LINEAR TRANSFORMATIONS FOR ELECTRONIC ACTIVE FILTER DESIGN

by

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ABSTRACT

In the present work inductorless electronic active filters of either active-RC or active-R form are designed from doubly resistive terminated LC ladder networks, based on the recently developed approach of Linear Transformations. The new approach is based on linearly transforming the port-variable vectors of a two-port network from the V-I domain to a new X-Y domain, in which the transformed two-port is then realized. By examining the ladder filter as a cascade connection of two-port subnetworks we obtain the linearly transformed active structure for each of those subnetworks and then interconnect them in an appropriate way to derive the Linear Transformation Active (LTA) filter simulating the original LC filters. This simulation can be carried out either by considering each element at a time, as has been done in earlier work, or by considering two elements a time, thereby embedding biquadratic sections as basic building blocks in the active structure. These blocks in fact serve as the simulation counterparts of inverted-L sections or indeed series or shunt tuned circuits.

It is shown that already existing methods such as the structural simulation or the operational simulation of LC ladders networks can be interpreted as special cases of the general approach.

The proposed techniques are extended to simulate LC ladders for active R structures using basic building blocks of 1st and 2nd order. It is shown that new and economical structures can be derived with satisfactory performance.

The sensitivity performance of the general LTA structures is also examined and some guidelines are given as to the choice of transformation matrices so as to improve such sensitivity performance in the active realization. Practical examples are also given which illustrate the flexibility, importance and usefulness of the new approach.

To all those (scientists, politicians, etc.) who have given and give their lives for a better world

and

To my wife ATHINA

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GLOSSARY OF SYMBOLS AND ABBREVIATIONS

C	:	Subset
$\vdash$	:	Belongs to
$\forall$	:	For each and every
$<\cdot\cdot>$	:	Inner product
$\|\|..\|\|$		Norm
Sup	:	Supremum
$\mathcal{A}_{1i}$	:	Nonsingular 2x2 left transformation matrix for the left port-variable vector of the two-port N_i
$\mathcal{A}_{2i}$	:	Nonsingular 2x2 right transformation matrix for the right port-variable vector of the two-port N_i
T_i	:	The modified chain matrix of the two-port N_i
$\mathcal{A}_i^{-1}$	:	The inverse matrix of $\mathcal{A}_i$
$\mathcal{A}_i^T$	:	The transpose of the matrix $\mathcal{A}_i$
$\mathcal{A}_i^C$	:	The compatible matrix of $\mathcal{A}_i$ under the cross-cascade connection
$\equiv$	:	"identically equal to"
$\overset{LT}{\equiv}$	:	"equivalent under linear transformation"

CHAPTER 1

ACTIVE-RC FILTERS: A SURVEY

1.1 SOME INTRODUCTORY COMMENTS ON FILTERS

1.2 ACTIVE FILTER DESIGN: TRANSFER FUNCTION SIMULATION

1.3 ACTIVE FILTER DESIGN: COMPONENT SIMULATION

1.3.1 Inductance Simulation Method

1.3.2 Impedance Scaling Method

1.4 ACTIVE FILTER DESIGN: OPERATIONAL SIMULATION

1.5 LINEAR TRANSFORMATION ACTIVE (LTA) FILTERS-A NEW APPROACH TO THE ACTIVE SIMULATION OF LC LADDERS

CHAPTER 1

ACTIVE-RC FILTERS: A SURVEY

1.1 SOME INTRODUCTORY COMMENTS ON FILTERS

Any communication system, by virtue of its nature, involves signal processing operations and employs several filters at the various stages within its structure. These filters may be classified according to the following criteria:

(i) Frequency range of operation (audio, radio and microwave filters).

(ii) Band selectivity (lowpass, highpass, bandpass, band-reject, all-pass).

(iii) Kind of elements employed (LC, distributed component, electromechanical, piezoelectric, magnetostrictive, resonators).

(iv) Filter topology (ladder, lattice).

(v) External power requirements (passive, active).

The initial information given to the filter designer refers to the first two criteria (i.e. the frequency range of operation and band selectivity) in the form of typical filter specifications. These filter specifications define the limits which the response of the filter must not exceed. The frequency response of a filter is described by its transfer function $T(j\omega)$, defined as $T(s) = \frac{Y(s)}{X(s)}\Big|_{s=j\omega}$, where $Y(s)$ and $X(s)$ are the Laplace transforms of the input signal $x(t)$ and output signal $y(t)$ respectively.

Once a transfer function $T(s)$ has been determined that meets the specifications, a synthesis procedure is then implemented for which the type of the filter, according to criteria (iii), (iv) and (v), must have been predetermined. This is done according to the capacity of each type of filter to

provide the maximum pole-Q for the given transfer function at the required frequencies. This is discussed in detail in references [ZV1], [MO2], from which we have the plot of Q versus frequency in Fig.1.1.1

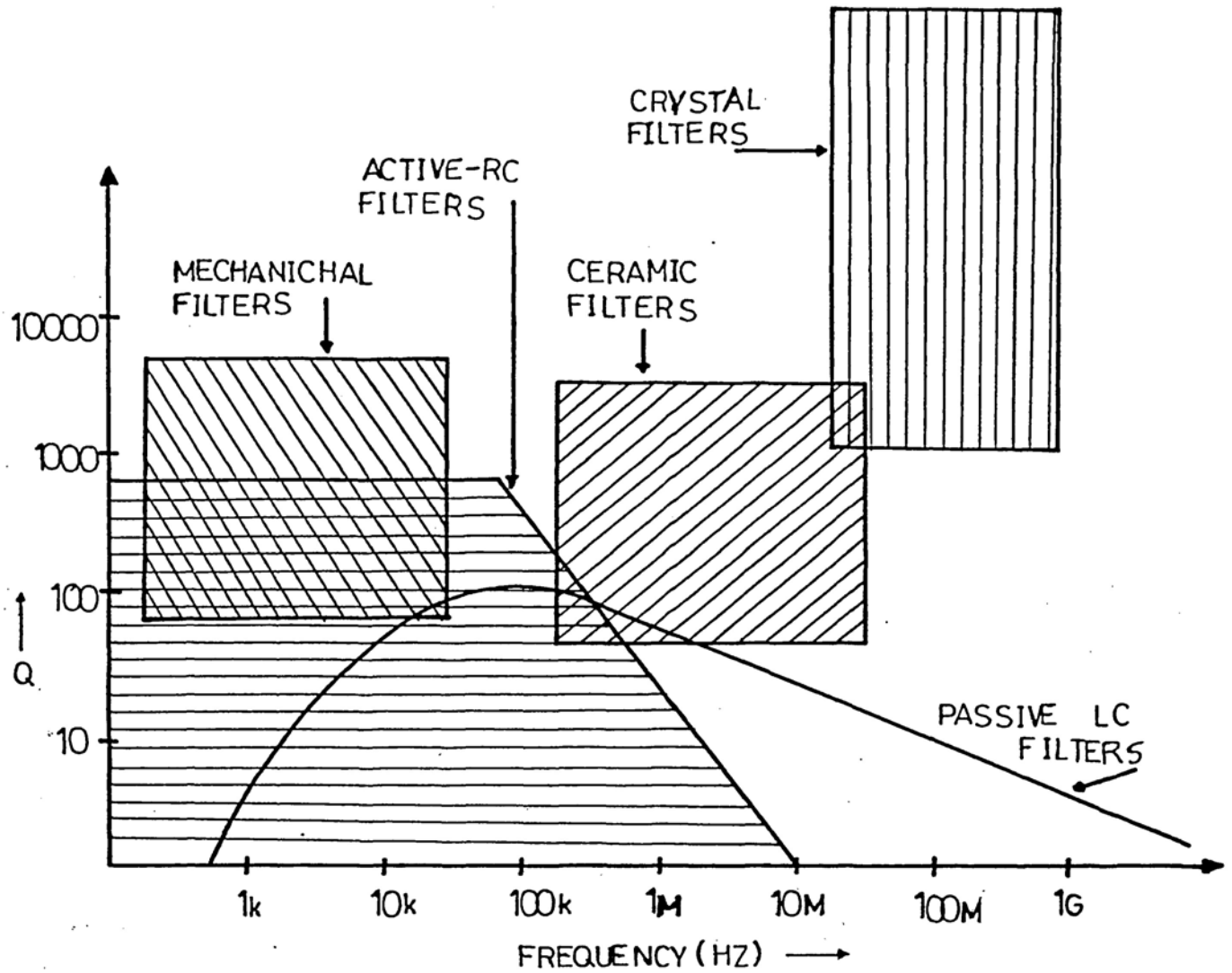


Fig. 1.1.1

The higher the pole-Q values are, more steep the filter cut-off may be. The mechanical, ceramic and crystal filters are not within the scope of this thesis, where we are entirely concerned with active-RC and active-R filters and particularly with a special class of such filters defined in Ref. [DI1], that simulate passive LC ladder networks. Our attention therefore, will be focused on the LC filters and

active-RC filters of Fig.1.1.1.

The weakness of the LC passive filters, to operate at very low frequencies, even with very low pole-Q values, is due among other reasons, to the fact that at those frequencies the inductors become very bulky and their inductive behaviour is dominated by the resistive behaviour due to their loss resistance R_L, shown in Fig. 1.1.2, as a series loss resistance.

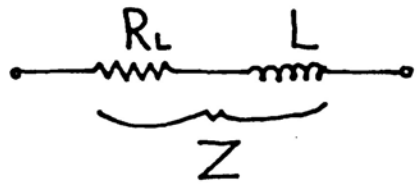


Fig. 1.1.2

Any attempt to reduce the value of R_L, increases the cost of the inductors and contributes rather negatively to the reduction of their size. So the case of active-RC filters use, at low-to-very-low frquencies appear without alternatives in the analogue domain.

Among the drawbacks of active-RC realizations is the finite bandwith of the active devices, which places a limit on the highest attainable pole frequency with present day technology and for most applications, reasonably good filter performance can be achieved for frequencies up to 100kHz with pole-Q requirements of up to several hundreds, while at higher frequencies, either active-R structures could be used, or passive LC filters are used as less objectionable at those frequencies, where inductors are less bulky. However many filtering applications are well within the frequency range up to 100kHz as voice or data transmittion. Active-RC filters provide a very powerful, elegant and pra-

ctically desirable solution to many of those filtering problems, whilst the development on new design methods and the continuous improvement of active components could conceivably extend the frequency limits to much higher levels.

1.2 ACTIVE FILTER DESIGN-TRANSFER FUNCTION SIMULATION

Once the decision on the use of an active-RC realization of the transfer function T(s) has been made, a synthesis procedure must be followed. The transfer function T(s) has, of course, been derived through some approximation procedure to satisfy the design specifications. This approximation procedure leads to a transfer function of the form:

$$T(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_o}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_o} , \quad m \leq n \qquad (1)$$

which may be written as the product of second order functions, as follows:

$$T(s) = \prod_{j=1}^{n/2} T_j(s) = \prod_{j=1}^{n/2} k_j \frac{(s-z_{j1})(s-z_{j2})}{(s-p_j)(s-p_j^*)} \qquad (2)$$

where z_j are the zeroes of the transfer function (roots of the nominator) and p_j, p_j^* are the conjugate pole pairs of T(s) (i.e. the roots of the denominator). Odd polynomials, of course, have at least single negative real root in addition to the conjugate pole pairs p_j, p_j^*. The above procedure is known as the decomposition of the transfer function T(s) into the second order subfunctions $T_j(s)$ which now can be realized individually by single-input-single-output second order active-RC networks [MO2], [DA1] which, when cascaded, form T(s).

This synthesis procedure is known as the transfer function simulation, by cascading second-order active-RC building blocks by several methods, such as those presented in refs. [MO3], [MO2], [LU1], are concerned with the grouping of the poles and zeroes of T(s), so that the resulting cascade structure is optimized with respect to sensitivity, dynamic range

and distortion. With the cascade structure, as a starting network, several feedback paths may be added intenting to reduce the sensitivity of the resulting structures. Such a modification, of course will change the overall transfer function, but it could be re-adjusted and identified within the desirable T(s). Various active structures of this kind have appeared in the literature, of which the most successful hitherto have been the Follow-the-Leader structures [LA2], [HU2] and the Shifted-Companion-Form structure [TO1].

The use of Negative Impedance Convertes (NIC's) for the synthesis of transfer impedances originated by Linvill [LI1], was of great interest in the early days of active-RC filter design (1950-1965) and many active-RC filter synthesis methods using NIC's have been proposed during that time [YA1], [SI4], etc. However, the NIC active-RC filters based on transfer function simulation are no longer of potential significance due mainly, to their large sensitivities with respect to element variations and also due to the excesive number of RC components needed for their realization [HE1].

Recently, as it will be discussed in the next section, NIC's are used for active-RC filter simulation of LC ladders, and this approach is very promising, in contrast to their use for transfer function simulation [SI1].

In conclusion it must be said that active-RC filters derivable through transfer function simulation are not of minimum sensitivity and, as a rule, they are more sensitive than structures obtainable through any ladder simulation method, as it has been shown in the refs. [DA1], [TU1].

1.3 ACTIVE FILTER DESIGN-THE COMPONENT SIMULATION

A transfer function T(s) that meets the filter specifications can be determined using the very familiar and systematic passive network synthesis techniques [WE1], [TE2], [ZV1]. The familiarity of the majority of filter designers with those methods as well as the availability of LC filter design tables and catalogues [ZV1], [SA10], can be beneficially used for the design of active-RC filters.

The above mentioned features of LC filters led designers to simulate LC filters for Active-RC by component simulation of inductors which are the most problematic components. The LC filter prototypes are chosen to be of ladder type and are doubly resistive terminated for low sensitivity.

They are of ladder type because these type of filters are having less disadvantages than lattice filters, which require a very sensitive bridge-balance condition to be satisfied to form a transmission zero. This transmission zero could be created in ladders only if either a series impedance or a shunt admittance becomes zero [TE1]. In that case the sensitivity of the LC ladder in the stopband makes this kind of filter highly suitable.

The LC filters are also doubly resistive terminated in order to have low sensitivity in their passband, which in fact is zero, at points of maximum power transfer, from source to load according to Orchard's principle [OR1]. A very elegant proof of this statetment is given in Ref. [TE1]. Those sensitivity zeroes at the maximum power transfer points, keep the sensitivity low within the passband.

For the above described reasons, doubly resistive terminated ladder LC filters with maximum power transfer

points, can be used as a design vehicle for active-RC filters. The various active-RC simulation methods are briefly described in the rest of this section.

1.3.1 Inductance Simulation Method

A doubly terminated LC ladder filter consists of resistors, capacitors and inductors [RLC network]. It is the presence of the inductor, as explained earlier, that makes these filters very problematic at certain frequencies and very objectionable at others. Therefore, if the inductive behaviour (i.e. a terminal relationship of the form V=sLxI) can be performed by an active-RC section, this simulated inductance can replace directly the inductors in the RLC ladder, turning it into an active-RC ladder filter leaving the ladder topology unaltered. Several active-RC simulated inductance circuits have been proposed [OR2], [RI1], with the gyration concept backed by a very sound theory. A gyrator is an actively realizable two-port device that is capable of converting a capacity load into a simulated inductance at its input terminals. Its schematic representation together with its Z-matrix is shown in Fig.1.3.1 where port 2 is terminated in a parallel combination of a resistance R_L and a capacitance C_L, producing the equivalent circuit shown.

It is observed that when $R_L=\infty$, the input impedance of the terminated gyrators becomes purely inductive. The simulation of a floating inductor is more complicated, requiring two gyrators in a back-to-back arrangements [HE1].

Several circuits for the active realization of the gyrator have been proposed as those in Refs. [AN2], [RI1].

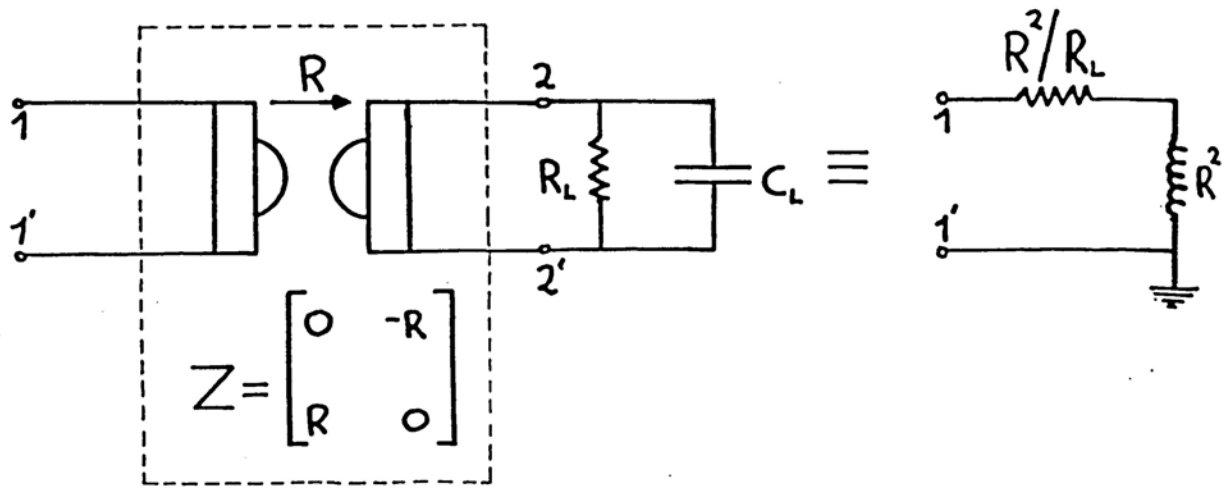


Fig. 1.3.1

Recently another method for inductance simulation has appeared based on NICs [SI1]. The general idea is to introduce in the passive ladder, passive and negative resistances and conductances, converting the resulting Πs of impedances into equivalent Ts (shown with dotted lines), and realize the negative components, either capacitors or resistors by NICs as generally shown in Fig.1.3.2 [SI1,2].
Generally as it is shown the resistors R_{L2} and $-R_{L2}$ can be absorbed by the adjacent arms of the ladder..

The above mentioned methods are in fact special cases of linear transformation approach introduced in [CO1], [DI1], as will be shown in Chapter 2.

1.3.2 The Impedance Scaling Method

On dividing all impedances of an LC doubly terminated filter by a function of the complex frequency X(s), the transfer function $T(s) = \frac{V_o(s)}{E(s)}$, in accordance with Fig.1.3.3 remains unchanged.

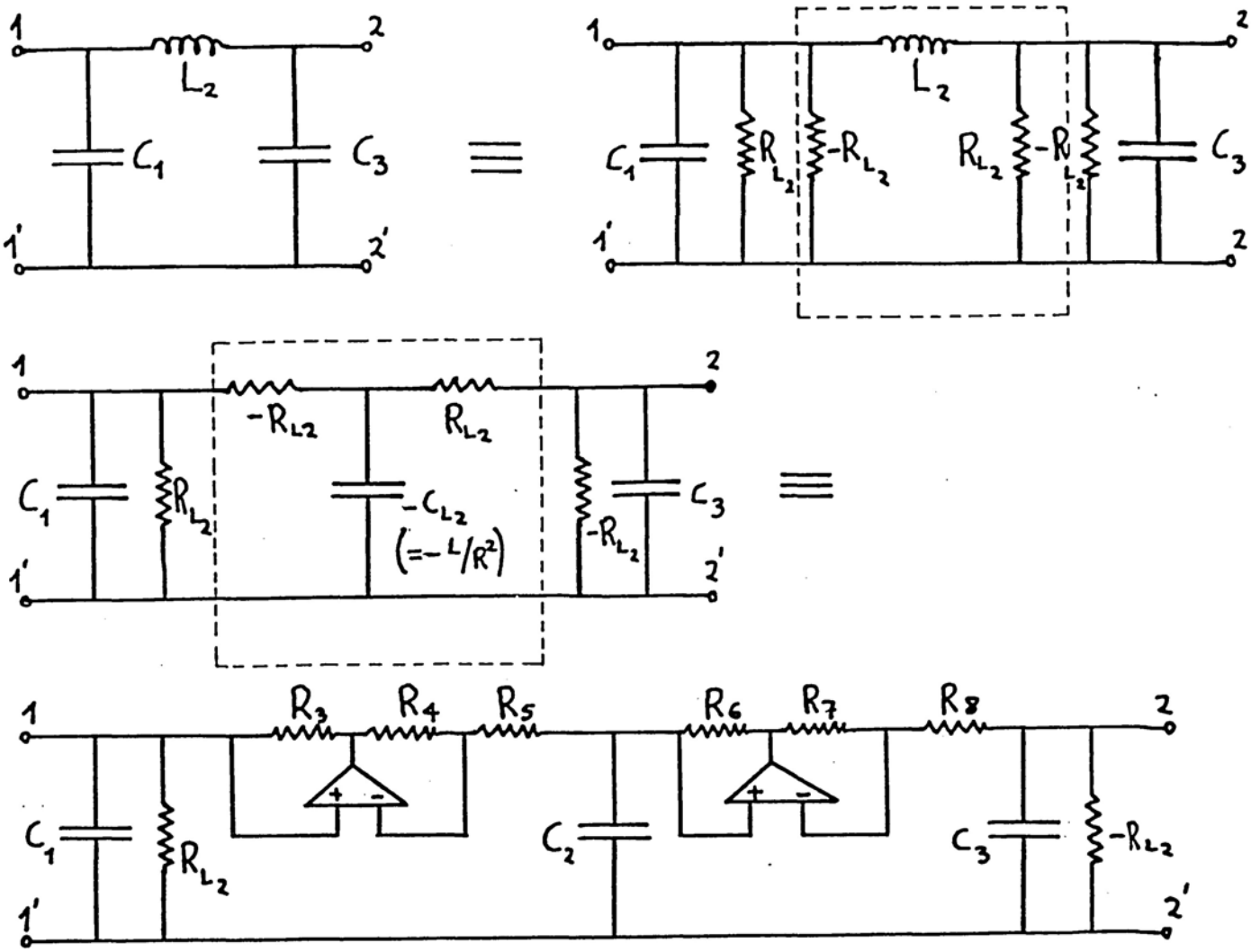


Fig. 1.3.2

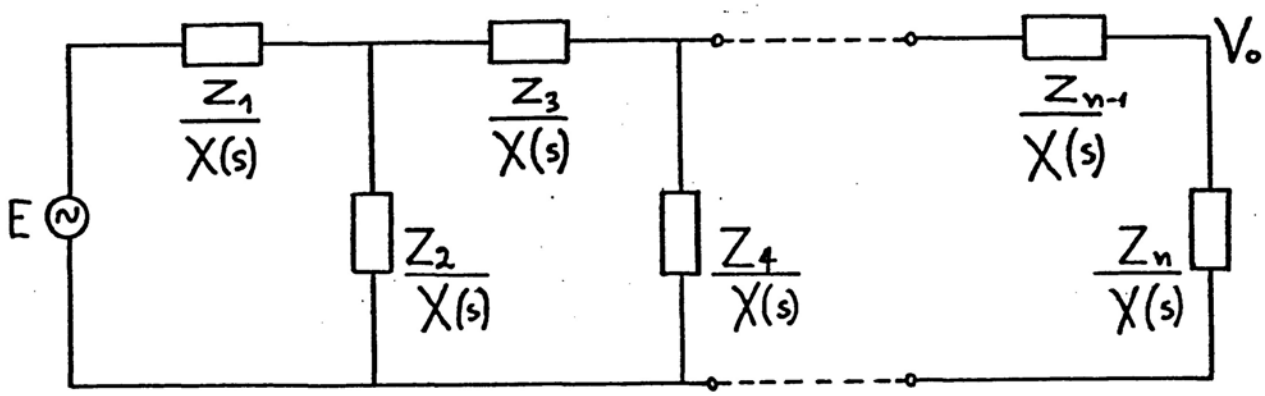


Fig. 1.3.3

This almost self evident statement which revolutionized active filter design was first presented, proved and applied to the simulation of LC ladder filters by L.Bruton [BR6], for X(s)=ks and has been generalized in Ref.[DI5]. The element with impedance of the form $1/s^2$ is referred to as supercapacitor or, rather mi sleadingly, as Frequency Dependent Negative Resistance (FDNR). Fig.1.3.4 gives the scaled ladder impedances for X(s)=ks.

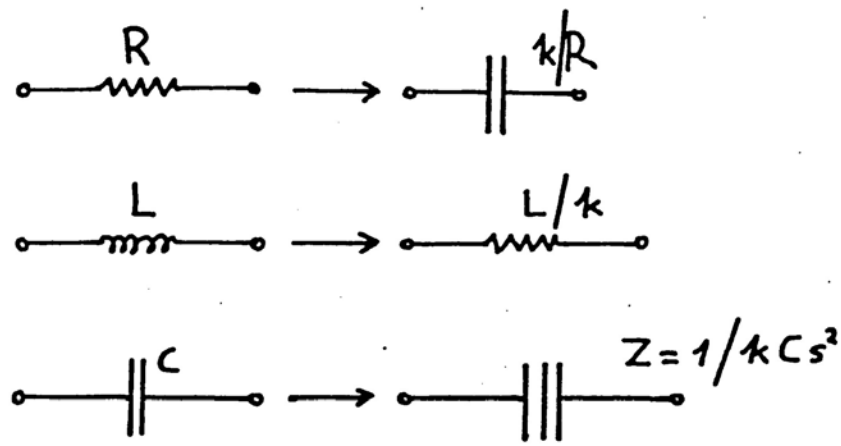


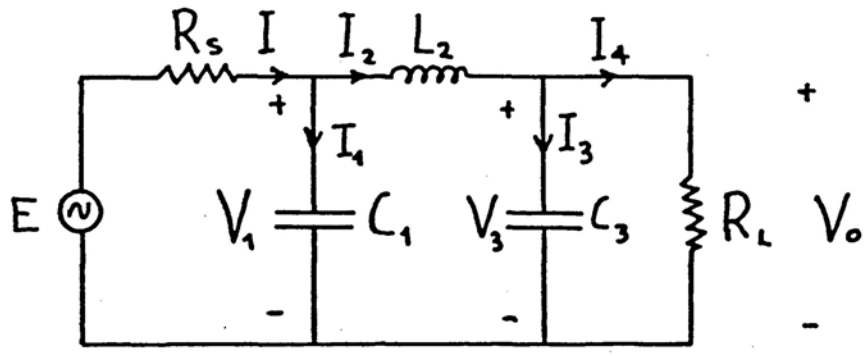
Fig. 1.3.4

Some realizations for FDNRs have been proposed by different designers, but the most useful realization is that given in Ref.[AN1]. Some modifications of the impedance scaling method have been proposed in Refs.[PA1], [SA6], to enable practical simulation of narrowband bandpass filters at high frequencies, as well as to minimise the number of capacitors and supercapacitors involved in the active-RC structures.

1.4 ACTIVE FILTER DESIGN-OPERATIONAL SIMULATION

In section 1.3.1, the inductive behaviour has been performed by active-RC sections, leaving the ladder topology unaltered. An alternative simulation method is to perform, by active-RC sections, the operational function of the ladder. Doing so, the ladder topology will be changed but the operational function of the active-RC filter will be the same as that of the LC prototype. In this operational simulation, the relations of the series and shunt arms by KVL and KCL, are simulated, in a way that the signal-flow-graph of the filter remains the same.

The basic operational simulation approach has been described by [GI1], shown in Figs.1.4.1-2, for the third order low-pass filter. The simulated equations are also shown.



$$E=V_1+IR_s,\quad I_2=I_3+I_4$$
$$I=I_1+I_2,\quad I_3=V_3\cdot sC_3$$
$$V_1=I_2L_2s+V_3,\quad I_4=\frac{V_3}{R_L}$$

$$\rightarrow \quad V_1=\frac{E-R_sI_2}{1+sC_1R_s},\quad I_2R_s=\frac{V_1-V_3}{\frac{L_2s}{R_s}}$$
$$V_3=\frac{R_s}{\frac{R_s}{R_L}+sC_3R_L}\,I_2$$

Fig.1.4.1

Other operational simulation approaches are the Direct SFG Simulation of LC ladder Networks with applications to Active Filter Design, described in [BR1], Signal-Flow-Filters [HA3], etc. As it has been shown [DI1-3], all those approaches can be interpreted as special cases of Linear Trans-

formation technique and proofs are given in Chapter 2 of this thesis.

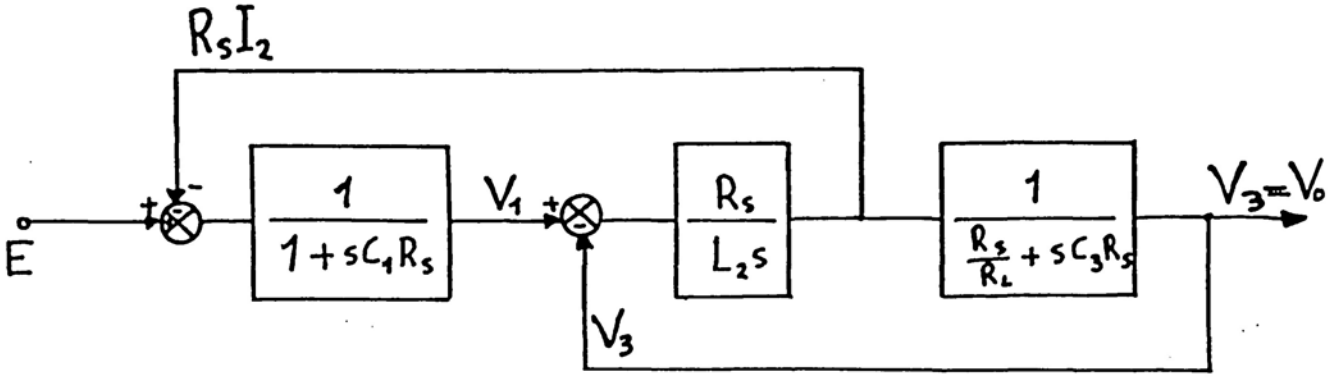


Fig. 1.4.2

1.5 LINEAR TRANSFORMATION ACTIVE (LTA) FILTERS-A NEW APPROACH TO THE ACTIVE-RC SIMULATION OF PASSIVE FILTERS [DI1-4], [CO2-4]

In this thesis, active-RC and active-R filters are disigned from doubly terminated LC ladder networks based on the newly developed approach of Linear Transformations [DI1-4], [CO2-4]. The new approach is based on linearly transforming the port-variables vectors of a two-port network from the V-I domain to a new X-Y domain, in which the two-port is then realized. By examining the ladder filter as a cascade connection of two-port subnetworks, we obtain the Linearly Transformed active structure for each of those subnetworks and then interconnect them in an appropriate way, to derive the Linear Transformation Active (LTA) filter, simulating the original doubly terminated LC filter. The element-by element linear transformation of the ladder filter ensures the low sensitivity of the derived structures due to the correspondence that exists between a ladder arm and the Linearly Transformed equivalent version of it. The new approach yield a powerful active-RC filter design technique which with the theoretical background provided, can be very useful to active filter designers.

The structure of this thesis is as follows:
In Chapter 2, the mathematical background of the Linear Transformation theory, as a Linear System Theory is presented based on Refs. [TZ1-4], [CO2-4], [DI1-5]. It is shown that all the previously existed methods like those in sections 1.3 and 1.4 can be interpreted as special cases of the new general approach. Finally a classification of LTA filters is given according to specific choices of Linear Transformation matrices. The Linear Transformation method, or rather

a design philosophy, can be classified as The Generalized Operational Simulation method of LC ladders, for active filter design, because it simulates the operational function of the ladder, in the new domain X-Y, which is a linear transformation of the V-I domain.

In Chapter 3, biquadratic sections are imposed to derive active structures, though Linear Transformation method, and the corresponding structures simulate either passive filters with series or shunt tuned circuits, or inverted-L polynomial and elliptic filters. In the remainding of the Chapter, off-the-shelf structures are embedded to design LTA filters.

In Chapter 4, the proposed technique is extented to simulate LC ladders for active-R structures, using basic building blocks of 1st and 2nd order, excisting in the literature. It is shown that new and economical structures could be derived with satisfactory performance.

Research on the sensitivity of the general LTA structures in order to express explicit sensitivity formulae in terms of the parameters of the transformations, is carried out in Chapter 5, where it is shown that specific choices of the elements in the transformation matrices result in less sensitive structures.

The LTA simulation approach, described in this thesis is put into practice in Chapter 6, where several ladder filters are simulated using the general acyclic LTA procedure. Those filters are then constructed and their complexity, sensitivity and high frequency performance are examined and compared to those of the original passive ladder, so as to show the flexibility of the LTA approach and the importance of the practical results.

The final conclusions concerning the practicality efficiency, flexibility and usefulness of the new approach are drawn and discussed in Chapter 7 where, in addition some directions for further research along the line of the LTA simulation concept are given.

CHAPTER 2

LINEAR TRANSFORMATION TECHNIQUE (LT)

2.1 LINEAR SYSTEM THEORY APPLIED TO N-PORTS

2.2 N-PORTS IN V-I DOMAIN

2.3 FROM V-I DOMAIN TO X-Y DOMAIN

- 2.3.1 Definition of the Σ-MATRIX
- 2.3.2 Relation between Σ and other network matrices
- 2.3.3 Relation between Σ and chain matrix
- 2.3.4 Passivity, losslessness, reciprocity
- 2.3.5 Dimensions of X and Y
- 2.3.6 Ladder elements considerations
- 2.3.7 Linearly transformed ladder constraints

2.4 LT, A GENERAL APPROACH TO LADDER SIMULATION

- 2.4.1 Ladder simulation
  - 2.4.1.1 Inductance simulation method special case 1
  - 2.4.1.2 Impedance scaling method special case 2
- 2.4.2 Operational simulation. Special case 3

2.5 CLASSIFICATION OF LTA STRUCTURES

- 2.5.1 Cyclic transformations
- 2.5.2 Low-complexity structures

CHAPTER 2

LINEAR TRANSFORMATION TECHNIQUE

In this thesis the design of Active-RC filters from doubly terminated LC ladder networks is presented based on a recently developed approach [DI1,2,3], [TZ1,2]. This new approach linearly transforms the port variable vectors of a two-port network to effect a transformation from the V-I domain to a new X-Y domain, in which the two-port is then realized actively.

Before introducing the linear transformation matrix [TZ1] which is the mathematical description of every structural analog building block, some basic definitions will be revised and theorems from Linear System Theory for the mathematical formulation of the approach. In the remaining of this Chapter it will be shown that most of the already existing approaches to Active-RC filter design can be interpreted as special cases of the general approach.

And finally, a classification of Linear Transformation Active Filters (LTA) is presented in a systematic way, for simple and low-complexity structures.

2.1 LINEAR SYSTEM THEORY APPLIED TO N-PORTS

The theory of linear n-ports have been deeply exproled during the last three decades. Among others, the work of Youla [YO], Newcomb [NE], Saeks [SA] and Anderson [AN], is worth mentioning which will be used as a basic reference, for the theoretical part of the present investigation.

Thus, with $x=x(t)$ and $y=y(t)$ denoting functions of the real variable t, will be established the following definitions.

Definition 2.1.1. An n-port is an ordered pair (Gn,N) where Gn is the graph of n disconneted edges and N is a relation on D_+^n (App.A), which defines a set of "admissible pairs"

$$(x,y) \vdash N \tag{2.1.1}$$

where x and y are n-dimensional vectors identified as input and output signals respectively (Fig.2.1).

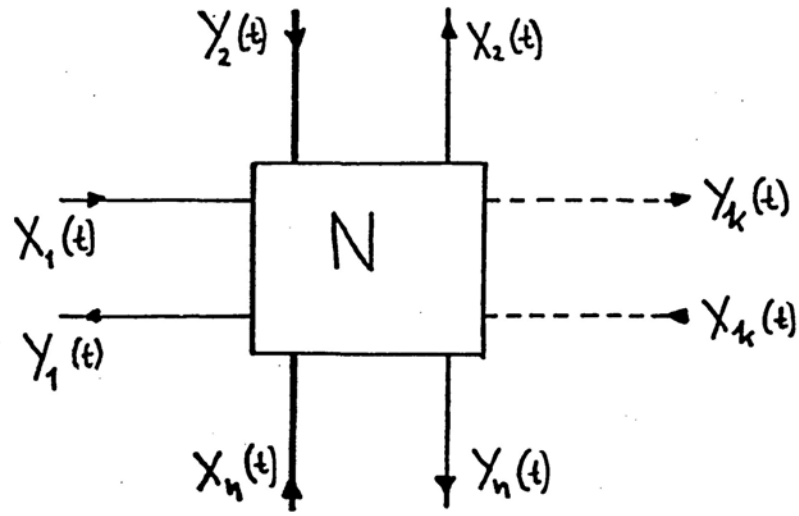


Fig.2.1: The n-port

Definition 2.1.2. An n-port is linear if for the given two sets of admissible pairs $(x_i,y_i) \vdash N$, i=1,2 and for all real numbers a and b, we have:

$$(ax_1+bx_2,\ ay_1+by_2) \vdash N \tag{2.1.2}$$

Definition 2.1.3. An n-port is causal, if whenever $(x_i,y_i) \vdash N$, i=1,2 and for every real T, the equality

$$x_1(t) = x_2(t) \qquad \forall t<T \tag{2.1.3a}$$

implies

$$y_1(t) = y_2(t) \qquad \forall t<T \tag{2.1.3b}$$

Moreover, if the n-port is linear the conditions for causality reduce to:

$$\text{if} \quad x(t) = 0 \quad \forall t<T$$

$$\text{then} \quad y(t) = 0 \quad \forall t<T \tag{2.1.4}$$

Definition 2.1.4. An n-port is time-invariant if for every $(x(t),y(t)) \vdash N$ and every real T, the pair $(x(t-T),y(t-T))$ is also in N.

Definition 2.1.5. An n-port is solvable if for every $f \vdash D_+^n$ there exists a unique $(x,y) \vdash N$ such that x=f, or equivalently, for each $x \vdash D_+^n$ there is one unique y. Moreover for x in L_2 then y is also in L_2.

Definition 2.1.6. An n-port is completely solvable (closed) if, in addition to Definition 2.1.5, a sequence of admissible pairs in N, (x_i,y_i), converges to a pair (x,y), then (x,y) is also in N.

Definition 2.1.7. A n-port is nonenergetic if there exists a finite real number M, such that for all $(x,y) \vdash N$ and all real t we have:

$$\int_{-\infty}^{t} (Mx^T x - y^T y)\,dt \geq 0 \tag{2.1.5}$$

If M=1, the n-port is passive.

Definition 2.1.8. An n-port is lossless if it is passive and for every pair $(x,y) \vdash L_2$ then:

$$\langle x,x\rangle - \langle y,y\rangle = 0 \qquad (2.1.6a)$$

Furthermore the n-port is instantaneously lossless if for every pair $(x(t),y(t)) \vdash N$ and for all real t:

$$x^T(t)x(t) - y^T(t)y(t) = 0 \qquad (2.1.6b)$$

Definition 2.1.9. An n-port is reciprocal if for every two pairs k and n and any admissible pairs in N having the form

$$(x_1,y_1) = (x_{1i}=0,\ i=1,n,\ i\neq k)$$

$$(x_2,y_2) = (x_{2i}=0,\ i=1,n,\ i\neq m) \qquad (2.1.7)$$

where

$$x_{1k} = x_{2m} \quad \text{then} \quad y_{1m} = y_{2k}$$

The definitions above are very important for network classification and will be used at a later stage to identify the analogue network under consideration. In the following we will refer to some theorems without proof, which are consequences from the above definitions.

Theorem 2.1.1. A causal solvable n-port is nonenergetic if and only if, there exists a finite real number M, such that for every (x,y) in N with $x \vdash L_2$

$$M\langle x,x\rangle - \langle y,y\rangle \geq 0 \qquad (2.1.8)$$

For M=1, the n-port is passive.

Clearly, a causal solvable n-port is nonenergetic (resp.passive) if and only if, it has a finite (resp.equal to unity)

norm, defined as:

$$||N|| = \sup \frac{||y||}{||x||} = \sup \left[\frac{\langle y,y\rangle}{\langle x,x\rangle}\right]^{1/2}$$

where

$0<||x||<\infty, \quad 0<||y||<\infty$

Theorem 2.1.2. Every linear nonenergetic n-port is causal. The above theorems assure the existence of an integral relation between the input and the output (Kernel's Theorem [SC1]).

Theorem 2.1.3. Every linear, nonenergetic (hence causal) and solvable n-port defined in terms of input-output pairs (x,y) has an integral representation.

$$y(t) = \int_{-\infty}^{\infty} k(t-\tau)x(\tau)d\tau = \int_{-\infty}^{t} k(t-\tau)x(\tau)d\tau \qquad (2.1.10)$$

since, for $t<\tau$, $k(t-\tau)=0$ in causal networks [SA4]. Notice, that in the above Theorem the n-port need not be passive.

2.2 N-PORTS IN V-I DOMAIN

The n-ports that are described in terms of (v,i) or (i,v) pairs as inputs and outputs are subject to the same definition as the n-ports described in the previous section except the energy is now defined in a different manner.

Definition 2.2.1 [AN2, NE1]. Let an n-port be in the v-i domain, i.e. is described by the (v,i) or (i,v) as input-output pairs (Fig.2.2). The energy delivered to the n-port is defined as:

$$E(t) = \int_{-\infty}^{t} v^T(t)i(t)dt \qquad (2.2.1)$$

for all real t.

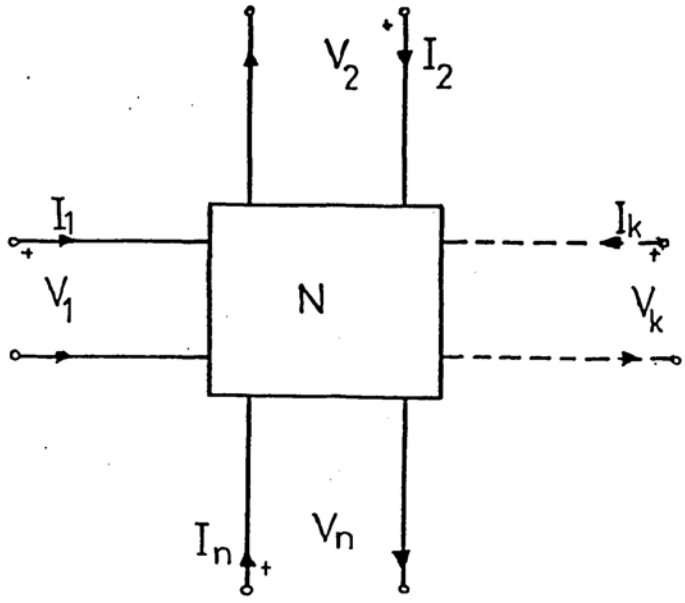


Fig.2.2: The v-i n-port

Furthermore, the n-port is:

nonenergetic if $+\infty > E(t) > -\infty$

passive if $+\infty > E(t) \geq 0$

lossless if $E(t)=0$

instantaneously lossless if $v^T(t)i(t)=0$.

Definitions 2.1.7 and 2.2.1, yields the following remarks:

a) In nonenergetic n-ports under Def.2.1.7, for inputs $x \vdash L_2$, the outputs y are also in L_2.

b) For a nonenergetic n-port under Def. 2.2.1, no such conclusion can be derived for inputs in L_2. Clearly, in view of the remarks a) and b), the energy defined by eqn. 2.2.1, although it has an obvious physical meaning, it is mathematically ill defined as far as the input-output representation of (v,i) or (i,v) pairs is concerned. Furthermore, no theorem for causality as Theor. 2.1.3 is deduced.

2.3 FROM V-I TO X-Y DOMAIN

In these sections the fundamental concepts and definitions of Linear Transformations as applied to linear time-invariant n-ports are examined. Their properties and existence criteria are derived by establishing a new matrix [TZ1] describing the network, which is called Σ matrix. Their implications in terms of retaining certain classical properties (e.g. passivity, losslessness, reciprocity) of the original network are derived.

2.3.1 Definition of the Σ matrix

Let a linear time-invariant n-port be described in terms of (v,i) admissible pairs as depicted in sec 2.2. Consider the equivalence shown in Fig.2.3.1 established via a Linear

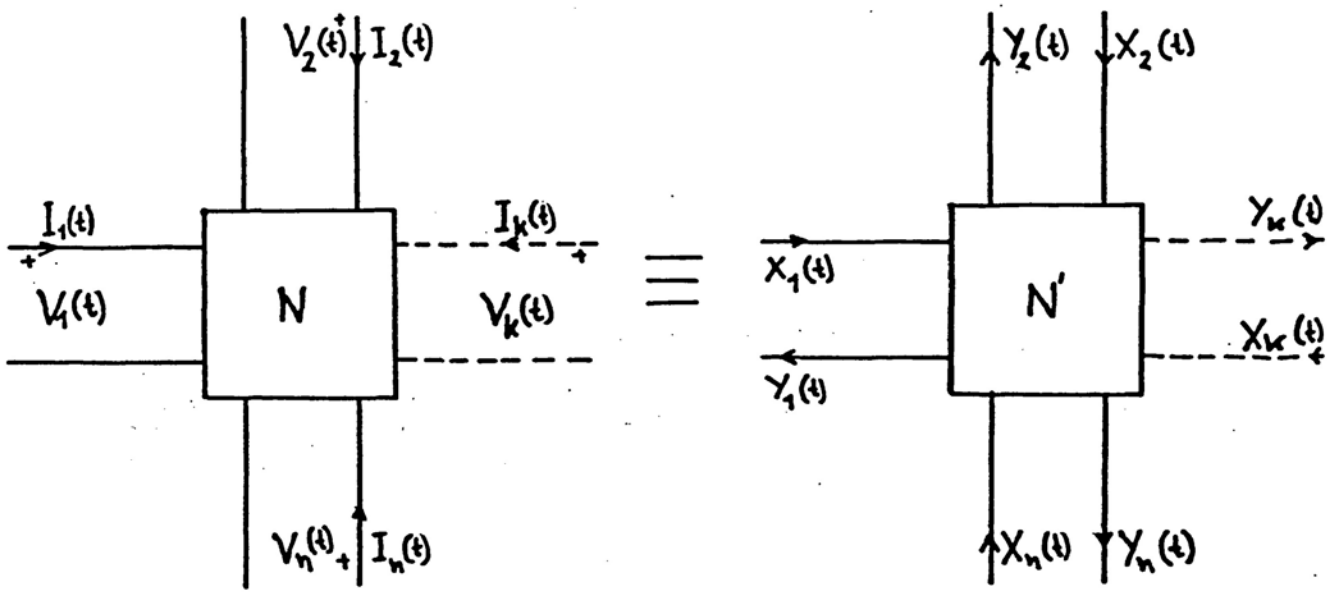


Fig.2.3.1: The Linearly Transformed n-port

transformation in the frequency domain of the voltage-current variables as indicated in eq.2.3.1.

$$\begin{bmatrix} X_k(s) \\ Y_k(s) \end{bmatrix} = \begin{bmatrix} \alpha_k & \beta_k \\ \gamma_k & \delta_k \end{bmatrix} \begin{bmatrix} V_k(s) \\ I_k(s) \end{bmatrix} = \mathcal{A}_k \begin{bmatrix} V_k(s) \\ I_k(s) \end{bmatrix} \qquad (2.3.1)$$

where $\Delta_k = \det \mathcal{A}_k = \alpha_k \delta_k - \gamma_k \beta_k \neq 0$.

The lower (upper) case letters are denoting variables in the time (frequency) domain. The α_k, β_k, γ_k, δ_k of the matrix $\mathcal{A}_k$ are, in general, assumed to be functions of frequency s.

By definition, it is assumed that N´ satisfies the following postulates:

a) The vectors x(t) and y(t) are the input and output vectors respectively. Moreover for x(t) real, y(t) is also real.

b) The input energy to the n-ports of Fig.2.3.1 is respectively:

$$E_a(t) = \int_{-\infty}^{t} v^T(t) i(t) dt, \quad E_b(t) = \int_{-\infty}^{t} (x^T(t)x(t) - y^T(t)y(t)) dt) \tag{2.3.2}$$

c) N´ is linear by Defn. 2.1.2.

d) N´ is time-invariant by Defn. 2.1.4.

e) N is causal by Defn. 2.1.3.

f) N´ is completely solvable by Defn. 2.1.6.

The requirement for complete solvability is very crucial for it is equivalent to the continuity of N´ considered as a linear valued operator (App.A) which in turn is the necessary condition for the existence of the input-output representation of N´ (Kernel's Theorem in App.A). Therefore the entries of $\mathcal{A}_k$ cannot have arbitrary values but, on the contrary, are subject to existence constraints emerging from consideration of the linear transformation in eqn. 2.3.1 in conjuction with the properties of the original network N. Unlike other properties, complete solvability is not an idealization. Indeed in view of the alternative forms for continuity of linear operators (App.A), complete solvability is equivalent to the finiteness of the norm of the network. This is in addition to causality, means (Theorem 2.1.1) N´

is nonenergetic, i.e. it exchanges with enviroment a finite ammount of energy. This is however always the case.

From the above, it is clear that N´ is solvable, nonenergetic, linear and time-invariant. Then by Theor. 2.1.3 the input-output relation is given by:

$$y(t) = \int_{-\infty}^{t} \Sigma(t-\tau)x(\tau)d\tau \qquad (2.3.3)$$

and the corresponding Laplace Transform is:

$$Y(s) = \Sigma(s)X(s) \qquad (2.3.4)$$

where X(s) and Y(s) are nx1 vectors and $\Sigma(s)$ is an nxn matrix with entries being functions of s. Clearly, the representation given by eqns. (2.3.3,4) exists for all matrices $\mathcal{A}_k$ for which N´ is completely solvable. In reference [TZ1], some criteria for existence of matrix Σ and therefore constraints on matrices $\mathcal{A}_k$, are described.

The Σ matrix is a generalization that, among others, includes all the known network matrices.
For example by setting:

$$\mathcal{A}_k = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad k=1,n$$

we derive the impedance matrix Z(s) of the n-port.
Similarly, a substitution

$$\mathcal{A}_k = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad k=1,n$$

gives the Scattering matrix S of the n-port with normalization equal to unity. Nevertheless, the utility of the

Σ-matrix in active filter design lies upon the fact that matrices other the conventional are employed [DI1,2,3]. The same holds for digital filters through Linear Transformations [TZ2,3].

2.3.2 Relation between the Σ matrix and other network matrices

With reference to Fig.2.3.1 we have the following well known n-port matrices [AN], for V-I domain

Impedance matrix Z: $V=ZI$

Admittance matrix Ψ: $I=\Psi V$

Scattering matrix S: $V^r=SV^i$

and Linear Transformation matrix Σ: $Y=\Sigma X$

for X-Y domain.

The matrices Z,Ψ,S and Σ are nxn matrices with entries, functions of s and V,I,V^r,V^i,Y,X, are nx1 vectors for the voltage, current, refrected voltage, incident voltage and linearly tranformed vectors. The relations between these four n-port matrices have been established [TZ1] and are given in Table 2.3.1.

Table 2.3.1

	Σ	S	Ψ	Z
Σ	Σ	$(C+D+(C-D)S)\cdot(A+B+(A-B)S)^{-1}$	$(C+D\Psi)(A+B\Psi)^{-1}$	$(CZ+D)(AZ+B)^{-1}$
S	$(D-C+\Sigma(A+B))^{-1}\cdot(D+C-\Sigma(A+B))$	S	$(1_n+\Psi)^{-1}(1_n-\Psi)$	$(Z+1_n)^{-1}(Z-1_n)$
Ψ	$(-D+\Sigma B)^{-1}(C-\Sigma A)$	$(1_n-S)(1_n+S)^{-1}$	Ψ	Z^{-1}
Z	$(C-\Sigma A)^{-1}(-D+\Sigma B)$	$(1_n+S)(1_n-S)^{-1}$	Ψ^{-1}	Z

The matrices A,B,C,D are defined as:

$A=\text{diag}\{\alpha_1,\alpha_2,\ldots\alpha_n\}$, $B=\text{diag}\{\beta_1,\beta_2,\beta_3,\ldots\beta_n\}$

$C=\text{diag}\{\gamma_1,\gamma_2,\ldots\gamma_n\}$, $D=\text{diag}\{\delta_1,\delta_2,\delta_3,\ldots\delta_n\}$

and $\Delta_i=\alpha_i\delta_i-\beta_i\gamma_i\neq 0$, $i=1,n$

2.3.3 Relation between Σ and the modified Chain Matrix T

[DI1,2]

Assume that the n-port is in fact such that n=2m ports, the first m being the input ports and the rest m ports considered as the output ports. The 2mx1 vectors $[V_1 I_1]^T$ and $[V_2 I_2]^T$ are taken as input and output vectors respectively. Consider that the n-port is consisting of m two-ports connected in cascade and each two port is having a modified chain matrix T as in Fig.(2.3.2).

