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**THE LARGE DEFLECTION BEHAVIOUR OF
STIFFENED DECKS UNDER LATERAL PATCH LOADING**

by

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*To my parents
and Theodora*

ABSTRACT

The large deflection behaviour of stiffened decks under lateral patch loading is studied by means of numerical and experimental methods. The behaviour of a bay forming part of a multi-bay assembly is investigated by studying two limiting cases; one bay fully restrained against rotation and in-plane movement at the supports, and another bay simply supported and free to draw-in.

Initially, the restrained deck is studied numerically and as a consequence the main actions are identified. The deck is then represented as a series of axially restrained beams subjected to two idealized loading patterns. First, the response of axially restrained beams on an elastic foundation to centrally applied loading is studied. A second analysis enables the prediction of the behaviour of axially restrained beams under symmetric distributed loading to be made. Comparisons between analytical and numerical results for both beam studies show very good agreement. As a result of these studies, simple bilinear mechanisms are proposed for use in the analysis of stiffened decks restrained against pull-in. Finally, the mathematical formulation of a proposed model for the restrained deck is presented, and its predictions again compare favourably with numerically generated results.

Next, the behaviour of three simply supported and free to draw-in decks is studied. For the first deck experimental evidence is used to validate the finite element modelling. Numerical results from all three decks are used to study the importance of membrane action and to propose a suitable model. Subsequently, the proposed model is presented analytically and its predictions show good correlation with the numerical results.

The analyses presented, can be used to calculate the ultimate limit state and energy absorption capacity of stiffened decks subjected to patch loading.

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NOTATION

A	cross-sectional area of a beam
a	distance of the spreading hinge tip from the support
a_0	value for a when $y_0(L)$ is a minimum
b	stiffener spacing
$b_{j,j+1}$	spacing between stiffeners j and j+1
B_1	coefficient defined by Eqn. (5.19)
B_2	coefficient defined by Eqn. (5.19)
C	coefficient used in Eqn. (4.7)
C_ℓ	$= \left(\frac{Zp}{AL} \right) (C_{0\ell}^s + (\lambda-1)C_{0\ell}^h) \quad , \quad \text{for } \ell = 2, 3, 4$
C_ℓ^*	coefficients C_ℓ modified according to Eqn. (4.10) ($\ell=1, 2, 3, 4$)
C_1	$= \left(1 + \frac{1}{\mu_1} \right) \frac{\sigma_y}{E}$
$C_{0\ell}$	coefficients used in Eqn. (4.3) ($\ell=2, 3, 4$)
$C_5(a)$	coefficient defined by Eqn. (C.3)
$\bar{C}_\ell$	coefficients defined by Eqn. (5.12) ($\ell=2, 3, 4$)
D	coefficient used in Eqn. (6.3)
$D_{j,j+1}$	coefficients used in Eqns (6.1) and (6.2)
D_3	coefficient used in Eqn. (8.3)
D_4	coefficient used in Eqn. (8.5)
E	elastic modulus
e	total extension of a beam centroidal axis
F	coefficient defined by Eqn. (5.8)
$f(x)$	mode-shape
$f'(x)$	$df(x)/dx$
H, h	depth of a rectangular cross-section
k	axial spring stiffness
k_0	initial stiffness at the centre of the beam for a given loading pattern
k_s	elastic foundation stiffness
L	half-span length of a beam/deck
ℓ	dummy index

M	bending moment
m	non-dimensionalized bending moment = M/M_0
M_0	plastic moment ($=\sigma_y Z_p$)
M_x	transverse bending stress resultant in the plate
M_{x_0}	M_x plastic value ($=\sigma_y t^2/4$)
M_y	longitudinal bending stress resultant in the plate
M_{y_0}	M_y plastic value ($=\sigma_y t^2/4$)
N	horizontal component of axial force
n	non-dimensionalized horizontal component of axial force = N/N_0
N_0	squash load ($=\sigma_y A$)
N_x	transverse membrane stress resultant in the plate
N_{x_0}	N_x plastic value ($=\sigma_y t$)
N_{xy}	membrane shear stress resultant in the plate
N_{xy_0}	N_{xy} plastic value ($=\sigma_y t/\sqrt{3}$)
N_y	longitudinal membrane stress resultant in the plate
N_{y_0}	N_y plastic value ($=\sigma_y t$)
P	concentrated or total patch load
P_0	limit load causing plastic collapse of a beam in bending
Q	shear force in a beam
q	characteristic distributed load intensity
$\bar{q}$	distributed load intensity
Q_0	Q plastic value ($=\sigma_y A/\sqrt{3}$)
q_0	characteristic distributed load intensity at limit load
R_f	reduction factor given by Eqn. (B.8)
s	arc length
t	plate thickness
x	beam longitudinal coordinate measured from support
x_γ	value of x , corresponding to the current γ value, defined in Section B.2
$\bar{y}$	distance from the transverse support as shown in Fig. 3.19(a)
$y(x)$	lateral deflection at x
$y_0(L)$	generalized deflection of segment following the mode-shape $f(x)$
$y_0(x)$	$= y_0(L) f(x)$
Z_p	plastic modulus
z	distance between the instantaneous centre of rotation and the centroidal axis

α_1	coefficient used in Eqn. (4.11)
α_2	coefficient used in Eqn. (C.5)
$\alpha_{j,j+1}$	angle of inclination for a transverse plate strip shown in Fig. 3.23
β	$= t/b$
β_1	patch load half-width non-dimensionalized with respect to the beam spacing
β_2	plate strip half-width non-dimensionalized with respect to the beam spacing
γ	scalar denoting the horizontal projection of the elastic segment of a beam on an elastic foundation, expressed as a fraction of L
γ_x	value of γ when the deflection at x ceases to increase, defined in Section B.2
Δ	elastic pull-in at the supports
$\bar{\delta}$	deflection at $x = a$
$\delta_{1,2}(\bar{y})$	differential deflection between stiffeners 1 and 2 at $\bar{y}$
δ_c	central deflection
δ_c^*	threshold central deflection for a beam on an elastic foundation
δ_{cx}	central deflection in a beam on an elastic foundation at which the deflection at x stops increasing
δ_j	central deflection for stiffener j
δ_{jcr}	central deflection for stiffener j when a plastic mechanism in stiffener $j+1$ is formed
ζ	scalar denoting the vertical projection of the elastic segment of a beam on an elastic foundation, expressed as a fraction of the central deflection δ_c
η_j	$= n_j/n_1$
θ	slope at supports
λ	denotes support conditions; $\lambda = 1, 2$ for pinned and clamped supports respectively
μ_1	$= \frac{k L}{E A}$
μ_2	scalar characteristic of the applied load pattern, used in Eqn. (5.1)
ρ	load factor ($=P/P_0$)
σ	stress
σ_y	yield stress
φ, ψ	defined in Fig. 6.5

Superscripts

e	elastic
h	hogging
i	denotes sign of the bending moment (i = s, h)
l	longitudinal
p	plastic
s	sagging
t	transverse

Subscript

c	central
cr	critical
f	flange
I	initial
j	index used to identify a particular beam
w	web

Abbreviations

BEF	Beam on an elastic foundation
FE	Finite element

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

Stiffened structures are widely used nowadays with a variety of applications in offshore, onshore, marine and aircraft structures. The effectiveness of the structure can be increased by rearranging the material available and using the concept of stiffening. This is also apparent in nature, in the corrugated or ribbed structure of some sea-shells and leaves for instance.

In man-made structures on the other hand, the stiffening will usually take the form of parallel and orthogonal stiffeners, attached to one side of a plate of uniform thickness. Very often, the spacing of the stiffeners will be quite different in the two orthogonal directions. In one direction relatively light stiffeners, the stringers or just stiffeners, will be spaced quite closely together, whereas in the other direction a much heavier stiffener, the cross-girder or web-frame, will be spaced more widely. The rectangular plate elements enclosed by these stiffeners are termed panels, and the group of adjacent panels enclosed by two orthogonal pairs of heavier stiffeners is referred to as a bay.

Extreme loads can occasionally be applied to stiffened structures by accident, such as impact loads. Under extreme loads structures experience large deformations which in many cases, particularly of metallic plated structures, give rise to significant membrane action. In such cases this membrane action may be the main contributor to the strength of plated structures at large deflections and cannot be neglected.

This dissertation contributes to the estimation of the ultimate strength of stiffened plating subjected to lateral patch static loading developing significant membrane action as they undergo large deflections.

1.2 SCOPE

The research presented in this thesis has two aims; to understand the large deflection response of stiffened decks under lateral patch load, with emphasis on the importance of the membrane action, and to propose simplified models to predict such behaviour.

To that end, the cases of single-bay decks fully restrained and unrestrained against in-plane movements at their supports are considered. These two cases bound the behaviour of an internal bay of a multi-bay assembly.

To gain insight into the behaviour, numerical results are generated through a finite element package and studied together with experimental results. By examining the observed behaviour, the decks are idealized as sets of parallel beams which form plastic mechanisms. The behaviour of such beam elements is studied analytically in detail and bilinear mechanisms are developed for use in a more composite model proposed to predict deck behaviour.

1.3 LAYOUT OF THE THESIS

In Chapter 2 the available literature concerning stiffened decks under lateral loading either alone or in combination with compression is reviewed. Literature on the behaviour of restrained beams is also studied in order to facilitate the development of analytical models for stiffened structures.

Chapter 3 investigates the behaviour of a fully restrained deck under patch loading. Numerical results are presented first and the main actions are identified. The deck is then idealized as a series of axially restrained beams of two types in terms of the applied loading pattern. The first type of beam (Type I) can be considered as a beam on an elastic foundation subjected to a centrally applied patch load, whereas the second type of beam (Type II) can be regarded as being subjected to a symmetric triangular loading distributed over the whole span. The former case is subsequently used to represent strips of stiffened deck under the patch load and the latter to represent strips of stiffened deck further removed from the patch load.

The behaviour of Type I beams, that is, axially restrained beams on an elastic foundation under centrally applied loading, is the subject of Chapter 4. An approximate bilinear model is proposed and its results are compared with

numerical results.

In Chapter 5 the behaviour of Type II beams, that is, axially restrained beams under symmetric distributed loading, is investigated. Initially, an extension of existing methods concerning uniform loading is made to cover beams under symmetric distributed loading. Subsequently, a bilinear model is proposed.

In Chapter 6 the restrained deck strip model identified in Chapter 3 is described analytically using the beam models developed in Chapters 4 and 5. The behaviour of the analytical model for the complete bay, is compared with numerical results for three different deck geometries.

Chapter 7 concerns the behaviour of unrestrained simply supported decks, beginning with a comparison between numerical and experimental results. These results, together with further numerical analyses, are used to identify the key parameters, which could usefully be incorporated in a simplified analytical model for unrestrained deck bays. Alternative methods to estimate the parameters involved in the model for other deck geometries are also discussed.

Chapter 8 proposes a simplified analytical model for the analysis of unrestrained deck. The mathematical formulation is followed by a parametric study investigating the sensitivity of the proposed model to variation in its controlling parameters. A comparison between the model and advanced elasto-plastic finite element results is also made.

Chapter 9 presents the conclusions drawn from the work and outlines areas for further research.

CHAPTER TWO

REVIEW OF THE LITERATURE

2.1 INTRODUCTION

The membrane action induced in plates and beams restrained against pull-in by changes in geometry, can increase their load carrying capacity to many times the limit load obtained in the presence of bending action alone. There is a fundamental difference, however, between plates and beams. The latter, require external axial restraints to develop membrane action when they experience large deflections. On the other hand, metal plates exhibit membrane action when they take on a non-developable shape, irrespective of the presence of any in-plane restraint provided at the boundary. This occurs because in the formation of the non-developable shape, some regions of the plate mid-surface are stretched at the expense of others that are compressed. The presence of any boundary restraint may, of course, make the membrane action more marked.

In this chapter, the literature on the large deflection behaviour of stiffened plates and restrained beams subjected to lateral loads, with or without in-plane loading, is reviewed.

2.2 UNSTIFFENED PLATES AND BEAMS UNDER LATERAL LOADING

2.2.1 Evolution of the elasto-plastic methods

The considerable reserve of strength which exists in plating loaded laterally beyond the elastic limit has been well understood since the beginning of this century, see Boobnov (1902) and Inglis (1925). Originally, interest was focused on slender structures, that is, structures with high span to depth ratios in the case of beams with rectangular cross-section, or high slendernesses in the case of plates. Specific attention was given to thin plates supported around their boundary by more rigid

elements such as stiffeners and restrained in their plane by the surrounding structure. Such structures exhibit some membrane behaviour while still elastic. Under increasing lateral load they are transformed into plastic membranes. Elastic deformations, however, may not be insignificant and an elasto-plastic analysis is often to be preferred to a rigid-plastic one.

The first attempt to exploit the post-elastic load carrying capacity of an initially flat clamped rectangular plate subjected to uniform lateral pressure, appears to be due to Hovgaard (1940), who produced empirical design curves. In the case of infinitely long plates, the one-dimensional nature of the problem considerably simplifies the analysis. A beam under Uniformly Distributed Loading (UDL) can be studied instead. It was for this type of plate that the first practically useful solutions were obtained by Clarkson (1956). These ideas were followed by Wah (1958) and Jaeger (1958) who produced similar types of analyses. Jaeger also indicated how a square plate could be treated as a long rectangular plate through modification of its thickness. With the exception of Jaeger's, the aforementioned studies extended the elastic large deflection analysis for long rectangular plates, for instance as described by Timoshenko & Woinowsky-Krieger (1959), up to the point where the central plastic hinge is formed.

Young (1959) presented an analysis which extended Clarkson's method for larger deflections up to an ultimate point where the long plate becomes a plastic membrane. The main contribution attributable to Young was the incorporation of the plastic extensions which take place at the plate plastic hinges, in addition to the extensions due to the elastic stretching. It can be said that Young presented the first complete treatment of long rectangular plates under distributed loading, by analysing and testing equivalent beams. Young's analysis can be considered as complete since it considered all three phases which a beam experiences up to and including the plastic membrane stage, with the incorporation of the plastic extensions that are very important during the third phase. These three phases are, the elastic non-linear range, which comes to an end with the formation of plastic hinges at the supports, a second phase that ends when an additional hinge is formed at the centre of the beam, and a third phase that ends once the beam becomes a plastic membrane. Young used two different analyses for the second and third phases. The absence of plastic extensions at the support hinges during the second phase causes a transitional jump in the load-deflection response, between the second and the third phase, in the solution presented by Young. It is worth mentioning that the more slender the plate, the more non-linear the first two phases become. For stockier plates or beams, on the other hand, the linear elastic response, governed by bending, is closely followed and plasticity reduces the

stiffness gradually.

For these stockier plates or beams, the extent of the second phase is much less than for more slender beams. The support and central hinges tend to occur simultaneously in the case of a very stocky structure subjected to symmetric loading. If such is the case, the first and third phases described above are those needed to describe the behaviour up to the stage the plate becomes a membrane. The linearity of the first phase, as a result of stockiness, makes the large deflection elastic treatment of the first phase unnecessary. In the third phase, elastic extensions may be only small fractions of their plastic counterparts, therefore they may be neglected. This last point justifies the evolution of rigid-plastic methods for the analysis of stocky beams rather than of generally slender plates. These rigid-plastic methods are discussed in the next section.

Although Young's analysis was, at that time, the best contribution to the analytical treatment of long rectangular plates, by considering slender beam behaviour, it was not relevant to plates with aspect ratios between one and two. Hook & Rawlings (1969a) however, provided experimental data for plates within that aspect ratio range.

Labib (1971) analysed clamped plate strips subjected to uniform pressure using dynamic relaxation. The analysis covered both elastic non-linear and elasto-plastic behaviour. Investigation into the beneficial effects of initial dishing and pre-stressing was also made. Campbell & Charlton (1973) considered a clamped beam under point loading in the elastic and elasto-plastic range. Backlund (1976) used Finite Elements (FE) in a flexibility approach to solve the elasto-plastic problem for beams and frames of any cross-section. Finally, Kamtekar (1981) extended much of this earlier work, in developing an elasto-plastic analysis of beams and plates using the collocation technique. Kamtekar's analysis also included imperfection effects, and the influence of movements at the supports.

2.2.2 Rigid-plastic methods

Following the early work on slender structures, attention was focused on stockier structures, for example rectangular beams with relatively low span to depth ratios. A stiffener and its associated plate in stiffened structures of typical geometries falls into this category, provided that the surrounding material still provides the axial restraint necessary for the development of membrane action. For such structures, the initial elastic deflections are reasonably small. Thus, very little membrane force

is built up in the elastic range. Subsequently, the extensions necessary for the build-up of the membrane forces take place mainly over the plastic segments of the structure. Therefore, a rigid-plastic analysis neglecting elastic bending and membrane effects may be used to represent the main characteristics of the behaviour. Its simplicity renders it an attractive option compared to other types of analysis.

In conventional small deflection analysis, equilibrium equations are developed for the original undeformed configuration of the structure. In large deflection analyses, geometric changes are taken into account in the equilibrium equations. As mentioned before, these geometric changes in axially restrained beams produce membrane forces which increase the load carrying capacity many times beyond the limit load.

The pioneer in the rigid-plastic analysis of axially restrained beams appears to be Haythornthwaite (1957), who analysed a rectangular beam subjected to a centrally applied point load. Haythornthwaite (1961) analysed a sandwich beam and a rectangular beam both subjected to uniformly distributed loading. The sandwich beam analysis took into account two distinct modes of deformation. During the first mode, the central plastic hinge at the limit load configuration splits into two hinges which travel outwards under increasing lateral deflection until they reach the third points of the span. At this stage the second mode begins. The travelling hinges split again and spread towards the supports and the middle, giving the intermediate plastic segments of the structure a parabolic shape. For the rectangular beam, Haythornthwaite assumed a single mode of deformation. During this mode, the central hinge, formed at the limit load, splits into two and spreads towards the supports under increasing lateral deflection. Small deflection assumptions are made by Haythornthwaite and the results are compared with experiments undertaken by Young (1959).

Martin (1962) analysed axially restrained grids under point loads. Martin also presents the analysis of beams under a number of evenly spaced lateral point loads. The method can be extended to predict the behaviour of beams under uniformly distributed loading by using a large number of point loads.

Gill (1973) followed Haythornthwaite (1961) in the two mode treatment for the sandwich beam, and analysed a rectangular beam under uniformly distributed loading having either rotationally restrained or unrestrained ends. Gill presumably thought that the two mode solution would improve the results for the rectangular section. The second mode, which is treated approximately, does not lead exactly

to pure membrane action at finite deflections. This problem, and the extent to which the two mode solution is an improvement on Haythornthwaite's single mode solution, for a rectangular beam under uniform loading, are further discussed in Chapter 5.

Gürkök & Hopkins (1973) presented the complete solution for a pin ended beam under uniformly distributed loading, using a Lagrangian formulation to monitor the changes in the geometry of the beam. This treatment allows all curvature changes along the curved segments to be measured and is the most rigorous analysis developed to date. Gürkök & Hopkins presented two solutions considering both small and large strain expressions in the strain-displacement relationship. Two modes similar to those considered by Gill were assumed. Although this analysis is very complex, it converges to the membrane solution at finite deflections, which both Haythornthwaite's and Gill's analyses are unable to predict. This solution shows that membrane yield occurs at about five times the limit load level, for the geometry considered.

Onat & Haythornthwaite (1956) appear to have been the first to incorporate the membrane action in the rigid-plastic analysis of plates. They considered the analysis of circular plates under lateral pressure loading. The geometry of the plate facilitated the incorporation of the membrane action in the plate very conveniently, with the deflected plate forming a shell of revolution.

Sawczuk (1964) presented a rigid-plastic analysis of polygonal plates under lateral pressure. The method is based on an admissible displacement field similar to that assumed in the yield line theory of plates, Johansen (1962). The energy dissipated along the yield lines is due to both bending and membrane action, while no energy is absorbed by the rigid parts of the plate contained within the yield lines. As an application, Sawczuk presented the analysis of a rectangular plate under uniform pressure with supports free to rotate but not allowed to draw-in. Sawczuk appears to be the first who presented a rigid-plastic analysis of unstiffened polygonal plates, with boundaries restrained against in-plane movements.

Jones & Walters (1971) developed Sawczuk's work and presented a rigid-plastic analysis of plates under static distributed loading. Jones (1971) presented an analysis of unstiffened plates under static and dynamic loading and extended the analysis to beams by considering them as long rectangular plates.

Dean (1975) presented a rigid-plastic analysis of plate panels subjected to various combinations of in-plane loading and boundary conditions. The analysis also

included lateral pressure loading and the effect of in-plane shear deformation along the yield lines.

In addition, some analyses are reviewed which can be classified as hybrid methods. They evolved from the necessity to incorporate pull-in at the supports. If an elastic spring law can be assigned to the support, then the methods can be said to be elasto-plastic. Jones (1973), Hodge (1974) and Ronalds (1985) have also considered the effects of partial restraint on beams under a centrally applied point load. Low (1981, 1982) analysed a beam loaded by a rigid circular indenter. Low considered the effects of partial axial restraint by incorporating a linear spring in the longitudinal direction at each support. Under increasing deflection the curved segment of the beam was assumed to follow the indenter's shape.

Finally, membrane action may also be present in concrete slabs, with the requirement that in-plane restraint exists at the plate boundary, see Park & Gamble (1980). It is of a different nature to that present in metallic plates and passes through two phases. The initial increase in strength is due to arching action (or jamming) of the concrete as the plate develops some out of plate deflection. After a fall off in strength to the yield line prediction, a second increase in strength occurs, due to the development of membrane action in the anchored reinforcement. This is not reviewed any further because of its lack of relevance to the problem under discussion.

2.2.3 Experimental work on beams under uniform loading

Few test results are available on the large deflection behaviour of rectangular beams under distributed loading. Young (1959) conducted a series of tests in which uniformly distributed loading was simulated by four point loads. The span to depth ratios were 48, 96 and 132. The results corresponding to the first of the geometries tested by Young can be compared with a rigid-plastic analysis, while the latter two geometries are slender enough to require predictions from an elasto-plastic analysis. Haythornthwaite (1961), found very good agreement between his rigid-plastic analysis and the results corresponding to the first of the geometries tested by Young.

Pang & Millar (1978) tested a rectangular aluminium beam with span to depth ratio of 16.7. The uniformly distributed loading was simulated by ten equal point loads. The results compare favourably with the rigid-plastic analyses by Haythornthwaite (1961), Gill (1973) and Jones & Walters (1971). It may be of

importance to note that the beam failed prematurely at the supports due to shear.

2.3 STIFFENED PLATING UNDER LATERAL LOADING

The collapse load of stiffened plates under patch or uniformly distributed loading is calculated by the use of the upper- and lower-bound theorems of plastic limit analysis. In such an analysis, the idealization of stiffened plating was found to be similar to the one used for elastic analysis of this type of structural form. Two main approaches, for this idealization, can be noted in the literature.

The first recognizes the discrete nature of the stiffening, and considers the deck as a grid of beams. Each beam is composed of the stiffener and the associated effective plate. For the latter the values suggested by Clarkson (1963) can be used. Heyman (1953), demonstrated the methods of plastic limit analysis leading to upper and lower bounds of the so called grillages. More recently, Grigorian (1972) and Chowdhury (1985) presented limit analyses for torsionless grillages.

In the second approach the deck is visualized as an orthotropic plate. This is achieved by smearing out the discrete properties of the stiffeners. In addition, the limit problem is treated along the lines of limit analysis of unstiffened plates, see Johansen (1962). Recently, Manolakos & Mamalis (1988) presented upper- and lower-bound solutions in which the torsional moments are also considered.

The approaches mentioned above disregard completely the presence of membrane action in the plate, as the deck experiences large deflections. In general they are more applicable to reinforced concrete rather than to metallic structures. In metal plates, membrane action is associated with geometry changes at large deflections.

There has been little progress in the design of laterally loaded plates towards the exploitation of the additional strength due to membrane action in the elasto-plastic range. Cox (1953) presented a method of analysis, based on the energy method, for stiffened decks under patch loading, which allowed for a moderate amount of inelastic action to occur. A simply supported uniaxially stiffened deck is viewed as an elastic plate strip spanning transversely across a limited number of stiffeners, while the stiffeners involved in the model are represented by bilinear springs. Each spring models the stiffener and an associated plate width which, for the geometries considered, was taken to be equal to the beam spacing. The transverse plate strip has the same thickness as the deck plate, but different effective widths are taken when calculating the flexural and membrane rigidities. This was necessary to

represent the experimental behaviour observed by Cox. Two cases were considered, one with the patch loading applied over a beam and the other with load applied to the plate midway between two beams. The method predicts the behaviour of the deck in the vicinity of the applied loading, from first yield of the beam close to or under the applied loading up to first yield of the next adjacent stiffener. Information is provided by the method about the load distribution (in the elasto-plastic range) between the two beams adjacent to the loading, and the maximum strains in the plate and the stiffener. In addition, the method predicts the total central deflection under the load and the permanent set on removal of load.

Cox also described the behaviour of tests in the plastic range up to collapse. For all the specimens, yielding of the central beams was followed by outward spreading of load as evidenced by the increased deflections of the beams adjacent to the central beams. The difference between deflection under load and permanent set became greater as the loads increased. The membrane stresses in the plate were tensile in the centre of the plate and compressive near the supports. These stresses are balanced if they are integrated along a centre line, as no in-plane loading is applied to the plate externally. The effective tensile width of plate in the transverse direction appeared to be approximately equal to the spacing of the beams. After yielding of the centre beams occurred, the membrane tensile strains rapidly increased and they became large in comparison with the flexural component of strain, thus indicating the importance of membrane action in helping to distribute load outward from the centre beams.

Each specimen was loaded until a maximum load was attained. This load was between 10 and 12 times the load causing first yield, which occurred either in the plate or in the beam depending on the position of the applied loading. Thus, application of the load midway between two beams caused first yield to occur in the plate. When the load was applied at midspan directly over a centre beam, yielding first began in the bottom fibres of the beam under the load. The specimens failed in a secondary manner by buckling of the plate over the end supports between the centre beams. The strains in the plate never became very large. On the contrary, beam strains attained very high values. When the load applied to a specimen was above about five times the load causing first yield, the large deformations had a drastic influence on the cross-sectional shape of the structure. Hence, the specimen began to behave somewhat as one simple beam whose moment of inertia continually changed with increased deflections. The neutral axis of the entire system rose above the top flanges of the centre beams, as was evidenced by the fact that the upper flanges of the centre beams were in

tension rather than compression for very large loads. The stiffening effect of the change in geometry explains why the maximum load taken by each specimen was considerably greater than the sum of the loads that would be required to produce a fully plastic moment in each of the beams made up from a stiffener and a fully effective plate.

Dowling (1968) tested a single-bay longitudinally stiffened deck under lateral patch loading positioned midway between the central stiffeners. The deck was supported at the four corners. Two very strong girders were used along the longitudinal edges, while for the transverse ends, cross-girders were used. Yield was initiated in the deck plate under the wheel loading. The bottom flanges of the loaded stiffeners yielded next. Following the collapse of the loaded stiffeners, the applied load was carried to the adjacent stiffeners by a combination of transverse bending and membrane action in the plate. A width of plate of the order of two to three times the spacing of the stringers was estimated to be under transverse membrane tension. These tensile membrane forces were balanced by compressive forces in the plate over the cross-girders. Buckling of the plate over the cross-girder was followed by the collapse of the entire deck due to the formation of plastic hinges at the centres of the cross-girders. The collapse load was approximately 10 times the load at first yield. It was the analysis of these experimental results that prompted the investigations contained in this thesis.

2.4 STIFFENED PLATING UNDER COMBINED LATERAL AND IN-PLANE LOADING

2.4.1 Combined compression and lateral uniform loading

The effect of lateral pressure on the ultimate strength of stiffened plates under combined compression and lateral pressure loading is distinctively marked between single- and multi-bay stiffened plates. In single-bay models the presence of lateral pressure of any magnitude generally reduces the ultimate compressive strength obtained in the presence of compression alone, see Webb & Dowling (1980) and Ostapenko & Lee (1960). In multi-bay models, the presence of a moderate lateral pressure may improve the compressive strength obtained in the absence of the lateral load, while at some higher lateral pressure a reduction in the compressive strength has been observed, Smith (1975). The increase in the compressive strength, in the presence of moderate pressure, is achieved as the application of the latter load component suppresses the buckling mode developed under the

action of compression alone. Pure compression acting alone causes bays to buckle up and down alternately, while lateral pressure applied alone imposes a single bow in all the bays in the direction of the pressure.

Dean (1975) tested three stiffened models representative of ship's bottom plating subjected to combined compression and lateral pressure. Each of the models had three bays with the plate in the end bays reinforced in order for the failure to occur in the centre bay. Stocky stiffener properties were used so that they failed in a stocky column mode. The modes of failure observed in the tests were used to calculate interaction curves between the applied load components. For this, the interaction between longitudinal forces and bending moments was used. Good agreement was found between the load component interaction curves produced and the test results (see also Dean & Dowling (1977)).

Carlsen (1978) compared numerical and analytical predictions of the collapse load of stiffeners subjected to compression and hydrostatic lateral load. The analysis presented gives conservative results with respect to the numerical predictions. In general two modes should be tried, one with pure compression (mode I) and the other with simultaneous action of both load components (mode II). The lower of the two strength predictions should be used. It was found that under dominating compression the lateral load can be neglected. This analysis is proposed by Carlsen (1977) for the prediction of the ultimate load in stiffened flanges subjected to combined compression and hydrostatic loading.

Webb & Dowling (1980) studied the behaviour of stiffened plates both experimentally and analytically. As an analytical tool, the theory of discretely stiffened plates was used. The governing large deflection equations were expressed in their finite difference forms and a dynamic relaxation numerical technique was employed to solve the equations. Plasticity in the plate was treated by a single-layer yield criterion, while for the stiffener a multi-layer approach was used. The bulk of the parametric study was devoted to the behaviour of a single-bay stiffened plate under in-plane loading. Two parameters were varied, that is, plate and stiffener slenderness ratios, and only flat bar stiffeners were considered.

From the main study one stiffened plate with plate slenderness, b/t , of 60 and stiffener slenderness, $2L/r$, of 30 was re-analysed under the combined action of lateral and in-plane loading. The lateral load was applied first, followed by increasing in-plane loading to failure. It was concluded that, for the single-bay studied, application of the lateral load leads to a reduction in the in-plane

capacity in proportion to the lateral load applied. This is because application of the lateral load does not alter the mode of deformation of the primary supporting elements, that is the stiffeners. In addition the mode of plate deformation resulting from compressive forces alone can be considerably modified by the application of lateral pressure.

The behaviour of stiffened flanges under combined loading was also investigated by Kristek et al (1982). These authors used large deflection folded plate theory. The parameter investigated was the stiffener configuration. It was found that closed-section stiffeners are beneficial to the post-buckling behaviour of the stiffened flange. This is due to the additional bending restraint offered to the unsupported width of panel by the enhanced torsional rigidity of this type of stiffener.

2.4.2 Combined compression and lateral patch loading

Wheel loading of the usual magnitude specified in codes of practice for bridge deck design such as BS 5400 (Part 2, 1978), has little adverse effect on the ultimate in-plane strength obtained under the sole action of compression. This is corroborated by tests undertaken by Chan et al (1977) on 1/4 scale models of stiffened steel decks representative of those met in bridges. BS 5400 (Part 3, 1982), states that the strength obtained by the Perry-Robertson formula, adopted in the code when compression acts alone, can be also used for the case of compression acting in combination with wheel loading. This is supported by the fact that the column approach ignores any redistribution of applied compression. Such redistribution may be considered by analysing the deck as an orthotropic deck. A rational approach for the case of combined loading that is consistent with the column model is not as yet available.

2.5 CONCLUDING REMARKS

This chapter has presented a review of the literature on beams and plates both stiffened and unstiffened developing membrane action at large deflections under lateral loading, or combined with compression in the case of stiffened plates.

First, the elasto-plastic methods for the analysis of unstiffened plates and slender beams under lateral load was reviewed. For the stockier beams, such as the stiffener and associated effective plating in a stiffened structure, rigid-plastic

methods are more readily applicable. Such methods are well developed and if necessary, can include the effects of elastic stretching and pull-in at the supports. The validity of the rigid-plastic analysis for beams is corroborated by available experimental evidence. Beam models following rigid-plastic material laws can be used as components in the analysis of more composite structures such as the stiffened plating.

Next, approaches based on limit analysis, as they have been used for the stiffened plating under lateral loads, were presented. The methods currently available appear to disregard completely the membrane action in the plate which exists at large deflections. Little progress has been made towards incorporating this membrane action in the analysis of plates, both stiffened and unstiffened. Membrane action has been included in the cases of unstiffened polygonal plates restrained against pull-in at their supports, and circular plates free to draw-in. Also for stiffened plating under patch loading, membrane action has been incorporated in the analysis concerning the very early stages of the elasto-plastic response.

Lateral distributed load on stiffened plating, such as bridge or ship decks, combined with predominant compression, has a marked effect on single- and multi-bay decks. However, localized loads like those applied by the wheels of vehicles, usually have a minor effect on the compressive strength of such decks.

A rigid-plastic analysis of stiffened decks subjected to predominant patch lateral load is presented in this thesis. The analysis includes the membrane action which is developed in the plate at large deflections. In this context, two analytical beam models are also presented. One generalizes previous work on axially restrained beams subjected to distributed loading and the other concerns elasto-plastic beams on an elastic foundation.

CHAPTER THREE

BEHAVIOUR OF STIFFENED DECKS RESTRAINED AGAINST PULL-IN

3.1 INTRODUCTION

The behaviour of stiffened decks which are fully restrained against pull-in in their plane, under concentrated loading, is investigated using an existing finite element package in this chapter. Membrane stresses are built up rapidly with increasing plate deflections, and membrane behaviour quickly prevails. This behaviour is used as a basis for a proposed model that treats the stiffened plate as a series of beams loaded with the vertical components of the transverse membrane forces and connected transversely by plate strips.

3.2 MODEL GEOMETRY

An eight stiffener single bay deck was analysed. Due to the presence of double symmetry on plan, only one quarter of the deck, as shown in Fig. 3.1(a) was modelled by non-linear finite elements.

The boundary conditions along all supported edges prevented any translation or rotation. Stocky stiffener proportions were employed in order to minimize reductions in strength due to local buckling effects occurring in practice. The stiffener cross-section is shown in Fig. 3.1(b). Details of the particular geometry examined in this chapter are given in Table 3.1.

3.3 NUMERICAL ANALYSIS

3.3.1 Description of finite element program

The numerical analysis was undertaken using the finite element program FINAS originally developed by Trueb (1983). The program is particularly suited to structural stability problems and has been extensively used for the modelling of structural components or their assemblages encountered in bridge or offshore structures. A variety of element modules are available in the package library, but only the isoparametric shell, isoparametric beam and bond-slip elements have been used.

The elements forming the structure can be divided into different element groups, each containing any number of elements of the same type. There is no limitation on the number of elements used or on the number of geometric and material properties used within an element. A variety of material laws such as linear elastic, elasto-plastic based on Ivanov's yield criterion, concrete material behaviour including cracking and crushing and the elasto-plastic multi-layer technique can be used.

The element stiffness matrices are generated by Gauss numerical integration with allowance for up to 10 integration stations in any one direction. The set of the resulting simultaneous equations is solved using Gauss elimination and it is not restricted to positive definite matrices. In this way, load-deflection paths can be followed into the unloading range.

Storage requirements are reduced by storing only the non-zero coefficients of the upper triangle in the stiffness matrix, while solution time is optimized by node re-numbering. In addition, the stiffness matrix is automatically subdivided into blocks that are stored on mass storage devices. Only the two blocks currently reduced are stored in-core. Therefore, the maximum problem which can be solved depends only on the out-of-core storage available.

Different solution strategies are available for tracing the equilibrium path which could be non-linear. Such non-linear techniques are based on the Newton-Raphson iterative technique and differ in the stage at which the stiffness matrix is reformed. No single strategy is suitable for the whole non-linear path and convergence difficulties are overcome by switching to a strategy more appropriate to the type of non-linearity, with simultaneous subdivision of the applied step size.

Switch back to the initial minimum solution strategy, specified by the user, takes place once the difficulty has been overcome.

Both load and displacement increments can be applied to the structure with the latter being necessary to trace unloading paths. The size of each increment may either be specified by the user or dynamically controlled by the program. Crossing of limit points requires a constraint equation to be satisfied in addition to the equilibrium equations. Three types of constraint equations are available in FINAS such as, displacement control, automatic selection of the constraint equation (displacement) and the constant arc-length method.

3.3.2 Description of the elements used

From the existing element library three elements, all of the isoparametric type were used, in this thesis. They include, the doubly curved 8-noded shell element, the 3-D beam element and the bond-slip element. The first two elements were developed by Bates (1987), while the third was developed by Elnashai (1984).

3.3.2.1 Shell element

The shell element was used in modelling the deck plate of the stiffened decks studied. It is a doubly curved 8-noded isoparametric element with six degrees of freedom per node. The element formulation is based on Mindlin's theory which assumes zero direct stress in the thickness direction and uniform transverse shear stress distribution through the thickness.

Shear locking is eliminated by preventing the transverse shear stiffness from exceeding a certain threshold value. Plasticity is based on the von Mises yield criterion and the Prandtl-Reuss flow rule with linear isotropic hardening. Large displacement and rotation effects are included through the coupling of membrane and bending terms. A constraint equation is used to express the incremental out-of-plane rotation as a function of the rotation of the mid-surface obtained in terms of the mid-surface displacements.

3.3.2.2 Beam element

This element was used either alone for the beam analyses undertaken or jointly with the shell element to model the stiffeners of the decks analysed. It is a 3-noded 3-D space beam suitable for large deflection elasto-plastic analysis. The kinematic model is based on a unified theory which combines Mindlin type models for transverse shear deformation with classical thin-walled beam theory. The element has seven degrees of freedom, that is, three translations, three rotations and a warping degree of freedom. As the kinematic models for the 8-noded shell described above and the 3-noded beam elements are compatible they are well suited for the analysis of stiffened plates or shells. In this way the modelling is very economical as the beam element shares the same nodes with the shell element. Only phenomena that cause distortion of the stiffener's initial cross-sectional geometry cannot be modelled.

The cross-section of the beam element is divided into straight segments each of uniform thickness. In such a way any open cross-section composed of up to 20 segments can be modelled. The cross-section of each segment is subdivided into stations through the thickness and along the length in order to improve modelling of plasticity based on the von Mises yield criterion with linear isotropic hardening. Geometric non-linearity is accurately predicted even under a combination of large rotations and translations. This is achieved with a formulation that allows third- as well as second-order terms to be included.

3.3.2.3 The bond-slip element

This bond-slip element due to Elnashai (1984) was originally used as a contact element between concrete and steel. Subsequently, it was modified by Burgan (1987) to a 2-D global spring element. The element can be assigned any non-linear law. In the present thesis, it was mainly used as a linear spring to represent the surrounding structure in the analysis of beams on an elastic foundations.

3.3.3 Mesh convergence study

The mesh chosen for the finite element analysis is shown in Fig. 3.2. Five 8-noded shell elements were used along the length and ten across the width of the plate. For the stiffeners, the 3-noded 3-D beam element was used. The two

elements are compatible as stated in Section 3.3.2.2. The 'two elements per buckling half-wave length' rule was used as a guide in the choice of the mesh in the transverse direction. This rule is frequently applicable when buckling of the plate elements is possible, although in this case buckling of the plate was not anticipated. In the longitudinal direction the first and the last elements were made shorter than the other elements, their length being equal to $0.1L$ in anticipation of plastic hinges forming at those positions.

As described in the following section, in the large deflection regime the plate forms kinks along stiffener lines as shown in Fig. 3.4. In investigating the significance of these localized curvatures, an alternative mesh, with four shell elements in the transverse direction between stiffeners and six elements along the length, was tried. The shell elements on either side of the stiffener were approximately $0.14b$ wide. Throughout the numerical analysis undertaken in this thesis the number of layers in the shell element was kept at six. As the differences in the load carrying capacity at any given central deflection of stiffener 1 in Fig. 3.1(a) were within 0.5%, the accuracy of the mesh in Fig. 3.2 for investigating the behaviour in the large deflection regime was established.

3.4 LOADING

The loading is applied over the two central stiffeners as a pair of patch line loads. Figure 3.1(a) shows the line load on the quarter of the deck analysed with the finite elements. Each of these loads has a total length of $1.26b$, where b is the spacing of the stiffeners. This spacing is uniform for all the decks examined in this thesis.

The load factor, ρ , that appears in Chapters 3, 4 and 6 expresses the applied load on a stiffener non-dimensionalized with respect to the limit load, P_0 , causing a mechanism in bending of an encastré beam. This beam cross-section comprises a stiffener and the associated fully effective deck plate. Subsequently, a beam with such a cross-section is referenced as 'notional beam'. Following the definition above, the total load carried at any instant by the deck is equal to 2ρ , since two line loads are applied on the deck.

3.5 OBSERVED BEHAVIOUR

The central deflections of the various stiffeners are plotted in Fig. 3.3 as functions of the applied load. Stiffener numbering increases by moving away from the longitudinal centre line towards the support, see Fig. 3.1(a). The load is non-dimensionalized with respect to the limit load – obtained assuming a plastic mechanism in bending – of the loaded stiffener plus its associated effective plating. The latter was taken to have a width equal to the beam spacing. The load versus stiffener 1 deflection curve is a continuously softening curve, shown in Fig. 3.3. Positions where it changes slope correspond to the incipient collapse of subsequent stiffeners 2, 3 and 4. Figure 3.4 illustrates the deformation of the deck in a schematic manner. Dashed lines depict the deck in the undeflected state. Stiffener positions are easily traced, as they cross the transverse central line at positions where the latter shows discontinuity in slope. The forces applied to the stiffener by the stretched plate are also shown.

3.5.1 Stiffener deflections

In Fig. 3.5, the central deflection of each stiffener is plotted versus the deflections of the adjacent one. Referring to Fig. 3.5(a) which shows the deformation histories of stiffeners 1 and 2, a slope of 1:2 is noticeable over the first part of this history. This indicates that when stiffeners 1 and 2 experience the slope 1:2, their incremental deflections lie on a straight line which pivots about stiffener 3 for as long as this ratio holds. Thereafter, a ratio of 2:3 is observed which signifies that stiffeners 1 and 2 pivot about stiffener 4, with corresponding incremental deflections in that ratio. This is further discussed, in Section 6.2.3.3.

Figure 3.5(b) presents the deflections of stiffeners 2 and 3. The slope 1:2 signifies that the incremental deflections of stiffeners 2 and 3 lie on a straight line which pivots with respect to stiffener 4.

3.5.2 Stress resultants in the plate

For a better understanding of the plate behaviour the stress resultants in the plate without the stiffeners were plotted (shown in figures 3.6 to 3.8, 3.10, 3.12 and 3.13) and the observed behaviour is discussed in the following sections. The general convention followed in the plots for the stress resultants produced, is that continuous lines are used for positive quantities and dashed lines for negative

values. In the plots of membrane stress resultants, tension is considered to be positive. Positive bending moment gives tension in the top plate fibres, and thus hogging moments are considered to be positive. All the plate stress resultants with their positive values are shown on Fig. 3.1(a). The total height of the box surrounding each plot is twice the scale value, where this is defined in each diagram. The plots are based on the value of the various quantities at Gauss point positions connected with straight segments, that is, no stress smoothing process was used. As the 2 by 2 integration rule was used in the shell elements, stiffeners are located after every fourth plotted line across the width. Stiffener positions are marked along the transverse central line. The stress resultants at different load steps are presented in the following sections, aiming to show the main trends in the behaviour. It should again be noted that, the values are plotted at the Gauss points which are located within the elements, and not at the nodes which are located at the element boundaries, as shown on Fig. 3.2.

3.5.2.1 Transverse membrane forces

The build-up of the transverse membrane tension N_x is shown in Figs 3.6(a) and 3.6(b), where N_{x_0} is the full plastic value of the stress resultant. Attention is drawn to the distribution of the transverse stresses along the longitudinal centre line. This distribution is the sum of two components, one consisting of tension in the middle of the plate and compression near the supports, and the other being uniform tension along the longitudinal centre line. The first component stems from the plate deflecting in a non-developable shape, which results in tensile radial stresses at the centre and compressive hoop stresses around the perimeter of the plate. The second component is due to overall stretching of the plate as it behaves like a membrane anchored at the boundary. The transverse edge tries to shorten as the plate deflects into a non-developable surface. The restraint of movements along the transverse edge is responsible for inducing an additional transverse tension along this support, as it prevents the plate from contraction.

3.5.2.2 Longitudinal membrane forces

Distributions of the longitudinal membrane forces in the plate are plotted at two load steps. One, early in the loading history, is shown in Fig. 3.7(a), while the second is at a load stage where all but the outmost stiffener have formed plastic mechanisms, Fig. 3.7(b). The scale used is the N_{y_0} value, which is equal to the stress resultant fully plastic value per unit plate width.

Figure 3.7(a) corresponds to the early bending-governed behaviour in the longitudinal direction with the plate acting as the top flange of the stiffener. In Fig. 3.7(b) the presence of stretching in the longitudinal direction, causing an overall tension, is evident.

3.5.2.3 Membrane shear forces

Membrane shear distributions are given in Figs 3.8, with N_{xy_0} being the stress resultant plastic value per unit plate width. As Fig. 3.8(a) shows, shear is mainly developed in the panel between stiffeners 1 and 2 at this early stage. Only stringer 1 experiences considerable deflection and the shearing is due to large deflection strains. The membrane shear pattern when more stringers experience substantial deflections is given in Fig. 3.8(b).

Shears in the panel between stiffeners 1 and 2 are shown schematically in Fig. 3.9(a). Their reactions cause an axial force distribution on stringer 1 of the type shown in Fig. 3.9(b). These membrane shears play an important role as they increase the stiffener axial force in the middle of the deck.

3.5.2.4 Transverse bending moments

Typical plots of transverse moments are shown in Fig. 3.10 where M_{x_0} is the plastic moment per unit plate width. At low loads the plate bends with a significant hogging moment at the location of stiffener 2 and a sagging moment at stringer 1, see Fig. 3.10(a). The load transfer between stiffeners occurs through transverse shear action in the plate. At a later stage, Fig. 3.10(b), the membrane action is highly developed and the bending contribution diminishes where the membrane forces are close to yield. Significant bending is still present near the outer stiffeners however, where the membrane stresses are less than yield.

An interesting feature is the scalloped shape of the bending moment distribution, which is very pronounced in Fig. 3.10(b). This is due to a $P-\delta$ effect in the plate. In Fig. 3.11(b) M_{x_j} and $M_{x_{j+1}}$ are the transverse bending moments in the plate at stiffener j and $j+1$ positions. As no load is applied externally to the plate between these two stiffeners, the bending moment between stiffeners j and $j+1$ should vary linearly. The plate is also under transverse membrane forces and its shape is locally curved between stiffeners j and $j+1$, as indicated in Fig. 3.11(a).

This results in a $P-\delta$ moment component between stiffener positions and the bending moment variation becomes curved as shown in Fig. 3.11(b). This $P-\delta$ effect is of a stabilizing nature as the tension applied to the ends of a curved plate segment tries to straighten it out.

3.5.2.5 Longitudinal bending moments

Longitudinal bending moment plots are given in Fig. 3.12. The scale value My_0 equals the plastic moment per unit width. The moments are generally distributed in a similar way to transverse bending moments.

3.5.2.6 Transverse shears

Finally, the pattern of transverse shears existing at a rather early stage is plotted in Fig. 3.13. It can be seen that the transverse shear relieves stiffener 1 and loads stiffener 2. When the membrane stresses are fully developed the contribution of transverse shears in transferring the load from stiffener to stiffener diminishes. In these later stages the load is mainly transferred via the vertical component of the transverse membrane force.

3.5.3 Stress resultants in the beams

The structure is now visualized as a series of longitudinal beams, each comprising a stiffener and the section of plate between the centre lines of the two adjacent plate panels. Reference to these beams is subsequently made by the term 'notional beams'. Stress resultants along these beams are presented in the next sections. The same sign convention as for the plate stress resultants is followed in these plots.

3.5.3.1 Axial forces

Axial force distributions along the 'notional beams' are shown in Figs 3.14(a) and 3.14(b). The scale value N_0 equals the squash load of the 'notional beam' cross-section. The first of the figures corresponds to an early stage in the load history. Stiffener 1 is shown loaded partially in tension and compression. This is due to the presence of shearing stresses in the plate, developed due to the larger deflection of stiffener 1 relative to stiffener 2. This behaviour was explained in

Section 3.5.2.3.

In Fig. 3.14(b) well-developed membrane action is seen, corresponding to a stage when all the stiffeners have developed plastic mechanisms. It is worth noting the non-uniform tension along the 'notional beams'. Axial displacements of the 'notional beams' are resisted not only by the supports but also by the transverse continuity of the plate. Any differential extension between the stiffeners, mainly due to differential central deflections, causes shearing in the plate and ultimately alters the axial force distribution along the stiffeners.

3.5.3.2 Bending moments

Bending moments along the 'notional beams' are presented in Figs 3.15(a) and 3.15(b), with M_0 being the plastic moment for the nominal beam section. Figure 3.15(a) corresponds to the early bending-governed phase in the plate-stiffener units. In Fig. 3.15(b) the bending moments in the presence of the axial forces shown in Fig. 3.14(b), are given. The reduction in the sagging moments near the centre of stiffeners 1 and 2 is in agreement with the interaction between the flexural and axial stress resultants for the 'notional beam' cross-section. This interaction is given in a non-dimensionalized form in Fig. 3.16. It is easy to verify that the axial forces and the bending moments in Figs 3.14 and 3.15 follow this interaction curve. Identical observations can be made for the hogging moments of the inner stiffeners.

It is worth noting the shape of the curve connecting the centre and the support moments of the stiffeners in Fig. 3.15(b). Along stiffener 1 the curve is convex, indicating the presence of an upwards distributed loading. This opposes the externally applied load, as seen in Fig. 3.17(a). In contrast, a downwards distributed loading is indicated on stiffener 2 and subsequent stiffeners, of the type drawn in Fig. 3.17(a). Bending moments and shear forces on a simply supported beam produced by the loading in Fig. 3.17(a) are illustrated in Figs. 3.17(b) and 3.17(c) respectively. The direction of loading on the stiffeners is corroborated by the shape of shear force distributions along the 'notional beams', as plotted in Fig. 3.18. The curved distribution of shear along stiffeners 2 to 4 shows that the net load applied along the stiffeners has a higher magnitude near the centre of the beams. This behaviour is very similar to the purely elastic behaviour of stiffened plates, as described by Scordelis & Meyer (1969) in the case of multi-cell bridge structures. The only difference is in the source of the distributed forces. In the elastic range the deflections are usually small and transverse shears

produce the distributed force. However, in a fully developed membrane state the vertical component of the membrane forces provides most of the loading, caused by the existence of differential deflections between the stiffeners. In general, the loading on the stiffeners is the sum of these two components.

3.5.4 Discussion of the observed behaviour

A fully restrained longitudinally stiffened deck of single bay appears to develop membrane action from the early stages of the loading history. The magnitude of the transverse membrane stress reaches the yield value in the middle region of the deck when the central deflection is about three times the plate thickness.

In the longitudinal direction, the plate behaves as the upper flange of the longitudinal stiffeners. If longitudinal beams are adopted to help visualize the structural response then their loading, and the behaviour of the 'notional beams' under this loading, is of interest. They appear to be subjected to distributed loading, as shown in Fig. 3.17(a). The distributed loading is initially due to transverse shear but at moderate deflections the vertical component of the transverse membrane force becomes dominant.

Based on the observed behaviour a new model is proposed in the following sections of this chapter.

3.6 PROPOSED SIMPLE MODEL FOR FULLY RESTRAINED LONGITUDINALLY STIFFENED DECK

A model that incorporates the main actions mentioned briefly in the previous section could treat the stiffened deck as a series of longitudinal beams which are fully clamped at their ends. Each beam comprises the stiffener and its effective plate. These beams are connected along their length by transverse plate strips, that is the plate longitudinal bending and extensional rigidities are neglected. As these plate strips are independent from one another, any shearing and torsion action which exists in the deck plate, is also neglected.

Under the action of lateral loading each of the longitudinal beams forms a plastic mechanism with the same mode. This is the characteristic three-hinged mechanism, occurring at limit load. With further loading, axial forces are built up in the beams due to changes in geometry and the extensions associated with these changes. The load on each beam is then carried by a combination of axial force and bending moment. Each of these longitudinal beams is loaded by the externally applied load and the reactions of the plate strips.

In the following sections all the assumptions made in developing the model are checked in turn, differences from the actual behaviour are discussed and further assumptions are made, where appropriate.

3.6.1 Behaviour of the plate between stiffeners

A strip of the deck plate spanning across three panels is shown in Fig. 3.19(a). If this strip is isolated from the rest of the deck plate by two transverse cuts, a model which expresses its behaviour is that shown in Fig. 3.19(b). The end springs in this figure model the axial rigidity of the adjacent panels plus some shearing restraint of the extension of this strip in the panels beyond stiffener 2, see Fig. 3.19(a).

Attention is drawn to the similarity of the systems in Figs 3.19(b) and 3.19(c). Under rigid-plastic assumptions they are identical, as they have the same compatibility and equilibrium equations. Because the rigid-plastic analysis commences at the limit load, the stress resultants at any point along the middle segment of the system in Fig. 3.19(b) are on the yield surface right from the beginning. As this middle segment experiences no changes in rotation, normality postulates that it experiences no changes in extension either. Therefore, the

contribution of the middle segment to the compatibility equation is zero and the system is identical to the one in Fig. 3.19(c).

If the end springs, in Fig. 3.19(c), are assumed to be stiff enough, such that pull-in is not allowed at the supports, the behaviour of such a strip can be found in the literature. Haythornthwaite (1957) and Campbell & Charlton (1973) undertook tests of axially fixed encastré beams of rectangular cross-section under a centrally applied loading. Load-deflection results obtained by Haythornthwaite (1957) are presented in Fig. 3.20(a). Both the applied load and the central deflection are presented non-dimensionalized with respect to the limit load and the beam depth, respectively. The membrane solution is also indicated in Fig. 3.20(a) by a dashed line. Test results corresponding to the span to depth ratio of 64, due to Campbell & Charlton (1973), are indicated by the small circles in Fig. 3.20(b). Comparison between the test results and the membrane solution shows that the latter is a good approximation of the behaviour of beam strips, similar to the one shown in Fig. 3.20(a), especially for span to depth ratios in the range of 20–65.

Whether the membrane solution represents the plate behaviour in the transverse direction is investigated next. For this, the load applied by the plate on a stiffener in deck R02 is calculated. From FINAS analyses the transverse shear and the vertical component of the transverse membrane force were summed up and plotted against the differential deflection between two adjacent stiffeners. For the membrane component, the shape of the deflected plate was considered to be linear between the stiffeners. Results are presented in Fig. 3.21(a). The data points in this figure correspond to various locations in the plate along stringer 1 at various load steps. The positions of the sampling points are given in Fig. 3.2. The deflection $\delta_{1,2}(\bar{y})$ on the horizontal axis in Fig. 3.21(a) is the differential deflection between stiffeners 1 and 2 at the longitudinal position $\bar{y}$ defined in Fig. 3.19(a). Six values are given for each point in Fig. 3.2, corresponding to the load factors 1.21, 1.32, 1.56, 2.35, 3.15 and 3.50.

The two straight lines also plotted in Fig. 3.21(a) are based on the assumption that the behaviour of deck plate follows the membrane solution in the transverse direction. The continuous line in this figure assumes that the transverse stress in the plate reaches the yield stress σ_y , while the dashed line assumes the stress reaches $1.155\sigma_y$. It is seen that the data points from the numerical analysis of deck R02 lie within the two lines when $\delta_{1,2}(\bar{y})$ is in excess of 2 to 3 times the plate thickness. This strongly supports the idea that the plate strips can be assumed to be represented by the membrane solution.

Figure 3.21(a), also shows that, the data points eventually converge on the dashed line. This is as a result of stress biaxiality in the plate. As shown in Fig. 3.21(b) a transverse stress equal to $1.155\sigma_y$ is reached on the yield locus at point A. Normality postulates that sampling point positions as shown in Fig. 3.2 experience excessive straining in the transverse direction as compared to the longitudinal direction. The last point supports the assumption of having transverse plate strips connecting the 'notional beams' spanning in the longitudinal direction.

Figure 3.6(b) shows the transverse membrane forces in the plate to be around the yield value Nx_0 . In the following the assumption is made that each of the transverse plate strips is modelled by the membrane solution and is under stress of σ^t .

3.6.1.1 Loading applied along the stiffener

Figure 3.22(a) shows, schematically, the complete load versus deflection relationship of a plate strip spanning between stiffeners, where the horizontal axis represents the differential deflection between the stiffeners, non-dimensionalized with respect to the plate thickness, at any longitudinal location of the beam strip. After the initial discrepancy the actual behaviour is close to the membrane solution. Provided that the stiffeners remain straight as they deflect, the load applied along the stiffener can easily be calculated using Fig. 3.22(a), as shown in Fig. 3.22(b).

3.6.1.2 Loading along a stiffener with increasing deflection

As the difference between the central deflections of two adjacent stiffeners becomes larger, the part of the plate near the centre of the stiffener attains the membrane solution. In this case the load pattern on the stiffener has the shape shown by the solid line on Fig. 3.22(c). It can be seen that the loading consists of two parts: a linearly varying part over the centre of the beam and a non-linear part near the support. In the proposed model the loading is assumed to be linearly distributed between the support and the centre line, as shown by the dashed line in Fig. 3.22(c). As the difference in the stiffener central deflections increases, a larger segment of the stiffener experiences the assumed loading. More importantly, the assumed membrane loading coincides with the actual load over the central part of the stiffener, which is the region contributing more significantly to the equilibrium equation. Thus, as the difference in the central deflections increases, the extent of the region over which the loading is overestimated is

reduced and its contribution to the equilibrium equation diminishes.

3.6.1.3 Plate reaction on the stiffeners

The assumption of purely membrane behaviour for the plate in the transverse direction has been seen to be close to the behaviour observed in numerical analysis. The effects of the assumption on the load applied by the plate to the stiffeners is now explained.

The plastic plate behaves like a membrane. It lacks flexural rigidity, but it has geometric stiffness, that is, stiffness due to its shape. Figure 3.23 shows a plate strip of unit width between two stiffeners. The vertical reaction q of such a strip is given by:

$$q = t \sigma^t \sin \alpha_{j,j+1} - t \sigma^t \sin(\tan^{-1}(\frac{\delta_j - \delta_{j+1}}{b_{j,j+1}})) \quad (3.1)$$

where δ_j and δ_{j+1} are the deflections of stiffeners j and $j+1$, $b_{j,j+1}$ is the spacing between stringers j and $j+1$ which is assumed to remain constant, and $\alpha_{j,j+1}$ is the angle of inclination of the plate in the transverse direction between stiffeners j and $j+1$. The plate thickness is t and σ^t is the plate stress in the transverse direction. For small transverse angles we have:

$$\sin(\alpha_{j,j+1}) \approx \alpha_{j,j+1}$$

Considering the horizontal equilibrium of plate forces on both sides of a stiffener, the two components cancel each other if the membrane stress is the same on both sides of the stiffener (small angle approximation).

The vertical component of the loading applied by a plate strip of unit width spanning between the centres of stiffeners j and $j+1$ can be approximated as:

$$q \approx \frac{t}{b_{j,j+1}} \sigma^t (\delta_j - \delta_{j+1}) \quad (3.2)$$

using the simplification made above, that is,

$$\sin(\alpha_{j,j+1}) \approx \frac{(\delta_j - \delta_{j+1})}{b_{j,j+1}} \quad (3.3)$$

Figure 3.24 shows that the effect of this approximation is to overestimate the vertical component of the transverse plate stress. However this overestimation is small, being essentially zero for small deflections and increasing to nearly 7% at:

$$\frac{\delta_j - \delta_{j+1}}{b_{j,j+1}} = 0.40$$

3.6.2 Bilinear mechanism assumption

Any difference between the assumed bilinear mechanism and the actual deflected shape of the stiffener has a double effect. It affects the way in which the axial forces are built up in the 'notional beams'. It also alters the distribution of the reacting forces along the stiffeners provided by the plating.

3.6.2.1 Effect on the axial forces in the beams

To enable comparison between the bilinear shape assumption made and the behaviour observed in the numerical analysis of deck R02, the deflected shapes of two adjacent stiffeners 1 and 2 are given in Fig. 3.25. It is clear that the bilinear mechanism is a simplification of the actual behaviour. For a particular central deflection the curved stiffener shapes in Fig. 3.25 need bigger extensions than those required by the bilinear mechanism. Therefore, one could expect the bilinear mechanism to predict smaller axial forces than those in the actual structure. However, it is rather inappropriate to draw any conclusion from this brief discussion. The effect of the 'notional beams' bilinear mechanisms is further discussed in Chapters 4 and 5.

3.6.2.2 Effect on the loading along the beams

In a simplified model, the bilinear mechanism is the simplest assumption that could be made. With this mechanism the difference in the deflections between two adjacent stiffeners is distributed triangularly along the stiffeners. It was pointed out in Section 3.6.1.3 that the vertical component of the transverse membrane force varies linearly with the differential deflection between two stiffeners, provided that

the differential deflections between the stiffeners are small. The implication of this linearity is that the loading along a stiffener is distributed triangularly.

The error in the loading calculated by adopting the bilinear mechanism is the difference between the δ_1 - δ_2 curve in Fig. 3.25 and a straight line connecting the central differential deflection with the support. Generally the load along the stiffener is slightly overestimated if the bilinear mechanism is adopted, for a particular difference in the central deflections.

3.6.3 Interaction between the stress resultants in the 'notional beams'

The plate is under plane stress conditions. Elastic stresses can be estimated from strains. Once the plate develops plasticity the incremental flow rule is adopted, implying that the ratio of the strain increments at a point defines the ratio of stresses. In this way biaxiality affects both the magnitude of the transverse membrane stresses and also the width of the effective plate which acts together with the stiffeners. The plate can be assumed to follow the membrane solution in the transverse direction, as was discussed in Section 3.6.1. The effect of stress biaxiality in the plate on the uniaxial interaction for the 'notional beam' cross-section represented in Fig. 3.16 by the continuous line, is discussed next.

The stress resultants for a stiffener and the associated plate are compared with the uniaxial interaction discussed in Section 3.5.3.2. This comparison is made in Fig. 3.16. Data points at five load steps corresponding to the four stiffeners are shown in this figure. The points on the lower part of this figure correspond to the Gauss points closer to the transverse central line, and are under sagging moments. On the upper part, the stress resultants of the Gauss points closer to the transverse support are shown. It can be concluded, that the uniaxial interaction between the stress resultants for a stiffener and the associated fully effective plate can still be used to represent the interaction between the stress resultants of the longitudinal stresses, even if the plate is under biaxial stress state.

Some of the points, located at the upper part of Fig. 3.16 and corresponding to Gauss points which are closer to the transverse support, lie just outside the uniaxial interaction. This is due to the biaxial stress state in the plate. Points in the plate, located close to the transverse support, experience nearly zero straining in the transverse direction. In the longitudinal direction, the plate is strained as it forms the top flange of the 'notional beams'. Excessive straining in the longitudinal direction relative to that in the transverse gives a stress point near

point B on the biaxial interaction between the principal stresses, as shown in Fig. 3.21(b). Point B has a longitudinal stress of 1.155 times the yield stress. The uniaxial interaction, shown on Fig. 3.16, has been drawn for the 'notional beam' cross-section on the assumption of a top flange width of b and stress block at the yield level. Accounting for stresses higher than yield, in the top flange, an alternative interaction can be drawn. This should consider the modified stress resultants obtained for a cross-section in which the top flange is $1.115b$ wide and the stress block at yield. If an interaction is drawn, in which the modified stress resultants are normalized with respect to their plastic values for the 'notional beam' cross-section, all the points shown on Fig. 3.16 are deemed to lie within it.

3.6.4 Structural elements in the longitudinal direction

The beams can be classified into two categories. The first comprises the externally loaded stiffeners and the second comprises the stiffeners loaded by the plate only.

3.6.4.1 Stiffeners with externally applied load

The central stiffeners experience two types of loading, one is the externally applied load and the other is the deck reaction. If an axially restrained beam is under a pattern of distributed loading it attains a given shape, which is function of the applied loading pattern. This is explained in Chapter 5. An additional problem with the system under consideration is that these two load components are not proportional. As the ratio of the magnitudes of the two components of the load changes, it is difficult to predict precisely the behaviour of such a system by assuming a single deformation mode for the structure. Under a continuously changing pattern of loading, there is no simple method, in terms of generalized coordinates, that could accurately predict the behaviour of the beam. For such a problem a method that could account for continuous mode-shape changes is more appropriate.

Assuming that the deflected shapes of both stiffeners 1 and 2 can be approximated by bilinear mechanisms, and further that the ratio of the central deflections δ_2/δ_1 is fairly constant throughout the deformation history, then the load applied by the deck on beam 1 as the vertical component of the transverse membrane forces, can be assumed to be given by linear springs. Then the analogy of an axially restrained beam on elastic foundation arises and this behaviour is studied in

Chapter 4. It is also shown in Chapter 4 that the bilinear mode assumption can be adopted for a simplified analysis.

3.6.4.2 Stiffeners not externally loaded

If all the stiffeners are assumed to form bilinear mechanisms as it was assumed above for the two inner stiffeners of the bay, this results in stiffeners loaded by triangularly distributed loading along their length. This triangular loading is applied on both sides of a stiffener and comes as a reaction of the plate strips. The stiffener is axially restrained and is loaded by triangularly distributed loading.

The behaviour of axially restrained beams under symmetric distributed loading is studied in Chapter 5. The case of triangular loading is studied as a special case, and further comments on the biaxial mode of deformations are made.

3.7 CONCLUSIONS

A fully restrained longitudinally stiffened bay exhibits similar structural behaviour in the large deflection regime as under small deflections. By this it is meant that a similar distributed loading exists along the beams, although the source of the loading differs in the small and large deflection regions.

The restraint against pull-in produces considerable membrane action. For example stresses of the order of the yield stress are generated at deflections of about twice the plate thickness.

Transverse shear forces and the vertical component of the transverse membrane force in the plate combine to transfer the load between adjacent stiffeners. The latter becomes dominant at deflections in excess of twice the plate thickness.

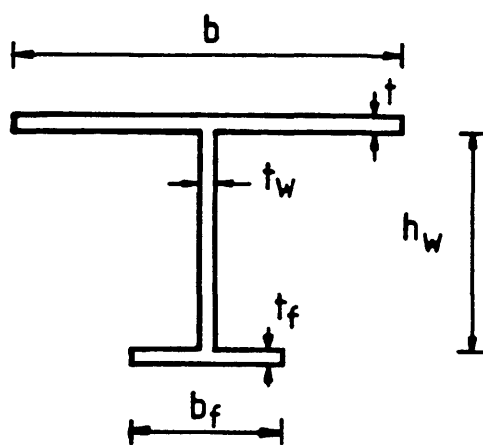
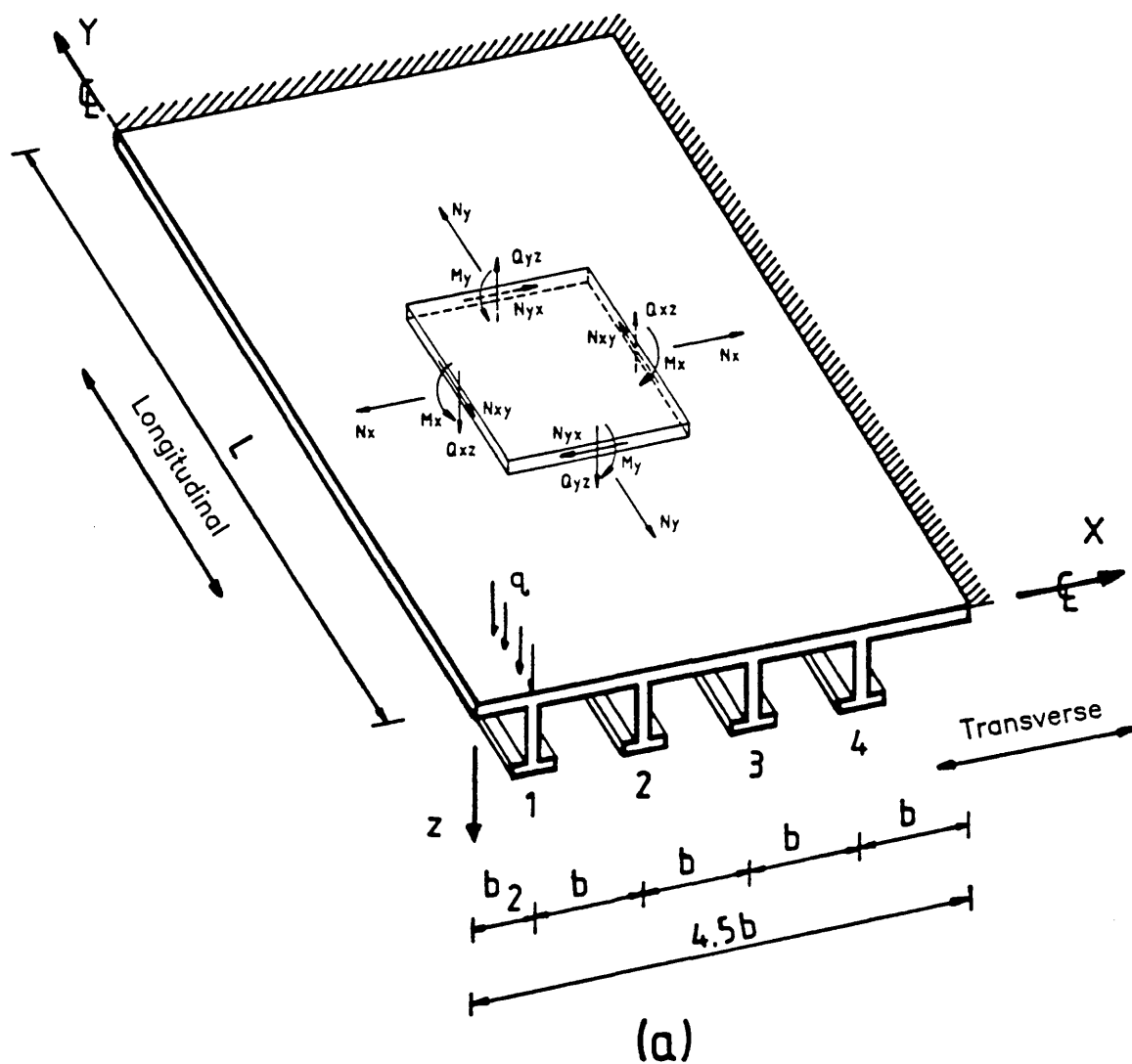
On the basis of these observations a simplified model of the behaviour was proposed. In this, the deck is considered as a series of longitudinal beams connected in the transverse direction by plate strips that follow the membrane solution. The assumptions made were discussed and compared with the behaviour observed in the finite element analysis, and the differences were found to be acceptable for a simplified analysis. The formulation of the proposed model and checks of its validity are presented in Chapter 6.

TABLE 3.1 Cross-sectional geometry

DECK	G E O M E T R Y					MATERIAL
No.	$t^{(1)}$ (mm)	b/t	h_w/t_w	b_f/t_f	L/b	E/σ_y
R02 ⁽²⁾	4.76	26.67	14.99	10.50	6	837

(1) $t_w = t_f = t$

(2) For notation see Fig. 3.1(b)



(b)

Fig. 3.1 Deck and stiffener geometry

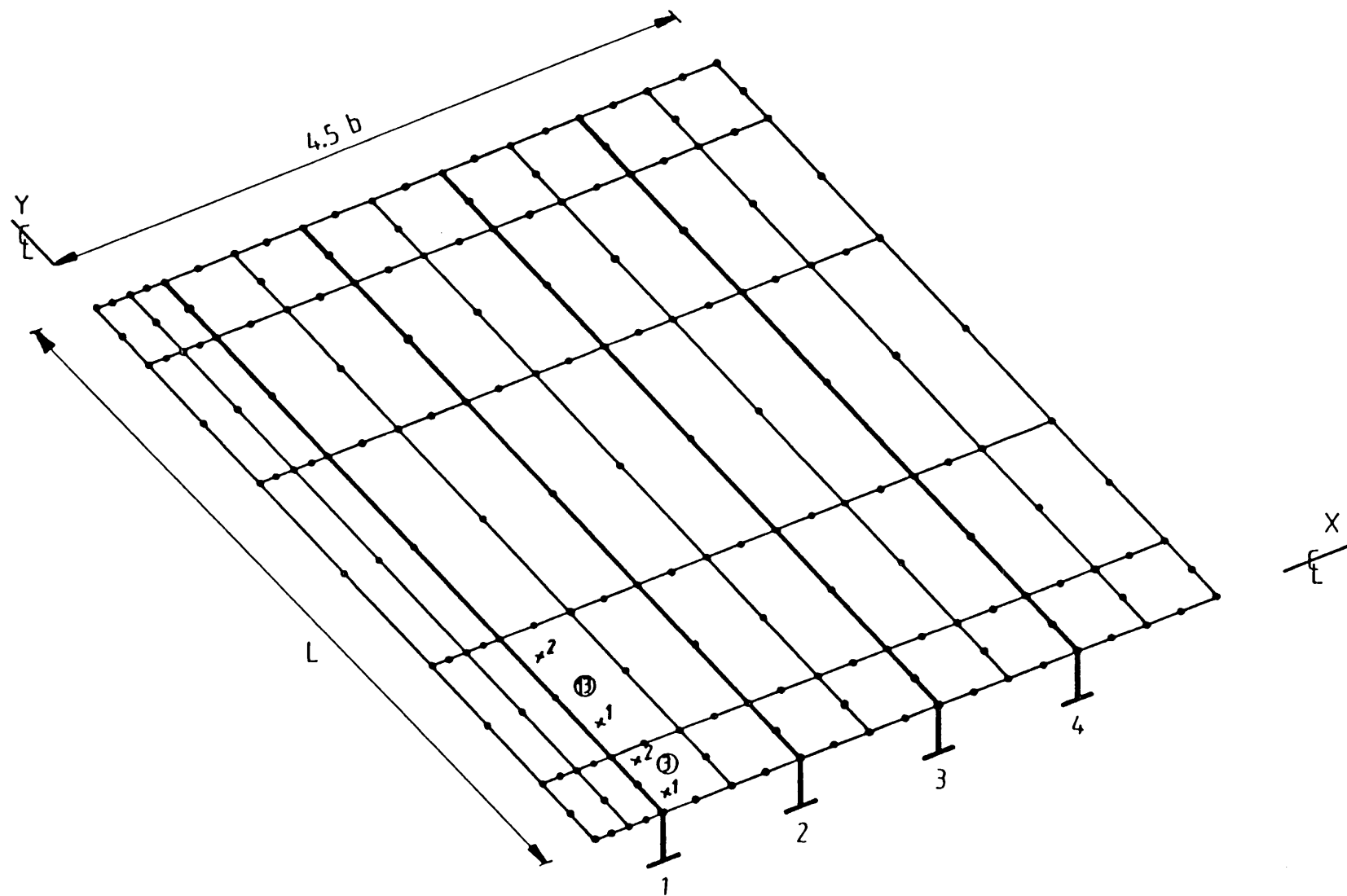


Fig. 3.2 FE mesh

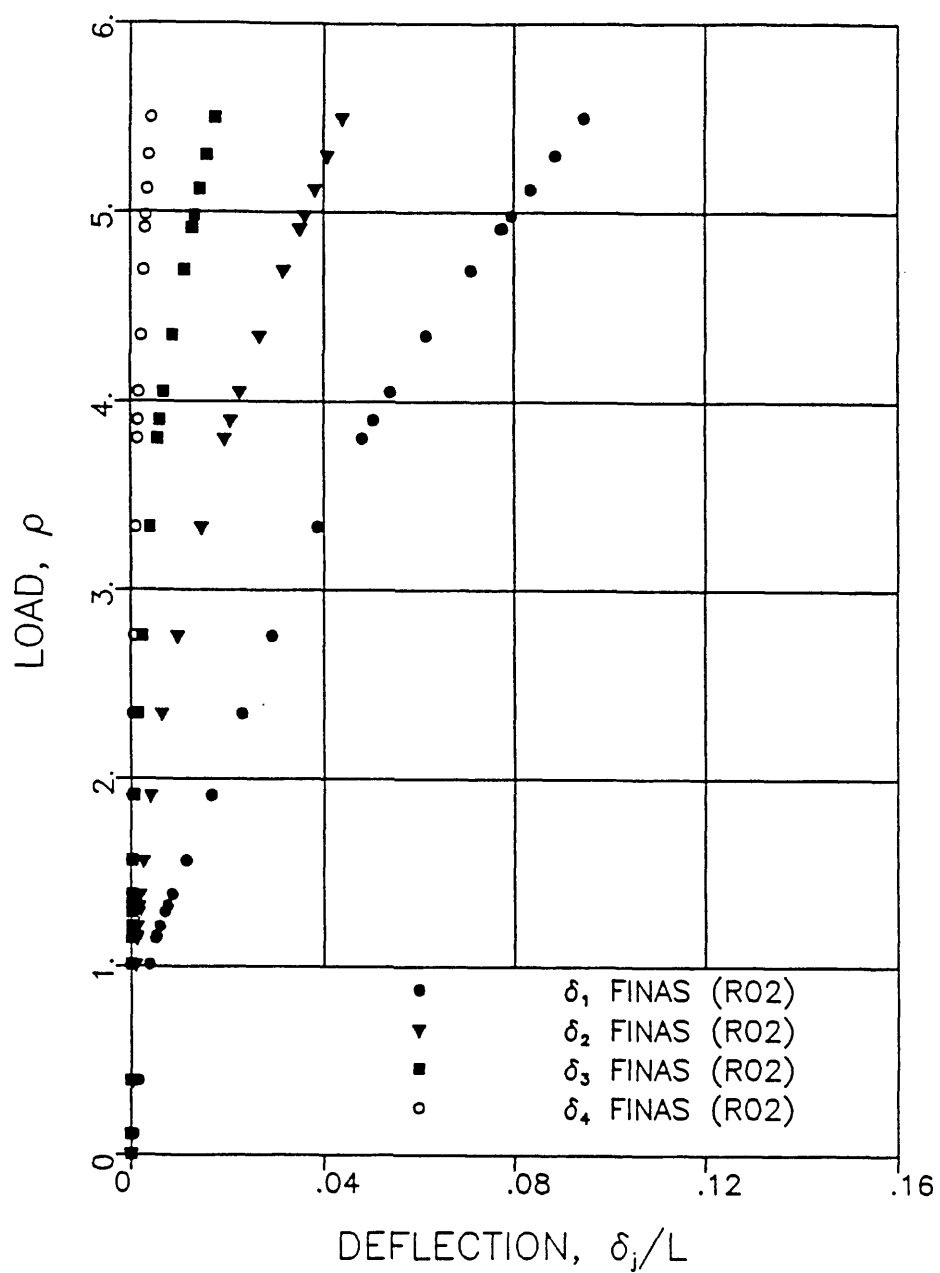


Fig. 3.3 Load vs. stiffener central deflections

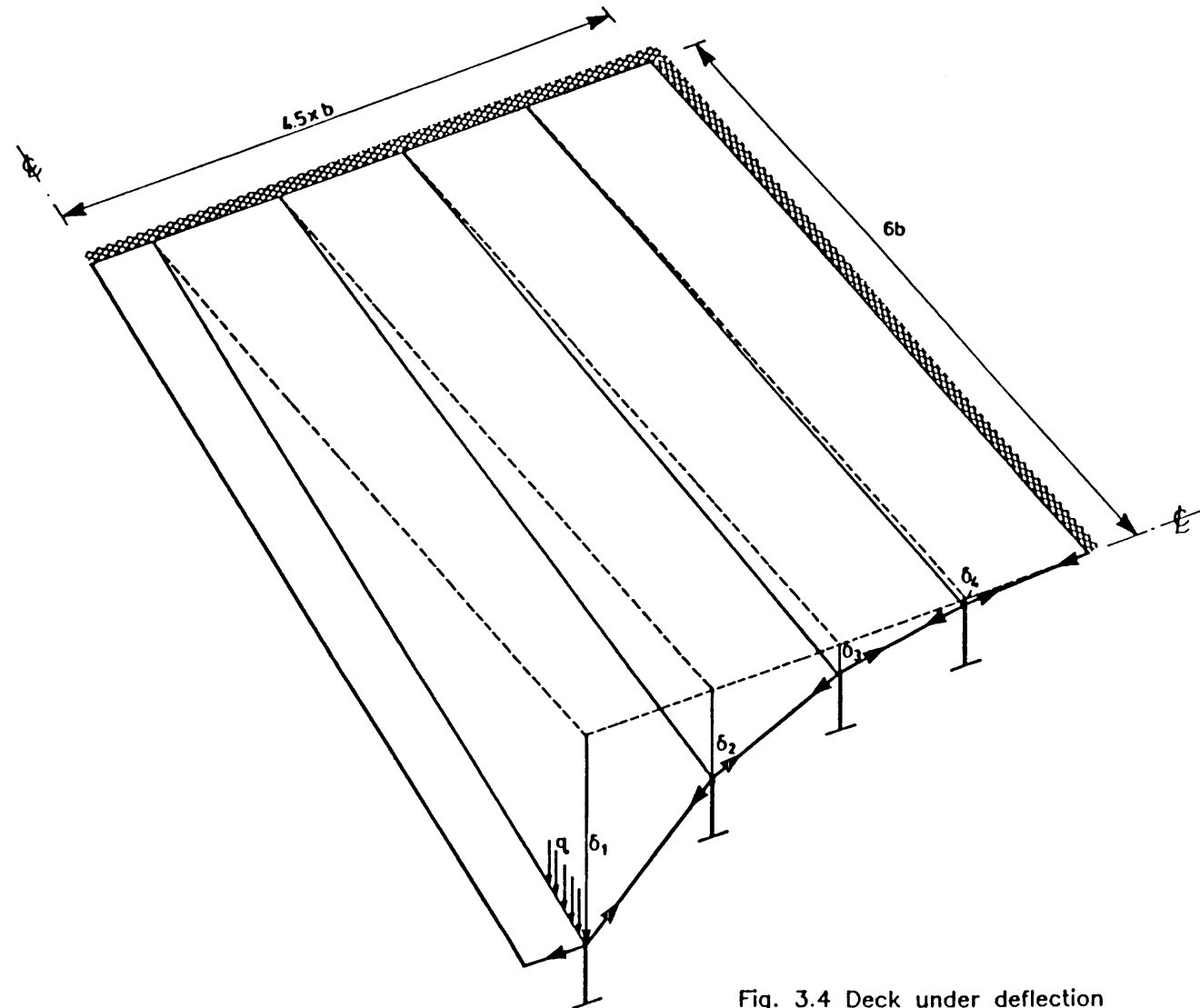


Fig. 3.4 Deck under deflection

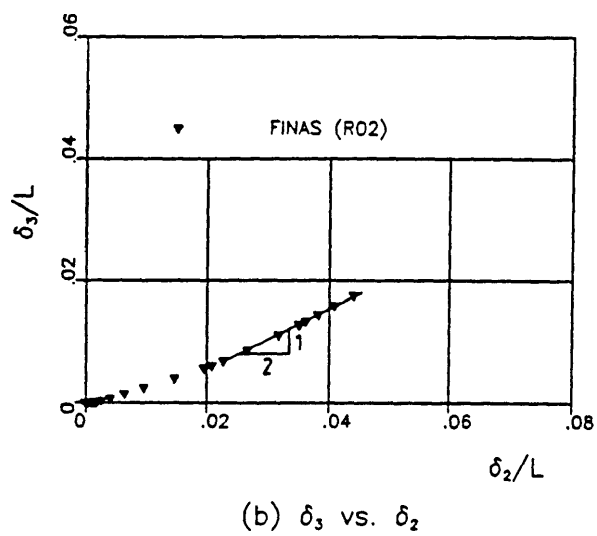
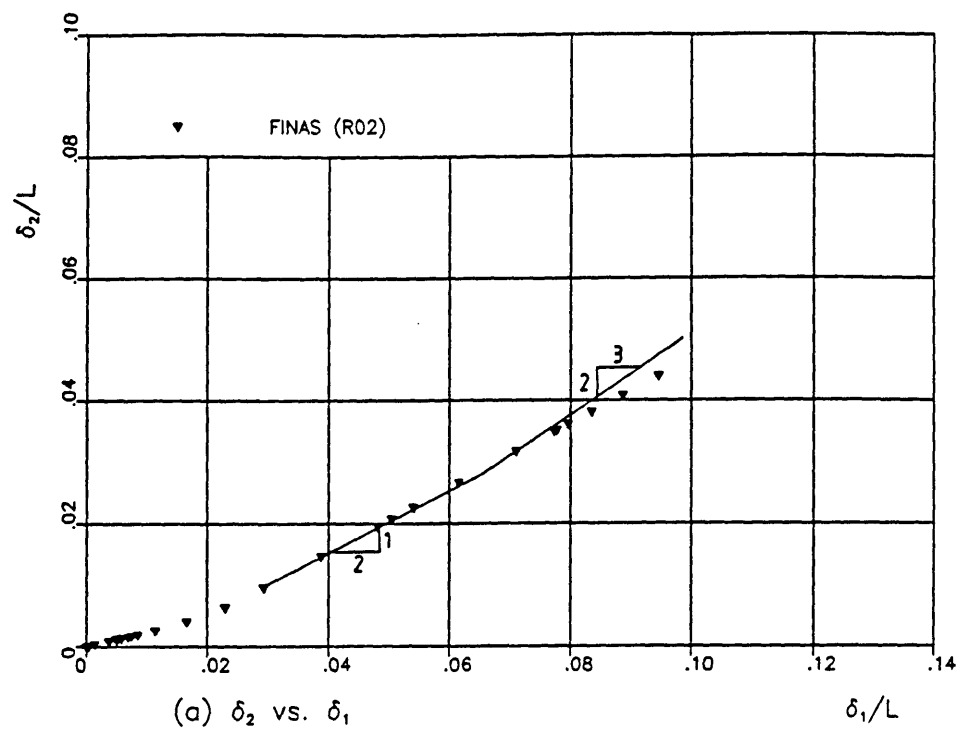


Fig. 3.5 Adjacent stiffener central deflections

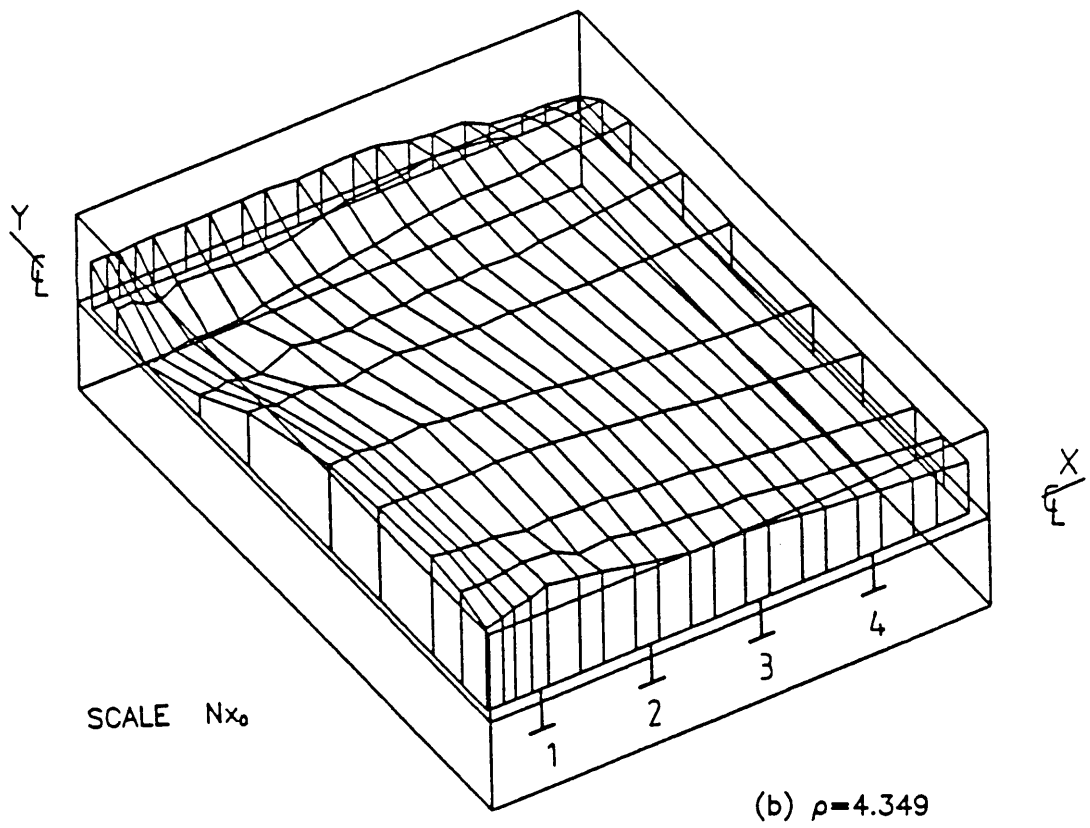
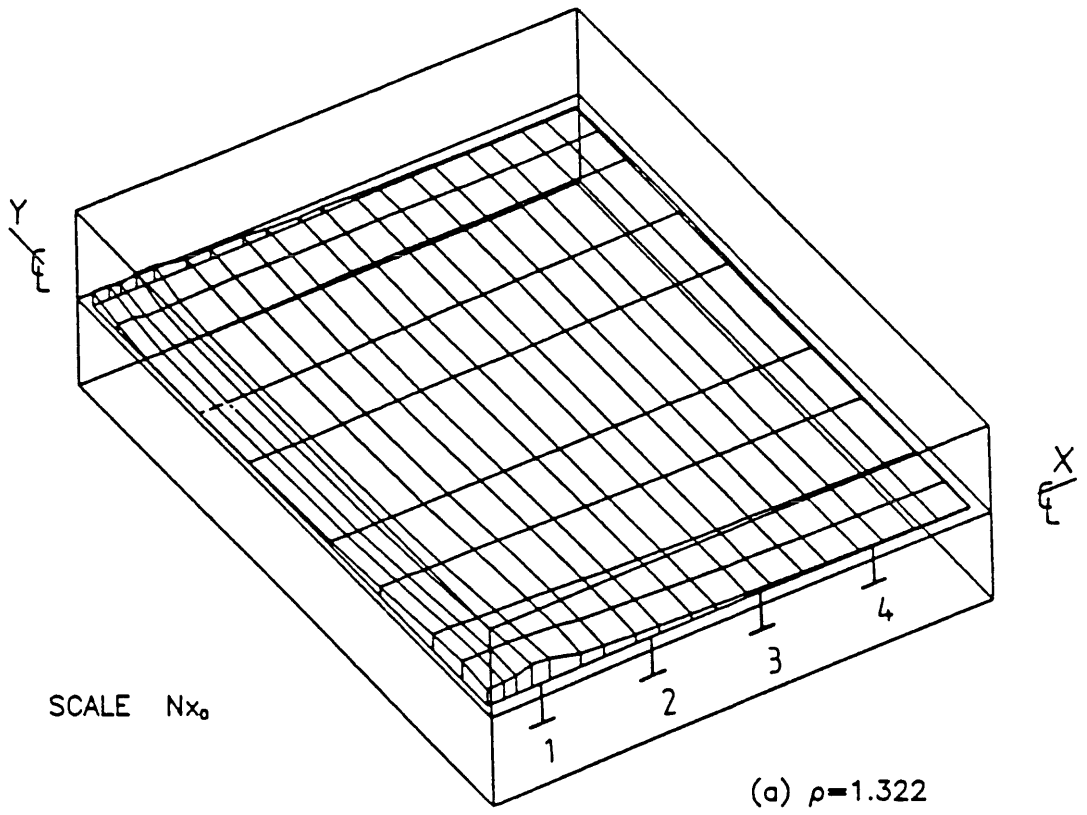


