

Seismic Response of Rock Towers at the Trona Pinnacles (U.S.A.) to the 2019 Ridgecrest Earthquake Sequence: Theory, Observations, and Models

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ABSTRACT

We analyze the seismic response of a class of fragile geologic features (FGFs), referred to as rock towers (RTs) at the Trona Pinnacles, a group of RTs in southern California that suffered strong shaking during the 2019 Ridgecrest earthquake sequence. FGFs, including RTs, may provide maximum constraints on past earthquake shaking intensity, and thereby support probabilistic seismic hazard assessments (PSHAs). In a rare case study, we explore the hypothesis that RT structural integrity is time dependent, as damage accumulates progressively. We develop finite-element method (FEM) models of the RTs using photogrammetric shape models. We validate the models by comparing numerical simulations of their response to broadband ground shaking with low-intensity seismic recordings obtained at the Pinnacles. Results of our simulations are in good agreement with the seismic recordings of actual earthquake aftershocks. We next use the results of the FEM models to analyze the response and evolution of RTs. Our analyses elucidate the influence of geometry over their seismic response, providing a rationale that may explain the rarity of slender RTs at Trona: high-aspect-ratio structures that respond in bending develop detrimental tensile stresses that crack the rock, whereas low-aspect-ratio ones' response also includes shearing, which does not compromise material integrity as much as tension. Field measurements with a rebound hammer support this finding, suggesting that the material around the base of slender rocks has been weakened relative to other parts of the RT. We also study how to define simplified mechanical models ("archetypes") to predict the natural frequencies of RTs. Results from our work illuminate the fundamental mechanisms of seismic response and progressive failure of RTs, and open new avenues of research to potentially incorporate these geologic features as long-return period constraints on PSHA, in ways analogous to those of the widely used precariously balanced rocks.



KEY POINTS

- Rock towers are a type of fragile geologic feature, which may be used to test and improve hazard models.
- High-aspect-ratio towers show evidence of seismic damage, consistent with predictions from theory and models.
- For hazard applications, the strength of these features may be considered time dependent.

INTRODUCTION

The Trona Pinnacles site (Fig. 1) is located about 20 km east of Ridgecrest, California, the town that lends its name to the July 2019 Ridgecrest earthquake sequence (Ponti *et al.*, 2020). The Pinnacles themselves are formed from calcareous tufa rock and range in size from small boulders to rock towers (RTs) more

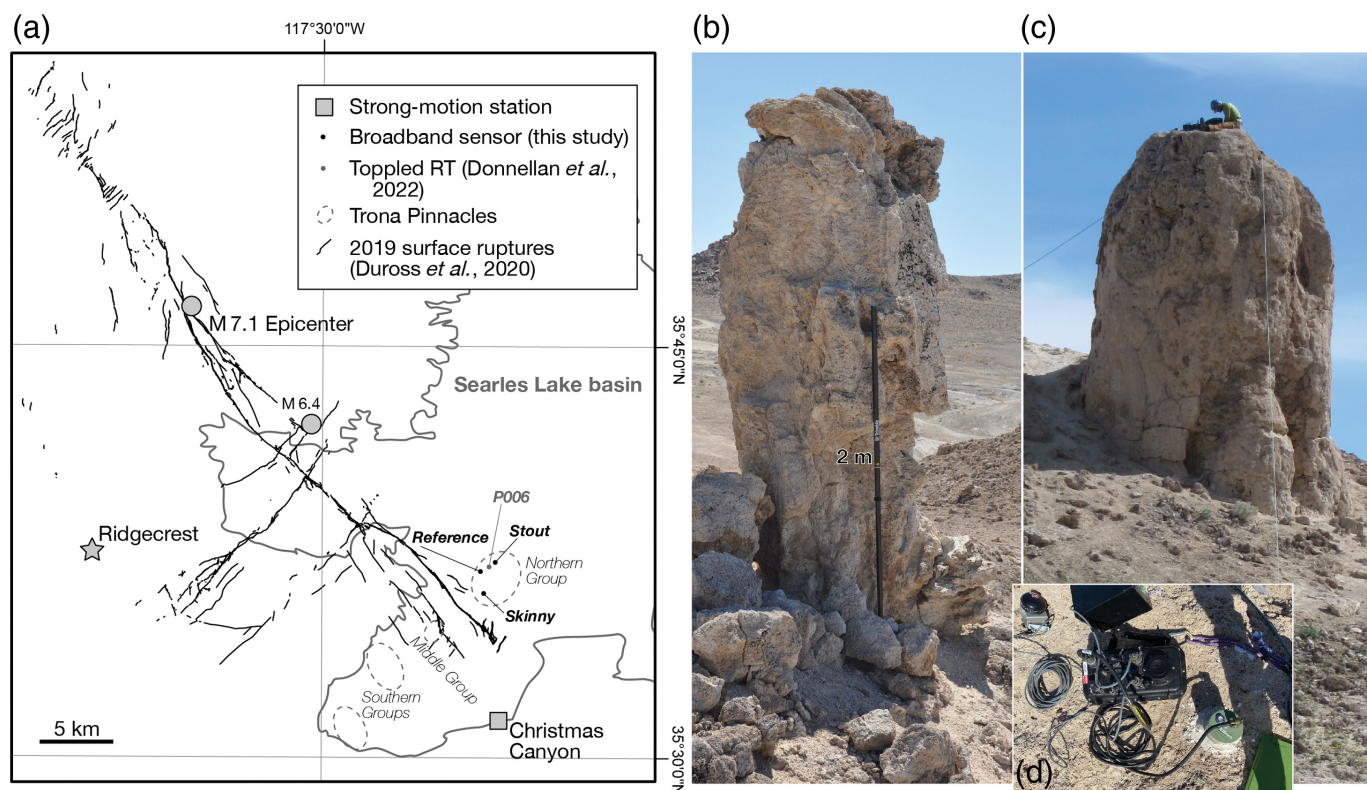
than 40 m tall; over 500 of these RTs cover an area of 14 square miles (Guo and Chafetz, 2012). This unusual landscape was declared a National Natural Landmark by the U.S. Department of the Interior in 1968. Several RTs were damaged during the

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Ridgecrest earthquake sequence (Goulet *et al.*, 2019; Donnellan *et al.*, 2022). This setting, following a large earthquake, provides a rare opportunity to understand the dynamic response of RTs to earthquake shaking.

The Trona Pinnacles and similar RTs are a category of so-called fragile geologic features (FGFs), defined as features of the landscape vulnerable to failure during earthquake shaking (Stirling *et al.*, 2020). Over the past 30 yr, published studies have shown that FGF data can improve probabilistic seismic hazard assessments (PSHAs), which forecast the frequency of exceedance of earthquake ground motions at various intensities (Brune *et al.*, 1996; Baker *et al.*, 2013; Lozos *et al.*, 2015; Rood *et al.*, 2020, 2023, 2024; Stirling *et al.*, 2021; McPhillips and Pratt, 2024). PSHAs are limited by sparse and scarce observational data (Field, 2015). For example, instrumental recordings of earthquake shaking are available only for the last hundred years or so, at best, and near-field data from larger earthquakes are especially limited (e.g., Frankel, 2016; Gerstenberger *et al.*, 2020). FGFs are a rare source of data to supplement seismic recordings in the near field and at long-return periods. The fragility of an FGF (i.e., the range of shaking intensities likely to cause failure) provides a maximum constraint on shaking intensity during the FGF's lifetime (typically thousands of years). The age of an FGF is often challenging and time-consuming to determine with precision (Balco *et al.*, 2011; Rood *et al.*, 2023). These constraints can be used to improve the quality of PSHA forecasts (e.g., Rood *et al.*, 2024) or reserved for independent testing and validation (e.g., Baker *et al.*, 2013).

Figure 1. (a) Map of the Trona Pinnacles in relation to the 2019 Ridgecrest earthquake sequence surface ruptures and the Pleistocene Searles Lake basin (Ponti *et al.*, 2020; Olson and Lowenstein, 2021). The Pinnacles form three broad groups, termed northern, middle, and southern (Scholl, 1960). Black circles indicate the locations of broadband seismometers deployed for this study. Mapped surface ruptures are attributed to the **M** 7.1 mainshock and the **M** 6.4 foreshock. In general, northwest–southeast-oriented ruptures and all ruptures in the vicinity of the Pinnacles are associated with the **M** 7.1 mainshock (Ponti *et al.*, 2020). (b) Image of the Pinnacle referred to as “Skinny.” (c) Image of the Pinnacle referred to as “Stout.” (d) Image of the seismometer deployed on top of Stout. RT, rock tower.

Most previous work has emphasized FGFs such as precariously balanced rocks (PBRs), and the category of RTs has received relatively little attention (Hall, 1996; Lignola *et al.*, 2010; Finzi *et al.*, 2020; McPhillips and Scharer, 2021; LaHusen *et al.*, 2025). The fundamental difference between PBRs and RTs is the fact that the latter are continuously connected to the ground, whereas the former are resting atop it, without a material connection. This difference makes their response to ground shaking radically different: whereas PBRs can be studied using rigid-body dynamics, RTs require the tools of continuum mechanics to understand their deformation, the associated internal forces, and their possible failure. As a result, the rich literature analyzing the fragility of PBRs does not apply to RTs (Housner, 1963; Anooshehpour *et al.*, 2004; Purvance *et al.*, 2008; Veeraraghavan *et al.*, 2016; Trugman *et al.*, 2023; Chen *et al.*, 2024). Further study of

the fragility of RTs is needed to improve the accuracy and precision of their ground-motion constraints (McPhillips and Scharer, 2021).

In a rare case study, Donnellan *et al.* (2022) documented a well-characterized RT at the Trona Pinnacles, which survived the 2020 M 7.1 Ridgecrest mainshock and subsequently failed during or shortly after a 2020 M 5.5 aftershock of the sequence. Dynamic modeling demonstrated that the M 7.1 mainshock shaking was likely sufficient to crack the rock at the base of the RT, whereas the M 5.5 aftershock, which shook the site with about half the intensity of the mainshock, was not. This observation led Donnellan *et al.* (2022) to suggest that rock damage inflicted during the mainshock left the RT especially vulnerable to toppling. More specifically, we (along with coauthors therein) posited a damage-accumulation process, for which successive earthquakes are envisioned to progressively damage the basal region of the RT. In extreme cases, over geologic time, the base of an RT may lose its transverse deformation resistance entirely, such that it becomes mechanically equivalent to a PBR. Prior to this work, the fragilities of FGFs have not been considered time-dependent, to our knowledge.

Here, we extend the previous case study to include additional surviving RTs, and we develop models and theory to better understand the fragility of RTs exposed to earthquake shaking. First, we present new field observations from the Trona Pinnacles, where we characterize the geometry of two intact RTs and their seismic responses to nearby earthquake aftershocks. We also report in situ rebound hammer measurements of rock strength as a proxy for damage accumulation. Second, we build finite-element method (FEM) numerical models of the dynamic response of 3D, intact RTs, which we validate by comparison to our field data. In addition to modal analysis of natural vibration frequencies, we also simulate the transient response of an RT to real, low-intensity earthquake shaking. Third, we use theory to develop simplified structural models of RTs and suggest how this “archetype” approach could be used to aid fragility calculations. Our work advances a mechanics-based framework to interpret the stability and failure of RTs and thereby supports both their conservation in culturally important landscapes and their application to support PSHA.

BACKGROUND

Geological setting

Between 10,000 and 32,000 yr ago, biologically assisted carbonate precipitation led to the underwater formation of tufa rock structures beneath the lakes that filled the Searles basin (Fig. 1; Scholl, 1960). Tufa is a form of porous travertine, and the Trona Pinnacles generally consist of concentric, overgrown deposits with heterogeneous character (Guo and Chafetz, 2012). The tufa generally lacks pervasive, planar discontinuities, such as bedding planes of foliation. Following the lake desiccation around 10,000 yr ago, the tufa structures became exposed as the Trona

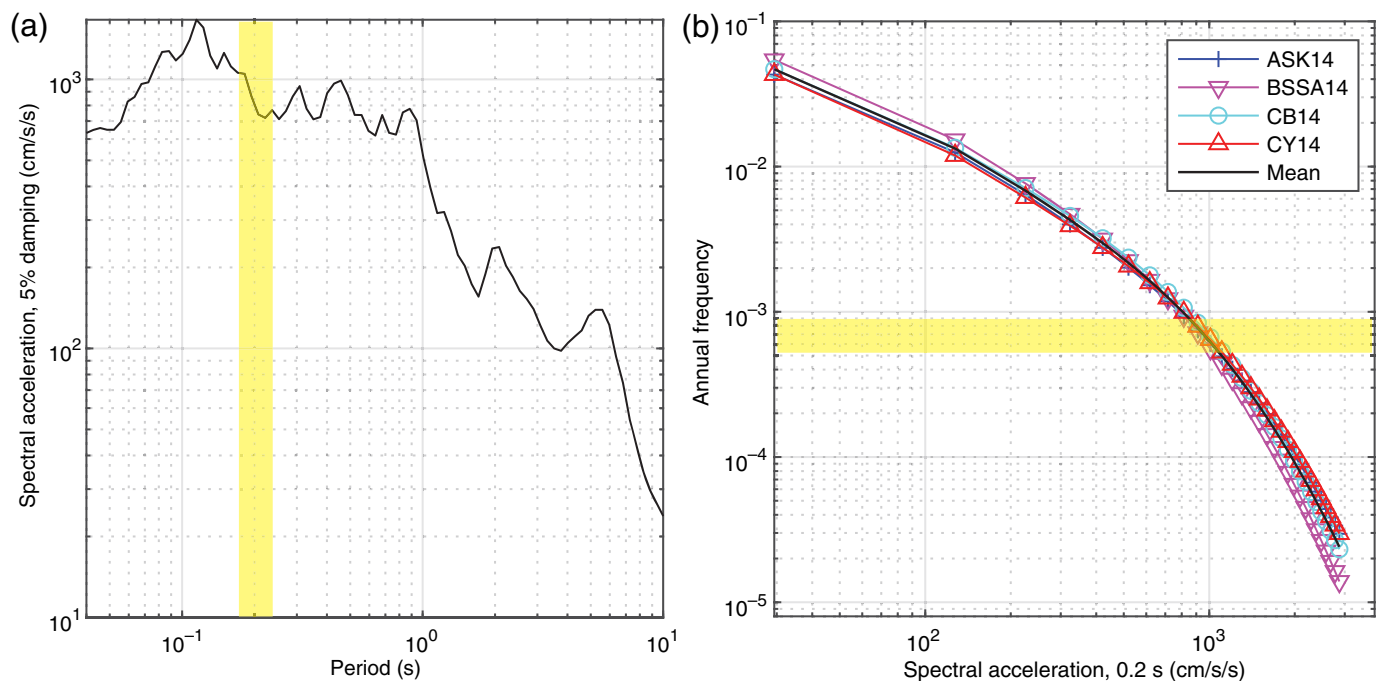
Pinnacles, which have been subject to erosion from weather and wind, as well as seismic shaking. Today, the Pinnacles come in different shapes: RTs, ridges, and mounds that are on average 3–12 m high with some reaching as high as 30–45 m. Base widths average 5–10 m, but some ridges exceed 100 m, and some slender RTs have widths < 1 m.

