

~~THE~~ FRACTURE MECHANICS

OF

CONCRETE

by

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A B S T R A C T

The work reported in this thesis represents an attempt to analyze the fracture processes of concrete in a rational and quantitative manner. This investigation was concerned primarily with the first "stage" of fracture, termed "fracture initiation".

Existing theories concerning the initiation of brittle fracture have had only limited success in the explanation of concrete fracture, primarily because its heterogeneous nature will produce a highly disturbed and complex system of local stresses or strains. To obtain information on the nature and magnitude of the local strains, an experimental program was undertaken which involved the testing of plate-models representing thin two-dimensional slices of concrete.

For these tests, the complex structure of concrete was simplified and represented as a two-phase system of large coarse aggregate particles embedded in a homogeneous mortar matrix. The local and average strains in the models were measured using both photo-elastic coating methods and electrical resistance strain gauges. The strain information obtained for models representing various volume fractions of aggregate was analyzed using an experimentally derived "fracture initiation envelope". This envelope defined the combinations of principal strains for which fracture would initiate within the mortar. Predictions were made concerning the position of the point from which fracture was likely to initiate, and the strains necessary to cause such initiation.

The fracture processes were observed experimentally using photo-elastic coatings, high speed photography and fluorescent dyeing techniques. Reasonable correlation was found between the predicted and the observed fracture events.

Tests of the three-dimensional concrete prototype were undertaken, and the results found to be in acceptable qualitative agreement with the results predicted from the plate-model tests. It was considered that the plate-model tests probably represented a "lower bound" solution for the strains at which fracture would initiate.

It was concluded that the analytical and experimental techniques used gave some rational insight to the fracture processes undergone by concrete.

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SIGN CONVENTION

The sign convention used throughout this thesis is as follows:

for principal stresses;

$$\sigma_1 > \sigma_2 > \sigma_3, \text{ tension positive.}$$

and for principal strains;

$$\epsilon_1 > \epsilon_2 > \epsilon_3, \text{ tension positive.}$$

CHAPTER I

Introduction

Over the years a large number of investigators have endeavoured to apply a variety of failure criteria to concrete. These criteria have ranged from the simple to the complex; they have been based upon theoretical calculations and upon empirical data, and have considered the microscopic and the macroscopic behaviour of the material. Several critical reviews of such criteria are available which discuss their applications and limitations¹⁻⁵. A detailed discussion of all possible criteria which may have application to the fracture of concrete is outside the scope of this Thesis. It is considered sufficient, in the context of this work, to note that any single criterion which has been proposed has had only limited validity or success in the explanation of the complete fracture process of concrete under all states of stress. This inability of any single criterion to adequately explain the complete fracture process experienced by concrete, has been ascribed to the experimental observation that the failure of concrete does not consist of any single mechanism, but rather is the result of a progressive series of mechanisms, the sum of these leading eventually to the complete disruption of the material⁵¹⁻⁵³.

That the failure of concrete does not consist of a single catastrophic event has been recognized for some years. Early work by Brandtzaey⁹² and Evans⁷ showed that cracking in concrete under load occurred prior to the ultimate failure of the material. More recent work, concerned with the monitoring of the acoustic emissions generated within a concrete specimen under load^{6-8,90,91}, has showed that a virtually continuous series of fracture events precedes the final failure of the concrete. Measurements of abrupt changes in the transmission time of an applied ultrasonic pulse^{1,9,10}; in the electrical resistivity¹¹, and in the volumetric

compressibility¹² of loaded samples have served to reinforce the hypothesis that concrete failure is composed of a progressive series of fracture events or mechanisms.

It has been considered⁵¹⁻⁵³, as a first approximation, that the overall failure of a concrete specimen may be broadly divided into three "zones or stages" as follows:

- 1) The local, perhaps microscopic, initiation of fracture at isolated points throughout the specimen.
- 2) The growth and inter-connection of these isolated fractures.
- 3) The final, macroscopic, disruption of the material.

If it is assumed, as seems reasonable, that the local initiation of fracture (Stage 1 above) always precedes macroscopic failure (Stage 3 above), then an important conclusion arises. This is that the material which eventually gives rise to the observed, macroscopic failure is a physically different material from that which first gave rise to the local initiation of fracture. In other words, by the time that macroscopic failure occurs, the material has already been severely disrupted internally and may no longer be considered, even roughly, as a continuous mass having certain predetermined material properties. It would appear, therefore, that to endeavour to explain the mechanism of final failure, without first explaining the mechanics of the fracture initiation process, would be a somewhat premature undertaking. Brandtzaeg⁹² noted, in 1927, that "The process of failure must necessarily be intimately connected with the properties of small parts of the material, in particular those small parts where failure first begins to develop. It would seem the individual rather than the average properties of the constituent parts of the material need to be considered".

It is of interest to note, in the context of local fracture, that observations of coarse aggregate particles removed from concrete specimens tested under uniaxial compression have shown a rather surprising feature^{1,5}.

Particles of different shapes and sizes, taken from concretes with very different mix parameters and exhibiting quite different macroscopic failures are shown in Figure I - 1. From this photograph it can be seen that the fracture surfaces of the adhering mortar all have a "cone-like" shape. It is considered that any phenomenon such as this, which is consistently and repeatably observed, should be capable of rational, quantitative, explanation.

A logical first step towards the rational description of the fracture mechanics of concrete was considered to be the determination of the position of the local points at which fracture was likely to initiate, and of the stage in the stress or strain history of the material at which such fractures would occur. Thus the work would be concerned primarily with the first stage of fracture previously mentioned.

It was apparent that the determination of information concerning the local initiation of fracture required the solution of two basic problems.

- 1) Dantu¹⁷ and others^{18,19} have demonstrated that the heterogeneous nature of concrete (shown in Figure II-1) will cause local strain concentrations upon the application of a load to the material. Thus, in order to analyze any particular point within the material, the magnitudes of the local strains or stresses at that point must first be determined.
- 2) A local "fracture initiation" criterion is necessary in order to determine whether or not any particular set of stresses or strains is likely to give rise to fracture and, if so, at what stage in the loading regime such fracture initiation is likely to occur.

The possible use of existing theories of brittle crack initiation and propagation are discussed in Chapter II. The development of an empirical fracture initiation criterion is outlined in Chapter III. In Chapter IV, the experimental determination of the local strain magnitudes in concrete under load is discussed. Succeeding Chapters deal with the tests that were



Aggregate Particles Removed from Concrete Specimens
Failed under Uniaxial Compression.

FIG. I-1

carried out, the analysis of the fracture initiation process, the predictions of behaviour based upon such analysis and the correlation between predicted and observed behaviour. Finally, in Chapter IX, the major conclusions derived from the results are summarized, and some comments on the influence of these conclusions upon previous work are given.

CHAPTER II

THE THEORETICAL ANALYSIS OF CRACK INITIATION AND PROPAGATION IN BRITTLE MATERIALS.

Section II - 1

Introduction

It is the object of this Chapter to determine whether or not any of the existing theories of brittle crack initiation and propagation can sensibly be used in the analysis of concrete under compressive and tensile stress states. A brief history of the development of the relevant theories is given and the shortcomings or advantages of each theory discussed with reference to the problem under consideration.

Section II - 2

The Griffith Theory of Tensile Fracture

In 1920, A.A.Griffith²⁰ submitted a paper to the Royal Society in which he sought to explain the vast discrepancy between the observed tensile strengths of brittle materials and their theoretical strengths, the latter being based upon consideration of the molecular cohesive strength of the material. Griffith postulated the existence of small cracks or flaws in the material which acted as local stress-intensifiers. Using the (elastic) solution of Inglis²¹ he calculated that the maximum tensile stress, for a material stressed in uniaxial tension, would occur at the tip of such a crack, approximated as an ellipse, lying perpendicular to the direction of applied stress, and would be of magnitude:

$$(\sigma_1)_{\max.} = 2(\sigma_1)a \left(\frac{c}{r}\right)^{\frac{1}{2}} \dots\dots\dots (1)$$

where:

$(\sigma_1)a$ = remotely applied uniaxial tensile stress.

c = length of semi-major axis of crack.

r = radius of curvature of crack tip

As there existed no facilities for directly measuring the value of "r", and in order to determine the value of $(\sigma_1)a$ at which failure or propagation of the crack would occur, Griffith used a method of energy balance.

The internal surfaces of a material are associated with a certain energy, this energy being in the form of free surface energy. The amount of such energy contained in a system of unit thickness having in its body a single crack of length $2c$ may be given by:

$$U = 4 c \gamma \dots\dots\dots (2)$$

where: $2c$ = length of crack

γ = specific surface energy of the material. For an incremental increase in crack length, dc , the increase in surface energy stored in the material is given by:

$$\frac{d}{dc} (U) = 4\gamma \dots\dots\dots (2a)$$

A specimen containing a crack of length $2c$ has a certain excess strain energy. That is, it contains more elastic energy than it contained prior to being cracked due to the displacements induced around the crack. It may be shown from consideration of the stress field around such a crack that this excess elastic energy is given, per unit thickness for the case of plane stress, by:

$$We = \pi c^2 (\sigma_1)^2 \dots\dots\dots (3)$$

where: $2c$ = length of crack

$(\sigma_1)a$ = remotely applied (tension) stress
 E = Young's Modulus

For an incremental increase in crack length, dc , the gain in excess strain energy is given by:

$$\frac{d}{dc} (We) = \frac{2\pi c (\sigma_1)^2}{E} \dots\dots\dots (3a)$$

Thus, the total gain in energy of the system, due to an incremental increase in crack length, may be written:

$$\text{Energy gain} = \frac{d}{dc} (U) + \frac{d}{dc} (We) \dots\dots\dots (4)$$

It is worthy of note that this equation has been extended by several investigations²²⁻²⁷ to include terms for plastic flow and kinetic energy. However, in the context of brittle fracture for an elastic material, such terms are of minor importance.

The energy lost by the system when a crack propagates is equal to the work done by the externally applied (static) load. Denoting this loss in energy per increment of crack extension by $\frac{d}{dc} (W)$, we have for equilibrium:

$$\frac{d}{dc} (W) = \frac{d}{dc} (U) + \frac{d}{dc} (We) \dots\dots\dots (5)$$

It has been shown²⁸ that the work done by the external forces as a crack extends is equal to twice the gain in excess elastic strain energy.

$$\text{Thus: } 2 \frac{d}{dc} (We) = \frac{d}{dc} (U) + \frac{d}{dc} (We) \dots\dots\dots (5a)$$

Until the above equation is at least satisfied the crack cannot extend, as to do so would tend the system to a higher level of potential energy - a violation of the second law of thermodynamics. However, once this energy balance is at least satisfied, then the system tends to have a lower level of potential energy, and the crack becomes self-propagating. From equations (2a), (3a) and (5a), for this condition:

$$\frac{2\pi c (\sigma_1)_a^2}{E} = 4\gamma \dots\dots\dots (6)$$

and:

$$(\sigma_1)_a = \left(\frac{2 \gamma E}{\pi c} \right)^{\frac{1}{2}} \dots\dots\dots (6a)$$

Thus, for a condition of plane stress, the criterion for which fracture will initiate and propagate to failure, is that the remotely applied tension stress, $(\sigma_1)_a$, has the value given by equation (6a).

This equation may be rewritten:

$$(\sigma_1)_a = 0.795 \left(\frac{\gamma E}{c} \right)^{\frac{1}{2}} \dots\dots\dots (6b)$$

If the preceding analysis were carried out for the case of plane strain, then the criterion becomes:

$$(\sigma_1)_a = \left[\frac{2 \gamma E}{\pi c(1-\nu^2)} \right]^{\frac{1}{2}} \dots\dots\dots (6c)$$

where : ν = Poisson's Ratio.

For a value of $\nu = -0.15$, an average value for normal concrete¹⁵, this equation becomes:

$$(\sigma_1)_a = 0.306 \left(\frac{\gamma E}{c} \right)^{\frac{1}{2}} \dots\dots\dots (6d)$$

which may be compared to equation (6b).

Returning to equation (1), it will be recalled that the maximum stress developed around an elliptical crack in a homogeneous, elastic and isotropic medium subjected to a remotely applied tension stress was given by:

$$(\sigma_1)_{\max.} = 2(\sigma_1)a \left(\frac{c}{r}\right)^{\frac{1}{2}}$$

Orowan²³ has suggested that the molecular cohesive strength of a material may be approximated by the equation:

$$\sigma_o = \left(\frac{2 \gamma E}{d}\right)^{\frac{1}{2}} \dots\dots\dots (7)$$

where: γ = specific surface energy

E = Young's Modulus

d = spacing of neighbouring atomic planes

If $(\sigma_1)_{\max.}$ becomes equal to σ_o , then it is reasonable to expect fracture to occur. As a limiting case it may be assumed that the radius of the crack tip " r " approaches the value of the atomic plane spacing " d ". Under these circumstances, from equations (1) and (7), the criterion for failure in tension is that the remotely applied stress reaches the value:

$$(\sigma_1)a = \left(\frac{\gamma E}{2c}\right)^{\frac{1}{2}} = 0.707 \left(\frac{\gamma E}{c}\right)^{\frac{1}{2}} \dots\dots\dots (8)$$

This equation may be compared with equation (6b), and is seen to give a lower value of remotely applied stress necessary to cause failure. This is primarily due to the assumption that the crack tip radius is equal to the atomic plane spacing.

Sack²⁹, in 1946, extended the analysis of internal flows to include the three-dimensional case of a very flat oblate ellipsoid or "penny-shaped" crack existing under uniaxial tension. His analysis, based on Neuber's solution of the equations of elastic equilibrium in oblate spheroidal co-ordinates is laborious and complex, and a similar analysis carried out by Sneddon³⁰ in cylindrical polar co-ordinates is recommended as being considerably easier to follow. The main points of interest of this analysis are given as follows:

- i) All isochromatics (lines of equal principal shear stress) pass through the end of the crack. Infinite shear stress, therefore, exists at these points, which means that any purely elastic solution must be approximate.
- ii) Superimposition of an intermediate tensile principal stress parallel to the plane of the crack does not materially alter the stress distribution in the vicinity of the crack. It should, however, be noted that superimposing such a stress will alter the statistical probability of failure occurring as there will then exist two planes in which a critical flaw may be found. Imposition of an intermediate principal stress will also alter the behaviour of a propagating crack when such a crack meets a plane of weakness lying perpendicular to its path³¹. This is of importance in heterophase systems such as concrete.
- iii) It was noticed that, very locally, the stress conditions could not be distinguished from a case of plane strain.

The magnitude of the major principal stress necessary to cause initiation of failure was given by:

$$(\sigma_1)_a = \left[\frac{\pi \gamma E}{2c(1-\nu)^2} \right]^{\frac{1}{2}}$$

or, for concrete with a Poisson's Ratio = 0.15:

$$(\sigma_1)_a = 1.28 \left(\frac{\gamma E}{c} \right)^{\frac{1}{2}} \dots\dots\dots (9)$$

Note that from (iii) above, this equation is valid for remotely applied conditions of either plane stress, or plane strain.

It is interesting, though not altogether surprising, that this equation differs only by a numerical factor from equations (6b), (6d) and (8), and denotes for the case of uniaxial tension a "stronger" material. This is due to the additional restraint placed on the crack by the three-dimensional existence of the surrounding material.

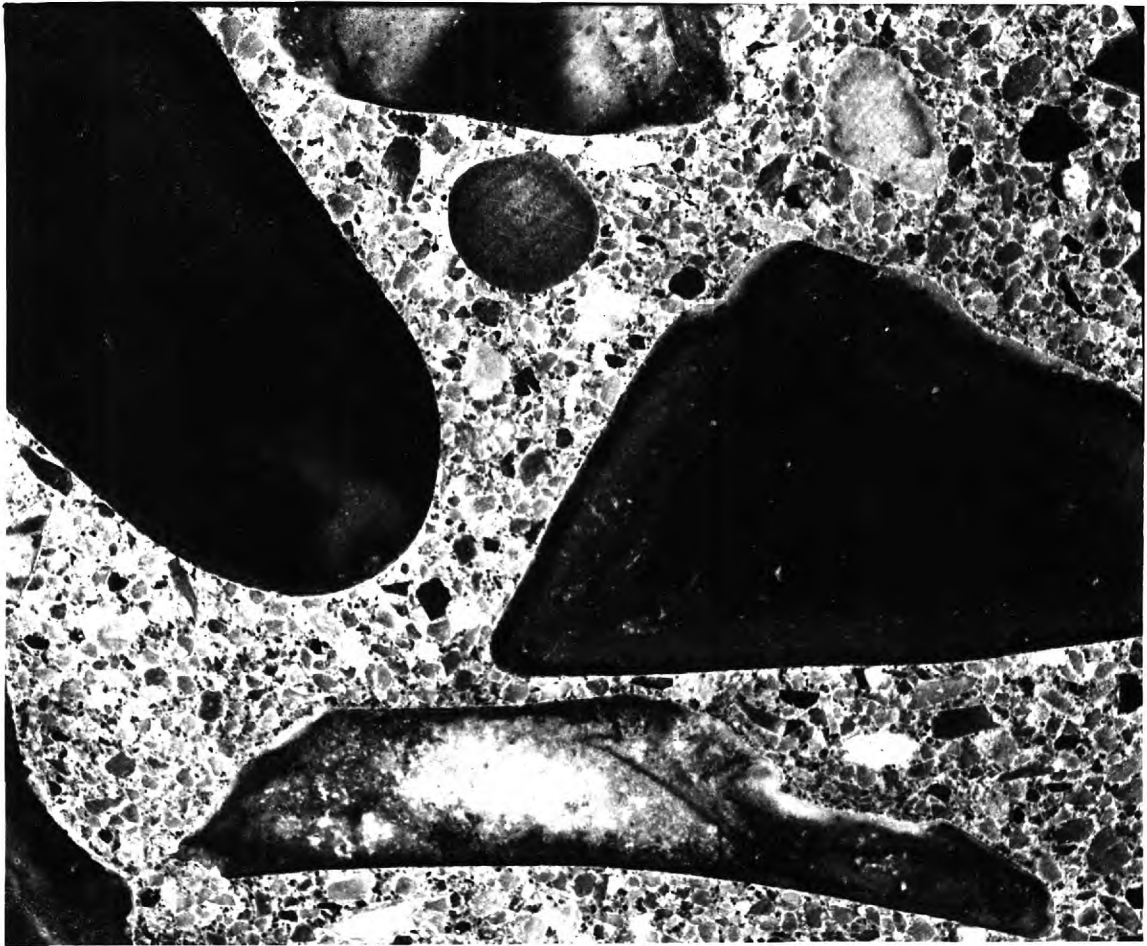
Section II - 2.1Discussion of Application to Concrete

All the equations derived in this section give, for various boundary conditions, a criterion of failure for a homogeneous, elastic and isotropic material, containing an initial crack or flaw, and existing under a state of uniaxial tension induced by a remotely applied force. The Griffith theory was an important step in the field of fracture mechanics, perhaps the most important, but for the problem under consideration here, it has two very severe drawbacks.

- i) The equations have all been developed for materials existing under a state of tension. Further developments, by Griffith and others, to include the case of compression have been undertaken and are reviewed in Section II - 4.
- ii) The heterogeneous nature of concrete is shown by the three photographs, taken at increasing magnifications, in Figures II - 1(a), (b) and (c). This heterogeneity means that the stress state within a concrete specimen is heavily disturbed. The work of Dantu¹⁷, Hawkes¹⁸ and Stephen and Pirtz¹⁹ has shown this most clearly. Thus, a crack must pass through zones of quite different stresses. The complex nature of concrete also provides crack arresting mechanisms, in the form of aggregate particles and air bubbles³¹, and contains zones having widely different surface energy characteristics. Thus even under a state of pure tension, it is unlikely that crack initiation and crack propagation will be coincident.

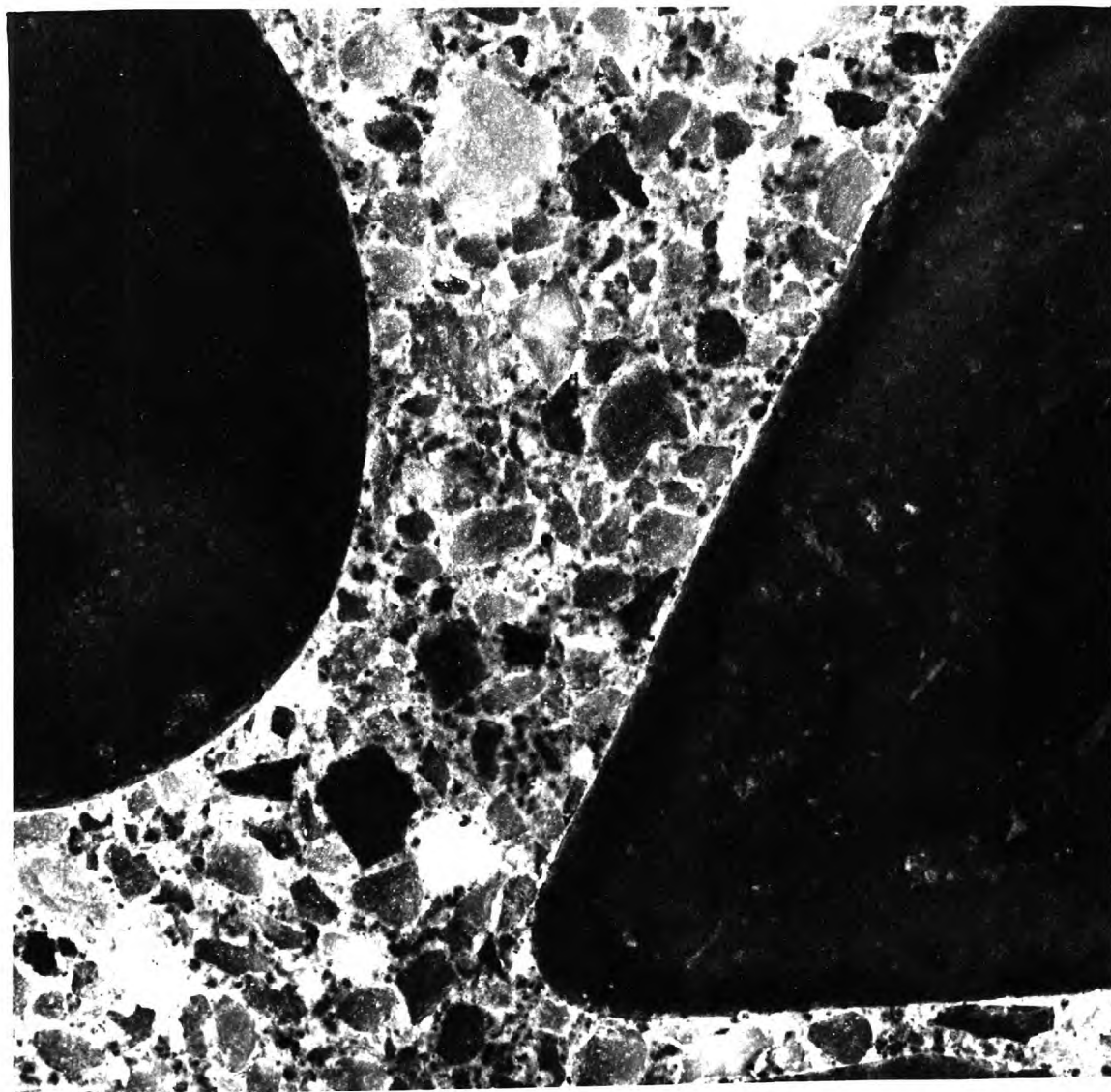
Section II - 3The Concept of the Critical Strain Energy
Release Rate

Discussion of this concept is included here as there have been attempts to apply it to concrete. In fact, the concept springs directly from the original Griffith theory - it is merely a different way of viewing the same approach.



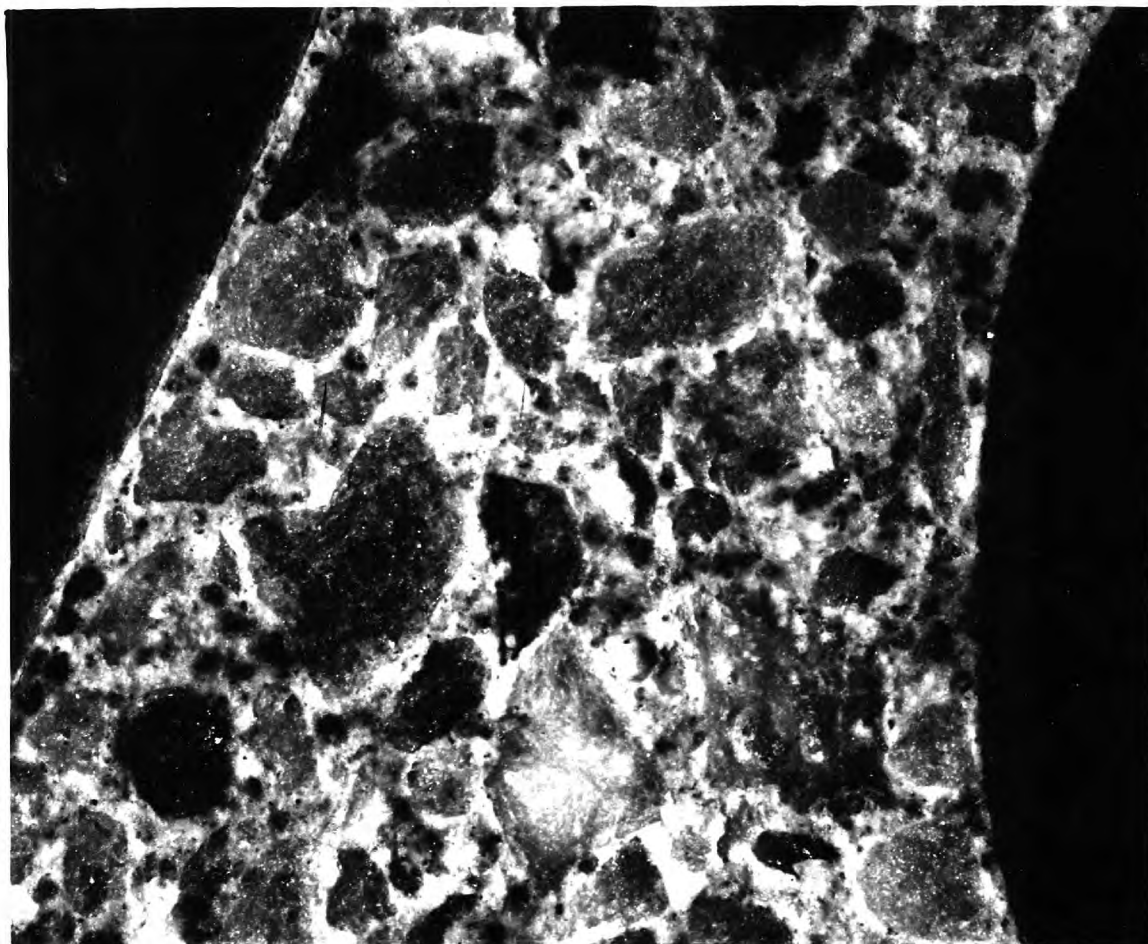
General View of Structure of Concrete

FIG.II - 1(a)



Magnified View of Structure of Concrete from
FIG.II - 1(a).

FIG.II - 1(b)



Magnified View of Structure of Concrete from
FIG.II - 1(b).

FIG.II - 1(c)

Equation (6) is reproduced here for reference:

$$\frac{2 \pi c(\sigma_1) a^2}{E} = 4\gamma$$

It would appear that the right-hand side of the above equation, which denotes the increase in surface energy of a material for an incremental increase in crack length, is a material constant, as γ is itself a material constant. Thus the left-hand side must also be a constant and is referred to as the "critical strain energy release rate", G_c . Thus:

Critical strain energy release rate - G_c - constant, and for the particular case of a material under uniaxial tension,

$$G_c = \frac{2 \pi c(\sigma_1) a^2}{E} \dots\dots\dots (10)$$

or, for the case of plane strain:

$$G_c = \frac{2 \pi c(\sigma_1) a^2}{E} (1 - \gamma^2) \dots\dots\dots (11)$$

As, ideally, the value of G_c remains constant whatever the stress-state, then if the value of G_c can be measured under one simple state of stress, and if expressions can be derived relating G_c to any arbitrary stress-state, then it should be possible to predict the magnitudes of the loads necessary to cause failure for any geometry of loading. Unfortunately, severe practical and theoretical difficulties arise, both from the measurement of a value for G_c and from the attempted application of this value to general states of stress.

Section II - 3.1

Discussion of Application to Concrete

General Comments

Values of G_c are probably most easily measured experimentally by testing specimens under uniaxial tension. Measurement of the crack length, $2c$, and the applied stress, $(\sigma_1)a$, at the onset of fracture, and knowledge of Young's Modulus (and Poisson's Ratio if the plane strain case is required) allows calculation of G_c from equations (10) and (11). In a heterophase material such as concrete, several problems immediately arise.

- i) The value of the stress field within which the crack exists must be known. In a homogeneous material this value may be taken as the remotely applied load divided by the cross-sectional area of the specimen. In a heterogeneous material stress concentrations arise due to the existence of phases of different mechanical properties¹⁷⁻¹⁹. Due to these concentrations it is difficult to assess the magnitude of the stress field exerted upon a particular crack or flaw. Thus the problem is one of defining the correct value of $(\sigma_1)a$ in the preceding equations.
- ii) As a crack advances through a heterophase material, its path will, in general, lie in zones of differing elastic and energy properties. That is, the value of G_c for a crack advancing along an interface between an aggregate particle and the surrounding mortar or paste, and the value appropriate to a crack passing through the paste or mortar, will be different³². In short, a heterophase material may be considered as consisting of several different materials, each with a particular G_c value, mixed together. Thus the overall or average value of G_c may only be considered a material constant as long as the proportions of the crack path lying in each phase do not change.
- iii) Glucklich³³ has suggested that where equations (10) and (11) are complicated for ductile materials by consideration of the plastic flow occurring near the tip of a crack, an analogous complication exists for the case of brittle materials due to a zone of "advance microcracking" which spreads out in a fan-like shape from the tip of a propagating crack. Thus the problem may be considered as one of definition of the true crack length or, more precisely, the total area of crack face. Glucklich felt that the total length of such advance micro-cracking might be far greater than the apparent length of the visible parent crack. He further felt that it was possible that the rate of increase of this advance zone of cracking would be non-linear, increasing more rapidly as the length of the parent

crack grew.

Previous Work on Measurement of G_c for Concrete

Two methods have been used, to the writer's knowledge, in an effort to determine a value of G_c for particular concretes. Both are subject to the above general comments. More detailed comments are given below.

Kaplan's experiments³⁴ were conducted on a notched beam. The depth of the notch was considered to be the crack length and the magnitude of the stress at the root of the notch, calculated from elastic beam formulae, to be the remotely applied tension stress. Equation (11) was then used to calculate a value of G_c . Kaplan points out that the actual crack length may have been greater than the notch depth, and that his results should, therefore, represent a lower limit for the value of G_c . A further complication arises from the fact that the beam notch existed in a stress gradient, in fact quite a steep one due to the small dimensions of the beams used. If cracking extended from the tip of the notch prior to failure, as seems probable, then the crack tip existed in a zone of stress much greater than that calculated by Kaplan. As the stress appears as a squared term in the equation for G_c , errors in the calculated value of G_c due to even small errors in the magnitude of stress used would be fairly considerable. This effect should again tend to make Kaplan's values a lower limit and may well have over-ridden the effect of using an incorrect crack length value. One might also criticize Kaplan's use of the elastic beam formulae, which can only be considered approximate, to determine the value of the stress to be used in equation (11). This equation was derived for the case of tension applied at infinity, existing uniformly over the length of the crack, and everywhere perpendicular to the major axis of the crack. Kaplan's results varied from 0.08 in. lb/in² to 0.12 in. lb/in².

The method used by Romualdi and Batson³⁵ was that of a "cut-plate" specimen. Cuts of lengths from two to twelve inches were introduced into plate specimens, which were then tested in uniaxial tension. The length of the cut was assumed to be the length of initial flaw, and the load at failure

divided by the original cross-sectional area to be the remotely applied stress. Equation (11) was then used to calculate G_c . Romualdi criticizes his own work in that no correction was made for possible crack extension prior to failure. He also states that the value of G_c was expected to increase with increasing cut length and that this was observed to be the case. The writer can think of no justification for such an expectation since the value of G_c should theoretically remain a constant. For very large cuts, such as those of 12 ins. length in a 24 ins. wide plate, the problem of defining $(\sigma_1)_a$ was not considered, but would appear to be of some importance. It is interesting to note that Romualdi's results gave values of G_c from 0.03 to 0.07 in - lbs/in² - considerably lower than the results of Kaplan which should have represented a lower bound. Direct comparison of results is, however, difficult as the materials tested appeared to be widely different.

It would seem, from these two attempts, that any sort of measurement of a value of the critical strain energy release rate is extremely difficult. The results of the tests mentioned are suspicious, as neither test simulates very closely those conditions for which the equations used were derived.

Effects of Non-homogeneity of Concrete and of Generalised

States of Stress

Even if the problems of determining accurately a value for G_c were overcome, could such a value then be assumed valid for concrete under all states of stress and hence used to predict failure? Some of the difficulties involved are given below.

- 1) As in the case of the original Griffith theory, the basic equations for the critical strain energy release rate were derived from consideration of a homogeneous, elastic material under tension. In theory the concept should be valid for any state of stress, providing a suitable equation linking G_c and (σ) applied can be devised. This rather presupposes that failure in compression is merely failure in tension with the sign changed. There has been

some evidence brought forward to support such a hypothesis^{32,36} pointing out that vertical splitting under uniaxial compression is similar to cleavage cracking in tension. However, McClintock and Walsh³⁷, Hoek³⁸, Walsh³⁹, Brace⁴⁰ and Jaeger⁴⁴ all present evidence to show that there is a fundamental difference between the fracture mechanisms operating under a tensile state of stress and those in operation under a compressive field. In this context it is important to make a clear distinction between fracture initiation and the propagation of a crack, or cracks, to failure.

In simple tension these two stages are coincident (apart from material heterogeneity effects) and need no distinction. In other words, the development of a local tensile stress sufficient to initiate fracture means that total failure will then occur, as the propagating crack becomes an increasingly "worse" case. Under other states of stress, it has been shown^{38,41,42} that the initiation of fracture may simply change the crack geometry and local stresses to produce a more stable crack than previously existed. In order to further propagate this new crack system another factor must be introduced, such as an increase in the remotely applied stresses. Thus, under states of stress such as these, a clear distinction between the initiation of fracture and the propagation of cracks to failure must be observed. This point is discussed further in Section II - 4.

The concept of the critical strain energy release rate was developed for the case under uniaxial tension, where the initiation of fracture occurs simultaneously with the onset of propagation to failure. The use of this concept for any case where these two stages are not simultaneous would, therefore, leave ground for considerable doubt.

- ii) As previously mentioned, different phases of a heterogeneous material will have different values of specific surface energy³². It appears that the proportions of the final failure path lying in each phase of

concrete may be stress dependent. High degrees of triaxial compression may force failure through aggregate particles¹², whereas uniaxial tension will not. Thus the value of G_c must be said to be stress dependent. This means that a unique value of G_c cannot be measured from one stress state and applied to any general stress state.

iii) Equations (10) and (11) will be complicated by additional energy terms for cracks that open or move other than in pure cleavage. For the case of an inclined crack in a compressive field shearing forces may cause differential movement of the crack faces, leading to the dissipation of energy through friction^{37,38}. Energy dissipation of this nature was not included in the original derivation of G_c . The value of such a term will depend upon the frictional characteristics at the faces of a microscopic crack. Such characteristics would not easily be determined.

Section II - 4 The Development of Critical Tensile Stresses under Combined States of Stress

In 1924 Griffith extended his original work on fracture to include the case of a crack existing in a material subjected to a general triaxial stress state⁴³. He postulated that fracture would occur when the maximum tensile stress generated at the tip of the most critically oriented crack exceeded the molecular cohesive strength of the material. The analysis was carried out in two dimensions, in the plane defined by the major and minor principal stresses. The basic assumption here is that the intermediate principal stress has no marked effect. Neglecting terms of secondary importance, a brief summary of his results, as simplified by Hoek³⁸, is given below:

For $\sigma_1 > \sigma_2 > \sigma_3$, tension positive.

Critical crack orientation:

$$\psi_c = \frac{1}{2} \cos^{-1} \left[\frac{\sigma_3 - \sigma_1}{2(\sigma_1 + \sigma_3)} \right] \dots\dots\dots (12)$$

For fracture initiation:

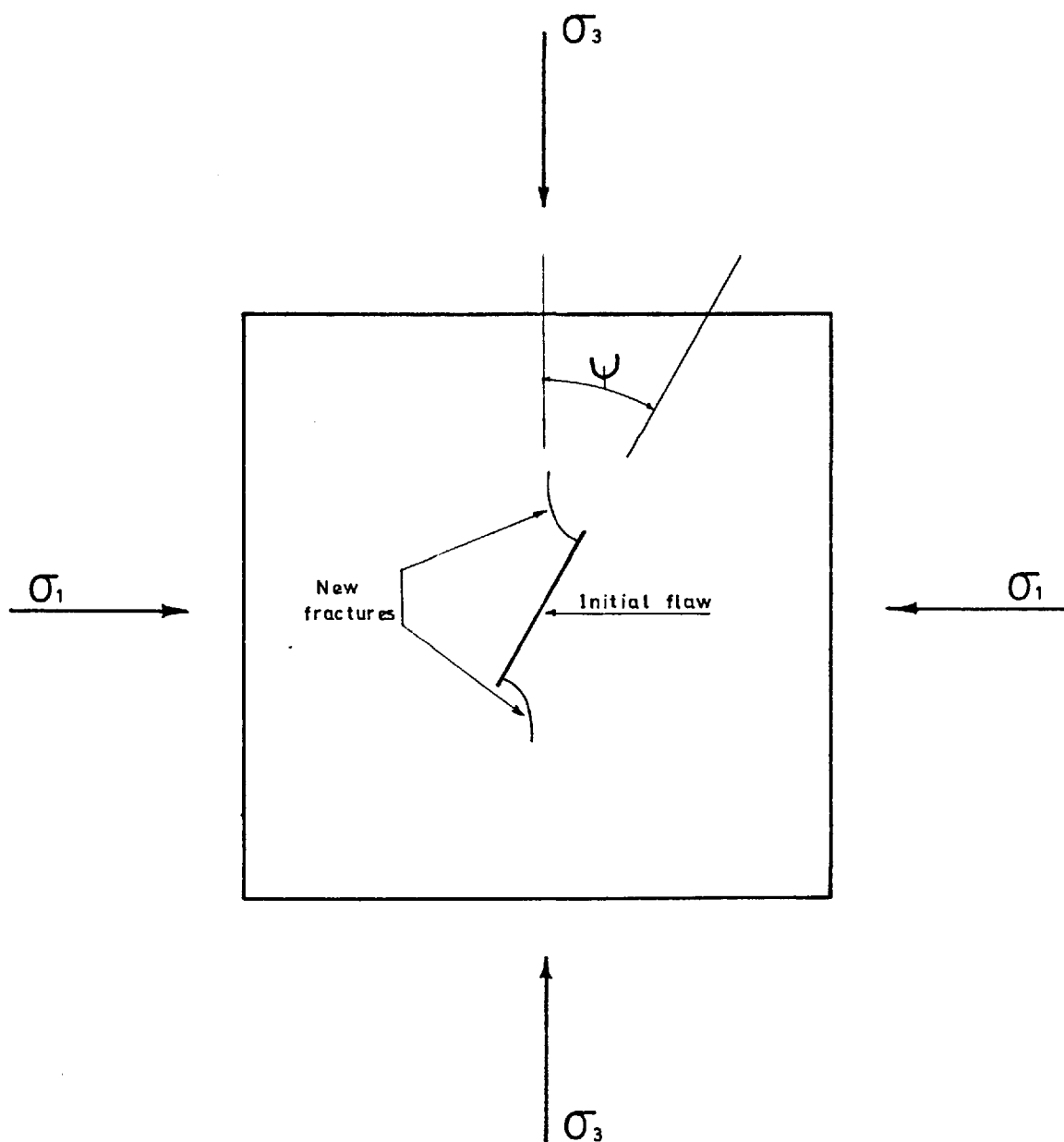
$$\frac{(\sigma_3 - \sigma_1)^2}{(\sigma_1 + \sigma_3)} = -8\sigma_t \quad \dots\dots\dots (13)$$

where σ_t = the uniaxial tensile strength of the material. It will be noticed that the above equations are only meaningful if the value of σ_1 , the most tensile applied stress, is less than $-1/3(\sigma_3)$. For values of (σ_1) greater than this the original Griffith theory of tensile fracture is applicable.

For some years it was assumed that once the above conditions were satisfied complete failure of the material occurred. However, recent work^{38,41,42} has shown that whereas in tension the attainment of a local tensile stress large enough to initiate fracture is a sufficient condition to generate total failure, in compression this is not the case.

Work by Hoek³⁸ on glass models containing critically oriented cracks has shown that once fracture initiates from the boundary of such cracks they take up a new configuration as shown in Fig. II-2, this configuration being extremely stable due to the redistribution of stresses. Either additional loading or the presence of other cracks and flaws is then required in order to generate total failure. The hypothesis that crack initiation and propagation are not in general coincident in compression, is given further weight by the experimental evidence of Brace^{41,42} and McClintock and Walsh³⁷.

Griffith's original hypothesis of tensile fracture defined both crack initiation and crack propagation, these being coincident. The extension of his work to cover compression only defined fracture initiation and does not necessarily have any validity in the context of total failure. Thus it is a misconception to state that the Griffith theory predicts an overall strength, which is governed by the propagation of cracks to failure, in uniaxial compression ($\sigma_1 = 0$ in equation (13)) of eight times the uniaxial tensile strength. It would also appear to be a common misconception that the Griffith theory predicts infinite strength under equal biaxial compression. Griffith's extension was based on the assumption that the intermediate principal stress had no effect. Thus his equations predict no increase in

Stable Crack ConfigurationFollowing the Initiation of Fracture(after Hoek³⁸)FIGURE : II-2

the stress at which fracture initiates for biaxial compression, and an infinite strength in equal triaxial compression.

Modifications to the Griffith extension taking into account the friction acting along the faces of a closed (inclined) crack were first suggested by McClintock and Walsh³⁷. Further extension of this work was carried out by Hoek³⁸ in 1965. Hoek derived a spectrum of fracture initiation equations, each equation having application within a certain range of remotely applied stress ratios and for a certain range of coefficients of internal friction, this friction being assumed to act at the faces of closed cracks. Obviously, the ratio of the principal stresses remotely applied will partially determine whether or not the existing cracks are open, closed, or in transition, and may also influence the actual value of the coefficient of friction. For a complete summary of these equations and the conditions under which they are valid, the reader is referred to Hoek's work³⁸.

Section II - 4.1

Discussion of Application to Concrete

Recent work, such as that of Hoek, represents a major step in the qualitative understanding of the fracture processes in brittle materials under compression. The practical difficulties of applying this work to a quantitative analysis of fracture initiation in concrete are still, unfortunately, considerable.

In order to carry out such an analysis, and as the stresses in a heterophase material will vary from point to point, a knowledge of the local stress conditions must be obtained. This requirement has already been mentioned in Chapter I and is a basic prerequisite for any type of analysis. Once such information is furnished, each point may then be analyzed, using the appropriate equation, to determine the orientation of the most critical flaw and the value of the stresses for which initiation will occur. However, in order to determine which equation is appropriate to the particular point under consideration it must be determined whether or not the flaw

is closed, or closing, and, if so, what is the value of the interfacial frictional coefficient. Determination of this value would appear to be somewhat difficult. Comparison of the results of the analysis at each point in question should allow determination of that point from which fracture would first initiate. Such an analysis would be concerned only with the first "zone" of fracture mentioned in Chapter I. That is, with the initiation of fracture. Once this first fracture initiates, the specimen must then be completely re-analyzed in order to determine the sequence of subsequent fractures as, for each initiation, the stress field will be markedly re-distributed^{38,42}.

An analytical process such as this, though undoubtedly tedious, would appear at least to have a reasonable chance of giving some meaningful results, provided that the material properties, notably the value of the interfacial coefficient of friction, could be accurately determined.

Section II - 5

Conclusions

The original Griffith theory, invaluable for its philosophy, is of limited practical use for the quantitative analysis of concrete. The same may be said of the concept of critical strain energy release rate. The application of these theories becomes more difficult and suspect when generalized stress states are considered.

The concept of critical tensile stress development, with all its recent modifications and additions is exciting in its prospects but, at the moment, rather fearsome in its difficulties of application. Perhaps the greatest benefit of this recent work lies in its contribution to the qualitative rationalization of the complex processes of fractures.

A further obstacle, of not inconsiderable proportions, lies in the path of the quantitative application of the theoretical work described in this Chapter. All this work has been developed in terms of stress. In order to determine the local stresses which exist in concrete it is necessary

to take experimental measurements, such measurements depending solely upon the strains which exist at the point under consideration. Whereas the conversion from strain to stress is simple and valid for a perfectly elastic material, it becomes considerably less so for a non-elastic material. For such a material, both Young's Modulus and Poisson's Ratio will be dependent upon the stress state and stress level^{1, 16}. In order to determine the values of these "constants" applicable at any point the principal stresses at that point must be known, and in order to determine the values of the principal stresses the values of Young's Modulus and Poisson's Ratio must be known. The problem is only soluble by virtue of an assumption of the values of these parameters, likely to be in error, or by a lengthy iterative process, tedious in the extreme and requiring comprehensive experimental data.

In short, it appears that recent developments in the theoretical analysis of crack initiation in brittle materials will swiftly make possible the quantitative application of such an analysis to the problem of concrete under compression, if the practical and experimental difficulties can be overcome.

CHAPTER III

THE FRACTURE INITIATION ENVELOPE FOR MORTAR

Section III - 1

Introduction

The difficulties of applying, to the problem of concrete under compression, any of the theoretical analyses discussed in the previous Chapter, lead one to a consideration of the manner in which experimentally derived data could be used. In general terms, the problem is one of determining experimentally whether or not any set of principal stresses or strains existing within, or imposed upon a material will result in the initiation of cracking.

Section III - 2

Definition of Problem

The heterogeneous nature of concrete may be approximated roughly as a two-phase mixture. The phases in this model consist of hard aggregate particles bonded together by an interconnected mortar matrix of hardened cement paste, fine sand, air and water. Such a model, in which the aggregate particles and mortar are considered to be homogeneous, has already been extensively used for quantitative explanations of the mechanical behaviour of concrete^{3,45-50}. As it appears unlikely, for the normal range of concretes being considered here, that fracture initiates within the hard particles of coarse aggregate, there are only two zones in which fracture may initiate. These two zones are the mortar matrix and the mortar-aggregate interface⁵¹⁻⁵⁴.

Experiments can be carried out on specimens of mortar, considered as homogeneous in the above model, under a variety of stress or strain regimes^{1,4,5,12}. Application of a particular type and magnitude of loading to a specimen of mortar will generate a certain strain response. The principal strains of the specimen may then be measured using electrical resistance strain gauge or other techniques⁵⁸. Use of one of the methods

outlined in Section III - 3, should then allow determination of the stage at which this particular strain response gives rise to the initiation of fracture within the mortar specimen. If one deals entirely with the measured principal strains, then the external loading system which produced such strains is of no particular consequence. The sole concern is that the measured principal strains, having a certain inter-relationship, give rise to fracture initiation when they reach a particular magnitude. Thus, one is not concerned with how a particular set of principal strains is generated, but only with the fact that such strains do exist. As the mortar has been considered as homogeneous, the measured principal strains are considered to exist unchanged at all points within the specimen. That is, the "average" strains and the "local" strains are equal. Thus, each mortar specimen would represent one element in the mortar phase of concrete, such an element existing under a particular state of strain.

Unfortunately, the experimental analysis of specimens simulating the mortar-aggregate interface which is present in concrete is considerably more difficult ^{53,56}. In order to define completely the state of strain in specimens containing such an interface, the principal strains in the adjacent mortar and their orientations with respect to the interface, as well as the principal strains and orientations on the aggregate side of the interface must all be known. The complexity of this problem arises from the fact that, for one particular external loading arrangement, the two-dimensional interfacial plane under consideration may exist in any one of an infinite number of three-dimensional orientations.

Because of the difficulty in examining experimentally the mortar-aggregate interface, it was decided to consider primarily the mortar. As the initial experimental work was to be limited to specimens of pure mortar, any analysis of concrete based on these results would of necessity be limited to the mortar phase of the concrete. It was fully realized that fracture in concrete might in fact initiate in the mortar-aggregate bond zone. There is some experimental evidence to show that such is likely to

be the case^{51,53,54}. However, as Glucklich points out³⁶, in order to cause complete failure of a concrete specimen it is necessary to fracture the mortar phase. It was, therefore, considered that analysis of the mortar phase in concrete, based on experimental work on specimens of pure mortar, was a justifiable approach. However, analysis of this phase alone can only be considered valid if one of the following requirements is fulfilled:

- i) If the experimental evidence shows that fracture in concrete does initiate in the mortar phase.

Then analysis of the mortar phase prior to fracture initiation, in order to determine at which point initiation will occur, is entirely valid.

- ii) When initiation at the mortar-aggregate interface is allowed to take place prior to analysis of the mortar phase being carried out.

If the experimental evidence shows that fracture initiates from the bond interface, such initiation will cause a redistribution of strains in the surrounding mortar³⁸. Analysis to determine the point of fracture initiation in the mortar would only be valid after any such redistribution had been allowed to take place.

Experimental evidence presented in Chapter VI, shows that for the particular materials used in the concrete model test specimens and for the particular configuration of aggregate particles used and external load applied, fracture did initiate in the mortar phase. As will be emphasized later, this must not be taken as a general conclusion that fracture will always initiate in the mortar phase of concrete. It is only valid for the particular tests done.

Section III - 3

Methods of Measurement of Fracture Initiation

A number of methods have been developed over the years for the determination of the average stresses or strains at which the initiation of

fracture in a material occurs. The following review of these methods is brief and covers only those thought to be of direct application to the problem under consideration. For a comprehensive review the reader is referred to the work of J.B.Newman¹² and of Robinson¹⁴.

Measurement of Ultra-Sonic Pulse Velocity.

A method was developed by Jones^{9,10} for the measurement of the changes in the velocity of an ultra-sonic pulse transmitted through a specimen under load. Jones and others^{1,12} found that the best results were obtained by application of the pulse in the direction of the major (most tensile) principal stress. It was found that the ultra-sonic pulse velocity remained largely constant until a particular point in the stress history of the specimen was reached. At this point a sharp decrease in the pulse velocity was observed and this was related to the formation of internal voids, cracks or flaws not previously present. This point has been variously termed the "onset of major micro-cracking"⁵ and the "point of discontinuity"¹⁶. Investigators¹² have found that the method appears to be moisture sensitive. Values of the stress at which fracture initiated varied from approximately 40% of ultimate load¹ for lean mixes to approximately 95% of ultimate load for rich mixes^{1,12}. Newman¹² has found considerable variation (standard deviations of the order of 400 p.s.i.) in values of the "discontinuity stress" obtained by this method for nominally identical concretes. He also found that other methods of the measurement of the onset of fracture gave consistently lower results for "discontinuity stress" than did the use of the ultra-sonic pulse technique. It would appear, therefore, that the point at which fracture occurs as measured by the ultra-sonic pulse technique is not necessarily the first point at which fracture is initiated. Thus, the detection of crack initiation will depend upon the sensitivity of the equipment available, as well as the method used.

Measurement of Acoustic Emissions.

Work by Davies^{6,7}, Robinson⁵ and Ruetz⁹⁰ on concrete, and by Franklin⁸ and Schultz¹³ on rocks has indicated that the acoustic emissions generated by a specimen under load may be correlated to the fracture events occurring within the specimen. Although of considerable interest, this method was not considered feasible for this investigation. The equipment required is extensive, highly specialized and extremely sensitive. Considerations of cost and of the non-availability of silent surroundings argued strongly against the use of such a method.

Measurement of Electrical Resistivity or Conductivity.

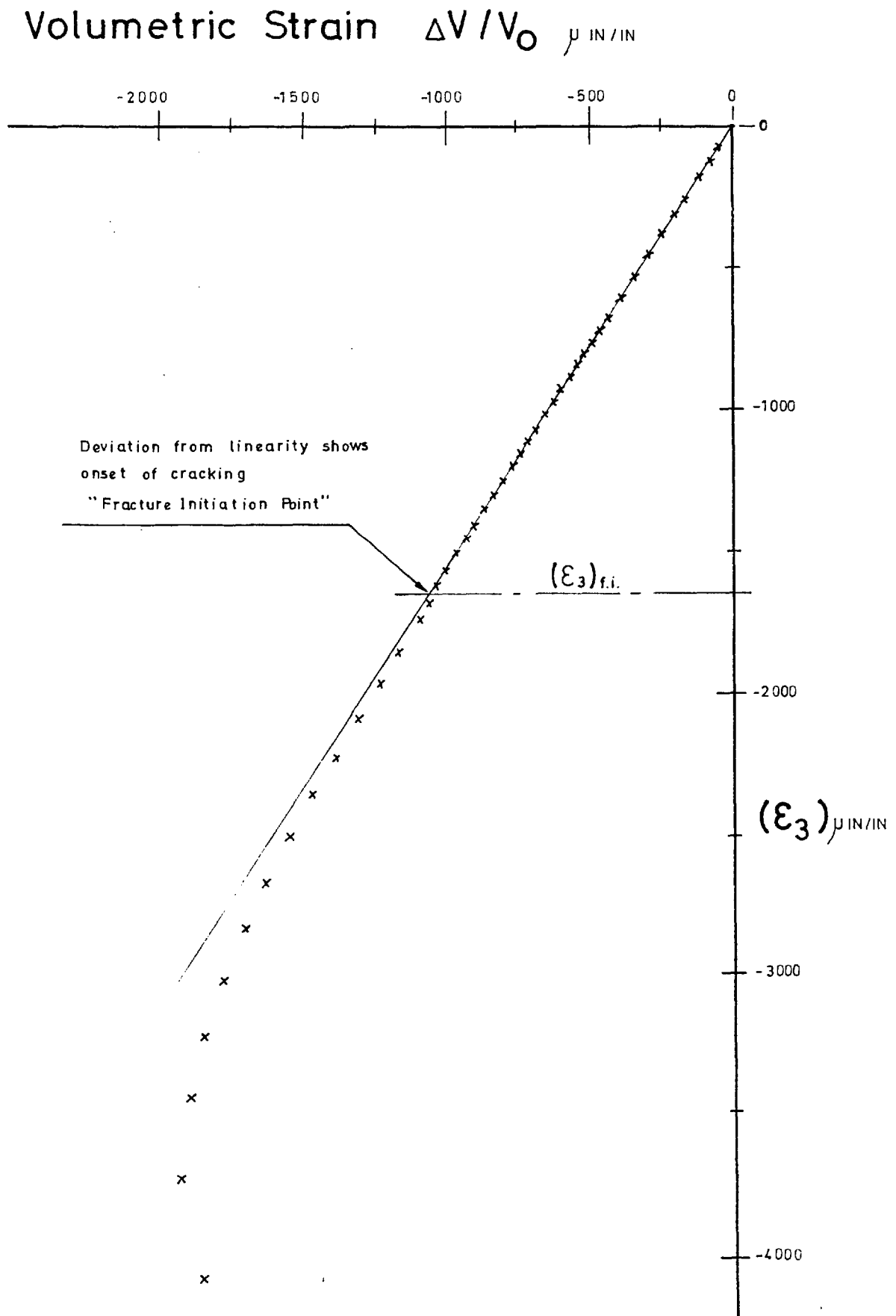
Brace and Orange¹¹ have measured the changes in the electrical resistivity of rock specimens due to loading. Abrupt changes in this property have been related to the formation of cracks or voids in the material. Unfortunately, the method is distinctly moisture sensitive, and the author knows of no published work on the application of this method to concrete.

Measurement of the Volume Change.

A method used in the field of rock mechanics to determine the onset of fracture is that of measuring the volume changes in terms of the average strain values, undergone by a specimen under load⁵⁷. Simple calculations will show that for any state of stress the volumetric strain, $\Delta V/V_0$, is given by the algebraic sum of the three principal strains, (ϵ_1) , (ϵ_2) and (ϵ_3) .⁵⁸ Thus:

$$\frac{\text{Change in volume}}{\text{Initial volume}} = \text{volumetric strain} = \frac{\Delta V}{V_0} = (\epsilon_1 + \epsilon_2 + \epsilon_3).$$

For a particular test this volumetric strain may be plotted against some other parameter such as one of the applied principal stresses or one of the measured principal strains. A typical resultant curve for mortar under uniaxial compression is shown in Figure III - 1, in which the volumetric strain ($\Delta V/V_0$) is plotted against the minor principal strain (ϵ_3). It will



Determination of the
Fracture Initiation Point

FIGURE : III-1

be seen that this curve has a linear portion corresponding to a linear volume compressibility of the material. In other words, over this portion of the curve, an incremental increase in the magnitude of the minor principal strain experienced by the material results in a constant increment of volume decrease of the material. In order for such behaviour to deviate from linearity a further factor must be added to the system. It may be seen from Fig. III - 1 that the direction in which the curve deviates from linearity is such as to diminish the effect of the incremental decrease in volume. Thus, the added factor must be such as to add a measure of volume increase which partially offsets the existing, linear, volume decrease. It has been suggested that the added factor must, therefore, be the introduction of some new internal volume, such volume existing as cracks^{57,12}. The point at which the curve deviates from linearity may then be considered as the point at which, within the sensitivity of measurement, fracture initiates. The principal strains at this point are designated as (ϵ_1) f.i., (ϵ_2) f.i. and (ϵ_3) f.i.

The original work on this method was concerned with rock specimens⁵⁷, and the curve plotted was that of volumetric strain against minor principal stress. As the analysis to be carried out by the author was entirely in terms of strain (Chapter II, Sect. II - 5), a volume strain against minor principal strain plot was used. This plot was found to give extremely consistent and repeatable results. An advantage to this method is that the external loading system is of no direct consequence, provided that the principal strain directions are known and that the magnitude of these strains can be determined. Again it is emphasized that the major concern is not how a particular set of strains is generated, but only that such strains do exist.

This method of volumetric strain measurement was found to be generally more repeatable, and to give consistently lower results, than those of the previously mentioned methods which were investigated¹². As this method

showed the initiation of fracture at an earlier stage in the stress/strain history of the material (in terms of average values) than other methods, it was considered to be more sensitive. It must be pointed out, however, that this method is still limited by the sensitivity of strain measurement possible, and does not necessarily indicate the very first fractures which form. A practical advantage of considerable benefit is that this method requires no special equipment other than the strain measuring equipment normally used.

Section III - 4

Determination of the Fracture Initiation

Envelope for Mortar

The volumetric strain measurement technique may be used to determine the average principal strains for which fracture will initiate within specimens of a particular material. For the problem under consideration (Section III - 2) it was decided that specimens of pure mortar would be tested, under various stress/strain regimes, these specimens representing elements from the mortar phase of concrete. The general philosophy was to submit specimens of the mortar under consideration to a variety of stress states. For each stress state the strain response, in the form of the three principal strains, was to be measured throughout the loading cycle. Plotting the calculated values of the volumetric strain, $\Delta V/V_0$, against the measured minor (most compressive) principal strain, ϵ_3 , would then allow determination of the fracture initiation strains in the mortar for each test. Thus, for each test, the three principal strains necessary to cause fracture initiation could be determined.

The stress states used were such that two of the principal strains were equal in each test. This limitation was imposed by the fact that no adequate means existed for the application of stress states having three independently variable principal stresses. Thus, the fracture initiation information furnished was such that either:

$$(\epsilon_1) \text{ f.i.} = (\epsilon_2) \text{ f.i.}, (\epsilon_3) \text{ f.i.}$$

or:

$$(\epsilon_1) \text{ f.i.}, (\epsilon_2) \text{ f.i.} = (\epsilon_3) \text{ f.i.}$$

where the suffix "f.i." refers to fracture initiation. As this information was intended for application to the mortar phase of two-dimensional models of concrete (Chapter IV), in which only the major and minor principal strains (ϵ_1) and (ϵ_3) were measured, the limitation was not severe. Obviously, for extension to the three-dimensional case the limitation will become severe unless it is assumed that the intermediate principal strain, (ϵ_2) , has no effect.

For tests covering a wide range of stress states (or more precisely, strain states) values of the major and minor principal strains necessary to cause fracture initiation within the mortar may then be plotted against each other. The plot may be made non-dimensional by dividing these values of $(\epsilon_1) \text{ f.i.}$ and $(\epsilon_3) \text{ f.i.}$ by the value of the minor principal strain required for fracture initiation under uniaxial compression, $[(\epsilon_3) \text{ f.i.}] \sigma_1 = 0$. The purpose of such a procedure is, in general, to allow comparison of the results from different materials. The factor used to non-dimensionalize the plot may, in general, be arbitrarily chosen, its choice being simply a matter of convenience. The reason for the choice of the particular factor used is explained in Chapter V. It is important to note that this procedure in no way changes the shape of the resultant curve, as both co-ordinates are divided by the same number.

A plot of these values,

$$\frac{(\epsilon_1) \text{ f.i.}}{[(\epsilon_3) \text{ f.i.}] \sigma_1 = 0} \quad \text{vs.} \quad \frac{(\epsilon_3) \text{ f.i.}}{[(\epsilon_3) \text{ f.i.}] \sigma_1 = 0}$$

is shown in Figure III - 2 for a particular mortar. The resultant curve passing through these experimentally determined points is referred to as the "Fracture initiation Envelope" of the mortar. Further details of the derivation of the particular Envelope used for analysis of the mortar phase

Fracture Initiation Envelope
for
Mortar Used in Plate Models

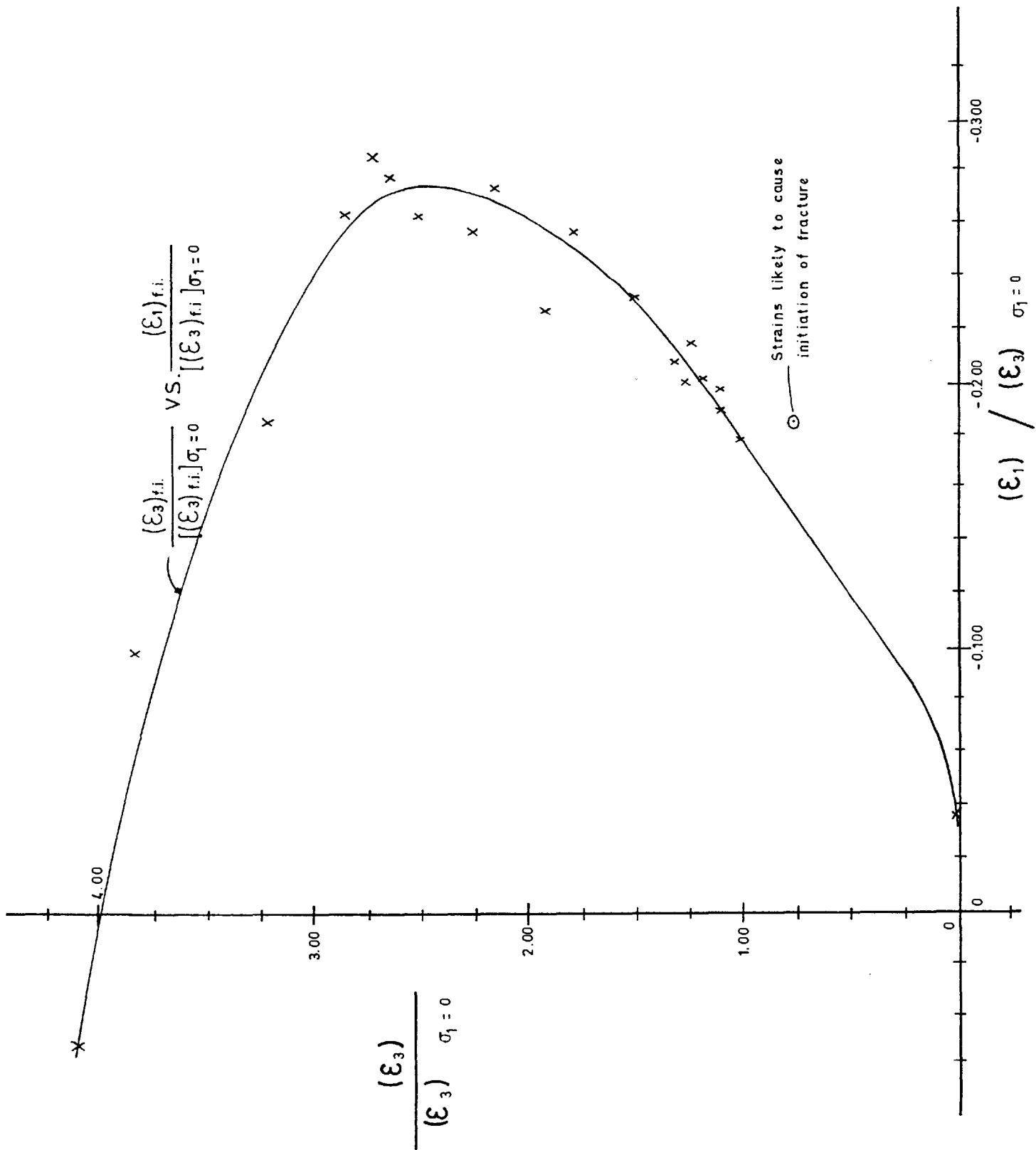


FIGURE : III - 2

of concrete are given in Chapter V.

Section III - 5

Meaning of the Fracture Initiation Envelope of Mortar

For the purposes of explanation, a comparison may be drawn between the meaning of the Fracture Initiation Envelope of mortar (shown in Fig. III - 2), and the meaning of a Mohr Failure Envelope for concrete, an example of which is shown in Fig. III - 3. If an element of concrete in a structure exists under a state of stress lying outside the Mohr Envelope applicable to that particular concrete, such as the point shown in Fig. III - 3, then it is probable that such a state of stress will cause that element of concrete to fail⁵⁸. Similarly, if an element of mortar within the structure of a concrete is subjected to a state of strain which lies outside the Fracture Initiation Envelope of that mortar, then it is probable that fracture will have been initiated in that element of mortar. Thus, just as the Mohr Envelope may be used to determine which concrete elements in a complex engineering structure are likely to be critical, so the Fracture Initiation Envelope may be used to determine which mortar elements in the complex structure of concrete are likely to initiate fracture. The use of these Envelopes requires a knowledge of the stresses, in the case of the Mohr Envelope, and of the strains, in the case of the Fracture Initiation Envelope, existing in the element under consideration.

Section III - 6

Conclusions

Use of the mortar Fracture Initiation Envelope allows analysis of any element of mortar in the complex structure of concrete, provided that the principal strains sustained by the element are known. Such analysis will allow determination of whether or not a particular set of strains sustained by a particular element of mortar is likely to give rise to the initiation of fracture.

The application of a Fracture Initiation Envelope is only strictly

Example of Mohr Failure Envelope for Concrete

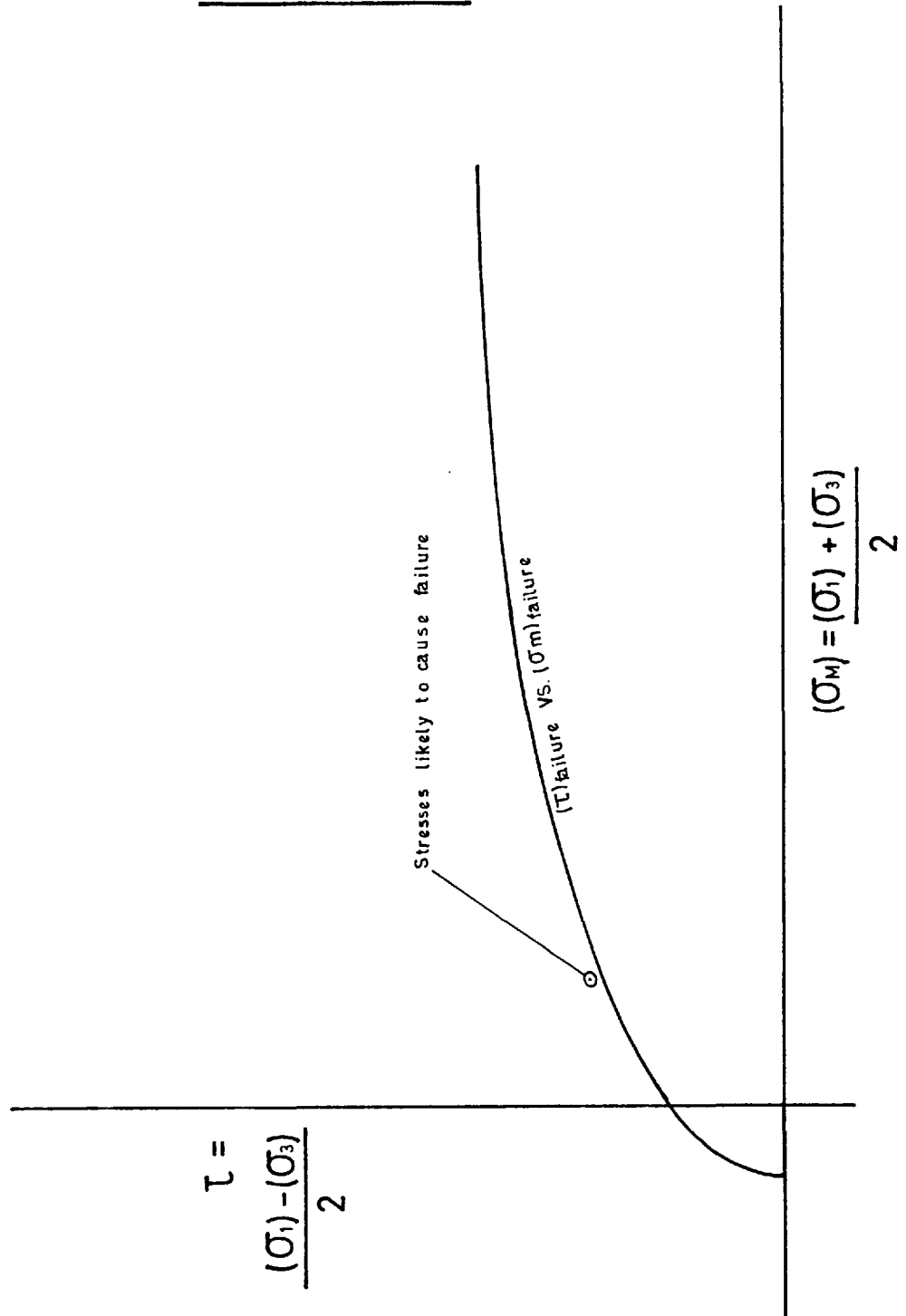


FIGURE : III-3

valid over the range of conditions under which the particular Envelope was derived. Thus, the Envelope for one material may not with impunity be applied to another material. Similarly, an Envelope derived from tests under short-term, static load conditions cannot be correctly applied to strains generated by long-term or dynamic loads.

Details of the tests used in the derivation of a Fracture Initiation Envelope are given in Chapter V with reference to the particular mortar used. Also given in Chapter V are more detailed comments on the precise use of the Envelope in the prediction of fracture initiations within the mortar phase of concrete.

CHAPTER IV

EXPERIMENTAL TECHNIQUES FOR PLATE MODEL TESTS

Section IV - 1

Introduction

The previous Chapter described a method whereby the analysis for fracture initiation of any element of mortar in the complex structure of concrete could be undertaken, provided that the principal strains of each element of mortar were known. Thus, the problem is to determine, upon the application of load, the local strain distribution in the mortar phase of concrete due to the heterogeneous nature of the concrete.

Section IV - 2

Determination of Experimental Parameters

Section IV - 2.1

Model Shape

It was preferable to be able to test the specimens under general triaxial states of stress. However, to allow measurement of the point-to-point strains in the models it was necessary to have a free face in order to attach, at any point, the strain measuring equipment used. Thus, the application of stress was limited to two directions. The models, therefore, represented a two-dimensional slice cut from the body of the concrete prototype and existing in a plane defined by two of the applied principal stresses. The results of each model may only be considered as being applicable to the particular plane represented.