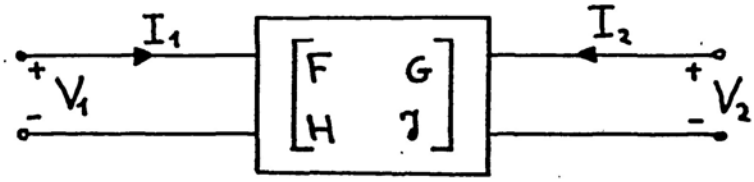


Fig.2.3.2: One of the m two-ports

The term "modified" is used to indicate that the current I_2 is taken with opposite to the conventional direction. Thus one has:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = T \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} F & G \\ H & J \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2.3.5)$$

The port vectors $\Lambda_1=[V_1\ I_1]^T$ and $\Lambda_2=[V_2\ I_2]^T$ are referred to as the left and the right port variable vectors respectively.

Recalling the eqn.(2.3.1):

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \mathcal{A}_1 \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \quad \text{with } \Delta_1 = \det \mathcal{A}_1 \neq 0 \tag{2.3.6}$$

and

$$\begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \mathcal{A}_2 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad \text{with } \Delta_2 = \det \mathcal{A}_2 \neq 0 \tag{2.3.7}$$

Matrix $\mathcal{A}_1$ is referred as the <u>left transformation matrix</u> and $\mathcal{A}_2$ as the <u>right transformation matrix</u>. The new port-vectors $[X_1\ Y_1]^T$ and $[X_2\ Y_2]^T$ are referred to as the <u>left</u> and <u>right Linearly Transformed port-vectors</u> respectively.

On substituting the port variables vectors in eqn.(2.3.5) by the new vectors, as defined by eqns.(2.3.6,7) the result can be obtained:

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \mathcal{A}_1 T \mathcal{A}_2^{-2} \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \tag{2.3.8}$$

The matrix

$$T' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mathcal{A}_1 T \mathcal{A}_2^{-1}$$

is called the linearly transformed chain matrix.

The elements of T' are determined through eqns.(2.3.5,6,7,8) to be as shown below:

$$a = \frac{1}{\Delta_2}\left[\alpha_1(\delta_2 F-\gamma_2 G)+\beta_1(\delta_2 H-\gamma_2 J)\right]$$

$$b = \frac{1}{\Delta_2}\left[\alpha_1(\alpha_2 G-\beta_2 F)+\beta_1(\alpha_2 J-\beta_2 H)\right]$$

$$c = \frac{1}{\Delta_2}\left[\gamma_1(\delta_2 F-\gamma_2 G)+\delta_1(\delta_2 H-\gamma_2 J)\right] \qquad (2.3.9)$$

$$d = \frac{1}{\Delta_2}\left[\gamma_1(\alpha_2 G-\beta_2 F)+\delta_1(\alpha_2 J-\beta_2 H)\right]$$

$$\Delta_2 = \alpha_2\delta_2-\beta_2\gamma_2$$

It can be easily found that:

$$\det T' = ad-bc = -\frac{\det \mathcal{A}_1}{\det \mathcal{A}_2} = -\frac{\Delta_1}{\Delta_2} \neq 0 \qquad (2.3.10)$$

As defined previously, we assume that X_1, X_2 will be the inputs of the linearly tranformed network N' and Y_1, Y_2 the outputs.

So one is interested far more in the matrix Σ rather than the matrix T'. From (2.3.8):

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \Sigma \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (2.3.11)$$

It has been found that the transfer functions K,L,M,N are given in terms of the elements of T' as:

$$K = \frac{d}{b} \qquad M = \frac{1}{b}$$

$$L = -\frac{ad-bc}{b} \qquad N = -\frac{a}{b} \quad \text{and} \quad \Sigma = \begin{bmatrix} \frac{d}{b} & -\frac{ad-bc}{b} \\ \frac{1}{b} & -\frac{a}{b} \end{bmatrix} \qquad (2.3.12)$$

The existence of such Linear Transformation requires $b \neq 0$. This constraint is explicity written, by eqn.2.3.9:

$$\frac{\alpha_1(\alpha_2 G-\beta_2 F)+\beta_1(\alpha_2 J-\beta_2 H)}{\Delta_2} \neq 0 \quad \text{or}$$

$$\alpha_1(\alpha_2 G-\beta_2 F)+\beta_1(\alpha_2 J-\beta_2 H) \neq 0 \quad \text{or not } \alpha_1=\alpha_2=0 \text{ for} \qquad (2.3.13)$$

series arm Z and not $\beta_1=\beta_2=0$ for shunt arm Y.

The eqns.(2.3.12) and (2.3.9) give an indirect way of relating the matrices T of the analogue two-port with Σ of the Linearly Transformed network, by relating their elements.

The representation of the oriented LT network described by eqn.2.3.11 is shown in Fig.(2.3.3). In this figure the inputs X_1 and X_2 are denoted by a small circle, whilst the outputs Y_1 and Y_2 are denoted by a small triangle appropriately oriented.

The transfer functions K,L,M,N which are rational functions of s, must be practically realizable under the technology in mind (e.g. active-RC or active-R) while their stability may or may not be necessary. This fact is not surprising since their connection to the LT structure, could give a stable realization.

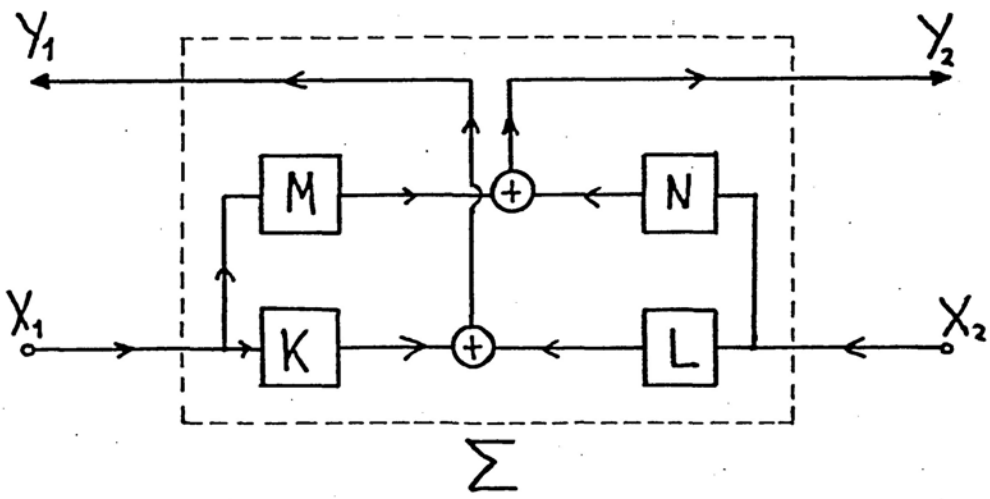


Fig.2.3.3: The LT two-port

2.3.4 Passivity, Losslessness, Reciprocity

The Linear Transformation in Sec 2.3.1 is an equivalence between two n-ports established in a very general manner, to provide a greater flexibility to the designer so as to allow the design of simple networks. Since the LT is by definition, a frequency transformation, it follows that if the original network is passive, or lossless, or reciprocal, the derived network N´ (Fig.2.3.1) is not, in general,expected to retain these properties [NE1].

So in order N´ to retain the properties of passivity, losslessness and reciprocity the following conditions, should be met:

Passivity. To check for passivity it is necessary to consider eqn.(2.1.5), which is the necessary and sufficient condition for passivity, in causal, solvable n-ports.
With reference to Fig.2.3.1 and by using Parseval's Theorem, eqn.(2.1.5) is interpeted on the jω-axis as:

$$X^*X-Y^*Y \geq 0\Big|_{s=j\omega}$$

which by virtue of Y=ΣX becomes:

$$I_n-\Sigma^*\Sigma \geq 0\Big|_{s=j\omega} \qquad (2.3.14)$$

where I_n is the nxn unity matrix.
The inequility (2.3.14) is the well known Bounded property of the scattering matrix. The condition for losslessness is derived by replacing the "≥" with "=" in eq.2.3.14. So:

N´ is lossless if $I_n-\Sigma^*\Sigma = 0$ (2.3.15)

Reciprocity as defined by Def.2.1.9, is satisfied by a symmetric matrix. Thus if $\Sigma = \{\sigma_{ik}, i,k=1,n\}$, the necessary and sufficient condition for the n-port N´ to be reciprocal is given by:

$$\sigma_{ik} = \sigma_{ki} \ , \quad i,k=1,n \tag{2.3.16}$$

Alternatively, one can define other less restrictive reciprocity modes than the one in Def. 2.1.9, by allowing the presence of a finite real constant factor λ in (2.3.16) i.e.

$$\sigma_{ik} = \lambda\sigma_{ki} \ , \ i,k=1,n \tag{2.3.17}$$

This relationship has no effect on the shape of the output, other than a constant level shift which of course can be taken into account.

2.3.5. Dimensions of X and Y. Selection of inputs and outputs

The dimensions of the new variables X and Y depend on the dimensions of the elements of the matrices $\mathcal{A}_1$ and $\mathcal{A}_2$. Indeed the transformation variables can represent any electrical quantity, e.g. voltage, current or even power. To illustrate this point it may be assumed that α's and γ's, for example, represent conductances, whereas β's and δ's may be taken to be dimensionless quantities.
In such a case X and Y have the dimension of electrical currents. So therefore, it can be classified five different types of electrical quantities represented by X's and Y's. These are shown in Table 2.3.2.

Table 2.3.2

Name of LT	Dimensions						Matrix
	X	Y	α	β	γ	δ	e.g.
Voltage-Voltage	Voltage	Voltage	Pure number	impedance	Pure number	impedance	$\begin{bmatrix} 1 & R_1 \\ 1 & R_2 \end{bmatrix}$
Voltage-Current	Voltage	Current	pure number	impedance	admittance	Pure number	$\begin{bmatrix} 1 & R_1 \\ G_2 & 1 \end{bmatrix}$
Current-Voltage	Current	Voltage	admittance	Pure number	Pure number	impedance	$\begin{bmatrix} G_1 & 1 \\ 1 & R_2 \end{bmatrix}$
Current-Current	Current	Current	admittance	pure number	admittance	pure number	$\begin{bmatrix} G_1 & 1 \\ G_2 & 1 \end{bmatrix}$
Power-Power	Power$^{1/2}$	Power$^{1/2}$	admittance$^{1/2}$	impedance$^{1/2}$	admittance$^{1/2}$	impedance$^{1/2}$	$\begin{bmatrix} R_1^{1/2} & R_2^{1/2} \\ R_3^{1/2} & R_4^{1/2} \end{bmatrix}$

However, in practice the nature, and hence the dimensions, of the variables X and Y is influenced by the active components to be used in the final implementation of the Linearly Transformed network. If voltage-controlled-voltage-source devices are used such as the operational amplifiers, it is reasonable to require the transformed variables to represent voltages. This implies that the a,b,c and d parameters of eqns.(2.3.8,9) represent voltage transfer ratios and hence they are dimensionless. This can be achieved by taking α's and γ's in the transformation matrices to be dimensionless (i.e. pure numbers, or voltage transfer ratios), and β's and δ's to have dimensions of impedances as it can be shown from the first row of Table 2.3.2.

As it will be shown later on, impedance scaling method for ladder simulation could be interpreted as voltage-current transformation.

The active realization of the LT network requires an oriented description of the network, as shown in Fig.2.3.3. It should be noted that the interdependence of V_1 and I_1 (V_2 and I_2) in the V-I domain, is transferred to X_1 and Y_1 (X_2 and Y_2) through the linear transformation, implying that X_1 and Y_1 (X_2 and Y_2) can not be taken simultaneously as inputs or outputs of the LT network. Therefore, only two divisions of the set of the transformed variables into input and output sets are possible as previously said, and they are:

(i) $\{X_1, X_2 | Y_1, Y_2\}$

and

(ii) $\{X_1, Y_2 | Y_1, X_2\}$

These two divisions, however, theoretically are not independent. It is apparent that by considering the first division and then interchanging the rows of λ_2, it can actually be obtained the second division, since an interchange of X_2 and Y_2 is effected thereby. Therefore the first division with X_1 and X_2 as inputs can be considered as a general case.

In fact, it can be taken the set X_1 and Y_1 as the inputs of the network and X_2 and Y_2 as the outputs, but as will be shown this will require additional active and passive elements than those the original network contained, and this fact is much undesirable. To have such a representation one has to eliminate the interdependence of V_1 and I_1 (V_2 and I_2), by introducing to the analogue network nullators and norators to the inputs and outputs, as it is shown in Fig.2.3.4.

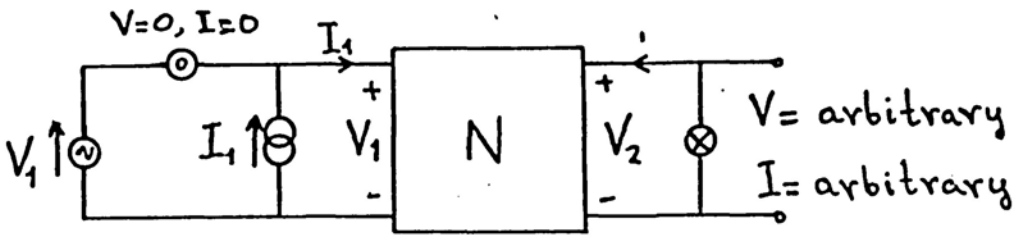
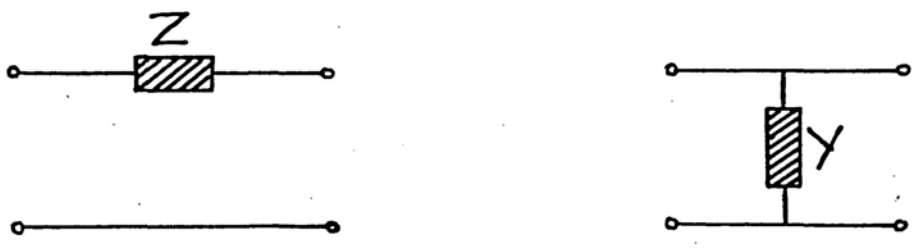


Fig.2.3.4

By linearly transforming the network N it could be derived the network N´ with X_1, Y_1 inputs and X_2, Y_2 outputs. It can be seen that this choice is undesirable, but further investigation could be done in order to draw specific conclusions.

2.3.6 Ladder Elements Considerations

Up to now it has been considered the n-port network consisting of m two-ports where n=2m, connected in cascade and we concluded some general results of the matrix. It can now be examined the internal configuration of each of the individual two-ports. In passive, doubly resistive terminated ladder LC filters, as this is the case, as explained in Chapter 1, there are two ports composed either by a series impedance Z, or by a shunt admittance Y with modified chain matrices T_Z or T_Y respectively. (Fig.2.3.5)



(a) (b)

$$T_Z = \begin{bmatrix} 1 & -Z \\ 0 & -1 \end{bmatrix} = T_Z^{-1} \qquad T_Y = \begin{bmatrix} 1 & 0 \\ Y & -1 \end{bmatrix} = T_Y^{-1}$$

Fig.2.3.5

This fact, the ladder type of the filter, and the form of chain matrices are very important contributing factors towards the simplification of the transform equations derived so far; for the individual series and shunt cases.

The transfer ratios of eqn. (2.3.12) or the elements of Σ matrix can be simplified and are given by eqns.(2.3.18,19):

For the series arm Z

$$K_Z = \frac{\gamma_1(\alpha_2 Z+\beta_2)+\delta_1\alpha_2}{\alpha_1(\alpha_2 Z+\beta_2)+\beta_1\alpha_2}\ , \qquad L_Z = -\frac{\Delta_1}{\alpha_1(\alpha_2 Z+\beta_2)+\beta_1\alpha_2}$$

$$M_Z = -\frac{\Delta_2}{\alpha_1(\alpha_2 Z+\beta_2)+\beta_1\alpha_2}\ , \qquad N_Z = \frac{\alpha_1(\gamma_2 Z+\delta_2)+\beta_1\gamma_2}{\alpha_1(\alpha_2 Z+\beta_2)+\beta_1\alpha_2} \qquad (2.3.18)$$

For the shunt arm Y

$$K_Y = \frac{\delta_1(\beta_2 Y+\alpha_2)+\gamma_1\beta_2}{\beta_1(\beta_2 Y+\alpha_2)+\alpha_1\beta_2}\ , \qquad L_Y = \frac{-\Delta_1}{\beta_1(\beta_2 Y+\alpha_2)+\alpha_1\beta_2}$$

$$M_Y = -\frac{\Delta_2}{\beta_1(\beta_2 Y+\alpha_2)+\alpha_1\beta_2}\ , \qquad N_Y = \frac{\beta_1(\delta_2 Y+\gamma_2)+\alpha_1\delta_2}{\beta_1(\beta_2 Y+\alpha_2)+\alpha_1\beta_2} \qquad (2.3.19)$$

In both cases $\Delta_i=\alpha_i\delta_i-\beta_i\gamma_i=\det\lambda_i\neq 0$, i=1,2.
K,L,M and N are in fact the transfer ratios to be actively realized within the LT structure to generate Y_1 and Y_2 from the inputs X_1 and X_2. Therefore, as already pointed out, they must be realizable and this requirement sets some constraints on the choice of the transformation matrices as it will be discussed elsewhere in this work.

Notice that it is used the same symbol Y for the admittance of the analogue prototype and the outputs of the LT network. We distinguish them by having two subscripts in the outputs of the LT network· e.g. Y_{11}, Y_{21} etc.

2.3.7 The Linearly Transformed Ladder Network. Some Constraints

The general form of a doubly terminated lossless ladder filter is shown in Fig. 2.3.6, where, as previously discussed, the two-port networks N_i, i=1,n are simple or composite series or shunt reactive arms connected in cascade to form the ladder filter.

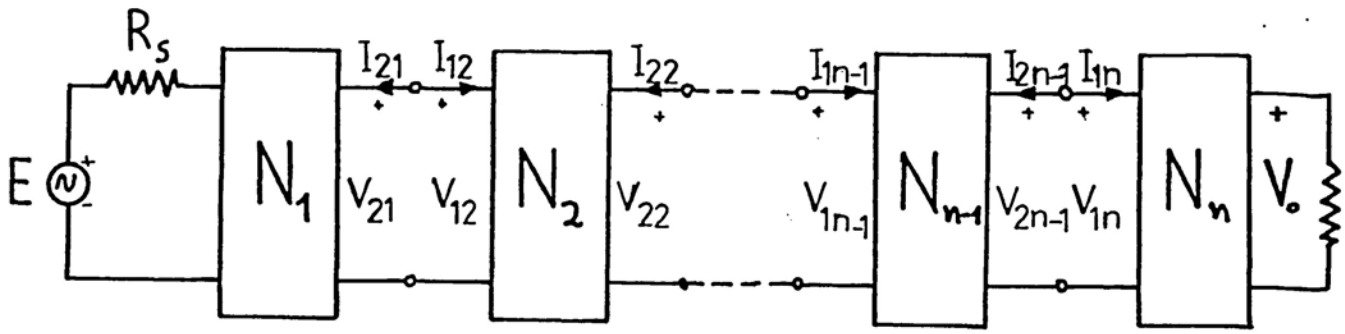


Fig. 2.3.6.

In the previous sections have been considered transformations of such two-port networks and in this section, the important question of their interconnection in the transformed domain is examined as in addition the transformation of the passive terminations (i.e. R_s in series with the voltage source E, and R_L) are considered so as to enable the complete formulation of the Linear Transform Active ladder filter (LTA).

Interconnection constraints [DI1,2,3]

The interconnection (i.e. cascading) of two ladder elements (e.g. a series arm connected to a shunt arm) involves certain constraints at the interconnection ports. The cascade connection of two passive two-ports in V-I domain shown in Fig.2.3.7. is described by eqn.(2.3.20) below:

$$\begin{bmatrix} V_{2i} \\ I_{2i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_{1j} \\ I_{1j} \end{bmatrix} \qquad (2.3.20)$$

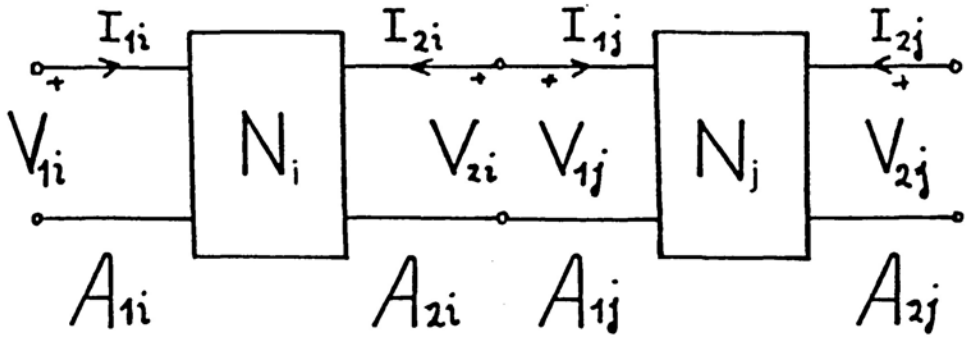


Fig. 2.3.7

The linear transformation $\{\mathcal{A}_{1i}, \mathcal{A}_{2i}\}$ is assumed to be applied to N_i, whilst $\{\mathcal{A}_{1j}, \mathcal{A}_{2j}\}$ is applied to N_j.
By definition:

$$\begin{bmatrix} X_{1j} \\ Y_{1j} \end{bmatrix} = \mathcal{A}_{1j} \begin{bmatrix} V_{1j} \\ I_{1j} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_{2i} \\ I_{2i} \end{bmatrix} = \mathcal{A}_{2i}^{-1} \begin{bmatrix} X_{2i} \\ Y_{2i} \end{bmatrix} \tag{2.3.21}$$

By combining (2.3.20,21) one has:

$$\begin{bmatrix} X_{1j} \\ Y_{1j} \end{bmatrix} = \mathcal{A}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathcal{A}_{2i}^{-1} \begin{bmatrix} X_{2i} \\ Y_{21} \end{bmatrix} \tag{2.3.23}$$

The matrix $I_{ij} = \mathcal{A}_{1j} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathcal{A}_{2i}^{-1}$ is called "the interconnection matrix" and it has been found [DI1,2] that the simplest possible interconnecting network is defined by the matrix:

$$I_{ij} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ i.e. } \begin{matrix} X_{1j}=Y_{2i} \\ X_{2i}=Y_{1i} \end{matrix} \tag{2.3.24}$$

This interconnection will be referred as the <u>cross-cascade interconnection</u>. (Fig.2.3.8)

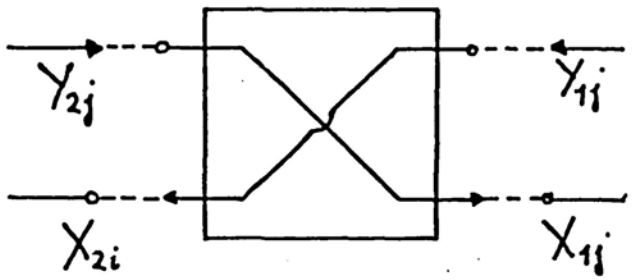


Fig. 2.3.8

By eqns. 2.3.23,24 it is true:

$$\mathcal{A}_{1j}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\mathcal{A}_{2i}^{-1} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \qquad (2.3.25)$$

This relationship will be reffered as the compatibility relationship, imposing the constraint that $\mathcal{A}_{2i}$ will be compatible to $\mathcal{A}_{1j}$, denoted by: $\mathcal{A}_{2i}=\mathcal{A}_{1j}^{c}$, where:

$$\mathcal{A}_{2i}=\mathcal{A}_{1j}^{c} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\mathcal{A}_{1j}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\alpha_{1j} & \beta_{1j}\\ \gamma_{1j} & \delta_{1j}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$$\text{or } \mathcal{A}_{2i}=\mathcal{A}_{1j}^{c} = \begin{bmatrix}\gamma_{1j} & -\delta_{1j}\\ \alpha_{1j} & -\beta_{1j}\end{bmatrix} \qquad (2.3.26)$$

Input termination constraint. [DI1,2]

To eliminate the number of transformation matrices used, it has been proposed [DI1,2] to transform the first reactive element of the ladder together with the resistive input termination. Doing so the LT network is represented by a two input-one output model (E and X_{21} inputs and Y_{21} output). This is shown explicitly in Fig.2.3.9 and by eqn. (2.3.27).

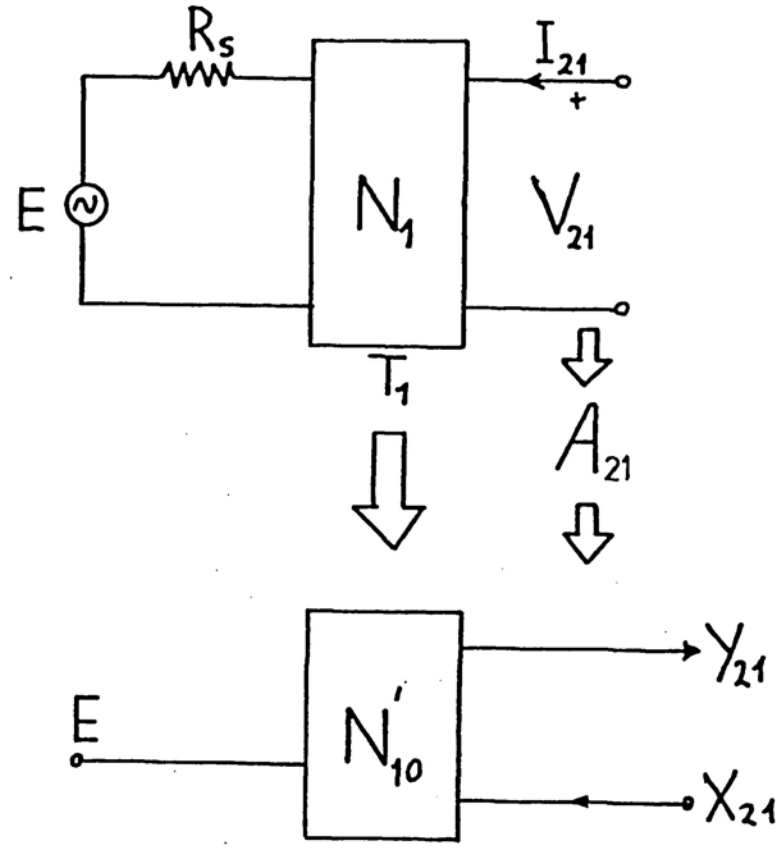


Fig. 2.3.9

$$\begin{bmatrix}1 & R_s\end{bmatrix} T_1 A_{21}^{-1} \begin{bmatrix}X_{21} \\ Y_{21}\end{bmatrix} = E \qquad (2.3.27)$$

The explicit forms of eqn.(2.3.27) are given separately below, when N_1 is taken for (i) a series arm of a ladder and (ii) a shunt arm of the ladder.

(i) For the series arm Z_1

$$Y_{21} = \frac{\beta_{21}\gamma_{21}-\alpha_{21}\delta_{21}}{\beta_{21}+\alpha_{21}(Z_1+R_s)} E + \frac{\delta_{21}+\gamma_{21}(Z_1+R_s)}{\beta_{21}+\alpha_{21}(Z_1+R_s)} X_{21} \qquad (2.3.28)$$

(ii) For the shunt arm Y_1

$$Y_{21} = \frac{\beta_{21}\gamma_{21}-\alpha_{21}\delta_{21}}{\alpha_{21}R_s+\beta_{21}(1+R_sY_1)} E + \frac{\gamma_{21}R_s+\delta_{21}(1+R_sY_1)}{\alpha_{21}R_s+\beta_{21}(1+R_sY_1)} X_{21} \qquad (2.3.29)$$

Output termination constraint

The same has been done with the last reactive arm N_n of the ladder filter, taken in conjuction with load termination R_L. (Fig. 2.3.10)

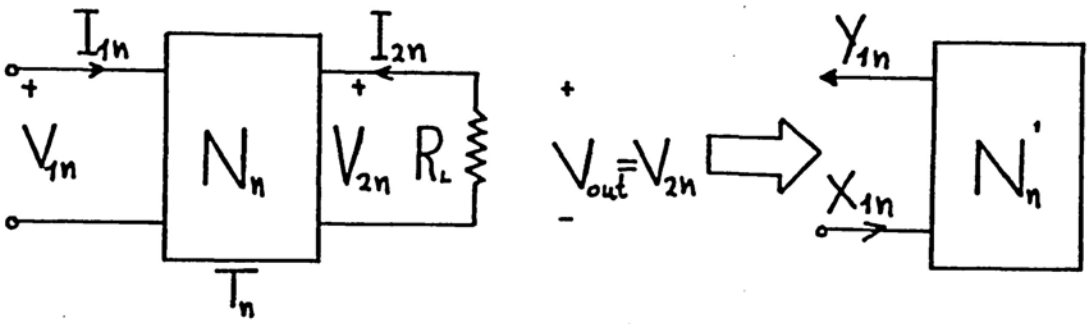


Fig. 2.3.10

$$[1 \quad R_L]\, T_n^{-1} \mathcal{A}_{1n}^{-1} \begin{bmatrix} X_{1n} \\ Y_{1n} \end{bmatrix} = 0 \qquad (2.3.30)$$

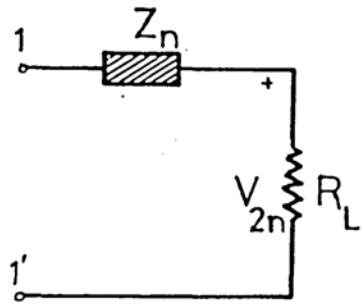
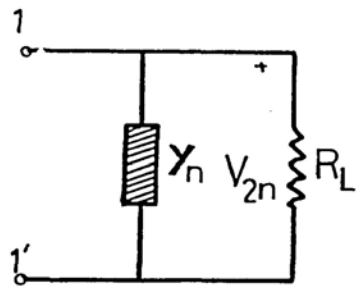
Eqn.(2.3.30) relates the new variables X_{1n} and Y_{1n} giving the network N_n'.

However, if this model for the terminated last reactice ladder arm is employed, the output Vout of the original passive ladder is no longer available explicitly. In such a case the output node of the LTA filter must be located from eqn.(2.3.31) with a proper choice for $\mathcal{A}_{1n}$.

$$\begin{bmatrix} X_{1n} \\ Y_{1n} \end{bmatrix} = \mathcal{A}_{1n} T_n \begin{bmatrix} V_{2n} \\ I_{2n} \end{bmatrix} = \mathcal{A}_{1n} T_n \begin{bmatrix} 1 \\ -G_L \end{bmatrix} \text{Vout} \qquad (2.3.31)$$

This constraint will allow specific choices of the elements of $\mathcal{A}_{1n}$ as shown in Table 2.3.3.

Table 2.3.3

TEPMINATED ARM	A_{1n}	LT DESCRIPTION
	$\begin{bmatrix} 0 & \beta_{1n} \\ \gamma_{1n} & \delta_{1n} \end{bmatrix}$	$Y_{1n}=\frac{\delta_{1n}+\gamma_{1n}(Z_n+R_L)}{\beta_{1n}}X_{1n}$ $X_{1n}=\beta_{1n}G_L V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & \beta_{1n} \\ 0 & \delta_{1n} \end{bmatrix}$	$X_{1n}=\frac{\delta_{1n}}{\beta_{1n}+\alpha_{1n}(Z_n+R_L)}X_{1n}$ $Y_{1n}=\delta_{1n}G_L V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & 0 \\ \gamma_{1n} & \delta_{1n} \end{bmatrix}$	$Y_{1n}=\frac{\gamma_{1n}R_L+\delta_{1n}(Y_nR_L+1)}{\alpha_{1n}R_L}X_{1n}$ $X_{1n}=\alpha_{1n}V_{2n}$
	$\begin{bmatrix} \alpha_{1n} & \beta_{1n} \\ \gamma_{1n} & 0 \end{bmatrix}$	$Y_{1n}=\frac{\alpha_{1n}}{\alpha_{1n}+\beta_{1n}(Y_nR_L+1)}X_{1n}$ $Y_{1n}=\gamma_{1n}V_{2n}$

2.4 LT, A GENERAL APPROACH TO LADDER SIMULATION

As it has been discussed in Chapter 1 in order to design an active filter, someone starts either from the transfer function of the filter to be simulated, or from the passive LC structure. Therefore there are three main ways of designing the desirable filter:

(i) By Transfer Function Simulation

(ii) By Ladder Simulation which contains the methods of

a) Inductance Simulation

b) Impedance Scaling and

(iii) By Signal-Flow-Graph Simulation or Operational Simulation

In this section it will be shown that most of the methods of Ladder Simulation and Operational Simulation can be interpreted as special cases of the general Linear Transformation technique.

2.4.1 Ladder Simulation

This type of Simulation includes the method of substituting the inductances of the passive-prototype by active-RC structures (inductance simulation method) and the method of transforming, in the frequency domain, all the elements of the ladder and realizing the transformed elements actively (impedance scaling method). Both methods are using the basic building block called Generalized Impedance Converter (GIC). This configuration could be taken as positive impedance converter (PIC), or negative impedance converter (NIC) and can be taken either as a gyrator (in inductance simulation method), or as an FDNR (in impedance scaling method). The GIC, therefore, is the most useful building block for ladder simulation. Practical realization for GIC has been proposed at first by Antoniou [AN1].

It has been proved [CO6], that the proposed configuration of Antoniou GIC, could be derived under linear transformations by transforming the variables in the V-I domain with matrices of the form:

$$\mathcal{A} = \begin{bmatrix} 1 & -Z_1 \\ 1 & \dfrac{Z_1 Z_3}{Z_2} \end{bmatrix} \qquad (2.4.1)$$

where Z_1, Z_2, Z_3 are appropriate impedances.

2.4.1.1 Inductance Simulation Method. Special Case 1

Although the inductance simulation method for deriving active-RC filter structures from doubly terminated ladders does not alter the port variables of the constituent sub-networks of the original ladder, the method described in section 2.4.1 may be used at the ports where inductors are embedded, and the Gorski -Popiel method with PIC's will appear

It should be noticed that these are not simulated structures in X-Y domain as LTA method requires, but Linear Transformation matrices are realized themselves. By imposing transformation matrices other than those described in (2.4.1) it could, possibly, be derived better and simpler structures. This interesting topic needs further investigation. Because of the form of LT in eqn.(2.4.1), it can be classified the impedance simulation circuirs as wave active filters [HA1], [CO1,2,3]. In Fig.2.4.1 the active-RC simulation of the 5th order low-pass elliptic filter is shown. This structure can be found in Ref.[A1]. That structure has been derived by embedding linear transformations, as previously described, at the points where the dotted line intersects the passive LC ladder as shown in Fig.2.4.1.

2.4.1.2 Impedance Scaling Method. Special Case 2

By impedance scaling method, the passive network, as discussed in Sec.1.3, consists of resistors, capacitors and frequency-dependent-negative resistors in the place of inductors, resistors and capacitors respectively. It will be shown, that the impedance scaling method is a special case of LT method, by considering voltage-current linear transformations. To illustrate this, consider the dual of the

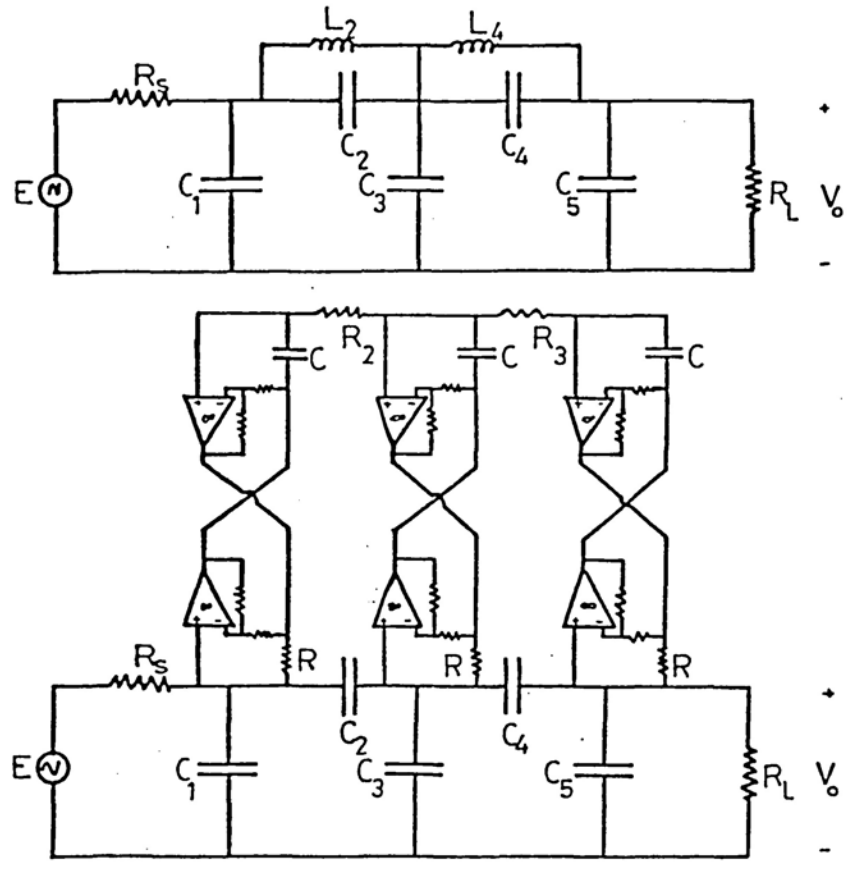


Fig. 2.4.1

third order elliptic filter in Fig.2.4.2a and the transformation matrices as shown.

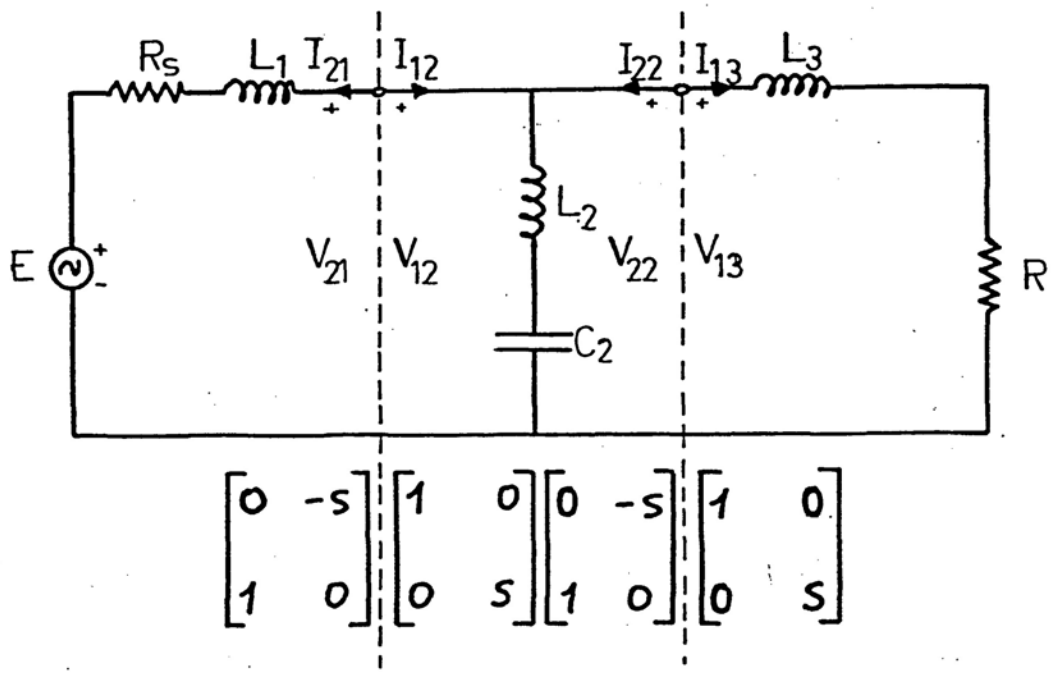


Fig. 2.4.2a

The variables in the X-Y domain: X_{21}, Y_{12}, X_{22}, Y_{12} represent currents and the Y_{21}, X_{12}, Y_{22}, X_{13} represent voltages. By the equations of Secs 2.3.6,7:

$$Y_{21} = E - L_1 X_{21} - \frac{R_s}{s} X_{21} \quad \text{for the input}$$

$$Y_{12} = \frac{R_s}{LCs^2+1} X_{12} + X_{22}, \quad Y_{22} = X_{12} \quad \text{for middle element and}$$

$$Y_{13} = \frac{R_s}{L_3 S + R_L} X_{13}, \quad Y_{13} = sG_L V_2 = \frac{1}{s\frac{1}{R_L}} V_2 \quad \text{for output}$$

Therefore the filter in X-Y domain is given as in Fig.2.4.2b.

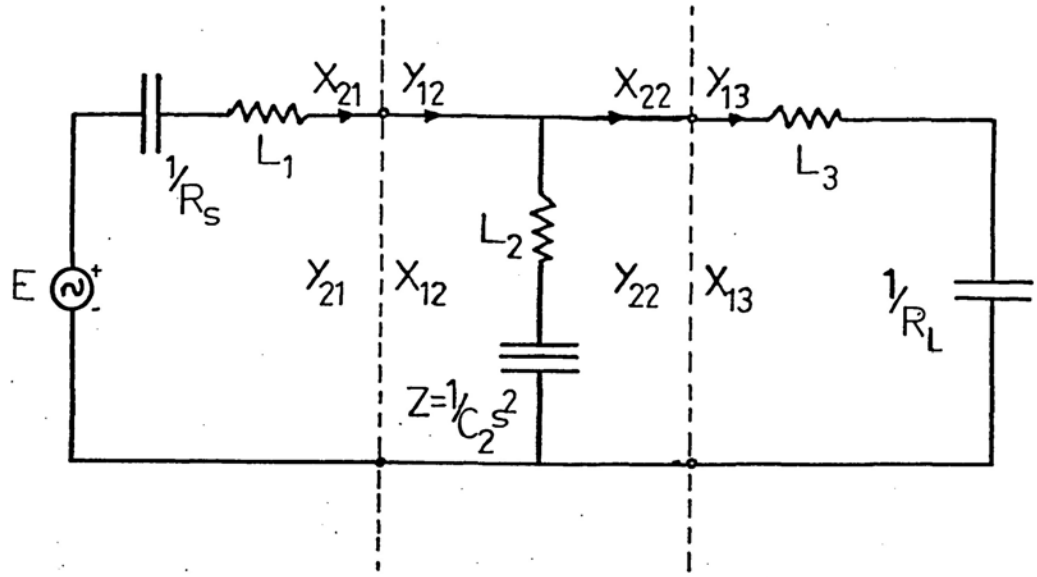


Fig. 2.4.2b

The establishment of a general voltage-current LTA procedure towards this end, instead of only giving an insight to the various impedance scaling techniques, will obtain some more interesting new methods and, perhaps give better circuits.

2.4.2 Operational Simulation. Special Case 3

This simulation in based on signal-flow-graph of the ladder. The relationships between the voltages and currents of the ladder are simulated and realized actively the result-

ing equations. This procedure leads to the well-known leapfrog-synthesis, proposed by Girling and Good [GI1,2]. It has been shown [DI1,3] that the leapfrog filter is a special case of LTA method if it is used series-shunt compatible transformations, as shown in Fig.2.4.3 for the fifth-order low-pass filter.

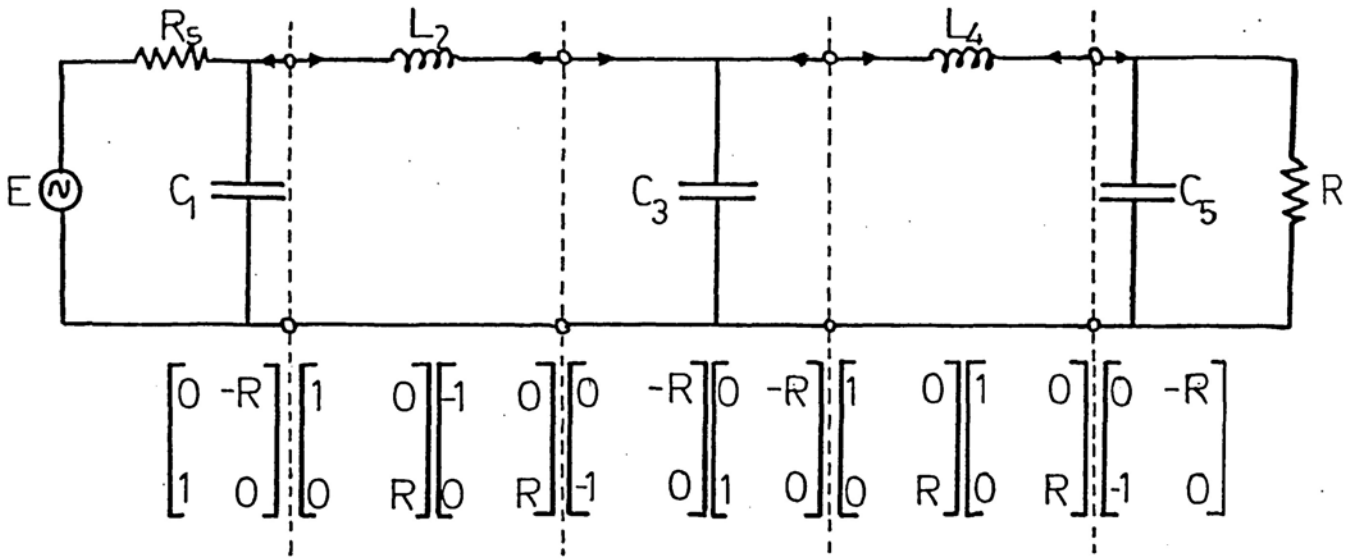


Fig. 2.4.3

It has been also shown [DI1] that more economical structures, than the leapfrog, could be obtained using the form-compatible linear transformation. The so called Wave Active Filters [CO1], [HA1] are also special cases of LTA [DE1].

The direct Signal Flow Graph [BR1] simulation has also been shown to be a form-compatible LTA case, where the application of the Frequency Dependent Linear Transformation (FDLT) proved to be more effective yielding more economic and, under certain conditions, canonic structures.

All the above cases belong to operational simulation method and are in fact special LTA cases. As LT simulates the voltages and currents of the ladder, it can be considered as Generalized Operational Simulation Technique. This Technique also gives multiple feedback active structures

which have been proved [TE1], [HU1], to give less sensitive structures than the cascaded ones.

The previous sections emphasized that LTA method could be interpreted as a general theory of structural simulation of passive filters to derive active-RC filters with the same properties. This fact in conjuction with the mathematical formulation described in Sec.2.3, could lead into a general synthesis procedure of Σ matrix, to derive realizable structures, as it has been done the last three decades for Z, Ψ and S matrices [BA], [VA], [AN1].

2.5 CLASSIFICATION OF LTA STRUCTURES

The classification of the active structures derived through Linear Transformations will be done by either some constraints that imposed to transformation matrices between the arms of the passive ladder, or by the form of Σ matrix of each arm itself. Those contraints will give simple and low-complexity structures.

Consider the nth order ladder with the appropriate transformation matrices as shown in Fig.2.5.1.

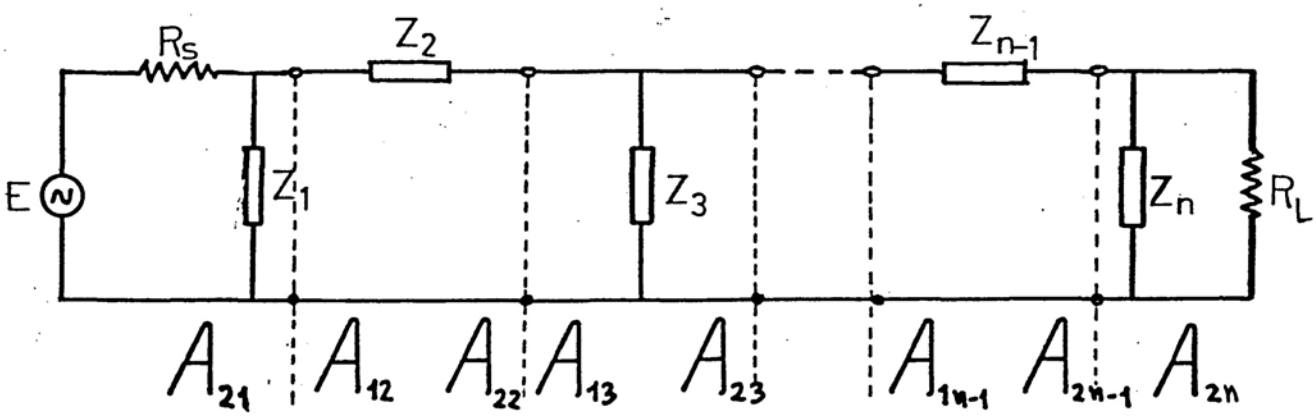


Fig. 2.5.1

The constraints of the 2(n-1) transformation matrices, which have been discussed so far is the compatibility relationship

(Sec.2.3.7) and the constraints in the output (Sec.2.3.7). The set $\{\mathcal{A}_{11},\mathcal{A}_{21}\}$, $\{\mathcal{A}_{12},\mathcal{A}_{22}\}\cdots\{\mathcal{A}_{1(n-1)},\mathcal{A}_{2(n-1)}\}$, $\{\mathcal{A}_{2n},\mathcal{A}_{1n}\}$ is called compatible set. So one element in transformation matrix $\mathcal{A}_{2n}$ is defined for direct output and the matrices $\mathcal{A}_{21},\mathcal{A}_{22},\cdots\mathcal{A}_{2(n-1)}$ are defined through the compatibility relationship: $\mathcal{A}_{21}=\mathcal{A}_{12}^{c},\ldots,\mathcal{A}_{2(n-1)}=\mathcal{A}_{1n}^{c}$. So there are $4(n-1)-1$ or $4n-5$ unknowns. According to constraints between different arms of the ladder, cyclic and acyclic transformations are obtained and according to the form of Σ matrix for each arm forms for low-complexity structures are derived.

