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Don't hurry, be happy! The bright side of late product release

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Abstract. When a firm releases its product later than its competitor, the firm loses sales because many consumers prefer to buy the competitor's product rather than wait for the firm's product release. Therefore, in the absence of any late-mover advantages, conventional wisdom suggests that competing firms will release their products as soon as possible to avoid losing customers if they were to enter later. But, when the market evolves over time and consumers are forward-looking, we demonstrate that this intuition fails and propose a new explanation for why a firm may strategically release its product later than its competitor. Namely, a firm's late entry can help alleviate price competition due to some consumers' decisions to wait for the firm's product release instead of buying a currently available product. We show that in markets where the growth rate is sufficiently high and differentiation between firms is not too low, the firm's profit gain from alleviated price competition dominates its profit loss from reduced sales, making the firm better off by releasing its product later than the competitor. Surprisingly, when the fraction of consumers who enter the market relatively early increases, the firm may have even greater incentives to release its product late. Finally, we consider markets where firms are differentiated both horizontally and vertically, with one firm having a higher quality (or stronger brand image) than its competitor. We find that high level of vertical differentiation will induce both high- and low-quality firms to rush to the market, releasing their products in the early period. However, when vertical differentiation is moderately high, the high-quality firm may choose to release late, whereas the low-quality firm will prefer to release its product early.

Key words: Market entry, entry timing, late entry, competitive strategy, pricing

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1. Introduction

Conventional wisdom suggests that in a competitive market, a firm will want to release its product as fast as possible to avoid giving its competitors a head start. Nevertheless, there are examples of firms that were not very keen on expediting their product releases, even though their competitors had already released their products. For example, in the U.S. market of gaming consoles, Sega was the first to release its CD-ROM-based Saturn console in May 1995. At that time, Sony had already released its own CD-ROM-based console (PlayStation) in some markets but decided to wait till September to release it in the U.S., giving Sega a head start (IGN 2012). In the smartwatch market, Apple had started working on a smartwatch early on (Slivka 2012), and media outlets reported that works were at an advanced stage, predicting Apple's smartwatch release by the end of 2013 (Burrows and Satariano 2013; Patel 2013). Nevertheless, Apple released its first smartwatch only in 2015, more than a year after its main competitor, Samsung, released its smartwatch. One can think of many other examples of late product releases such as Samsung's first tablet Galaxy Tab (Ganapi 2010), Apple's introduction of AirPods (Kahn 2016), etc.

Past literature points out that a firm may benefit by releasing its product late in markets with substantial uncertainty or when the firm has an opportunity to significantly improve its product before releasing it to the market (e.g., Kim et al. 2022, Narasimhan and Zhang 2000, Bayus et al. 1997). Our paper differs from the prior literature in several key ways. First, we abstract away from factors like market uncertainty, product improvement, etc. Second, our analytical setting takes into account the evolution of the market over time and consumers' dynamic tradeoffs whereby a consumer may choose to wait for a more desirable product/brand instead of buying a currently available product.[2] Third, we demonstrate that when a firm releases its product later than its competitor, some consumers' decision to wait for the firm's product release will influence both firms' pricing decisions, making price competition less intense than when each firm rushes to the market early on. Fourth, we show that the profit gain from alleviated price competition can lead a firm to release its product later than its

[2] In reality, consumers often become aware of a brand's future product release well in advance due to media coverage, pre-announcements and teaser promotions, and leaks from insiders or supply chain sources (Lilly and Walters 2003, Hannigan et al. 2018). This allows consumers to make more informed decisions by accounting for their option value of waiting.

competitor, even when there is no uncertainty in the market or quality improvement to be made, thus providing a new rationale for late entry into a market.[3]

We start with the premise that markets evolve over time and consumers have the ability and choice in deciding when to purchase a product. Using these characteristics, we develop an analytical model with two horizontally differentiated firms selling a durable product. Each firm's product is ready to be released in the first period, but a firm can choose to delay its product release till the second period. At the start of each period, the firms that have already released their products simultaneously choose their prices. To capture the reality that in most product categories, the market expands over time, we assume that new consumers enter the market in both periods. After observing the first-period prices of available products, a consumer can choose to buy one of the products, but may also choose to wait if, for example, the consumer wants to buy a product that is not yet available or if the consumer anticipates the prices to be lower in the second period. Thus, our model captures the dynamic tradeoffs that consumers often face in real life. We solve for a rational-expectations equilibrium where consumers' expectations about the future are fulfilled. Our analysis reveals a number of important insights about the firms' pricing and product release timing strategies.

Although late product release reduces a firm's equilibrium sales, we find that it can also have a strategic benefit for the firm. Releasing its product later leads to less intense price competition, allowing both firms to charge higher equilibrium prices than when they release their products simultaneously. Intuitively, when a firm decides to release its product in the second period, some first-period consumers with a stronger preference for this firm's product are more inclined to wait for the firm's product release. These consumers' decisions to wait skews the distribution of consumers in the second period, giving the firm an incentive to set a high price. This in turn allows its competitor (the early entrant) to also charge a high second-period price, extracting more surplus from the segment of new consumers who enter the market in the second period. Hence, price competition becomes less intense, benefitting both firms. Note that late product release leads to lower equilibrium sales for the firm because the competitor

[3] We acknowledge that in real-life situations, firms' decisions may be affected by multiple factors. Our analysis provides an alternative rationale that could have been a contributing factor. Further, our results reveal prescriptive insights that can help managers improve firm performance. We discuss these insights in Section 6.

exploits the firm's absence and captures extra market share in the first period. Nevertheless, can the strategic competition-alleviation effect induce a firm to release its product late? We demonstrate that late product release will occur in markets where growth rate is sufficiently high and differentiation between firms is not too low.

When the fraction of consumers entering the market in the first period increases, one might expect that each firm will gain more incentives to release its product in the first period to avoid losing a large number of customers. We find that, counterintuitively, more consumers' entering in the first period can give a firm more incentives to delay its product release, making late product release more likely to happen in equilibrium. Moreover, the increase in the fraction of first-period consumers tends to strengthen the competition-alleviation effect, increasing the late entrant firm's profits even though its sales decline. These findings highlight the importance of market research for managers to evaluate how fast consumers would develop an interest in the product category, entering the market.[4] The literature on product adoption and diffusion offers some insights about how managers can do this; see the Discussion and Conclusion section for a detailed overview.

In many markets, firms have different levels of R&D expertise, which leads to some firms offering higher quality products than their competitors. Similarly, some firms tend to have stronger brand image and reputation, which is also a source of vertical advantage for such firms. Does vertical differentiation between firms (due to differences in quality or brand strength) make late product release more likely to occur in the market? Would a high-quality firm prefer to release its product early and would a low-quality firm prefer late product release? To answer these questions, we extend our core model to investigate the role of ex-ante vertical (quality) differences on the firms' product release times. We find that when one firm has a significant quality advantage over its competitor, then in equilibrium, both high- and low-quality firms gain more incentives to release their products in the first period. Thus, very high quality differences tend to make late product release less likely to occur as both firms rush to the

[4] Consumers' gradual entry into the market over time has been incorporated in many studies, including those that investigate product introduction timing strategies (e.g., Cohen et al. 1996, Klastorin and Tsai 2004). However, in these studies, it is typically assumed that consumers enter the market in equal numbers across periods. By contrast, our setting allows us to capture more realistic situations where relatively fewer consumers enter the market in the early period, and we are able to draw important insights about the firms' product release timing and pricing strategies in such markets.

market. However, in markets where consumers enter relatively fast (i.e., the fraction of consumers entering the market in the first period is sufficiently large) and firms' quality differences are moderate, we find that late product release is more likely to occur than when the firms offer similar quality. Specifically, in the unique equilibrium, the high-quality firm prefers to release its product in the second period, whereas the low-quality firm releases in the first period. Hence, our results provide an explanation why some firms with strong brand image and/or quality advantage do not always rush to start selling their products in new categories.

The rest of the paper is organized as follows. In section 2, we discuss the related literature. Section 3 describes the analytical model, and section 4 provides the analysis of the firms' equilibrium pricing and product release timing strategies. Section 5 examines a market where the firms are differentiated both horizontally and vertically. Finally, in section 6, we summarize our results, highlight their managerial implications and discuss directions of future research.

2. Background and Positioning of the Paper

Below we review the prior literature pertinent to our work. We start by sketching the literature that deals with factors that lead to delayed entry, followed by literature on pricing when consumers are forward looking and conclude with the literature on firms employing other strategies to alleviate price competition and increase profits.

Rationale for late entry

Both in marketing and management, streams of the literature have studied how entry timing influences firm strategy. For example, Carpenter and Nakamoto investigate the rationale for first-mover advantage (1989) and late entrant's advertising and pricing strategies (1990), De Castro and Chrisman (1995) study the late entrant's financial performance depending on whether the entrant prioritizes cost efficiency or product differentiation, and Klingebiel and Joseph (2015) explore how late entrant's product features differ from those of a first entrant. Note that these studies do not analyze whether a firm benefits from being late to the market, focusing instead on the firm's optimal strategy *given that* the firm is late. By contrast, some studies have investigated the firms' endogenous market entry timing decisions, focusing on factors such as investment in product performance (Bayus et al. 1997), market

uncertainty and first (late) mover advantages (disadvantages) (Narasimhan and Zhang 2000, Miller and Folta 2002), product release preannouncement (Su and Rao 2011; Ofek and Turut 2013), and entry deterrence (Shen and Villas-Boas 2010). These studies use reduced-form models where firms do not make any pricing decisions. In our setting, the firms' pricing decisions are endogenous, allowing us to explore the relationship between product release timing and the intensity of price competition. Klastorin and Tsai (2004) and Wu and Lai (2019) have analyzed a setting with myopic consumers where late product release gives a firm a vertical advantage in terms of increased product quality (Klastorin and Tsai 2004) or being perceived as more stat-of-the-art (Wu and Lai 2019), allowing the firm to charge a higher price and earn more profits. In contrast to these studies, we analyze forward-looking consumers who can choose to delay making a purchase in order to buy from a more desirable brand and/or at a lower price. We find that consumers' decisions to wait for a firm's product release give rise to a strategic competition-alleviation effect, and show that under specific market conditions, due to the competition-alleviation effect, late product release can emerge as an equilibrium outcome even if the firm does not obtain any vertical advantages by delaying its product release.

Intertemporal trade-off and strategic price competition

Our paper is related to the literature on pricing of durable goods. In a competitive market where undifferentiated firms compete over infinite periods, Gul (1987) and Ausubel and Deneckere (1987) show that high prices can be sustainable on the equilibrium path due to the existence of infinitely many grim-trigger type of equilibria where any deviation from high prices is punished by a switch to the zero-price equilibrium for the rest of the game. Thus, in Gul (1987) and Ausubel and Deneckere (1987), the equilibrium construct itself (i.e., grim trigger) allows the firms to achieve a collusive outcome. By contrast, our paper analyzes a finite-horizon market where competing firms are horizontally differentiated. Our setting precludes the existence of grim-trigger type of equilibria analyzed by Gul (1987) and Ausubel and Deneckere (1987), and we are able to characterize the uniquely optimal equilibrium prices based on the firms' concave profit optimization problems that do not hinge upon an equilibrium construct like grim-trigger. Further, note that in these studies, competing firms' products are simultaneously released to the market, whereas in our paper, equilibrium prices are influenced by the differences in the firms' product release times. More related to our paper, Eliashberg and Jeuland

(1986) and Rubel (2003) study a firm facing entry into its market. In a reduced-form demand model, Eliashberg and Jeuland (1986) show that whereas entry makes price competition more intense, the time of entry itself does *not* influence the intensity of competition. Rubel (2003) has a similar finding in a setting with uncertainty about the length of product lifetime. Hence, in Eliashberg and Jeuland (1986) and Rubel (2003), if the entrant could endogenously choose its market entry timing, then it would benefit from entering as early as possible. By contrast, by accounting for consumers' intertemporal tradeoffs, we show that a firm may want to delay its product release because doing so can alleviate the intensity of price competition.

Alleviating price competition

Prior literature has identified settings where firms' asymmetric strategies can dampen price competition, resulting in higher profits. One stream of this literature has shown how commitment to target different consumer segments can alleviate competition and improve profits. For example, Choudhary (2010) shows that price competition is less intense when one firm adopts per-unit pricing to target light-user segment whereas the competitor adopts fixed-fee pricing to target heavy users. In Harutyunyan et al. (2017), the firm that charges a lower markup than its competitor captures the segment of fairness-concerned consumers, which can lead to higher equilibrium prices and profits for both firms. Fazli and Shulman (2018) find that price competition is dampened when one firm exits a market that generates negative spillovers on the primary market, whereas the competing firm chooses to operate in the spillover-generating market. While these studies underscore the role of serving some segments and not the others, our analysis highlights the importance of the time in which a firm serves a segment. More specifically, we show that a firm's decision to release its product later than its competitor allows each firm to serve its respective share of the first-period consumer segment in different time periods (rather than in the same period) due to some first-period consumers' decisions to wait for the firm's product release in the second period. Such a difference in timing can make price competition less intense because upon releasing its product to the market, the firm has an incentive to charge a high price to exploit the segment of first-period consumers who have been waiting to buy its product. Because consumers who waited have a strong preference in favor of the firm, the firm's competitor chooses not

to compete for these consumers but to focus on the new consumers entering the market. The firm's high price allows the competitor to also charge a high price, dampening price competition.

Another stream of the literature has identified strategies that can help a firm establish more differentiation from competitors, enabling the firm to charge a higher price. For example, delaying product release can allow the firm to increase its quality and price (Klastorin and Tsai 2004, Wu and Lai 2019). When facing a high-quality competitor, a firm may benefit by reducing its product quality (Shaked and Sutton 1982, Amaldoss and Shin 2011). A multi-product firm may limit its product varieties when competing against a mass-customization firm (Alptekinoğlu and Corbett 2008), and a dominant retailer may allow the competing retailer to carry a greater assortment because doing so increases the competitor's costs, making competition less intense (Dukes et al. 2009). In our setting, the firms are horizontally differentiated, and delaying product release does not allow a firm to further increase differentiation or gain a cost advantage. Further, when the firms are differentiated not only horizontally but also vertically, late product release can become even more likely by the high-quality firm. Hence, whereas prior studies explore firm strategies to increase differentiation to make price competition less intense, we demonstrate that the existence of sufficient differentiation can make late product release a viable strategy to further alleviate competition.

Other works have shown how strategies like the adoption of most-favored-customer pricing (Cooper 1986), reduction in the firm's targeting accuracy (Chen et al. 2001), provision of hidden product benefits (Harutyunyan and Jiang 2017), and hiring of irrationally optimistic decision-makers (Jiang and Liu 2018) can all lead to less intense price competition. Consistent with this stream, we show how the decision to release its product late could be driven by the firm's desire to maximize expected profit by blunting the severity of price competition in a new product context. Thus, we add to this literature while at the same time point out how factors like growth of the market over time, consumers' forward-looking behavior, and firms' vertical and horizontal differentiation interact in interesting ways on firms' entry timing strategies.

Finally, it is well known that sequential decision-making may allow firms to achieve a better outcome than when the firms make their decisions simultaneously (von Stackelberg 1934, Belleflamme and Peitz 2015). Further, Gal-Or (1985) shows that in settings where decisions are made sequentially,

the firm that decides later can have a late-mover advantage, earning a greater payoff than the firm that decides earlier. Unlike these studies, in our setting, after releasing their products, the firms simultaneously decide their prices at the beginning of each period. Hence, the competition-alleviation effect that we find is not driven by sequential decision-making. Further, the firm that chooses to release its product relatively late earns a lower profit than the firm that releases earlier. Hence, a firm's equilibrium strategy to release its product later is not driven by any late-mover advantages.

Positioning of our paper

Our goal in this paper is to offer an explanation for a firm to delay entry to maximize its profit. Our contrast with the past literature in advancing our result is to show this in the most simplistic setting by incorporating few key features of what happens in real markets. Firms are differentiated, the market evolves over time, consumers are heterogeneous in their preferences and can choose when to buy a new product by engaging in intertemporal tradeoffs. Consumers' forward-looking behavior has been documented in experiments and on the field (e.g., Holak et al. 1987, Jacobson and Obermiller 1990, Chevalier and Goolsbee 2009) and plays an important role in dynamic settings (e.g., Gul 1987, Su 2007, Aviv and Pazgal 2008, Cachon and Swinney 2009, Rao and Turut 2019). Hence, when consumers anticipate a firm's product release in the future, consumers may consider the option to wait to buy the firm's product. While past literature has advanced trading off first-mover advantage, uncertainty about market acceptance and product improvement, our point is that even in the absence of such tradeoffs, when consumers are aware of the value of the firms' products and are willing to wait to buy their preferred product, a firm needs to consider delaying its entry under specific market conditions. This is the primary contribution of our paper.

3. Model

Two firms, 1 and 2, compete in a horizontally differentiated market, selling one durable product each. To capture differentiation between the firms, we use the standard Hotelling model (Hotelling 1929) by assuming that the firms are located at the opposite ends of the unit line $[0,1]$ (e.g., Jain 2008, Chen et al. 2009, Desai et al. 2016). Each firm has a constant marginal cost c, which is assumed to be zero without loss of generality. There are two time periods. At the beginning of the first period, both firms'

products are fully developed and are ready to be released immediately. However, a firm can also choose to release its product later (in the second period), and we let $R_i \in \{1,2\}$ denote firm i's product release period. To economize on notation and to simplify the analysis, we assume that the firms have a discount factor one. In the Web Appendix B, we demonstrate the robustness of our main results when the firms' discount factor is less than one.

New consumers enter the market in both periods. We normalize the measure of consumers entering the market to one, and we let $\alpha \in (0,1)$ be the fraction of consumers who enter in the first period, with the remaining $(1-\alpha)$ fraction entering in the second period. Consumers are heterogeneous in their preferences for the firms, which is reflected by their locations on the interval $[0,1]$. Namely, a randomly chosen consumer location is uniformly distributed on $[0,1]$. The consumer's valuation of firm i's product is given by $V - \theta d_i$, where V is the perceived value of the product if it perfectly matches the consumer's preference and $d_i \in [0,1]$ is the consumer's distance from firm i, representing the mismatch between the firm's product and the consumer's ideal.[5] The parameter $\theta > 0$ shows the consumer's sensitivity with respect to preference mismatch. Equivalently, θ captures the differentiation level between the firms (Belleflamme and Peitz 2015). We let p_{it} denote firm i's price in period $t \in \{1,2\}$. Thus, if a consumer purchases firm i's product in period $t \in \{1,2\}$, then the consumer receives a utility $V - \theta d_i - p_{it}$. Consumers also have an outside option which gives zero utility. Further, each consumer has unit demand and exits the market after purchasing a product (e.g., Moorthy and Png 1992, Klastorin and Tsai 2004). A consumer who enters the market in the first period has the option to wait till the second period to buy a product; we let $\delta \in (0,1)$ be the rate at which the consumer discounts future utility. Hence, when making a decision in the first period, the consumer evaluates the utility from purchasing firm i's product in the second period to be $\delta(V - \theta d_i - p_{i2}^e)$, where p_{i2}^e is the second-period price that the consumer expects firm i to charge. We analyze a rational-expectations equilibrium, i.e., the prices that consumers expect the firms to charge in the second period will be consistent with the

[5] Our results continue to hold if we explicitly model the value that a product provides in each period over a certain time horizon—we provide such a robustness check in Web Appendix B. The modeling approach in our main model has been used in many other studies on durable goods (e.g., Ausubel and Deneckere 1987, Gul 1987, Moorthy and Png 1992, Dhebar 1994, Klastorin and Tsai 2004), which allows to simplify the analysis and avoid making additional assumptions about the product and its consumption pattern.

firms' second-period equilibrium prices. The analysis focuses on $\theta < \bar{\theta}$, where $\bar{\theta} = \frac{V(1-\delta)}{2-\delta} < \frac{V}{2}$. Our main findings can hold if we use a less stringent upper bound on θ. However, $\theta < \bar{\theta}$ ensures two things. First, when both firms' products are released to the market, the market is fully-covered, giving rise to direct competitive friction between the firms. Second, the equilibrium with full market coverage is unique.

Below we summarize the timing of events.

- ***Firms' product release timing decisions***. The firms simultaneously decide when to release their products.
- ***Firms' pricing decisions***. After releasing its product, each firm chooses its price at the beginning of each period. If both firms have already released their products, then they choose their prices simultaneously.
- ***Consumers' purchase decisions***. After entering the market and observing the firms' current prices, consumers make their purchase decisions.

Note that in our main model, the firms correctly assess their differentiation level and consumers are aware about their preferences for the firms' products. Such awareness can be based on, for example, past experience with the firms' brands or their products in other categories. Further, well before releasing their products, firms often present them at conventions and special events, receiving extensive coverage from media outlets, bloggers, etc. Such exposure allows consumers to learn about their preferences and competitors to evaluate their differences. In the Web Appendix, we show the robustness of our results in a market where consumers still face some level of preference uncertainty.

4. Analysis

There are four subgames based on the firms' product release times: $(R_1, R_2) \in \{(1,2),(2,1),(1,1),(2,2)\}$. To find which would be an equilibrium and under what conditions, we will first obtain the firms' equilibrium profits in each subgame. Let $\pi_{it}^{(R_1,R_2)}$ denote firm i's profit in period t when the firms' product release times are (R_1, R_2). Similarly, we let $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \pi_{i2}^{(R_1,R_2)}$ be the sum of firm i's profits across periods.

4.1. Products Released in the Same Period: $(R_1, R_2) = (2, 2)$ or $(R_1, R_2) = (1, 1)$

First, consider the subgame where both firms release their products in the second period, $(R_1, R_2) = (2, 2)$. In the first period, consumers cannot buy any of the products and have to wait till the next period. In the second period, both firms release their products and simultaneously choose their prices to compete for a unit measure of consumers. Because $\theta < \bar{\theta}$, in equilibrium, all consumers will buy a product in the second period and the marginal consumer will be indifferent between the two products: $V - \theta d - p_{12} = V - \theta(1 - d) - p_{22}$, where d is the consumer's distance from firm 1. Solving this equation, we find $d = \frac{\theta - p_{12} + p_{22}}{2\theta}$. Hence, firm 1's and firm 2's second-period subgame equilibrium prices, $p_{12}^{(2,2)}$ and $p_{22}^{(2,2)}$, will maximize the firms' respective profits, $\pi_{12}^{(2,2)} = \frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22}^{(2,2)} = (1 - \frac{\theta - p_{12} + p_{22}}{2\theta}) p_{22}$. Solving the first-order conditions ($\frac{d\pi_{12}^{(2,2)}}{dp_{12}} = 0$ and $\frac{d\pi_{22}^{(2,2)}}{dp_{22}} = 0$), we find that $p_{12}^{(2,2)} = \theta$ and $p_{22}^{(2,2)} = \theta$. The analysis for the subgame with $(R_1, R_2) = (1, 1)$ is similar and we provide the details in the Web Appendix A. Lemma 1 summarizes the equilibrium prices and respective profits of the firms.

LEMMA 1. The subgame equilibrium prices and profits of firm $i \in \{1,2\}$ are as follows:

- *If* $(R_1, R_2) = (2, 2)$, *then* $p_{i2}^{(2,2)} = \theta$, $\pi_{i1}^{(2,2)} = 0$ *and* $\pi_{i2}^{(2,2)} = \frac{\theta}{2}$;
- *If* $(R_1, R_2) = (1, 1)$, *then* $p_{i1}^{(1,1)} = p_{i2}^{(1,1)} = \theta$, $\pi_{i1}^{(1,1)} = \alpha \frac{\theta}{2}$ *and* $\pi_{i2}^{(1,1)} = (1 - \alpha) \frac{\theta}{2}$.

All proofs are provided in the Web Appendix A. Note that in the subgame where the firms release their products in the first period, $(R_1, R_2) = (1, 1)$, the firms' second-period equilibrium prices are the same as in the first period. Therefore, in equilibrium, all consumers who enter the market in the first period prefer to buy a product in that period instead of waiting till the second period to buy. In the Web Appendix B, we explore the possibility of an alternative equilibrium where the firms charge high prices in the first period to serve only a fraction of the market, inducing some consumers to wait till the second period to buy a product. We demonstrate that such an equilibrium does not exist in the region of the parameters that our analysis focuses on, i.e., when the level of differentiation between the firms is not too high ($\theta < \bar{\theta}$). Hence, when both firms release their products in the first period, in the unique

equilibrium characterized in Lemma 1, the market is fully covered in each period, giving rise to direct competitive interaction between the firms. This well-known equilibrium with full market coverage in the Hoteling framework serves as a natural benchmark for investigating the implications of late product release. Let us now turn to the subgames where one of the firms releases its product later than its competitor.

4.2. Products Released in Different Periods: $(R_1, R_2) = (1, 2)$ or $(R_1, R_2) = (2, 1)$

Consider the subgame where firm 2 releases its product relatively late: $(R_1, R_2) = (1, 2)$. To find the equilibrium, we will use backward induction. Specifically, we will first obtain second-period subgame equilibrium prices and profits, after which we will find firm 1's first-period equilibrium price.

In the second period, the firms compete for the segment of new consumers entering the market, as well as the segment of first-period consumers who had chosen to wait till the second period to buy a product. Therefore, to find the firms' second-period subgame-equilibrium prices, we first need to characterize the first-period consumers who will still be shopping for a product in the second period. Namely, this segment consists of consumers who anticipated to receive a higher utility by purchasing a product in the second period than by purchasing firm 1's product in the first period, i.e.,

$$\max\{\delta(V - \theta d - p_{12}^e), \delta(V - \theta(1-d) - p_{22}^e)\} > V - \theta d - p_{11}, \quad (1)$$

where $d \in [0,1]$ is the consumer's distance from firm 1 on the Hotelling line and p_{i2}^e is the price that consumers expect firm i to charge in the second period. The left-hand side of (1) is the utility that the consumer anticipates receiving if she makes a purchase in the second period. The right-hand side of (1) is the consumer's utility when the consumer purchases firm 1's product in the first period. Each triplet $(p_{11}, p_{12}^e, p_{22}^e)$ leads to a certain second-period subgame, because p_{11}, p_{12}^e and p_{22}^e determine which first-period consumers postpone their purchases till the second period. In the Web Appendix A, we show that the inequality (1) reduces to the following:

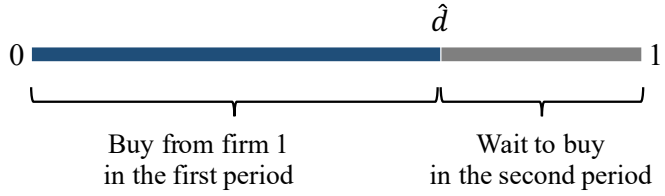
$$d > \hat{d}, \quad (2)$$

where

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta \\ \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)} & \text{if } V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq V(1-\delta) + \delta p_{12}^e \end{cases} \quad (3)$$

The four possible values that $\hat{d}$ can take capture four possible scenarios that may occur in a subgame equilibrium. First, if p_{11} is very low, then all consumers prefer to buy firm 1's product in the first period instead of waiting till the second period. Hence, $\hat{d} = 1$. Second, if p_{11} is in the lower intermediate region, then most consumers buy firm 1's product in the first period, whereas the remaining consumers prefer to wait till the second period to buy firm 2's product. Hence, the marginal consumer with $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)}$ is indifferent between buying firm 1's product in the first period and buying firm 2's product in the second period. Third, if p_{11} is in the upper intermediate region, then some of the consumers who prefer not to purchase in the first period anticipate to buy firm 1's product in the second period (expecting a price reduction) whereas others anticipate to buy firm 2's product. The marginal consumer with $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$ is indifferent between buying firm 1's product in the first period and buying it in the second period. Finally, when p_{11} is very high, then all consumers prefer to wait till the second period to buy a product, i.e., $\hat{d} = 0$. Figure 1 graphically illustrates the consumers' decisions in the first period.

Figure 1 Consumers' Decisions in the First Period



Let us now turn to the firms' second-period pricing decisions. The firms compete for the $(1-\alpha)$ measure of new consumers entering the market, as well as the $\alpha(1-\hat{d})$ measure of first-period consumers who had chosen not to buy in the first period. Note that the second-period distribution of consumers is skewed to the right because first-period consumers who chose to wait tend to be located

closer to firm 2. We let $p_{i2}^{(1,2)}$ denote firm i's second-period subgame equilibrium price, and we look for a rational-expectations equilibrium where $p_{i2}^{(1,2)} = p_{i2}^{e}$ for $i = 1, 2$.

LEMMA 2. *Suppose that* $(R_1, R_2) = (1, 2)$. *If* $0 < \alpha < \bar{\alpha}$ *and* $\underline{\delta} < \delta < 1$, *then there exists a rational-expectations second-period subgame equilibrium* $(p_{12}^{(1,2)}, p_{22}^{(1,2)})$ *where*

$$p_{12}^{(1,2)} = p_{12}^{e} = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2\alpha(p_{11} - V(1-\delta)) + \theta(3-\alpha)(1+\delta)}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11} - V(1-\delta)) + 3\theta(1-\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{4}$$

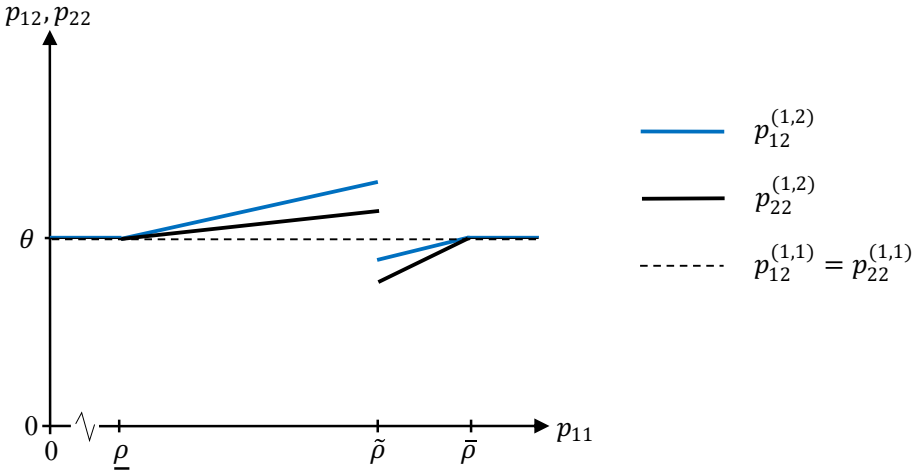
$$p_{22}^{(1,2)} = p_{22}^{e} = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2\alpha(p_{11} - V(1-\delta)) + \theta(3-(3-2\alpha)\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{5}$$

where $\bar{\alpha} \approx 0.64$, $\underline{\delta} \equiv \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$, $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$.

The conditions on α and δ in Lemma 2 are necessary to ensure the existence of a rational-expectations subgame equilibrium for *each* possible $p_{11} \geq 0$. If these conditions do not hold, then there exists a small interval of first-period prices for which there is no corresponding rational-expectations equilibrium in second-period prices, i.e., for some p_{11}, there does not exist a corresponding pair (p_{12}^{e}, p_{22}^{e}) satisfying $(p_{12}^{(1,2)}, p_{22}^{(1,2)}) = (p_{12}^{e}, p_{22}^{e})$. Hence, in the rest of the paper, we assume that the conditions on α and δ in Lemma 2 are satisfied. Since our main results occur strictly within the interval $[0, \bar{\alpha})$, focusing on $\alpha < \bar{\alpha}$ does not affect our key findings or managerial insights.

Figure 2 shows that when $(R_1, R_2) = (1, 2)$, second-period price competition can be more intense or less intense than when $(R_1, R_2) = (1, 1)$. The relative intensity of price competition will depend on firm 1's first-period price, p_{11}.

Figure 2 Second-Period Subgame Equilibrium Prices [6]



To see how firm 1's first-period price p_{11} affects the second-period prices $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$, first note that when p_{11} is very low ($p_{11} < \underline{\rho}$), in equilibrium, all consumers who enter the market in the first period prefer to buy firm 1's product instead of waiting till the second period to buy. By contrast, when p_{11} is very high ($p_{11} > \bar{\rho}$), all consumers prefer to wait and purchase a product in the second period. In either case, the distribution of consumers in the second period is the same—uniform on the interval $[0,1]$—which is the reason why the second-period equilibrium prices are constant on these intervals: $p_{12}^{(1,2)} = p_{22}^{(1,2)} = \theta$.[7] By contrast, when firm 1's price is neither too low nor too large ($\underline{\rho} \leq p_{11} \leq \bar{\rho}$), some but not all consumers prefer to wait till the second period to buy a product. This changes the distribution of consumers in the second period, influencing the firms' equilibrium prices.

Next, consider p_{11} in the intermediate region $[\underline{\rho}, \bar{\rho}]$. If p_{11} is in the lower intermediate region ($p_{11} \in [\underline{\rho}, \tilde{\rho}]$), then second-period price competition is alleviated, whereas if p_{11} is the upper intermediate region ($p_{11} \in (\tilde{\rho}, \bar{\rho}]$), then second-period competition becomes more intense. This is where the interesting effect of consumers' waiting decisions is revealed. First, when p_{11} is in the lower

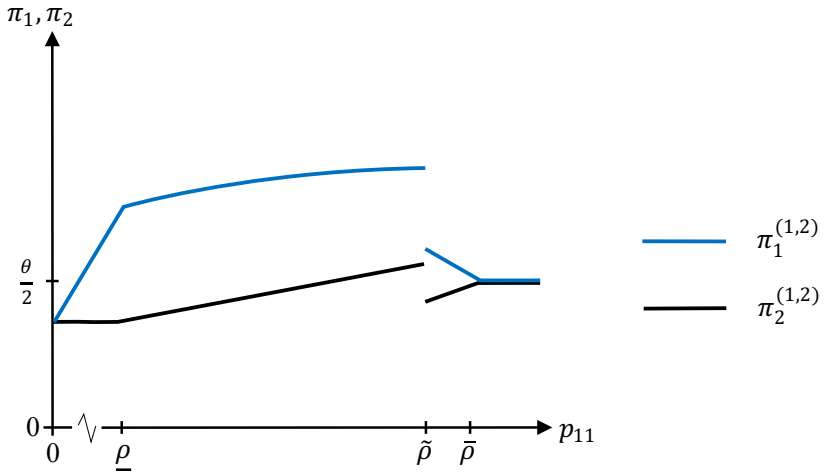
[6] Figure 2 is obtained using the following numerical values: $\alpha = 0.5$, $\theta = 0.3$, $\delta = 0.95$, $V = 10$. Under these values, $\underline{\rho} \approx 0.48$, $\tilde{\rho} \approx 0.71$ and $\bar{\rho} \approx 0.78$. Also, recall from Lemma 1 that $p_{12}^{(1,1)} = p_{22}^{(1,1)} = \theta$.

[7] A similar situation occurs in the subgame with $(R_1, R_2) = (1,1)$. We know from Lemma 1 that $p_{11}^{(1,1)} = p_{21}^{(1,1)} = \theta$. If firm 1 charges some $p_{11} \neq \theta$, then given the pair (p_{11}, θ) of first-period prices, all consumers rationally prefer to buy in the first period rather than wait (see the proof of Lemma 1). Therefore, the distribution of consumers in the second period stays uniform, leading to constant second-period equilibrium prices: $p_{12}^{(1,1)} = p_{22}^{(1,1)} = \theta$. This is represented by the dotted line in Figure 2.

intermediate region, most consumers prefer to buy firm 1's product in the first period except consumers with a very strong preference for firm 2's product (i.e., consumers located close to firm 2), who choose to wait till the second period to buy from firm 2. In the second period, the presence of first-period consumers with a strong preference for its product gives firm 2 an incentive to charge a high price to extract more surplus from this segment of consumers. Firm 1 does not want to compete for these consumers, knowing that they have a very strong preference in firm 2's favor. Instead, firm 1 focuses on new consumers entering the market, and firm 2's high price allows firm 1 to also charge a high price. Hence, second-period price competition becomes less intense. Second, when firm 1's first-period price is in the upper intermediate region (i.e., $p_{11} \in [\tilde{\rho}, \bar{\rho}]$), only a small number of consumers (located very close to firm 1) buy its product in the first period, while most consumers prefer to wait till the second period to buy either from firm 1 (at a lower price) or from firm 2. Given the large number of first-period consumers who chose to wait, in the second period, firm 1 wants to compete for a positive market share in this segment of consumers. Moreover, since firm 1 has already served those first-period consumers who have a high willingness to pay for its product, firm 1 has an incentive to compete more aggressively on price in the second period in order to attract the remaining consumers with relatively lower willingness to pay (located further away from firm 1). Firm 1's charging a low price compels firm 2 to also charge a low price in order not to lose too many sales. Hence, when $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, second-period price competition becomes more intense, leading to low prices for both firms' products.

Using the second-period equilibrium prices in Lemma 2, we can obtain the firms' second-period subgame equilibrium profits ($\pi_{12}^{(1,2)}$ and $\pi_{22}^{(1,2)}$), as well as the cutoff $\hat{d}$ in equation (3) which characterizes firms 1's sales in the first period; the mathematical expressions are provided in the Web Appendix A. In the first period, firm 1 chooses its price p_{11} to maximize its overall profit $\pi_1^{(1,2)} = \pi_{11}^{(1,2)} + \pi_{12}^{(1,2)}$, where $\pi_{11}^{(1,2)} = \alpha \hat{d} p_{11}$ is firm 1's first-period profit and $\pi_{12}^{(1,2)}$ is firm 1's second-period subgame equilibrium profit. Figure 3 illustrates firm 1's and firm 2's profits depending on firm 1's first-period price, p_{11}.

Figure 3 Effect of First-Period Price on Profits [8]



First, we can see from Figure 3 that, as one would expect, firm 1's profits exceed firm 2's profits due to firm 1's earlier product release. Second, notice that when firm 1's first-period price increases above $\tilde{\rho}$, firm 1's profits jump down. This is because, as we discussed earlier, second-period price competition is more intense when firm 1's first period price is in the upper-intermediate region. Further, consumers rationally anticipate that second-period prices will be low, gaining more incentives to wait instead of buying firm 1's product in the first period. Thus, charging a price in the upper-intermediate region ($p_{11} \in [\tilde{\rho}, \bar{\rho}]$) significantly reduces firm 1's first-period sales and profits. Lemma 3 shows that in equilibrium, firm 1 strategically chooses its price in the lower-intermediate region.