Fig. 3.6 Transverse membrane forces in the plate

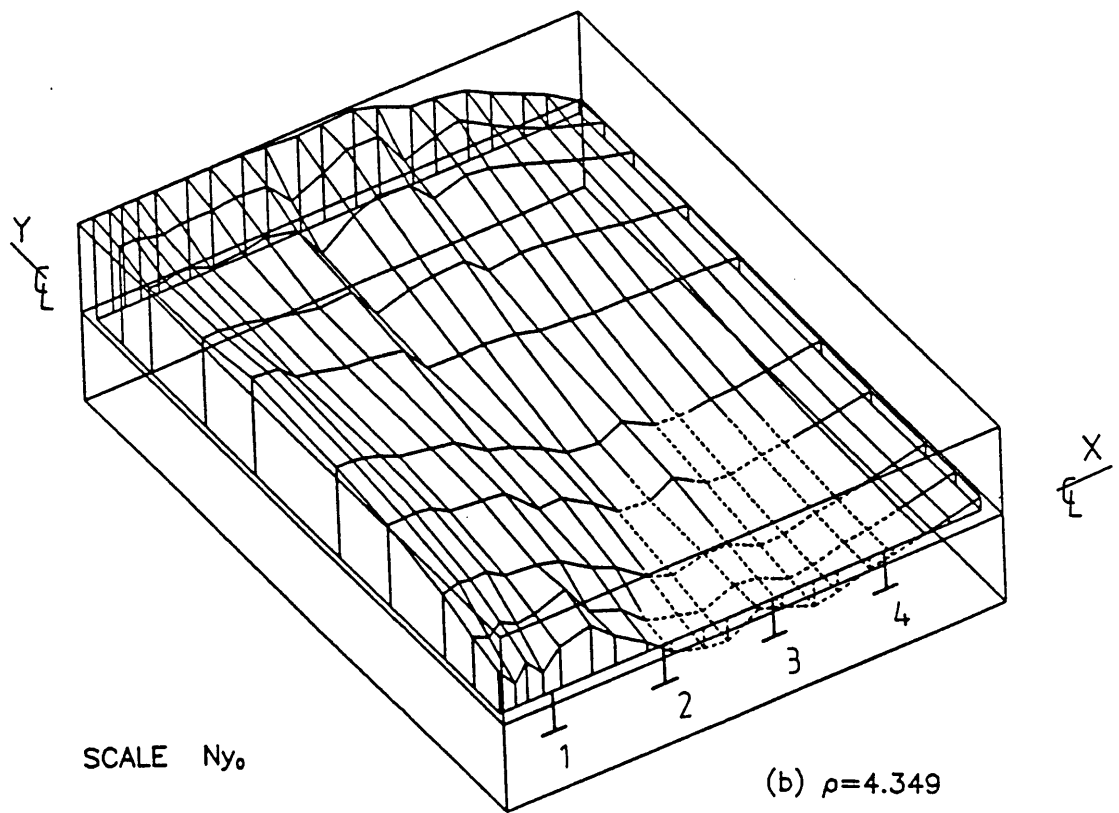
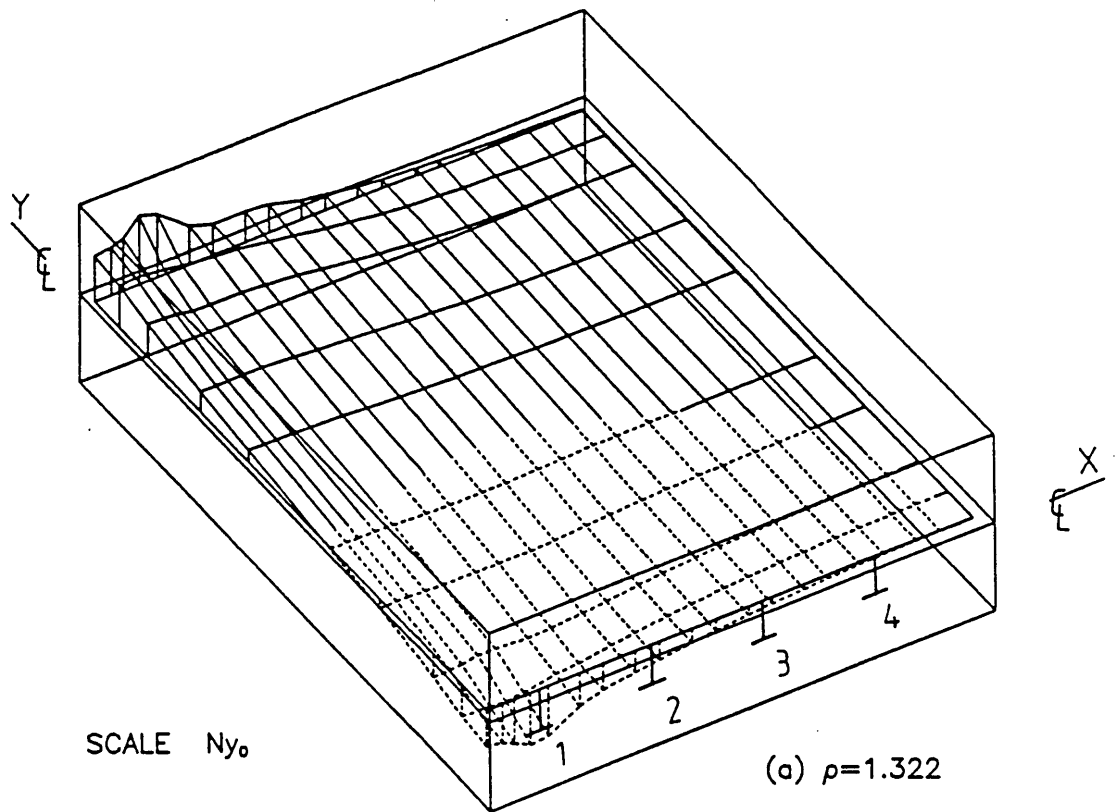


Fig. 3.7 Longitudinal membrane forces in the plate

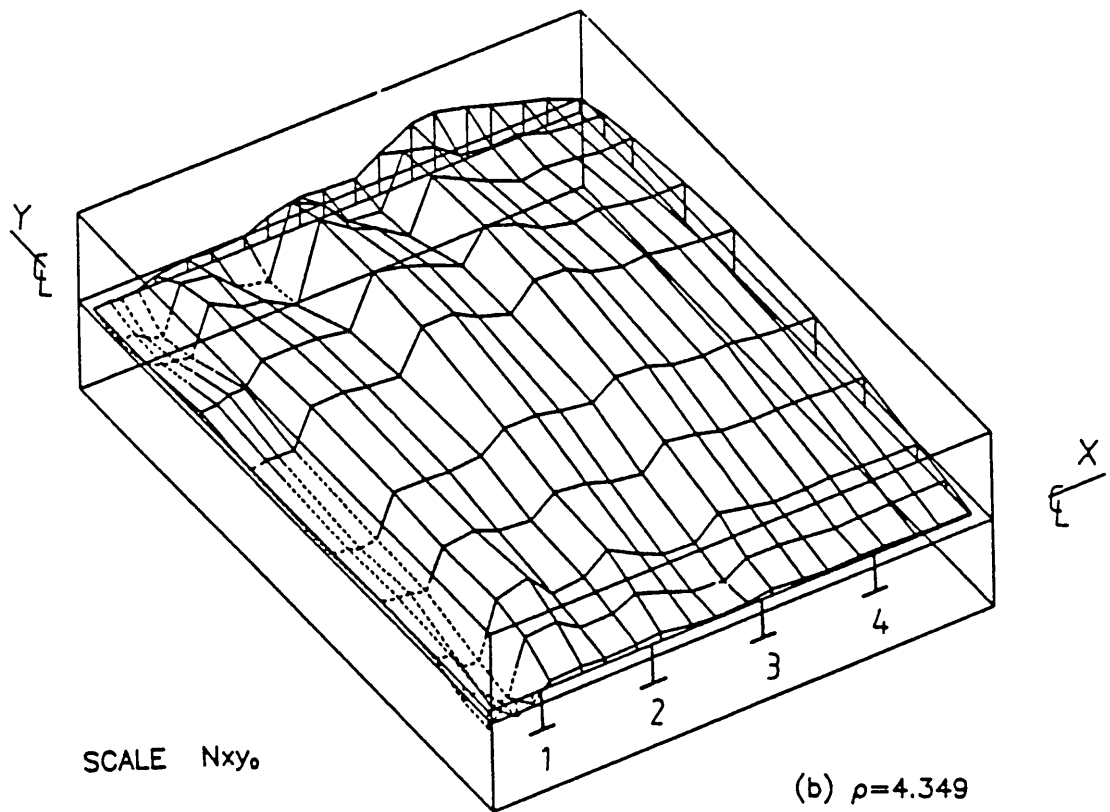
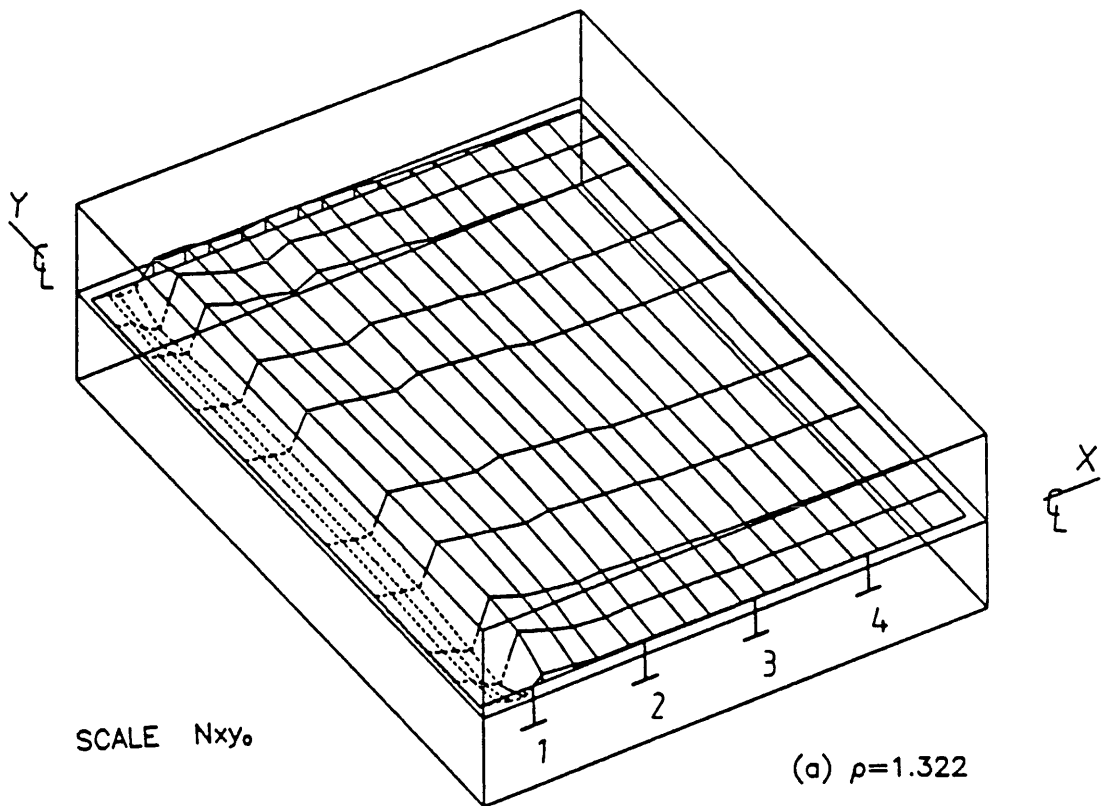
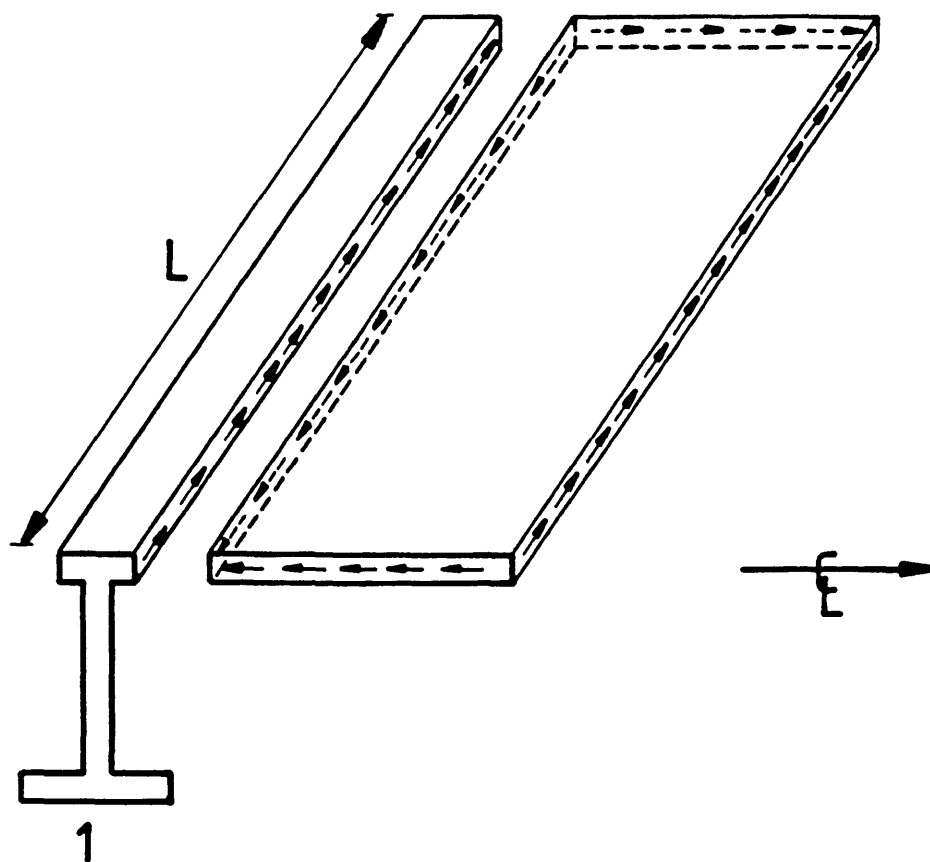
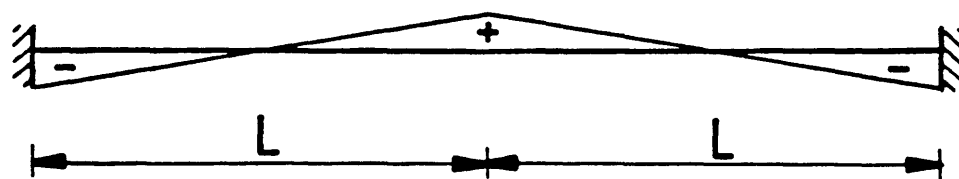


Fig. 3.8 Membrane shear forces in the plate



(a) Shearing in the plate loads adjacent stiffeners



(b) Axial forces in stiffener 1 due to shearing in the plate

Fig. 3.9 Effect of shearing in the plate on stiffener 1

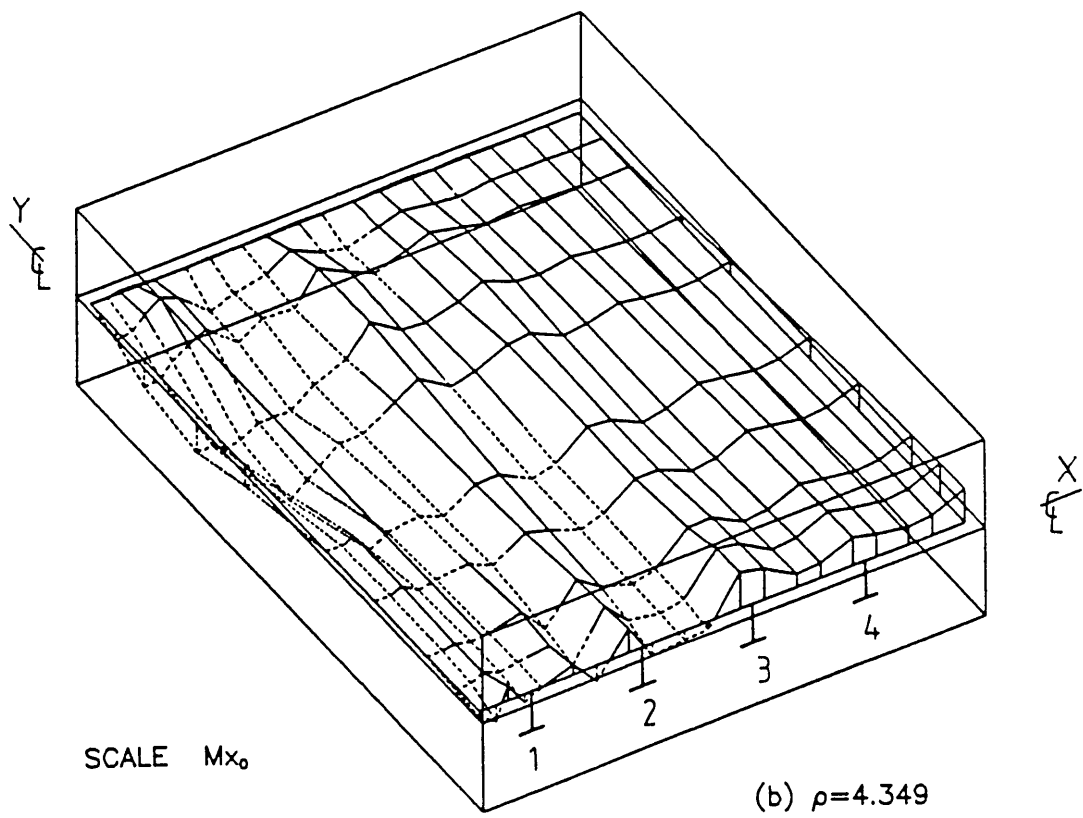
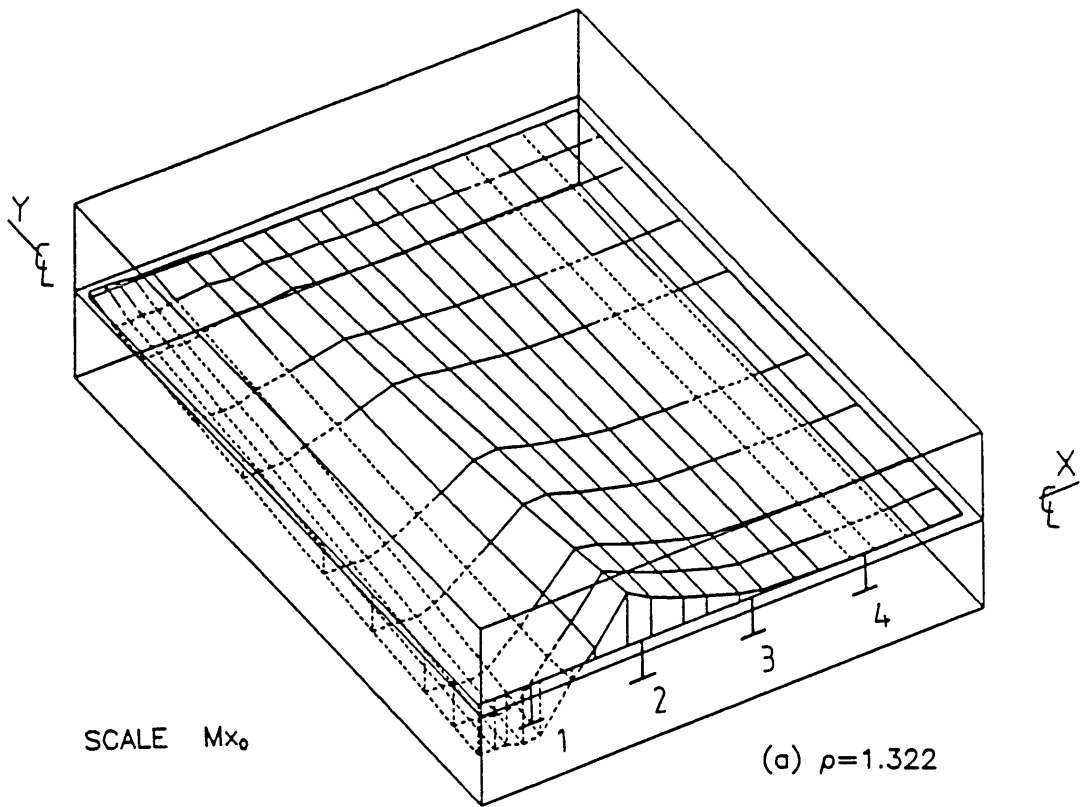
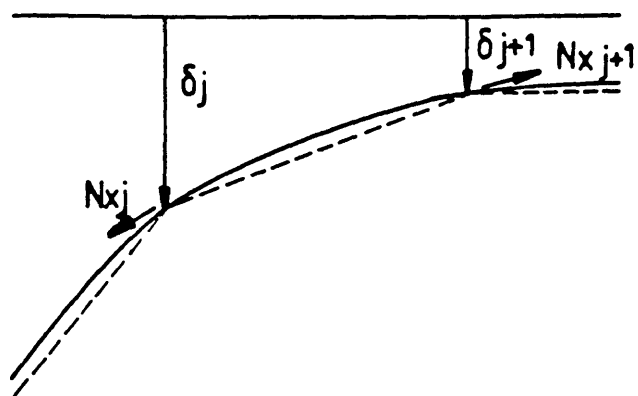
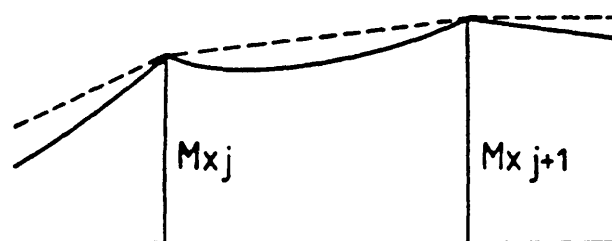


Fig. 3.10 Transverse bending moment distributions in the plate



(a) Deflected shape



(b) Bending moments

Fig. 3.11 $P-\delta$ effect in the plate

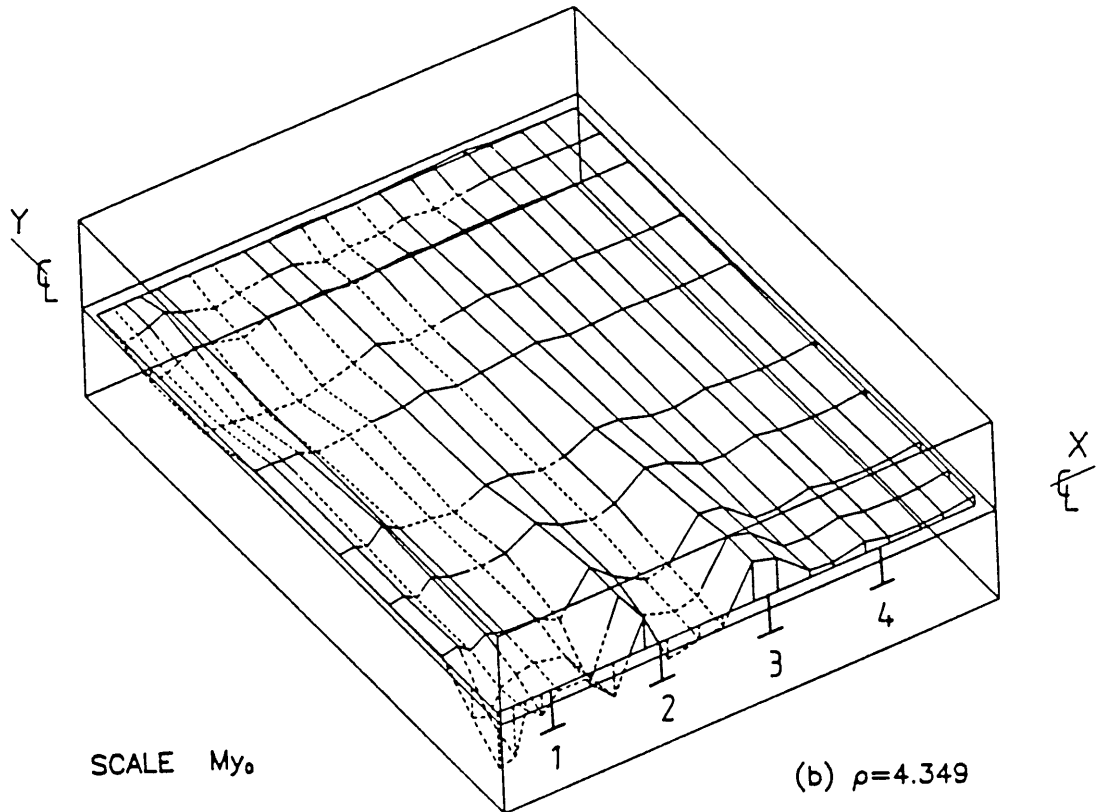
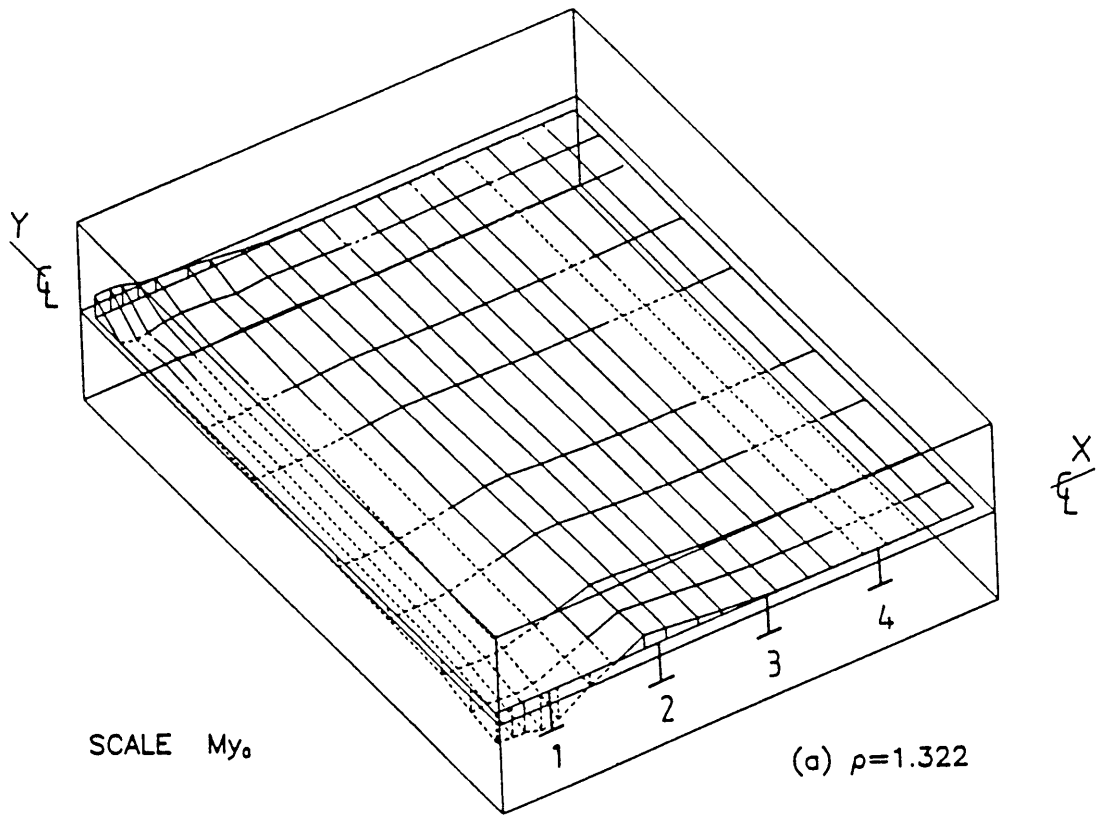


Fig. 3.12 Longitudinal bending moment distribution in the plate

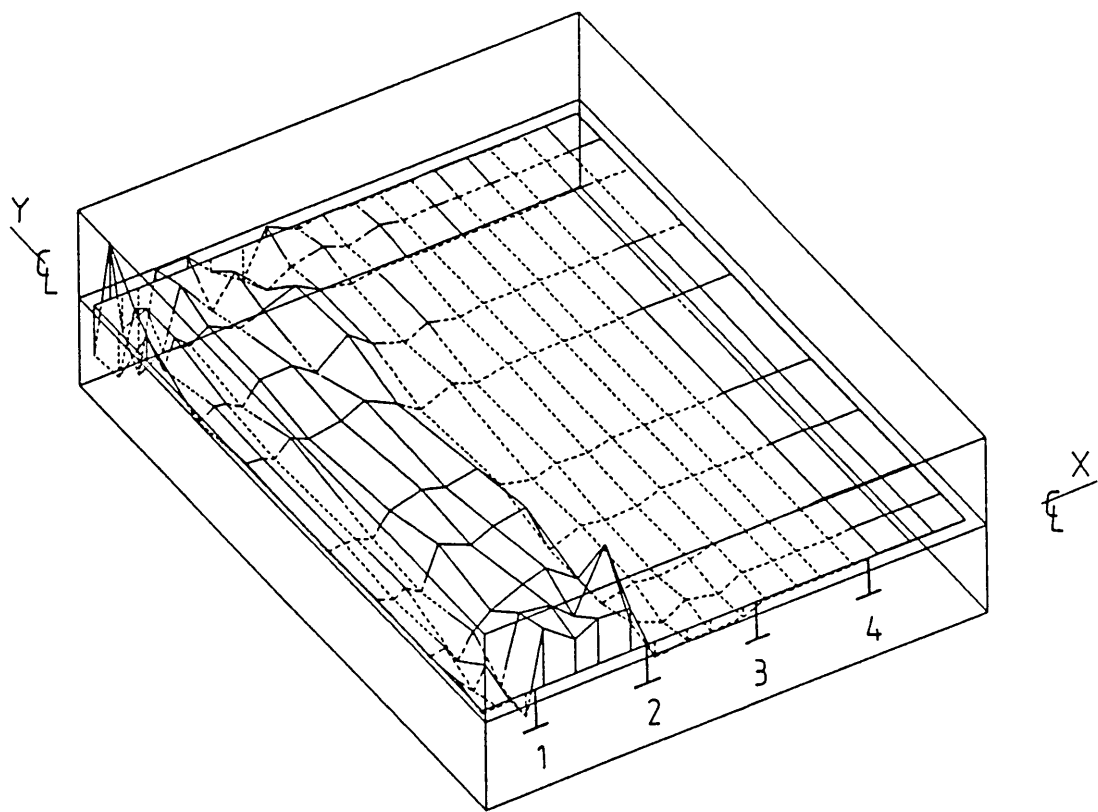


Fig. 3.13 Transverse shear pattern at $\rho=1.322$

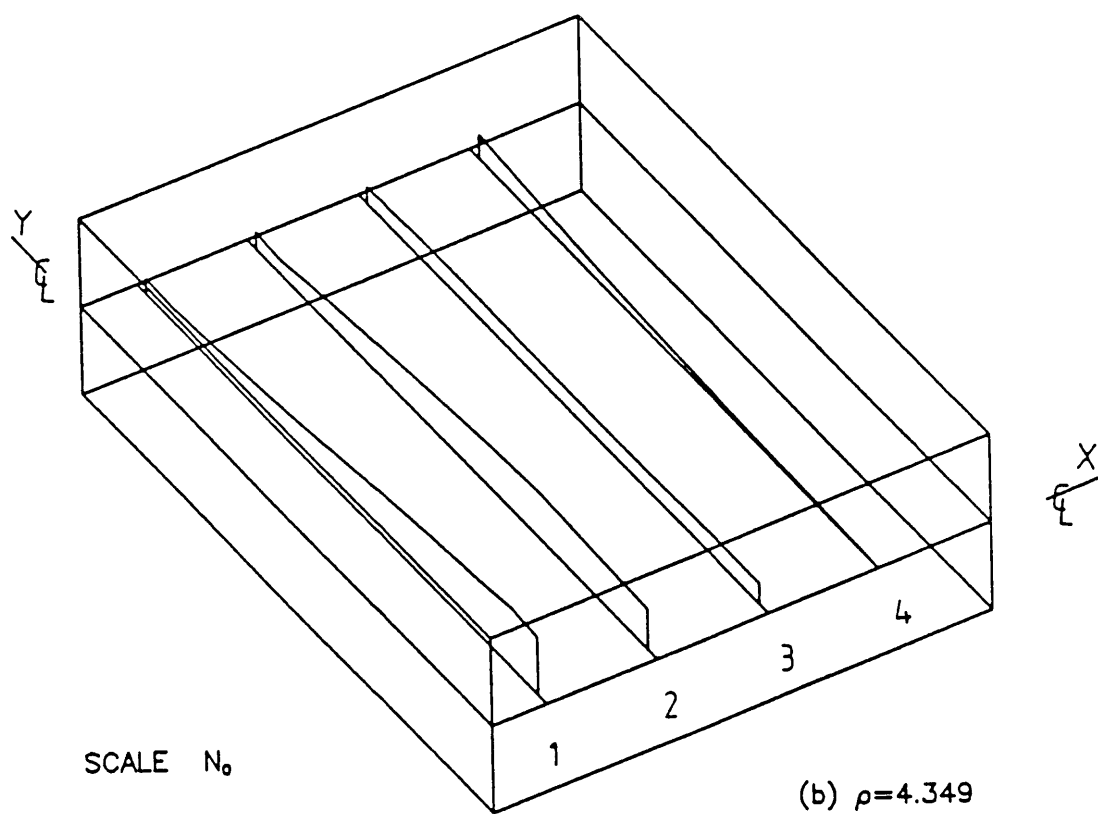
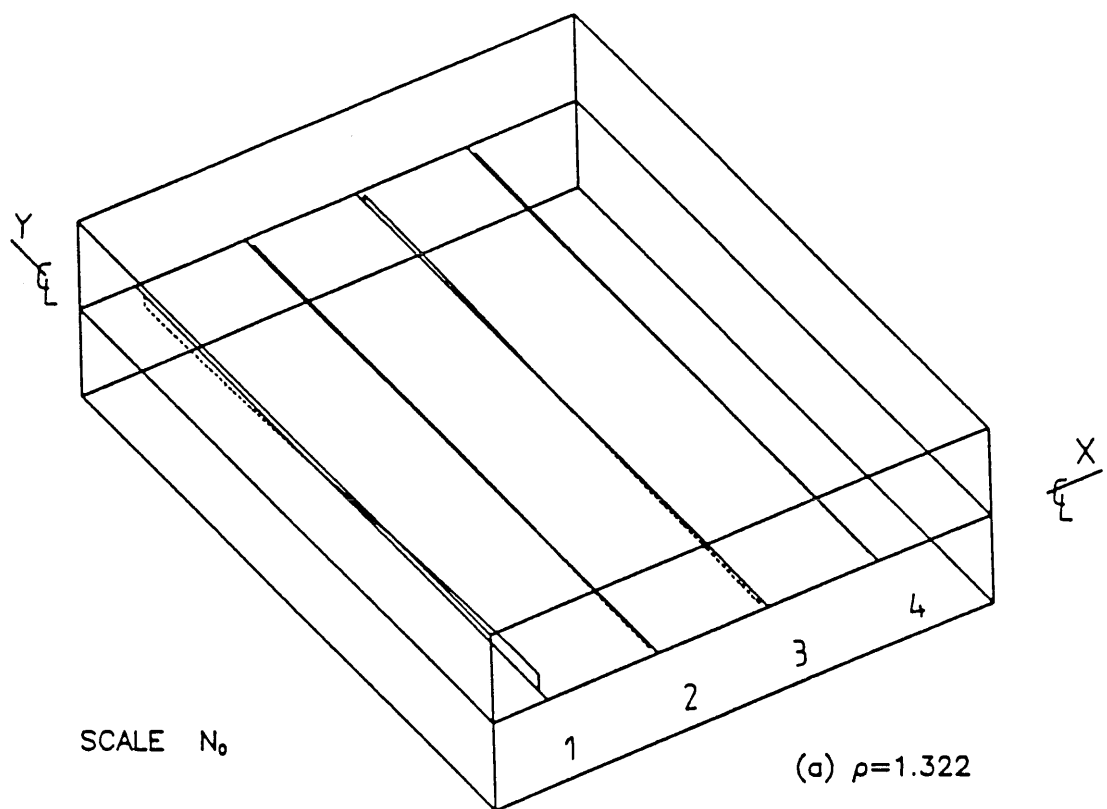


Fig. 3.14 Axial forces along the notional beams

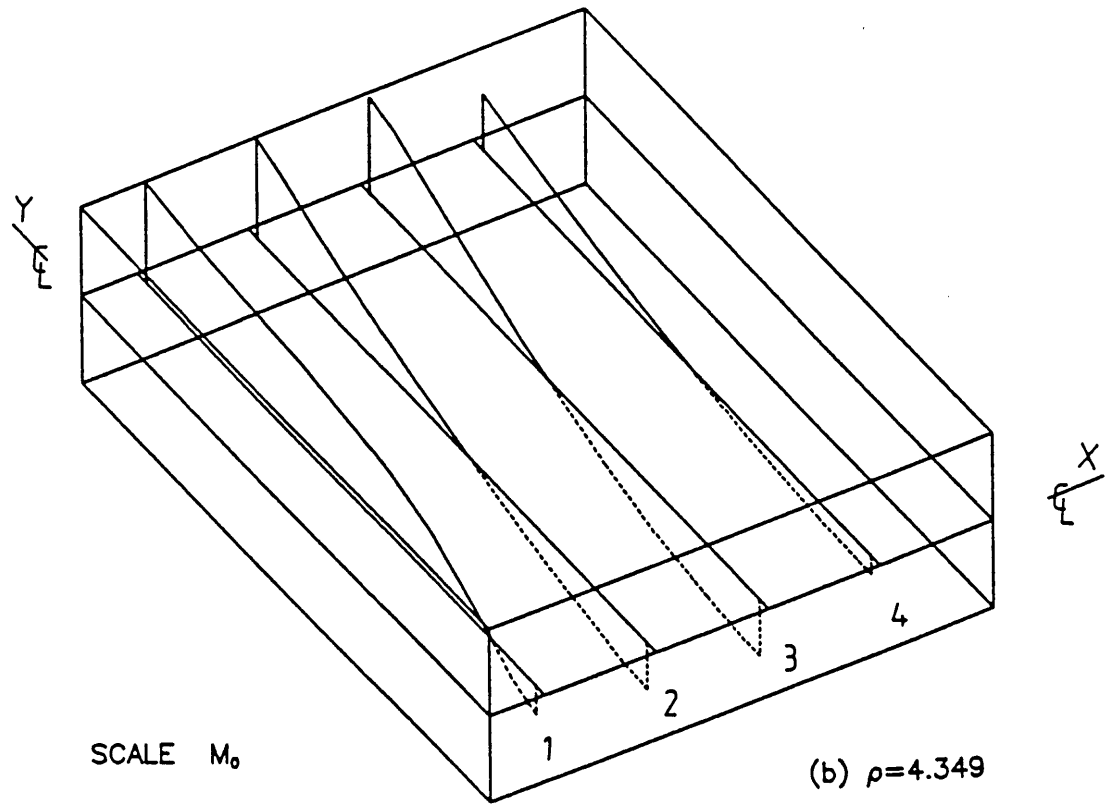
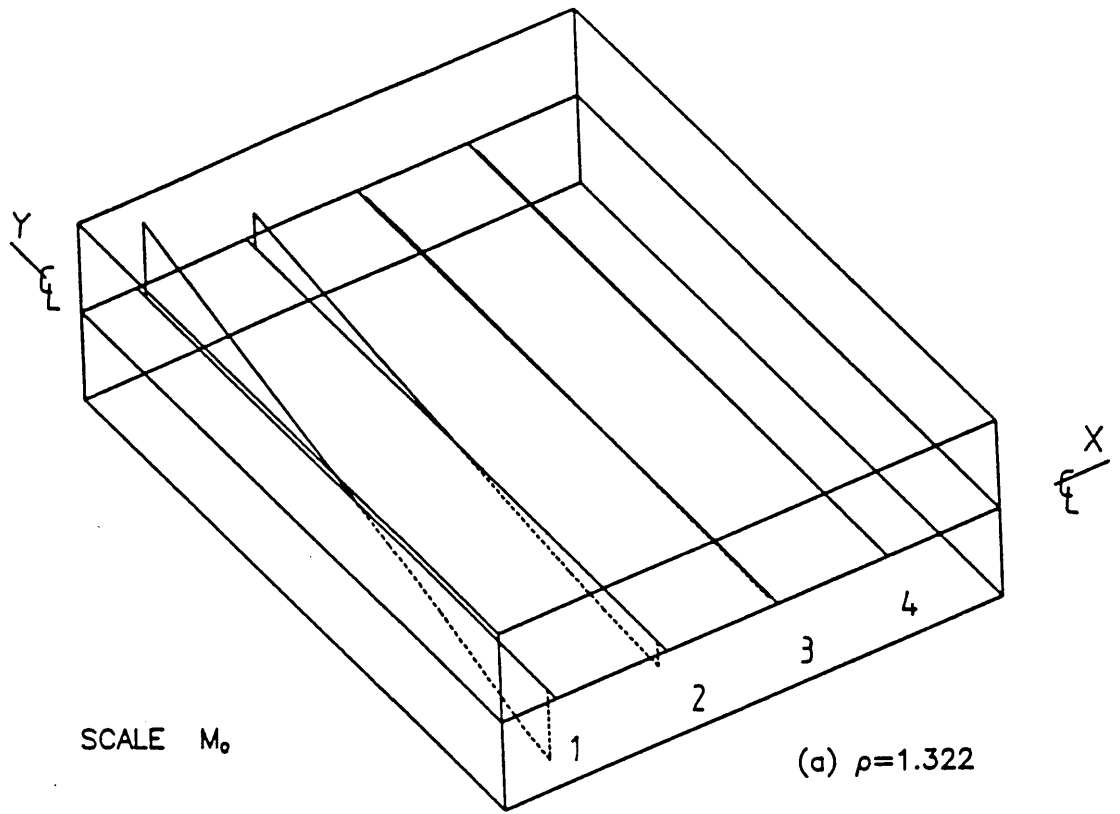


Fig. 3.15 Bending moments along the notional beams

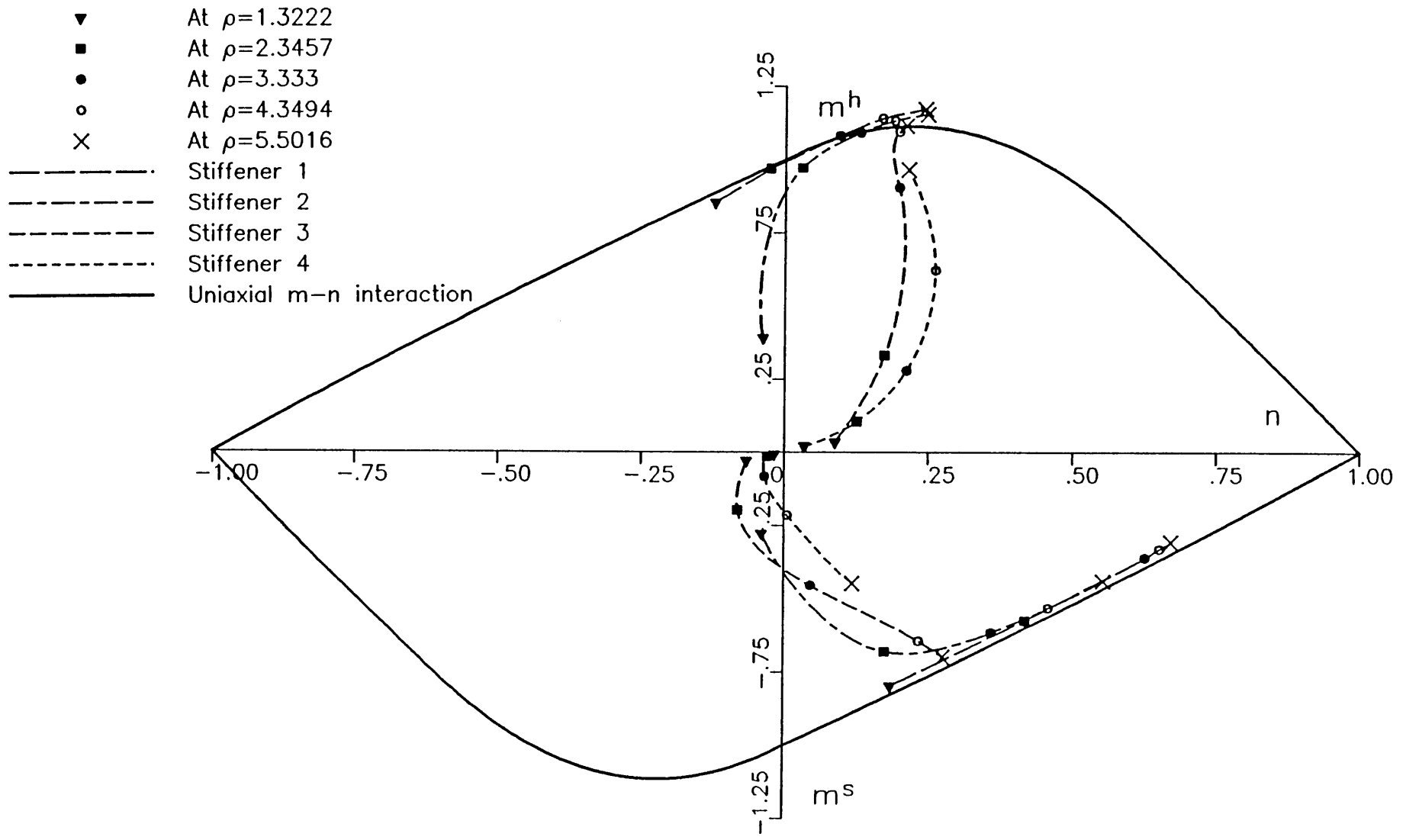
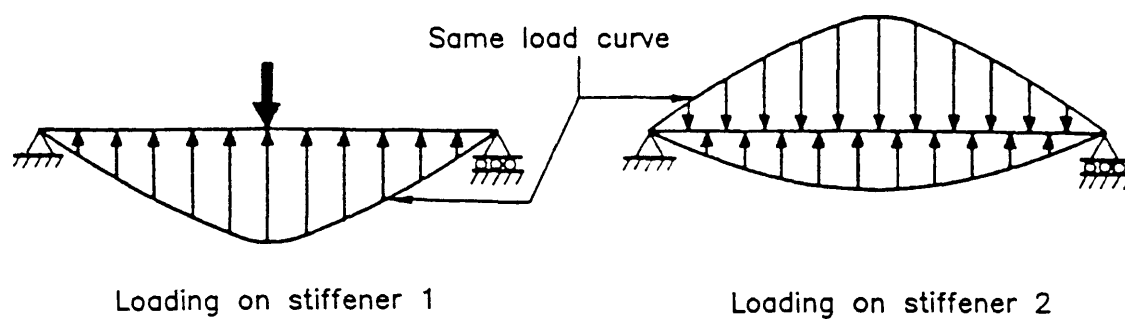
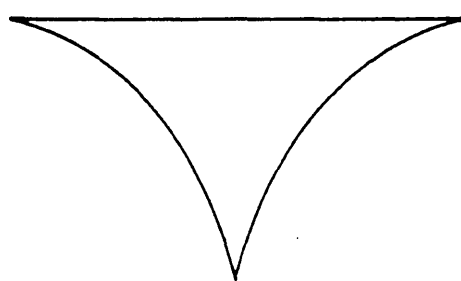


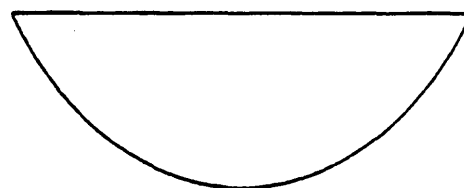
Fig. 3.16 Stress resultants interaction for the notional beam cross-section



(a)

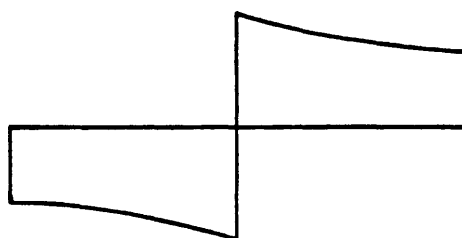


Moment diagram along 1

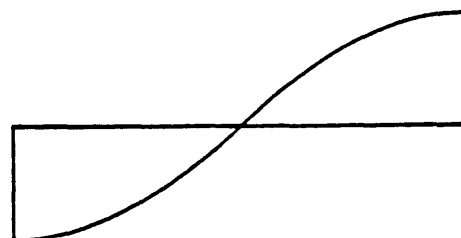


Moment diagram along 2

(b)



Shear diagram along 1



Shear diagram along 2

(c)

Fig. 3.17 Typical load transfer

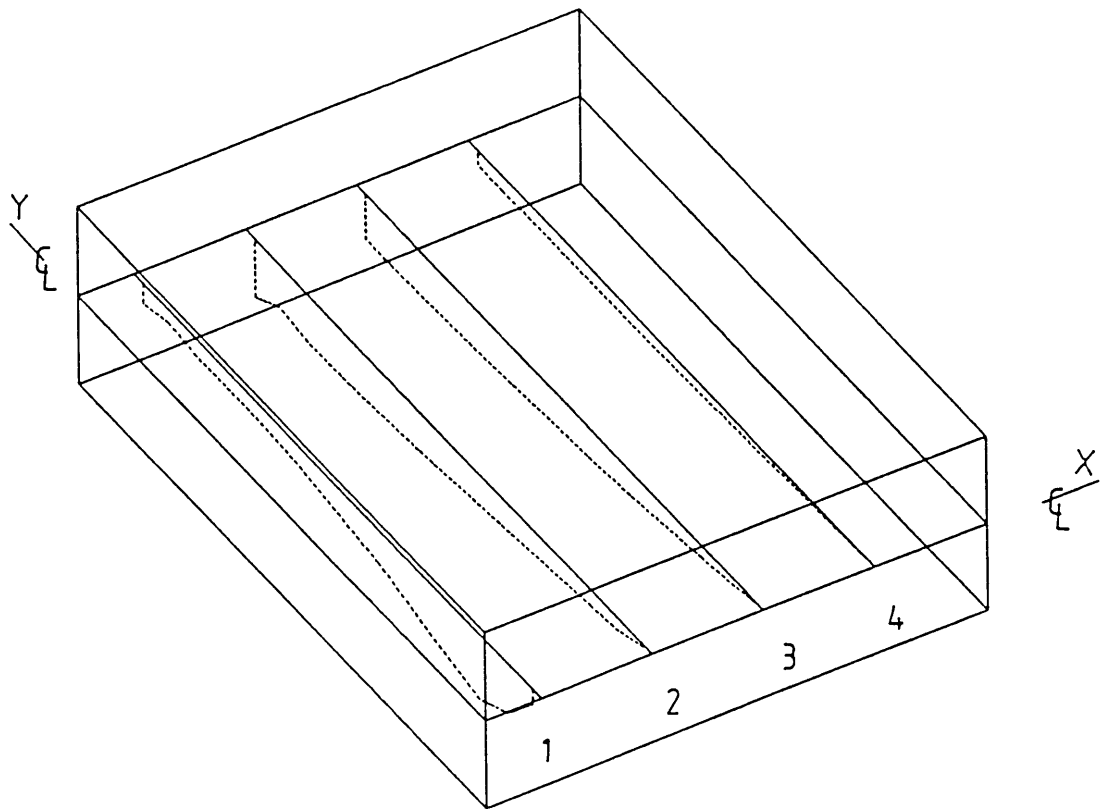


Fig. 3.18 Shear forces along the notional beams at $\rho=4.349$

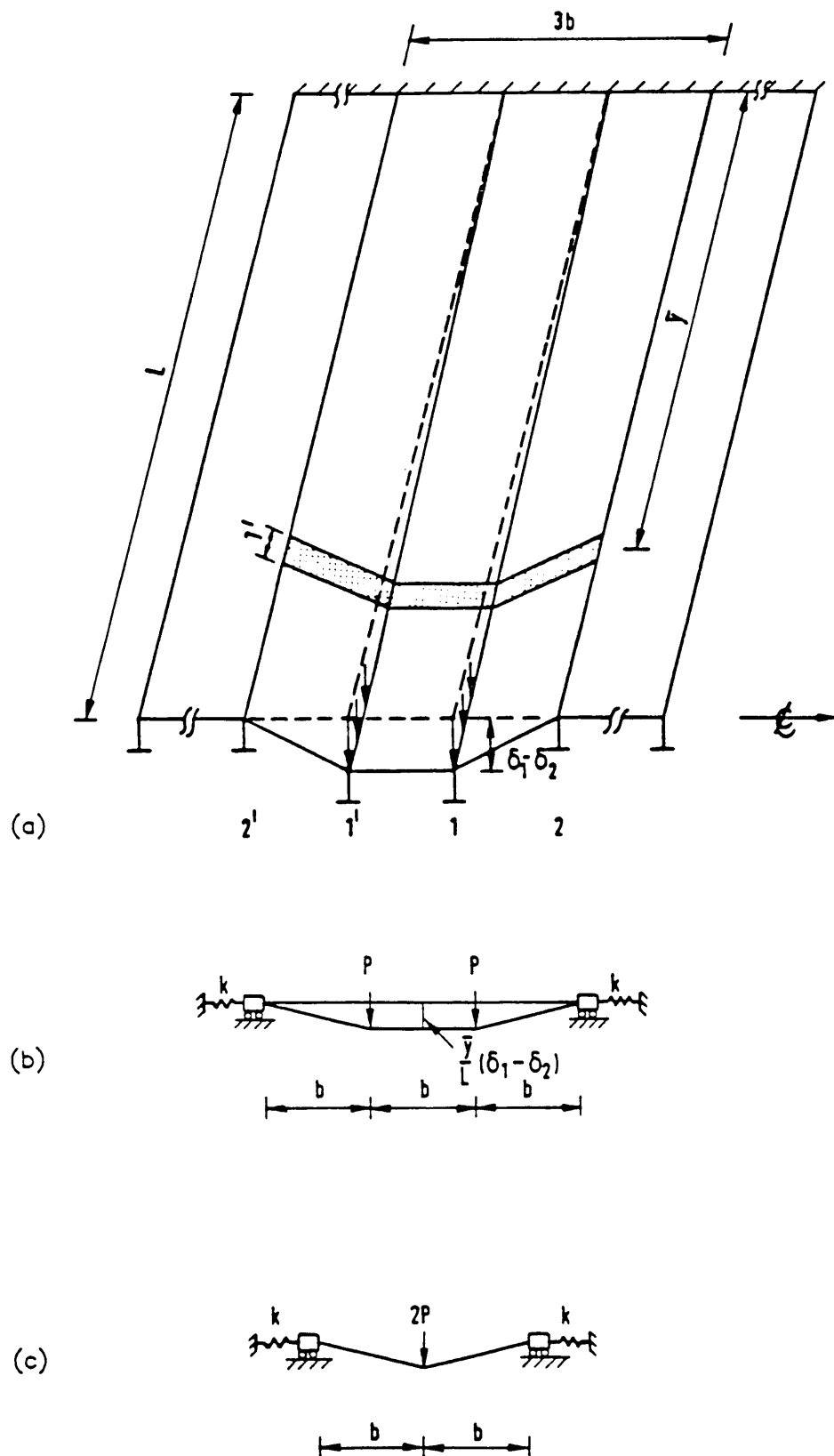
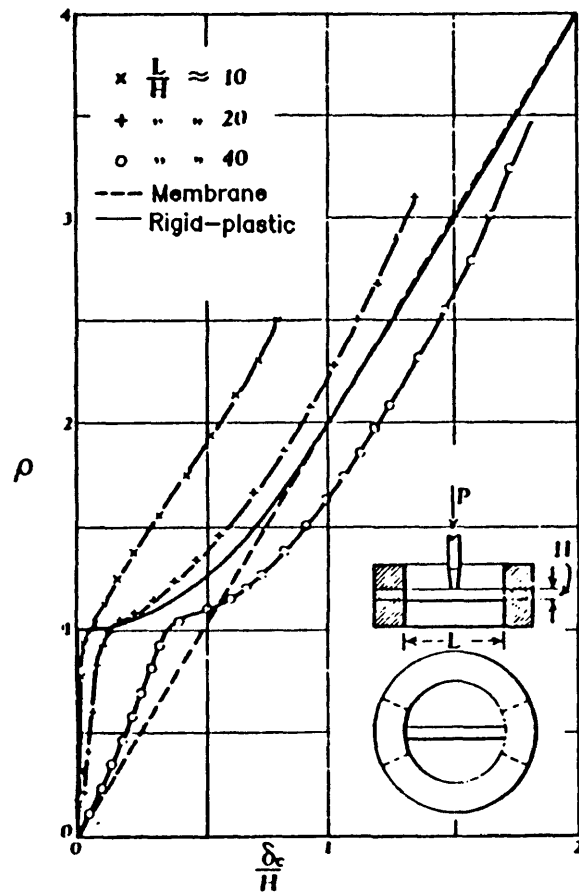
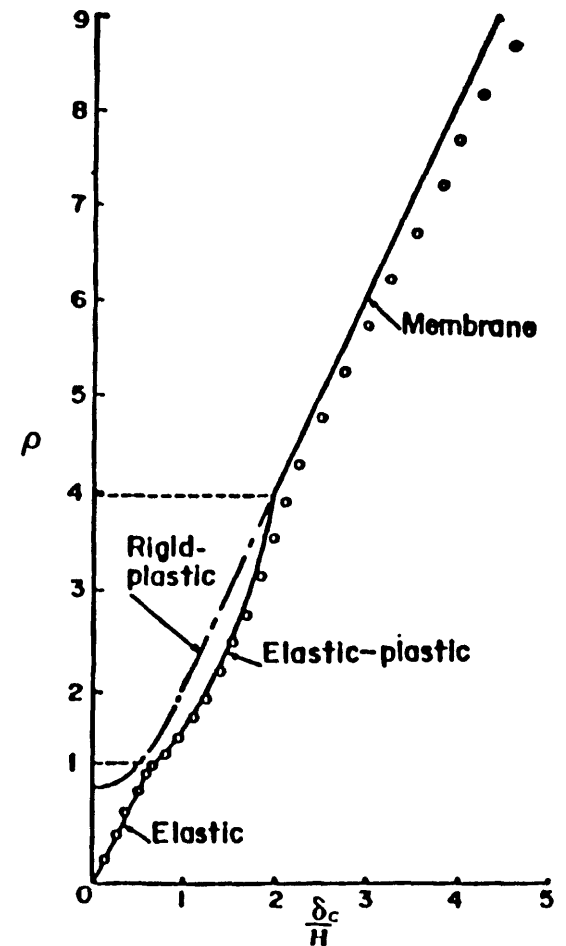


Fig. 3.19 Plate idealization in the transverse direction



(a) Haythornthwaite (1957)



(b) Campbell & Charlton (1973)

Fig. 3.20 Load-deflection predictions for an encastre beam under point load.

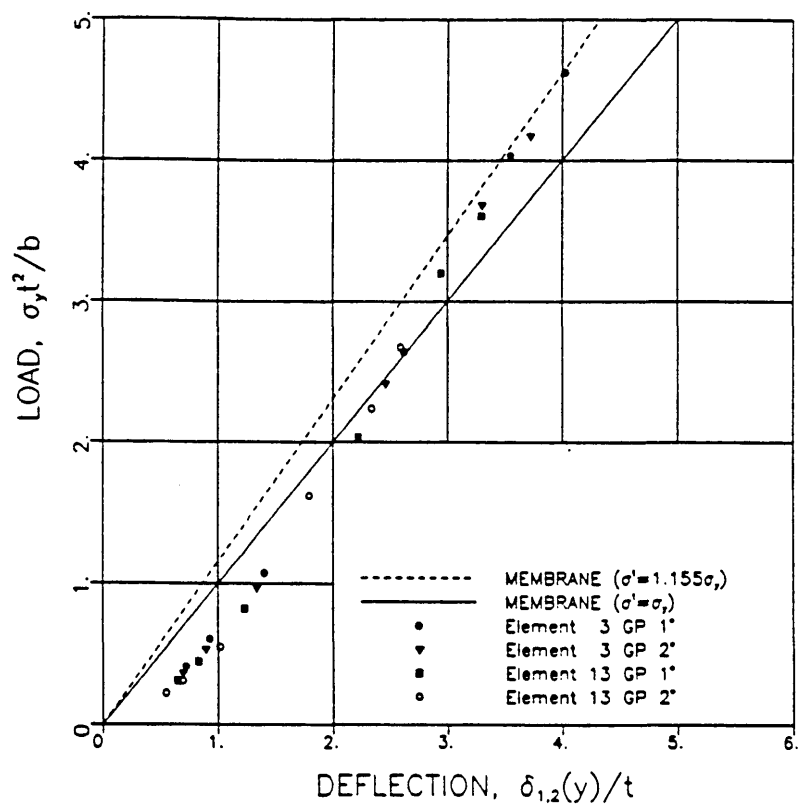


Fig. 3.21(a) Load on stiffener 1 vs. differential deflection

• See Fig. 3.2

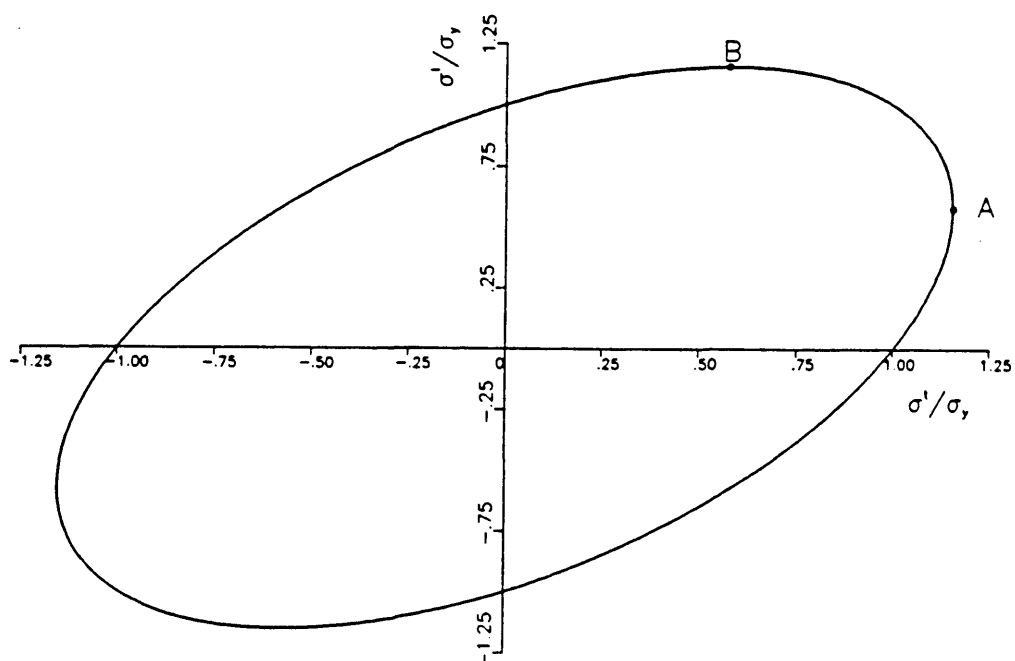


Fig. 3.21(b) Biaxial interaction between the principal stresses

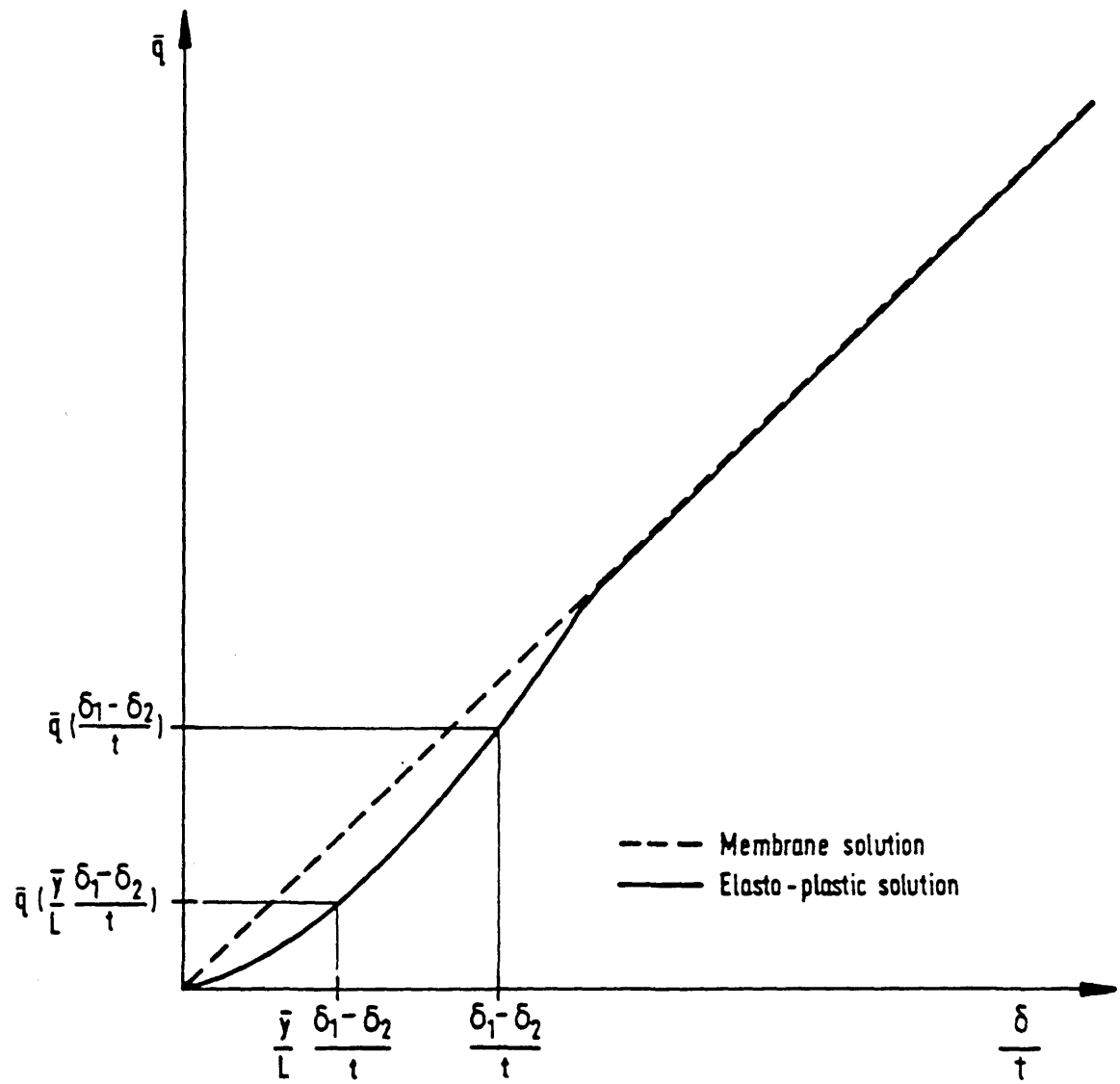


Fig. 3.22(a) Load vs. differential deflection of the unit width strip

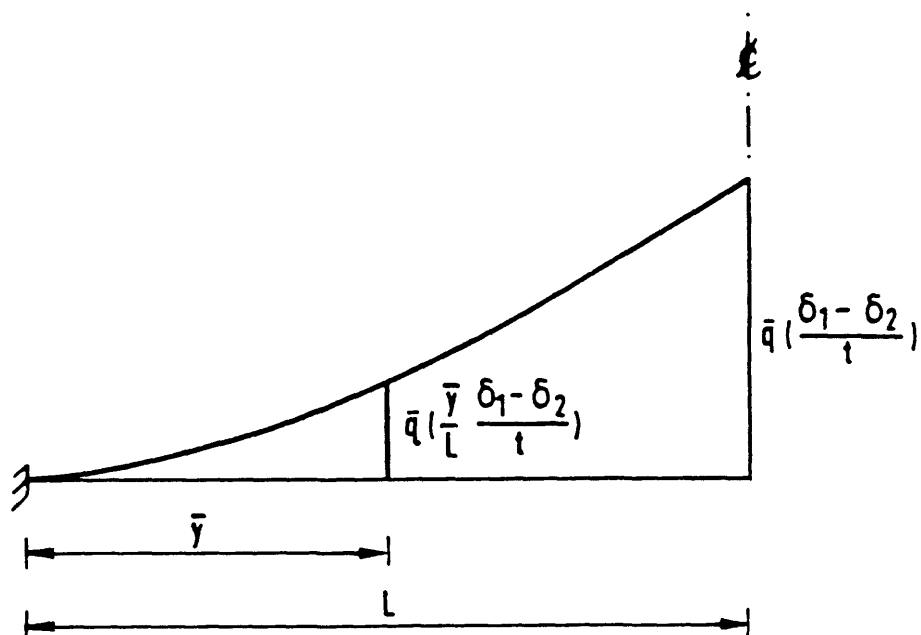


Fig. 3.22(b) Load applied on a stiffener

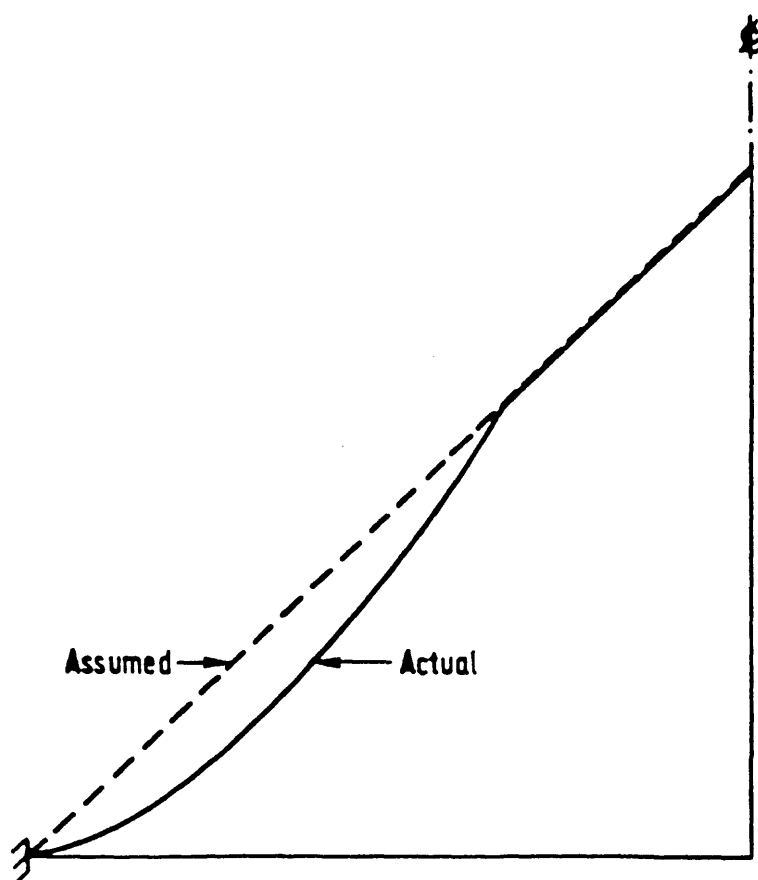


Fig. 3.22(c) Load on the stiffener after the plate strips close to the middle of the beam attain the membrane solution.

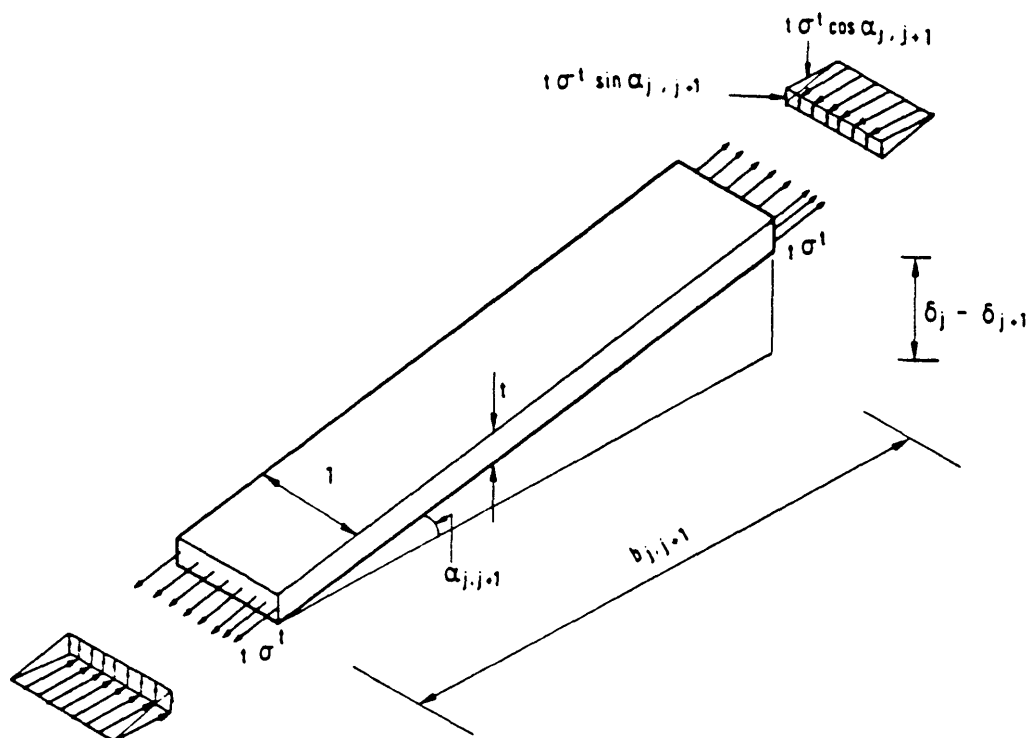


Fig. 3.23 Unit width plate strip under σ^t

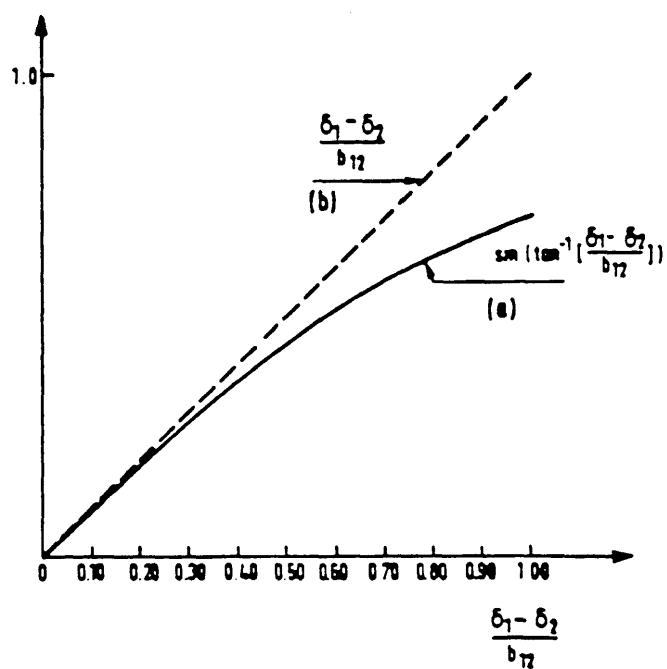


Fig. 3.24 Investigation into the actual angle effects

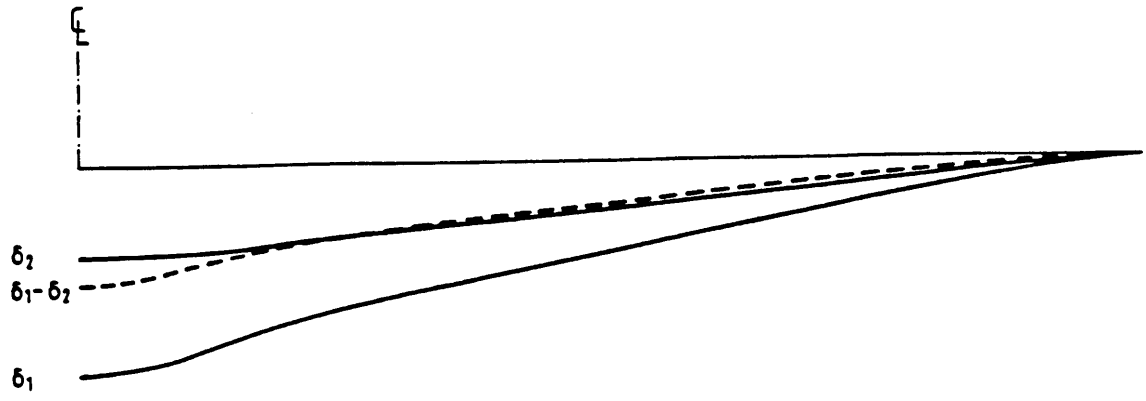
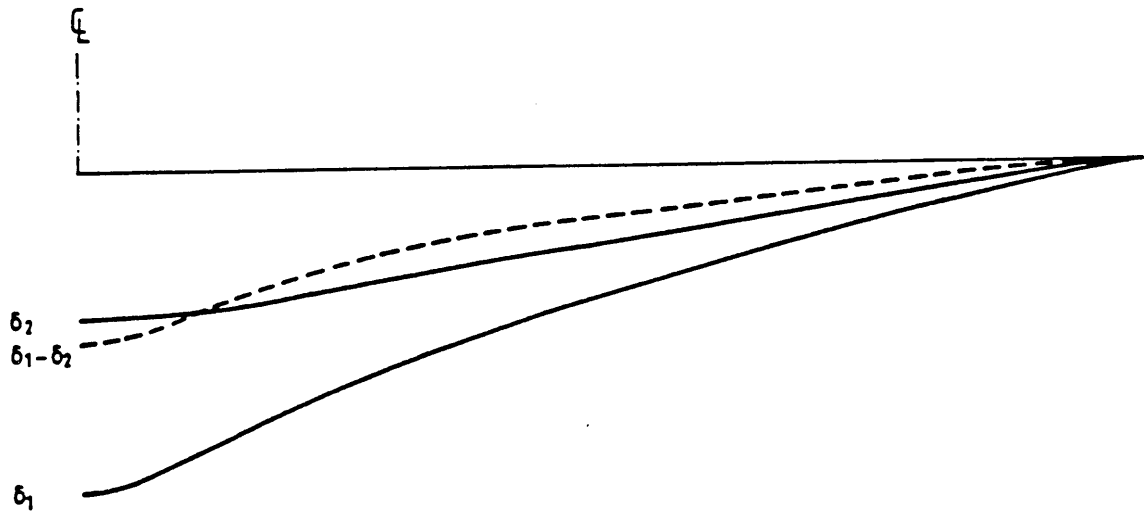
(a) $\rho = 4.349$ (b) $\rho = 5.502$

Fig. 3.25 Deflected shapes for stiffeners 1 and 2

CHAPTER FOUR

AXIALLY RESTRAINED BEAMS ON AN ELASTIC FOUNDATION

4.1 INTRODUCTION

If a stiffened deck restrained against pull-in is subjected to a centrally applied patch loading, the stiffeners directly under the patch loading can be viewed as beams on elastic foundations as discussed in Chapter 3. The objective of this chapter is to obtain an insight into the behaviour of axially restrained Beams on an Elastic Foundation (BEFs), under centrally applied lateral patch loading using finite elements as a reference and comparing the results with those obtained from an analysis assuming a bilinear deformation mode as discussed in Section 3.6.

4.2 CHOICE OF THE GEOMETRY

Three geometries concerning BEFs are considered in this chapter. The first, corresponds to the cross-sectional configuration of a notional beam in deck R02, analysed in Chapter 3. This geometry is called BEFR02 and its cross-sectional configuration is shown in Fig. 3.1(b) with the proportions given in Table 3.1. The other two geometries, correspond to typical plate-stiffener units existing in decks R03 and R04 considered in Section 6.7. These two geometries are called BEFR03 and BEFR04 respectively. The cross-sectional configuration of BEFR03 is shown in Fig. 4.1, with the proportions given in Table 4.1, while the BEFR04 configuration is similar to BEFR02, but with the proportions given in Table 4.2.

The elastic foundation simulates the reaction of surrounding deck plate on the stiffener and may be modelled in the finite element analysis via linear springs of constant stiffness distributed along the beam length. The distributed spring stiffness k_s is derived from Eqn. (3.2), which involves the deflections of stiffeners 1 and 2 with the notation shown in Fig. 3.4. As the present analysis involves the central deflection of stiffener 1 only, an average value for the δ_2/δ_1 ratio was determined after the analyses of decks R02, R03 and R04 had been undertaken. The

distributed spring stiffness was calculated using the formula:

$$\frac{k_s}{\sigma_y} = \frac{t}{b_{1,2}} \frac{\sigma^t}{\sigma_y} \left(1 - \frac{\delta^2}{\delta_1^2}\right)$$

with the notation as described in Sections 3.6.1 and 3.6.1.3, and values for k_s corresponding to the various beams analysed are shown in Table 4.3.

As the distributed springs model the reactions of the deck plate on the externally loaded stiffener 1, according to the notation in Fig. 3.4, the overall load versus central deflection behaviour of the decks studied in Chapter 3 and 6 can be predicted by analysing this BEF model. Comparisons between the response of the deck and the BEF representation are presented later in this chapter.

The patch loadings applied to the BEFs analysed by finite elements were identical to those applied in decks R02, R03 and R04. In the analytical model presented this loading is considered as a point loading.

4.3 NUMERICAL ANALYSIS

The finite element program FINAS, described in Section 3.3, was used for these analyses. The 3-D beam element, developed by Bates (1987), was chosen to model the beams. For the modelling of the springs, the spring element, implemented initially by Elnashai (1984) as bond-slip or contact element, was used.

The mesh chosen in the present analyses corresponds to the one shown in Fig. 3.2 along the stiffeners. Mesh convergence studies were also undertaken with different meshes having five, six and twelve elements along the semi-span analysed. The twelve element mesh was uniform, as opposed to those with five and six elements. In the graded mesh cases the nodal spring stiffnesses were properly weighted in order to give uniform stiffness along the beam length. It was found that the difference in the load versus central deflection predictions between the five and twelve element meshes was less than 0.5% throughout the range. Therefore the five element mesh, as shown in Fig. 4.2(b), was adopted for the BEF study. The cross-section is provided with a number of sampling points (stations) through its depth for the better modelling of plasticity. These stations are also shown in Fig. 4.2(a) for the two cross-sectional configurations considered

in this chapter.

4.4 ANALYTICAL MODEL

Under the centrally applied point load the BEF is assumed to deform following the incipient velocity field, that is, the collapse mode at limit load. This is a bilinear mode with a central plastic hinge and hinges at the supports. These hinges may be either real hinges at the ends of a simply supported beam or be plastic hinges which would form at encastré supports.

4.4.1 Assumptions

The following assumptions are made in this first derivation (although some will be revised in a later section):

- 1) As customary in this type of analysis, a rigid-plastic material law for plastic hinges is adopted.
- 2) The rotations undertaken by the longitudinal beam segments, in order to form the bilinear mechanism, are assumed to be small.
- 3) The beam deforms in a single mode, described by the bilinear mechanism discussed in Section 3.6.

Several assumptions can be made in the formulation of the compatibility equation of the beam. In increasing order of complexity these are:

- 4a) The ends are fully restrained and there are only rigid-plastic contributions to the compatibility equation.
- 4b) The ends are fully restrained and both elastic extensibility of the beam between plastic hinges and rigid-plastic contributions are included in the compatibility equation.
- 4c) The ends can pull-in elastically, in addition to the allowances made in (b).

4.4.2 Compatibility equation

Assumption 4(c) is made here. In this derivation Ronalds' (1985) approach, after the analyses presented by Jones (1973) and Hodge (1974), is used.

For the bilinear mechanism shown in Fig. 4.3(a), the spring stiffness k models the stiffness of the support. Simple geometry gives:

$$(L-\Delta)^2 + \delta_c^2 = (L+e)^2$$

where

$$e = e^e + e^p$$

After expansion in a Taylor series and truncation we get the approximation:

$$\begin{aligned} \frac{e^p}{L} &\approx \frac{1}{2} \left(\frac{\delta_c}{L} \right)^2 - \frac{\Delta}{L} - \frac{e^e}{L} \\ &= \frac{e^s + (\lambda-1)e^h}{L} \end{aligned} \quad (4.1)$$

Here Δ is the elastic pull-in at the support, e^e is the elastic extension of the beam between the support and the central hinge, δ_c is the central deflection, L is the half-span of the beam and e^s , e^h are the plastic extensions at the sagging and hogging plastic hinges, respectively, which together give the total plastic extension e^p . Finally, λ is the number of plastic hinges ($\lambda=1, 2$, for simple supports and encastré supports respectively).

The axial force in the beam is approximately equal to its horizontal component, N , shown on Fig. 4.4. This force is non-dimensionalized with respect to the squash load $N_0 = \sigma_y A$ and denoted by n . Here A is the beam cross-sectional area and σ_y is the yield stress. The combined elastic contributions in the compatibility Eqn. (4.1) are

$$e^e + \Delta = n \frac{\sigma_y}{E} \left(1 + \frac{1}{\mu_1} \right) L$$

where E is the elastic modulus and μ_1 is defined as:

$$\mu_1 = \frac{k L}{E A}$$

Differentiating Eqn. (4.1) with respect to δ_c yields:

$$\left(1 + \frac{1}{\mu_1}\right) \frac{\sigma_y}{E} \frac{dn}{d\delta_c/L} + \frac{de^s + (\lambda-1)de^h}{(d\delta_c/L)L} - \frac{\delta_c}{L} = 0 \quad (4.2)$$

The interaction between the axial and flexural stress resultants at a plastic hinge can be expressed as

$$m^i = C_{02}^i n^2 + C_{03}^i n + C_{04}^i \quad (4.3)$$

where C_{02}^i , C_{03}^i , C_{04}^i are coefficients depending on the cross-sectional geometry, as described in Appendix A. Equation (4.3) is applicable to cross-sections made up by the assembly of up to three rectangles, I-sections for example. Expressions for the coefficients are given by Ronalds (1985, 1990). The superscript i takes the values s and h for sagging and hogging plastic hinges respectively. Finally, m^i is defined as follows:

$$m^i = \frac{M^i}{M_0}$$

with M^i being the moment at a plastic hinge i and M_0 the plastic moment of the beam cross-section in the absence of axial force. The ratio

$$\frac{de^i}{d\delta_c/L}$$

is the ratio of the centroidal axis extension/contraction increment at a plastic hinge i to the corresponding infinitesimal change of beam inclination $d\delta_c/L = d\theta$. As shown in Fig. 4.3(b)

$$\frac{de^i}{d\delta_c/L} = z^i$$

where z^i is the distance between the instantaneous centre of rotation and the centroidal axis. This approach is based on the incremental theory of plasticity, Hill (1950). From normality,

$$z^i = - \frac{dM^i}{dN} = - \frac{Z_p}{A} \frac{dm^i}{dn} \quad (4.4)$$

In the above expression Z_p is the cross-sectional plastic modulus. Equation (4.2) then becomes:

$$\left(1 + \frac{1}{\mu_1}\right) \frac{\sigma_y}{E} \frac{dn}{d\delta_c/L} + \frac{z^{s+(\lambda-1)}z^h}{L} - \frac{\delta_c}{L} = 0 \quad (4.5)$$

Using Eqn. (4.3) and defining the coefficients as:

$$C_1 = \left(1 + \frac{1}{\mu_1}\right) \frac{\sigma_y}{E}$$

$$C_\ell = \left(\frac{Z_p}{AL}\right) (C_{0\ell}^s + (\lambda-1)C_{0\ell}^h) \quad , \quad \text{for } \ell = 2, 3, 4$$

Equation (4.5) becomes:

$$C_1 \frac{dn}{d\delta_c/L} - 2C_2 n - C_3 - \frac{\delta_c}{L} = 0 \quad (4.6)$$

making use of Eqn. (4.4) as well. Equation (4.6) is the full compatibility equation, with general solution of the form:

$$n = C \exp\left(\frac{2C_2}{C_1} \frac{\delta_c}{L}\right) - \frac{1}{2C_2} \frac{\delta_c}{L} - \frac{C_3}{2C_2} - \frac{C_1}{4C_2^2} \quad (4.7)$$

where, C is a coefficient calculated from the initial values for the horizontal component of the axial force, n_I , and the central deflection, δ_{IC} . In the particular case of rigid supports which cannot pull-in (assumption 4(b) in Section 4.4.1) then:

$$\mu_1 \rightarrow \infty \quad \text{and} \quad C_1 = \sigma_y/E$$

If, further, only rigid-plastic extension contributions are considered (assumption 4(a) in Section 4.4.1) then

$$C_1 = 0$$

and Eqn. (4.6) can be solved to give the n value:

$$n = - \frac{1}{2C_2} \left(\frac{\delta_c}{L} + C_3 \right) \quad (4.8)$$

For supports rigid against pull-in, the most suitable choice between (4.6) and (4.8) is dependent on the cross-sectional geometry of the beam. If elastic extensions are not very small in comparison with plastic extensions throughout the deformation history Eqn. (4.6) should be used. Two cases were experienced by the author where Eqn. (4.6) was significantly more accurate than Eqn. (4.8). The first is an encastré beam with asymmetric I-section deforming from the initial condition $n_I = 0$. In this case the extensions at the support and centre hinges are of opposite sign at early stages of the behaviour and, therefore, the elastic extensions may be considerable in comparison with the net plastic extensions. The second case is a cruciform section with wide horizontal segments, see Fig. 4.1. As this cross-section is symmetric, the centroidal axis lies inside the horizontal segment. For much of the response, the instantaneous centre of rotation also lies inside the horizontal part of the section and thus its distance from the centroidal axis is small. Therefore the increment of plastic extensions corresponding to a small increase in the deflection δ_c , that is,

$$(z^s + (\lambda-1)z^h)(d\delta_c/L)$$

is proportional to the term

$$z^s + (\lambda-1)z^h$$

which is small and therefore the plastic extensions are comparable with their elastic counterparts.

Finally, an extreme case is mentioned. It occurs when only one plastic hinge is present and the beam centroidal axis contracts at the hinge location. This happens, for example, in the case of a simply supported beam of an asymmetric I-section having the bottom (tension) flange bigger than the top one, while $n_I \approx 0$. Here elastic extensions must be considered. The axial force build-up shifts the plastic neutral axis towards the centroidal axis. Plastic extensions, therefore, become increasingly less for a given change in the rotation of the mechanism, as the centroidal axis is approached. The conclusion drawn from this discussion is

that rigid-plastic analysis, without elasticity effects, should be used with care, as its accuracy is very dependent on the geometry under consideration.

It is noted that in all the above remarks concerning the choice between Eqns (4.6) and (4.8) a bilinear mode is assumed. As discussed in later sections this mode is an approximation for BEFs.