On 4 July 2019, the Ridgecrest earthquake sequence began (Baltzopoulos *et al.*, 2020). A M 6.4 left-lateral foreshock ruptured southwestward toward the town of Ridgecrest (Barnhart *et al.*, 2019; Brandenburg *et al.*, 2019; Ross *et al.*, 2019). About 34 hr later, on 5 July 2019, the M 7.1 right-lateral mainshock ruptured bilaterally northwest and southeast along a 48-km-long surface trace (Fig. 1). Thousands of smaller foreshocks and aftershocks completed the sequence. The southern termination of the M 7.1 rupture is located 4 km southwest of the Trona Pinnacles National Monument (Fig. 1).

At the Christmas Canyon China Lake seismic station, located just 10 km south of the Pinnacles, the peak horizontal acceleration during the M 7.1 mainshock was 0.567g, whereas the spectral acceleration of the corresponding component reached values that exceeded 1g at periods near 0.2 s (Center for Engineering Strong Motion Data, 2022). Short-period shaking typically approximates the natural periods of RTs (McPhillips and Scharer, 2021). Response spectra and the site-specific hazard forecast for the Pinnacles (disaggregated using the Uniform California Earthquake Hazard Forecast Version 3; Biasi and Anderson, 2016) are shown in Figure 2. Notably, the return time for high-frequency ground shaking as intense as was observed during the Ridgecrest mainshock is likely several thousand years (Fig. 2), suggesting that the RTs at the Pinnacles have experienced that level of shaking on multiple occasions within their lifetimes. Paleoseismic investigation on the Paxton Ranch fault, which ruptured in the 2019 mainshock, indicates that the penultimate earthquake occurred between the mid-Holocene and late Pleistocene (Kozaci *et al.*, 2021). In this case, previous strong shaking at the Trona Pinnacles had different fault sources, likely including the Garlock fault. The disaggregation includes all fault sources (Fig. 2).

RTs and their application in PSHAs

To provide useful input to PSHAs, both the age and the fragility of an FGF must be determined (Stirling *et al.*, 2020). FGF age is most often determined by geochronology, and the fragility is typically interpreted as constant over that interval (Balco *et al.*, 2011; Baker *et al.*, 2013; Rood *et al.*, 2023). To calculate fragility, quasi-static estimates are often useful for preliminary surveys (Anooshehpour *et al.*, 2004, 2013, 2020; McPhillips and Scharer, 2021). These estimates have limited precision, and FGF fragility is best-described probabilistically, perhaps jointly in terms of multiple intensity measures (e.g., peak ground acceleration and the ratio of peak ground velocity and peak ground acceleration; Baker *et al.*, 2013; Rood *et al.*,



2020, 2024; Stirling *et al.*, 2021; Trugman *et al.*, 2023). For PBRs, probabilistic fragility functions are often estimated from empirical relations principally derived from shake-table experiments (Purvance *et al.*, 2008; Wittich and Hutchinson, 2015; Klaboe *et al.*, 2018), and recent numerical modeling provides a promising complement (Purvance *et al.*, 2008; Veeraghavan *et al.*, 2016; Chen *et al.*, 2024). However, these results are inappropriate for calculating the fragilities of other categories of FGFs, such as RTs.

RTs differ from PBRs because they are mechanically connected to the bedrock at their base (pedestal for PBRs) and thus experience dynamic amplification when excited by earthquake ground shaking or environmental noise, similarly to man-made building structures (Clough and Penzien, 2004; Finnegan *et al.*, 2022). As a simple mechanical analog, the seismic response of a slender RT has been approximated by an Euler–Bernoulli cantilever beam (Mendecki and Szczygiel, 2019; Finzi *et al.*, 2020). This approach yields an estimate of the frequency range for which an RT is most vulnerable to failure. When combined with a quasi-static estimate of the acceleration required to open a new crack, this approach also yields a first-order estimate for the fragility of an RT (Hall, 1996; Anooshehpour *et al.*, 2020; Finzi *et al.*, 2020; McPhillips and Scharer, 2021; LaHusen *et al.*, 2025).

A recent compilation of measurements from 32 mostly slender RTs shows that fundamental frequency scales with the ratio of width to height squared, as expected from Euler–Bernoulli beam theory (Finnegan *et al.*, 2022). This study gives credence to the suitability of this simplified model for slender RTs, subject to several caveats. First, the data show considerable scatter, which Finnegan *et al.* (2022) attribute to differences in rock properties such as elastic modulus and density. Second, not

Figure 2. (a) Spectral acceleration for the 2019 M 7.1 Ridgecrest mainshock, recorded at the Christmas Canyon station (Center for Engineering Strong Motion Data, 2022). (b) Spectral acceleration hazard curves for northern Trona Pinnacles, determined by disaggregation of the Uniform California Earthquake Rupture Forecast Version 3 (Biasi and Anderson, 2016). The disaggregation uses 760 m/s for the site condition, which is appropriate for weathered bedrock. Individual curves are calculated using four ground-motion prediction equations (ASK14, BSSA14, CB14, and CY14, respectively; Abrahamson *et al.*, 2014; Boore *et al.*, 2014; Campbell and Bozorgnia, 2014; Chiou and Youngs, 2014). The color version of this figure is available only in the electronic edition.

all RTs are uniform in cross-section and slender, and their response depends on their shape and composition, which, furthermore, determine the frequencies at which the shaking will be amplified, and the distinct mode of deformation at each frequency (Moore *et al.*, 2019). For example, RTs with aspect ratios of approximately one feature strong coupling between shear and bending, and require a more sophisticated description than Euler–Bernoulli theory, such as the Timoshenko beam theory (Timoshenko, 1922). Finally, external factors may also contribute to an RT’s dynamic response and energy radiation during seismic shaking: RTs are frequently situated on bedrock slopes, ridge tops, or other complex bases, and seismic shaking may exhibit directionality, for example.

Although simplified models have benefits, numerical methods may be necessary to obtain accurate estimates of an RT’s stability, frequency response, and fragility. There are a limited number of RTs and other FGFs in the landscape, and accurate fragility calculations make best use of the FGFs that are available. Numerical methods, such as the FEM analyses used herein, may incorporate complex, realistic geometries and material properties

observed in the RTs and their bases (Hall, 1996; Lignola *et al.*, 2010; Valentin *et al.*, 2018; Moore *et al.*, 2019). Numerical methods also allow for the investigation of an FGF's response to realistic earthquake shaking, as opposed to generic vibrational analysis (Purvanche *et al.*, 2008; Chen *et al.*, 2024). Finally, numerical methods are useful for validating a variety of simplified models, which remain important for survey-scale analyses or situations for which the resources for numerical simulations are lacking.

FIELD WORK

Seismic instrumentation

To study the differences in ground-shaking response likely associated with RT aspect ratio, we present two case studies, both from actual RTs from Trona: (a) Skinny corresponds to a feature of height-to-basal-width ratio of 3:1 approximately (see Fig. 1b). In this case, we expect a bending-dominated response; (b) Stout corresponds to a ratio of 1:1 approximately (see Fig. 1c), for which we expect substantial shearing coupled with bending.

We measured their seismic responses on 4–5 March 2020, using three Nanometrics Trillium Compact 120s broadband seismometers with 24-bit RefTek RT130(s) data loggers recording at 200 Hz. We deployed a seismometer on top of both Skinny and Stout (Fig. 1b,c) and the third, reference seismometer on flat terrain, approximately 700 m from Skinny and 200 m from Stout (see map in Fig. 1a). All three seismometers were leveled, oriented to magnetic north, affixed to intact tufa rock using plaster, and covered with buckets. The Skinny and Stout instruments were recorded overnight, but the reference instrument's data logger failed after approximately two hours. During the interval when all three instruments were recording, two earthquakes occurred with epicentral distances between 10 and 15 km: **M** 1.2 at 02:22:39 (UTC) and 6.5 km depth; and **M** 1.9 at 03:03:59 (UTC) and 2.3 km depth (McPhillips, 2022; USGS, 2025).

To estimate the spectral content of the data, we converted counts to ground velocity using a bitweight of 1.58997×10^{-6} V/counts (RT130) or 2.742×10^{-6} V/counts (RT130s; Earthscope Primary Instrument Center [formerly Passcal], 2020); we removed the mean and trend from each trace; we band-pass filtered the data between 0.05 and 80 Hz; and finally selected the most appropriate time window to focus the analysis on the earthquake signals before computing the fast Fourier transform of the said time window. We also calculated amplification ratios in the frequency domain, comparing RT records to the reference record (e.g., Molnar *et al.*, 2022). We focus our analysis on the **M** 1.9 earthquake, but note that analysis of the full record yields similar patterns of spectral amplification (McPhillips, 2022).

Relative to the reference instrument, data from Skinny show a clear amplification of shaking in both horizontal directions in the vicinity of 15 Hz (Fig. 3). By contrast, the vertical

displacements are barely amplified. The spectra from Skinny reveal two peaks at 13.5 and 17.9 Hz, which we interpret as the first- and second-modal frequencies of the RT (i.e., resonant frequencies; Fig. 3).

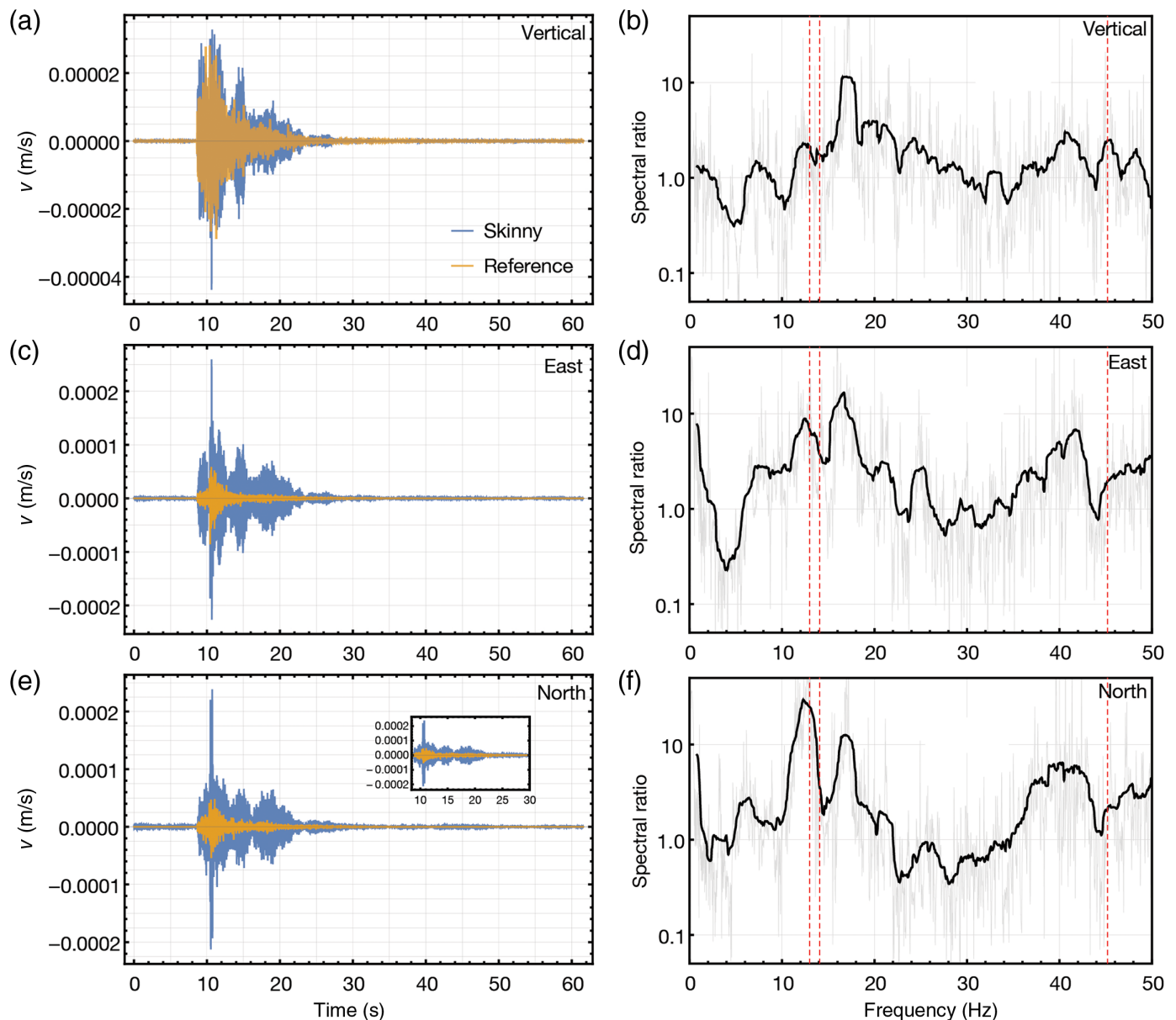
Stout shows less amplification in the horizontal directions (Fig. 4). In the vertical direction, the velocity amplitude time series are damped on top of the feature compared to the reference condition, whereas the spectral ratio shows small peaks consistent with deflection (Fig. 4). Altogether, the data suggest that the RT does not deform so much relatively to the base.

