AN ANALOGUE BANDPASS FILTER REALISATION OF THE CONTINUOUS WAVELET TRANSFORM

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1 Motivation

- Lightweight, portable devices must be physically small.
- They can thus have only small, limited capacity batteries making power consumption the key system design parameter.
- In addition, it is often desirable to carry out onboard signal processing.
- Low power, low computational complexity signal processing methods are thus required.
- The Continuous Wavelet Transform (CWT) is one such signal processing technique. It is used by the portable system described in [1] and so a low power implementation requires investigation.

2 Objectives

- To illustrate a wavelet approximation procedure termed the Low Power CWT (LPCWT) as it is suitable for low power implementation. The method is similar to that in [2] and related papers, and can be implemented using any desired circuit topology.
- To act as a step-by-step wavelet design guide, illustrating the trade-offs that are possible. This could be useful to engineers with similar signal processing requirements.
- To give application specific performance metrics for the approximation method. This allows the performance of the LPCWT in a real usage situation to be found which is more meaningful than an arbitrary error norm between the wanted and approximated wavelet shapes.

3 Wavelet design procedure

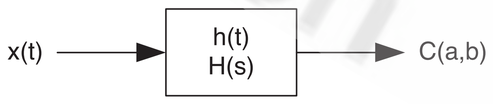
- The CWT of signal $x(t)$ at a scale a, time b and with mother wavelet $\psi(t)$ is defined as

$$C(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^* \left(\frac{t-b}{a}\right) dt. \quad (1)$$

- This is equivalent to the convolution of the signal $x(t)$ with an impulse response

$$h(t) = \frac{1}{\sqrt{a}} \psi \left(\frac{-t}{a}\right). \quad (2)$$

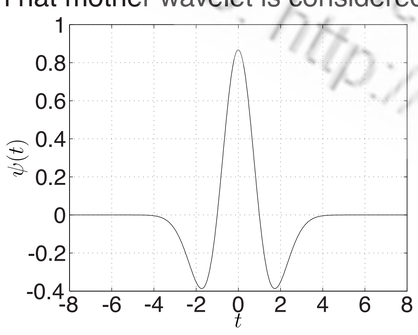
- All that is needed to implement the CWT at scale a is thus a filter with impulse response $h(t)$, or the corresponding $H(s)$.



The design procedure to generate $H(s)$ is summarised as:

1. Based on the application determine the mother wavelet required. The Mexican hat wavelet is the example used here.
2. Choose the scales at which analysis must be carried out. Reducing the number of scales used will reduce the circuitry required and minimise the power consumption.
3. Mathematically approximate the wavelet shape using the procedure outlined here, exploiting the mathematical form if possible.
4. Determine a time delay T to ensure that the wavelet approximation is realisable.
5. Use the dynamic range and other application specific criteria to select the order of the approximation required.
6. Generate the required filter transfer functions and implement these in a suitable circuit.

- The Mexican hat mother wavelet is considered here:



- This has the Fourier Transform:

$$H(j\omega) = \pi^{1/4}\sqrt{\frac{8}{3}a^5\omega^2 e^{-a^2\omega^2/2}}. \quad (3)$$

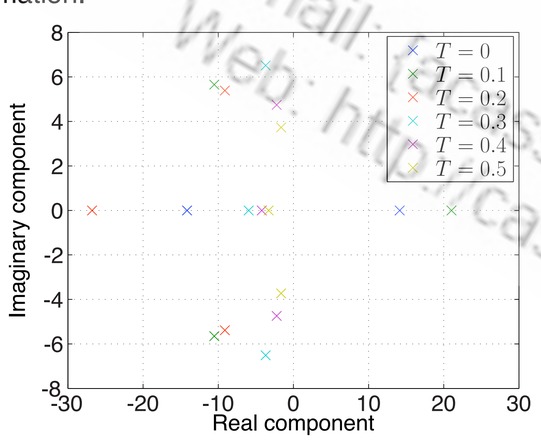
- But $\psi(t)$ is symmetrical around $t = 0$ so this is not stable.
- To correct for this a time delay T is introduced shifting $\psi(t)$ to the right. Letting $j\omega \rightarrow s$:

$$H(s) = -\pi^{1/4}\sqrt{\frac{8}{3}a^5 s^2 e^{a^2s^2/2}e^{-sT}} = \frac{-\pi^{1/4}\sqrt{\frac{8}{3}a^5 s^2}}{e^{sT-a^2s^2/2}}. \quad (4)$$