2.5.1 Cyclic Transformations

In Cyclic Transformations, the set of Transformation matrices $\{\mathcal{A}_{1i},\mathcal{A}_{2i}\}$, for the ith arm appears more than once in the compatible transformation set. The period of the appearance of the set is the order of cycle. Thus one has cycle 1, cycle 2, etc. The compatible transformation set for the nth-order filter which is of cycle n belongs to acyclic transformations. The more useful transformations are obtained from cycles 1 or 2

Cyclic 2. Definition 2.5.1.1

A compatible set of linear transformation pairs

$$\{\mathcal{A}_{11},\mathcal{A}_{21}\},\{\mathcal{A}_{12},\mathcal{A}_{22}\},\{\mathcal{A}_{13},\mathcal{A}_{23}\},\{\mathcal{A}_{1(n-1)},\mathcal{A}_{2(n-1)}\},\{\mathcal{A}_{1n},\mathcal{A}_{2n}\}$$

used to transform a ladder $\{N_1,N_2,\ldots N_n\}$, is called series-shunt compatible, if and only if, the following conditions are satisfied:

$$\mathcal{A}_{1i} = \mathcal{A}_{1(i+2)}$$

$$\forall\ i=1,2,3,\ldots(n-2) \qquad (2.5.1)$$

$$\mathcal{A}_{2i} = \mathcal{A}_{2(i+2)}$$

These conditions imply that all series arms are transformed using the same pair $\{\mathcal{A}_{1Z}\,\mathcal{A}_{2Z}\}$, whilst all shunt arms are transformed using the same pair $\{\mathcal{A}_{1Y}\,\mathcal{A}_{2Y}\}$, where $\mathcal{A}_{1Z}$ is compatible to $\mathcal{A}_{2Y}$ and $\mathcal{A}_{1Y}$ is compatible to $\mathcal{A}_{2Z}$ and:

$$\mathcal{A}_{1Z} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix}, \quad \mathcal{A}_{2Z} = \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix}$$

$$\mathcal{A}_{1Y} = \begin{bmatrix} \gamma_2 & -\delta_2 \\ \alpha_2 & -\beta_2 \end{bmatrix}, \quad \mathcal{A}_{2Y} = \begin{bmatrix} \gamma_1 & -\delta_1 \\ \alpha_1 & -\beta_1 \end{bmatrix} \qquad (2.5.2)$$

The elements K,L,M,N of the transformation matrix are therefore, for all Z's and Y's:

Series arm Z:

$$K_Z = \frac{\gamma_1\alpha_2 Z+\gamma_1\beta_2+\delta_1\alpha_2}{\alpha_1\alpha_2 Z+\alpha_1\beta_2+\beta_1\alpha_2}, \quad L_Z = \frac{-\Delta_1}{\alpha_1\alpha_2 Z+\alpha_1\beta_2+\beta_1\alpha_2}$$

$$M_Z = \frac{-\Delta_2}{\alpha_1\alpha_1 Z+\alpha_1\beta_2+\beta_1\alpha_2}, \quad N_Z = \frac{\alpha_1\gamma_2 Z+\alpha_1\delta_2+\beta_1\gamma_2}{\alpha_1\alpha_2 Z+\alpha_1\beta_2+\beta_1\alpha_2} \qquad (2.5.3a)$$

Shunt arm Y:

$$K_Y = \frac{\delta_1\beta_2 Y-\gamma_1\beta_2-\delta_1\alpha_2}{\delta_1\delta_2 Y-\gamma_1\delta_2-\delta_1\gamma_2}, \quad L_Y = \frac{-\Delta_2}{\delta_1\delta_2 Y-\gamma_1\delta_2-\gamma_2\delta_1}$$

$$M_Y = \frac{-\Delta_1}{\delta_1\delta_2 Y-\gamma_1\delta_2-\delta_1\gamma_2}, \quad N_Y = \frac{\beta_1\delta_2 Y-\alpha_1\delta_2-\beta_1\gamma_2}{\delta_1\delta_2 Y-\gamma_1\delta_2-\delta_1\gamma_2} \qquad (2.5.3b)$$

Thus the structures derived using cyclic 2 transformations depend on eight unknowns which should be identified

for simple and realizable structures. Some additional constraints are given therefore: [DI1]

Self Duality. Deifinition 2.5.1.2: A set of linear transformation pairs, $\{\mathcal{A}_{11},\mathcal{A}_{21}\},\ldots\{\mathcal{A}_{1i},\mathcal{A}_{2i}\},\ldots\{\mathcal{A}_{1n},\mathcal{A}_{2n}\}$ used for the LTA simulation of a ladder $\{N_1,\ldots,N_i,\ldots N_n\}$ is said to be self-dual if and only if:

(i) It is series-shunt compatible.

(ii) The LT structures of a series arm Z_μ (shunt arm Y_λ) is identical to that of its dual shunt arm $Y_\mu = \frac{2\mu}{R_o^2}$ (series arm $Z_\lambda = Y_\lambda R_o^2$), where R_o is a convenient resistance value.

Such a transformation yields active-RC filters which be referred to as self-dual LTA filters [DI1].

The following explicit conditions have been found from self-duality:

$$\frac{\delta_1\delta_2}{\alpha_1\alpha_2} = \frac{\beta_1\beta_2}{\gamma_1\gamma_2} = \frac{\beta_1\delta_2}{\alpha_1\gamma_2} = \frac{\delta_1\beta_2}{\gamma_1\alpha_2} = -R^2, \quad \Delta_1 = -\Delta_2 \neq 0 \tag{2.5.4}$$

where R is a convenient resistance value used for the definition of duality between a series impedanze Z and a shunt admittance: $Y = Z/R^2$.

With this constraint five elements could be evaluated. So three elements must be identified.

Linear constraint between inputs and outputs. Definition 2.5.1.3

The LT two-port is described by the equations:

$$Y_1 = KX_1 + LX_2$$

$$Y_2 = MX_1 + NX_2 \tag{2.5.5}$$

It would be convenient if it was possible to realize actively only one of (2.5.5), the other being evaluated as a linear combination of this output and the inputs. At this end the following constraint is imposed:

$$Y_1+\varepsilon Y_2 = \varepsilon_1 X_1+\varepsilon_2 X_2 \qquad (2.5.6)$$

with

$$\varepsilon_1 = 1,0,-1, \quad \varepsilon_2 = 1,0,-1, \quad \varepsilon = \pm 1$$

to be hold for every ladder arm, and for every set of inputs X_1 and X_2.
The constraints for this relation are:

$$\gamma_1 = \varepsilon_1\alpha_1 \qquad \gamma_2 = \frac{\varepsilon_2}{\varepsilon}\alpha_2 \qquad (2.5.7)$$

$$\alpha_1\delta_2+\delta_1\alpha_2 = 0$$

$$\text{with} \quad \alpha_1\alpha_2\delta_1\delta_2 \neq 0$$

$$\alpha_1\delta_2(\varepsilon-\varepsilon_1\varepsilon_2) = \Delta_1$$

$$\text{and} \quad \varepsilon \neq \varepsilon_1\varepsilon_2$$

By combining the self-duality constraints with the above, one has the practical self-dual LTA filters [DI4].

The Rotation or R-property. Definition 2.5.1.4

A linear transformation is said to possess the rotation or R-property if the RC-actively realized output Y_i of the LT structure for a series arm Z (shunt arm Y) being $Y_i=A(s)X_1+B(s)X_2$ the RC-actively realized output Y_i' of the LT structure of the series arm $Z' = \frac{R^2}{Z}$ (shunt arm $Y' = \frac{1}{YR^2}$) is:

$$Y_i = A(s)X_2+B(s)X_1 \qquad (2.5.8)$$

The definition of the R-property implies that the LT structures of the highpass arms can be realized from the LT structures of the corresponding lowpass arms by interchanging the inputs X_1 and X_2. A similar state of affairs exists between the bandreject and the bandpass ladder arms. By definition:

$$Y_{1Z} = K(Z)X_1+L(Z)X_2 \equiv K(\frac{R^2}{Z})X_2+L(\frac{R^2}{Z})X_1 \rightarrow$$

$$\gamma_1^2\alpha_2^2 = \frac{\Delta_1^2}{R^2}\ , \quad \gamma_1\beta_2+\delta_1\alpha_2 = 0\ , \quad \alpha_1\beta_2+\alpha_2\beta_1 = -\frac{\alpha_1\Delta_1}{\gamma_1}$$

These constraints combined with self-duality constraints give the R-Self-Dual LTA filters [DI5].

Cyclic 1. Definition 2.5.1.5

A compatible set of linear transformation pairs

$$\{\mathcal{A}_{11},\mathcal{A}_{21}\},\ldots\{\mathcal{A}_{1i},\mathcal{A}_{2i}\},\ldots\{\mathcal{A}_{1n},\mathcal{A}_{2n}\}$$

used for the transformation of a ladder $\{N_1,N_2,\ldots N_n\}$ is defined as self-compatible if and only if

$$\mathcal{A}_{1i} = \mathcal{A}_1 \ \wedge\ \mathcal{A}_{2i} = \mathcal{A}_2, \qquad \forall\ i=1,2,\ldots n$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are two nonsingular compatible square matrices. Thus if we set the matrix $\mathcal{A}_1$ to be:

$$\mathcal{A}_1 = \begin{bmatrix} \alpha & +\beta \\ \gamma & +\delta \end{bmatrix} \qquad (2.5.9a)$$

then from the compatibility relationship it could be found that:

$$\mathcal{A}_2 = \begin{bmatrix} \gamma & -\delta \\ \alpha & -\beta \end{bmatrix} \qquad (2.6.9b)$$

Moreover $\det\mathcal{A}_1 = \det\mathcal{A}_2 = \Delta \neq 0$

The elements of the matrix Σ, K,L,M,N can now be re-expressed as follows:

Series arm Z:

$$K_Z = \frac{\gamma^2 Z}{\alpha\gamma Z - \Delta} \quad , \quad L_Z = \frac{-\Delta}{\alpha\gamma Z - \Delta}$$

$$M_Z = \frac{-\Delta}{\alpha\gamma Z - \Delta} \quad , \quad N_Z = \frac{\alpha^2 \Delta}{\alpha\gamma Z - \Delta}$$

Shunt arm Y:

$$K_Y = \frac{\delta^2 Y}{\beta\delta Y + \Delta} \quad , \quad L_Y = \frac{\Delta}{\beta\delta Y + \Delta}$$

$$M_Y = \frac{\Delta}{\beta\delta Y + \Delta} \quad , \quad N_Y = \frac{\beta^2 Y}{\beta\delta Y + \Delta}$$

In the above case it cannot be defined self-duality and the linear constraint between outputs and inputs because there are only 4 unknows and the conditions described either by 2.5.4 or 2.5.7 are conflicting. We could however define the rotation or R-property in the series or shunt arms.

Rotation or R-Property in the series arm:

In order to maintain the same structures for highpass arms and low-pass arms in the series arms it must be:

$$Y_{1Z} = K(Z)X_1+L(Z)X_2 \equiv K(\frac{R^2}{Z})X_2+L(\frac{R^2}{Z})X_1$$

and (2.5.10)

$$Y_{2Z} = M(Z)X_1+N(Z)X_2 \equiv M(\frac{R^2}{Z})X_2+N(\frac{R^2}{Z})X_1$$

Solving eqns. (2.5.10) :

$$\alpha = \gamma \ , \quad (\beta-\delta)^2 = \alpha^2R^2 \tag{2.5.11}$$

For the case when $\beta=-\delta$ and $\alpha=\gamma=1$:

$$\alpha=\gamma=1 \quad 4\beta^2=R^2 \rightarrow \beta=\pm\frac{R}{2}$$

and the wave-active case [CO], [HA], is obtained.
In a similar way .. can be defined the
Rotation or R-property in the shunt arm, for which we find that the conditions
$\beta=\delta$ and
$\beta^2 = R^2(\alpha-\gamma)^2$ must hold. (2.5.12)

From (2.5.11) it is observed that in wave-active case:

$$\mathcal{A}_1 = \begin{bmatrix} \alpha & \beta \\ \alpha & -\beta \end{bmatrix} = \mathcal{A}_2$$

Other constraints which can be imposed on the Σ-matrix rather than the transformation matrices could be the following.

2.5.2 Low-Complexity Constraints

The description of the oriented LT structure corresponding to a two-port N under linear transformation set $\{\mathcal{A}_1, \mathcal{A}_2\}$

given in Sec.2.3 (eqn.2.3.11) is repeated here for convenience:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \Sigma \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (2.5.13)$$

Two simple constraints for the Σ matrix could be drawn:

a) Choosing the transformation matrices in such a way that one or more of the transfer ratios becomes zero, or Σ matrix have one or more zero entries.

This will be referred as the Transfer Ratio Elimination approach (TRE). In such a case $\Sigma = \begin{bmatrix} 0 & L \\ M & N \end{bmatrix}$ or $\Sigma = \begin{bmatrix} K & L \\ M & O \end{bmatrix}$

b) Choosing the transformation matrices so that some or all of the transfer ratios of eqn. (2.5.13) become identical. This will be referred to as the Transfer Ratio Identification approach (TRI).

In such a case it can be:

$$\Sigma = \begin{bmatrix} \lambda M & \lambda N \\ M & N \end{bmatrix} \quad \text{or} \quad \Sigma = \begin{bmatrix} \lambda & \lambda \\ 1 & 1 \end{bmatrix} M$$

where λ is a real number.

The conditions for those constraints are summarized in Table 2.5.1 and 2.5.2 for series and shunt arms respectively.

Finally the classification of LTA filters as in the previous Section described is given in Diagram 1 and the special cases for practical filters are also shown.

Series arm Z:

Table 2.5.1

	A_1 A_2	LT STRUCTURE DESCR.	FURTHER SIMPLIFICATION
SERIES ARM Z	TRI $\begin{bmatrix}\alpha_1 & \beta_1\\ 0 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ 0 & \delta_2\end{bmatrix}$	$y_1 = -\frac{\delta_1}{\delta_2} y_2$ $y_2 = \frac{-\alpha_2\delta_2}{\alpha_1\beta_2+\beta_1\alpha_2+\alpha_1\alpha_2 Z}\left(x_1-\frac{\alpha_1}{\alpha_2}x_2\right)$	$\delta_1=-\delta_2$, $\alpha_1=-\alpha_2$ for $y_1 = y_2$ $y_2=\frac{-\delta_2(x_1+x_2)}{\beta_1-\beta_2+\alpha_1 Z}$ $\alpha_1(\beta_1-\beta_2)\geqslant 0$
	TRE N≡0 $\begin{bmatrix}0 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ 0 & \delta_2\end{bmatrix}$	$y_1=\frac{\gamma_1\alpha_2 Z+\gamma_1\beta_2+\delta_1\alpha_2}{\beta_1\alpha_2}x_1+\frac{\gamma_1}{\alpha_2}x_2$ $y_2=-\frac{\delta_2}{\beta_1}x_1$	$\beta_1=-\delta_2$; $\gamma_1=\alpha_2$ for $y_2=x_1$ $y_1=\frac{\alpha_2 Z+\delta_1+\beta_2}{\beta_1}x_1+x_2$
	TRE K≡0 $\begin{bmatrix}\alpha_1 & \beta_1\\ 0 & \delta_1\end{bmatrix}\begin{bmatrix}0 & \beta_2\\ \gamma_2 & \delta_2\end{bmatrix}$	$y_1= -\frac{\delta_1}{\beta_2}x_2$ $y_2=\frac{\gamma_2}{\alpha_1}x_1+\frac{\alpha_1\gamma_2 Z+\alpha_1\delta_2+\beta_1\gamma_2}{\alpha_1\beta_2}x_2$	$\delta_1=-\beta_2$, $\gamma_2=\alpha_1$ for $y_1= x_2$ $y_2=\frac{\alpha_1 Z+\delta_2+\beta_1}{\beta_2}x_2+x_1$
	$\begin{bmatrix}0 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}0 & \beta_2\\ \gamma_1 & \delta_1\end{bmatrix}$	INADMISSIBLE	

Shunt arm:

Table 2.5.2

	$\mathcal{A}_1$ $\mathcal{A}_2$	LT STRUCTURE DESCR.	FURTHER SIMPLIFICATION
SHUNT ARM Y	TRI $\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & 0\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & 0\end{bmatrix}$	$y_1=\frac{\gamma_1}{\gamma_2}y_2$ $y_2=\frac{\beta_2\gamma_2}{\alpha_1\beta_2+\alpha_2\beta_1+\beta_1\beta_2 Y}\left(x_1+\frac{\beta_1}{\beta_2}x_2\right)$	$\gamma_1=\gamma_2$, $\beta_1=\beta_2$ for $y_1=y_2$ $y_2=\frac{\gamma_2(x_1+x_2)}{\alpha_1+\alpha_2+\beta_1 Y}$ $\beta_1(\alpha_1+\alpha_2)\geqslant 0$
	TRE $N\equiv 0$ $\begin{bmatrix}\alpha_1 & 0\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & \beta_2\\ \gamma_2 & 0\end{bmatrix}$	$y_1=\frac{\delta_1\beta_2 Y+\delta_1\alpha_2+\gamma_1\beta_2}{\alpha_1\beta_2}x_1-\frac{\delta_1}{\beta_2}x_2$ $y_2=\frac{\gamma_2}{\alpha_1}x_1$	$\gamma_2=\alpha_1$, $\delta_1=-\beta_2$ for $y_2=x_1$ $y_1=\frac{\delta_1 Y+\gamma_1-\alpha_2}{\alpha_1}x_1+x_2$
	TRE $K\equiv 0$ $\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & 0\end{bmatrix}\begin{bmatrix}\alpha_2 & 0\\ \gamma_2 & \delta_2\end{bmatrix}$	$y_1=\frac{\gamma_1}{\alpha_2}x_2$ $y_2=-\frac{\delta_2}{\beta_1}x_1+\frac{\beta_1\delta_2 Y+\beta_1\gamma_2+\alpha_1\delta_2}{\beta_1\alpha_2}x_2$	$\gamma_1=\alpha_2$, $\delta_2=-\beta_1$ for $y_1=x_2$ $y_2=x_1+\frac{\delta_2 Y+\gamma_2+\alpha_1}{\alpha_2}x_2$
	$\begin{bmatrix}\alpha_1 & 0\\ \gamma_1 & \delta_1\end{bmatrix}\begin{bmatrix}\alpha_2 & 0\\ \gamma_2 & \delta_2\end{bmatrix}$	INADMISSIBLE	

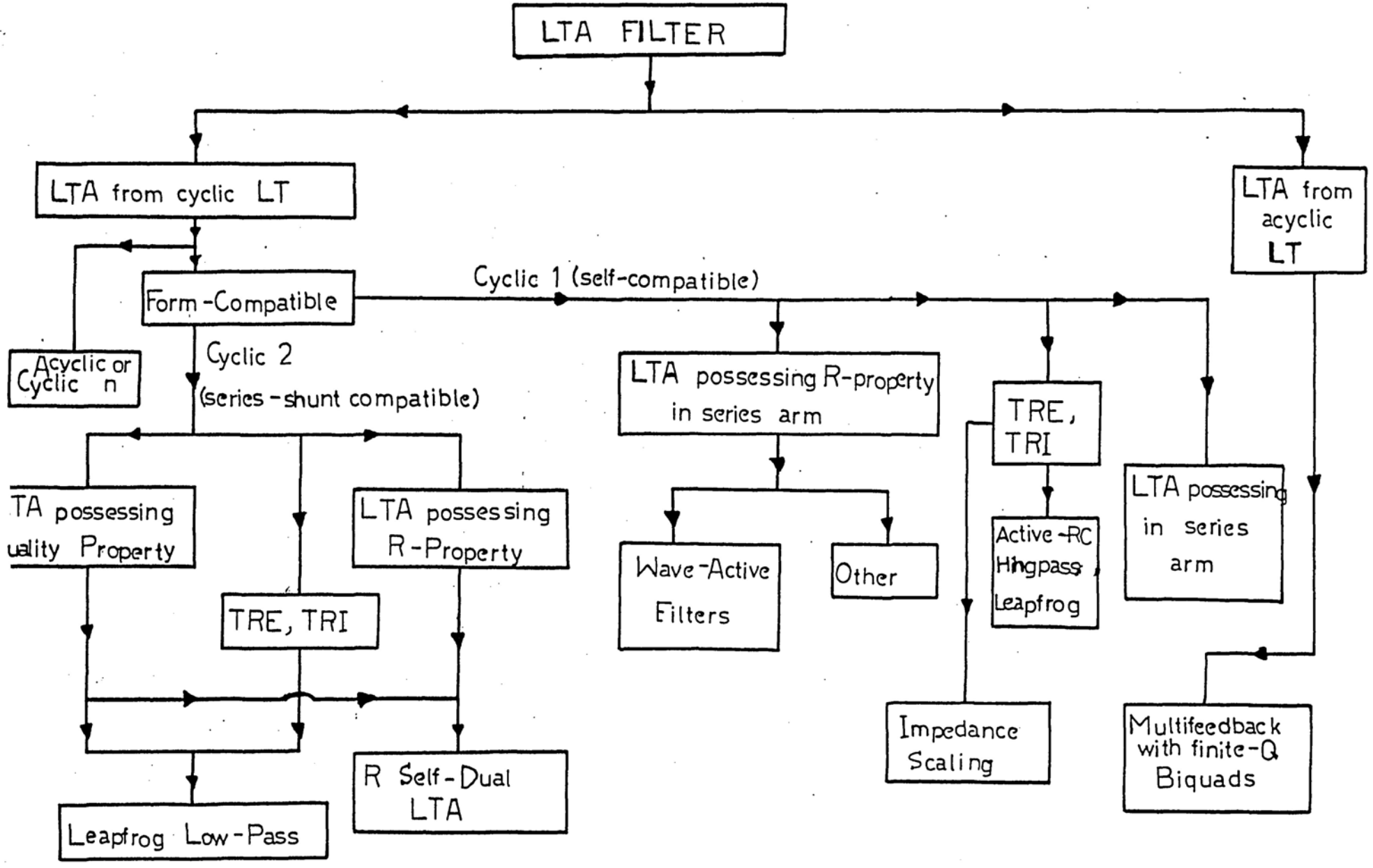

LTA FILTER
LTA from cyclic LT
LTA from acyclic LT
Form-Compatible
Cyclic 1 (self-compatible)
Acyclic or Cyclic n
Cyclic 2
(series-shunt compatible)
LTA possessing R-property in series arm
TRE, TRI
LTA possessing in series arm
TA possessing uality Property
LTA possessing R-Property
Wave-Active Filters
Other
Active-RC Hingpass Leapfrog
TRE, TRI
Impedance Scaling
Multifeedback with finite-Q Biquads
R Self-Dual LTA
Leapfrog Low-Pass

Diagram 1

CHAPTER 3

BIQUAD LTA FILTERS

3.1 FILTERS CONTAINING TUNED CIRCUITS

3.1.1 Lowpass (Elliptic)

3.1.2 Bandpass Filters (Polynomial)

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3.3.2 Design Examples

3.3.3 Conclusions

CHAPTER 3

BIQUAD LTA FILTERS

The design of LTA filters has been carried out so far, by embedding first order structures in a multiple feedback topology, to realize active-RC filters from LC ladder prototypes [DI1-4],[CO2]. In this chapter it will be tried to impose biquadratic sections either in negative feedback topology [DE2], [FR1] or in positive feedback topology [SA7],[SA9], as basic building blocks, to the realizations, in order to minimise the number of operational amplifiers used. These biquads will be of finite-Q only and in most cases of single amplifier. Firstly they will be used to simulate filters with tuned circuits and secondly to simulate polynomial filters or as the so called "Inverted-L" filters by transforming linearly two adjacent arms of the LC ladder together.

Alternatively, integrated circuits realizing biquads could be used, as they recently appeared in the industry. Their implementations are given in Sec.3.3.

3.1 FILTERS CONTAINING TUNED CIRCUITS [CO3]

In this section we are dealing with filters containing tuned circuits and simulate them by biquadratic sections. Those filters could be elliptic filters of all types and bandpass or bandreject filters evaluated through their lowpass or highpass analogs by frequency transformations. The process to be followed will be the acyclic linear transformation. The use of the acyclic LT requires considerable

experience on behalf of the designer who must be very familiar with the general LTA method, otherwise the resulting LTA structures will be very complex indeed. However, one can draw some useful rules for using the acyclic case, to ensure that certain conditions are automatically met. Thus one should start from the matrix $\mathcal{A}_{1n}$ which ensures the existence of a convenient output node, in accordance with the result of section 2.3.7, so that a practical LT structure may be derived for the last reactive arm of the ladder, taken in conjuction with the load resistance. With $\mathcal{A}_{1n}$ specified, its adjacent right transformation matrix $\mathcal{A}_{2(n-1)}$ is automatically defined by the compatibility relationship. Next, the matrix $\mathcal{A}_{1(n-1)}$ can now be so chosen, that the transformation pair $\{\mathcal{A}_{1(n-1)}, \mathcal{A}_{2(n-1)}\}$ leads to a practical and easily realizable LT structure for the reactive arm N_{n-1}. This case can be checked using the general expressions for the transfer ratios K,L,M and N (sections 2.3.3 and 2.5.2). Once the matrix $\mathcal{A}_{1(n-1)}$ has been determined, its adjacent matrix $\mathcal{A}_{2(n-1)}$ is also automatically known and the acyclic procedure continues on the basis so that the LT structure realizations can be tailored to the needs of the designer. It will be shown that the application of acyclic transformation will give very simple structures, which to the designer knowledge are using the minimum number of op-amps, than all the well-known simulation methods (e.g. GIC, leapfrog, etc.) except a recently appeared method using NIC's [SI1].

3.1.1 Lowpass Elliptic Filters

It will be considered here the well-known lowpass filters which are doubly resistive terminated with elliptic characteristics, consisting of capacitors, inductors and tuned circuits. The

tuned circuits are simulated by finite-Q single-amplifier-biquads, while capacitors and inductors by first order structures, which by proper choice of transformation matrices, can be chosen to be quite simple.

Consider in Fig.3.1.1 the fifth order low-pass Elliptic filter to be simulated by acyclic linear transformation.

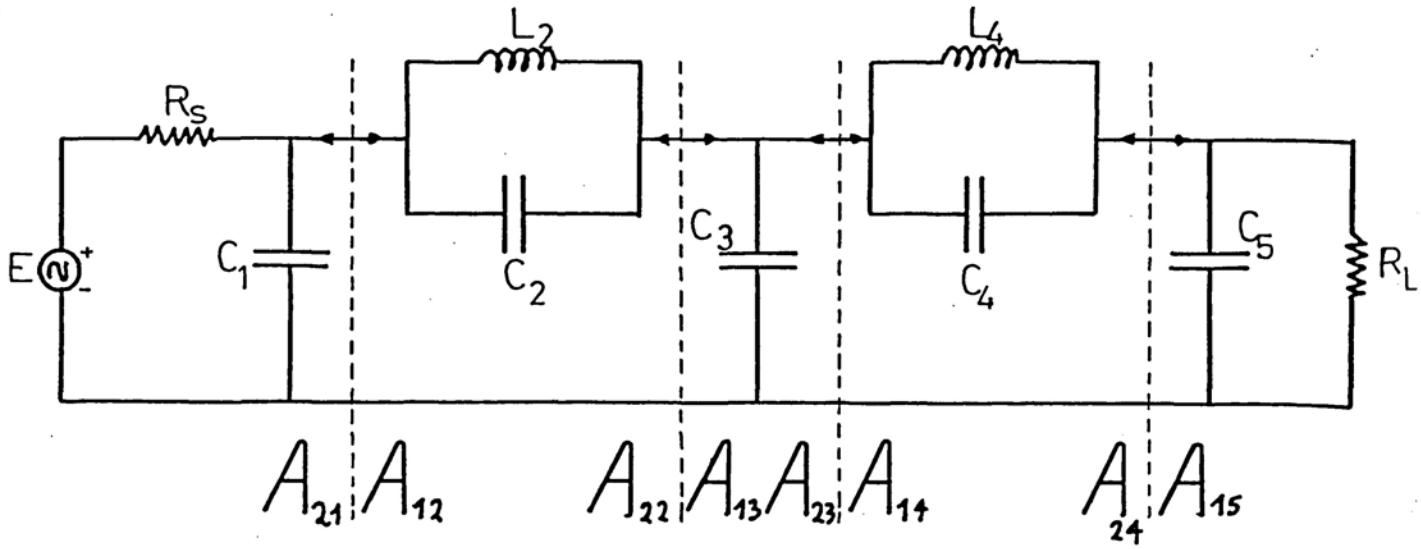


Fig. 3.1.1

Starting from the output terminated capacitor C_5 taken in conjuction with load resistor R_L, by using the transformation matrix

$$A_{15} = \begin{bmatrix} 0 & \varepsilon R \\ \varepsilon & 0 \end{bmatrix} \quad \text{where} \quad \varepsilon=\pm 1 \tag{3.1.1}$$

$\delta_{15}=0$, for direct output as explained in Sec.2.5.2. It must also be $\gamma_{1n}=\varepsilon$ in order that $Y_{15}=\varepsilon V_o$. It can be chosen $\alpha_{15}=0$, $\beta_{15}=\varepsilon R$ where R is a resistance value for simplicity. If $\varepsilon=\pm 1$ one has for the LT structure:

$$Y_{15} = \frac{R_L}{R(Y_5 R_L+1)} X_{15}, \qquad Y_{15}=\varepsilon V_o \tag{3.1.2}$$

which can be realized by the structure in Fig.3.1.2.

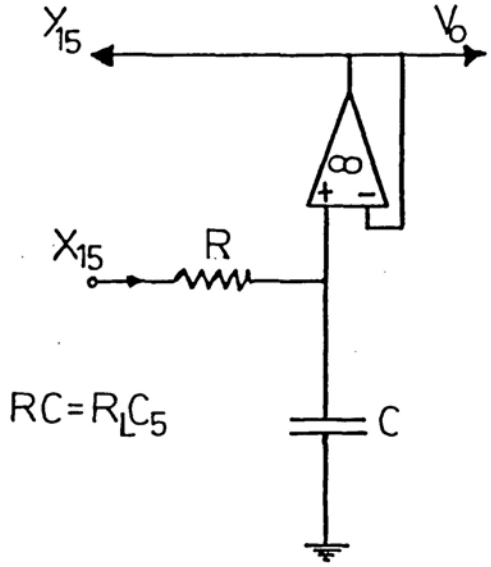


Fig. 3.1.2

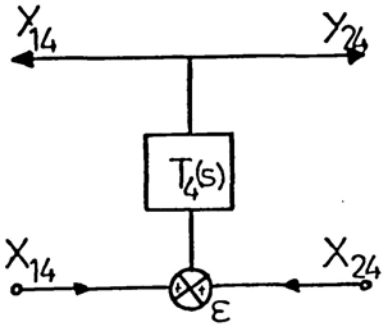
The value of ε will be determined by LT structures of other than the last arm. The $\mathcal{A}_{24}$ matrix for the transformation of the series tuned circuit is readily found from $\mathcal{A}_{15}$ by interchanging its rows and changing the sign of the second column (compatibility relationship). So: $\mathcal{A}_{24} = \begin{bmatrix} \varepsilon & 0 \\ 0 & -\varepsilon R \end{bmatrix}$ The matrix $\mathcal{A}_{14}$ is then so chosen as to have the simplest LT structure for the series tuned circuit. It will be desirable to have one single-amplifier-biquadratic section with finite-Q for this arm. To do so from Table 2.5.2 it should be chosen:

$$\mathcal{A}_{14} = \begin{bmatrix} \pm\varepsilon & \pm\varepsilon R \\ 0 & \varepsilon R \end{bmatrix} \quad \text{or} \quad \gamma_{14}=\gamma_{24}=0, \quad \delta_{14}=-\delta_{24}$$

It must also be $\beta_{14}\neq 0$ and of the same sign as a_{14} (TRI case, Table 2.5.2). The LT structure is given by:

$$Y_{14} = Y_{24} = \frac{\varepsilon R}{\pm\varepsilon R+\varepsilon Z_4}\,(X_{14}\pm\varepsilon X_{24}) \qquad (3.1.3)$$

In order to realize eqn.(3.1.2) by the negative feedback biquad ([DA1], [FR1]), it should be $\beta_{14}=\varepsilon R$, $\alpha_{14}=\varepsilon$. The configuration is shown in Fig.3.1.3.



$$\mathcal{A}_{14}=\begin{bmatrix}\varepsilon & \varepsilon R\\ 0 & \varepsilon R\end{bmatrix},\ \mathcal{A}_{24}=\begin{bmatrix}\varepsilon & 0\\ 0 & -\varepsilon R\end{bmatrix}$$

Fig. 3.1.3

The transfer function $T_4(s)=\dfrac{RC_4L_4s^2+R}{RC_4L_4s^2+L_4s+R}$.

The matrix $\mathcal{A}_{23}$ is then automatically defined by the matrix $\mathcal{A}_{14}$ as:

$$\mathcal{A}_{23}=\begin{bmatrix}0 & -\varepsilon R\\ \varepsilon & -\varepsilon R\end{bmatrix}$$ and the matrix $\mathcal{A}_{13}$

is taken

$$\mathcal{A}_{13}=\begin{bmatrix}0 & -\varepsilon R\\ \varepsilon & 0\end{bmatrix}$$ yielding:

$$Y_{13}=-\frac{1}{RC_3s}(X_{13}+X_{23}),\quad Y_{23}=-\frac{1}{RC_3s}(X_{13}+X_{23})+X_{23} \qquad (3.1.4)$$

and is realized as in Fig.3.1.4 in conjuction with the previous arm.

From figure 3.1.4 it can be seen that it must be chosen $\varepsilon=-1$ in order to use the virtual earth point of the differential amplifier in LT structure of the tuned circuit to perform the summing operation of the adder that appears in the LT structure of C_3. Thus a reduction of one active element takes place. With $\varepsilon=-1$ the matrices $_{13}$, $_{23}$ are:

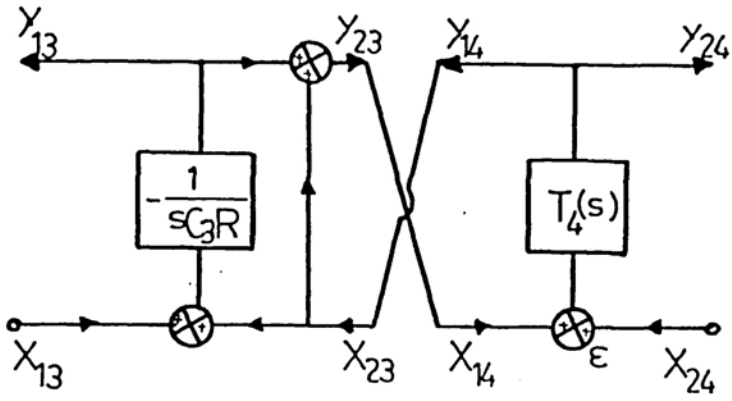


Fig. 3.1.4

$$\mathcal{A}_{13} = \begin{bmatrix} 0 & R \\ -1 & 0 \end{bmatrix} \quad , \quad \mathcal{A}_{23} = \begin{bmatrix} 0 & R \\ -1 & R \end{bmatrix}$$

Again from the matrix $\mathcal{A}_{13}$, it can found its adjacent matrix $\mathcal{A}_{22}$ for the transformation of the tuned circuit in the second arm of the ladder. It will be, again, desirable to have one single-amplifier-biquad with finite-Q. Following the same rules as before:

$$\mathcal{A}_{22} = \begin{bmatrix} -1 & 0 \\ 0 & -R \end{bmatrix} \quad \text{and} \quad \mathcal{A}_{12} = \begin{bmatrix} 1 & R \\ 0 & R \end{bmatrix}$$

Notice that this time there is no problem with the input stage, having an adder as previously. Therefore it can be $\alpha_{12}=-\alpha_{22}$ and thus saving the differential amplifier. In that case:

$$Y_{12} = Y_{22} = \frac{RC_2L_2s^2+R}{RC_2L_2s^2+L_2s+R} (X_{12}+X_{22}) \qquad (3.1.5)$$

Finally the transformation of the capacitor C_1 taken in conjuction with the source resistance can be obtained from

$\mathcal{A}_{12}$ by compatibility relationship. The matrix $\mathcal{A}_{21} = \begin{bmatrix} 0 & -R_s \\ 1 & -R_s \end{bmatrix}$, where is taken $R=R_s$ (input resistance) for simpler structure. For the input termination it will be obtained:

$$Y_{21} = \frac{1}{1+sR_sC_1} E + \frac{sR_sC_1}{1+sR_sC_1} X_{21} \qquad (3.1.6)$$

The complete filter is shown in Fig.3.1.5.
From Fig. 3.1.5 it can be seen that for the fifth order lowpass Elliptic filter,6 op-amps (2 buffers) are used, less than GIC method using 7 op-amps (with a buffering output).
To design highpass elliptic filters a similar procedure is followed based,again on acyclic linear transformations.

3.1.2. Bandpass-Filters (Polynomial)

This type of filter is derived from lowpass prototype using lowpass to highpass transformations. To realize such a filter using LT, it is inserted a third entry in the transformation matrices of the appropriate arms as analyzed earlier, to produce finite-Q biquads. By third entry it is meant that it is chosen $\beta_{1i}\neq 0$ or $\beta_{2i}\neq 0$ where i is the order of the arm, i=2,...(n-1).
For the sixth order BP prototype shown in Fig.3.1.6. a suitable set of transformation matrices for the series tuned circuit is:

$$\mathcal{A}_{1Z} = \begin{bmatrix} -1 & -R_s \\ 0 & -R_s \end{bmatrix} \quad , \quad \mathcal{A}_{2Z} = \begin{bmatrix} 1 & 0 \\ 0 & R_s \end{bmatrix} \quad .$$

where R_s is the input resistance.
The input-output relationships of the three sections are:

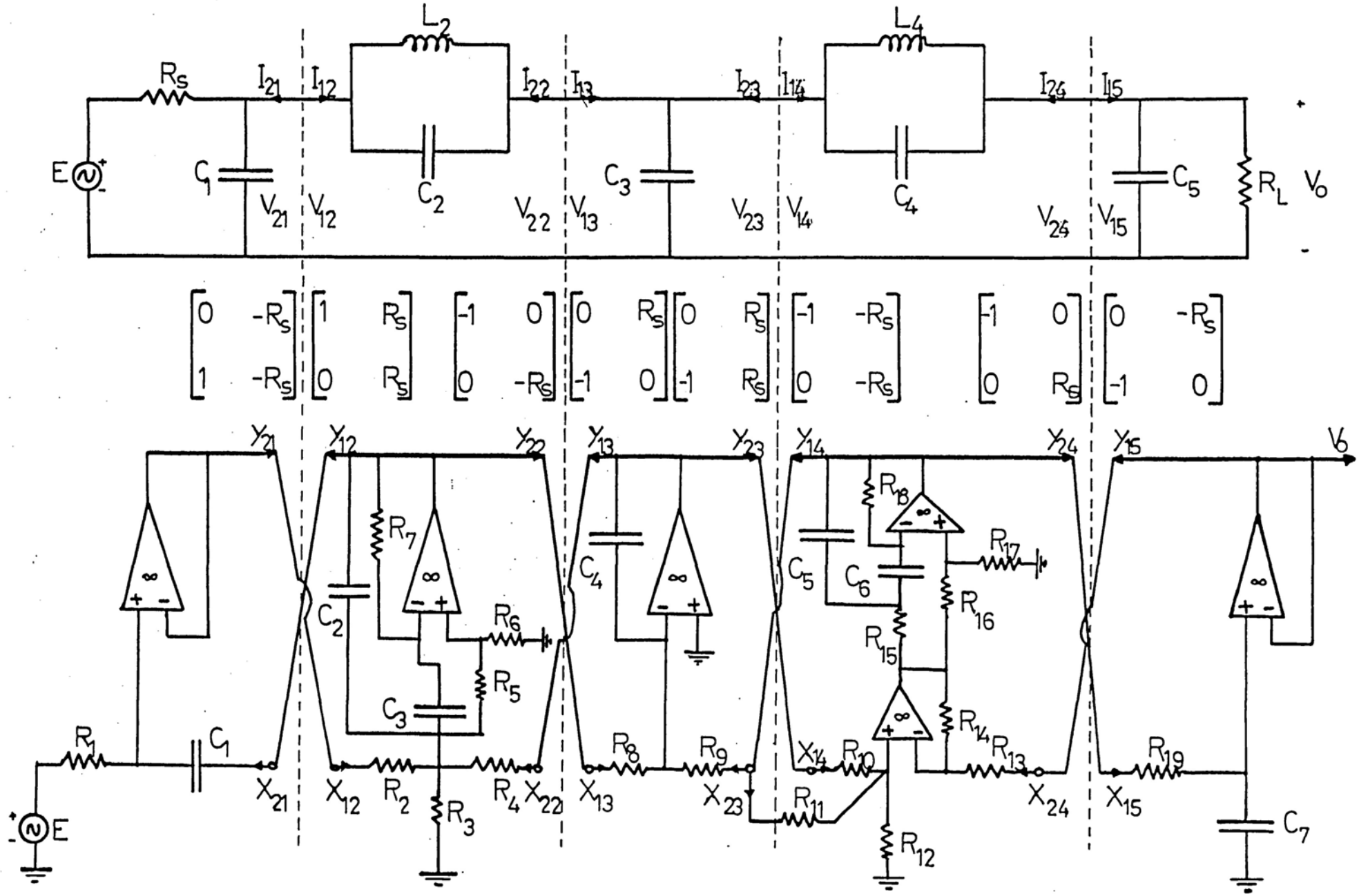

L2
L4
Rs
I21
I12
I22
I13
I23
I14
I24
I15
E
C1
C2
C3
C4
C5
RL
Vo
V21
V12
V22
V13
V23
V14
V24
V15
Y21
Y12
Y22
Y13
Y23
Y14
Y24
Y15
R7
R6
R5
R8
R9
R18
R17
R16
R15
R14
R13
R10
R11
R12
R19
C6
C7
R1
R2
R3
R4
X21
X12
X22
X13
X23
X14
X24
X15

Fig.3.1.5

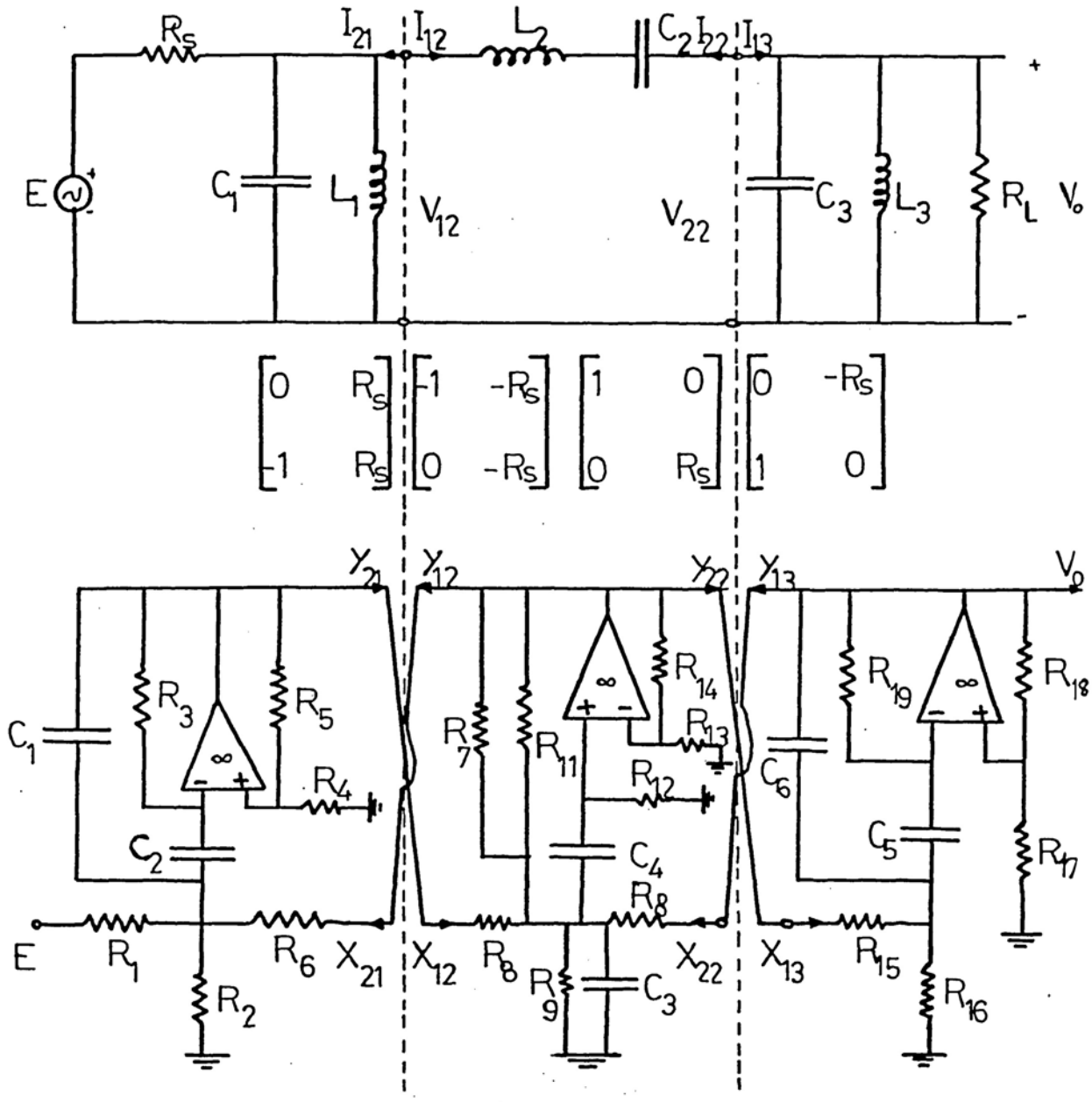


Fig. 3.1.6: 6th order Bandpass filter

$$Y_{21} = -T_1(s)\,(E+X_{21})+X_{21}$$

$$Y_{12} = Y_{22} = T_2(s)\,(X_{12}+X_{22}) \qquad (3.1.7)$$

$$Y_{13} = -T_3(s)X_{13},\ Y_{13} = \text{Vout}$$

where:

$$-T_1(s) = -T_3(s) = \frac{-L_1 s}{R_s C_1 L_1 s^2 + L_1 s + R_s}$$

$$T_2(s) = \frac{R_2 C_2 s}{C_2 L_2 s^2 + R_s C_2 s + 1}$$

The first and third section could be realized using the Deliyiannis single-amplifier biquad [DE2], while the second could be realized using the Sallen and Key positive feedback single-amplifier biquad [SA9].

The topology of this simulation is shown in Fig.3.1.7.

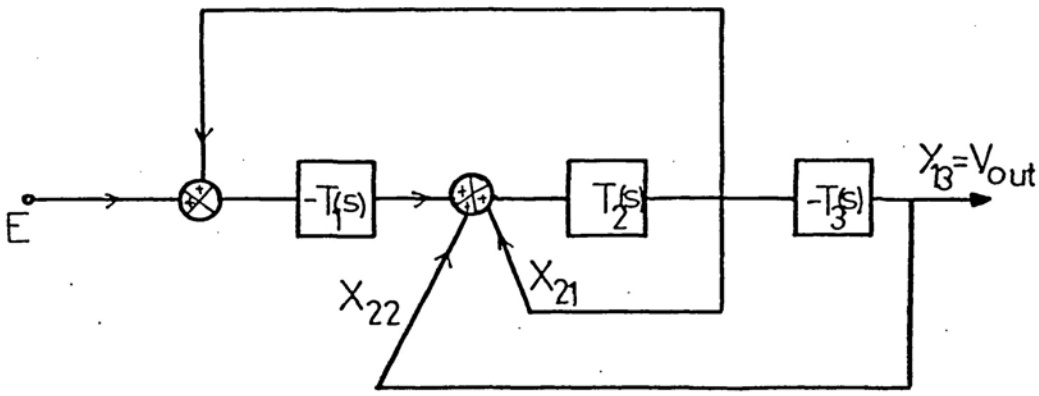


Fig. 3.1.7

This multiple-feedback structure looks like the leapfrog structure with one additional loop around the middle biquad. All biquads have finite-Q which is very desirable when someone uses biquads with low-Q. This configuration could be seen as the leapfrog method with the middle biquad having infinite-Q and using Q-enhancement adding some negative feedback to maintain finite-Q and so to adjust Q [DA1].

The above structure, however, results through a general theory.

For an eight order BP filter is found the topology shown in Fig.3.1.8 where, again, T_1, T_2, T_3, T_4 are biquads with finite-Q.

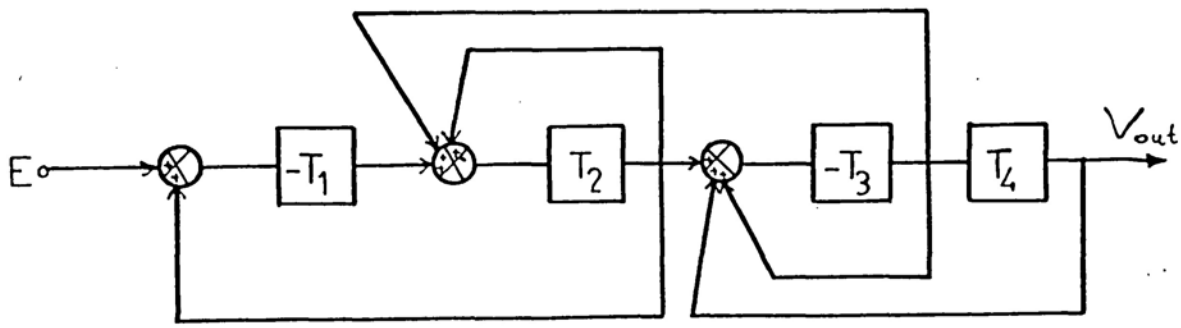


Fig. 3.1.8

In this case a differential amplifier is required in the third section. Generally for 2Nth order filter it is required $n = \frac{3N-4}{4}$ amplifiers if N=even and $n = \frac{3N-6}{4}$, if N=odd.

3.1.3. R-Self-Dual Biquad LTA filters

In the previous sections, acyclic linear transformations have been used to obtain Biquad LTA filters. It will be tried now to impose the constraints of self-duality and the rotation or R-property to the transformation matrices as defined in Sec. 2.5.1, to get the R-Self-Dual Biquad LTA filters. According to reference [DI3] a specific pair of transformation matrices can be chosen in order that the structures retain these properties. So, for all series Z and shunt Y arms of the filter must be: (case 5 in [DI3])

$$\mathcal{A}_{1Z} = \begin{bmatrix} 1 & R \\ 0 & R \end{bmatrix}, \quad \mathcal{A}_{2Z} = \begin{bmatrix} 1 & 0 \\ 1 & -R \end{bmatrix}$$

$$\mathcal{A}_{1Y} = \begin{bmatrix} 1 & R \\ 1 & 0 \end{bmatrix}, \quad \mathcal{A}_{2Y} = \begin{bmatrix} 0 & -R \\ 1 & -R \end{bmatrix}$$

In this case we realize four arms by the same structure, achieving thereby a high degree of modularity in the structures. For both series and shunt arms the output Y_{2i} could be formed from the input X_{2i} and the actively realized output Y_{1i} using the relationship:

$$Y_{2i} = Y_{1i}+X_{2i}$$

The output Y_{1Z} for the series arm Z case is:

$$Y_{2Z} = \frac{R}{R+Z}\ (X_{1Z}-X_{2Z}) \qquad \text{and for shunt Y:}$$

$$Y_{1Y} = \frac{1}{RY+1}\ (X_{1Z}-X_{2Z}) \qquad (3.1.8)$$

These expressions become identical for Y_{1Z} to that of Y_{1Y} for $Y = \frac{Z}{R^2}$ as expected for a Self-Dual transformation. The LT structures of the four passive arms and their descriptions are given in Table 3.1.

It has been seen that every LT structure of all arms require one differential amplifier, one biquadratic section, which for lowpass arms and their duals should be of a band stop function while for highpass and dual its of bandpass function, and one adder to give output Y_{2i}. It will be shown that in any case this adder can be substituted with the virtual earth of the next section. This will be emphasized in considering the fifth-order low-pass elliptic filter in Fig. 3.1.10a.