Rebound hammer measures of hardness and damage

Based on modal analyses of natural RTs (Moore *et al.*, 2019; Finnegan *et al.*, 2022) and our previous FEM modeling (Donnellan *et al.*, 2022), we develop the hypothesis that RTs are vulnerable to damage accumulation, which is controlled by the character of their dynamic response. Specifically, for RTs with similar material properties, high-aspect-ratio RTs (such as Skinny) are expected to deform by full-height bending, whereas low-aspect-ratio RTs (such as Stout) deform in coupled bending and shear (Fig. 5). Low-aspect-ratio RTs are also sensitive to shorter period shaking (≤ 0.1 Hz), which may not be the dominant frequency content of typical strong ground motions. We expect that bending-mode deformation creates high tractions at the base of high-aspect-ratio RTs, which produce tensile stresses that approach critical values, and likely develop mode-I fractures (Donnellan *et al.*, 2022). Cyclic loading due to the amplification of environmental noise may also drive subcritical crack growth near the base of slender RTs (Atkinson, 1984; Eppes and Keanini, 2017; Finnegan *et al.*, 2022). In contrast, deformation by internal shear (shear coupled to bending) that characterizes low-aspect-ratio RTs distributes the resulting stress throughout the RT's volume, and, as a consequence, fracture intensity factors remain low even during extreme events such as earthquakes.

To test this hypothesis, we measured rock strength in situ and compared values from the base with values from the midsections of the RTs spanning a range of aspect ratios at Trona. For this purpose, we used a Schmidt L-type rebound hammer. Rebound hammers measure surface hardness, which scales closely with rock compressive strength (Wang *et al.*, 2017). Rebound hammers are widely used to measure rock strength in situ and have also been used successfully as a proxy for rock damage by cracking (Shobe *et al.*, 2017; Eppes *et al.*, 2018).

The RTs ranged from 3 to 10 m in height. At each of these RTs, we made two groups of rebound hammer measurements on approximately orthogonal faces, at high and low positions, for a total of four groups per RT. At each rock face, we made one group of measurements near the soil interface, usually within 1 m of the ground. We made another group at least 2 m higher than the first group. This sampling strategy was designed to identify potential differences in hardness between the bases of the RTs and the midsections. Each group consisted

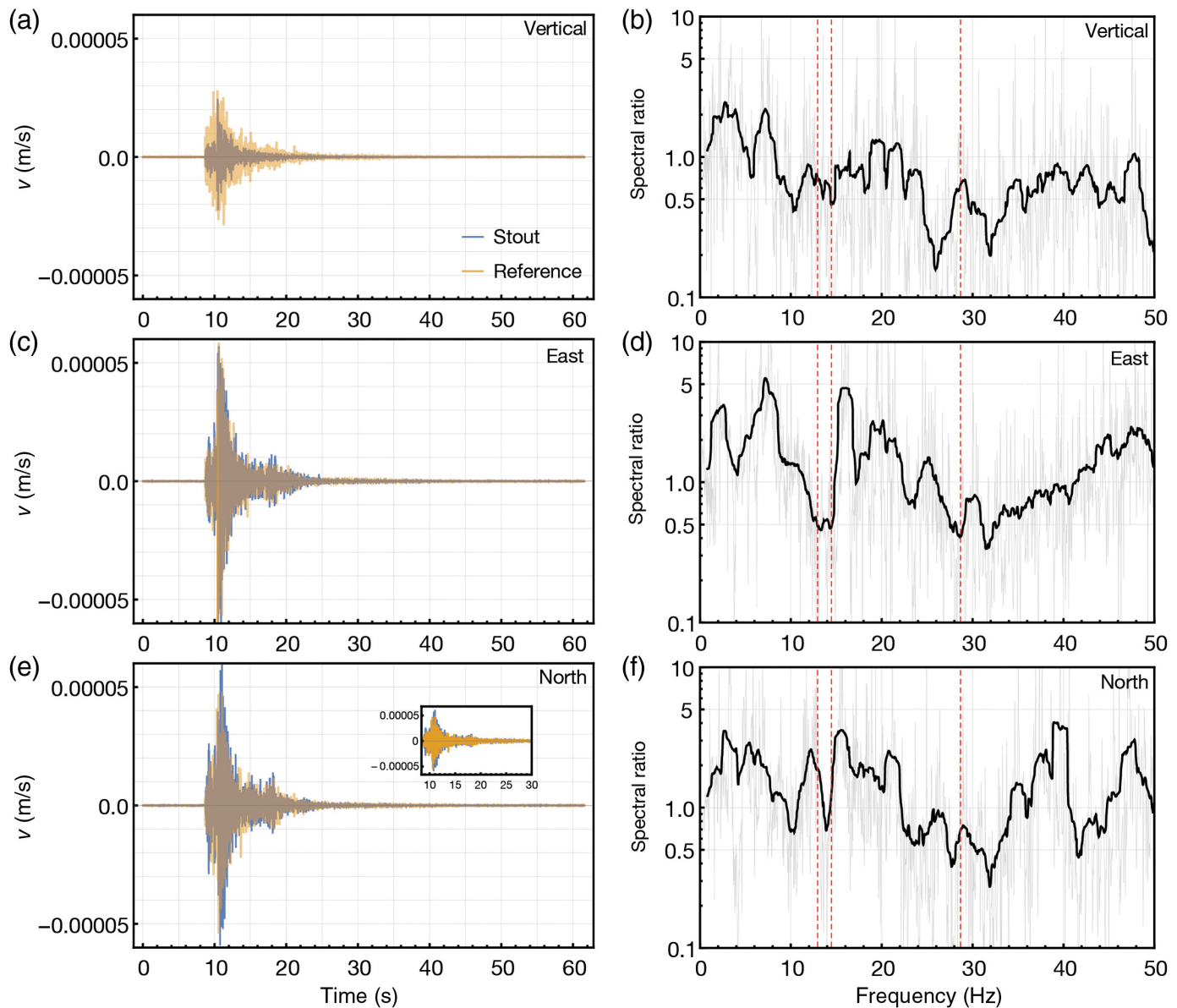


of 10 measurements that we made using a slightly modified version of ASTM-D5873-14 (2014). The single difference was that no effort was made to polish the tufa rock surface before measurement. Tufa is a porous rock, and polishing was not a reliable method for achieving a flat, clean rock face for contact with the rebound hammer. Instead of polishing, extreme care was taken to identify areas of the rock face where the existing tufa was clean and smooth. Care was also taken to ensure that the tufa rock facies were identical for the base and midsection groups. These considerations precluded the possibility of making measurements on similarly oriented faces. The orientations of the measured rock faces varied widely from RT to RT. Measurements were corrected to the horizontal measurement angle. Over 560 rebound hammer strikes were recorded on 14 RTs (McPhillips, 2022).

Figure 6 plots the ratios of rebound numbers at the midsection to rebound numbers at the base as a function of the RT

Figure 3. Seismic observations from Skinny and the reference station. (a,c,e) Velocity records during the nearby M 1.9 earthquake were used in subsequent simulations. The time interval that is simulated is shown in the inset (McPhillips, 2022). (b,d,f) Spectral ratio of the records from the same time interval (Skinny's over reference locations). The gray background line represents raw results, and the bold black line represents their moving average using a 1.6 Hz window (McPhillips, 2022). Red vertical dashed lines represent the position of vibration modes predicted by finite-element method (FEM) models with a compliant base (see the Numerical modeling of RT dynamic response section). Notice the vertical axis in a logarithmic scale. The color version of this figure is available only in the electronic edition.

aspect ratio. We here define the aspect ratio of an RT as the quotient of the height of the center of mass and the short radius of the best-fit ellipse at the base. Measurements were made using publicly available light detection and ranging (lidar) data (see Data and Resources; Hudnut et al., 2020), following the



methods described in [McPhillips and Scharer \(2021\)](#). Overall, the midsection-to-base rebound number ratio varies by about a factor of two. The aspect ratios of the RTs we measured ranged over a factor of five. Plotted together, the measurements show a clear trend toward relatively weaker rock at the base of relatively slender RTs (Fig. 6). There is a large gap in the measurements that reflects the absence of extant RTs with aspect ratios higher than 2.5 at the Trona Pinnacles, with Skinny being the only exception that we could identify.

We calculated tensile strength values from the rebound hammer data using the method described in [McPhillips and Scharer \(2021; Fig. 7\)](#). The ratios of tensile strength values from the midsections and bases are similar to the rebound number ratios. Although the midsections are only about 10% stronger on average, the ratios range over about a factor of 2. For comparison, [Sopaci and Akgün \(2015\)](#) provide laboratory measurements of tensile strength for similar tufa facies from Antalya,

Figure 4. Seismic observations from Stout and the reference station. (a,c,e) Velocity records during the nearby M 1.9 earthquake were used in subsequent simulations. The time interval that is simulated is shown in the inset ([McPhillips, 2022](#)). (b,d,f) Spectral ratio of the records from the same time interval (Stout's over reference locations). The gray background line represents raw results, and the bold black line represents their moving average using a 1.6 Hz window ([McPhillips, 2022](#)). Red-vertical-dashed lines represent the position of vibration modes predicted by FEM models with a compliant base (see the [Numerical modeling of RT dynamic response](#) section). Notice the vertical axis in a logarithmic scale. The color version of this figure is available only in the electronic edition.

Türkiye. The mean laboratory measurement is 1.8 ± 1.1 MPa, and our rebound hammer data yield a mean value of 1.7 MPa. It is important that these strength measurements almost certainly overestimate the true strength of RTs because of a scale effect ([Yoshinaka et al., 2008](#)). We omitted the scale-effect

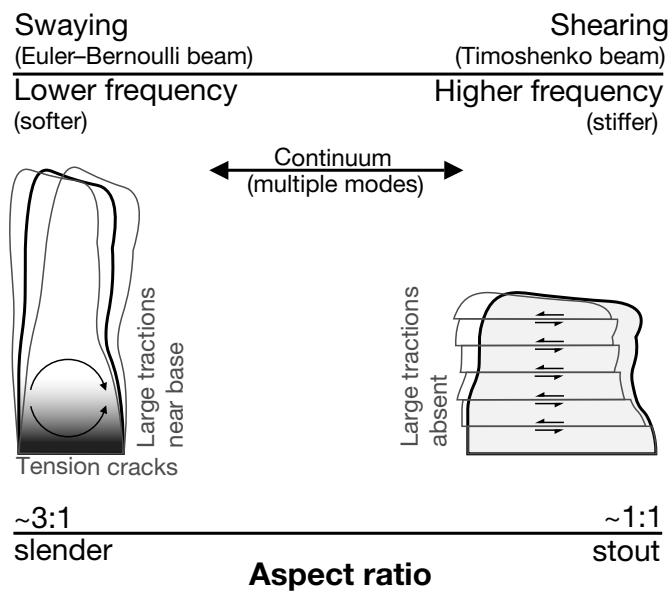


Figure 5. Schematic illustration of the seismic response of high- and low-aspect-ratio RTs.

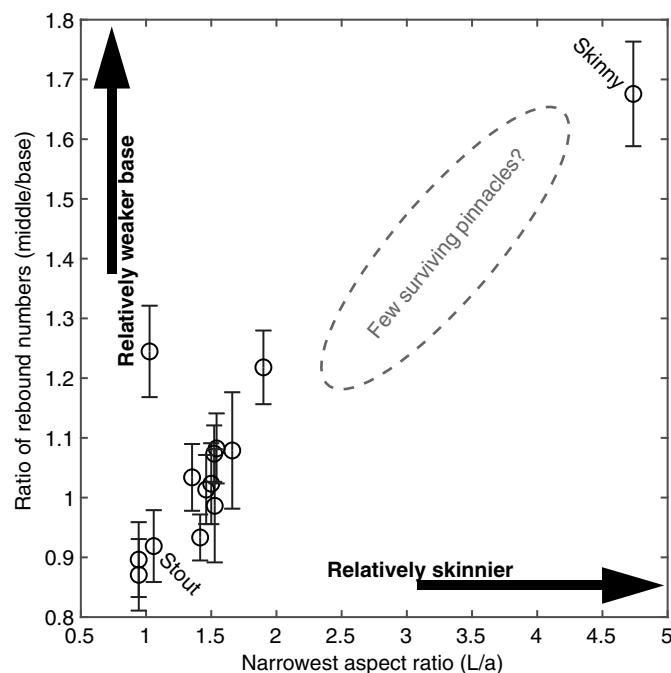


Figure 6. Rebound hammer results as a function of aspect ratio, in which L is the height to the center of mass and a is the short radius of the best-fit ellipse approximating the basal cross section (McPhillips, 2022). Error bars are the standard deviation of the rebound number ratios. Higher aspect ratio RTs consistently show relatively weaker bases. These data are a sample of the Northern Group Pinnacles, and many more existing Pinnacles were not measured.

