

Operational optimization of networks of compressors considering condition-based maintenance

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Abstract

The paper presents a mixed integer linear programming optimization model which deals with the optimal operation and maintenance of networks of compressors of chemical plants. This optimization model considers a condition-based maintenance model which involves the degradation of the condition of the compressors. The paper focuses on online and offline washing, two different cleaning procedures which reduce the extra power used by the compressors due to fouling. The state-of-the-art has demonstrated the optimal schedule of the maintenance of a single compressor neglecting the interactions between operation and maintenance of more than one compressor. The suggested optimization model studies a compressor station with multiple compressors and provides their optimal schedule and the best decisions for their washing. Different case scenarios examine the influence of different types of washing methods on the total costs of operation and maintenance. The paper demonstrates the benefits of the optimization and proves that maintenance and operation have to be examined simultaneously and not separately, in contrast to common industrial practice and previous literature.

Keywords: mixed integer programming, condition based maintenance, washing schedule, compressor washing, fouling

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1 Nomenclature

Indices/Sets

$e \in E$	products, i.e. O ₂ , N ₂
$i \in I$	compressors
$j \in J$	headers
$n \in N$	process plants
$t \in T$	time periods
$u \in U$	air separation columns
$z \in Z$	storage tanks

Subsets

I_l	large compressors
I_s	small compressors
I^{NOFF}	compressors which are not under offline maintenance before optimization
$J_{(i)}$	headers connected with compressor i

Subscripts

aux	auxiliary
of	offline
on	online

Superscripts

c	clean
max	maximum
no	nominal
o	initial state

Parameters

C_D	change header cost [m.u./change]
$C_{f(i)}$	shut down cost [m.u./shut down]
$C_{of(i)}$	offline washing episode cost [m.u./wash]
$C_{on(i)}$	online washing episode cost [m.u./wash]
$C_{O(e,t)}$	purchase cost of product e in period t [m.u./kg]
$C_{st(i)}$	start up cost [m.u./start up]
d_c	duration of discretized time period [d]

$d_{of(i)}^o$	duration of offline washing of compressor started before optimization [d]
$d_{on(i)}^o$	time period between the latest online washing and the beginning of optimization [d]
$o(i)$	maximum running time [d]
R	recovery factor of online washing model [-]
t_S	number of days before the end of the time horizon [d]
$W_{el,l/s}^{No}$	nominal electrical power of a motor of a large (l) or small (s) compressor [kW]
$\gamma(i)$	minimum time between two consequent online washing episodes [d]
$\delta S_{(i)}^o$	initial duration of cumulative days of operation before optimization [d]
$\epsilon(i)$	degradation rate of compressor i [kW/d]
λ_k	minimum number of offline washing episodes [-]
λ_{off}	maximum number of offline washing episodes in each time period [-]
λ_{on}	maximum number of online washing episodes in each time period [-]
$\nu(i)$	duration of maintenance [d]
$\xi_{(e,z)}$	capacity of storage tanks [kg]
$\pi(i)$	pressure ratio of compressor [-]
$\rho(i)$	mass flow rate of compressor [kg/s]
$\psi_{(i)}$	minimum shut down time [d]
$\tilde{\psi}_{(i)}$	continuous offline time before optimization [d]
$\omega_{(i)}$	minimum online time after the latest start up of compressor i [d]
$\tilde{\omega}_{(i)}$	continuous online time before optimization [d]
$\Omega_{(i)}$	maximum extra power because of degradation [kW]

Continuous non-negative variables

$A_{(e,z,t)}$	amount of product e from storage tank z at t [kg]
$B_{(e,z,t)}$	amount of stored product e in tank z at t [kg]
$C_{(e,u,t)}$	rate of product e from air separation column u at t [kg/s]
$L_{(e,u,z,t)}$	amount of product e from air separation column u provided to storage tank z at t [kg]
$M_{(i,j,t)}$	flow rate of compressed air of compressor i delivered to header j at t [kg/s]

$O_{(e,t)}$	amount of product e purchased from external sources at t [kg]
$P_{(i,j,t)}$	discharge pressure of compressor i connected with header j at t [bar]
$W_{el(i,t)}$	power of a motor of compressor i in period t [kW]
$W_{el(i,t)}^c$	power consumed when compressor i is not fouled in period t [kW]
$\Delta S_{(i,t)}$	cumulative time of the operation after the last maintenance of compressor i at t [d]
ΔS_{aux}	auxiliary variable to linearize the non-linear online washing model [-]
$\Delta S_{on,aux}$	auxiliary variable to linearize the non-linear combined online and offline washing model [-]
$\Delta W_{(i,t)}$	extra power consumed from the motor of compressor i at t [kW]

Binary variables

$D_{(i,t)}$	change header status of a compressor i in period t
$F_{(i,t)}$	shut down status of a compressor i in period t
$K_{of(i,t)}$	finish status of offline washing of compressor i in period t
$S_{(i,t)}$	start up status of a compressor i in period t
$U_{of(i,t)}$	offline washing status of compressor i in period t
$U_{on(i,t)}$	online washing status of compressor i in period t
$W_{of(i,t)}$	beginning of offline washing of compressor i in period t
$X_{(i,t)}$	operational status of compressor i in period t
$Y_{(i,j,t)}$	connection between compressor i and header j in period t

1. Introduction

Energy intensive process plants use multiple compressors in parallel to increase total available capacity. Spare compressors are used to improve the flexibility of the plants and avoid unsatisfied production deliveries, according to Kurz et al. (2012). The users of the plants, therefore, have to take decisions to optimally exploit this redundancy and coordinate the operation to achieve the minimum power consumption and minimum wear of the compressors while at the same time the plant meets its production targets in the long term. Examples of these decisions are the selection of the online and offline compressors.

Moreover, the maintenance of the compressors increases the complexity of the decision-making as more degrees of freedom have to be considered. Therefore there is a need for decisions which minimize operational costs, increase the lifetime of the compressors and their motors, while also ensuring that the plant meets its demand.

Boyce (2003) explains that the maintenance of a compressor may involve several actions such as bearing and rotor repairs, coupling and gear box maintenance, and inspections of the sealings and the motors. The compressors which are placed in process gas applications mainly suffer from fouling (Aretakis et al., 2012). Fouling is the depositions of particles in the fluid to the airfoil. The depositions increase the roughness of the surfaces of the internal mechanical components of the compressors (e.g. impeller and diffuser area) and restrict the passages areas of the fluid. An example of a fouled impeller can be seen in Fig. 1. Thus, the result of the fouling is a decrease in performance and increase in power consumption for the same load compared to a

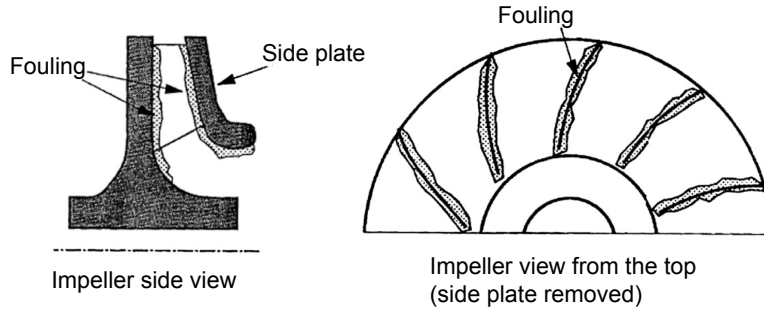


Figure 1: Side and top view of a fouled impeller (Forsthoffer, 2011)

27 non-fouled compressor.

28 There are two common strategies to deal with the problem of fouling,
29 namely offline and online washing. The offline washing takes place when a
30 compressor is not operating and the online washing cleans the compressor
31 online without interrupting its operation.

32 An Offline Washing Episode (OFWE) is complete after several cleaning
33 steps when the compressor is in offline mode. The purpose of the washing
34 of these cleaning steps is to recover the efficiency of the compressor which
35 results in decreasing the extra power consumed. Fabbri et al. (2011) reports
36 that supplementary maintenance tasks, such as mechanical and electrical
37 inspections, can be included during a typical OFWE, hence the total duration
38 of this OFWE can increase up to a few days.

39 Another method for improving the performance of a compressor is to in-
40 ject a cleaning solution inside the compressor while it is operating online. The
41 advantage of this method is that less power is consumed without shutting
42 down the compressor to wash. However, the recovery of the performance
43 is smaller than in the case of an offline washing episode. Figure 2 shows
44 the qualitative trends of the efficiencies when a compressor is not washed,
45 is washed online exclusively and is cleaned with offline and online washing.
46 According to Fabbri et al. (2011) an Online Washing Episode (ONWE) can-
47 not take place if the ambient temperature is less than 14°C because ice is
48 created which can damage the blades of the compressor.

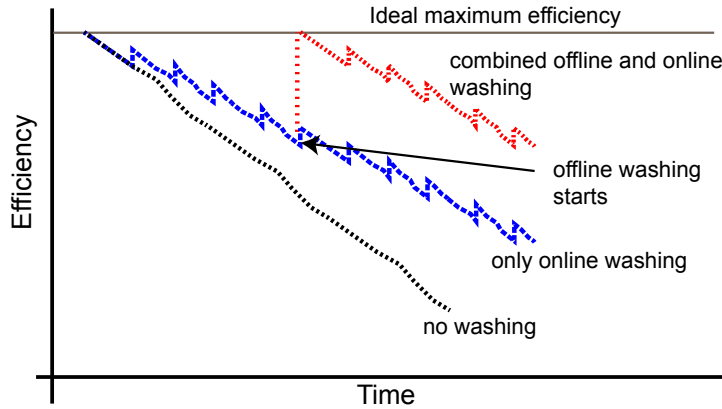


Figure 2: The qualitative trend of the efficiency considering different types of washing methods.

49 2. Background and context

50 In the last half decade there has been interest in the optimization of com-
51 pressor stations. Xenos et al. (2015a) presented a survey on the optimization
52 of compressor stations in natural gas networks, utilities, gas storage and other
53 applications. The key points of the survey were that most of the studies on
54 the optimization of compressors focus on the operation of gas networks in-
55 cluding compressors and the optimization disregards important aspects of
56 operation of compressors such as start up and shut down actions.

57 The optimal operation of gas compressors in different process systems
58 has been studied by many researchers, for example Sun et al. (2014), Hasan
59 et al. (2009) and Han et al. (2004). These researchers employed a steady-
60 state optimization analysis (solution for one set of operational conditions)
61 compared to the scheduling approach from van den Heever and Grossmann
62 (2003). The latter study focused on the pipeline network and not on the op-
63 erational aspects of the compressors such as shut down and start up actions.
64 Other studies focused on the optimal control of the compressors. These stud-
65 ies employed model predictive control strategies and recent examples are the
66 papers by Gopalakrishnan et al. (2013) and Zavala (2014).

67 There is research on the estimation of the performance of compressors
68 considering degradation due to fouling, especially on axial compressors, an
69 example is the study from Boyce et al. (2007). There are also many authors
70 who studied the performance deterioration of the compressor of a gas turbine
71 for power generation. Aretakis et al. (2012) carried out an economic analysis
72 of a single gas turbine to estimate the optimal number of the offline washings
73 for one year. The authors considered costs such as capital, washing, start
74 up and fuel costs. Li et al. (2009) described a prognostic approach to esti-
75 mate the remaining time of a gas turbine until its next maintenance episode.
76 Other thermodynamic and economic analysis were presented by Sánchez et
77 al. (2009) and Fabbri et al. (2011).