4.4.3 Equilibrium equation

The structure under loading is shown in Fig. 4.4. Consideration of the equilibrium of moments with respect to the support yields

$$\frac{PL}{2} = \frac{k_s \delta_c L^2}{3} + M^s + (\lambda - 1)M^h + \delta_c N$$

because of no shear at mid-span due to symmetry in both the structure and the loading. Here P is the externally applied concentrated loading. If we non-dimensionalize the applied load with respect to

$$P_0 = \frac{2\lambda M_0}{L},$$

where P_0 is the limit load of the beam under central point loading, we obtain:

$$\rho = \frac{1}{\lambda} \left[\frac{k_s L^3}{3\sigma_y Z_p} \frac{\delta_c}{L} + \frac{AL}{Z_p} (C_2 n^2 + C_3 n + C_4 + \frac{\delta_c}{L} n) \right] \quad (4.9)$$

4.5 COMPARISON BETWEEN ANALYTICAL AND NUMERICAL RESULTS

The analytical model proposed in Section 4.4 was used to predict the behaviour of the BEF geometries mentioned in Section 4.2. The results obtained from the analytical models are compared with numerical results generated using finite element analysis. This comparison concerns the applied load histories and the axial force with increasing central deflection.

4.5.1 Applied load versus central deflection

The predicted strength increases with increasing deflection for the three beam geometries considered are illustrated in Figs 4.5 to 4.7. The results of the analysis are plotted as continuous lines, while the numerical predictions for the BEF are represented by full circles. Both compare very well at the early stages of the behaviour, up to a deflection of the order of 6% of the half-beam length L . Subsequently, the two predictions diverge with the analysis always predicting higher values than those obtained by finite elements. This phenomenon is investigated in detail in a later section of this chapter.

Comparison between the behaviour of a deck and its corresponding BEF model is also made. The central deflection from a BEF analysis is compared with the central deflection of a loaded stiffener in the deck (stiffener 1). Results corresponding to the deck behaviour is plotted using full squares. The comparison shows that at moderately large central deflections, that is, in the range of deflections $0.02L$ – $0.06L$, the deck exhibits slightly higher strength than the corresponding BEF model. At deflections in excess of $0.10L$ approximately, however, the deck response falls closer to the numerical predictions of the BEF.

The elastic prediction, as obtained from Hetenyi (1946) after modification of the central deflection due to transverse shear deformation, is also given in these plots by a line under the key 'elastic'. For the geometries studied here the transverse shear deformation appeared to be a significant percentage of the total elastic deflection, accounting for as much as 15% of the deflection in one of the cases studied.

4.5.2 Axial force along the beams

Figures 4.8 to 4.10 illustrate the manner in which axial force builds up in the BEFs under increasing central deflection. The same symbols are used as in Figs 4.5 to 4.7. The agreement between the analysis and the finite element results is very good for central deflections less than about $0.06L$, but at higher deflections the divergence can become considerable, especially for the cruciform geometry BEFR04, as shown in Fig. 4.10. The source of the discrepancy between the predictions for the axial force is further investigated in the next sections.

4.6 DEFLECTED SHAPE PLOTS

To understand the discrepancies which occur in the predicted growth of axial force with deflection, it is worth re-examining the assumptions made in deriving the compatibility equations and comparing them with those implicit in the numerical analysis. The assumption concerning the deflected shape is re-examined in this section.

The deflected shapes at the various load levels for the three BEFs analysed are shown in Figs 4.11 to 4.13, corresponding to numerical analyses BEFR02, BEFR03 and BEFR04, respectively. These figures show a gradual departure from the assumed bilinear mode-shape as the deflection increases. Following the initial stage of bilinear deformation, the segments near the supports cease to deflect. This results in a shorter length of the beam experiencing rotation, as illustrated schematically in Fig. 4.14(a). These observations suggest that the support hinge is spreading towards the middle of the beam. The elastic segment between the beam centre and the inner edge of the plastic segment experiences higher incremental rotation than that assumed by the bilinear mechanism. This incremental rotation is schematically shown in Fig. 4.14(b). The increment of rotation at the ends of the elastic segment BC having a horizontally projected length equal to γL ($\gamma < 1$), is a $1/\gamma$ multiple of the rotation increment experienced by segment AC.

The changing geometry is considered in Fig. 4.14(b). Assuming that the curved segment AB, is plastic and does not experience changes in its shape, it can be postulated from normality that no extensions take place along its length. Thus, from simple geometry:

$$(\gamma L - \Delta)^2 + (\xi \delta_c)^2 = (\gamma L + e)^2$$

with ξ defined in Fig. 4.14(b). With reference to the same figure, and following a procedure similar to that outlined in Section 4.4.2, an equation can be derived equivalent to Eqn. (4.2):

$$\frac{\gamma}{\xi^2} \left(\gamma + \frac{1}{\mu_1} \right) \frac{\sigma_y}{E} \frac{dn}{d\delta_c/L} + \frac{\gamma}{\xi^2} \frac{de^s + (\lambda - 1)de^h}{(\delta_c/L)L} - \frac{\delta_c}{L} = 0$$

As the incremental rotation is $d\theta/\gamma = (d\delta_c/\gamma L)$, the ratio of the extensions at a plastic hinge to this rotation is

$$\frac{de^i}{d\delta_c/\gamma L} = z^i$$

with z^i as in Section 4.4.2.

Finally, an equation similar to (4.6) is obtained but with the C_ℓ coefficients replaced by the following C_ℓ^* values:

$$C_1^* = \frac{\gamma}{\xi^2} \left(\gamma + \frac{1}{\mu_1} \right) \frac{\sigma_y}{E} \quad (4.10(a))$$

$$C_\ell^* = \frac{1}{\xi^2} C_\ell, \quad \text{for } \ell = 2, 3, 4 \quad (4.10(b))$$

The axial force expression (Eqn. (4.7)) is similarly modified by updating the C_ℓ values.

4.7 DETERMINING THE SPREADING OF THE CURVED SEGMENT

In this section a simplified relationship which allows linear spreading of the support hinge is proposed. By defining a threshold level, δ_c^* , to represent the limiting value of central deflection beyond which the bilinear mode is no longer applicable, the parameter γ can be written as:

$$\gamma = 1 - \alpha_1 \left(\frac{\delta_c}{L} - \frac{\delta_c^*}{L} \right) \quad (4.11)$$

Various approaches can be used to define the unknown values for the scalar α_1 and δ_c^*/L in Eqn. (4.11). It could even be assumed that the point about which the elastic part pivots moves away from the support right from the beginning of the central deflection history, that is, $\delta_c^*/L = \delta_{IC}/L$.

The aim of the approach followed here is to trace the threshold deflection δ_c^*/L and to give a more rational method for the prediction of the spreading hinge process. The bilinear mode is assumed and the bending moment diagram between the support and the central hinges predicted by the analytical method of Section 4.4 is plotted at various load steps in Fig. 4.15. In this figure, three typical

bending moment diagrams are shown. The first, Fig. 4.15(a), concurs with the bilinear mode assumed above. However Figs 4.15(b) and 4.15(c) show that the assumption of the bilinear mode becomes invalid after some central deflection δ_c'/L is exceeded because yield is violated. It is noted here that yield is violated at any position where the moment exceeds the current m^s and m^h values. Figures 4.15(b) and 4.15(c) also show that the point of maximum moment travels from the support towards the centre of the beam. If the value of $(1-\gamma)L$ is assigned to the distance between the support and the position of maximum moment, or the distance over which yield is violated, and these values are plotted against the central deflection, a series of curves, as shown in Fig. 4.16, is obtained. Any linear or piecewise linear curve can be fitted to the curves in Fig. 4.16, and values for α_1 and δ_c'/L in Eqn. (4.11) can be determined. This procedure was followed with $(1-\gamma)L$ being the distance between the support and a point defined as the mid-point between the position of peak hogging moment, $m_{\max}^h$, and the position where the moment is equal to the support moment, m^h (Fig. 4.15(c)). The values for α_1 and δ_c'/L after a straight line was fitted to each set of data points, as shown in Fig. 4.16, are given in Table 4.4.

4.8 RESULTS FROM THE MODIFIED COMPATIBILITY EQUATIONS

This section discusses the predictions obtained by the preceding analysis using the compatibility Eqn. (4.6) modified to account for the curvilinear shape of the deformation mode. This modification has been presented in Section 4.6 in relation to the parameters C_ρ^* and γ . Values for the latter are obtained from Eqn. (4.11), using the values given in Table 4.4.

Plots of the axial forces versus central deflection are given in Figs 4.8 to 4.10 by dashed lines. They closely follow the finite element predictions for the BEF and, particularly in Fig. 4.10, remove the large discrepancy between the numerical prediction and the analysis using the C_ρ values. These axial force values were further implemented in the strength Eqn. (4.9) but with the bilinear mode assumption retained in the calculation of the component accounting for the spring forces, that is, the first term within the square brackets. It was found that although the improvement in the prediction of strength was proportional to the improvement in the n value, the analysis still substantially overestimated the numerical prediction for strength at very large deflections. Therefore, this overestimation cannot be attributed to inaccuracies in the prediction of the axial force only.

As mentioned above, the first term inside the square brackets of the equilibrium Eqn. (4.9) accounts for the spring forces generated by a bilinear mode-shape. If, however, the shape postulated by (4.11) is adopted, then this term has to be modified. This is discussed in the following section, leading to re-calculation of the spring forces acting on the curved beam shape.

4.9 USE OF NON-LINEAR MODE-SHAPE IN THE EQUILIBRIUM EQUATION

The shape of the beam, as predicted by Eqn. (4.11) for the values of α_1 and δ_c'/L given in Table 4.4, is calculated by the method described in Appendix B. Calculating the shape from Eqn. (4.11) results in lower values for the spring forces corresponding to the same central deflection δ_c/L . The contribution of these forces to the equation of equilibrium is less than the value given by Eqn. (4.9) (the first term inside the square brackets). To account for this, the reduction factor R_f calculated in Appendix B is used and Eqn. (4.9) becomes:

$$\rho = \frac{1}{\lambda} \left[R_f \frac{k_s L^3}{3\sigma_y Z_p} \frac{\delta_c}{L} + \frac{AL}{Z_p} (C_2^* n^2 + C_3^* n + C_4^* + \frac{\delta_c}{L} n) \right] \quad (4.12)$$

The predictions obtained from Eqn. (4.12) are represented in Figs 4.5 to 4.7 by dashed lines. It is remarkable how close these predictions are to those obtained by the finite analysis, in all three cases considered. The difference between the two predictions of strength given by the continuous and the dashed lines in Figs 4.5 to 4.7 is mainly due to the modification of the first term in Eqn. (4.12) by the reduction factor R_f . As shown in Chapter 5 the strength is almost insensitive to inaccuracies in the axial force value.

In general, this modified prediction can be considered to be very accurate over the entire large deflection range. In addition it is conservative with respect to the overall behaviour of stiffened decks if the dashed line predictions are compared with the full squares in Figs 4.5 to 4.7.

The deflected shapes, as predicted in Appendix B, have been compared with the shapes given by the numerical analysis. This comparison is made at the central deflection corresponding to the final step of the numerical analysis for which results appear in Figs 4.5 to 4.10. Figures 4.17, 4.18 and 4.19 illustrate this comparison for beams BEFR02, BEFR03 and BEFR04, respectively. The deflected

shapes compare favourably given the approximate relationship described by Eqn. (4.11). Near the support, the spread of the hinge is predicted very accurately in all three plots. The length of the curved plastic segment is predicted slightly less accurately, but nonetheless the main features of the shape are clearly present.

Further refinement of an approximate relationship like Eqn. (4.11) is not worthwhile. The further discussion and modification of the analysis presented originally in Section 4.4 had the aim of achieving a better understanding of the phenomena occurring, and shows how a very simple model, after some modification, can represent all the important facets of the actual behaviour. To deal with the behaviour of such BEF models, where the shape changes continuously due to changes in the pattern of loading, a method is required that can predict all these changes (as explained in Section 3.6.4.1). Such facilities exist in a finite element or finite difference package.

In practice, even the simple analysis presented in Section 4.4 can be used for predicting the strength corresponding to a given central deflection. There are two reasons for this. Firstly, the accuracy of the strength prediction at moderate deflections is acceptable for preliminary analysis. Secondly, impending material rupture due to strains occurring at very large deflections limits the maximum deflections, even for ductile materials. Thus, accurate predictions at very large deflections are unnecessary.

As discussed in Chapter 6, the main issue is to predict the strength at a given central deflection. For this, the approximate analysis presented in Section 4.4 can be used. Of course, the refined method presented in Sections 4.6 to 4.9 can also be used if the additional effort is warranted. If queries concerning the history of axial force versus central deflection arise, the refinement of Section 4.7 should be used.

4.10 SPREADING OF THE SUPPORT HINGE

In order to investigate the possibility of hinge spreading in axially restrained beams, it is helpful to consider the bending moment diagram of an equivalent simply supported beam under the externally applied load. In the case of a beam on an elastic foundation this should also include the spring forces. Hinge spreading is expected to start from the region where the bending moment diagram of the equivalent simply supported beam is nearly flat and to progress towards the position where the moment diagram exhibits its maximum slope. In the case of a

beam on an elastic foundation the moment diagram is nearly flat in the region of the supports after some initial central deflection (see Fig. 4.15) and the hinge spreads from the support towards the centre of the beam. This process is corroborated by the bending moment and shear force diagrams obtained by numerical analysis of model BEFR03 presented in Figs 4.20 and 4.21, respectively. In these figures the notation adopted in Chapter 3 is used, that is, the total height of the frame surrounding these plots equals twice the plastic values for the bending moment and shear force corresponding to the cross-sections under consideration. As shown in Fig. 4.20, cases (a) through (c), the initially localized support hinge spreads towards the beam centre line. Over the plastic segment the bending moment is virtually constant. Therefore the shear force along this segment, seen in Figs 4.21(b) and 4.21(c), is zero. As the beam is nearly horizontal under the assumption of small rotations, the axial force along the plastic segment is also approximately constant.

Hinge spreading in axially restrained beams subjected to a symmetrically distributed loading is discussed in Chapter 5. The remarks made above concerning the spreading of plastic hinges hold for this case as well. However, for plates this may not be the rule. Hook & Rawlings (1969b) illustrate the process of spreading plasticity in an unstiffened plate under distributed lateral pressure which is restrained against pull-in. These authors show the spreading process occurring from the support towards the centre of the plate, while in the case of a beam under uniformly distributed loading, the spreading would have started from the centre.

4.11 CONCLUSIONS

A simple bilinear mode-shape mechanism has been presented in this chapter to predict the behaviour of BEFs restrained at their supports against pull-in. The agreement between the analysis and corresponding finite element results is very good, especially for strength predictions.

In its original form, the analysis failed to adequately predict the history of the axial force along the beam under increasing central deflection. This weakness was found to be associated with higher rotational increments experienced at the ends of the elastic segment, and their effect was incorporated into the compatibility equation. Predictions for the axial force based on this modified expression compare favourably with the numerical results.

As an additional refinement to the method, an alternative shape to the originally assumed bilinear mode-shape was considered in both the equilibrium and compatibility equations. The prediction for strength based on this shape follows the finite element results very closely.

Under distributed loading, axially restrained beams exhibit the phenomenon of spreading plasticity along their length. In a BEF under centrally applied patch or point loading, plasticity spreads from the supports towards the beam centre line.

Finally, this BEF model can be used to predict the load-deflection history of an axially restrained single bay stiffened deck under centrally applied patch loading, provided the proper distributed spring stiffness is used, as discussed in Sections 4.2 and 4.5.1.

TABLE 4.1 Cross-sectional geometry

DECK	G E O M E T R Y					MATERIAL
No.	t (mm)	t_w (mm)	h_w/t_w	b/t	L/b	E/σ_y
R03 ⁽¹⁾	4.76	8.72	6.72	26.67	6	837

(1) For notation see Fig. 4.1(b)

TABLE 4.2 Cross-sectional geometry

DECK	G E O M E T R Y						MATERIAL
No.	t (mm)	⁽¹⁾ t_w (mm)	b/t	h_w/t_w	b_f/t_f	L/b	E/σ_y
R04 ⁽²⁾	4.76	8.72	26.67	8.18	5.73	6	837

(1) $t_f = t_w$

(2) For notation see Fig. 3.1(b)

TABLE 4.3 Distributed spring stiffness

BEAM No.	$k_s/\sigma_y \times 10^3$
BEFR02	21.0
BEFR03	23.4
BEFR04	21.5

TABLE 4.4 Values for the parameters in Eqn. (4.11)

BEAM No.	α_1	δ'_c/L
BEFR02	3.500	0.000
BEFR03	3.182	0.030
BEFR04	3.278	0.050

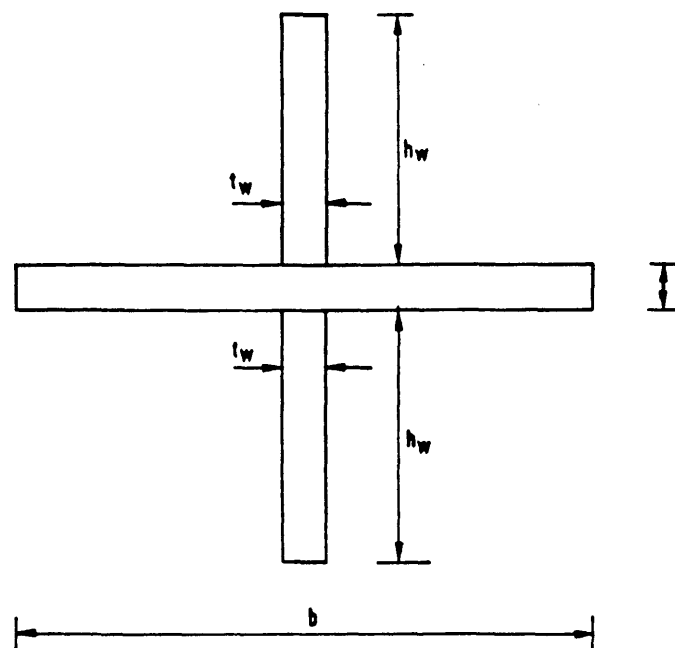
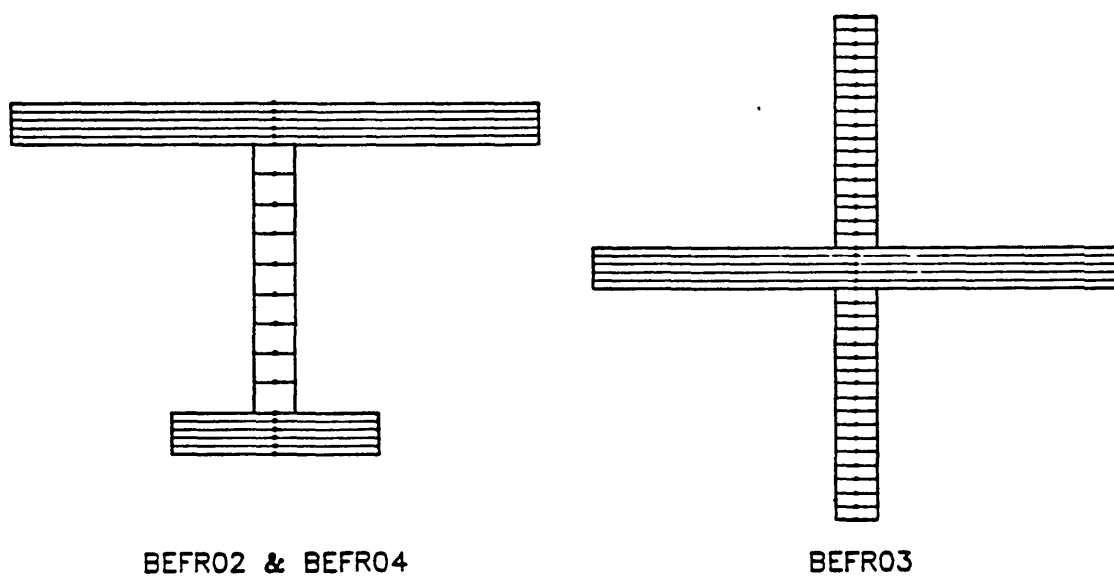
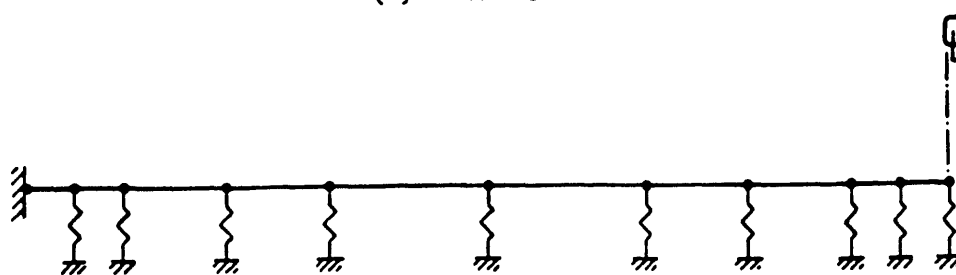


Fig. 4.1 Cross-sectional geometry for BEFR03



(a) Stations



(b) Mesh

Fig. 4.2 Finite element modelling

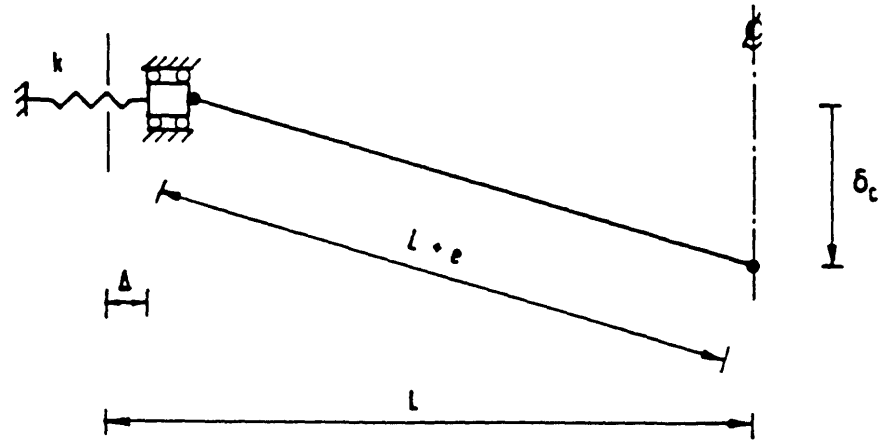


Fig. 4.3(a) Beam deformation geometry

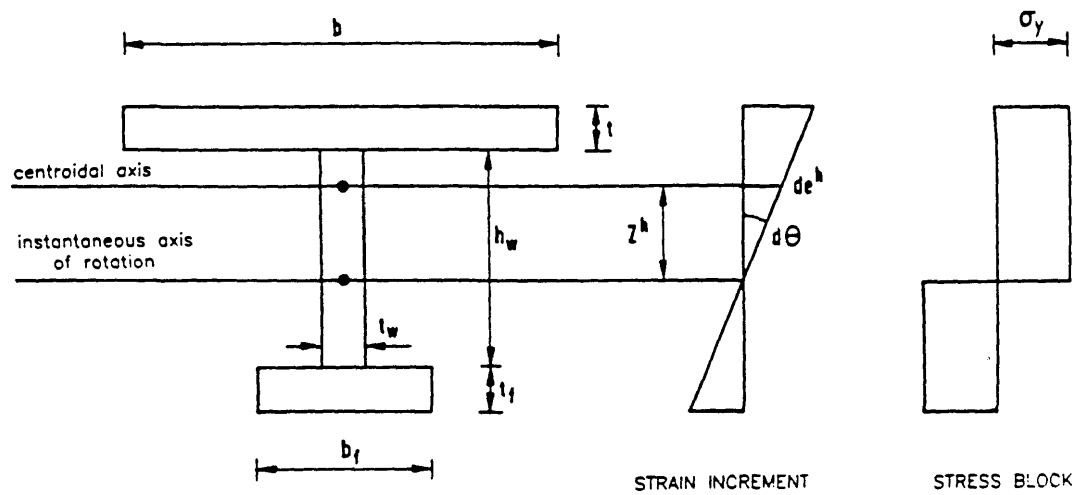


Fig. 4.3(b) Cross-sectional deformations and resulting stress block

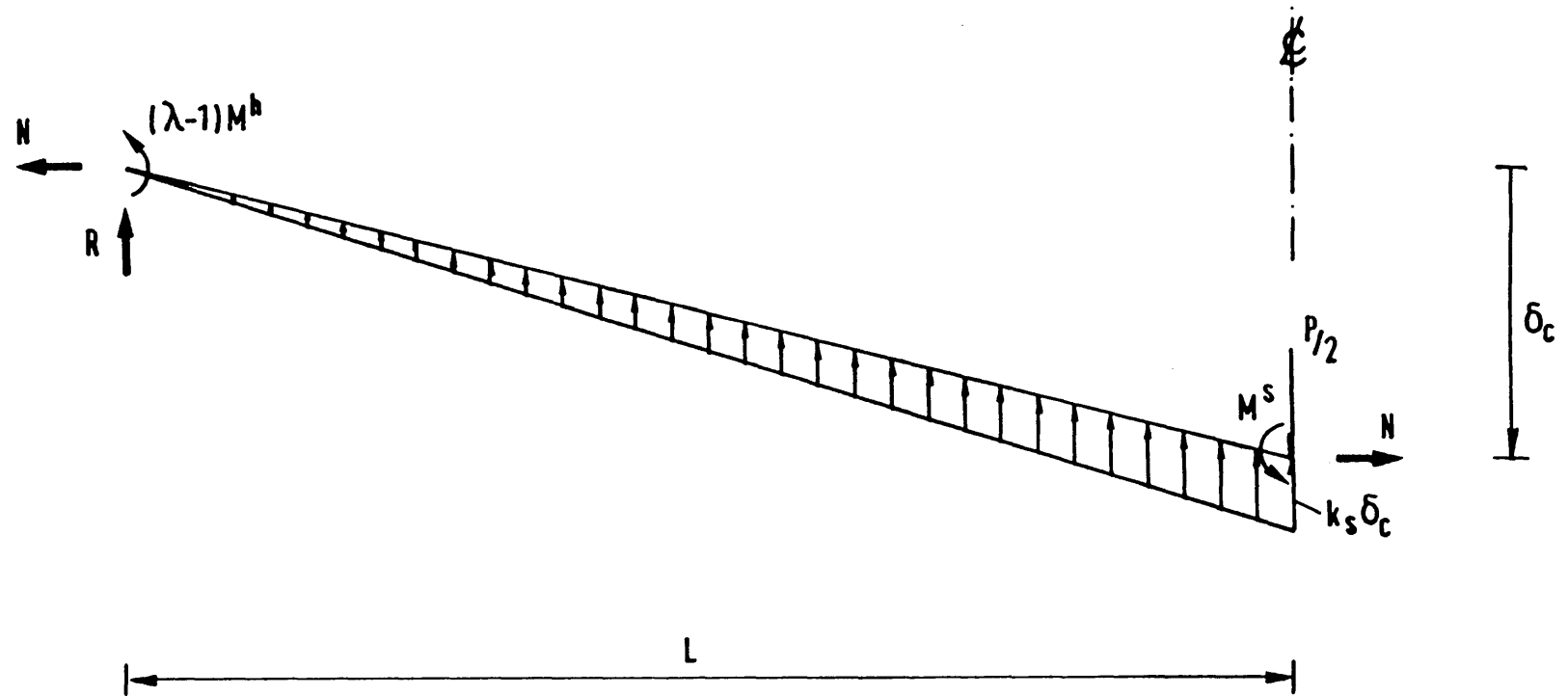


Fig. 4.4 Forces acting on the beam

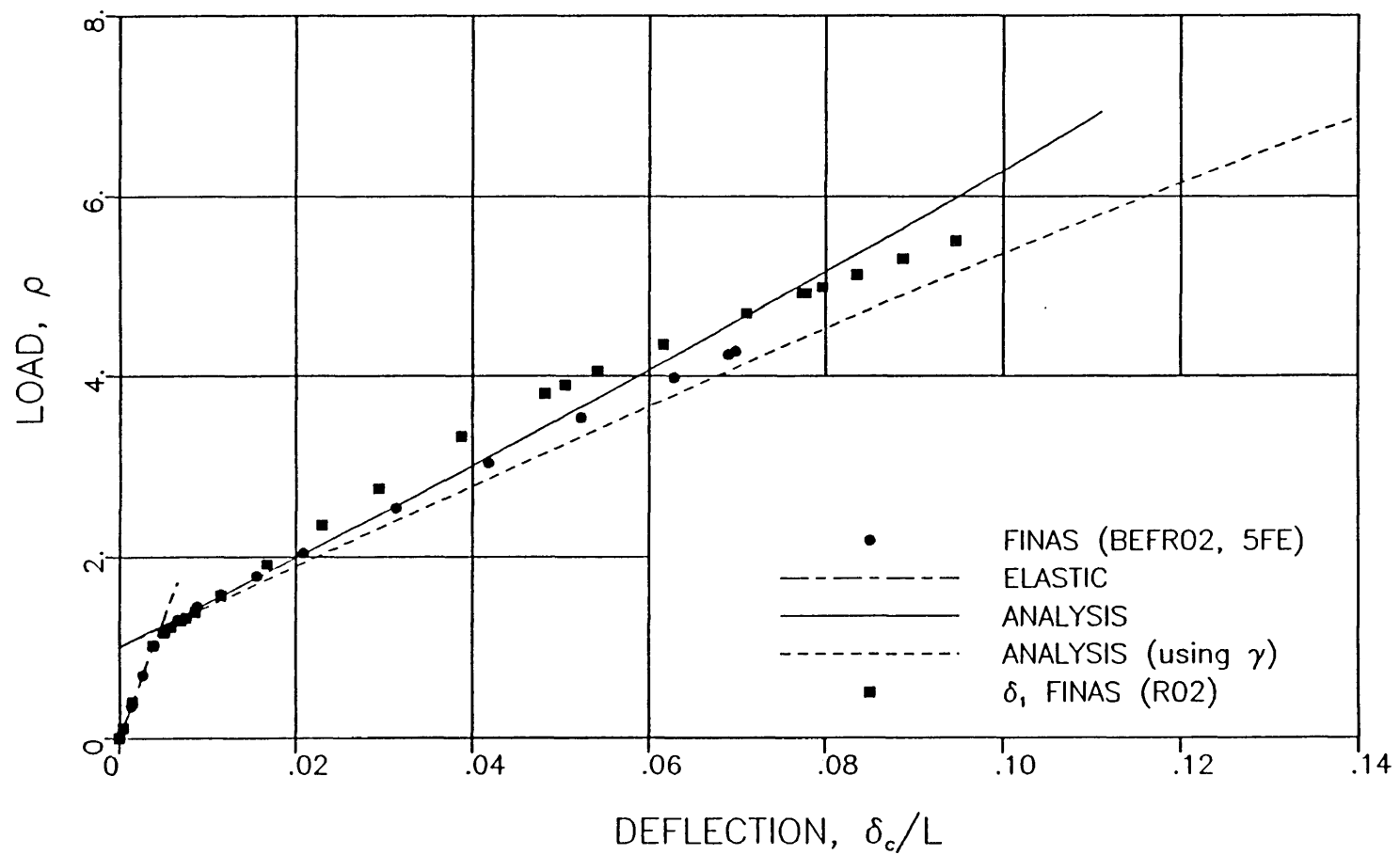


Fig. 4.5 Load vs. deflection for the BEF corresponding to R02

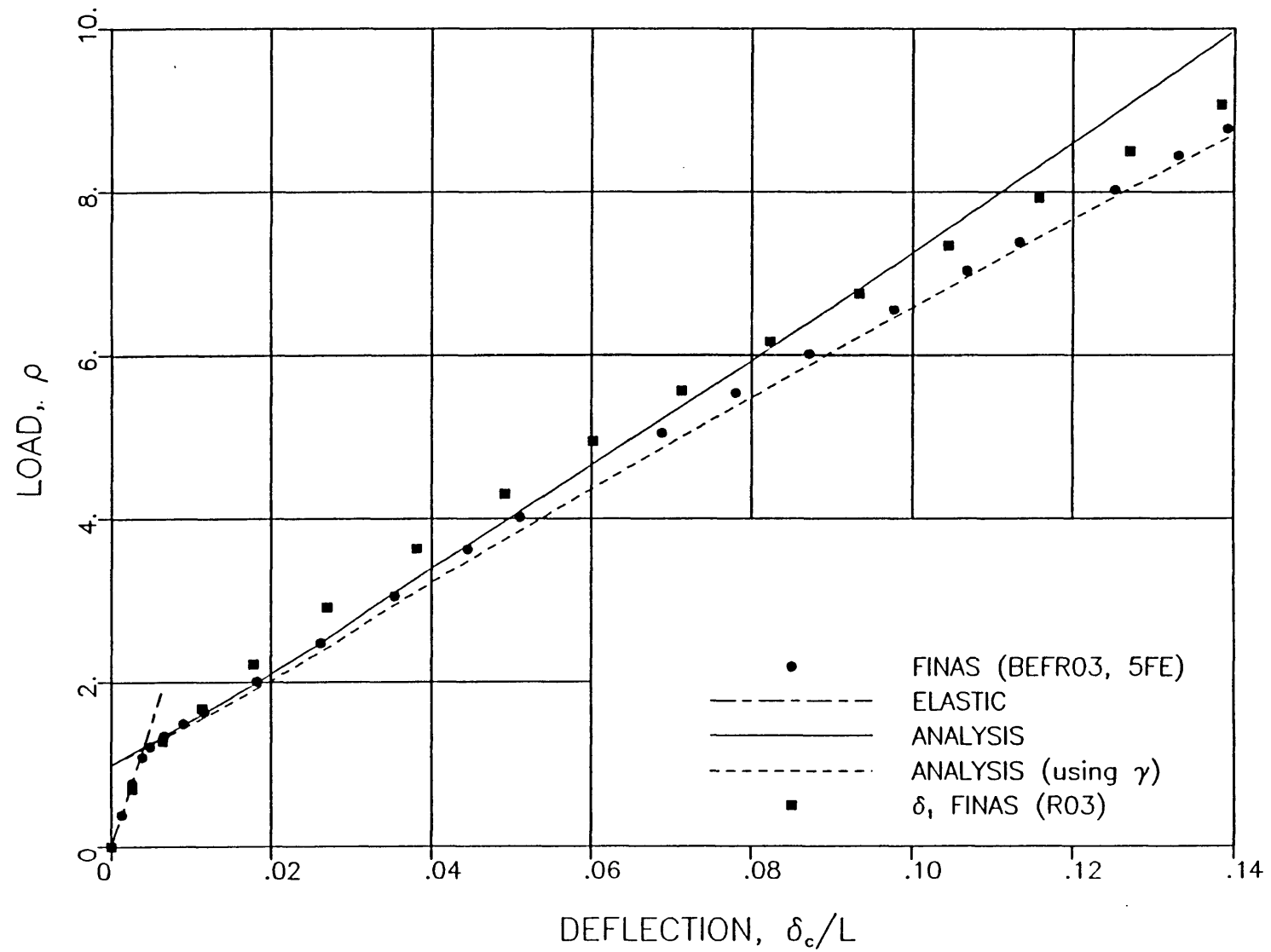


Fig. 4.6 Load vs. deflection for the BEF corresponding to R03

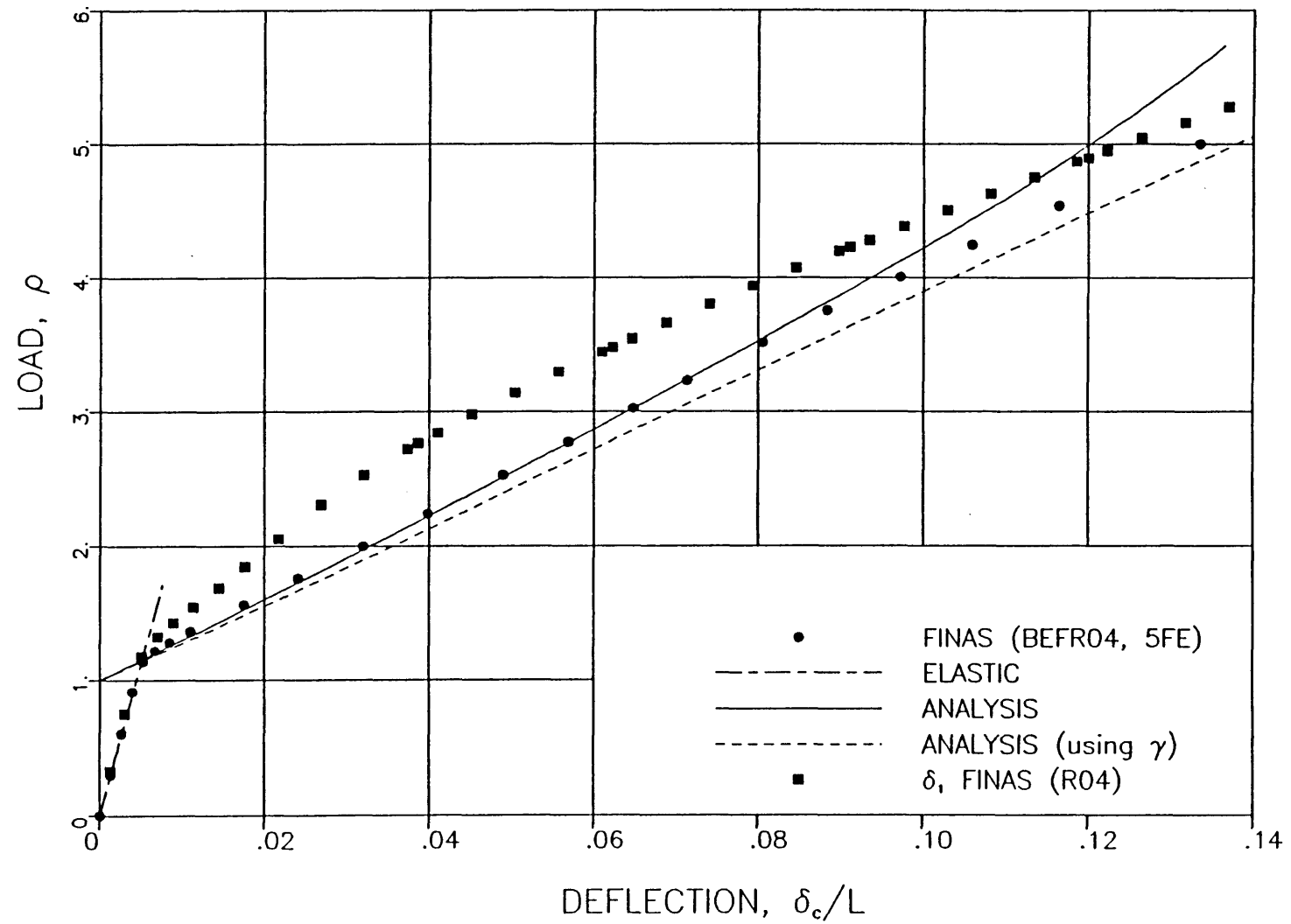


Fig. 4.7 Load vs. deflection for the BEF corresponding to R04

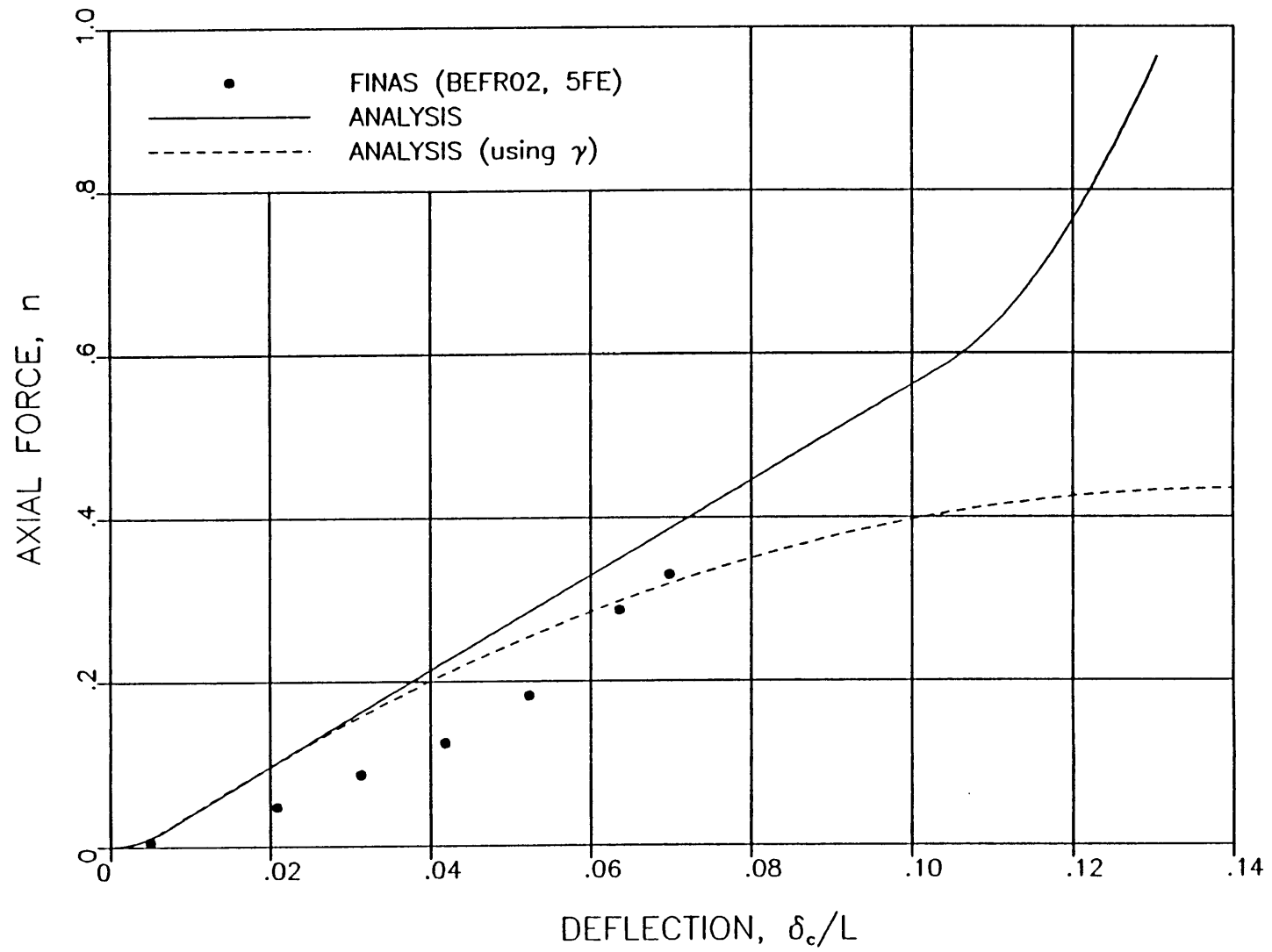


Fig. 4.8 Axial force vs. deflection for the BEF corresponding to R02

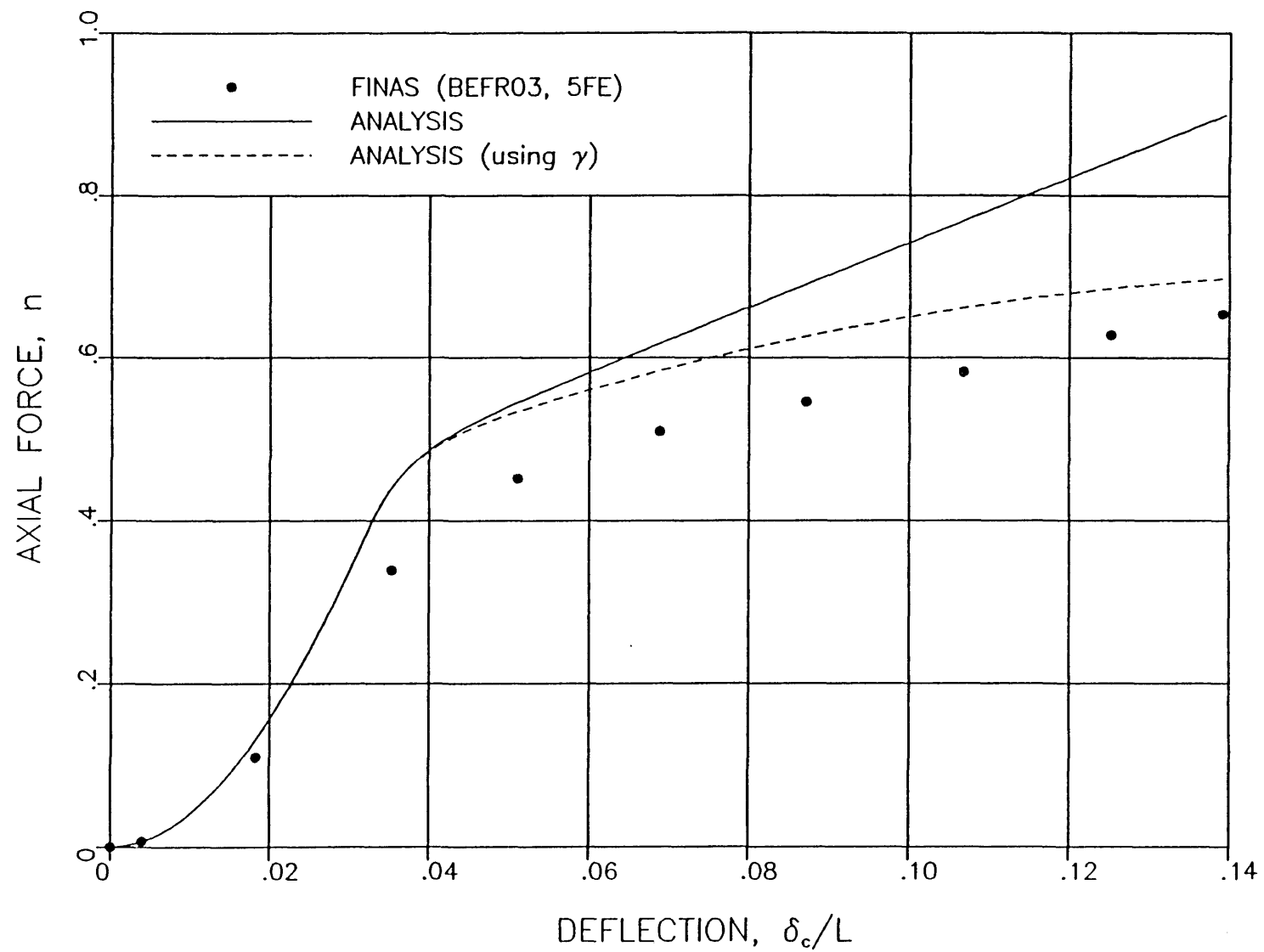


Fig. 4.9 Axial force vs. deflection for the BEF corresponding to R03

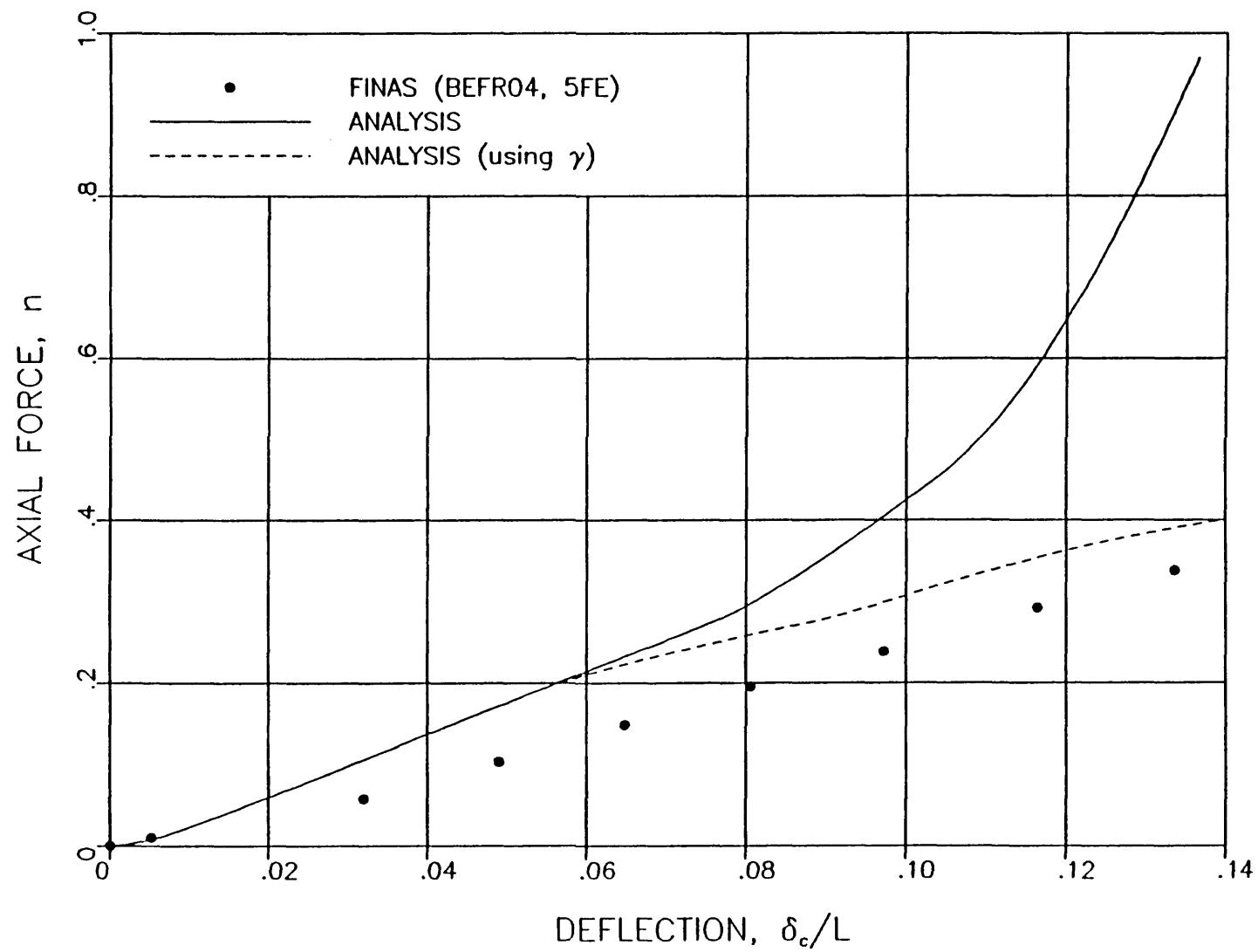


Fig. 4.10 Axial force vs. deflection for the BEF corresponding to R04

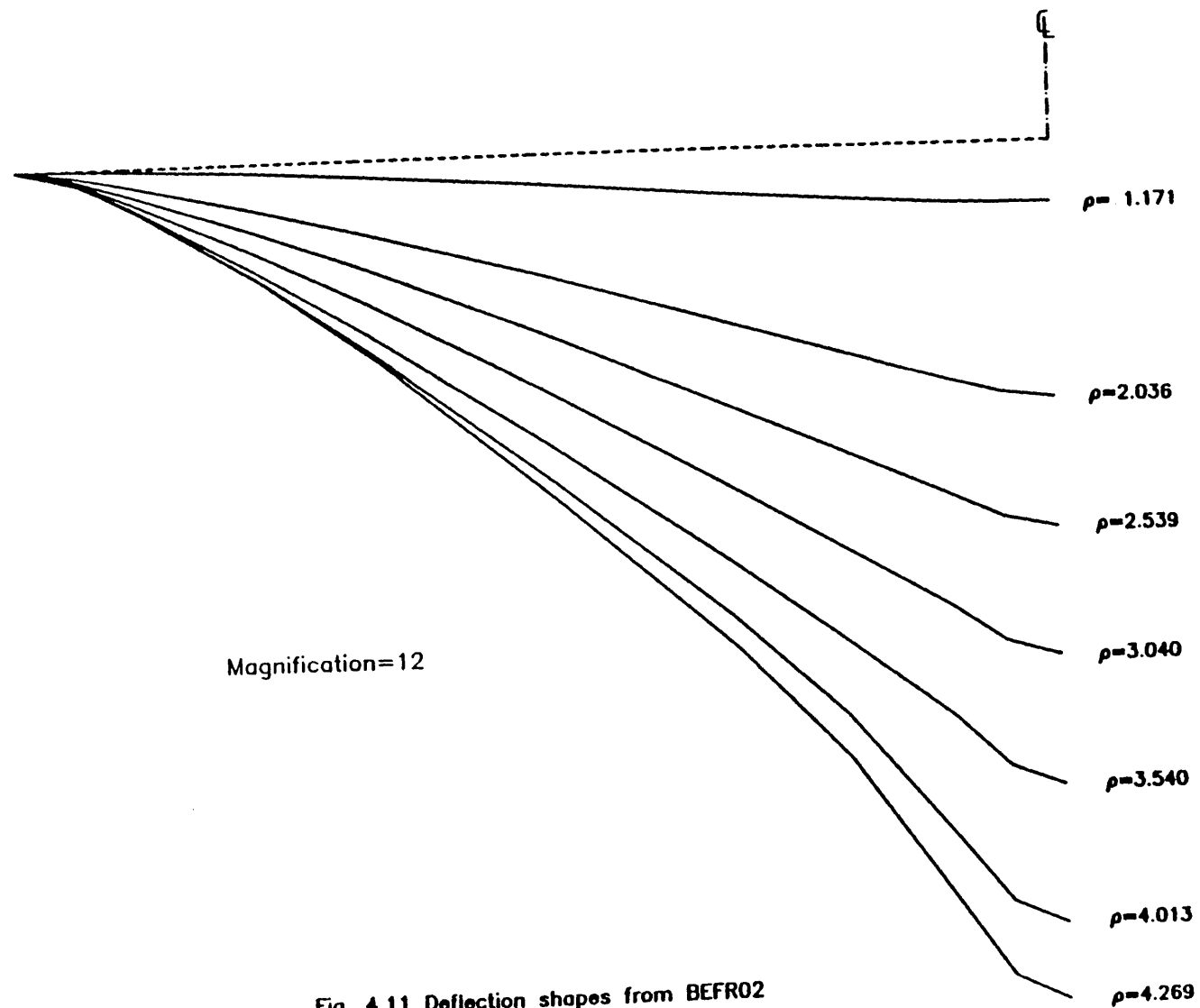


Fig. 4.11 Deflection shapes from BEFR02

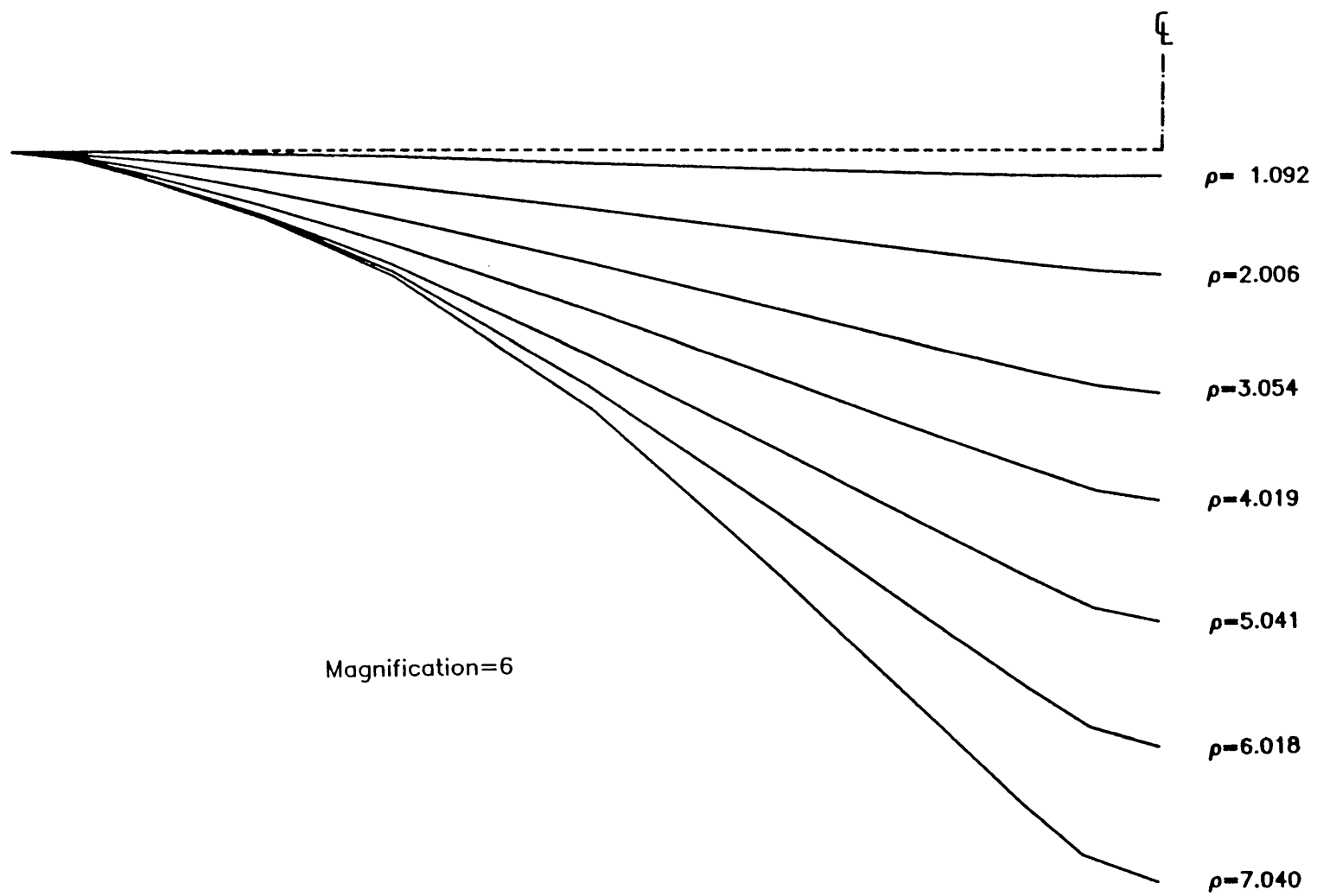


Fig. 4.12 Deflection shapes from BEFR03

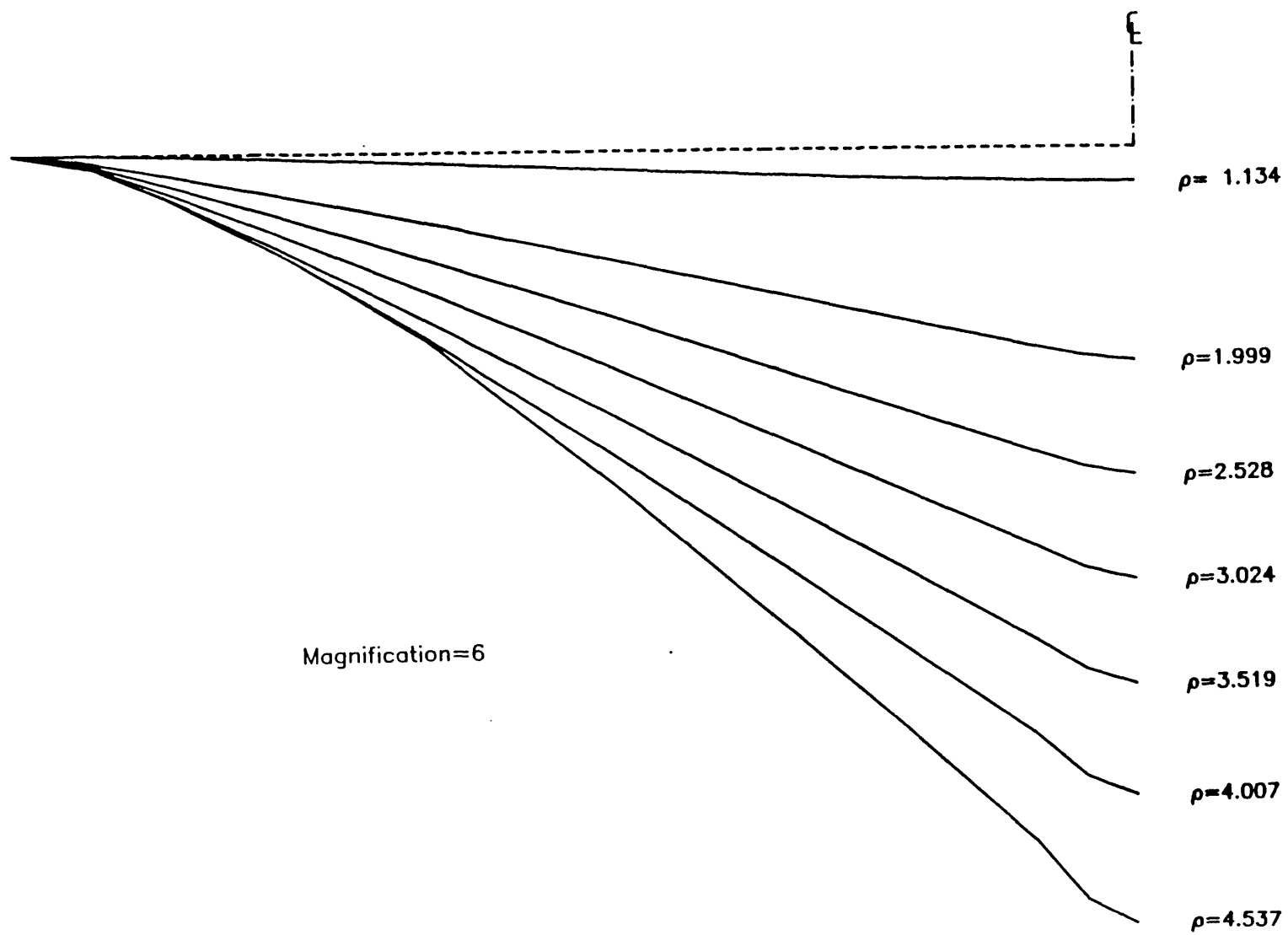


Fig. 4.13 Deflection shapes from BEFR04

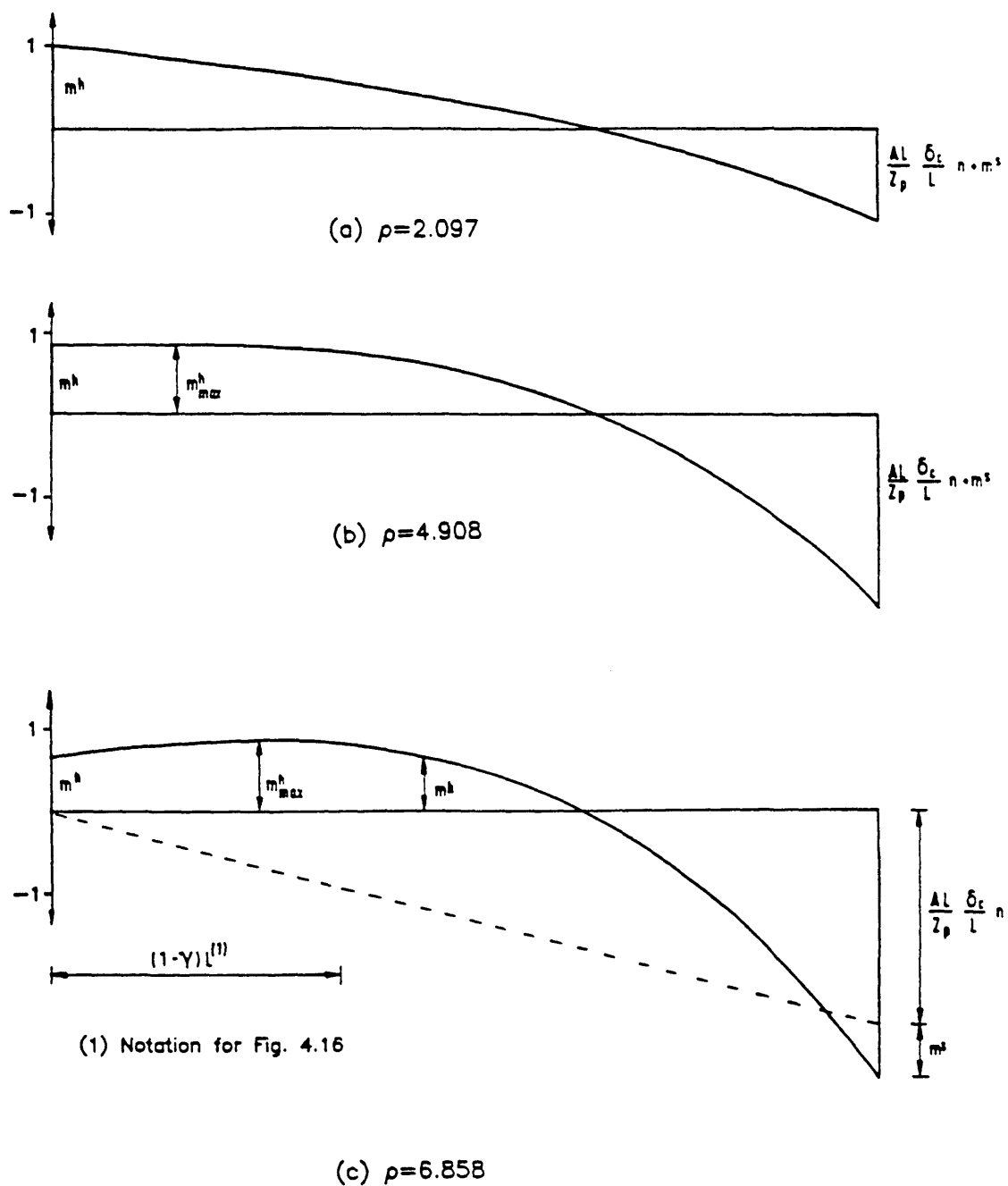


Fig. 4.15 Typical BMDs for BEFR03 if the bilinear mode-shape is assumed

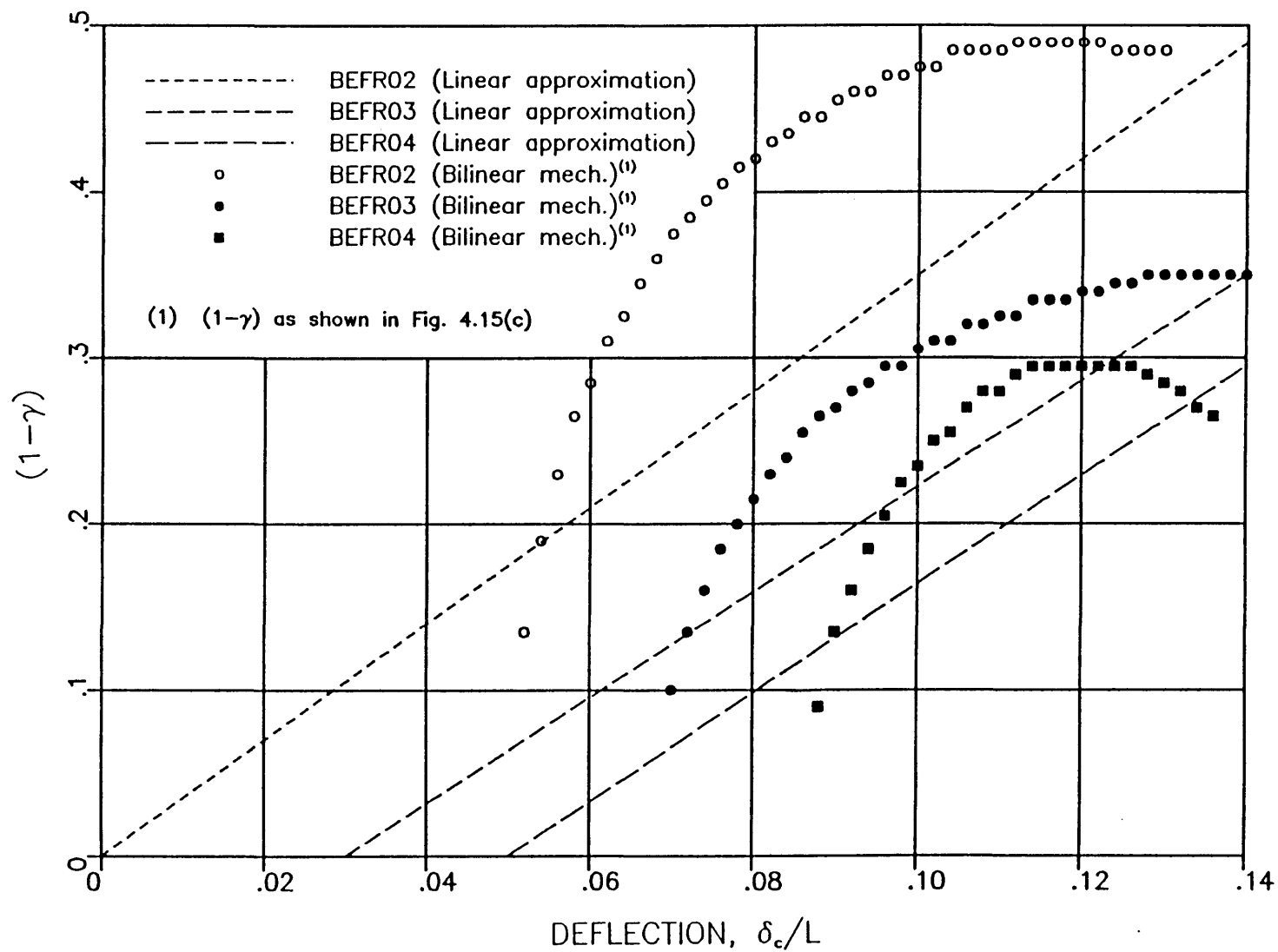


Fig. 4.16 Estimation of the parameters in Eqn. (4.11)

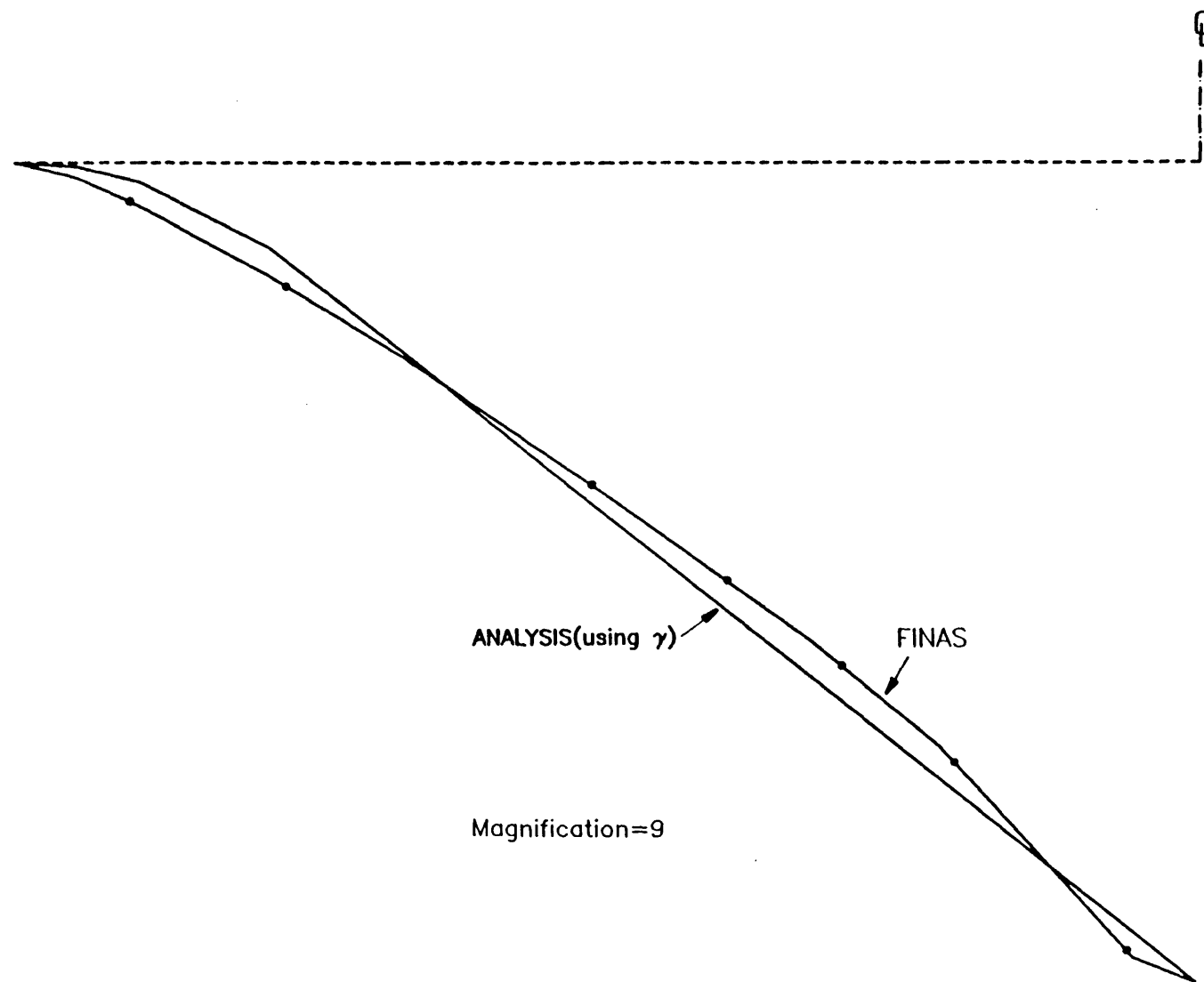


Fig. 4.17 Comparison of deflected shapes for BEFR02 at $\delta_0/L=0.0698$

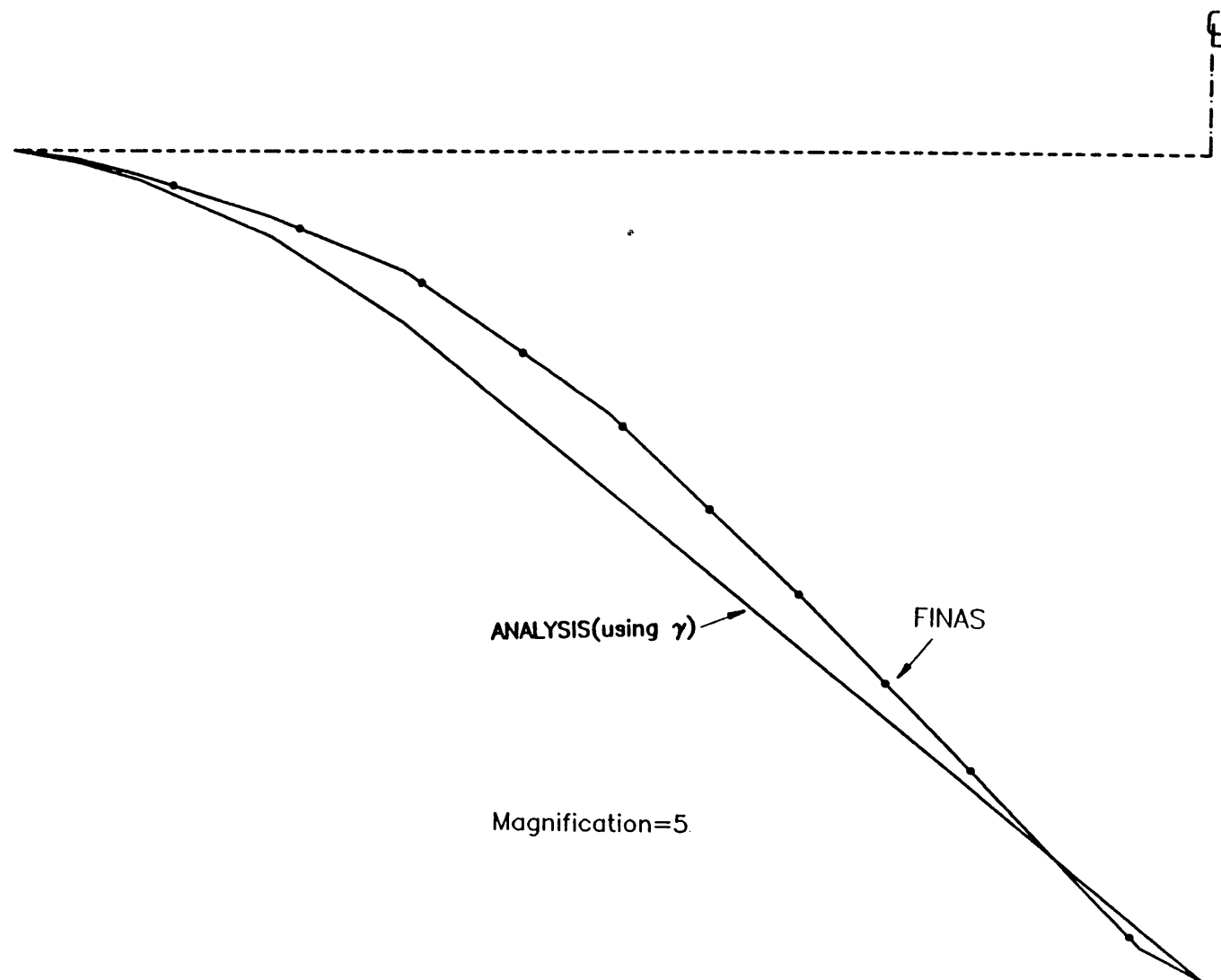


Fig. 4.18 Comparison of deflected shapes for BEFR03 at $\delta_0/L=0.1392$

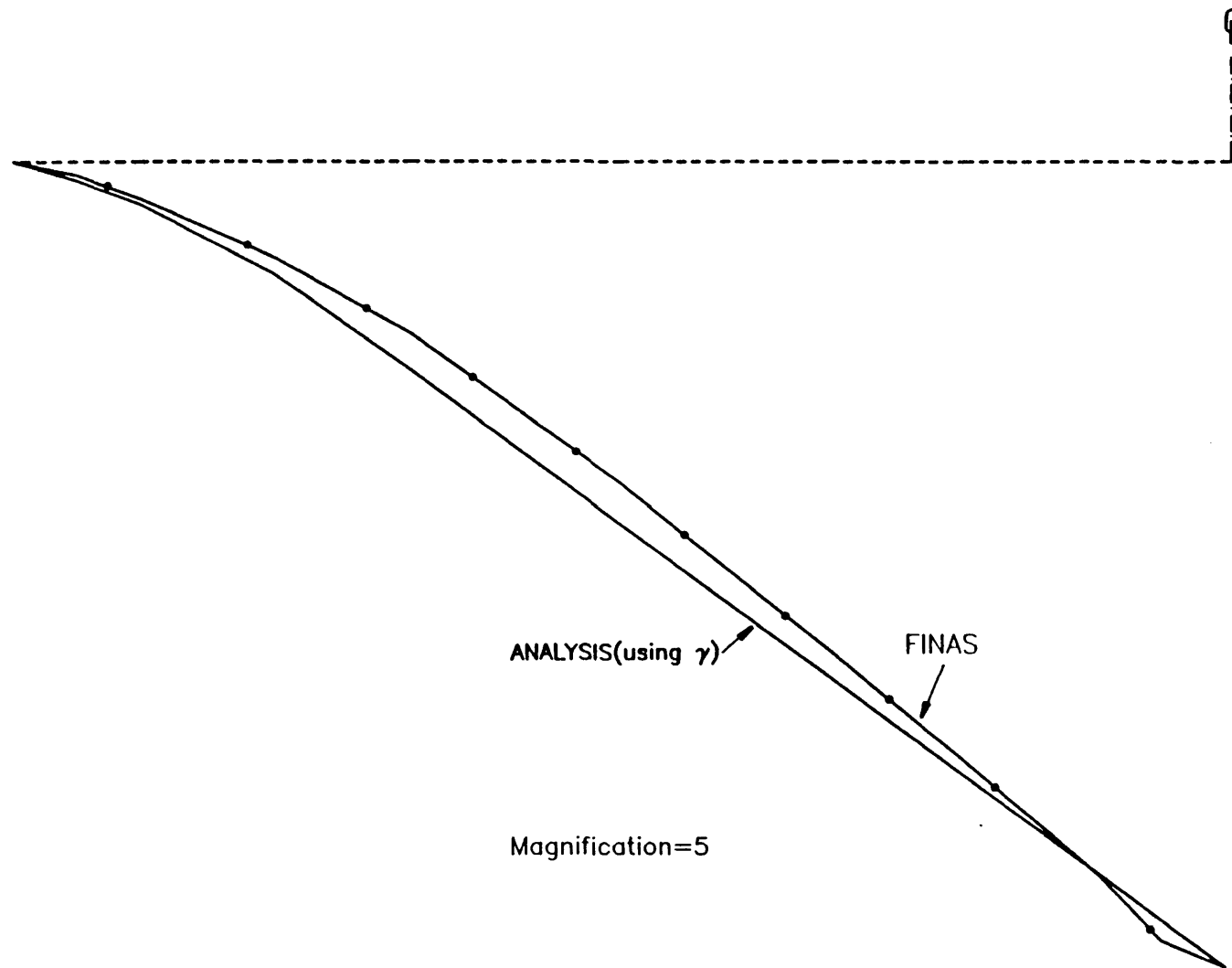
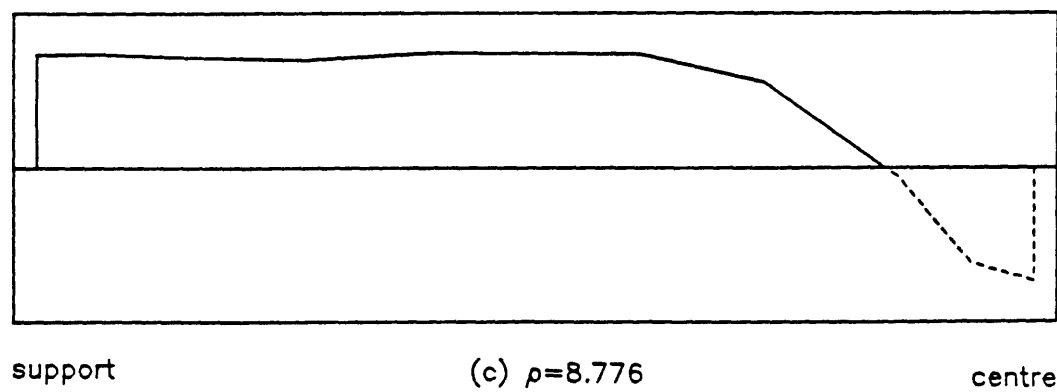
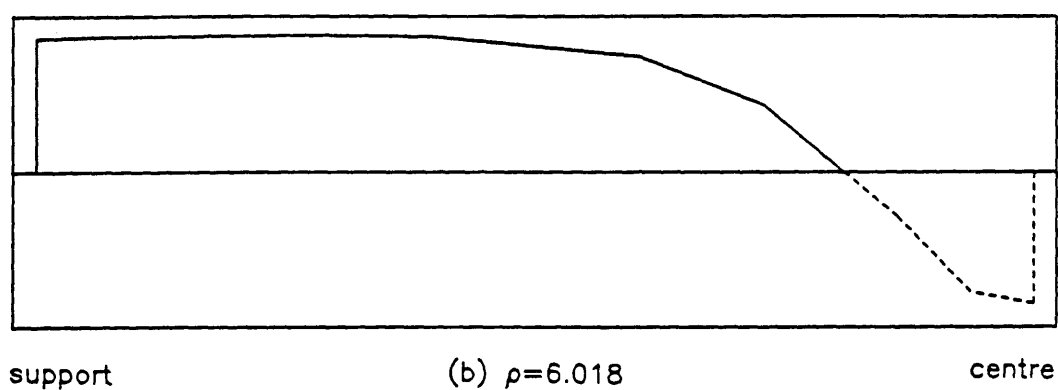
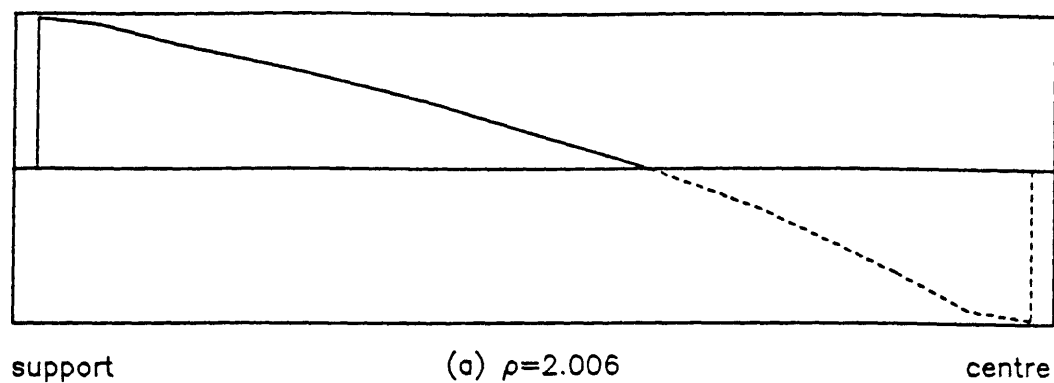


Fig. 4.19 Comparison of deflected shapes for BEFR04 at $\delta_0/L=0.1336$



L

SCALE M_0

Fig. 4.20 Bending moment diagrams for BEFR03 showing spreading of plasticity (from FINAS)

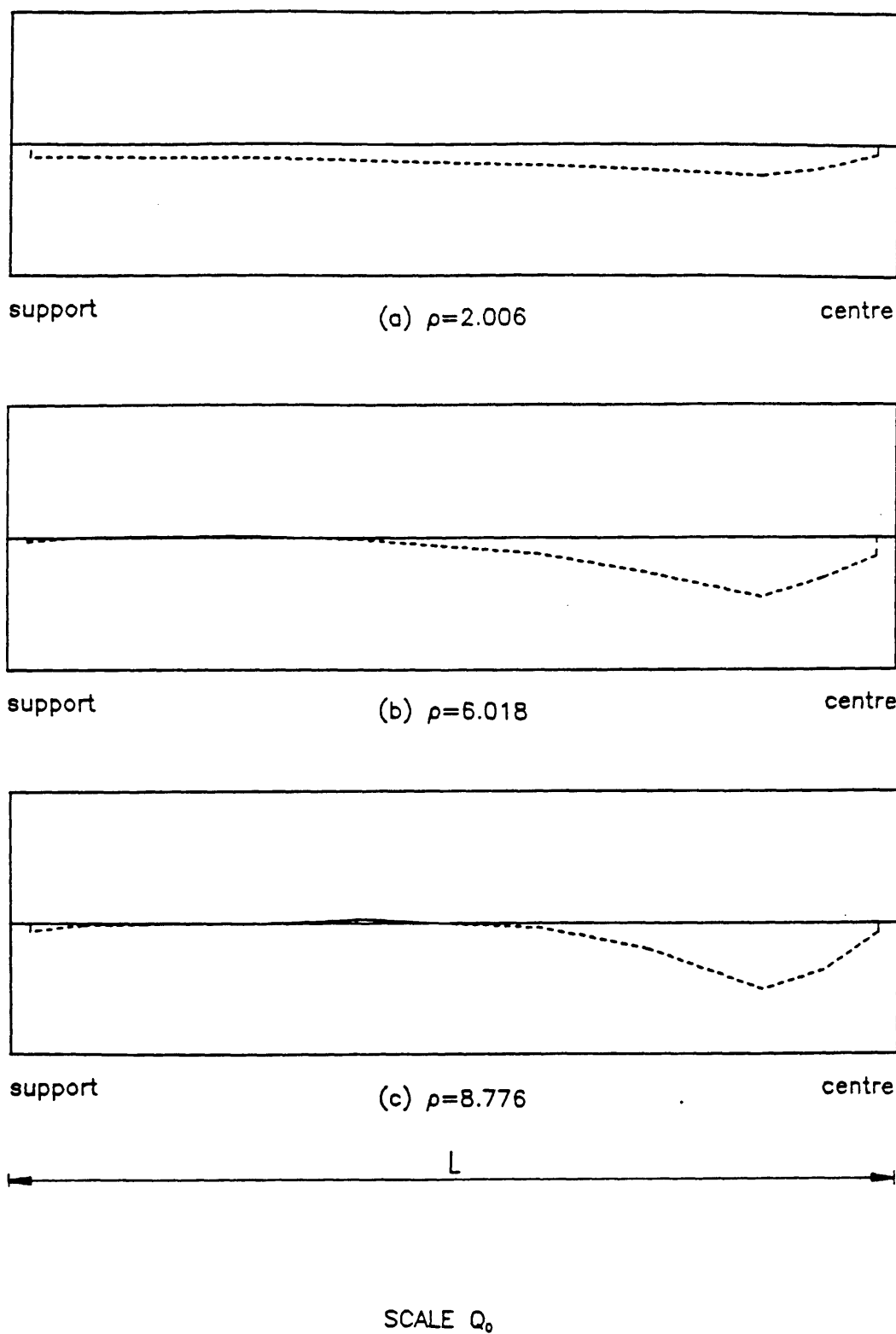


Fig. 4.21 Shear force diagrams for BEFR03 corresponding to the moment distributions shown in Fig. 4.20 (from FINAS)

CHAPTER FIVE

AXIALLY RESTRAINED BEAMS UNDER SYMMETRIC DISTRIBUTED LOADING

5.1 INTRODUCTION

As explained in Section 3.6 the stiffeners and the plate in a longitudinally stiffened deck bay with full end restraint can be regarded as a series of clamped beams. The vertical component of the transverse membrane forces in the plate is distributed in a triangular fashion along the stiffeners remote from the patch loading. Axially restrained beams under triangularly distributed loading are therefore examined in some detail within this chapter.

A generalization of Haythornthwaite's (1961) rigid-plastic analysis for rectangular beams under uniformly distributed loading is described. It is applicable to axially restrained beams under various patterns of symmetric distributed loading and with various end conditions. The prediction of membrane yield at infinite deflections by Haythornthwaite's (1961) and Gill's (1973) analyses is explained. The results of the various analyses are compared with experimental and numerical results.

5.2 GENERAL ASSUMPTIONS

For the beam under consideration, shown in Fig. 5.1(a), it is assumed that:

- 1) The material law is rigid-plastic.
- 2) The applied loading is conservative. This load applied on a simply supported substitute structure produces a moment distribution with the maximum value in the middle of the beam.
- 3) Under the action of the externally applied load, the structure changes shape in a single deformation mode starting from the limit load configuration with three hinges; the central plastic hinge spreads towards the supports altering the shape of the structure.

- 4) During the spreading hinge phase, the shape of the structure is composed of two segments; one elastic and straight and one plastic and curved, shown in Fig. 5.1(b). The curved segment follows the mode-shape of the moment produced on a substitute simply supported beam.
- 5) The angles are small, unless otherwise stated.
- 6) First order approximations to changes in geometry are sufficient.
- 7) Transverse shear effects are negligible.

5.3 MODE-SHAPE CHOICE

Assumption 4 is singled out from those outlined above due to its importance in the ensuing analysis. The changing shape of the beam follows from the assumption that at the membrane state, for small angles, the external loading is carried by membrane action only. The shape at the membrane state can be related to the moment produced along a simply supported substitute structure by the externally applied load, with the relationship:

$$N y_0(x) = M(x)$$

where $y_0(x)$ is the deflected shape and N is the horizontal component of the axial force, assumed to be constant along the structure for small angles. Linear relationship between $y_0(x)$ and $M(x)$ implies that both follow the same mode-shape $f(x)$, that is:

$$M(x) = M(L) f(x) \quad , \quad \text{for } 0 \leq x \leq L \quad (5.1(a))$$

$$y_0(x) = y_0(L) f(x) \quad , \quad \text{for } 0 \leq x \leq L \quad (5.1(b))$$

Where $M(L)$ is the maximum value of the bending moment in the middle of the beam and $f(x)$ is the mode-shape of $M(x)$, normalized with respect to the ordinate in the middle ($f(L)=1$). The shape, $y_0(x)$, is followed by the curved segment of the structure with $y_0(L)$ its maximum ordinate in the middle of the beam.