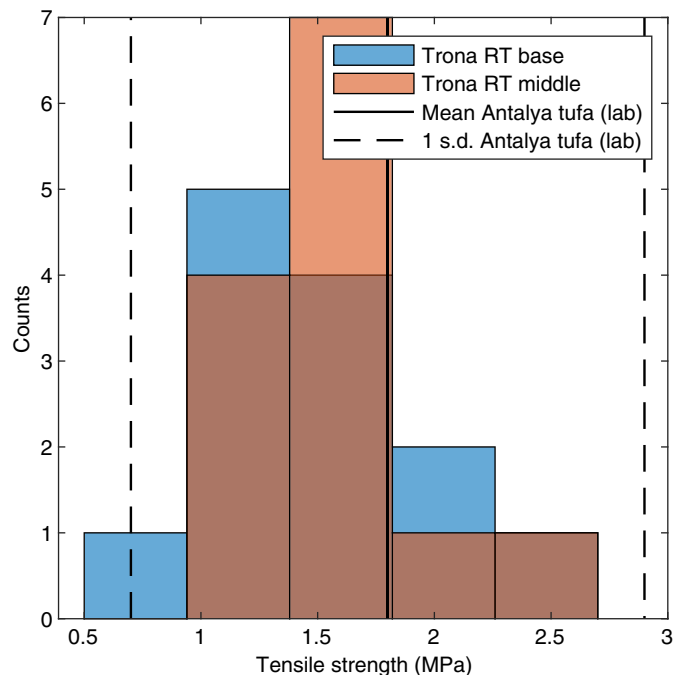


Figure 7. Comparison of tensile strength results from the Trona Pinnacles and Antalya tufa (Sopaci and Akgün, 2015; McPhillips, 2022). Histogram bars are semitransparent, showing the overlap among midsection and base RT measurements at the Pinnacles. The tufa rock at both sites exhibits similar facies characteristics (Guo and Chafetz, 2012; Sopaci and Akgün, 2015). The tensile strength of the Antalya tufa has been characterized with standard laboratory methods, whereas the strength of the Pinnacles tufa was calculated from in situ rebound hammer measurements. The color version of this figure is available only in the electronic edition.

correction from our calculations here to compare our results on a similar basis; both types of measurements interrogate similar volumes of rock (McPhillips and Scharer, 2021).

Photogrammetric point clouds

To model the seismic response of the RTs, we create detailed 3D models using photogrammetry. We used different datasets for the RTs Skinny and Stout because of their different scales. For Stout, we use the point cloud derived from aerial imagery described in Donnellan *et al.* (2022). For Skinny, we followed the methods described in Reitman *et al.* (2015), using Agisoft Photoscan (Agisoft, 2019) to build a dense point cloud from many overlapping photographs taken using a consumer-grade camera. We then built finite-element meshes from the point clouds, as described in Appendix A (McPhillips, 2022).

NUMERICAL MODELING OF RT DYNAMIC RESPONSE

Mechanical properties

We next developed numerical models of the two RTs to study their overall deformation response when subjected to seismic shaking and qualitatively test our progressive failure hypothesis.

We validate our numerical models by comparing the field-measured vibration characteristics and aftershock response to our numerical predictions. For model building, slender RTs can be considered homogeneous calcium bicarbonate formations (tufa). Larger, low-aspect-ratio towers are often consistent of two different tufa facies on the interior and exterior with somewhat different mechanical properties (Guo and Chafetz, 2012). Because we cannot easily observe the RT interiors, we modeled both Skinny and Stout as uniform materials. The laboratory work by Sopaci and others (Sopaci and Akgün, 2015; Sopaci et al., 2019) serves as a reference to compare with and complement our field measurements: they reported, for several similar facies of tufa in the region under study, values of Young's modulus of 4.2 ± 3.0 GPa, whereas Poisson's ratio was 0.08 ± 0.04 . From Sopaci and Akgün (2015), the average density (unit weight) was reported as 1950 kg/m^3 , the average porosity being 14.7%. The consistency of our in situ tensile strength estimates with these laboratory data supports their applicability to the Pinnacles (Fig. 7). In the following set of elastodynamic analyses of the RT models, we used the elastic properties in the range reported by Sopaci and Akgün (2015) and Sopaci et al. (2019): $\rho = 1700 \text{ kg/m}^3$ (density), $\nu = 0.15$ (Poisson's ratio), and $E = 1.7 \text{ GPa}$ (Young's modulus).

Natural frequencies and vibration modes

As a first step, we made modal analyses of the finite-element meshes to examine the natural frequencies of the RTs. The natural vibration of a structure indicates the deformation field (vibration mode) and the rhythm of oscillation (natural period) at which the structure tends to vibrate when excited dynamically. The vibration mode and natural period typically control the amount of deformation that a structure sustains in response to seismic excitation. If the excitation aligns with the vibration mode and the frequency content of the excitation overlaps with the natural period, the structure may experience the phenomenon of dynamic amplification (termed "resonance" in the most extreme versions; Clough and Penzien, 2004). In this case, the structure will deform much more than predicted by quasi-static loading at the same intensity. This larger deformation entails larger internal forces in the system, more material degradation, and a higher probability of structural collapse.

We used the Mathematica (Wolfram, 2000) function NDEigensystem with default settings to extract the natural vibration information. This approach assumes a linear elastic isotropic material response. Although this assumption is not accurate for progressive failure, it is a good first approximation to describe what the RT experiences when subject to low-to-medium intensity shaking, such as small-magnitude aftershocks.

High-aspect-ratio RT (Skinny). We first modeled this high-aspect-ratio RT as an isolated deformable body attached to a rigid base. The implication of this decision, which is the most

TABLE 1
Influence of Base Compliance on Skinny's Vibration Modes

Model Comparison	First Mode	Second Mode	Third Mode
Without pedestal	16.61 Hz	18.07 Hz	50.50 Hz
With pedestal	12.99 Hz	15.07 Hz	45.20 Hz
Difference	27.8%	28.4%	11.7%

typical approach for this class of problems, is that once the vibrational energy—seismic, environmental, or otherwise—enters the system, the only way for it to be attenuated is through internal damping mechanisms such as microcracks that can lead to rock degradation. Values of the first three natural frequencies are listed in Table 1. Because it can be readily seen in Figure 8, the two first vibration shapes correspond to bending modes, whereas the third corresponds to a torsion mode.

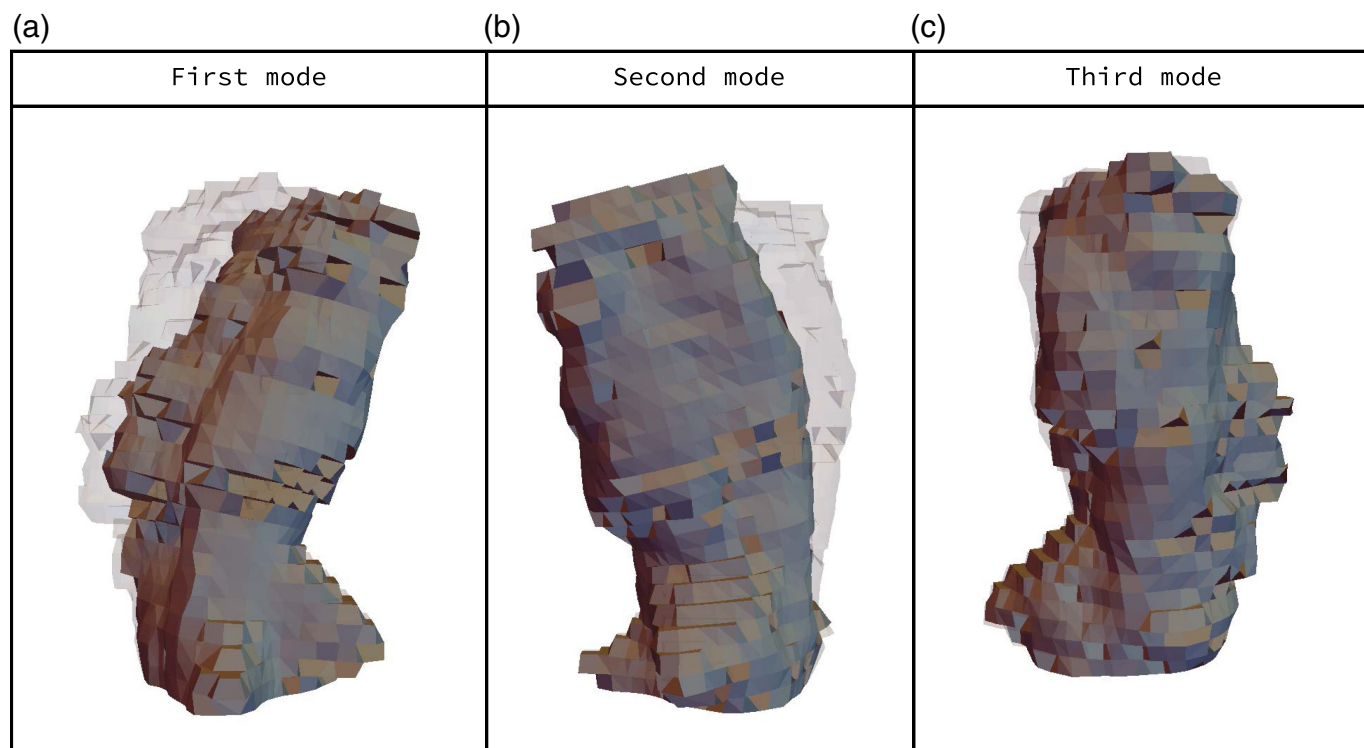
We also create a more realistic model representation of the RT, which acknowledges that the RT is attached to an extensive tufa rock deposit at its base. This bedrock is similar to the RT itself and, therefore, somewhat compliant. To capture this reality, we next modeled the RT and a cuboid pedestal of 6×6 square meters cross-section (centered at the RT's center of mass) with 4 m depth. The material is assumed to be the same as the RT. The boundary conditions are clamped faces at the bottom and the flanks of the cuboid pedestal.

Taking the model with the compliant pedestal as the true value, Table 1 shows that removing the surrounding tufa increases the first two natural frequencies: the deviation magnitude is more than 25% compared to the value with the pedestal. The third (torsional) mode, while it also increases, it only does by a 12% approximately, which can be explained if one can visualize how much less mass from the pedestal participates in the torsional vibration of the RT compared to its transverse vibration.

The differences between the two models reflect the classic phenomenon in soil-structure interaction termed "period lengthening" (Kausel, 2010): the surroundings add more mass than stiffness to the overall system, increasing the natural period (lowering the natural frequency).

We also point out that the fundamental-mode shape in Figures 8 and 9 barely changes. In other words, the inclusion of the far field in the model reduces the natural vibration frequency, but it does not change the mode shapes (i.e., in mathematical terms, we would say that the eigenvalue decreases, and the eigenvector remains the same).

We also modeled the low-aspect-ratio RT using both a rigid and a compliant base. Similar to Skinny, the first two modes of the rigid-base model of Stout represent bending modes (Fig. 10). These bending modes were accompanied by more shearing in this case. The third mode is a so-called "breathing" mode: it features "inflation and deflation" of the upper portion of the RT as if it were a balloon.



Low-aspect-ratio RT (Stout). Unlike Skinny, however, for which the surroundings are fairly leveled, Stout sits on a slope (Figs. 1c, 11), which adds complexity to its modeling. Fortunately, the slope barely affects its natural vibration features because they depend on the mass and stiffness distribution, not on the relative inclination of the structure with respect to the flat ground. On the other hand, it would require extra care to set up a model to be subjected to transient ground shaking because how waves arrive at the base of the structure and the shaking they induce depends on the orientation with respect to the vertical.

Results with and without ground compliance are shown in Table 2. The magnitude of the difference after adding ground compliance is greater for Stout than for Skinny. The first two modes remain of similar shape, but the frequencies decrease substantially: the values without the pedestal are no less than 45% lower than the ones where the surroundings are accounted for.

However, the third mode changes shape as well as frequency: it goes from a breathing mode to an extensional mode along the longitudinal axis of the RT. Thus, the addition of extra compliance around the base plays a critical role for this mode because it changes both the shape and the frequency of the deformation.

Model validation using field records. Next, we validated our numerical models using data from the M 1.9 aftershock from the [Seismic instrumentation](#) section to assess the concordance between the structural vibration measured in the field and the natural frequencies obtained numerically. The event

Figure 8. Results of modal analyses: (a) first, (b) second, and (c) third vibration shapes of the RT Skinny, when modeled with an isolated base. The natural vibration modes depicted here are exaggerated for emphasis and do not represent the amplitude of the actual deformation during forced shaking. The color version of this figure is available only in the electronic edition.

was chosen because it was high in signal-to-noise ratio yet small in magnitude, so it would properly reflect the linear elastodynamic response of the RTs modeled in our simulations. This is a test from the [Model validation using field records](#) section.

Skinny's spectral amplitude ratio shows two peak components within the interval 10–20 Hz (Fig. 3). The first peak occurs at around 13.5 Hz, close to the numerically predicted value for the fundamental frequency in Table 1. However, there is a second peak at about 17.9 Hz, which better matches the value for the second mode without pedestal than the model with pedestal (Table 1). We suggest that this discrepancy may be associated with the choice of the pedestal shape or material properties because the model with a compliant pedestal is likely more realistic than a model with a rigid (i.e., isolated) base.