In order to simulate a two-dimensional "slice" of concrete, it was decided to use prismatic plate specimens having one dimension (i.e. the "thickness" of the slice) approximately an order of magnitude less than the other two dimensions. In this way, the probability of significant major flaws or strain disturbances existing in the third dimension would be kept as low as possible, and the model more closely represent an infinitesimally thin plane. The dimensions in the plane of the model 10 ins. x 10 ins.,

were determined by the dimensions of the existing testing equipment. As it was required to be able to apply two independently variable stresses in the plane of the model, the biaxial loading frame designed by Vile¹ was used. After tests to determine the effect of buckling of the specimens had been carried out (Sect. IV - 3.1), the thickness of the models was fixed at $\frac{3}{4}$ ".

Thus the dimensions of the models to be used were determined as 10 ins. x 10 ins. x $\frac{3}{4}$ in., giving a maximum to minimum dimension ratio of 13.3.

Section IV - 2.2

Loading Arrangement

It is debatable⁵⁸ whether the interior of the prototype concrete more closely simulates conditions of plane strain, or those of plane stress. In order to achieve conditions of plane strain for the model tests, a restraining force across the thickness of the specimen was recognized as being necessary. However, application of such a force would disallow the use of surface strain measuring techniques and was not considered feasible. The models were therefore tested under conditions of plane stress.

The errors involved in applying the results of plane stress tests to a case where some condition intermediate between plane stress and plane strain exists are difficult to assess. As a very approximate guide, it may be taken that the solution of the plane stress problem will give results containing a factor of (1 - Poisson's Ratio) with respect to the plane strain solution⁵⁸. Thus, for concrete with a Poisson's Ratio of approximately 0.15, a solution under perfectly plane stress might be expected to give values of individual strains from 85% to 115% of the values derived from a solution under perfectly plane strain conditions. As the actual conditions are unlikely to fit either case exactly, the errors involved in the application of results from the models to the prototype would be correspondingly less.

As the model tests were planned to include processes of fracture, it was desirable to use a system which would apply as uniform a load as

possible throughout the test. The "piston platen" shown in Figures IV - 1 and IV - 2 was accordingly designed and built, the major features of which are:

- i) The pistons are connected by a common oil gallery, ensuring that the load on each piston is identical.
- ii) A clearance gap was left between the edges of each piston and its corresponding loading shoe. A "frictionless" P.T.F.E. shim was introduced between the face of each piston and its corresponding shoe. Thus, independent and largely unrestrained lateral movement of each shoe was possible.

This "piston platen" was used to apply load to one edge of the models in uniaxial compression. The platen used for the other edge consisted of a solid steel block. This solid platen was re-ground frequently to maintain a smooth finish. Between the solid platen and the specimen edge, a P.T.F.E. shim covered by a thin (0.030") aluminium strip was placed.

This platen arrangement was used for all the model tests reported in this Thesis and was found to give generally acceptable results.

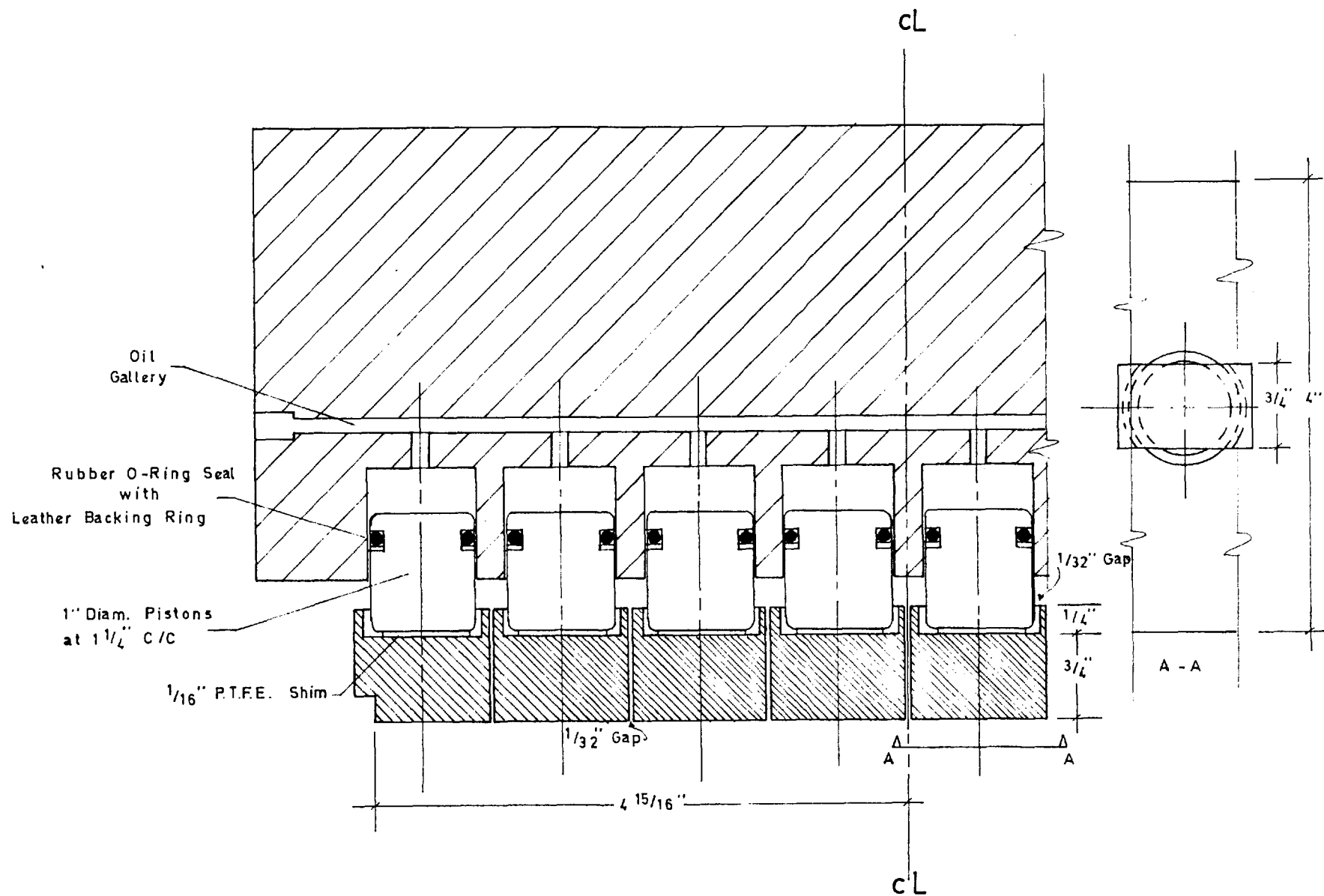
Section IV - 2.3

Materials

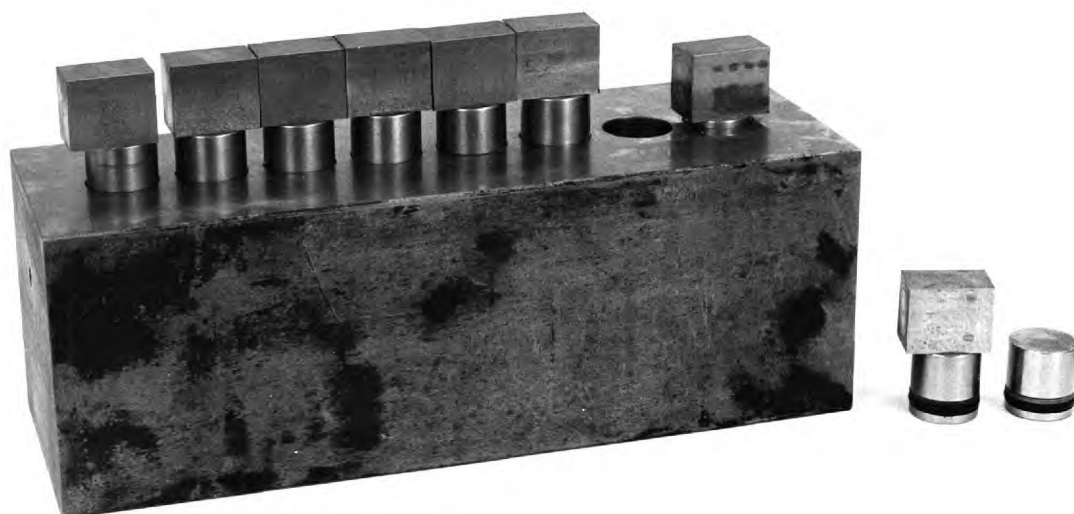
The choice of the materials used in the models was chiefly determined by the following requirements.

- 1) The materials used in the models must closely simulate the materials used in the "real" prototype concrete.
- 2) From Chapter III, Section III-2, it was decided that, as a first assumption, concrete could be considered as a two-phase material, in which large particles of hard coarse aggregate are embedded in a softer, interconnecting, homogeneous mortar matrix. It was further decided to simplify the geometry of the system by assuming that the coarse aggregate particles existed as single-sized spheres. It was considered essential that, for the first steps at least, a

FIGURE: IV-1



General Arrangement of
Linked-Piston Platen



Piston Platen in Process of Manufacture

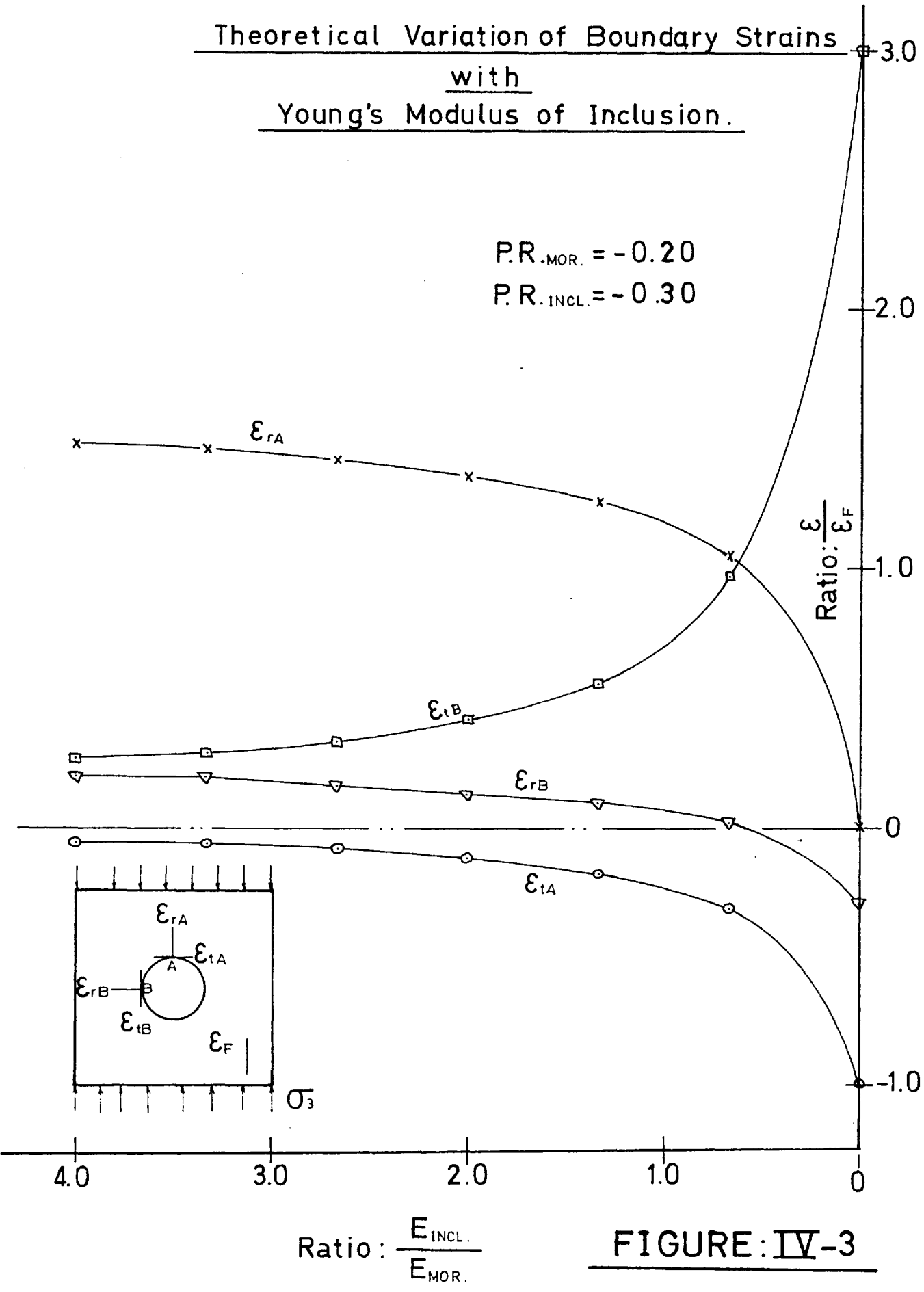
FIG. IV - 2

clear dimensional distinction between the phases was maintained. In other words, the maximum particle size present in the mortar matrix was to be kept small in relation to the size of coarse aggregate. Such a material was not felt to be too unrealistic, having a reasonable similarity to gap-graded concrete.⁶⁰

- 3) The equations of Muskhelishvili⁵⁹ were used to calculate the theoretical strains existing in a matrix surrounding a single inclusion due to the application of a uniaxial compressive load.* The strains at isolated points in the matrix were calculated for various ratios of the Young's Modulus of the inclusion to the Young's Modulus of the matrix. The concentration factor of these isolated strains, over and above the strains in the matrix remote from the inclusion, was plotted against the Modular Ratio, $E_{\text{inclusion}}/E_{\text{matrix}}$. One such plot is shown in Figure IV - 3. It may be seen from this plot that the concentration factor of the local strains considered remains largely constant for a modular ratio greater than three. It was, therefore, felt that as long as the modular ratios employed in the models and in the prototype concretes were equal to or greater than three, only small errors would be introduced by the application of results from one to the other.

An interesting sidelight stemming from these calculations concerned the qualitative mechanics of the fracture processes in concretes with various aggregate to mortar modular ratios. As the local strain field was seen to be largely invariant for modular ratios greater than three, it might be reasonably expected that the fracture processes involved would also be largely invariant. However, for modular ratios less than three, abrupt variation in the local strain field was noticed. It was, therefore, concluded that for normal concretes containing hard aggregates the fracture process was unlikely to change abruptly for changes in the mechanical properties of the aggregates.

* These equations are shown in Table IV - 2.



For concretes containing softer aggregates, including light-weight materials, the fracture process might be expected to be both qualitatively and quantitatively different from that experienced by normal concretes, and to be extremely sensitive to small changes in the Young's Modulus of the aggregate.

The equations derived by Muskhelishvili were for the case of a plate of infinite extent composed of a single elastic inclusion, perfectly bonded to an elastic matrix. As such, it would be dangerous to apply quantitative results obtained from his equations to the case of mortar and aggregate. However, it was felt that qualitative application to determine rough trends was justified, and it was, therefore, decided to choose materials for the plate model tests such that the ratio of $E_{\text{aggregate}}/E_{\text{mortar}}$ was at least equal to three.

- 4) The materials chosen must have properties which are consistent between batches.

With the above points in mind, the materials chosen were as follows:

Mortar

In order to keep the maximum particle size as small as possible, sand in the size range from British Standard Sieve No. 25 to No. 100 was used. A typical sieve analysis of the sand is given in Table IV-1.

After a number of trials, the mix details finally chosen for the mortar were:

Sand size - #25 - #100

Cement - O.P.C.

Sand/cement ratio (by weight) = 1.40

Water/cement ratio (by weight) = 0.39

Details of the casting and curing techniques are given in Section IV - 2.5.

Figure IV - 4 shows a typical stress-strain curve for this mortar under uniaxial compression obtained from short-term static

T A B L E IV - 1

Sieve Analysis of Sand used in Plate Model Mortar

<u>B.S.Sieve No.</u>					<u>Approx. % Passing</u>
14	...	...	...	...	100
25	...	...	...	...	100
52	...	...	...	...	39
100	...	...	...	...	3
200	...	...	...	...	1

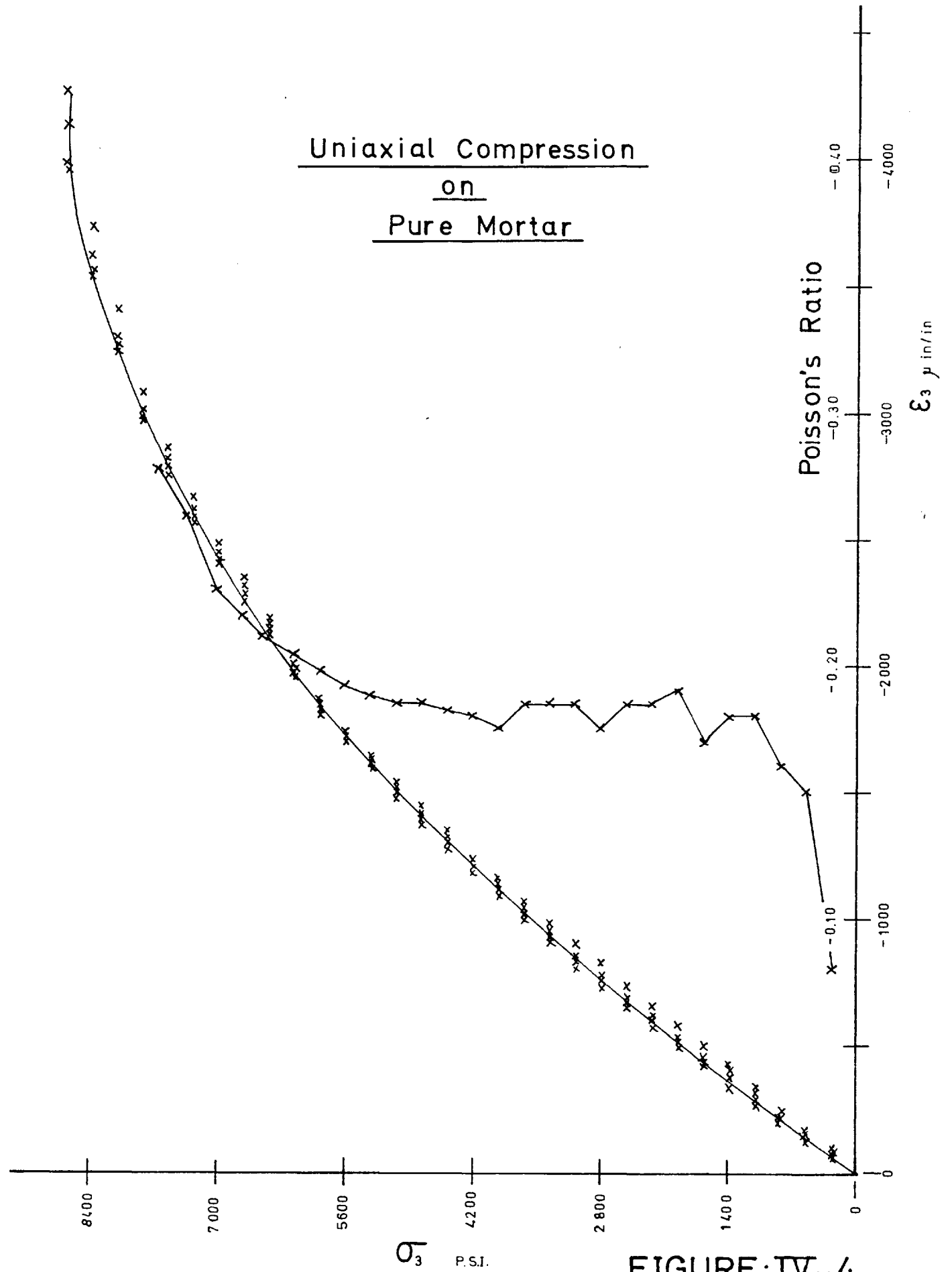


FIGURE: IV-4

tests on cylinders 4 ins. in diameter by 10 ins. high. An approximate value for the tangent Young's Modulus at the stress level anticipated for the plate model tests (Section IV - 4) is 3.5×10^6 p.s.i. As a measure of the repeatability of the mortar properties between batches, the standard deviation of the tangent Young's Modulus at 1000 p.s.i. was calculated and found to be of the order of 0.07×10^6 p.s.i.

Aggregate

Choice of rock type

An initial fairly random choice was made of six rock types. Specimens of these rocks were supplied by a monumental mason (Frank England Co. Ltd.) and their trade names are listed here for reference.

Spec. A.	-	"Ebony granite"
Spec. B.	-	"White Silica" (Marble)
Spec. C.	-	"Portland Stone"
Spec. D.	-	"York Stone"
Spec. E.		"Irish Limestone"
Spec. F.	-	"Sandstone"

Uniaxial compression cylinders, 2 ins x 4 ins., were tested for each rock type, and the resulting stress-strain curves are given in Figure IV - 5. It should be noted that, due to scale difficulties in plotting, the test results shown are not necessarily taken to complete failure. From these results it was decided to use rock of the "Ebony Granite" (Spec.A.) variety. This rock gave a consistently linear stress-strain curve having a Young's Modulus of 13.2×10^6 p.s.i. It may be seen that this gives a modular ratio, $E_{agg.}/E_{mortar}$, of approximately 3.75. Variation in the value of the Young's Modulus between batches was so slight that no standard deviation could be determined for this value. The rock structure was of a very fine grained, densely packed type, and,

Uniaxial Compression Curves

for
Various Rock Types

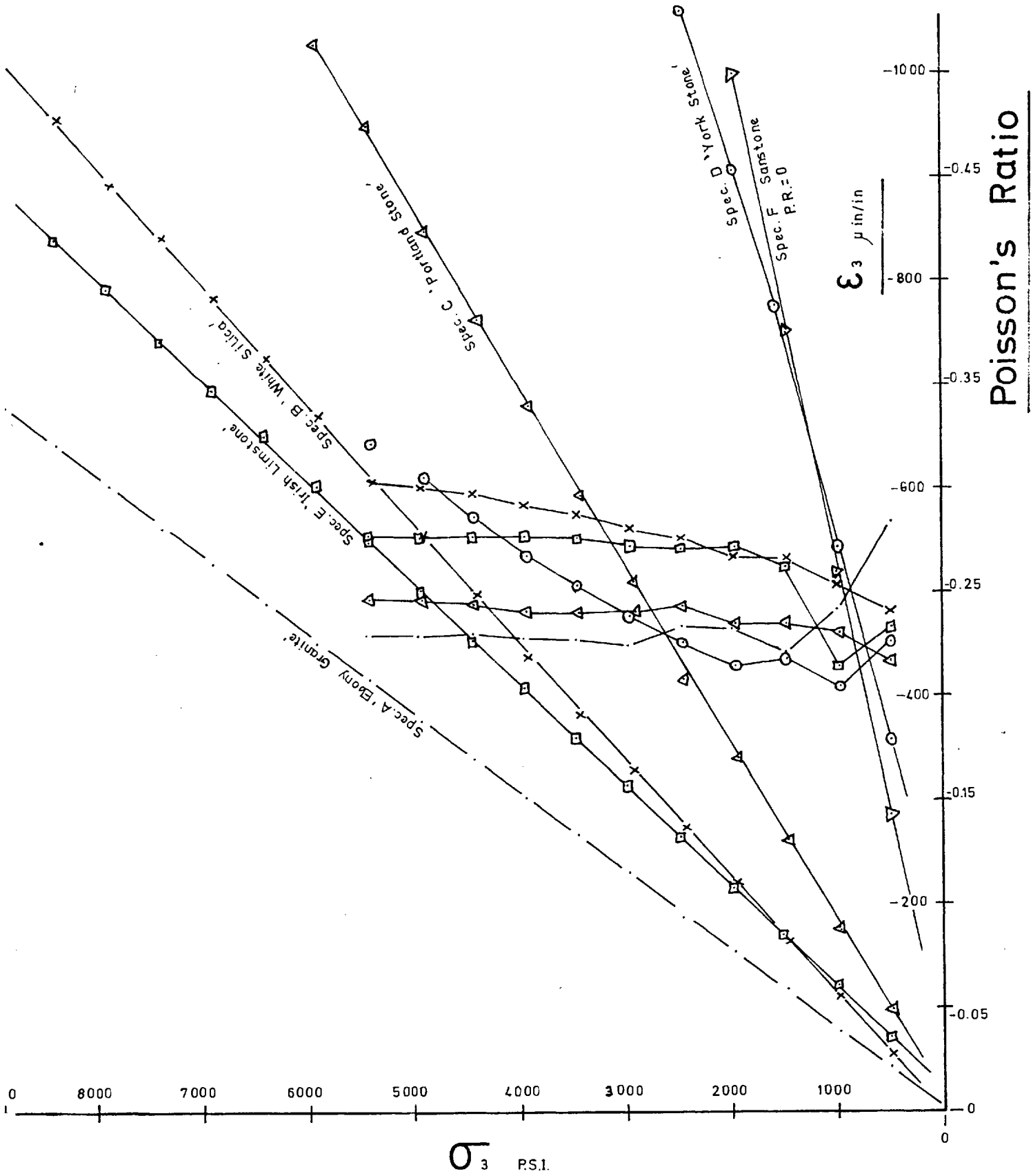


FIGURE: IV-5

although several attempts were made to determine the precise mineralogical nature of the rock, expert opinion was divided and no firm conclusion was reached. It was agreed that it was of a granitic type (igneous) and probably composed of fine grained feldspar - of which there are twelve basic types.

Choice of Shape and Size

As previously mentioned, the shape of the aggregate particles was simplified, and such particles were assumed to consist of single-sized spheres. For two-dimensional model work, it was, therefore, necessary to use cylinders of aggregate to simulate equatorial slices of such spheres. The diameter of cylinders used was determined by two factors.

- i) The ratio of the diameter of the cylinders to the planar dimensions of the models (10" x 10"). For cylinder diameters greater than one-quarter of the least planar dimension, it was expected that interference would occur in the local strain field due to edge effects and irregularities of loading⁶¹. As it was planned to use groups of aggregate cylinders in a single model, it was considered that a cylinder diameter of one inch, one-tenth of the least planar model dimension, should not be exceeded if possible.
- ii) The physical extent of the local strain field generated by an inclusion is a function of the inclusion diameter⁵⁹. It was apparent that the point-to-point measurement of strains would be considerably easier, the greater the extent of the strain disturbance. Thus, there was a strong argument against the use of cylinders with diameters smaller than those absolutely essential from (i) above.

The final deciding factor for the precise diameter used, $1\frac{3}{16}$ ins., was the availability of the diamond-tipped coring tools necessary for the production of the cylinders.

Choice of surface texture

For this type of rock, the surface finish left by a diamond tipped

coring tool was extremely smooth. A smooth, impermeable surface such as this was not considered to be a good enough approximation to the surface finish of aggregates normally used for the production of concrete⁵⁵. Also, for future work, it was desirable that the surface texture of the aggregate should be variable.

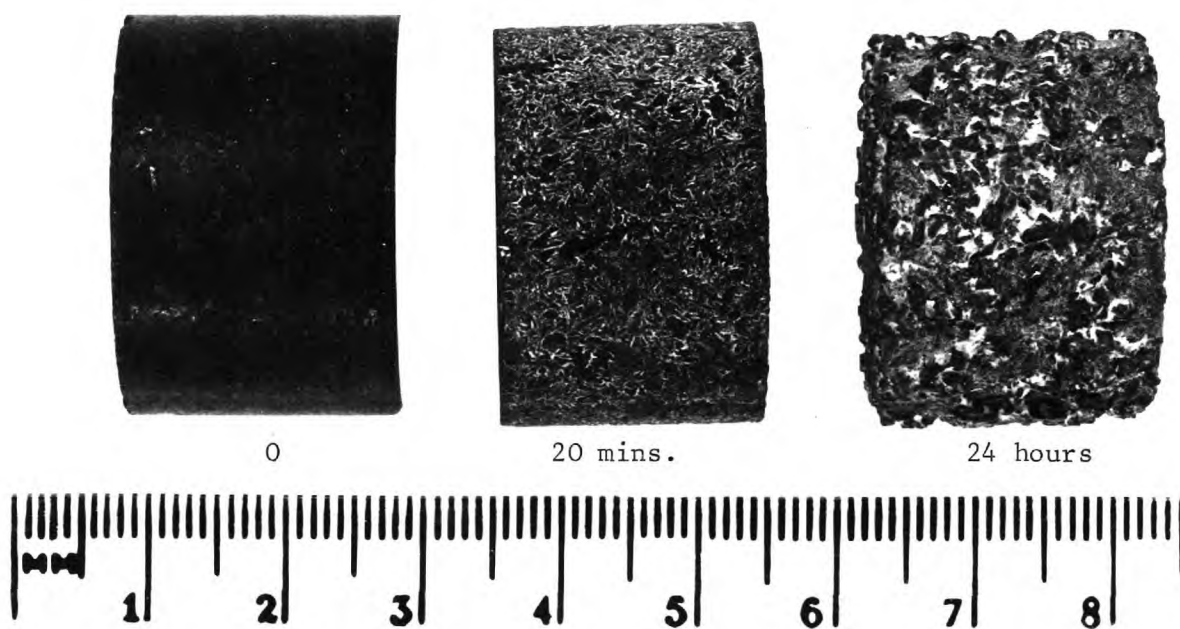
First attempts at imposing a particular surface texture upon the rock aggregate cylinders consisted of chipping with tungsten-carbide tools. Unfortunately, the effect on the tools was generally greater than the effect on the rock specimens.

The method finally devised and used was that of etching with hydrofluoric acid. An ordinary laboratory solution (10%) of HF1 was used. The ends of the rock cylinders were coated with vaseline such that only the peripheral (bond) area was affected by the acid. After immersion in HF1 the specimens were removed, thoroughly washed, wire brushed and re-washed. The effect on the surface texture due to varying the time of immersion may be roughly judged from Figure IV - 6. It was decided that, for the first series of plate model tests, the surface texture given by a twenty-minute immersion in acid would be used.

An advantage of surface etching such as this (apart from the obvious advantage of being able to vary the surface texture at will) was that the surface roughness could be precisely defined in terms of immersion time, and thus could be repeatably produced.

Section IV - 2.4 Arrangement of Aggregate Particles

In order to allow the meaningful comparison of results between different plate model tests, it was necessary to arrange the cylindrical aggregate particles in some form of regular array. The array chosen must be correlated to a three-dimensional packing of spheres. There appeared to be three basic types of regular, three-dimensional packing of spheres which were applicable to the problem⁶².



Surface Roughness Achieved by Various Times of
Immersion in Hydro-Fluoric Acid.

FIG. IV-6

- 1) A $30^\circ - 60^\circ$ spatial pyramid. Although this type of packing seems realistic, there exists no single two-dimensional plane which may be considered representative of the three-dimensional case. Thus, its application to model work is difficult.
- 2) A face-centered cubic packing, which has the advantage of being readily transferred to a representative two-dimensional slice.
- 3) A body-centered cubic packing, which also has the advantage of easy representation in two-dimensions.

It was decided to use a two-dimensional aggregate array representing the three-dimensional case of a face-centered cube. The type of array used is shown in Fig. IV - 7 and may be seen to compare favourably with the particle packing displayed by the ground end of a cylinder cast with glass marble aggregate shown in Fig. IV - 8. It should be clearly understood that the choice of this type of packing was in some ways arbitrary and does not necessarily represent the type of packing most likely to occur in concrete. It will be emphasized later that all the results derived from the plate model tests must be considered within the context of the particular aggregate packing used.

Calculations based on the geometry of the three-dimensional packing and of its two-dimensional representation give a relation between the particle diameter and spacing in the model and the total, three-dimensional, volume fraction of coarse aggregate.

Thus:

$$V_f = 2.1 \frac{D^3}{S^3} \times 100 \quad \dots\dots\dots (1)$$

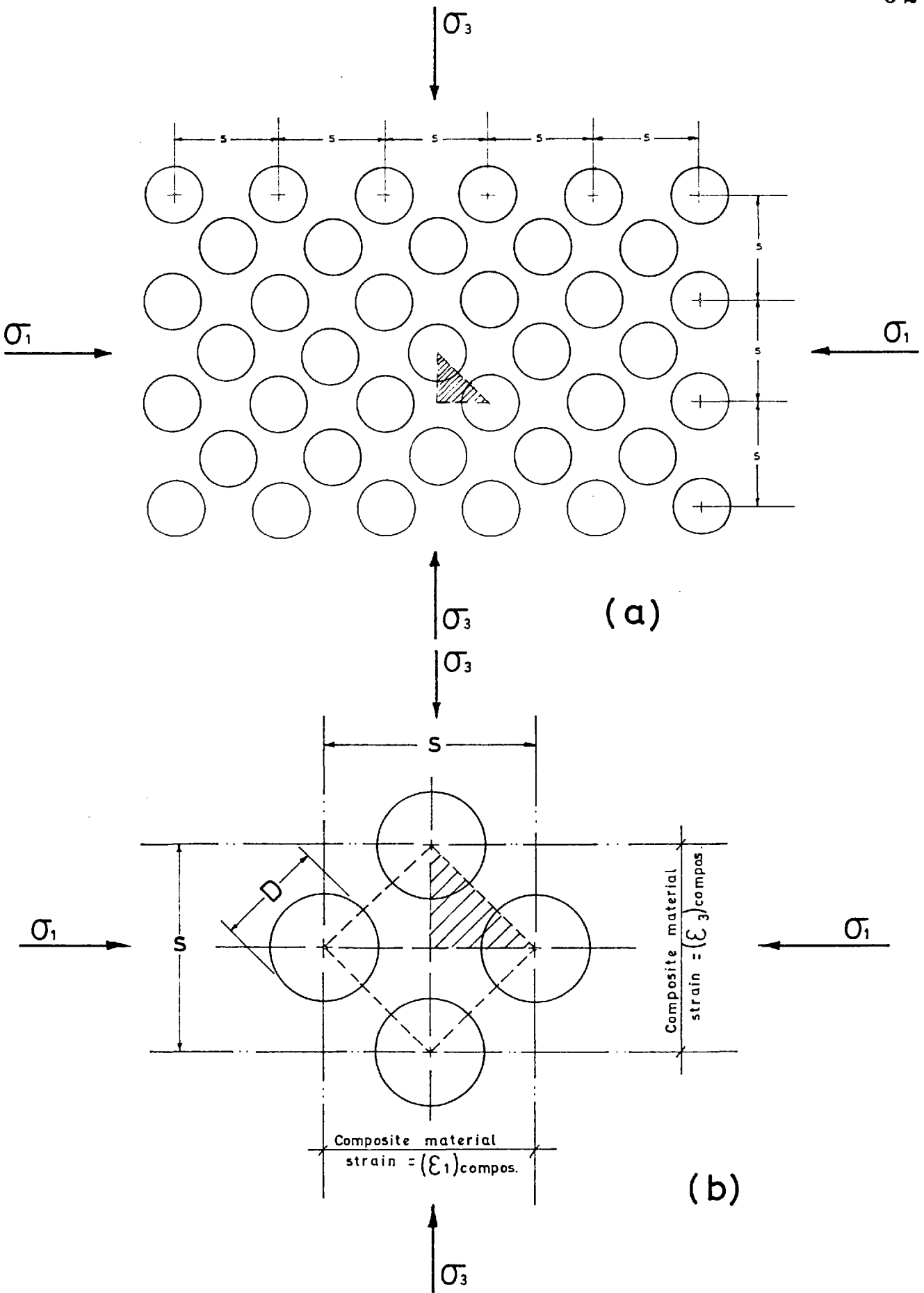
where:

V_f = the three-dimensional volume fraction of coarse aggregate as represented by the model.

D = diameter of coarse aggregate particle.

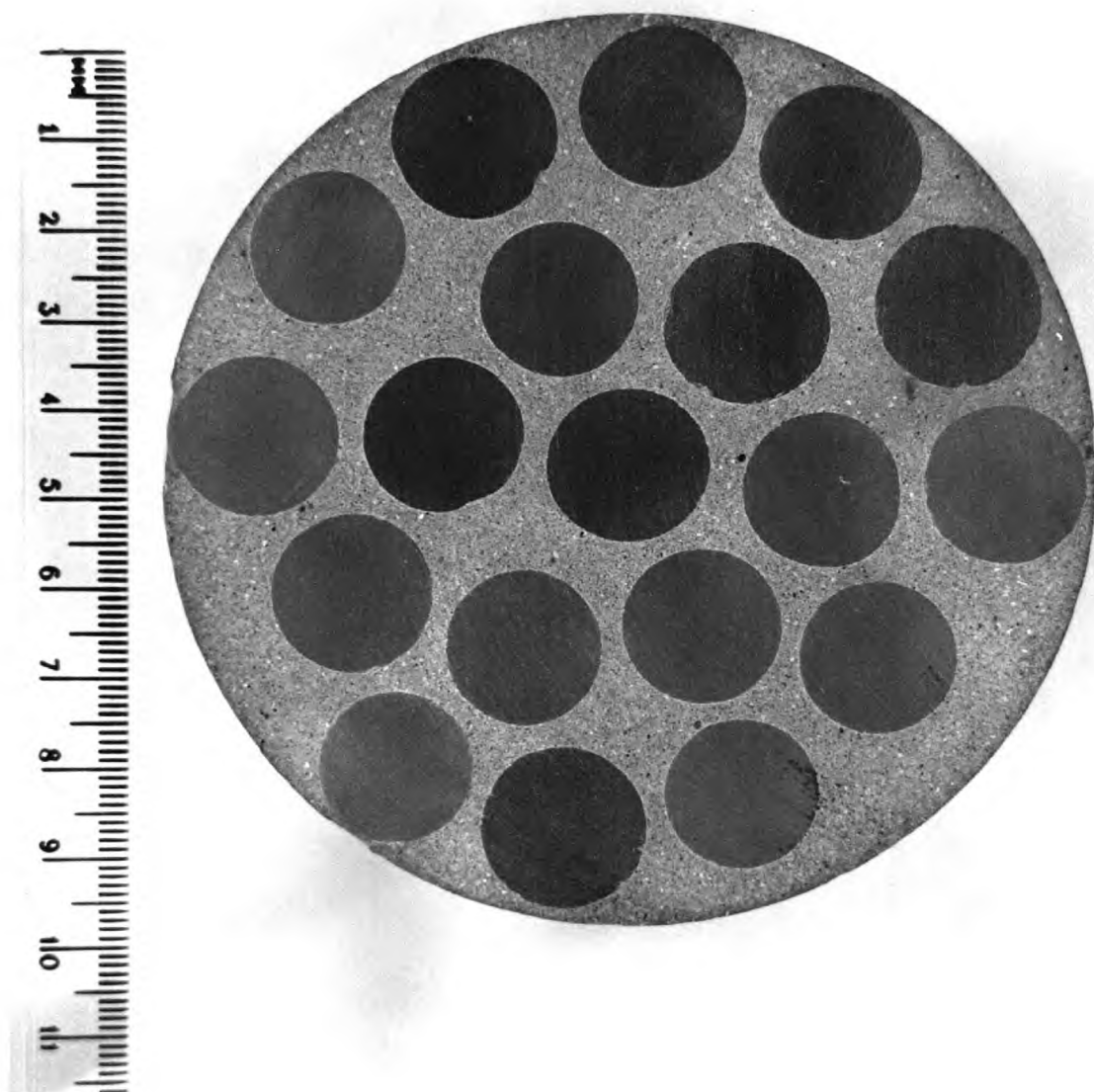
S = center to center spacing of coarse aggregate particles.

This spacing is measured as shown in Figure IV - 7 and is the length of any side of the face-centered cube under consideration.



Two-Dimensional Representation of Face-Centered
Cubic Packing

FIGURE: IV-7



Aggregate packing displayed by ground end of
concrete cylinder containing glass marbles

FIG. IV-8

At the risk of over-emphasis, it must be clearly understood that this equation relates to the volume fraction present in the three-dimensional material. The actual volume fraction of the rock cores with respect to the two-dimensional model strictly has no meaning, as a two-dimensional plane cannot contain a volume. Therefore, for a particular model, this equation may be used to determine the volume fraction of coarse aggregate represented by, but not actually contained in, that model.

It may be seen from the geometry of the system that for a generalized stress state where $\sigma_1 \neq \sigma_3$ the smallest infinitely repeatable unit, in terms of the strain response of the system, is the triangle shown in Fig. IV - 7(a). It may also be seen that the least number of particles which may be considered to bound this unit is four.

Use of this number of particles in a model requires that several points be borne in mind.

- i) In the general case of loading where $\sigma_1 \neq \sigma_3$, there exist within the packing four identical triangular areas. One such area is shown cross-hatched in Fig. IV - 7(b). Thus, resolution of the strains within any one triangle gives complete resolution of the entire strain field within the packing.
- ii) The aggregate packing is intended to simulate an infinite array of such particles. Therefore, as far as analysis of the strains within such a packing is concerned, only the area bounded by the dotted lines in Fig. IV - 7(b) need be considered.
- iii) The use of four particles to simulate an infinite array will only give results due to first order effects. The use of sixteen particles as shown in Figure IV - 9 would give more realistic results, as second order effects would be included. A number of specimens containing a sixteen-particle array were cast, but were found to be impractical. Figure IV - 9 shows one such specimen. The internal restraint provided by this number of particles, and the relation of the

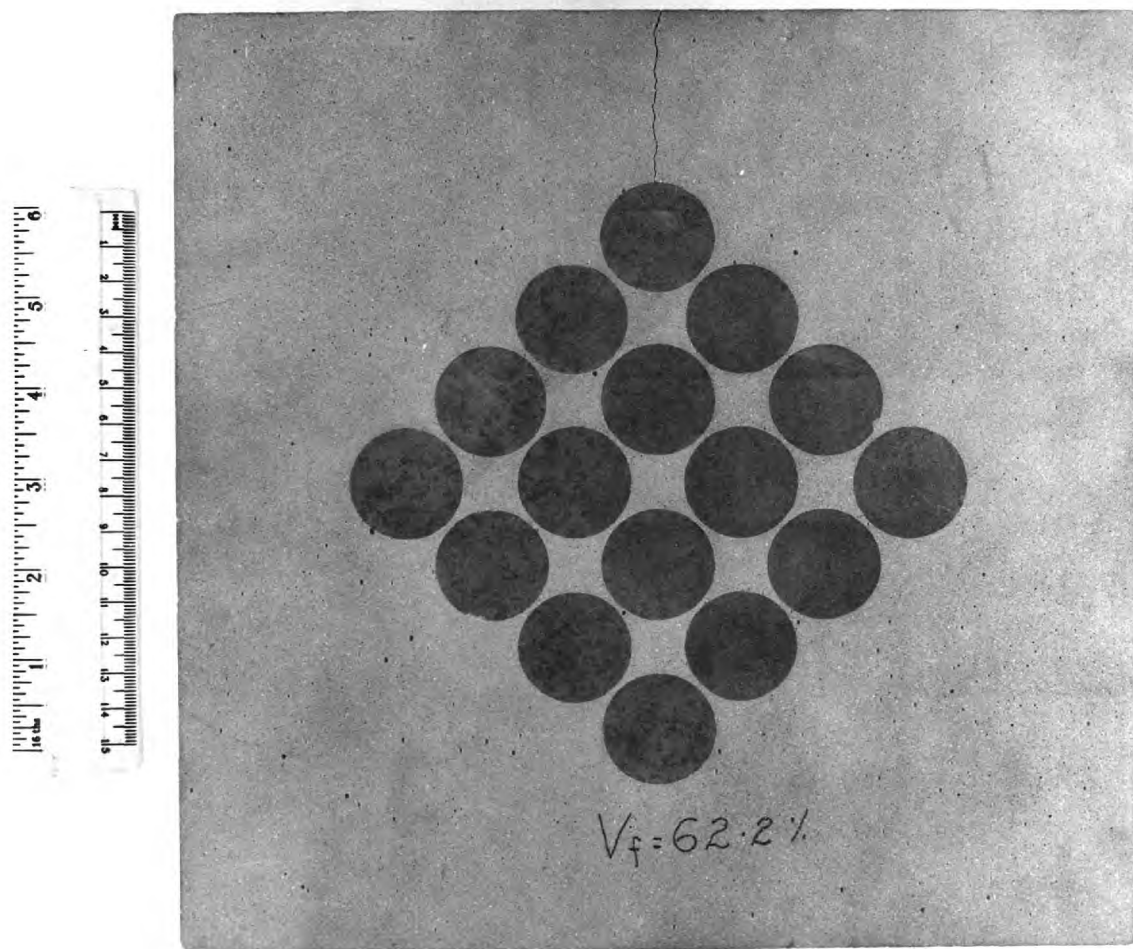


Plate-Model Specimen Containing a Sixteen Particle Array

FIG.IV-9

overall size of the array to the model dimensions, caused extensive shrinkage cracking. Various curing and drying regimes were tried, with no beneficial result. Unfortunately, cracking was so severe that no idea of the magnitude of the second order effects could be obtained. It is, therefore, of importance to remember that the results of the plate model tests reported in this thesis only take into account the first order effects of aggregate packing.

It should also be noted from Fig. IV - 7 that the principal strains of the composite (rock and mortar) material will be given by the average strains between the centerlines of the particles as shown.

Section IV - 2.5

Casting and Curing Procedure

Steel moulds of 10" x 10" x $1\frac{1}{2}$ " were constructed. The purpose of the additional depth of the mould was to allow the specimens to be cast oversize in thickness, usually 1 inch, and then ground to the required $\frac{3}{4}$ ins. Details of this procedure are given in Section IV - 2.7.

The center-to-center spacing of the particles, s , to be used in a particular model was first decided. A grid was then scribed on the base plate of a mould using a Vernier height gauge. The purpose of the grid was to allow the rock cores to be accurately positioned for the particular value of " s " chosen. It was, of course, necessary to regrind the base plate of the mould after each use, in order that a new grid could be scribed. The surfaces of the rock cores to be used were then etched with HF1 as described in Section IV - 2.3. After the etched surfaces had been thoroughly cleaned, one end face of each rock cylinder was coated with Durofix, and the cylinder attached to the base plate. This method of fixing was found to be entirely satisfactory. The cylinders were not displaced by vibration, yet the specimen could be easily removed from the mould after casting.

The dry ingredients of the mortar were mixed in an Eirich 2 cu. ft. horizontal pan mixer for two minutes. The water was then added and the

mortar mixed for a further two minutes. The mortar was placed in the mould around the rock cylinders in two layers of approximately $\frac{1}{2}$ inch, each layer being vibrated briefly. The time of vibration was sufficient to ensure good compaction without segregation. No attempt was made to strike off the top surface of the cast specimen.

The specimens were left under damp sacking for 48 hours after casting, before being de-moulded. All specimens were then cured under water for a further 26 days according to British Standard 1881⁶³.

Further stages in the treatment of the specimens are described in Section IV - 2.7.

Section IV - 2.6Measurement of Strains

As outlined in Section IV - 1, the object of the experimental work was to determine the surface strains existing in each plate model due to the application of a load and to the physical nature of the model. The choice of the strain measuring system used was governed by three general criteria.

- 1) Complete resolution of the strain field was required.

To fulfil this requirement it was necessary to determine the magnitudes and directions of the principal strains at every point on the surface of the model.

- 2) It was desirable to devise a method whereby the actual point or location at which fracture initiated could be observed.

This would allow it to be determined whether fracture initiated from the mortar-aggregate interface, or from within the mortar itself. (See Chapter III, Sect. III - 2). Observation of the fracture initiation point was also essential for comparison with the predicted point of fracture initiation from the theoretical analysis.

- 3) Some means of observing the path of fracture propagation was considered desirable.

Although the analysis was concerned solely with the initiation of fracture, observation of the path of propagation would be of great interest in establishing the mode of ultimate failure.

A) Comparison of the Available Methods of Strain Measurement

With the above general criteria in mind, several strain measuring systems were examined. The following review of the main methods investigated is brief, more detailed information being provided in the quoted references.

The Moiré Fringe Technique^{61, 64-70}

The moiré fringe technique was developed from the observed phenomenon

that alternate dark and light fringes appear when two grids or gratings with different spacings are superimposed. This phenomenon, known in ancient China as an artistic device, has recently found application in the field of experimental mechanics. In its simplest form, two initially identical grids, composed of alternate opaque and transparent bars, are used. One such grid is placed on the specimen under consideration. When this grid is stretched, due to straining of the specimen, and viewed through the other, "master", grid, fringes are observed. These black fringes occur due to the changed spacing of the specimen grid lines and may, therefore, be related to the displacements undergone by the specimen perpendicular to the direction of the grid ruling.

In its more sophisticated form, multi-directional grids may be used, giving values of the displacements in several directions. Use of such grids, aided by methods of frequency or rotational mismatch, will allow measurements to be taken which will give, after calculation, complete resolution of the strain field at any point. However, as Post⁶⁷ points out, such methods are both tedious and cumbersome. The basic difficulty is that the moiré fringe method is fundamentally a method for measuring the components of displacement normal to certain arbitrary directions. Thus, in order to determine the principal strains and directions at a particular point, lengthy calculations are required.

In order to obtain the required degree of strain resolution, i.e. approximately $\pm 10 \mu$ in/in., grid rulings approaching 1000 lines per inch would be required together with interpolation to $\frac{1}{100}$ of a fringe. Fringe interpolation of this order could only be performed satisfactorily using a photo-electric sensing device. Furthermore, the use of grids of this fineness requires a test set-up of virtually optical precision. This was clearly out of the question for the particular loading frame used.

Moiré grids are normally superimposed on the face of a specimen by photographic techniques^{64,70}. As the faces of the mortar and stone models

used would inevitably be pockmarked by air voids, such techniques become difficult, and the grid sensitivity suffers.

The cost of diffraction gratings or grids was found to be considerable. The equipment required to carry out an acceptable strain survey of the models was estimated as costing in the region of £2,000.

It appeared that use of the moiré fringe technique by itself to resolve the strain field on the models was expensive and cumbersome. The method would nevertheless appear to be a powerful experimental tool, and use of it in conjunction with other methods⁷¹ may eventually prove to be the "ultimate" solution to many problems of strain field resolution.

Strain Gauge Rosettes

The use of strain gauge rosettes to determine the principal strains and directions at a point is well-known and needs no detailed explanation. The theory involved is covered in any standard text on the theory of elasticity⁵⁸.

One of the principal drawbacks of the use of strain gauge rosettes is that they have a finite size. The result of a strain-gauge rosette reading can be considered only as the average value of the strains existing over the length of the gauge. The smallest commercially available rosette, of the "three crossed" type, had (at that time), overall dimensions of 11 m.m. x 11 m.m. Thus, the distance between the points of measurement would have a minimum value of almost $\frac{1}{2}$ inch (12.5 m.m.). This distance is considerable when viewed in the context of the strain field generated by a particle $1^3/16$ th in diameter.

The cost of using rosettes was also found to be considerable. An expenditure of £40 per specimen was estimated, and this was not considered justifiable in terms of the information derived. In particular, information concerning the point of fracture initiation and the path of fracture

propagation would only be furnished by coincidental placing of the gauges.

Brittle Lacquers^{58 72}

The use of brittle lacquers in determining the surface strains of a specimen has been established for some time. A coating of a brittle lacquer is applied to the specimen under consideration. Upon loading, the brittle lacquer will exhibit a pattern of cracking which will be related in some fashion to the strains existing in the specimen. However, the precise meaning of, and information which may justly be derived from, the use of brittle coatings is still a subject of considerable debate⁵⁸. It would certainly appear that their use to determine the stress trajectories, and hence the principal stress directions, in a specimen under load is relatively simple. However, the determination of any further quantitative information from brittle lacquer techniques has been termed by one investigator to be ".....endlessly tedious and highly suspicious" ⁷². Durelli, Phillips and Tsao⁵⁸ compiled a list of forty factors which would influence to a greater or lesser extent the behaviour of a brittle coating applied to a smooth, non-porous surface. In addition to these factors, the models used in this Thesis have a pockmarked, porous surface. Therefore, the use of brittle lacquers was dismissed as impractical.

The Conducting Paper Analogy^{38 71}

The conducting paper analogy depends upon the fact that the voltage distribution in a uniformly conducting plane is governed by the equation of Laplace³⁸. It may be demonstrated^{58,59} that the sum of the principal stresses in a planar, isotropic and elastically stressed body (free from body forces) is also governed by the Laplace equation. Consequently, if voltages corresponding to "applied load" are applied to the boundary of a uniformly conducting sheet which is cut into the shape of the stressed model, the voltage at any point in the conducting paper may be correlated

to the sum of the principal stresses in the model.

This method requires that some other method be used in conjunction with it to separate the values of the principal strains. The primary reason for discarding the use of this method was that the conditions at any boundary outside or inside the model must be known, in order to determine the voltage values to be applied. It was considered that any assumption concerning boundary conditions at the mortar-aggregate interface had still to be justified. If such assumptions were made, the whole purpose of using realistic materials for the models was lost. Any assumption of boundary conditions would allow theoretical analysis of the entire problem, and there would be no necessity for carrying out model tests.

Photo-Elastic Coatings

There has been a wealth of material written on the theory of photoelasticity^{73,78}, and it is not proposed to enter upon a detailed discussion of this theory. In principle, the method is based on the phenomenon of "bi-refringence" exhibited by certain materials under load. In such materials, the transmission of light at any point is governed by the orientation of the principal strain axes and by the relative magnitudes of the principal strains acting along these axes. The transmission of polarized light through such a material will result, at any point, in two mutually perpendicular light vectors, lying in the planes of the principal strains, and having a certain "relative retardation" due to any non-equality of the two principal strains. The directions of the principal strain axes (isoclinics) and the values of the principal strain differences (isochromatics) may be determined at each point, therefore, by analysis of the light vector resulting from the transmission of polarized light through any bi-refrangent material under load.

In the last fifteen years, an experimental photo-elastic technique has been developed for use when opaque or non-birefringent materials

are to be examined^{19,79-82}. The basis of this technique is the adhesion of a birefringent layer of plastic to the material under consideration. If a reflective backing is placed behind the plastic, then equipment can be used which will shine polarized light through the plastic, normal to the plane of the coating, and analyze the reflected result. This technique is referred to as the "normal incidence" method. The theory for photo-elastic coatings in no way differs from the standard theory of transmitted polarized light.

The use of such birefringent layers of plastic, or photo-elastic coatings, in conjunction with normal-incident polarizing and analyzing equipment may furnish, at any point, the following information:

- 1) The values of the isoclinics or directions of principal strain at the point. Determination of the isoclinic values depends upon the achievement of light "extinction" at a point as the polarizing and analyzing units are rotated with respect to some arbitrary axis. As the two principal strains at a point on a surface are mutually perpendicular⁵⁸, determination of the orientation of one principal strain with respect to some arbitrary axis automatically determines the orientation of the other. Throughout this Thesis any isoclinic given, represents the angle subtended between the direction of the applied minor (most compressive) principal stress and the direction of the minor (most compressive) principal strain at the point.
- 2) The value of the isochromatic passing through each point. The isochromatic value is determined by the amount of interference between the two light vectors at a point, this interference being a function of the relative retardation of the vectors. The colour of the light pattern seen at a point is used to measure this interference, and points of equal colour are termed isochromatics. From the theory of photo-elasticity, the value of the isochromatic is directly related to the value of the principal strain difference, or shear strain

at the point. The factor relating the two values is the fringe coefficient of the birefringent layer used. Through general usage, the word "isochromatic" has come to mean any line along which the value of the principal strain (or stress) difference is equal i.e. a line of constant shear.

- 3) Information of a qualitative nature may also be obtained concerning the initiation and propagation of fracture^{38 78}. It is necessary to assume that the overall pattern of a local strain field will remain unchanged with increasing load unless there is some alteration to the geometry of the system. In other words, the relationship between the strain values at any two points in the model are assumed to remain constant as long as the physical geometry of the specimen is unchanged. Such an assumption is only valid if the Young's Modulus and the Poisson's Ratio of the material are not strain-level dependant. Such an assumption does not appear completely valid for concrete or mortar (see Chapter II, Section II - 5). Nevertheless, it would appear reasonable to suppose that abrupt alteration of the overall strain pattern corresponds to some abrupt mechanism of change in geometry. Such a mechanism may be considered as being the formation of a crack. Further discussion on this topic is given in Section IV - 3.2.

It is not proposed to consider, in detail, the precise mechanics of the use of photo-elastic coatings to determine isoclinic and isochromatic values. These are fully discussed in the trade literature of various equipment manufacturers.

The important points, in the context of this discussion, are that the use of photo-elastic coatings provide a means for the rapid assessment of the values of the isoclinics and of the isochromatics in any two-dimensional strain field. Use of this method may also provide qualitative information concerning the point of fracture initiation and the path of fracture propagation. Advantages of this method are that "whole-field" cover is possible and that, with experience, areas of interest may be quickly

detected by visual observation. The basic equipment required is simple, if somewhat expensive (approx. £300). Disadvantages include the cost of photo-elastic coatings, which are generally non-recoverable; the reinforcing effect of the coating⁸², and the "shear-lag" experienced through the thickness of the coating⁸³. Further error is introduced by the small angle subtended between the polarizing and analyzing units. This angle should theoretically be zero. However, the primary disadvantage to the method is that some additional method is required in order to obtain the individual values of the principal strains from the principal strain difference.

Methods for use in Conjunction with Photoelastic Coatings

Any of the previously mentioned strain measuring techniques could be used in conjunction with photo-elastic coatings to separate the values of the principal strains and thus give complete resolution. Each method would still be subject to its particular limitations. Further possibilities are discussed below.

Oblique-Incidence

It would seem logical to use as an additional method some technique employing the photo-elastic coating. The previous discussion was concerned solely with the use of equipment which applied polarized light normal to the face of the coating. Such light would also be reflected normal to the coating face. Use of this equipment gave values of the fringe order at a point, this fringe order being some function of the principal strain difference. It has long been recognized⁷³ that use of oblique-incident polarized light will also give values of fringe order at a point, these values being some other function of the principal strain difference, depending upon the angles of incidence and reflection used. Thus the two methods in conjunction allow the solution of two simultaneous equations in terms of the principal strain difference. Solution of these equations, of the type:

$$n_1 = f(\epsilon_1 - \epsilon_3)$$

$$n_2 = g(\epsilon_1 - \epsilon_3)$$

will give values for each of the principal strains.

The method has been developed for use with photoelastic-coatings, primarily by the Budd Co. in the U.S.A. A special instrument, shown in Fig. IV - 10, is required. This instrument transmits polarized light to the coating at an angle of 45° to the plane of the coating, and analyzes the reflected result. The resulting fringe order is related to the principal strain difference by an equation of the form shown above, the particular equation depending upon the particular equipment used.

Readings of the isoclinic and isochromatic values at the point under consideration must first be obtained using the standard normal incident equipment and techniques, such as that shown in Fig. IV - 11. The "oblique incidence meter" is then positioned over the point, in intimate contact with the coating, and aligned along one of the principal strain directions. Isochromatic readings are taken using the meter, and the two equations derived from the two isochromatic readings may then be simultaneously solved.

As in the available testing machine the plate models were tested in a vertical plane, it was necessary to construct a special traversing frame for the oblique incidence meter. This frame had to be capable of both vertical and horizontal movement. In addition some provision was necessary for correctly aligning the meter along one of the principal strain directions, and for pressing the meter into intimate contact with the coating. A frame was accordingly designed and constructed and is shown in Figure IV - 11. Due to the geometry of the testing machine used, it was also necessary to construct an "optical extension" tube for the meter, which can also be seen in Fig. IV - 11.

Pilot tests of plate specimens containing a central hole (see also Section IV - 3.2) were carried out to determine whether such a method was



Oblique-Incidence Meter

FIG. IV-10



Test Set-Up Showing Normal and Oblique Incidence
Photo-elastic Equipment.

FIG.IV-11

feasible. For several reasons it was regretfully decided that it was not.

Firstly, it was found to be virtually impossible to accurately position the meter over the point under consideration. Precise alignment of the meter along a principal strain direction was also found to be extremely difficult. In addition the geometry of the testing machine only allowed analysis over the central position of the model. Then, in order to obtain optical contact with the coating it was necessary to introduce a film of cedarwood oil between the meter and the coating. Such a film placed on the vertical model face merely ran down the coating and optical contact was lost. In order to regain this contact, the meter was pulled away from the coating face and the cedarwood oil film replenished. Replacement of the meter then meant complete re-alignment.

Use of the normal and oblique incident methods in conjunction meant that any errors introduced were cumulative. Certain errors in the isoclinic and isochromatic readings from the normal incident method were inevitable. (see Part B). Alignment of the oblique incidence meter was governed by this value of the isoclinic at the point. Intrinsic errors in the use of oblique incidence methods⁵⁸ were multiplied by the practical difficulties mentioned above. Solution of the two simultaneous equation again multiplied any errors present.

It was found that values of the principal strains derived from this method differed by up to $\pm 200\%$ from the theoretically derived values for the strains around a hole (Section IV-3.2 and Table IV - 2). Differences of this magnitude could not be acceptably explained by any differences between the case from which the theoretical solution was derived and the actual case under consideration. Use of this method was, therefore, discontinued.

Strain Gauges

The view of several experienced stress analysts who were approached

was that the most practical solution to the problem of separating the principal strain values would be to use short-length electrical resistance strain gauges. This method required that values of the isoclinics at each point be determined by use of the normal incident photo-elastic equipment. The position of each point of interest would then be duplicated on the "back" face of the specimen, i.e. the face not covered by the coating, and short strain gauges placed at each point, aligned along one of the principal axes. Thus, direct measurement of one principal strain would be possible. Simple subtraction of the value of this measured strain from the measured value of the isochromatic at the point would give resolution of the strains. Obviously the accuracy of such a method for resolution of an entire strain field depends on the number of points within the field which it is physically possible to gauge.

The use of gauges was subject to the same limitation of finite size as with strain gauge rosettes. However, as only uni-directional gauges were to be used, extremely small sizes could be obtained. The method also pre-supposes that the state of strain would be the same on both free faces of the model. A means of correcting for non-equality of the strains on each face is given in Section IV - 3.1.

After development tests had been carried out, (Section IV - 3), it was decided to use this method of gauging, in conjunction with the normal incident use of photo-elastic coatings, as an experimental strain measuring system.

B) Details of Equipment Chosen.

Normal Incidence Photo-elastic Equipment.

The equipment used was a "reflection polariscope", Model 030, manufactured by Photoelastic Inc., U.S.A. Equipment is supplied in this country by Horstman Ltd. As the points of analysis in the models were of small extent, a telemicroscope attachment, Model 037, was used.

Photoelastic Coatings

After investigation of the reinforcing⁸² effects of coatings, and taking into consideration the sensitivity required, a photo-elastic coating was chosen. The coating was manufactured by Photoelastic Inc. and is designated PS-1A. Thickness of the coating was 0.125 ins.[±] 0.01 ins., with a fringe sensitivity of the order of 600 μ in/in/fringe. Figures for the exact fringe sensitivity were quoted for each coating supplied. The recommended adhesive, PC-1 reflective cement, was used for attaching the coatings.

The above equipment was calibrated using the standard cantilever-beam method⁸⁰. A strip of coating was attached to a steel cantilever beam, and a known load applied to the beam. The values of the difference of principal strains (isochromatics) obtained were correlated with the strains predicted by elastic beam formulae and found to agree within $\pm$ 2%. It should be noted that as the method depends upon colour recognition, individual calibration of the equipment by each operator using the equipment is essential.

Shaping of Coatings

The method is only valid for use near any discontinuous face if the geometry of the free coating edges corresponds to the geometry of the free edges of the model⁸¹. If it is not possible to fulfil this requirement, the coating edges must be remote from the model edges by at least four times the coating thickness. In such a case the coating edges should be bevelled at 45° if any measurements are to be taken near the coating edges.

Coatings for the models were cut and bevelled using a band-saw and special jig. As the coatings were known to be heat sensitive it was important to use extremely sharp tools and plenty of coolant. The coolant used was turpentine, as the coatings are moisture sensitive.

Reference Grid

A reference grid was scribed on the coatings using well sharpened drafting implements. As cylindrical aggregate inclusions were to be used, a grid in polar co-ordinates was utilized. In general, the grid was scribed after attachment of the coating to the specimen in order that the location of the inclusions with respect to the coating could be accurately determined. (Fig. VI - 1 shows a typical grid).

Attachment of Coating

The model specimens were first prepared as in Section IV - 2.7. For porous materials, the coating manufacturers recommended that an initial sealing layer of adhesive be used. This was found to be unnecessary. Normal manufacturers instructions were otherwise followed. The adhesive was allowed to cure for at least 24 hrs. at ambient conditions. As freshly cut edges of the coatings were found to be moisture sensitive, care was exercised in building up a layer of adhesive over such edges.

Figure IV - 12 shows a coating attached to a plate specimen with a central hole.

Gauges

Electrical resistance strain gauges were manufactured to the author's specification by the Showwa Sokki Co. Ltd. of Japan. (Distributors Graham and White, St. Albans). The gauges were designated as Fb - 2 and were of the "foil" type. Gauge length was 2 m.m. with a resistance of 120 ohms $\pm$ 0.1 ohm and a gauge factor of approximately 2. Details of the positioning of the gauges are given in Sections IV - 3 and IV - 4.

Section IV - 2.7

Preparation of Specimens

The method of preparation of the plate model specimens was governed

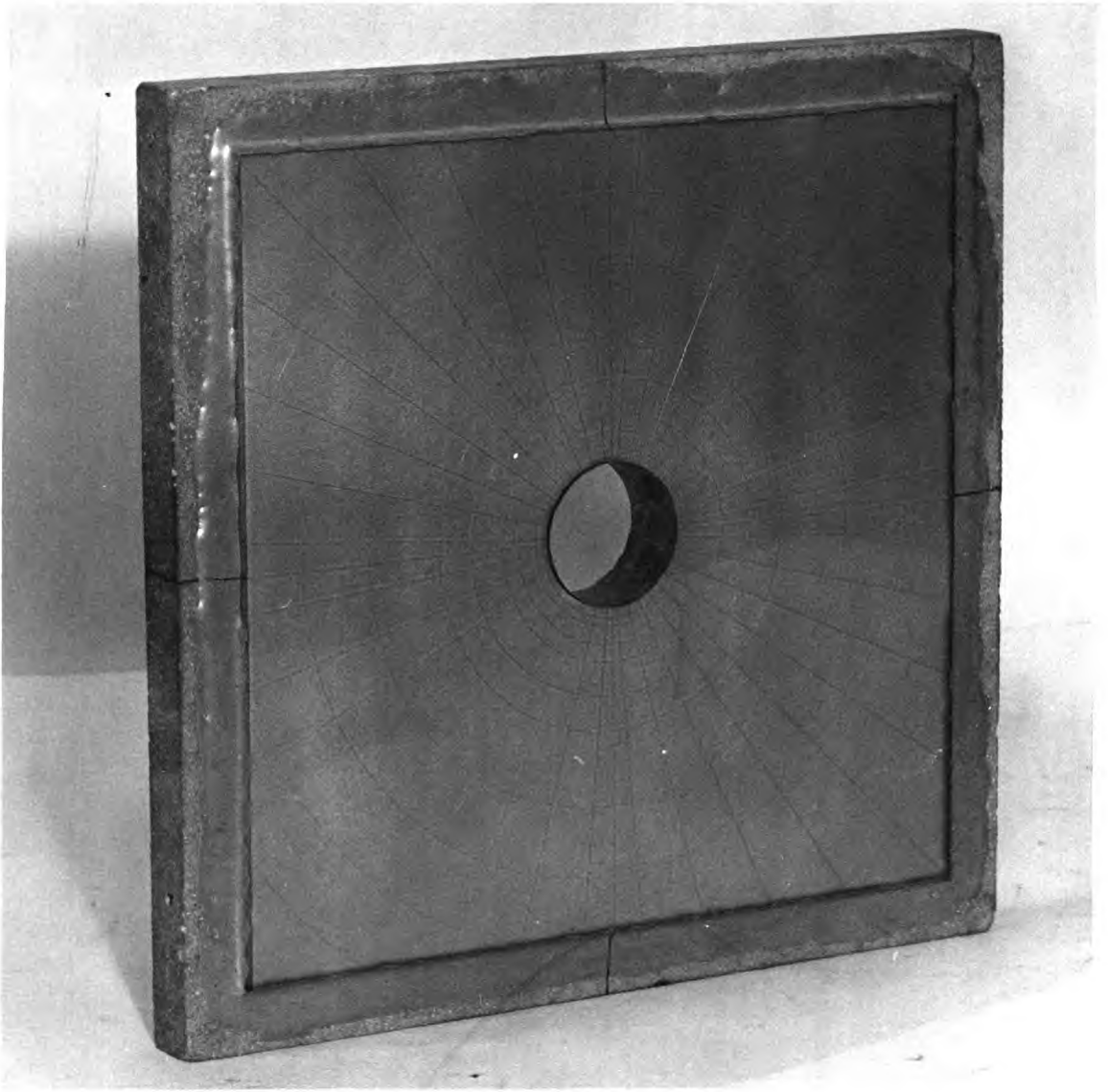


Plate-Model Specimen with Central Hole

FIG.IV-12

primarily by the experimental strain measuring system chosen.

In order to achieve models having uniform properties across their thickness, and to produce flat surfaces suitable for the application of a photo-elastic coating, it was decided to cast the specimens oversize, approximately 1 inch, and then to grind the specimens to the required thickness of $\frac{3}{4}$ ins. (see Section IV - 2.5). For this purpose an Abwood vertical head grinding machine was obtained. The grinding wheel used was a diamond impregnated lense-polishing wheel. Use of this equipment resulted in specimens that were dimensionally correct to within ± 0.001 ins. and perfectly square within the accuracy of measurement. All faces and edges were perfectly flat within the sensitivity of measurement, except for the inevitable presence of small air voids. As only very shallow cuts, approximately 0.020 ins. were used in the presence of plenty of coolant (water), it was considered that no harmful deterioration of the specimen was caused by this grinding process. Observations before and after grinding, using the method outlined in Section IV - 2.8, confirmed this belief.

After grinding, the specimens were air dried for at least two weeks before placing the photo-elastic coatings. This was necessary as the adhesive was known to be moisture sensitive. Further discussion of this point is given in Section IV - 3.1.

Section IV - 2.8

Observation of Fracture

The possible use of photo-elastic coatings to observe the onset and propagation of fracture has already been mentioned in Section IV - 2.6 and further discussion on this point is given in Section IV - 3.2. It was considered that some supplementary method of observation would be of value.

A method used for many years in the metallurgical field was that of a "wash" composed of fine particles of fluorescent material suspended in a light oil. Such particles are concentrated by capillary action in

fine cracks, which then showed as bright lines when viewed under ultra-violet light. This method, which has been utilized to determine the extent of cracking in concrete specimens⁸⁷, was found to give excellent results for the visual observation of cracks which had formed. However, it was found to be extremely difficult to photograph the results. The determination that a crack existed at a point was based upon an observed difference in the intensity of fluorescence. In other words, cracks showed as "brighter" lines of fluorescence than that given off by the background material. It appeared that whereas the eye could distinguish this difference quite clearly, the camera could not. It was, therefore, necessary to pencil-in the observed cracks before they were photographed. Further work is presently being undertaken on this problem. In this Thesis, photographs of the crack patterns of failed specimens were all derived by this method.

From a consideration of the particle size present in the fluorescent wash used, it was thought that cracks of a width from 5 to 10 micro-inches could be detected.

The equipment used was:

"Cermor" Fluorescent Ink, supplied by the Manchester Oil Refineries (Sales) Ltd., Manchester.

Ultra-violet "black light" bulb, MBW-11, 125 watt and Ballast for ultra-violet bulb, L4125 supplied by the Phillips Electrical Co. Ltd.

A general view of the equipment is shown in Figure IV - ¹³14.

