Three Essays in Financial Economics: Systemic Risk, Pension Economics and Income Inequality

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Acknowledgements and Declaration

Originality Declaration Except where otherwise indicated, this thesis is my own original work.

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2. Part III of this thesis is a follow-up work from my previous publication [Biffis et al. (2014) (104)] with my advisor, Dr. Enrico Biffis, Associate Professor at Imperial College Business School. I have worked with Enrico since 2008/2009 on longevity and catastrophe linked securities. I thank Enrico very much for his patience and trust for me to take on this research project. He has also provided generous financial support [through research fund OASIS] and continuous guidance for the development of my entire thesis. I have obtained copyright permission from the publisher [John Wiley and Sons] to reproduce several important figures in this thesis. Permission certificate is attached with thesis. This paper [Biffis et al. (2014) (104)] is available online via several links below:

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Abstract This thesis is composed of three parts, under the main theme of Extreme Value Theory, applied to various areas in financial economics. Tail events are low probability, high impact phenomena. Therefore, understanding distributions of various financial economics dataset enables us to understand the underlying risks.

Part I explores a banking network in an advanced emerging economy, in the post-crisis period. Although static in time, it captures the dynamics among factors such as balance sheet size, interconnectedness and systemic importance. Network structure is examined by classic network theory, with reference to extreme value theory. It worths noting that interbank network structure sometimes exhibits (double) heavy-tailedness, which resembles an un-balanced interconnectedness among nodes (banks). Network structure may have an impact on network (in)stability. This is realized by a simulation, using a defined a sequential default mechanism, taking into account all risks, arising from any single (or multiple) bank(s) default(s). Results show credit risk (often referred as first round effect) does not post threat to the banking sector, as much as fire sales and liquidity shortage (often referred to as secondary effects).

Part II studies the household income and consumption inequality of the country Spain. It firstly estimates the lognormal body, as well as both tails of the income and consumption distributions, across pre, during and post-crisis periods, using household finance data collected by Bank of Spain. It then examines the Income Elasticity of Spending (consumption) on various goods, including (a) food, (b) other non-durable goods, (c) vehicle, (d) other durable goods. Methodology adopted included the random effect and fixed effect models. Results show that income inequality causes heterogeneity in household consumption behaviours on various goods. Household belonged to different income percentiles display asymmetric impacts on their consumption on various goods, due to income inequality caused by crisis in 2007-2008.

Part III studies a UK occupational pension fund. It shows the pension income displays classic Pareto behaviour, as pointed by previous literature studying income distribution. It then shows the negative relationship between pension income and mortality experiences. Applying Expectation Maximization (EM), the pensioner population can be broken down into several sub-populations, according to pension income size. This pensioner bucketing strategy can be used in pension de-risking solution - longevity swaps. In the emerging market for longevity risk transfers, bespoke solutions [swaps written on the pension fund that provides perfect hedges] have so far played a major role, as they allow pension funds and annuity providers to hedge the risk of mortality perfectly. Indexed instruments, such as swaps written on the mortality experience of a reference population, represent a cost-effective alternative to bespoke solutions, but have so far been less popular. One of the main reasons is that they give rise to basis risk, the risk of mismatch between the mortality experience of the hedger and the one tracked by the index. The pensioner bucketing strategy developed in this paper quantifies basis risk, at the same time, shows that the quality of a hedge can only be assessed by jointly considering the cashflows from the hedging instrument and its mark-to-market/model dynamics. Once such broader perspective is adopted, the true magnitude of basis risk can appear substantially lower than expected. Indexed based longevity solution becomes practical.

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Part I.

Financial Networks and Interconnectedness in an Advanced Emerging Market Economy

Chapter 1

Financial Crisis: Causes and Evolution

Financial crises, unfortunately, tend to occur more frequently than expected and seemingly without prior warning. During the post-second world war period, the global economy has been shaken by several financial crises, including but not limited to the debt crisis in Latin America in the 1980s, the breakdown of the European Monetary System and the collapse of the Scandinavian banking system in the early 1990s, the Asian crisis in 1997, followed by the Russian debt default in 1998. The later contributed to the demise of Long Term Capital Management (LTCM), a relative value hedge fund, prompting its bailout by its major private sector counterparties. The bailout, organized by the Federal Reserve Bank of New York, aimed at assuaging concerns about the negative spillovers from a disorderly unwinding of LTCM's highly levered positions and its contagion on related counterparties, through losses arising from direct exposures, and also on unrelated counterparties, from the downward pressure on funding liquidity affecting the market value of securities holdings and the ability to roll over short-term funding (Brunnermeier and Pedersen, 2009 (18)).

The demise of LTCM was akin to the global financial crisis in 2007-8, which highlighted dramatically the risks of interconnectedness pose to the financial system. The origins of the crisis could be traced back to a surge of defaults in the subprime mortgage market in the United States, following a decline in house prices in major urban areas. Given the relatively small size of the market, the policymaking community believed that problems were likely to be contained in the area, as Chairman Bernanke suggested in his testimony to the U.S. Congress in 2007. With the benefit of hindsight, this sanguine assessment ignored several factors, all of them linked to interconnectedness in the financial system, that amplified the initial shocks and led to a major crisis which ultimately engulfed all major financial institutions. These factors included:

1. Increased cross-border linkages, which facilitated major international shock spillovers. Haldane (2009) (45), documents a fourteen-fold increase in cross-border stocks of external assets and liabilities in eighteen advanced economies, measured relative to GDP, from 1985 to 2005.

2. Heavy reliance on complex financial instruments, which contributed to higher leverage in the financial system and the amplification of default losses. In the run-up to the global

recession, the prevailing low rate environment encouraged market participants to boost the profitability of their investment strategies through the use of collateralized lending including repurchase agreement (REPO) and reverse REPO operations, collateralized debt obligations (CDO), mortgage-backed securities (MBS), and asset-backed securities (ABS). This trend, in turn, led to heightened complexity in financial markets, making it more difficult for regulators to assess the extent of exposures and embedded leverage both at the system or macro-level, and at the firm or micro-level.

3. Heightened homogeneity, driven by the adoption of similar investment strategies as market participants searched for yield, which was further encouraged by regulatory requirements inducing similar risk management practices across institutions. In the face of an adverse shock, homogeneous firms react similarly. The ensuing herd behavior could lead to a disorderly unwinding of exposures, and the fire-sale of assets would amplify the initial shock, therefore affect market and funding liquidity adversely. As pointed out by Tiziano et al. (2013) (61), using standardized analysis in a homogeneous network setting would lead to late detection of the 2008 crisis, while the more realistic heterogeneous one would identify early warning signal 3 years earlier before the crisis, using Dutch interbank data.
4. Inadequate understanding of the interaction between credit risk, market risk and funding risk, which prevented market participants and authorities from assessing adequately default contagion and its impact on liquidity and funding conditions. The situation was aggravated by the complexity of the instruments being used, and no prior similar historical experience.

The linkage between financial crises and interconnectedness risk motivates this paper, and the analysis is done in the context of an advanced emerging market economy. We approach the analysis from two different perspectives. The first perspective emphasizes the topological properties of the financial network in the economy; the second perspective uses a simulation approach to analyze default contagion based on balance-sheet network analysis.

1.1. Systemic Risk and Interconnectedness

The concept of systemic risk is largely associated with the fallout of financial crises, with the latter closely associated with banking panics and the resulting drying up of liquidity in the banking system, and its ultimate impact on real economic activity (Friedman and Schwartz, 1963 (38)). Gorton (2010) (41) offered a more recent iteration, accounting for the interaction between the banking system and the shadow banking system. Along this line of reasoning, Franklin and Mishkin (1995) (32) defined a financial crisis as "(an event that) involves sharp declines in asset prices, failures of large financial and non-financial firms, deflation or disinflation, disruptions in foreign exchange markets." This idea feeds naturally in the definition of systemic risk, advanced in Mishkin (1994) (54)) and Bank of International Settlements (BIS, 1994 (12)), as the risk of potential disruptions to the funding channel and/or to the payment system in the event of a financial crisis triggered by the default or failure of a market participant to honor its obligations. This definition highlights interconnectedness as a source of systemic risk. Along

this perspective, the U.S. Commodity Futures Trading Commission defines systemic risk as "the risk that a default by one market participant will have repercussions on other participants due to the interlocking nature of financial markets. Similarly, Kaufman (1995) (49) defined systemic risk as the "chain reaction of falling interconnected dominoes." Finally, the U.S. Federal Reserve relates systemic risk to the disruption of payments system: "... an institution participating on a private large-dollar payments network were unable or unwilling to settle its net debt position ... Serious repercussions could, ... spread to other participants ... , to other depository institutions not participating in the network, and (Board of Governors of the Federal Reserve System (2001) (15)).

But more generally, at least among macroeconomists, disruptions to payment systems or to the financial system may not represent systemic risk, as long as the real economy continues functioning normally. Short-lived stock market crashes such as the one experienced in the United States in October 1987 (Black Monday), or the trillion dollar Flash Crash in May 2010, attributed to high frequency trading (Kirilenko et al, 2014 (50)), did not constitute systemic risk events. Accounting for the impact on the real sector, the Group of Ten (2001) (44) defines systemic risk as "... the event will trigger a loss of economic value ... have significant adverse effects on the real economy." The European Central Bank (ECB), likewise, defines systemic risk as a financial instability risk that is "so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially (ECB, 2010 (31)). Billio et al. (2010) (11) define systemic events as "any set of circumstances that threatens the stability of or public confidence in the financial system" while the G-20 defines systemic risk as "disruptions to financial services... could affect the real economy." We conclude this discussion by noting that under a systemic risk perspective, financial regulation and supervision needs to adopt a macro-prudential approach aimed at reducing risks to the financial system as a whole, instead of the traditional micro-prudential approach aimed at mitigating the risks of individual institutions on a stand-alone basis (see Brunnermeier et al, 2009a).

Once we settle on a definition of systemic risk, the next challenge is to measure it. Bisias et al. (2012)(14) documented more than 30 different ways to measure systemic risk. They can be roughly classified into two categories depending on the nature of the shocks and/or factors affecting the financial system:

1. Type I systemic risk measures assume that financial instability arises endogenously, and can be captured ex-ante by analyzing the interaction and feedback between economic and financial factors. In principle, these measures could help finding early warning signals based on the identification of behavioral patterns in the financial system, including price and trading volume dynamics, that characterizes the build-up of systemic risk.

2. Type II systemic risk measures assume that the failure of single firm or a group of firms generates financial instability though knock-off effects on other firms, due to direct and indirect exposures. Since defaults are typically considered unpredictable events, the measurement of systemic risk in this category relies on what-if scenario analysis attempting to capture the impact of the default of a firm on others.

Within the Type II measures, there are two categories that are of interest from a regulatory

standpoint. The first one is the category of Too-Big-to-Fail (TBTF) measures, which evaluate a firm's size relative to the domestic and/or global financial market where it operates, focusing on its market share and substitutability, i.e. whether other firms could provide similar services at a similar price if the firm were to cease operations. TBTF measures assume implicitly that the failure of a dominant firm could pose a substantial threat to the financial system and the markets where the firm operates, given its linkages to other firms, both financial and non-financial, and the importance of the services it provides, (Laconte, 2015 (52), BCBS, 2013 (10)).

The second category comprises Too-Interconnected-to-Fail (TICTF) measures which attempt to capture the likelihood of the failure of a single firm and its effect on the financial sector and the real economy. A similar concept was mentioned earlier by Battiston et al (2016) (9) as Too Central to Fail (TCTF), which states network topology (formally defined in section 2.2) and node position, or centrality (formally defined in section 2.2.5) matter to system stability; capturing these effects is essential to quantify individual nodes' systemic importance in the network. Despite the implementation difficulties, focus on these type of measures seems justified, as concerns about interconnectedness risk have shaped the response of government officials in recent crisis episodes, as it was the case of the bailout of the insurance firm American International Group, Inc (AIG) in the aftermath of the bankruptcy of Lehman Brothers in September 2008.

The remainder of the paper, which focuses on TICTF measures, is organized as follows. Section 1.2 frames the paper's contribution in the extant literature. Section 1.2.3 presents the key findings. Detailed analysis on network properties can be found in section 2, and default mechanism can be found in section 3. Section 3.3 concludes. The Appendix (Sec 3.3) provides additional information.

1.2. Contribution

1.2.1. Microprudential and Macroprudential Approaches

To better understand systemic risk, it is appropriate to analyze it from both the micro and macroprudential perspectives. At the microprudential level, the systemic importance of an individual firm in the financial network depends on the size of its balance sheet and its capital reserves relative to the firm's overall exposure to other firms (or interbank exposure if we restrict the analysis to the banking system). At the macroprudential level, which emphasizes the stability of the system as a whole, the topological properties of the network, or interconnectedness among the firms, determine how likely domino effects are, and if triggered, what the potential damages to the system are. For simplicity, the defaults that trigger the domino effects are assumed as exogenous events in this paper.

When studying the banking network, we used a *model-free* approach to examine the microprudential aspect of this banking network. This approach assesses how the attributes of individual firms, such as capital buffers and the size of firms, influenced network properties such as the distribution of connectivity measures. As suggested by its name, this *model-free* approach, requires

no modelling of a clearly defined default mechianism or a contagion chain, which makes it particularly useful when detailed balance sheet data and interbank exposure data are unavailable and a specific default contagion mechanism is either unobservable or difficult to discern.

We complemented the microprudential analysis with a macroprudential *model-based* analysis that captured the domino effect or contagion chain triggered by the default of a firm. The default mechanism, based on the observed behavior of financial firms in past crisis events, incorporated various factors contributing to a cascade of failures following an initial default, such as direct counterparty credit exposures, market shocks and liquidity shortages. Hence, the approach captured the macroprudential dimension associated with interconnectedness.

1.2.2. Related Literature

Several strands of the academic literature and current policy debates influence our study. The *model-free* approach builds on the early work on modern network theory of Newman (2003) (58), which set the foundations for the analysis of the topological properties of financial networks. Together with Boss et al. (2004) (16) and Cont et al (2012) (27), this paper is among the few that analyze the empirical structure of a financial system using real-world, highly disaggregated data on the banking sector of an advanced (emerging) market economy. Having access to disaggregated and complete exposures helps the paper stand out from earlier work using aggregate exposure data. By necessity, the latter relied on maximum entropy methods to infer unobserved bilateral interbank exposures from aggregate counterparty exposures (Sheldon and Maurer, 1998 (63); Upper and Worms, 2004 (66); Wells, 2004 (69); Elsinger et al., 2006a,b (34; 35); Degryse and Nguyen, 2007 (29)). But as pointed out by Cont et al (2012), maximum entropy methods typically assume exposures are shared equally among all counterparts, which is at odds with data on reported interbank exposures. As a result, maximum entropy-based studies could underestimate the likelihood and severity of contagion and domino effects in financial networks.

Following the earlier network literature, the *model-based* approach builds on a numerical simulation framework, first developed by Eisenberg and Noe (2001) (33), who assumed a fictitious default mechanism to model domino effects; and Furfine (2003) (40), who assumed a more natural sequential default mechanism, i.e. firms default one by one if the losses incurred exhaust their safety buffers. In Eisenberg and Noe (2001) (33), following an initial default, the existence of a unique equilibrium requires all surviving firms to liquidate their cross holdings to determine their equilibrium values, assuming all liabilities receive equal priority. Cont et al. (2012) (27) argue that such equilibrium is unrealistic. Rather, it may be more appropriate to assume that when firms facing a liquidity shortage unwind their portfolios, the recovery rate could be as low as zero in the short-run. This assumption seems consistent with the protracted and lengthy bankruptcy procedures observed in practice, which forces creditors looking forward to cut their exposures rapidly to accept low recovery rates. The sequential default algorithm of Furfine (2003) (40) bypasses Eisenberg and Noe's equilibrium conditions, modeling instead the potential cascade of failures that a single default could trigger. Furfine applied his algorithm to analyze the U.S. federal funds market. Since this market only comprises around one-seventh

of the U.S. wholesale interbank market, the results might have underestimated systemic risk in the U.S. financial system (Upper and Worm, 2004 (66), Armantier and Copeland, 2012 (5)).

Our analysis goes some steps beyond the earlier literature discussed above. As in previous studies, it captures the systemic risk associated with credit risk driven contagion, or *primary effects*. Following a round of initial defaults, or *fundamental default,* the transmission of the initial shock relies only on the credit losses resulting from the failure of debtor banks to honor their obligations. The initial round of credit losses could cause additional banks to fail, triggering subsequent defaults. But our analysis also looks at the *secondary effects* associated with the funding risk and liquidity shortages when creditor banks demand immediate repayment from failed or stressed debtor banks. The later set of banks, facing higher funding costs, may be forced to sell their assets at fire sale values, which in turn would reduce the fair asset and equity value of banks holding similar assets, creating an adverse positive feedback effect or liquidity spiral (Brunnermeier et al, 2009 (17), Caccioli et al., 2015 (19)). We capture both *primary* and *secondary effects*, the latter linking funding costs and fire-sale losses to the solvency of individual firms, using an extension of the balance sheet network models of Jo (2012)(48) which in turn builds on the earlier work of Chan-Lau (2010)(20).

Last, while this study benefits from data availability on interbank exposures, we would like to discuss briefly alternative approaches based on market price data that could make up for the lack of exposure data. For instance, Acharya et al. (2010) (1) and Zhou et al. (2009) (70)construct systemic risk measures based on high frequency historical market data on credit default swap spreads and/or equity return volatility. Chan-Lau et al (2016) (21) identify systemically important banks using partial correlation networks, where the links between firms correspond to the partial correlation of projected probabilities of default. These projections use information from market prices and balance sheet and income statements. One assumption underlying these measures is that markets are sufficiently efficient, ensuring that market prices capture well information on the risk profile of the firms analyzed, including their potential interaction during periods of distress.

1.2.3. Selected Results

While results will be discussed in detail later, this section offers a glimpse of the main results. In this advanced emerging market economy, the network comprised twenty five banks, and any two banks were pairwise connected if there were counterparty exposures between them. These exposures could be related to any of twenty seven different asset classes (see Table 2.1.2), differentiated by type of contract, currency of denomination, and maturity. In the analysis, we used counterparty exposure data as of end-September 2010.

From the microprudential perspectives, we have examined several properties of this interbank network:

1. Network topology reveals a semi-complete network exhibiting high network density (formally introduced in section 2), which is defined by the number of edges currently exisit in the network, divided by the maximum number of possible edges (Chinazzi et al., 2013

(24)). Higher network density echos completeness, or full interconnectedness, and can be a double-edged sword: Allen and Gale (2000) (3) argue that completeness makes the financial network more resilient by creating multi-dimensional connections among firms, and hence more credit channels; at the same time, this property creates additional transmission channels for shocks during adverse market condition, with one firm's default affecting several firms at the same time. On the other hand, high interconnectedness makes adverse shocks to dissipate quicker. This theory has been realized by Chinazzi et al. (2013) (24), using a multi-layered (disaggregate across assets) international financial network (IFN) data on roughly 70 countries' complete bilateral data during 2001-2010.

2. Compared to other financial networks, banks in this economy tend to cluster closely together, exhibiting the *small world* property. The global clustering coefficient of this network, which measures the number of groups of three banks connected to each other, is around$\sim$0.53, well above what has been found for Austria ($\sim$0.12) (Boss et al, 2004 (16)). Boss et al. (2004) (16) also points out that the Austrian network is a *small-world* network, with *three degrees of separation*, meaning the path lengths among the nodes are smaller than three. Originally described by Watts and Strogatz (1998) (68), a *small-world* network can be created by randomly rewiring several links among the nodes of a regular ring lattice graph; these links become the short-cuts among the originally distant nodes, which creates a short average path length (formally defined in section 2) of the network. Compared to the Austrian interbank network, another emerging market economy, Brazil, described in Cont et al. (2012) (27), presents decaying local clustering coefficient, across nodes with increasing degrees of connection. Cont et al (2012) (27) argue that, a small diameter (path length) is not sufficient to characterize the *small-world* property. Indeed, the Brazilian financial network does not enjoy the *small-world* property since the distribution of the local clustering coefficients across all nodes is not bounded away from zero. In comparison, our network can be characerized as a *small-world* netowrk: it has a significantly higher local clustering coefficient (see Table 2.2.2), only slightly larger path length than that of its random graph counterpart, and *two degrees of separation*, meaning every bank in this network connects to its neighbour through one other bank at most.

3. The financial network exhibits (double) heavy-tailedness (or *scale-free* property): The interbank network can be naturally separated into a core, comprising a reduced number of institutions, and a periphery, comprising the rest, which suggests a multi-money centre structure as described in the early theoretical work by Allen and Gale (2000) (3), and later examined in real world overnight money market network (Fricke et al., 2015 (39)). This is a natural outcome for a network that exhibits a two-sided heavy-tailed distribution, where the degree measures the number of connections a bank has. The left tail comprises banks with a small number of connections and distant from the core, and mainly groups foreign bank branches. The right tail comprises banks with a large number of connections, and mainly comprises domestic banks. Within the banking system, the right tail banks are the most important (systemic) ones from an economic viewpoint. Two-sided heavy tail distributions also support the existence of a *preferential attachment* effect in the network, or in other words, highly connected notes tend to get more connected as the network expands

(Barabasi and Albert, 1997(7)), although later Guido et al. (2002) (43) augue that for some network systems the *scale-free* behaviour has no direct relationship with the preferential attachment when the information of degree connection of nodes is missing. Our network connectivity and exposure distributions are similar to those of Austrian network (Boss et al. (2004) (16)), which also have double heavy-tailedness. In comparison, Brazilian interbank network connectivity and exposure distributions seem to have significant right tail only (Cont et al., 2012) (27). In comparison, German interbank network connectivity and exposure distributions, do not exhibit significant heavy-tailedness (Roukny et al., 2014 (62)). On the other hand, Our study can be compared to Roukny et al (2013), which examines optimal network architecture, by analyzing interplay of several factors such as network topology, capital ratios and market illiquidity, using Italian interbank money market data from 1999-2011. It shows topology matters when market is illiquid, and scale-free network (discussed below in point 3 in section 1.2.3; formerly defined in section 2.3.2) makes the network both more robust and fragile than homogeneous ones.

4. Capital ratio: Using capital ratio requirement proposed by Basel III, all banks in this network seem to have adequate buffer. However, when applying relative exposure measures described in Cont et al. (2012) (27), we identify several institutes at the borderline of capital adequacy. Centrality measures also capture the SIFIs in the network in the debt and credit dimensions; in other words, the important debtors and creditors in the banking system. Alter et al (2015) also used centrality based measure to analyze capital allocation.

5. Size and Centrality are positively correlated in our network. Size as a single predictor have over 60% explanatory power to Centrality, and the result is statistically significant. However, there are several small peripheral (non money center) banks that also have high centrality values, possibly due to the large bilateral exposures, which makes them systemically important in our system. This phenomenon is comparable to the finding of Chinazzi et al. (2013) (24). In their IFN, a node that does not belong to a "rich club", however being central in the system, poses adverse threats in times of crisis.

The microprudential analysis identifies a set of core banks and several peripheral ones systemically more important than others based on their counterparty exposures. The macroprudential analysis then allows us to evaluate whether the failure of a systemic bank could impair the banking system. The sequential default analysis, based on numerical simulations, suggests that despite the core-periphery structure and the incompleteness of the network, bank defaults are likely to remain isolated events and would not trigger a cascade of failures, due to credit risk alone. This result follows from the structure of the banking system, where banks' counterparty exposures are small relative to the assets in their balance sheet, and their capital buffers. Credit losses, or *primary effects*, do not seem to be the main source of losses to banks, but rather the losses associated with the *secondary effects*, or fire sale of assets when firms face liquidity shortages. This finding appears consistent with empirical evidence, albeit yet scant, that fire sale of assets tend to drive losses during crises (Mitchell and Pulvino, 2010 (55); Merrill et al, 2013 (37)), and supports the lender of last resort role played by the central bank. In comparison, it is worth mentioning that Upper and Worm (2004) (66) estimate the bilateral exposure

in the German banking system, using balance sheet information, and measure the credit risk due to bilateral counterparty exposure. Their results show despite the safety net of institutional guarantees for saving banks and cooperative banks, which reduces contagion risks significantly, one single bank default can cause as much as 15% assets destroyed in the entire banking system.

Chapter 2

Banking Network Topology

2.1. Data

Before delving into details, let's formally define a bank's stylized balance sheet properties as in figure 2.1.1. From perspectives of commercial banks, assets include physical assets and financial assets such as reserves, loans, investment securities, government and corporate bonds, etc. Banks' assets also include interbank assets, which are their claims on their counterparties. These generate interest revennues for banks. Liabilities of banks include other entities' claims on banks, such as deposits from savings account, and certificate of deposites, as well as central bank loans and interbank liabilities which are their financial liabilities to their counterparties. In this paper, we focus on systemic risk stems from interbank exposures. One bank's interbank asset is the other's interbank liability.

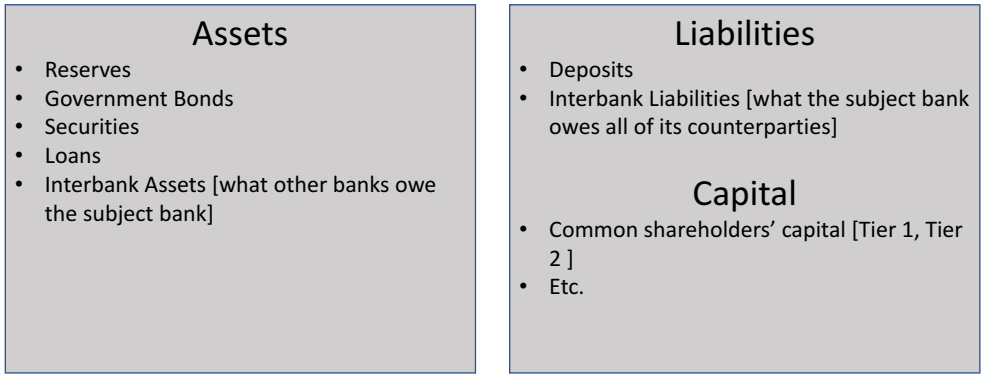
The interbank data used in this paper is collected by Banco Central de Chile (Central Bank of Chile) on a regular basis. The dataset comprises 25 financial institutions, including 12 domestic banks (DBs) and 13 foreign bank branches (FBs) operating in the domestic market. Among the DBs we included several globally operating foreign banks since the nature of their operations resemble those of domestic banks. This choice was justified when later on, the analysis showed their network properties were quite similar to those of domestic banks. Table (2.1.1)shows the market share in the banking sector of domestic banks and foreign bank branches as of end-September 2010. The choice of the time examined in the paper (end of Sep 2010) is due to its resemblance to the financial environment during the crisis period.

Table 2.1.1.: Assets by different types of institutions, end-September 2010

	Sep 2010 in local currency Million (MM)	%
Domestic Banks	95,255,691	81.16%
Foreign bank branches	22,106,349	18.84%
Total Banking system	117,362,039	100%

For these banks there was detailed information on interbank exposures disaggregated into 27 different asset classes as of end September 2010, as shown in Table 2.1.2. The different asset categories distinguish between:

Figure 2.1.1.: Stylized Bank Balance Sheet



- Overnight deposits (denominated in domestic and foreign currencies)
- Short-term deposits up to one year (denominated in domestic and foreign currencies)
- Repos (denominated in domestic and foreign currencies)
- Derivatives (denominated in domestic and foreign currencies)
- Interbank claims (denominated in domestic and foreign currencies)

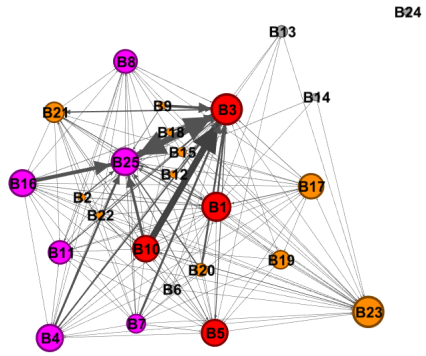
Within these categories, there were additional subcategories corresponding to different maturities: overnight, less or equal to one year, or more than one year. The dataset, hence, consists of a three-dimensional matrix with dimensions $25 \times 25 \times 27$, which is similar to the multi-layerd (disaggregate across assets) IFN on countries' bilateral lending and borrowing relationships in Chinazzi et al. (2013) (24). In addition, we have the capital levels and detailed balance sheet structures of all the institutions in the network shown in Table 2.1.2. In analysis on network connectivity and exposure distribution in Section 2.2, the two-dimensional matrix 25×25 is created by aggregating all 27 asset classes.

Figure 2.1.2, 2.1.3, 2.1.4, 2.1.5, 2.1.6, 2.1.7, 2.1.8, 2.1.9 and 2.1.10 illustrate interbank networks of disaggregate asset classes as described in Table 2.1.2. Figure 2.1.11 provides a graphical visualization of the entire banking network of the country, aggregate across all asset classes in Table 2.1.2. All these network figures are directed (shown by arrows) and weighted by the volumes of transactions. A quick glance at Figure 2.1.11 highlights how closely this network resembles a multi-money center network (Allen and Gale, 2000,(3)), that is, a network where a

Table 2.1.2.: Asset classes, maturity and currency denomination

Exposure type	Maturity	Currency of denomination	Type of instruments
1	Overnight	Domestic, nominal	Checking accounts, other sight deposits, in-due process
2	Overnight	Domestic, UF denominated	
3	Overnight	Foreign currency	
4	Less or equal to a 1 year	Domestic, nominal	Term deposits, in-due-process
5	Less or equal to a 1 year	Domestic, UF denominated	Term deposits, in-due-process
6	Less or equal to a 1 year	Foreign currency	Term deposits, in-due-process
7	More than 1 year	Domestic, nominal	Term deposits, in-due-process
8	More than 1 year	Domestic, UF denominated	Term deposits, in-due-process
9	More than 1 year	Foreign currency	Term deposits, in-due-process
10	Over night	Domestic, nominal	Repos
11	Over night	Domestic, UF denominated	Repos
12	Over night	Foreign currency	Repos
13	Less or equal to 1 year	Domestic, nominal	Repos
14	Less or equal to 1 year	Domestic, UF denominated	Repos
15	Less or equal to 1 year	Foreign currency	Repos
16	Less or equal to 1 year	Domestic, nominal	Derivatives
17	Less or equal to 1 year	Domestic, UF denominated	Derivatives
18	Less or equal to 1 year	Foreign currency	Derivatives
19	More than 1 year	Domestic, nominal	Derivatives
20	More than 1 year	Domestic, UF denominated	Derivatives
21	More than 1 year	Foreign currency	Derivatives
22	Less or equal to 1 year	Domestic, nominal	Interbank claims
23	Less or equal to 1 year	Domestic, UF denominated	Interbank claims
24	Less or equal to 1 year	Foreign currency	Interbank claims
25	More than 1 year	Domestic, nominal	Interbank claims
26	More than 1 year	Domestic, UF denominated	Interbank claims
27	More than 1 year	Foreign currency	Interbank claims

Figure 2.1.2.: Network of Overnight Checking Account and Sight Deposits



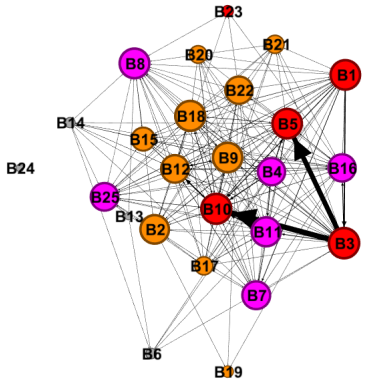
(a) Overnight checking account and sight deposits [Asset type 1-3 in Table 2.1.2]

Key network stats	
Average Degree	8.08
Network Diameter	4
Graph Density	0.337
Avg. Path Length	1.621

(b) Key network stats

few banks, i.e. banks B1, B3, B5, B7, B9 and B10, have a central role connecting all the banks in the network. A couple of banks are just weakly connected to the network, i.e. B13 and B6, while one bank is completely disconnected (B24). While we analyze it formally later, the graphical evidence already suggests a network with a high degree of clustering and density. The node colours indicate the sizes of the banks, with red being the largest banks, with risk-weighted assets (RWA) over 10,000,000 MM, magenta being the ones with RWA over 1,000,000 MM, orange being the ones with RWA over 100,000 MM, and grey are the smallest ones with RWA under 100,000MM. The node sizes are ranked by In-degrees, which is the number of directed links going into the nodes, or number of lenders in our case (formally defined in section 2.2). Node sizes ranked by other criteria such as modularity and degrees (formerly defined in section 2) do not change the rankings significantly. Please also note that in Figure 2.1.5, Overnight Repo exposure network, the links are not in existence; however, the sizes of the nodes are not the same. This is because although there are links among the nodes, their weights are zeros. This may be due to the dataset is a snapshot at a time, and some or all links may exist in historical time. The zero-weighted links exist in other disaggregated and the aggregated network; however, the numbers are not large and most measures we use here are weighted, hence these zero-weighted links do not affect our results. The key network statistics table beside each network figure summarizes key properties for comparison. For formal definitions of these concepts, please refer to section 2.2. All figures 1-10 are produced using software Gephi.

Figure 2.1.3.: Network of Deposit with maturity < 1 yr

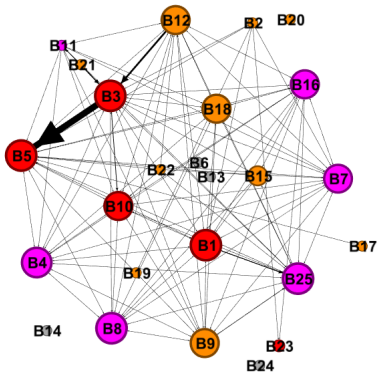


(a) Deposit with Maturity < 1 yr [Asset Type 4-6 in Table 2.1.2]

Key network statistics	
Average Degree	12.4
Network Diameter	3
Graph Density	0.517
Avg. Path Length	1.41

(b) Key network stats

Figure 2.1.4.: Network of deposits with maturity $>$ 1yr

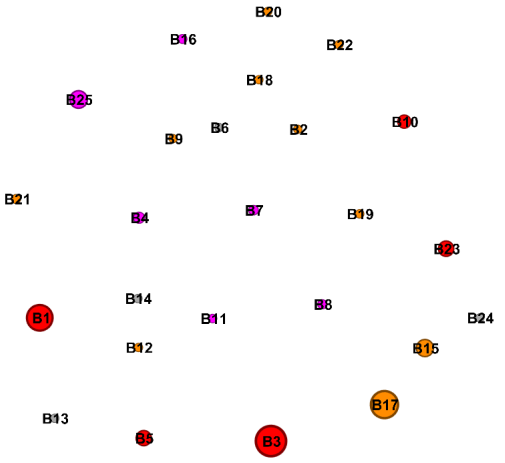


(a) Deposits with maturity $>$ 1 yr [Asset Type 7-9 in Table 2.1.2]

Key network statistics	
Average Degree	6.4
Network Diameter	3
Graph Density	0.267
Avg. Path Length	1.361

(b) Key network stats

Figure 2.1.5.: Network of Overnight Repo exposure



(a) Overnight Repo exposure [Asset type 10-12 in Table 2.1.2]

Key network statistics	
Average Degree	8.08
Network Diameter	4
Graph Density	0.337
Avg. Path Length	1.621

(b) Key network stats. Please note that the links in this network all have weights zero. This may mean the linkes exist in historical times.

Table 2.1.3 shows the banks' regulatory capital ratios, or equity to risk-weighted assets. All banks in our network exceed the minimum regulatory requirement of 8 percent proposed by the Basel Committee (whereas investors typically favour 10% minimum). Government bonds, which carry a zero risk weight, comprise the larger share of assets held by the banks. Following Cont et al (2012) (27) we also report other useful size and several relative exposure (RE) measures. These measures include:

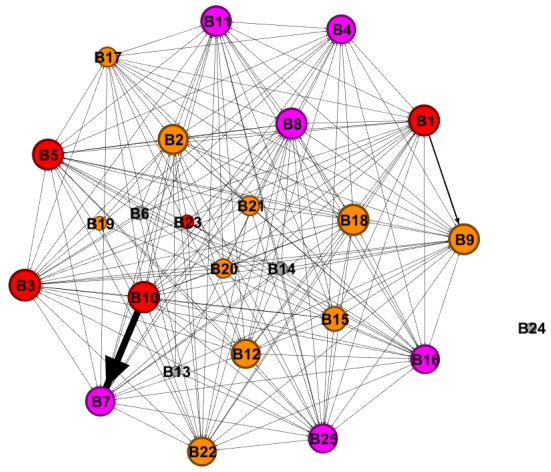
1. Total Capital (TC_i)= Tier 1 + Tier 2 Capital for bank i. E_i is the total exposure of bank i , essentially the total interbank liabilities of bank i in the network. TC = Core Capital + Provisions + Subordinated bonds + Minority Interest. Total Capital Ratio $TCR_i = {}^{TC_i}/_{RWA_i}$. RWA_i is the risk-weighted Assets of bank i.

2. Tier 1 Capital Ratio $T1CR_i = {}^{Tier\,1\,Capital_i}/_{RWA_i}$. Prudential regulations stipulate that this ratio exceeds the minimum regulatory capital requirement.

3. Basic Capital Ratio$BCR_i = {}^{Tier\,1\,Capital_i}/_{A(i)}$. $A(i)$ is the un-weighted assets of bank i.

4. The Cont et al (2012)(27) measures of relative exposure are: (1).$RE_i^1 = {}^{TC_i}/_{E_i}$ (2). $RE_i^2 = {}^{Tier1C_i}/_{max\,Ei,j}$. (3).$RE_i^3 = {}^{Tier1C_i}/_{E_i+RWA_i}$.

Figure 2.1.6.: Network of Repo maturity > 1yr

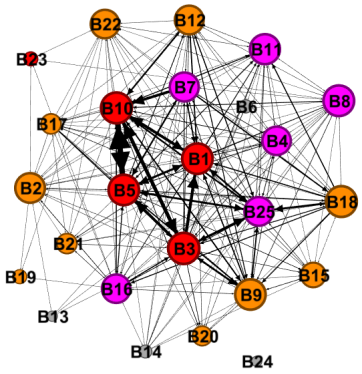


(a) Repo Maturity > 1 yr [Asset type 13-15 in Table 2.1.2]

Key network statistics	
Average Degree	12.4
Network Diameter	3
Graph Density	0.517
Avg. Path Length	1.41

(b) Key network stats

Figure 2.1.7.: Network of Derivative with maturity < 1 yr

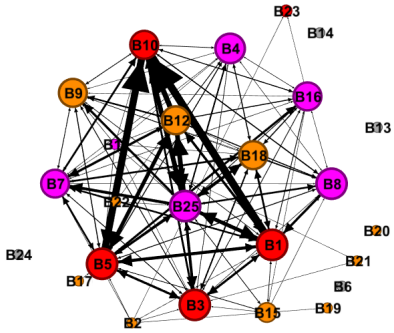


(a) Derivatives with maturity < 1 yr [Asset type 16-18 in Table 2.1.2]

Key network statistics	
Average Degree	12.4
Network Diameter	3
Graph Density	0.517
Avg. Path Length	1.41

(b) Key network stats

Figure 2.1.8.: Network of Derivatives with maturity > 1 yr

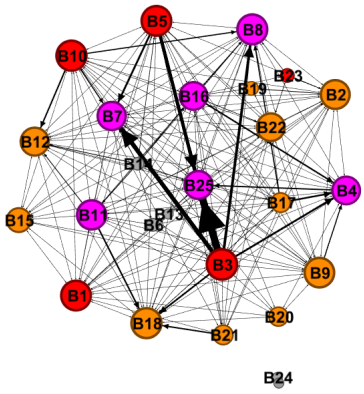


(a) Derivatives with maturity > 1 yr [Asset type 19-21 in Table 2.1.2]

Key network statistics	
Average Degree	6.4
Network Diameter	3
Graph Density	0.267
Avg. Path Length	1.361

(b) Key network stats

Figure 2.1.9.: Network of Interbank claims with maturity < 1yr

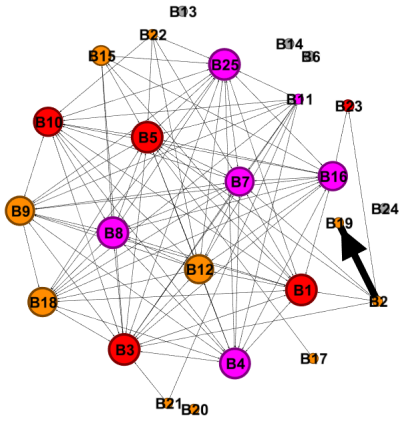


(a) Interbank claims with maturity < 1 yr [Asset type 22-24 in Table 2.1.2]

Key network statistics	
Average Degree	12.4
Network Diameter	3
Graph Density	0.517
Avg. Path Length	1.41

(b) Key network stats

Figure 2.1.10.: Network of Interbank claims withmaturity > 1 year

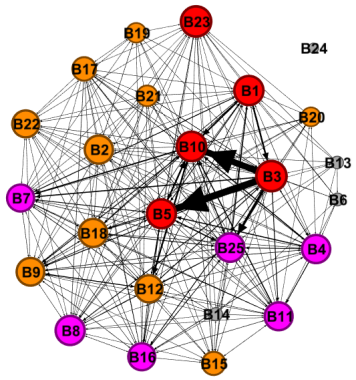


(a) Interbank claims withmaturity > 1 year [Asset type 25-27 in Table 2.1.2]

Key network statistics	
Average Degree	6.4
Network Diameter	3
Graph Density	0.267
Avg. Path Length	1.361

(b) Key network stats

Figure 2.1.11.: Aggregate interbank asset exposure [Aggregate all assets in Table 2.1.2]



(a) Aggregate Network Visualization

Key network statistics	
Average Degree	14.76
Network Diameter	2
Graph Density	0.583
Avg. Path Length	1.332

(b) Key Network Stats

5. We calculate bank size as percent of assets in the whole banking network, as comparison. Please note B24 has no interbank exposure.

Conflicting conclusions can be drawn from observations: B12, 13 and 14 have very large TCR, and T1CR ratios. This is likely because they held large amount of government bonds as assets, which carried zero or low risks. Non-targeted capital ratios adopted by regulators (TCR, T1CR and BCR) do not show insufficient capital buffer in the banking network. However, relative exposure measure RE^1 show B3, 9, 12, 14 and 18 have relatively thin capital (RE^1<1), indicating the capital not enough to cover its total exposure. Criteria ($RE^2 > 1$) shows all banks have sufficient capital to cover its largest counterparty exposure. Using RE^3 as an alternative capital ratio to target high concentration nodes, there are quite a few nodes that have relatively thin capital ratio ($RE^3 < 10\%$), e.g., B1, 2, 3, 5, 7, 8, 16, 20 and 25. Among these nodes, B1, 3, 5, 10 are large in size (each of which with $Size\% > 10\%$ of the total banking system), and in total they account for almost 70% of the total assets.

2.2. Network Properties

2.2.1. Degrees of Connection

This section analyzes the topological characteristics of the interbank network. An interbank network can be modeled as a weighted directed graph, defined as two dimensional variable $I = (V, E)$,where:

- V is a set of financial institutions, each is denoted by number $n = 1, 2, 3, ...$
- E denotes a 25×25 matrix that represents a bilateral exposure data between each pair of the counterparties. This matrix is created by aggregating all 27 asset classes in Table 2.1.2. To be clear, the calculation of network properties in this section refer to this aggregated interbank network matrix. $E_{i,j}$ represents the exposure of bank i on bank j, in other words, how much bank j owes bank i. It is the short term maximum loss of bank i caused by an exogenous default of bank j. In interbank network studies, interbank asset of bank i often refers to the total exposure of it, which is the total amount of all its counterparties owe it; the total interbank liabilities of bank i often refers to the opposite, which is the total amount of bank i owes all of its counterparties.

In a directed graph, the *degree* of a node is defined as the number of edges directed towards or originated from a node. If the edge originates from a node it is counted as $out - degree$, K_{out}, if the edge is directed towards the node, it is counted as $in - degree$, K_{in}. For $i, j \in V$,

- $K_{in}(i) = \sum_{j \in V} \mathbf{1}_{E_{ij}>0}$ measures the number of institution i's debtors.
- $K_{out}(i) = \sum_{j \in V} \mathbf{1}_{E_{ji}>0}$ measures the number of institution i's creditors.
- The degree of connectivity is measured by $K_i = K_{in}(i) + K_{out}(i)$.
- The total interbank assets for institution i is denoted as $A(i) = \sum_{j \in V} E_{ij}$
- The total interbank liabilities for institution i is denoted as $L(i) = \sum_{j \in V} E_{ji}$

The summary statistics are presented in the table below.

Table 2.1.3.: Capital and Relative Exposure Measures, end-September 2010. Critical values are highlighted: Blue highlights mean sufficient or large. Red highlights mean insufficient or small. We **bold font** the Domestic Banks (**DBs**), most of which are the core (nodes that have large connectivity and exposure) of the network shown in Figure 2.1.11. Here, for simplicity, we define money centre with connectivity of 75 percentile or above. We denote the core with a star (*). Please note that some foreign bank branches that have strong presence in this economy also act like core, e.g. B4*. They are consistent in the paper, in Table 2.2.2, Figure 3.2.1 and Figure 3.2.3.

	TCR (%)	T1CR(%)	BCR (%)	RE_i^1	RE_i^2	RE_i^3(%)	Size(%)
B1*	13.64	8.79	6.72	5.6295	14.58	**8.58**	**17.45**
B2*	11.74	7.43	5.24	1.8454	6.39	**6.99**	**0.75**
B3*	11.85	8.01	4.8	**0.9941**	1.96	**7.16**	**16.50**
B4*	15.42	12.60	9.09	2.8797	10.02	11.96	4.73
B5*	13.64	9.38	7.01	4.4535	10.64	**9.10**	**12.24**
B6	**59.21**	**58.95**	**54.18**	**19.8519**	67.14	**57.25**	**0.03**
B7*	12.82	8.76	6.76	4.5874	14.58	**8.52**	6.46
B8	12.22	9.23	6.74	8.8715	24.03	**9.11**	2.77
B9	13.45	13.45	6.37	**0.4977**	2.52	10.59	1.25
B10*	14.52	10.50	7.2	5.2123	18.80	10.22	**20.80**
B11*	13.88	12.50	9.08	3.3736	6.38	12.01	3.23
B12	**126.39**	**126.39**	29.75	**0.7344**	3.41	**46.45**	**0.47**
B13	**124.36**	**124.36**	64.55	2.6744	5.46	**84.89**	**0.02**
B14	**115.79**	**115.79**	22.83	**0.7847**	4.32	**46.77**	**0.08**
B15	**88.91**	**88.78**	**37.97**	**3.6271**	11.65	**71.30**	**0.29**
B16*	12.96	8.77	6.22	2.2496	8.17	**8.29**	2.75
B17	19.37	14.08	10.27	1.9690	5.78	12.81	**0.83**
B18*	**28.9**	**28.90**	8.26	**0.4861**	2.98	18.12	**0.86**
B19	16.44	16.42	13.36	**76.4129**	86.48	16.38	**0.21**
B20	10.75	10.75	6.4	1.3462	3.18	**9.96**	**0.51**
B21	25.52	25.52	13.68	1.2815	3.14	21.28	**0.33**
B22	47.12	47.40	19.42	2.8666	9.16	**40.67**	**0.22**
B23	15.04	15.04	13.06	3.3545	13.06	14.40	**0.17**
B24	**192.43**	**192.43**	70.53	-	-	**192.43**	**0.16**
B25*	13.13	8.57	6.26	3.3710	9.54	**8.25**	6.88

Table 2.2.1.: Summary of connectivity measures

Summary	$k_{in}(i)$	$k_{out}(i)$	$k(i)$	***A(i) MM$***	***L(i) MM$***
Mean	14	14	28	171500.65	171500.65
Standard deviation	7	6	12	281567.60	242075.25
Minimum	0	0	0	0.00	0.00
10th percentile	3	5	8	846.36	185.82
30th percentile	10	11	22	28043.90	8811.81
50th percentile	17	16	33	70042.82	85702.52
75th percentile	18	19	36	219226.28	234782.75
Maximum	23	22	45	1374956.94	929915.91

2.2.2. Network characteristics

In this section, we defined a common framework for a network of size of n nodes (or vertices) and m existing links. The maximum possible number of edges exist in this network is $n(n-1)$. In this section onwards, all network characteristics calculation are defined on the network by aggregating all assets (Figure 2.1.11).

Density: *Density* (defined as *den*) refers to the ratio of the number of existing links m to the maximum number of possible edges $n(n-1)$ (Chinazzi et al., 2013 (24)). The *connectivity* of a network, defined as *den*, is the unconditional probability that two nodes share one link. $den = \frac{m}{n(n-1)}$. $0 \leq den \leq 1$. At its smallest value, 0, the network is completely disconnected. At its peak value, 1, the network is a complete network, meaning every node has interbank exposure in both directions (in and out) to every other node in the network. For example, in the aggregated network of all assets (shown in figure 2.1.11), $n = 25$, $m = 350$, $den = \frac{m}{n(n-1)} = \frac{350}{25\times24} = 0.5833$.

Reciprocity: *Reciprocity* (defined as r) refers to the probability of any out-going link is reciprocated. The *weighted reciprocity* (defined as $wgt - r$) is considered with the volume of transaction (Chinazzi et al., 2013 (24)). In the aggreagated interbank network (shown in figure 2.1.11), binary reciprocity $r = \frac{89}{350} = 25.43\%$. This means about one quarter of the links are reciprocated. Chinazzi et al. (2013) (24) considers reciprocity a measure of (a)symmetry: the lower the value of r or $wgt - r$, the more asymmetric the network is, hence the more unbalance in the bilateral relationship.