78 The latter studies did not incorporate maintenance actions into their op-
79 timization formulation. Boyce (2003) referred to a concept of integration
80 between operation and maintenance, the author called this integration Per-
81 formance Based Total Productive Maintenance (PTPM), however this refer-
82 ence did not present a method to achieve this integration. Recent works on
83 networks of compressors by Kopanos et al. (2015) and Xenos et al. (2015b)
84 presented the simultaneous optimization of operation and maintenance. Both
85 studies included maintenance tasks which deal with preventive maintenance

tasks, however the cost of the maintenance was not included into the objective function.

There are also papers which studied the maintenance scheduling of single or parallel units in other applications such as biopharmaceutical manufacturing (Liu et al., 2014), wind turbine farms (Gustavsson et al., 2014), process plants (Sequeira et al., 2001; Hazaras et al., 2012; Biondi et al., 2015) and power plants with gas engines (Castro et al., 2014). The article by Castro et al. (2014) deals with the maintenance scheduling of parallel gas engines and the work demonstrates many similarities with the topic of the current paper. Castro et al. (2014) employed a mixed integer linear programming model which considered shut downs and start ups of the engines, and constraints which involved industrial restrictions related to maintenance, for example where there were limited maintenance resources. However, the model considered an industrial case with identical engines and their power consumption was independent of time and operational conditions.

According to the previous statements, the literature from one side examines the maintenance of a single compressor considering its performance, and on the other side examines the optimization of the operation of networks of compressors neglecting their maintenance tasks. Hence, the optimization of a compressor station with multiple (not necessarily similar) compressors in parallel considering their maintenance is still an open question.

For these reasons, this work suggests a mathematical framework for the optimal operation of a network of compressors considering condition-based maintenance. The framework includes a Mixed Integer Linear Programming (MILP) model. The model involves the basic operational constraints of the compressors considering the prediction of power consumption depending on operational conditions, the extra power consumption due to degradation, and minimum running and minimum shut down times of the compressors. The model also includes the offline and online washing maintenance which is the key contribution of the current study.

Section 3 presents the MILP optimization models including constraints for the online, offline and combined washing. Section 4 describes an industrial case study of an air separation plant. Section 5 demonstrates and discusses the results of the paper and Section 6 presents the conclusions.

3. Methodology

The paper focuses on the optimal operation of an air separation plant which involves an air compressor station. Figure 3 shows a simplified description of the plant. The compressor station comprises of multiple compressors with dissimilar characteristics and they operate in parallel. The air compressors can connect with different headers. Moreover, the headers are connected with downstream processes, for example air separation columns, and process plants which require compressed air for utilities. The air separation columns separate the compressed air into nitrogen, N_2 , and oxygen, O_2 , which are the main products of the plant. These products can be stored in buffer tanks. At the exit of the plant both O_2 and N_2 are provided to internal or external customers, an internal customer may be a process plant on the same site which uses oxygen for its own processes.

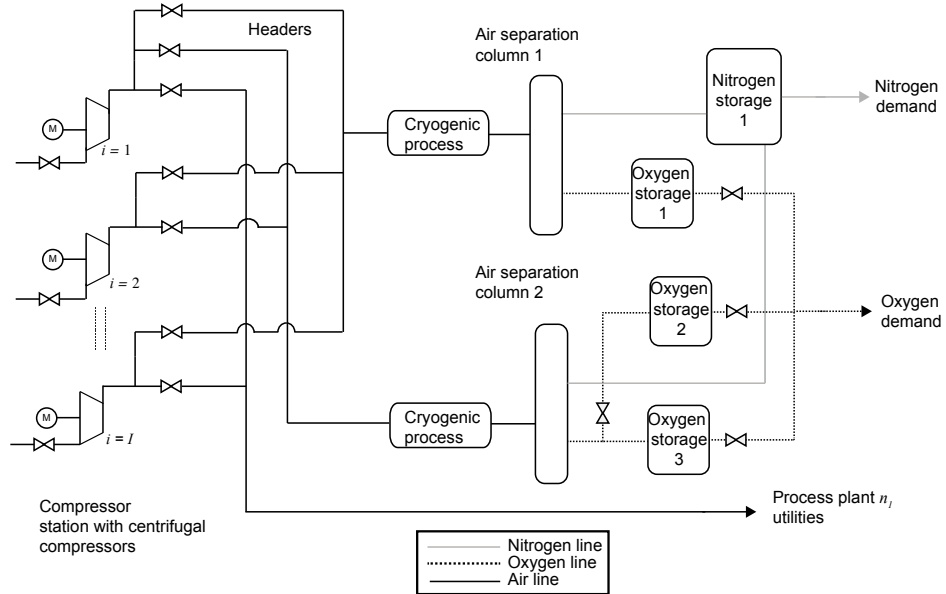


Figure 3: An air separation plant with an air compressor station.

The proposed optimization framework optimizes the operation and at the same time considers a condition-based maintenance model. The proposed mathematical model is based on the operational model of networks of compressors developed by Kopanos et al. (2015). The equations of the

operational model are summarized in Tables 6, 7 and 8 which can be found in the Appendix.

The optimization model of the current framework includes a set of compressors $i \in I$, a set of headers $j \in J$, a set of air separation columns $u \in U$, a number of tanks $z \in Z$ and set of gas products $e \in E$. The optimization considers a finite time horizon which is uniformly discretized in time periods $t \in T$. The main binary variables used in the optimization problem are presented below:

- $X_{(i,t)} = 1$, if a compressor i is operating during time period t ,
- $S_{(i,t)} = 1$, if a compressor i starts up at the beginning of t ,
- $F_{(i,t)} = 1$, if a compressor i shuts down at the beginning of t ,
- $Y_{(i,j,t)} = 1$, if a compressor i is serving header j at the beginning of t ,
- $D_{(i,t)} = 1$, if a compressor i changes header at the beginning of t ,
- $W_{of(i,t)} = 1$, if a compressor i starts offline washing at t ,
- $U_{of(i,t)} = 1$, if a compressor i is under offline washing during t ,
- $K_{of(i,t)} = 1$, if the offline washing of a compressor i has already finished at the beginning of t ,
- $U_{on(i,t)} = 1$, if online washing of a compressor i occurs in time period t .

Sections 3.1 – 3.5 provide the extension of the operational model of the compressors implementing the models of the offline, online and combined washing maintenance. The combined washing strategy considers the option of selecting online or offline washing. The resulting integrated framework determines optimal operation and provides the best decisions of the maintenance tasks.

3.1. Offline washing maintenance

Figure 4 illustrates an example of an episode of offline maintenance, i.e. offline washing. The offline washing starts at time period $t3$ and it lasts until $t5$. Therefore, the compressor is available from period $t6$ and after. In this example the compressor remains off for periods $t6$ and $t7$ and starts up in period $t8$.

Equations (1) – (4) connect the basic variables of an Offline Washing Episode (OFWE). The duration of an OFWE is described by parameter $\nu_{(i)}$ and it varies according to the type of the compressor and the supplementary maintenance actions that are related to the washing maintenance (e.g. mechanical inspection). When an offline washing episode starts, i.e. $W_{of(i,t)} = 1$,

174 then the compressor must stay offline during this period t , i.e. $X_{(i,t)} = 0$.
 175 This is modeled through Eq. (5) which also satisfies the case when a com-
 176 pressor has to be switched off, i.e. $X_{(i,t)} = 0$, for reasons other than washing,
 177 therefore binary variable $W_{of(i,t)}$ can take the value zero.

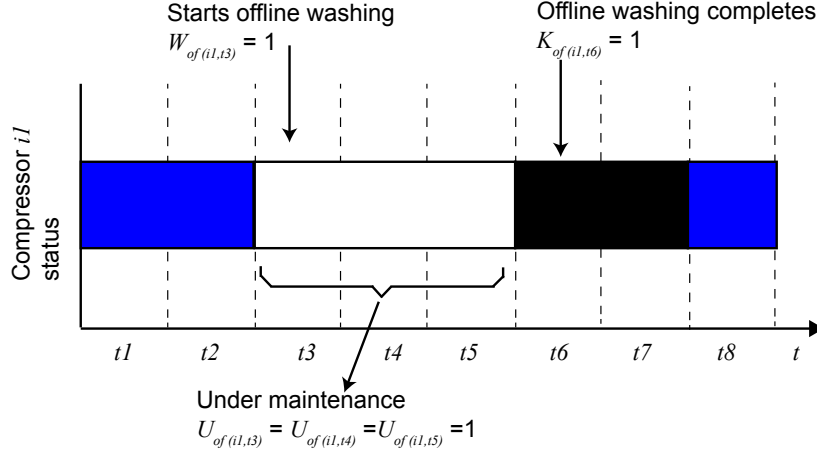


Figure 4: Offline washing episode and binary variables explanation.

178 The optimization model considers offline washing episodes (OFWEs) that
 179 occurred before the start of the optimization time window. This information
 180 is part of the input of the model and describes the initial state of the system.
 181 The significance of proper consideration of the initial state of the system has
 182 been highlighted in the work by Kopanos et al. (2014). The parameter $d_{of(i)}^o$
 183 gives the duration of a compressor which has already been in maintenance
 184 before the beginning of the time horizon of the optimization. If $d_{of(i)}^o > 0$
 185 then the compressor has to continue being maintained up to the time period
 186 $(\nu_{(i)} - d_{of(i)}^o)$. Equation (6) describes this constraint.

$$W_{of(i,t)} - K_{of(i,t)} = U_{of(i,t)} - U_{of(i,t-1)},$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t > 1) \quad (1)$$

$$W_{of(i,t)} - K_{of(i,t)} = U_{of(i,t)},$$

$$\forall i \in I, d_{of(i)}^o = 0, t = 1 \quad (2)$$

$$W_{of(i,t)} + K_{of(i,t)} \leq 1,$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t \in T) \quad (3)$$

$$1 - U_{of(i,t)} + \sum_{t'=\max\{1, t-\nu_{(i)}+1\}}^t W_{of(i,t')} = 1,$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t \in T) \quad (4)$$

$$X_{(i,t)} \leq 1 - U_{of(i,t)},$$

$$\forall i \in I, t \in T \quad (5)$$

$$U_{of(i,t)} = 1,$$

$$\forall i \in I, d_{of(i)}^0 > 0, 1 \leq t \leq (\nu_{(i)} - d_{of(i)}^o) \quad (6)$$

Equation (7) models the maximum number λ_{off} of maintenance episodes which can take place in one time period. The simultaneous maintenance episodes in one time period (e.g. in one day) may be restricted due to industrial policies.

$$\sum_{i \in I} U_{of(i,t)} \leq \lambda_{off}, \quad \forall t \in T \quad (7)$$

187 The constraints described by Eqs. (8) – (12) provide the models of the
 188 degradation rate and recovery of the extra power consumption after an of-
 189 fline washing episode. The main assumptions of the degradation and power
 190 recovery model are summarized below:

191

192 1) The degradation rate of a compressor depends on the type of the com-
 193 pressor and on the cumulative time of operation after the last maintenance.
 194 A linear function between extra power consumption due to degradation ΔW
 195 and cumulative operational time ΔS is employed: $\Delta W_{(i,t)} = \epsilon_{(i)} \Delta S_{(i,t)}$, where

196 $\epsilon_{(i)}$ is the degradation rate of each compressor.