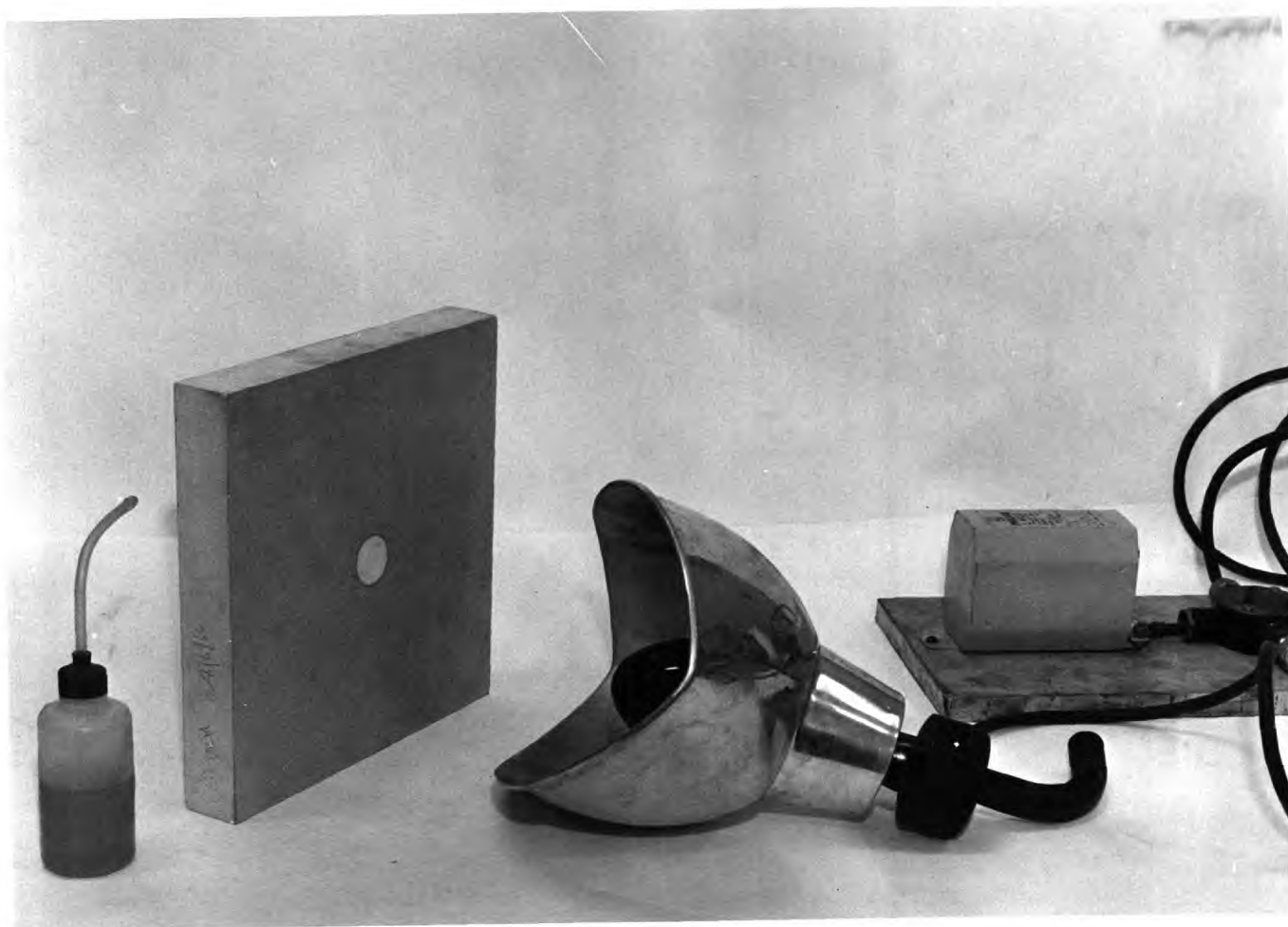
Section IV - 3

Development Tests for Experimental Technique

Section IV - 3.1

Buckling Tests

As concluded from Section IV - 2.1, plate models were required having the minimum thickness practical. Accordingly, tests were performed to determine whether or not buckling effects were of importance.



Ultra-Violet Equipment Used for Crack Detection

FIG. IV-13

The first series of tests was carried out on specimens with thicknesses from $1\frac{1}{2}$ ins. to $\frac{3}{4}$ in. Both faces of the specimens were strain gauged, and the results at equivalent points on the two faces, compared. A further test series utilized photo-elastic coatings on each face of the specimens.

From such tests it emerged that extremely careful centering and positioning of the specimens with respect to the testing frame was necessary. If the required care was exercised, the strains at equivalent points on opposite faces were found to be generally compatible within $\pm 10\%$. This figure was not found to be improved by use of the thicker, $1\frac{1}{2}$ in., specimens. It was, therefore, decided to use specimens of $\frac{3}{4}$ in. thickness.

Face Correction Factor

In order to help reduce the errors due to any non-equality of the strains at corresponding points on opposite faces of the specimens, a "face correction factor" was used. It was decided that the shear strains at four points remote from the inclusions would be determined on the photo-elastic coating face of a specimen. Readings of the shear strain at corresponding points on the gauged face of the specimen would be determined by the use of pairs of crossed gauges. The ratio of the average result of these "field" strains from each face was termed the "face correction factor" for the specimen. Thus:

$$\frac{\text{avg. } [(\epsilon_3 - \epsilon_1)F] \text{ gauged face}}{\text{avg. } [(\epsilon_3 - \epsilon_1)F] \text{ coating face}} = \text{face correction factor.}$$

where suffix "F" refers to "field".

This factor was found in general to vary between 0.90 and 1.10. This correction factor was then applied to any isochromatic values obtained from the photo-elastic coating.

It was fully realized that such a factor, derived from a ratio of

the "field" shear-strains, might not be strictly valid for application to the shear-strains determined at any point on the model face. However, it was considered that use of such a factor would at least help to minimise the errors due to any non-equality of strains between faces.

Section IV - 3.2

Plate Specimens with Central Hole

It was necessary to determine if the experimental strain measuring technique chosen gave reasonable results. For this purpose tests were carried out on plate specimens containing a central hole (see Fig. IV - 12). It was considered that such specimens should relate closely to theoretical models. Accordingly, the equations of Muskhelishvili⁵⁹ were used to analyze the theoretical strain field existing around a hole in an infinite plate, composed of an elastic homogeneous material loaded under uniaxial compression. These equations are given in Table IV - 2. The values of the local strains thus calculated were non-dimensionalized with respect to the "field" strains in the material remote from the hole. The results of these theoretical calculations are shown in Figures IV - 14 to IV - 16. It should be noted that the isoclinics given represent the angle between the applied minor principal stress and the minor principal strain at the point in question.

Specimens were cast containing central holes varying from $\frac{3}{4}$ in. to $1\frac{1}{2}$ ins. in diameter. After preparation (Section IV - 2.7), a photo-elastic coating was placed on the specimen. The strain measuring system to be used, that of a coating in conjunction with gauges, required that readings of the isoclinics first be determined from the coating in order to be able to position the gauges correctly. It was immediately noticed that the isoclinic values were markedly different from those determined theoretically.

Treatment of Specimens Governed by Isoclinic Determination.

It was found that values of the isoclinics were extremely difficult to read and were often of different sign from those theoretically derived.

TABLE IV - 2

Theoretically Derived Equations Governing the Strain Distribution in an Infinite Elastic Plate Containing a Bonded Elastic Inclusion (after Muskhelishvili⁵⁹)

Radial Strain

$$(\epsilon_r) (E) = \left(\frac{\sigma_P + \sigma_Q}{2} \right) (1 - \gamma a) + \left(\frac{\sigma_P - \sigma_Q}{2} \right) (1 - 2\beta a - 3\delta a^2) \cos 2\theta$$

Tangential Strain

$$(\epsilon_\theta) (E) = \left(\frac{\sigma_P + \sigma_Q}{2} \right) (1 + \gamma a) - \left(\frac{\sigma_P - \sigma_Q}{2} \right) (1 - 3\delta a^2) \cos 2\theta$$

Shear Strain

$$(\tau_{r\theta}) (E) = - \left(\frac{\sigma_P - \sigma_Q}{2} \right) (1 + \beta a + 3\delta a^2) \sin 2\theta$$

where

$$\gamma = \frac{G(\eta_0 - 1) - G_0(\eta - 1)}{2G_0 + G(\eta_0 - 1)}$$

$$\beta = \frac{-2(G_0 - G)}{G + G_0 \eta}$$

$$\delta = \frac{G_0 - G}{G + G_0 \eta}$$

$$a = (R/r)^2$$

and

$$\eta = 3 - 4\mu ; \eta_0 = 3 - 4\mu_0$$

$$G = E/2(1 + \mu) ; G_0 = E_0/2(1 + \mu_0)$$

E = Young's Modulus of plate matrix

E₀ = Young's Modulus of inclusion

μ = Poisson's Ratio of plate matrix

μ₀ = Poisson's Ratio of inclusion

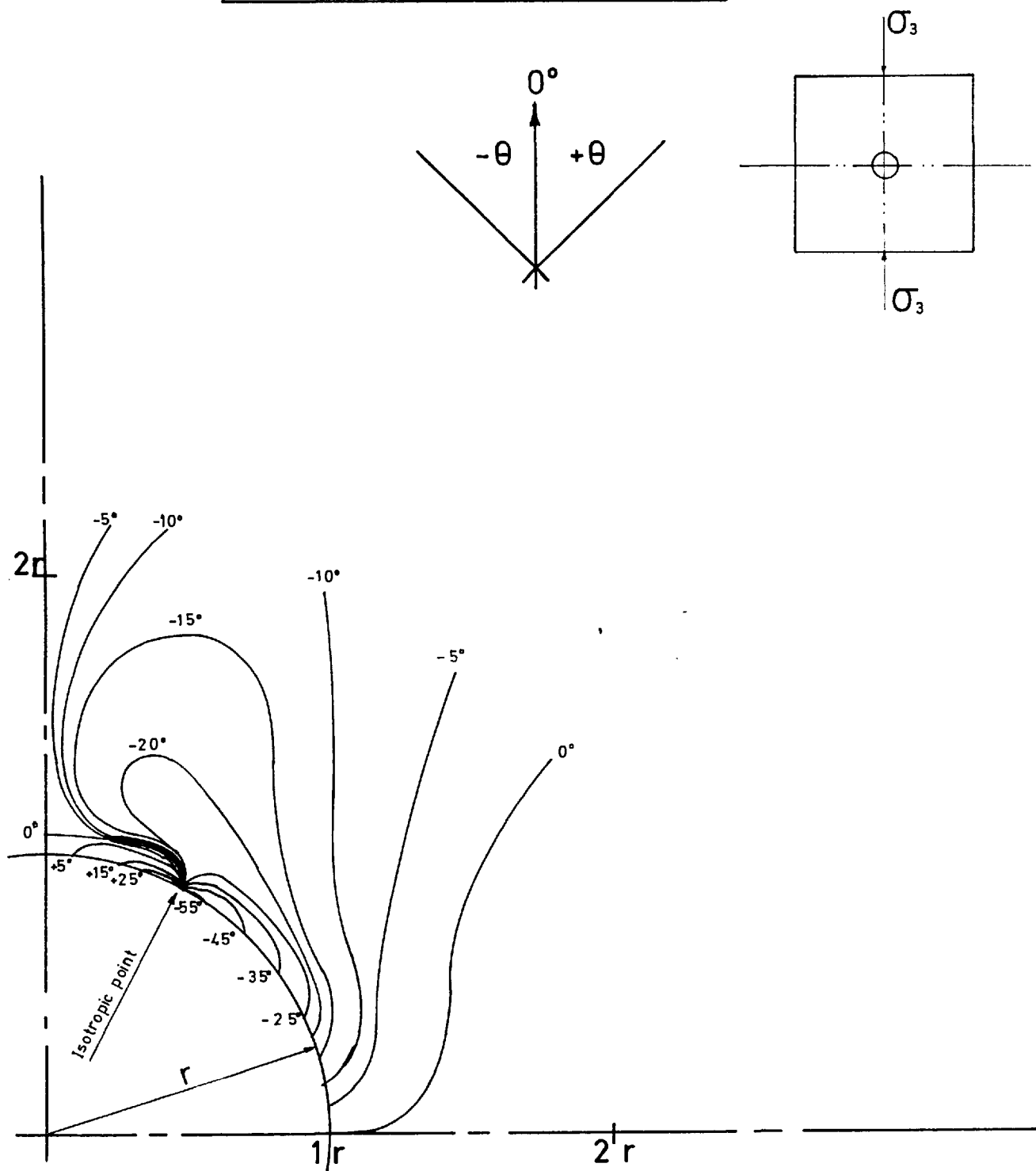
R = radius of inclusion.

r, θ = polar co-ordinates of point

σ_P = major principal stress remotely applied

σ_Q = minor principal stress remotely applied

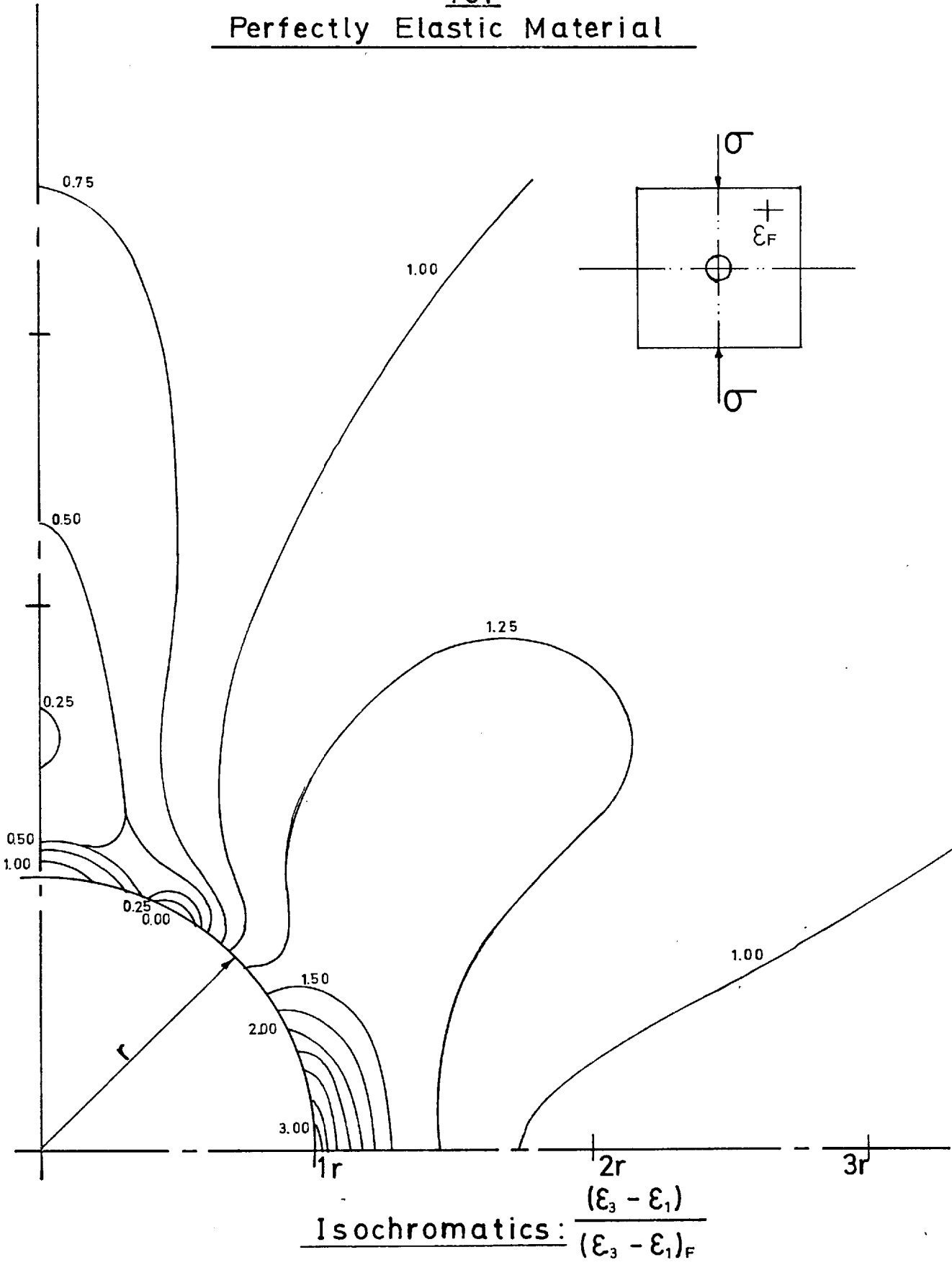
Theoretical Calculation
for
Perfectly Elastic Material



Isoclinics
Specimen with Central Hole

FIG. IV-14

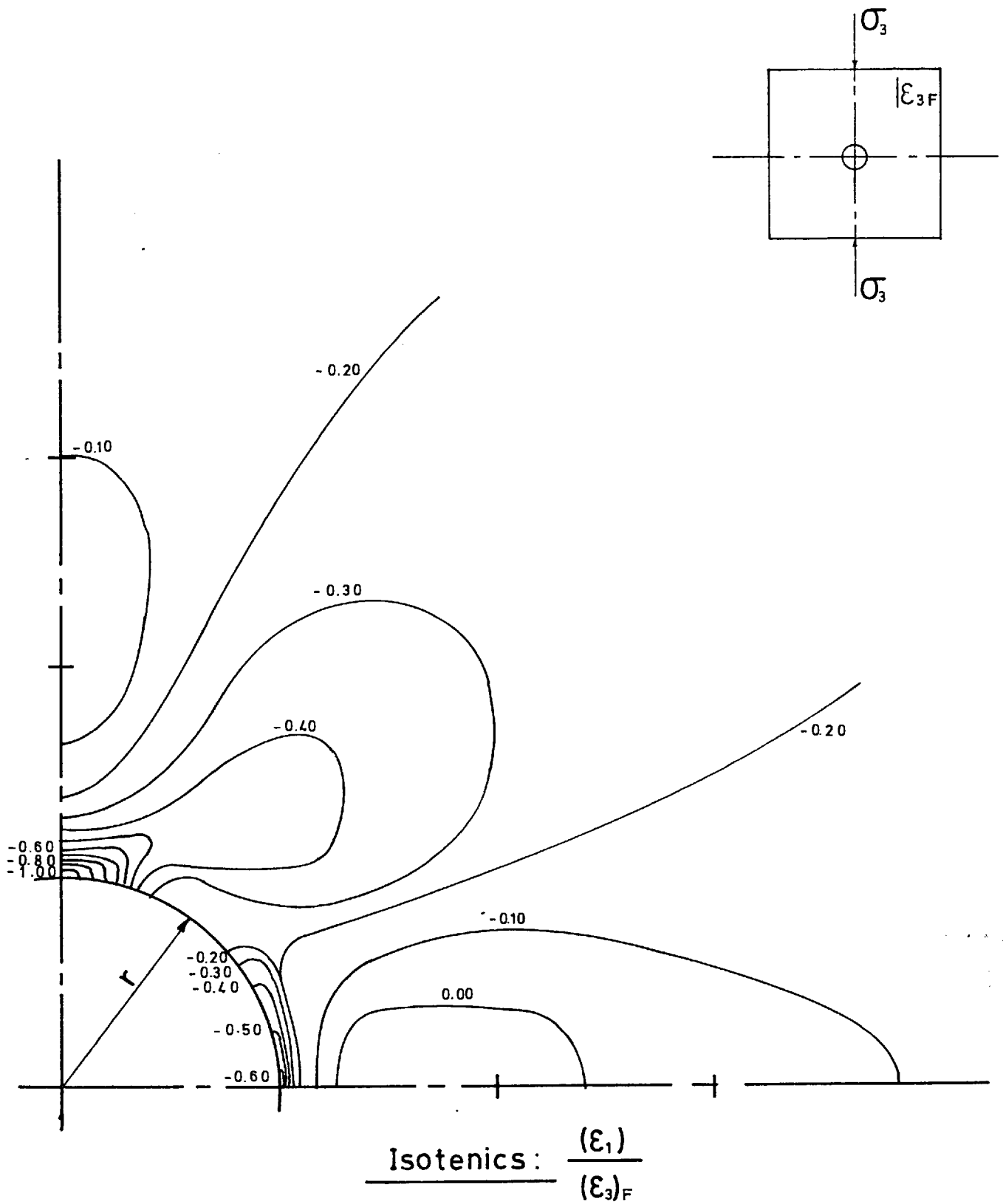
Theoretical Calculation
for
Perfectly Elastic Material



Specimen with Central Hole

FIG. IV-15

Theoretical Calculation
for
Perfectly Elastic Material



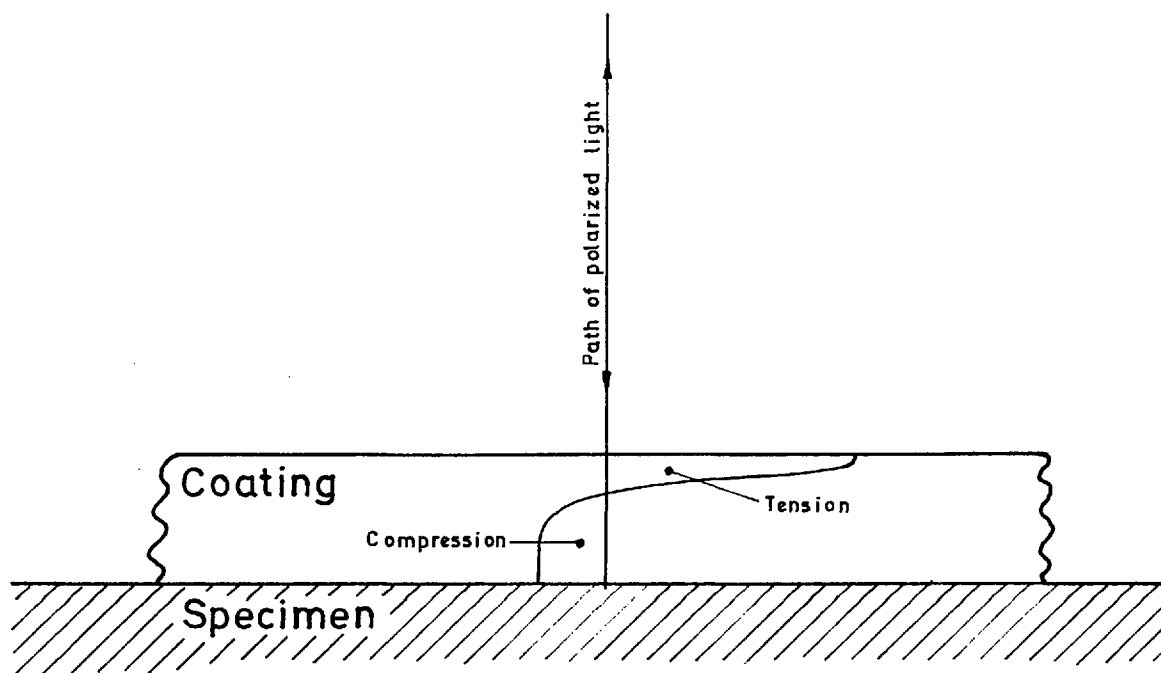
Specimen with Central Hole

FIG.IV-16

It was considered that discrepancies of this magnitude could not be adequately explained by physical differences between the theoretical and experimental models, and must indicate some fault in the experimental measuring technique. Added weight was given to this premise by the observation that the isochromatics correlated well with theory - only the isoclinics seemed badly in error.

After discussion with several stress analysts⁸⁴⁻⁸⁶, an explanation of this phenomenon was proposed which seemed to explain the observed facts. It was known that both the photo-elastic coating and the adhesive used were moisture sensitive. Application of these materials to dry impermeable surfaces, such as of metals, generally only resulted in moisture errors after long periods of time. However, use of these materials on concrete or mortar would appear to set up, virtually instantaneously, a moisture gradient across the thickness of the coating. The hygroscopic nature of the materials appeared to attract water from the structure of the cement gel phase. The result of such a moisture gradient is shown schematically in Figure IV - 17. As the net resultant of the forces across the coating section must, from equilibrium, be zero, net values of the principal stress difference (isochromatics) at a point on the specimen should be unaffected. Such was observed to be the case. However, the direction of the principal stresses due to the moisture gradient could be expected to vary throughout the thickness of the coating. Thus, the net isoclinic value at the point would be due partially to the stress directions in the specimen and partially to the stress directions in the coating, resulting in hazy isoclinics and giving erroneous readings.

In an effort to overcome these difficulties, further tests were performed in which specimens were used saturated (with the addition of a silane bonding compound to the coating adhesive); oven dried to constant weight, and air dried to varying degrees of saturation. In none of these different cases were the measured isoclinic values any closer to those



Stresses in Coating
Due to
Moisture Differential

FIGURE: IV-17

theoretically determined.

It was eventually discovered that if a specimen, with a coating attached, was allowed to stand at ambient conditions for a period of six to eight weeks, the isoclinics became sharp, easy to read, and with values close to those theoretically determined. It was thought that this was due to the eventual attainment of moisture equilibrium across the coating thickness. It was, therefore, decided to follow this procedure for all specimens, and allow them to stand for approximately six weeks after attaching the coating before any tests were run.

It was also found that reading of the isoclinics was made considerably easier if the loads applied to the specimen for this purpose were kept to a minimum. Stresses in the range 500-600 p.s.i. were generally found to be most suitable.

Positioning of Gauges

The tests on specimens with a central hole were re-run after following the above procedure. Measurement of the isoclinic values then allowed the positioning of the gauges to be determined. Initial tests were conducted with the gauges aligned at each point in the direction of the minor principal strain, ϵ_3 . As the arithmetic value of this strain (the most compressive) was often an order of magnitude greater than the value of the major (most tensile) principal strain, ϵ_1 , it was found that considerable errors could be developed. As illustration, consider a point where the actual values of principal strain should have been;

$$(\epsilon_3) = -200 \text{ } \mu\text{in/in}$$

$$\text{and } (\epsilon_1) = +20 \text{ } \mu\text{in/in}$$

The shear strain value read from the coating would thus give (assuming for a moment that no error is involved in this measurement);

$$(\epsilon_3 - \epsilon_1) = -220 \text{ } \mu\text{in/in.}$$

For a gauge aligned in the (ϵ_3) direction, an error of $\pm 10\%$ could easily be developed due to slight misalignment or gauge errors. Thus, the measured value of (ϵ_3) might be:

$$(\epsilon_3) = -180 \mu\text{in/in, or, } -220 \mu\text{in/in.}$$

Calculation of the value of (ϵ_1) from the shear strain reading would then give:

$$(\epsilon_1) = +40 \mu\text{in/in, or, } 0 \mu\text{in/in.},$$

producing an error in this strain of $\pm 100\%$. For a gauge aligned in the (ϵ_1) direction, and following the same argument, the measured values of (ϵ_1) would be:

$$(\epsilon_1) = +18 \mu\text{in/in or } + 22 \mu\text{in/in}$$

and the calculated value of (ϵ_3) ,

$$(\epsilon_3) = -202 \mu\text{in/in or } -198 \mu\text{in/in.}$$

giving errors of $\pm 1\%$ in the value of (ϵ_3) .

It was obvious, therefore, that considerable advantage was to be gained by aligning the gauges in the direction of the major (most tensile) principal strain, ϵ_1 .

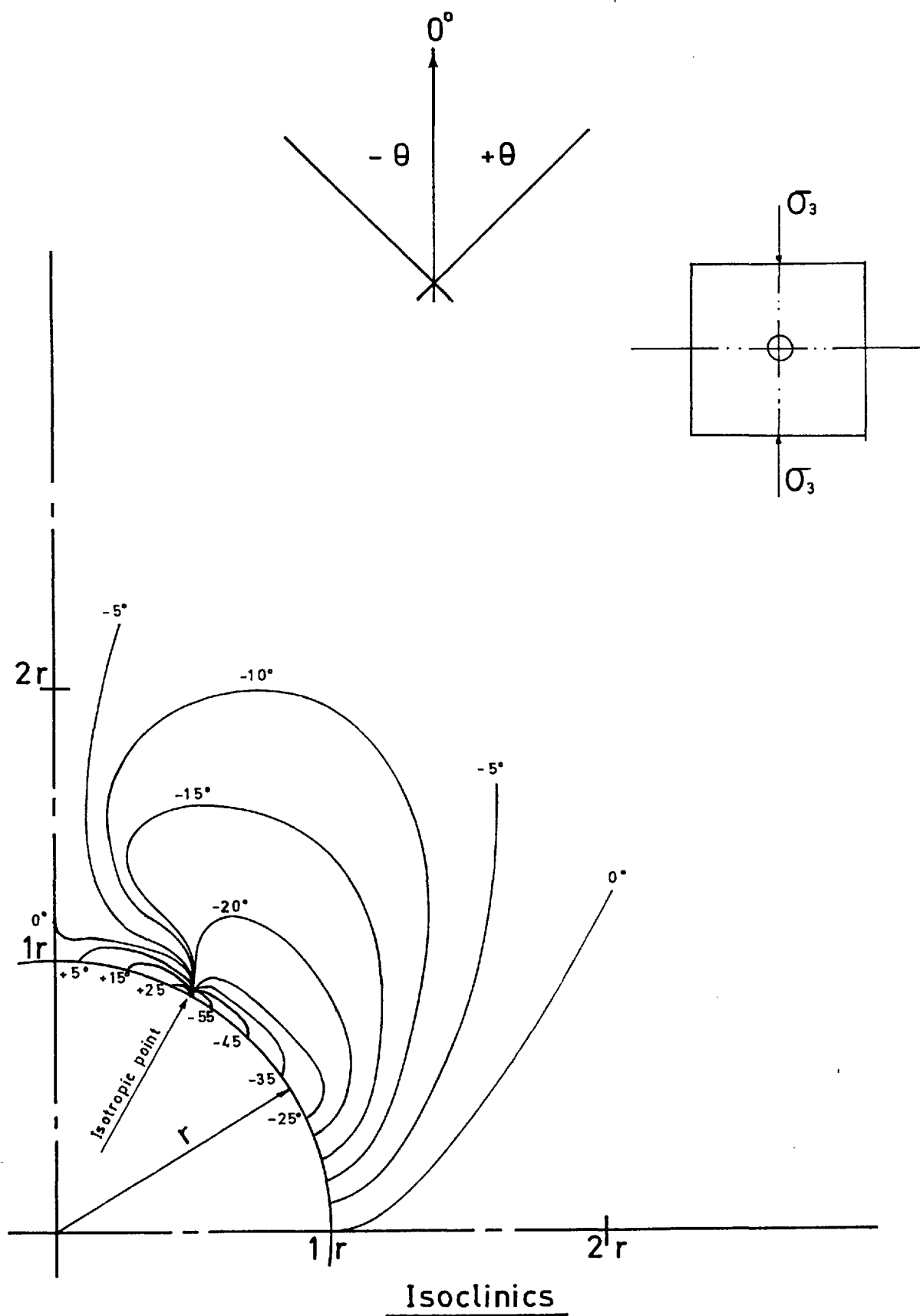
Results of Tests

Following the procedures outlined above, the central-hole specimens were retested. Typical results of these tests are shown in Figures IV - 18 to IV - 20 and may be compared with the theoretical results shown in Figures IV - 14 to IV - 16. The correlation of results was found to be acceptable. The general trends and magnitudes of the results were seen to be closely similar, and well within the limits of the validity of application of the theoretical results derived from a highly idealized situation.

It was considered, therefore, that the experimental strain measuring technique could be expected to give results which were meaningful.

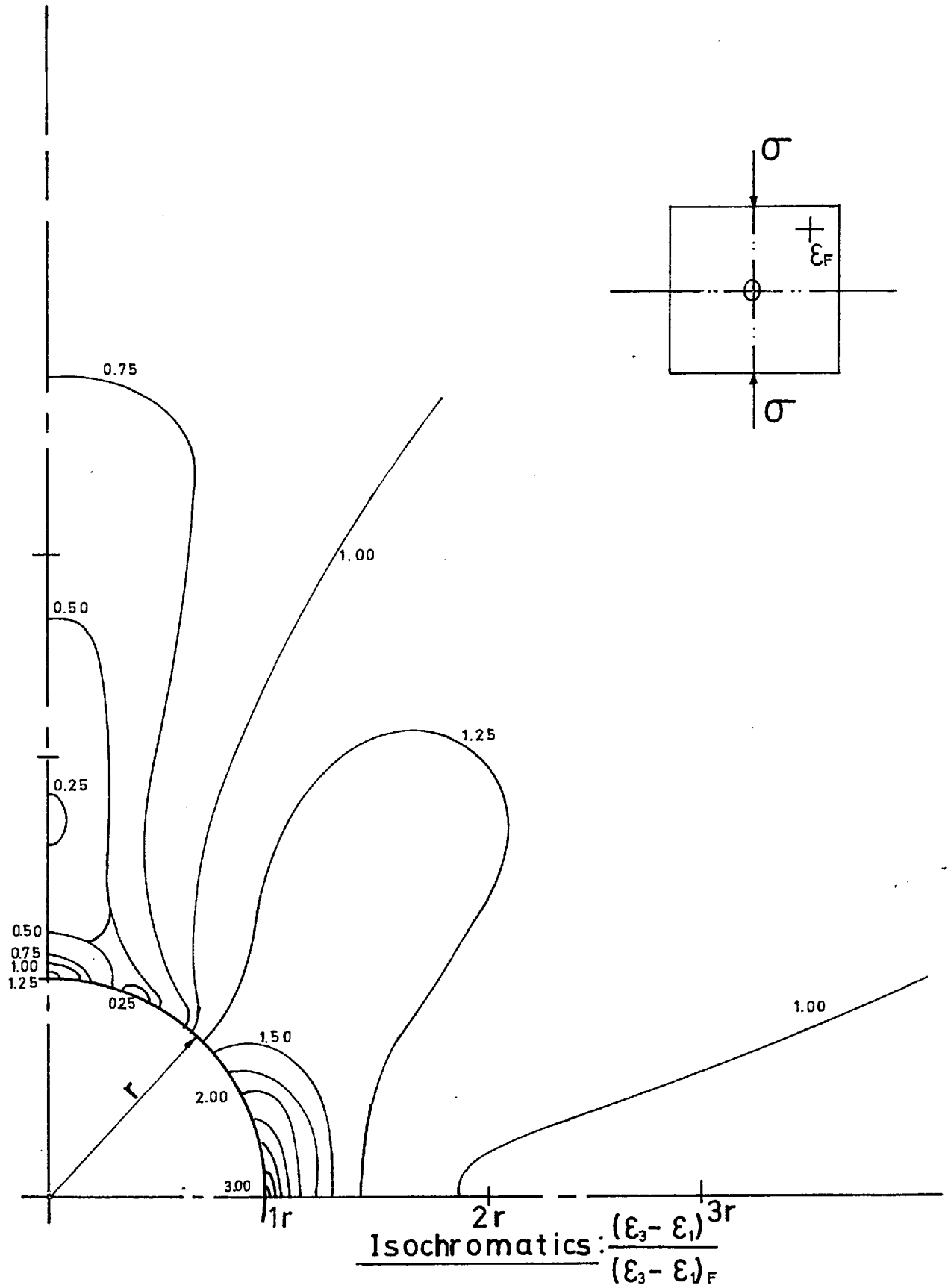
The Observation of Fracture Initiation and Propagation.

A brief discussion of the use of photo-elastic coatings in observing



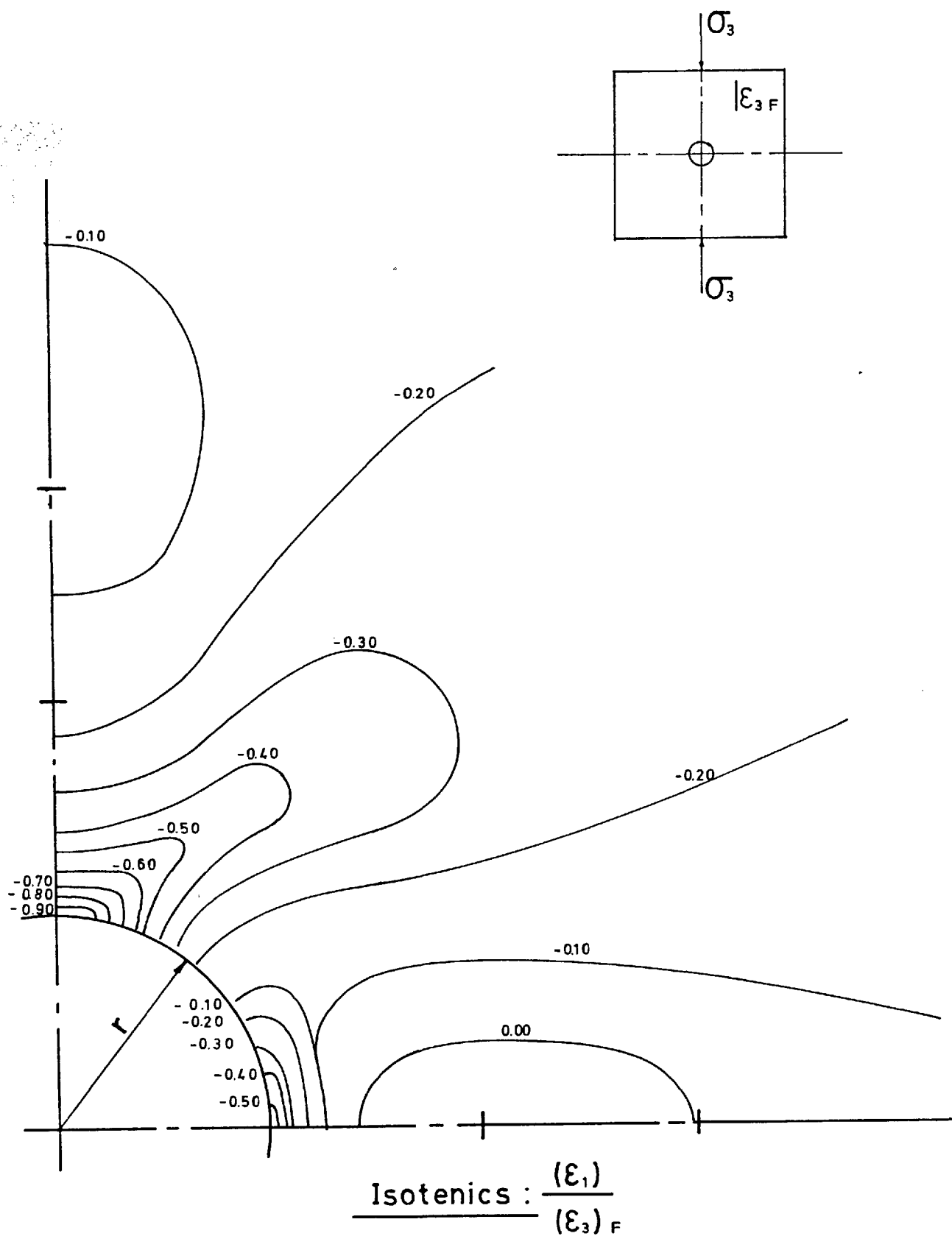
Specimen with Central Hole

FIGURE: IV-18



Specimen with Central Hole

FIGURE : IV-19



Specimen with Central Hole

FIGURE:IV-20

fracture processes was given in Section IV - 2.6. Hoek³⁸ has analysed the fracture processes around a circular excavation (hole) in a plate model. From his analysis, the sequence of fracture processes was as follows:

- 1) Due to the high tensile strains at the roof and floor of such an excavation, fracture initiated at very low loads at these points. Such fractures became extremely stable after propagation for a short distance.
- 2) Due to the redistribution of strains following these first fractures, fracture then initiated from four points symmetrically placed around the hole, but remote from it (approximately $\frac{1}{2}$ radius).
- 3) The further application of load propagated these "remote" fractures until failure was reached. The initial roof and floor fractures remained stable throughout this process.

In order to determine if similar fracture processes could be observed, the central-hole plate specimens were loaded to failure. A Schackman camera was used to monitor the isochromatic pattern of the photo-elastic coating during loading. This camera recorded the events at $\frac{1}{4}$ second intervals. Standard 35 mm. colour transparency film was used with the camera operating at $\frac{1}{200}$ th of a second at f4. A drawback of the method was that total running time of the camera was approximately 13 seconds. Thus, it was necessary to load the specimens to failure within this period.

Results of this technique for one specimen are shown in Figure IV -21. Unfortunately, due to the extremely low loads necessary to initiate the roof and floor fractures this specimen was slightly damaged during the initial testing. This was not considered to be serious as the only object of these tests was to determine if any fracture initiation and propagation processes could be determined.

Figure IV - 21 shows selected frames from a film strip of the isochromatic pattern generated around a hole by increasing the loads to failure.

Frame (a) shows an initial application of load. The overall pattern of the isochromatics should be noted. Small fractures at the roof and floor, due to initial testing, may be seen by the sharp concentration of fringes at these points.

Frame (b) shows the further application of load. The fringe orders have, of course, increased, but it may be seen that the overall pattern is unchanged from Frame (a).

Frame (c) taken $\frac{1}{4}$ second later, shows the sudden change of pattern at the point indicated. This sudden change corresponds to the initiation of fracture at this point.

Frame (d) shows the growth of the fracture indicated by frame (c) and the further initiation of fracture at the points indicated.

Frame (e) shows the result of these processes just prior to failure. Fracture has initiated at four points around, but remote from the hole boundary. The fractures have propagated. It may be seen by examination of the photograph that the roof and floor fractures initially present have not markedly increased in length.

Frame (f) shows the "frozen" isochromatic pattern after failure has occurred. This pattern clearly shows the path of propagation of the fractures which were initiated remote from the hole boundary. The small roof and floor fractures may also be seen. This photograph was taken from a different specimen from that represented by the previous frames, due to the total disintegration of that specimen at failure.

Figure IV - 22 shows the resultant crack pattern after failure. This crack pattern was observed by use of the fluorescent dyeing technique outlined in Section IV - 2.8, and can be seen to correlate with the "frozen" isochromatic pattern shown in Figure IV - 21, Frame (f).

It must be clearly understood that quantitative analysis from photographs such as these is not valid. Once a fracture has initiated, the free-edge geometry of the coating no longer corresponds to the free-edge geometry of the model. As the faces of a fracture discontinuity move



(a)



(b)



(c)

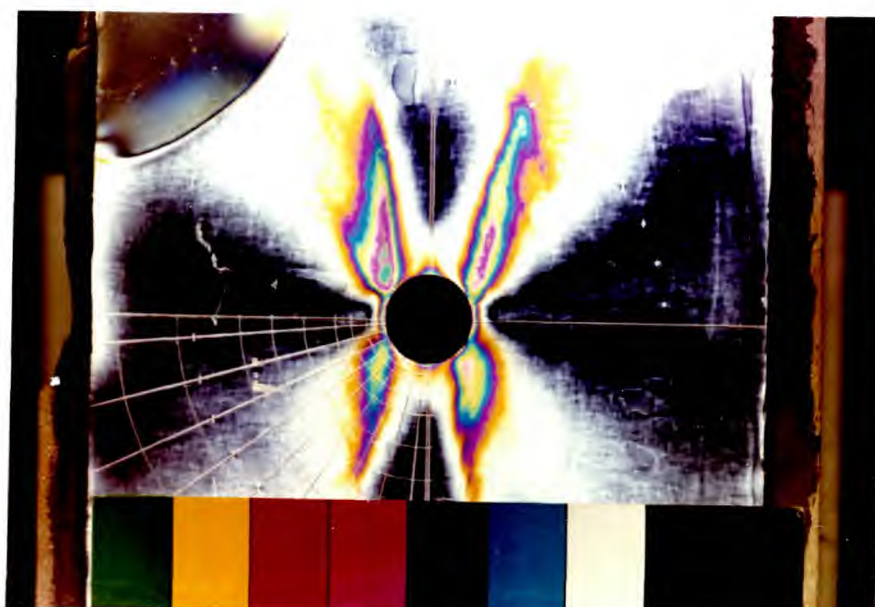
FIG. IV-21



(d)

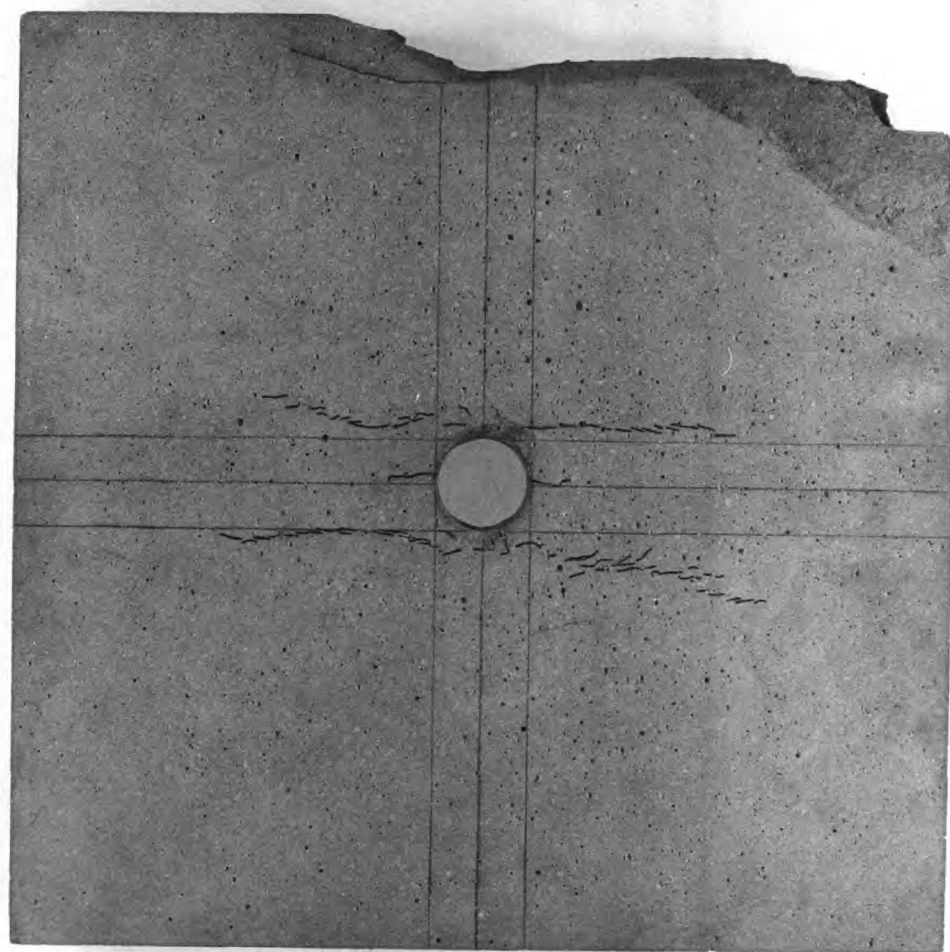


(e)



(f)

FIG. IV-21
(Contd.)



Final Crack Pattern in Plate Specimen
with Central Hole

FIG. IV-22

relative to one another, a massive concentration of isochromatics will be developed at the area in the coating covering such a discontinuity. Readings of shear-strain within such an area have no direct physical meaning, since strain cannot exist in a non-existent material i.e. between the faces of a discontinuity.

It was concluded from these tests that the method could be used to give a qualitative observation of the fracture processes in the models.

Section IV - 3.3

Single "Bonded" Inclusion Tests.

It was considered that the case of a single hard inclusion cast into a mortar plate model might correlate closely with the theoretically derived case of a perfectly bonded elastic inclusion existing in an infinite matrix of elastic material. If close correlation were found, then the theoretical analysis of the local strain distribution around aggregate particles might be considered justifiable.

The equations of Muskhelishvili⁵⁹ were used to determine the theoretical local strain field around a single bonded inclusion. The equations used are shown in Table IV - 2 and the results of these calculations are shown in Figures IV - 23 to IV - 26.

Plate models containing a single aggregate inclusion were cast and analyzed following the procedures previously outlined. Results of these tests are shown in Figures IV - 27 to IV - 30.

Comparison of the theoretically and experimentally derived results showed major differences. It was, therefore, decided that theoretical analysis was not justified and that the plate model tests were necessary to arrive at a realistic analysis of the local strain fields and fracture processes around aggregate particles.

Section IV - 4

Final Experimental Procedure

The investigations and tests outlined resulted in a final experimental

Theoretical Calculation
for
Perfectly Bonded Inclusion

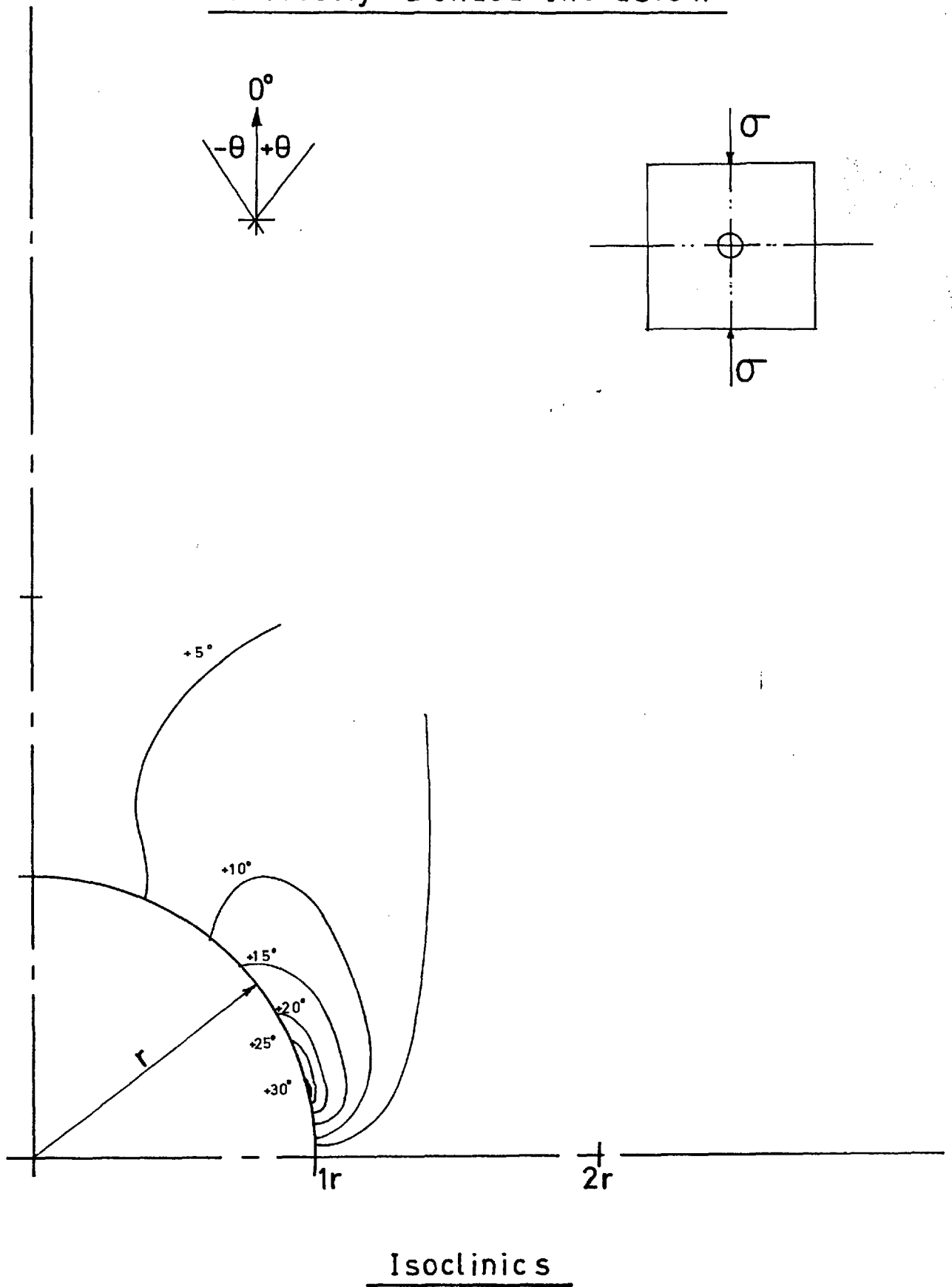


FIGURE : IV-23

Theoretical Calculation
for
Perfectly Bonded Inclusion

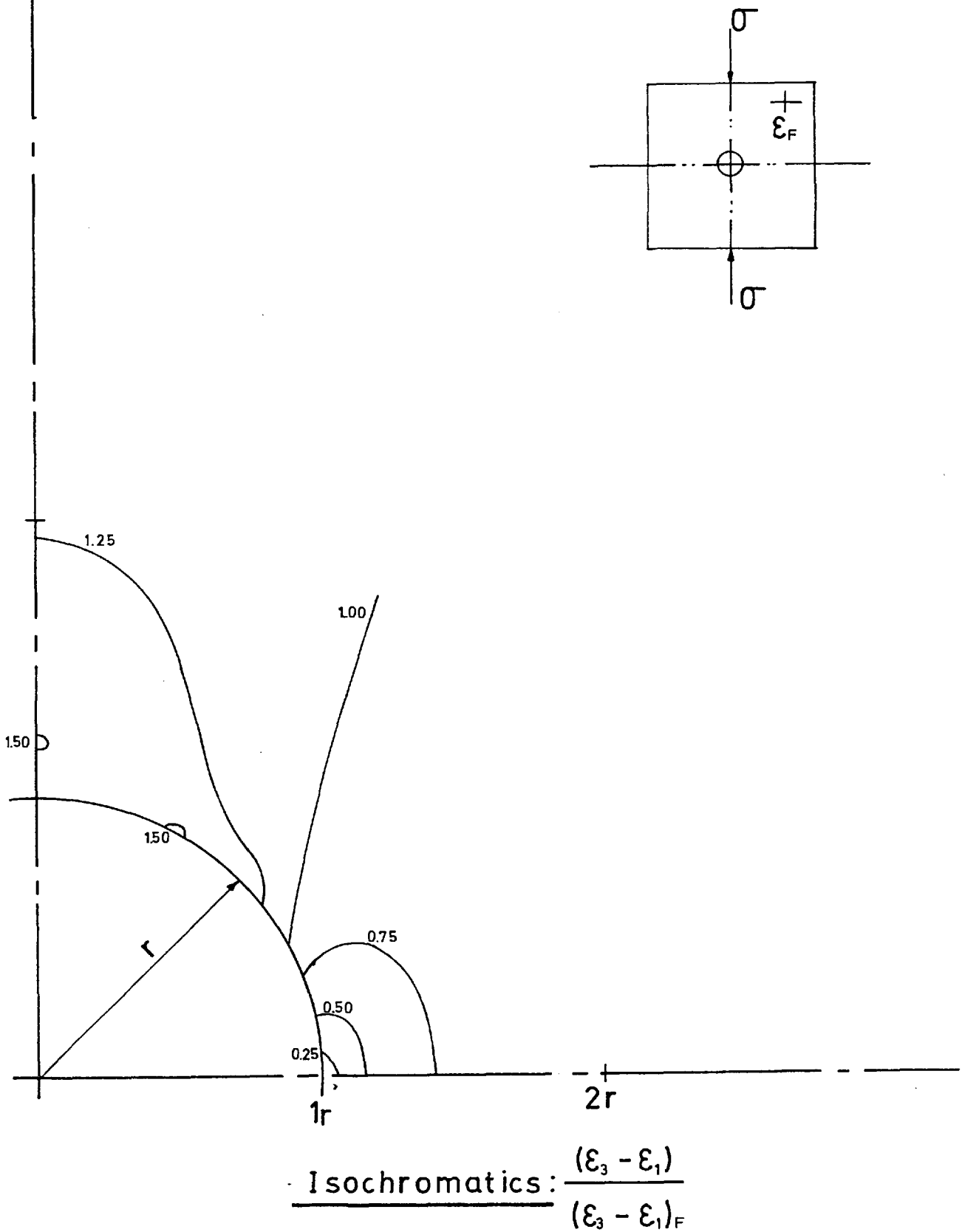


FIGURE: IV-24

Theoretical Calculation
for
Perfectly Bonded Inclusion

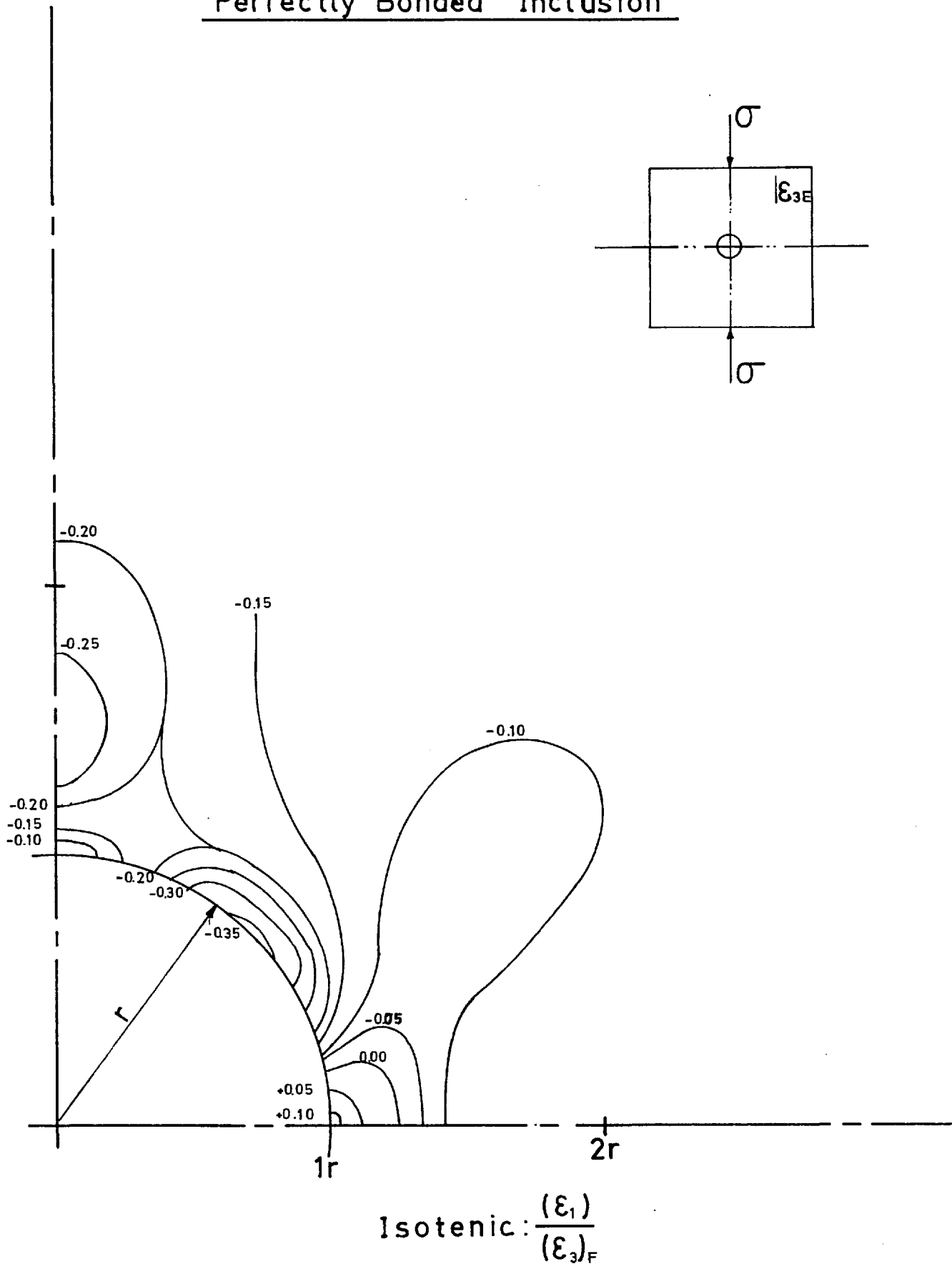


FIGURE : IV-25

Theoretical Calculation
for
Perfectly Bonded Inclusion

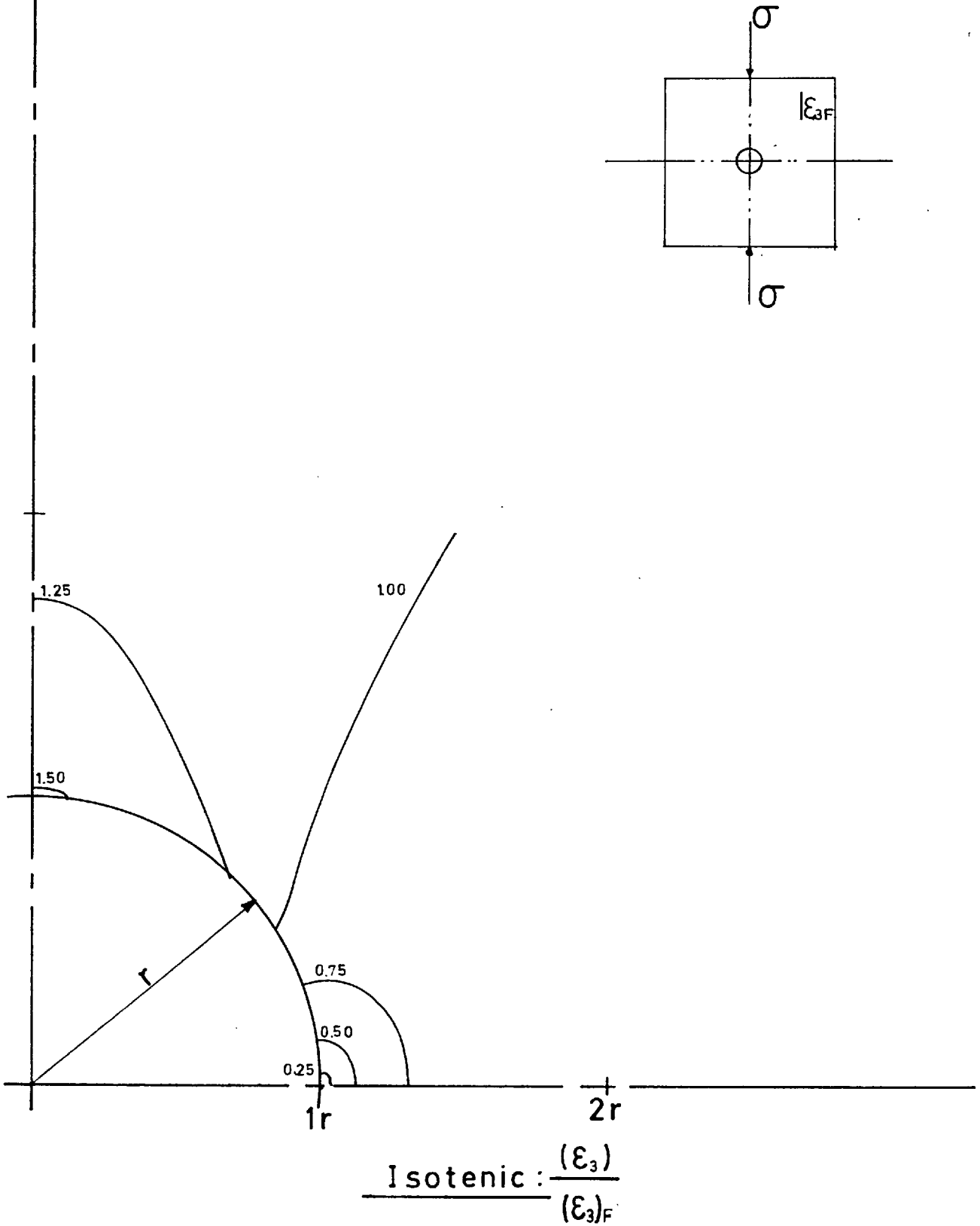


FIGURE : IV - 26

DRM -1 : Single Inclusion

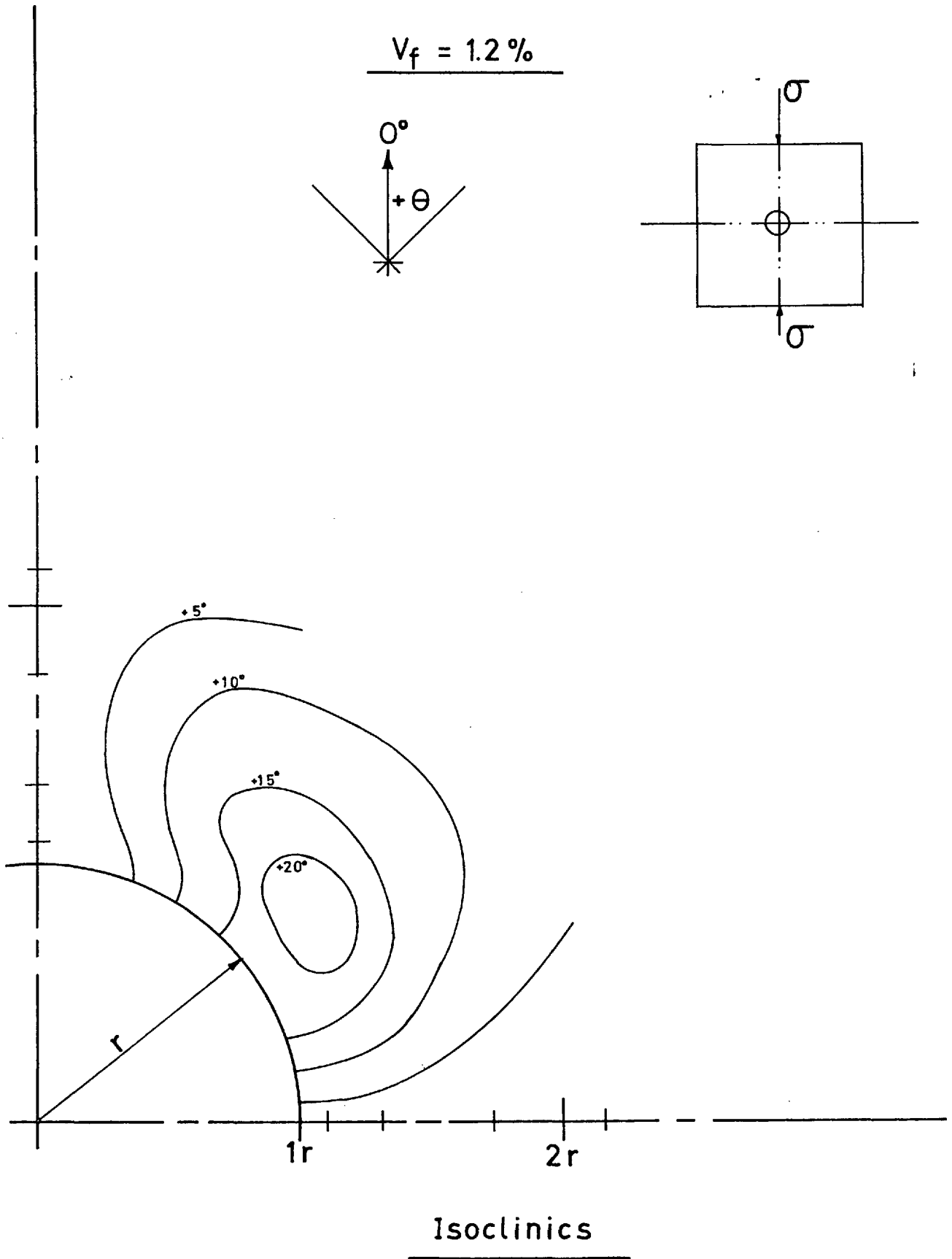
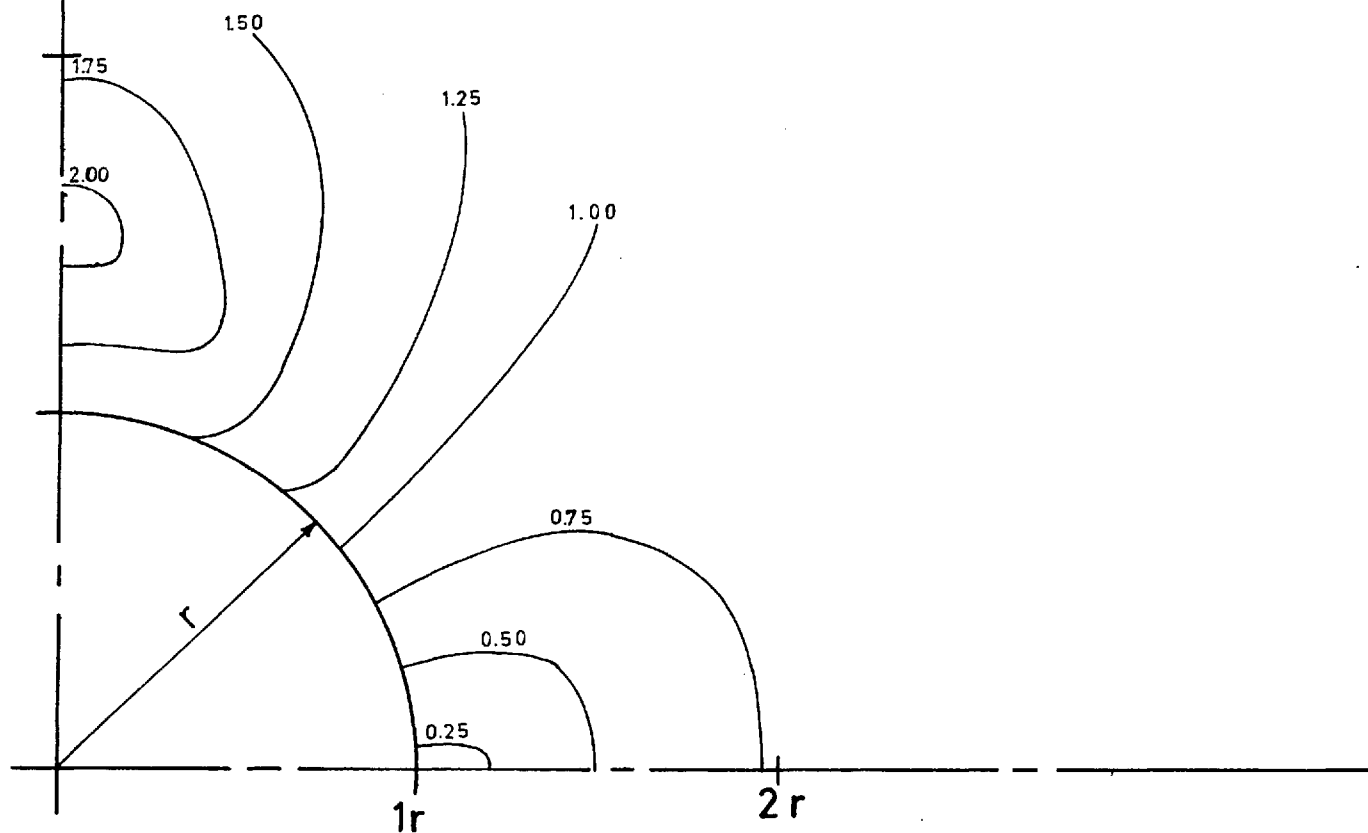
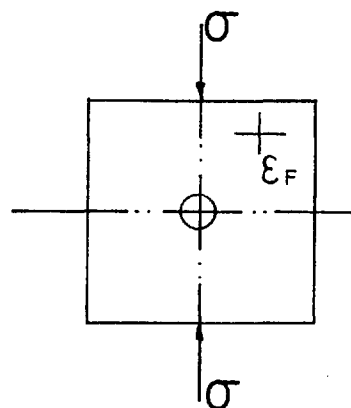


FIGURE :IV-27

DRM - 1 : Single Inclusion

$$V_f = 1.2\%$$

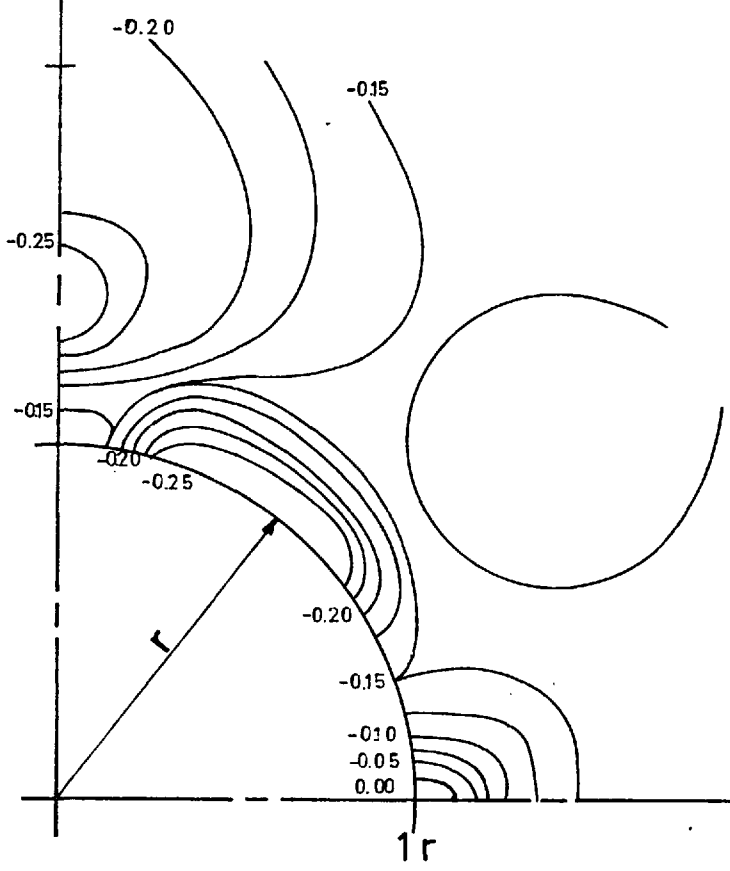
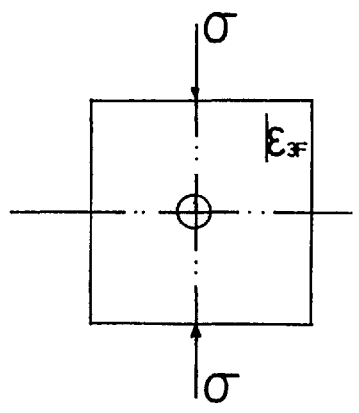


$$\text{Isochromatics: } \frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$$

FIG. IV-28

DRM - 1 : Single Inclusion

$V_f = 1.2\%$



Isotenic : $\frac{(\epsilon_1)}{(\epsilon_3)_F}$

FIGURE: IV - 29

DRM-1: Single Inclusion

$V_f = 1.2\%$

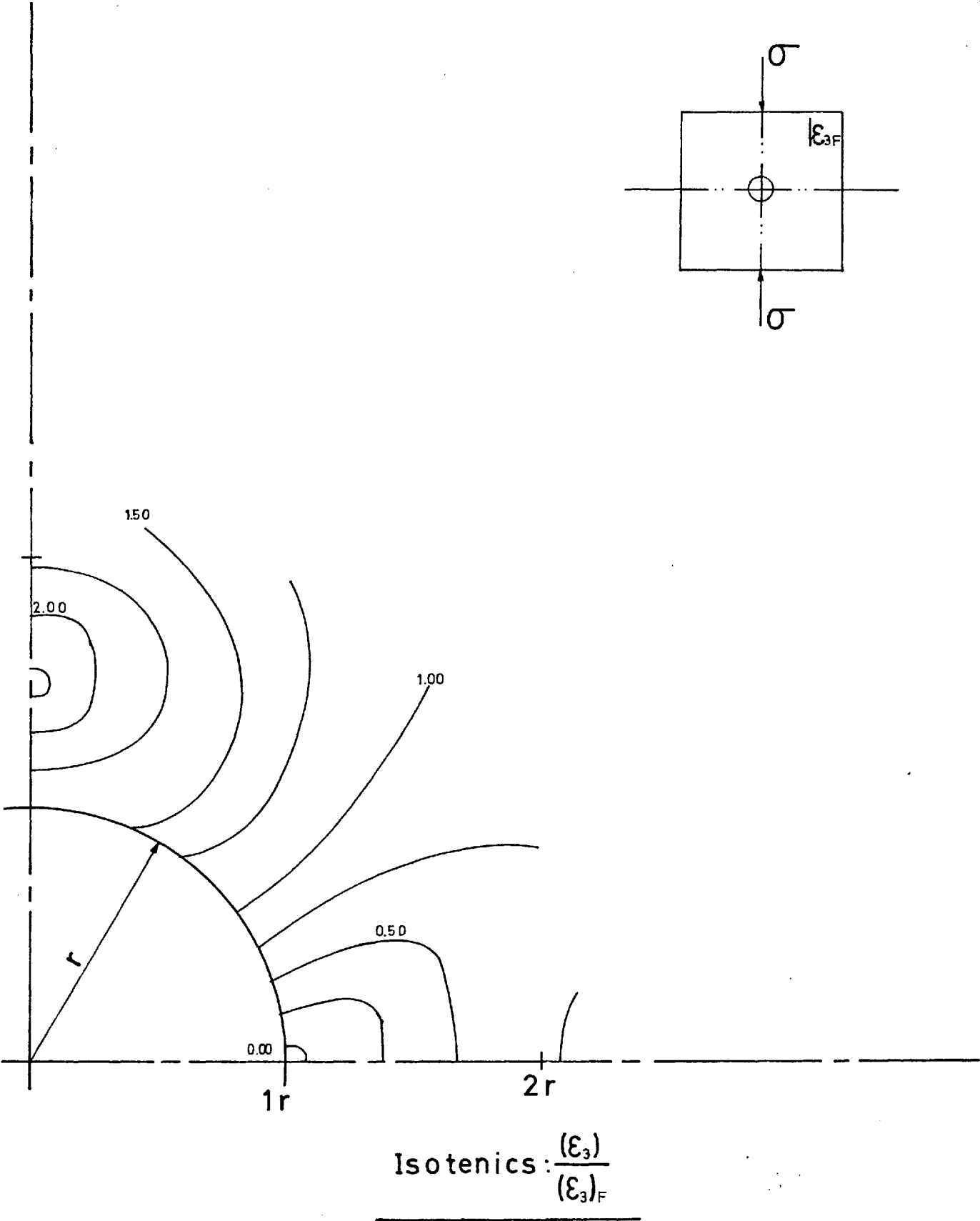


FIGURE: IV-30

procedure for the analysis of plate models as given below.