Distance *Distance* (defined as d) refers to the length of the shortest path between any two nodes. For example, *distance* between node i and node j, defined as $d_{i,j}$, which is the length of the shortest path between them. If there is a link directed from node i to node j, $d_{i,j} = 1$. Table 3.3.1 shows the shortest paths between all pairwise combinations of banks. Since $d_{i,j}$ for most pair of banks is either 1 or 2, the banking network is well connected.

Eccentricity and Diameter: The *eccentricity* of node i represents the maximum *distance* (d) to any other node in the network, is denoted as $\varepsilon_i = max_j d_{ij}$. In other words, this is the number of interim counterparties node i needs to connect to in order to connect to node$j$. In our

network, since bank 24 is not accessible for all the other banks in the network, the *eccentricity* of any bank i becomes ∞. In this case, the conventional solution is to exclude the pair(s) of nodes that are not connected in the calculation of APL (Newman (2003) (58) p181). From hereafter, we calculate network characteristics excluding B24 where appropriate. Hence, if we consider a sub-graph of our aggregated network (shown in figure 2.1.11) except for node 24, the eccentricity of each bank i can be calculated (shown in Table 3.3.1). The *diameter* of a network is defined as $Dia = max_i \varepsilon_{ij}$. Similarly to the case of eccentricity, since bank 24 is not accessible from other banks, Dia is ∞. Hence, we consider a sub-graph of our aggregated network (shown in figure 2.1.11) except for node 24, $Dia = 2$. The results are presented in Table 2.2.2.

Shortest Path Length The *shortest path length* (SPL) of node i within the network is defined as $l_i = {}^1/_{(n-1)} \sum_{j \neq i} d_{i,j}$. In the aggregated banking network (shown in figure 2.1.11), the average path length is Inf, as there is a disconnected bank, B24. Similar to the case of *eccentricity* and *Diameter*, we form a sub network excluding B24, the average path length for each node i, l_i, can be calculated, as shown in Table 2.2.2. The average shortest path length (APL) for all nodes in this connected sub-network (excluding B24), defined as $\bar{L} = \frac{1}{n} \sum l_i$, is **1.3659**. This is slightly larger than its random graph counterpart (a random graph with the same number of nodes) $L_{random,n=24,k=14}$ =1.2042. Please note that this is only descriptive, not statistically significance. This is not strictly defined in previous academic literature.

2.2.3. Clustering Coefficient

Classic graph theory refers *clustering coefficient* to the degree to which vertices in a graph cluster together, that is to concentrate in groups with a high density of links. Empirical research show that in most real-world complex networks such as genetic networks, social networks and internet, vertices tend to have higher clustering coefficients than in the random network of Erdős and Rényi (1959), or ER network(36) (Watts and Strogatz, 1998 (68)). There are several different measures of clustering coefficients:

Global Clustering Coefficient It measures the overall clustering effect in the network (see Luce and Perry (1949) (53) and Wasserman and Faust (1994) (67)). It is calculated as follows:

$$C_{global} = \frac{3\times\text{number of triangles}}{\text{number of connected triplets of vertices}} = \frac{\text{number of closed triplets}}{\text{number of connected triplets of vertices}}$$

The *global clustering coefficient* result for our network (excluding node 24) is **0.5279.** This shows our network is higher clustered compared to the Austrian network analyzed by Boss et al. (2004) (16), which has $C_{global} = 0.12 \pm 0.01$ (mean and standard deviation across 10 datasets).

Local Clustering Coefficient (LC1) It is a local version of the above global clustering coefficient measure (Newman, 2003 (58)) and defined as follows: $LC1_i = \frac{\text{number of triangles connected to vertex i}}{\text{number of triples centered on vertex i}}$

Then the average LC1 is $L\bar{C}1 = \frac{1}{n} \sum_{i=1}^{n} LC1_i$. Results are presented in Table 2.2.2 (LC1).

Local Clustering Coefficient (LC2) This is another measures of local clustering coefficient developed by Watts and Strogatz (1998) (68), which is calculated as follows: Suppose that a vertex i has k_i neighbours; then at most $k_i(k_{i-1})/2$ edges can exist between each pair of

two neighbours in the network. In other words, that is when every neighbour of vertex i is connected to every other neighbour of vertex i. Let $LC2_i$ denote the fraction of these maximum possible edges that actually exist, $LC2_i = \frac{m}{k_i \times (k_i - 1)/2}$, and average local clustering coefficient, $L\bar{C}2 = \frac{1}{n}\sum_{i=1}^{n} LC2_i$

Please note that B24 is not accessible at all, hence $LC2_{24}$ does not exist.

Table 2.2.2 (LC2) presents the results for the local clustering coefficients. The average local clustering coefficient of our network is 0.8267 (except for bank 24). Including Bank 24, the average local clustering coefficient is 0.7936. This is significantly higher than if the graph is a random graph of the same vertex set as $LC2_{random} \approx \frac{k}{n}$. $LC2_{random,n=25,k=14} = 0.56$, $LC2_{random,n=24,k=14} = 0.5833$. A network of n nodes and k links is a *small-world* network if it has approximately the same APL, and significantly higher local clustering coefficient as defined by $L\bar{C}2$ above, which can be expressed as $\bar{L}_{n,k} \gtrsim \bar{L}_{random,\,n\,k}$, and $L\bar{C}2_{n,\,k} > L\bar{C}2_{random\,n\,k}$.

Weighted Clustering Coefficient Previous three clustering coefficient measures (C_{global}, $LC1$ and $LC2$) are so far unweighted measures, analyzed using binary adjacency matrix of the aggregated interbank network data. Defined by Barrat et al. (2004) (8) and Giorgio (2007) (42), weighted (local) clustering coefficient for node i is defined as: $lc_i^w = \frac{1}{s_i(k_i-1)}\sum_{j,\,h}\frac{(w_{ij}+w_{ih})}{2}a_{ij}a_{ih}a_{jh}$

where $s_i = \sum_{j=1}^{N} a_{ij}w_{ij}$ which measures node strength by extending the degrees of node $i(K_i)$

a_{ij} is the adjacency matrix of the network, whose elements are binary, with 1 means connection exists from node i to j, and 0 otherwise.

w_{ij} is the actual volume from node i to j.

The normalization factor $\frac{1}{s_i(k_i-1)}$ calculates the fraction of maximum possible triplets the weighted edge may participate, and hence ensures $0 < lc_i^w < 1$. Results are shown in 2.2.2. Please note that in our case, average weighted local clustering coefficient, $\bar{lc} = \frac{1}{n}\sum_{i=1}^{n} lc_i^w = 0.3806$, which is larger than the unweighted local clustering coefficient defined as $L\bar{C}1$, 0.1775. According to Barrat et al (2004) (8), in very large random (uncorrelated) network, $L\bar{C}1 \approx \bar{lc}$. In our case, weighted coefficient is larger than the unweighted one, $\bar{lc} > L\bar{C}1$. This means the clustering effects (intercoonected triplets) are more likely to be formed by higher weighted edges, vice versa. This reflects the high heterogeneity in weights of edges (Exposures) which will be discussed in section 2.3.2 in details.

2.2.4. Assortativity

Assortativity mixing or homophily (ρ, and in some previous literature called *pearson coefficient*) in a network with n nodes and m edges is defined as follows in Newman (2002) (57) as follows:

$\rho = \frac{m^{-1}\sum_i j_i k_i - [m^{-1}\sum_i \frac{1}{2}(j_i+k_i)]^2}{m^{-1}\sum_i \frac{1}{2}(j_i^2+k_i^2) - [m^{-1}\sum_i \frac{1}{2}(j_i+k_i)]^2}$

where j_i, k_i are the degrees of vertices at the ends of the ith edge, with $i = 1, \ldots m$. In the aggregated network shown in fugure 2.1.11, $\rho = -0.1607$. A negative ρ implies dis-assortative

mixing, or disassortivity, which is a phenomenon that high-degree nodes tend to connect to low-degree ones. Disassortativity is known in technology and biological networks, whereas assortative mixing, meaning highly connected nodes tend to connected to highly connected ones, exists in social networks (Newman, 2002 (57)). It is worth mentioning that our result (disassortativity) is similar to the German credit and derivative network presented in Roukny et al (2014) (62).

A **weighted** version of this coefficient, defined by Leung and Chau (2006) (23) as belows:

$\rho^w = \frac{H^{-1}\sum_i w_i j_i k_i - [H^{-1}\sum_i \frac{1}{2} w_i (j_i + k_i)]^2}{H^{-1}\sum_i \frac{1}{2} w_i (j_i + k_i^2) - [H^{-1}\sum_i \frac{1}{2} w_i (j_i + k_i)]^2}$

$0 < \rho^w < 1$. w_i is the volumn of ith link, H is the total weight of all links in the network. If ρ^w is positive, then the network is weighted assortative, which means nodes with similar weighted links tend to be connected to each other. If ρ^w is negative, the network is disassortative. In the aggregated network, $\rho^w = 0.1805$. This is consistent with the findings in Leung and Chau (2006) (23) that although unweighted assortativity can be positive and negative for different types of network, all networks are tend to be weighted assortative, that is $\rho^w > 0$ for most networks discovered. Moreover, if $\rho^w > \rho$, a high weighted link tend to connect to two similar degree nodes together. This is the so-called "coarse-graining" network, resembling a structure with several core nodes (money centers) connected by high weighted links, while each of the money center connects to multiple pheripheral nodes with only light weighted links.

2.2.5. Centrality

Eigenvector centrality is measured as follows:

Consider a graph defined in section 2.2.1, $I = (V, E)$ with n number of nodes or vertices, where $E_{i,j}$ is exposure of bank i on bank j, or how much bank j owes bank i. Let $A = (a_{i,j})$ be the adjacency matrix. If $a_{i,j} = 1$, vertex i is linked to vertex j, and $a_{i,j} = 0$ otherwise. The weighted adjacency matrix represent cashflows between each pair of the vertex, e.g.

Suppose that a vertex i has k_i neighbours . The centrality score of vertex i can be defined as: $x_i = \frac{1}{\lambda}\sum_{j \in k(i)} x_j = \frac{1}{\lambda}\sum_{t \in G} a_{i,j} x_j$ where $k(i)$ is a set of the neighbors of i and λ is a constant. This can be rearranged and rewritten in vector notation as the eigenvector equation $\mathbf{Ax} = \lambda\mathbf{x}$. Centrality measures the relative importance of nodes within a network.

We calculate several centrality measures as follows (all are shown in Table 2.2.2):

(1) Eigenvector Centrality of un-weighted and un-directed graph $C^1_{undir/w}$: This centrality measure assesses each of the banks' importance score using the un-directed, un-weighted adjacency matrix. It is symmetric and square and where there is a link between vertex i and vertex j, $a_{i,j} = 1$, otherwise, 0. However, this measure is not sensitive to weights,

(2) Eigenvector Centrality of gross bilateral exposure matrix C^2_g: This centrality measure assesses each of the banks' importance score using the weighted adjacency matrix of gross exposure, each of whose element is the total interbank asset and liability of each vertex i: $a_{i,j} = E_{i,j} + E_{j,i}$. This matrix is symmetric and square. However, this measures does not reveal the net exposure information.

(3) Eigenvector Centrality of net bilateral exposure matrix C_n^3: This centrality measure assesses each of the banks' importance score using the weighted adjacency matrix of net exposure, each of whose element is the net interbank asset or liability of each vertex i: $a_{i,j} =\mid E_{i,j} - E_{j,i} \mid$. This matrix is symmetric and square. However, this measures reports the bank's importance within the network in the debtor and creditor space, as long as it has a net exposure, either it is debt or credit.

(4) - (7) Binary Authority and Hub (defined as $C_{b,aut}^4$ and $C_{b,hub}^5$) scores, and their weighted versions (defined as $C_{w,aut}^6$ and $C_{w,hub}^7$), defined as belows:

Ideally our goal is to analyze the banks' importance in the debtor and creditor space separately. However, for asymmetric matrices resembling directed network, eigenvector centrality is known to give inaccurate results. HITS (hyperlink-Induced Topic Search) algorithm developed by Kleinberg in 1990s (51) can be employed as folows:

Hubs in our interbank network are the banks lend out to many good authority banks. In other words, important **creditors**. **Authorities** (authority banks) in our interbank network are those who receive funds from good hubs. In other words, the important **debtors**. Each node i in our directed network is assigned an **Authority weights** , defined as x_i, and a **Hub weight,** defined as y_i, both of which have initial arbitray non-zero values. Then weights are updated as follows:

$x_i^{(k)} = \sum_{j:(j,i)\in E} y_i^{(k-1)}$ and $y_i^{(k)} = \sum_{j:(i,j)} x_j^{(k)}$ for $k = 1, 2, 3 \ldots$. Please note that k is the iterative round.

Weights are normalized as $\sum_j (x_j^{(k)})^2 = 1$ and $\sum_j (y_j^{(k)})^2 = 1$. Both vectors $x^{(k)}$ and $y^{(k)}$ converge as $k \longrightarrow \infty$. In summary, HITS calculates the largest nonnegative eigenvectors of the matrices of $A^T A$ (authority matrix) and AA^T (hub matrix).

According to Benzi et al. (2013) (13), a reformulation can improve the HITS estimates as follows: Let $\mathbb{A} = \begin{pmatrix} 0 & A \\ A^T & 0 \end{pmatrix}$. Note $\mathbb{A}$ is symmetric and square, $\mathbb{A}^T = \mathbb{A}$. $\mathbb{A}^T\mathbb{A} = \mathbb{A}\mathbb{A}^{TT}$. $\mathbb{A}\mathbb{A}^T = \mathbb{A}^2$. and $\mathbb{A} \in \mathbb{R}^{2n\times 2n}$.

$\mathbb{A}^2 = \begin{pmatrix} AA^T & 0 \\ 0 & A^T A \end{pmatrix}, \mathbb{A}^3 = \begin{pmatrix} 0 & AA^T A \\ A^T AA^T & 0 \end{pmatrix}., \ldots,$ $\mathbb{A}^{2k} = \begin{pmatrix} AA^T & 0 \\ 0 & A^T A \end{pmatrix}$; $\mathbb{A}^{2k+1} = \begin{pmatrix} 0 & A(A^T A)^k \\ (A^T A)^k A^T & 0 \end{pmatrix}$.

Appling HITS to $\mathbb{A}$, the dominant eigenvector (defined as $u^{(k)} = \begin{pmatrix} y^{(k)} \\ x^{(k)} \end{pmatrix}$ for $k = 1, 2, 3 \ldots$).

Each of the vector $y^{(k)}$ and $x^{(k)}$ are of size $n \times 1$. $y^{(n)}$, the first n entries of $u^{(k)}$ is the hub scores of each node i, and $x^{(k)}$, the last n entries of $u^{(k)}$ is the authority scores of each node i.

The weighted version results are obtained from applying this method to weighted adjacency matrix.

Table 2.2.2.: Network Property Summary Table. Significant values are highlighted in red. Eccentricity (Ecc). Average Path Length (APL): Core money centres of the network (APL<1.20) include B1, 4, 5, 8, 10, 11, 25. The rest are peripheral nodes. Local clustering coefficients (LC1 and LC2). Centrality measures $C^1 - C^7$: importance rankings of banks in creditor and debit space, using gross and net weighted, and binary and weighted adjacency matrices. Most significant ones are highlighted in **red**, and secondary importance are highlighted in **blue**.

	Ecc.	APL	LC1	LC2	lc_i^w	$C^1_{undir/w}$	C^2_g	C^3_n	$C^4_{b,aut}$	$C^5_{b,hub}$	$C^6_{w,aut}$	$C^7_{w,hub}$
B1*	2	**1.0435**	0.1696	0.8211	0.2747	0.2524	**0.2014**	**0.1258**	0.1832	0.1796	**0.0413**	**0.1050**
B2*	2	1.2174	0.205	0.9265	0.4539	0.2259	0.0154	0.0033	0.1661	0.1628	0.0047	0.0061
B3*	1	1	0.154	0.7100	0.1753	0.2524	**0.6012**	**0.6779**	0.1877	0.1787	**0.0365**	**0.6737**
B4*	2	**1.1739**	0.2112	0.9000	0.3107	0.2165	0.0996	**0.0957**	0.1638	0.1521	**0.0315**	**0.0799**
B5*	2	**1.1304**	0.1795	0.7895	0.6487	0.2483	**0.4513**	**0.5206**	0.1775	0.1759	**0.4837**	**0.0897**
B6	2	1.8261	0.0972	1.0000	0.3235	0.0785	0.0004	0.0003	0.0287	0.0535	0.0001	0.0001
B7*	2	1.2609	0.1857	0.8480	0.5468	0.2355	**0.1484**	0.0729	0.1639	0.1731	**0.0730**	**0.0513**
B8	2	**1.1304**	0.2472	0.9619	0.6792	0.2232	0.0417	0.0416	0.1705	0.1515	**0.0345**	0.0090
B9	2	1.2174	0.2268	0.9167	0.5346	0.2136	0.1229	0.0239	0.1577	0.1572	**0.0499**	**0.0441**
B10*	2	**1.1304**	0.1776	0.8211	0.6137	0.2421	**0.4885**	**0.4468**	0.1765	0.1800	**0.4640**	**0.0788**
B11*	2	**1.1739**	0.2095	0.8382	0.4456	0.2334	0.0695	0.0703	0.1745	0.1579	**0.0381**	**0.0427**
B12	2	1.2609	0.2379	0.9619	0.5088	0.2129	**0.1577**	0.0596	0.1637	0.1519	**0.0367**	**0.0585**
B13	2	1.913	0.0238	0.8000	0.0001	0.0732	0.0008	0.0007	0.0214	0.0449	0.0000	0.0008
B14	2	1.8696	0.0321	0.8444	0.0088	0.1212	0.0048	0.0059	0.0276	0.0935	0.0000	0.0040
B15	2	1.4348	0.2482	0.9273	0.3965	0.1649	0.0159	0.0109	0.1272	0.1067	0.0043	**0.0113**
B16*	2	1.2609	0.1849	0.8480	0.4120	0.2297	0.0681	0.0445	0.1640	0.1723	**0.0379**	**0.0258**
B17	2	1.4348	0.129	0.8596	0.0861	0.2308	0.0159	0.0137	0.1180	0.1786	0.0034	0.0076
B18*	2	1.2174	0.2317	0.9667	0.5420	0.2228	0.1147	0.0359	0.1701	0.1586	**0.0555**	**0.0403**
B19	2	1.6087	0.3516	1.0000	0.8859	0.1422	0.0003	0.0002	0.0877	0.0454	0.0001	0.0000
B20	2	1.6522	0.1333	1.0000	0.2310	0.1330	0.0172	0.0151	0.0818	0.0747	0.0034	**0.0100**
B21	2	1.5652	0.1759	1.0000	0.2517	0.1725	0.0195	0.0129	0.1046	0.1319	0.0070	0.0035
B22	2	1.3043	0.1989	0.8750	0.4872	0.2220	0.0098	0.0051	0.1542	0.1616	0.0039	0.0047
B23	1	1	0.2316	0.9286	0.1011	0.1405	0.0011	0.0014	0.0861	0.0740	0.0000	0.0013
B24	Inf	-	0	-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
B25*	2	**1.1304**	0.1956	0.8954	0.5969	0.2408	**0.2407**	**0.1570**	0.1778	0.1698	**0.1741**	**0.0429**
mean(ex. B24)	1.9167	1.3315	0.1849	0.8576	-	-	-	-				
mean (inc. B24)	Inf	-	0.1775	-	0.3806	-	-	-				

2.3. Distribution of connectivity and size measures

2.3.1. Parametric distribution fitting

Analyzing the distribution of connectivity and exposure further our understanding on inter-bank network structure. For K_{in} ,K_{out} , K and Exposure A we fit the following parametric distributions:

A. Lognormal distribution with mean $\mu$and standard deviation σ

pdf: $y = f(x|\mu,\, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(ln\, x-\mu)^2}{2\sigma^2}}$

B. Extreme Value (EV) Distribution: location parameter μ and scale parameter σ;

pdf [1]: $y = f(x|\mu,\, \sigma) = \sigma^{-1}\, exp(\frac{x-\mu}{\sigma})\, exp(-exp(\frac{x-\mu}{\sigma}))$

C. Weibull Distribution: location parameter a and scale parameter b. Please note that if T has a Weibull distribution with parameters a and b, then $log(T)$ has an extreme value distribution with parameters $\mu = log(a)$ and $\sigma = 1/b$.

pdf: $y = f(x|a,\, b) = \frac{b}{a}(\frac{x}{a})^{b-1} e^{-(x/a)^b}$

D. Truncated Pareto distribution fitting, which is a semi-parametric fitting process, discussed in detail in the section 2.3.2

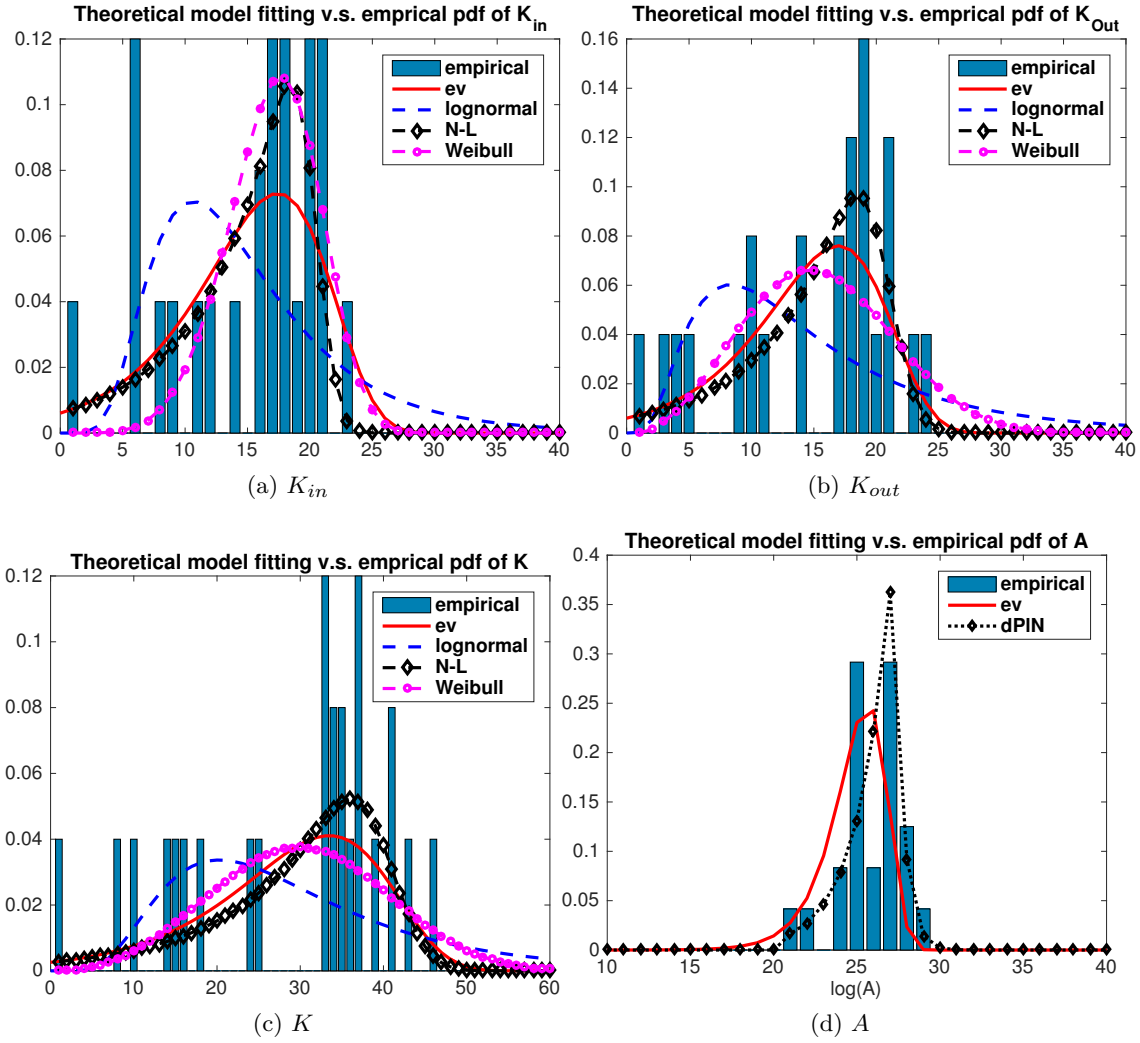
E. Normal-Laplace (N-L) and Double Pareto Lognormal (dPlN) Distributions, discussed in detail in the Appendix.

Figure 2.3.1 shows the fitted distributions with the empirical pdf, while the estimated parameters are presented in Table 2.3.1. The data suggests, as demonstrated by the titling point in the plots reported in Figure 2.3.2 that the dPlN distribution best fit the data. This distribution falls somewhere in between lognormal and Pareto distribution (see Reed and Jorgensen, 2004, (101) and the statistical appendix). Our choice of the dPlN distribution is inspired by the core-peripheral network structure displayed in Figure 2.1.11, suggesting possible two tail in both ends of the connectivity and exposure distributions. This distribution was first discovered by Mitzenmacher (2001) that "have a lognormal body and Pareto tail" (Reed and Jorgensen, 2004) (101).

Equation 3.3.24 and Equation3.3.25 show the limiting cases for the paretian tail indices when the distribution approaches right (∞) and left (0), respectively. The results of our network distribution are presented in Table 2.3.2d. We can view these results as the paretian tail index estimates in the limiting cases (should the network expands), and they are comparable to the results presented in the truncated Pareto case in Section 2.3.2. This is an evidence of possible community structure and hierarchy within our network.

[1] This form of EV distribution is suitable to measure the minimum values of a distribution whose tail decays exponentially fast, such as normal distribution. To model the largest values, one may use the negative of the original values.

Figure 2.3.1.: Fitted lognormal v.s. empirical pdf of $k_{in}(i)$,$k_{out}(i)$, $k(i)$ and Exposure A $(in\, 10^{10}\, local\, currency)$



(a) K_{in} (b) K_{out}

(c) K (d) A

Table 2.3.1.: Distribution parameter estimates of K_{in}, K_{out}, K and exposure A_i (*in 10^{10} of domestic currency). CI refers to confidence intervals. *nll* refers to negative log-likelihood. The returned value of $AD\,h = 0$ is the hypothesis test result, which indicates that Anderson-Darling test fails to reject the null hypothesis of the respective assumption of distributions at the default 5% significance level, and vice versa. *cv* refers to AD critical values.

	μ	$CI_{\alpha=5\%}$	σ	$CI_{\alpha=5\%}$	nll	AD h	$p_{AD,\,5\%}$	AD statistics	cv
K_{in}	2.7298	[2.5894, 2.8702]	0.3084	[0.2360, 0.4453]	61.9218	1	0.009	1.0073	0.7553
K_{out}	2.5354	[2.2622, 2.8085]	0.6469	[0.5028, 0.9075]	83.9508	1	0.0092	1.0131	0.7273
K	3.2712	[3.0547, 3.4879]	0.5129	[0.3986, 0.7195]	96.0405	1	0.0037	1.1668	0.7262
A	24.7964	[23.9938, 25.599]	1.9007	[1.4772, 2.6662]	644.0808	1	0.0000	2.8928	0.7262

(a) Log-normal distribution fitting: As expected at 5% significant level, A-D test reject the hypothesis of lognormal distribution for K_{in} , K_{out} , K and A.

	a	$CI_{\alpha=5\%}$	b	$CI_{\alpha=5\%}$	nll	AD h	$p_{AD,\,5\%}$	AD stat	cv
K_{in}	17.4285	[15.9514, 19.0424]	5.0391	[3.5145, 7.2250]	57.9402	1	0.0026	1.2296	0.7387
K_{out}	16.2917	[13.9821, 18.9827]	2.71171	[1.9056, 3.8589]	77.5211	1	0.0013	1.3490	0.7393
K	32.6124	[28.5729, 37.2229]	3.1478	[2.2181, 4.4671]	91.1131	1	0.0000	1.4109	0.7387
A	13.6895*	[7.3731*, 25.4171*]	0.6807	[0.4989, 0.9288]	642.1669	0	0.7832	0.2440	0.7387

(b) Weibull distribution fitting: The A-D test shows strong evidence to reject the null hypothesis of Weibull distribution in the case of K_{in} , K_{out} , K. However, fail to reject the hypothesis of Weibull distribution in the case of Exposure A. We further tests goodness-of-fit of the extreme value distribution to the distribution of log(A) in tab 2.3.2c.

	μ	$CI_{\alpha=5\%}$	σ	$CI_{\alpha=5\%}$	nll	AD h	$p_{AD,\,5\%}$	AD stat	cv
K_{in}	16.662	[14.883 18.441]	4.3245	[3.1260 5.9827]	76.9965	0	0.1163	0.5984	0.7393
K_{out}	16.9521	[14.9619, 18.9422]	4.8353	[3.5034, 6.6735]	79.6616	0	0.1172	0.5972	0.7393
K	33.4891	[29.8050, 37.1733]	8.9545	[6.4834, 12.3676]	95.1290	0	0.0511	0.7358	0.7393
A	34.3361*	[15.2187*, 53.4534*]	45.6317*	[35.9967*, 57.8456*]	705.5769	1	0.0000	4.8015	0.7393
$log(A)$	25.6425	[25.0237, 26.2613]	1.4691	[1.0767, 2.0044]	47.0536	0	0.7832	0.2440	0.7387

(c) Extreme value distribution fitting: The A-D test does not reject the null hypothesis of extreme value distribution with moderate evidence (p-value at 5%) in the cases of K_{in} , K_{out} , K, whereas, it rejects the null hypothesis in the case of Exposure A, with strong evidence. For the distribution of log(A), A-D test shows no evidence of rejection. Together with evidence shown in sub-table 2.3.2b, it is intuitive that this specific class of extreme value distribution fits well with the logarithmic form of the exposure distribution, and the distributions of in and out-degree and connectivity.

	α	β	ν	τ	nlog-likelihood
K_{in} **(NL)**	3.9189	0.1604	20.1082	1.3260	457.3975
K_{out} **(NL)**	2.9628	0.1604	20.4802	1.7901	273.0211
K **(NL)**	1.1735	0.0887	39.5836	3.3891	235.0851
A (dPlN)	1.8732	0.5210	26.1820	0.1651	1.6204e+03

(d) N-L and dPlN distributions fitting using MLE. Please note that, as we understand, and evidences shown in sub-table 2.3.2b and 2.3.2c, we are fitting distribuiton of in and out-degree and connectivity to Normal-Laplace distribution, and we are fitting Exposure to Double Pareto Lognormal distribution. Please note that dPlN is the exponential form of the N-L distribution.

2.3.2. Semi-parametric heavy-tail distribution fitting

The failure to fit a lognormal distribution to the connectivity measures steers the analysis towards the detection of heavy tails, or the so-called *scale-free* property, which can be assessed from an analysis of log-log plots of the complementary cumulative distribution function (CCDF), which is simply given by $\mathbb{P}(K_{out} \geq k)$, $\mathbb{P}(K_{in} \geq k)$, $\mathbb{P}(K \geq k)$ and $\mathbb{P}(A \geq a)$ for the out-degree, in-degree, degrees of connectivity and exposure respectively. It shows the *scale-free* property occurs quite often in nature, i.e. earthquake intensity, and man-made phenomena, i.e. city population, income distribution, etc. Many known large complex networks such as internet, actors collaboration, genetic networks, exhibit *scale-free,* or *power-law* distribution in their connectivity. This is due to two generic characteristics: (i) networks expand continuously as new vertices enter the system, and (ii) new vertices have *preferential attachment*s (7)). In interbank networks the *scale-free* property is present in the Austrian and Brazilian networks (see Boss et al. (2004) (16) and Cont et al. (2012) (27)). *Preferential attachments,* if shown in presence in historical time, in interbank network refers to the phenomenon that new banks entering the network tend to connect to banks that already have more links to other neighbours. Guido et al. (2012) argue that *preferential attachment* is not related to *scale-free* properties by origin, strictly speaking. They proposed a mechanism by firstly assigning each node an importance rank, accounting for its *intrinsic fitness,* which is defined by some characters such as influence, or degree connectivity, then simulate network formation process by various network systems such as BA or ER models. These lead to the result that even non-scale-free fitness distribution network can generate scale-free networks. This mechanism is particularly useful when the degree connectivity is not publicly available. While this is not the focus of our paper, we would like to conduct future work when the historical data becomes available.

In a *scale-free* network, the first two moments of the distribution, the mean and standard deviation, are not enough to represent the empirical distribution of the data since the density of the distribution is heavily weighed in the right tail, as implied by the *power-law* distribution: $p(x) \sim x^{-\alpha}$, where α is the *exponent* or *scaling* parameter. Normally, $2 < \alpha < 3$, with some exceptions. The analysis of empirical datasets finds that above a minimum threshold value, x_{min}, the *tail* of the distribution follows a *power-law* (Choromanski et al., 2013 (25); Onnela et al., 2007(59)).

Detecting heavy tails and estimating the parameters of a power law distribution is relatively difficult. Traditional least squared fitting, developed by Pareto's work in 19th century (Arnold, 1983 (6)), can produce inaccurate results. Clauset et al (2009) (26) have suggested combining maximum likelihood estimation (MLE) with goodness-of-fit tests such as Kolmogorov-Smirnov (K-S) statistic and likelihood ratios.

We follow Clauset et al. (2009) (26) in our analysis, and use MLS with K-S Statistics, with adjustment to finite sample. Table 2.3.2 shows the key statistics, and Figure 2.3.2 shows the log-log plots of the CCDF of K_{in},K_{out},K and exposures$A$, and the corresponding fitted power law distribution. According to Clauset et al. (2009) (26), when the sample size $n \lesssim 100$, and p-values for the tested distributions are above our threshold of **0.1**, the power-law hypothesis cannot be ruled out. Our sample size, however, is very small which makes difficult for the

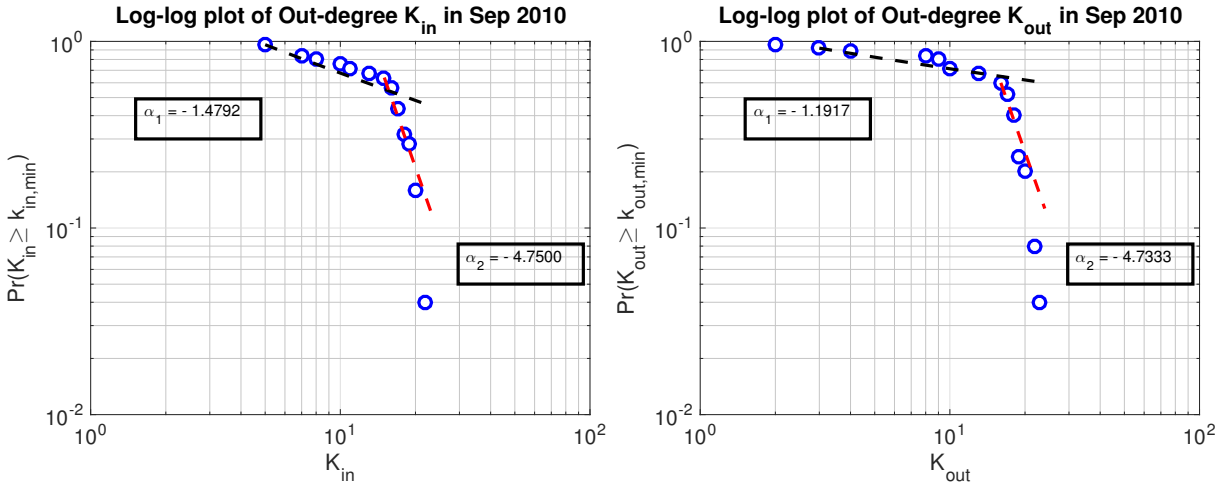
Table 2.3.2.: Key Statistics of MLE and K-S for $K_{in}(i)$,$K_{out}(i)$, $K(i)$ and$A(i)$ (**in 10^8 local currency). This table is accompanying Figure 2.3.2. $k_{in,\,min}$, $k_{out,\,min}$, k_{min} are selected by the fitted distributions that have the minimum of K-S goodness-of-fit statistics (D). We use the hatted parameters for estimation of the (unhatted) true (unobserved) variables; subscript 1 and 2 denote left and right tail statistics respectively: $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the estimated left and right tail indices, $\hat{\sigma}(\hat{\alpha}_1)$ and $\hat{\alpha}_2$ are the estimated standard deviation of $\hat{\alpha}_1$ and $\hat{\alpha}_2$. This is equivalent to the Hill (1975) (46) estimator. $Limit_1$ is the upper limit of the data analysed as this is a truncated Pareto distribution (selected based on observation of Figure 2.3.2), and it is only defined for the left tail in this test. The right tail is the natural upper limit of the entire distribution. L_1 and L_2 are the log-likelihood estimates for left tail and right tail distribution. p_1 and p_2 are the p-values of K-S test (when larger than **0.1** - significant*). Large $\hat{\alpha}$ and $\hat{\sigma}(\hat{\alpha})$ may be caused by the small sample bias.

Left tail statistics					Right tail statistics				
	$k_{in}(i)$	$k_{out}(i)$	$k(i)$	A_i^{**}		$k_{in}(i)$	$k_{out}(i)$	$k(i)$	A_i^{**}
$\hat{\alpha}_1$	1.4792	1.1917	1.2875	1.1949	$\hat{\alpha}_2$	4.7500	4.7333	4.7500	2.5421
$\hat{\sigma}(\hat{\alpha}_1)$	0.3038	0.6825	0.6685	1.1554	$\hat{\sigma}(\hat{\alpha}_2)$	0.6787	0.6542	0.6744	1.1813
$\hat{k}_{in,min\,1}$	5	2	7	4**	$\hat{k}_{in,min\,2}$	15	16	32	1630.1722
$Limit_1$	12	15	30	1630.1722**	$Limit_2$	-	-	-	-
L_1	-92.2793	-110.2908	-110.5894	-215.2695	L_2	-58.4769	-45.9618	-44.0562	-61.4525
p_1	**0.7430***	**0.2150***	**0.3030***	0.0000	p_2	0.002	0.007	0.0660	**0.9430***
D_1	0.4479	0.5791	0.3189	0.3646	D_2	0.1882	0.1940	0.2423	0.1005

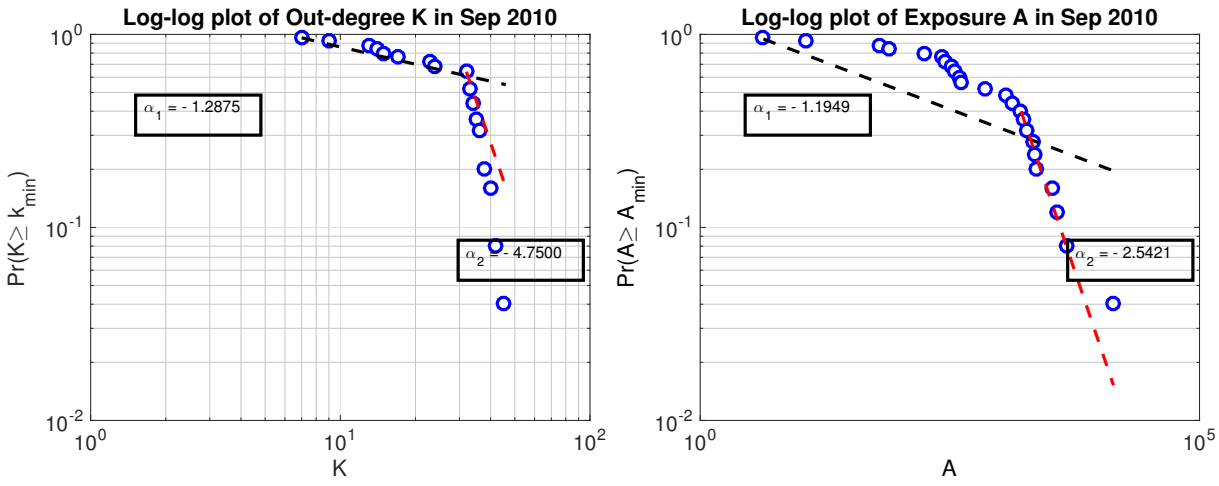
test to single out the true power-law distribution from others, e.g. exponential, lognormal etc. This problem disappears as the sample size increases since the p-values for the non-power-law distributions drop off. Hence, even if the dataset is sampled from a power law distribution, the small data sample prevents a good fit, which is evident by the large standard deviations of the parameter estimates in Table 2.3.2. Nonetheless, the clear and distinct double tail features shown in Figure 2.3.2, exhibit two regions in connectivity and exposure measures that could be fitted with power. Overall, the tail indices in our interbank network where prominent with statistical significance (K-S test p value > **0.1***).

In addition to connectivity and exposures, we also analyze size effect. It appears to have a single right tail only. Result is shown in Figure 2.3.3. The Pareto index for size is around 1.4267, with a K-S p value = 0.1741, which is statistically significant.

Figure 2.3.2.: Log log plot of ccdf of in-degree, out-degree and degree of connectivity and exposure in Sep 2010



(a) K_{in} (b) K_{out}

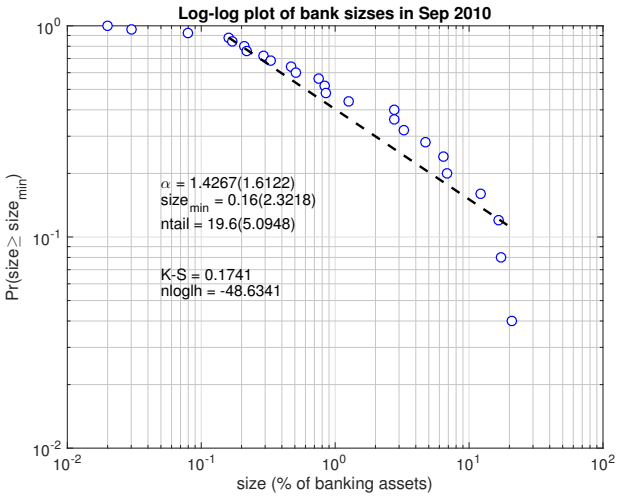


(c) K (d) Exposure (A)

2.4. Size and Centrality

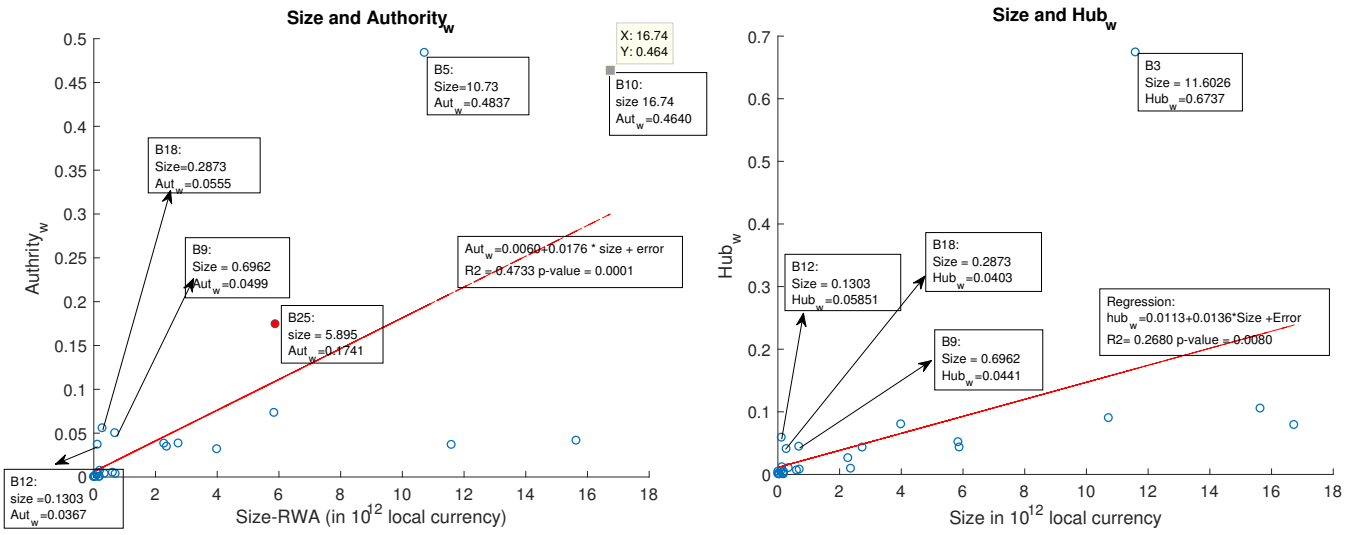
Some previous papers argue centrality to be a better measure of systemic risk than size: TICTF is more appropriate than TBTF. We found empirical evidence of strong correlation between size and centrality. In this section, we analyze the correlation between bank size (as shown in Table 2.1.3), weighted Authority and Hub scores, C_w^6 in Table 2.2.2). Size is correlated with authority and hub ranking scores, with statistical significance. The important message here is that the banks in the northwest corner of the sub-figures below (for example, B9, B12 and B18), are

Figure 2.3.3.: Pareto Analysis of Bank Size (% of assets). Numbers in brackets are standard deviations of the estimated parameters.



smaller but central - important debtors and creditors.

Figure 2.4.1.: Size and Debtor and Creditor Importance Ranking



(a) Size and Authority (Debtor) Ranking (C_w^6 in Table 2.2.2) Regression

(b) Size and Hub (Creditor) Ranking (C_w^7 in Table 2.2.2) Regression

Chapter 3

Systemic Risk Analysis in a Balance-Sheet Framework

While an examination of the topological properties of the interbank network is useful to determine the degree of interconnectedness among banks, and to make qualitative inferences about systemic risk, it is not enough to quantify systemic risk. Henceforth, we shift the focus to the analysis of systemic risk within a balance-sheet network framework, which can capture how connectivity and exposure affect the transmission of fundamental defaults within the network.

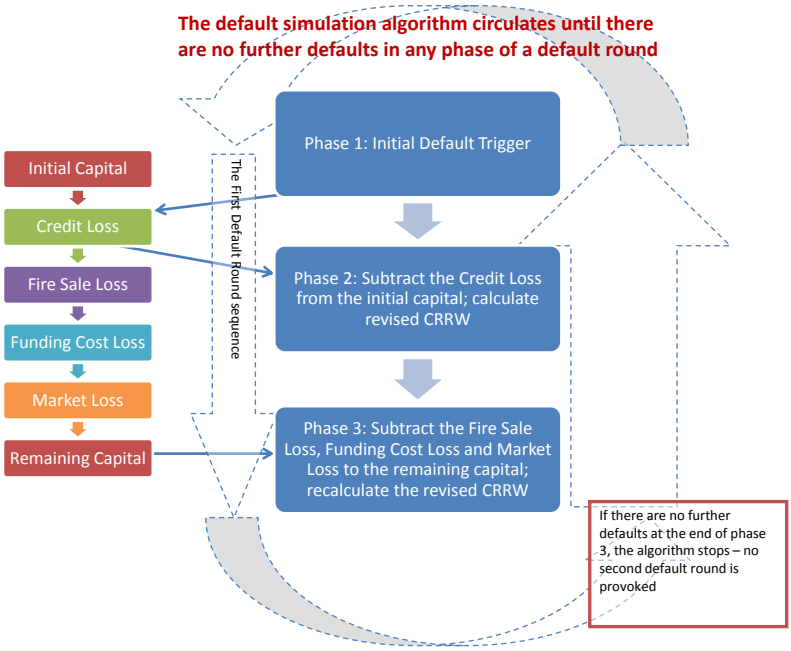
3.1. Default mechanism

A default event occurs when a bank is unable to fulfill its financial obligation, i.e. failure to repay principal or interests on contractual obligations, or inability to provide a scheduled loan as agreed with a counterparty. In theory, a institution fails when it becomes insolvent, i.e. its capital buffer is wiped out by credit and/or market losses. These losses can be triggered by exogenous events or macroeconomic shocks affecting the institution's earnings.

However, corrective action frameworks and central bank intervention may prevent insolvency, while still triggering contagion (Chan-Lau and Sy, 2007 (22)). This suggests that for practical purposes, a default event is better defined as the point when the capital reserves of a bank fall below a minimum requirement established by the regulatory authority. In our model, default is defined as failure to meet an regulatory minimum capital requirement of 7 percent.

Related Literature: It worths to mention that there are mainly three methods to model default dependence structure: (1) *Conditionally independent defaults* (CID) approach, which generates credit risk dependence among firms through the dependence of the firms' intensity processes, use a common set of state variables. Its drawback is that its generated default rate is lower than empirical one. Extensions of CID models by Duffie and Singleton (1999b) incorporate joint jump diffusion into default dependencies. However, this approach makes the model hard to calibrate. (2) The second approach is the *contagion models* extending the CID model to incorporate empirical default clustering experiences. The intuition behind this is that there are period of time when there are default clustering events taking place, possibly due to their financial interconnectedness (Jarrow and Yu 2001), or generally economy states deterioate (Davis and Lo 1999). (3) *Copula approach*: Copula approach Rogge and Schonbucher takes a marginal default probabilities of a group of companies as default dependence structure.

Figure 3.1.1.: Default contagion mechanism. CRRW (Capital ratio risk weighted)



these periods of default clustering is the existence of credit risk contagion mechanisms between firms, either because of their commercial or financial relationships (Jarrow and Yu 2001) or simply because the general level of default risk in the economy has increased (Davis and Lo 1999). Contagion models try to include, on the specification of default intensities, the existence of such sources of contagion.

The balance-sheet network analysis presented here follows Jo (2012) (48), which expands on Chan-Lau (2010) (20). The key differences between Jo (2012) and our work here are as follows: (1) Jo (2012) analyzed intrasectoral default contagion effects among sectors (domestic banks, foreign banks, credit agencies and insurance companies, etc.), while our analysis uses interbank exposure data collected by central bank without estimation. (2) Jo (2012) assumes recovery rate is 0 due to slow bankcruptcy procedure as suggested by Cont et al (2012). In our future work, we may relax this assumption by varying the degree of the recovery rate in our future work. We may also consider different degrees of market illiquidity by varying the estimations of fund replacement coefficient(γ in step 5 below) and incremental funding cost coefficient (μ, defined in step 6 below).