LEMMA 3. *When* $(R_1, R_2) = (1, 2)$, *firm 1's first-period equilibrium price satisfies* $p_{11}^{(1,2)} \in [\underline{\rho}, \tilde{\rho}]$ *and is given by*

$$p_{11}^{(1,2)} = \begin{cases} \underline{\rho} & \text{if } 0 < \theta \leq \dot{\theta} \\ \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \tilde{\rho} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad (6)$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$, $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$, $\underline{\rho} = (1-\delta)(V-\theta)$, *and* $\tilde{\rho} = (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$.

Upon plugging firm 1's first period price in the equation (6) into the firms' second-period subgame equilibrium prices in the equations (4)–(5), we obtain the second-period prices on the equilibrium path:

[8] Figure 3 is obtained using the following numerical values: $\alpha = 0.3$, $\theta = 0.4$, $\delta = 0.95$, $V = 10$.

$$p_{12}^{(1,2)} = \begin{cases} \theta & \text{if } 0 < \theta \leq \dot{\theta} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3V(1-\alpha)\alpha(1-\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad (7)$$

$$p_{22}^{(1,2)} = \begin{cases} \theta & \text{if } 0 < \theta \leq \dot{\theta} \\ \frac{9\theta(1+\delta)-6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad (8)$$

Using the firms' equilibrium prices in the equations (6)-(8), we can readily obtain the firms' equilibrium profits, which we provide in the Web Appendix A. Note that since the firms are symmetric, when $(R_1, R_2) = (2, 1)$, the firms' equilibrium prices and profits are the reverse of their prices and profits when $(R_1, R_2) = (1, 2)$.

4.3. Equilibrium Product Release Times

We now turn to the firms' product release timing decisions. As one would expect, at least one of the firms will prefer to release its product in the first period. Put differently, in equilibrium, both firms will not want to wait till the second period to start selling their products. This is intuitive; if one firm waits till the second period to release its product, then the other firm can capture more sales and charge a higher price by releasing its product in the first period, earning a greater profit than if it were to also wait till the second period. For expositional convenience, let us assume that firm 1 will release its product in the first period (i.e., $R_1 = 1$) and explore firm 2's decision when to release its product.

Because firm 2's product is ready for a release, one may intuit that firm 2 will prefer to release in the first period because otherwise, firm 2's competitor, firm 1, will have an advantage in the first period, capturing a bigger market share. Proposition 1 shows that, surprisingly, firm 2 can be better off by releasing its product in the second period.

PROPOSITION 1.

a) *If the fraction of consumers entering the market in the first period is not too high and the differentiation level between the firms is not too low, then in equilibrium, firm 2 releases its product later than firm 1. Mathematically, there exist* $0 < \hat{\alpha} < \bar{\alpha}$ *and* $0 < \hat{\theta} < \bar{\theta}$*, such that* $(R_1^*, R_2^*) = (1, 2)$ *if and only if* $0 < \alpha < \hat{\alpha}$ *and* $\hat{\theta} \leq \theta < \bar{\theta}$.

b) When firm 2 releases its product later than firm 1, second-period price competition is less intense than when firm 2 also releases its product in the first period, i.e., $p_{12}^{(1,2)} > p_{12}^{(1,1)}$ *and* $p_{22}^{(1,2)} > p_{22}^{(1,1)}$.

Proposition 1 makes clear when a firm would delay entry. But why would a firm enter late foregoing sales in the first period? Intuitively, firm 2 correctly anticipates that upon its late product release, firm 1 will strategically choose its first-period price to be neither too high nor too low (as shown in Lemma 3) so that firm 1 will serve most of the first-period market except those consumers who have a very strong preference for firm 2's product; these consumers will, in equilibrium, choose to wait till the second period to buy from firm 2. As we discussed earlier, consumers waiting to buy firm 2's product helps alleviate price competition because they skew the distribution of consumers in the second period in firm 2's favor, giving firm 2 an incentive to charge a higher price than under simultaneous entry. Firm 2's high price allows firm 1 to also charge a high price, extracting more profits from new consumers entering the market in the second period. Thus, as part *b*) in Proposition 1 shows, firm 2's late product release provides a strategic benefit in terms of higher equilibrium prices. Part *a*) in Proposition 1 shows that firm 2's profit gain from high equilibrium prices can more than compensate for the negative effect from reduced sales. However, as we explain below, this effect is not ubiquitous.

As the number of consumers entering in the first period (α) increases, the negative pressure on firm 2 also increases both due to the lost sales in the first period and the decrease in the potential sales ($1-\alpha$) in the second period. Hence, when $\alpha > \hat{\alpha}$, the strong negative effect of late product release on firm 2's equilibrium sales dominates the gains due to alleviated price competition, making firm 2 worse off if it releases its product late. By contrast, when $\alpha < \hat{\alpha}$, late product release does not cost firm 2 too many sales, and firm 2's profit gain from less intense price competition can potentially dominate its profit loss from reduced sales. In this respect, the level of differentiation between the firms (θ) plays a crucial role. Note that in general, the lower θ, the higher the intensity of price competition. Hence, one might intuit that late product release, as a strategy to mitigate competition, would be most attractive to firm 2 when the differentiation level θ is low. This logic, however, ignores firm 1's strategic decisions. Specifically, when the differentiation level between the firms is small, firm 1 is able to easily capture

extra market share in the first period by slightly reducing its first-period price, exploiting firm 2's late product release. Thus, in equilibrium, very few (if any) consumers wait till the second period to buy firm 2's product. This has a strong negative impact on firm 2's sales and also weakens the competition-alleviation effect, making firm 2 worse off than when it releases in the first period. By contrast, when the differentiation level between the firms is high ($\theta > \hat{\theta}$), consumers tend to have a stronger incentive to wait for firm 2's product, making it more difficult for firm 1 to capture extra market share in the first period. Therefore, with high θ, the number of first-period consumers choosing to wait till the second period to buy firm 2's product is also high. This helps reduce the negative effect of late product release on firm 2's sales to an extent that firm 2's profit loss from reduced sales becomes dominated by firm 2's profit gain from alleviated price competition, i.e., firm 2 becomes better off by releasing its product in the second period instead of doing so in the first period.

When firm 2 releases its product in the second period, the higher the fraction of consumers entering the market in the first period (α), the lower firm 2's equilibrium sales. Hence, one might presume that firm 2 would benefit from fewer consumers entering the market in the first period, or equivalently, from a larger proportion of consumers entering in the second period. Proposition 2 reveals that, surprisingly, the opposite may hold: firm 2 can be better off if the fraction of consumers who enter the market in the first period increases.

PROPOSITION 2. *Suppose that* $0 < \alpha < \hat{\alpha}$ *and* $\hat{\theta} < \theta < \bar{\theta}$*, so that in equilibrium* $(R_1^*, R_2^*) = (1, 2)$*. Then, the larger the fraction of consumers entering the market in the first period* (*i.e., the higher* α)*, the lower firm 2's equilibrium sales. However, firm 2's profits are non-monotone in* α*, and an increase in* α *can make firm 2 better off provided that* α *is not too high.*

To understand the rationale behind Proposition 2, note that a greater fraction of consumers entering the market in the first period helps further alleviate price competition in the second period. This happens because when the fraction of first-period consumers increases, the number of consumers who choose to wait till the second period to purchase firm 2's product also increases. As a result, given the larger number of consumers waiting to buy its product, firm 2 gains an incentive to charge a higher price in the second period, which, in turn, allows firm 1 to do the same, thereby mitigating second-period price

competition even further. Proposition 2 shows that firm 2's profit gain from less intense price competition can outweigh its losses from decreased sales, ultimately improving firm 2's profitability.

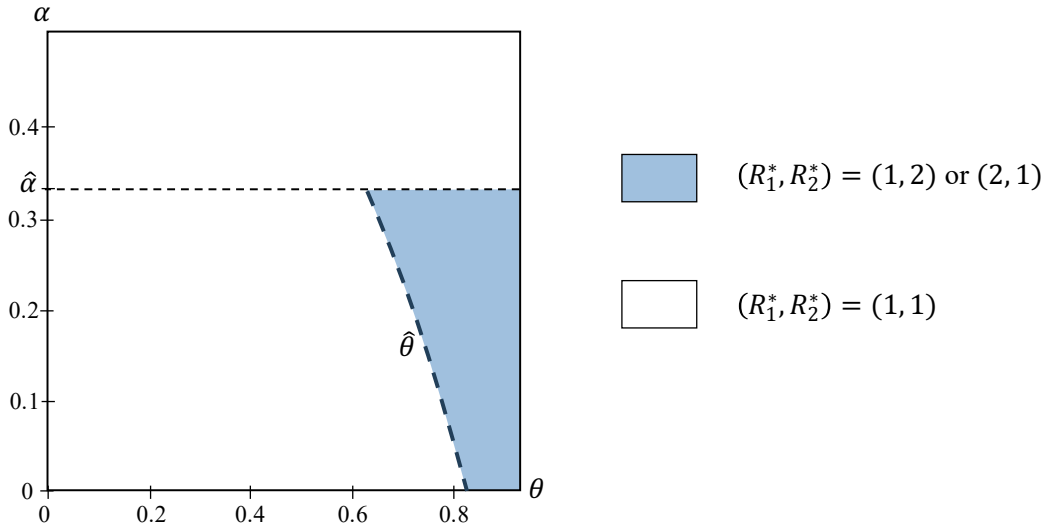
When more consumers enter the market in the first period, one might expect that both firms will be more likely to rush to the market, releasing their products in the first period. Proposition 3 demonstrates that this intuition may not always hold.

PROPOSITION 3. *An increase in the fraction of consumers who enter the market in the first period makes firm 2 more likely to release its product in the second period, i.e., the parameter region on which* $(R_1^*, R_2^*) = (1, 2)$ *expands as* α *grows as long as* $\alpha < \hat{\alpha}$. *Mathematically,* $\frac{d\hat{\theta}}{d\alpha} < 0$ *at any* $\alpha \in (0, \hat{\alpha})$.

It would be helpful to first demonstrate the mathematical reasoning behind Proposition 3 and then provide an intuitive explanation. For a given $0 < \alpha < \hat{\alpha}$, the cutoff $\hat{\theta}$ satisfies $\pi_2^{(1,2)}|_{\theta=\hat{\theta}} - \pi_2^{(1,1)}|_{\theta=\hat{\theta}} = 0$, making firm 2 indifferent between releasing in the first period or doing so in the second period. Totally differentiating with respect to α, we find that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} + \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} - \frac{d\pi_2^{(1,1)}}{d\alpha}|_{\theta=\hat{\theta}} - \frac{d\pi_2^{(1,1)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} = 0$. Recall from Lemma 1 that $\pi_2^{(1,1)} = \frac{\theta}{2}$, which implies that $\frac{d\pi_2^{(1,1)}}{d\alpha} = 0$ and $\frac{d\pi_2^{(1,1)}}{d\theta} = \frac{1}{2}$. Hence, $\frac{d\hat{\theta}}{d\alpha} = \frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} \left(\frac{1}{2} - \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}}\right)^{-1}$. First, one can show that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$. Second, $\frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} > \frac{d\pi_2^{(1,1)}}{d\theta}|_{\theta=\hat{\theta}} = \frac{1}{2}$. The inequalities $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} > \frac{1}{2}$ imply that $\frac{d\hat{\theta}}{d\alpha} < 0$. Now, let us explain the intuition behind the mathematical proof. When the differentiation level between the firms (θ) decreases, firm 2's profits drop regardless of its product release time. However, the reduction in its profits is higher when firm 2 releases its product in the second period than when firm 2 releases in the first period. This is because late product release puts firm 2 at a more vulnerable position as it cannot directly compete for the first-period consumers, and a decrease in firm differentiation aggravates firm 2's vulnerability. Hence, the lower the differentiation level, the weaker firm 2's incentives to release its product in the second period. By contrast, an increase in the fraction of first-period consumers (α) strengthens the competition-alleviation effect, giving firm 2 stronger incentives to release in the second period. Therefore, when α grows, firm 2's profit gain due to a

stronger competition-alleviation effect can make late product release an optimal strategy at lower levels of θ, i.e., less differentiation is required to induce firm 2 to release its product in the second period. The result in Proposition 3 is illustrated in Figure 4 which shows the firms' equilibrium product release times in the (θ, α) parameter space.

Figure 4 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [9]



Note that due to the symmetry between the firms, there also exists an equilibrium with $(R_1^*, R_2^*) = (2, 1)$ on the same parameter region where $(R_1^*, R_2^*) = (1, 2)$ is an equilibrium. These equilibria are qualitatively similar and allow us to make a clear prediction: the competing firms will avoid releasing their products in the same period, and hence, one of the firms will end up releasing later than its competitor. Put differently, in a market where one firm has just released its product and the competitor can also do so immediately, our analysis shows that the competitor may choose to strategically delay its product release. In addition, note that some firms tend to have more efficient managerial structure and decision-making processes than others, enabling them to make their decisions faster. In our setting, if firm 1 decides its product release time R_1 before firm 2 decides R_2, then firm 1 will optimally choose $R_1 = 1$ because $\pi_1^{(1,2)} > \pi_1^{(2,1)}$, inducing firm 2 to choose $R_2 = 2$. Thus, with sequential decision making—which is often the case in reality—we can identify a unique equilibrium in product release times, and all insights from our main model continue to hold.

[9] Figure 4 is drawn using the following parameter values: $V = 10$ and $\delta = 0.9$.

5. Effect of Vertical Differentiation on Release Timing

Some firms excel their competitors in terms of R&D expertise and capabilities, which allows such firms to develop higher quality products than competitors. Similarly, a firm may have stronger brand image or reputation, which is another source of vertical advantage akin to product quality. How do vertical differences influence the firms' product release timing strategies? Which firm is more likely to release late/early? To answer these questions, we extend our main model to analyze a market where firms are differentiated both vertically and horizontally. Specifically, if a product perfectly matches a consumer's horizontal preference, then the consumer's valuation for firm 2's product is $V + \mu$, whereas the valuation for firm 1's product is V. The parameter $\mu \geq 0$ represents firm 2's advantage over firm 1 which may stem from, for example, firm 2's higher product quality. We designate firm 1 as the "*low-quality firm*" and firm 2 as the "*high-quality firm*". We assume that $\mu < 3\theta$ to ensure that, in equilibrium, firm 1 obtains a positive market share. All remaining assumptions are the same as in the main model, and consistent with the main model, we focus on $0 < \alpha < \bar{\alpha}$, $\underline{\delta} < \delta < 1$, and $\theta < \bar{\theta}$, where $\bar{\alpha} \approx 0.64$, $\underline{\delta} \equiv \max\{\frac{5(1-\alpha)}{2(3-\alpha)}, \frac{18-10\alpha}{36-\alpha(45-11\alpha)}\}$ and $\bar{\theta} = \frac{V(1-\delta)}{2-\delta}$. The technical analysis is available in the Web Appendix B.

Proposition 4 describes the firms' product release timing strategies based on the market characteristics in terms of the fraction of consumers entering the market early and the two dimensions of firm differentiation (vertical and horizontal).

Proposition 4.

a) *The low-quality firm releasing its product later than the high-quality firm*—$(R_1^*, R_2^*) = (2, 1)$—*is an equilibrium if and only if the fraction of first-period consumers is low* $(0 < \alpha < \hat{\alpha})$, *vertical differentiation is not too high* $(0 < \mu < \hat{\mu}_1)$, *and horizontal differentiation is sufficiently high* $(\hat{\theta}_1 < \theta < \bar{\theta})$.

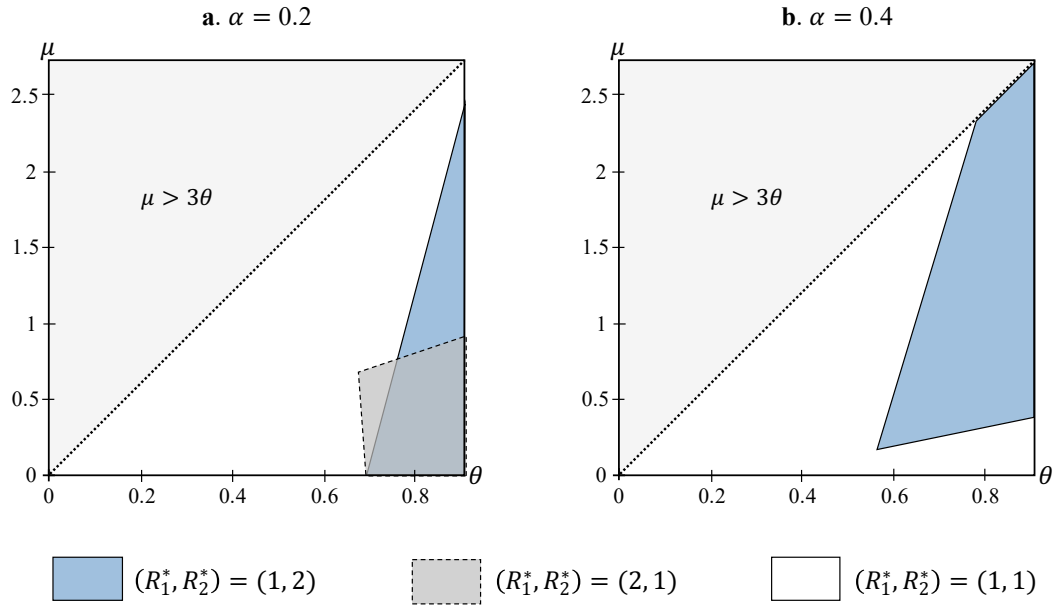
b) *The high-quality firm releasing its product later than the low-quality firm*—$(R_1^*, R_2^*) = (1, 2)$—*is an equilibrium if and only if one of the following conditions holds*:

i. the fraction of first-period consumers is low $(0 < \alpha < \hat{\alpha})$, *vertical differentiation is not too high* $(0 < \mu < \hat{\mu}_2)$, *and horizontal differentiation is sufficiently high* $(\hat{\theta}_2 < \theta < \bar{\theta})$.

ii. the fraction of first-period consumers is moderately high ($\hat{\alpha} < \alpha < \tilde{\alpha}$), *vertical differentiation is neither too high nor too low* ($\tilde{\mu}_2 \leq \mu < \hat{\mu}_2$), *and horizontal differentiation is sufficiently high* ($\hat{\theta}_2 < \theta < \bar{\theta}$).

All cutoffs in Proposition 4 are explicitly defined in its proof in the Web Appendix B. The results in Proposition 4 are graphically illustrated in Figure 5.

Figure 5 Effect of Vertical Differentiation on Product Release Times [10]



In the previous section, we already explained why horizontal differentiation (θ) must be sufficiently high for late product release to be optimal. Hence, let us focus on the role of vertical differentiation (μ). The first part of Proposition 4 is intuitive. Specifically, if firm 1's quality disadvantage is very high ($\mu > \hat{\mu}_1$), then very few consumers will wait for firm 1's product release in the second period. Further, as we previously discussed, the smaller the number of consumers waiting for the firm's product release, the weaker the competition-alleviation effect in the second period. Hence, when μ is large, firm 1's gains from delaying its product release are small, whereas its losses from reduced sales are significant. For this reason, late product release can be optimal for the low-quality firm (firm 1) only if its quality disadvantage is not too large ($\mu < \hat{\mu}_1$).

[10] Figure 5 is drawn using the following parameter values: $V = 10$, $\delta = 0.9$. In Figure 5a, $\alpha = 0.2 < \hat{\alpha}$, whereas in Figure 5b, $\alpha = 0.4 > \hat{\alpha}$.

We now turn to the equilibrium where the high-quality firm (firm 2) releases its product late. One might expect that when firm 2's quality advantage is very high, firm 2 would have even more incentives to delay its product release in order to alleviate price competition, knowing that consumers have stronger incentives to wait for its product due to its much superior quality. Contrary to this intuition, Proposition 4 shows that when its quality advantage is very high ($\mu > \hat{\mu}_2$), firm 2 actually prefers to release its product in the first period. To see the rationale, note that its high quality allows firm 2 to charge a high price. Hence, each sale is very profitable for firm 2, making firm 2 unwilling to give up any sales by delaying its product release even if doing so would have allowed firm 2 to charge a higher price in the second period. By contrast, when $\mu < \hat{\mu}_2$, its small quality advantage allows firm 2 to increase its price by only a small amount. In this situation, firm 2 can be better off by releasing its product late, giving up some market share but gaining more from the strategic competition alleviation effect due to its late product release. Interestingly, Part *b.ii* in Proposition 4 reveals that, unlike the low-quality firm, the high-quality firm may have an incentive to release its product even when the fraction of consumers entering in the first period is moderately high ($\hat{\alpha} < \alpha < \tilde{\alpha}$). This is illustrated in Figure 5b. To see the intuition, first recall from Proposition 1 that in the absence of any vertical differentiation ($\mu = 0$), when $\alpha > \hat{\alpha}$, both firms release their products in the first period to avoid losing a large number of customers. This is still the reason why the low-quality firm chooses to release in the first period when $\alpha > \hat{\alpha}$. However, from the high-quality firm's perspective, having a sufficiently large quality advantage ($\mu > \tilde{\mu}_2$) allows the firm to avoid losing too many customers if it releases its product late. This is because consumers would be willing to wait to buy a product that has a significantly higher quality. Thus, when the fraction of consumers is moderately high ($\hat{\alpha} < \alpha < \tilde{\alpha}$), unlike the low-quality firm, the high-quality firm will prefer to release in the second period in order to alleviate price competition provided that the firm's quality advantage is neither too low nor too high ($\tilde{\mu}_2 < \mu < \hat{\mu}$).

Our findings in part *b* in Proposition 4 contrast with the established perspective in the extant literature about the relationship between a firm's product quality and its product release timing strategy. Specifically, many studies have shown that a firm may want to release its product late in order to

improve it and obtain a quality advantage. By contrast, our analysis reveals that having an *ex-ante* quality advantage could give a firm an incentive to release its product relatively late.[11]

6. Discussion and Conclusion

In this section, we summarize our contributions and findings, key insights, discuss boundary conditions and directions for future research.

Summary and prescriptive insights. Market evolution over time and consumers deciding when to buy are common characteristics of many markets. In such a setting, we studied competition between two firms that have fully-developed products and are ready to release them to the market. Our results reveal that release time is a strategic choice even when there is no uncertainty about market acceptance, opportunity to improve a product or targeting specific segments. We show that a firm may choose to release its product later than its competitor instead of rushing into the market, thus offering a new rationale for late entry. This is because late product release gives rise to a strategic benefit by alleviating the intensity of price competition. By entering late, a firm captures those consumers who would prefer to wait and buy its product which skews the distribution of the consumers in its favor, providing an incentive to charge a higher price than otherwise. This competition-alleviation effect can offset the profit reduction from lost sales due to late product release, leading to an equilibrium of late entry. We explicitly characterize the firm- and consumer-related market conditions under which late product release arises as an equilibrium. Specifically, this happens when the horizontal differentiation between the firms is sufficiently high and the fraction of consumers entering the market relatively early is not too high. Interestingly, more consumers entering the market early on can strengthen the competition-alleviation effect, giving the firm even more incentives to delay its product release despite higher losses in sales.

In an extension, we demonstrate that the existence of a significant quality difference can make both high- and low-quality firms more likely to release their products in the first period, making price competition more intense. However, provided that the quality differences are not too large, we show

[11] In the Web Appendix B, we demonstrate the robustness of our findings when delaying product release allows a firm to further improve its product quality.

that the existence of quality differentiation can actually make late product release more likely to occur in equilibrium. Counterintuitively, in a unique equilibrium, the high-quality firm is the one that chooses to release its product relatively late.

Our findings offer important prescriptive insights to managers when making product release timing decisions. To estimate whether the benefits from late product release can outweigh the losses, our analysis highlights the importance of market research to evaluate the level of differentiation between competing firms, as well as how fast consumers would develop an interest in the product category, entering the market. Hence, our analysis shows how combining insights from competitor and consumer analysis can lead to better managerial decisions. We discuss these variables in more detail below.

Past research has found evidence that both demographic (e.g., age, income, education) and psychographic (e.g., innate innovativeness) characteristics of a consumer influence how fast the consumer will develop an innovation adoption intention (e.g., Manning et al. 1995, Im et al. 2003, Rogers 2010). For example, consumers with a more innovative psychographic profile tend to enter the market earlier. Consistent with these insights, in our model, we assume that only a fraction $\alpha \in (0,1)$ of consumers enter the market in the first period. The firms can assess the profiles of relevant consumer segments and estimate the share of consumers with characteristics that are predictive of a high likelihood of early adoption. Another approach is to use past data combined with survey data to assess how large the market is likely to be when the firm plans to enter. If the diffusion were to follow the Bass model, having the best estimates for the parameters in the model, one can forecast how fast or slow the market is likely to evolve over time. Knowing when the firm is likely to enter as a late entrant, it can estimate the potential size $(1 - \alpha)$ of the market when it enters. In this context, Sultan et al. (1990) and Van den Bulte and Stremersch (2004) provide the methodologies for using meta-analytic data to predict model parameters. Van den Bulte (2002) provides a very nice practical illustration of how this can be done across geographical markets and categories using past data. Lilien et al. (2000) provide the model parameter estimates from several categories, and a firm can use these to find similar markets to its own. Finally, if attribute information about the category can be collected using survey prior to launch, these can be included to obtain forecast before launch (see for example Lee et al. 2014, Orbach and

Fruchter 2011, Urban et al. 1990 among others).[12] Such analysis may help the managers estimate the negative effect on sales due to late product release and compare it to the anticipated upside due to higher price margin. In our main model with symmetric firms, the critical threshold of α below which late product release (given sufficient product differentiation) can occur is slightly above 32% (i.e., $\hat{\alpha} \approx 0.32$). Low values of α are relevant to product categories that are more innovative/technological (Rogers 2010).[13] Lilien et al. (2000) show that the average estimate of the innovation coefficient across more than 50 product categories is 3.7%. This is well within the above threshold. Importantly, our results reveal that more consumers' entering the market early on does not necessarily imply that the managers should rush releasing their products. In fact, the opposite may hold because of the strengthening of the competition-alleviation effect. Thus, instead of hastening the release of the product, the managers may want to focus on communication with consumers to persuade them to wait for the firm's product release.

Another important parameter for our results is the differentiation level between the firms. In practice, market research can help managers understand the competitors' positioning, identify differences between the competitors and the firm, and evaluate consumers' perception of these differences, giving managers a sense about the perceived differentiation level of the firms within the spectrum of consumers' preferences or tastes (MacMillan and McGrath 1997). In our model, the differentiation parameter θ can be interpreted as the price difference needed for a firm to persuade its competitor's customers to buy its product. In our analysis, we identify the expression for the critical level of differentiation ($\hat{\theta}$) that is needed to justify late product release. We provide this expression in the Web Appendix (equation A20). Using market research data, managers can estimate $\hat{\theta}$, as well as conduct conjoint analysis or surveys to find the actual θ.

If the market conditions are appropriate for following a late product release strategy, managers should realize the importance that consumers who enter the market earlier and whose preferences match the firm's product/brand choose to wait for the firm's product instead of opting for an alternative. To

[12] We are very grateful to Christophe Van den Bulte and Seethu Seetharaman for their generous help in providing us with examples of the relevant research.

[13] For example, smartwatch adoption rate in the U.S. was only 3–4% in 2015 but reached 12% in 2016 as consumers became more interested in wearable technologies (Deloitte 2018).

this end, managers should clearly communicate information regarding their upcoming products to the relevant consumer segments, including the key points of difference from competitors and the time when the product will become available for sale. For example, many firms provide early samples to influencers and media outlets, present their upcoming products at conventions and special events, or launch informative advertising campaigns. Further, the firms can preannounce their product release times in advance during special media events, which help disseminate information. Such actions can increase the likelihood that consumers who expect to find a good fit with the firm's product (or brand) will choose to wait for it, increasing the firm's sales and strengthening the strategic competition-alleviation effect.

Our findings allow us to make several interesting predictions about market outcomes, which we formulate as hypotheses.

- *Competing firms' price mark-ups are likely to be higher when the firms release their products in different time periods. The firm that releases its product first reduces its markup following the competitor's product release. After the reduction, the firm's markup is lower than that of the competitor.*
- *Firms are more likely to release their products in different time periods in markets where*
    - *the differentiation level between firms is moderately high,*
    - *a significant fraction of consumers enters late.*
- *In markets where the perceived difference in qualities is very high, both high- and low-quality firms are likely to release their products early. However, if the quality difference is moderately high, the high-quality firm is more likely to release later than the low-quality firm, provided that a significant fraction of consumers enters late.*

We hope empirical research will be able to test these hypotheses about the firms' product release timing and pricing strategies, and their relationship with the market factors that drive our results.

Boundary conditions. We outline two boundary conditions for our results. First, our main analysis focuses on markets where the horizontal differentiation level between the firms is not too high, giving rise to direct competitive friction between the firms once they release their products to the market. By

contrast, in markets where the differentiation level is very high, the firms can avoid competition, essentially acting like local monopolists. In such markets, late product release will not have a competition-alleviation effect, thus making it more profitable for the firms to release their products as early as possible. Hence, *high level of horizontal differentiation* between the firms is a boundary condition for our results. Second, our insights may not apply to *repeat-purchase product categories*, where consumers can have demand for the product in every period, thus returning to the market even if they have recently purchased a product. In such markets, a firm's product release timing decision and consumers' waiting decisions do not influence the distribution of consumers in the market, removing the mechanism for price competition alleviation that we find in this paper. Thus, firms selling repeat purchase products are less likely to experience alleviated price competition by delaying their product release times.

Directions for further research. In some product categories (especially for big-ticket items), used-goods markets play an important role. In our analytical setting, the existence of a used-goods market could prompt some first-period consumers to buy firm 1's product to use it until firm 2 releases its product at which point these consumers would resell firm 1's product to buy from firm 2. Such behavior would increase the number of first-period consumers wishing to buy firm 2's product. This is because firm 2's product will be demanded by not only consumers who did not buy a product in the first period but also the ones who resold firm 1's product to buy from firm 2. Hence, firm 2 may have more incentives to charge a high price in the second period. Firm 1, on the other hand, will have to compete with the used-goods market in the second period, and therefore, may not be able to charge as high a price as without a used-goods market. If the size of the used-goods market is small (e.g., due to transaction costs), then its negative effect on firm 1's second-period price should also be small, making it likely that we will still observe a competition-alleviation effect similar to our main model. We hope future research can formally tackle this interesting problem. It is likely that some aspects of our setting will need to be abstracted away from or simplified in order to make the technical analysis tractable in a rational expectations framework.

Our analysis focused on finding a rational-expectations equilibrium, similar to numerous prior studies. Future research can explore firms' product release timing decisions in a setting where

consumers are boundedly rational. Bounded rationality could manifest itself, for example, in the formation of consumers' expectations about future prices whereby consumers would rely on a simple heuristic to form their expectations.[14] Our analysis of a rational-expectations equilibrium serves as an important benchmark for understanding the effect bounded rationality.

Future analytical research can investigate the role of price preannouncement on product release timing. On the one hand, by preannouncing a low price, the firm can avoid losing too many sales due to its late product release. On the other hand, the firm's ability to preannounce a low price can weaken the competition-alleviation effect, inducing both firms to charge lower prices than without preannouncement. One can explore the interplay between these effects and study when a firm should preannounce its price if it plans to release its product relatively late. Further, empirical research can investigate a number of interesting hypotheses based on our findings. We outlined several such hypotheses earlier in this section. We think that testing the relationship between product release timing and prices may be difficult as we do not observe what the firms' prices would have been if their product release times were different. However, in many product categories, we observe new generations of products being introduced over time by competing firms. In the future, researchers might have enough cross-generational data on firms' product release times and price mark-ups to investigate the relationship between the two.

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WEB APPENDIX A

PROOF OF LEMMA 1. Consider the subgame where $(R_1, R_2) = (1,1)$. We solve the game by backward induction using the guess-and-verify approach. Specifically, we start with a guess that in the first period, the market is fully covered, and hence, no consumer chooses to wait till the second period to buy a product (we will later verify our guess).[15] Mathematically, this is equivalent to

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\} \quad \text{(A1)}$$

for all $d \in [0,1]$, where p_{12}^e and p_{22}^e are the prices that first-period consumers expect firms 1 and 2 to charge in the second period, respectively.

If the inequality (A1) is satisfied, then in the second period, the firms compete for the $(1-\alpha)$ measure of new consumers who enter the market. Given the firms' second-period prices p_{12} and p_{22}, a consumer located at a distance d from firm 1 will prefer firm 1's product over firm 2's if and only if $V - p_{12} - \theta d > V - p_{22} - \theta(1-d)$, or equivalently, $d < \frac{\theta - p_{12} + p_{22}}{2\theta}$. Hence, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta})p_{22}$. Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$), we find that $p_{12}^{(1,1)} = p_{22}^{(1,1)} = \theta$. The corresponding second-period profits are given by $\pi_{12}^{(1,1)} = \pi_{22}^{(1,1)} = \frac{(1-\alpha)\theta}{2}$. Rational-expectations condition requires that $p_{12}^e = p_{22}^e = \theta$. Plugging these prices into (A1), the inequality (A1) becomes

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \max\{\delta(V - \theta - \theta d), \delta(V - \theta - \theta(1-d))\} \quad \text{(A2)}$$

Hence, if a given pair (p_{11}, p_{21}) of first-period prices satisfies the inequality (A2), then *a*) $p_{12}^e = p_{22}^e = \theta$ are rational expectations about the second-period prices, and *b*) all first-period consumers rationally choose to buy in the first period rather than wait till the second period.

In the first period, the firms choose their prices to maximize the sum of their profits over the two periods. If prices p_{11} and p_{21} satisfy (A2), then the firms' profits are given by $\pi_1 = \pi_{11} + \pi_{12}^{(1,1)} =$

[15] In Web Appendix B, we show that there does not exist an alternative equilibrium in which some of the consumers who enter the market in the first period prefer to wait till the second period to buy a product.

$\alpha\frac{\theta-p_{11}+p_{21}}{2\theta}p_{11}+\frac{(1-\alpha)\theta}{2}$ and $\pi_2=\pi_{21}+\pi_{22}^{(1,1)}=\alpha(1-\frac{\theta-p_{11}+p_{21}}{2\theta})p_{21}+\frac{(1-\alpha)\theta}{2}$. Solving the first-order conditions, we find that $p_{11}^{(1,1)}=p_{21}^{(1,1)}=\theta$. Firm i's corresponding profits in each period are given by $\pi_{i1}^{(1,1)}=\frac{\alpha\theta}{2}$ and $\pi_{i2}^{(1,1)}=\frac{(1-\alpha)\theta}{2}$.

Because $\delta<1$, it is easy to see that the price pair $(p_{11},p_{21})=(\theta,\theta)$ indeed satisfies the inequality (A2). Hence, our initial guess that first-period consumers will not wait till the second period is verified.

Finally, let us check that firm i does not have any non-local profitable deviations from its first-period price $p_{i1}^{(1,1)}=\theta$. Given firm 2's first-period price $p_{21}^{(1,1)}=\theta$, suppose that firm 1 deviates to a price p_{11}'. Because $\theta<\bar{\theta}=\frac{V(1-\delta)}{2-\delta}$, one can show that the inequality (A2) is satisfied for any pair (p_{11}',θ). If first-period prices satisfy (A2), then as we discussed earlier, all first-period consumers would rationally choose to purchase in the first period instead of waiting till the second period. It follows that firm 1's deviation to p_{11}' affects firm 1's first-period market share but does not induce any consumer to wait till the second period instead of buying in the first period. If none of the first-period consumers waits till the second period, then we know that firm 1's second-period subgame equilibrium profits are $\pi_{12}=\frac{(1-\alpha)\theta}{2}$. Hence, firm 1's deviation profit is given by $\pi_1'=\alpha\frac{\theta-p_{11}'+\theta}{2\theta}p_{11}'+\frac{(1-\alpha)\theta}{2}$. From earlier analysis, we know that π_1' is maximized when $p_{11}'=\theta$. Therefore, firm 1's deviation to a price $p_{11}'\neq\theta$ cannot be profitable.

The analysis for the subgame $(R_1,R_2)=(2,2)$ is provided in the main text. ■

PROOF OF LEMMA 2. We solve the game using backward induction. In the second period, the firms compete for $(1-\alpha)$ new consumers entering the market, as well as those first-period consumers who had decided not to buy in the first period but wait. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize those first-period consumers who will prefer not to buy in the first period but wait. Specifically, a consumer will choose to wait if and only if

$$\max\{\delta(V-p_{12}^e-\theta d),\delta(V-p_{22}^e-\theta(1-d))\}>V-p_{11}-\theta d. \tag{A3}$$

The left-hand-side of the inequality shows the utility that a consumer anticipates to receive if she chooses to wait till the second period to buy. The right-hand-side is the utility that the consumer can

obtain by purchasing firm 1's product in the first period. One can readily show that $\max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\} - (V - p_{11} - \theta d)$ is increasing in d. Therefore, if the inequality (A3) is satisfied for some $d \in [0,1]$, then there must exist a unique $\hat{d}$ such that the inequality holds if and only if $d \in (\hat{d}, 1]$. Hence, to know which first-period consumers will still be shopping in the second period, we need to find $\hat{d}$. There are four possible situations that may happen in equilibrium.

i. *No consumer chooses to wait*. In this case, $\hat{d} = 1$. Therefore, it must be that even the consumer located furthest away from firm 1 prefers to buy firm 1's product in the first period rather than wait till the second period: $V - p_{11} - \theta > \max\{\delta(V - p_{12}^e - \theta), \delta(V - p_{22}^e)\}$. Because, in equilibrium, firm 2 must have a positive market share in the second period and consumers' price expectations must be rational, we have $V - p_{12}^e - \theta < V - p_{22}^e$, i.e., the consumer with $d = 1$ would buy from firm 2 in the second period.[16] Hence, $V - p_{11} - \theta > \max\{\delta(V - p_{12}^e - \theta), \delta(V - p_{22}^e)\} = \delta(V - p_{22}^e)$, which is equivalent to $p_{11} < V(1-\delta) + \delta p_{22}^e - \theta$.

ii. *A positive measure of consumers chooses to wait and all of them anticipate purchasing firm 2's product in the second period*. Hence, the consumer with $d = \hat{d}$ must be indifferent between buying firm 1's product in the first period and buying firm 2's product in the second period. Solving the equality $V - p_{11} - \theta\hat{d} = \delta(V - p_{22}^e - \theta(1-\hat{d}))$, we obtain $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)}$. It must be that $0 < \hat{d} < 1$ and $V - p_{22}^e - \theta(1-\hat{d}) \geq V - p_{12}^e - \theta\hat{d}$. These inequalities hold when $V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2}$.

iii. *A positive measure of consumers chooses to wait; some of these consumers anticipate purchasing firm 1's product in the second period, while others expect to buy firm 2's product*. In this case, the marginal consumer with $d = \hat{d}$ must be indifferent between purchasing firm 1's product in the first period and purchasing it in the second period. Solving the equality $V - p_{11} - \theta\hat{d} =$

[16] More formally, we know that both firms will have positive market share in the second period. Therefore, one can show that their second-period prices will satisfy $0 < \frac{\theta - p_{12}^* + p_{22}^*}{2\theta} < 1$, where $\frac{\theta - p_{12}^* + p_{22}^*}{2\theta}$ is the location of the marginal consumer who is indifferent between the two products. Rational expectations require that $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$, which implies that $0 < \frac{\theta - p_{12}^e + p_{22}^e}{2\theta} < 1$. Finally, the last inequality implies that $V - p_{12}^e - \theta < V - p_{22}^e$.