- 1) Specimens were cast (IV - 2.5) with approximate dimensions 10"x10"x1" (IV - 2.1). Each specimen contained a predetermined arrangement of particles (IV - 2.4)
- 2) Specimens were cured in a standard way (IV - 2.5)
- 3) Upon removal from water after 28 days the specimens were ground (IV - 2.7) to the exact dimensions required (10" x 10" x $\frac{3}{4}$ ") leaving all faces plane and square (or parallel).
- 4) Specimens were air-dried under ambient conditions for two weeks.
- 5) The photo-elastic coating to be used (IV - 2.6 B), was cut to a size of 9" x 9" and left with a 45° chamfer on all edges (IV - 2.6 B).
The coating was attached to the specimen using reflective epoxy resin cement and allowed to cure at ambient conditions for 24 hours (IV-2.6B).
- 6) After curing a polar-coordinate reference grid was scribed on the coating (IV - 2.6B).
- 7) The coated specimen was allowed to stand at ambient conditions for a period of six weeks to two months in order for moisture equilibrium through the thickness of the coating to be reached (IV - 3.2).
- 8) After moisture equilibrium was attained, the specimen was placed in the loading frame (IV - 2.2) and stresses of the required configuration applied. In order to obtain stable gauge readings, the specimens were load cycled six times. Readings of the isoclinic values at each point on the reference grid were taken (IV - 2.6A and IV - 3.2).
- 9) The specimen was removed from the loading frame. The positions of the points to be gauged (IV - 2.6A and B) were determined, governed chiefly by the physical size of the gauges used. These positions, and the orientation of the major principal strain at the point, were reproduced on the "back", uncoated, face of the specimen.
- 10) 2 mm. gauges (IV - 2.6B) were placed at each position and aligned in the

- direction of the major (most tensile) principal strain at the point (ϵ_1). Standard techniques were used for the cementing of the gauges.
- 11) Pairs of crossed 20 m.m. gauges were placed at four points on the model remote from any aggregate inclusions (IV - 3.1). Such gauges were normally placed two inches in from each corner of the model, and represented the uniform "field" strains in the model.
 - 12) The specimen was replaced in the loading frame (III - 2.2) and stresses of the required configuration applied. Isochromatic readings and gauges readings were taken at each point in the model. Applied stresses in the range 900 - 1000 p.s.i. were found to be generally acceptable for this purpose.
 - 13) The isochromatic pattern of each specimen was photographed using standard colour negative film in a Leica camera with a 45 m.m. lens. A full power electronic flash was used with the camera aperture at f.2.6.
 - 14) The specimen was removed from the loading frame and all 2 m.m. gauges removed. (Note that the pairs of 20 m.m. crossed gauges were not removed).
 - 15) Strain gauges were then attached between the centerlines of the aggregate inclusions in order that values of the average principal strains of the composite material could be obtained (see IV - 2.4 and Figure IV - 7.).
 - 16) The specimen was replaced in the loading frame and the required stresses once again applied. Gauge readings were taken of the "composite material strains" and the remote "field strains".
 - 17) The specimen was removed, all gauges removed, and the uncoated face of the specimen reground (IV - 2.7) to give a smooth surface.
 - 18) The uncoated face was washed with fluorescent dye and viewed under ultra-violet light to determine any initial cracking due to shrinkage, specimen preparation, prior testing etc. (IV - 2.8)

- 19) The specimen was replaced in the testing frame and loaded to failure. The isochromatic pattern was monitored using a Shackman high speed camera (IV - 3.2).
- 20) After failure, provided that the specimen was still reasonably intact, the uncoated face was again washed with fluorescent dye to determine the path of fracture propagation (IV - 2.8).

In some tests, steps (19) and (20) were changed slightly. In these tests the loading was stopped when fracture was thought to have initiated, and step (20) followed to determine the location of the point of initiation.

Section IV - 5. Information Furnished by Experimental Technique

The basic information furnished by the use of this experimental technique was as follows:

- 1) The Directions of the principal strains at all points on the model surface.

It should be noted that in this Thesis all isoclinic results shown denote the angle subtended between the applied minor (most compressive) principal stress and the minor principal strain at the point. The major principal strain, therefore, exists at 90° to the values shown.

- 2) The magnitude and sign of the principal strain difference ($\epsilon_3 - \epsilon_1$) at each point on the model surface.

Note that values of this shear strain were obtained both for points within the particle packing (Fig. IV - 7) and for points in the mortar "field" of the models, remote from the particle packing (Section IV-3.2)

- 3) The magnitude and sign of the major principal strain (ϵ_1) at gauged points on the model surface.

The magnitude and sign of the minor principal strain at each point could then be calculated from the value of the shear strain at the point.

Once again values both of "local" strains at points within the particle packing and of the "field" strains existing remote from the aggregate inclusions were determined.

- 4) The magnitudes and signs of the principal strains of the composite material.

These "composite material strains" were composed of the average strains of the aggregate particles and of the mortar matrix along the axes of symmetry of the particle packing (Fig.IV - 7).

- 5) A visual observation of any initial cracking due to shrinkage etc.
- 6) A photographic record of the isochromatic pattern from the application of load through to failure.

Qualitative analysis of these photographs allowed determination of the approximate position of the fracture initiation point and the approximate path of fracture propagation.

- 7) A visual observation of the final crack pattern, where a failed specimen remained reasonably intact.

Section IV - 6

Model Parameters to be Investigated

Having established a suitable experimental technique, it was necessary to decide which model parameters should first be investigated.

As has already been mentioned the prime object of these tests was to allow national analysis of the fracture initiation process around aggregate particles. It was recognized that a large number of parameters might reasonably be expected to affect this process. Variation in the geometry of loading; in the properties of the mortar matrix or of the aggregate particles, and in the geometry of particle packing used, were all considered to be potentially important to the analysis of the fracture initiation process. It was decided that the first investigation would be to determine the effect of variation in the spacing of the aggregate particles.

Thus all tests were run under the same type of applied stress, uniaxial

compression, where $\sigma_1 = \sigma_2 = 0$. The same basic shape of particle packing was used, representing a face-centered cubic packing in three dimensions (IV - 2.4). The materials for use in the models were not changed (IV - 2.3). The surface texture of the aggregate was always precisely repeated (IV - 2.3). The only variable to be allowed was the value of the center-to-center spacing of the aggregate particles "s". From Section IV - 2.4 it will be recalled that the relationship between the three-dimensional volume fraction of coarse aggregate represented by a model and the particle diameter and spacing used was given by equation (1), Section IV - 2.4.

Thus, the aim of the initial test series was to determine any variation in the fracture initiation process due to an increasing volume fraction of coarse aggregate for one type of loading, one particular type of aggregate packing, and one particular set of model materials.

Typical models representing various volume fractions of coarse aggregate are shown in Figures IV - 31 to IV - 33.

Section IV - 7

Conclusions

After fairly extensive development work, experimental techniques were devised whereby some quantitative idea of the principal strains of any element of mortar in the structure of concrete considered in two dimensions could be obtained. Such techniques must be recognized as giving only a reasonable approximation to reality. The representation of three-dimensional concrete by two-dimensional models is obviously an approximation. The measuring techniques used were all subject to errors of equipment and of operation. Furthermore, any results obtained could only be considered as approximately correct within the limits of the particular materials used and the particular stress state applied.

Techniques for the qualitative observation of fracture initiation and propagation were also determined.

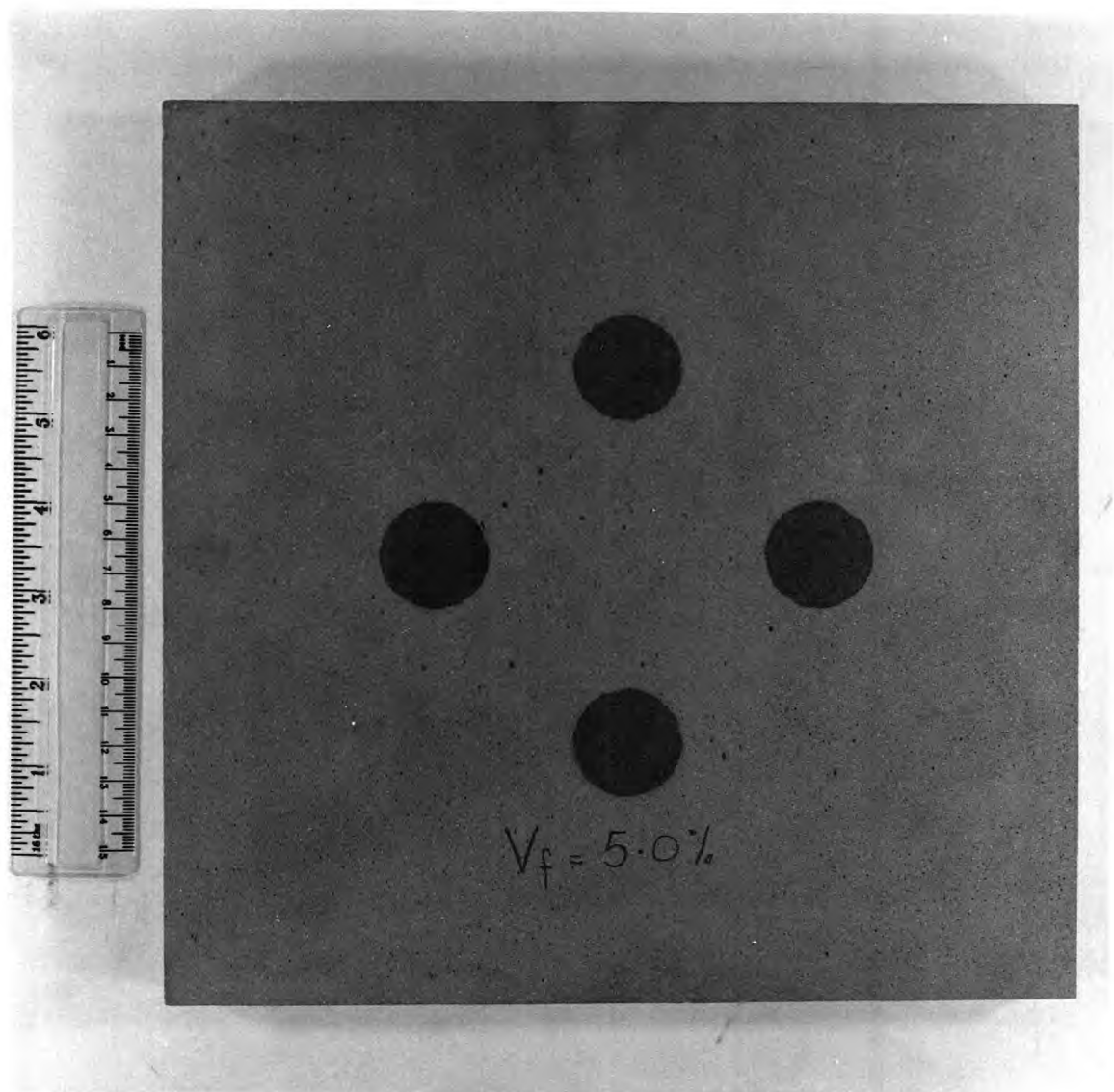


Plate-Model Representing Coarse Aggregate
Volume Fraction of 5.0%

FIG.IV-31

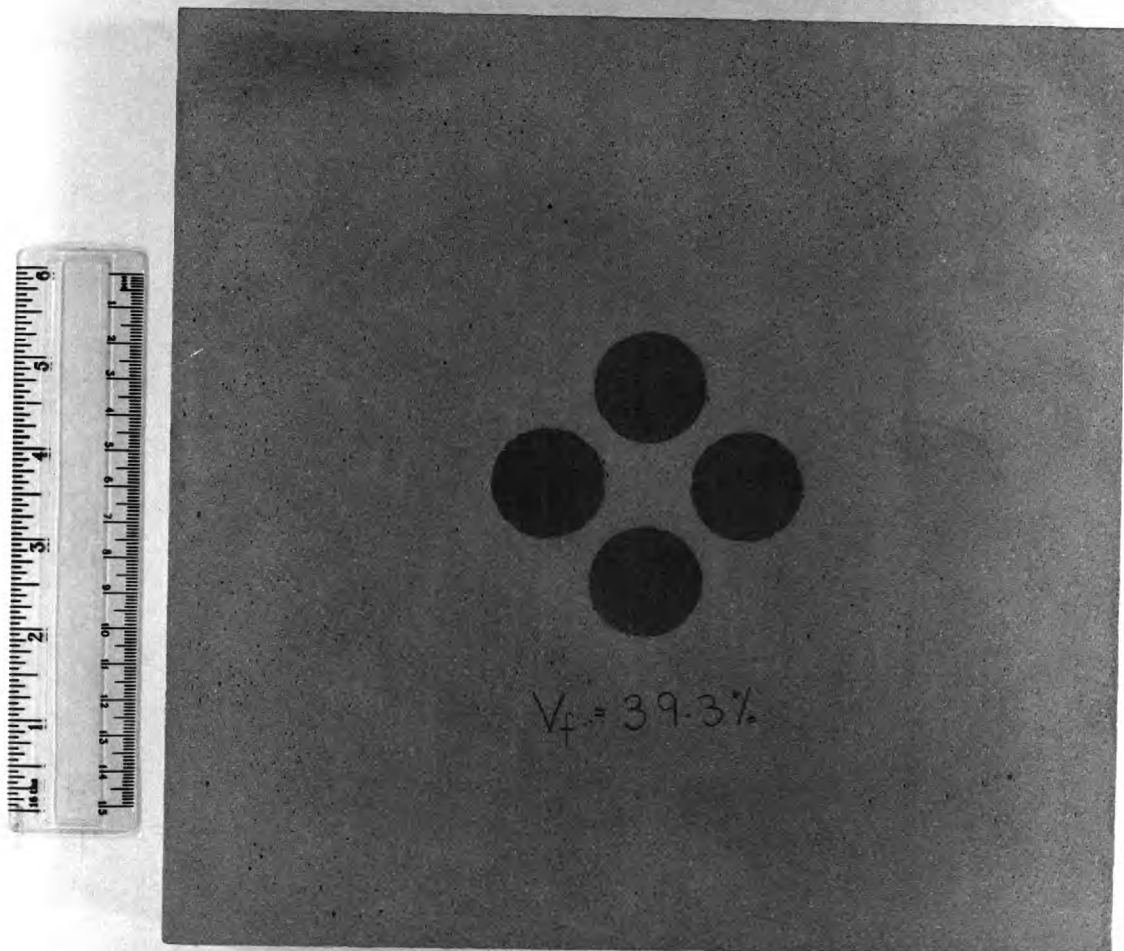


Plate-Model Representing Coarse Aggregate
Volume Fraction of 39.3%

FIG.IV-32

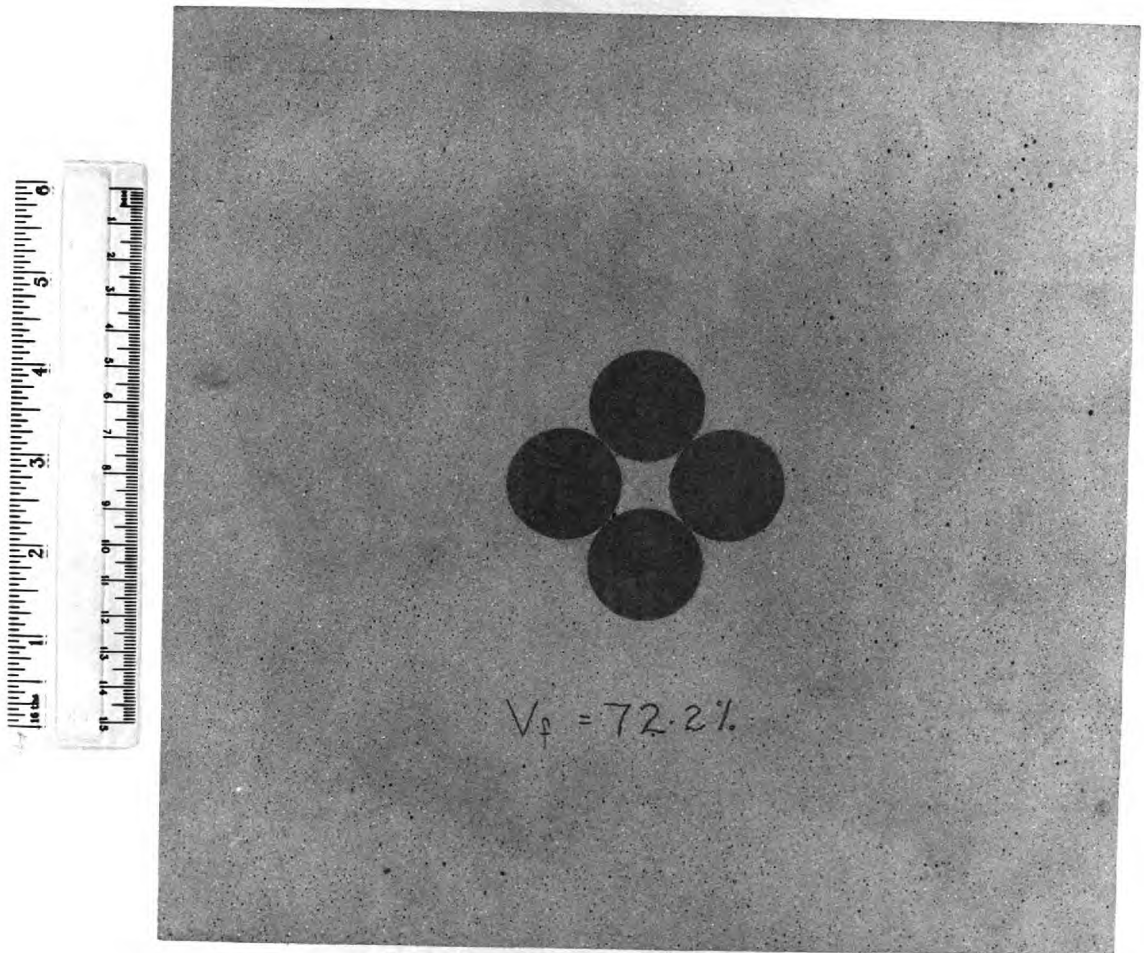


Plate-Model Representing Coarse Aggregate Volume
Fraction of 72.2%

FIG. IV-33

CHAPTER V

Determination and Analysis of Critical Points in Plate Models

Section V - 1

Introduction

In Chapter III a method for determining the "fracture initiation envelope" of a material was outlined. The application of such an envelope to the problem of analyzing the fracture initiation process within the mortar phase of concrete was briefly discussed. Chapter IV has described experimental techniques for obtaining quantitative information concerning the state of strain of any element of mortar within the structure of concrete, represented in two-dimensions. This Chapter deals with a method of using the fracture initiation envelope, in conjunction with the mortar strains derived from the plate-model tests, to determine those points in the mortar of a plate model at which fracture is likely to initiate. The area in the model containing all such points was referred to as the "critical zone" of that model. The point at which it was most probable that fracture would first initiate was also determined and referred to as the "most critical point".

Section V - 2

Fracture Initiation Envelope for

Plate-Model Mortar

As mentioned in Chapter III, a fracture initiation envelope may only be considered valid when applied to the particular material from which it was derived. It was necessary, therefore, to determine a fracture initiation envelope for the mortar to be used in the plate model tests (Chapter IV, Section IV - 2.3).

It will be recalled (Chapter IV, Section IV - 2.1) that the plate-models represented an infinitessimally thin slice taken from the

body of the prototype concrete, and existing in a plane defined by two of the applied principal stresses. It was apparent that the correlation between the two-dimensional model and the three-dimensional concrete would be greatly enhanced if the concrete prototype existed under a state of stress (or strain) such that two of the principal stresses (or strains) were equal. In such a case if the plate model applicable to that concrete were tested under the same conditions of major and minor principal stress as existed on the concrete, the results would be applicable to an infinite number of planes in the concrete defined by the major and minor principal stresses. In the generalized case, where the three stresses applied to the concrete prototype are not equal, the results of a single model test would only be applicable to the single plane in the prototype defined by the two stresses applied to the model.

Following a similar line of argument for the development of a fracture initiation envelope, it was decided that the two principal strains measured at a particular point in the mortar matrix of a plate model would correlate more closely with the strains measured on three-dimensional mortar specimens if one of the following requirements were fulfilled during the testing of the mortar specimens.

- i) $\sigma_3, \sigma_1 = \sigma_2$ = negative (triaxial compression)
- ii) $\sigma_3, \sigma_1 = \sigma_2$ = 0 (uniaxial compression)
- iii) $\sigma_3, \sigma_1 = \sigma_2$ = positive (compression-tension = tension)
- iv) $\sigma_1, \sigma_2 = \sigma_3$ = negative (tension-compression = compression)
- v) $\sigma_1, \sigma_2 = \sigma_3$ = 0 (uniaxial tension)
- vi) $\sigma_1, \sigma_2 = \sigma_3$ = positive (triaxial tension)

Of the stress-states listed above, only three could be applied using the available equipment. These were triaxial compression (i), uniaxial compression (ii) and uniaxial tension (v). The deficiencies of the derived fracture initiation envelope due to the non-availability of the other stress

states listed are discussed later in this section.

In order to apply triaxial compression to specimens of the plate-model mortar, a triaxial cell designed and built by Newman¹² was used. Specimens used in the cell were cylinders of the mortar, 4 inches in diameter by 10 inches high. A full description of this cell, shown in Figure V-1, and of the testing techniques required, is beyond the scope of this Thesis and is fully covered in the quoted reference. The main features of the cell, in the context of these tests, were that "lateral" pressures could be applied hydraulically from 0 p.s.i. to 20,000 p.s.i.; "vertical" pressures could be mechanically applied up to 100,000 p.s.i., and, most important, that the resulting specimen strains could be directly measured using special "high-yield" electrical resistance strain gauges.

A testing frame developed by Ward⁸⁸ was used to apply uniaxial tension to necked prismatic specimens having a central portion 3 inches by 4 inches and an overall height of 20 inches. For uniaxial compression cylindrical specimens, 4 inches in diameter by 10 inches high, were tested in an Amsler (300-ton) testing frame.

All specimens tested were cast from the same mortar as that to be used in the plate models (Chapter IV, Section IV - 2.3) and cured in a standard way. All cylinders were ground such that the ends of each specimen were plane, parallel, and square to the long axis of the cylinder. The flat faces of each tension specimen were also ground. The curved faces of the test cylinders were sandblasted to remove the exterior "latence". Removal of this latence facilitated drying of the specimens (in order to simulate the drying experienced by the plate-models) and the attachment of electrical resistance strain gauges. As the mortar was considered to be homogeneous (Chapter III, Section III-1), the average strains of each specimen were to be measured and would be assumed as constant throughout the specimen, i.e. equal to the "local" strains. For the purposes of strain



General View of Triaxial Cell

FIG. V - 1

measurement, foil gauges 60 m.m. in length were attached to each specimen. For the uniaxial tension and uniaxial compression specimens standard techniques were used for the adhesion of these gauges. The special techniques required for use with the triaxial cell are covered in the quoted reference¹². After grinding and surface preparation, each specimen was allowed to stand at ambient conditions for at least two months prior to testing in order to simulate the treatment of the plate-models (Chapter IV, Sections IV - 3.2 and IV -4).

For each stress-state used, the principal strains of each specimen were measured throughout the loading cycle. The volumetric strains, $\Delta V/V_0$, were calculated at each load stage from the algebraic sum of the principal strains, ϵ_1 , ϵ_2 and ϵ_3 . (Note, that for the tests carried out, either $\epsilon_1 = \epsilon_2$ or $\epsilon_2 = \epsilon_3$). Following the method outlined in Chapter III, the volumetric strains were then plotted against (in general) the minor principal strains throughout the test. Deviation of this curve from linearity then determined the strains at which fracture was considered to have been initiated (Chapter III). These strains were designated (ϵ_1) f.i., (ϵ_2) f.i. and (ϵ_3) f.i., where the suffix "f.i." refers to fracture initiation. As the intermediate principal strain, ϵ_2 , was always equal to one of the other principal strains, and as the information was intended for application to two-dimensional models, only the major (most tensile) and minor (most compressive) principal strains were of direct relevance. Values of these strains, (ϵ_1) f.i. and (ϵ_3) f.i., were computed by interpolation between the averaged values of the strains (from correspondingly positioned gauges) at each load stage and are shown in the Figures to the nearest micro-strain. It should not be assumed from these figures that the author considered all test results to be accurate within one micro-strain. In general, strain readings between corresponding gauges were found to be within $\pm 5\%$ of the average value.

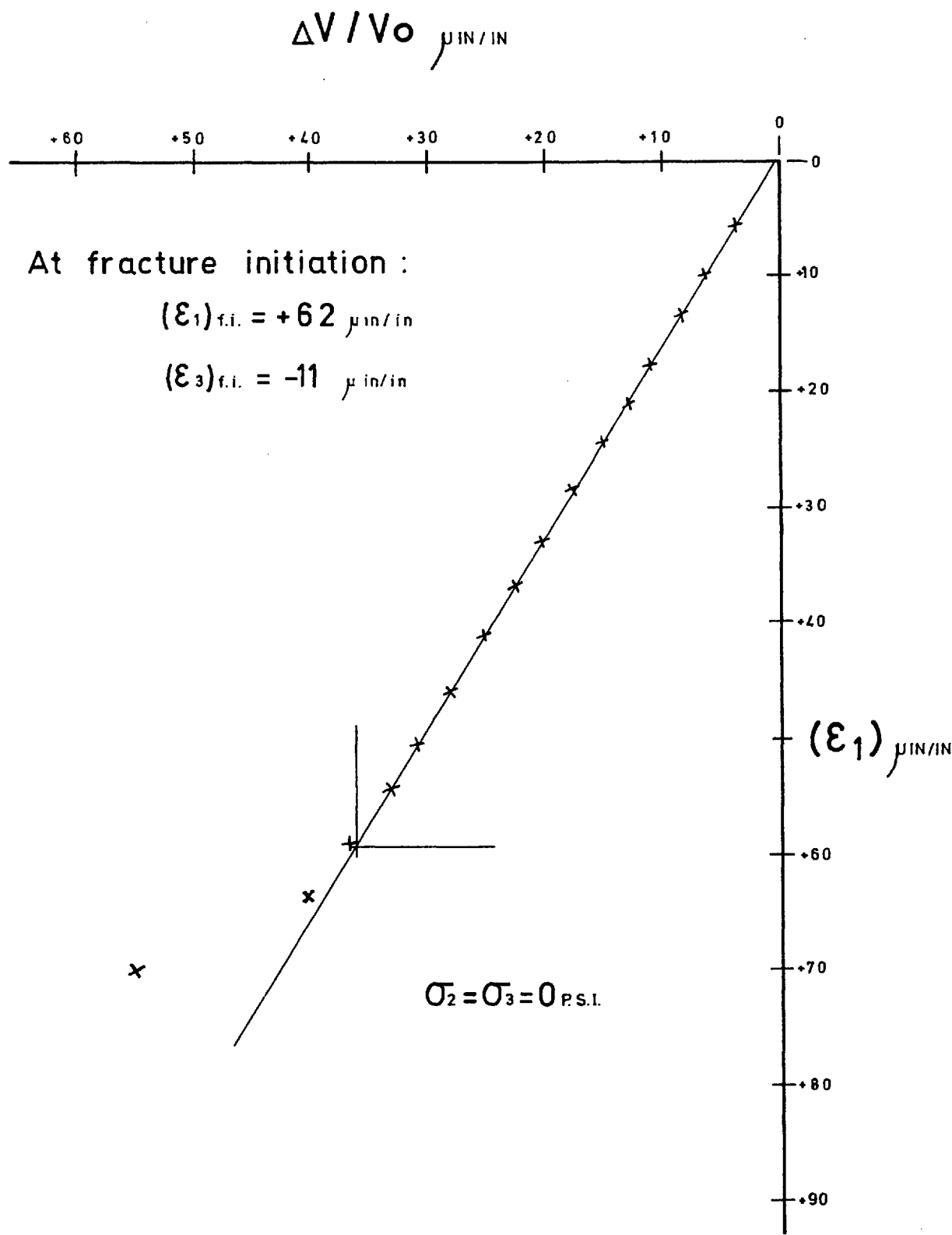
Typical test results are shown as volumetric strain plots in Figures

V - 2 (uniaxial tension), V - 3 (uniaxial compression and V - 4 to V - 7 (triaxial compression). It will be noted that for uniaxial tension tests the volumetric strain was plotted against the major principal strain (ϵ_1), whereas for compression tests it was plotted against the minor principal strain (ϵ_3). This procedure was dictated by scale difficulties in plotting and in no way alters the validity of the method.

As the fracture initiation information derived from these tests was intended for application to the plate-model tests, it was necessary to non-dimensionalize the strain information with respect to a common factor. From Chapter IV, Section IV -6, it will be recalled that all the plate-model tests were to be run under uniaxial compression. Thus, the strains generated in the mortar "field", remote from any disturbance due to the aggregate particles, would result from a uniaxial compression state of stress. The fracture initiation information derived from the tests described in this Chapter was, therefore, non-dimensionalized with respect to the minor principal strain necessary to cause fracture initiation under uniaxial compression (Fig. V-3). The importance and meaning of this method of non-dimensionalizing is discussed in Section V-3.

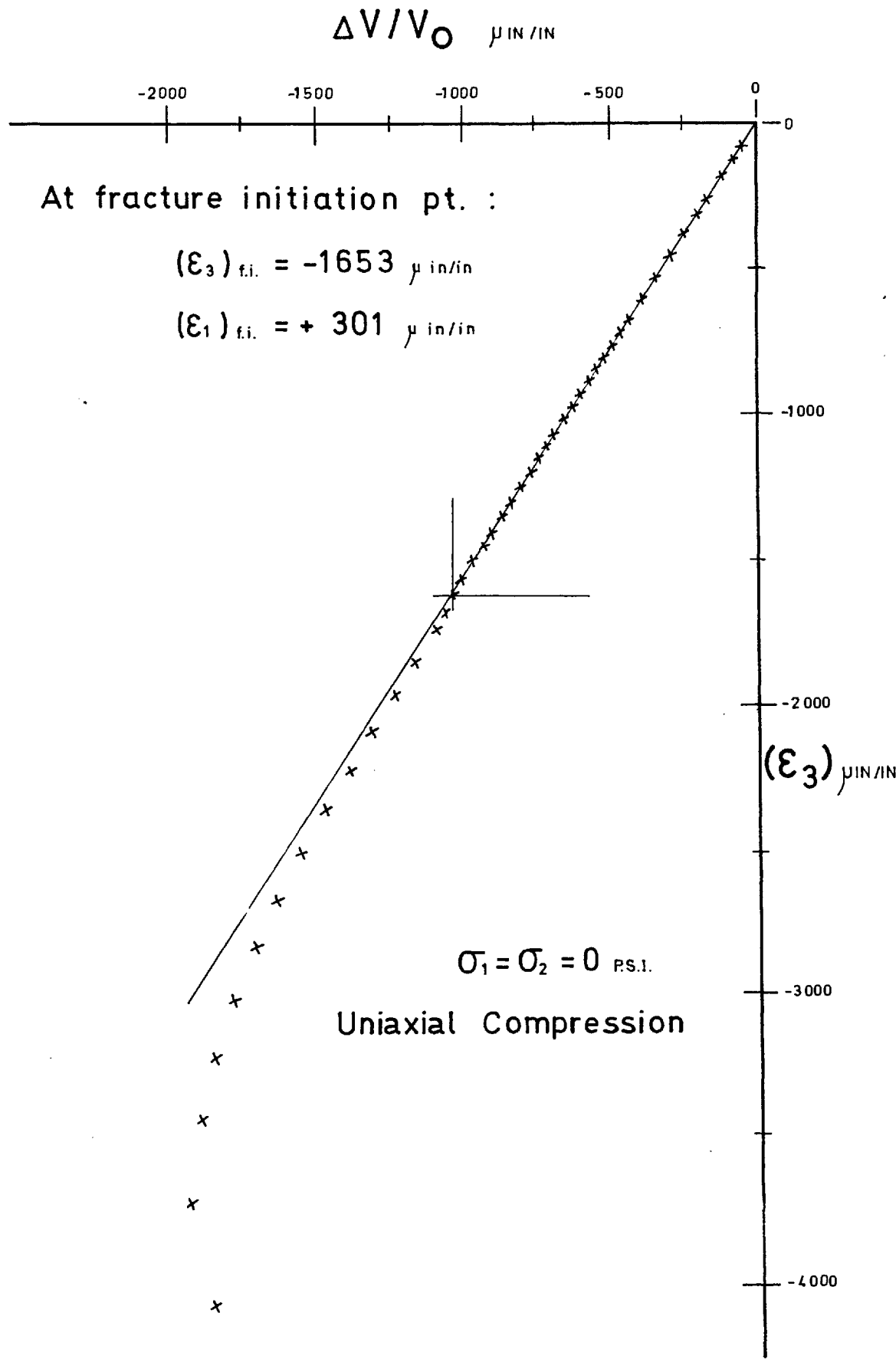
Typical experimental points obtained for the strains necessary to cause fracture initiation are shown, non-dimensionalized, in Figure V-8.

It may be seen from Figure V-8 that no additional experimental points were determined between the "uniaxial compression point" and the "uniaxial tension point". From the previous discussion, it appeared that stress states of compression-tension = tension (iii), or tension-compression = compression (iv), were required in order to determine such points. It appeared that the development of a testing technique capable of applying these states of stress would be a lengthy process. The development of a satisfactory triaxial compression testing technique had already consumed a considerable amount of time. The further expenditure of time upon the experimental derivation of one portion of the fracture initiation envelope



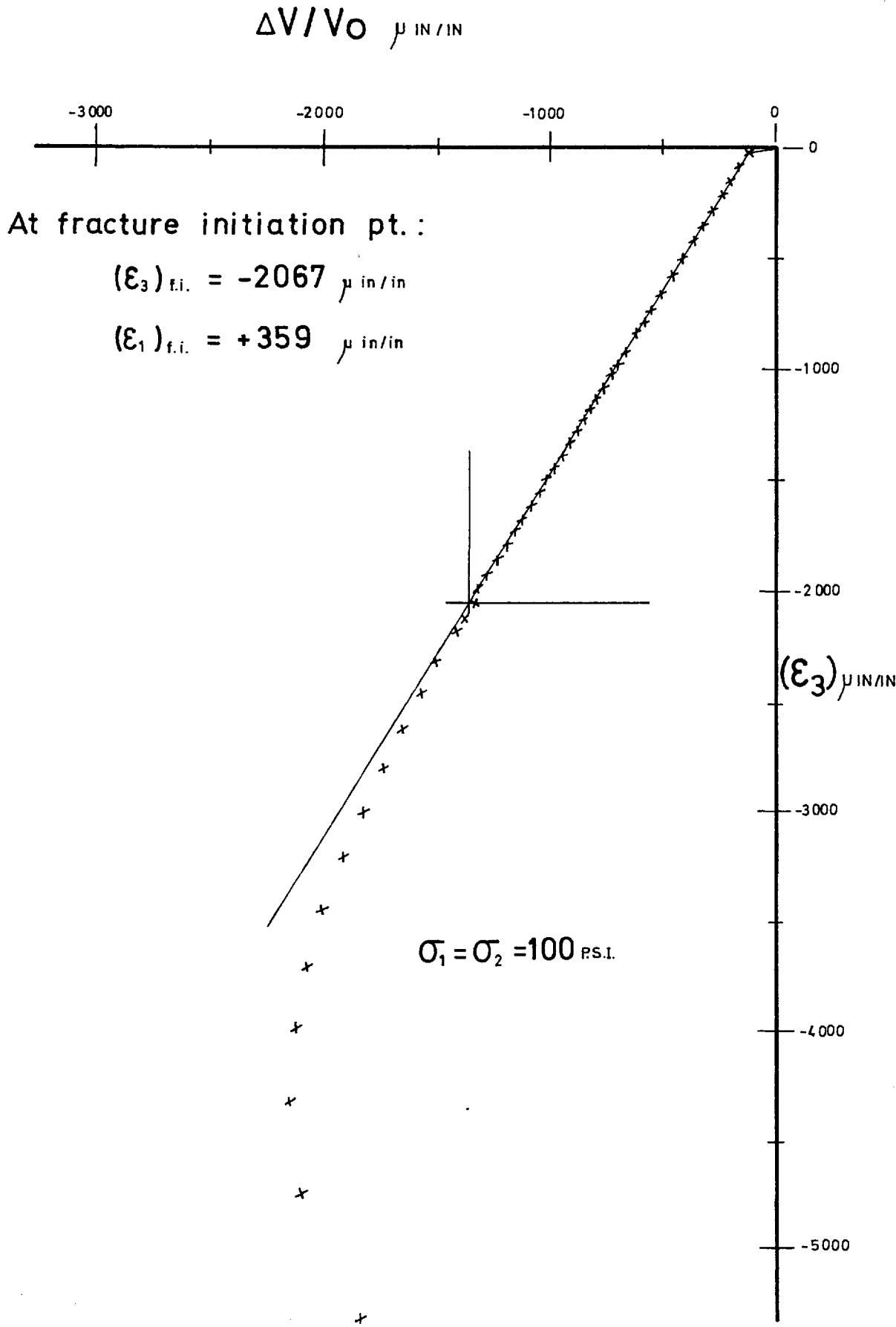
Uniaxial Tension Specimen

FIGURE : V- 2



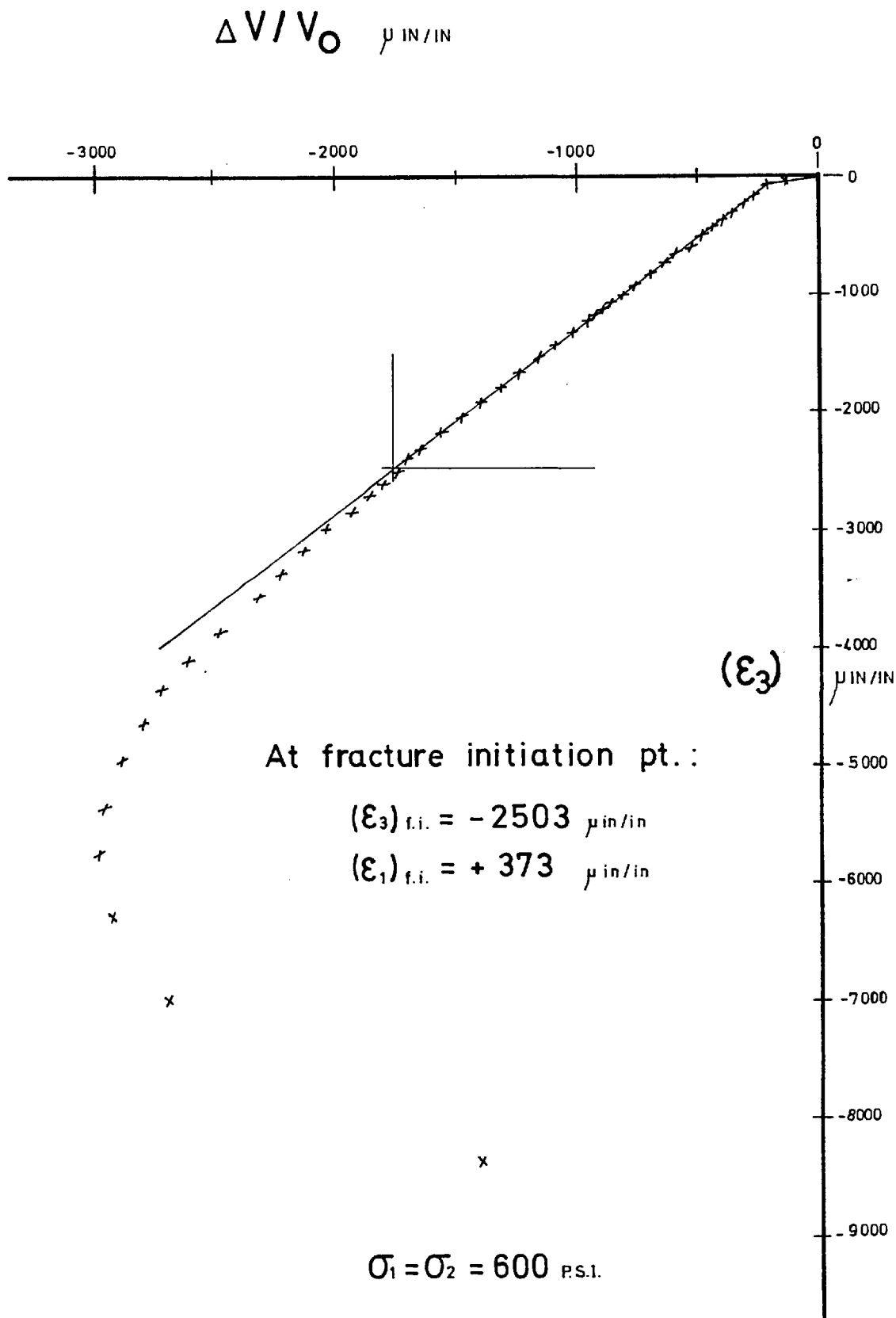
Specimen : DRM-1/0

FIGURE : V-3



Specimen : DRM-1/100

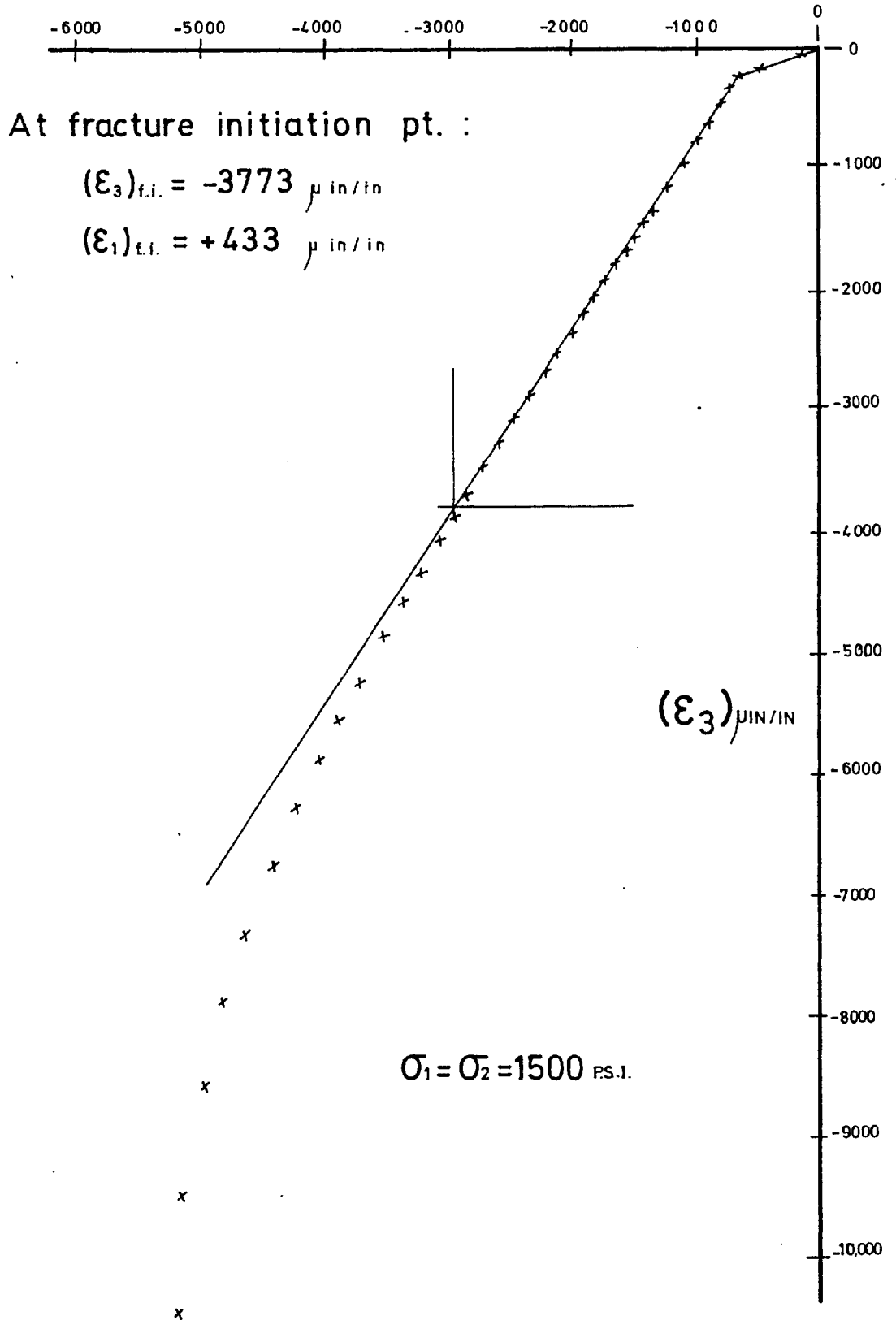
FIG. V-4



Specimen: DRM -1/600

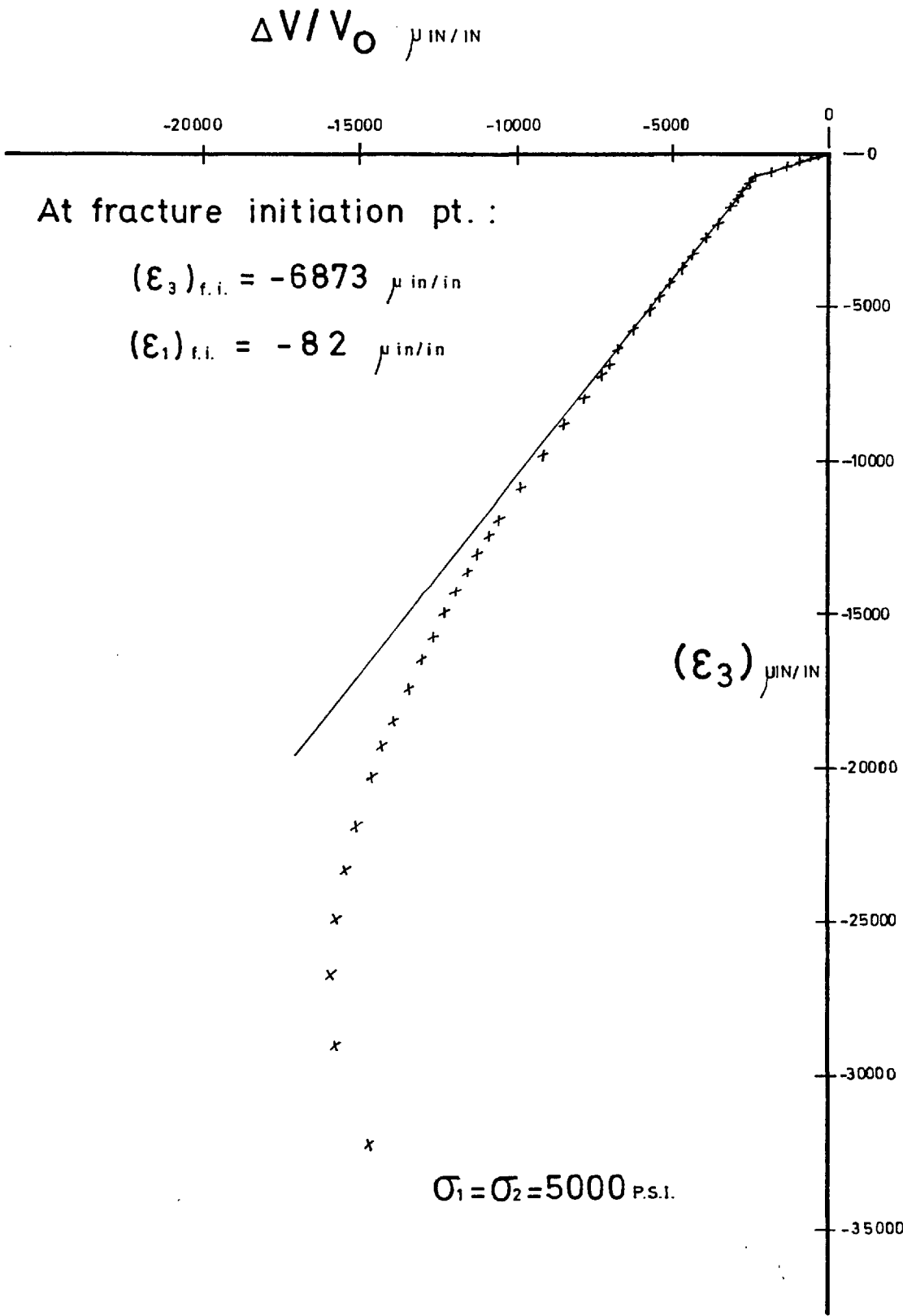
FIGURE: V-5

$$\Delta V / V_0 \text{ } \mu\text{IN/IN}$$



Specimen: DRM-1/1500

FIG. V-6



Specimen : DRM-1/5000

FIG. V -7

Fracture Initiation Envelope
for
Plate-Model Mortar

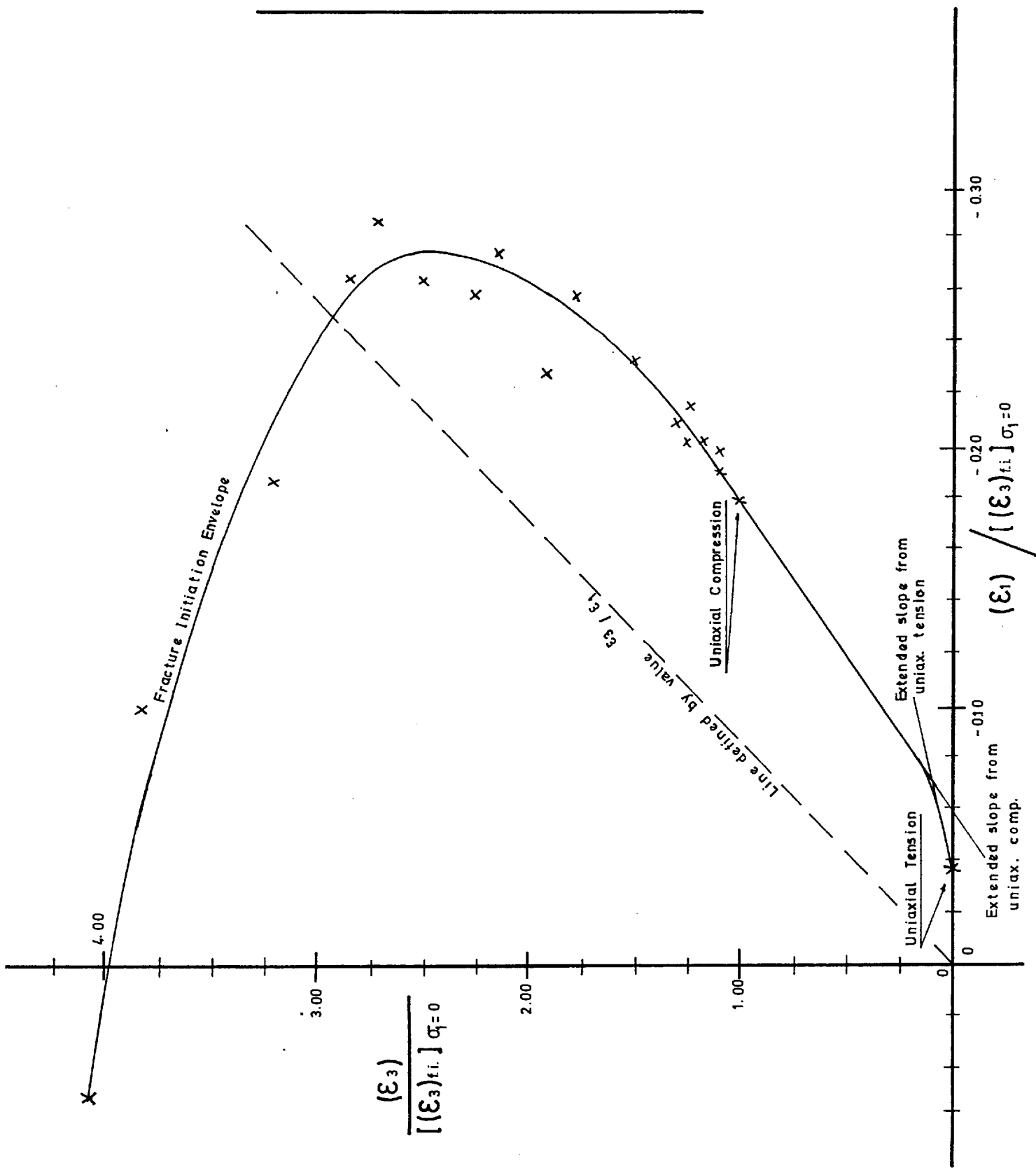


FIGURE : V- 8

was not considered to be justified. Communication with Hoek⁸⁹ suggested that this portion of the envelope, between the uniaxial compression and uniaxial tension points, could be approximated as a bi-linear curve without the likelihood of serious error.

The slope of the mean line through the experimental points was well defined in the region of uniaxial compression (Fig. V-8). A line of this slope was extended from the uniaxial compression point in the direction of the uniaxial tension point. This line is shown in Figure V-8 as "Extended line from uniax. comp.". The slope of this line was measured and related to the value of the principal strain ratio at fracture initiation, $(\epsilon_3)_{f.i.} / (\epsilon_1)_{f.i.}$ at the uniaxial compression point.

Thus:

$$\begin{aligned} \text{(slope of mean line through uniaxial compression point)} = \\ (\text{constant}) \times (\text{value of ratio } (\epsilon_3)_{f.i.} / (\epsilon_1)_{f.i.} \text{ at uniaxial} \\ \text{compression}). \end{aligned}$$

Uniaxial tension tests were carried out (Fig. V - 2) and the value of the ratio $(\epsilon_3)_{f.i.} / (\epsilon_1)_{f.i.}$ calculated for this stress-state. Use of the value of this ratio in the preceding equation allowed calculation of an approximate value for the slope of the mean line passing through the experimental points in the vicinity of uniaxial tension. A line of this calculated slope was extended towards the uniaxial compression point and is shown in Figure V-8 as "Extended slope from uniaxial tension". The intersection between the two extended "slope lines" was rounded off to give a smooth curve running between the uniaxial compression and uniaxial tension points.

This method of completing the fracture initiation envelope contains a basic assumption. This assumption is that the general nature of the strain response of the material, up until fracture initiates, is of an identical type under either uniaxial compression or uniaxial tension. An estimate of the validity of this assumption may be obtained from a comparison

of the Poisson's Ratio of the material under these two testing conditions.

From Figure V-2, the Poisson's Ratio of the mortar under uniaxial tension had, at fracture initiation, the value:

$$\text{Poisson's Ratio} = \frac{(\epsilon_3)_{f.i.}}{(\epsilon_1)_{f.i.}} = \frac{-11 \mu\text{in/in}}{62 \mu\text{in/in}} = -0.178$$

From Figure V-3, the value of the Poisson's Ratio at fracture initiation under uniaxial compression was given by:

$$\text{Poisson's Ratio} = \frac{(\epsilon_1)_{f.i.}}{(\epsilon_3)_{f.i.}} = \frac{301 \mu\text{in/in}}{-1653 \mu\text{in/in}} = -0.182$$

From consideration of these two values it appeared that the nature of the strain response prior to fracture initiation did not vary markedly between uniaxial compression and uniaxial tension. Thus, this method of completing the fracture initiation envelope between these two points was considered to be a reasonable approximation.

The curve resulting from the mean line through the plotted experimental points and from the calculations described is shown in Figure V-8 and is referred to as the fracture initiation envelope for the plate-model mortar.

Section V - 3 Prediction and Analysis of Critical Zone

It is important that the meaning of the fracture initiation envelope be clearly understood. The axes of the fracture initiation diagram, Figure V-8, have been non-dimensionalized with respect to the minor (most compressive) principal strain necessary to cause fracture initiation under uniaxial compression. Thus, each axis may be considered as showing the value of the "strain concentration factor" of a particular strain with respect to the minor principal strain at fracture initiation under uniaxial compression. If it is assumed that the material behaves elastically prior to fracture initiation, then the value of this "strain concentration factor" for a particular strain will remain unchanged until fracture initiates.

Examination of Figure V-8 will show that the position of a point plotted on the non-dimensionalized fracture initiation diagram may be considered to be a function of two parameters.

These parameters will be the ratio of the two principal strains being considered, $(\epsilon_3 / \epsilon_1)$, and the value of the concentration factor of one of the strains with respect to the minor principal strain necessary to cause fracture initiation under uniaxial compression.

Section V - 3.1

Determination of Extent of Critical Zone

From the plate-model tests (Chapter IV) the values of the principal strains were determined at each point of measurement within the aggregate packing (Fig. IV - 7). These strains were referred to as the "local" strains (ϵ_1) local and (ϵ_3) local. The values of the principal strains existing homogeneously in the mortar, remote from the aggregate particles, were also determined (Chapter IV, Section IV-5) and termed the "field" strains (ϵ_1) field and (ϵ_3) field. Thus, at each position of measurement, the ratios of the local strains within the aggregate packing to the field strains existing remote from the aggregate could be determined. In order that this information could be used in conjunction with the fracture initiation diagram, the denominator of these ratios was chosen to be the minor principal field strain, $(\epsilon_3)_F$. Thus, for each point on the model at which strain measurements were taken, the ratios:

$$\frac{(\epsilon_1)}{(\epsilon_3)_F} \quad \text{and} \quad \frac{(\epsilon_3)}{(\epsilon_3)_F} \quad \dots\dots\dots(1)$$

were determined. As the "local" strains are the strains under examination, no suffix has been attached to these values. Thus, (ϵ_1) and (ϵ_3) represent any two principal strains being considered, and the suffix "F" refers to the field strain.

The ratios shown in (1) give the value of the intensification of each

local strain with respect to the minor principal strain existing in the mortar remote from the aggregate inclusions. As this field strain, $(\epsilon_3)_F$, was undisturbed by any effects due to the aggregate inclusions, it represents precisely the minor principal strain existing in pure mortar due to the application of uniaxial compression.

Thus:

$$(\epsilon_3)_F \equiv (\epsilon_3)_{\sigma_1=0} = 0 \quad \dots\dots\dots(2)$$

The ratios given in (1) may be expressed, therefore, in the form:

$$\frac{(\epsilon_1)}{(\epsilon_3)_{\sigma_1=0}} \quad \text{and} \quad \frac{(\epsilon_3)}{(\epsilon_3)_{\sigma_1=0}} \quad \dots\dots\dots(3)$$

Assuming that the material behaves elastically prior to the initiation of fracture, the ratios given in (3) may be seen to be similar in form to the ratios:

$$\frac{(\epsilon_1)}{[(\epsilon_3)_{f.i.}]_{\sigma_1=0}} \quad \text{and} \quad \frac{(\epsilon_3)}{[(\epsilon_3)_{f.i.}]_{\sigma_1=0}} \quad \dots\dots\dots(4)$$

which define the axes of the fracture initiation diagram. Although these ratios (3) and (4) are of similar type, they are obviously not identical. Thus, the justification and implications of plotting a point defined by the plate-model ratios (3) on the fracture initiation diagram are not obvious and require some discussion.

From Figure V-8, it may be seen that any straight line drawn from the origin of the fracture initiation diagram is defined solely by a particular value of the principal strain ratio $(\epsilon_3)/(\epsilon_1)$. The magnitudes of the principal strains necessary to cause fracture initiation for this particular value of $(\epsilon_3)/(\epsilon_1)$ may be calculated from the co-ordinates of the intersection point between the straight line and the fracture initiation envelope. Consider that, in a particular plate-model, several points exist having identically the same value for the principal strain ratio $(\epsilon_3)/(\epsilon_1)$.

However, at each point, the value of the strain concentration factor

$(\epsilon_3)/(\epsilon_1)_{\sigma_1=0}$ or $(\epsilon_1)/(\epsilon_3)_{\sigma_1=0}$ differs. Thus, all these points exhibit the same type of strain response, $(\epsilon_3)/(\epsilon_1)$, but the strains are of different magnitudes with respect to the applied field strain due to the particular, local, concentration factors. For such a case it is apparent that the strain magnitudes necessary to cause fracture initiation will first be achieved at the point exhibiting the greatest strain concentration factor.

However, it was necessary to compare points at which both the principal strain ratios $(\epsilon_3)/(\epsilon_1)$ and the strain concentrations (3) were different. Three such points are shown plotted in Figure V-9. Each Point was plotted from the values of the ratios (3) existing at the position in the plate-model represented by the Point. The meaning of Point 1 was that the state of strain existing at the equivalent position on the plate-model was such that fracture initiation was likely to occur within the remote field mortar before the strains at Point 1 could reach sufficient magnitude to initiate fracture. However, at Point 2 or 3, the local state of strain was such that fracture initiation at the Point was likely to occur prior to the initiation of fracture within the remote field mortar. Thus, the fact that these Points fell "above" the envelope merely indicated that fracture initiation at the positions on the plate-model represented by these Points was more probable than fracture initiation within the remote field mortar. It must be clearly understood that the use of the strain ratios (3), (derived from the plate-model tests) to plot points on the fracture initiation diagram, was simply a device for determining the relative probability of fracture initiation with respect to the remote field mortar. Thus, a point falling "above" the envelope did not indicate that fracture had initiated at that point, but simply that initiation was more probable at that point than at a point within the remote field mortar.

From the plate-model tests, the ratios given in (3), representing the state of strain at each point of measurement, were used to plot a point

on the non-dimensionalized fracture initiation diagram. The plotted points falling "above" the fracture initiation envelope (Point 2 and 3 in Fig. V-9) then represented positions on the plate-models at which the state of strain was such that fracture was likely to initiate at these positions before fracture could be initiated in the "field" mortar. All such points were termed "critical points" and the area of the plate-model containing all the critical points was termed the "critical zone" of that model. The extent of the "critical zone" for various plate models is shown, in Chapter VI, in Figures VI - 8, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39 and 42.

Section V - 3.2

Determination of Most Critical Point

It is important to recall that the initiation of fracture at any single point in a model will cause a redistribution of the local strains³⁸. Thus, any quantitative analysis based upon the strain distribution derived from the plate-model tests prior to fracture initiation could only be conducted upon this first point of fracture or "most critical point".

Section V - 3.1 discussed a method whereby all those points (within the limit of the number of points at which measurements were taken) at which the state of strain was likely to initiate fracture prior to initiation occurring in the "field", could be determined for a particular model. Thus, the problem was to determine which of these "critical points" was likely to be most critical, i.e. first give rise to fracture initiation as the magnitudes of the strains were increased.

It has been mentioned in Section V-3.1 that the state of strain which gave rise to the plotted Point 1 in Figure V-9 could be considered "non-critical". However, both Points 2 and 3 must be considered as "critical"

Point 2 represented a state of strain in which the two principal strains had a certain relationship $(\epsilon_3)/(\epsilon_1)$. The locus of all points representing a state of strain having this same value for the principal strain ratio will be a straight line from the origin of the fracture initiation

diagram and passing through the plotted point. The parameter which distinguished the particular Point 2 was the strain concentration factor at the position in the model represented by this Point. Thus, other states of strain could exist having the same value of the principal strain ratio but nevertheless being non-critical due to the presence of lower strain concentration factors. An analogous argument was followed for Point 3.

An estimate of the "degree" of criticality of a particular Point was obtained by consideration of the amount by which the Point was displaced "above" the fracture initiation envelope due to the particular strain concentration factors which it represented. For this purpose the locus of constant $(\epsilon_3)/(\epsilon_1)$ was drawn for each Point plotted on the fracture initiation diagram. These loci are shown as dashed lines for Points 2 and 3 in Figure V-9. For each point the proportion of the line falling "below" the fracture initiation envelope, with respect to the total length of the line from the origin to the Point, was calculated. Comparison of these calculated values showed that Point 3 was proportionally farther removed above the fracture initiation diagram than was Point 2. Thus, it was considered that the state of strain represented by Point 3 was "more critical" than that represented by Point 2. The position in the plate model represented by Point 3 was, therefore, considered to be the "most critical point" in the model, from which fracture was first likely to initiate.

Section V-4

Analysis of Most Critical Point

Having determined the point from which fracture was first likely to initiate (the most critical point) it was then possible to determine the absolute values of the strains necessary, at that point, to initiate fracture. It may be seen from Figure V-9 that the locus defining the principal strain ratio at Point 3 cut the fracture initiation envelopes at co-ordinates:

$$\frac{(\epsilon_3)_{f.i.}}{[(\epsilon_3)_{f.i.}] \sigma_1 = 0} = 0.65$$

and

$$\frac{(\epsilon_1)_{f.i.}}{[(\epsilon_3)_{f.i.}]_{\sigma_1=0}} = -0.13$$

The value of the minor principal strain necessary to cause fracture initiation under uniaxial compression (the denominator of the above ratios) is shown in Figure V-3 as:

$$[(\epsilon_3)_{f.i.}]_{\sigma_1=0} = -1653 \mu\text{in/in.}$$

Thus, for the example given in Figure V-9, the strains required at the most critical point represented by Point 3, in order to initiate fracture were given by:

$$(\epsilon_3)_{f.i.} = 0.65 (-1653) = -1078 \mu\text{in/in}$$

and:

$$(\epsilon_1)_{f.i.} = -0.13 (-1653) = +215 \mu\text{in/in.}$$

It should be clearly understood that these were values of the local strains necessary at the most critical point in order to initiate fracture.

Section V-5

Conclusions

The methods described in this Chapter provided a means for the determination of those points on a particular plate-model at which the strain state that existed was likely to cause fracture to initiate when the magnitude of the strains became sufficiently large. A method for the determination of that particular point from which fracture was first likely to initiate was also described. The magnitudes of the strains necessary to generate this first fracture were calculated.

The precise validity of the methods discussed was difficult to determine. Several basic assumptions concerning the material behaviour were made. In order to complete the portion of the fracture initiation diagram between the experimental uniaxial compression and uniaxial tension points, it was assumed that the nature of the material's strain response

was similar for the two tests. This assumption was discussed in the relevant Section (V-2) and considered to be reasonably valid.

Determination and analysis of the critical points in the plate-models required an assumption that the material behaved elastically until the initiation of fracture, i.e. that the state of strain at a point was independent of the strain magnitudes at the point. The linearity of the volumetric strain plots (Figs. V-2 to V-7) prior to fracture initiation supported this assumption.

While it was considered that these assumptions were unlikely to be a source of serious error, it was recognized nevertheless that any quantitative results should be viewed with some suspicion. However, as the same procedure was to be followed for the analysis of all the plate-models tested, it was considered that any trends indicated, due to variation in the volume fraction of coarse aggregate between models, should be valid.

CHAPTER VI

ANALYSIS AND RESULTS OF PLATE-MODEL TESTS

Section VI -1

Introduction

In Chapter IV the experimental techniques to be used for the measurement of strains from the plate-model tests were discussed. It was concluded that these techniques could furnish quantitative information concerning the strain distribution within the mortar phase of the models, which represented a thin slice cut from the body of a concrete. Qualitative information on the position of the point from which fracture initiated could also be obtained.

In Chapter V a method of analytically determining the point from which it was probable that fracture would first initiate in a particular model was discussed. It was concluded that this point could be located and that the strains at this point necessary to initiate fracture could be calculated.

It is proposed in this Chapter to consider in detail the results and analysis of one typical plate model. For other plate-models only the results are presented, since the methods of obtaining these and analyzing the plate-models was identical for every model tested.

For detailed discussion, a plate-model representing a fairly low volume fraction of coarse aggregate has been chosen (Chapter IV, Section IV - 2.4). For models representing high volume fractions the spacing of the aggregate particles becomes small. The number of strain gauges which may be placed within the area bounded by the particles (Fig. IV - 7) decreases correspondingly and, in general, the detailed presentation and discussion of results becomes more difficult.