5.4 ANALYSIS OF SPREADING HINGE PHASE

The analysis commences from the bilinear configuration at limit load of intensity, q_0 . Applying the principle of virtual work to the structure in Fig. 5.1(a), q_0 is obtained as:

$$q_0 = \frac{\mu_2 \lambda M_0}{L^2} \quad (5.2)$$

where μ_2 is a scalar quantity related to the pattern of loading and q is a characteristic load ordinate. In Fig. 5.1(a), for example, q is the magnitude of the loading in the middle of the beam.

5.4.1 Geometry

From the bilinear configuration at limit load, the structure is transformed under increasing deflection to a plastic membrane with shape dictated by the mode-shape $f(x)$. Due to symmetry, only half of the beam is considered. The deflection shape of the structure at some stage of this transformation, shown in Fig. 5.1(b), can be expressed as:

$$y(x) = \frac{x}{a} \bar{\delta} \quad , \quad \text{for } x \leq a \quad (5.3(a))$$

$$y(x) = \bar{\delta} + y_0(L) [f(x) - f(a)] \quad , \quad \text{for } a \leq x \leq L \quad (5.3(b))$$

where $y_0(L)$ is the instantaneous depth of the curved segment measured from the reference level $y_0(0)$. From Eqn. (5.3(b)) it follows that the central deflection is:

$$y(L) = \delta_c = \bar{\delta} + y_0(L) [1 - f(a)] \quad (5.4)$$

Continuity of the slope at $x=a$, the junction between the straight and curved parts, yields:

$$\frac{\bar{\delta}}{a} = y_0(L) f'(a) \quad (5.5)$$

Changes in geometry and associated extensional requirements of the beam centroidal line are considered next. The extensions required for the structure to

follow the shape assumed above are treated separately for the elastic and plastic parts of the beam.

A small change in geometry is shown in Fig. 5.2(a). Close inspection of the regions OA''EA'O and OE'A'O in Fig. 5.2(b) shows that the latter can be used for the evaluation of the extension, de_1 , associated with an increase $d\theta$ in the angle of segment OA'. Considering the triangle D'E'A':

$$\tan \theta = \frac{\overline{D'E'}}{\overline{A'D'}} = \frac{de_1}{\sqrt{a^2 + \bar{\delta}^2} d\theta}$$

However,

$$\tan \theta = \frac{\bar{\delta}}{a}$$

therefore:

$$\frac{de_1}{d\theta} = \sqrt{1 + (\bar{\delta}/a)^2} \bar{\delta} \quad (5.6(a))$$

For small slopes Eqn. (5.6(a)) simplifies to:

$$\frac{de_1}{d\theta} = \bar{\delta} \quad (5.6(b))$$

The main difference between a point hinge of zero length and a hinge of finite length, is that the latter may have extensional requirements if curvature changes take place over its finite length. Extensional requirements for the curved length A'B' in Fig. 5.2(a) are based on the need for the curved segment to increase its length under increasing curvature. The incremental extensions over the curved segment, de_2 , are considered in Appendix C and are expressed by Eqn. (C.5) as:

$$\frac{de_2}{d\theta} = \alpha_2 C_5(a) \bar{\delta}$$

Summation of Eqns (5.6(b)) and (C.5) gives the total incremental extension to be:

$$\frac{de}{d\theta} = (1 + \alpha_2 C_5(a)) \bar{\delta} \quad (5.7)$$

where $C_5(a)$ accounts for the extensional requirements over the curved length and α_2 is a parameter. Both quantities are explained in Appendix C.

Defining

$$F = 1 + \alpha_2 C_5(a) \quad (5.8)$$

then,

$$\frac{de}{d\theta} = F \bar{\delta} \quad (5.9)$$

Two assumptions can be made about the parameter F . The first is that as the curved length extends towards the support the previously developed curved profile does not change. This corresponds to $F=1$ and is the assumption made by Haythornthwaite (1961). The second assumption is that some curvature changes occur over the curved length and, therefore, $F \neq 1$.

5.4.2 Normality

In a rigid-plastic analysis, where elastic extensions are disregarded, the extensions required for changes in geometry take place at or along plastic hinges. Normality postulates that:

$$\frac{de^i}{d\theta} = - \frac{dM^i}{dN} \quad (5.10)$$

The interaction between the flexural and axial stress resultants at a plastic hinge, i , has been expressed by Eqn. (4.3) as:

$$m^i = C_{02}^i n^2 + C_{03}^i n + C_{04}^i$$

The total extension is then given as:

$$\frac{de}{d\theta} = - \frac{Z}{A} P (2\bar{C}_2 n + \bar{C}_3) \quad (5.11)$$

where

$$\bar{C}_\ell = C_{0\ell}^s + (\lambda-1)C_{0\ell}^h, \quad \text{for } \ell = 2, 3, 4 \quad (5.12)$$

Superscripts s and h refer to sagging and hogging hinges respectively.

5.4.3 Compatibility

For compatibility the extensions provided at the plastic hinges will balance the extensions required for changes in geometry. Combining Eqn. (5.9) with Eqn. (5.11) gives:

$$F \bar{\delta} = - \frac{Z}{A} P (2\bar{C}_2 n + \bar{C}_3) \quad (5.13)$$

5.4.4 Equilibrium

A combination of bending moment and axial force in the beam resists the applied load. Moment equilibrium of the half structure, Fig. 5.3(a), gives:

$$M(L) = \rho \lambda M_0 = M^s + (\lambda-1)M^h + \delta_c N \quad (5.14)$$

Here ρ is the load factor, with $\rho = 1$ at the limit load. The bending moment is envisaged as being constant for $x \geq a$.

Because $M(x)$ and $y_0(x)$ have the same shape the following expression may be derived (using the equations of Section 5.3):

$$y_0(L) = \frac{M(L)}{N} = \frac{\rho \lambda M_0}{N} \quad (5.15)$$

5.4.5 Solution

The previously derived equations are further processed to arrive at a solution. Combining Eqns (5.5) and (5.15) gives:

$$\rho = \frac{A \bar{\delta}}{\lambda Z_p} n \frac{1}{af'(a)} \quad (5.16)$$

Using Eqns (5.4), (5.15) and the interaction formula (4.3), Eqn. (5.14) becomes:

$$\rho f(a) = -\frac{1}{\lambda} (\bar{C}_2 n^2 + \bar{C}_3 n + \bar{C}_4) + \frac{A \bar{\delta}}{\lambda Z_p} n \quad (5.17)$$

Substituting Eqn. (5.16) into Eqn. (5.17) gives:

$$B_1 f(a) - B_2 a f'(a) = 0 \quad (5.18)$$

where,

$$B_1 = \frac{\bar{\delta} A n}{\lambda Z_p} \quad (5.19)$$

$$B_2 = B_1 + \frac{1}{\lambda} (\bar{C}_2 n^2 + \bar{C}_3 n + \bar{C}_4)$$

The previous equations can be solved in the following order:

For a particular value of the axial force, n , the deflection at the junction between the elastic and plastic segments, $\bar{\delta}$, follows from Eqn. (5.13). Consequently, Eqn. (5.18) yields the distance of the spreading hinge tip from the support, a . The values of the load factor, ρ , the deflection, $y_0(L)$, and the central deflection, δ_c , follow from Eqns (5.16), (5.5) and (5.4) respectively.

5.4.6 Remarks

- 1) Haythornthwaite's (1961) solution is obtained from the above for the case of uniformly distributed loading with $F=1$ in Eqn. (5.8). Solutions with $F=1$ and

loading other than uniformly distributed will also be termed 'Haythornthwaite solutions' in this thesis for convenience.

- 2) The extensions occurring within the curved segment are considered in Appendix C. These extensions can be taken into account by altering the F value, as shown by Eqn. (5.8). Solutions with $F \neq 1$ are referred to as 'analysis ($F \neq 1$)'.
- 3) Either value of F ($F=1$ or $F \neq 1$) can be used up to the point where the predicted response crosses the membrane solution given by Eqns (5.20) and (5.21), derived in the following section.

5.5 ANALYSIS OF MEMBRANE PHASE

Under increasing deflection, during the spreading hinge phase, the beam becomes plastic over its entire length and arrives at the membrane phase. Due to this finite deflection, the angles are no longer small. Their actual values are therefore considered in this section.

During the membrane phase shown in Fig. 5.3(b), the beam over its entire length is assumed to develop the shape dictated by the mode-shape $f(x)$. The beam behaves essentially as a string, due to the loss of flexural stiffness caused by extensive plastification. Figure 5.3(b) shows the axial force in the middle of the beam to be lower than the squash load, N_0 , as the angle there is less than the angle at the support. Therefore, for the structure to be plastic in this region, bending should coexist with tension to satisfy the yield condition. In summary, during the membrane phase the parts of the beam where the angle of inclination, θ , is maximum are plastic under axial force only, while the rest of it is plastic under combined axial force and bending moment.

From the condition of full membrane yield at the supports and using Eqn. (5.1(b)), the horizontal component of the axial force is given by:

$$n \approx \cos (\tan^{-1}(f'(0) \delta_C)) \quad (5.20)$$

In deriving Eqn. (5.20) the shear at the support was approximated to be zero. According to the assumptions made about the shape of the beam during the membrane phase, both the shear at the support and its horizontal component should be almost zero when the deflections are small. As deflection increases, both values are increased and the approximation becomes gradually inaccurate. On the other hand, the finite element analyses undertaken show the assumption for

negligible shear at the supports to be a good one for the entire range of deflections for which results are presented in this chapter (up to four times the beam depth).

The load factor, ρ , is evaluated from equilibrium considerations similar to Eqn. (5.17). It is evident that $M^h=0$ in this case. Hence,

$$\rho = \frac{1}{\lambda} (C_{02}^s n^2 + C_{03}^s n + C_{04}^s) + \frac{A\delta_c}{\lambda Z_p} n \quad (5.21)$$

Equations (5.20) and (5.21) can be solved for particular values of the central deflection, δ_c . It is apparent from Eqn. (5.20) that the horizontal component of the axial force, n , decreases with increasing δ_c .

Under the assumption of small deflections Eqn. (5.21) simplifies to the linear equation

$$\rho = \frac{A\delta_c}{\lambda Z_p} \quad (5.22)$$

because $n=1$ and $M^s=0$. This is the linear membrane solution and is presented in Figs 5.4 to 5.11.

5.6 NUMERICAL ANALYSIS

The finite element program FINAS described in Section 3.3 was used to undertake large deflection analyses of beams under different loads and boundary conditions. The numerical results are first checked against existing experimental and analytical results and then compared with the results of the preceding analysis.

5.6.1 Choice of element type

By restricting the analysis to model the behaviour in one plane, either plane stress elements or beam elements would be used. The choice was made on the basis of computational efficiency for a required accuracy of results. For the rectangular cross-section analysed, the key requirement for accuracy is to provide the structure with a sufficient number of sampling points (stations) through the depth to

adequately model the plastic hinges. To comply with this requirement a number of plane stress elements need to be used through the depth since each element has only two sampling points in this direction. On the other hand, a beam element allows for a very refined mesh of sampling points through the depth. It was thus preferable to the plane stress elements and it was selected for the numerical analysis. An additional advantage of the beam element is that it allows sampling points at the extreme fibres, while for plane stress elements the sampling points are the nearest Gauss points.

This beam element cannot, however, model certain secondary effects. For example, it has been observed in tests (Murray (1973), Ronalds (1986)) that when the beam comprises a plate segment and an attached flat-bar stiffener the latter may tend to trip locally where the beam experiences localized curvatures and the stiffener outstand is in compression. A second effect not modelled in these analyses is any twisting of the beam as the elements were restrained to deflect in a vertical plane.

5.6.2 Mesh-refinement study

Once the element type was chosen, the required mesh size was investigated. One half-span was analysed due to the symmetry of the structure and the loading.

Different number of elements along the length of the half-span and different number of stations through the depth were considered, as well as both uniform and graded meshes. The number of elements was varied between 5 to 20 along the half-span. The number of stations through depth was varied between 9 and 39.

Comparison of the various analyses showed that in the range of large deflections even the smallest number of elements tried, that is 5, can predict the behaviour with very good accuracy. The only requirement for such a coarse mesh is to place short elements, less than 5% of the span, where localized curvatures occur, that is at plastic hinge locations at limit load. Thus a similar mesh to the one shown in Fig. 4.2 is established, but without the spring elements. It was also found that, a suitable number of stations through the depth for rectangular beams is about 20.

5.7 COMPARISON OF DIFFERENT PREDICTIONS AND FINITE ELEMENT RESULTS

There are solutions for the case of uniformly distributed loading in the literature (Haythornthwaite (1961), Gill(1973), Gürkök & Hopkins (1973)). The particular case of a beam with pinned supports and uniformly distributed loading was used to compare different predictions since this was the only case for which Gürkök & Hopkins (1973) developed a rigorous solution. All the analyses are based on rigid-plastic theory, and therefore they should ideally be compared with numerical results assuming the same rigid-plastic material law. This was achieved in the finite element analysis by specifying a very high value for the elastic modulus. This results in the elimination of early elastic deflections due to bending and transverse shear, and also of axial extensions in the elastic segments between plastic hinges. The central deflection is non-dimensionalized with respect to the beam depth h throughout.

Following the comparison between the various predictions, information about the elasto-plastic behaviour and the spreading of plasticity obtained from the numerical analysis, is presented.

5.7.1 Uniform loading

5.7.1.1 Beams with pinned supports

Existing predictions for the load history are shown in Fig. 5.4(a) for uniformly distributed loading and pinned supports. The numerical predictions with a rigid material law, that is when the elastic modulus has a very high value, can be regarded as the reference solution of the problem for correlation with the various rigid-plastic analyses. It can be seen, firstly, that all approaches are capable of predicting the behaviour with considerable accuracy. It should be noted that, in figures 5.4 to 5.11 predictions are only shown in the region where they do not coincide with Haythornthwaite's (1961) solution. In particular, Haythornthwaite's (1961) and Gill's (1973) predictions are indistinguishable in Figs 5.4(a) and 5.6(a), therefore, they are represented by one curve.

Figure 5.4(a) shows also that there is negligible difference in the overall load-deflection prediction between a structure made of a material that follows a rigid-plastic law and one with an elasto-plastic law, provided the latter has

adequately high elastic stiffness. This is measured by the parameter k_0 , which is defined as the initial stiffness at the beam centre. The beam geometry for which results are illustrated in Figs 5.4(a) and 5.4(b) corresponds to a non-dimensionalized value for k_0/σ_y of 0.0161. A rigid-plastic analysis will give similar accuracy when k_0/σ_y is about 0.0067. For beams with lower k_0/σ_y values, the accuracy is reduced and an elasto-plastic analysis is more suitable. The membrane solution given by Eqn. (5.21), however, is fairly accurate for all geometries.

At small deflections Gürkök & Hopkins' (1973) prediction is the most accurate and, being rigid-plastic, it always lies just slightly to the left of the responses obtained by the finite elements. The same trend is exhibited by the present analysis when extensions over the curved length are included as described in Appendix C, shown in Fig. 5.4(b) as 'analysis (F≠1)'. Thus the present small deflection analysis, combined with the membrane solution given by Eqn. (5.21), gives very accurate results throughout the response. Haythornthwaite's (1961) and Gill's (1973) predictions cross the FE results and so do not exhibit any fixed trend in under- or over-estimating the numerically estimated behaviour.

In Fig. 5.5 the trends of the different predictions become clearer. The present analysis is approximate but it exhibits the trends of the reference solution, as well as being considerably simpler. It is also obvious that Gill's modification of Haythornthwaite's original analysis produces virtually no alteration to the response. As shown in this figure, Gill's prediction is slightly different from Haythornthwaite's only when $n \approx 0.60$. At higher axial force values the two curves are coincident. This is because Gill's second mode is identical with the single mode assumed by Haythornthwaite. The extra computational effort of Gill's two mode analysis is therefore unwarranted. The large deflection membrane solution correlates extremely well with the numerical analysis as seen in Figs 5.4(b) and 5.5. This membrane solution given by Eqns (5.20) and (5.21) was not considered by any of the previous authors, as they produced small rotation analyses.

5.7.1.2 Beams with fixed supports

The load versus deflection responses for uniformly distributed loading and fixed supports are illustrated in Figs 5.6(a) and 5.6(b). The predictions are corroborated by tests on an aluminium beam carried out by Pang & Millar (1978) with k_0/σ_y of 0.0303 and on a steel beam by Young (1959) with k_0/σ_y of 0.0087. Tests, finite elements and theory are in very good agreement. It can be seen that

Young's beam is near the lower end of k_0/σ_y values, for a rigid-plastic analysis to give comparable results. The membrane solution given by Eqn. (5.21) presented in Fig. 5.6(b) corresponds to the geometries analysed by finite elements which had $2L/h=12.5$ but various k_0/σ_y values, as different values were assigned to the elastic modulus.

The development of membrane tension is plotted against deflection in Fig. 5.7. Haythornthwaite's and Gill's predictions correlate very well with the finite element results in this case. As seen in this figure the analysis presented in this thesis, including the effects of extension changes over the curved length, does not improve the Haythornthwaite prediction, although the opposite can be observed in Fig. 5.5.

It is worth mentioning that the aluminium beam tested by Pang & Millar, under this type of loading and boundary conditions, ruptured at the supports. Material rupture limits the range of deflections attained in practice and is further discussed in Section 5.11.

5.7.2 Triangular loading

The analysis was also used to predict the behaviour of a beam under triangularly distributed loading, indicated in Fig. 5.8. As mentioned above Haythornthwaite's solution is based on the mode-shape $f(x)$ given by Eqn. (5.2) for triangular loading, and setting $F=1$. Predictions for the load and the axial force versus central deflection for a pin ended beam are illustrated in Figs 5.8 and 5.9 respectively, while Figs 5.10 and 5.11 correspond to fixed supports. The same conclusions, as for the uniformly distributed loading case, can be drawn concerning the relative accuracy of the analyses presented.

5.8 DEVELOPMENT OF PLASTICITY

Figures 5.12(a) through 5.12(d) show the gradual development of plasticity with increasing deflection in a typical encastré beam under uniformly distributed loading.

By studying these diagrams it is possible to verify the assumptions made regarding travelling and spreading hinges. In Fig. 5.12(a) it is seen that the location of one of the plastic hinges is close to the centre of the beam. Under increasing loading,

Fig. 5.12(b), the hinge position moves towards the support while the centre of the beam begins to unload in bending. The depth of the plastic zone in tension at the hinge position is increasing due to increasing axial tension. In Fig. 5.12(c) it is observed that a significant length of the beam is now a plastic sagging hinge. Thus, a change of mode from travelling to spreading plastic hinge has taken place somewhere between the stages in Figs 5.12(b) and 5.12(c). This is consistent with Gill's (1973) and Gürkök & Hopkins' (1973) predictions that the mode change occurs at $\rho=1.66$.

This mode change from travelling hinge to spreading hinge can also be predicted by the approximate analysis presented here. With reference to Fig. 5.1(b), the value of deflection $y_0(L)$ decreases as the curved segment expands from the centre towards the supports. When the change in the mode occurs $y_0(L)$ is a minimum as discussed in Appendix C. Plots of $y_0(L)$ versus the non-dimensionalized horizontal projection of the curved segment, $1-a/L$, are shown in Figs 5.13(a) and 5.13(b) for uniform and triangular loading, respectively. The first of these figures agrees with Gill's and Gürkök & Hopkins' predictions, that the change in mode for a beam under uniformly distributed loading occurs at $a/L=0.667$. Figure 5.13(b) shows that under triangularly distributed loading the mode changes when the hinge position is at $a/L=0.73$. This corresponds to $\rho=1.55$.

Finally, Fig. 5.12(d) illustrates the spreading of plasticity to the supports with further increase in the axial tension. This figure corresponds to the membrane phase, as indicated by the load factor, ρ . It is evident that the support section is under pure membrane yield whereas a small amount of bending coexists with the axial tension elsewhere in the beam. This agrees with the assumptions made in Section 5.5 regarding the horizontal component of the axial force.

It may be noted that the plastic hinges at the supports do not exhibit any spreading. From observations on the behaviour of axially restrained beams under other types of loading (Section 4.10), it may be concluded that spreading or travelling is likely to occur from positions where the bending moment diagram on the simply supported substitute beam has zero or nearly zero slope. Thus, plasticity can readily spread where the bending moment is nearly constant.

5.9 BENDING MOMENT DISTRIBUTION

In addition to the information presented in the preceding section, it is useful to illustrate the moment distribution along the beams. Such information is given in

Figs 5.14 and 5.15. The first figure corresponds to the encastré beam under uniformly distributed loading and the second figure to the beam with pinned supports and the same pattern of loading. Each figure relates to a given load factor.

Comparison of Haythornthwaite's analysis and of the present analysis modified to include extensions over the curved part with the numerical results, shows that the assumption of constant moment along the curved part is a good one. The degree of accuracy with which the extent of the spreading of the curved segment is predicted is also remarkable.

On close study, very small differences can be seen between the predictions of Haythornthwaite and the finite elements. In Fig. 5.14(c) it is seen that Haythornthwaite predicts the bending moment obtained by the numerical analysis very accurately. The present analysis which includes the extensions occurring over the curved length, indicated by 'analysis ($F \neq 1$)', slightly underestimates the moment. In contrast, for the pin ended beam, both Haythornthwaite and the preceding analysis exhibit slight discrepancies from the numerical results, seen in Figs 5.15(b) and 5.15(c). The smaller moment predicted by the 'analysis ($F \neq 1$)' corresponds to the stiffer axial force versus central deflection relationship predicted by this analysis as compared to the finite element solution.

The observed drop in the moment computed by finite elements in the middle of the beam is a result of a travelling hinge. This behaviour corresponds to the first mode of deformation assumed by Gill (1973) and Gürkök & Hopkins (1973). The travelling hinge leaves the curved segment with a shape different from that dictated by the mode-shape $f(x)$ which produces constant moment on the structure.

5.10 SIMPLIFIED MODE OF DEFORMATION

An estimation of the accuracy of the simplified bilinear mode of deformation proposed in Chapter 3 for the indirectly loaded stiffeners in a fully restrained stiffened plate under lateral load is carried out in this section.

Comparison of the bilinear mode results with the other available predictions is made in Fig. 5.16. As seen in this figure even a rough estimation of the deformation mode gives acceptable accuracy. The bilinear mode of the deformation overestimates the load by 5% with respect to the Haythornthwaite's solution, when the central deflection equals the depth of the rectangular cross-section. At

deflections in excess of the beam depth the solution coincides with the small rotation membrane solution given by Eqn. (5.22).

This comparison helps to explain why Gill (1973) did not improve Haythornthwaite's (1961) single mode prediction for the rectangular beam, during the first mode assumed. The first mode assumed by Gill ends at a central deflection of 0.866 times the beam depth. This is less than the deflection at which the cruder bilinear mode differs from the reference analysis by just 5%. In addition, Haythornthwaite's parabolic shape assumption was closer to the actual curved shape than the one by the bilinear mechanism.

This insensitivity occurs because at the early stages of the deformation of a beam, which can be analysed by a rigid-plastic analysis, the behaviour is governed by bending. The build-up of axial force in the beam is mostly affected by the assumed mode of deformation.

It is worth mentioning that Haythornthwaite (1961) also outlined an analysis based on a bilinear deformation mechanism to predict the behaviour. Relevant to the approximation of bilinear mode considered in this section, is also the work by Jones (1971).

The same investigation was also undertaken for the 'notional beam' cross-section considered in Chapter 3 under triangular loading. The error made when the bilinear mode of deformation is assumed is approximately 2% compared to the numerical solution when the central deflection equals the beam depth. Thus the bilinear mechanism proposed in Section 3.6 for the beams remote from the patch load is a suitable approximation at realistic deflections.

5.11 MATERIAL RUPTURE

Material rupture limits the range of permissible deflections. The aluminium beam tested by Pang & Millar (1978) failed quite early at the supports in a shearing mode. Liu & Jones (1987) differentiate between tensile and shear failure with respect to the flatness of the broken section. In a tensile tearing, the two faces of the broken section have a convex-concave appearance. A shear failure lay in a single plane.

Failure at the supports has also been reported by Hook & Rawlings (1969a) who undertook experiments on clamped rectangular plate panels loaded by lateral

pressure. McDermott et al (1974) present test results on rectangular beams loaded at midspan by an indenter. These results show that as the indenter becomes increasingly blunt, the position of rupture changes from just under the indenter to the supports. For a beam with rectangular cross-section, loaded by a patch or distributed loading, rupture is more likely to occur at the supports. For asymmetric cross-sections the position of maximum distance between the initial neutral axis and the extreme fibre in tension should be considered as well.

Knowledge of the permissible strain causing material rupture, requires calculation of extreme fibre strains. Extreme fibre extensions can be calculated using a mechanism approach. Thus, calculation of the extreme fibre strains requires a notional hinge length. To that end the work by Ronalds (1990) may be useful. A discussion on the permissible rupture strains is presented in Section 8.7.

5.12 CONCLUSIONS

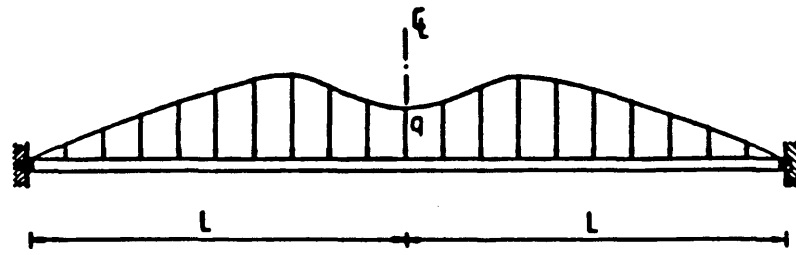
The rigid-plastic mechanism approach presented in this chapter extends Haythornthwaite's (1961) analysis of the large deflection behaviour of beams under uniformly distributed loading to any symmetrically distributed loading on a beam of arbitrary cross-section with encasté or pinned supports.

Two very simple analyses, the 'Haythornthwaite solution' and the 'membrane solution' expressed by Eqn. (5.21), give excellent agreement with the finite elements over the complete load versus deflection response. Similar agreement is also found in comparisons of the development of membrane tension. The latter is very sensitive to assumptions made in the analysis and is thus the best measure of the accuracy of the various methods. If additional refinement is needed in the analysis, especially for pin ended cases, the extensions associated with curvature changes over the curved segment can be incorporated using the method given in Appendix C.

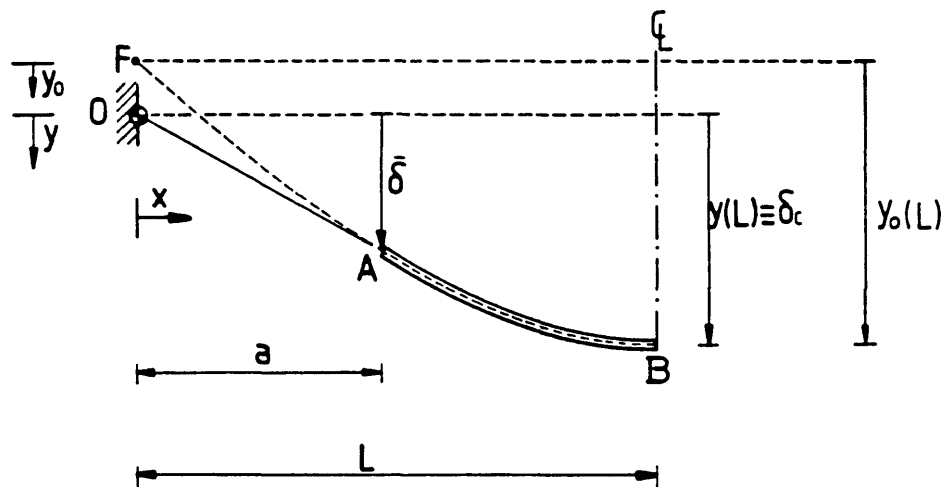
The differences between the rigid-plastic predictions of the analyses and the elasto-plastic behaviour predicted by finite elements is negligible for the geometries analysed. The structures were rectangular beams with span to depth ratios, $2L/h$, of 12.5 and k_0/σ_y between 0.0161 and 0.1123, but the analysis can be used without significant loss of accuracy for values of k_0/σ_y as low as 0.0067. For beams with more slender proportions, with lower k_0/σ_y values, the accuracy of a rigid-plastic analysis decreases. An elasto-plastic analysis becomes more suitable in these cases. Though, it is the simplicity of the rigid-plastic analysis that makes it

attractive.

Finally, detailed finite element studies verify the assumptions made by Gill and Gürkök & Hopkins concerning travelling and spreading hinges. For a simplified model as the one proposed in Chapter 3, a bilinear mode for the deformation of the indirectly loaded beams can be adopted without significant loss of accuracy.

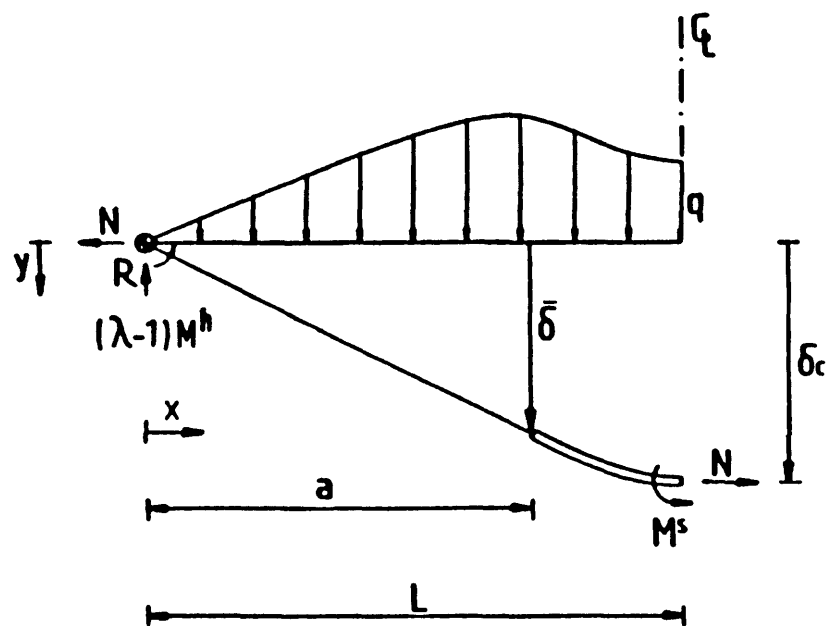


(a) Structure under loading

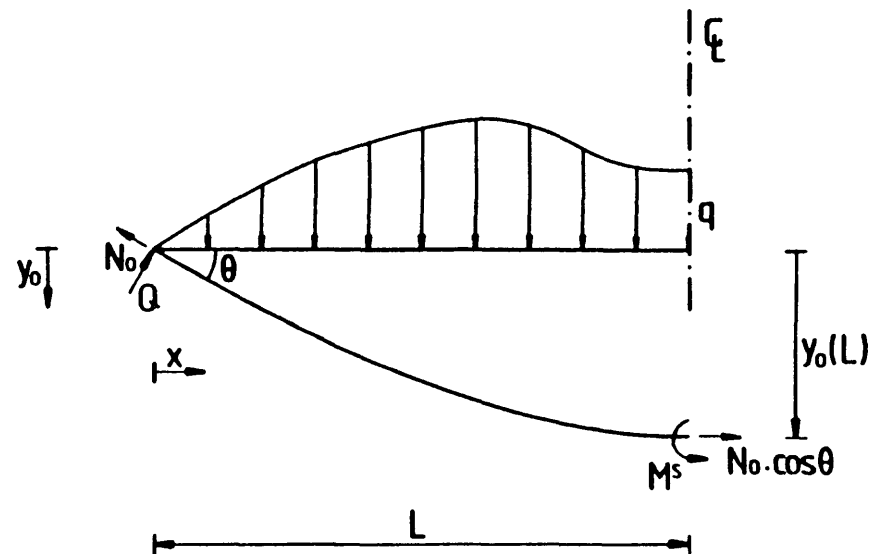


(b) Deflected shape

Fig. 5.1 Deflections under loading



(a) During transformation



(b) During membrane phase

Fig. 5.3 Applied forces

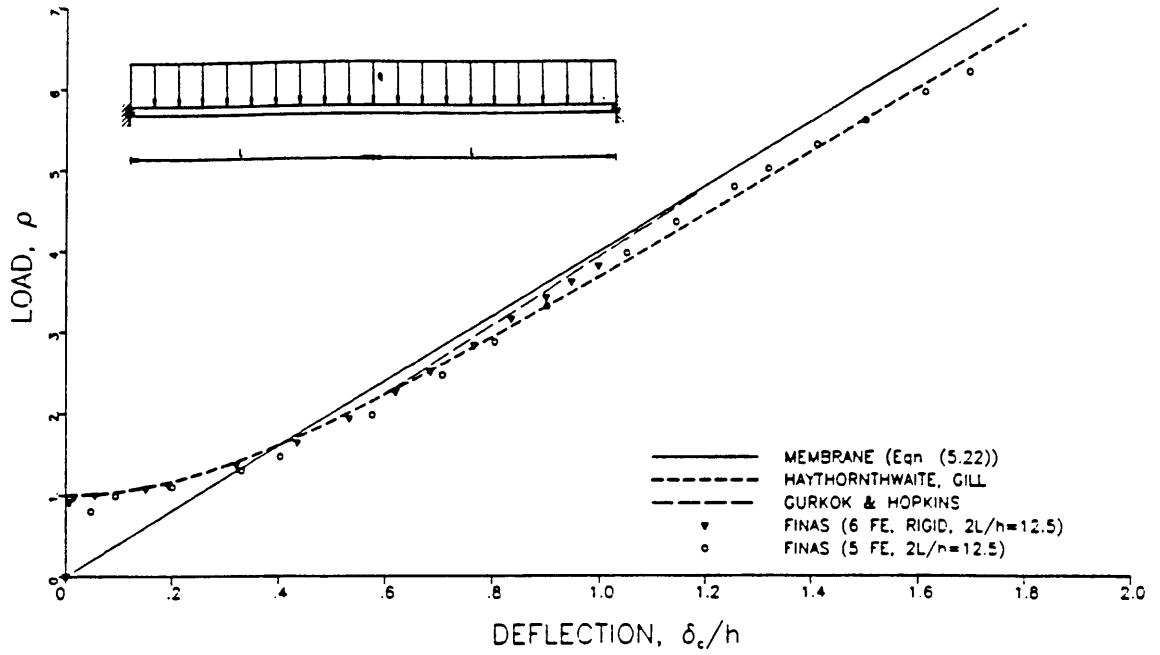


Fig. 5.4(a) Load vs. deflection (pinned supports)

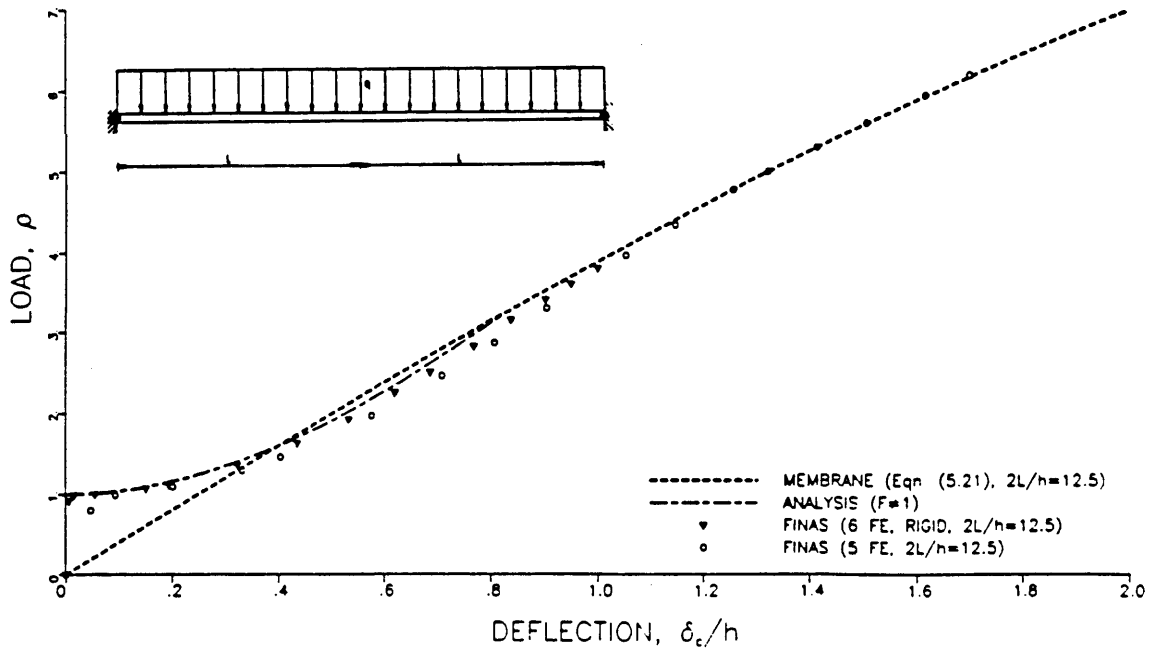


Fig. 5.4(b) Load vs. deflection (pinned supports)

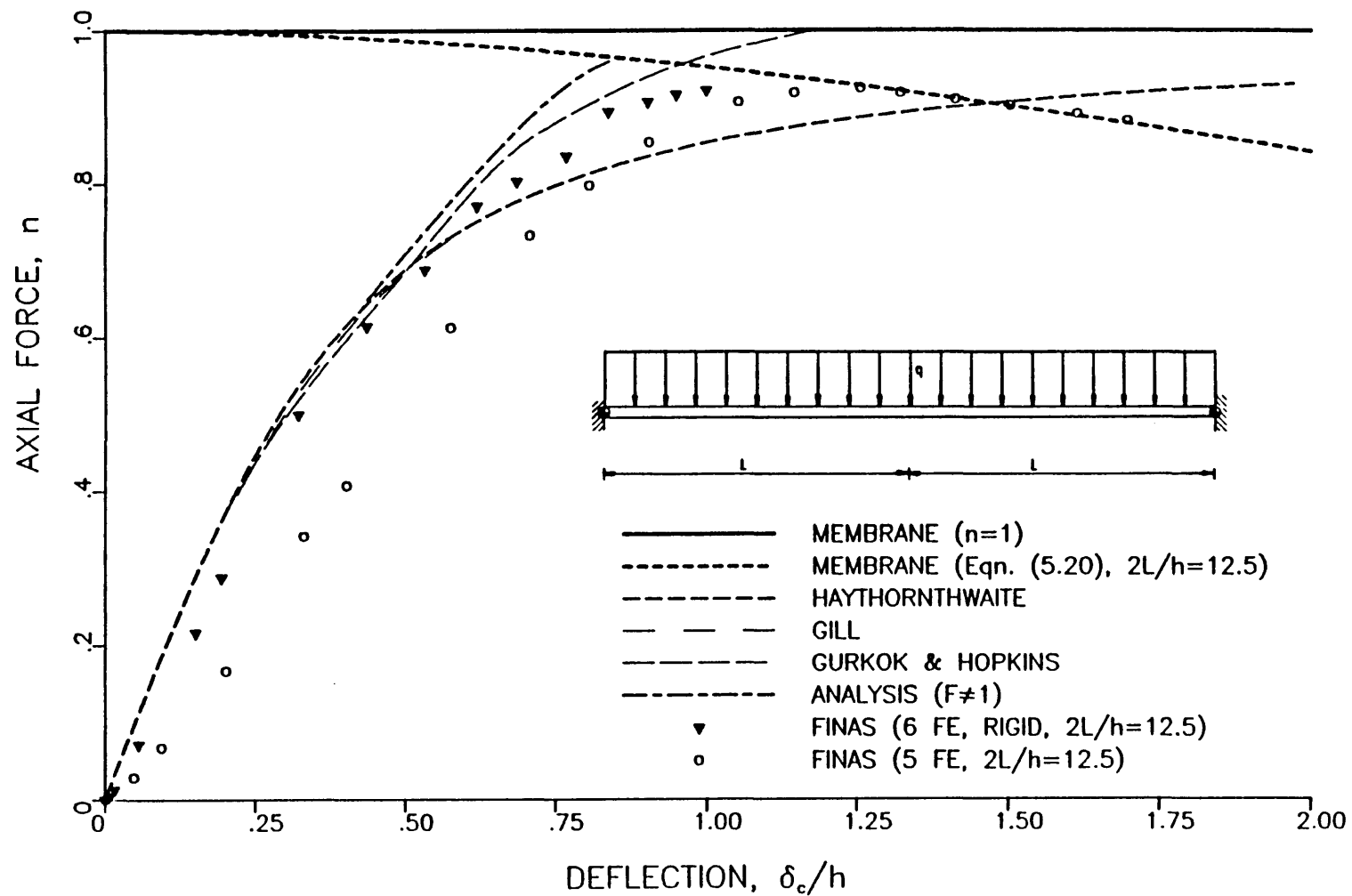


Fig. 5.5 Axial force vs. deflection (pinned supports)

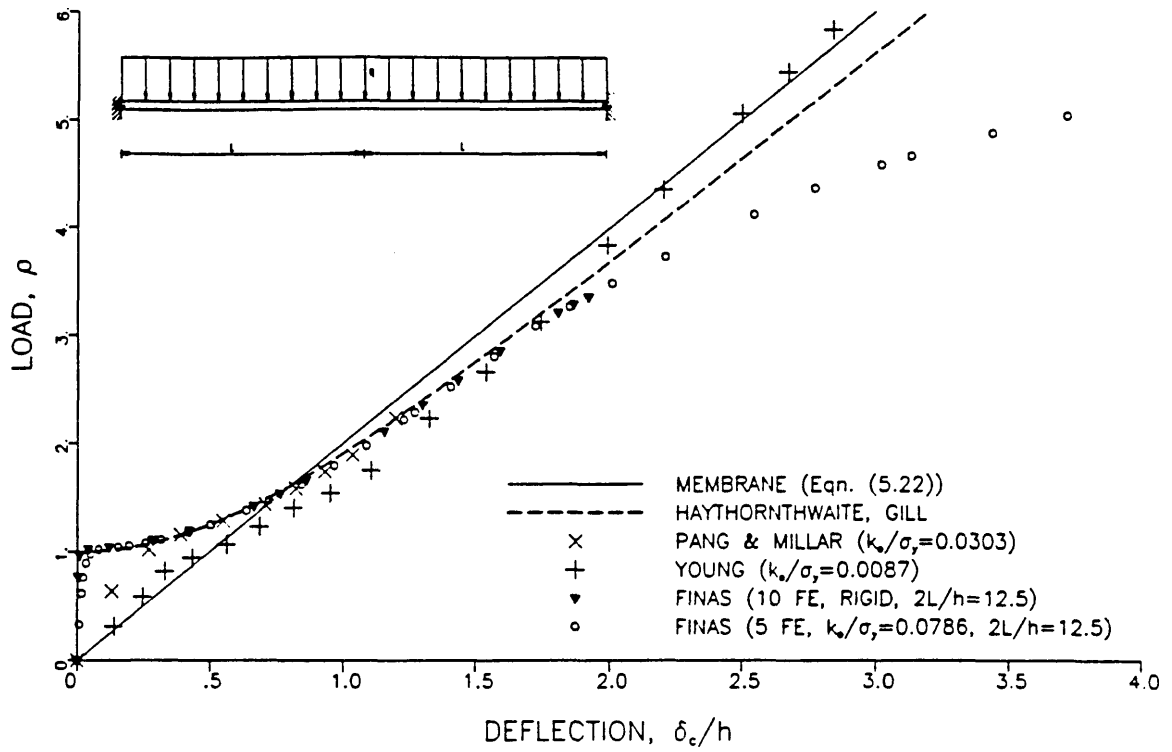


Fig. 5.6(a) Load vs. deflection (fixed supports)

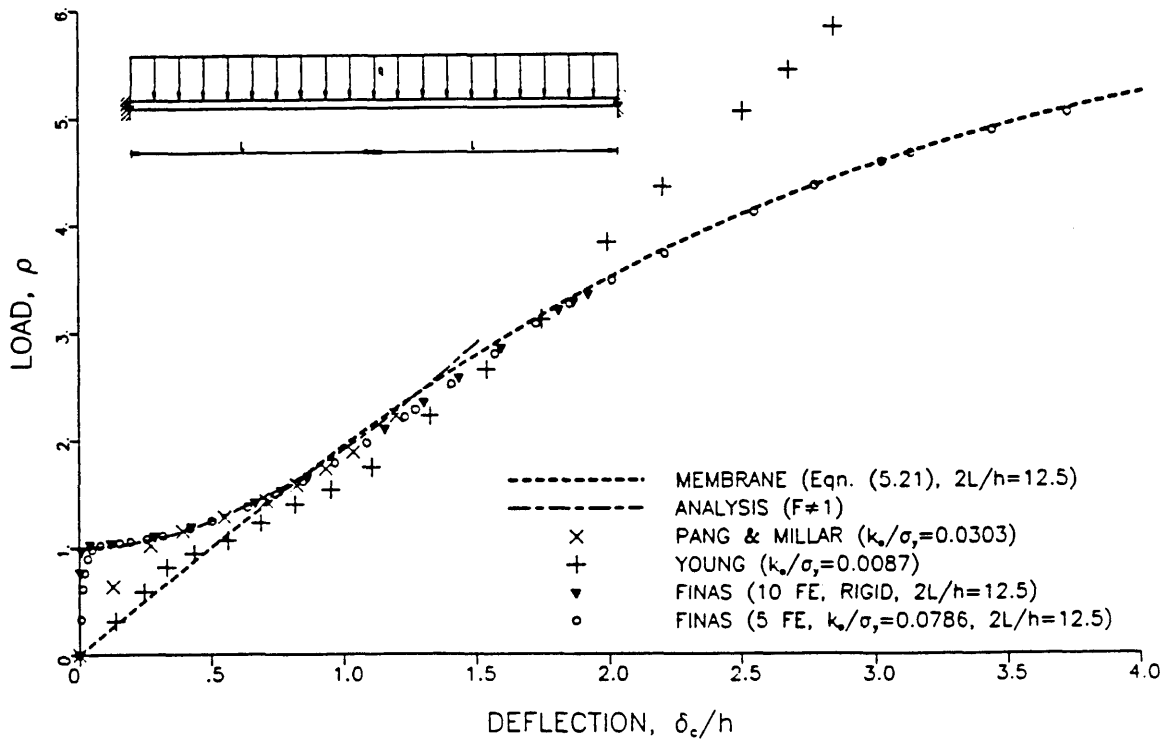


Fig. 5.6(b) Load vs. deflection (fixed supports)

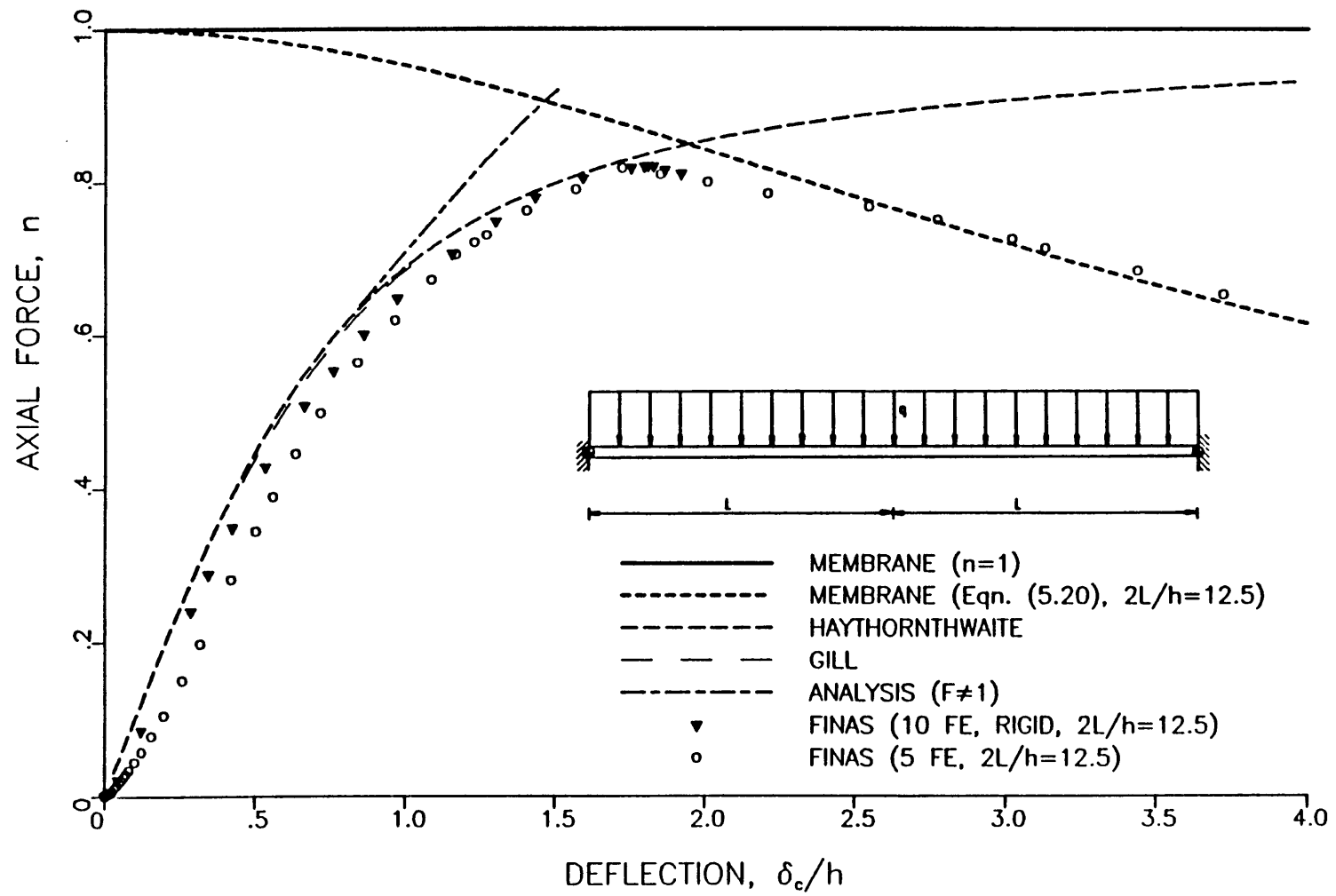


Fig. 5.7 Axial force vs. deflection (fixed supports)

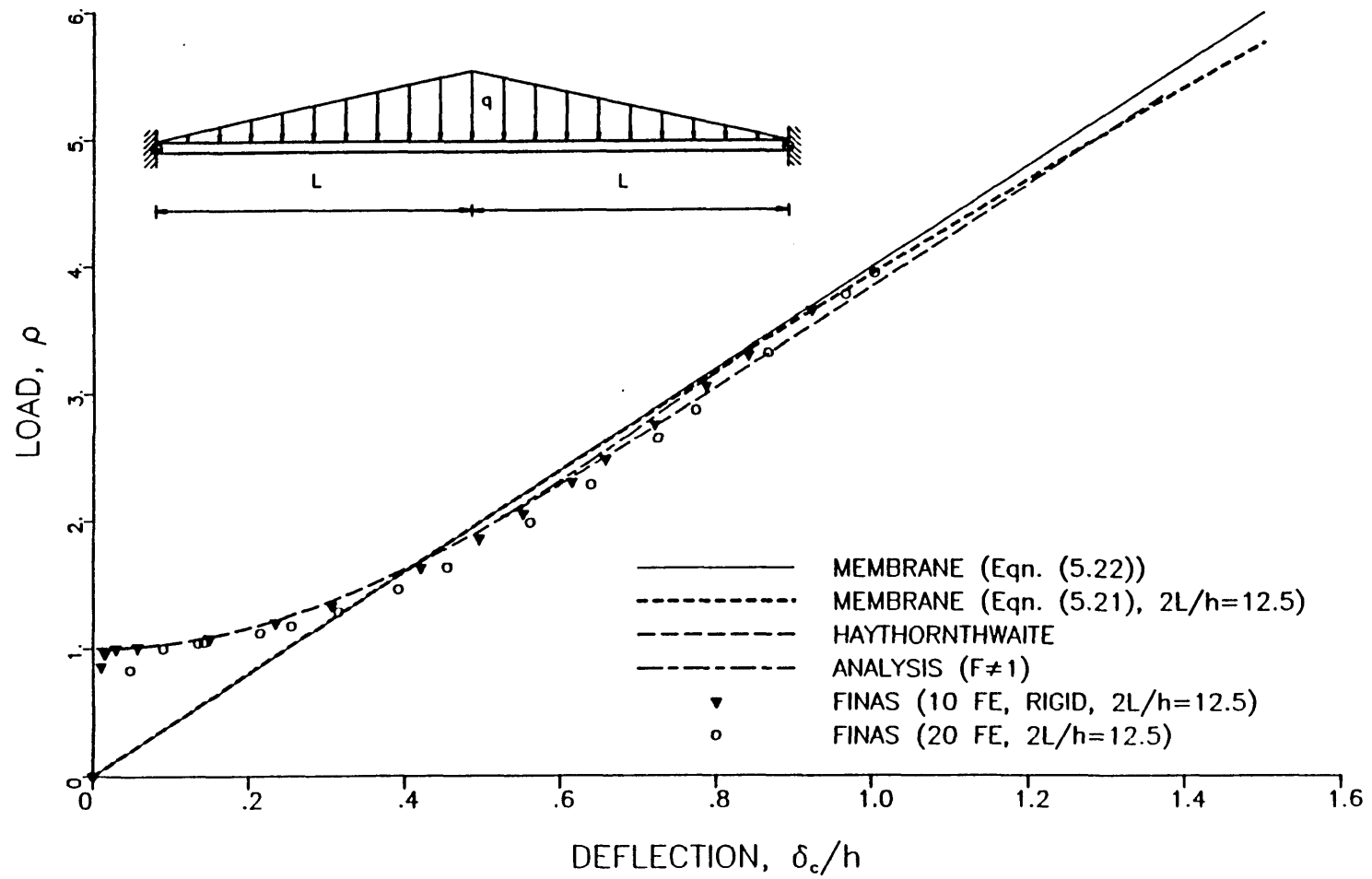


Fig. 5.8 Load vs. deflection (pinned supports)

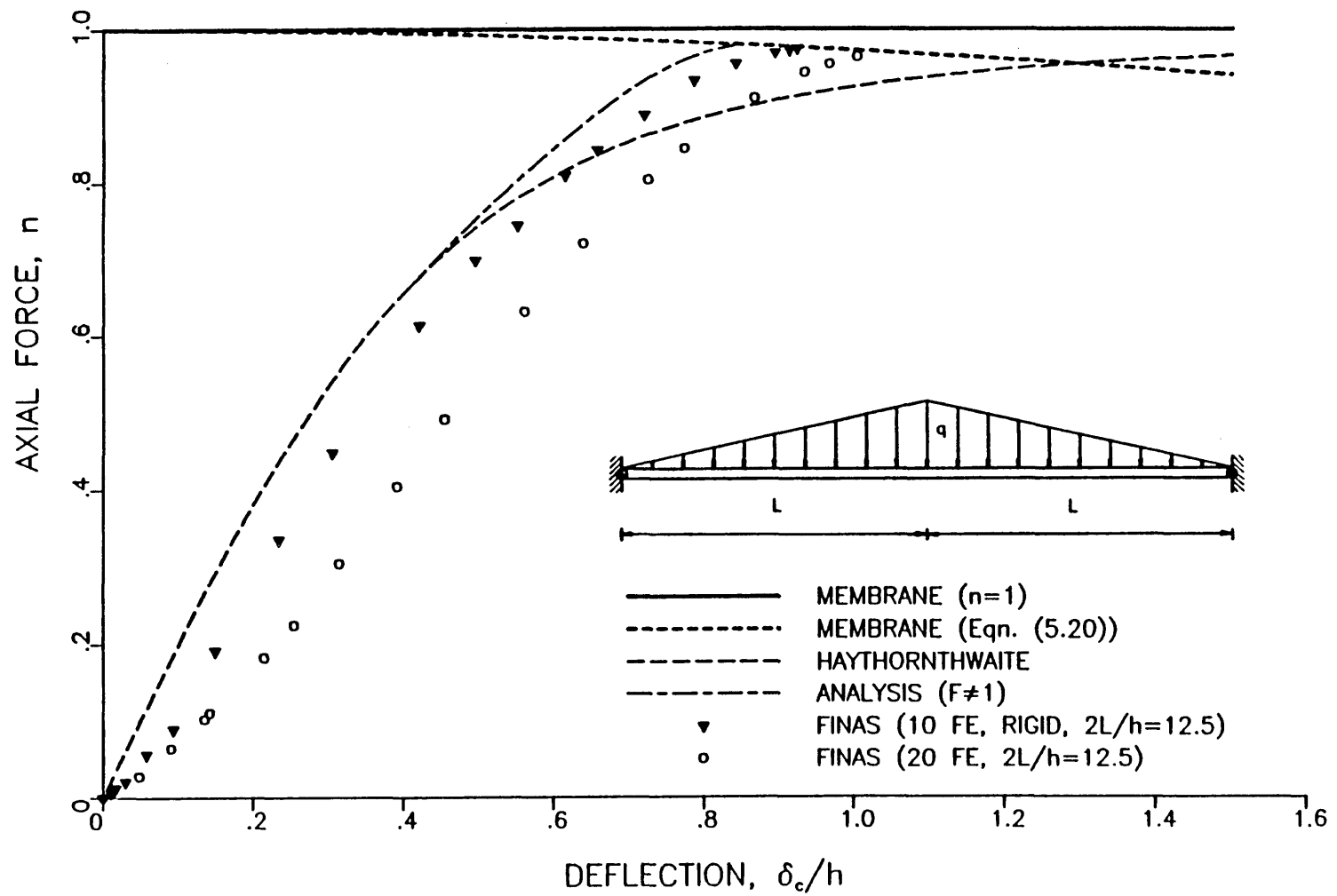


Fig. 5.9 Axial force vs. deflection (pinned supports)

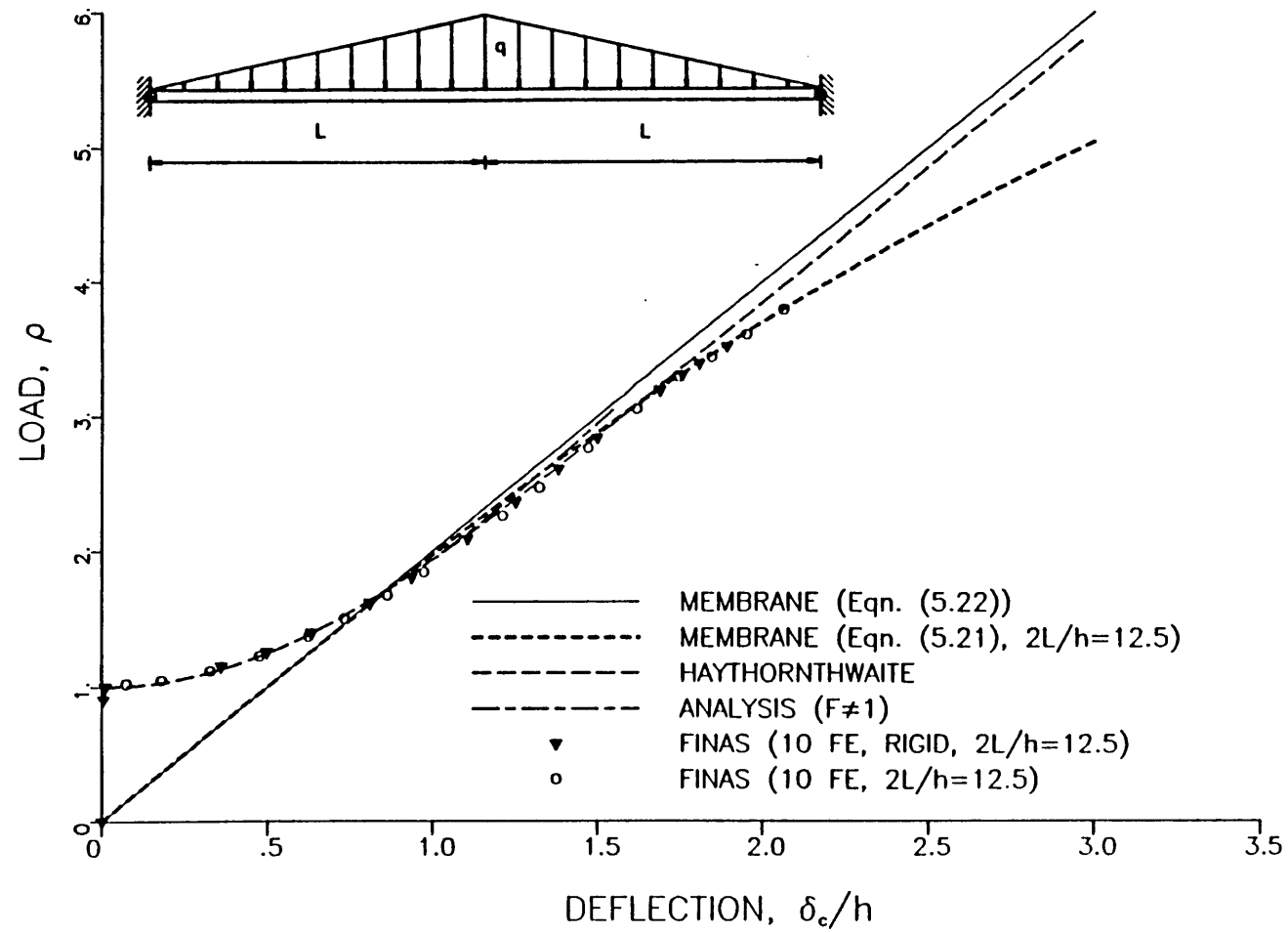


Fig. 5.10 Load vs. deflection (fixed supports)

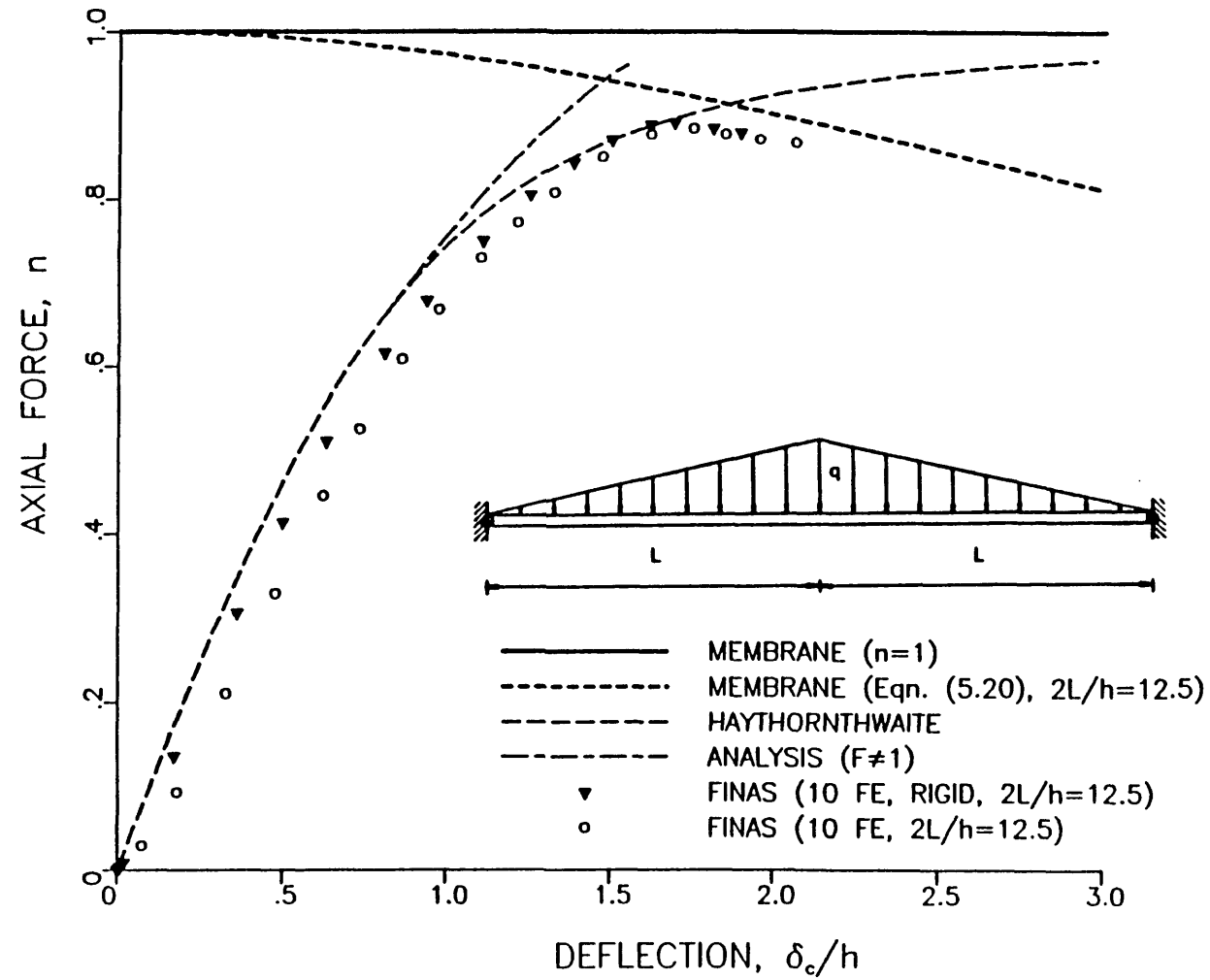


Fig. 5.11 Axial force vs. deflection (fixed supports)

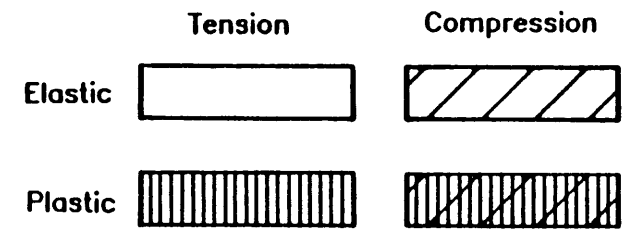
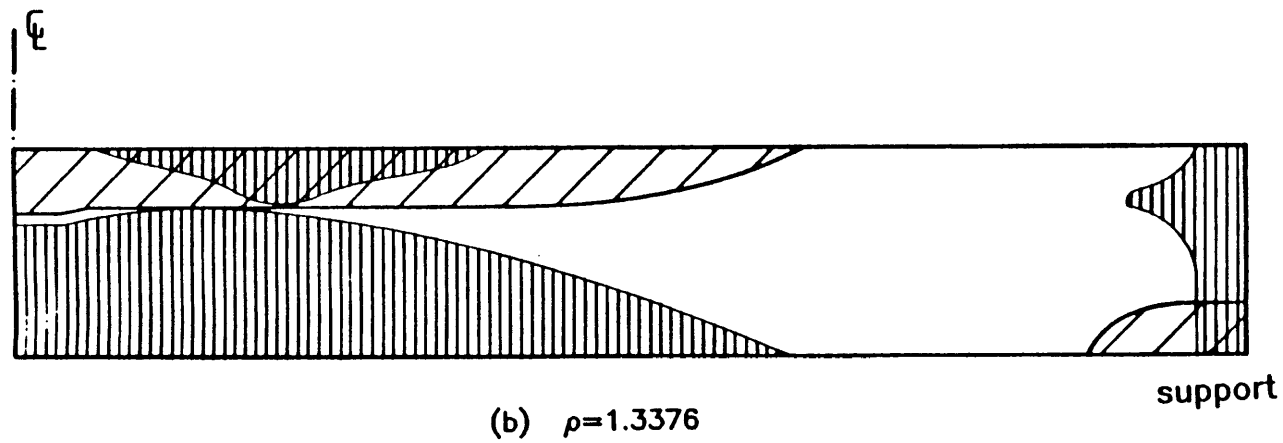
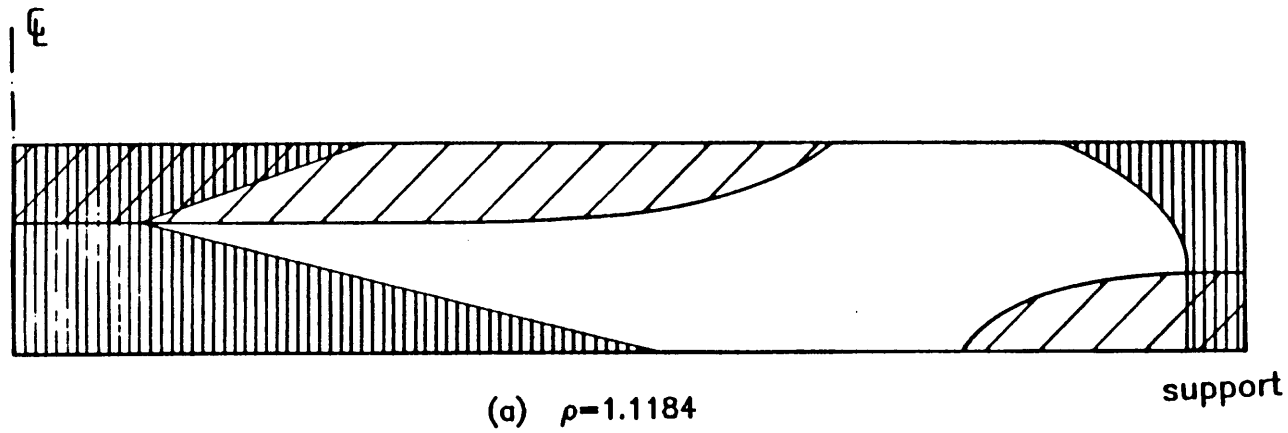


Fig. 5.12 Spreading of plasticity in an encastré beam under uniformly distributed loading

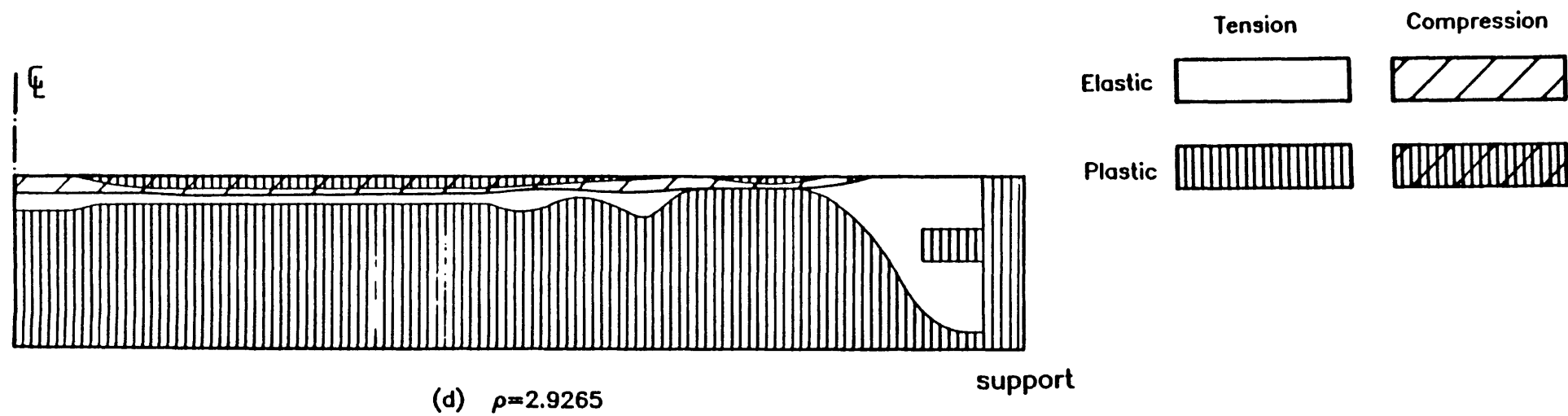
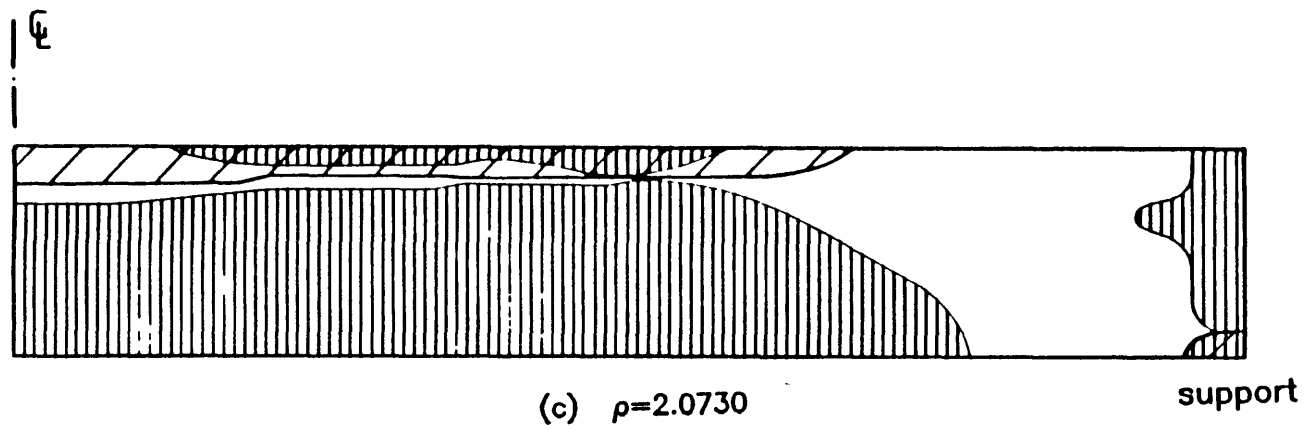


Fig. 5.12 Spreading of plasticity in an encastré beam under uniformly distributed loading

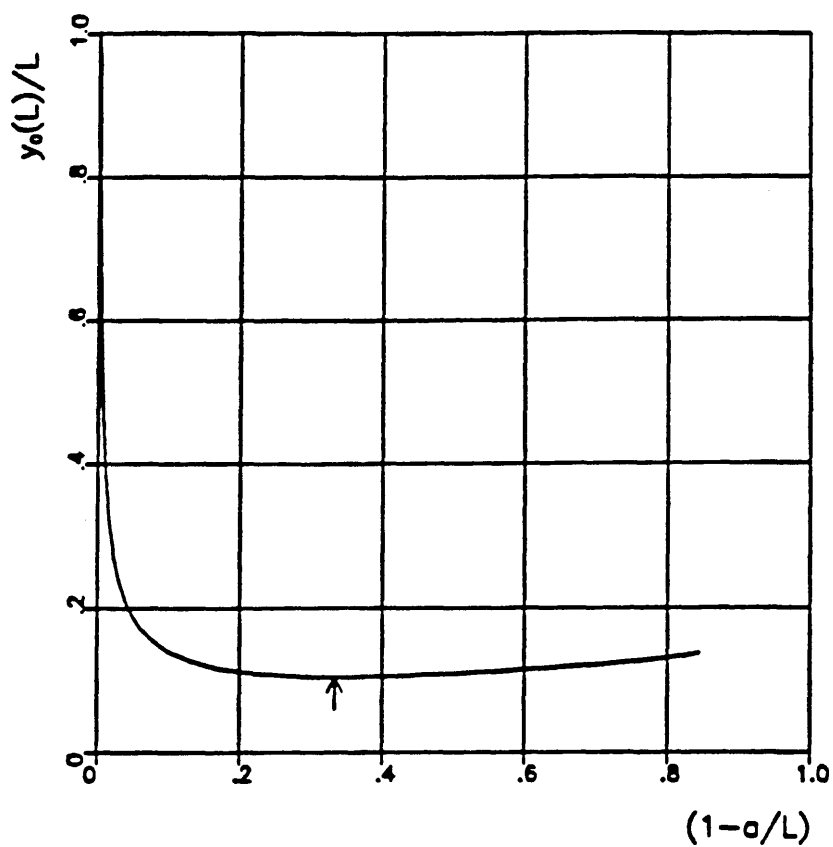


Fig. 5.13(a) Relationship between the deflection $y_0(L)$ and the horizontal projection of the curved segment $(L-a)$ for uniformly distributed loading

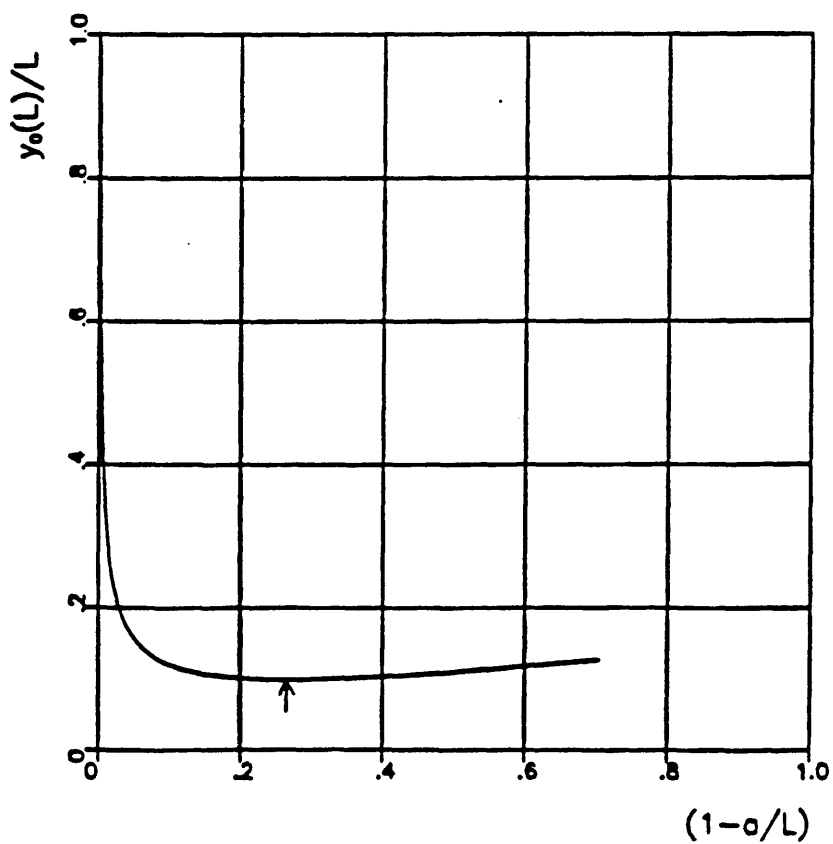


Fig. 5.13(b) Relationship between the deflection $y_0(L)$ and the horizontal projection of the curved segment $(L-a)$ for triangularly distributed loading

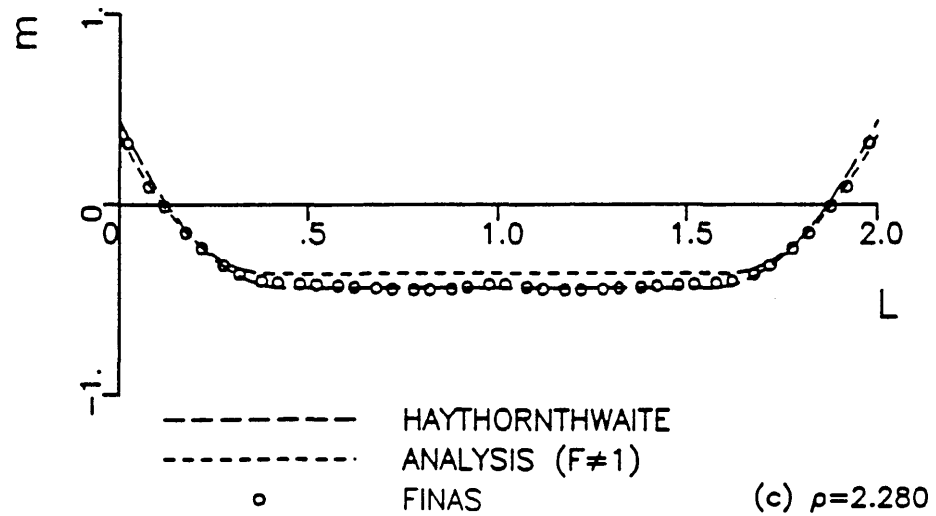
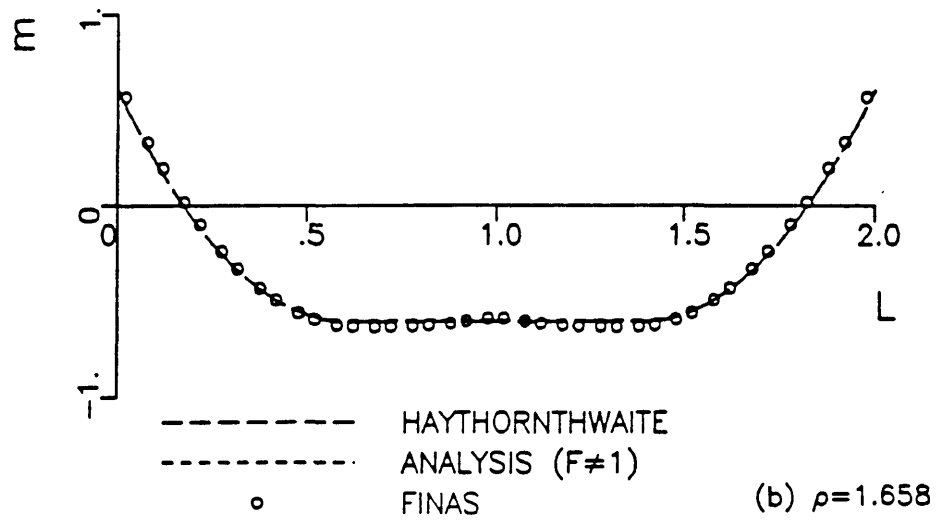
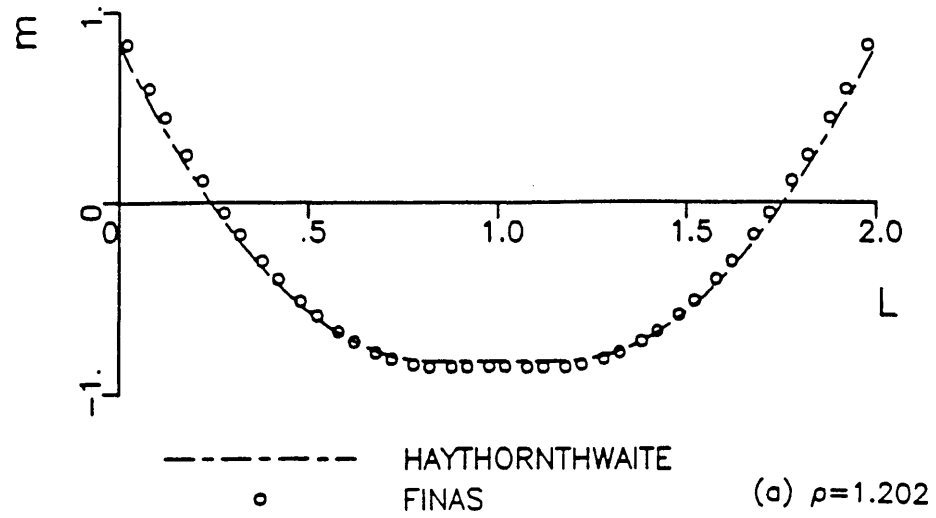


Fig. 5.14 Bending moment diagrams of an encastre beam under uniformly distributed loading

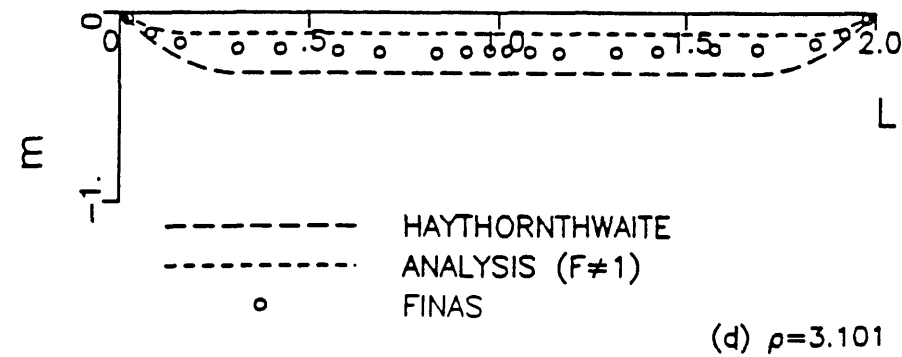
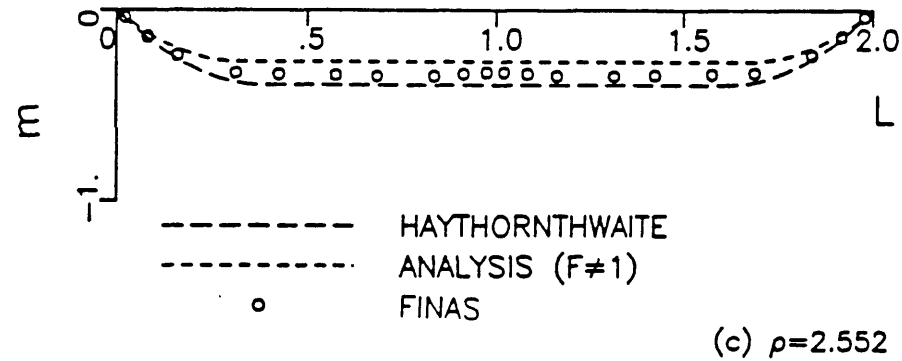
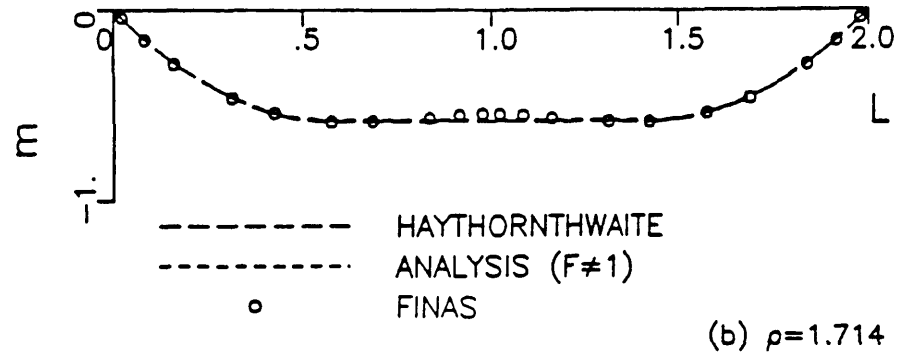
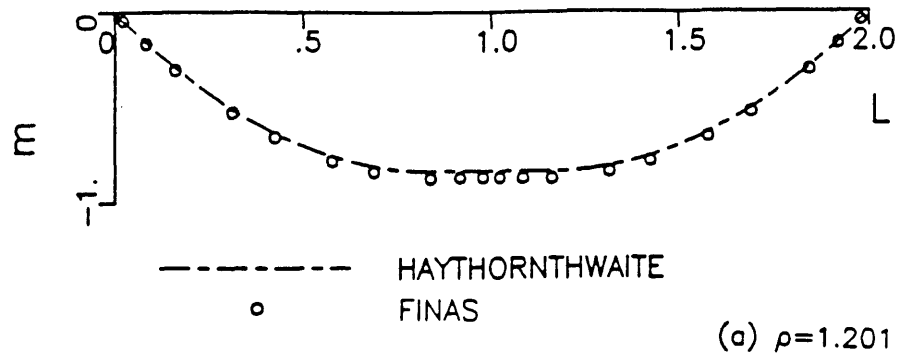


Fig. 5.15 Bending moment diagrams of a pin ended beam under uniformly distributed loading

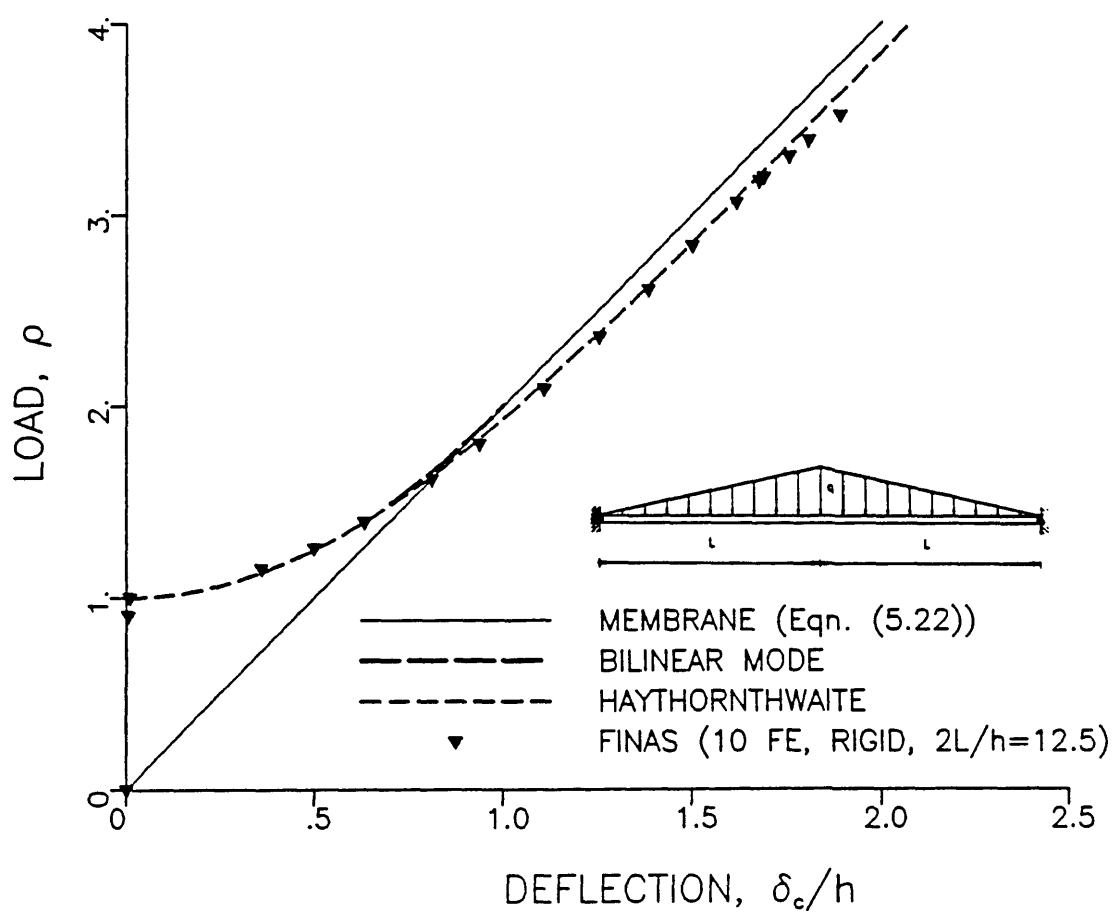


Fig. 5.16 Load vs. deflection (fixed supports)

CHAPTER SIX

ANALYTICAL MODEL FOR STIFFENED DECKS RESTRAINED AGAINST PULL-IN

6.1 INTRODUCTION

A qualitative model of a fully restrained deck with longitudinal stiffeners under patch lateral loading was proposed in Chapter 3. Figure 6.1 shows a quarter model of the deck, which consists of two types of axially restrained beams. Type I beams represent those loaded by the patch loading and the reactions of the transverse plate strips. One such beam is shown as beam 1 in Fig. 6.1. Type II beams are those remote from the patch loading and are shown as beams 2 to 4 in Fig. 6.1. As explained in Chapter 3, Type I beams can be idealized as beams on an elastic foundation and Type II beams can be envisaged as being under triangularly distributed loading. The analysis of Type I beams has been given in Chapter 4 while Type II beams have been investigated in Chapter 5.