In contrast to Skinny, Stout's spectral amplitude ratio reveals deamplification in the horizontal components at frequencies consistent with the predicted vibration modes (Table 2). We speculate that deamplification could be due to those modes being excited at frequencies much higher than their vibration frequency. However, the seismic data also reveal amplification peaks that do not match the natural frequency predictions. These peaks occur both at low frequencies

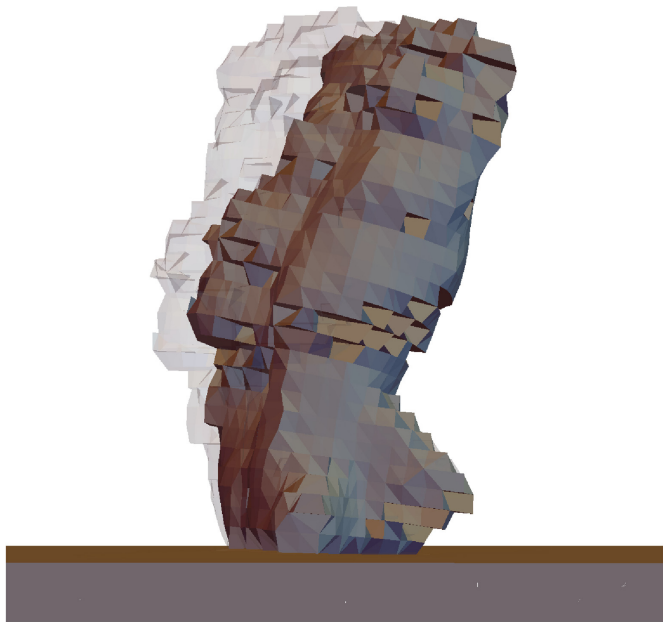


Figure 9. Results of modal analyses: first, second, and third vibration shapes of Stout, when modeled with an isolated base. The natural vibration modes depicted here are exaggerated for emphasis and do not represent the amplitude of the actual deformation during forced shaking. The color version of this figure is available only in the electronic edition.

(~2 and ~8 Hz) and at higher frequencies (~39 and ~48 Hz; Fig. 4). The magnitudes of these dynamic amplifications are much smaller than observed for Skinny (around $\times 5$ vs. $\times 10$), but their presence suggests clear limitations in the numerical model for Stout, in particular. These limitations could result in part from a geometric reconstruction from a relatively sparse

topographic dataset (Donnellan *et al.*, 2022). We also recall that the larger RTs at Trona, such as Stout, are not typically uniform and consist of distinct interior and exterior tufa facies (Guo and Chafetz, 2012). Therefore, the model for Stout appears less reliable than Skinny's.

Transient analysis

To further explore the discrepancies between our field measurements and modal analysis (described earlier), we used transient numerical simulations of the slender RT (Skinny). We focus on Skinny because our previous analysis suggests that Skinny's dynamic response is better captured with numerical models. The transient simulations were carried out in Seismo-VisLab (Kusanovic *et al.*, 2023) under the assumption of linear elastic isotropic material response. In these simulations, we used the same material parameters as in the modal analysis described in the Natural frequencies and vibration modes section. The simulations were carried out using an implicit solver (Newmark-beta method; Cook *et al.*, 2007). The timestep and simulation span are consistent with the data recorded with the seismometers. The domain-reduction method (Bielak *et al.*, 2003; Yoshimura *et al.*, 2003) was used to propagate the incoming wavefield from the far field toward the ground surface, which represents an advantage compared to other software packages. The incoming wavefield amplitudes were assumed to be one-half of the displacement recorded by the free-field seismometer during the first after-shock event (M 1.9) introduced in Figure 3 (to account for the "free-surface effect," Kramer, 1996) and comprised of vertically propagating, decoupled P and S components. Lysmer dashpots (Lysmer and Kuhlemeyer, 1969) were installed at the boundaries numerical domain to prevent reflections of

First mode	Second mode	Third mode

Figure 10. Fundamental mode shape of Skinny for a modal analysis with a compliant pedestal (not fully pictured). The natural vibration modes depicted here are exaggerated for emphasis and do not represent the amplitude of

the actual deformation during forced shaking. The color version of this figure is available only in the electronic edition.

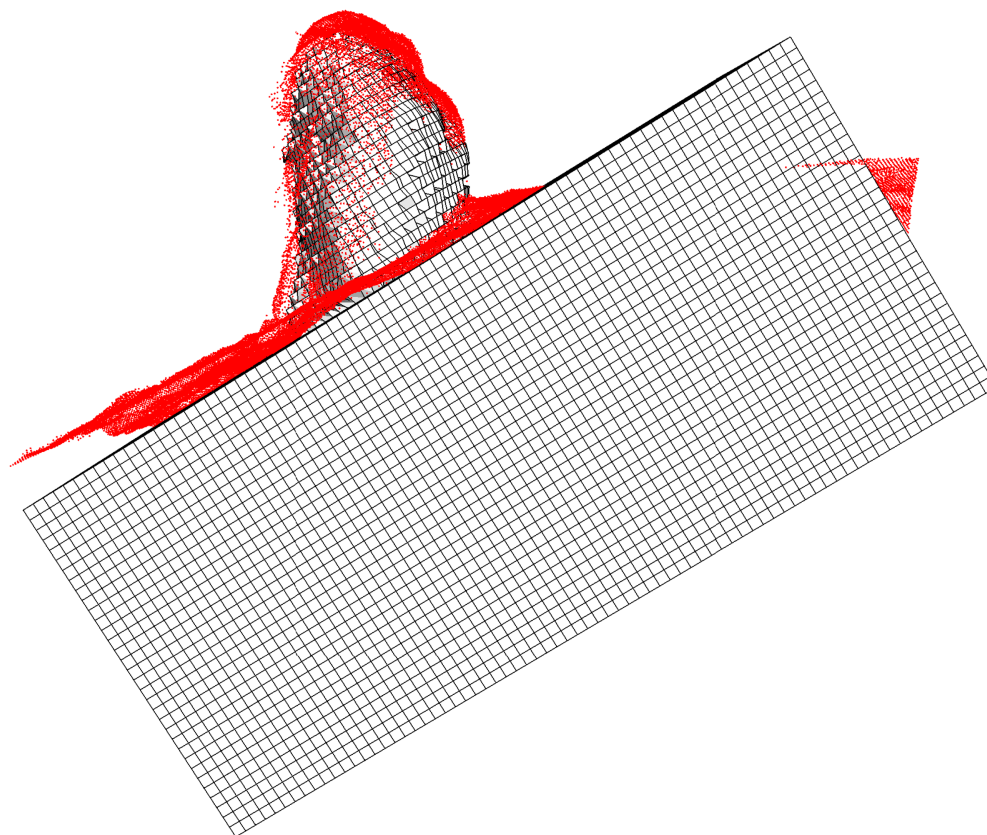


Figure 11. Original point cloud overlying Stout's final FEM model mesh (inclination in the figure reproduces the real one, see Fig. 1c). The mesh surface that defines the RT approximates the photogrammetric point cloud, which includes noise in the form of spurious points. The rectangular-prismatic pedestal mesh approximates the first-order ground surface slope in the point cloud. The color version of this figure is available only in the electronic edition.

the scattered wavefield, thus properly accounting for radiation damping. The mesh of this model comprises 90,619 elements, whereas the number of degrees of freedom (dof) is 294,195. See Figure 12 for illustration. Parallelization using multi-frontal massively parallel solver (MUMPS, Amestoy *et al.*, 2001, 2006) and message passing interface (MPI; Gabriel *et al.*, 2004; Hursey *et al.*, 2007; Broquedis *et al.*, 2010) is employed.

Results

We compare the transient model results with the seismic recording by plotting the velocity output from a model node at the top of the RT, at approximately the same position as the broadband seismometer (Fig. 13). There are several important observations.

tions that overlaps with the frequency of the most prominent peak in the seismic recording. The transient model results also match its amplitude. However, the transient model results typically fail to capture the details of the seismic recording, which shows multiple lower-amplitude peaks.

We note that there are no fit parameters in this model, unlike prior work (e.g., Finnegan *et al.*, 2022). In this light, the comparison between the spectral content of the response in the horizontal directions up to approximately 30 Hz (the frequency range of engineering interest) is quite good. Some tuning to better match the true material properties could improve model performance. For example, the addition of material damping could increase the high-frequency decay of the simulated time series and bring the results closer to the measured data.

TABLE 2
Influence of Base Compliance on Stout's Vibration Modes

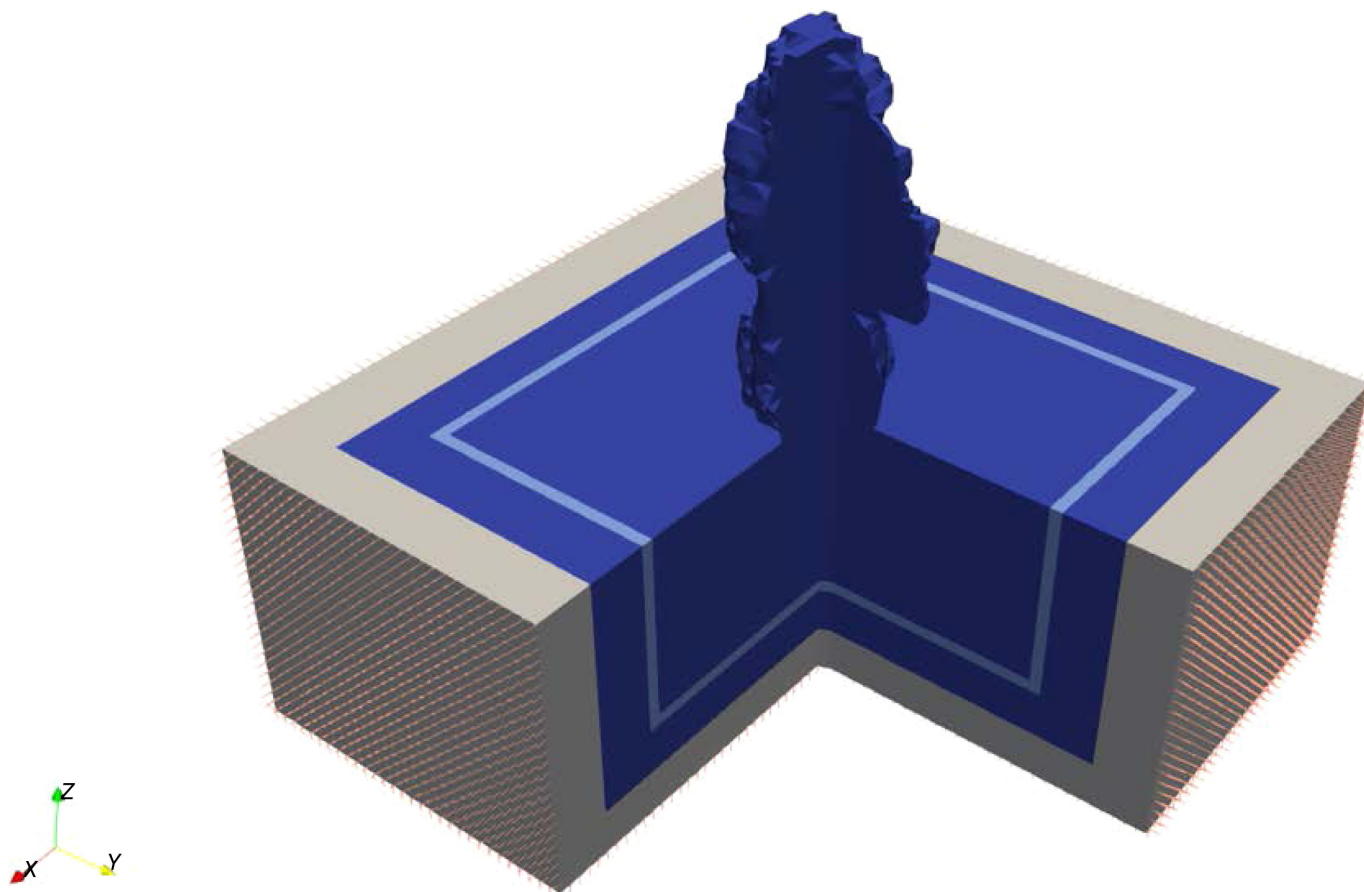
Model Comparison	First Mode	Second Mode	Third Mode
Without pedestal	18.76 Hz	21.03 Hz	37.09 Hz
With pedestal	12.94 Hz	14.46 Hz	28.67 Hz
Difference	45.0%	45.4%	29.4%

First, the response envelope recorded in the field is generally captured by the simulations, albeit with a slight time offset. Second, the transient model results underestimate the amplitude of maximum velocity in both horizontal directions (located at around 2 s). Third, the breathing phenomenon appears in the FEM response due to the existence of two dominant modes that partially overlap (i.e., provoke deformation along the same direction with very similar frequencies), recall the results presented in the High-aspect-ratio RT (Skinny) section. The real response does not present this phenomenon, at least not markedly, as the two bending modes are more "distanced" from each other than in the numerical model.

The Fourier amplitude spectra of both time responses support these observations (Fig. 13). The transient model results show a single, broad peak in the horizontal direc-

SEISMIC ARCHETYPES OF RTs: A SIMPLIFIED APPROACH

Numerical analyses of rock formations have proved to be valuable tools when it comes to assessing the response of both PBRs (Anooshehpour *et al.*, 2004, 2013) and RTs (Lignola *et al.*, 2010; Moore *et al.*, 2019) subjected to ground shaking. In prior sections, we illustrated how simulations can reveal the main



traits of the dynamic response observed in the field in some cases. However, simulations of the kind that are warranted to reflect the seismic response of RTs are time-consuming (even when using heavy parallelization, it still takes hours to run a transient simulation as those shown already) and require expert users to prepare and execute them. A simplified approach for understanding the evolution of these features is, logically, highly desirable.

As a compromise between accuracy and simplicity (in terms of geometry, material, and mechanical response, as well as analysis method), we present below the definition of “seismic archetypes,” which display the same principal traits of the more complex numerical simulations while confronting the problem analytically. A reasoned application of dimensional analysis and Buckingham theorem (Appendix B; Buckingham, 1914) leads to the conclusion that, under assumptions of geometric regularity (cylindrical shape) and small strains, a functional form of the kind of

$$\frac{\omega_n}{\sqrt{\mu/\rho/\ell_1}} = \bar{g}\left(\nu, \frac{L}{\ell_1}, \frac{\ell_2}{\ell_1}\right), \quad (1)$$

can be argued, in which ω_n is the fundamental frequency, μ , ν , and ρ are the shear modulus, Poisson’s ratio and the density of tufa, respectively, L is the height of the pillar, ℓ_1 and ℓ_2 are two dimensions assumed to suffice to describe the constant cross section of

Figure 12. Three-fourths of the FEM of Skinny for time-history transient simulations. The orange line elements at the edges represent Lysmer dashpots, whereas the lighter blue cubes within the box are the domain-reduction elements. The color version of this figure is available only in the electronic edition.

