

Optimal operation of compressors in terms of power and efficiency

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Abstract—Two main approaches to optimisation of load allocation for a system of compressors working in parallel rely on either minimisation of power or maximisation of efficiency. This paper shows that the two approaches are not equivalent. An example with two compressors confirms that the two approaches do not give the same solution.

Index Terms—compressors, optimization, power, efficiency, convex functions

I. INTRODUCTION

The objective of a compressor station is to transport gas. Typically, a compressor station consists of two or more compressors working or in parallel. The mass flow through each compressor is called the *load* and there exist several approaches to assigning the loads to the compressors.

If it can be assumed that the compressors operating in parallel are identical, optimal load-sharing can be done by assigning the same load to each unit [1]. However, it is almost impossible to have two units with the same characteristics due to age, wear, or design differences. Most common approaches include assigning the loads in the order of decreasing efficiency of the units or assigning loads to minimise the overall power consumption of the compressors.

The authors of [2] and [3] indicated that the power to run a compressor is a nonlinear function of the mass flow rate. Thus, the minimisation of power can be formulated as a nonlinear programming problem. Furthermore, [4] used the nonlinear optimisation as a first step in a two-step control system to adjust the loads according to individual characteristics of the compressors. The individual characteristics of the compressors were also considered by the authors of [5] who used nonlinear optimisation to allocate the loads. They showed that depending on the chosen objective function, ‘cheaper’ compressors were run at capacity. The term ‘cheaper’ meant ‘more efficient’ if power was minimised, and ‘less expensive’ if the overall cost was taken into account.

Nonlinear programming was also applied to optimisation of a compressor station by [6] and [7] who included an

external demand requirement for the compressor station in the optimisation. Recently, [8] presented an overview of methods used for solving the problem of optimisation of the cost or power consumption in a compressor station with the focus on their computational efficiency.

An optimisation approach was also used by [9], who maximised the efficiency of the compressors in parallel taking compressor dynamics into account. The efficiency of a compressor was approximated as a second order polynomial function of the mass flow rate. Efficiency maximisation with compressor dynamics was also considered in [10] and [11], whereas the authors of [12] minimised the power consumption calculated from estimated efficiency.

This paper shows that maximisation of efficiency is not equivalent to minimisation of the power to run the compressors due to differences in the properties of the functional forms of the power curves and efficiency curves. It is shown that the power is a convex and strictly increasing function of the flow, whereas the efficiency is only strictly concave.

The rest of the paper is structured as follows. Section II describes the characteristics of the compressors and in particular gives an analysis of the power to run the compressors. An industrial application in Section III confirms that power minimization and efficiency maximization depend on the functional form of the power consumption curve. Finally, Section IV concludes the paper.

II. COMPRESSOR STATION

A. Operation of a compressor station

An example of a compressor station with two centrifugal compressor units in parallel is shown in Fig. 1. Each unit A and B is characterized by a *compressor map*, which expresses the relationship between the suction and discharge pressures, compressor speed, mass flow, and compressor efficiency. The load-sharing module allocates the loads to each compressor, so that the demand for the gas is satisfied. The demand might come from the final customers or from other compressor stations [13]. The system from Fig. 1 includes flow controllers FC which receive their set-points from the load-sharing module. The flow controllers then provide set-points to the respective speed controllers SC [10]. The discharge

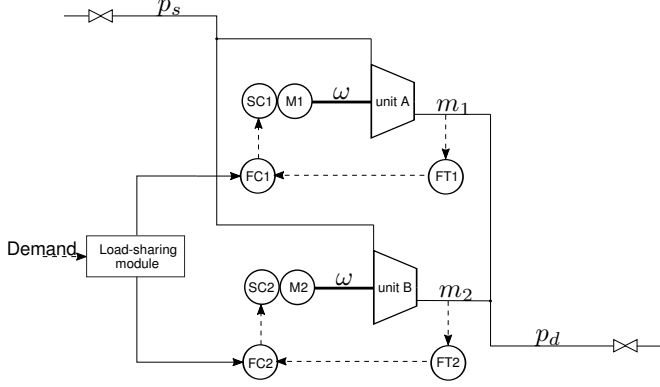


Fig. 1: A compressor station with two compressors working in parallel

pressure p_d is fixed by the export pipeline and the suction pressure p_s is fixed by an upstream pressure controller or by the supply pressure.

