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High efficiency transfer of quantum information and multiparticle entanglement generation in translation-invariant quantum chains

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Abstract. We demonstrate that a translation-invariant chain of interacting quantum systems can be used for high efficiency transfer of quantum entanglement and the generation of multiparticle entanglement over large distances and between arbitrary sites without the requirement of precise spatial or temporal control. The scheme is largely insensitive to disorder and random coupling strengths in the chain. We discuss harmonic oscillator systems both in the case of arbitrary Gaussian states and in situations when at most one excitation is in the system. The latter case, which we prove to be equivalent to an xy-spin chain, may be used to generate genuine multiparticle entanglement. Such a 'quantum data bus' may prove useful in future solid state architectures for quantum information processing.

The realization of quantum communication and computation requires at various stages the mapping between stationary and flying qubits and subsequent transfer of quantum information between different units of our quantum information processing devices. Traditionally, the stationary forms of qubits are massive systems such as atoms, ions, quantum dots or Josephson junctions, while the flying qubit is a photon, i.e. radiation. Photons might be optimal when considering long distance communication where they may travel through free space or optical

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fibres. In very small quantum information processing devices such as condensed matter systems, however, this is difficult as the length scale of both the component parts and their separation will generally be below optical wavelengths. In this situation, it is worth considering novel approaches for the communication of quantum information and the generation of entanglement. To this end, it is of interest to consider the properties of interacting quantum systems. An interesting class of interacting quantum systems is formed by harmonic oscillator systems that are realized in various condensed matter physics settings such as nano-mechanical oscillators. While static harmonic (or spin) systems near their ground state do not exhibit long distance entanglement [1], the situation changes drastically when considering time-dependent properties of interacting quantum systems [2]. Indeed, solid state devices such as arrays of nano-mechanical oscillators, described as interacting harmonic oscillators, allow for the generation [3], transfer and manipulation of entanglement [4] with a minimum of spatial and temporal control. The Landau–Zener mechanism [6] may also be exploited to move electrons in arrays of quantum dots and then to transfer information between two-level systems [5]. However, in translation-invariant systems, the efficiency for this transfer decreases with distance. This can be overcome either by making the coupling strengths between neighbouring systems position-dependent [4, 7] or by active steps such as quantum repeater stages [8] or conclusive transfer [9]. Nevertheless, active steps or the fabrication of precisely manufactured spatially dependent couplings are difficult in practice and will require a significant degree of control. Furthermore, the precise value of the coupling parameters and the timing of the operations will depend on the distance across which one aims to transfer quantum information. Consequently, it would be desirable to achieve high efficiency transmission of quantum information between arbitrary places and distances with minimal spatial and temporal control. In the following, we show that this is indeed possible by employing translation-invariant chains of interacting quantum systems with stationary couplings.

We first describe the system, termed a quantum data bus, and demonstrate its functionality by numerical examples. Then we present an approximate analytical model that reveals the basic physical mechanism that is utilized in the operation of the quantum data bus. This model then allows us to maximize entanglement transfer efficiency and transmission speed of the quantum data bus by adjusting the eigenfrequencies of the sender and receiver system. It also explains why the transmission is largely insensitive to disorder and random coupling strengths in the ring. We discuss the scaling behaviour of the time that is required for the transfer between distant sites at a given efficiency. Finally, we show that in the regime where at most one excitation is in the system, a quantum data bus made of interacting harmonic oscillators becomes equivalent to an interacting spin chain. A further application for such a chain is the generation of three or multiparticle entangled states as we also show in this paper. The model we have in mind is depicted in figure 1. A ring of interacting quantum systems (blue circles) forms the quantum data bus. At arbitrary position on the ring, two further quantum systems (red) may be coupled weakly to the ring. The subsequent time evolution will allow for high efficiency transfer of entanglement between the two distinguished quantum systems. The following ideas are not restricted to the specific ring-like geometry presented here. The depicted geometry has been chosen because it simplifies the analytical treatment of the problem.