$\delta(V - p_{12}^e - \theta\hat{d})$, we obtain $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$. It must be that $0 < \hat{d} < 1$ and $V - p_{12}^e - \theta\hat{d} > V - p_{22}^e - \theta(1 - \hat{d})$. These inequalities are equivalent to $\frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e$.

iv. *All consumers choose to wait till the second period*. In this case, $\hat{d} = 0$. That is, the consumer with $d = 0$ prefers to wait instead of buying firm 1's product in the first period which implies that all other consumers also prefer to wait. Mathematically, $V - p_{11} \leq \max\{\delta(V - p_{12}^e), \delta(V - p_{22}^e - \theta)\}$. Because firm 1 will have positive market share in the second period and consumers' price expectations are rational, it must be that $V - p_{12}^e > V - p_{22}^e - \theta$, i.e., the consumer with $d = 0$ would buy from firm 1 in the second period. Therefore, $V - p_{11} \leq \delta(V - p_{12}^e)$, which is equivalent to $p_{11} \geq V(1-\delta) + \delta p_{12}^e$.

To summarize the above, we considered the four possible situations that may happen in equilibrium and characterized $\hat{d}$ in each situation. Further, in each case, we obtained the conditions on p_{11}, p_{12}^e and p_{22}^e to ensure that consumers' decisions to wait are optimal given their expectations about second-period prices. Specifically, $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta \\ \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)} & \text{if } V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq V(1-\delta) + \delta p_{12}^e \end{cases} \quad \text{(A4)}$$

Outline of solution steps. To find the second-period subgame equilibrium prices that satisfy the rational expectations conditions, we will use the standard "guess and verify" approach. *Step 1*: we start by making a guess that p_{11} satisfies the conditions corresponding to one of the four possible cases outlined above. *Step 2*: solving the first-order conditions, we obtain the firms' second-period subgame equilibrium prices p_{12}^* and p_{22}^* as a function of p_{12}^e, p_{22}^e and p_{11}. *Step 3*: because we are looking for a rational-expectations equilibrium, we solve the equations $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ to find the price pair (p_{12}^e, p_{22}^e) that satisfies the condition for rational-expectations: $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$. *Step 4*: to verify the guess that we started with, we plug the prices $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$ back into the condition that we assumed to hold in Step 1. This allows us to find conditions on p_{11} under which the triplet

$(p_{11}, p_{12}^e, p_{22}^e) = (p_{11}, p_{12}^*, p_{22}^*)$ satisfies the conditions of the case, verifying our initial guess. *Step 5*: given the conditions on p_{11} that we obtained in Step 4, we check whether any firm has a profitable non-local deviation. We repeat Steps 1-5 for each of the four possible cases.

Case i: $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$. In this case, $\hat{d} = 1$, i.e., all first-period consumers buy firm 1's product in the first period. Hence, in the second-period, firms compete for the new consumers entering the market. Firms 1 and 2 choose prices to maximize $(1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $(1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta})p_{22}$, respectively. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations require that $p_{i2}^e = p_{i2}^* = \theta$. Note that Case *i* assumes that $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$. We need to check that this inequality is satisfied. Substituting $p_{22}^e = \theta$ into the inequality $p_{11} \leq V(1-\delta) + \delta p_{22}^e - \theta$, it follows that the condition for case *i* is satisfied when $p_{11} \leq \underline{\rho}$, where $\underline{\rho} \equiv (1-\delta)(V-\theta)$. It is straightforward to show that the firms' profit functions are quasi-concave, and thus, the firms do not have any profitable non-local deviations.

Case ii: $V(1-\delta) + \delta p_{22}^e - \theta < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2}$. In this case, we have $\hat{d} = \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)}$. In the first period, consumers with $d \leq \hat{d}$ bought firm 1's product, while the remaining consumers with $d > \hat{d}$ preferred to wait till the second period, anticipating to buy firm 2's product. Since we are solving for a rational-expectations equilibrium, the firms' second-period subgame equilibrium prices must indeed be such that consumers with $d > \hat{d}$ buy firm 2's product. This happens when firm 2's second-period price is sufficiently low: $p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$. In addition, to have a positive market share, firm 1's price must not be too high: $p_{12} < p_{22} + \theta$, or equivalently, $p_{22} > p_{12} - \theta$. When $p_{12} - \theta \leq p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$, the firms' second-period profits are

$$\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$$

$$\pi_{22} = (\alpha(1-\hat{d}) + (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}))p_{22}$$

Solving the first-order conditions ($\frac{d\pi_{12}}{dp_{12}} = 0$ and $\frac{d\pi_{22}}{dp_{22}} = 0$), we obtain the candidate equilibrium prices: $p_{12}^* = \frac{2\alpha(p_{11} - V - p_{22}^e\delta + V\delta) + \theta(3(1+\delta) - \alpha - 3\alpha\delta)}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \frac{4\alpha(p_{11} - V - p_{22}^e\delta + V\delta) + \theta(3(1+\delta) + \alpha(1-3\delta))}{3(1-\alpha)(1+\delta)}$. To

ensure that the rational-expectations condition is satisfied, we need that $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$. Solving these equations, we find that $p_{12}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^e = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. Plugging p_{12}^e and p_{22}^e into the expressions for p_{12}^* and p_{22}^*, we verify $p_{12}^* = p_{12}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = p_{22}^e = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. For future reference, using p_{22}^e, we find that $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p_{22}^e+\theta\delta}{\theta(1+\delta)} = \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))}$.

Next, we need to verify the assumptions that we used in the analysis. First, case *ii* assumes that p_{11} satisfies $V(1-\delta)+\delta p_{22}^e-\theta < p_{11} \leq \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2}$. Plugging in the expressions for p_{12}^e and p_{22}^e, we find that $V(1-\delta)+\delta p_{22}^e-\theta < p_{11} \leq \frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2}$ is equivalent to $\underline{\rho} < p_{11} \leq \frac{2V(3-2\alpha)(1-\delta)+\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, where $\underline{\rho} = (1-\delta)(V-\theta)$. Second, recall that to ensure that both firms have positive market shares and that consumers with $d > \hat{d}$ indeed buy firm 2's product, we assumed that the firms' prices would satisfy $p_{12}-\theta \leq p_{22} \leq p_{12}-\theta+2\theta\hat{d}$. One can readily show that when $\underline{\rho} < p_{11} \leq V(1-\delta)+\frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, the equilibrium prices p_{12}^* and p_{22}^* indeed satisfy $p_{12}^*-\theta \leq p_{22}^* \leq p_{12}^*-\theta+2\theta\hat{d}$.

Firm 1's profit function is not necessarily quasi-concave because firm 1 may potentially want to make a non-local deviation to a much lower price in order to capture some of the consumers with $d > \hat{d}$. Hence, we need to rule out such deviations. To serve some of the consumers located on $(\hat{d}, 1]$, firm 1 will need to deviate to a price $p_{12}' < p_{22}^*+\theta-2\theta\hat{d}$, in which case firm 1's deviation profit will be $\pi_{12}' = (\alpha(\frac{\theta-p_{12}'+p_{22}^*}{2\theta}-\hat{d})+(1-\alpha)\frac{\theta-p_{12}'+p_{22}^*}{2\theta})p_{12}'$. Note that π_{12}' is concave. Evaluating the left-derivative of π_{12}' at the point $p_{12}' = p_{22}^*+\theta-2\theta\hat{d}$, we find that $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} = \frac{V(6-7\alpha+3\alpha^2)(1-\delta)-p_{11}(6-7\alpha+3\alpha^2)-\theta(3-\alpha)(1-3\delta+2\alpha\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))}$. Next, $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} \geq 0$ if and only if $p_{11} \leq \tilde{\rho}$, where $\tilde{\rho} \equiv (1-\delta)V+\frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ satisfies $\underline{\rho} < \tilde{\rho} < V(1-\delta)+\frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$. Since π_{12}' is concave, $\frac{\partial_-\pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} \geq 0$ implies that π_{12}' is increasing in p_{12}' at any $p_{12}' < p_{22}^*+\theta-2\theta\hat{d}$. Since firm 1's profit function is continuous at the point $p_{22}^*+\theta-2\theta\hat{d}$, it follows that a deviation

to $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$ cannot be profitable whenever $\underline{\rho} < p_{11} \leq \tilde{\rho}$. Because $\tilde{\rho} < V(1-\delta) + \frac{\theta(3-\alpha)(3\delta-1)}{6-4\alpha}$, our initial guess holds. Therefore, the prices $p_{12}^* = p_{12}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = p_{22}^e = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$ constitute a rational-expectations subgame equilibrium for any given $p_{11} \in (\underline{\rho}, \tilde{\rho}]$.

Case iii: $\frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e$. In this case, $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p_{12}^e}{\theta(1-\delta)}$. In the first period, consumers with $d < \hat{d}$ bought firm 1's product. The marginal consumer with $d = \hat{d}$ preferred to wait because the consumer anticipated that buying firm 1's product in the second period will give greater utility. So, in a rational-expectations equilibrium, firm 1 must have a positive market share among consumers who are located on the interval $(\hat{d}, 1]$, which happens when firm 1's second-period price is low enough: $p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$. Also, firm 2 will have a positive market share only if $p_{22} < p_{12} + \theta$, or equivalently, if $p_{12} > p_{22} - \theta$. When $p_{22} - \theta < p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$, the firms' second-period profit functions are as follows:

$$\pi_{12} = (\alpha(\frac{\theta-p_{12}+p_{22}}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta-p_{12}+p_{22}}{2\theta})p_{12}$$

$$\pi_{22} = (\alpha\frac{\theta-p_{22}+p_{12}}{2\theta} + (1-\alpha)\frac{\theta-p_{22}+p_{12}}{2\theta})p_{22}$$

Solving the first-order conditions ($\frac{\partial\pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial\pi_{22}}{\partial p_{22}} = 0$) and plugging in $\hat{d} = \frac{V(1-\delta)-p_{11}+\delta p_{12}^e}{\theta(1-\delta)}$, we obtain the subgame equilibrium prices: $p_{12}^* = \theta - \frac{4\alpha(V-V\delta-p_{11}+p_{12}^e\delta)}{3(1-\delta)}$ and $p_{22}^* = \theta - \frac{2\alpha(V-V\delta-p_{11}+p_{12}^e\delta)}{3(1-\delta)}$. To ensure that the rational-expectations condition is satisfied, we need that $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$. Solving these equations, we find that $p_{12}^e = \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p_{22}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$. Plugging p_{12}^e and p_{22}^e back into the expressions for p_{12}^* and p_{22}^*, we verify that $p_{12}^* = p_{12}^e = \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p_{22}^* = p_{22}^e = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$. For future reference, plugging p_{12}^e into the expression for $\hat{d}$, we find that $\hat{d} = \frac{3(V-V\delta-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)}$.

Next, we need to verify the assumptions that we used in the analysis. First, case *iii* assumes that p_{11} satisfies $\frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e$. Plugging in the expressions for p_{12}^e and

p_{22}^e , we find that $\frac{(2V-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta)+\delta p_{12}^e$ is equivalent to $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{\rho}$, where $\bar{\rho} \equiv (1-\delta)V + \delta\theta$. Second, recall that to ensure that both firms have positive market shares and that firm 1 serves some of the consumers located on $(\hat{d}, 1]$ we assumed that the firms' prices would satisfy $p_{22} - \theta < p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$. Using the expressions for p_{12}^* and p_{22}^*, one can readily show that when $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{\rho}$, we indeed have $p_{22}^* - \theta < p_{12}^* \leq p_{22}^* + \theta - 2\theta\hat{d}$.

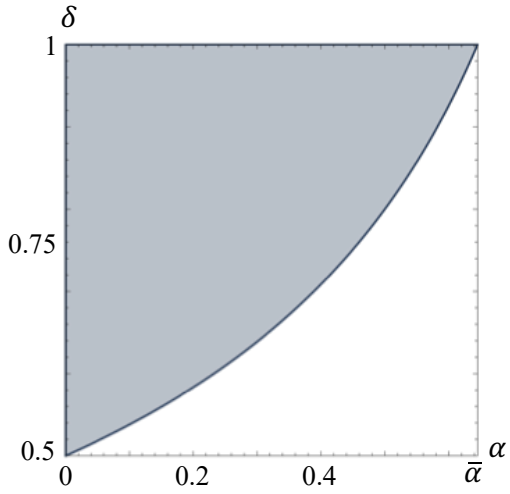
As in the previous case, firm 1's profit function is not necessarily quasi-concave, and hence, we need to check that there are no non-local deviations. It is easy to check that deviations to lower prices will not be profitable. However, firm 1 may potentially deviate to a higher price to serve only the new consumers who entered the market in the second period, giving up its market share in the segment of first-period consumers who waited to buy a product in the second period. Specifically, if $p_{12}' > p_{22}^* + \theta - 2\theta\hat{d}$, then firm 1's deviation profit is $\pi_{12}' = (1-\alpha)\frac{\theta - p_{12}' + p_{22}^*}{2\theta}p_{12}'$. Notice that π_{12}' is concave. One can show that the right derivative of π_{12}' at $p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}$ is given by $\frac{\partial_+ \pi_{12}'}{\partial p_{12}'}|_{p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}} = \frac{(1-\alpha)(V(6+\alpha)(1-\delta) - p_{11}(6+\alpha) - 3\theta(1-(3-\alpha)\delta))}{\theta(3-(3-4\alpha)\delta)}$. Straightforward algebra shows that $\frac{\partial_+ \pi_{12}'}{\partial p_{12}'}|_{p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}} \leq 0$ if and only if $p_{11} \geq \hat{\rho}$, where $\hat{\rho} \equiv (1-\delta)V + \frac{3\theta((3-\alpha)\delta - 1)}{6+\alpha}$ satisfies $\frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)} < \hat{\rho} < \bar{\rho}$. Since π_{12}' is concave, $\frac{\partial_+ \pi_{12}'}{\partial p_{12}'}|_{p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}} \leq 0$ implies that π_{12}' is decreasing in p_{12}' when $p_{12}' > p_{22}^* + \theta - 2\theta\hat{d}$. Hence, continuity of firm 1's profit function at the point $p_{22}^* + \theta - 2\theta\hat{d}$ implies that deviations to $p_{12}' > p_{22}^* + \theta - 2\theta\hat{d}$ will not be profitable for firm 1 provided that $\hat{\rho} \leq p_{11} < \bar{\rho}$. Because $\hat{\rho} > \frac{2V(3-\alpha)(1-\delta)-\theta(3-(9-6\alpha)\delta)}{2(3-\alpha)}$, our initial guess holds when $p_{11} \in (\hat{\rho}, \bar{\rho})$. Therefore, the prices $p_{12}^* = p_{12}^e = \frac{4\alpha(p_{11} - V(1-\delta)) + 3\theta(1-\delta)}{3-(3-4\alpha)\delta}$ and $p_{22}^* = p_{22}^e = \frac{2\alpha(p_{11} - V(1-\delta)) + \theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta}$ constitute a rational-expectations subgame equilibrium for any given $p_{11} \in (\hat{\rho}, \bar{\rho})$.

***Case iv**: $p_{11} \geq V(1-\delta) + \delta p_{12}^e$.* In this case, $\hat{d} = 0$, i.e., all first-period consumers decided not to purchase a product in the first period but wait. In the second period, firm i will choose its second-period

price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = \frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = \frac{\theta - p_{22} + p_{12}}{2\theta} p_{22}$. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations imply that $p_{i2}^e = p_{i2}^* = \theta$. Note that Case *iv* assumes that $p_{11} \geq V(1-\delta) + \delta p_{12}^e$. We need to check that this inequality is satisfied. Substituting $p_{12}^e = \theta$ into the inequality $p_{11} \geq V(1-\delta) + \delta p_{12}^e$, it follows that the condition for case *iv* is satisfied when $p_{11} \geq \bar{\rho}$, where recall that $\bar{\rho} \equiv (1-\delta)V + \delta\theta$. It is straightforward to show that the firms' profit functions are quasi-concave, and thus, the firms do not have any profitable non-local deviations.

Since firm 1 chooses $p_{11} \geq 0$ in the first period to maximize the sum of its first- and second-period profits, we need to know the second-period subgame equilibrium outcome corresponding to each $p_{11} \geq 0$. Our analysis in cases *i*, *ii*, *iii* and *iv* characterizes and shows the existence of the second-period subgame equilibrium outcome for p_{11} contained in the intervals $[0, \underline{\rho}]$, $(\underline{\rho}, \tilde{\rho}]$, $(\hat{\rho}, \bar{\rho})$, and $[\bar{\rho}, \infty)$, respectively. Notice that $[0, \underline{\rho}] \cup (\underline{\rho}, \tilde{\rho}] \cup (\hat{\rho}, \bar{\rho}) \cup [\bar{\rho}, \infty) = [0, \infty]$ if and only if $\tilde{\rho} \geq \hat{\rho}$, where recall that $\tilde{\rho} = (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\hat{\rho} = (1-\delta)V + \frac{3\theta((3-\alpha)\delta-1)}{6+\alpha}$. The inequality $\tilde{\rho} \geq \hat{\rho}$ is satisfied if and only if $\underline{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$ where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$. The shaded region in Figure A1 graphically illustrates the parameter conditions on δ and α.

Figure A1 Parameter Region where $\underline{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$



Note that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, there are two possible second-period rational-expectations equilibria corresponding to the cases *ii* and *iii* that we analyzed earlier. The selection of which equilibrium occurs

will not have a qualitative impact on our results. Since both firms' profits are higher when the equilibrium from case *ii* is played (i.e., it is the focal equilibrium), we assume that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, the subgame equilibrium prices will be the ones from case *ii*, i.e., $p_{12}^* = \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^* = \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$.

Hence, to summarize our analysis, it follows that when $(R_1, R_2) = (1, 2)$, given any $p_{11} \geq 0$, the firms' second-period prices in a rational-expectations subgame equilibrium are as follows:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{A5}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{A6}$$

where $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$. This finishes the proof of Lemma 2. For future reference, let us derive the firms' equilibrium profits and sales. Specifically, upon plugging the prices in (A5) and (A6) into the respective profit functions (explicitly provided in earlier cases *i–iv*), we obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1, 2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A7}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(V(1-\delta)-p_{11})-\theta(3(1+\delta)+\alpha(1-3\delta)))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{A8}$$

Using the prices in (A5)-(A6), we can also find the firms' second-period unit sales:

$$S_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)}{2\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(A9)}$$

$$S_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\theta(3+\alpha)+4\alpha(p_{11}-V(1-\delta))+3\theta(1-\alpha)\delta)}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta)}{2\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(A10)}$$

To find firm 1's first-period unit sales, $S_{11}^{(1,2)}$, recall that consumers with $d < \hat{d}$ buy firm 1's product in the first period, where $\hat{d}$ is given in the equation (A4). Since consumers are uniformly distributed, $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^{e} = p_{12}^{(1,2)}$ and $p_{22}^{e} = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{11}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(A11)}$$

■

PROOF OF LEMMA 3. In the first period, firm 1 chooses its price to maximize its overall profit over the two periods: $\pi_1 = S_{11}^{(1,2)}p_{11} + \pi_{12}^{(1,2)}$, where $S_{11}^{(1,2)}$ is firm 1's first-period unit sales as in equation (A11) and $\pi_{12}^{(1,2)}$ is firm 1's second-period subgame equilibrium profit as in equation (A7).

$$\pi_1 = \begin{cases} \alpha\, p_{11} + \frac{(1-\alpha)\theta}{2} & \text{if } p_{1_1} \in [0, \underline{\rho}) \\ \alpha\frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))}p_{1_1} + \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{1_1} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)}p_{1_1} + \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{1_1} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{1_1} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(A12)}$$

where $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$.

It is easy to see that π_1 is increasing in p_{11} at any $p_{11} < \underline{\rho}$. Since π_1 is continuous at $\underline{\rho}$, it must be that the optimal price is above $\underline{\rho}$. Further, if $p_{11} > \bar{\rho}$, then π_1 is constant. Since π_1 is continuous at $\bar{\rho}$, the optimal price will not be above $\bar{\rho}$. Hence, the optimal price is either in $I_1 \equiv [\underline{\rho}, \tilde{\rho}]$ or $I_2 \equiv (\tilde{\rho}, \bar{\rho}]$. Let p_{I_k} denote the locally optimal price within the interval I_k, where $k = 1,2$. We will first find p_{I_1} and p_{I_2}, after which we will compare the corresponding profits to determine the globally optimal price.

First, when $p_{11} \in I_2$, we have

$$\pi_1 = \alpha \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{11} + \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2}$$

It is easy to show that $\frac{d^2\pi_1}{d(p_{11})^2} < 0$, i.e., π_1 is concave.

- If $0 < \theta \leq \dot{\theta}$, then π_1 is decreasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Hence, the locally optimal price is at the left corner: $p_{I_1} = \underline{\rho} = (1-\delta)(V-\theta)$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{2V\alpha(1-\delta)+\theta(1-\alpha(3-2\delta))}{2}$.
- If $\dot{\theta} < \theta < \ddot{\theta}$, then the local maximizer is in the interior of the interval I_1, where $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Solving $\frac{d\pi_1}{dp_{11}} = 0$, we find the interior maximizer: $p_{I_1} = \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$. The corresponding profit is
$\pi_1(p_{I_1}) = \frac{9V^2(1-\alpha)^2\alpha(1-\delta)^2-4\theta V(3-\alpha)(1-\alpha)\alpha(1-\delta)(1-3\delta)+2\theta^2(3-\alpha)^2(1+\delta-\alpha(1-\delta)(1+2\delta))}{4\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$.
- If $\ddot{\theta} \leq \theta < \bar{\theta}$, then π_1 is increasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, and hence, the locally optimal price is at the right corner: $p_{I_1} = \tilde{\rho} = (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{(3-\alpha)(2V\alpha(6-\alpha(7-3\alpha))(1-\delta)+\theta(3-\alpha)(4-\alpha(10-6\delta-\alpha(5-\alpha-4\delta))))}{2(6-\alpha(7-3\alpha))^2}$.

Second, when $p_{1_1} \in I_2$, we have

$$\pi_1 = \alpha \frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} p_{1_1} + \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2}$$

Because $\underline{\delta} < \delta < 1$, $0 < \alpha < \bar{\alpha}$ and $\theta < \bar{\theta}$, one can readily show that $\frac{d\pi_1}{dp_{11}} = \frac{\alpha(V(1-\delta)(9-9\delta-4\alpha(4-3\delta))-2p_{11}(9(1-\delta)-4\alpha(2-3\delta))+3\theta(4-\delta(1+3\delta-4\alpha\delta)))}{\theta(3-(3-4\alpha)\delta)^2} < 0$ for all $p_{11} \in I_2$. That is, π_1 is decreasing in p_{11} on I_2. Since $I_2 = (\tilde{\rho}, \bar{\rho}]$, π_1 does not have a local maximizer on I_2 because the point $\tilde{\rho}$ is not included in I_2. However, this does not create problems for the existence of a global maximizer of π_1. Specifically, one can readily show that $\pi_1(\tilde{\rho}) > \lim_{p_{11} \to \tilde{\rho}^+} \pi_1$, i.e., the right limit of π_1 at $p_{11} = \tilde{\rho}$ is strictly lower than $\pi_1(\tilde{\rho})$. In other words, π_1 discretely jumps down at the point $\tilde{\rho}$. Since $\frac{d\pi_1}{dp_{11}} < 0$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, it follows that $\pi_1(\tilde{\rho}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$, it follows that $\pi_1(p_{I_1}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Hence, the optimal price within the interval I_1 yields greater profit than any price $p_{11} \in I_2$. Therefore, we conclude that the global maximizer of π_1 is contained in the interval I_1. That is, $p_{11}^{(1,2)} = p_{I_1}$. Hence,

$$p_{11}^{(1,2)} = \begin{cases} (1-\delta)(V-\theta) & \text{if } \theta \leq \dot{\theta} \\ \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad \text{(A13)}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$.

For future reference, let us use the expression of $p_{11}^{(1,2)}$ in (A13) to obtain the firms' profits and second-period prices on the equilibrium path.

Upon plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from Lemma 2, we can find the firms' second-period prices on the equilibrium path.

$$p_{12}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \dot{\theta} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3V(1-\alpha)\alpha(1-\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad \text{(A14)}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \leq \dot{\theta} \\ \frac{9\theta(1+\delta)-6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases} \quad \text{(A15)}$$

Using the equilibrium prices in (A13)-(A15), we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) & \text{if } \theta \le \dot{\theta} \\ \frac{\alpha(9V(1-\alpha-\delta+\alpha\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{(3-\alpha)\alpha(V(6-\alpha(7-3\alpha))(1-\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \quad \text{(A16)}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{(3V\alpha(1-\alpha-\delta+\alpha\delta)-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \quad \text{(A17)}$$

$$\pi_{21}^{(1,2)} = 0 \quad \text{(A18)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \quad \text{(A19)}$$

where $A \equiv \frac{V(1-\alpha)(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$. ■

PROOF OF PROPOSITION 1. We will first prove part b) of Proposition 1 and then proceed to the proof of part a).

When $(R_1, R_2) = (1,1)$, Lemma 1 showed that the firms' equilibrium prices will be $p_{i2}^{(1,1)} = \theta$. When $(R_1, R_2) = (1,2)$, the firms' equilibrium second-period prices are given in (A14)-(A15).

Clearly, if $\theta \le \dot{\theta}$, then $p_{i2}^{(1,2)} = p_{i2}^{(1,1)} = \theta$ for each firm $i = 1,2$.

Next, if $\dot{\theta} < \theta < \ddot{\theta}$, we have $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3V(1-\alpha)\alpha(1-\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$ and $p_{22}^{(1,2)} = \frac{9\theta(1+\delta)-6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)}$. Straightforward algebra shows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ if and only if $\theta > \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Recall that $\dot{\theta} \equiv \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Hence, when $\dot{\theta} < \theta < \ddot{\theta}$, we have $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$.

Finally, if $\ddot{\theta} \leq \theta < \bar{\theta}$, then $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)}$ and $p_{22}^{(1,2)} = \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)}$. Since $(3-\alpha)(2-\alpha) > 6-\alpha(7-3\alpha)$ and $6-\alpha(3+\alpha) > 6-\alpha(7-3\alpha)$ for any $\alpha \in [0, \bar{\alpha})$, it readily follows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$.

The above shows that $p_{i2}^{(1,2)} \geq p_{i2}^{(1,1)}$. Next, let us prove part a) of Proposition 1. For future reference, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $\underline{\delta} < \delta < 1$, $0 \leq \alpha < \bar{\alpha}$, and $\theta < \bar{\theta}$, where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$, $\bar{\alpha} \approx 0.64$ and $\bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

Firm i chooses its product release period $R_i \in \{1,2\}$ to maximize its overall profit $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \pi_{i2}^{(R_1,R_2)}$. The pair (R_1, R_2) is an equilibrium if no firm has any profitable deviations.

First, one can readily show that $(R_1, R_2) = (2,2)$ is not an equilibrium because firm 1 will benefit by deviating to $R_1 = 1$, i.e., $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, where $\pi_1^{(2,2)} = \frac{\theta}{2}$. To see this, notice that if firm 1 deviates to $R_1 = 1$ and sets an extremely large price ($p_{11} \geq \bar{\rho}$), then its overall profit will be $\pi_1(\bar{\rho}) = \frac{\theta}{2}$, which is given in equation (A12). However, we know from Lemma 3 that firm 1's unique optimal price is $p_{11}^{(1,2)} < \bar{\rho}$ with a corresponding profit of $\pi_1^{(1,2)}$. Hence, $\pi_1^{(1,2)} > \pi_1(\bar{\rho})$. Since $\pi_1(\bar{\rho}) = \pi_1^{(2,2)} = \frac{\theta}{2}$, it follows that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Thus, in equilibrium, either one firm releases its product in the first period and the other firm does so in the second period, or both firms release their products in the first period.

Next, let us demonstrate that a pure strategy equilibrium exists. If $(R_1, R_2) = (1,2)$ is an equilibrium, then the proof is finished. If $(R_1, R_2) = (1,2)$ is not an equilibrium, then let us show that $(R_1, R_2) = (1,1)$ must be an equilibrium. Since $(R_1, R_2) = (1,2)$ is not an equilibrium, one of the firms must have a profitable deviation. Firm 1 will not want to deviate to $R_1 = 2$ because $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Hence, it must be that firm 2 has a profitable deviation from $R_2 = 2$ to $R_2 = 1$, which means that $\pi_2^{(1,1)} > \pi_2^{(1,2)}$. By symmetry between firms 1 and 2, we know that $\pi_2^{(1,2)} = \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} = \pi_1^{(1,1)}$. Hence, $\pi_2^{(1,1)} > \pi_2^{(1,2)}$ implies that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$. Then, it must be $(R_1, R_2) = (1,1)$ is an equilibrium because we found that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} > \pi_2^{(1,2)}$.

Now, let us find conditions under which $(R_1, R_2) = (1, 2)$ is an equilibrium. If $R_1 = 1$, then firm 2's best response is $R_2 = 2$ if and only if $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$, where $\pi_2^{(1,1)} = \frac{\theta}{2}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)}$ is given in equations (A18)-(A19). Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)\theta}{2} - \frac{\theta}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{\theta}{2} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{\theta}{2} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Note that $\Delta\pi_2$ is a continuous.

When $\theta \le \dot{\theta}$, clearly, $\Delta\pi_2 < 0$. Hence, consider $\theta > \dot{\theta}$. The following two results about the sign of $\frac{d\Delta\pi_2}{d\theta}$ are important for the analysis.

First, when $\dot{\theta} < \theta < \ddot{\theta}$, the function $\Delta\pi_2$ is increasing in θ, i.e., $\frac{d\Delta\pi_2}{d\theta} > 0$. To see this, note that $\frac{d^2\Delta\pi_2}{d\theta^2} = \frac{36V^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta^3(9-11\alpha+9\delta+3\alpha\delta)^2} > 0$, i.e., $\Delta\pi_2$ is convex. Further, $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\dot{\theta}} = \frac{\alpha(23-9\delta-\alpha(21-13\delta))}{2(9-11\alpha+9\delta+3\alpha\delta)} > 0$, where the inequality follows because $\delta > \underline{\delta} \ge 0.5$. The inequalities $\frac{d^2\Delta\pi_2}{d\theta^2} > 0$ and $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\dot{\theta}} > 0$ imply that $\frac{d\Delta\pi_2}{d\theta} > 0$ for any $\dot{\theta} < \theta < \ddot{\theta}$.

Second, when $\ddot{\theta} \le \theta < \bar{\theta}$, we have $\frac{d\Delta\pi_2}{d\theta} = \frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{1}{2} > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ is the solution to $\frac{(1-\hat{\alpha})(6-\hat{\alpha}(3+\hat{\alpha}))^2}{2(6-\hat{\alpha}(7-3\hat{\alpha}))^2} - \frac{1}{2} = 0$, or equivalently, $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17 + \hat{\alpha})) = 0$.

We will separately analyze the cases with $\alpha \le \hat{\alpha}$ and $\alpha > \hat{\alpha}$. Specifically, we will show that if $\alpha > \hat{\alpha}$, then $\Delta\pi_2 < 0$, but if $0 < \alpha \le \hat{\alpha}$, then $\Delta\pi_2 > 0$ for θ large enough.

Case 1: $\alpha > \hat{\alpha}$. When $\alpha > \hat{\alpha}$, one can show that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$.[17] We showed earlier that $\Delta\pi_2$ is increasing when $\dot{\theta} < \theta < \ddot{\theta}$ and decreasing when $\ddot{\theta} \le \theta < \bar{\theta}$. Since $\Delta\pi_2$ is

[17] To see this, note that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ if and only if $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta} < 0$. Using the fact that $\delta > \underline{\delta} \ge 0.5$, one can show that the function $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta}$ is decreasing in α. Plugging in $\alpha = 0.3$, one can show that $(\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta})|_{\alpha=0.3} < 0$.

continuous, it follows that $\Delta\pi_2|_{\theta=\bar{\theta}} > \Delta\pi_2$ for any $\dot{\theta} \le \theta < \bar{\theta}$ and $\theta \neq \ddot{\theta}$. Straightforward algebra shows that $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{3V(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))(1-\delta)}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, and $\Delta\pi_2|_{\theta=\ddot{\theta}} < 0$ if and only if $12 - \alpha(52-(3-\alpha)\alpha(17+\alpha)) > 0$. The last inequality is equivalent to $\alpha > \hat{\alpha}$, where recall that $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52-(3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$. Hence, when $\alpha > \hat{\alpha}$, we have $\Delta\pi_2 \le \Delta\pi_2|_{\theta=\ddot{\theta}} < 0$ for any $\dot{\theta} \le \theta < \bar{\theta}$.

Case 2: $\alpha < \hat{\alpha}$. When $\alpha < \hat{\alpha}$, we showed earlier that $\Delta\pi_2$ is increasing in θ at any $\dot{\theta} \le \theta < \bar{\theta}$. Further, we can show that $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$, where recall that $\ddot{\theta} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \bar{\theta}\}$. Specifically, if $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, then we already know from Case 1 that $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{3V(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))(1-\delta)}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} > 0$ when $\alpha < \hat{\alpha}$. Next, if $\ddot{\theta} = \frac{V(1-\delta)}{2-\delta}$, then $\Delta\pi_2|_{\theta=\ddot{\theta}} = \frac{V\alpha(1-\delta)((3+\alpha)^3\delta^2+2(1-\alpha)(3+\alpha)(3+7\alpha)\delta-9+63\alpha-103\alpha^2+49\alpha^3)}{2(1-\alpha)(2-\delta)(9-11\alpha+3(3+\alpha)\delta)^2} > 0$, where the inequality follows because $\delta > \underline{\delta} \ge \frac{1}{2}$. Because $\Delta\pi_2$ is increasing in θ, $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$ implies that $\Delta\pi_2 > 0$ for all $\ddot{\theta} \le \theta < \bar{\theta}$. Further, because $\Delta\pi_2|_{\theta=\dot{\theta}} = -\frac{3V(1-\alpha)\alpha(1-\delta)}{8(2-\alpha(2-\delta))} < 0$, $\Delta\pi_2|_{\theta=\ddot{\theta}} > 0$ and $\Delta\pi_2$ is a continuous increasing function, it follows that there exists a unique $\dot{\theta} < \hat{\theta} < \ddot{\theta}$, such that $\Delta\pi_2|_{\theta=\hat{\theta}} = 0$ and $\Delta\pi_2 < 0$ if $\dot{\theta} < \theta < \hat{\theta}$ and $\Delta\pi_2 > 0$ if $\hat{\theta} < \theta < \ddot{\theta}$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))} \quad \text{(A20)}$$

To summarize the above, we found that if $0 < \alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\Delta\pi_2 > 0$. Thus, in this parameter region, firm 2 will not deviate from $R_2 = 2$. Since $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, firm 1 also has no incentives to deviate from $R_1 = 1$. It follows that $(R_1, R_2) = (1, 2)$ is a Nash equilibrium. ■

Hence, $\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V(1-\delta)}{2-\delta} < 0$ for all $\alpha > 0.3$. Since $\hat{\alpha} > 0.3$, it follows that $\ddot{\theta} = \frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ when $\alpha > \hat{\alpha}$.

PROOF OF PROPOSITION 2. First, let us show that firm 2's equilibrium unit sales, $S_2^{(1,2)}$, are decreasing in α. Note that $S_2^{(1,2)} = S_{22}^{(1,2)}$, where $S_{22}^{(1,2)}$ is given in the equation (A10). Upon plugging in firm 1's first-period equilibrium price $p_{11}^{(1,2)}$, we find firm 2's sales on the equilibrium path:

$$S_2^{(1,2)} = \begin{cases} \frac{1-\alpha}{2} & \text{if } \theta \le \dot{\theta} \\ \frac{9\theta(1+\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)-6V\alpha(1-\delta)(1-\alpha)}{2\theta(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \dot{\theta} < \theta < \ddot{\theta} \\ \frac{(1-\alpha)(6-\alpha(3+\alpha))}{2(6-\alpha(7-3\alpha))} & \text{if } \ddot{\theta} \le \theta < \bar{\theta} \end{cases} \quad \text{(A21)}$$

where $\dot{\theta} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\ddot{\theta} = \min\{\frac{3V(1-\alpha)\left(6-\alpha(7-3\alpha)\right)(1-\delta)}{2(3-\alpha)\left(5-3\delta-\alpha\left(6-8\delta-\alpha(1-3\delta)\right)\right)}, \bar{\theta}\}$. Note that $S_2^{(1,2)}$ is continuous. We first need to show that if $0 < \alpha < \hat{\alpha}$, then $\frac{dS_2^{(1,2)}}{d\alpha} < 0$.