This chapter presents an analysis of the model for a single bay stiffened deck fully restrained against pull-in under patch lateral loading. Two additional simplified analyses are also given. The simplifications concern the assumptions made about the axial forces in the beams. The first simplified analysis assumes zero axial forces in the beams, while presence of axial force in the loaded beam only gives the second simplified analysis. Predictions obtained from all three alternative analyses are compared with the numerical results of the deck examined in Chapter 3. Further comparisons are made for two additional decks with different geometries.

6.2 DEVELOPMENT OF ANALYTICAL MODEL

6.2.1 Assumptions

The following assumptions are made in addition to those outlined in Sections 3.6 for deck analysis and 4.4.1 for beam analysis:

- 1) The beams develop independent mechanisms, that is the corresponding compatibility equations are independent. The only interaction between adjacent mechanisms is due to the vertical component of the transverse membrane force in the plate.
- 2) Shearing in the plate, caused by differential deflections between adjacent stiffeners, is neglected. The effect of the shearing in the plate was discussed in Sections 3.5.2.3 and 3.5.3.1.

6.2.2 Compatibility equations

The compatibility equations for individual beams have been presented in Chapter 4. Since adjacent beams are assumed to form bilinear mechanisms, the compatibility Eqn. (4.6) applies to all the beams. Depending on the assumptions made, the resulting axial forces can be derived either from Eqn. (4.7) or (4.8). In this chapter the axial force is taken from Eqn. (4.7) as:

$$n = C \exp\left(\frac{2C_2}{C_1} \frac{\delta_c}{L}\right) - \frac{1}{2C_2} \frac{\delta_c}{L} - \frac{C_3}{2C_2} - \frac{C_1}{4C_2^2}$$

because of its wider application. The subscript c, in this equation used to indicate the central deflection, is substituted by the beam index j (j=1,2,...) using the layout shown in Fig. 3.1(a). The same index is also assigned to the beam axial forces.

6.2.3 Equilibrium equations

6.2.3.1 Directly loaded beams

The total patch loading applied to the deck is carried by longitudinal action of the loaded beams and transverse action of the plate which then transfers load to the unloaded beams. Figure 6.2(a) shows stiffener 1 under loading. The width of the patch load is $2\beta_1 b$, where b is the spacing of the stiffeners. Moment equilibrium of the applied forces with respect to the support gives:

$$\frac{P(L - \frac{\beta_1 b}{2})L}{2} = \frac{1}{3} \frac{t_{1,2}}{b_{1,2}} \sigma^t (\delta_1 - \delta_2) L^2 + M_1^s + (\lambda - 1) M_1^h + \delta_1 N_1$$

where $t_{1,2}$ is the plate thickness between stiffeners 1 and 2, $b_{1,2}$ has been defined in Section 3.6.1.3 for $j=1$, σ^t was defined in Section 3.6.1 and P is the total patch load. Non-dimensionalizing the applied loading with respect to

$$P_0 = \frac{2\lambda M_{01}}{(L - \frac{\beta_1 b}{2})}$$

where P_0 is the limit load for beam 1 when loaded centrally with a patch load of width $2\beta_1 b$, the above equation gives:

$$\rho = \frac{1}{\lambda} \left[D_{1,2} \left(\frac{\delta_1}{L} - \frac{\delta_2}{L} \right) + \frac{A_1 L}{Z_{p1}} (C_{21} n_1^2 + C_{31} n_1 + C_{41} + \frac{\delta_1}{L} n_1) \right] \quad (6.1)$$

In Eqn. (6.1), the value for $D_{1,2}$ is calculated from the general expression for $D_{j,j+1}$ given by:

$$D_{j,j+1} = \frac{1}{3} \frac{t_{j,j+1}}{b_{j,j+1}} \frac{\sigma^t}{\sigma_y} \frac{L^3}{Z_{p1}}$$

for $j=1$.

6.2.3.2 Indirectly loaded beams

The loading system for stiffener 2 and subsequent stiffeners ($j \geq 2$) is shown in Fig. 6.2(b). Taking moments about the support:

$$D_{j-1,j} \frac{\delta_{j-1}}{L} - (D_{j-1,j} + D_{j,j+1}) \frac{\delta_j}{L} + D_{j,j+1} \frac{\delta_{j+1}}{L} = \frac{A_j L}{Z_p} (C_{2j} n_j^2 + C_{3j} n_j + C_{4j} + \frac{\delta_j}{L} n_j) \quad (6.2)$$

The formulation of the above equation in terms of the coefficient $D_{j,j+1}$ allows variations in plate thickness, stiffener spacing and transverse membrane force from panel to panel to be modelled. In the case where the value of the transverse membrane force also varies σ^t should be replaced by $\sigma_{j,j+1}^t$.

6.2.3.3 Further considerations

When the transverse membrane stress, σ^t , equals the yield stress, the plate thickness is uniform and stiffeners are evenly spaced, Eqns (6.1) and (6.2) may be simplified as follows:

$$\rho = \frac{1}{\lambda} \left[D \left(\frac{\delta_1}{L} - \frac{\delta_2}{L} \right) + \frac{AL}{Z_p} (C_{21} n_1^2 + C_{31} n_1 + C_{41} + \frac{\delta_1}{L} n_1) \right] \quad (6.3(a))$$

$$D \left[\frac{\delta_{j-1}}{L} - 2 \frac{\delta_j}{L} + \frac{\delta_{j+1}}{L} \right] = \frac{AL}{Z_p} (C_{2j} n_j^2 + C_{3j} n_j + C_{4j} + \frac{\delta_j}{L} n_j) \quad , \quad \text{for } j \geq 2 \quad (6.3(b))$$

where

$$D = \frac{1}{3} \frac{t}{b} \frac{L^3}{Z_p}$$

The term in square brackets on the left hand side of Eqn. (6.3(b)) indicates that, if the central deflections of three successive stiffeners lie on a straight line, there is no loading on the middle stiffener since the loads cancel out as shown in Fig. 6.3. The right hand side of Eqn. (6.3(b)) gives the load carrying capacity of the beam comprising both bending and longitudinal membrane action. As shown in Fig. 5.16, at early stages of the behaviour, before significant membrane action builds up, the strength in a beam forming a longitudinal mechanism is not significantly higher than the limit load value. When the strength does not vary with increasing deflection the following equation relating incremental deflections holds:

$$\frac{d\delta_{j-1}}{L} - 2\frac{d\delta_j}{L} + \frac{d\delta_{j+1}}{L} = 0 \quad (6.4)$$

The above equation implies that the incremental deflections of adjacent stiffeners lie on a straight line. This is characteristic of a bending-governed behaviour occurring at early stages of the deflection history. If such a behaviour is assumed, then the slopes, expected in Fig. 3.5(a), are those represented by continuous lines.

Figure 6.4 shows schematically the load-deflection behaviour of an axially restrained beam under transverse loading. The increase in stiffener strength with increasing deflection indicates that incremental deflections of adjacent stiffeners do not satisfy Eqn. (6.4), which holds for constant strength only. Figure 6.4 also shows that for a stiffener, the larger the central deflection, the higher the increase in the strength due to an increase in the central deflection. The incremental deflection of stiffener $j-1$ is made up of three components, while stiffener ℓ has not yet formed a mechanism, as shown in Fig. 6.5. The first component follows Eqn. (6.4), the second corresponds to equal increases in the strength of stiffeners j and $j+1$, and the third is due to different increases in the strength of stiffener j and $j+1$. Therefore, the expected ratio of the incremental central deflection of stiffeners $j-1$ and j is:

$$\frac{d\delta_{j-1}/L}{d\delta_j/L} = \frac{(\ell-j+1)/(\ell-j) + 3\varphi + \psi}{1 + \varphi} \quad (6.5(a))$$

$$\frac{d\delta_{j-1}/L}{d\delta_j/L} > \frac{\ell-j+1}{\ell-j} \quad (6.5(b))$$

According to expression (6.5(b)) the deflection curves for the actual deck may be expected to be under the continuous lines drawn in Fig. 3.5(a). These are observed to be so.

6.2.4 Initiation of mechanism in indirectly loaded beams

The rigid-plastic analysis of indirectly loaded beams ($j \geq 2$) commences when the loading on the directly loaded beam ($j=1$) reaches the limit load. Hence, the deck starts to deflect under initial conditions:

$$\rho = \rho_I, \quad \delta_j = \delta_{Ij}, \quad n_j = n_{Ij}$$

where δ_{Ij} and n_{Ij} are the initial values for deflection and prestress respectively of beam j . The initial value for the load factor, ρ_I , is obtained from consideration of Eqn. (6.1) using the initial conditions.

Stiffener $j+1$ ($j \geq 1$) forms a mechanism when the central deflection in stiffener j obtains the critical value, δ_{jcr} . This value can be determined by consideration of Eqn. (6.2) for stiffener $j+1$, giving:

$$\frac{\delta_{jcr}}{L} = 2 \frac{\delta_{I,j+1}}{L} - \frac{\delta_{I,j+2}}{L} + \frac{A_{j+1}L}{D_{j,j+1} Z_{p1}} \left[C_{2,j+1} n_{I,j+1}^2 + C_{3,j+1} n_{I,j+1} + C_{4,j+1} + \frac{\delta_{I,j+1}}{L} n_{I,j+1} \right] \quad (6.7)$$

The state of the deflections can be expressed as:

$$\text{if } \delta_j \leq \delta_{jcr} \text{ then } \delta_\ell = \delta_{I\ell}, \quad \text{for } \ell = j+1, j+2 \dots$$

6.2.5 Method of solution

The solution technique is given below:

- 1) The analysis starts assuming that stiffener 1 has formed a mechanism and pivots with respect to stiffener 2 which is intact. Steps 2 to 5 are followed starting with $j=1$.

- 2) An increment, $d\delta_j$, in the central deflection of stiffener j which pivots with respect to the stiffener $j+1$, is applied. The total deflection of stiffener j should not exceed the critical deflection, δ_{jcr} , indicating collapse of stiffener $j+1$ and given by Eqn. (6.7). If δ_{jcr} is exceeded, the index j is advanced by one, and this step is repeated. When the outmost stiffener collapses it pivots with respect to the support and the check through Eqn. (6.7) is no longer applicable.
- 3) Equation (4.7) yields the axial force in stiffener j , n_j , which upon substitution into Eqn. (6.3(b)) gives the deflection of stiffener $j-1$, δ_{j-1} .
- 4) Step 3 is repeated until the values for the central deflection and the axial force in the loaded stiffener 1, δ_1 and n_1 , are found.
- 5) Upon substitution of n_1 and δ_1 in the equilibrium equation (6.3(a)) the value for the load factor, ρ , is obtained.

6.3 FIRST SIMPLIFIED METHOD OF ANALYSIS

This simplified analysis is based on the assumption that no axial tension is developed along the beams (in the longitudinal direction). It is still assumed that the beams remain straight between the support and central hinges. The behaviour of the plate is represented by the membrane solution in the transverse direction and all the assumptions made in Section 6.2 are valid.

Stiffener $j+1$ starts to deflect once the deflection of stiffener j attains the critical value δ_{jcr} , given by Eqn. (6.7). As no additional loading in excess of that initiating the plastic mechanism can be carried by the beams, each of them is equally loaded once lateral deflections take place.

It was pointed out in Section 6.2.3.3 that the condition necessary for no additional loading on a stiffener which has formed a mechanism, is given by Eqn. (6.4). Assuming that stiffener $j+1$ has not experienced any deflection while stiffeners j , $j-1$, $j-2$, etc. have formed mechanisms, consideration of equations similar to (6.4) for the deflecting stiffeners gives:

$$\frac{d\delta_{j+1}}{L} = 0$$

$$\frac{d\delta_{j-\ell-1}}{L} = 2 \frac{d\delta_{j-\ell}}{L} - \frac{d\delta_{j-\ell+1}}{L} \quad , \quad \text{for } \ell = 0, \dots, j-2$$

Application of these equations to all the stiffeners results in:

$$\frac{d\delta_{j-\ell-1}}{L} = (\ell+2) \frac{d\delta_j}{L}, \quad \text{for } \ell = 0, 1, 2, \dots, j-2$$

The incremental deflections of stiffeners 1 to j between the incipient collapse of stiffeners j and $j+1$ can be obtained from the above equation by setting:

$$\frac{d\delta_j}{L} = \frac{\delta_{jcr}}{L}$$

The load carrying capacity when a mechanism is formed in stiffener $j+1$, is the sum of the limit loads of the stiffeners forming mechanisms, which is equal to $(j+1)P_0$. Once all the stiffeners have collapsed, the incremental central deflections are pivoting with respect to the supports. Reference to this analysis is made by the term 'simplified 1'.

6.4 SECOND SIMPLIFIED METHOD OF ANALYSIS

Figure 6.4 shows that the biggest increment in strength due to longitudinal membrane action occurs for beam 1. Since the compatibility equations for the beams are independent, it can be assumed that one or two of them develop longitudinal membrane action, whereas no such action takes place in the remainder of the beams.

This simplified analysis is based on the assumption of longitudinal membrane action in beam 1 only. The remaining assumptions and all the equations set in Section 6.3 are still valid. The only difference with the previously described simplified analysis is that the load carrying capacity is derived by consideration of the equilibrium of stiffener 1, that is, from Eqn. (6.1) or (6.3(a)). In the later sections, the present analysis is referenced as 'simplified 2'.

6.5 COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS

6.5.1 Load-deflection predictions

Figures 6.6(a) to 6.6(d) show the load versus central deflection response for the stiffeners, determined by finite elements as well as the three analyses.

In Fig. 6.6(a) the predictions for stiffener 1 are presented. The initial linear elastic behaviour of the stiffener can be predicted using a grillage analysis (Clarkson (1959)). Beyond the linear elastic region, the responses predicted by the finite elements and the analysis presented in Section 6.2, are comparable. Initially, the strength is overestimated by the analysis. At the point of maximum discrepancy, when the deflection of stiffener 1 is approximately two times the plate thickness or $0.013L$, the finite elements predict 22% less load than that expected by the analysis. This overestimation of strength is attributed to the assumption that the transverse membrane stress is equal to the yield stress. The rigid-plastic analysis is more applicable to the large deflection region where it becomes slightly conservative with a maximum difference of the order of 8%.

The response predicted by the first simplified analysis, is initially identical to that which considers longitudinal membrane action. As the deflection increases, longitudinal membrane action becomes increasingly important, and therefore the behaviour predicted by the simplified analysis diverges. However, this analysis has its own merit in being simple and giving a conservative strength estimate at deflections in excess of several times the plate thickness.

The response predicted by the second simplified analysis, is virtually identical to the result obtained by the full analysis presented in Section 6.2. This can be explained by referring to Fig. 6.4. For the range of deflections shown in figures 6.6, only stiffener 1 attains a significant enhancement in strength above that corresponding to the limit load due to longitudinal membrane action. At very high deflections however, the two predictions begin to differ as membrane action begins to take effect in stiffener 2.

The responses of indirectly loaded stiffeners are compared in Figs 6.6(b) to 6.6(d). The same general conclusions as for stiffener 1 can be drawn about the degree of conservatism in the predicted behaviour by the analyses. As shown in these figures, the predictions for the indirectly loaded stiffeners show higher discrepancies than those for stiffener 1. However, the accuracy with which the response of the

directly loaded stiffeners is predicted, is of primary importance.

In general, the second simplified analysis seems the most attractive of all. It has the advantage of avoiding complexity of the analysis and improving the accuracy over the first simplified analysis.

6.5.2 Adjacent stiffener deflections

The estimated deflections of the adjacent stiffeners are compared in Figs 6.7. Figure 6.7(a) shows stiffeners 1 and 2 pivoting with respect to stiffener 3 for a significant part of the deflection history presented. The simplified analyses predict slopes 1:2, 2:3 and 3:4 for stiffeners 1, 2 and 3 respectively as there is no increase in the strength of beams 2, 3 etc. above their limit load value. Therefore, the slopes can be predicted by using the limiting value of the relationship (6.5(b)). The full analysis predicts some differential increase in the strength of the beams and therefore the predicted slopes are slightly altered but in accordance to (6.5(a)). The data points obtained from the finite element analysis of the deck R02 show the stiffeners 1 and 2 pivoting with respect to stiffener 3 for a considerable part of the deformation history. This can be explained by the increased axial force in both stiffeners 1 and 2 in R02 due to shearing in the deck plate. This in turn causes the increase in strength in the same stiffeners.

Good agreement between the theoretical analyses and the numerical results is shown in Figs 6.7(b) and 6.7(c) in the range of deflections where results from all the analyses exist. This is because the outer stiffeners have much smaller deflections than the inner one, and therefore, their strengths are close to those at limit load.

6.5.3 Axial forces in the stiffeners

Axial forces in the middle of the stiffeners are plotted against stiffener central deflections in figures 6.8. Figure 6.8(a) shows the response for stiffener 1, where the axial force predicted by the finite element solution grows faster than that predicted analytically. At a value of axial force slightly higher than 60% of the squash load, the predictions follow two completely opposite trends. The initially higher axial force values in the numerical model for deck R02 may be a consequence of plate shearing which is not considered in the analysis. The following plateau in the numerical prediction for deck R02 is probably due to

plastification in the plate panel between stiffeners 1 and 2.

The axial force for stiffener 2 is given in Fig. 6.8(b). At small deflections, the axial force in the centre of the stiffener can be compressive due to shearing in the plate panel between stiffeners 1 and 2. The higher values of axial force in R02 observed thereafter, are due to plate shearing in the panel between stiffeners 2 and 3.

Although the discrepancies in the axial force values are considerable at small deflections their effect on the strength is not significant because their contribution to the product $n\delta/L$ is small though. When the values of $n\delta/L$ become higher, the discrepancies in n are smaller, particularly for stiffener 1.

6.6 MODEL IMPLEMENTATION USING FINITE ELEMENTS

In order to isolate the influence of the assumption of a bilinear mode mechanism the proposed model was generated using finite elements. By using flexible beam finite elements to model the longitudinal beams, any difference in shape from the straight line assumed in the analytical model can be followed. To model the vertical component of the transverse membrane force, the linear spring element was used. As explained in Section 3.6.1.3, the vertical component of the transverse membrane force is assumed to vary linearly with the differential deflection between two adjacent stiffeners at the same longitudinal position.

The load-deflection response as predicted by both finite elements and the analytical model is shown in Fig. 6.9, where it can be seen that the agreement is very good. The more flexible response of the finite element model is anticipated as a result of elasticity effects incorporated in the beam element. Therefore, the analytical model presented in this chapter is acceptable, for the range of deflections shown in Fig. 6.9, even if the bilinear mode of deformation is assumed.

The build-up of axial force with increasing deflection for stiffener 1 is shown in Fig. 6.8(a). The agreement with the full analysis is good up to a value of n higher than 60% of the squash load. Thereafter the predictions diverge. Comparison of the predicted deflection responses for stiffener 2 in Fig. 6.8(b) shows fairly good agreement both in terms of values and general trends.

The observed discrepancy in the values of the axial force is due to the difference in the mode-shapes assumed, as explained in Chapter 4. Additional evidence showing that the beams do not follow the bilinear mode assumed in the analysis, is given in Figs 6.10 and 6.11. These plots illustrate the mode-shapes for stiffener 1 at various stages of the deformation history. Figure 6.10 is produced from the numerical analysis of deck R02, while Fig. 6.11 is produced from the numerical analysis of model MR02. Comparison of the two figures shows that both mode-shapes undergo incremental rotations in excess of those assumed by the bilinear mode-shape.

6.7 COMPARISONS WITH OTHER DECK GEOMETRIES

Two other decks with different geometries were also analysed. The first deck, namely R03, had four longitudinal stiffeners across the entire width of the deck. The stiffeners were flat bar sections symmetrically placed with respect to the plate mid-surface. This results in 'notional beams' of cruciform cross-section. The parameters of this cross-sectional geometry were given in Table 4.1. The various predictions for the central deflection of the loaded stiffener (stiffener 1) are plotted in Fig. 6.12(a). The agreement between the numerical analysis and the analytical model is excellent. As in Section 6.5.1, the same general conclusions can be drawn for the simplified analyses. Analytical predictions for stiffener 2 are given in Fig. 6.12(b) and have similar degree of accuracy as for stiffener 1. Predictions of adjacent stiffener deflection for the two stiffeners in this quarter-deck are given in Fig. 6.12(c). As in the case of deck R02, stiffener 1 deflects more relative to stiffener 2 than predicted by considering both stiffeners to pivot with respect to the support.

The second deck, namely R04, had a similar cross-sectional geometry to deck R02, but different proportions for the 'notional beams', as given in Table 4.2. The predictions for the load versus stiffener central deflections are given in Figs 6.13(a) to 6.13(d). Good agreement is observed between the analyses and the finite element results. Finally, the deflections of adjacent stiffeners are given in Figs 6.13(e) to 6.13(g). The same conclusions as for the previous decks can be drawn for the deflections of adjacent stiffeners.

6.8 CONCLUSIONS

A fully restrained deck with longitudinal stiffeners can be visualized as a series of longitudinal beams and transverse plate strips.

Three types of analysis were presented. A common assumption in all analyses was that the behaviour of the plate strips is represented by the membrane solution in the transverse direction. The first analysis considered longitudinal axial force development in the beams and gave a very good agreement with the finite element results for the behaviour of the deck. The second analysis disregarded the longitudinal axial force development and resulted in a simple prediction which diverged from the analysis with increasing deflection. The third analysis considered longitudinal membrane force development in the directly loaded beam only and could be regarded as an optimum, with regard to complexity and accuracy.

The decks considered in this chapter help verify the wider applicability of the proposed analysis, although a full parametric investigation is beyond the scope of this thesis. An insight into the phenomena occurring in this type of deck under the localized loads considered and an attempt to model the main actions governing the behaviour of a simple model were given primary importance.

The load history of restrained decks can be estimated using the analyses presented. Other limit analyses, which disregard the membrane action, give a single value for the ultimate load sustained by the stiffened plating. Therefore, the advantage of the analyses presented in this thesis is that the plastic energy absorbed during the deformation of the deck can be calculated. Furthermore, the load carrying capacity estimated without including the contribution of membrane action in the decks, is expected to be different to that predicted by the present analyses.

The range of deflections over which the analyses are applicable was shown to be of the order of twenty five times the thickness of the deck plate and probably even higher. This range is enormous and its upper end may not be of practical interest. At very large deflections, of the order of many times the plate thickness, it is likely that material rupture due to excessive straining will govern the behaviour. This limits the range of acceptable deformations, and therefore, any extension of the analyses to account for even higher deflections, is rather unnecessary.

Finally, it is worth mentioning that two aspects of the actual deck behaviour were not modelled in any of the analyses presented. The first aspect is shearing in the

plate that results from the deck taking on a non-developable shape. The effect of plate shearing is to enhance the longitudinal tension force near mid-span. The second aspect is that the directly loaded stiffener does not follow exactly the bilinear mechanism shape assumed. In Chapter 4 an approximate method to consider this effect was discussed. Both aspects could be the subject of further research.

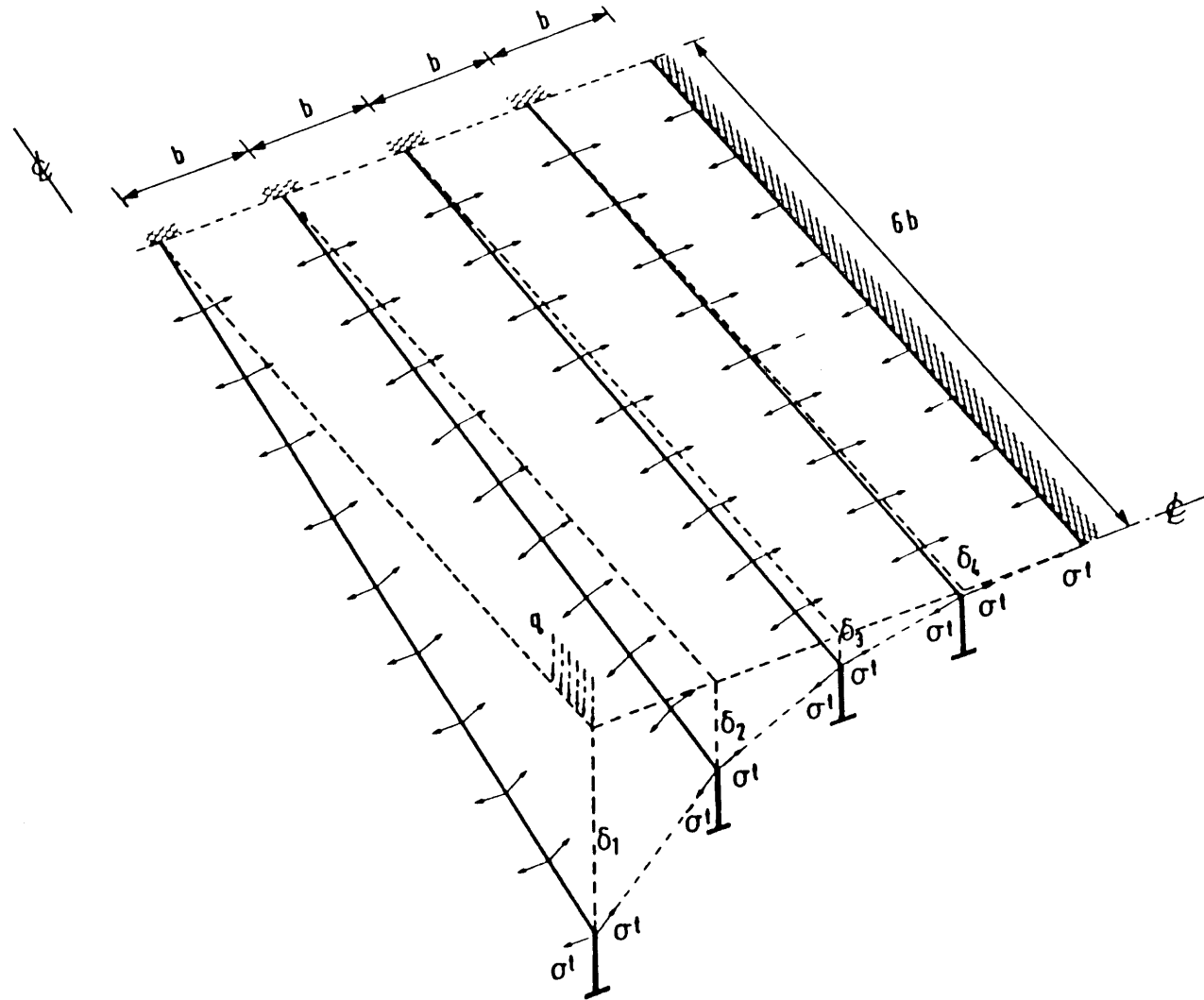


Fig. 6.1 Proposed model for restrained deck

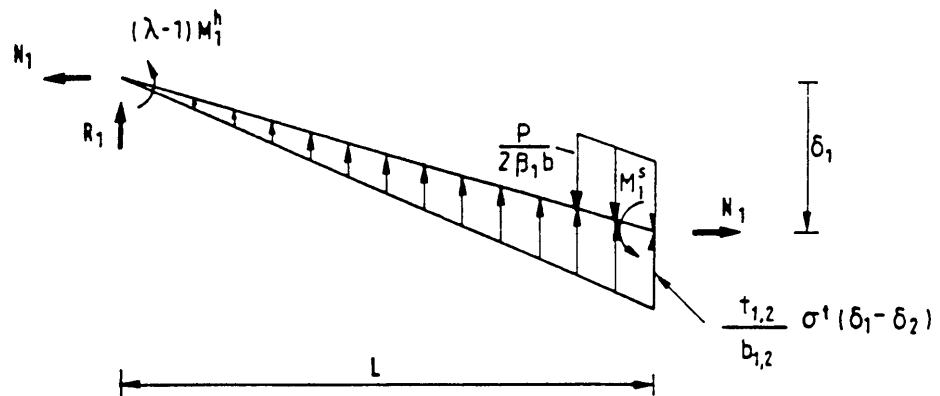


Fig. 6.2(a) Forces acting on stiffener 1

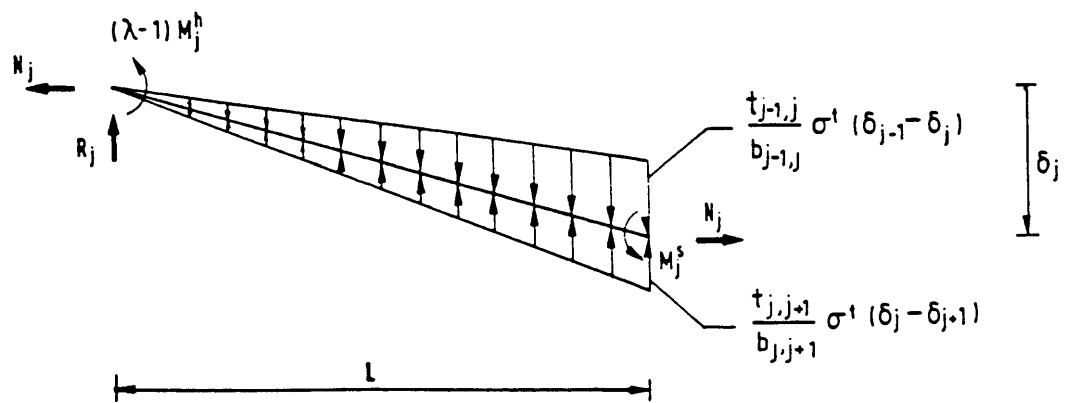


Fig. 6.2(b) Forces acting on stiffeners other than 1

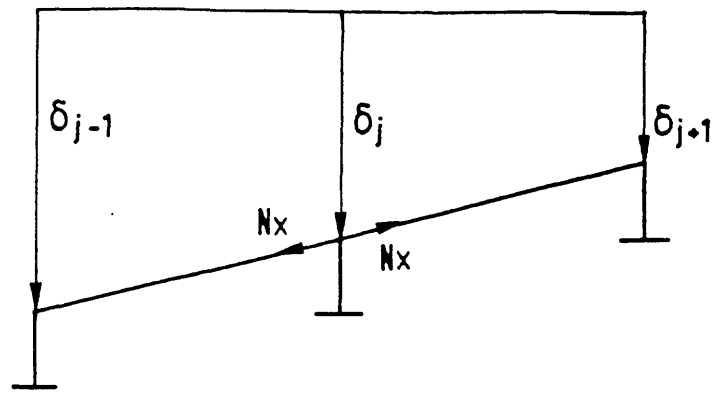


Fig. 6.3 Load applied by the plate on stiffener j

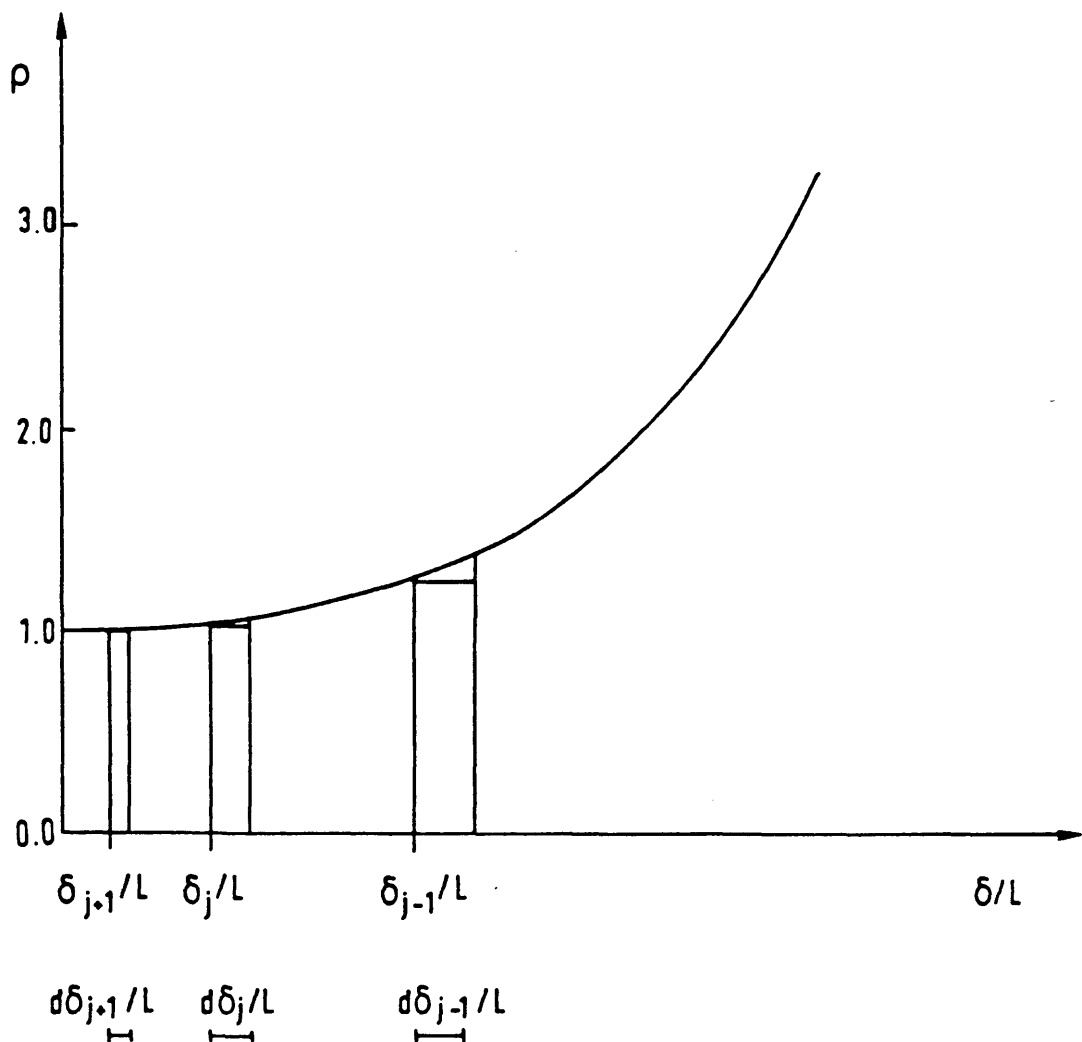


Fig. 6.4 Strength increments of longitudinal mechanisms

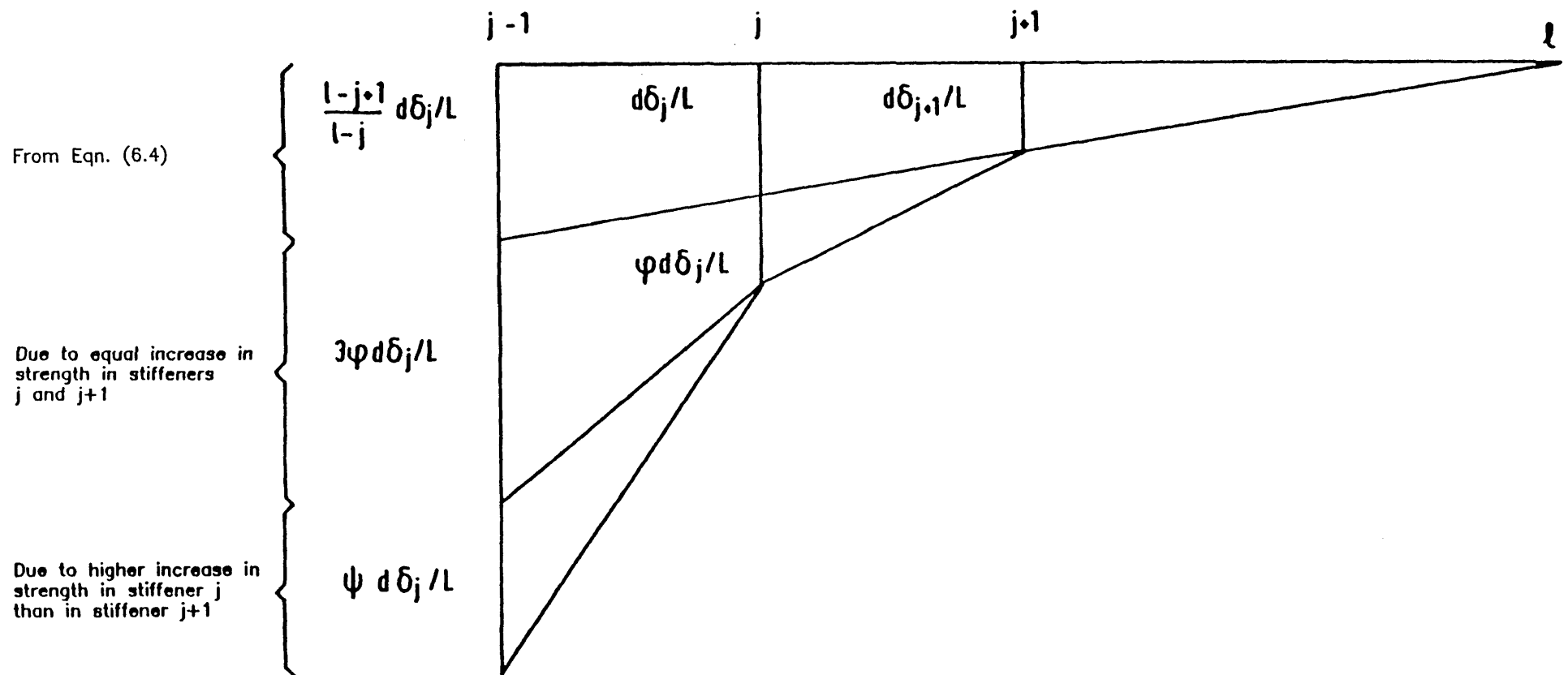


Fig. 6.5 Components of stiffener deflection increment

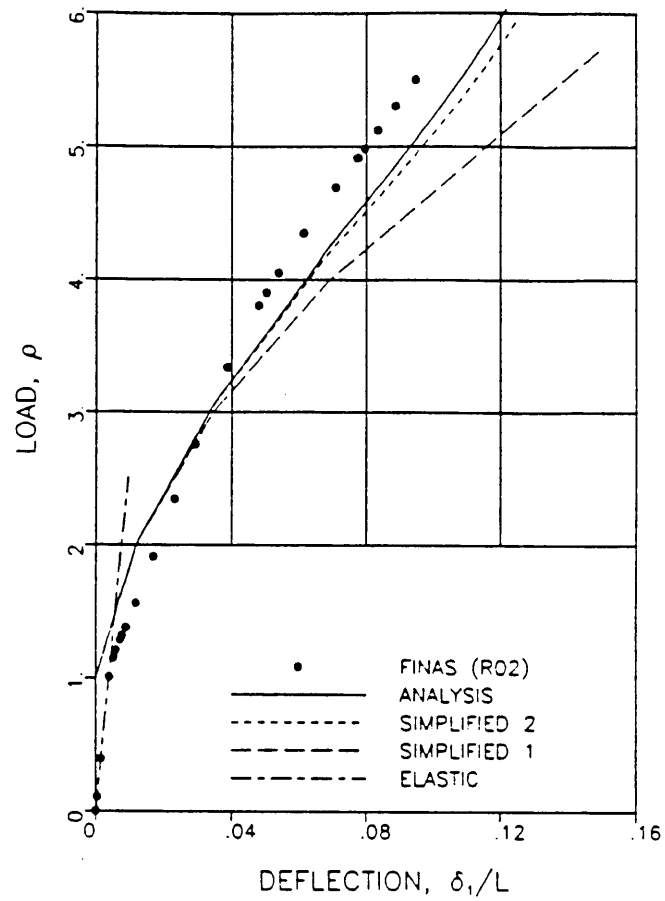


Fig. 6.6(a) Load vs. stiffener 1 central deflection

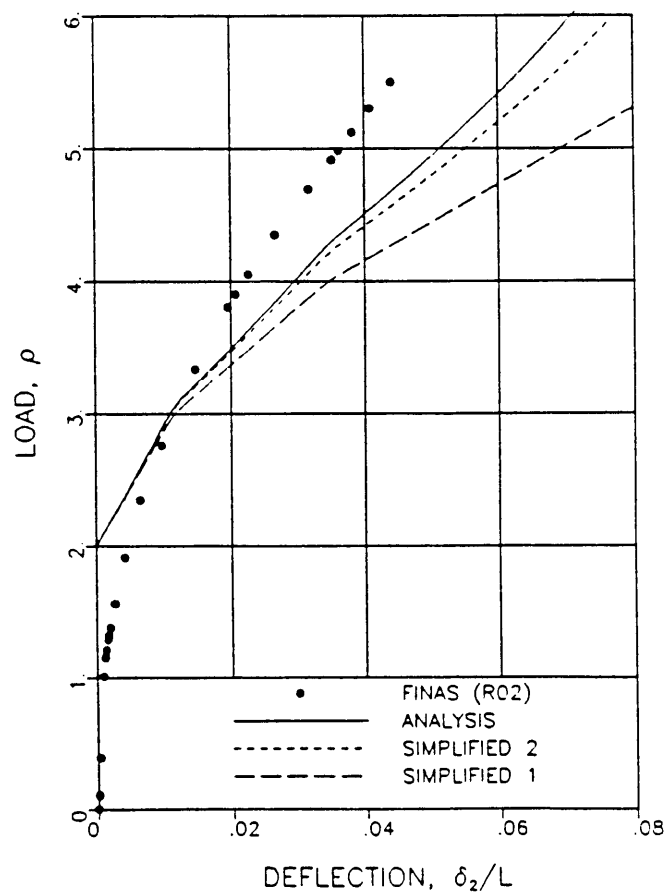


Fig. 6.6(b) Load vs. stiffener 2 central deflection

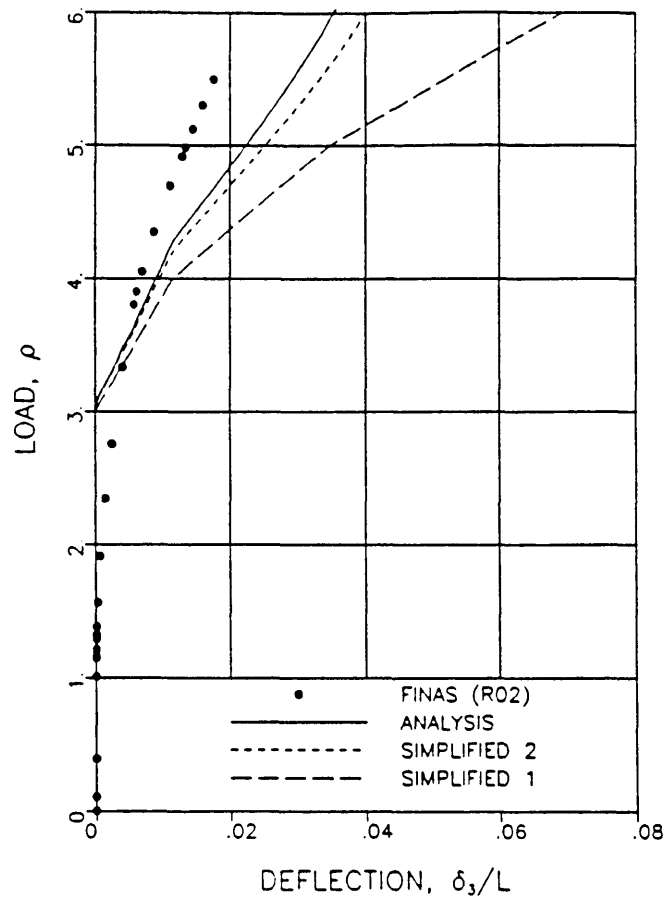


Fig. 6.6(c) Load vs. stiffener 3 central deflection

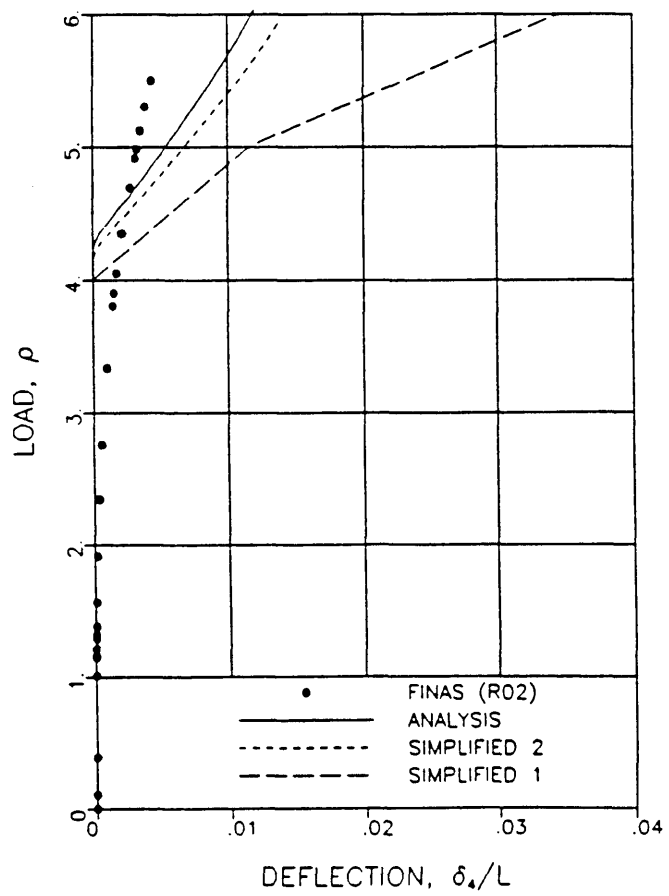


Fig. 6.6(d) Load vs. stiffener 4 central deflection

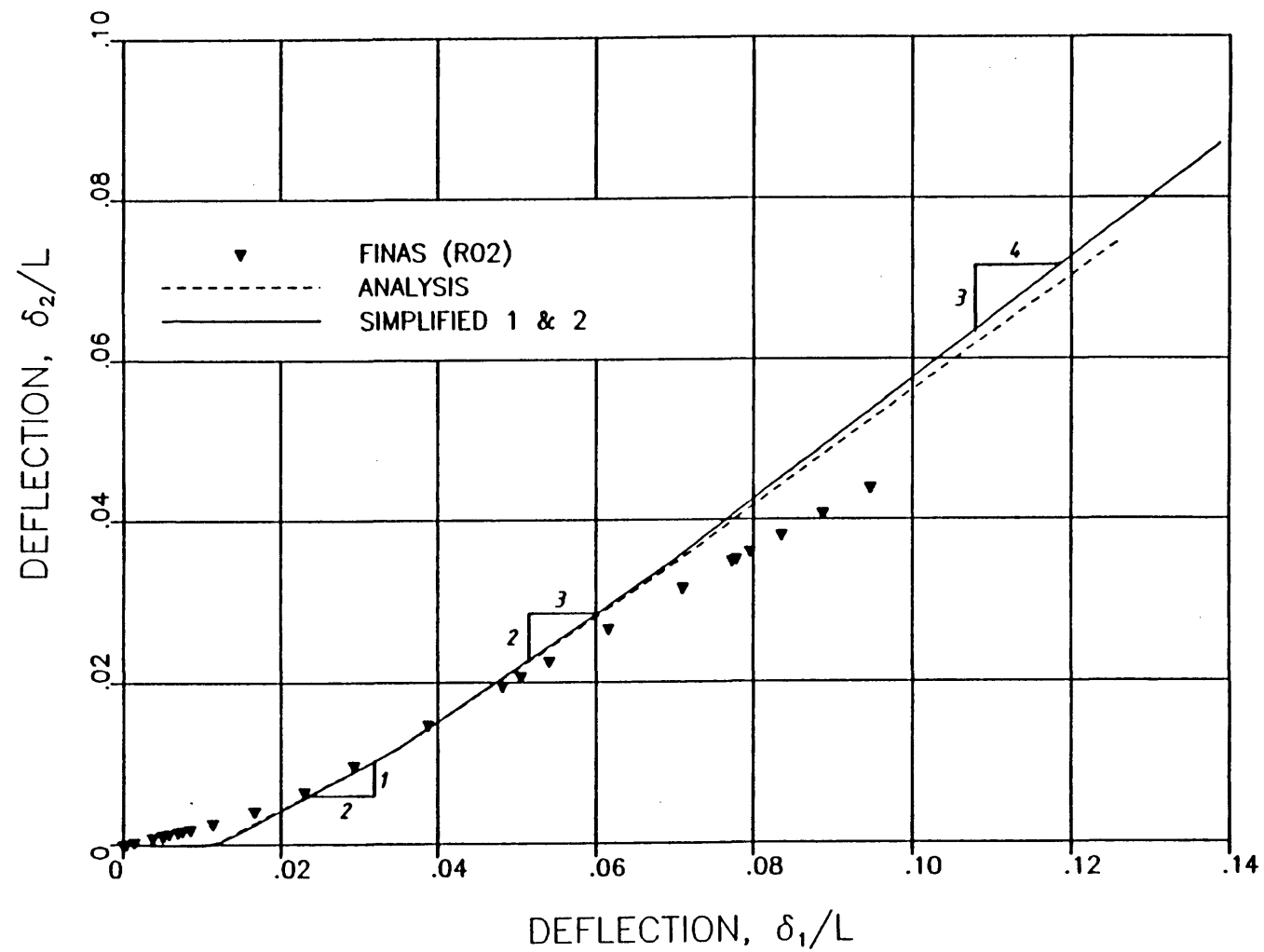


Fig. 6.7(a) Central deflection in stiffener 2 plotted against central deflection in stiffener 1

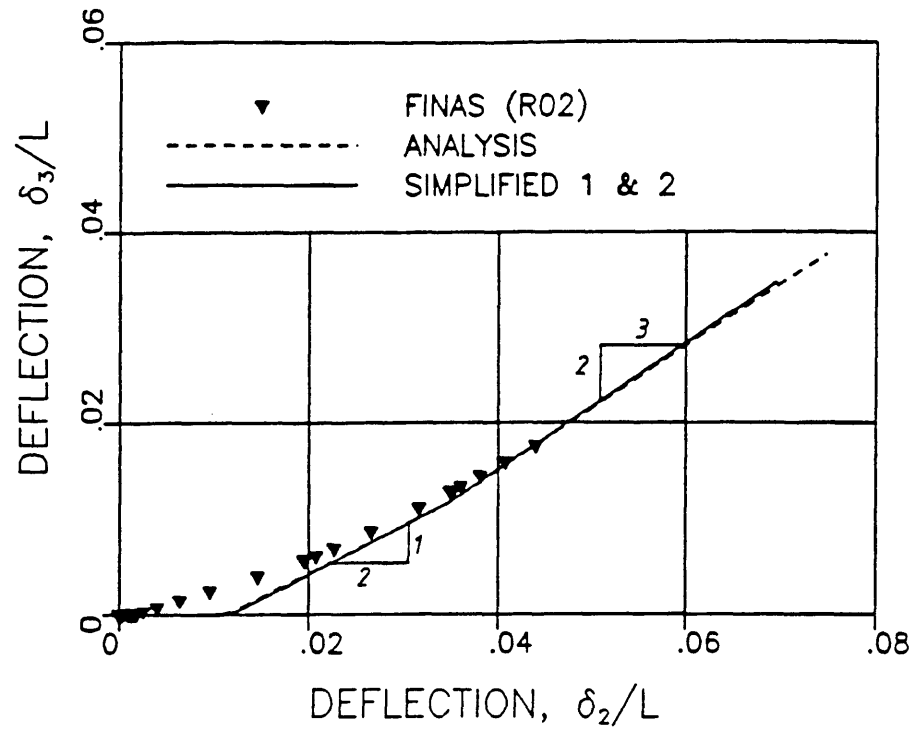


Fig. 6.7(b) Central deflection in stiffener 3 plotted against central deflection in stiffener 2

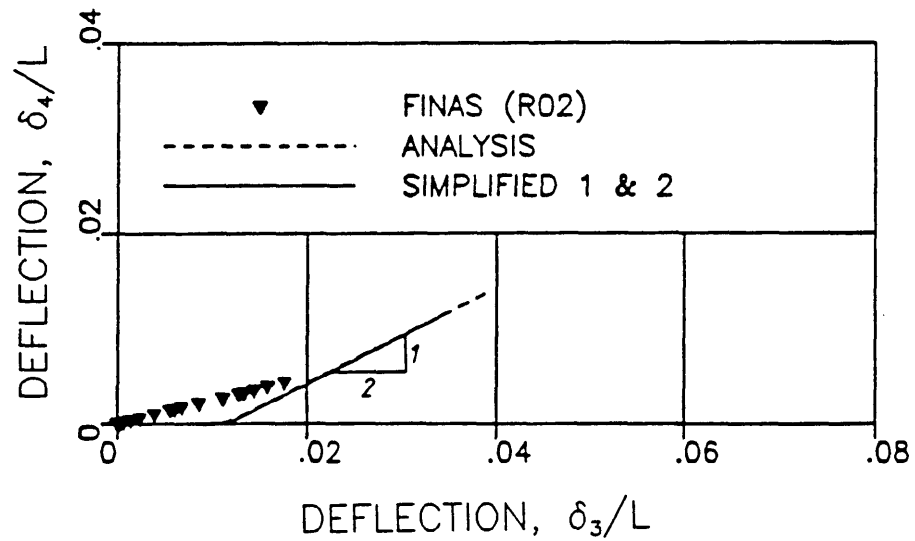


Fig. 6.7(c) Central deflection in stiffener 4 plotted against central deflection in stiffener 3

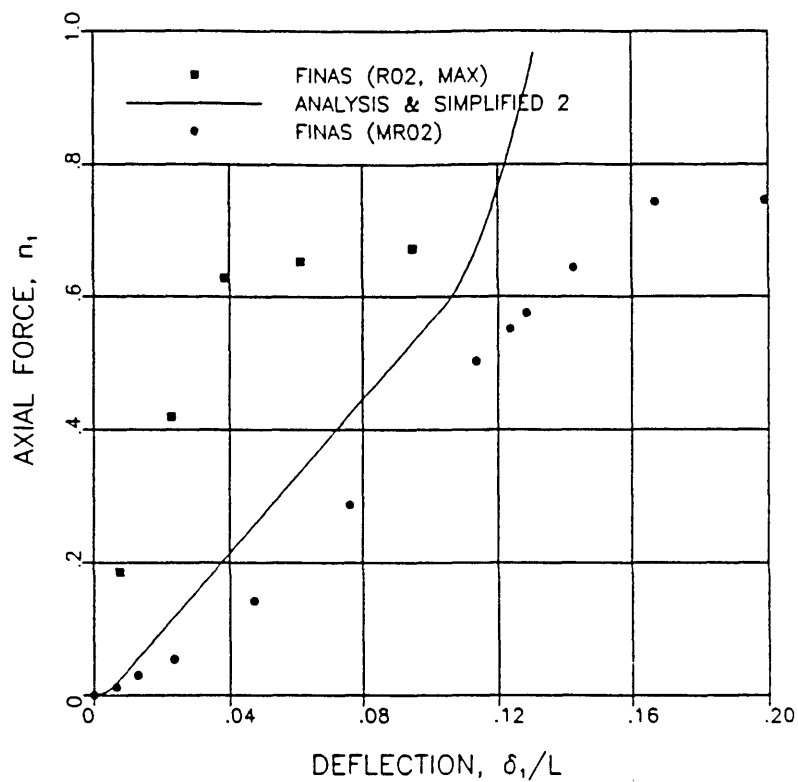


Fig. 6.8(a) Axial force vs. deflection for stiffener 1

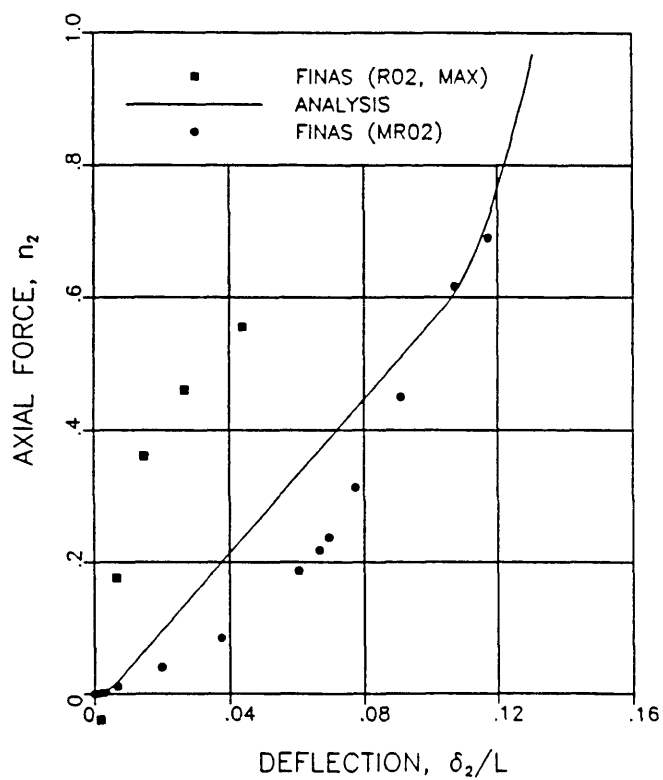


Fig. 6.8(b) Axial force vs. deflection for stiffener 2

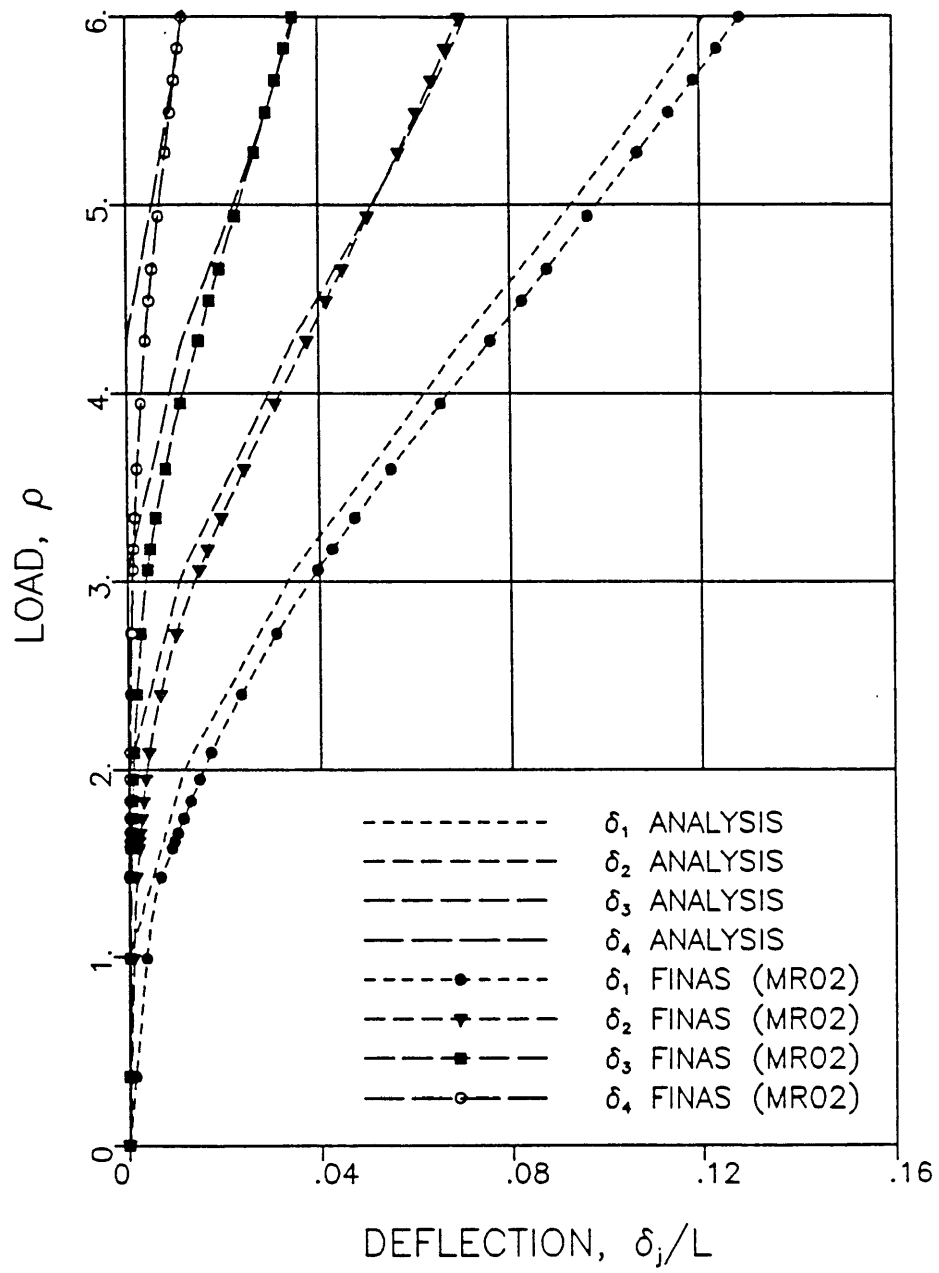


Fig. 6.9 Load vs. stiffener central deflections for deck R02

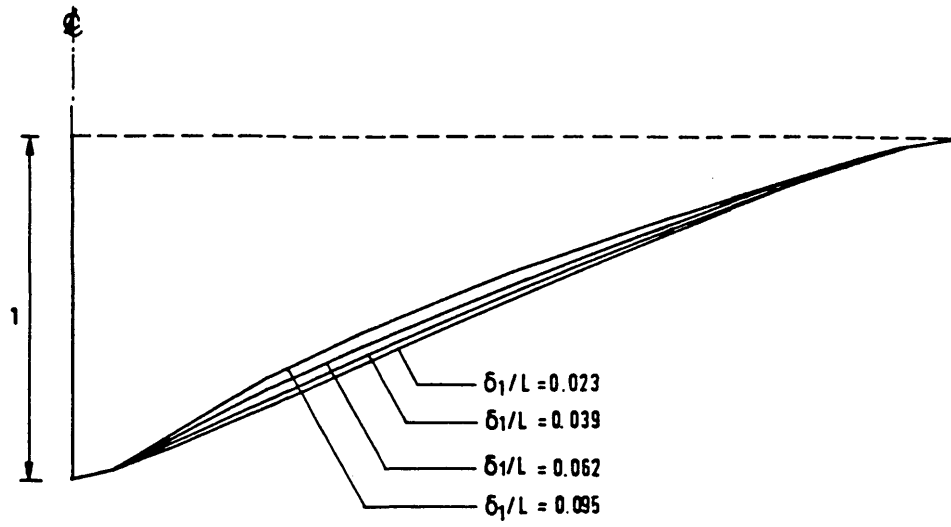


Fig. 6.10 Stiffener 1 mode-shapes from R02

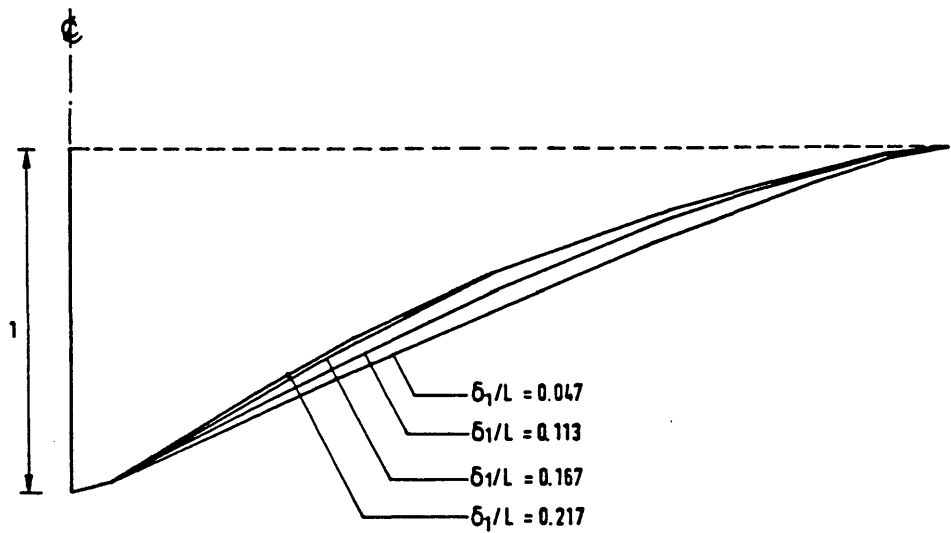


Fig. 6.11 Stiffener 1 mode-shapes from MR02

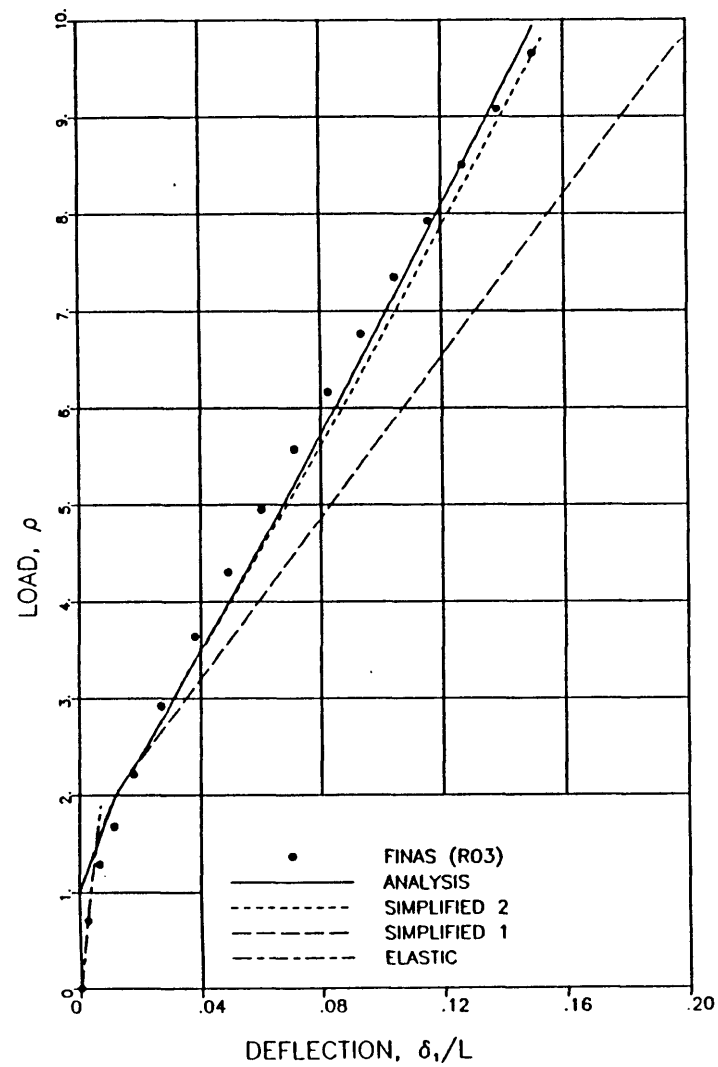


Fig. 6.12(a) Load vs. stiffener 1 central deflection

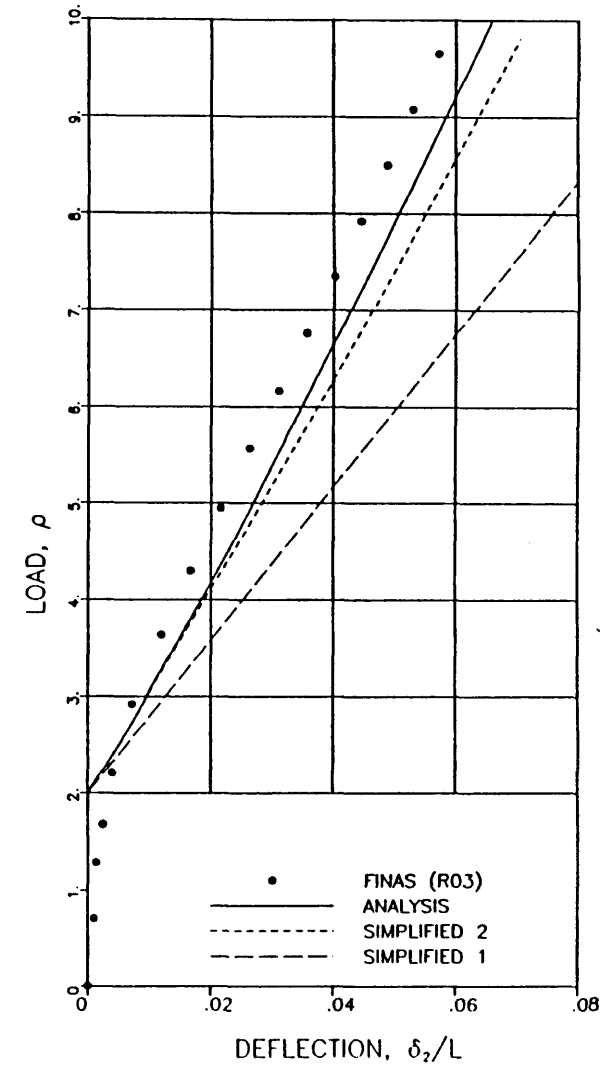


Fig. 6.12(b) Load vs. stiffener 2 central deflection

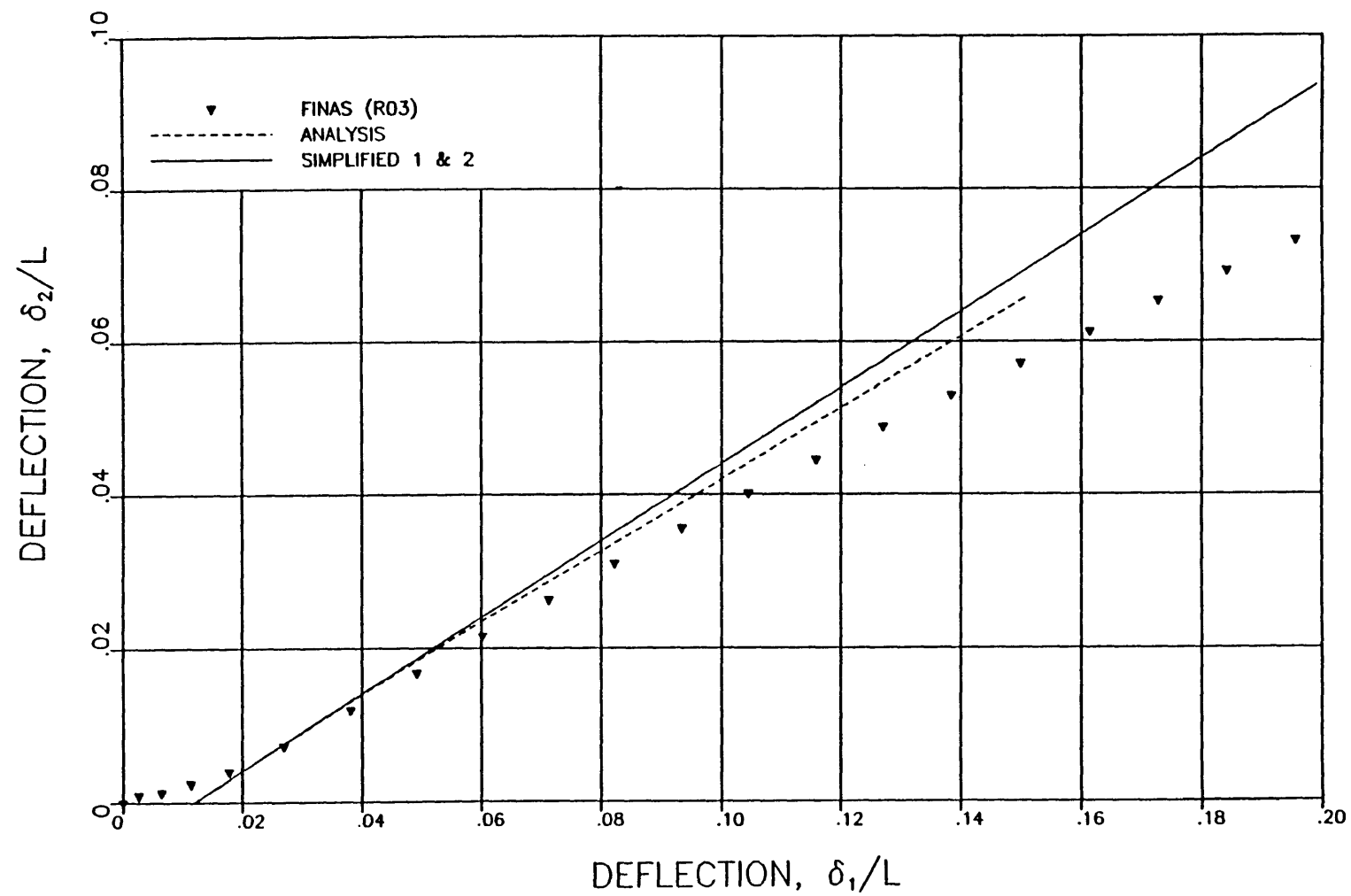


Fig. 6.12(c) Central deflection in stiffener 2 plotted against central deflection in stiffener 1

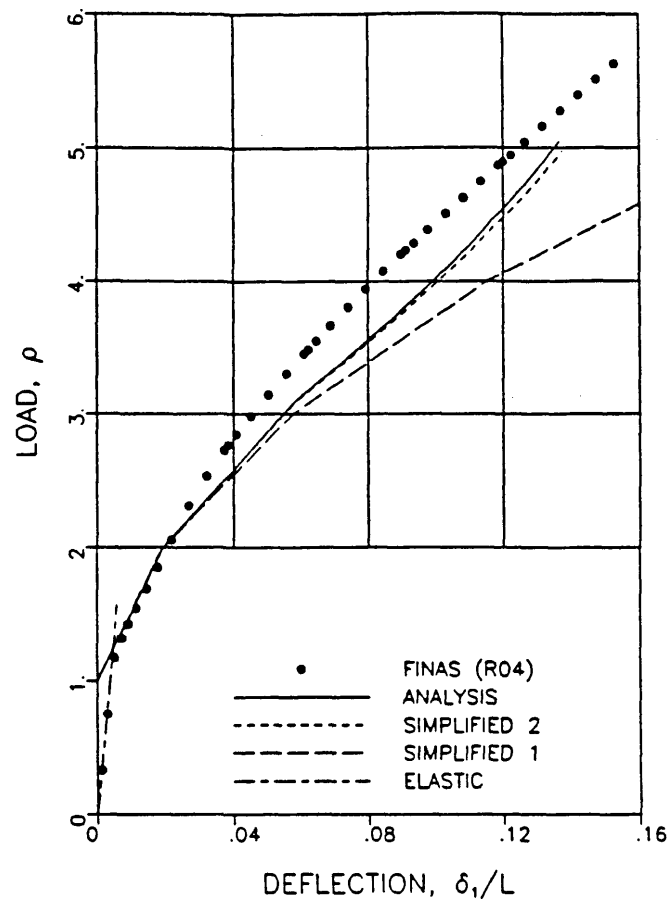


Fig. 6.13(a) Load vs. stiffener 1 central deflection

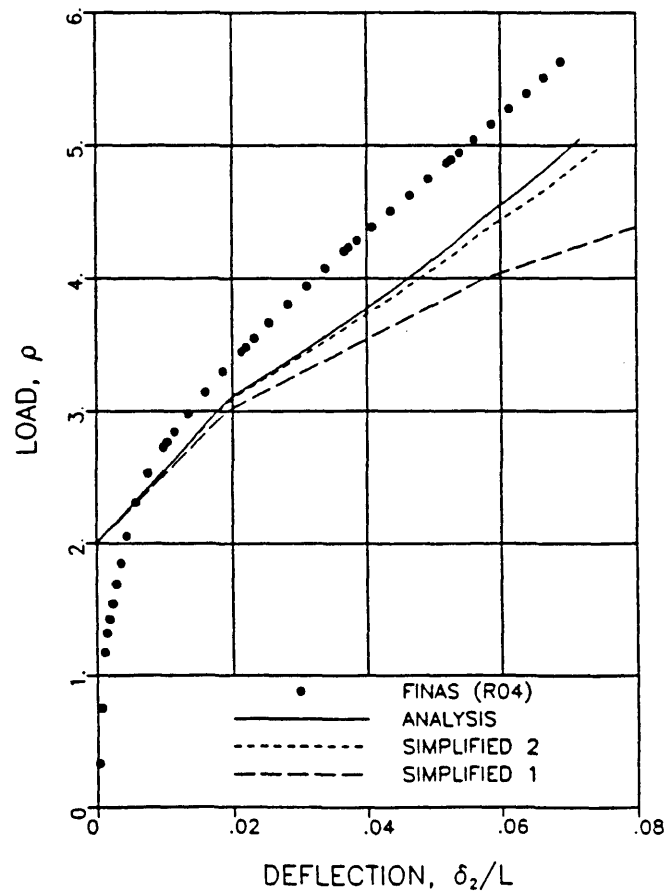


Fig. 6.13(b) Load vs. stiffener 2 central deflection

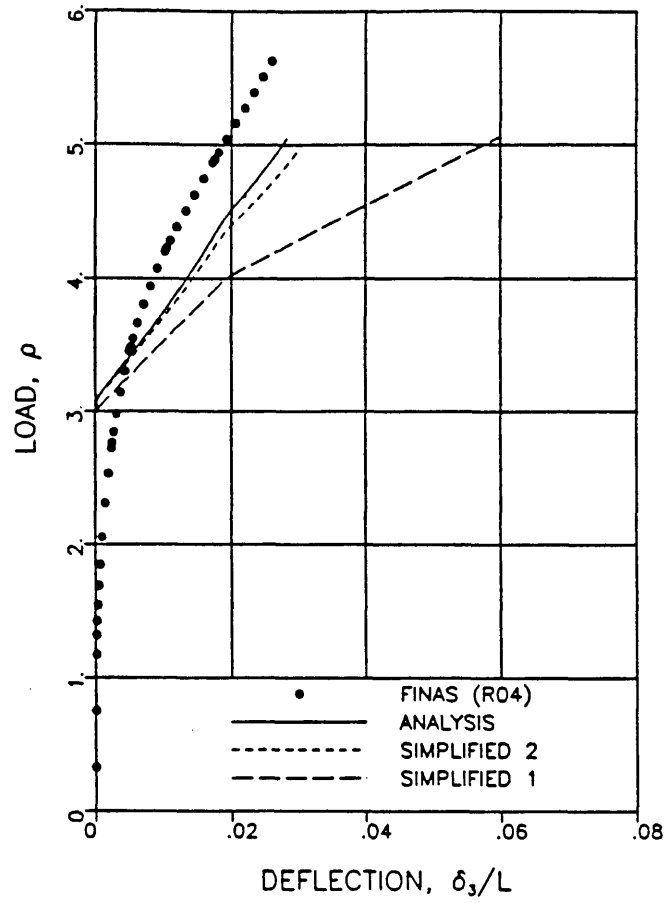


Fig. 6.13(c) Load vs. stiffener 3 central deflection

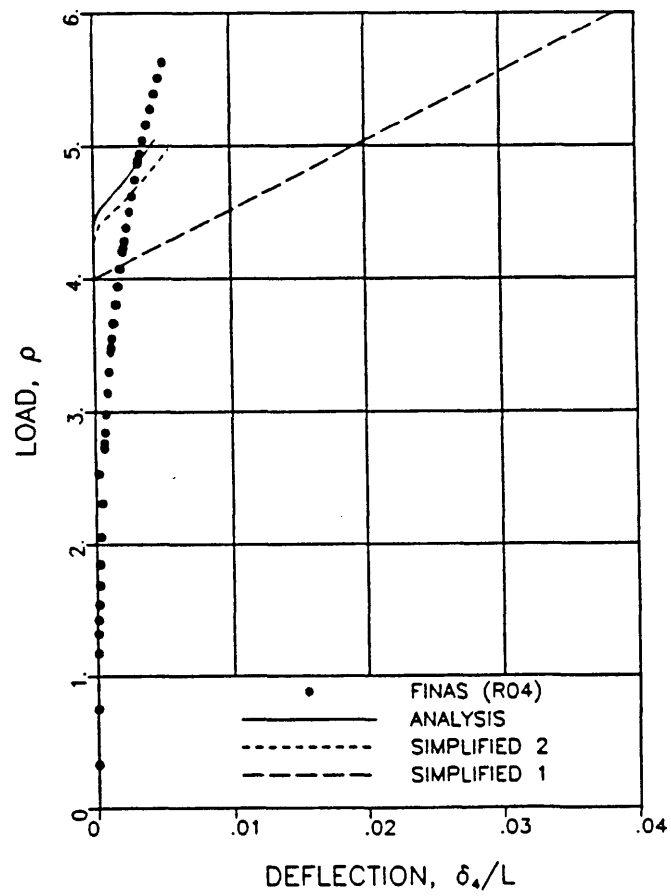


Fig. 6.13(d) Load vs. stiffener 4 central deflection

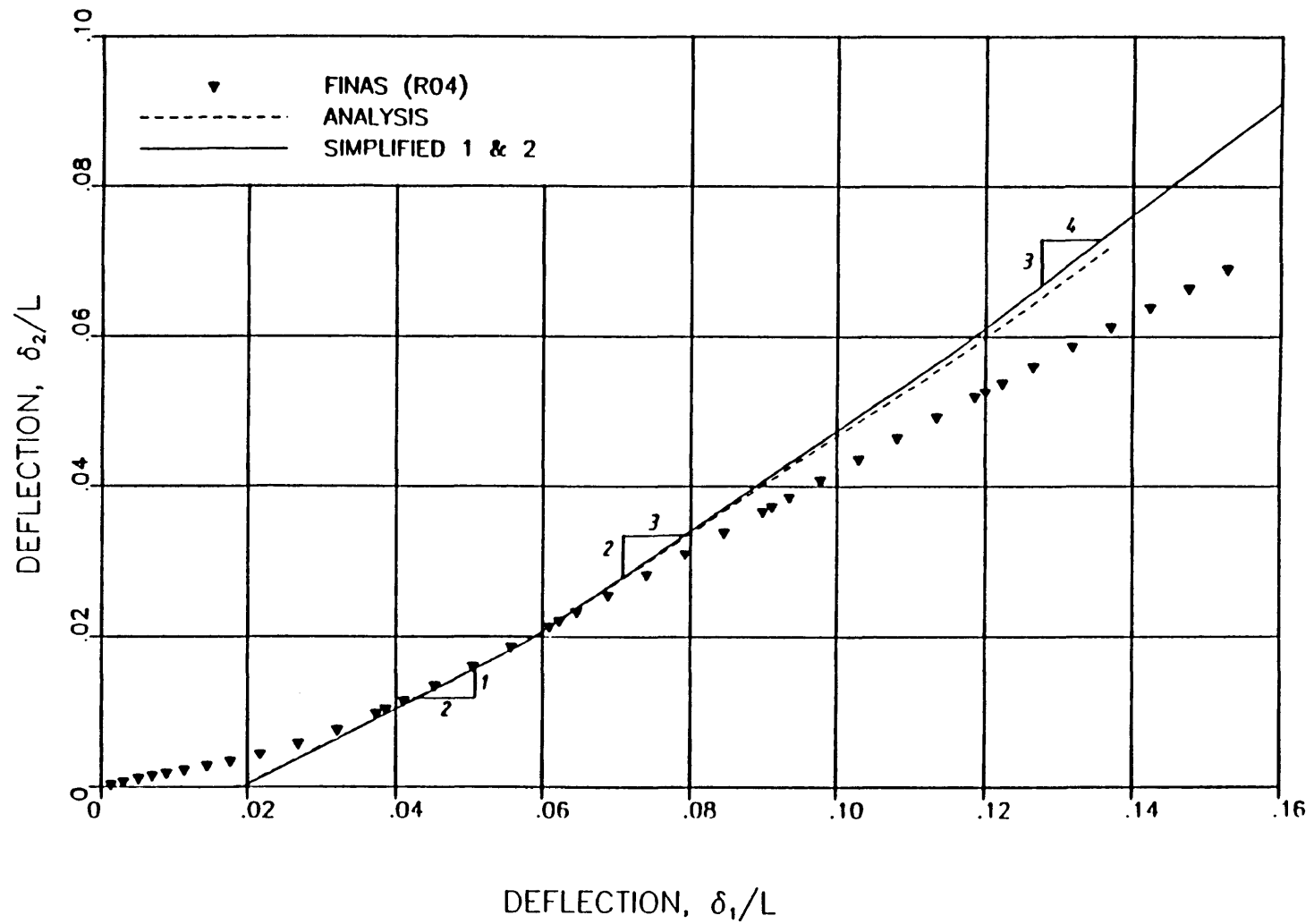


Fig. 6.13(e) Central deflection in stiffener 2 plotted against central deflection in stiffener 1

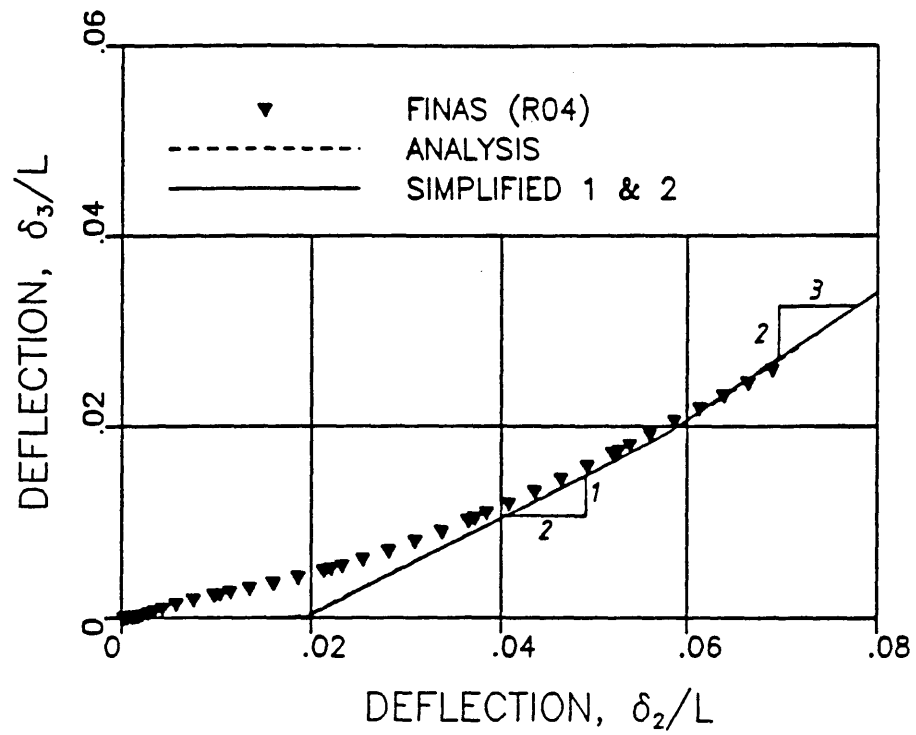


Fig. 6.13(f) Central deflection in stiffener 3 plotted against central deflection in stiffener 2

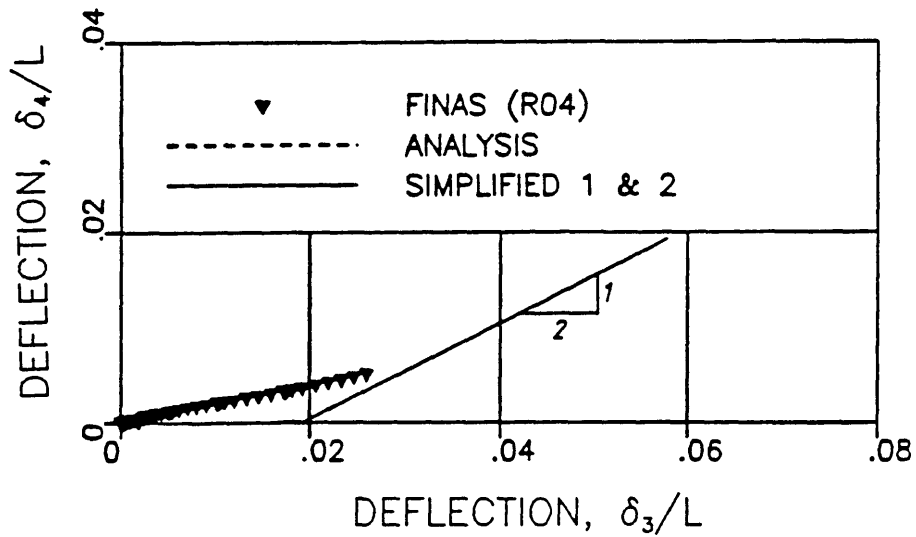


Fig. 6.13(g) Central deflection in stiffener 4 plotted against central deflection in stiffener 3

CHAPTER SEVEN

BEHAVIOUR OF UNRESTRAINED STIFFENED DECKS UNDER PATCH LOADING

7.1 INTRODUCTION

In Chapters 3 to 6 the behaviour of a single-bay stiffened deck with full in-plane restraint under lateral patch loading was studied and an analytical model was proposed. The behaviour of an unrestrained stiffened deck is studied in this chapter, whilst an analytical model describing this type of end conditions is developed in Chapter 8. In practice, the behaviour of a bay incorporated in a multi-bay assembly lies between these two extreme cases studied in this chapter.

Three simply supported and free to pull-in single-bay deck geometries subjected to lateral patch loading are studied in the inelastic range by means of numerically generated data and, in one case, available test results. The effect of bay width to length ratio, called bay aspect ratio, on the distribution of the membrane action is primarily investigated. The three geometries considered have bay aspect ratios of 9:12, 12:12 and 15:12 and are labelled as S1, S2 and S3 respectively. Two alternative geometries are used for the stiffeners, with geometries S1 and S3 having identical stiffeners. The panel aspect ratio, $b/2L$, and the panel slenderness, b/t , are kept constant.

The increasing contribution of the membrane action to the load carrying capacity of the deck as deflection increases, is studied. Methods to predict such action are also investigated.

7.2 ANALYSIS OF DECK S2

Dowling (1968) performed a series of tests on simply supported single bay decks under centrally applied patch loading. Most of the tests were carried out to investigate the differences in load distribution between adjacent stiffeners in the

elastic range caused by the variation in deck flexural and torsional rigidities. Of interest to the work in this thesis is a half-scale model which was initially tested elastically but was subsequently loaded in the inelastic range up to collapse. This geometry is referred to as deck S2.

7.2.1 Geometric details of the model

The main feature of the model deck tested by Dowling was that the torsional rigidity of the model should be varied without significantly altering the longitudinal and transverse flexural rigidities. For this reason the rather unusual stiffener geometry (wide flanges), shown in Fig. 7.1, was chosen.

The deck cross-section was originally of the twin-skin type, in which the vertical stiffeners were confined by the top and bottom plates, with high torsional rigidity. After this model had been tested in the elastic range, the longitudinal welds joining the bottom flange panels of the inverted T-sections were opened by sawing. This modified model was then loaded in the elastic and inelastic range up to collapse.

The deck plate was 4.76mm (3/16") thick, spanning over the stiffeners spaced at 127mm (5") centres, giving a panel slenderness for the deck plate of 26.7. The stiffeners spanned over flexible cross-girders 1524mm (60") apart. The depth to span ratio for the stiffeners was 1:20 and both the flanges and the webs had thicknesses of 2.54mm (0.1"). Table 7.1 gives the proportions of the cross-sectional configuration for a 'notional beam' in deck S2.

Flexible cross-girders were used, comprising inverted T-sections, spanning over a distance of 1397mm (4' 7") between the supports at the corners of the model and giving a depth to span ratio of 1 in 8.8. The web was cut from a 152.4mm x 4.76mm (6" x 3/16") plate and the flanges were 63.5mm x 6.35mm (2.5" x 0.25") in cross-section. A particular feature was the openings in the cross-girder web between the stiffener support positions. These were 101.4mm x 50.8mm (4" x 2") each and mounted access for the strain gauge wiring. In the longitudinal direction deep thin girders supported the deck edges. The overall deck geometry is shown in Fig. 7.2.