The default simulation, illustrated in Figure 3.1.1 runs as follows. We follow notations in Jo (2012) and present only key equations for readers' convenience. For details, please refer to section 2.2 on page 6-12 of Jo (2012)):

- A set of fundamentally defaulted institution $D = \{i : C_i - L_i < C_i^r(1 + b_i)\}$. C_i - capital held by the institution i; L_i - the institution's loss; C_i^r- minimum regulatory required

capital for i, and b_i - additional capital buffer.

- Credit loss of institute i is CL_i, which arises with counterparty defaults: $CL_i = \sum_{h \in D} E_{ih}\delta)$, where the *loss given default (LGD)*, δ is computed through exogenous default mechanism by Chan-Lau (2010) (20), later this assumptions is relaxed and we compute other scenarios when LGD equals 0, as suggested by Cont et al (2012) (27) due to slow bankruptcy procedure.
- Funding losses FL_i: To repay claims from defaulted (creditor) institutions, debtor institutions may be able to refinance mainly through two sources: (1) sell part of its assets, or (2) get fundings from other alternative creditors. It makes a difference when switching market regimes: Normal market conditions assumes debtors refinance easily. However, stress scenarios complicate re-financing process due to additional funding cost. The FL_i is computed as follows:

1. After initial *fundamental defaults*, from defaulted set defined by D, institution i would collect $\sum_{h \in D} E_{hi}$ from D (assumed withdrawn completely).
2. Institute i replaces cash outflow partially by $\gamma \sum_{h \in D} E_{hi}\, (0 \leq \gamma \leq 1)$, where γ is the replacement rate.
3. Fire-sale events happen when the sale prices are significantly lower than the assets' book values. Let q denote fire-sale loss rate for liquid assets, and z define loss rate for illiquid assets[1]. Cash generated from fire-sales of liquid assets a_i^q is $a_i^q(1-q)$.

   If the cash outflow cannot be recovered by selling liquid assets: $(1-\gamma)\sum_{h \in D} E_{hi} > a_i^q(1-q)$, then institution i has to sell illiquid assets[2]Equation (1) on page 9 on Jo (2012) states the following:

$$FSL_i = Min\left[(1-\gamma)\sum_{h \in D} E_{hi},\ a_i^q(1-q)\right] \cdot \frac{q}{1-q} + Max\left[(1-\gamma)\sum_{h \in D} E_{hi} - a_i^q(1-q),\ 0\right] \cdot \frac{z}{1-z} \quad (3.1.1)$$

4. Funding cost loss (FCL): Jo (2012) (48) assumes that short-term debts are from two sources: (1) from defaulted set D, calculated as $\gamma \sum_{h \in D} E_{hi}$; (2) from the non-defaulted institutes, as $\sum_{k \notin D} E_{ki}^S$. Liquidity risks surge for the institutes that rely on short-term loans. Equation (2) on page 9 of Jo (2012) states the following:

$$FCL_i = \sum_{h \in D} E_{hi}\gamma\mu + \sum_{k \notin D} E_{ki}^S\mu \quad (3.1.2)$$

5. Calibration of the *replacement rate* γ*:* Jo (2012) assumes there exists a normal capital ratio, λ_0, at which an institution can completely rollover or find alternative creditors (during normal market conditions). Fund replacement becomes exceedingly difficult as capital ratio falls between λ_0 and the minimum regulatory requirement, $\bar{\lambda}$. Below $\bar{\lambda}$, refinancing is not possible. Equation 3 on page 10 of Jo (2012) states the following:

$$\gamma(\lambda) = \begin{cases} 1 & if\ \lambda_0 < \lambda \\ 1 - \frac{(\lambda_0 - \lambda)^2}{(\lambda_0 - \bar{\lambda})^2} & if \quad \bar{\lambda} < \lambda \leq \lambda_0 \\ 0 & otherwise \end{cases} \quad (3.1.3)$$

[1]It is reasonable to assume $q < z$

[2]It is reasonable to assume that illiquid assets should be sold only after all liquid assets are sold.

We use historical balance sheet data to construct the normal capital ratios for each institute in our network and calibrate this relationship accordingly. For details please refer to Appendix section 3.3.4.

6. Calibration of funding cost loss ratio μ: The results by Jo(2012) (48) for Korea suggest that the credit spread is proportional to the third power of a decline in the capital ratio[3]. This relationship is shown by equation (4) on page 11 of Jo (2012):

$$\mu = \begin{cases} 0 & if\ \lambda_0 < \lambda \\ a \cdot (\lambda_0 - \lambda)^3 & if\ \bar{\lambda} < \lambda \leq \lambda_0 \end{cases} \tag{3.1.4}$$

where a is the proportional constant obtained by the regression. In this economy, this is relatively small, less than 1%. For details please refer to Appendix section 3.3.4.

7. By combining Equations 3.1.1, 3.1.2, 3.1.3, and 3.1.4, the total *funding loss* can be expressed as below (first equation on page 12 of Jo (2012)):

$$\begin{aligned} FL_i &= FSL_i + FCL_i \\ &= Min\left[(1-\gamma(\lambda))\sum_{h\in D} E_{hi},\ a_i^q(1-q)\right]\frac{q}{1-q} + Max\left[(1-\gamma(\lambda))\sum_{h\in D} E_{hi} - a_i^q(1-q),\ 0\right]\frac{z}{1-z} \\ &\quad + \sum_{h\in D} E_{hi}\gamma(\lambda)\mu(\lambda) + \sum_{h\notin D} E_{ki}^S\mu(\lambda) \end{aligned} \tag{3.1.5}$$

3.2. Default simulation results (exogenous case)

For parameter calibrations of μ, $a$and λ, as in Equations 3.1.3 and 3.1.4, please see Appendix 3.3.4. The estimation of funding cost loss ratio in our economy is $\mu = \begin{cases} 0 & if\ \lambda_0 < \lambda \\ 0.0006495 \times (\lambda_0 - \lambda)^3 & if\ \bar{\lambda} < \lambda \leq \lambda_0 \end{cases}$

We also make the following assumptions, which are suggested by previous literature (Jo (2012) (48) and several others).

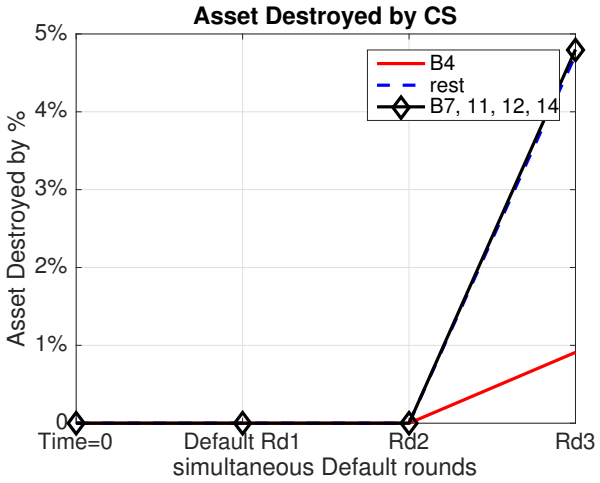
- The the fire sale loss rate for liquid assets:$q = 0.2$
- The fire sale loss rate for illiquid assets: $z = 0.7$
- Fair Value loss rate $FVLR = 0.2$.

Discussion: The results of the default simulations are summarized in Table 3.2.1 and Figure 3.2.1, 3.2.2 and 3.2.3.

1. Using simulation analysis, any one single bank's default does not cause contagion due to direct counterparty exposure in the first round of the simulation. Majority of the damage is caused by fire-sales of assets and liquidity shortage. These in turn, would trigger second round default of several institutes with thin capital buffers, and so on and so forth. We assume multiple rounds of default simulations take place simultaneously.

[3]This can be re-calibrated to fit each country's specific market characteristics.

Figure 3.2.1.: Asset Destroyed% by Direct Counter Party Exposure (Credit Shock). Due to the high capital level of the institutes within the network, defaults caused by credit risk is not high, only less than 5% assets are destroyed due to credit risk.



2. Small banks (Table 2.1.3) default may cause domino effects as much as larger banks' defaults, due to fire-sale of assets and liquidity shortage in the stress scenario, especially if these small banks have large interbank exposures (see point 5).

3. By examining capital adequacy regarding bilateral exposures, relative exposure RE^3 in Table 2.1.3 shows that B1, 2, 3, 4* 5, 7, 8, 9* 10*, 11*, 16, 17* 20, 25 (*banks at borderlines) are the ones with the thin capital buffer ($RE^3 < 10\%$).

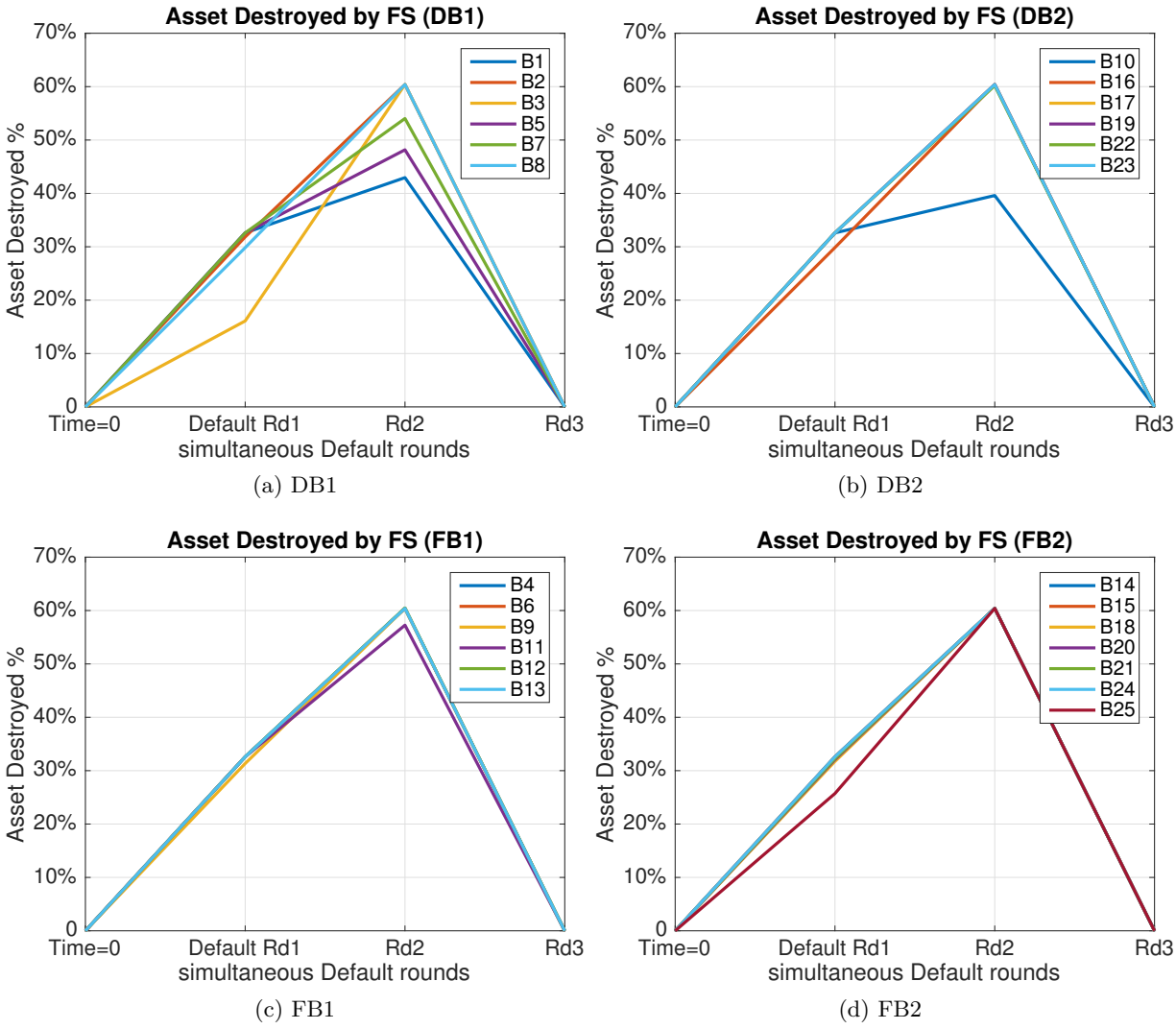
4. Results from the default simulation (Table 3.2.1 and Figure 3.2.1, 3.2.2 and 3.2.3) show similar results to the relative exposure measure above: The first round defaults due to secondary effects (fire-sale and funding loss in stress market conditions) are consistent: Bank 2, 3, 8, 9, 16, 18, 20, 21, and 25. The second round defaulted banks due to secondary effects are also consistent: 1, 5, 7, 10, 11, 22, with occassionally 4 or 14 also defaulted in this round. The third round, defaults due to credit loss are 4 and 14, with bank 17 only defaulted once, triggered by initial default of bank 4.

5. After examination (Table 2.1.3), we found the relative exposure of Bank 9, 12, 14, 18, 20 and 21, $RE_i^1 \preceq 1$, which means their total Tier 1 capital is just enough to cover its total interbank exposure. At the same time, their sizes are small ($size_i \preceq 1\%$ of the total banking sector assets). In addition, bank 22 shares similar characteristics, with $RE_{22}^1 = 2.87$ and $size_i \preceq 1\%$. These all make the above mentioned nodes vulnerable in the network. Hence, the micro and macro analysis have identified the same group of vulnerable nodes in this banking network.

Table 3.2.1.: Default simulation result (exogenous case). Credit Shock (CS), Funding Shock (FS). Results show Direct counterparty exposure do not cause as much damage as Funding (liquidity) shock.

	Default round 1		Default round 2		Default round 3	
Initial trigger	CS	FS	CS	FS	CS	FS
Bank 1	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	5, 7, 10, 11, 22	4, 14	n/a
Bank 2	n/a	3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 3	n/a	2, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 4	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	14, 17	n/a
Bank 5	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 7, 10, 11, 22	4, 14	n/a
Bank 6	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 7	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 10, 11, 14, 22	4	n/a
Bank 8	n/a	2, 3, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 9	n/a	2, 3, 8, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 10	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 11, 22	4, 14	n/a
Bank 11	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 14, 22	4	n/a
Bank 12	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 14, 22	4	n/a
Bank 13	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 14	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4	n/a
Bank 15	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 16	n/a	2, 3, 8, 9, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 17	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 18	n/a	2, 3, 8, 9, 16, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 19	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 20	n/a	2, 3, 8, 9, 16, 18, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 21	n/a	2, 3, 8, 9, 16, 18, 20, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 22	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11	4, 14	n/a
Bank 23	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 24	n/a	2, 3, 8, 9, 16, 18, 20, 21, 25	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a
Bank 25	n/a	2, 3, 8, 9, 16, 18, 20, 21	n/a	1, 5, 7, 10, 11, 22	4, 14	n/a

Figure 3.2.2.: Asset Destroyed% by Funding (Liquidity) Shock. DB-Domestic Banks, FB - Foreign Bank Branches



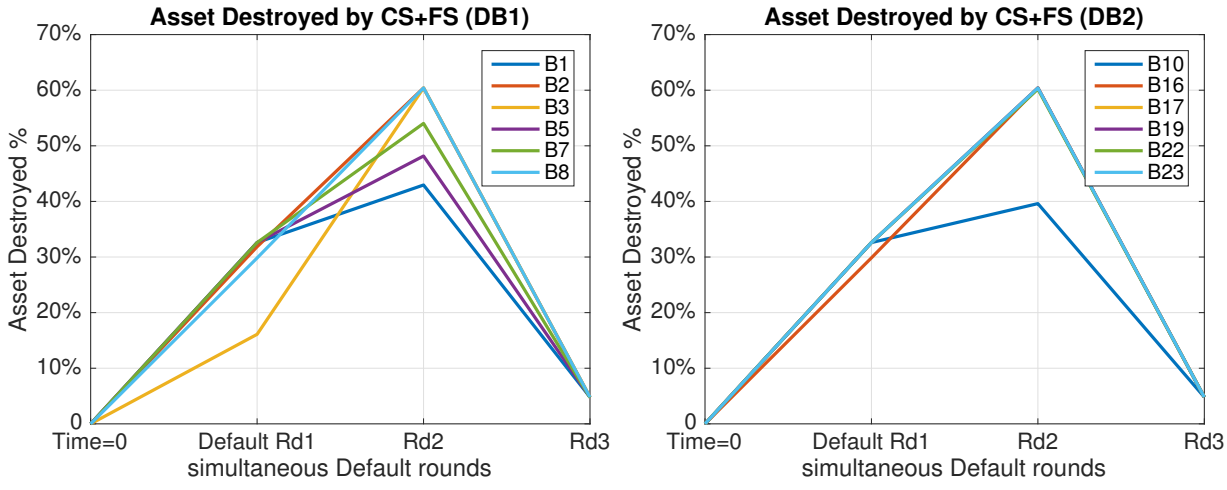
(a) DB1

(b) DB2

(c) FB1

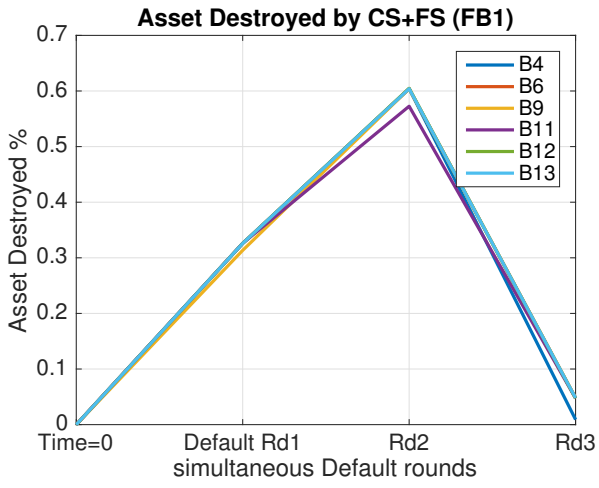
(d) FB2

Figure 3.2.3.: Comprehensive Default Simulation Illustration (CS + FS)

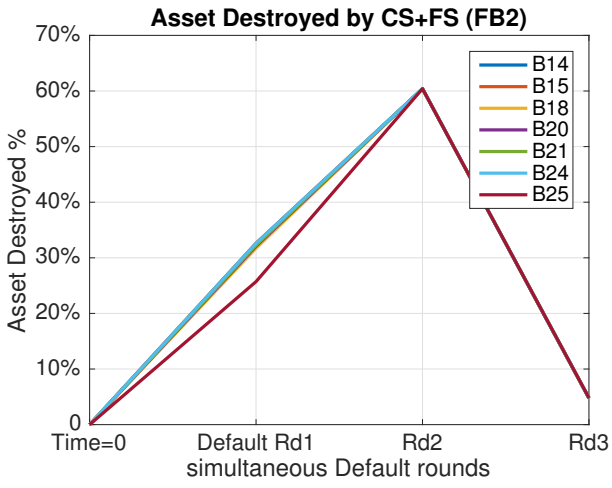


(a) DB1

(b) DB2



(c) FB1



(d) FB2

3.3. Conclusions

We used two complementary approaches to analyze systemic risk in a banking sector in an advanced emerging economy. The first, the *model-free* approach, focuses on the topological properties of the network by examining interbank connectivity and exposure using network theory, which does not rely on granular balance sheet information or the funding cost structure faced by the banks. There exist two tails in the connectivity and exposure distribution, which indicates two groups of banks within the network: the well connected money centre banks, and the distant peripheral banks. By the non-targeted, homogeneous capital ratio currently pledged by Basel Committee and implemented by Central banks, banks in our network are solvent and well capitalized. However, using *relative exposure*, a measure developed by Cont et al (2012) (27) to target exposure concentrated banks (SIFIs), a few banks are identified as vulnerable nodes in the network. Also, centrality measures shows the important debtors and creditors in the network. Size is highly correlated with centrality, with some exceptions - small banks (B9, 12 and 18) with high bilateral exposures can be central nodes, too. These small but central banks cause at least as much as credit and market loss (in nominal monetary terms) to their counterparties when they default. In relative terms, even higher, as shown in Table 3.2.1, 3.2.2 and 3.2.3.

While topological analysis is useful for identifying core and peripheral banks, qualitative measures such as relative exposure and centrality, systemic risk assessment also requires quantifying the risk. This was accomplished by the *model-based* approach, employing a balance-sheet network framework, to define a specific default mechanism. In this emerging economy, counterparty credit risk, or the *primary effect*, in the interbank market is relatively small, and the default of a single bank does not lead to successive defaults. However, counterparty credit risk is just one of many risk factors driving systemic risk. Results indicate that *secondary effects* such as fire-sale losses, fair value market losses, funding losses and liquidity shortages could lead to widespread failures in the banking network, inducing further defaults by the domino effect.

The two complementary approaches single out the same group of SIFIs. This finding suggests that a network topology approach could be used when granular public information on balance sheet, securities prices, and credit ratings is missing. In the case that this information is available, the balance-sheet network approach and default simulations could provide complementary insights on systemic risk. Finally, the analysis highlights the importance of heterogeneous interconnectedness in the banking system as a driver of systemic risk, supporting policy efforts towards incorporating the interconnectedness dimension in the regulatory framework.

Appendix I Figures and Tables

Table 3.3.1.: Shortest path length of the banking network, end-September 2010. The number in cell (i, j) in the table refers to the shortest path length between bank i and bank j. If it is ∞, it means bank i has no access to bank j.

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25
B1	**0**	1	1	1	1	2	1	1	1	1	1	1	2	2	1	1	1	1	1	1	1	1	2	∞	1
B2	1	**0**	1	1	1	2	1	1	1	1	1	1	2	2	2	1	1	1	2	2	1	1	1	∞	1
B3	1	1	**0**	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	∞	1
B4	1	1	1	**0**	1	1	1	1	1	1	1	1	1	2	2	1	1	1	2	1	2	1	1	∞	1
B5	1	1	1	1	**0**	2	1	1	1	1	1	1	2	1	1	1	1	1	2	1	1	1	2	∞	1
B6	2	2	1	2	2	**0**	2	2	2	2	1	2	1	1	2	1	1	1	2	2	2	2	2	∞	1
B7	1	1	1	1	1	2	**0**	1	1	1	1	1	2	1	2	1	1	1	2	2	1	1	2	∞	1
B8	1	1	1	1	1	2	1	**0**	1	1	1	1	1	1	1	1	1	1	2	1	1	2	1	∞	1
B9	1	1	1	1	1	2	1	1	**0**	1	1	1	2	2	1	1	1	1	2	2	1	1	2	∞	1
B10	1	1	1	1	1	2	1	1	1	**0**	1	1	2	2	1	1	1	1	2	2	1	1	2	∞	1
B11	1	1	1	1	1	2	1	1	1	1	**0**	1	2	2	1	1	2	1	2	2	2	1	2	∞	1
B12	1	1	1	1	1	2	1	1	1	1	1	**0**	2	2	1	1	2	1	2	2	2	1	2	∞	1
B13	2	2	1	2	1	2	2	2	2	2	2	2	**0**	2	2	2	2	2	2	2	2	2	1	∞	1
B14	2	2	1	2	2	2	2	2	2	2	2	2	2	**0**	1	1	2	2	2	2	2	2	1	∞	1
B15	1	2	1	2	1	2	2	1	1	1	1	1	2	1	**0**	2	2	1	2	2	2	2	1	∞	1
B16	1	1	1	1	1	2	1	1	1	1	1	1	2	1	2	**0**	1	1	2	2	1	1	2	∞	1
B17	1	1	1	1	1	2	1	2	2	1	2	2	2	2	2	1	**0**	1	1	2	1	2	1	∞	1
B18	1	1	1	1	1	2	1	1	1	1	1	1	2	2	1	1	1	**0**	2	1	1	1	2	∞	1
B19	1	1	2	2	2	2	1	2	2	1	2	2	2	2	2	2	2	2	**0**	2	2	1	1	∞	1
B20	1	2	1	1	1	2	2	2	1	1	1	2	2	2	2	2	2	2	2	**0**	2	2	2	∞	2
B21	1	1	1	2	1	2	1	2	2	1	2	2	2	2	2	1	1	2	2	2	**0**	1	2	∞	1
B22	1	1	1	2	1	2	1	1	1	1	1	1	2	2	2	1	1	1	2	2	1	**0**	1	∞	1
B23	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**0**	∞	1
B24	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	**0**	∞
B25	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	2	∞	**0**

Appendix II Statistical Methodology

3.3.1. Normal Laplace distribution and double pareto-lognormal distribution

The *dPlN* distribution has four parameters, similar to log-hyperbolic disbution, both of which exhibit power-law behavior in both tails. These two distributions are derived from mixture of lognormal distributions. The difference between the two is that *dPlN* arises from the distribution of a final state of a *Geometric Brownian Motion (GBM)* with a constant killing rate, while the hyperbolic distribution has a more complicated killing rate functions. Consider a *Geometric Brownian Motion (GBM)* defined by *Itô stochastic differential equation (SDE)* as follows:

$$dX = \mu X dt + \sigma X dw \tag{3.3.1}$$

It has an initial state $X(0) = X_0$ distributed lognormally, $logX_0 \sim N(\upsilon, \sigma^2)$. After T units of time, $X(T)$ will have a lognormal distribution as follows:

$$logX(T) \sim N(\upsilon + (\mu - \sigma^2/2)T, \tau^2 + \sigma^2 T) \tag{3.3.2}$$

Supposedly at time T, the *GBM* process is observed has an exponentially distributed random variable with density $f_T(t) = \lambda e^{-\lambda t}$, $t > 0$. λ is refereed as the "killed" rate rate, and $k(X) \equiv \lambda$. The distribution of the state $\hat{X}$ at the time of the observation of the "killing", is a mixture of lognormal random variables 3.3.1with mixing parameter T.

Taking logarithmic of X, $\hat{Y} = log(\hat{X})$ so that $\hat{Y}$ is the state of an ordinary Brownian motion after an exponentially distributed time. Y is the sum of independent variables W and Z. $Z \sim N(\upsilon, \tau^2)$; W is a skewed Laplace distribution as described by , with a *pdf* as follows:

$$fw(\omega) = \begin{cases} \frac{\alpha\beta}{\alpha+\beta} e^{\beta\omega}, \ for\, \omega \leq 0 \\ \frac{\alpha\beta}{\alpha+\beta} e^{-\alpha\omega}, \ for\, \omega \geq 0 \end{cases} \tag{3.3.3}$$

where α and β are roots of characteristic function

$$\frac{\sigma^2}{2} z^2 + (\mu - \frac{\sigma^2}{2}) z - \lambda = 0 \tag{3.3.4}$$

Hence, the distribution of $\hat{Y}$ is a convolution of Laplace and normal distributions. We introduce the concept of *Mill's ratio* for convenience as follows:

$$R(z) = \frac{\Phi^c(z)}{\phi(z)} \tag{3.3.5}$$

where $\Phi^c(z)$ is the complementary cumulative density function of normal distribution, and $\phi(z)$ is the normal pdf. We can then express the density function of $\hat{Y}$ as follows:

$$g(y) = \frac{\alpha\beta}{\alpha+\beta} \phi(\frac{y-\upsilon}{\tau}) [R(\alpha\tau - (y-\upsilon)/\tau) + R(\beta\tau + (y-\upsilon)/\tau)] \tag{3.3.6}$$

This is a *Normal-Laplace (N-L)* distribution. In other words, $Y \sim NL(\alpha, \beta, \nu, \tau^2)$.

Since a Laplace random variable can be expressed as the difference between two exponentially distributed variates, we can rewrite Y as follows:

$$Y \doteq \nu + \tau Z + {}^{E_1}/_{\alpha} - {}^{E_2}/_{\beta} \tag{3.3.7}$$

where E_1 and E_2 are standard independent exponentiate deviates, and Z is a standard normal deviate, which is independent of E_1 and E_2. One can use this equation to generate pseudo-numbers from N-L distribution.

We then deduce the *pdf* of $\hat{X}$ as follows:

$$f(x) = \frac{1}{x} g(log(x)) \tag{3.3.8}$$

That is, in the form of *ccdf* $\phi_c$and *pdf* ϕ of standard normal distribution $N(0,1)$ as follows:

$$f(x) = \frac{\alpha\beta}{\alpha+\beta}[A(\alpha,\upsilon,\tau)x^{-\alpha-1}\Phi(\frac{logx-\upsilon-\alpha\tau^2}{\tau}) \ + x^{\beta-1}A(-\beta,\upsilon,\tau)\Phi^c(\frac{logx-\upsilon+\beta\tau^2}{\tau})] \tag{3.3.9}$$

where

$$A(\theta,\upsilon,\tau) = exp(\theta\upsilon + {}^{\alpha^2\tau^2}/_2) \tag{3.3.10}$$

This is a double Pareto lognormal distribution. In other words, $X \sim dPlN(\alpha,\beta,\upsilon,\tau^2)$. From 3.3.1, we can express X as follows:

$$X \doteq {}^{UV_1}/_{V_2} \tag{3.3.11}$$

Where U follows a lognormal distribution that $log(U) \sim N(\upsilon,\tau^2$, and $V_1$and V_2 are Pareto distributed and with a *pdf* as follows:

$$f(\upsilon) = \theta\upsilon^{-\theta-1}, \upsilon > 1 \tag{3.3.12}$$

with $\theta = \alpha$ and $\theta = \beta$ for $V_1$and V_2, respectively. We can rewrite the expression for X as

$$X \doteq UQ \tag{3.3.13}$$

where $Q = \frac{V_1}{V_2}$, and has *pdf* as follows:

$$f(q) = \begin{cases} \frac{\alpha\beta}{\alpha+\beta}q^{\beta-1} & for\, 0 < q < 1 \\ \frac{\alpha\beta}{\alpha+\beta}q^{-\alpha-1} & for\, q < 1 \end{cases} \tag{3.3.14}$$

This is referred to as the *double pareto-lognfrmal (dPlN) distribution*. One can generate pseudo-numbers from dPlN distribution by exponentiating the N-L pseudo-numbers generated using 3.3.1.

3.3.2. Properties of N-L distribution

There are two special cases of the N-L distribution:

1. When $\alpha = \infty$. This corresponds to the case of N-L distribution is the difference between the independent normal and exponential components. It exhibits extra normal behavior in the lower tail. It is also referred to as the *right-handed normal-exponential distribution*, and is expressed as $Y \sim NE_r(\beta,\ \upsilon,\ \tau^2)$. Its *pdf* is reduced from the original 3.3.1:

$$g_1(y) = \beta\phi(\frac{y-\upsilon}{\tau})R(\beta\tau + (y-\upsilon)/\tau) \tag{3.3.15}$$

2. When $\beta = \infty$. This corresponds to the case of N-L distribution is the sum of the independent normal and exponential components. It exhibits extra normal behavior in the upper tail. It is often referred to as *left-handed normal-exponential distribution*, and is expressed as $Y \sim NE_l(\alpha,\ \upsilon,\ \tau^2)$. Its *pdf* is

$$g_2(y) = \alpha\phi(\frac{y-\upsilon}{\tau})R(\alpha\tau + (y-\upsilon)/\tau) \tag{3.3.16}$$

The $N-L(\alpha,\ \beta,\ ,\upsilon\ ,\tau^2)$ distribution can be represented as a mixture of the *right-handed* and *left-handed normal-exponential distributions*, and its pdf can be written as follows:

$$g(y) = \frac{\beta}{\alpha+\beta}g_1(y) + \frac{\beta}{\alpha+\beta}g_2(y) \tag{3.3.17}$$

The *cumulative density function (cdf)* of the $N-L(\alpha,\ \beta,\ ,\upsilon\ ,\tau^2)$ can be expressed as follows:

$$G(y)=\Phi(\frac{y-\upsilon}{\tau})-\phi(\frac{y-\upsilon}{\tau})\frac{\beta R(\alpha\tau-(y-\upsilon)/\tau)+\alpha R(\beta\tau+(y-\upsilon)/\tau)}{\alpha+\beta} \tag{3.3.18}$$

The *moment generating function (mgf)* of the $N-L(\alpha,\ \beta,\ ,\upsilon\ ,\tau^2)$ is the product of the *mgf* normal and exponential components, and can be expressed as follows:

$$M_Y(s)=\frac{\alpha\beta\, exp(\upsilon s+\tau^2 s^2/2)}{(\alpha-s)(\beta+s)} \tag{3.3.19}$$

3.3.3. The properties of dPlN distribution

Corresponding to the two cases (right-handed and left-handed) of the *N-L distribution* described as above, we deduce the *right-handed* and *left-handed dPlN distribution pdf* as follows:

1. When $\alpha=\infty$. This limiting *single Pareto-lognormal* distribution only exhibits heavy tail behavior in the lower tail.
$$f_1(x)=\beta x^{\beta-1}A(\beta,\ \upsilon,\ \tau)\Phi(\frac{log\, x-\upsilon+\beta\tau^2}{\tau}) \tag{3.3.20}$$
2. When $\beta=\infty$. This limiting *single Pareto-lognormal* distribution only exhibits heavy tail behavior in the upper tail.
$$f_2(x)=\alpha x^{-\alpha-1}A(\alpha,\ \upsilon,\ \tau)\Phi(\frac{log\, x-\upsilon-\alpha\tau^2}{\tau}) \tag{3.3.21}$$
where $A(\theta,\upsilon,\tau)=exp(\theta\upsilon+{}^{\alpha^2\tau^2}/_2)$ as in the case of *N-L* distribution.
3. The *dPlN* distribution *pdf* can be represented as a mixture of *right-handed* and *left-handed* pareto-lognfrmal distributions:
$$f(x)=\frac{\beta}{\alpha+\beta}f_1(x)+\frac{\alpha}{\alpha+\beta}f_2(x) \tag{3.3.22}$$
The cdf of the dPlN distributions can be written as $F(x)=G(exp(x))$, where G is equal to 3.3.18, or it can be expanded in the forms of ϕ and Φ^c as follows:

$$F(x)=\Phi(\frac{logx-\upsilon}{\tau})-\frac{1}{\alpha+\beta}[\beta x^{-\alpha}A(\alpha,\ \upsilon,\ \tau)\Phi(\frac{logx-\upsilon-\alpha\tau^2}{\tau})+\alpha x^{\beta}A(-\beta,\ \upsilon,\ \tau)\Phi^c(\frac{logx-\upsilon-\beta\tau^2}{\tau})] \tag{3.3.23}$$

The $dPlN(\alpha,\ \beta,\ \upsilon,\ \tau^2)$ exhibits heavy tail behavior at both tails in the sense that

$$f(x)\sim k_1x^{-\alpha-1}(x\to\infty) \tag{3.3.24}$$

and

$$f(x)\sim k_2x^{\beta-1}(x\to 0) \tag{3.3.25}$$

where

$$k_1=\alpha A(\alpha,\ \upsilon,\ \tau) \tag{3.3.26}$$

and

$$k_2=\alpha A(\beta,\ \upsilon,\ \tau) \tag{3.3.27}$$

The maximum likelihood function of *dPlN* and the *N-L* distributions are the same, in the sense that if the dataset $(x_1,\ x_2,\ x_{3,\ldots}\, x_n)$ is assumed to follow a *dPlN*, then its logarithmic form $(y_1,\ y_2,\ ,\ldots,y_n)$ is assumed to follow a *N-L* distribution. The log likelihood function is as follows:

$$l(\alpha,\ \beta,\ \tau)=nlog\alpha\ +nlog\beta\ -nlog(\alpha+\beta)+\sum_{i=1}^{n}log[R(\alpha\tau-\frac{y_i-\bar{y}+{}^1/_\alpha+{}^1/_\beta}{\tau})+R(\beta\tau+\frac{y_i-\bar{y}+{}^1/_\alpha+{}^1/_\beta}{\tau})] \tag{3.3.28}$$

and υ can be solved analytically as $\hat{\upsilon}=\bar{y}-{}^1/_\alpha+{}^1/_\beta$.

3.3.4. Parameter Calibration

The funding cost loss function needs to be calibrated according to each economy's funding characteristics. In Jo (2012) (48), the calibration results are based on data before Lehman bankruptcy (2007Q3 to 2010 Q3), and the regression is fitted as follows:

$$spread_t = 0.04 \times (14.62 - Basel\, ratio_t)^3 + 0.5 \tag{3.3.29}$$

The incremental funding cost function is estimated as $\mu(\lambda) = 0.04(14.62 - \lambda)^3$.

For our network here, there is limited data for calibrating the incremental funding cost function. We estimated a panel regression using July 2013 data of option-adjusted spreads (OAS) for eight institutions and the government yield curve as the risk-free rate to estimate the equation:

$$\mu = \begin{cases} 0 & if\ \lambda_0 < \lambda \\ a \cdot (\lambda_0 - \lambda)^r & if\ \bar{\lambda} < \lambda \leq \lambda_0 \end{cases}$$

where $\lambda_{0,i}$ is calculated as the historical average capital ratio before the crisis for each institution i. We found the following incremental funding cost loss equation in our market is $\mu = 0.0006495 \times (\lambda_0 - \lambda)^3$, which was used in the analysis of default contagion. Although the government yield curve is not strictly risk-free, it is the best estimate in this exercise. It may not be appropriate to use a safer assets such as US or UK government bonds as the base rate in this case, due to the different market environments. In future research work, we may consider using different parameters of the equations to simulate different liquidity environements in distress time.

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Part II.

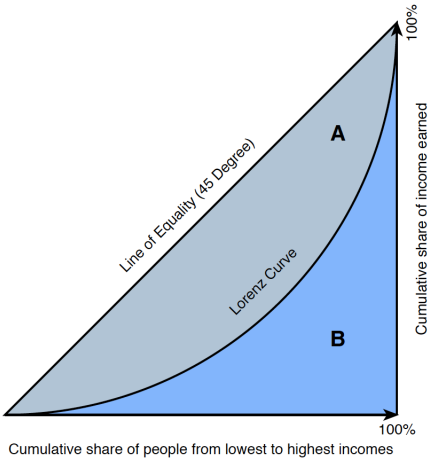
Household Income Inequality and Elasticity of Spending Across the Recession: A Case Study of Spain

Introduction

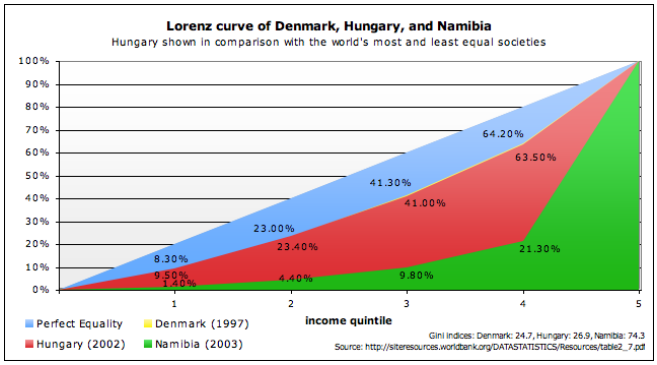
4.1. Background

Generally speaking, the income distribution in a nation represents how the Gross Domestic Product (GDP) is distributed among the citizens and residents living in the country at a certain time period. Income and its distribution have always been a central topic in the economic theory and policy making arena, mainly because it has an intimate relationship with economic development in the long run. Earlier seminal work to measure income distribution such as Lorenze Curve in Lorenze (1905) (92) as well as Gini index (Gini (1909) (84) and Gastwirth (1972) (83)) have been cited greatly. In short, Lorenze Curve is s graphical illustration of inequality, most notably in income and wealth. The curve is a graph showing the proportion of overall income or wealth assumed by the bottom x% of the population, loosely speaking without discrimination of a finite sample size. Please see figure 4.1.2a for an illustration of a typical Lorenz Curve and figure 4.1.2b for an application to income inequality in various countries.

Figure 4.1.1.: Example of Lorenze Curve



(a) A Typical Lorenz Curve



(b) A Lorenz Curve Application to Income Inequality Among Countries [Source: World Bank Dataset]

In Figure 4.1.2a, the Gini coefficient or Gini index can then be expressed as the ratio of the area that lies between the line of equality and the Lorenz curve (marked A in the figure) over the total area under the line of equality (marked A and B in the figure); i.e., G = A / (A + B). Generally speaking, the higher the gini index or coefficient, the more unequal the income (or other variables in research) is. As shown in Figure 4.1.2b, Hungary in 2002 and Denmark in 1997 are more or less similar in their income distribution among its citizens and residents [please note that overlapping of Hungary and Denmark in the figure], where almost 63-64% total income are earned by the 80% of citizens and residents in these two countries. Hungary and Denmark are more equal than Namibia, in terms of income distribution. In the latter, only 21.3% of total income are earned by 80% of its citizens and residents in 2003.

Among OECD countries, in the late 20th century, considering the effect of taxes and transfer payments through social security programs and assistance from other family members, the income Gini coefficient ranged between 0.24 and 0.49, with Slovenia (and in general Eastern European and Scandinavian countries) being the lowest, and Chile (and in general Latin America and Africa) the highest. African countries had the highest pre-tax Gini coefficients in 2008–2009, with South Africa the world's highest, variously estimated to be 0.63 to 0.7,[7][8] These figures drop to ~0.52 after transfers are taken into account, and drops again to 0.47 after taxation.[9] . The global income Gini coefficient in 2005 has been estimated to be between 0.61 and 0.68 by various organizations such as World Bank and Central Intelligence Agency (CIA)[1]. In this paper, I analyze the household incomes before and after social transfers and several key consumption (spending) variables. Results on Gini indices can be found in Table 5.2.1. However,

[1]Please refer to the World Fact Book published by CIA at https://www.cia.gov/library/publications/the-world-factbook/rankorder/2172rank.html

due to limitation of data, the before and after tax information is not available (or convenient to obtain for the country of Spain at time of writing), I have to leave this question for my future research agenda. Gini index or coefficient is a snapshot of the income or wealth distribution at a point in time, hence has the drawback not being able to capture the changes of demographic structure within the population over time. Factors such as population age and mobility within income class are not reflected in Gini index. An economy with high Gini index at a point in time, may be due to a higher percentage of younger population, who have lower income and wealth than the older population. However, if the life-span earnings of citizens and residents in this economy is taken into account for income and wealth inequality, this economy may not be as unequal as it seems measured by Gini index. Another example is population structural and living condition changes. For example, families may split into smaller units when children grow up, and when the elderly move into nursing houses and long-term cares.

The academia and economic policy makers have seen a resurgence of interests on global income and wealth inequality since the Global Financial Crisis (GFC) rooted from the US Subprime Mortgage Crisis that started in late 2007. There are several main reasons. Firstly, The income gap between the top docile and bottom one is the widest in developed countries in decades, whereas income inequality trend has sent mixed signals in emerging and developing countries (EMDCs). However, in some cases of EMDCs, although income inequalities are shrinking over decades, the rights to access to social security, education and resources for development diverge for different income groups. This has contributed to the slow economic growth as well as the sustainability of the recovery after the Great Financial Crisis (GFC) rooted from the 2007-2008 US Subprime Mortgage Crisis (IMF, 2015 (91)). So far, majority of the inequality studies have been focused on the high income groups, rather than the middle and lower income groups. However, according to IMF (2015) (91), within the countries in their research sample, there is a negative correlation between the growth of the top 20% income docile and the growth of GDP; at the same time, there is a positive relationship between the growth of the bottom 20% income docile and the GDP growth. These relationships suggest the middle and lower income groups are the drivers of the economic development. Allison et al (2014) (73) have come to similar conclusion to IMF (2015) (91), on the income inequality trends in developed countries and EMDCs over the past decades. The paper also summarizes the key trends and drivers of income inequality across the globe and over time. While the income inequality measured by per capita income gap has decreased *between* countries over the past decades, which is the first time in centuries, income inequality has increased *within* countries in most developed countries and some EMDCs. In the future, it is expected the economic development measured by income per capita is driven by income inequality *within* countries. Global income inequality may decrease if economy grows faster, while it may increase if *within* country income inequality increases. These all points to the importance of the topic in research here - income distribution and inequality.

Dataset In this paper I investigate the income inequality and its impacts on income elasticity of spending (consumption) on goods for households in the European countries across the crisis period, which sheds lights on income inequality of demands of various goods. In the current version of this paper, I use the household income and wealth, consumption on various goods and savings in the country of Spain. This survey is conducted by Bank of Spain, the dataset

is called EFF from hereafter. These income and wealth survey data in European countries are collected by their respective central banks or university research institutes. Since they are done by research organizations independent of each other, they have different research horizon, frequencies, features and focuses, according to each country's characteristics. Sometimes it is difficult to do cross-country or panel analysis. The recent effort by European Central Bank (ECB) of Household Finance and Consumption Survey (HFCS) is to consolidate these individual country surveys to a standardized form, which aims to provide a comparison to the Consumer Expenditure Survey (CES) in the US.

Research Motivation So far most previous literature have been focused on the upper tail of the income inequality, rather than lower tail. However, significance of the lower tail has been found in many populations. Seminal work have argue whether consumption inequality has indeed tracked income inequality. Most work have been done using data of US Consumption Expenditure Survey (CES). This paper provides a comparison using a case study of Spain. Secondly, The income inequality within population may have **heterogeneous** impacts on consumptions on various goods, across the GFC period. In other words, this translates into the **asymmetric** reflections on households' spending behaviours, on essential household goods, such as food, vehicles and other durable and non-durable goods. At the time of writing, there has not been significant work to address this question. To elaborate further on the concept of asymmetric impacts, I mean the unequal impacts of households in different income brackets may make different decisions in spending on necessities and durable goods (non-food related). For example, a richer family may reduce food spending less than a poorer family. However, both rich and poor family may not differ in their spending on vehicles prior or post crisis. This may be because cars have longer useful lives and are necessities. The results of this paper may shed light on how fast and sustainable is the recovery post GFC, and whether households are affected through their spending on the main categories of goods. In this paper, I use Spain as a case study. This framework can be used in other countries to do further cross-country comparison analysis with a longer history.

Methodology Firstly, I analyze the upper and lower tail indices and lognormal properties (mean and standard deviation) of the household income (before and after transfers due to social security and relatives from other households) and wealth distribution, as well as spending (consumption) on various goods, taking into account population sample weights. The distribution I adopt to measure these variables is the Double Pareto Lognormal distribution (dPlN from hereafter) described in Reed (2003) (94) which is discussed in more details in the related literature and section 5.1 I also analyze Gini index of all the income, wealth and spending (consumption) variables. Secondly, I analyze different income percentiles' elasticity of spending (consumption) on various goods, including [1] food, [2] other non-durable goods, [3] vehicles and [4] other durable goods. These variables are analyzed all across the period from 2001-2011, which is an interesting time reflecting before, during and after the GFC. This will provide insights on how households make spending and consumption decisions on various goods, taking effects of various factors such as their income and wealth, social security, etc. These result would also provide insights on how the post-GFC economic recovery on household level, from the consumption's perspective. I use the fixed-effect (fe) and random-effect (re) regressions described in details in

section 6.1.

Layout of the rest of this paper The next section provides an overview of related literature and how this research contributes to the current understanding of this topic. Chapter 5 discusses the dataset in more details, and provides a preliminary statistical analysis on properties of the key variables such as income, wealth and spending (consumption) on various goods. It estimates all the parameters of dPlN of these variables and Gini index too. Chapter 6 quantifies the relationship between household income and consumption expenditure on various goods, using fixed effect (fe) and random effect (re) models (explained in the relevant chapter in more details). Chapter 7 concludes and summarizes key findings.

4.2. Related Literature

This paper is related to several strands of academic literature. Firstly, it is related to extreme value theory applied to measurement of income and wealth inequality. Earlier seminal work by Neal and Rosen (2000) (123) argues that the labour income depends on occupation, education, experience and other observed traits. They found labour income has long right tails and typically the mean exceeds median. It provides a theoretical framework to explain how earnings diverge due to various factors such as endowment, significant skill investment (human capital development). They do not mention lower tail significance in labour incomes. It worths to mention that Kleiber and Kotz (2003) (118) provides a first complete overview of all parametric statistical models developed over the past century, on income, wealth and other similar variables, and systematically investigate each of their advantages and disadvantages. Chotikapanich (2008) (110) is another comprehensive review on modeling income distributions and the goodness-of-fit properties.

Until lately, most of previous papers only examine the upper tail of the distribution. Reed (2003) (94) and Reed and Jorgensen (2004) (94), discover the lower tail of income distribution is also prominent in several countries, such as Canada personal earnings in 1996, US Household income 1997, and Sri Lanka Household income 1981 and Bohemia Personal Income 1993. Their work provides an overview of dPlN model, which provides an estimate of both lower and upper tail indices and the lognormal body parameters. This is particularly useful because of the convenience to analyze tail index changes to capture inequality at extreme values, and economic mobility over time in a society. Latest paper by Toda (2012) (131) also supports the evidence of dPlN fit to personal labour income using US data (2000 to 2009 of the Current Population Survey), with further proof on goodness-of-fit criteria.

Some literature have used percentiles and quantiles to measure incomes, which is a relative primitive measure, assuming there is no difference within the income quantiles. This is in contradiction to real world observations. Vermeulen (2015) (100) has discovered tail behaviour even among the wealthiest. This is why we use a dPlN distribution analysis to measure inequality within population (households in this paper). When there are tail index movements, upper and/or lower tails, one can easily see the changes of inequality within the household population. If inequality within population increases, it shows social mobility possibility, but in both

directions: poor can become poorer, rich can become richer. If inequality within household population decrease, it shows stability of social-economic classes. This may provide evidence to IMF (2015) (91) and Allison (2014) (73)'s argument on income inequality and economic development, if further incorporating the country's economic recovery from GFC and future economic development into the research.

This paper is also related to theory papers, Ibragimov and Ibragimov (2007) (86) and Ibragimov et al. (current) (87), on income inequality and demand elasticity of goods. In this paper, the research question is on income elasticity of spending (consumption) on various household essential goods. This is due to the unavailability of demand data of these goods. However, spending can be viewed a a proxy of demand.