The LT structures are described by equations (3.1.9).

$$Y_{21} = \frac{1}{1+R_sC_1s}\ E + \frac{sR_sC_1}{1+R_sC_1s}\ X_{21} \qquad (3.1.9)$$

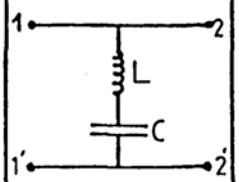
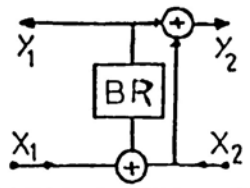
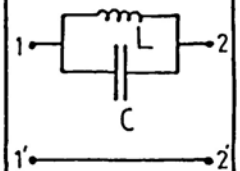
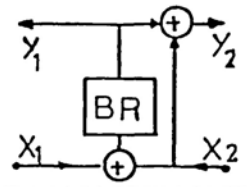
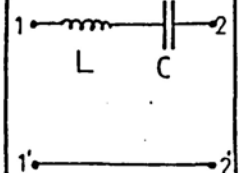
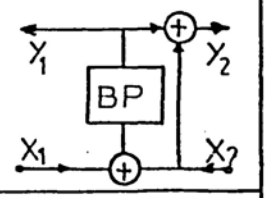
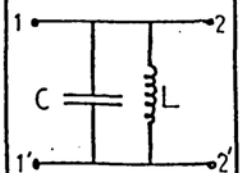
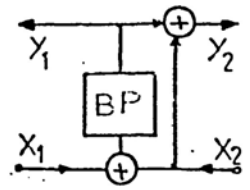
PASSIVE	$\mathcal{A}_1$ $\mathcal{A}_2$	LT STR. DESCR.	LT STRUCTURE
	$\begin{bmatrix}1 & R\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -R\\ 1 & -R\end{bmatrix}$	$Y_1=\frac{1}{RY\cdot 1}(X_1-X_2)$ $Y_2=Y_1\cdot X_2$	
	$\begin{bmatrix}1 & R\\ 0 & R\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & -R\end{bmatrix}$	$Y_1=\frac{R}{Z\cdot R}(X_1-X_2)$ $Y_2=Y_1\cdot X_2$	
	$\begin{bmatrix}1 & R\\ 0 & R\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & -R\end{bmatrix}$	$Y_1=\frac{R}{Z\cdot R}(X_1-X_2)$ $Y_2=Y_1\cdot X_2$	
	$\begin{bmatrix}1 & R\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & -R\\ 1 & -R\end{bmatrix}$	$Y_1=\frac{R}{RY\cdot 1}(X_1-X_2)$ $Y_2=Y_1\cdot X_2$	

Table 3.1

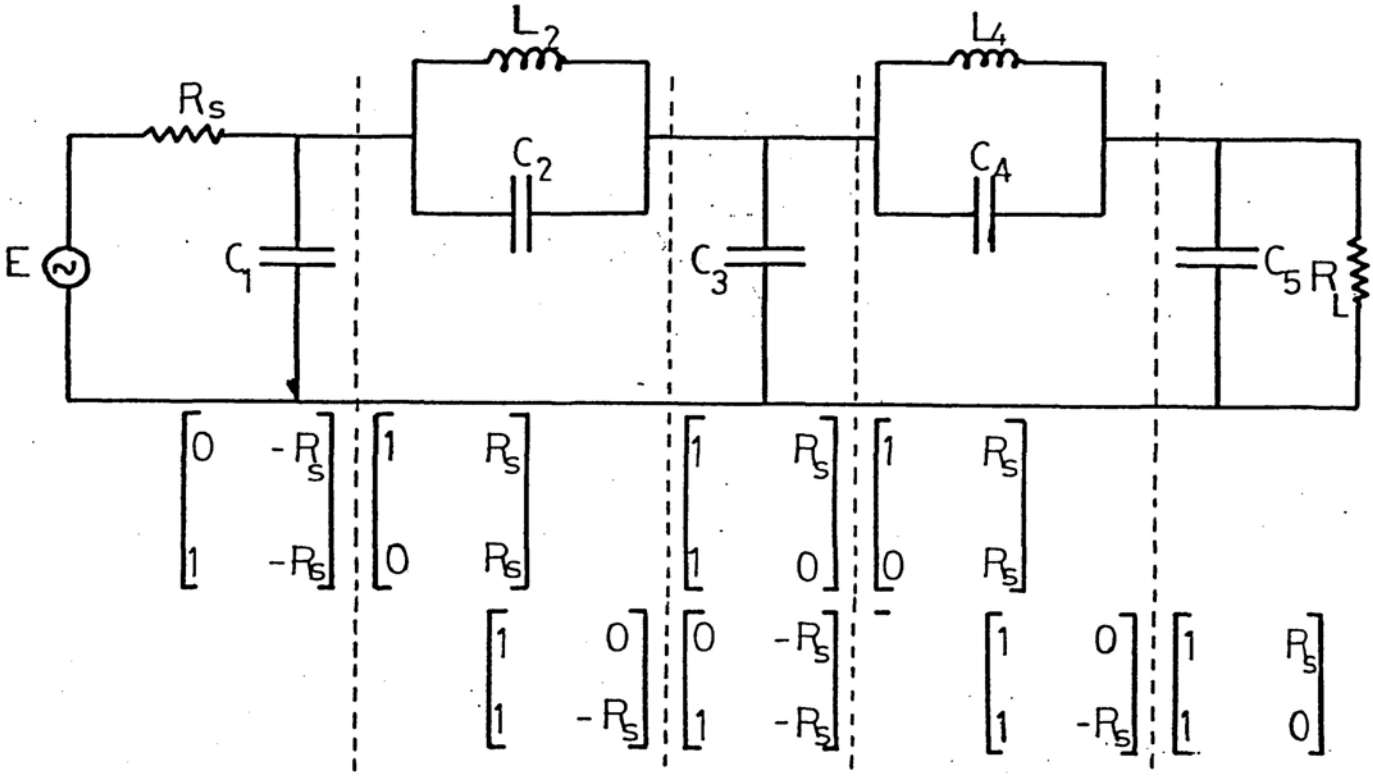


Fig. 3.1.10a

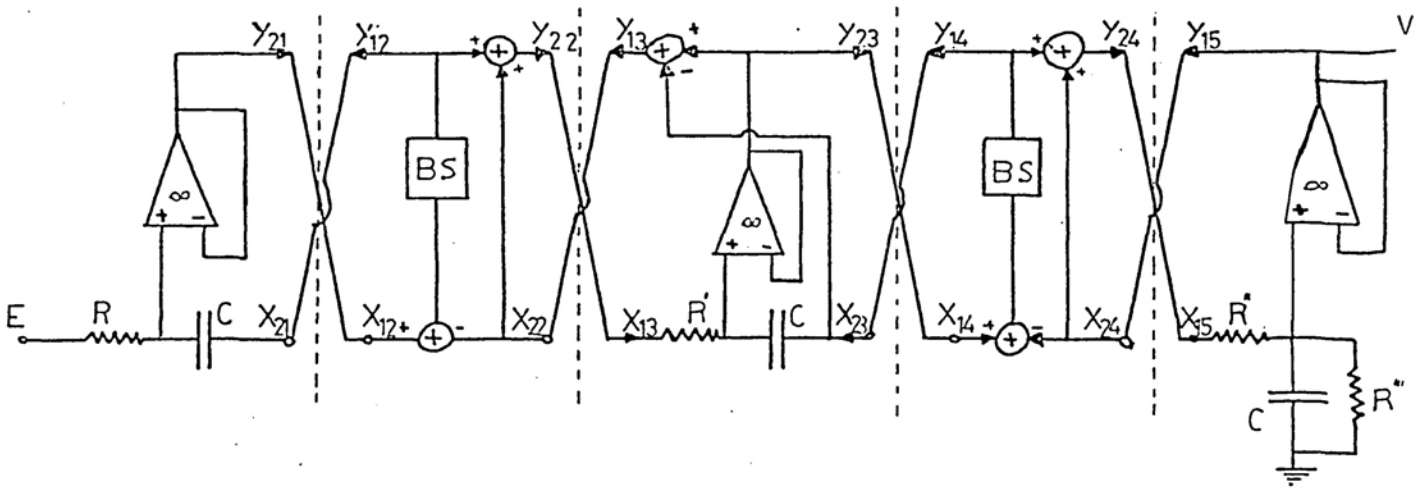
$$Y_{12} = \frac{R_s L_2 C_2 s^2 + R_s}{R_s L_2 C_2 s^2 + L_2 s + R_s} (X_{12} - X_{22}), \qquad Y_{22} = Y_{12} + X_{22}$$

$$Y_{23} = \frac{1}{1+R_s C_3 s} X_{13} + \frac{sR_s C_3}{1+R_s C_3 s} X_{23}, \qquad Y_{13} = Y_{23} - X_{23} \qquad (3.1.9)$$

$$Y_{14} = \frac{R_s L_4 C_4 s^2 + R_s}{R_s L_4 C_4 s^2 + L_4 s + R_s} (X_{14} - X_{24}), \qquad Y_{24} = Y_{14} + X_{24}$$

$$Y_{15} = \frac{R_L}{1 + \frac{R_L}{R_s} + R_L C_s s} X_{15} = V_o$$

The realization of the filter is shown in Fig.3.1.10b and in modified form in Fig.3.1.10c.



$RC = R_s C_1$ $\qquad\qquad$ $R'C = R_s C_3$

Fig.3.1.10b

This configuration needs 8 op-amps (3 buffers and 3 differential amplifier), if single-amplifier biquads will be used. The realizations shown in Figrs.3.1.10b and c are the same for the fifth order highpass elliptic filter and their dual except that bandpass biquads are used at the places of bandreject biquads.

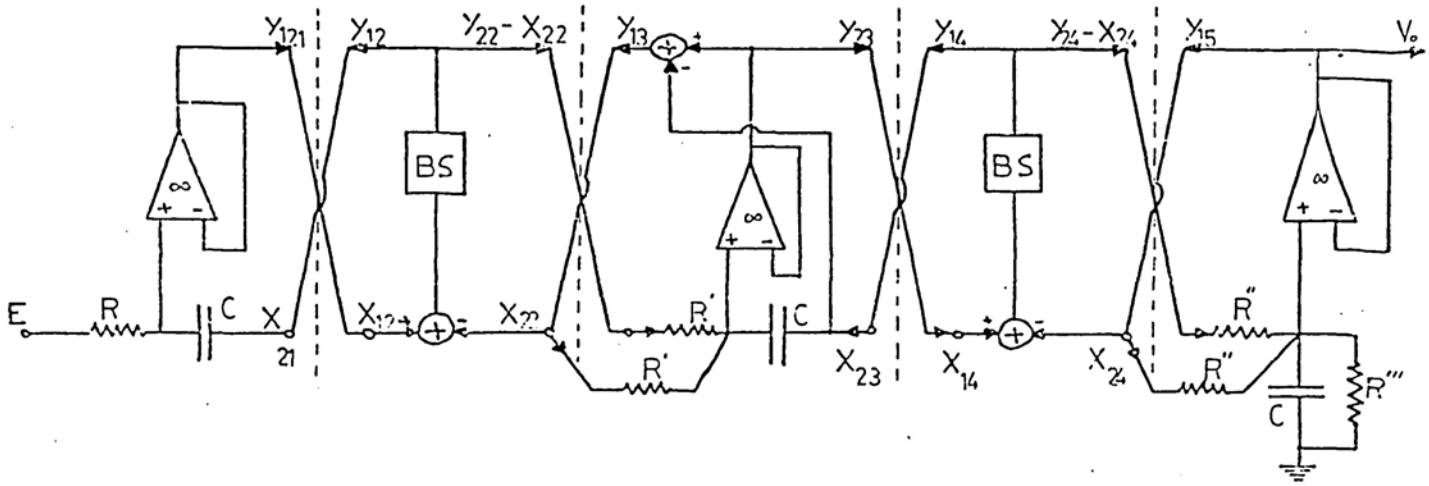


Fig. 3.1.10c

3.2 INVERTED-L FILTERS

In this section, polynomial LC ladder filters of the Inverted-L type are simulated via Linear Transformations, to produce Active-RC filters with biquads as basic building blocks, so as to minimize the number of op-amps used. The resulting structures are effectively embedded in multiple-feedback topologies with the biquads having finite-Q only. It is shown that it is possible in many cases to have canonic structures and considerable reduction of the number of operational-amplifiers employed. Specific choices of the Linear Transformation matrices, allow the realization of lowpass and highpass filters as well as unconventional bandpass filters. For the realization of Inverted-L filters it will be used the acyclic frequency indepented or frequency dependent linear transformation method.

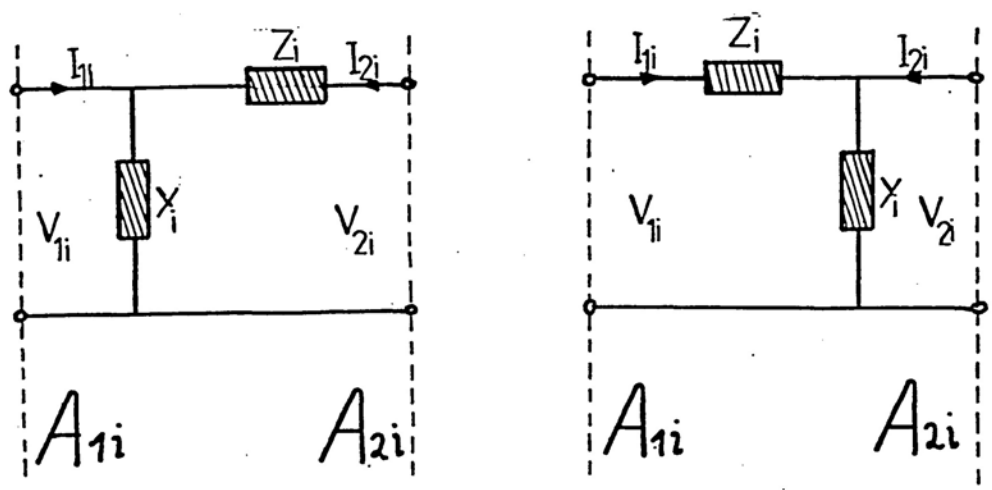
3.2.1 Linearly Transformed ladder network

So far the simulation of ladders has been, only, restricted to each and every one arm of the ladder, resulting

consequently to coupled-first order section structures or, as in Sec.3.1 described, to first and second order structures in a multiple feedback configuration.

Another possibility of simulation is by taking into account two-arms at a time and embedding biquads to realize the corresponding structures coming under Linear Transformations. The biquads which will be used will have finite-Q only, to enable us to use practical single-amplifier biquads [DA1]. The resulting Biquad Linear Transformation Active Filter (BLTA) will contain the minimum number of op-amps, if good sensitivity behaviour is to be maintained, because sensitivity increases rapidly by having coupled-structures of higher than second degree [MO1]. Moreover by substituting some relationships of low-complexity structures, it could be achieved simple and canonic structures, especially for the unconventional filters containing capacitive loops or inductive cut sets.

The n-th order ladder is considered of cascading of 2-port sections of the Inverted-L type (it's shape is like the inverted capital letter L) shown in Fig.3.2.1 and appears in two forms A and B.



Form A

Form B

Fig. 3.2.1

This section is treated as a two-port network with port variables $\left[V_{1i}I_{1i}\right]^T$ and $\left[V_{2i}I_{2i}\right]^T$ respectivily. With these two-ports, two nonsingular matrices $\mathcal{A}_{1i}$ and $\mathcal{A}_{2i}$ are associated, to produce two transformed port rectors $\left[X_{1i}Y_{1i}\right]^T = \mathcal{A}_{1i}\left[V_{1i}I_{1i}\right]^T$ and $\left[X_{2i}Y_{2i}\right]^T = \mathcal{A}_{2i}\left[V_{2i}I_{2i}\right]^T$ as described earlier in Sec.2.3. With T_i the modified chain matrix of the inverted-L section generally will be:

$$\begin{bmatrix} X_{1i} \\ \\ Y_{1i} \end{bmatrix} = \mathcal{A}_{1i} T_i \mathcal{A}_{2i}^{-1} \begin{bmatrix} X_{2i} \\ \\ Y_{2i} \end{bmatrix} = T_{LT_i} \begin{bmatrix} X_{2i} \\ \\ Y_{2i} \end{bmatrix} = \begin{bmatrix} a_i & b_i \\ \\ c_i & d_i \end{bmatrix} \begin{bmatrix} X_{2i} \\ \\ Y_{2i} \end{bmatrix} \quad (3.2.1),$$

(see also 2.3.1)

The modified-chain matrix is given for the forms A and B as in (3.2.2) respectively:

$$T_{Y_iZ_i} = \begin{bmatrix} 1 & -Z_i \\ \\ Y_i & -1-Z_iY_i \end{bmatrix}, \quad T_{Z_iY_i} = \begin{bmatrix} 1+Z_iY_i & -Z_i \\ \\ Y_i & -1 \end{bmatrix} \quad (3.2.2)$$

It is worth observing that $T_{Z_iY_i}^{-1} = T_{Y_iZ_i}$

For $\mathcal{A}_{ji} = \begin{bmatrix} \alpha_{ji} & \beta_{ji} \\ \\ \gamma_{ji} & \delta_{ji} \end{bmatrix}$, $\quad j=1,2, \quad i=1,2,\ldots$

where α_{ji}, γ_{ji} are dimensionless numbers and β_{ji}, δ_{ji} have dimensions of impedance, eqn.(3.2.1) could be solved for form A sections. The elements a_i, b_i, c_i, d_i of matrix T_{LT_i} are given by eqns.(3.2.3):

$$a_i = \frac{1}{\Delta_{2i}} \{\beta_{1i}\gamma_{2i}Z_iY_i + \alpha_{1i}\gamma_{2i}Z_i + \beta_{1i}\delta_{2i}Y_i + \alpha_{1i}\delta_{2i} + \beta_{1i}\gamma_{2i}\}$$

$$b_i = -\frac{1}{\Delta_{2i}} \{\beta_{1i}\alpha_{2i}Z_iY_i + \alpha_{1i}\alpha_{2i}Z_i + \beta_{1i}\beta_{2i}Y_i + \alpha_{1i}\beta_{2i} + \beta_{1i}\alpha_{2i}\}$$

$$c_i = \frac{1}{\Delta_{2i}} \{\delta_{1i}\gamma_{2i}z_i y_i + \gamma_{1i}\gamma_{2i}z_i + \delta_{1i}\delta_{2i}y_i + \delta_{1i}\gamma_{2i} + \gamma_{1i}\delta_{2i}\}$$

$$d_i = \frac{1}{\Delta_{2i}} \{\delta_{1i}\alpha_{2i}z_i y_i + \gamma_{1i}\alpha_{2i}z_i + \delta_{1i}\beta_{2i}y_i + \gamma_{1i}\beta_{2i} + \delta_{1i}\alpha_{2i}\} \qquad (3.2.3)$$

where $\Delta_{2i}=\alpha_{2i}\delta_{2i}-\beta_{2i}\gamma_{2i}$.
It is worth observing that $\Delta T_{LTi} = a_i d_i - b_i c_i = \frac{\Delta_{1i}}{\Delta_{2i}}$. Similar equations are derived considering form B sections. It can be described eqn.(3.2.1) in terms of the inputs X_{1i}, X_{2i} and the outputs Y_{1i}, Y_{2i} as:

$$\begin{bmatrix} Y_{1i} \\ Y_{2i} \end{bmatrix} = \begin{bmatrix} K_i & L_i \\ M_i & N_i \end{bmatrix} \begin{bmatrix} X_{1i} \\ X_{2i} \end{bmatrix} \qquad (3.2.4)$$

where K_i, L_i, M_i, N_i are voltage transfer ratios and given by:

$$K_i = \frac{d_i}{b_i}, \quad L_i = -\frac{a_i d_i - b_i c_i}{b_i} = \frac{\Delta_{1i}}{b_i \Delta_{2i}}, \quad M_i = \frac{1}{b_i}, \quad N_i = -\frac{a_i}{b_i}$$

The above expressions need to be constrained in order that a reduction in the active realizations is achieved.
Suitable constraints could be: (Section 2.3.7)

I) $K_i = L_i = M_i = N_i$

II) $K_i + \varepsilon M_i = \varepsilon_1$

and

$L_i + \varepsilon N_i = \varepsilon_2$, $\varepsilon = 1, -1$, $\varepsilon = -1, 0, 1$, $\varepsilon_2 = -1, 0, 1$

Input Termination

The input E of the filter is taken into account with the first section, to minimise the number of op-amps used.

Thus for the input it can be used the one-output-two-input model, in comparison of the intermediate stages where the two-input-two-output model is applied. Therefore, for filters with form A sections will be:

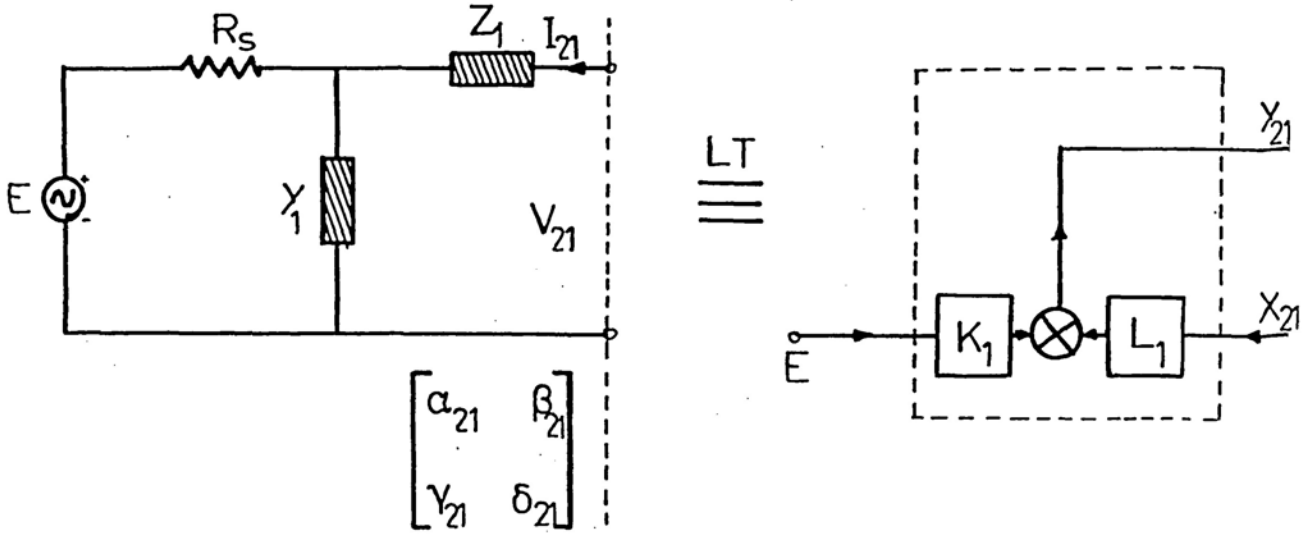


Fig. 3.2.2

The analysis of input termination gives:

$$Y_{21} = \frac{-\alpha_{21}\delta_{21}+\beta_{21}\gamma_{21}}{\beta_{21}(1+R_sY_1)+\alpha_{21}(R_sY_1Z_1+Z_1+R_s)} E + \frac{\delta_{21}(1+R_sY_1)+\gamma_{21}(R_sY_1Z_1+Z_1+R_s)}{\beta_{21}(1+R_sY_1)+\alpha_{21}(R_sY_1Z_1+Z_1+R_s)} Y_{21} \qquad (3.2.5)$$

or $Y_{21}=K_1E+L_1X_{21}$.
where K_1, L_1 are appropriate tranfer ratios.
Incidently, there are few restrictions to be applied to this model, in order to have low-complexity structures. Almost any choice of matrix elements effect in realization of biquadratic sections.

Output Termination

It will be considered the terminating resistance in conjuction with the last section. It will also be considered analysis for both of forms A and B sections:

Form A:

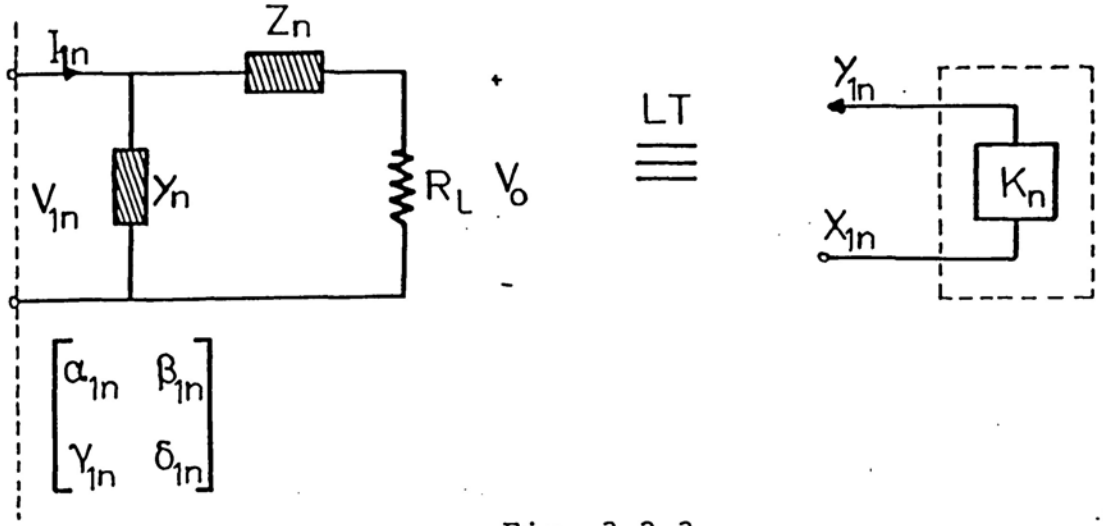


Fig. 3.2.3

The analysis of Fig.3.2.3 gives:

$$\begin{bmatrix} X_{1n} \\ Y_{1n} \end{bmatrix} = \begin{bmatrix} (\alpha_{1n}+\beta_{1n}Y_n)(1+G_L Z_n)+\beta_{1n}G_L \\ (\gamma_{1n}+\delta_{1n}Y_n)(1+G_L Z_n)+\delta_{1n}G_L \end{bmatrix} V_o \qquad (3.2.6)$$

The only constraint to be put here will be having direct output either from X_{1n} or Y_{1n}. For this constraint to be satisfied it must be:

I) either $\alpha_{1n}=-\beta_{1n}Y_n \rightarrow X_{1n}=\beta_{1n}G_L V_o$, β_{1n}= freq.independent

II) or $\gamma_{1n}=-\delta_{1n}Y_n \rightarrow Y_{1n}=\delta_{1n}G_L V_o$ $\delta_{1'n}$=freq.indepentent

III) or $\delta_{in}=0$, $\gamma_{1n}=\frac{1}{1+G_L Z_n} \rightarrow Y_{1n}=V_o$

IV) or $\beta_{in}=0$, $\alpha_{1n}=\frac{1}{1+G_L Z_n} \rightarrow X_{1n}=V_o$

Similarly as above it can be found for form B sections:

$$\begin{bmatrix} X_{1n} \\ Y_{1n} \end{bmatrix} = \begin{bmatrix} (\alpha_{1n}Z_n+\beta_{1n})(G_L+Y_n)+\alpha_{1n} \\ (\gamma_{1n}Z_n+\delta_{1n})(G_L+Y_n)+\gamma_{1n} \end{bmatrix} V_o \qquad (3.2.7)$$

Form B:

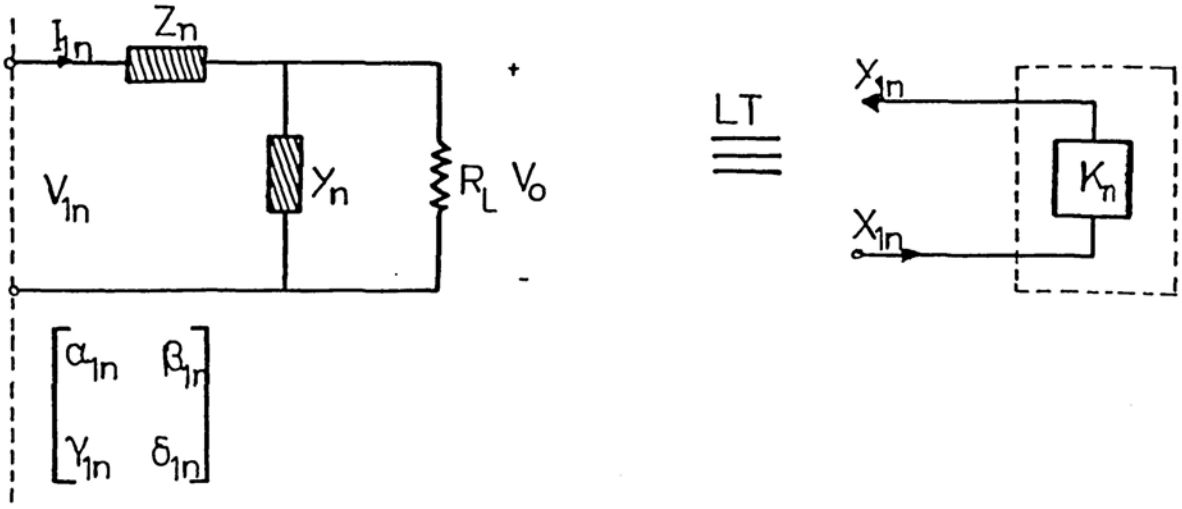


Fig. 3.2.4

The conditions for direct output are:

I) $\delta_{1n}=-\gamma_{1n}Z_n \rightarrow Y_{1n}=\gamma_{1n}V_o$ III) $\gamma_{1n}=0,\ \delta_{1n}=\frac{1}{G_L+Y_n} \rightarrow Y_{1n}=V_o$

II) $\beta_{1n}=-\alpha_{1n}Z_n \rightarrow X_{1n}=\alpha_{1n}V_o$ IV) $\alpha_{1n}=0,\ \beta_{1n}=\frac{1}{G_L+Y_n} \rightarrow X_{1n}=V_o$

In the analysis above, the basic equations and constraints for designing inverted-L filters have been drawn. Although in most of the times one has only inverted-L sections in filters with even order, it is desirable sometimes, for simplicity, to consider single arms as well, for series impedances or shunt admittances together with the L-sections. That will depend on the order and type of the filter aswill be shown in the following design examples.

3.2.2 Design Examples

In the following it will be given some design examples for lowpass and bandpass filters. This method can also be applied to design lowpass and highpass Elliptic filters. It is shown that in such a case we end up with canonic structures.

Low-pass Polynomial filters

A. 3rd order filter

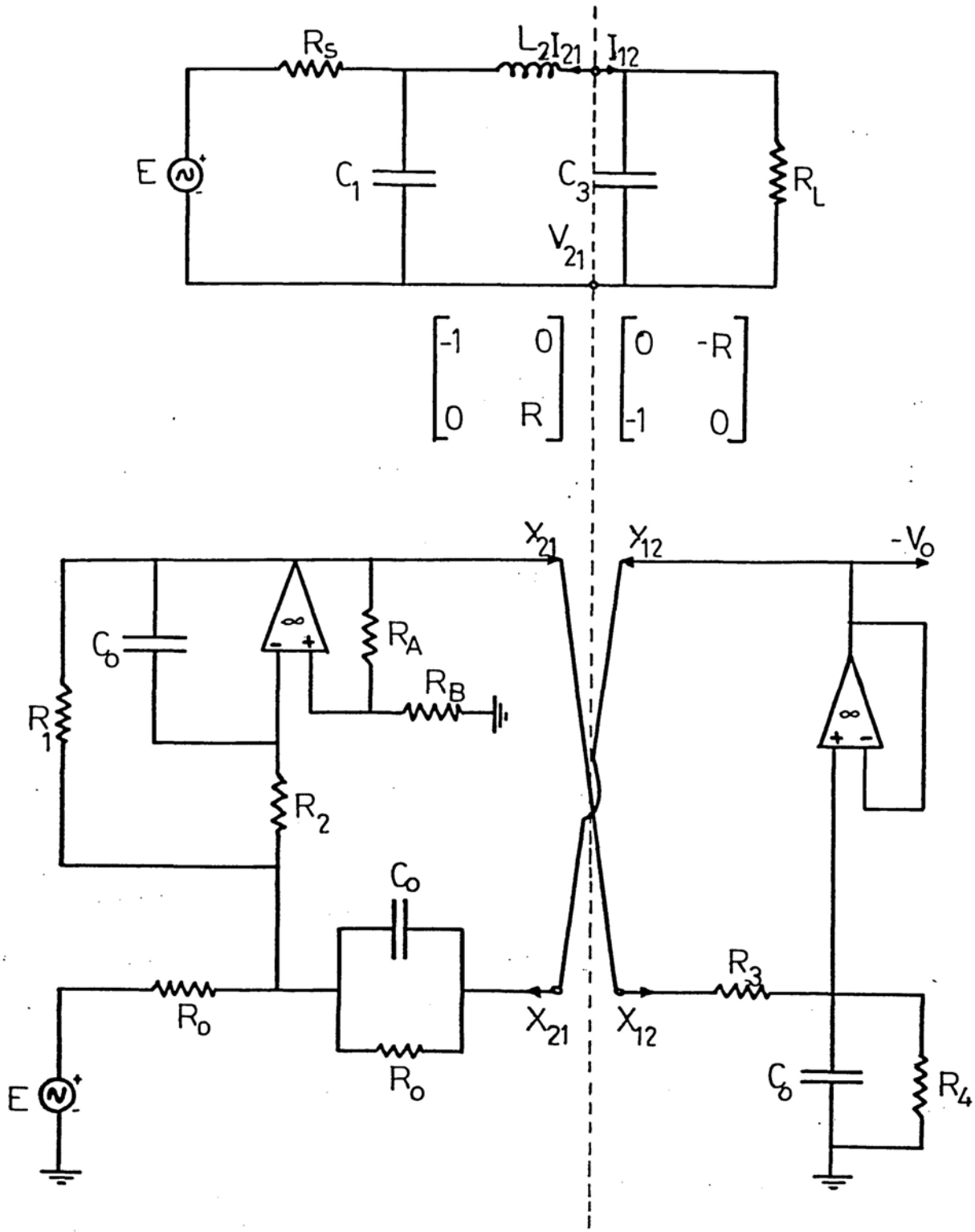


Fig. 3.2.5

B. 4rth order

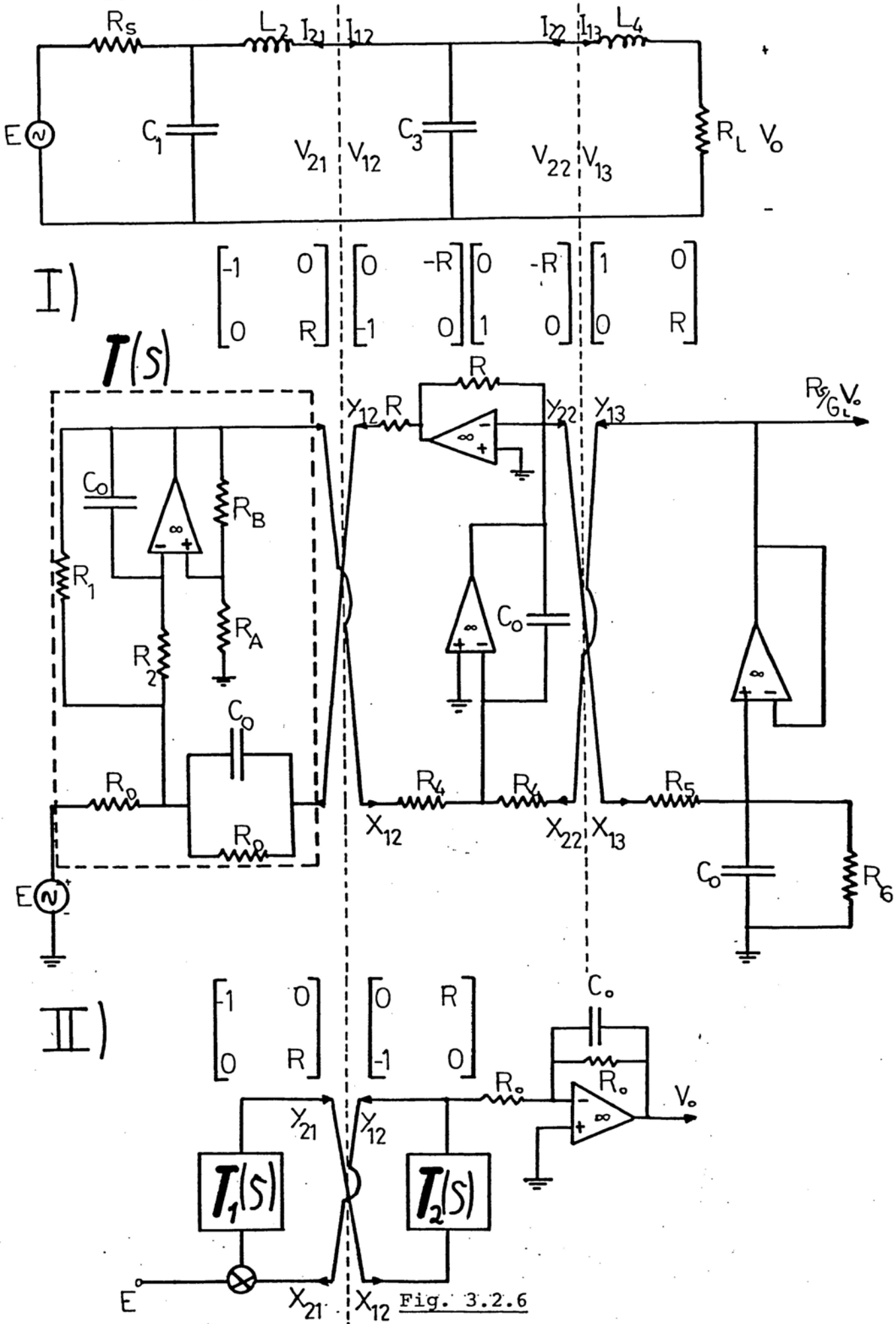


Fig. 3.2.6

In the case B.II) above the conditions applied in (3.2.6), are violated to have direct output, ending with an extra lossy integrator to get the output V_o from Y_{12}. So the structure is non-canonic but is uses 3 op-amps instead of 4 of case B.I).

C. 5th order

For the fifth order filter, by similar statements the LT matrices shown in Fig.3.2.7, can be used with appropriate structures.

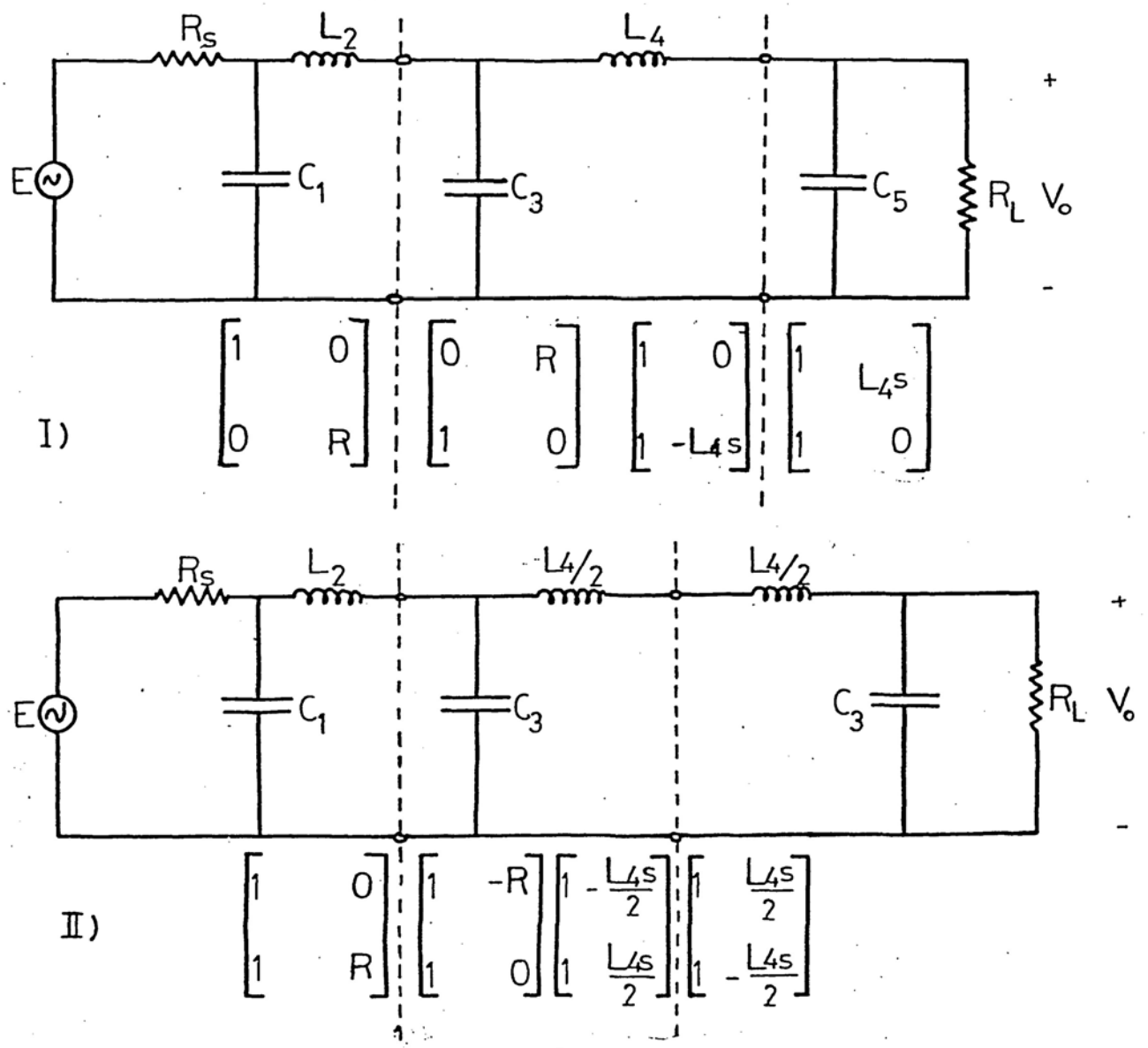


Fig.3.2.7

Lowpass Eiliptic Filters

4rth-order Elliptic Filter

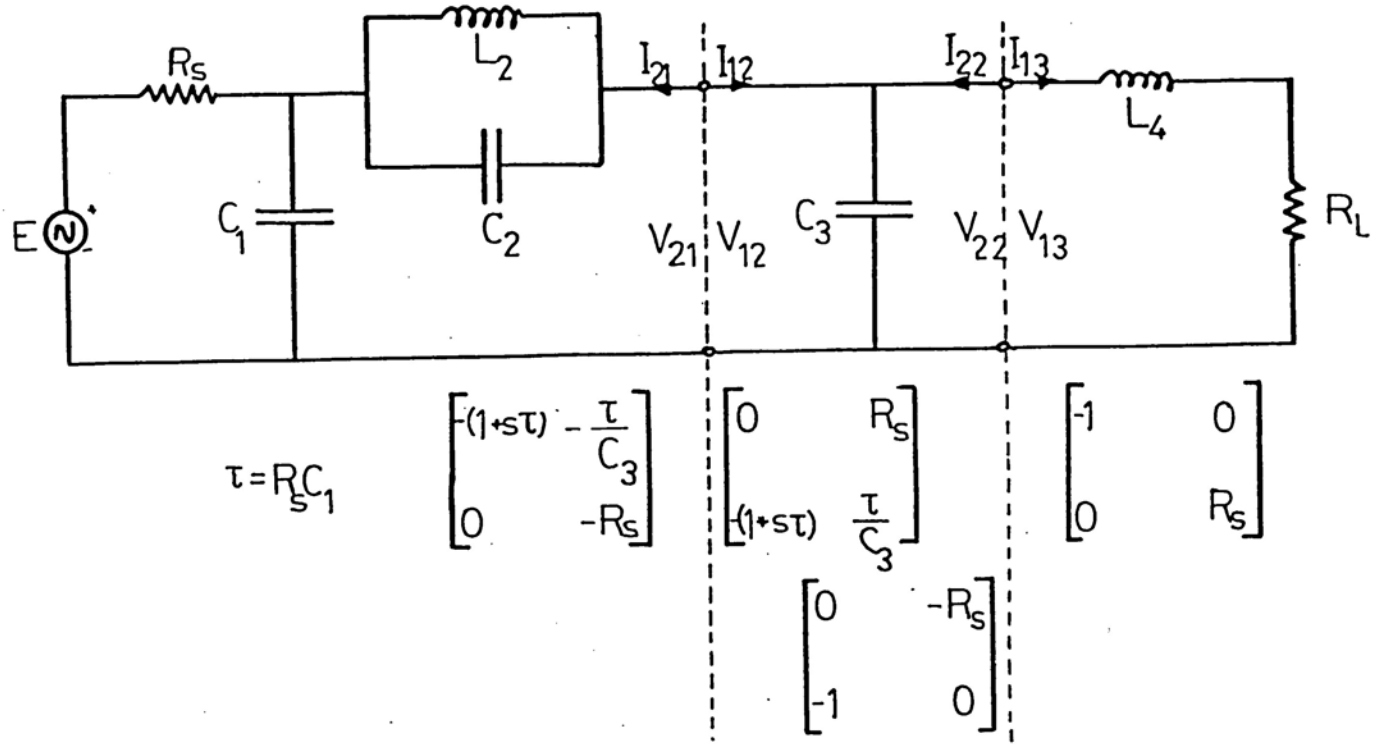


Fig. 3.2.8a

The analysis of this circuit gives:

$$Y_{21} = \frac{L_2C_2s^2+1}{s^2(2C_1L_2+C_2L_2) + \frac{L_2}{R_s}s + 1 + \frac{C_1}{C_3}} [E+X_{21}] = T_2(s)(E+X_{21})$$

$$Y_{12} = \frac{1}{R_sC_3s}(X_{12}-X_{22}) + \frac{C_1}{C_3}X_{22}$$

$$Y_{22} = -\frac{1}{R_sC_3s}(X_{12}-X_{22})$$

$$Y_{13} = -\frac{R_s}{L_4s+R_L}X_{13} \quad , \quad Y_{13} = \frac{R_s}{R_1}V_o$$

Its realization is shown in Fig.3.2.8b, where

T(s) is a transfer function of the form:

$$T(s) = \frac{L_2C_2s^2+1}{s^2(2C_1L_2+C_2L_2) + \frac{L_2}{R_s}s + 1 + \frac{C_1}{C_3}}$$

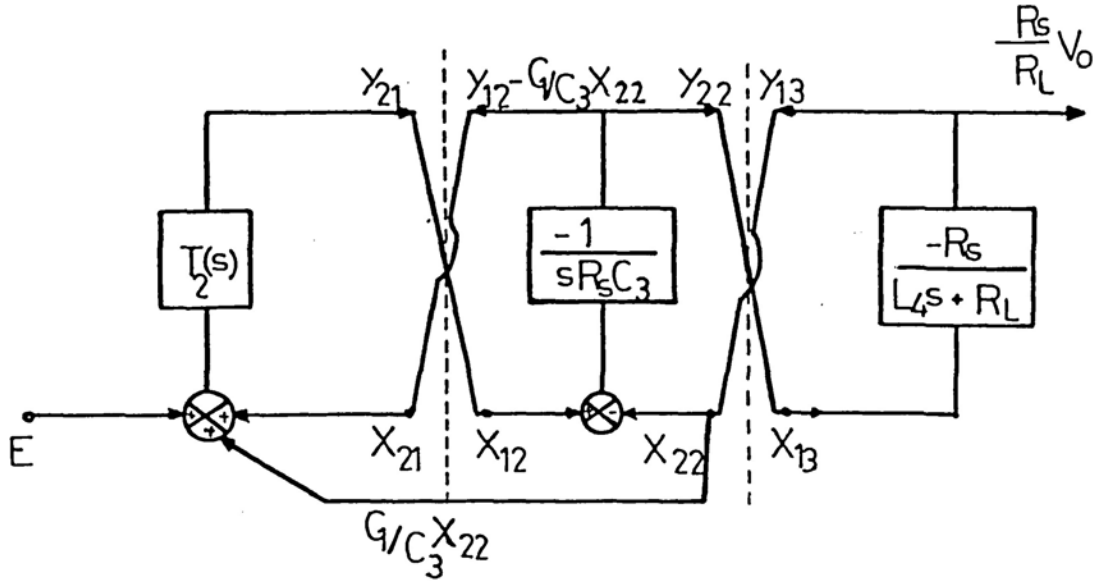


Fig. 3.2.8b

Bandpass filters

The proposed method is, also, useful for bandpass filters where capacitive loops or inductive cut-sets are included.

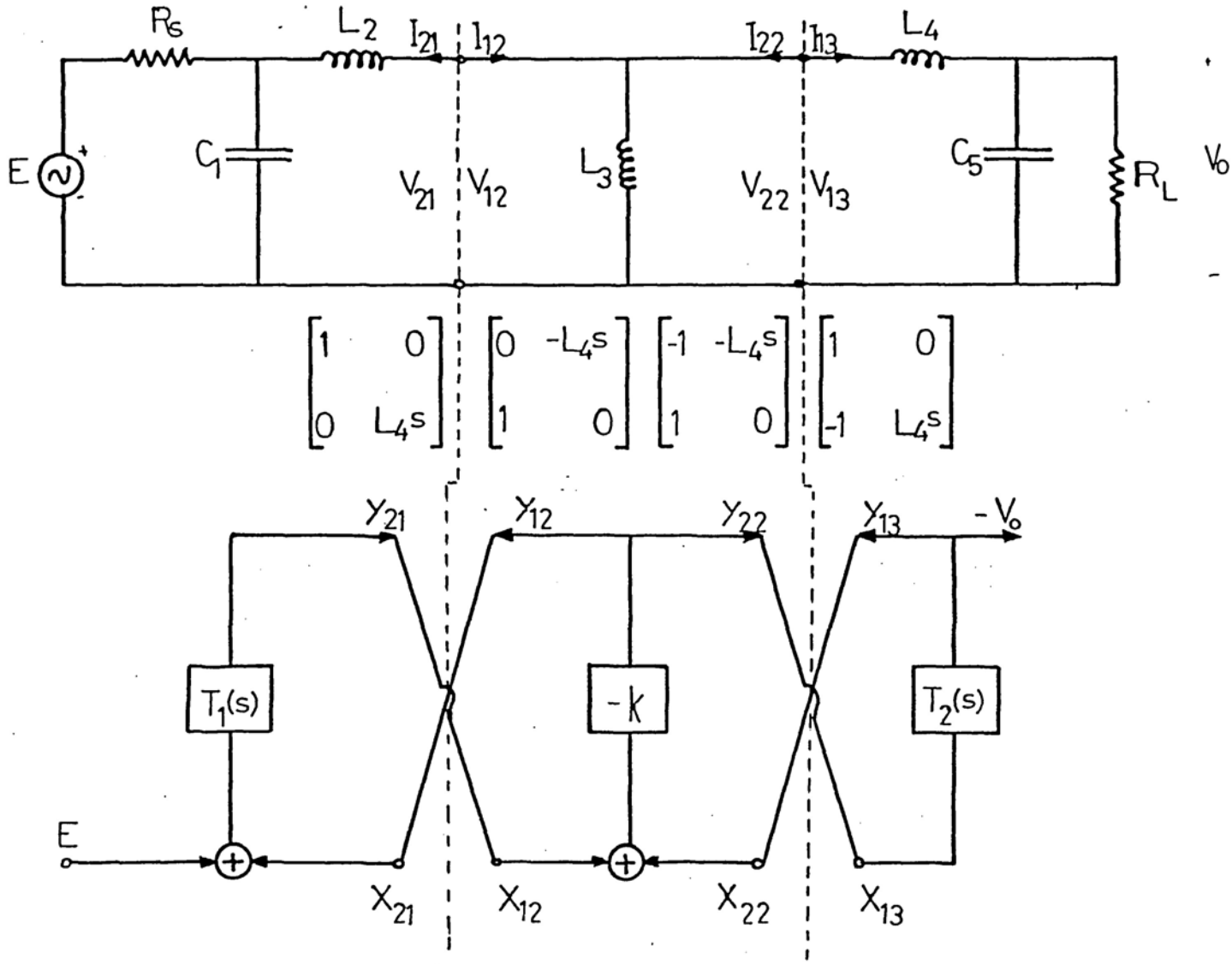
By applying the frequency-dependent Linear Transformation [DI1] the number of frequency-dependent elements could be reduced and be made equal to the order of the filder.

