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A Microwave Trap for Atoms and Molecules

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Abstract

Trapping particles is a key starting point for a range of physics experiments. The ability to confine, manipulate and interrogate a trapped species enables an invaluable degree of control and precision. Each trap offers different advantages and possibilities, and emerging experiments often demand different methods of trapping.

This thesis describes a new three-dimensional microwave trap for atoms and molecules. Like an optical dipole trap, it can trap ground state polar molecules, but its depth and volume are larger by factors of 10^3 and 10^9 respectively.

The experiment consists of a lithium oven, Zeeman slower, magneto-optical trap (MOT), a moving magnetic trap, and a high-quality Fabry-Perot cavity which serves as the microwave trap. Starting from a MOT of 1×10^8 ^7Li atoms, the atomic cloud is cooled from a temperature of around 1 mK to $41(1) \mu\text{K}$ with the recently developed Raman grey molasses method. Next, the population is optically pumped to a magnetically trappable state, and up to $85(2)\%$ are loaded into a quadrupole magnetic trap. These atoms are then transferred to a moving magnetic trap and translated 600 mm to the centre of the microwave cavity. After optimizing the transport to minimize atom heating and loss, we can produce samples of 2×10^7 atoms in the microwave chamber, at a phase space density of $3.58(15) \times 10^{-7}$. Finally, loading of the microwave trap is demonstrated and its basic properties studied. When the input power to the cavity is 610 W, the electric field amplitude at the centre reaches roughly 20 kV cm^{-1} . At this input power, centre-of-mass oscillation frequencies in the microwave trap were measured as $28.55(5) \text{ Hz}$ along the axis of the cavity, and $8.81(8) \text{ Hz}$ along the orthogonal direction. The observed lifetime was $1.76(12) \text{ s}$, limited by the vacuum quality. The atomic cloud expands and cools to $22(3) \mu\text{K}$ when loaded into the microwave trap, with a phase space density of $4(1) \times 10^{-7}$. This is consistent with adiabatic loading from the magnetic trap.

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Chapter 1

Introduction

In this chapter, I explain the opportunities presented by cold molecule science, which motivates the work of this thesis. I describe the experimental challenges associated with cooling molecules and concentrating a molecular ensemble into a small number of internal states. Several important strides have been made along these two avenues, and I review these to aid the reader in understanding the motivation and context of this work. I then discuss sympathetic cooling of molecules with atoms, and explain why this is challenging with current methods of trapping. Finally, I present a microwave resonator as a trap with unique advantages for multi-species trapping and sympathetic cooling.

1.1 Motivation: Physics with Cold Molecules

Cooling matter close to zero kelvin brings new physics into sharp focus. The true potential of low temperatures was first revealed in 1911, when Onnes observed superconductivity in mercury at 4.19 K by cooling it with liquid helium. This observation prompted a forty year journey culminating in the BCS theory of superconductivity, and an ongoing search to bring such behaviour to ambient temperature and pressure. Moreover, the stark differences between the two isotopes of helium at cryogenic temperatures is a fundamentally quantum mechanical phenomenon, resulting from their classification by intrinsic angular momentum into fermions (^3He) and bosons (^4He).

By cooling a physical system, we are decreasing its entropy¹; the accompanying increase in the system order is a valuable resource for doing physics. Within the context of cold atom traps, it is useful to introduce the concept of *phase-space density* as a direct measure of the entropy of the system. Each particle i has respective position and momentum coordinates $\mathbf{x}_i$ and $\mathbf{p}_i$; the combination of $\mathbf{x}_i$ and the wavevector $\mathbf{k}_i = \mathbf{p}_i/\hbar$ together define its coordinates in phase-space. The phase-space density of the ensemble of trapped particles, $\rho(\mathbf{x}, \mathbf{k})$, is the number of particles per unit volume of phase space. For an ideal gas at uniform density n and temperature T , the phase-space density in three dimensions is,

$$\rho = n\Lambda^3 \quad . \quad (1.1)$$

Here, $\Lambda = h/\sqrt{2\pi mk_B T}$ is the de Broglie wavelength for a typical particle at temperature T , and is called the thermal wavelength. For gas at ambient temperature and pressure, $\rho \sim 10^{-9}$. Increasing the phase-space density of the gas is equivalent to decreasing its entropy. This is appealing in much the same way a laser is appealing when compared to a light bulb: it allows details to emerge which are otherwise hidden. For example, when ρ reaches unity, the typical distance between particles has approached their thermal wavelength, and quantum statistics begins to influence the physics. Bringing samples to extremely low temperatures allows samples to remain dilute, which has the advantage of making them simpler to understand, and also provides a valuable increase in the time available for many experiments.

Although modern $^3\text{He}/^4\text{He}$ dilution refrigerators can reach temperatures of a few millikelvin, to access the microkelvin regime new methods were needed. The advent of laser cooling, first demonstrated by Bill Phillips and co-workers in 1982 [1], provided such a technique. Since then, a vast avenue of research into cooling and trapping atomic gases emerged, and opened the door to the nanokelvin regime. This enabled the dramatic demonstration of Bose-Einstein condensation [2], the archetypal quantum statistical phase transition, and studies of the superfluid-Mott insulator [3] and Berezinskii-Kosterlitz-Thouless [4] theories. As well as exposing new physics, ultracold temperatures facilitate exceptionally precise metrology. Ultracold atoms are competing to become the standard for measuring time and

¹More strictly, we transfer entropy out the system without decreasing the entropy of the universe.

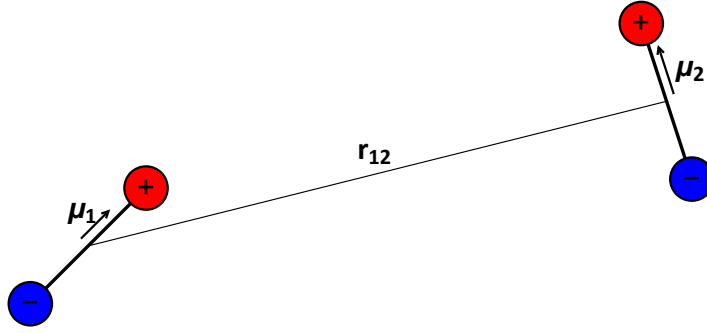


Figure 1.1: Two interacting electric dipoles.

frequency [5, 6], gravity [7], acceleration [8] and magnetic fields [9].

The transition from cold atoms to cold molecules is a major advance: molecules are systems with an internal structure whose complexity far exceeds that of an atom. The presence of rotational and vibrational degrees of freedom leads to a rich hierarchy of energy scales, and a vastly expanded number of internal states. This, in turn, gives rise to a wealth of new phenomena and applications to explore. In addition, a heteronuclear molecule usually has a far greater polarisability than an atom, due to its internal electric dipole moment, μ , that lies parallel to its internuclear axis. Two such molecules, with electric dipoles μ_1 and μ_2 whose centres are separated by r_{12} are shown in figure 1.1. Their interaction energy, U_{Int} , is given by,

$$U_{\text{Int}} = \frac{1}{4\pi\epsilon_0 r_{12}^3} \left[\mu_1 \cdot \mu_2 - \frac{3(\mu_1 \cdot r_{12})(\mu_2 \cdot r_{12})}{r_{12}^2} \right] . \quad (1.2)$$

Equation 1.2 shows that the dipole-dipole interaction is both long range ($\sim 1/r_{12}^3$), and anisotropic. Apart from ultracold molecules, these interactions have so far only been observable with ultracold gases of the highly magnetic atoms, or Rydberg atoms. In the former case, the interactions are still relatively weak, and in the latter case, the Rydberg states are relatively short-lived.

Interest in the field of cold molecules has rapidly expanded since the new millennium. As an example of this, figure 1.2 plots the number of articles published by Physical Review Letters featuring the keyword “ultracold molecules” from 1995 onwards. The sudden increase from 2001 was prompted in part by the successful association of cold atoms to form diatomic

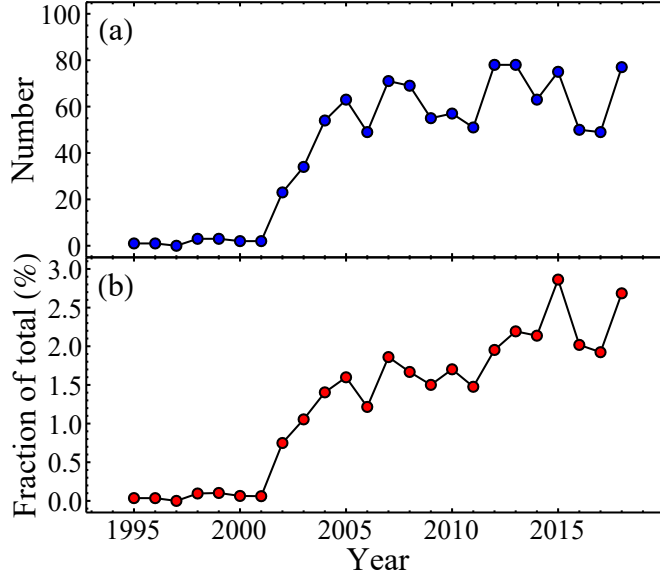


Figure 1.2: Physical Review Letter articles returned by a keyword search for “ultracold molecules” for each year between 1995 and 2018. (a) Number of articles. (b) Fraction of the total number of articles.

molecules, first by photoassociation of Cs_2 molecules in 1998 [10] and then by magnetoassociation in 2003 [11]. Since then, activity has remained strong, although the nature of cold molecule research has evolved. In the following, I summarise some research topics in which ultracold molecules are playing, or are expected to play, an important role.

Quantum Simulation

One of the most challenging areas of study in physics is the study of strongly-interacting, many-body quantum systems. These systems are often responsible for intriguing phenomena, such as superconductivity. However, most quantum many-body problems are impossible to solve analytically, and quickly become computationally impractical to simulate with current technology. It is hoped that by simulating such systems experimentally, we can gain important insights into their behaviour.

Polar molecules in particular possess many favourable properties for this application. Like atoms, molecules are a “clean” system: each molecule is identical and immune from much of the environmental inhomogeneity which complicates solid state experiments. Further, the dipole-dipole interaction between molecules is long range ($\sim 1/r^3$), anisotropic, and

readily tuneable with external fields. At a temperature $T = 1\text{ }\mu\text{K}$, the interaction between two dipoles of magnitude 5 D is equal to $k_B T$ at a separation of 560 nm , which is around the length scale of an optical lattice. There has been extensive theoretical work studying arrays of molecules confined in optical lattices [12], with predictions of exotic new phases of matter. Examples of these include the so-called supersolid phase in which there is both dissipationless flow and long-range order, and striped/checkerboard phases where the system takes on long range order to minimise repulsive interactions. In [13], it is shown that a variety of spin-lattice models can be engineered with high tuneability using polar molecules. For an extensive review on this subject, the reader is directed to [14].

Fundamental Physics

Experiments testing fundamental physics generally split into two categories. One approach is to collide particles together at extreme energies, breaking matter into its constituent parts. This has been the overriding approach of the high energy physics community, and gave rise to the enormous success of the Standard Model (SM). A complementary approach is to make measurements with matter at low energy, but with extremely high precision ². It turns out that certain molecules are an extremely sensitive testing ground for various theories, meaning we can push the limits of current understanding. Cooling molecules to the ultracold regime would improve the statistical sensitivity of many experiments, by increasing the interaction time for spectroscopy and reducing finite temperature effects.

One key area of study is symmetry breaking. An important shortcoming of the Standard Model is its failure to account for the degree of asymmetry between matter and antimatter in the observable universe. This implies violation of charge-conjugation and parity (CP) symmetry, at a level orders of magnitude above that allowed by the SM. Many extensions to the SM exist which attempt to remedy this problem, and they all predict much larger electric dipole moments (EDMs) of fundamental particles such as the electron. Attempts to measure the electron EDM (eEDM) initially focused on heavy atoms, where the relativistic motion of the electron makes a measurement possible ³. An experiment using ^{205}Tl constrained the

²An extensive review of this research area is given in [15]

³For a non-relativistic electron, an external electric field cannot produce a first order interaction with the eEDM because of Schiff's theorem [16].

eEDM as less than $1.6 \times 10^{-27} \text{ ecm}$, limited in part by electric field strengths reachable in the laboratory [17].

In heavy polar molecules, there is both a large internal electric field along the internuclear axis, and relativistic enhancement of the energy shift caused by the eEDM interacting with this field. When the molecular axis is aligned in the laboratory frame, one expects a non-zero eEDM to cause a difference in the spin precession frequency in a magnetic field applied parallel or anti-parallel to the internal electric field. This method was first demonstrated with YbF in our group [18], and a similar experiment using ThO [19] currently sets the most stringent limit on the eEDM, at $1.1 \times 10^{-29} \text{ ecm}$, which already rules out many of the most popular supersymmetric extensions to the SM.

In addition, many polyatomic molecules exhibit chirality, meaning they exist as two mirror images (enantiomers) which cannot be superposed onto one another. Looking for differences in the spectra of each enantiomer is a direct means of studying parity violation in the weak interaction, first observed in the radioactive β decay of ^{60}Co [20]. An ongoing collaboration hopes to achieve this aim with high resolution laser spectroscopy of organic transition metal complexes [21].

Another application exists in the search for varying fundamental constants. Many extensions to the SM predict that fundamental constants are actually functions of spacetime coordinates or the local matter density. The molecule CH, a radical present in the interstellar media, has energy levels termed Λ -doublets which are particularly sensitive to the value of the fine structure constant, α and the proton-electron mass ratio, m_p/m_e . By measuring transition frequencies in the laboratory with a cold CH source, comparisons with astronomical data have tested for differences in the observed values of α and m_p/m_e , [22]. A Hertz-level precision was achieved by Ramsey interferometry of the molecules, driving transitions in a microwave transmission line; an upper limit found for the fractional variation of α was $(0.3 \pm 1.1) \times 10^{-7}$, a record for an astronomical comparison at the time of publication. In another example, a vibrational transition in SF_6 molecules were used to constrain the fractional variation in m_p/m_e to $(-3.8 \pm 5.6) \times 10^{-14}/\text{year}$, by comparison with a Cs atomic clock. Astronomical observation of H_2 absorption lines has enabled a stringent limit of 5×10^{-6} on the variation of m_e/m_p across the universe, for look-back times of 10 to 12.4

billion years into cosmic history [23].

Cold Chemistry and Collisions

Chemical reaction studies are normally limited in resolution by thermal broadening, both in terms of the collision energy of interacting particles and in the population of their internal states. With cold molecules it is possible to control these parameters and therefore gain greater understanding of the chemistry of, for example, interstellar gas clouds. The development of crossed molecular beam experiments since the 1950s has enabled studies with tuneable collision energies in the 10 – 200K range, with resolutions of a few kelvin. More recently, a molecular synchrotron has allowed storage of packets of molecules in a ring trap [24, 25], which goes some way to improving the available time for collisions. Similar enhancement is also possible with merged beam experiments [26, 27, 28, 29].

Below temperatures of a few kelvin, it becomes possible to resolve the contribution of individual partial waves to scattering phenomena. This gives rise to low energy scattering resonances, such as the orbital “shape” resonances observed in collisions between O_2 and H_2 [30], and spin dependence of these resonances in He-H_2 collisions [27]. In addition, at millikelvin temperatures, one enters the regime in which the de-Broglie wavelength of a molecule exceeds the typical length scale for intermolecular forces. It is expected that quantum tunnelling and many-body collective effects will affect the chemical dynamics.

For an overview on this topic, the reader is referred to [31].

Quantum Information

When it comes to potential applications of quantum physics, there are few more desirable than a so-called quantum computer. There exist classical algorithms where the scaling of computation time with problem size is exponential, while the equivalent quantum algorithms show polynomial dependence. Implementing quantum algorithms will thus give rise to a significant increase in computational power, with potentially far reaching consequences for the transfer and processing of information.

There are many proposed architectures for a quantum computer currently under research, each with its own advantages and drawbacks. Ultracold molecules offer many long-lived

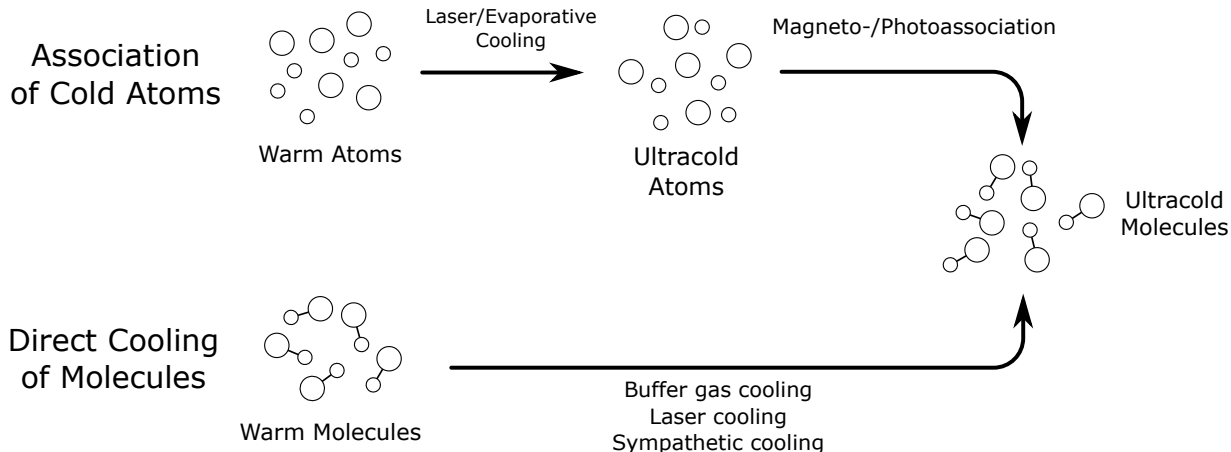


Figure 1.3: Methods for producing ultracold molecules.

internal states, and ease of interaction with external fields. A key paper by DeMille [32] describes how an optical lattice of dipolar molecules in an inhomogeneous electric field could be utilised in quantum information processing. By taking dipole alignment relative to the field as a qubit, the author describes a design capable of achieving 10^5 CNOT operations in 5 s. A more recent paper proposes the use of a self-assembled molecular dipole crystal as a processor, with gate operations performed by localised phonon coupling, achieved using a movable “marker” molecule which can address individual molecules [33].

1.2 Making Cold Molecules

The production of ultracold molecules can be split into two complementary methods, illustrated schematically in figure 1.3. The first is to associate ultracold atoms into molecules, by means of photo- or magnetoassociation; the second is to directly cool molecules to the ultracold regime.

Magnetoassociation is currently more successful in producing pure samples of ground state molecules at high phase-space density, and has been applied to various combinations of the alkali atoms which are readily laser coolable. Magnetoassociation relies on the phenomenon of Feshbach resonances, in which a loosely bound molecular state lies close in energy to an unbound state describing free atoms. If the two states are coupled and have different magnetic g-factors, then by tuning an external magnetic field one can traverse an

avoided crossing between the unbound and bound states. In this way, highly vibrationally excited molecules can be adiabatically formed from free atoms. To transfer molecules to the vibrational ground state, the technique of Stimulated Raman Adiabatic Passage (STIRAP) is used. This technique utilises a Raman transition to the ground state of the molecule, via an electronically excited state. By applying a “reverse” pulse sequence, one coherently populates the ground state without significantly populating the electronic excited state. Observed STIRAP efficiencies of better than 90% are possible [34, 35] and do not heat the molecular ensemble, so that molecules produced in this way are already ultracold. Typically sample sizes are $\sim 10^4$ molecules and temperatures are $\sim 1\text{ }\mu\text{K}$.

Producing molecules by association relies on a strong collaboration between theory and experiment, since this method normally requires accurate calculation of interspecies scattering potentials, which have to be calibrated by experiment. Since there are stringent requirements on the precursor gases, so far the method has been chiefly applied to alkali atom combinations. In most cases the molecules are not directly observable and so the STIRAP process must be reversed to make experimental observations possible.

Progress in directly cooling molecules has advanced rapidly in the last five years. Most experiments begin with a molecular beam, usually cooled using supersonic expansion from a pulsed source, or by cryogenic cooling with a carrier “buffer gas”. In a supersonic expansion, isentropic expansion of the gas molecules from high to low pressure converts internal energy into forward motion; this produces a spatially compact packet of molecules which is translationally and internally cold (typically around 1 K), but with a forward velocity of several hundred metres per second. Buffer gas beams [36], on the other hand, cool the molecules by collisions with a cold carrier gas. This reduces the forward velocity and internal temperature of the beam. Properties of buffer gas sources in particular are not fully understood and still the subject of active research.

Early experiments focussed on decelerating supersonic pulses with electric and magnetic fields, known as Stark and Zeeman deceleration respectively. The approach here is to use time-varying fields to ensure that molecules repeatedly climb potential energy hills, which reduces their forward kinetic energy. Multi-stage Stark deceleration was first implemented successfully for CO [37], and was later applied to a number of species, including CaF [38], and

LiH [39] amongst others; see ref. [40] for an exhaustive list. Multi-stage Zeeman deceleration was initially applied to H and D atoms [41], and has subsequently been demonstrated with O₂ [42] and metastable He₂ molecules [43]. More recently, traveling wave decelerators have been developed in which particles are decelerated in a single moving trap [44, 45]; this generally improves the phase-stability of the decelerator, meaning that a greater fraction of the initial distribution is successfully slowed. However, decelerating does not cool, and so the final temperature of the molecular sample is determined largely by the source; selecting low energy molecules rapidly diminishes the number available, so even with an intense molecular beam temperatures are ~ 1 K or above. Still, the versatility of decelerators means they are used as precursor loading stages in many experiments.

The coldest directly cooled molecules yet have been produced by laser cooling. It was initially thought that this method could never work for molecules, because scattering a large number of photons tends to populate other vibrational states of the electronic ground state which are not addressed by the cooling light. However, it was pointed out by Di Rosa in 2004 [46] that a subset of molecules have excited electronic states whose interatomic potentials have almost the same equilibrium separation and curvature as that of the ground state. For these molecules, optical pumping to other vibrational levels is highly suppressed, and by careful consideration of the selection rules for rotational angular momentum it is possible to scatter many photons. This realisation has led to three-dimensional molecular MOTs of SrF [47, 48], CaF [49], and YO [50], even reaching sub-Doppler temperatures in the case of CaF [51] and SrF [52]. One-dimensional cooling has been reported in beams of YbF [53, 54] and SrOH [55], and a recent paper [56] shows how polyatomics can be slowed and cooled by attaching laser-coolable functional groups. Unfortunately, many of these molecules are radicals, and must be produced in-situ via laser ablation. This adds an additional complication to the experiments, although understanding of these sources is constantly improving.

Another recent approach to cooling molecules has combined a static electric trap with an optoelectric Sisyphus effect, cooling trapped formaldehyde to 420 μ K [57]. In this experiment, the molecules are pumped into strongly trapped states in the uniform potential of a box-like static electric trap. When they reach the walls of the trap, they experience a strong

electric field gradient, which provides both a potential energy slope and changes the transition frequencies between the Stark shifted levels. Molecules with sufficient kinetic energy travel far enough along the electric field gradient that they are driven to more weakly trapped states with radio-frequency fields, removing some kinetic energy. By repeating this process many times, ensembles as large as 3×10^5 have been cooled, without reaching a fundamental limit. The pumping to strongly trapped states requires only a single infra-red laser and three microwave frequencies, driving transitions between the two lowest vibrational levels of the electronic ground state. Hence the problem of vibrational branching, the primary obstacle for laser cooling molecules, is neatly sidestepped. In practice, the cooling rate is limited by the spontaneous emission lifetime of the vibrationally excited state, which for most molecules is greater than 0.1 s^4 .

1.3 Project Aim: Sympathetic Cooling in a Microwave Trap

As we have seen, there are many reasons for producing samples of cold and ultracold molecules. However, it remains experimentally challenging to do this, and it is likely that new methods of cooling and trapping will be essential to broaden the range of ultracold species. One proposed method being explored in our group is sympathetic cooling of molecules with ultracold atoms, illustrated in figure 1.4. This approach aims to take advantage of the high phase-space densities available with cold atom technology⁵: by co-trapping two species and allowing them to thermalise, it should be possible to make ultracold molecules. In our project we aim to use lithium as the atomic refrigerant, motivated by two factors. First, the low atomic mass increases the angular momentum barrier for scattering events involving higher partial waves; this suppresses the likelihood of inelastic collisions which could eject molecules from the trap. Second, ultracold lithium is experimentally more convenient than the other low-mass coolants proposed, magnesium [58] and atomic hydrogen [59]. Whilst Mg has been laser cooled and trapped in a MOT [60], this requires ultraviolet light and the

⁴Formaldehyde has an anomalously short vibrational lifetime of about 0.017 s , which is an important advantage in the experiment.

⁵In particular, evaporative cooling which benefits greatly from a large initial number of atoms, at a high initial density.

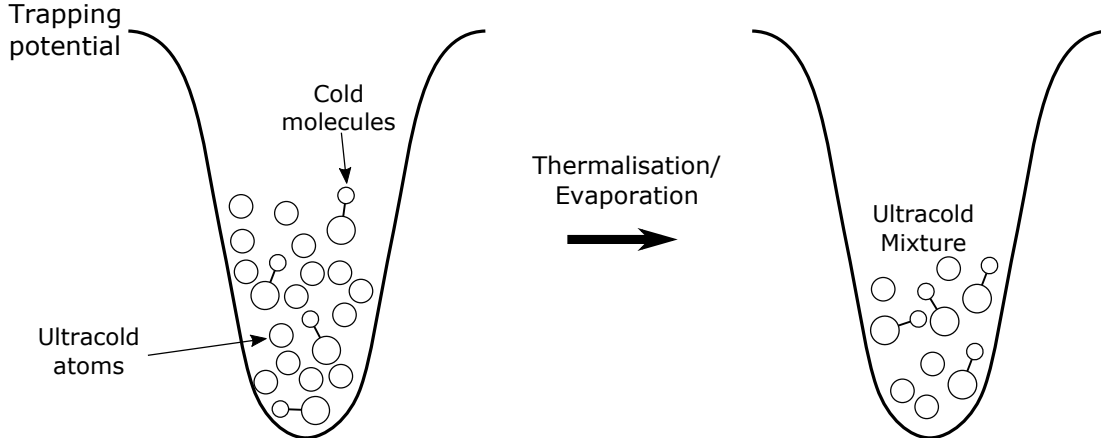


Figure 1.4: Sympathetic cooling of molecules with atoms.

number of atoms is small ($\sim 10^6 - 10^7$). Hydrogen, unlike magnesium, can be magnetically trapped, and has previously been evaporatively cooled to quantum degeneracy [61]. However, the small elastic scattering cross-section made this extremely challenging, and probing the atoms in-situ requires two-photon ultraviolet spectroscopy.

For sympathetic cooling to be possible, the trap must satisfy a number of criteria:

- The trap **must** confine atoms and molecules in the same region of space. This is to ensure collisions are possible.
- The trap **must** confine molecules in the rotational ground state, to suppress inelastic (state-changing) collisions which could transfer molecules to untrapped states.
- The trap **should** confine atoms in a way that is robust to atom-atom and atom-molecule collisions.
- The trap **should** be at least as large as the volume occupied by samples of cold atoms and molecules currently available.
- The trap **should** be as deep as possible for molecules, given that only a few species are available in the ultracold regime.
- The trap **should** allow straightforward loading of multiple species.

An important result for atomic and molecular traps is Earnshaw’s theorem, concerning the magnitude of the electric (or magnetic) field in free space. For a free space field maximum to exist, we require,

$$\nabla^2|\boldsymbol{\mathcal{E}}^2| = \nabla^2(\boldsymbol{\mathcal{E}} \cdot \boldsymbol{\mathcal{E}}) < 0 \quad . \quad (1.3)$$

From the rules of vector calculus, it can then be shown,

$$\nabla^2|\boldsymbol{\mathcal{E}}^2| = 2[(|\boldsymbol{\nabla}\mathcal{E}_x|^2 + |\boldsymbol{\nabla}\mathcal{E}_y|^2 + |\boldsymbol{\nabla}\mathcal{E}_z|^2) + \boldsymbol{\mathcal{E}} \cdot \nabla^2\boldsymbol{\mathcal{E}}] < 0 \quad . \quad (1.4)$$

The term in round brackets in 1.4 is a modulus squared and is therefore always positive. The second term in free space is proportional to $\boldsymbol{\mathcal{E}} \cdot \partial_t^2\boldsymbol{\mathcal{E}}$, as dictated by Maxwell’s equations. It follows that the sum of the two terms can never be positive if the electric field is constant in time, which means that a static electric field maximum cannot be formed in free space.⁶ The important consequence of Earnshaw’s theorem is that the absolute ground state cannot be trapped in a static trap, because this state is strong-field seeking.

Given the requirements of our trap, and armed with Earnshaw’s theorem, I will now discuss a number of trapping methods which have been applied to molecules and atoms. I will then present the microwave trap as a means of overcoming the disadvantages of conventional methods.

Static magnetic and electric traps

Magnetic trapping of molecules was first demonstrated with CaH [62], which was loaded from an ablation source and cooled with ³He buffer gas. Since this achievement, many more molecules have been magnetically trapped, including NH [63], CaF [64], SH [65], SrF [66] and methyl (CH₃) [67] radicals. Recently co-trapping of O₂ with lithium atoms [68] was demonstrated, loading from a dual species Zeeman decelerator. Whilst magnetic trapping is extremely versatile and in principle applicable to many atomic and molecular species, Earnshaw’s theorem means that only weak-field seeking states are trappable. The consequence of this is that species can collide and transfer to untrapped (high-field seeking) states, without

⁶Identical logic applies to the magnetic field in free space.

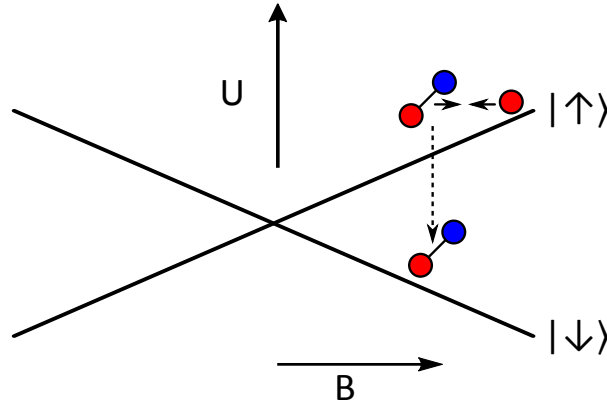


Figure 1.5: Spin relaxation of molecules in a magnetic trap.

an energy barrier inhibiting this process. This loss mechanism, illustrated in figure 1.5, was initially investigated theoretically in a series of papers [58, 69, 70] in the context of sympathetic cooling. Studies led the authors to suggest the light alkaline earth metals Be and Mg, as refrigerants. The advantage of choosing an alkaline earth atom is the reduced anisotropy in the atom-molecule interaction potential which leads to an improvement in the ratio of elastic to inelastic collisions [70]. These atoms have only a nuclear magnetic moment and in practice would have to be trapped via a separate means. However, there is experimental evidence that even then spin relaxation is difficult to overcome. For example, in the NH experiments carried out by the Doyle group, it was possible to observe spin relaxation by collisions with ^3He buffer gas. At temperatures in the hundred millikelvin range, the spin relaxation was found to limit the trap lifetime at high buffer gas densities. Furthermore, the spin relaxation rate is likely to be considerably higher for molecules co-trapped with paramagnetic atoms. As such, sympathetic cooling to the ultracold regime is likely to be difficult for molecules in a magnetic trap.

Static electric traps have been used extensively for confining polar molecules via the Stark effect, and have been successfully used to trap the OH radical [71], ammonia [72], NH [73], and CO [74] molecules amongst others. For diatomic molecules, the electric field mixes rotational levels, whereas for ammonia molecules there are opposite parity inversion states as a result of nitrogen tunneling through the hydrogen-containing plane. Once again, Earnshaw's theorem means the ground state cannot be trapped, and so inelastic collisions (e.g. to a lower rotational state) are always possible. Clear experimental evidence for this was

reported in a study of electrostatically trapped ND_3 molecules colliding with magnetically trapped Rb atoms [75]. For collision energies ~ 100 mK, enhanced loss rates of ND_3 were observed which increased with the Rb density, and the inelastic cross-sections were found to be enhanced by the electric field. Therefore a static electric trap is unsuitable for our purpose.

Optical Dipole Traps

Optical dipole traps are extensively used in cold atom physics, and relatively straightforward to implement. The trapping mechanism is the dipole force arising from the light-atom interaction. The oscillating electric field induces an oscillating electric dipole in the atom, and this then interacts with the oscillating electric field, producing an energy shift. For red-detuned light, the dipole oscillates in phase with the field, resulting in attraction towards the peak intensity of the laser beam for the ground state. Most optical dipole traps are based on a single focused gaussian beam. However, a blue-detuned repulsive “box-trap” has also been demonstrated [76], and by interfering multiple laser beams it is possible to create lattice structures. Molecules have also been loaded into optical dipole traps, including a recent demonstration of single molecules trapped in tightly focused dipole traps known as tweezer traps [77].

The key difficulty with an optical dipole trap is that in addition to the dipole force, there is a significant contribution from the scattering force caused by spontaneous emission. The latter is not a conservative force and so heats the atoms, and this reduces the effective trap depth and lifetime. The solution is to use a far-off resonance trap (FORT) [78], where the light is detuned from the atomic resonance by several nm. This suppresses the scattering force relative to the dipole force. However, since the dipole force also decreases with the detuning of light, an effective trap requires high laser power (often exceeding 10 W) and a tightly focused beam waist. Current traps are typically ~ 1 mK deep with waist sizes in the range $1 - 50 \mu\text{m}$, meaning they are difficult to load and only possible with a high phase space density. Consequently, trapping a range of molecular species is likely to be very difficult with this type of trap.

The ac Electric Trap

Another trap devised for molecules is the ac electric trap, which was first demonstrated with deuterated ammonia (ND_3) [79]. In this method, a time-varying potential is created by switching between two electric field configurations. Individually, neither field configuration forms a 3D trap, instead confining (repelling) along one axis and repelling (confining) along the others. By appropriately switching the potentials as molecules are moving, focusing towards the trap centre can occur. The ac trap used for ND_3 was demonstrated for both strong and weak field seeking states, meaning the rotational ground state can be confined in such a trap. However, there are a number of complications with this method. First, the switching frequency is a critical parameter in the trap operation; switching too slowly means the range of stable trajectories is small, and switching fast causes the focusing effect to diminish. This makes it difficult to confine more than one species in the same trap. Second, when the possibility of collisions was examined in simulations [80], it was found that the randomising of trajectories pushes molecules onto unstable trajectories, leading to rapid loss from the trap. As such, the ac electric trap is not considered a viable option for sympathetic cooling experiments.

The Solution: A Microwave Trap

The work of this thesis is to demonstrate a trap which does satisfy the sympathetic cooling criteria, a microwave trap. This method of trapping molecules was first proposed in 2004 by DeMille [81], and is illustrated in figure 1.6a. A microwave cavity is excited with input power at its resonance frequency, creating a strong electric field inside, $\mathcal{E}$. Atoms and molecules are polarised by this field and confined to the position where the electric field is strongest.

Atoms in the cavity experience an energy shift by the Stark effect, and are trapped according to their static polarisability α . The cycle-averaged energy shift ΔE is $-\frac{1}{4}\alpha|\mathcal{E}|^2$, which is general and applicable to any atom. Chapter 5 gives a detailed treatment of the trap potential for lithium used in our experiment.

In a rigid rotor molecule dipole, the microwave field mixes the rotational levels, labelled with total angular momentum quantum number J and projection m_J onto the quantization

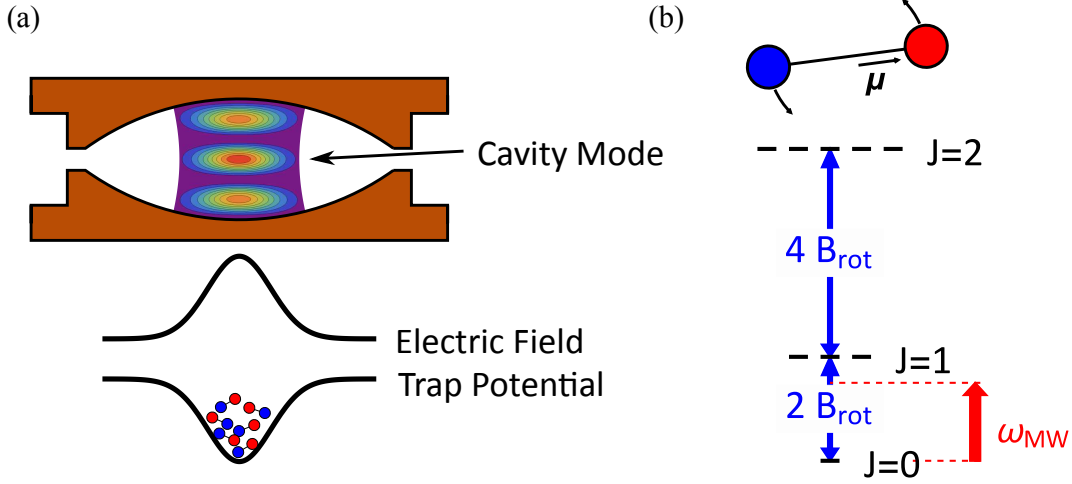


Figure 1.6: (a) Microwave trapping with a resonant cavity. (b) Low-lying rotational levels of a rigid rotor molecule.

axis. The field-free energy spectrum is given by $E(J) = B_{\text{rot}}J(J+1)$, with B_{rot} termed the rotational constant. To trap a molecule in the rotational ground state, the microwave frequency must be below the $J = 0 \rightarrow 1$ transition frequency as shown in figure 1.6b. In this case the energy shift, ΔE , is given approximately by second order perturbation theory:

$$\Delta E = - \sum_m |\langle J=1, m_J=m | \frac{\mu \cdot \mathcal{E}}{2} | J=0, m_J=0 \rangle|^2 \left[\frac{1}{2B_{\text{rot}} - \hbar\omega_{\text{MW}}} + \frac{1}{2B_{\text{rot}} + \hbar\omega_{\text{MW}}} \right]. \quad (1.5)$$

Here, $\mathcal{E}$ is the microwave electric field amplitude, ω_{MW} is the microwave frequency, and the sum includes all projections of the angular momentum, m_J , onto the quantization axis. Equation 1.5 is strictly only valid when the perturbation is small compared with the rotational splitting, and when multi-photon coupling to higher rotational states is negligible ($\omega_{\text{MW}} \sim 2B_{\text{rot}}$). In reality our trap is designed to operate in the strong field regime, and far detuned from the first rotational transition; a more general calculation is presented in chapter 5. What is important is that the trap method is general, and therefore multiple species can be confined simultaneously.

To illustrate the wide applicability of microwave trapping to polar molecules, it is worth visualising some molecular data. Figure 1.7 plots the electric dipole moment and rotational constant for a selection of diatomic molecules, taken from [82]. Those which have a large elec-

tric dipole and rotational splitting are favourable for trapping in the ground state. Molecules are labelled according to the properties of their electronic ground state, Λ, Σ , which refer to the projection onto the internuclear axis of the orbital angular momentum and electron spin respectively. Those with $\Lambda = 0$ are to a good approximation described by a rigid rotor model. Molecules with both $\Lambda, \Sigma = 0$ have only a nuclear magnetic moment, which effectively prevents them from being magnetically trapped.

Our cavity operates with a resonance at 14.3 GHz, and so we require $B_{\text{rot}} > 7.15$ GHz to trap the ground state. Equation 1.5 provides a good approximation to the trap depth, U_{trap} , provided $U_{\text{trap}} \ll \hbar\omega_{\text{MW}}$, which for our parameters corresponds to $k_B \times 690$ mK. The shading in figure 1.7 marks the regions where U_{trap} is greater than 10 mK and 100 mK, for a cavity electric field $\mathcal{E} = 30$ kV cm⁻¹. Of the 71 molecules shown, 33 have a predicted trap depth greater than 100 mK, and 28 of these have $(\Lambda, \Sigma) = 0$.

This trap can satisfy all the above criteria for sympathetic cooling. A microwave field is not bound by Earnshaw’s theorem and can exhibit a maximum in free space, meaning ground state atoms and molecules can be trapped at the same place. In the ideal case this means that inelastic processes all become endothermic, and therefore can be suppressed if the initial thermal energies are low enough. The trap volume is on the order of a microwave wavelength, and therefore at least as large as currently available traps. Furthermore, rotationally excited states of molecules are extremely long-lived, with spontaneous emission lifetimes of seconds to hours. As a result the scattering force, and associated heating, are almost zero, in contrast with an optical dipole trap.