197

198 2) It is assumed that the effects from the surroundings do not change abruptly
199 during a short time period. For instance, the construction of a new plant close
200 to the compressor station might increase the dust intake in the compressors
201 and the fouling would increase. The linear correlation between extra power
202 consumption and cumulative time period of operation would not hold true.
203 An advanced performance monitoring diagnostic tool could deal with this
204 problem in a rolling horizon scheduling approach similar to that of Kopanos
205 et al. (2015).

206

207 3) When a compressor is switched off and remains offline without mainte-
208 nance the condition of the compressor stays as it was before the shut down.
209 In reality, corrosion can continue between the fouling deposits and the blades,
210 also during stand-by of the machine according to Meher-Homji et al. (2001).

211

212 4) The additional power consumed due to degradation is assumed constant
213 during the period t , and it does not depend on the operating point.

214

215 5) When a compressor is washed offline it is assumed that the efficiency is
216 fully recovered, in other words the extra power consumption due to degrada-
217 tion becomes zero immediately after the washing, see Fig. 2. According to
218 Diakunchak (1992) a residual decrement of 1% of the degradation might still
219 be observed after offline washing, however for the purposes of the illustration
220 of the framework this residual is not modeled.

221

222 The assumption of a linear correlation between extra power consumption
223 due to the accumulation of fouling and time holds true up to a particular
224 amount of fouling (Li et al., 2009). The current paper employs values of the
225 degradation rate $\epsilon_{(i)}$ derived from the analysis of Ciccotti et al. (2014) on
226 the same air separation case study.

227

228 Although there is not a well defined model of the prediction of the ac-
229 cumulation of fouling at the moment, it is possible that parameters apart
230 from the cumulative time increase the accumulation of fouling. Sánchez et
231 al. (2009) presented a list of factors which influence the build up of fouling,
232 however the authors in the same work expressed the fouling accumulation
233 as a function of time. Fouling, erosion and corrosion are the main causes of
increased roughness of the surfaces (e.g. blades surfaces) of the compressors.

234 This increase in roughness is the major factor of reduced performance. The
 235 rate at which the performance deteriorates is influenced by a large number
 236 of factors, this fact makes the prediction of the degradation rate impossible
 237 without infield observations of past behaviour of the compressor.

238 Hence, the main objective of this paper is to show the practical use of
 239 the framework and its potential use when advanced performance monitoring
 240 methods and explicit models of the accumulation of the fouling are available
 241 in the future, therefore the assumption that the extra power consumption
 242 due to degradation depends on the cumulative time of operation is sufficient
 243 for the current study.

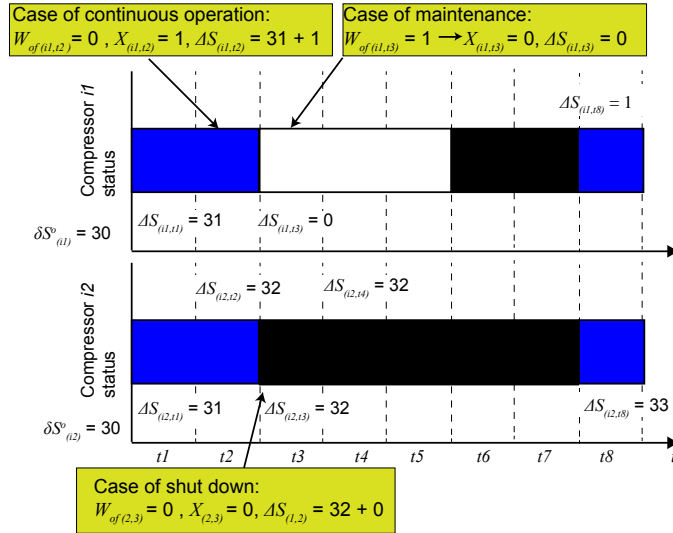


Figure 5: Three different case scenarios of the status of a compressor: continuous operation, maintenance and shut down.

244 As the time is discretized uniformly in time periods $t \in T$, a continuous
 245 variable $\Delta S_{(i,t)}$ is used to enumerate the cumulative time of operation at
 246 each period t . Figure 5 explains graphically the function of this variable.
 247 Equations (8) and (9) give the mathematical description of the $\Delta S_{(i,t)}$. When
 248 a compressor is already being maintained, i.e. $d_{of(i)}^o > 0$, then Eq. (10) sets
 249 $\Delta S_{(i,t)}$ equal to zero for the remaining time period $(\nu_{(i)} - d_{of(i)}^o)$. The $\delta S_{(i)}^o$
 250 gives the duration of a compressor which has been operated before the start
 251 of the optimization. This parameter is part of the input of the initial state
 252 of the problem.

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - W_{of(i,t)}),$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t > 1) \quad (8)$$

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)})(1 - W_{of(i,t)}),$$

$$\forall i \in I, d_{of(i)}^o = 0, t = 1 \quad (9)$$

$$\Delta S_{(i,t)} = 0,$$

$$\forall i \in I, d_{of(i)}^o > 0, 1 < t \leq (\nu_{(i)} - d_{of(i)}^o) \quad (10)$$

$$\Delta W_{(i,t)} = \epsilon_{(i)} \Delta S_{(i,t)} X_{(i,t)},$$

$$\forall i \in I, t \in T \quad (11)$$

$$\Delta W_{(i,t)} \leq \Omega_{(i)},$$

$$\forall i \in I, t \in T \quad (12)$$

Equation (11) gives the expression of the extra power a compressor consumes because of the degradation. The degradation rate $\epsilon_{(i)}$ depends on the size of the compressor. The parameter $\Omega_{(i)}$, in Eq. (12), is a maximum boundary restricting the extra power consumption due to degradation. This parameter is derived from the relationship of the extra power and cumulative days, $\Omega_{(i)} = \epsilon_{(i)} o_{(i)}$, where $o_{(i)}$ is the maximum running time.

Equations (8) and (9) involve bilinear terms some comprising multiplication of binary with binary variables and others the multiplication of binary with continuous variables. The following set of equations, which use the principles of the Big-M formulation (Vecchietti et al., 2003), convert these nonlinear constraints into linear. The reason is to relax the optimization problem and avoid to solve a hard mixed integer non-linear programming optimization model.

$$\Delta S_{(i,t)} \leq o_{(i)}(1 - W_{of(i,t)}),$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t \in T) \quad (13)$$

$$\Delta S_{(i,t)} = 0,$$

$$\forall i \in I, d_{of(i)}^o > 0, 1 \leq t \leq (\nu_{(i)} - d_{of(i)}^o) \quad (14)$$

$$\Delta S_{(i,t)} \leq \delta S_i^o + X_{(i,t)} + o_{(i)}W_{of(i,t)}, \quad \forall i \in I, d_{of(i)}^o = 0, t = 1 \quad (15)$$

$$\delta S_i^o + X_{(i,t)} - o_{(i)}W_{of(i,t)} \leq \Delta S_{(i,t)}, \quad \forall i \in I, d_{of(i)}^o = 0, t = 1 \quad (16)$$

$$\Delta S_{(i,t)} \leq \Delta S_{(i,t-1)} + X_{(i,t)} + o_{(i)}W_{of(i,t)},$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t > 1) \quad (17)$$

$$\Delta S_{(i,t-1)} + X_{(i,t)} - o_{(i)}W_{of(i,t)} \leq \Delta S_{(i,t)},$$

$$\forall i \in I, (d_{of(i)}^o > 0, (\nu_{(i)} - d_{of(i)}^o) < t \leq T) \vee (d_{of(i)}^o = 0, t > 1) \quad (18)$$

Moreover, Eq. (11) involves the multiplication of the continuous variable $\Delta S_{(i,t)}$ with the binary variable $X_{(i,t)}$. The reason for this multiplication is that the extra power due to degradation, $\Delta W_{(i,t)}$, at a period t should not be added into the objective function when a compressor i is offline but not washed. On the other hand, the past time period a compressor has been operating has to be registered through $\Delta S_{(i,t)}$ for each period t . This leads to a non-linear equation, where the following formulation can convert the constraints into linear constraints:

$$\Delta W_{(i,t)} \leq \Omega_{(i)}X_{(i,t)}, \quad \forall i \in I, t \in T \quad (19)$$

$$\Delta W_{(i,t)} \leq \epsilon_{(i)}\Delta S_{(i,t)} + \Omega_{(i)}(1 - X_{(i,t)}), \quad \forall i \in I, t \in T \quad (20)$$

$$\epsilon_{(i)}\Delta S_{(i,t)} - \Omega_{(i)}(1 - X_{(i,t)}) \leq \Delta W_{(i,t)}, \quad \forall i \in I, t \in T \quad (21)$$

266 As discussed in the assumptions, if the model considered the assumption
 267 of Meher-Homji et al. (2001) that a compressor continues being fouled while
 268 it is offline, then Eqs. (8) and (9) should exclude the $X_{(i,t)}$ in the first
 269 parenthesis. This would lead to an easier mathematical problem to solve.
 270 The current optimization problem considers the case that a compressor is
 271 not fouled during offline mode.

272 3.2. Online washing

The injection of a cleaning solution into the compressor can have unwanted side effects such as corrosion of the blades if done too frequently. For this reason, it is assumed a minimum time period $\gamma_{(i)}$ of each compressor i between two consequent Online Washing Episodes (ONWEs):

$$U_{on(i,t)} = 0, \quad \forall i \in I, d_{on(i)}^o < \gamma_{(i)}, 1 \leq t \leq (\gamma_{(i)} - d_{on(i)}^o) \quad (22)$$

$$\sum_{t'=\max\{t-\gamma_{(i)}+1,1\}}^t U_{on(i,t')} \leq 1, \quad \forall i \in I, (d_{on(i)}^o \geq \gamma_{(i)}, t \in T) \vee (d_{on(i)}^o < \gamma_{(i)}, (\gamma_{(i)} - d_{on(i)}^o) < t \leq T) \quad (23)$$

273 Equation (22) is valid when a compressor has been washed online for time
 274 period $d_{on(i)}^o$ before the start of the optimization and the $d_{on(i)}^o$ is smaller than
 275 the $\gamma_{(i)}$. For any other case, Eq. (23) holds true.