So far we have not specified any particular type of quantum systems nor their mutual interaction. In the following, we will consider, as an example, the case of coupled harmonic oscillators which may be realized in various solid state settings [3]. Towards the end of this paper, we will also consider a setting which is equivalent to a spin chain with an xy-interaction. Let us assume that the ring consists of M harmonically coupled identical oscillators. We set

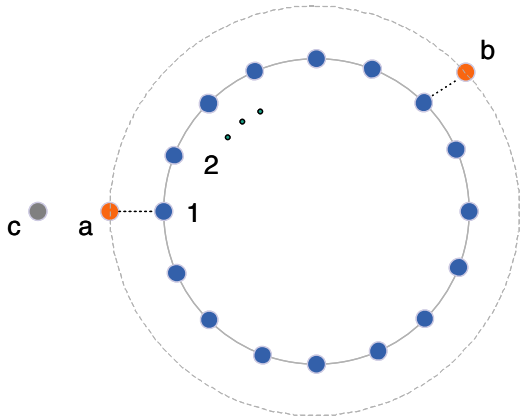


Figure 1. A ring of interacting quantum systems (blue circles) forms the quantum data bus. Two further quantum systems a and b (red) may couple weakly at arbitrary positions to the ring. The subsequent time evolution will allow for the transfer of quantum information or entanglement that exists between systems a and the decoupled system c from system a to system b. System c is decoupled but might be entangled with a to study transfer of entanglement.

$\hbar = \omega = m = 1$ and denote the coupling strength in the ring by f. We assume that oscillator a (b) couples to oscillator 1 (k) with strength ϵ, so that the Hamilton operator of this system including the oscillators a, b and c is given by

$$H(\epsilon) = \frac{1}{2}\left[\sum_{k=1}^{M} p_k^2 + \sum_{k,l=1}^{M} x_k V_{kl} x_l + \sum_{i=a,b,c} (x_i^2 + p_i^2)\right] + \frac{\epsilon}{2}[(x_a - x_1)^2 + (x_b - x_k)^2] \tag{1}$$

with the potential matrix V given by $V_{kk} = 1 + 2f$ and $V_{k,k+1} = V_{k+1,k} = -f$ and $V_{1M} = V_{M1} = -f$ for all k and zero otherwise.

In the following, we will demonstrate that it is indeed possible to transmit quantum information through this system with high efficiency but minimal spatial and temporal control. For the moment we are focusing on Gaussian states, i.e. states whose characteristic function or Wigner function is Gaussian. The characteristic function determines the quantum state and as any Gaussian is determined by its first and second moments, the same applies to the corresponding quantum state [10]. In the present setting, the first moments will not be directly relevant (they correspond to biases that can be removed by redefining the coordinate origin). Therefore, the state of the system is determined by second moments that can be arranged in the symmetric $2N \times 2N$-covariance matrix $\Gamma_{R,S} = 2\mathrm{Re}\langle (R - \langle R\rangle)(S - \langle S\rangle)\rangle$, where R and S stand for the canonical operators $x_1, \ldots, x_n$ and $p_1, \ldots, p_n$. Employing the Hamiltonian operator equation (1), we can now study numerically the quality of the entanglement transfer between oscillators a and b. Let us consider the situation where the harmonic oscillator a and c are initially in a pure entangled two-mode squeezed state

$$|\psi\rangle = \sqrt{1 - q^2} \sum_{n=1}^{\infty} q^n |n\rangle |n\rangle \tag{2}$$

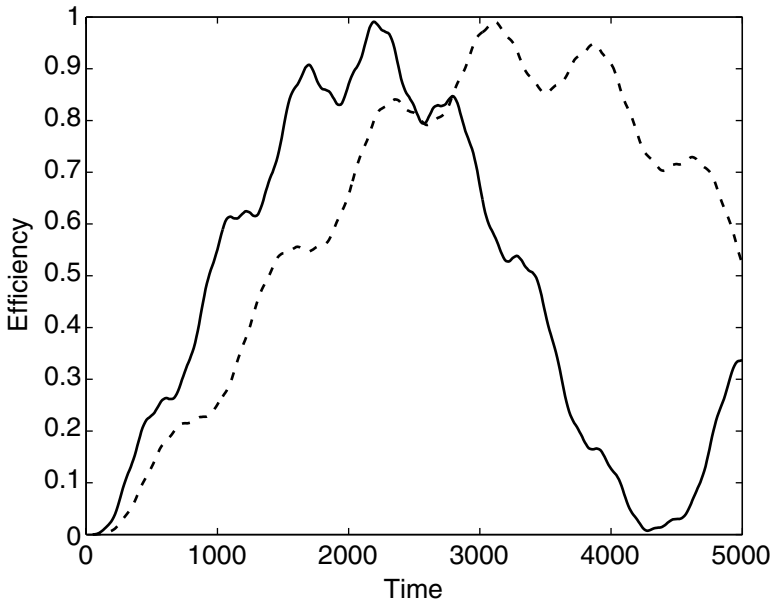


Figure 2. The efficiency of entanglement transfer, defined as ratio of transmitted entanglement to initial entanglement, between the oscillators a and b at opposite ends of a quantum data bus consisting of 20 oscillators and a nearest-neighbour coupling strength of $f = 1$. The coupling strength of the oscillators a and b to the quantum data bus is $\epsilon = 0.015$ (dashed line) and $\epsilon = 0.021$ (solid line). The speed of propagation is proportional to the coupling strength ϵ and in both cases we observe maximal efficiency exceeding 0.99.