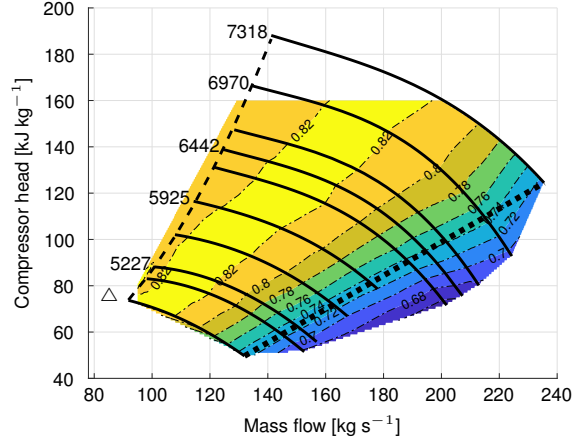
B. Compressor map

Typically, a compressor map shows the compressor head, obtained using the pressures p_s and p_d , as a function of the mass flow rate and the speed ω of a compressor. The efficiency of a compressor is shown as a function of the compressor head and the mass flow rate, and superimposed on the compressor map as *efficiency contours*. A compressor map is normally provided by the manufacturer, or it may be derived from first principles [14, 15]. Figure 2 shows the compressor maps used in this work. The solid lines denote the *speed lines*. The solid lines labelled with 4740 and 7318 denote the minimal and maximal speed of the compressor. Together with the surge and choke constraints (dashed and dotted lines), the minimal and maximal speed lines indicate the operating range for a compressor. The efficiency contours are superimposed on top of the speed lines, with yellow denoting the highest efficiency. This work assumes that the operating ranges of Compressor A and B are the same, but the efficiencies are different. The data for the compressor map from Fig. 2a were obtained from [16] using the software from [17]. The efficiency of Compressor B in Fig. 2b is obtained as 90% of efficiency of Compressor A. The white areas above 160 kJ kg^{-1} are due to physical constraints imposed on the real industrial compressor station used by [16].

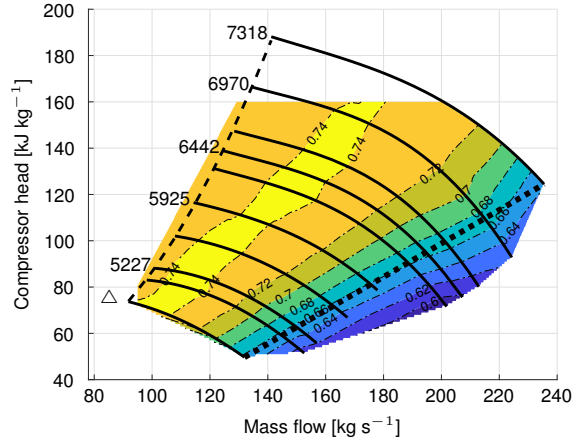
C. Compressor efficiency

Figure 3a shows the efficiency from Fig. 2a as a function of the mass flow for a fixed head. The red dotted lines indicate the efficiency obtained from [16], whereas the black solid lines denote the quadratic approximation, as suggested by [10]. For a given head, the efficiency of a compressor can be approximated with a quadratic polynomial of the mass flow m :

$$\eta = am^2 + bm + c \quad (1)$$



(a) Compressor map of Compressor A



(b) Compressor map of Compressor B, with efficiency obtained as 90% of the efficiency of Compressor A

Fig. 2: Compressor map with speed lines and efficiency contours. The line labelled with a triangle denotes the minimal speed 4740 rpm. The white area above 160 kJ kg^{-1} is due to physical constraints imposed on the system used by [16]

where a , b , and c are constant parameters. In particular, $a < 0$, which indicates that the efficiency from Eq. (1) is a concave function of the mass flow. This approach is also considered in this work.