Section VI - 2Details of Results and Analysis of Plate-Model

$$\text{DRM} - 4 : V_f = 13.4\%$$

Section VI - 2.1Strain Distribution

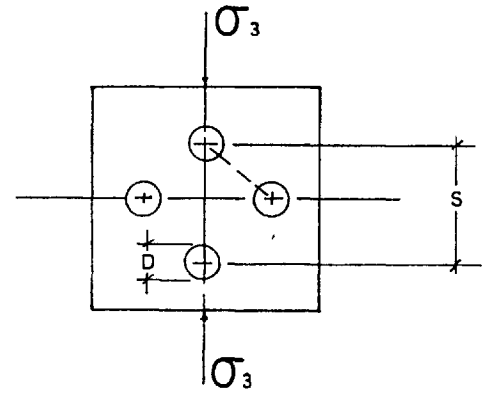
The necessary treatment and preparation of the specimen prior to testing was completed (see Chapter IV, Section IV - 4 stages (1) to (7)). The type of reference grid scribed on the photo-elastic coating is shown in Figure VI - 1 for one quadrant of the area within the aggregate particle packing. As previously discussed (Chapter IV, Section IV - 2.4) four identical areas, in terms of the strain response of the specimen, exist within the aggregate particle packing. In all the results presented, only one such area is shown, the results within this area being the average of the results from equivalent points in all four areas.

Method of Loading

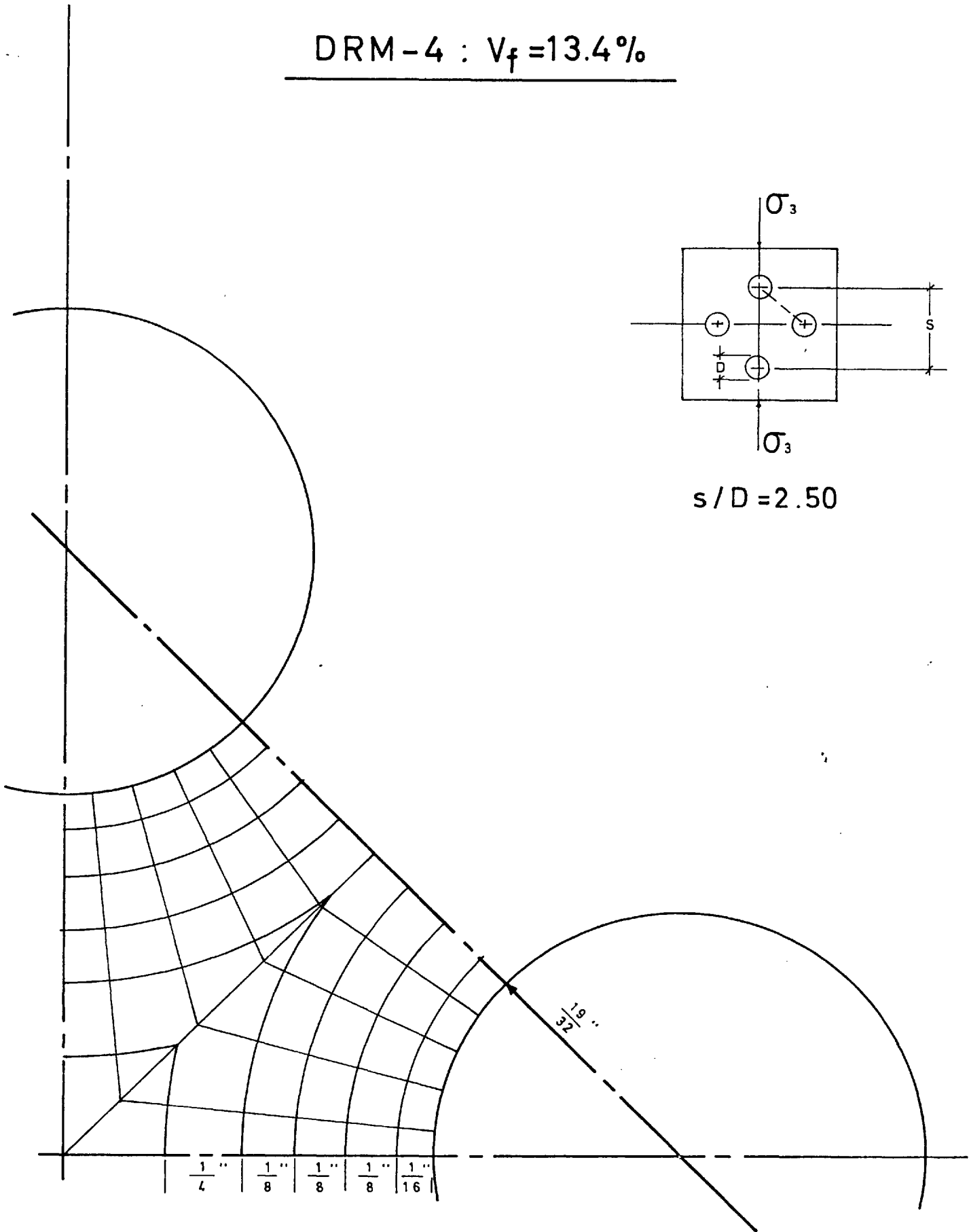
The specimen was allowed to stand at ambient conditions for six to eight weeks in order to attain moisture equilibrium across the coating thickness (Chapter IV, Section IV - 3.2). After this period, prior to being loaded, the photo-elastic coating was photographed in order to record any pre-existing isochromatic pattern. Frame (a) of Figure VI-9 shows the resulting photograph for this model. Any isochromatic pattern prior to loading was attributed to the introduction of shrinkage strains (stresses) during the drying period of the specimen. Further discussion of this effect is given in Section IV - 4.

In order to determine the directions of the principal strains at each point due to the application of a load, it was obviously necessary to apply a load of sufficient magnitude to "overcome" the directions of the strains already existing due to shrinkage effects. However, application of too great a load made the isoclinics (black lines) difficult to distinguish from the isochromatics (coloured lines). Furthermore, as the specimen was to be reloaded several times (Chapter IV, Section IV - 4), and repeatable

DRM-4 : $V_f = 13.4\%$



$$s/D = 2.50$$



Scribed Reference Grid

FIGURE : VI-1

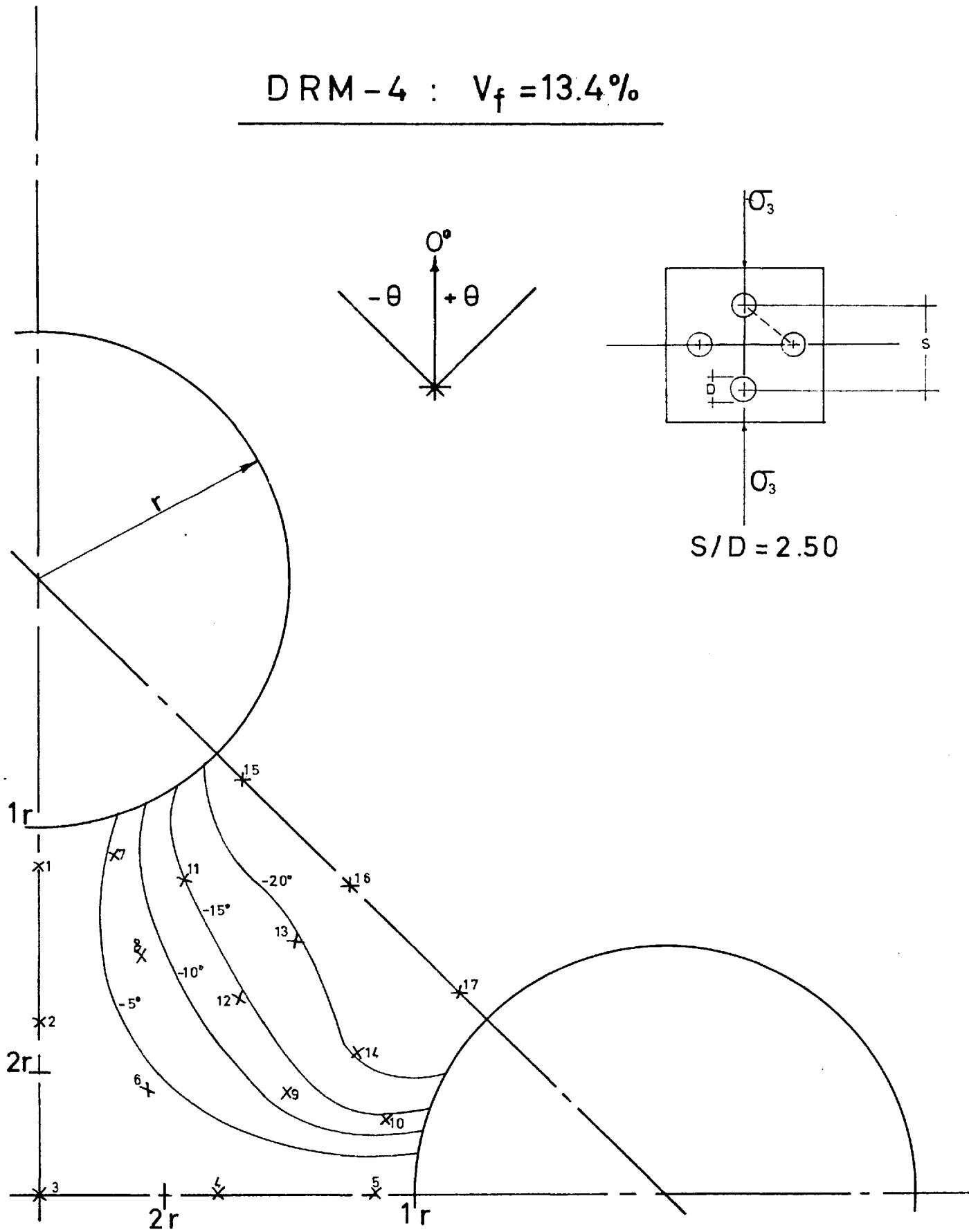
results were required, it was necessary to ensure that the applied loads were small enough to be within the "elastic" range of the specimen and not cause fracture to initiate. After several trials it was concluded that applied stresses of approximately 500 p.s.i. to 600 p.s.i. were sufficient for the measurement of the isoclinics. In order to generate reasonably measurable strains, applied stresses from 900 p.s.i. to 1000 p.s.i. were used for the determination of the isochromatics (shear strain) and of the strains at the gauged points (major principal strains). In all cases, prior to testing, the specimen was "exercised" six times to the required stress level.

Determination of Isoclinics (Directions of Principal Strain) at Each Point.

After allowing moisture equilibrium across the coating to be achieved (Chapter IV, Section IV - 3.2), the specimen was placed in the testing frame and a uniaxial compression load of 500 p.s.i. to 600 p.s.i. applied. At each intersection point of the reference grid the values of the isoclinics were read using normal incident photo-elastic equipment (Chapter IV, Section IV - 2.6B). From considerations of the total area within the aggregate packing and the need to cover as many points as possible with the size of gauge available the points to be gauged were selected. The orientation of the gauges (in the direction of the major, most tensile, principal strain at each point) were calculated from the isoclinic value at each point. The isoclinics determined represented the angle, measured to the nearest degree, between the direction of loading (σ_3) and the direction of the minor principal strain (ϵ_3) at each point. Thus, in order to determine the orientation of each gauge, in the direction of the major principal strain (ϵ_1), the addition of 90° to the measured isoclinic value was necessary.

Values of the isoclinics at the grid intersection points are shown, for this model, in Figure VI - 2(a). Contours of the isoclinics derived from these values are shown for this model in Figure VI-2(b). The gauge positions used are also shown and are numbered for later reference. The

DRM-4 : $V_f = 13.4\%$



Isoclinics and Gauge Positions

FIGURE: VI - 2(b)

degree of accuracy of the isoclinic contours may be judged from comparison between Figures VI - 2(a) and 2(b).

Determination of the Local Major Principal Strains (ϵ_1) and the Field Strains (ϵ_1)F and (ϵ_3)F.

Electrical resistance strain gauges of the "foil" type, 2 m.m. in length (Chapter IV, IV - 2.6B), were placed on the uncoated face of the specimen at the points corresponding to the grid intersection points selected as gauge positions. Each gauge was aligned in the direction of the major (most tensile) principal strain at the point, determined from the measured isoclinic value. The positioning and aligning of each gauge was considered to be accurate within $\pm \frac{1}{2}^\circ$ and ± 0.1 m.m. Crossed pairs of 20 m.m. gauges were placed at four points remote from the disturbing effect of the aggregate particles. These gauges were aligned to measure both the major and minor principal strains and were normally placed 2 ins. in from the corners of the model. These gauges were to be used to measure the "field" strains in the mortar, (ϵ_1)F and (ϵ_3)F.

The specimen was replaced in the testing frame and average stresses of the order 900 p.s.i. to 1000 p.s.i. applied in uniaxial compression. After "exercising" of the specimen, gauge readings were taken using a Peekel Data Logger. The measured value of the major principal strain at each point was non-dimensionalized by dividing by the average value of the minor principal strain in the "field" mortar (Chapter V, Section V - 3.1). Thus, values of:

$$\frac{(\epsilon_1)}{(\epsilon_3)F}, \quad \text{where suffix "F" refers to "Field"}$$

were obtained. Values of this ratio are shown, for each gauge point, in Figure VI - 3. From these values, contours of the above ratio were drawn and maybe seen in Figure VI-4. These contours are referred to as the "isotomics" of the above ratio. An idea of the accuracy of the contours

DRM-4 : $V_f = 13.4\%$

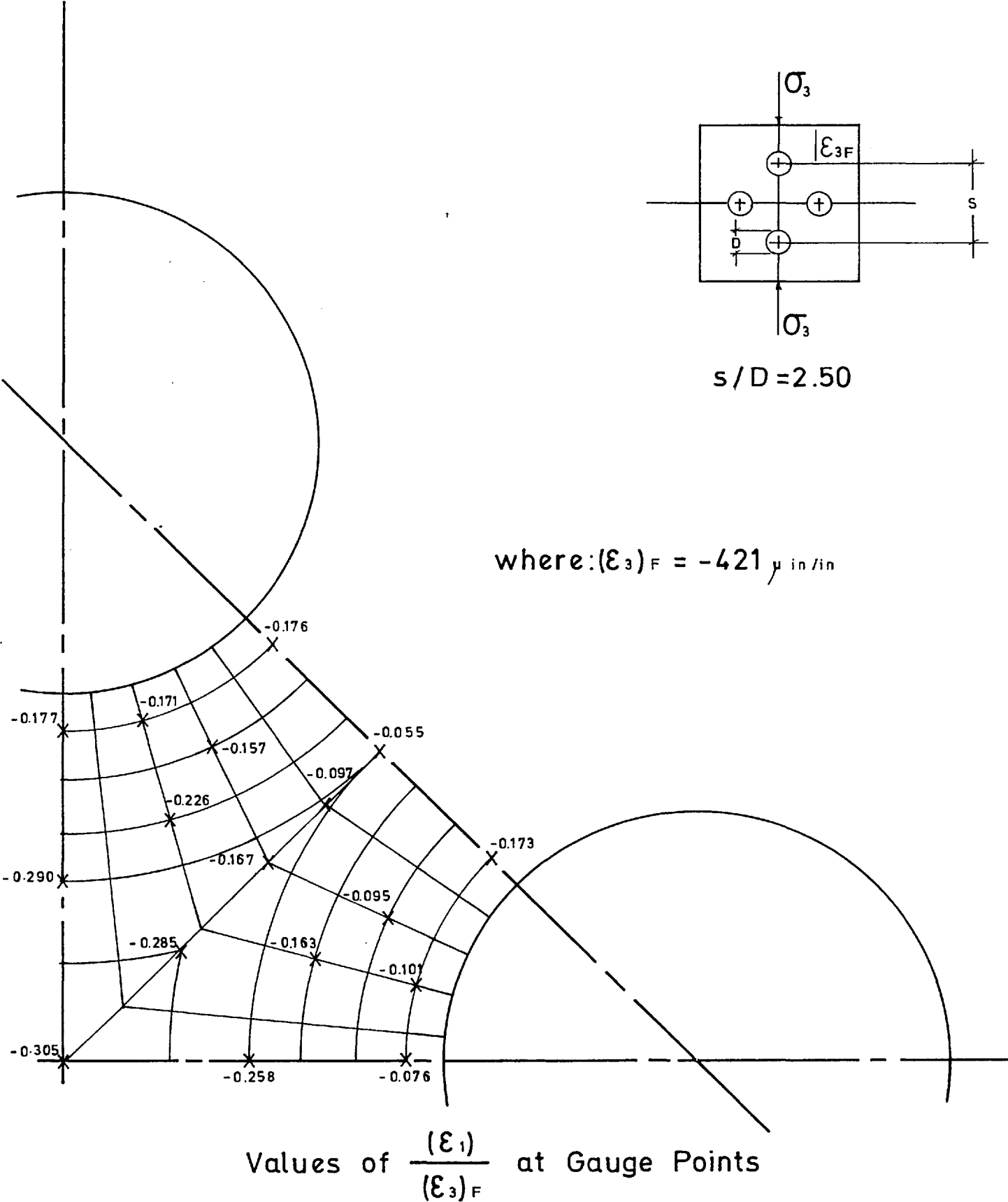


FIGURE : VI - 3

DRM-4 : $V_f = 13.4\%$

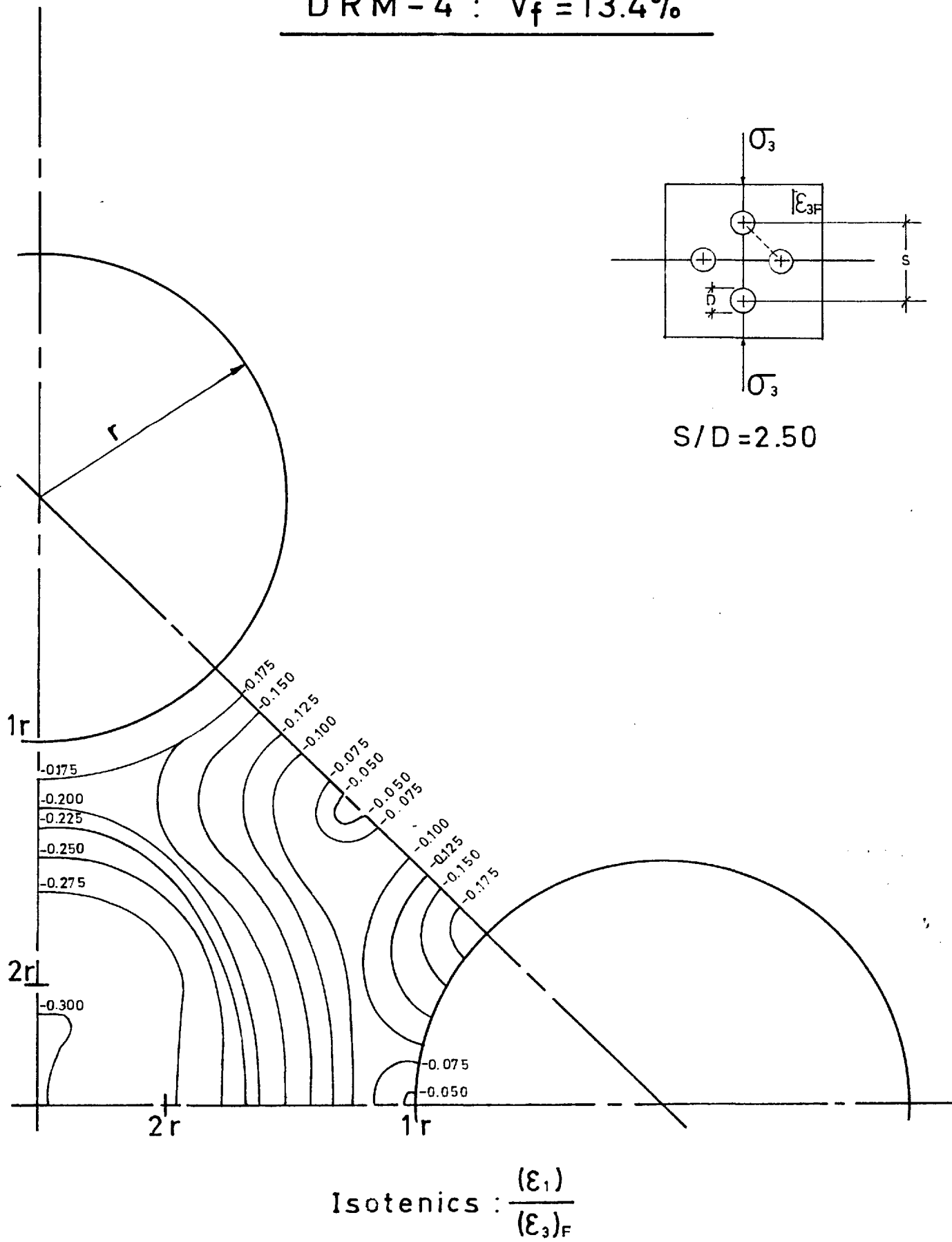


FIG. VI- 4

may be obtained from comparison of Figures VI - 3 and VI - 4. (Contours of isoclinics or of isochromatics were drawn from values measured at every point on the reference grid and are correspondingly more accurate).

From the crossed pairs of gauges placed in the mortar "field", values of the field shear strain were calculated. Thus, from the four gauged field positions, an average value of:

$$\left[(\epsilon_3 - \epsilon_1)F \right]_{\text{gauge}}$$

was obtained.

Determination of the Isochromatics and Face Correction Factor.

With the specimen still under load (as above for the measurement of the major principal strains) values of the fringe order at each point on the reference grid, and at the four "field" positions, were measured from the isochromatic pattern exhibited by the photo-elastic coating. From a knowledge of the fringe sensitivity of the particular coating used (supplied by manufacturer), the magnitude of the shear strain at each point was calculated. Thus, values of:

$$(\epsilon_3 - \epsilon_1) \text{ uncorrected, at each (local) point}$$

$$\text{and; } \left[(\epsilon_3 - \epsilon_1)F \right]_{\text{coating, from the four remote "field" points,}}$$

were obtained.

From the gauge and coating readings, the ratio:

$$\frac{\left[(\epsilon_3 - \epsilon_1)F \right]_{\text{gauge}}}{\left[(\epsilon_3 - \epsilon_1)F \right]_{\text{coating}}}$$

was determined and termed the "Face Correction Factor" (Chapter IV, Section IV - 3.1). For the particular model under consideration here;

$$\text{Face Correction Factor} = 1.084$$

All isochromatic (shear strain) readings were then multiplied by this correction factor in order to ensure as far as possible, the comparability

of strain readings from the two faces (coated and gauged) of the specimen. In all the results shown, values of the isochromatics given, $(\epsilon_3 - \epsilon_1)$, are the corrected values.

For the purposes of plotting, and in order that the results from different models could be compared, the shear strain at each point was non-dimensionalized with respect to the "field" value. Figure VI - 5 shows the resulting isochromatic contours for this model.

Determination of the Minor Principal Strains

From the corrected values of the isochromatics, $(\epsilon_3 - \epsilon_1)$, and the value of the major principal strain at each gauged point, (ϵ_1) , the magnitude of the minor principal strain (ϵ_3) , at each gauge position was calculated. This value was non-dimensionalized with respect to the average minor principal strain in the field, $(\epsilon_3)F$ (Chapter V, Section V - 3.1). For this particular model, the resulting isotenics of:

$$\frac{(\epsilon_3)}{(\epsilon_3)F}$$

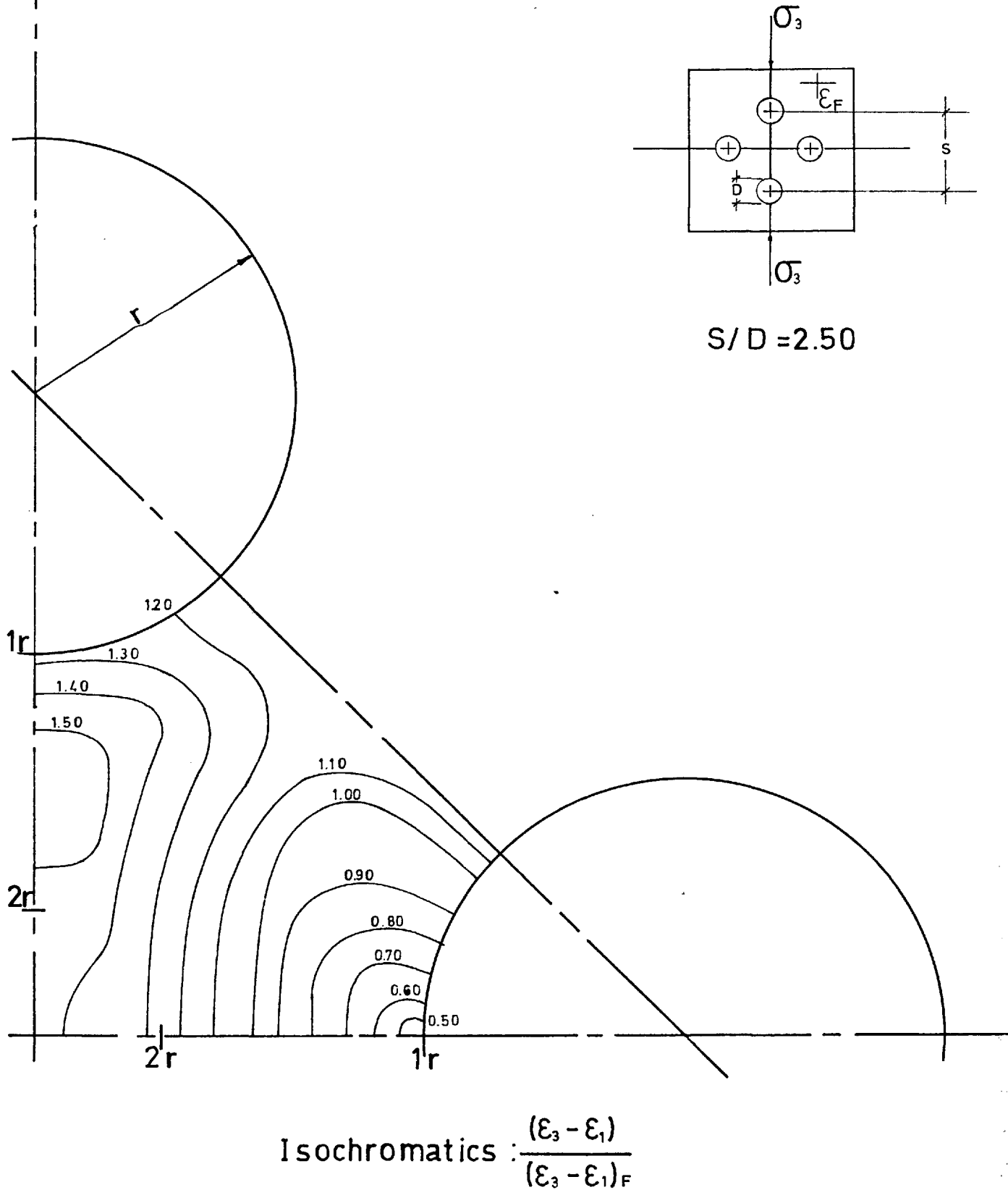
are shown in Figure VI - 6.

As previously mentioned, the pre-existing isochromatic pattern due to shrinkage was photographed prior to the specimen being loaded. Frame (a) in Figure VI - 9 shows this shrinkage pattern for this particular model. Upon the application of the load used to measure the isochromatics and the major principal strains, the resulting isochromatic pattern was again photographed and is shown in Frame (b) of Figure VI - 9. Similar photographs for other models are shown in Figures VI - 43, VI - 44 and VI - 45.

Determination of the Composite Material Strains

The specimen was removed from the loading frame and all 2 m.m. gauges

DRM - 4 : $V_f = 13.4\%$



$S/D = 2.50$

FIG. VI-5

DRM-4 : $V_f = 13.4\%$

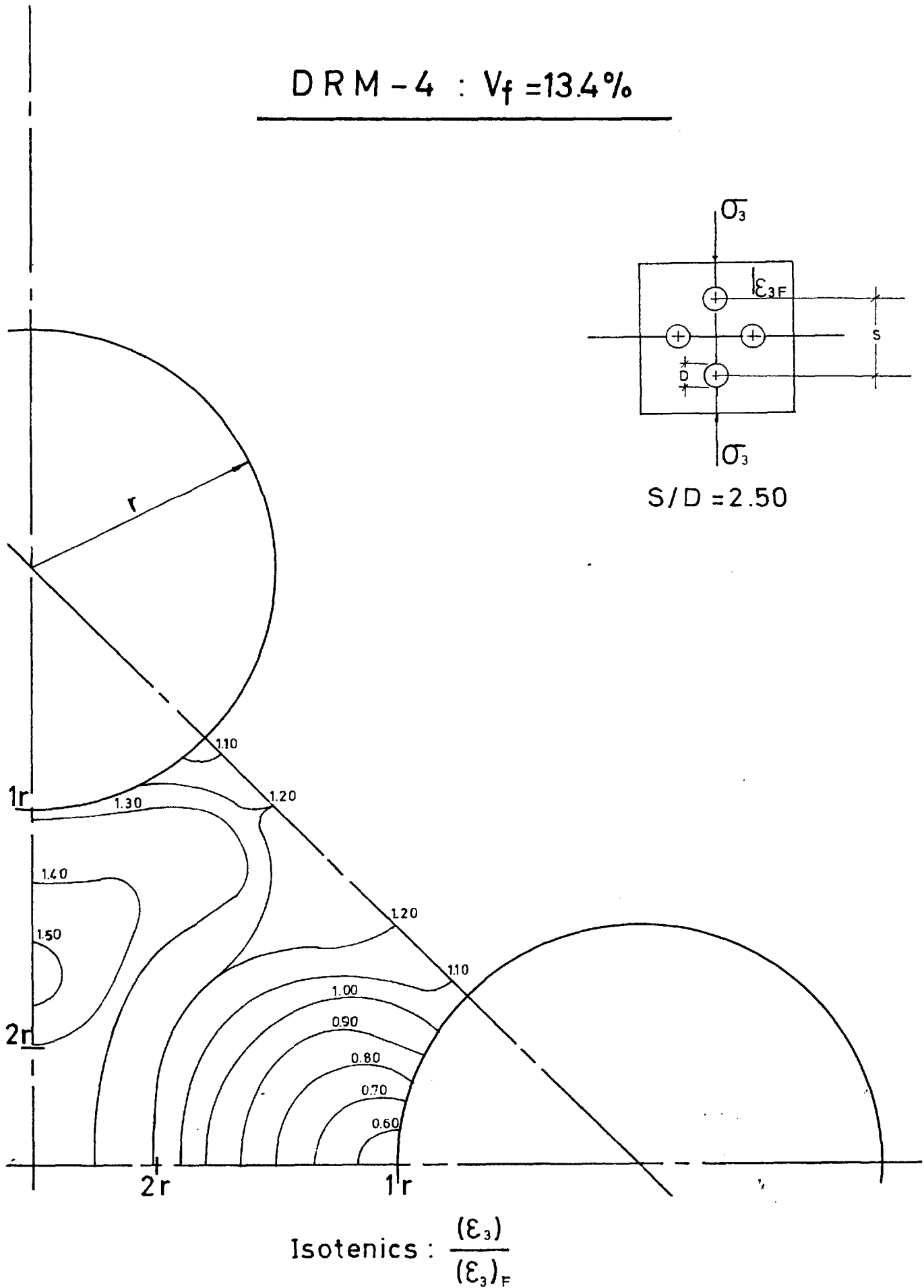


FIG. VI-6

removed. The 20 m.m. field gauges were not removed, however. The specimen was regauged between the centerlines of the aggregate particles in order that the strains of the composite material (aggregate and mortar) could be obtained (Fig. IV - 7). The specimen was replaced in the testing frame and reloaded until the values of the field strains were approximately equal to the values previously measured. The composite material strains were read, and the ratios:

$$\frac{(\epsilon_3) \text{ compos.}}{(\epsilon_3) F} \quad \text{and} \quad \frac{(\epsilon_1) \text{ compos.}}{(\epsilon_3) F} \quad \text{where suffix "compos." refers to "composite material".}$$

were calculated. The value of these ratios for particular models are given in Table VI - 1. For the model being considered here,

$$\frac{(\epsilon_3) \text{ compos}}{(\epsilon_3) F} = 0.988$$

and:

$$\frac{(\epsilon_1) \text{ compos}}{(\epsilon_3) F} = - 0.183$$

Section VI - 2.2 Critical Point Analysis

In Chapter V a method of critical point analysis was discussed. It was shown that the strain ratios calculated for each point on the plate-models:

$$\frac{(\epsilon_1)}{(\epsilon_3) F} \quad \text{and} \quad \frac{(\epsilon_3)}{(\epsilon_3) F} \quad \dots\dots\dots (1)$$

were of similar form to the ratios used to plot the fracture initiation diagrams:

$$\left[\frac{(\epsilon_1)}{(\epsilon_3) f.i.} \right] \sigma_1 = 0 \quad \text{and} \quad \left[\frac{(\epsilon_3)}{(\epsilon_3) f.i.} \right] \sigma_1 = 0 \quad \dots\dots (2)$$

The ratios in (1) could, therefore, be plotted on the fracture initiation diagram of the particular mortar and used to determine the extent of the "critical zone" in each model (Chapter V, Section V - 3.1).

For the model being considered, the strain ratios (1) at each gauge point are shown plotted in Figure VI - 7, on the fracture initiation diagram for the plate-model mortar. The points falling outside the fracture initiation envelope determine the boundaries of the critical zone (Chapter V, Section V - 3.1). The critical zone for this plate-model is shown in Figure VI - 8 as a shaded area. This drawing also shows both isotenics,

$\frac{(\epsilon_3)}{(\epsilon_3)_F}$ and $\frac{(\epsilon_1)}{(\epsilon_3)_F}$, from which the fracture initiation diagram co-ordinates

at any point in the model may be determined. For ease of reference, only the gauge points of the model are shown plotted on the fracture initiation diagram in Fig. VI - 7.

In order to determine the position of the point within the critical zone from which fracture was first likely to initiate, the procedure outlined in Chapter V, Section V - 3.2 was followed. For each Point lying outside the fracture initiation envelope in Fig. VI - 7, a line was drawn from the origin of the plot to that Point. The ratios of the path length within the "safe" portion of the envelope, A, to the total path length, L, were calculated for each Point. For the four critical Points shown in Fig. VI-7, representing gauge positions (2), (3), (4) and (6), (see Fig. VI - 2), the calculated ratios were of value:

$$(A/L)_2 = 0.489$$

$$(A/L)_3 = 0.422$$

$$(A/L)_4 = 0.438$$

$$(A/L)_6 = 0.439$$

Comparison of these ratios showed that Point (3) had the smallest proportion of its path length within the "safe" zone of the fracture initiation diagram. It was considered, therefore, that fracture would probably first initiate

from the position represented by this Point. The position on the plate model represented by this Point was termed the "most critical point" and is shown cross-hatched in Fig. VI - 8.

As outlined in Chapter V, Section V-4, the strains at the most critical point necessary to initiate fracture were then determined. From Fig. VI - 7, the fracture initiation strains at the Point (3) were given by:

$$(\epsilon_3) \text{ f.i.} = 0.510 \left[(\epsilon_3) \text{ f.i.} \right]_{\sigma_1 = 0} \dots (3)$$

and:

$$(\epsilon_1) \text{ f.i.} = -0.122 \left[(\epsilon_3) \text{ f.i.} \right]_{\sigma_1 = 0}$$

From Figure V-3, the value of the minor principal strain required to initiate fracture under uniaxial compression for the plate-model mortar was:

$$\left[(\epsilon_3) \text{ f.i.} \right]_{\sigma_1 = 0} = -1653 \mu \text{ in/in.}$$

Thus, for this plate-model, the strains required at the most critical point in order to initiate fracture were given by:

$$(\epsilon_3) \text{ f.i.} = 0.510 (-1653) \mu \text{ in/in} = -846 \mu \text{ in/in} \dots (4)$$

and:

$$(\epsilon_1) \text{ f.i.} = -0.122(-1653) \mu \text{ in/in} = +202 \mu \text{ in/in.}$$

Section VI - 2.3

Determination of Composite Material Strains Required to Initiate Fracture

The average strains which are measured on a concrete specimen under uniaxial compression are composed of strains both in the aggregate and in the mortar. Thus, they may be considered as the average strains of the composite (aggregate and mortar) material. In order to apply the results of the (uniaxial) plate-model tests to the uniaxial testing of concrete

specimens, it was desirable, therefore, to determine the strains of the composite material necessary to initiate fracture at the most critical point. In general, it was possible to deal either in terms of the major, (ϵ_1) , on the minor, (ϵ_3) , principal strain of the composite material. It was decided that the minor (most compressive) principal strain would be utilized.

As discussed in Section VI - 2.1, the strain ratio

$$\frac{(\epsilon_3)}{(\epsilon_3)_F}, \quad \text{where suffix "F" refers to "field"}$$

could be calculated at each gauge point. As the most critical point in the model under discussion was also a gauge point (Figs. VI - 8 and VI - 2), a "precise" value of this ratio could be calculated. For this model,

$$\left[\frac{(\epsilon_3)}{(\epsilon_3)_F} \right]_{\text{most critical point}} = 1.328 \quad \dots\dots\dots(5)$$

Inverting this ratio:

$$\left[\frac{(\epsilon_3)_F}{(\epsilon_3)} \right]_{\text{most critical point}} = 0.754 \quad \dots\dots\dots(6)$$

Also from Section VI - 2.1, for this model:

$$\frac{(\epsilon_3)_{\text{compos.}}}{(\epsilon_3)_F} = 0.988 \quad \dots\dots\dots(7)$$

where suffix "compos." refers to "composite material".

Combining (6) and (7):

$$\left[\frac{(\epsilon_3)_F}{(\epsilon_3)} \right]_{\text{most crit. point}} \times \frac{(\epsilon_3)_{\text{compos.}}}{(\epsilon_3)_F} = \left[\frac{(\epsilon_3)_{\text{compos.}}}{(\epsilon_3)} \right]_{\text{most crit. point}} = 0.745 \quad \dots\dots\dots(8) \text{ pt.}$$

(N.B. It should be noted that the similarity between the values calculated from equations (6) and (8) is entirely coincidental)

Thus, the value of the minor principal strain of the composite material is

0.745 times the value of the minor principal strain at the most critical point. Assuming that this ratio remains constant until fracture initiates at the most critical point, the composite material strain necessary to cause fracture initiation may then be calculated. From (8):

$$(\epsilon_3)_{\text{compos.}} = 0.745 (\epsilon_3)_{\text{most c. it. point.}}$$

and, at fracture initiation,

$$\left[(\epsilon_3)_{\text{compos.}} \right]_{\text{f.i.}} = 0.745 \left[(\epsilon_3)_{\text{most crit. pt.}} \right]_{\text{f.i.}} \dots (9)$$

Thus, from (4) and (9), the minor principal strain of the composite material which is necessary to cause fracture to initiate at some point within the composite material (most critical point), is given, for this particular model, by:

$$\left[(\epsilon_3)_{\text{compos.}} \right]_{\text{f.i.}} = 0.745 (-846) \mu \text{ in/in} = -630 \mu \text{ in/in.} \dots (10)$$

This calculation is based upon the assumption that the ratio of the composite material strain to the strain at a particular point within the aggregate packing will remain unchanged until the geometry of the system changes i.e. until fracture initiates. It was recognized that this assumption would create errors if the mortar exhibited strain-level dependent creep. Tests conducted at various strain levels could not detect any such behaviour and it was considered, therefore, that errors due to this assumption were likely to be small.

Values of the minor composite material strain required to initiate fracture at some point (the most critical point) within the composite material are shown in Table VI - 1 for models representing various volume fractions of coarse aggregate.

Section VI - 2.4

Observations upon Loading to Failure.

Following the procedure outlined in Chapter IV Section IV - 2.8, the un-coated face of the plate-model specimen was washed with a fluid suspension of fluorescent particles. When viewed under ultra violet light,

initial cracking due to shrinkage (or other causes) could be detected, and was marked in with pencil (cracks of this nature may be seen in Figures VI - 46 to VI - 49). The specimen was then placed in the testing frame and loaded to failure. A Shackman high-speed camera was used to photograph the isochromatic pattern in the photo-elastic coating at intervals of $\frac{1}{4}$ second. The resulting film strip could then be analysed to determine approximately the position of the point of fracture initiation (Chapter IV, Section - 3.2). Selected frames from the film strip obtained for the model being considered here are shown in Figure VI - 9.

Frame (a) shows the initial isochromatic pattern due to shrinkage. The fact that the coating was placed on the specimen and the specimen then left to dry for six to eight weeks prior to testing, meant that any shrinkage strains generated in the model during this period showed as an initial isochromatic pattern on the photo-elastic coating. Shrinkage shear strains of the order of 0.1 fringes (approx. 60μ in/in) were seen to exist in this model.

Frame (b) shows the isochromatic pattern resulting from the application of a small load (approx. 200 p.s.i.). The overall pattern or shape of the isochromatics should be noted.

Frame (c) shows the isochromatic pattern resulting from the further application of load (approx. 1300 p.s.i.). As expected, the magnitudes of the isochromatics had increased due to the increased load. However, the important point to note in this context is that the overall shape of the isochromatic pattern had remained largely unchanged from that seen in Frame (b). It was considered, therefore, that the geometry of the system had remained unchanged with respect to the formation of any new internal boundaries i.e. cracks.

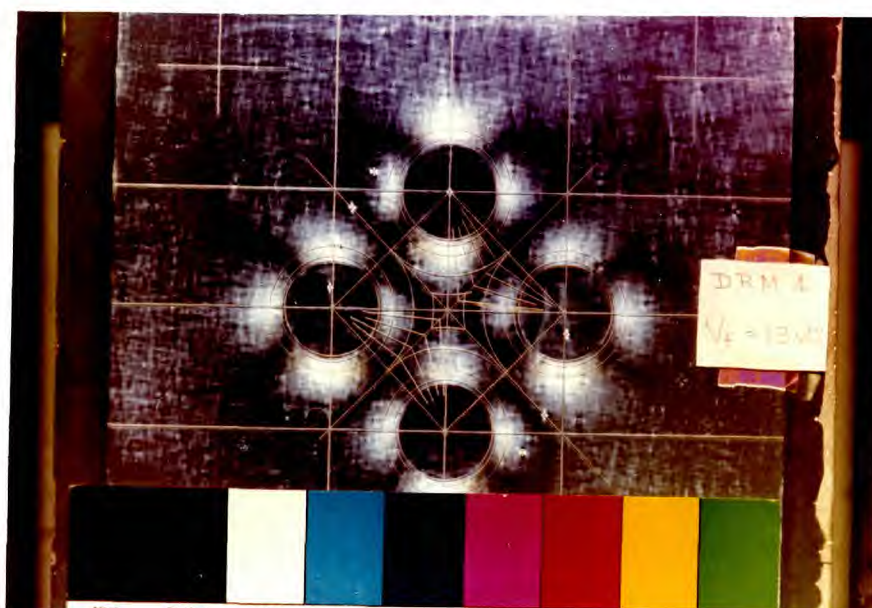
Frame (d) shows the result of further load application. It may be seen that the isochromatic pattern at the point marked has changed. The isochromatics formed a closed ring around this point and had increased markedly in value with respect to the isochromatics at other points in the

field. It was considered, therefore, that cracking had initiated at the point marked. Unfortunately, as no direct link between the camera and the testing machine could be made, the load at which fracture initiation occurred was difficult to determine, but was thought to be in the region of 2,500 p.s.i. The "most critical point" marked in Figure VI - 8 and the point of initiation marked in this Frame may be compared. Comparison of these points showed that the analytically determined point of initiation correlated extremely well with the experimentally observed point, within the limited accuracy of the experimental observation.

It was noted that no evidence of bond failure between the mortar and the aggregate inclusions could be detected from this Frame. The importance of this experimental observation has been discussed in Chapter III, Section III - 1. It is again emphasized that the conclusion that fracture initiates within the mortar phase of concrete, rather than at the mortar-aggregate interface, must be limited to the particular materials, model configuration and loading used in these tests. In all the tests carried out, it was observed that fracture initiated in the mortar phase. It was concluded, therefore, that for these particular tests, analysis of the mortar phase to determine the point of fracture initiation was justified.

Frame (e) shows the isochromatic pattern just prior ($\frac{1}{4}$ second) to failure. Extension of the zone in which fracture had occurred was noted from the concentration of isochromatics at various points. It may be seen that fracture had extended from the point of initiation in the direction of the applied minor (compressive) principal stress. It was further noted that fracture around the aggregate particles appeared now to be in progress, i.e. bond failure had occurred.

Provided that the specimen was still reasonably intact after failure, the un-coated face was re-washed with the fluorescent dye and examined under ultra-violet light. The crack pattern was marked. Unfortunately, the



(a)



(b)



(c)

FIG.VI-9



(d)



(e)

specimen being considered in this section had by this time completely shattered and could not be examined in this fashion. Results of this type of examination for various other specimens are shown in Figures VI - 46 to VI - 49 and are discussed in Section VI - 3.

Section VI - 2.5

Correlation of Analysis and Observations

The analysis of the plate-model determined the point at which fracture was likely to initiate. This point was seen to relate closely to the experimentally observed point of fracture initiation. The observation of the failure processes detected no bond separation between the aggregate and the mortar until just prior to failure. This point is discussed in Section VI - 4. The quantitative correlation between the predicted strains necessary to cause fracture initiation and the actual strains required to initiate fracture in "real", three-dimensional, concrete is discussed in Chapters VII and VIII.

Section VI - 3

Results of Plate-Model Tests

The results of the experimental strain analysis conducted on each plate-model are given graphically in Figures VI - 10 to VI - 42. For each model, contours of the isoclinics, of the isochromatics, and of the two principal isotenics are given. Also presented are the results of each critical zone and critical point analysis. Table VI - 1 gives a synopsis of these results and of the calculated value of the minor principal composite material strain necessary to cause fracture initiation within each model. Photographs of the shrinkage and initial load isochromatics are shown for two models in Figures VI - 43 and VI - 44. Selected frames from the film strip taken during loading to failure are shown for one model ($V_f = 50.0\%$) in Figure VI - 45. Also shown is the "frozen" isochromatic pattern after load was removed. Figures VI - 46 to VI - 49 show the crack patterns exhibited by various specimens. Note that the model shown in Fig. VI - 46

was only loaded to the point at which fracture was thought to have initiated. The model was then examined using the ultra-violet light and fluorescent dyeing technique (Chapter IV, Section IV - 2.8) to locate the position of the point at which fracture had initiated. This position was compared to the position of the fracture initiation point determined for similar models from examination of the isochromatic record of the isochromatic pattern during loading to failure. From this comparison it was concluded that the examination of the isochromatic pattern could be used with reasonable validity to determine the position of the fracture initiation point.

Section VI - 4

Discussion of Results

Position of Most Critical Point

For a model representing an extremely low volume fraction of coarse aggregate (5%), the most critical points were seen to lie approximately 1.7 radii from the edge of each aggregate particle, along a line in the direction of the applied compressive stress (Fig. VI - 12). For all other models tested (V_f from 13.4% to 72.2%) the critical point was determined, both from analytical and experimental evidence, as being at the center of symmetry of the aggregate packing.

Composite Material Strains Required to Cause Fracture Initiation at Most Critical Point.

As previously mentioned (VI - 2.1) it was decided to deal in terms of the minor, most compressive, principal strain. From Table VI - 1, values of the ratio:

$$\frac{(\epsilon_3)_{\text{compos.}}}{(\epsilon_3)_F}$$

are shown for each model tested. It may be seen that this ratio is slightly greater than unity at low volume fractions, and less than unity at high

volume fractions. It would appear that the value of this ratio was governed by the interaction of two opposing effects. Firstly, as more aggregate was introduced, a higher proportion of the composite material was composed of the hard, stiff, aggregate particles. Thus the strains of the composite material might be expected to decrease in relation to the remote mortar "field" strains. However, the increased stiffness of the composite section of the plate-model caused load channelling to occur. That is, the composite section of the model carried greater overall loads than the pure mortar section. Thus, the average stresses on the composite section were higher than those on the mortar section, and the composite material strains could be expected to increase accordingly with respect to the mortar field strains. It was apparent (Table VI - 1) that at low volume fractions the increase in the composite material strains due to the second, "load-channelling", effect was more pronounced than the decrease in the strains due to the stiffness effect. At higher volume fractions (approximately greater than 5%) the "priorities" were reversed and the composite material strains were seen to decrease with respect to the mortar field strains.

In Chapter V, it was noted that the "criticality" of a particular point was a function of the state of strain at that point with respect to the state of strain existing in the remote field mortar. Thus, the local strains at the most critical point required to initiate fracture would be a function of the principal strain ratio at the point, $(\epsilon_3)/(\epsilon_1)$, and the concentration factor of one strain with respect to the mortar field strain, $(\epsilon_3)/(\epsilon_3)F$. In all the plate models tested, the state of strain at the most critical point was such that the local minor principal strain required for fracture initiation, $(\epsilon_3)f.i.$, was less than that required by a mortar element existing purely under uniaxial compression (represented in the first line of Table VI - 1 as $V_f = 0\%$). Furthermore, the value of this strain, $(\epsilon_3)f.i.$, was seen to generally decrease as the volume fraction of aggregate was increased. This was due to the changing values of the

principal strain ratio at the most critical point, and to the changing strain concentration factors.

The net results of the interaction of all the above effects upon the composite material strain required to initiate fracture at the most critical point may be seen from Table VI - 1 and from Figure VI - 50. Figure VI - 50 shows values of the minor principal composite material strain required to cause fracture initiation plotted against the volumes of coarse aggregate represented by the models. The strain axis of this plot has been non-dimensionalized with respect to the minor principal strain necessary to cause fracture initiation in a pure mortar specimen ($V_f = 0\%$). From this Figure it may be seen that as the aggregate volume fraction represented by the models was increased, crack initiation was calculated as occurring at progressively lower values of the overall minor principal strain of the composite material.

As previously mentioned the addition of increasing quantities of hard coarse aggregate to a mortar matrix will cause the resulting composite material (concrete) to become increasingly stiff, i.e. the net Young's Modulus of the material is increased. Thus, for the application of a constant stress to the composite material, the average strains generated will decrease in accordance with the increasing Young's Modulus. A "line of constant stress", showing the consequent strain decrease is shown plotted in Figure VI - 50. It can be seen that the average strains of the composite material, computed from the model tests as being necessary to cause cracking, were very much lower than could be explained by any increase in stiffness of the material due to the addition of coarse aggregate. In fact, this effect was primarily due to the increasing magnitude of the local strain concentrations and to the different values of the local, most critical, principal strain ratios, as the volume fraction of coarse aggregate in the material was increased.

It may be seen from Figure VI - 50 that the average strain of the

composite material necessary to cause fracture initiation dropped sharply in magnitude with the addition to pure mortar of a small volume fractions of coarse aggregate. This was due to the concentration and change in nature (i.e. (ϵ_3/ϵ_1) value at local points) of the local strains, around the aggregate particles, in relation to the strains which existed in the pure, homogeneous mortar. This effect may be clearly seen by the isotonic values in Figure VI - 12 for a model representing a coarse aggregate fraction of 5.0%. The addition of more aggregate further increased the magnitude of the local strain concentrations (Figs. VI - 13 to VI - 42 and Table VI - 1), thus lowering the composite material strain necessary to cause crack initiation.

Load Transference between Aggregate Particles.

Other than the initial "movement" of the position of the most critical point, there did not appear to be any fundamental difference in behaviour, in the context of fracture initiation, between models representing widely different aggregate volume fractions. Extrapolation of the theoretical equations of Muskhelishvili⁵⁹ (Table IV - 2) showed that the degree of criticality of the most critical point could be expected to increase with increasing aggregate content. As discussed in the previous section, this was observed to be the case. It was noted, however, that there was a certain qualitative difference between the response of low volume fraction models and those representing higher volume fractions.

Figure VI - 9 shows, in frames (b) and (c), the isochromatic pattern in a loaded model, this model representing a volume fraction of 13.4%. In a general sense, one can see from the pattern that the method of load transference between particles is of a basically "columnar" type, and is in the direction of the applied principal stress. Figure VI - 44 shows a model representing a higher volume fraction (46.9%), from which it may be seen that the method of load transference between particles is largely

of the "ring" type along the 45° line joining the particle centers. Figure VI - 43 depicts an approximately median case ($V_f = 39.2\%$) in which the interparticle load transferrance was apparently shared between the above two methods. It appeared from the tests that the transition from one method to the other was gradual and apparently began to occur when the particle edges were separated by approximately one particle diameter. This configuration represented a volume fraction of 26.3% ($S/D = 2.00$), and the strain response of the corresponding model is shown in Figures VI - 16 to VI - 18. No marked quantitative effect upon the most critical point could be traced to this gradual transition in the method of interparticle load transferrance. This phenomenon is of interest in the context of structural models depicting the behaviour of concrete, such as the Baker Lattice⁵⁰. For such a lattice, it would appear that the relative stiffnesses of the various "struts" between aggregate particles may be considered to be a function of the particle spacing to particle diameter ratio - in other words, to be a function of the volume fraction of coarse aggregate.

Influence of Pre-existing Shrinkage Cracks

It was noted that the mortar in the area denoted by the shortest line between any two particle centers (45° line), generally existed under states of biaxial (triaxial if extended to three dimensions) compressive strain. This observation was of importance when considering the effect of shrinkage cracking upon the fracture initiation process. In Figures VI - 46 to VI - 49, the initial cracking due to shrinkage may be seen for various models. This cracking generally followed the 45° line between particle centers. As the mortar in this zone was subjected, for the aggregate configuration used, to biaxially compressive strains when a uniaxial load was applied to the model, the shrinkage cracks closed and became extremely stable. Thus, they played no part in the initiation of fracture. The part played by these shrinkage cracks in the overall failure process was impossible to

determine quantitatively. However, examination of the final crack patterns exhibited by various failed specimens, indicated that the initial shrinkage cracking had not extended (Figs. VI - 46 to VI - 49).

It is of importance to note that the quantitative strain analysis in the mortar zone along the 45° inter-particle line must be viewed with suspicion. If the closure of pre-existing shrinkage cracks occurred, then the strain readings obtained in this zone could not be strictly considered as representative of the mortar strains. In such a case, they merely indicated an approximate value and direction of the displacement of the crack faces relative to one another.

Initiation of Fracture within the Mortar and Failure of the Mortar-Aggregate Bond.

Initiation of fracture was observed to occur within the mortar phase of the composite material for all the models tested and to correlate closely with the predicted point of fracture initiation. Failure of the aggregate-mortar bond did not appear to occur until very near the ultimate strength of each specimen. It was considered, therefore, that the analysis of the mortar phase of the material was justified. It is of interest to speculate on the reasons why bond failure did not occur prior to fracture within the mortar. It would appear⁵⁵, for the particular aggregates and particular surface roughness used in these tests, that the aggregate-mortar bond strength in direct tension could be expected to be of the same order of magnitude as the direct tensile strength of the pure mortar. For the particular particle configuration and load geometry used, the results in Figs. VI - 10 to VI - 42 show that the tensile strain at the most critical point was generally five to ten times greater than the tensile strain at the mortar-aggregate interface. Although the author is well aware that the magnitude of the tensile strain at a point is not necessarily the sole criterion of fracture, it was considered that differences in tensile strains of this order could account for the fact that fracture did not initiate at

the mortar-aggregate interface. It is appreciated that for different materials used in different configurations, very different results might have been obtained.

Section VI - 5

Conclusions

The plate-model tests indicated a gradual decrease in the average composite material strain necessary to cause fracture initiation as the volume fraction of coarse aggregate was increased. This decreasing strain was not explicable solely in terms of the increased "stiffness" of the material due to the addition of coarse aggregate, but was seen to be a function of the increasing "criticality" of the most critical point in the model. This was because the magnitudes of the local strain concentrations were increased and the values of the local principal strain ratios were changed by increasing the volume fraction of coarse aggregate.

It was concluded that, in general, the most critical point in each model was at the center of symmetry of the aggregate packing. For models representing low volume fractions (approx. 5.0%) the position of the point was slightly different (Fig.VI - 12).

No marked change in the quantitative behaviour of the models was noticed as the volume fraction of aggregate was increased above 5%. Qualitative transition from a "columnar" to a "ring" method of interparticle load transferrance was noted and appeared to begin when the spacing to diameter ratio of the particle packing was less than 2.00.

For the particle configuration used, shrinkage cracks appeared to generally exist in zones of biaxial (triaxial in three dimensions) compression and, therefore, did not contribute to the fracture initiation process. The contribution of such cracks to the total failure of the specimen appeared to be minimal.

Observations of the specimens when tested to failure confirmed that fracture initiated within the mortar phase of the composite material.

However, fracture of the aggregate-mortar interface was not observed until disruption of the specimen was almost complete.

The curve plotted in Figure VI - 50 shows the variation in the minor principal composite material strain (computed from the plate model tests and the mortar fracture initiation diagram (Section VI - 2.3) necessary to cause fracture to initiate at some point within the composite material. As each point on this curve was computed from a plate-model test, and as each test utilized the same experimental and analytical methods, the relationship between any two points on the curve should be relatively valid. In other words, although the quantitative values of any one point on the curve may be in error due to the assumptions made and the techniques used (see previous Chapters), the relative values between any two points should be reasonably accurate. Thus, the behaviour of concretes containing various proportions of coarse aggregate could be expected to follow the same type of curve as that shown in Figure VI - 50. The various limitations of applying any two-dimensional model to a three-dimensional prototype, as outlined in Chapter IV would, of course, apply. Nevertheless, if the plate model technique, both experimental and analytical, was to be considered as having given meaningful results, the shape of the curve shown in Fig. VI --50 (i.e. the qualitative trend of the curve) should closely duplicate the shape of a similar curve derived from three-dimensional concretes composed of similar materials and tested under uniaxial compression.

TABLE VI - 1

SYNOPSIS OF PLATE-MODEL TEST RESULTS

Vf%	(ϵ_1) compos.						$[(\epsilon_3)$ compos.] f.i.
	(ϵ_3) F	(ϵ_3) compos.	At "most critical point"				
			(ϵ_3) F		(ϵ_3) f.i.		
			$(\epsilon_1)/(\epsilon_3)$ F	$(\epsilon_3)/(\epsilon_3)$ F	(ϵ_1) f.i. μ in/in	(ϵ_3) f.i. μ in/in	
0	-0.182	1.000	-0.182	1.000	+301	-1653	-1653
1.2	-0.146	1.007	-0.247	1.153	+221	-1030	- 894
5.0	-0.180	1.021	-0.267	1.146	+220	- 940	- 839
13.4	-0.183	0.988	-0.305	1.328	+202	- 846	- 630
20.0	-0.156	0.915	-0.276	1.114	+189	- 764	- 618
26.3	-0.211	0.972	-0.420	1.114	+156	- 413	- 362
30.6	-0.165	0.855	-0.365	1.082	+159	- 470	- 371
34.9	-0.185	0.983	-0.344	1.338	+187	- 717	- 526
39.2	-0.186	0.840	-0.477	1.188	+139	- 347	- 246
46.9	-0.158	0.885	-0.400	1.000	+139	- 348	- 307
50.0	-0.179	0.806	-0.549	0.993	+114	- 206	- 168
54.6	-0.157	0.872	-0.351	1.130	+164	- 528	- 407
67.2	-0.162	0.760	-0.366	1.126	+152	- 466	- 325
72.2	-0.160	0.676	-0.390	0.961	+134	- 330	- 232

where:

Vf = volume of coarse aggregate represented by model.

(ϵ_1) = major (most tensile) principal strain

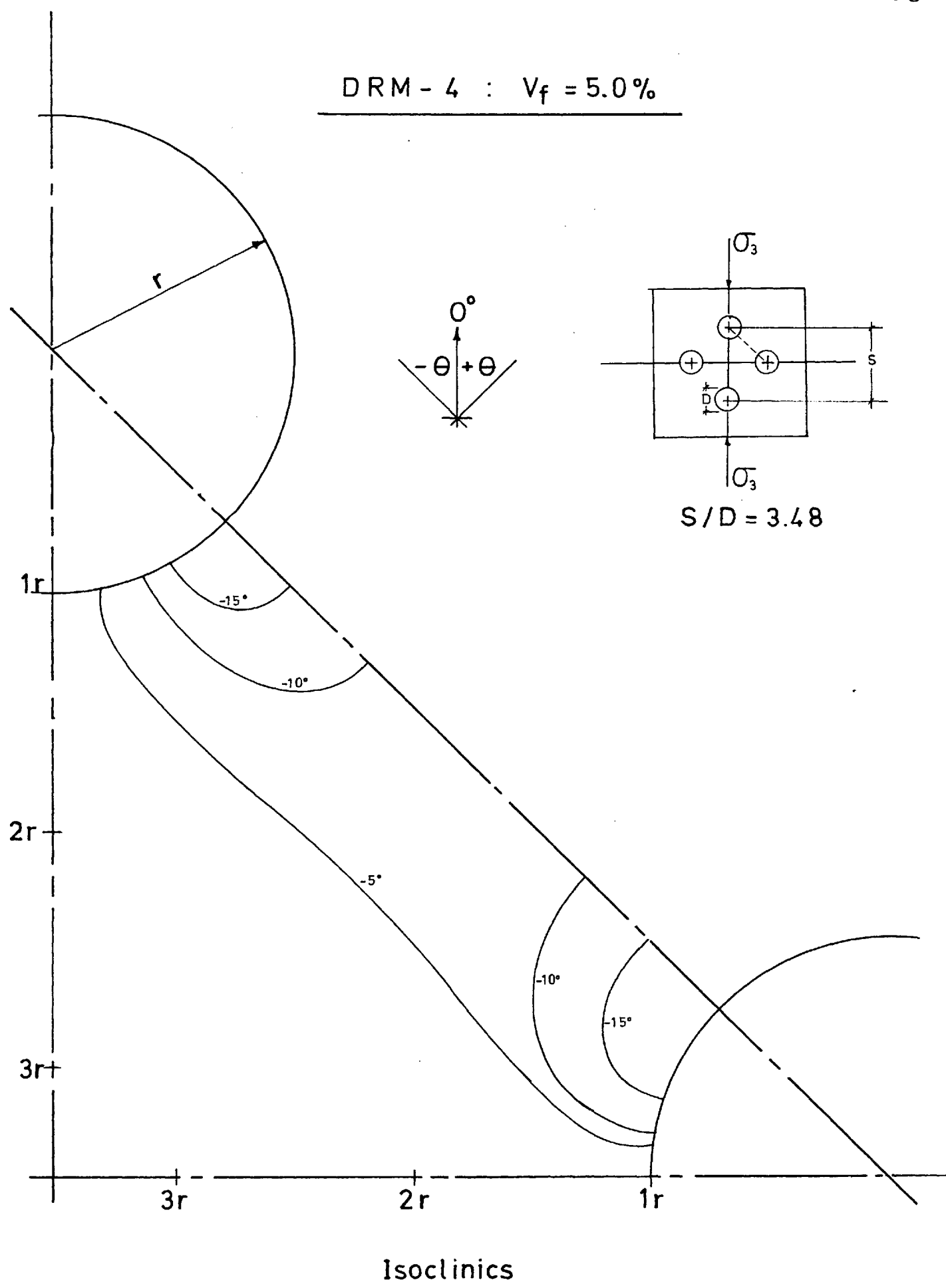
(ϵ_3) = minor (most compressive) principal strain

suffix "compos." refers to "composite material"

suffix "F" refers to "field"

suffix "f.i." refers to "fracture initiation".