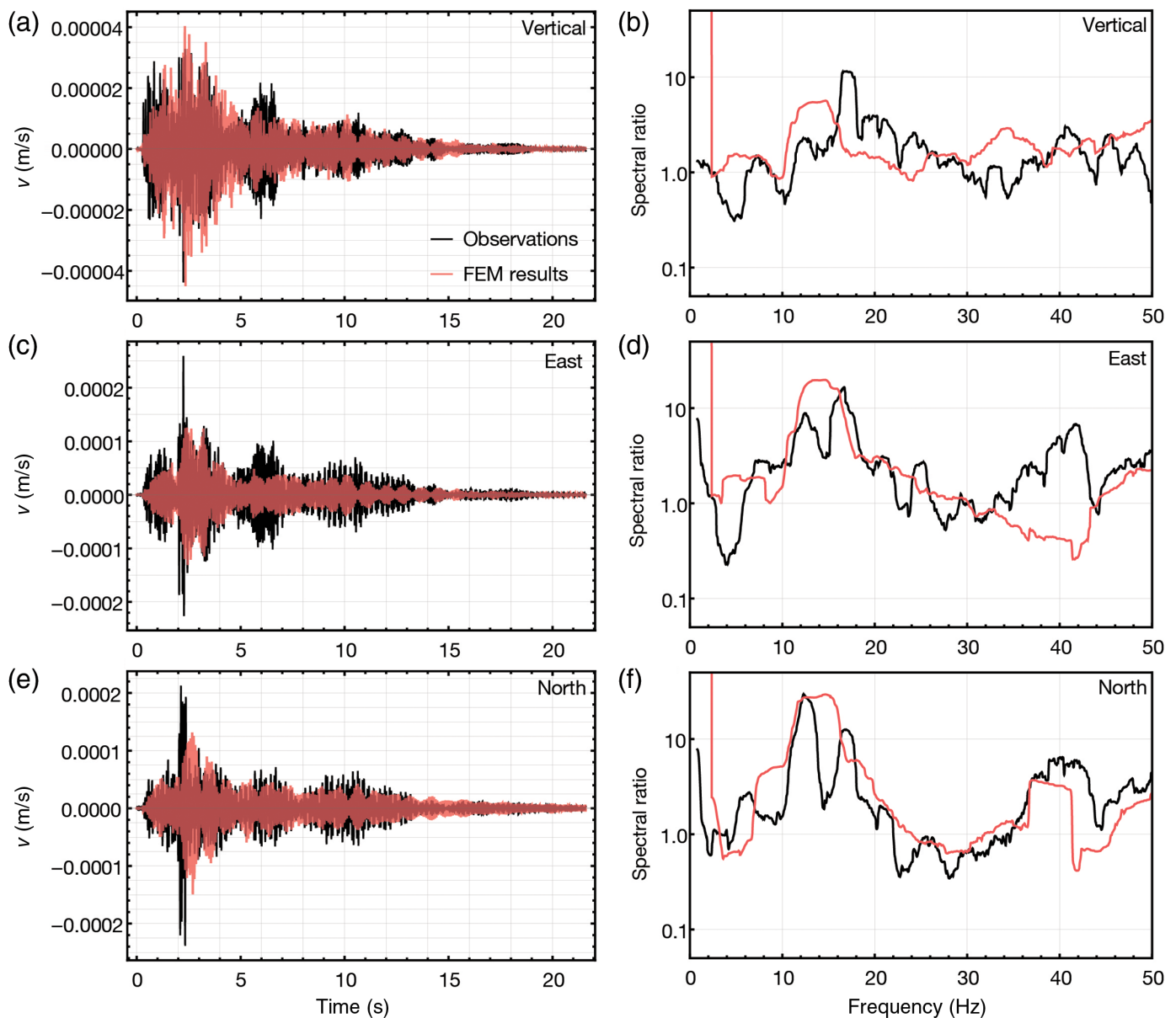
the archetype (e.g., these could be thought as the respective lengths of the two axes of an ellipse). See Appendix B for all the details.

We define an archetype as an RT that has the same materials parameter values as the reconstructed shape, and resembles the inertia and stiffness distribution of the real shapes so that the archetype’s natural frequencies are comparable to the original RT characteristics. In the two paradigmatic cases we are to discuss (Skinny and Stout), visual inspection suggests that using a cylindrical shape can be a good first-order approximation, so a circular cylinder archetype ($\ell_1 = \ell_2$) will be adopted.

Granting a circular cylinder archetype, its height is chosen to be the same as the RT’s at hand L_r , and the last parameter ℓ_1 , which in this case is identified with the radius of the circular cross section, is chosen so that the volume (mass) of the archetype is the same as the RT’s:

$$\ell_1 = R = \sqrt{V_r/\pi L_r}, \quad (2)$$

in which ω_n represents the volume of the RT as computed from the analysis of the point cloud.



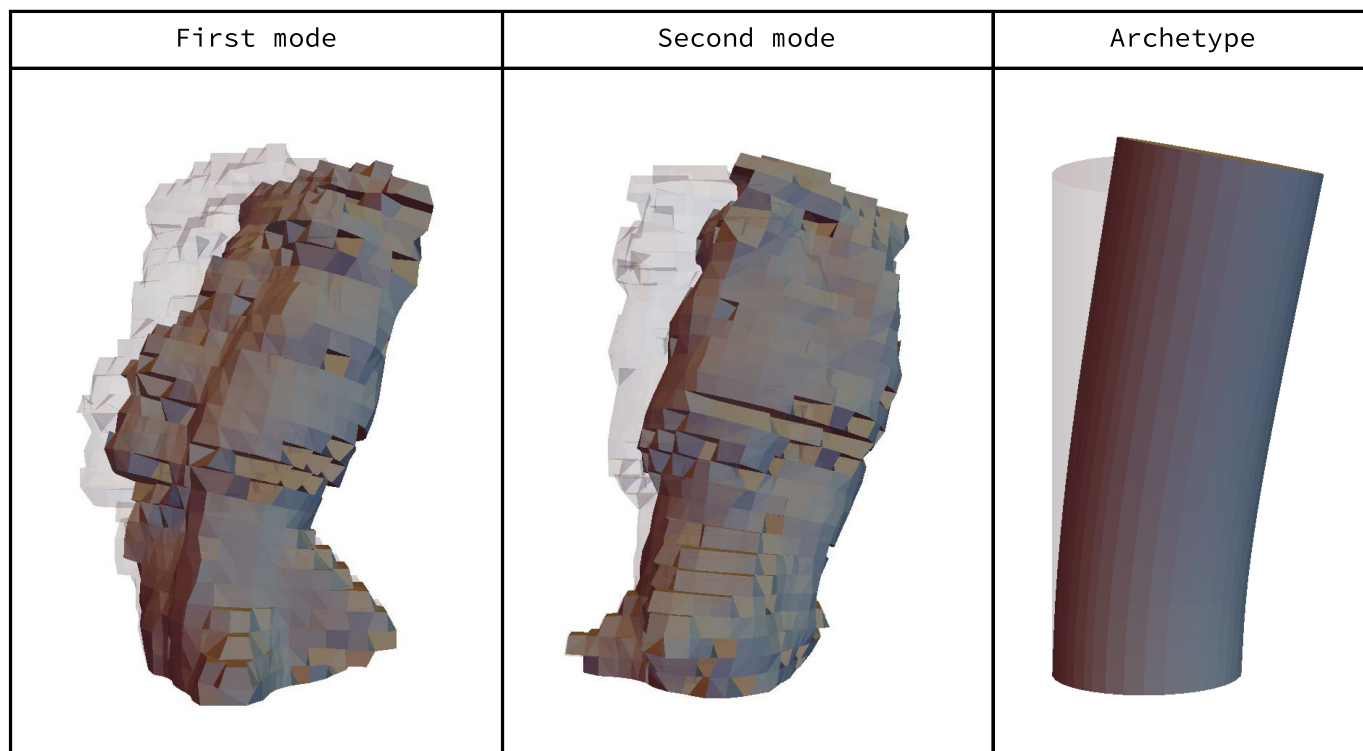
Skinny

Based on the reconstructed model, we assert that the height in this case is $L_r = 2.8$ m, whereas $V_r = 2.52$ m³. Once the geometry is defined, the cylinder is discretized in a finite-element mesh and the natural frequencies and vibrational modes are computed (Figs. 14, 15). The comparison to the values obtained from the reconstructed shape (using a mesh of quadratic-interpolation elements in both cases) is presented in Table 3.

To capture the phenomenon termed “period lengthening” (i.e., frequency reduction), which was already observed in the FEM study (see the [High-aspect-ratio RT \(Skinny\)](#) and the [Low-aspect-ratio RT \(Stout\)](#) sections), we use the theory of soil-structure interaction (see [Kausel, 2010](#), and references therein), which lumps the compliance provided by the surroundings as an equivalent spring constant. Consider that the resistance of the beam to bending was described by a

Figure 13. Results of the transient numerical analysis. (a,c,e) Velocity response of a node near the top of the RT Skinny, in the three principal directions (red lines). For comparison, the observed seismic record of Skinny’s response is also shown (black lines). Both the numerical analysis and the seismic observation show the response to the same low-magnitude earthquake. (b,d,f) Results as spectral ratios (Skinny’s response over reference station’s, see Fig. 1a). For comparison, the observed spectral ratios are also plotted (black lines). Both data sets have been smoothed with a moving average using a 1.6 Hz window. Note vertical axis has a logarithmic scale. The color version of this figure is available only in the electronic edition.

lumped parameter k_r . Mechanics of materials calculations provide $k_r = 3EI/2L$, in which E is the Young’s modulus of the material, I is the cross-section inertia, L is the RT height; k_r establishes the proportionality between rotation of the beam’s free end to the bending moment at its clamped end (assuming this moment was generated by a constant distributed load).



The “rocking impedance” K_r for a cylindrical foundation of radius R represents the proportionality factor between a moment applied on a circular path of the surface of a half-space and the rotation experienced at the same point in the surface, and it is given by $K_r = 8\mu R^3/3(1 - \nu)$, μ and ν representing the shear modulus and the Poisson’s ratio of the ground surface material, respectively (recall that we observe that the RT and the immediate surroundings are made of tufa).

Abstracting the whole system of structure plus ground compliance into a one-degree-of-freedom system, we can compute its natural frequency using the two springs acting in parallel, so the formula for the shortened natural frequency $\tilde{\omega}_n$ is

$$\tilde{\omega}_n = \frac{\omega_n}{\sqrt{1 + k_r/K_r}}, \quad (3)$$

in which the last denominator is but the period-lengthening factor (see [Veletsos and Meek, 1974](#)). The factor can be computed easily, taking advantage of the fact that the material in structure and ground is the same, yielding $\sqrt{(1 + k_r/K_r)} \approx 1.47$, which translates into about 50% “lengthening” of the natural period, 30% “shortening” of the fundamental frequency, thus $\tilde{\omega}_n = 0.68 \times \omega_n = 0.68 \times 17.82 \text{ Hz} = 12.15 \text{ Hz}$. This last value compares noticeably well with the values obtained for the first, 12.99 Hz and second frequency 14.07 Hz (Table 1).

This exercise illustrates the capacity of the archetype model to capture not only the structural response but also the influence of the surrounding ground on the dynamic trait of the RT.

Figure 14. Comparison of the first and second modes of the reconstructed shape to the first and second deformation modes of the archetype. The color version of this figure is available only in the electronic edition.

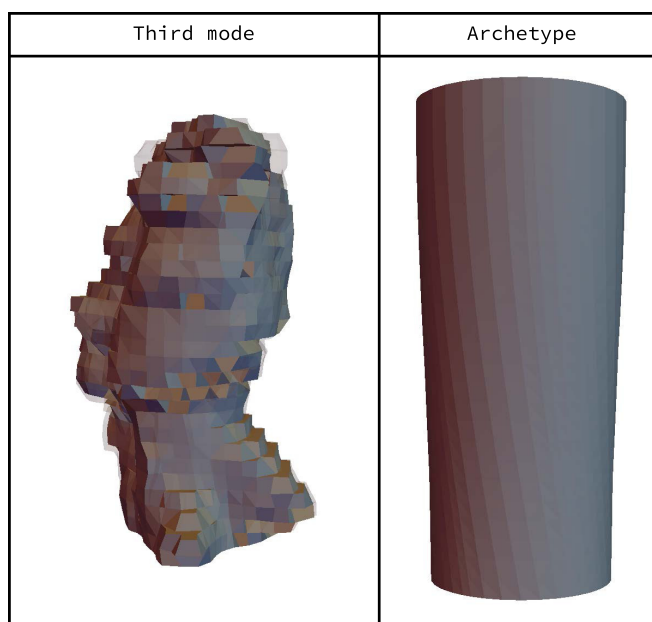


Figure 15. Comparison of the third mode of the reconstructed shape to the third mode of the archetype. The color version of this figure is available only in the electronic edition.

TABLE 3
Comparison of Real-Shape Values to Archetype's (Quadratic Meshes)

Model Comparison	First Mode	Second Mode	Third Mode
Point cloud	16.22 Hz	17.63 Hz	49.40 Hz
Archetype	17.82 Hz	17.82 Hz	58.90 Hz
Difference	9.9%	1.1%	19.4%

Stout

The aptitude of the simple archetype to capture bending behavior has been attested in the prior section that focused on the slender RT. Next, we move to the bulky RT and to show how the archetype can help explain the dynamic deamplification along the vertical direction that is observed in the field measurements amply discussed in the [Natural frequencies and vibration modes](#) section.