Equation (24) describes the fact that there is also a maximum λ_{on} ONWEs of different compressors which can take place in each time period of the time horizon. The reason for considering this constraint is that it is possible the online washing infrastructure could not support simultaneous online washing as described by Boyce et al. (2007). In any other case, this constraint can be omitted.

$$\sum_{i \in I} U_{on(i,t)} \leq \lambda_{on}, \quad \forall t \in T \quad (24)$$

An online washing episode cannot take place when the compressor is offline. Equation (25) describes that if a compressor i is online at period t , i.e. $X_{(i,t)} = 1$, then it has the option to be washed, i.e. $U_{on(i,t)} = 1$ or it can be decided to operate without being washed, i.e. $U_{on(i,t)} = 0$. However, if the compressor is offline, i.e. $X_{(i,t)} = 0$, variable $U_{on(i,t)}$ has to be equal to zero in this time period.

$$U_{on(i,t)} \leq X_{(i,t)}, \quad \forall i \in I, t \in T \quad (25)$$

The recovery model for online washing, which is described in Eqs. (26) and (27), assumes that $\Delta S_{(i,t)}$ is reduced by a recovery factor R . For instance, if the recovery factor is $R = 0.2$ and the operating period is $\Delta S_{(i,t)} = 100$ d, the final operating period after an online washing would be $\Delta S_{(i,t)} \cdot (1 - R) =$

80 d. Then the $\Delta W_{(i,t)}$ is estimated based on the duration of this period. The variable $\Delta S_{(i,t)}$ represents the Equivalent Operating Time (EOT) presented by de Backer (2000) and Bohrenkamper et al. (2000). The online washing model completes with the consideration of the degradation model, which is the same one considered in the case of the offline washing in the previous Section 3.1 and it is defined by Eqs. (11) and (12).

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - R \cdot U_{on(i,t)}), \quad \forall i \in I, t > 1 \quad (26)$$

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)})(1 - R \cdot U_{on(i,t)}), \quad \forall i \in I, t = 1 \quad (27)$$

The formulation of the constraints in Eqs. (26) and (27) shows that the multiplication of binary and continuous variables is more complicated compared to the corresponding case of the offline washing. To overcome this complexity, the linearization of these constraints employ an auxiliary variable $\Delta S_{aux(i,t)}$. The linearized constraints of Eqs. (26) and (27) can be seen below:

$$\Delta S_{(i,t)} = (\delta S_i^o + X_{(i,t)}) - \Delta S_{aux(i,t)}, \quad \forall i \in I, t = 1 \quad (28)$$

$$\Delta S_{(i,t)} = (\Delta S_{(i,t-1)} + X_{(i,t)}) - \Delta S_{aux(i,t)}, \quad \forall i \in I, t > 1 \quad (29)$$

$$\begin{aligned} \Delta S_{aux(i,t)} &= 0, \\ \forall i \in I, d_{on(i)}^o &< \gamma_{(i)}, 1 \leq t \leq \gamma_{(i)} - d_{on(i)}^o \end{aligned} \quad (30)$$

$$\begin{aligned} \Delta S_{aux(i,t)} &\leq o_{(i)} U_{on(i,t)}, \\ \forall i \in I, (d_{on(i)}^o &\geq \gamma_{(i)}, t \in T) \vee (d_{on(i)}^o < \gamma_{(i)}, t > (\gamma_{(i)} - d_{on(i)}^o)) \end{aligned} \quad (31)$$

$$\begin{aligned} \Delta S_{aux(i,t)} &\leq (\Delta S_{(i,t-1)} + X_{(i,t)})R + (1 - U_{on(i,t)})o_{(i)}, \\ \forall i \in I, (d_{on(i)}^o &\geq \gamma_{(i)}, t > 1) \vee (d_{on(i)}^o < \gamma_{(i)}, t > \gamma_{(i)} - d_{on(i)}^o) \end{aligned} \quad (32)$$

$$\begin{aligned} (\Delta S_{(i,t-1)} + X_{(i,t)})R - (1 - U_{on(i,t)})o_{(i)} &\leq \Delta S_{aux(i,t)}, \\ \forall i \in I, (d_{on(i)}^o &\geq \gamma_{(i)}, t > 1) \vee (d_{on(i)}^o < \gamma_{(i)}, t > \gamma_{(i)} - d_{on(i)}^o) \end{aligned} \quad (33)$$

$$\begin{aligned} \Delta S_{aux(i,t)} &\leq (\delta S_i^o + X_{(i,t)})R + (1 - U_{on(i,t)})o_{(i)}, \\ \forall i \in I, d_{on(i)}^o &\geq \gamma_{(i)}, t = 1 \end{aligned} \quad (34)$$

$$\begin{aligned} (\delta S_i^o + X_{(i,t)})R - (1 - U_{on(i,t)})o_{(i)} &\leq \Delta S_{aux(i,t)}, \\ \forall i \in I, d_{on(i)}^o &\geq \gamma_{(i)}, t = 1 \end{aligned} \quad (35)$$

276 The degradation model used in the online washing considers the lineariza-
277 tion of the constraint of Eq. (11) which was described earlier when discussing

278 Eqs. (19) – (21) exactly as in the case of the offline washing degradation
279 model.

280 3.3. Combined online and offline washing

281 Section 3.3 describes the scenario when offline and online washings are
282 available to clean the compressors. In this case, the optimization model
283 employs all the binary variables $W_{of(i,t)}$, $U_{of(i,t)}$, $K_{of(i,t)}$ and $U_{on(i,t)}$. The
284 optimization model includes the constraints in Eqs. (1) – (7) from the offline
285 washing formulation in Section 3.1 and the constraints in Eqs. (22) – (25)
286 from the online washing model in Section 3.2.

The degradation and recovery model is a combination of the two different washing models. This combined model is described by the constraints in the general form in Eqs. (11) – (12) and the linearized form in Eqs. (19) – (21). The recovery model in the combined online and offline washing scenario is given by Eqs. (36) and (37).

$$\begin{aligned} \Delta S_{(i,t)} &= (\Delta S_{(i,t-1)} + X_{(i,t)})(1 - W_{of(i,t)})(1 - R \cdot U_{on(i,t)}), \\ \forall i \in I, t > 1 \end{aligned} \quad (36)$$

$$\begin{aligned} \Delta S_{(i,t)} &= (\delta S_i^o + X_{(i,t)})(1 - W_{of(i,t)})(1 - R \cdot U_{on(i,t)}), \\ \forall i \in I, t = 1 \end{aligned} \quad (37)$$

Equations (36) and (37) can be linearized with the use of an auxiliary variable $\Delta S_{on,aux(i,t)}$ as follows:

$$\begin{aligned} \Delta S_{(i,t)} &\leq o_{(i)}(1 - W_{of(i,t)}), \\ \forall i \in I, (d_{of(i)}^o = 0, t \in T) \vee (d_{of(i)}^o > 0, t > (\nu_{(i)} - d_{of(i)}^o)) \end{aligned} \quad (38)$$

$$\begin{aligned} \Delta S_{(i,t)} &= 0, \\ \forall i \in I, d_{of(i)}^o > 0, 1 \leq t \leq (\nu_{(i)} - d_{of(i)}^o) \end{aligned} \quad (39)$$

$$\begin{aligned} \Delta S_{(i,t)} &\leq \Delta S_{on,aux(i,t)} + o_{(i)}W_{of(i,t)}, \\ \forall i \in I, (d_{of(i)}^o = 0, t > 1) \vee (d_{of(i)}^o > 0, t > (\nu_{(i)} - d_{of(i)}^o)) \end{aligned} \quad (40)$$

$$\begin{aligned} \Delta S_{on,aux(i,t)} - o_{(i)}W_{of(i,t)} &\leq \Delta S_{(i,t)}, \\ \forall i \in I, (d_{of(i)}^o = 0, t \in T) \vee (d_{of(i)}^o > 0, t > (\nu_{(i)} - d_{of(i)}^o)) \end{aligned} \quad (41)$$

$$\begin{aligned}\Delta S_{on,aux(i,t)} &= (\Delta S_{(i,t-1)} + X_{(i,t)}) - \Delta S_{aux(i,t)}, \\ \forall i \in I, (d_{of(i)}^o = 0, t > 1) \vee (d_{of(i)}^o > 0, t > (\nu_{(i)} - d_{of(i)}^o))\end{aligned}\quad (42)$$

$$\begin{aligned}\Delta S_{on,aux(i,t)} &= (\delta S_i^o + X_{(i,t)}) - \Delta S_{aux(i,t)}, \\ \forall i \in I, d_{of(i)}^o = 0, t = 1\end{aligned}\quad (43)$$

$$\begin{aligned}\Delta S_{aux(i,t)} &\leq o_{(i)} U_{on(i,t)}, \\ \forall i \in I, (d_{on(i)}^o \geq \gamma_{(i)}, t \in T) \vee (d_{on(i)}^o < \gamma_{(i)}, t > \gamma_{(i)} - d_{on(i)}^o)\end{aligned}\quad (44)$$

$$\begin{aligned}\Delta S_{aux(i,t)} &= 0, \\ \forall i \in I, d_{on(i)}^o < \gamma_{(i)}, 1 \leq t \leq \gamma_{(i)} - d_{on(i)}^o\end{aligned}\quad (45)$$

$$\begin{aligned}\Delta S_{aux(i,t)} &\leq (\Delta S_{(i,t-1)} + X_{(i,t)})R + (1 - U_{on(i,t)})o_{(i)}, \\ \forall i \in I, (d_{on(i)}^o \geq \gamma_{(i)}, t > 1) \vee (d_{on(i)}^o < \gamma_{(i)}, t > \gamma_{(i)} - d_{on(i)}^o)\end{aligned}\quad (46)$$

$$\begin{aligned}(\Delta S_{(i,t-1)} + X_{(i,t)})R - (1 - U_{on(i,t)})o_{(i)} &\leq \Delta S_{aux(i,t)}, \\ \forall i \in I, (d_{on(i)}^o \geq \gamma_{(i)}, t > 1) \vee (d_{on(i)}^o < \gamma_{(i)}, t > \gamma_{(i)} - d_{on(i)}^o)\end{aligned}\quad (47)$$

$$\begin{aligned}\Delta S_{aux(i,t)} &\leq (\Delta S_{(i,t-1)} + X_{(i,t)})R + (1 - U_{on(i,t)})o_{(i)}, \\ \forall i \in I, d_{on(i)}^o \geq \gamma_{(i)}, t = 1\end{aligned}\quad (48)$$

$$\begin{aligned}(\delta S_i^o + X_{(i,t)})R - (1 - U_{on(i,t)})o_{(i)} &\leq \Delta S_{aux(i,t)}, \\ \forall i \in I, d_{on(i)}^o \geq \gamma_{(i)}, t = 1\end{aligned}\quad (49)$$

287 3.4. Objective function

288 The objective function is given by Eq. (50). The form of the objective
 289 function is similar to that of the Unit Commitment optimization problems,
 290 e.g. Marcovecchio et al. (2014). The objective function in that case includes
 291 the operating costs, and start up and shut down costs. The current article
 292 considers costs of operation, start up and shutdown, header changes, and
 293 maintenance.

294 The first term represents the total electricity cost. The $W_{el(i,t)}$ gives the
 295 power used by compressor i in time period t . This power is equal to the
 296 summation of: (a) the power consumed when the compressor is clean, $W_{el(i,t)}^c$,
 297 which depends on the operating conditions for example mass flow rate and
 298 pressure, and (b) the extra power consumption $\Delta W_{(i,t)}$ due to degradation.