with $q = \tanh \frac{r}{2}$ which possesses the covariance matrix

$$\Gamma_{x_a x_c p_a p_c} = \begin{pmatrix} \cosh r & 0 & -\sinh r & 0 \\ 0 & \cosh r & 0 & \sinh r \\ -\sinh r & 0 & \cosh r & 0 \\ 0 & \sinh r & 0 & \cosh r \end{pmatrix}. \tag{3}$$

The entanglement as quantified by the logarithmic negativity [11] of this state is then given by $E_N(|\psi\rangle) = r$. The time evolution of the entanglement between the oscillators a and b at opposite ends of a quantum data bus consisting of 20 oscillators and a nearest neighbour coupling strength of $f = 1$ is given in figure 2. The propagation speed is proportional to ϵ and the efficiency of entanglement transfer decreases weakly with increasing speed. In both cases we find a maximal efficiency,[4] defined as the ratio of transmitted entanglement to initial entanglement, exceeding 0.99. It should be noted that the time required for the transfer of entanglement is independent of the distance of places where the oscillators a and b couple to the ring. The high quality of the entanglement transfer and its independence of the position of sender and receiver will be successfully explained by the following model that encapsulates the essential physics in the system.

[4] Note that for a total system in a Gaussian state, the reduced state of a subsystem is also Gaussian because the partial trace preserves the Gaussian character of a quantum state.

We first observe that the unitary matrix Ω with matrix elements

$$\Omega_{kl} = \frac{1}{\sqrt{M}} \mathrm{e}^{\frac{2\pi i k l}{M}}, \tag{4}$$

achieves $V = \Omega^{\dagger} \Lambda^2 \Omega$ with a diagonal matrix Λ such that

$$\Lambda_{kk}^2 = 1 + 2f - 2f \cos \frac{2\pi k}{M}. \tag{5}$$

Then we can define the normal mode variables

$$X_k = \sum_{l=1}^{M} \Omega_{kl} x_l, \qquad P_k = \sum_{l=1}^{M} \Omega_{kl}^* p_l, \tag{6}$$

which ensure that $[X_k, P_l] = [x_k, p_l] = \mathrm{i}\delta_{kl}$, i.e. the canonical commutation relations are preserved. Note that we will use the convention $X_0 \equiv X_M$ and $P_0 \equiv P_M$, which reflects the periodic boundary conditions of the quantum data bus. Furthermore, denote $X_{a,b,c} = x_{a,b,c}$ and $P_{a,b,c} = p_{a,b,c}$ to make the notation more uniform. In these normal modes, we can write the Hamiltonian equation (1) as

$$H(\epsilon) = \frac{1}{2}\left[\sum_{k=1}^{M} P_k^{\dagger} P_k + \Lambda_{kk}^2 X_k^{\dagger} X_k + \sum_{i=a,b} P_i^2 + (1+\epsilon) X_i^2\right] + \frac{1}{2}(X_c^2 + P_c^2) - \epsilon X_a \sum_{l=1}^{M} \Omega_{1l}^* X_l$$
$$- \epsilon X_b \sum_{l=1}^{M} \Omega_{kl}^* X_l + \frac{\epsilon}{2} \sum_{lm} (\Omega_{l1}^* \Omega_{m1}^* + \Omega_{lk}^* \Omega_{mk}^*) X_l X_m.$$

Defining the annihilation operators

$$A_k = \frac{\Lambda_{kk} X_k + \mathrm{i} P_k^{\dagger}}{\sqrt{2\Lambda_{kk}}} \qquad \text{and} \qquad A_{a,b,c} = \frac{X_{a,b,c} + \mathrm{i} P_{a,b,c}}{\sqrt{2}},$$

we can rewrite the Hamiltonian operator in terms of the A_k. Indeed, shifting the zero of energy and moving to an interaction picture with respect to

$$H = \frac{1}{2} \sum_{k=1}^{M} P_k^{\dagger} P_k + \Lambda_{kk}^2 X_k^{\dagger} X_k + \frac{1}{2} \sum_{i=a,b,c} P_i^2 + X_i^2,$$