When $\theta < \dot{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{1}{2} < 0$.

When $\dot{\theta} < \theta < \ddot{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = \frac{\Psi}{\theta(9-11\alpha+3(3+\alpha)\delta)^2}$, where

$\Psi = \frac{6V(1-\delta)(\alpha(18-11\alpha+3(6+\alpha)\delta)-9(1+\delta))+\theta(5\alpha^2(1-\delta)(11-3\delta)+9(1+\delta)(7-9\delta)-90\alpha(1-\delta^2))}{2}$. Clearly, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ if and only if $\Psi < 0$. Hence, we need to show that $\Psi < 0$. First, note that Ψ is concave in α because $\frac{d^2\Psi}{d\alpha^2} = -2(1-\delta)(6V - 5\theta(1-\delta))(11-3\delta) < 0$.[18] Second, because $\theta < \bar{\theta}$, one can show that $\Psi|_{\alpha=0} = -\frac{9(6V(1-\delta)-\theta(7-9\delta))(1+\delta)}{2} < 0$ and $\Psi|_{\alpha=0.5} = -\frac{6V(1-\delta)(11-3\delta)-\theta\left(127-\delta(142+129\delta)\right)}{8} < 0$. Concavity of Ψ, together with $\Psi|_{\alpha=0} < 0$ and $\Psi|_{\alpha=0.5} < 0$, implies that $\Psi < 0$ for any $0 < \alpha < 0.5$. Since $\hat{\alpha} \approx 0.32 < 0.5$, it follows that $\Psi < 0$ for any $0 < \alpha < \hat{\alpha}$. Thus, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ when $\dot{\theta} < \theta < \ddot{\theta}$.

When $\ddot{\theta} \le \theta < \bar{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{12+\alpha(12-\alpha(31-\alpha(14-3\alpha)))}{2(6-\alpha(7-3\alpha))^2}$. Straightforward algebra shows that $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ for any $0 < \alpha < \hat{\alpha}$.

[18] To see that $\frac{d^2\Psi}{d\alpha^2} < 0$, note that $11 - 3\delta > 0$ and $6V - 5\theta(1-\delta) > 0$ because $\theta < \bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

To prove the second part of the proposition, we will show that there exists $\tilde{\alpha} < \hat{\alpha}$, such that if $\alpha < \tilde{\alpha}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$. Since we are focusing on $\theta > \hat{\theta}$ and since $\dot{\theta} < \hat{\theta} < \ddot{\theta}$, it follows from the equation (A19) that $\pi_2^{(1,2)} = \pi_{22}^{(1,2)}$ is given by

$$\pi_2^{(1,2)} = \begin{cases} \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \hat{\theta} < \theta < \ddot{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \ddot{\theta} \leq \theta < \bar{\theta} \end{cases}$$

Upon differentiating $\pi_2^{(1,2)}$ with respect to α and evaluating at $\alpha = 0$, we find that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta(23-9\delta)-12V(1-\delta)}{18(1+\delta)}$ when $\hat{\theta} < \theta < \ddot{\theta}$, and $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta}{6}$ when $\ddot{\theta} < \theta < \bar{\theta}$. Clearly, when $\ddot{\theta} < \theta < \bar{\theta}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$. Further, when $\hat{\theta} < \theta < \ddot{\theta}$, it is easy to show that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$ if and only if $\theta > \frac{12V(1-\delta)}{23-9\delta}$. Note that $\hat{\theta}|_{\alpha=0} = \frac{12V(1-\delta)}{23-9\delta}$. By continuity of $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}$, it follows that there exists $\tilde{\alpha} \in (0, \hat{\alpha}]$ such that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$ if $\alpha < \tilde{\alpha}$ and $\theta > \hat{\theta}$. ■

PROOF OF PROPOSITION 3. For a given $0 < \alpha < \hat{\alpha}$, the cutoff $\hat{\theta}$ satisfies $\pi_2^{(1,2)}|_{\theta=\hat{\theta}} - \pi_2^{(1,1)}|_{\theta=\hat{\theta}} = 0$. Totally differentiating with respect to α, we find that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} + \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} - \frac{d\pi_2^{(1,1)}}{d\alpha}|_{\theta=\hat{\theta}} - \frac{d\pi_2^{(1,1)}}{d\theta}|_{\theta=\hat{\theta}} \frac{d\hat{\theta}}{d\alpha} = 0$. Because $\pi_2^{(1,1)} = \frac{\theta}{2}$, we have $\frac{d\pi_2^{(1,1)}}{d\alpha} = 0$ and $\frac{d\pi_2^{(1,1)}}{d\theta} = \frac{1}{2}$. Hence, $\frac{d\hat{\theta}}{d\alpha} = \frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} \left(\frac{1}{2} - \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}}\right)^{-1}$. To show that $\frac{d\hat{\theta}}{d\alpha} < 0$, we will demonstrate that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}} > \frac{1}{2}$.

As we showed in the proof of Proposition 1, $\hat{\theta}$ is given by $\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$ and satisfies $\dot{\theta} < \hat{\theta} < \ddot{\theta}$. Further, $\pi_2^{(1,2)}$ is given by $\pi_2^{(1,2)} = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2}$. One can readily show that

$\frac{d\pi_2^{(1,2)}}{d\theta} = \frac{(6V(1-\alpha)\alpha(1-\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)+9\theta(1+\delta))(9\theta(1+\delta)-\theta\alpha(4+5\alpha(1-\delta)+6\delta)-6V\alpha(1-\alpha-\delta+\alpha\delta))}{2\theta^2(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2}$, and $\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$ if and only if $\theta > \frac{6V(1-\alpha)(1-\delta)\sqrt{\alpha}}{\sqrt{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}}$. Using $\alpha < \hat{\alpha} \approx 0.32$, one can show that $\frac{6V(1-\alpha)(1-\delta)\sqrt{\alpha}}{\sqrt{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}} < \dot{\theta}$.

Because $\hat{\theta} > \dot{\theta}$, it follows that $\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$ when $\theta = \hat{\theta}$.

Next,

$\frac{d\pi_2^{(1,2)}}{d\alpha} = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))}{(1-\alpha)^2}\left(\frac{6V(1-\alpha)(1-\delta)(18(1+\delta)-\alpha(27-11\alpha+3(9+\alpha)\delta))}{2\theta(9-11\alpha+3(3+\alpha)\delta)^3} + \frac{(5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta)))}{2(9-11\alpha+3(3+\alpha)\delta)^3}\right)$. Denote $A = \frac{(\theta\alpha(4+5\alpha(1-\delta)+6\delta)-9\theta(1+\delta)+6V\alpha(1-\alpha-\delta+\alpha\delta))}{(1-\alpha)^2}$ and $B = \frac{6V(1-\alpha)(1-\delta)(18(1+\delta)-\alpha(27-11\alpha+3(9+\alpha)\delta))}{2\theta(9-11\alpha+3(3+\alpha)\delta)^3} + \frac{(5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta)))}{2(9-11\alpha+3(3+\alpha)\delta)^3}$. Hence, $\frac{d\pi_2^{(1,2)}}{d\alpha} = AB$. We will show that if $\theta = \hat{\theta}$, then $A < 0$ and $B < 0$, which will imply that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$. First, one can show that $A < 0$ if and only if $\theta > \frac{6V(1-\alpha)\alpha(1-\delta)}{9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)}$. Because $\hat{\theta} > \dot{\theta} > \frac{6V(1-\alpha)\alpha(1-\delta)}{9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)}$, it follows that $A|_{\theta=\hat{\theta}} < 0$. Second, $B < 0$ if and only if

$$\theta > \frac{6V(1-\alpha)(1-\delta)(-18(1+\delta)+\alpha(27-11\alpha+3(9+\alpha)\delta))}{5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta))}.$$

Because $\hat{\theta} > \frac{6V(1-\alpha)(1-\delta)(-18(1+\delta)+\alpha(27-11\alpha+3(9+\alpha)\delta))}{5\alpha^3(1-\delta)(11-3\delta)+63\alpha(1+\delta)(7-5\delta)-9(1+\delta)(23-9\delta)-\alpha^2(289-\delta(86+123\delta))}$ when $\alpha < \hat{\alpha}$, it follows that $B|_{\theta=\hat{\theta}} < 0$. The inequalities $A|_{\theta=\hat{\theta}} < 0$ and $B|_{\theta=\hat{\theta}} < 0$ imply that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} = (AB)|_{\theta=\hat{\theta}} > 0$.

We showed that $\frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\theta} > \frac{1}{2}$. Hence, $\frac{d\hat{\theta}}{d\alpha} = \frac{d\pi_2^{(1,2)}}{d\alpha}|_{\theta=\hat{\theta}}\left(\frac{1}{2} - \frac{d\pi_2^{(1,2)}}{d\theta}|_{\theta=\hat{\theta}}\right)^{-1} < 0$.

■

WEB APPENDIX B

1. UNIQUENESS OF EQUILIBRIUM IN LEMMA 1

Lemma 1 shows the existence of a rational-expectations equilibrium in which the market is fully covered in the first period, i.e., in equilibrium, first-period consumers buy a product in the first period instead of waiting. In this section, we consider the possibility that other equilibria may exist where, on the equilibrium path, some first-period consumers prefer not to purchase in the first period but wait. We will show that such equilibria do not exist when the differentiation level between the firms is not too high (i.e., $\theta < \bar{\theta}$). Hence, the equilibrium from Lemma 1 is unique.

Given the firms' first-period prices, a consumer located at $d \in [0,1]$ will prefer to wait till the second period to buy a product if and only if the following inequality holds:

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} < \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}, \text{ (B1)}$$

where p_{12}^e and p_{22}^e are the second-period prices that the consumer expects the firms to charge. One can readily show that if the above inequality is satisfied for some $d \in [0,1]$, then the set of such d's must constitute a connected set, i.e., an interval $[\hat{d}_1, \hat{d}_2]$.[19] There are three possible cases.

i) $\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1-\hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1-\hat{d}_2))$, i.e., some of the consumers who prefer not to buy in the first period expect to purchase firm 1's product in the second period, while others expect to purchase firm 2's product.

ii) $\delta(V - p_{12}^e - \theta\hat{d}_1) < \delta(V - p_{22}^e - \theta(1-\hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1-\hat{d}_2))$, i.e., all consumers who prefer not to buy in the first period expect to purchase firm 2's product in the second period.

[19] This can be shown using a proof by contradiction. Specifically, if the set of d's satisfying inequality (B1) is not connected, then there must exist $d_1 < d_2 < d_3$ such that d_1 and d_3 satisfy (B1) but d_2 does not. There are two possible cases: $d_2 < \frac{\theta - p_{11} + p_{21}}{2\theta}$ or $d_2 \geq \frac{\theta - p_{11} + p_{21}}{2\theta}$. Without loss of generality, assume that $d_2 < \frac{\theta - p_{11} + p_{21}}{2\theta}$. Then, $\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} = V - p_{11} - \theta d$ for $d = d_1, d_2$. Because $\delta < 1$, notice that $V - p_{11} - \theta d - \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}$ is decreasing in d. Therefore, if $V - p_{11} - \theta d_2 > \max\{\delta(V - p_{12}^e - \theta d_2), \delta(V - p_{22}^e - \theta(1-d_2))\}$, then it must be that $V - p_{11} - \theta d_1 > \max\{\delta(V - p_{12}^e - \theta d_1), \delta(V - p_{22}^e - \theta(1-d_1))\}$. This contradicts the assumption that d_1 satisfies (B1).

iii) $\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_1))$ and $\delta(V - p_{12}^e - \theta\hat{d}_2) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_2))$, i.e., all consumers who prefer not to buy in the first period expect to purchase firm 1's product in the second period.

We will search for an equilibrium using the guess-and-verify approach. Specifically, we start by guessing that the inequalities characterizing one of the above three cases are satisfied. Then, using first-order conditions, we will find the firms' second- and first-period prices. Using these prices, we will check whether our initial guess is indeed satisfied, as well as other necessary and sufficient conditions for the prices to constitute an equilibrium.

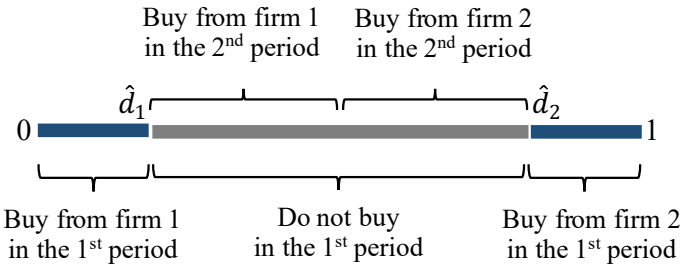
Case i). We guess that the following inequalities will be satisfied in equilibrium:

$$\delta(V - p_{12}^e - \theta\hat{d}_1) > \delta(V - p_{22}^e - \theta(1 - \hat{d}_1)) \quad \text{(B2)}$$

$$\delta(V - p_{12}^e - \theta\hat{d}_2) < \delta(V - p_{22}^e - \theta(1 - \hat{d}_2)) \quad \text{(B3)}$$

The above inequalities imply that among first-period consumers who choose to wait till the second period to buy, some expect to purchase from firm 1 in the second period, while others expect to purchase from firm 2. In a rational-expectations equilibrium, the decision that a consumer expects to make in the second period must be consistent with the consumer's equilibrium choice. Figure B1 graphically illustrates first-period consumers' decisions.

Figure B1 Purchase Decisions of Consumers Entering the Market in the 1st Period



Note that the consumer located at $\hat{d}_1$ must be indifferent between buying firm 1's product in the first period and buying it in the second period. Mathematically, $V - p_{11} - \theta\hat{d}_1 = \delta(V - p_{12}^e - \theta\hat{d}_1)$, which implies that $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$. Similarly, the consumer located at $\hat{d}_2$ must be indifferent

between buying firm 2's product in the first period and doing so in the second period. Mathematically, $V - p_{21} - \theta(1 - \hat{d}_2) = \delta(V - p_{22}^e - \theta(1 - \hat{d}_2))$, which implies $\hat{d}_2 = \frac{p_{21} - \delta p_{22}^e - (V-\theta)(1-\delta)}{\theta(1-\delta)}$.

In the second period, the firms compete for the $(1 - \alpha)$ new consumers entering the market, as well as first-period consumers located on $[\hat{d}_1, \hat{d}_2]$ who did not buy a product in the first period. If a rational-expectations equilibrium exists, then the firms' second-period prices, p_{12} and p_{22}, must be such that the consumer with $d = \hat{d}_1$ prefers to buy firm 1's product, whereas the consumer with $d = \hat{d}_2$ prefers to buy firm 2's product.[20] Therefore, the firms' second-period profit functions are as follows:

$$\pi_{12} = \left(\alpha\left(\frac{\theta - p_{12} + p_{22}}{2\theta} - \hat{d}_1\right) + (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}\right)p_{12}, \tag{B4}$$

$$\pi_{22} = \left(\alpha\left(\hat{d}_2 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right) + (1-\alpha)\left(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right)\right)p_{12}, \tag{B5}$$

where recall that $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + \delta p_{12}^e}{\theta(1-\delta)}$ and $\hat{d}_2 = \frac{p_{21} - \delta p_{22}^e - (V-\theta)(1-\delta)}{\theta(1-\delta)}$. Using the first-order conditions, we find the firms' profit-maximizing second-period prices: $p_{12}^* = \theta - \frac{2\alpha(3V(1-\delta) + (2p_{12}^e + p_{22}^e)\delta - 2p_{11} - p_{21})}{3(1-\delta)}$ and $p_{22}^* = \theta - \frac{2\alpha(3V(1-\delta) + (p_{12}^e + 2p_{22}^e)\delta - p_{11} - 2p_{21})}{3(1-\delta)}$. Rational-expectations condition requires that $p_{12}^* = p_{12}^e$ and $p_{22}^* = p_{22}^e$. Solving these equalities, we find that $p_{12}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21}))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$ and $p_{22}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$. Upon plugging p_{12}^e and p_{22}^e back into p_{12}^* and p_{22}^*, one can verify that

$$p_{12}^* = p_{12}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21}))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}, \tag{B6}$$

$$p_{22}^* = p_{22}^e = \frac{\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta))}{(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}. \tag{B7}$$

The firms' corresponding second-period profits are given by

$$\pi_{12}^* = \frac{(\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(2p_{11}(1-(1-\alpha)\delta) - (1-\delta)(3V - V(3-2\alpha)\delta - p_{21})))^2}{2\theta(3-(3-2\alpha)\delta)^2(1-(1-2\alpha)\delta)^2}, \tag{B8}$$

$$\pi_{22}^* = \frac{(\theta(1-\delta)(3-(3-2\alpha)\delta) + 2\alpha(p_{11}(1-\delta) + 2p_{21}(1-(1-\alpha)\delta) - V(1-\delta)(3-3\delta+2\alpha\delta)))^2}{2\theta(3-(3-2\alpha)\delta)^2(1-(1-2\alpha)\delta)^2}. \tag{B9}$$

[20] If otherwise, then the prices p_{12} and p_{22} would violate either the conditions (B2)-(B3) or the rational-expectations condition $p_{12} = p_{12}^e$ and $p_{22} = p_{22}^e$.

Next, let us obtain the conditions under which our initial guess is satisfied. Upon plugging p_{12}^e and p_{22}^e into the inequalities (B2) and (B3), these inequalities reduce to the following conditions, respectively:

$$p_{11} > \frac{\theta(1-(3-2\alpha)\delta)(3-(3-2\alpha)\delta)+2(p_{21}\alpha(1-3\delta+2\alpha\delta)-V(1-\delta)(3-3\delta+2\alpha\delta))}{2(\alpha-3+(1-\alpha)(3-2\alpha)\delta)} \tag{B10}$$

$$p_{21} > \frac{\theta(1-(3-2\alpha)\delta)(3-(3-2\alpha)\delta)+2(p_{11}\alpha(1-3\delta+2\alpha\delta)-V(1-\delta)(3-3\delta+2\alpha\delta))}{2(\alpha-3+(1-\alpha)(3-2\alpha)\delta)} \tag{B11}$$

Hence, after finding the firms' first-period equilibrium prices p_{11}^* and p_{21}^*, we will need to verify that they satisfy (W10) and (W11). In the first period, the firms maximize the sum of their profits over the two periods:

$$\pi_1 = \pi_{11} + \pi_{12}^*,$$

$$\pi_2 = \pi_{21} + \pi_{22}^*,$$

where $\pi_{11} = \alpha\, \hat{d}_1 p_{11}$, $\pi_{21} = \alpha(1-\hat{d}_2)p_{21}$, $\hat{d}_1 = \frac{(1-\delta)V-p_{11}+\delta p_{12}^e}{\theta(1-\delta)}$ and $\hat{d}_2 = \frac{p_{21}-\delta p_{22}^e-(V-\theta)(1-\delta)}{\theta(1-\delta)}$. Upon plugging in the expressions for p_{12}^e and p_{22}^e, we have $\hat{d}_1 = \frac{3V-p_{11}(3-(3-4\alpha)\delta)+\delta(3\theta-6V+2(p_{21}+V)\alpha+(V-\theta)(3-2\alpha)\delta)}{\theta(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$ and

$\hat{d}_2 = \frac{(6V-2(p_{11}+V)\alpha-p_{21}(3-4\alpha)-\theta(9-8\alpha))\delta-3(V-p_{21}-\theta)-(V-2\theta(1-\alpha))(3-2\alpha)\delta^2}{\theta(3-(3-2\alpha)\delta)(1-(1-2\alpha)\delta)}$. Plugging $\hat{d}_1$ and $\hat{d}_2$ into the profit functions and solving the first-order conditions ($\frac{d\pi_1}{dp_{11}} = 0$ and $\frac{d\pi_2}{dp_{21}} = 0$), we obtain the firms' first-period prices:

$$p_{11}^* = p_{21}^* = \frac{V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4+\delta(4\alpha-5-2\delta+4\alpha\delta+(3-4(2-\alpha)\alpha)\delta^2))}{2(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}. \tag{B12}$$

Using the expression for p_{11}^* and p_{21}^*, we find that

$$\hat{d}_1 = \frac{(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)}{2\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}, \tag{B13}$$

$$\hat{d}_2 = 1 - \frac{(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)}{2\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}, \tag{B14}$$

$$\pi_{11}^* = \pi_{21}^* =$$

$$\frac{\alpha(1-\delta)(3V-4\theta-(V-\theta)(3-4\alpha)\delta)(V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4+\delta(4\alpha-5-2\delta+4\alpha\delta+(3-4(2-\alpha)\alpha)\delta^2)))}{4\theta(1-(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2}, \tag{B15}$$

$$\pi_{12}^* = \pi_{22}^* = \frac{(V\alpha(1-\delta)(3-(3-4\alpha)\delta)-\theta(3-6(1-\alpha)\delta+(3-2(3-\alpha)\alpha)\delta^2))^2}{2\theta(1+(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2}. \quad \text{(B16)}$$

The firms' overall profits are $\pi_1^* = \pi_{11}^* + \pi_{12}^*$ and $\pi_2^* = \pi_{21}^* + \pi_{22}^*$.

The prices p_{11}^* and p_{21}^* in (B12) must satisfy the inequalities (B10) and (B11) because otherwise, our initial guess would be violated. Moreover, a necessary condition for the existence of this equilibrium is that $0 < \hat{d}_1 < \hat{d}_2 < 1$. We find that the inequalities (B10), (B11) and $0 < \hat{d}_1 < \hat{d}_2 < 1$ are satisfied if and only if $\frac{V(1-\delta)(3-(3-4\alpha)\delta)}{7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2} < \theta < \bar{\theta}$, where recall that $\bar{\theta} = \frac{V(1-\delta)}{2-\delta}$.

Denote $\tilde{\theta} = \frac{V(1-\delta)(3-(3-4\alpha)\delta)}{7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2}$. Note that the inequality $\tilde{\theta} < \bar{\theta}$ can hold if and only if $0 < \delta < \frac{1}{3}$ and $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$.

Because our main analysis focuses on $\delta > \underline{\delta} \geq \frac{1}{2}$, it follows that the equilibrium from Case *i*) cannot exist as an alternative equilibrium on the parameter region that we analyze. Furthermore, as we demonstrate below, even when $0 < \delta < \frac{1}{3}$, the equilibrium from Case *i*) still fails to exist due to the existence of non-local profitable price deviations in the first period.

Let us show that when $\tilde{\theta} < \theta < \bar{\theta}$, firm 2 has a profitable non-local deviation in the first period. Specifically, suppose that firm 2 deviates to a lower price: $p_{12}' = \theta$. Under the price pair (p_{11}^*, θ), first-period consumers rationally choose to buy a product in the first period instead of waiting.[21] Hence, after its deviation to $p_{12}' = \theta$, firm 2's first-period sales are given by $\alpha(1-\tilde{d})$, where $\tilde{d}$ satisfies $V - p_{11}^* - \theta\tilde{d} = V - \theta - \theta(1-\tilde{d})$ and is given by

$$\tilde{d} = \frac{V(1-\delta)(3(1-\delta)^2-8\alpha(1-\delta)^2-4\alpha^2(2-\delta)\delta)+\theta(4-\delta(5-4\alpha+2\delta-4\alpha\delta-(3-4(2-\alpha)\alpha)\delta^2))}{4\theta(1-(1-\alpha)\delta)(3-3\delta-\alpha(4-6\delta))}.$$

Because none of the first-period consumers waits till the second period to buy, in the second-period, the firms' equilibrium prices are $p_{12} = p_{22} = \theta$, with corresponding profits of $\pi_{12} = \pi_{22} = \frac{(1-\alpha)\theta}{2}$

[21] We showed this in the proof of Lemma 1. Namely, for a given pair of first-period prices (p_{11}, θ), consumers' rational beliefs about second-period prices are $p_{12}^e = p_{22}^e = \theta$. Given these beliefs, buying in the first period is preferable because $\max\{V - p_{11} - \theta d, V - \theta - \theta(1-d)\} \geq \max\{\delta(V - p_{12}^e - \theta d), \delta(V - p_{22}^e - \theta(1-d))\}$, where the inequality holds because $\theta < \bar{\theta} = \frac{V(1-\delta)}{2-\delta}$. If all consumers buy in the first period, then we know that the equilibrium second-period prices are $p_{12}^* = p_{22}^* = \theta$. Hence, consumers' price expectations and purchase decisions are indeed rational.

(see the proof of Lemma 1). Therefore, by deviating to $p'_{12} = \theta$, firm 2 obtains a profit of $\pi'_2 = \alpha(1 - \tilde{d})\theta + \frac{(1-\alpha)\theta}{2}$. It remains to demonstrate that $\pi'_2 > \pi^*_2 = \pi^*_{21} + \pi^*_{22}$, where π^*_{21} and π^*_{22} are given in equations (B15) and (B16), respectively. Denote $\Delta\pi = \pi'_2 - \pi^*_2$. Then, $\Delta\pi$ is concave because $\frac{d^2\Delta\pi}{d\theta^2} = -\frac{V^2\alpha(1-\delta)^2(3-(3-4\alpha)\delta)\left(3-2\alpha-2(3-5\alpha)\delta+(3-4(2-\alpha)\alpha)\delta^2\right)}{2\theta^3(1-(1-\alpha)\delta)^2\left(3-3\delta-\alpha(4-6\delta)\right)^2} < 0$. Further,

$\Delta\pi|_{\theta=\tilde{\theta}} = \frac{V(1-\alpha)\alpha(1-\delta)(3-(3-4\alpha)\delta)}{2(7-4\alpha-13\delta+(17-4\alpha)\alpha\delta+(6-\alpha(13-6\alpha))\delta^2)} > 0$ and

$\Delta\pi|_{\theta=\bar{\theta}} = \frac{V\alpha(1-\delta)(2(1-\delta)^3(4-3\delta)+4\alpha^4(2-\delta)\delta^2(2-3\delta)+8\alpha^3(1-\delta)\delta\left(4-\delta(13-6\delta)\right)-\alpha(1-\delta)^2\left(24-\delta(65-33\delta)\right)+\Omega)}{2(2-\delta)(1-(1-\alpha)\delta)^2(3-3\delta-\alpha(4-6\delta))^2}$

where $\Omega = \alpha^2(1-\delta)(16-\delta(108-\delta(163-63\delta)))$. Recall that we must have $0 < \delta < \frac{1}{3}$, $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$ and $\tilde{\theta} < \theta < \bar{\theta}$ to ensure that the necessary condition for the existence of the equilibrium is satisfied. Using the conditions $0 < \delta < \frac{1}{3}$ and $0 < \alpha < \frac{9(1-\delta)\delta-4+\sqrt{16-(1-\delta)\delta(56-9(1-\delta)\delta)}}{4\delta(2-3\delta)}$, one can show that $\Delta\pi|_{\theta=\bar{\theta}} > 0$. Given the concavity of $\Delta\pi$ at any $\theta \in (\tilde{\theta}, \bar{\theta})$ and $\Delta\pi|_{\theta=\tilde{\theta}} > 0$ and $\Delta\pi|_{\theta=\bar{\theta}} > 0$, it follows that $\Delta\pi > 0$ for any $\theta \in (\tilde{\theta}, \bar{\theta})$. Hence, this implies that firm 2's deviation to $p'_{21} = \theta$ is profitable, which, in turn, implies that p^*_{12} and p^*_{21} in (B12) cannot constitute an equilibrium.

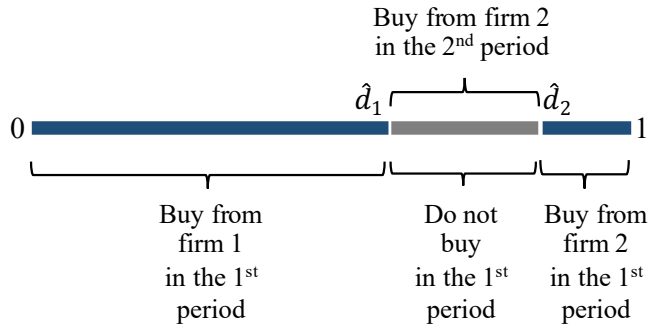
Case ii). We guess that the following inequalities will be satisfied in equilibrium:

$$\delta(V - p^e_{12} - \theta\hat{d}_1) < \delta(V - p^e_{22} - \theta(1 - \hat{d}_1)), \tag{B17}$$

$$\delta(V - p^e_{12} - \theta\hat{d}_2) < \delta(V - p^e_{22} - \theta(1 - \hat{d}_2)). \tag{B18}$$

In words, first-period consumers who do not buy in the first period expect to purchase firm 2's product in the second period. In a rational-expectations equilibrium, the decision that a consumer expects to make in the second period must be consistent with the consumer's equilibrium choice. Figure B2 illustrates first-period consumers' decisions.

Figure B2 Purchase Decisions of Consumers Entering the Market in the 1st Period



The consumer located at $\hat{d}_1$ is indifferent between purchasing firm 1's product in the first period and purchasing firm 2's product in the second period, whereas the consumer located at $d = \hat{d}_2$ is indifferent between purchasing firm 2's product in the first period and purchasing it in the second period. Mathematically, $V - p_{11} - \theta\hat{d}_1 = \delta(V - p_{22}^e - \theta(1 - \hat{d}_1))$ and $V - p_{21} - \theta(1 - \hat{d}_2) = \delta(V - p_{22}^e - \theta(1 - \hat{d}_2))$. Solving these equations, we find $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + (p_{22}^e + \theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21} - (1-\delta)(V-\theta) - p_{22}^e\delta}{\theta(1-\delta)}$. In a rational-expectations equilibrium, the firms' second-period prices must be such that all consumers located on $[\hat{d}_1, \hat{d}_2]$ purchase firm 2's product because otherwise the inequalities (B17) and (B18) will be violated. The firms' second-period profit functions are as follows:

$$\pi_{12} = (1 - \alpha)\frac{\theta - p_{12} + p_{22}}{2\theta}p_{12}, \tag{B19}$$

$$\pi_{22} = \left(\alpha\left(\hat{d}_2 - \hat{d}_1\right) + (1 - \alpha)\left(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}\right)\right)p_{22}, \tag{B20}$$

where $\hat{d}_1 = \frac{(1-\delta)V - p_{11} + (p_{22}^e + \theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21} - (1-\delta)(V-\theta) - p_{22}^e\delta}{\theta(1-\delta)}$. Solving the first-order conditions, we obtain $p_{12}^* = \theta + \frac{2(p_{21} - p_{22}^e)\alpha}{3(1-\alpha)(1-\delta)} + \frac{2(p_{11} + p_{22}^e + \theta - 2V)\alpha}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \theta + \frac{4(p_{21} - p_{22}^e)\alpha}{3(1-\alpha)(1-\delta)} + \frac{4(p_{11} + p_{22}^e + \theta - 2V)\alpha}{3(1-\alpha)(1+\delta)}$. The rational-expectations condition requires that $p_{12}^* = p_{12}^e$ and $p_{22}^* = p_{22}^e$. Solving these equalities using the expressions for p_{12}^* and p_{22}^*, we find that $p_{12}^e = \frac{2\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1+\delta)(3 - \alpha - 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}$ and $p_{22}^e = \frac{4\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1-\delta)(3 + \alpha + 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}$. Upon plugging the expression for p_{12}^e and p_{22}^e back into p_{12}^* and p_{22}^*, we verify that

$$p_{12}^* = p_{12}^e = \frac{2\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1+\delta)(3 - \alpha - 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}, \tag{B21}$$

$$p_{22}^* = p_{22}^e = \frac{4\alpha(p_{21}(1+\delta) - (2V - p_{11})(1-\delta)) + \theta(1-\delta)(3 + \alpha + 3(1-\alpha)\delta)}{3 - 3\delta^2 - \alpha(3+\delta)(1-3\delta)}. \tag{B22}$$

The firms' second-period profits are given by

$$\pi_{12}^{*} = \frac{(1-\alpha)(2\alpha(p_{21}(1+\delta)-(2V-p_{11})(1-\delta))+\theta(1+\delta)(3-\alpha-3(1-\alpha)\delta))^2}{2\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))^2}, \tag{B23}$$

$$\pi_{22}^{*} = \frac{(1-\alpha)(4\alpha(p_{21}(1+\delta)-(2V-p_{11})(1-\delta))+\theta(1-\delta)(3+\alpha+3(1-\alpha)\delta))^2}{2\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))^2}. \tag{B24}$$

Using the expressions for p_{12}^{e} and p_{22}^{e}, we obtain the cutoffs $\hat{d}_1 = \frac{(1-\delta)V-p_{11}+(p_{22}^{e}+\theta)\delta}{\theta(1+\delta)}$ and $\hat{d}_2 = \frac{p_{21}-(1-\delta)(V-\theta)-p_{22}^{e}\delta}{\theta(1-\delta)}$ as functions of the first-period prices:

$$\hat{d}_1 = \frac{3V(1-\alpha)(1-\delta)^2+4\alpha\delta p_{21}+2\theta\delta(3-\alpha-3(1-\alpha)\delta)-p_{11}(3-3\alpha-3\delta+7\alpha\delta)}{\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))}, \tag{B25}$$

$$\hat{d}_2 = \frac{3(p_{21}+\theta-V)(1-\alpha)+\delta(p_{21}(3+\alpha)-4p_{11}\alpha-\theta(3-7\alpha))+3(V-2\theta)(1-\alpha)\delta^2}{\theta(3-3\delta^2-\alpha(3+\delta)(1-3\delta))}. \tag{B26}$$

A necessary condition for the existence of the equilibrium is that $\hat{d}_1 < \hat{d}_2$. Further, p_{12}^{e} and p_{22}^{e} must satisfy the inequalities (B17) and (B18). We find that these conditions can be satisfied only if the firms' first-period prices satisfy the following inequality:

$$\max\left\{\frac{3(V-\theta)(1-\alpha)\delta^2-3(V-p_{21})(1-\alpha)+(p_{21}-\theta)(3+\alpha)\delta}{4\alpha\delta}, \frac{(1-\delta)(2V-3\theta+p_{21})-2(p_{21}-\theta)}{1-\delta}\right\} < p_{11} < \frac{(2V(1-\delta)-\theta(1-3\delta))(3-\alpha-3(1-\alpha)\delta)-2p_{21}\alpha(1-3\delta)}{6(1-\delta)-4\alpha(1-3\delta)} \tag{B27}$$

After finding the firms' first-period prices, we will need to verify that these prices satisfy the inequality (B27).

In the first-period, consumers with $d < \hat{d}_1$ buy firm 1's product, whereas consumers with $d > \hat{d}_2$ buy firm 2's product. The firms' equilibrium first-period prices must maximize the sum of their profits over the two periods:

$$\pi_1 = \alpha\hat{d}_1 p_{11} + \pi_{12}^{*}, \tag{B28}$$

$$\pi_2 = \alpha(1-\hat{d}_2)p_{21} + \pi_{22}^{*}. \tag{B29}$$

We assume that $\delta > \frac{1}{2}$ to ensure that the profit functions are concave. Solving the first-order conditions ($\frac{d\pi_1}{dp_{11}} = 0$ and $\frac{d\pi_2}{dp_{21}} = 0$), we obtain

$$p_{11}^{*} = \frac{\theta(9-(16-7\alpha)\alpha+\alpha(17-5\alpha)\delta-9(1-\alpha)\delta^2)((1-\delta)(1+\delta)(1+3\delta)-\alpha(1+3\delta(1-\delta(3+\delta))))+\Sigma_1}{3(1-\alpha)^2(9-19\alpha)+6(1-\alpha)\alpha(21-31\alpha)\delta-6(1-\alpha)(9-\alpha(23+16\alpha))\delta^2-2\alpha(63-\alpha(116-5\alpha))\delta^3+9(1-\alpha)(3-\alpha(6+\alpha))\delta^4}$$

$$p_{21}^{*} = \frac{\frac{\theta(9-\alpha(4+5\alpha)+2\alpha(5+7\alpha)\delta-9(1-\alpha)^2\delta^2)((1-\delta)(1+\delta)(4+3\delta)-\alpha(4+3\delta(1-\delta(4+\delta))))}{2}+\Sigma_2}{3(1-\alpha)^2(9-19\alpha)+6(1-\alpha)\alpha(21-31\alpha)\delta-6(1-\alpha)(9-\alpha(23+16\alpha))\delta^2-2\alpha(63-\alpha(116-5\alpha))\delta^3+9(1-\alpha)(3-\alpha(6+\alpha))\delta^4}$$

where $\Sigma_1 = \frac{V(1-\alpha)(1-\delta)(27(1-\delta^2)^2-12\alpha(1-\delta)(1+\delta)(8-3\delta(3+\delta))+\alpha^2(69-\delta(204-\delta(34+60\delta+9\delta^2))))}{2}$ and $\Sigma_2 = \frac{V(1-\alpha)(1-\delta)(27(1-\delta^2)^2-12\alpha(1-\delta)(1+\delta)(11-9\delta-6\delta^2)+\alpha^2(105-\delta(252+\delta(118-3\delta(52+15\delta)))))}{2}$.

A necessary condition for the prices p_{11}^* and p_{21}^* to constitute an equilibrium is that the inequality (B27) is a satisfied. Upon plugging the prices p_{11}^* and p_{21}^* into the inequality (B27) and simplifying the expressions, one can show that $\frac{(1-\delta)(2V-3\theta+p_{21}^*)-2(p_{21}^*-\theta)}{1-\delta} > p_{11}^*$ for any $\delta > \frac{1}{2}$ and $\theta < \bar{\theta}$. Therefore, the prices p_{11}^* and p_{21}^* cannot satisfy (B27). It follows that the necessary conditions for p_{11}^* and p_{21}^* to be an equilibrium fail to hold. Intuitively, to induce some of the first-period consumers to wait till the second period, the firms' first-period prices need to be high. However, if firm 1 sets a high first-period price, then firm 2 gains more profit by reducing its first-period price and capturing more customers in the first period. As a result, our initial guess that some of the first-period consumers would wait till the second period to buy fails to hold on the "equilibrium" path.

