Universal continuous-variable state orthogonalizer and qubit generator

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We experimentally demonstrate a universal strategy for producing a quantum state that is orthogonal to an arbitrary, infinite-dimensional, pure input one, even if only a limited amount of information about the latter is available. Arbitrary coherent superpositions of the two mutually orthogonal states are then produced by a simple change in the experimental parameters. We use input coherent states of light to illustrate two variations of the method. However, we show that the scheme works equally well for arbitrary input fields and constitutes a universal procedure, which may thus prove a useful building block for quantum state engineering and quantum information processing with continuous-variable qubits.

In the quantum mechanical context, states $|\psi\rangle$ and $|\psi_\perp\rangle$ are said to be orthogonal when the overlap between the two state vectors is zero, *i.e.*, $\langle\psi|\psi_\perp\rangle = 0$. Two states, which are orthogonal to each other, are then maximally discernible [1, 2]. In a classical binary system, states denoted by 0 and 1 are orthogonal and perfectly distinct to each other, and the NOT operation will switch between them. In quantum mechanics, a simple swap of 0 and 1 does not bring one state to its orthogonal state, due to the superposition principle. As an extreme case, a superposition state of $1/\sqrt{2}(|0\rangle + |1\rangle)$ is unchanged by such a swap operation. The universal quantum NOT operation, which is defined as an operation to bring a state to its orthogonal one, is indeed not possible without some prior knowledge of the state [3], just like it is impossible to perfectly and deterministically clone or amplify a quantum state [4, 5] without prior information. An orthogonalizer with minimal information required was demonstrated for states living in a limited two-dimensional Hilbert space [6, 7].

When a general quantum state in an infinite-dimensional Hilbert space is concerned, the situation is even more complicated because a given state will then have infinite orthogonal states. Despite this fundamental limitation, it has been recently proposed for an optomechanical system by Vanner et al. [8] that a perfect orthogonalizer can be in principle realized even if only some very limited preliminary information about the input state is available. One can generalize the proposal of [8] as follows: Given an arbitrary operator $\hat{C}$, as we know its mean value $\langle\hat{C}\rangle$ for the input state $|\psi\rangle$, the operation

$$\hat{O}_C \equiv \hat{C} - \langle\hat{C}\rangle\hat{\mathbb{1}} \tag{1}$$

converts the original state to an orthogonal state, where $\hat{\mathbb{1}}$ is the identity operator. It is straightforward to see that when $\hat{O}_C$ is applied to the input state, the resulting state $|\psi_\perp\rangle = \mathcal{N}\hat{O}_C(|\psi\rangle)$, where $\mathcal{N}$ is the normalization factor, is orthogonal to $|\psi\rangle$. This is equally applicable to pure states existing within finite or infinite Hilbert spaces, but it does not generally work for mixed states. Although the operator $\hat{C}$ can be in principle arbitrary, the above procedure cannot be applied if the input states are among its eigenstates because the success probability of the operation drops to zero [9].

Here we propose and demonstrate the generalized orthogonalization procedure of Eq.(1) to infinite-dimensional, continuous-variable (CV) states of light. In addition to this, we show that this approach also naturally leads to a method for producing arbitrary coherent superpositions of orthogonal quantum states out of any input pure one. For any complex number c, the (unnormalized) superposition

$$c|\psi\rangle + |\psi_\perp\rangle = (c\hat{\mathbb{1}} + \mathcal{N}\hat{O}_C)|\psi\rangle \tag{2}$$

$$= [\mathcal{N}\hat{C} + (c - \mathcal{N}\langle\hat{C}\rangle)\hat{\mathbb{1}}]|\psi\rangle \tag{3}$$

is realized by applying the same operation of Eq.(1) to the input state, but with an appropriate change in the weight of the identity operator. Therefore, once the orthogonalizer is in operation, any quantum superposition of $|\psi\rangle$ and $|\psi_\perp\rangle$, which constitutes a general arbitrary CV qubit, can also be straightforwardly realized.

In principle, a full tomographic reconstruction of a given input state would allow one to design a specific setup to generate the orthogonal one (and, possibly, a superposition of the two). However, such a strategy would be very inefficient and far from 'universal'. State tomography involves the measurement of observable probabilities for a large number of experimental settings, thereby requiring many identical copies of the input state for an accurate estimation after a numerical processing of the measured data sets. Furthermore, once each state is reconstructed, one should design an 'ad hoc' experimental scheme to generate the orthogonal one, and this might in general be far from trivial.