Let us begin by introducing the vertical stiffness of the RT. This computation is easy using the cylindrical archetype and assuming that it behaves as a single-degree-of-freedom system when stretched vertically, which can be described with mechanics of materials theory too, thus its stiffness to this kind of deformation is $k_z = EA_{\text{arch}}/L_{\text{arch}}$. The impedance for a circular foundation, radius R , was found by Boussinesq's solution to be $K_z = (4\mu R)(1 - \nu)$ (Kausel, 2010).

Similar arguments can be used to find natural frequency estimates. Plugging values ($V_r = 84.3 \text{ m}^3$, $L_r = 6.0 \text{ m}$, and $A_r = 14.1 \text{ m}^2$) we find $\tilde{\omega}_n = 21.4 \text{ Hz}$, a value even smaller than the one obtained for the third mode in Table 2 using an FEM that includes a portion of the surroundings. This value should be taken as an upper bound of the frequency, which could be even smaller in reality, because the mass of the soil that is mobilized during the modal deformation is not properly accounted for in Figure 3. For instance, if the soil mass engaged during deformation was five times that of the RT, the natural frequency would change to $\tilde{\omega}_n = 8.7 \text{ Hz}$.

We hypothesize that one possible explanation for the observed deamplification could be the soil-structure interaction phenomenon, along with the low value of the natural frequency. The system pillar plus surrounding rock behaves as a unit deformable block, for which the natural frequency happens to be smaller than the range of frequencies of excitation (either ambient noise or aftershock) tends to cluster. It is known from fundamental structural dynamics (Clough and Penzien, 2004) that in these cases the system responds to the dynamic loading in the so-called high-frequency regime: the structure vibrates at the frequency of the excitation but displaying an amplitude smaller than that of the excitation itself.

DISCUSSION

Seismic response of the Trona Pinnacles

The proximity of the Trona Pinnacles to the 2019 Ridgecrest earthquake sequence provides a rare opportunity to study the

dynamic response of RTs to earthquake shaking. Our observations differ in that regard from previous observations, which utilized ambient environmental noise (e.g., Lignola *et al.*, 2010; Finzi *et al.*, 2020; Finnegan *et al.*, 2022). We also know that the RTs analyzed here survived a sequence of well-characterized, damaging earthquakes, likely in their present form (Goulet *et al.*, 2019; Donnellan *et al.*, 2022). However, this natural laboratory is limited by the complex, heterogeneous character of the tufa rock (Guo and Chafetz, 2012). An ideal site would have consisted of a simpler lithology. Although we believe we captured the average material properties of the tufa in our models, supported by the good agreement in mean tensile strength values from our in situ rebound hammer measurements and compiled laboratory data, there is clearly variability among the RTs and even within individual RTs. With this variability in mind, we chose not to tune the material property parameters in our models, instead evaluating how effective a simple approximation could be.

Despite this challenge, the observations in this case study support several inferences from the model and theory. The seismometer on the top of Skinny shows a clear velocity amplification compared to the reference station nearby (Fig. 3). The spectral ratio reveals that amplification occurs at specific frequencies (Fig. 3). In contrast, Stout shows a much more muted response within the range of observed frequencies, with modest amplification and deamplification in specific frequency bands (Fig. 4).

The modal FEM analyses show mixed success at reproducing the specific frequencies of amplification. For Skinny, the modal analysis correctly identifies the fundamental frequency between 12 and 18 Hz in the horizontal directions (Fig. 3). This is consistent with a bending mode vibration, as observed in other similar analyses (Fig. 8; Moore *et al.*, 2019). The modal analysis of Stout shows a poor fit in both horizontal directions (Fig. 4). The seismic response in the vertical dimension is poorly predicted for both RTs. The misfits for both RTs are likely due in part to the complexity of the tufa material, which is not captured by the model. In addition, our sensitivity analysis shows the importance of correctly describing the base (or pedestal) of an RT in the modeling. This is especially true for Stout because both the modal frequencies and the deformation shape change when a realistic, sloping base is considered (Fig. 11).

The modal analysis misfit for Stout can be further interpreted as an outcome of effective damping mechanisms, which were not present for Skinny; such a mechanism has to do with the low-aspect ratio, which allows for energy to be more easily dissipated away. The vertical displacement at the RT top displays a lesser magnitude than the ones at the free field, strikingly enough at first. We suggest that this observation is related to the presence of the vertical mode of vibration, yet the location in the spectrum seems inconsistent with the value put forward by our FEM model; this mismatch could be due to

numerical limitations arising from having to keep a sensible model size, too small to capture all the mass mobilized by this mode.

The transient FEM model reproduces many important features of the observations at Skinny (Fig. 12). In the horizontal, it captures the strong amplification between 12 and 18 Hz and modest deamplification between 20 and 30 Hz. The north dimension also reproduces the smaller amplification peak at 36–44 Hz, although the east dimension does not. A portion of this misfit may be attributed to our implementation of ground shaking in SeismoVLab, which adds uncertainty to the model result. In particular, we chose to impose ground shaking as decoupled plane waves that impinge perpendicularly on the free surface of the model domain. This choice is perhaps the simplest method to replicate the recordings of the far-field seismometer (Fig. 1a), but other implementations are also possible.

A consistent discrepancy between our FEM models and observations is the separation between the first and second natural vibration frequencies observed in the horizontal dimensions: neither the modal analysis nor the transient model captures the separation between a pair of prominent peaks observed between 12 and 18 Hz in the seismic data (Figs. 3, 13). We posit that the disagreement of the frequency values of the two bending modes could be due to an incomplete resolution of the geometry of the RT around the base: the point cloud obtained from field photogrammetry captures not only the RT but also various other rock fragments (Fig. 1b). Although the point cloud was trimmed to remove nearby rocks, these rocks obscure the camera view and thereby reduce the point cloud density near the base of the RT. As a result, the geometry of the lower RT has limited resolution in the point cloud, which could lead to errors in the construction of the FEM mesh. The lower region of the RT has the highest curvature, so its response has a strong influence on the deformation of the structure as a whole. Errors in the FEM mesh, smoothing the shape or size of the base, could reduce the number of vibration modes in the model, for example. Future work may consider how to best characterize the basal geometry; a combination of photogrammetry, lidar, and detailed field measurements can provide more robust data (e.g., McPhillips and Pratt, 2024).

Progressive damage and failure

By validating the transient FEM model with observations, we support previous modeling by Donnellan *et al.* (2022). The previous work examined the dynamic response of another RT (P006), also using SeismoVLab. The model inputs consisted of RT geometry and a generic estimate of material properties, in addition to the 2019 M 7.1 Ridgecrest mainshock seismogram, as recorded at a nearby station. P006 survived the mainshock but toppled during or just after a 2020 M 5.5 aftershock with roughly half the shaking intensity. The

dynamic modeling showed that peak tensile stresses at the base of P006 approached the tensile strength of the tufa rock. As a result, Donnellan *et al.* (2022) proposed that the basal material was sufficiently damaged during the mainshock that the RT was vulnerable to failure during subsequent, weaker shaking.

Here, we further develop a hypothesis for progressive damage and failure of RTs subject to earthquake shaking. From our conceptual model, we expect that aspect ratio has an important control on damage accumulation because of its role in determining both resonant frequencies and deformation modes (Fig. 5). As discussed earlier, our observations and model results support previous work, for which high-aspect-ratio RTs behave as Euler–Bernoulli beams and bending-mode deformation imposes large tractions at their bases, which may contribute to the formation of tensile cracks (Figs. 3, 8; Finzi *et al.*, 2020; Donnellan *et al.*, 2022; Finnegan *et al.*, 2022). We also show that an opposite end member, such as Stout, behaves differently. This low-aspect-ratio RT shows much less amplification of bending-mode vibrations (Fig. 3), and other deformation modes may be important (Fig. 9). In Timoshenko beam theory, deformation may be dominated by internal shearing, and such beams therefore experience smaller tractions (and less tensile crack development) than Euler–Bernoulli beams when exposed to similar shaking intensities. These results suggest that less intense crack development occurs at the bases of low-aspect-ratio RTs.

We test these inferences about crack development using *in situ* rebound hammer measurements of rock strength, which correlates with rock damage by cracking (Eppes *et al.*, 2018). By comparing rebound data from different elevations on multiple RTs, we show that higher aspect ratio RTs have relatively weaker bases. Specifically, the rebound number ratio appears to scale linearly with aspect ratio between about one and two. Very few RTs with aspect ratios greater than two are preserved at the Trona Pinnacles, and Skinny, with an aspect ratio of about 4.5, is unusual, although it is also consistent with the linear scaling. In our interpretation, higher aspect ratio RTs experience stronger amplification of bending-mode deformation, which drives mode-I crack development preferentially at their bases. The Trona Pinnacles experience strong shaking, comparable to the 2019 M 7.1 mainshock, roughly every thousand years (Fig. 2). We suggest that many high-aspect ratio RTs (>2.5) that were exposed subaerially in the Searles Lake bed in the late Pleistocene have subsequently failed due to the progressive damage of their bases. In contrast, lower aspect ratio RTs are less vulnerable to earthquake shaking. Finally, although we focus on aspect ratio, other geometric characteristics may also influence seismic response (e.g., Housner, 1963).

Large earthquakes likely produce critical tensile stresses in high-aspect-ratio RTs, but theory suggests that cracks also grow under subcritical loads imposed by both earthquakes and environmental noise (Eppes and Keanini, 2017). For the Trona Pinnacles, we suggest that earthquakes are the most important driver of crack growth. Our reasoning is based on Paris' Law

for the crack growth under cyclical loading, which describes the rate of crack growth as a power-law function of stress (Paris *et al.*, 1961). Estimates of the value of the power-law exponent are consistently large: Eppes and Keanini (2017) cite a range from 58 to 78 from Nara *et al.* (2013). As a result, the large stresses imposed by rare earthquake shaking likely yield orders of magnitude greater crack length than lower stresses that might be imposed orders of magnitude more frequently, by gusting winds, for example, White (2015) and Donnellan *et al.* (2022). Elsewhere, however, subcritical crack growth may contribute significantly to damage accumulation, especially where there is larger magnitude environmental noise, such as ocean waves (e.g., McPhillips and Scharer, 2021).

Implications for RT fragility measurements

The deformation and toppling of an RT exposed to earthquake shaking is a complex process, which begins in the domain of an elastic continuum and evolves into a rigid body problem (e.g., Finzi *et al.*, 2020). At the moment that an RT finally topples, its fragility may be best described in terms of a PBR because it no longer shares a material connection with its base. To date, fragility analyses of RTs have focused on the initial elastic deformation because a finite tensile strength always stabilizes an RT relative to a PBR with a similar geometry. These analyses infer that the transformation from RT to PBR occurs almost instantaneously, and earthquake shaking that is strong enough to crack its base is also sufficient to topple it (Anooshehpour *et al.*, 2013; Finzi *et al.*, 2020; McPhillips and Scharer, 2021). Although this inference may be justified in many situations, the recent case study from the Trona Pinnacles demonstrates that damage and toppling need not occur in the same earthquake (Donnellan *et al.*, 2022).

This study advances the measurement of RT fragility by explicitly considering damage accumulation. Our field measurements show evidence of widespread damage at the bases of RTs, and the geometry and material properties of the RTs predict the likely degree of damage. Our results suggest that future fragility calculations should consider damage. For example, if rebound hammer measurements are used to estimate in situ tensile strength, then measurements from higher up on the RTs, outside the damage zone, will better reflect the long-term material properties. On the other hand, progressive damage by subcritical crack growth may weaken RTs, leading to their eventual failure. In this scenario, extant RTs may yield fragility estimates consistent with stronger peak shaking intensities than the site actually experiences, leading to a very conservative maximum constraint on PSHA hazard curves. At other locations, pervasive planes of weakness, such as bedding and fracture surfaces, may also reduce tensile strength locally (Shang *et al.*, 2016). These large-scale features contribute to the scale effect in tensile strength measurement (Yoshinaka *et al.*, 2008), which may be considered when applying laboratory or rebound hammer measurements to RTs (McPhillips and

Scharer, 2021; Donnellan *et al.*, 2022). In the future, fragility models may benefit from considering the time evolution of damage and rock strength.

This study also extends the modeling of fragilities by validating the transient FEM simulation with SeismoVLab. As computational methods become more powerful and efficient, numerical analyses will likely replace quasi-static fragility approximations for many PSHA applications. This transition is already occurring for PBRs (Purvance *et al.*, 2008; Veeraraghavan *et al.*, 2016; Chen *et al.*, 2024). Numerical methods are better able to consider realistic seismic shaking in three dimensions. For RTs, numerical methods may be uniquely able to consider the transition from elastic to brittle or plastic deformation that precedes toppling. This study shows that the FEM model can capture essential elements of the seismic response, given a detailed geometry and approximate material properties, potentially even for heterogeneous RTs.

Finally, analytical or semi-analytical approximations of fragility will likely always have a role in the initial survey of RTs and other FGFs. There remains the need to screen candidates for modeling. The “archetype” analysis presented here provides a method to carry out accurate vibrational analyses using simplified geometries. This approach saves the need to build and run complex FEM models for every FGF in a survey.

CONCLUSIONS

There are RTs in many regions and landscapes. At the Trona Pinnacles and elsewhere, RTs are culturally important features (Lignola *et al.*, 2010; Moore *et al.*, 2019). They are also FGFs, which can be used to test and improve PSHA (Finzi *et al.*, 2020; McPhillips and Scharer, 2021). For both of these applications, fragilities may be calculated to describe the probability of toppling at given shaking intensities. Although simple, first-order estimates may be useful, the process of failure and toppling is complex. At the Trona Pinnacles, seismic data show that the seismic response of RTs depends on aspect ratio. Modeling sheds light on the frequencies and deformational modes of the seismic response. In situ rock strength measurements show evidence of damage accumulation, and the patterns are consistent with a hypothesis that bending-mode deformation preferentially develops mode-I fractures near the bases of higher aspect ratio RTs. These results suggest that RT fragility is time-dependent because it loses strength as damage accumulates with successive seismic excitations. This case study at the Trona Pinnacles may provide valuable insight for future improvements in fragility estimation.