1.4 Thesis Overview

This thesis is divided into four main chapters. In chapter 2, I explain the progress prior to my own work on this project, and describe improvements made to the experiment. Chapter 3 describes the setup used to implement and optimise Raman grey molasses cooling for ⁷Li. Chapter 4 details the procedure used to magnetically trap and transport atoms to the centre of the microwave cavity. Chapter 5 explains the work carried out to test, and subsequently load, the microwave trap.

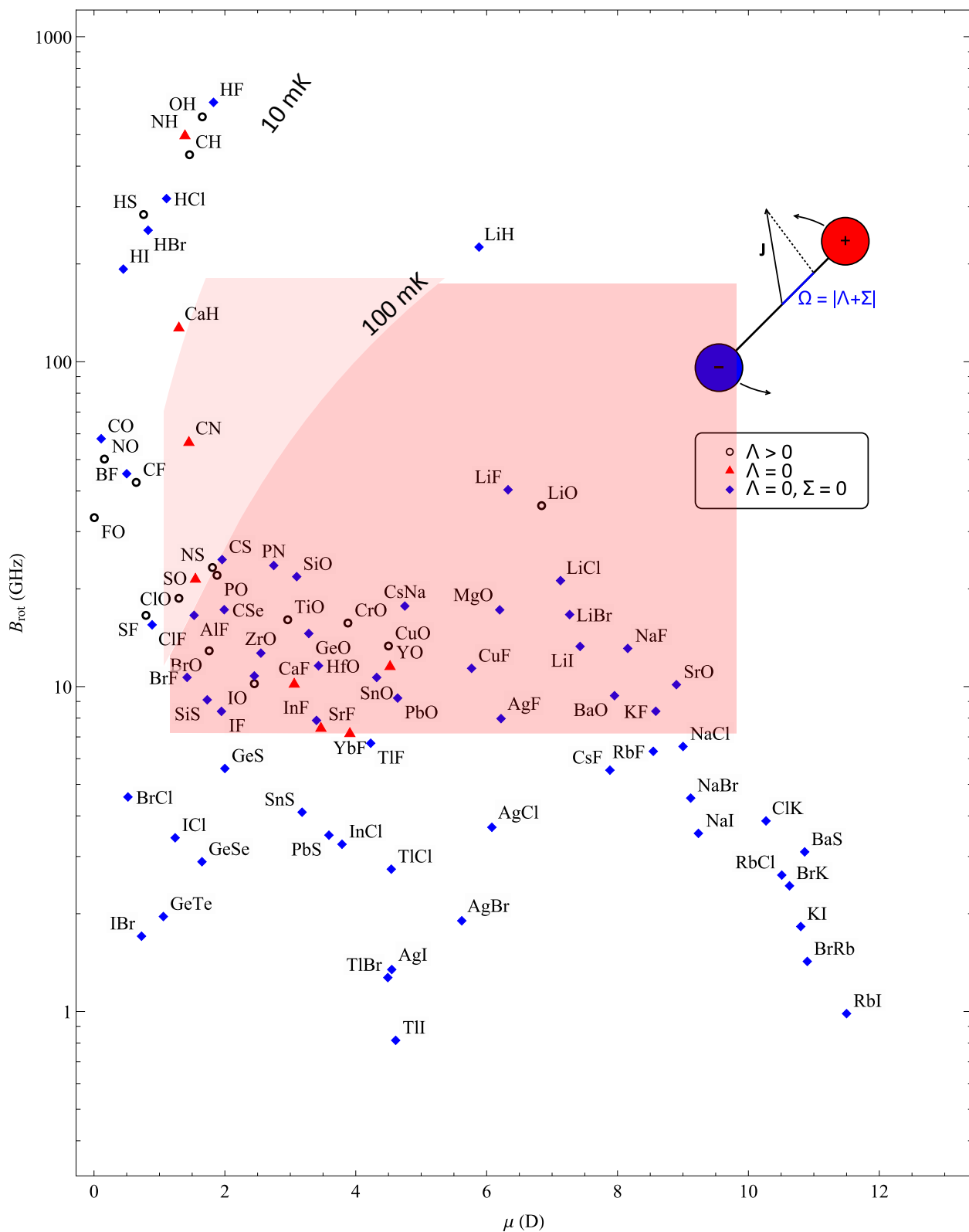


Figure 1.7: Rotational constants and electric dipole moments for a selection of diatomic molecules in the electronic ground state. The bounding of the shaded regions is discussed in the text.

Chapter 2

Experiment Overview

In this chapter I give an overview of our experiment, describing the essential features and typical operating conditions. I shall describe the state of the experiment in 2015 when I began work, and detail improvements and modifications I made in order to implement sub-Doppler cooling.

2.1 Previous Work

The Lithium MOT and Magnetic Transport System

Prior to my own efforts, work on our lithium experiment was predominantly done by Jacob Stack, succeeded by Aisha Kaushik, each of whom defended a thesis focussed on this side of the project [83, 84]. Jacob built our first lithium source, designed and demonstrated the Zeeman slower (currently still in use), and made our first lithium MOT. Aisha subsequently improved the MOT, implemented magnetic trapping, and transported the cloud to a separate “science” chamber which is now used for microwave trapping. I shall summarise Aisha’s findings in relation to the lithium experiment, since these are more pertinent to the following sections.

Firstly, the temperature of atoms in the MOT was around 1.3 mK, measured by ballistic expansion following their release from the trap. This is similar to other lithium MOTs, and significantly above the Doppler cooling limit for the lithium D_2 line. By implementing

a compressed MOT (CMOT) stage, the cloud was cooled to around 0.75 mK. Next, the cloud was loaded directly into a quadrupole magnetic trap by shutting off the laser light to the experiment. The magnetic trapping coils were externally mounted and used for both the MOT and magnetic trapping stages. To transport atoms to a second chamber, the coils were translated with a motorised stage, passing the cloud through a stainless steel connecting tube.

The $1/e$ lifetime for atoms in the magnetic trap was found to be 1 – 3 seconds in the MOT chamber, and 18.5 (0.7) s in the science chamber. The lifetime in the MOT chamber was a factor ~ 10 shorter than a prediction based on the background gas pressure. However, this was complicated by a thermally activated leak in the lithium oven, and the proximity of the MOT chamber pressure gauge to the vacuum pump. The pressure in the science chamber was below the limit of the measurement gauge (1×10^{-9} mbar), and the lifetime was approximately half the prediction based on this limit.

An important finding from Aisha’s thesis is that the externally mounted coils tend to induce eddy currents in the MOT chamber when switched rapidly (> 40 A/ms). These eddy currents give rise to a time-varying induced magnetic field at the position of the MOT, detuning the atoms from the probe beam by the Zeeman effect. As a result, the measured atom number from absorption images decreases as the cloud was allowed to expand. Absorption spectra were taken during the first 1.5 ms of expansion, and compared with calculations to estimate the induced magnetic field at the MOT centre. The estimated magnetic field 1.4 ms after release from the CMOT was 19 (2)G, and directed along the axis of the trapping coils.

The Microwave Trap

Previous work on the microwave trap was carried out by Lyra Dunseith and Stefan Truppe. This included the choice and design of the cavity, and investigations concerning the delivery of microwave power to the desired mode through a small hole in one of the mirrors. The electric field amplitude of the cavity mode was simulated and measured experimentally, and the quality factor was measured as a function of the temperature of the mirrors and the radius of the coupling hole. Loss from the cavity was found to be dominated by resistive loss in the mirrors, rather than diffractive loss from the open walls of the cavity. These findings

are published in [85], and discussed further in chapter 5.

The aforementioned experiments were carried out using ~ 10 mW of microwave power, whereas the intended operating power for the trap is 1 kW. On one occasion about 700 W was coupled into the cavity, but the vacuum assembly was found to leak due to thermal strain introduced by cryogenic cooling of the mirrors. The high power properties of the cavity were not studied, and so the effect of a high microwave load on the cavity quality factor was not known. These two factors meant that loading of atoms into the cavity was not attempted.

2.2 The Current Experiment

Figure 2.1 shows the main components of the lithium source, Zeeman slower, MOT chamber, transport system, and the microwave trap. The coordinate system adopted throughout the remaining chapters is shown on the left-hand side of the figure, and we take the origin to be the centre of the MOT chamber.

The lithium source section is shown in the top right of the figure, and comprises an effusive lithium oven, pneumatic linear motion feedthrough, and photomultiplier tube (PMT). The linear motion feedthrough is driven with compressed air and used to block the atomic beam from reaching the MOT chamber once a MOT has been loaded. The PMT is used for fluorescence detection, to monitor the output flux from the oven. The pressure in this chamber is maintained by a turbomolecular pump (TMP), and typically stabilises between $10^{-7} - 10^{-8}$ mbar when the oven is operational.

The atomic beam leaves the source chamber and enters the Zeeman slower section. This section is separated from the source by a gate valve, enabling maintenance of the oven whilst the rest of the experiment is under vacuum. Vacuum is maintained at a pressure $\sim 2 \times 10^{-8}$ mbar by a Varian Noble diode ion pump; a 100 mm long differential pumping tube of inner diameter 5 mm maintains the pressure difference with the source chamber. The Zeeman slower itself is composed of 14 magnetic field coils, powered by 4 current supplies which are adjusted to optimise the number of atoms loaded into the MOT.

The Zeeman slower attaches directly onto the MOT chamber, where our atomic samples

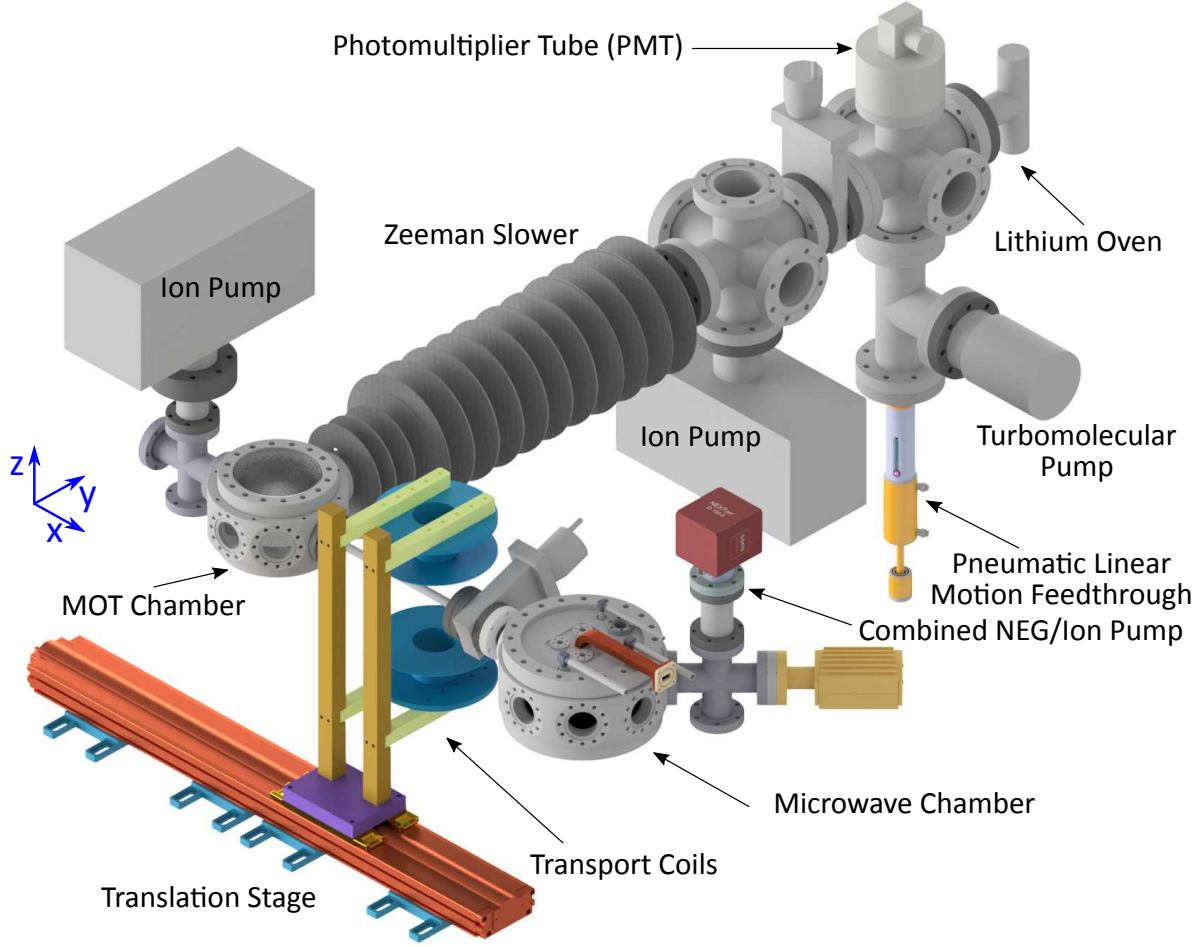


Figure 2.1: The current experimental setup, showing the important vacuum apparatus, external transport coils, and translation stage.

are trapped and laser cooled. The vacuum chamber is a 6-inch spherical octagon chamber made from 304L stainless steel. The pressure in this chamber is typically 1×10^{-9} mbar, maintained by a second ion pump. Visible in the bottom left of figure 2.1 is the translation stage (Parker 404XR), onto which the external trapping coils are mounted. The translation stage moves the centre of a magnetic trap along a 250 mm long stainless steel transport tube of inner diameter 9 mm. For the coils to pass over the microwave chamber, their inner separation had to be increased from 140 mm (used for the experiments in [84]) to 180 mm; this reduces the field gradient per unit current by a factor 2, to $0.50 \text{ G cm}^{-1} \text{ A}^{-1}$ along the z -axis.

Attached to the opposite end of the transport tube is the microwave chamber, visible

in the bottom centre of figure 2.1. The vacuum chamber is an 8 inch spherical octagon, made from 304L stainless steel. Between the microwave chamber and the transport tube is a gate valve and short bellows section; these enable installation and testing of the microwave system whilst the MOT chamber is under vacuum. The pressure in the microwave chamber is typically $1-2 \times 10^{-9}$ mbar, and is maintained by a combined non-evaporable getter and ion pump (SAES NexTorr D100-S). Details on the microwave chamber and assembly are provided in chapter 5.

Figure 2.2 shows a schematic illustrating a typical experimental sequence, along with the number of atoms and temperature of the cloud at each stage in this sequence. During the first 7 s, the MOT is loaded with 1×10^8 atoms at a temperature of 1 mK. Next, we implement a Raman grey molasses stage, which takes 3 ms and cools the atoms to around 50 μ K. The atoms are then optically pumped, and loaded into a magnetic quadrupole trap for transport to the microwave chamber. The transport requires around 2 s and we retain 56% of the atoms loaded into the magnetic trap. Finally, atoms are transferred from the magnetic trap to the microwave trap, typically for 1 – 2 s.

2.3 Redesigning the Optical System

Figure 2.3 shows the energy levels of ^7Li which are relevant to our experiment, along with the main transitions addressed when laser slowing and cooling. We use the D_2 line for cooling and trapping atoms in the MOT, and for absorption imaging; for sub-Doppler cooling and optical pumping we use the D_1 line, whose frequency is lower by approximately 10.05 GHz. This frequency difference is too large to generate the required power from a single laser. We use two separate systems, hereon referred as the D_2 and D_1 lasers respectively. In addition to the MOT beams, we also require Zeeman slowing and repump frequencies, which are detuned by around 400 MHz relative to their counterparts in the MOT light. This detuning is determined by the magnetic field profile of the Zeeman slower.

The original optical system for the experiment is illustrated in figure 2.4a, and described in detail in [84]. The MOT laser is locked at the cross-over resonance using polarisation spectroscopy, and light at this frequency is used as a repump for the Zeeman slower. All

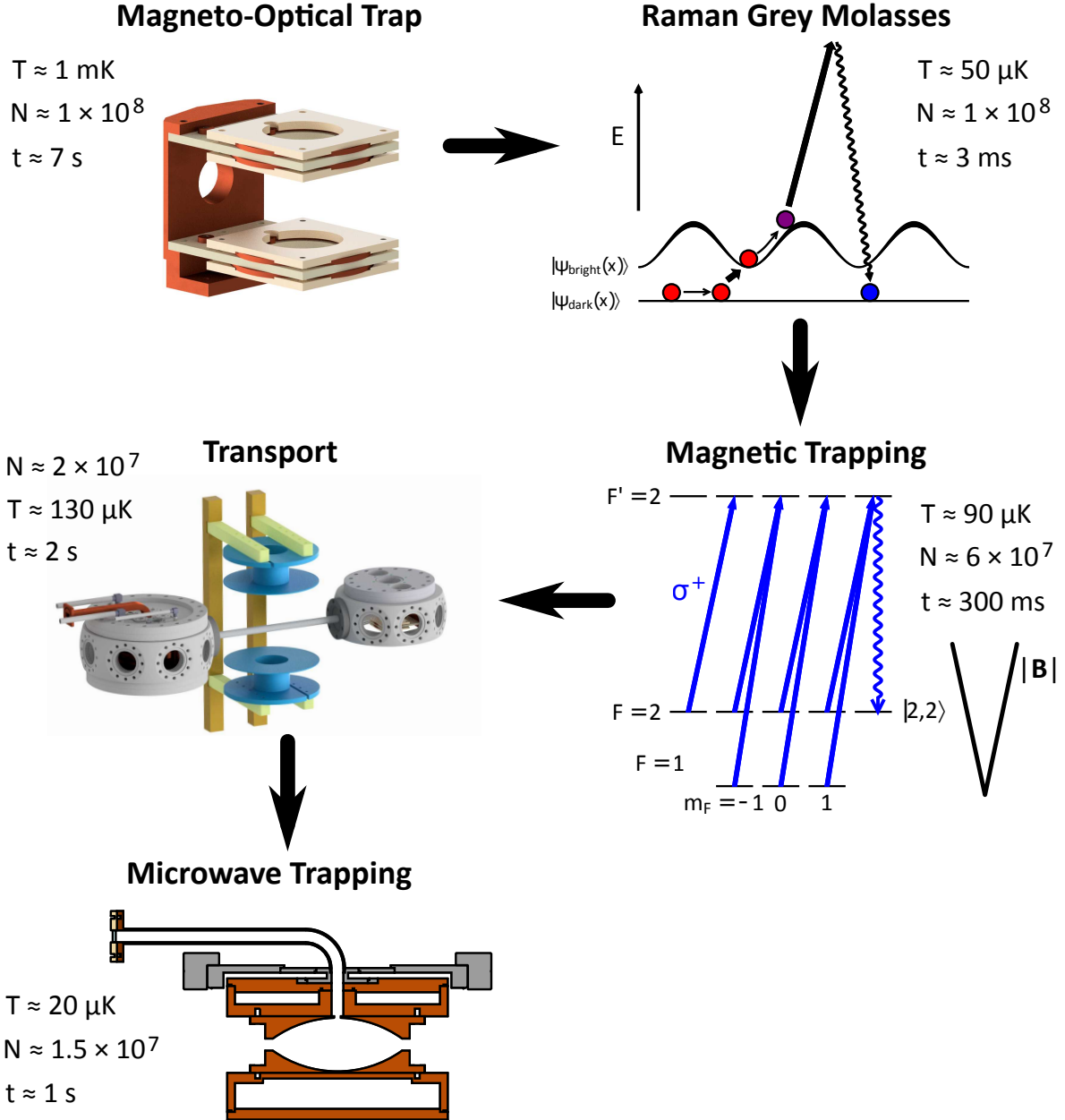


Figure 2.2: A schematic of the main stages of our experiment. The approximate number of atoms, temperature and time required are indicated beside each illustration.

the necessary frequency components for making a MOT are generated by acousto-optic modulators (AOMs). A plot showing the distribution of the D_2 laser power is shown in the lower half of figure 2.4a. The D_1 light is combined with the trapping light, following it through the optical system to the MOT chamber. To ensure the required power is available, a separate tapered amplifier (TA) is required.

This setup suffered from several drawbacks:

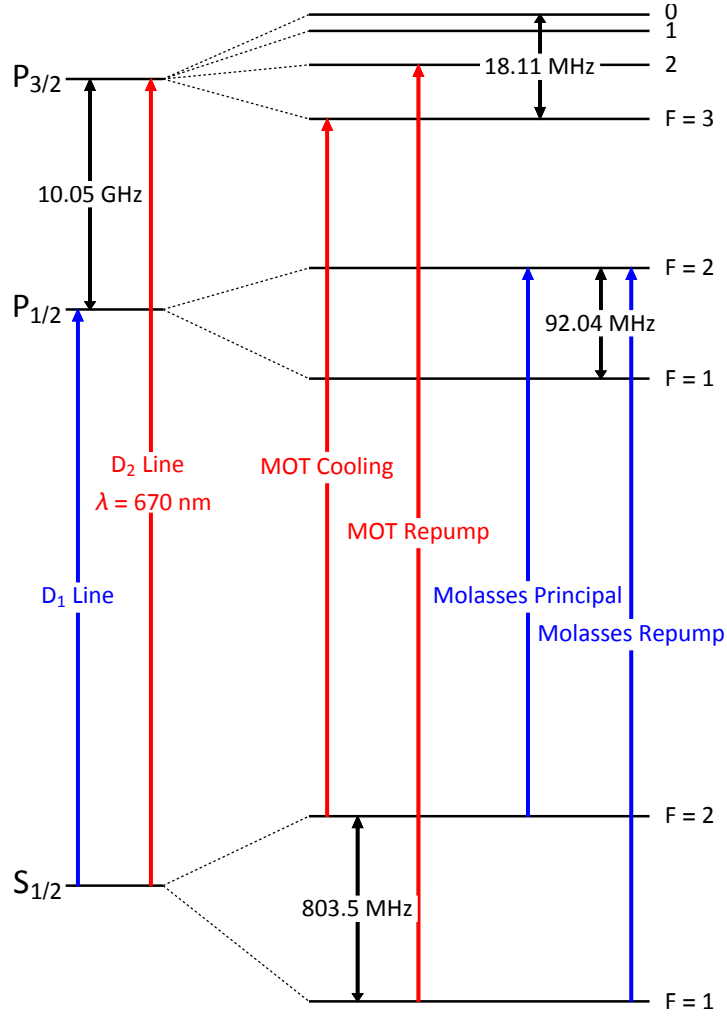


Figure 2.3: Energy levels in ^7Li relevant to our experiment, and the transitions that we drive.

- The D_2 laser is locked at a frequency where relatively little power is needed. The majority of the D_2 light has to be frequency shifted by the AOMs, with significant accompanying power loss.
- For both the Zeeman and MOT light, the repump and cooling beams have to be spatially overlapped before delivery to the experiment, which is imperfect and results in some loss of useful power.
- For the MOT and molasses light to pass through the same optical setup, the seeding to the TA must be rapidly switched during the experiment. As well as requiring the

introduction of three switching AOMs, the performance and stability of the TA is potentially compromised during the grey molasses stage.

- The molasses beam size is fixed to that of the MOT beams, which reduces the intensity achievable for Raman molasses.

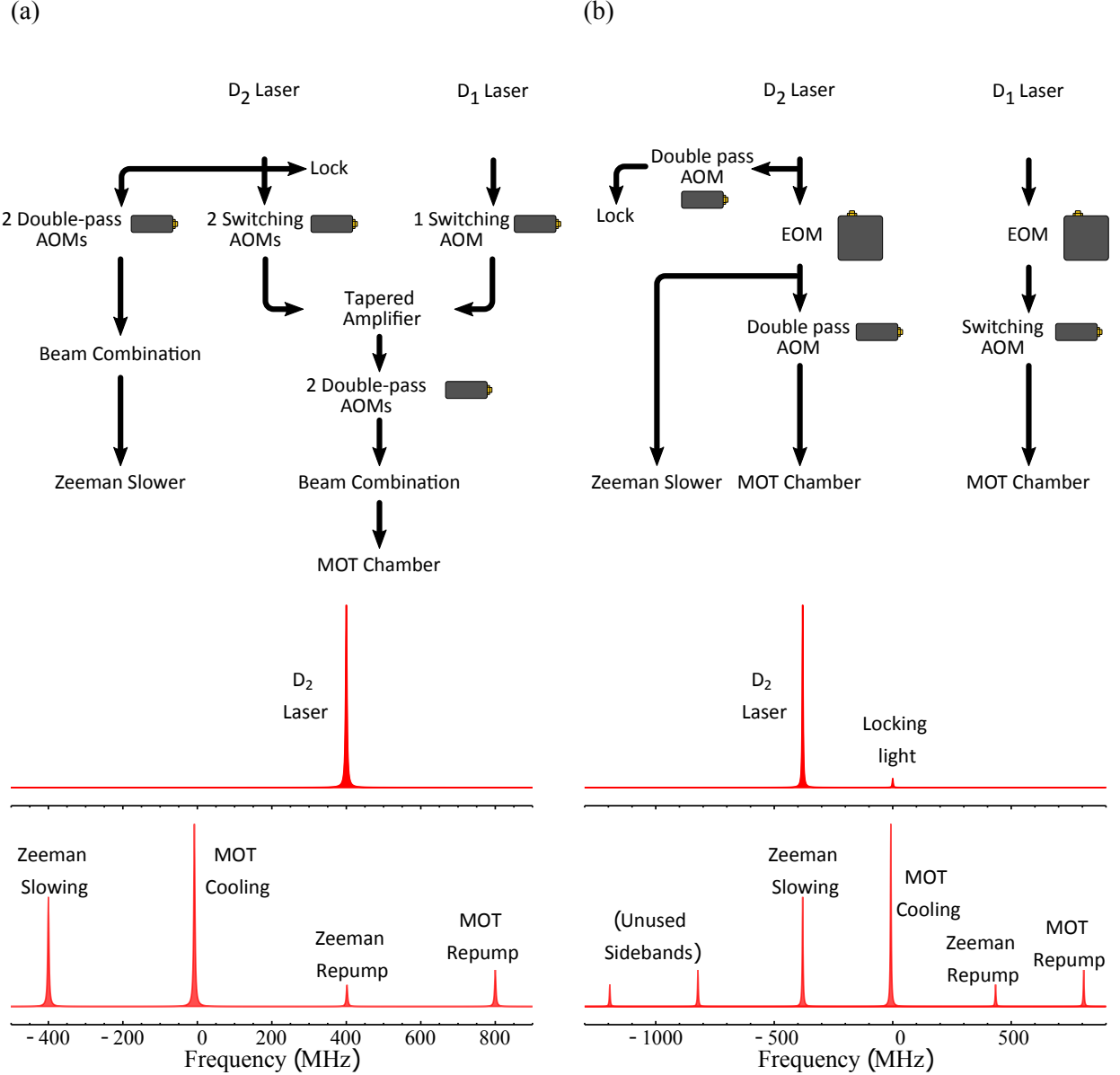
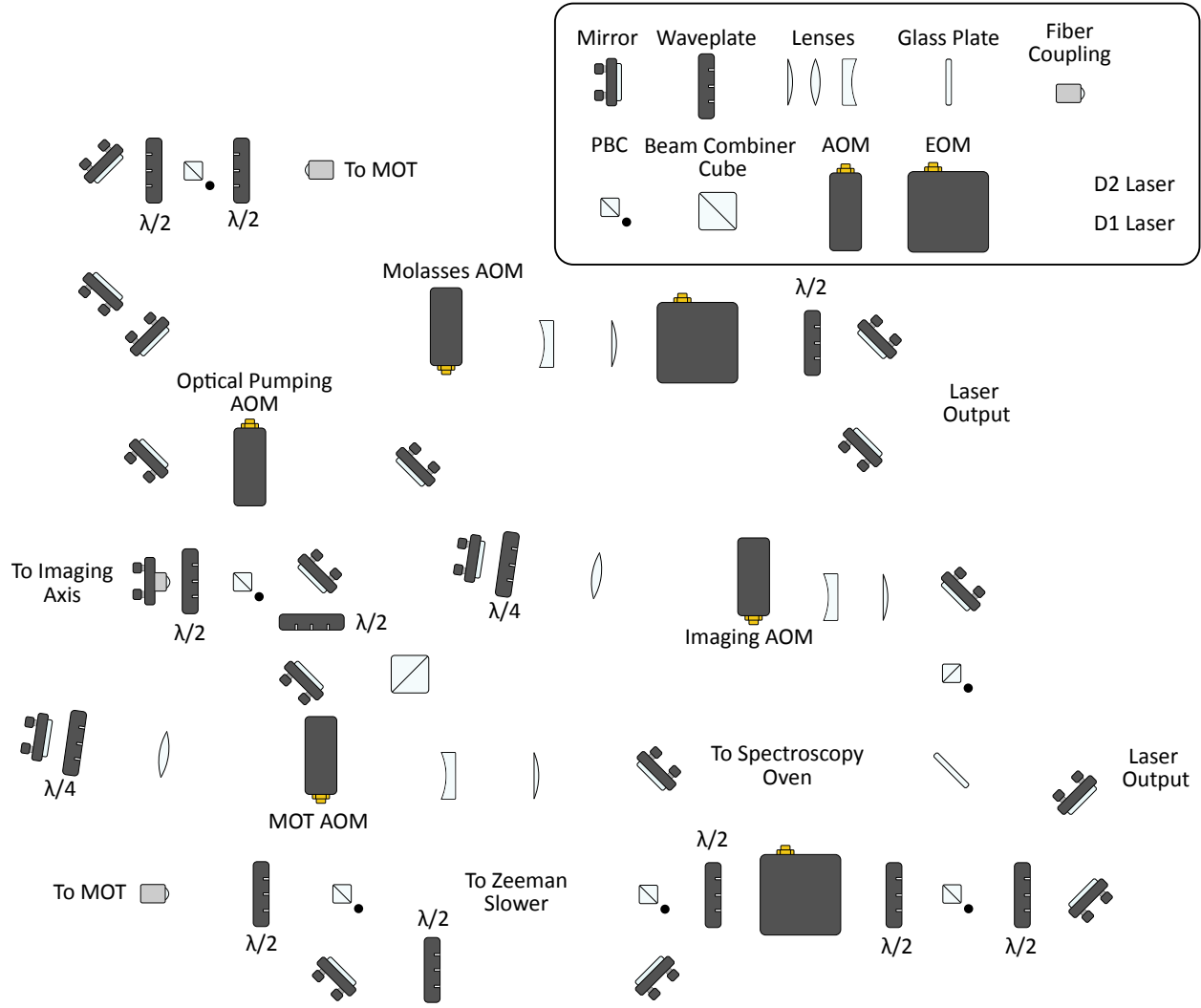


Figure 2.4: Comparison of the original optical system (a), and the re-designed version (b). The upper plots show the output frequency of the D₂ laser when locked to the master laser output, and the lower spectrum shows the distribution of laser power used in the experiment. Frequencies are referenced to the $F = 2 \rightarrow 3$ D₂ laser cooling transition.

The new optical scheme, illustrated in figure 2.4b, is designed to benefit from two properties of our lithium MOT [84]. Firstly, whilst sensitive to repump power, the number of atoms in the MOT is relatively insensitive to its detuning. Secondly, the optimal cooling-repump detuning is the same for both the MOT and Zeeman light. Therefore it is possible to generate the frequency difference required for all of the repump light with a single EOM operating at 814.5 MHz, with the added benefit that the repump automatically overlaps with the cooling light. There is an additional frequency sideband generated by the EOM, but this is a small portion of the overall power and does not interfere with the experiment. The D_2 laser is locked at the frequency required for Zeeman slowing, by up-shifting the locking light with an AOM, which saves power compared with the original system. The combination of these improvements allowed us to remove the TA, significantly reduced the alignment work, and resulted in a more robust system for making MOTs. The molasses light now has a separate optical system, shown in the upper half of figure 2.5. Here, a single EOM provides the necessary weak “repump” sideband, and a single-pass AOM acts as an intensity switch.

Figure 2.5 shows a more detailed layout of the current optical setup, showing both the D_1 and D_2 lasers. The output power of the D_2 laser is typically 450 mW. About 20 mW is used for locking, before the EOM adds repump sidebands. The Zeeman light is separated from the MOT light at a polarising beam splitter (PBS) and 40 mW is sent to the experiment. The MOT light is frequency shifted using a double-passed AOM in cat’s eye configuration [86] and fibre coupled for delivery to the experiment. The output power of the D_1 laser is typically 330 mW; an EOM adds the repump sideband, followed by a single pass switching AOM and then fibre coupling to deliver to the experiment for molasses. The unused light from the molasses AOM is sent to a separate AOM and used for optical pumping. This light is combined with the imaging light from the D_2 laser and delivered to the experiment along the same optical fibre.

Both the MOT and molasses beams are delivered to the experiment using polarisation maintaining fibre splitters (Evanescence Optics Inc.), which efficiently couple the light from one fibre into three output fibres. The MOT and molasses light is combined separately for each of the three optical axes, using a polarising beam cube (PBC). The co-travelling beams subsequently pass through a quarter waveplate, after which they have circular polarisations



of opposite handedness. This setup is shown in in figure 2.6.

Our EOMs were purchased from Qubig GmbH, (EO-Li7M3-VIS), each with a resonance bandwidth of 2.7 MHz. The resonance frequency is manually tunable over the range 685-905 MHz, via a tuning screw. The EOMs are driven with standard RF driver circuits with amplitude and frequency modulation, and the frequency sidebands are monitored using a scanning Fabry-Perot cavity. For the MOT, we operate at 814.5 MHz with carrier-to-sideband ratio of approximately 1:5 (approximately 0.8 rad of modulation depth). For the molasses light, the frequency is close to the 803.5 MHz ground state hyperfine splitting, and the carrier-to-sideband ratio is a few per cent.

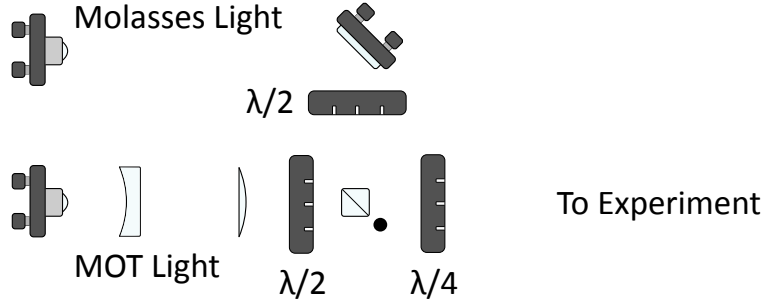


Figure 2.6: Combining the MOT and molasses light at the experiment.

2.4 Improving the Lithium Source

Lithium has a negligible vapour pressure ($\ll 10^{-11}$ mbar) at room temperature, and so most lithium MOTs are loaded from an oven operating at around 400 - 500 °C. The original experiment used an oven with a circular exit aperture of diameter 0.7 mm to produce an effusive beam. Maintenance and inspection of the oven was difficult and time-consuming, since this section of the experiment had to be first cooled and then vented with inert gas to avoid oxidation of the lithium. There was a recurrent problem of insufficient lithium flux reaching the MOT chamber, with the aperture becoming increasingly blocked up with solid lithium. Refilling the oven required opening up a CF flange typically caked with solid lithium, and often resulted in leaks upon reassembly. Hence opening the oven to inspect or refill, although an infrequent occurrence, was highly undesirable.



Figure 2.7: The capillary array used as an oven aperture. There are 55 capillaries of inner diameter 0.58 mm and length 20 mm.

To distinguish between oven problems and other problems that reduce the number of

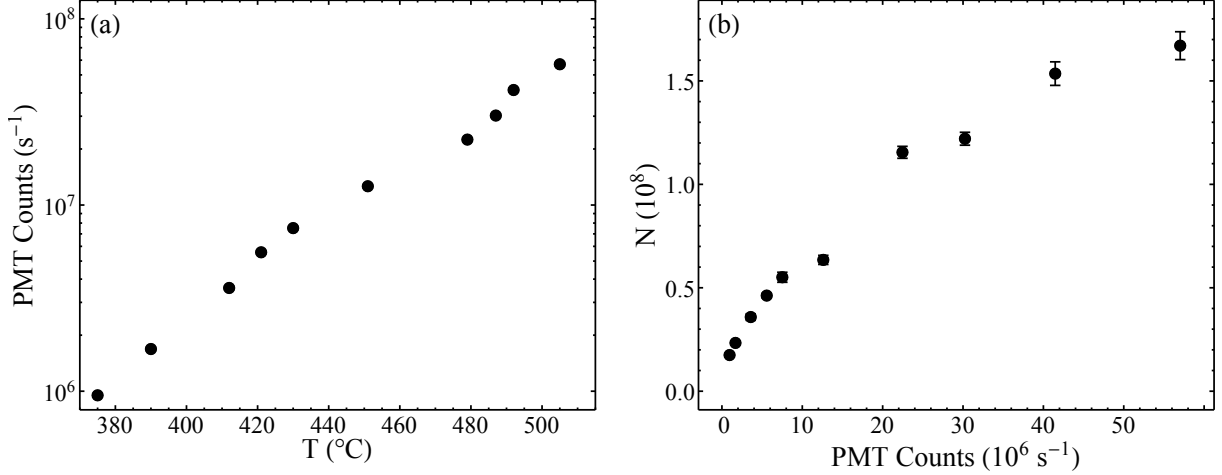


Figure 2.8: (a) PMT count versus oven temperature. (b) Number of atoms in the MOT versus the PMT count rate.

atoms in the MOT, it was decided to directly monitor its output flux. We added a photomultiplier tube (PMT; see figure 2.1) to detect laser induced fluorescence of the atomic beam, using probe light intersecting the beam perpendicular to its travel direction. By calibrating the fluorescence against the oven temperature and resultant MOT size, it was then possible to quickly and non-invasively verify the flux entering the Zeeman slower.

We also modified the oven exit aperture, replacing the single effusive hole with an array of capillaries. The capillaries cover a circular aperture of diameter 5 mm and are secured into a mount which can be removed from the oven. The hole beneath the capillaries can be used to feed granular lithium into the oven, meaning the CF flange is no longer needed. The mounted capillaries are shown in figure 2.7.

To measure the relative flux exiting the oven, we observe laser induced fluorescence on the lithium D₂ line by exciting the $F = 2 \rightarrow F' = 3, 2, 1$ transitions. The probe beam has a power 10 μ W, a $1/e^2$ diameter of 4 mm and intersects the atomic beam 130 mm downstream from the exit of the capillary array. Figure 2.8a shows the fluorescence signal at the PMT as the oven temperature is varied. For these parameters, the peak intensity of the probe is 0.3 mW cm⁻², 0.12 times the saturation intensity of the D₂ transition. The PMT was calibrated by comparing the photon-counting and current-mode readings for counts between $1 - 5 \times 10^6$ s⁻¹. Figure 2.8b shows the number of atoms in the MOT against the PMT count rate, which can now be used to diagnose the state of the oven.

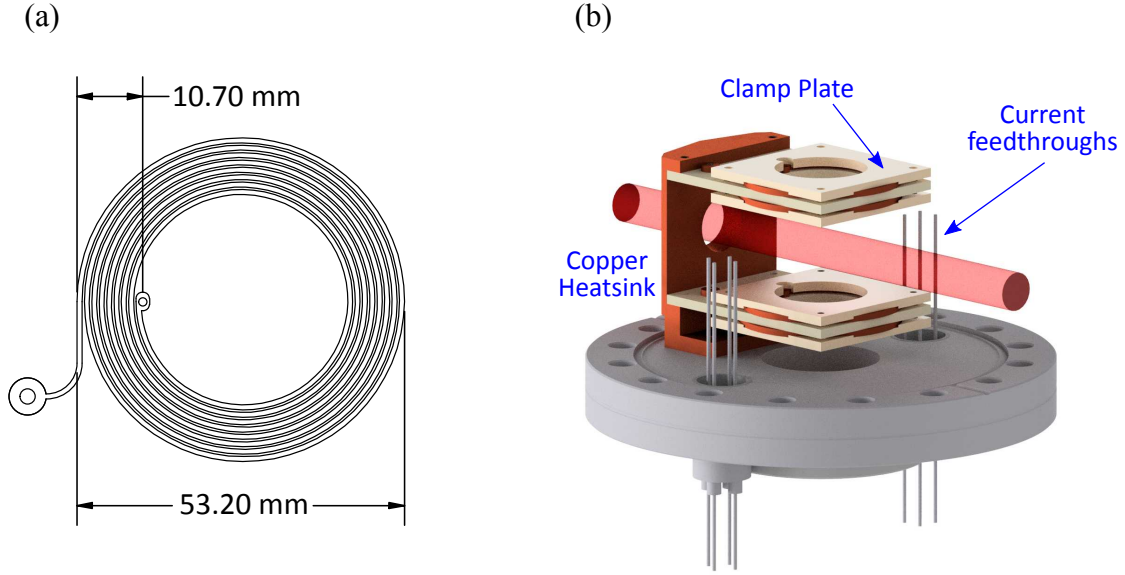


Figure 2.9: The in-vacuum coil design. (a) Copper spirals used to generate the magnetic field profile. (b) Full assembly.