On the contrary, even if the measurement of the mean value of operator $\hat{C}$ in our scheme ideally requires a large number of identical copies of the input state, a measure of accuracy may be found. Specifically, when there are only N copies, the estimation of the mean value is accurate by $\Delta C/\sqrt{N}$, where ΔC is the standard deviation of

the measurement outcomes for the expectation value of the operator $\hat{C}$. Thus, even though we do not claim that this approach requires the least amount of resources, it is certainly much more resource-efficient than those based on a full reconstruction (see a discussion on the level of observation vs. the amount of information about a state in [10]). More importantly, a single 'universal' experimental apparatus, only depending on a single parameter (the mean value $\langle\hat{C}\rangle$) is necessary for our scheme to orthogonalize arbitrary states and produce arbitrary quantum superpositions. In the particular case of a set of input states for which $\langle\hat{C}\rangle$ is the same, no change at all is needed in the apparatus to process all the elements of the set.

Both the orthogonalizer and the procedure for producing coherent superpositions are general enough to work based on any operator $\hat{C}$ and with any pure input state $|\psi\rangle$. Choosing a particular operator implies designing the experimental setup accordingly, but a specific apparatus would then work for arbitrary input states, with the proper adjustment of its parameters. To concisely demonstrate the functioning of the above methods and their generality, in the following we present the experimental analysis of specific examples based on two different $\hat{C}$ operators (therefore implying two completely different experimental setups). We experimentally test both systems with one kind of input states, but we also show that they are able to deal equally well with arbitrary ones.

A particularly simple and interesting case is obtained when $\hat{C} = \hat{a}^\dagger$, the bosonic creation operator, which has no eigenstates and can thus be safely applied independently of the arbitrary state at the input. Here, one just needs to know the mean value of $\hat{a}^\dagger$ on the particular input state to construct the $\hat{O}_{a^\dagger} \equiv \hat{a}^\dagger - \langle\hat{a}^\dagger\rangle\hat{\mathbb{1}}$ orthogonalizer or a general superposition as prescribed by Eq.(3). Both these operations can be experimentally implemented by extending some of the tools recently developed in our group. In particular, the photon creation operator can be conditionally realized by means of stimulated parametric down-conversion (PDC) in a nonlinear crystal seeded by the optical input state in the signal mode [12–14]. The coherent superposition of this operation and the identity can be realized by mixing the (herald) idler PDC mode with a coherent light field on an unbalanced beamsplitter that erases the information about the origin of a click in the heralding single-photon detector at one of its outputs. By simply controlling both the relative phase between the input and the coherent state impinging on the beam-splitter, and the reflectivity of the latter, different superpositions of $\hat{a}^\dagger$ and $\hat{\mathbb{1}}$ can be obtained, in particular those corresponding to the orthogonalizer and to arbitrary CV superposition states. A simplified scheme of the experimental setup is shown in Fig.1a (also see [9]). Similar techniques, involving phase-space displacement on the herald mode of conditional state generation,

