad (5.14)$$

where ω represents the angular velocity (deg/sec) of the array about the reference point. Here $\underline{r}_x(t_l), \underline{r}_y(t_l) \in \mathcal{R}^{N \times 1}$ denote the instantaneous x and y array positions respectively. The known transition matrix $\mathbb{F} \in \mathcal{R}^{2N \times 2N}$ is determined by the known forces and the physics of the medium. The errors in the array positions namely $\tilde{\underline{r}}_x(t_l), \tilde{\underline{r}}_y(t_l) \in \mathcal{R}^{N \times 1}$ arise from approximations in the mobility model and other external disturbances such as drifts/winds not modelled by the transition

matrices and the driver terms. It is assumed that the first and second order statistics of these noise vectors is available. However, please note that although a relatively simple mobility model is chosen here, it may be modified to suit other class of mobility models as well. For example, for the third class of random mobility models, the transition matrix would be a random matrix and this may be further rewritten as a known transition matrix (obtained from the mean of the random transition matrix) and an error component whose second order statistics would be state dependent. This can be handled with an additional recursive update step of the extended Kalman filter to update the covariance matrix.

5.3.3 Target Constant Velocity Mobility Model

In this chapter, the targets are assumed to move along a path with constant angular velocity. However, perturbations in angular velocity are also modelled as perturbations in the angular acceleration. Thus, the unknown target parameters for the i -th user may be grouped into a state vector $\underline{b}_i(t_l) \in \mathcal{R}^{2 \times 1}$ given by

$$\underline{b}_i(t_l) = \begin{bmatrix} \theta_i(t_l) \\ v_{\theta_i}(t_l) \end{bmatrix}, \quad (5.15)$$

where $\theta_i(t_l)$ and $v_{\theta_i}(t_l)$ denote the azimuth and azimuthal angular velocity of the i -th user, respectively. As per the constant velocity motion model, the discrete kinematic model may be written as

$$\underline{b}_i(t_l) = \mathbb{G}\underline{b}_i(t_{l-1}) + \tilde{\underline{b}}_i(t_l), \quad (5.16)$$

where

$$\mathbb{G} = \begin{bmatrix} 1, & T_s \\ 0, & 1 \end{bmatrix}, \quad (5.17)$$

and the perturbations $\tilde{\underline{b}}_i(t_l)$ are modelled as zero mean with the covariance matrix $\mathbb{Q}_i$ given by

$$\mathbb{Q}_i = \sigma_{v_{\theta_i}}^2 \begin{bmatrix} \frac{T_s^3}{3}, & \frac{T_s^2}{2} \\ \frac{T_s^2}{2}, & T_s \end{bmatrix}, \quad (5.18)$$

where $\sigma_{v_{\theta_i}}^2$ denotes the continuous time model intensity in the azimuthal acceleration of the i -th target [112]. Thus, the composite state vector $\underline{b}(t_l)$ consisting of the unknown parameters of the M targets is given by

$$\underline{b}(t_l) = \begin{bmatrix} \underline{b}_1(t_l) \\ \underline{b}_2(t_l) \\ \vdots \\ \underline{b}_M(t_l) \end{bmatrix} \in \mathcal{R}^{2M \times 1}, \quad (5.19)$$

and the composite kinematic model may be written as

$$\underline{b}(t_l) = (\mathbb{I}_M \otimes \mathbb{G}) \underline{b}(t_{l-1}) + \tilde{\underline{b}}(t_l), \quad (5.20)$$

where the perturbations $\tilde{\underline{b}}(t_l)$ are zero mean with the covariance matrix $\mathbb{Q} \in \mathcal{R}^{2M \times 2M}$ that is a block diagonal matrix consisting of the individual covariance matrices $\mathbb{Q}_i$ of each of the users.

5.4 Arrayed-State Space Model and Extended Kalman Framework for Flexible Arrays

Using the models presented in Section 5.3, in this section, a state space model framework that enables the simultaneous tracking of targets and the flexible array sensor locations is described. The signal $\underline{x}(t_l)$ received at the flexible array at an instant t_l in the observation interval, as given by Eq. 5.1, can be written as

$$\underline{x}(t_l) = \mathbb{H}(t_l) \underline{m}(t_l) + \underline{n}(t_l), \quad (5.21)$$

where $\mathbb{H}(t_l) \in \mathcal{C}^{N \times M}$ is the channel matrix given by