There is another strand of literature on relationship between consumption inequality and income inequality. This is due to the belief that consumption represent a permanent income hypothesis. Slesnick (2001) (96) (on US) and Cutler and Katz (1992) (81) (on UK) are among the first to analyze the consumption and income trends. An earlier influential work by Krueger and Perri (2006) (89), which is built upon Slesnick (2001) (96), use the CES data and argue that consumption inequality has not kept up pace with income inequality. The latest paper by Aguiar and Bils (2015) (72) suggest the opposite to Krueger and Perri (2006) (89), arguing consumption inequality among households have indeed tracked income inequality more closely than previously believed, using an innovative methodology called the double differencing method, which corrects the measurement errors that can vary over time by good and income when using CES. Last but not least, Fisher et al. (2013) (88) provides an overview of consumption and income inequality development up to most recently.

This paper, although uses a shorter history of consumption expenditure on various goods, sheds light on how income inequality and elasticity of consumption expenditure on various goods changes across the GFC period in Spain, as a case study of Europe. Compared to the conservative measurement of income inequality in the papers by Aguiar and Bils (2015) (72) and others, I use the latest discovered dPlN distribution which exhibites goodness-of-fit to income and wealth distributions (Reed, 2003 (94) Toda (2012) (131) etc.). This provides a holistic understanding on the extremism of income distribution on both tails, as well as the lognormal mean and standard deviation of the entire income (wealth and spending) distribution. This paper also quantify the consumption changes in respect to the changes in household incomes over the period of GFC. It is the first to provides such granular analysis, and especially during the crisis period.

Chapter 5

Case Study: Spain

For Spain, I use the Spanish Survey of Household Finance (EFF) 2002, 2005, 2008 and 2011 waves. This is an equivalent wealth survey to the United States Survey of Consumption and Finance (SCF) and Italian wealth survey (SHIW), which will be used to do country comparison analysis for future work. Bover (2008) (77) states the key motivation of EFF is debt information; hence more attention is devoted to debt classification, similar to SCF. However, less detailed asset categorization in EFF is more in line with SHIW. Spain and Italy share some similar family characteristics; e.g. more extended family line living together. Hence, more questions on labour status for each family member to account for complexed situation. Several important features and special care need to be taken when using the EFF dataset due to its structure and designs. I listed below for future research reference:

Imputation: Bover (2004) suggests EFF dataset provides five imputations for each of the variables with "No Answer" or "Don't know" replies from the interviewees. The multiple imputation (MI) method is described in Barcelo (2005). For simplicity, I only use one imputation (implicate) for my analysis in this paper, as the methodologies can be applied to all five imputations easily. For a holistic unbiased use of multiple imputation data, one can combine five imputations into one dataset and then calculate key statistics (mean, variance, tails etc) from this complete dataset. However, care must be taken when calculating variance and tails. For a detailed discussion, please see Appendix 7.2 on page 115.

5.1. Upper and lower tail estimates on income, wealth and spending variables

For household income, I use the variable renthog (annual household gross income) in the EFF dataset. For details on variables used in this analysis, please refer to Appendix Section 7.1. There are several advantages of using an income distribution analysis of Double-pareto lognormal (dPlN) which is described in Reed and Jorgensen (2004) (101). I only include the key equations below for easy references. For details, please refer to Appendix section in Chapter 3.3. The dPlN distribution has four parameters, and is a mixture of lognormal distributions, arised from the final state of geometric Brownian motion (GBM), killed with a constant killing rate,

or in other words, the state of the GBM after an exponentially distributed period of time. This flexibility appears to be the reason of its perfect fit to many empirical data, such as income, wealth, sizes, etc. From hereafter, I define this distribution as below:

$$X \sim dPlN(\alpha,\, \beta,\, \nu,\, \tau^2) \tag{5.1.1}$$

Such distribution can also be written as $X \sim UV_1/V_2$, where U, V_1, V_2 are independent from each other, and U is lognormally distributed as $logU \sim N(\nu,\, \tau^2)$, with V_1, *and* V_2 are Pareto distributions with parameters α *and* β. Their pdf can be written as

$$f(v) = \theta v^{-\theta-1},\ v > 1 \tag{5.1.2}$$

with $\theta = \alpha$ and $\theta = \beta$ respectively. Alternatively we can write $X \sim UQ$, where $Q = V_1/V_2$, and Q has pdf as

$$f(q) = \begin{cases} \frac{\alpha\beta}{\alpha+\beta} q^{\beta-1} & for\, 0 < q < 1 \\ \frac{\alpha\beta}{\alpha+\beta} q^{-\alpha-1} & for\, q < 1 \end{cases} \tag{5.1.3}$$

The right-hand limiting case and left-hand limiting cases of the dPlN distribution have the following pdfs:

$$f_2(x) = \alpha x^{-\alpha-1} A(\alpha,\, \upsilon,\, \tau)\Phi(\frac{log\, x - \upsilon - \alpha\tau^2}{\tau}) \tag{5.1.4}$$

$$f_1(x) = \beta x^{\beta-1} A(\beta,\, \upsilon,\, \tau)\Phi(\frac{log\, x - \upsilon + \beta\tau^2}{\tau}) \tag{5.1.5}$$

Hence its tail behaviour can be described in the sense that $f(x) \sim k_1 x^{-\alpha-1} (x \to \infty)$; $f(x) \sim k_2 x^{\beta-1} (x \to 0)$.

Slopes: on log scale, $dPlN(\alpha,\, \beta,\, \nu,\, \tau^2)$ is similar to hyperbola, with the asymptotes of slope of $-(\alpha+1)$ and $\beta - 1$.

This means when $\beta > 1$, $dPlN(\alpha,\, \beta,\, \nu,\, \tau^2)$'s pdf is unimodal; when $0 < \beta < 1$, it is monotonically decreasing.

In the following exercises, I calibrate the parameters of $dPlN$ for the following variables (all variables are described in more details in section 6.1.1) in all the years during the surveys were conducted [2001/2002, 2004/2005, 2007/2008 and 2010/2011, 4 waves (observations) in total]:

(1) Labour income variable `labinchhm` Reported (pre-tax) income and post social security transfer labour income, including salaries from employment and self-employment business incomes, as well as social securities such as subsidies from government, and charity, scholarships, financial assistance from relatives who belong to other households; excluding non-labour incomes such as

incomes from holding financial securities and rental incomes, etc. In this analysis, `labinchhm` can be compared to the total household income variable, `renthog`(explained below), to show the difference before and after social transfers as well as asset (capital) returns.

(2) Total labour and non-labour income `renthog` - total labour (salaries and family business enterprise income) and non-labour (financial asset return and rental income) household annual income. This is supposed pre-tax income, as in the survey code book or report by Bank of Spain, it is not clear whether these figures are pre-tax or post-tax.

(3) Spending (consumption) on food `alim`.

(4) Spending on other non-durable goods `nodur`

(5) Spending on vehicles `gimpvehic`

(6) Spending on other durable goods `p2_70`

(7) Total net wealth (total real and financial net assets) `riquezanet`

(8) Net financial wealth (total financial net assets) `actfinanc`

I firstly examine the tail behaviours of the total distribution, taking sample weights into consideration. However, some controversy on when weights should be used in regressions has been discussed in Deaton (1997 , Chapter 2 (82)) Cameron and Trivedi (78) (2005, Chapter 24). For a application discussion of weights used in STATA, please refer to STATA Manual (98).

Simply speaking, sample weights are estimated when survey is done on a selective subjects, while the whole population is unobservable. In our case, the Bank of Spain (and several other central banks in our broader research data pool) selected several thousands of household as representatives of the whole household population, and conduct interviews on a low frequency bases, every 2-3 years. Then they estimated how many families with similar household structure (number of family members, level of social security, financial and non-financial wealth, etc.) and most importantly, household income level that exist in the country. As the last step, researchers assign the interviewee household with a estimated sample (population) weights. As a result, in this paper, I estimate tail of the distribution by taking account of the population weights explained just above. Please note that the un-weighted household population would inevitably under or over estimate the tails, as unweighted household population would assume each of the household in the sample is equivalently weighted in the country household population, and this is incorrect. The weighted distribution tail estimate results are presented Table 5.1.1, 5.1.2 etc.

Table 5.1.1.: Spanish Household Labour Income (post-transfers): Weighted Results Tail Indices Estimates. `p6_labinchhm` in the code book in section 6.1.1.

	α	β	ν	τ
2002	0.4556	2.5959	4.0009	0.2950
2005	0.4561	1.9542	6.7515	0.4219
2008	0.4278	2.6189	12.8546	0.6797
2011	0.4038	2.5094	15.6271	0.7797

(a) Spanish Household Labour Income (post-transfers): Weighted Results Tail Indices Estimates

	$-(\alpha+1)$ [right tail slope]	$\beta-1$ [left tail slope]
2002	-1.4556	1.5959
2005	-1.4561	0.9542
2008	-1.4278	1.6189
2011	-1.4038	1.5094

(b) Spanish Household Labour Income (post-transfer) Tail Indices Estimate Slopes

Table 5.1.2.: Spanish Household Labour and Non-Income (before-transfer): Weighted Results Tail Indices Estimates. `renthog` in the code book in section 6.1.1.

	α	β	ν	τ
2002	0.4111	2.5956	12.1858	0.3180
2005	0.3947	1.6320	20.1472	0.3863
2008	0.3950	2.0550	20.6623	0.5729
2011	0.3689	1.8325	28.4891	0.6699

(a) Spanish Household Total Labour and Non-Labour Income before transfers Tail index Estimates

	$-(\alpha+1)$ [right tail slope]	$\beta-1$ [left tail slope]
2002	-1.4111	1.5956
2005	-1.3947	0.6320
2008	-1.3950	1.0550
2011	-1.3689	1.8325

(b) Spanish Household Labour Income (post-transfer) Tail Indices Estimate Slopes

After examination of income inequality by estimating the lower and upper tail indices of Spain household income, I observe the following:

1. For both the Labour income (post-transfers) variable and Total Labour `p6_labinchhm` and non-labour income (pre-transfer) variable `renthog`. The inequality trend is consis-

tent. Both income variables increase in their log-normal body mean, νin Table 5.1.1 and Table 5.1.2. This suggests overall income of the household population increases over time. Standard deviation (τin 5.1.1 and 5.1.2) of their lognormal body increase over time, which suggest inequality within the entire household population increases.

2. The upper tail of both income variables behave similarly. αin Table 5.1.1 and Table 5.1.2, stably increases over the period 2002-2011. In 2005, αincreases very mildly from 2002, meaning the inequality effect of the wealthiest population is reduced in the upper tail of the distribution. However, in 2008, αdecreases significantly, which means the richer households are getting richer faster among those wealthiest Spanish households. GFC timing correlates with upper income earners getting richer faster.

3. The lower tail of both income variables (denoted as β in Table 5.1.1 and Table 5.1.2), behave similarly in 2002-2011. β first decreases over the period 2002-2005. This means the poorer part of the household become more unequal in incomes. There is a divergence among the poorer parts of the households during this period. Then in 2008 β increases sharply from 2005, then decreases again from 2008-2011. In the long term, GFC has made the poorer part of the households poorer. However, β of `renthog` decreases more than `labinchhm`, this means financial returns in the renthog variable may decrease household incomes. which suggest the social transfers smooth our the income inequality in the poorer part of the household population.

4. It seems like during the period of 2001-2011, the income of the whole Spanish households diverges overall. The richer households has a tendency to get richer, while the poorer households first diverges then converges in incomes.

All other variables's tail indices are as follows (for simplicity, I omit the slope estimates as they are easily converted from the tail indices):

Table 5.1.3.: Spending (consumption) on food `alim`.Weighted Results Tail Indices Estimates.

	α	β	ν	τ
2002	0.4698	2.4871	-0.2976	0.2171
2005	0.4510	2.5799	-0.2092	0.4602
2008	0.4465	2.6034	0.1147	0.6514
2011	0.4116	2.5302	0.0674	0.6684

(a) Spending (consumption) on food `alim`.Tail index Estimates

Table 5.1.3 shows the extremism on both tails of food consumption during 2002-2011. The upper tail index α is decreasing during this period, which means richer households are spending more and more on food during this period. The lower tail index β first increases during 2002-2008, which means the poorer households are converging on food consumption. β decreases from 2005-2008, which means the poorer households are diverging in their food consumption during this period. The lognormal body of the food consumption distribution shows the mean increases

during 2002-2008, and then decreases slightly in 2008. The standard deviation of the lognormal body increases during the entire period.

Table 5.1.4.: Spending (consumption) on other non-durable goods `nodur`. Weighted Results Tail Indices Estimates.

	α	β	ν	τ
2002	0.4434	1.8307	0.3014	0.8136
2005	0.5173	3.5208	-1.1337	1.3917
2008	0.5049	3.3732	-0.6559	1.4268
2011	0.3750	1.6342	1.8293	0.5541

(a) Spending (consumption) on other non-durable goods `nodur`. Tail index Estimates

Table 5.1.4 shows the consumption on other non-durable goods during 2002-2011. Upper tail index αincreases from 2002-2005, and then decreases from 2005-2011. This means the richer households are spending more and more on other non-durable goods during this period. Lower tail index β exhibits similar behaviour to α, this means the poorer households also diverge in non-durable goods consumption during 2005-2011. This distribution over the period gets more and more unequal on both tails. The lognormal body of the distribution shows decreases from 2002-2005, and then increases from 2005-2011. Standard deviation shows increases from 2002-2008, and decrease from 2008-2011.

Table 5.1.5.: Spending on vehicles `gimpvehic`

	α	β	ν	τ
2002	0.4981	3.4991	0.3014	1.2224
2005	0.4486	0.9052	3.3819	0.3670
2008	0.7372	2.4511	4.1277	2.0429
2011	0.5844	2.5071	4.3533	2.1776

(a) Spending on vehicles `gimpvehic`Tail index Estimates

Table 5.1.5 shows the upper tail index α shows some fluctuation during 2002-2011 period. Overall it increases. This means the richer households consumption on vehicles converges over time. The lower tail index β decreases sharply in 2005, and then picks up again in 2008-2011. This is an interesting fact. It reflects the poorer households suddenly diverge in their spending on vehicles in 2005, however, converge to long-run equilibrium in 2008-2011. Over time, lognormal body mean increases and standard deviation increases too.

Table 5.1.6.: Spending on other durable goods `p2_70`

	α	β	ν	τ
2002	0.4480	5.1355	-1.4575	0.9737
2005	0.5173	3.5208	-1.1337	1.3917
2008	0.4296	1.6616	0.6859	0.7969
2011	0.4535	3.4865	-1.0452	1.3647

(a) Spending on other durable goods `p2_70`Tail index Estimates

Table 5.1.6 shows the spending on other durable goods (except for vehicles) during 2002-2011. The upper tail α first increases and then decreases to its original level. This means the richer households diverges in their spending on other durable goods in 2008, the start of GFC period, and then returns to original level of 2002 in 2011. The lower tail β decreases from 2002-2008, sharply, which means the poorer households diverges in their spending on other durable goods during this period, however, it picks up to its inequality level of 2005 in 2011. The lognormal mean firstly increases from 2002-2008 and then decreases from 2008-2011. It 2011 level was even lower than 2002. It seems the GFC does have negative impact on durable goods consumption. Standard deviation first increases from 2002-2005, later decreases from 2005-2008, and then picks up again in 2011. Over the entire period 2002-2011, the standard deviation increases. The spending on other durable goods diverges during the research period.

Table 5.1.7.: Total net wealth (total real and financial net assets) `riquezanet`

	α	β	ν	τ
2002	0.3623	1.2549	177.1179	0.2984
2005	0.3364	0.6167	601.2848	0.3249
2008	0.3288	0.6216	972.8515	0.3158
2011	0.2973	0.6111	1282.25482	0.3487

(a) Total net wealth (total real and financial net assets) `riquezanet` Tail index Estimates

Table 5.1.7 shows the total net wealth variable distribution during 2002-2011. Its upper tail index α decreases during this period, which means the richer households gets more and more net total wealth over the research period. The lower tail index β also has a long term trend of decreases, which means the poorer households are getting poorer and poorer over the period. Lognormal normal body mean is increasing, which means average households are getting more and more wealth. Standard deviation is increasing over the period, which means wealth distribution is diverging.

Table 5.1.8.: Net financial wealth (total financial net assets) `actfinanc`

	α	β	ν	τ
2002	0.3998	2.0852	149.9179	2.3822
2005	0.3511	1.4289	209.7891	2.4173
2008	0.4384	1.7319	238.1213	2.3809
2011	0.4110	1.6212	486.2042	3.0628

(a) Net financial wealth (total financial net assets) `actfinanc` ail index Estimates

Table 5.1.8 shows net financial wealth distribution during 2002-2011 period. Its upper tail index α is fluctuating during the period, but over time increases. This means the richer households are having not growing richer in financial asset holding steadily. The lower tail index β is decreasing over time in general, this means poorer families are doing worse in their financial asset holdings. Lognormal body mean is increasing, which means average households are doing better over time. Standard deviation is increasing over time, which means the financial wealth level is diverging.

5.2. Gini indices of income, wealth and spending variables

Table 5.2.1.: Gini Indices

variable code book	`p6_labinchhm`	`renthog`
	labour income post-transfers	labour and non-labour income before transfers
2002	0.3945	0.4144
2005	0.4069	0.4480
2008	0.4324	0.4185
2011	0.4426	0.4395

variable code book	`alim`	`nodur`	`gimpvehic`
	spending on food	other non-durable goods	spending on vehicles
2002	0.2889	0.4637	0.3784
2005	0.3058	0.4683	0.4169
2008	0.3065	0.4540	0.4571
2011	0.3020	0.4203	0.4983

variable code book	`p2_70`	`riquezanet`	`actfinanc`
	other durable goods	real net wealth	financial net wealth
2002	0.5865	0.5616	0.7990
2005	0.6197	0.5411	0.7928
2008	0.6009	0.5581	0.7917
2011	0.6338	0.5763	0.8087

Gini indices for both income variables `p6_labinchhm` and `renthog` are increasing in the 2002-2011 period. This in general means more income **inequality** over time. Except for the `renthog` gini decreases a little bit in 2008. As `renthog` includes financial asset returns, this may be due to GFC, financial assets such as stocks and bonds drop in value in general.

Food consumption (`alim`) gini is increasing over time, except for in 2001 it decreases slightly. This means this variable gets more **unequal** over time.

Other non-durable goods (`nodur`) gini is decreasing over time. This mean this variable gets more and more **equal** over time.

Vehicle spending (`gimpvehic`) gini is increasing over time strictly. This means this variable gets more and more **unequal** over time.

Other durable good spending (`p2_70`) gini is increasing over time, although in 2008 GFC period, it stalls. This means this variable is getting more and more **unequal** over time.

Real net wealth (`riquezanet`) gini is increasing over time, except for in 2005, it decreases slightly. This mean this variable is getting more and more **unequal** over time.

Financial net wealth (`actfinanc`) gini is decreasing strictly over time. This means this variable is getting more and more **equal** over time. This is the only key variable that smooths out over time in our research. This may be due to GFC period, financial assets drop in values.

Chapter 6

Income Elasticity of Spending on Goods Analysis

Income Elasticity of Demand is defined as follows:

$$e_i = \frac{\frac{\delta Q}{Q}}{\frac{\delta I}{I}} \tag{6.0.1}$$

The income data I gathered is collected at low frequency, e.g. once every 2 or 3 years. In the original formula elasticity in equation on the current page, the numerator is change of quantity. In my dataset, I only know how much the households spend on each good, and the categories of goods are not specific. At the time of writing, I use the following regression method as a proxy of cross-sectional income elasticity of spending on goods in this paper. Please note that the term "income elasticity of good X" is used in this paper for simplicity.

$$y_{it} = \alpha + x_{it}\beta + \epsilon_{it} \tag{6.0.2}$$

where the LHS variable, y_{it} is the spending of household $i$in year t. The RHS variable αmay be a constant term, x_{it} is the household i' income in year t. ϵ is the error term. Hence, please note that results derived in table on page 111 reflect the coefficient β in the above equation. It is not the elasticity measure described in equation on the current page.

6.1. Fixed Effect and Random Effect Models

I performed the following tests to make sure that the model I am applying is a good fit for the data. This means I need to decide firstly whether there is indeed a panel effect in the data, meaning whether there is heterogeneity among households spending on goods. If there is no heterogeneity in household spending in a specific good among all the income percentiles, then random effect (re) model is a good fit. This simply means the spending on this good is not sensitive to household incomes. Otherwise, if heterogeneity in household spending on a specific

good exists, then fixed effect (fe) model is a good fit. This just means spending on this good is sensitive to different household income percentiles. Random and fixed effects regressions are explained in more details later in this section.

My ultimate goal is to analyze the income elasticity of household spending on various goods across the period the EFF survey covers. Due to the **unbalanced** panel (old households dropped out from the panel, and new households were added in to maintain the total) and changing population weights nature, I can only do panel regression on a single [household-year] basis, but not across the horizon, since fixed regression (fe) across years require constant population weights for econometrics reasons. Please note that "weights" mean the sample weights. For a detailed discussion, please see the technical document on EFF surveys by Bover (2004) (77).

I include a short description below on three settings for the models using STATA: Random Effect (re), Fixed Effect (fe) and Between Effect (be). Please refer to the Baltagi (2013, chap. 2) (74) and Wooldridge (2013, chap. 14) (102) for more details on `fe` and `re` models using STATA. For technical details, please see STATA manual (97).

.

Typical panel data assumes the following form:

$$y_{it} = \alpha + x_{it}\beta + \nu_i + \epsilon_{it} \tag{6.1.1}$$

Where subscript i represents units. In the case of this paper, this is households or income quantile groups. Subscript t represents time. In the case of this paper, I don't have continuously yearly data, due to the low frequency of the survey. $\nu_i + \epsilon_{it}$is the error term; ν_i is the unit specific error term. ϵ_{it} is the "usual" error term as appeared in many econometric analysis, with its "normal properties": mean 0 and uncorrelated with itself and x, and uncorrelated with the unit difference error term ν, and homoskedastic. My goal is to estimate β without the bias from differences result from units.

Regardless of the properties of the error terms ν_i and ϵ_{it}, rewrite 6.1.1 by taking the mean of dependent and independent variables, it becomes

$$\bar{y_i} = \alpha + \bar{x_i}\beta + \nu_i + \bar{\epsilon_i} \tag{6.1.2}$$

where $\bar{y_i} = \sum_t y_{it}/T_i$, $\bar{x_i} = \sum_t x_{it}/T_i$ and $\bar{\epsilon_i} = \sum_t \epsilon_{it}/T_i$. Subtracting 6.1.2 from 6.1.1, it arrives at

$$(y_{it} - \bar{y_i}) = (x_{it} - \bar{x_i})\beta + (\epsilon_{it} - \bar{\epsilon_i}) \tag{6.1.3}$$

While these above three equations estimate β, my goal is to estimate the fixed-effect estimator, also called within estimator, using STATA command `xtreg, fe` , which estimates the coefficient in 6.1.3 using Ordinary Least Square (OLS). On the other hand, `xtreg, be` estimates coefficient (also called between-estimator) in 6.1.2 using OLS; `xtreg, re` estimates coefficient (also called random-effect estimator) in 6.1.1.

I divided the households in the sample (weighted) into the quantiles defined in empirical iterations such as Bover (2008) (77) described as below:

- 1st quantile $< 20th\,percentile$
- 2nd quantile $20th\,percentile - 40th\,percentile$
- 3rd quantile $40th\,percentile - 60th\,percentile$
- 4th quantile $60th\,percentile - 80th\,percentile$
- 5th quantile $80th\,percentile - 90th\,percentile$
- 6th quantile $90th\,percentile - 100th\,percentile$

6.1.1. Income elasticity on spending variables in EFF dataset

I run random effect and fixed effect models as introduced above, and bivariate (single variable household gross income) and multivariate models (household income, financial and non-financial wealth, household head age) and select the best model to estimate income elasticity of food and durable goods in the survey year 2001/2002, 2004/2005, 2008 and 2011. Please see all the variables used in the following analysis from the original Code book (excerpt) below (except for `p6_labinchhm`, which is by author's own calculation, all others are from the original EFF dataset):

- `h_2002` is the household identifier in 2002 survey
- `bage` - household head age
- `np1` - number of family members in each family
- `imp` - imputation number (between 1 - 5). In this current version of paper, I use one imputation for the analysis
- `facine3` - population weights (sampling weights)
- `percrent` - quantiles (ranges from 1-6)
- `renthog` - total labour (salaries and family business enterprise income) and non-labour (financial asset return and rental income) household annual income
- `p6_labinchhm` - author's own calculations: total labour income (only salaries and family business enterprise income), excluding non-labour income such as financial and real asset returns; plus total social transfer such as job seekers' unemployment allowance; scholarships; child assistances; social securities programs such as pension and annuities incomes; external transfers from families and relatives not residing in the same households
- `alim` - annual food consumption
- `nodur` - annual non-durable goods purchases
- `gimpvehic` - annual vehicle purchases

- `p2_70` - other durable goods purchases
- `riquezanet` - the **net wealth**, Total Assets (both real assets and financial assets) - Total Debts.
- `actfinanc` - total financial net wealth.

6.1.2. Year 2001 Income Elasticity of Food Results

In order to find the best fitted models to explain the Income elasticity of spending on various goods, I follow the following steps. From all the goods and years onwards, I follow the same steps as below.

1. Load the dataset in the following format in Table 6.1.1. The data looks like the following format. Please note that the survey takes place in 2002 while asking questions on income and wealth of households in 2001. Year 2002 and 2001 are used interchangeably in this paper. Variable `percrent` is the income percentile groups, ranging from 1-6, from 1 being the lowest quantile, and 6 the highest. Eyeballing the dataset, it is easy to spot that with each percentile, for example, the following table the bottom two lines of families 3 and 4 as listed in `h_2002` [family id], family 4 has household income 14870 (`renthog`), larger than family 3, which has an income of 10700. Intuitively, family 4's consumption on food (`alim`) is proportionally larger than family 3's, which are 4320 and 3600, respectively. This is also true in the non-durable goods consumption (`nodur`). Variable `facine3`is the sample weights, it is the inverse of the probability of each household exists in the nation.

Table 6.1.1.: EFF 2001/2002 data (eff_2002_imp1_ssub), subset

h_2002	imp	facine3	percrent	renthog	alim	nodur
1	1	5799.7153	4	38898	4320	10080
2	1	137.81359	3	17129	7200	7200
3	1	243.75424	2	10700	4320	4200
4	1	23.322895	2	14870	3600	2400

2. Run random effect (re) regression. The random effect does not allow weights option. This is because random effect model assumes no heterogeneity in household spending on good (food). Hence, weights do not play a role in the results. Results are in table 6.1.2. **Discussion:** Random effect regression is equivalent to a normal GLS regression. Important statistics are highlighted in bold texts. For example, p-values show both variable (renthog - annual household income) and constant (meaning certain amount must be spent on food for each of the household in the regression) are significantly different from zero. The prob>chi2 = **0.0000**, which is <0.05. This mean statistically significant. This is a F-Test to check whether all coefficients in the model are different from zero. Please note that this is a bivariate model, which may give rise to misspecification risk on the model. Please see multivariate models in Table 6.1.8 onwards.

Table 6.1.2.: 2001 RE Regression Results [Income Elasticity of Food]

```
. xtreg alim renthog, re

Random-effects GLS regression                   Number of obs     =      5,143
Group variable: percrent                        Number of groups  =          6

R-sq:                                           Obs per group:
     within  = 0.0200                                         min =        552
     between = 0.8645                                         avg =      857.2
     overall = 0.1287                                         max =      1,003

                                                Wald chi2(1)      =     111.82
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =     0.0000
```

alim	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
renthog	.0072921	.0006896	10.57	0.000	.0059406	.0086437
_cons	5770.157	356.8697	16.17	0.000	5070.706	6469.609
sigma_u	862.88005					
sigma_e	3170.1136					
rho	.06897808	(fraction of variance due to u_i)				

3. Testing whether panel effect exists in our dataset, meaning if there is any unit differences in our data - families make similar decision on food consumption regardless of their income.

Breusch and Pagan Lagrangian multiplier test for random effects

$$alim[Percrent, t] = Xb + u[percrent] + e[percrent, t] \tag{6.1.4}$$

Results are as follows. **Discussion**:This is a test to check whether there is any panel effect in the data. The p-value Prob > chibar2 = **0.0000,** which means statistically significant. This proves panel effect exists in our dataset, meaning there are differences consumption habits between income groups. Random effect model does not fit the data well.

Table 6.1.3.: 2001 Breusch and Pagan Lagrangian multiplier test [Income Elasticity of Food].

```
. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        alim[percrent,t] = Xb + u[percrent] + e[percrent,t]

        Estimated results:
                     |       Var     sd = sqrt(Var)
             ---------+-----------------------------
                alim |   1.41e+07       3754.958
                   e |   1.00e+07       3170.114
                   u |     744562       862.8801

        Test:   Var(u) = 0
                            chibar2(01) = 47637.92
                          Prob > chibar2 =   0.0000
```

4. I run a fixed effect model as a comparison to the random effect model in step 2: Results are as below. Discussion: Here I run a unweighted fixed effect regression as a comparison to the previous [unweighted] random effect regression. Please again note that this is a bivariate model that may give rise to misspecification risk to the model selection. Please see multivariate model in Table 6.1.8 onwards.

Table 6.1.4.: 2001 FE 1 Results (unweighted) [Income Elasticity of Food]

```
. xtreg alim renthog, fe

Fixed-effects (within) regression               Number of obs      =       5,143
Group variable: percrent                        Number of groups   =           6

R-sq:                                           Obs per group:
     within  = 0.0200                                         min =         552
     between = 0.8645                                         avg =       857.2
     overall = 0.1287                                         max =       1,003

                                                F(1,5136)          =      104.94
corr(u_i, Xb)  = 0.4779                         Prob > F           =      0.0000

------------------------------------------------------------------------------
        alim |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0070721   .0006903    10.24   0.000     .0057187    .0084255
       _cons |    5632.71   52.04235   108.23   0.000     5530.685    5734.735
-------------+----------------------------------------------------------------
     sigma_u |  1839.9638
     sigma_e |  3170.1136
         rho |  .25198696   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(5, 5136) = 230.03                     Prob > F = 0.0000
```

5. I run the following code to run a Fixed effects regression using Least Squares Dummy Variables model (LSDV), and obtained the following results. **Discussion**: Results show renthog (household income) is statistically significant, p-value<0.05. Income percentile dummies are also statistically significant, with p-value<0.05. These are in line with the Breusch and Pagan test conducted above.

Table 6.1.5.: 2001 Fixed Effect II Regression (no options for weights) [Income Elasticity of Food]

```
. regress alim renthog i.percrent
```

Source	SS	df	MS	Number of obs	=	5,143
				F(6, 5136)	=	346.38
Model	2.0886e+10	6	3.4810e+09	Prob > F	=	0.0000
Residual	5.1615e+10	5,136	10049620.1	R-squared	=	0.2881
				Adj R-squared	=	0.2872
Total	7.2501e+10	5,142	14099711.5	Root MSE	=	3170.1

alim	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
renthog	.0070721	.0006903	10.24	0.000	.0057187	.0084255
percrent						
2	1230.2	146.0264	8.42	0.000	943.926	1516.474
3	1977.315	145.2432	13.61	0.000	1692.576	2262.053
4	2701.455	146.4477	18.45	0.000	2414.355	2988.555
5	3784.713	170.3505	22.22	0.000	3450.754	4118.673
6	5165.33	166.5633	31.01	0.000	4838.795	5491.864
_cons	3305.547	100.2304	32.98	0.000	3109.053	3502.041

6. To test whether our data fits fixed effect model or random effect model the best, I run a Hausman test by Greene (2008) (85). Null hypothesis is that random effect is the preferred model. It tests whether the unit error term ν_i are correlated with the regressor, renthog (household gross income). The results are as follows. **Discussion: p<0.05,** It rejects the null hypothesis that random effect model fits our data. Fixed effect model is more appropriate than random effect for our data. Panel effect exists in our data: different income groups have different consumption habits on food. Please note that this test does not have weights option. However, weights options do not affect the conclusion.

Table 6.1.6.: 2001 Hausman Test (no weights option) [Income Elasticity of Food]

	Coefficients			
	(b) fixed	(B) random	(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
renthog	**.0070721**	**.0072921**	**-.0002201**	**.0000325**

b = consistent under Ho and Ha; obtained from xtreg
B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(**1**) = (b-B)'[(V_b-V_B)^(-1)](b-B)
= **45.92**
Prob>chi2 = **0.0000**

7. Using weights, running a fixed effect regression, result are as bellow. **Discussion**: Prob > F = 0.0001, which is <0.05. This means the joint coefficients is statistically significantly different from zero. Coefficient of renthog (household annual income) is .0210591, This is different from previous random and fixed effect coefficients. This is because of the weights options. Some of the household takes a higher weights in the survey than others. The regression result indicates household income increases by 1 unit, food consumption demand increase by 0.0210591. This result already consider income group differences. This is the coefficient of income elasticity of demand on food. It is less than 1, and hence it is inelastic. The graphical results are presented in the next section Figure 6.1.1, 6.1.2, 6.1.4a and 6.1.4b.

Table 6.1.7.: 2001 `areg` result (with weights option) [sum of wgt is 1.3971e+07] [Income Elasticity of Food]

```
. areg alim renthog [pw=facine3], a(percrent) cl(percrent)
(sum of wgt is   1.3971e+07)

Linear regression, absorbing indicators         Number of obs     =      5,143
                                                F(   1,      5)   =     136.45
                                                Prob > F          =     0.0001
                                                R-squared         =     0.2480
                                                Adj R-squared     =     0.2471
                                                Root MSE          =  2663.8723

                                (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
        alim |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0210591   .0018028    11.68   0.000     .0164249    .0256933
       _cons |   4849.426   52.81795    91.81   0.000     4713.653    4985.199
-------------+----------------------------------------------------------------
    percrent |   absorbed                                     (6 categories)
------------------------------------------------------------------------------
```

8. In order to test whether there are more than one variables to affect food consumption, I also run a multi-variate regression as follows: **Discussion**: Results show variable renthog [household income] is statistically significant, p-value<0.05. bage [household head age] is at a borderline of statistically significant, p-value close to 0.05. np1 [number of household members] is also statistically significantly, which is intuitive. actreales [value of real assets] is not statistically significant. actfinanc [value financial assets] is not. riquezabr [net assets] is collinear with other variables, hence omitted. nodur [non-durable goods consumption] is moderately statistically significant, with p-value = 0.022.

Table 6.1.8.: 2001 Multi-variate regression on Food Consumption (weighted) [Income Elasticity of Food]

```
(sum of wgt is   1.3971e+07)
note: riquezabr omitted because of collinearity

Linear regression, absorbing indicators         Number of obs     =      5,143
                                                F(   5,      5)   =          .
                                                Prob > F          =          .
                                                R-squared         =     0.3300
                                                Adj R-squared     =     0.3285
                                                Root MSE          =  2515.4476

                                (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
        alim |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0163428   .0015722    10.40   0.000     .0123014    .0203842
        bage |    178.711    69.7625     2.56   0.051      -.61926    358.0412
         np1 |   778.0364   43.37617    17.94   0.000     666.5344    889.5384
   actreales |   .0001127   .0001097     1.03   0.351    -.0001693    .0003947
   actfinanc |   .0000265   .0001765     0.15   0.887    -.0004272    .0004802
   riquezabr |          0  (omitted)
       nodur |    .05145   .0157585     3.26   0.022     .0109414    .0919586
       _cons |    1884.08   333.5882     5.65   0.002     1026.565    2741.596
------------------------------------------------------------------------------
     percrent |   absorbed                                     (6 categories)
```

Income Elasticity of Spending on Food 2001 (Figures)

The following figures 6.1.1 and 6.1.2 are results for year 2001.

Figure 6.1.1.: EFF 2001 imp1 Scatter Plot of Household Income and Food Consumption by Income groups (1-6) [same scale]

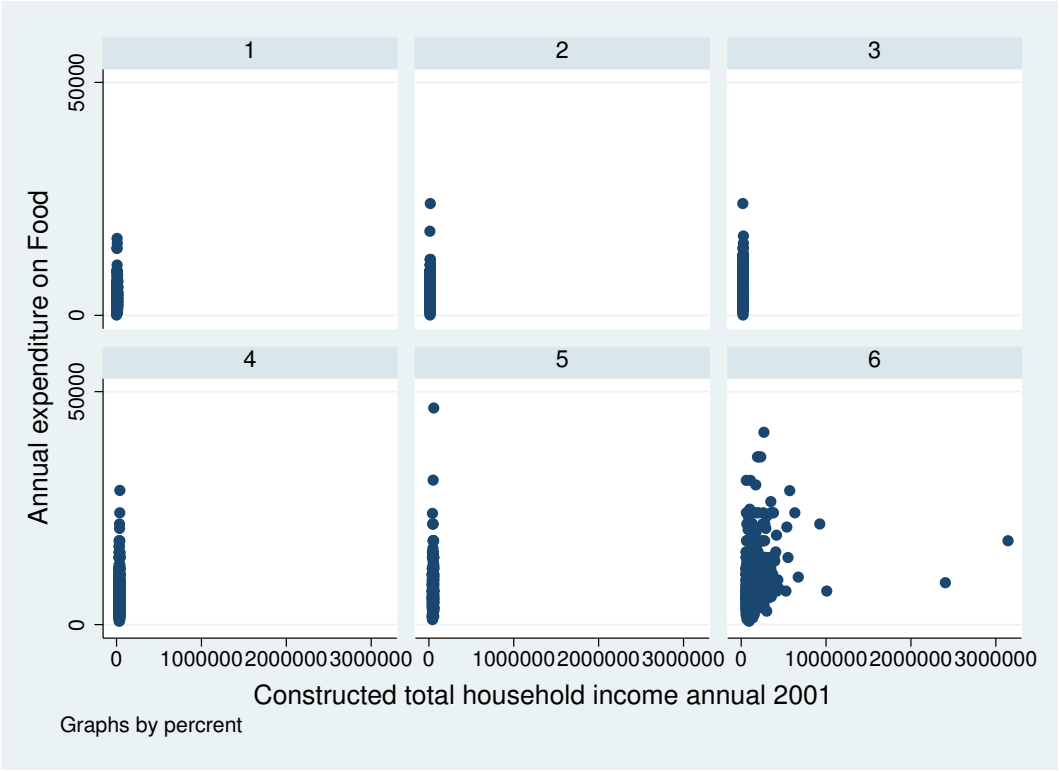
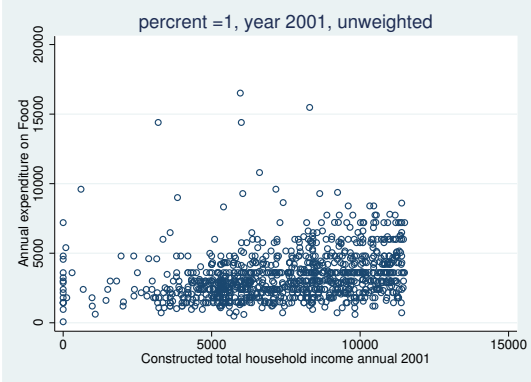
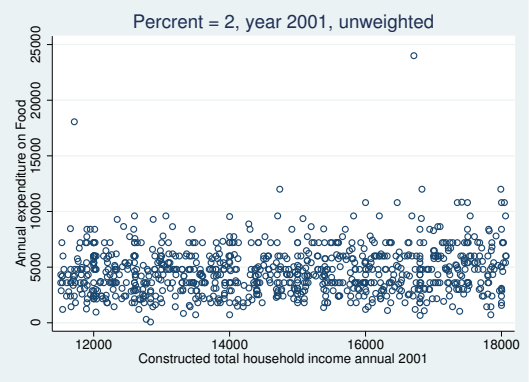


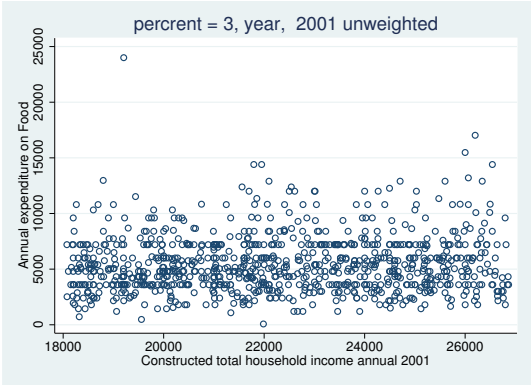
Figure 6.1.2.: EFF 2001 imp1 Scatter Plot of Household Income and Food Consumption by Income Group (1-6) [closer snapshot]



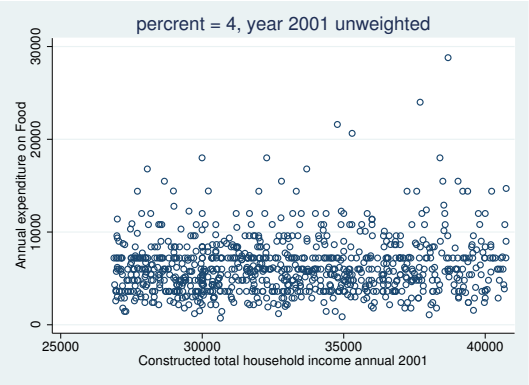
(a) Group 1 (1-20th percentile)



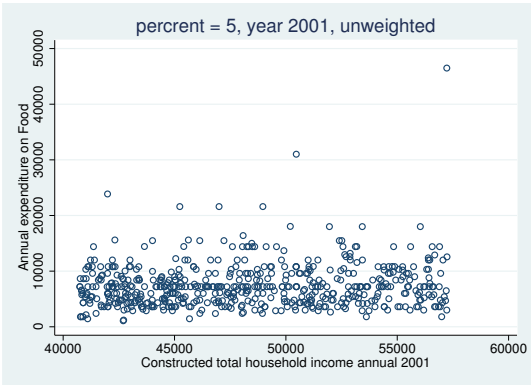
(b) Group 2 (20-40th percentile)



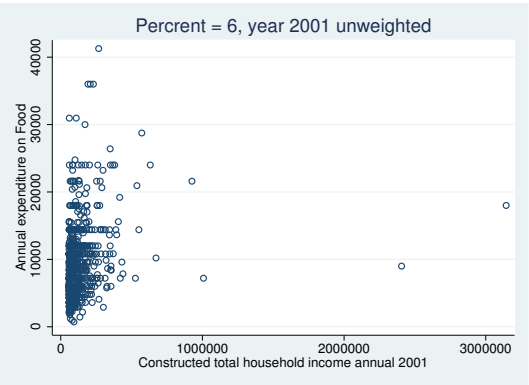
(c) Group 3 (40-60th percentile)



(d) Group 4 (60-80th percentile)

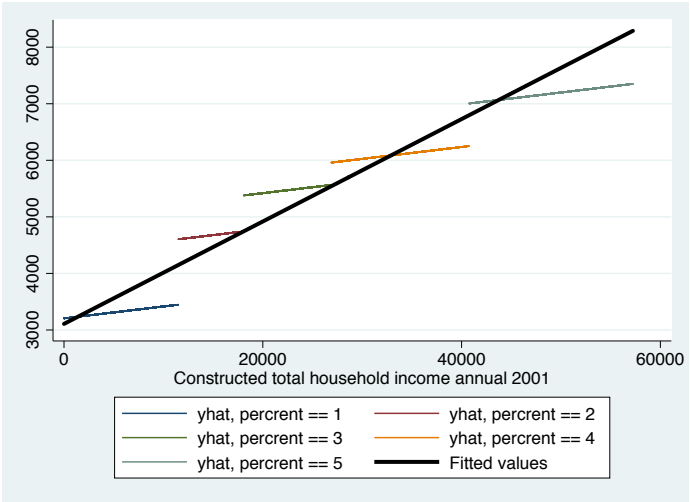


(e) Group 5 (80-90th percentile)

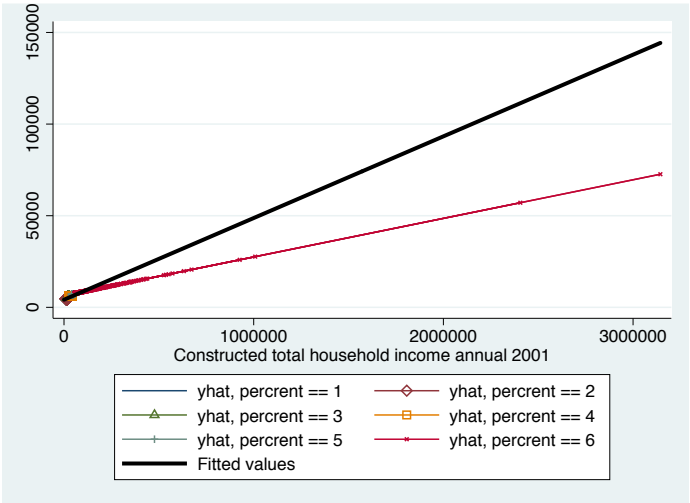


(f) Group 6 (90-100th percentile)

Figure 6.1.3.: 2001 EFF Estimated Consumption using `areg` regression coefficient as in Table 6.1.7



(a) Estimated Consumption Group 1-5. This is to present a closer shot of percentile 1st - 90th percentiles. [weighted estimates and fitted values].



(b) Estimated Consumption Group 1-6. This is to show the entire scale of the income distribution of all survey data. [weighted estimates and fitted values]

6.1.3. Year 2001/2002 Income Elasticity of Other Non-durable Goods Spending

From hereafter, I only show the key results. Please note that I follow all the steps as in section 6.1.2.

Discussion: renthog [household income] is statistically significant' constant term is statistically significant. Both p-values<0.05. Breusch and Pagan test shows random effect is not appropriate for our data, with prob>chi2 = 0.000 < 0.05. This means panel effect [heterogeneity among households of different income percentiles] does exist. Fixed effect regression is more appropriate.

Table 6.1.9.: 2001 Household Annual Income and Other Non-Durable Goods Spending Random Effect Regression Results

```
. xtreg nodur renthog, re

Random-effects GLS regression                   Number of obs     =       5,143
Group variable: percrent                        Number of groups  =           6

R-sq:                                           Obs per group:
     within  = 0.0372                                         min =         552
     between = 0.9842                                         avg =       857.2
     overall = 0.1371                                         max =       1,003

                                                Wald chi2(1)      =      313.15
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =      0.0000

------------------------------------------------------------------------------
       nodur |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |     .02734    .001545    17.70   0.000     .0243119    .0303681
       _cons |   4853.254   227.1486    21.37   0.000      4408.05    5298.457
-------------+----------------------------------------------------------------
     sigma_u |  463.55687
     sigma_e |  7285.8107
         rho |  .00403177   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        nodur[percrent,t] = Xb + u[percrent] + e[percrent,t]

        Estimated results:
                     |       Var     sd = sqrt(Var)
             ---------+------------------------------
               nodur |   6.74e+07       8207.798
                   e |   5.31e+07       7285.811
                   u |     214885       463.5569

        Test:   Var(u) = 0
                            chibar2(01) =  9625.68
                        Prob > chibar2 =   0.0000
```

Discussion: Fixed effect regression results show `renthog` [household labour and non-labour income (before transfers)] and constant term are statistically significant, with p-value<0.05. Coefficient of fixed effect regression is slightly different from random effect regression in previous table.

Table 6.1.10.: 2001 Household Annual Income and Other Non-durable Goods Fixed Effect (I) Regression Results

```
. xtreg nodur renthog, fe

Fixed-effects (within) regression               Number of obs      =       5,143
Group variable: percrent                        Number of groups   =           6

R-sq:                                           Obs per group:
     within  = 0.0372                                         min =         552
     between = 0.9842                                         avg =       857.2
     overall = 0.1371                                         max =       1,003

                                                F(1,5136)          =      198.63
corr(u_i, Xb)  = 0.5155                         Prob > F           =      0.0000

------------------------------------------------------------------------------
       nodur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0223614   .0015866    14.09   0.000      .019251    .0254719
       _cons |   4944.416   119.6079    41.34   0.000     4709.934    5178.899
-------------+----------------------------------------------------------------
     sigma_u |  2826.7379
     sigma_e |  7285.8107
         rho |  .13083332   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(5, 5136) = 99.00                     Prob > F = 0.0000
```

Discussion: All constant terms for all income percentiles are statistically significant. This confirms heterogeneous impacts of household income have on demand on other non-durable consumption.

Table 6.1.11.: 2001 Household Annual Income and Other Non-durable Goods Fixed Effect (II) Regression Results

```
. regress nodur renthog i.percrent
```

Source	SS	df	MS	Number of obs	=	5,143
				F(6, 5136)	=	231.62
Model	7.3771e+10	6	1.2295e+10	Prob > F	=	0.0000
Residual	2.7263e+11	5,136	53083037.7	R-squared	=	0.2130
				Adj R-squared	=	0.2120
Total	3.4641e+11	5,142	67367940.2	Root MSE	=	7285.8

nodur	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
renthog	.0223614	.0015866	14.09	0.000	.019251	.0254719
percrent						
2	1057.262	335.6096	3.15	0.002	399.3241	1715.2
3	1724.986	333.8096	5.17	0.000	1070.577	2379.395
4	2879.194	336.578	8.55	0.000	2219.358	3539.03
5	3921.98	391.5132	10.02	0.000	3154.448	4689.513
6	7971.986	382.8091	20.82	0.000	7221.517	8722.455
_cons	2167.581	230.3577	9.41	0.000	1715.982	2619.18

Discussion: **p<0.05,** It rejects the null hypothesis that random effect model fits our data. Fixed effect model is more appropriate than random effect for our data. Panel effect exists in our data: different income groups have different consumption habits on other non-durable goods. Please note that this test does not have weights option. However, weights options do not affect the conclusion.