Case iii). The analysis of Case *iii*) is similar to that in Case *ii*) because of the symmetry between the firms.

The above analysis suggests that on the parameter region that our analysis focus on, the equilibrium in Lemma 1 is unique. However, for very large values of θ, there can also exist an equilibrium where the firms avoid competitive friction and stay as local monopolists. In this situation, the firms would want to enter the market early instead of delaying. Thus, high level of differentiation between the firms acts as a boundary condition for our results; we discuss this in the Conclusion section in the main paper.

2. PROFIT COMPARISON WHEN $(R_1, R_2) = (1, 2)$

As we point out in the main text, when $(R_1, R_2) = (1, 2)$, firm 1's equilibrium profit exceeds firm 2's profit: $\pi_1^{(1,2)} > \pi_2^{(1,2)}$. Below we prove this.

Using firm 1's first-period equilibrium price $p_{11}^{(1,2)}$ in Lemma 3, we can plug it into the equations (A9), (A10), (A11), (A14) and (A15) to obtain the firms' first- and second-period sales and prices *on the equilibrium path*. To simplify notation, we will write $S_{it}^{*(1,2)}$ to denote firm i's sales in period t on the equilibrium path, i.e., $S_{it}^{*(1,2)} = S_{it}^{(1,2)}|_{p_{11}=p_{11}^{(1,2)}}$. Similarly, we will write $p_{i2}^{*(1,2)}$ to denote firm i's second-period price on the equilibrium path: $p_{i2}^{*(1,2)} = p_{i2}^{(1,2)}|_{p_{11}=p_{11}^{(1,2)}}$.

Using the above notation, the firms' equilibrium profits can be written as follows: $\pi_1^{(1,2)} = S_{11}^{*(1,2)} p_{11}^{(1,2)} + S_{12}^{*(1,2)} p_{12}^{*(1,2)}$ and $\pi_2^{(1,2)} = S_{22}^{*(1,2)} p_{22}^{*(1,2)}$. Note that $S_{11}^{*(1,2)} + S_{12}^{*(1,2)} + S_{22}^{*(1,2)} = 1$ because the measure of consumers entering the market is one. Then, we can write $\pi_1^{(1,2)} - \pi_2^{(1,2)} = S_{11}^{*(1,2)} p_{11}^{(1,2)} - (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)} + S_{12}^{*(1,2)} p_{12}^{*(1,2)} - (1 - \alpha - S_{12}^{*(1,2)}) p_{22}^{*(1,2)}$. One can readily show that $S_{11}^{*(1,2)} > \frac{\alpha}{2}$ and $S_{12}^{*(1,2)} \geq \frac{1-\alpha}{2}$. Further, $p_{11}^{(1,2)} > p_{22}^{*(1,2)}$.[22] Hence, we must have $S_{11}^{*(1,2)} p_{11}^{(1,2)} > (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)}$. Now, given firm 1's first-period price $p_{11}^{(1,2)}$, we know that $p_{12}^{*(1,2)}$ and $p_{22}^{*(1,2)}$ are the firms' second-period prices on the equilibrium path. Suppose that in the second-period, firm 1 deviates to $p_{12}^{d} = p_{22}^{*(1,2)}$, i.e., charges the same price as firm 2. Then, firm 1 will capture exactly half of the new consumers entering the market in the second period, obtaining a second-period profit of $\frac{1-\alpha}{2} p_{22}^{*(1,2)}$. This deviation profit must be less than what firm 1 earns by charging its equilibrium price $p_{12}^{*(1,2)}$, i.e., $\frac{1-\alpha}{2} p_{22}^{*(1,2)} < S_{12}^{*(1,2)} p_{12}^{*(1,2)}$. Because $S_{12}^{*(1,2)} \geq \frac{1-\alpha}{2}$, it follows that $S_{12}^{*(1,2)} p_{12}^{*(1,2)} > \frac{1-\alpha}{2} p_{22}^{*(1,2)} > (1 - \alpha - S_{12}^{*(1,2)}) p_{22}^{*(1,2)}$. The inequalities $S_{11}^{*(1,2)} p_{11}^{(1,2)} > (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)}$ and

[22] To see this, note that for a given $p_{11} \in [\underline{\rho}, \tilde{\rho}]$, Lemma 2 shows that $p_{22}^{(1,2)} = \frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. Because $\theta < \bar{\theta}$, straightforward algebra shows $\frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)} < p_{11}$ for all $p_{11} \in [\underline{\rho}, \tilde{\rho}]$. From Lemma 3, we know that $p_{11}^{(1,2)} \in [\underline{\rho}, \tilde{\rho}]$. Hence, on the equilibrium path, $p_{11}^{(1,2)} > p_{22}^{*(1,2)}$.

$S_{12}^{*(1,2)} p_{12}^{*(1,2)} > (1-\alpha-S_{12}^{*(1,2)}) p_{22}^{*(1,2)}$ imply that $\pi_1^{(1,2)} - \pi_2^{(1,2)} = S_{11}^{*(1,2)} p_{11}^{(1,2)} - (\alpha - S_{11}^{*(1,2)}) p_{22}^{*(1,2)} + S_{12}^{*(1,2)} p_{12}^{*(1,2)} - (1-\alpha-S_{12}^{*(1,2)}) p_{22}^{*(1,2)} > 0$. ■

3. EFFECT OF VERTICAL DIFFERENTIATION

We will first obtain the firms' equilibrium pricing strategies and profits in each subgame $(R_1, R_2) \in \{(1,1),(2,2),(1,2),(2,1)\}$. Then, we will characterize market conditions under which $(R_1^*, R_2^*) = (1,2)$ and/or $(R_1^*, R_2^*) = (2,1)$ constitute an equilibrium. We assume that $\mu < 3\theta$ to ensure that, in equilibrium, firm 1 has a positive market share.

Subgame with $(\boldsymbol{R_1, R_2}) = (\mathbf{1,2})$

The analysis follows similar steps as in the proof of Lemma 2. We solve the game by backward induction. In the second period, the firms compete for the new consumers entering the market, as well as consumers who entered the market in the first period but did not buy a product in that period. Therefore, to obtain the firms' second-period prices, we first characterize those first-period consumers who would still be shopping in the second period. Namely, in the first period, a consumer prefers to wait till the second period to buy if and only if

$$V - p_{11} - \theta d < \max\{\delta(V - p_{12}^e - \theta d), \delta(V + \mu - p_{22}^e - \theta(1-d))\}. \tag{B30}$$

Similar analysis as in the main model shows that (B30) reduces to

$$d > \hat{d}. \tag{B31}$$

The cutoff $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \le V(1-\delta) + \delta(p_{22}^e - \mu) - \theta \\ \frac{V(1-\delta) - p_{11} + (p_{22}^e + \theta - \mu)\delta}{\theta(1+\delta)} & \text{if } V(1-\delta) + \delta(p_{22}^e - \mu) - \theta < p_{11} \le \frac{(2V+\mu-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V+\mu-p_{22}^e-\theta)(1-\delta)+p_{12}^e(1+\delta)}{2} < p_{11} < V(1-\delta) + \delta p_{12}^e \\ 0 & \text{if } p_{11} \ge V(1-\delta) + \delta p_{12}^e \end{cases} \tag{B32}$$

As in the proof of Lemma 2, we separately analyze the four possible cases. We find that the second period equilibrium prices are as follows:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta - \frac{\mu}{3} & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2\alpha(p_{11} - V(1-\delta)) + \theta(3-\alpha)(1+\delta) - \mu(1-\alpha)(1+\delta)}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11} - V(1-\delta)) + 3\theta(1-\delta) - \mu(1-\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B33}$$

$$p_{22}^{(1,2)} = p_{22}^{e} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4\alpha(p_{11} - V(1-\delta)) + \theta(3+\alpha+3\delta(1-\alpha)) + \mu(1+\delta-\alpha(1-3\delta))}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta+\mu+2\alpha(\mathrm{p}_{11}-V)+(2(\theta+V+\mu)\alpha-3\theta-\mu)\delta}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \quad \text{(B34)}$$

where $\underline{\rho} = (1-\delta)(V-\theta) - \frac{2\mu\delta}{3}$, $\tilde{\rho} \equiv \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv V(1-\delta) + \theta\delta - \frac{\mu\delta}{3}$. Consistent with the main model, to ensure the existence of the second-period subgame equilibrium for any given first-period price $p_{11} \geq 0$, we assume that $\alpha < \bar{\alpha}$ and $\delta > \underline{\delta}$, where $\bar{\alpha} \approx 0.64$, $\underline{\delta} \equiv \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$.

For future reference, we use the prices in (B33) and (B34) to obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1, 2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta)-\mu(1-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta)-\mu(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{(3\theta-\mu)^2}{18} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B35)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(p_{11}-V(1-\delta))+\theta(3+\alpha+3\delta-3\alpha\delta)+\mu(1-\alpha+\delta+3\alpha\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(3\theta+\mu+2\alpha(p_{11}-V)+(2(\theta+V+\mu)\alpha-3\theta-\mu)\delta)^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{(3\theta+\mu)^2}{18} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B36)}$$

Firm 1's first-period sales are given by $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^{e} = p_{12}^{(1,2)}$ and $p_{22}^{e} = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{12}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{3(V-p_{11})(1-\alpha)+2\theta(3-\alpha)\delta-(3V+2\mu)(1-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3V-3p_{11}-(3V-3\theta+\mu)\delta}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B37)}$$

In the first period, firm 1 chooses its price to maximize its overall profit over the two periods: $\pi_1 = S_{11}^{(1,2)} p_{11} + \pi_{12}^{(1,2)}$. One can show that the price $p_{11}^{(1,2)}$ maximizing π_1 is contained in the interval $[\underline{\rho}, \tilde{\rho}]$.

Define $\dot{\mu}_a = \min\{\frac{12\theta(2-\alpha(2-\delta))+9V(-1+\alpha+\delta-\alpha\delta)}{2(1-\alpha)(1-3\delta)}; 3\theta\}$ and $\ddot{\mu}_a = \min\{\max\{0, \frac{\theta(3-\alpha)}{1-\alpha} +$

$\frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10}\}; 3\theta\}$. Further, let $\Sigma_{a1} = \{(\theta,\mu)|\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, $\Sigma_{a2} = \{(\theta,\mu)|\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\mu < \ddot{\mu}_a\}$ and $\Sigma_{a3} = \{(\theta,\mu)|\ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta\} \cup \{(\theta,\mu)|\ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\ddot{\mu}_a \leq \mu < 3\theta\}$. Then, we find that

$$p_{11}^{(1,2)} =$$

$$\begin{cases} (1-\delta)(V-\theta) - \frac{2\mu\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)-2\mu(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B38}$$

Upon plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$, we find the firms' second-period prices on the equilibrium path.

$$p_{12}^{(1,2)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)(\theta(3-\alpha)-\mu+\mu\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-(1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B39}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\theta(6-\alpha(3+\alpha))+\mu(2-(1-\alpha)\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{(1-\alpha)(3\mu(1+\delta)-6V\alpha(1-\delta)-5\mu\alpha(1-\delta))+\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B40}$$

Using the equilibrium prices in (B38)-(B40), we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) - \frac{2\mu\alpha\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\alpha(\theta(3-\alpha)+\mu\alpha-\mu)(V(1-\delta)(6-\alpha(7-3\alpha))+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+2\mu-6\mu\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B41}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)+\mu\alpha-\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \tag{B42}$$

$$\pi_{21}^{(1,2)} = 0 \tag{B43}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases} \quad \text{(B44)}$$

where $A \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)-2\mu(1-\alpha+4(1-\alpha)\delta+(3+\alpha)\delta^2))+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period in equations (B41)-(B44), we can readily find firm i's overall profit in the subgame with $(R_1, R_2) = (1, 2)$: $\pi_i^{(1,2)} = \pi_{i1}^{(1,2)} + \pi_{i2}^{(1,2)}$.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{1})$

The analysis is very similar to that in the subgame with $(R_2, R_2) = (1, 2)$. Therefore, we will directly provide the equilibrium prices and profits to avoid repetitiveness.

Define $\dot{\mu}_b = \max\{\frac{12\theta(2-\alpha(2-\delta))-9V(1-\alpha-\delta+\alpha\delta)}{(1-\alpha)(7-3\delta)}; 0\}$ and $\ddot{\mu}_b = \max\{0, \frac{2\theta(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))-3V(1-\alpha)(1-\delta)(6-\alpha(7-3\alpha))}{(1-\alpha)(8-12\delta-\alpha(9-5\delta-\alpha(7-3\delta)))}\}$. Further, let $\Sigma_{b1} = \{(\theta,\mu)| \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))}$ and $\dot{\mu}_b < \mu < 3\theta\} \cup \{(\theta,\mu)| \ \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $3\theta(1-\alpha) \leq \mu < 3\theta\}$, $\Sigma_{b2} = \{(\theta,\mu)| \ \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $\ddot{\mu}_b < \mu < 3\theta(1-\alpha)\}$ and $\Sigma_{b3} = \{(\theta,\mu)| \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))}$ and $0 \leq \mu < \dot{\mu}_b\} \cup \{(\theta,\mu)| \ \frac{3V(1-\alpha)(1-\delta)}{1+3\delta+\alpha(6-2\delta-\alpha(7-3\delta))} < \theta < \bar{\theta}$ and $0 \leq \mu < \ddot{\mu}_b\}$. We find that firm 2's optimal first-period price is given by

$$p_{21}^{(2,1)} = \begin{cases} (V-\theta)(1-\delta) + \frac{3-\delta}{3}\mu & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(5-\alpha(2-\alpha)(3-\delta)-3\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)+\mu(11+8\delta-3\delta^2-\alpha(15-8\delta+\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B45)}$$

The firms' second-period prices on the equilibrium path are given by

$$p_{12}^{(2,1)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{6\theta - \theta\alpha(3+\alpha) - \mu(2-\alpha+\alpha^2)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)) - (1-\alpha)(\mu(3+\alpha)+6V\alpha(1-\delta)+\mu(3-\alpha)\delta)}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B46)}$$

$$p_{22}^{(2,1)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(\theta(3-\alpha)+\mu(1-\alpha))(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)) - (1-\alpha)(2\mu\alpha(3-\delta)+3V\alpha(1-\delta)-3\mu(1+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B47)}$$

Using the equilibrium prices, we obtain the firms' equilibrium profits in each period.

$$\pi_{11}^{(2,1)} = 0 \quad \text{(B48)}$$

$$\pi_{12}^{(2,1)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha)) - \mu(2-(1-\alpha)\alpha))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+\mu(3+\alpha)+\mu(3-\alpha)\delta) - \theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B49)}$$

$$\pi_{21}^{(2,1)} =$$

$$\begin{cases} \alpha(V-\theta)(1-\delta) + \alpha\mu\frac{3-\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{\alpha(\theta(3-\alpha)-\mu\alpha+\mu)(V(1-\delta)(6-\alpha(7-3\alpha)) + \mu(5-3\delta-(2-\alpha)\alpha(3-\delta)) - \theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+\mu(7-3\delta)) - 2\theta(3-\alpha)(1-3\delta))B}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B50)}$$

$$\pi_{22}^{(2,1)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{b1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)-\mu\alpha+\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{b2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)-3\mu(1+\delta)+2\mu\alpha(3-\delta)) - \theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{b3} \end{cases} \quad \text{(B51)}$$

where $B \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)+\mu(11-15\alpha+8(1+\alpha)\delta-(3+\alpha)\delta^2)+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period in the equations (B48)-(B51), we can readily obtain firm i's overall profit in the subgame with $(R_1, R_2) = (2,1)$: $\pi_i^{(2,1)} = \pi_{i1}^{(2,1)} + \pi_{i2}^{(2,1)}$.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{1})$

We solve the game by backward induction, following similar steps as in the main model. Specifically, we start with a guess that in the first period, the market is fully covered, and hence, no consumer will

choose to wait till the second period to buy a product (we will later verify our guess).[23] Mathematically, this is equivalent to

$$\max\{V - p_{11} - \theta d, V + \mu - p_{21} - \theta(1-d)\} \geq \delta \max\{V - p_{12}^e - \theta d, V + \mu - p_{22}^e - \theta(1-d)\} \quad \text{(B52)}$$

for all $d \in [0,1]$, where p_{12}^e and p_{22}^e are the prices that first-period consumers expect firms 1 and 2 to charge in the second period, respectively.

If the inequality (B52) is satisfied, then in the second period, the firms compete for the $(1-\alpha)$ measure of new consumers who enter the market. Given the firms' second-period prices p_{12} and p_{22}, a consumer located at a distance d from firm 1 will prefer firm 1's product over firm 2's if and only if $V - p_{12} - \theta d > V + \mu - p_{22} - \theta(1-d)$, or equivalently, $d < \frac{\theta - \mu - p_{12} + p_{22}}{2\theta}$. Hence, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = (1-\alpha)\frac{\theta - \mu - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = (1-\alpha)(1 - \frac{\theta - \mu - p_{12} + p_{22}}{2\theta})p_{22}$. Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$), we find that $p_{12}^{(1,1)} = \theta - \frac{\mu}{3}$ and $p_{22}^{(1,1)} = \theta + \frac{\mu}{3}$. The corresponding second-period profits are given by $\pi_{12}^{(1,1)} = \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta}$ and $\pi_{22}^{(1,1)} = \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta}$. Rational-expectations condition implies that $p_{12}^e = \theta - \frac{\mu}{3}$ and $p_{22}^e = \theta + \frac{\mu}{3}$. Plugging these prices into (B52), the inequality (B52) becomes

$$\max\{V - p_{11} - \theta d, V - p_{21} - \theta(1-d)\} \geq \delta \max\{(V - \theta + \tfrac{\mu}{3} - \theta d), \delta(V + \tfrac{2}{3}\mu - \theta - \theta(1-d))\} \quad \text{(B53)}$$

Hence, if a given pair (p_{11}, p_{21}) of first-period prices satisfies the inequality (B53), then *a*) $p_{12}^e = \theta - \frac{\mu}{3}$ and $p_{22}^e = \theta + \frac{\mu}{3}$ are rational expectations about the second-period prices, and *b*) all first-period consumers rationally choose to buy in the first period rather than wait till the second period.

In the first period, the firms choose their prices to maximize the sum of their profits over the two periods. If prices p_{11} and p_{21} satisfy (B53), then the firms' profits are given by $\pi_1 = \pi_{11} + \pi_{12}^{(1,1)} = \alpha\frac{\theta - \mu - p_{11} + p_{21}}{2\theta} p_{11} + \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta}$ and $\pi_2 = \pi_{21} + \pi_{22}^{(1,1)} = \alpha(1 - \frac{\theta - \mu - p_{11} + p_{21}}{2\theta})p_{21} + \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta}$. Solving the first-order conditions, we find that $p_{11}^{(1,1)} = \theta - \frac{\mu}{3}$ and $p_{21}^{(1,1)} = \theta + \frac{\mu}{3}$. It follows that the

[23] Using a similar argument as in the main model, one can show that this equilibrium is unique when $\theta < \bar{\theta}$.

firms' first-period profits are given by $\pi_{11}^{(1,1)} = \frac{\alpha(3\theta-\mu)^2}{18\theta}$ and $\pi_{21}^{(1,1)} = \frac{\alpha(3\theta+\mu)^2}{18\theta}$. Their overall profits across the two periods are $\pi_1^{(1,1)} = \frac{(3\theta-\mu)^2}{18\theta}$ and $\pi_2^{(1,1)} = \frac{(3\theta+\mu)^2}{18\theta}$.

Because $\delta < 1$, it is easy to see that the price pair $(p_{11}, p_{21}) = (\theta - \frac{\mu}{3}, \theta + \frac{\mu}{3})$ indeed satisfies the inequality (B53). Hence, our initial guess that first-period consumers will not wait till the second period is verified.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{2})$

The analysis is straightforward. One can show that $p_{12}^{(2,2)} = \theta - \frac{\mu}{3}$ and $p_{22}^{(2,2)} = \theta + \frac{\mu}{3}$, and the firms' profits are given by $\pi_1^{(2,2)} = \frac{(3\theta-\mu)^2}{18\theta}$ and $\pi_2^{(2,2)} = \frac{(3\theta+\mu)^2}{18\theta}$.

Having obtained the firms' equilibrium profits in each subgame, we can find their equilibrium product release times.

PROOF OF PROPOSITION 4. First, let us characterize market conditions under which the high-quality firm (firm 2) releases its product later than firm 1.

Equilibrium where $(\boldsymbol{R_1^*}, \boldsymbol{R_2^*}) = (\mathbf{1}, \mathbf{2})$

$(R_1^*, R_2^*) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} > \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Using a similar argument as in the proof of Proposition 1, one can easily show that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. It remains to check whether $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

If $(\theta, \mu) \in \Sigma_{a1} = \{(\theta, \mu) | \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, then $\Delta\pi_2 = \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} - \frac{(3\theta+\mu)^2}{18\theta}$. Clearly, $\Delta\pi_2 < 0$. Therefore, $(R_1, R_2) = (1, 2)$ cannot be an equilibrium.

Next, consider $(\theta, \mu) \in \{(\theta, \mu) | \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta\} \subseteq \Sigma_{a3}$. This subset of Σ_{a3} is non-empty (i.e., $\dot{\mu}_a < 3\theta$) if and only if $\frac{3V(1-\alpha)(1-\delta)}{6(1+\delta)-2\alpha(3+\delta)} < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Assuming θ satisfies these conditions, $\Delta\pi_2$ is given by

$\Delta\pi_2 = \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta}$. We find that $\Delta\pi_2 <$ 0 when $\frac{3V(1-\alpha)(1-\delta)}{6(1+\delta)-2\alpha(3+\delta)} < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta$.[24]

The above analysis suggests that if $\theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$, then $\Delta\pi_2 < 0$ for all $\mu < 3\theta$. Hence, we may have $\Delta\pi_2 > 0$ only if $\theta \geq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Denote $\dot{\theta}_{a1} = \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$. Conditional on $\dot{\theta}_{a1} \leq \theta < \bar{\theta}$, the expression for $\Delta\pi_2$ is as follows:

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} - \frac{(3\theta+\mu)^2}{18\theta} & \text{if } 0 < \mu \leq \ddot{\mu}_a \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta} & \text{if } \ddot{\mu}_a \leq \mu < 3\theta \end{cases}.$$

Note that $\Delta\pi_2$ is a continuous function. Using the expression of $\ddot{\mu}_a$, one can readily show that

$$\ddot{\mu}_a = \begin{cases} 0 & \text{if } \dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2} \\ \frac{\theta(3-\alpha)}{1-\alpha} + \frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10} & \text{if } \dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3} \\ 3\theta & \text{if } \dot{\theta}_{a3} \leq \theta < \bar{\theta} \end{cases}$$

where $\dot{\theta}_{a2} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}; \bar{\theta}\}$ and $\dot{\theta}_{a3} = \min\{\frac{3V(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{4\alpha(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}; \bar{\theta}\}$. We will separately analyze the cases with *a*) $\dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2}$, *b*) $\dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3}$, and *c*) $\dot{\theta}_{a3} \leq \theta < \bar{\theta}$.

a) Consider $\dot{\theta}_{a1} \leq \theta \leq \dot{\theta}_{a2}$. On this parameter range,

$\Delta\pi_2 = \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{(3\theta+\mu)^2}{18\theta}$ for all $\mu \in [0, 3\theta)$.

Because $\alpha < \bar{\alpha}$ and $\delta > \underline{\delta}$, one can show that $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$, i.e., $\Delta\pi_2$ is concave in μ. Further, if $\Delta\pi_2|_{\mu=0} < 0$, then one can show that $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} < 0$, which implies—due to concavity of $\Delta\pi_2$—that $\Delta\pi_2 < 0$ for all $\mu \in [0, 3\theta)$. Therefore, we may have $\Delta\pi_2 > 0$ only if $\Delta\pi_2|_{\mu=0} \geq 0$. The latter inequality is equivalent to $\theta > \hat{\theta}_2$, where

$\hat{\theta}_2 = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$. On a side note, notice that $\hat{\theta}_2$ is the same as $\hat{\theta}$ in Proposition 1. One can readily show that $\Delta\pi_2|_{\mu\to\infty} < 0$. Therefore, when

[24] To demonstrate this, first note that $\Delta\pi_2$ is concave as a function of μ, i.e., $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$. Further, we find that $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\dot{\mu}_a} < 0$. The inequalities $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\dot{\mu}_a} < 0$ imply that $\frac{d\Delta\pi_2}{d\mu} < 0$ for all $\mu \geq \dot{\mu}_a$. Finally, because $\Delta\pi_2|_{\mu=\dot{\mu}_a} < 0$, it follows that $\Delta\pi_2 < 0$ for all $\mu \geq \dot{\mu}_a$.

$\hat{\theta}_2 < \theta \leq \dot{\theta}_{a2}$, the inequalities $\Delta\pi_2|_{\mu=0} \geq 0$ and $\Delta\pi_2|_{\mu\to\infty} < 0$, together with the concavity of $\Delta\pi_2$, imply that there exists a unique $\hat{\mu}_{2a} > 0$, such that $\Delta\pi_2 > 0$ if and only if $\mu \in [0, \hat{\mu}_{2a}]$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\mu}_{2a} = \frac{3(-6V(1-\alpha)(1-\delta)(9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(3\delta+15\sqrt{1-\alpha}\delta-11-15\sqrt{1-\alpha}))+\theta(9(1+\delta)(20+3\sqrt{1-\alpha}-3(4-\sqrt{1-\alpha})\delta)+F_1+F_2))}{\sqrt{1-\alpha}(153-374\alpha+225\alpha^2(1-\delta)^2+9(2-15\delta)\delta+6\alpha\delta(64+9\delta))},$$

where $F_1 = 5\alpha^2(1-\delta)(44+15\sqrt{1-\alpha}-3(4+5\sqrt{1-\alpha})\delta)$ and $F_2 = 2\alpha(-200-53\sqrt{1-\alpha}+48(2+\sqrt{1-\alpha})\delta+9(8-3\sqrt{1-\alpha})\delta^2)$. Because we are focusing on $\mu < 3\theta$, it follows that when $\hat{\theta}_2 < \theta \leq \dot{\theta}_{a2}$ and $\mu \leq \min\{\hat{\mu}_{2a}, 3\theta\}$, we have $\Delta\pi_2 > 0$. Note that the interval $(\hat{\theta}_2, \dot{\theta}_{a2}]$ is non-empty (i.e., $\hat{\theta}_2 < \dot{\theta}_{a2}$) if and only if $\alpha < \hat{\alpha}$, where recall from the proof of Proposition 1 that $\hat{\alpha} \approx 0.32$ is the solution to the equality $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$.

b) Consider $\dot{\theta}_{a2} \leq \theta \leq \dot{\theta}_{a3}$. Let $\tilde{\alpha}$ be the solution to the equality $6 - 11\tilde{\alpha} + \left(3 - 2\sqrt{1-\tilde{\alpha}}\right)\tilde{\alpha}^2 = 0$. One can show that $\tilde{\alpha} \approx 0.6$. We will separately analyze three subcases: $\alpha < \hat{\alpha}$, $\hat{\alpha} < \alpha < \tilde{\alpha}$ and $\tilde{\alpha} < \alpha < \bar{\alpha}$.

First, if $\alpha < \hat{\alpha}$, then $\Delta\pi_2|_{\mu=0} > 0$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} > 0$ for $\mu \in [0, \ddot{\mu}_a)$. Because $\frac{d^2\Delta\pi_2}{d\mu^2} > 0$ at any $\mu \in [0, \ddot{\mu}_a)$, the inequality $\frac{d\Delta\pi_2}{d\mu}|_{\mu=0} > 0$ implies that $\Delta\pi_2$ is increasing in μ on the interval $[0, \ddot{\mu}_a)$. Therefore, the inequalities $\Delta\pi_2|_{\mu=0} > 0$ and $\frac{d\Delta\pi_2}{d\mu} > 0$ imply that $\Delta\pi_2 > 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Next, for $\mu \in [\ddot{\mu}_a, 3\theta)$, $\Delta\pi_2$ takes the same functional form as in *a*). Using the same argument, we can show that $\Delta\pi_2 > 0$ if $\ddot{\mu}_a < \mu < \min\{\hat{\mu}_{2a}, 3\theta\}$, and $\Delta\pi_2 < 0$ if $\min\{\hat{\mu}_{2a}, 3\theta\} < \mu < 3\theta$. Note that $\ddot{\mu}_a < \hat{\mu}_{2a}$. To summarize, when $\alpha < \hat{\alpha}$, we have $\Delta\pi_2 > 0$ if and only if $\mu \in [0, \min\{\hat{\mu}_{2a};\ 3\theta\})$.

Second, if $\hat{\alpha} < \alpha < \tilde{\alpha}$, then $\Delta\pi_2|_{\mu=0} < 0$. Define $\tilde{\theta} = \frac{3V(1-\alpha)(6-\alpha(5-6\sqrt{1-\alpha}+(3+6\sqrt{1-\alpha})\alpha))(1-\delta)}{4(3-4\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$. One can show that if $\dot{\theta}_{a2} < \theta < \tilde{\theta}$, then $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$. Convexity of $\Delta\pi_2$ on the interval $[0, \ddot{\mu}_a)$ implies that $\Delta\pi_2 < 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Moreover, because $\frac{d^2\Delta\pi_2}{d\mu^2} < 0$ on $(\ddot{\mu}_a, 3\theta)$ and $\frac{d\Delta\pi_2}{d\mu}|_{\mu=\ddot{\mu}_a} < 0$, it follows that $\frac{d\Delta\pi_2}{d\mu} < 0$ for all $\mu \in [\ddot{\mu}_a, 3\theta)$. Hence, $\Delta\pi_2 < \Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ for all $\mu \in (\ddot{\mu}_a, 3\theta)$. In sum, when $\dot{\theta}_{a2} < \theta < \tilde{\theta}$, we have $\Delta\pi_2 < 0$ for any $\mu \in [0, 3\theta)$. Next, if $\tilde{\theta} < \theta \leq \dot{\theta}_{a3}$, then

$\Delta\pi_2|_{\mu=\ddot{\mu}_a} > 0$. Hence, because $\Delta\pi_2$ is a strictly convex function on $[0, \ddot{\mu}_a)$, there must exist a unique $\tilde{\mu}_2 \in (0, \ddot{\mu}_a)$ such that $\Delta\pi_2 > 0$ if $\mu \in (\tilde{\mu}_2, \ddot{\mu}_a)$ and $\Delta\pi_2 < 0$ if $\mu \in [0, \tilde{\mu}_2)$. Solving $\Delta\pi_2 = 0$, we find that

$$\tilde{\mu}_2 = \frac{3\theta(-12+4\sqrt{1-\alpha}\alpha(6-\alpha(7-3\alpha))-\alpha(-28+3\alpha(5-(2-\alpha)\alpha)))}{12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha))}.$$

On the interval $(\ddot{\mu}_a, 3\theta)$, we know that $\Delta\pi_2$ is concave. Because $\Delta\pi_2|_{\mu=\ddot{\mu}_a} > 0$, it follows that $\Delta\pi_2 > 0$ if $\mu \in [\ddot{\mu}_a, \min\{\hat{\mu}_{2a}, 3\theta\})$ and $\Delta\pi_2 < 0$ if $\mu \in [\min\{\hat{\mu}_{2a}, 3\theta\};\ 3\theta)$, where recall that $\hat{\mu}_{2a}$ satisfies $\Delta\pi_2|_{\mu=\hat{\mu}_{2a}} = 0$. In sum, when $\tilde{\theta} < \theta \leq \dot{\theta}_{a3}$, we have $\Delta\pi_2 > 0$ if and only if $\mu \in (\tilde{\mu}_2, \min\{\hat{\mu}_{2a}, 3\theta\})$.

Third, if $\tilde{\alpha} < \alpha < \bar{\alpha}$, then $\Delta\pi_2|_{\mu=0} < 0$ and $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$. Convexity of $\Delta\pi_2$ on the interval $[0, \ddot{\mu}_a)$ implies that $\Delta\pi_2 < 0$ for all $\mu \in [0, \ddot{\mu}_a)$. Moreover, $\Delta\pi_2$ is decreasing on the interval $[\ddot{\mu}_a, 3\theta)$. Therefore, $\Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ implies that $\Delta\pi_2 < \Delta\pi_2|_{\mu=\ddot{\mu}_a} < 0$ for all $\mu \in (\ddot{\mu}_a, 3\theta)$. In sum, $\Delta\pi_2 < 0$ for any $\mu \in [0, 3\theta)$.

c) Consider $\dot{\theta}_{a3} < \theta < \bar{\theta}$. On this interval, we have $\Delta\pi_2 = \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} - \frac{(3\theta+\mu)^2}{18\theta}$ for all $\mu \in [0, 3\theta)$. Similar analysis as in *b*) shows that $\Delta\pi_2 > 0$ provided that $\mu \in (\min\{\tilde{\mu}_2, 3\theta\}, 3\theta)$. Note that the interval $(\min\{\tilde{\mu}_2, 3\theta\}, 3\theta)$ is non-empty (i.e., $\tilde{\mu}_2 < 3\theta$) if and only if $\alpha < \tilde{\alpha}$, where recall that $\tilde{\alpha} \approx 0.6$.

Define $\hat{\mu}_2 = \min\{\hat{\mu}_{2a}, 3\theta\}$. The analyses in *a*), *b*), and *c*) can be summarized as follows: $\Delta\pi_2 > 0$ (and thus, $(R_1^*, R_2^*) = (1, 2)$ is an equilibrium) if and only if

i. $0 < \alpha < \hat{\alpha}$, $\hat{\theta}_2 < \theta < \bar{\theta}$ and $0 \leq \mu \leq \hat{\mu}_2$ or

ii. $\hat{\alpha} \leq \alpha < \tilde{\alpha}$, $\tilde{\theta} < \theta < \bar{\theta}$ and $\tilde{\mu}_2 \leq \mu \leq \hat{\mu}_2$

where $\hat{\alpha} \approx 0.32$ is the same as in Proposition 1, $\tilde{\alpha} \approx 0.6$ is the solution to the equality $6 - 11\tilde{\alpha} + \left(3 - 2\sqrt{1-\tilde{\alpha}}\right)\tilde{\alpha}^2 = 0$, $\hat{\theta}_2 = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$,

$\tilde{\theta} = \frac{3V(1-\alpha)(6-\alpha(5-6\sqrt{1-\alpha}+(3+6\sqrt{1-\alpha})\alpha))(1-\delta)}{4(3-4\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, $\hat{\mu}_2 = \min\{\hat{\mu}_{2a}, 3\theta\}$,

$$\hat{\mu}_{2a} =$$

$$\frac{3(-6V(1-\alpha)(1-\delta)(9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(3\delta+15\sqrt{1-\alpha}\delta-11-15\sqrt{1-\alpha}))+\theta(9(1+\delta)(20+3\sqrt{1-\alpha}-3(4-\sqrt{1-\alpha})\delta)+F_1+F_2))}{\sqrt{1-\alpha}(153-374\alpha+225\alpha^2(1-\delta)^2+9(2-15\delta)\delta+6\alpha\delta(64+9\delta))},$$

$$\tilde{\mu}_2 = \frac{3\theta(-12+4\sqrt{1-\alpha}\alpha(6-\alpha(7-3\alpha))-\alpha(-28+3\alpha(5-(2-\alpha)\alpha)))}{12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha))}.$$

Equilibrium where $(\boldsymbol{R_1^*, R_2^*}) = (\mathbf{2}, \mathbf{1})$

The analysis follows similar steps as for $(R_1^*, R_2^*) = (1, 2)$. Therefore, we directly provide the conditions under which $(R_1, R_2) = (2, 1)$ is an equilibrium. Namely, $(R_1^*, R_2^*) = (2, 1)$ if and only if the following conditions are satisfied:$0 < \alpha < \hat{\alpha}$, $0 < \mu < \hat{\mu}_1$ and $\hat{\theta}_1 < \theta < \bar{\theta}$, where

$$\hat{\theta}_1 = \begin{cases} \hat{\theta}_{b1} & if\ 0 < \mu < \hat{\mu}_{b1} \\ \hat{\theta}_{b2} & if\ \hat{\mu}_{b1} < \mu < \hat{\mu}_1 \end{cases}$$

$$\hat{\mu}_1 = \frac{3V(12-4\alpha\sqrt{1-\alpha}(6-\alpha(7-3\alpha))-\alpha(28-3\alpha(5-(2-\alpha)\alpha)))(1-\delta)}{(12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha)))(2-\delta)},$$

$$\hat{\mu}_{b1} =$$

$$\frac{9V(1-\alpha)(-1+\delta)(12-9\alpha^5(1-\delta)+36\delta-\alpha^2(315-184\sqrt{1-\alpha}-(203-216\sqrt{1-\alpha})\delta)-\alpha^4(81-8\sqrt{1-\alpha}-(9-24\sqrt{1-\alpha})\delta)+C_2)}{81\alpha^6(1-\delta)^2-12(1+3\delta)^2-4\alpha(1+3\delta)(59-51\delta)+\alpha^2(153+(926-375\delta)\delta)+4\alpha^5(97+33(2-3\delta)\delta)+8\alpha^3(125-\delta(18+91\delta))-C_1}$$

$$C_1 = 2\alpha^4(687 + \delta(154 - 489\delta)) \quad , \quad C_2 = 4\alpha(23 - 30\sqrt{1-\alpha} - 9(5 - 2\sqrt{1-\alpha})\delta) + \alpha^3(301 - 72\sqrt{1-\alpha} - (93 - 136\sqrt{1-\alpha})\delta),$$

$$\hat{\theta}_{b1} =$$

$$\frac{\mu\sqrt{1-\alpha}(166\alpha+9\alpha^2(1-\delta)^2-6\alpha\delta(8+9\delta)-9(1+\delta)(19-21\delta))+18V(1-\alpha)(1-\delta)(-9(1+\sqrt{1-\alpha})(1+\delta)+\alpha(11-3\sqrt{1-\alpha}+3(-1+\sqrt{1-\alpha})\delta))}{3(\alpha(40+178\sqrt{1-\alpha}-6\delta((6+9\sqrt{1-\alpha})\delta-10(1-\sqrt{1-\alpha})))-9(1+\delta)(2+21\sqrt{1-\alpha}+3(2-5\sqrt{1-\alpha})\delta)-\alpha^2(1-\delta)(22-15\sqrt{1-\alpha}-3(2-5\sqrt{1-\alpha})\delta))},$$

$$\hat{\theta}_{b2} = \frac{\mu(12-\alpha(4+3\alpha(7-3(2-\alpha)\alpha)))}{36+3\alpha(-4(7+6\sqrt{1-\alpha})+\alpha(15+28\sqrt{1-\alpha}+3\alpha(-2-4\sqrt{1-\alpha}+\alpha)))}. \blacksquare$$

4. PRODUCT IMPROVEMENT OVER TIME

We extend our main model to study situations in which a firm can improve its product to deliver a greater value, but this necessitates a delayed entry.[25] To capture such product improvement effects in this extension we assume that by delaying its product release till the second period, a firm can increase the valuation of its product to $V + \mu$, where $\mu \geq 0$ represents the product improvement effect. We assume that $\mu < 3\theta$ to ensure that each firm has a positive market share regardless of its product release time. To keep the model as simple as possible, without loss of generality, the product improvement is assumed to be costless for the firm. For example, the firm may have a technology that is already developed but requires additional time to be finalized and integrated into the product. By contrast, if the firm releases its product in the first period, then consumers' product valuation stays constant at the level V, i.e., the firm is unable to sell an improved version of the product in the second period (Bayus et al. 1997). This is reasonable because, in practice, modifying a product that is already released to the market is likely to require substantial time and resources to change the product design, manufacturing process, supply chain, and marketing strategy. Finally, for a more interesting analysis, we assume that the product improvement effect is not excessively high ($\mu < \bar{\mu}$) to avoid both firms choosing to wait till the second period in order to improve their products.