$$\mathbb{H}(t_l) = [\underline{S}_1(t_l), \underline{S}_2(t_l), \dots, \underline{S}_M(t_l)], \quad (5.22)$$

and $\underline{m}(t_l) \in \mathcal{C}^{M \times 1}$ encompasses the message, path fading and Doppler coefficients and is given as

$$\underline{m}(t_l) = \begin{bmatrix} \beta_1(t_l) m_1(t_l) \exp(j2\pi l \mathcal{F}_1 T_s) \\ \beta_2(t_l) m_2(t_l) \exp(j2\pi l \mathcal{F}_2 T_s) \\ \vdots \\ \beta_M(t_l) m_M(t_l) \exp(j2\pi l \mathcal{F}_M T_s) \end{bmatrix}. \quad (5.23)$$

The next stage of building the state space model is to group all the unknowns of the problem under consideration into a state vector $\underline{z}(t_l)$ given by

$$\underline{z}(t_l) = \begin{bmatrix} r_x(t_l) \\ r_y(t_l) \\ b(t_l) \end{bmatrix} \in \mathcal{R}^{2(N+M) \times 1}. \quad (5.24)$$

Thus, Eqs. 5.4 and 5.20 may be combined to yield an evolution equation for the state vector $\underline{z}(t_l)$ as follows

$$\underline{z}(t_l) = \mathbb{F}_{\text{all}} \underline{z}(t_{l-1}) + \underline{z}_{\text{d}}(t_l) + \tilde{\underline{z}}(t_l), \quad (5.25)$$

where the state transition matrix $\mathbb{F}_{\text{all}} \in \mathcal{R}^{2(N+M) \times 2(N+M)}$ is given by

$$\mathbb{F}_{\text{all}} = \begin{bmatrix} \mathbb{F}, & \mathbb{O}_{2N \times 2M} \\ \mathbb{O}_{2N \times 2M}, & \mathbb{I}_M \otimes \mathbb{G} \end{bmatrix}. \quad (5.26)$$

In Eq. 5.25, the term $\underline{z}_{\text{d}}(t_l)$ denotes the known driver term that is dictated by the flexible array application under study and is given by

$$\underline{z}_{\text{d}}(t_l) = \begin{bmatrix} \underline{u}_x(t_l) \\ \underline{u}_y(t_l) \\ \underline{0}_{2M} \end{bmatrix}. \quad (5.27)$$

The term $\tilde{\underline{z}}(t_l) \in \mathcal{R}^{2(N+M) \times 1}$ incorporates all the uncertainties that arise due to approximations in the mobility model or due to the presence of forces unaccounted for in the mobility model, such as gusts of wind, and can be represented as follows

$$\tilde{\underline{z}}(t_l) = \begin{bmatrix} \tilde{r}_x(t_l) \\ \tilde{r}_y(t_l) \\ \tilde{\underline{b}}(t_l) \end{bmatrix}. \quad (5.28)$$

Equations 5.21 and 5.25 constitute an arrayed-state space model that describes the signal received at the flexible array in terms of the unknown target parameters and unknown flexible array sensor positions encompassed in the state vector. Thus, the state space model can be written as

$$\begin{cases} \underline{z}(t_l) = \mathbb{F}_{\text{all}} \underline{z}(t_{l-1}) + \underline{z}_d(t_l) + \tilde{\underline{z}}(t_l) \\ \underline{x}(t_l) = \mathbb{H}(\underline{z}(t_l)) \underline{m}(t_l) + \underline{n}(t_l) \end{cases}. \quad (5.29)$$

Also, the mean of the state vector $\underline{z}(t_l)$ is given by

$$\begin{aligned} \mathcal{E}\{\underline{z}(t_l)\} &= \mathcal{E}\{\mathbb{F}_{\text{all}} \underline{z}(t_{l-1}) + \underline{z}_d(t_l)\} \\ &= \mathbb{F}_{\text{all}}^l \underline{z}(t_0) + \sum_{m=1}^l \underline{z}_d(t_m), \end{aligned} \quad (5.30)$$

assuming $\mathcal{E}\{\tilde{\underline{z}}(t_l)\} = \underline{0}_{2(N+M)}$. Also, the covariance matrix of $\underline{z}(t_l)$ denoted by $\mathbb{R}_{zz}(t_l) \in \mathcal{R}^{2(N+M) \times 2(N+M)}$ is given by