we find that

$$H_I = \frac{\epsilon}{2} A_a^{\dagger} A_a + \frac{\epsilon}{2} A_b^{\dagger} A_b - \epsilon \sum_{l=1}^{M} \frac{\Omega_{1l}^* (A_a \mathrm{e}^{-\mathrm{i}t} + A_a^{\dagger} \mathrm{e}^{\mathrm{i}t}) + \Omega_{kl}^* (A_b \mathrm{e}^{-\mathrm{i}t} + A_b^{\dagger} \mathrm{e}^{\mathrm{i}t})}{2\sqrt{\Lambda_{ll}}} (A_l \mathrm{e}^{-\mathrm{i}\Lambda_{ll} t} + A_{M-l}^{\dagger} \mathrm{e}^{\mathrm{i}\Lambda_{ll} t})$$
$$+ \sum_{lm=1}^{M} \frac{\epsilon (\Omega_{l1}^* \Omega_{m1}^* + \Omega_{lk}^* \Omega_{mk}^*)(A_l \mathrm{e}^{-\mathrm{i}\Lambda_{ll} t} + A_{M-l}^{\dagger} \mathrm{e}^{\mathrm{i}\Lambda_{ll} t})(A_m \mathrm{e}^{-\mathrm{i}\Lambda_{mm} t} + A_{M-m}^{\dagger} \mathrm{e}^{\mathrm{i}\Lambda_{mm} t})}{2\sqrt{\Lambda_{ll}\Lambda_{mm}}}. \tag{7}$$

In this interaction picture, we find for $\epsilon = 0$ that $A_k^I(t) = A_k^I(0)$ while for finite ϵ we have $\mathrm{d}(A_k^I(t))/\mathrm{d}t = \mathrm{i}[H_I, A_k^I(t)]$. In time-dependent perturbation theory using

$$\frac{A_k^I(t+\Delta t) - A_k^I(t)}{\Delta t} \cong \frac{\mathrm{i}}{\Delta t}\int_t^{t+\Delta t} \mathrm{d}s_1[H_I(s_1), A_k^I(t)]$$

and setting $\frac{1}{f} \ll \Delta t \ll \frac{1}{\epsilon}$ we find that to first order in ϵ, the modes described by A_a and A_b are only coupled to one collective mode, namely the centre-of-mass mode, described by A_M. Therefore, we can ignore the contributions from all other eigenmodes. Shifting the zero of energy again, we finally obtain the simplified set of equations

$$H_{\text{approx}} = \frac{\epsilon}{2}A_a^\dagger A_a + \frac{\epsilon}{2}A_b^\dagger A_b + \frac{2\epsilon}{M}A_M^\dagger A_M - \frac{\epsilon}{2\sqrt{M}}[(A_a + A_b)A_M^\dagger + (A_a^\dagger + A_b^\dagger)A_M]. \tag{8}$$

Now we write this Hamiltonian again in the quadrature components. Defining $P \equiv (P_c, P_a, P_b, P_M)^T$ and $X \equiv (X_c, X_a, X_b, X_M)^T$ we find

$$H_{\text{approx}} = P^T V P + X^T V X,$$

where

$$V = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\epsilon}{2} & 0 & -\frac{\epsilon}{2\sqrt{M}} \\ 0 & 0 & \frac{\epsilon}{2} & -\frac{\epsilon}{2\sqrt{M}} \\ 0 & -\frac{\epsilon}{2\sqrt{M}} & -\frac{\epsilon}{2\sqrt{M}} & \frac{2\epsilon}{M} \end{pmatrix}.$$

The above set of approximate equations of motion can be understood intuitively by a simple mechanical model. Indeed they describe the motion of a very heavy central pendulum (corresponding to the oscillators in the quantum data bus) that is coupled weakly to two comparatively light oscillators. From undergraduate mechanics, we know that if one of the light pendula is initially oscillating then, after some time, it would have stopped oscillating, while the other light pendulum is now oscillating with almost the same amplitude while the heavy central pendulum remains essentially at rest. From this simple mechanical picture, the dynamics in the quantum setting that is described below can be understood quite intuitively.

That the above approximate Hamiltonian represents a good approximation to the true dynamics can be seen from a comparison of the exact time evolution with that generated by the approximate Hamiltonian. In figure 3 we chose a ring with 20 oscillators, $f = 10$ and $\epsilon = 0.015$. The observable mismatch between the frequencies is due to second-order corrections to the approximate model. Furthermore, as the excitation of a quantum in the centre-of-mass mode corresponds to the simultaneous in-phase motion of all the oscillators in the quantum data bus and the fact that the oscillators a and b couple predominantly to the centre-of-mass mode M also explains why the entanglement transfer is distance-independent. The frequency of the centre-of-mass mode is independent of both the coupling strengths between oscillators in the chain and possible disorder in which oscillators are coupled as long as there is a connected path of coupled oscillators between a and b. Therefore, the transmission of quantum information between a and b will depend only weakly on disorder and randomness. The main correction arises when the frequency separation between the lowest two eigenmodes becomes small so that