2.5 In Vacuum Magnetic Trapping Coils

As discussed in section 2.1, the switching time of the external transport coils is limited by induction in the MOT chamber. This can cause a number of significant problems. One is that the coils turn off too slowly to make a reliable ballistic expansion temperature measurement of magnetically-trapped atoms. Another is that the molasses cooling fails in the presence of the slowly-decaying magnetic field. To resolve these issues, we decided to use in-vacuum trapping coils for loading the MOT. Our design is shown in figures 2.9a-b. Each coil is formed from two laser cut copper spirals, and secured onto a central mounting plate with two clamping slabs. The mounting plate is made from Aluminium Nitride, which is both a good electrical insulator and an excellent thermal conductor. Each mounting plate is secured to a copper heatsink which provides thermal contact with the upper 6 inch flange; a hole is cut in the heatsink to allow access for the imaging light. Electrical connection is made to the coils using vacuum feedthroughs rated for 20 A, and the total resistance across the input and output terminals of the feedthrough is $0.1\ \Omega$. The final assembly is shown in figure 2.9b.

A difficulty with the spiral design was the risk of contacting adjacent loops, short-circuiting part of the coil and distorting the field profile. To test for this issue, the magnetic field profile of the combined coil pair was measured along the radial direction. Figure 2.10a

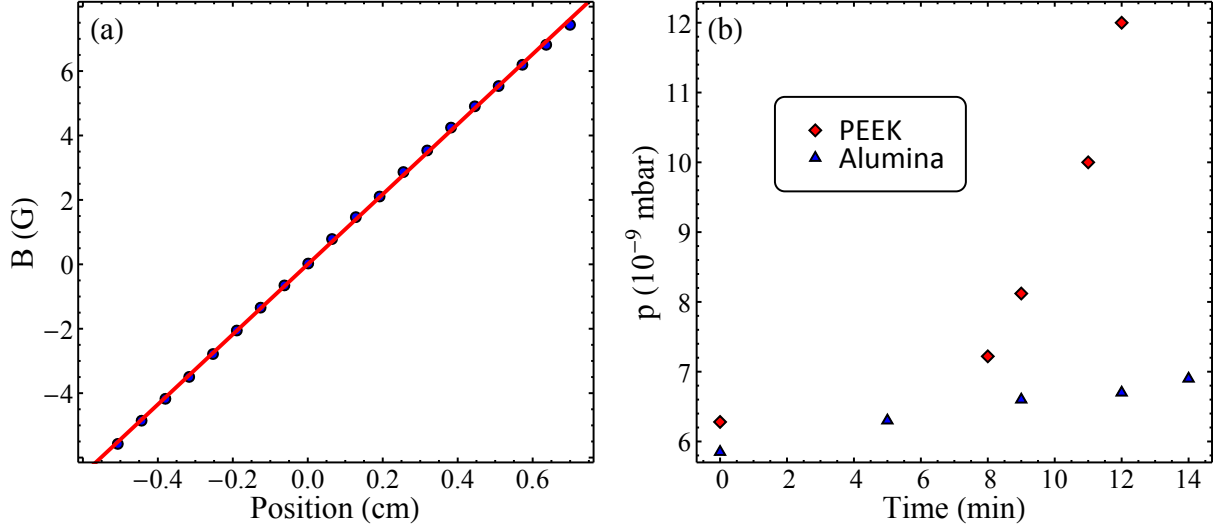


Figure 2.10: (a) Radial magnetic field measurements around the magnetic field minimum of the in-vacuum coils ($I = 10.08$ A). (b) Comparison of the test chamber pressure with a fixed current $I = 10$ A through the coils, using Alumina and PEEK clamping plates.

shows the measured field profile with a current $I = 10.08$ A per coil, along with a linear fit to the data. The profile is close to linear over at least ± 5 mm around the centre of the coils, and the calculated axial field gradient is 1.08 G/cm per coil. This is within 5% of the expected value for our coil configuration, and therefore we can be confident that there is at most one contact across the four spirals.

An important concern with the new design was vacuum compatibility; previous design iterations in our group used PEEK clamping plates, but these were thought to outgas when running the coils for long periods. Before opening the main experiment, a test setup was assembled, with one coil clamped using Alumina and the other with PEEK. Figure 2.10b shows the pressure measured when operating with a DC current of 10 A for 14 minutes. The pressure begins to degrade after around 8 minutes of continuous operation when using the coil clamped with PEEK, and so we opted to use Alumina clamping plates in the experiment.

Figure 2.11 compares switching the magnetic field using the internal (a) and external (b) coils. In both cases, the current switches off in under 1 ms, as shown by the inset to each plot. However, the magnetic field decays over a few hundred microseconds when switching the internal coils, whereas a large field persists for at least 10 ms when switching the external coils. The time available for Raman molasses is only a few milliseconds, and we were unable

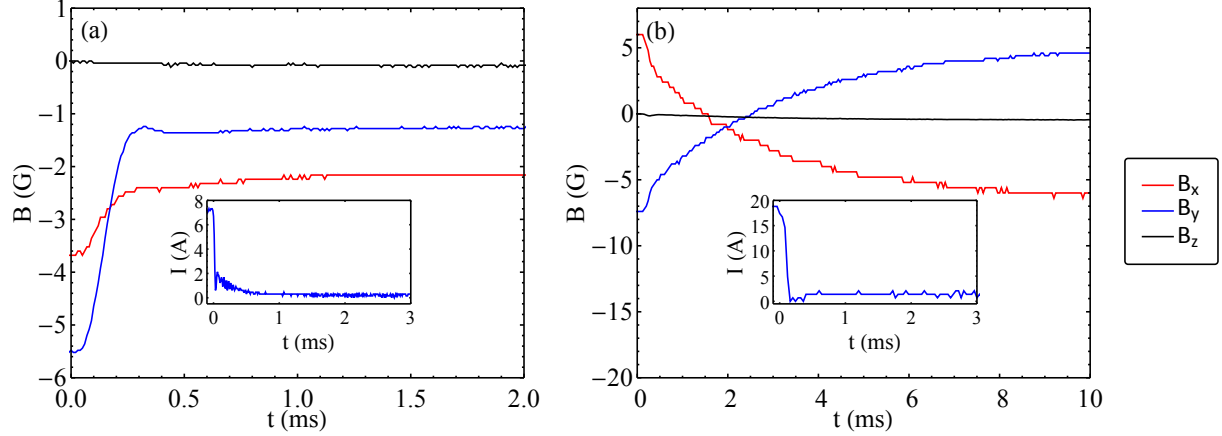


Figure 2.11: Magnetic field measured close to the MOT position after switching the coil currents to zero. (a) In-vacuum coils (b) External transport coils, when positioned over the MOT chamber. Insets: current monitor trace.

to observe effective cooling when using the external coils for the MOT.

2.6 Imaging

To detect atoms, we use absorption imaging on the D_2 line. Figure 2.12 illustrates the imaging setup. A probe beam with an intensity 0.025 mW/cm^2 (an on-resonance saturation parameter $s \approx 0.01$) illuminates the atoms, and the resultant absorption shadow is imaged onto a CCD camera using a $4f$ -imaging configuration with unit magnification. The atomic cloud is positioned at the focal point of the first lens, and in the focal plane shared by the lenses, the fourier transform of its shadow is produced. The inverse fourier transform is returned a distance f from the second lens, imaging the atomic shadow onto the CCD. The numerical aperture presented by the first lens is ~ 0.05 , which results in a diffraction-limited resolution of about $10 \mu\text{m}$. The atomic cloud is several hundred microns across, and so this is sufficient for our purposes. The pixel size of the CCD camera (AVT Pike) is $6.45 \mu\text{m}$; the illumination region is restricted by the transport tube, and a square range of interest for analysis can be at most $6.45 \times 6.5 \text{ mm}$. The optics collect a negligible contribution ($< 0.1\%$) of the scattered light.

In each experiment, three images are taken to construct a final absorption image. First, the probe beam is flashed on with the atoms present, and an image is taken. The probe is

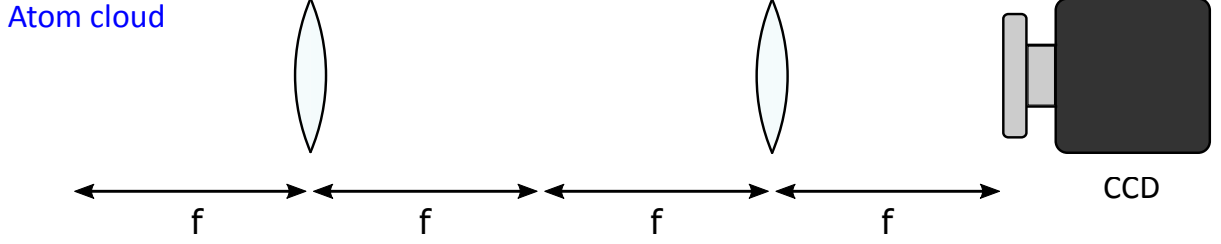


Figure 2.12: The $4f$ imaging configuration.

flashed a second time after allowing all atoms to leave the probe region, which provides the reference image for absorption measurements. Finally, a third image is taken without the probe beam which allows us to correct for any background sources of light. The intensity values measured in each image are I_{atoms} , I_{probe} and $I_{\text{background}}$ respectively. These are then used to calculate the absorption, A_i , at each pixel i in the image, as

$$A_i = -\ln \left[\frac{I_{\text{atoms}} - I_{\text{background}}}{I_{\text{probe}} - I_{\text{background}}} \right] . \quad (2.1)$$

When the probe saturation parameter $s \ll 1$, absorption by a sample with fixed absorption cross-section σ should follow the Lambert-Beer law. As such, $A(x, z) = \sigma n_{2D}(x, z)$, where $n_{2D} = \int n(x, y, z) dy$ is the 2D density profile perpendicular to y , the propagation direction of the light.

There are a number of considerations when trying to deduce the density and atom number from absorption images. First, the sample must not be too optically thick, since digitization of the pixel values results in a maximum absorption which can be measured; for our camera the images are 14-bit and so we can in principle measure $A \leq 10$. This is not a limit reached in the experiments. Second, the absorption cross-section for an atom depends on the linewidth of the exciting light and the atomic velocity distribution. For lithium atoms at 1 mK, the typical speed of an atom is 1.1 m s^{-1} , corresponding to a Doppler detuning of 1.6 MHz; this is 0.27Γ , smaller than the transition linewidth. A more important effect is the recoil caused by successive scattering events. Absorption of a photon results in a recoil velocity of 0.085 m s^{-1} , and after around 23 absorption events an atom will be Doppler shifted by $\Gamma/2$. With a probe saturation parameter s , the scattering rate for an atom should be at most $s\Gamma/2$, and with $s = 0.01$ it takes on average $62 \mu\text{s}$ to scatter 23 photons. For all absorption images in this thesis, the camera exposure time was fixed to $59 \mu\text{s}$, to reduce this

effect.

The second complication for lithium is the unresolved structure of the $^2P_{3/2}$ excited state. In principle we should be able to drive the $F = 2, m_F = 2 \rightarrow F = 3, m_F = 3$ stretched transition, in which case we realise a simple two level system with a well-defined scattering cross-section $\sigma_0 = 3\lambda^2/2\pi$. However, this requires establishing pure circularly polarised light, and a magnetic field which defines the quantization axis. For the molasses experiments in chapter 3, it was decided to image in zero magnetic field, since this is where the cooling is most effective. Averaging over all $F = 2 \rightarrow 3$ transitions reduces the cross-section to $7\sigma_0/15$. We must also consider the possibility of driving transitions to $F = 2$ excited states, which can decay to $F = 1$ ground states. For a probe beam resonant with the $2 \rightarrow 3$ transition, the detuning $2 \rightarrow 2$ is -1.5Γ , and the average on-resonance transition strength is $5/14\sigma_0$. This predicts a $2 \rightarrow 2$ relative excitation rate of 3.5%. The $F = 2$ excited states decay to $F = 1, 2$ ground states with equal probability, meaning that on average 1.75% of the absorption events pump the atom to $F = 1$. Assuming this loss probability, then 67% of a population initially in the $F = 2$ ground states will survive 23 absorption events, and on average 19 photons will be scattered per atom. Therefore the off-resonant optical pumping is comparable to the recoil Doppler shift problem.

For these reasons, it is difficult to determine accurately the number of atoms in our experiment. We assume an absorption cross section $\sigma = 7\sigma_0/15$ when imaging in zero magnetic field. This probably yields an underestimate of the true atom number. After optical pumping was implemented for magnetic trapping (chapter 4), we assume $\sigma = \sigma_0$, and image in a 1 G magnetic field.

2.7 Ballistic Expansion

Unless otherwise stated, temperature measurements in this thesis are made by ballistic expansion in free space. We assume a velocity distribution function, $P(\mathbf{v}) \sim \exp(-\frac{m\mathbf{v}^2}{2k_B T})$, which is independent of position at $t = 0$. If the atomic cloud is released at time $t = 0$, the density profile at time t , $n(\mathbf{r}, t)$ is given by,

$$n(\mathbf{r}, t) = \int n(\mathbf{r}', 0) P(\mathbf{v} = \frac{\mathbf{r} - \mathbf{r}'}{t}) d^3r' . \quad (2.2)$$

Thus the evolution of the density profile is given by the convolution of the initial density profile with that of an expanding point source at temperature T . We assume the initial density profile is a gaussian function, so that

$$n(\mathbf{r}, 0) = n(0, 0) \exp \left[- \left(\frac{x^2}{2\sigma_x(0)^2} + \frac{y^2}{2\sigma_y(0)^2} + \frac{z^2}{2\sigma_z(0)^2} \right) \right] , \quad (2.3)$$

where $\sigma_{x,y,z}(0)$ are the RMS widths along the cartesian axes. With this initial distribution, the density profile remains gaussian at all times. The RMS widths evolve as,

$$\sigma_{x,y,z}^2(t) = \sigma_{x,y,z}^2(0) + \frac{k_B T}{m} t^2 . \quad (2.4)$$

Plotting measurements of σ^2 against t^2 allows us to extract the temperature from the gradient of a linear fit to equation 2.4. Resonant absorption imaging heats the cloud, so we build an expansion sequence by repeating experiments with different expansion times. Each sequence provides a temperature measurement for given experimental conditions, and by repeating sequences (typically 5-10 times) a mean temperature and uncertainty is deduced.

Figures 2.13a shows a set of absorption images taken 0.2 – 1.5 ms after releasing atoms from a compressed-MOT (CMOT). In the CMOT step, the (axial) magnetic field gradient was ramped linearly from 14 G/cm to 32 G/cm in the 3 ms before release, the detunings of the trapping light were switched to -2.7Γ (cooling) and -2.4Γ (repump), and the total laser power was reduced by 60%. The ballistic expansion data are shown in figure 2.13b; fitting to equation 2.4 provides $T_z = 1.01(1)$ mK and $T_y = 0.693(8)$ mK. This compares with $T_z = 1.18(2)$ mK and $T_y = 1.04(4)$ mK after release from a MOT. In figure 2.13c the number of atoms in the field of view is plotted against expansion time. If there are no artefacts in the measurement, this number should be a constant. In reality, we see a slight increase from $t = 0.2$ ms – 0.5 ms, which we attribute to the residual magnetic field present at the atoms at these early times after turning off the coils.. From $t = 0.5$ ms onwards the number of atoms remains constant within about 10%, confirming that the magnetic field is well controlled.

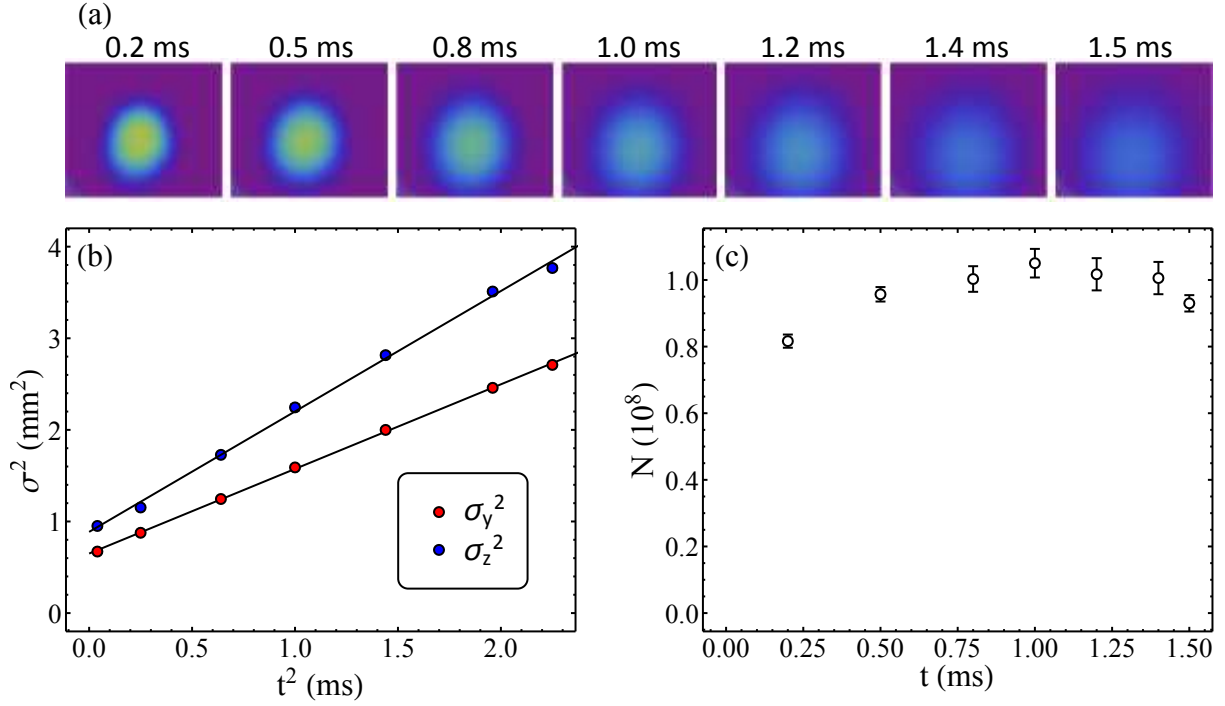


Figure 2.13: Ballistic expansion of the atomic cloud from the compressed MOT. (a) Absorption images labelled by time after release. (b) Spatial widths of the cloud. Solid lines show fits to equation 2.4 (c) Number of atoms in the field of view.

When releasing from a MOT made with the external coils, the observed number of atoms fell by a factor 5 between 0.2 and 1.5 ms as a result of the induced eddy currents.

2.8 Summary

This chapter has described improvements and modifications to the lithium MOT. A redesign of the optical setup reduced the overall laser power required for the experiment, and simplified alignment work. An improved oven design was implemented, and fluorescence detection was installed to calibrate and monitor the atomic beam. In-vacuum coils were designed and installed to allow rapid switching of the magnetic fields at the MOT location, which is essential for effective sub-Doppler cooling. After releasing from a compressed-MOT, there are typically 1×10^8 atoms available, with temperatures $T_x = 0.693(8)$ mK and $T_y = 1.01(1)$ mK. These are the starting conditions for the experiments in the following chapters.

Chapter 3

Raman Grey Molasses

In this chapter, I motivate our use of Raman Grey Molasses for sub-Doppler cooling ^7Li , and give an overview of laser cooling with dark states. I describe our experimental procedure and results, and link these to an intuitive picture using eigenstates of the atom-light Hamiltonian.

3.1 Motivation

There are two important reasons for using Raman grey molasses in our experiment. Firstly, the potential depth of the microwave trap for lithium is around $250\,\mu\text{K}$ when using the maximum intended input power of 1 kilowatt ¹. This is a factor 4 below the temperature of atoms in the MOT. Second, if we are to use lithium as a sympathetic coolant, then a high initial phase-space density is favourable. Whilst a narrow line MOT has been demonstrated for ^6Li at a temperature of $60\,\mu\text{K}$ [87], this requires light in the ultraviolet where laser power is limited. Sub-Doppler cooling on the D_2 line has also been demonstrated with far-detuned light [88], but only achieves temperatures of $100\,\mu\text{K}$ and cools at most 45% of atoms loaded into the MOT. Since Raman molasses is both efficient and convenient, it is our method of choice.

¹The details of the trapping potential are explained in section 5.3.

3.2 Background on Laser Cooling

3.2.1 Doppler Cooling

Laser cooling of atoms was initially proposed and understood in the so-called Doppler cooling regime. In this picture, photons provide discrete momentum kicks to the atoms. Absorption of a photon with wavevector $\mathbf{k}$ provides a momentum kick $\hbar\mathbf{k}$, and a spontaneous emission provides a momentum kick which can be in any direction, but the probability distribution is inversion symmetric in momentum space.

In one dimension, a pair of counter-propagating, red-detuned beams, results in a damping force. This is caused by the Doppler effect; an atom with a non-zero velocity preferentially absorbs from the counter-propagating beam, which it sees as closer to resonance with the transition. When the Doppler shift kv is small compared to the laser detuning δ , the damping force is well described by $F = -\beta v$ [89], where

$$\beta = -\hbar k^2 \frac{8s_0\delta/\Gamma}{(1 + s_0 + (2\delta/\Gamma)^2)^2} . \quad (3.1)$$

Here, s_0 is the on-resonance saturation parameter I/I_s , with I_s the light intensity at which the excited state population reaches 1/4. This cooling process must be considered alongside the random momentum kicks provided by scattering processes, which we can think of as a random walk in momentum space. By balancing these two processes, one can derive the minimum temperature achievable, T_D , known as the Doppler cooling limit:

$$k_B T_D = \frac{\hbar\Gamma}{2} , \quad (3.2)$$

where Γ is the spontaneous emission rate for the cooling transition. For the alkali atom D_2 lines, Γ is around $2\pi \times 6$ MHz, and for lithium $T_D = 142 \mu\text{K}$.

Doppler cooling requires only a two-level atomic system, and the cooling mechanism arises from preferential scattering of Doppler shifted light beams. Due to the multi-level nature of atoms, and the polarisation dependence of the light-matter interaction, it is possible to cool below the Doppler limit. I shall now outline some of the known mechanisms of sub-Doppler cooling.

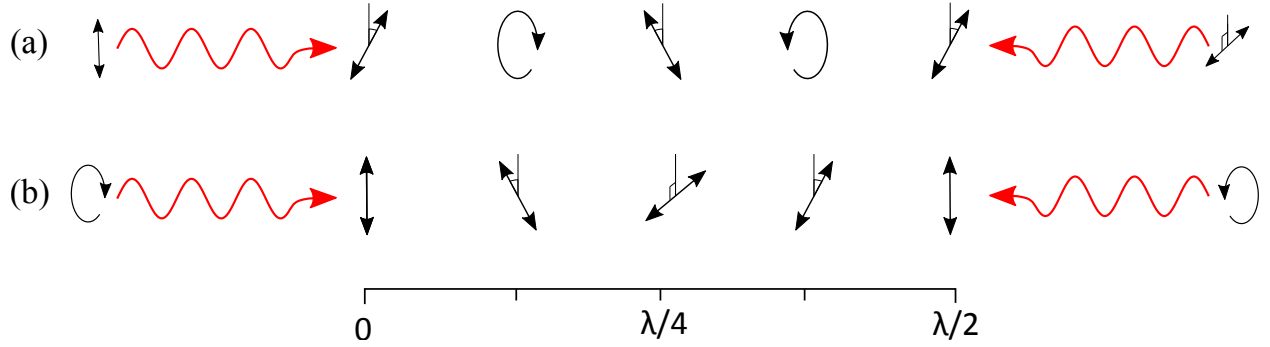


Figure 3.1: Polarisation configurations in one dimension. (a) The lin- $\perp$ -lin configuration. (b) The σ^+/σ^- configuration.

3.2.2 Sub-Doppler Cooling

The first experiments demonstrating laser cooling and trapping of atoms were performed on sodium atoms [90, 91] and achieved temperatures close to (but above) the Doppler limit. However, in 1988 a key paper by Bill Phillips and co-workers showed strong evidence that the Doppler cooling limit had been violated by nearly an order of magnitude in a sodium optical molasses [92].

Further experiments followed in 1989 [93], and a thorough explanation of the mechanisms involved was delivered by Dalibard and Cohen-Tannoudji [94], and by Chu and coworkers [95] in the same year. They considered two configurations of laser polarisation for a one-dimensional system, comprising two counter-propagating beams. The two scenarios are illustrated in figure 3.1. In the σ^+/σ^- configuration, the beams have opposite circular polarisation, resulting in a polarisation which is linear everywhere but whose direction rotates about the beam axis. In the lin- $\perp$ -lin configuration, the beams have orthogonal linear polarisations; this results in a polarisation which oscillates between linear and circular. Although both can lead to sub-Doppler cooling, there are subtle differences in the mechanisms.

The lin- $\perp$ -lin configuration leads to so-called Sisyphus cooling, and is best illustrated by a $J = 1/2 \rightarrow J' = 3/2$ system as shown in figure 3.2. The laser light is red-detuned from the transition, and the two Zeeman levels of the ground state have a light shift which is negative everywhere. Since the polarisation of the laser field varies between linear and circular, and the transition strengths depends on the polarisation, this light shift oscillates with position. As a result, an atom with $m_J = +1/2$ at a position of σ^+ polarisation is made to climb a

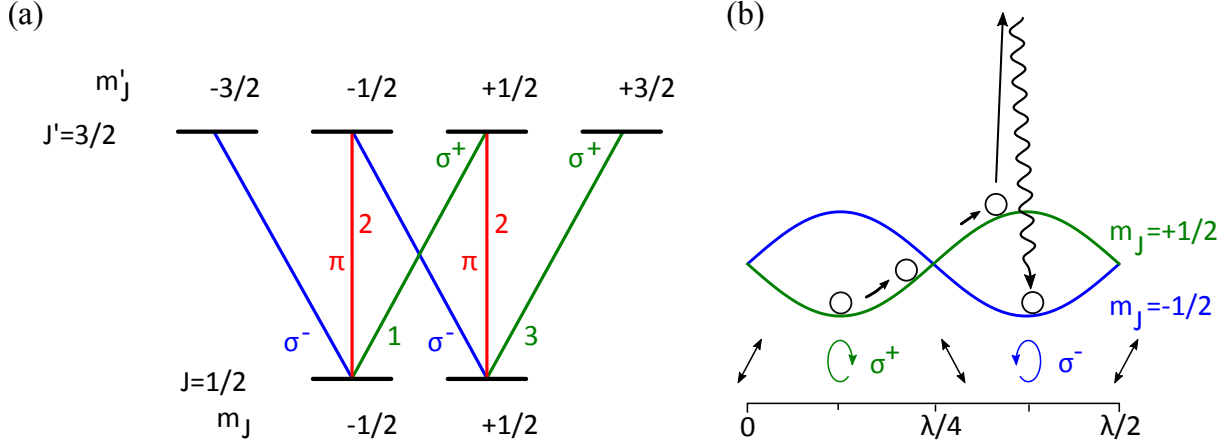


Figure 3.2: The Sisyphus cooling process in a $J = 1/2, J' = 3/2$ system. (a) Transitions that can be driven, together with their relative strengths. (b) Light shifts of the $m_J = \pm 1/2$ ground states as a function of position in the light field, illustrating how the Sisyphus mechanism works.

potential energy hill, losing kinetic energy. At the top of the hill, the laser polarisation is σ^- and the atom is optically pumped back to the $m_J = -1/2$ state. The σ^- polarisation corresponds to a potential minimum for the $m_J = -1/2$ state, and so there has been a net removal of kinetic energy from the atom. Figure 3.2b demonstrates this idea. In reality, the cooling process only works because optical pumping takes time so lags behind the motion of the atom - the atom is not usually in the state to which it would be optically pumped if it were at rest. Since the momentum loss does not come from the absorption of photons, the limiting temperature can be below the Doppler limit and is instead given by the recoil momentum of a photon.

The mechanism for cooling in the σ^+/σ^- configuration is different and requires at least three Zeeman sub-levels in the ground state; the level scheme is illustrated in figure 3.3. Since the laser polarisation is linear everywhere, there is no spatially varying light shift in the ground state (the states change, but their energies do not). An atom at zero velocity is pumped to a superposition with equal populations in the $m_J = \pm 1$ states, so that it scatters equally from the two laser beams. Because the polarisation changes through space, a moving atom does not reach this equilibrium population, and instead there is an imbalance in the $m_J = \pm 1$ populations. It turns out that an atom moving towards the σ^+ (σ^-) beam, the population is biased towards $m_J = +1/2$ ($-1/2$). From the relative transition strengths shown in figure 3.3, we see that population in the $m_J = +1$ state will scatter six times

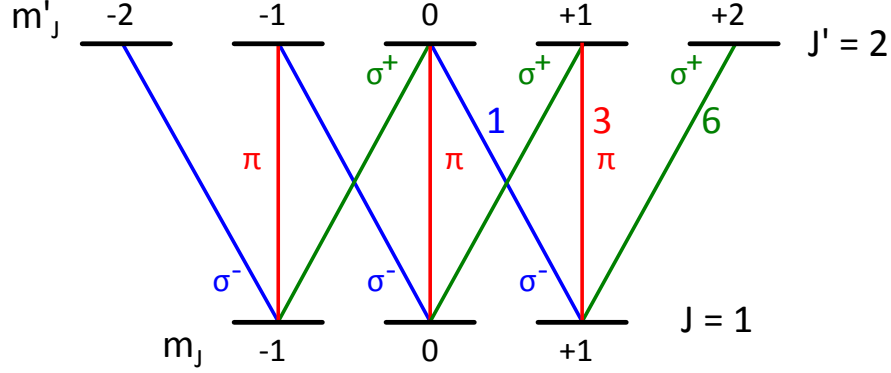


Figure 3.3: The $J = 1, J' = 2$ level system, the simplest for which polarisation gradient cooling is effective. Relative transition strengths for each polarisation are shown for the $m_J = +1$ ground state.

as many photons from the σ^+ beam compared with the σ^- beam. The net result is an additional damping force for atoms at low velocity, which can cool atoms below the Doppler temperature. The limit, like the Sisyphus cooling mechanism, is the recoil of a single photon.

3.3 Dark State Cooling

The use of dark states in laser cooling is motivated by two findings. Firstly, it was shown that metastable helium can be cooled below the recoil limit by velocity-selective coherent population trapping (VSCPT), by accumulating atoms in dark states that cannot couple to the light (dark states) provided the atoms have a velocity very close to zero [96, 97]. Second, Ketterle and co-workers showed in 1993 that the density in a MOT can be increased by an order of magnitude in a “dark-SPOT” trap, where atoms are decoupled from the cooling light at its centre [98]. This suppresses density-limiting mechanisms in a conventional MOT, where at high densities there are appreciable probabilities for re-absorption of scattered photons, or for collisions involving electronically excited atoms. Recently in our group, a similar density enhancement was obtained in a MOT with a scheme involving dark states, using light blue-detuned from the main cooling transition [99].

Before proceeding, we should first define what is meant by a dark state. Consider a set of ground and excited states $|g_i\rangle$ and $|e_j\rangle$, which are coupled by the dipole interaction $\mathcal{H}_{\text{light}}$ as shown in figure 3.4. The coupling coefficient between g_i and e_j is $\langle e_j | \mathcal{H}_{\text{light}} | g_i \rangle = c_{ij}$. A dark state, $|g_{\text{dark}}\rangle$, is defined as some superposition of the ground states which is an eigenstate of

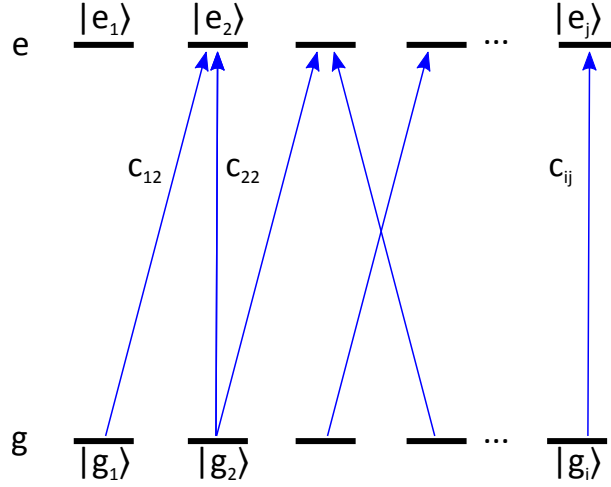


Figure 3.4: A selection of ground and excited states linked by the electric dipole interaction.

the Hamiltonian, where for all excited states $|e_j\rangle$,

$$\langle e_j | \mathcal{H}_{\text{light}} | g_{\text{dark}} \rangle = 0 \quad . \quad (3.3)$$

This is to say that $\mathcal{H}_{\text{light}}$ does not couple $|g_{\text{dark}}\rangle$ to any of the excited states, and so the dark state is fully decoupled from the light field. It is straightforward to show that there will always be dark states for a system of energy levels in which there are more ground states than excited states [100]. Writing $|g_{\text{dark}}\rangle = \sum_i a_i |g_i\rangle$, to satisfy 3.3 we require

$$\sum_i a_i c_{ij} = 0 \quad . \quad (3.4)$$

for all excited states $|e_j\rangle$. This constitutes a system of N_e equations for the N_g unknown coefficients a_i , where $N_g(N_e)$ is the number of ground (excited) states. Changing the polarisation of the light field just changes the coupling coefficients c_{ij} , and therefore the composition of the dark states. If we have $N_e > N_g$, then the system is over-constrained unless a sufficient number of equations are equivalent. In this case all the ground states are coupled to at least one excited state, and there are no dark states.

3.3.1 Grey Molasses

The mechanism of grey molasses cooling was first proposed by Weidemuller et al. [101] in 1994, who considered light *blue*-detuned from a $1 \rightarrow 1$ system. This means that the ground state ac Stark shifts are always positive, opposite to the red-detuned molasses discussed earlier. In the Doppler regime, blue-detuned light leads to heating, but the existence of dark states leads to the emergence of a sub-Doppler cooling mechanism. The $1 \rightarrow 1$ mechanism requires both polarisation and intensity gradients, which can be generated using a pair of counter-propagating linearly-polarised laser beams, with one polarization at an angle ϕ to the other (we call this lin- ϕ -lin). Atoms are optically pumped to dark states $|g_{\text{dark}}\rangle$ at all positions of the laser field, but can make non-adiabatic transitions back to a bright state, $|g_{\text{bright}}\rangle$. An approximate expression for the probability, P , of making such a transition can be found by following the formalism given in Messiah's book [102]:

$$P \approx \frac{|\hbar \langle g_{\text{dark}} | \frac{d}{dt} | g_{\text{bright}} \rangle|^2}{\Delta E^2} , \quad (3.5)$$

where ΔE is the energy difference between the dark and bright states. Substituting $\frac{d}{dt} = v \frac{d}{dx}$, we find that

$$P \approx v^2 \frac{|\hbar \langle g_{\text{dark}} | \frac{d}{dx} | g_{\text{bright}} \rangle|^2}{\Delta E^2} . \quad (3.6)$$

From equation 3.6, we see that non-adiabatic transitions tend to occur where the energies of the dark and bright states are closest. This means that, when the light is blue-detuned, atoms tend to lose kinetic energy before being pumped back into a dark state. In addition to this effect, slowly moving atoms tend to remain in dark states for longer, because $P \sim v^2$; this leads to an accumulation of atoms at low velocity in dark states, which represents a cooling force. This Sisyphus-like mechanism is illustrated in figure 3.5a.

It should be pointed out that for $\phi = 0$ or $\phi = \frac{\pi}{2}$ the cooling force vanishes; in the former case, there are no polarisation gradients, and so the transfer probability vanishes because the dark state is the same everywhere; in the latter case the light shift of the coupling states are the same everywhere and so no kinetic energy is removed ². However, in three dimensional

²Strictly this is only true for the $1 \rightarrow 1$ system

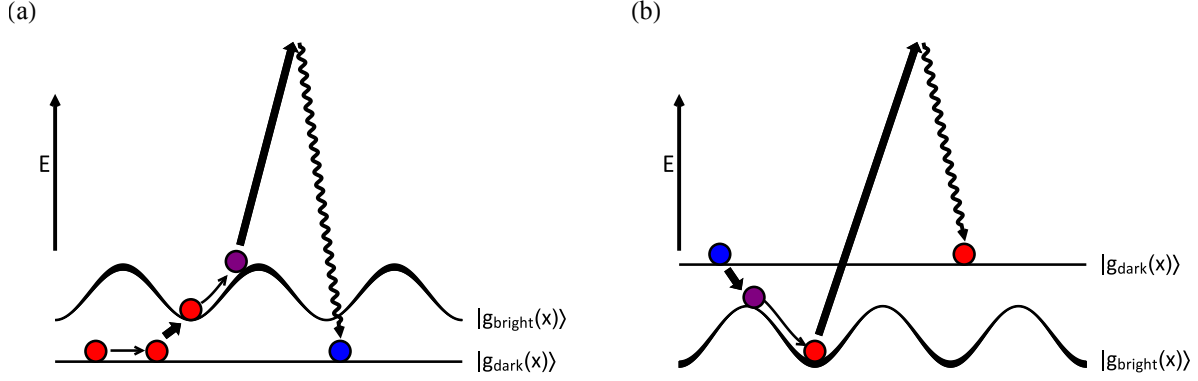


Figure 3.5: The grey molasses mechanism. (a) cooling with a blue-detuned molasses. (b) heating with a red-detuned molasses.

laser fields there are always gradients in both polarisation and intensity, meaning the cooling process can operate. With the laser field red-detuned (shown in figure 3.5b), the ground state ac Stark shifts are negative. Atoms tend to *gain* kinetic energy before being pumped back to bright states, resulting in heating.

Grey molasses was implemented first in 1995 with cesium [103], and temperatures ultimately reached as low as 1.1 μK , 2.4 times the recoil temperature [104]. Cesium has a closed $F = 3 \rightarrow F' = 2$ transition on the D_2 line, meaning the molasses light can be delivered by the MOT lasers; a repump laser was used to prevent optical pumping by off-resonant transitions. Interestingly, in [104], the authors note that a laser setup with 100 kHz linewidths gave measurably lower temperatures, than an 8 MHz linewidth system, by approximately 0.4 μK (36%).

3.4 Raman Grey Molasses

For the grey molasses mechanism to work effectively, atoms need to be reliably pumped to dark superpositions within the same hyperfine manifold. This requires a well-resolved $F \rightarrow F - 1$ transition from the ground state level with the smallest total angular momentum F_{min} , which ensures that scattering events can only return atoms to sublevels of F_{min} . For the alkali atoms, this can never work on the D_1 line because each hyperfine ground state has a corresponding excited state with the same value of F . On the D_2 line, this is possible

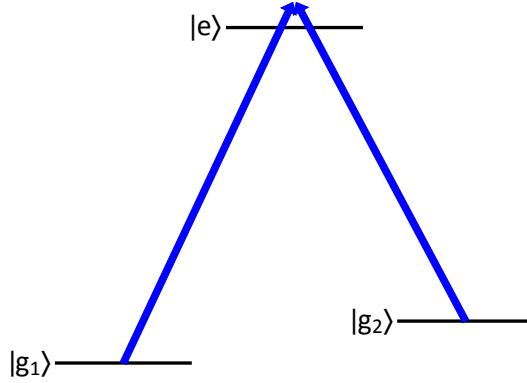


Figure 3.6: The general Λ system used for Raman grey molasses.

provided the nuclear spin $I \geq 3/2$, and the hyperfine splitting is large compared with the excited state linewidth. Only ^{87}Rb and Cs satisfy these criteria.

Raman molasses solves the problem of pumping to unwanted hyperfine ground states, by bringing these states into the cooling scheme with a second laser frequency. This method was originally applied using to ^{40}K [105] and ^{39}K [106], followed by ^7Li [107], ^6Li [108], Na [109], ^{87}Rb [110] and Cs [111].

The key building block of the cooling is the so-called Λ -system³ illustrated in figure 3.6, in which two ground states are linked to an excited state by separate laser frequencies. We know from section 3.3 that there must be a dark state $|g_{\text{dark}}\rangle$, and this is given by $\frac{1}{\sqrt{|a_1|^2 + |a_2|^2}} [a_1|g_2\rangle - a_2|g_1\rangle]$. With the lasers tuned closed to the Raman condition, the cooling mechanism in figure 3.5a can be reproduced. I shall illustrate this for ^7Li in the following sections.