Table 6.1.12.: 2001 Hausman Test and Sargan-Hansen test, Household Income and Other Non-durable Goods

□

```
. hausman fixed random
```

	Coefficients			
	(b) fixed	(B) random	(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
renthog	**.0223614**	**.02734**	**-.0049785**	**.0003611**

b = consistent under Ho and Ha; obtained from xtreg
B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(**1**) = (b-B)'[(V_b-V_B)^(-1)](b-B)
= **190.06**
Prob>chi2 = **0.0000**

Discussion: Both renthog [household income] and constant term (absorbing all constant dummies for all income percentiles) are statistically significant.

Table 6.1.13.: 2001 Household Income and Other Non-durable Goods,

```
. areg nodur renthog [pw=facine3], a(percrent) cl(percrent) robust
(sum of wgt is   1.3971e+07)

Linear regression, absorbing indicators         Number of obs     =      5,143
                                                F(   1,      5)   =    1871.91
                                                Prob > F          =     0.0000
                                                R-squared         =     0.1876
                                                Adj R-squared     =     0.1867
                                                Root MSE          =  4525.6915

                                  (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
       nodur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0349019   .0008067    43.27   0.000     .0328283    .0369756
       _cons |   3680.168   23.63431   155.71   0.000     3619.415    3740.922
-------------+----------------------------------------------------------------
    percrent |   absorbed                                     (6 categories)
------------------------------------------------------------------------------
```

Discussion: `areg` multivariate regression I. It shows collinearity among predictor variables. Variable bage [household head age] is statistically significant. This means different ages of household heads make different decisions on how much households spend on other non-durable goods. variable np1 [number of household members], actreals [real assets] and a are not statistically significant. Finally, variable actfinanc [financial assets] is statistically significant. 6.1.13 is still best model.

Table 6.1.14.: 2001 Household Income and Other Non-durable Goods, Multivariate Regression (with weights option).

```
. areg nodur renthog bage np1 actreales actfinanc riquezabr [pw=facine3], a(percrent)
> cl(percrent) robust
(sum of wgt is   1.3971e+07)
note: riquezabr omitted because of collinearity

Linear regression, absorbing indicators         Number of obs     =      5,143
                                                F(   5,      5)   = 2480874.86
                                                Prob > F          =     0.0000
                                                R-squared         =     0.1986
                                                Adj R-squared     =     0.1970
                                                Root MSE          =  4496.8215

                                (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
      nodur |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    renthog |   .0317192   .0010054    31.55   0.000     .0291347    .0343036
       bage |  -292.126    28.61241   -10.21   0.000    -365.6765   -218.5755
        np1 |   72.13679   57.25112     1.26   0.263    -75.03191    219.3055
   actreales |  -.0002088   .0001118    -1.87   0.121    -.0004963    .0000786
   actfinanc |   .0016533   .0002854     5.79   0.002     .0009197    .0023869
   riquezabr |          0  (omitted)
       _cons |   4529.607   229.4375    19.74   0.000     3939.819    5119.395
-------------+----------------------------------------------------------------
    percrent |   absorbed                                     (6 categories)
------------------------------------------------------------------------------
```

6.1.4. Year 2001/2002 Income Elasticity of Vehicles Purchased

Discussion: The vehicle purchase is positively correlated with household income, and it is statistically significant with p-value<0.05. However, the constant term is not statistically significant, with p-value = 0.228 > 0.05. Breusche and Pagan test for random effect shows the prob>chi2 = 1.0000 > 0.05. This means random effect model is appropriate for our data. Heterogeneity among household decisions to spend on vehicle purchases does not exist.

Table 6.1.15.: 2001 Household Annual Income and Vehicles Purchases Random Effect Regression Results, using General Least Square [gls] option.

```
. xtreg gimpvehic  renthog, re

Random-effects GLS regression                   Number of obs     =       5,143
Group variable: percrent                        Number of groups  =           6

R-sq:                                           Obs per group:
     within  = 0.0328                                         min =         552
     between = 0.9976                                         avg =       857.2
     overall = 0.0390                                         max =       1,003

                                                Wald chi2(1)      =      208.71
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =      0.0000
```

gimpvehic	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
renthog	.0615324	.0042593	14.45	0.000	.0531844	.0698804
_cons	-436.5692	361.9554	-1.21	0.228	-1145.989	272.8503
sigma_u	0					
sigma_e	22941.812					
rho	0	(fraction of variance due to u_i)				

```
. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        gimpvehic[percrent,t] = Xb + u[percrent] + e[percrent,t]

        Estimated results:
                         |       Var     sd = sqrt(Var)
                ---------+------------------------------
                gimpvehic |   5.47e+08       23395.77
                        e |   5.26e+08       22941.81
                        u |          0              0

        Test:   Var(u) = 0
                                chibar2(01) =     0.00
                            Prob > chibar2 =   1.0000
```

Discussion: Weights option, pweight [population weights] is not used in random effect regression with maximum likelihood estimates [mle]. iweight [Important weight] is a general weights option used in case of other weights options are not available. For details on weights, please see STATA 13 Manual (97; 98). In order to fit random model regression with weights, weights have to be constant within each unit of panel variable. Hence, I need to use the household id as panel variable instead of income percentile (quantile). This model reflects our data the best. By "best", I mean first of all, I need to determine whether the dependent variable, spending on goods are sensitive to income percentiles (quantiles). If the sensitivities are not statistical significant, random effect model is more appropriate. Otherwise, fixed effect model should be used instead.

Table 6.1.16.: 2001 Household Annual Income and Vehicles Purchases Random Effect Regression using Maximum Likelihood Estimates [mle], with iweights options [importance weights].

```
. xtreg gimpvehic renthog [iweight = facine3], mle

Fitting constant-only model:
Iteration 0:   log likelihood = -1.412e+08

Fitting full model:
Iteration 0:   log likelihood = -1.411e+08

Random-effects ML regression                    Number of obs     =      5,143
Group variable: h_2002                          Number of groups  =      5,143

Random effects u_i ~ Gaussian                   Obs per group:
                                                              min =          1
                                                              avg =        1.0
                                                              max =          1

                                                LR chi2(1)        =  299846.15
Log likelihood  = -1.411e+08                    Prob > chi2       =     0.0000
```

gimpvehic	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
renthog	.0291595	.000053	550.53	0.000	.0290557	.0292633
_cons	798.9906	2.211095	361.36	0.000	794.6569	803.3243
/sigma_u	0	4162.907			.	.
/sigma_e	5887.24	1.113753			5885.057	5889.423
rho	0	(omitted)				

```
LR test of sigma_u=0: chibar2(01) = 0.00                 Prob >= chibar2 = 1.000
```

Discussion: I run the fixed effect regression for a comparison to the random effect regression above. Results do not change much. However, as pointed above, random effect is more appropriate. This can be shown in further analysis below.

Table 6.1.17.: 2001 Household Annual Income and Vehicles Purchases Fixed Effect (I) Regression Results

```
. xtreg gimpvehic  renthog, fe

Fixed-effects (within) regression               Number of obs     =      5,143
Group variable: percrent                        Number of groups  =          6

R-sq:                                           Obs per group:
     within  = 0.0328                                         min =        552
     between = 0.9976                                         avg =      857.2
     overall = 0.0390                                         max =      1,003

                                                F(1,5136)         =     174.05
corr(u_i, Xb)  = -0.5177                        Prob > F          =     0.0000

------------------------------------------------------------------------------
   gimpvehic |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0659102    .004996    13.19   0.000     .0561159    .0757045
       _cons |  -610.7425   376.6255    -1.62   0.105    -1349.089    127.6039
-------------+----------------------------------------------------------------
     sigma_u |  683.09065
     sigma_e |  22941.812
         rho |  .00088576   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(5, 5136) = 0.58                     Prob > F = 0.7178
```

Discussion: I run the following fixed effect regression for a comparison to the random effect regression above. Coefficient of renthog [household income] does not change much from the random effect regression. However, as pointed above, random effect is more appropriate. In this fixed effect regression, I add the dummy variables with all income percentiles. Results show all dummies are not statistically significant. This is in line with the Breusche and Pagan test in Table 6.1.15.

Table 6.1.18.: 2001 Household Annual Income and Vehicles Purchases Fixed Effect (II) Regression Results

```
. regress gimpvehic renthog i.percrent
```

Source	SS	df	MS	Number of obs	=	5,143
				F(6, 5136)	=	35.25
Model	1.1132e+11	6	1.8554e+10	Prob > F	=	0.0000
Residual	2.7032e+12	5,136	526326734	R-squared	=	0.0396
				Adj R-squared	=	0.0384
Total	2.8145e+12	5,142	547362048	Root MSE	=	22942

gimpvehic	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
renthog	.0659102	.004996	13.19	0.000	.0561159	.0757045
percrent						
2	54.52924	1056.779	0.05	0.959	-2017.208	2126.267
3	-170.5519	1051.111	-0.16	0.871	-2231.178	1890.074
4	-332.5983	1059.828	-0.31	0.754	-2410.314	1745.117
5	-366.2256	1232.81	-0.30	0.766	-2783.059	2050.607
6	-1783.73	1205.403	-1.48	0.139	-4146.833	579.3723
_cons	-187.8771	725.3582	-0.26	0.796	-1609.888	1234.134

Discussion: Hausman Test p-value = 0.0936 > 0.05. The null hypothesis that random effect model is not rejected. Random effect model is appropriate for our data. Panel effect [heterogeneity among units (household income percentile groups)] does **not** exist. Random Effect Regression results should be used in the analysis.

Table 6.1.19.: 2001 Hausman Test and Sargan-Hansen test

```
. hausman fixed random

                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |    fixed        random       Difference          S.E.
-------------+------------------------------------------------------------
     renthog |   .0659102     .0615324        .0043778        .0026112
------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        2.81
                Prob>chi2 =      0.0936
```

6.1.5. Year 2001/2002 Income Elasticity of Spending on Other Durable Goods

Table 6.1.20.: 2001 Household Annual Income and Vehicles Purchases Random Effect Regression Results

```
. xtreg p2_70      renthog, re robust

Random-effects GLS regression                   Number of obs     =      1,674
Group variable: percrent                        Number of groups  =          6

R-sq:                                           Obs per group:
     within  = 0.0024                                         min =        179
     between = 0.8296                                         avg =      279.0
     overall = 0.0178                                         max =        406

                                                Wald chi2(1)      =      36.86
corr(u_i, X)   = 0 (assumed)                    Prob > chi2       =     0.0000

                              (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
      p2_70  |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0029063   .0004787     6.07   0.000      .001968    .0038445
       _cons |   1824.638   301.4502     6.05   0.000     1233.806    2415.469
-------------+----------------------------------------------------------------
     sigma_u |  305.90595
     sigma_e |  3480.8175
         rho |  .00766429   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xttest0

Breusch and Pagan Lagrangian multiplier test for random effects

        p2_70[percrent,t] = Xb + u[percrent] + e[percrent,t]

        Estimated results:
                     |       Var     sd = sqrt(Var)
             ---------+------------------------------
               p2_70 |   1.27e+07       3559.155
                   e |   1.21e+07       3480.817
                   u |   93578.45       305.9059

        Test:   Var(u) = 0
                             chibar2(01) =   105.51
                          Prob > chibar2 =   0.0000
```

Table 6.1.21.: 2001 Household Annual Income and Vehicles Purchases Fixed Effect (I) Regression Results

```
. xtreg p2_70     renthog, fe

Fixed-effects (within) regression               Number of obs     =      1,674
Group variable: percrent                        Number of groups  =          6

R-sq:                                           Obs per group:
     within  = 0.0024                                         min =        179
     between = 0.8296                                         avg =      279.0
     overall = 0.0178                                         max =        406

                                                F(1,1667)         =       4.07
corr(u_i, Xb)  = 0.4211                         Prob > F          =     0.0438

------------------------------------------------------------------------------
       p2_70 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0019514   .0009674     2.02   0.044      .000054    .0038488
       _cons |    1948.76   98.49586    19.79   0.000     1755.571    2141.948
-------------+----------------------------------------------------------------
     sigma_u |  732.97066
     sigma_e |  3480.8175
         rho |  .04245884   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0: F(5, 1667) = 10.19                     Prob > F = 0.0000
```

Table 6.1.22.: 2001 Household Annual Income and Vehicles Purchases Fixed Effect (II) Regression Results

```
. regress p2_70 renthog i.percrent
```

Source	SS	df	MS	Number of obs	=	1,674
				F(6, 1667)	=	13.69
Model	995345780	6	165890963	Prob > F	=	0.0000
Residual	2.0198e+10	1,667	12116090.4	R-squared	=	0.0470
				Adj R-squared	=	0.0435
Total	2.1193e+10	1,673	12667584.3	Root MSE	=	3480.8

p2_70	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
renthog	.0019514	.0009674	2.02	0.044	.000054	.0038488
percrent						
2	316.0153	345.3904	0.91	0.360	-361.4294	993.4599
3	616.9705	327.2603	1.89	0.060	-24.91402	1258.855
4	852.3399	325.095	2.62	0.009	214.7024	1489.977
5	1546.468	352.2563	4.39	0.000	855.5562	2237.379
6	1919.117	333.7435	5.75	0.000	1264.516	2573.717
_cons	955.9302	260.2765	3.67	0.000	445.427	1466.433

Hausman Test p-value is 0.0001, < 0.05. The null hypothesis that random effect model is rejected. Random effect model is **not** appropriate for our data. Panel effect [heterogeneity among units (household income groups)] exists. Fixed Effect Regression results should be used in the analysis.

Table 6.1.23.: 2001 Hausman Test and Sargan-Hansen test.

```
. hausman fixed random
```

	Coefficients			
	(b) fixed	(B) random	(b-B) Difference	sqrt(diag(V_b-V_B)) S.E.
renthog	**.0019514**	**.0029063**	**-.0009549**	**.0002376**

b = consistent under Ho and Ha; obtained from xtreg
B = inconsistent under Ha, efficient under Ho; obtained from xtreg

Test: Ho: difference in coefficients not systematic

chi2(**1**) = (b-B)'[(V_b-V_B)^(-1)](b-B)
= **16.15**
Prob>chi2 = **0.0001**

Table 6.1.24.: 2001 `areg` result (with weights option)

```
. areg p2_70 renthog [pw=facine3], a(percrent) cl(percrent) robust
(sum of wgt is   4.9478e+06)

Linear regression, absorbing indicators         Number of obs     =      1,674
                                                F(   1,      5)   =     271.04
                                                Prob > F          =     0.0000
                                                R-squared         =     0.0383
                                                Adj R-squared     =     0.0348
                                                Root MSE          =  2636.8840

                                (Std. Err. adjusted for 6 clusters in percrent)
```

p2_70	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
renthog	**.0072426**	**.0004399**	**16.46**	**0.000**	**.0061117**	**.0083734**
_cons	**1538.141**	**15.69975**	**97.97**	**0.000**	**1497.784**	**1578.499**
percrent	absorbed				(6 categories)	

Discussion: It shows collinearity among predictor variables. Also, it produced counter-intuitive results on renthog (household income) and gimpvehic (vehicle purchase), likely due to adding statistically insignificant variables. Hence, it does not improve the model presented in Table 6.1.20.

Table 6.1.25.: 2001 Multivariate Regression (with weights option). `areg` multivariate regression I.

```
. areg p2_70 renthog i.np1 [pw=facine3], a(percrent) cl(percrent) vce(cluster percrent
> )
(sum of wgt is   4.9478e+06)

Linear regression, absorbing indicators         Number of obs     =      1,674
                                                F(   5,      5)   =     190.01
                                                Prob > F          =     0.0000
                                                R-squared         =     0.0406
                                                Adj R-squared     =     0.0348
                                                Root MSE          =  2636.8773

                                (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
      p2_70  |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     renthog |   .0075219   .0005155    14.59   0.000     .0061967    .0088472
             |
         np1 |
          2  |   139.4601   230.0826     0.61   0.571    -451.9861    730.9063
          3  |   218.0848   359.6445     0.61   0.571    -706.4107     1142.58
          4  |    61.8421   216.9375     0.29   0.787    -495.8135    619.4978
          5  |  -212.5632   284.5944    -0.75   0.489    -944.1364    519.0101
             |
       _cons |   1446.017   221.6972     6.52   0.001     876.1261    2015.907
-------------+----------------------------------------------------------------
    percrent |   absorbed                                     (6 categories)
------------------------------------------------------------------------------
```

Discussion: It does not improve the R square, and the statistical significance of both the household income and its squared term deteriorate. Hence, the best model is the one presented in Table 6.1.24.

Table 6.1.26.: 2001 Multivariate Regression (with weights option). `areg` multivariate regression II.

```
. areg p2_70 renthog bage actfinanc [pw=facine3], a(percrent) cl(percrent) robust
(sum of wgt is   4.9478e+06)

Linear regression, absorbing indicators         Number of obs     =      1,674
                                                F(   3,      5)   =     128.05
                                                Prob > F          =     0.0000
                                                R-squared         =     0.0590
                                                Adj R-squared     =     0.0545
                                                Root MSE          =  2609.8617

                                (Std. Err. adjusted for 6 clusters in percrent)
------------------------------------------------------------------------------
             |               Robust
      p2_70  |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    renthog  |   .0051442    .00059     8.72   0.000     .0036274    .0066609
       bage  |  -217.4934   76.7095    -2.84   0.036    -414.6815   -20.30541
   actfinanc |   .0006774   .0000426    15.91   0.000     .0005679    .0007868
       _cons |   2205.924   208.7648    10.57   0.000     1669.277    2742.571
-------------+----------------------------------------------------------------
    percrent |   absorbed                                     (6 categories)
------------------------------------------------------------------------------
```

6.2. Summary of Results

To save space, and the similarity of results in different years and goods, I only include the most relevant results in Table 6.2.1. I found that coefficient of non-durable goods such as food consumption responds well with the lower tail indices presented in Table 5.1.1 and 5.1.2.

Table 6.2.1.: Summary on Income Elasticity of Spending on Various Goods

	food	other non-dur	vehicle	other dur
2001	0.0211 [areg, wgt]	0.0349 [areg, wgt]	0.0292 [re, mle]	0.0072 [areg, wgt]
2005	0.0097 [areg, wgt]	0.0350 [areg, wgt]	0.0028 [areg, wgt]	0.0095 [areg, wgt]
2008	0.0169 [areg, wgt]	0.0194 [areg, wgt]	0.0176 [areg, wgt]	0.0070 [areg, wgt]
2011	0.0201 [areg, wgt]	0.0298 [areg, wgt]	0.0094 [areg, wgt]	0.0076 [areg, wgt]

Discussion: 2008 other durable goods analysis [highlighted in blue] shows conflicting results with Breusche and Pagan v.s. hausman test. I treated it as fixed effect regression. I highlight significant decreases in red, significant increases in green. I also include the model selected for the criteria in the brackets beside the statistics.

Chapter 7

Conclusion

There are several key findings:

Income Inequality trend:

1. The Spanish household income and wealth on average, have indeed grown over the period of 2001-2011, shown by the lognormal body of the distribution mean. However, the income and wealth also have dispersed further apart within the country, which is shown by the growing standard deviation of the lognormal body of the distribution.
2. The upper and lower tails of the income and wealth distributions, in general, have decreased over time. This means the poorer gets poorer, and richer gets richer. However, the richer households in Spain gets richer slower than the poorer households get poorer. This is shown by the smaller upper tail movement than lower tail movement in the Labour income post transfer variable `p6_labinchhm`, in table 5.1.1. The total income (Labour and non-labour, pre-transfer) variable, `renthog`, as presented in table 5.1.2, exhibits similar trends as `p6_labinchhm`. However, with the growth of non-labour income such as financial returns and rental incomes, the upper tail movement is larger than lower tail movement. The richer is getting richer faster. This would mean the transfer indeed smooth out income distribution.
3. All income and wealth variables become more and more unequal over time, which is shown by the Gini indices in table 5.2.1.

Spending variables:

1. Most spending variables have increased on average and also diverged further over time. These can be seen in the growing lognormal body mean and standard deviation of the distributions.
2. Most spending variables have become more and more unequal over time, which is reflected by the Gini indices in table 5.2.1.

Income inequality and consumption sensitivity:

The Income Elasticity of goods are responds to to tail indices changes asymmetrically:

1. Regarding food, when lower tail index increases in year 2005 (highlighted in red in Table 5.1.3), meaning poorer population is getting poorer, food elasticity in 2005 decreases

(highlighted in red in Table 6.2.1). Food is a life necessity, when population gets poorer, it is less elastic, which is intuitive.

2. For other non-durable goods, the elasticity measures decreases in 2008 (highlighted in red in Table 6.2.1). At the moment of writing, it still needs to be investigated what has caused it. It may be either or both the tail indices changes in the lower and upper tails of the distribution. It may simply because other non-durable goods have a longer useful lives, e.g. clothing as a non-durable item lasts several years, and hence, it picks up the lower tail changes one period later than food, which has a much shorter useful life.

3. For vehicles, the elasticity measure decreases in 2005 sharply, and then rise in 2008 and then decreases in 2011 again. This may be due to the multiple years lifespan of vehicles. It takes several years to pick up the income effects.

4. For other durable goods, the elasticity measures stay stable over the period 2001-2011. This is in line with intuition for goods that have longer life span, the spending on such goods do not change with income increases/decreases.

Appendix

7.1. Details on variables used from the EFF Dataset

I also use several variables defined in the EFF database (Variable names and definitions adapted from definitions.doc on Bank of Spain EFF survey information online):

- h_2002, h_2005, h_2008, h_2011, the household identity number
- facine3, the population weights, which is the inverse of the probability of a household exist in the sample designed in this survey. This is crucial as the unweighted results differ largely from the weighted results. In this paper, I focus on the weighted sample population inequality measurements. I firstly normalize the weights, which means proportionate the weights to the sum of one.
- imp, the imputation series, ranges from 1-5
- renthog, this is the **total income** (labour and non-labour, gross, before tax) of all the family members. In the case households do not provide answer to any of the survey question related income, then a direct imputation is applied to the total.
- riquezanet, this is the **net wealth**, Total Assets (both real assets and financial assets) - Total Debts. This is defined after defining all relevant asset and debt variables as follows:
- Real Assets as below:

1. p2_5, main residence value - real asset,
2. otraspr, other real estate asset value - real asset,
3. p2_84, value of Jewelry, work of art, antiques - real asset,
4. valhog, value of business related assets

- Financial Assets as below:

1. p4_7_3, deposits and accounts for payments - financial asset,
2. p4_15, value of listed shares - financial asset,
3. p_4_24, value of unlisted shares and other equities - financial asset,
4. p4_35, fixed income securities - financial asset,
5. allf, total mutual fund value - financial asset,
6. salcuentas, savings account used for home purchase - financial asset
7. valor, pension scheme - financial asset,

8. valseg, life insurance - financial asset
9. valpenseg = valor + valseg, pension scheme - financial asset,
10. odeuhog,

7.2. Multiple Imputation (MI)

The key point to note for MI is, if the research interest is a point estimate of mean, median, regression parameter (denoted by Q for example), for each of the five implicates, the MI estimate $\bar{Q} = \frac{1}{5}\sum_{i=1}^{5} \hat{Q}_i$. The variance of this estimate $\bar{Q}$ has two components:

1. Within imputation sampling variance W, the average of the five variance estimates $(\hat{V_i})$: $W = \frac{1}{5}\sum_{i=1}^{5}(\hat{V_i})$
2. Between imputations variance that reflects the variability due to imputation uncertainty: $B = \frac{1}{4}\sum_{i=1}^{5}(\hat{Q}_i - \bar{Q})^2$

Total variance of $\bar{Q}$ then can be written as $T = W + (\frac{6}{5})B$.

Alternatively, to obtain MI estimates, it may be possible to do the following: If only mean of the statistics are of interest, instead of analyzing the five implicates separately, one can construct a complete dataset by combining the five implicates successively. This is equivalent to construct a unique dataset with 5 times of the actual number of respondents. The calculation can be done by dividing the weight variables (facine3, discussed immediately below) by five, and calculate the desired statistic. Carlin et al. (2003) (79) describes the procedure for analysis using MI dataset by STATA.

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Part III.

Gauging basis risk of longevity risk solutions

Chapter 8

Introduction

Background Being the first country to witness the development of both pension liability buy-in/outs and longevity-linked securities, UK has become one of the most important global centers of pension longevity markets during the past two decades. In today's low interest rate regime, coupled with slow recovery of asset prices to pre-crisis period, as well as stable, long-term longevity improvement, pension schemes may face large funding deficits. On the other hand, Solvency II regulatory reform has created uncertainties to measurement of pension scheme's balance sheet, which motivates pension industry to adopt a de-risking strategy. These factors all contribute to the historical peak of the global pension risk transfer market in the past decade. According to Aon Hewitt (2015) (103), by the end of 2014, the global pension risk transfer market has grown to the size of £35 billion, with £22 billion of these transactions being longevity swaps (£8.8bn in 2013), and bulk annuities reached around £13.2 billion (£7.8bn in 2013). Bulk annuities, or buy-in/outs transactions are indemnity-based solutions which allow the pension sponsors to transfer part or all pension liabilities to a buyer, usually a specialty insurance company. Through these instruments, pension schemes sell their liabilities outwards by paying a buy-out premium. An alternative hedging solution for the pension schemes is to implement their own Asset-Liability Management (ALM) strategy, with longevity swaps being part of it to hedge the unfavourable longevity improvements. Currently there are mainly two types of longevity derivatives: Among the recent market transactions of longevity securities, the majority of those transactions are *bespoke*, or *customised, indemnity-based* longevity swaps that hedge the longevity risks of specific pension schemes or annuity providers' portfolios, which are similar to the annuity reinsurance offered in the reinsurance markets currently[1]. The other type of longevity swaps are linked to a population index such as the LifeMetrics index developed by JP Morgan (see Coughlan et al (2007)(112)). They are referred to as the *indexed-based*, or *standardized* longevity swaps. Please see Tables A.0.1 A.0.2 and A.0.3 in Appendix for a list of recent longevity transactions.

[1]It is worth noting that the fundamental difference between the bespoke longevity swap and a traditional reinsurance contract is that the longevity swaps are usually backed up by collateral, whereas reinsurances are not. The lack of collateral associated with the traditional annuity reinsurance contracts are possibly due to the fact that longevity risk transfer through reinsurance programme is part of the global de-risking strategy of the pension plans or annuity providers, before the innovation of longevity swaps

8.1. *Bespoke* v.s. *index-based* longevity swaps

Early examples of ***bespoke***, or ***customized*** longevity swaps are the transactions agreed between Babcock International and Credit Suisse in 2009. The cash flows exchanged between the hedgers (pension plans and annuity providers, etc) and the hedge suppliers (specialty (re)insurers, investment banks and hedge fund etc.) are designed to match exactly the survival index of the hedgers' pensioners/annuitants portfolio. To sum up, the advantages of the *bespoke* longevity swaps are significant: They provide a perfect hedge to the longevity risk hedgers. However, the costs associated with setting up such transactions are usually high. This is because the hedge suppliers need to put up substantial regulatory capital to ensure the pension liabilities can be met with a high probability. At the same time, the hedge suppliers expect the buy-out price includes a premium relative to the longevity risks being transferred. Moreover, these bespoke securities are viewed as static, buy-and-hold hedges that sit on the hedgers' balance sheets which do not generate further market transactions.

In contrast, the other type of longevity securities is ***index-based***, or ***index-linked***, ***standardized*** longevity swaps. For example, pension sponsors can structure such longevity swaps by combining several *q-forwards* (112) designed by JP Morgan. These instruments exchange cash flows between the hedgers and hedge suppliers that linked to a broader index (e.g. a national population), usually larger and more reliable than the hedgers' portfolios. This design makes the *index-based* transactions more transparent, and easier to manage and structure, which induces lower set-up cost and better liquidity than the *bespoke* ones. Moreover, some pension scheme is too large to hedge with bespoke solutions. This makes *indexed* longevity hedge is currently the most practical and only available solution for hedging the longevity risk associated with deferred pensions and deferred annuities, as well as those super large schemes. Moreover, the specialist longevity insurance companies may use indexed longevity swaps to hedge their exposure instead of purchasing reinsurance as a cost-effective solution.

Nonetheless, the disadvantages of these *index-based* longevity hedge are obvious, which is the mismatch of the survival experience between the hedgers' own portfolios and the reference population, which would cause mismatch of cash flows. The *index-based* hedges are imperfect, leaving a residual amount of risk that cannot be hedged. This specific risk is called ***basis risk***.

8.2. Basis risk

Basis risk can arise when there are differences, errors, or mismatches between the cash flows linked to the underlying hedged instrument and hedging instrument. In the context of longevity risk, it arises when the pension schemes or annuity providers (exposed or hedged population) have a different demographic characteristics from the population associated with the hedging instrument (hedging population). Previous literature (Cairns et al. (2011) (107), Richards and Jones (2004) (126) and Li and Hardy (2009) (121)) list a exhaustive factors of longevity basis risk. We summarize them as follows:

- Gender mismatch

- Age mismatch
- Geographical mismatch (country mismatch)
- Subpopulation mismatch (mortality experience differences between a specific pension fund and the national population)
- Social economic status, lifestyle, etc.

We differentiate the definition of *basis risk* and *sampling risk*. As mentioned in Coughlan et al. (2011) (114), the latter refers to the demographic experience differences due to size of the population. For example, smaller pension fund may experience larger variations in higher age groups due to the number of pensioners is in each age group is smaller. This is also called *"random risk"*. Due to the existence of *basis risk*, when the hedgers contemplate whether to hedge their longevity risks, or what instruments they should use (either the *customized* or *indexed-based* hedge), the foremost and essential step should be to evaluate and quantify of the *basis risk*, then compare the longevity risk reduction by the hedges and the costs associated. The *basis risk* arise from gender and age can be dealt with, without too much troubles by index-based longevity hedge, if structured properly. However, the *basis risk* due to sub-population and social-economic status differences are more challenging to evaluate and hedge. In this paper, we would like to address this question formally.

8.3. Value hedge v.s. cash flow hedge

Value hedging is fundamentally different from ***cash flow hedging***. Cairns et al. (2011) (107) briefly discusses the two concepts.

A *value hedge* is used to hedge the present value of a series of future pension liability cashflows. Value hedge is important to in the situation when short-term solvency requirements is more onerous than meeting cashflows in the long term. In case of mergers and acquisitions, pension plans' present values are more important relative to the operation business. Therefore, value hedge can be very effective.

A *cash flow hedge* is designed to deliver a series of future cash flows to match the pension plan's liability cash flows. Cash flow hedge strategy often involves defining a specific asset strategy to fund the liabilities. Our paper takes into account the cash flow fluctuations of the pension fund balance sheets due to the longevity hedging instruments. This gives us a holistic view of the hedge effectiveness of the instrument across the hedging horizon.

8.4. Previous literature on basis risk modeling

This paper is linked to several strands of academic literature. Firstly, there are several papers on longevity basis risk modelling. Cairns et al. (2011) (107) set up a framework for assessment of longevity basis risk.It provides a case study to examine the basis risk between the mortality

experiences of the assured lives of Continuous Mortality Investigation (CMI) index and the national population (English and Welsh, or E&W) index. The results show that the the two populations have a stable, long-term correlated trend, with the CMI population being a subset of the E&W one. The results of this UK case study show that the basis risk between two correlated populations can be hedged effectively with a appropriately designed, static, index-based longevity swap. This paper considers value-hedging instead of cash-flow hedge.

Using the same dataset of the E&W and CMI populations, Salhi and Loisel (2012) (127) propose multivariate approach for forecasting pairwise mortality rates of related populations. They focuse on modeling the long-term behaviours between the two correlated populations (one is the sub-set of the other), assuming that the two mortality time-series cannot wander off in opposite directions in the long term, without mean reverting, on grounds of biological reasonableness. They aim to propose a consistent approach to forecast pairwise mortality to assess basis risk embedded in the index-based longevity securities.

Dowd et al. (2011) (111) (see also Jarner and Kryger, 2011) design the so-called *gravity mortality model* for two related but different sized populations. The larger population is modeled independently, while the sub-population is modeled in terms of the spreads to the larger one. Based on the Age-Period-Cohort (APC) mortality model, the spreads between the two populations depend on the gravity (or spread reversion parameters) for the period and cohort effects[2]. This model is also tested on the E&W and CMI population data.

Several other authors investigate basis risk of two correlated populations or a group of populations in various ways. Hardy and Li (2011) (121) propose an augmented common factor model for a group of populations, imposing a common mortality age factor, but allowing each population to have its own mortality age pattern and level of mortality. They use data of the female populations of Canada and the United States, and show the goodness-of-fit and ex-post forecasting results suggest that the augmented common factor model performs better than others. They use this model to quantify the basis risk in a longevity hedge of 65-year-old Canadian females structured using a portfolio of q-forward contracts predicated on U.S. female population. Their results show significant hedging effectiveness and longevity risk reduction of the indexed-based longevity instruments. However, this paper makes a strong assumption that the annuity amount of the hedged population is homogenous ($1), which is a highly hypothetical case.

Cox and Lin (2007) (109) investigate the basis risk between populations associated with annuity portfolios and life insurance portfolios, and their results suggested that this basis risk is manageable. Yang and Huang (2013) (132) propose an optimal longevity hedging strategy not only on the internal natural hedging between the annuity portfolio and the life insurance portfolio, but also on external hedging using longevity-linked securities. They propose an optimal hedging strategy based on the insurer' profit function, based on immunization theory. They address the

[2]Their arguments lie in the sizes of the two related populations, recourse to the planetary analogy that "a binary star system with two stars of more or less equal mass exerting similar gravitational pulls on each other". If these are two similar sized populations, the gravitational pulls between the two populations are equivalent relatively. If one population is significantly smaller than the other, and is a subset of the larger one, its gravitational pull on the larger one is negligible. Biological reasonableness suggests that the two related populations do not have mortality experience that deviate from each other for a prolonged period.

basis risk problem by applying Yang and Wang (2013)(133)'s multi-population mortality model to capture the mortality experience differences for the annuity and life insurance policy holders.

8.5. Research contribution

Our research contributes to the current academic literature and pension industry practices in various ways. Firstly, most of the literature modelling basis risk use a theoretical pension plan data, e.g. a pooled assured lives in a country, rather than a real occupational pension plan. This may underestimate the basis risk because pooled assured population is much larger than any specific pension plan, which may smooth out the mortality improvement characteristics. To the best of our knowledge, our paper is the first one to link mortality experience of a pension scheme to sizes of annuities, and set up a formal framework to hedge longevity*basis risk*.

Secondly, through the literature survey on the most recent papers on longevity basis risk quantification and hedging, we found that previous literature mainly focus on *value hedging* rather than real *cash flow* dynamics across the hedging horizon of the pension scheme. A design of longevity hedging instrument based on real cash flow and Marking-to-Market (MTM) dynamics provides more realistic insights on the hedge effectiveness of the instruments across the hedging horizon.

Also, previous papers make explicit or implicit assumption of homogeneous pension annuity payments to each pensioner, which could render hedge effectiveness results less robust. In our paper, we firstly examined the pension income heterogeneity within our pension scheme, rooted from the income distribution theory established by Pareto (1896a, b) (124), (125)[3]. The heterogeneity of the pension income across ages and years suggest there may exist multiple sub-populations within pensioner population, which may be linked to their respective mortality experience curves. Our paper divide pensioners into their respective age-pension[4]-year buckets and discover systematic mortality differentials across groups and with the national index, which subsequently can be applied to effective longevity basis risk hedging strategy.

Our framework provides a thorough overview of the asset-liability management strategies for pension plans and annuity providers. Once basis risk can be measured and managed, the longevity risk transfer market could become popular for trading parties rather than a pure hedgers-led market as it is now.

8.6. Outline of the rest of the paper

Chapter 9 provides a mortality experience study which compares the mortality experience between the UK pension scheme and the national population. It firstly examine the opposite re-

[3]For a brief review of the recent development on income distribution models, please see Neal and Rosen (2000) (123), Kleiber and Kotz (2003) (118), and Chotikapanich (2008) (110), and the latest article by Toda (2012) (131). These literature suggest income and wealth may be power-law distributed in both the upper and the lower tails.

[4]This refers to the size of pension annuities.

lationship between pension annuities and mortality experiences, using a simple Cox Regression (Cox 1972 (108)). We then set up a dynamic pensioner bucketing strategy based on Gaussian mixture models (based on income distribution models, see Colombi (1990) (106)) and Expectation Maximization algorithm (developed by Dempster et al. (1977) (115)). Our model breaks down pensioners into different sub-populations, according to their age-pension-year index. The best mixture model of the pension population is then selected based on the *Bayesian Information Criterion (BIC)* developed by Schwarz (1978) (130). The mortality differentials of each sub-population to the national population can be quantified.

In Chapter 10, we construct a portfolio of q-forwards (developed by JP Morgan) to hedge each of the sub-populations, based on time-varying hedge ratios suggested by the mortality differentials in the previous step. Section 10.1 presents the *Marking to market* (MTM) and *Profit and loss*(*P&L*) dynamics of the pension scheme's portfolios. We consider *cash-flow* based measurement to provide a holistic view of the hedge effectiveness of the *index-based* longevity instruments.

Chapter 10.2.1.3 concludes and summarizes the main results.

Chapter 9

Pension Data Analysis

9.1. Pension mortality experience studies

The mortality data of English and Welsh (E&W hereafter) population from Human Mortality database (HMD hereafter) for the period of 1930-2012 for both sexes is available. This is the national population index used in this paper, We refer this population as *population 1 (pop 1)*, or simply *E&W* hereafter. We refer the pension scheme population as *population 2 (pop 2)*, or simply *Scheme* hereafter. *Pop 2* is a sub-population of *pop 1.*

The pension scheme in our research has around 40,000 lives, and was established in early 1900s, and reached a mature period since 1990s, with the highest number of pensioners reaching retirement age (65). The mortality data is available from 2004 - 2009 for both sexes.

Below is a sample of scheme data:

Table 9.1.1.: Sample Pension Scheme Data

Ref	Sex	Date of Birth	Date of Retired	Date of Death	Pension Annuity
1	M	01/01/1939	01/01/2004	01/04/2009	£1,300
2	M	01/01/1939	01/01/2004	01/04/2010	£1,900
3	M	01/01/1939	01/01/2004	01/07/2009	£500
4	F	01/01/1949	01/01/2003	01/01/2010	£1,000
5	M	01/01/1940	01/01/2002	01/04/2009	£8,900

With these information, we calculate the death counts in each calendar year (t) and each age (x), $D_{x,t}$, and exposure to death, $E_{x,t}$, and estimate the raw mortality rates, $m_{x,t}$ by the relationship $m_{x,t} = \frac{D_{x,t}}{E_{x,t}}$. We include the graphical results comparing *pop 1* and *pop 2* with age brackets of 5 years in Figure 9.1.1. At first glance, the pensioners have generally lower mortality experience than the national population, both males and females. This is in line with the assumption the pensioners in occupational pension scheme have better access to medical care and health services, higher incomes from their secure employment, which could possibly lead to better longevity improvements than the national population. This is similar to the findings in Coughlan et al. (2011) (114) that the assured population collected by the Continuous Mortality Index (CMI) generally have lower mortality experience than the national population.

9.1.1. Cox regression: annuity and mortality

Figure 9.1.1 suggests scheme members have generally lower mortality rates than the UK national population in our research horizon. To quantify the relationship between mortality rates and annuity incomes, we firstly run Cox (1972) (108) regressions as follows.

Cox (1972) (108) regression is originally used to analyze survival functions of different patient groups undertaking different treatments in the medical science area. The purpose of this model is to simultaneously explore the effects of several covariates (variables) on the survival rates of the populations. The advantage of Cox Regression is that it tests the effects of each of the independent covariates simultaneously. If the covariate has a positive regression coefficient, it means that the hazard risk of this covariate is higher if the values of it is higher, and vice versa. In simple terms, Cox regression is a parametric model based on the exponential survival function as follows:

$$log\, h_i(t) = \alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik} \tag{9.1.1}$$

Or, it can be rewritten as

$$h_i(t) = exp(\alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik}) \tag{9.1.2}$$

Cox (1972) (108) regression can be viewed as a linear model for the log-hazard function as in Equation 9.1.1, or as a multiplicative model for the hazard function as in 9.1.2. The letter i in the above equations refer to each individual patient in the treatment group, which is individual pensioners in our pension population. The letter $x's$ are the covariates. The constant α refers to a log-baseline models based on the assumption of the survival distribution such as *Compertz*, *Weibulls*, etc. In contrast, Cox Regression is a *semi-parametric* procedure because the baseline hazard function, $\alpha(t) = log(h_o(t)$ is unspecified. This makes the Cox regression flexible and widely used.

We run two Cox (1972) (108) regressions on our pensioners to prove there exists a negative relationship between pensioner annuities and their mortality rates. Pension annuity can be an effective identifier of social-economic characteristics that lead to mortality differentials, with absence of other data such as occupation, postcodes, medical history, lifestyles, etc.

Cox Regression 1 Regress mortality rate on covariates of *Ages*, *Sexes* and *Annuity amount (divided by £1000)*. Results are summarized in Section 9.1.2

Table 9.1.2.: Cox (108) Regression 1 pensioners in £1,000 bucket

Covariates	Coefficient	Standard error	z-stat	p-value
Age	0.0238	0.0006	41.9746	0
Sex $(0-M, 1-F)$	-0.1196	0.0109	-10.9513	$6.55e^{-28}$
Annuity in $£1K$ bucket	-0.0055	0.00095	-5.8265	$5.66e^{-09}$
Covariates	Hazard Ratio	Survival Ratio	Lower bound	Upper Bound
Age	1.0241	0.9765	0.9754	0.9775
Sex $(0-M, 1-F)$	0.8873	1.1271	1.1032	1.1514
Annuity in $£1K$ bucket	0.99454	1.0056	1.0037	1.0074

Conclusion:

- The coefficient of the covariate *Age* is positive, 0.0238. As pensioners'*ages* increase, the higher hazard risks

they face. The relative risk for *Age* is $e^{0.0238} = 1.024$. Thus the mortality rate for pensioners one year older is 1.024 times higher on average, after adjustment for the effects of all other covariates in the model.

- The coefficient of the covariate *Sex* is negative. Please note that we denote males as 0, and females as 1. Hence, the higher the value of this covariate, the lower the hazard risk is. In other words, being female has a better chance to survive in this particular population. The relative risk for the covariate *Sex* is $e^{-0.1196} = 0.8873$. The mortality rate for female is 88.73% of that of a male, after adjustment for the effects of all other covariates in the model.
- Similar to *Sex*, the coefficient of the covariate *Pension Amount (£1,000 bucket)* is negative. The relative risk for the covariate *Pension Amount (£1,000 bucket)* is $e^{-0.0055} = 0.9945$. Thus the mortality rate for pensioners with £1,000 more in their pension amount tends to decrease to 99.45% of that of the group of pensioners in a lower pension amount bracket, after adjustment for the effects of all other covariates in the model.
- The p-values for all three covariates are all significant. The analysis is valid.

Cox Regression 2 In order to examine the effects of the pension amount with different buckets ($£4,000$), we run Cox Regression with a different set of covariates as follows (Results are summarized in the Table 9.1.3):

Table 9.1.3.: Cox (108) Regression 2 (pensioners in £4,000 bucket)

Covariates	Coefficient	Standard error	z-stat	p-value
Age	0.0238	0.0006	39.5956	0
$G1(0-others, 1-PM < £4K)$	0.0335	0.0119	2.8066	0.0050
$G2(0-others, 1-PM£4K-8K)$	−0.1045	0.0141	−7.4198	$1.17e^{-13}$
$G3(0-others, 1-PM > £8K)$	−0.1645	0.0169	−9.7367	$2.10e^{-22}$
Covariates	Hazard Ratio	Survival Ratio	Lower bound	Upper Bound
Age	1.0224	0.9781	0.9770	0.9791
$G2(0-others, 1-PM£4K-8K)$	1.0340	0.9671	0.9448	0.9900
$G3(0-others, 1-PM > £8K)$	0.8483	1.1788	1.1404	1.2185

- *Age*: The coefficient of covariate Age is 0.0222. The relative risk for this covariate is $e^{0.0222} = 1.0224$. This means the hazard rate of the pensioners suffer a mortality rate of 1.0224 times if they are one year older, after adjustment for the effects of the other covariates in the model.
- *Sex*: Similar to Cox Regression model 1, hence can be ignored here.
- *Group* 1 *Pensioner annuity size* $£0-4,000$ (27,628 lives): The coefficient of covariate *Group 1* is 0.0335. The relative risk for this covariate is $e^{0.0335} = 1.0340$. This means the pensioners in this group has a mortality rate of 1.0340 times higher than those who are not in this group, after adjustment for the effects of the other covariates in the model. This is in line with our intuition since this is the lowest band of the pension amount buckets.
- *Group* 2 *Pensioners annuity size* $£4,000-8,000$ (6,181 lives): The coefficient of covariate *Group 2* is −0.1045. The relative risk for this covariate is $e^{-0.1045} = 0.9008$. This means that the pensioners in this group has a mortality rate of 90.08% of those who are not in this group, after adjustment for the effects of all other covariates in the model. This is also intuitive since this is the middle band of the pension annuity buckets.
- *Group* 3 *Pensioners annuity size* $£8,000+$ (4,037 lives): The coefficient of covariate *Group 3* is −0.1645. The relative risk for this covariate is $e^{-0.1645} = 0.8483$. This means that the pensioners who are in this group has a mortality rate of 84.83% of those who are not in this group, after adjustment for the effects of the other covariates in the model. This is intuitive because this group is the highest band of the pension

amount buckets. The pensioners in this group are supposed to be wealthiest, and healthiest, and should have lowest mortality rate among all three pension amount buckets.

- p-values of all covariates are significant - The regression results are valid.
- We select the £4,000 bucketing strategy as an example to identify the negative relationship with mortality experiences and pension annuity amounts. We found similar results when dividing pensioners using £5,000 buckets. However, without further analysis, Cox Regression alone cannot identify the optimal age-pension-year bucketing strategy. We need to use a more rigorous method to identify sub-populations according to their pension annuity incomes in each year across our research horizon.

9.1.2. Pension scheme cash flow analysis

As the UK pension scheme used in our research was established in early 1900s, and reached its maturity in 1980s - 1990s. By maturity, we mean the number of retirees reaches the highest. We expect the cash flows due to liabilities would fluctuate during our research horizon due to various factors. Firstly, the individual annuity amounts are highly heterogeneous as discovered in many income and wealth distributions found in various countries (See Colombi (1990) (106) for example in the UK income distribution). A panel study on our UK pension scheme annuity cash flow across our research horizon (2004-2009) can be found in Figure 9.1.2. In the left sub-figure of 9.1.2, we observe multiple (possibly three or more) modes in the distribution, which means there may exist multiple sub-populations within our pension scheme. In the right sub-figure of 9.1.2, we observe heavy-tailedness in the pension annuity distribution, with a tail index of around 3.14. This is consistent to the results suggested in previous income distribution literature. The estimation method is the standard *Kolmogorov-Smirnov (K-S)* Goodness-of-Fit test, details of which can be found in Chakravart, Laha and Roy (1967) (113). In simple terms, K-S test is used to determine if a sample statistics is from a population with a specific distribution. The p-value shows that the heavy-tailedness is significant and the result is valid.

The second factor that contributes to the fluctuation of the cash flow due to liabilities are the age-year bracket. Granular results of 2004 can be found in Figure 9.1.3. We observe that the lower age groups (age 65-69, age 70-74 and age 75-79) have more pronounced effects of multiple sub-populations by pension annuities, while for the higher age groups (age 80-84, 85-89 and 90-94), this effect is less pronounced. This suggests we need a consistent and robust methodology to divide pensioners into age-pension-year groups to effectively hedge the basis risk arising from mortality differentials with regard to the national population. Moreover, it is well known that the number of male senior executives are larger than females. Hence, it makes sense to break down the analysis for both sexes. The results for these analyses can be found in figure 9.1.2.

9.1.3. Pensioner bucketing strategy: income distribution approach

From the previous two sections, we draw two conclusions: Firstly, Section 9.1.1 suggests there is a negative relationship between pension annuity and longevity. In other words, the higher the annuity income is, the longer the pensioner survives. Secondly, Section 9.1.2 suggests that there exists multiple sub-populations in the scheme based on the (pension) income distribution analysis.

Therefore, we can make use of existing *income distribution models* based on mixture models (Flachaire and Nunez (2007)(116)) to identify pensioners belonged to their respective subpopulations by sizes of their annuity incomes (as a proxy of their social-economic characteristics with absence of other variables), which then links to their respective mortality curves. This approach is pioneering in comparison to the traditional actuarial approach so far has been introduced to price longevity risk related instruments. The reason is that, majority of the pricing model assumes homogeneous pension payout, and pensioner bucketing strategy does not take into account such differences. Practitioners bucket pensioners into different groups due to their socio-economic characteristics. However, there is no consensus and a scientifically robust method to do that. Hence, our approach provides evidence that income distribution is a convenient method in case all other social-economic characteristics are not available. Please note that, our approach will be further refined if other social-economic characteristics and medical history of pensioners are available.

We followed the notation used by Flachaire and Nunez (2007)(116) and set up a mixed populations model as follows:

We assume that this scheme population can be broken down into k sub-populations whose incomes (in this case the annuities) are characterized by log-normal distributions with mean μ_k and standard deviation σ_k. The density function can be expressed as

$$f(y) = \sum_{k=1}^{K} p_k \wedge (y;\, \mu_k,\, \sigma_k) \tag{9.1.3}$$

where $\wedge(\centerdot; \mu, \sigma)$ is the lognormal distribution with mean μ and standardiviation σ. A point worth noting is that number of modes used to detect heterogeneity, the number of subpopulations defined, K, could change over time. Alternatively, a conditional model can be defined as below by letting the mixing probabilities vary with exogenous individual characteristics:

$$f(y|X) = \sum_{k=1}^{K} p_k(X) \wedge (y;\, \mu_k,\, \sigma_k) \tag{9.1.4}$$

where $p_k(X)$ is the probability of a pensioner belonging to a homogeneous subpopulation k. It is assumed that these mixing probabilities depend on a linear index of X. In reality, the optimal number of subpopulations can be chosen to minimize some criterion, e.g. BIC criterion by Schwarz (1978) (130).