Two test coupons, taken from the deck plate, gave an average yield stress of 229N/mm² and an elastic modulus of 195kN/mm². The yield stress for the cross-girder was higher and a value of 340N/mm² is used in the numerical model described in the following section.

7.2.2 Finite Element modelling

The finite element package FINAS, described in Section 3.3, was employed. The 8-noded isoparametric shell element and the 3-noded isoparametric space beam element were selected from the element library. The deck plate was modelled with shell elements while the beam element was used for the stiffeners and the cross-girder. The longitudinal-girder was assumed to respond stiffly and remain elastic throughout the load history. Therefore, it was excluded from the finite element model and its location was modelled as being rigid to out-of-plane deflections but rotationally free.

The quarter deck analysed by finite elements is shown in Fig. 7.3. Two rows of shell elements, each having six elements along the length, were used between the stiffeners. Two rows of elements were also used between the longitudinal centre line and the adjacent stiffener, while one row of shell elements was employed for the overhanging portion of the deck over the longitudinal-girder. The layout of shell elements is shown in Fig. 7.3.

Proper modelling of the openings in the cross-girder web requires the use of shell elements. Such a model, however, is very costly in computational time, especially when non-linearity in the material is also modelled. In addition, numerical difficulties are frequently associated with elasto-plastic finite element analyses involving a large number of admissible degrees of freedom. It was decided therefore to use the beam element for the cross-girder. While the hanging portion of the deck over the cross-girder contributes fully to the cross-girder top flange, its contribution to the behaviour of the longitudinal stiffeners is negligible. Since the behaviour of the stiffeners relative to that of the cross-girder was to be compared, this portion of the deck was not modelled.

As part of a mesh convergence study the number of shell elements along the length was increased to eight by using shorter elements close to the cross-girder. Modelling of the longitudinal-girder with beam elements was also undertaken. These alterations to the mesh had very little effect on the central deflections of the stiffeners relative to those at their supports on the cross-girder. Therefore, the simpler shell element layout first described above and shown in Fig. 7.3, was adopted for detailed examination of the deck behaviour.

Finally, a model using shell elements to model the cross-girder webs, including their openings was devised and compared with the above model and the test in the elastic range only. This model also incorporated the portion of the deck hanging over the cross-girder. Reference to this model is subsequently made by the term 'elastic', in contrast with the model described above for use in the elasto-plastic range, labelled 'S2'.

7.2.3 Loading

The total loading in the test was a central patch covering an area 152.4mm wide by 31.75mm long. On the quarter deck, shown in Fig. 7.3, the loaded area is shown hatched. This results in three partially loaded shell elements. The equivalent nodal loads were found by performing the appropriate integrations. Load increments are applied as a function of the load factor, ρ , defined as the lateral patch load acting on half deck divided by the limit load of a simply supported stiffener and its associated fully effective plating spanning between cross-girders.

7.2.4 Comparison of experimental and numerical results

7.2.4.1 Load histories of the stiffener central deflections

The growth in the central deflections of the three inner-most stiffeners (stiffeners 1 to 3 in Fig. 7.3) under increasing loading is illustrated in Figs 7.4 to 7.6. The elastic response, obtained from the numerical model in which the openings in the cross-girder were modelled, is shown by a continuous line. There is excellent agreement between the 'elastic' numerical model and the early experimental response. The elasto-plastic finite element analysis for deck S2, however, gave an unconservative response with respect to the test results in both the elastic and elasto-plastic ranges for stiffeners 1 and 2. For stiffener 3 the numerical analysis was unconservative in the elasto-plastic range only. This may be attributed to the distortion of the cross-sections of the stiffeners from their initial shape. Such change in the cross-section cannot be accounted for when the stiffener is modelled with beam elements. The distortion of the stiffener from its initial shape is expected to be significant, especially when the stiffener has very thin and wide flanges. The loss of stiffness observed in the load-deflection history of stiffener 1 due to successive collapse of the stiffeners across the width of the deck is also observed.

In general, it may be concluded that a very good agreement between the finite elements and the experimental results is observed in Figs 7.4 to 7.6.

7.2.4.2 Relative stiffener deflections

The central deflections of stiffeners 1 and 2 are plotted against the central deflections of adjacent stiffeners in Figs 7.7 and 7.8. Slopes of 1:2 and 2:3 are observed in Fig. 7.7, indicating that stiffener 1 pivots with respect to stiffeners 3 and 4 respectively. As stated earlier in Section 6.2.3.3, this behaviour is characteristic of stiffener responses governed by bending. The load spreads outwards by means of combined transverse shear and transverse membrane mechanism. However, the longitudinal membrane distribution across the deck width cannot increase the load significantly above the plastic collapse load value of a stiffener and its associated plating.

7.2.4.3 Transverse membrane action in the plate

The growth of transverse membrane action with increasing deflection is shown in Figs 7.9(a) to 7.9(d). In these and all the following figures concerning stress resultants in the plate, the notation assumed is the one adopted in Section 3.5.2 and shown on Fig. 3.1(a). It is worth noting that the transverse compressive forces do not balance the transverse tensile forces along a longitudinal section of the plate because the cross-girder action contributes to the former. At higher loads, the maximum tensile stress resultant per unit width corresponds to membrane yield in the deck plate. Another interesting point is that the effective width of the tensile transverse membrane action (on the quarter of the deck shown in these figures) appears to increase from b to $1.5b$ as the load is increased.

7.2.4.4 Transverse bending moments

Transverse bending moment distributions in the plate at various load levels are illustrated in Figs 7.10(a) to 7.10(d). Two trends can be noted by close inspection of these figures. The first is the overall growth of bending moments, and the second is the reduction of bending action near the transverse centre line. The latter is due to the development of high longitudinal and transverse membrane action in the plate at large deflections.

A particular feature not present in the transverse bending moment plots presented in Section 3.5.2.4, is the opposite sign of the bending moment values at either side of a stiffener. This is because of the torsional restraint that the cross-girder provides at the end of the stiffeners. On the other hand, the scalloped pattern described in 3.5.2.4 can also be observed in this case.

7.2.4.5 Bending moments in the beams

The same idealization, as in Section 3.5.3, where each stiffener and its associated fully effective plate is viewed as a 'notional beam', is used here. Figures 7.11(a) and 7.11(b) show the bending moment distribution along such beams at two load levels. The distribution at an early stage is shown in Fig. 7.11(a), with most of the load carried by stiffeners 1 and 2. As the applied load increases in magnitude, load is spread to the other stiffeners, as seen in Fig. 7.11(b). It is worth noting that the support moments of these 'notional beams' are not zero. This is due to the rotational restraint, applied by the cross-girder as an eccentric force at the beam ends.

7.2.4.6 Axial forces in the beams

The distribution of the axial force along the 'notional beams', normalized with respect to the squash load, is shown in Figs 7.12(a) and 7.12(b). The axial force distribution at an early stage is shown in Fig. 7.12(a), where only stiffener 1 is subjected to a modest axial force. At higher loads (Fig. 7.12(b)), there is significant membrane action, with the value of the axial force in beam 1 being much higher than elsewhere. The compressive force existing near the end of beam 1 is applied by the cross-girder.

7.2.4.7 Interaction between stress resultants in the 'notional beams'

The theoretical fully-plastic interaction between the non-dimensionalized stress resultants on a 'notional beam' cross-section under a uniaxial stress state is plotted in Fig. 7.13 with a continuous line. The stress resultants of the various stiffeners observed at the Gauss point closest to the transverse central line are also plotted for comparison. It can be seen that, once a 'notional beam' becomes plastic, the uniaxial stress resultant interaction is followed fairly closely.

7.2.5 Discussion

The comparison presented above between numerical analysis and the test undertaken by Dowling (1968), is a validation of the finite element package FINAS for this type of analysis. Additional features of deck behaviour, including the effect of cross-girders were also illustrated. In the test deck S2 failed at $\rho \approx 5.5$ due to the collapse of the cross-girder. Assuming a rigid cross-girder, the deck would experience larger relative deflections and the phenomena occurring in this range are investigated in the following.

7.3 THE INFLUENCE OF BAY ASPECT RATIO ON MEMBRANE ACTION

The aim of the two analyses presented here, is to obtain further information concerning the development of membrane action in a deck, and to investigate ways in which the behaviour of such a deck can be predicted analytically. Both decks analysed in this section are simply supported along all four boundaries and free to draw-in. This investigation is not a parametric study of membrane action, but an attempt to devise a procedure by which the behaviour of any given deck can be predicted. It was thought that one of the main parameters affecting the distribution of the membrane forces in the deck is the bay aspect ratio. Thus the width to length ratio of the entire bay is the only variable in the finite element analyses presented next.

7.3.1 Geometries analysed

The aspect ratio of the bay was altered by varying the number of stiffeners across the width. The two configurations S1 and S3 had 8 and 14 stiffeners across the entire width respectively, while the stiffeners spanned over a distance of twelve times the stiffener spacing. The stiffeners comprised inverted tee sections. Typical geometric parameters, defining the cross-section of a stiffener and associated plate are given in Table 7.2. The finite element mesh and the loading applied to deck S1 are identical to those in deck R02 (shown in Figs 3.2 and 3.4 respectively), while the mesh in deck S3 contained two rows of shell elements for each additional panel.

7.3.2 Deck S1

In the following, attention is drawn to the overall behaviour of the deck under coexisting membrane and bending action. The deck is visualized as a series of 'notional beams', each having the cross-section of a stiffener and the associated plate. The investigation is focused on the behaviour exhibited by the 'notional beams' and the deck plate. Information on both issues is obtained from deck S1 which had the advantage that its strength, achieved by the numerical analysis, was in excess of the load carried by the 'notional beams' under bending action only.

7.3.2.1 Transverse membrane action in the plate

The development of transverse membrane action in the plate at four stages of the load history is shown in Figs 7.14. Where plots similar to those in the preceding chapters are made, the same notation is adopted. Comparison between Figs 7.14(a) and 7.14(b) shows that the transverse membrane action tends to increase in magnitude at any point along the transverse centre line approaching tensile yield near the centre of the deck. At points further away from the centre of the deck in the transverse direction, the membrane action is reduced with zero value at the edge of the deck. In attempting to quantify an effective width for the tensile membrane forces along the longitudinal centre line, a value between b and $1.3b$ can be assigned for the quarter deck shown in Figs 7.14.

Comparison of Figs 7.14(a) and 7.14(b) with Figs 7.14(c) and 7.14(d) shows a different trend in the values of the transverse membrane action along the transverse central line. In the latter two figures, the values for the transverse membrane action near the intersection between the transverse central line and the longitudinal support drop to zero. This tendency spreads towards the centre of the deck with increasing loading, as indicated by further comparison between Figs 7.14(c) and 7.14(d). It can be attributed to a secondary phase in the large deflection behaviour of the deck. During the first phase, transverse membrane and transverse bending action facilitate the outward distribution of the loading. In the following phase, which commences with the collapse of the outermost stiffener, a different process takes place and this is discussed further in Section 7.3.2.6.

7.3.2.2 Transverse bending moments in the plate

Distributions of the transverse bending moments in the deck plate are illustrated in Figs 7.15. The first two figures are associated with the spreading phase of the loading away from the loaded stiffener, while the last two correspond to the secondary phase of the deck load history at very large deflections.

7.3.2.3 Bending moments in the 'notional beams'

The bending moments along the 'notional beams' are presented in Figs 7.16. The load levels used are identical with those in Figs 7.14 and 7.15. The loading phase of the stiffeners in bending is shown in Figs 7.16(a) and 7.16(b). In the last two figures 7.16(c) and 7.16(d), development of further bending action is hardly seen, as compared to the first two plots. Therefore, the increase in strength is not associated with bending acting in the beams.

7.3.2.4 Axial forces in the 'notional beams'

Longitudinal membrane action in the deck is better presented by the axial force distribution along the 'notional beams' in Figs 7.17. The first two figures, (a) and (b), show the growth in longitudinal membrane action. As no load is applied externally in the longitudinal direction, the resulting force in this direction should be zero from equilibrium requirements. However, the same forces result in an in-plane moment applied on the quarter deck shown in these figures. This in-plane bending should be balanced by its counterpart, produced by the transverse membrane forces acting along the longitudinal centre line, shown in Figs 7.14. Comparison between the first two and last two plots in Figs 7.17, shows a trend for increasing membrane action in stiffener 1 with the appropriate redistribution of the forces in the other stiffeners. This trend is in contrast to the one observed from comparison between Figs 7.17(a) and 7.17(b) showing growth in the stiffener axial forces across the width. Hence, during the second phase, at very large deflections the tension in the loaded stiffener can keep increasing, with the necessary redistribution of the axial forces in the other beams.

7.3.2.5 Interaction between stress resultants in the 'notional beams'

It is worth developing the yield locus, obtained from the interaction between the axial and bending stress resultants, assuming uniaxial stress state in the 'notional beams'. Such a locus, obtained for the geometry of a typical beam in deck S1, is shown by the continuous line in Fig. 7.18. The interaction between the stress resultants in the 'notional beams', obtained at the Gauss points closer to the transverse centre line, is also shown. It is remarkable to observe how closely the uniaxial yield surface is followed. Hence, the biaxiality in the plate hardly affects the uniaxial interaction between the stress resultants in the 'notional beams'. This is a useful observation if a plastic analysis employing the 'notional beams' is to be used.

7.3.2.6 Contribution of stiffeners to the total strength

The maximum load sustained by deck S1, as calculated from the last step of the numerical analysis, was about 7.3 times the limit load carried by a 'notional beam' forming a plastic hinge at the centre. An upper bound to the load factor can be estimated by assuming that all the 'notional beams' form plastic mechanisms in bending with hinges lying along the transverse central line. As there are four such beams across the deck half-width, the load factor predicted by such analysis should be just in excess of 4, since some reactions are applied along the longitudinal support.

Considering moment equilibrium for the quarter deck with respect to a transverse axis passing through the supports of the stiffeners, it is of interest to determine the contribution to the strength of every stiffener across the width. As equilibrium of the entire deck is being considered, any interaction between the 'notional beams' through the deck plate is neglected. The actions contributing to the equilibrium are the axial force and bending moment in the 'notional beams', and the vertical reactions along the longitudinal edge of the deck. For stress resultants, the values obtained at the Gauss points nearest the transverse centre line, were used to approximate the values on the centre line. This results in a very small error. Considering the equilibrium of stiffener j alone, the following expression is obtained for the contribution to the total strength made by each stiffener:

$$\rho_j = m_j^s + \frac{A_j L}{Z_{p1}} n_j \frac{\delta_j}{L} \quad (7.1)$$

Columns (2) to (5) in Table 7.3 present the values, given by Eqn. (7.1), for each stiffener at various load levels. Column (1) shows the overall load factors in deck S1, as obtained from the finite element analysis. In column (6) the summation of the values in columns (2) to (5) is given. Comparing values in columns (1) and (6), good correlation is noted in the first five load steps. Thereafter, an additional contribution due to the formation of a flexural plastic hinge in the outer panel along its intersection with stiffener 4 must be accounted for. This contribution exists only after the collapse of stiffener 4. From such a consideration, the maximum reaction which can be developed along the edge of the deck gives an additional contribution of 0.802 to the total value of the load factor. Thus, an amount of 0.802 has been added on the last three values in column (6) and the sum is printed in column (7). Comparison of the results presented in the first and sixth columns shows that nearly all the strength has been taken into account.

Considering the contributions of individual stiffeners to the total strength it is shown that strength is mobilized to the loaded stiffener, not only by the action of the plate in the transverse direction but also by the axial forces in the 'notional beams'. Examining columns (2) to (5) it is noted that whereas column (2) indicates a monotonically increasing contribution to the total strength, values in columns (3) to (5) show a peak followed by a fall off in the individual contributions to strength. The turning point (about the fifth value), in column (5) coincides with the collapse of stiffener 4, thus differentiating between two phases in the behaviour of the deck.

During the first phase, the increase in strength comes from the outward spreading of the load, and to some extent from the increase in the axial force in the loaded stiffener (stiffener 1). After the outmost stiffener (stiffener 4) has collapsed, any additional strength comes from the increased axial force in the loaded stiffener, while the role played by the plate diminishes dramatically. An indication of the load carrying capacity being transferred to the loaded stiffener can be obtained by the summation of the load values in columns (3) to (5), shown in column (8). Monitoring of the maximum value in column (8) can also be used as an indicator between the two phases in deck behaviour.

The last load level in Table 7.3 is clearly within the second phase. Comparison between the values in columns (2) and (8) at this level shows that most of the strength has been mobilized on the loaded stiffener by the redistribution of the axial forces across the width. This is supported by the fact that the value in column (2) comes almost entirely from the second term on the right hand side of

Eqn. (7.1). Hence, the strength of the deck above that predicted by pure bending action is attributed to the development of the longitudinal membrane forces in the deck.

As mentioned above, the axial force in the loaded stiffener increases monotonically, while the axial force in the outer stiffener increases during phase 1 and decreases within phase 2, as shown in Figs 7.17. During the first phase, the tensile and compressive forces increase, as indicated in Figs 7.17(a) and 7.17(b). On the contrary, the compressive axial forces in the outer stiffener do not increase in magnitude, while the axial force in the loaded stiffener is still increasing, as Figs 7.17(c), 7.17(d) and 7.18 show.

In conclusion, it can be argued that the additional strength above that based purely on bending action comes from the membrane action in the deck and the way in which it is distributed across the transverse centre line.

7.3.3 Deck S3

Following the discussion above, it becomes clear that central to a successful prediction of the large deflection response of stiffened decks is the history of the membrane forces. Having covered deck aspect ratios of less or equal to one by analysing decks S1 and S2 respectively, data for a deck with bay aspect ratio greater than one is obtained from the analysis of deck S3. Additional information obtained from deck S3, pertaining to aspects of the behaviour already presented for deck S1, follows next. The information from all three decks is subsequently used to study the build-up of membrane forces with increasing deflection and the influence of the deck aspect ratio on their distribution.

The transverse membrane action in the plate at two stages in the loading history, is shown in Figs 7.19. The first of the figures corresponds to an intermediate load level while the second corresponds to the last load step in the numerical analysis. A larger effective width for the tensile membrane forces than that suggested for deck S1 can be seen, with a value of about $2b$.

Figures 7.20 present the distribution of the transverse bending moments in the plate at the same load levels as Figs 7.19. It can be seen from Fig. 7.20(b) that the bending phase in the plate has not fully developed across the entire width. This is an indicator that the second phase has not yet been reached.

Bending action along the 'notional beams' is shown in Figs 7.21. The first of the figures shows significant bending in stiffener 2 while, in the second figure, load is transferred by means of the transverse mechanism further away from the centre.

Distributions of the axial forces along the 'notional beams' at the two load steps are illustrated in Figs 7.22. Once more, stiffener 1 attracts the maximum tensile forces while, it is stiffener 5 and not the outermost stiffener which is under the maximum compressive force.

From the behaviour shown above, it becomes clear that the second phase has not been reached by deck S3. Effort was made to pursue the numerical analysis to higher load, though unsuccessful due to numerical difficulties. In this respect, since the second phase (following the collapse of the outmost stiffener) has not been reached for this wide deck, it is not possible to determine its onset as a function of aspect ratio.

7.4 THE DISTRIBUTION OF AXIAL FORCES IN THE 'NOTIONAL BEAMS'

Information concerning the longitudinal membrane forces in the 'notional beams', may be obtained by studying the development of the axial force in the loaded stiffener with increasing deflection and the distribution of forces across the width. In the following, the axial forces are non-dimensionalized with respect to the squash load and further normalized with respect to the axial force in the loaded stiffener.

7.4.1 Axial force in the loaded beam

Results from all three numerical analyses undertaken are used in Fig. 7.23 to devise a relationship between the axial force in the loaded stiffener, n_1 , and the central deflection, δ_1 , non-dimensionalized with respect to the plate thickness. All three analyses predict points distributed close enough for the relationship to be approximated by a straight line up to membrane yielding. Alternatively, a piecewise linear curve can be fitted. From Fig. 7.23 it appears that the axial force developed in the loaded stiffener, under increasing deflection, is almost independent of the deck aspect ratio, at least for values of n_1 less or equal to 0.75.

7.4.2 Axial force in other beams

Distributions of the longitudinal forces in the stiffeners across the deck width are illustrated in Figs 7.24(a), 7.24(b) and 7.24(c) for the decks S1, S2 and S3 respectively. In these figures, the stiffener index is plotted along the horizontal axis. Along the vertical axis the axial force in the stiffeners is plotted at various load levels normalized with respect to the axial force in stiffener 1. The objective of this discussion is to show how the distribution of these forces varies during the loading process.

Figure 7.24(a) shows the distribution of the longitudinal forces at $\rho=5.342$ and also at the last load step of the numerical analysis, corresponding to $\rho=7.357$. The broken line corresponds to the stage on deck S1 when it changes phase, while the continuous line corresponds to the second phase. The distribution of forces at $\rho=5.342$ exhibits a strong linearity implying that the deck experiences in-plane bending as a deep beam. The span and depth of this deep beam seem to be equal to the length and half-width of the deck respectively. At $\rho=7.357$ a relative reduction in the compressive forces in stiffener 4, associated with the reduction of tension in stiffener 2 can be observed. This redistribution is associated with the second phase in deck behaviour and may not occur for decks which do not reach this phase. This phase is more likely to occur in narrow decks with aspect ratios less than one.

The distribution of longitudinal forces in deck S2 is shown in Fig. 7.24(b). At $\rho=2.298$, stiffener 3 is under the maximum compressive force, while at $\rho=4.547$ stiffener 4 is under maximum compression. The redistribution occurring is attributable to plasticity, that is gradual development of plastic mechanisms across the width.

Distributions for deck S3 are shown in Fig. 7.24(c). Stiffener 5 is seen to succeed stiffener 4 as the most compressed stiffener, whilst stiffeners 6 and 7 are comparatively under less compression. This is in sharp contrast to the behaviour of the outmost stiffeners in the narrow deck shown in Fig. 7.24(a).

This seems to indicate that, in wide decks, the stiffeners under maximum compression are confined to be within a region of deck which is approximately of square plan, having sides equal to the bay length. The correctness of this statement must be validated by further numerical or experimental evidence.

Following this discussion, two alternative methods for predicting the membrane forces across the width, other than by performing an elasto-plastic analysis, will be investigated. The first examines the distribution of forces across the width as a geometrically non-linear elastic problem, and the second investigates the distribution of forces as an analogy of the in-plane problem of the deck to a cantilevered beam loaded in the plane of the plate. This investigation is carried out on the geometry of deck S3.

7.4.3 Geometrically non-linear elastic analysis

An elastic, but geometrically non-linear analysis may give, in some cases, guidance on the distribution of stresses in the elasto-plastic range. Deck S3 was analysed allowing for geometric non-linearity but neglecting material plasticity. Figure 7.25 compares distributions of the longitudinal forces across the width of the deck obtained from the various analyses. Comparison between the distribution labelled 'non-linear elastic' and the distribution at $\rho=6.548$ obtained from the elasto-plastic analysis of deck S3, indicates the role of plasticity in the redistribution process. These distributions have been plotted for the same central deflection in the loaded stiffener.

7.4.4 In-plane analysis

The transverse membrane forces, acting along the longitudinal centre line of the quarter deck analysed, apply a stress resultant similar to in-plane bending of a flat deck. The load distribution, idealized in Fig. 7.26, applies an in-plane bending moment on the quarter deck as indicated in this figure. The resulting distribution of longitudinal membrane forces across the width is shown in Fig. 7.25. This distribution, labelled 'in-plane', has a different shape than that shown for deck S3 at $\rho=6.548$, with the maximum compression in the position of the outer stiffener (stiffener 7), as expected. However, this analysis predicts the magnitudes of the forces in stiffeners 2 and 3, relative to those in stiffener 1, fairly accurately.

7.5 CONCLUSIONS

A simply supported longitudinally stiffened single-bay deck, subjected to lateral patch loading, may experience two distinct phases at large deflections. During the first phase, the load is shed outwards, initially by means of the transverse bending

action in the plate and subsequently by means of its transverse membrane action. The strength in the deck is primarily due to bending action in the beams but the longitudinal membrane forces also contribute. The latter become of increasing importance as the deflection increases. This phase ends, in a narrow deck, with the collapse of the outer stiffener. In the second phase, the load carrying capacity is increased primarily by the increasing longitudinal membrane force and deflection in the loaded beam, while the contribution to the strength from the other stiffeners diminishes. It is possible that the second phase may occur only in narrow decks.

A simply supported single-bay deck with longitudinal stiffening loaded to collapse by Dowling (1968), compares favourably with a numerical model. This deck geometry was used to validate the application of the finite element program FINAS to this type of analysis.

Two more decks were analysed, which had the same panel slenderness and aspect ratio, while the deck aspect ratio was variable. A relationship between the axial force in the loaded stiffener and its central deflection was found. This relationship appears to be largely independent of the deck aspect ratio and the way in which the patch loading is applied. Its dependence on the aspect ratio and slenderness of the panel remains to be investigated.

The importance of the longitudinal membrane action in enhancing the total load carrying capacity of the deck above the limit load, derived from a yield line type of analysis ignoring such action, was recognized. Ways to predict the distribution of longitudinal membrane action other than by performing a full elasto-plastic analysis were investigated.

Based on the observed behaviour two analytical models are proposed in the following chapter.

TABLE 7.1 Cross-sectional geometry

DECK	G E O M E T R Y						MATERIAL
No.	t (mm)	$t_w^{(1)}$ (mm)	b/t	h_w/t_w	b_f/t_f	L/b	E/σ_y
S2 ⁽²⁾	4.76	2.54	26.67	30	48.75	6	852

(1) $t_f = t_w$

(2) For notation see Fig. 3.1(b)

TABLE 7.2 Cross-sectional geometry

DECK	G E O M E T R Y					MATERIAL
No.	$t^{(1)}$ (mm)	b/t	h_w/t_w	b_f/t_f	L/b	E/σ_y
S1 ⁽²⁾ , S3 ⁽²⁾	4.76	26.67	8.57	10.50	6	837

(1) $t_w = t_f = t$

(2) For notation see Fig. 3.1(b)

TABLE 7.3 Strength contributions in deck S1

ρ	ρ_1	ρ_2	ρ_3	ρ_4	$\Sigma \rho_j$	$\Sigma \rho_j + 0.802$	$\rho_2 + \rho_3 + \rho_4$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.359	0.786	0.351	0.142	0.057	1.336	—	0.550
2.458	1.011	0.835	0.349	0.158	2.353	—	1.342
3.385	1.263	0.961	0.660	0.267	3.160	—	1.897
4.465	1.691	1.054	0.904	0.416	4.065	—	2.374
5.342	2.254	1.158	0.900	0.478	4.790	—	2.536
6.236	3.215	1.196	0.735	0.315	5.461	6.263	2.246
6.890	4.144	1.116	0.552	0.098	5.910	6.712	1.766
7.357	5.072	0.971	0.422	-0.150	6.315	7.117	1.243

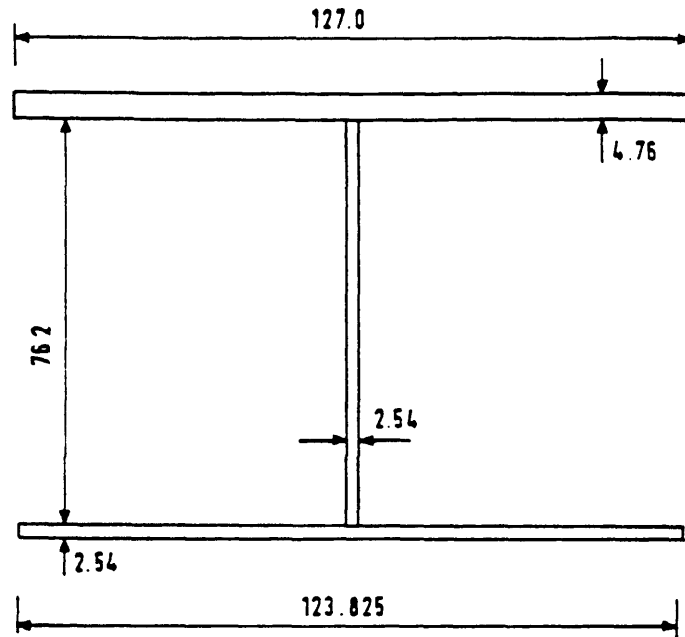


Fig. 7.1 Cross-sectional geometry for a stiffener and associated plate in deck S2 (dimensions in mm)

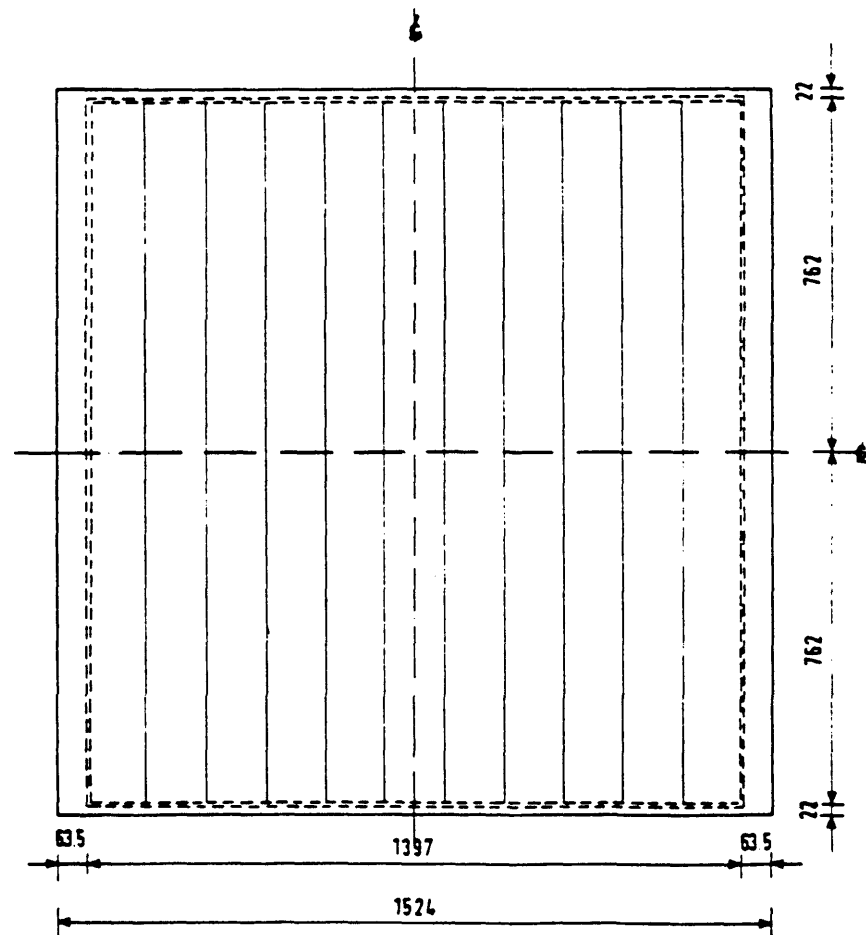


Fig. 7.2 Test bay on plan

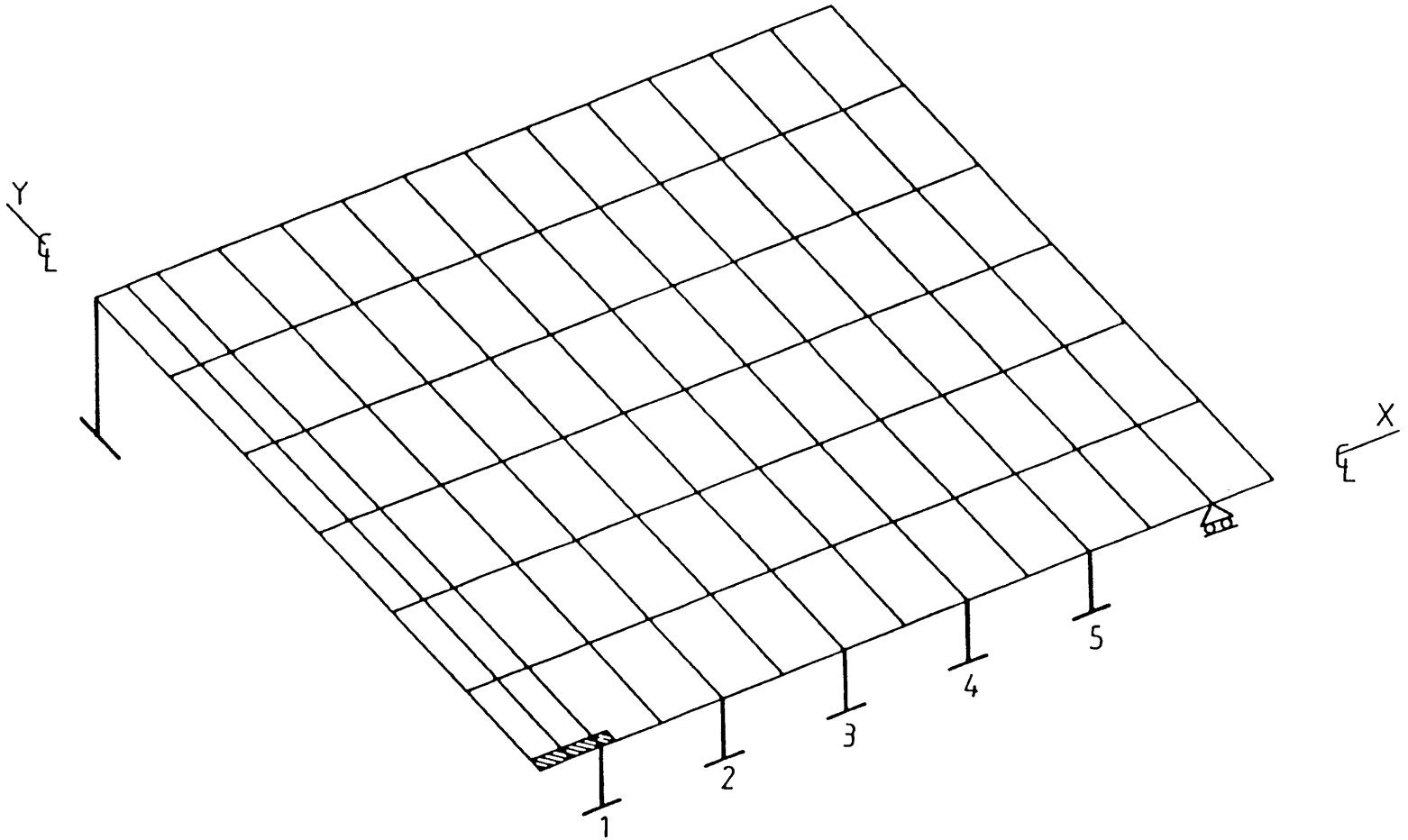


Fig. 7.3 Shell element mesh for the deck plate. The area loaded by the patch loading appears hatched.

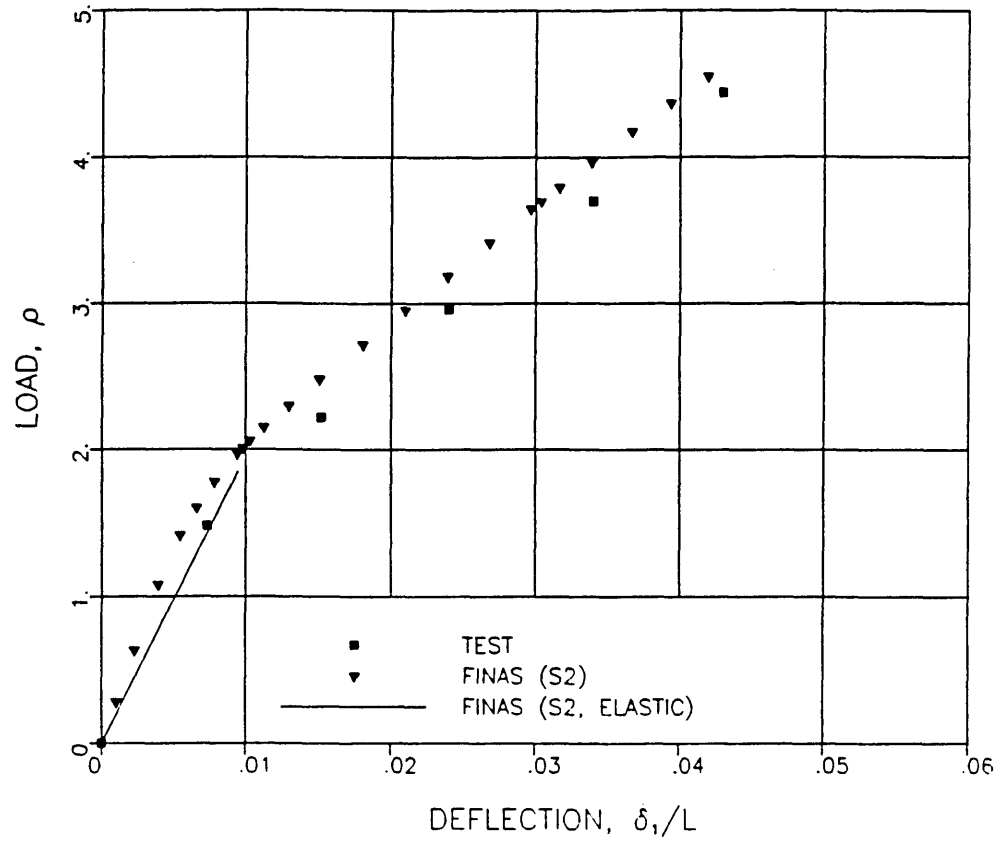


Fig. 7.4 Load vs. deflection for stiffener 1

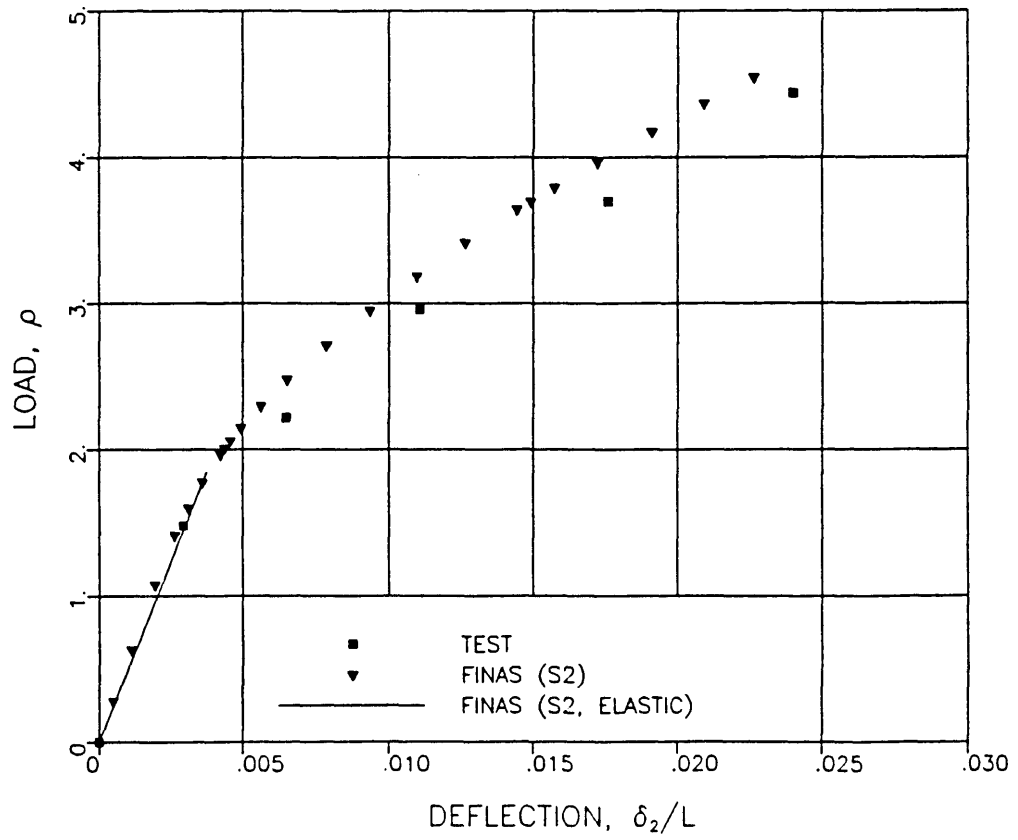


Fig. 7.5 Load vs. deflection for stiffener 2

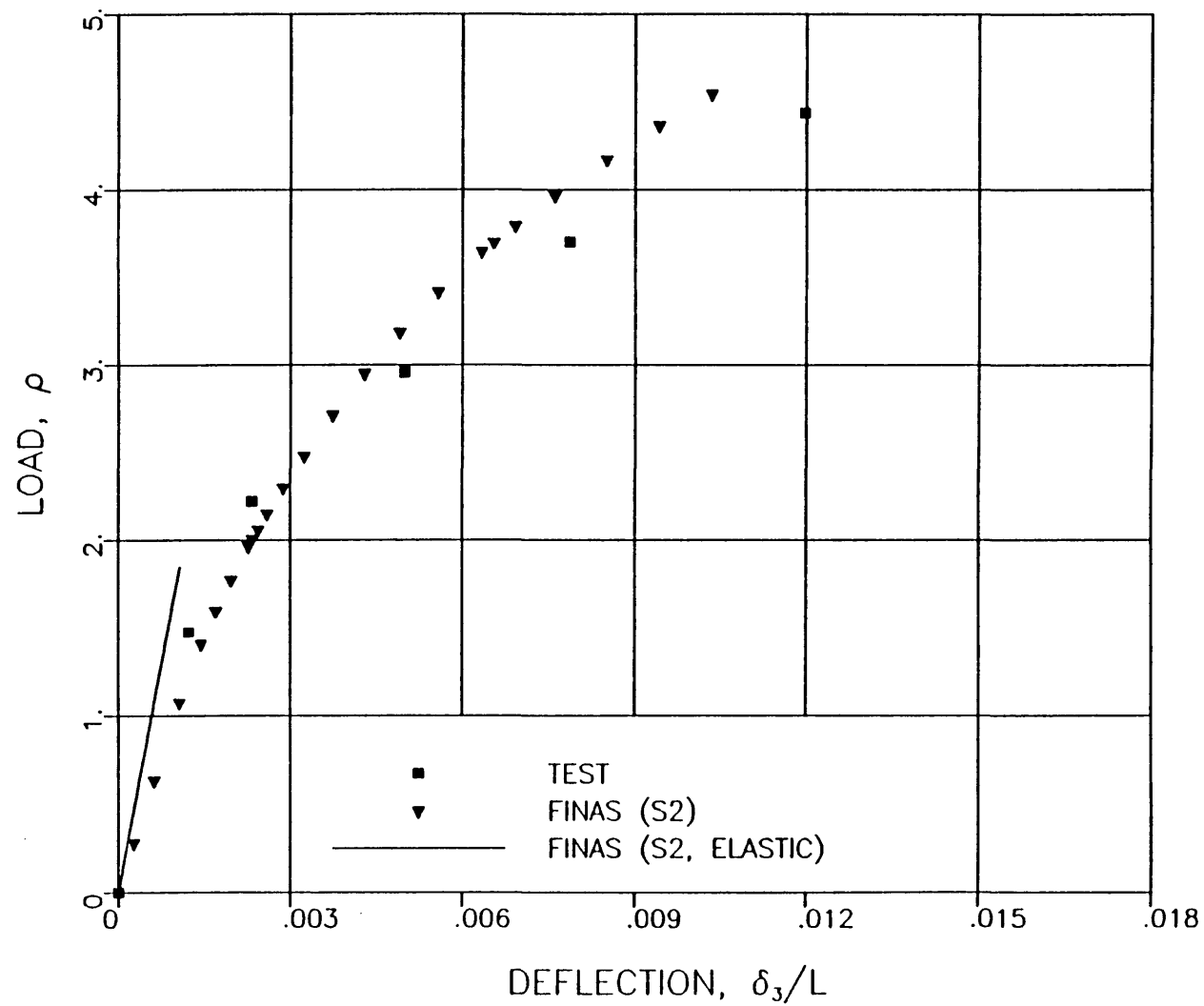


Fig. 7.6 Load vs. deflection for stiffener 3

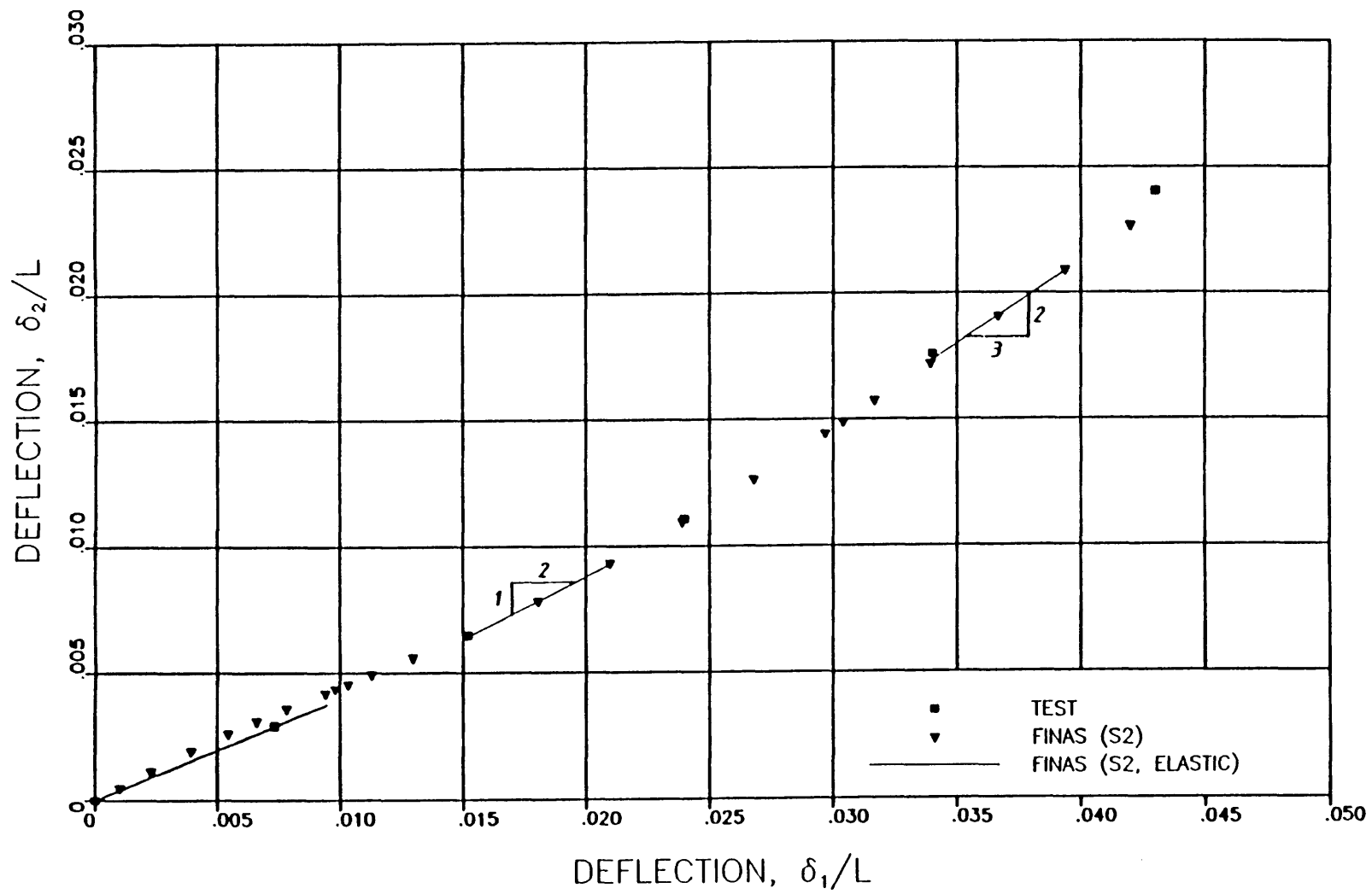


Fig. 7.7 Central deflection in stiffener 2 plotted against central deflection in stiffener 1

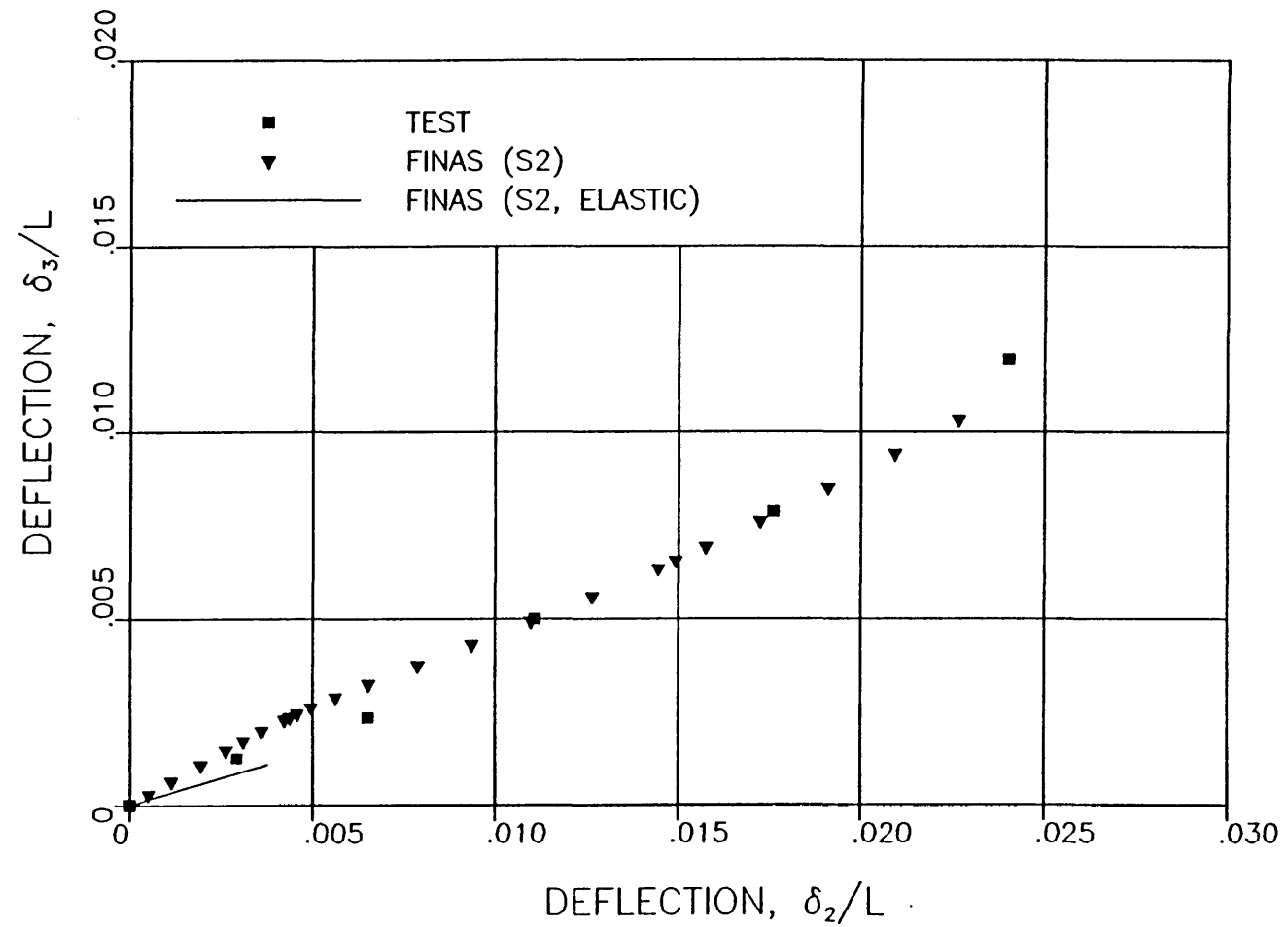


Fig. 7.8 Central deflection in stiffener 3 plotted against central deflection in stiffener 2

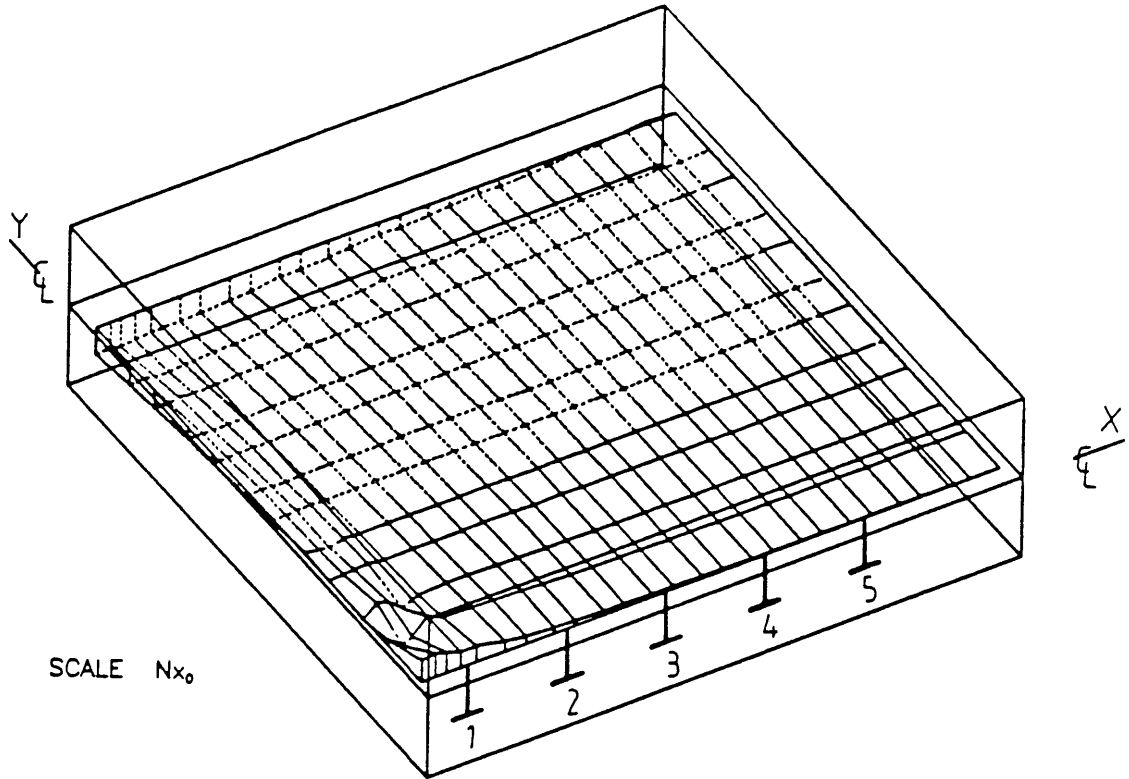


Fig. 7.9(a) Transverse membrane forces in deck plate S2, at $\rho=1.601$

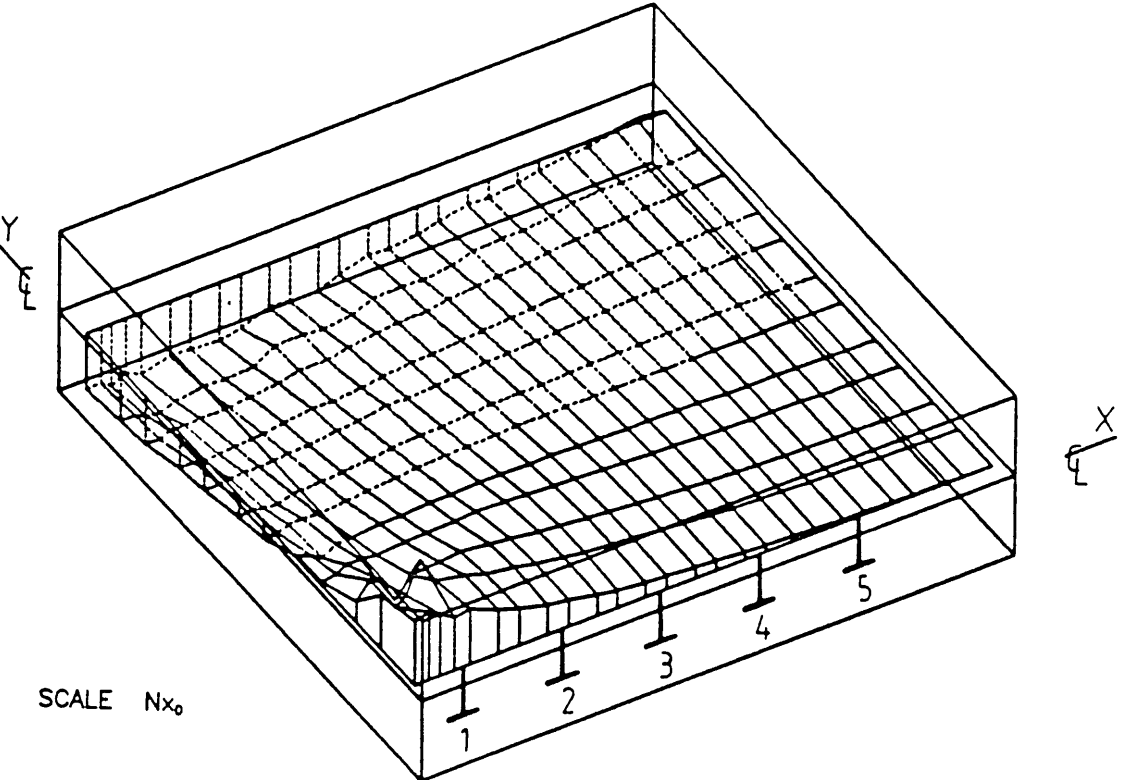


Fig. 7.9(b) Transverse membrane forces in deck plate S2, at $\rho=3.186$

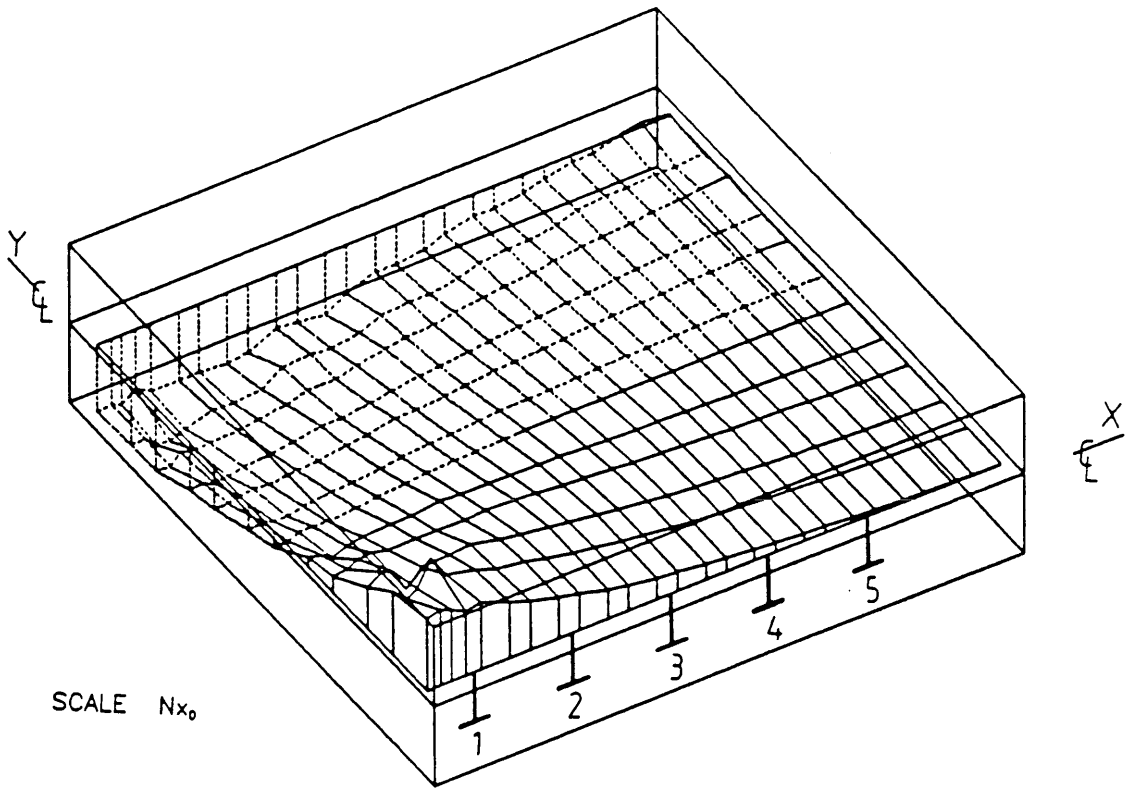


Fig. 7.9(c) Transverse membrane forces in deck plate S2, at $\rho=3.795$

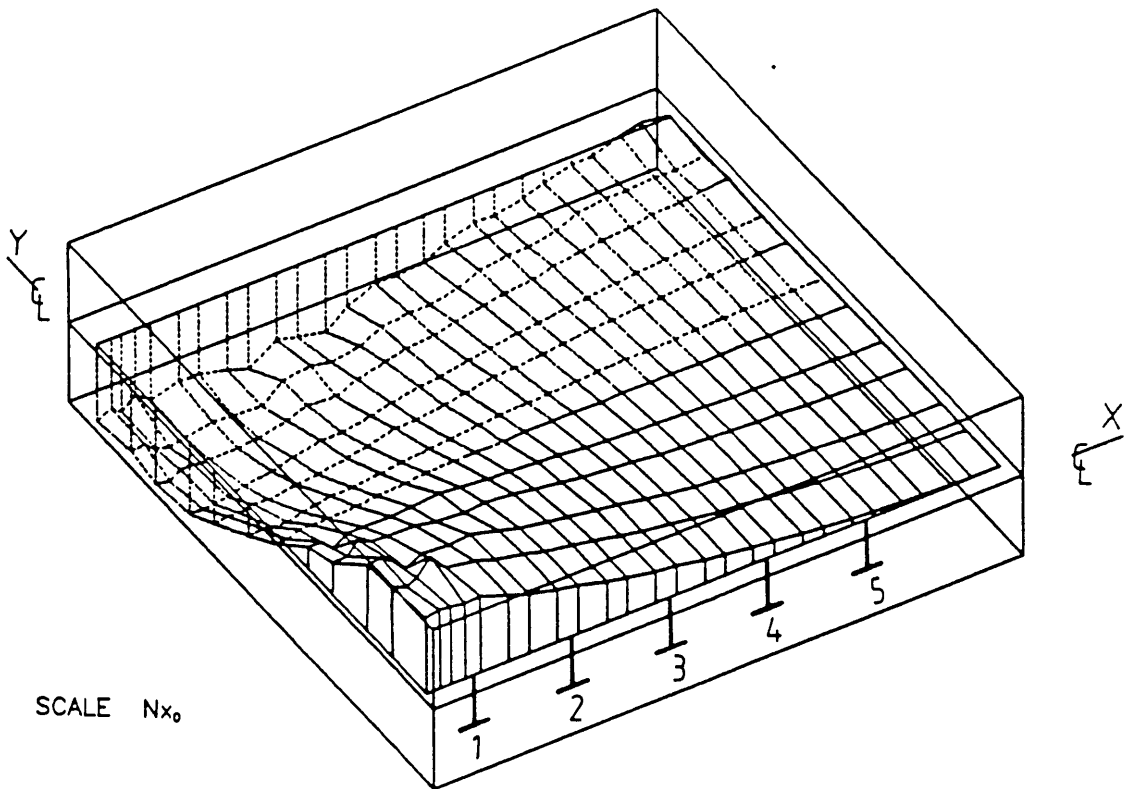


Fig. 7.9(d) Transverse membrane forces in deck plate S2, at $\rho=4.547$

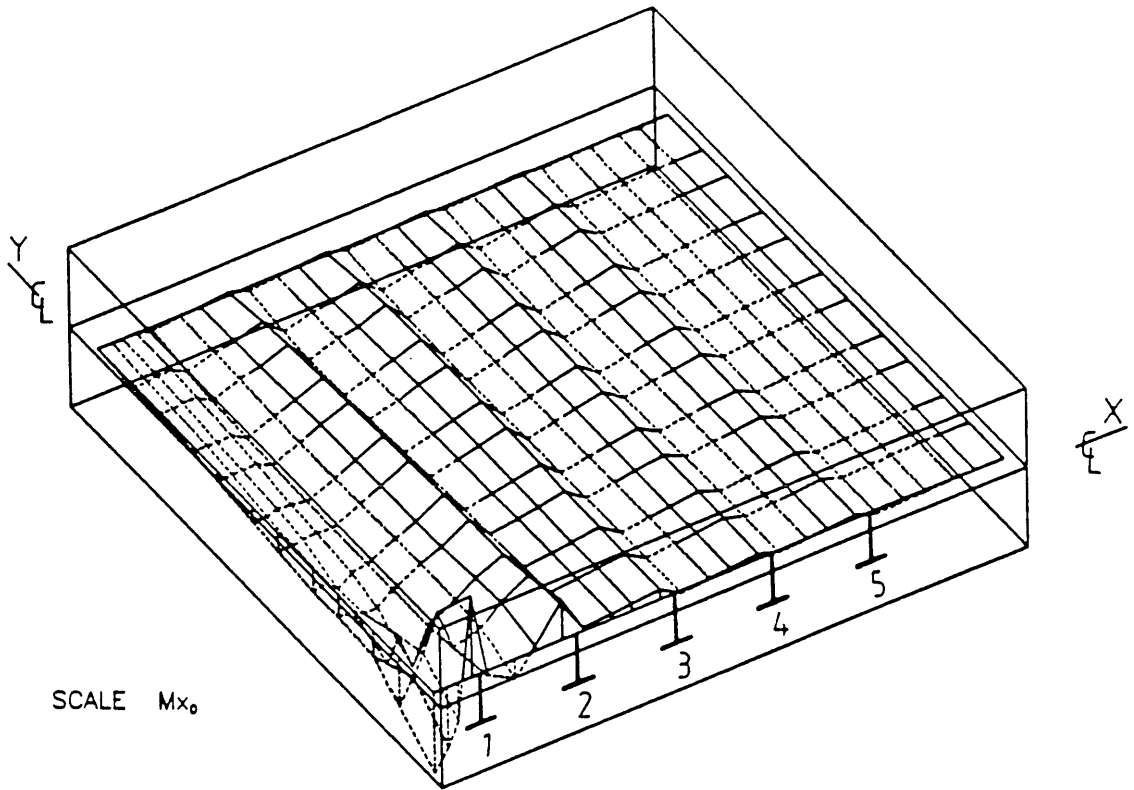


Fig. 7.10(a) Transverse bending distribution in deck plate S2, at $\rho=1.601$

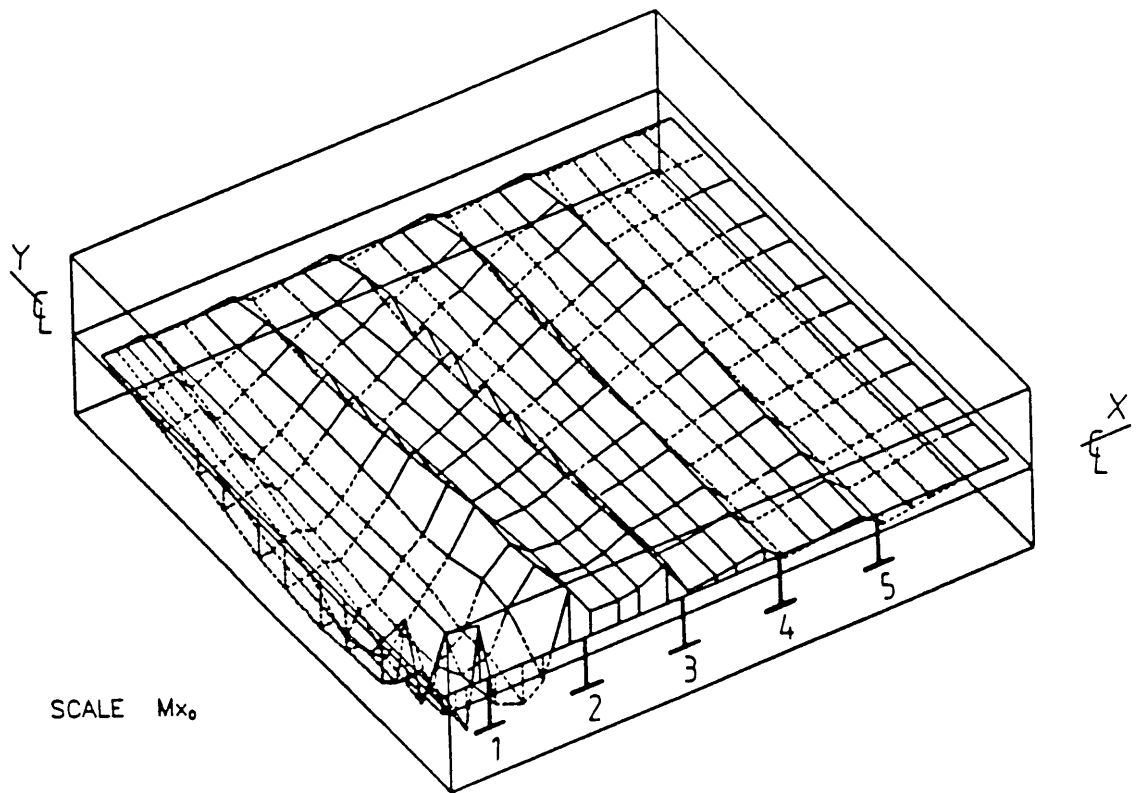


Fig. 7.10(b) Transverse bending distribution in deck plate S2, at $\rho=3.186$

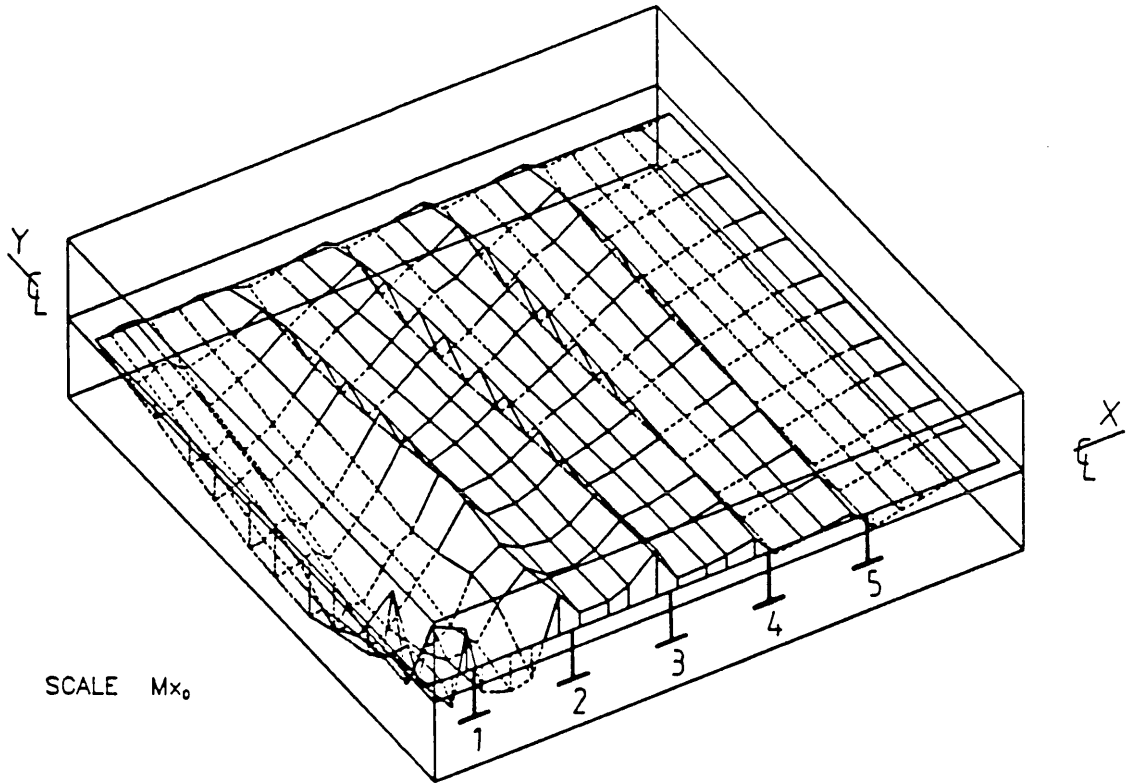


Fig. 7.10(c) Transverse bending distribution in deck plate S2, at $\rho=3.795$

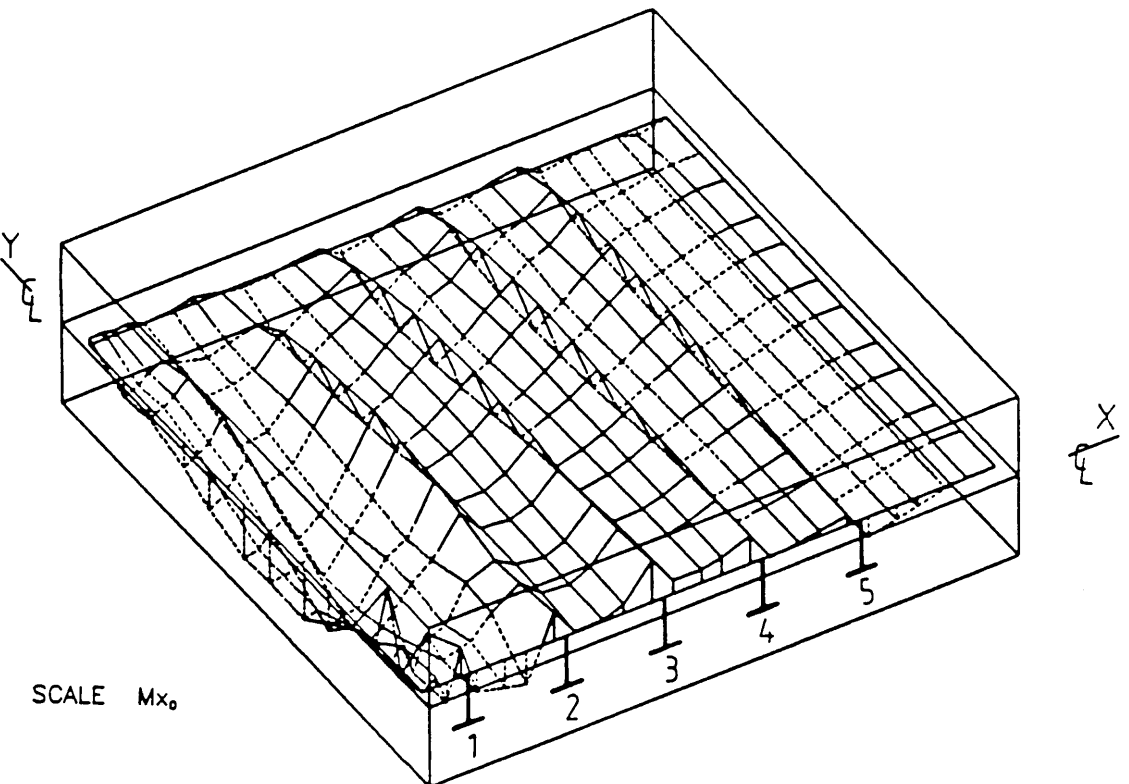


Fig. 7.10(d) Transverse bending distribution in deck plate S2, at $\rho=4.547$

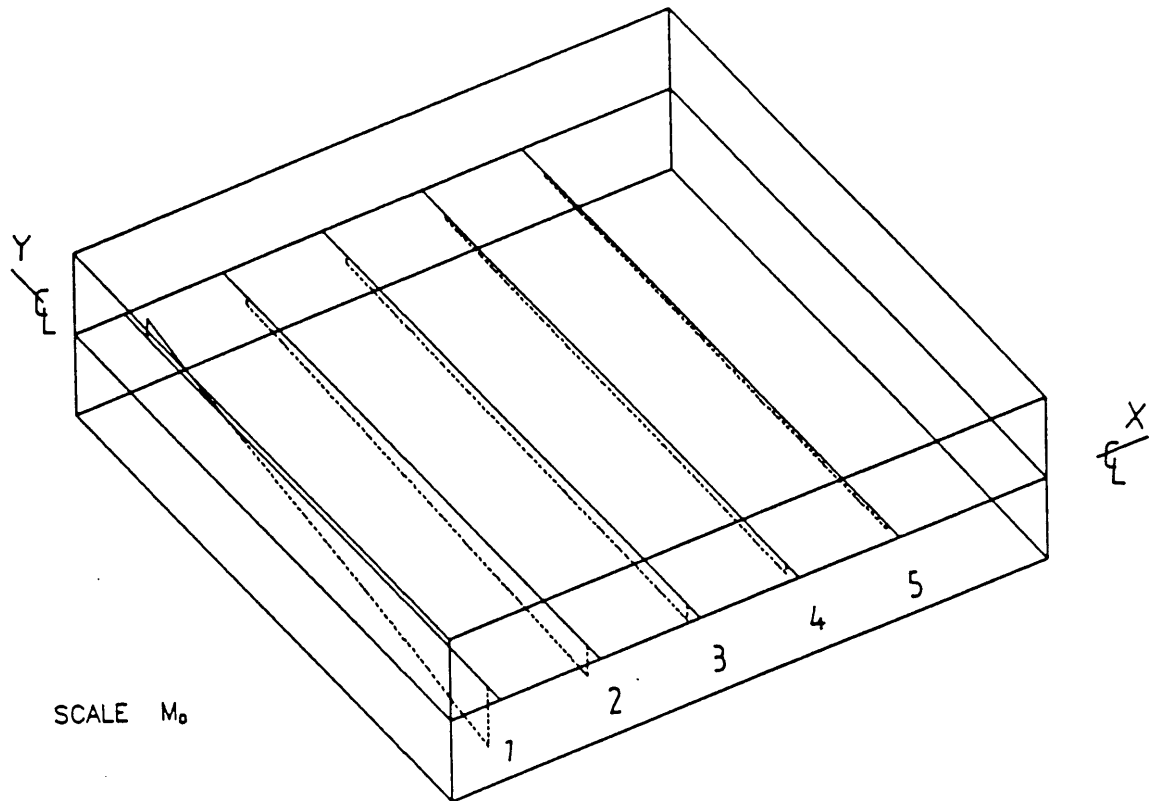


Fig. 7.11(a) Bending moment along the notional beams in S2, at $\rho=1.601$

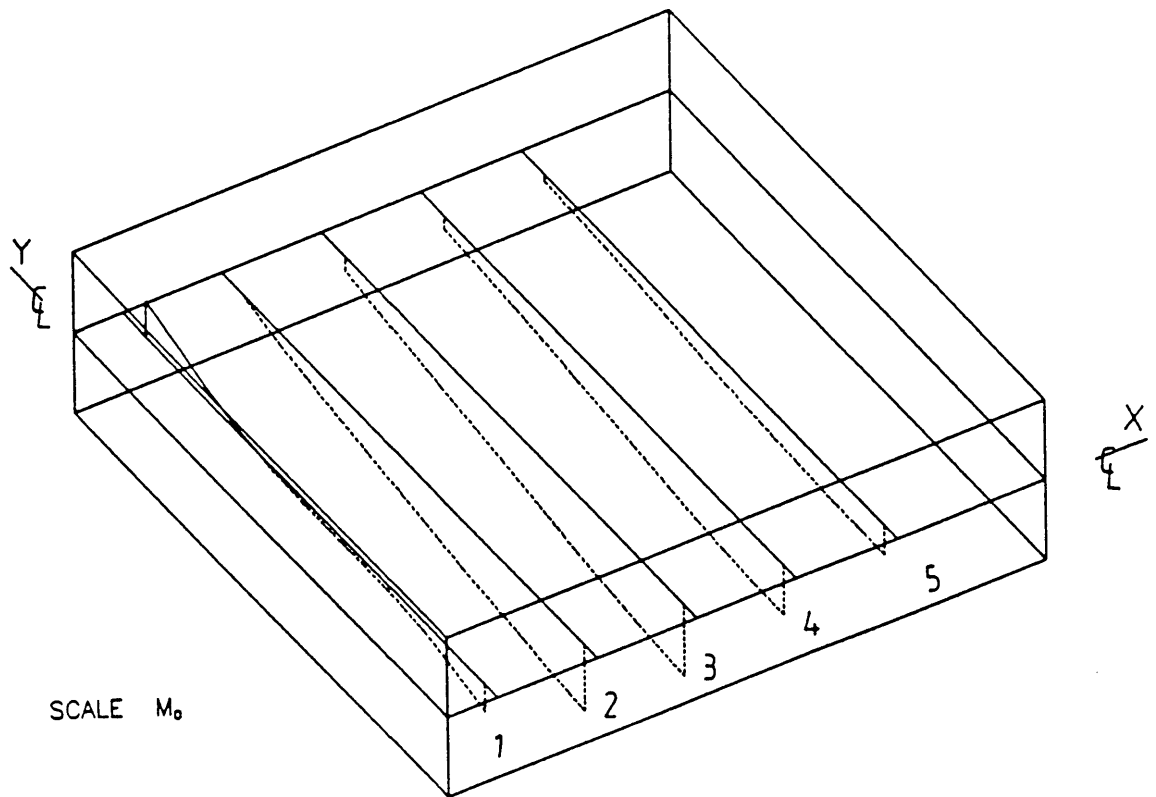


Fig. 7.11(b) Bending moment along the notional beams in S2, at $\rho=4.547$

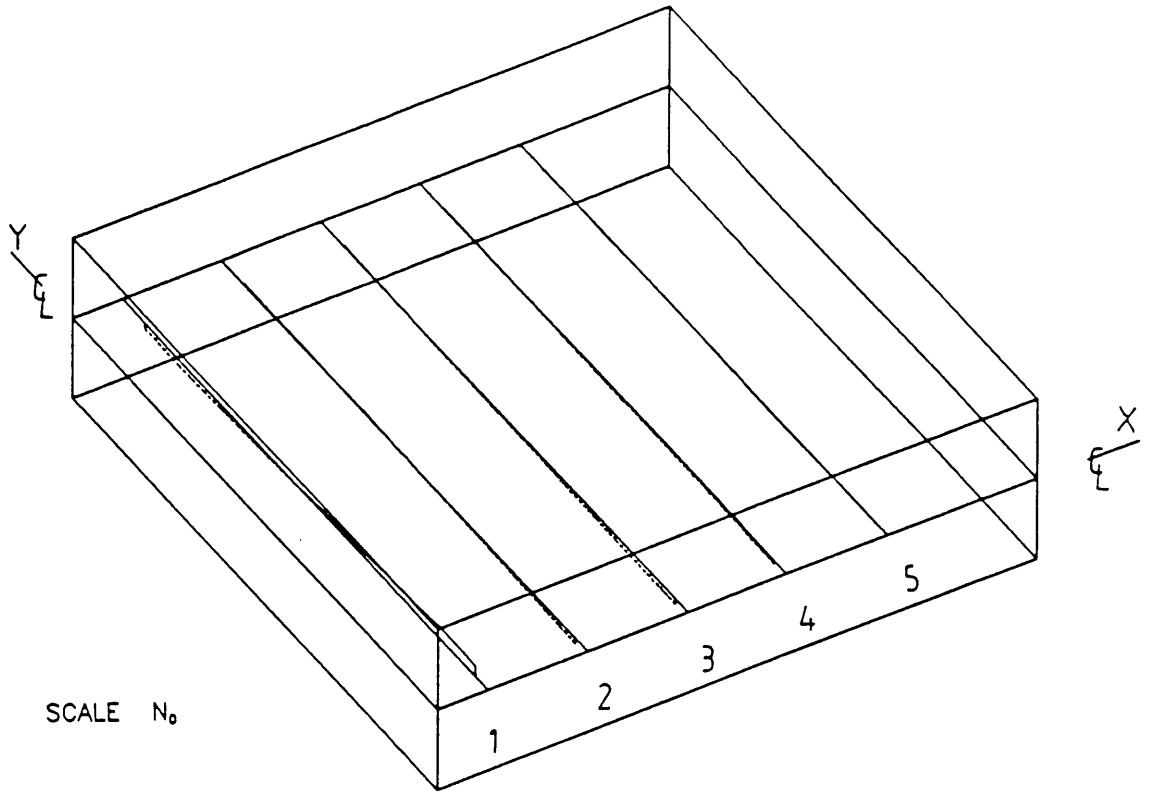


Fig. 7.12(a) Axial force along the notional beams in S2, at $\rho=1.601$

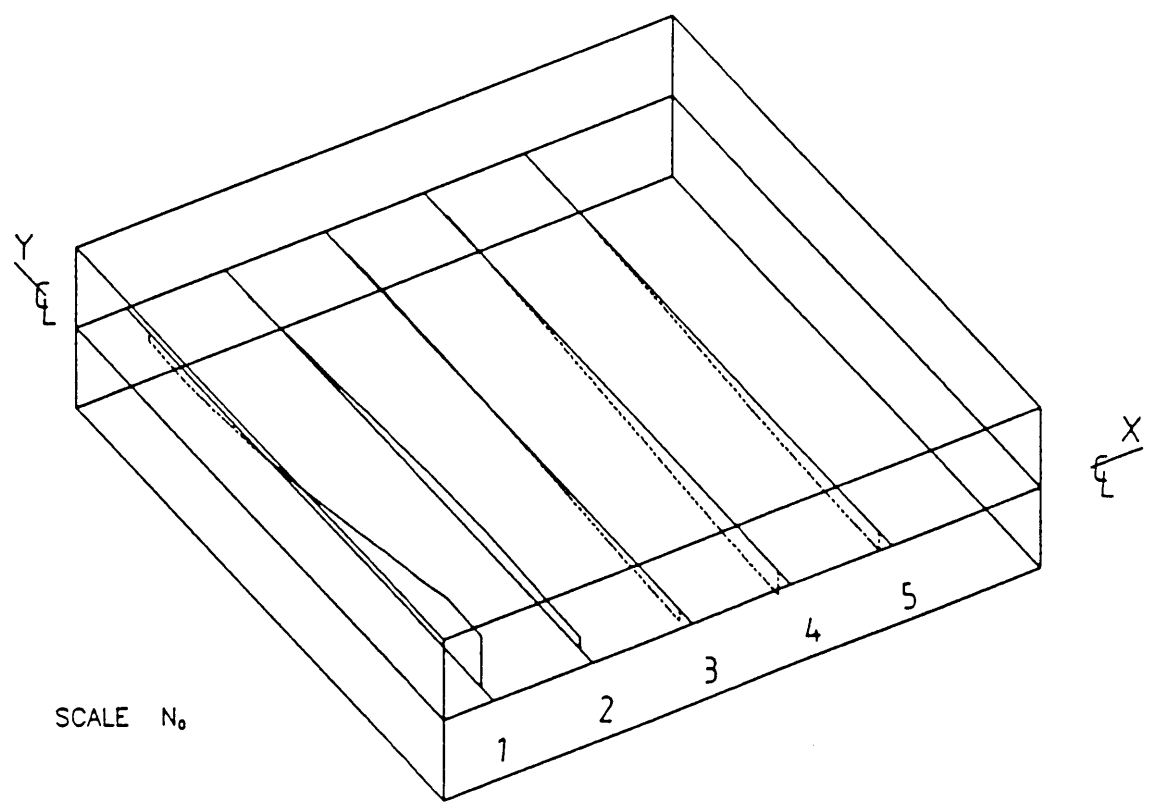


Fig. 7.12(b) Axial force along the notional beams in S2, at $\rho=4.547$

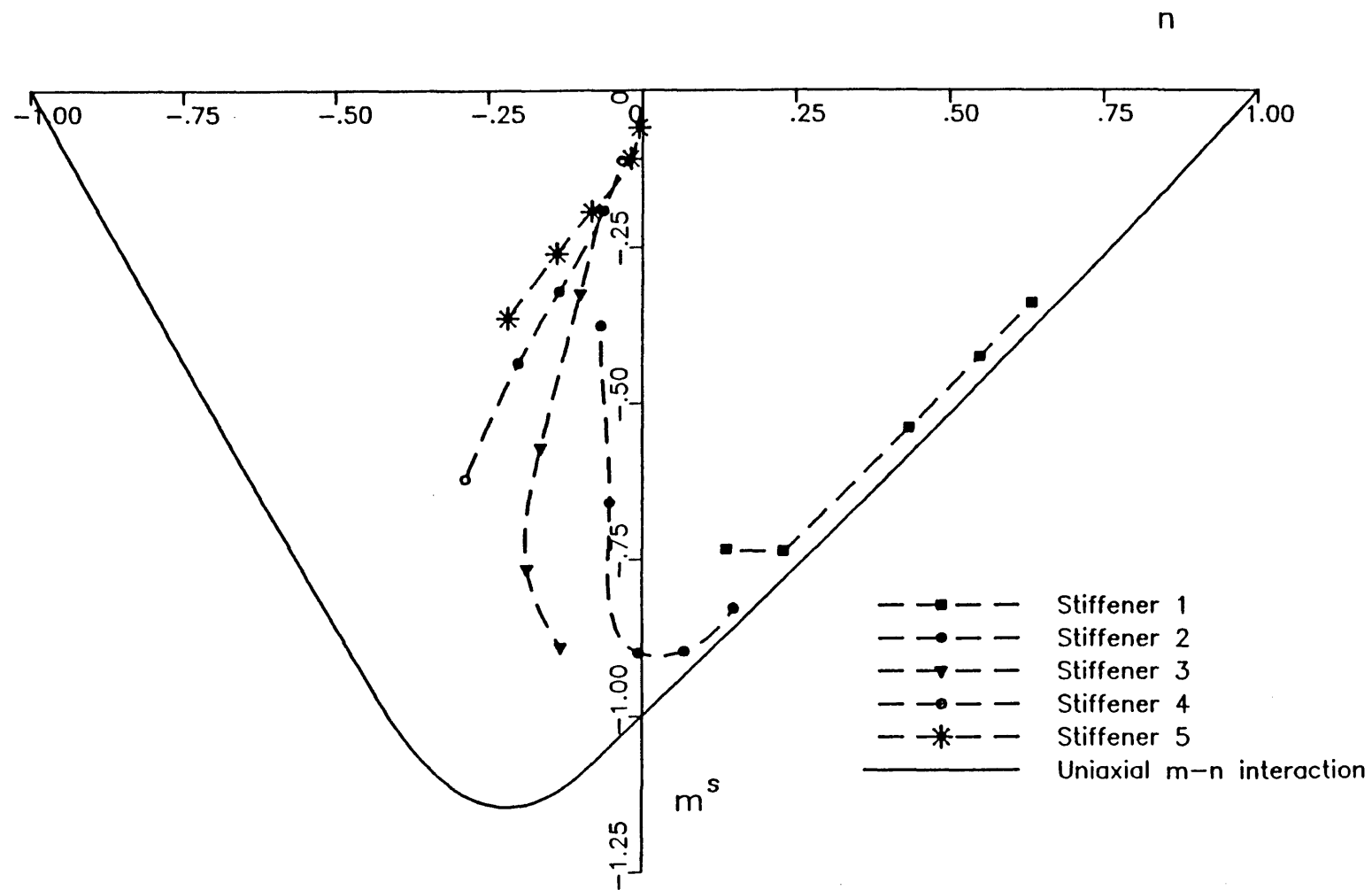


Fig. 7.13 Comparison between the stress resultants in deck S2 and the uniaxial m - n interaction for the notional beam cross-section