$$\begin{aligned} \mathbb{R}_{zz}(t_l) &= \mathcal{E}\left\{(\underline{z}(t_l) - \mathcal{E}\{\underline{z}(t_l)\})(\underline{z}(t_l) - \mathcal{E}\{\underline{z}(t_l)\})^T\right\} \\ &= \mathbb{F}_{\text{all}} \mathbb{R}_{zz}(t_{l-1}) \mathbb{F}_{\text{all}}^T + \mathcal{E}\{\tilde{\underline{z}}(t_l) \tilde{\underline{z}}^T(t_l)\} \\ &= \mathbb{F}_{\text{all}} \mathbb{R}_{zz}(t_{l-1}) \mathbb{F}_{\text{all}}^T + \mathbb{R}_{\tilde{z}\tilde{z}}, \end{aligned} \quad (5.31)$$

where $\mathbb{R}_{\tilde{z}\tilde{z}} \in \mathcal{R}^{2(N+M) \times 2(N+M)}$ is given as

$$\mathbb{R}_{\tilde{z}\tilde{z}} = \begin{bmatrix} \sigma_{r_x}^2 \mathbb{I}_N & 0_{N \times N} & 0_{N \times 2M} \\ 0_{N \times N} & \sigma_{r_y}^2 \mathbb{I}_N & 0_{N \times 2M} \\ 0_{2M \times N} & 0_{2M \times N} & \mathbb{Q} \end{bmatrix}, \quad (5.32)$$

where the perturbations in the flexible array geometry are modelled as white Gaussian noise with zero mean and intensity of $\sigma_{r_x}^2$ and $\sigma_{r_y}^2$ along the x and y axes respectively. Note that Eq. 5.32 has been assumed for simplicity and is not a constraint. For instance, Eq. 5.32 can be suitably modified for the case of correlated noise in the state vector across the x and y antenna elements. Therefore, the state space model presented by Eq. 5.29 forms the input to an arrayed-extended Kalman filter (EKF) that tracks and adaptively estimates the state vector $\underline{z}(t_l)$. An extended Kalman filter is employed since the transition matrix $\mathbb{H}(\underline{z}(t_l))$ is non-linear. In order to estimate $\underline{z}(t_l)$ adaptively in the extended Kalman framework, the algorithm summarised in Table 5.1 may be employed where the notations are simplified as $\underline{z}_l \triangleq \underline{z}(t_l)$, $\underline{z}_{d,l} \triangleq \underline{z}_d(t_l)$, $\mathbb{H}_l \triangleq \mathbb{H}(\underline{z}(t_l))$, $\underline{\hat{m}}_l \triangleq \underline{\hat{m}}(t_l)$ and $\underline{x}_l \triangleq \underline{x}(t_l)$.

Table 5.1: Summary of steps to estimate target parameters and the flexible array positions using the extended Kalman filter

Initialization
$\hat{\underline{z}}_{0 0} = \mathcal{E}\{\underline{z}_0\} + \underline{z}_{d,0}$
$\mathbb{P}_{0 0} = \mathcal{E}\left\{(\underline{z}_0 - \hat{\underline{z}}_{0 0})(\underline{z}_0 - \hat{\underline{z}}_{0 0})^T\right\}$
For $l = 1, \dots, L$
$\hat{\underline{z}}_{l l-1} = \mathbb{F}_{\text{all}} \hat{\underline{z}}_{l-1 l-1} + \underline{z}_{d,l}$
$\mathbb{P}_{l l-1} = \mathbb{F}_{\text{all}} \mathbb{P}_{l-1 l-1} \mathbb{F}_{\text{all}}^T + \mathbb{R}_{\underline{z}\underline{z}}$
$\underline{\hat{m}}_l = \mathbb{H}_{l l-1}^H \left(\mathbb{R}_{\text{nn}} + \mathbb{H}_{l l-1} \mathbb{H}_{l l-1}^H \right)^{-1} \underline{x}_l$
$\mathbb{D}_{l l-1} = \nabla_{\underline{z}_l} (\mathbb{H}_{l l-1} \underline{\hat{m}}_l) _{\hat{\underline{z}}_{l l-1}}$
$\mathbb{K}_l = \mathbb{P}_{l l-1} \mathbb{D}_{l l-1}^H \times$
$\quad \left(\mathbb{D}_{l l-1} \mathbb{P}_{l l-1} \mathbb{D}_{l l-1}^H + \mathbb{R}_{\text{nn}} \right)^{-1}$
$\hat{\underline{z}}_{l l} = \hat{\underline{z}}_{l l-1} + \text{Re}\left\{ \mathbb{K}_l (\underline{x}_l - \mathbb{H}_{l l-1} \underline{\hat{m}}_l) \right\}$
$\mathbb{P}_{l l} = \left(I_{4M} - \mathbb{K}_l \mathbb{D}_{l l-1} \right) \mathbb{P}_{l l-1}$