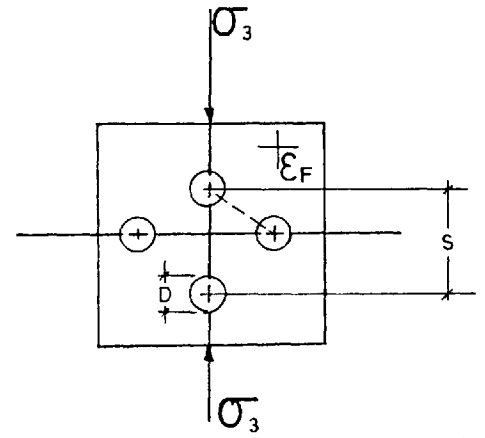
DRM - 4 : $V_f = 5.0\%$



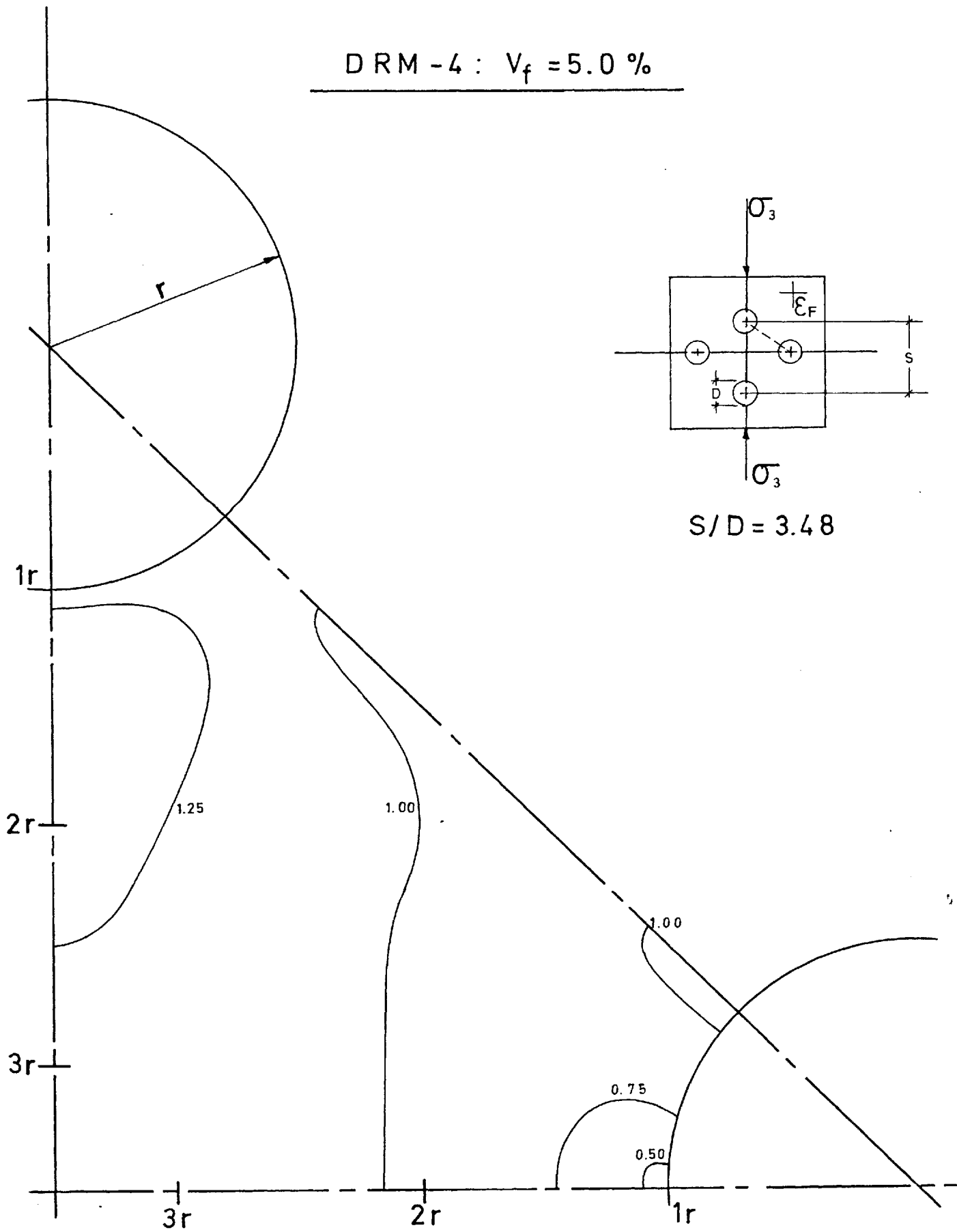
Isoclinics

FIGURE : VI-10

DRM -4 : $V_f = 5.0\%$



$$S/D = 3.48$$



Isochromatics : $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG. VI-11

DRM - 4 : $V_f = 5.0\%$

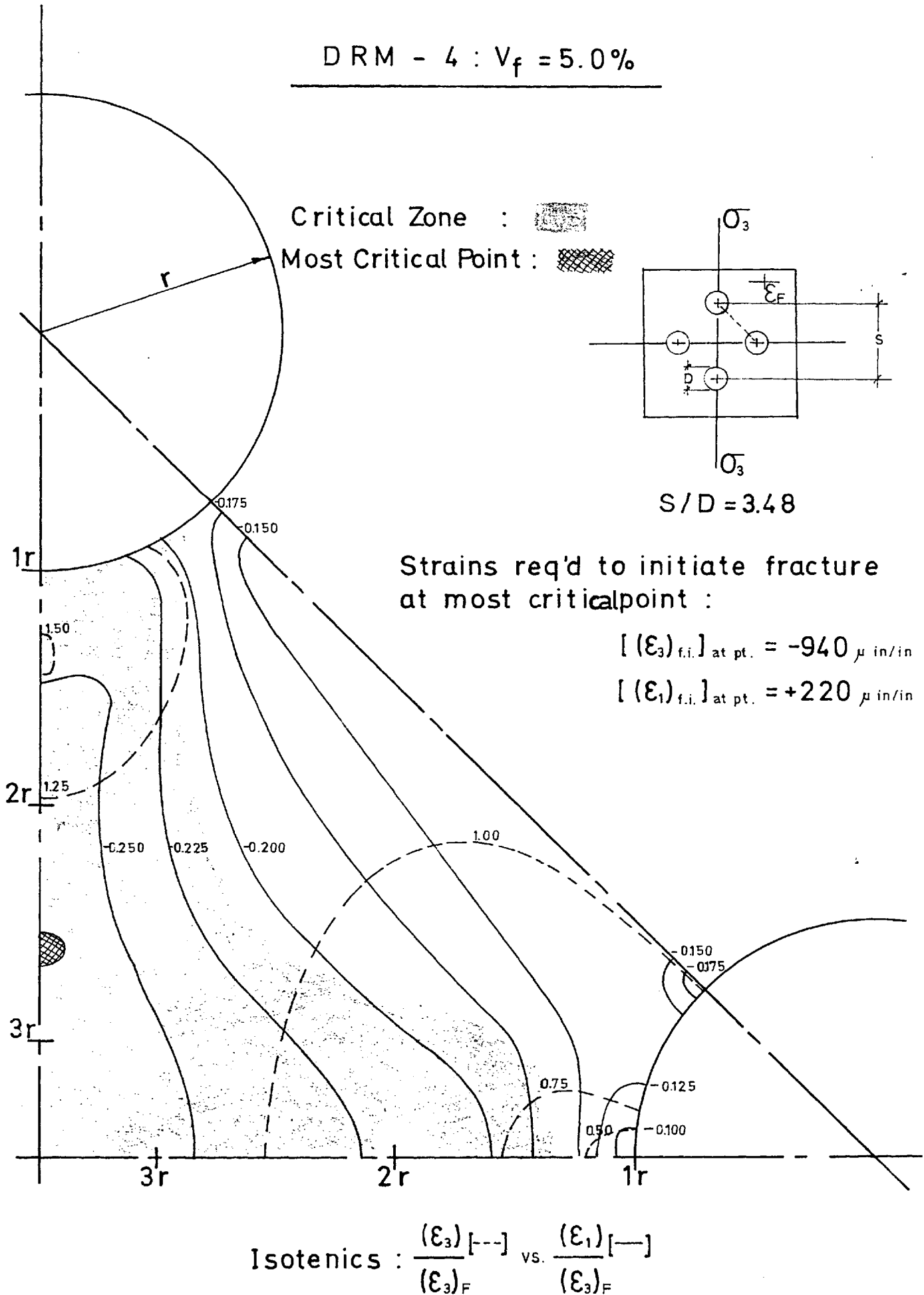


FIG. VI-12

DRM-3 : $V_f = 20.0\%$

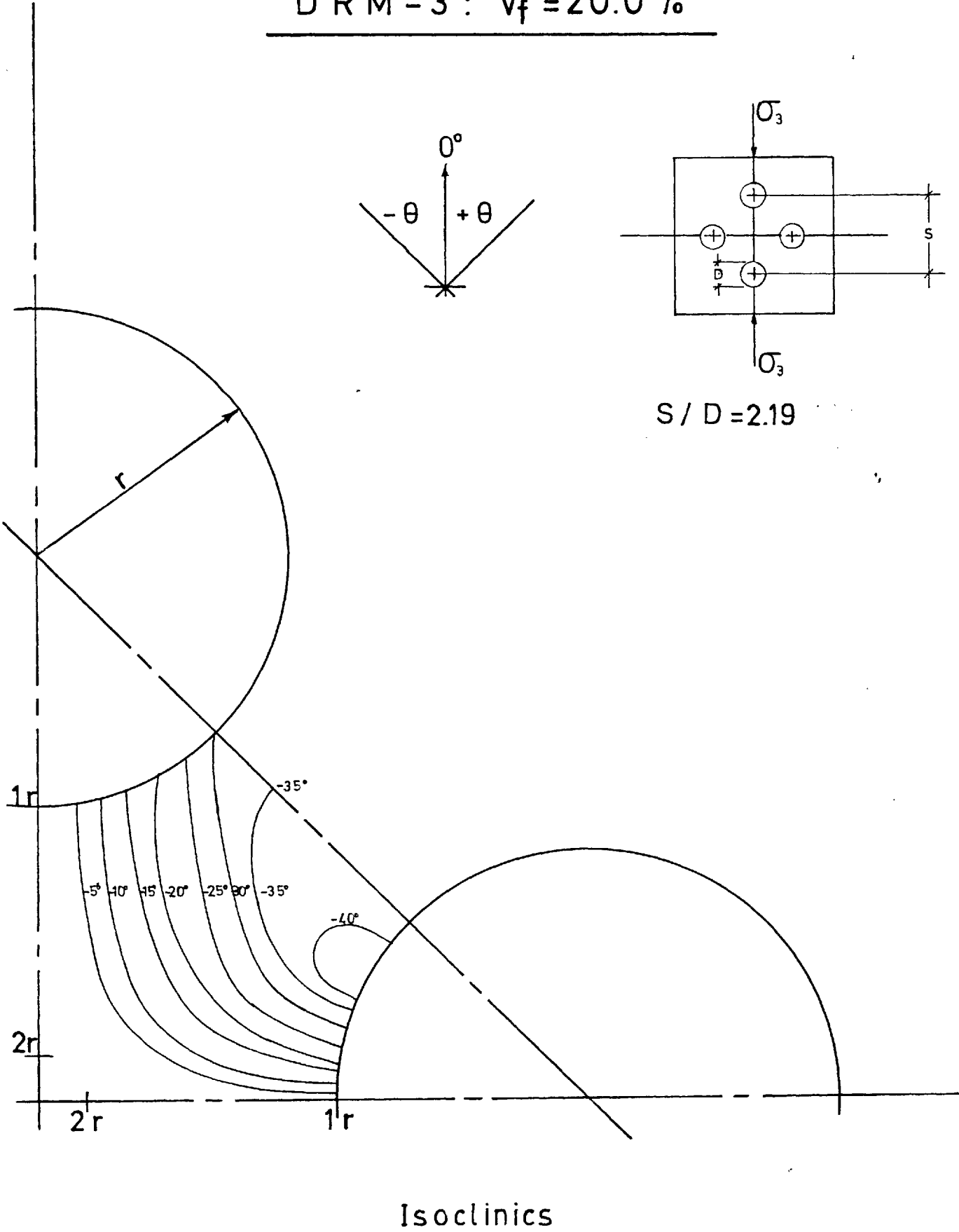
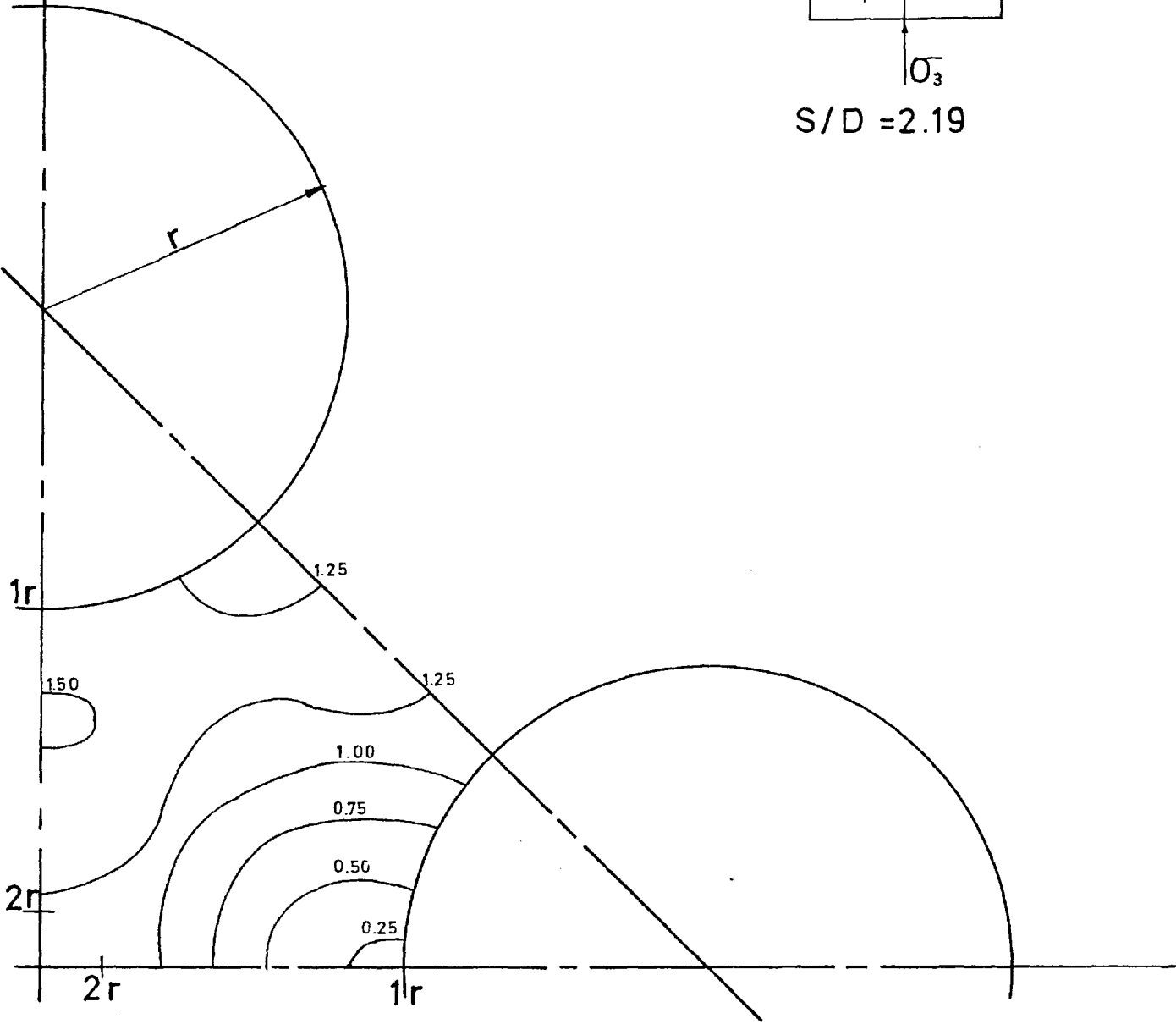
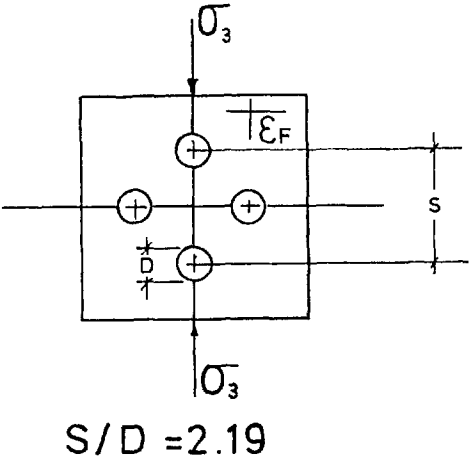


FIG. VI-13

DRM-3 : $V_f = 20.0\%$



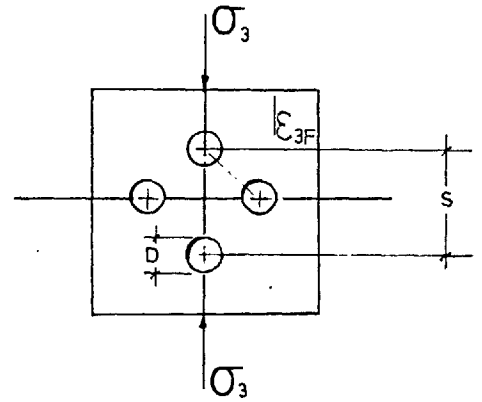
Isochromatics : $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG.VI-14

DRM -3 : $V_f = 20.0\%$

Critical Zone : 

Most Critical Point : 

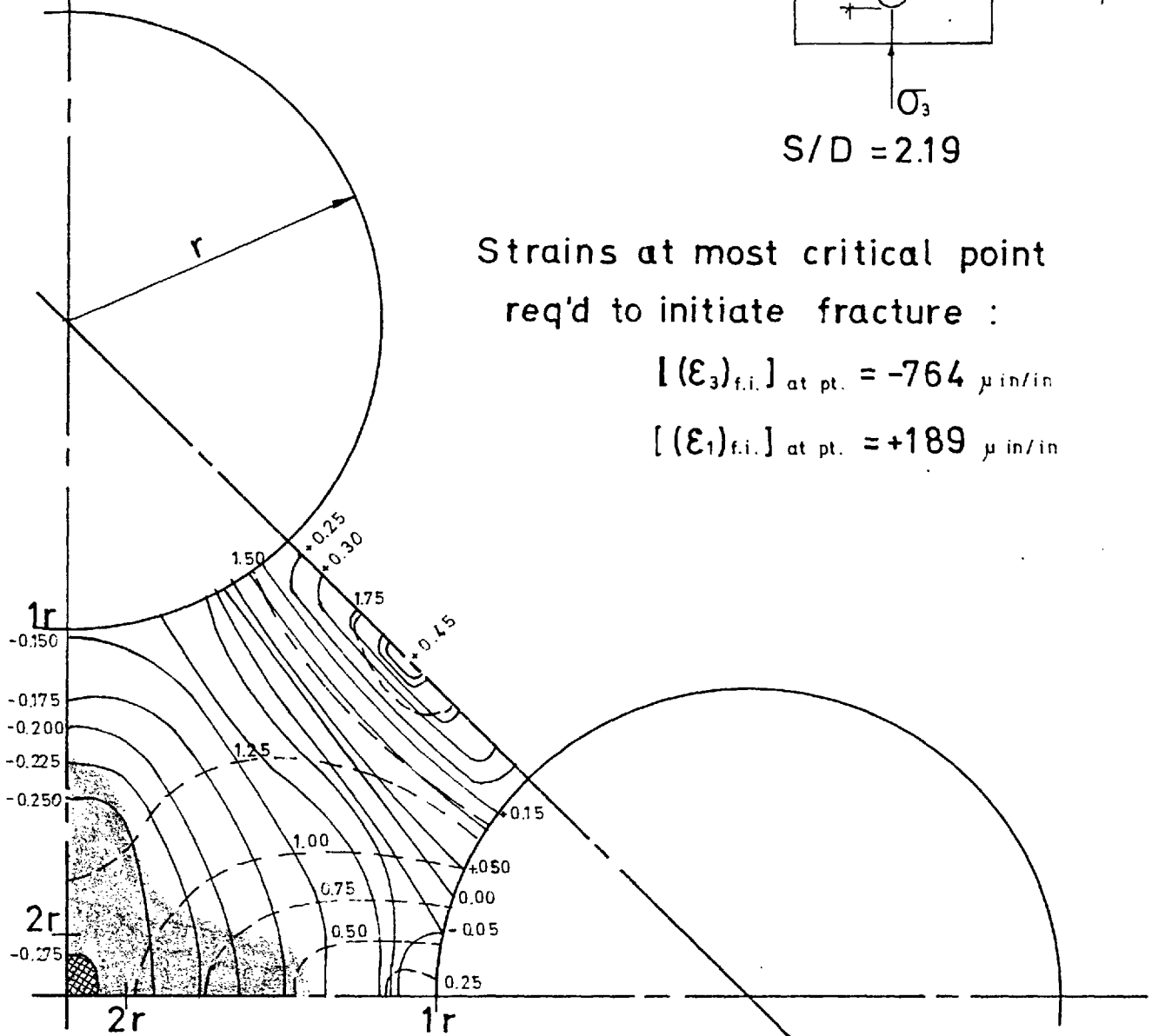


$$S/D = 2.19$$

Strains at most critical point
req'd to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{at\ pt.} = -764 \mu in/in$$

$$[(\epsilon_1)_{f.i.}]_{at\ pt.} = +189 \mu in/in$$



$$\text{Isotomics : } \frac{(\epsilon_3)}{(\epsilon_3)_F} [---] \text{ vs. } \frac{(\epsilon_1)}{(\epsilon_3)_F} [—]$$

FIG. VI-15

DRM - 4 : $V_f = 26.3\%$

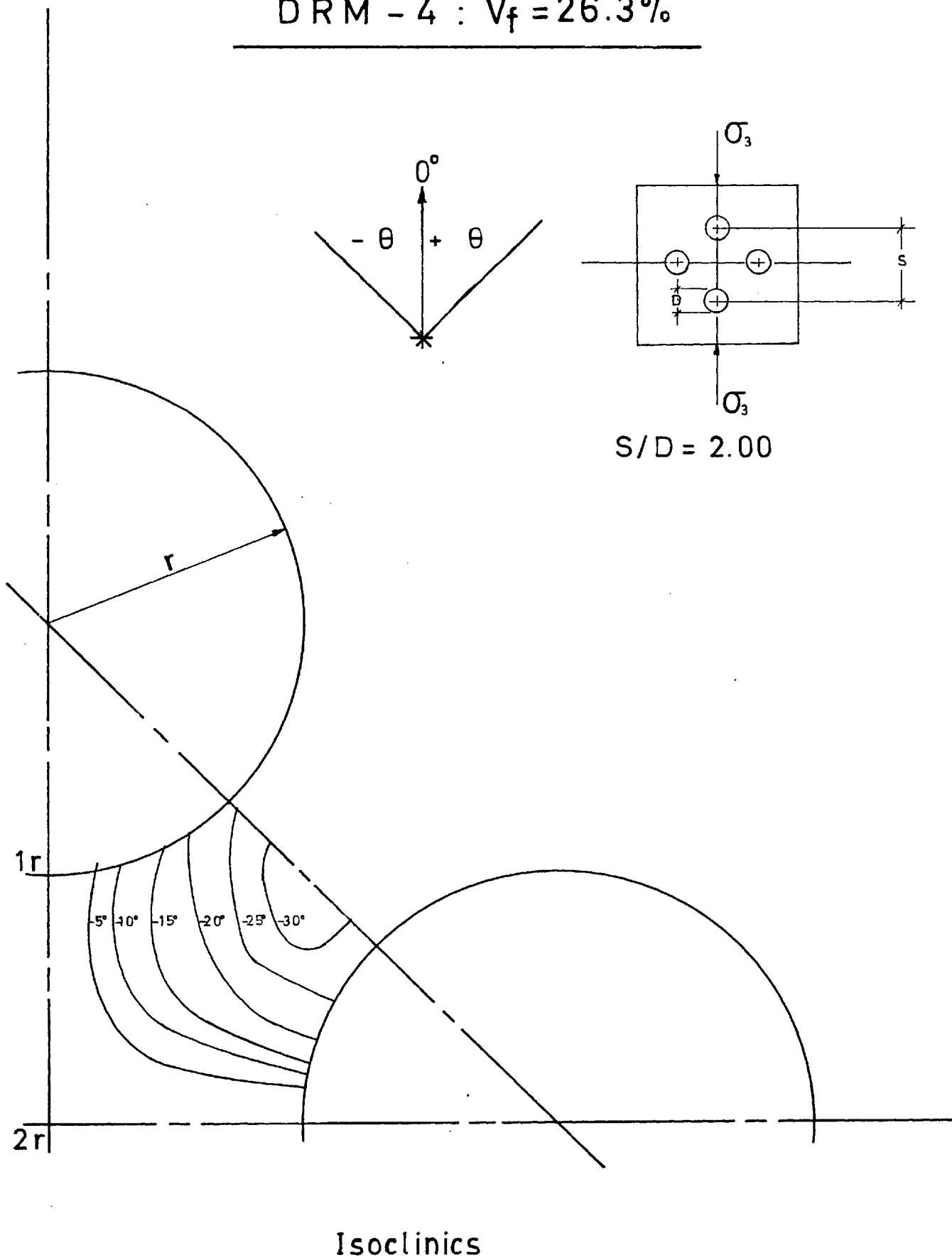
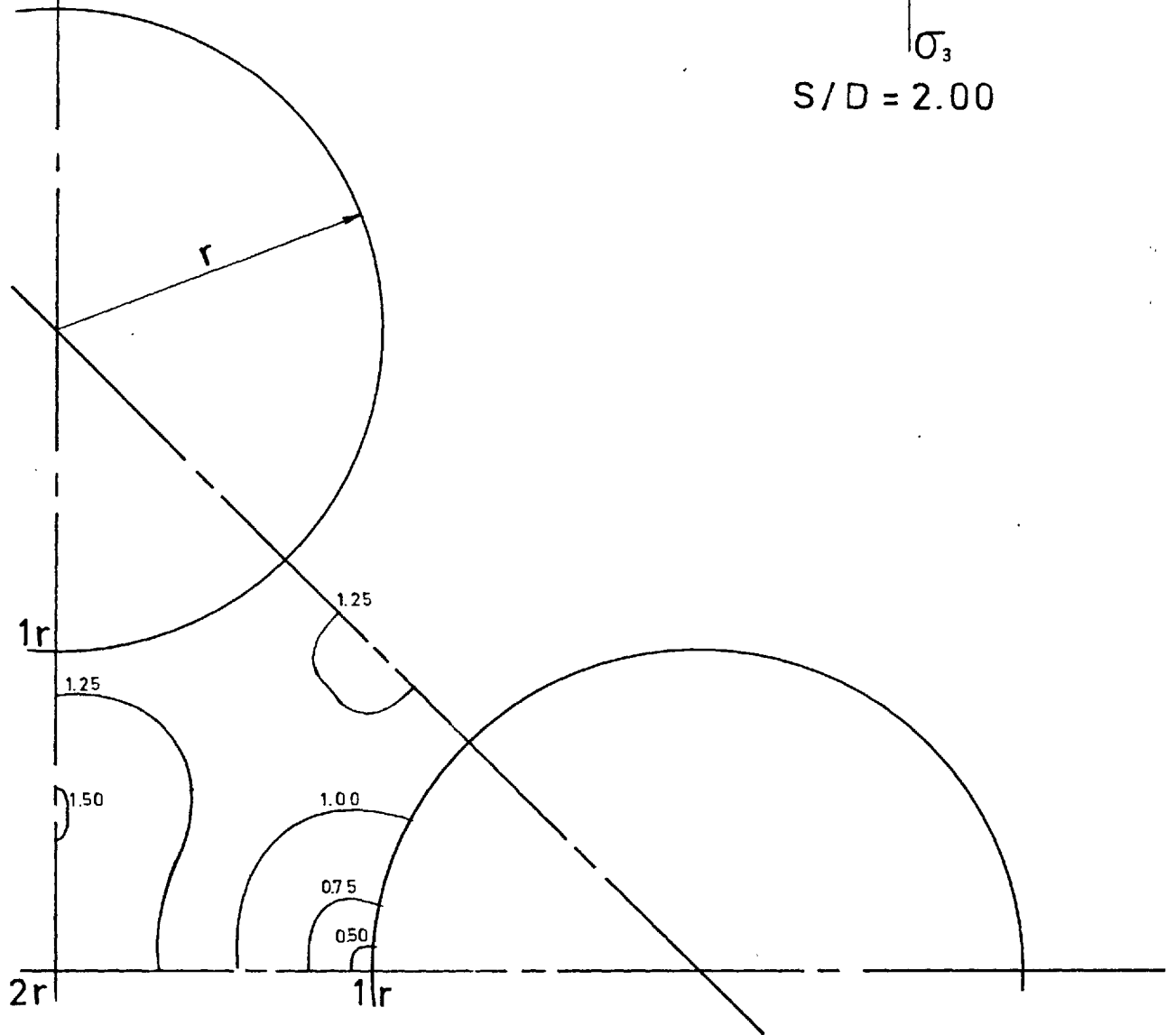
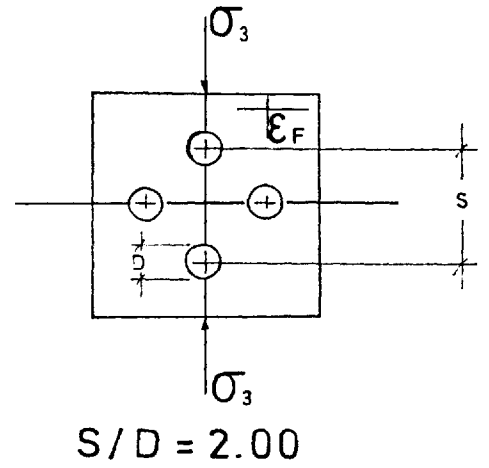


FIG. VI-16

DRM -4 : $V_f = 26.3\%$



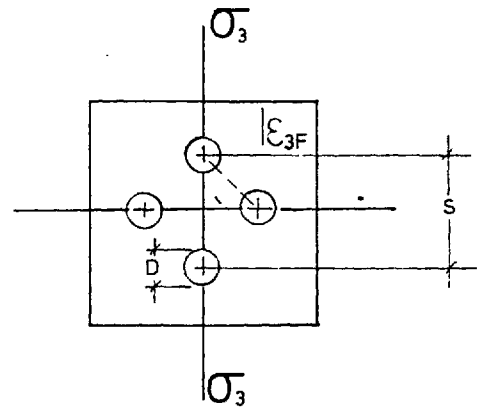
Isochromatics : $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG. VI-17

DRM -4 : $V_f = 26.3\%$

Critical Zone : 

Most Critical Point : 

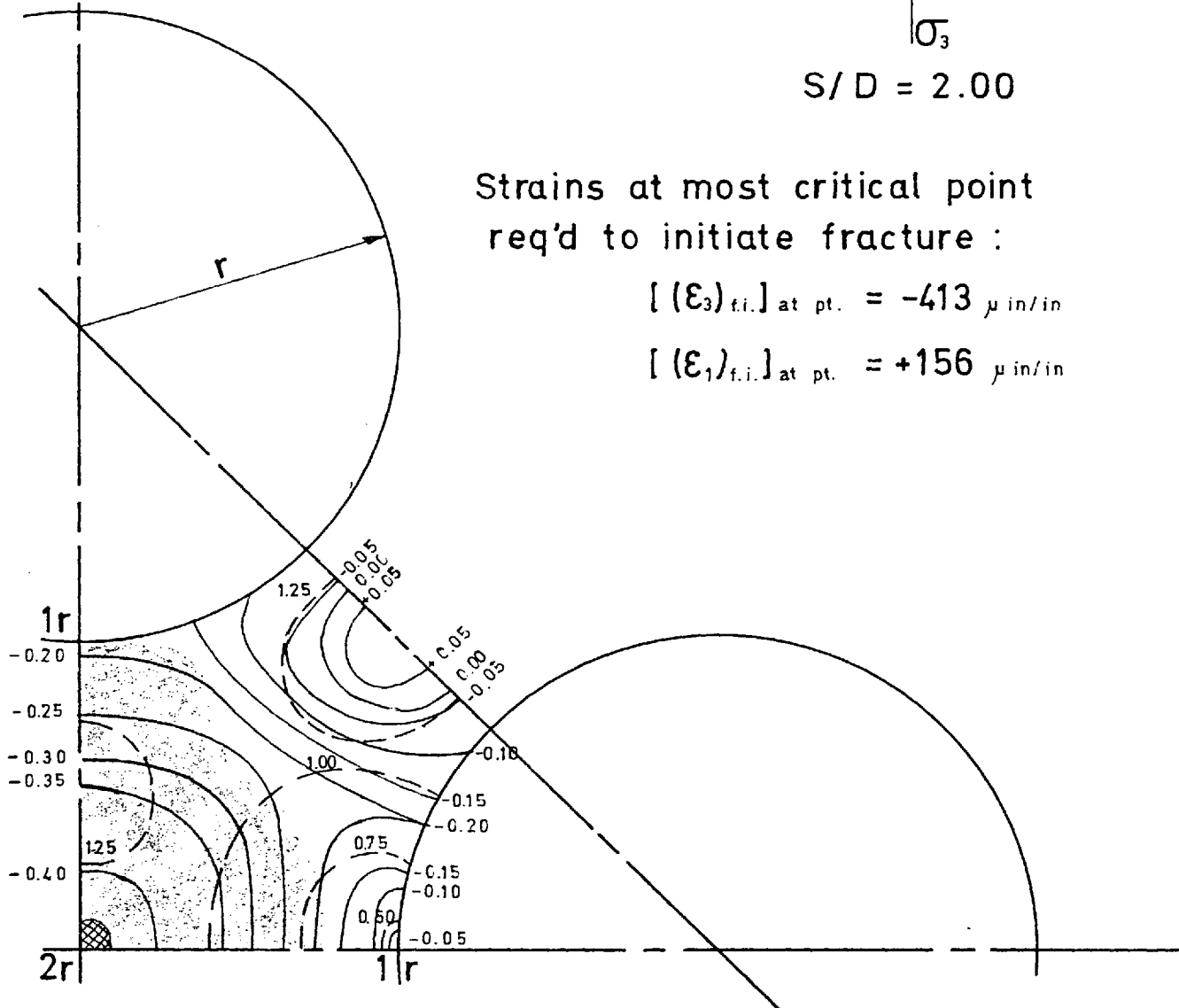


$$S/D = 2.00$$

Strains at most critical point
req'd to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{\text{at pt.}} = -413 \mu \text{ in/in}$$

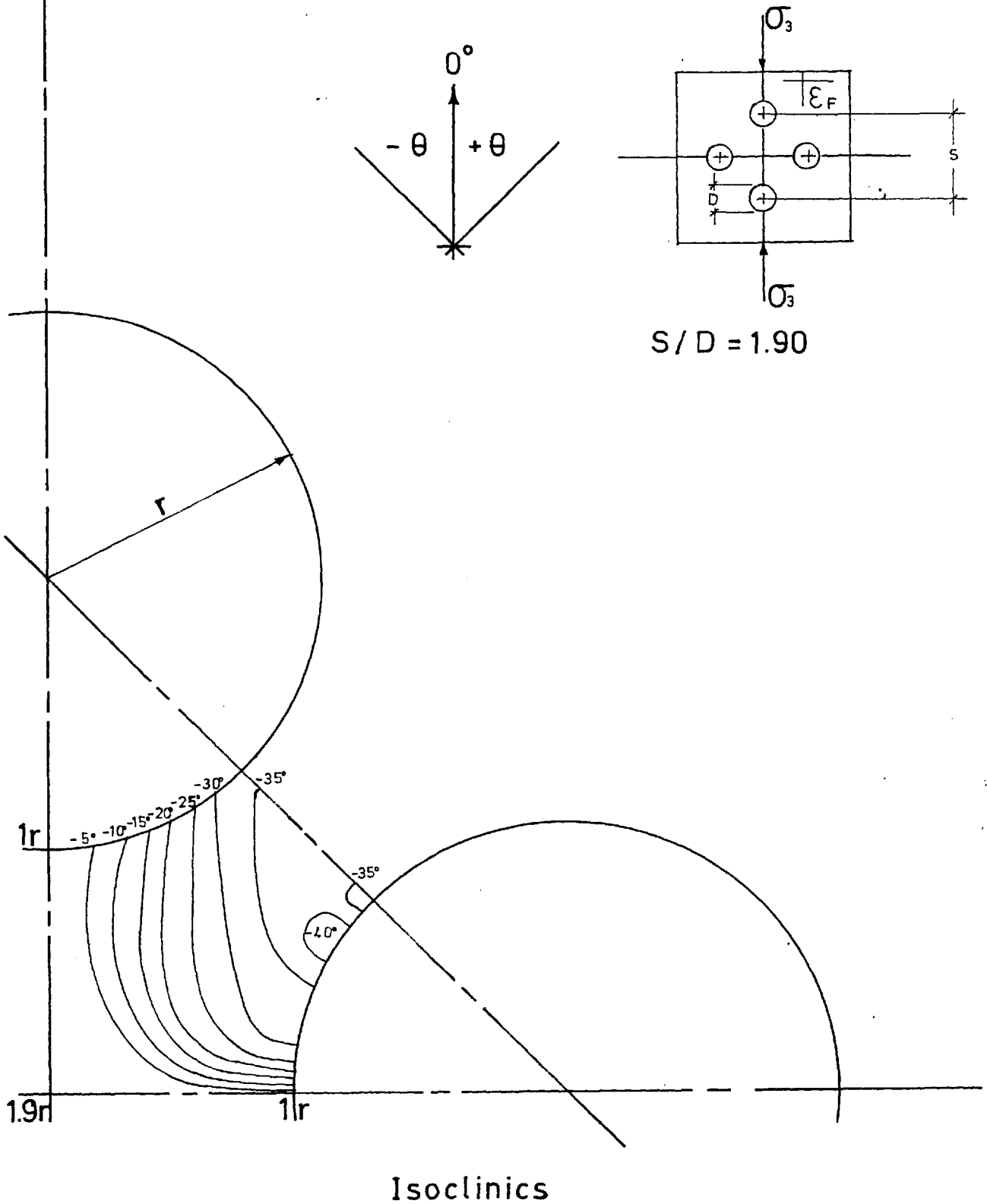
$$[(\epsilon_1)_{f.i.}]_{\text{at pt.}} = +156 \mu \text{ in/in}$$



$$\text{Isotonics: } \frac{(\epsilon_3)}{(\epsilon_3)_F} \text{ [---]} \text{ vs. } \frac{(\epsilon_1)}{(\epsilon_3)_F} \text{ [—]}$$

FIG.VI-18

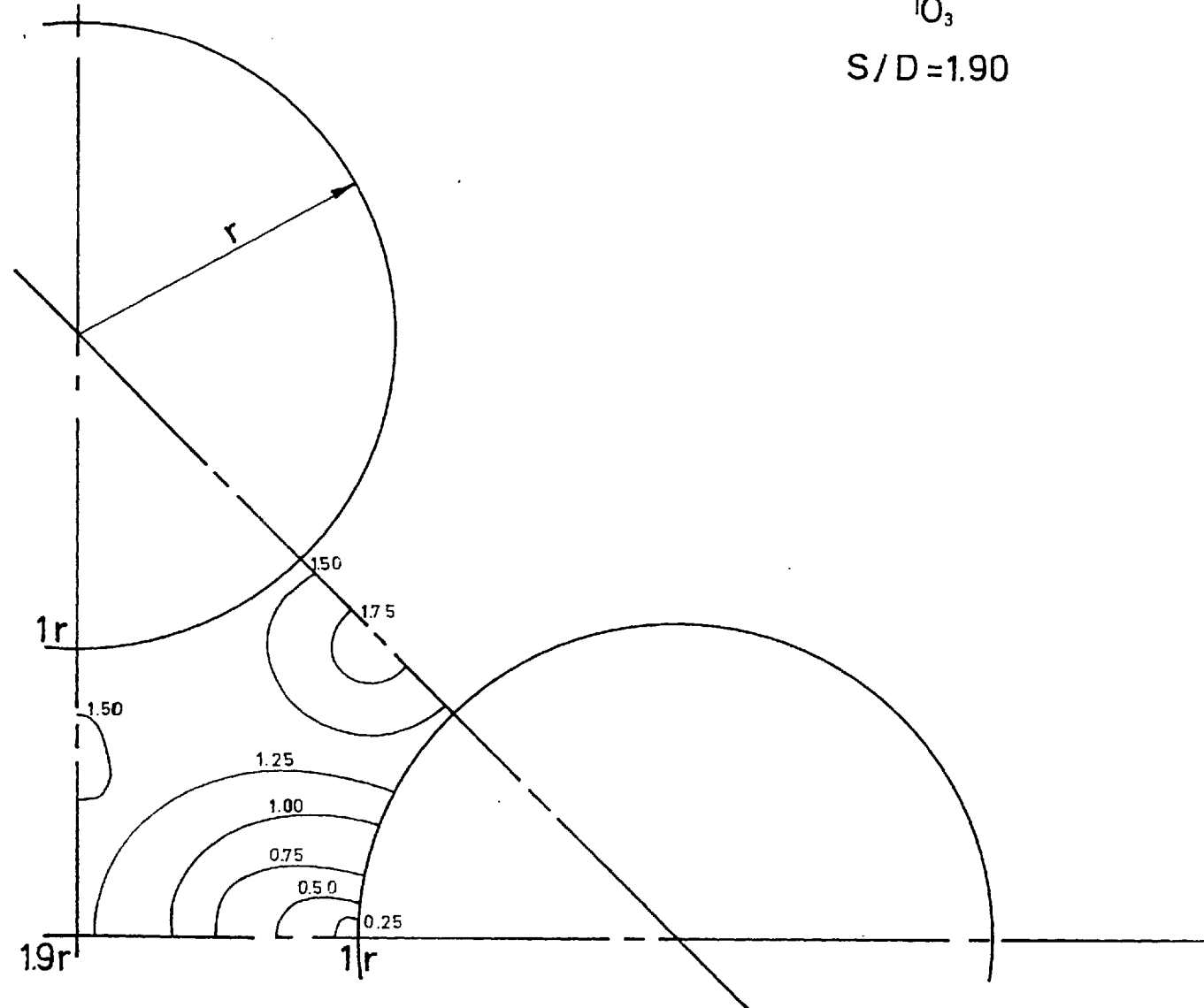
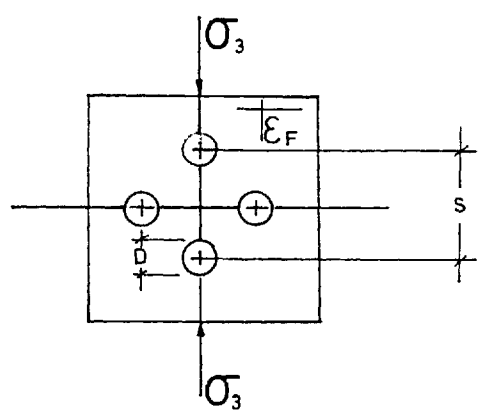
DRM - 3 : $V_f = 30.6\%$



$S/D = 1.90$

FIG. VI-19

DRM-3 : $V_f = 30.6\%$



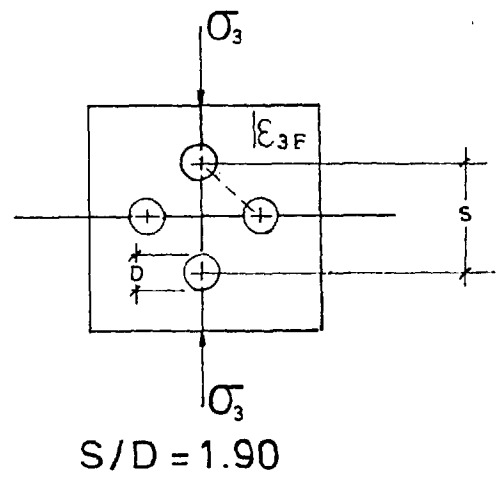
Isochromatics :
$$\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$$

FIG.VI-20

DRM-3 : $V_f = 30.6\%$

Critical Zone : 

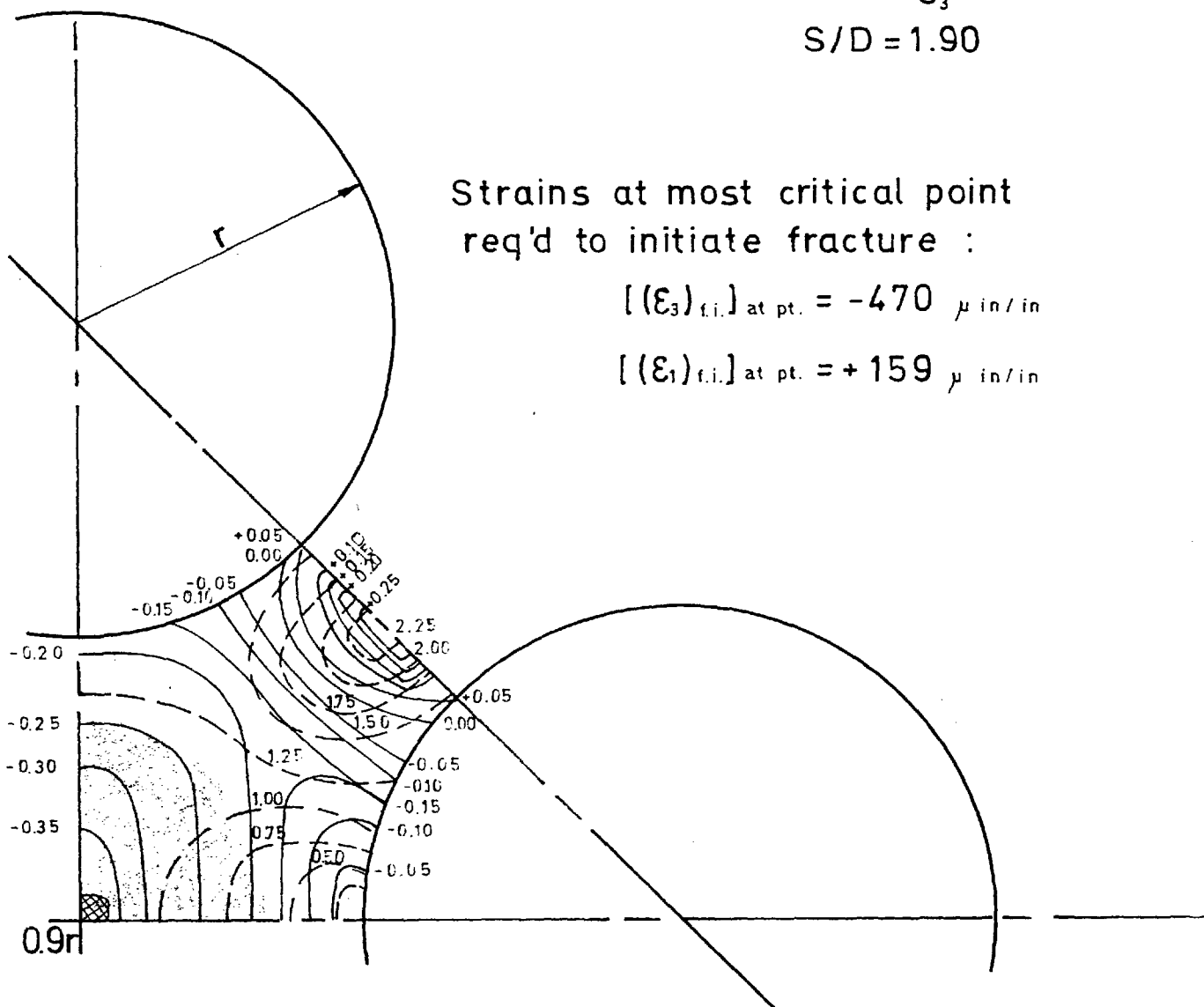
Most Critical Point : 



Strains at most critical point
req'd to initiate fracture :

$$[(\epsilon_3)_{f.i.}] \text{ at pt. } = -470 \text{ } \mu \text{ in/in}$$

$$[(\epsilon_1)_{f.i.}] \text{ at pt. } = +159 \text{ } \mu \text{ in/in}$$



Isotonics : $\frac{(\epsilon_3)}{(\epsilon_3)_F} \text{ [---]} \text{ vs. } \frac{(\epsilon_1)}{(\epsilon_3)_F} \text{ [—]}$

FIG.VI-21

DRM-3 : $V_f = 34.9\%$

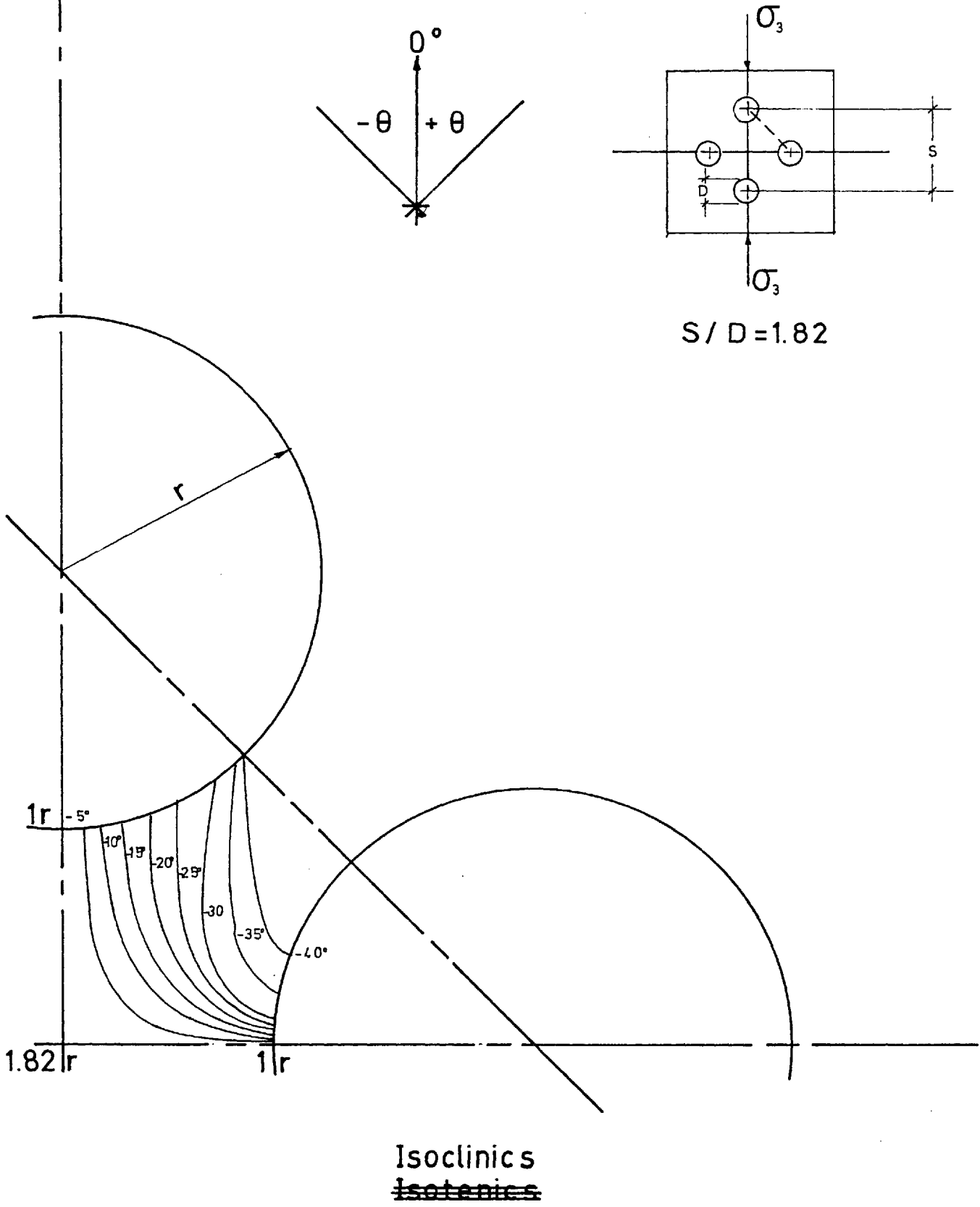


FIG.VI-22

DRM - 3 : $V_f = 34.9\%$

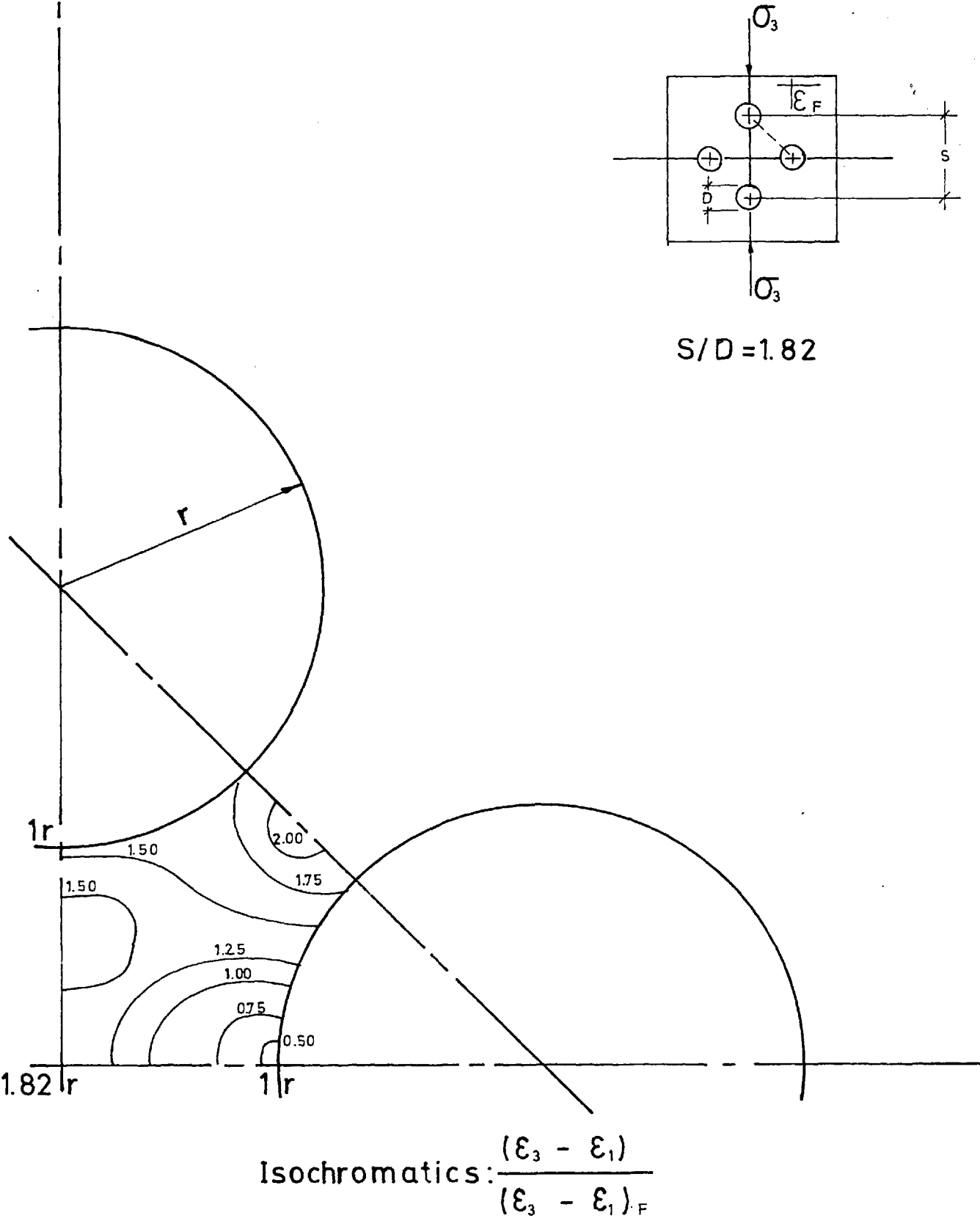
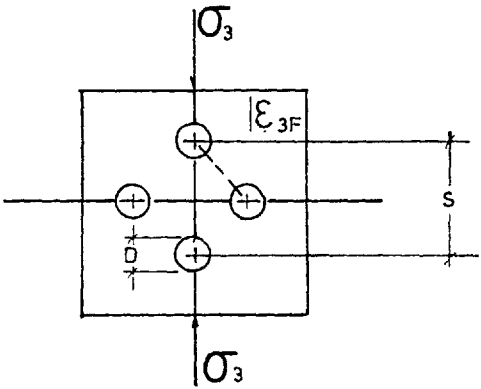


FIG. VI-23

DRM -3 : $V_f = 34.9\%$

Critical Zone :

Most Critical Point :

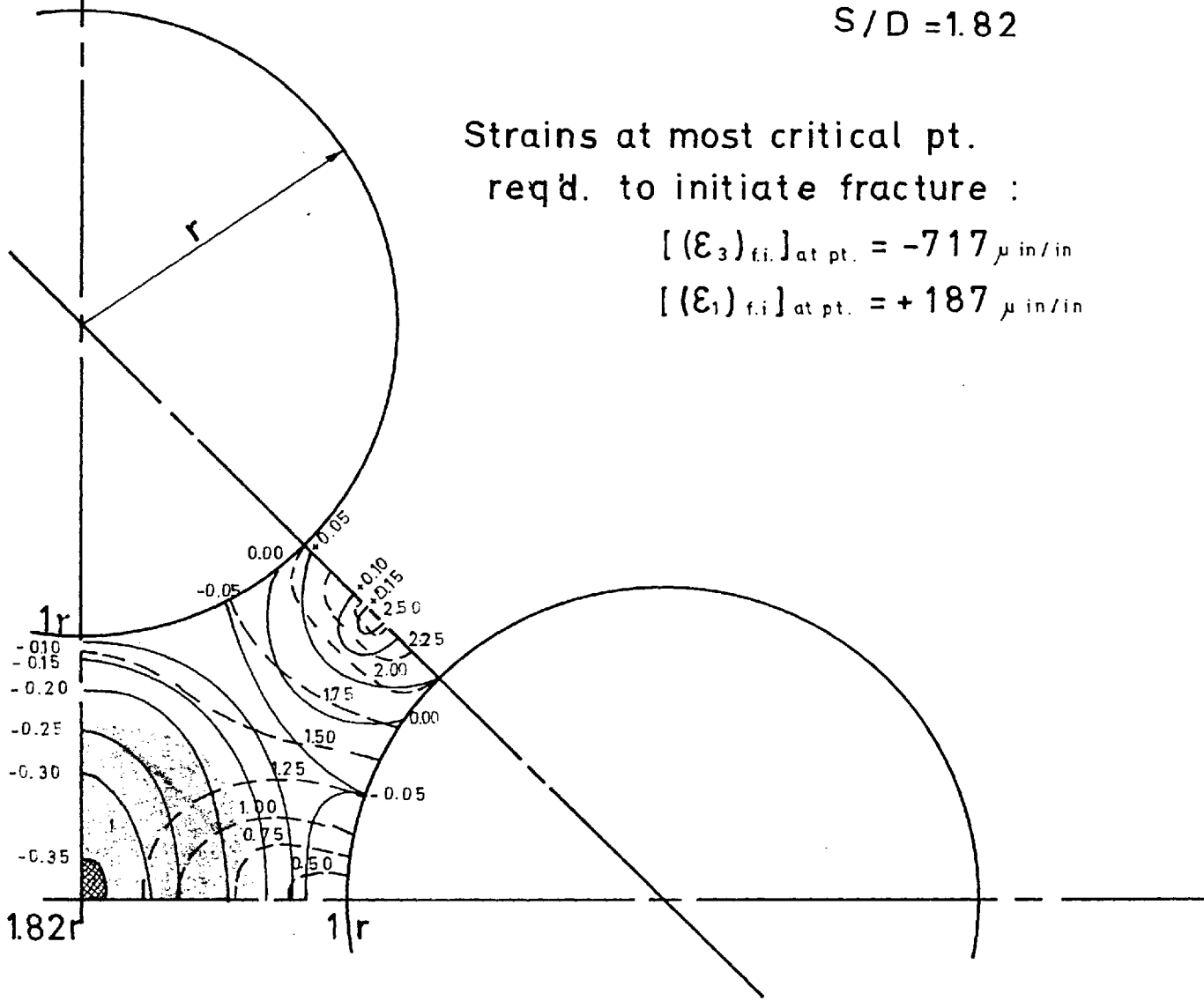


$S/D = 1.82$

Strains at most critical pt.
req'd. to initiate fracture :

$[(\epsilon_3)_{f.i.}]_{at\ pt.} = -717\ \mu\text{ in/in}$

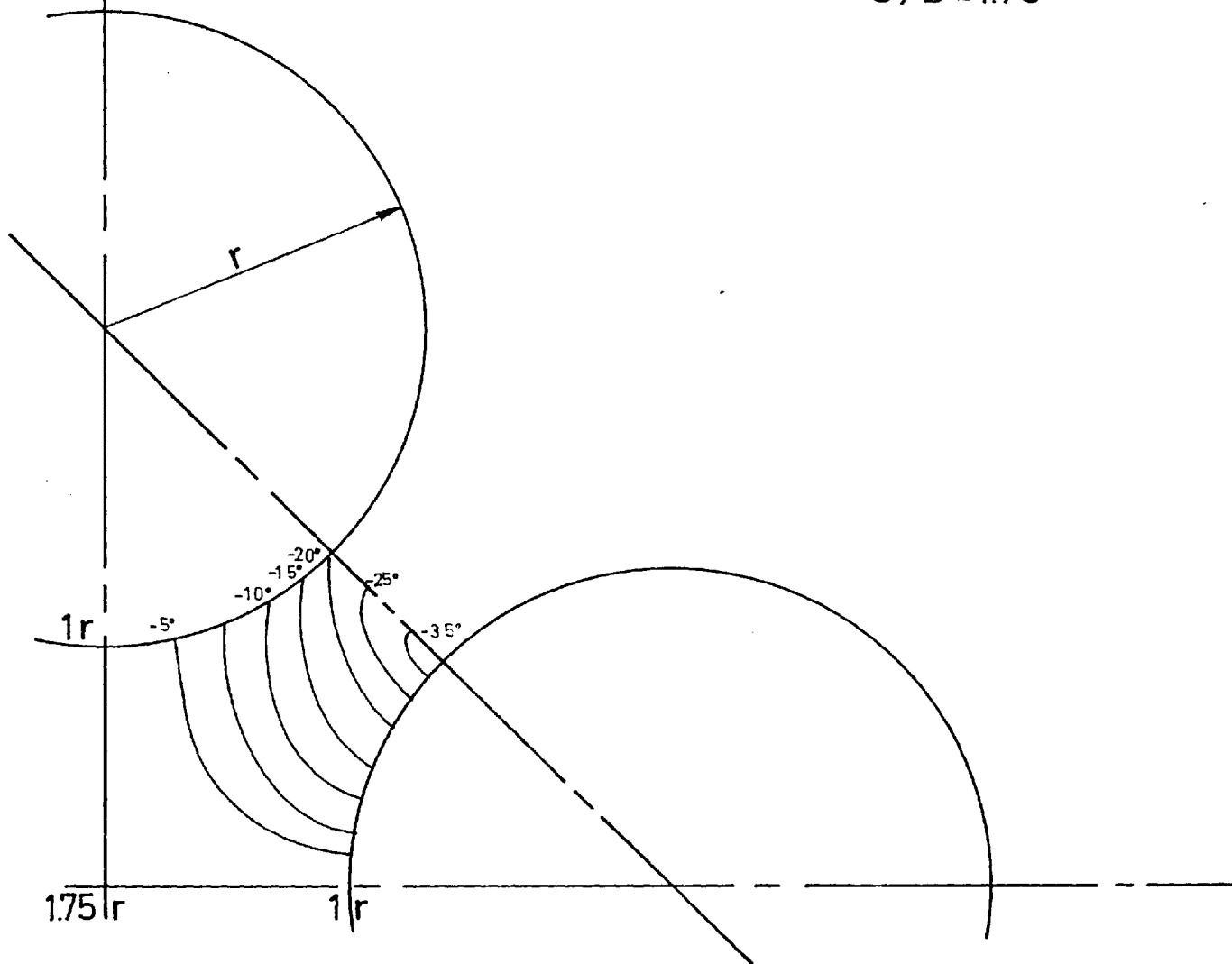
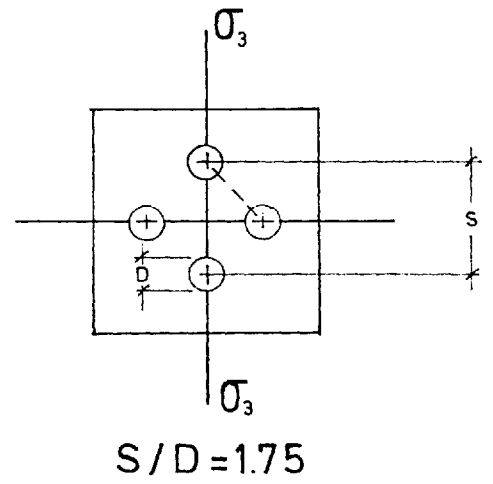
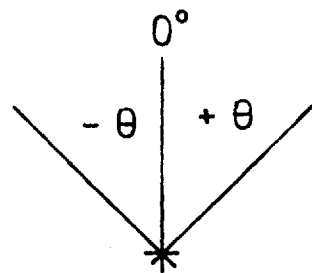
$[(\epsilon_1)_{f.i.}]_{at\ pt.} = +187\ \mu\text{ in/in}$



Isotenics : $\frac{(\epsilon_3)}{(\epsilon_3)_F} [---] \text{ vs. } \frac{(\epsilon_3)}{(\epsilon_3)_F} [—]$

FIG.VI-24

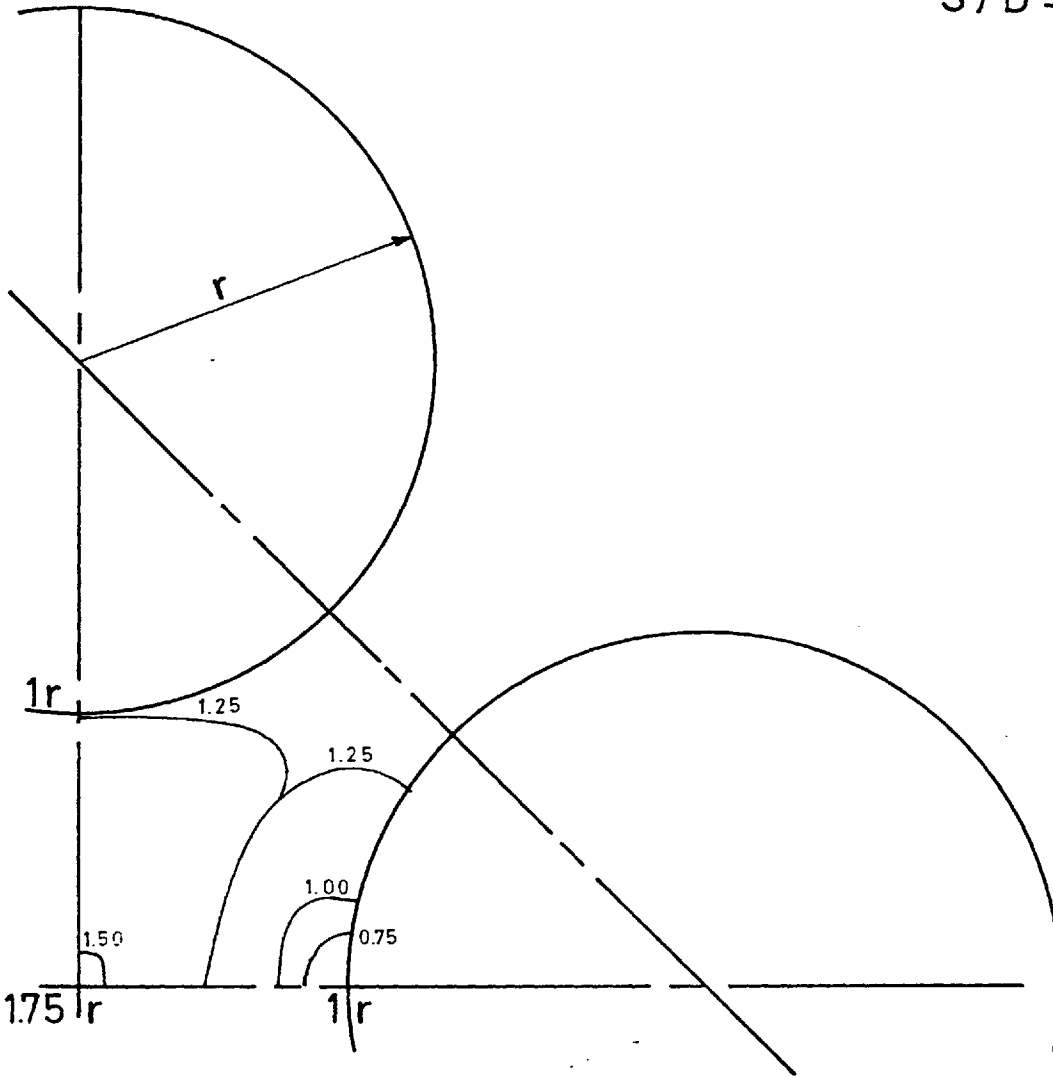
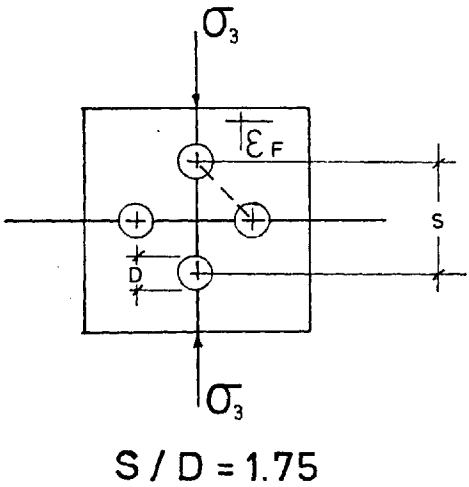
DRM - 4 : $V_f = 39.2\%$



Isoclinics

FIG. VI-25

DRM-4 : $V_f = 39.2$



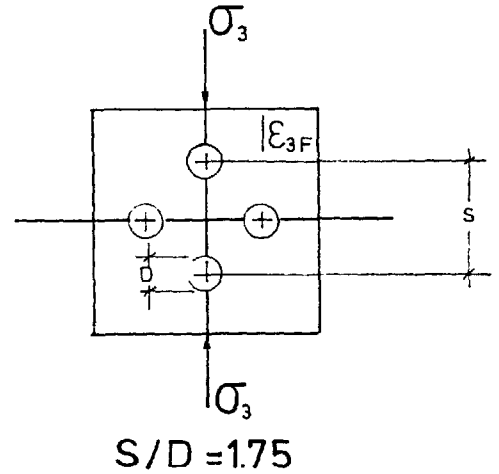
Isochromatics :
$$\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$$

FIG.VI-26

DRM-4 : $V_f = 39.2\%$

Critical Zone :

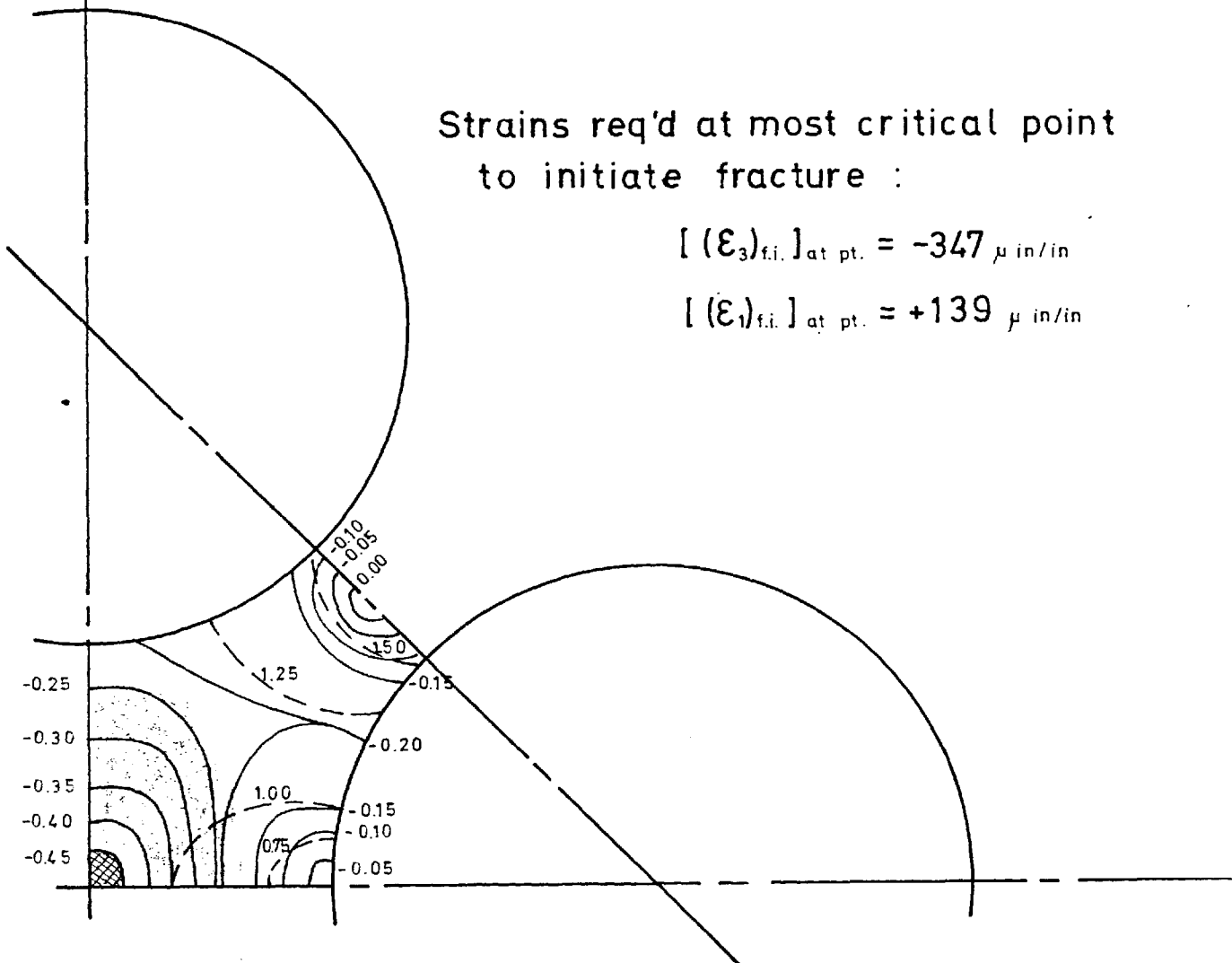
Most Critical Point :



Strains req'd at most critical point
to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{at \ pt.} = -347 \mu \text{ in/in}$$

$$[(\epsilon_1)_{f.i.}]_{at \ pt.} = +139 \mu \text{ in/in}$$



Isotenics : $\frac{(\epsilon_3)}{(\epsilon_3)_F} [---]$ vs. $\frac{(\epsilon_1)}{(\epsilon_3)_F} [—]$

FIG.VI-27

DRM-3: $V_f = 46.9\%$

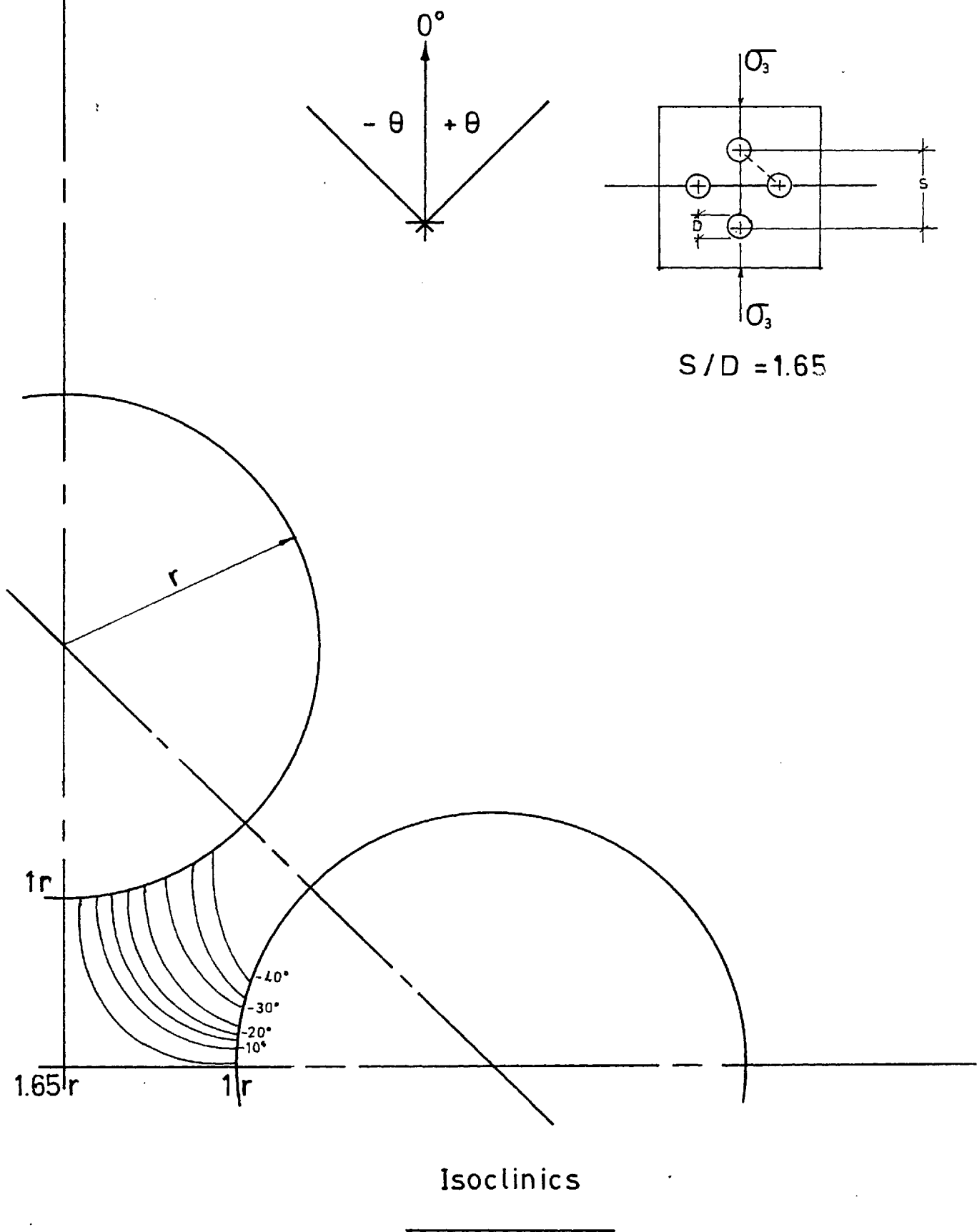
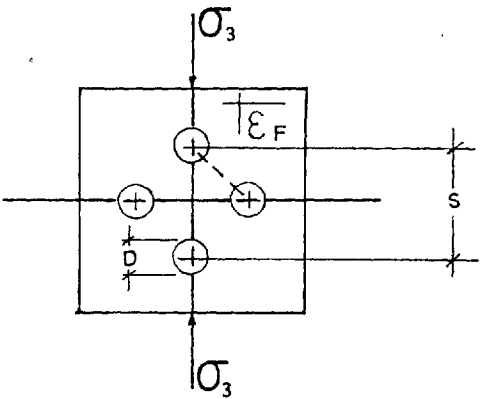
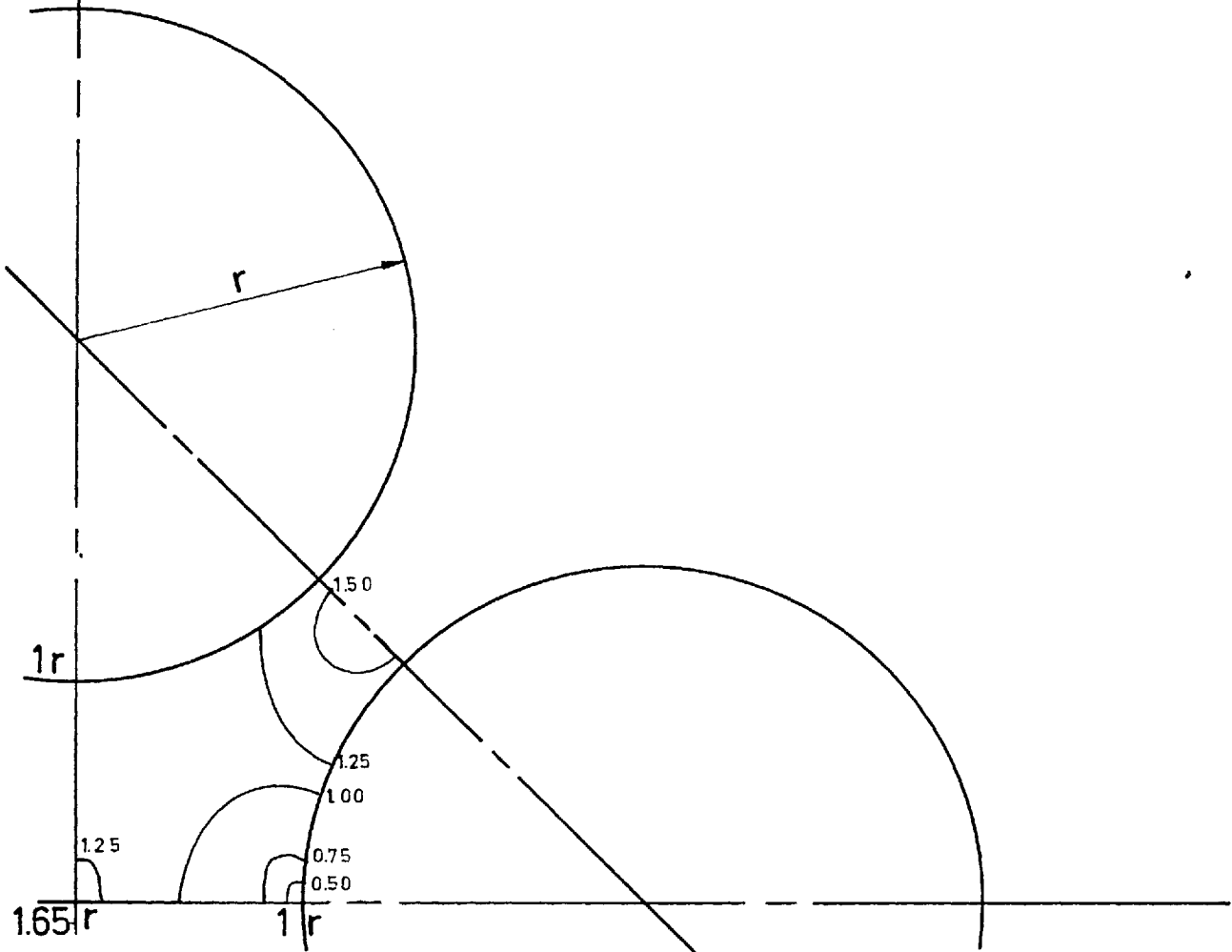