For a fixed number of components K, we can estimate $f(y)$ by maximum likelihood. An alternative conditional model can be set up as follows according to the intuition of individual characteristics influence the magnitude of the group-specific pension income μ_k. Subsequently the individual characteristics could be used to model the mean of the subpopulations rather than the probabilities of belonging to a subpopulation. This conditional model is as below:

$$f(y|X) = \sum_{k=1}^{K} p_k \wedge (y;\, \mu_k(X),\, \sigma_k) \tag{9.1.5}$$

where the conditional mean Equation 9.1.5 can be assumed to be linearly dependent on X, for instance, $\mu_k(X) = X\beta_k$, and they are used to analyze the intra-group variability, whereas conditioning probabilities in Equation 9.1.4applies when the research is focused on inter-group variability.

In our case, if we want to detect inequality in pension incomes among groups such as "high income group", "medium income group", and "low income group", then Equation 9.1.4 has a higher potential to provide an answer. If we would like to analyse the income inequality within each group, then Equation 9.1.5 is more appropriate.

9.1.4. The mixture model

Our objective is to explain the distribution of individuals across groups using the means of the individual characteristics, similar to regression analysis. We set up the following model as in Flachaire and Nunez (2007) (116):

$$Y_i = X_i^c\beta + \epsilon_i \ (i = 1, 2, , \dots, n) \tag{9.1.6}$$

where X_i^c is a centered vector of individual characteristics.β is an l-vector of parameters. ϵ_i is i.i.d. random variable which has a normal distribution with mean of 0 and standard deviation of 1, $N(0, 1)$.

For $k = 1, 2, , \dots K$, we define a dummy variable Z as belows:

$$Z_{ik} = \begin{cases} 1 & if\, U_i \in [\gamma_{k-1},\ \gamma_k] \\ 0 & if\, U_i \notin [\gamma_{k-1},\ \gamma_k] \end{cases} \tag{9.1.7}$$

where $-\infty = \gamma_0 < \gamma_1 < \dots < \gamma_{K-1} < \gamma_K = +\infty$. In our case, we have a few candidates models with different pension income brackets. We will elaborate that in details later.

The vector $Z_i = (Z_{i1}, Z_{i2}, , \dots, Z_{ik})$ has value of 1 in the ikth entry of the matrix of Z_{ik}, and a value of 0 otherwise.

We assume that given vector Z_i, the logarithmic transformation of pension income Y_i, y_i has a density function as follows:

$$f(y_i|Z_i) = \sum_{k=1}^{K} Z_{ik}\varphi(y_i;\ \mu_k,\ \sigma_k) \tag{9.1.8}$$

Where $\varphi(\cdot;\ \mu,\ \sigma)$ is the normal distribution density function with mean μ and standard deviation of σ.

We follow the identifiability constraint and impose that $\mu_1 < \mu_2 < \dots < \mu_K$ to avoid the classic label switching problem mentioned by Redner and Walker (1984).

$Z_i's$ are assumed to be independent and follows a multinomial distribution of $M(1;\ p_{ik},\ p_{i2},\ \dots,\ p_{iK})$, where

$$p_{ik} \equiv E(Z_{ik}) = \Phi(\gamma_k - X_i^c\beta) - \Phi(\gamma_{k-1} - X_i^c\beta) \tag{9.1.9}$$

where $\Phi(\cdot)$ is the cumulative distribution function (cdf) of the normal distribution. This means the probability any pensioner belonging to the k-th group is the probability that a standard normal variable belongs to a certain interval with bounds depending on the values of that individual's characteristics.

Hence, we conclude that the logarithmic transformation ofY_i, y_i follows the mixture densities as follows:

$$f(y_i|X_i) = \sum_{k=1}^{K} p_{ik}\varphi(y_i;\, \mu_i,\, \sigma_k) \tag{9.1.10}$$

Consequently, if we define $\mu = (\mu_1,\, \mu_2, \ldots, \mu_K)$, $\sigma = (\sigma_1,\, \sigma_2,\, ...,\, \sigma_K)$, $\gamma = (\gamma_1,\, \gamma,\, ...,\, \gamma_{K-1})$ and $\theta = (\mu,\, \sigma,\, \gamma,\, \beta)$', the log-likelihood function of y_i can be written as follows:

$$l_n(\theta,\, y) = \sum_{i=1}^{n} log\, f(y_i|X_i) \tag{9.1.11}$$

The *Maximum Likelihood Estimate (MLE)* can be calculated by equating the first derivatives of the log-likelihood function 9.1.11 to zero, with respect to all the parameters. Since there is no analytical solution can be found, an iterative algorithm can be used to solve such equation.

9.1.5. Estimation of the optimal mixture model

Since equation 9.1.11 does not necessarily globally concave, with respect to unknown parameters, iterative algorithm like Newton Raphson method can produce diverging results. Alternatively, *Expectation-maximization (EM)* algorithm developed by Dempster et al.(1977) (115) often exhibits slow linear convergence. Since the methodology used in this case is not the focus of this paper, we choose *EM* as an implementation example, and our results show consistence and convergence among most of the *age-annuity-year* groups. In short, *EM* algorithm is an iterative method to find maximum likelihood or maximum a posteriori estimates of parameters in statistical models, where the model depends on unobserved latent variables. The EM iteration alternates between performing an expectation (E) step and maximization (M) step. This E step creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters, while the M step following computes parameters maximizing the expected log-likelihood found on the previous E step. These parameter-estimates are then used to determine the distribution of the latent variables in the next E step. It continues to reiterate until it reaches a stable maximum of log-likelihood function.

The full log-likelihood of our model can be written as below:

$$l_n(\theta,\, Z,\, y) = \sum_{i=1}^{n}\sum_{k=1}^{K} Z_{ik}(log\, \varphi(y_i;\, \mu_k,\, \sigma_k) + log\, p_{ik}) \tag{9.1.12}$$

To maximize the log-likelihood function, we apply the *EM* method as described above. In practice, the optimal number of sub-populations is the unknown variable, hence we need to select the best mixture model based on criterion like *Baysian Information Criterion (BIC)* by Schwarz (1978) (130). The *BIC* of our log-likelihood function can be written as follows:

$$BIC = -2l_n(\hat{\theta}, y) + (3K - 1 + l)\log n \tag{9.1.13}$$

We present the ***EM* and *BIC* Results** as follows:

- **Total Scheme Population (males & females)**: As discussed in 9.1.2, since the pension income distribution in males and females diverge significantly across *age-year* bucket, it makes sense to run analysis for sexes separately. Hence, we omit the results for the total populations hereafter.
- **Male pensioners:** The BIC converges to its minimum value around $4-8$ number of components (sub-populations by pensioners' annuity amounts) for most *age-year* groups. The BIC statistics simulation results can be found in B.0.2 and B.0.3. We visualize the sub-populations and their distributions by *EM* algorithm as follows:
  - B.0.5 (age $55-59$) - **BIC not converging at its minimum value**. It means the BIC is not stably decreasing to a minimum value, there may exist several possible numbers of sub-populations that is viewed as statistical significance. The result of BIC not converging may be due to one or more of the following reasons: The variable (pension income) probably is not enough to capture the longevity variation. An alternative explanation would be that this set of pension income and longevity data does not exhibit the desired statistical property very well. It may also be due to the lower number of pensioners in this bucket, or size of pension scheme is not large enough.
  - B.0.6 (age $60-64$), B.0.7 (age $65-69$), B.0.8 (age $70-74$), B.0.9 (age $75-79$), B.0.10 (age $80-84$) - **BIC converging at its minimum value**.
  - B.0.11 (age $85-89$), B.0.12 (age $90-94$) and B.0.13 (age $95-99$) - **BIC not converging**.
- **Female pensioners:** Similar to the male pensioners' cases, the *BIC* converges to its minimum value around 4-8 number of components (sub-populations). The BIC simulation results can be found in B.0.4. We visualize the sub-populations and their distributions with *EM* algorithm as follows:
  - B.0.14 (age $55-59$) - **BIC not converging at its minimum value**.
  - B.0.15 (age $60-64$), B.0.16 (age $65-69$), B.0.17 (age $70-74$), B.0.18 (age $75-79$), B.0.19 (age $80-84$), B.0.20 (age $85-89$), B.0.21 (age $90-94$) and B.0.22 (age $95-99$) - **BIC converging at its minimum value**.

We follow the analysis using the groups that have the most significant income distribution patterns and where the BIC is converging to a stable minimum value.

A summary table with key statistics of *EM* and *BIC* and the number of sub-populations can be found in A.0.4, A.0.5

In this paper, we arbitrarily choose the optimal number of sub-populations is 3. In many cases, the BIC converges to a stable minimum value at a higher number (»3) of optimal number of sub-populations. However, as the number of sub-populations goes up, the number of pensioners remain in each sub-population goes down naturally. This gives rise to sampling risk, which is unfavourable for our analysis. The problem is particularly serious in higher annuity sub-population groups and old age groups, in which the number of pensioners is very small (only a dozen of pensioners).

Figure 9.1.1.: Scheme Mortality Experience Comparison v.s. E&W

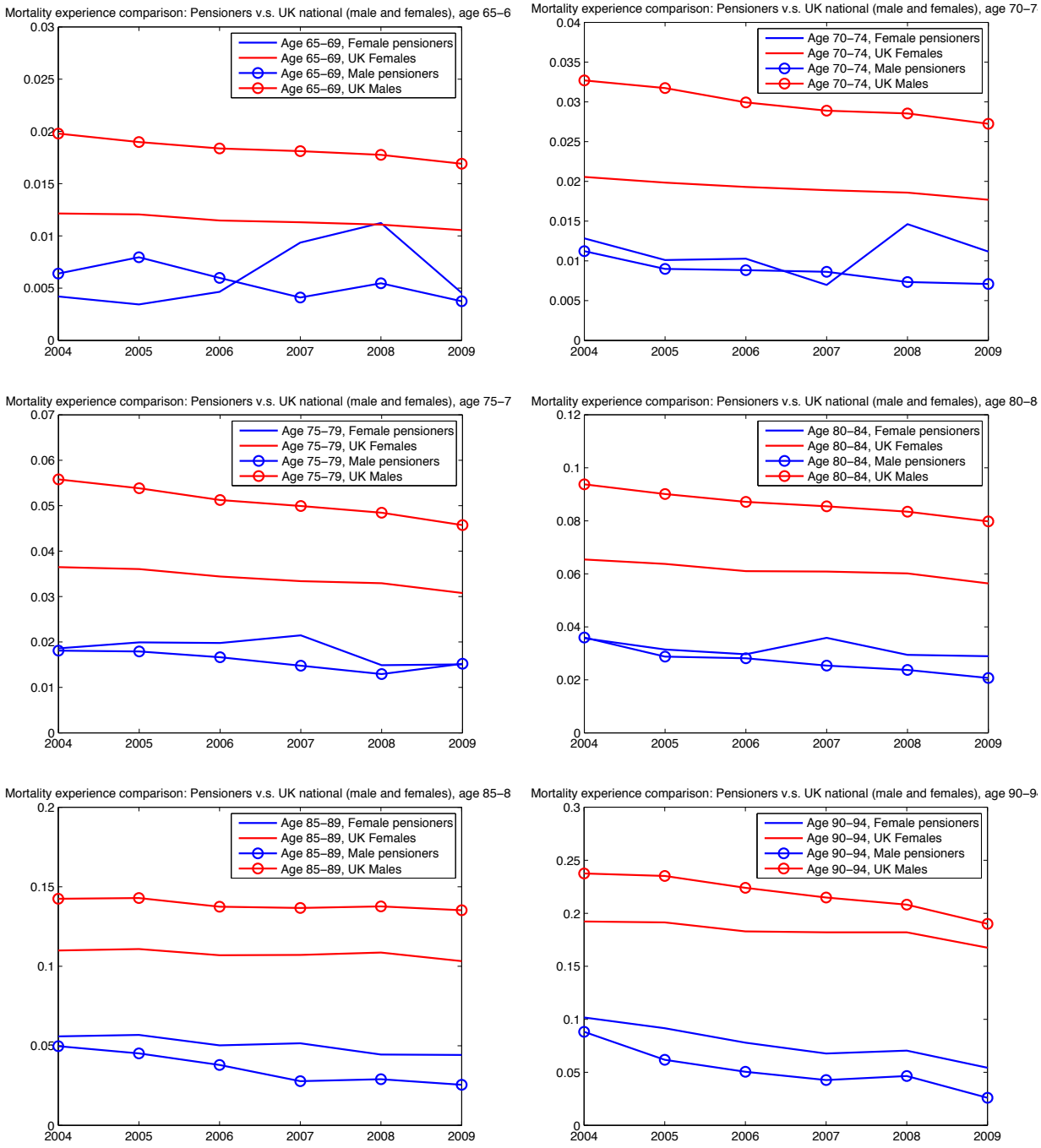


Figure 9.1.2.: Histogram and log-log plot of pension annuity of the entire scheme from inception (1950s) to present (2012)

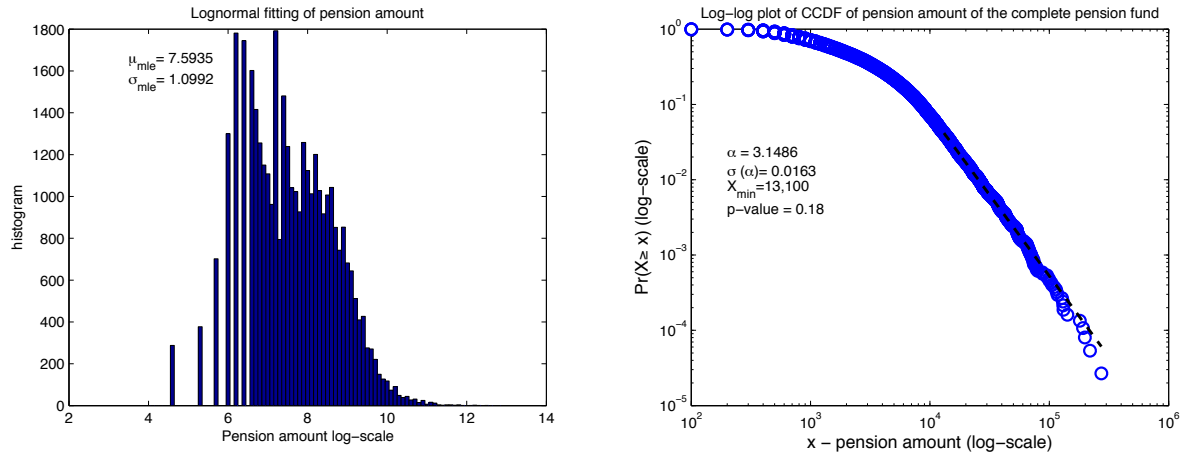
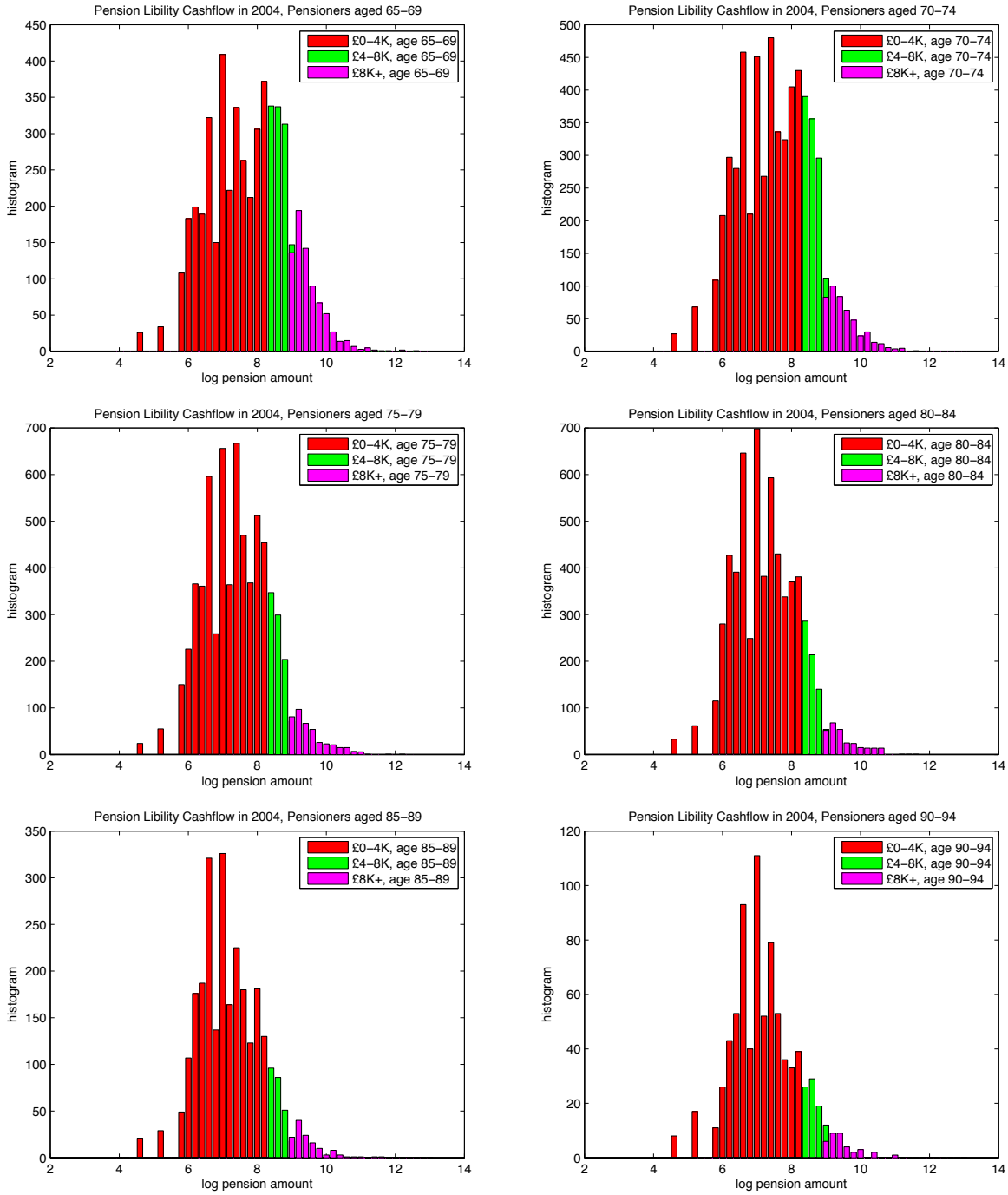


Figure 9.1.3.: Pension cash outflow using age-year bucketing strategies based on Cox regression



Chapter 10

Longevity Swaps

Previous sections examined the granular details on the relationship between annuity sizes and mortality experiences of the UK pension scheme in our study. We bucketed pensioners in this UK pension schemes into three sub-populations according to their demographic characteristics (*age-annuity-year*). In this section, we are going to study the *Marking-to-market*, or *MTM* and *Profit and Loss*, *or P&L* of longevity swaps written to hedge those sub-populations' longevity improvements, using the national index (E&W population).

Longevity swap is a form of instrument that ties the cash flows to the survival rates of specific populations. There are two trading parties involved in the structure of the longevity swaps, the hedgers and the hedge suppliers. Currently, there is no widely accepted *MTM* framework on longevity swaps, the counter-parties of the longevity risk transactions look for reference model from similar markets, such as insurance swap markets. The details of setup of longevity swaps can be found in Biffis et al. (2014) (104). We adopted the same notations in Biffis et al. (2014) (104) for simplicity.

Setup Suppose that the hedgers have obligations to pay a stream of cash flows to the policy holders, $X_{T1}, X_{T2}, ...X_{Tn}$, which are linked to the actual survival experience of the hedgers' portfolio. The stream of cash flows also depend on the interest rates and inflation rates. In this paper, we assume that interest rates and inflation rates are hedged and fixed the interest rate at 5% flat across the horizon. We define the liability of longevity hedgers at a payment date $T > 0$ is given by the random variable $(n - N_T)X_T$, where n refers to the number of lives in the population at the start of the hedge, and N_T refers the number of deaths occurs during the period $[0, T]$. The form of hazard function is unknown, and we assume that death times have common intensity $(\mu_t)_{t\geq 0}$. Thus, the expected number of survivors at time T can be written as

$$E^{\mathbb{P}}[n - N_T] = np_T \tag{10.0.1}$$

- p_T denotes the survival probability of the reference population, denoted as below:

$$p_T = E^{\mathbb{P}}[exp(-\int_0^T \mu_t dt)] \tag{10.0.2}$$

- p_T can be calculated using Lee-Carter mortality model (see Lee and Carter (1992) (119))
- $\mathbb{P}$ denotes the real world probability measure, or the *physical probability measure*

In the financial markets, as previous academic literature has suggested, it is commonly believed asset pricing follows a *risk-neutral* pricing probability measure (denoted by $\tilde{\mathbb{P}}$) rather than the real world probability measure, or the *physical probability* (denoted by $\mathbb{P}$). Hence the expected market value of the aggregate liability of a pension plan or annuity provider that has the survival probability p_T can be expressed as follows (without making assumption on the pricing measure for now):

$$E^{\tilde{\mathbb{P}}}\left[\sum_i exp\left(-\int_0^{T_i} r_t dt\right)(n - N_{T_i})\, X_{T_i}\right] = n\sum_i E^{\tilde{\mathbb{P}}}\left[exp\left(-\int_0^{T_i}(r_t+\mu_t)dt\right)X_{T_i}\right] \tag{10.0.3}$$

As illustrated in previous sections, there are two types of longevity securities available for the hedgers to choose, the *customised* longevity swaps and the *index-based* ones. One point worth noting is that in longevity swaps, the fixed rate refers to a swap curve, which represents the best estimated survival rates across the hedging horizon, rather than one single (fixed) rate as in interest swaps. At each point of the longevity swap curve, the survival rate is pertained to its respective individual payment date.

In the ***customised* longevity swap case**, the hedger pays a *fixed* survival rate (a series of swap rates), $\bar{p}^N \in (0,1)$ against the *realized (floating)* survival rate experienced by the hedged population during $T(0 < T < 1)$. The net cash flow to the hedger at time T is

$$n(\frac{n - N_T}{n} - \bar{p}^N) \tag{10.0.4}$$

Hence the market value at inception of the longevity swap can be expressed as follows:

$$S_0 = nE^{\tilde{\mathbb{P}}}\left[exp\left(-\int_0^T r_t dt\right)\left(\frac{n-N_T}{n} - \bar{p}^N\right)\right] = nE^{\tilde{\mathbb{P}}}\left[exp\left(-\int_0^T (r_t+\mu_t)dt\right)\right] - nB(0,T)\bar{p}^N \tag{10.0.5}$$

where $B(0,T)$ denotes the present value of a zero coupon bond with a maturity of T years.

By the definition of swap rate, the swaps are priced at time 0 at the inception using the fair priced swap rate. Setting $S_0 = 0$, the swap rate (curve) can be calculated as follows:

$$\bar{p}^N = \tilde{p}_T + B(0,T)^{-1} Cov^{\tilde{\mathbb{P}}}\left(exp\left(-\int_0^T r_t dt\right), exp\left(-\int_0^T \mu_t dt\right)\right) \tag{10.0.6}$$

$\tilde{\mathbb{P}}_t$ refers to the risk adjusted survival probability as defined in equation 10.0.2, with the expectations under *risk-neutral* pricing measure $\tilde{\mathbb{P}}$.

The structure of ***index-based* longevity swaps** are very similar to the *bespoke* ones. In the *indexed* longevity swap case, the hedger needs to pay a fixed rate that is linked to a specific index[1], denoted by $\bar{p}^I \in (0,1)$, against the actual survival experience of the index population.

[1] examples are the HMD, Lifemetrics developed by JP Morgan or the Life expectation index developed by Deutsche Borse

Assuming the reference population has mortality intensity denoted as $\mu^I_{t\,t\leq 0}$, the survival probability of the index population can be expressed as $I_t = exp(-\int_0^t \mu^I_s ds)$. The fixed swap curve, $\bar{P}^I$ can be denoted as in equation 10.0.6, replacing μ and $\tilde{P}_T$, with μ^I and $\tilde{P}^I_T$, respectively.

10.1. *MTM* and *P&L* of Longevity Swaps

Generally speaking, in the case of *customised/bespoke longevity hedge*, where *basis risk* is not present, at each payment date, the difference between the pre-set mortality rate and the estimated mortality rate using the most recent mortality data reflects the cash inflow or outflow for the hedger. This is also called pure "cashflow" hedge. Though these *customised* hedges can provide perfect hedge to the hedgers, in abnormal financial market conditions or in the case when mortality improvement trend shifts significantly in direction and magnitude, the hedge effectiveness of such instruments could be dampened. Biffis et al. (2014) (104) illustrate some numerical examples of the *MTM* and *P&L* effects of a bespoke longevity hedge on a hypothetical population based on the English and Wales males aged 65 years at the year 1980, with a maturity of 25 years, assuming the interest rate is hedged away through interest rate swaps, flat at 5% annually. The figures are included in Appendix B.0.1 on page 166. The market value at time t of a *customised* longevity swap in a longevity hedger's view can be expressed as below:

$$S_t = nE_t^{\tilde{\mathbb{P}}}\left[exp\left(-\int_t^T r_s ds\right)\left(\frac{n-N_t}{n}exp\left(-\int_t^T \mu_s ds\right)\right)\right] - nB(t,T)\bar{p}^N \tag{10.1.1}$$

with $nE_t^{\tilde{\mathbb{P}}}\left[exp\left(-\int_t^T r_s ds\right)\left(\frac{n-N_t}{n}exp\left(-\int_t^T \mu_s ds\right)\right)\right]$ representing the floating rate payments from the hedge suppliers to the hedger at time evaluated at time t, and $nB(t,T)\bar{p}^N$ representing the fixed swap curve linked to payments from the hedger to hedge supplier. $E_t^{\mathbb{P}}[*]$ represents the conditional expectation under risk-neutral pricing measure $\mathbb{P}$, given all information available at time t. $B(t,T)$ represents the market value of a zero-coupon bond with *time-to-maturity* of $T-t$. The *MTM* process for indexed longevity swap is similar to *customised* ones, simply replacing μ and $\tilde{P}_T$, with μ^I and $\tilde{P}^I_T$, respectively.

10.2. Index-based longevity swaps on UK pension scheme

In this paper, we are going to use the granular real world pension data to set up the *MTM* and *P&L* results of the *index-based* longevity swaps. We set up the *index-based* longevity swaps as below:

- The reference population is the national population, English and Welsh populations, males and females, respectively.
- The hedged population is the male pensioners and females pensioners in the UK pension scheme.
- For male and female pensioners, we divided them into groups according to the *EM* and *BIC* results, and the age band is 5 years. For instance, age $65-69$, age $70-74$, age $75-79$, age $80-84$. We are focusing on these four most populated age groups there are four age groups for both males and females.

Table 10.2.1.: Detail example statistics: Female pensioners Group I age $65-69$, pension amount £0-£1,700

Years	2004	2005	2006	2007	2008	2009
E&W female $m_{65-69,\,t}$	0.01215	0.01206	0.01147	0.01131	0.01108	0.01055
Female pensioners $m_{65-69,\,t,\,G1}$	0.0042	0.0018	0.0024	0.0065	0.0092	0.0026

- Both male pensioners and female pensioners can be divided into three pension annuity groups accordingly to the *EM* and *BIC* results. [2]. We denote these three annuity groups as ***High-income (H)***, ***Middle-income (M)*** and ***Low-income (L)***, respectively.

Female pensioners can be divided into the following three pension annuity groups. The pension annuity is modeled as a log-normal distribution:

- Group 1***Low-income (L)*** : $£0-£1,700$, $n=9,982$. with $\mu=844$, $\sigma=381$
- Group 2 ***Middle-income (M)***: $£1,700-£6,800$. $n=5,489$. $\mu=2,884$, $\sigma=1,781$
- Group 3 ***High-income (H)***: $£6,800-£117,500(max)$. $n=608$. $\mu=12,856$, $\sigma=14,284$

10.2.1. MTM and P&L Main Results

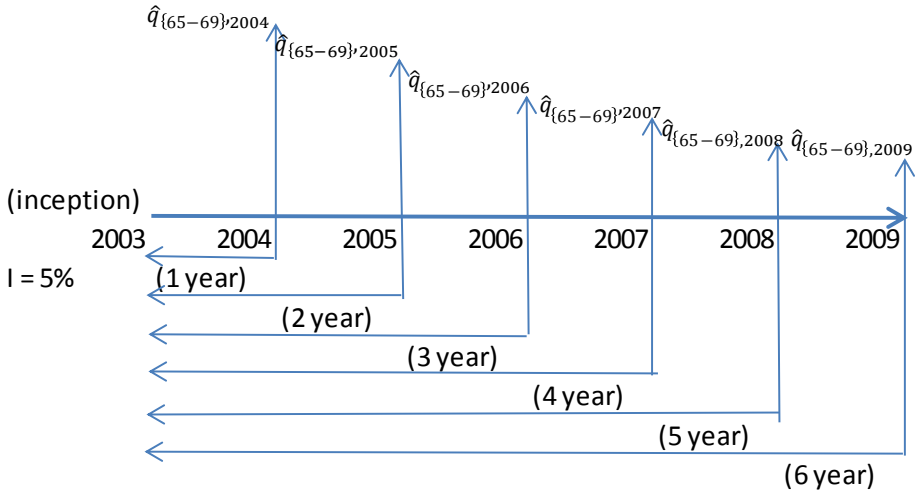
10.2.1.1. Detailed Example I Female pensioners G1 (£0-£1,700), aged $65-69$

Hedging Female Pensioners Group 1 aged 65-69 (*FP G1 (£0-£1,700), aged 65-69*) using national female population, see mortality experience comparision between the national E&W female population and female pensioners, age 65-69, with different bench marks at hedge ratios $h1 = 0.3939$, and $h2 = 0.2318$ in 10.2.2.

- The average hedge ratio (denoted as δ) is calculated as the average fraction of mortaltiy rate of the female pensions and the national female population as in 10.2.1 and 10.2.2. The hedge ratio is denoted by $h_1 = 0.3939$. This means on average of the research period $2004-2009$, the female pensioners (aged $65-69$, in $G1$)' mortality is around 39% of the national female population.
- We also calculated the hedge ration $h_2 = 0.2318$ using solver to minimize the standard deviation of the MTM results across the hedgin horizon.
- We hedge the pension annuity portfolio of female pensioners age $65-69$ in Group 1, using a portfolio of 6 q-forwards (See LLMA (2010) (122)) based on the national index of female pensioners aged $65-69$.
- The hedge horizon for the pension fund is 6 years, from 2004 to 2009.
- The inception date of 6 q-forwards is the end of year 2003, 2004, 2005, 2006, 2007, 2008.
- The notional amount is the mean of the pension annuities viewed as a normal distribution, which is £844, with standard deviation of £381. The number of contract is the number of the pensioners in this group and age band, which is $9,982$.
- Our hedging portfolio include the following 6 components (Illustrative diagram as in 10.2.1):

[2] In some cases of the less populated age groups, the *BIC* does not converge. The BIC converges and reaches minimum around 6 components (sub-populations) in most cases. However, in this case, the number of pensioners in each sub-population is relatively small, in some cases only a dozen of lives. This would give rise to unwanted *sampling risk* rather than *basis risk*. Hence we would like to keep the optimal number of the sub-populations as 3 as an example. We will leave this issue to further research

Figure 10.2.1.: Hedging Portfolio Design: Female Pensioners age 65-69, G1



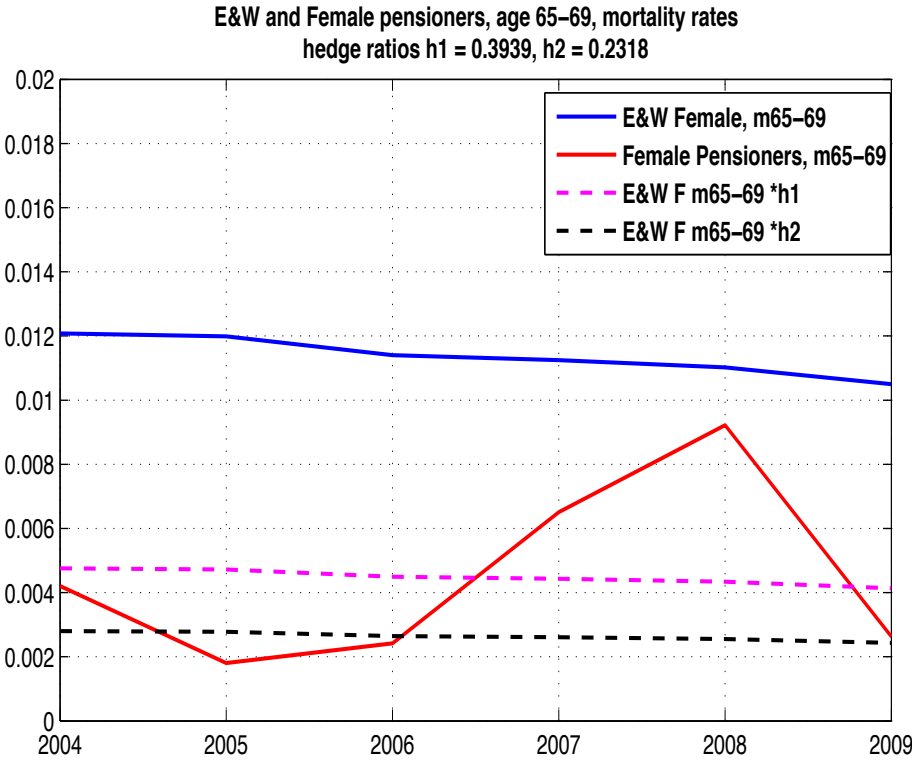
- 1 Q-forward with inception date at the end of year 2003, and maturity of 1 year (terminates at the end of year 2004). The forward rate is $\hat{q}_{65-69,\,2004}$[3].
- 1 Q-forward with inception date at the end of year 2004, and maturity of 1 year (terminates at the end of 2005). The forward rate is $\hat{q}_{65-69,\,2005}$[4].
- 1 Q-forward with inception date at the end of year 2005, and maturity of 1 year, (terminates at the end of 2006). The forward rate is $\hat{q}_{65-69,\,2006}$
- 1 Q-forward with inception date at the end of year 2006, and maturity of 1 year (terminates at the end of 2007). The forward rate is $\hat{q}_{65-69,\,2007}$
- 1 Q-forward with inception date at the end of year 2006, and maturity of 1 year (terminates at the end of 2008). The forward rate is $\hat{q}_{65-69,\,2008}$
- 1 Q-forward with inception date at the end of year 2007, and maturity of 1 year (terminates at the end of 2009). The forward rate is $\hat{q}_{65-69,\,2009}$

- The MTM and P&L results are illustrated in 10.2.3

[3]The notation $\hat{q}$ refers to the best estimate of mortality rate of age $65-69$, at the end of year 2004, using historical mortality experience data. In our case, we use $1961-2003$ as our sample period for mortality projection. We project mortality rates using standard classic Lee-Carter (1992) (119) mortality model. The MTM and P&L results may be different depending on the mortality models adopted. We are not claiming Lee-Carter model is the best model here. However, this is not the focus of our paper.

[4]Similar to $\hat{q}_{65-69,\,2004}$, we estimate $\hat{q}_{65-69,\,2005}$ using historial mortailty experience data from $1961-2004$. Alternatively, rolling window sample period $1962-2004$ can be used

Figure 10.2.2.: E&W and Female Pensioners Group 1, age 65-69, hedge ratios h1 and h2



10.2.1.2. Detailed Example II Male & Female pensioners (Moving Average Hedge Ratio cases)

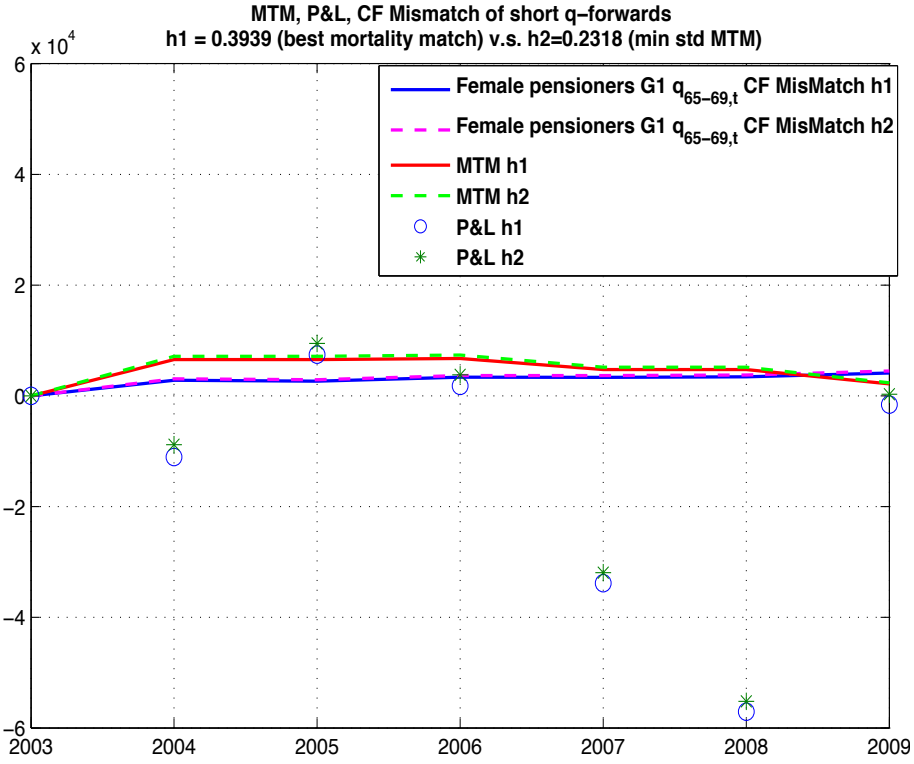
In this section, we showcase some examples of male *age-pension-year* groups hedging results using a index-linked longevity instrument. We choose the pensioners groups that are ***effective***. By saying effective, we mean that based on all selection criteria as follows:

- By applying EM algorithm, the Gaussian Mixture models divided pensioners into their respective sub-populations, so that the BIC converges to a minimum in the simulation exercises (Refer to A.0.4 for BIC convergence results).
- Each sub-populations within the *age-pension-year* group are well populated, with no less than 30 pensioners (Refer to A.0.4 for details). This makes sure that our analysis is valid, with no *sampling risk* due to sample size too small.

The *age* groups that meet these above criteria for male pensioners are age 70-74, age 75-79, and age 80-84. The set-up of the index-based longevity instrument are designed as follows:

- Use Lee-Carter (1992) (119) as a benchmark model for mortality rate projections for the E&W national population (index)
  - Use 1961-2003 as a starting in-sample to project the mortality rate in year 2004, $index_{2004,\,LC}$
  - Use 1961-2004 as in-sample to project the mortality rate in year 2005, ...
  - ...

Figure 10.2.3.: Female Pensioners Group 1 age 65-69, Short q-forwards MTM, P&L, and Cash-flow mismatch of q-forwards, hedge ratios



  - Use 1961-2008 as in-sample to project mortality rate in year 2009

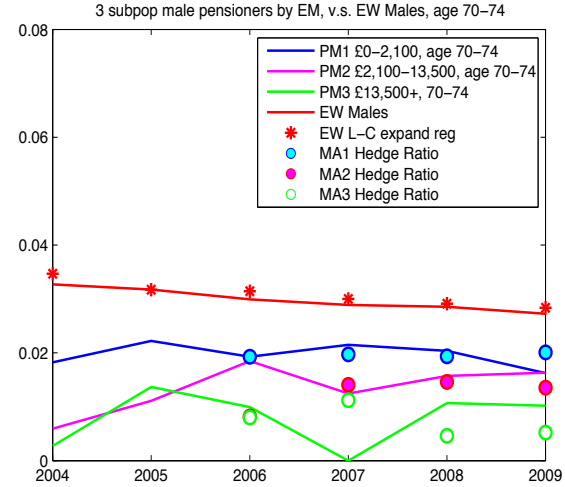
- Use EM algorithm to divide pensioners according their respective sub-populations (sub-pop1 - Low-income; sub-pop 2 - Middle-income; sub-pop 3 - High-income).
- Calculate the mortality experience (Mortality Rate, MR hereafter) during 2004 – 2009 for sub-populations
- Calculate the Hedge Ratios (HR) using a simple Moving Average(MA(2)) Process such that
  - $MAHR_{pop(i),2006} = Avg[\frac{MR_{pop(i),\,y2004}}{index_{2004}}, \frac{MR_{pop(i),\,y2005}}{index_{2005}}] \times index_{2006,\,LC}$
  - $MAHR_{pop(i),2007} = Avg[\frac{MR_{pop(i),\,y2005}}{index_{2006}}, \frac{MR_{pop(i),\,y2006}}{index_{2006}}] \times index_{2007,\,LC}$
  - ...
  - $MAHR_{pop(i),2009} = Avg[\frac{MR_{pop(i),\,y2007}}{index_{2007}}, \frac{MR_{pop(i),\,y2008}}{index_{2008}}] \times index_{2009,\,LC}$
- Then calculate the MTM and P&L using the $index \times MAHR$ to hedge the respective sub-population mortality experiences (Results in Figures 10.2.8, 10.2.9 10.2.10 10.2.7 etc...)
- The portfolio consists of the following q-forwards
  - 1-year Q-forward inception in 2004, does not exchange cashflow in 2005 because we do not have the date to calculate the MAHR in 2005.
  - 2-year Q-forward inception in 2004, exchange cashflow in 2006, the projected mortality rate is $MAHR_{pop(i),2006}$, and the realized mortality rate is the sub-populations' actual mortality experience in 2006

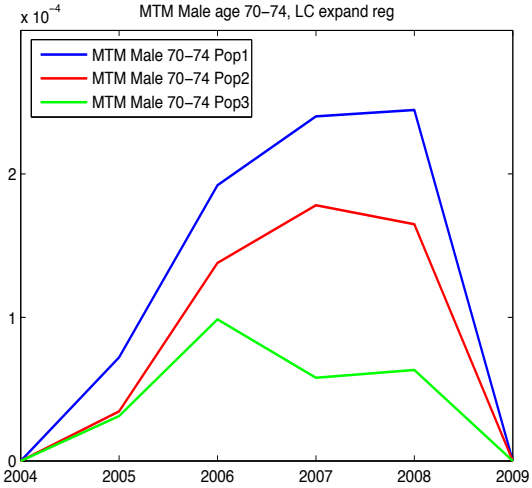
– 3-year Q-forward inception in 2004, exchange cashflow in 2007, the projected mortality rate is $MAHR_{pop(i),2007}$, and the realized mortality rate is the sub-populations' actual mortality experience in 2007

– 4-year Q-forward inception in 2004, exchange cashflow in 2008, the projected mortality rate is $MAHR_{pop(i),2008}$, and the realized mortality rate is the sub-populations' actual mortality experience in 2008

– 5-year Q-forward inception in 2004, exchange cashflow in 2009, the projected mortality rate is $MAHR_{pop(i),2009}$, and the realized mortality rate is the sub-populations' actual mortality experience in 2009

Figure 10.2.4.: Male Pensioners age 70-74 index-linked security MTM, P&L, hedge ratios





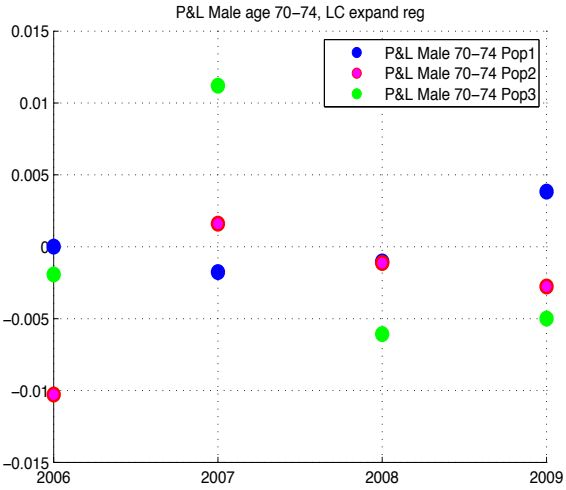
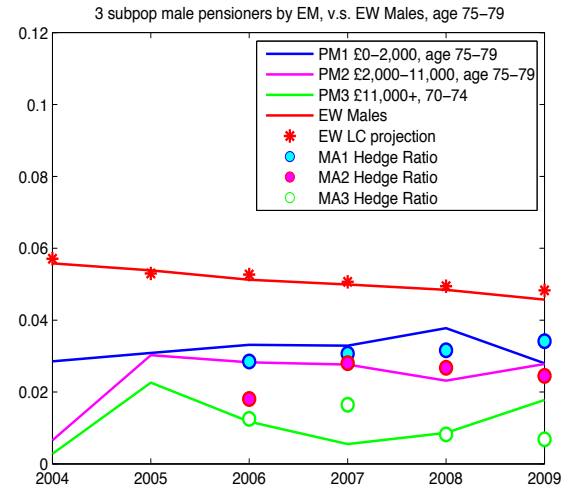
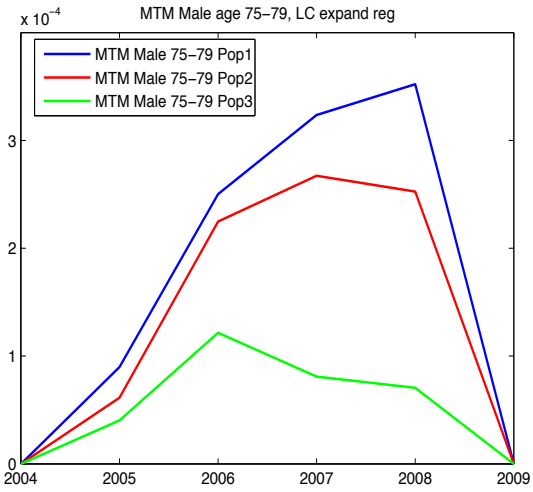


Figure 10.2.5.: Male Pensioners age 75-79 index-linked security MTM, P&L, hedge ratios





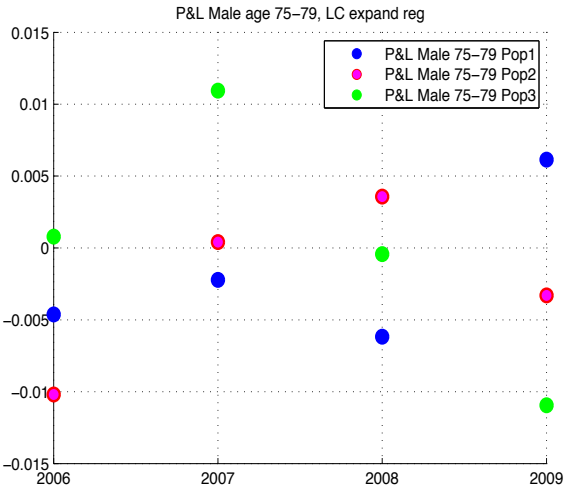
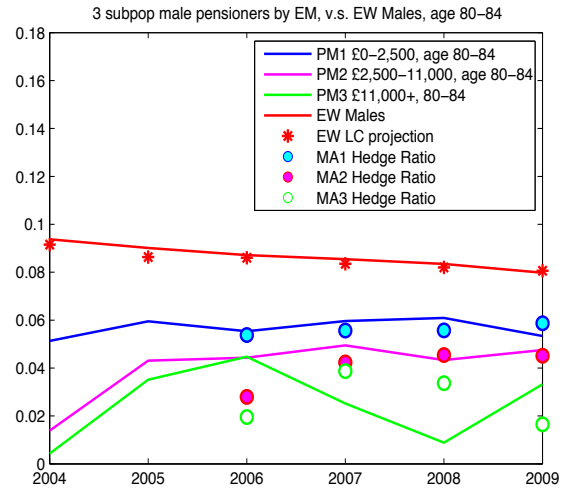
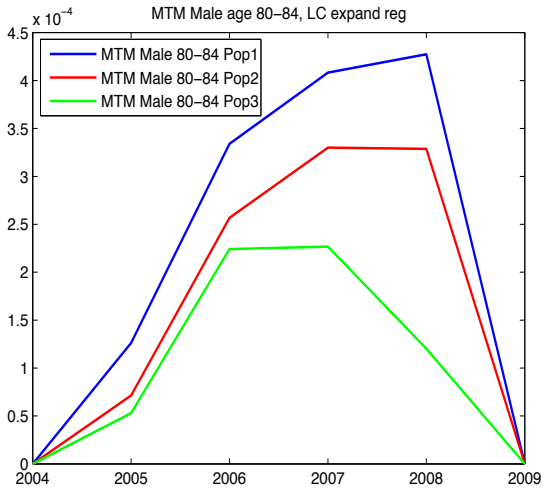


Figure 10.2.6.: Male Pensioners age 80-84 index-linked security MTM, P&L, hedge ratios