Thus the predicted state vector $\hat{\underline{z}}_l$ consists of all the parameters to be estimated, namely, the unknown flexible array sensor locations and the parameters of the targets. Note that the initial covariance matrix $\mathbb{P}_0$ can be tuned according to the level of uncertainty in the initial estimate $\hat{\underline{z}}_0$. For instance, for high levels of

uncertainty in the initial estimates, $\mathbb{P}_0$ may be set to $\mu \mathbb{I}_{2(N+MK)}$, where μ is an arbitrarily large real number indicating the confidence level in the estimate. Note that an estimate of the unknown message is obtained at each step. At low SNR, this may be further refined by substituting the updated estimate at the end of any iteration and may be repeated as many times as desired.

5.5 Simulation Studies

In this section, the proposed flexible array signal processing model and the corresponding state space framework described in Sections 5.3 and 5.4, are examined via computer simulations. Two specific flexible array applications are examined namely towed arrays and a cluster of UAVs. For both applications, two narrow-band far field targets are assumed to be present in the environment. The signals of the targets are assumed to be independent identically distributed Gaussian sources of zero mean and unity variance. The input SNR is assumed to be 20 dB. Tracking is assumed to be carried out over an interval of 50 s with a sampling interval $T_s = 0.01$ s (i.e. 5000 snapshots). The rest of the simulation environment specific to the application and the tracking results of each of the applications are presented in the following subsections.

5.5.1 Cluster of UAVs

The carrier frequency is assumed to be 50 MHz. Figures 5.4 and 5.5 illustrate the true and tracked azimuth and azimuth angular velocity trajectories of the two moving targets. It is clear from these figures that the target parameters are indeed tracked successfully. The continuous time model intensity $\sigma_{v_\theta}^2$ in the azimuth tracking model is chosen to be 10 (deg/s)². Also, the intensities of the perturbations along the x and y axis, are set as follows $\sigma_{r_x}^2 = \sigma_{r_y}^2 = 1.8$ (m/s)². Furthermore, the angular velocity of the array ω is chosen to be 1 deg/s. The covariance matrix $\mathbb{P}_0$

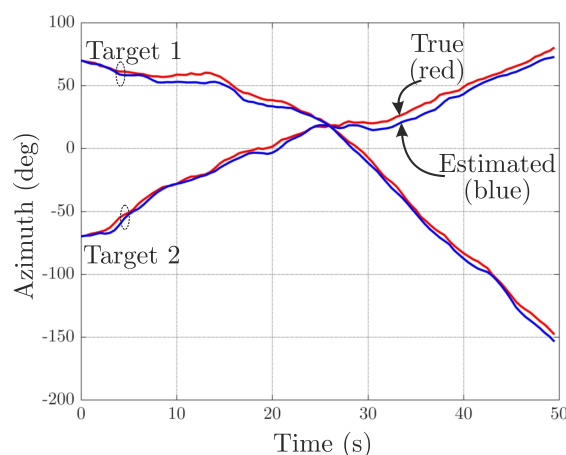


Figure 5.4: True (red) and tracked (blue) azimuth trajectories of the two targets in the environment of the UAV cluster application

is set to $\mu \mathbb{I}_{2(N+M)}$, where $\mu = 10^{-4}$, which indicates a reasonably high confidence in the accuracy of the initial estimates. For the purpose of simulations, the UAV cluster is assumed to consist of $N = 7$ elements. As described in Section 5.3.2.2, the UAV cluster is assumed to begin with a circular geometry $\mathbf{r}$ given as follows in metres

$$\mathbf{r}^T = \begin{bmatrix} 3.2, & 1.1, & -1.9, & -3.4, & -2.4, & 0.4, & 2.9 \\ 1.2, & 3.2, & 2.9, & 0.3, & -2.7, & -3.4, & -1.8 \\ 0 & 0 & 0 & 0, & 0, & 0, & 0 \end{bmatrix}. \quad (5.33)$$