Consider the 4rth order unconventional pandpass filter shown in Fig.3.2.9.

With those matrices, the frequency dependent matricies of the middle arm (L_3) cancel the admittance L_3s and a frequency-independent structure in the middle arm is obtained.

3.2.3 Conclusions

A method has been presented for simulation of LC filters through Linear Transformations that employs biquads of finite-Q in the realizations. Although the sensitivity of the new structures is expected not to be as good as of the structures resulting by simulation of one-arm at a time, it is found that is low-enough, according to Orchard



where $T_1(s)$, $T_2(s)$ are biquads and -k an inverter.

Fig. 3.2.9

Principle [OR1] of maximum power tranfer in doubly terminated filters. It has been shown that considerable reduction in the numbers of op-amps employed, has been done and in many cases canonic structures, are obtained as e.g. in elliptic filters.

The only method, so far, that realize elliptic filters with canonic structures is in reference [PS1], but it uses more op-amps than the presented method

Finally, research should be done, in order to be employed in the realization the best biquads as far as sensitivity is concerned, to achieve better performances.

3.3 Off-the-shelf Building Blocks

In the two previous sections of Chapter 3, it has been used single-amplifier biquads to realize active-RC filters through LTA method, from their passive, counterparts, in order to minimise the number of op-amps used. Sometimes in the applications of active-RC filter design, the number of op-amps embedded in the realizations is not so important factor as the easy controllability and tuning. For those applications, the three-amplifier biquads are used. Such biquads having appeared in the literature, are the Kerwin-Huelsman-Newcomb biquad [KE1], B.Tarmy and M.S.Ghausi [TA1], Fleisher and Tow biquad [FL1] and many others described in [DA1], [HU1].

The main advantages of the three-amplifier biquads over the single-amplifier ones are:

I) They realize the general biquadratic function, so that the same topology can be used for generating all the filter functions.

II) Tuning is easier and more economical.

III) The one-to-one relationship between the biquadratic parameters and circuit resistors, allow their use in switchable-filter applications.

Therefore, for stringent filter applications requiring tuning, the advantages of the three amplifier biquads will

often offset the cost of the extra op-amps and their higher sensitivity, due to more components in the circuit realizations. All those features have led the industry to manufacture the three-amplifier biquad and espessially Kerwin-Huelsman-Newcomb biquad [KHN] in integrated form. In the following it will be tried to realize active filters, through LT technique using that IC chip as a basic building block.

3.3.1 IC AF150 as a basic building block

Recently, National Semiconductor company has manufactured in a chip an integrated circuit, called AF150, realizing lowpass, highpass and bandpass functions of the second order, at separate outputs of the chip. Notch and allpass functions can be formed by summing the lowpass and highpass outputs or by using RC input instead of resistive. The internal configuration of the chip is based on KHN biquad [KE1] and its connection diagram is shown in Fig. 3.3.1 [NS]:

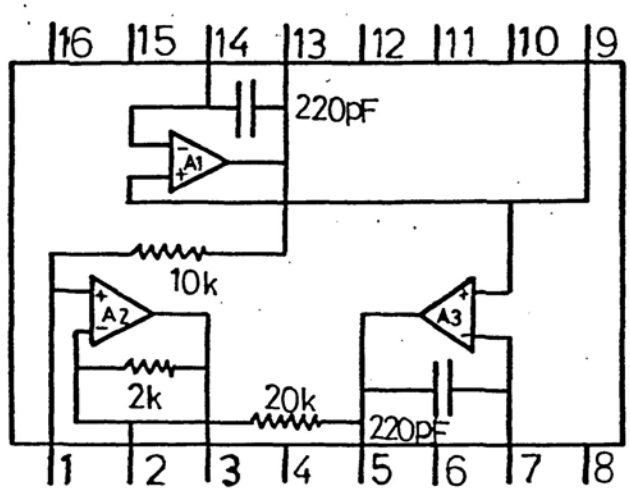


Fig. 3.3.1

The numbers above indicate the pin numbers of the chip. By setting two equal value resistors between pins 3 and 14 and 13 and 7 (Fig.3.3.2), it could be adjusted to frequencies above 1kHz. For frequencies below 1kHz, a T resistive tuning is required [NS]. The relation between the frequency tuning resistors and the frequency f_o is given by tuning resistors $R_{f1,2}$ of value:

$$R_{f1,2} = \frac{228.8 \times 10^6}{f_o} \ \Omega \qquad (3.3.1)$$

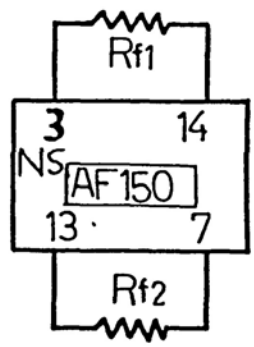


Fig. 3.3.2

Setting the Q requires one resistor from either pin 1 or pin 2 to ground. The value of Q setting resistor depends on the input connection and input resistance, as well as the value of Q. The Q will be inversely proportional to the resistance from pin 1 to ground and directly proportional to resistance from pin 2 to ground. For the non-inverting connection we choose a value of input resistor R_{IN} and calculate the factor Q_{MIN} by the eqn:

$$Q_{MIN} = \frac{1 + \frac{10^4}{R_{IN}}}{3.48} \qquad (3.3.2)$$

If the Q required in the circuit is greater than Q_{MIN} the configuration of Fig.3.3.3a is used to calculate R_Q and R_{IN} resistors. If the Q of the circuit is less than Q_{MIN}, the configuration of Fig.3.3.3b is used.

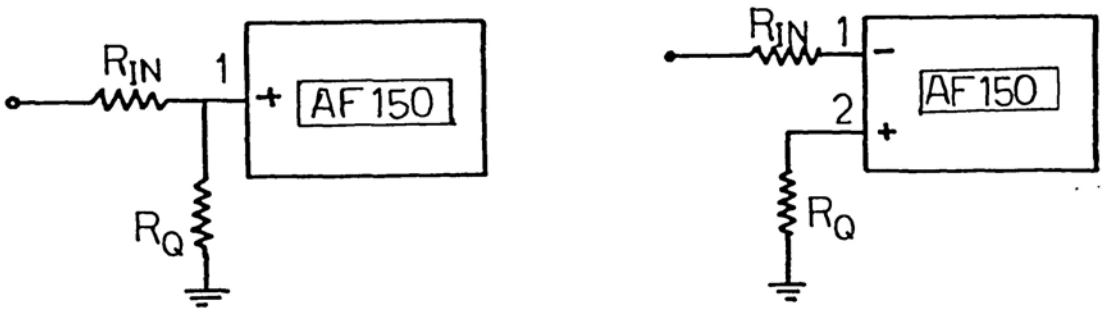


Fig. 3.3.3

For the inverting connection, for any Q required, the configuration of Fig.3.3.4 is used.

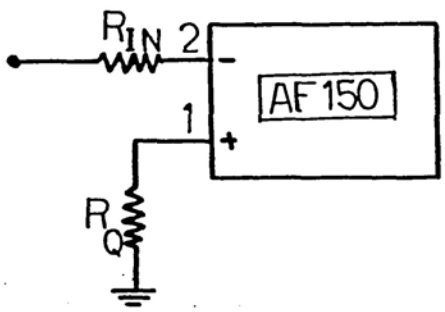


Fig. 3.3.4

The value of resistor R_Q is evaluated for specific R_{IN}, by eqns.(3.3.2a,b,c) [NS] for the non-inverting mode with $Q>Q_{MIN}$ or $Q<Q_{MIN}$ and inverting-mode respectively.

$$R_Q = \frac{10^4}{3.48Q - 1 - \frac{10^4}{R_{IN}}} \ \Omega \qquad (3.3.2a)$$

$$R_Q = \frac{2\times10^3}{0.3162\,\frac{Q}{(1+\frac{10^4}{R_{IN}})} - 1.1} \ \Omega \qquad (3.3.2b)$$

$$R_Q = \frac{10^4}{3.16Q(1.1 + \frac{2x10^3}{R_{IN}}) - 1} \ \Omega \qquad (3.3.2c)$$

In the above equations, as mentioned earlier, R_{IN} is the input resistance of the section and is evaluated by specific formulas given in [NS] for center frequency gain adjustments and summarized below:

Non-inverting input bandpass transfer function

i) $Q>Q_{MIN}$

$$\frac{E_b}{E_{IN}} = \frac{-s\omega_1\left[\frac{1.1}{1 + \frac{R_{IN}}{10^4} + \frac{R_{IN}}{R_Q}}\right]}{\Delta} \qquad \text{where} \qquad (3.3.3a)$$

$$\Delta = s^2 + s\left[\frac{1.1}{1 + \frac{10^4}{R_Q} + \frac{10^4}{R_{IN}}}\right]\omega_1 + 0.1\omega_1\omega_2, \quad \omega_1 = \frac{10^{12}}{R_{f1}\cdot 220}, \quad \omega_2 = \frac{10^{12}}{R_{f2}\cdot 220}$$

$$R_{f1} = R_{f2}$$

ii) $Q<Q_{MIN}$

$$\frac{E_b}{E_{IN}} = \frac{-s\omega_1\left[\frac{1.1 + \frac{2x10^3}{R_Q}}{1 + \frac{R_{IN}}{10^4}}\right]}{\Delta} \qquad (3.3.3b)$$

Inverting input bandpass transfer function

$$\frac{E_b}{E_{IN}} = \frac{s\omega_1(\frac{2x10^3}{R_{IN}})}{\Delta} \qquad (3.3.3c)$$

In the following, active-RC realizations will be carried out using the IC AF150 chip as the basic building block.

3.3.2 Design Examples

A. Consider the second order lowpass passive filter shown in Fig.3.3.5a and the corresponding 4rth order bandpass filter after applying the lowpass to bandpass transformation shown in Fig.3.3.5b.

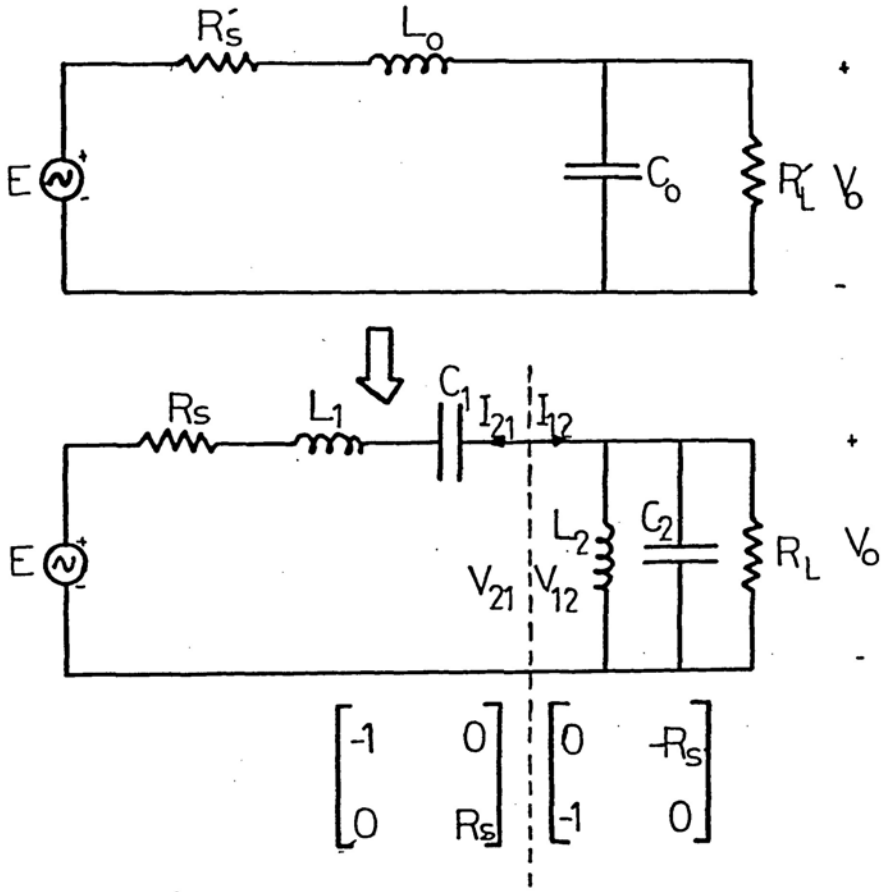


Fig. 3.3.5

The Linear Transformation matrices shown are specifically chosen as to give the following transfer functions for the input and output sections:

$$Y_{21} = -\frac{R_s C_1 s}{L_1 C_1 s^2 + R_s C_1 s + 1}\,(E + X_{21})$$

$$Y_{12} = \frac{L_2 s}{R_s L_2 C_2 s^2 + R_s C_2 L_2 s + R_s}\,X_{12}, \qquad Y_{12} = -V_o \tag{3.3.4}$$

Comparing eqns.(3.3.4) with (3.3.3) it can be seen that the input section could be realized by the noninverting configuration, while the output stage could be realized by

the inverting configuration. The complete filter is shown in Fig.3.3.6.

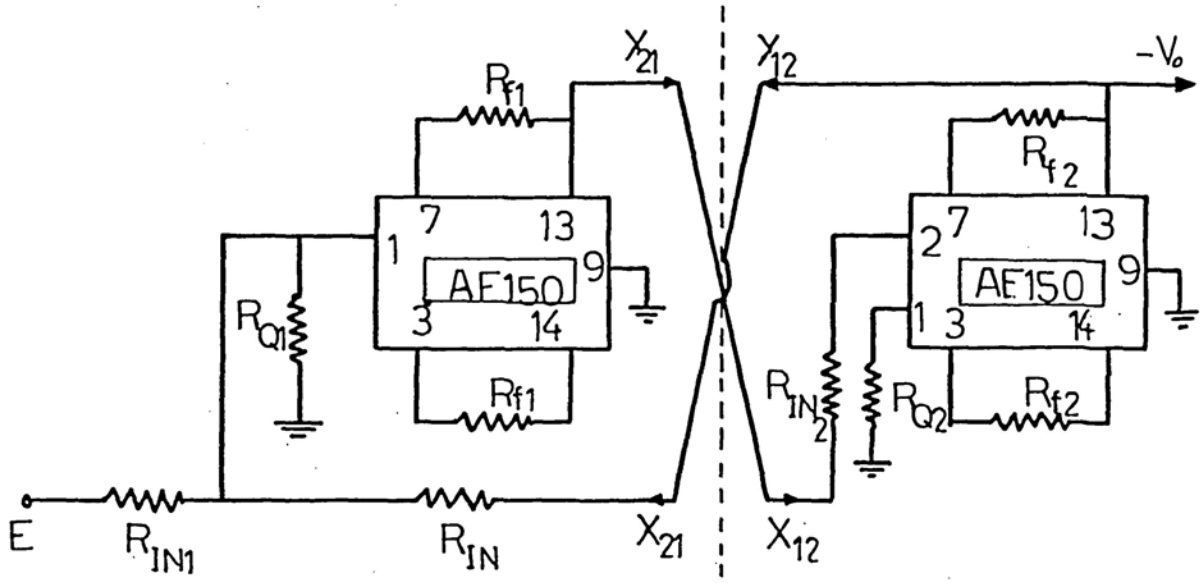


Fig. 3.3.6

B. Consider the 6th order Bandpass filter in Fig.3.3.7a with the assossiated linear transformation matrices shown. The matrices are the same as those in the example in section 3.1. There is a third entry in the matrices of the middle arm to give an additional loop in order finite-Q biquad, as that of IC AF150 chip, could be embedded in the realization of the series tuned circuit of the filter. Thus the first and third section are realized by the non-inverting configuration while the second section is realized by the inverting configuration. The complete filter is shown in Fig. 3.3.7b where the resistors R_{IN} are the input resistors to the sections and the resistors R_Q set the Q's required while R_f set the center frequency. Numerical examples for the section will be given in Chapter 6, in practical realizations.

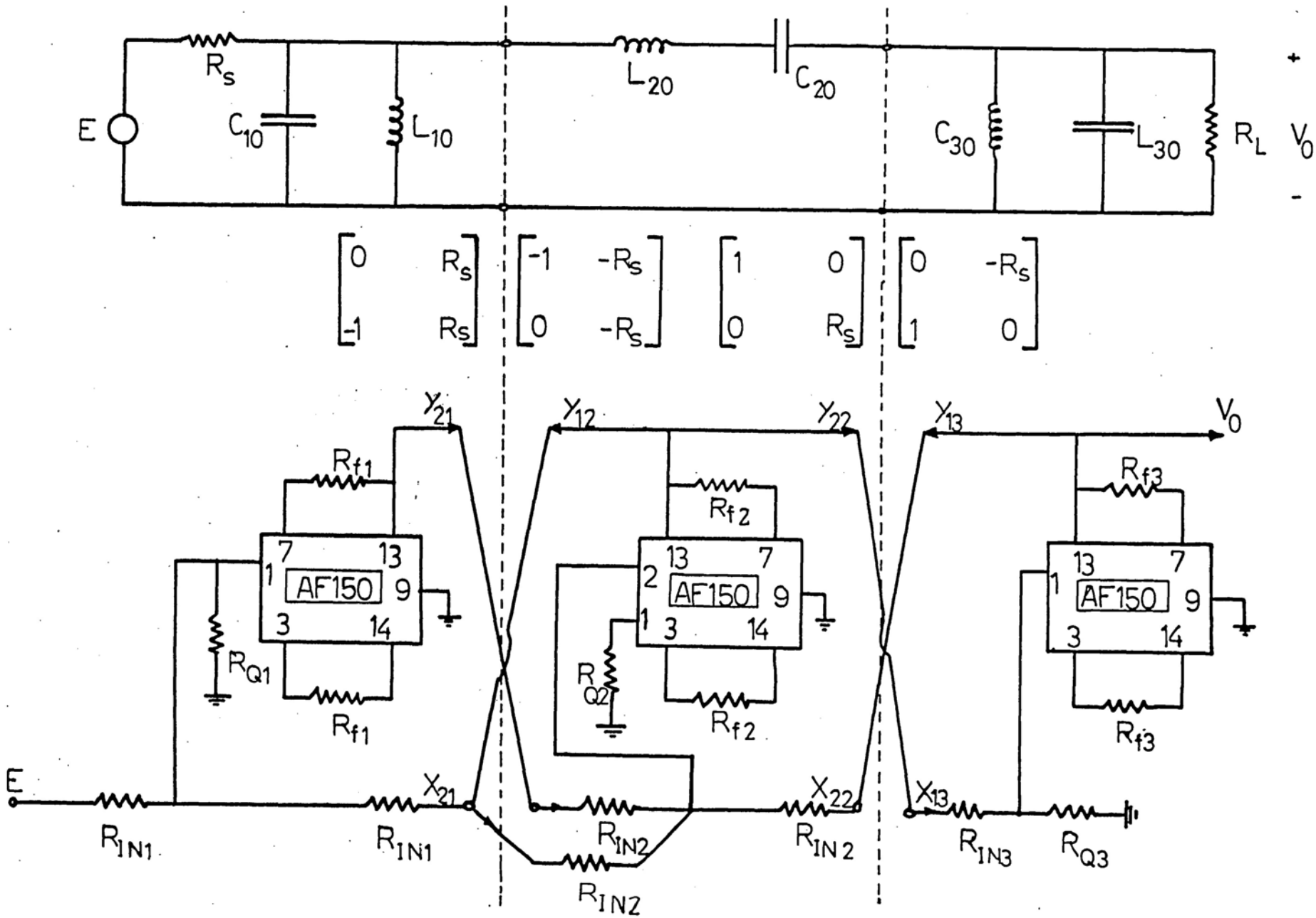

E
R_s
C_{10}
L_{10}
L_{20}
C_{20}
C_{30}
L_{30}
R_L
V_0
+
-
$\begin{bmatrix} 0 & R_s \\ -1 & R_s \end{bmatrix}$
$\begin{bmatrix} -1 & -R_s \\ 0 & -R_s \end{bmatrix}$
$\begin{bmatrix} 1 & 0 \\ 0 & R_s \end{bmatrix}$
$\begin{bmatrix} 0 & -R_s \\ 1 & 0 \end{bmatrix}$
Y_{21}
Y_{12}
Y_{22}
Y_{13}
V_0
R_{f1}
R_{f2}
R_{f3}
AF150
R_{Q1}
R_{Q2}
R_{Q3}
X_{21}
X_{22}
X_{13}
R_{IN1}
R_{IN2}
R_{IN3}

Fig. 3.3.7

3.3.3 Conclusions

In this section integrated circuits realizing the general biquadratic function have been embedded in multiple feedback structures, to realize active-RC filters from LC passive prototypes through Linear Transformations. Although only Bandpass filters have been simulated using those IC's, lowpass and highpass structures can be obtained and notch and allpass functions can be formed by summing the outputs with an external amplifier. This wide band active filter is a general second order RC network in IC form, requiring only four external resistors to program it and is called AF150 manufactured by National Semiconductors. The Inverted-L filters could also be realized according to instructions in section 3.2.

The interesting features of this chip are:

i) Independent Q, frequency and gain adjustments.

ii) Low sensitivity to external component variation.

iii) Separate lowpass highpass, bandpass outputs.

iv) Inputs may be differential, inverting or non-inverting.

v) Operates to 100kHz.

vi) Q range to 500.

vii) High accuracy $\pm$1% unadjusted and

viii) Q frequency product 2×10^5.

CHAPTER 4

LINEAR TRANSFORMATION ACTIVE-R FILTERS WITHOUT EXTERNAL CAPACITORS

4.1 OPERATIONAL-AMPLIFIER FINITE GAIN-BANDWIDTH CONSIDERATIONS

4.1.1 One-pole roll-off model

4.1.2 Active 1st and 2nd order sections using the operational-amplifier pole

4.2 ACTIVE-R FILTERS FROM LC LADDERS

4.2.1 Filters in literature as special cases of Linear Transformation approach

4.2.2 Active-R Filters with tuned circuits

4.2.3 Inverted-L Active-R filters

4.3 DISCUSSIONS AND CONCLUSIONS

CHAPTER 4

LINEAR TRANSFORMATION ACTIVE-R FILTERS WITHOUT EXTERNAL CAPACITORS

4.1 OPERATIONAL-AMPLIFIER FINITE GAIN-BANDWIDTH CONSIDERATIONS

In the previous sections is has been assumed that the gain of the operational-amplifiers used in Active-RC filter Design is infinite, so the op-amps has been considered to be ideal. This fact is not true in practice and the actual operational-amplifier differs in many ways from the ideal. Foremost among these, is the frequency-dependent nature of the gain. As a result the gain of the op-amp is no longer infinite but A(s), and the input voltage is no longer zero but $V_a(s)$ as shown in Fig.4.1.1.

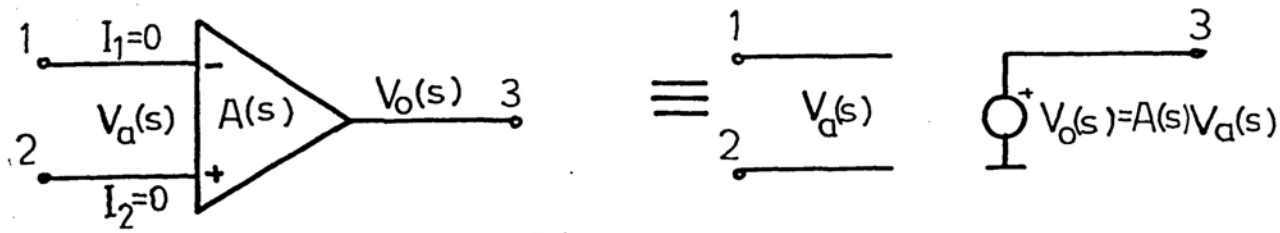


Fig. 4.1.1

As A(s) approaches infinity, the input voltage, $V_a(s)$ approaches zero and the output voltage $V_o(s)$ remains finite and dependent upon the external circuits parameters.

The amplifier gain A(s), for a typical op-amp, can be approximated by a 3-pole function of the form:

$$A(s) = 10^5 \frac{2\pi\cdot 10^3}{(s+2\pi\cdot 10^3)} \cdot \frac{2\pi\cdot 10^5}{(s+2\pi\cdot 10^5)} \cdot \frac{2\pi\cdot 5(10^6)}{(s+2\pi 5\cdot 10^6)} \qquad (4.1.1)$$

The dc gain is $A_o=10^5$ or 100db. The low-frequency poles at 1000 Hz and 100kHz are due to stray capacitors at high im-

pedance points of the transistor circuit comprising the op-amp. The additional high frequency roll-off represented by the pole at 5MHz, is attributed to the finite time taken by the minority carries to travel accross the bases of the transistors. Usually op-amps are frequently compensated internally and therefore represented by one roll-off model or two-pole roll-off model corresponding to single pole compensation and double pole compensation respectively.

Because of the presence of the pole in the actual operational amplifiers, designers have tried either to eliminate its effect temporarily by the method of predistortion [DA1], predistorting the frequency and Q-factor, or permanently by external compensation using capacitors or resistors in appropriate points [SI2].

Another method is taking into account the effect of the pole and eliminate the number of capacitors involved in active-RC structures [RA1-2], or not using capacitors at all, in active-R structures: [SO1-3], [BR1-3], [SC1-2], etc. It will mainly be discussed in this section the active-R structures and how they are derived by Linear Transformation technique.

4.1.1 One-pole Roll-Off model

Most op-amps are designed so that, for small signal operation, A(s) can be represented by a dominant pole, that is:

$$A(s) = -\frac{A_o\omega_a}{s+\omega_a} = -\frac{GB}{s+\omega_a} \qquad (4.1.2)$$

where $A_o \triangleq$dc gain, $\omega_a \triangleq$3db bandwidth of the op-amp and dominant pole frequency, $GB \triangleq A_o\omega_a$=gain-bandwidth product.

Typical values for op-amp μA741 are:

$A_o \simeq 10^5$, $\omega_a \simeq 2\pi.10$r/sec, $GB \simeq 2\pi.20^6$r/sec

In many applications at frequencies $\omega >> \omega_a$ the following approximation is true:

$$A(s) \simeq - \frac{GB}{s}$$

Usually, practical Active-R filters are designed for $\omega > 2\pi \cdot 20$kHz.

Gain and Bandwidth of the non-inverting amplifier

The configuration in the non-inverting mode of the amplifier is shown in Fig.4.1.2.

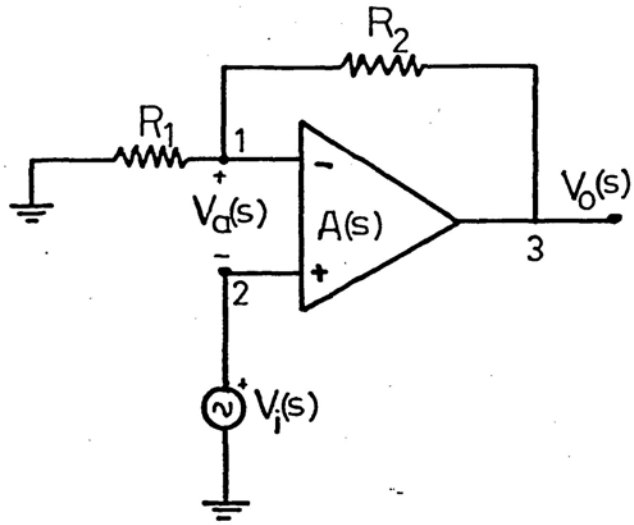


Fig. 4.1.2

The following relationships are valid:

$$V_a(s) = V(1) - V(2), \quad V_o(s) = A(s)V_a(s), \quad A(s) = - \frac{GB}{s+\omega_a}$$

$$\text{or } V_a(s) = V_a(s)A(s) \frac{R_1}{R_1+R_2} - V_i(s) = K_o \frac{-V_i(s)}{K_o - A(s)}, \quad K_o = 1 + \frac{R_2}{R_1}$$

$$\text{and } V_o(s) = K_o \frac{-A(s)}{K_o - A(s)} V_i(s)$$

$$\text{or} \quad \frac{V_a(s)}{V_i(s)} = -\frac{s+\omega_a}{s+\omega_a+\frac{GB}{K_o}} \quad \text{and} \quad \frac{V_o(s)}{V_i(s)} = \frac{GB}{s+\omega_a+\frac{GB}{K_o}} \tag{4.1.3}$$

$$\text{with } K_o = 1+\frac{R_2}{R_1}$$

As long as $K_o << \frac{GB}{\omega_a} = A_o$ the equations (4.1.3) reduce to eqns.(4.1.4)

$$\frac{V_a(s)}{V_i(s)} = \frac{V_1-V_2}{V_i(s)} = \frac{V_1(s)-V_i(s)}{V_i(s)} = -\frac{s+\omega_a}{s+\frac{GB}{K_o}} \tag{4.1.4}$$

$$\text{and} \quad \frac{V_o(s)}{V_i(s)} = \frac{GB}{s+\frac{GB}{K_o}}$$

Gain and Bandwidth of the inverting amplifier

By having connected the input voltage to the inverting mode of the op-amp, one has the configuration of Fig.4.1.3.

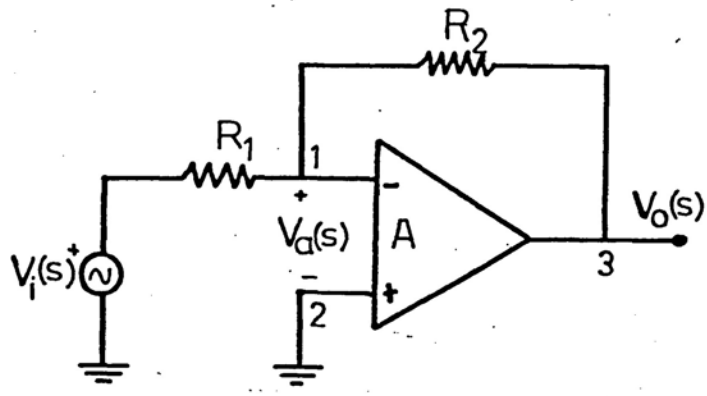


Fig. 4.1.3

Similarly working as previously the following relations could be obtained:

$$\frac{V_a(s)}{V_i(s)} = \frac{V_1(s)}{V_i(s)} = \frac{s+\omega_a}{s+\omega_a+\frac{GB}{1+K_o}}, \quad \frac{V_o(s)}{V_i(s)} = -\frac{K_o}{1+K_o}\,\frac{GB}{s+\omega_a+\frac{GB}{1+K_o}} \tag{4.1.5}$$

By using two inputs in the inverting mode:

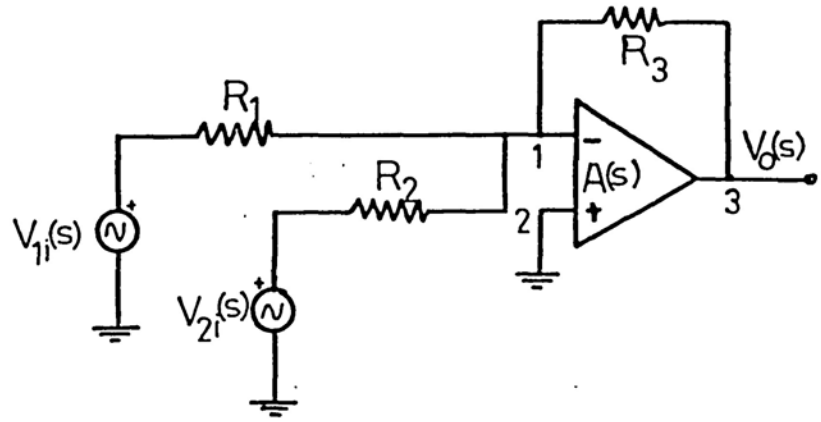


Fig. 4.1.4

$$V_o(s) = - \frac{K_o}{(1+K_o)(1+K_1)} \cdot \frac{GB}{s+\omega_a + \frac{GB}{1+K_o}} (K_1 V_1 + V_2) \qquad (4.1.6)$$

where:

$$K_o = \frac{R_3}{R_2 || R_1}, \qquad K_1 = \frac{R_2}{R_1}$$

Gain and Bandwidth in mixed configuration

By using both inverting and non-inverting modes of the operational amplifier the configuration of Fig.4.1.5 is obtained:

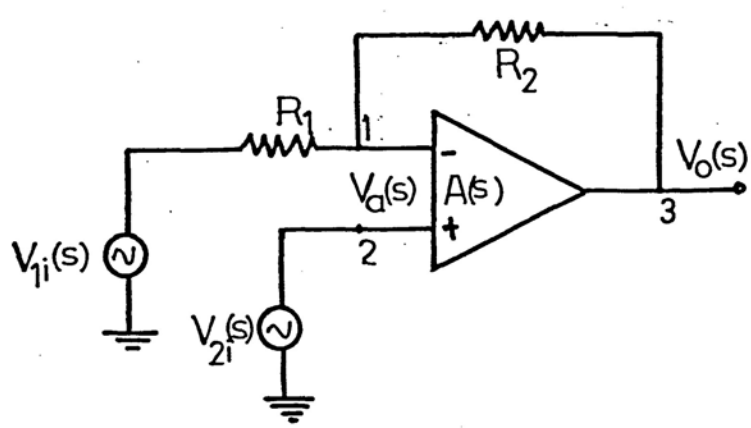


Fig. 4.1.5

$$K_o = \frac{R_2}{R_1}$$

$$V_o(s) = \frac{GB}{s+\omega_a + \frac{GB}{1+K_o}} \left(V_{2i}(s) - \frac{K_o}{1+K_o} V_{1i}(s)\right) \qquad (4.1.7)$$

4.1.2 Active 1st and 2nd order sections using the operational amplifier pole

Having in mind the above configurations the integrated summer, shown in Fig.4.1.6, can be implemented using an operational amplifier with gain A_1 and 3-db frequency [BR4].

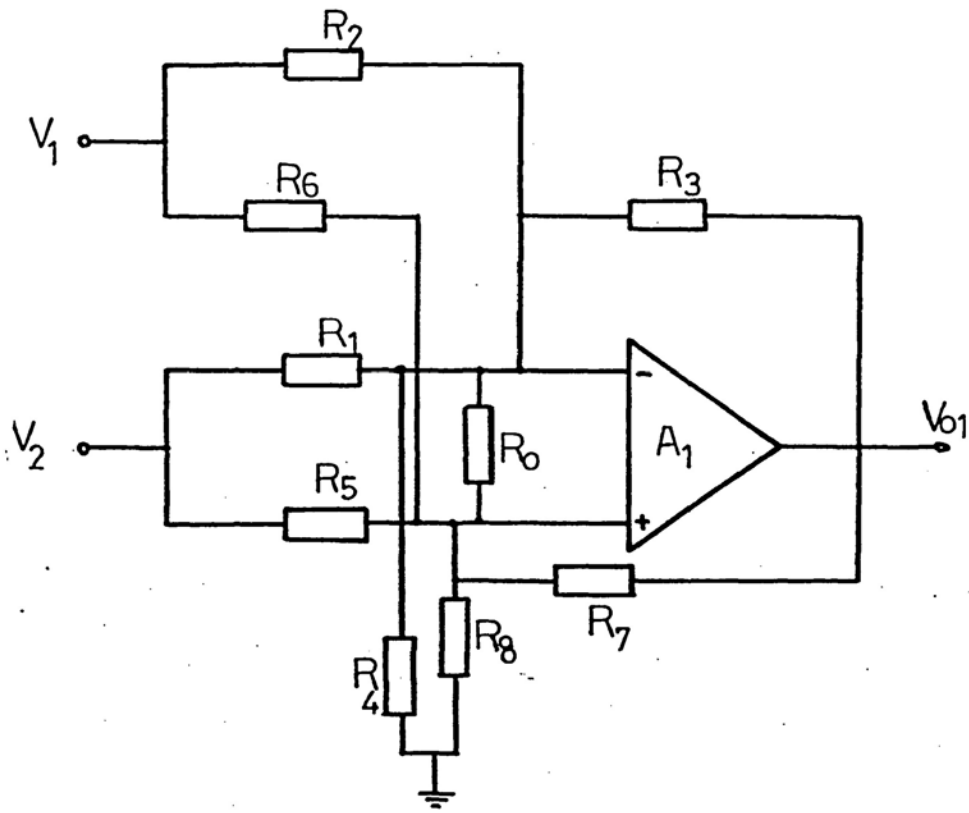


Fig. 4.1.6

where $V_{o1} = \frac{C_1}{1/A_1+C_o} V_1 + \frac{C_o}{1/A_1+C_o} V_2$ (4.1.8)

and the following abbreviations have been introduced:

$C_o=b_3-b_7$, $C_1=b_5-b_1$, $C_2=b_6-b_2$

$$b_1 = \frac{a_1+a_{on}a_5}{1-a_{on}a_{op}}, \quad b_2 = \frac{a_2+a_{on}a_6}{1-a_{on}a_{op}}, \quad b_3 = \frac{a_3+a_{on}a_7}{1-a_{on}a_{op}}$$

$$b_5 = \frac{a_5+a_{op}a_1}{1-a_{on}a_{op}}, \quad b_6 = \frac{a_6+a_{op}a_2}{1-a_{on}a_{op}}, \quad b_7 = \frac{a_7+a_{op}a_3}{1-a_{on}a_{op}} \qquad (4.1.9)$$

$$a_{on} = \frac{G_o}{G_{Tn}}, \quad a_i = \frac{G_i}{G_{Tn}} \ (i=1,2,3) \qquad G_{Tn} = \sum_{i=0}^{4} G_i$$

$$a_{op} = \frac{G_o}{G_{TP}}, \quad a_i = \frac{G_i}{G_{TP}} \ (i=5,6,7) \qquad G_{TP} = G_o + \sum_{i=5}^{8} G_i$$

This section can give a 1st order lowpass circuit with negative or positive gain. In addition a number of second-order lowpass or bandpass filters may be derived from this circuit by replacing one of the resistors by a capacitor, (e.g. [RA3], [MI1]).

By employing another block similar to that in Fig.4.1.6 and interconnecting them can be obtained the general active-R biquadratic block which is shown in Fig.4.1.7 in simplified form by eliminating some resistors [BR3] and which can be realize lowpass and bandpass functions. This configuration can be found also in References ([MI2], G_5=0), [KI1].

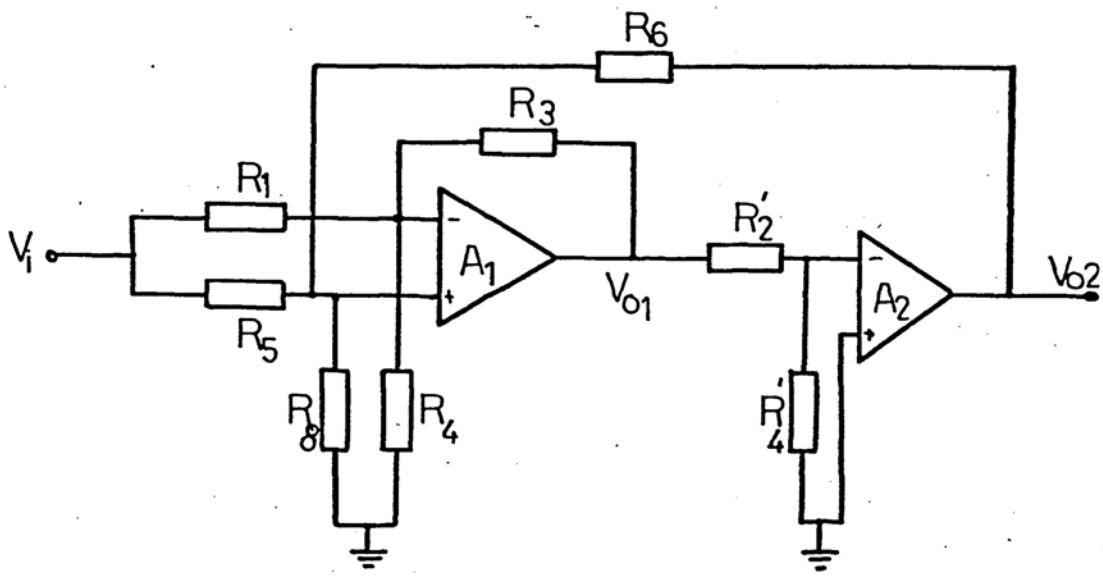


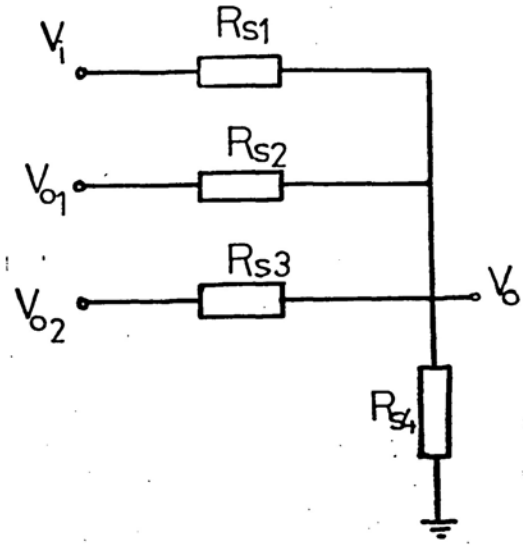
Fig. 4.1.7

where

$$\frac{V_{o1}}{V_i} = \frac{sGB_1(a_5-a_1)}{s^2+sGB_1a_3+GB_1GB_2a_6a_2}$$

$$\frac{V_{o2}}{V_i} = \frac{-GB_1GB_2(a_5-a_1)a_2}{s^2+sGB_1a_3+GB_1GB_2a_6a_2} \qquad (4.1.10)$$

and a_i are given by eqns.(4.1.9) and GB_1, GB_2 are the gain-bandwidth products of the amplifiers A_1,A_2 respectively. In the above circuit there is: $R_o=R_2=R_7=a_{on}=a_{op}=a_2=a_7=a_6=0$. It could be realized also highpass, allpass and notch filters by using the resistive summer shown in Fig.4.1.8 [BR4]:



$$\frac{V_o}{V_i} = k_1+k_2\frac{V_{o1}}{V_i}+k_3\frac{V_{o2}}{V_i}$$

$$k_i = \frac{G_{si}}{\sum_{j=1}^{4} G_{sj}} \quad .(i=1,2,3)$$

Fig. 4.1.8

4.2 ACTIVE-R FILTERS FROM LC LADDERS

4.2.1 Filters in literature as special cases of Linear Transformation approach

In this section we are using circuits of Figs. 4.1.6 and 4.1.7 as basic building blocks for simulating LC doubly terminated ladders via the Linear Transformation method. It will be shown that the realization of Soderstand [SO1-3] using the basic units of Fig. 4.1.6 as active integrators simulating the passive Ls and Cs of the ladder can be interpreted as special cases of the Linear Transformation Technique. Con-

sider the 4rth order lowpass polynomial filter shown in Fig. 4.2.1 and the transformation matrices shown below.

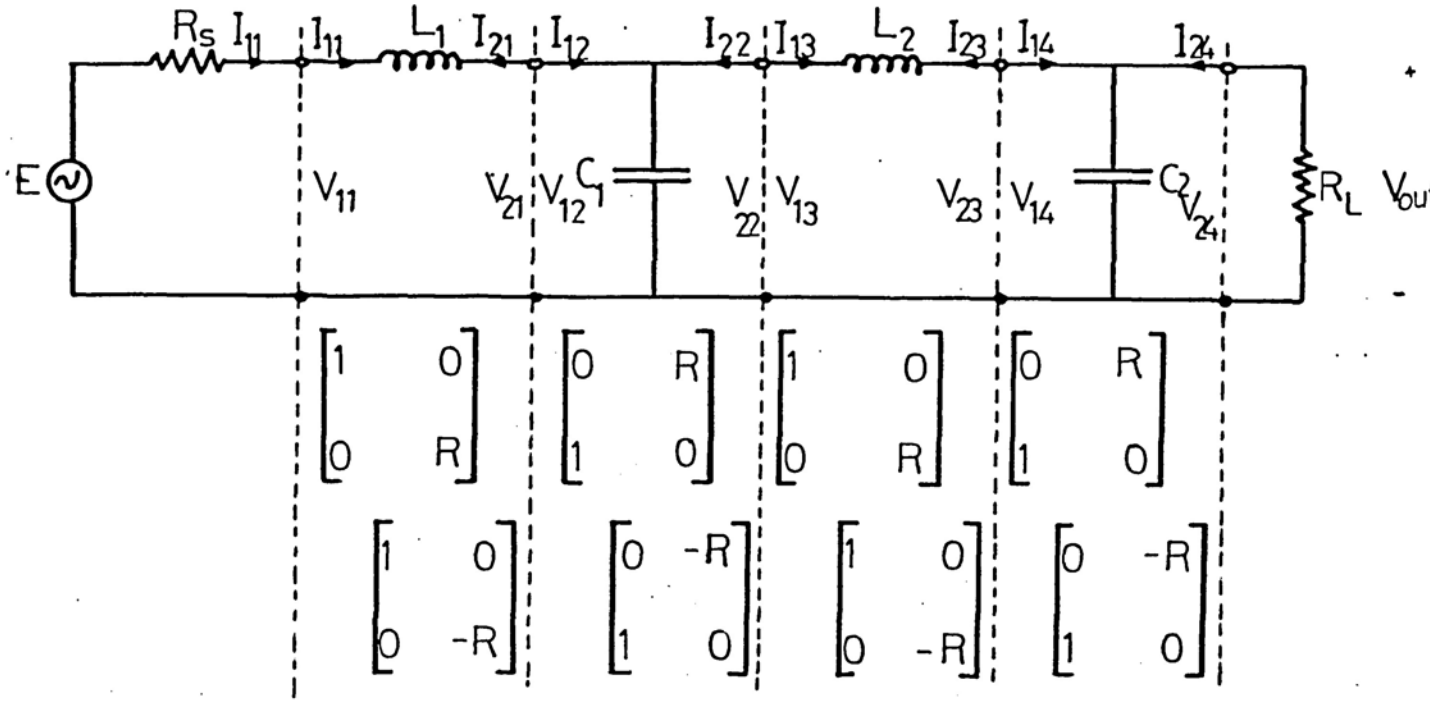


Fig. 4.2.1

This transformation is a self-dual transformation described as case 12 in Table I of Ref. [DI1]. Therefore it is expected that the active structure of the series inductor will be identical with that of the shunt capacitor. The equations for each arm of the ladder, of Xs and Ys in Linear Transformation domain are given by eqns. 4.2.1.

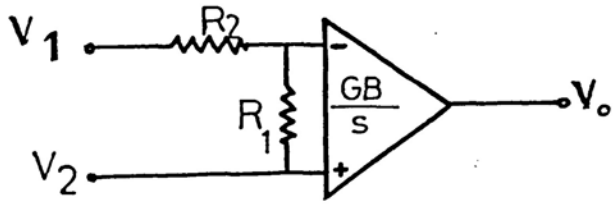
$$X_{11} = E - \frac{R_s}{R} Y_{11}, \qquad Y_{11} = Y_{21} = \frac{R}{sL_1} (X_{11} - X_{21})$$

$$Y_{12} = Y_{22} = \frac{1}{RC_1 s} (X_{12} - X_{22}), \quad Y_{13} = Y_{23} = \frac{R}{sL_2} (X_{13} - X_{23}) \qquad (4.2.1)$$

$$Y_{14} = Y_{24} \quad \frac{1}{RC_4 s} (X_{14} - X_{24}), \quad Y_{24} = V_{24} = Vout, \quad X_{24} = \frac{R}{R_L} Y_{24}$$

It can be selected $R=R_L$ so that $Y_{24} = X_{24} = V_{24} = Vout$. The realization of the input is given by an ideal summer which, however imposes an undesirable pole. To remove this pole we will apply the two-input, one-output model for the

active counterpart of the input termination taken with the first reactive element as always has been done for active-RC filters. However, this approach will give identical structures with those of Soderstand [SO1-3]. Each of the remaining transfer functions in eqns. (4.2.1) is realized by the active block shown in Fig.4.2.2 which is a special case of the block in Fig.4.1.6 if $R_1=R_3=R_4=R_5=R_6=R_7=R_8=0$



$$V_o = \frac{GB}{s}\,\frac{(V_1-V_2)}{1+\frac{R_2}{R_1}} \qquad (4.2.2)$$

Fig. 4.2.2

The structure of the filter is shown in Fig.4.2.3.

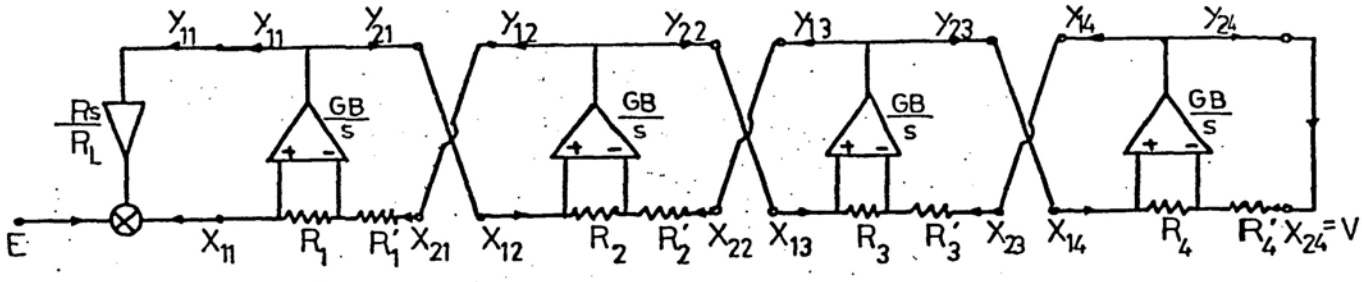


Fig. 4.2.3

The design eqns are, for $R=R_L=1\Omega$:

$$\frac{R_1'}{R_1}=L_1GB-1,\quad \frac{R_2'}{R_2}=C_1GB-1,\quad \frac{R_3'}{R_3}=L_2GB-1,\quad \frac{R_4'}{R_4}=C_2GB-1 \qquad (4.2.3)$$

By taking into account the input termination with the first reactive element (L_1) could be saved the input ideal summer that the above realization requires [HO1]. The input

termination will be simulated by the circuit in Fig.4.2.4, the rest of the structure being the same.