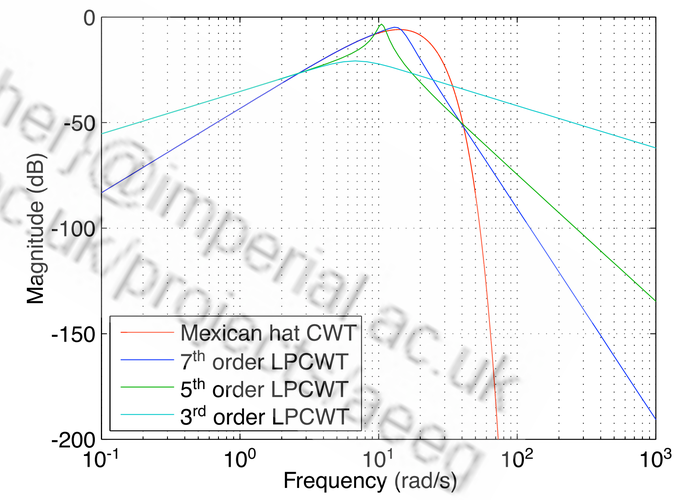
- This is in rational form, but requires a finite order denominator for implementation.
- Thus a Maclaurin series expansion is used to give the final generalised transfer function:

$$H(s) = \frac{-\pi^{1/4}\sqrt{\frac{8}{3}a^5 s^2}}{1 + sT + \left(\frac{T^2}{2} - \frac{a^2}{2}\right)s^2 + \left(\frac{T^3}{6} - \frac{Ta^2}{2}\right)s^3 + \cdots}. \quad (5)$$

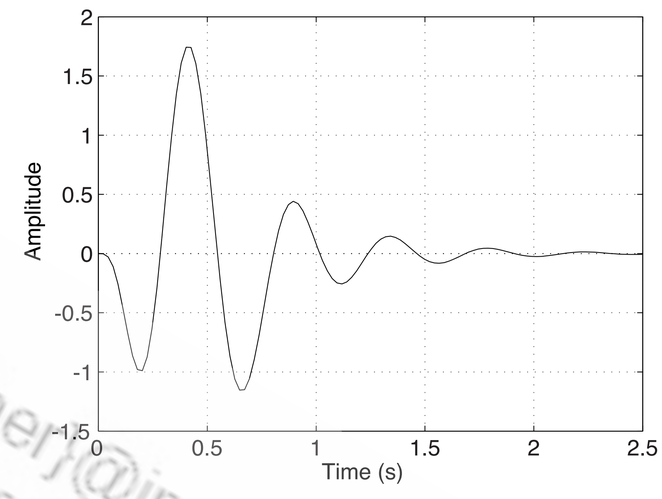
- T can be varied to help place the poles in desired positions. The figure below illustrates the case for a 3rd order approximation.



- The filter order, which determines the quality of the approximation, can then be chosen. For $a = 0.1$ and $T = 0.4$:



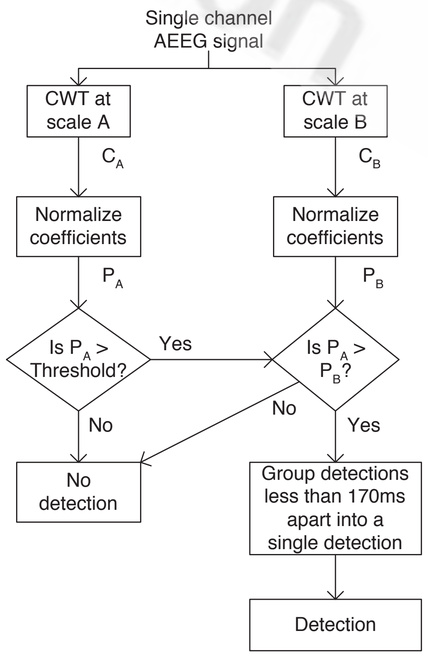
- Note that the bode plots only need to match over the dynamic range of the situation under consideration.
- Full numerical transfer functions ready for implementation can now be found.
- Substituting $a = 0.1$ and $T = 0.4$ into the 7th order approximation gives the impulse response below.