The relative error of the approximation for the data from Fig. 3a is calculated according to formula

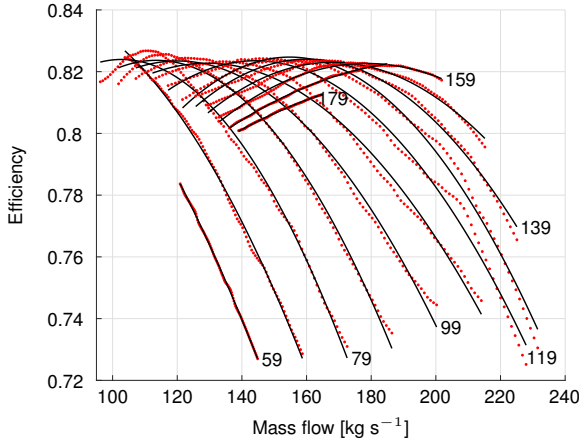
$$\epsilon = \frac{|\eta_{\text{real}} - \eta_A|}{\eta_{\text{real}}} \quad (2)$$

where η_{real} is obtained from [16] and η_A is calculated from Eq.(1). The error ϵ is smaller than 1%, so the quadratic approximation is used in further calculations in Section III.

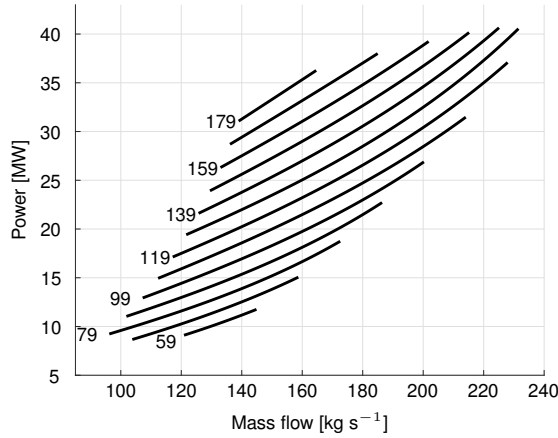
D. Compressor power

The power necessary to run a compressor with efficiency η at the head H and the mass flow m is given as [3]:

$$W = \frac{Hm}{\eta} \quad (3)$$



(a) Compressor efficiency from [16] (dotted red) with the quadratic approximation (black solid)



(b) Compressor power calculated for approximated efficiency

Fig. 3: Compressor efficiency and power as functions of the flow for fixed head

Figure 3b presents the power to run a compressor with efficiency approximated by Eq. (1) for the data obtained from [16].

Analysis of Fig. 2 indicates that the maximal efficiency is obtained inside the operating range for a compressor (yellow area is inside the operating range). At the same time, Fig. 3 shows that the minimal power to run is required for the minimal flow. It will be shown in Section III that the mass flow for which the maximal efficiency is obtained is not the same as the flow for which the power to run a compressor is minimal.

III. MAXIMISATION OF EFFICIENCY AND MINIMISATION OF POWER CONSUMPTION

A. Optimisation formulation

The problem of optimal load-sharing in a compressor station is formulated as a nonlinear convex optimisation problem. If the objective is to minimise the power necessary

to run a compressor station with N compressors [3], the optimisation problem becomes:

$$\min \sum_{i=1}^N W_i \quad (4)$$

where W_i is calculated according to Eq. (3). Conversely, if the objective is to maximise the efficiency shown in [10], the objective function is

$$\min \left(- \sum_{i=1}^N \eta_i \right) \quad (5)$$

where η_i is calculated as a quadratic function according to Eq. (1).