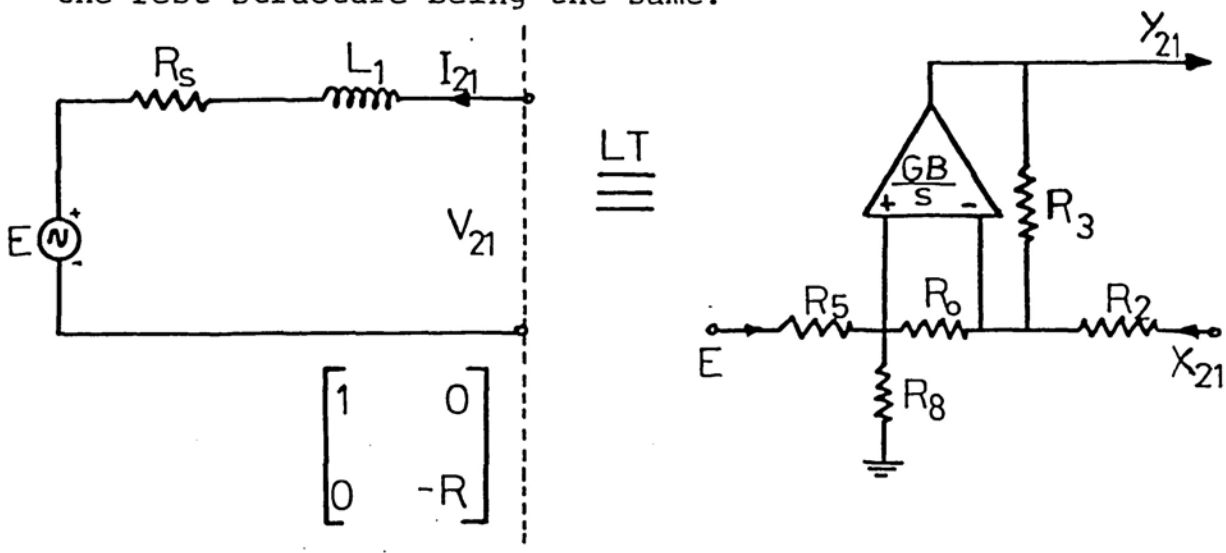


Fig. 4.2.4

The design equations are:

$$\frac{R_8}{R_5+R_8} = \frac{R_3}{R_2+R_3}\ , \quad \frac{R_3}{R_2} = \frac{R_L}{R_s}\ , \quad 1+\frac{R_3}{R_2}+\frac{R_3}{R_o}=\frac{L_1}{R_s}\,GB \qquad (4.2.4)$$

By similar procedure it can be shown that the active realization of Soderstrand [SO1-2] for highpass polynomial filters is a self-compatible transformation where the Linear Transformation matrices are the same for each and every series and shunt arms:

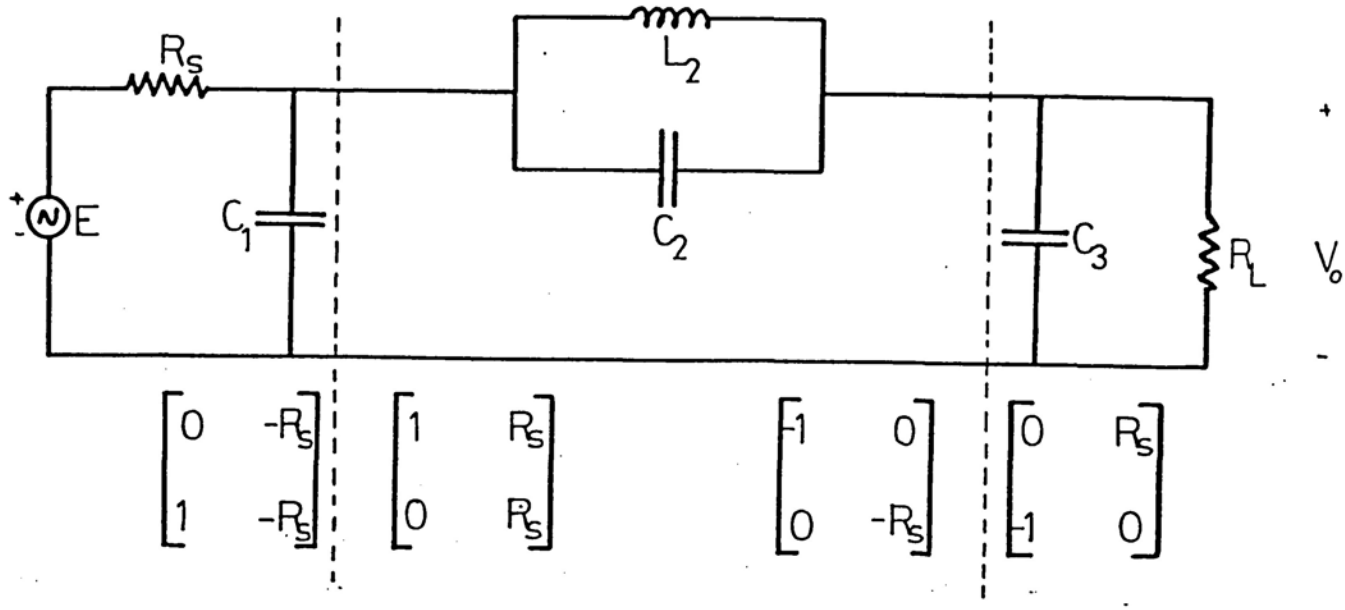
$$\mathcal{A}_1 = \begin{bmatrix} 0 & -R \\ 1 & 0 \end{bmatrix}\ , \quad \mathcal{A}_2 = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix}$$

This transformation is not self-dual as it was in lowpass case. It is possible, as discussed above, take into consideration the input termination in conjuction with the first reactive element to eliminate the ideal summer required at the input.

4.2.2 Active-R Filters with tuned circuits

As it has been shown previously Soderstrand techiques for simulating LC ladder for Active-R filters are special cases of Linear Transformation Technique. However, these sets of Transformations matrices if applied to filters which contain tuned circuits as series or shunt tanks, result in very complicated realizations requiring a number of ideal summers [SO1]. This difficulty could be overcome by specific choice of Linear Transformation matrices, derived under acyclic Linear Transformation to meet the required specifications. For that purpose will be followed techniques described in section 3.2. Consider the third-order Elliptic lowpass filter shown in Fig.4.2.5 and its realization under the specific Transformation matrices shown.

For the realization 5 operational amplifiers are required (with same GB), 4 of which are represented by the one-pole roll off model and the other assumed to be ideal. The summation of the voltage X_{21} into the output Y_{21}-X_{21} of the 1st section is given in the summation point of the second section with the voltages X_{12} and X_{22}, by appropriate choices of resistors R_3 and R_4. The simulation of the series tuned circuit in the middle arm of the passive has been carried out by the circuit of Fig.4.1.7 (with R_5=0) giving a bandpass output V_1. To realize the notch function required by eqn.4.2.5 for the series tuned circuit an ideal summer is embedded giving the notch output V_1' (see also Fig.4.1.8) and the required outputs of the section Y_{12}=Y_{22}. The 3rd section is realized in a similar way to that of the 1rst section. So the design equations of the three sections are:



$$Y_{21}=\frac{1}{1+sR_sG}\,(E-X_{21})+X_{21} \qquad Y_{12}=Y_{22}=\frac{R_sC_2L_2s^2+R_s}{R_sC_2L_2s^2+L_2s+R_s}\,(X_{12}+X_{22}) \qquad Y_{13}=-V_o=-\frac{1}{\frac{R_s}{R_L}+sR_sC_3}\,X_{13}$$

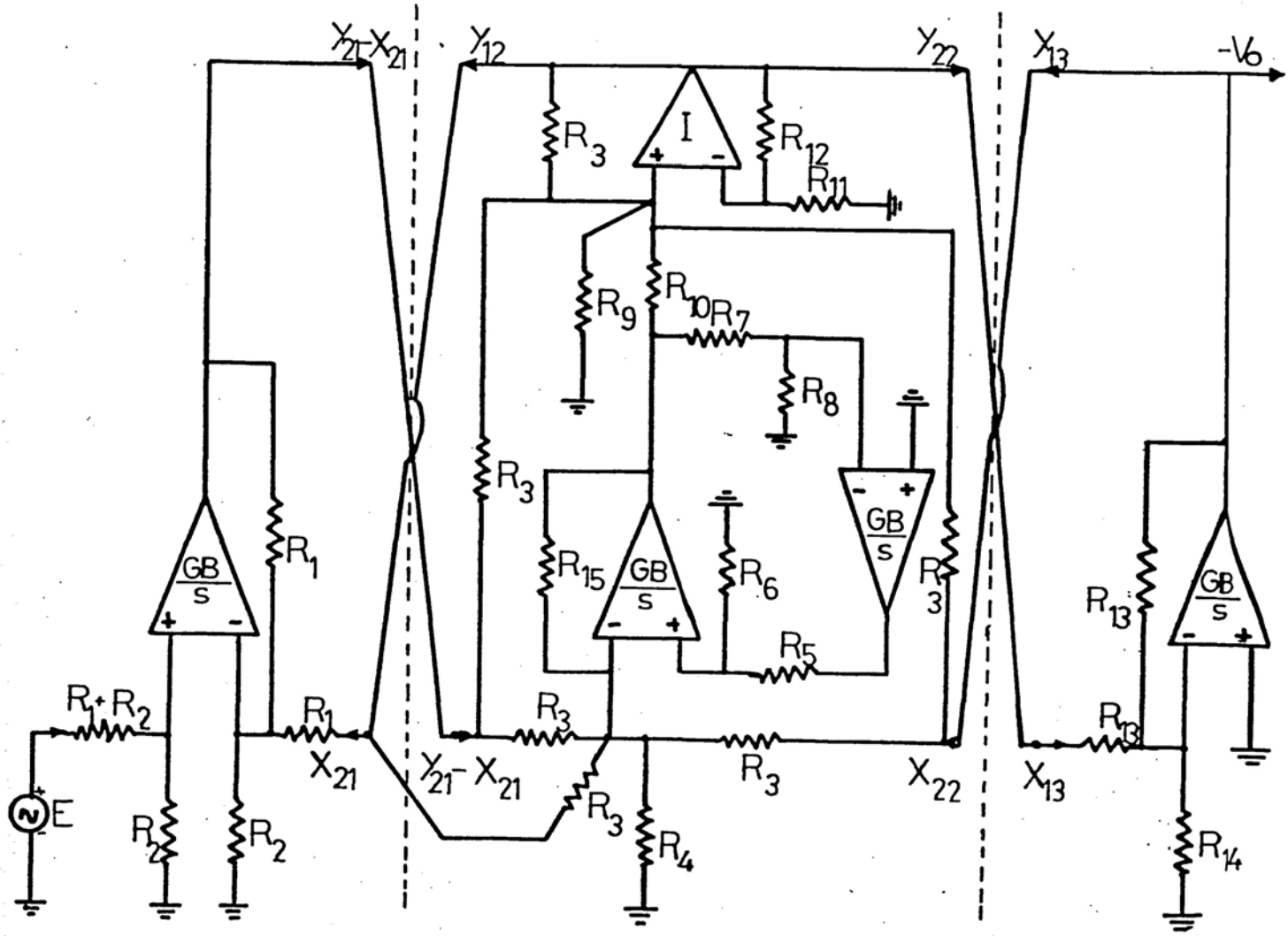


Fig. 4.2.5

input:

$$Y_{21}-X_{21} = \frac{1}{1+s\frac{(2+R_1/R_2)}{GB}}(E-X_{21}) \equiv \frac{1}{1+sR_sC_1}(E-X_{21}) \qquad (4.2.6)$$

Comparing eqns.(4.2.5) and (4.2.6):

$$\frac{R_1}{R_2} = (GB)R_sC_1-2 \qquad (4.2.7)$$

middle element: (Fig.4.1.7, $R_5=0$ and Fig.4.1.8)

$$V_1 = \frac{s(GB)(\alpha_5-\alpha_1)(X_{12}+X_{22})}{s^2+s(GB)\alpha_3+(GB)^2\alpha_6\alpha_2}, \quad V_1' = k_1(X_{12}+X_{22})+k_2V_1 \qquad (4.2.8)$$

where for $R_5=0 \rightarrow \alpha_5=0$

and $\alpha_1 = \frac{G_3}{3G_3+G_4+G_{15}}$, $\alpha_2 = \frac{G_7}{G_7+G_8}$, $\alpha_3 = \frac{G_{15}}{3G_3+G_4+G_{15}}$

$\alpha_6 = \frac{G_5}{G_5+G_6}$, $k_1 = \frac{G_3}{3G_3+G_9+G_{10}}$, $k_2 = \frac{G_{10}}{3G_3+G_9+G_{10}}$

Therefore:

$$V_1' = k_1(X_{12}+X_{22}) + k_2 \left(-\frac{(GB)\alpha_1 s}{s^2+s(GB)\alpha_3+\alpha_6\alpha_2(GB)^2}\right)(X_{12}+X_{22})$$

$$\text{or: } V_1'=k_1\frac{s^2+s(GB)\alpha_3-\frac{k_2}{k_1}\alpha_1(GB)s+\alpha_6\alpha_2(GB)^2}{s^2+s(GB)\alpha_3+\alpha_6\alpha_2(GB)^2}(X_{12}+X_{22}) \qquad (4.2.9)$$

The required outputs Y_{12} and Y_{22} are given by an ideal summer of gain $1+\frac{R_{12}}{R_{11}} = \frac{1}{k_1}$

So the design equations will be for that arm:

$$Y_{12} = Y_{22} = \frac{s^2+\alpha_6\alpha_2(GB)^2}{s^2+s(GB)\alpha_3+\alpha_6\alpha_2(GB)^2}(X_{12}+X_{22}) \equiv \frac{s^2+\frac{1}{L_2C_2}}{s^2+\frac{1}{R_sC_2}s+\frac{1}{L_2C_2}}$$

(4.2.10)

This is true by choosing:

$$\alpha_3 = \frac{k_2}{k_1}\alpha_1$$

For the output termination similarly as the input it could be derived:

$$\frac{R_{13}}{R_{14}} = (GB)R_1C_3-2 \qquad (4.2.11)$$

Band-reject filters could also be obtained by using the circuit in Fig.4.1.7 as basic building block. Consider the 4rth order band-reject filter shown in Fig.4.2.6a and it's realization under the transformation matrices shown below. In that case 4 gain amplifiers and 2 ideal are required for the summation points, to give the notch characteristics and the negative and positive gains in the transfer functions of the two sections, in contrast with Soderstrand circuit described in [SO1], requiring 4 gain amplifiers and 7 ideal summers. In fact in the proposed realization the summer amplifier of the first section, required to give the notch characteristics of the series tuned circuit could be avoided, by summing the voltages X_{21} and E in the non-inverting mode of the gain amplifier of the second section as indicated in Fig. 4.2.6, where BP means bandpass and BS means bandstop t.f. From eqns. (4.2.9) and (4.2.10) and the required transfer functions in Fig. (4.2.6) it must be true:

$$Y_{21} = -\frac{R_{s2}}{R_{s_1}}(E+X_{21} + \frac{-s(GB)\alpha_1}{s^2+s(GB)\alpha_3+(GB)^2\alpha_6\alpha_2}(E+X_{21})) \equiv$$

$$\equiv -\frac{s^2+\frac{1}{L_1C_1}}{s^2+\frac{1}{R_sC_1}S+\frac{1}{L_1C_1}}(E+X_{21}) \qquad (4.2.12)$$

$$\text{or} \quad \frac{R_{s2}}{R_{s_1}} \frac{s^2+s(GB)\alpha_3+(GB)^2\alpha_6\alpha_2-sGB}{s^2+s(GB)\alpha_3+(GB)^2\alpha_6\alpha_2} 1 \equiv \frac{s^2+\frac{1}{L_1C_1}}{s^2+\frac{1}{R_sC_1}s+\frac{1}{L_1C_1}}$$

Therefore, it must be chosen:

$$\alpha_1=\alpha_3 \text{ (or } R_1=R_3), \quad R_{s2}=R_{s1} \text{ and } (GB)^2\alpha_6\alpha_2=\frac{1}{L_1C_1}, \; (GB)\alpha_3=\frac{1}{R_sC_1}$$

$$\text{where } \alpha_6=\frac{G_6}{G_6+G_8}, \; \alpha_2=\frac{G_2}{G_2+G_7}, \quad \alpha_1=\alpha_3=\frac{G_1}{3G_1+G_4}, \quad R_{s1}=R_{s2} \qquad (4.2.13)$$

Eqns.(4.2.13) are the design equations for the first section. For the second section from eqns.(4.2.9) and transfer function in Fig.4.2.6, it must be true:

$$Y_{22}=\frac{1}{\frac{R_{20}}{R_{19}}+1} \cdot k_1 \cdot \frac{s^2+\alpha_6\alpha_2(GB)^2}{s^2+s(GB)\alpha_3+\alpha_6\alpha_2(GB)^2} X_{12} \equiv$$

$$\equiv \frac{R_L}{R_s} \frac{s^2+\frac{1}{L_2C_2}}{s^2+\frac{R_L}{L_2}s+\frac{1}{L_2C_2}} X_{12}$$

$$\text{with } \alpha_3'=a_1\frac{k_2}{k_1}, \quad \alpha_3'=\frac{G_{15}}{G_{15}+G_9+G_{10}}, \quad \alpha_1'=\frac{G_9}{G_{15}+G_9+G_{10}}$$

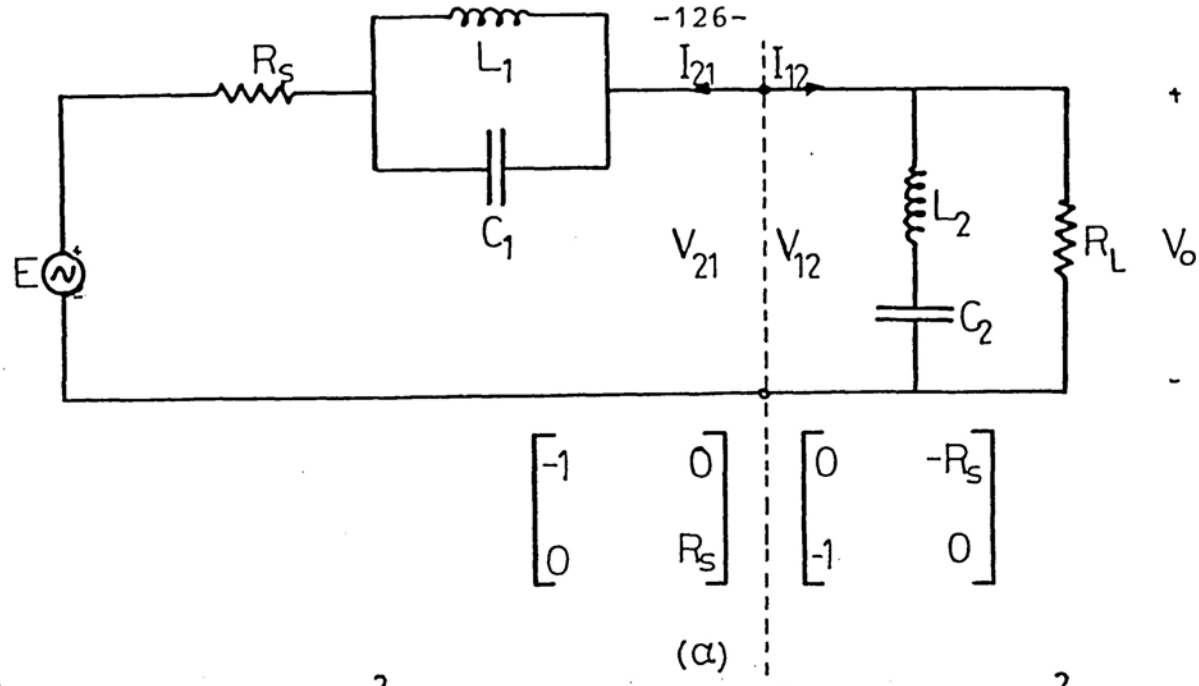
(4.2.14)

$$\alpha_6'=\frac{G_{12}}{G_{12}+G_{11}}, \quad \alpha_2'=\frac{G_{13}}{G_{13}+G_{14}}, \quad k_2=\frac{G_{16}}{G_{16}+G_{17}+G_{18}}, \; k_2=\frac{G_{17}}{G_{16}+G_{17}+G_{18}}$$

Eqns.(4.2.14) give the design eqns. for the second section, from which components values could be evaluated.

4.2.3. Inverted-L Active-R filters

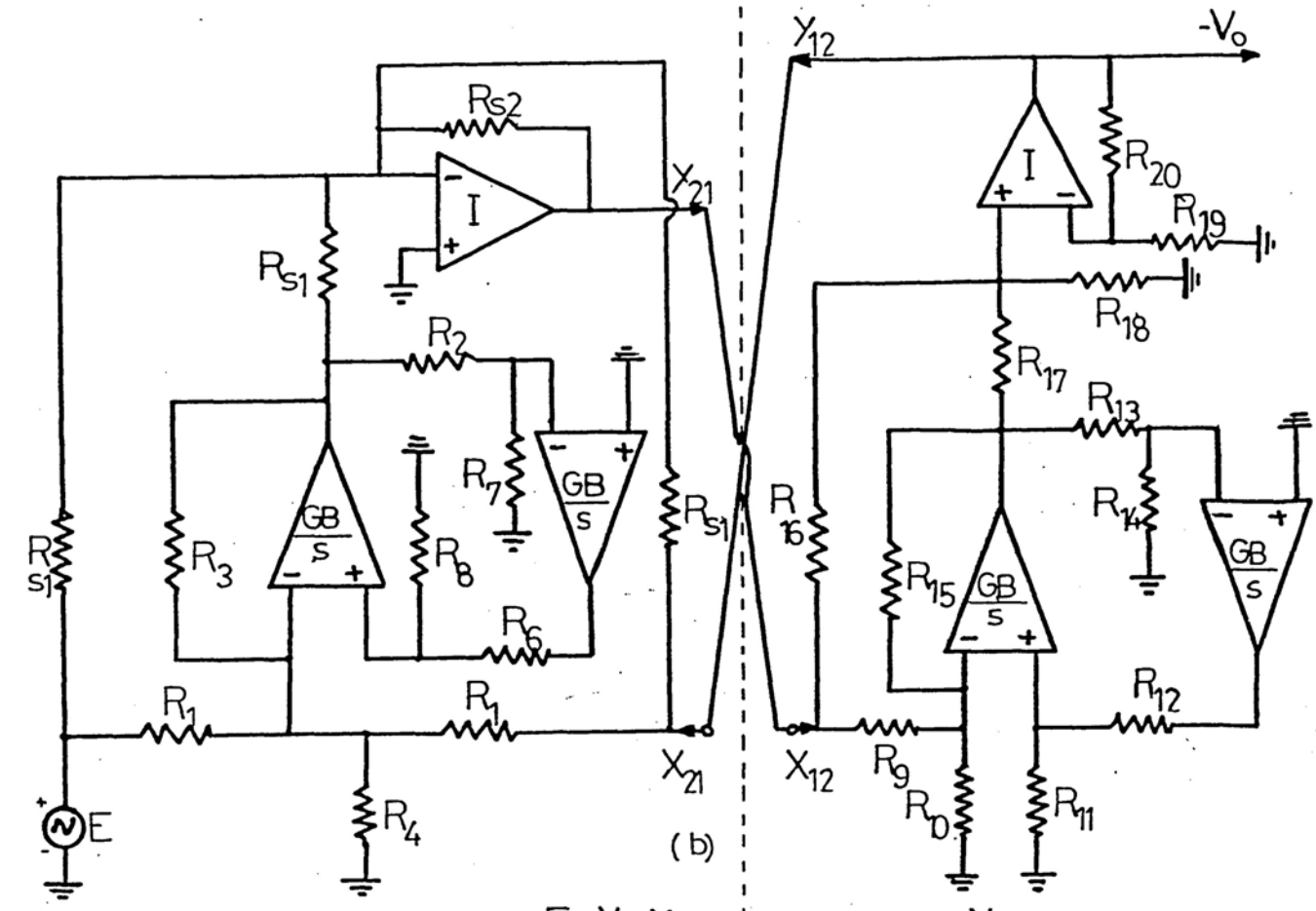
By the term Inverted-L filters it is meant, as discussed in section 2.4, the passive LC ladders consinsing of



$$\begin{bmatrix} -1 & 0 \\ 0 & R_s \end{bmatrix} \quad \begin{bmatrix} 0 & -R_s \\ -1 & 0 \end{bmatrix}$$

(a)

$$Y_{21} = -\frac{R_s C_1 L_1 s^2 + R_s}{R_s C_1 L_1 s^2 + L_2 s + R_s}\,(E + X_{21}) \qquad Y_{12} = -V_o = \frac{R_L}{R_s} \cdot \frac{C_2 L_2 s^2 + 1}{C_2 L_2 s^2 + C_2 R_L s + 1}\, X_{12}$$



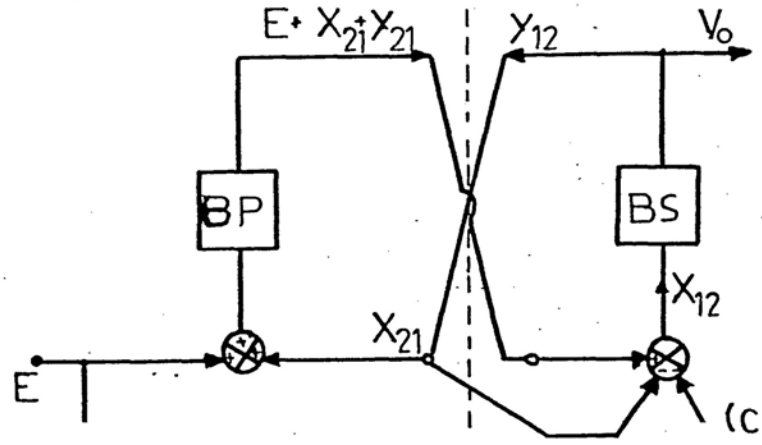
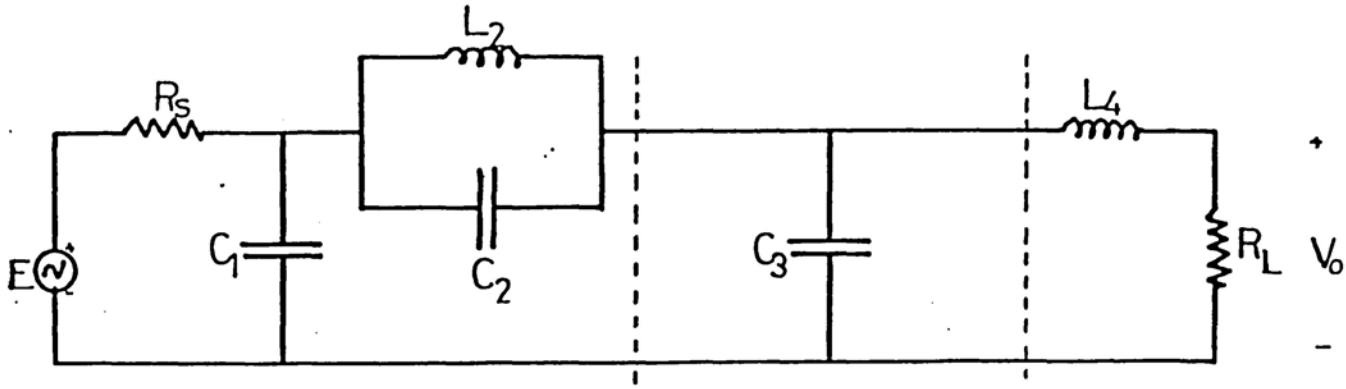


Fig. 4.2.6

sections having the form of the inverted capital letter L. Such filters are the polynomial lowpass and highpass filters and the elliptic filters. In this section it will be tried to design Active-R filters, to simulate Inverted-L elliptic filters in order to minimise the number of operational amplifiers employed. In this case the order of the elliptic filter is less than the number of the reactive elements employed. So there is a way to reduce the number of op-amps being equal the order of the filter. Therefore, the techniques described in section 2.4 will be reused for this purpose. Consider the 4rth order lowpass elliptic filter shown in Fig. 4.2.7a with the linear transformation matrices below. It is taken the series-tank in conjuction with the first reactive element and the input termination. The remainding elements are taken separetely, each at a time. The realization of Active-R counterpart is shown below in Fig.4.2.7b. In this design 4 gain-amplifiers and 1 ideal summer are used, while BR means second-order bandreject transfer function.

4.3 DISCUSSIONS AND CONCLUSIONS

In this chapter Active-R filters has been designed from LC ladders prototypes by using Linear Transformation Technique. It has been proved that circuits simulating the LC ladders' existing in the literature can be interpeted as special cases of a general approach. It has also been presented a number of different methods for designing Active-R filters. Those filters differ from each other by the number of op-amps used, number of ideal summers used, number of resistors etc. A complete study of them will be needed, therefore, for comparison reasons in order to find out the best one from sensitivity point of view, from variations in tempera-



$$\tau = R_s C_1 \qquad \begin{bmatrix} -(1+s\tau) & -\frac{\tau}{C_3} \\ 0 & -R_s \end{bmatrix} \begin{bmatrix} 0 & R_s \\ (1+s\tau) & \frac{\tau}{C_3} \end{bmatrix} \begin{bmatrix} 0 & -R_s \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & R_s \end{bmatrix}$$

$$Y_{21} = \frac{L_2 C_2 s^2 + 1 \,[E + X_{21}]}{s^2 (2C_1 L_2 + C_2 L_2) + \frac{L_2}{R_s} s + 1 + \frac{C_1}{C_3}} \qquad Y_{22} = \frac{1}{R_s C_3 s} (X_{22} - X_{12})$$

$$Y_{12} = Y_{22} + \frac{C_1}{C_3} X_{22} \qquad Y_{13} = \frac{R_s}{R_L} V_o = - \frac{R_s X_1}{(L_4 s + R}$$

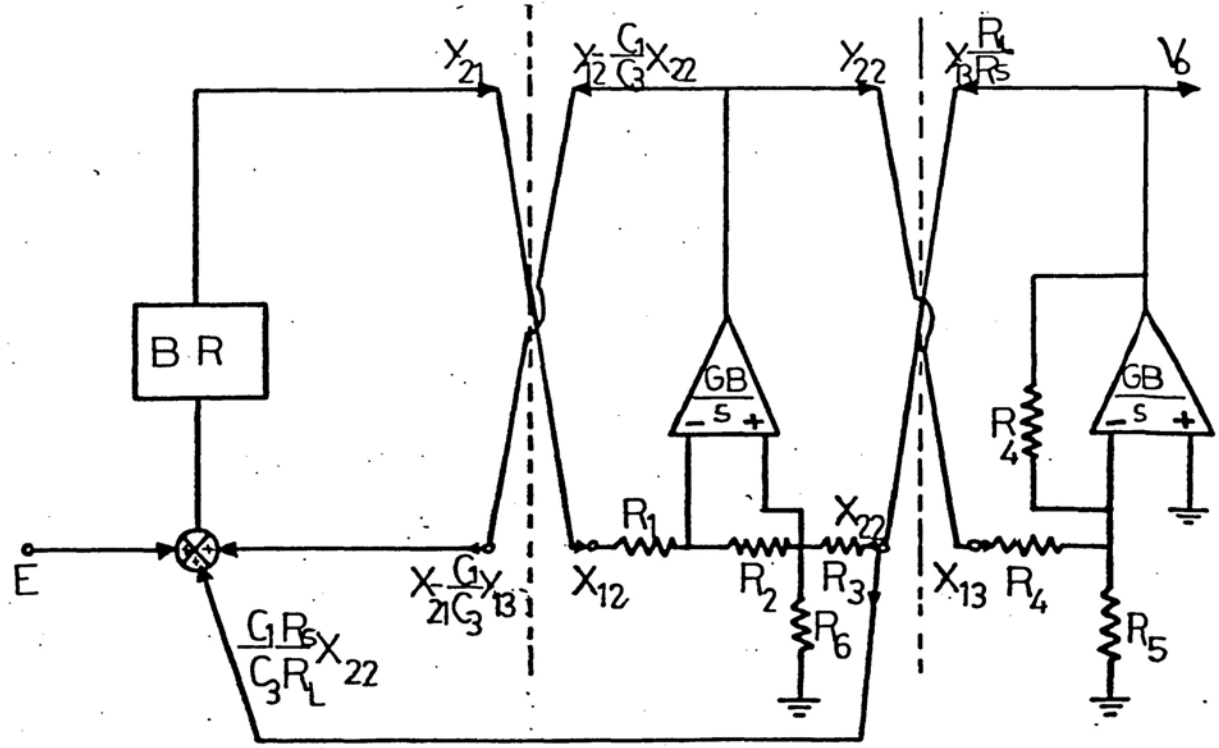


Fig. 4.2.7

ture or dc voltage point of view and so forth.

The behaviour of Active-R filters deviates in a number of ways from the ideal operation described by the equations in section 4.1. All of these effects can be traced to operational amplifier imperfections, and can in our opinion not be significantly affected by improved filter design methods. Nevertheless, the nonideal operation should be clearly understood so that, as far as possiple, corrective measures can be taken. The nonideal operation of the op-amps are due mainly to temperature drifts, excess phase of the op-amp, limited dynamic range, etc. In order to minimize or complete eliminate those nonideal operations, it has been proposed by many designers the method of phase-lock stabilization technique for compensating the temperature drifts, the predistortion technique of resistive tuning for correcting ω_o, Q of 2nd order block in the presence of the excess phase of the op-amp and others. A complete review of those techniques could be found in Ref. [BR4].

Having in mind all those compensation schemes, Active-R filters will be the ideal alternative to Active-RC filters in high-frequency signal processing with integrated circuits.

CHAPTER 5

SENSITIVITY OF THE LTA STRUCTURES

5.1 GENERAL THEORY

5.2 LINEAR TRANSFORMATION AS A LINEAR MAPPING

5.2.1 Forward and Inverse Linear Transformation

5.2.2 Analysis Procedure

5.2.3 Synthesis Procedure

5.3 CONCLUSIONS

CHAPTER 5

SENSITIVITY OF LTA STRUCTURES

It has been given, up to now in this thesis different LTA structures that simulate doubly resistive terminated ladder LC filters. In this section will be discussed the sensitivity performance of those structures and various methods will be proposed of compensation for minimizing the sensitivities of the gain due to component variations. This proposal for minimization will result from choosing the elements of the transformation matrices accordingly, as to substitute the changes of various components of the specific filter under consideration.

5.1 GENERAL THEORY

As it has be seen previously one has a choice of many circuit realizations for a prescribed filter function. Given perfect components there would be little difference among the various realizations. In practice, real components will deviate from their nominal values due to their initial tolerances associated with their manufacture; the enviromental effects of temperature and humidity; and chemical changes due to the aging of the components. As a consequence, the performance of the built filters will differ from the nominal design. One way to minimize this difference is to choose components with small manufacturing tolerances, and with low temperature, aging and humidity coefficients. However this approach will usually result in a circuit that is more expensive than is necessary. A more practical solution is to select a circuit that has a low sensitivity to these

changes. The lower the sensitivity of the circuit, the less will its performance deviate because of element changes. Stated differently, the lower the sensitivity the less stringent will the requirements on the components be and, accordingly, the circuit becomes cheaper to manufacture. For this reason, sensitivity is one of the more important criteria used for comparing various circuit realizations. There are many different types of sensitivities which will be given below. For the biquadratic filter function,6 different sensitivities could be derived in terms of the parameters ω_p, ω_z, Q_p, Q_z, K and gain $G(\omega)$ of eqn.(5.1.1):

$$T(s) = K \frac{s^2 + \frac{\omega_z}{Q_z} s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2} \qquad (5.1.1)$$

Let first consider the sensitivity of the pole frequency ω_p to a change in a resistor R, embedded in the realization of transfer function T(s).

Pole sensitivity is defined as the per-unit change in the pole frequency, $\frac{\Delta\omega_p}{\omega_p}$, caused by a per-unit change in the resistor, $\frac{\Delta R}{R}$. Mathematically:

$$S_R^{\omega_p} = \lim_{\Delta R \to 0} \frac{\frac{\Delta\omega_p}{\omega_p}}{\frac{\Delta R}{R}} = \frac{R}{\omega_p} \cdot \frac{\partial \omega_p}{\partial R} = \frac{\partial (\ell n \omega_p)}{\partial (\ell n R)} \qquad (5.1.2)$$

Note that the cost of manufacturing a component is a function of the percentage change $(100 \times \frac{\Delta R}{R})$ rather than the absolute change (ΔR) of the component. For this reason it is desirable to measure sensitivity in terms of the relative changes in components, as is done in (5.1.2). The sensitivities of the parametes ω_z, Q_p, Q_z and K to any element of the network are defined in a similar way:

$$S_C^{\omega_p} = \frac{C}{\omega_p} \cdot \frac{\partial \omega_p}{\partial C} \ , \ S_R^{Q_p} = \frac{R}{Q_p} \cdot \frac{\partial Q_p}{\partial R} \ , \quad S_R^K = \frac{R}{K} \cdot \frac{\partial K}{\partial R} \ , \text{ etc.} \qquad (5.1.3)$$

The gain in dB of t.f T(s) is given by:

$$G(\omega) = 20\log_{10}|T(j\omega)| \qquad (5.1.4)$$

The gain sensitivity is defined as the change in gain in db due to a per-unit change in an element (or parameter x):

$$S_x^{G(\omega)} = \frac{\partial G(\omega)}{\frac{\partial x}{x}} = x \frac{\partial G(\omega)}{\partial x} \text{ dB} \qquad (5.1.5)$$

or

$$S_x^{G(\omega)} = \frac{\partial(20\log_{10}|T(j\omega)|)}{\partial x} = 8.686 S_x^{|T(j\omega)|} \qquad (5.1.6)$$

Therefore, the sensitivity of the gain function $S_x^{G(\omega)}$ is proportional to the per-unit sensitivity of the magnitude function, $S_x^{|T(j\omega)|}$. The per-unit change in the element x, is known as the variability of x and represented by: $V_x = \frac{\Delta x}{x}$.

often of interest is the multi-element deviations of all the circuit elements x_j and the deviation of one paretemer due to those deviations is examined. The change $\Delta\omega_p$ for example may be obtained by expanding in a Taylor series for small deviations Δx_1, Δx_2,...,Δx_m:

$$\Delta\omega_p = \frac{\partial \omega_p}{\partial x_1} \Delta x_1 + \frac{\partial \omega_p}{\partial x_2} \Delta x_2 + \ldots + \frac{\partial \omega_p}{\partial \omega_m} \Delta x_m + \text{second and higher-order terms} \qquad (5.1.7)$$

where m is the total number of elements in the circuit. Since the changes in the components Δx_j are assumed to be small, the second and higher-order terms can be ignored. Thus:

$$\Delta\omega_p \cong \sum_{j=1}^{m} \frac{\partial\omega_p}{\partial x_j} \Delta x_j = \sum_{j=1}^{m} S_{x_j}^{\omega_p} V_{x_j} \omega_p \qquad (5.1.8)$$

It can be shown [DA1] that in the LC ladders, at the frequencies of maximum power transfer points where the power P, defined as the ratio of the power delivered into the load resistor of the filter over the available maximum power from the source, takes the value of unity [OR1] and:

$$\frac{\partial P}{\partial x} = 0 \rightarrow S_x^{|T(j\omega)|} = \frac{x}{|T(j\omega)|} \cdot \frac{\partial |T(j\omega)|}{\partial x} = 0 \qquad (5.1.9)$$

for any element L or C of the passive filter.
The following relationships are also true:

$$S_{R_s}^{|T(j\omega)|} = \frac{R_s}{|T(j\omega)|} \cdot \frac{\partial |T(j\omega)|}{\partial R_s} = -1/2$$

$$S_{R_L}^{|T(j\omega)|} = \frac{R_L}{|T(j\omega)|} \cdot \frac{\partial |T(j\omega)|}{\partial R_L} = \frac{1}{2} \qquad (5.1.10)$$

For the active RC filters, it can be shown that the dimensional homogeneity property exists [DA1] as:

$$\sum S_{R_i}^{Q_p} = \sum S_{C_i}^{Q_p} = 0$$

$$\sum S_{R_i}^{\omega_p} = \sum S_{C_i}^{\omega_p} = -1 \qquad \forall \text{ element } i. \qquad (5.1.11)$$

In the following it will be tried to outline an indirect way relating the sensitivity formulae in terms of the parameters of the transformations so as to establish optimum, if possible, sensitivity performance structures within the LTA concept.

5.2 LINEAR TRANSFORMATION AS A LINEAR MAPPING

Any Linear Transformation is a linear mapping from a domain W to a domain Z, where W≡Z, that is a linear mapping from a vector space W into itself. In this case every vector ξ of W is transformed into another vector $\Phi(\xi)$ in the same space W, where Φ is the Linear Transformation.
An n-port network defines an n-dimensional vector space W over the field of complex numbers C, that is over the complex frequency s.

A basis for the n-dimensional vector space should be defined which must consists of n-vectors, linear and independent. Since the port variables of the network are the currents and voltages at each port, it has to be found the basis of the vector space out of the set of 2n vectors $(V_1,...V_n, I_1,...I_n)$ using any combination. It could be found that the only linear independent sets, except of course if some sets be made externally independent as discussed in section 2.3.5, are the sets: $\{V_1,...V_n\}$ and $\{I_1,...I_n\}$. All the other possible sets are dependent, because V_i and I_i of the port-i are related by the Ohm's Law.

But it is of interest to study n-port networks which are ladders and could be formed by cascading n 2-port subnetworks, such as that shown in Fig.5.2.1.

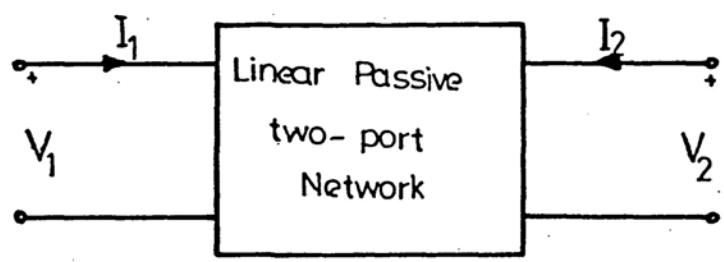


Fig. 5.2.1

The base of the 2-dimensional vector space defined by this 2-port sub-network of the ladder should then be $\{V_1, V_2\}$, over the field of complex frequency s, because $V_1(s)$ and $V_2(s)$ are linear independent vectors. The set $\{V_1, V_2\}$ could be taken as the input vector of the network and the set $\{I_1, I_2\}$ as the output vector. These two sets are related by the admittance matrix Y as:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5.2.1}$$

Because the V-I domain for the 2-port network is described in the 2-dimensional vector space W by the vectors $\begin{bmatrix} V_i \\ I_i \end{bmatrix}$, i=1,2. These vectors have to be expressed in terms of the base of the vector space W. Therefore:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ y_{11} & y_{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{5.2.2}$$

The X-Y domain in the same 2-dimensional vector space W, is defined by the vectors $[X_i \quad Y_i]^t$, i=1,2 which are the linear transformations of the left and right port variable vectors:

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} \triangleq \mathcal{A}_1 \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ y_{11} & y_{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

and (5.2.3)

$$\begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \triangleq \mathcal{A}_2 \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Eqns.(5.2.3) give the expressions for the new transformed variables in terms of the base $[V_1 \quad V_2]^t$ of the vector space W. As discussed before the set $[X_1, X_2]$, being independent from $[Y_1, Y_2]$, could be chosen as the new base of the vector space W, giving the new set $[Y_1, Y_2]$. Thus:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \gamma_1+\delta_1 y_{11} & \delta_1 y_{12} \\ \delta_2 y_{21} & \gamma_2+\delta_2 y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} =$$

$$= \begin{bmatrix} \gamma_1+\delta_1 y_{11} & \delta_1 y_{12} \\ \delta_2 y_{21} & \gamma_2+\delta_2 y_{22} \end{bmatrix} \begin{bmatrix} \alpha_1+\beta_1 y_{11} & \beta_1 y_{12} \\ \beta_2 y_{21} & \alpha_2+\beta_2 y_{22} \end{bmatrix}^{-1} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (5.2.4)$$

$$\text{or} \quad \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \sum \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (5.2.5)$$

The new base of W is related to the old one by eqn.(5.2.6):

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \alpha_1+\beta_1 y_{11} & \beta_1 y_{12} \\ \beta_2 y_{21} & \alpha_2+\beta_2 y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (5.2.6)$$

The LT description of (5.25) could be derived by the alternative way, as already mentioned, using the chain matrix of the passive 2-port as:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{d}{b} & -\frac{ad-bc}{b} \\ \frac{1}{b} & -\frac{a}{b} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (5.2.7)$$

where:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix}^{-1}, \quad T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Eqns.(5.2.7) and (5.2.4) are equivalent as it can be seen by substituting in (5.2.4) the equivalents of the elements in the admittance matrix Y with those of chain matrix T.

5.2.1 Forward and Inverse Transformation

It has been observed in the previous section, that the Linear Transformation of port variables of a 2-port network is nothing else but a change of base of the 2-dimensional vector space W defined by the 2-port network that is, base $\{V_1, V_2\}$ chances into $\{X_1, X_2\}$. The dependent vectors which are defined are called currents I_i, i=1,2 in the V-I domain with base $\{V_1, V_2\}$ and are the variables $\{Y_1, Y_2\}$ in the X-Y domain with base $\{X_1, X_2\}$. The above described transformation is called forward linear transformation Φ:

$$W_V \xrightarrow{\Phi} W_X$$

It could define the inverse linear transformation Φ^{-1} of W with base X, to itself with base V:

$$W_X \xrightarrow{\Phi^{-1}} W_V$$ Therefore:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{Y_{ij}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \underset{\Phi^{-1}}{\overset{\Phi}{\rightleftarrows}} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} K & L \\ M & N \end{bmatrix}}_{\sum} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (5.2.8)$$

It is known that each transformation could be represented by a matrix, e.g. Φ. The matrix Φ is called nonsingular

if $\Phi\ \Phi^{-1} = I$, where I is the unit transformation. With the above description, variations in Σ matrix could change the admittance matrix Y_{ij} in V-I domain. Suppose that Σ is changing to $\Sigma' = \Sigma+P$.

In that case:

$$Y_{ij} = \Phi^{-1}(\Sigma') = \Phi^{-1}(\Sigma+P) = \Phi^{-1}(\Phi(Y_{ij})+P) = Y_{ij}+\Phi^{-1}(P) \quad (5.2.9)$$

Eqns.(5.2.9) give the new description of the passive 2-port that results from the variations in the active structure. This description will be used only for mathematical formulation of the approach, since there is an easier description through the chain matrix of the network, as always been the case in the present work.

In order to find the chain-matrix description of the new passive network, resulting from the variations of the active structure eqn.(5.2.7) should be recalled:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \frac{d}{b} & -\frac{ad-bc}{b} \\ \frac{1}{b} & -\frac{a}{b} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (5.2.10)$$

By changing the voltage transfer ratios to K´,L´,M´,N´ the dependent transformed vector in X-Y domain is given by:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} K' & L' \\ M' & N' \end{bmatrix} \begin{bmatrix} X_2 \\ X_2 \end{bmatrix} = \begin{bmatrix} \frac{d'}{b'} & -\frac{a'd'-b'c'}{b'} \\ \frac{1}{b'} & -\frac{a'}{b'} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (5.2.11)$$

The chain matrix of the new passive filter after taking the inverse transformation of eqn.(5.2.11) will be:

$$T' = \mathcal{A}_1^{-1} \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \mathcal{A}_2 \qquad (5.2.12)$$

Assuming that the elements a,b,c,d are perturbed by the ammounts Δa, Δb, Δc, Δd, respectively, giving a', b', c', d':

$$T' = \mathcal{A}_1^{-1} \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \mathcal{A}_2 = \mathcal{A}_1^{-1} \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \right) \mathcal{A}_2 \quad \text{or}$$

$$T' = \mathcal{A}_1^{-1} \left(\mathcal{A}_1 T \mathcal{A}_2^{-1} + \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \right) \mathcal{A}_2 = T + \mathcal{A}_1^{-1} \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \mathcal{A}_2$$

$$T' = T + \mathcal{A}_1^{-1} \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \mathcal{A}_2 \qquad (5.2.13)$$

Eqn. (5.2.13) gives an alternative description, than that of eqn.(5.2.9) for the new passive network. This relation could be seen in two ways. For specific choices of the perturbations Δa, Δb, Δc, Δd, eqn.(5.2.13) gives the description of the two-port after taking those perturbations under consideration. Alternatively, for predetermined description of the chain matrix T', the perturbations could be calculated as to have specific values in terms of the elements of matrices $\mathcal{A}_1$ and $\mathcal{A}_2$. It will be referred to those two alternative ways as analysis and synthesis procedure respectively. In the analysis case, for given transformation matrices $\mathcal{A}_1$, $\mathcal{A}_2$ and given perturbations Δa, Δb, Δc, Δd it will be determined the description of the network by T'. Thus a chain matrix for the series impedance Z, under those conditions,

could be modified into a chain matrix of either a series impedance Z´ or an inverted-L section. For Z=sL it could be result that Z´=sL´+R or to be an inverted-L section with admittance G and impedance sL´´. In the synthesis case, for specific choice of T´, normally to be of the same form as T, it could be selected transformation matrices $\mathcal{A}_1$ and $\mathcal{A}_2$ and perturbations Δa, Δb, Δc, Δd to meet this requirement. So, e.g., for shunt lossless admittances it could be required shunt lossless admittances, after the perturbations.

5.2.2 Analysis procedure

Suppose that the 2-port network is consisting of a series inductor of value L as in Fig. 5.2.2.

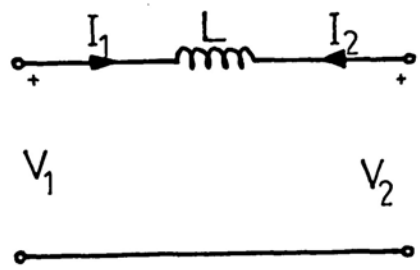


Fig. 5.2.2

Let's transform this series inductor under the set:

$$\mathcal{A}_1 = \begin{bmatrix} 1 & R \\ 0 & -R \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -1 & 0 \\ 0 & R \end{bmatrix}$$

The LT description will be:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{1+\frac{L}{R}s} & -\frac{1}{1+\frac{L}{R}s} \\ -\frac{1}{1+\frac{L}{R}s} & -\frac{1}{1+\frac{L}{R}s} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad (5.2.14)$$

In that case:

$$K(s) = L(s) = M(s) = N(s) = -\frac{1}{1+s\frac{L}{R}} \quad \text{and}$$

$$a=-1, \quad d=1, \quad c=0, \quad b=-1-\frac{L}{R}s$$

The realization of (5.2.14) for the active-RC case with ideal op-amp is given in Fig.5.2.3a. For the active-R case is given in Fig.5.2.3b.

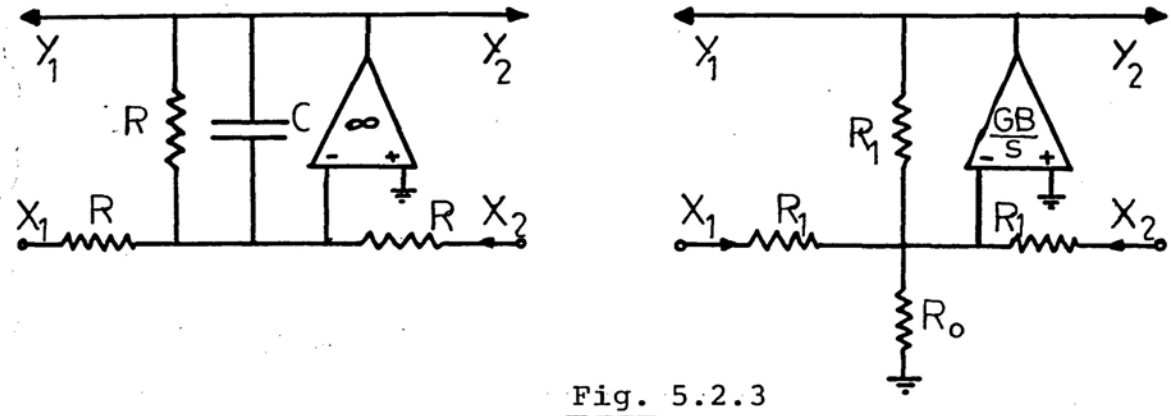


Fig. 5.2.3

where for Fig.5.2.3b:

$$\frac{R_1}{R_o} = (GB)\frac{L}{R} - 3 \qquad (5.2.15)$$

Let's assume first the active-RC case. By changing the feedback resistor from value R to R_1 the per-unit change is: $V_R = \frac{|\Delta R|}{R} = \frac{|R-R_1|}{R}$ and the sensitivity of transfer function in (5.2.14):

$$S_R^{|Y_1|} = S_R^{(1+(\frac{L}{R}\omega)^2)^{-1/2}} = -\frac{1}{2} S_R^{1+(\frac{L}{R}\omega)^2} =$$

$$= -\frac{1}{2}\,\frac{1 \cdot S_R^1 + \frac{L^2}{R^2}\omega^2 S_R^{\frac{L^2\omega^2}{R^2}}}{1+\frac{L^2\omega^2}{R^2}}$$

$$\text{or } S_R^{|Y_1|} = -\frac{1}{2}\,\frac{\frac{L^2}{R^2}\,\omega^2(S_R^{L^2\omega^2}-S_R^{R^2})}{1+\frac{L^2\omega^2}{R^2}} = -\frac{1}{2}\,\frac{\frac{L^2}{R^2}\,\omega^2(0-2)}{1+\frac{L^2\omega^2}{R^2}} \text{ or}$$

$$S_R^{|Y_1|} = \frac{L^2\omega^2}{R^2+L^2\omega^2} < 1 \qquad (5.2.16)$$

By that change of resistor R, eqn.(5.2.14) changes to:

$$Y_1=Y_2=-\frac{1}{\frac{R}{R_1}+\frac{L}{R}\,s}(X_1+X_2) \text{ so } K'(s)=L'(s)=M'(s)=N'(s)=-\frac{1}{\frac{R}{R_1}+\frac{L}{R}\,s}$$

and $b'=-(\frac{R}{R_1}+\frac{L}{R}\,s)$, $a'=-1$, $d'=1$, $c'=0$, or

$$\Delta a=0, \quad \Delta b=1-\frac{R}{R_1}, \quad \Delta c=0, \quad \Delta d=0.$$

Eqn.(5.2.13) gives:

$$T' = \begin{bmatrix} 1 & -Ls \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & R \\ 0 & -R \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1-\frac{R}{R_1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & R \end{bmatrix} \text{ or}$$

$$T' = \begin{bmatrix} 1 & -Ls \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{R^2}{R_1} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -Ls-\frac{R^2}{R_1}+R \\ 0 & -1 \end{bmatrix} \qquad (5.2.17)$$

which is a chain matrix of a lossy inductor, shown in Fig. 5.2.4.