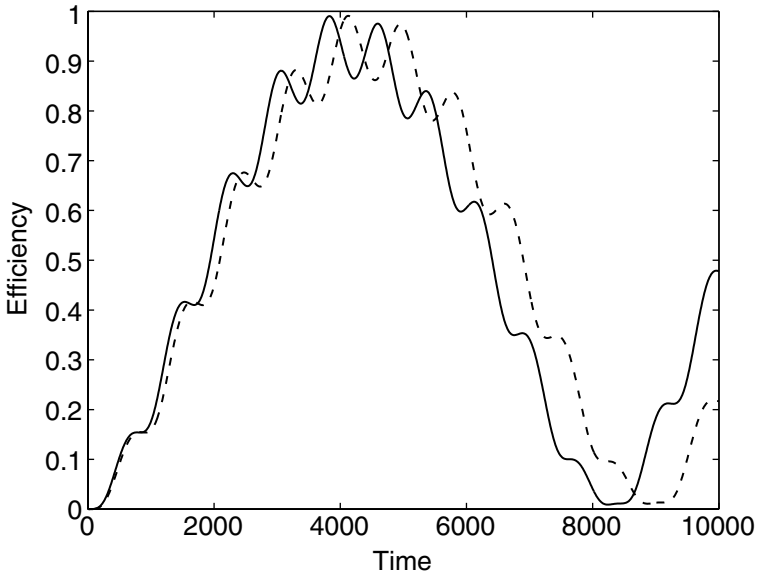


Figure 3. Comparison of the approximation and the exact time evolution for a ring of 20 oscillators with $f = 10$ and $\epsilon = 0.015$. The dotted line is the approximation and the solid line is exact.

off-resonant couplings of the oscillators a and b to modes other than the centre-of-mass mode become non-negligible. In the derivation of equation (8), we have neglected many terms that led to oscillating contributions in the Hamiltonian. These neglected terms will lead to corrections whose size will depend on the length M of the quantum data bus and will affect both the speed of propagation and also its efficiency. As we can always adjust the waiting time, corrections to the propagation speed are less relevant. An efficiency reduction due to population losses is more serious as it will require the application of error correction methods. For the setting described by the Hamiltonian equation (1), we will now present an estimate of the size of these corrections. Indeed, following equation (7) we neglect all terms that couple the modes a and b non-resonantly to eigenmodes different from the centre-of-mass mode. For a small coupling strength ϵ, the rapid oscillations will reduce the population in these modes significantly. The mode, other than the centre-of-mass mode, with the smallest oscillation frequency, will be the mode $M-1$. For large M, the frequency difference to the centre-of-mass mode is given by $\Delta = |\Lambda_{M,M} - \Lambda_{M-1,M-1}| \simeq 2\pi^2 f/M^2$ and the coupling strength to this mode is of the order of $\epsilon/2\sqrt{M}$. The loss of population into this mode, which is of the order of the square of the ratio of coupling strength to frequency separation $\sim (\frac{\epsilon}{2\sqrt{M}})^2(\frac{2\pi^2 f}{M^2})^{-2}$, should be small, i.e.

$$\frac{\epsilon}{f} \ll \frac{4\pi^2}{M^{3/2}}. \tag{9}$$

From equation (8) we find that the relevant timescale for evolution is proportional to $\epsilon/(2M)$. With equation (9) and for a prescribed transfer efficiency the transmission time therefore scales as

$$T \sim \frac{2M}{\epsilon} \sim \frac{M^{5/2}}{2\pi^2 f}. \tag{10}$$

The error source that enters this scaling is the population loss to off-resonant modes. This suggests that this scaling can be improved considerably when one allows for a fixed frequency difference of the oscillators a and b compared to the oscillators in the quantum data bus such that the oscillators a and b become resonant with a different collective mode whose frequency difference to the neighbouring modes is as large as possible. Indeed, if we couple a and b to the mode described by $A_{M/4}$, i.e. if we shift the eigenfrequencies of a and b by $2f$, then we find that the next mode is separated by a frequency difference $\Delta = 2f\pi/M$ and the coupling strength to this neighbouring mode is again of the order of $\epsilon/2\sqrt{M}$. As a consequence, the population loss into other modes is again of order of the square of the ratio of coupling strength to frequency separation to the next mode, i.e. $\sim [\epsilon\sqrt{M}/(4\pi f)]^2$ and we only need to ensure that

$$\frac{\epsilon}{f} \ll \frac{4\pi}{\sqrt{M}}, \tag{11}$$

so that the propagation time for fixed transfer efficiency scales as

$$T \sim \frac{2M}{\epsilon} \sim \frac{M^{3/2}}{2\pi f}. \tag{12}$$

This improved scaling has been achieved by coupling the oscillators a and b, but it should be noted that unlike the centre-of-mass mode M this mode will have nodes. As a consequence, there will be relative positions of oscillators a and b such that they will not couple, namely, when one is sitting at a node while the other is at an anti-node. Indeed, the $M/4$ mode has a node at every second oscillator of the ring, so that the oscillators a and b couple only when their distance is an even number of oscillators.