The objective function from Eq. (4) is convex, as shown in Fig. 3b and the fact that a sum of convex functions is convex. It is assumed that the efficiency is strictly concave with respect to the flow, so the objective function from Eq. (5) is therefore convex, because it has a negative sign. Therefore, the optimisation problem is convex and can be solved using the approaches described by [18]. Further theoretical analysis of convexity and monotonicity of the objective functions from Eq. (4) and (5) was done in [19].

Regardless of the objective function, the compressor station must satisfy the demand:

$$\sum_{i=1}^N m_i = M \quad (6)$$

The load assigned to each compressor must be within the operating range defined by the compressor map:

$$m_i^{\min} \leq m_i \leq m_i^{\max} \quad (7)$$

The values $m_i^{\min}$ and $m_i^{\max}$ in the inequality (7) are obtained for a fixed value of head from the compressor map from Fig. 3. The head is constant, because the suction and discharge pressure are constant, as indicated in Section II.

B. Application

For this work, a compressor station with two compressors was chosen as an example, and hence $N = 2$. The characteristics of both compressors are shown in Fig. 4 with the efficiency in Fig. 4a and power in Fig. 4b obtained for $H = 119 \text{ kJ kg}^{-1}$. The flows through each compressor must be in the range $116 \text{ kg s}^{-1} \leq m_i \leq 228 \text{ kg s}^{-1}$. The solid lines in Fig. 4 come from [16] and represent the characteristics of Compressor A. The dash-dotted lines correspond to Compressor B and were obtained as 90% of characteristics of Compressor A. The efficiency for Compressor A is given as:

$$\eta_A = -1.3126 \times 10^{-5} m_A^2 + 0.0038 m_A + 0.5508 \quad (8)$$

with m_A denoting the mass flow through Compressor A. The efficiency for Compressor B is given as:

$$\eta_B = 0.9(-1.3126 \times 10^{-5} m_B^2 + 0.0038 m_B + 0.5508) \quad (9)$$

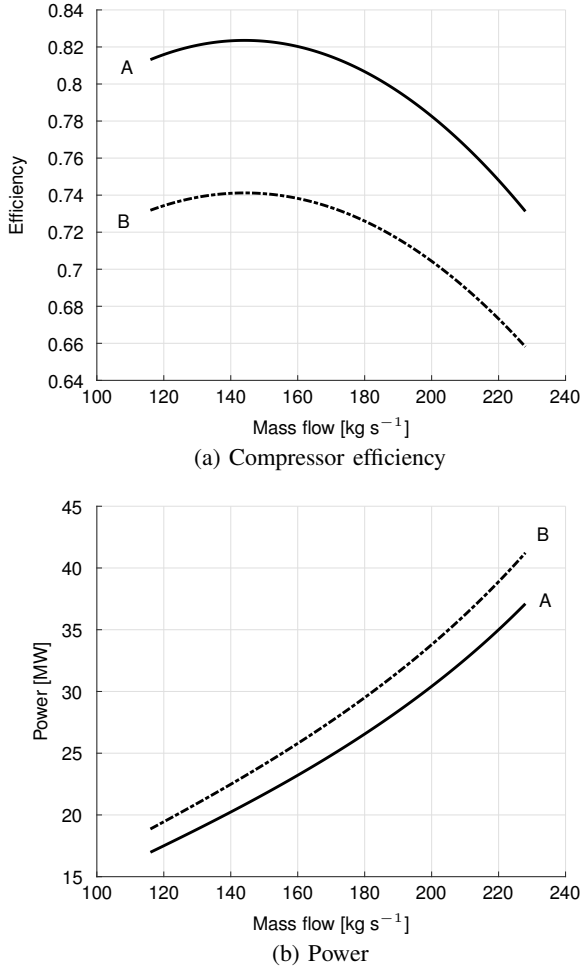


Fig. 4: Compressor characteristics as a function of the mass flow for fixed head $H = 119 \text{ kJ kg}^{-1}$, chosen for further analysis. The solid line corresponds to Compressor A directly from [16], the dash-dotted line corresponds to Compressor B with characteristics obtained as 90% of characteristics of Compressor A

with m_B denoting the mass flow through Compressor B.