299 The extra power consumption depends on the cumulative time periods of
 300 operation. The parameter d_c is the duration of the time period of the finite
 301 time horizon and $C_{el(t)}$ is the electricity price in [m.u./kWh], where m.u. is
 302 the monetary units. The equation of the power consumption $W_{el(i,t)}^c$ is given
 303 by the following equation:

$$W_{el(i,t)}^c = \sum_{j \in J(i)} (\delta_{(1,i)} Y_{(i,j,t)} + \delta_{(2,i)} M_{(i,j,t)} + \delta_{(3,i)} P_{(i,j,t)}), \quad \forall i \in I, t \in T$$

304 where parameters $\delta_{(1..3,i)}$ correspond to normalized coefficients of the power
 305 consumption of every compressor i . These parameters mainly depend on
 306 information related to cooling water consumption and ambient conditions of
 307 each compressor. The estimation of the parameters and the validity region
 308 of this data-driven expression has been discussed in Kopanos et al. (2015).
 309 The power consumption could be modeled as a quadratic function to reduce
 310 errors, but at the expense of increased computational time. However, for the
 311 purposes of demonstration of the framework in the current study the use of
 312 the linear approximation is acceptable.

The $C_{st(i)}$, $C_{f(i)}$ are the costs related to start up and shut down of compressor i respectively. The units of the costs are in [m.u./event], where the event corresponds either to a start up or a shut down. The $C_{of(i)}$ and $C_{on(i)}$ are the costs of the offline and online washings, the units are in [m.u./wash]. The C_D is the cost when a compressor changes header, in units [m.u./change]. The $C_{O(e,t)}$ is the cost to purchase product $e = \{O_2, N_2\}$ at period t , [m.u./kg of product e] from external sources in the case that the demand cannot be satisfied from the air separation plant.

$$\begin{aligned} \min \quad & \sum_{t \in T} C_{el(t)} \sum_{i \in I} d_c \cdot W_{el(i,t)} + \sum_{t \in T} \sum_{i \in I} (C_{st(i)} S_{(i,t)} + C_{f(i)} F_{(i,t)} \\ & + C_{of(i)} W_{of(i,t)} + C_{on(i)} U_{on(i,t)}) + \sum_{t \in T} \sum_{i \in I} C_D D_{(i,t)} \\ & + \sum_{t \in T} \sum_{e \in E} C_{O(e,t)} O_{(e,t)} \end{aligned} \quad (50)$$

313 Equation (50) presents the complete objective function when online and
 314 offline washings are available. When the analysis of a compressor station
 315 involves only one of the two washings then the objective function considers
 316 the respective terms of costs. This objective function is an extension of the
 317 objective function used in Xenos et al. (2014) and Kopanos et al. (2015).

318 3.5. Terminal constraints

319 The optimization determines a solution which minimizes the total cost
 320 of the operation of the plant. The operation is expected to run close to its
 321 boundaries. For instance, the optimal operation for one month may suggest
 322 running the compressors for as long as possible without cleaning them to
 323 avoid shut down and washing costs. However, the current optimization does
 324 not consider the requirements for a following scheduling problem and it is
 325 possible to have an infeasible solution or a non-optimal solution compared
 326 to the case where two consequent scheduling problems have been solved si-
 327 multaneously. The reason for using a relatively short time horizon of one
 328 month is to reduce the uncertainty in the forecast of parameters, such as the
 329 production targets.

For this reason, the optimization has to consider boundary conditions at the end of the time horizon. These boundary conditions can be called terminal constraints. These constraints take into account the requirements of a following scheduling problem. Equation (51) guarantees that the contents of the tanks $z \in Z$ are full at the end of the time horizon, $t = T$.

$$B_{(e,z,t)} = \xi_{(e,z)}^{max}, \quad \forall e \in E, z \in Z, t = T \quad (51)$$

Moreover, Eqs. (52) and (53) give the constraints regarding the offline washing of compressors. They describe that a predefined number λ_k of Offline Washing Episodes (OFWEs) have to take place within the time period $[T - t_S^*, T]$, where $t_S^* = t_S + \max\{\nu_{(i)}\}$ and t_S is a number of days before the end of the time horizon.

$$\sum_{t'=T-t_S^*}^T \sum_{i \in I} K_{of(t',i)} = \lambda_k \quad (52)$$

$$\sum_{t'=T-t_S^*}^T K_{of(t',i)} \leq 1, \quad \forall i \in I^{NOFF} \quad (53)$$

330 The set $I^{NOFF} \subseteq I$ involves all the compressors which have not been
 331 under offline maintenance before the start of the optimization. Equation
 332 (53) guarantees that a compressor will be maintained only once in the time
 333 window $[T - t_S^*, T]$.

334 The terminal constraints have to be investigated using heuristics from
 335 historical operation and literature. The decisions of these constraints are
 336 strongly related with the particular configuration of the compressors.

337 4. Description of case study

Table 1: Specifications of compressors i related to constraints (59) and (60).

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$	$i7$	$i8$	$i9$	$i10$	$i11$
$\rho_{(i)}^{min}$	41.6	34.0	38.0	35.5	34.9	58.2	47.5	48.0	55.0	48.8	53.6
$\rho_{(i)}^{max}$	58.5	55.7	55.2	55.8	56.5	88.4	87.6	83.7	83.4	87.0	87.7
$\pi_{(i)}^{min}$	52.6	44.2	49.5	50.3	48.4	53.7	46.3	50.1	52.1	45.7	47.4
$\pi_{(i)}^{max}$	68.9	64.8	70.0	59.8	62.6	64.9	69.3	69.3	66.5	69.2	69.8

338 Section 4 considers the industrial air separation plant of BASF in Lud-
 339 wigshafen, Germany. This plant encompasses eleven multi-stage centrifugal
 340 compressors working in parallel. The compressors supply three headers with
 341 compressed air. There are five small compressors, $i \in I_s = \{i1, i2, i3, i4, i5\}$,
 342 driven by electrical motors with nominal power of some MW, $W_{el,s}^{No}$. More-
 343 over, there are six large compressors, $i \in I_l = \{i6, i7, i8, i9, i10, i11\}$ with
 344 power of more than ten MW each, $W_{el,l}^{No}$, and $W_{el,l}^{No} = 2W_{el,s}^{No}$. The actual
 345 nominal power of the motors cannot be presented due to confidentiality re-
 346 strictions. Table 1 gives the feasible region, i.e. boundaries of mass flows and
 347 pressures, of the compressors derived from data analysis of past operation.

348 The description of the plant has been described in Section 3 in which Fig.
 349 3 gives the schematic of the plant. The first header $j1$ collects the compressed
 350 air for utilities in the industrial complex of BASF. The other two headers $j2$
 351 and $j3$ are connected with two air separation columns $u1$ and $u2$ respectively.

352 The optimization model uses a finite time horizon of thirty days and
 353 uniform time periods of one day each. Table 2 provides the main parameters
 354 of the optimization model. The minimum running time $\omega_{(i)}$ is nine days
 355 for the large compressors and for the small compressors are five, six, seven,
 356 six and five days for $i1$, $i2$, $i3$, $i4$ and $i5$ respectively. The values of the
 357 maximum running times and the rest of the parameters such as $\nu_{(i)}$, $\gamma_{(i)}$ and
 358 λ_{on} are selected to reflect typical industrial practices. The value of the R
 359 has been chosen based on a reasonable average recovery factor according to
 360 observations in literature.

361 A typical duration for offline washing with other minor maintenance ac-
 362 tions, such as inspections and corrective repairing, is three days. This value
 363 has been used for all the compressors apart from $i1$ and $i5$ for which the time
 364 periods of the maintenance actions account for two and five days respectively.

Table 2: Main parameters.

Symbol	Units	Values	Comment
t	d	1	time period
T	d	30	total periods of time horizon
$\omega_{(i)}$	d	5-7	minimum run time of small compressors
$\omega_{(i)}$	d	9	minimum run time of large compressors
$o_{(i)}$	d	65	maximum run time of small compressors
$o_{(i)}$	d	75	maximum run time of large compressors
$\nu_{(i)}$	d	2-5	duration of OFWEs
$\gamma_{(i)}$	d	10	minimum gap between two ONWEs
R	-	0.2	recovery factor after an ONWE
λ_{on}	-	1	maximum ONWEs in a time period
λ_{off}	-	3	maximum OFWEs in a time period
λ_k	-	2	parameter of Eq. (52)
t_s	d	10	parameter related to Eqs. (52) and (53)

365 The ratio between extra power consumption and nominal power for one
 366 month, $\Delta W/W_{el}^{No}$ is 4.5% and 3.3% of the small and large compressors re-
 367 spectively. These values are based on the work from Ciccioiti et al. (2014)
 368 which used a detailed first principles model of an industrial compressor to
 369 estimate the profile of the degradation over time. The degradation rates are
 370 calculated based on monitoring parameters (for example performance) of the
 371 compressor using a linear approximation.

Table 3: Cost parameters of operation and maintenance.

Costs	Units	Values	Comment
$C_{st(i)}$	m.u./start up	44.55	start up cost of small compressors
$C_{st(i)}$	m.u./start up	89.10	start up cost of large compressors
$C_{of(i)}$	m.u./wash	4.32	offline wash. cost of small compressors
$C_{of(i)}$	m.u./wash	8.60	offline wash. cost of large compressors
$C_{on(i)}$	m.u./wash	0.86	online wash. cost of small compressors
$C_{on(i)}$	m.u./wash	1.72	online wash. cost of large compressors
C_D	m.u./change	2.27	cost for changing header

372 Table 3 provides the values of all the major parameters of the costs of
 373 the objective function of Eq. (50). The start up costs for small and large

374 compressors were assumed equal to the energy consumed from the motors of
375 the compressors for half a day. The current work assumed that there is not
376 a major energy consumption for switching off a compressor, therefore this
377 cost is zero. The washing costs are estimated using the assumptions from
378 Aretakis et al. (2012) and Fabbri et al. (2011). Therefore, Table 3 gives the
379 values of the costs of a washing episode in this particular case study of the
380 compressors of BASF. The cost for changing header has been explained by
381 Kopanos et al. (2015).

Table 4: Initial state of the system.

Compressor	<i>i1</i>	<i>i2</i>	<i>i3</i>	<i>i4</i>	<i>i5</i>	<i>i6</i>	<i>i7</i>	<i>i8</i>	<i>i9</i>	<i>i10</i>	<i>i11</i>
Header	-	<i>j1</i>	-	-	<i>j2</i>	<i>j2</i>	-	<i>j1</i>	-	<i>j3</i>	<i>j3</i>
$\tilde{\omega}_{(i)}$	0	6	0	0	40	22	0	20	0	40	55
$\tilde{\psi}_{(i)}$	30	0	18	20	0	0	30	0	30	0	0
$\delta S_{(i)}^o$	20	6	50	0	40	22	20	20	0	40	55
$d_{of(i)}^o$	1	0	0	3	0	0	0	0	1	0	0
$d_{on(i)}^o$	-	2	-	-	-	-	-	-	-	1	5

382 A base case study is formulated to examine the optimal operation and
383 maintenance plan of compressors with different types of washings. This base
384 case uses the information from Tables 1, 2 and 3. Moreover, the information
385 of the initial state of the system is provided by Table 4. The base case employs
386 modified production targets for O₂, N₂ and compressed air for utilities from
387 real industrial data. Figure 6 gives the production targets for O₂ and N₂ in
388 kg of compressed air (scaled units).

389 5. Results and discussions

390 5.1. Base case study

391 All given data and reported results are normalized and made dimension-
392 less due to confidentiality reasons. All optimization problems have been
393 solved in GAMS/CPLEX 11.1, under default configurations, in an Intel(R)
394 Core(TM) i7-2600 CPU @3.4 GHz with 8 GB RAM. A zero optimality gap
395 has been imposed in all problems instances.