We provide the technical analysis below. We find that, as expected, the ability to improve its product over time gives a firm even more incentives to release it in the second period because by doing so a) the firm can gain a competitive advantage in the second period due to its improved product, b) the firm does not lose so many sales because, in the first period, consumers anticipate the firm's product improvement, becoming more willing to wait for it, and c) the increased number of consumers who wait for the firm's product tends to reinforce the price-competition alleviation effect, providing an additional benefit to the firm. Thus, the possibility of product improvement over time expands the parameter region where, in equilibrium, one firm prefers to wait till the second period to release its product whereas the competitor releases in the first period.

[25] Note that even if there is no improvement in the product's functionality, a product that is released later may be perceived by consumers as newer or more state-of-the-art, and such perceptions may increase consumers' willingness to pay for the product.

We will first obtain the firms' equilibrium pricing strategies and profits in each subgame $(R_1, R_2) \in \{(1,1),(2,2),(1,2),(2,1)\}$. Then, we will characterize market conditions under which $(R_1^*, R_2^*) = (1,2)$ is an equilibrium.

Subgame with $(\boldsymbol{R_1, R_2}) = (\mathbf{1}, \mathbf{2})$

In this subgame, firm 2 is able to improve its product, increasing consumers' valuation for the product to $V + \mu$. The analysis of this subgame is identical to that in the previous section with vertically differentiated firms. Hence, we will directly provide the equilibrium prices and profits.

Recall that $\dot{\mu}_a = \min\{\frac{12\theta(2-\alpha(2-\delta))+9V(-1+\alpha+\delta-\alpha\delta)}{2(1-\alpha)(1-3\delta)}; 3\theta\}$ and $\ddot{\mu}_a = \min\{\max\{0, \frac{\theta(3-\alpha)}{1-\alpha} + \frac{3V(6-\alpha(7-3\alpha))(1-\delta)}{6\delta+2\alpha(6-8\delta-\alpha(1-3\delta))-10}\}; 3\theta\}$. Further, $\Sigma_{a1} = \{(\theta,\mu) | \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\mu < \dot{\mu}_a\}$, $\Sigma_{a2} = \{(\theta,\mu) | \ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\mu < \ddot{\mu}_a\}$ and $\Sigma_{a3} = \{(\theta,\mu) | \ 0 < \theta \leq \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)}$ and $\dot{\mu}_a \leq \mu < 3\theta\} \cup \{(\theta,\mu) | \ \frac{3V(1-\alpha)(1-\delta)}{8-4\alpha(2-\delta)} < \theta < \bar{\theta}$ and $\ddot{\mu}_a \leq \mu < 3\theta\}$. Then, from (B38), we know that firm 1's equilibrium first-period price is given by

$$p_{11}^{(1,2)} =$$

$$\begin{cases} (1-\delta)(V-\theta) - \frac{2\mu\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{V(6-\alpha(7-3\alpha))(1-\delta)+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{2\theta(3-\alpha)(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2))+(1-\alpha)(V(1-\delta)(9-13\alpha+9\delta+3\alpha\delta)-2\mu(1+4\delta+3\delta^2-\alpha(1+4\delta-\delta^2)))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

The firms' second-period prices on the equilibrium path are given in (B39)-(B40):

$$p_{12}^{(1,2)} = \begin{cases} \theta - \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)(\theta(3-\alpha)-\mu+\mu\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-(1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta + \frac{\mu}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\theta(6-\alpha(3+\alpha))+\mu(2-(1-\alpha)\alpha)}{6-\alpha(7-3\alpha)} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{(1-\alpha)(3\mu(1+\delta)-6V\alpha(1-\delta)-5\mu\alpha(1-\delta))+\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta))}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

The firms' equilibrium profits in each period are given in (B41)-(B44).

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(1-\delta)(V-\theta) - \frac{2\mu\alpha\delta}{3} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{\alpha(\theta(3-\alpha)+\mu\alpha-\mu)(V(1-\delta)(6-\alpha(7-3\alpha))+\mu(1-\alpha)(1-3\delta+2\alpha\delta)-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{\alpha((1-\alpha)(9V(1-\delta)+2\mu-6\mu\delta)-2\theta(3-\alpha)(1-3\delta))A}{(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta-\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(2-\alpha)^2(1-\alpha)(\theta(3-\alpha)+\mu\alpha-\mu)^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(3V\alpha(1-\delta)+3\mu(1+\delta)-\mu\alpha(3+\delta))-\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

$$\pi_{21}^{(1,2)} = 0$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)(3\theta+\mu)^2}{18\theta} & \text{if } (\theta,\mu) \in \Sigma_{a1} \\ \frac{(1-\alpha)(\theta(6-\alpha(3+\alpha))+\mu(2-\alpha+\alpha^2))^2}{2\theta(6-\alpha(7-3\alpha))^2} & \text{if } (\theta,\mu) \in \Sigma_{a2} \\ \frac{((1-\alpha)(6V\alpha(1-\delta)+5\mu\alpha(1-\delta)-3\mu(1+\delta))-\theta(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } (\theta,\mu) \in \Sigma_{a3} \end{cases}$$

where $A \equiv \frac{(1-\alpha)(V(1-\delta)(9-13\alpha+3(3+\alpha)\delta)-2\mu(1-\alpha+4(1-\alpha)\delta+(3+\alpha)\delta^2))+2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))}{4\theta}$.

Using the profits obtained in each period, we can readily find firm i's overall profit in the subgame with $(R_1, R_2) = (1, 2)$: $\pi_i^{(1,2)} = \pi_{i1}^{(1,2)} + \pi_{i2}^{(1,2)}$.

Subgame with $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{1})$

Due to symmetry between the firms, the equilibrium profits in this subgame are the reverse of profits in the subgame with $(R_1, R_2) = (1, 2)$. That is, $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

Subgame where $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{1}, \mathbf{1})$

Since both firms release their products in the first period, neither firm's product is improved. Therefore, the analysis of this subgame is identical to the one in the main model (see Lemma 1). Specifically, firm i's equilibrium price in period t is given by $p_{it}^{(1,1)} = \theta$, where $i = 1,2$ and $t = 1,2$. Firm i's corresponding profits in the first and second periods are given by $\pi_{i1}^{(1,1)} = \frac{\alpha\theta}{2}$ and $\pi_{i2}^{(1,1)} = \frac{(1-\alpha)\theta}{2}$, respectively.

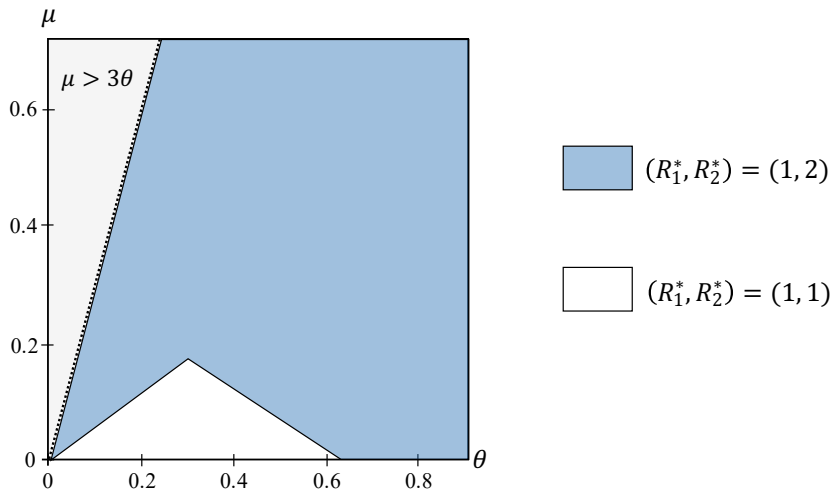
Subgame where $(\boldsymbol{R_1}, \boldsymbol{R_2}) = (\mathbf{2}, \mathbf{2})$

Because both firms wait till the second period to release their products, each firm improves its product, increasing consumers' valuation to $V + \mu$. Since the firms are symmetric, one can easily show that second-period equilibrium prices and profits are given by $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \theta$ and $\pi_{12}^{(2,2)} = \pi_{22}^{(2,2)} = \frac{\theta}{2}$. Because first-period profits are zero, it follows that the firms' overall profits are the same as what they receive in the second period: $\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta}{2}$.

Equilibrium Product Release Times

$(R_1^*, R_2^*) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. $(R_1^*, R_2^*) = (1, 1)$ is an equilibrium if and only if $\pi_1^{(1,1)} \geq \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} \geq \pi_2^{(1,2)}$. Using the subgame equilibrium profits that we characterized in the earlier analysis, we graphically illustrate the firms' equilibrium product release times in Figure B3.

Figure B3 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [26]



Notice from Figure B3 that as μ increases above zero, in equilibrium, firm 2 releases its product later than firm 1 (i.e., $(R_1^*, R_2^*) = (1, 2)$) not only when the level of horizontal differentiation between the firms is high (i.e., θ is large) but also when the differentiation level is low. Intuitively, when θ is

[26] Figure 5 is drawn using the following parameter values: $V = 10$, $\alpha = 0.3$ and $\delta = 0.9$. Under these values, $\mu < \bar{\mu} \approx 0.73$ ensures that, in equilibrium, at least one of the firms releases its product in the first period. Further, we focus on $\mu < 3\theta$ to guarantee that each firm has a positive market share regardless of product release times.

small, consumers tend to care more about vertical attributes (value differences) than horizontal ones (distance from the firm). Therefore, in such a situation, the increase in product value that firm 2 obtains due to its late product release allows firm 2 to significantly increase its market share, as well as its price. In other words, even a small value of μ can make a big difference when there is not much horizontal differentiation between the firms. For this reason, even when θ is very small, firm 2 may optimally choose to release its product in the second period in order to improve its product and gain an advantage over firm 1. However, it is important to note that when θ is in the lower region, firm 2's late product release is more driven by the direct benefit from product improvement than by the strategic benefit from alleviated price competition. This is because when θ is small, in the first period, firm 1 exploits the lack of horizontal differentiation between the firms and captures a very large fraction of the marker. Hence, very few first-period consumers choose to wait till the second period to buy firm 2's product, which dampens firm 2's incentives to charge a high price, thus weakening the competition-alleviation effect. By contrast, when θ is large, the competition-alleviation-effect is much stronger and plays an important role in inducing firm 2 to release its product later than firm 1 even when the product improvement effect is very weak (i.e., μ is small). Hence, we can make the following distinction: in markets where horizontal differentiation between the firms is low, product improvement effect has a stronger influence on product release timing decisions than price competition alleviation effect. By contrast, when the differentiation level is high, the price competition alleviation effect tends to play a more important role, making late product release preferable even in the absence of any product improvement effects.

5. CONSUMER PREFERENCE UNCERTAINTY

In practice, firms often provide detailed information about their products before launching them. For example, tech companies typically present the products at special events and tech conventions (e.g., the International Consumer Electronics Show). Popular magazines, bloggers and other media outlets closely follow such events, providing consumers with detailed information about the firms' products. However, in some other markets (especially for new product categories), consumers may still lack full knowledge about their tastes or preferences prior to purchasing a product (e.g., Gu and Xie 2013, Shulman et al. 2015). So, we extend our core model by assuming that consumers' knowledge about their own preferences is "noisy." In particular, at the aggregate level, consumers' true preferences are assumed to be uniformly distributed on the unit line $[0,1]$, which is consistent with our core model. However, a randomly chosen consumer does not know her exact location on the line. Instead, each consumer receives a signal $s \in [0,1]$ about her location. Provided that the consumer's true location is $l \in [0,1]$, the distribution of s is as follows: $s = l$ with probability γ, and with probability $1-\gamma$, the signal s is randomly drawn from uniform distribution on $[0,1]$. The parameter γ captures the level of uncertainty that a given consumer faces.

Proposition WA5.1 shows the robustness of our main results in the presence of consumers' preference uncertainty and discusses how such uncertainty affects the firms' decisions and profits.

PROPOSITION WA5.1.

a) *There exist* $0 < \hat{\alpha} < \bar{\alpha}$, $0 < \hat{\theta} < \bar{\theta}$ *and* $0 < \hat{\gamma} < 1$ *such that if* $0 < \alpha < \hat{\alpha}$, $\hat{\theta} < \theta < \bar{\theta}$ *and* $\hat{\gamma} < \gamma \leq 1$, *then, in equilibrium, firm 2 will release its product later than firm 1*: $(R_1^*, R_2^*) = (1, 2)$.

b) *Consumers' preference uncertainty reduces firm 2's gains from releasing its product later, i.e.,* $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} \geq 0$.

c) *Both firms' equilibrium profits are increasing in* γ.

The first part of the proposition shows that firm 2 may prefer to release its product late even if consumers face some uncertainty about their preferences. However, as the second part of the proposition

suggests, consumers' preference uncertainty makes it less likely that firm 2 will choose to release its product in the second period rather than in the first period. Put differently, preference uncertainty makes the parameter region where $(R_1^*, R_2^*) = (1, 2)$ smaller. To see the intuition, note that if consumers face preference uncertainty, then fewer consumers will be inclined to wait for firm 2's product. For example, even if a consumer receives a signal $s = 1$, there is still some chance that firm 2's product is not a good fit with the consumer's true preference. The existence of such uncertainty makes the consumer less willing to wait for firm 2's product. Fewer consumers waiting for firm 2's product has two negative effects on firm 2's profits. First, firm 2's equilibrium sales decrease. Second, price competition becomes more intense in the second period because firm 2 gains more incentives to compete for the segment of new consumers entering the market rather than focus on the consumers who have been waiting for its product release. For these reasons, consumers' preference uncertainty makes it less attractive for firm 2 to release its product in the second period. Finally, the third part of Proposition WA5.1 shows that a reduction in consumers' preference uncertainty will benefit not only firm 2 but also firm 1. One reason is that an increase in γ can induce firm 2 to release its product in the second period instead of the first period, which will alleviate price competition and allow firm 1 to capture a bigger market share. Another reason is that an increase in γ makes consumers less price sensitive as consumers become more certain in their preferences for one or the other firm – this makes price competition less intense, allowing both firms to charge higher prices. Therefore, our results suggest that well before releasing their products to the market, the firms would benefit from revealing information about their planned products and provide opportunities to different media outlets to try early samples and share information about these products with consumers, allowing them to better understand their needs and preferences.

PROOF OF PROPOSITION WA5.1. We will first solve for the firms' subgame equilibrium profits for each $(R_1, R_2) \in \{(1, 1), (2, 2), (1, 2), (2, 1)\}$.

Subgame with $\boldsymbol{(R_1, R_2) = (1, 2)}$

We solve the game by backward induction, starting with the second period. In the second period, the firms compete for the $(1 - \alpha)$ new consumers entering the market, as well as any first-period

consumers who decided not to buy a product in the first period. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize the segment of first-period consumers who wait till the second period to buy. In the first period, a consumer who received a signal $s \in [0,1]$ will prefer to wait till the second period to purchase a product if and only if the following condition holds:

$$\delta \max\{V - p_{12}^e - \theta\mathbb{E}(d|s)), V - p_{22}^e - \theta\big(1 - \mathbb{E}(d|s)\big)\} > V - p_{11} - \theta\mathbb{E}(d|s) \tag{B54}$$

Using $\mathbb{E}(d|s) = \gamma s + (1-\gamma)\frac{1}{2}$, one can readily show that (B54) is equivalent to

$$s > \hat{s} \tag{B55}$$

where

$$\hat{s} \equiv \begin{cases} 1 & \text{if } p_{11} \leq \frac{2((1-\delta)V + p_{22}^e\delta) - \theta(1+\gamma-(1-\gamma)\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{22}^e + \theta\delta}{\theta(1+\delta)} & \text{if } \frac{2((1-\delta)V + p_{22}^e\delta) - \theta(1+\gamma-(1-\gamma)\delta)}{2} < p_{11} \leq \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} \\ \frac{V(1-\delta) - p_{11} + \delta p_{12}^e}{\theta(1-\delta)} & \text{if } \frac{(2V - p_{22}^e - \theta)(1-\delta) + p_{12}^e(1+\delta)}{2} < p_{11} < \frac{2((1-\delta)V + p_{12}^e\delta) + \theta(\delta(1-\gamma) - 1 + \gamma)}{2} \\ 0 & \text{if } p_{11} \geq \frac{2((1-\delta)V + p_{12}^e\delta) + \theta(\delta(1-\gamma) - 1 + \gamma)}{2} \end{cases}$$

Having characterized the first-period consumers who wait till the second period to buy a product, we can now solve for the second-period subgame equilibrium prices that satisfy the rational-expectations condition. The analysis is very similar to the cases *i-iv* in the proof of Lemma 2. To avoid repetitiveness, we directly provide the second-period rational-expectations subgame equilibrium prices:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \theta\gamma & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty] \\ \frac{2p_{11}\alpha - 2V\alpha(1-\delta) + \theta(3\gamma(1+\delta) + \alpha(1-2\gamma-\delta))}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4p_{11}\alpha - (4V\alpha - \theta(2\alpha(1-\gamma) + 3\gamma))(1-\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B56}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \theta\gamma & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty] \\ \frac{4p_{11}\alpha - 4V\alpha(1-\delta) + \theta(3\gamma(1+\delta) + \alpha(2-\gamma-2\delta-\gamma\delta))}{3(1+\delta) - \alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2p_{11}\alpha - 2V\alpha(1-\delta) + \theta(\alpha+3\gamma-\alpha\gamma-\alpha\delta-3\gamma\delta+3\alpha\gamma\delta)}{3 - (3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B57}$$

where $\underline{\rho} \equiv \frac{(2V - \theta - \theta\gamma)(1-\delta)}{2}$,

$\tilde{\rho} \equiv \frac{2V(6-\alpha(7-3\alpha))(1-\delta) + \theta(\alpha(7-3\alpha(1-\gamma)-5\gamma) - 6(1-\delta) + (12\gamma - \alpha(7+11\gamma-\alpha(3+\gamma)))\delta)}{2(6-\alpha(7-3\alpha))}$,

$\hat{\rho} \equiv \frac{2V(6+\alpha)(1-\delta)-\theta(\alpha(1-\gamma-\delta+7\gamma\delta)+6(1-\delta-2\gamma\delta))}{2(6+\alpha)}$ and $\bar{\rho} \equiv \frac{2V(1-\delta)-\theta(1-\gamma-\delta-\gamma\delta)}{2}$. Note that similar to the main model, we assume that $\underline{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$ to ensure the existence of a subgame equilibrium for every $p_{11} \geq 0$, where $\underline{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Using the second-period subgame equilibrium prices in (B56)-(B57), we obtain the firms' second-period subgame equilibrium profits:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta\gamma(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2V\alpha(1-\delta)-2p_{11}\alpha-3\theta\gamma(1+\delta)-\theta\alpha(1-2\gamma-\delta))^2}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4p_{11}\alpha-(1-\delta)(4V\alpha-\theta(2\alpha(1-\gamma)+3\gamma)))^2}{2\theta\gamma(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta\gamma}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B58}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{\theta\gamma(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4V\alpha(1-\delta)-4p_{11}\alpha-3\theta\gamma(1+\delta)-\theta\alpha(2-\gamma-(2+\gamma)\delta))^2}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2p_{11}\alpha-2V\alpha(1-\delta)+\theta(\alpha+3\gamma(1-\delta)-\alpha\delta-\alpha\gamma(1-3\delta)))^2}{2\theta\gamma(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta\gamma}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B59}$$

Firm 1's first-period profit is given by $\pi_{11} = \alpha\, \hat{s}\, p_{11}$. Plugging the prices from (B56)-(B57) into the expression for $\hat{s}$, we can obtain π_{11}:

$$\pi_{11} = \begin{cases} \alpha\, p_{11} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{6V(1-\alpha)(1-\delta)-6p_{11}(1-\alpha)+\theta(\alpha(3-3\gamma-3\delta-\gamma\delta)+3(\gamma+\delta+3\gamma\delta-1))}{2\theta\gamma(3(1+\delta)-\alpha(3-\delta))} p_{11} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(2V(1-\delta)-2p_{11}+\theta(\gamma+\delta+\gamma\delta-1))}{2\theta\gamma(3-(3-4\alpha)\delta)} p_{11} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B60}$$

In the first period, firm 1 chooses its price to maximize its overall profit $\pi_1 = \pi_{11} + \pi_{12}^{(1,2)}$. Similar analysis as in the proof of Lemma 3 shows that firm 1's optimal price belongs to the interval $[\underline{\rho}, \tilde{\rho}]$ and is given by:

$$p_{11}^{(1,2)} = \begin{cases} \frac{(2V-\theta-\theta\gamma)(1-\delta)}{2} & \text{if } 0 < \gamma \leq \underline{\gamma} \\ \frac{(1-\alpha)\big((2V-\theta)(9-13\alpha)+\theta(21-17\alpha)\gamma\big)+16(1-\alpha)(2V\alpha+3\theta\gamma-\theta\alpha)\delta-(3+\alpha)(3(2V-\theta)(1-\alpha)-\theta(9-\alpha)\gamma)\delta^2}{4(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)} & \text{if } \underline{\gamma} < \gamma \leq \bar{\gamma} \\ \frac{2V(6-\alpha(7-3\alpha))(1-\delta)+\theta(\alpha(7-3\alpha(1-\gamma)-5\gamma)-6(1-\delta)+(12\gamma-\alpha(7+11\gamma-\alpha(3+\gamma)))\delta)}{2(6-\alpha(7-3\alpha))} & \text{if } \bar{\gamma} < \gamma \leq 1 \end{cases} \tag{B61}$$

where $\underline{\gamma} = \min\{1, \frac{3(2V-\theta)(1-\alpha)(1-\delta)}{\theta(13+3\delta-\alpha(13-5\delta))}\}$ and $\bar{\gamma} = \min\{1, \frac{3(2V-\theta)(1-\alpha)(6-\alpha(7-3\alpha))(1-\delta)}{\theta(42-18\delta+\alpha(69\delta-53+\alpha(6-38\delta+\alpha(5+3\delta))))}\}$.

Upon plugging $p_{11}^{(1,2)}$ into equations (B58)-(B60), we find the firms' profits on the equilibrium path.

$$\pi_{11}^{(1,2)} =$$

$$\begin{cases} \frac{\alpha(2V-\theta-\theta\gamma)(1-\delta)}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{\alpha((1-\alpha)(16(\theta\alpha-2V\alpha-3\theta\gamma)\delta-(2V-\theta)(9-13\alpha)-\theta(21-17\alpha)\gamma)+(3+\alpha)(3(2V-\theta)(1-\alpha)-\theta(9-\alpha)\gamma)\delta^2)E}{(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{(3-\alpha)\alpha(2V(6-7\alpha+3\alpha^2)(1-\delta)-\theta(\alpha^2(3-3\gamma-3\delta-\gamma\delta)+6(1-\delta-2\gamma\delta)-\alpha(7-5\gamma-7\delta-11\gamma\delta)))}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases}$$

(B62)

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)\gamma}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{(6V(1-\alpha)\alpha(1-\delta)+\theta(\alpha^2(3-\gamma(9-\delta)-3\delta)-18\gamma(1+\delta)-3\alpha(1-\delta-3\gamma(3+\delta))))^2}{8\theta(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2\gamma}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases} \quad \text{(B63)}$$

$$\pi_{21}^{(1,2)} = 0 \quad \text{(B64)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta\gamma}{2} & \text{if } 0<\gamma\leq\underline{\gamma} \\ \frac{(6V(1-\alpha)\alpha(1-\delta)-\theta(1-\alpha)(9\gamma+\alpha(3+2\gamma))-\theta\delta(9\gamma-\alpha(3-3\alpha+3\gamma-2\alpha\gamma)))^2}{2\theta(1-\alpha)\gamma(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\gamma}<\gamma\leq\bar{\gamma} \\ \frac{\theta(1-\alpha)(6-3\alpha-\alpha^2)^2\gamma}{2(6-7\alpha+3\alpha^2)^2} & \text{if } \bar{\gamma}<\gamma\leq 1 \end{cases} \quad \text{(B65)}$$

where $E = \frac{-18V(1-\alpha)(1-\delta)+\theta(9-9\alpha+3\gamma+5\alpha\gamma-3(3+9\gamma-\alpha(3+\gamma))\delta)}{16\theta}$. The firms' overall profits in the subgame $(R_1, R_2) = (1,2)$ are $\pi_1^{(1,2)} = \pi_{11}^{(1,2)} + \pi_{12}^{(1,2)}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)} = \pi_{22}^{(1,2)}$. Note that since the firms are symmetric, their profits in the subgame $(R_1, R_2) = (2,1)$ are given by $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

***Subgame with* $(\boldsymbol{R_1, R_2}) = (\mathbf{1,1})$ or $(\boldsymbol{R_1, R_2}) = (\mathbf{2,2})$**

The analysis of the subgames with $(R_1, R_2) = (1,1)$ or $(R_1, R_2) = (2,2)$ follows similar steps as in the main model. Namely, if $(R_1, R_2) = (2,2)$, then consumers who enter the market in the first period wait till the second period to buy a product. Hence, in the second period, firms compete for a unit measure of consumers. A consumer who received a signal $s \in [0,1]$ will prefer to buy firm 1's product if and only if

$$V - p_{12} - \theta\mathbb{E}(d|s) > V - p_{22} - \theta\big(1 - \mathbb{E}(d|s)\big) \quad \text{(B66)}$$

where $\mathbb{E}(d|s)$ is the consumer's expected location on the Hotelling line conditional on the signal s. Since the consumer does not know the actual value of l, she makes her purchase decision to maximize

expected utility. Note that $\mathbb{E}(d|s) = \gamma s + (1-\gamma)\frac{1}{2}$. Using this expression in (B66), it follows that the consumer will buy from firm 1 if and only if $s \leq \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$. Note that the probability of $s \leq \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$ is $\frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}$. Hence, firms' profits are given by $\pi_{12} = \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma} p_{12}$ and $\pi_{22} = (1 - \frac{\theta\gamma - p_{12} + p_{22}}{2\theta\gamma}) p_{22}$. Solving the first-order conditions, we find the firms' second-period equilibrium prices: $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \theta\gamma$. The firms' equilibrium profits are

$$\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta\gamma}{2} \tag{B67}$$

The analysis of the subgame with $(R_1, R_2) = (1,1)$ follows similar steps as in the main model. Specifically, one can show that firm i's equilibrium price are given by $p_{i1}^{(1,1)} = p_{i2}^{(1,1)} = \theta\gamma$, with corresponding profits of $\pi_{i1}^{(1,1)} = \alpha\frac{\theta\gamma}{2}$ and $\pi_{i2}^{(1,1)} = (1-\alpha)\frac{\theta\gamma}{2}$ in the first and second periods, respectively. Thus, the firms' overall profits in the $(R_1, R_2) = (1,1)$ subgame are given by

$$\pi_1^{(1,1)} = \pi_2^{(1,1)} = \frac{\theta\gamma}{2} \tag{B68}$$

Having characterized the firms' subgame equilibrium profits, we proceed to finding the firms' equilibrium product release times. A similar proof as in Proposition 1 shows that a pure strategy equilibrium in product release times exists and in equilibrium, at least one of the firms will release its product in the first period. Hence, in equilibrium, either $(R_1, R_2) = (1,1)$ or $(R_1, R_2) = (1,2)$.

Let us first demonstrate that $\frac{d\pi_i^{(R_1,R_2)}}{d\gamma} > 0$ for all (R_1, R_2). If $(R_1, R_2) = (1,1)$, then $\pi_i^{(1,1)} = \frac{\theta\gamma}{2}$, and it is clear that $\frac{d\pi_i^{(1,1)}}{d\gamma} = \frac{\theta}{2} > 0$. Similarly, if $(R_1, R_2) = (2,2)$, then $\frac{d\pi_i^{(2,2)}}{d\gamma} = \frac{\theta}{2} > 0$. Next, if $(R_1, R_2) = (1,2)$, then we will analyze each of the possible intervals to which γ can belong: $(0, \underline{\gamma}]$, $(\underline{\gamma}, \bar{\gamma}]$ and $(\bar{\gamma}, 1]$.

First, if $\gamma \in (0, \underline{\gamma}]$, then $\frac{d\pi_1^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha(2-\delta))}{2} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha)}{2} > 0$, where the first inequality follows because $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$.

Second, if $\gamma \in (\underline{\gamma}, \bar{\gamma}]$, then note that $\frac{d^2\pi_1^{(1,2)}}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha(1-\delta)^2}{8\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)} > 0$ and $\frac{d\pi_1^{(1,2)}}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta(1-\alpha(2-\delta))}{2} > 0$, which implies that $\frac{d\pi_1^{(1,2)}}{d\gamma} > 0$ for all $\gamma \in (\underline{\gamma}, \bar{\gamma}]$. Next, $\frac{d^2\pi_2^{(1,2)}}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)^2} > 0$ and $\frac{d\pi_2^{(1,2)}}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta(9(1+\delta)+\alpha(6-\alpha(15-7\delta)))}{18-22\alpha+6(3+\alpha)\delta} > 0$, which implies that $\frac{d\pi_2^{(1,2)}}{d\gamma} > 0$ for all $\gamma \in (\underline{\gamma}, \bar{\gamma}]$.

Third, if $\gamma \in (\bar{\gamma}, 1]$, then $\frac{d\pi_1^{(1,2)}}{d\gamma} = \frac{\theta(3-\alpha)(12+\alpha(4(-7+3\delta)+\alpha(18-11\delta-\alpha(5-\alpha-\delta))))}{2(6-\alpha(7-3\alpha))^2} > 0$, where the inequality follows because $\bar{\delta} < \delta < 1$ and $0 \le \alpha < \bar{\alpha}$. Similarly, $\frac{d\pi_2^{(1,2)}}{d\gamma} = \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} > 0$.

Hence, the above three cases all indicate that $\frac{d\pi_i^{(1,2)}}{d\gamma} > 0$. Since $\pi_i^{(1,2)}$ is a continuous function that is differentiable at all but a finite number of points, it follows that $\pi_i^{(1,2)}$ is increasing in γ. The proof for the subgame $(R_1, R_2) = (2, 1)$ is identical to the one for $(R_1, R_2) = (1, 2)$.

Next, we will show that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then there exists $\hat{\gamma} \in (0,1)$ such that $(R_1, R_2) = (1, 2)$ is an equilibrium if and only if $\gamma > \hat{\gamma}$. As we discussed earlier, firm 1 will not want to deviate from $R_1 = 1$. Hence, we need to show that firm 2 will not want to deviate from $R_2 = 2$, which is the case when $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. Note that if $\gamma \in [0, \underline{\gamma}]$, then $\pi_2^{(1,2)} < \pi_2^{(1,1)}$. Hence, let us focus on $\gamma \in (\underline{\gamma}, 1]$. When $\gamma \in (\underline{\gamma}, \bar{\gamma}]$, then $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} > 0$ because $\frac{d^2(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma^2} = \frac{9(2V-\theta)^2(1-\alpha)\alpha^2(1-\delta)^2}{\theta\gamma^3(9-11\alpha+3(3+\alpha)\delta)^2} > 0$ and $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma}|_{\gamma=\underline{\gamma}} = \frac{\theta\alpha(17-3\delta-\alpha(15-7\delta))}{18-22\alpha+6(3+\alpha)\delta} > 0$, where the last inequality holds since $\bar{\delta} < \delta < 1$ and $0 \le \alpha < \bar{\alpha}$. When $\gamma \in (\bar{\gamma}, 1]$, we have $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} = \frac{\theta}{2}\left(\frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{(6-\alpha(7-3\alpha))^2} - 1\right) > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52 - (3 - \hat{\alpha})\hat{\alpha}(17 + \hat{\alpha})) = 0$. Note that this cutoff is the same as in Proposition 1 (see the proof). Let us analyze the two possible cases: $\alpha < \hat{\alpha}$ and $\hat{\alpha} < \alpha < \bar{\alpha}$.

If $\alpha < \hat{\alpha}$, then we showed in the previous paragraph that $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at any $\gamma \in (\underline{\gamma}, 1]$. Hence, the maximal value is obtained at $\gamma = 1$. We find that $(\pi_2^{(1,2)} - \pi_2^{(1,1)})_{\gamma=1} > 0$ if and only if $\theta > \hat{\theta}$, where

$\hat{\theta} = \frac{6(V(1-\alpha)^{3/2}(1-\delta)(9-11\alpha+3(3+\alpha)\delta)+V(1-\alpha)(1-\delta)(9(1+\delta)-\alpha(4+5\alpha(1-\delta)+6\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))}$ is the same as in Proposition 1. Assuming that $\theta > \hat{\theta}$, it is straightforward to show that $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$ if and only if $\gamma > \hat{\gamma}$, where

$$\hat{\gamma} = \frac{3(2V-\theta)(1-\delta)((1-\alpha)^{3/2}(9-11\alpha+3(3+\alpha)\delta)+(1-\alpha)(\alpha(-7-2\alpha(1-\delta)-3\delta)+9(1+\delta)))}{\theta(4\alpha^3(1-\delta)^2+9(1+\delta)(17-3\delta)+\alpha^2(149-\delta(82+3\delta))+18\alpha(-17+\delta(-2+5\delta)))} \quad \text{(B69)}$$

Note that $\hat{\gamma} < 1$.

If $\hat{\alpha} < \alpha < \bar{\alpha}$, then $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at $\gamma \in (\underline{\gamma}, \bar{\gamma}]$ and decreasing at $(\underline{\gamma}, 1]$. Hence, the maximal value is obtained when $\gamma = \bar{\gamma}$. One can show that $(\pi_2^{(1,2)} - \pi_2^{(1,1)})_{\gamma=\bar{\gamma}} < 0$ when $\alpha > \hat{\alpha}$. Therefore, it follows that $\pi_2^{(1,2)} - \pi_2^{(1,1)} < 0$ for all $\gamma \in [0,1]$.

Finally, let us prove that $\frac{d(\pi_2^{(1,2)} - \pi_2^{(1,1)})}{d\gamma} \geq 0$. We already proved that $(R_1^*, R_2^*) = (1,2)$ if and only if $\alpha < \hat{\alpha}$, $\theta > \hat{\theta}$ and $\gamma > \hat{\gamma}$. Further, we showed that if $\alpha < \hat{\alpha}$, then $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing in γ at any point $\gamma \in (\underline{\gamma}, 1]$. Since $\hat{\gamma} \in (\underline{\gamma}, 1]$, it follows that $\pi_2^{(1,2)} - \pi_2^{(1,1)}$ is increasing at any $\gamma > \hat{\gamma}$. Hence, when $(R_1^*, R_2^*) = (1,2)$, firm 2 is better off as γ increases. ■

6. FIRMS' DISCOUNT FACTOR

To simplify the analysis, in our main model, we assumed that the firms' discount factor is one. We will demonstrate that our qualitative results can still hold even if the firms' discount factor is below one. Let us denote the firms' discount factor by $\delta_f \in (0,1)$. To make the algebra less cumbersome, we consider a numerical example with $V = 10$, $\delta = 0.95$ and $\alpha = 0.2$. To find the equilibrium outcome, we need to find the firms' equilibrium profits in each subgame corresponding to the firms' product release times $(R_1, R_2) \in \{(1,1), (2,2), (1,2), (2,2)\}$. Let $\pi_i^{(R_1,R_2)}$ represent the discounted sum of firm i's first- and second-period profits in the subgame with (R_1, R_2), i.e., $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \delta_f \pi_{i2}^{(R_1,R_2)}$.

First, let us analyze the subgame where $(R_1, R_2) = (1, 2)$. The discount factor δ_f clearly does not affect the firms' second-period pricing decisions, and the analysis for finding the second-period subgame equilibrium prices is the same as in our core model. More specifically, the firms' second-period prices and profits in a rational-expectations subgame equilibrium are the same as in Lemma 2 in the main paper. The corresponding second-period profits are provided in the equations (A7) and (A8) in the Web Appendix A, which we reproduce below for convenience:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4\alpha(p_{11}-V(1-\delta))+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4\alpha(V(1-\delta)-p_{11})-\theta(3(1+\delta)+\alpha(1-3\delta)))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

where recall that $\underline{\rho} = (1-\delta)(V-\theta)$, $\tilde{\rho} \equiv (1-\delta)V + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv (1-\delta)V + \delta\theta$.

In the first period, firm 1 chooses its first-period price p_{11} to maximize the discounted sum of its first- and second-period profits, $\pi_1^{(1,2)} = S_{11}^{(1,2)} p_{11} + \delta_f \pi_{12}^{(1,2)}$, where $S_{11}^{(1,2)}$ is firm 1's first-period unit sales as in the equation (A11) in the Web Appendix A.