For $R=R_1$, $Z=Ls$, for $R_1<R$, $\frac{R^2}{R_1}-R>0$, $Z=Ls+\frac{R^2}{R_1}-R$

and for $R_1>R$, $\frac{R^2}{R_1}-R<0$, $Z=Ls+\frac{R^2}{R_1}-R$

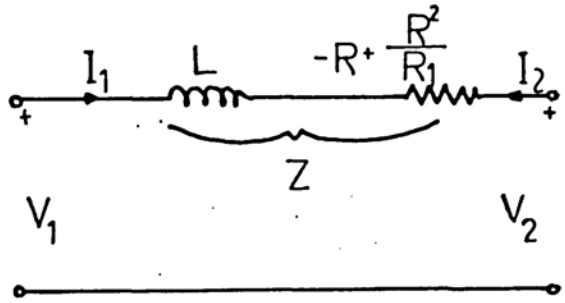


Fig. 5.2.4

Similarly by changing the input resistors from R to R_2 it is found a lossy inductor with value $L'=L+\frac{L}{R}(R-R_2)$, $R_L=R-R_2$. By changing the value of the capacitor $C=\frac{L}{R^2}$ to C_1 it can be found a lossless inductor with value $L''=2L-C_1R^2$. Therefore the change of the frequency dependent element, such as the capacitor does not change the lossless behaviour of the initial reactive element. Only its value is changed. The only changes are due to the resistors in the active structure. Now, if the set of transformation matrices is been changed to:

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -R \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -1 & 0 \\ 0 & R \end{bmatrix}$$

it could be found that the LT description is given by Fig. 5.2.5.

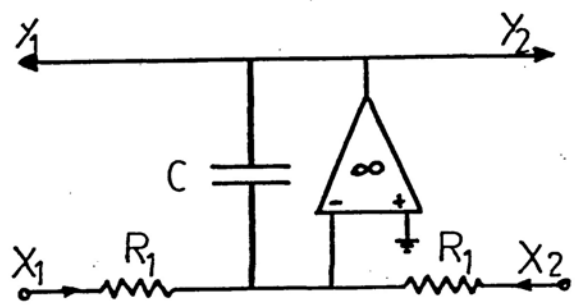


Fig. 5.2.5

By changing the capacitor value C to C_1 it is found as before no changes of the losslessness of the inductor, only the value

changes to $L'=L\frac{C_1}{C}$. By changing one or both of the resistors R_1 to value R_2 it is also found no changes of the losslessness as well, the value now becoming $L''=L\frac{R_2}{R_1}$. These results are expected to be so, since both the capacitive value C and the resistive value R_1 are proportional to the inductive value L, since $CR_1=\frac{L}{R}$.

Therefore, the choice of transformation matrices is crucial for the sensitivities of the overall structures.

Up to now, it has been assumed that the gain of the operational amplifier is infinite. In fact it is not true and it is frequency dependent:

$$A(s) = \frac{GB}{s+\omega_a} = \frac{GB}{s} \text{ for } \omega >> \omega_a$$

Having this in mind the transfer function in Fig.(5.2.5) becomes:

$$Y_1 = Y_2 = -\frac{(X_1+X_2)}{sR_1C(1+\frac{s}{GB})} \qquad (5.2.18)$$

for $(GB) >> \frac{1}{R_1C}$ as usually is the case.
In order to obtain a transfer function like that, it has to be imposed frequency dependent elements in the transformation matrices and the finite gain-band width product as well. The set of transformation matrices:

$$\mathcal{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{ks} \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} -1 & -L(GB) \\ 0 & \frac{1}{ks} \end{bmatrix}$$

give for the inductor:

$$Y_1 = Y_2 = -\frac{(X_1+X_2)}{skL(GB)(1+s\frac{1}{(GB)}} \qquad (5.2.19)$$

Comparing eqns.(5.2.19) with (5.2.18) it must be true that

$$R_1 C = kL(GB) \qquad (5.2.20)$$

$$\text{with} \quad GB >> \frac{1}{R_1 C}$$

where k is a constant number with dimensions of capacitance. Therefore, by the specific choice of transformation matrices, the active-RC structure for the inductor simulation will be insensitive to either the passive elements or the active ones. This specific choice will of course affect the adjacent elements of the filter, because the presence of frequency dependent elements in the transformation matrices could be transformed with the compatibility relationship. Let's assume that the previous arm to the inductor is a shunt capacitor C_1 with the input termination and input resistor R_s as in Fig.5.2.6.

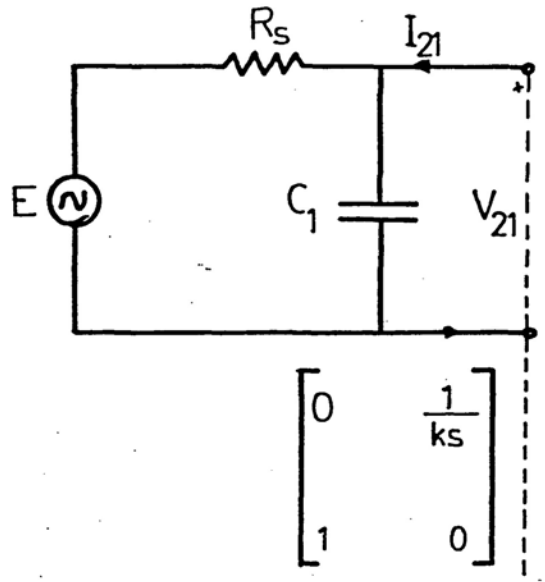


Fig. 5.2.6

It could be selected the above matrix for the input termination in order the compatibility relationship to be held. For this matrix the LT structure will be described by the equation:

$$Y_{21} = \frac{\frac{1}{ks}}{\frac{1}{ks}(1+sC_1R_s)} E + \frac{R_s}{\frac{1}{ks}(1+sC_1R_s)} X_{21} \text{ or}$$

$$Y_{21} = \frac{1}{1+sC_1R_s}(E+skR_sX_{21}) \qquad (5.2.21)$$

This eqn. could be written in alternative form by substituting eqn.(5.2.19) with it. From cross-cascading connection it could be obtained $X_{21}=Y_{12}=Y_1$ of the next arm (the inductor arm).

So:

$$X_{21} = Y_1 = -\frac{X_1+X_2}{skL(GB)(1+s\frac{1}{(GB)})} \rightarrow skR_sX_{21} = -\frac{(X_1+X_2)R_s}{L(GB)(1+\frac{s}{GB})}$$

therefore:

$$Y_{21} = \frac{1}{1+sC_1R_s}(E - \frac{(X_1+X_2)R_s}{L(GB)(1+\frac{s}{GB})}) \qquad (5.2.22)$$

It could either be realized Y_{21} by eqn.(5.2.21) or (5.2.22) as shown in Fig.5.2.7a and 5.2.7b respectively.
In Fig. 5.2.7b one amplifier isabuffer with gain 1 and another is inverter with gain 2, while the third is a buffer again. So this circuit can go up to very high frequencies.
In Fig.5.2.7a, the op-amp is buffer, so this circuit is good for high frequencies and since $RC=R_sC_1$, the sensitivities of the structure to variations in R or C are very low, according to Orchard's principle.
Let's be assumed now that the next arm to the inductor is a shunt capacitor C_2 with the load termination R_L. With this termination could be chosen the matrix:

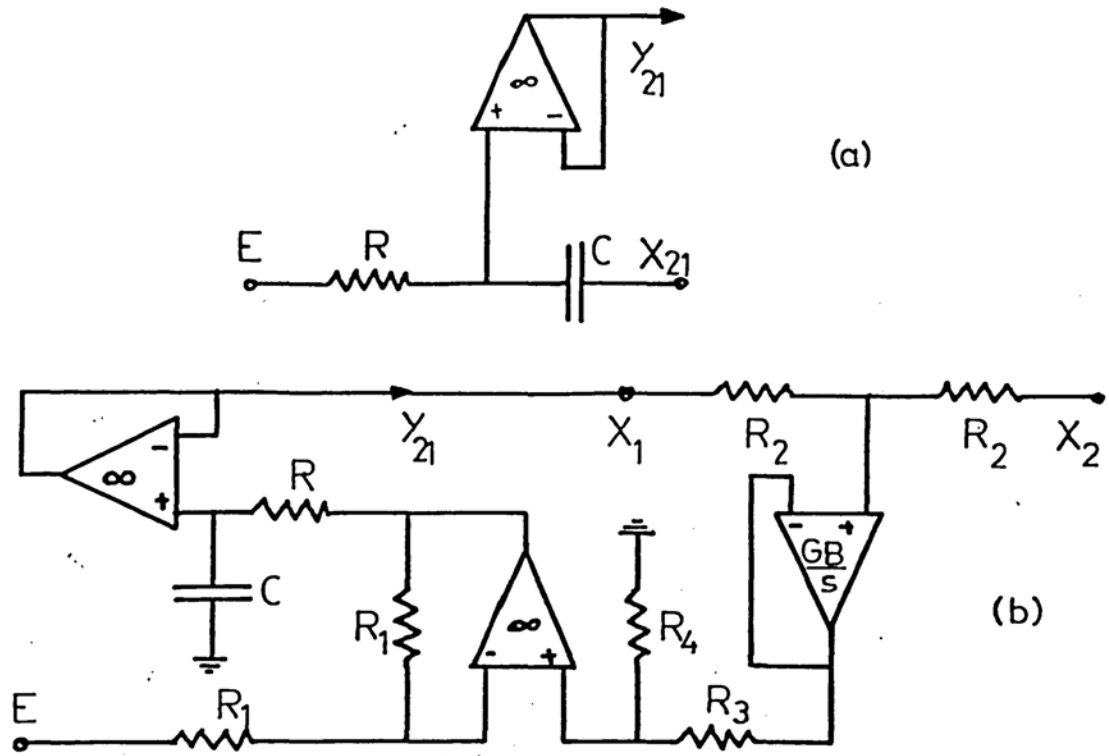
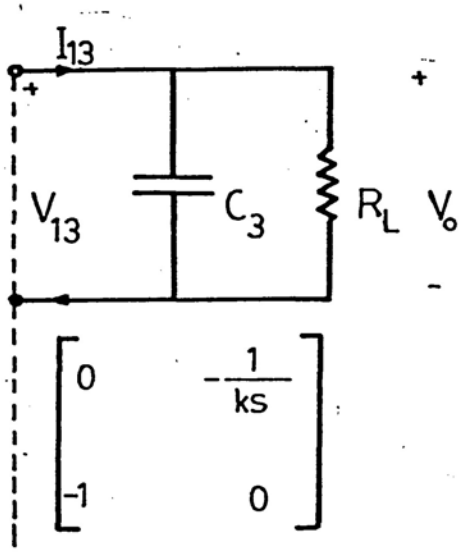


Fig. 5.2.7

$$\mathcal{A}_{13} = \begin{bmatrix} 0 & -\frac{1}{ks} \\ -1 & 0 \end{bmatrix}$$

in order to have direct output [DI1]. This is shown in Fig. 5.2.8.



$$Y_{13} = -V_o$$

$$Y_{13} = \frac{-R_L}{-\frac{1}{ks}(sC_3R_L+1)} X_{13}$$

Fig. 5.2.8

The interconnection with the previous arm is not the cross-cascade one but it can be found as follows:

$$\begin{bmatrix} X_{13} \\ Y_{13} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{sk} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -L(GB) \\ 0 & \frac{1}{ks} \end{bmatrix}^{-1} \begin{bmatrix} X_{22} \\ Y_{22} \end{bmatrix} \quad \text{or}$$

$$\begin{bmatrix} X_{13} \\ Y_{13} \end{bmatrix} = -sk \begin{bmatrix} 0 & \frac{1}{sk} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{ks} & L(GB) \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_{22} \\ Y_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & skL(GB) \end{bmatrix} \begin{bmatrix} X_{22} \\ Y_{22} \end{bmatrix}$$

$$\text{or } X_{13} = Y_{22}, \quad Y_{13} = X_{22} + skL(GB)Y_{22} \tag{5.2.23}$$

By substituting eqn.(5.2.19) again into (5.2.23) it is obtained:

$$X_{13}=Y_{22}, \quad Y_{13}=X_{22} - \frac{X_1+X_2}{1+s\frac{1}{(GB)}}, \quad \text{or } X_{22}=Y_{13} + \frac{X_1+X_2}{1+\frac{s}{GB}} \tag{5.2.24}$$

Note that $X_2 \equiv X_{22}$.

Complete realization is shown in Fig.5.2.9.

This filter will work up to very high frequencies because there are 5 op-amps, 3 of which are buffers, one is used as a nonideal integrator, and the 5th in non-inverting adder. The sensitivity of the resistors and capacitors in the realizations will be very low, as it has been discussed. The resistors embedded, to realize the interconnecting network do not have any equivalent in the passive network so the sensitivity due to them will be not low enough.

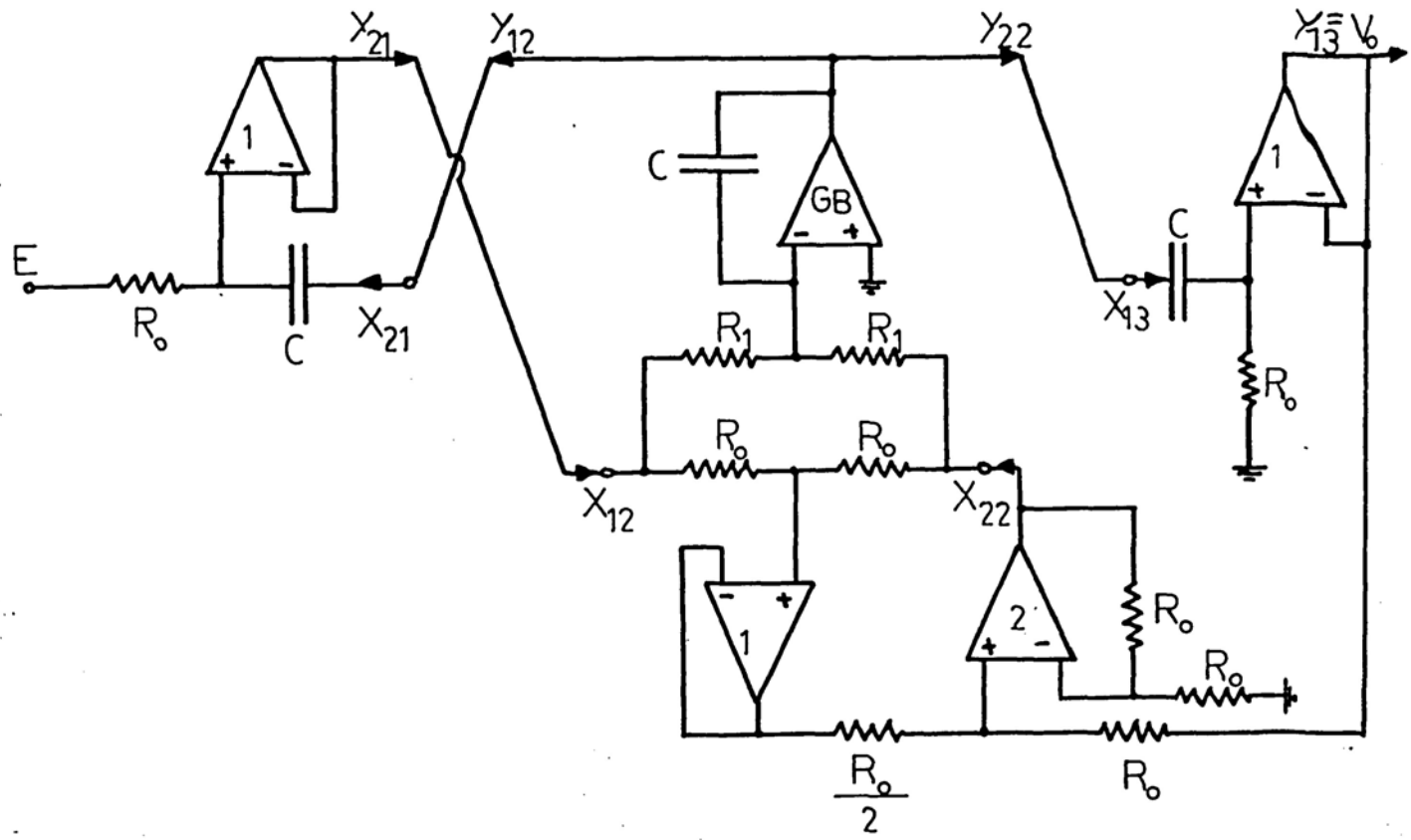


Fig. 5.2.9

5.2.3 Synthesis procedure

If eqn.(5.2.13) is recalled:

$$T' = T + \mathcal{A}_1^{-1} \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \mathcal{A}_2 \qquad (5.2.25)$$

In the synthesis case, as discussed above, the matrices $\mathcal{A}_1$ and $\mathcal{A}_2$ are evaluated for prescribed chain matrix T'. It is required that the chain matrix of the new passive arm to be of the same form as that of T. For series impedance Z the chain-matrix is:

$$T = \begin{bmatrix} 1 & -Z \\ 0 & 1 \end{bmatrix} \qquad (5.2.26a)$$

and it is required:

$$T' = \begin{bmatrix} 1 & -Z' \\ 0 & -1 \end{bmatrix} \qquad (5.2.26b)$$

where Z' is of the same form as Z, e.g. if $Z=Ls$ then Z' must be $L's$, in the sense that lossless inductors are corresponding to lossless inductors by changing one or more elements in the active structure. By those requirements one has:

$$\begin{bmatrix} 1 & -Z' \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -Z \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix}^{-1} \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix}$$

or

$$\begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix}^{-1} \begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix} = \begin{bmatrix} 0 & Z-Z' \\ 0 & 0 \end{bmatrix} \qquad (5.2.27)$$

From eqn.(5.2.27) could be obtained:

$$\begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 \\ \gamma_1 & \delta_1 \end{bmatrix} \begin{bmatrix} 0 & Z-Z'=Z'' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_2 & \beta_2 \\ \gamma_2 & \delta_2 \end{bmatrix}^{-1} \quad \text{or}$$

$$\begin{bmatrix} \Delta a & \Delta b \\ \Delta c & \Delta d \end{bmatrix} = \begin{bmatrix} 0 & \alpha_1 Z'' \\ 0 & \gamma_1 Z'' \end{bmatrix} \underbrace{\begin{bmatrix} \delta_2 & -\beta_2 \\ -\gamma_2 & \alpha_2 \end{bmatrix}}_{\Delta_2} = \frac{1}{\Delta_2} \times \begin{bmatrix} -\alpha_1\gamma_2 Z'' & \alpha_1\alpha_2 Z'' \\ -\gamma_1\gamma_2 Z'' & \alpha_2\gamma_1 Z'' \end{bmatrix}$$

Therefore in order the passive arm retain its nature, it must be:

$$\Delta a = -\frac{\alpha_1\gamma_2 Z''}{\Delta_2}, \qquad \Delta b = \frac{\alpha_1\alpha_2 Z''}{\Delta_2}$$

$$\Delta c = -\frac{\gamma_1\gamma_2 Z''}{\Delta_2}, \qquad \Delta d = \frac{\alpha_2\gamma_1 Z''}{\Delta_2} \tag{5.2.28}$$

where $\Delta_2=\alpha_2\delta_2-\beta_2\gamma_2=\det \mathcal{A}_2$.

The values of a,b,c and d from eqn.(5.2.7) are:

$$a = \frac{1}{\Delta_2}(\alpha_1\gamma_2 Z+\alpha_1\delta_2+\beta_1\gamma_2), \qquad b= \frac{1}{\Delta_2}(-\alpha_1\alpha_2 Z-\alpha_1\beta_2-\delta_1\alpha_2)$$

$$c = \frac{1}{\Delta_2}(\gamma_1\gamma_2 Z+\gamma_1\delta_2+\delta_1\gamma_2), \qquad d = \frac{1}{\Delta_2}(-\gamma_1\alpha_2 Z-\gamma_1\beta_2-\delta_1\alpha_2) \tag{5.2.29}$$

It can be seen from (5.2.28) and (5.2.29) that the perturbations must be proportional to the coefficients of the frequency dependent elements in eqns.(5.2.29), as it would be expected in order to have the same expressions as in (5.2.29) for the element Z'. It has be seen from the analysis procedure that the best sensitivities, changing the elements in the active structures, could be obtained, when the time constants of those elements are proportional to the value of their passive reactive counterparts.

This condition with the constraints described in eqns. (5.2.28) lead to the conclusion that one must have present, if possible, only the term which is frequency dependent in the expression of b in eqn.(5.2.29) and the terms which are frequency independent in the expressions of a, c and d. By doing so the expressions of voltage transfer ratios K,L,M,N will be of the form:

$$K = \frac{e}{\lambda Z}, \quad L = \frac{g}{\lambda Z}, \quad M = \frac{h}{\lambda Z}, \quad N = \frac{n}{\lambda Z} \tag{5.2.30}$$

Those constraints could be met, where e, g, h, n, λ are constants by having $\gamma_1=\gamma_2=0$ and $\beta_1=\beta_2=0$.
Therefore, in that case:

$$K = \frac{d}{b} = \frac{\delta_1\alpha_2}{\alpha_1\alpha_2 Z} = \frac{\delta_1}{\alpha_1 Z}\ , \quad L = \frac{-ad+bc}{b} = -\frac{\Delta_1}{\alpha_1\alpha_2 Z} = \frac{-\alpha_1\delta_1}{-\alpha_1\alpha_2 Z} \quad \text{or}$$

$$L = \frac{-\delta_1}{\alpha_2 Z} \tag{5.2.31}$$

$$M = \frac{1}{b} = \frac{-\Delta_2}{\alpha_1\alpha_2 Z} = \frac{-\alpha_2\delta_2}{\alpha_1\alpha_2 Z} = \frac{-\delta_2}{\alpha_1 Z}\ , \quad N = \frac{-d}{b} = \frac{\alpha_1\delta_2}{\alpha_1\alpha_2 Z} = \frac{\delta_2}{\alpha_2 Z}$$

The perturbations, therefore, required will be of the form:

$$\Delta a = 0, \quad \Delta b = \frac{\alpha_1\alpha_2(Z-Z')}{\alpha_2\delta_2} = \frac{\alpha_1(Z-Z')}{\delta_2}\ , \quad \Delta c = 0, \quad \Delta d = 0 \tag{5.2.32}$$

The conditions in eqns.(5.2.31) and (5.2.32) give the neccessary conditions in order a ladder series arm retain its nature, in respect of frequency, when the active components in the simulation structure vary by a certain percentage. Similar expressions could be derived for the shunt admittance Y. A special case of the expressions described in (5.2.31) will be the case of TRI (Transfer elimination approach) [DI1] when K(s)=M(s)=N(s)=L(s), an example of which described in section 5.2.2. An alternative way to obtain expressions like those in eqn.(5.2.30) for all the voltage transfer ratios or some of them is to choose:

$\beta_1=\beta_2=0$, $\delta_1=kZ$, $\delta_2=\lambda Z$ where λ,k are constants.

In that case:

$$K = \frac{d}{b} = \frac{-\gamma_1\alpha_2{}^Z - \alpha_2 k^Z}{-\alpha_1\alpha_2} = \frac{\gamma_1+k}{\alpha_1}$$

$$L = -\frac{\Delta_1}{b} = \frac{-\alpha_1 k^Z}{+\alpha_1\alpha_2{}^Z} = -\frac{k}{\alpha_2} \qquad (5.2.33)$$

$$M = -\frac{\Delta_2}{b} = \frac{-\alpha_2\lambda^Z}{+\alpha_1\alpha_2{}^Z} = -\frac{\lambda}{\alpha_1}$$

$$N = \frac{-a}{b} = \frac{\alpha_1\gamma_2{}^Z + \alpha_1\lambda^Z}{\alpha_1\alpha_2{}^Z} = \frac{\gamma_2+\lambda}{\alpha_2}$$

Furthermore for $k=-\alpha_2$, $\lambda=-\alpha_1$, $\gamma_2+\lambda=\alpha_2=\gamma_2-\alpha_1$

and $\gamma_1+k=\alpha_1=\gamma_1-\alpha_2 \rightarrow k=-\alpha_2$, $\lambda=-\alpha_1$, $\gamma_2=\gamma_2=\alpha_1+\alpha_2$

or: $\beta_1=\beta_2=0$, $\delta_2=-\alpha_2 z$, $\delta_2=-\alpha_1 z$, $\gamma_1=\gamma_2=\alpha_1+\alpha_2$

it will be: $K(s)=L(s)=M(s)=N(s)=1$

The structure in the active domain will then be of zero order. This specific choice of transformation matrices will complicate the adjacent arms of the present series arm. However if the adjacent arms will be of the same form as that of the present arm, in a way as to form a Π of capacitors or a T of inductors, this specific choice of transformation matrices will reduce the order of the structure. Alternatively, if any Π or T is not formed the adjacent arms will become structures of second order. This fact is very attractive since it can be selected by this way the matrices as to give zero-order active structures for the inductors and second-order structures for the capacitors in a minimum capacitor filter. The other way round could be done in minimum inductor filters, e.g. having zero-order structures for the capacitors and second-order structures for the inductors. This kind of approach is very interesting and needs to be developed, since new circuits will come out, probably better than the existing ones.

This approach has been used to design Digital Filters via Frequency Dependent Linear Transformations [TZ1] and has given extremely good results. In order to classify if a filter is a minimum inductor or a minimum capacitor, the number of capacitors must be at least the number of inductors plus two in the first case and the number of inductors must be at least the number of capacitors plus two in the second case.

5.3 CONCLUSIONS

In this chapter some guidelines have been given for selective choosing of the transformation matrices in such a way as to give less sensitive structures when the components in the active filter are changing by a certain percentage. This aspect needs further investigations so as to give practical results and lead, if possible to insensitive structures. To have a structure that deviates very little from its nominal design is much desirable from the manufacture point of view, because in those cases wide tolerance components could be used without any serious deviation from the actual design. Most of the times, designers are interested to minimize the sensitivity of the overall structure of the filter, but optimizing ω or Q pole sensitivities will many times be of great interest since their optimization, reduces the overall structural sensitivity in an indirect way.

CHAPTER 6

PRACTICAL REALIZATIONS

6.1 3rd-ORDER ELLIPTIC LOWPASS FILTER

6.2 6th-ORDER BANDPASS FILTER

6.3 3rd-ORDER LOWPASS POLYNOMIAL FILTER

6.4 4rth-ORDER BANDPASS FILTER USING THE AF150 BIQUAD AS THE BASIC BUILDING BLOCK

6.5 3rd-ORDER ELLIPTIC LOWPASS ACTIVE-R FILTER

CHAPTER 6

PRACTICAL REALIZATIONS

The general LTA approach for the simulation of passive ladder networks has been presented in Chapter 2, while in Chapter 3 this approach has been used to design filters with biquadratic sections as basic building blocks. In Chapter 4, the LT technique has been extented as to design Active-R filters.

It is the aim of this Chapter to apply the above theory to practice by simulating several ladder filters. The frequency responses of the LTA structures, so derived, as well as their sensitivities are examined in each case, showing the advantages of LTA technique over some existing ladder simulation methods.

The reactive ladder filters used here are so designed, as to match the source resistance R_S to the load R_L at certain frequencies within the passband, so providing maximum power transfer points at which the first order sensitivities of their amplitude characteristic with respect to a reactive component are zero. This is known why the simulation of these doubly terminated LC ladders is desirable.

Ideally, RC-active simulation of such filters should possess this low sensitivity property with respect to most, if not all, of their passive (R,C) elements. However, in practice, only some of the passive elements of the simulated RC-active structure obey the Orchard's Principle, so that the sensitivity of the filter attenuation characteristic with respect to those elements will be very low within the

passband, due to the zero sensitivities at the maximum power transfer points.

In the present designs, because of the biquadratic sections used, there is not always one-to-one correspondance between all the elements of the active structure and those of the passive. Therefore, the variations of the frequency response due to the elements deviation from their nominal value will be in some cases more severe than if first-order structures could be used. However, it is shown that those variations are well acceptable in many applications.

6.1 3rd-ORDER ELLIPTIC LOWPASS FILTER

The general acyclic transformation has been applied for the simulation of the 3rd order elliptic Chebyshev lowpass filter (ripple 0.28db) shown in Fig.6.1.1a ($R_S'=R_L'=1\Omega$, $C_{10}=C_{30}=1.2030F$, $L_{20}=0.9624H$, $C_{20}=0.2016F$). The choice of the transformation matrices has been carried out according to the rules of section 3.1.1, so as simple first-order structures to be used for the realization of the input and output termination, in conjuction with the terminating capacitors, and a biquad of finite-Q (negative-feedback) to be used for the series tuned circuit in the middle arm of the ladder. For those reasons a third entry in the transformation matrices of the tuned circuit is used and the overall active-RC structure is given in Fig.6.1.1b.

The resulting LTA structure, after denormalization for $f_c=10kHz$ and $R=1k\Omega$, was constructed with equal value polystyrene capacitors (1nF, 2% tolerance) and 5% tolerance resistors adjusted in such a way as to give time constants accuracy of 0.1%. The op-amps used for the realization have a gain-bandwith product of 5MHz. The frequency response is given in Fig.6.1.2, from which it is observed that the zero at 22.7kHz have more than 100db attenuation and the point of minimum attenuation is at 38kHz approximately. The passband response is also given and the maximum power transfer occurs at zero frequency and at approximately $\frac{\omega}{\omega_c} = 0.85$. The values of the components are given by equations (6.1.1):

$$R_1=R_{10}=19146\Omega,\ R_3=R_5=3387\Omega,\ R_2=R_4=1k\Omega,\ R_6=7658\Omega$$
$$R_g=15317\Omega,\ R_7=R_8=6417\Omega \text{ and } C_1=C_2=C_3=C_4=1nF \qquad (6.1.1)$$

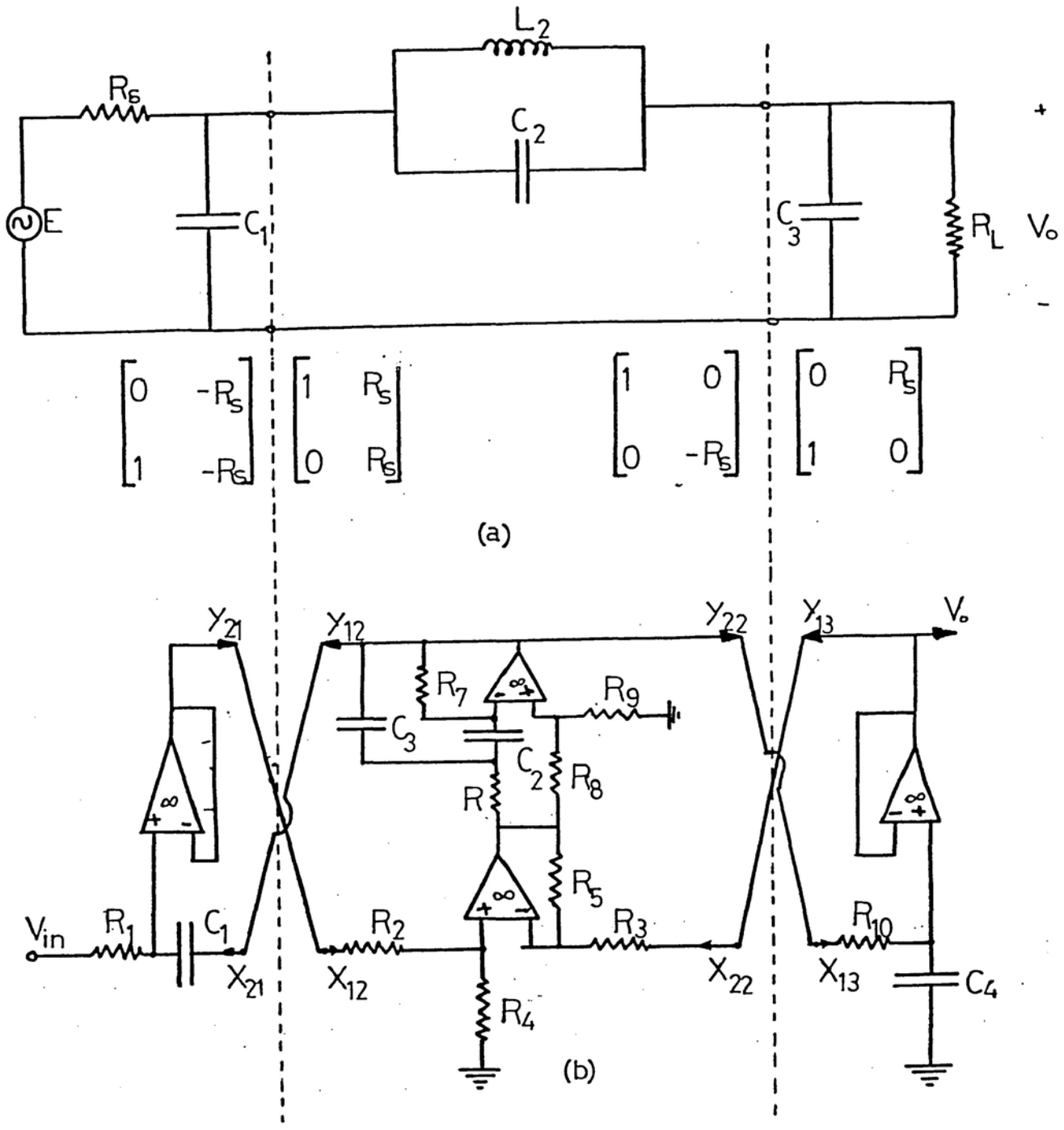

L2
C2
R6
E
C1
C3
RL
Vo
+
-
0 -Rs
1 -Rs
1 Rs
0 Rs
1 0
0 -Rs
0 Rs
1 0
(a)
Y21
Y12
Y22
Y13
Vo
R7
R9
C3
C2
R8
R
R5
Vin
R1
C1
R2
R3
R10
X21
X12
X22
X13
C4
R4
(b)

Fig. 6.1.1

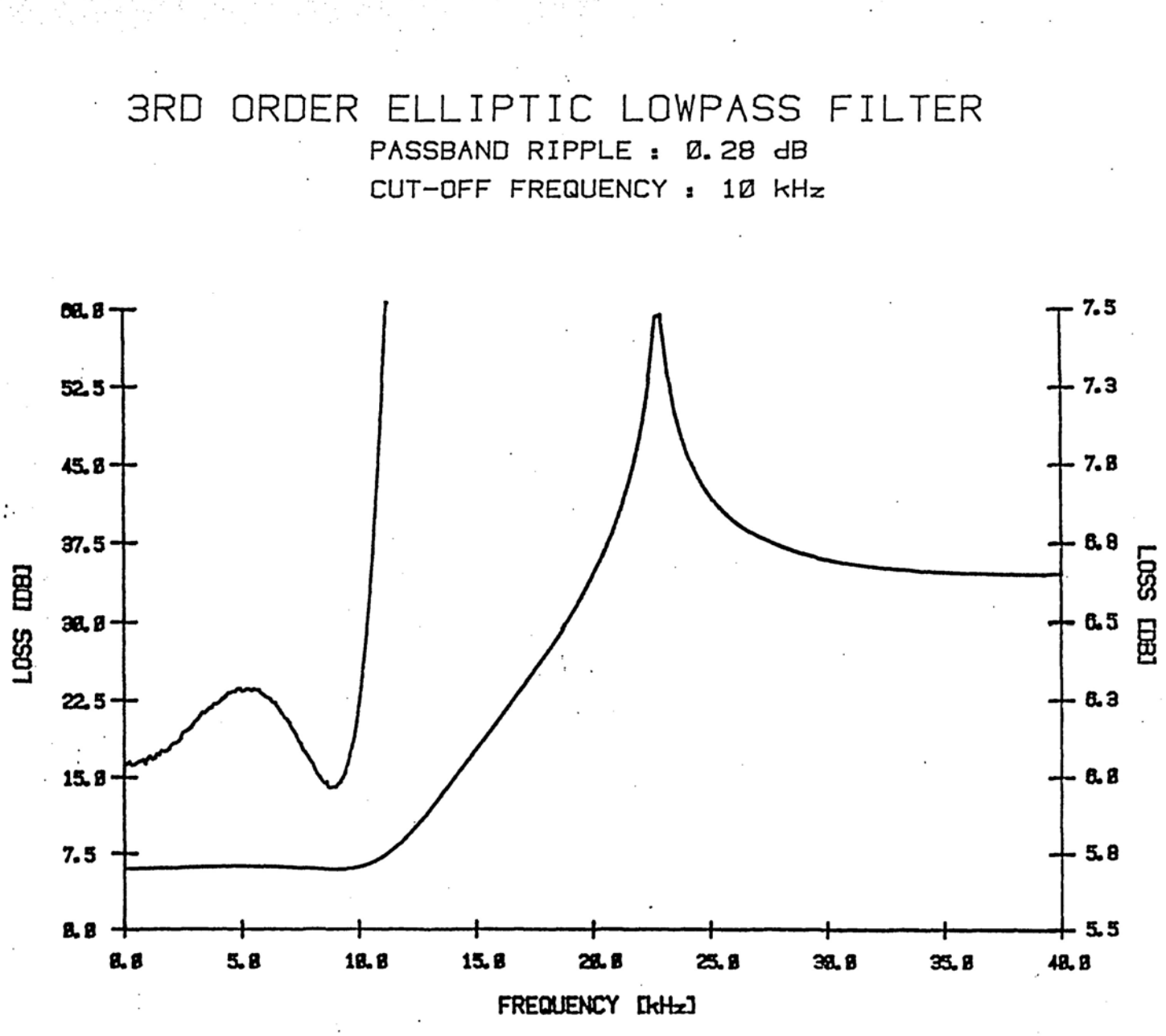

3RD ORDER ELLIPTIC LOWPASS FILTER
PASSBAND RIPPLE : 0.28 dB
CUT-OFF FREQUENCY : 10 kHz
60.0
52.5
45.0
37.5
30.0
22.5
15.0
7.5
0.0
LOSS [DB]
7.5
7.3
7.0
6.8
6.5
6.3
6.0
5.8
5.5
LOSS [DB]
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
FREQUENCY [kHz]

Fig. 6.1.2

6.2 6th-ORDER BANDPASS FILTER

This type of filter is derived from lowpass prototype using lowpass to bandpass transformations. The transformation matrices have been chosen following the procedure described in Sec.3.1.2 in such a way as to give an LTA canonic realization consisting of three single-amplifier biquads of finite-Q embedded in a multiple feedback topology. The complete structure is shown in Fig.6.2.1, where the input and output termination sections are realized with the Deliyiannis negative beedback biquad [DE2] while for the middle tuned circuit, the Sallen and Key [SA2] positive feedback biquad is used.

The complete LTA structure simulating the passive ladder with passband ripple of 0.5db and Bandwidth of 10% at 10kHz center frequency has been simulated on the computer using op-amps with Gain-Bandwith product of 20MHz. The frequency response is given in Fig.6.2.2 where no deviation from the ideal response was observed.

The variation with respect to component values are shown in Fig.6.2.3 and 6.2.4 for $\pm$1% change of resistors R_1,R_4,R_6, R_7,R_8,R_9 where it is shown that this change has an effect of adding less than 0.1db insertion loss. The variation of R_3 by $\pm$1% and capacitors C_1,C_2 by $\pm$2% are also shown in Fig. 6.2.4 which are acceptable for most applications. The sensitivities of the elements in the output termination section are expected to be similar if not the same with those given above due to symmetry of the structure.

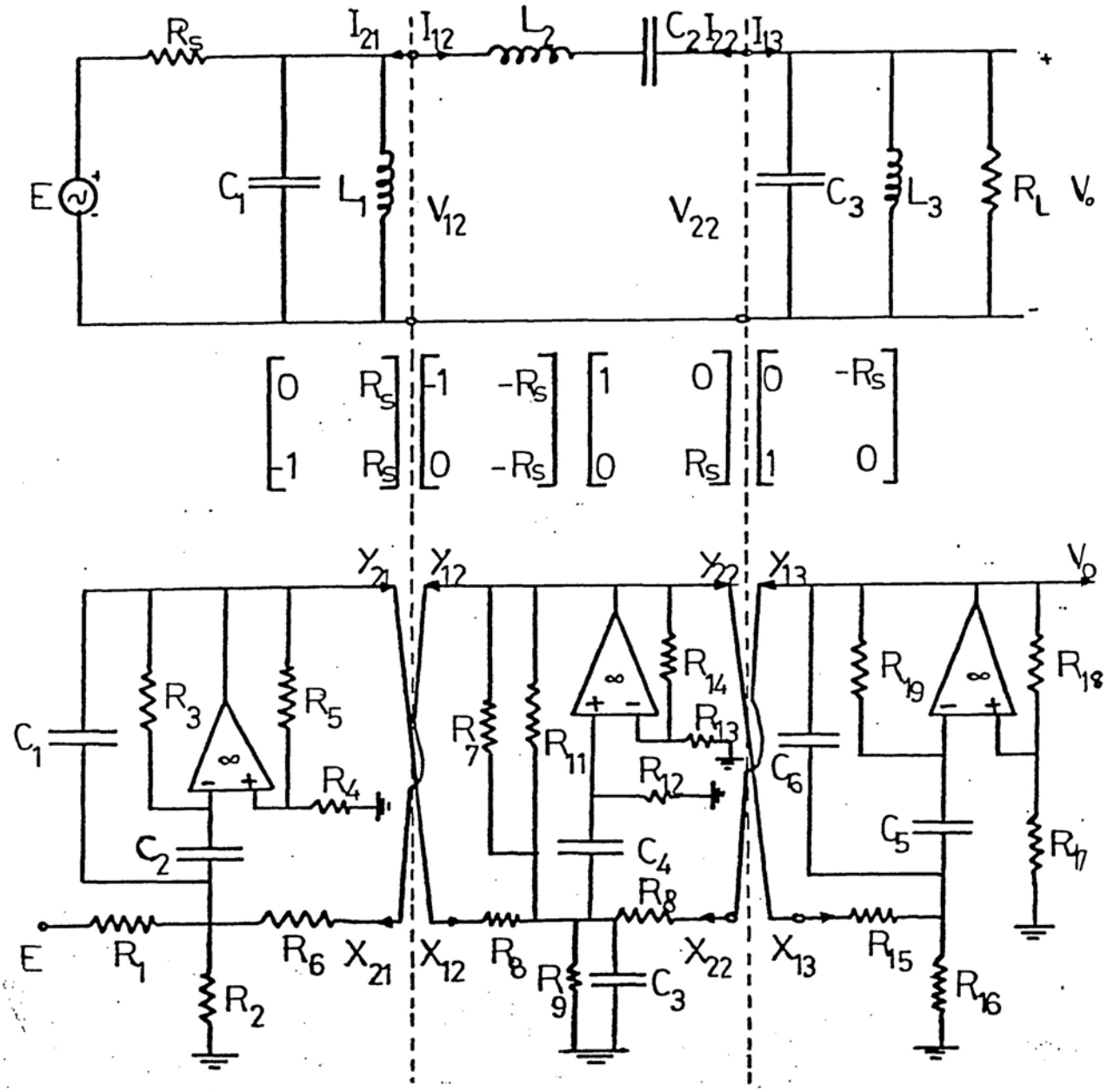

Rs
I21
I12
L2
C2
I22
I13
E
C1
L1
V12
V22
C3
L3
RL
Vo
+
-
[0 Rs; 1 Rs] [1 -Rs; 0 -Rs] [1 0; 0 Rs] [0 -Rs; 1 0]
Y21
Y12
Y22
Y13
Vo
R3
R5
R4
C1
C2
R7
R11
R14
R13
R12
C4
R8
C6
R19
R18
C5
R17
E
R1
R6
X21
X12
R10
R9
C3
X22
X13
R15
R16
R2

Fig. 6.2.1

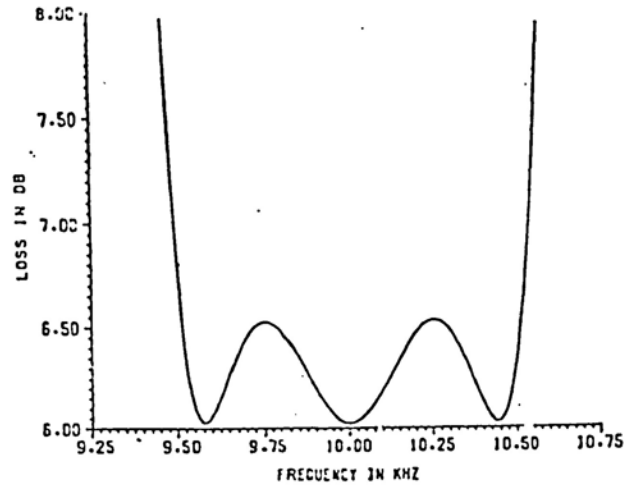


Fig. 6.22.

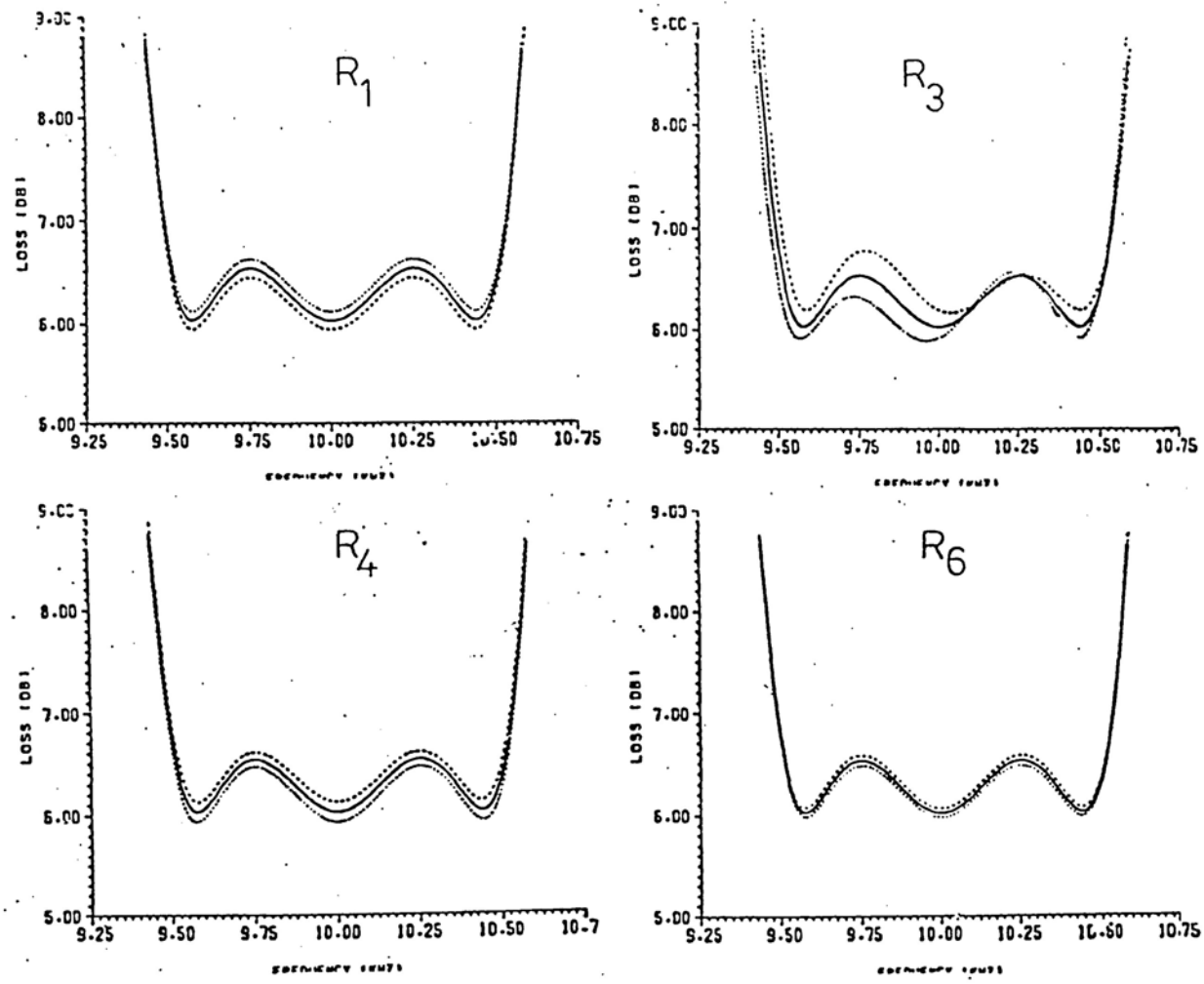


Fig 6.2.3

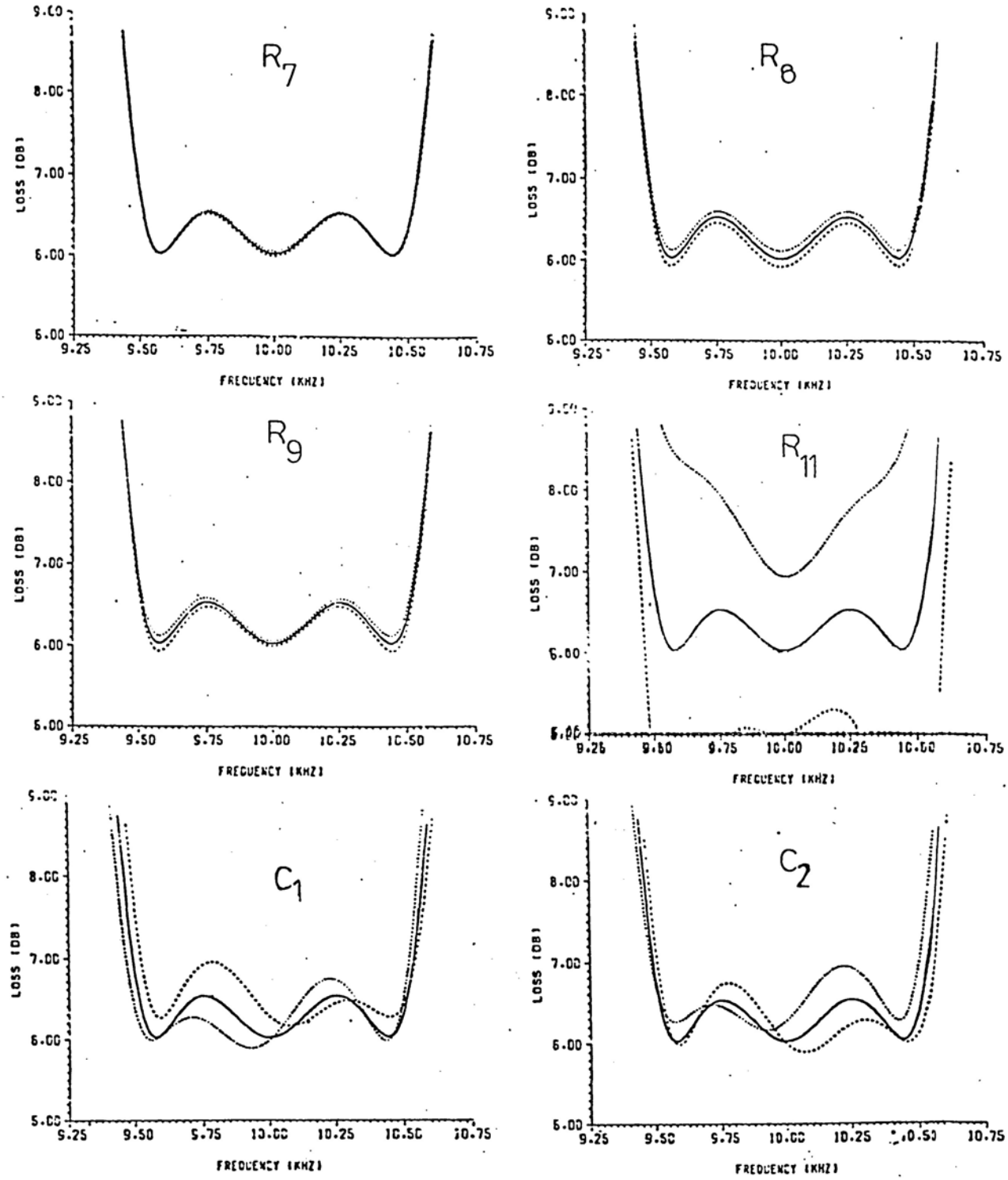

R7
R8
R9
R11
C1
C2
LOSS (DB)
FREQUENCY (KHZ)
9.00
8.00
7.00
6.00
5.00
9.25
9.50
9.75
10.00
10.25
10.50
10.75

Fig 6.24

6.3 3rd-ORDER LOWPASS POLYNOMIAL FILTER

In Sec.3.2, inverted-L filters have been simulated by transforming two arms of the ladder together, forming an inverted-L section under LT technique and different realizations have been derived shown in Figrs.3.2.5,6,7,8,9. In this section the 3rd order lowpass polynomial filter with 0.5db ripple in the passband and cut-off frequency at 10kHz shown in Fig. 6.3.1 has been simulated. The variations of the response due to element tolerances by $\pm$1% for the resistors and $\pm$2% for the capacitors are shown in Fig.6.3.2a,b. It is observed that resistors R_3 and R_4 and capacitor C_3 have sensitivities of the order of the passive filter, because there is one to one correspodence between them and their passive counterparts, according to Orchard's principle. The sensitivities for the resistors R_o,$R_o(C_o)$,R_1,R_A and R_B (where $S_{R_A}^{|T(s)|} = S_{R_B}^{|T(s)|}$) are giving only an insertion loss while the sensitivity for R_2 is more cumbersome. So resistor R_2 has to be very accurate, as well as the two capacitors $C_{1,2}$ in the biquad configuration.