A question that arises naturally concerns the purity of the entangled state involving oscillators b and c after the distribution. In order to investigate the purity of the state, various measures such as the entropy $S = -\mathrm{tr}\,\rho_{bc}\log_2\,\rho_{bc}$ or the linear entropy $S_L = 1 - \mathrm{tr}\,\rho_{bc}^2$, where ρ_{bc} is the reduced density matrix for the oscillators b and c obtained from tracing out the remaining oscillators, may be used. In the following we will concentrate on the entropy which can be determined directly from the four symplectic eigenvalues μ_i of the covariance matrix describing the state of particles b and c [12]

$$S = \sum_{i=1}^{4}\left(\frac{\mu_i+1}{2}\log\frac{\mu_i+1}{2} - \frac{\mu_i-1}{2}\log\frac{\mu_i-1}{2}\right). \tag{13}$$

In figure 4 we plot the entropy of the state of particles b and c for the same parameters used in figure 3 and one can see that for optimal times it comes close to zero indicating that the transferred entanglement resides in an essentially pure entangled state. This fact further corroborates the view that the data bus is a suitable tool for pure entanglement transfer.

Finally, we demonstrate that the above considerations are not restricted to the continuous variable regime and the properties of Gaussian continuous variable states. Indeed, we can equally consider a situation in which we restrict our dynamics to the subspace spanned by the vacuum and those states that correspond to a single excitation. The derivation of the approximate model starting from the Hamiltonian equation (1) remains of course valid. In the basis represented by the states $\{|00\rangle_{ab}|0\rangle_M, |10\rangle_{ab}|0\rangle_M, |01\rangle_{ab}|0\rangle_M, |00\rangle_{ab}|1\rangle_M\}$, where $|1\rangle_M \equiv \frac{1}{\sqrt{M}}\sum_{r=1}^{M} a_r^\dagger|\mathrm{vacuum}\rangle$,

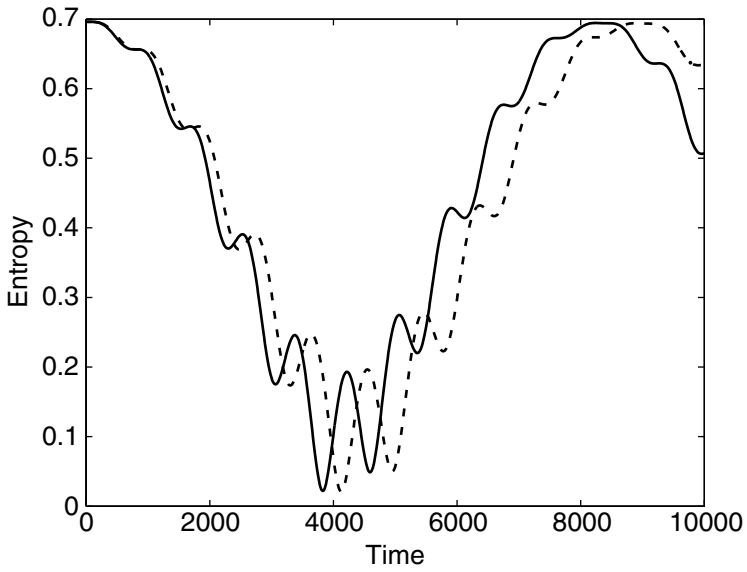


Figure 4. Entropy of the density matrix of the oscillators b and c for the same parameters as in figure 3. Again the dotted line is the approximation and the solid line is exact. The curves approach zero near the optimal time indicating that the entanglement transfer leads to an almost pure state of particles b and c.

we can rewrite the Hamiltonian equation (8) as

$$H_{\text{approx}} = \frac{1}{2}\left(\frac{2\epsilon}{M} - \frac{\epsilon}{2}\right)(\mathbb{1} + \sigma_z^M) - \frac{\epsilon}{4\sqrt{M}}[(\sigma_x^a + \sigma_x^b)\sigma_x^M + (\sigma_y^a + \sigma_y^b)\sigma_y^M], \tag{14}$$

which corresponds to a spin chain with an xy-interaction [13] under the same constraint of considering at most a single excitation. This similarity between a harmonic oscillator systems and a spin chain is not due to the approximations in the derivation of equation (8), but a generic feature when one limits the number of excitations to at most one. A simple computation shows that generally a harmonic chain in the rotating wave-approximation and the single excitation regime will be equivalent to a spin chain with xy-interaction in the same regime.