DATA AND RESOURCES

The Mathematica (Wolfram, 2000) notebooks used to implement calculations of natural frequencies of Skinny are available on GitHub (github.com/jgarciasuarez, last accessed February 2025) in the repository titled “trona-pinnacles.” Field data, including seismic recordings, light detection and ranging (lidar) point clouds, and rebound hammer measurements, are available in the U.S. Geological Survey (USGS)

data release at doi: [10.5066/P9EBIJG5](https://doi.org/10.5066/P9EBIJG5). The Seismo-VLab code and documentation are available at www.seismovlab.com (last accessed February 2025). OpenTopography website is available at OpenTopography.org (last accessed April 2021).

DECLARATION OF COMPETING INTERESTS

The authors acknowledge that there are no conflicts of interest recorded.

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APPENDIX A

Finite-element models (FEMs)

Generation of FEM mesh from point cloud

The meshing process is carried out in Mathematica (Wolfram, 2000). Other options were considered and discarded, based on the following premises.

- We favor hexahedral meshes over tetrahedral ones. Most of the third-party meshing software for general geometries only enables the latter.

- We require a platform that permits full control over the process from point cloud to meshed geometry because not only are we interested in finite-element simulations, but also in analyzing the geometry itself (e.g., asymmetry of cross sections).
- We prefer to have an easy-to-share platform (notebooks). Jupyter notebooks (Kluyver *et al.*, 2016) were the first choice, but the toolbox available in Mathematica proved to be, in our estimation, better suited for the task at hand.

Because the geometry of the rock towers (RTs) is fairly regular, developing a meshing framework is not as involved as for other, more general shapes. The meshing pipeline includes the following steps.

- Centering the point cloud.
- When necessary, filtering out those points that represent other rocks surrounding the RT.
- Select element size: it must be consistent with the desired level of resolution (surface rugosity to be captured in the model).
- Consolidate the points at different heights consistent with the element mesh that is to be used.
- At each of these levels, interpolate a curve among the points of the point cloud.
- Create a regular grid of hexahedra, defined according to the choice of element size, of the volume that engulfs the shape.
- Embedding the interpolating curves into this discretized volume, keep the elements that are either interior or intersect the cross-section interpolants.
- Correct the position of the nodes of those elements that are only partially contained within the inner volume associated with the point cloud.
- Add a surrounding soil box and connect it to the RT.

Element-type sensitivity

We analyze first the differences between the results obtained using linear-interpolation elements and those equipped with quadratic interpolation. For this analysis, only the spire is used (the surroundings are not introduced in the model). The base is supposed to be rigidly fixed to the ground, thus defining the boundary condition for the model. The error introduced in the natural frequencies by the use of linear elements (considering the quadratic elements as the baseline true response) also provides insight into the time-domain error associated with the use of a linear mesh. Should the error in the natural frequency be small, we can expect the error to be small in the transient time-domain simulations as well.

Skinny. Both linear and quadratic models (of the isolated RT) utilize the same mesh of 2051 elements. The linear one has 8715 degrees of freedom (dofs), whereas the quadratic one has 32,103. Results for the first two modes are displayed in Table A1. Taking the quadratic results as baseline truth, we can acknowledge that

TABLE A1
Influence of Element Type on Skinny's Vibration Modes

Model Comparison	First Mode	Second Mode	Third Mode
Linear	16.61 Hz	18.07 Hz	49.30 Hz
Quadratic	16.22 Hz	17.63 Hz	50.50 Hz
Difference	2.4%	2.5%	2.3%

TABLE A2
Influence of Element Type on Stout's Vibration Modes

Model Comparison	First Mode	Second Mode	Third Mode
Linear	18.76 Hz	21.03 Hz	37.10 Hz
Quadratic	17.71 Hz	19.90 Hz	35.50 Hz
Difference	1.8%	1.6%	1.6%

TABLE A3
Sensitivity to Volume of Pedestal Included in the Mesh on Skinny's Vibration Modes

Model Comparison	First Mode	Second Mode	Third Mode
6 m × 6 m × 4 m	12.99 Hz	14.07 Hz	45.20 Hz
6.7 m × 6.7 m × 5.7 m	12.99 Hz	14.07 Hz	45.20 Hz
Difference	<0.1%	<0.1%	<0.1%

the linear elements do not introduce meaningful differences: the discrepancies amount to less than 3%. As mentioned earlier, this outcome is encouraging insofar it indicates that the usage of a linear mesh does not stiffen the system perceptibly (Cook *et al.*, 2007), hence “cheaper” simulations, in terms of computational resources, using the linear mesh are warranted.

Stout. We also ascertain that the difference drawn by different element type in “Stout” is minimal, see Table A2.

Pedestal-size sensitivity (Skinny). The following is a verification that enough pedestal volume was included in the previous model of the slender RT. If that were not the case, increasing the size of the pedestal would reveal a meaningful decrease in the eigenvalues.

To pursue this verification, we use Skinny's mesh, solving a similar eigenvalue problem, the only difference being an enlargement of the cuboid, for which the volume now is $6.7 \times 6.7 \times 5.7 \text{ m}^3$, which increases the mass of the underlying soil introduced in the model by 44%. The number of elements in this model is 129,687, whereas the number of dofs in this case is 416,919.

Table A3 shows these results. Clearly, the effect of augmenting the volume of surroundings included in the mesh is negligible; hence, we can conclude that the original model is

already converging to the final result; in other words, all the mass and stiffness of the surroundings engaged during the natural mode deformation were already present in the smaller model.

APPENDIX B

Dimensional analysis

During the abstraction from reality into a model, one has to establish which are the parameters and the variables the solution for the response depends. The variables in the problem are space and time, and we used the opportune coordinates to describe them (Cartesian coordinates and time coordinate). Similarly, there are a number of governing parameters in the problem.

- The structures are made of rock, density ρ assumed to be constant (homogeneous material), and also to be linear-elastic (to first approximation which can be satisfactory for low-intensity loading), characterized by two constants: Poisson's ratio ν and shear modulus μ . This can be regarded as a satisfactory model when it comes to considering low-intensity loading conditions.
- The features of the seismic excitation need a sizeable number of parameters to be fully characterized: one may choose to describe the source instead of the seismic wavefield, as the former is the cause of the latter. For the following discussion, we will avoid delving into the influence of the fine details of the excitation; instead, it will be simply approximated using a characteristic frequency, $\bar{\omega}$ (which would represent the order of magnitude of the frequency content of earthquakes), and a characteristic ground acceleration $\ddot{X}_g$ (peak ground acceleration, which would represent the intensity of the earthquake as experienced at the ground level).
- The shape of the rock is challenging to describe, due to the rugosity of the surface, let alone the inner structure, which includes crack systems and porosity. The number of parameters required to describe the geometry depends on the level of detail wished to attain. In its rawest version, the shape can be described by a certain height L , and two characteristic cross-section lengths ℓ_1 and ℓ_2 . The texture of the exterior face can be described simply with a "perturbation wavelength," λ (which quantifies the difference between the real shape with irregularities and the "average smooth shape" we adopt when using the three aforementioned parameters); the amplitude of these perturbations can also be considered to be of order λ .
- The surrounding topography, at the scale of many multiples of the RT height, can impact the response as well. Despite assuming that the mechanical properties of the surrounding soil and bedrock do not differ from those of the tufa the RTs are made of, one would require a myriad of measurements to describe every detail of the geometry of the ground surface around the RT. For the sake of simplicity, we shall assume a priori that the topography has a secondary effect on the response of the RTs we are considering; evidently, this

may not always be the case because it is well known already (Asimaki and Mohammadi, 2018).

Once the survey of governing parameters is completed, Dimensional analysis (Simon *et al.*, 2017) posits a functional relation between the outputs of the system (its response) and the system parameters. For instance, we could consider the response (output) to be the maximum RT deformation (in time and space) $u_{\max}$, thus we would write the following functional relationship:

$$u_{\max} = f(\rho, \mu, \nu, L, \ell_1, \ell_2, \lambda, \ddot{X}_g, \bar{\omega}). \quad (\text{B1})$$

Buckingham's theorem (Buckingham, 1914) states that equation (B1) can be expressed in dimensionless form as some function of the following dimensionless ratios:

$$\frac{u_{\max}}{\rho \ddot{X}_g \ell_1^2 \mu^{-1}} = f\left(\frac{\mu}{\rho \ddot{X}_g \ell_1}, \nu, \frac{L}{\ell_1}, \frac{\ell_2}{\ell_1}, \frac{\lambda}{\ell_1}, \frac{\bar{\omega}}{\sqrt{\mu/\rho \ell_1^{-1}}}\right). \quad (\text{B2})$$

Each dimensionless ratio of the expression depends on has a physical interpretation.

- The ratio $\mu/\rho \ddot{X}_g \ell_1$ represents the inverse of a characteristic value of the strain in the RT induced by the seismic load. Hence, the linear-elastic hypothesis implies $\mu/\rho \ddot{X}_g \ell_1 \gg 1$. This also implies automatically that the response is linear, meaning that it scales linearly proportionally to the magnitude of the external force (Conti *et al.*, 2017).
- The Poisson's ratio ν quantifies the compressibility of the rock.
- The aspect ratio of the RT is quantified by L/ℓ_1 : the higher the value, the more dominant the vertical length is over all the others. In practice, for all RTs one finds in nature $L/\ell_1 \sim 1$, meaning that both quantities range in the order of meter(s), yet not much larger or smaller, nor substantially larger than the other.
- As a first attempt to capture the shape irregularity of the RT, two characteristic cross-section lengths were introduced, ℓ_1 and ℓ_2 . This choice is motivated by a simple fact: most spires present a cylinder-like form. The cross-section of the RT cannot be described with just one length (one parameter would describe only either a circle or a square), as there will clearly be directions of deformation more favorable than others, that is, (assuming at this point bending-dominated deformation), there could be directions with less cross-section inertia than others. Hence, our model allows for two different bending axes, and if we take $\ell_1 \sim \ell_2$, or, in other words, $\ell_1/\ell_2 \sim 1$, we allow in our framework for consideration of a strong axis and a weak axis but of similar magnitude.
- The quotient ℓ_1 can be understood as representing the magnitude of all the other quantities necessary to define the outer shape of the RT, divided by the diameter of a characteristic

cross section. The possibility of defining a proper average shape requires $\ell_1 \ll 1$, that is, no abrupt surface dents, so an appropriate definition of “characteristic cross-section length” can be put forward to begin with. Another physical argument to argue that λ can only represent second-order effects is in terms of scattering: small features that perturb the average shape would introduce scattered fields for which wavelength is proportional to the one of the feature; therefore, the extra scattering introduced must be small and will not perturb the “average” wave propagating and the global response. This argument is also connected to the one we are about to present to argue that $\omega \ell_1 / \sqrt{\mu/\rho} \ll 1$, which, combined with $\ell_1 \ll 1$, means $\omega \lambda / \sqrt{\mu/\rho} \ll 1$.

- Finally, $\omega \ell_1 / \mu/\rho$ represents the ratio between the characteristic length of the RT geometry to a characteristic wavelength of the waveform propagating in the soil toward the surface: see that, first, $\sqrt{\mu/\rho} = V_s$ (the shear-wave velocity in the material), and then $\omega / \sqrt{\mu/\rho} = 1/\lambda$, this meaning finally that $\omega \ell_1 / \sqrt{\mu/\rho} = \ell_1/\lambda$. This ratio represents how much the load changes over length spans of the order of the size of the RT; if the load changes over spans much greater than the size of the rock, $\ell_1/\lambda \ll 1$, meaning that the whole system adapts to the load variation in less time than the one that the load takes to change its magnitude perceptibly, we would therefore see that the whole RT responds to the loading as a coherent unit. Conversely, when $\ell_1/\lambda \sim 1$ the load changes significantly over distances comparable to the size of the rock, meaning that, in one given instant, different parts of it experience different loading conditions, so the response is strongly local, as it can vary significantly within the RP. If we take realistic values of geometrical dimensions,

characteristic frequency content of earthquakes and material properties, only in exceptional occasions we will have anything other than $\ell_1/\lambda \ll 1$. Hence, one can surmise that the response of the RT can be modeled as a horizontal body force that induces a global response as that we see in, for instance, buildings during earthquakes. The aforementioned exception would involve the largest spires being accosted by high-frequency seismic pulses.

According to the prior discussion, we could take the following limits to narrow down the type of systems under consideration:

$$\frac{u_{\max}}{\rho \ddot{X}_g \ell_1^2 \mu^{-1}} = f\left(\frac{\mu}{\rho \ddot{X}_g \ell_1} \rightarrow \infty, \nu, \frac{L}{\ell_1}, \frac{\ell_2}{\ell_1}, \frac{\lambda}{\ell_1} \rightarrow 0, \frac{\omega}{\sqrt{\mu/\rho} \ell_1^{-1}} \rightarrow 0\right) \\ = \tilde{f}\left(\nu, \frac{L}{\ell_1}, \frac{\ell_2}{\ell_1}\right). \quad (\text{B3})$$

Thus, the response of the system must depend, under these assumptions, on the material compressibility ν , the aspect ratio L/ℓ_1 , and on the direction of the loading, as ℓ_2/ℓ_1 implies the possibility of a weak and a strong axis.

If we were to consider a free-vibration scenario, we would find a similar expression for the fundamental frequency of the system ω_n using the same reasoning:

$$\frac{\omega_n}{\sqrt{\mu/\rho} \ell_1^{-1}} = \tilde{g}\left(\nu, \frac{L}{\ell_1}, \frac{\ell_2}{\ell_1}\right). \quad (\text{B4})$$

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