396 The base case study considers three different optimization problems with
397 respect to each available washing method: only online washing (ON), only

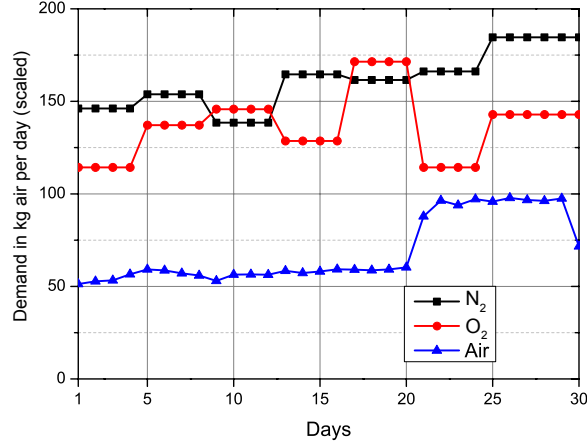


Figure 6: Production targets of the air separation plant for thirty days.

Table 5: Problem specifications, and value of objective function in scaled monetary units of each scenario.

Cases	eqns	bin vars	cont vars	nodes	CPU (s)	Obj
ON	14267	2640	4380	514	51	983
OFF	14638	3297	4050	749	137	1022
ON+OFF	16601	3627	4707	522	85	996

398 offline (OFF) and both washings (ON+OFF). The specifications of each op-
399 timization problem and the values of the objective functions of each solution
400 can be seen in Table 5.

401 The number of variables of the OFF case is larger than the ON case and
402 the number of variables in the ON+OFF case is considerably larger than in
403 the OFF case due to the consideration of the extended mathematical model
404 described in Section 3.3. Furthermore, Table 5 shows that the problem in the
405 OFF case takes relatively more time than in the other two cases to be solved.
406 The reason for this is that in the OFF case a compressor can be washed only
407 if it is offline. Therefore, the offline washing cost is implicitly connected with
408 the cost of a start up. Indeed, if a compressor which has been washed after
409 a shut down has to to operate again, then it has to start up with a start up
410 penalty. On the other hand in the ON case the online washing can prolong
411 the operational time of the compressor without shutting it down.

5.1.1. Offline washing results

Figure 7a gives the Gantt chart of the scheduling of the compressors of the OFF case. The figure gives the operational status of each compressor (online or offline mode), the connections between compressors and headers, and the washing status over a time horizon of thirty days.

The general trend of the solution of the optimization is that the compressors should operate as much as possible providing compressed air to the headers with which the compressors are connected in the first time period. A remark is that the connections compressors-headers in the first period is not necessarily the same with those of the initial state as can be seen in Table 4. For example, compressor $i8$ can satisfy the load of header $j1$ without the contribution of another compressor as the load of this header is relatively low compared to the load of the air separation columns connected with headers $j2$ and $j3$ (see Fig. 6). In the latter cases, more than one compressor is needed to meet the demand at each header.

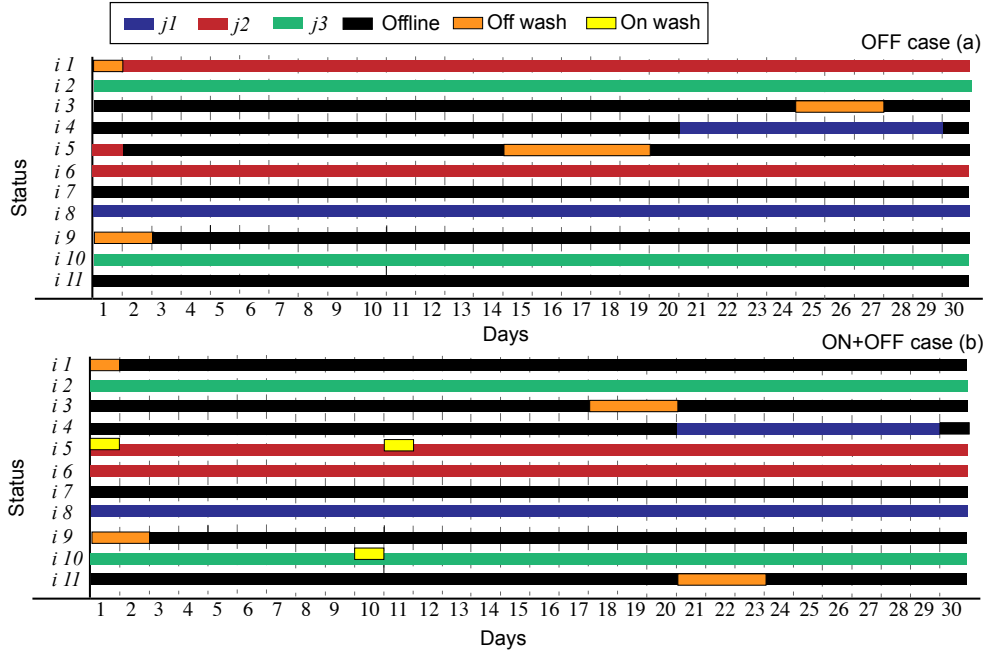


Figure 7: Gantt charts of the scheduling of the compressors for the base case study with only offline washing (a), and both online and offline washing (b).

As can be observed from the results, the initial state influences the op-

428 timal solution. For instance, Fig. 7a shows that in the beginning of the
 429 optimization compressor $i11$ is switched off and it is not used at all. In addi-
 430 tion, in the beginning of optimization compressor $i2$ has changed from header
 431 $j1$ to $j3$. The reason for this is that compressor $i11$ has higher minimum limit
 432 of mass flow rate than compressor $i2$, therefore compressor $i11$ switches off
 433 and compressor $i2$ satisfies the load of header $j3$ along with compressor $i10$.
 434 These changes support the further decisions which switch off compressor $i5$
 435 at day 2 and start up compressor $i1$ connecting to header $j2$.

436 The optimization suggests maintaining compressors $i3$ and $i5$ which have
 437 been operated more than the others as can be seen from the values of $\delta S_{(i)}^o$
 438 in Table 4. Hence, this decision leads to lower maintenance costs compared
 439 to the scenario which could have decided the washing of a larger compressor
 440 such as compressor $i11$. Table 3 shows that the offline washing cost of a small
 441 compressor is lower than the cost of a large compressor. In the section of
 442 the time horizon between Day 15 and Day 30, the two washing episodes of
 443 compressors $i3$ and $i5$ are derived from the terminal constraints in Eqs. (52)
 444 and (53) with the consideration of $\lambda_k = 2$ and $t_S^* = 15$ d.

445 The previous statements show that there are many factors which influence
 446 the best decisions for optimal operation of the compressors and these factors
 447 depend on the knowledge of the future information such as forecast of the
 448 demand and the decisions for the succeeding time periods. Moreover, the
 449 values of the parameters of the problem influence these decisions. Examples
 450 of these parameters are the initial operating and shut down times, and the
 451 values of the penalty costs.

452 Thus, even the most well trained and experienced team of managers and
 453 operators could find it difficult to take the best decisions without having a
 454 sophisticated optimization model based on the systematic use of the previous
 455 mentioned information. The nature of the problem is combinatorial involving
 456 a large number of scenarios, and therefore it is impractical and probably
 457 impossible to identify the optimal patterns for the best decisions without the
 458 use of optimization.

459 5.1.2. Results from the combination of offline and online washing

460 Figure 7b illustrates the Gantt chart of the scheduling of the compressors
 461 of the base case with available both online and offline washings, ON+OFF
 462 case. The solution of the ON case results in the same Gantt chart as in the
 463 ON+OFF case apart from the existence of offline washing episodes, therefore
 464 the ON Gantt chart is not presented. Additionally, the numbers and times

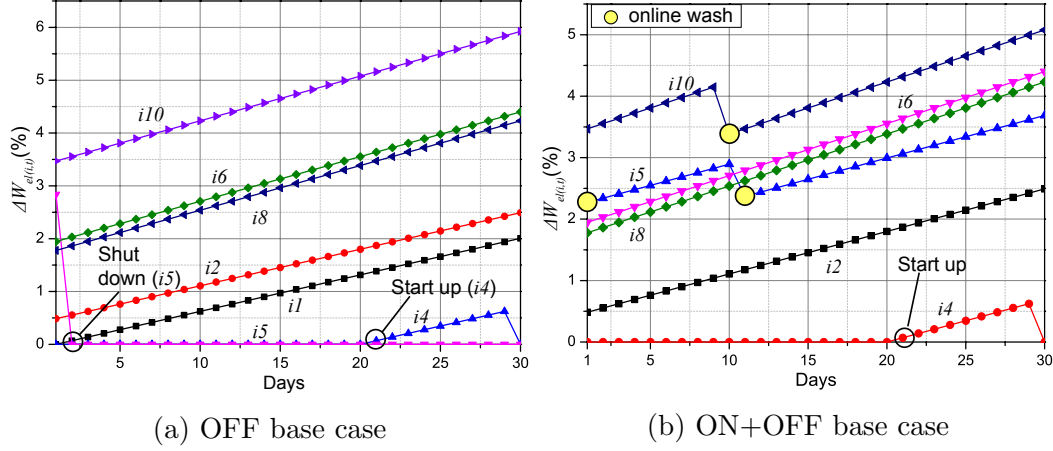


Figure 8: Extra power consumption (scaled) over time.

of the online washing episodes (ONWEs) are the same in both cases.

The results from the schedule of the ON+OFF case show that online washing is used along with offline. In this case compared to the OFF case, compressor $i5$ is not switched off but it is washed online. Hence, both schedules included cleaning of compressor $i5$, the OFF case with offline washing, but the ON+OFF with online and keeping the compressor on. The reason is that this small compressor is the second most fouled one after compressor $i11$. As the schedule of the OFF case switched off compressor $i11$ the same can be observed in the schedule of the ON+OFF case. In the latter case, compressor $i5$ remains online and therefore only compressors $i7$ and $i11$ can be washed offline. The other smaller compressors cannot be washed offline as $i1$, $i4$ have been washed before the beginning of the optimization and the other compressors $i2$, $i5$ remain online over the whole time horizon.

Figure 8 illustrates the extra power consumption $\Delta W_{el(i,t)}$ of each compressor i in scaled units, this means that if the real extra power is $\Delta W_{el(i,t)}^*$ in kW and a fixed parameter $\Delta W_{(i)}^s$ is given in kW, then the vertical axis shows the scaled variable $\Delta W_{el(i,t)} = \Delta W_{el(i,t)}^* / \Delta W_{(i)}^s$ with no units.