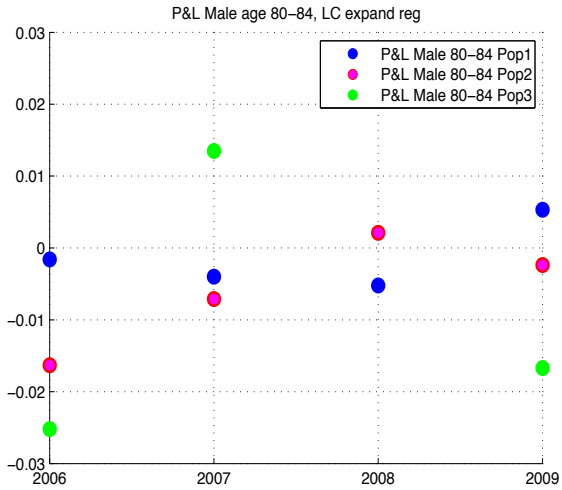
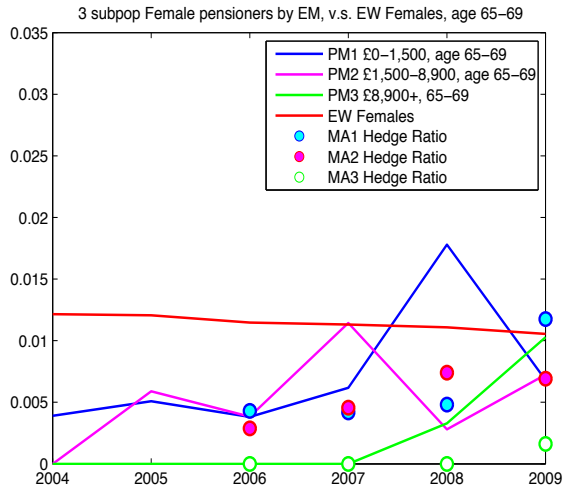
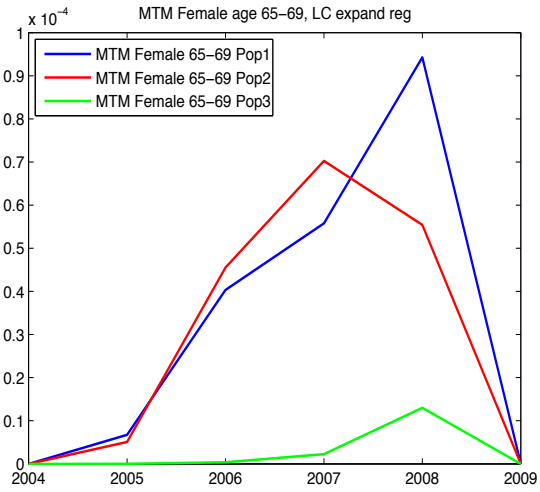


Figure 10.2.7.: Female Pensioners age 65-69 index-linked security MTM, P&L, hedge ratios





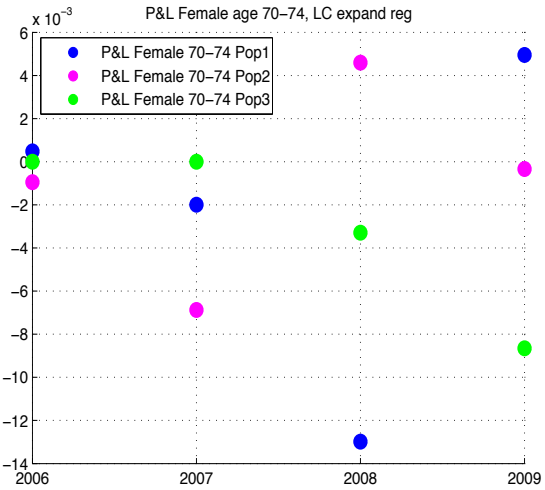
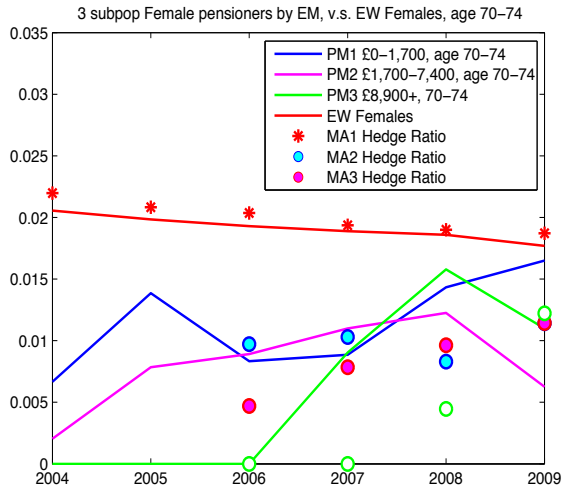
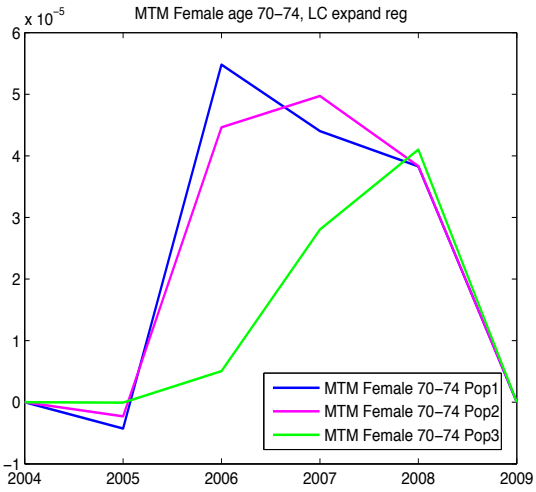


Figure 10.2.8.: Female Pensioners age 70-74 index-linked security MTM, P&L, hedge ratios





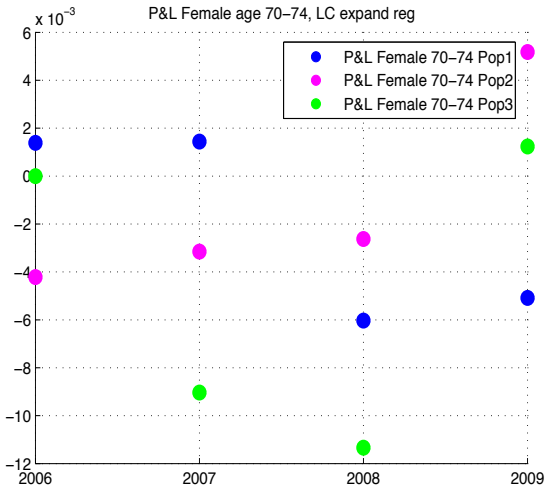
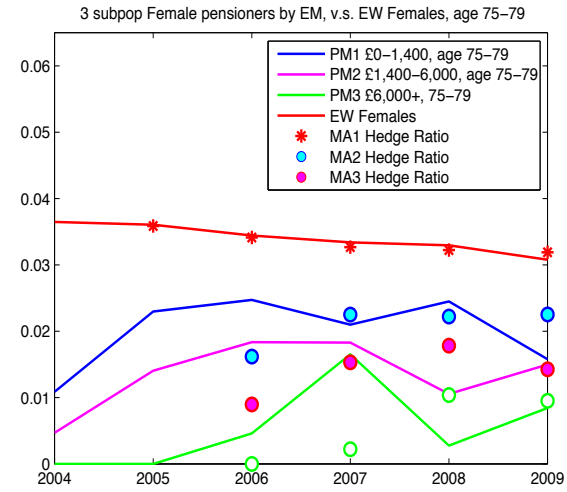
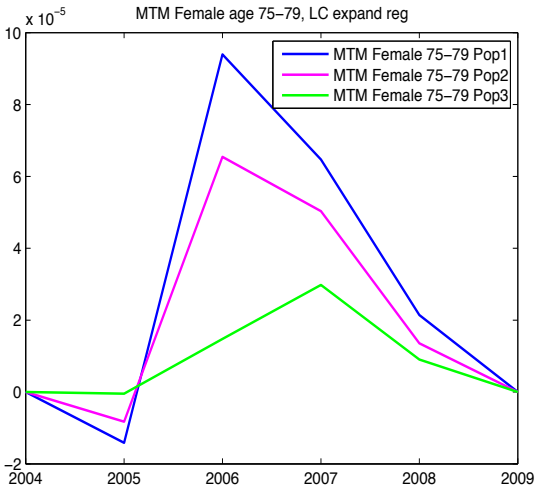


Figure 10.2.9.: Female Pensioners age 75-79 index-linked security MTM, P&L, hedge ratios





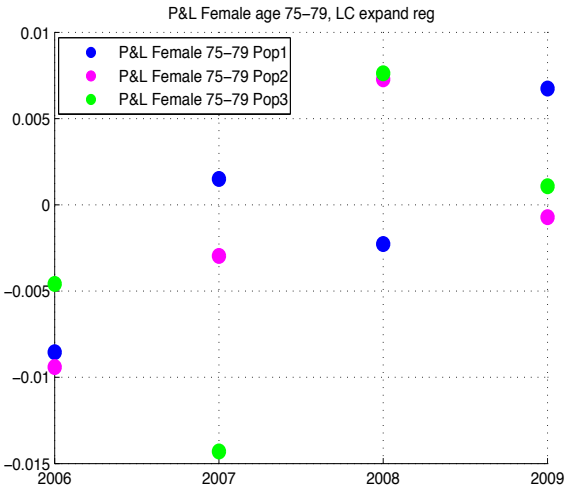
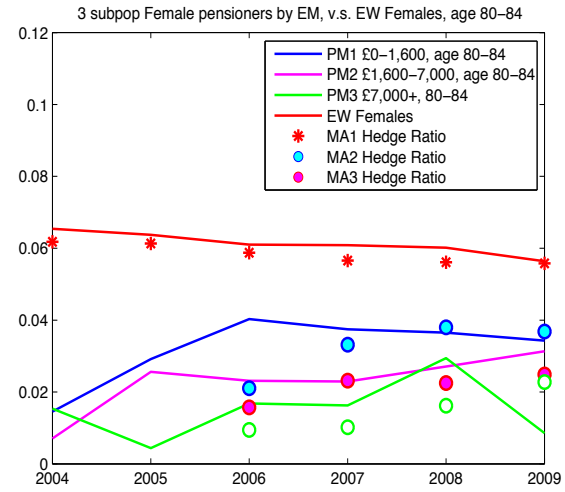
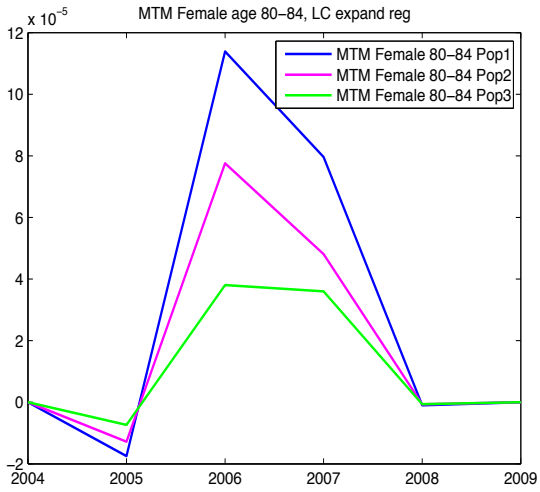


Figure 10.2.10.: Female Pensioners age 80-84 index-linked security MTM, P&L, hedge ratios





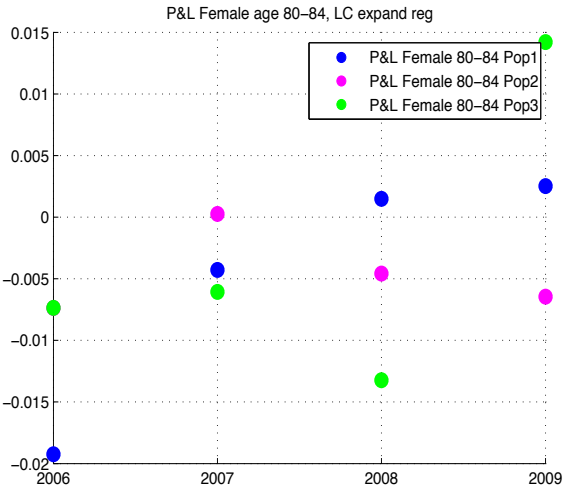
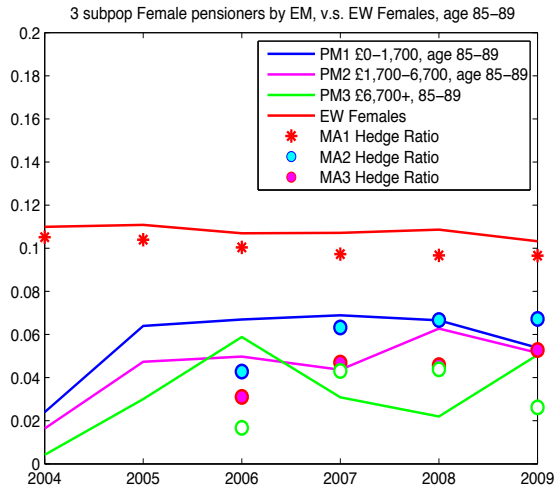
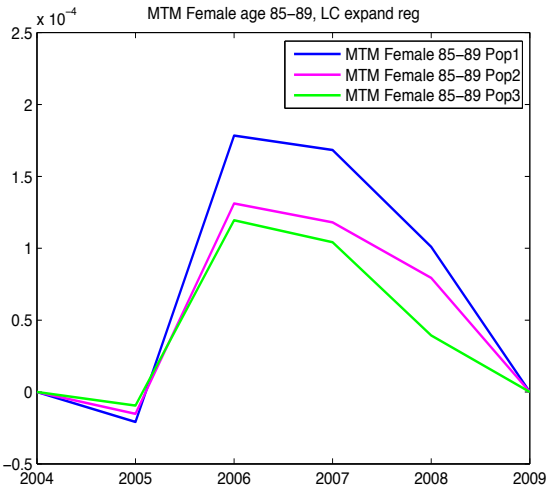


Figure 10.2.11.: Female Pensioners age 85-89 index-linked security MTM, P&L, hedge ratios





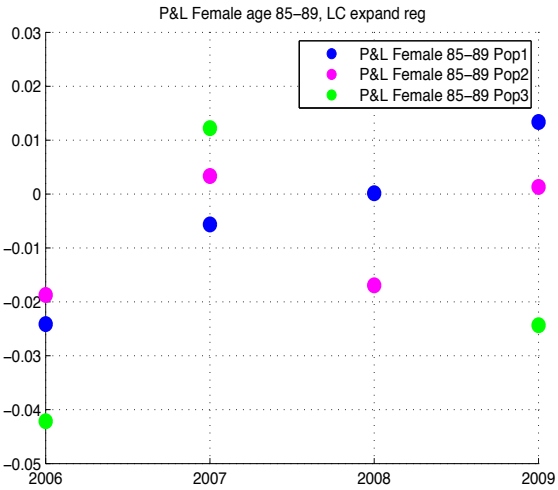
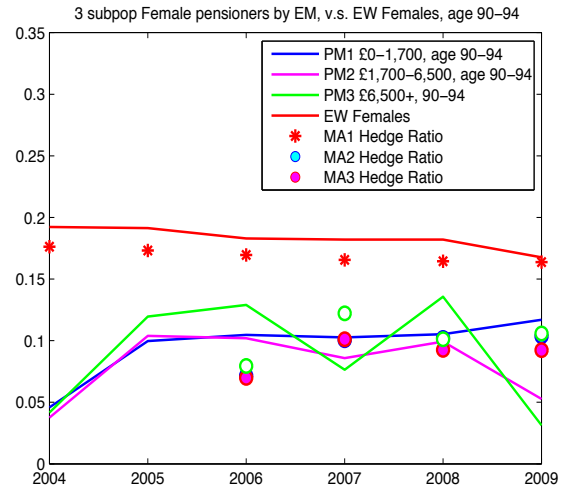
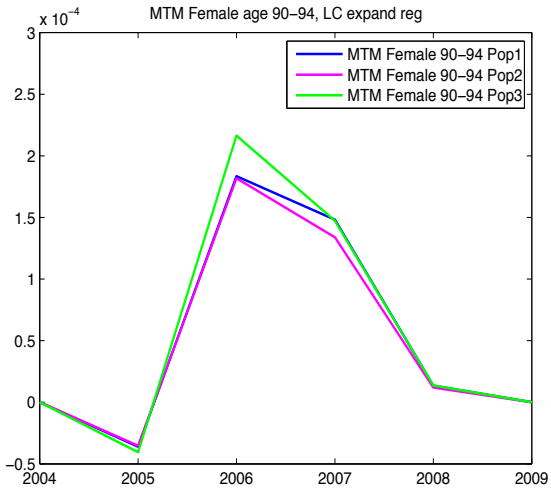
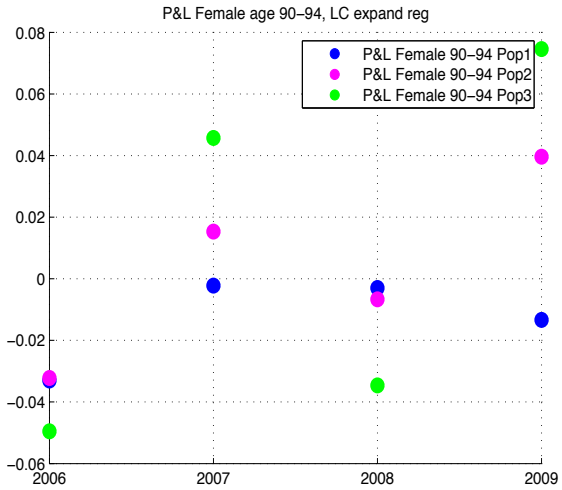


Figure 10.2.12.: Female Pensioners age 90-94 index-linked security MTM, P&L, hedge ratios







10.2.1.3. Detailed examples: Male pensioners (Quadratic Hedging)

In this section, we implement a hedging strategy based on Biffis et al. (2014) (104) example 4.2 on page 19. We summarize the hedging strategy as below:

- Considering the pension scheme is facing a time-T liability arising from a portfolio of pure endowments issued to m individuals with common-$\mathbb{G}$-predictable intensity of mortality μ.
- Denote Hedging portfolio as $H_T \in F_T$ Let $H_T = \Sigma_{i=1}^{m} \mathbf{1}_{\tau_i} f(P_T^1)$, with $f(\cdot)$ a bounded positive function
- Consider a self-financing strategy $\pi = (\pi^1, \pi^2)'$ where π_t^j denotes the amount of wealth invested in asset P^j at time t in the class of square-integrable, $\mathbb{F}$-predictable process solving the optimization problem
$$min_\pi E[(H_T - X_T^{x,\pi})^2] \tag{10.2.1}$$
- $X^{x,\pi}$ is the wealth process associated with the investment strategy π starting from a given initial wealth level $x > 0$

In our example, as a common first step, we use Lee-Carter (119) model to estimate E&W mortality experiences from year 2004 to 2009. Note that the in-sample period is expanding as in the previous section[5]. Taking the age-band $65-69$ as an example, the hedge portfolio X_T consists of the following 6 instrument:

- 1-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2004, denoted by $q^i_{65-69,2003-2004}$, where i denotes the pension income group (High-income, middle-income and low-income)
- 2-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2005, denoted by $q^i_{65-69,2003-2005}$,
- 3-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2006, denoted by $q^i_{65-69,2003-2006}$,
- 4-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2007, denoted by $q^i_{65-69,2003-2007}$,
- 5-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2008, denoted by $q^i_{65-69,2003-2008}$,
- 6-year maturity q-forward, with inception at the end of year 2003, matures at the end of year 2009, denoted by $q^i_{65-69,2003-2009}$,

The results are as follows:

[5] Using $1961-2003$ year's data to predict year 2004's mortality rate, using year $1961-2004$'s data to predict year 2005's data, ... etc

Figure 10.2.13.: Male Pensioners age 70-74 index-linked security MTM, P&L, quadratic hedging ratios

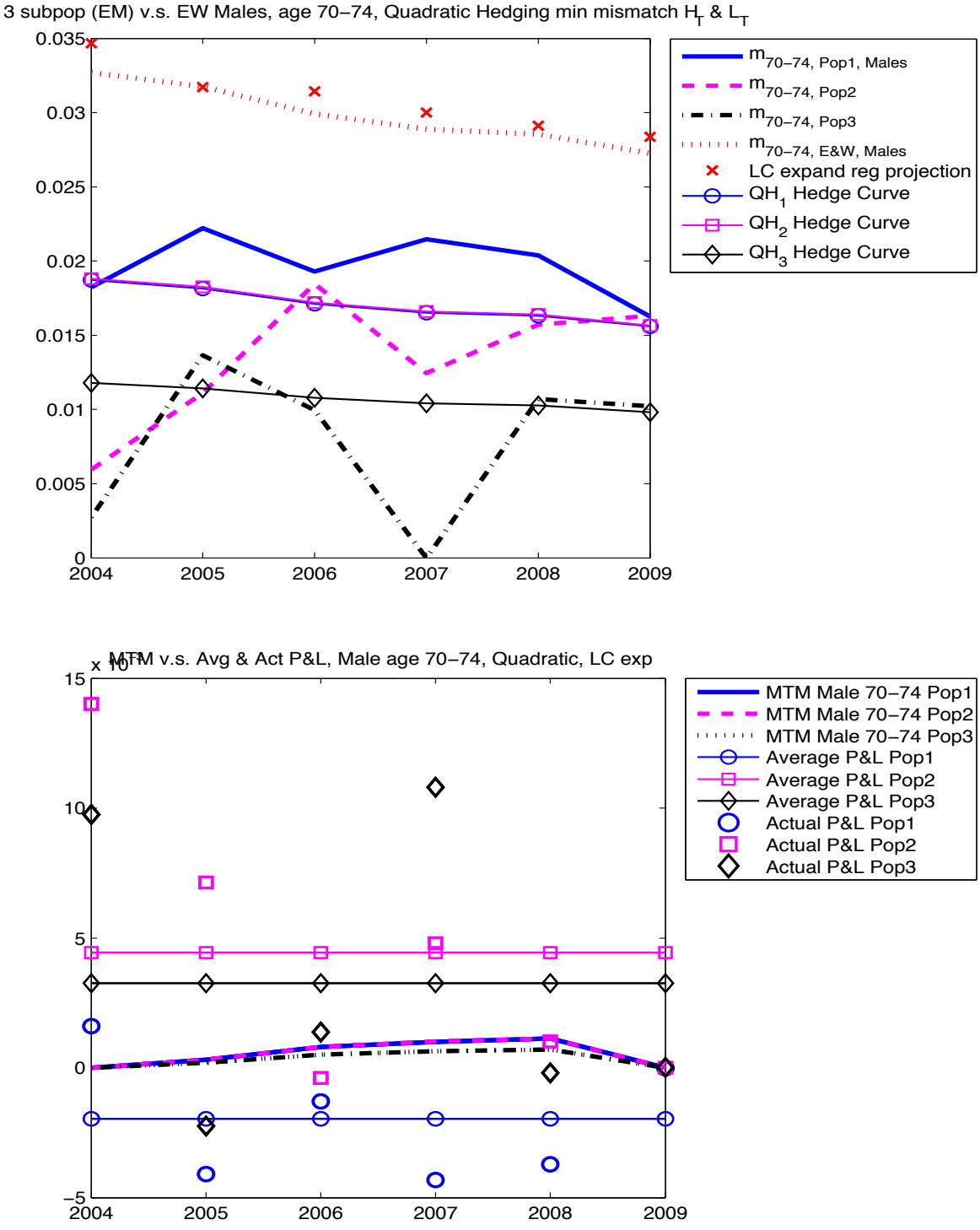


Figure 10.2.14.: Male Pensioners age 75-79 index-linked security MTM, P&L, quadratic hedging ratios

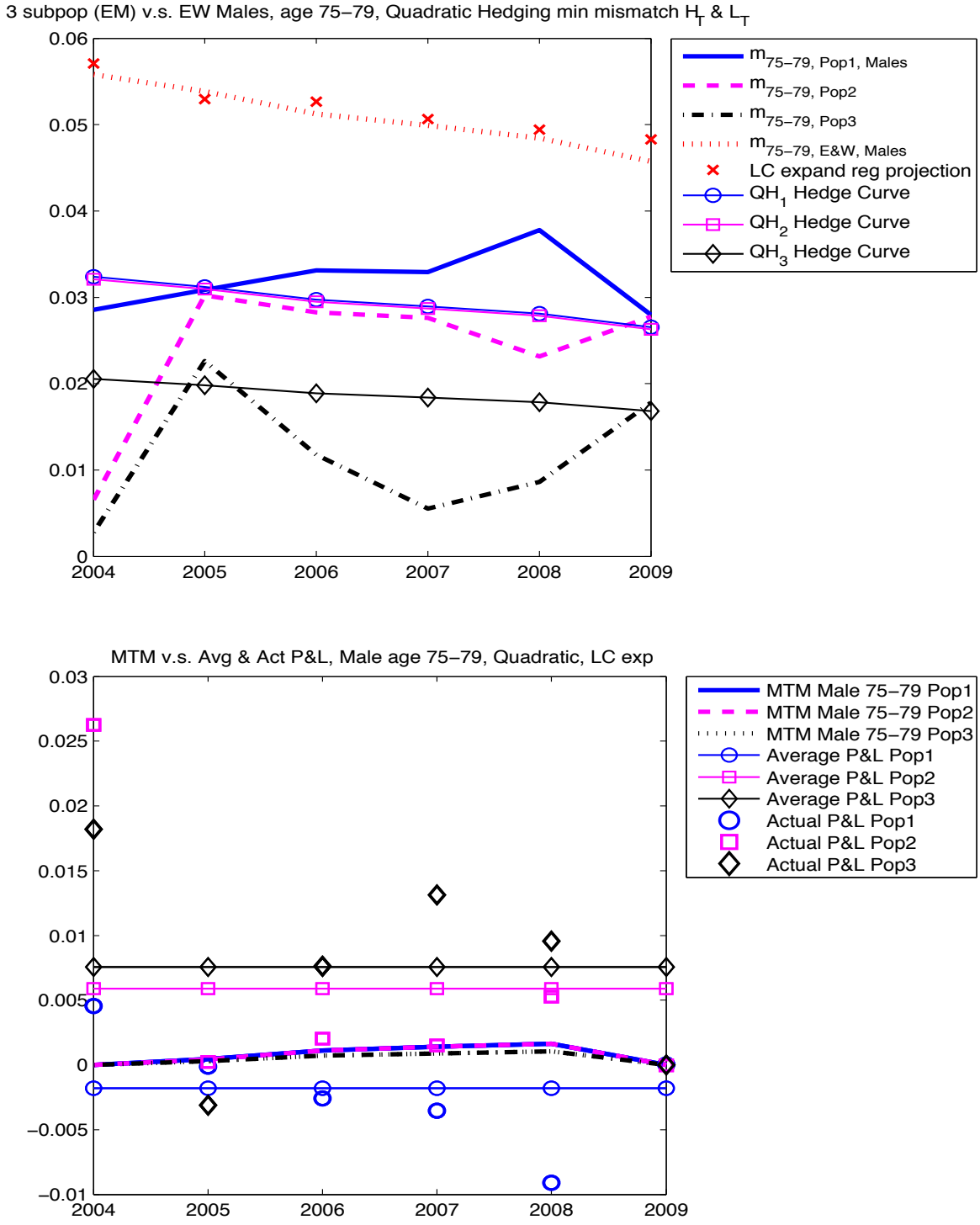


Figure 10.2.15.: Male Pensioners age 80-84 index-linked security MTM, P&L, quadratic hedging ratios

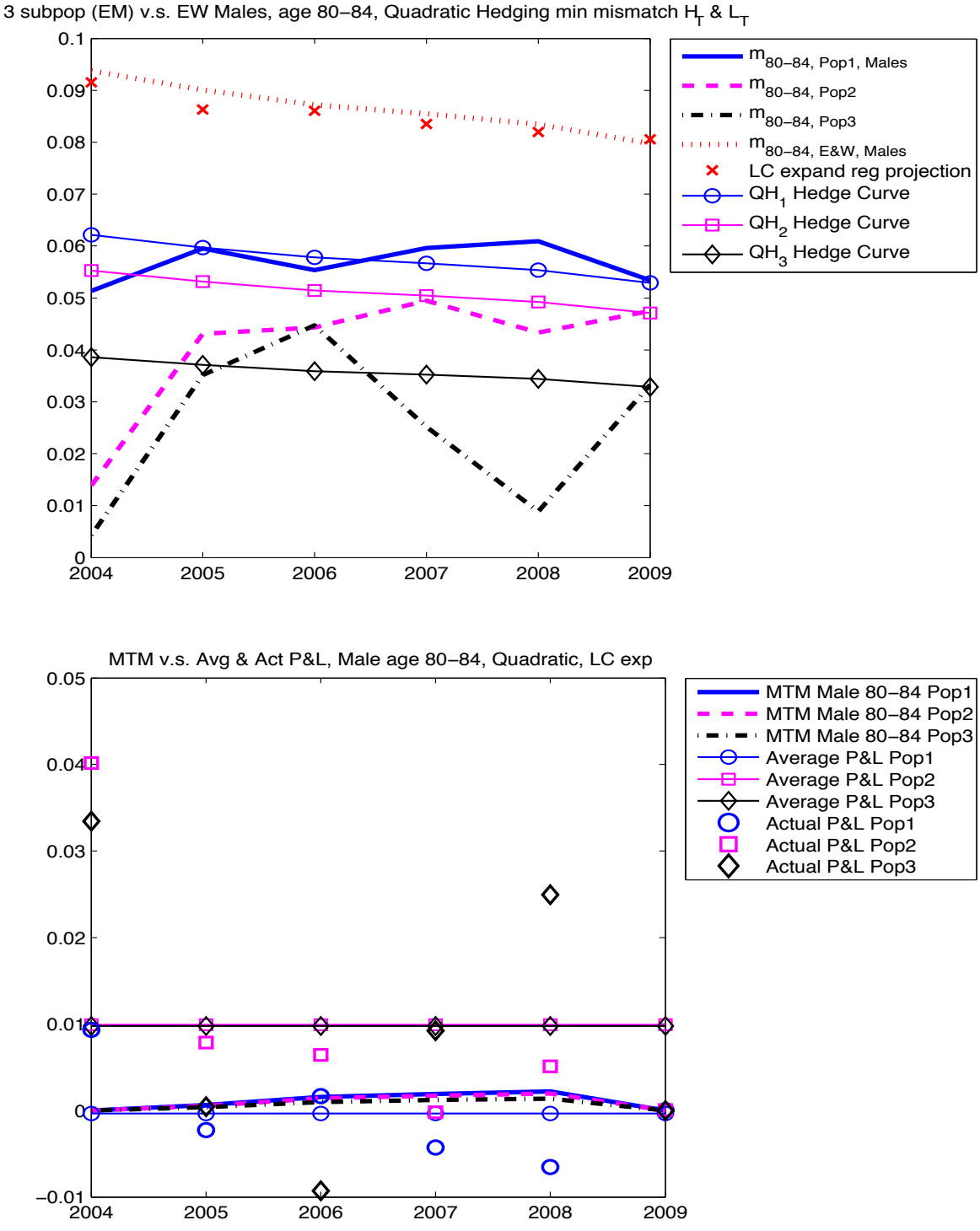


Figure 10.2.16.: Male Pensioners age 85-89 index-linked security MTM, P&L, quadratic hedging ratios

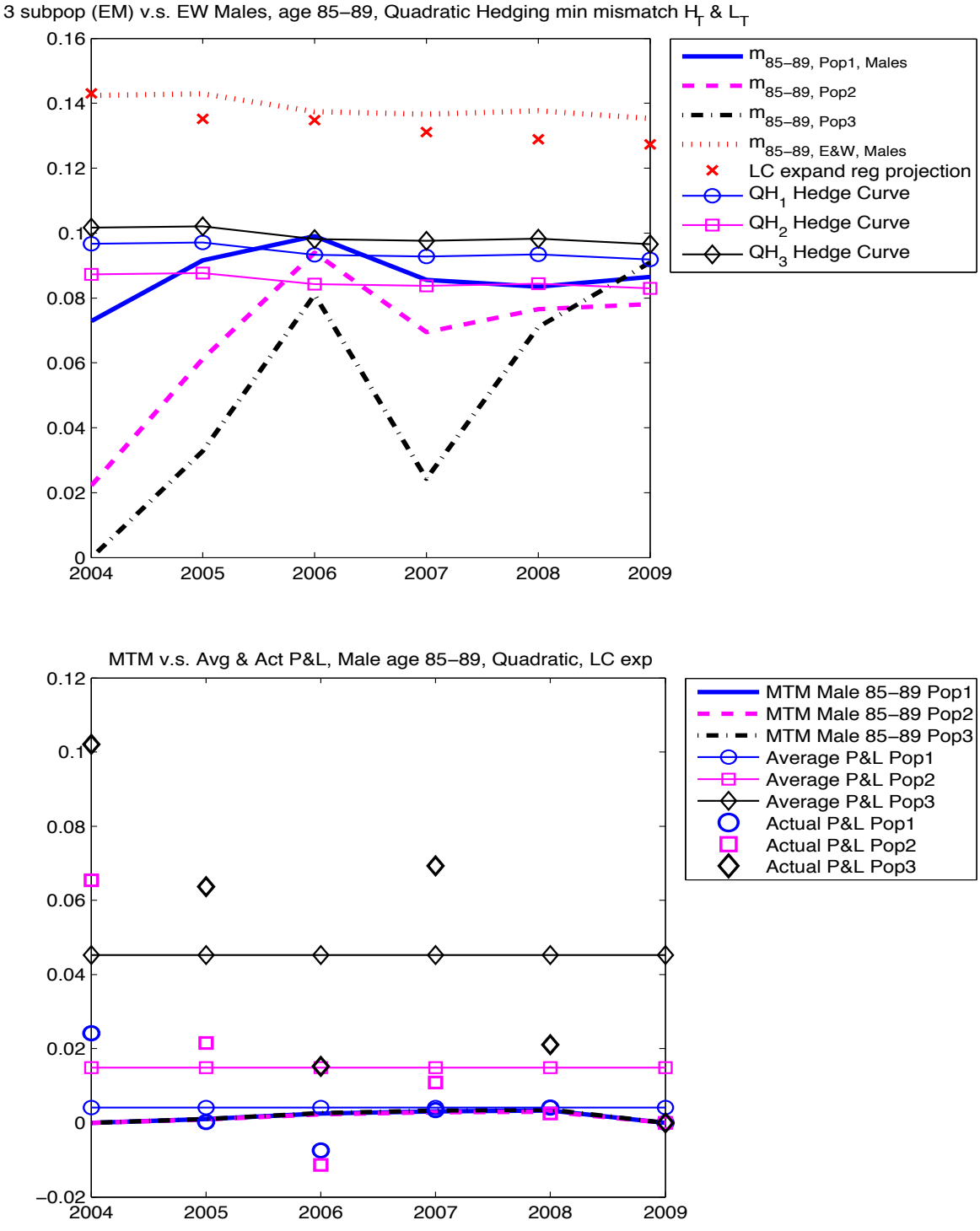
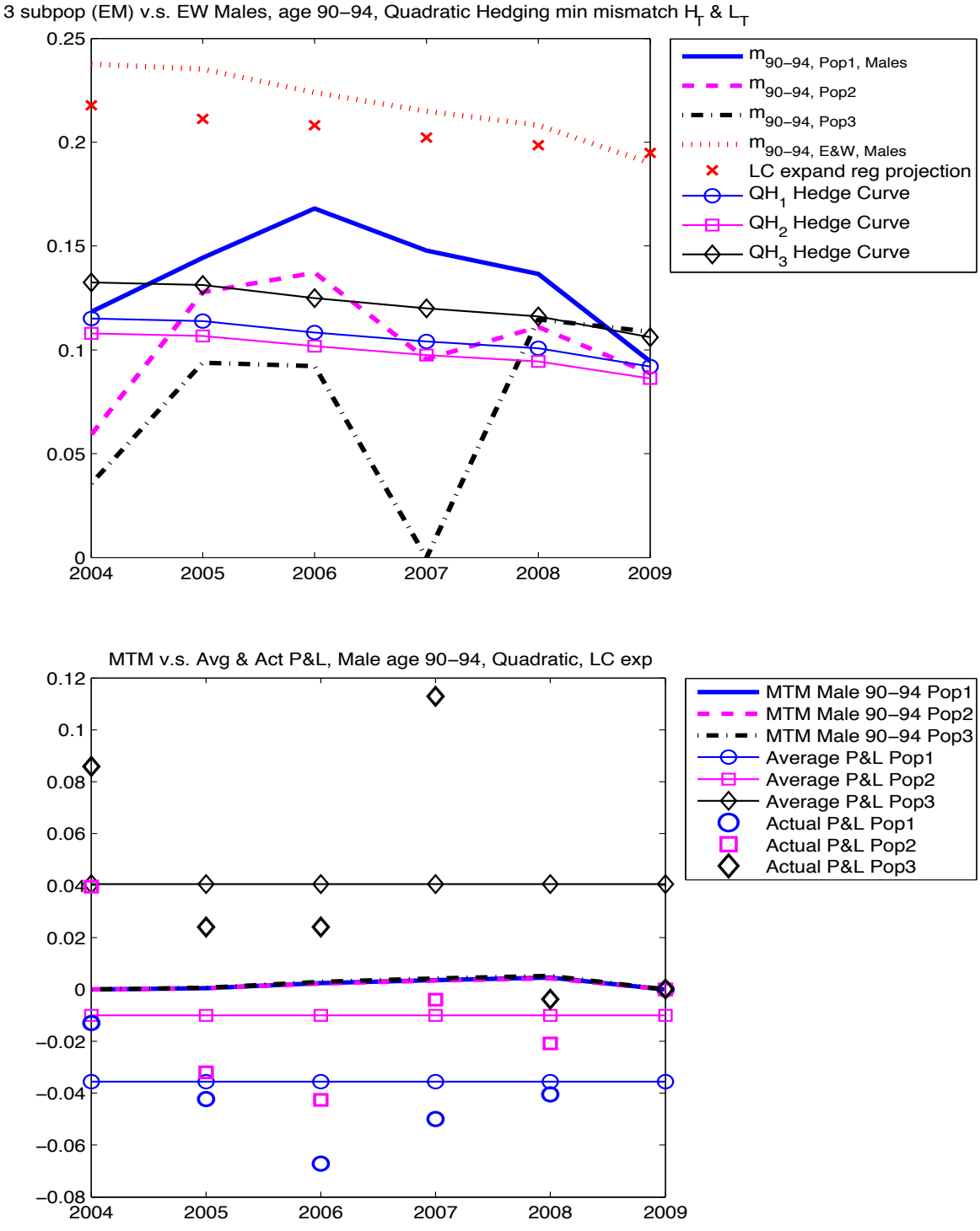


Figure 10.2.17.: Male Pensioners age 90-94 index-linked security MTM, P&L, quadratic hedging ratios



Conclusion

The paper is the first to use a scientifically robust methodology to analyze pensioner's income (and potential other socio-economic variables) and longevity improvements. In real business world, pension fund or annuity providers, and/or longevity hedge providers may have more information on pensioner' socioeconomics status, e.g. post-code, home value, real asset and financial assets, or tax return information, medical history, health status. These variables all contribute to the pensioners' mortality/longevity experiences. The Expectation Maximization methods allow multiple variables as inputs. These are going to improve the estimation. However, using only one variable, pensioner' income, our paper is able to derive three (or more) clear sub-populations that have varying levels (high, middle and low) of mortality experience curves.

Adopting this pensioner bucketing strategy, the hedger and hedger providers are both better off using index-based longevity swaps, due to their cost-effectiveness. Cashflow can be measured precisely across the hedging horizon, assuming various hedging strategies. For majority of the cases analyzed in this study, MTM is mostly positive across the hedging horizon, while PL may be positive or negative. This is because we use Lee-Carter (119) as baseline projecting model, it is known to underestimate longevity risk, especially for the very old ages. This is the situation many pension funds are facing at the UK now. The Figures in Biffis et al. (2014) (104) (Figure B.0.1 on 166) have similar patterns to the findings of this paper. The difference is time scale. As the swap in Biffis et al. (2014) (104) is of longer time horizon, the cash flow mismatch and hence MTM and PL can be more pronounced. This is the case with potential longevity deals as pension fund cares about long-term impacts on their portfolio and balance sheet.

Tables

Date	Fund\Hedger	Hedge Provider	Approx size	Solution
1. Nov 2016	Canadian Bank Note Company, Ltd	Canada Life Assurance	$35M	Longevity swap & reinsurance
2. Nov 2016	AXA France	RGA Re	€1.3B	Longevity swap & reinsurance
3. Oct 2016	Unnamed DB pension scheme	Zurich / Pacific Life Re	£50M	Longevity swap & reinsurance
4. Aug 2016	Two Pirelli pension plans	Zurich / Pacific Life Re	£600M	Longevity swap
5. Aug 2016	Manweb (ScottishPower)	Abbey Life	£1B	Longevity swap
6. Jun 2016	Pension Insurance Corp.	Prudential Insurance Company of America	£1.1B	Longevity reinsurance
7. Apr 2016	Legal & General	Prudential	?	Longevity reinsurance
8. Dec 2015	Unnamed UK pension plan	Zurich / Pacific Life Re	£90M	Longevity swap & reinsurance
9. Nov 2015	RAC (2003) pension scheme	SCOR SE	£900M	Longevity swap & reinsurance
10. Nov 2015	Philips UK pension fund	Pension Insurance Corp. & Hannover Re	£2.4B	Buyout & Longevity & reinsurance
11. Sep 2015	Scottish & Newcastle pension fund	Friends Life (plus Swiss Re)	£2.4B	Longevity swap & reinsurance
12. Aug 2015	Legal & General	Prudential	£2.9B	Longevity reinsurance
13. Jul 2015	Aegon	Canada Life Re	€6B	Longevity swap & reinsurance
14. Jul 2015	AXA UK Pension Scheme	RGA Re	£2.8B	Longevity swap
15. Jun 2015	Pension Insurance Corp.	Prudential Insurance Company of America	£1.6B	Longevity reinsurance
16. Jun 2015	Delta Lloyd	RGA Re	€12B	Index-based Longevity swap

Table A.0.1.: Longevity Market Deals 2008 - 2016(1) - continued on next page. Deals highlighted in red are index-based longevity swaps. Cyan ones are longevity solution structured to target capital market investors. Blue one is the first longevity deal originated in North America.

Date	Fund\Hedger	Hedge Provider	Approx size	Solution
17. Apr 2015	Pension Insurance Corp.	Prudential Insurance Company of America	?	Longevity reinsurance
18. Mar 2015	Bell Canada Pension Plan (BCE Inc.)	Sun Life Financial Inc. (plus SCOR & RGA Re)	CAD$5B	Longevity swap & reinsurance
19. Feb 2015	ScottishPower	Abbey Life	£2B	Longevity swap
20. Jan 2015	Rothesay Life	Prudential	$450M	Longevity reinsurance
21. Jan 2015	MNOPF	Pacific Life Re	£1.5B	Longevity swap & reinsurance via incorporated cell
22. Dec 2014	Rothesay Life	Pacific Life Re	£1B	Longevity reinsurance
23. Oct 2014	Legal & General	Prudential	£2.2B	Longevity reinsurance
24. Aug 2014	Delta Lloyd	RGA Re	€12B	Index-based longevity swap
25. Aug 2014	Phoenix Group	Phoenix Life	£900M	Longevity swap with simultaneous quota share
26. Aug 2014	Rothesay Life	Prudential	£1B($1.7B)	Longevity reinsurance transaction
27. Aug 2014	AXA France	Hannover Re	€750M	Longevity swap
28. Jul 2014	BT Pension Scheme	Prudential Insurance Company of America	£16B	Pensioner bespoke longevity swap
29. Jun 2014	Total UK pension plan	Pension Insurance Corporation/Hannover Re	£16B	Buy-in and longevity reinsurance transaction
30. May 2014	Royal London	RGA International Reinsurance Co.	£1B	Longevity reinsurance transaction
31. Mar 2014	AkzoNobel	Legal & General /Prudential	£3.6B	Buy-in, bulk annuity and longevity reinsurance
32. Mar 2014	Aviva	Swiss Re / Munich Re /SCOR	£5B	Pensioner bespoke longevity swap
33. Dec 2013	BAE Systems	Legal & General /SCOR	£1.7B	Pensioner bespoke longevity swap
34. Dec 2013	Astra Zeneca	Deutsche Bank & Abbey Life	£2.5B	Pensioner bespoke longevity swap
35. Dec 2013	Carillion	Deutsche Bank & Abbey Life	£1B	Pensioner bespoke longevity swap
36. Dec 2013	Aegon	Société Générale CIB / SCOR	€Euro1.4B	Longevity swap to capital market investors & reinsurers
37. Nov 2013	Pension Insurance Corp.	Reinsurers	£1.4B	Longevity reinsurance
38. Oct 2013	Philips Pension Fund	Rothesay Life	£0.484B	Pension insurance including longevity risk
39. Aug 2013	Cobham plc	Rothesay Life	£280M	Bulk annuity and longevity insurance
40. Jul 2013	EMI Group Pension Fund	Pension Insurance Corp.	£1.5B	Buy-out of all liabilities including longevity risk
41. Jun 2013	Canadian Wheat Board	Sun Life Assurance Company of Canada	$150M	Annuity buy-in and Asset Transfer
42. Jun 2013	Bentley	Deutsche Bank /Abbey Life	£0.4B	Annuity buy-in and Asset Transfer
43. Apr 2013	Abbey Life /Rothesay Life	Hannover Re	£1B	Longevity reinsurance
44. Feb 2013	BAE Systems	Legal & General /Hannover Re	£3.2B	Pensioner bespoke longevity swap
45. Dec 2012	LV	Swiss Re	£0.8B	Pensioner and all members over 55 longevity swap
46. Dec 2012	Pension Insurance Corp.	Munich Re	£0.4B	Longevity reinsurance transaction
47. Oct 2012	Verizon	Prudential	£7B	Pension liability buy-out
48. Jul 2012	Pension Insurance Corp.	Munich Re	£0.3B	Longevity reinsurance transcation
49. Jun 2012	General Motors Corp.	Prudential	$26B	Pension liability buy-out
50. May 2012	AkzoNobel Corp.	Swiss Re	£.1.4B	Pension bespoke longevity swap

Table A.0.2.: Longevity Market Deals 2008 - 2016(2) - continued on next page. Deals highlighted in red are index-based longevity swaps. Cyan ones are longevity solution structured to target capital market investors. Blue one is the first longevity deal originated in North America.

Date	Fund\Hedger	Hedge Provider	Approx size	Solution
51. Apr 2012	Berkshire Council	Swiss Re	£100M	Pensioner bespoke longevity swap
52. Feb 2012	Aegon	Deutsche Bank	€Euro 12B	Capital Markets longevity swap
53. Dec 2011	Pilkington	Legal & General Hannover Re	£1B	Pensioner bespoke longevity swap
54. Dec 2011	British Airways	Goldman Sachs /Rothesay Life	£1.3B	Pensioner bespoke longevity swap
55. Nov 2011	Rolls-Royce	Deutsche Bank	£3Bn	Pensioner bespoke longevity swap
56. Aug 2011	ITV	Credit Suisse	£1.7Bn	Pensioner bespoke longevity swap
57. Feb 2011	Pall	J P Morgan	£0.1B	Non-pensioners index-based longevity hedge
58. Jul 2010	British Airways	Goldman Sachs /Rothesay Life	£1.3B	Synthetic buy-in (longevity swap plus asset swap)
59. Feb 2010	BMW	Abbey Life /Deutsche Bank	£3B	Pensioner bespoke longevity swap
60. Dec 2009	Royal Berkshire	Swiss Re	£1B	Pensioner bespoke longevity swap
61. Jul 2009	RSA Insurance	Goldman Sachs / Rothesay Life	£1.9B	Synthetic buy-in (longevity swap plus asset swap)
62. May 2009	Babcock	Credit Suisse	£0.5B	Pensioner bespoke longevity swap (three schemes)
63. Sep 2008	Canada Life UK arm	JP Morgan	£0.5B	Pensioner bespoke longevity swap
64. Feb 2008	Lucida	JP Morgan	?	Index-based longevity swap

Table A.0.3.: Longevity Market Deals 2008 - 2016(3). Deals highlighted in red are index-based longevity swaps. Cyan ones are longevity solution structured to target capital market investors. Blue one is the first longevity deal originated in North America.