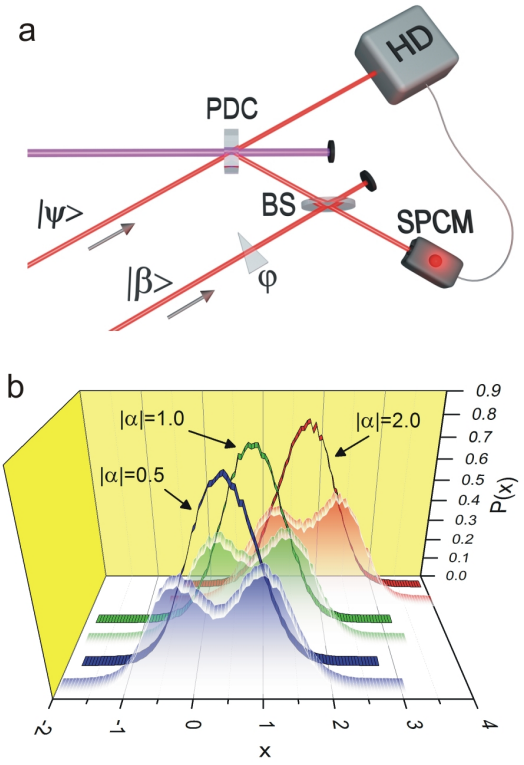

FIG. 1: (color online) State orthogonalizer and CV qubit generator based on the photon creation operator $\hat{a}^\dagger$. a) Conceptual experimental scheme of the orthogonalizer and CV qubit generator based on photon addition by heralded stimulated PDC. A click in the single-photon-counting module (SPCM) normally heralds a single photon addition to the generic input $|\psi\rangle$ state. However, if the PDC idler mode is mixed with a coherent state $|\beta\rangle$ on a beam-splitter (BS) prior to detection, a superposition of the photon creation operator and the identity with adjustable weights and phases can be obtained. In the actual experiments we used coherent states $|\alpha\rangle$ as the input states and the operator superposition was implemented by using polarization modes (see Supplemental Material [9] for details). HD is a time-domain homodyne detector triggered by SPCM clicks [11]. b) Raw measured x quadrature distributions (marginals of the Wigner function) for the input coherent states (ribbon-style curves) and for the corresponding results of the orthogonalization procedure (shaded areas) with $|\alpha| = 0.5, 1.0, 2.0$.

have been recently used for quantum state engineering up to two photons [15], and for the generation of optical CV qubits made of superpositions of squeezed vacuum and squeezed single-photon states [16].

We tested the concept presented above by using coherent states $|\alpha\rangle = \hat{D}(\alpha)|0\rangle$ as the input (where $\hat{D}(\alpha)$ is the displacement operator [17]). In this special case, the orthogonalizer operator based on photon creation becomes

$$\hat{O}_{a^\dagger}(\alpha) = \hat{a}^\dagger - \alpha^*\hat{\mathbb{1}}, \tag{4}$$

and it is easy to see that, when applied to $|\alpha\rangle$, this results in the displaced Fock state $\hat{D}(\alpha)|1\rangle$, which is clearly

orthogonal to $|\alpha\rangle$ [9]. Fig.1b illustrates the result of the application of the orthogonalizer to coherent states of different initial amplitudes. The x quadrature distributions show that the Wigner functions of the orthogonal states are differently-displaced versions of a single-photon Wigner function [18]. A full tomography of the input and output states for the case with $|\alpha| = 1.0$ results in a mutual fidelity of 0.4. The discrepancy from an ideal value of zero comes from different sources of experimental imperfections that limit the purity of the prepared and measured states and, as a consequence, their orthogonality (a detailed discussion is presented in the Supplemental Material [9]).

A simple adjustment of the parameters in the beam-splitter placed in the idler mode allows one to produce various CV qubit states. In Fig.2 we show the measured Wigner functions for different equal-weight superposition states of an input coherent state with $|\alpha| = 1.0$ and its orthogonal state. In the different plots, the phase of the resulting CV qubit is simply varied by properly controlling the relative phase between the input and the displacement coherent states and the reflectivity of the idler beam-splitter. The fidelities of the reconstructed states to the ideal superpositions of $|\alpha\rangle$ and $|\alpha_\perp\rangle$ are all quite large, and of the order of 90%.

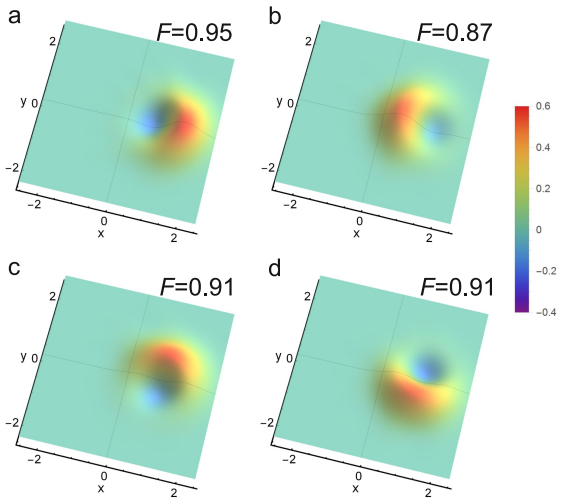

FIG. 2: (color online) Wigner functions of arbitrary CV qubit states. Wigner functions for different balanced superpositions of states $|\alpha\rangle$ and $|\alpha_\perp\rangle$ with $|\alpha| = 1.0$, as reconstructed by correcting for a detection efficiency of 70%. a) and b) correspond to states $1/\sqrt{2}(|\alpha\rangle \pm |\alpha_\perp\rangle)$, c) and d) correspond to states $1/\sqrt{2}(|\alpha\rangle \pm i|\alpha_\perp\rangle)$, respectively. The experimental fidelities F of the generated states to the ideal CV superpositions are also shown. Different values of F are mainly connected to the different level of stability in the experimental superposition phase for the different states.