The proposed arrayed EKF algorithm also simultaneously tracks the sensor/UAV positions as shown in Fig. 5.6. The initial estimates of the array locations are set to the known circular geometry is shown in Fig. 5.6a at $t = 0T_s$. As dictated by the mobility model in Section 5.3.2.2, this geometry evolves to the perturbed geometry illustrated in Figs. 5.6b, 5.6c and 5.6d at time $t = 500T_s$, $t = 1000T_s$ and $t = 3000T_s$ respectively. The black markers indicates the positions predicted by the rotation alone in the mobility model without accounting for the noise. The blue markers indicates the estimated positions of the UAVs while the red markers indicate the true positions. It is clear that the proposed algorithm indeed tracks the array locations iteratively even though the deviations from the mobility model

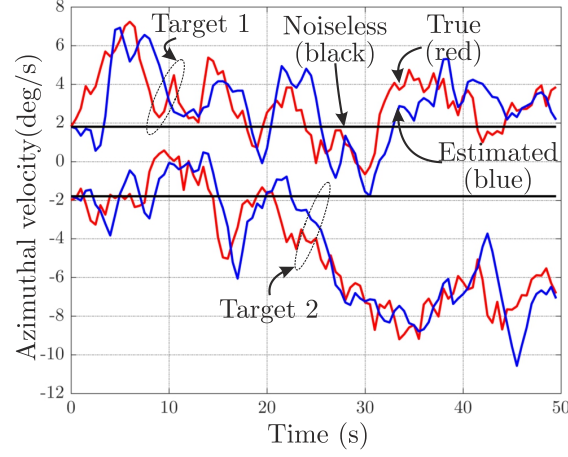


Figure 5.5: True (red) and tracked (blue) azimuth angular velocity trajectories of the two targets in the environment. The black line indicates the azimuthal velocity if no process noise were present in the mobility model.