FIG. VI-28

DRM - 3 : $V_f = 46.9\%$



$S / D = 1.65$



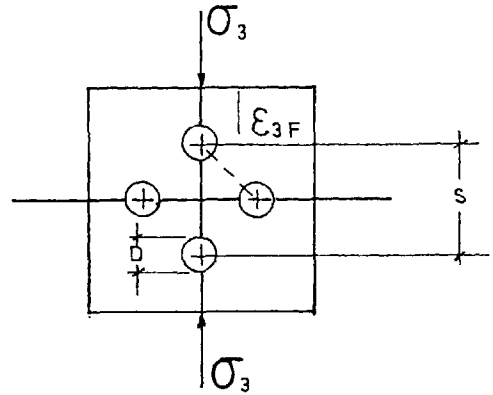
Isochromatics : $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG.VI-29

DRM-3 : $V_f = 46.9\%$

Critical Zone : 

Most Critical Point : 

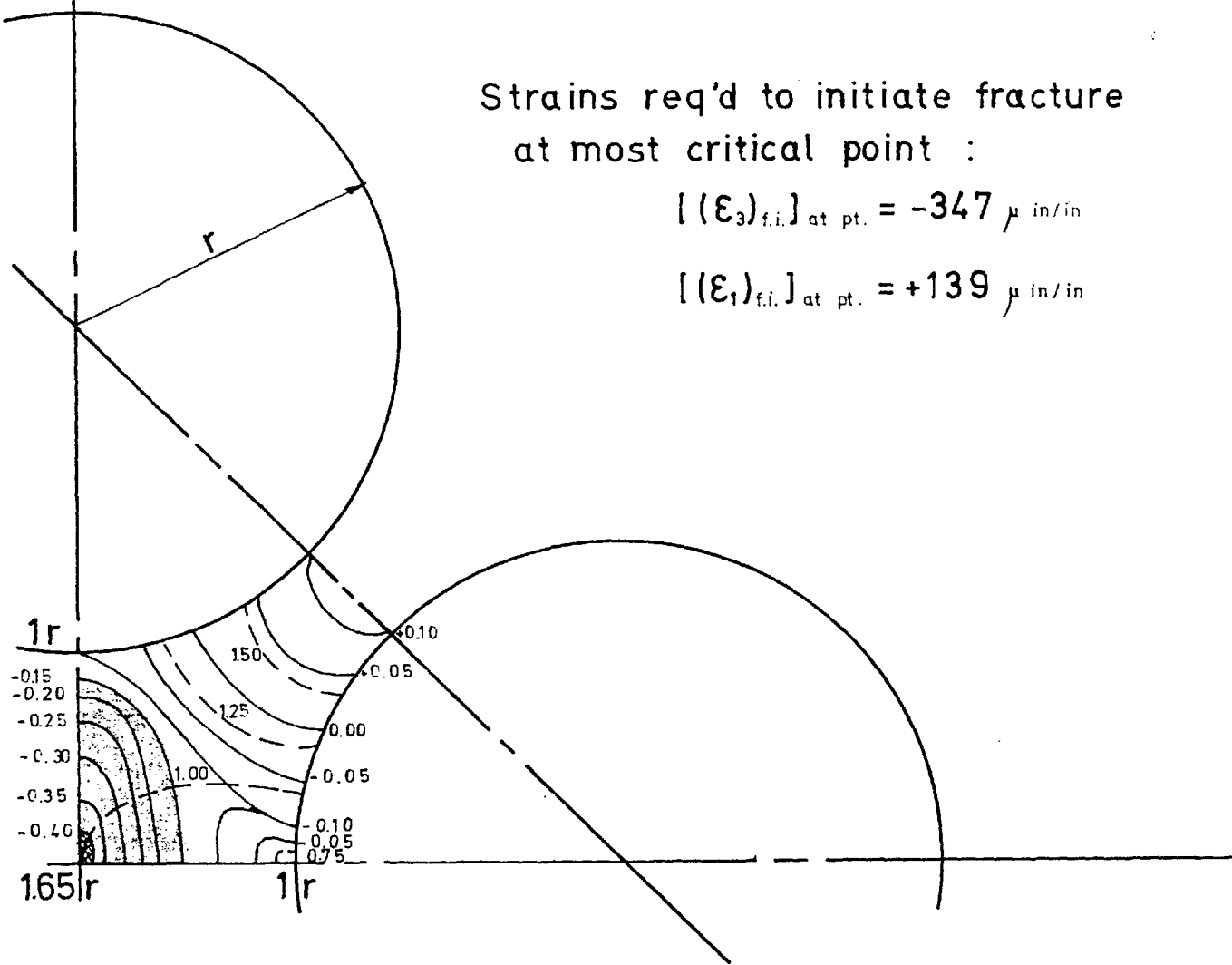


$S/D = 1.65$

Strains req'd to initiate fracture
at most critical point :

$$[(\epsilon_3)_{f.i.}]_{at \ pt.} = -347 \mu \text{ in/in}$$

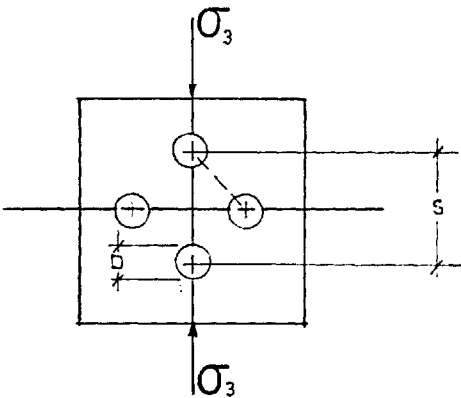
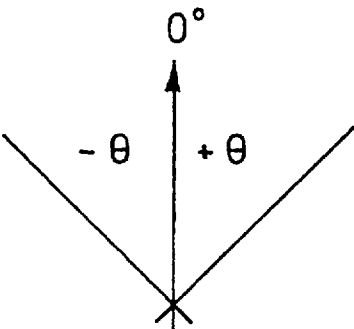
$$[(\epsilon_1)_{f.i.}]_{at \ pt.} = +139 \mu \text{ in/in}$$



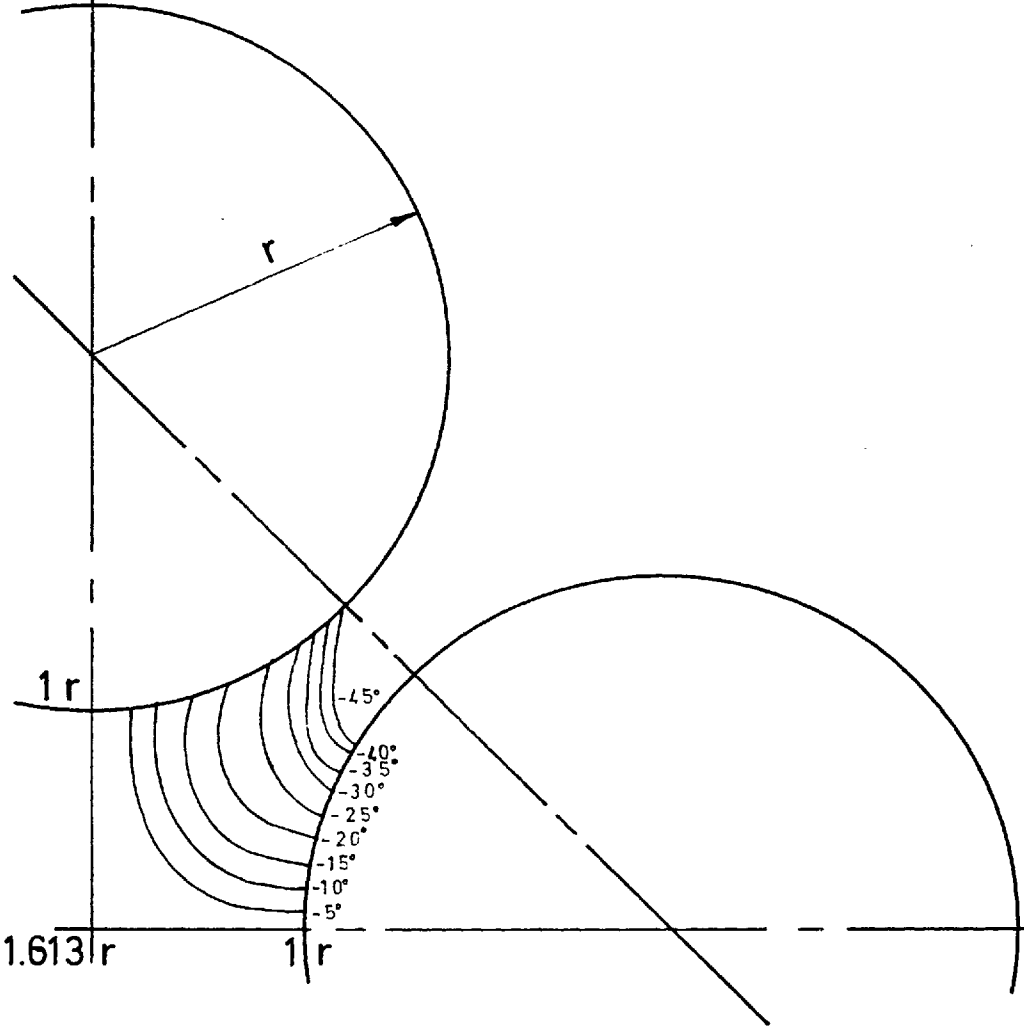
Isotenics : $\frac{(\epsilon_3)}{(\epsilon_3)_F} [----]$ vs. $\frac{(\epsilon_1)}{(\epsilon_3)_F} [—]$

FIG. VI-30

DRM - 4 : $V_f = 50.0\%$



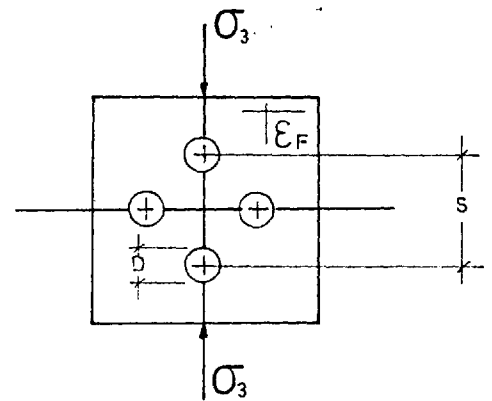
$S/D = 1.613$



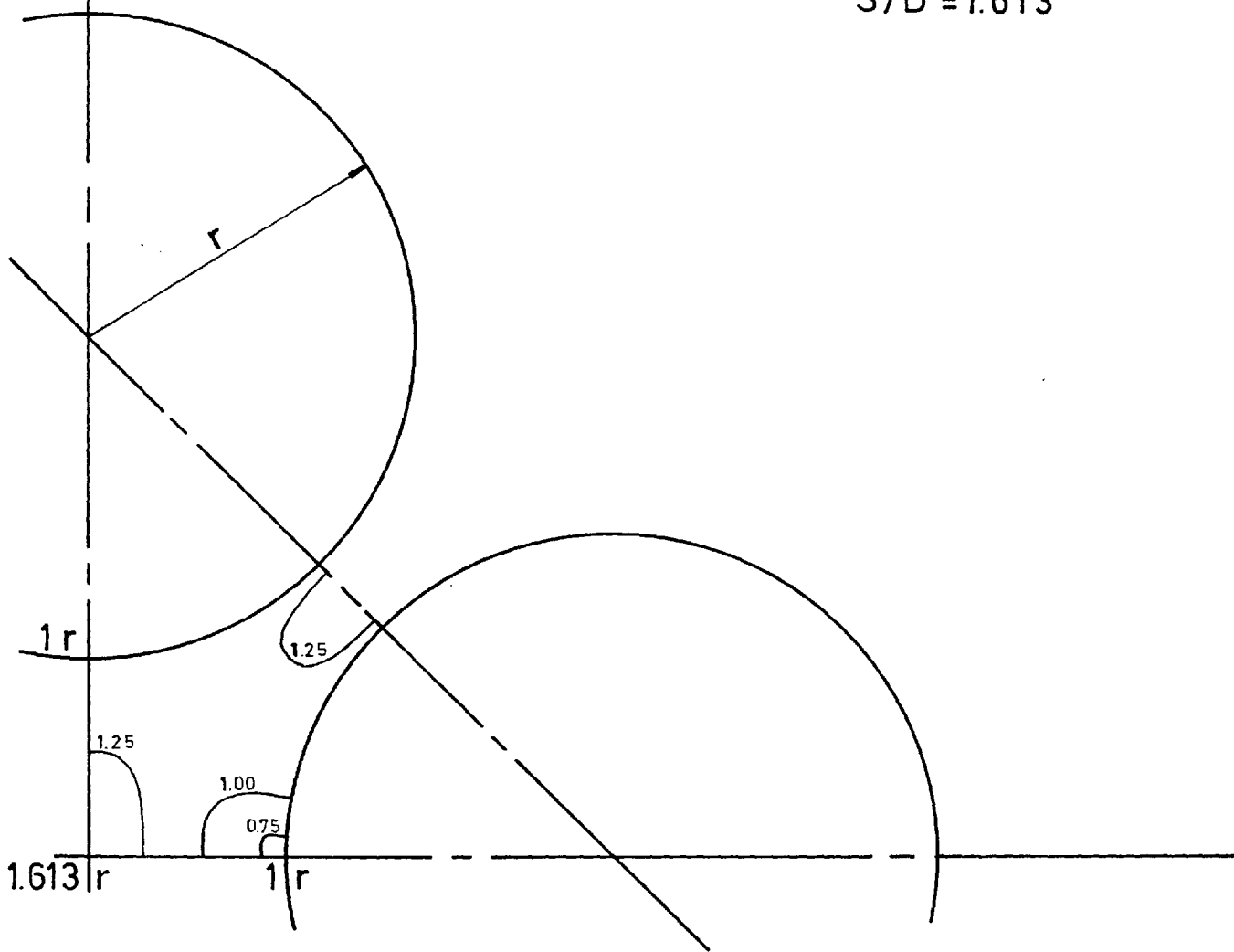
Isoclinics

FIG.VI - 31

DRM-4 : $V_f = 50.0\%$



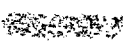
$$S/D = 1.613$$

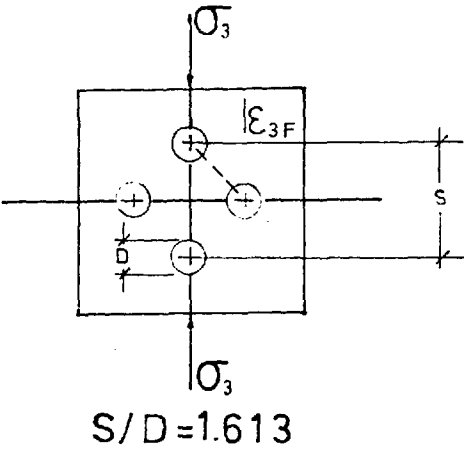


$$\text{Isochromatics : } \frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$$

FIG.VI-32

DRM-4 : $V_f = 50.0\%$

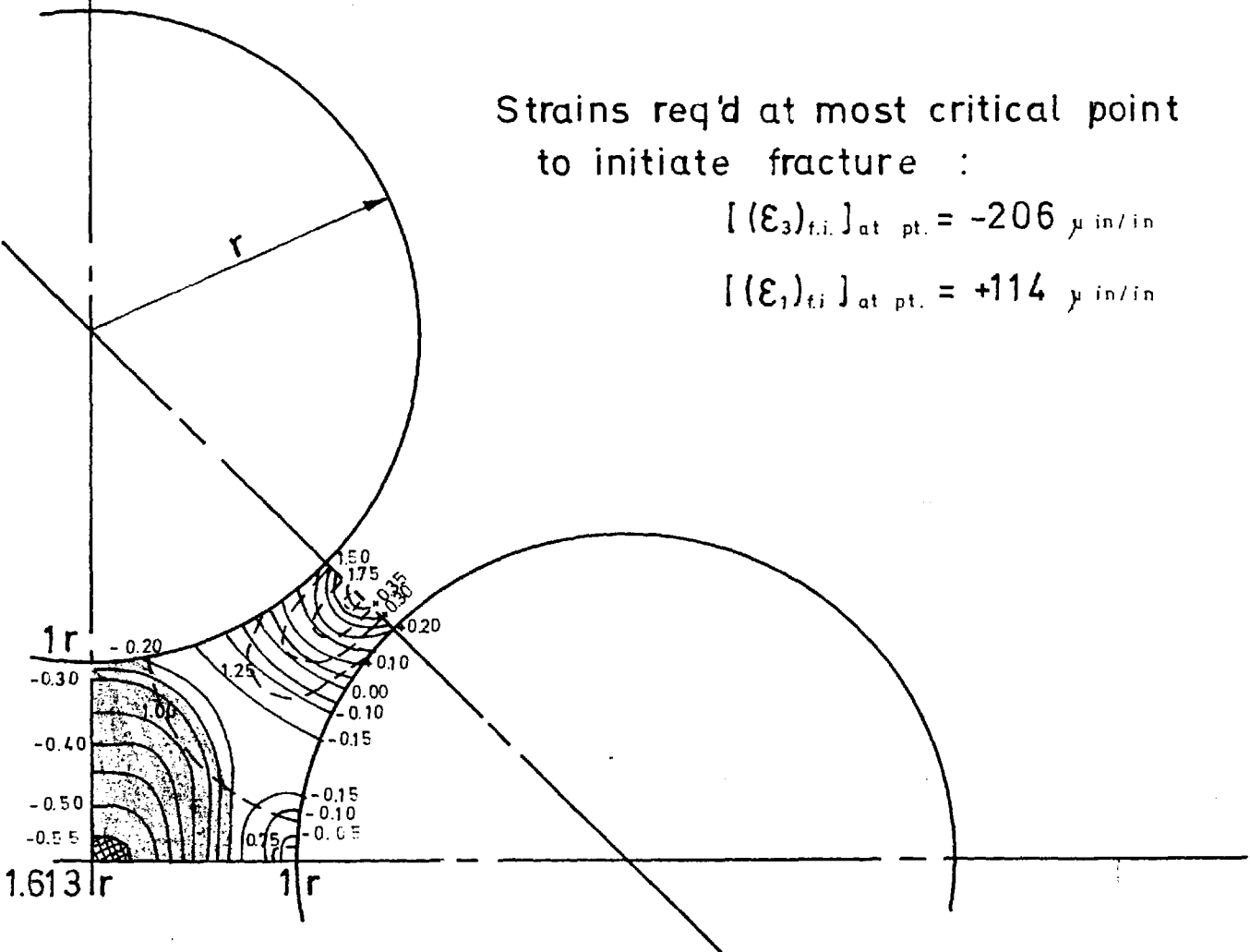
Critical Zone : 
 Most Critical Point : 



Strains req'd at most critical point
 to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{at \ pt.} = -206 \ \mu \text{ in/in}$$

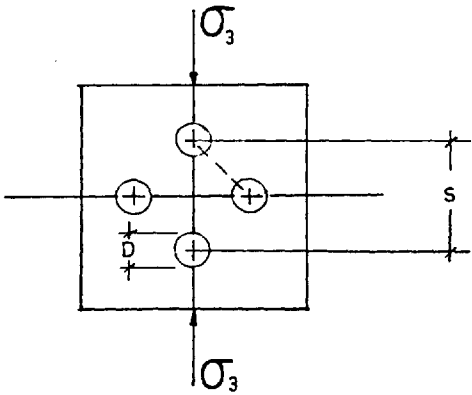
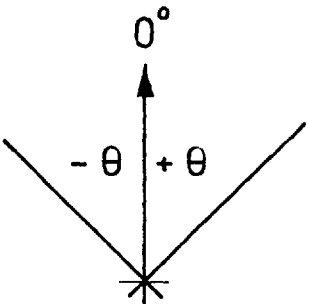
$$[(\epsilon_1)_{f.i.}]_{at \ pt.} = +114 \ \mu \text{ in/in}$$



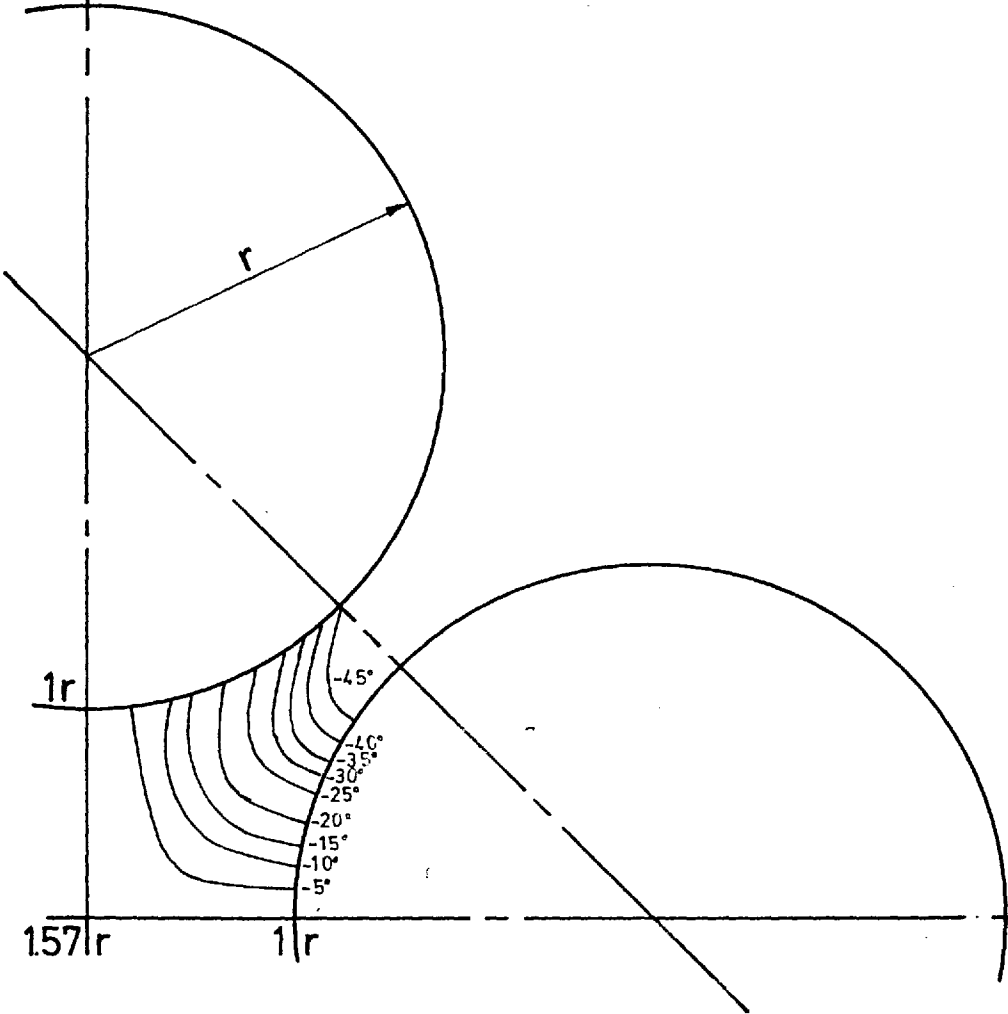
Isotonics : $\frac{(\epsilon_3)}{(\epsilon_3)_F}$ [---] vs. $\frac{(\epsilon_1)}{(\epsilon_3)_F}$ [—]

FIG.VI-33

DRM -3 : $V_f = 54.6\%$



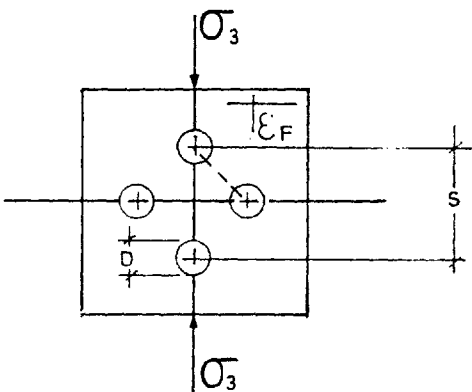
$S/D = 1.57$



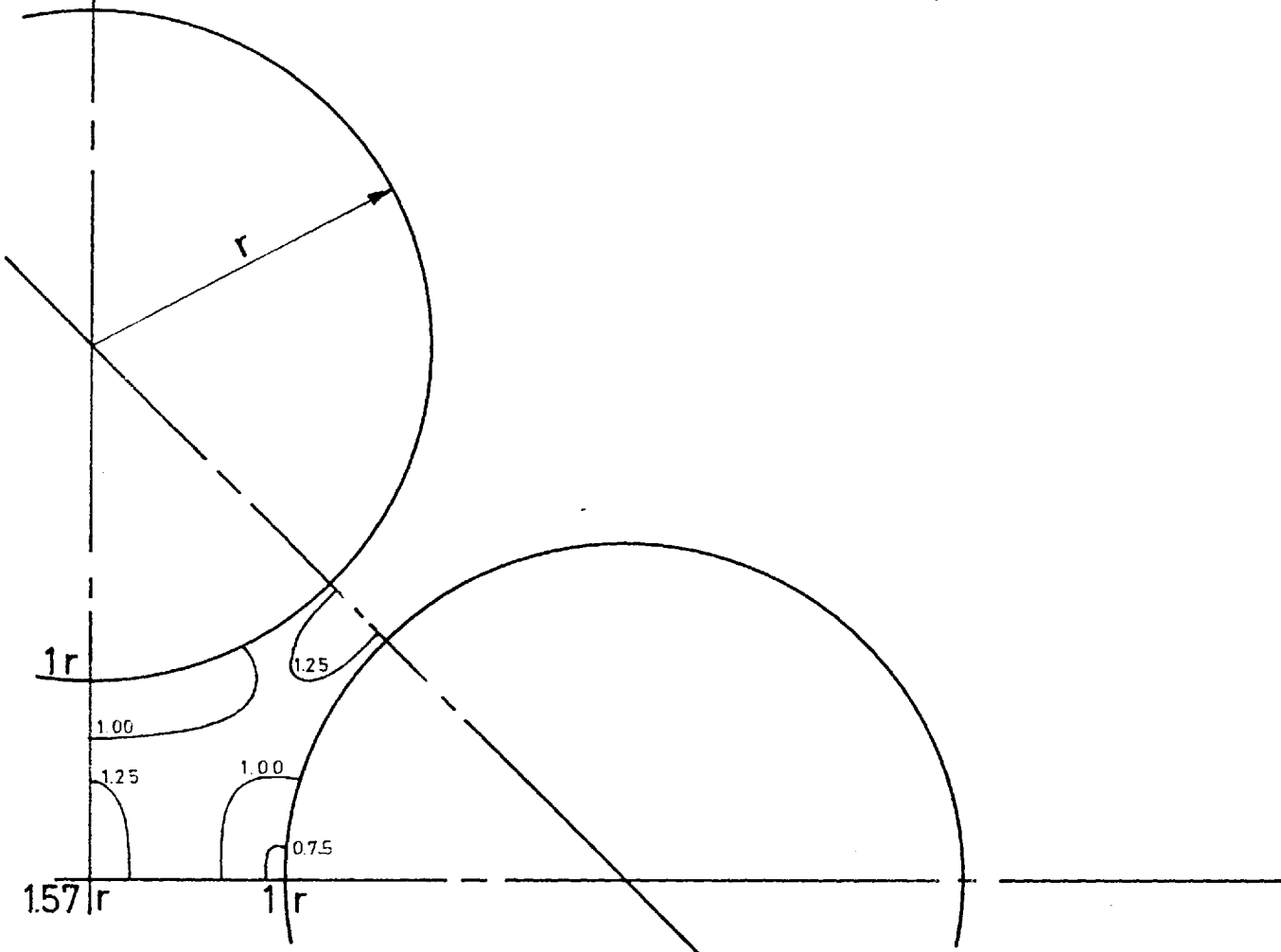
Isoclinics

FIGURE : VI -34

DRM - 3 : $V_f = 54.6\%$



$S/D = 1.57$

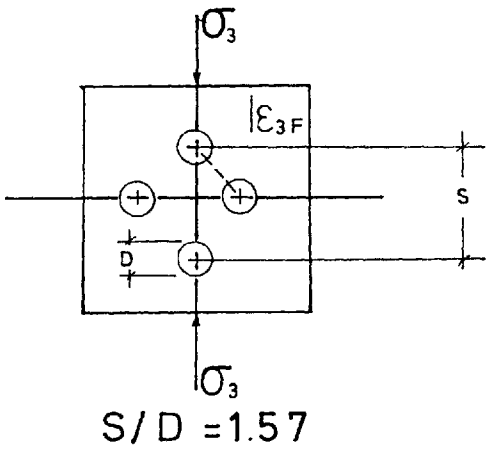


Isochromatics: $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG. VI- 35

DRM - 3 : $V_f = 54.6\%$

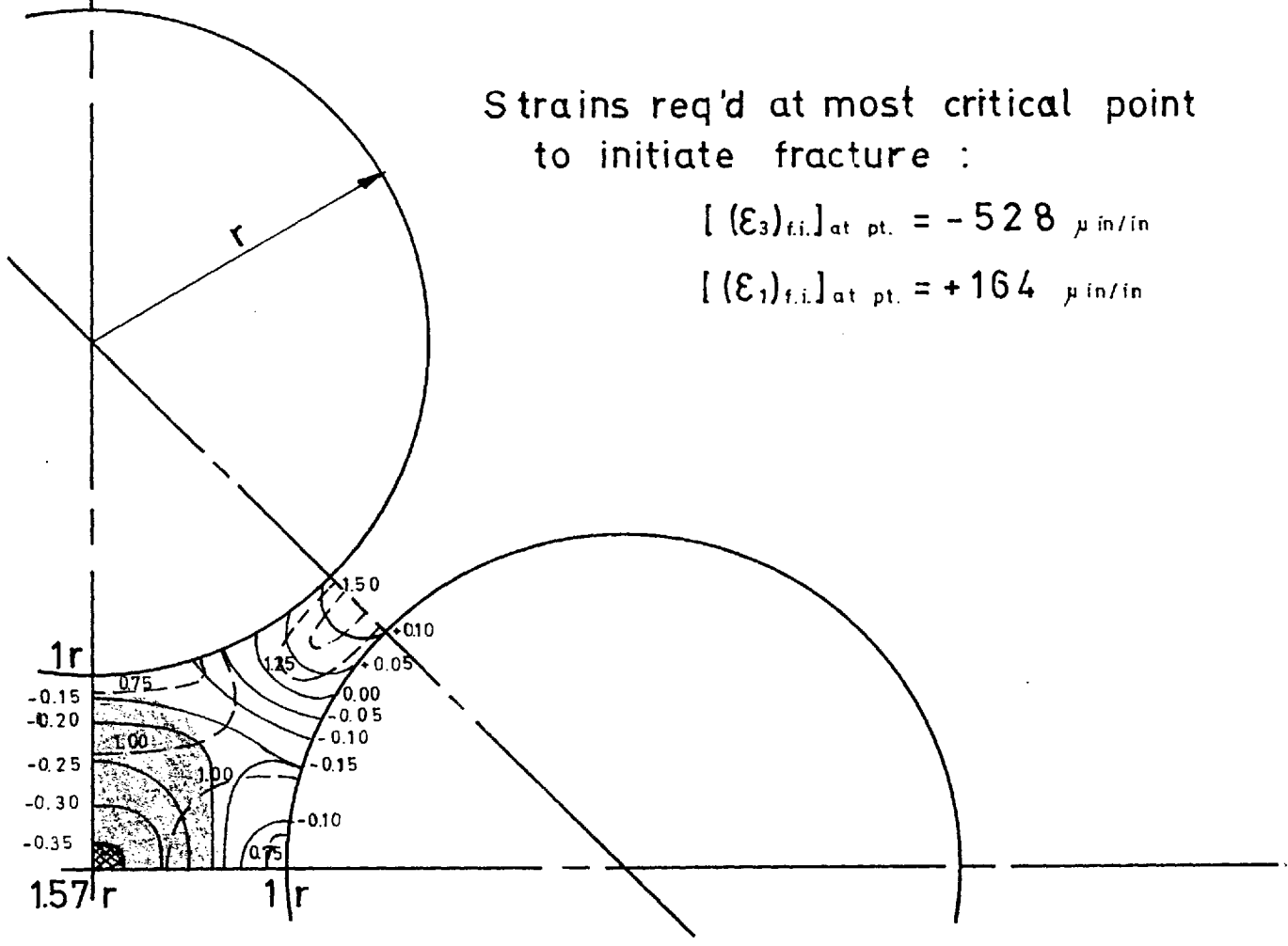
Critical Zone : 
 Most Critical Point : 



Strains req'd at most critical point to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{at \ pt.} = -528 \ \mu in/in$$

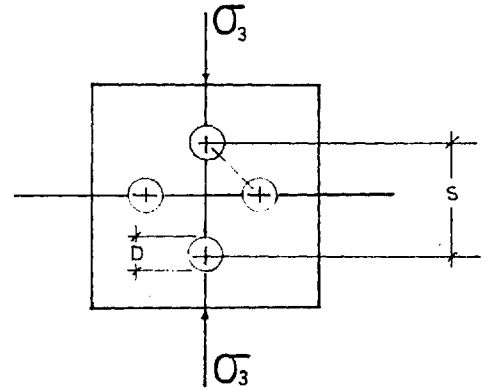
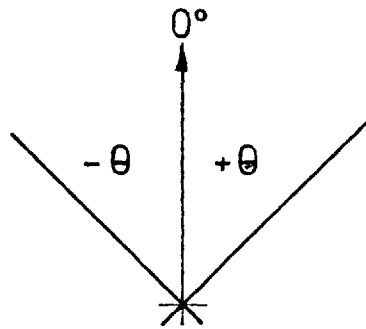
$$[(\epsilon_1)_{f.i.}]_{at \ pt.} = +164 \ \mu in/in$$



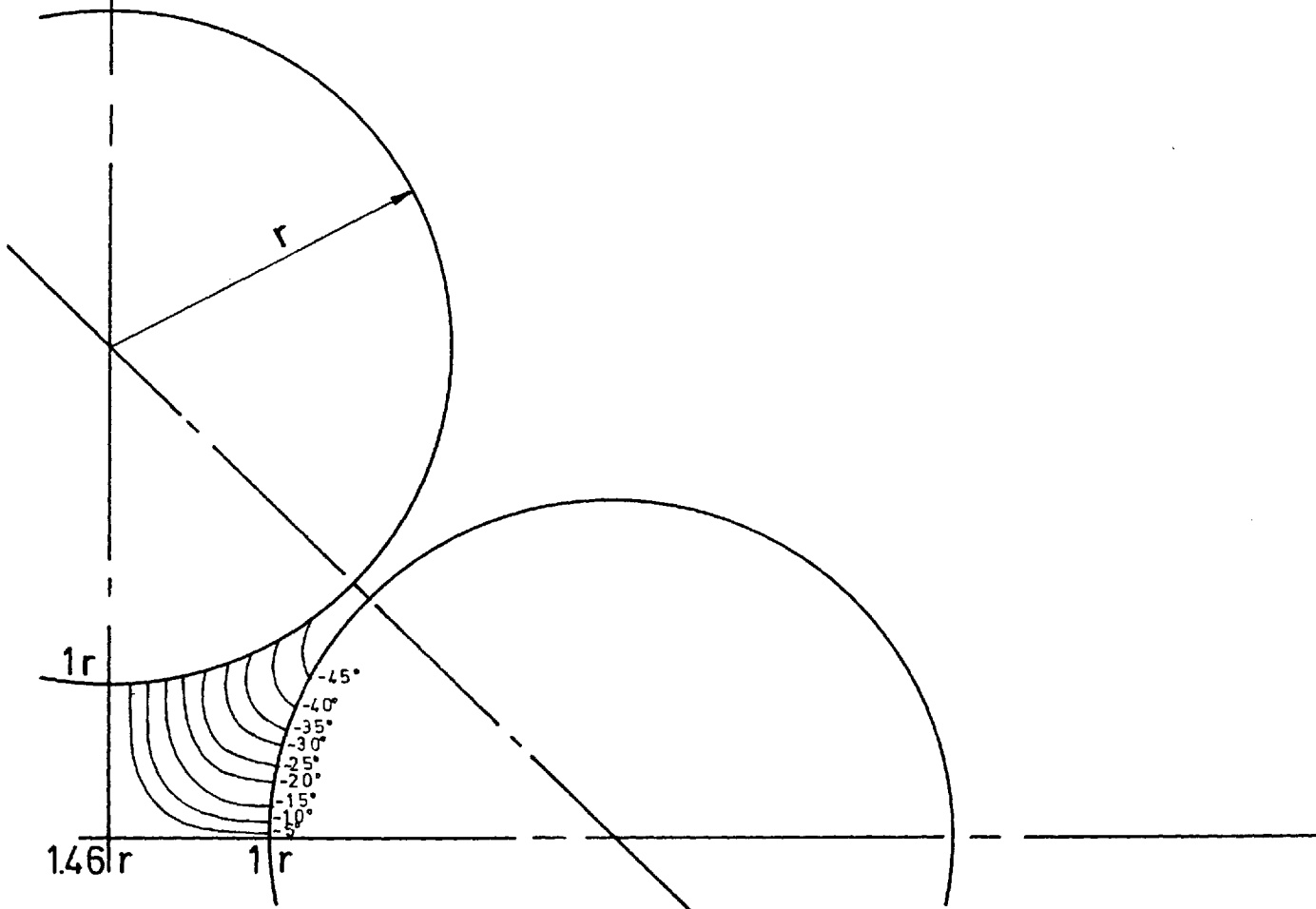
Isotenics : $\frac{(\epsilon_3)}{(\epsilon_3)_F} [---]$ vs. $\frac{(\epsilon_1)}{(\epsilon_3)_F} [—]$

FIG.VI-36

DRM-3 : $V_f = 67.2\%$



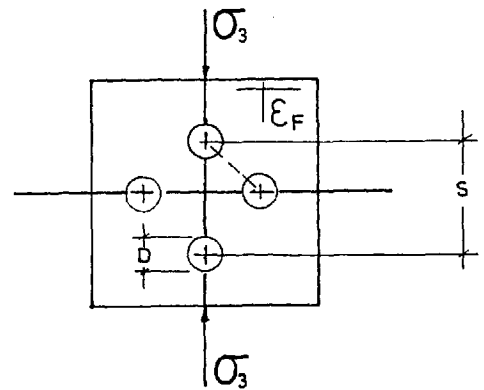
$S/D = 1.46$



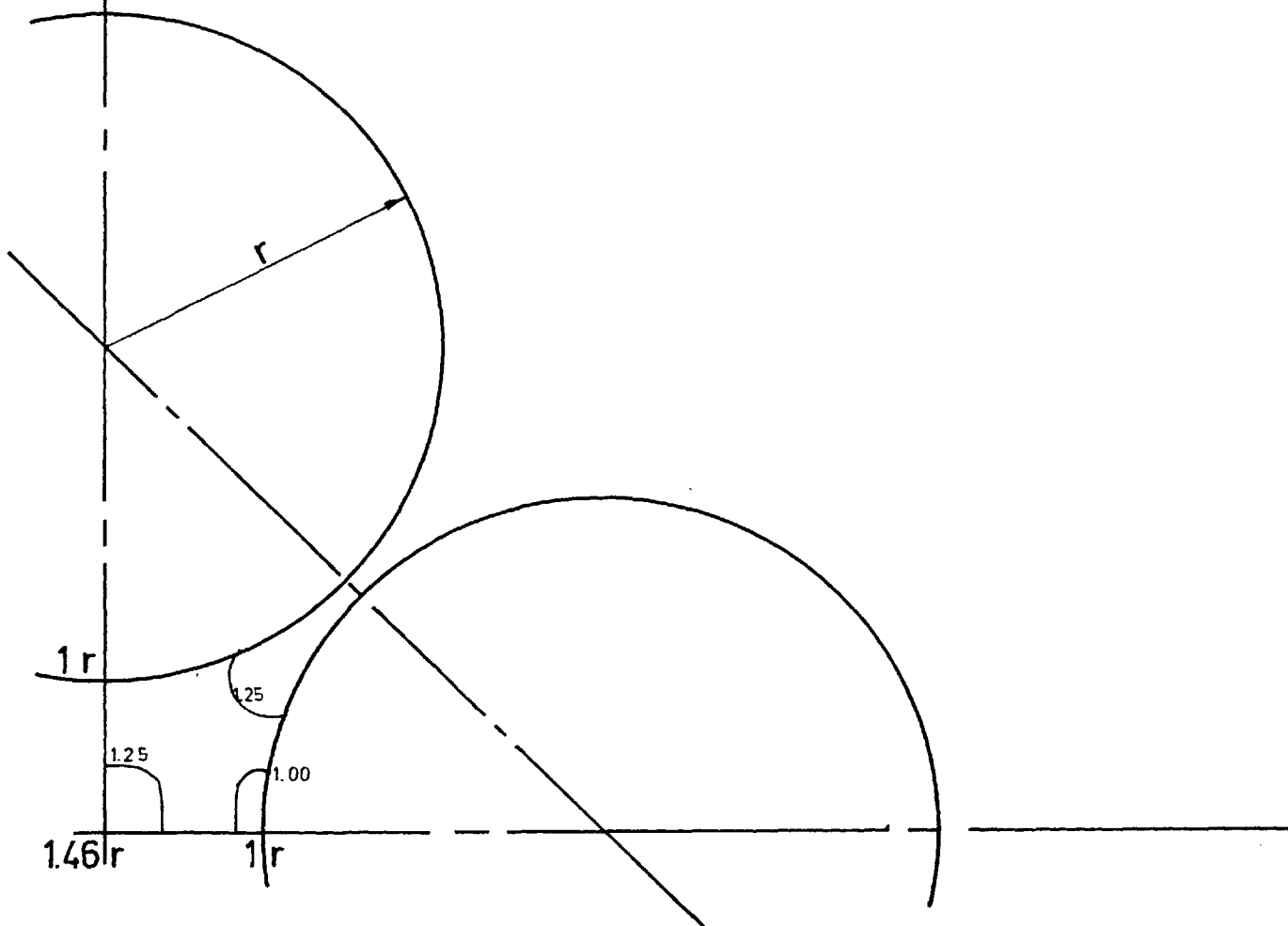
Isoclinics

FIG. VI-37

DRM -3 : $V_f = 67.2\%$



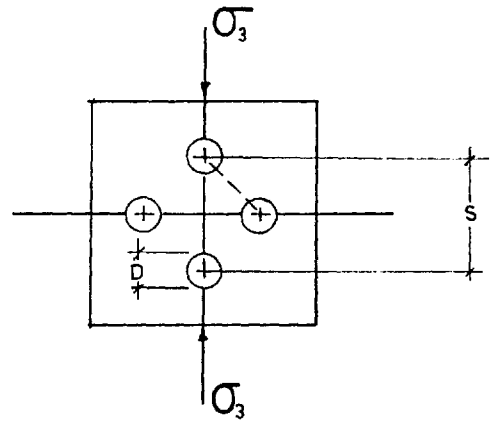
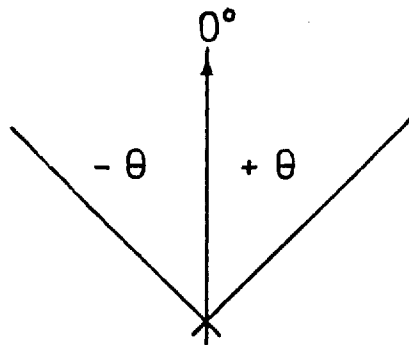
$$S/D = 1.46$$



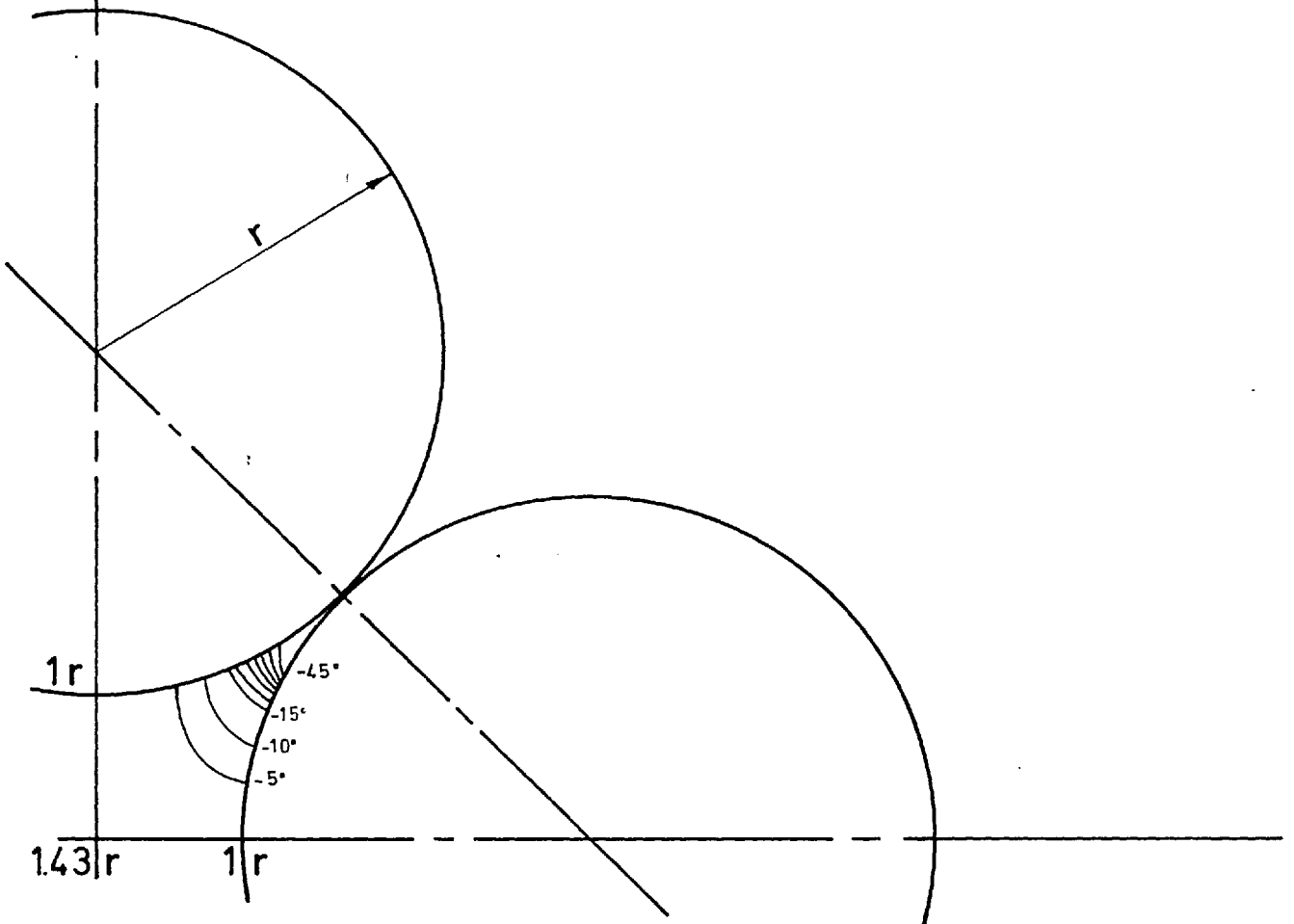
$$\text{Isochromatics : } \frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$$

FIG.VI-38

DRM - 4 : $V_f = 72.2\%$



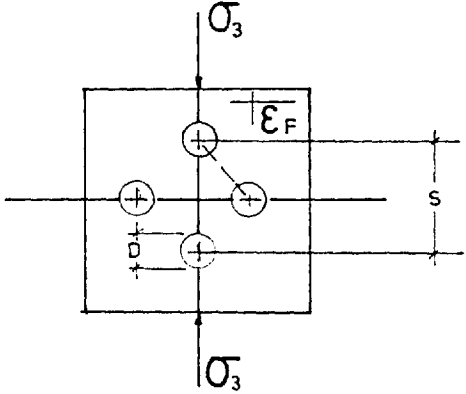
$S/D = 1.43$



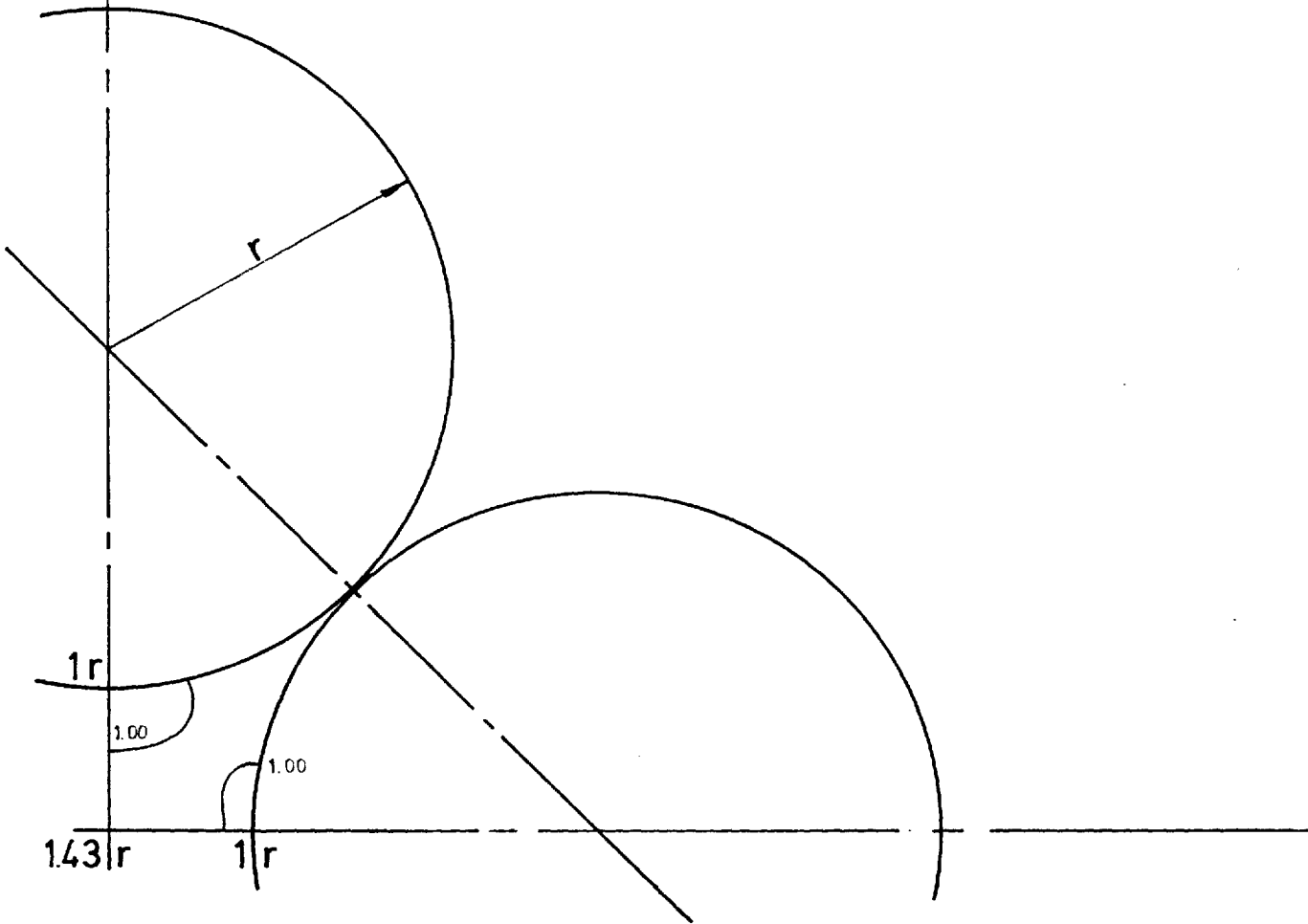
Isoclinics

FIGURE: VI-40

DRM - 4 : $V_f = 72.2\%$



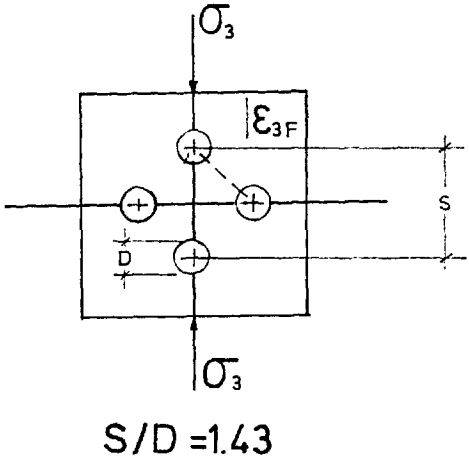
$S/D = 1.43$

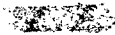


Isochromatics : $\frac{(\epsilon_3 - \epsilon_1)}{(\epsilon_3 - \epsilon_1)_F}$

FIG.VI-41

DRM -4 : $V_f = 72.2\%$



Critical Zone : 
Most Critical Point : 

Strains req'd at most critical point
to initiate fracture :

$$[(\epsilon_3)_{f.i.}]_{at\ pt.} = -330 \text{ u in/in}$$

$$[(\epsilon_1)_{f.i.}]_{at\ pt.} = +134 \text{ u in/in}$$

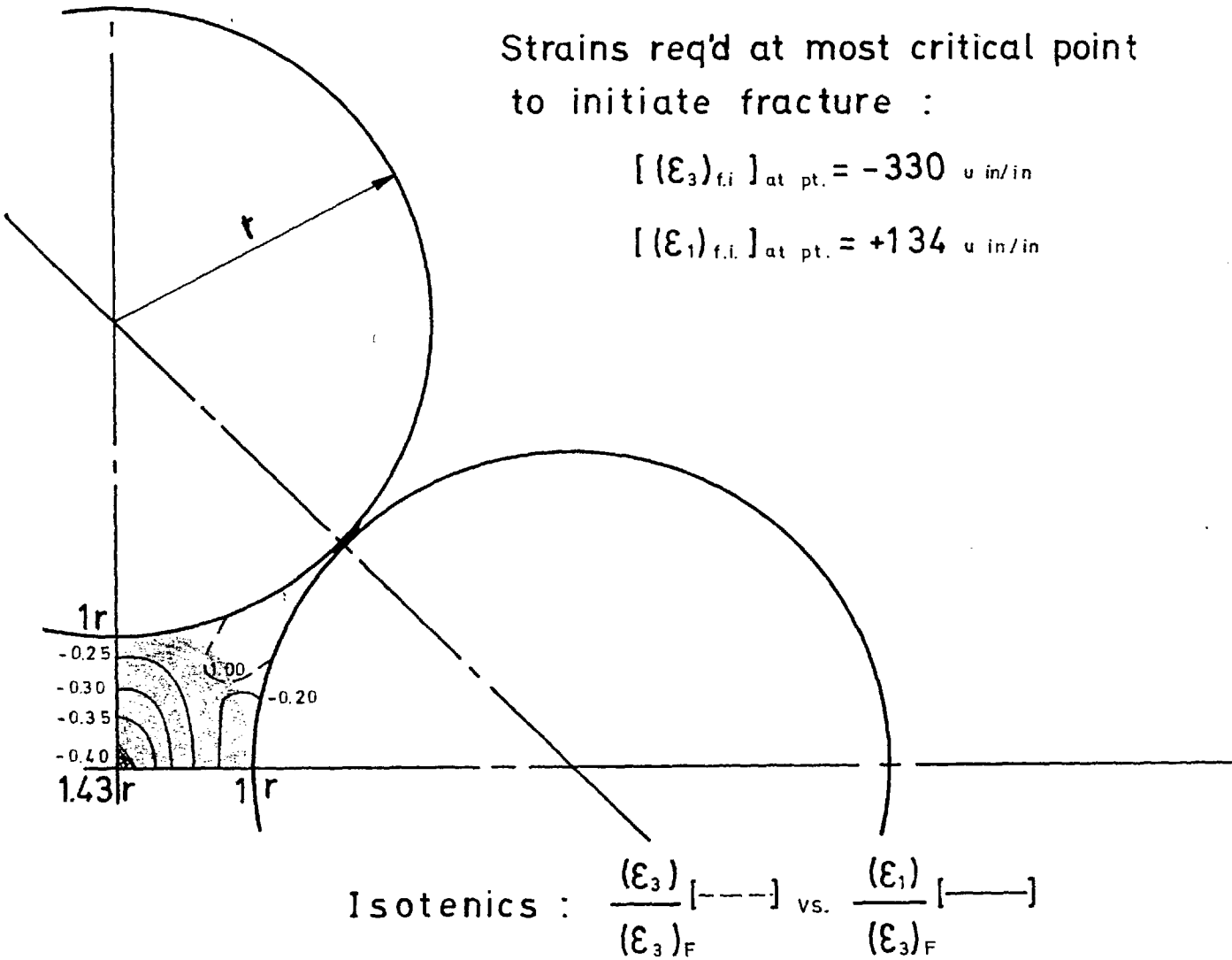
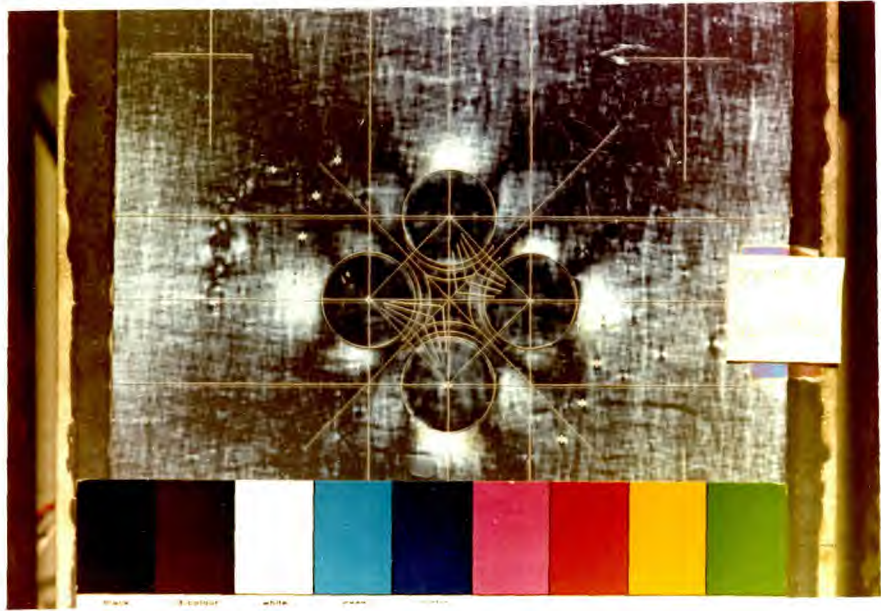
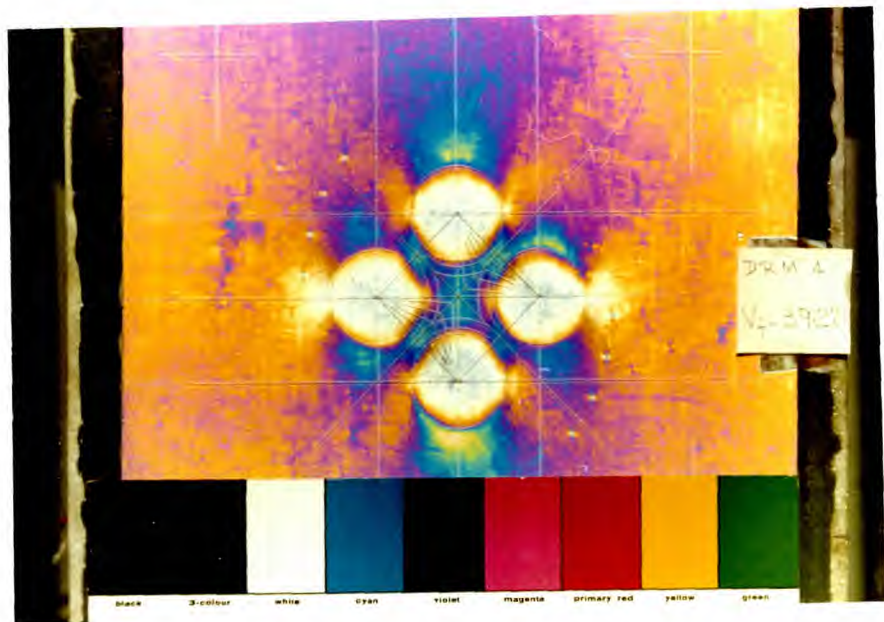


FIG.VI-42

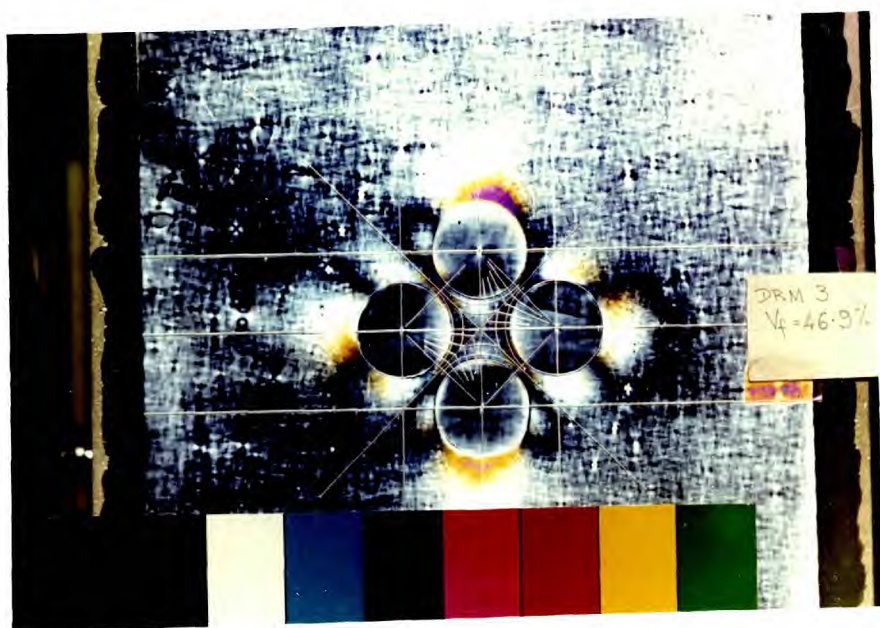


(a)

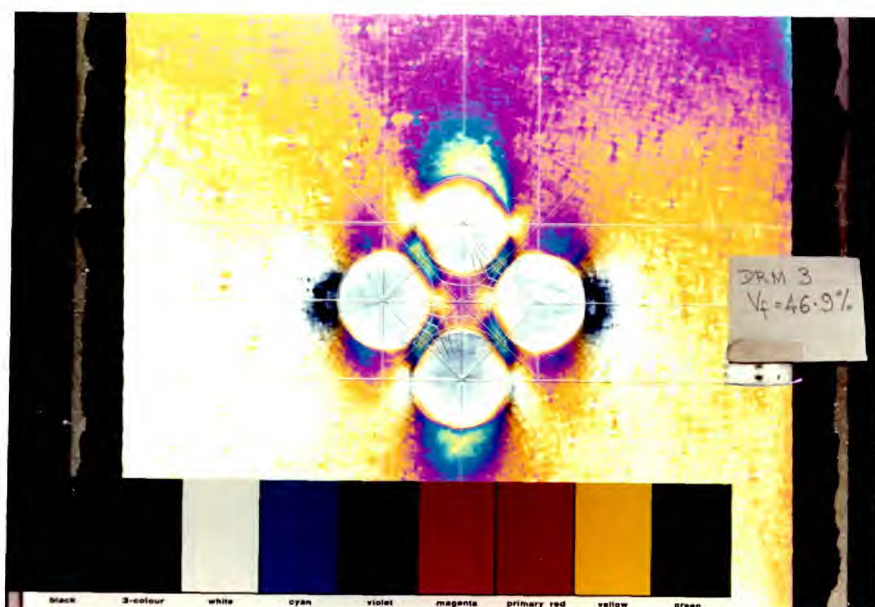


(b)

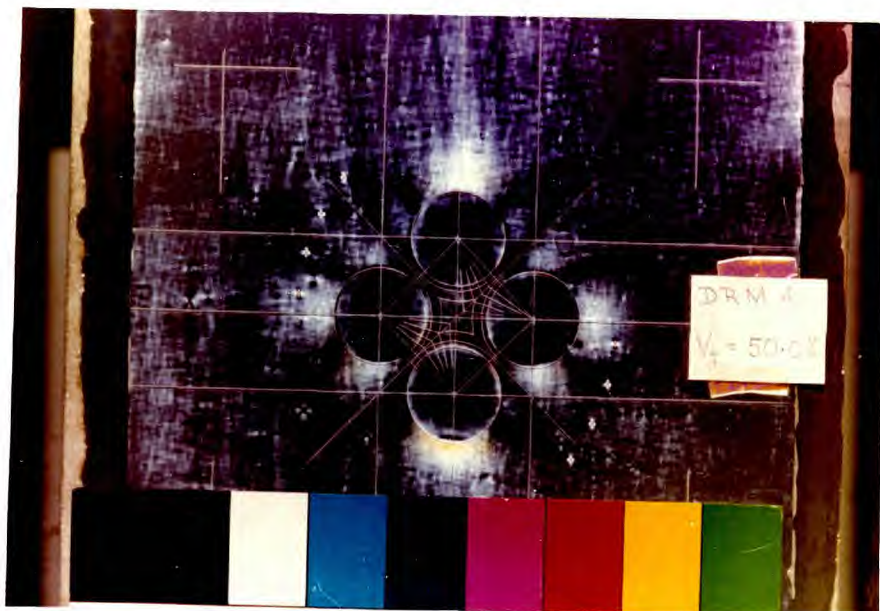
FIG.VI-43



(a)



(b)



(a)



(b)



(c)

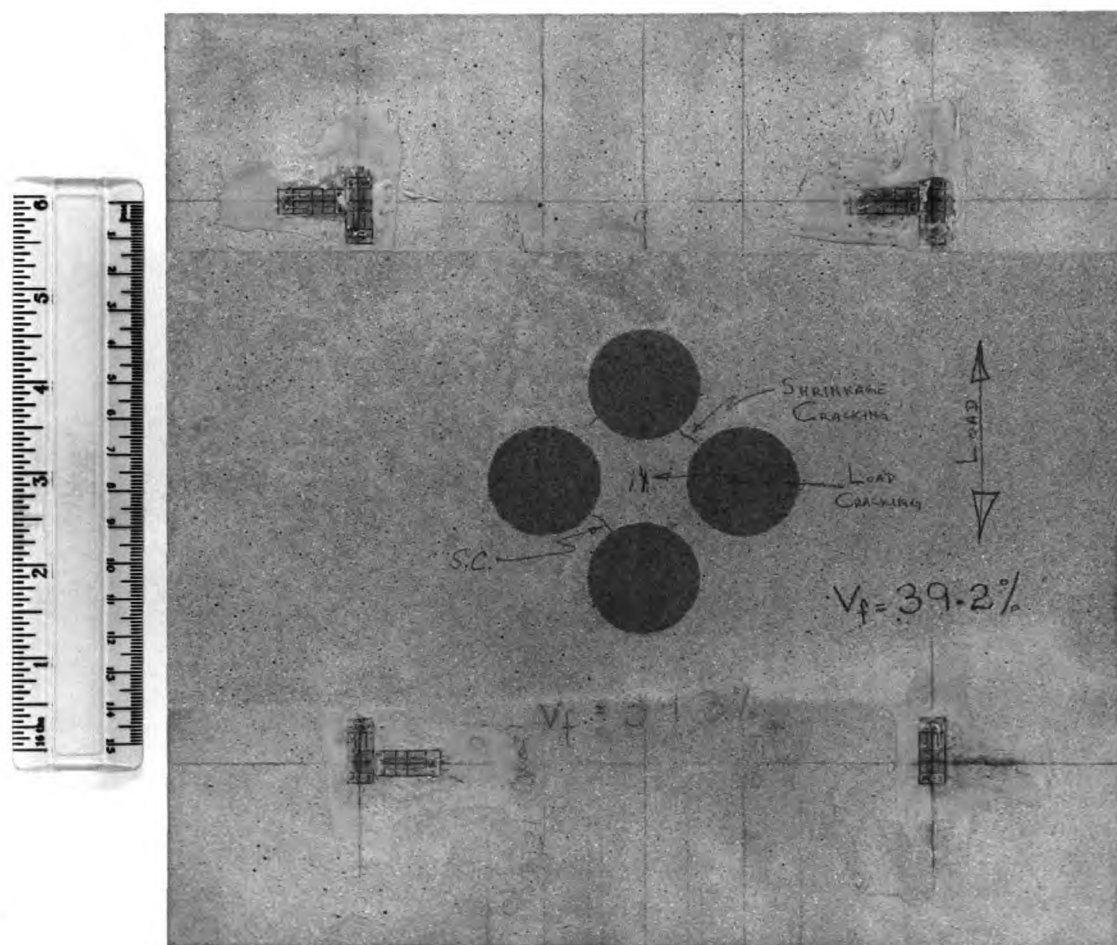
FIG.VI-45



(d)

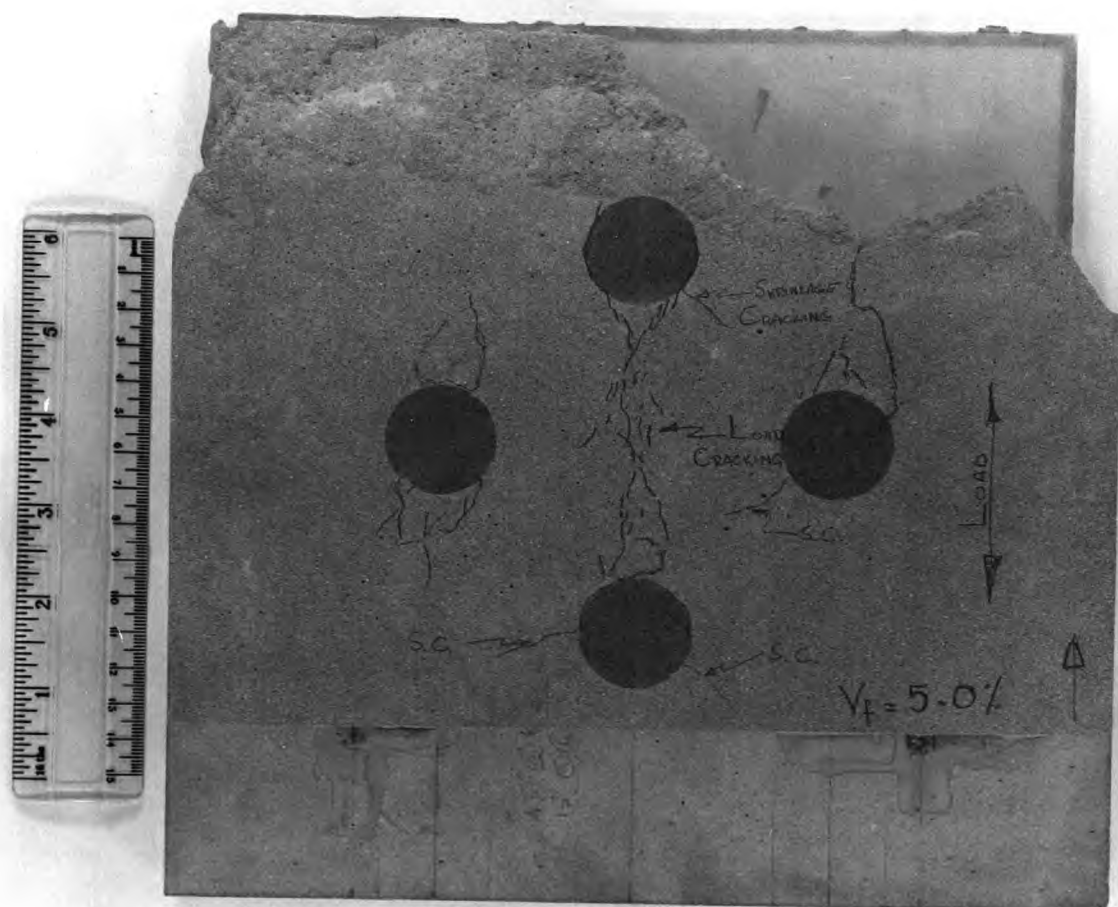


(e)