$\pi_1^{(1,2)} =$

$$\begin{cases} \alpha\, p_{1_1} + \delta_f \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha \frac{3V(1-\alpha)(1-\delta)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{1_1} + \delta_f \frac{(1-\alpha)(2\alpha(p_{11}-V(1-\delta))+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha \frac{3(V(1-\delta)-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} p_{1_1} + \delta_f \frac{(2\alpha(p_{11}-V(1-\delta))+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \delta_f \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}$$

After plugging in the numerical values for the parameters, $\pi_1^{(1,2)}$ becomes

$$\pi_1^{(1,2)} = \begin{cases} \frac{1}{4} p_{11} + \delta_f \frac{3}{8}\theta & \text{if } p_{11} \in [0, \frac{10-\theta}{20}) \\ \frac{3p_{11}(10-20p_{11}+19\theta)}{88\,\theta} p_{11} + \delta_f \frac{(10-20p_{11}-3\theta)^2}{968\,\theta} & \text{if } p_{11} \in [\frac{10-\theta}{20}, \frac{71+121\,\theta}{142}] \\ \frac{45-90p_{11}+209\theta}{854\,\theta} p_{11} + \delta_f \frac{3(20-40p_{11}-429\theta)^2}{1458632\,\theta} & \text{if } p_{11} \in (\frac{71+121\,\theta}{142}, \frac{10+19\,\theta}{20}] \\ \delta_f \frac{1}{2}\theta & \text{if } p_{11} \in [\frac{10+19\,\theta}{20}, \infty) \end{cases}$$

Firm 1 chooses its first-period price p_{11} to maximize $\pi_1^{(1,2)}$. Consistent with the main model, we focus on $\theta < \frac{V(1-\delta)}{2-\delta} = \frac{10}{21}$. One can show that the optimal p_{11} is as follows:

$$p_{11}^* = \begin{cases} \frac{10-\theta}{20} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{427(45+209\,\theta)-60(20-429\,\theta)\delta_f}{76860-2400\,\delta_f} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{71+121\,\theta}{142} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

Upon plugging p_{11}^* into $\pi_1^{(1,2)}$ and $\pi_2^{(1,2)} = \delta_f \pi_{22}^{(1,2)}$, we find the firms' subgame equilibrium profits:

$$\pi_1^{(1,2)} = \begin{cases} \frac{10-\theta(1-30\,\delta_f)}{80} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{(45+209\theta)^2-330\theta(20-429\theta)\delta_f}{240\theta(1281-40\,\delta_f)} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{11(284+11\theta(44+147\,\delta_f))}{40328} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

$$\pi_2^{(1,2)} = \begin{cases} \frac{3\theta\,\delta_f}{8} & \text{if } \theta \in (0, \frac{45}{218+60\,\delta_f}) \\ \frac{25\delta_f(36-\theta(943+24\,\delta_f))^2}{24\theta(1281-40\delta_f)^2} & \text{if } \theta \in [\frac{45}{218+60\,\delta_f}, \min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}) \\ \frac{20667\,\theta\,\delta_f}{40328} & \text{if } \theta \in [\min\{\frac{3195}{3949+4620\,\delta_f}, \frac{10}{21}\}, \frac{10}{21}) \end{cases}$$

When $(R_1, R_2) = (2, 1)$, the symmetry between the firms implies that $\pi_1^{(2,1)} = \pi_2^{(1,2)}$ and $\pi_2^{(2,1)} = \pi_1^{(1,2)}$.

When $(R_1, R_2) = (1, 1)$, given our focus on θ not too high, a similar argument as in the main model can show that, in equilibrium, consumers who enter the market in the first period will buy a product in that period instead of waiting till the second period to buy. In this situation, the firms' subgame equilibrium profits are given by $\pi_{11}^{(1,1)} = \pi_{12}^{(1,1)} = \alpha\frac{\theta}{2}$ and $\pi_{12}^{(1,1)} = \pi_{22}^{(1,1)} = (1-\alpha)\frac{\theta}{2}$. The discounted sum of the profits is given by $\pi_1^{(1,1)} = \pi_2^{(1,1)} = \alpha\frac{\theta}{2} + \delta_f(1-\alpha)\frac{\theta}{2}$.

Finally, when $(R_1, R_2) = (2, 2)$, clearly, the firms' discount factor does not influence their pricing decisions in the second period. Therefore, the subgame equilibrium prices are the same as in the main model. The corresponding profits are given by $\pi_1^{(2,2)} = \pi_2^{(2,2)} = \delta_f\frac{\theta}{2}$.

Now, let us find the firms' equilibrium product release times. $(R_1, R_2) = (1, 2)$ is an equilibrium if and only if $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$ and $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. Because $\pi_1^{(1,2)} \geq \pi_1^{(2,2)}$, firm 1 will not benefit by deviating from $R_1 = 1$ to $R_2 = 2$. It remains to check whether $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$. Straightforward algebra shows that $\pi_2^{(1,2)} \geq \pi_2^{(1,1)}$ holds when $\hat{\delta}_f < \delta_f \leq 1$ and $\hat{\theta} < \theta < \frac{10}{21}$, where $\hat{\theta} \equiv \frac{180\delta_f}{-1281\sqrt{3\delta_f(1+3\delta_f)}+5\delta_f(943+24\delta_f+8\sqrt{3}\sqrt{\delta_f(1+3\delta_f)})}$ and $\hat{\delta}_f \equiv \frac{5041}{5544} \approx 0.91$.

Recall that we assumed the consumers' discount factor is $\delta = 0.95$. Since $\hat{\delta}_f < \delta$, the above analysis shows that late product release can be optimal for firm 2 even if the firms' discount factor is less than that of consumers. ■

7. ALTERNATIVE MODELING OF CONSUMPTION VALUE

As a robustness check, we show that our main results continue to hold when we explicitly model the stream of consumption values that a durable product provides to consumers. Specifically, in section 7.A., we assume that upon purchasing the firm's product in period t, the cosumer obtains a value v in each period up to some period $T > 2$. Thus, the utility from purchasing the firm's product is given by $V_{it} - p_{it} - \theta d_i$, where $V_{it} = \sum_{j=t}^{T} \delta^{j-t} v$. In section 7.B., we show that our results will hold if the consumer receives $v - \theta d_i$ in each period, i.e., horizontal preferences influence the consumer's consumption value in each period. All remaining assumption are the same as in our main model. Further, we focus on $\theta < \frac{V_2}{2}$ to ensure that the market is fully covered so that, in equilibrium, each consumer makes a purchase either in the first or in the second period.

7.A. $V_{it} = \sum_{j=t}^{T} \delta^{j-t} v$

Subgame with $(R_1, R_2) = (1, 1)$ **or** $(R_1, R_2) = (2, 2)$

The analyses of these subgames is very similar to those in the main model. One can show that the firms' subgame equilibrium profits are given by $\pi_i^{(1,1)} = \pi_i^{(2,2)} = \frac{\theta}{2}$ for each firm $i = 1, 2$.

Subgame with $(R_1, R_2) = (1, 2)$

We solve the game using backward induction. In the second period, the firms compete for $(1 - \alpha)$ new consumers entering the market, as well as those first-period consumers who had decided not to buy in the first period but wait. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize those first-period consumers who will prefer not to buy in the first period but wait. In the main text, we showed that a consumer will choose to wait if

$$\max\{\delta(V_2 - p_{12}^e - \theta d), \delta(V_2 - p_{22}^e - \theta(1 - d))\} > V_1 - p_{11} - \theta d,$$

which reduces to

$$d > \hat{d} \tag{B69}$$

The cutoff $\hat{d}$ is given by

$$\hat{d} \equiv \begin{cases} 1 & \text{if } p_{11} \leq v + \delta p_{22}^e - \theta \\ \frac{v - p_{11} + \delta p_{22}^e + \delta\theta}{\theta(1+\delta)} & \text{if } v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} \\ \frac{v - p_{11} + p_{12}^e \delta}{\theta(1-\delta)} & \text{if } \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e \\ 0 & \text{if } p_{11} \geq v + \delta p_{12}^e \end{cases} \tag{B70}$$

Note that $\hat{d}$ depends on the prices p_{12}^e and p_{22}^e that consumers expect the firms to charge in the second period, and it takes a different functional form depending on p_{11}. We will separately analyze each of the four intervals into which p_{11} can fall. In each case, we will first find the firms' second-period subgame equilibrium prices p_{12}^* and p_{22}^* as a function of p_{12}^e, p_{22}^e and p_{11}. Since we are looking for a rational-expectations equilibrium, we will solve the equations $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ to find the price pair (p_{12}^e, p_{22}^e) that satisfies the condition for rational-expectations: $(p_{12}^e, p_{22}^e) = (p_{12}^*, p_{22}^*)$.

Case i: $p_{11} \leq v + \delta p_{22}^e - \theta$. In this case, $\hat{d} = 1$, i.e., all first-period consumers buy firm 1's product in the first period. Hence, in the second-period, firms compete for the new consumers entering the market. Firms 1 and 2 choose prices to maximize $(1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $(1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta})p_{22}$, respectively. Solving the first-order conditions, we find that $p_{12}^* = p_{22}^* = \theta$. Rational expectations require that $p_{i2}^e = p_{i2}^* = \theta$. Substituting $p_{i2}^e = \theta$ into inequality $p_{11} \leq v + \delta p_{22}^e - \theta$, it follows that the condition for case *i* is satisfied when $p_{11} \leq \underline{\rho}$, where $\underline{\rho} \equiv v - \theta(1-\delta)$.

Case ii: $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2}$. In this case, we have $x = \frac{v - p_{11} + \delta p_{22}^e + \delta\theta}{\theta(1+\delta)}$. In the first period, consumers with $d \leq \hat{d}$ bought firm 1's product, while the remaining consumers with $d > \hat{d}$ preferred to wait till the second period in order to buy firm 2's product. Since we are solving for a rational-expectations equilibrium, the firms' second-period subgame equilibrium prices must indeed be such that consumers with $d > \hat{d}$ actually buy firm 2's product. This happens when firm 2's second-period price is sufficiently low: $p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$. In addition, to have a positive market share, firm 1's price must not be too high: $p_{12} < p_{22} + \theta$, or equivalently, $p_{22} > p_{12} - \theta$. When $p_{12} - \theta \leq p_{22} \leq p_{12} - \theta + 2\theta\hat{d}$, the firms' second-period profits are

$$\pi_{12} = (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$$

$$\pi_{22} = (\alpha(1-\hat{d}) + (1-\alpha)(1 - \frac{\theta - p_{12} + p_{22}}{2\theta}))p_{22}$$

Solving the first-order conditions ($\frac{d\pi_{12}}{dp_{12}} = 0$ and $\frac{d\pi_{2_2}}{dp_{2_2}} = 0$), we obtain the equilibrium prices: $p_{12}^* = \frac{\theta(3(1+\delta)-\alpha-3\alpha\delta)-2\alpha(v-p_{11}+p_{22}^e\delta)}{3(1-\alpha)(1+\delta)}$ and $p_{22}^* = \frac{\theta(3+\alpha)+3\theta(1-\alpha)\delta+4\alpha(p_{11}-v-p_{22}^e\delta)}{3(1-\alpha)(1+\delta)}$. Solving $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ together, we find that $p_{12}^e = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^e = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$. Plugging p_{12}^e and p_{22}^e into the expressions for p_{12}^*, p_{22}^* and $\hat{d}$, we verify that $p_{12}^* = p_{12}^e = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$, $p_{22}^* = p_{22}^e = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $\hat{d} = \frac{3v(1-\alpha)+2\theta(3-\alpha)\delta-3p_{11}(1-\alpha)}{\theta(3(1+\delta)-\alpha(3-\delta))}$.

We need to check that case *ii*'s condition $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2}$ is satisfied. Using the expressions for p_{12}^e and p_{22}^e, one can readily show that $v + \delta p_{22}^e - \theta < p_{1_1} \leq \frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2}$ is equivalent to $\underline{\rho} < p_{11} \leq \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$, where recall that $\underline{\rho} = v - \theta(1-\delta)$. Note that when $\underline{\rho} < p_{11} \leq \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$, one can show that the prices p_{12}^* and p_{22}^* satisfy $p_{12}^* - \theta \leq p_{22}^* \leq p_{12}^* - \theta + 2\theta\hat{d}$, which ensures that both firms have positive market shares and that consumers with $d > \hat{d}$ indeed buy firm 2's product.

Firm 1's profit function is not necessarily quasi-concave because firm 1 may potentially want to decrease its price to capture some of the consumers with $d > \hat{d}$. Specifically, to capture some of the consumers located on $(\hat{d}, 1]$, firm 1 will need to deviate to a price $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$, in which case firm 1's deviation profit will be $\pi_{12}' = (\alpha(\frac{\theta-p_{12}'+p_{22}^*}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta-p_{12}'+p_{22}^*}{2\theta})p_{12}'$. Note that π_{12}' is concave. Evaluating the left-derivative of π_{12}' at the point $p_{12}' = p_{22}^* + \theta - 2\theta\hat{d}$, we find that $\frac{\partial_- \pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} = \frac{v(6-7\alpha+3\alpha^2)-p_{11}(6-7\alpha+3\alpha^2)-\theta(3-\alpha)(1-3\delta+2\alpha\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))}$. One can readily show that $\frac{\partial_- \pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} > 0$ if and only if $p_{11} < \tilde{\rho}$, where $\tilde{\rho} \equiv v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ satisfies $\underline{\rho} < \tilde{\rho} < \frac{6v-4v\alpha-\theta(3-\alpha)(1-3\delta)}{6-4\alpha}$. Since π_{12}' is concave, $\frac{\partial_- \pi_{12}'}{\partial p_{12}'}|_{p_{12}'=p_{22}^*+\theta-2\theta\hat{d}} > 0$ implies that π_{12} is increasing in p_{12}' at any $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$. Since firm 1's profit function is continuous at the point $p_{22}^* + \theta - 2\theta\hat{d}$, it follows that a deviation to $p_{12}' < p_{22}^* + \theta - 2\theta\hat{d}$ cannot be profitable as long as $p_{11} < \tilde{\rho}$.

Case iii: $\frac{2v-(1-\delta)(p_{22}^e+\theta)+(1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e$. In this case, $\hat{d} = \frac{v-p_{11}+p_{12}^e\delta}{\theta(1-\delta)}$. In the first period, consumers with $d < \hat{d}$ bought firm 1's product. The marginal consumer with $d = \hat{d}$ preferred

to wait till the second period to buy firm 1's product. So, in a rational-expectations equilibrium, firm 1 must have a positive market share among consumers who are located on the interval $(\hat{d}, 1]$, which happens when firm 1's second-period price is low enough: $p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$. Also, firm 2 will have a positive market share only if $p_{22} < p_{12} + \theta$, or equivalently, if $p_{12} > p_{22} - \theta$. When $p_{22} - \theta < p_{12} \leq p_{22} + \theta - 2\theta\hat{d}$, the firms' second-period profit functions are as follows:

$$\pi_{12} = (\alpha(\frac{\theta - p_{12} + p_{22}}{2\theta} - \hat{d}) + (1-\alpha)\frac{\theta - p_{12} + p_{22}}{2\theta})p_{12}$$

$$\pi_{22} = (\alpha\frac{\theta - p_{22} + p_{12}}{2\theta} + (1-\alpha)\frac{\theta - p_{22} + p_{12}}{2\theta})p_{22}$$

Solving the first-order conditions ($\frac{\partial \pi_{12}}{\partial p_{12}} = 0$ and $\frac{\partial \pi_{22}}{\partial p_{22}} = 0$) and plugging in $\hat{d} = \frac{v - p_{11} + p_{12}^e\delta}{\theta(1-\delta)}$, we obtain the subgame equilibrium prices: $p_{12}^* = \theta - \frac{4\alpha(v - p_{11} + p_{12}^e\delta)}{3(1-\delta)}$ and $p_{22}^* = \theta - \frac{2\alpha(v - p_{11} + p_{12}^e\delta)}{3(1-\delta)}$. Solving $p_{12}^e = p_{12}^*$ and $p_{22}^e = p_{22}^*$ together, we find that $p_{12}^e = \frac{3\theta(1-\delta) + 4\alpha(p_{11} - v)}{3 - (3-4\alpha)\delta}$ and $p_{22}^e = \frac{2(p_{11} - v)\alpha + \theta(3 - (3-2\alpha)\delta)}{3 - (3-4\alpha)\delta}$. Plugging p_{12}^e and p_{22}^e back into the expressions for p_{12}^*, p_{22}^* and $\hat{d}$, we verify that $p_{12}^* = p_{12}^e = \frac{3\theta(1-\delta) + 4\alpha(p_{11} - v)}{3 - (3-4\alpha)\delta}$ and $p_{22}^* = p_{22}^e = \frac{2(p_{11} - v)\alpha + \theta(3 - (3-2\alpha)\delta)}{3 - (3-4\alpha)\delta}$ and $\hat{d} = \frac{3(v - p_{11} + \theta\delta)}{\theta(3 - (3-4\alpha)\delta)}$.

Using the expressions for p_{12}^e and p_{22}^e, we find that *case iii*'s condition $\frac{2v - (1-\delta)(p_{22}^e + \theta) + (1+\delta)p_{12}^e}{2} < p_{11} < v + \delta p_{12}^e$ is satisfied if and only if $\frac{2(3-\alpha)v - 3(\theta - \theta(3-2\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{\rho}$, where $\bar{\rho} \equiv v + \delta\theta$. Note that when $\frac{2(3-\alpha)v - 3(\theta - \theta(3-2\alpha)\delta)}{2(3-\alpha)} < p_{11} < \bar{\rho}$, the prices p_{12}^* and p_{22}^* indeed satisfy $p_{22}^* - \theta < p_{12}^* < p_{22}^* + \theta - 2\theta\hat{d}$, which ensures that both firms have positive market shares and that the marginal first-period consumer located at $\hat{d} = \frac{3(v - p_{11} + \theta\delta)}{\theta(3 - (3-4\alpha)\delta)}$ buys firm 1's product in the second period.

As in the previous case, firm 1's profit function is not necessarily quasi-concave, and hence, we need to check that there are no non-local deviations. It is easy to check that deviations to lower prices will not be profitable. However, firm 1 may potentially deviate to a higher price to serve only the new consumers who entered the market in the second period, giving up its market share in the segment of first-period consumers who waited to buy a product in the second period. Specifically, if $p_{12}' > p_{22}^* + \theta - 2\theta\hat{d}$, then firm 1's deviation profit is $\pi_{12}' = (1-\alpha)\frac{\theta - p_{12}' + p_{22}^*}{2\theta}p_{12}'$. Notice that π_{12}' is concave. One

can show that the right derivative of π'_{12} at $p'_{12} = p^*_{22} + \theta - 2\theta\hat{d}$ is given by $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} = -\frac{(1-\alpha)(p_{11}(6+\alpha)-v(6+\alpha)+3\theta(1-(3-\alpha)\delta))}{\theta(3-(3-4\alpha)\delta)}$. Straightforward algebra shows that $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} < 0$ if and only if $p_{11} > \hat{\rho}$, where $\hat{\rho} \equiv \frac{v(6+\alpha)-3\theta(1-(3-\alpha)\delta)}{6+\alpha}$ satisfies $\frac{2(3-\alpha)v-3(\theta-\theta(3-2\alpha)\delta)}{2(3-\alpha)} < \hat{\rho} < \bar{\rho}$. Since π'_{12} is concave, $\frac{\partial_+ \pi'_{12}}{\partial p'_{12}}|_{p'_{12}=p^*_{22}+\theta-2\theta\hat{d}} < 0$ implies that π'_{12} is decreasing in p'_{12} when $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$. Hence, if $p_{11} > \hat{\rho}$, then continuity of firm 1's profit function at the point $p^*_{22} + \theta - 2\theta\hat{d}$ implies that deviations to $p'_{12} > p^*_{22} + \theta - 2\theta\hat{d}$ will not be profitable because firm 1 will gain by reducing its price.

***Case iv**: $p_{11} \geq v + \delta p^e_{12}$.* In this case, $\hat{d} = 0$, i.e., all first-period consumers will decide not to purchase a product in the first period, but wait. In the second period, firm i will choose its second-period price p_{i2} to maximize its profit, π_{i2}, where $\pi_{12} = \frac{\theta - p_{12} + p_{22}}{2\theta} p_{12}$ and $\pi_{22} = \frac{\theta - p_{22} + p_{12}}{2\theta} p_{22}$. Solving the first-order conditions, we find that $p^*_{12} = p^*_{22} = \theta$. Rational expectations imply that $p^e_{i2} = p^*_{i2} = \theta$. Substituting $p^e_{i2} = \theta$ into the inequality $p_{11} \geq v + \delta p^e_{12}$, it follows that the condition for case *iv* is satisfied if and only if $p_{11} \geq \bar{\rho}$, where recall that $\bar{\rho} = v + \delta\theta$.

Our analysis in cases *i*, *ii*, *iii* and *iv* characterizes the second-period subgame equilibrium prices for every p_{11} contained in the intervals $[0, \underline{\rho}]$, $(\underline{\rho}, \tilde{\rho}]$, $(\hat{\rho}, \bar{\rho})$, and $[\bar{\rho}, \infty]$, respectively. To find the first-period equilibrium price, we need to know the second-period subgame equilibrium outcome for each $p_{11} \geq 0$. That is, to have a full characterization of second-period subgame equilibria, we need that $[0, \underline{\rho}] \cup (\underline{\rho}, \tilde{\rho}] \cup (\hat{\rho}, \bar{\rho}) \cup [\bar{\rho}, \infty] = [0, \infty]$, which can happen only if $\tilde{\rho} \geq \hat{\rho}$, where recall that $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\hat{\rho} = \frac{v(6+\alpha)-3\theta(1-(3-\alpha)\delta)}{6+\alpha}$. The inequality $\tilde{\rho} \geq \hat{\rho}$ is satisfied if and only if $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$ where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Note that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$, there are two possible second-period rational-expectations equilibria corresponding to the cases *ii* and *iii* that we analyzed earlier. The selection of which equilibrium occurs will not have a qualitative impact on our results. Since both firms' profits are higher when the equilibrium from case *ii* is played (i.e., it is the focal equilibrium), we assume that when $p_{11} \in (\hat{\rho}, \tilde{\rho})$,

the subgame equilibrium prices will be the ones from case *ii*, i.e., $p_{12}^{*} = \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)}$ and $p_{22}^{*} = \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)}$.

Hence, to summarize our analysis, it follows that when $(R_1, R_2) = (1, 2)$, the firms' second-period prices in a rational-expectations subgame equilibrium are as follows:

$$p_{12}^{(1,2)} = p_{12}^{e} = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta(1-\delta)+4\alpha(p_{11}-v)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B71}$$

$$p_{22}^{(1,2)} = p_{22}^{e} = \begin{cases} \theta & \text{if } p_{11} \in [0, \underline{\rho}) \cup [\bar{\rho}, \infty) \\ \frac{4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta)}{3(1+\delta)-\alpha(3-\delta)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{2(p_{11}-v)\alpha+\theta(3-(3-2\alpha)\delta)}{3-(3-4\alpha)\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \end{cases} \tag{B72}$$

where $\underline{\rho} = v - \theta(1-\delta)$, $\tilde{\rho} \equiv v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} \equiv v + \delta t$. For future reference, let us derive the firms' equilibrium profits and sales. Specifically, upon plugging the prices in (B71) and (B72) into the respective profit functions (explicitly provided in earlier cases *i–iv*), we obtain the firms' second-period subgame equilibrium profits when $(R_1, R_2) = (1,2)$:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B73}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4(p_{11}-v)\alpha+\theta(3+\alpha+3\delta-3\alpha\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(2(p_{11}-v)\alpha+\theta(3-(3-2\alpha)\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \tag{B74}$$

We can also find the firms' second-period unit sales:

$$S_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{4\alpha(p_{11}-v)+3\theta(1-\delta)}{6\theta(1-\delta)+8\theta\alpha\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B75)}$$

$$S_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(4(p_{11}-v)\alpha+3\theta(1+\delta)+\theta\alpha(1-3\delta))}{2\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3\theta(1-\delta)+2\alpha(p_{11}-v)+2\theta\alpha\delta}{6\theta(1-\delta)+8\theta\alpha\delta} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{1}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B76)}$$

To find firm 1's first-period unit sales, $S_{11}^{(1,2)}$, recall that consumers with $d < \hat{d}$ buy firm 1's product in the first period, where $\hat{d}$ is given in the equation (B70). Since consumers are uniformly distributed, $S_{11}^{(1,2)} = \alpha\hat{d}$. Plugging $p_{12}^e = p_{12}^{(1,2)}$ and $p_{22}^e = p_{22}^{(1,2)}$ into the expression for $\hat{d}$, we find that

$$S_{11}^{(1,2)} = \begin{cases} \alpha & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(v-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B77)}$$

In the first period, firm 1 chooses its price to maximize $\pi_1 = \pi_{11}^{(1,2)} + \pi_{12}^{(1,2)}$, where $\pi_{11}^{(1,2)} = S_{11}^{(1,2)}p_{11}$. Note that $\pi_{12}^{(1,2)}$ and $S_{11}^{(1,2)}$ are given in equations (B73) and (B77), respectively.

$$\pi_1 = \begin{cases} \alpha\, p_{11} + \frac{(1-\alpha)\theta}{2} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \alpha\frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))}p_{11} + \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \alpha\frac{3(v-p_{11}+\theta\delta)}{\theta(3-(3-4\alpha)\delta)}p_{11} + \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta}{2} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases}, \quad \text{(B78)}$$

where $\underline{\rho} = v - \theta(1-\delta)$, $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$ and $\bar{\rho} = v + \delta\theta$.

Note that π_1 is continuous everywhere except at $p_{11} = \tilde{\rho}$, where π_1 jumps down. It is easy to see that π_1 is increasing in p_{11} at any $p_{11} < \underline{\rho}$, and hence, the optimal price will be above $\underline{\rho}$. Further, if $p_{11} > \bar{\rho}$, then π_1 is constant. Since π_1 is continuous at $\bar{\rho}$, the optimal price will not be above $\bar{\rho}$ either.

Hence, the optimal price is either in $I_1 \equiv [\underline{\rho}, \tilde{\rho}]$ or $I_2 \equiv (\tilde{\rho}, \bar{\rho}]$. Let p_{I_k} denote the locally optimal price within the interval I_k. We will first find p_{I_1} and p_{I_2}, after which we will compare the corresponding profits to determine the globally optimal price.

First, consider $p_{11} \in I_1$. On this interval, we have

$$\pi_1 = \alpha \frac{(3v(1-\alpha)-3p_{11}(1-\alpha)+2\theta(3-\alpha)\delta)}{\theta(3(1+\delta)-\alpha(3-\delta))} p_{11} + \frac{(1-\alpha)(2(p_{11}-v)\alpha+\theta(3-\alpha)(1+\delta))^2}{2\theta(3(1+\delta)-\alpha(3-\delta))^2}.$$

One can readily show that π_1 is concave because $\frac{d^2\pi_1}{dp_{11}^2} < 0$.

If $0 < \theta \leq \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$, then π_1 is decreasing in p_{11} at any $\underline{\rho} < p_{11} < \tilde{\rho}$, and hence, the locally optimal price is at the left corner: $p_{I_1} = \underline{\rho}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{2v\alpha+\theta(1+\alpha(-3+2\delta))}{2}$.

If $\frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta} < \theta < \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$, then the local maximizer is in the interior of the interval $[\underline{\rho}, \tilde{\rho}]$. Using the fist-order condition ($\frac{d\pi_1}{dp_{1_1}} = 0$), we find the interior maximizer: $p_{I_1} = \frac{v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)+\alpha^2(-1-4\delta+\delta^2)-3(1+4\delta+3\delta^2))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$. The profit is $\pi_1(p_{I_1}) = \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1-3\delta))(v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)-\alpha^2(1+4\delta-\delta^2)-3(1+4\delta+3\delta^2)))}{4\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2}$.

If $\min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\} \leq \theta < \frac{V_2}{2}$, then π_1 is increasing in p_{1_1} at any $\underline{\rho} < p_{1_1} < \tilde{\rho}$, and hence, the locally optimal price is at the right corner: $p_{I_1} = \tilde{\rho}$. The corresponding profit is $\pi_1(p_{I_1}) = \frac{(3-\alpha)(2v\alpha(6-\alpha(7-3\alpha))+\theta(3-\alpha)(4-\alpha(10-6\delta-\alpha(5-\alpha-4\delta))))}{2(6+\alpha(-7+3\alpha))^2}$.

Second, consider $p_{11} \in I_2$. On this interval, we have

$$\pi_1 = \alpha \frac{3(v-p_{11}+t\delta)}{\theta(3-(3-4\alpha)\delta)} p_{11} + \frac{(4(p_{11}-v)\alpha+3\theta(1-\delta))^2}{2\theta(3-(3-4\alpha)\delta)^2}.$$

It is easy to show that $\frac{d^2\pi_1}{dp_{11}^2} < 0$, implying that π_1 is concave.

If $\theta \leq \tilde{\theta}$, then π_1 is decreasing in p_{11} at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, where $\tilde{\theta} = \frac{3v(6-\alpha(7-3\alpha))(3-(3-4\alpha)\delta)}{2(63-\alpha(75-26\alpha))-(234-\alpha(417-(213-32\alpha)\alpha))\delta+3(3-4\alpha)(12-(11-\alpha)\alpha)\delta^2}$. Hence, a local maximum does not exist.

If $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$, then π_1 has a local maximizer in the interior of the interval $(\tilde{\rho}, \bar{\rho}]$. Using the first-order condition, we find that $p_{I_2} = \frac{3\theta(4-\delta-(3-4\alpha)\delta^2)+v(9-9\delta-4\alpha(4-3\delta))}{2(9-9\delta-4\alpha(2-3\delta))}$. The corresponding profit is $\pi_1(p_{I_2}) = \frac{9v^2\alpha-6\theta v\alpha(4-3\delta)+9\theta^2(2-2\delta+\alpha\delta^2)}{4\theta(9-9\delta-4\alpha(2-3\delta))}$.

If $\frac{3v}{4-3\delta} < \theta < \frac{V_2}{2}$, then π_1 is decreasing in p_{11} at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Hence, the local maximizer is at $p_{I_2} = \bar{\rho}$, with a corresponding profit of $\pi_1(p_{I_2}) = \frac{\theta}{2}$.

Next, let us find the globally optimal price, $p_{11}^{(1,2)}$.

When $\theta \leq \tilde{\theta}$, π_1 is decreasing at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since π_1 jumps down at $p_{11} = \tilde{\rho}$ (i.e., $\lim_{p_{11}\to\tilde{\rho}^-} \pi_1 > \lim_{p_{11}\to\tilde{\rho}^+} \pi_1$) and $\frac{d\pi_1}{dp_{11}} < 0$ at any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$, it follows that $\pi_1(\tilde{\rho}) > \pi_1(p_{11})$ for any $p_{11} \in (\tilde{\rho}, \bar{\rho}]$. Since $\tilde{\rho} \in I_1$ and $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$, it follows that $p_{11}^{(1,2)} = p_{I_1}$.

When $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$, we have $\pi_1(p_{I_2}) = \frac{9v^2\alpha-6\theta v\alpha(4-3\delta)+9\theta^2(2-2\delta+\alpha\delta^2)}{4\theta(9-9\delta-4\alpha(2-3\delta))}$. One can show that $\pi_1(\tilde{\rho}) > \pi_1(p_{I_2})$.[27] Since $\tilde{\rho} \in I_1$, it must be that $\pi_1(p_{I_1}) \geq \pi_1(\tilde{\rho})$. Hence, by transitivity, it follows that $\pi_1(p_{I_1}) > \pi_1(p_{I_2})$, which implies that $p_{11}^{(1,2)} = p_{I_1}$.

Finally, when $\frac{3v}{4-3\delta} < \theta < \frac{V_2}{2}$, we have $\pi_1(p_{I_2}) = \frac{\theta}{2}$. Again, straightforward algebra shows that $\pi_1(\tilde{\rho}) > \pi_1(p_{I_2})$, and by the same argument as in the previous paragraph, it follows that $p_{11}^{(1,2)} = p_{I_1}$.

Hence, we found that firm 1's optimal first-period price belongs to the interval I_1, and is given by

$$p_{11}^{(1,2)} = \begin{cases} \underline{\rho} & \text{if } \theta \leq \underline{\theta} \\ \frac{v(1-\alpha)(9-13\alpha+9\delta+3\alpha\delta)-2\theta(4\alpha(1+4\delta)-\alpha^2(1+4\delta-\delta^2)-3(1+4\delta+3\delta^2))}{2(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \tilde{\rho} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases} \quad \text{(B79)}$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$, $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$, $\underline{\rho} = v - \theta(1-\delta)$ and $\tilde{\rho} = v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1)}{6-\alpha(7-3\alpha)}$.

[27] To see this, note that the function $\pi_1(\tilde{\rho}) - \pi_1(p_{I_2})$ is concave in θ because $\frac{d^2(\pi_1(\tilde{\rho})-\pi_1(p_{I_2}))}{d\theta^2} = -\frac{9v^2\alpha}{2\theta^3(9-9\delta-4\alpha(2-3\delta))} < 0$ for any $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$. Further, $(\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}))|_{\theta=\tilde{\theta}} > 0$ and $(\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}))|_{\theta=\frac{3v}{4-3\delta}} > 0$. Hence, it must be that $\pi_1(\tilde{\rho}) - \pi_1(p_{I_2}) > 0$ for any $\tilde{\theta} < \theta < \frac{3v}{4-3\delta}$.

For future reference, by plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from Lemma 2, we can find the firms' second-period prices on the equilibrium path:

$$p_{12}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \le \underline{\theta} \\ \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)} & \text{if } \bar{\theta} \le \theta < \frac{V_2}{2} \end{cases} \tag{B80}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta & \text{if } \theta \le \underline{\theta} \\ \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-\alpha(4+6\delta))-6v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)} & \text{if } \bar{\theta} \le \theta < \frac{V_2}{2} \end{cases} \tag{B81}$$

Using the equilibrium prices, we obtain the firm's equilibrium profits in each period. Specifically,

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(v-\theta(1-\delta)) & \text{if } \theta \le \underline{\theta} \\ \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1-3\delta))(v(1-\alpha)(9-13\alpha+3(3+\alpha)\delta)+A)}{4\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{(3-\alpha)\alpha(v(6-\alpha(7-3\alpha))-\theta(3-\alpha)(1-(3-2\alpha)\delta))}{(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{V_2}{2} \end{cases} \tag{B82}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \underline{\theta} \\ \frac{(3(1-\alpha)(v\alpha-\theta(3-\alpha))-\theta(3-\alpha)^2\delta)^2}{2\theta(1-\alpha)(9-11\alpha+3(3+\alpha)\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-5\alpha+\alpha^2)^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{V_2}{2} \end{cases} \tag{B83}$$

$$\pi_{21}^{(1,2)} = 0 \tag{B84}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta}{2} & \text{if } \theta \le \underline{\theta} \\ \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{V_2}{2} \end{cases} \tag{B85}$$

where $A \equiv 2\theta(3-\alpha)((1+\delta)(1+3\delta)-\alpha(1+(4-\delta)\delta))$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$.

PROPOSITION 7.1. *If firm 2 releases its product later than firm 1, then second-period price competition is less intense than when firm 2 also releases its product in the first period, i.e.,* $p_{12}^{(1,2)} \ge p_{12}^{(1,1)}$ *and* $p_{22}^{(1,2)} \ge p_{22}^{(1,1)}$.

PROOF OF PROPOSITION 7.1. Before we proceed to the proof, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $t < \frac{V_2}{2}$, $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$, where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

When $(R_1, R_2) = (1, 1)$, Lemma 1 showed that the firms' equilibrium prices will be $p_{i2}^{(1,1)} = \theta$.

When $(R_1, R_2) = (1, 2)$, the firms' equilibrium second-period prices are given in equations (B80)-(B81). We need to show that $p_{i2}^{(1,2)} \geq p_{i2}^{(1,1)}$. Clearly, if $\theta \leq \underline{\theta}$, then $p_{i2}^{(1,2)} = p_{i2}^{(1,1)} = \theta$ for each firm $i = 1, 2$.

Next, if $\underline{\theta} < \theta < \bar{\theta}$, we have $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(3(1+\delta)-\alpha(3+\delta))-3v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$ and $p_{22}^{(1,2)} = \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-\alpha(4+6\delta))-6v(1-\alpha)\alpha}{(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)}$. Straightforward algebra shows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ if and only if $\theta > \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$. Recall that $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$. Hence, $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$ when $\underline{\theta} < \theta < \bar{\theta}$.

Finally, if $\bar{\theta} \leq \theta < \frac{V_2}{2}$, then $p_{12}^{(1,2)} = \frac{\theta(3-\alpha)(2-\alpha)}{6-\alpha(7-3\alpha)}$ and $p_{22}^{(1,2)} = \frac{\theta(6-\alpha(3+\alpha))}{6-\alpha(7-3\alpha)}$. Since $(3-\alpha)(2-\alpha) > 6-\alpha(7-3\alpha)$ and $6-\alpha(3+\alpha) > 6-\alpha(7-3\alpha)$ for any $\alpha \in [0, \bar{\alpha})$, it readily follows that $p_{12}^{(1,2)} > \theta$ and $p_{22}^{(1,2)} > \theta$. ■

PROPOSITION 7.2. *There exist* $0 < \hat{\alpha} < \bar{\alpha}$ *and* $0 < \hat{\theta} < \frac{V_2}{2}$, *such that if* $\alpha < \hat{\alpha}$ *and* $\theta > \hat{\theta}$, *then in equilibrium, firm 2 will release its product later than firm 1, i.e.,* $(R_1^*, R_2^*) = (1, 2)$.

PROOF OF PROPOSITION 7.2. For future reference, recall that to ensure the existence of a rational-expectations equilibrium with full market coverage, we are assuming that $t < \frac{V_2}{2}$, $\bar{\delta} < \delta < 1$ and $0 \leq \alpha < \bar{\alpha}$, where $\bar{\delta} = \frac{18-10\alpha}{36-\alpha(45-11\alpha)}$ and $\bar{\alpha} \approx 0.64$.