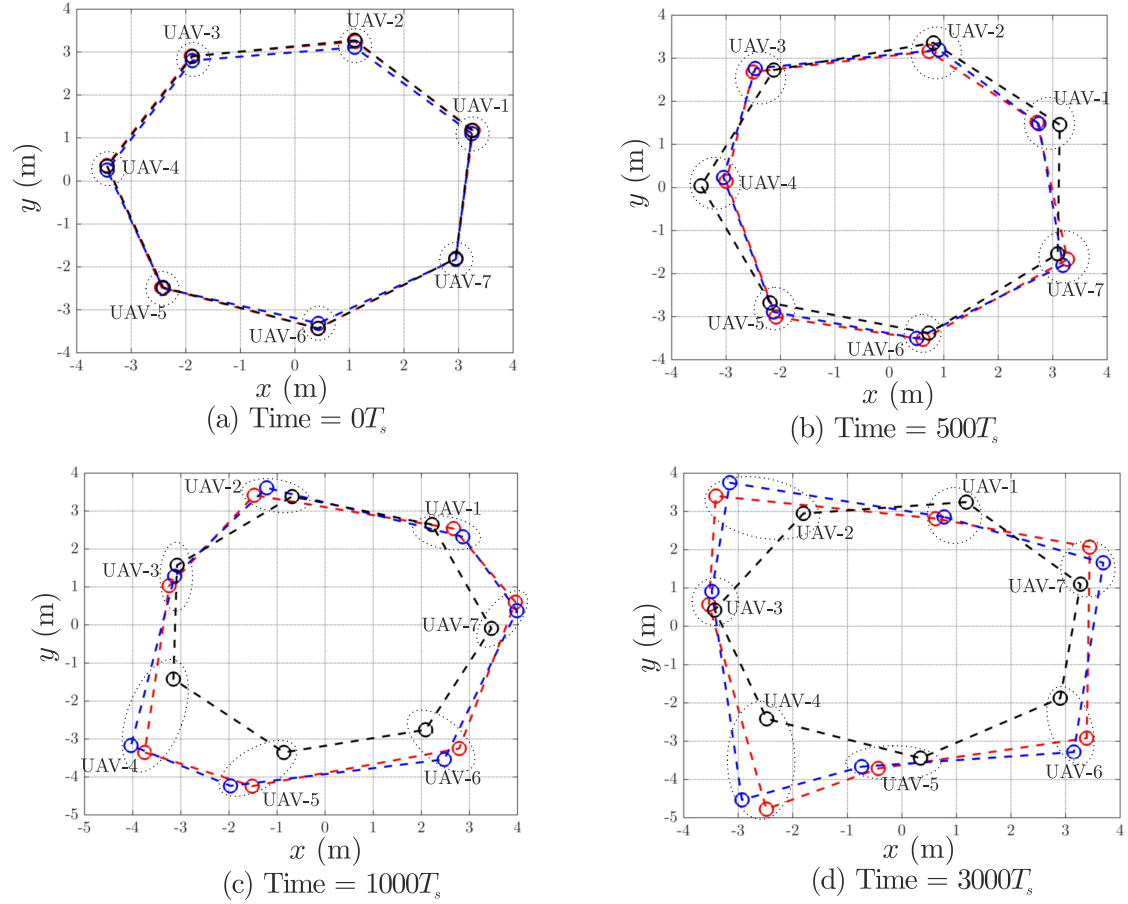


Figure 5.6: True (red), estimated (blue) and no-noise mobility model predictions (black) of the UAV cluster array's sensor locations at (a) $t = 0T_s$ (b) $t = 500T_s$ (c) $t = 1000T_s$ and (d) $t = 3000T_s$. The input SNR is set to 20 dB.

are quite high. This illustrates that the algorithm is robust to mitigating the errors in the mobility model arising from factors such as winds or controller imperfections. Table 5.2 demonstrates the difference in the mean square error (MSE) in predictions from the noiseless mobility model and the tracked state estimates provided by the proposed algorithm. The results are obtained from Monte-Carlo simulations carried out over 100 iterations. For instance, a 20 dB suppression ability in azimuth angle implies that an azimuthal root mean square error of 10 degrees from a prediction made by the mobility model with no knowledge of the state and environmental noise would be brought down to about 1 degree by the proposed algorithm.

Table 5.2: Difference in MSE of the predictions from the noiseless mobility model and the proposed algorithm

Parameter	MSE Difference
Azimuth	20.2 dB
Azimuthal velocity	4.0 dB
UAV sensor positions	5.2 dB

5.5.2 Towed Arrays

The proposed arrayed-EKF algorithm was also evaluated for the application of towed arrays consisting of $N = 11$ sensor elements. For the purpose of simulations, a simple scenario of the ship travelling linearly is considered. In such a scenario, conventionally the towed array is expected to remain linear. However, due to external factors such as water drag and turbulence, this may not be the case. This is modelled by the perturbations in the mobility model described in Section 5.3.2.1. Note that turns made by the towing ship may also be modelled by the driver term described in the model. The fixed intersensor spacing in the model is assumed to be 0.5 metres. Figures 5.7 and 5.8 illustrate the true and tracked azimuthal and azimuthal velocity trajectories of two moving targets. It is

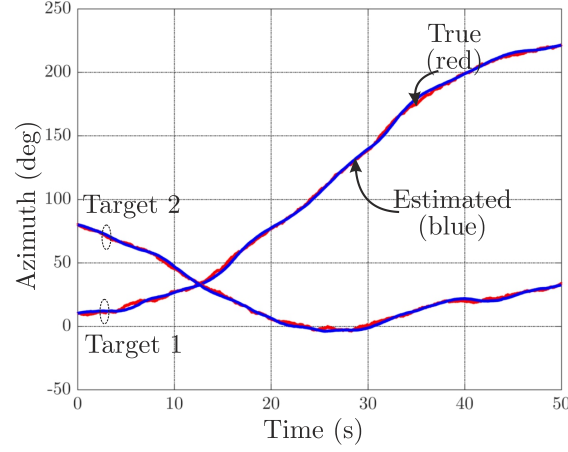


Figure 5.7: True (red) and tracked (blue) azimuth trajectories of the two targets in the environment of the towed array environment

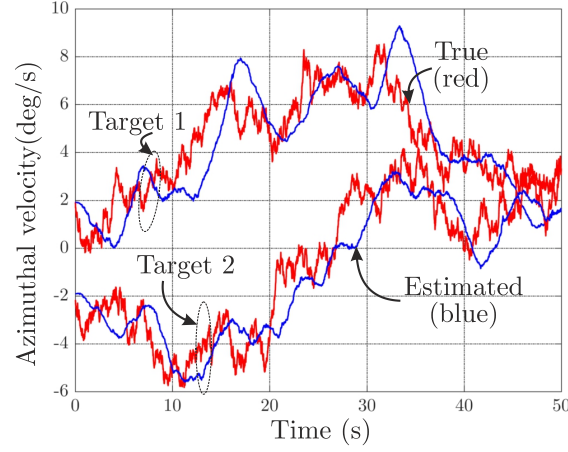


Figure 5.8: True (red) and tracked (blue) azimuthal velocity trajectories of the two targets in the environment of the towed array.