Figure 5 shows the two objective functions from Eq. (4) (colour) and (5) (dark blue) with the operating range for Compressor A in black and for Compressor B in grey. The dark blue lines show the contours of the objective function used for maximisation of efficiency and the largest value of efficiency corresponds to the smallest circle. Conversely, the coloured lines correspond to the contours of the objective function for minimisation of power, with the yellow line in the top right corner denoting the largest value and the blue line in the bottom left corner denoting the lowest value of power to run the two compressors. The dashed lines show the minimal flows for both compressors, with black corresponding to Compressor A and grey corresponding to Compressor B. Similarly, the solid black and grey lines show the maximal flows for both compressors. The pink lines

correspond to two values of demand M from Eq. (6) and will be analysed in Section III-D.

C. No specified value for the flow

If there is no specified value for the flow, for instance if the compressors are only used to maintain the downstream pressure p_d , the minimisation of power would make the compressors work at their lower limit (thick black asterisk in Fig. 5). In contrast, maximising the efficiency would place the compressors within the operating range at the point marked with a thick black circle.

This fact indicates that if the flow is not specified, there is no demand constraint from Eq. (6) and the minimisation of power is not equivalent to maximisation of efficiency.

D. With demand for the flow - efficiency maximisation

Figure 5 shows also that the solutions obtained from minimisation of power (asterisk) would be different than obtained from maximisation of efficiency (circle) for a fixed value of demand. The dashed pink line shows the demand constraint for $M = 250 \text{ kg s}^{-1}$ and the dash-dotted pink line for $M = 400 \text{ kg s}^{-1}$. The values were chosen to put the equal load on both sides of the maximal efficiency $\eta^{\max}$ in Fig. 4a. The grey-black rectangle shows the operating range for the station, with dashed lines showing the minimal and the solid lines the maximal flows for each compressor. Black lines show the limits for Compressor A and grey showing the limits for compressor B.

The flows assigned to each compressor if the efficiency is maximised are indicated by the open circle symbols in Fig. 5. They are:

$$\begin{aligned} m_A &= \frac{1.8aM - 0.1b}{3.8a} \\ m_B &= \frac{2aM + 0.1b}{3.8a} \end{aligned} \quad (10)$$

Equation (10) was obtained by solving a quadratic optimisation problem with the objective function from Eq. (5). Analysing the solution from Eq. (10) and indicated by open circle symbols in Fig. 5 reveals that for $M < 289.5$ the value assigned to Compressor A will be smaller than the value assigned to Compressor B. Conversely, for $M > 289.5$ the value assigned to Compressor A will be larger than the value assigned to Compressor B. If $M = 289.5$, then the compressors will have the same mass flow rate, $m_A = m_B$, because it corresponds to the maximum of efficiency.

E. With demand for the flow - power minimisation

To find the flows assigned to each compressor if the power is minimised, it is necessary to solve a nonlinear optimisation problem with the objective function from Eq. (4) using Eq. (3). It was shown in [3] the relationship between power consumption and flow can also be approximated by a second order polynomial. It was shown in [3] that the power can be approximated by a second order polynomial as well

$$W_i = a_i m_i^2 + b_i m_i + c \quad (11)$$

where a_i , b_i , and c_i are constant parameters, $i \in \{A, B\}$. Following the assumption that the efficiency of Compressor B is 90% of the efficiency of Compressor A and using Eq. (3) indicates that the power to run Compressor B is 111% of the power to run Compressor A. For the power curves from Fig. 4b, it is obtained that:

$$W_A = 510m_A^2 + 6300m_A + 11 \times 10^6 \quad (12)$$

with m_A denoting the mass flow through Compressor A. The power for Compressor B is given as:

$$W_B = 1.11(510m_A^2 + 6300m_A + 11 \times 10^6) \quad (13)$$

with m_B denoting the mass flow through Compressor B. Solving a quadratic optimisation problem with the objective function from Eq. (4) yields:

$$\begin{aligned} m_A &= \frac{2.11a_A M + 0.11b_A}{4.22a_A} \\ m_B &= \frac{2a_A M - 0.11b_A}{4.22a_A} \end{aligned} \quad (14)$$