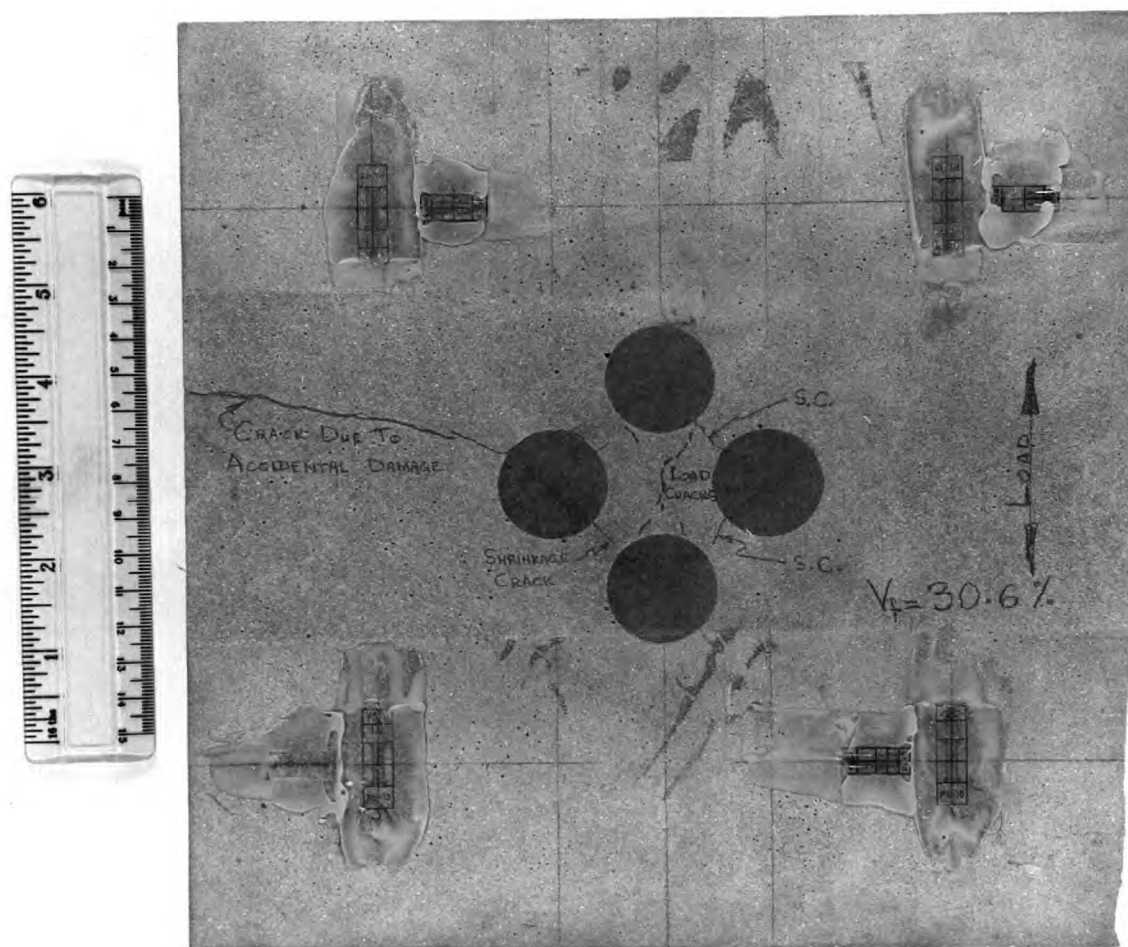
Crack Pattern due to Initiation of Fracture
under Load for Model: $V_f = 39.2\%$

FIG. VI-46



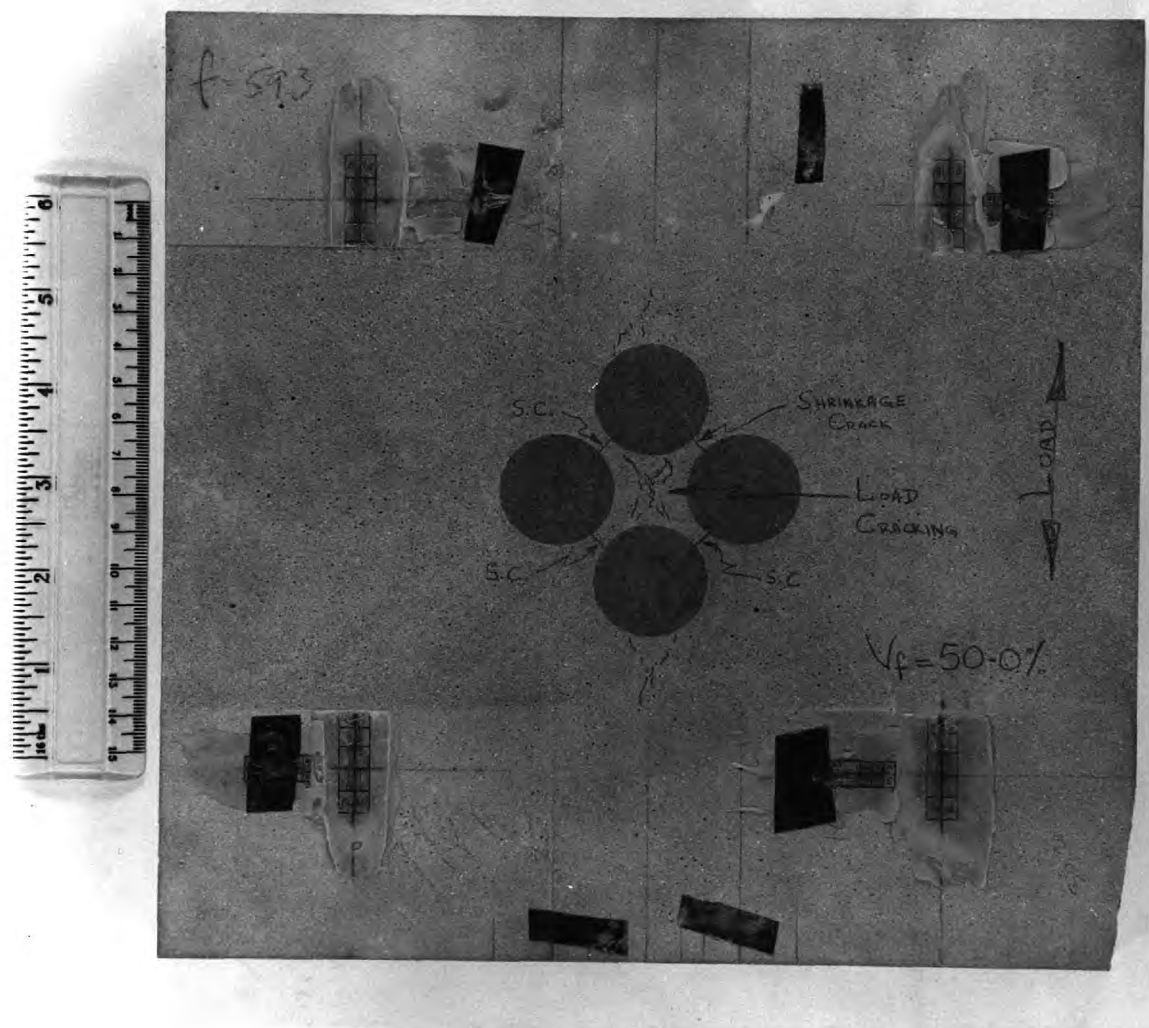
Final Crack Pattern after Failure for
Model: $V_f = 5\%$

FIG. VI-47



Final Crack Pattern after Failure for
Model: $V_f = 30.6\%$

FIG. VI-48



Final Crack Pattern after Failure for
Model: $V_f = 50\%$

FIG. VI-49

Results of Plate-Model Tests

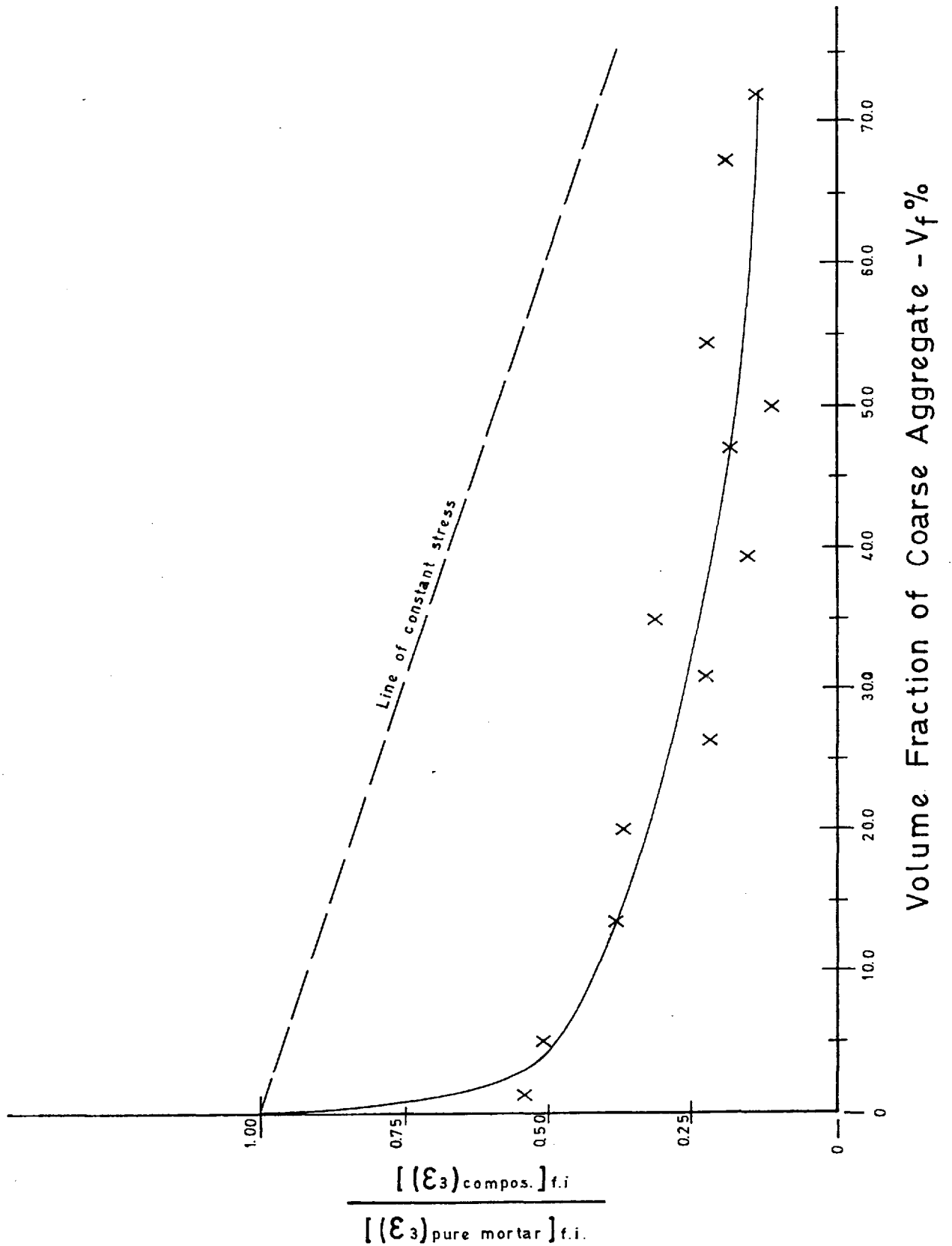


FIGURE : VI - 50

CHAPTER VII

TESTS ON THE PROTOTYPE CONCRETE

Section VII - 1

Introduction

It was necessary to determine whether the experimental testing and analysis of the plate-model specimens had given results which were meaningful and could be related to the behaviour of a "real" three-dimensional prototype concrete. It was decided, therefore, to test concrete specimens under uniaxial compression, and to determine the average strains at which fracture initiation occurred. As previously mentioned, the average strains of a concrete specimen are composed of both the aggregate strains and the mortar strains. Thus, it was considered that the composite material strains at fracture initiation measured from a particular plate-model test, should be related to the average strains exhibited at fracture initiation, by a concrete specimen composed of the same materials and containing the same volume fraction of coarse aggregate, tested under uniaxial compression.

In Chapter III, a method for determining the average strains at which fracture initiation occurred within a particular material was outlined. This method was based on the calculation of the volumetric strains of the material under test. In Chapter V, this method was used to determine the average strains at which fracture initiated within mortar specimens. It was proposed to use the same method for the testing of concretes under uniaxial compression. Whereas in Chapter V the variable parameter was the state of strain or stress applied to each specimen, the variable parameter in the testing of concretes was to be the volume fraction of coarse aggregate.

Section VII - 2

Scope and Philosophy of Tests

The prime object of these tests was to determine experimentally the variation in the average, "composite material", strains at fracture initiation for concretes containing various volume fractions of coarse aggregate.

It was decided that cylinders of concrete, 4 ins. in diameter and 10 ins. high would be tested. Previous work⁹³ had shown that the central 4 ins. of such a specimen could reasonably be considered to exist under a state of uniaxial compression, undisturbed by platen restraints, when loaded. Each specimen was cast containing a predetermined volume fraction of coarse aggregate (see also Section VII - 5). The materials used are described in Section VII - 3. Each specimen was tested under short-term uniaxial compression in an Amsler 300-ton testing frame, and the average strains measured at each load stage (Section VII - 5). A plot of volumetric strain against minor principal strain was used to determine the strains at fracture initiation (Chapter III).

Thirty-six specimens were cast in three batches of twelve. Only two specimens containing any one volume fraction of aggregate were cast, and the intervals of volume fraction between specimens were chosen arbitrarily. In this way, it was considered that any errors due to the casting technique would be randomized as far as possible.

Section VII - 3

Choice of Materials

In order to obtain a reasonable correlation, the materials used in the manufacture of the concrete prototype specimens were chosen to simulate, as closely as possible, the materials used in the plate-model tests.

Mortar

The mortar mix used was identical in all respects to that used in the plate models (Chapter IV, Section IV - 2.3). Thus, the matrix of the two-phase concrete model (Chapter III, Section III-2) was unchanged between the plate-models and the concrete prototype.

Aggregate

In Chapter IV, Section IV - 2.3, it was assumed that coarse aggregate particles existed as single sized spheres within a homogeneous matrix. As the rock type used for the aggregate particles in the plate-model tests could not be obtained in a spherical form, it was decided to use glass marbles

as the aggregate in the concrete prototype. From Chapter IV, Section IV - 2.3, it will be recalled that the Young's Modulus of the aggregate was chosen such that a ratio $E_{agg.}/E_{mortar}$ of at least three was obtained. The actual value of this ratio was 3.75. For the glass marbles used, the value of the Young's Modulus of the glass, quoted by the manufacturer, was 11.0×10^6 p.s.i., giving a ratio $E_{agg.}/E_{mortar}$ equal to 3.14 for the concrete specimens. A theoretical calculation⁵⁹ (Table IV - 2 and Fig. IV - 3) showed that the local strain magnitudes were not expected to vary markedly with changes in the aggregate modulus provided that a ratio $E_{agg.}/E_{mortar}$ greater than three existed. It was considered that the ratios given above should provide reasonable correlation between the plate-models and the concretes.

In order to obtain a surface roughness on the glass marbles roughly equivalent to that achieved on the rock cylinders used in the plate models, a "tumble" mill was used. Immersion of the glass marbles in hydrofluoric acid could not be used for this purpose as an insufficient degree of differential etching was obtained. Due to the homogeneous nature of the glass, the marbles merely dissolved slowly without producing a satisfactory degree of surface roughness. It was determined that "tumbling" the marbles for approximately two hours in a mill containing No.46 carborundum powder produced a surface texture which was considered to be roughly equivalent to that exhibited by the rock cores used in the plate-models (Chapter IV, Section IV - 2.3)

The glass marbles used were $\frac{3}{4}$ " in diameter, this being dictated by the maximum size of the marbles commercially available in bulk quantities. The size of marble used did not affect the volume fractions of coarse aggregate which could be achieved, as this fraction was solely a function of the diameter to spacing ratio, (D/S) (Chapter IV, Section IV - 2.3). It is recognized, however, that the use of smaller coarse aggregate particles than those represented by the rock cores in the plate-model tests might

affect any correlation between the two types of test.

Section VII - 4

Preparation of Specimens

The mixing techniques used for the concrete specimens were not changed from those outlined in Chapter IV, Section IV - 2.5, for the plate-model specimens. The moulds used were 4 ins. diameter by 10 in. high cylinders, in which the concrete was placed and vibrated in three layers. The consistency of the mortar was such that no evidence of segregation was observed. As the preparation of the specimens prior to test included the grinding off of the ends of each specimen, segregation was not considered to be a serious problem. However, it was recognized that the entrainment of any air, due to the presence of the coarse aggregate particles, could seriously affect the measurement of the strains at fracture initiation. Accordingly, each batch was tested using a pressure operated air-entrainment meter to determine the air content of the mix. From the values of the total air content of the mix the air content of the mortar matrix was determined as a percentage of the volume of mortar present. Typical values for various concretes are shown in Table VII - 1. It was seen that the air content of the mortar decreased slightly with increasing coarse aggregate content, this probably being due to the additional vibration required to compact the leaner mixes. It was considered from such results that errors were not being introduced due to the entrainment of additional air voids.

The curing regime outlined in Chapter IV, Section IV - 2.5, was followed.

After curing, the ends of each specimen were ground to ensure that they were plane, parallel, and square to the long axis of the cylinder. (Chapter IV, Section IV - 2.7). The curved faces of each cylinder were sandblasted to remove the excess mortar "latence".

After preparation, each specimen was allowed to stand at ambient conditions for at least eight weeks (Chapter IV, Section IV - 3.3 and IV - 4).

TABLE VII - 1AIR CONTENT OF MORTAR FOR VARIOUS CONCRETES

Nominal

<u>Vf%</u>	<u>Percentage Air in Mortar</u>
0	5.50
5	5.32
10	5.28
34	4.86
46	4.43
58	3.36

where V_f = volume fraction of coarse aggregate in concrete.

Section VII - 5Test Method

Electrical resistance strain gauges, 60 m.m. in length, were attached to each specimen using standard glueing techniques. For each specimen two "vertical" (ϵ_3) and two "horizontal" (ϵ_1) gauges were placed.

All specimens were tested under uniaxial compression in a 300-ton Amsler frame. Testing rates of approximately 1000 p.s.i./minute were used, and the average strains of the specimens read, from the gauges, using a Peekel Data Logger at intervals of approximately 350 p.s.i. For all tests a subsidiary ball-seating was used to ensure the uniformity of load and the comparability of strains on opposing faces of the specimen.

Tests were run to the point of incipient failure, determined by a rapid increase in the recorded strain values. After test, the "central" portion of each specimen (that portion over which the 60 m.m. gauges were placed) was cut out using a diamond tipped saw, and its volume determined. This portion was then broken open by hand and the volume of (glass marble) aggregate within the gauge span measured. The precise value of the volume fraction of coarse aggregate present in that portion of each specimen covered by the gauges was then calculated.

Section VII - 6Test Results

The results from typical specimens are shown in Figures VII - 1 to VII - 7 in which the volumetric strains are plotted against the minor principal strains. To allow comparison between specimens, all plots have been drawn to the same scale. The average strains at which fracture initiation was considered to have occurred from the volumetric strain plot (Chapter III) are given in each Figure. The results from the complete series of tests are shown in Figure VII - 8. The average minor principal strain of the concrete at fracture initiation, non-dimensionalized with respect to the strain of pure mortar ($V_f = 0\%$) at fracture initiation, is shown plotted against the volume fraction of coarse aggregate within the concrete.

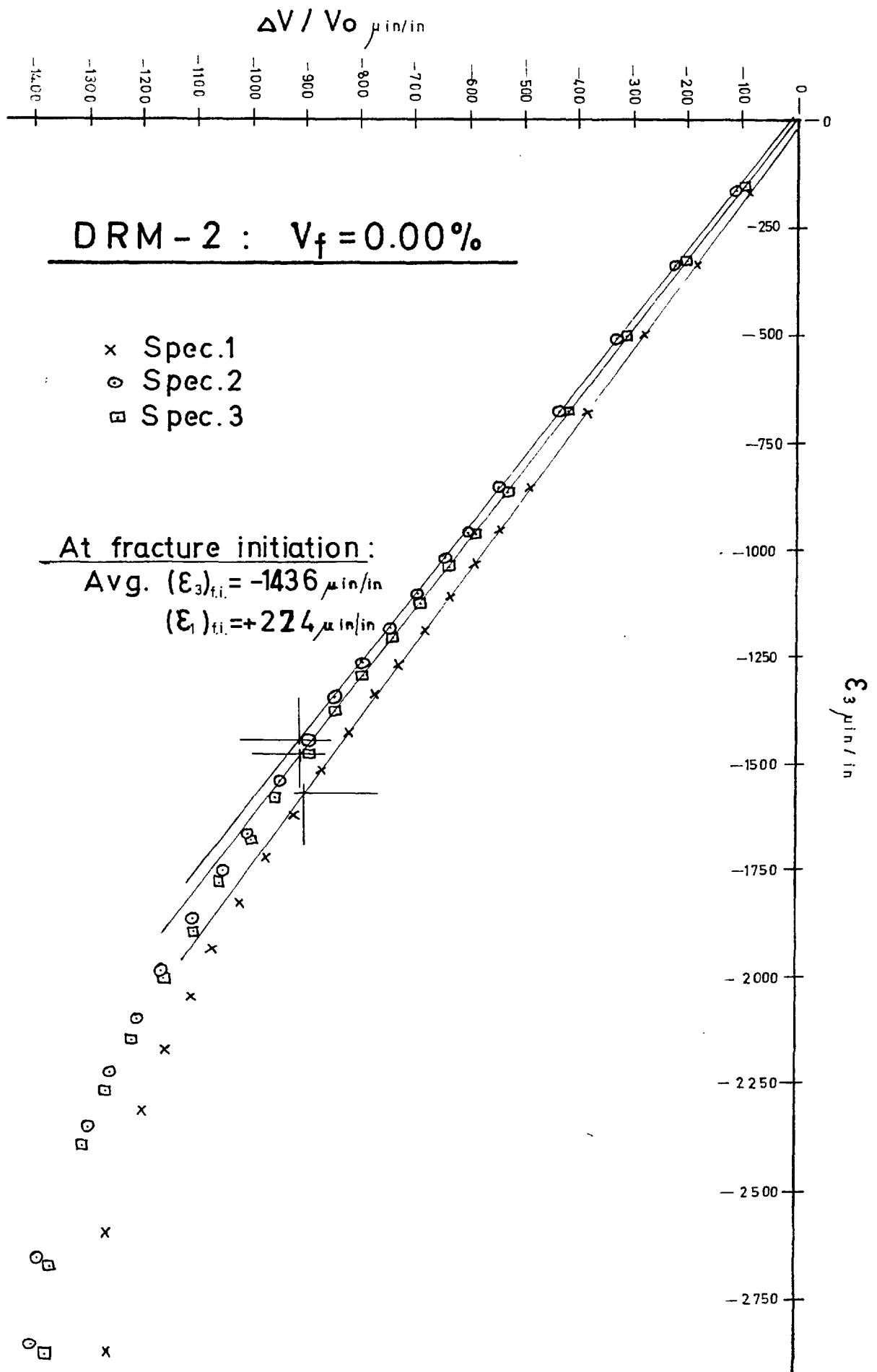


FIGURE: VII -1

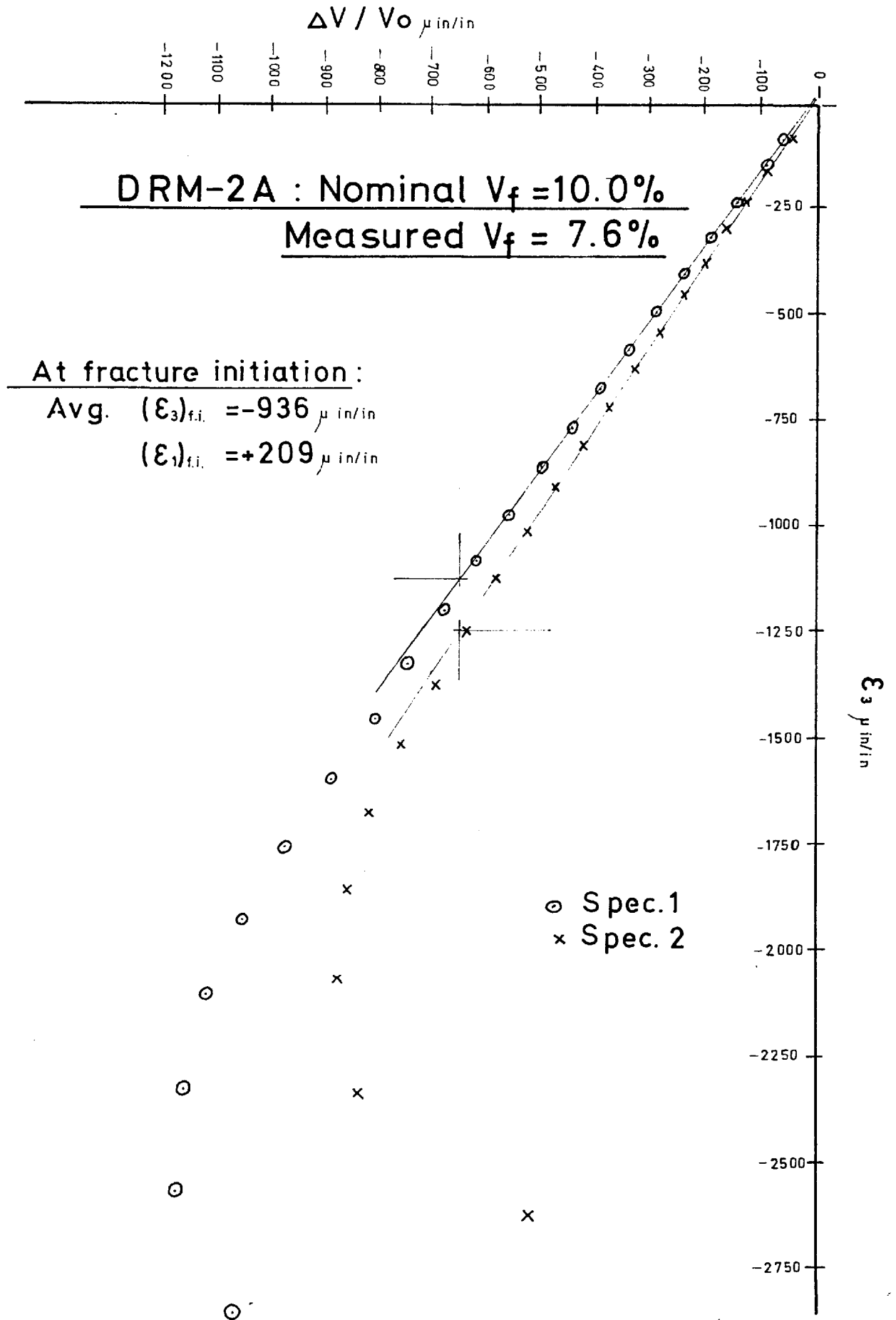


FIGURE : VII-2

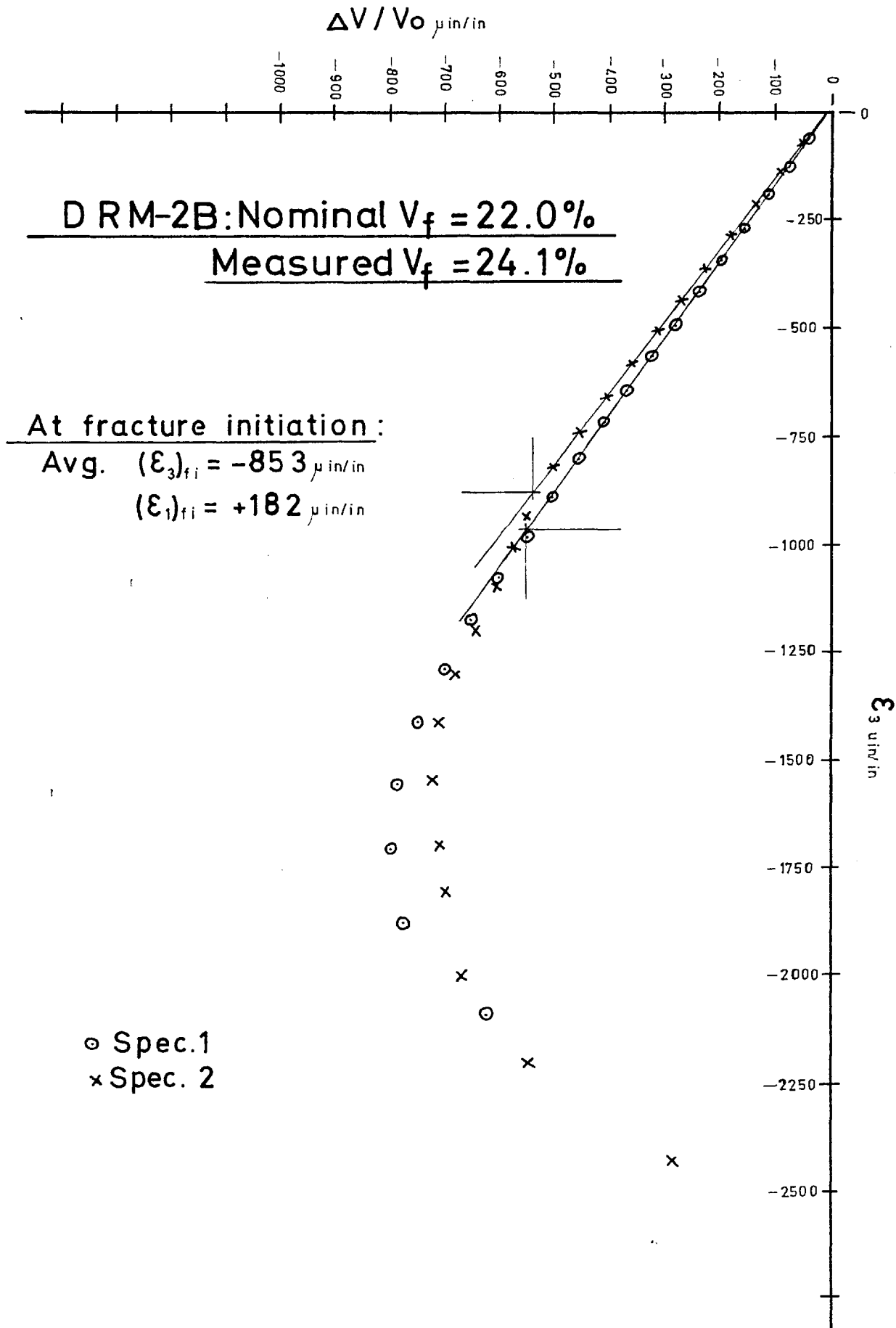


FIGURE:VII-3

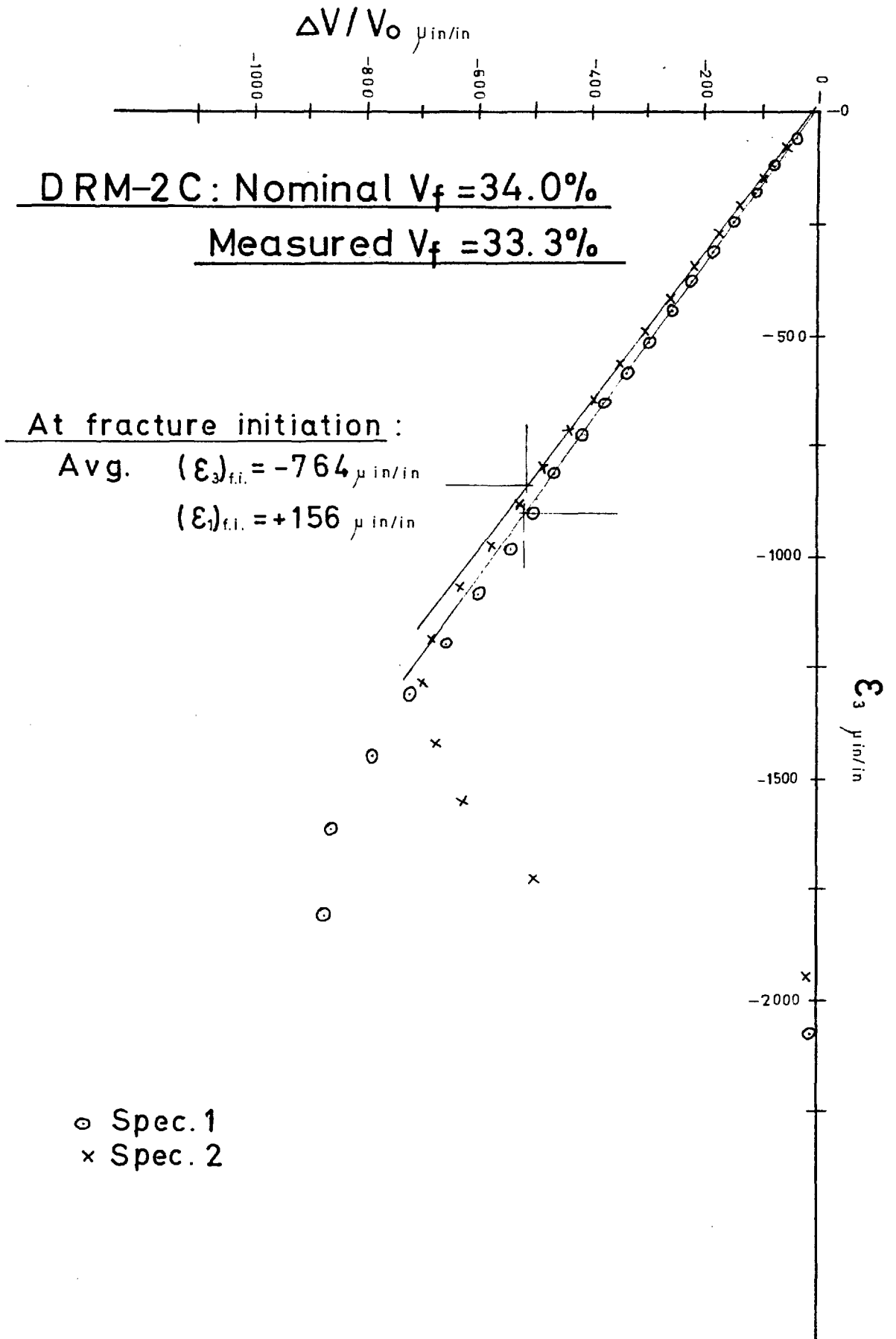


FIGURE: VII-4

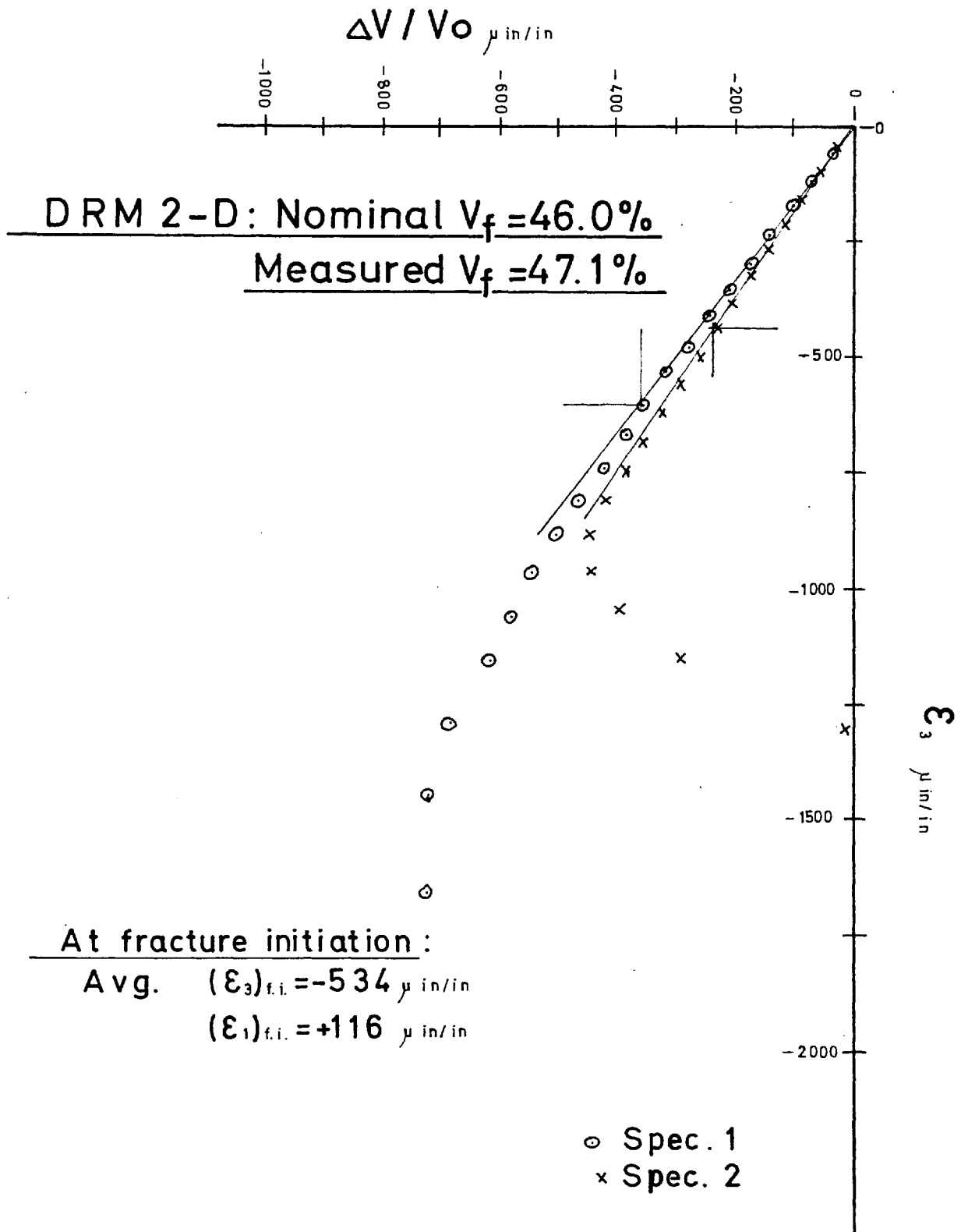


FIGURE : VII-5

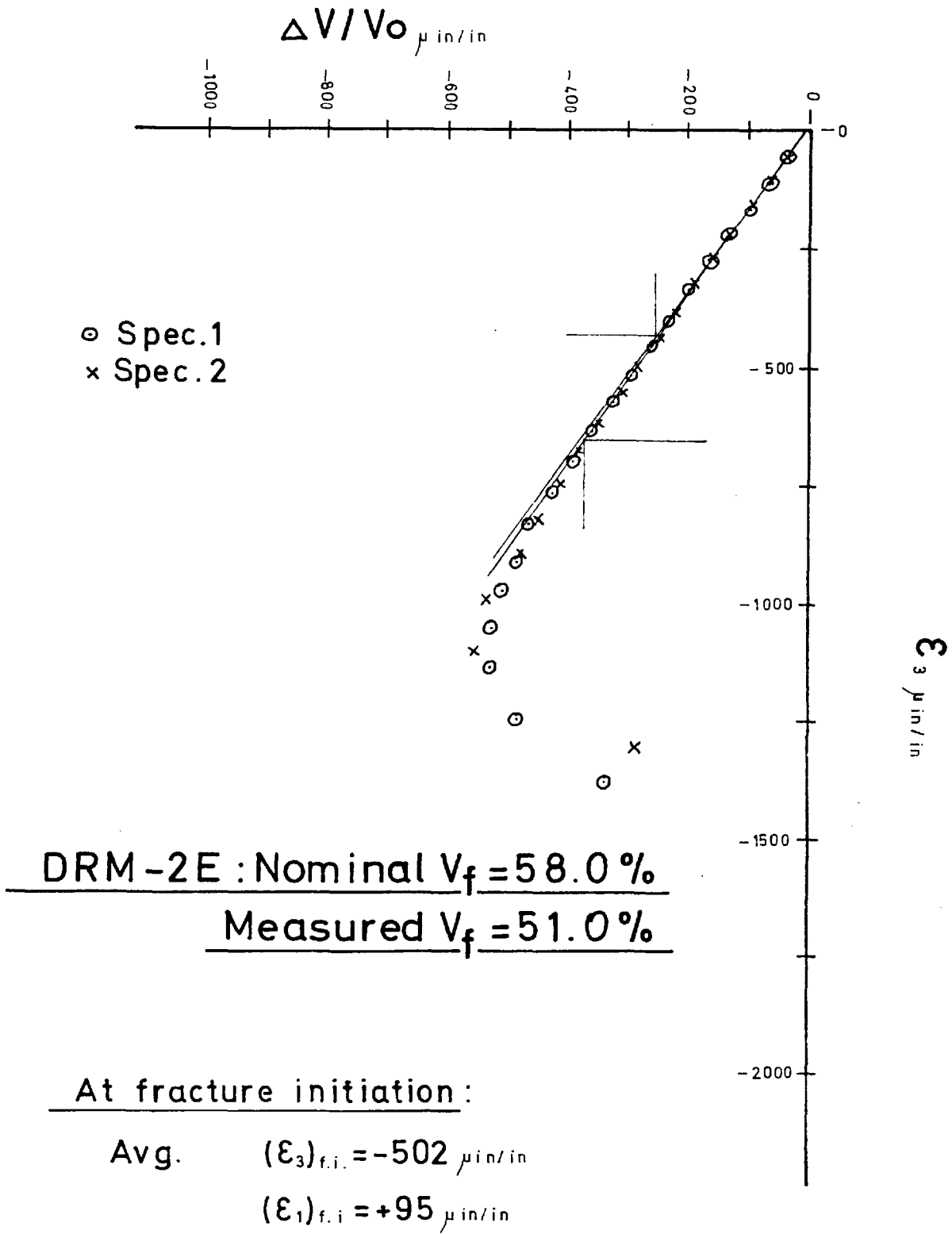


FIGURE:VII-6

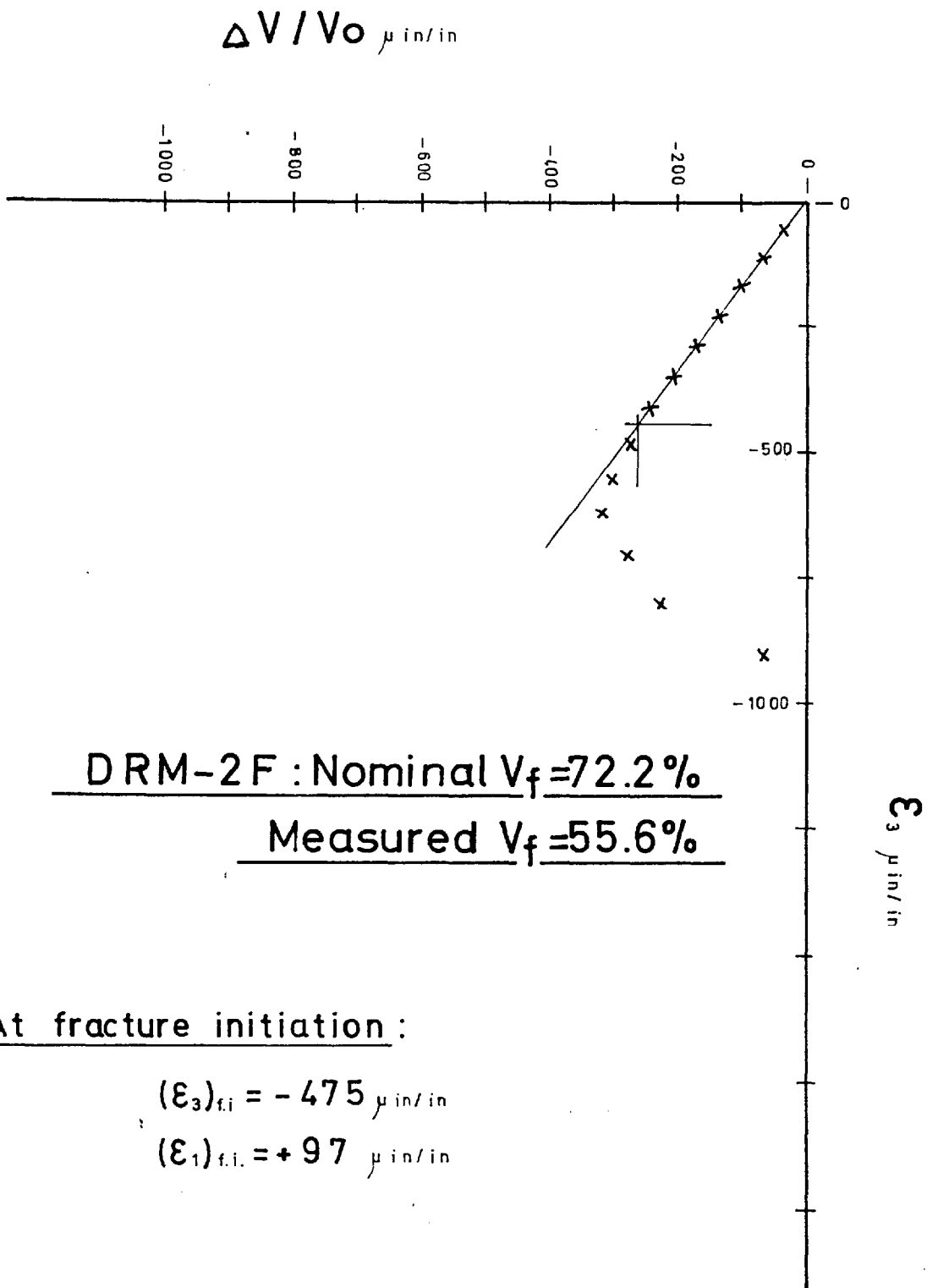


FIGURE: VII-7

Results of
Glass-Marble Concrete Tests

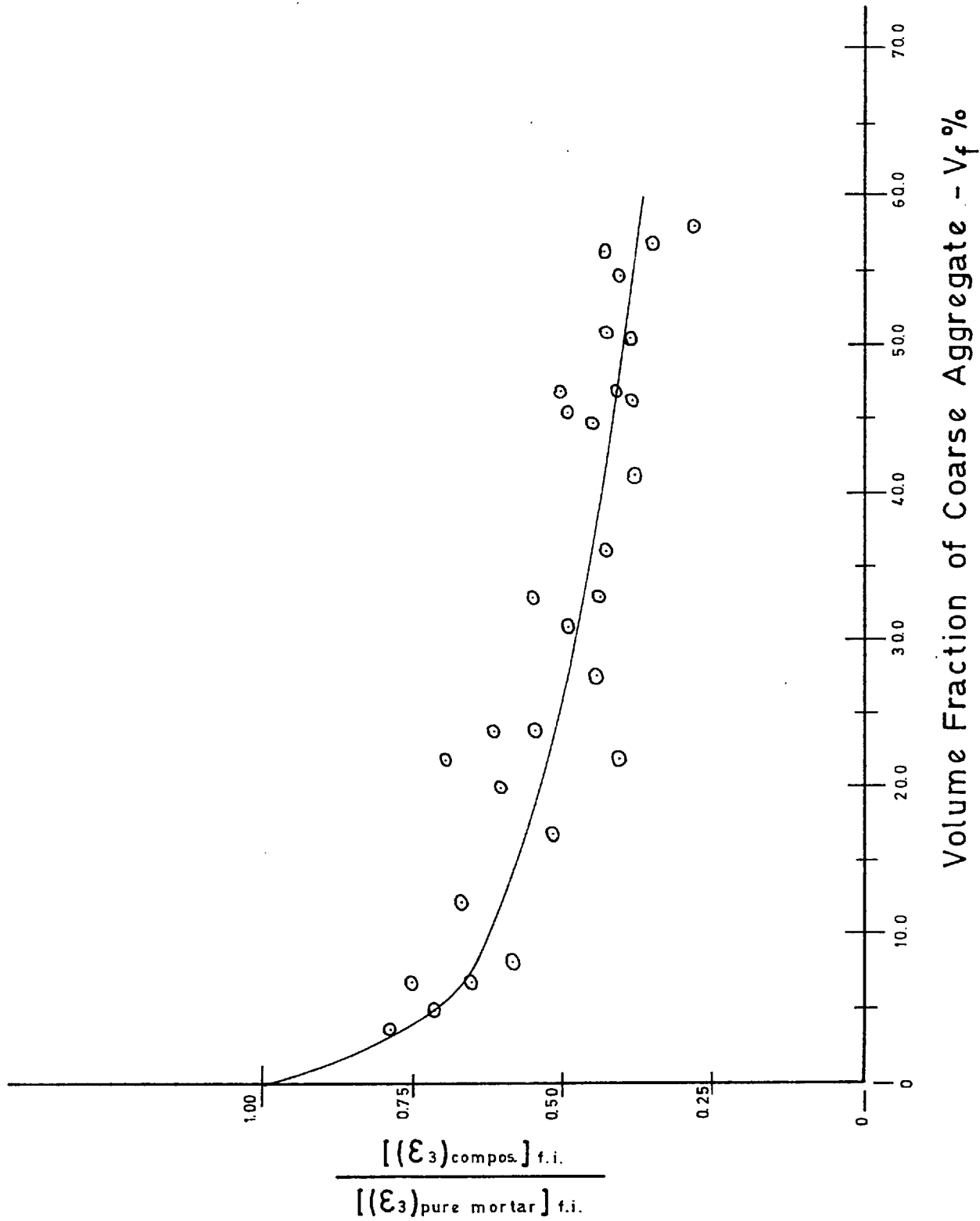


FIGURE : VII- 8

Coarse aggregate volume fractions greater than 60% could not be achieved, even by pre-packing of the aggregate into the moulds. For a face-centered cubic packing, the theoretical maximum may be calculated (Chapter IV, Section IV - 2.4) as 72.2%. In practice it was not possible to achieve intimate contact between aggregate particles. During compaction a thin film of mortar formed around each particle, lowering the volume fraction of coarse aggregate in the specimen.

Section VII - 7

Discussion of Results

As pointed out in Chapter III, the use of the volumetric strain plot will not necessarily detect the stage at which the first fractures are initiated. The method is limited by the sensitivity of strain measurement possible. Errors in the absolute values measured will occur due to all the normal testing errors of operator technique, strain measurement etc. However, as the same techniques were used for each specimen, the trend of the results should be valid and comparable to the trend of the results predicted from the plate-model tests.

CHAPTER VIII

CORRELATION OF RESULTS FROM PLATE MODEL TESTS AND PROTOTYPE
(GLASS MARBLE) CONCRETE TESTS.

Section VIII - 1.

Discussion of Correlation

The results from the plate-model tests and from the glass-marble concrete tests are shown superimposed in Figure VIII - 1. From this Figure it may be seen that the quantitative result derived from a plate model test representing a particular volume fraction of coarse aggregate differed considerably from the result derived from a concrete containing that volume fraction. The value of the minor principal composite material strain at fracture initiation predicted from the plate-model tests was seen to be, in general, approximately 50% of the value measured from the glass-marble concrete tests.

However, the trend of the two curves was seen to be closely similar. Some idea of this similarity in trend of the two curves was obtained from an examination of the relationship between two points on each curve. Thus, for the glass-marble concretes:

$$\frac{(\epsilon_3) \text{ f.i. at } Vf = 40\%}{(\epsilon_3) \text{ f.i. at } Vf = 20\%} = 0.729$$

and for the plate-model tests:

$$\frac{(\epsilon_3) \text{ f.i. at } Vf = 40\%}{(\epsilon_3) \text{ f.i. at } Vf = 20\%} = 0.683$$

giving a variation about the mean value of approximately $\pm 3\%$. It was considered, therefore, that the results of the plate-model tests gave a valid indication of the trend of the onset of fracture which might be expected due to variation in the volume fraction of coarse aggregate. Thus, errors due to the assumptions made during the analysis of the plate-

Results of Plate-Model Tests and Glass-Marble Concrete Tests

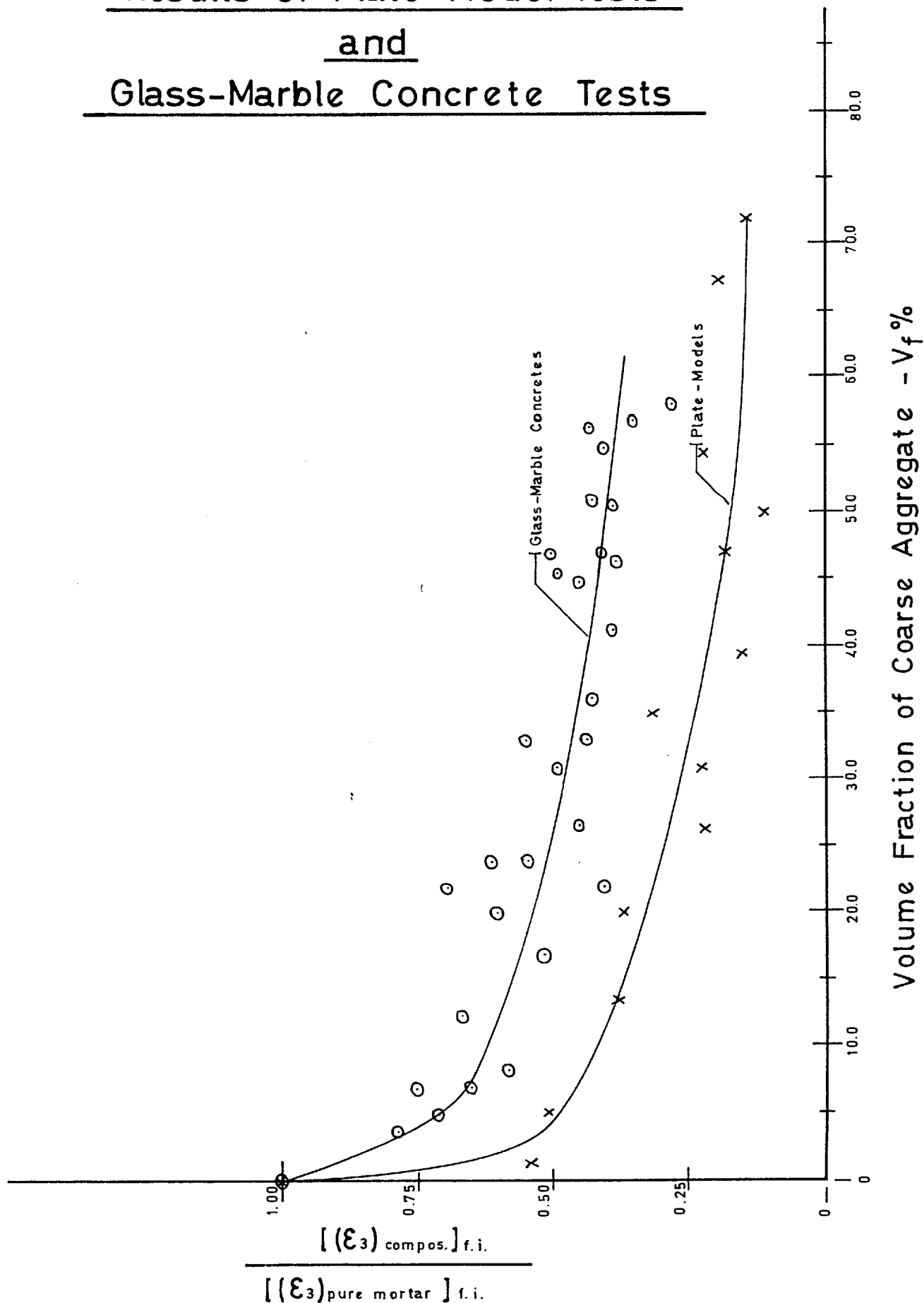


FIGURE: VIII - 1

models (Chapter V) appeared at least to have been of constant magnitude for each test.

There are a number of basic differences between the two types of test, which would tend to displace the two curves of results relative to one another.

- 1) The plate-model tests were conducted in two-dimensions.

As discussed in Chapter IV, a two-dimensional model was necessary in order to allow detailed strain measurements and observations. The correlation factor between two-dimensional results and three-dimensional tests is difficult to assess⁵⁸. In general, if the intermediate principal strain, absent in the two-dimensional case, is approximately equal to the major principal strain, only small differences between the two cases should result. Under uniaxial compression this is normally the case, as $\epsilon_1 = \epsilon_2 \neq \epsilon_3$. However, as the critical points within the plate models did not, in general, appear to exist under a state of uniaxial compression (at the local point), the above strain equality may not be valid in the three-dimensional case. It was expected that this source of error might, in the limit, displace the curve of results by, very approximately, $\pm 15\%$ ⁵⁸.

- 2) The plate-models contained a particle array representing a face-centered cube (Chapter IV, Section IV - 2.4).

In practice, it is probable that spheres in edge contact will pack in a rhomboidal ($30^\circ - 60^\circ$) form^{46,60}. For spheres not in contact, it would appear that one type of packing is as likely as another. Although the magnitude of this source of error could not be evaluated, it was thought that relative displacement of the two curves of results (Fig. VIII - 1) would be greater at high volume fractions where edge contact between the spheres might occur.

- 3) A regular array of particles was used in the plate-models.

In practice there is no reason for the spherical particles to form a regular array, unless they are in contact. Thus any relative displacement of the two curves due to non-regularity of the aggregate packing was expected to be greatest at low volume fractions. This effect may diminish to some extent the effect of (2) above.

- 4) The results from the plate-models were derived from tests under low stresses (approx. 1000 p.s.i.).

The comparison of these results with the measured, concrete, fracture initiation results assumed that the ratios of the strains within a plate-model specimen would remain constant until fracture initiated. This assumption has already been discussed in Chapters V and VI and was thought to be of minor importance.

- 5) For the plate-models, fracture was observed to initiate in the mortar phase.

The analysis of the plate-models was concerned, therefore, solely with the mortar matrix. As previously mentioned, the use of different aggregates packed in a different configuration may well initiate fracture at the aggregate-mortar interface. As the concretes tested contained glass aggregates (instead of granite) which may have existed in other types of arrays (see 2 and 3 above), the two curves of results might be expected to have some relative displacement due to the initiation of fracture within different zones.

- 6) The plate-models gave results due solely to the first-order effects of aggregate packing.

As discussed in Chapter IV, Section IV - 2.4, the number of particles used in the models was the minimum necessary. Unfortunately, due to the effects of extensive shrinkage cracking in plate-models containing sixteen particles (Fig. IV -a), no information concerning the second order effects of the aggregate packing could be obtained.

However, it seemed fairly certain that such second-order effects would exist, and would, therefore, cause the two curves of results to have some relative displacement.

- 7) The analysis of the plate-models was concerned with the initiation of the first crack.

It was considered to be extremely unlikely that the volumetric strain plot technique was capable of detecting the first crack to form. Thus, in the glass marble concretes, the measured strains at which fracture was considered to have initiated were almost certainly higher than the average strains at which the first fracture initiated. In other words, the volumetric strain technique was only capable of detecting cracking when such cracking was far enough advanced to have a cumulative effect which was measurable in terms of total volumetric strain. It was considered that the analytical plate-model technique should, therefore, represent a lower bound limit to the value of the composite strain at which fracture would initiate.

- 8) The diameter of the coarse aggregate particles used in the glass-marble concretes was less than that used for the plate-model tests.

As the physical extent of the local strain field generated by a particle is a direct function of the particle size⁵⁹, the probability of finding a critical flaw or weakness within the local, intensified, strain field diminishes as the particle size decreases. Thus, the plate-model tests could again be expected to give lower results than the glass-marble concrete tests.

From examination of these sources of difference it was concluded that the plate-model tests could reasonably be expected to give results, in terms of the composite material strains necessary to cause fracture initiation, representing a lower bound.

Section VIII - 2Conclusions

The principal point of comparison between the results of the glass-marble concrete tests and those of the plate-model tests was that the trend of the strains necessary for fracture initiation, with increasing content of coarse aggregate, was closely similar for both series of tests. Quantitative differences in the absolute magnitude of the result derived from each series for a particular volume fraction of coarse aggregate were noted. It was thought that this difference in magnitude was not unreasonable when the basic differences between the two types of test were considered.

CHAPTER IX

C O N C L U S I O N S

Section IX - 1

Validity of Plate-Model Tests and Analyses

It must be emphasized that the scope of this investigation was limited, and therefore, that any conclusion drawn must be viewed within its narrow context, (Chapter IV). Furthermore, several assumptions concerning the material behaviour prior to the initiation of fracture had to be made in order to allow analysis of the models (Chapter V).

In Chapter VIII, the correlation between results from the plate-model tests and results from tests of the concrete prototype were discussed. From this discussion, two general conclusions concerning the validity of the plate-model testing technique emerged.

- 1) From the plate-model tests, it was concluded that the overall (composite material) strain, necessary to generate fracture initiation, decreased with increasing volume fraction of coarse aggregate (Chapter VI). It was further concluded that this effect was not explicable solely by consideration of the increased Young's Modulus of the mixture due to the inclusion of coarse aggregate particles, but was due to variation in the magnitudes of the local strains caused by the hard inclusions. This predicted trend, from the idealized models, was closely similar to the trend of the measured results from tests of the three-dimensional concrete prototype (Fig. VIII - 1). It was concluded, therefore, that the plate-model tests and analyses had given a valid indication of the variation in the overall composite material strain necessary to cause fracture initiation, due to an increasing content of coarse aggregate.
- 2) Although qualitative agreement between the predicted plate-model results and the measured concrete prototype results was noted, a significant quantitative difference was seen to exist (Chapter VIII). From discussion of the basic differences between the two types of

test, it was concluded that the plate-model results may well represent a "lower bound" solution, in terms of the absolute magnitude of the composite material strain required to cause fracture initiation, for a particular volume fraction of coarse aggregate. This was primarily due to the fact that the plate-models were analyzed to determine the strains at which the first fracture would form. For the concrete prototype tests, fracture initiation could only be detected when the cumulative effect of cracking was large enough to influence the volumetric strains of the concrete. Thus, the "fracture initiation" strains of the glass-marble concretes may, in fact, represent a more advanced stage in the fracture process (see also Section IX - 2).

Section IX - 2

Fracture Processes

In Chapter I, the fracture processes of concrete were broadly divided into three "zones or stages". It was pointed out that the work to be undertaken would be concerned primarily with the first of these stages, - the initiation of fracture at local, isolated, points within the material. The experimental techniques used (Chapter IV) were such that some qualitative information concerning the subsequent failure processes could also be obtained (Chapter VI). Several conclusions concerning the fracture processes were drawn from consideration of the plate-model test results.

- 1) The plate-model tests indicated, both from analysis and observation, that under uniaxial compression fracture initiated from within the mortar zone of the concrete rather than at the mortar-aggregate interface. It is again emphasized that this conclusion may only be valid for the particular aggregate packing and materials used (Chapter IV), and should not be taken as a general conclusion applicable to all concretes.
- 2) The position of the point from which fracture initiated was observed experimentally by two independent methods (Chapter IV). Photographic recording of the isochromatic patterns during loading (Figs. VI - 9 and VI - 45) and fluorescent dyeing techniques (Figs. VI - 46 to VI - 49)

were used. For models representing coarse aggregate contents greater than 5%, the analytically determined position of the fracture initiation point correlated well with the observed position, and was at the center of symmetry of the aggregate packing adopted (Fig. IV - 7).

- 3) It was noted that an initial isochromatic pattern existed prior to loading of the models. This pattern was attributed to the introduction of strains due to drying shrinkage of the models. It was further noted that initial cracking could be generated by those shrinkage strains due to the restraining effect of the aggregate inclusions (Figs. IV - 9, VI - 46 to VI - 49). It was observed that, for the particular particle configuration used, shrinkage cracks were generally closed upon the application of a uniaxial compression load to the model, since the cracked zones were subjected to biaxially compressive strain states. It was concluded that such cracks did not play a significant part in the fracture initiation process. Furthermore, observation of the final crack patterns exhibited by various specimens after failure (Figs. VI - 46 to VI - 49), showed no significant increase in the length of any pre-existing shrinkage cracking. It appeared, therefore, that such cracks did not contribute significantly to the final failure process.
- 4) Examination of the final crack configurations and of the photographic records of the isochromatic patterns, indicated that the general direction of the failure path was in the direction of the applied compressive load.
- 5) It was noted that the fracture path "divided" in the vicinity of the coarse aggregate particles and travelled around, rather than through, the hard strong particles (Figs. VI - 46 to VI - 49). The resultant fracture surface was observed to be "cone-like" in shape in the vicinity of the aggregate particles. This phenomenon has already been mentioned in Chapter I (Fig. I-1) and has been observed by several investigators^{1,5}. For the particular tests carried out, it appeared that these fracture "cones" were not formed until total disruption of the specimen was

almost complete.

- 6) From analysis of the plate-models (Chapter VI), it was concluded that an increase in the coarse aggregate content caused an increase in the "criticality" of the local strains. This increased "criticality" was due to variation in the nature of the local strain fields (i.e. the value of the ratio of the principal strains at a point) and in the magnitudes of the local strain concentrations. This increased, local, "criticality", caused the overall strains necessary to generate fracture initiation to decrease with increasing aggregate content. The correlation of this predicted effect with the effect measured from the concrete prototype tests has already been discussed in the previous Section.
- 7) Detailed examination of the final crack patterns (Figs. IV -22, VI -46 to VI - 49) indicated that the final "path" of failure was not composed of a single crack which had merely extended with increasing load. This final "path" appeared to be composed of a series of small, interconnected, individual cracks. Thus, the overall failure process appeared to consist of several stages. A primary fracture initiation (at the most critical point - Chapter V) was followed by a redistribution of the local strains. Fracture was then initiated at some other point. This process of multiple fracture initiation continued until some critical stage was reached, whereupon the isolated fractures became interconnected, and total disruption of the material followed. It must be emphasized that the quantitative analyses discussed in this thesis dealt only with the first fracture to initiate, and that this conclusion concerning the overall fracture process is merely qualitative, based upon experimental observation. No quantitative information on the mechanics of the eventual interconnection of the isolated fractures, or of the final total disruption of the material was obtained.

i) "Stages" of Failure

In Chapter I, a broad division of the failure processes into three "zones or stages" was given. These stages are generally considered⁵¹ to be "initiation", "propagation" and "total disruption". Although the plate-model tests were concerned primarily with the "initiation" stage, some qualitative information concerning the other stages was obtained. From this information it was concluded that stages consisting of "initiation", "multiplication of isolated fractures", "interconnection of isolated fractures" and "total disruption" might be more realistic.

ii) Griffith Theory

The extension of the original Griffith theory to include the case of a material under compression was discussed in Chapter II. It was noted that this theory was concerned solely with the initiation of fracture from an existing flaw, and that only for the special case of uniaxial tension did it (necessarily) have any validity in the context of total disruption of the material. The plate-model tests reinforced the view that fracture initiation and total failure are not coincident under compressive stress fields, and that the local strains were the factor governing the initiation of fracture. No evidence was obtained which altered the conclusion, reached in Chapter II, that the modified Griffith theory was theoretically capable of explaining the fracture initiation processes in concrete. However, the difficulties of defining the necessary material parameters and local stresses are considerable, and a great deal of further work is necessary before this theory may be used with any exactitude.

iii) Structural Models

Structural models, such as the Baker lattice⁵⁰, are useful tools for the general description of the overall load-bearing and deformational characteristics of concrete. As discussed in Chapter VI, the plate-model tests

showed that the "stiffness" assigned to any strut of a structural model cannot be considered a constant, but will vary with the modular ratio of the phases (aggregate and mortar) and with the interparticle spacing, the latter being a function of the volume fraction of coarse aggregate. Furthermore, the plate-model tests showed that the local strain fields in a concrete will be highly disturbed, due simply to the inter-action of the two major phases, aggregate and mortar, present. Thus, the use of structural models, in which stresses of constant magnitude are assumed to exist in each strut, to analyse the fracture processes in concrete, may only be considered to give quantitatively, an approximate solution.

Section IX - 4 Suggested Refinements to Techniques

It has not been the intention to suggest that any of the techniques outlined, either experimental or analytical, comprised the "ultimate" solution to each particular problem. Thus, all the techniques are open to improvement and refinement. However, there were two notable areas in which improvement to the techniques used could have resulted in major improvements in the quantitative results obtained.

- 1) The development of a more sensitive method for the detection of the onset of fracture in a material would allow the more accurate determination of a fracture initiation envelope. This envelope (Chapter III) was the basis for the determination and analysis of the "critical points" in the models (Chapters V and VI). Any improvement in the sensitivity of the derivation of this envelope would contribute greatly to the accuracy of the model results. In particular, tests of pure mortar specimens under triaxial stress states of "compression-tension = tension" or "tension-compression = compression" (Chapter V) would define the shape of the envelope between uniaxial compression and uniaxial tension more accurately than the empirical method used.

- 2) The strain measuring system used for the plate-model tests could be improved. As previously mentioned (Chapter IV) the use of Moiré grids in conjunction with photo-elastic coatings would allow point-to-point determination of the surface strains without the averaging effects due to the use of gauges of finite length. The experimental use of Moiré grids is developing rapidly and may soon provide an attractive alternative to the system used.

Moreover, it was recognized that several other refinements were desirable. The use of sixteen rather than four aggregate particles (Chapter IV) to include second order effects; the use of a smaller testing frame, more suited to the physical size of the plate-models; the development of a precise method for the centering and aligning of the specimens and loading apparatus, and the provision of some quantitative correlation between the photographed fracture events and the magnitude of the generated composite material strains, would all have helped to reduce the quantitative errors induced. It was particularly noted that the entire plate-model technique was an extremely involved and protracted one and, therefore, prone to an accumulation of errors due to the number of operations which were carried out. It was considered that any simplification of this process would, therefore, be beneficial.

Section IX - 5

Extension of Work

It was apparent that the use of the techniques outlined could be extended, virtually without limit, to investigate the effects of combinations of various other parameters. Some of the parameters which might be expected to have an effect are grouped into three broad classes as follows:

- 1) Variations in the state of stress imposed.

It was discussed in Chapters IV and V that the plate-models represented planar slices from the body of a concrete, the particular plane represented being defined by the two principal stresses applied to the

model. For all the tests described in this thesis, the state of stress used was that of uniaxial compression (σ_3 neg., $\sigma_1 = 0$). The testing frame used¹ was capable of applying biaxial compression (σ_1 neg., σ_3 neg.), uniaxial tension (σ_1 pos., $\sigma_3 = 0$), or biaxial compression-tension (σ_1 pos., σ_3 neg.). It should be noted that for the uniaxial compression tests described, the use of the fracture initiation envelope depended upon the non-dimensionalization of this envelope with respect to the strain necessary for fracture initiation under uniaxial compression (Chapter V). Tests carried out under other states of stress would require that the corresponding non-dimensionalizing factor be used.

2) Variations in the geometry of the aggregate packing.

It has been previously mentioned (Chapter VI) that very different results concerning the point of fracture initiation may have been obtained if some aggregate packing other than face-centered cubic (Chapter IV) had been used. The geometry of the system might also be altered to determine the effects of aggregate grading and of aggregate shape.

3) Variations in the material properties.

Here the combinations are almost endless. The mechanical and chemical properties of the aggregates, the mix parameters of the mortar matrix and the shape and surface texture of the aggregates might all be reasonably expected to exert an influence on the fracture processes.

It should be noted that further tests under different states of stress (notably tension), or conducted upon different model types, may show fracture initiation at the mortar-aggregate interface. In such a case, experimental data concerning the properties of such an interface would need to be obtained in order to determine the criterion governing local fracture initiation at the bond interface (Chapter III).

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