In order to demonstrate the effectiveness and generality of the proposed approach, we now consider another scheme to realize the orthogonalizer. Since the mean number of photons in a state is a parameter often easy to determine experimentally, one may insert such a mean photon number $\bar{n}$ and the number operator $\hat{n} \equiv \hat{a}^\dagger \hat{a}$ into Eq.(1), which thus becomes

$$\hat{O}_n \equiv \hat{n} - \bar{n}\hat{\mathbb{1}}. \tag{5}$$

We put this scheme to an experimental test with the setup of Fig.3a, which is similar to the one first developed for testing the bosonic commutation relation [19] (see the Supplemental Material [9] for a more detailed description).

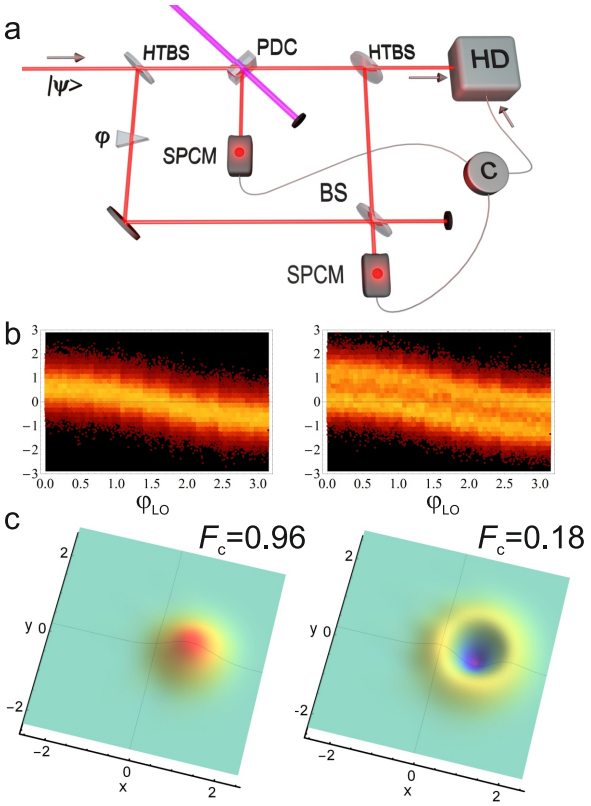

FIG. 3: (color online) State orthogonalizer based on the photon number operator $\hat{n}$. a) Conceptual experimental scheme for the orthogonalizer based on the photon number operator. HTBS are high-transmittivity beam-splitters, C is a coincidence logic circuit. b) Experimental homodyne detection traces for the original input coherent state $|\alpha\rangle$ (left panel) and for the state obtained by the orthogonalization procedure (right panel). The input amplitude was $|\alpha| = 0.78$, and 10 different values of the local oscillator phases were used. c) Wigner functions of the input coherent state and of its orthogonal (a displaced single-photon Fock state) as reconstructed from the homodyne data after correcting for the limited (70%) detection efficiency.

Such a setup conditionally produces the arbitrary superposition of operators

$$A\hat{a}^\dagger \hat{a} + B\hat{a}\hat{a}^\dagger, \tag{6}$$

which is seen to be proportional to $\hat{n} + \frac{B}{A+B}\hat{\mathbb{1}}$ using the bosonic commutation relation. Therefore, the generic orthogonal state to one of mean photon number $\bar{n}$ can be straightforwardly implemented by adjusting the setup of Fig.3a so that $\frac{B}{A+B} = -\bar{n}$. It is seen that, when used in combination to input coherent states $|\alpha\rangle$, this scheme results in the same orthogonal state as for the previous example. Note that we do not assume any knowledge about the initial state except its mean photon number. Thus, simply choosing another coherent state $|\alpha'\rangle$ as the orthogonal one, is not a valid alternative. In any case, the overlap of two coherent states $|\langle\alpha|\alpha'\rangle|^2 = e^{-|\alpha-\alpha'|^2}$ is never zero, especially in a regime of low intensities. Figs.3b and c show the measured quadrature distributions and the reconstructed Wigner functions for such input and orthogonal output states, whose measured mutual fidelity is $F = 0.34$, while fidelities F_c to an ideal coherent state $|\alpha = 0.78\rangle$ are 0.96 and 0.18, respectively (see the Supplemental Material [9] for more details).

To summarize, we have shown the first experimental application of a universal orthogonalization procedure to arbitrary CV optical states. Relying on a very limited amount of preliminary information about the input states, we verified the effectiveness and generality of this powerful technique through two illustrative examples based on the photon creation ($\hat{a}^\dagger$) and number ($\hat{n}$) operators. Simple modifications in the experimental parameters also allowed us to produce CV qubits based on the superposition of an arbitrary input state $|\psi\rangle$ and its orthogonal $|\psi_\perp\rangle$.