3.4.1 The ^7Li D₁ Line

We now focus on the ^7Li D₁ line, for which Raman molasses has already been investigated by the group of Christophe Salomon [107]. Figure 3.7a shows the full hyperfine structure of this system. The two ground state manifolds $F = 2$ and $F = 1$ have a hyperfine splitting of 803.5 MHz, much larger than the hyperfine splitting of the $F' = 1, 2$ excited states, which is 91.8 MHz. In the original experiments, a bichromatic laser field addresses predominantly

³For this reason, the cooling is often termed Λ -enhanced grey molasses.

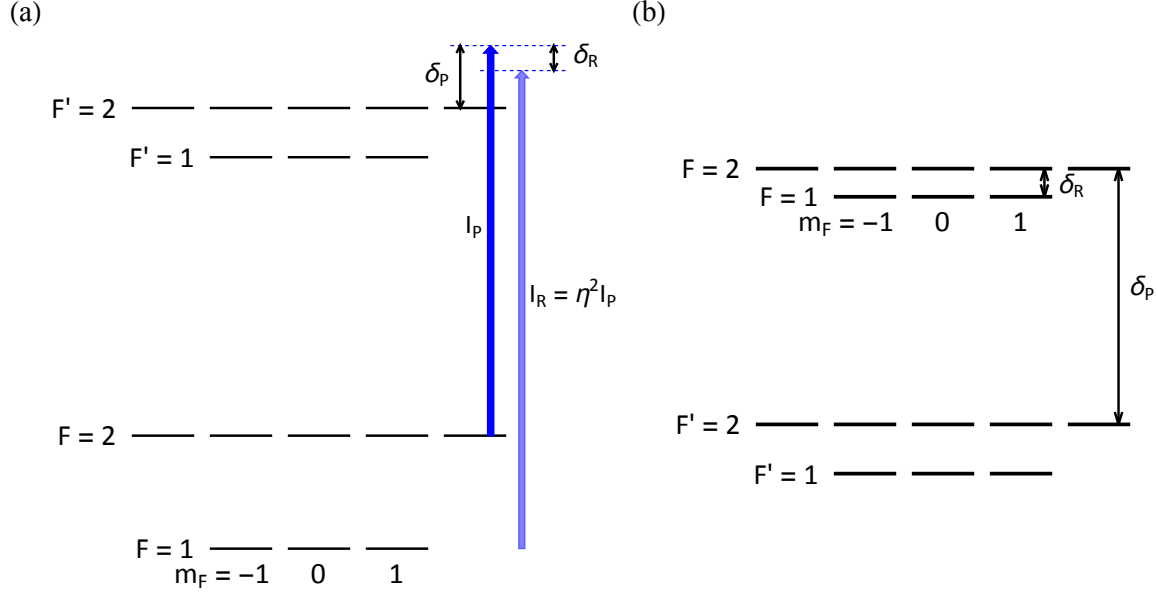


Figure 3.7: (a) Level scheme for Raman molasses in ^7Li . (b) Energy levels after transforming to the dressed basis.

the $F = 1, 2 \rightarrow F' = 2$ transitions. The $2 \rightarrow 2$ laser has higher intensity, and is labelled the “principal” beam with intensity I_P . The $1 \rightarrow 2$ light is labelled the “repumper” with intensity $I_R = \eta^2 I_P$. The detuning of the principal light is labelled δ_P , and the two-photon detuning is labelled δ_R , which is measured relative to the Raman condition. For optimal Raman molasses $|\delta_R| < \Gamma$. Since the hyperfine splitting in the ground state is much larger than the laser detunings, or the excited state splittings, we assume that the principal (repump) light addresses only the $F = 2(1)$ ground states.

In figure 3.7b, the same levels are shown in the dressed atom picture. The essence of this transformation is that the number of photons in the light field is incorporated into the state vector, which raises the ground state energies by the energy of the photon interacting with the state. When the laser detunings are positive, then dressed ground states are higher in energy than the excited states. This picture is used later to illustrate the cooling mechanism.

Dark States on the D_1 line

For now, we ignore the $F' = 1$ excited states, and calculate the dark states when the two lasers are exactly at the Raman condition $\delta_R = 0$. Figure 3.8a shows the relative coupling strengths for σ^+ polarised light. In this case, the dark states are

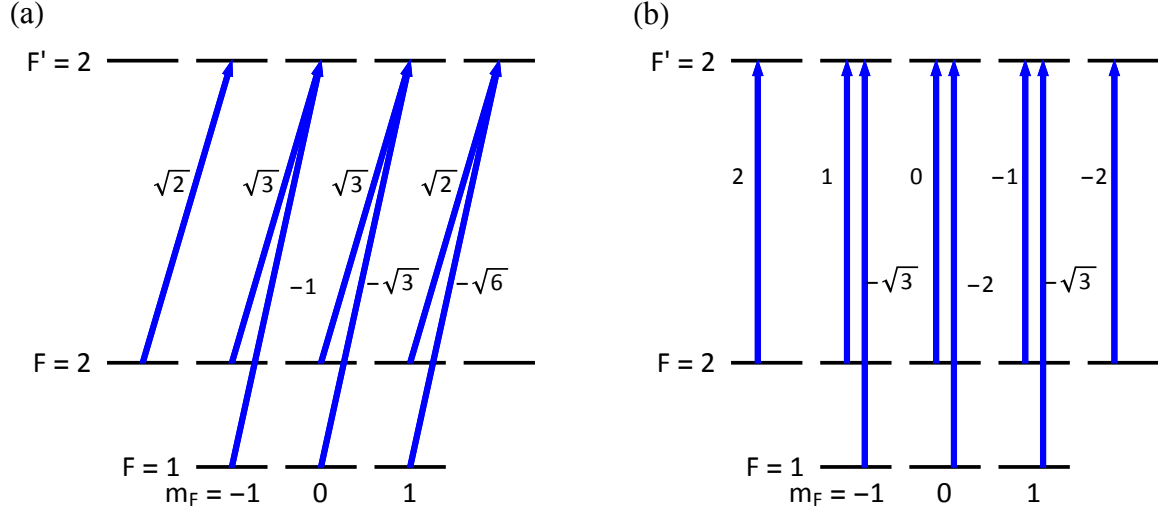


Figure 3.8: Coupling strengths between the $F = 1, 2$ ground states and the $F' = 2$ excited state on the lithium D_1 line. (a) σ^+ polarisation. (b) π polarisation.

$$|g_{\text{dark}}\rangle = \frac{1}{\sqrt{1+3\eta^2}}(|1, 1\rangle + \sqrt{3}\eta|2, 1\rangle) \quad , \quad (3.7)$$

$$\frac{1}{\sqrt{1+\eta^2}}(|1, 0\rangle + \eta|2, 0\rangle) \quad , \quad (3.8)$$

$$\frac{1}{\sqrt{\eta^2+3}}(\sqrt{3}|1, -1\rangle + \eta|2, -1\rangle) \quad , \quad (3.9)$$

$$|2, 2\rangle \quad . \quad (3.10)$$

Figure 3.8b shows the same concept for π polarised light relative to the z -axis. There are now three dark states:

$$|g_{\text{dark}}\rangle = \frac{1}{\sqrt{3\eta^2+1}}(|1, 1\rangle - \sqrt{3}\eta|2, 1\rangle) \quad , \quad (3.11)$$

$$|2, 0\rangle \quad , \quad (3.12)$$

$$\frac{1}{\sqrt{3\eta^2+1}}(|1, -1\rangle + \sqrt{3}\eta|2, -1\rangle) \quad . \quad (3.13)$$

It is possible to derive an analytic expression for the general case of elliptically polarised light, but it is cumbersome and not instructive. However, it is worth stating that circularly

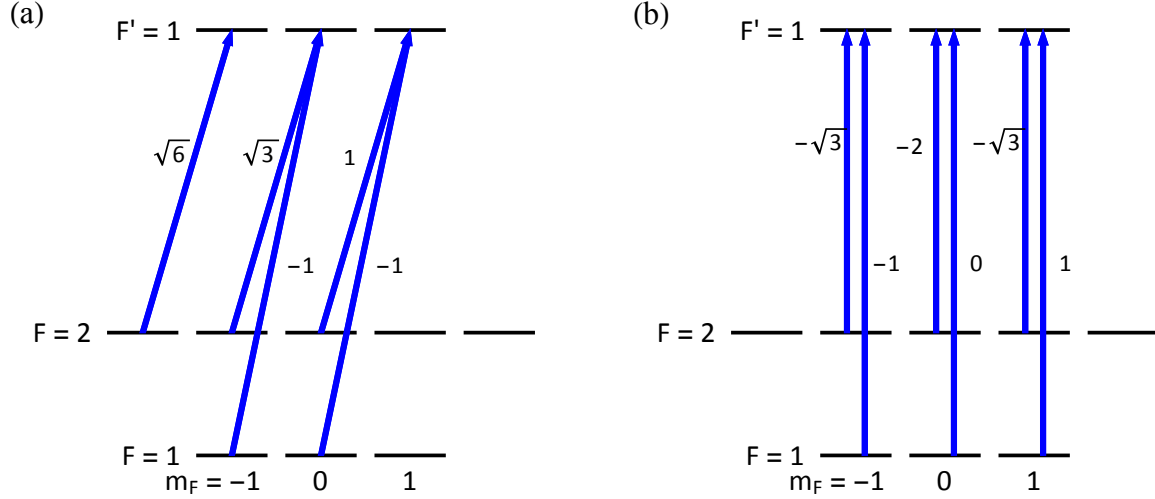


Figure 3.9: Coupling strengths between the $F = 1, 2$ ground states and the $F' = 1$ excited state on the lithium D_1 line. (a) σ^+ polarisation. (b) π polarisation.

polarised light represents a special case, and in the general case of elliptically polarised light there are three dark states.

Next, we consider the inclusion of the $F' = 1$ excited states. These levels are further detuned from the laser light by about 15Γ , which is not far enough that we can fully neglect their effect. Figures 3.9a-b show the various couplings from the ground states to the $F' = 1$ excited states, scaled for direct comparison with figures 3.8a-b. There are multiple interactions to consider for each ground state, and so it would appear that finding the dark states is much more difficult. However, we can make progress by continuing with the reasoning of the previous section. The laser field does not directly connect any two excited states, and so we can effectively treat each set independently. Hence it follows that if a state is dark when coupled to both excited state manifolds, it must be dark when coupled to each set individually. For this to be true, we require that the relative transition strengths satisfy

$$\frac{\langle e_1 | \mathcal{H}_{\text{light}} | g_2 \rangle}{\langle e_1 | \mathcal{H}_{\text{light}} | g_1 \rangle} = \frac{\langle e_2 | \mathcal{H}_{\text{light}} | g_2 \rangle}{\langle e_2 | \mathcal{H}_{\text{light}} | g_1 \rangle} \quad (3.14)$$

or, for the example in question:

$$\frac{\langle 1', m' | \mathcal{H}_{\text{light}} | 2, m \rangle}{\langle 1', m' | \mathcal{H}_{\text{light}} | 1, m \rangle} = \frac{\langle 2', m' | \mathcal{H}_{\text{light}} | 2, m \rangle}{\langle 2', m' | \mathcal{H}_{\text{light}} | 1, m \rangle} \quad (3.15)$$

In words, the relative transition strengths of $g_1 \rightarrow e_1$ and $g_2 \rightarrow e_1$ must be the same as the relative strengths of $g_1 \rightarrow e_2$ and $g_2 \rightarrow e_2$. If this is not the case, then we cannot form a superposition state of g_1 and g_2 which is truly dark. This rule is simply a restatement of the conclusions of section 3.3; the common ratio reduces the number of unique equations, meaning a solution is possible. In practice it is only applicable when the energy splitting of the excited states is small or comparable to the laser detuning(s), because otherwise the couplings that destroy the dark state are too far off resonance to be relevant.

Writing the matrix elements in terms of the Wigner 3-j and 6-j symbols, we obtain from (3.14),

$$\frac{\begin{Bmatrix} J' & F'_1 & I \\ F_2 & J & 1 \end{Bmatrix} \times \begin{pmatrix} F'_1 & 1 & F_2 \\ -m'_{F1} & q & m_{F2} \end{pmatrix}}{\begin{Bmatrix} J' & F'_1 & I \\ F_1 & J & 1 \end{Bmatrix} \times \begin{pmatrix} F'_1 & 1 & F_1 \\ -m'_{F1} & q & m_{F1} \end{pmatrix}} = \frac{\begin{Bmatrix} J' & F'_2 & I \\ F_2 & J & 1 \end{Bmatrix} \times \begin{pmatrix} F'_2 & 1 & F_2 \\ -m'_{F2} & q & m_{F2} \end{pmatrix}}{\begin{Bmatrix} J' & F'_2 & I \\ F_1 & J & 1 \end{Bmatrix} \times \begin{pmatrix} F'_2 & 1 & F_1 \\ -m'_{F2} & q & m_{F1} \end{pmatrix}} . \quad (3.16)$$

Here, I and J refer to the nuclear spin and total electronic angular momentum respectively, with q describing the polarisation of the light field.

For the D₁ line in ⁷Li, $J' = J = 1/2, I = 3/2, F_2 = F'_2 = 2, F'_1 = F_1 = 1$. With linearly polarised light, we set $q = 0$, and find that (3.16) cannot be satisfied. Therefore there can be no truly dark eigenstates if the laser polarisation is linear. For circularly polarised light, $q = \pm 1$, and we find that (3.16) is satisfied for all relevant transitions. The dark states are identical to those given by equations 3.7-3.10.

Eigenstate calculations

To illustrate the Raman molasses mechanism, we work in the basis where the field-free atomic states $|F, m_F, I, J, L, S\rangle$ are dressed by the photons in the light field. The electron's orbital, spin, and total angular momentum quantum numbers are denoted by L, S, J respectively; I refers to the nuclear angular momentum and F is total atomic angular momentum quantum number, with z-axis projection m_F . For clarity, we refer to these states as $|F', m'\rangle$ and

$|F, m\rangle$, with the primes used to differentiate a $^2P_{1/2}$ excited state from a $^2S_{1/2}$ ground state.

We describe the Hamiltonian for the system with three terms:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 \quad . \quad (3.17)$$

The first term represents the dressed “zero-field” energies of the system, which is diagonal in our chosen basis:

$$\begin{aligned} \frac{\mathcal{H}_0}{\hbar} = & \sum_{F,m,F',m'} \delta_P |F=2, m\rangle \langle F=2, m| \\ & + (\delta_P + \delta_R) |F=1, m\rangle \langle F=1, m| \\ & - \delta_{\text{HFS}} |F'=1, m'\rangle \langle F'=1, m'| \quad . \end{aligned} \quad (3.18)$$

This measures the energies of the dressed atomic levels relative to the $^2P_{1/2}, F'=2$ states; $\delta_{\text{HFS}} = 91.8 \text{ MHz}$ is the hyperfine splitting of the $^2P_{1/2}$ state. The second and third terms are the dipole interaction terms for the strong and weak beams respectively:

$$\mathcal{H}_1 = \sum_{F',m,m'} \hbar \Omega_{F,F',m,m'} |F', m'\rangle \langle F=2, m| + \text{h.c.} \quad , \quad (3.19)$$

$$\mathcal{H}_2 = \eta \times \sum_{F',m,m'} \hbar \Omega_{F,F',m,m'} |F', m'\rangle \langle F=1, m| + \text{h.c.} \quad . \quad (3.20)$$

Equations 3.19 and 3.20 are constructed such that the strong (weak) beams only address the $F=2$ ($F=1$) ground states. The parameter η fixes the ratio of the laser field amplitudes, such that $\eta^2 = I_2/I_1$. The couplings $\Omega_{F,F',m,m'}$ are the Rabi frequencies between the ground and excited states:

$$\hbar \Omega_{F,F',m,m'} = \boldsymbol{\mathcal{E}} \cdot \langle F', m'| \boldsymbol{d} |F, m\rangle \quad . \quad (3.21)$$

Here the dipole moment $\boldsymbol{d} = -e\boldsymbol{r}$ is the dipole operator, and $\boldsymbol{\mathcal{E}}$, e and $\boldsymbol{r}$ refer to the electric field vector, the electron charge, and the electron position vector respectively.

The coupling terms are found by expressing the dipole $\mathbf{r}$ in terms of rank 1 spherical tensors C_q^1 :

$$\mathbf{d} = \begin{pmatrix} \frac{C_{-1}^1 - C_1^1}{\sqrt{2}} \\ \frac{C_{-1}^1 + C_1^1}{\sqrt{2}i} \\ C_0^1 \end{pmatrix} d \quad . \quad (3.22)$$

Note that $C_q^1(\theta, \phi) = \sqrt{\frac{4\pi}{3}} Y_q^1(\theta, \phi)$, $Y_q^1(\theta, \phi)$ being the normalised spherical harmonics. Application of the Wigner-Eckart theorem gives the result for each tensor operator:

$$\begin{aligned} \langle F', m', I', J', L', S' | dC_q^1 | F, m, I, J, L, S \rangle &= (-1)^{F'-m'} \begin{pmatrix} F' & 1 & F \\ -m' & q & m \end{pmatrix} \\ &\times \langle F', m', I', J', L', S' || dC^{(1)} || F, m, I, J, L, S \rangle \quad , \end{aligned} \quad (3.23)$$

where the first factor is a 3j-symbol, and the second is a reduced matrix element. Because the operator acts only in the space of the electronic angular momentum, the reduced matrix element can be reduced further:

$$\begin{aligned} \langle F', m', I', J', L', S' || dC^{(1)} || F, m, I, J, L, S \rangle &= (-1)^{F+I'+J'+1} \times \sqrt{(2F+1)(2F'+1)} \begin{Bmatrix} J' & F' & I \\ F & J & 1 \end{Bmatrix} \\ &\times (-1)^{J+L'+S'+1} \sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix} \\ &\times \langle L' || dC^{(1)} || L \rangle \times \delta_{S,S'} \delta_{I,I'} \quad . \end{aligned} \quad (3.24)$$

This procedure is broken into two steps: decoupling F back into the nuclear and electronic angular momentum I and J , and then decoupling J back into orbital and spin angular momentum of the electron, L and S . This leaves the reduced matrix element $\langle L' || dC^1 || L \rangle$, which we can relate to the spontaneous decay rate of the excited state, $\Gamma_{L' \rightarrow L}$:

$$|\langle L' || dC^{(1)} || L \rangle| = \sqrt{2L' + 1} \sqrt{\frac{3\epsilon_0 \hbar \lambda^3 \Gamma_{L' \rightarrow L}}{8\pi^2}}. \quad (3.25)$$

Crucially here, $L' = 1$ refers to the excited state; λ is the wavelength of the transition in vacuum. Appendix A gives a derivation of this relation.

Putting all this together, we arrive at the expression

$$\begin{aligned} \langle F', m'_F, I', J', L', S' || dC_q^1 || F, m_F, I, J, L, S \rangle &= (-1)^{F+F'-m'_F} \\ &\times \sqrt{(2F+1)(2F'+1)(2J+1)(2J'+1)} \\ &\times \begin{Bmatrix} J' & F' & I \\ F & J & 1 \end{Bmatrix} \times \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix} \times \begin{pmatrix} F' & 1 & F \\ -m'_F & q & m_F \end{pmatrix} \times 3 \sqrt{\frac{\epsilon_0 \hbar \lambda^3 \Gamma}{8\pi^2}}. \end{aligned} \quad (3.26)$$

In the above, factors of (-1) which are common to all matrix elements in the basis are omitted and the δ -functions are omitted because the basis satisfies these by construction.

Equation (3.26) provides the expectation value of a spherical tensor operator between two of the angular momentum states. The laser polarisation picks out the relative contributions of each tensor operator by its scalar product with the position vector. We describe the electric field of a single laser beam, of frequency ω and propagating along the z-axis, with two counter-rotating waves:

$$\mathcal{E}(r, t) = \frac{1}{2} [\mathcal{E} e^{i(\omega t - kz)} + \mathcal{E}^* e^{-i(\omega t - kz)}]. \quad (3.27)$$

In terms of the light-matter interaction, the first term represents emission of a photon, and the second represents absorption. We make the rotating wave approximation, since we are near resonant with the transition ($\delta_{P,R}/\omega_{P,R} \ll 1$). The result of this is that for a given matrix element, only one of the terms in equation (3.27) contributes, depending on whether the states in question are linked by absorption or emission of a photon. In practice, it is sufficient to find the absorption terms permitted by the rotating wave approximation, and then impose the condition of hermiticity,

$$\langle \psi_1 | \mathcal{H} | \psi_2 \rangle = \langle \psi_2 | \mathcal{H}^\dagger | \psi_1 \rangle^* \quad , \quad (3.28)$$

in order to complete the interaction Hamiltonian.

Once the full Hamiltonian has been assembled, diagonalisation will yield the eigenstates and energies. To adiabatically follow the states, we start at an interaction strength of zero, and increment the field in steps up to the desired value $\mathcal{E}$. For the calculations presented in this chapter, we assume each laser beam has an intensity 55 times the saturation intensity of the ^7Li D₂ line, which is 2.56 mW/cm² [112]. This is the effective intensity provided by a single pair of counter-propagating beams in the experiments. Next, the interaction strength is fixed, and the polarisation is varied. For a 1D lin-perp-lin model, moving along the direction of propagation of the laser light varies the phase difference between the two counter-propagating components. For light propagating along the z -axis, the electric field polarisation written in Cartesian coordinates is

$$\mathcal{E} = \frac{|\mathcal{E}|}{\sqrt{2}}(e^{i\phi}, e^{-i\phi}, 0) \quad . \quad (3.29)$$

where $\phi = kz$. Figure 3.10a shows the calculated ac Stark shifts of the ground states, as a function of ϕ , for $\delta_P = +13\Gamma$, $\delta_R = 0$, and $\eta^2 = 0.04$. For simplicity, we have only included the $F' = 2$ excited states. When the phase ϕ is a multiple of $\frac{\pi}{2}$, the polarisation is linear, and there are 4 distinct sets of eigenstates. The three dark states have $\Delta E = 0$, since these are uncoupled from the laser field; there is a single weakly shifted state which is weakly coupled, and two pairs which are more strongly shifted. As the polarisation shifts, becoming circular at $\phi = \pi/4$, the weakly coupled state becomes truly dark, as expected from the discussion of figure 3.8.

In figure 3.10b, the results of the same calculation are plotted with inclusion of the $F' = 1$ levels. The ac Stark shifts are qualitatively different. As explained in the discussion of figure 3.9, none of the states are totally decoupled from the laser field at positions of linear polarisation, although there are still two states which are weakly coupled. The Stark shifts are identical at linear polarisation for the two most strongly shifted states, because these cannot couple to $F' = 1$ states by a π transition. At circular polarisation positions,

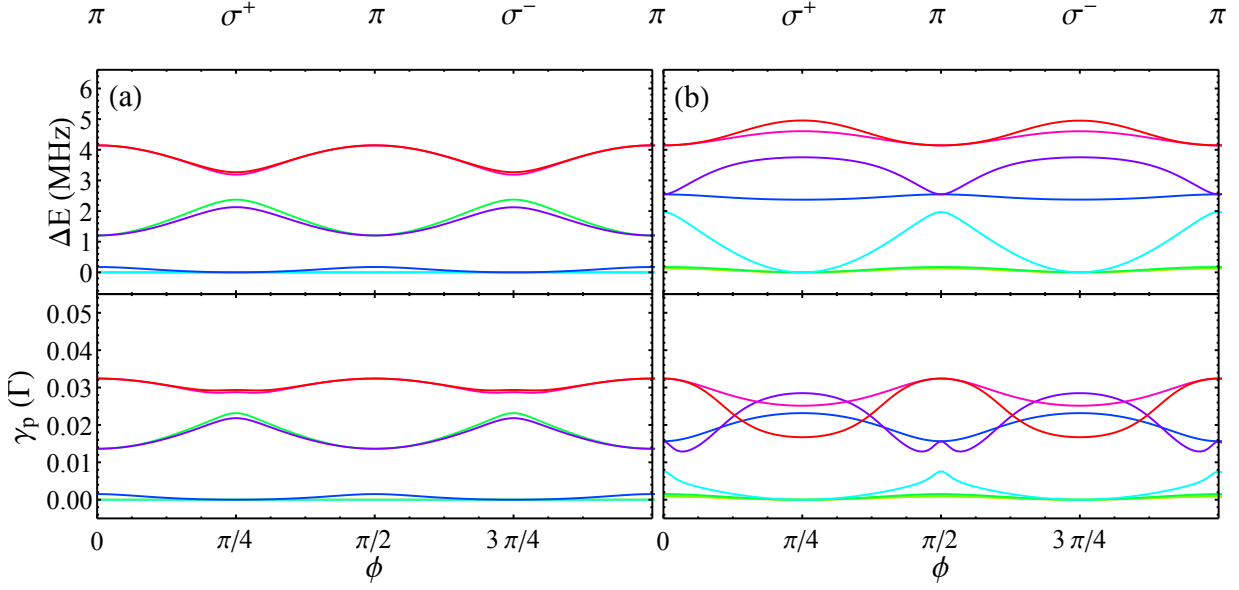


Figure 3.10: Ground state ac Stark shifts, ΔE , and optical pumping rates, γ_p , for ^7Li using a 1D lin- $\perp$ -lin model. (a) coupling only the $F' = 2$ excited states. (b) coupling both the $F' = 1$ and 2 excited states.

we recover four dark states, and the remaining four bright states are separated in energy.

At first, it may seem surprising that the $F' = 1$ levels have such an influence on the eigenvalues, because these states are detuned by an additional 15Γ compared to the $F' = 2$ states. To explain this, consider the energy shift predicted by second order perturbation theory scales as I/Δ , where I is the intensity of the laser beam and Δ is the detuning from the relevant excited state. The shift from coupling to the $F' = 1$ excited states is $\sim I_P/(28\Gamma)$ for our parameters, which is about a factor 10 larger than the shift due to the weak repump beam, $\sim \eta^2 I_P/(13\Gamma)$.

To elucidate the grey molasses mechanism, we follow the method of Kuhr, and calculate the optical pumping rate as a function of position in the laser field. The total scattering rate for an eigenstate $|\psi\rangle$, γ_0 , is given by its excited state fraction multiplied by the spontaneous emission rate:

$$\gamma_0 = \Gamma \sum_e |\langle e|\psi\rangle|^2. \quad (3.30)$$

Next, we find the branching ratio for decay back to the original state $|\psi\rangle$, P_ψ :

$$P_\psi = \frac{\sum_{e,q} |\langle \psi | e \rangle \langle e | d_q | \psi \rangle|^2}{\sum_{e,q,\phi_g} |\langle \psi | e \rangle \langle e | d_q | \phi_g \rangle|^2} . \quad (3.31)$$

The sums in equation 3.31 runs over all polarisation components q , and their totals compare the relative rate for decay to $|\psi\rangle$ with decay to all ground eigenstates $|\phi_g\rangle$, as given by the dipole operator d_q . The optical pumping rate to the other ground states, γ_p , is then found as

$$\gamma_p = \gamma_0(1 - P_\psi) . \quad (3.32)$$

The pumping rates γ_p of the dressed ground states of ^7Li are plotted in the lower panels of figure 3.10. When considering only the $F' = 2$ excited states, as in figure 3.10a, the situation is as follows. The weakly coupled states show weak optical pumping rates, since they have close to zero excited state population. Importantly, the remaining strongly coupled states show a positive correlation between ΔE and γ_p ; that is to say an atom in one of these states tends to be optically pumped at positions where its ac Stark shift is largest. Since this is true of all the bright states, we can see that on average, an atom will lose kinetic energy before being pumped to one of the dark states. This, in combination with the non-adiabatic transitions from bright to dark states, underpins the cooling process. Inclusion of the $F' = 1$ excited states, in figure 3.10b, complicates the situation somewhat. However, the correlation between ΔE and γ_p remains true for the strongly coupled state which lies closest in energy to the weakly coupled states, and so cooling can still occur.

3.5 Experiments

Frequency Offset Lock

To vary the single photon detuning of the molasses light, we use a frequency offset lock (FOL) [113], which stabilises the frequency difference between two lasers by measuring their optical beat note. A schematic of the electronics is illustrated in figure 3.11. Light from the MOT and molasses lasers is combined in a fibre splitter, and incident on a fast photodiode (Hamamatsu G4176-03). The beat note ($\sim 10\text{ GHz}$) in the laser intensity is picked up

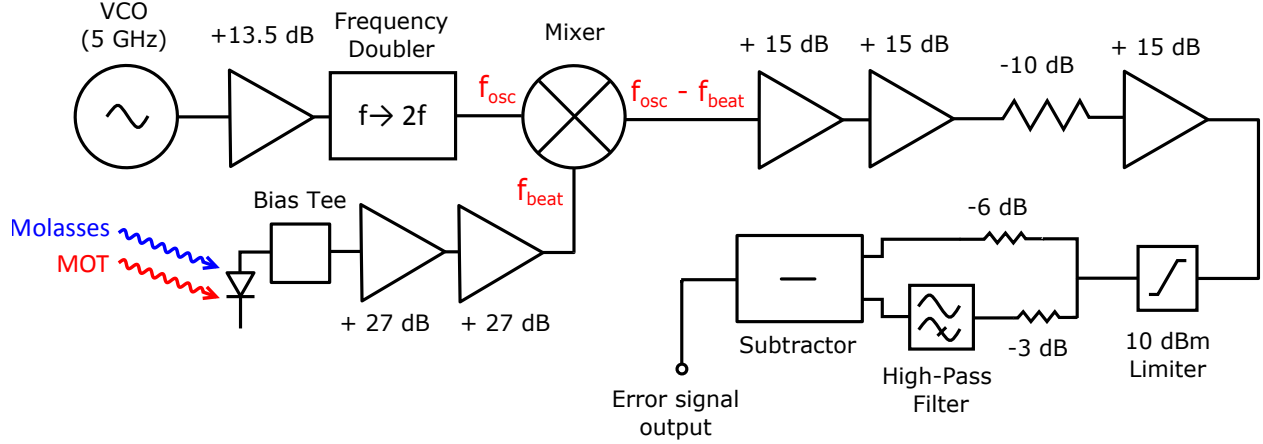


Figure 3.11: Schematic of the frequency offset lock electronics.

by the photodiode, amplified, and then mixed down with the reference oscillator frequency (Minicircuits ZX95-5400+, frequency doubled). The frequency difference between the optical beat note and the reference oscillator is locked to the 3dB point of a high pass filter. This locks the frequency offset between the lasers to a value determined by the reference oscillator, which can be tuned by several hundred megahertz with an external voltage. Since the MOT laser is stabilised by polarisation spectroscopy, the FOL allows robust and independent tuning of the molasses light over a broad range.

Figure 3.12 shows the error signal obtained from the FOL against the frequency at the mixer output. The narrow zero crossing at 171 MHz is determined by the filter properties. With a fixed external tuning voltage, the stability of the oscillator is better than 0.3 MHz over a 30 minute period, sufficient for our purposes. The total laser power at the photodiode is typically less than 0.1 mW.

Molasses Beam Alignment

For the experiments in this chapter, the molasses light comprised of six beams in σ^+/σ^- configuration, each beam having a $1/e^2$ radius 2.6 mm. Experiments typically operated with 30 mW per beam, which provides an on-resonance saturation intensity of $\sim 55I_{\text{sat}}$ when averaged over this radius⁴.

To optimise the alignment of the molasses beams I found the most effective method was to load a MOT and scan the molasses laser frequency over one of the D_1 transitions, periodically

⁴Here, $I_{\text{sat}} = 2.56 \text{ mW/cm}^2$, as given in appendix D of [112].

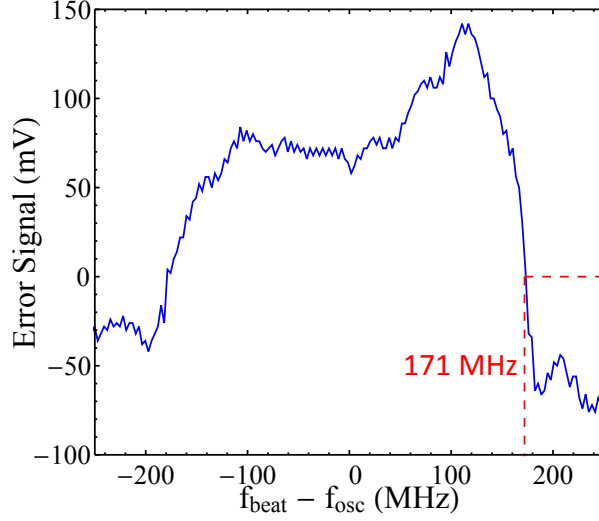


Figure 3.12: Error signal obtained from the frequency offset lock.

heating and disturbing the atomic cloud. An aperture was placed around each molasses beam before the first pass through the MOT chamber, reducing the beam size to below that of the MOT. In this way, only atoms in the beam path were disturbed. Alignment was achieved by blocking two of the beam axes, centering the forward beam, and then ensuring the returning beam was well overlapped.

3.5.1 Results from Raman Molasses Using the D_1 Line

The first experiments focussed on Raman molasses using the scheme illustrated in figure 3.7a. The timeline of the experiments is illustrated in figure 3.13. In each experimental run the MOT was allowed to load for 7 s before a CMOT stage was implemented as described in section 2.7. The MOT repump light was removed 0.1 ms prior to releasing the cloud, which was verified as sufficient to pump the entire population to the $F = 1$ ground states. Following a 0.3 ms wait to allow the trap magnetic fields to decay, the molasses light was pulsed on for a time τ_{mol} . Absorption images were taken after allowing the cloud to expand for a further 2–4 ms, to measure the temperature and number of atoms remaining. Ideally, this would be preceded by optically pumping all population out of the $F = 1$ ground states. However, this would require additional laser power and another AOM in the optical setup. Instead, the population was redistributed amongst the $F = 1, 2$ ground states by a 0.1 ms pulse of MOT and MOT repump light, at an intensity 2 mW/cm². The combination of optical pumping

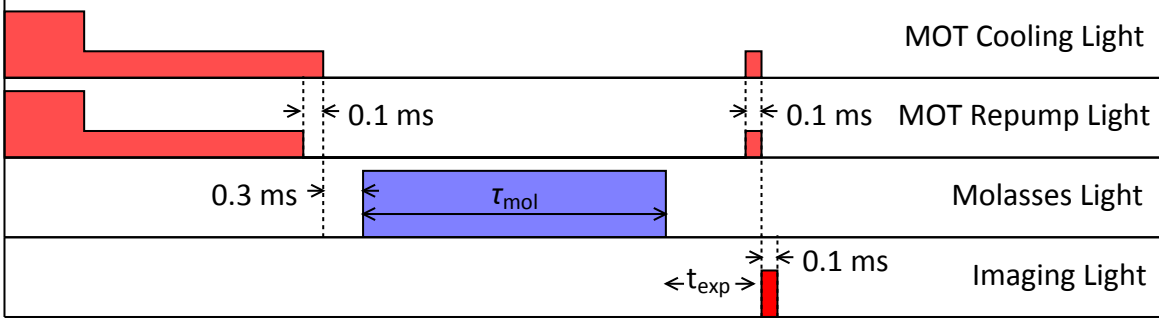


Figure 3.13: Timeline for molasses experiments.

and redistribution was found to cause a small change in the number of atoms visible by absorption imaging, reducing the measured atom number to 91(4)%.

Figure 3.14 shows the number of atoms and the temperature against the molasses pulse duration. The number of atoms was measured 4 ms after the molasses pulse was applied, at least 4.5 ms after release from the MOT. By this time, around 90% of the cloud is outside of the probe region if the cloud expands according to the temperature in the MOT. However, the atoms are quickly captured and cooled by the molasses meaning a greater number are visible. The temperature of the cloud is around 80 μK even after 100 μs of molasses, and roughly independent of the pulse length thereafter. The number of captured atoms appears to initially increase with pulse length before saturating after $\tau_{\text{mol}} = 1$ ms. In combination, these two results are surprising, as we might expect that cooling is not complete until the number of captured atoms saturates. In other studies of Raman molasses, there are a range of different observations; for ^{40}K [105] with a cooling intensity $\sim 14I_{\text{sat}}$, the molasses temperature was observed to decrease over the first 6 ms, whilst the number of captured atoms saturated over the same period. For experiments with Na [109] using a cooling light intensity of $\sim 30I_{\text{sat}}$, the temperature stabilised over 200 μs , but the number of captured atoms required about a factor 2 longer to stabilise. Given that our experiments use a higher intensity and a lighter atomic species, we might expect the cooling to occur more rapidly.

Figures 3.15a and b show the effect on the temperature of applying a bias magnetic field, along the z - and y - directions respectively. For these experiments, $\tau_{\text{mol}} = 2$ ms. Quadratic fits to the data provide temperature coefficients of $d^2T/dB_z^2 = 52(8)\mu\text{K G}^{-1}$ and $d^2T/dB_y^2 = 78(9)\mu\text{K G}^{-1}$. Following [51], we can model the effect of magnetic field on the

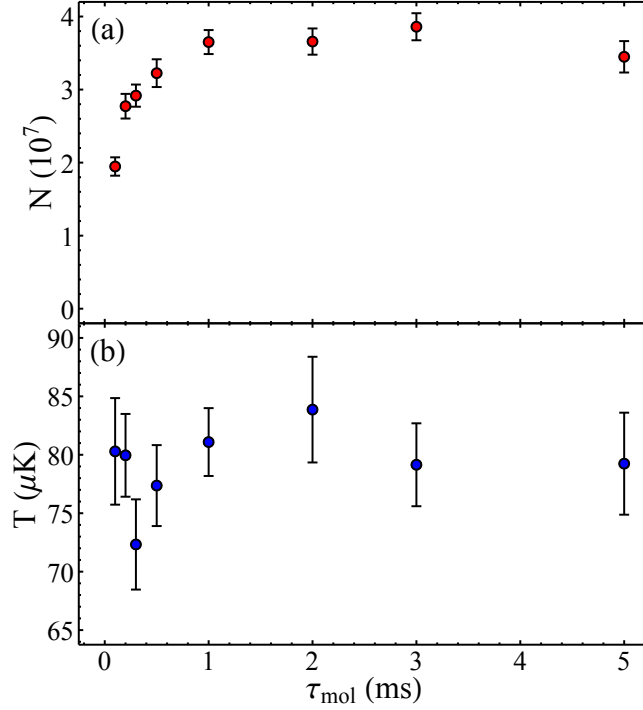


Figure 3.14: Performance of the D₁ Raman molasses as a function of the molasses duration. (a) Number of atoms remaining. (b) Temperature.

molasses temperature by considering the destabilisation of dark states. A magnetic field will cause spin precession which transfers atoms from dark to bright states at roughly the larmor precession frequency $\mu B/h$. This effectively limits the velocity of an atom to at least $\frac{\mu B \lambda}{4h}$, where $\lambda/4$ is the length scale over which the non-adiabatic transitions occur. From this, we can fit an approximate model for the temperature, $T(B) = T_{\text{min}} + \frac{\pi m}{8} (\frac{\mu B \lambda}{4h})^2 (B - B_0)^2$. Assuming the above model, we expect $d^2T/dB^2 \sim 37 \mu\text{K G}^{-1}$, which is within a factor two of the measurements.

The remaining experiments used the optimised magnetic field values, and $\tau_{\text{mol}} = 4$ ms. To measure the capture efficiency of the molasses, atoms were prepared in the $F = 1$ ground states and released from the CMOT, then redistributed and imaged 0.5 ms later. These absorption images were then compared with images taken 0.5 ms after a molasses pulse to measure the fraction captured.