It can be shown using Eq. (14) that Compressor B will always get a smaller load than Compressor A, because the gradient of its power curve with respect to the flow is larger than that of Compressor A. Because of the convexity of power, the gradient is larger in the whole operating range, so Compressor B will always get less load, regardless of the value of the demand. Physically, if compressor A is more efficient than compressor B, then the power consumption can be reduced by giving more flow to Compressor A than to Compressor B. This is true even if the extra flow means compressor A is not working at its own maximum efficiency.

F. With demand for the flow - comparison

Figure 5 shows that the results obtained from maximisation of efficiency (circles) are different than the results obtained from minimisation of power (asterisks). These findings are further confirmed in Fig. 6a and 6b, which show enlarged areas around the solutions obtained for the demand $M = 400 \text{ kg s}^{-1}$ and $M = 250 \text{ kg s}^{-1}$. In both cases, the solution maximising the efficiency and denoted with a circle lies on the dark blue efficiency contour, but not on the power contour (yellow in Fig. 6a and light blue in Fig. 6b). Conversely, the solution obtained for minimisation of power consumption and denoted with the asterisk lies on the power contour, but not on the efficiency contour.

In particular, Figure 6b shows that the solution obtained for minimisation of power (asterisk) lies on the boundary of the operating range of Compressor B. Conversely, the solution obtained for maximisation of efficiency (circle) lies inside the operating range. This fact confirms the observation from Section II, which indicated that a compressor attains its maximal efficiency inside the operating range, but it will require minimal power if the flow is minimal. Therefore, the minimisation of power consumption in a compressor station is not equivalent to maximisation of the efficiency of each compressor.

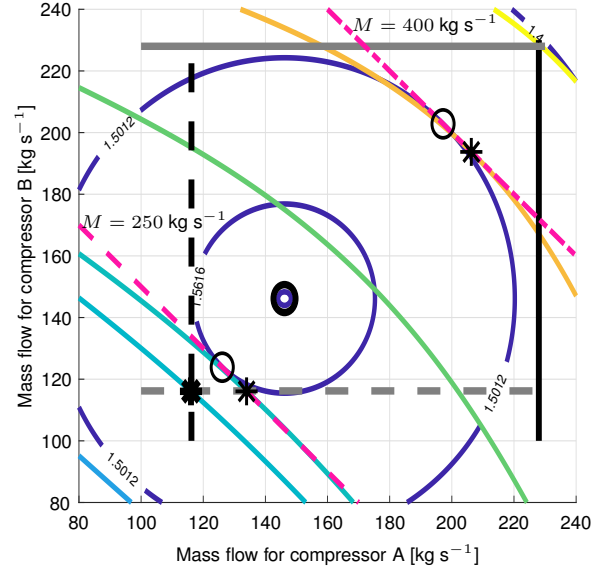


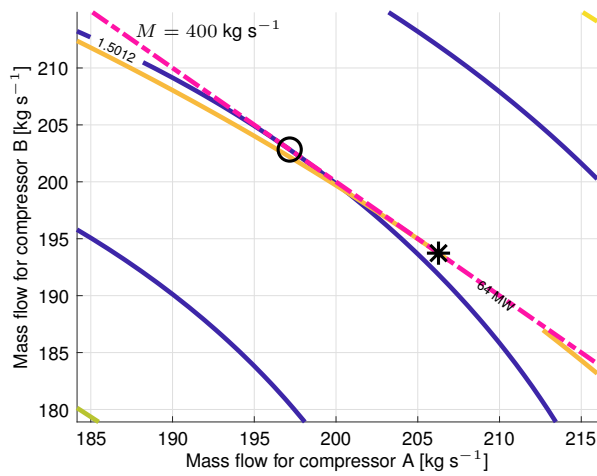
Fig. 5: Two objective functions from Eq. (4) (colour) and (5) (dark blue) with the operating range for Compressor A in black and for Compressor B in grey. Maximal efficiency corresponds to the smallest dark blue circle. Minimal power is shown with light blue curve in bottom left corner and yellow corresponds to the largest power. Optimal solutions are shown for two values of demand: $M = 250 \text{ kg s}^{-1}$ (dashed line) and $M = 400 \text{ kg s}^{-1}$ (dash-dotted line) obtained for power minimisation (thin asterisks) and efficiency maximisation (thin black circles). Thick black circle and thick asterisk show the optimal solution if no demand for the flow is required