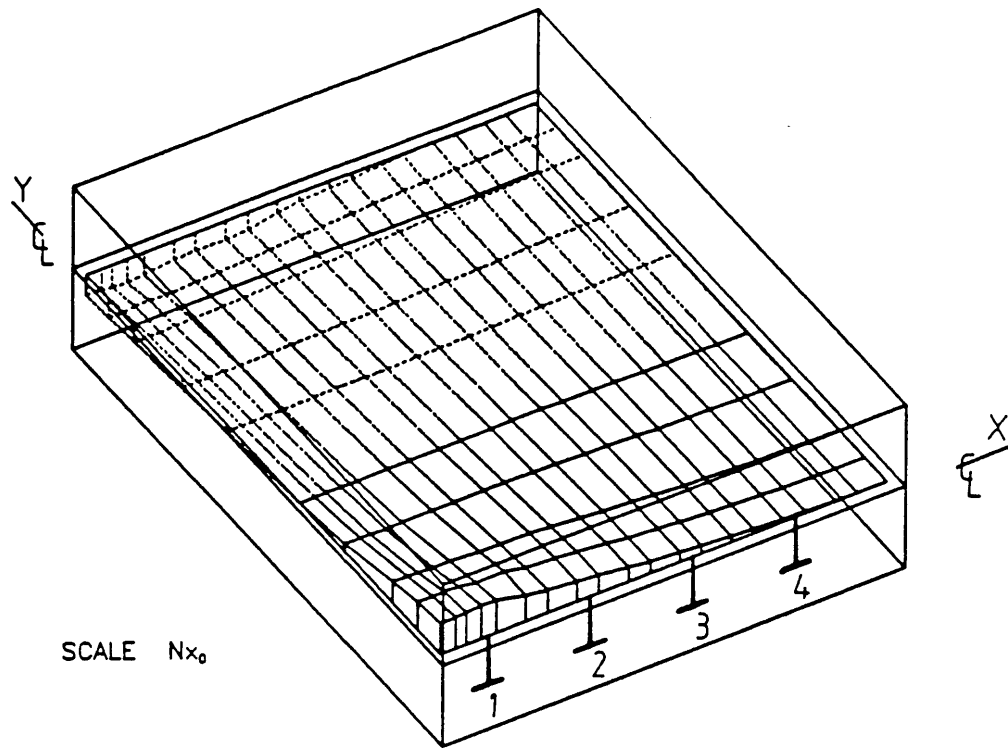


Fig. 7.14(a) Transverse membrane forces in deck plate S1, at $\rho=2.458$

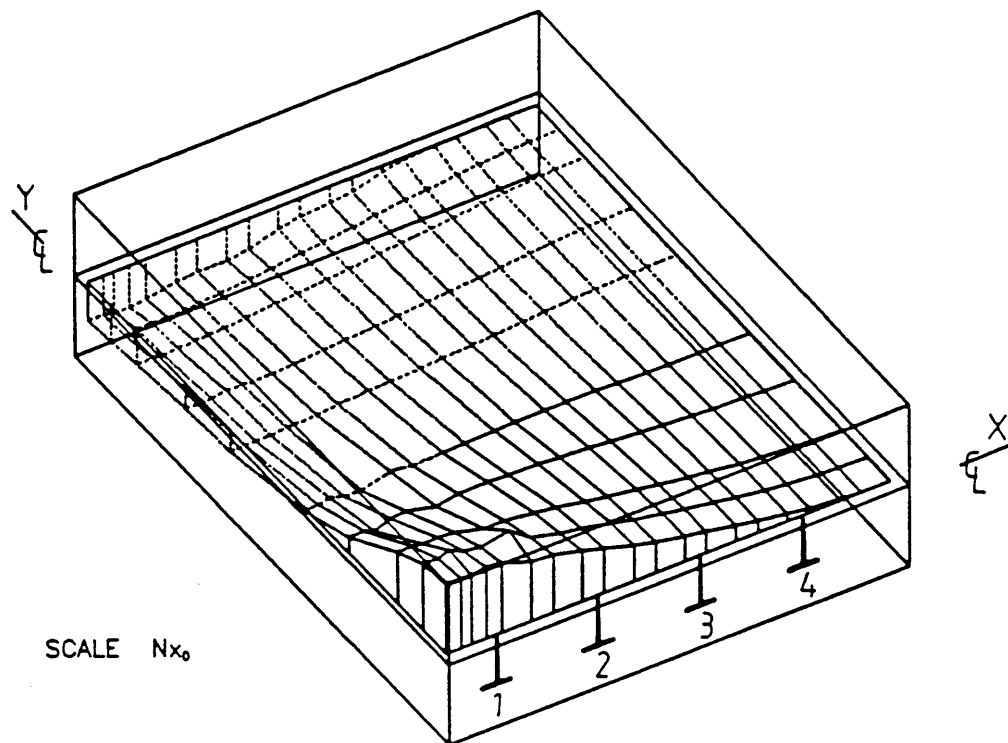


Fig. 7.14(b) Transverse membrane forces in deck plate S1, at $\rho=4.465$

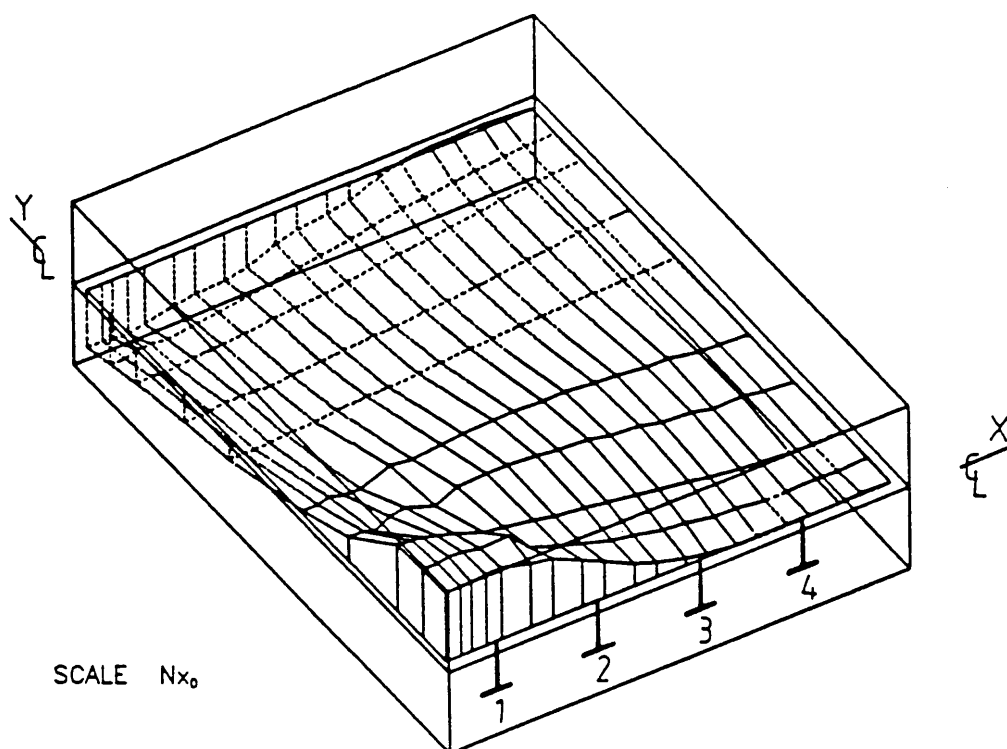


Fig. 7.14(c) Transverse membrane forces in deck plate S1, at $\rho=6.236$

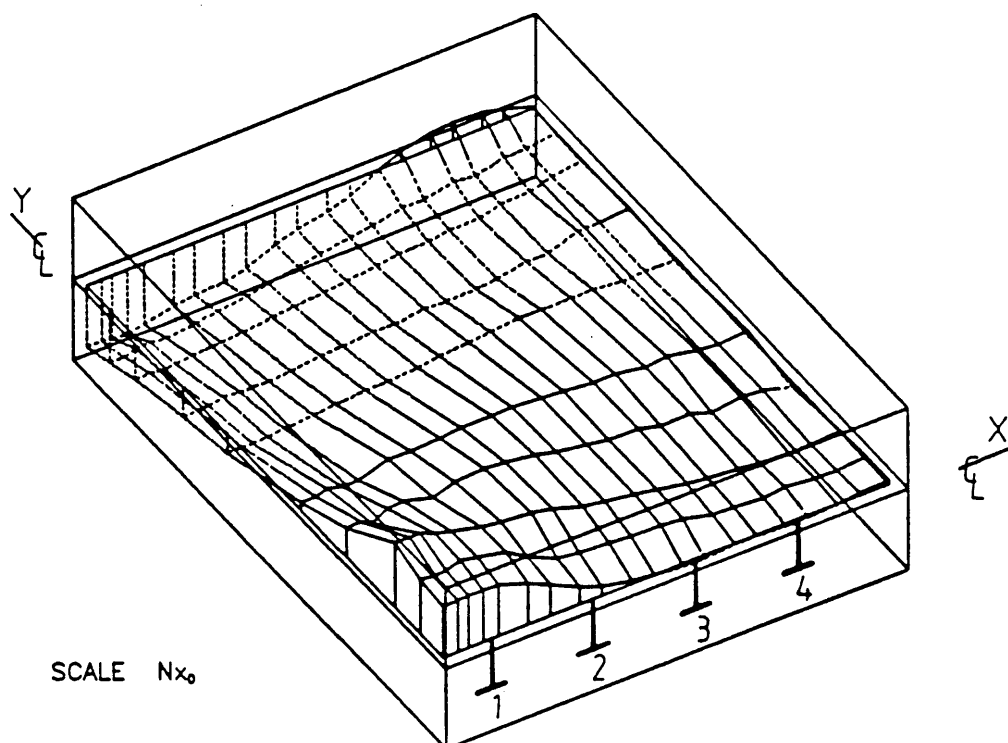


Fig. 7.14(d) Transverse membrane forces in deck plate S1, at $\rho=7.356$

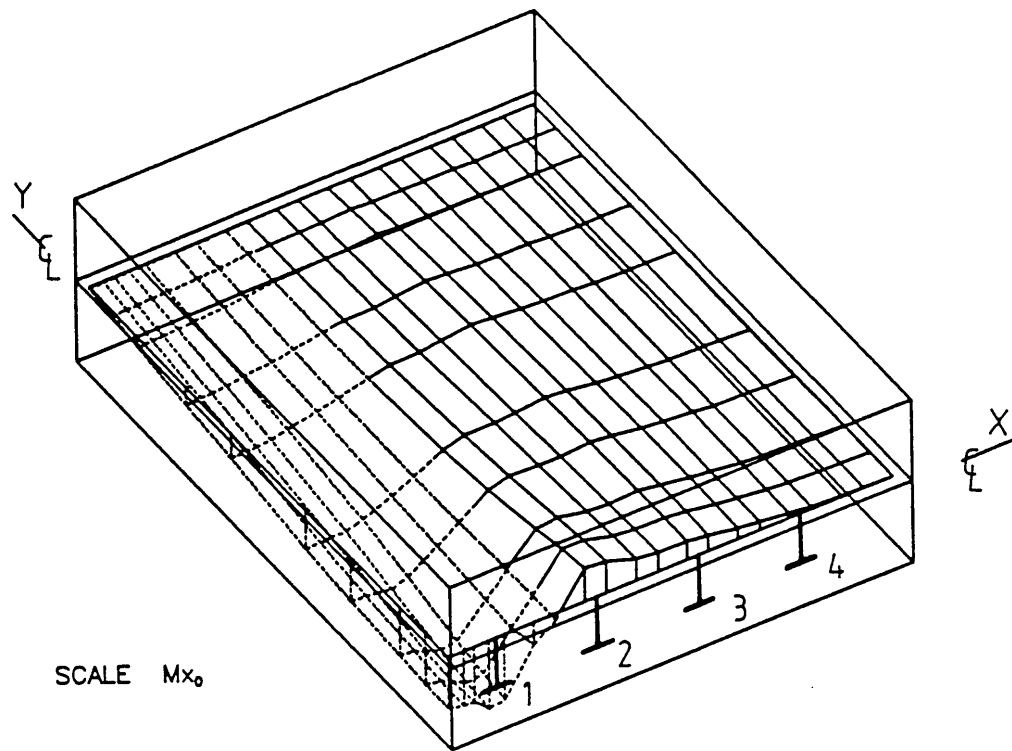


Fig. 7.15(a) Transverse bending distribution in deck plate S1, at $\rho=2.458$

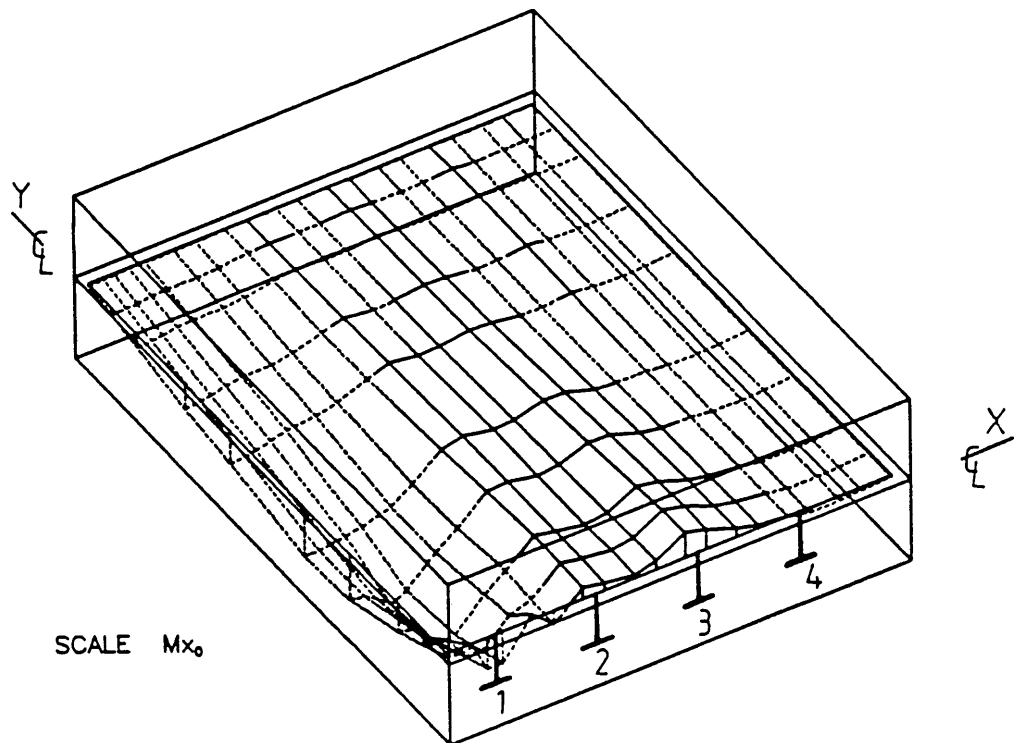


Fig. 7.15(b) Transverse bending distribution in deck plate S1, at $\rho=4.465$

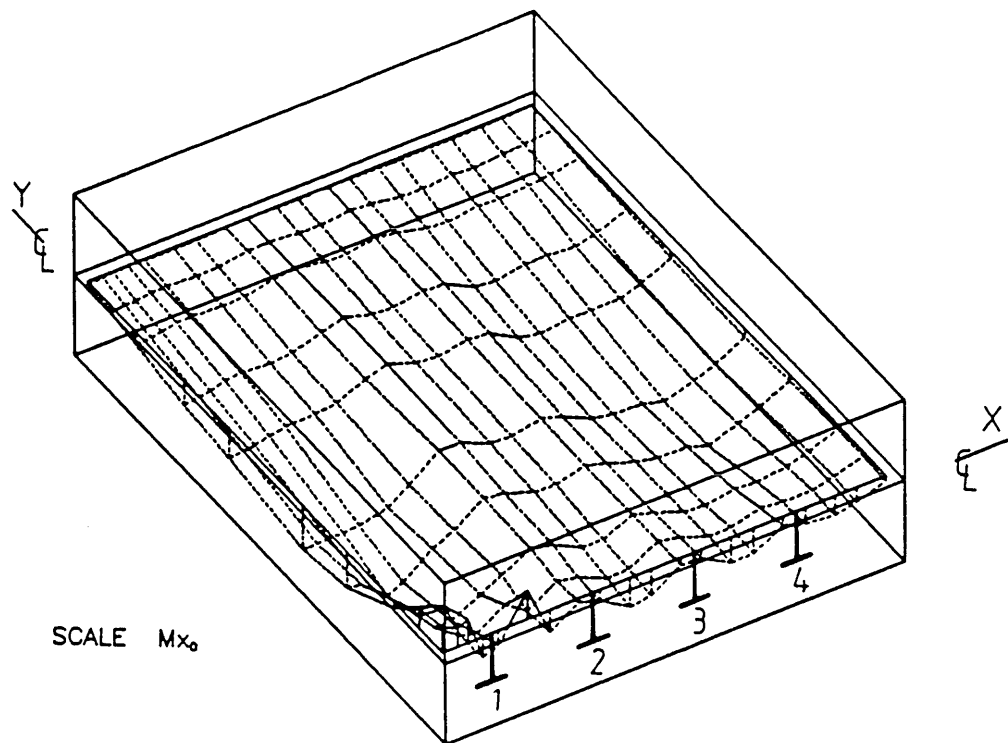


Fig. 7.15(c) Transverse bending distribution in deck plate S1, at $\rho=6.236$

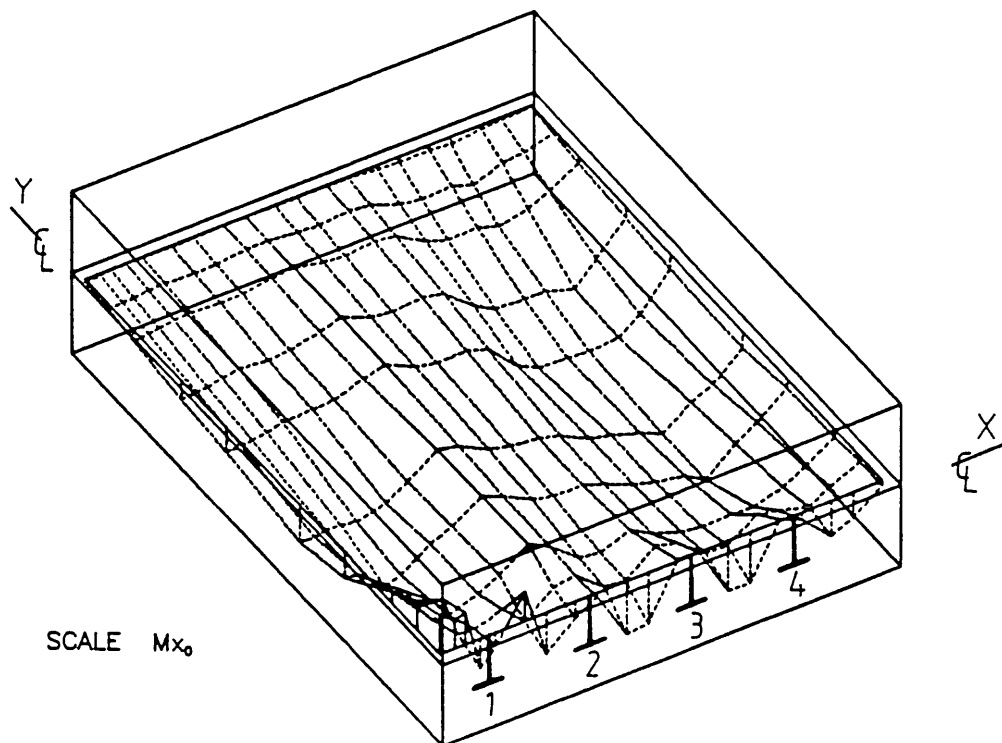


Fig. 7.15(d) Transverse bending distribution in deck plate S1, at $\rho=7.356$

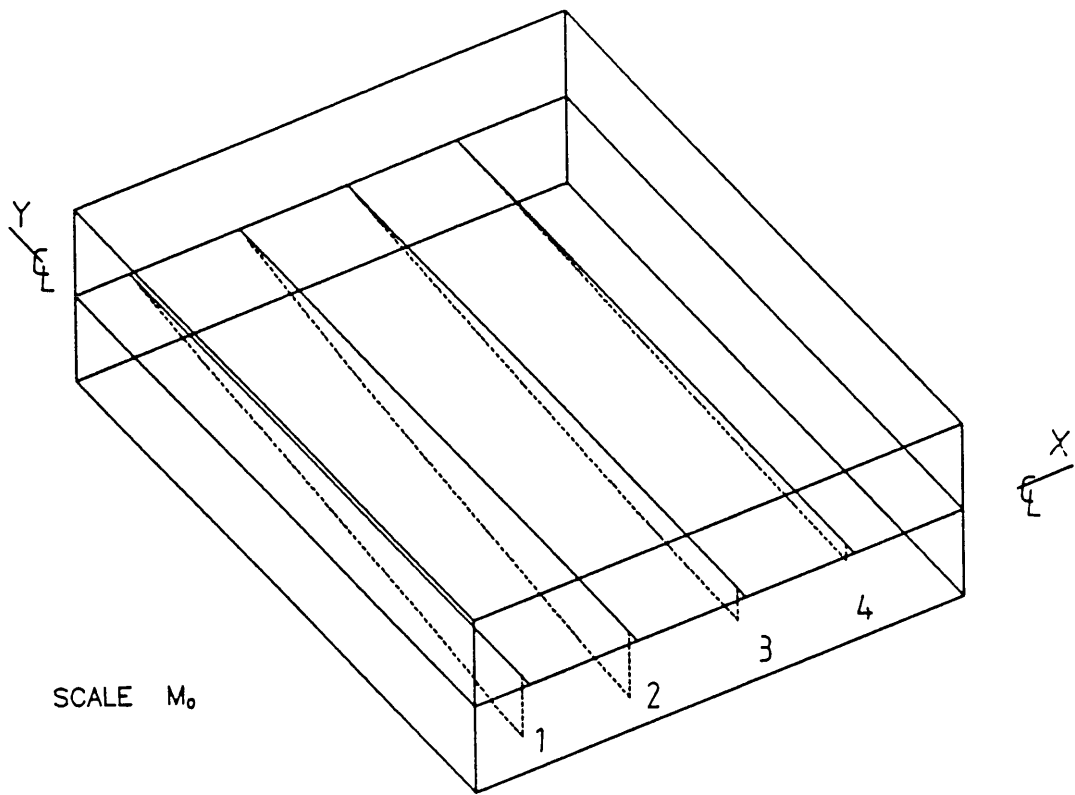


Fig. 7.16(a) Bending moment along the notional beams in S1, at $\rho=2.458$

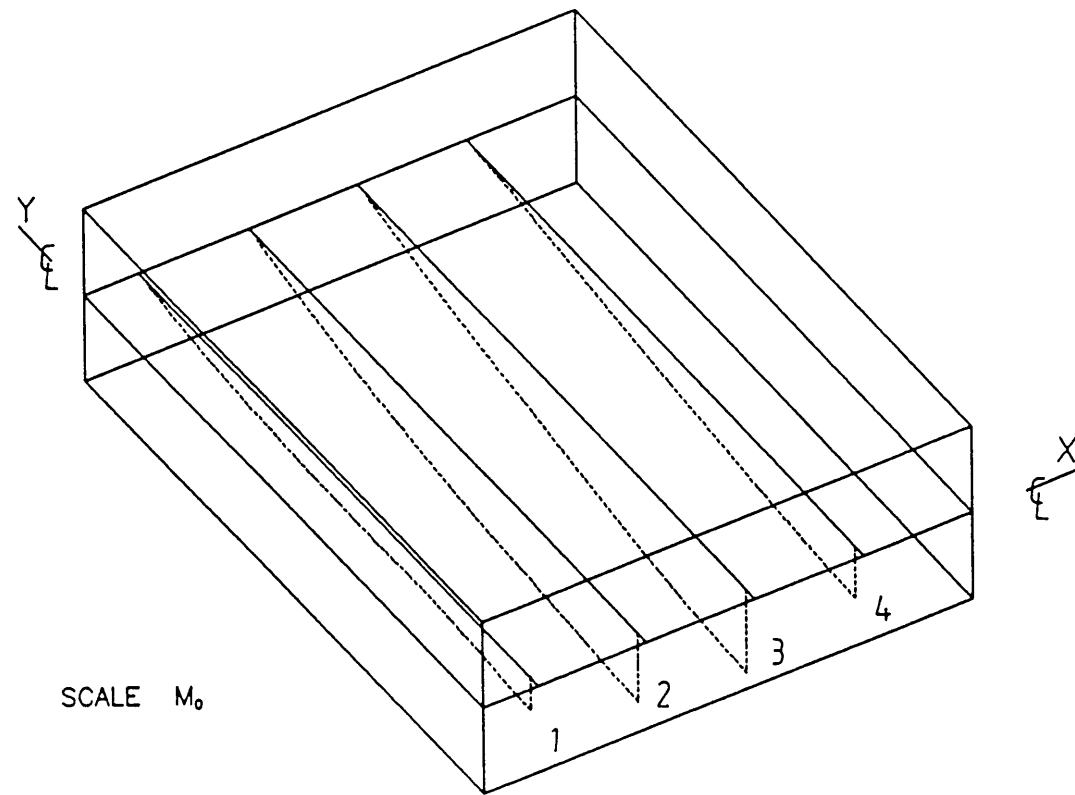


Fig. 7.16(b) Bending moment along the notional beams in S1, at $\rho=4.465$

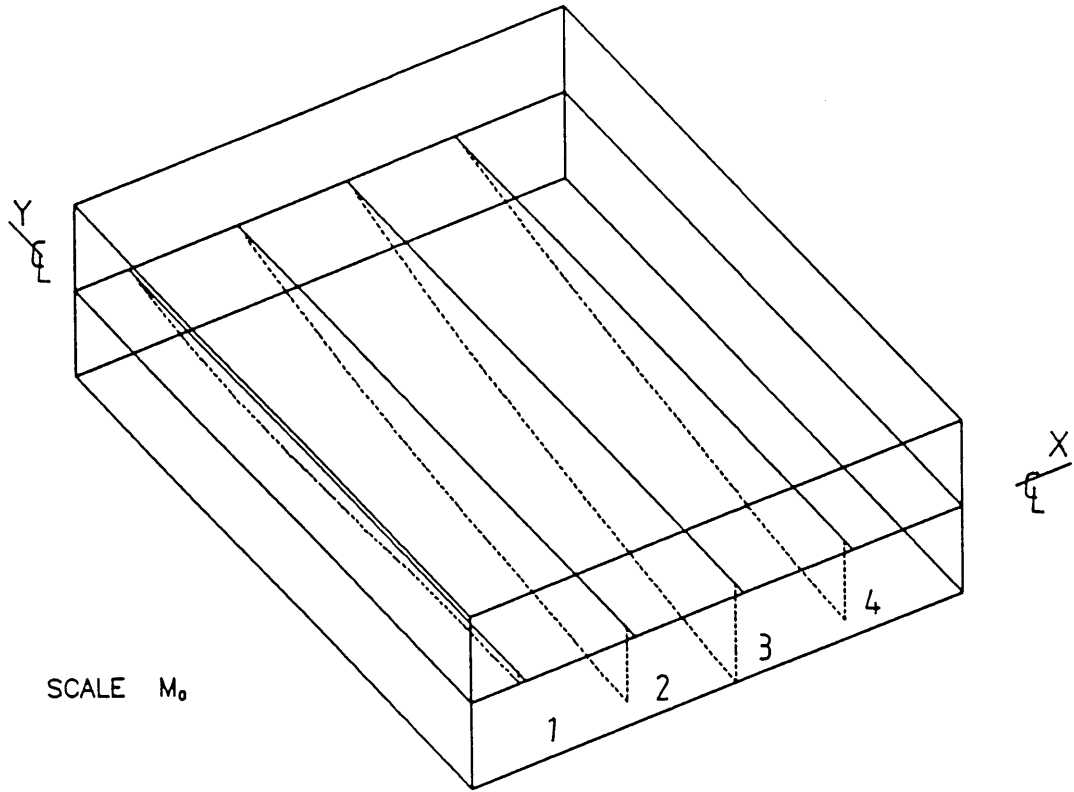


Fig. 7.16(c) Bending moment along the notional beams in S1, at $\rho=6.236$

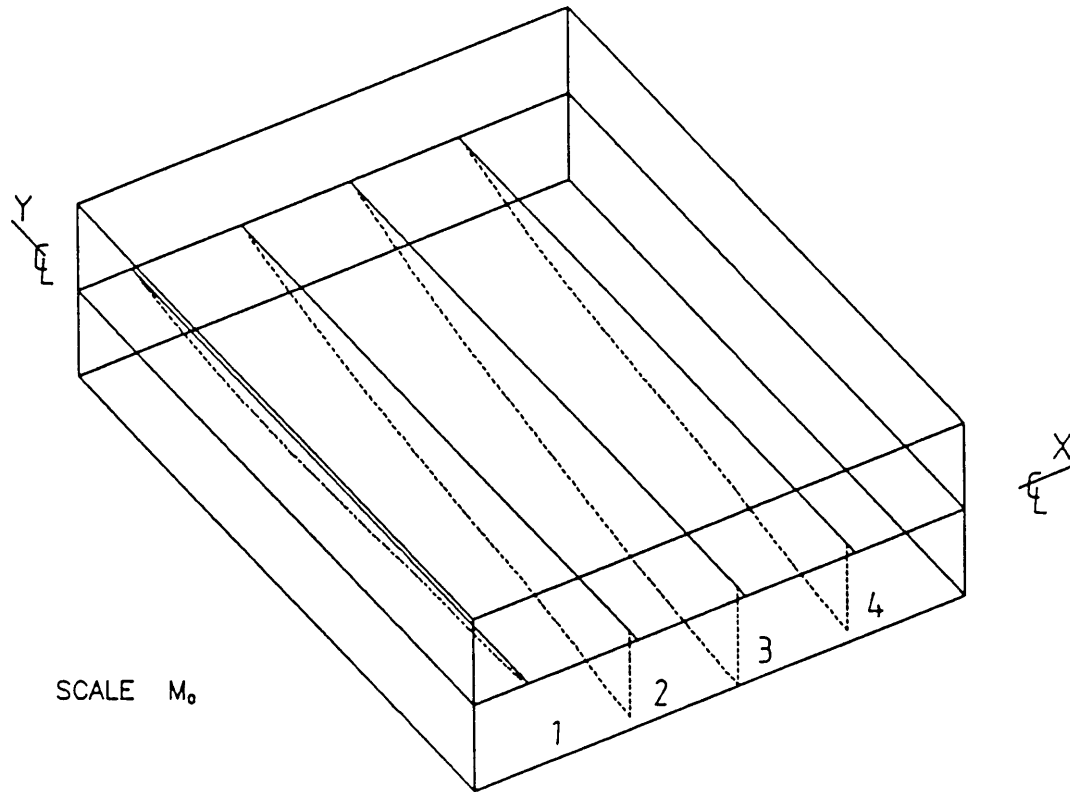


Fig. 7.16(d) Bending moment along the notional beams in S1, at $\rho=7.356$

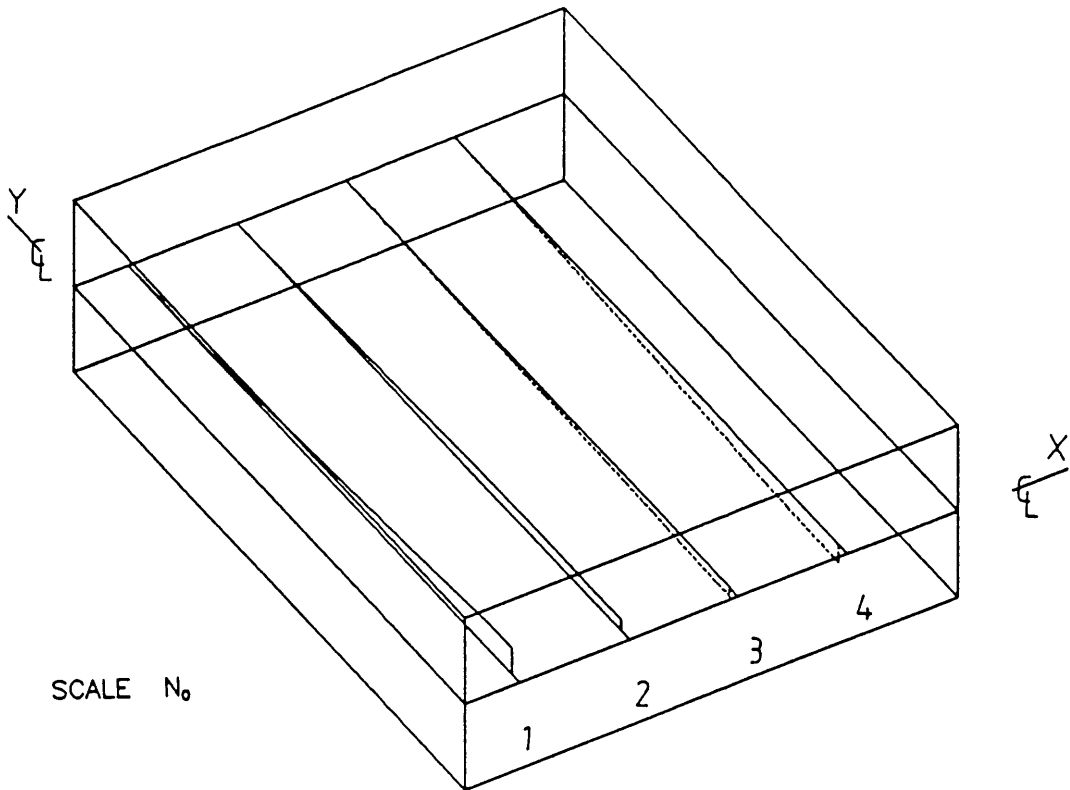


Fig. 7.17(a) Axial force along the notional beams in S1, at $\rho=2.458$

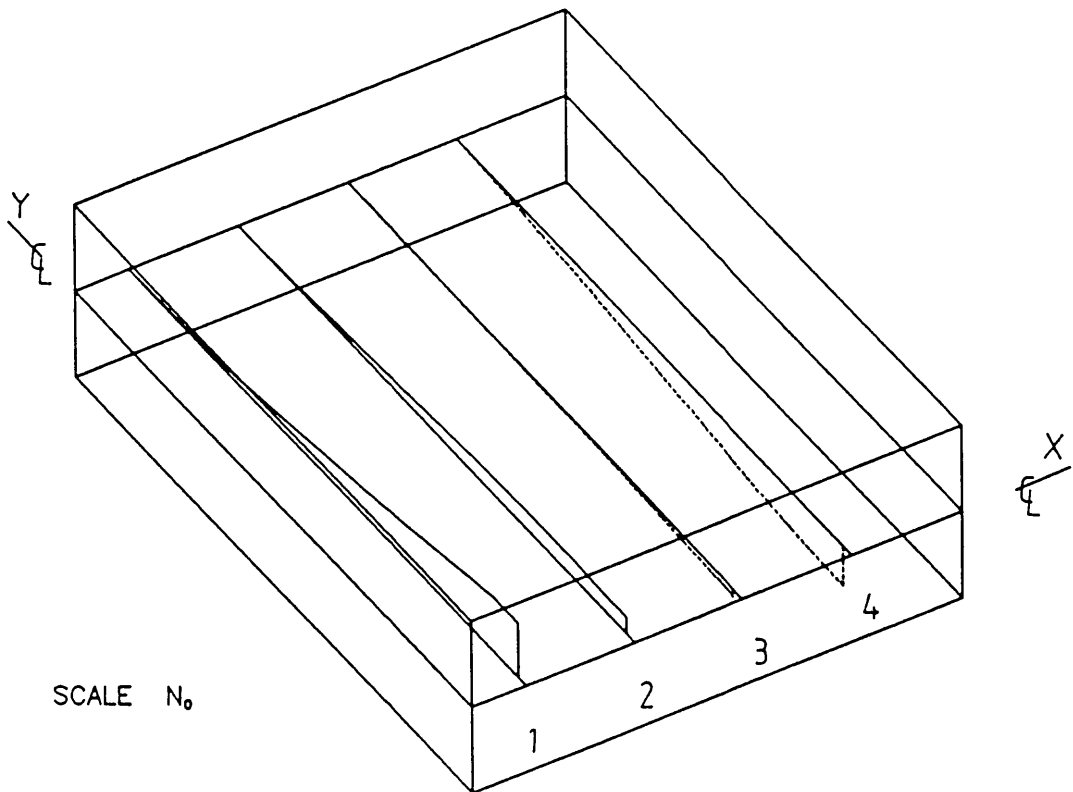


Fig. 7.17(b) Axial force along the notional beams in S1, at $\rho=4.465$

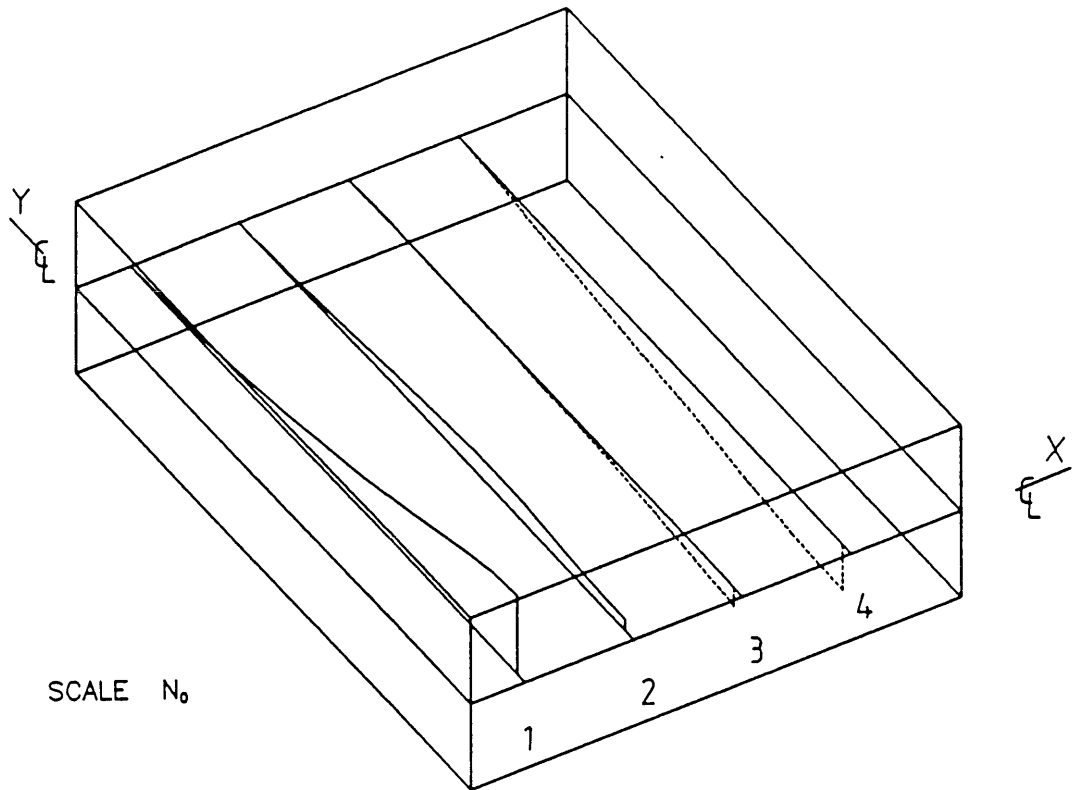


Fig. 7.17(c) Axial force along the notional beams in S1, at $\rho=6.236$

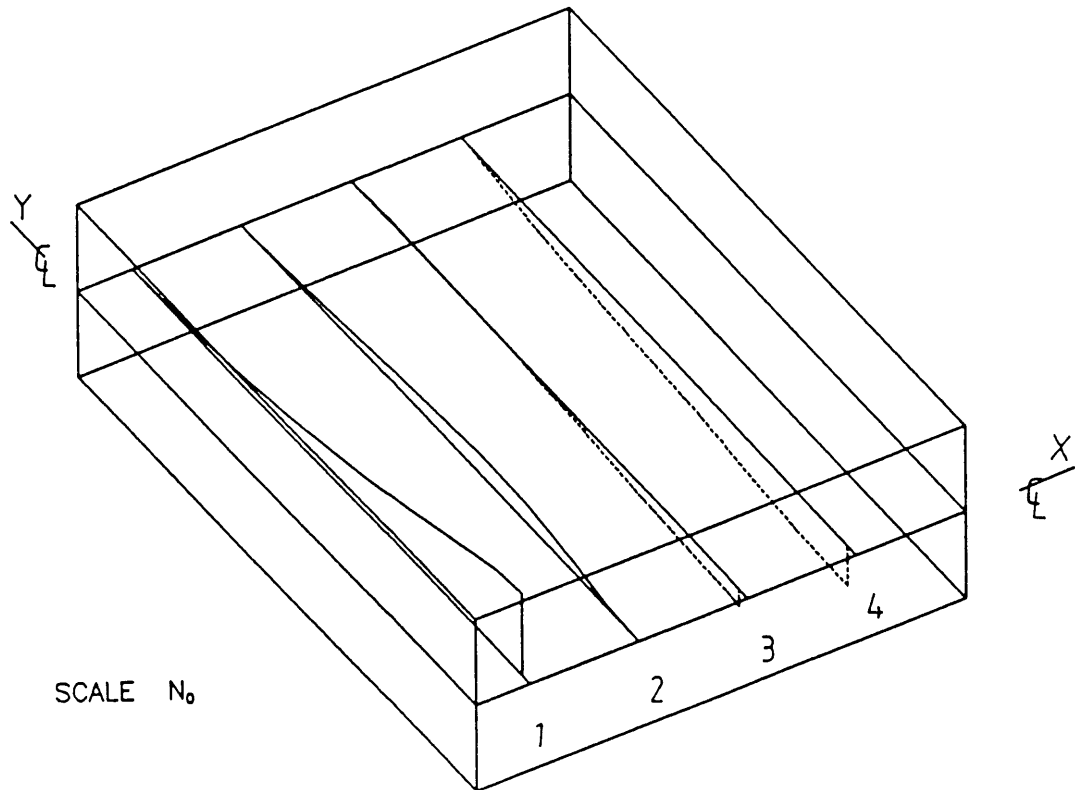


Fig. 7.17(d) Axial force along the notional beams in S1, at $\rho=7.356$

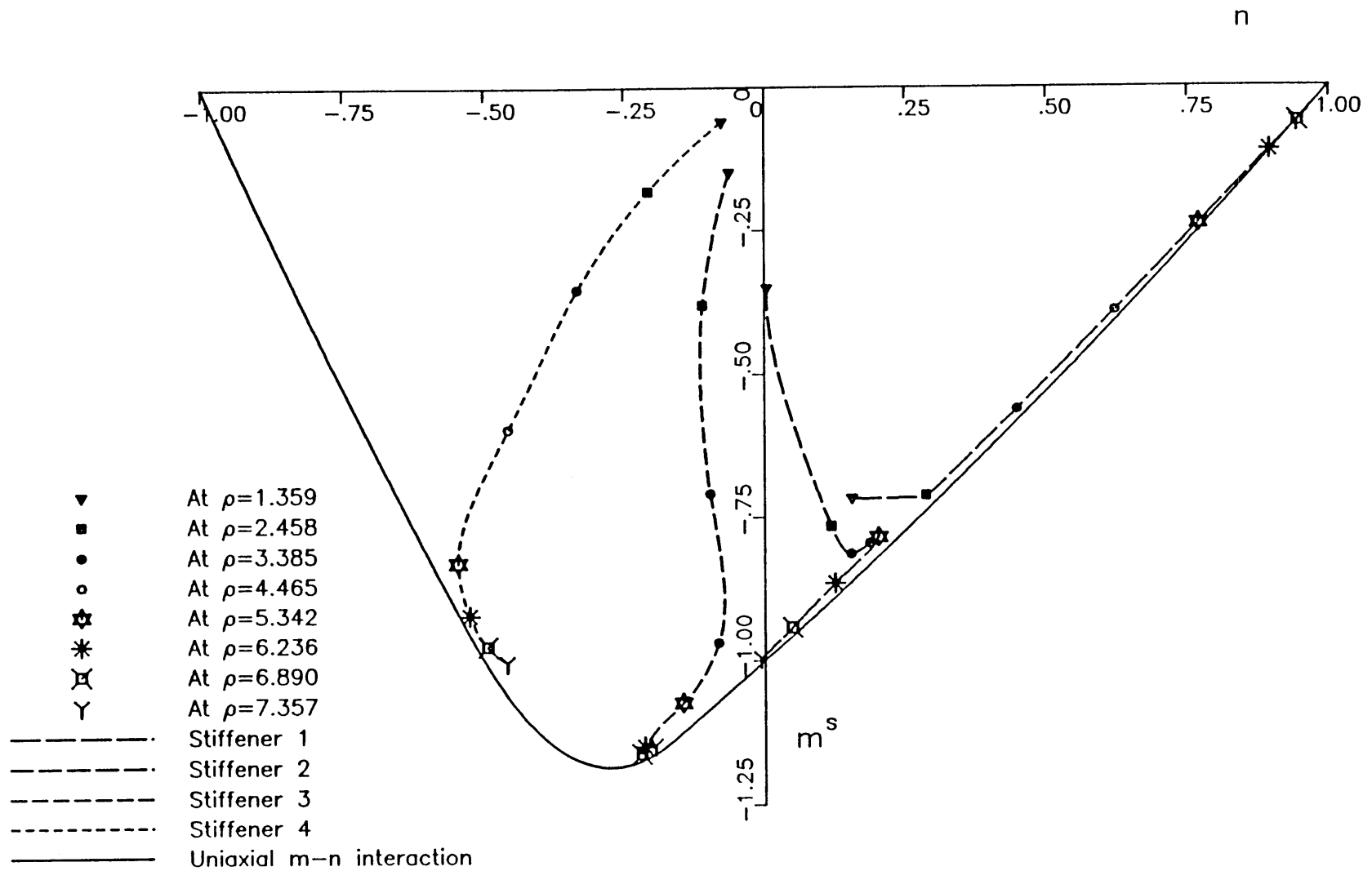


Fig. 7.18 Comparison between the stress resultants in deck S1 and the uniaxial m - n interaction for the notional beam cross-section

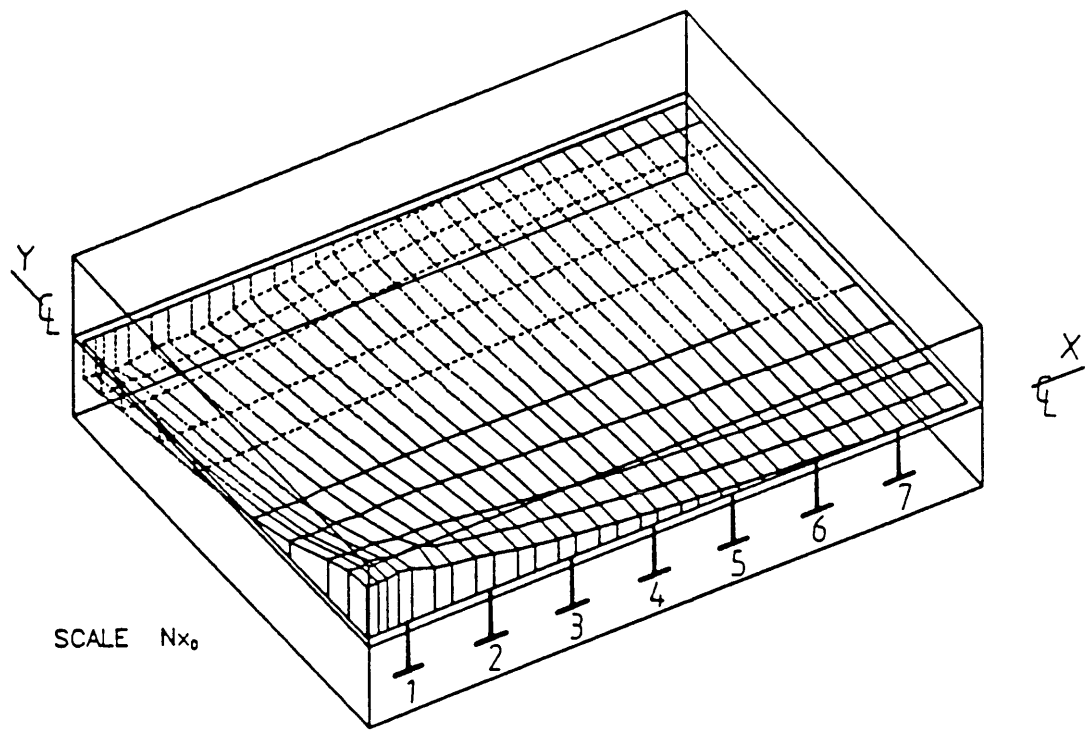


Fig. 7.19(a) Transverse membrane forces in deck plate S3, at $\rho=3.271$

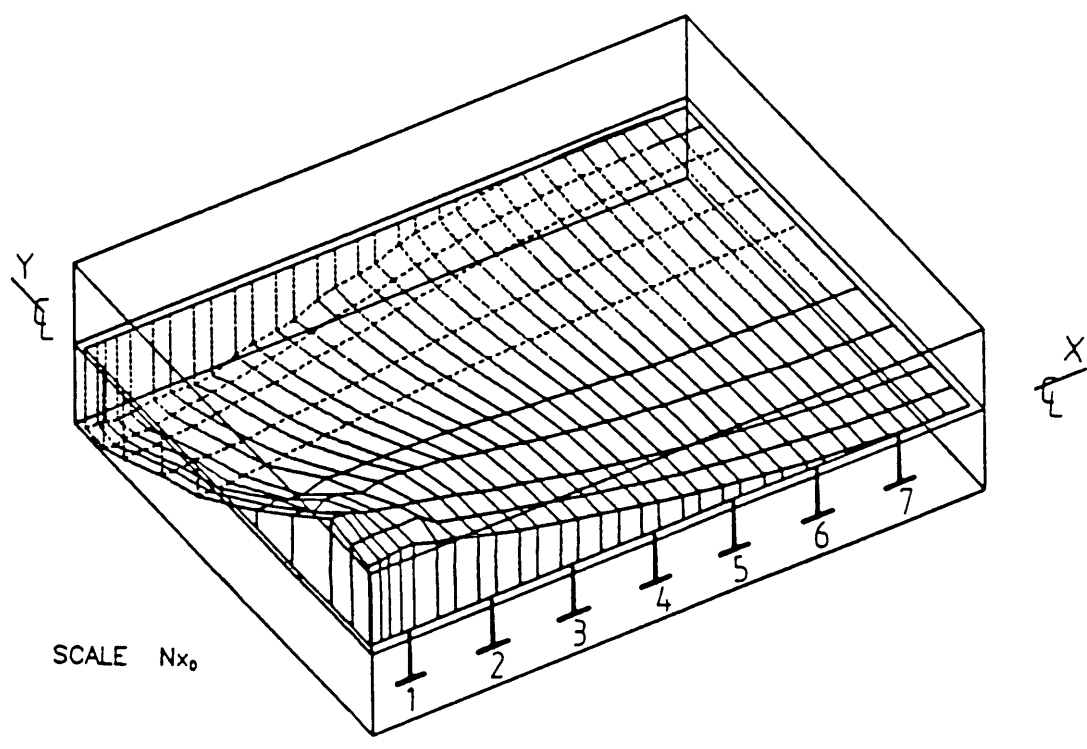


Fig. 7.19(b) Transverse membrane forces in deck plate S3, at $\rho=6.548$

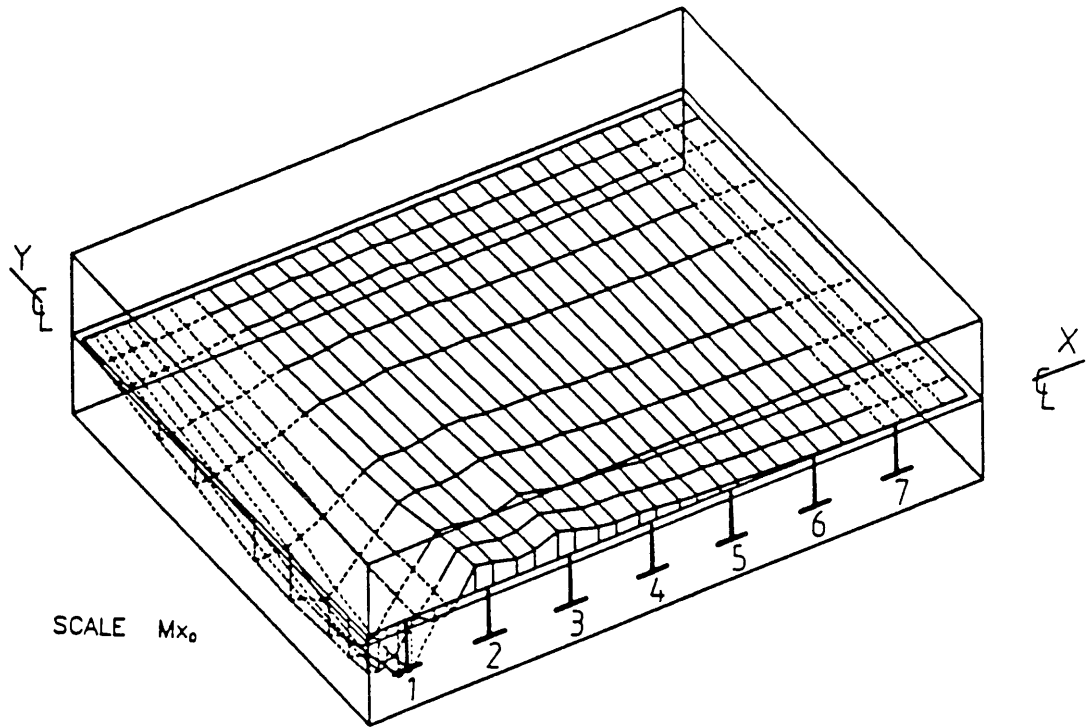


Fig. 7.20(a) Transverse bending distribution in deck plate S3, at $\rho=3.271$

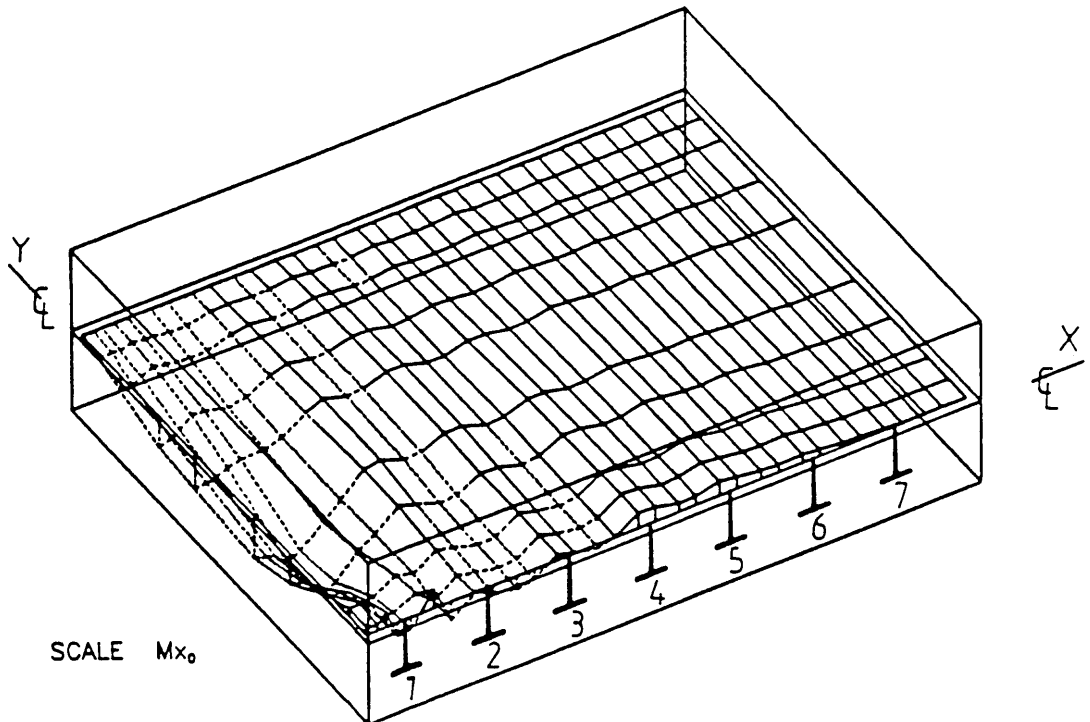


Fig. 7.20(b) Transverse bending distribution in deck plate S3, at $\rho=6.548$

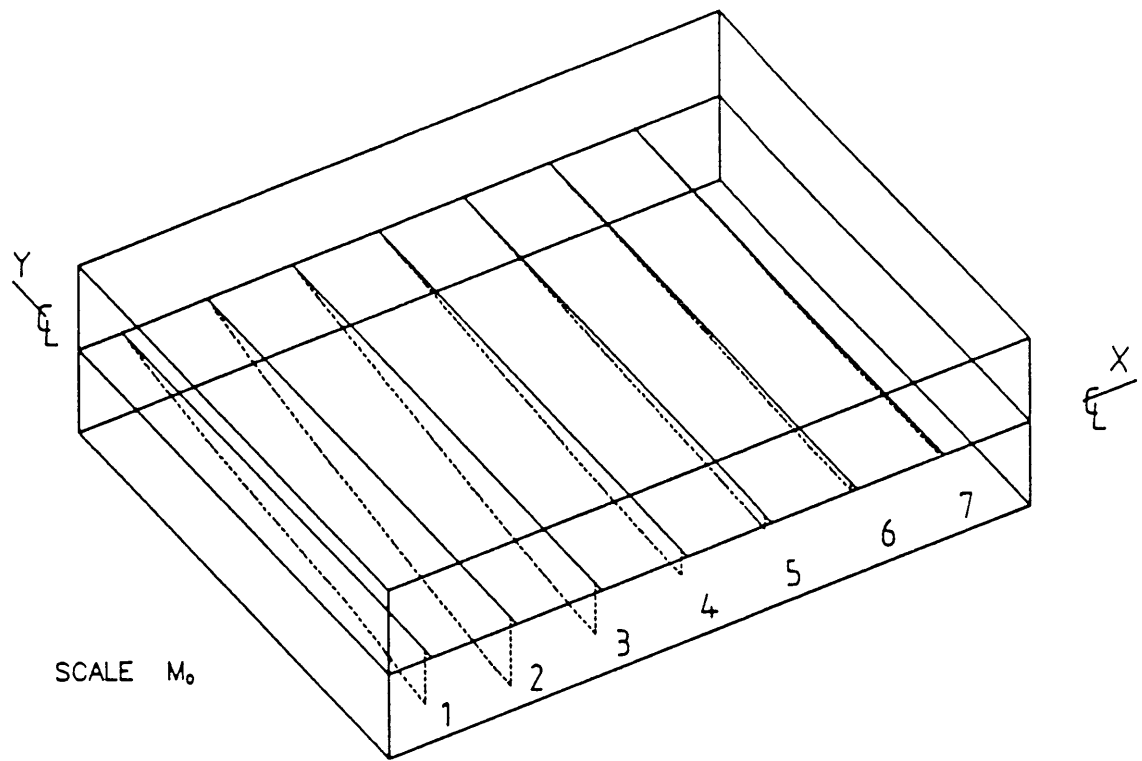


Fig. 7.21(a) Bending moment along the notional beams in S3, at $\rho=3.271$

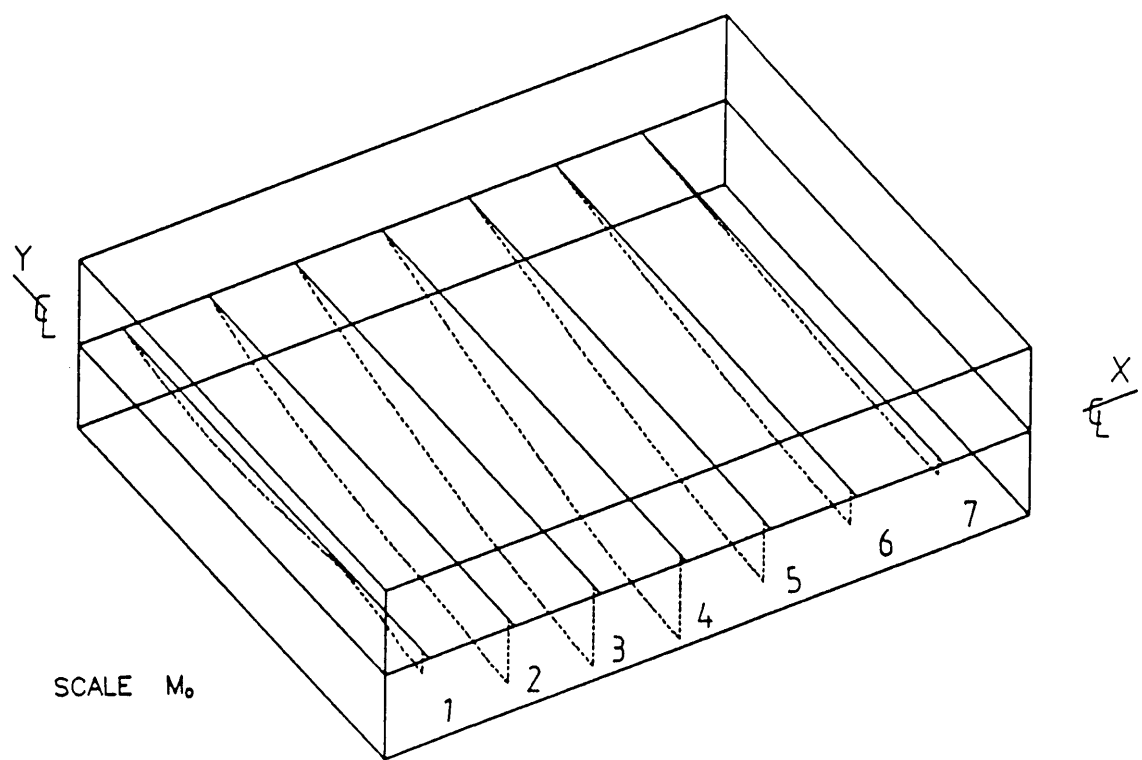


Fig. 7.21(b) Bending moment along the notional beams in S3, at $\rho=6.548$

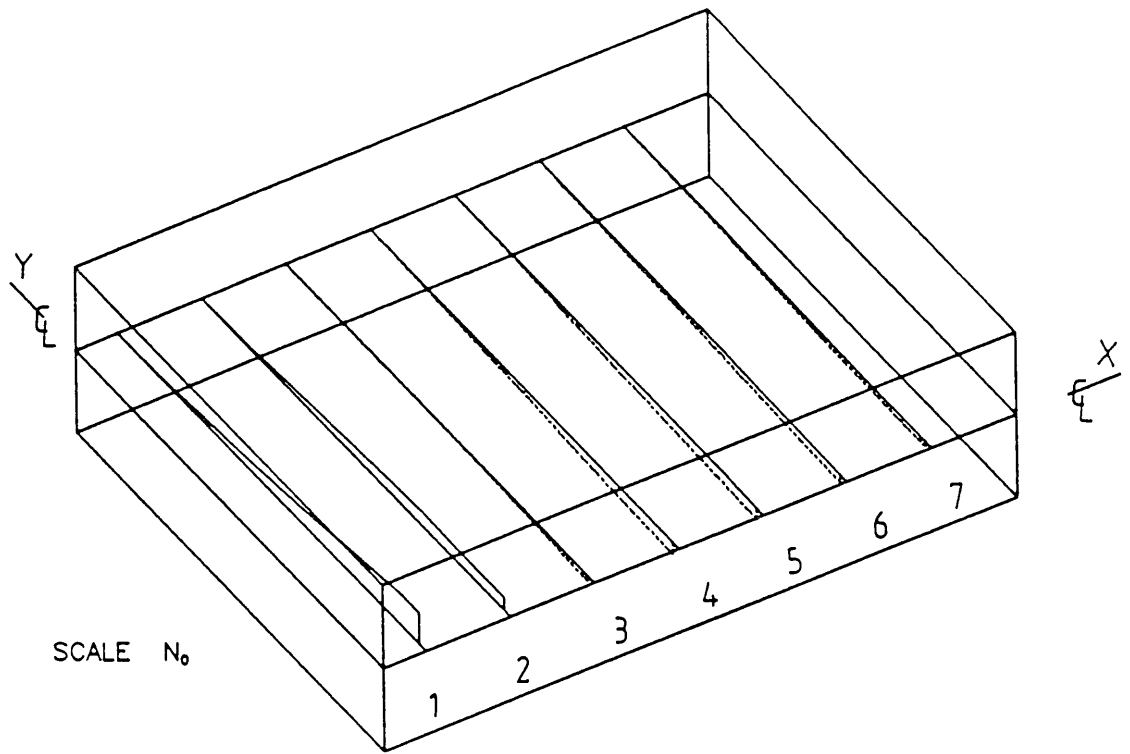


Fig. 7.22(a) Axial force along the notional beams in S3, at $\rho=3.271$

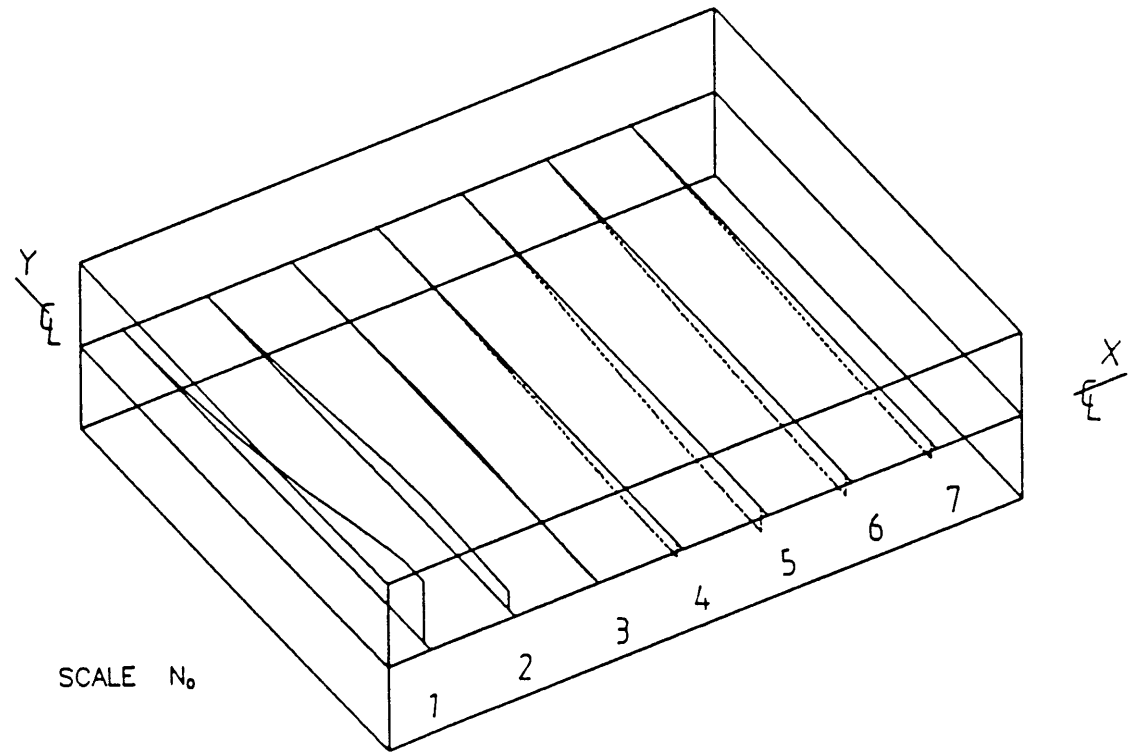


Fig. 7.22(b) Axial force along the notional beams in S3, at $\rho=6.548$

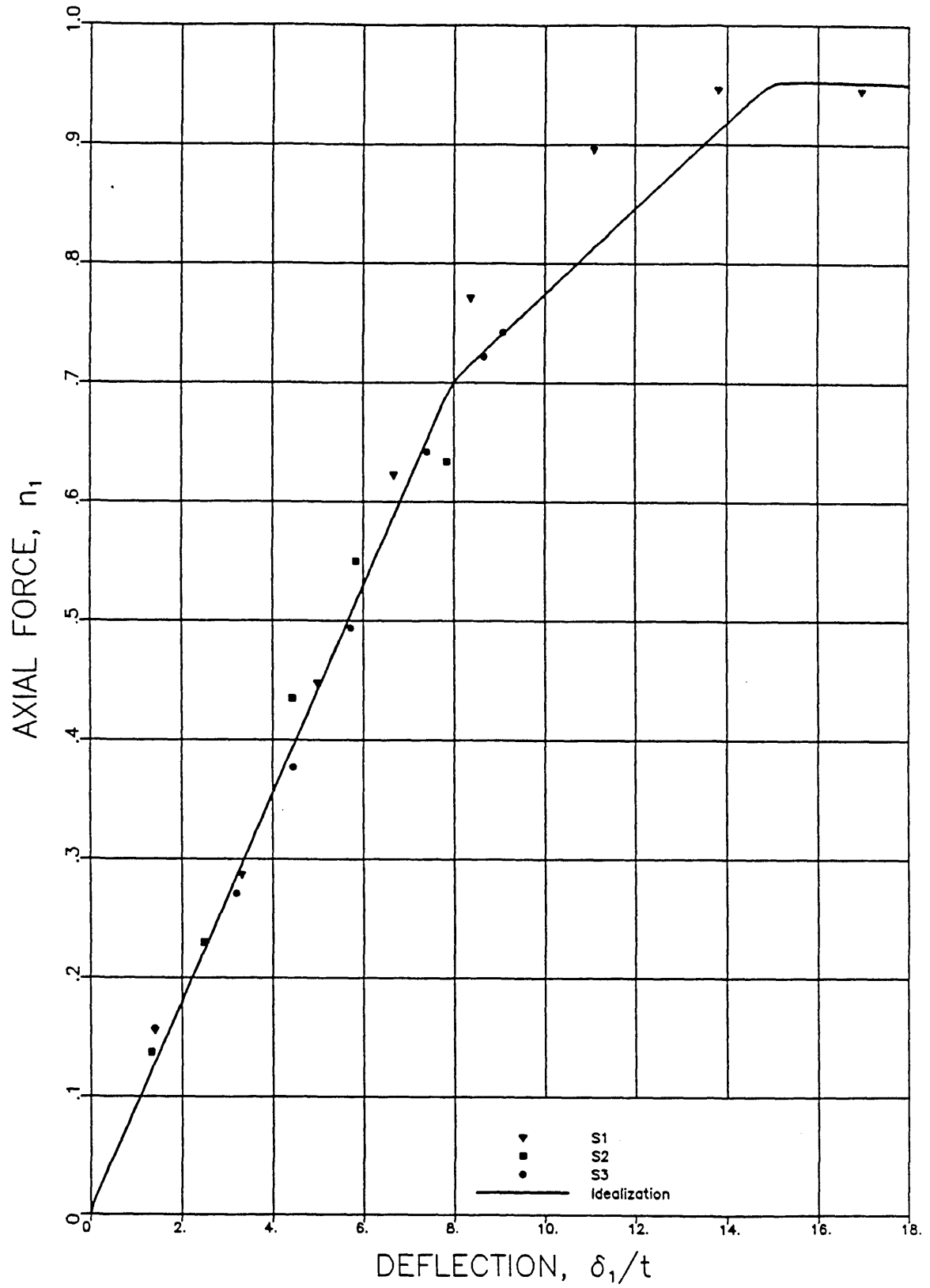


Fig. 7.23 Axial force in stiffener 1 with increasing deflection

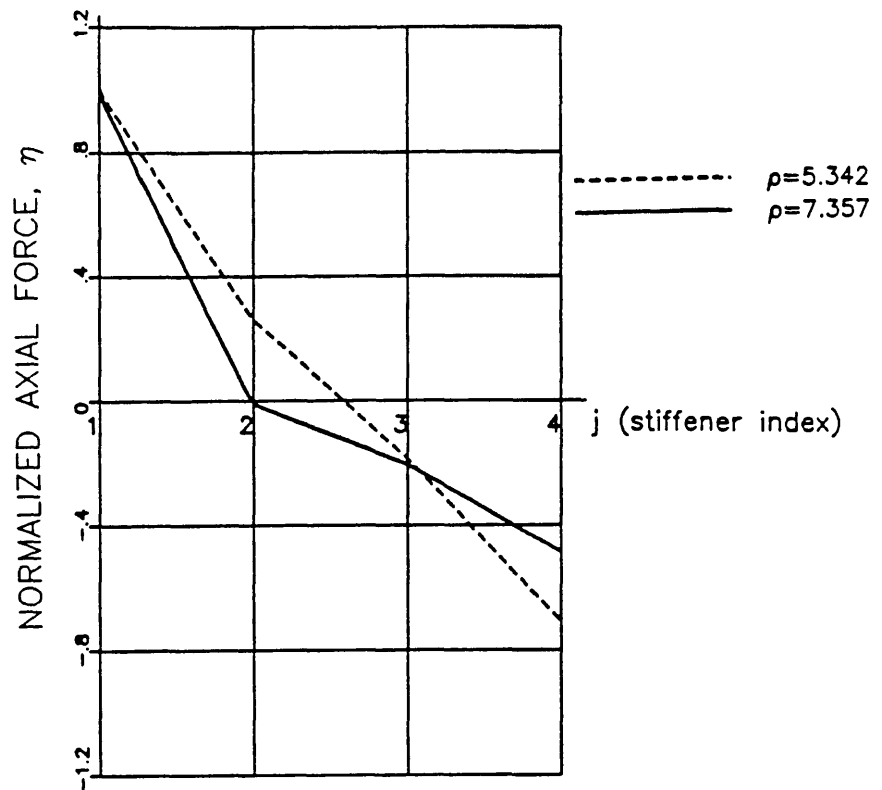


Fig. 7.24(a) Distribution of longitudinal membrane forces across the width in deck S1

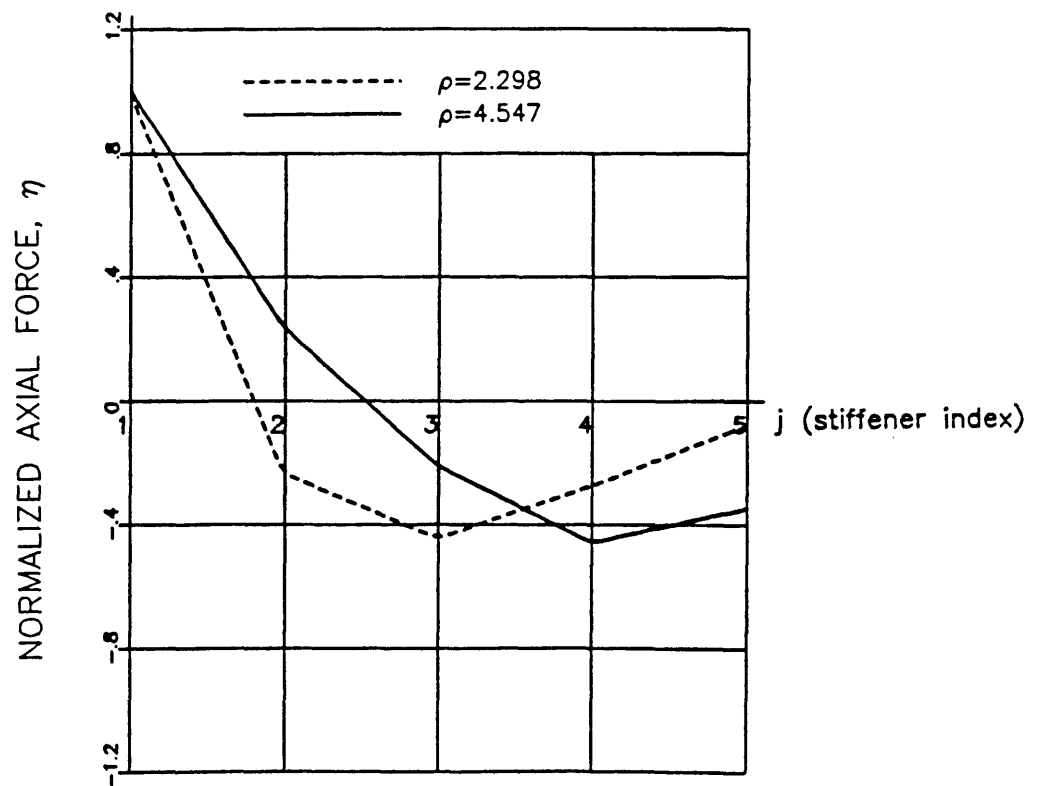


Fig. 7.24(b) Distribution of longitudinal membrane forces across the width in deck S2

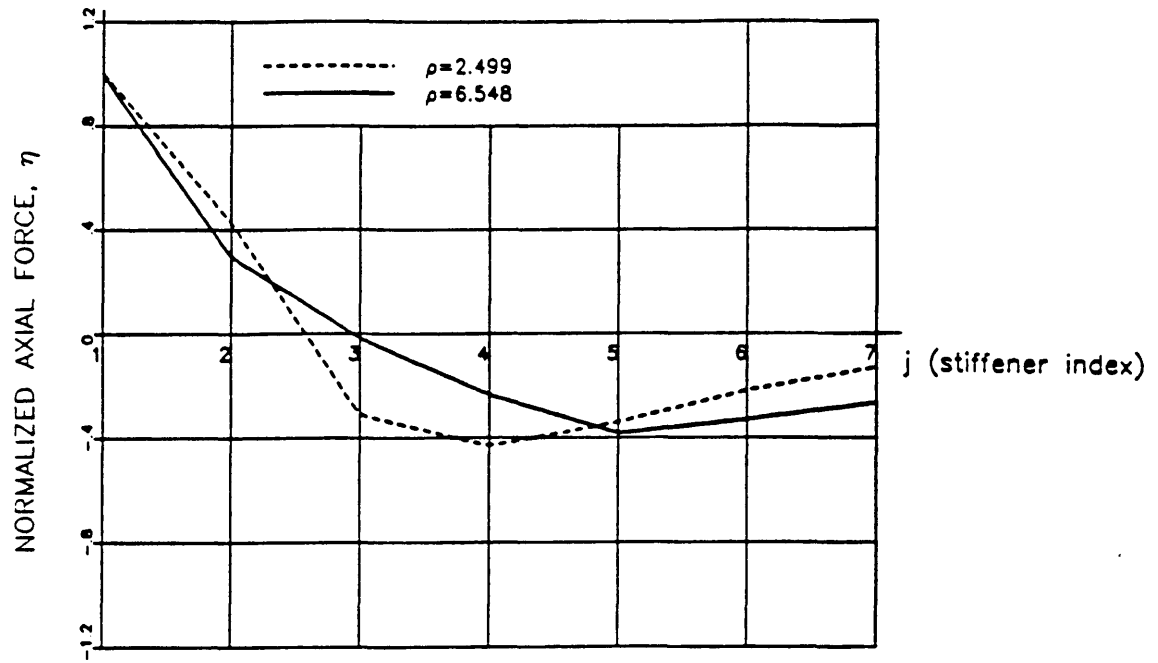


Fig. 7.24(c) Distribution of longitudinal membrane forces across the width in deck S3

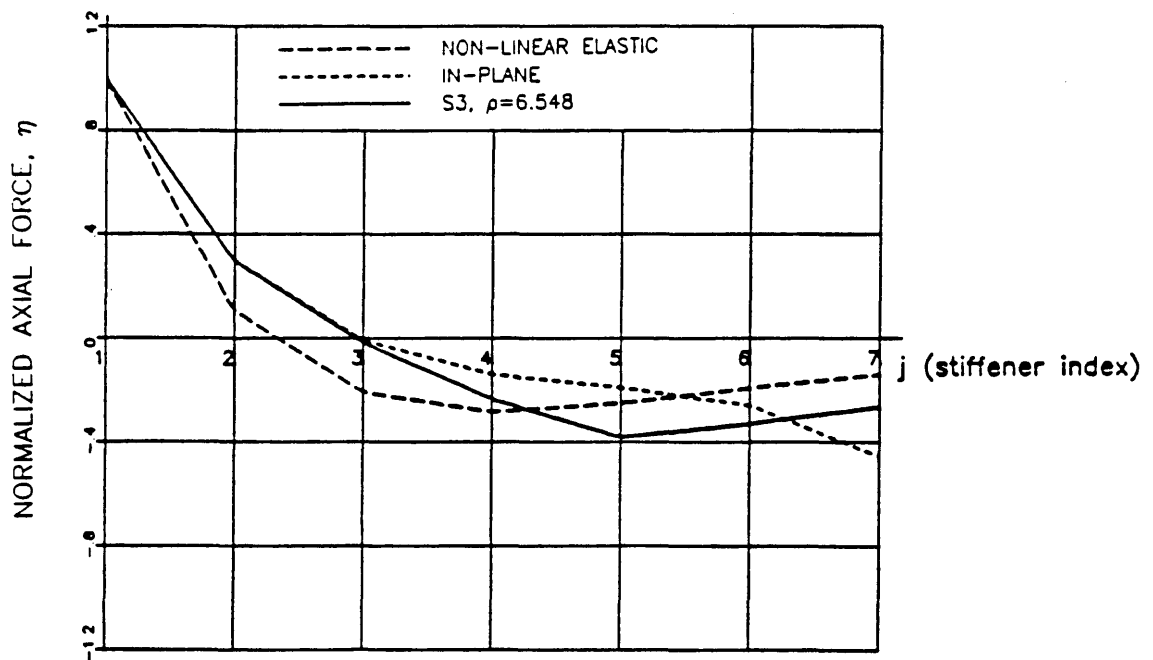


Fig. 7.25 Distribution of longitudinal membrane forces across the width in deck S3

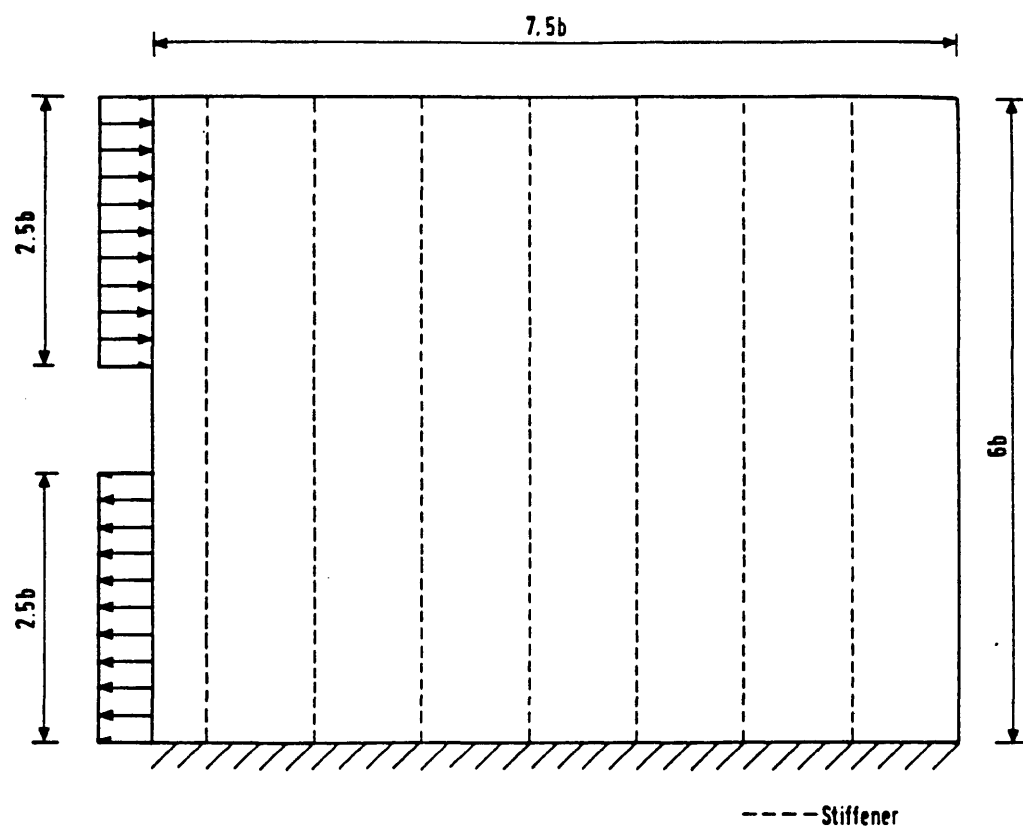


Fig. 7.26 Deck S3 under in-plane bending loading
analysed as a deep cantilevered beam

CHAPTER EIGHT

ANALYTICAL MODEL FOR UNRESTRAINED STIFFENED DECKS

8.1 INTRODUCTION

The behaviour of a single-bay longitudinally stiffened deck under lateral patch loading was studied in the previous chapter. An analytical model for this type of deck is proposed in this chapter.

The mathematical formulation is followed by a study of the model sensitivity to some of its parameters. As a result of this study, the parameters necessary for a successful prediction for other geometries are identified. In addition, a simplified version of the model similar to the one presented in Section 6.3 is proposed and its results are compared with numerical data obtained in the previous chapter. The closing section of the chapter is concerned with a discussion on the permissible strains which limit the mechanism analyses presented in this thesis.

8.2 PROPOSED MODEL

A model of the unrestrained simply supported bay comprises a plate strip running in the transverse direction and supported by springs which represent simply supported beams, as illustrated in Fig. 8.1. The strip is required to transfer loading transversely and can be considered as a membrane strip, anchored at the outer panels. Each beam comprises a stiffener and its associated effective plate, and forms a plastic mechanism under coexisting bending moment and axial force (at the central hinge). The stress resultants at such a hinge follow the uniaxial yield surface, discussed in Sections 7.2.4.7 and 7.3.2.5.

The principal phenomenon during the first phase of deck response discussed in the previous chapter, is the sequential failure of stiffeners across the width. During mode j only stiffeners with indices less than or equal to j experience deflection increments.

8.3 DEVELOPMENT OF ANALYTICAL MODEL

8.3.1 Assumptions

The following assumptions were utilized in formulating the problem:

- 1) The analysis is based on a rigid-plastic material law.
- 2) The stiffener spacing, b , is uniform.
- 3) The plate strip that transfers load to the stiffeners is assumed to follow the membrane solution and has a width of $2\beta_2 b$ and a thickness of t .
- 4) Under the action of the external loading, P , of patch width $2\beta_1 b$, applied at the centre of stiffener 1, the stiffeners collapse successively across the width.
- 5) The distribution of axial forces in the stiffeners across the width is assumed to be known.
- 6) The only interaction between adjacent stiffeners is through the membrane plate strip.
- 7) From the two phases identified in Section 7.3.2.6, only the first phase is considered in this model.

8.3.2 Equilibrium equations

8.3.2.1 Equilibrium of the loaded stiffener

Stiffener 1 is shown in Fig. 8.2(a) under the action of the applied patch loading and the vertical reaction of the transverse membrane plate strip. Consideration of equilibrium with respect to the line of support at the end of the bay gives:

$$- \frac{P \left(L - \frac{\beta_1 b}{2} \right)}{2} + \frac{\delta_1 - \delta_2}{b} t \sigma^t \beta_2 b \left(L - \frac{\beta_2 b}{2} \right) +$$

$$M_1^s + (\lambda - 1) M_1^h + \delta_1 N_1 = 0 \quad (8.1)$$

The limit load for a 'notional beam' under a centrally applied patch load of total width $2\beta_1 b$, obtained by the application of virtual work, is:

$$P_0 = \frac{2 \lambda M_0}{L - \beta_1 b/2} \quad (8.2)$$

Assuming that the stress in the transverse strip, σ^t , equals the yield stress, σ_y , and that the interaction formula between the stress resultants is given by Eqn. (4.3), equations (8.1) and (8.2) lead to

$$-\rho + \frac{1}{\lambda} \left[D_3 \left(\frac{\delta_1}{L} - \frac{\delta_2}{L} \right) + \frac{AL}{Z_p} (C_{21} n_1^2 + C_{31} n_1 + C_{41} + \frac{\delta_1}{L} n_1) \right] = 0 \quad (8.3)$$

where the coefficient D_3 is given by:

$$D_3 = \frac{\beta_2 t L (L - \frac{\beta_2 b}{2})}{Z_p}$$

Equation (8.3) is similar to Eqn. (6.3(a)) derived for the restrained bay, differing only to the latter by the coefficient D_3 . Both coefficients D and D_3 reflect the pattern of plate strip reaction along the stiffeners, corresponding to triangular and central patch patterns respectively.

8.3.2.2 Equilibrium of a stiffener away from the load

Figure 8.2 illustrates any stiffener not directly subjected to externally applied loading, (that is $j \neq 1$), but acted upon by the loads transferred by the transverse membrane strip. Consideration of equilibrium with respect to the line of the supports of the beams will lead to:

$$\frac{\delta_{j-1} - 2\delta_j + \delta_{j+1}}{b} t \sigma^t \beta_2 b (L - \frac{\beta_2 b}{2}) - M_j^s - (\lambda - 1) M_j^h - \delta_j N_j = 0$$

Using the coefficient D_3 , the interaction formula between the stress resultants at a plastic hinge as expressed by Eqn. (4.3) and the assumption that the membrane stress in the plate strip is at yield, the above equation can be rewritten as:

$$\begin{aligned}
& - D_3 \left[\frac{\delta_{j-1}}{L} - 2 \frac{\delta_j}{L} + \frac{\delta_{j+1}}{L} \right] \\
& + \frac{AL}{Z_p} (C_{2j} n_j^2 + C_{3j} n_j + C_{4j} + \frac{\delta_j}{L} n_j) = 0, \quad \text{for } j \geq 2
\end{aligned} \quad (8.4)$$

Equations (8.4) and (6.3(b)) are similar differing only in the definition of the coefficients D and D_3 .

8.3.2.3 Last stiffener in the model

In a deck with bay aspect ratio less or equal to one ℓ is the outer stiffener, while in a wide deck (aspect ratio greater than one) ℓ is the outermost stiffener contained in a square plan. An additional term can be assumed contributing to the strength of the outer stiffener in the series. This additional strength stems from the transverse action in the plate. It is assumed that the variation of the transverse bending across the outermost panel, assumed in the model, equals its plastic value. This transverse membrane action applies an upward transverse shear along stiffener ℓ , having magnitude equal to:

$$\frac{t^2}{4} \frac{\sigma_y}{b}$$

per unit length of the stiffener. Equilibrium of moments with respect to the transverse line of support results in:

$$\begin{aligned}
& \frac{\delta_{\ell-1} - \delta_{\ell}}{b} t \sigma_y \beta_2 b \left(L - \frac{\beta_2 b}{2} \right) - M_{\ell}^s - (\lambda - 1) M_{\ell}^h - \delta_{\ell} N_{\ell} - \\
& \frac{t^2}{4} \frac{\sigma_y}{b} \frac{L^2}{2} = 0
\end{aligned} \quad (8.5)$$

Introducing the coefficient

$$D_4 = \frac{t^2 L}{8 b A}$$

and following the same procedure as in deriving Eqn. (8.4), Eqn. (8.5) becomes:

$$- D_3 \left[\frac{\delta_{\ell-1}}{L} - \frac{\delta_{\ell}}{L} \right] + \frac{AL}{Z_p} (C_{2\ell} n_{\ell}^2 + C_{3\ell} n_{\ell} + C_{4\ell} + \frac{\delta_{\ell}}{L} n_{\ell} + D_4) = 0 \quad (8.6)$$

Inclusion of the term D_4 in Eqn. (8.6) with its full value may violate yield in some cases. This is because the panel to which this transverse bending distribution is applied, is also under longitudinal compression. Under such circumstances, it becomes more appropriate to reduce the contribution of the term D_4 by a certain percentage, in order for yield not to be violated anywhere in the outmost panel assumed in the model.