Figure 3.16 shows the dependence of the molasses capture efficiency and temperature on single photon detuning, δ_P . As the molasses light approaches resonance with the $2 \rightarrow 2'$ transition, the temperature increases sharply and the capture efficiency drops. We can

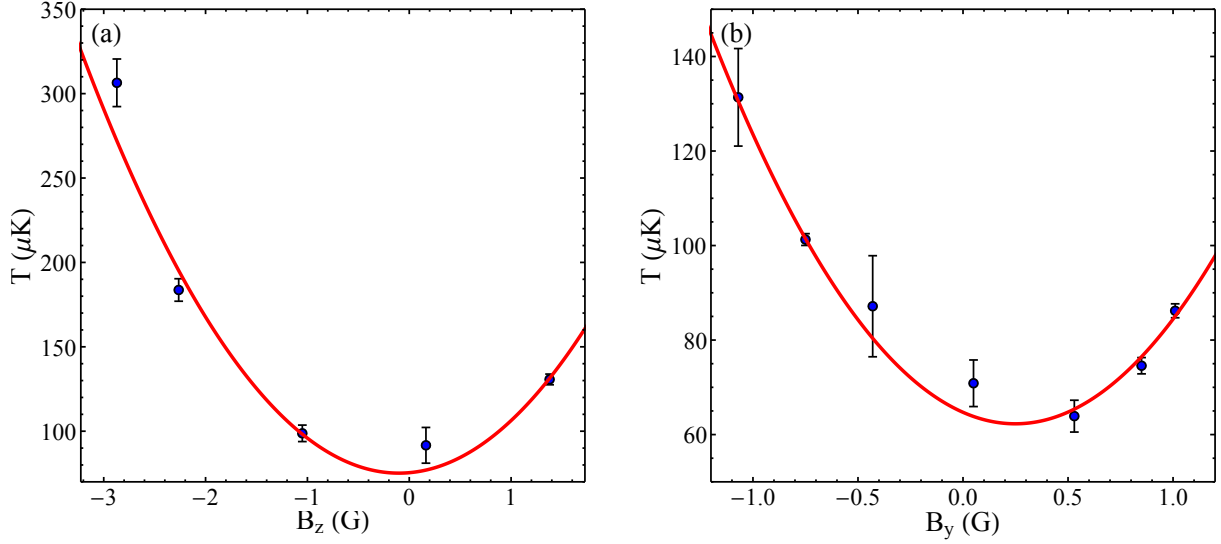


Figure 3.15: Temperature in the molasses versus applied offset magnetic fields. (a) Varying the axial magnetic field. (b) Varying the magnetic field along the axis that connects the MOT chamber to the microwave trap chamber. Solid red lines are fits to a quadratic function.

understand this as being the result of the photon scattering rate increasing, which heats the atoms. The peak capture efficiency occurs when $10\Gamma < \delta_P < 15\Gamma$, and then begins to fall, although the capture efficiency is still 20% at a detuning of 50Γ . The temperature remains below the Doppler limit for all $\delta_P > 5\Gamma$. Together, these results can be explained by noting that as the detuning increases, the excited state population decreases, and eventually the optical pumping processes are too slow to capture the entire atomic cloud.

Figure 3.17 shows the effect of the two-photon Raman detuning δ_R . We see here the narrow Raman resonance phenomenon, where the molasses mechanism is most effective. Close to $\delta_R = 0$, the temperature falls below the Doppler limit and the capture efficiency peaks, reaching 91(2)% at $\delta_R = +0.1\Gamma$. The temperature in the region around the Raman condition is shown in figure 3.17c; at $\delta_R = +0.17\Gamma$ the temperature is 41(1) μK . This minimum temperature is in agreement with that previously measured with ^7Li [107]. Increasing δ_R eventually leads to strong heating and an abrupt fall in the capture efficiency. For $\tau_{\text{mol}} = 4 \text{ ms}$, no atoms remain in the molasses between $1.5\Gamma < \delta_R < 2.5\Gamma$. In the dressed state picture, this can be understood as being the result of the weakly coupled states lying higher in energy than the bright states, meaning the non-adiabatic transfer tends to occur when at the top of ac Stark potential hills. Away from the Raman condition, either at negative or positive

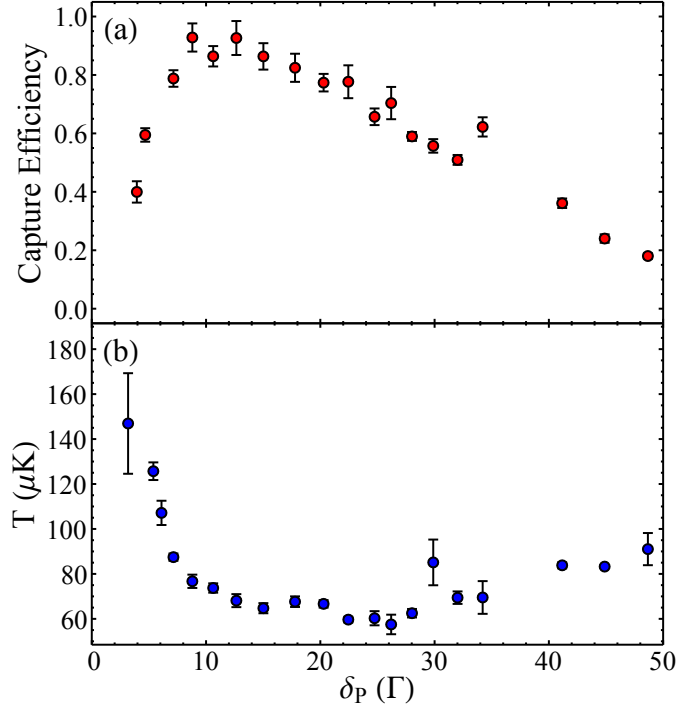


Figure 3.16: Dependence of the capture efficiency and temperature of the molasses stage on the single photon detuning δ_P . For these experiments, $\delta_R = +0.1\Gamma$.

δ_R , the molasses still cools the atoms to around 200 μK , a factor 5 colder than in the MOT. In this regime, the energy difference between the bright and dark states becomes too large for coherent population transfer, and instead we can consider the strong principal laser as producing a blue-detuned molasses, with the repump acting incoherently. In this case we still expect some cooling to occur, because optical pumping to the weakly coupled states still occurs preferentially at positions where the ac Stark shift is large and positive. However, the capture efficiency is much lower, around 30% for these data.

Figure 3.18 shows the effect of the total molasses power on the cooling process, for fixed η^2 . The temperature falls with intensity, at the expense of capture efficiency; the lowest temperature measured is 28(4) μK , but the capture efficiency is below 40% here. One might expect that by ramping the molasses intensity down from an initial “capture” value, we can decrease the final temperature without loss. However, I did not find improvement with this method, and so subsequent experiments were completed with the intensity held constant.

Figure 3.19 shows the effect of changing the ratio of the repump/principal powers, η^2 whilst fixing the total molasses power. The optimal temperature occurs at $\eta^2 \approx 0.01$, but

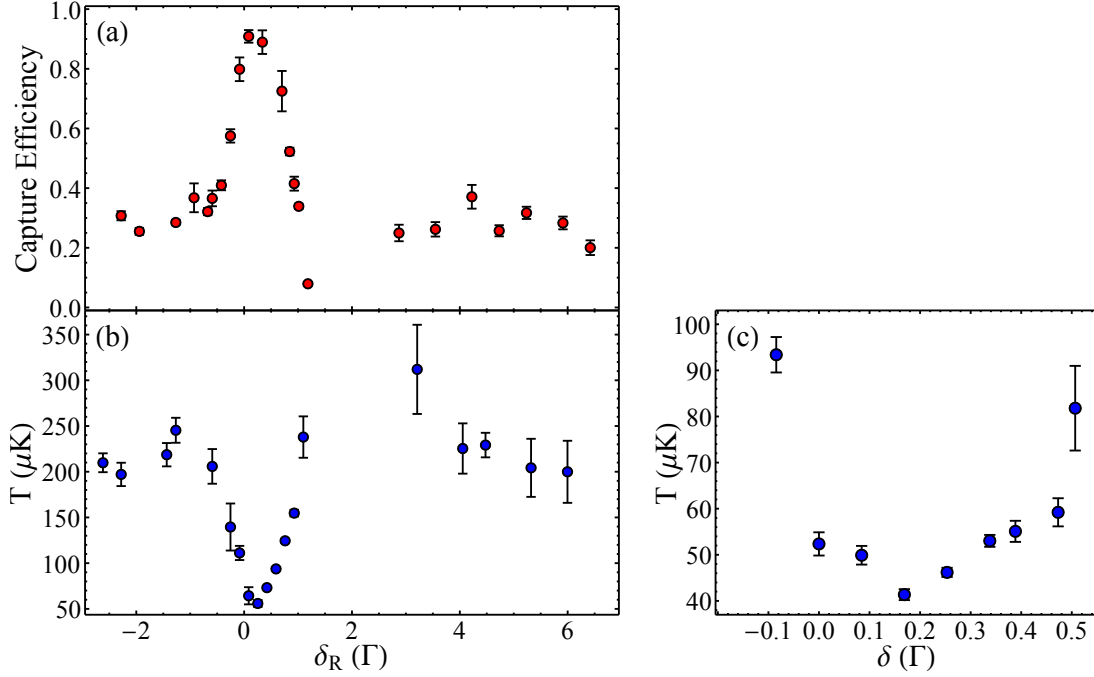


Figure 3.17: Dependence of the capture efficiency and temperature of the molasses on the Raman detuning δ_R . For these experiments, $\delta_P = +13.5\Gamma$. (c) Temperature measurements close to the Raman condition.

remains below the Doppler limit for all the data. This is in contrast with reference [107], where sub-Doppler cooling was only achieved for $\eta^2 < 3\%$. The capture efficiency is relatively independent of the power ratio for $\eta^2 > 0.01$, but drops when $\eta^2 < 0.01$.

3.5.2 Diffusion Measurements

By extending the hold time in the molasses, it was possible to observe diffusive behaviour of the atomic cloud due to the viscous confinement. Figure 3.20 shows data obtained with $\delta_P = +15\Gamma$, $\delta_R = +0.1\Gamma$. For $0 < \tau_{\text{mol}} < 100$ ms, the widths of the cloud (figure 3.20a) follow the diffusion law $\sigma^2 = 2D\tau_{\text{mol}}$, where D is the spatial diffusion constant. The values of D determined by fitting straight lines to the data at short τ_{mol} in the radial and axial directions are $D_x = 0.0277(7) \text{ cm}^2 \text{ s}^{-1}$ and $D_z = 0.030(1) \text{ cm}^2 \text{ s}^{-1}$ respectively. During this time the cloud is confined within the effective molasses volume, and the number of atoms remains constant. As the cloud expands beyond the region addressed by the molasses beams, the number of atoms begins to fall, and the observed width stops expanding. As can be seen

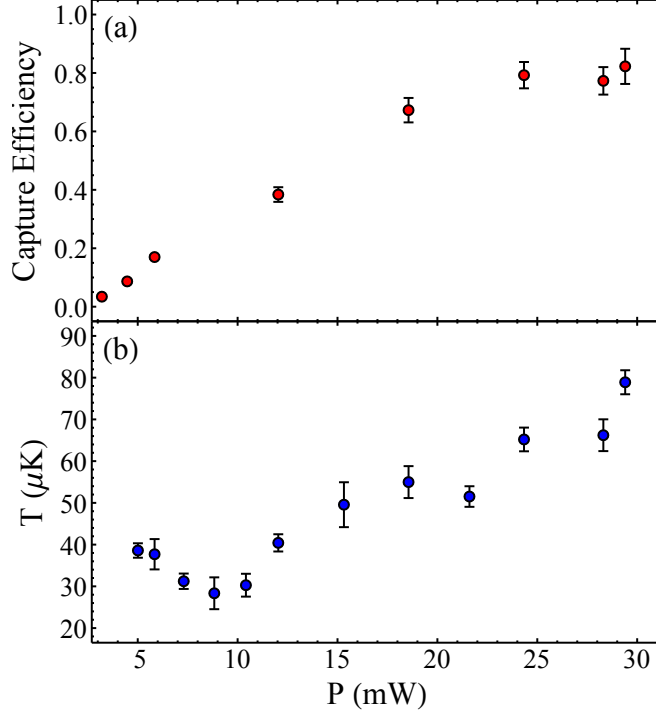


Figure 3.18: Dependence of the capture efficiency and temperature of the molasses on the total molasses power. For these experiments, $\delta_P = +13.5\Gamma$.

in figure 3.20c, there is a small drift velocity whilst the cloud is trapped in the molasses, of $0.77(2) \text{ cm s}^{-1}$ in the axial direction, and $0.117(5) \text{ cm s}^{-1}$ radially.

Diffusion in optical molasses has been studied in bright optical molasses in detail [114, 115]. We can make an analogy with Brownian motion and relate the measured spatial diffusion coefficient to the momentum diffusion coefficient, D_p , and the damping coefficient, α :

$$D_p = \frac{(k_B T)^2}{D_x} \quad , \quad (3.33)$$

$$\alpha = \frac{k_B T}{D_x} \quad . \quad (3.34)$$

Using the data in figure 3.20, and assuming a temperature of $60 \mu\text{K}$, we find $D_p/m^2 = 1.26 \times 10^3 \text{ m}^2 \text{ s}^{-3}$ and a velocity damping rate $\alpha/m = 2 \times 10^4 \text{ s}^{-1}$. The corresponding velocity decay time is $50 \mu\text{s}$, which supports the rapid cooling observed in figure 3.14. If the atoms are falling under gravity then we expect a critical velocity along z of $mg/\alpha = 0.5 \text{ cm s}^{-1}$,

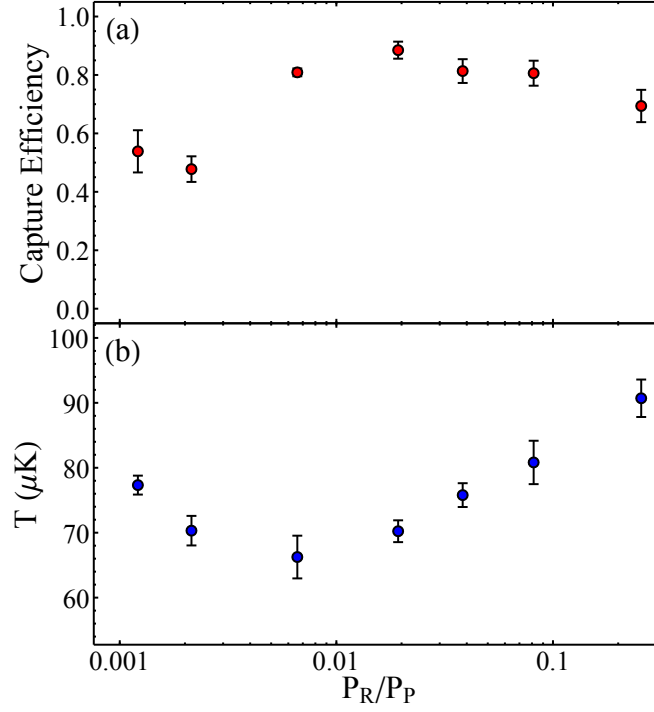


Figure 3.19: Dependence of the capture efficiency and temperature of the molasses on the repump power ratio. For these experiments, $\delta_P = +15\Gamma$, $\delta_R = +0.1\Gamma$.

but the drift velocity of the cloud is significantly larger than this along both axes. This is probably caused by an intensity imbalance between counter-propagating beams which can easily result in a net acceleration that is much larger than gravity.

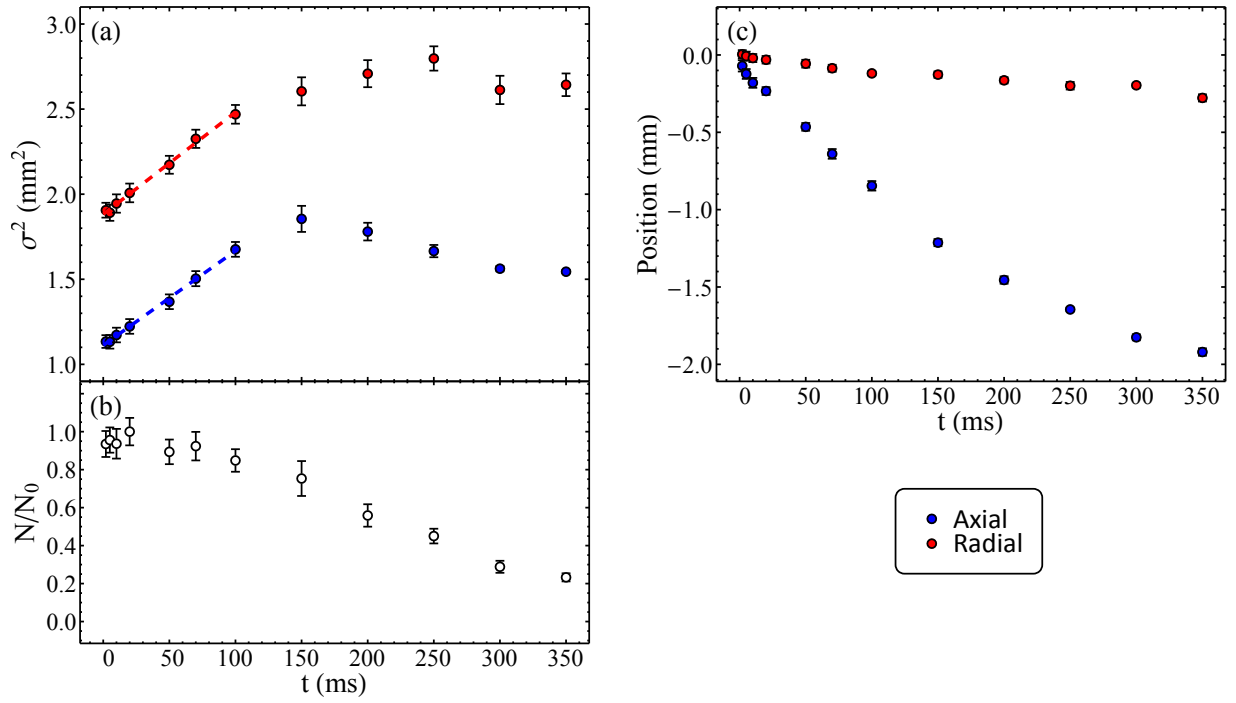


Figure 3.20: Diffusion of the atomic cloud in the molasses. (a) Mean square radii vs hold time. Dashed lines are linear fits to the data in these regions. (b) Fraction of atoms remaining. (c) Centre-of-mass coordinates.

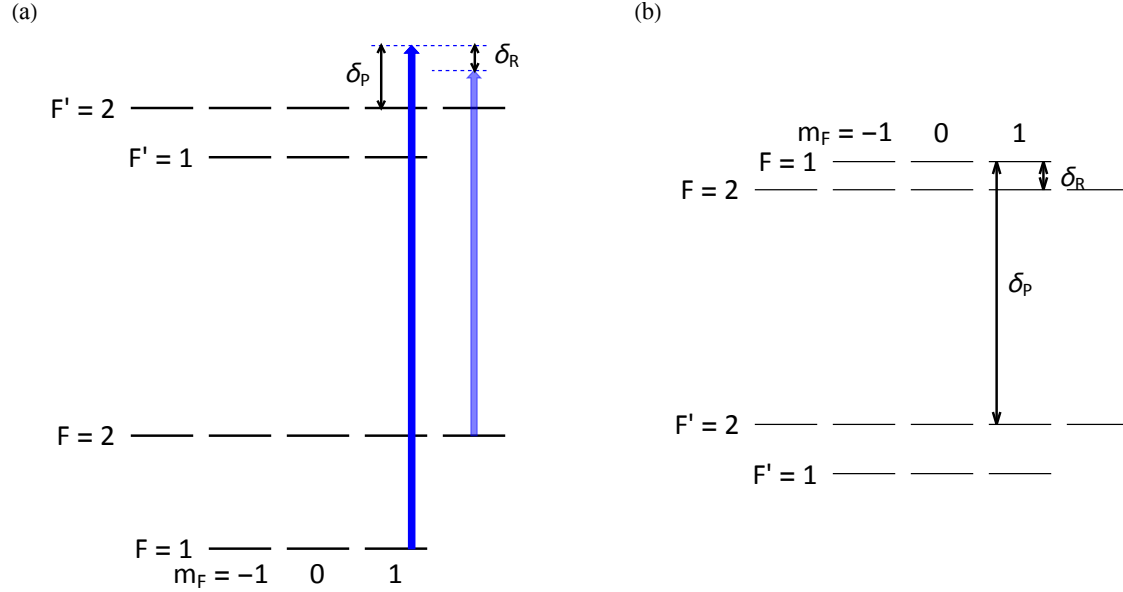


Figure 3.21: Energy scheme for inverted Raman molasses. (a) Bare atomic states and laser frequencies. (b) Dressed state transformation.

3.5.3 Results from a Molasses with an Inverted Power Ratio

Having studied the established Raman molasses process, I decided to investigate other frequency configurations, to see if it might be possible to reproduce the cooling mechanism without the idealised Λ -like system. The first attempt was to swap the frequencies of the principal and repump beams, so that $\eta^2 > 1$. In the experiments, this was done by increasing the molasses laser frequency by ~ 803.5 MHz, the ground state hyperfine splitting. The alternate (lower frequency) EOM sideband then acts as the repump component. The laser configuration, and equivalent dressed state transformation, are illustrated in figure 3.21a,b respectively. Compared with the conventional D_1 molasses, there are now three strongly coupled states of predominantly $F = 1$ character, and five weakly coupled states of predominantly $F = 2$ character. Figure 3.22 shows the results of an eigenstate calculation with $\delta_R = 0$, $\delta_P = 4.5\Gamma$ for a lin-perp-lin polarisation configuration. Considering the lowest lying strongly coupled state in the figure, we can see that the optical pumping rate also shows a positive correlation with the ac Stark shift, so that an atom in this state will tend to lose kinetic energy before being pumped to one of the weakly coupled states.

Figure 3.23 shows the effect of scanning the Raman detuning δ_R for the inverted molasses,

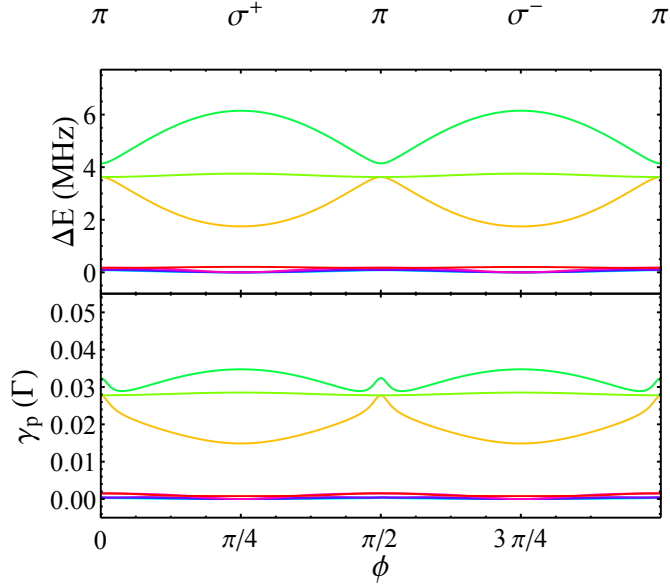


Figure 3.22: ac Stark shifts, ΔE , and optical pumping rates, γ_p , for the inverted Raman molasses.

with $\delta_p = +15.2\Gamma$, $1/\eta^2 = 0.006$. Close to the Raman condition, there is a similar peak in the capture efficiency and sub-Doppler temperatures are reached. However, in this case the cooling and capture was only found to be effective over a narrow range of δ_R , and no atoms were captured for $\delta_R > 0.5\Gamma$. We can understand this sensitivity by considering the competition between Doppler and sub-Doppler cooling; when far from the Raman condition, the repump light acts incoherently, and the principal light acts as a blue-detuned type-I molasses. This Doppler-heats the atomic cloud, and so it is only when coherent dark states can be formed that the sub-Doppler cooling can become effective.

Figure 3.24 shows the dependence of temperature on the power ratio $1/\eta^2$; similar to the ordinary molasses, the minimum temperature of about $50\mu\text{K}$ occurs when the ratio of the beam powers is close to 1%, and remains below the Doppler limit at $\eta^2 = 0.1$.

Figure 3.25 shows the effect of scanning the single photon detuning, with $\delta_R = +0.1\Gamma$. The capture efficiency is near 100% around $\delta_p = +7\Gamma$, where the temperature is $94(4)\mu\text{K}$. At $\delta_p = +15.2\Gamma$, the capture efficiency is 60% and consistent with the Raman detuning data in figure 3.23a. Thus we conclude the sub-Doppler cooling is similarly effective in the inverted molasses configuration, albeit more sensitive to the two-photon detuning.

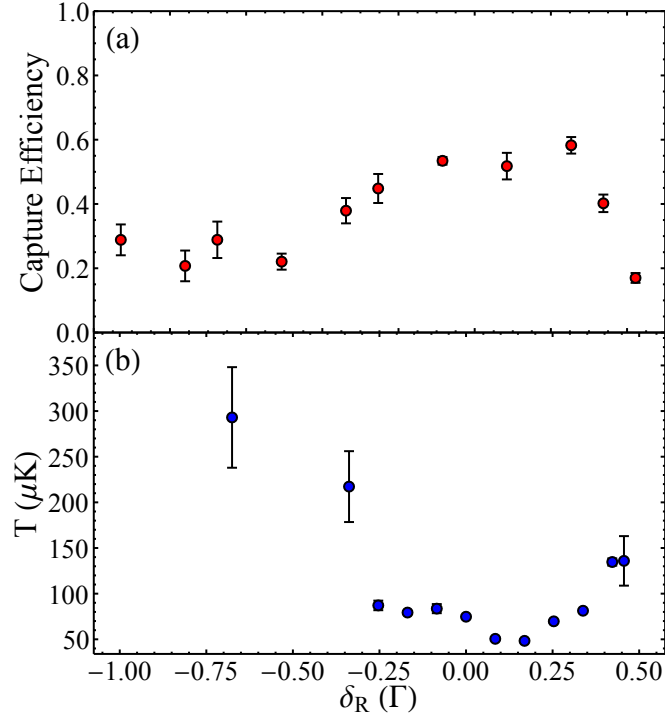


Figure 3.23: Dependence of the capture efficiency and temperature of the molasses on the two-photon Raman detuning δ_R , in the inverted molasses on the D_1 line. For these experiments, $\delta_P = +15.2\Gamma$, and $1/\eta^2 = 1\%$.

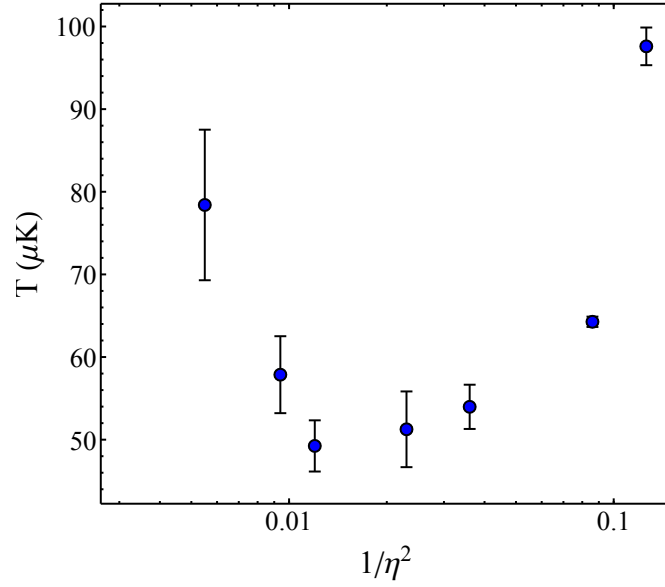


Figure 3.24: Temperature versus the principal-repump ratio $1/\eta^2$ in the inverted molasses. For these data, $\delta_P = +15.2\Gamma$ and $\delta_R = +0.1\Gamma$.

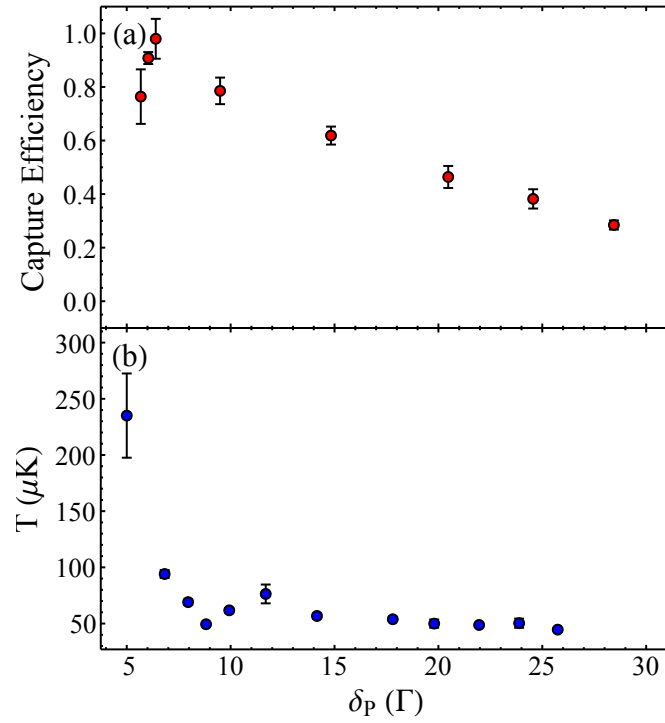
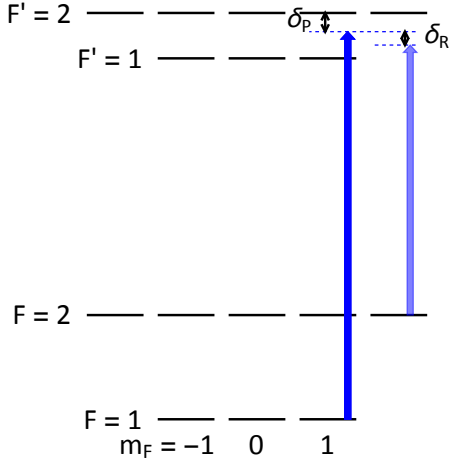


Figure 3.25: Dependence of the capture efficiency and temperature of the molasses on the single photon detuning δ_P , for the inverted molasses on the D_1 line. For these experiments, $\delta_R = +0.1\Gamma$, $\tau_{\text{mol}} = 4\text{ ms}$.

(a)



(b)

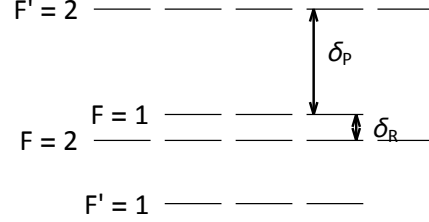


Figure 3.26: Inverted Raman molasses with the single photon detuning set between the two $P_{1/2}$ excited states. (a) Laser detunings. (b) Dressed state transformation.

Exploring $\delta_P < 0$

I also found it was possible to capture atoms in the inverted molasses with the lasers red-detuned from the $1 \rightarrow 2$ transition, but blue-detuned from the $1 \rightarrow 1$ transition. This scenario is illustrated by figure 3.26. In this case, the light shifts from the $F' = 1$ and $F' = 2$ levels are comparable. Figure 3.27 shows the results of eigenstate calculation with $\delta_P = -8\Gamma$, $\delta_R = -\Gamma$. There is a Sisyphus effect possible for the lowest energy strongly coupled state, but the optical pumping rate away from this state almost independent of polarisation. Thus we have, in effect, removed one of the ingredients to the Raman molasses mechanism.

Figure 3.28 shows the effect of the Raman detuning with $\delta_P = -10.8\Gamma$, which places the laser roughly in between the two excited states. Over the range $-1.5\Gamma < \delta_R < -\Gamma$, the temperature is close to the Doppler limit, and the capture efficiency is around 0.5. At $\delta_R = -0.8\Gamma$ the capture efficiency is close to 1, but the temperature is roughly twice the Doppler limit.

Figure 3.29 shows the effect of varying the single photon detuning with $\delta_R = -1.3\Gamma$ and $1/\eta^2 = 0.04$. Around $-11\Gamma < \delta_P < -10\Gamma$ the temperature remains close to the Doppler limit. The capture efficiency falls to zero as we approach resonance with the $1 \rightarrow 1$ transition ($\delta_P = -15.5\Gamma$). Increasing δ_P improves the capture efficiency, whilst also raising the temperature.

Figure 3.30 shows the effect of varying the intensity ratio $1/\eta^2$, with $\delta_P = -10.8\Gamma$ and

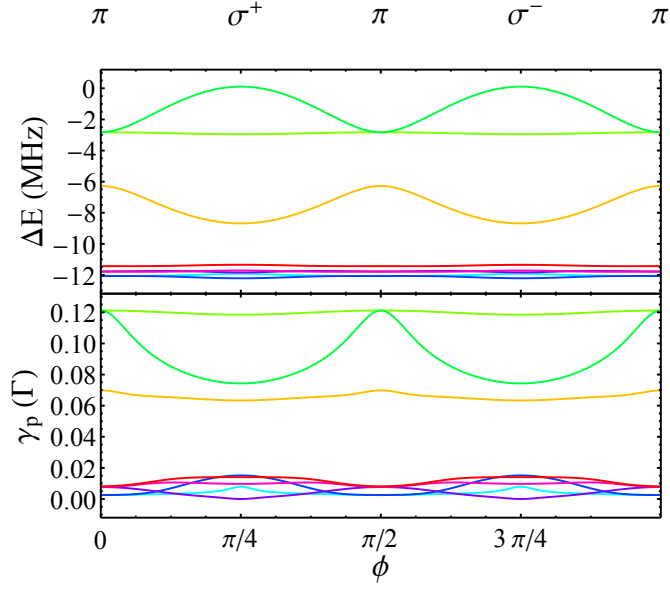


Figure 3.27: Eigenstate calculation for the inverted molasses with $\delta_P = -8\Gamma$, $\delta_R = -\Gamma$. Optical pumping rates are shown in the lower panel.

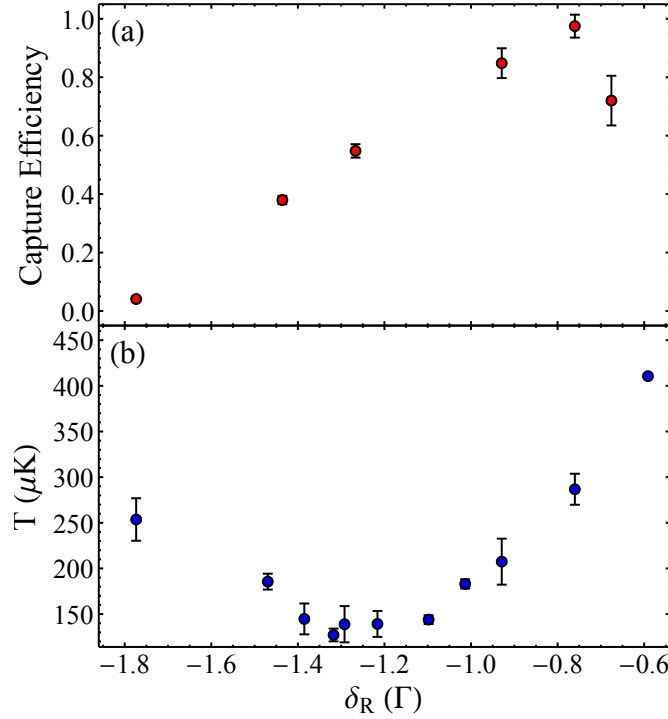


Figure 3.28: Capture efficiency and temperature versus two photon detuning for the red-detuned inverted molasses. For these data, $\delta_P = -10.8\Gamma$, $1/\eta^2 = 0.04$.

$\delta_R = -1.3\Gamma$. The temperature is remarkably robust for $1/\eta^2 \gtrsim 0.1$, remaining close to the Doppler limit.

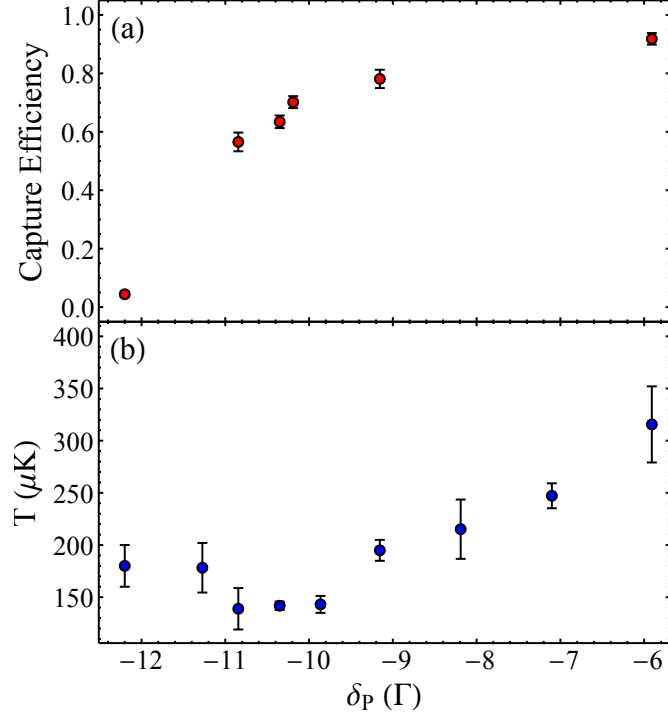


Figure 3.29: Capture efficiency and temperature versus the single photon detuning for the red-detuned inverted molasses. For these data, $\delta_R = -1.3\Gamma$, $1/\eta^2 = 0.04$.

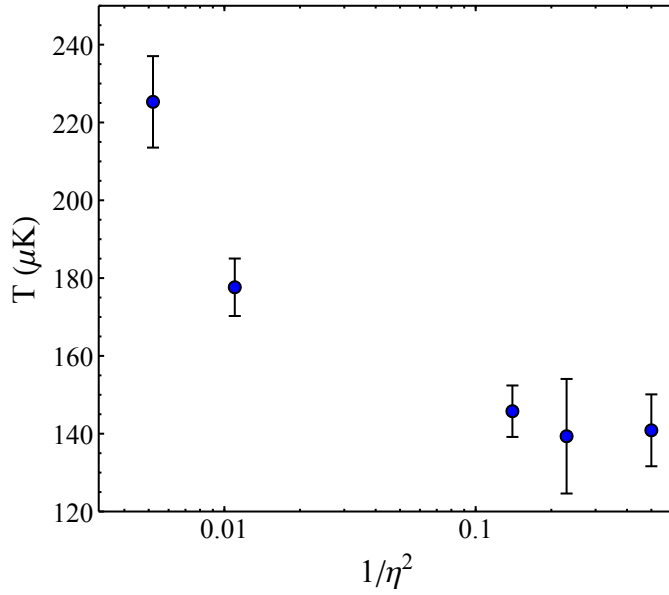


Figure 3.30: Temperature versus the power ratio $1/\eta^2$ for the red-detuned inverted molasses, with $\delta_P = -10.8\Gamma$, and $\delta_R = -1.3\Gamma$.

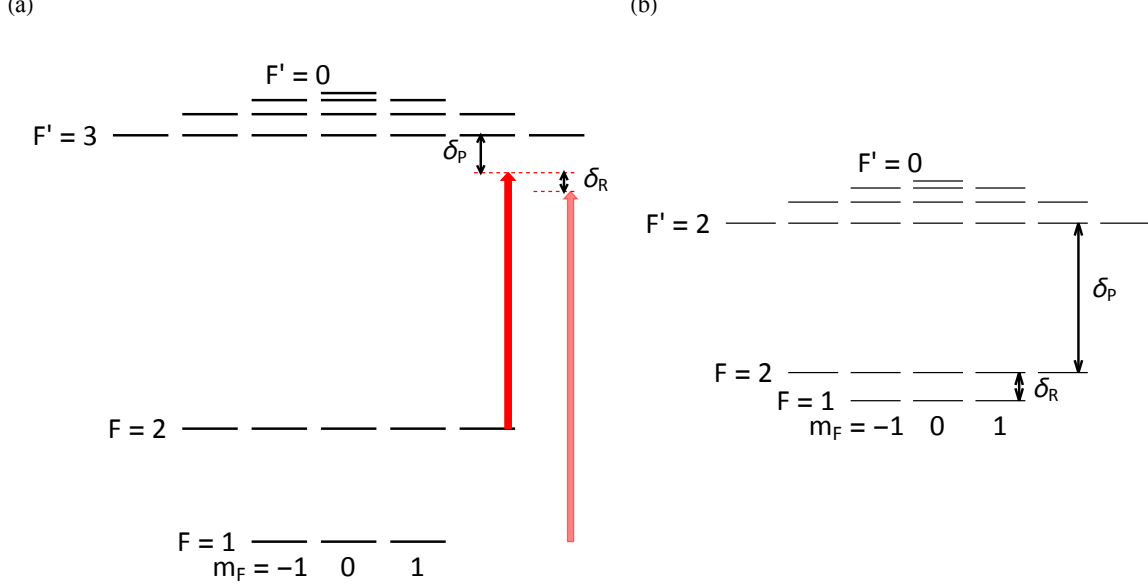


Figure 3.31: Energy level scheme for red-detuned Raman molasses on the D₂ line.

3.5.4 Raman Molasses on the D₂ Line

Raman molasses is almost always applied on the D₁ line, which can be inconvenient because it typically requires a separate laser system to the MOT. However, it was demonstrated recently that the cooling method can be applied to a type-I transition in ⁴⁰K [116], using the D₂ line which is used to load a MOT. The situation for potassium is similar to lithium; the unresolved hyperfine structure in the ²P_{3/2} excited state precludes effective sub-Doppler cooling via bright optical molasses. Therefore we decided to see if the cooling mechanism was similarly applicable in ⁷Li. For these experiments, a separate frequency offset lock was made so that the molasses laser could be tuned close to the D₂ line.