Continuing in this setting of a spin chain, we now demonstrate that multiparticle entanglement [14] can be generated with a quantum data bus extending the ideas employed in the paper so far. While we focus on the discrete case, i.e. the spin Hamiltonian, one may carry out a similar investigation for the harmonic oscillator case. For the purpose of the generation of entangled states, there is no need of including the decoupled oscillator in the discussion. While the ideas presented below may easily be generalized to many oscillators, let us, for simplicity, consider three oscillators coupled to the chain. We assume that the oscillator a (b, c) couples to oscillator 1 (k_1, k_2) of the quantum data bus with strength ϵ, so that the Hamilton operator of this system is a generalization of equation (14) and is given by

$$H = \frac{1}{2}\left(\frac{3\epsilon}{M} - \frac{\epsilon}{2}\right)(\mathbb{1} + \sigma_z^M) - \frac{\epsilon}{4\sqrt{M}}[(\sigma_x^a + \sigma_x^b + \sigma_x^c)\sigma_x^M + (\sigma_y^a + \sigma_y^b + \sigma_y^c)\sigma_y^M]. \tag{15}$$

Let us now suppose that the initial state of the system is

$$|\psi(0)\rangle = |1\rangle_a|0\rangle_b|0\rangle_c|0\rangle_M \equiv |1000\rangle, \tag{16}$$

where a single excitation is initially present in oscillator a. The evolved state according to the Hamiltonian equation (15) may then be written as

$$|\psi(t)\rangle = a(t)|0001\rangle + b(t)|1000\rangle + c(t)|0100\rangle + d(t)|0010\rangle, \tag{17}$$

where, with the scaled time $\tau \equiv \frac{\epsilon t}{2}$ and the constant $\alpha \equiv \frac{3-M}{M}$, as well as

$$C(M,\tau) = \cos\left(\sqrt{\frac{12+\alpha^2}{M}}\tau/2\right) \qquad \text{and} \qquad S(M,\tau) = \sin\left(\sqrt{\frac{12+\alpha^2}{M}}\tau/2\right), \tag{18}$$

the time-dependent coefficients are given by

$$\begin{aligned} a(\tau) &= \mathrm{i}\frac{\mathrm{e}^{-\mathrm{i}\alpha\tau/2}}{\sqrt{12+M\alpha^2}}C(M,\tau), \\ b(\tau) &= \frac{2}{3} + \frac{\mathrm{e}^{-\mathrm{i}\alpha\tau/2}}{3}\left[C(M,\tau) + \mathrm{i}\frac{S(M,\tau)}{\sqrt{12+M\alpha^2}}\right], \\ c(\tau) &= d(\tau) = -\frac{1}{3} + \frac{\mathrm{e}^{-\mathrm{i}\alpha\tau/2}}{3}\left[C(M,\tau) + \mathrm{i}\frac{S(M,\tau)}{\sqrt{12+M\alpha^2}}\right]. \end{aligned} \tag{19}$$

From equation (19) we can see that in the limit of large M, the coefficient $a(\tau)$ tends to zero indicating that the quantum data bus disentangles from the three oscillators and the result may be a W state of the form

$$|W_{x,y,z}\rangle \otimes |0\rangle_M = (x|100\rangle + y|010\rangle + z|001\rangle) \otimes |0\rangle_M. \tag{20}$$

In order to see that this is the case, we plot the overlap between the W state equation (20) and the evolved state obtained with the use of equations (17) and (19). For the case of 70 oscillators in the chain, and the W state with $x = -y = -z = 1/\sqrt{3}$, it is shown in figure 5 that the overlap is about 96%. The overlap is not complete due to fact that the population of the state $|0001\rangle$ does not vanish and that the coefficients of equation (19) have a small imaginary part. It should be noted that the second of these effects can be reduced significantly by employing subsequent local unitary rotations which are irrelevant when considering entanglement properties of a state. In figure 6 we show the populations as a function of the scaled time τ. A more general situation would allow for the variation of the coupling constants independently. In this case, different W states could be generated by carefully choosing the coupling constants of the three oscillators. The system Hamiltonian for this general setting is given by