The ON case results in minimum total cost compared to the other two cases as can be seen in Fig. 9a. However, the ON case does not include terminal constraints as cases OFF and ON+OFF considering them. At the end of the optimization the total Equivalent Operating Time (EOT) in days of the compressors are 405, 321 and 280 in ON, OFF and ON+OFF cases respectively. Therefore, the ON+OFF case achieves 2.5% lower total cost

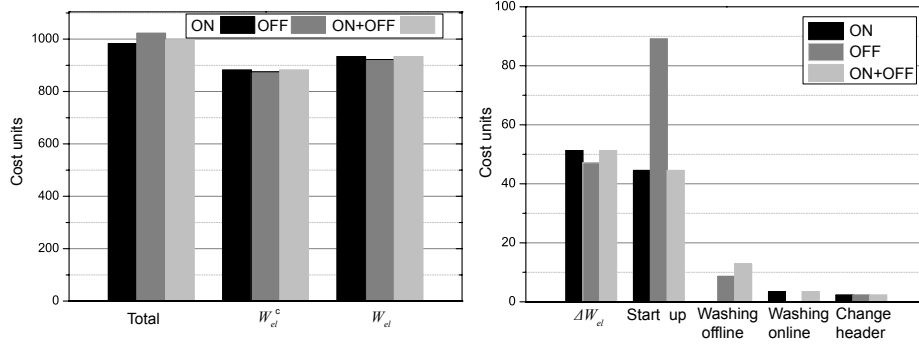


Figure 9: Total and electricity costs (a) and other costs (b) for thirty days.

than the one in the OFF case and the compressors have been operated 41 days less. Therefore, the former case achieved to meet the demand with decreased operational costs and to wear less the compressors due to less equivalent operating time compared to the latter case.

Figure 9b explains that in the case of ON+OFF the total cost is lower, even if the costs of the washings are higher, than in the case of OFF. However, the total electricity W_{el} consumed in the ON+OFF is higher than in the case of the OFF. This proves that the maintenance strategy considerably influences the operation. The main reason that the schedule in the ON+OFF case is less expensive than the OFF is that the online washing complements to the offline, thus this results in fewer start up events. The results which show that a compressor has to operate online continuously as much as possible is mostly in line with the industrial policy. This is justified as Fig. 7a shows that compressor *i*5 shuts down after 41 days and Fig. 7b shows that the same compressor operates for 70 days equal to 53.2 equivalent operating days.

5.2. Different degradation rates

5.2.1. Description of case studies

Section 5.2 examines the influence of different degradation rates on the scheduling of the compressors. In the base case study the degradation rate is based on 4.5% and 3.3% extra power consumption per month of the small and the large compressors respectively in the air separation plant in BASF, Ludwigshafen, Germany. These degradation rates define the low degradation rate case (Low case). A Medium and a High case consider 6% and 9% extra power consumption per month for small and large compressors. Aretakis et

al. (2012) stated that 10% extra power consumption per month is relatively significant but it can also be realistic.

The structure of the network of compressors and headers, the air separation columns, storage tanks and customers are the same as in those of the base case. Moreover, the input from the Tables 1, 2, 3 and 4 remain the same.

5.2.2. Online washing

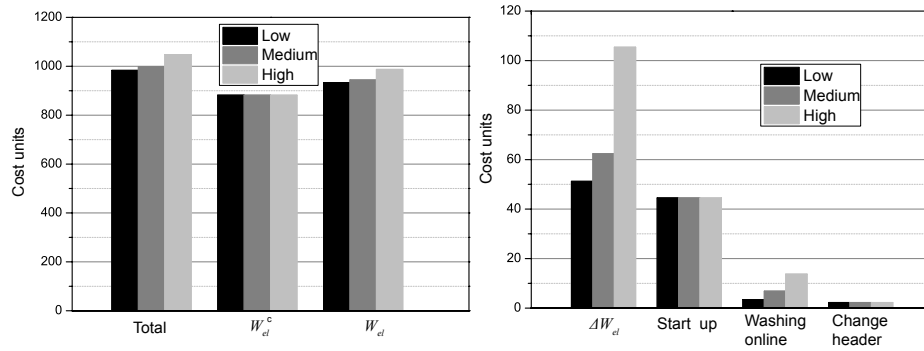


Figure 10: Total cost and electricity costs (a) and other costs (b) for different degradation rates when there is only online washing for thirty days.

Figure 10 presents the total cost and distribution of costs per case with different degradation rates, Low, Medium and High, when there is only online washing. According to Fig. 10a, the higher the degradation rate is, the more the total cost increases. Figure 10b shows that the increase in total cost is a result of the increased cost of the total extra power consumption and the washing costs. The difference in total extra power, ΔW_{el} , between High and Medium is much higher than in the difference between Medium and Low cases. The number of online washings are three, five and ten for Low, Medium and High cases respectively. The power of the cleaned compressors, W_{el}^c , is the same for all three cases and this means that different degradation rates do not influence the operation. This is explained from the fact that the compressors do not have differences in configuration in the three schedules according to optimization output, and the start up and change header costs are the same.

5.2.3. Offline washing

Figure 11 illustrates the total cost, electricity cost distribution and other costs when only offline washing is available. The decisions of operation and maintenance of the Medium and High cases are the same with those of the base case. For this reason the W_{el}^c and offline washing costs are the same for all three cases. The difference in total extra power between medium and low degradation rate is 31.3% and the difference between high and medium is 91.6%. This results in higher total cost in the High case. Moreover, the optimization suggests keeping the online compressors operating as much as possible to avoid start up costs or possible change header costs.

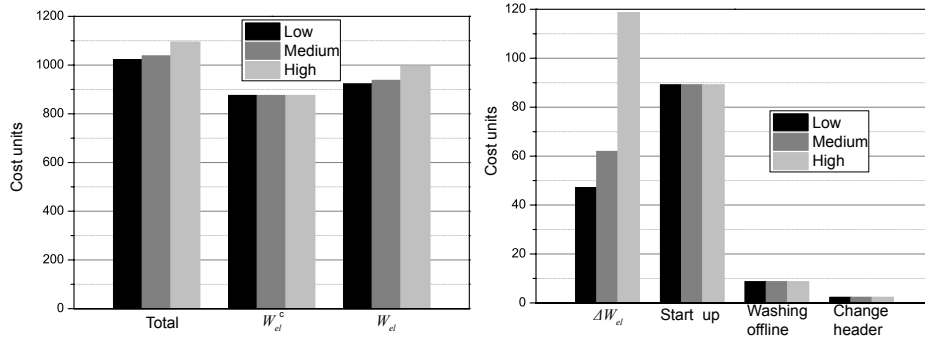


Figure 11: Total cost and electricity costs (a) and other costs (b) for different degradation rates when there is only offline washing for thirty days.

5.2.4. Both online and offline washing

Figure 12 shows that the combination of online and offline washings achieve reduced total costs by 2.5%, 2.5% and 3.0% in each degradation rate case respectively compared to the OFF case presented in Section 5.2.3. However, only in the High case the extra power ΔW_{el} cost is lower in the case of the ON+OFF case than in the OFF case. The total cost of the OFF case includes increased start up costs, on the other hand the ON+OFF case is more flexible with lower total start up cost. The previous observations support the use of both online and offline washing, especially when the Equivalent Operating Time (EOT) of the ON+OFF case is reduced by 35% compared to this of the OFF case. This shows that the operation of compressors in the former case would be more flexible for a following scheduling problem as the compressors have been maintained in a more optimal way in the current scheduling problem.

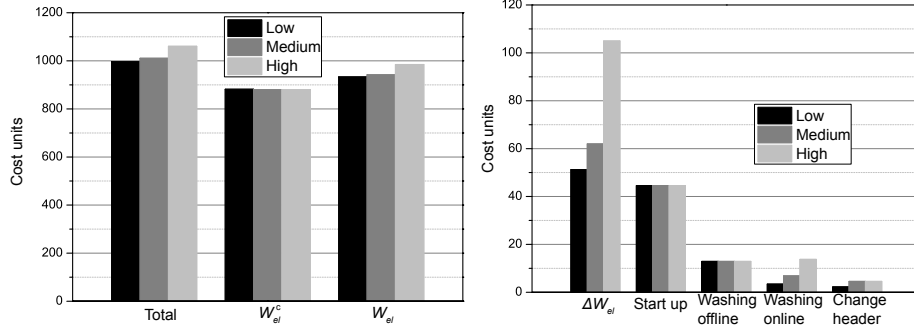


Figure 12: Total cost and electricity costs (a) and other costs (b) for different degradation rates when both offline and online washing are considered for thirty days.

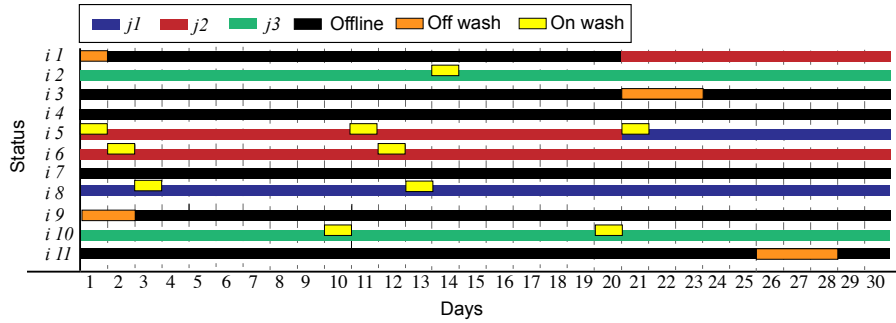


Figure 13: Scheduling of compressors with online and offline washing, and high degradation rates.

Figure 13 shows the scheduling of the compressors and the online and offline washing episodes timings in the case of the high degradation rate (High case). The resulting schedule is different than the schedule when the degradation rate is lower, see schedule of base case in Fig. 7b. The optimization does not suggest washing a compressor after the minimum possible period, $\gamma_{(i)}$ days, as can be seen in the case of compressor $i2$. This proves that a periodic fixed schedule, which is usually used in a typical industrial preventive maintenance strategy, would increase the total costs if maintenance and operation had not been integrated in the optimization model. Figure 13 shows that compressor $i5$ should be maintained in a periodic way. This does not apply to the maintenance frequency of the other compressors according to the figure.

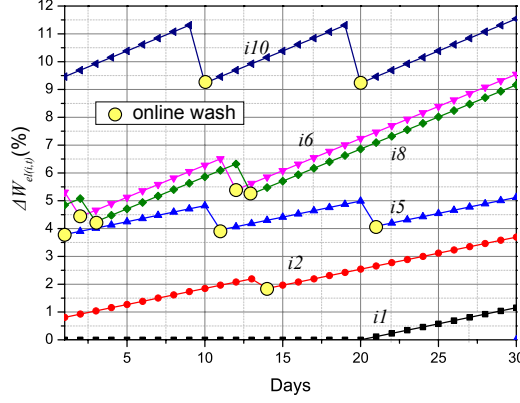


Figure 14: Extra power consumption of compressors with online and offline washing, and high degradation rates.

Figure 14 gives the profile of the extra power ΔW_{el} of each compressor over the time horizon of the optimization. The online washing model assumed that the application of online washing recovers a fixed percentage of the extra power of a compressor at the period the washing occurs. This means that for higher values of extra power consumption, the effect of the washing becomes greater. Moreover, Figure 13 shows that the optimization suggests washing the more fouled compressors (see Table 4), $i5$, $i6$, $i8$ and $i10$, with a high frequency. Compressor $i5$ is the most fouled and it is washed three times, which is the maximum possible number of washing episodes in the total duration of the optimization. Finally, Compressor $i2$ is the least fouled, and therefore the optimization suggests that only one washing is worthwhile as the fouling accumulation is not high.