Figure 3.32a-b shows the results of eigenstate calculations applied to the D₂ line, with $\delta_P = -11\Gamma$, $\delta_R = -2.5\Gamma$ and $\eta^2 = 0.04$. Because the laser field is red-detuned, ac Stark shifts of the ground state are negative; the strongly coupled states are predominantly $F = 2$ character, and the $F = 1$ states addressed by the repumper light have almost zero light shift. According to [116], the cooling mechanism proceeds as follows: an atom in one of the weakly coupled states makes a non-adiabatic transition to the bright state as per the usual non-adiabatic transition probability. This is most likely when the polarisation is $\sigma^+(\sigma^-)$, when the most strongly shifted bright state, shown in light blue in the figure, is close in energy

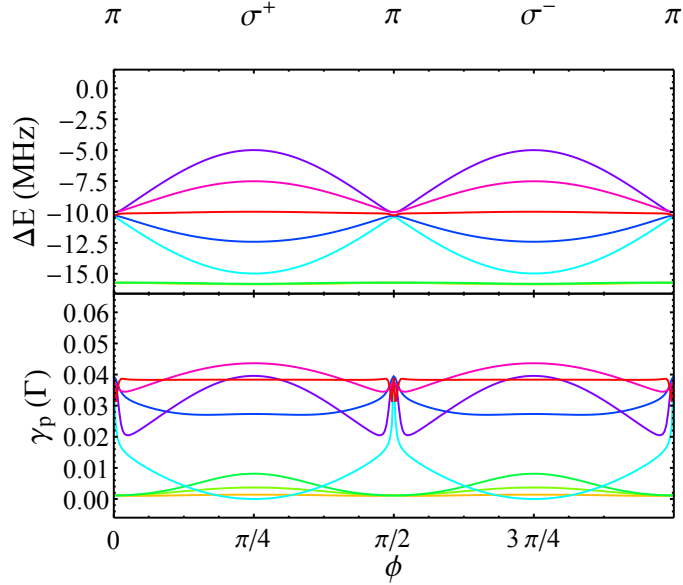


Figure 3.32: Ground state ac Stark shifts, ΔE , and optical pumping rates, γ_p , for Raman molasses on the D_2 line. For this calculation, $\delta_P = -11\Gamma$, $\delta_R = -2.5\Gamma$, and $\eta^2 = 0.04$.

to the weakly coupled states. The bright state at this position is a linear combination of the $|2, +2\rangle$ ($|2, -2\rangle$) and $|3', +3'\rangle$ ($|3', -3'\rangle$) states. While the polarisation is circular, optical pumping away from this state is not possible since the laser excites a “stretched” transition. However, as the polarisation changes, pumping back to the weakly coupled states becomes possible, and is strongly peaked around linear polarisation. This means that on average, an atom will lose potential energy before returning to one of the weakly coupled states.

Figure 3.33 shows the effect of scanning the Raman detuning with $\delta_P = -11\Gamma$ and $\eta^2 = 0.02$. From the capture efficiency in 3.33a we see a similar signature to that observed in the D_1 molasses, capturing more than 80% of the atoms from the MOT around $\delta_R = -1.5\Gamma$. The temperature (figure 3.33b) falls below the Doppler limit for $-1.8\Gamma < \delta_R < -0.8\Gamma$, and at $\delta_R = -1.1\Gamma$ the temperature is $105(2)$ μK with a capture efficiency of $89(4)\%$. Moving closer to the Raman condition reduces the temperature of the cloud, but at the expense of capture efficiency; at $\delta_R = -0.8\Gamma$, the temperature is $67(3)$ μK but the capture efficiency has fallen to $42(1)\%$. From the eigenstates picture we would expect the optimal δ_R to be negative, because the laser field is red-detuned, meaning the bright states gain a large and negative ac Stark shift. Qualitatively this is consistent with the data. At $\delta_R > +\Gamma$, it became

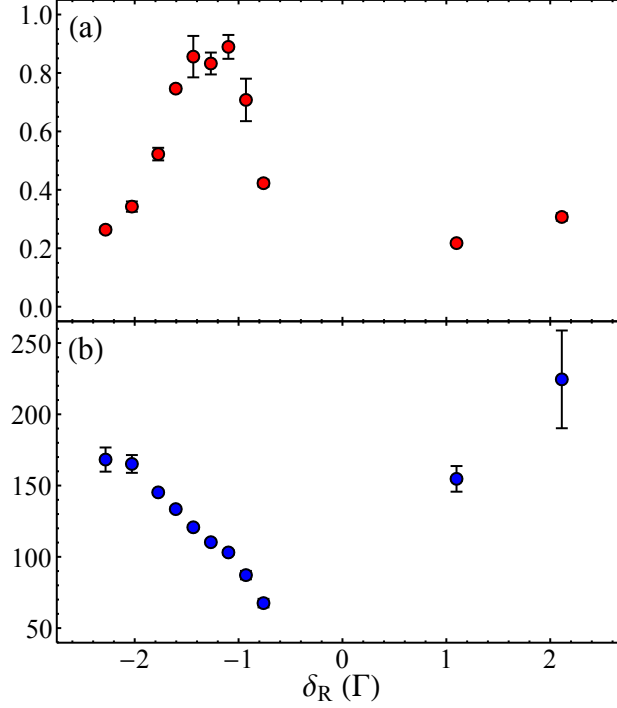


Figure 3.33: Scanning the two photon detuning in red-detuned D_2 Raman molasses, with $\delta_P = -11\Gamma$ and $\eta^2 = 0.02$. (a) Capture efficiency. (b) Temperature.

possible to capture atoms in the molasses once more, although the temperature is greater than the Doppler limit and the capture efficiency remains below 40%. This is qualitatively similar to the conventional D_1 molasses. Whilst the D_2 molasses is clearly less efficient than the conventional D_1 system, a substantial fraction of the atoms are cooled below the Doppler limit. Since the D_1 molasses requires a separate optical system to the MOT, whereas the D_2 molasses does not, D_2 molasses may be useful in certain scenarios.

3.6 Conclusions

This chapter has demonstrated sub-Doppler cooling of our lithium cloud using Raman grey molasses. Applying this method on the D_1 line, with $\delta_R = +0.17\Gamma$, $\delta_P = +13.5\Gamma$, 91(2)% of the atoms are captured and cooled to 41(1) μK . Compared with the CMOT, this represents an overall increase of a factor 40 in phase space density, and makes it possible to load a reasonable fraction of the atoms into the microwave trap.

Cooling is most effective on the D_1 line, where high intensity light blue-detuned from

the $2 \rightarrow 2$ transition combines with a weak sideband near the Raman condition, creating coherent dark states. These dark states are velocity selective, such that slow moving atoms remain decoupled from the laser field for longer, which gives rise to the cooling effect. If the relative intensities (and therefore the effective role) of the two frequency components are reversed, we can recover a Raman molasses effect and also cool to the sub-Doppler regime. However, the sub-Doppler cooling at low velocity competes with Doppler heating at higher velocities, making the cooling overall less effective and more sensitive to the experimental parameters.

Motivated by the demonstration of a Raman molasses on the D_2 line of ^{40}K , we investigated the possibility of a similar scheme for ^7Li . Indeed, we find sub-Doppler cooling is effective when addressing the $2 \rightarrow 3$ transition with a red-detuned molasses, provided the repump light is tuned appropriately. For this system, the optimal cooling does not coincide with the optimal capture efficiency, although we are able to capture 89(4)% of the atoms at a temperature of 105(2) μK . Optimal cooling is found when $\delta_R \lesssim -\Gamma$, but at this detuning the capture efficiency is below 50%.

For the experiments in the following chapters, we use the D_1 molasses, with $\eta^2 = 0.01$, $\delta_P = +13.5\Gamma$ and $\delta_R = +0.17\Gamma$.

Chapter 4

Magnetic Trapping and Transport

This chapter describes magnetic trapping of our ultracold cloud, and subsequent transport to the microwave cavity. Atoms are first trapped using the in-vacuum coils, then transferred to the externally mounted transport trap. By appropriate translation of the trap with a motorised stage, the cloud is delivered from the MOT chamber to the centre of the microwave cavity. I describe problematic sources of atom loss and heating which were encountered, a result of induction and magnetic field defects at the transport tube entrance. I show how to minimise these issues by careful refinement of the trapping currents when loading and moving the external trap. Using trajectory simulations, I show that ballistic expansion measurements will reveal any substantial heating along the transport direction, even though these measurements cannot directly determine the translational temperature along that direction.

4.1 A Brief History of Magnetic Trapping

Magnetic forces have been used in atomic physics since the seminal Stern-Gerlach experiment in 1922 ¹, which proved beyond doubt that angular momentum orientation is quantized. Deflection of atomic beams was then the method of choice for measuring magnetic moments, and later a number of proposals emerged for trapping neutral particles, for example Heer’s toriodal “magnetic bottle” idea [118]. Slow neutrons were first confined in this type of trap

¹An interesting discussion of the history and nuances of this discovery is provided in [117]

in 1978 [119], and storage times were long enough to be limited by the neutron half-life (~ 15 min).

The first demonstration of magnetically trapped neutral atoms was published in 1985 by Bill Phillips and co-workers, by laser slowing a beam of sodium atoms to rest and then loading into a trap [120]. In this experiment, a pair of current carrying coils in anti-Helmholtz configuration generated a potential 17 mK deep, with a vacuum-limited lifetime of 0.83 s. Subsequently a number of variations have been investigated, such as permanent magnet traps [121], and surface chip traps in which atoms are confined in a microscopic setting [122]. The long confinement times possible with magnetic trapping has enabled spectroscopy of antihydrogen to probe any differences between matter and antimatter [123].

Magnetic trapping proved to be instrumental in the realisation of Bose-Einstein Condensation (BEC) [2], where runaway evaporative cooling was shown to be highly effective in increasing phase-space density to quantum degeneracy. For this to work, new traps were devised to suppress Majorana “spin-flip” losses, which occur at the centre of a quadrupole trap and become increasingly likely at low temperatures. In the first atomic BEC, Cornell and co-workers used a Time-averaged Orbiting Potential trap [124], superposing a quadrupole trap with a small uniform field which rotates quickly compared with the atomic motion. The uniform field shifts the magnetic field zero, meaning it rotates around the quadrupole centre. Atoms see the time-averaged effect of this, which is a harmonic trap without any zero field region, and this suppresses Majorana losses. Another class of trap originated from the work of plasma physicist Mikhail Ioffe, and was developed by Dave Pritchard, and has become known as the Ioffe-Pritchard trap [125, 126, 127]. This approach superposes different DC coil configurations to ensure a constant offset field at the centre of the trap. There are a number of configurations, such as the cloverleaf configuration [128], and the quadrupole Ioffe-Pritchard configuration (QUIC) [129]. In the same year as Cornell’s group observed the first atomic BEC, much larger condensates were generated by optically “plugging” the Majorana loss region of a quadrupole trap with blue-detuned light [130].

4.2 The Zeeman Effect

A magnetic trap employs the Zeeman effect, which shifts the energies of atomic states in a magnetic field. In the following discussion we assume a magnetic field along the z -axis, $\mathbf{B} = B\mathbf{z}$, and a magnetic moment of the electron of $\boldsymbol{\mu}$. The Zeeman interaction Hamiltonian, $\mathcal{H}_Z$, is given by,

$$\mathcal{H}_Z = -\boldsymbol{\mu} \cdot \mathbf{B} \quad . \quad (4.1)$$

Here, $\boldsymbol{\mu} = (g_L\mathbf{L} + g_S\mathbf{S})\mu_B$, where $\mathbf{L}$ is the orbital angular momentum operator, $\mathbf{S}$ is the electronic spin operator, and $g_{L,S}$ are called the g-factors for the respective angular momenta. We neglect nuclear spin here, whose contribution is smaller by roughly the proton to electron mass ratio. For laboratory fields the Zeeman interaction is normally small compared to the spin-orbit coupling, but can be comparable or large compared to the hyperfine interaction. It is convenient to rewrite equation 4.1 in terms of the total electronic angular momentum, $\mathbf{J} = \mathbf{L} + \mathbf{S}$, whose projection onto the z -axis is m_J . In this case we can write

$$\mathcal{H}_Z = \mu_B g_J B \hat{J}_z \quad , \quad (4.2)$$

where the g-factor g_J is defined as

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad . \quad (4.3)$$

For atoms with a nuclear spin $\mathbf{I}$, the electronic angular momentum interacts with the nuclear magnetic spin. This results in the “bare” atom Hamiltonian $\mathcal{H}_{\text{atom}} = A\mathbf{I} \cdot \mathbf{J}$, where A is a constant. To find the energy shift in a magnetic field, we must diagonalize the total Hamiltonian $\mathcal{H}_{\text{tot}} = \mathcal{H}_Z + \mathcal{H}_{\text{atom}}$. The Zeeman Hamiltonian in equation 4.2 shares eigenstates with J_z , whereas $\mathcal{H}_{\text{atom}}$ includes hyperfine structure and shares eigenstates of F_z . If we choose to work in the F, m_F basis, then we can calculate the terms of $\mathcal{H}_Z$ by projecting each $|F, m_F, J, I\rangle$ state into eigenstates of J^2 and J_z :

$$|F, m_F, I, J\rangle = \sum_{J', I', m_{J'}, m_{I'}} C_{J', I', m_{J'}, m_{I'}, F, m_F} |J', m_{J'}, I', m_{I'}\rangle \quad . \quad (4.4)$$

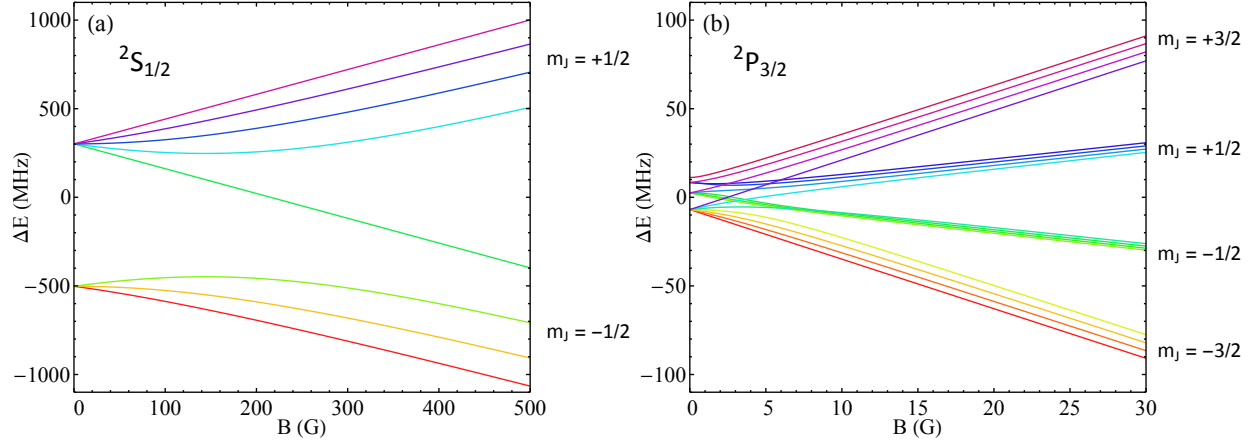


Figure 4.1: Zeeman shifts of ^7Li . (a) $^2S_{1/2}$ states (b) $^2P_{3/2}$ states. The axes are chosen to show the crossover from eigenstates of F^2, F_z to J^2, J_z ; labels to right of each plot are the eigenvalues of J_z in the strong field regime.

The coefficients C_{J,I,m_J,m_I,F,m_F} are the Clebsch-Gordon coefficients for coupling composite angular momentum quantum numbers J, m_J and I, m_I into total angular momentum quantum numbers F, m_F . Having found $\mathcal{H}_{\text{tot}}$ as a function of B , we can then find the eigenenergies and calculate the Zeeman shifts of our atomic hyperfine states.

Figure 4.1 shows the Zeeman shifts of the hyperfine states within the $^2S_{1/2}$ and $^2P_{3/2}$ levels of ^7Li . The cross-over between the F, m_F eigenstates and the J, m_J eigenstates occurs as the Zeeman interaction energy becomes comparable to the hyperfine splitting. Labels to the right of each plot denote the eigenvalues in the strong field regime. From figure 4.1, it is clear that atoms in certain states will be attracted towards a strong magnetic field (strong-field seekers), while others will be attracted to regions where the field is weak (weak-field seekers). However, it is not possible to trap strong-field seekers with a static field, and so only weak-field seekers are useful.

Figure 4.2 shows the Zeeman shifts of the $^2S_{1/2}$ states for $0 < B < 5$ G, which is the relevant range for our atomic cloud. In this regime, the Zeeman shifts of the $^2S_{1/2}$ states can be well described by perturbation theory, so that

$$\Delta E = g_F m_F \mu_B B \quad , \quad (4.5)$$

where g_F is the magnetic g-factor for the total angular momentum. For the $F = 2$ states

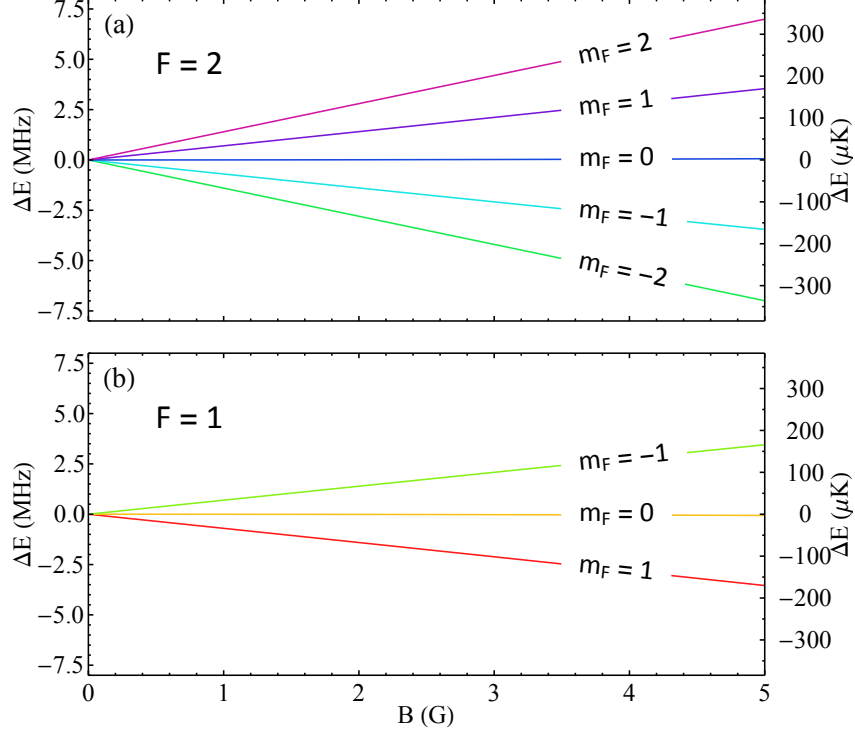


Figure 4.2: Zeeman shifts of the $2S_{1/2}$ ground states for $B < 5$ G. (a) $F = 2$ states. (b) $F = 1$ states.

(4.2a), $g_F = +1/2$; for the $F = 1$ states (figure 4.2b) $g_F = -1/2$. The most strongly confined state is the $|2, 2\rangle$ state, with a Zeeman shift of 1.40 MHz/G ($67 \mu K G^{-1}$). It therefore makes sense to prepare atoms in this state before loading the trap.

4.3 The Magnetic Quadrupole Trap

The simplest magnetic trap is the quadrupole trap, which can be made straightforwardly by arranging a pair of coils in anti-Helmholtz configuration. This is the configuration used in all of the experiments described in this work. To describe the magnetic field profile, we use coordinates with the z -axis parallel to the axis of the coil pair, with the origin midway between the coils. From Maxwell's equation $\nabla \cdot \mathbf{B} = 0$, and using cylindrical symmetry, it follows that the magnetic field close to trap centre is given by

$$\mathbf{B} = \frac{b}{2}(x\mathbf{x} + y\mathbf{y} - 2z\mathbf{z}) \quad . \quad (4.6)$$

Here, b refers to the magnetic field gradient along the axial direction. For the quadrupole field to form a trap, atoms must remain in weak-field seeking states as they move through space, which requires that their spins follow the field adiabatically. If this is true, then the atomic spin precession follows the local field direction which defines our quantization axis. The magnetic trap potential, U_{MQT} depends only on the magnitude of $\mathbf{B}$ and not its direction:

$$U_{\text{MQT}}(x, y, z) = \mu b \sqrt{z^2 + \frac{x^2 + y^2}{4}} \quad . \quad (4.7)$$

To simplify the notation for the subsequent sections, we made the replacement $\mu = \mu_B g_F m_F$; for the $|2, 2\rangle$ state, $\mu = \mu_B$.

Partition Function and Thermodynamics

A useful concept for us to consider is the partition function for a particle in the quadrupole trap Z_{MQT} :

$$Z_{\text{MQT}} = \frac{1}{(2\pi\hbar)^3} \int e^{-\beta \frac{p^2}{2m}} dp_x dp_y dp_z \int e^{-\beta U_{\text{MQT}}} dx dy dz = \frac{1}{(2\pi\hbar)^3} Z_{\text{KE}} \times Z_{\text{PE}} \quad . \quad (4.8)$$

Equation 4.8 essentially measures the volume of our particle in the phase space of position and momentum coordinates, (x, y, z, p_x, p_y, p_z) , with $\beta = 1/k_B T$. The first integral is the kinetic energy term, Z_{KE} , and evaluates to

$$Z_{\text{KE}} = (2\pi m k_B T)^{\frac{3}{2}} \quad . \quad (4.9)$$

The second integral is the potential energy term, Z_{PE} , and by substituting equation 4.7 for the quadrupole potential, we find²:

$$Z_{\text{PE}} = 32\pi \left(\frac{k_B T}{\mu b} \right)^3 \quad . \quad (4.10)$$

²To evaluate the integral, one can make the substitution $z' = 2z$, which means the integrand can be made spherically symmetric. The limits of the integral extend out to infinity, and they are unchanged by the substitution.

Combining the two terms, we get

$$Z_{\text{MQT}} = \frac{32\pi}{\hbar^3} \left(\frac{m}{2\pi} \right)^{3/2} \frac{(k_B T)^{9/2}}{(\mu b)^3} . \quad (4.11)$$

It is now straightforward to derive the equilibrium thermodynamic properties of a gas in the magnetic trap. To find the ensemble averaged energy of a trapped particle, $\langle E \rangle$ we use

$$\langle E \rangle = -\frac{1}{Z_{\text{MQT}}} \frac{\partial Z_{\text{MQT}}}{\partial \beta} = \frac{9}{2} k_B T . \quad (4.12)$$

We could have applied this method separately to Z_{KE} and Z_{PE} , and find the average potential and kinetic energies:

$$\langle KE \rangle = \frac{3}{2} k_B T , \quad (4.13a)$$

$$\langle PE \rangle = 3 k_B T . \quad (4.13b)$$

From equations 4.13, we find the virial relation for the quadrupole trap, which relates the average potential and kinetic energies:

$$\langle PE_{\text{MQT}} \rangle = 2 \langle KE_{\text{MQT}} \rangle . \quad (4.14)$$

The equivalent relation for a harmonic trap states that the average potential and kinetic energies are equal.

The equilibrium density profile in the quadrupole trap, $n(x, y, z)$ is given by,

$$n(x, y, z) = n_0 e^{-\beta U_{\text{MQT}}} = n_0 \exp \left[-\beta \mu b \sqrt{z^2 + \frac{x^2 + y^2}{4}} \right] . \quad (4.15)$$

If we restrict ourselves to considering the z -axis, the density distribution in equation 4.15 follows e^{-z/λ_z} , where the length scale λ_z is given by

$$\lambda_z = \frac{k_B T}{\mu b} . \quad (4.16)$$

Along the two orthogonal directions, the length scale is $\lambda_{x,y} = 2\lambda_z$. These length scales define

the size of our cloud in the quadrupole trap. Assuming $T = 100 \mu\text{K}$, and $b = 30 \text{ G cm}^{-1}$, then $\lambda_z = 0.5 \text{ mm}$ for the $|2, 2\rangle$ state. With N atoms in the trap, the peak density $n_0 = \frac{N}{32\pi\lambda_z^3}$; for 10^8 atoms this corresponds to $n_0 = 8 \times 10^9 \text{ cm}^{-3}$.

The force on a particle in the quadrupole potential is neither harmonic nor central. As a result, there is no well defined oscillation frequency, and in general particle paths are not closed orbits. However, we can find a useful estimate for the timescale of motion in the trap by considering a particle in a circular orbit in the $z = 0$ plane. The force on such a particle acts towards the trap centre, with magnitude $\mu b/2$; therefore the orbital time at radius λ_z is given by

$$\tau = 2\pi \sqrt{\frac{2m\lambda_z}{\mu b}} . \quad (4.17)$$

Substituting our estimate for λ_z , we find $\tau = 13 \text{ ms}$ for ^7Li .

Another useful relation is the relation for adiabatic compression in the trap. If the trapping gradient is changed slowly relative to the orbital motion of atoms in the trap, then we can treat the process as an adiabatic compression and conserve the phase-space volume of a trapped particle. By considering equation 4.11, it follows that

$$\frac{(k_B T)^{9/2}}{(\mu b)^3} = \text{const.} \quad (4.18)$$

We can restate this relation in terms of the size of the cloud by substituting for $k_B T$ in terms of λ_z using equation 4.16:

$$(\mu b)\lambda_z^3 = \text{const.} , \quad (4.19)$$

which is equivalent to conserving the peak phase-space density at the trap centre. From equations 4.18 and 4.19, it follows that the temperature of the cloud scales as $b^{2/3}$ and its width as $b^{-1/3}$, and therefore increasing the trapping gradient heats the cloud while compressing it rather weakly.

Collision rate in the magnetic trap

The elastic scattering rate in the trap, R_{el} , can be related to the density, n and the scattering cross section, σ_{el} , by

$$R_{\text{el}} = \sigma_{\text{el}} n \langle v \rangle , \quad (4.20)$$

where $\langle v \rangle = \sqrt{\frac{2k_B T}{m}}$ is the mean relative velocity of colliding particles. For ^7Li in the $|2, 2\rangle$ state, we can use the measured s-wave scattering length, $a = -27.3a_0$ [131], and the relation for identical bosons, $\sigma_{\text{el}} = 8\pi a^2$, to find $\sigma_{\text{el}} = 5.1 \times 10^{-13} \text{ cm}^2$. Taking our estimated peak density n_0 , and $T \sim 100 \mu\text{K}$, the mean time between collisions is $1/R_{\text{el}} = 4.9 \text{ s}$. It is on this timescale that the cloud will reach true thermal equilibrium.

Majorana Transitions

Since the quadrupole trap has a magnetic field zero at its centre, there is a small volume in which atoms undergo so-called Majorana transitions to untrapped states, and leave the trap. A Majorana transition can occur when the local Larmor precession frequency, $\Omega_{\text{L}} = \frac{\mu B}{\hbar}$ shows a rate of change comparable to the precession frequency itself. Physically, we require

$$\frac{1}{\Omega_{\text{L}}} \frac{d\Omega_{\text{L}}}{dt} \sim \Omega_{\text{L}} , \quad (4.21)$$

from which it follows that Majorana losses are restricted to a region near the trap centre. If the atoms are moving with a typical velocity v , it can be shown [124] that the width of this region, r , is given by,

$$r \sim \sqrt{\frac{\hbar v}{\mu b}} . \quad (4.22)$$

The Majorana loss rate, Γ_{M} , should be proportional to the flux of atoms entering this volume, which leads to the relation,

$$\Gamma_{\text{M}} \propto \frac{\hbar}{m} \left(\frac{\mu b}{k_B T} \right)^2 . \quad (4.23)$$

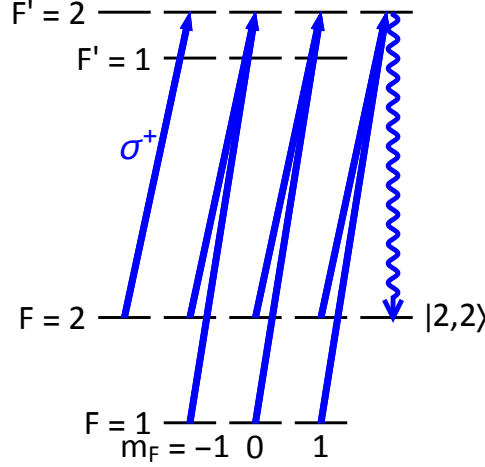


Figure 4.3: Optical pumping to the $|F, m_F\rangle = |2, 2\rangle$ state using the D_1 line.

The coefficients of proportionality have been estimated as 0.87 [132] for ^{87}Rb , and 0.14 for sodium [133]. Assuming a proportionality constant of 1, then with $b = 30 \text{ G cm}^{-1}$ we would have $\Gamma_M = 0.036 \text{ s}^{-1}$. This is small compared with the losses from background gas collisions in our experiment. However, should the atomic cloud require evaporative cooling in future, then Γ_M will increase as the temperature falls.

4.4 Loading the Internal Magnetic Trap

At the end of the Raman molasses stage, most of the atomic population is in the $F = 1$ manifold. Although atoms in the $|F = 1, m_F = -1\rangle$ state are trappable, we would rather prepare the atoms in $|F = 2, m_F = 2\rangle$, since it has the largest Zeeman shift. This state is dark to σ^+ light on the D_1 transition, and so the state can be efficiently prepared by optical pumping. Figure 4.3 shows the pumping scheme; we apply circularly polarised light resonant with the $2 \rightarrow 1$ and $2 \rightarrow 2$ transitions. The pumping light is derived from the molasses laser, and is delivered along the imaging axis. The $1 \rightarrow 2$ light is provided by the molasses EOM sideband; during this stage the modulation depth was increased to give a sideband-carrier intensity ratio of 50%.

Although the cloud is a factor ~ 20 colder after the molasses than in the MOT, the lithium atoms are still moving at an RMS velocity of around 0.25 m s^{-1} , and so the cloud will double in size in a few milliseconds if left to expand. This is insufficient time to turn on either the

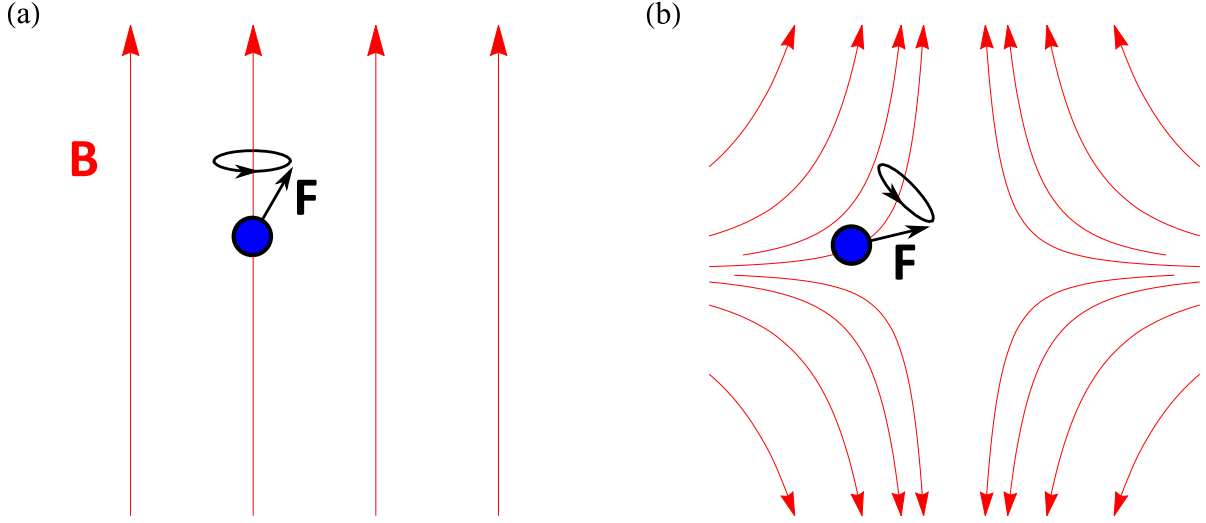


Figure 4.4: Spin precession in (a) a bias field used to prepare the atoms for magnetic trapping, and (b) in the quadrupole trap, shown in the $y - z$ plane.

externally mounted trapping coils, or the shim coil used to define the polarisation of the optical pumping light. Therefore we must apply an additional uniform magnetic field during the MOT and molasses stage, and first load the internal magnetic trap before transferring to the external transport trap. For the first experiments, a 0.2 ms optical pumping pulse was applied immediately after the molasses stage, and followed by a 1 ms linear ramp of the trapping currents.

The optical pumping is performed with a uniform field, and polarises the spin precession about the direction of the bias field. However, in the trap the direction of the magnetic field varies in space and trapped atoms will precess about the local field direction. These scenarios are illustrated in figures 4.4a and 4.4b respectively. When switching on the trap, we assume that the atomic spins adiabatically follow the change in field direction and remain in the $|2, 2\rangle$ state. This will be true for most atoms since for a 1 G bias field the spin precession time is $\sim 0.7 \mu\text{s}$, much faster than we are able to switch on the trapping coils. However, there will always be a region of space where Majorana losses are likely, and this region moves through space as the trap is switched on.

4.4.1 Imaging in the Magnetic Trap

The local magnetic field direction varies over space in the quadrupole trap, and so the quantization axis defining the polarisation of our imaging light is position dependent. As a consequence, absorption imaging on the $|2, 2\rangle \rightarrow |3, 3\rangle$ (σ^+) transition is only possible for a subset of atoms in the trap, and in general we will also drive π - and σ^- - transitions. Furthermore, the detuning of an atom from each allowed transition varies across the trap due to the Zeeman shift. Figure 4.5 illustrates the Zeeman shifts of the $^2S_{1/2}, F = 2$ ground states and the $^2P_{3/2}$ excited states at low magnetic field, and labels the transitions allowed under these circumstances.

Whilst the situation looks complicated, we can make a few simplifications and make progress. Firstly, given the temperature range for our atomic cloud, the Zeeman shifts of the various states are small compared to the excited state linewidth. This means that we can reasonably assume the transitions to the $F' = 3$ states are strongest, and optical pumping to the $F = 1$ ground states occur should remain small. As a first approximation, let us assume that the transition strength, averaged along the probe beam direction, is approximately constant. With this simplifying assumption, the integrated absorption densities along the y - and z - directions, $A_\Sigma(z)$ and $A_\Sigma(y)$, can be calculated from equation 4.15:

$$A_\Sigma(z) = 8\pi n_0 \bar{\sigma} \lambda_z^2 \exp\left[-\frac{|z|}{\lambda_z}\right] \left(1 + \frac{|z|}{\lambda_z}\right) , \quad (4.24)$$

$$A_\Sigma(y) = 4\pi n_0 \bar{\sigma} \lambda_z^2 \exp\left[-\frac{|y|}{2\lambda_z}\right] \left(1 + \frac{|y|}{2\lambda_z}\right) . \quad (4.25)$$

Figure 4.6 shows a typical absorption image in the magnetic trap, along with the line-integrated data. Equations 4.24 and 4.25 appear to fit the data well, suggesting that our assumption of a uniform transition strength across the cloud strength is a reasonable approximation. As a comparison, Gaussian fits are shown in figure 4.6, which underestimate the density at the centre and at the edges of the cloud. The fitted values of λ_z and λ_x are 0.27 mm and 0.47 mm respectively, and from the axial trapping gradient $b = 36 \text{ G cm}^{-1}$ we can estimate the temperature of the atomic cloud from equation 4.16, providing 58 μK (radial) and 67 μK (axial). However, it is still possible that systematic effects on the absorption

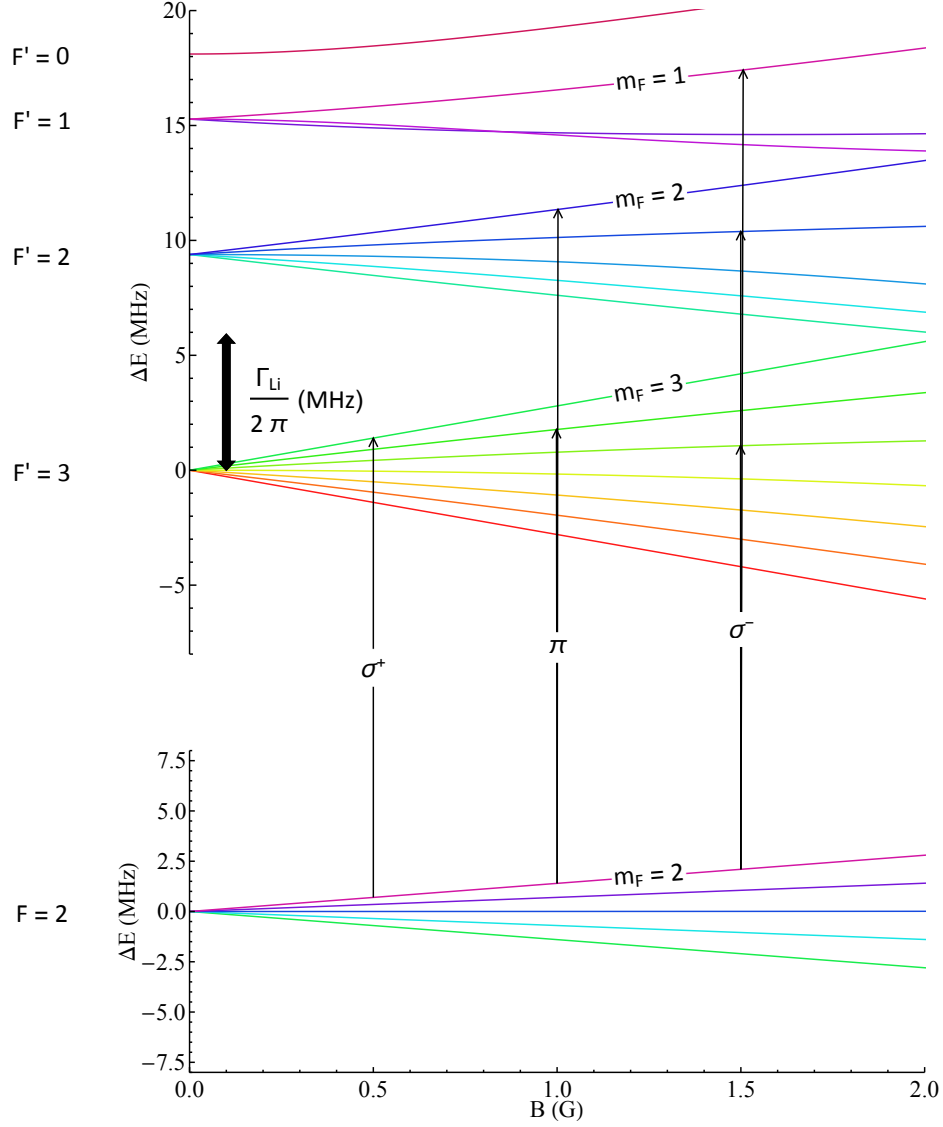


Figure 4.5: Zeeman shifts of the $^2S_{1/2}, F = 2$ and $^2P_{3/2}, F = 3, 2, 1, 0$ states for $0 < B < 2$ G. Allowed transitions from the magnetically trapped $|2, 2\rangle$ state are labelled by polarisation.

strength will distort our temperature measurements, so we should consider other methods for measuring the temperature.

One such method would be to compare the absorption profiles in the trap to those measured immediately following release. In the latter case, the imaging scenario is well controlled and we can be more confident of measuring the density profile faithfully. Figure 4.7 shows an absorption image taken 0.5 ms after release from the trap into the 1 G bias field, at which time we can be confident the trapping fields have decayed. The absorption signal is a factor 2.92(4) larger here, because the imaging light primarily drives the stretched

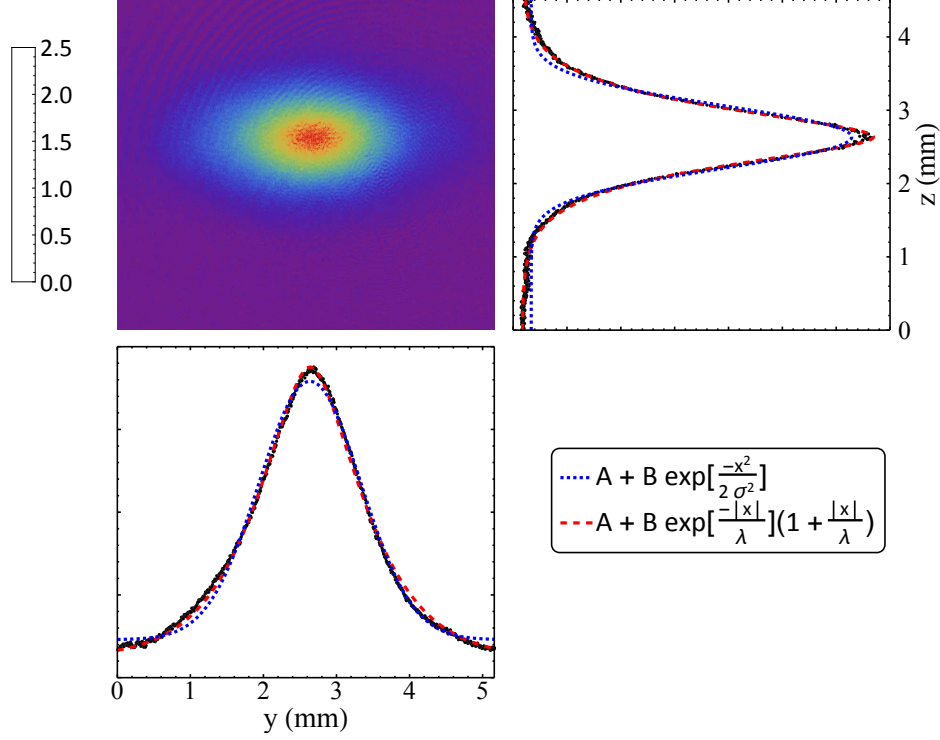


Figure 4.6: An absorption image of the cloud taken 50 ms after loading the internal magnetic trap, with the integrated row and column absorption data. Colour bar: absorption at each pixel in the image.