This filter has been constucted using equal value polysterine capacitors (10nF, 2% tolerance) and 5% tolerance resistors selected in such a way as to give time constants accuracy of 0.1%. Two operational amplifiers with Gain-Bandwidth product of 5MHz as those of the type LF356 have been used. The measured response is given at the bottom of Fig. 6.3.2b and is found to be very reasonable.

The values of the components are given by eqns (6.3.1).

$$R_1=R_2=537\Omega,\ R_o=2540.5\Omega=R_A,\ R_B=1148.8\Omega$$
$$R_3=26516\Omega,\ R_4=2809.7\Omega,\ C_1=C_2=C_3=C_o=10nF \qquad (6.3.1)$$

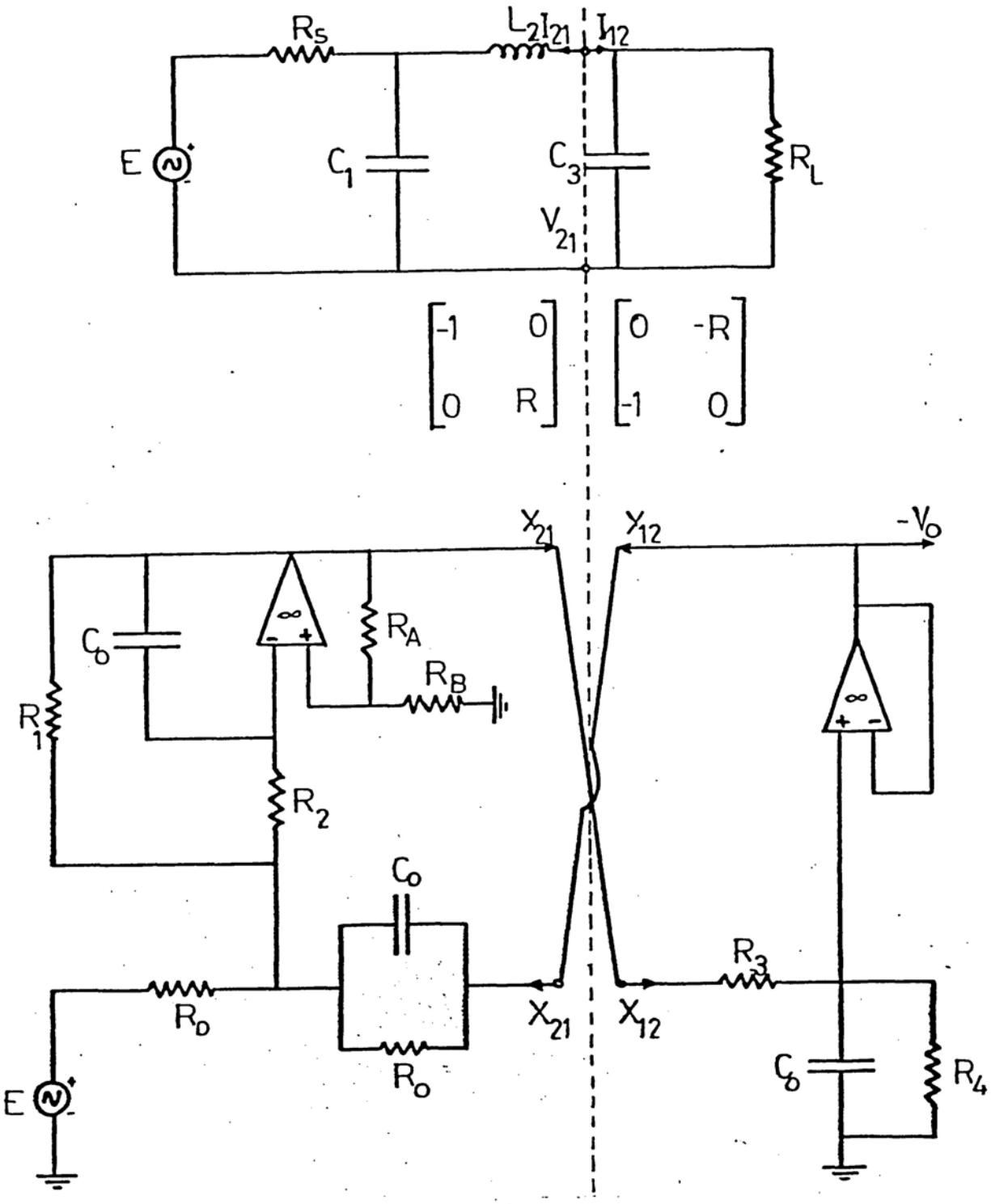
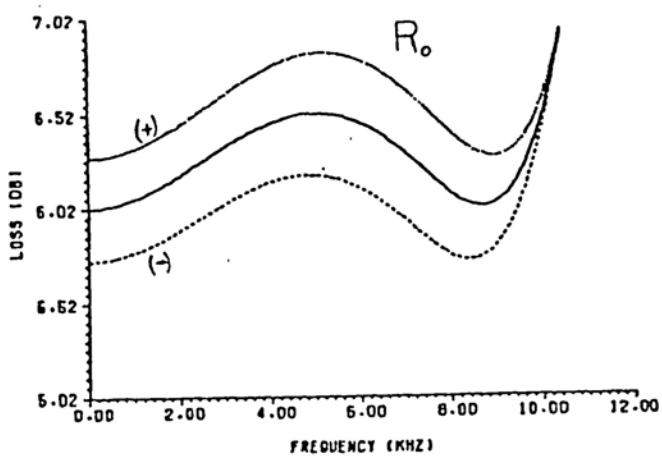
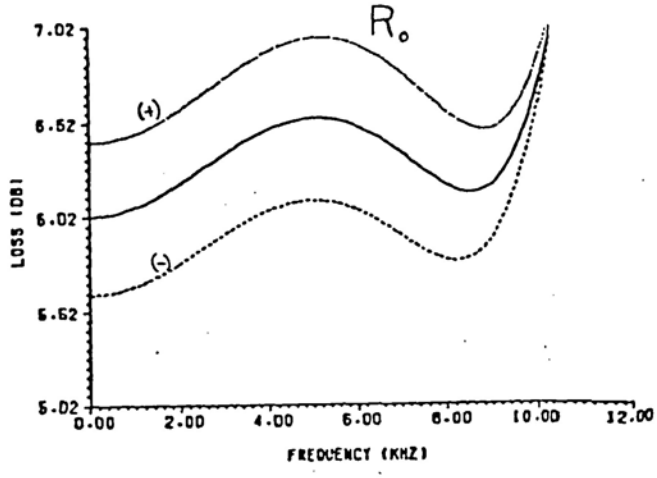
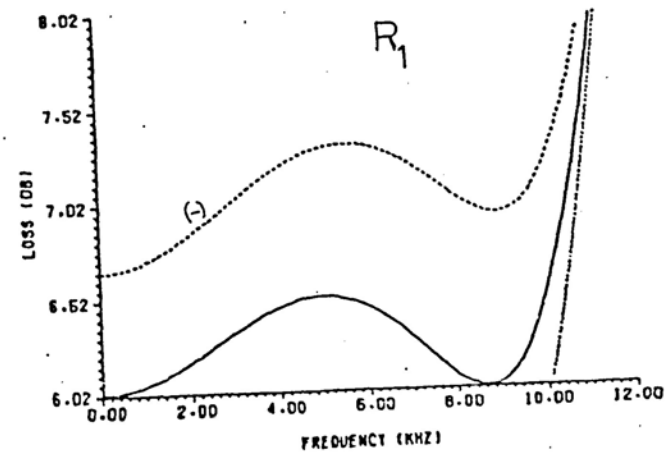
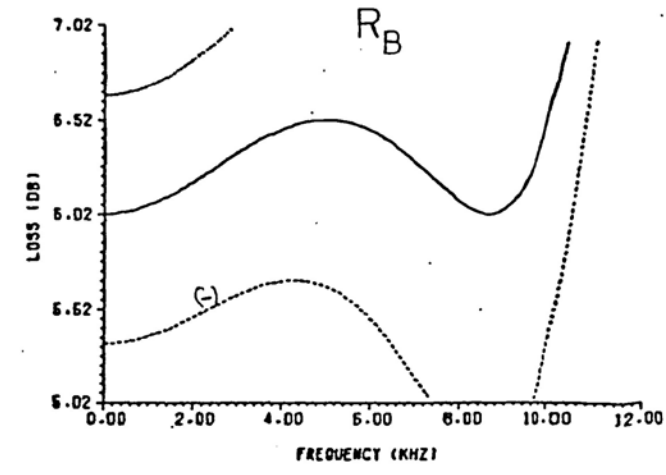

R_S
L_2 I_21
I_12
E
C_1
C_3
R_L
V_21
[-1 0; 0 R]
[0 -R; -1 0]
X_21
X_12
-V_o
C_o
R_A
R_B
R_1
R_2
C_o
R_o
R_o
X_21
X_12
R_3
C_o
R_4
E

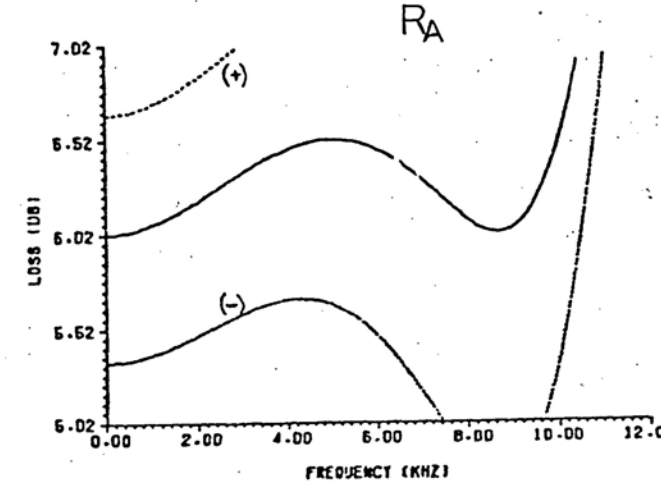
Fig. 6.3.1


R_o
(+)
(-)
LOSS (DB)
7.02
6.52
6.02
5.52
5.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)


R_o
(+)
(-)
LOSS (DB)
7.02
6.52
6.02
5.52
5.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)


R_1
(-)
LOSS (DB)
8.02
7.52
7.02
6.52
6.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)


R_B
(-)
LOSS (DB)
7.02
6.52
6.02
5.52
5.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)


R_A
(+)
(-)
LOSS (DB)
7.02
6.52
6.02
5.52
6.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)

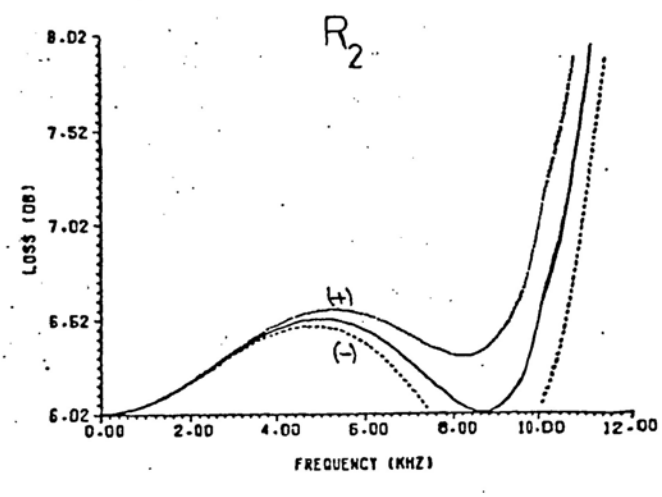

R_2
(+)
(-)
LOSS (DB)
8.02
7.52
7.02
6.52
6.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
FREQUENCY (KHZ)

Fig. 6.3.2a

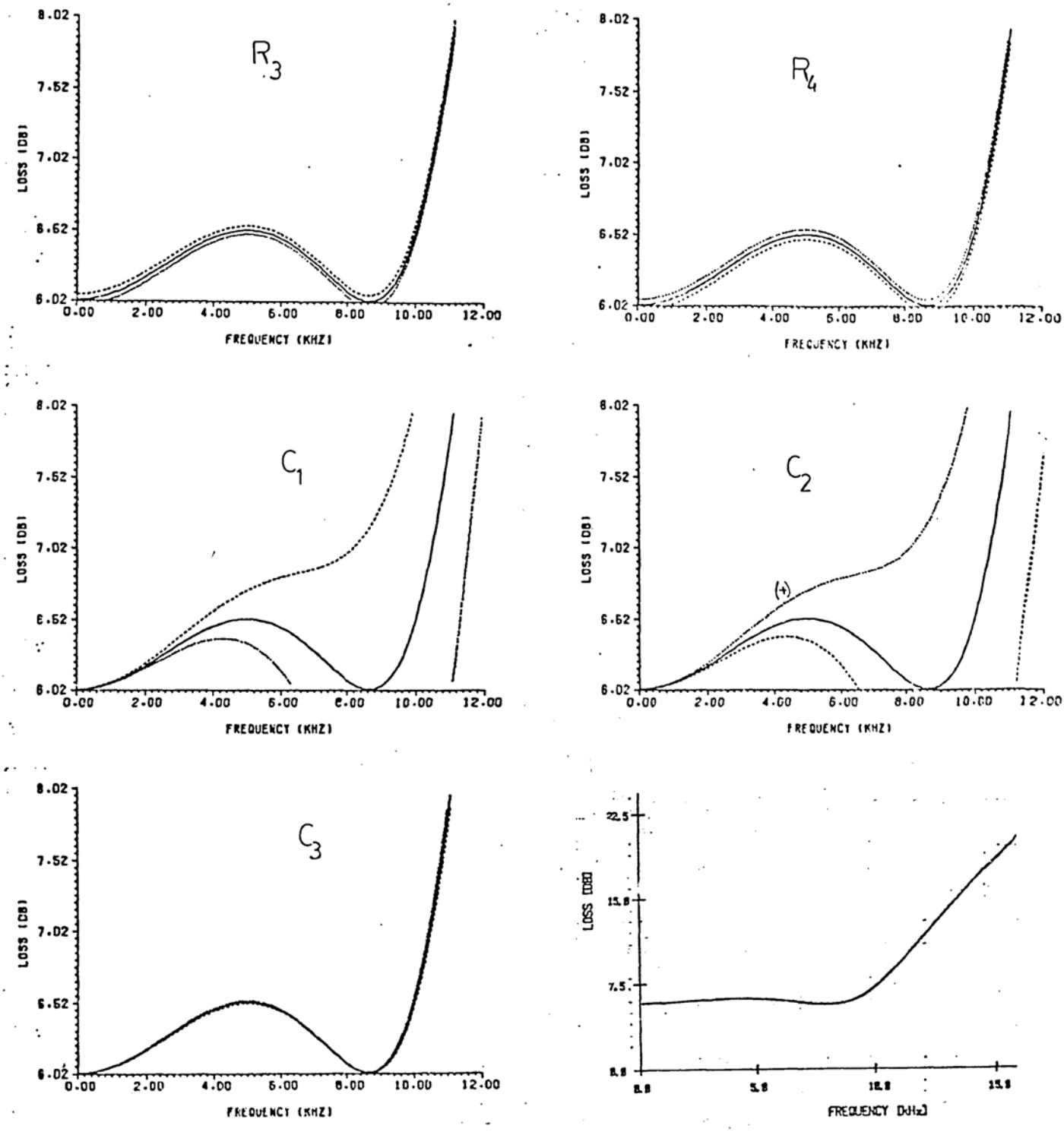

R3
R4
C1
C2
C3
(+)
LOSS (DB)
FREQUENCY (KHZ)
8.02
7.52
7.02
6.52
6.02
0.00
2.00
4.00
6.00
8.00
10.00
12.00
22.5
15.0
7.5
0.0
5.0
10.0
15.0
FREQUENCY (kHz)

Fig. 6.3.2b

6.4 4rth-ORDER BANDPASS FILTER USING THE AF150 BIQUAD AS THE BASIC BUILDING BLOCK

Off-The-Shelf Building Blocks have been described in Sec.3.3 recently appeared in industry. The most suitable for active simulation as that one of National Semiconductor, the ICAF150 biquad, has been used for the simulation of the 4rth-Order Chebyshev Bandpass filter with 0.1db passband ripple and 10% Bandwith at 10kHz center frequency shown in Fig.6.4.1.

Two of those ICS have been used, for the realization of the filter. The measured response is shown in Fig.6.4.2 when the components (in that case only resistors) are unadjusted and after adjustment for 1% tolerance.

The values of the resistors are given as:

$R_{In1}=R_{IN}=10^4 k\Omega$, $R_{Q1}=365.7\Omega$, $R_{f1}=22880\Omega$ and $R_{f2}=22880$, $R_{Q2}=337.15\Omega$, $R_{IN2}=39310.8\Omega$.

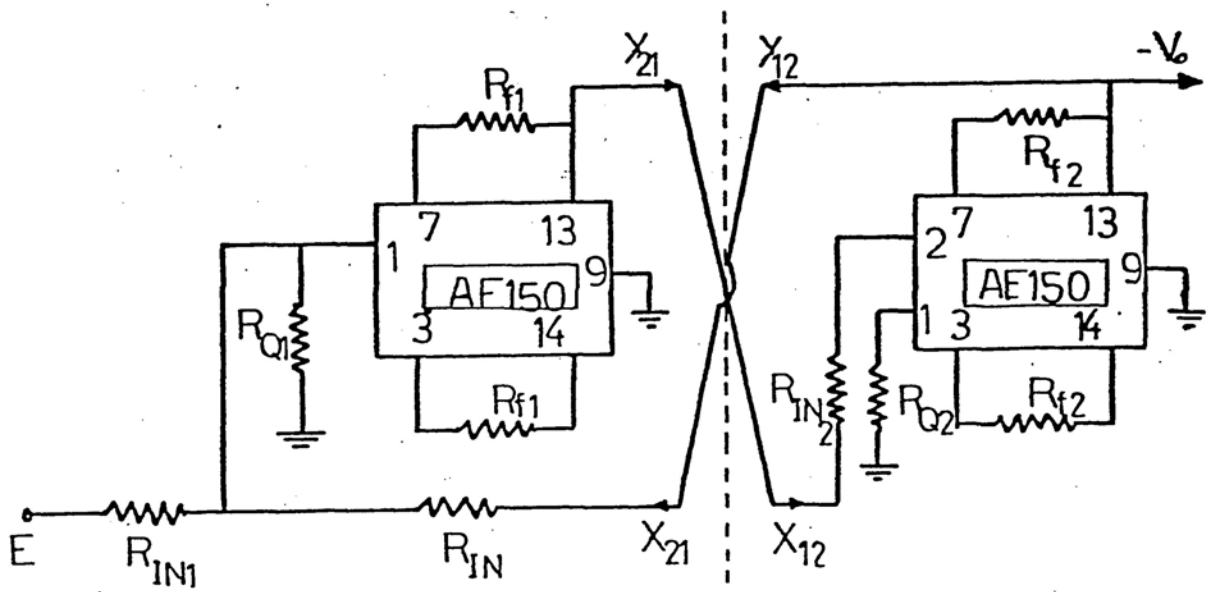
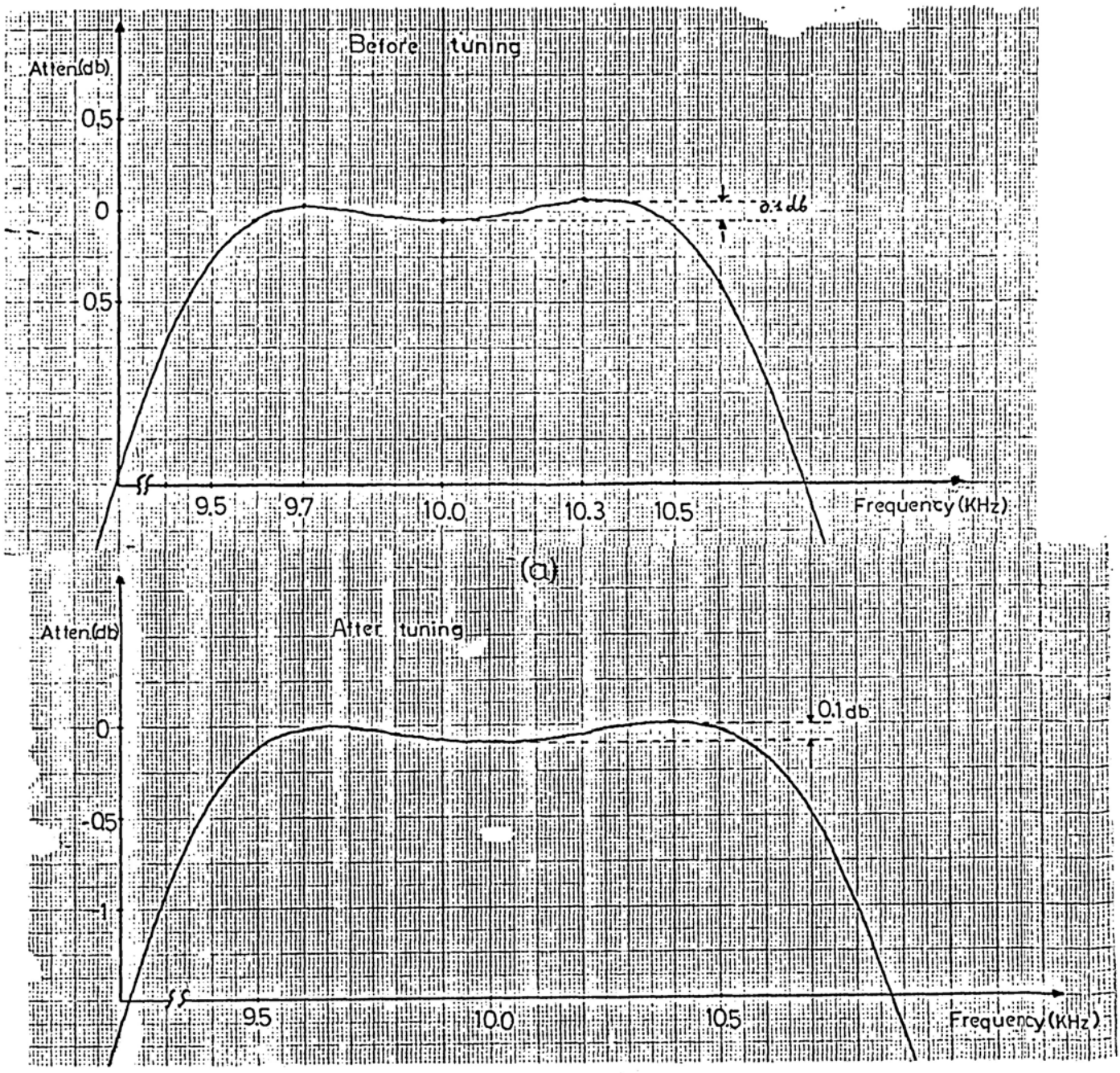


Fig. 6.4.1


Before tuning
Atten(db)
0.5
0
-0.5
0.1 db
9.5
9.7
10.0
10.3
10.5
Frequency (KHz)
(a)
After tuning
Atten(db)
0
-0.5
-1
0.1 db
9.5
10.0
10.5
Frequency (KHz)

(b)

Fig. 6.4.2

6.5 THIRD-ORDER ELLIPTIC LOWPASS ACTIVE-R FILTER

In Chapter 4, active-R structures have been derived through Linear Transformations without external capacitors. Those structures are taking into consideration the finite Gain-Bandwith of the operational amplifiers, and are designed, generally, for frequencies above 20kHz.

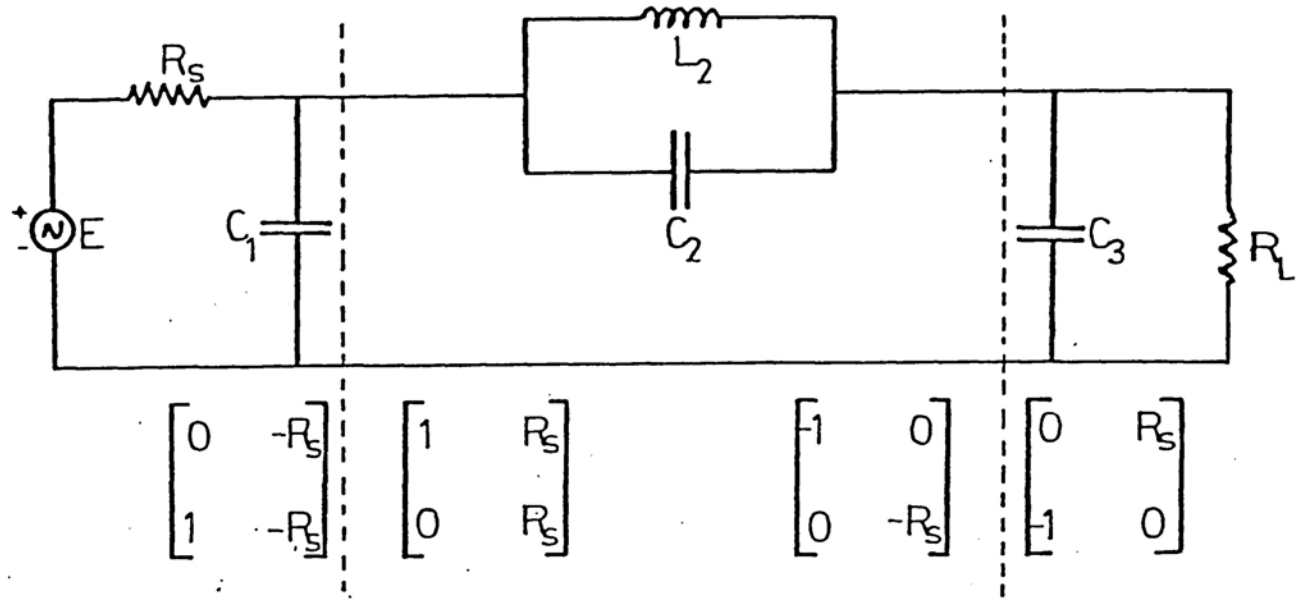
In section 4.2.2, the 3rd-order lowpass active-R filter is designed as shown in Fig.4.2.5 and in Fig.6.5.1 again, for convenience. For this structure 5 operational amplifiers are used, 4 of which are represented by the one-pole roll-off model and the other assumed to be ideal. Therefore, the design uses 5 frequency dependent elements while it is of third-order and it can be classified as a non-canonic structure without external capacitors.

The proposed configuration has been simulated on the computer with resistors of 1% tolerance, 4 μA741 operational amplifiers with GBW product of 1MHz and 1 LF356 op-amp with GBW product of 5MHz, simulating the passive filter of 0.28db passband ripple and cut-off frequency of 100kHz. The computed frequency response shown in Fig.6.5 2 gives perfect result, with the stopband zero at 230kHz approximately and the point of minimum attenuation at 340kHz. The resistor values for the design are given below:

$R_4 = 20607\Omega$, $R_9 = 62k$, $R_7 = 3733k$,

$R_{10} = R_6 = R_1 = 10k\Omega$, $R_8 = R_2 = 1k\Omega$, $R_3 = 3.1k\Omega = R_5$

The variations of the passband response due to variations of the resistors by 1% are given in Fig.6.5.3 and 6.5.4,5.



$$Y_{21}=\frac{1}{1+sR_sG}(E-X_{21})+X_{21} \qquad Y_{12}=Y_{22}=\frac{R_sC_2L_2s^2+R_s}{R_sC_2L_2s^2+L_2s+R_s}(X_{12}+X_{22}) \qquad Y_{13}=-V_o=-\frac{1}{\frac{R_s}{R_L}+sR_sC_3}X_1$$

Fig 6.5.1

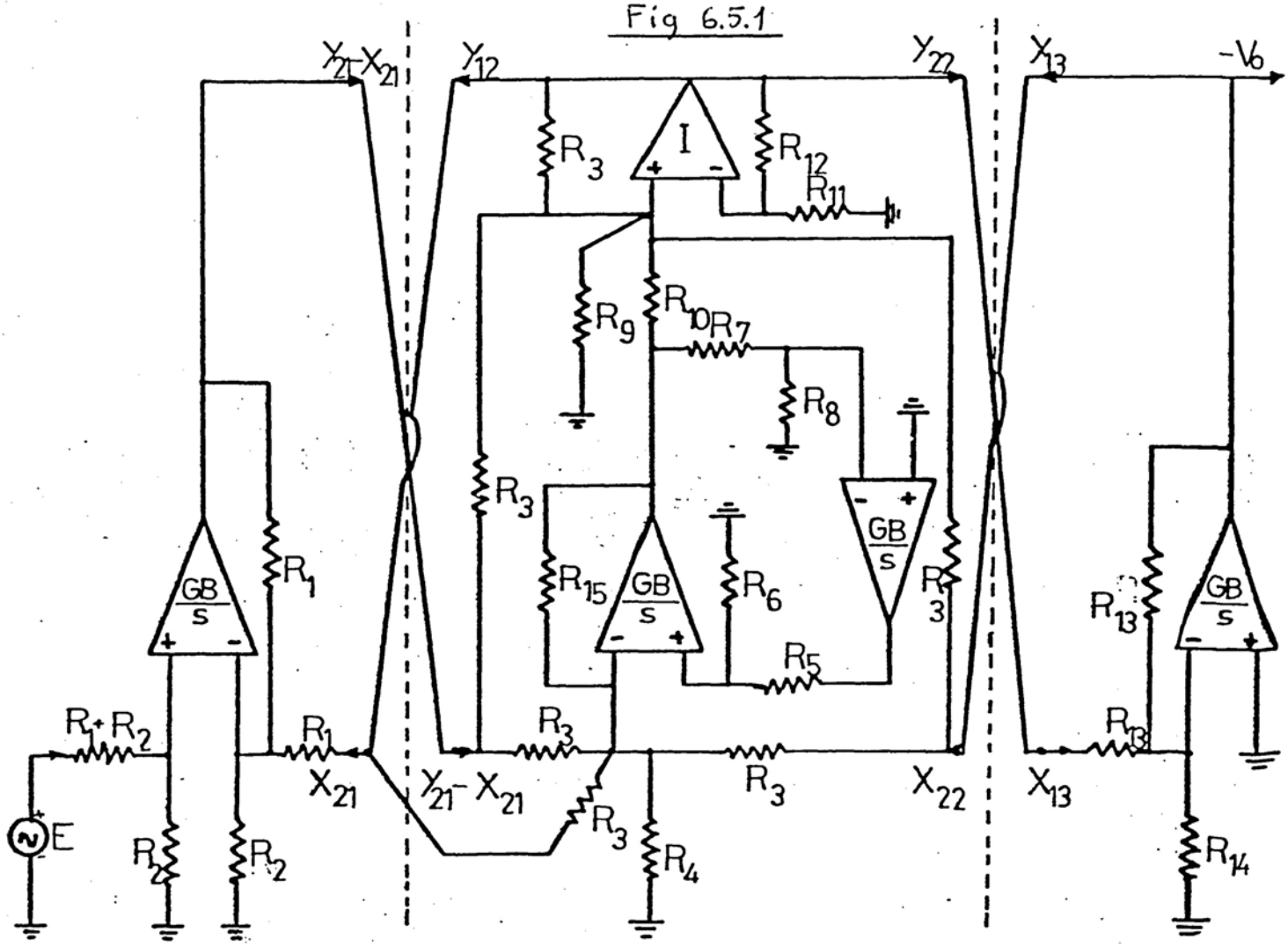


Fig. 6.5.2

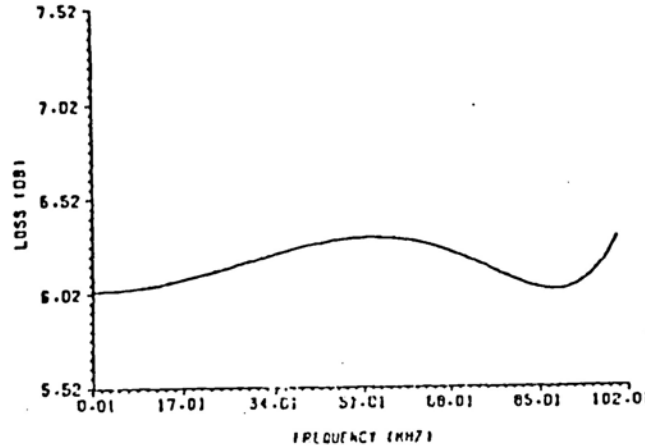


Fig. 6.5.2

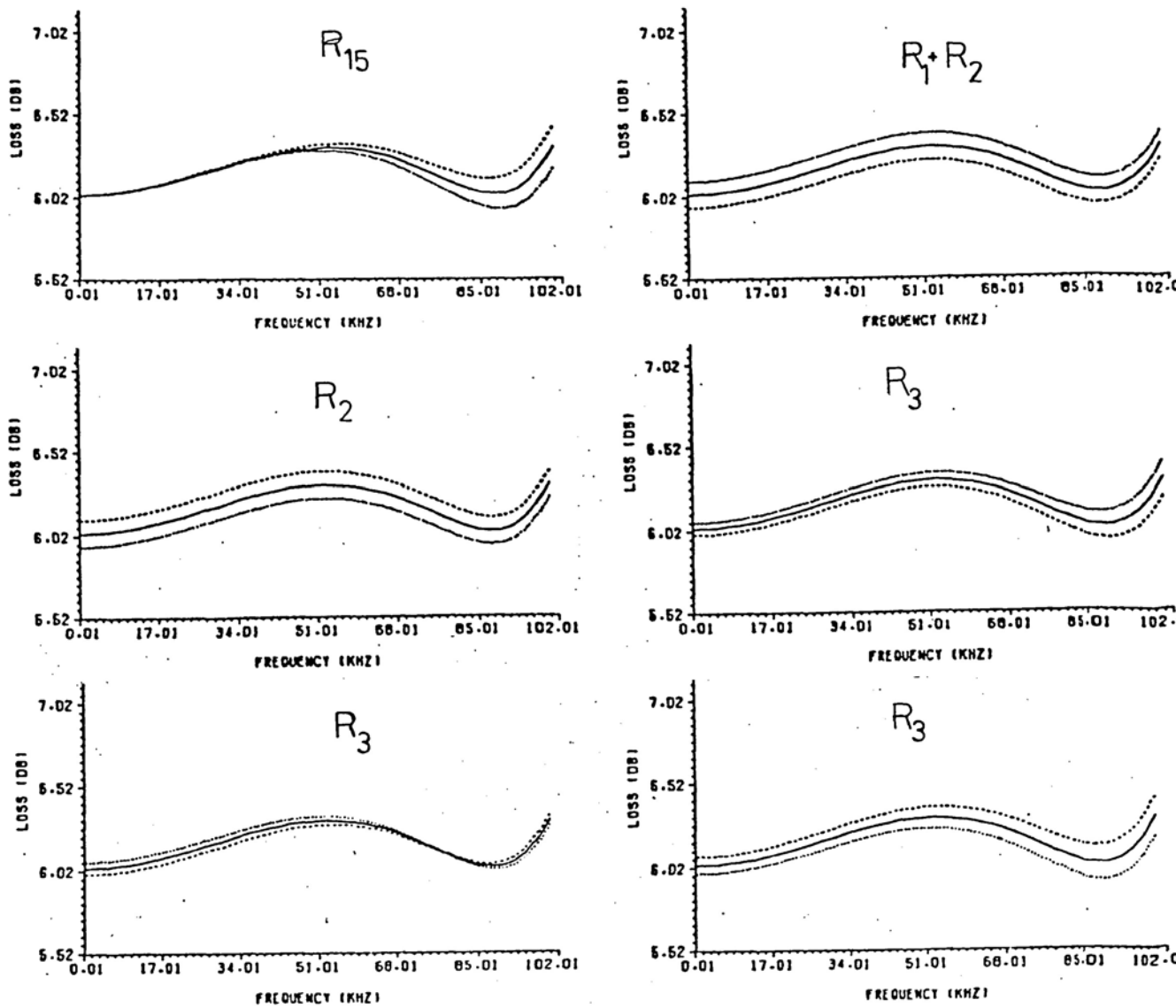


Fig. 6.5.3

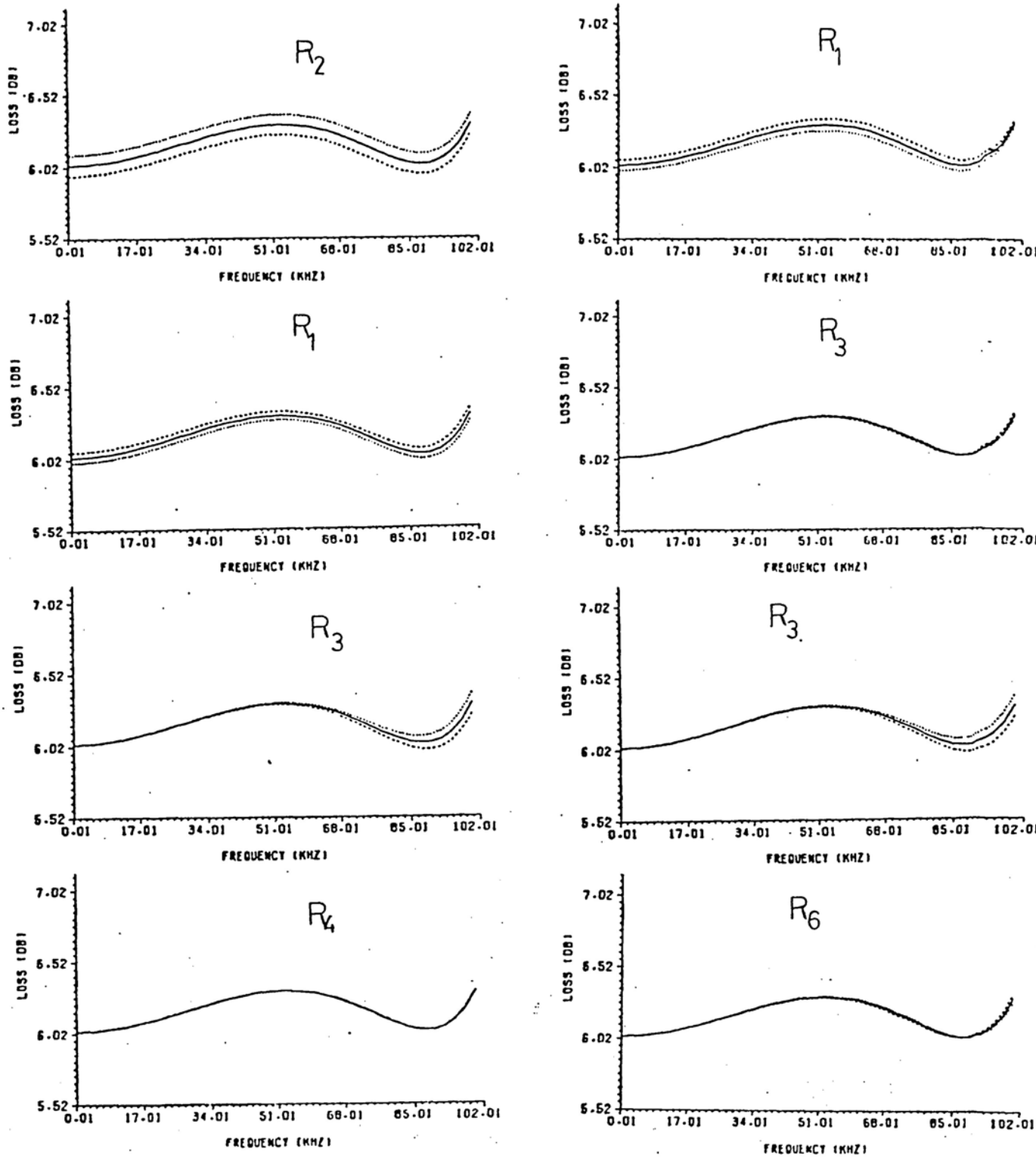

R2
R1
R1
R3
R3
R3
R4
R6
LOSS (DB)
7.02
6.52
6.02
5.52
0.01
17.01
34.01
51.01
68.01
85.01
102.01
FREQUENCY (KHZ)

Fig. 6.5.4

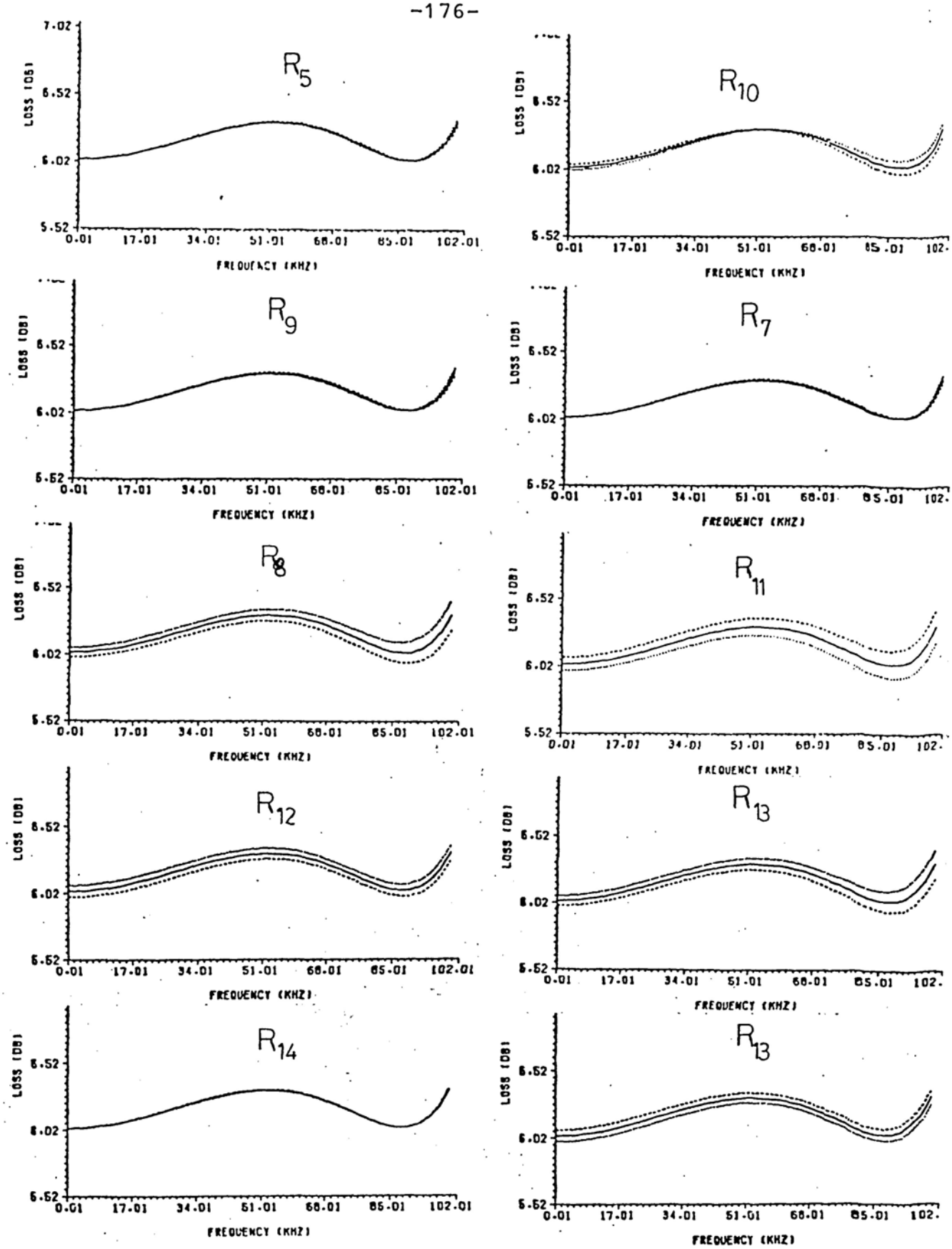

R5
R10
R9
R7
R8
R11
R12
R13
R14
R13
LOSS (DB)
FREQUENCY (KHZ)
0.01
17.01
34.01
51.01
68.01
85.01
102.01

Fig. 6.5.5

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS

FOR FURTHER RESEARCH

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The research work contained in this thesis has been primarily concerned with the development of a general theory of active filter design by employing linear transformations applied to reference analogue networks, with particular emphasis to doubly terminated ladder lossless filters. The nature of the Σ matrix established to describe each constituent network part was investigated in terms of its existence conditions and realizability conditions in the active domain.

The new approach is based on linearly tranforming the port-variable vectors of a two-port network from the V-I domain to a new X-Y domain, in which the transformed two-port is then realized. By examining the ladder filter as a cascade connection of a two-port subnetworks, the linearly transformed active structure is obtained for each of those subnetworks and by interconnecting them in an appropriate way, the Linear Transformation Active (LTA) filter simulating the original LC filter is derived.

New structures have been derived at various stages in this work as those described in Chapters 3 and 4. In Chapter 2 it is shown that already excisting methods such as the structural simulation or the operational simulation of LC ladder networks can be interpreted as special cases of the general approach.

The sensitivity performance of the general LTA structures is also examined, in Chapter 5, and some guidelines are given as to the choice of transformation matrices so as to improve such sensitivity performance in the active realiza-

tion. Practical examples are given in Chapter 6 which illustrate the flexibility, importance and usefulness of the new approach.

In the consluding parts of each chapter the questions remaining to be answered were stated in detail. Thus for example the voltage-current transformation as was described in Section 2.4 is entirely unexplored. Linear Transformation techniques, real and frequency dependent, as applied to reference networks by the one-port approach as described in Ref [TZ1] need investigation.

Although sensitivities formulae in terms of the parameters of the transformations have be given in Chapter 5, further investigation must be carried out as to express more accurately those formulae in a way for establishing optimum sensitivity performance structures within the LTA concept which, as already pointed out, . . includes almost all of the ex isting simulation methods.

The analysis procedure described in section 5.2.2 leads to transformation matrices with elements analogous to the gain-band width products of the operational amplifiers. This feature will be an interesting topic for further research.

APPENDIX A

APPENDIX A

In Linear Systems Theory, signals are functions of time which according to their properties are classified into several sets of functions as follows:

D: The set of infinitely continuously differentiable real valued functions of compact support, i.e. that are zero for values of t outside some bounded set.

D_+: The set of infinitely continuously differentiable real valued functions which are zero until some finite time, i.e. their support is bounded to the left.

D': The set of all functions f for which the inner product

$$<f,\varphi> = \int_{-\infty}^{\infty} f(t)\varphi(t)\, dt \qquad (A.1)$$

for every $\varphi \in D$ satisfies the following relations

$$<f,k_1\varphi_1+k_2\varphi_2> = k_1<f,\varphi_1>+k_2<f,\varphi_2> \qquad (A.2)$$

for k_1, k_2 real constants and

$$\lim_{i\to\infty} <f,\varphi_i> = 0 \quad \text{if} \quad \varphi_1 \to 0 \qquad (A.3)$$

L_2: The set of all square integrable functions, i.e. functions for which

$$<f,f> = M = \text{finite} \qquad (A.4)$$

When a superscript n is attached to any of those sets, i.e. D^n, D_+^n, $D^{n'}$, L_2^n, it denotes an n-dimensional vector space whose entries are taken from D, D_+, D', L_2, respectively.

Norm: The norm, when it exists, is defined as

$$||f|| = <f,f>^{1/2} \qquad (A.5)$$

Continuous Operators

Let V and W be two spaces where the norm is assured to exist and T a single valued linear operator from V to W. Then the following statements are equivalent:

a) T is continuous

b) for any $\varepsilon>0$ there is a $\delta>0$ such that for every $x,y \in V$ and $||x-y||>\varepsilon$, then $||Tx-Ty||_V<\delta$.

c) The norm of the operator T, defined as $||T|| = \text{supr.}(||Tx||/||x||)$, is finite, i.e. $||T||<\infty$.

d) if a sequence of vectors $\{x_i\}$ converges uniformly to x, then the sequence $\{Tx_i\}$ converges uniformly to Tx.

Kernels Theorem [SC1]

Every linear continuous operator from D to D' may be represented by a unique integral operator

$$y(t) = \int_{-\infty}^{\infty} H(t,\tau)x(\tau)d\tau \quad \forall\ x \in D,\ y \in D' \qquad (A.6)$$

If the operator is time invariant then $H(t,\tau)=H(t-\tau)$ and moreover if it is causal for $\tau>t, H(t-\tau)=0$ [SA4].

The Kernels Theorem also stated in a somewhat different form in Sect.1.1, which assures the existence of an integral relation between input and output, is well defined for inputs $x(t) \in D$ and outputs $y(t) \in D'$. This mapping of inputs in D to outputs in D' is quite restrictive since D is a set of ideal functions which by no means includes all the functions that appear in practice. Nevertheless, the theorem can be extended for inputs in D_+ by recalling the Extension Theorem [YO2, NE1] since $D \subset D_+ \subset D'$. Further, more, Newcomb [NE1, p.33] has shown that the integral representation exists for those inputs $x \in D'$ that can be defined by approaching x by a sequence x_i with $x_i \in D$.

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