Table A.0.4.: Key Statistics for Gaussian Mixture Model Selection, Male pensioners, k=3

k=3	M, age 55-59	M, age 60-64	M, age 65-69	M, age 70-74	M, age 75-79
logllh					
Low-income range $G1$	£0-3,300	£0-9,700	£0-13,000	£0-2,100	£0-2,000
Medium-income range $G2$	£3,300-18,000	£9,700-22,000	£13,000-22,000	£2,100-13,500	£2,000-11,000
High-income range $G3$	£18,000+	£22,000+	£22,000+	£13,500+	£11,000+
w_1	0.26	0.36	0.37	0.31	0.36
w_2	0.68	0.60	0.59	0.6	0.56
w_3	0.06	0.04	0.04	0.08	0.08
μ_1	£1,585	£1,314	£1,169	£987	£1,024
μ_2	£7,400	£7,865	£6,855	£4,992	£4,234
μ_3	£26,645	£30,601	£29,948	£20,612	£18,512
σ_1	£981	£911	£751	£510	£546
σ_2	£3,727	£4,291	£4,082	£2,758	£2,233
σ_3	£19,723	£24,518	£29,467	£20,286	£16,436
BIC Converge (**Yes**) v.s. (**No**)	**No**	**No**	**No**	**Yes**	**Yes**

k=3	M, age 80-84	M, age 85-89	M, age 90-94	M, age 95-99
logllh				
Low-income range $G1$	£0-2,500	£0-2,500	£0-2,500	£0-1,900
Medium-income range $G2$	£2,500-11,000	£2,500-10,600	£2,500-11,500	£1,900-8,500
High-income range $G3$	£11,000+	£10,600+	£11,500+	£8,500+
w_1	0.50	0.60	0.68	0.66
w_2	0.45	0.36	0.29	0.30
w_3	0.05	0.04	0.03	0.04
μ_1	$£1,179$	$£1,185$	$£1,177$	$£983$
μ_2	$£4,195$	$£4,279$	$£5,033$	$£4,855$
μ_3	$£19,310$	$£18,455$	$£21,779$	$£19,985$
σ_1	$£688$	$£745$	$£818$	$£709$
σ_2	$£2,238$	$£2,308$	$£2,746$	$£2,608$
σ_3	$£18,463$	$£15,578$	$£18,761$	$£13,401$
BIC Converge (**Yes**) v.s. (**No**)	**Yes**	**No**	**No**	**No**

Table A.0.5.: Key Statistics for Gaussian Mixture Model Selection, Female (denoted by F in the table) pensioners, k=3

k=3	F, age 55-59	F, age 60-64	F, age 65-69	F, age 70-74	F, age 75-79
logllh					
Low-income range $G1$	£	£0-1,900	£0-1,500	£0-1,700	£0-1,400
Medium-income range $G2$	£	£1,900-10,000	£1,500-8,900	£1,700-7,400	£1,400-6,000
High-income range $G3$	£	£10,000+	£8,900+	£7,400+	£6,000+
w_1	0.56	0.53	0.46	0.46	0.50
w_2	0.41	0.43	0.48	0.49	0.45
w_3	0.03	0.04	0.06	0.05	0.05
μ_1	$£849$	$£855$	$£1,169$	$£987$	$£1,024$
μ_2	$£3,919$	$£3,783$	$£6,855$	$£4,992$	$£4,234$
μ_3	$£20,375$	$£19,882$	$£29,948$	$£20,612$	$£18,512$
σ_1	$£532$	$£496$	$£378$	$£357$	$£369$
σ_2	$£2,138$	$£2,151$	$£1,693$	$£1,431$	$£1,255$
σ_3	$£20,305$	$£20,046$	$£13,870$	$£10,547$	$£8,727$
BIC Converge (**No**) v.s. (**Yes**)	**No**	**Yes**	**Yes**	**Yes**	**Yes**

k=3	F, age 80-84	F, age 85-89	F, age 90-94	F, age 95-99
logllh				
Low-income range $G1$	£0-1,600	£0-1,700	£0-1,700	£0-1,900
Medium-income range $G2$	£1,600-7,000	£1,700-6,700	£1,700-6,500	£1,900-7,300
High-income range $G3$	£7,000+	£6,700+	£6,500+	£7,300+
w_1	0.59	0.60	0.68	0.66
w_2	0.36	0.36	0.29	0.30
w_3	0.05	0.04	0.03	0.04
μ_1	£845	£1,185	£1,177	£983
μ_2	£2,610	£4,279	£5,033	£4,855
μ_3	£11,526	£18,455	£21,779	£19,985
σ_1	£402	£393	£396	£452
σ_2	£1,290	£1,344	£1,355	£1,460
σ_3	£9,250	£9,132	£8,147	£12,203
BIC Converge (**Yes**) v.s. (**No**)	**Yes**	**Yes**	**Yes**	**Yes**

Appendix B

Figures

Figure B.0.1.: MTM P&L customized Longevity swap written on National Population [adapted from Biffis et al (2014) (104)]

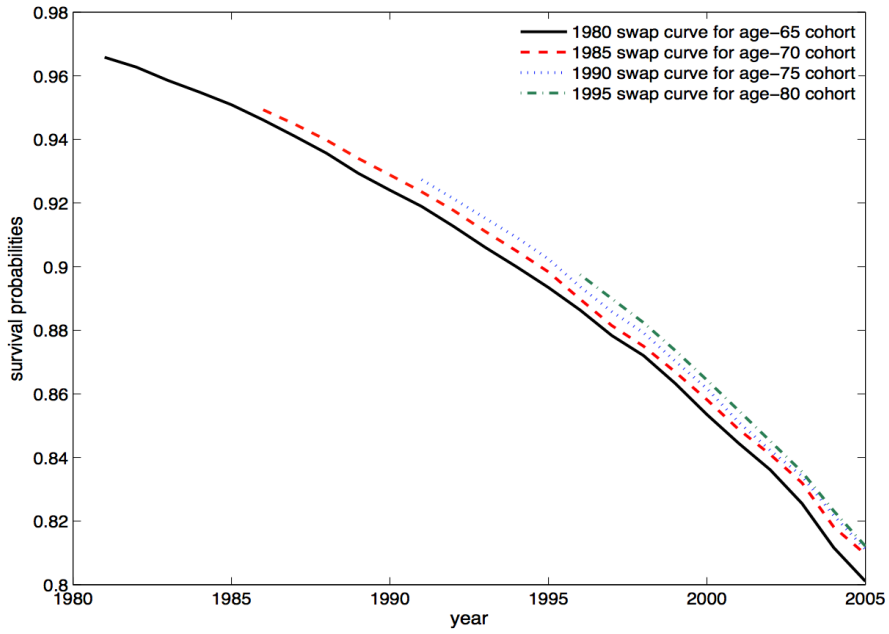


Figure 1: Survival curves computed at the beginning of year $t = 1980, 1985, 1990, 1995$ for England & Wales males aged $65 + t - 1980$ in year t. Forecasts are based on the Lee-Carter mortality projection model using the latest Human Mortality Database data available at the beginning of each year t.

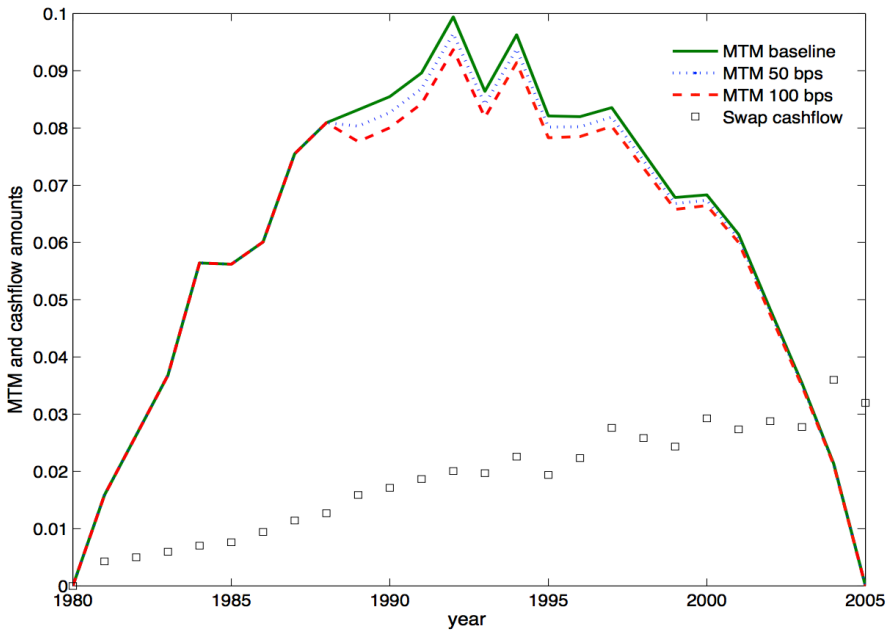


Figure 2: Mark-to-market value of the longevity swap in the baseline case and with the credit spread of counterparty HS widening by 50 and 100 basis points over 1988-2005. In the absence of default, the net payments from the swap are insensitive to credit spread changes.

Figure B.0.2.: Male panel, 2004-2009, 1D extended k

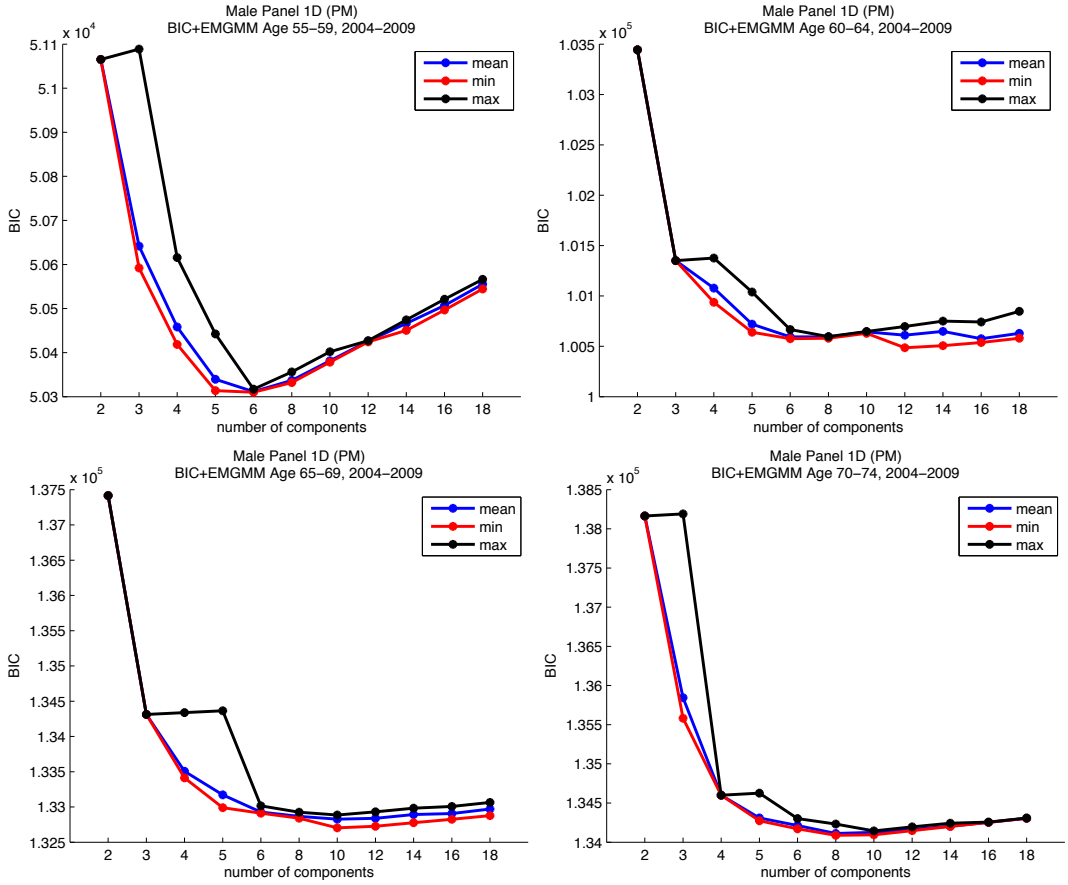


Figure B.0.3.: Male panel, 2004-2009, 1D extended k

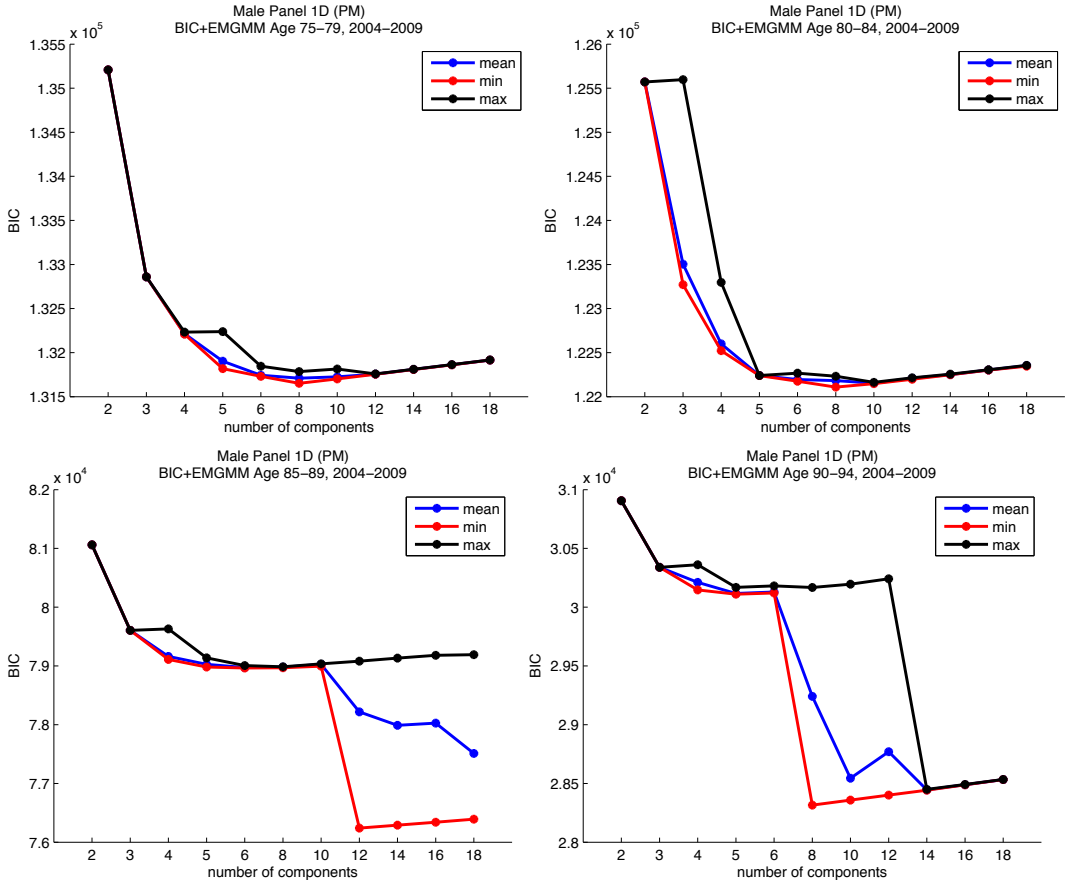


Figure B.0.4.: Female Panel, 2004-2009, 1D pension amount

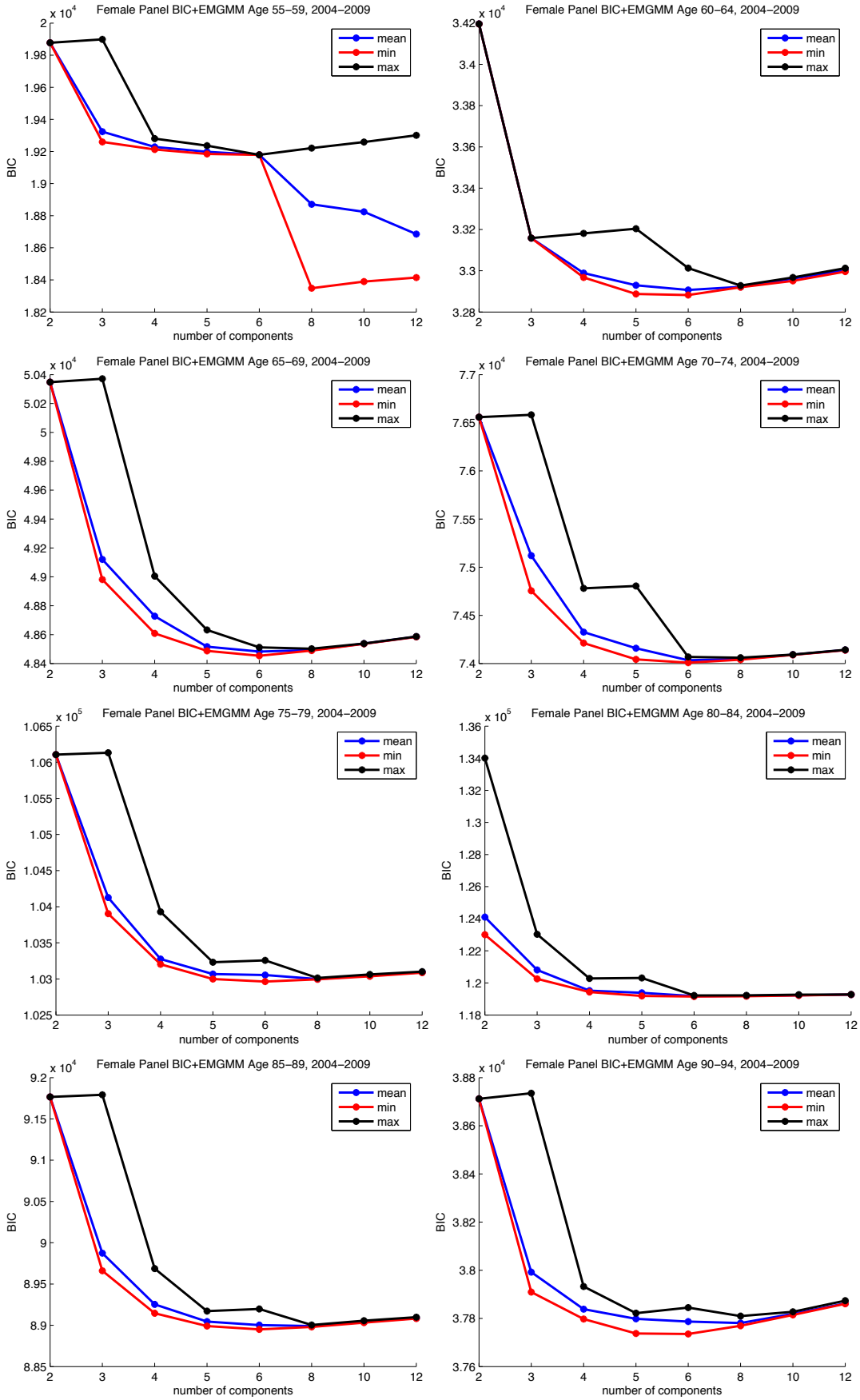


Figure B.0.5.: Male Panel, 2004-2009, age 55-59, k=3

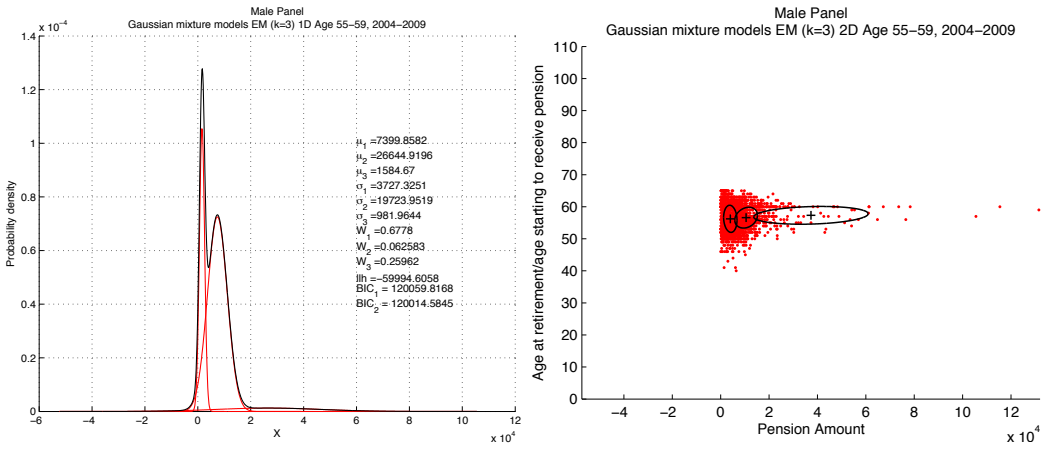


Figure B.0.6.: Male Panel, 2004-2009, age 60-64, k=3

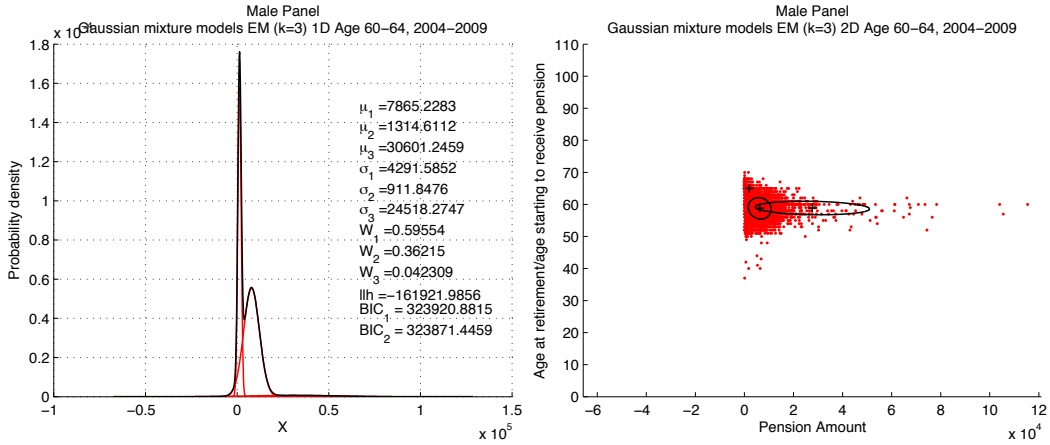


Figure B.0.7.: Male Panel, 2004-2009, 65-69, k = 3

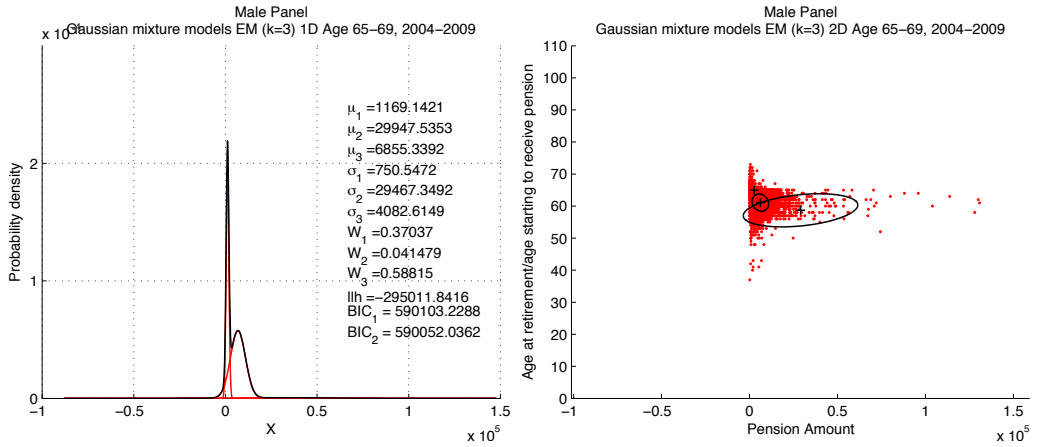


Figure B.0.8.: Male Panel, 2004-2009, 70-74, k=3

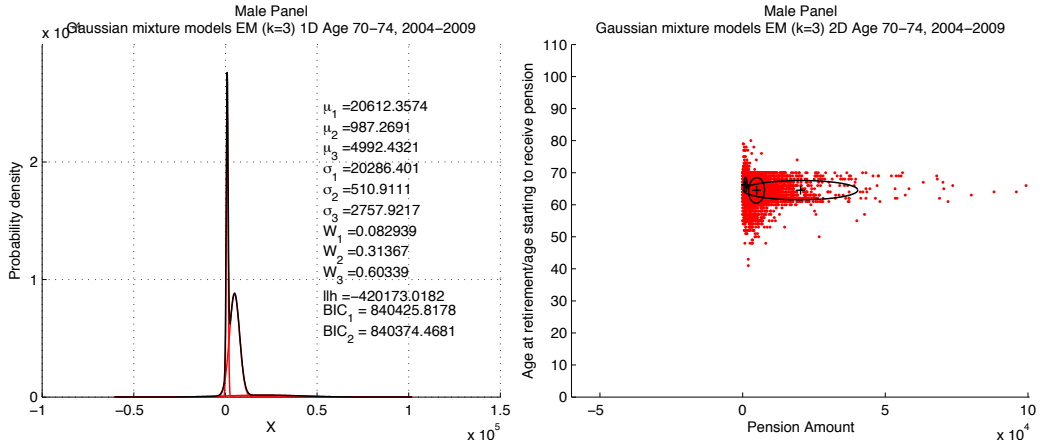


Figure B.0.9.: Male panel, 2004-2009, 75-79. k=3

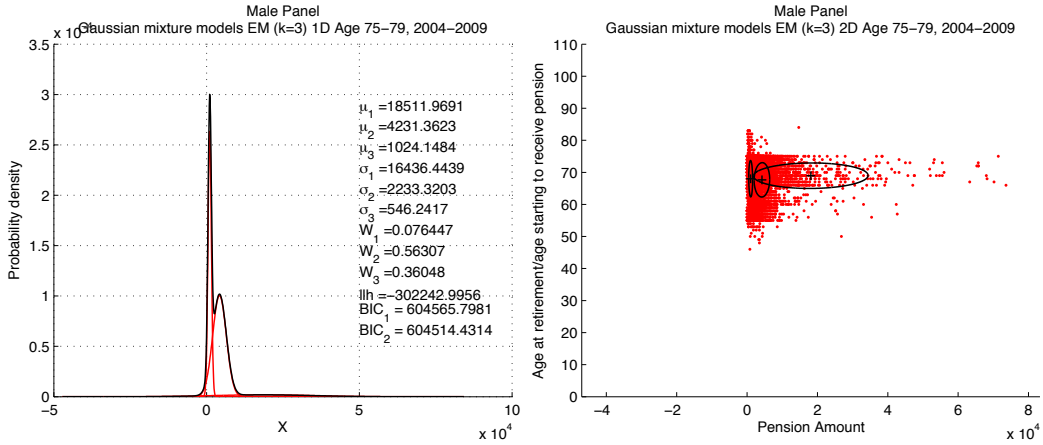


Figure B.0.10.: Male panel, 2004-2009, 80-84. k=8

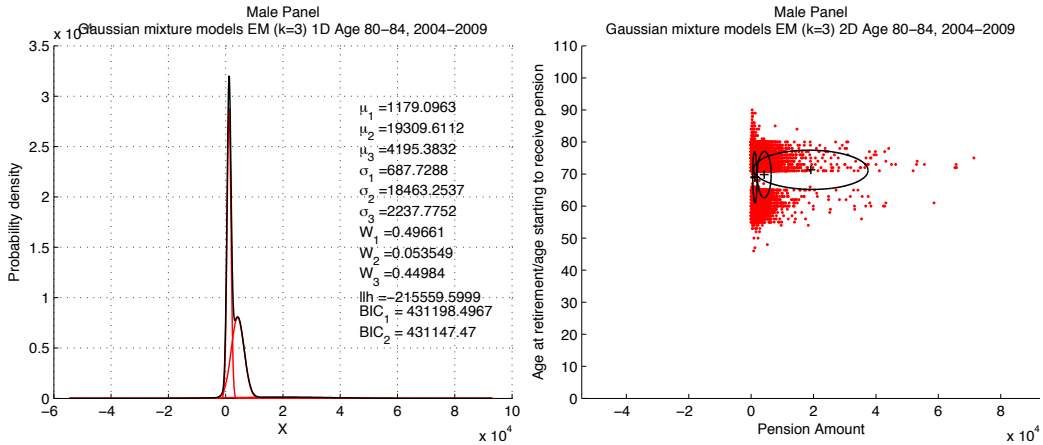


Figure B.0.11.: Male panel, 2004-2009, 85-89 k = 3

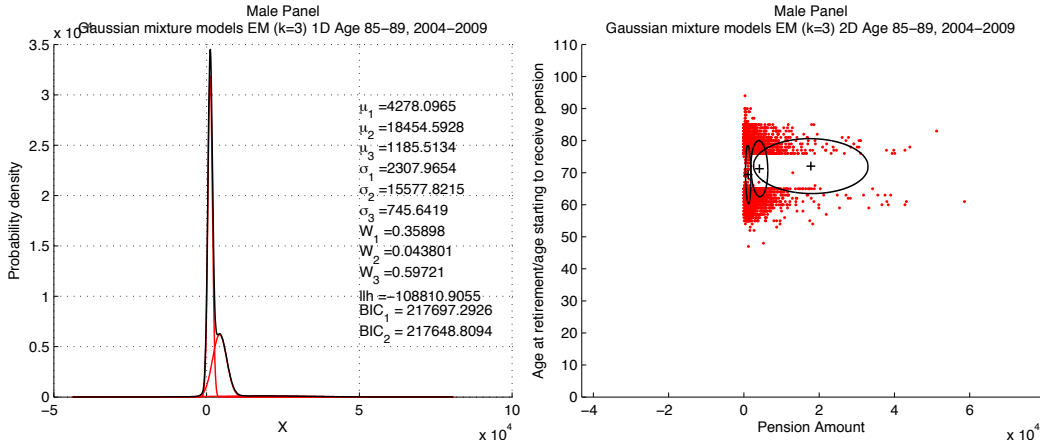


Figure B.0.12.: Male panel, 2004-2009, 90-94 k = 3

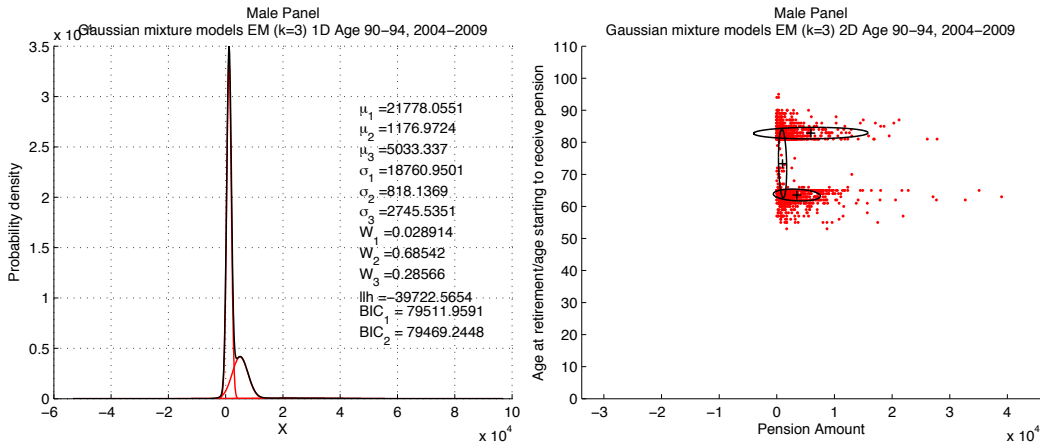


Figure B.0.13.: Male panel, 2004-2009, 95-99, k=3

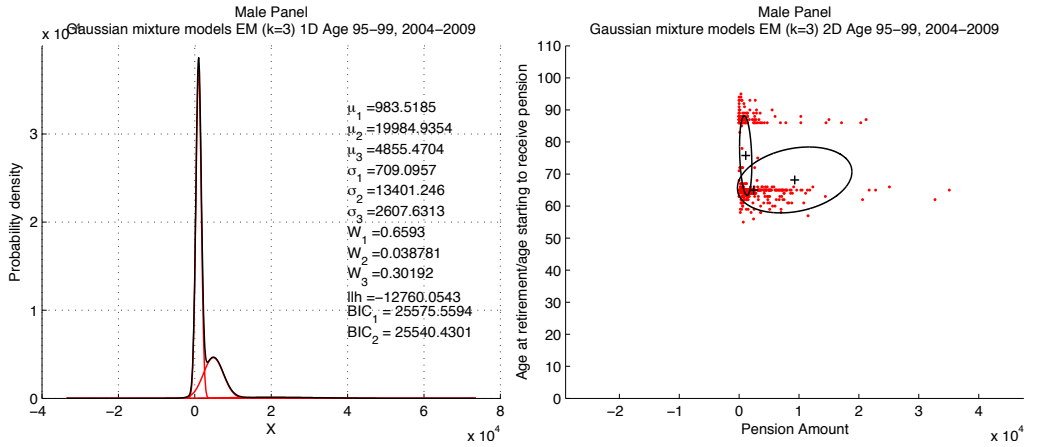


Figure B.0.14.: Female panel, 2004-2009, 55-59, k=3

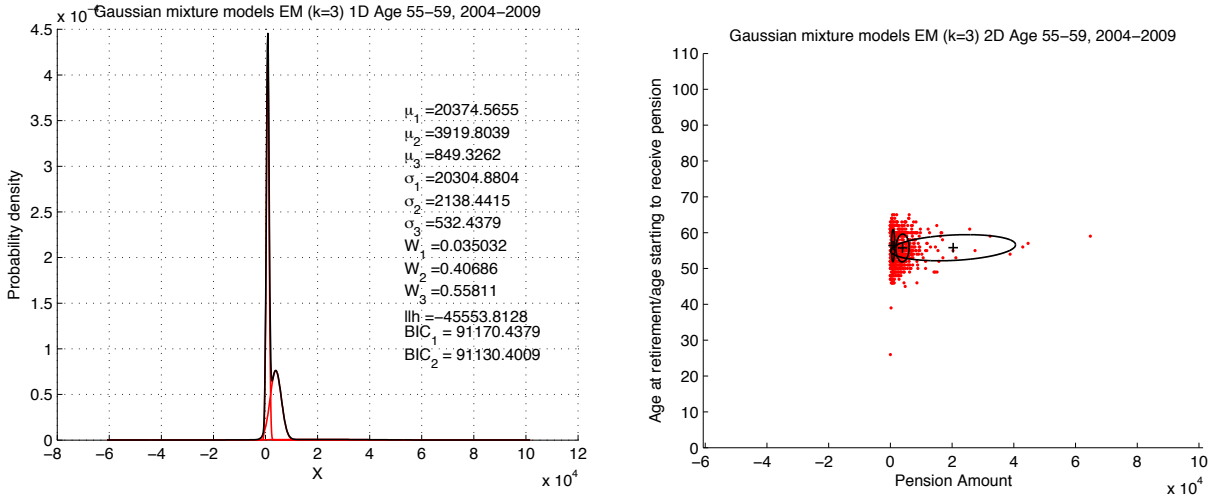


Figure B.0.15.: Female panel, 2004-2009, 60-64, k=3

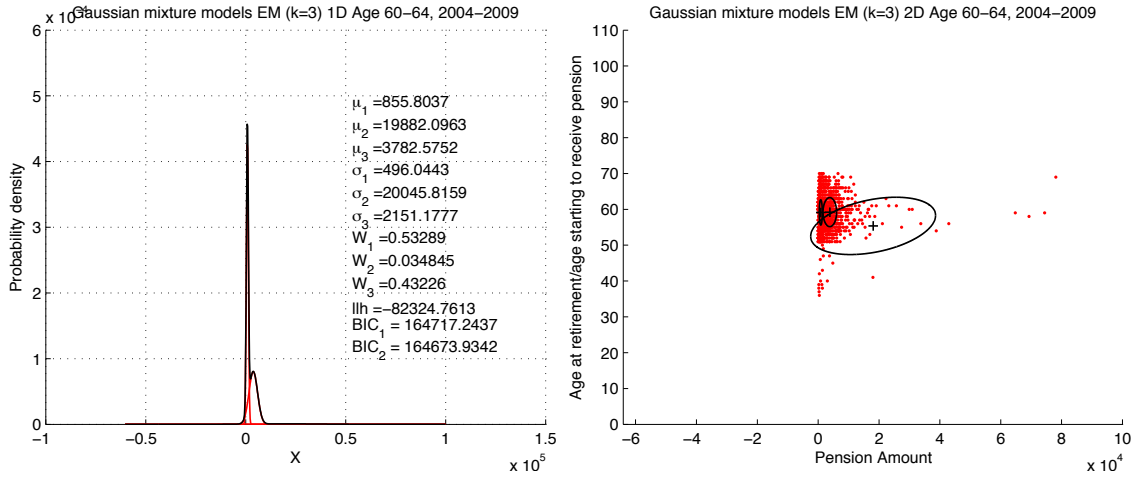


Figure B.0.16.: Female panel, 2004-2009, 65-69, k=3

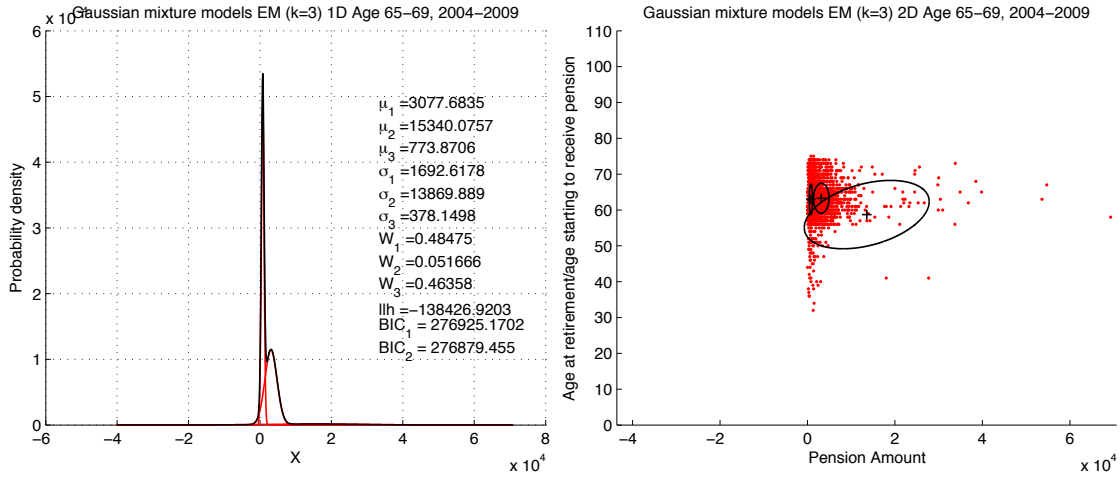


Figure B.0.17.: Female panel, 2004-2009, 70-74, k=3



Figure B.0.18.: Female panel, 2004-2009, age 75-79, k=3

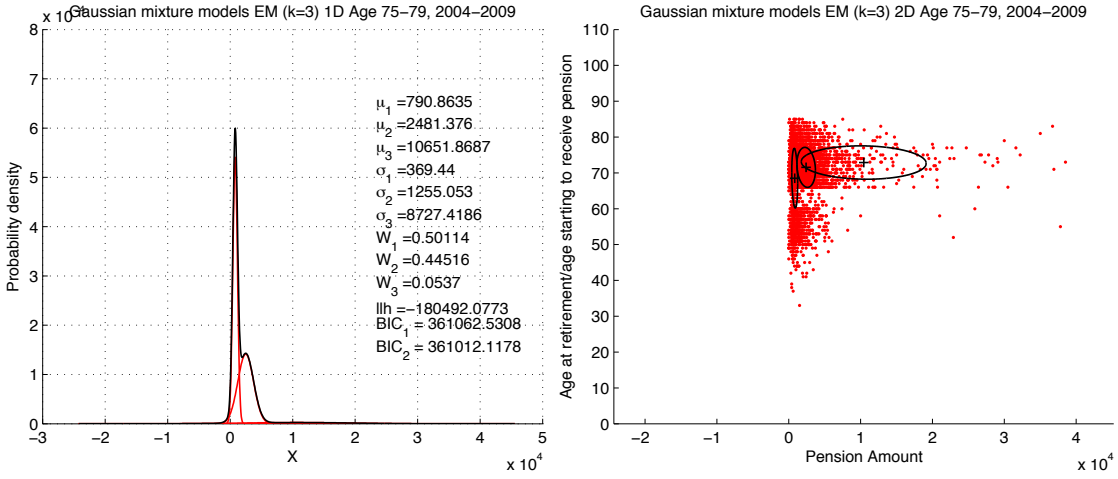


Figure B.0.19.: Female panel, 2004-2009, age 80-84, k=3

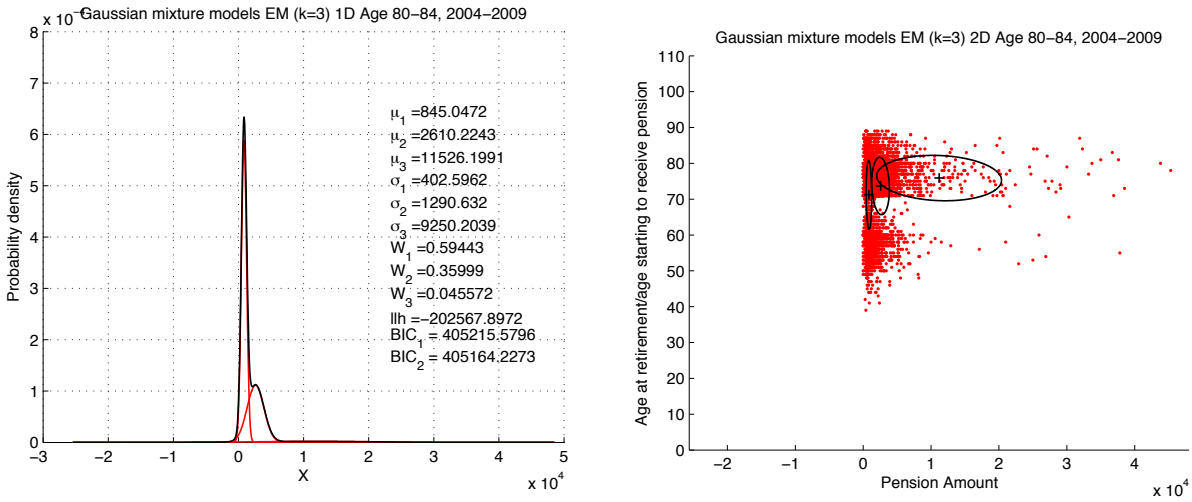


Figure B.0.20.: Female panel, 2004-2009, age 85-89, k = 3

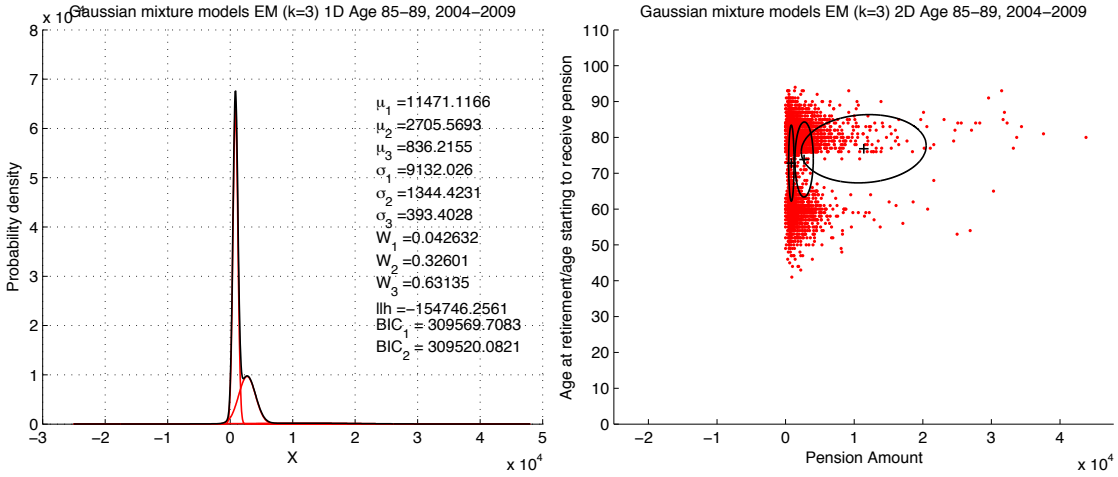


Figure B.0.21.: Female panel, 2004-2009, 90-94, k=3

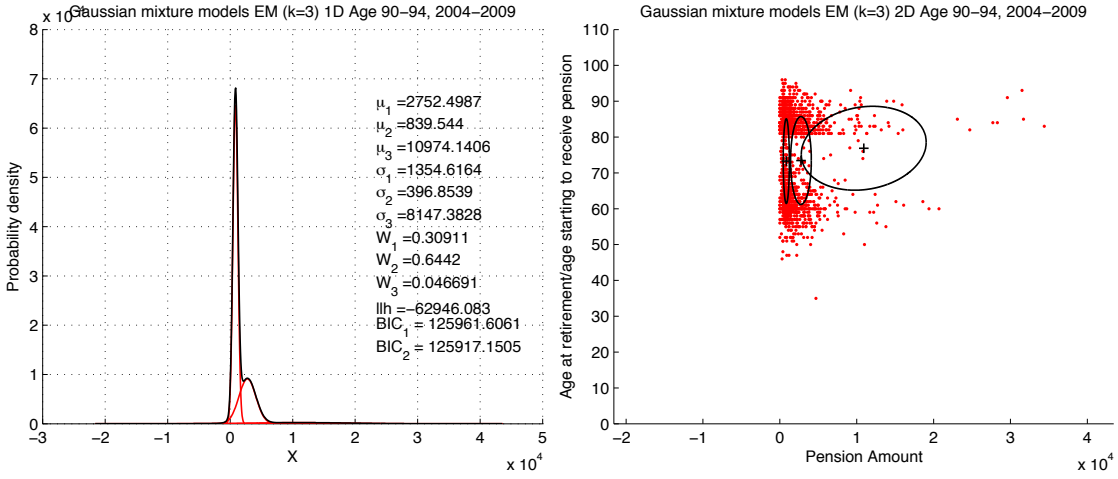
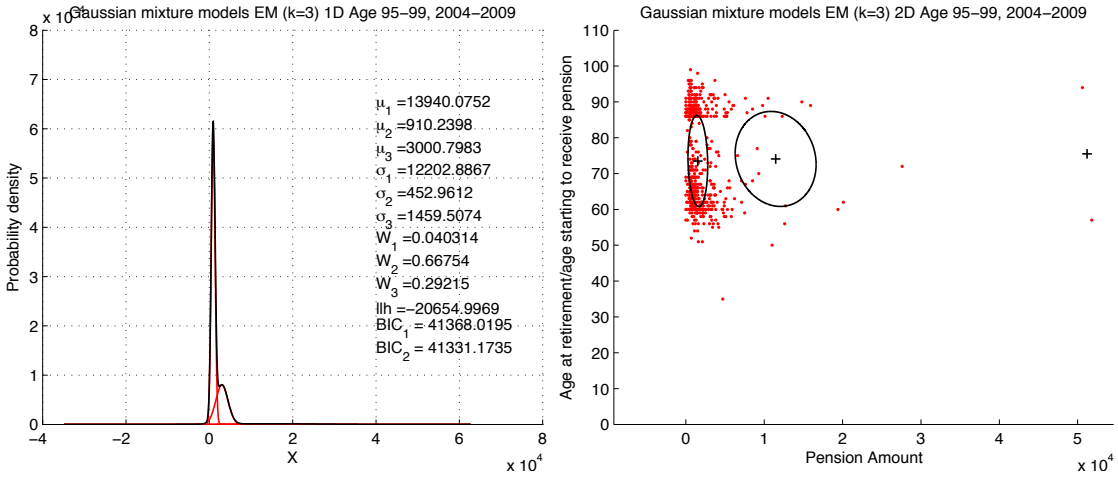


Figure B.0.22.: Female panel, 2004-2009, age 95-99, k=3



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Thesis Conclusion

This thesis is composed of three parts, each of which studies an aspect of EVT used in finance area. In particular, the phenomena of heavy-tailedness in several distributions examined in this thesis has shaped our understanding of the respective matters.

Part I studies an interbank network of an advanced emerging economy. It provides heavy-tailedness evidence of network structure, as opposed to early pioneer researchers' poisson-distributed network distribution assumed in social or biological network studies by Erdos and Rényi (1959). This is important because the structure of network have potential impacts on its (in)stability. Heavy-tailedness in network structure means nodes (financial institutions) in the network have asymmetric trading relationship with their neighbours. This is similar to advanced economy of Austria and another emerging economy, Brazil. Some nodes are better connected than other ones. Those in the "core" are mainly composed of domestic banks, and the ones at the peripheral are mainly composed of foreign banks. This so-called *preference attachment* may cause new banks enter into the market to connect to the already well connected banks.

The paper also examines whether TBTF is related to TICTF. This is done by a simple regression by linking the size of institutions and its importance measured by centrality. Results show bigger institutes are more central than smaller ones. However, there are exceptions: The smaller but well connected banks should be paid attention to, from a regulatory point of view. TBTF has its merits.

The paper shows interconnectedness risk in the real world could be a major source to contagion among financial institutions, which may cause further contagion to other sectors and real economy. It simulates default events led by ideosyncratic risks of each institute in this network, using a pre-defined sequential default mechanism. Results show that credit risk is not a major source of contagion, while second round effect, such as fire sale loss and liquidity problem seem to contribute more.

Limitation: This paper has limited period of data, hence calibration of the default mechanism using real world data becomes difficult. In may require further sensitivity analysis to test how the model results change in respect to the varying parameters. On the other hand, using public data set may gives alternative estimation of such parameters.

Part II studies Spanish household income inequality and a (proxy) elasticity measure of consumption of various goods in the decade of 2001-2011. The paper estimates both the lower and upper tails of the income distribution, and found that in 2005, the poorer households are getting poorer, with their incomes diverge, while in the long-term, the income of the poorer households

converge. The richest households behave opposite in their income. In 2005, the richest families get richer [only mildly as the tail index of α only increases slightly], while in the long term, their incomes are diverging.

These trends in income inequality of both tails have asymmetric impacts on the respective households' consumption behaviour on various goods. The goods with shortest useful lives (food) is most responsive to lower tail movement, with its' coefficient to household income dropped significantly from 0.0211 to 0.0097, in 2001 and 2005, respectively. While other non-durable goods respond one period later, as its' useful lives are normally longer.

Durable goods such as vehicles, have varying coefficients with household incomes. It first decreases from 0.0292 to 0.0028, from 2001 to 2005, respectively. Then it increases to 0.0176 in 2008, then decreases again in 0.0094. One of the possible reasons could be vehicles have long life span, hence it does not require replacement frequently. However, It requires further analysis to draw a solid conclusion. Other durable goods's coefficients with household income does not change significant during 2001-2011. This may be because they have very long life span, even longer than the observation period, hence they do not require replacement. This also needs further analysis.

Here EVT enhances our understanding of the income inequality in both tails of the distribution. It also shows its lognormal mean is moving to the right, meaning general households' incomes are increasing. It's lognormal volatility is also increasing, meaning households' incomes are diverging over time.

Part III analyzes basis risk of a longevity swap written on a real world UK pension portfolio, referenced to the national population. Heavy-tailedness in pensioner income distribution can be viewed as important input as modelling assumptions. Extreme large pension income may cause pension fund's funding deficit, given its longevity exposure. This factor has not been taken into consideration by previous papers or practitioners' modelling.

According to income distribution theory developed by Colombi (1990) (106), as an extension from the previous step, the paper then divide pensioners according to their pension incomes, then set up hedging instruments [index-longevity swap] to hedge the different sub-populations, referenced to the national population. It shows similar MTM and P&L results to my previous publication, Biffis et al (2014) (104). Basis risk can be managed and index-based longevity swap can be implemented to hedge pension fund's longevity exposure.

Limitation: Calculating MTM and P&L may not be enough to make solid conclusion on hedge-effectiveness. It requires quantification of risk reduction in the context of accounting standards.

All three parts show EVT can be used to measure different finance and economic events, especially as the first sign of problems.