$|2, 2\rangle \rightarrow |3, 3\rangle$ transition. Equation 4.25 appears to fit the data well along the y -direction, but along the z -direction it now appears that a Gaussian describes the data better. The fitted values of λ_z and λ_x , are 0.43 mm and 0.63 mm respectively, corresponding to 78 μK (radial) and 105 μK (axial). Expansion of the cloud during the 0.5 ms wait time can only account for around 5 – 20% of the increase in size measured after release from the trap. Therefore, the absorption cross-section varies significantly across the trap, and in-trap absorption imaging can only be used to estimate the temperature of the atoms to about a factor two. However, we can still track the position of the cloud by imaging in this way, estimate the number of atoms in the external trap during and after transport.

Ballistic Expansion from the quadrupole trap

If we assume a density distribution given by equation 4.15 at $t = 0$, then we can predict its ballistic expansion from equation 2.2 and compare with our measurements to estimate the temperature. To do this, we can fit to the function,

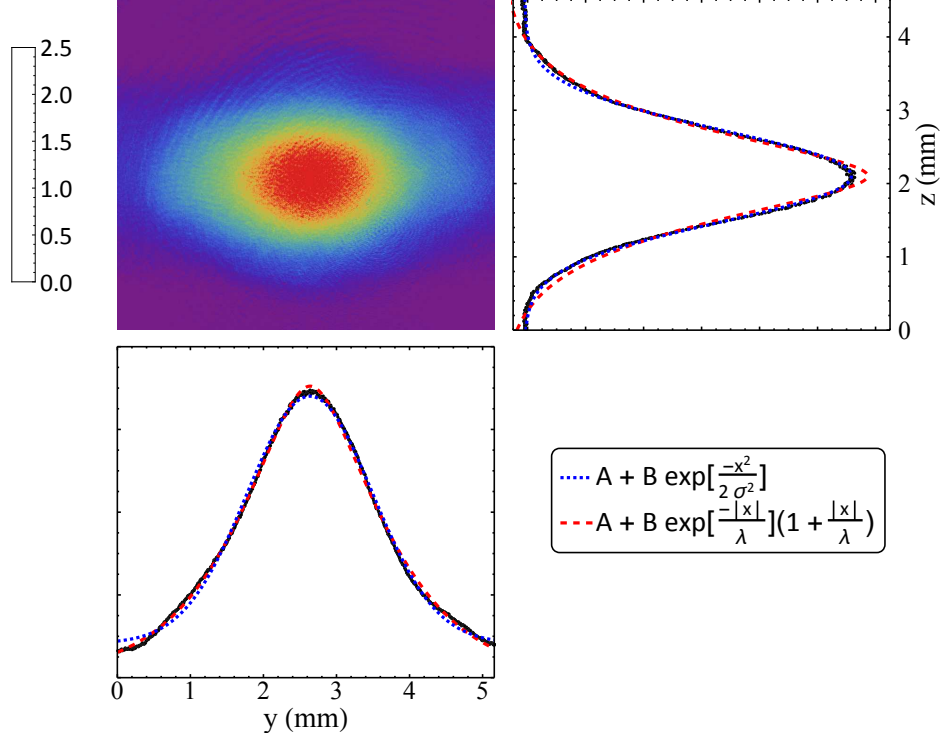


Figure 4.7: An absorption image of the cloud taken 0.5 ms after release from the magnetic trap. Colour bar: pixel absorption.

$$f(r, t) = A + B \int \exp\left[\frac{-|r'|}{\lambda}\right] \left(1 + \frac{|r'|}{\lambda}\right) \exp\left[\frac{(r - r')^2}{2\sigma_v^2 t^2}\right] dr' \quad . \quad (4.26)$$

to find best estimates for A , B , λ and σ_v . The fitted values of λ and σ_v are both dependent on the temperature. Therefore if the model describes the data well, the T estimated from λ should agree with T estimated from σ_v . However, I found that this was not the case, and that along each measurement axis the extracted values disagreed by a factor 3 or more. Instead, for the following experiments we assume a Gaussian density profile and fit the ballistic expansion data according to equation 2.4. The validity of this assumption improves as the cloud expands, since at very long times the density profile becomes independent of the initial spatial distribution. However, this is difficult to ensure in the experiments because of the restricted field of view. Therefore temperatures measured by ballistic expansion are reported alongside with those deduced from the width of the cloud, measured 0.5 ms after release from the trap.

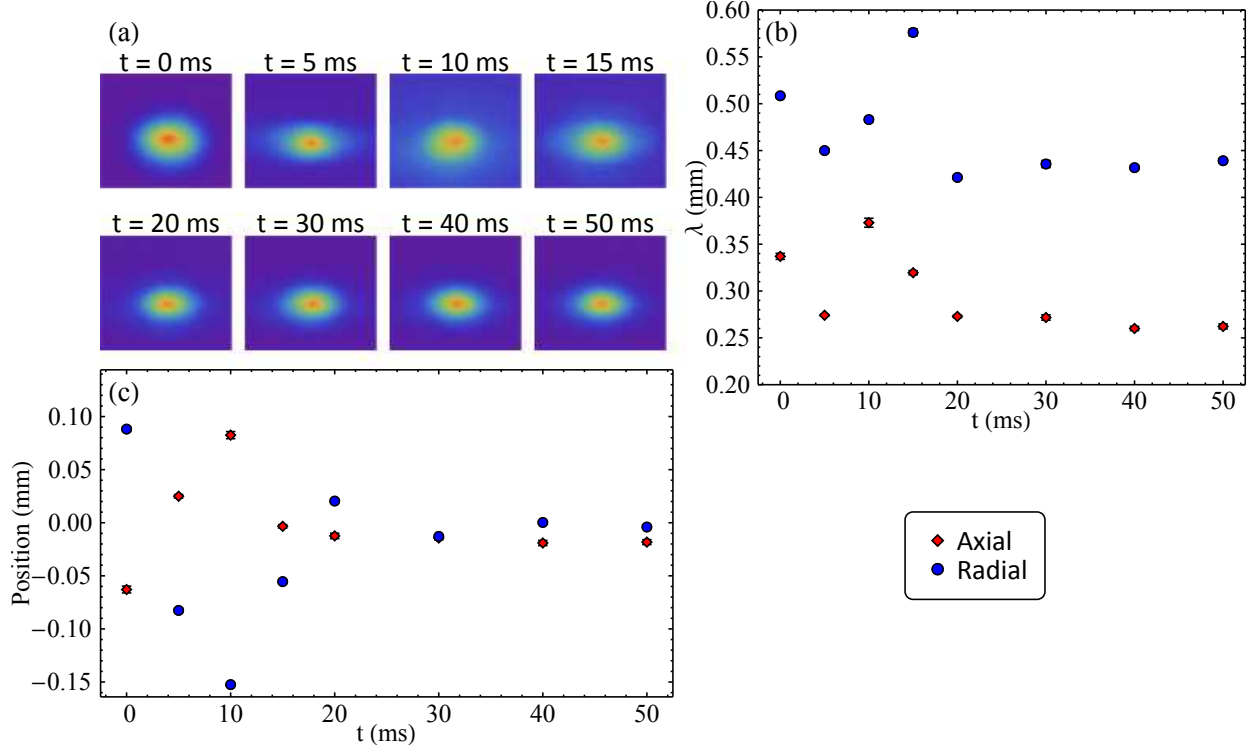


Figure 4.8: Behaviour of the cloud after loading the magnetic quadrupole trap. (a) Absorption images taken with the cloud in the magnetic trap. (b) Width of the cloud in the magnetic trap. (c) Centre of mass coordinates.

4.4.2 Investigating the Loading Process

Figure 4.8 shows the behaviour of the cloud after loading into the quadrupole trap with $b = 36 \text{ G cm}^{-1}$. There is some initial transient oscillation in the centre of mass position and width of the cloud, and after around 20 ms the density profile appears to stabilise. Comparing the data at $t = 0$ and $t = 50$ ms, the centre of mass coordinates shift by 0.09 mm (radial) and 0.05 mm (axial), and the widths decrease by around 20%. The centre of mass offset of the cloud is about 20% of its size, and about a factor 2 larger than we would expect from the drift velocity in the molasses. For the experiments in the following sections, absorption images were taken at $t = 50$ ms to allow time for the density profile to stabilise.

Optical Pumping

To maximise the loading efficiency, we want to ensure as much population as possible accumulates in the $|2, 2\rangle$ state, whilst minimising heating of the cloud. This depends on the

power and polarisation of the optical pumping light. The optical pumping light follows the same path to the experiment as the imaging light, so that changing the optical pumping polarisation changes the imaging polarisation. To take account of this fact, images were taken in the magnetic trap with and without optical pumping applied prior to loading, and the total absorption signal compared. Figure 4.9a shows the effect of varying the quarter waveplate angle θ , at a fixed intensity $I = 0.11 \text{ mW/cm}^2$. Without optical pumping, the absorption signal depends only weakly on the polarisation of the imaging light; when optical pumping is applied, there is a clear optimum around $\theta = -45^\circ$ from the factory-set fast axis. At this angle the light is σ^+ polarised, and we would expect to pump the entire population into the $|2, 2\rangle$ state. To deduce the capture efficiency, absorption images were taken with $\theta = -47^\circ$, after release from the quadrupole into the 1 G bias field. These images were compared with images taken before switching on the trap, so that the fraction of captured atoms could be calculated. When the light is σ^+ polarized, the fraction of atoms captured into the magnetic trap is 77(1)%.

When the light is σ^- polarised, atoms are optically pumped to the untrapped $|2, -2\rangle$ state, and we would expect the number of atoms captured to fall to zero. However, the minimum capture efficiency measured is 14(2)%. This may be caused by imperfect alignment between the laser light and the direction of the bias field, or by transfer of atoms to trapped states as the coils are switched on.

In figure 4.9b, the quarter waveplate angle was fixed to $\theta = -47^\circ$, and the intensity of the optical pumping light varied. The fraction of atoms captured saturates at 85(2)% when the total intensity of the light is 0.23 mW cm^{-2} . Since the $F = 1, 2$ ground states are addressed by separate frequencies, this corresponds to an effective intensity 0.045 times the saturation intensity of the D_2 line. The absorption cross-section, averaged over the transitions in figure 4.3, is $0.24\sigma_0$. From this, we can deduce that the mean time between photon scattering events is $2.5 \mu\text{s}$. Each absorbed photon moves the atom closer to our target state provided the emission produces a photon with different polarisation; averaged over the $F' = 2$ states excited in figure 4.3, this occurs with probability $\frac{17}{30}$. From this, we can estimate that an atom in the $|F = 2, m_F = -2\rangle$ state will scatter about 10 photons before reaching the target state, taking about $25 \mu\text{s}$. This is a factor 8 shorter than the applied pulse. However, in

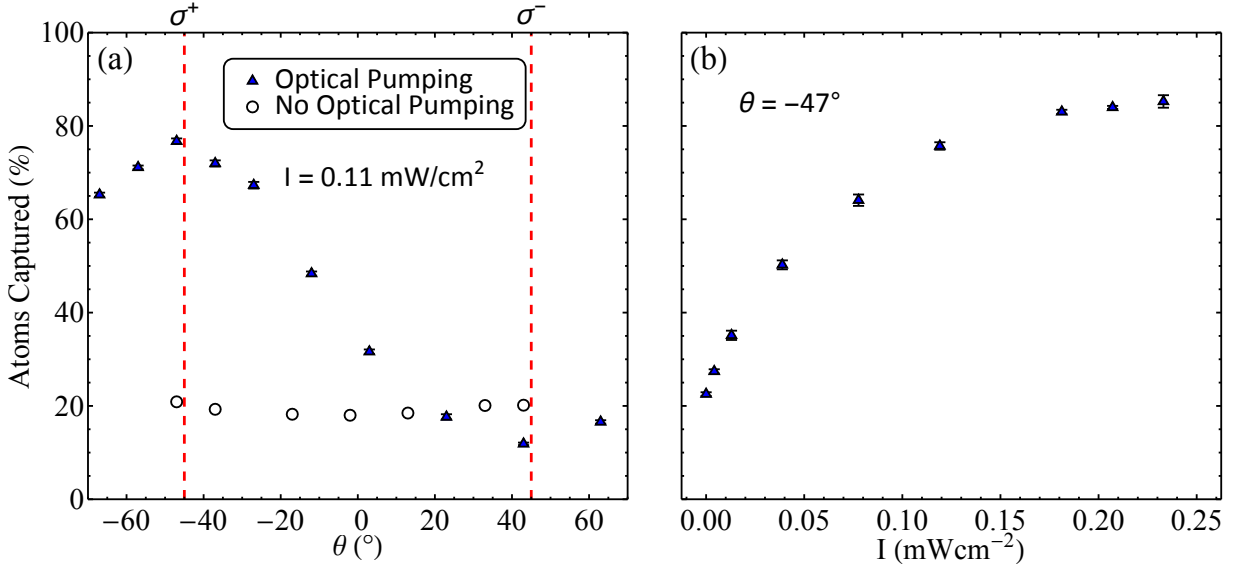


Figure 4.9: Percentage of atoms captured into the quadrupole trap as a function of (a) polarisation and (b) intensity of the optical pumping light.

practice the optical pumping will be inhibited by rescattering of emitted photons, which increases the number of absorption events required to reach the dark state.

Trap Gradient

Next, the effect of the final trapping gradient on the loading process was investigated. Figure 4.10a shows that the number of atoms captured increases as b increases, and begins to saturate at 36 G cm^{-1} . The temperature measurements are shown in figure 4.10b. The values measured from the axial ballistic expansion are close to those deduced from the width of the cloud. In the radial direction, the width measurements suggest a temperature increase, whereas the expansion data appears to predict approximately constant temperatures. Overall, loading a tighter trap tends to heat the cloud, and the heating appears most clearly in the axial velocity distribution. This can be understood as the increase in the potential energy associated with increasing the trap stiffness.

Ramp Time

Another parameter to investigate is the duration over which the trap fields are ramped up to their full value, which we call the ramp time. Figure 4.11a and 4.11b show the

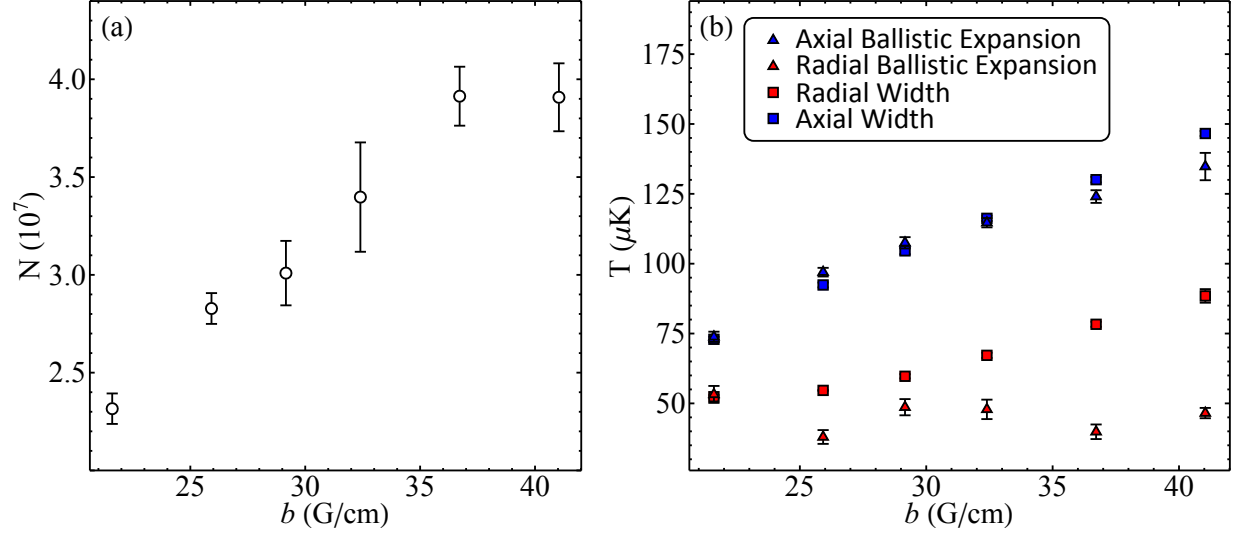


Figure 4.10: Loading the quadrupole trap with different axial magnetic field gradients b . (a) Number of atoms captured. (b) Temperature in the axial and radial directions determined by two different methods.

number of atoms and temperature as a function of the ramp time when loading the trap with $b = 34 \text{ G cm}^{-1}$. It appears from figure 4.11a that switching over 2 ms or longer reduces the number of atoms successfully loaded. The temperature measurements are more difficult to evaluate. The overall trend is that longer loading times tend to result in a colder cloud, but fewer atoms in the trap. At a temperature of $100 \mu\text{K}$, atoms undergoing 10 ms of ballistic expansion would have an RMS radius of 3.43 mm , so that it is unlikely many have left the volume of the trap. A possible explanation for the observed behaviour is that when the trapping gradient is weak, the Majorana loss volume in the trap is large, and that atoms are more likely to transfer to untrapped states when this region is larger for longer. Therefore, to maximise the number of atoms for loading the microwave trap, the ramp time was fixed to 1 ms for all future experiments.

Lifetime in the Internal Trap

An important parameter for our experiment is the magnetic trap lifetime, since we want to maximise the number of atoms available to load the microwave trap. To measure the lifetime, absorption images were taken after release from the magnetic trap, for various trapping times. Figures 4.12a-b show the data obtained under different trapping conditions.

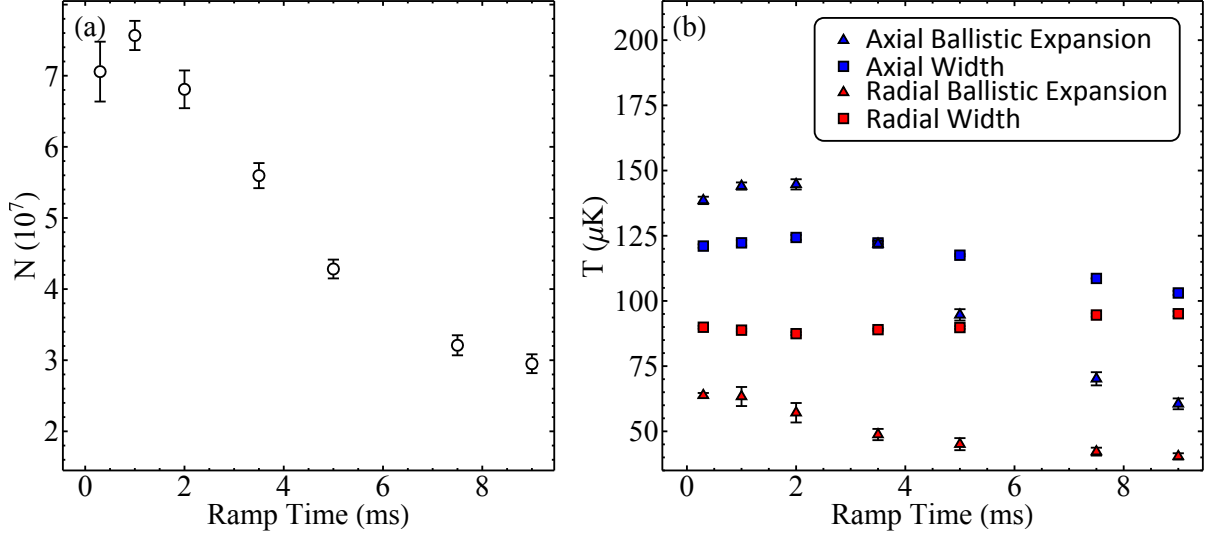


Figure 4.11: Loading the magnetic trap with different linear ramp times. (a) Number of atoms captured. (b) Temperature in the axial and radial directions determined by two different methods.

The molasses light is switched with an AOM, and the driving electronics used to control this AOM provide only 47 dB of attenuation. The small leakage of molasses light to the experiment removes atoms from the trap by optical pumping to untrapped states. As shown in figure 4.12a, the trap lifetime is 0.115(6) s without mechanical shuttering of the molasses light. Once a mechanical shutter was installed, this improved to 1.57(6) s.

Figure 4.12b shows lifetime measurements obtained for different trapping currents. For these data, the lithium oven was blocked using the atomic beam shutter 0.2 s before release from the MOT. For $0 < t < 1.5$ s the loss rate is approximately constant, and the $1/e$ lifetime with $I = 17$ A is 1.77(9) s for these data. This is an improvement of 0.2(1) s on leaving the MOT exposed to the atomic beam, and can be explained by the corresponding increase ($\sim 5\%$) in the MOT chamber pressure when the atomic beam shutter is open. For longer trap times $t > 1.5$ s, heat dissipated in the in-vacuum coils causes outgassing which increases the background pressure, with a corresponding increase in the loss rate. This is particularly evident for the data obtained with $I = 19$ A. With $I = 17$ A, repeating experiments was found to cause a gradual increase in the pressure, degrading the vacuum quality over 30 minutes or so. For this reason, subsequent trapping data were obtained using a trapping current of 15 A, corresponding to $b = 32 \text{ G cm}^{-1}$.

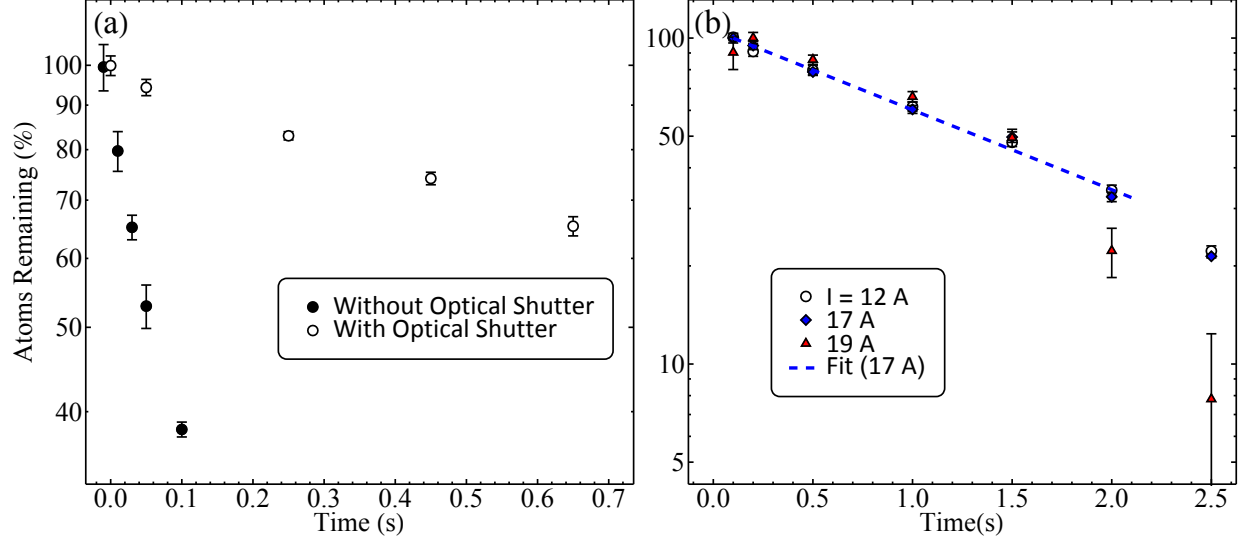


Figure 4.12: Lifetime measurements in the internal quadrupole trap. (a) Comparison of measurements with and without mechanical shuttering of the molasses light. (b) Comparison for different trapping currents I . Dashed line: fit to $A \exp[-\frac{t}{\tau_i}]$, from which τ_i is measured as $1.77(9)$ s for $0 < t < 2$ s.

4.5 Transfer to the External Trap

To transport the cloud, we must transfer atoms to the external trap. The external coils are in approximate quadrupole configuration, so that close to the trap centre we can describe the magnetic field with equation 4.6. However, we must account for the spatial offset between the internal and external traps. Let us assume that the z -axes of the two traps are parallel, and that the centre of the external trap is displaced by $\mathbf{r}_e = (x_e, y_e, z_e)$ from the origin. If the axial field gradients applied by the internal and external trap are b_i and b_e respectively, the resultant magnetic field is

$$\mathbf{B} = \frac{b_i + b_e}{2} \left[(x - \alpha x_e) \mathbf{x} + (y - \alpha y_e) \mathbf{y} - 2(z - \alpha z_e) \mathbf{z} \right] \quad . \quad (4.27)$$

The total field profile is quadrupolar with axial gradient $(b_i + b_e)$, but the field zero is offset by $\alpha \mathbf{r}_e$, with $\alpha = b_e / (b_i + b_e)$. Therefore a sensible method for transferring between traps is to increase b_e continuously from zero while keeping $(b_i + b_e)$ constant. This smoothly moves the trap centre whilst maintaining a constant trap stiffness. For the first transfer experiments, atoms were transferred to the external trap over 100 ms using linear current

ramps, as illustrated in figure 4.13a. After holding for 100 ms in the external trap, the transfer process was reversed so that temperature measurements could be made by ballistic expansion from the internal trap.

Figure 4.13b shows the position of the cloud measured from absorption images in the trap. In the radial direction, the cloud moves as we would expect, translating linearly from one trap centre to the other. However, the cloud appears to jump along the axial direction over around 10 ms at the beginning and end of each ramp stage. A sudden shift in the position of the cloud suggests the presence of an offset field during the ramp stage, which moves the position of the trap centre. The size of the jump, 0.5 mm, corresponds to an axial field of 1.6 G. This is too large to be explained by a delay between the current ramps applied to the external coils (measured as $< 100 \mu\text{s}$), suggesting that eddy currents are being induced in the MOT chamber by the external coils.

The eddy current induced by a change in the external coil currents is proportional to $\frac{dI}{dt}$, and so when the current ramp is reversed during the second ramp stage we expect to see the offset of the cloud reverse direction; this effect is clearly seen from figure 4.13a. The direction of the observed shift is consistent with dominant induction by the bottom trapping coil, which is mounted closer to the chamber. Figure 4.13b shows the result of repeating the transfer experiment with a 50 ms transfer time; the size of the positional jumps approximately doubles, again consistent with there being an induction field present during the current ramps. A sudden shift in the trap centre position will tend to heat the cloud, and I measured temperatures of 164(3) μK and 62(4) μK in the axial and radial directions after carrying out the linear ramp procedure in figure 4.13a.

One approach to mitigating this problem is to apply a compensation field during the ramp stages to cancel the induction field. However this method relies on the field-switching being well synchronised, which is difficult to ensure since our shim coils are external to the vacuum chamber and so will also induce eddy currents.

Another approach is to ramp the current smoothly between the initial and final values, such that the rate of change $\frac{dI}{dt}$ is continuous during the ramp stage. The resulting induced field will therefore vary continuously, making it easier to cancel. I decided to apply the ramp profile shown in figure 4.14a, whose ramp rate dI/dt is a symmetrical triangle function. The

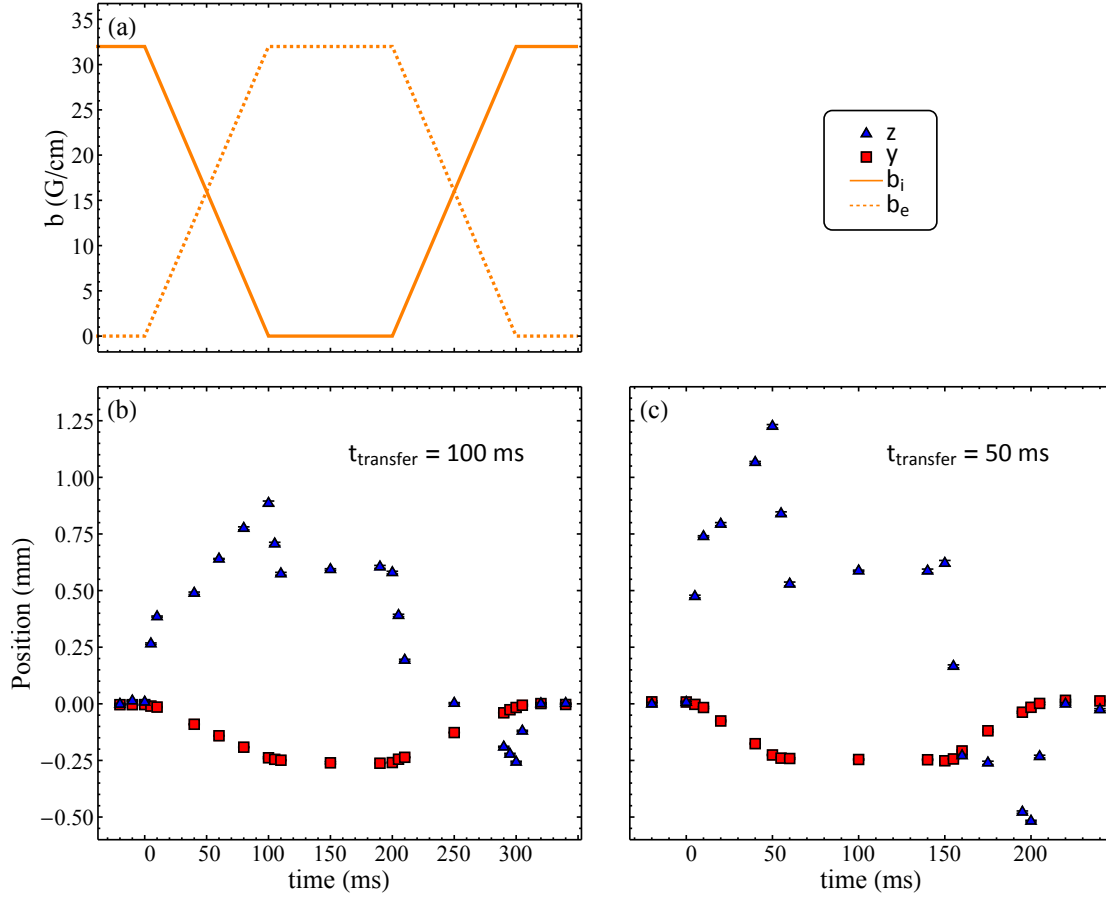


Figure 4.13: Transfer between the magnetic traps using linear current ramps such that $b_i + b_e$ is constant. (a) The transfer testing protocol. (b) Centre-of-mass of the cloud with a 100 ms transfer time. (c) Centre-of-mass of the cloud with a 50 ms transfer time.

maximum $\frac{dI}{dt}$ is twice that of an equivalent linear ramp, so that at its maximum the induced field is twice as large.

Figure 4.14b shows the cloud position when transferring between traps using the smooth ramp procedure, with a ramp time of 100 ms. The cloud position varies smoothly and reaches a turning point close to the midpoint of the ramp, which is qualitatively what we expect. For the data in figure 4.14c, the same transfer ramp was used, with additional adjustment of the bias coil currents to try to minimise displacement of the cloud. The cloud temperature 100 ms after returning to the internal trap was $114(3)$ μK (axial) and $63(3)$ μK (radial) when measured with ballistic expansion. When measuring from the width of the cloud, the values were $110(1)$ μK (axial) and $80(1)$ μK (radial). Our present bias coil drivers are unipolar, and so it was not possible to completely cancel induction by the external coils without introducing

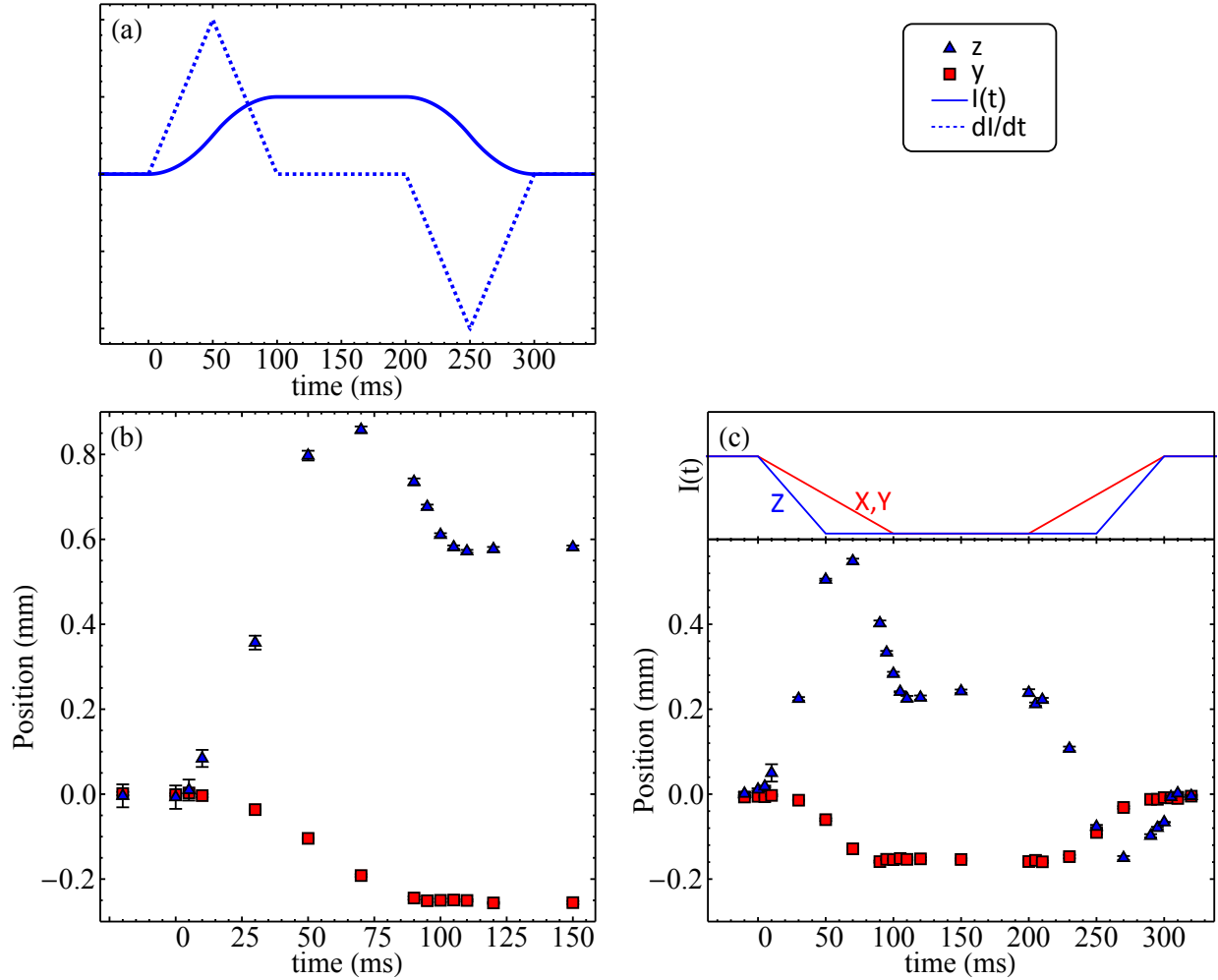


Figure 4.14: Trap transfer with a smooth current ramp. (a) Ramp profile $I(t)$ and dI/dt . (b) Transfer to the external trap, without adjustment of the shim coils during the ramp. (c) Round-trip transfer, with adjustment of the shim coil currents (upper panel).

an additional compensation coil external to the chamber. Nonetheless, the smooth ramp procedure reduces the observed heating in accordance with our expectations.

Lifetime In the External Trap

Having improved the trap transfer procedure, it was then possible to measure the properties of the external quadrupole trap. The important parameters for our purposes are the lifetime for atoms in the trap, and the heating rate.

Figure 4.15a shows a measurement of the trap lifetime. Fitting the data to $N = N_0 e^{-\frac{t}{\tau_e}}$ provides $\tau_e = 2.47(15)$ s, which is about 40% longer than measured with the internal

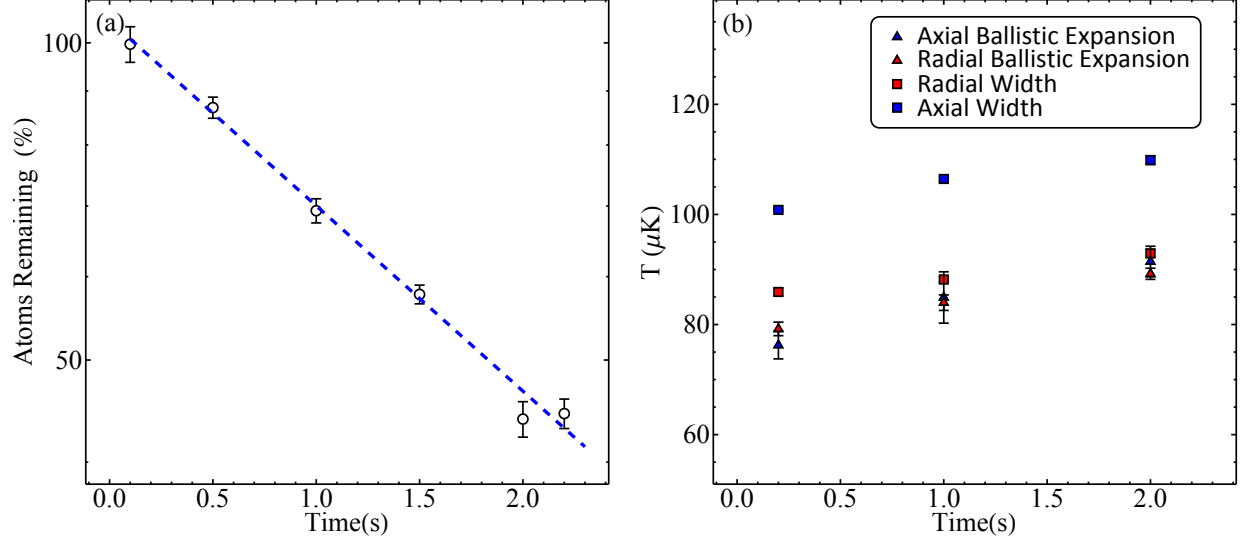


Figure 4.15: Measurements in the external magnetic trap. (a) Percentage of atoms remaining in the trap as a function of time. Dashed line: fit to an exponential decay. (b) Temperature against hold time.

magnetic trap. Given that the MOT chamber pressure typically increases by about 40% over a day of experiments, partly as a result of running the internal coils for long periods, this difference is not unexpected.