8.3.2.4 Solution of the governing equations

Equation (8.3), along with $(j-1)$ equations similar to (8.4) form a system of j simultaneous equations. There are $j+1$ unknowns (that is, the j stiffener deflections and the load factor ρ) and a solution can be obtained by increasing the value of one of them in steps. Increasing the value of δ_1 enables the calculation of the remaining unknown quantities during mode j .

At first, a mode is assumed, starting with mode 1. The value of δ_1 is increased, and a solution for the j unknowns considered in mode j is obtained. The solution arrived at is checked for mode violation. Considering the equilibrium equation for stiffener $j+1$, as given by Eqn. (8.4), the mode is not violated provided the left hand side of Eqn. (8.4) is non-negative. If the currently assumed mode j is violated, it is updated to mode $j+1$, and the above process is repeated. This solution strategy can be applied until collapse is attained of the outermost stiffener assumed in the model. In narrow decks, this marks the end of the first phase.

The resulting simultaneous equations can be solved using any of the available techniques. Reduction of the equations through Gauss elimination is probably the most appropriate technique, and was the approach used.

8.3.3 Prediction of forces in the beams

The force in the loaded stiffener ($j=1$) was found to vary with deflection following the piecewise linear curve, shown in Fig. 7.23. This relationship is used in

association with different patterns of axial force distribution across the width, as those shown in Figs 7.24 and 7.25, to define the actual forces in the stiffeners. The axial force n_j in stiffener j is obtained as:

$$n_j = n_1 \eta_j$$

Here η_j is the value of the axial force in stiffener j normalized with respect to the axial force in stiffener 1. Values for the η_j should be taken from the analyses undertaken in Chapter 7, as discussed further in Section 8.5.

8.4 COMPARISON OF THE ANALYSIS WITH NUMERICAL RESULTS

In this section, the predictions of the proposed analytical model are compared with the numerical analyses and test results presented in Chapter 7. The sensitivity of the model to the various parameters involved is also investigated.

8.4.1 Load versus deflection of the loaded stiffener

8.4.1.1 The influence of plate effective width

The load histories of the loaded stiffeners in decks S1, S2 and S3 are compared and the sensitivity of the predictions to the assumed value for the effective width of the transverse membrane strip is examined. Five values of β_2 were chosen for the comparison, corresponding to effective width values in the range from b to $3b$. The distributions of membrane force across the width are those given by continuous lines in Figs 7.24(a), 7.24(b) and 7.24(c) for decks S1, S2 and S3 respectively.

Figure 8.3 shows the predictions of the analytical model for deck S1. An increase in the value of β_2 gives a stiffer response, but also reduces the maximum deflection at the end of the first phase considered here. This is because higher β_2 values facilitate the rapid spread of the loading across the width. Comparison of the predictions with the numerical results shows that a value of β_2 between 1 and 1.25 is the most appropriate. It is remarkable how similar in shape the finite element prediction and the curve corresponding to $\beta_2=1.25$ are. The deflection predicted by the model, which is based on a rigid-plastic material law, should always be less than the deflection obtained by elasto-plastic finite element analysis.

Referring to Fig. 8.3, it can be seen that the numerically predicted curve can be divided into two parts. The first part corresponds to the first phase, while the change into the second phase occurs at a deflection of $\delta_1/L \approx 0.05$. It is reiterated that the scope of analytical predictions is limited to the first phase. The second phase is, however, important for narrow decks only, having few stiffeners across the width.

In Fig. 8.4, comparison of analytical, numerical and experimental results for deck S2 is made. The analytical predictions in Fig. 8.4 indicate a softer response throughout the β_2 range, when compared with the finite element and test results. This is because of the presence of the cross-girder in both the test and the numerical model, which applies a significant hogging moment at the supports of the stiffener 1 (as shown in Figs 7.11). Therefore, direct comparison is not strictly possible. Alternatively, two curves for each β_2 value, could have been presented. The first would assume pin supports, which is in fact the assumption made for the curves in Fig. 8.4, whilst the other would consider rotationally fixed supports. Both numerical and test results would lie between the two predictions. The range of the elasto-plastic model proposed by Cox (1953) is also shown in Fig. 8.4. As can be seen, the model proposed in this thesis is valid over a much wider range of deflections.

Finally, comparison between the model and the numerical results for deck S3 is made in Fig. 8.5. The finite element analysis, labelled as S3, did not continue into the very large deflection range and therefore the prediction presented is only part of the full response.

From all three comparisons, presented in this section, it is evident that β_2 is a very important parameter affecting the results of the proposed model. Cox (1953) proposed a value of $\beta_2 \approx 0.50$, which seems rather low in the light of the numerical analyses. Dowling (1968) suggests a value for β_2 between 1.0 and 1.5. A value of $\beta_2 \approx 1.0$ is advocated as a starting value.

8.4.1.2 The influence of the assumed membrane force distribution

Various distributions of the membrane forces across the width of the deck were presented in Chapter 7 for deck S3. The sensitivity of the model to those distributions is investigated next.

Figure 8.6 presents the behaviour predicted by the model for $\beta_2=1.0$, based on the four distributions of the membrane forces across the width of the deck, shown in Figs 7.24(c) and 7.25. The model seems rather insensitive to these distributions. This opposes the argument, made in the previous chapter, that accurate representation of the distribution of the membrane forces is central to a successful prediction of the deck behaviour. However it should be noted that all the distributions assume the same axial force history in the loaded stiffener. As the actual distribution across the width is not very important, it is recommended that the simplest method, that is the 'in-plane' prediction, should be used to obtain the η_j coefficients for other deck aspect ratios.

8.4.2 Relative stiffener deflections

The deflections of adjacent stiffeners are given in Figs 8.7 to 8.10. This comparison helps in understanding the growth of stiffener central deflections across the width.

The first two figures correspond to deck geometry S1. Figure 8.7 illustrates the deflection of stiffener 2 relative to that for stiffener 1. Three model predictions corresponding to values for β_2 of 0.75, 1 and 1.25 are given. Comparison with the finite element results shows agreement with slopes of 1:2 and 2:3, indicating pivoting of the two stiffeners with respect to stiffeners 3 and 4 respectively. It is also shown, from the numerical results, that the second phase results in both stiffeners increasing their deflections equally.

Figure 8.8 shows the deflections for stiffeners 2 and 3 for the same three β_2 values. The slope of 1:2 is achieved during mode 3, while both stiffeners pivot with respect to stiffener 4.

Similar comparisons are made in Figs 8.9 and 8.10 for the geometry of deck S3. Four values of β_2 are used and, in this case, $\beta_2=1.50$ seems the most appropriate. The higher value for the plate effective width in S3 compared to S1 has been also observed in Chapter 7. It can be argued that the higher the value of β_2 , the more rapid the outward spreading of the load, and consequently, the first phase ends at lower deflections.

The distributions of the membrane forces across the width, used in Figs 8.7 to 8.10, were identical with those used for obtaining the load histories in Figs 8.3 and 8.5, for decks S1 and S3 respectively.

8.5 REMARKS ON THE RANGE OF APPLICABILITY OF THE PROPOSED MODEL

For the prediction of the behaviour of a deck with a geometry differing from those studied, the distribution of the axial forces across the width is needed. These forces can be obtained using the axial force history for the loaded stiffener and the normalized axial force values, η_j , for the other stiffeners.

Following the discussion in Section 8.4.1.2, it can be argued that the actual η_j values are not crucial. Therefore, values from Figs 7.27(a), 7.27(b) and 7.27(c) can be used for decks with aspect ratios less than one, one and greater than one, respectively.

To determine how the axial force develops in the loaded stiffener under increasing central deflection, a parametric study needs to be undertaken. Two parameters should be varied. One is the panel slenderness and the other is the panel aspect ratio. The stiffener size and the bay aspect ratio should be invariant. It is not necessary for this parametric study to be very detailed as the proposed analysis is approximate. Three values for each of the parameters involved in the study are deemed to be sufficient to cover the range of values for these parameters met in practice. Having this information, the behaviour of any geometry can be predicted.

There is an additional restriction for decks with aspect ratios greater than one (wide decks). Figure 8.11 shows the stress blocks of the in-plane forces on the quarter deck considered. A maximum value of

$$\frac{t L^2 \sigma_y}{4}$$

for the in-plane moment acting at the narrow side of the deck can be assumed. A reduction must be imposed to the allowable in-plane moment if the plate slenderness is such that the compression on the narrow side cannot reach this value.

8.6 DEVELOPMENT OF A SIMPLIFIED ANALYTICAL MODEL

The preceding analysis requires knowledge of the axial force growth in the loaded stiffener with increasing deflection. As explained in Section 8.5 information about the axial forces in the beams can be obtained through a parametric study. However, it is of interest to investigate the predictions obtained from the preceding analysis assuming that the axial forces in the beams may be ignored. A similar simplification has also been proposed for the restrained bay in Section 6.3.

8.6.1 Modification of the preceding analysis

In the following analysis the assumptions listed in Section 8.3.1 are still valid. However, since the axial forces in the stiffeners are assumed to be zero, assumption 5 is no longer relevant. The equilibrium equations developed in Sections 8.3.2.1, 8.3.2.2 and 8.3.2.3 are still valid and can be further simplified as follows:

Equation (8.3), which is for the loaded stiffener ($j=1$), becomes:

$$-\rho + \frac{1}{\lambda} D_3 \left(\frac{\delta_1}{L} - \frac{\delta_2}{L} \right) + 1 = 0 \quad (8.7)$$

Equation (8.4), which is for a stiffener remote from the applied load, is simplified to:

$$-D_3 \left[\frac{\delta_{j-1}}{L} - 2 \frac{\delta_j}{L} + \frac{\delta_{j+1}}{L} \right] + \lambda = 0 \quad (8.8)$$

Equation (8.6), which is for the outmost stiffener ℓ involved in the model, can be written as:

$$-D_3 \left[\frac{\delta_{\ell-1}}{L} - \frac{\delta_{\ell}}{L} \right] + \lambda + \frac{A L}{Z_p} D_4 = 0 \quad (8.9)$$

The above equilibrium equations can be solved in a simpler way than that described in Section 8.3.2.4. This simplified approach has been described in Section 6.3 for the restrained deck.

Use of the technique described in Section 6.3, requires estimation of the critical deflection, δ_{jcr} , experienced by stiffener j , which has formed a mechanism, necessary to cause the formation of a plastic mechanism in stiffener $j+1$. The critical deflection can be obtained from Eqn. (6.7) after replacing coefficient $D_{j,j+1}$ by D_3 .

Alternatively the critical deflection can be obtained from the simplified Eqn. (8.8), written for stiffener $j+1$. If the deck is further assumed to be flat, without initial geometric imperfections, Eqn. (8.8) results in:

$$\frac{\delta_{jcr}}{L} = \frac{\lambda}{D_3}$$

The increment in deflection δ_{jcr} is experienced by stiffener j between the formation of mechanisms in stiffeners j and $j+1$. When the latter stiffener has just formed a mechanism, the total load carried by the deck is $(j+1)P_0$. During mode j , characterized by incremental deflections in j stiffeners only, the increments in the deflections of stiffeners 1 to j lie on a straight line pivoting with respect to stiffener $j+1$. This results in total incremental deflections during mode j , of $j\delta_{jcr}$ in stiffener 1, $(j-1)\delta_{jcr}$ in stiffener 2, $(j-2)\delta_{jcr}$ in stiffener 3, etc.

8.6.2 Discussion of the results

Results obtained by the use of the simplified analysis are compared with numerical and previous analytical results. For this comparison, only two values for the effective width of the transverse membrane forces, β_2 , are considered. Figures 8.12, 8.13 and 8.14 illustrate the comparison for the deck geometries S1, S2 and S3 respectively. The maximum load which is predicted by the simplified analysis is equal to the sum of the limit loads of the stiffeners involved in the model. Initially, the simplified analysis overestimates slightly the strength. This is more evident in Fig. 8.13. The overestimation occurs as the assumed axial force in the loaded beam, N_1 , reduces the sagging moment, M_1^S , more than the moment expressed by the term $N_1\delta_1$ enhances the strength, in Eqn. (8.1). The reduction in the sagging moment can be seen from Figs 7.13 and 7.18. As deflection increases further, the contribution of term $N_1\delta_1$ to the equilibrium equation (8.1) overcomes the reduction in the sagging moment, M_1^S . Therefore, the analysis of Section 8.3 predicts higher strength than its simplified version.

The simplified analysis can be used to obtain a conservative estimation of the load-deflection behaviour for the unrestrained bay compared with the behaviour predicted by the analysis of Section 8.3. It is also more attractive to the latter since knowledge of the axial forces in the beams are not needed.

8.7 LIMITATIONS DUE TO ALLOWABLE STRAINS

A further limitation to the analyses presented in this thesis based on a mechanism approach is the magnitude of allowable strains. From the analyses presented, it can be concluded that stiffened plates under lateral patch loading exhibit a stable load path even at very large deflections. This could happen in a real structure, provided that the deck plate does not fail in the compressive zones and the rupture strain is not exceeded. In the following a brief discussion of the latter is presented.

It is first assumed that the maximum strain can be achieved under ideal conditions. A flawless steel coupon in a tensile test ruptures at strains of 0.2~0.3. This strain is gauge length dependent, as McDermott et al (1974) remark. They suggest a local strain of 0.49 at rupture. This is corroborated by test results, undertaken by Ronalds (1985, 1990), showing rupture strains of the order of 0.80 for welded specimens. Therefore, it can be argued that under ideal conditions very large rupture strains can be achieved and that the rupture strain obtained from a tensile test is a minimum value.

On the other hand, defects in the weld metal, low ambient temperatures and high rates of strains can severely curtail the aforementioned ductilities, as Valsgård & Pettersen (1982) and Jones (1984) state. It is important to examine the significance of the above three factors in relation to the mechanism analysis.

Defects in the weld metal can cause premature fracture. As nothing is flawless, a crack of a given size may be assumed. The methods of Fracture Mechanics can be employed to indicate whether a crack is critical. If the flaw is of considerable size the member could fracture before developing a mechanism. In a welded structure, therefore, it is more important to determine whether a mechanism will develop in a member, rather than whether the member would fracture after a mechanism has developed.

The significance of fracture in a member depends on redundancy and reliability considerations. In a system with members assembled in series, failure of one of

the members causes global collapse. Parallel systems will benefit from redundancy so that fracture in some of the members that make up an assembly, allows the remaining members to develop mechanisms. It can be concluded, therefore, that for structures made up by welding, the number of members which develop mechanisms is influenced by Fracture Mechanics considerations.

Low temperatures can alter the energy absorbed during rupture, so that steels which are ductile at 'normal' room temperatures can become brittle at 'low' temperatures. It is part of the designer's task to select an appropriate high toughness steel which performs in a sufficiently ductile energy-absorbing manner. This was achieved, in the past, by specification of the amount of energy stored at a given temperature during a Charpy-V test, and more recently by specification of the fracture toughness.

Steel and titanium are notorious for their strain rate sensitivity. High rate of strain causes increase of the yield stress to its dynamic value with simultaneous reduction in ductility. Although effort has been expended in mathematically quantifying the dynamic yield stress as a function of the applied rate of strain (Symond (1965)), there is no equivalent analysis for the strains causing rupture. This may indicate that the reduction in the strain causing rupture is balanced by an increase in the yield stress and thus, the energy absorbed is not significantly different.

It can be argued that in the absence of a flaw of considerable size in a welded structure, the task of the designer is to ensure rupture strains of the order of those obtained from a tensile test.

8.8 CONCLUSIONS

A model for the prediction of the large deflection behaviour of simply supported stiffened decks which are free to pull-in under centrally applied patch loading has been presented. This model was analytically developed including the axial forces in the longitudinal stiffeners. Comparison of the predictions with numerical and experimental results showed a fair agreement. It showed also that the assumed value for the effective width of the transverse tensile forces in the plate affects significantly the results. On the other hand, the distribution of axial force in the stiffeners is not very important for the model. However, the relationship between the axial force and the deflection in the loaded stiffener is needed for use in the analysis of Section 8.3. This relationship can be obtained by a limited parametric study where the panel slenderness and aspect ratio are varied over the range of

practical importance.

A simplified version of the analytical model has also been presented. This analysis assumed no axial forces in the longitudinal stiffeners and gave slightly unconservative results initially. At higher loads conservative results were obtained compared with the results of the more detailed analysis. It is a very simple analysis and enables the load-deflection for the loaded stiffener to be predicted. Either analysis can be used up to a load level at which strains would limit the development of the assumed mechanisms.

Rupture strain values, obtained from tests on steel coupons, can be used to estimate limiting maximum strains developed in a mechanism analysis. Provided the rupture strains are known, the designer can ensure the development of the mechanism.

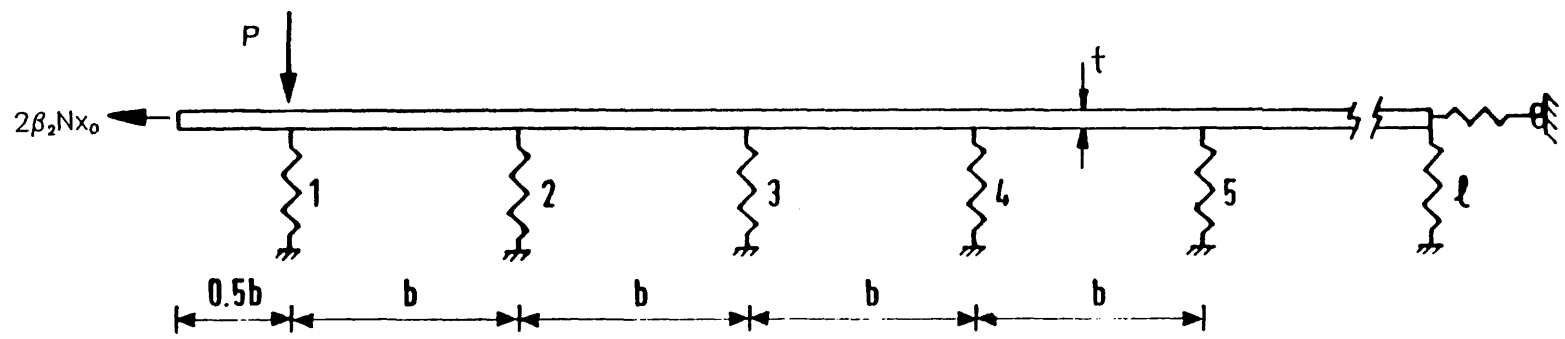


Fig. 8.1 Proposed model for deck free to draw-in

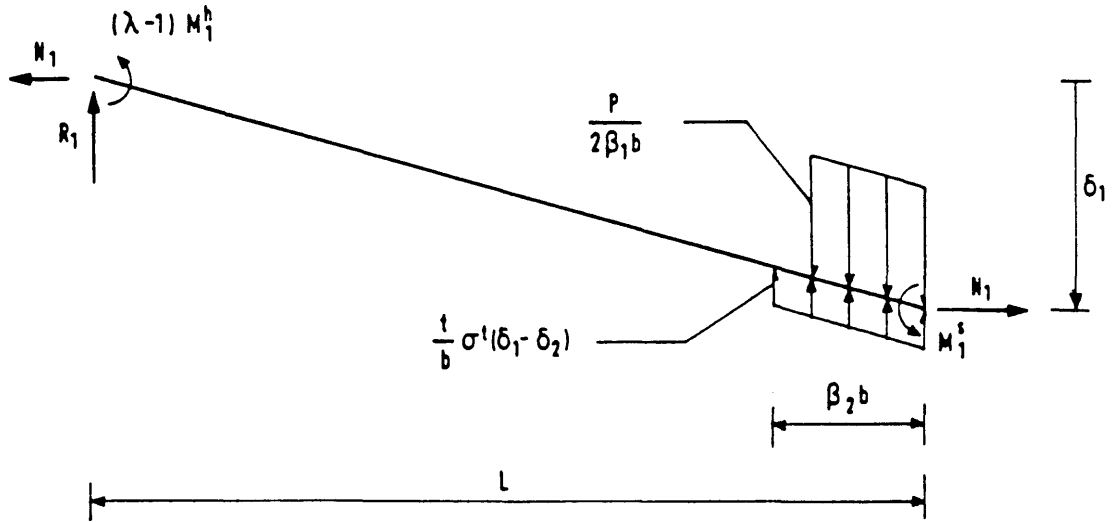


Fig. 8.2(a) Forces acting on stiffener 1

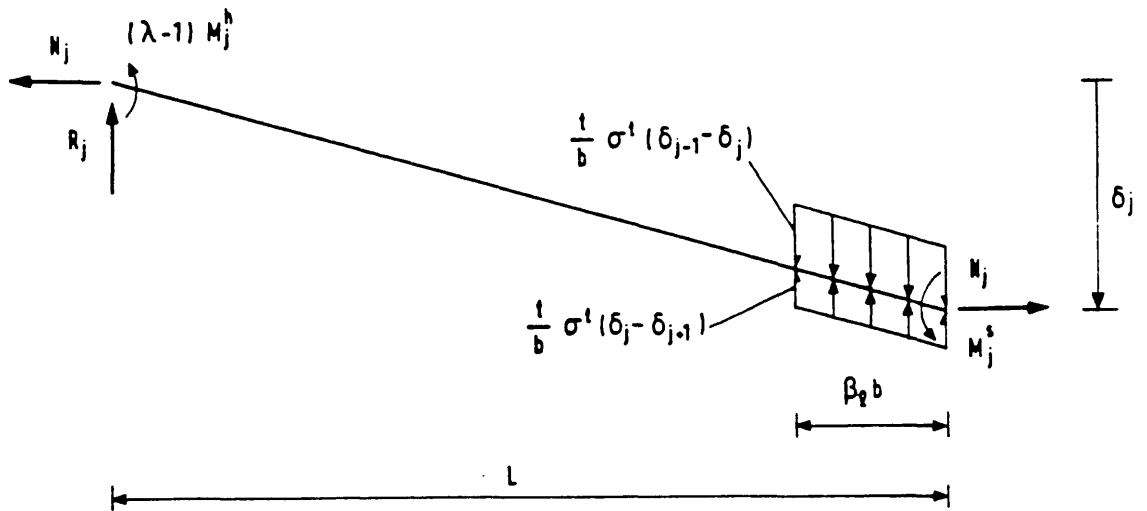


Fig. 8.2(b) Forces acting on stiffeners other than 1

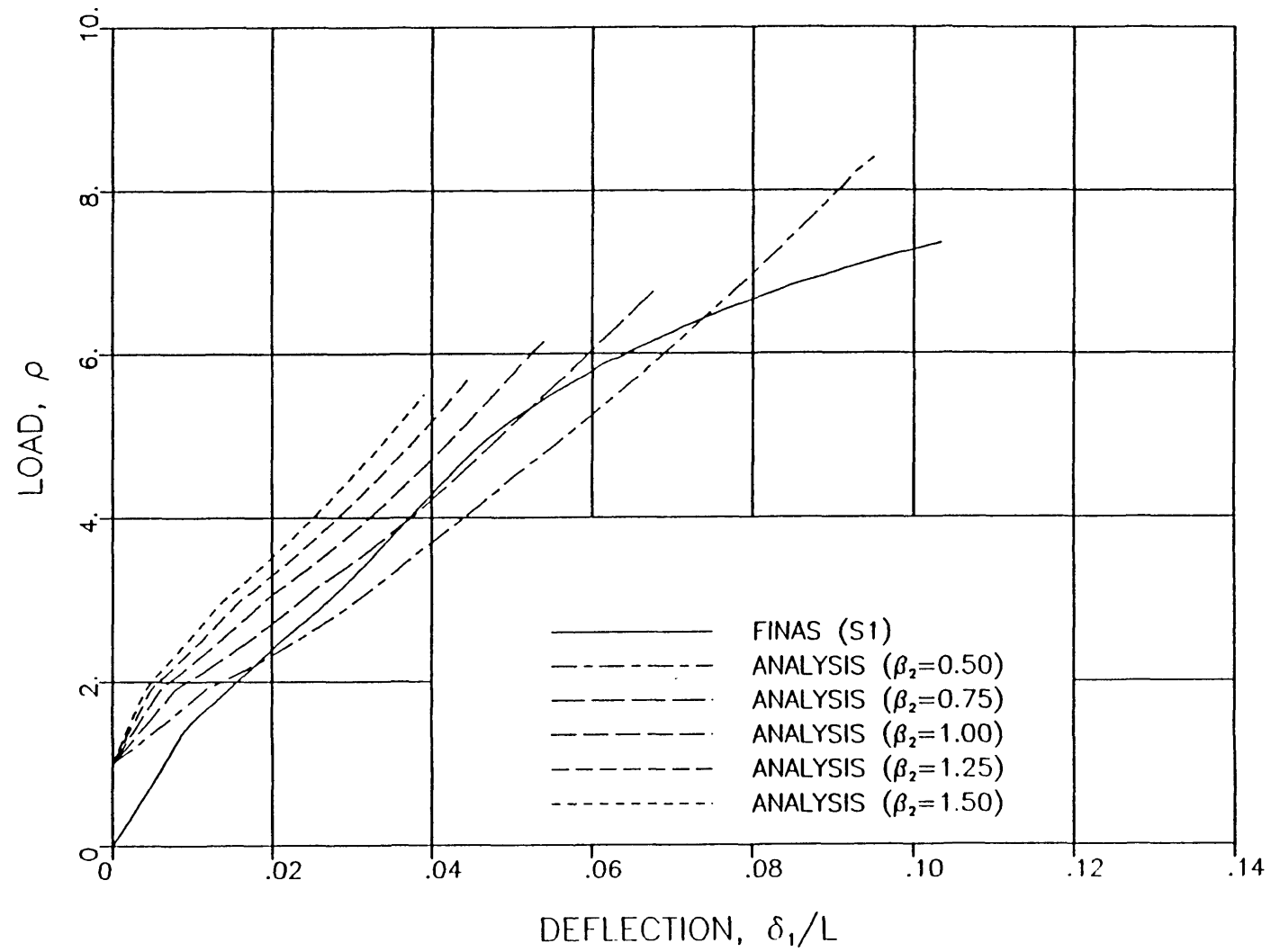


Fig. 8.3 Load vs. deflection for stiffener 1, deck S1

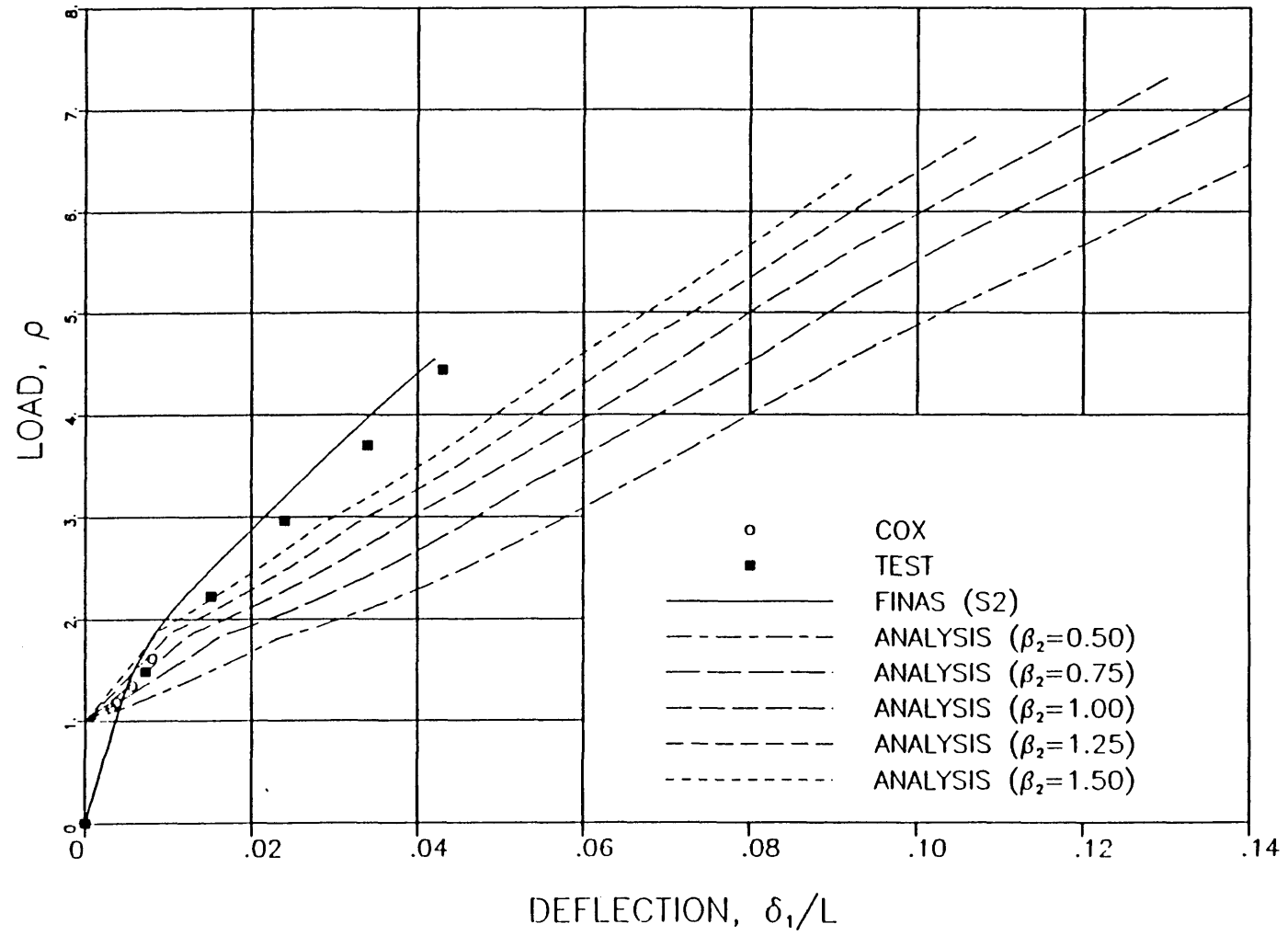


Fig. 8.4 Load vs. deflection for stiffener 1, deck S2

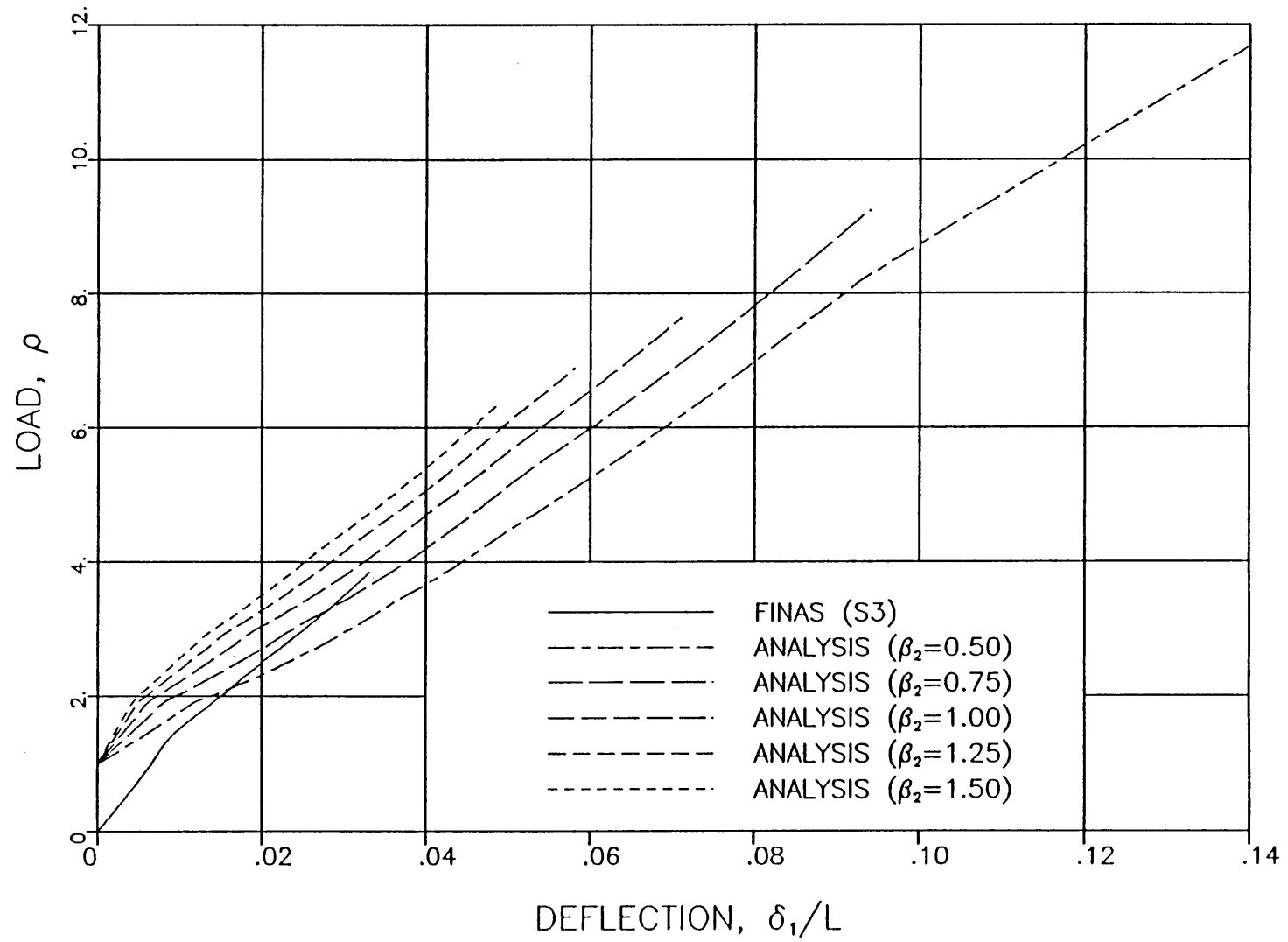


Fig. 8.5 Load vs. deflection for stiffener 1, deck S3

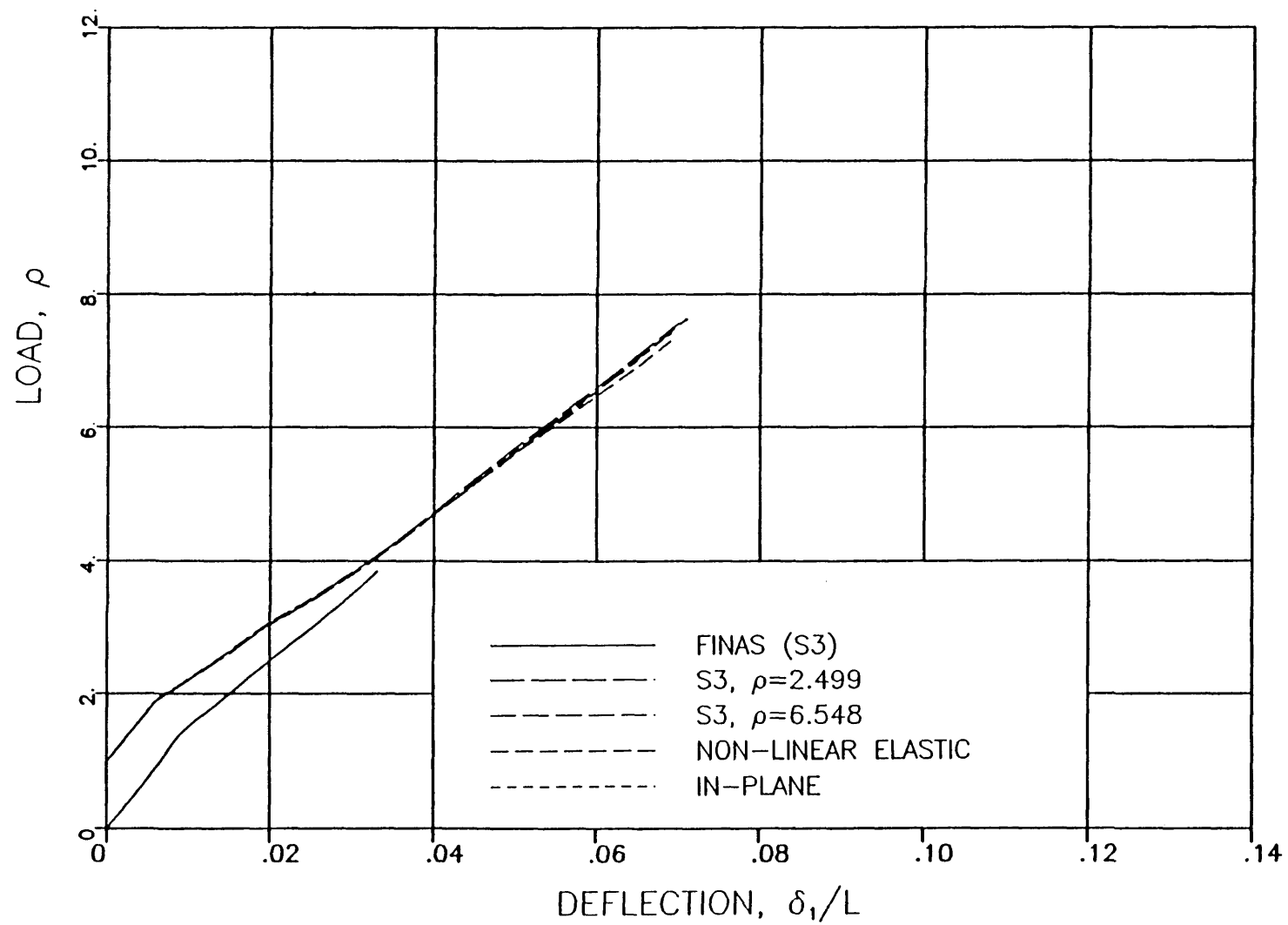


Fig. 8.6 Load vs. deflection for stiffener 1, deck S3.
(Model predictions for $\beta_2=1.0$)

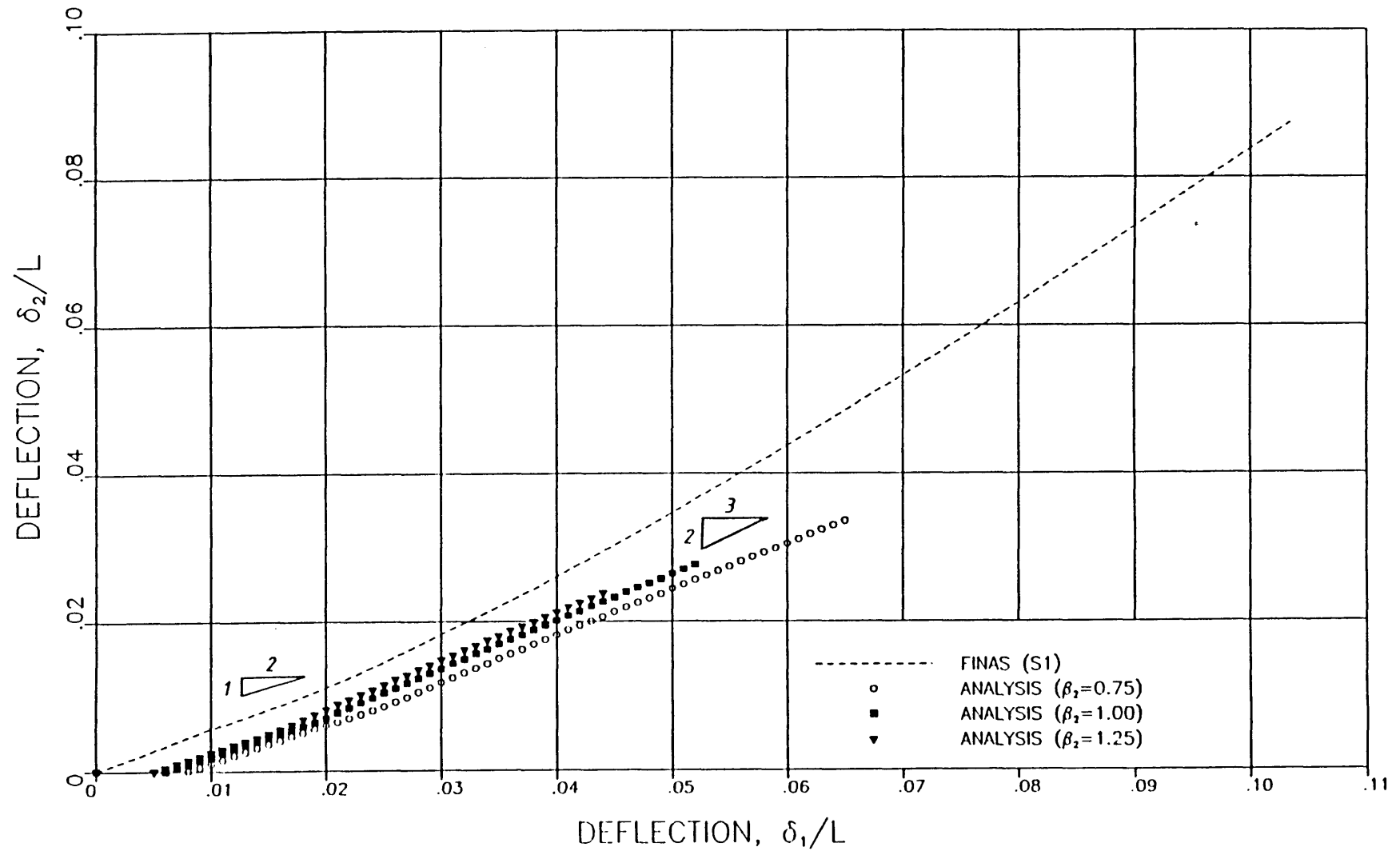


Fig. 8.7 Central deflection in stiffener 2 plotted against central deflection in stiffener 1, deck S1

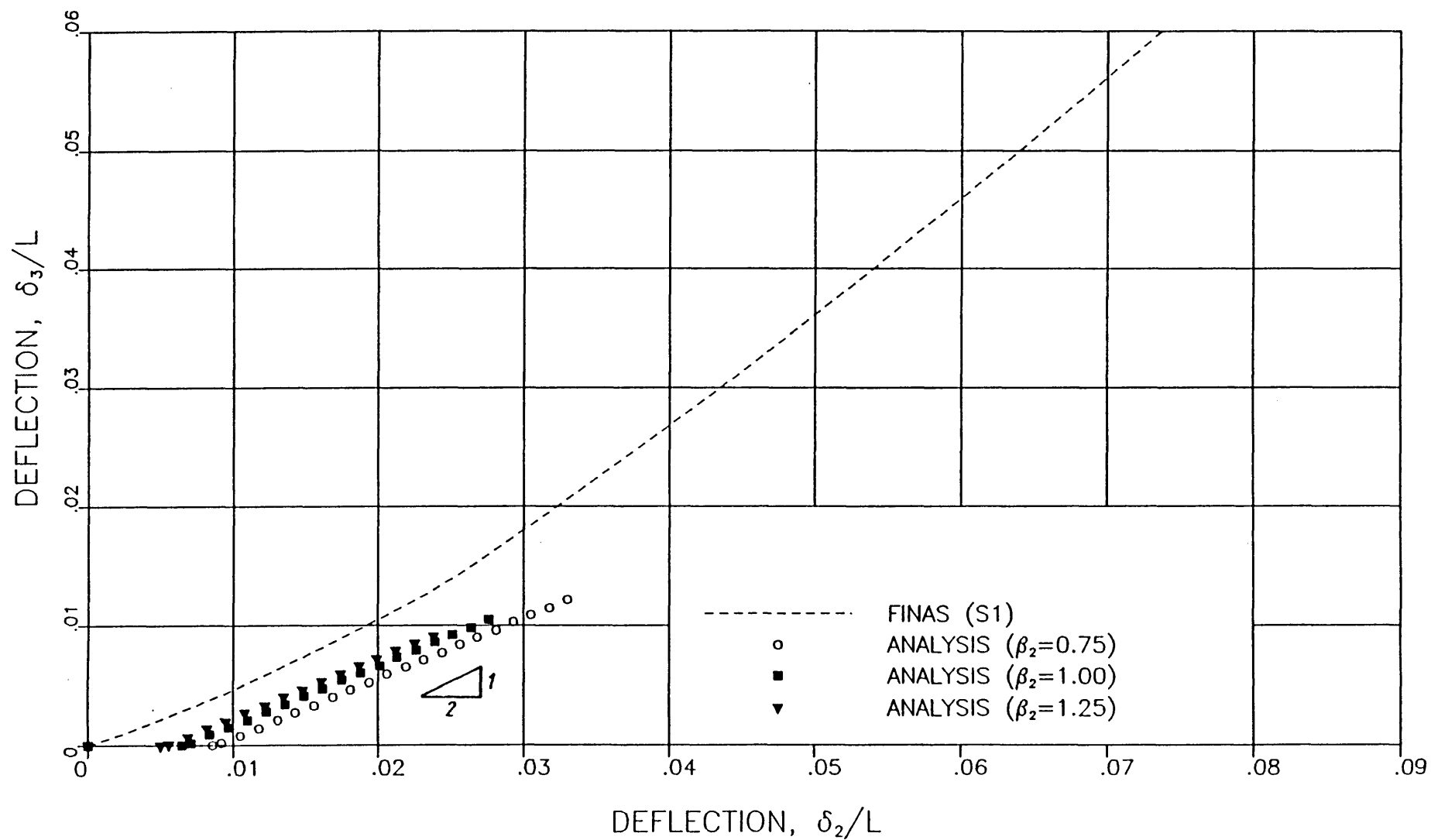


Fig. 8.8 Central deflection in stiffener 3 plotted against central deflection in stiffener 2, deck S1

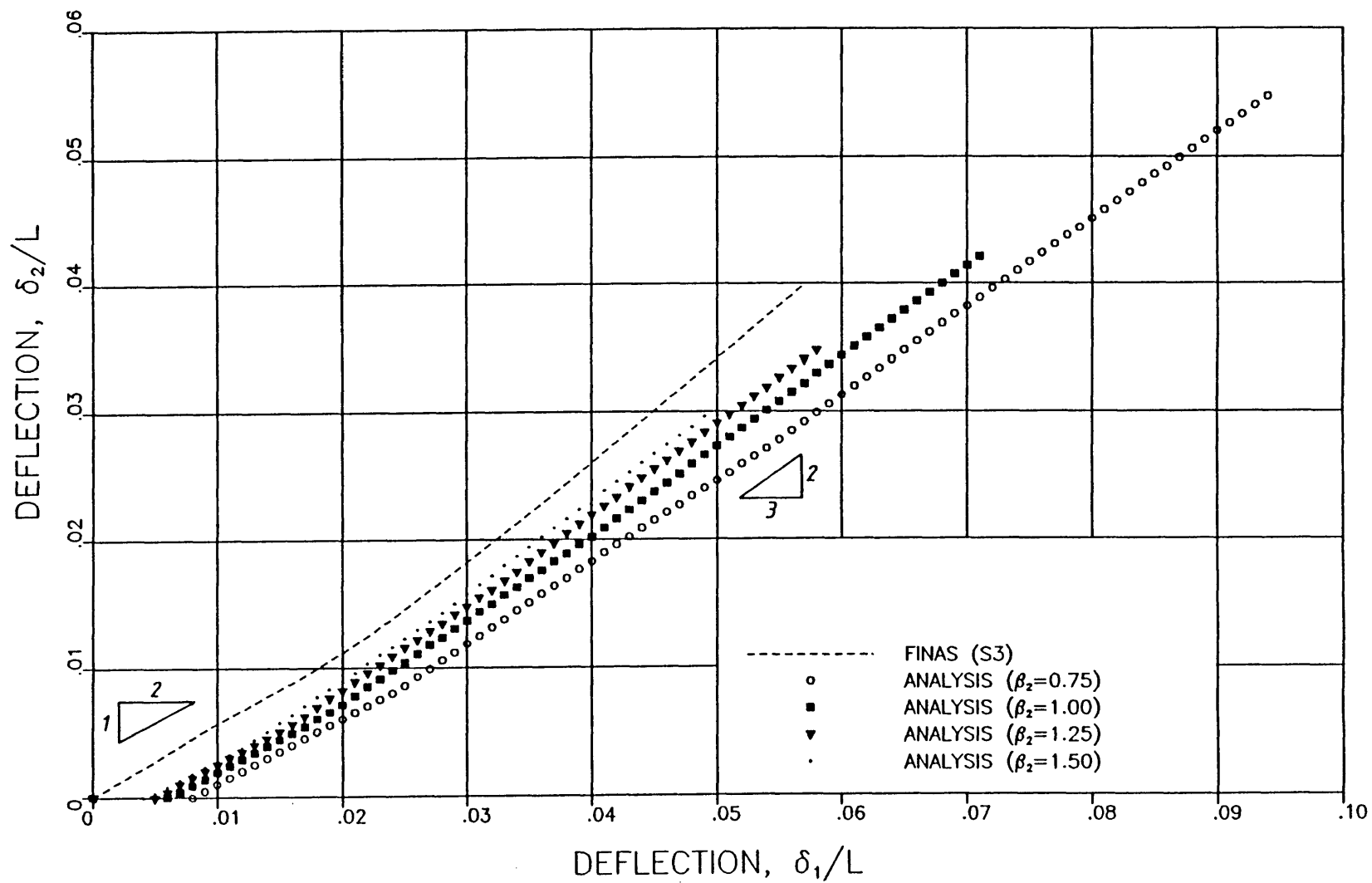


Fig. 8.9 Central deflection in stiffener 2 plotted against central deflection in stiffener 1, deck S3

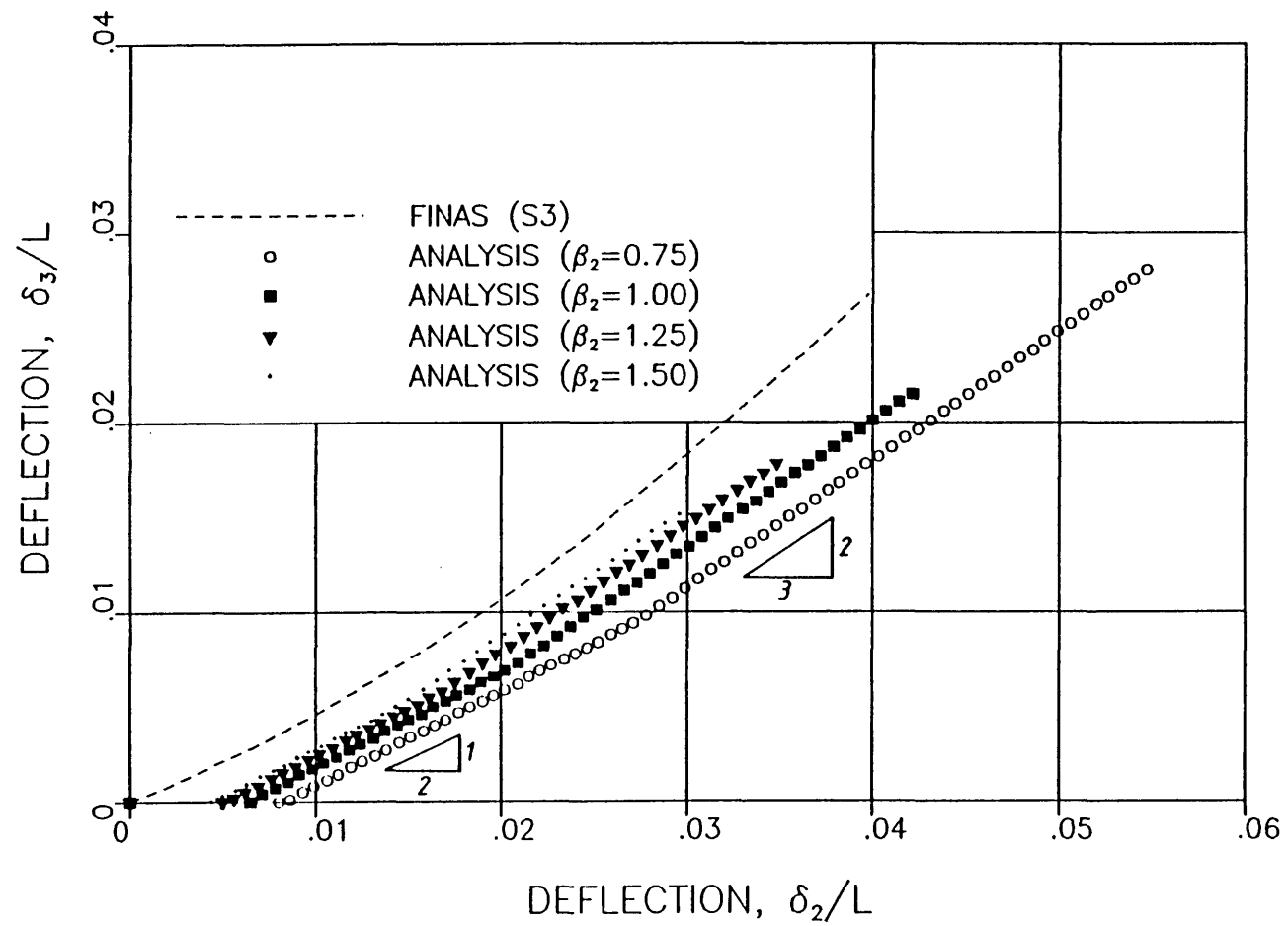


Fig. 8.10 Central deflection in stiffener 3 plotted against central deflection in stiffener 2, deck S3

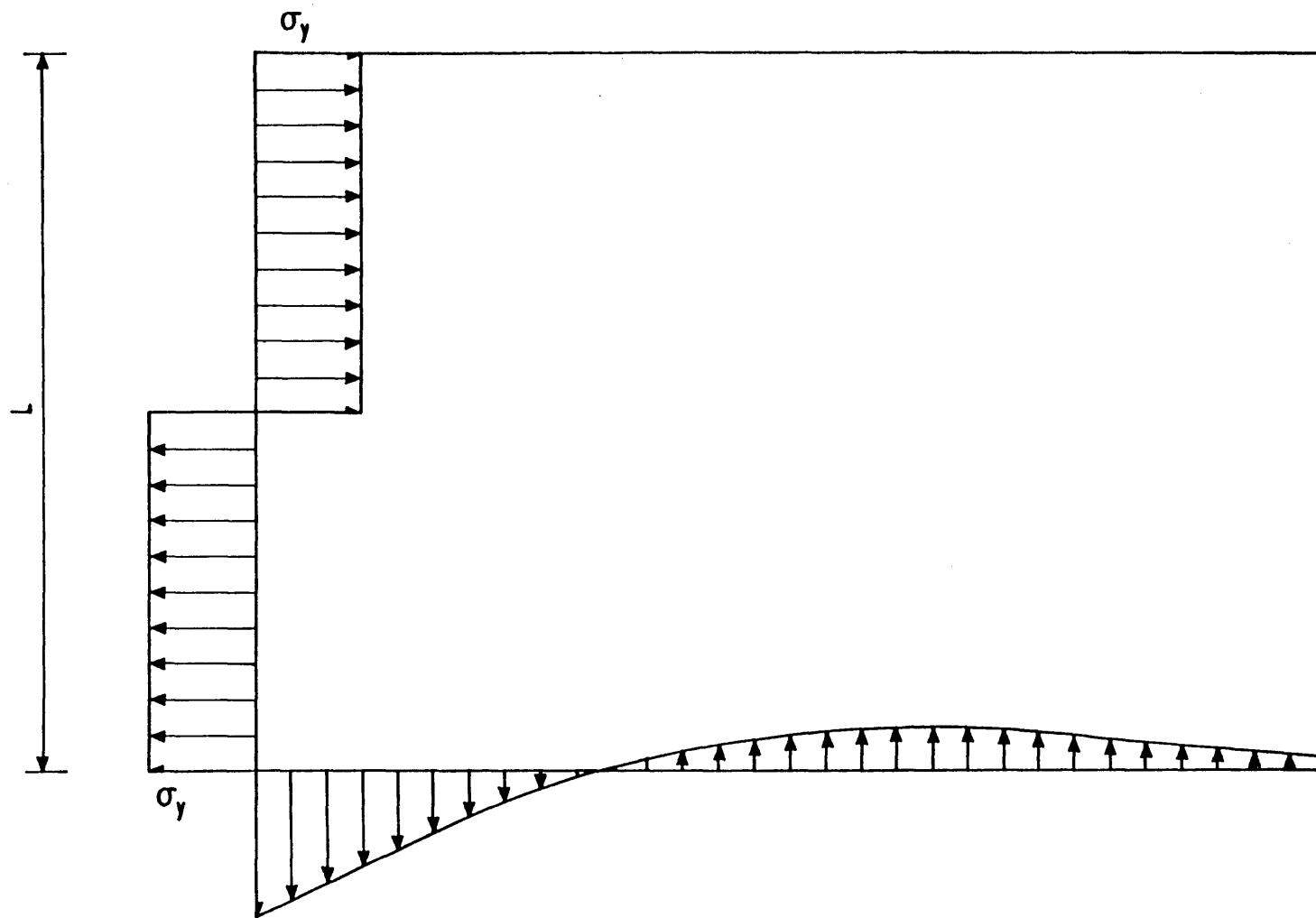


Fig. 8.11 Balanced in-plane membrane action

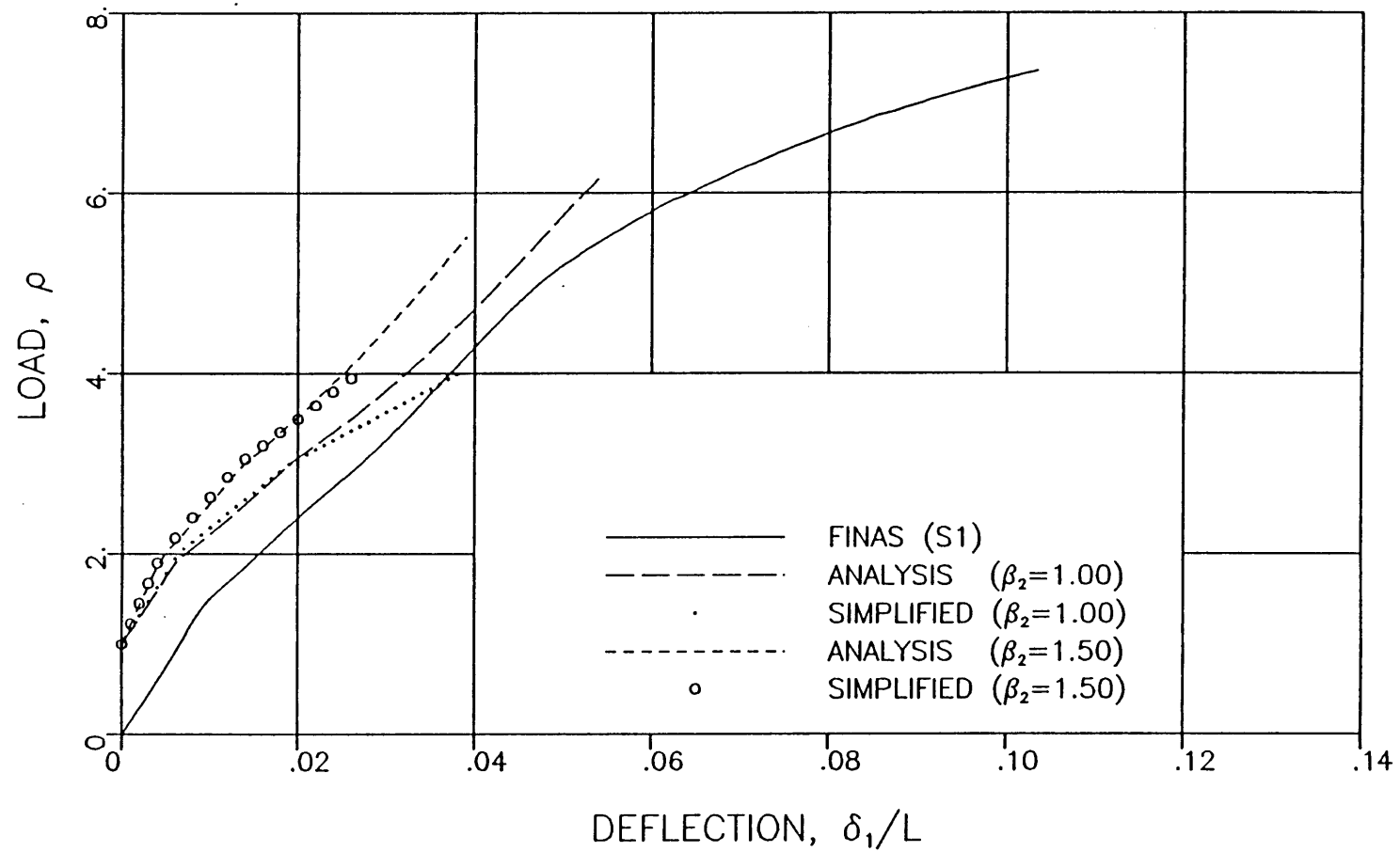


Fig. 8.12 Load vs. deflection for stiffener 1, deck S1

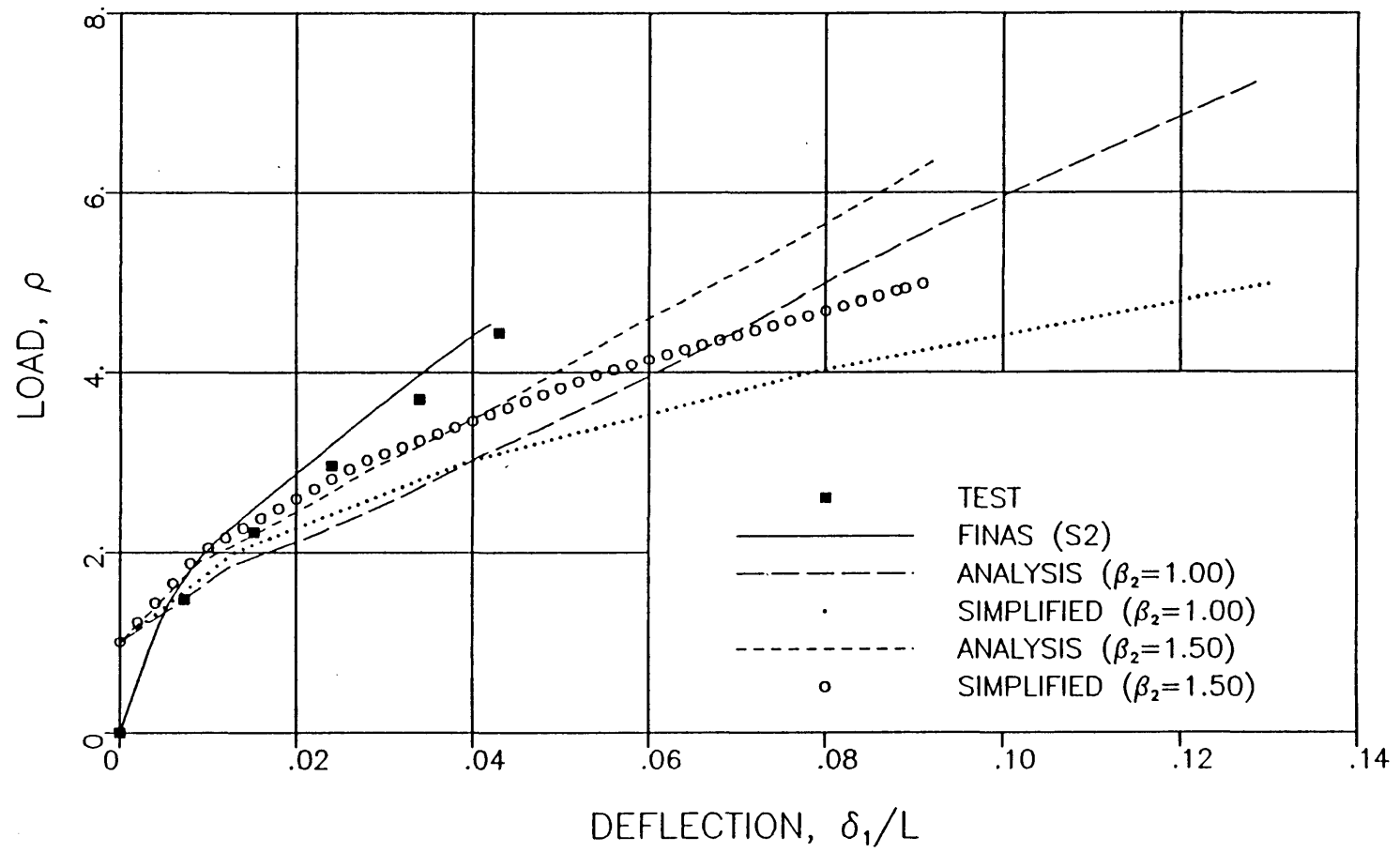


Fig. 8.13 Load vs. deflection for stiffener 1, deck S2

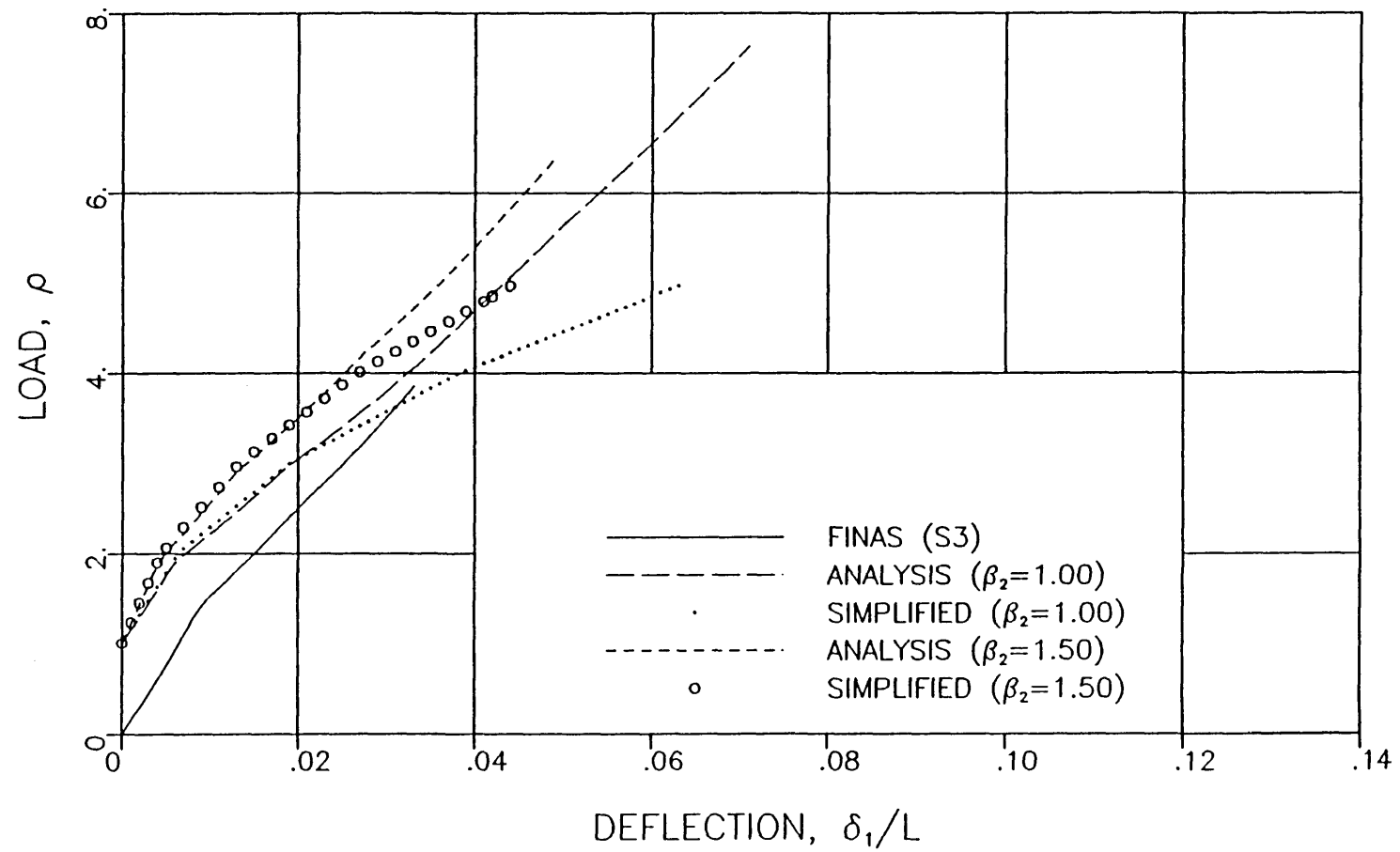


Fig. 8.14 Load vs. deflection for stiffener 1, deck S3

CHAPTER NINE

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

9.1 GENERAL CONCLUSIONS

The large deflection behaviour of a single-bay longitudinally stiffened deck under lateral patch loading has been studied. This single bay is either the only element of the deck under consideration, or may be part of a multi-bay assembly. In general, the restraint conditions at the boundaries of any bay will lie between the cases of clamped boundaries restrained against pull-in and simply supported boundaries free to draw-in. Simplified models for both cases have been proposed.

9.1.1 Fully restrained bay

In a restrained bay, the deck plate follows closely the membrane solution after a deformation in excess of two to three times the plate thickness has been reached. This can be exploited to devise a simplified model based on the stiffeners acting as a series of beams interacting via the deck plate which follows the membrane solution. If the stiffeners can be assumed to form bilinear mechanisms, that is the deformed shape of a beam varies linearly between the support and the central hinges, the load applied by the deck plate on the stiffeners is related to the differential deflection between adjacent stiffeners with an approximately linear relationship and has a triangular distribution along the deck. Two models of axially restrained beams were evolved which were subsequently used to predict the response of the entire bay. One was a beam on elastic foundation under centrally applied patch loading, and the other was a beam under triangularly distributed loading.

9.1.1.1 Axially restrained beams on an elastic foundation

The behaviour of axially restrained beams on elastic foundation was studied using the finite element method and was compared with the proposed model based on a bilinear mechanism. It has been shown that this model gives good predictions provided that a limitation is set to the maximum deflections allowed. For the geometries considered, limiting the central deflection to 5% of the span limits the error made in estimating the strength within the 10% of the numerical prediction. In order to expand the scope of this model a modified version is proposed to cope with phenomena occurring at higher deflections.

9.1.1.2 Restrained beams under triangular loading

The problem was set more generally in an attempt to devise an analytical model for beams under various patterns of symmetric distributed loading. This has led to an expansion of Haythornthwaite's method for the case of uniformly distributed loading. After developing an analytical solution based on a detailed deformation mode, a bilinear mechanism model was compared as a simplified alternative. It has been shown that the latter is a good approximation for central deflections up to 8% of the span.

9.1.1.3 Bay model

Using the beam models summarized above, an analytical model for the entire deck was developed consisting of a series of struts and plate strips. It was shown that the results of the proposed model for the large deflection behaviour of the stiffened bay compare favourably with results obtained by the finite element analysis. Two simplified versions of analysis were also proposed and can be used when accuracy is not of primary importance.

9.1.2 Unrestrained bay

An experimental model of a simply supported single-bay deck tested to collapse was used to validate the finite element program FINAS. Subsequently, the program was used to produce further numerical data on other deck geometries. The behaviour of these decks was studied up to very large deflections and it was found that the distribution of membrane forces in the deck is central to the formulation

of a suitable analytical model.

Study of decks with identical panel parameters but different bay aspect ratios, shows that the relationship between the axial force in the loaded stiffener and its central deflection is independent of bay aspect ratio. This feature was used to devise a model for these decks. The model, though semi-empirical in the sense that it requires prior knowledge of the axial force history of the loaded stiffener, gives a good first estimate of the behaviour of unrestrained decks. A proposed simplified version of the analysis can be used to provide a quick answer to the loading history.

9.1.3 Partially restrained bays

The above single-bay models can be used for predicting the load-deflection history of a multi-bay deck. One aspect that gives additional flexibility to the models is the treatment of the support moments. It is possible to choose the most appropriate flexural boundary conditions to obtain reasonable lower- and upper-bounds for bays with intermediate boundary conditions.

9.2 FUTURE RESEARCH

There is scope for additional development in devising more accurate models for axially restrained beams on an elastic foundation. In the more general case axially restrained beams subjected to non-proportional loading need to be analysed.

The treatment of the unrestrained decks presented in this thesis can be further developed. This is supported by the semi-empirical nature of the proposed model. In its present form the model needs additional data for bays having different panel parameters than those studied here. This can be obtained from a numerical parametric study in which the panel parameters are systematically varied.

Both restrained and unrestrained deck models can be further developed to include the effect of shearing in the deck plate between stiffeners.

For the unrestrained decks there is need to investigate the effect of the bay aspect ratio on the second phase of the behaviour, as noticed in the narrow deck analysed herein.

There is need for further experimental evidence concerning the maximum strains which can be developed and how this information can be used in association with the mechanism analysis.

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APPENDIX A

INTERACTION FORMULAE FOR AN ASYMMETRIC I-SECTION

A.1 Introduction

The interaction formulae between the bending and axial stress resultants for a cross-section formed by up to three rectangles under uniaxial stress state are presented here. These formulae have been derived by Ronalds (1985, 1990) and the cross-section assumed is an asymmetric I-section with the notation shown in Fig. 3.1(b).

A.2 Geometric properties for the cross-section

Three expressions for the plastic modulus are derived with respect to the location of the equal area axis. The position of the latter can be determined by the relative size of the panel area, A_p , the web area, A_w and the area of the bottom flange, A_f .

The axis is in the panel, if: $A_p > A_w + A_f$

$$Z_{pp} = \frac{1}{4} \frac{A^2}{b} + \frac{1}{2} (A_w + A_f)^2 \left[\frac{1}{t_w} - \frac{1}{b} \right] - \frac{1}{2} A_f^2 \left[\frac{1}{t_w} - \frac{1}{b_f} \right]$$

The axis is in the web, if: $A_p \leq A_w + A_f$ and $A_f \leq A_w + A_p$

$$Z_{pw} = \frac{1}{4} \frac{A^2}{t_w} - \frac{1}{2} A_p^2 \left[\frac{1}{t_w} - \frac{1}{b} \right] - \frac{1}{2} A_f^2 \left[\frac{1}{t_w} - \frac{1}{b_f} \right]$$

The axis is in the flange, if: $A_f > A_w + A_p$

$$Z_{pf} = \frac{1}{4} \frac{A^2}{b_f} - \frac{1}{2} A_p^2 \left[\frac{1}{t_w} - \frac{1}{b} \right] + \frac{1}{2} (A_w + A_p)^2 \left[\frac{1}{t_w} - \frac{1}{b_f} \right]$$

A.3 Interaction formulae

The interaction between the stress resultants at a plastic hinge i is given as:

Neutral axis location	Axial force	Moment interaction
panel	$-1 \leq n \leq K_p$	$m^i = \frac{Z_{pp}}{Z_p} \left[-g_p n^2 + (-1)^{\delta_{is}} (1 - g_p) n + 1 \right]$
web	$K_p \leq n \leq K_f$	$m^i = \frac{Z_{pw}}{Z_p} \left[-g_w n^2 + (-1)^{\delta_{is}} \left\{ 1 - g_w + \frac{A_p^2}{Z_{pw}} \left[\frac{1}{t_w} - \frac{1}{b} \right] \right\} n + 1 \right]$
flange	$K_f \leq n \leq 1$	$m^i = \frac{Z_{pf}}{Z_p} \left[-g_f n^2 + (-1)^{\delta_{ih}} (1 - g_f) n + 1 \right]$

Where Z_p is the plastic modulus for the cross-section, i takes the values s and h for sagging and hogging moment respectively and δ_{is} , δ_{ih} are Kronecker deltas ($\delta_{ss}=1$, $\delta_{hh}=1$, $\delta_{sh}=0$). The other coefficients involved in the table above are defined as follows:

$$K_p = \frac{A_p - A_w - A_f}{A}$$

$$g_p = \frac{1}{4} \frac{A^2}{b Z_{pp}}$$

$$K_f = \frac{A_p + A_w - A_f}{A}$$

$$g_w = \frac{1}{4} \frac{A^2}{t_w Z_{pw}}$$

$$g_f = \frac{1}{4} \frac{A^2}{b_f Z_{pf}}$$

APPENDIX B

PREDICTION OF THE DEFLECTION SHAPE

B.1 Introduction

The deflection shape predicted by Eqn. (4.11) is considered here. Its contribution to the equilibrium Eqn. (4.9) is also calculated.

B.2 The deflection profile

The deflection at a distance x from the support is calculated in this section and is denoted by $y(x)$.

1. For $\delta_c/L < \delta'_c/L$

$$\frac{y(x)}{L} = \frac{x}{L} \frac{\delta_c}{L} \quad (\text{B.1})$$

2. For $\delta_c/L \geq \delta'_c/L$,

there is always a curved segment in the range

$$0 \leq x \leq (1-\gamma)L \quad (\text{B.2})$$

and a straight segment in the range

$$(1-\gamma)L < x \leq L \quad (\text{B.3})$$

- (i) The deflection of any point x in the range described by Eqn. (B.2) is

$$\frac{y(x)}{L} = \frac{x}{L} \frac{\delta_c}{L} + \int_{\delta'_c/L}^{\delta_c x/L} \frac{x-(1-\gamma)L}{\gamma L} \frac{d\delta_c}{L} \quad (\text{B.4})$$

where δ_{cx} is the central deflection at which the deflection at point x stops increasing, obtained by equating

$$\frac{x}{L} = 1 - \gamma_x$$

with

$$1 - \gamma_x = \alpha_1 \left(\frac{\delta_{cx}}{L} - \frac{\delta_c'}{L} \right)$$

(from Eqn. 4.11), which gives:

$$\frac{\delta_{cx}}{L} = \frac{1}{\alpha_1} \frac{x}{L} + \frac{\delta_c'}{L} \quad (\text{B.5})$$

γ_x in the above expression is the value of γ when the deflection at x ceases to increase. Using Eqns (4.11) and (B.4), and performing the integration indicated in the latter, yields:

$$\frac{y(x)}{L} = \frac{x}{L} \frac{\delta_c'}{L} + \left(\frac{\delta_{cx}}{L} - \frac{\delta_c'}{L} \right) + \frac{1}{\alpha_1} \ln \left(1 + \alpha_1 \left(\frac{\delta_c'}{L} - \frac{\delta_{cx}}{L} \right) \right) \left(1 - \frac{x}{L} \right)$$

By substituting for δ_{cx}/L from Eqn. (B.5), the following expression is obtained

$$\frac{y(x)}{L} = \frac{x}{L} \left(\frac{\delta_c'}{L} + \frac{1}{\alpha_1} \right) + \frac{1}{\alpha_1} \left(1 - \frac{x}{L} \right) \ln \left(1 - \frac{x}{L} \right) \quad (\text{B.6})$$

(ii) The deflection of any point x in the range described by Eqn. (B.3) is given as

$$\frac{y(x)}{L} = \frac{y(x_\gamma)}{L} + \frac{(x - x_\gamma)}{(L - x_\gamma)} \left[\frac{\delta_c}{L} - \frac{y(x_\gamma)}{L} \right] \quad (\text{B.7})$$

where x_γ is the distance from the support corresponding to the current value of γ , taken as:

$$\frac{x_\gamma}{L} = 1 - \gamma$$

$$= \alpha_1 \left(\frac{\delta_c}{L} - \frac{\delta'_c}{L} \right)$$

The value for $y(x_\gamma)$ in Eqn. (B.7) is obtained from Eqn. (B.6) for $x=x_\gamma$.

B.3 Modification of the spring force contribution in Eqn. (4.9)

The ratio of the contribution of the actual spring forces in the equilibrium Eqn. (4.9) to the contribution obtained if the bilinear shape is assumed, denoted as R_f , is obtained as:

$$R_f = \frac{\int_0^L y(x) k_s x dx}{\frac{1}{3} k_s \delta_c L^2}$$

$$= \frac{\int_0^1 \frac{y(x)}{L} \frac{x}{L} \frac{dx}{L}}{\frac{1}{3} \frac{\delta_c}{L}} \quad (\text{B.8})$$

The numerator on the right hand side of Eqn. (B.8) was calculated numerically using Simpson's rule and the R_f values were implemented in the modified equilibrium Eqn. (4.12).

APPENDIX C

EXTENSIONS OVER THE PLASTIC BEAM SEGMENT

C.1 Introduction

A hinge of finite length may have extensional requirements if curvature changes take place over its finite length. In this appendix, such requirements are calculated for a curved beam segment representing a spreading hinge, as shown in Fig. 5.2(a), and an example for the case of triangular loading is given.

C.2 Geometric extensions at spreading plastic hinge tips

The geometric extensions over the curved length caused by an increment in the maximum ordinate $y_0(L)$ are now considered, Fig. C.1. Within the assumption of small rotations the inclined length of an element dx in its deformed position is:

$$ds \approx \left(1 + \frac{1}{2} \left(\frac{dy_0}{dx}\right)^2\right) dx$$

Here the slope dy_0/dx is limited to a value less than 0.5 to limit the error in the evaluation of the extension of an element dx at the steepest position $x=a$ to below 6%. The average error is obviously less than this. The extension over the curved length between $x=a$ and $x=L$, due to increase in the depth from 0 to $y_0(L)$, is:

$$e_2 = \int_{x=a}^{x=L} (ds - dx) = \frac{1}{2} y_0(L)^2 \int_{x=a}^{x=L} f'(x)^2 dx \quad (C.1)$$

Differentiation of (C.1) gives:

$$de_2 = dy_0(L) y_0(L) \int_{x=a}^{x=L} f'(x)^2 dx$$

It can be seen that if the entire rotation increment takes place over the previously curved segment

$$d\theta = dy_0(L) f'(a)$$

and combining the last two equations yields:

$$\frac{de^2}{d\theta} = a y_0(L) f'(a) C_5(a) \quad (C.2)$$

where

$$C_5(a) = \frac{\int_{x=a}^{x=L} f'(x)^2 dx}{a f'(a)^2} \quad (C.3)$$

and θ is the angle of inclination of the straight segment, as shown in Fig. 5.2(a).

Substitution of Eqn. (5.5), ensuring continuity of slope between straight and curved segments, into Eqn. (C.2) gives:

$$\frac{de^2}{d\theta} = C_5(a) \bar{\delta} \quad (C.4)$$

The equation above gives the extensional requirement of the curved segment assuming that the entire incremental rotation has occurred over this segment. However, most of the incremental rotation is accommodated by the spreading of the curved segment. Therefore, only a fraction of the extensions calculated by Eqn. (C.4) should be included. The fraction of these extensions can be expressed as:

$$\frac{de^2}{d\theta} = \alpha_2 C_5(a) \bar{\delta} \quad (C.5)$$

where the reduction factor, α_2 , is discussed in the next section.

C.3 Reduction of the extensions calculated over the curved segment

An important objective of the analysis presented in Section 5.4 is to maintain a relatively simple solution similar to Haythornthwaite's (1961) and yet determine the relative importance of the enhancements made later by Gill (1973) and Gürkök &

Hopkins (1973). Therefore, simplified assumptions are made about different aspects of the behaviour. According to Gill's and Gürkök & Hopkins' analyses the central hinge travels, but it does not spread until it approaches the third points of the span for the case of uniform loading. In the present analysis the assumption of a spreading rather than a travelling hinge is made. If $y_0(L)$ is reducing in magnitude, as shown in Figs 5.13(a) and 5.13(b), the extensions associated with the curved segment are not accounted for. This behaviour corresponds to the first mode in Gill (1973). Once $y_0(L)$ starts to increase in magnitude, these extensions are incorporated to account for changes in geometry associated with the spreading of the hinge towards the centre of the beam. This spreading occurs during the second mode according to Gill (1973) and Gürkök & Hopkins (1973). However, incorporation of the total extension implied by increasing values of $y_0(L)$ as calculated by Eqn. (C.4) corresponds to the case where all the rotational increment takes place over the previously developed curved length. That is, there is no spreading of the plastic segment ($a=\text{constant}$) leading to an over stiff solution.

The change in slope along the beam represented by the difference in the rotation along the beam for successive pairs of deflected shapes, is plotted in Fig. C.2. The three curves presented were obtained from a finite element analysis of an encastré beam under uniformly distributed loading with the corresponding ρ values indicated. It is shown that the significant change in rotation is due to the spreading of the plastic hinge. However, this does not account for the total change in rotation. Some change occurs along the curved segment, with increasing contribution as the curved segment spreads. When the membrane phase is reached, the rotational increment is distributed along the length without the localizations shown in Fig. C.2. At this phase, the cumulative rotational increment for uniformly distributed loading is given by a straight line with zero value at the centre of the beam and maximum value at the support. This is because the mode-shape $f(x)$ corresponding to the uniform loading is a parabola.

For these reasons the parameter $\alpha_2=(1-a/a_0)$, where $\alpha_2 \leq 1$, has been introduced since it is compatible with the limiting results at the two ends of the spreading hinge phase. Here a_0 is the length of the straight segment when $y_0(L)$ is minimum, as shown in Figs 5.13. The important point is that additional extensions take place over and above those taken into account by $F=1$, that is, by assuming that the curved segment has a fixed shape.

C.4 Example

For the beam under triangular loading shown in Fig. 5.8 the application of the principle of virtual work gives

$$q_0 = \frac{3 \lambda M_0}{L^2}$$

that is $\mu_2=3$ in Eqn. (5.1). The free bending moment on a substitute simply supported beam yields:

$$M(x) = M(L) [1.5 (x/L) - (x/L)^3] \quad , \quad \text{for } 0 \leq x \leq L$$

and therefore the mode-shape $f(x)$ defined by Eqn. (5.1(a)) is expressed as:

$$f(x) = 1.5 (x/L) - 0.5 (x/L)^3$$

Finally, if the expression for F given by Eqn. (5.8), is used, the required $C_5(a)$ value is given by:

$$C_5(a) = \frac{(L-a) - (2/3)(L^3 - a^3)/L^2 + 0.2(L^5 - a^5)/L^4}{a[1 - (a/L)^2]^2}$$

The predictions are shown in Figs 5.8 through 5.11.

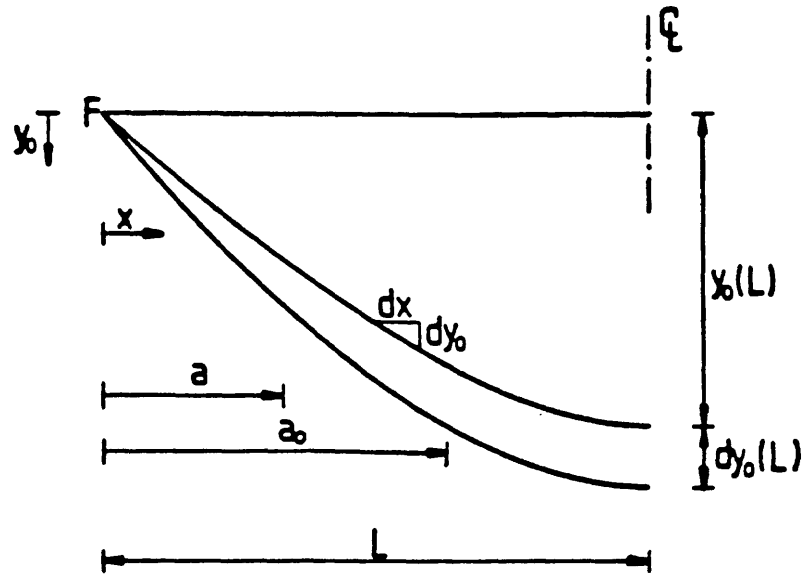


Fig. C.1 Assumed shape

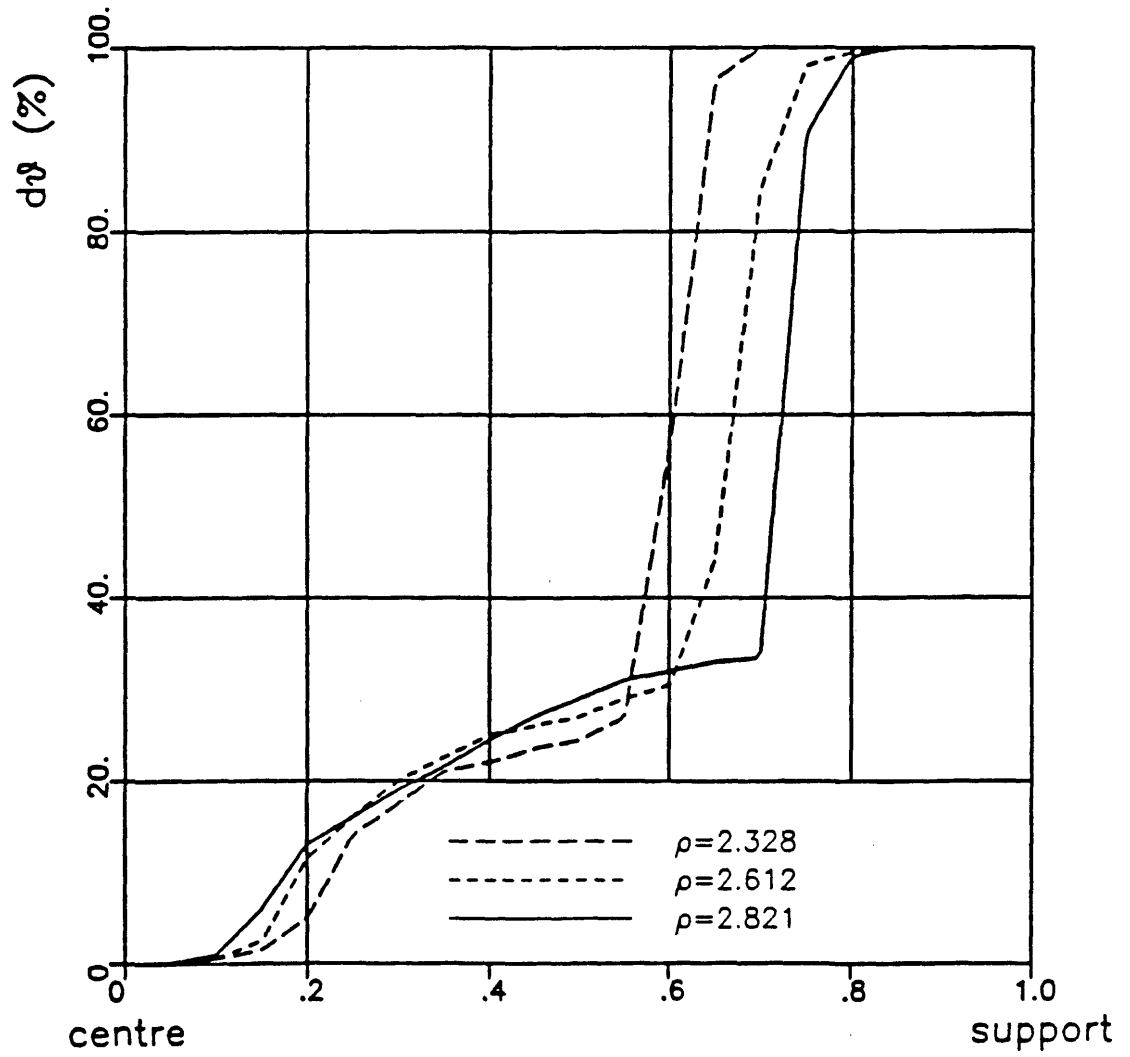


Fig. C.2 Cumulative change in rotation along the beam