IV. DISCUSSION AND CONCLUSIONS

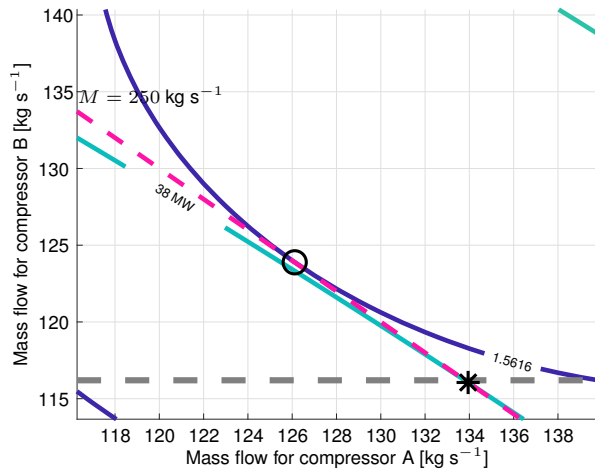
The paper shows that optimal load allocation in a compressor station yields different results if efficiency is maximised or if the power to run the station is minimised. The two approaches are not equivalent both in the case when the overall mass flow is not specified and when it is.

If the demand is not specified, then the maximisation of efficiency will assign the loads inside the operating range which correspond to the maximal efficiency of each compressor. The minimisation of power will assign the lowest possible loads to each compressor and therefore the two approaches are not equivalent. It can be observed, however, that the two approach would assign the same flows if the maximal efficiency for each compressor is obtained for the lowest flow.

If the mass flow is specified and the compressor station has to satisfy the demand, the values assigned to each compressor depend on the demand and on the gradient of the efficiency and the power. In the case of maximisation of efficiency, the less efficient compressor does not necessarily get a lower load, because the efficiency is a concave function of the mass flow. The fact that the efficiency is concave is illustrated



(a) Optimal solutions for $M = 400 \text{ kg s}^{-1}$



(b) Optimal solutions for $M = 250 \text{ kg s}^{-1}$

Fig. 6: Optimal solution for two values of the demand obtained for power minimisation (thin asterisks) and efficiency maximisation (thin circles)

in Fig. 5 for the two values of demand on both sides of the maximum. Since Compressor A has a larger value of efficiency, the flow assigned to Compressor A will be closer to the flow for which the efficiency is maximal, here larger than the flow assigned to Compressor B. Conversely, for demand smaller than 289.5 kg s^{-1} , the efficiency of both compressor decreases. Again, since Compressor A will be assigned a flow closer to the flow for which maximal efficiency is obtained, in this case the flow is smaller than the flow assigned to Compressor B. The power, on the other hand, is a strictly increasing and convex function of the flow as shown in Fig. 4b. Therefore, from Eq. (14) it is obtained that Compressor B will always get a lower flow.

Finally, both the maximisation of efficiency and the minimisation of power are formulated as quadratic optimisation problems. Moreover, the power and the efficiency of a

compressor are strictly related to each other, which suggests that one can be obtained if the other is known. Therefore, even though the two approaches are not equivalent, they both enable optimisation of a compressor station taking compressor performance into account.

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