clear from these figures that the target parameters are indeed tracked successfully. The continuous time model intensity in the azimuth tracking model is chosen to be similar to that of the UAV cluster. Also, the intensities of the perturbations along the x and y axis, are set as follows $\sigma_{r_x}^2 = \sigma_{r_y}^2 = 0.3 \text{ (m/s)}^2$. The covariance matrix $\mathbb{P}_0$ is set to $\mu \mathbb{I}_{2(N+M)}$ where $\mu = 10^{-4}$ which indicates a reasonably high confidence in the accuracy of the initial estimates. As described in Section 5.3.2.1, the towed array is assumed to be with a uniform linear array geometry with halfwavelength spacing. The proposed arrayed-EKF algorithm also simultane-

ously tracks the towed array sensor positions. However, the performance tracking of the towed array sensor positions were found to be not on par with the performance of the UAV sensor positions, i.e. the filtering operation does not bring in substantial improvement over the sensor position estimates directly provided by the mobility model. However, the arrayed-EKF algorithm still substantially improves the azimuth and azimuth angular velocity tracking results. The sensor tracking performance may be improved by employing alternate filters such as the H^∞ -filter that are more suited for perturbations to the mobility models that are not circularly Gaussian (The perturbations are not circularly Gaussian due to the $\mathbb{L}$ matrix in the mobility model described in Section 5.3.2.1).

5.6 Summary

In this chapter, an algorithm to track far field fast-moving targets using a flexible array was presented. The proposed algorithm comprised of a fully parametric array channel model incorporating the flexible geometry of the array, mobility model of the flexible array and the mobility model of the targets in conjunction with an extended Kalman filter to simultaneously predict the unknown array locations and target parameters. Simulation studies to illustrate the performance of the proposed algorithm were presented for two flexible array applications namely UAV clusters and towed arrays. The results illustrated that the proposed algorithm successfully tracks the target parameters for both applications, whilst the array shape tracking performance was found to be inferior for towed arrays.

Chapter 6

Conclusions and Further Work

This thesis explored techniques to exploit and extend degrees of freedom in antenna arrays with time-invariant and time-variant array geometry, also known as “flexible” arrays. The first part of the thesis presented novel algorithms and receiver structures to extend the degrees of freedom in typical antenna arrays, with time-invariant array geometry, in the context of future wireless communication systems. This addressed some of the crucial challenges in 5G wireless communication systems such as high user density, high path loss and the need for increased interference suppression abilities. The second part of this thesis dealt with “flexible” array signal processing, i.e. signal processing on arrays with time-varying array geometry, for the applications of a cluster of UAVs/drones and towed arrays. An extended Kalman filter was proposed in conjunction with a novel arrayed-state space model that encompassed a flexible arrayed parametric channel model, a flexible array mobility model and a target mobility model.

In Chapter 2, the concept of the array manifold and the parametric channel model for a MIMO channel was presented. Equipped with this channel model and tools from differential geometry, the array manifold was utilised to study the impact of array geometry on the antenna array’s capabilities of channel detection, estimation and capacity. The study was carried out using a family of grid arrays

for a worst-case scenario of two users spatially very close to each other. A novel concept of “capacity loss” was presented that measures the degradation in the capacity offered by an antenna array due to users placed spatially close to each other. Analytical expressions that illustrate the relationship between the array geometry and array performance criterion such as detection and resolution thresholds and capacity loss were obtained. The results illustrated that the performance of the antenna array system may indeed be improved by an intelligent choice of geometry based on the task at hand. Moreover, the analytic expressions obtained could be utilised as cost functions to dynamically select and design antenna arrays based on the requirements of the application.

In Chapter 3, the spatial-only array system was extended to a spatiotemporal array system to extend the DoF without increasing the number of antennas. Based on the parametric channel model introduced in Chapter 2, a spatiotemporal receiver, comprising of an intelligent transmitter, a spatiotemporal multiparameter channel estimator and beamformer, was presented. The proposed spatiotemporal system was evaluated with varying levels of noise and interference, and the results illustrated that the proposed receiver that utilises a “small” array is capable of delivering an antenna array gain and interference mitigation performance equivalent to a “massive” array. The chapter also illustrated a representative example where the proposed receiver consisting of only 9 Rx antenna elements outperforms a spatial-only massive MIMO receiver consisting of 80 antenna elements. Moreover, this enabled the system to operate in the absence of power control to combat the infamous near-far problem. Thus, such a spatiotemporal beamformer would be an apt solution for future 5G mm-wave systems that are required to operate within power, cost and space constraints whilst employing highly intelligent digital beamforming solutions.