- This is now suitable for performance testing via MATLAB simulations.

4 Performance analysis

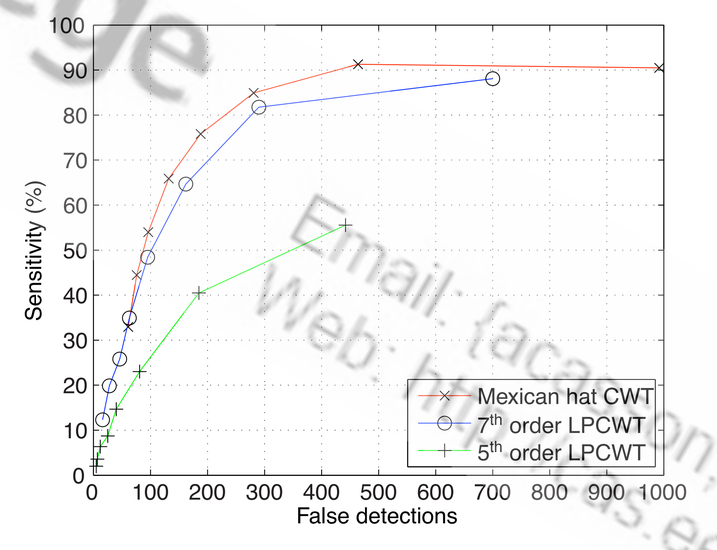
- Testing is carried out in the algorithm from [1], shown below.
- This gives an accurate measure of how well the approximation procedure operates in a real-usage situation.



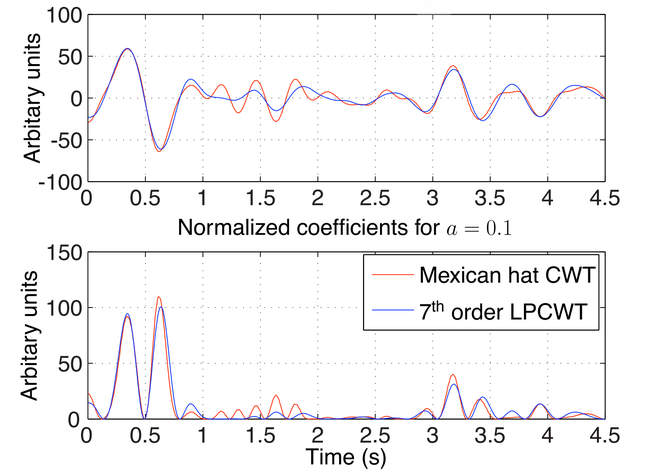
- The CWT is carried out in 4.5 s blocks and is normalized as:

$$P(a,b) = \frac{C^2(a,b)}{\sigma^2}. \quad (6)$$

- $a = 0.025$, $T = 0.1$ and $a = 0.1$, $T = 0.4$ are used for analysis at scales A and B respectively.
- The aim of the algorithm is to identify spikes in the ambulatory electroencephalogram (AEEG) [1].
- The performance metrics of interest are thus the sensitivity (the number correct detections over the total number of events present), and the number of false detections.
- A data set 22 minutes long with 252 expert marked events is analysed. Results are shown below.



- The 7th order LPCWT achieves sensitivities that are within a few percent, and so comparable to, the CWT function.
- These sensitivities are achieved with fewer false detections occurring which represents an improvement in performance.
- The detailed performance of the LPCWT compared to the MATLAB CWT function is shown below.
- The two are seen to be very similar.



5 Conclusion

- A wavelet approximation method suitable for implementing as an analogue filter circuit has been described.
- When applied to an actual usage situation simulations showed that the LPCWT is capable of giving results comparable to the MATLAB CWT function.
- As the choice of mother wavelet used is essentially arbitrary it is possible for a non-ideal approximation of it to achieve improved results.
- This could be used to further optimize the design procedure.

[1] A. J. Casson, D. C. Yates, S. Patel, and E. Rodriguez-Villegas, "Algorithm for AEEG data selection leading to wireless and long term epilepsy monitoring", in IEEE Eng. Med. Biol. conference, Lyon, August 2007.
[2] J. M. H. Karel, R. L. M. Peeters, R. L. Westra, and S. A. P. Haddad, "Wavelet approximation for implementation in dynamic translinear circuits", in Proc. of the IFAC world congress, Prague, July 2005.