Firm i chooses its product release period $R_i \in \{1,2\}$ to maximize its overall profit $\pi_i^{(R_1,R_2)} = \pi_{i1}^{(R_1,R_2)} + \pi_{i2}^{(R_1,R_2)}$. The pair (R_1, R_2) is an equilibrium if no firm has any profitable deviations.

First, one can readily show that $(R_1, R_2) = (2,2)$ is not an equilibrium because firm 1 will benefit by deviating to $R_1 = 1$, i.e., $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, where $\pi_1^{(2,2)} = \frac{\theta}{2}$. To see this, notice from (B78) that if firm 1 deviates to $R_1 = 1$ and sets an extremely large price ($p_{11} \geq \bar{\rho}$), then its overall profit will be $\pi_1(\bar{\rho}) = \frac{\theta}{2}$. However, we know from Lemma 3 that firm 1's unique optimal price is $p_{11}^{(1,2)} < \bar{\rho}$ with a corresponding profit of $\pi_1^{(1,2)}$. Hence, $\pi_1^{(1,2)} > \pi_1(\bar{\rho})$. Since $\pi_1(\bar{\rho}) = \pi_1^{(2,2)} = \frac{\theta}{2}$, it follows that $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Therefore, in equilibrium, either one firm releases its product in the first period and the other firm does so in the second period, or both firms release their products in the first period.

Next, let us demonstrate that a pure strategy equilibrium exists. If $(R_1, R_2) = (1,2)$ is an equilibrium, then the proof is finished. If $(R_1, R_2) = (1,2)$ is not an equilibrium, then let us show that $(R_1, R_2) = (1,1)$ must be an equilibrium. Since $(R_1, R_2) = (1,2)$ is not an equilibrium, one of the firms must have a profitable deviation. Firm 1 will not want to deviate to $R_1 = 2$ because $\pi_1^{(1,2)} > \pi_1^{(2,2)}$. Hence, it must be that firm 2 has a profitable deviation from $R_2 = 2$ to $R_2 = 1$, which means that $\pi_2^{(1,1)} > \pi_2^{(1,2)}$. By symmetry between firms 1 and 2, we know that $\pi_2^{(1,2)} = \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} = \pi_1^{(1,1)}$. Hence, $\pi_2^{(1,1)} > \pi_2^{(1,2)}$ implies that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$. Then, it must be $(R_1, R_2) = (1,1)$ is an equilibrium because we found that $\pi_1^{(1,1)} > \pi_1^{(2,1)}$ and $\pi_2^{(1,1)} > \pi_2^{(1,2)}$.

Now, let us find conditions under which $(R_1, R_2) = (1,2)$ is an equilibrium. If $R_1 = 1$, then firm 2's best response is $R_2 = 2$ if and only if $\pi_2^{(1,2)} - \pi_2^{(1,1)} > 0$, where $\pi_2^{(1,1)} = \frac{\theta}{2}$ and $\pi_2^{(1,2)} = \pi_{21}^{(1,2)} + \pi_{22}^{(1,2)}$ is given in equations (B84)-(B85). Define $\Delta\pi_2 \equiv \pi_2^{(1,2)} - \pi_2^{(1,1)}$.

$$\Delta\pi_2 = \begin{cases} \frac{(1-\alpha)\theta}{2} - \frac{\theta}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} - \frac{\theta}{2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{\theta}{2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases},$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$ and $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Note that $\Delta\pi_2$ is a continuous function. When $\theta \leq \underline{\theta}$, clearly, $\Delta\pi_2 < 0$. Hence, consider $\theta > \underline{\theta}$.

First, let us show that when $\underline{\theta} < \theta < \bar{\theta}$, the function $\Delta\pi_2$ is increasing in θ, i.e., $\frac{d\Delta\pi_2}{d\theta} > 0$. To see this, note that $\frac{d^2\Delta\pi_2}{d\theta^2} = \frac{36v^2(1-\alpha)\alpha^2}{\theta^3(9-11\alpha+9\delta+3\alpha\delta)^2} > 0$, i.e., $\Delta\pi_2$ is convex. Further, $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\underline{\theta}} = \frac{\alpha(23-9\delta-\alpha(21-13\delta))}{2(9-11\alpha+9\delta+3\alpha\delta)} > 0$, where the inequality follows because $\delta > \bar{\delta} \geq 0.5$. The inequalities $\frac{d^2\Delta\pi_2}{d\theta^2} > 0$ and $\frac{d\Delta\pi_2}{d\theta}|_{\theta=\underline{\theta}} > 0$ imply that $\frac{d\Delta\pi_2}{d\theta} > 0$ for any $\underline{\theta} < \theta < \bar{\theta}$.

Second, when $\bar{\theta} \leq \theta < \frac{V_2}{2}$, we have $\frac{d\Delta\pi_2}{d\theta} = \frac{(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} - \frac{1}{2} > 0$ if and only if $\alpha < \hat{\alpha}$, where $\hat{\alpha} \approx 0.32$ satisfies $\frac{(1-\hat{\alpha})(6-\hat{\alpha}(3+\hat{\alpha}))^2}{2(6-\hat{\alpha}(7-3\hat{\alpha}))^2} - \frac{1}{2} = 0$, or equivalently, $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$.

We will separately analyze the cases with $\alpha \leq \hat{\alpha}$ and $\alpha > \hat{\alpha}$. Specifically, we will show that if $\alpha > \hat{\alpha}$, then $\Delta\pi_2 < 0$, but if $\alpha \leq \hat{\alpha}$, then $\Delta\pi_2 > 0$ for θ large enough.

Case 1: $\alpha > \hat{\alpha}$. When $\alpha > \hat{\alpha}$, one can show that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$.[28] We showed earlier that $\Delta\pi_2$ is increasing when $\underline{\theta} < \theta < \bar{\theta}$ and decreasing when $\bar{\theta} \leq \theta < \frac{V_2}{2}$. Since $\Delta\pi_2$ is continuous, it follows that $\Delta\pi_2|_{\theta=\bar{\theta}} > \Delta\pi_2$ for any $\underline{\theta} < \theta < \frac{V_2}{2}$ and $\theta \neq \bar{\theta}$. Straightforward algebra shows that $\Delta\pi_2|_{\theta=\bar{\theta}} = \frac{3v(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, and $\Delta\pi_2|_{\theta=\bar{\theta}} < 0$ if and only if $12 - \alpha(52 - (3-\alpha)\alpha(17+\alpha)) > 0$. The last inequality is equivalent to $\alpha > \hat{\alpha}$, where recall that $\hat{\alpha} \approx 0.32$ satisfies $12 - \hat{\alpha}(52 - (3-\hat{\alpha})\hat{\alpha}(17+\hat{\alpha})) = 0$. Hence, when $\alpha > \hat{\alpha}$, we have $\Delta\pi_2 \leq \Delta\pi_2|_{\theta=\bar{\theta}} < 0$ for any $\underline{\theta} < \theta < \frac{V_2}{2}$.

Case 2: $\alpha < \hat{\alpha}$. When $\alpha < \hat{\alpha}$, we showed that $\Delta\pi_2$ is increasing in θ at any $\underline{\theta} < \theta < \frac{V_2}{2}$. Further, we can show that $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$, where recall that $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Specifically, if $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$, then we already know from Case 1 that $\Delta\pi_2|_{\theta=\bar{\theta}} =$

[28] To see this, note that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ if and only if $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2} < 0$. Using the fact that $\delta > \bar{\delta} > 0.5$, one can show that the function $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2}$ is decreasing in α. Plugging in $\alpha = 0.3$, one can show that $(\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2})|_{\alpha=0.3} < 0$. Hence, $\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} - \frac{V_2}{2} < 0$ for all $\alpha > 0.3$. Since $\hat{\alpha} > 0.3$, it follows that $\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}$ when $\alpha > \hat{\alpha}$.

$\frac{3v(1-\alpha)\alpha(12-\alpha(52-(3-\alpha)\alpha(17+\alpha)))}{4(3-\alpha)(6-\alpha(7-3\alpha))(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))} > 0$ when $\alpha < \hat{\alpha}$. Next, if $\bar{\theta} = \frac{V_2}{2}$, then first note that $V_2 = \sum_{k=2}^{T} \delta^{k-2} v$ is increasing in T. Since $T \geq 3$, it follows that $V_2 \geq V_2|_{T=3} = v(1+\delta)$. Since we already showed that $\Delta\pi_2$ is increasing in θ at any $\theta > \underline{\theta}$ and since $\frac{v(1+\delta)}{2} > \underline{\theta}$, it follows that $\Delta\pi_2|_{\theta=\frac{V_2}{2}} > \Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}}$. $\Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} = \frac{v(9(1+\delta)^2+\alpha^2(7+5\delta^2)-2\alpha(8+\delta(5+3\delta)))^2}{4(1-\alpha)(1+\delta)(9-11\alpha+3(3+\alpha)\delta)^2} - \frac{v(1+\delta)}{4}$, and since $\delta > \bar{\delta} > 0.5$, one can show that $\Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} > 0$. Hence, $\Delta\pi_2|_{\theta=\bar{\theta}} = \Delta\pi_2|_{\theta=\frac{V_2}{2}} > \Delta\pi_2|_{\theta=\frac{v(1+\delta)}{2}} > 0$.

Since $\Delta\pi_2$ is increasing in θ, $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$ implies that $\Delta\pi_2 > 0$ for all $\bar{\theta} \leq \theta < \frac{V_2}{2}$. Further, since $\Delta\pi_2|_{\theta=\underline{\theta}} = -\frac{3v(1-\alpha)\alpha}{8(2-\alpha(2-\delta))} < 0$, $\Delta\pi_2|_{\theta=\bar{\theta}} > 0$ and $\Delta\pi_2$ is a continuous increasing function, it follows that there exists a unique $\underline{\theta} < \hat{\theta} < \bar{\theta}$, such that $\Delta\pi_2|_{\theta=\hat{\theta}} = 0$ and $\Delta\pi_2 < 0$ if $\underline{\theta} < \theta < \hat{\theta}$ and $\Delta\pi_2 > 0$ if $\hat{\theta} < \theta < \bar{\theta}$. Solving $\Delta\pi_2 = 0$, we find that

$$\hat{\theta} = \frac{6(v(1-\alpha)^{3/2}(9-11\alpha+3(3+\alpha)\delta)-v(1-\alpha)(\alpha(4+5\alpha(1-\delta)+6\delta)-9(1+\delta)))}{25\alpha^3(1-\delta)^2+9(1+\delta)(23-9\delta)+\alpha^2(161-\delta(46+51\delta))-3\alpha(131+\delta(10-57\delta))} \qquad \text{(B86)}$$

To summarize the above, we found that if $0 < \alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\Delta\pi_2 > 0$. Thus, in this parameter region, firm 2 will not deviate from $R_2 = 2$. Since $\pi_1^{(1,2)} > \pi_1^{(2,2)}$, firm 1 also has no incentives to deviate from $R_1 = 1$. It follows that $(R_1, R_2) = (1,2)$ is a Nash equilibrium. ■

PROPOSITION 7.3. *Suppose that $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, so that in equilibrium $(R_1^*, R_2^*) = (1,2)$. Then, firm 2's equilibrium sales are decreasing in α. However, firm 2's profits are non-monotone in α, and an increase in α can make firm 2 better off as long as α is not too high.*

PROOF OF PROPOSITION 7.3. First, let us show that firm 2's equilibrium unit sales, $S_2^{(1,2)}$, are decreasing in α. Note that $S_2^{(1,2)} = S_{22}^{(1,2)}$, where $S_{22}^{(1,2)}$ is given in the equation (B76). Upon plugging in firm 1's first-period equilibrium price, we find firm 2's sales on the equilibrium path:

$$S_2^{(1,2)} = \begin{cases} \frac{1-\alpha}{2} & \text{if } \theta \leq \underline{\theta} \\ \frac{\theta(9(1+\delta)-5\alpha^2(1-\delta)-2\alpha(2+3\delta))-6v(1-\alpha)\alpha}{2\theta(9-11\alpha+9\delta+3\alpha\delta)} & \text{if } \underline{\theta} \leq \theta \leq \bar{\theta} \\ \frac{(1-\alpha)(6-\alpha(3+\alpha))}{2(6-\alpha(7-3\alpha))} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases}$$

where $\underline{\theta} \equiv \frac{3v(1-\alpha)}{8(1-\alpha)+4\alpha\delta}$ and $\bar{\theta} \equiv \min\{\frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-3\delta-\alpha(6-8\delta-\alpha(1-3\delta)))}, \frac{V_2}{2}\}$. Note that $S_2^{(1,2)}$ is continuous. When $\theta < \underline{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{1}{2} < 0$. When $\underline{\theta} < \theta < \bar{\theta}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = \frac{\Psi}{\theta(9-11\alpha+3(3+\alpha)\delta)^2}$, where $\Psi = \frac{6v(\alpha(18-11\alpha+3(6+\alpha)\delta)-9(1+\delta))+\theta(5\alpha^2(1-\delta)(11-3\delta)+9(1+\delta)(7-9\delta)-90\alpha(1-\delta^2))}{2}$. Clearly, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ if and only if $\Psi < 0$. Hence, we need to show that $\Psi < 0$. First, note that Ψ is concave in α because $\frac{d^2\Psi}{d\alpha^2} = -(6v - 5\theta(1-\delta))(11-3\delta) < 0$.[29] Second, since $\theta < \frac{V_2}{2}$, one can show that $\Psi|_{\alpha=0} = -\frac{9(1+\delta)(6v-\theta(7-9\delta))}{2} < 0$ and $\Psi|_{\alpha=0.5} = -\frac{6v(11-3\delta)-\theta(127-142\delta-129\delta^2)}{4} < 0$.[30] Concavity of Ψ, together with $\Psi|_{\alpha=0} < 0$ and $\Psi|_{\alpha=0.5} < 0$, implies that $\Psi < 0$ for any $0 < \alpha < 0.5$. Since $\hat{\alpha} \approx 0.32 < 0.5$, it follows that $\Psi < 0$ for any $0 < \alpha < \hat{\alpha}$. Thus, $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ when $\underline{\theta} < \theta < \bar{\theta}$. Finally, when $\bar{\theta} \leq \theta < \frac{V_2}{2}$, we have $\frac{dS_2^{(1,2)}}{d\alpha} = -\frac{12+\alpha(12-\alpha(31-\alpha(14-3\alpha)))}{2(6-\alpha(7-3\alpha))^2}$. One can easily show that $\frac{dS_2^{(1,2)}}{d\alpha} < 0$ for any $0 < \alpha < \hat{\alpha}$. This proves the first part of the proposition.

Next, let us show that there exists $\tilde{\alpha} < \hat{\alpha}$, such that if $\alpha < \tilde{\alpha}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$. Since we are focusing on $\theta > \hat{\theta}$ and since $\underline{\theta} < \hat{\theta} < \bar{\theta}$, it follows from the equation (B85) that $\pi_2^{(1,2)} = \pi_{22}^{(1,2)}$ is given by

$$\pi_2^{(1,2)} = \begin{cases} \frac{(6v(1-\alpha)\alpha+\theta(5\alpha^2(1-\delta)-9(1+\delta)+2\alpha(2+3\delta)))^2}{2\theta(1-\alpha)(9-11\alpha+9\delta+3\alpha\delta)^2} & \text{if } \hat{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \leq \theta < \frac{V_2}{2} \end{cases}$$

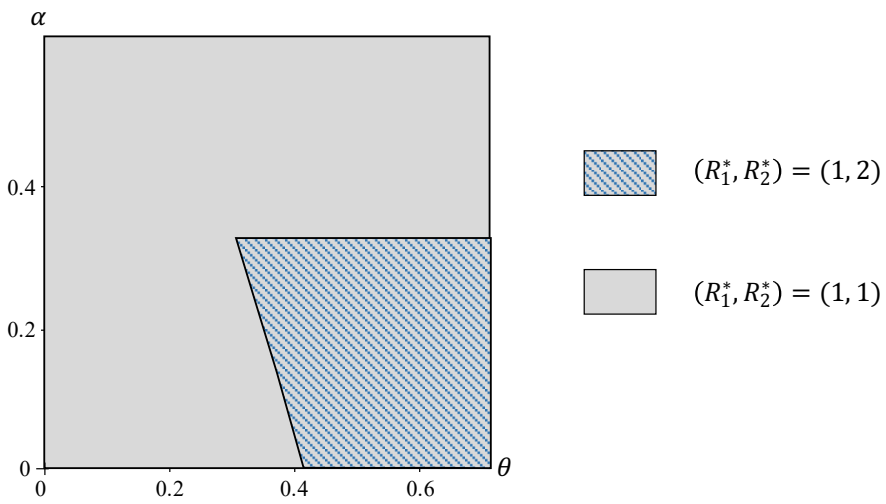
Differentiating $\pi_2^{(1,2)}$ with respect to α and evaluating at $\alpha = 0$, we find that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{23v-12v-9\theta\delta}{18(1+\delta)}$ when $\hat{\theta} < \theta < \bar{\theta}$, and $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} = \frac{\theta}{6}$ when $\bar{\theta} < \theta < \frac{V_2}{2}$. It is clear that if $\bar{\theta} < \theta < \frac{V_2}{2}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$. It is also easy to see that if $\hat{\theta} < \theta < \bar{\theta}$, then $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}|_{\alpha=0} > 0$ if and only if $\theta > \frac{12v}{23-9\delta}$. Note that $\hat{\theta}|_{\alpha=0} = \frac{12v}{23-9\delta}$. By continuity of $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha}$, it follows that there exists $\tilde{\alpha} \in (0, \hat{\alpha}]$ such that $\frac{\partial \pi_2^{(1,2)}}{\partial \alpha} > 0$ if $\alpha < \tilde{\alpha}$ and $\theta > \hat{\theta}$. ■

[29] To see that $\frac{d^2\Psi}{d\alpha^2} < 0$, note that $11 - 3\delta > 0$ and $6v - 5t(1-\delta) > 0$ because $\theta < \frac{V_2}{2}$.

[30] This is easily shown using the fact that $\theta < \frac{V_2}{2} < \frac{v}{2(1-\delta)}$, where the last inequality follows from taking $T \to \infty$.

Figure B4 shows the firms' equilibrium product release times in the (θ, α) parameter space.

Figure B4 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [31]



7.B. $V_{it} = \sum_{j=t}^{T} \delta^{j-t}(v - \theta d_i)$

In this subsection, we consider the scenario where consumers' horizontal preferences influence the consumption value of the durable product in each use period. Purchasing the firm i's product in period t gives the consumer a utility of $V_{it} - p_{it}$, where $V_{it} = \sum_{j=t}^{T} \delta^{j-t}(v - \theta d_i)$. To ensure full market coverage, we focus on θ sufficiently small: $\theta < \frac{v}{2}$.

***Subgame with* $(R_1, R_2) = (1, 1)$ or $(R_1, R_2) = (2, 2)$**

First, consider the subgame with $(R_1, R_2) = (2, 2)$. Consumers who enter the market in the first period wait till the second period to buy a product. Hence, in the second period, firms compete for a unit measure of consumers. A consumer who is located at a distance d from firm 1 will buy firm 1's product if and only if

$$V_{12} - p_{12} > V_{22} - p_{22} \tag{B87}$$

[31] Figure B4 is drawn using the following parameter values: $v = 0.5$, $\delta = 0.95$ and $T = 4$. To ensure full market coverage, we focus on $\theta < \frac{V_2}{2} \approx 0.71$.

Using the expression for V_{12} and V_{22} and simplifying the above inequality, we find that a consumer will buy from firm 1 if and only if $d \leq \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)}$. Hence, the firms' profits are given by $\pi_{12} = \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)}p_{12}$ and $\pi_{22} = (1 - \frac{\theta(\delta-\delta^T)-p_{12}(1-\delta)\delta+p_{22}(1-\delta)\delta}{2\theta(\delta-\delta^T)})p_{22}$. Solving the first-order conditions, we find the firms' second-period equilibrium prices: $p_{12}^{(2,2)} = p_{22}^{(2,2)} = \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta}$. The firms' equilibrium profits are

$$\pi_1^{(2,2)} = \pi_2^{(2,2)} = \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} \tag{B88}$$

The analysis of the subgame with $(R_1, R_2) = (1, 1)$ is similar. Specifically, one can show that firm i's equilibrium price are given by $p_{i1}^{(1,1)} = p_{i2}^{(1,1)} = \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta}$, with corresponding profits of $\pi_{i1}^{(1,1)} = \alpha\frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta}$ and $\pi_{i2}^{(1,1)} = (1-\alpha)\frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta}$ in the first and second periods, respectively. Thus, the firms' overall profits in the $(R_1, R_2) = (1, 1)$ subgame are given by

$$\pi_1^{(1,1)} = \pi_2^{(1,1)} = \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} \tag{B89}$$

Subgame with $(\boldsymbol{R_1, R_2}) = (\mathbf{1, 2})$

We solve the game by backward induction, starting with the second period. In the second period, the firms compete for the $(1-\alpha)$ new consumers entering the market, as well as any first-period consumers who decided not to buy a product in the first period. Hence, to find the firms' second-period subgame equilibrium prices, we first need to characterize the segment of first-period consumers who wait till the second period to buy. Namely, in the first period, a consumer who is located at a distance d from firm 1 will prefer to wait till the second period to purchase a product if and only if

$$\delta \max\{V_{12} - p_{12}^e, V_{22} - p_{22}^e\} > V_{11} - p_{11} \tag{B90}$$

Using the expressions for V_{11}, V_{12}, and V_{22}, we find that (B90) is equivalent to

$$d > \hat{d} \tag{B91}$$

where

$\hat{d} \equiv$

$$\begin{cases} 1 & \text{if } p_{11} \leq \frac{(1-\delta)(v+p_{22}^e\delta)-\theta(1-\delta^T)}{1-\delta} \\ \frac{(v-p_{11})(1-\delta)+p_{22}^e(\delta-\delta^2)+\delta\theta-\theta\delta^T}{\theta(1+\delta-2\delta^T)} & \text{if } \frac{(1-\delta)(v+p_{22}^e\delta)-\theta(1-\delta^T)}{1-\delta} < p_{11} \leq \frac{(2v-\theta)(\delta-\delta^T)-p_{22}^e(1-\delta)\delta+p_{12}^e\delta(1+\delta-2\delta^T)}{2(\delta-\delta^T)} \\ \frac{v-p_{11}+p_{12}^e\delta}{\theta} & \text{if } \frac{(2v-\theta)(\delta-\delta^T)-p_{22}^e(1-\delta)\delta+p_{12}^e\delta(1+\delta-2\delta^T)}{2(\delta-\delta^T)} < p_{11} < v+\delta p_{12}^e \\ 0 & \text{if } p_{11} \geq v+\delta p_{12}^e \end{cases}$$

We can now solve for the second-period subgame equilibrium prices that satisfy the rational expectations condition. Since $\hat{d}$ takes different functional forms depending on the interval to which p_{11} belongs, we will separately analysis each of the four possible cases. The analysis is very similar to the cases *i-iv* in the proof of Lemma 2. To avoid repetitive analysis, we will directly provide the second-period subgame equilibrium prices. Specifically, one can show that if the following condition holds,

$$v+\frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^T)}{(6-7\alpha+3\alpha^2)(1-\delta)} \geq v+\frac{\theta(3(3-\alpha)\delta-3-3(2-\alpha)\delta^T)}{(6+\alpha)(1-\delta)} \tag{B92}$$

then the prices in (B93) and (B94) constitute a rational-expectations subgame equilibrium:

$$p_{12}^{(1,2)} = p_{12}^e = \begin{cases} \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta} & \text{if } p_{11} \in [0,\underline{\rho}) \cup [\bar{\rho},\infty] \\ \frac{(\delta-\delta^T)(\theta(3-\alpha)(1+\delta-2\delta^T)-2(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho},\tilde{\rho}] \\ \frac{(3\theta-4(v-p_{11})\alpha)(\delta-\delta^T)}{\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho},\bar{\rho}] \end{cases} \tag{B93}$$

$$p_{22}^{(1,2)} = p_{22}^e = \begin{cases} \frac{\theta(\delta-\delta^T)}{(1-\delta)\delta} & \text{if } p_{11} \in [0,\underline{\rho}) \cup [\bar{\rho},\infty] \\ \frac{(\delta-\delta^T)(\theta(3+\alpha+3\delta-3\alpha\delta-2(3-\alpha)\delta^T)-4(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta+2(-3+\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho},\tilde{\rho}] \\ \frac{(\delta-\delta^T)(\theta(3-(3-2\alpha)\delta-2\alpha\delta^T)-2(v-p_{11})\alpha(1-\delta))}{(1-\delta)\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho},\bar{\rho}] \end{cases} \tag{B94}$$

where $\underline{\rho} = v-\theta$, $\tilde{\rho} = v+\frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^T)}{(6-7\alpha+3\alpha^2)(1-\delta)}$, and $\bar{\rho} = v+\frac{\theta(\delta-\delta^T)}{1-\delta}$. The condition (B92) is akin to the existence condition that we found in Lemma 3 in the main model. (B92) is equivalent to $\bar{\delta} < \delta \leq 1$ and $0 < \alpha < \bar{\alpha}$, where $\bar{\alpha} = \frac{45\delta-10-35\delta^T-\sqrt{100-108\delta+441\delta^2-92\delta^T+433\delta^{2T}-774\delta^{1+T}}}{22(\delta-\delta^T)}$ and $\bar{\delta} \geq \frac{1}{2}$ is implicitly defined from the equation $\bar{\alpha} = 0$.

Using the prices in (B93)-(B94), we obtain the firms' second-period subgame equilibrium profits:

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\delta-\delta^T)(\theta(3-\alpha)(1+\delta-2\delta^T)-2(v-p_{11})\alpha(1-\delta))^2}{2\theta(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(3\theta+4(p_{11}-v)\alpha)^2(1-\delta)(\delta-\delta^T)}{2\theta\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B95)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{(1-\alpha)\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{(1-\alpha)(\delta-\delta^T)(\theta(3+\alpha+3\delta-3\alpha\delta-2(3-\alpha)\delta^T)-4(v-p_{11})\alpha(1-\delta))^2}{2\theta(1-\delta)\delta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)^2} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{(\delta-\delta^T)(2(v-\mathrm{p}_{11})\alpha(1-\delta)-\theta(3-(3-2\alpha)\delta-2\alpha\delta^T))^2}{2\theta(1-\delta)\delta(3-(3-4\alpha)\delta-4\alpha\delta^T)^2} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ \frac{\theta(\delta-\delta^T)}{2(1-\delta)\delta} & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B96)}$$

Firm 1's first-period profit is given by $\pi_{11} = \alpha\, \hat{d}\, p_{11}$. Plugging the prices from (B93)-(B94) into the expression for $\hat{d}$, we can obtain π_{11}:

$$\pi_{11} = \begin{cases} \alpha\, p_{11} & \text{if } p_{11} \in [0, \underline{\rho}) \\ \frac{p_{11}\alpha(3v(1-\alpha-\delta+\alpha\delta)+2\theta(3-\alpha)(\delta-\delta^T)-3p_{11}(1-\alpha)(1-\delta))}{\theta(3-3\alpha+3\delta+\alpha\delta-2(3-\alpha)\delta^T)} & \text{if } p_{11} \in [\underline{\rho}, \tilde{\rho}] \\ \frac{3p_{11}\alpha((v-p_{11})(1-\delta)+\theta(\delta-\delta^T))}{\theta(3-(3-4\alpha)\delta-4\alpha\delta^T)} & \text{if } p_{11} \in (\tilde{\rho}, \bar{\rho}] \\ 0 & \text{if } p_{11} \in [\bar{\rho}, \infty) \end{cases} \quad \text{(B97)}$$

In the first period, firm 1 chooses its price to maximize its overall profit $\pi_1 = \pi_{11} + \pi_{12}^{(1,2)}$. To simplify the analysis, we assume that $T = 5$.[32] Similar analysis as in the proof of Lemma 3 shows that firm 1's optimal price belongs to the interval $[\underline{\rho}, \tilde{\rho}]$ and given by:

$$p_{11}^{(1,2)} = \begin{cases} v - \theta & \text{if } 0 < \theta \leq \underline{\theta} \\ \frac{v(9-13\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(5+\delta(5+\delta(5+3\delta))))-G}{2(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \leq \bar{\theta} \\ v + \frac{\theta(3-\alpha)((3-2\alpha)\delta-1-2(1-\alpha)\delta^5)}{(6-7\alpha+3\alpha^2)(1-\delta)} & \text{if } \bar{\theta} < \theta \leq \frac{v}{2} \end{cases} \quad \text{(B98)}$$

where $G = \frac{2\theta(3-\alpha)(1+\delta)(1+\delta^2)(\alpha-(1+3\delta)(1+2\delta(1+\delta)(1+\delta^2))+\alpha\delta(5+2\delta(2+\delta(2+\delta(2+\delta)))))}{1-\alpha}$,

$\underline{\theta} = \frac{3v(1-\alpha)}{8(1-\alpha)-2\alpha\delta(2+\delta(2+\delta(2+\delta)))+2\delta(4+\delta(4+\delta(4+3\delta)))}$ and

$\bar{\theta} = \frac{3v(1-\alpha)(6-\alpha(7-3\alpha))}{2(3-\alpha)(5-6\alpha+\alpha^2(1+\delta)(1-3\delta)(1+\delta^2)+2\delta(1+\delta+\delta^2)+\alpha\delta(2+\delta(2+\delta(2+5\delta))))}$.

[32] We have conducted the analysis using different values of $T \geq 3$ and results have been very similar. We provide the analysis for $T = 5$ as a representative case.

By plugging $p_{11}^{(12)} \in [\underline{\rho}, \tilde{\rho}]$ into $p_{12}^{(1,2)}$ and $p_{22}^{(1,2)}$ from (B93) and (B94), we can find the firms' second-period prices on the equilibrium path. Specifically,

$$p_{12}^{(1,2)} = \begin{cases} \theta(1+\delta)(1+\delta^2) & \text{if } 0 < \theta \le \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(\theta(3-\alpha)(3+6\delta(1+\delta)(1+\delta^2)-\alpha(3+4\delta(1+\delta)(1+\delta^2)))-3v(1-\alpha)\alpha)}{(1-\alpha)(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \le \bar{\theta} \\ \frac{\theta(6-5\alpha+\alpha^2)(1+\delta+\delta^2+\delta^3)}{6-7\alpha+3\alpha^2} & \text{if } \bar{\theta} < \theta \le \frac{v}{2} \end{cases} \quad \text{(B99)}$$

$$p_{22}^{(1,2)} = \begin{cases} \theta(1+\delta)(1+\delta^2) & \text{if } 0 < \theta \le \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta)))))-6v(1-\alpha)\alpha)}{(1-\alpha)(9-11\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(4+\delta(4+\delta(4+3\delta))))} & \text{if } \underline{\theta} < \theta \le \bar{\theta} \\ \frac{\theta(6-3\alpha-\alpha^2)(1+\delta+\delta^2+\delta^3)}{6-7\alpha+3\alpha^2} & \text{if } \bar{\theta} < \theta \le \frac{v}{2} \end{cases} \quad \text{(B100)}$$

Next, using the equilibrium prices, we can obtain the firm's equilibrium profits in each period. Specifically,

$$\pi_{11}^{(1,2)} = \begin{cases} \alpha(v-\theta) & \text{if } \theta \le \underline{\theta} \\ \frac{\alpha(9v(1-\alpha)-2\theta(3-\alpha)(1+\delta)(1-3\delta)(1+\delta^2))(K-L)}{4\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{(3-\alpha)\alpha(v(6-\alpha(7-3\alpha))-\theta(3-\alpha)(1-2(1-\alpha)\delta(1+\delta)(1+\delta^2)))}{(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{v}{2} \end{cases} \quad \text{(B101)}$$

$$\pi_{12}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)(1+\delta)(1+\delta^2)}{2} & \text{if } \theta \le \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(3v(1-\alpha)\alpha-\theta(3-\alpha)(3+6\delta(1+\delta)(1+\delta^2)-\alpha(3+4\delta(1+\delta)(1+\delta^2))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(3-\alpha)^2(2-\alpha)^2(1-\alpha)(1+\delta)(1+\delta^2)}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{v}{2} \end{cases} \quad \text{(B102)}$$

$$\pi_{21}^{(1,2)} = 0 \quad \text{(B103)}$$

$$\pi_{22}^{(1,2)} = \begin{cases} \frac{\theta(1-\alpha)(1+\delta)(1+\delta^2)}{2} & \text{if } \theta \le \underline{\theta} \\ \frac{(1+\delta)(1+\delta^2)(6v(1-\alpha)\alpha-\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta))))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} & \text{if } \underline{\theta} < \theta < \bar{\theta} \\ \frac{\theta(1-\alpha)(6-\alpha(3+\alpha))^2(1+\delta)(1+\delta^2)}{2(6-\alpha(7-3\alpha))^2} & \text{if } \bar{\theta} \le \theta < \frac{v}{2} \end{cases} \quad \text{(B104)}$$

where $K = \frac{v(1-\alpha)(9-13\alpha+18\delta(1+\delta)(1+\delta^2)-2\alpha\delta(5+\delta(5+\delta(5+3\delta))))}{1-\alpha}$ and

$L = \frac{2\theta(3-\alpha)(1+\delta)(1+\delta^2)(\alpha-(1+3\delta)(1+2\delta(1+\delta)(1+\delta^2))+\alpha\delta(5+2\delta(2+\delta(2+\delta(2+\delta)))))}{1-\alpha}$. Note that since the firms are symmetric, when $(R_1, R_2) = (2,1)$, the firms' profits are reversed, i.e., $\pi_{1t}^{(2,1)} = \pi_{2t}^{(1,2)}$ and $\pi_{2t}^{(2,1)} = \pi_{1t}^{(1,2)}$.

A similar proof as in Proposition 1 shows that a pure strategy equilibrium in product release times exists and in equilibrium, at least one of the firms will release its product in the first period. Hence, in

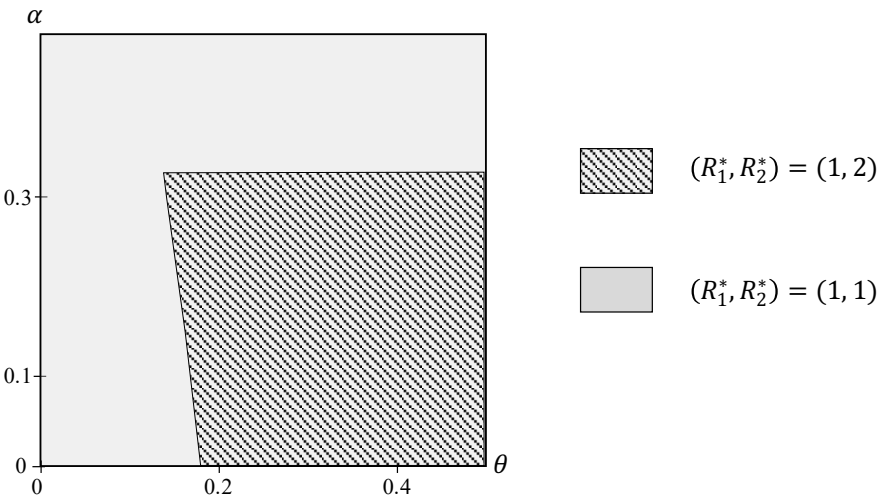
equilibrium, either $(R_1, R_2) = (1, 1)$ or $(R_1, R_2) = (1, 2)$. Let us show that there exist $\hat{\alpha} \leq \bar{\alpha}$ and $\hat{\theta} < \frac{v}{2}$ such that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $(R_1, R_2) = (1, 2)$ is an equilibrium.

To prove that $(R_1, R_2) = (1, 2)$ is an equilibrium, we need to demonstrate that neither firm has a profitable deviation. If $R_2 = 2$, then firm 1 will not want to deviate from $R_1 = 1$. Hence, we need to show that firm 2 will not want to deviate from $R_2 = 2$, which is the case when $\pi_2^{(1,2)} > \pi_2^{(1,1)}$. The technical analysis is very similar to that in the proof of Proposition 1. Specifically, one can show that if $\alpha < \hat{\alpha}$ and $\theta > \hat{\theta}$, then $\pi_2^{(1,2)} > \pi_2^{(1,1)}$, where $\hat{\alpha} \approx 0.32$ and

$$\hat{\theta} = \frac{6v((1-\alpha)^{3/2}(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))+(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta)))))}{\alpha^3(5-2\delta^4)^2+9(1+2\delta(1+\delta)(1+\delta^2))(23+2\delta(7+\delta(7+\delta(7+3\delta))))-3\alpha(131+4\delta(68+\delta(89+\delta(110+3\delta(40+\delta(22+\delta(3+\delta)(5+\delta)))))))+M}$$

and $M = \alpha^2(161 + 4\delta(69 + \delta(85 + \delta(101 + \delta(107 + \delta(46 + \delta(30 + (14 - 3\delta)\delta)))))))$. Note that $\underline{\theta} < \hat{\theta} < \bar{\theta}$ and satisfies $\frac{(1+\delta)(1+\delta^2)(6v(1-\alpha)\alpha-\theta(9+18\delta(1+\delta)(1+\delta^2)-\alpha^2(5-2\delta^4)-2\alpha(2+\delta(5+\delta(5+\delta(5+6\delta))))))^2}{2\theta(1-\alpha)(9+18\delta(1+\delta)(1+\delta^2)-\alpha(11+2\delta(4+\delta(4+\delta(4+3\delta)))))^2} = \frac{\theta(\delta-\delta^5)}{2(1-\delta)\delta}$ (i.e., $\pi_2^{(1,2)} = \pi_2^{(1,1)}$). Figure B5 graphically illustrates the region where $(R_1^*, R_2^*) = (1, 2)$.

Figure B5 Firms' Equilibrium Product Release Times (R_1^*, R_2^*) [33]



[33] To obtain Figure B5, the following numerical values are used: $v = 1$, $\delta = 0.95$, and $T = 5$.