The spatiotemporal system presented in Chapter 3 still failed to exploit all spatial domains since vital information of the Tx array geometry was lost. Thus,

in Chapter 4, the spatiotemporal system was enhanced to a spatiotemporal-virtual array system that extends the degrees of freedom further, by an enhanced Tx design that enables the extraction of the information about the Tx array geometry at the receiver. Results illustrated that the proposed receiver has the ability to accommodate a higher number of signals in the environment, making it suitable for future wireless communication environments where massive user densities are forecasted. Moreover, the proposed beamformer eliminated the antenna array's performance on the Tx array geometry unlike earlier spatial-only and spatiotemporal systems.

In Chapter 5, a unique class of applications known as “flexible” arrays, i.e. arrays that possess time-varying array geometry were studied. Two specific applications namely UAV clusters and towed arrays were considered. However, such algorithms are also applicable to scenarios in wireless communication applications, such as sensor arrays that span across multiple mobile and static platforms, especially in the context of internet of things. Thus, towards such applications, a novel algorithm consisting of a fully parametric array channel model incorporating the flexible geometry of the array, mobility model of the flexible array and the mobility model of the targets, in conjunction with an extended Kalman filter, to simultaneously predict the unknown array locations and target parameters was presented. Simulation studies to evaluate the performance of the proposed algorithm were presented for the two flexible array applications under consideration. Results illustrated that the proposed algorithm successfully tracks the target parameters and flexible array sensor positions supporting high levels of target mobility, variation in array geometry and imperfections in the mobility models.

Thus, in conclusion, this thesis presented some novel algorithms and insights to increase the degrees of freedom in antenna arrays with time-invariant and time-variant array geometry. The proposed algorithms and system design solutions may be utilised across a wide variety of applications for future 5G wireless communi-

cations, IoT applications and vehicular cluster applications.

6.1 List of Contributions

The following is a list of novel contributions in this thesis

1. Design of a parametric channel model for a MIMO system that may be utilised for applications ranging from mm-wave communications to flexible array applications.
2. Defining and deriving a new metric for antenna arrays termed as “capacity loss” to measure the degradation in channel capacity as a function of the array geometry and the spatial distribution of users.
3. Investigating the impact of array geometry on the signal processing abilities of an array.
4. Developing a spatiotemporal receiver comprising of a novel transmitter, spatiotemporal multiparameter channel estimator and Rx weight design algorithm, to extend the degrees of freedom beyond the spatial domain.
5. Comparing and investigating the effects of varying noise and interference levels on the performance of the spatiotemporal receiver with other traditional state of the art receivers.
6. Developing a spatiotemporal-virtual receiver that extends the degrees of freedom further to incorporate the Rx and Tx spatial domains and the temporal domain.
7. Comparing and investigating the effects of increasing number of signals in the system and the impact of Tx array geometry on the performance of the spatiotemporal-virtual receiver against spatial-only and spatiotemporal receivers.
8. Developing a novel algorithm to perform target estimation and tracking for a new class of arrays called “flexible” arrays.

9. Evaluating the proposed flexible array signal processing solution for two applications namely, a cluster of UAVs and towed arrays.

6.2 Suggestions for Further Work

The research presented in this thesis can be the basis of future research and the following list presents some ideas for the same

- Chapter 2 illustrated the importance of array geometry by comparing a set of five array geometries with respect to different criteria, namely detection and estimation threshold and the capacity loss. This may be enhanced by designing an algorithm that utilises these thresholds to design a cost function that dynamically selects the array geometry, given a grid of sensors.
- The techniques presented in Chapters 2, 3 and 4, are evaluated only at the receiver, although they may be utilised at the transmitter as well. Thus, the proposed algorithms may be enhanced by an appropriate design of the Tx weight and utilising the algorithms at the Tx and Rx.
- Chapter 5 dealt with an algorithm to perform target estimation and tracking with flexible arrays. The array sensor tracking results indicate that the proposed algorithm may be enhanced with alternative filters such as the H^∞ filter that may perform better with correlated noise in the flexible array mobility model. Also, the algorithm may be further enhanced to tackle clutter and distinguish multipath signals.
- Chapter 5 explored channel estimation and tracking techniques with flexible arrays, albeit only in the spatial domain. To enable flexible array operation at lower SNRs and obtain asymptotically low state estimation errors, the proposed solution may be extended to a spatiotemporal arrayed-state space model.

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