5.3. Case with a less flexible system

Section 5.3 investigates the case with a reduced number of installed compressors compared to the base case study. The motivation of this problem instance is that compressor stations in the industry may not involve a large number of spare compressors. This study examines the behaviour of the operation and maintenance of a less flexible system considering different methods to wash the compressors.

The system of the base case which includes all the compressors is called Flexible System (FS) and the system with the reduced number of compressors is called Less Flexible System (LFS). The LFS considers three small

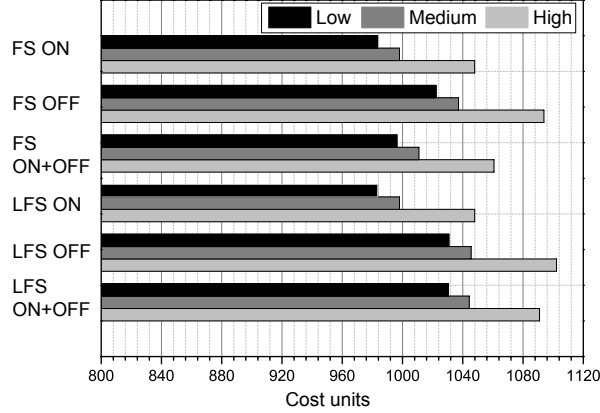


Figure 15: Comparison of total costs between a Flexible System (FS) and a Less Flexible System (LFS)

compressors $i \in I_s = \{i2, i4, i5\}$ and four large compressors $i \in I_l = \{i6, i8, i10, i11\}$.

Figure 15 shows the results from the Less Flexible System (LFS) and Flexible System (FS) study for each washing method used, i.e. ON, OFF and ON+OFF, and for each degradation rate case, i.e. Low, Medium and High. The results illustrate that the less flexible system is associated with higher total costs in all cases compared to the Flexible system, apart from the cases with the only available online washing (case ON). Indeed, the costs of both systems in the ON case are exactly the same.

The less flexible system has higher total cost in the OFF scenario and in all degradation rate cases by 0.8% and increased total cost in the ON+OFF case by approximately 3%. There is no difference in the ON case. This explains that the online washing along with the offline and with the use of spare compressors (i.e. flexible system) improves significantly the operation compared to the less flexible system case. This remark is important as compressors eventually must be washed offline at some point.

The impact of the flexibility of a system of compressors, which is associated with the increased number of spare compressors, on the total cost can be seen in the case of the Flexible System (FS) where the ON+OFF washing reduces the total cost compared to the OFF washing case. However, this reduction is less important in the case of the Less Flexible System (LFS).

The capital costs in the case of a compressor station with a reduced num-

ber of parallel units are lower compared a compressor station with a number of spare units. According to literature (Xenos et al., 2015b; Saidur et al., 2010), the capital cost of a compressor is significantly lower than the operational and maintenance costs in its lifecycle which is usually more than 30 years. Therefore, a framework to determine optimal operation and maintenance of an existing installation of compressors is necessary to reduce operational costs independently if the selection of the number of the compressors and the decision of the proper sizing of the compressors was successful at the front end design of the plant. A sophisticated approach which examines the simultaneous optimal selection of the installed compressors and their sizing, and the operational planning of the plant can be a future topic of research.

6. Conclusions

This paper presented an integrated optimization framework which can be used to optimize operation and maintenance of multiple compressors with different types of washings, namely offline and online. The optimization framework determines when and how often to wash the compressors using each washing method. The results showed that the best maintenance strategy to decrease the total costs and at the same time to minimize the wear of the compressors is a combination of offline and online washing. In the case which a compressor network involves spare compressors, the use of both washings decreases the total costs and increases the availability of the compressor significantly. The results have shown that the optimization framework can provide enhanced decision support leading to optimal operation and maintenance. By the modification of the constraints (i.e. compressor and models of the units of the plant), the optimization framework can be applied to other systems which include parallel compressors with large power consumption.

7. Acknowledgments

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762 Appendix

763 The basic operational model of networks of compressors connected with
 764 different headers and downstream processes can be seen in Tables 6, 7 and
 765 8. The logical constraints (54) – (56) connect the binary variables of start
 766 up $S_{(i,t)}$ and shut down $F_{(i,t)}$ with the binary variable which defines the
 767 status of the compressor $X_{(i,t)}$. The parameter $\bar{\chi}_{(i)}$ is the initial status of the
 768 compressors exactly before the beginning of the optimization. Constraints
 769 (54) – (56) are related to constraints (57) and (58) which define the minimum
 770 start up and shut down times of the compressors.

Table 6: Operational model and basic constraints (Kopanos et al., 2015).

Logical constraints of start up and shut down decisions:	
$S_{(i,t)} - F_{(i,t)} = X_{(i,t)} - \bar{\chi}_{(i)},$	$\forall i \in I, t = 1 \quad (54)$
$S_{(i,t)} - F_{(i,t)} = X_{(i,t)} - X_{(i,t-1)},$	$\forall i \in I, t > 1 \quad (55)$
$S_{(i,t)} + F_{(i,t)} \leq 1,$	$\forall i \in I, t \in T \quad (56)$
Minimum start up/shut down times:	
$X_{(i,t)} \geq \sum_{t'=\max\{1,t-\omega_{(i)}+1\}}^t S_{(i,t')},$	$\forall i \in I, t \in T : \omega_{(i)} > 1 \quad (57)$
$1 - X_{(i,t)} \geq \sum_{t'=\max\{1,t-\psi_{(i)}+1\}}^t F_{(i,t')},$	$\forall i \in I, t \in T : \psi_{(i)} > 1 \quad (58)$

771 Equations (59) and (60) in Table 7 define the feasible window of operation
 772 of compressor $i \in I$. The expression of the power consumption is valid within
 773 these operational bounds defined by the two equations. The parameters $\rho_{(i)}$
 774 and $\pi_{(i)}$ define the minimum and maximum boundaries of the decision vari-
 775 ables mass flow and outlet pressure of the compressors. The logical constraint
 776 (61) restrict online compressors to provide compressed air to more than one
 777 header. The constraints (62) – (63) model the change $D_{(i,t)}$ of a compressor
 778 to different headers. The parameter $\bar{\phi}_{(i,j)}$ denotes the initial connections of
 779 the compressors $i \in I$ with headers $j \in J_{(i)}$ exactly before the beginning of
 780 the optimization. The relation between outlet pressure of a compressor $P_{(i,j,t)}$

Table 7: Operational model and basic constraints (Kopanos et al., 2015).

Operating bounds of the compressors:	
$\rho_{(i)}^{min} Y_{(i,j,t)} \leq M_{(i,j,t)} \leq \rho_{(i)}^{max} Y_{(i,j,t)},$	$\forall i \in I, j \in J_{(i)}, t \in T \quad (59)$
$\pi_{(i)}^{min} Y_{(i,j,t)} \leq P_{(i,j,t)} \leq \pi_{(i)}^{max} Y_{(i,j,t)},$	$\forall i \in I, j \in J_{(i)}, t \in T \quad (60)$
Logical constraints and discharge pressure at the grids:	
$\sum_{j \in J_{(i)}} Y_{(i,j,t)} = X_{(i,j,t)},$	$\forall i \in I, t \in T \quad (61)$
$D_{(i,t)} \geq Y_{(i,j,t)} - \bar{\phi}_{(i,j)} - S_{(i,t)},$	$\forall i \in I, j \in J_{(i)}, t = 1 \quad (62)$
$D_{(i,t)} \geq Y_{(i,j,t)} - Y_{(i,j,t-1)} - S_{(i,t)},$	$\forall i \in I, j \in J_{(i)}, t > 1 \quad (63)$
$P_{(i,j,t)} = a_{(j)} \bar{M}_{(i,j,t)} + b_{(j)} Y_{(i,j,t)}$	$\forall i \in I, j \in J_{(i)}, t \in T \quad (64)$

781 and total mass flow in the header $\bar{M}_{(i,j,t)}$ is described by Eq. (64). This is
 782 a linear relation with parameters $a_{(j)}$ and $b_{(j)}$ estimated by fitting industrial
 783 data.

784 Table 8 gives the mass balances and operational boundaries of the air
 785 separation columns $u \in U$ and storage tanks $z \in Z$. The network of com-
 786 pressors has to satisfy the compressed air for utilities with demand $\theta_{(n,t)}$ for
 787 every process plant $n \in N$ and this is given by Eq. (65). Equations. (66) –
 788 (68) describe the mass balance and capacity boundaries of the air separation
 789 columns. The outlet mass flow of product e is given from Eq. (66) where
 790 the parameter $\sigma_{(e)}$ is the volumetric fraction of the primary component of
 791 air e . The variable $L_{(e,u,z,t)}$ is the total mass amount of product e provided
 792 to each storage tank z . Equations (69) – (72) give the mass balances in the
 793 storage tanks and for the end-users with demand $\zeta_{(e,t)}$. The variable $A_{(e,z,t)}$
 794 represents the amount of product e delivered to storage tank z , the variable
 795 $B_{(e,z,t)}$ is the amount of stored product e of storage tank z per period t , and
 796 the variable $O_{(e,t)}$ is the amount of product e purchased from external sources
 797 per period t . Parameters $c_{(e,u)}$ and $\xi_{(e,z)}$ are the boundaries of air separation
 798 columns and storage tanks. Finally, parameter $\bar{\beta}_{(e,z)}$ is the initial inventory
 799 of product e of tank z before optimization.

Table 8: Mass balances, and capacity and storage boundaries (Kopanos et al., 2015).

Demand of compressed air:

$$\sum_{i \in I} \sum_{(j \in J_n \cap J_{(i)})} M_{(i,j,t)} \geq \theta_{(n,t)} \quad \forall n \in N, t \in T \quad (65)$$

Air separation columns operation:

$$C_{(e,u,t)} = \sigma_{(e)} \sum_{i \in I} \sum_{(j \in J_{(u)} \cap J_{(i)})} M_{(i,j,t)} \quad \forall u \in U, t \in T \quad (66)$$

$$\sum_{z \in Z_{(e)}} L_{(e,u,z,t)} = k_t C_{(e,u,t)} \quad \forall e \in E, u \in U, t \in T \quad (67)$$

$$c_{(e,u)}^{min} \leq C_{(e,u,t)} \leq c_{(e,u)}^{max} \quad \forall e \in E, u \in U, t \in T \quad (68)$$

Storage tanks and end-customers (demand side) mass balances, $\forall e \in E$:

$$\sum_{z \in Z_{(e)}} A_{(e,z,t)} + O_{(e,t)} = \zeta_{(e,t)}, \quad \forall t \in T \quad (69)$$

$$B_{(e,z,t)} = \bar{\beta}_{(e,z)} + \sum_{u \in U_z} L_{(e,u,z,t)} - A_{(e,z,t)}, \quad \forall z \in Z_{(e)}, t = 1 \quad (70)$$

$$B_{(e,z,t)} = B_{(e,z,t-1)} + \sum_{u \in U_z} L_{(e,u,z,t)} - A_{(e,z,t)}, \quad \forall z \in Z_{(e)}, t > 1 \quad (71)$$

$$\xi_{(e,z)}^{min} \leq B_{(e,z,t)} \leq \xi_{(e,z)}^{max}, \quad \forall z \in Z_{(e)}, t \in T \quad (72)$$