Note that our goal is not that of producing a specific quantum state, which could be prepared with simpler methods anyhow, but rather to demonstrate a universal scheme for producing orthogonal and CV qubit states starting from arbitrary inputs. Coherent states were used here only for their ease of preparation. Adjusting a single parameter (for example the reflectivity of a beamsplitter) to accommodate for a different $\langle\hat{C}\rangle$ in the chosen experimental setup is sufficient for orthogonalizing and creating quantum superpositions out of any input pure state.

To better illustrate this, we give in the following (theoretical) examples of the generation of orthogonal states using the same orthogonalizing operators $\hat{O}_{a^\dagger}$ and $\hat{O}_n$. We simulate the output orthogonal state using exactly the same schemes analyzed in our experiments for two other initial states which are representative cases of highly non-classical states: an even cat state, $|\alpha\rangle + |-\alpha\rangle$, and a squeezed vacuum $\hat{S}(\zeta)|0\rangle$ (normalization omitted in the following for simplicity), where the single-mode squeezing operator is given by

$$\hat{S}(\zeta) = e^{-\frac{\zeta}{2}\hat{a}^{\dagger 2} + \frac{\zeta^*}{2}\hat{a}^2}, \tag{7}$$

with $\zeta = re^{i\phi}$ is the squeezing parameter. Fig. 4 shows the theoretical simulation of the Wigner functions for the orthogonal states generated by our experimental setups. Note that, while the application of a generic orthogonalizing operator will always take any input state to its orthogonal version, in general it is not the case that different choices of the operator $\hat{C}$ used in the $\hat{O}_C$ orthogonalizer map to the same orthogonal state. This is evident from the Wigner function plots in the figure: while both the states in Fig. 4 ($a2$) and ($a3$) are orthogonal to the original cat state $|\alpha\rangle + |-\alpha\rangle$ shown in Fig. 4 ($a1$) (just as states in Fig. 4 ($b2$) and ($b3$) are orthogonal to the squeezed vacuum $\hat{S}(\zeta)|0\rangle$ of Fig. 4 ($b1$)), they are neither the same orthogonal state nor are they orthogonal to each other.

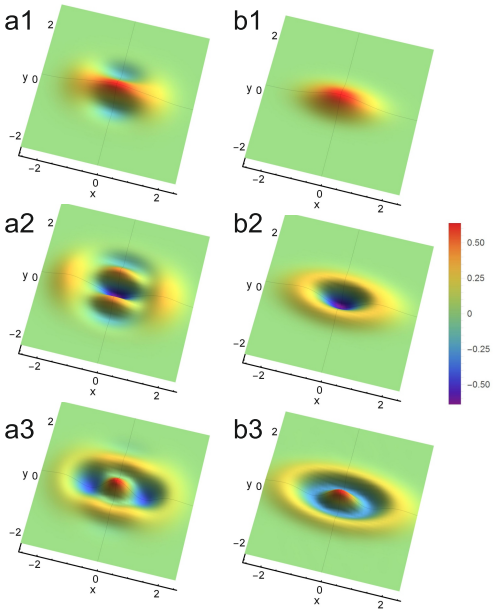

FIG. 4: (color online) Calculated Wigner functions of results of applying different orthogonalizing operations to cat and squeezed states. a1) $|\alpha\rangle + |-\alpha\rangle$, a2) $\hat{O}_{a^\dagger}(|\alpha\rangle + |-\alpha\rangle)$, a3) $\hat{O}_n(|\alpha\rangle + |-\alpha\rangle)$; b1) $\hat{S}(\zeta)|0\rangle$, b2) $\hat{O}_{a^\dagger}\hat{S}(\zeta)|0\rangle$, b3) $\hat{O}_n\hat{S}(\zeta)|0\rangle$. The colour scheme illustrates the value of the Wigner function at a given point, with the blue being most negative and red, most positive. For plots $a1-a3$ we take $\alpha = 1$, while for plots $b1-b3$ we take $\zeta = 0.4$. Here the normalization is omitted in the expressions, but the figures are properly normalized.

This technique can become a useful tool for quantum-state engineering, to produce custom-made quantum states. Future goals should include testing the system with different input states and extending its experimental application to larger intensity input fields. Beyond pure photonics, our scheme can also be applied to various physical systems including phononic states of ions in a trap and nanomechanical oscillators [8]. Even though our demonstration here is for relatively small-amplitude coherent states, the method itself can be applied to bigger systems, and can be thus used for the study of foundational issues, like the test of quantum-to-classical transition models [20].

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