$$\begin{aligned} H = {} & \frac{\epsilon_a}{4}(\mathbb{1}+\sigma_z^a) + \frac{\epsilon_b}{4}(\mathbb{1}+\sigma_z^b) + \frac{\epsilon_c}{4}(\mathbb{1}+\sigma_z^c) + \frac{\epsilon_a+\epsilon_b+\epsilon_c}{2M}(\mathbb{1}+\sigma_z^M) - \frac{\epsilon_a}{4\sqrt{M}}(\sigma_x^a\sigma_x^M + \sigma_y^a\sigma_y^M) \\ & - \frac{\epsilon_b}{4\sqrt{M}}(\sigma_x^b\sigma_x^M + \sigma_y^b\sigma_y^M) - \frac{\epsilon_c}{4\sqrt{M}}(\sigma_x^c\sigma_x^M + \sigma_y^c\sigma_y^M). \end{aligned} \tag{21}$$

The procedure for choosing the appropriate parameters for the generation of a particular quantum state is described in the following. By solving the equations of motion using the Hamiltonian (21),

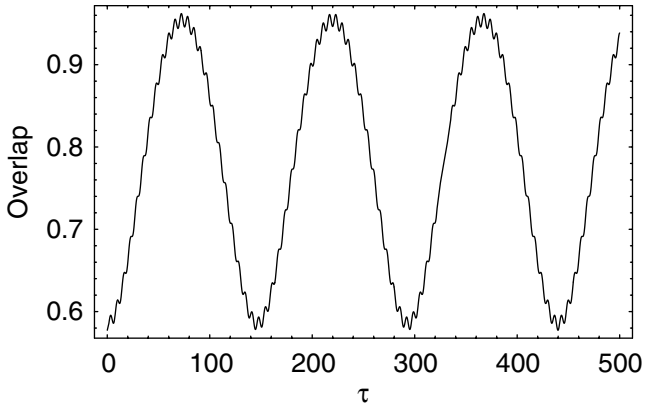


Figure 5. Overlap between the W state with $x = -y = -z = 1/\sqrt{3}$ and the evolved state of the system. The ring consists of 70 oscillators.

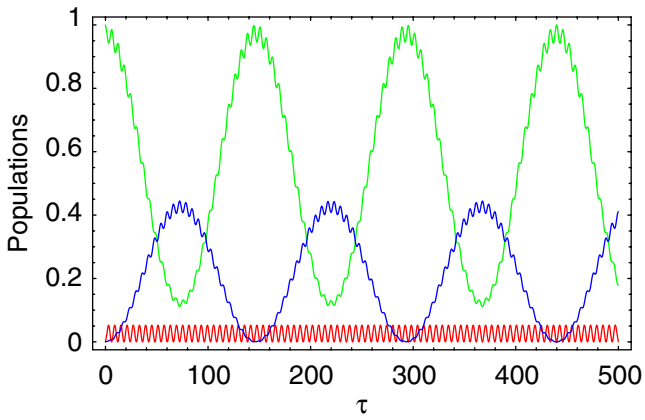


Figure 6. Time evolution of the populations, $|a(\tau)|^2$ in red, $|b(\tau)|^2$ in green and $|c(\tau)|^2 = |d(\tau)|^2$ in blue. The ring consists of 70 oscillators.

one obtains $a(\tau)$, $b(\tau)$, $c(\tau)$ and $d(\tau)$ defined in (17). They appear similar to equations (19) but contain more terms due to the different frequencies involved in the time-evolution. The next step is to choose the state $|W\rangle_{x,y,z}$ imposing specific x, y and z in equation (20) and ensure that they are proportional to $b(\tau)$, $c(\tau)$ and $d(\tau)$, respectively, while at the same time minimizing $a(\tau) = \delta$ with $\delta \ll 1$. The problem is then reduced to numerically solving a system of nonlinear coupled equations (using Newton's method, for instance). Due to the symmetry of the Hamiltonian (21), not all $|W\rangle_{x,y,z}$ can be generated as not all phases between x, y and z can be generated. This is related to the existence of solutions for this nonlinear system of equations. Finally, we also would like to remark that the generation of bipartite and multipartite entanglement in spin chains is also possible using other physical mechanisms such as defects in the XXZ model [15], for instance.

In summary, we have demonstrated that it is possible to transfer quantum information with high efficiency but with minimal spatial and temporal control between arbitrary sites of a translation-invariant chain of quantum systems. We have shown that this process works in the continuous variable regime but also in the single excitation regime when the system becomes equivalent to the dynamics exhibited by a single excitation in a spin chain with xy-interaction.

This interaction may be generalized to include more oscillators coupled to the chain allowing the generation of multiparticle entangled states. All these suggest that translation-invariant chains of interacting quantum systems are promising candidates for the transport of quantum information in solid state realizations of quantum information processing devices.

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