The coils for the external trap are controlled by separate current supplies, and therefore current noise is not common mode and translates onto jitter in the trap position. Noise on timescales comparable to atomic orbits in the trap will tend to heat the cloud over long periods. To characterise this effect, temperature measurements were made by ballistic expansion from the internal trap after different hold times in the external trap. Initially, I found that the temperature measured by ballistic expansion indicated a heating rate of $45 \mu\text{K s}^{-1}$ in the axial velocity distribution. I found that the main sources of current noise entered through our feedback control circuits. Firstly, there was 50 Hz pickup at the current monitors, which verifiably translated into real noise on the current through the coils. This was improved by ensuring a consistent single grounding point between the hardware control unit and the current drivers. Second, the analog output card (PXI-6723) used to control our current supplies has a large channel-to-channel update glitch, resulting in $2 \mu\text{s}$ long, 100 mV transients across all analog channels at the beginning of each clock cycle of the pattern generator (100 μs). This effect only occurs during an experimental run, where the pattern

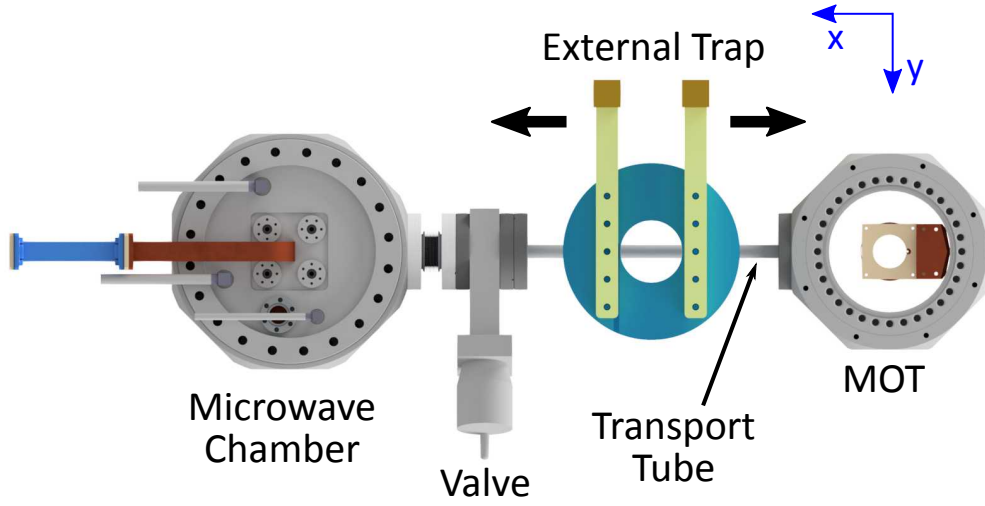


Figure 4.16: Top view of the transport scenario.

generator is in use. Whilst fast compared to the motion of atoms in the quadrupole trap, these transients did affect the tuning of the feedback circuits. After applying appropriate filtering to the control circuit inputs, the transients were reduced by a factor 20 without compromising the response time of the current drivers. An upgrade of the analog control hardware in future would further reduce this problem.

Having dealt with sources of current noise, heating of the cloud was re-examined. As shown in figure 4.15b, the temperature increase is small, about $5\mu\text{K s}^{-1}$. Since this was relatively small compared to the heating encountered in the transport stage, I made no further attempts at improvement.

4.6 Transporting the Cloud

In our experiment, atoms need to be moved from the MOT chamber to the centre of the chamber housing the microwave cavity, so that the microwave trap can be loaded. To do this, the external quadrupole trap is translated a total distance 602 mm along the x -axis with a motorised stage. The atoms pass along a hollow tube of length 250 mm and inner diameter 9 mm, which we call the transport tube. This scenario is shown in figure 4.16.

Background on magnetic transport

The first magnetic transport experiments with ultracold atoms were motivated by the experiments on Bose-Einstein condensation. The evaporative cooling process used requires large samples to work efficiently, and this generally requires a larger vapour pressure of the atoms being used. However, a higher background pressure reduces the available time for evaporation in a trap, because collisions with background atoms reduces the density. By transporting atoms from one chamber to another, one can overcome these competing priorities.

There are two established methods for magnetic transport. The first is to use a series of adjacent traps, and pass atomic samples down a chain by carefully switching the individual trap currents. This method was pioneered by the Hansch group in the early 2000s [134], using a sequence of nine quadrupole coil pairs to transport ^{87}Rb . By careful consideration of the transfer process, the authors were able to reduce the heating to $19\,\mu\text{K}$ and transfer the atoms 33 cm around a 90° corner to a second chamber.

A disadvantage of the multi-trap method is that the number of coils required scales with the transport distance, with an accompanying increase in control electronics. A simpler approach, first used by the Cornell group, is to translate a single trap using a motorised stage [135]. This method is much more scalable and takes advantage of well established commercial technology, making it straightforward to carry out arbitrary transport profiles. Today it is used in a variety of BEC experiments [136, 137, 138].

Our Translation Stage

The translation stage we use is a Parker 404XR linear motion stage (standard precision), which is controlled with a VIX500IE servo driver. Table 4.1 summarises the properties of our model. To operate the stage in an experimental run, it is pre-programmed with a sequence of instructions (hereon referred to as move profiles), each initiated using a digital trigger. A move profile is defined by its associated displacement x , acceleration a , and a peak velocity v ; there are two distinct categories. If $v > \sqrt{ax}$, then the stage cannot reach v before the midway point of the move, leading to a “triangular” velocity-time curve. Otherwise, the stage maintains velocity v until the deceleration stage, leading to a “trapezoidal” velocity

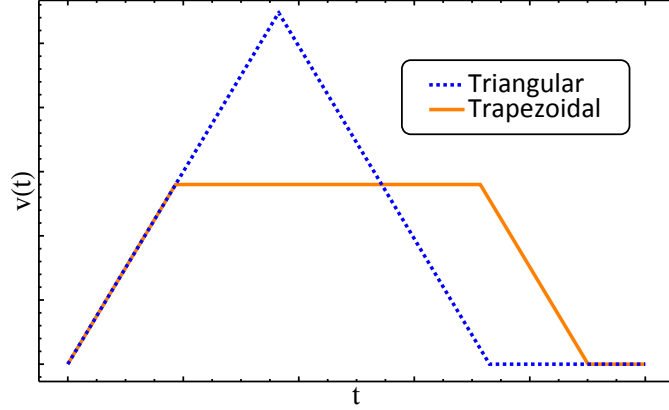


Figure 4.17: Move profiles available for the translation stage.

Property	Value
Bidirectional Repeatbility	$\pm 3.0 \mu\text{m}$
Maximum Acceleration	20 m s^{-2}
Rated Normal Load	1.67 kN
Rated Axial Load	0.88 kN

Table 4.1: Properties of the translation stage.

time profile. These scenarios are illustrated in figure 4.17.

4.6.1 Initial Testing

The majority of the transport experiments were carried out with a setup that omitted the microwave cavity and its vacuum housing, as shown in figure 4.18. This setup allowed for greater optical access along the transport direction, since the cavity mirrors reduce the size of the probe region. For the first tests, atoms were transferred to the external magnetic trap with $b_e = 32 \text{ G cm}^{-1}$ for 2 s, during which time a move profile was executed by the translation stage. Prior to the stage moving, currents in the bias coils were ramped to zero over 50 ms. All move profiles returned the external coils to the MOT position, and the atoms were then returned to the internal quadrupole trap. To measure the efficiency of the transport, the MOT was reloaded and absorption images taken after release; experiments of the same duration were repeated without transport, and the results compared. Temperature measurements were taken by releasing from the internal trap directly.

Figure 4.19a shows the percentage of atoms retained after transport for different accelerations. The stage was moved forwards a distance of 30 mm (meaning the trap centre was

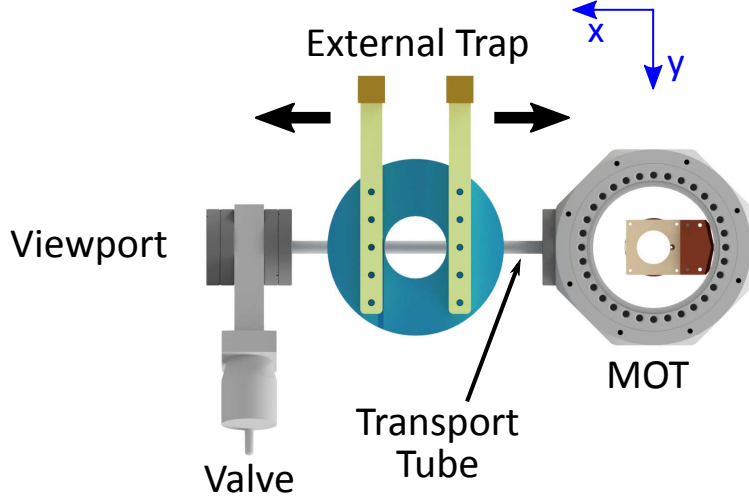


Figure 4.18: Top view of the setup used to test transport.

never close to the transport tube), using a trapezoidal move profile with a fixed peak velocity of 30 mm s^{-1} . We retain 95 – 100% of the atoms without any noticeable change in loss up to 0.75 m s^{-2} . Figure 4.19b shows the dependence of transport efficiency on distance, using $a = 0.75 \text{ m s}^{-2}$ and $v = 0.5 \text{ m s}^{-1}$. We abruptly lose around half of the atoms at $x = 90 \text{ mm}$, close to the entrance of the transport tube (marked by the dashed line in figure 4.19b), but transporting further does not result in additional losses.

To investigate directly what was happening to the cloud during the transport process, absorption images of the cloud were taken using a collimated imaging beam along the transport tube, without imaging optics. First, the transport parameters were set to $a = 0.75 \text{ m s}^{-2}$ and $v = 0.5 \text{ m s}^{-1}$, and the transport distance was varied from 0 to 120 mm. Absorption images were taken 0.95 s after the forward motion was initiated; in the case of $x = 120 \text{ mm}$, this allows 0.15 s between the coils coming to rest before imaging. Figure 4.20a shows a set of absorption images, and figure 4.20b shows the centre of mass coordinates of the cloud. Surprisingly, there is a large ($> 5 \text{ mm}$) shift in the axial position of the cloud as the coils move. Between 40 and 70 mm, the cloud moves out of the probe region entirely. By adjustment of the trapping currents, I found that the position of maximum displacement was $x = 60 \text{ mm}$. To ensure the cloud was central in the probe region here, it was necessary to increase the bottom coil current to 68 A. This corresponds to an axial magnetic field offset of around

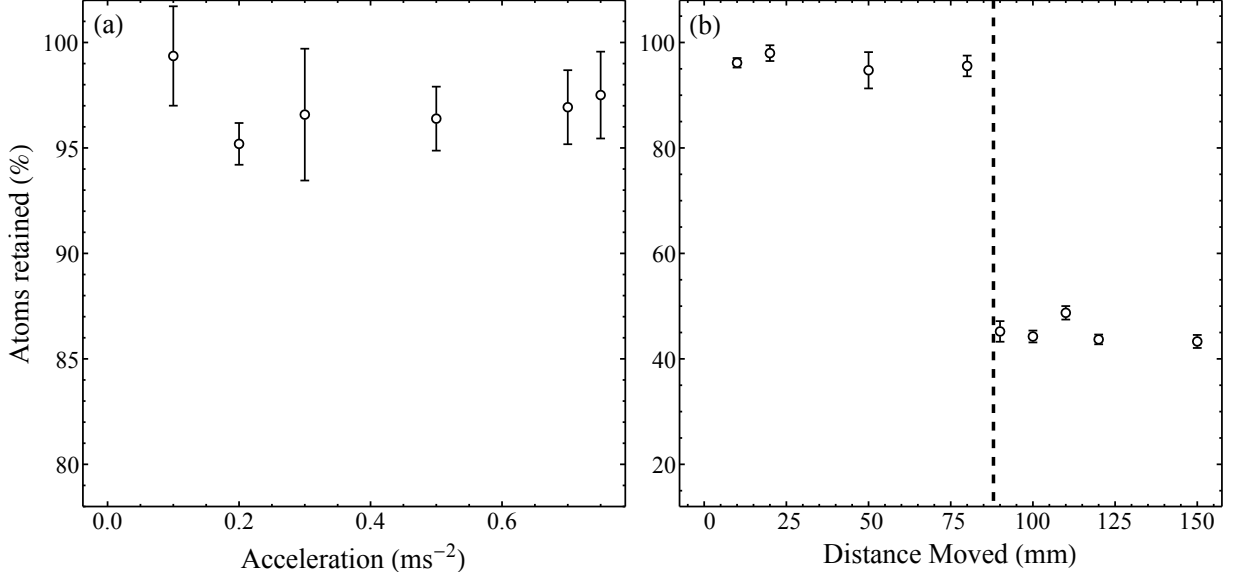


Figure 4.19: Percentage of atoms returned to the MOT for different transport move parameters. (a) $x = 30$ mm, varying the acceleration. (b) fixed acceleration $a = 0.75 \text{ ms}^{-2}$, varying the distance travelled; the dashed line marks the entrance of the transport tube.

20 G at $x = 60$ mm.

It is unlikely that such a large positional shift is caused by eddy currents, since for the data at $x = 40$ mm, the coils were at rest for 0.5 s before imaging. We would expect induced currents to have mostly dissipated by this time. To rule out induction as a cause, the cloud was imaged during two different motion profiles, both transporting the coils 40 mm. The velocity profiles were both triangular, but the first used $a = a_0 = 0.75 \text{ ms}^{-2}$, and the second used $a = a_0/4$. Since the second profile moves the coils more slowly, we would expect any induced field to be smaller. Figure 4.20c shows position measurements obtained; the time axis is scaled by the total motion time for each profile, t_{motion} , so that each position on the time axis represents the same position of the trapping coils along the translation stage. The two datasets overlap, from which we conclude that the cloud displacement is caused by a permanent magnetic field inhomogeneity in the chamber, rather than by induction from moving the coils. The most likely culprit is the weld at the transport tube entrance. Magnetic field measurements outside the chamber in the vicinity of the weld gave around 3 G, so that the field inside the chamber would need to be around a factor 6 larger than outside. Replacing the weld would require a re-bake of the MOT chamber and Zeeman slower, and so we decided to proceed regardless.

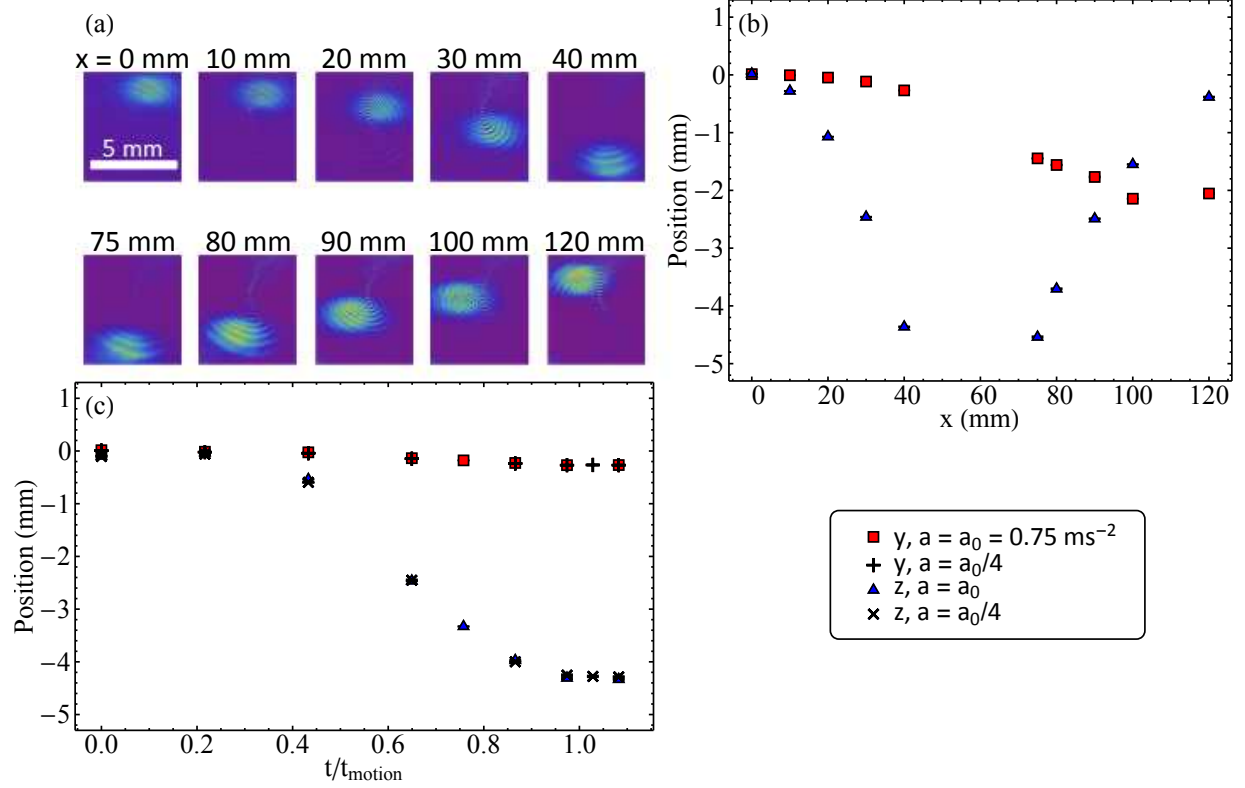


Figure 4.20: Measurements of the cloud centre-of-mass position during motion. (a) Absorption images taken after transporting various distances x , with $a = a_0 = 0.75 \text{ m s}^{-2}$. (b) Centre of mass coordinates for the experiments in (a). (c) Centre of mass coordinates during transport to $x = 40 \text{ mm}$, using $a = a_0$ and $a = a_0/4$.

As well as the undesirable loss of atoms, the sudden movement of the cloud as it enters the transport tube might heat the cloud, possibly making it too hot to load into the microwave trap. To investigate the temperature after transport, atoms were returned to the internal trap following a move profile, and the temperature measured by ballistic expansion. After transporting the cloud 150 mm with $b_e = 32 \text{ G cm}^{-1}$ and $a = 750 \text{ mm s}^{-2}$ the temperature measurements were $211(13) \mu\text{K}$ and $132(7) \mu\text{K}$ in the radial and axial directions respectively. As a comparison, the temperature data after simply holding for 2 s in the external magnetic trap was measured to be $96(6) \mu\text{K}$ in the radial direction and $98(6) \mu\text{K}$ in the axial direction. Thus, we see that there is significant heating from the transport.

There are several heating sources which are interrelated in the transport stage, and depend on the particular move profile chosen. For example, the amount of jerk of the trap centre will depend on the acceleration used; the acceleration time depends on the final

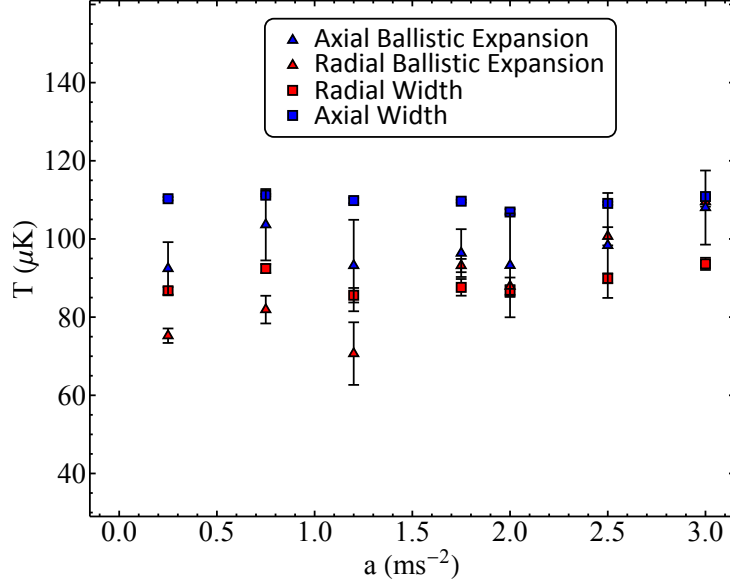


Figure 4.21: Temperature versus transport acceleration, for $x = 30$ mm.

velocity, and the effects of the defect field around the weld depend on the velocity and acceleration as the coils move towards the transport tube. These effects are difficult to disentangle completely, but we can make attempts to isolate or remove individual causes to gain some insight.

First, I tested the effect of the transport acceleration by fixing $x = 30$ mm, meaning the cloud does not significantly experience the effect of the weld. Figure 4.21 shows that accelerations up to 3 ms^{-2} cannot account for the heating observed after transport into the tube. Therefore it appears that heating is entirely due to moving through the defect field around the weld.

To examine the effect of the weld, the transport distance was set to 100 mm, so that the cloud travelled over the bulk of the defect field. Two velocity profiles were tested, shown in figure 4.22; both profiles use a linear acceleration of 0.75 ms^{-2} , but one has a trapezoidal profile with peak velocity 140 mm s^{-1} , meaning it passed over the defect field more slowly. The axial temperature dropped from $139(6) \mu\text{K}$ to $117(5) \mu\text{K}$ when using the trapezoidal profile, which is consistent with the idea that moving too quickly over the defect field heats the cloud. The radial temperature was unaffected by the choice of move profile; I measured $132(3) \mu\text{K}$ with the trapezoidal move profile, versus $135(4) \mu\text{K}$ with the triangular profile.

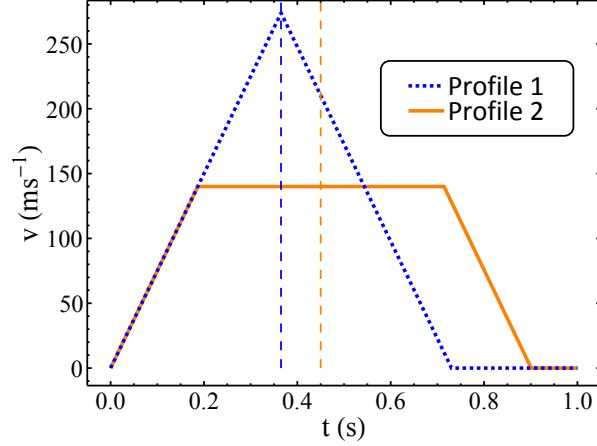


Figure 4.22: Move profiles used to test the effect of travelling over the defect field region around the tube weld. Blue, solid line: a triangular move profile; Orange, dashed line: a trapezoidal move profile with peak velocity 140 mm s^{-1} . Dashed lines: the time at which the stage passes 50 mm.

4.6.2 Improving the Transport Procedure

By referring back to figure 4.19b, it is clear that the displacement of the cloud does not cause a loss of atoms until the cloud is around the tube entrance. A reasonable way to proceed is to alter the coil currents as the translation stage moves, with the aim of maintaining alignment with the tube axis. To develop a suitable current-time profile, the axial position of the cloud was measured at various transport distances, as a function of the current imbalance between the coils. From these data, it was then possible to calculate the current imbalance required to maintain a fixed cloud position in the axial direction.

Whilst moving the transport coils does not cause significant induction in the MOT chamber, switching the currents does. We must remember the induction effects shown in figure 4.20, and avoid jerking the cloud by ramping the currents smoothly. To choose the ramp parameters, images of the cloud were taken at various transport distances, and the current balance of the trapping coils was adjusted such that the cloud remained roughly central in the probe beam. Figure 4.23a shows the current imbalance as a function of translation stage position, which can then be transformed using the chosen move profile to produce our desired current-time ramp. The move parameters were set to $a = 0.75 \text{ m s}^{-2}$, $x = 150 \text{ mm}$, and images were then taken throughout the transport stage and used to track the cloud centre position. The result of these measurements is shown in figure 4.23c. It is clear that

the cloud remains reasonably well aligned with the tube, and that the loss of atoms at the entrance tube is less severe than before. Nevertheless, around 50% of the atoms are still lost after moving well beyond the entrance of the tube. The current ramp, whilst ensuring the cloud remains within the field of view, doesn't fully stabilise the axial motion around the entry point. The temperatures measured following this move profile were $136(4)\mu\text{K}$ (radial) and $120(10)\mu\text{K}$ (axial), which are close to those measured with the trapezoidal move profile. Therefore we conclude that the shim technique is worthwhile in terms of reducing heating.

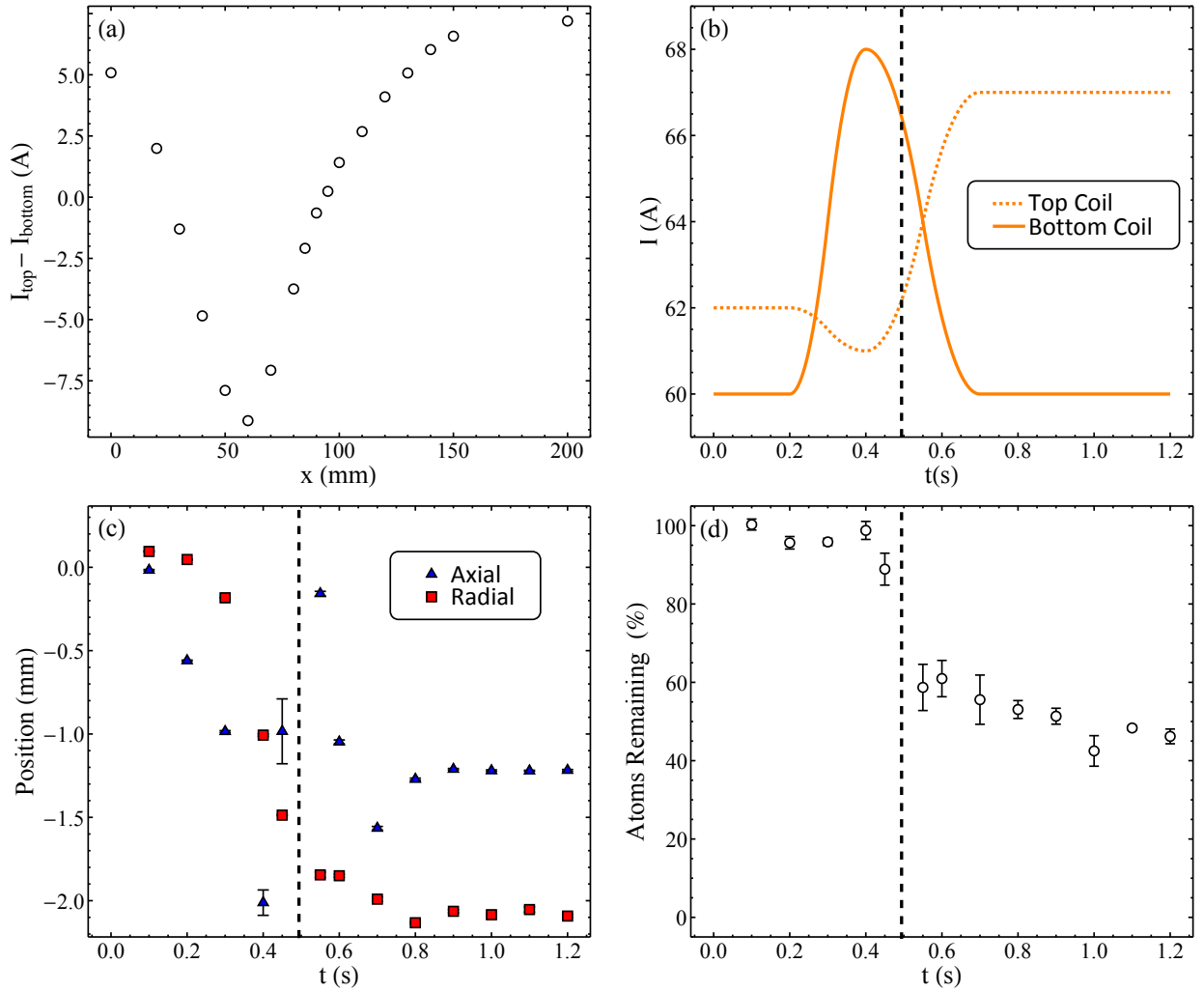


Figure 4.23: Designing a current ramp for transport around the entrance weld. (a) Current imbalance required to maintain the axial centre position at different transport distances. (b) The current-time ramp applied during transport into the tube. (c) Centre of mass coordinates during transport with the current ramping applied. (d) Fraction of atoms remaining during transport. Dashed lines mark the entry time into the transport tube.

Another attempt for reducing the heating effect is to use a tighter trap, which reduces the shift in the trap centre when a stray field is present. To test this approach, the external trap was loaded using $b_e = 50 \text{ G cm}^{-1}$ (100 A), rather than 32 G cm^{-1} . The transfer time was extended to 150 ms to avoid increasing the eddy currents induced by the external coils. From equation 4.18, the temperature of the cloud should increase by a factor 2 in the external trap, but if the compression is adiabatic then the phase-space density is conserved. I found that using this tighter trap improved the return transport efficiency to 70%, when transporting 150 mm from the MOT position.

One might hope that by both tightening the trap and compensating the field around the weld, we can improve the transport efficiency and keep the atoms cold. In order to test this, the procedure for imaging the cloud during the transport was repeated with $b_e =$, and a suitable current ramp designed. We used $x = 150 \text{ mm}$ and $a = 0.75 \text{ m s}^{-2}$ for direct comparison with the results at 32 G cm^{-1} . Figure 4.24a shows the coil currents used during the ramp, along with the translation stage velocity as reported by its servo driver. The ramping keeps the cloud within $\pm 1 \text{ mm}$ of its initial location in the axial direction. Figure 4.24c shows the ballistic expansion data obtained after transport, with and without the current ramping applied. The cloud temperature with fixed coil currents is 175 (6) μK (radial) and 141 (5) μK (axial); with the current ramping we find 134 (4) μK (radial) and 123 (6) μK (axial). The optimised ramp was found to further improve the transport efficiency to 85%.

Having improved the transport from the MOT position to inside the tube, the next step was to transport the cloud along the entire distance of the tube. For the first tests, I utilised the shim profile in 4.24a for entry in to the transport tube, and then fixed the trapping currents at the final values for the remainder of the move profile.

Figure 4.25a shows the cloud centre position versus distance, for $150 < x < 390 \text{ mm}$. There appears to be some misalignment between the translation stage and tube axis, which explains the slow drift in the position for $150 < x < 300 \text{ mm}$. From the data we can estimate an upper bound of 1.2° , which is approximately the same in each of the measurement directions. The tube exit position is at $x = 338 \text{ mm}$, marked by the dashed line. Here we see a sudden jump of around 0.3 mm in the axial position, which we interpret as being due to

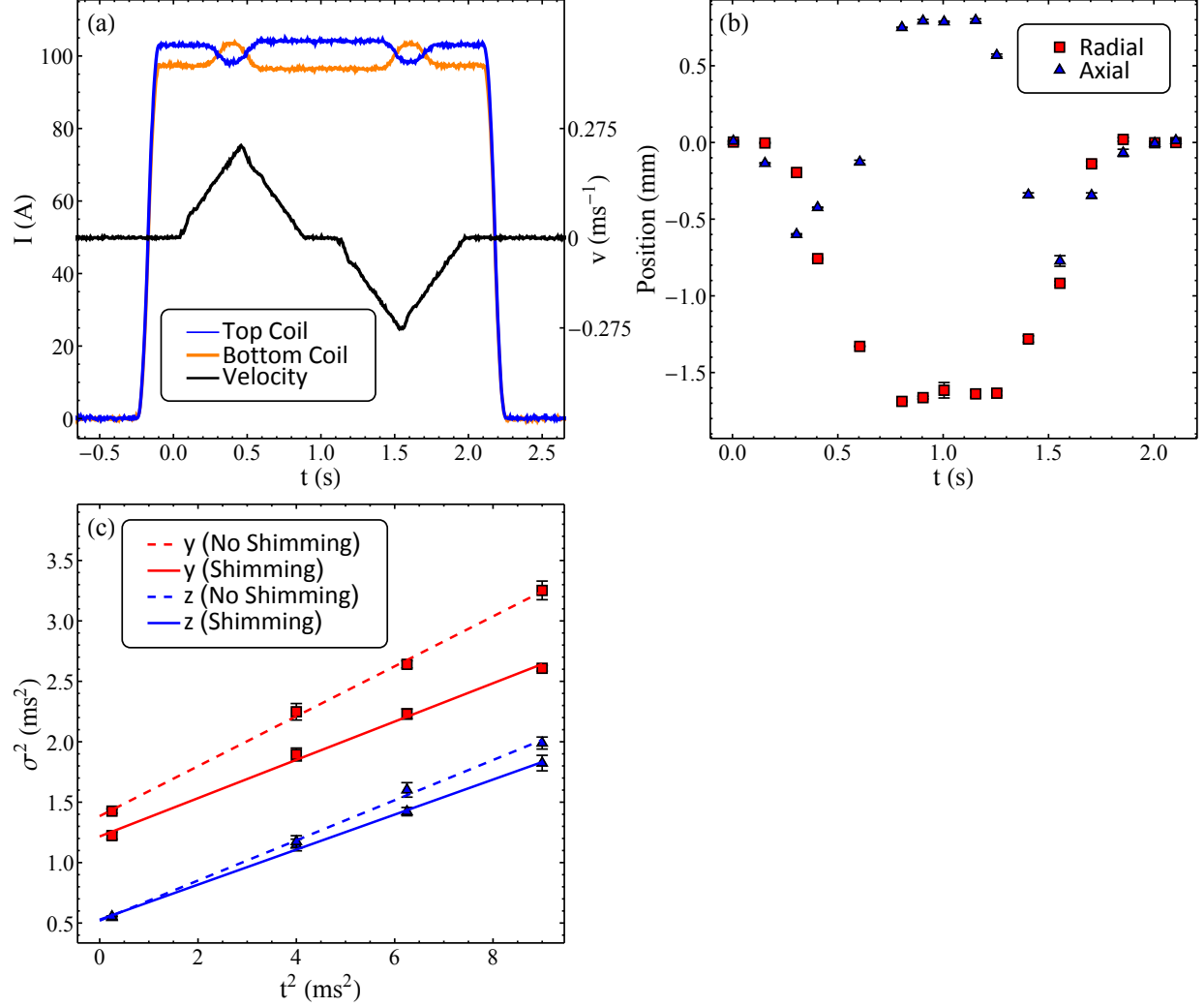


Figure 4.24: Improved transport scheme, with $b_e = 50 \text{ G cm}^{-1}$ and including axial shimming during motion. (a) external trap currents (left axis) and translation stage velocity (right axis). (b) cloud position during the transport. (c) Ballistic expansion measurements after transport. Dashed (solid) lines are linear fits to the data along y and z with (without) the compensation shimming applied during transport.

the exit weld; based on our trapping field gradient of 50 G cm^{-1} , the size of this field is 1.5 G. This is a factor ~ 10 smaller than around the entrance point of the tube. Between $x = 350$ and 390 mm the cloud centre shifts more abruptly. This suggests either that there is a defect field from the gate valve, or may indicate some confinement of the magnetic field from the exit to the gate valve opening. If we assume the latter, then we would estimate a magnetic field defect of 3.5 G and 1.3 G along the axial and radial directions respectively. This is a factor ~ 5 lower than at the entrance weld. The magnetic field outside the viewport was

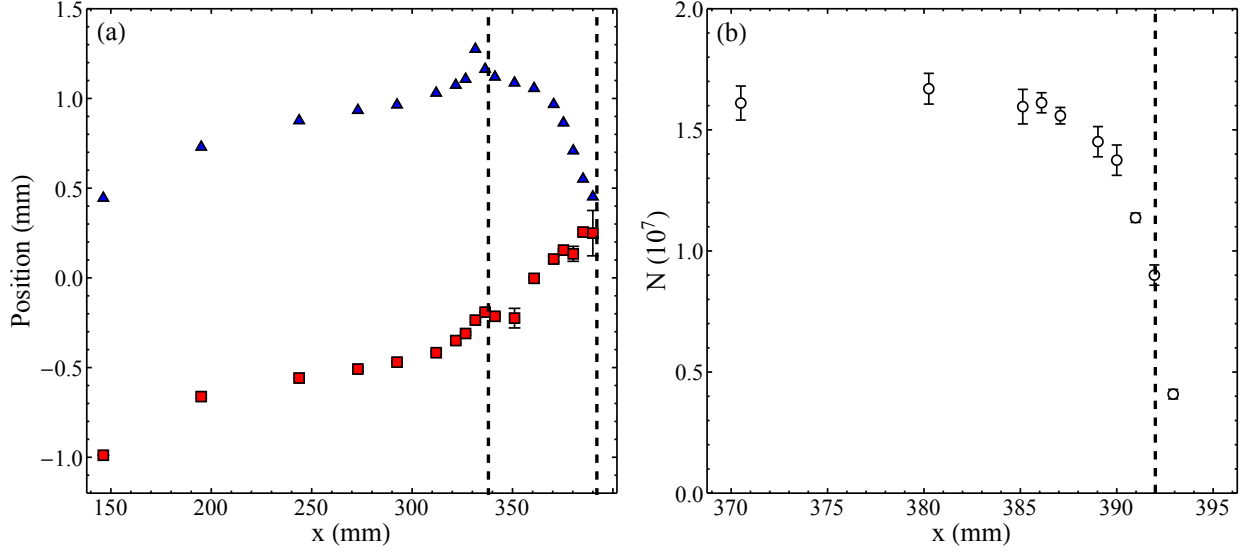


Figure 4.25: Transport through the exit weld of the tube. (a) cloud position measurements for $150 \text{ mm} < x < 390 \text{ mm}$. Black, dashed lines: location of the tube exit and end viewport. (b) number of atoms measured in the trap on approach to the end viewport. Black, dashed line: approximate position of the inner face of the viewport.

measured with a lakeshore 421 gaussmeter, providing upper bounds in the field variation of 1.5 G (along the tube axis), 0.5 G (along the axis of the magnetic trap), and 0.3 G (along the Zeeman slower direction) over a few centimeters around the tube axis. Figure 4.25b shows the number of atoms observed in the external trap as its centre approaches the viewport. The dashed line marks the position of the inner face of the viewport glass on the vacuum side, assuming that $x = 0$ corresponds to the centre of the MOT chamber. The number of atoms falls over a 10 mm range, and we observed a complete loss of signal at $x = 394 \text{ mm}$, which we can understand as the trap centre being inside the viewport. Although this is 2 mm further than expected, this could be explained by an offset in the original start position of the coils, or by a stray field around the gate valve. Assuming it is completely due to the stray magnetic field, we would estimate 5 G along the tube axis. This supports the conclusion of measurements outside the viewport, that the exit of the tube is less problematic than the entrance.

To correct for the axial misalignment between the transport tube and the stage, we can apply an additional current ramp whilst the cloud moves along the tube. Figure 4.26 shows position measurements during motion, using $x = 370 \text{ mm}$, $v = 500 \text{ mm s}^{-1}$, $a = 750 \text{ m s}^{-2}$.

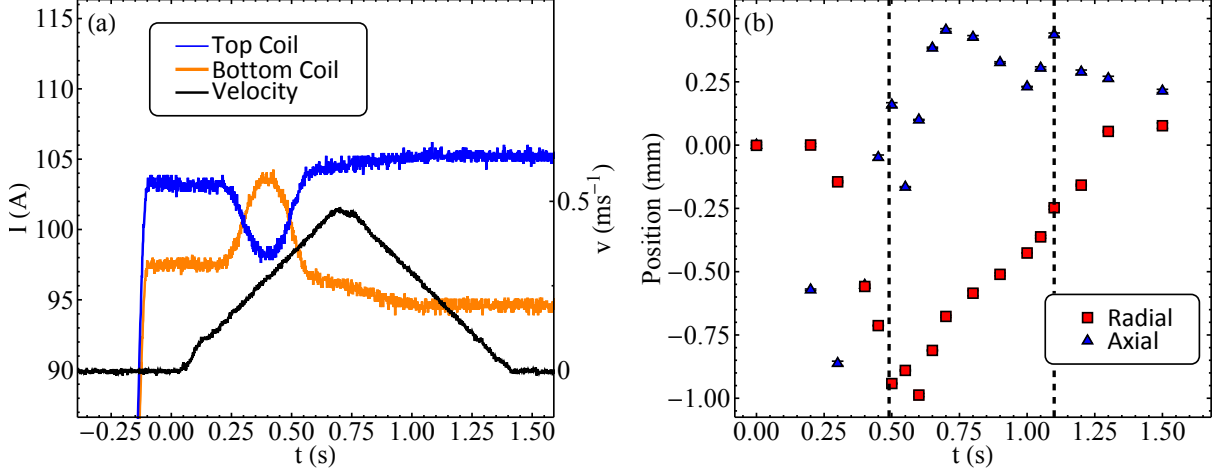


Figure 4.26: Transport through the entire transport tube. (a) external trap currents (left axis) and translation stage velocity (right axis). (b) cloud position during the transport; dashed lines mark the entrance and exit from the tube. The move profile was triggered at $t = 0$.

For $0.6 \text{ s} < t < 1.0 \text{ s}$, a linear ramp was applied to the coil currents, shown in figure 4.26a. As a result, the overall shift in the cloud centre from $x = 0$ to $x = 370 \text{ mm}$ is less than 0.15 mm , meaning we could comfortably transport to the cavity centre. At this point, the microwave chamber was attached to the main experiment.

4.6.3 Transport into the Microwave Chamber

Transport and Transfer Considerations

The cavity assembly is made almost entirely from copper, and so we might expect induction to cause problems when moving the coils over the cavity, and when switching off the coils to load the microwave trap. Figure 4.27a shows the z -component of the magnetic field (measured with a fluxgate magnetometer) close to the cavity centre, as the coils are moved to the cavity centre position with $I = 10 \text{ A}$ per coil. The acceleration of the move profile was $a = 2 \text{ m s}^{-2}$, meaning the coils were moving at 0.7 m s^{-1} when 12.5 cm from the cavity centre. Introducing the cavity changes the magnetic field measurements by less than 50 mG , and so we do not expect induction to cause problems on approach to the microwave trap.

Figure 4.27b shows the magnetic field components close to the cavity centre after a sudden switch off from $b_e = 5 \text{ G cm}^{-1}$ at $t = 0$. After an initial transient, the magnetic field requires about 40 ms to turn off. This is about a factor 4 longer than for switching over the MOT

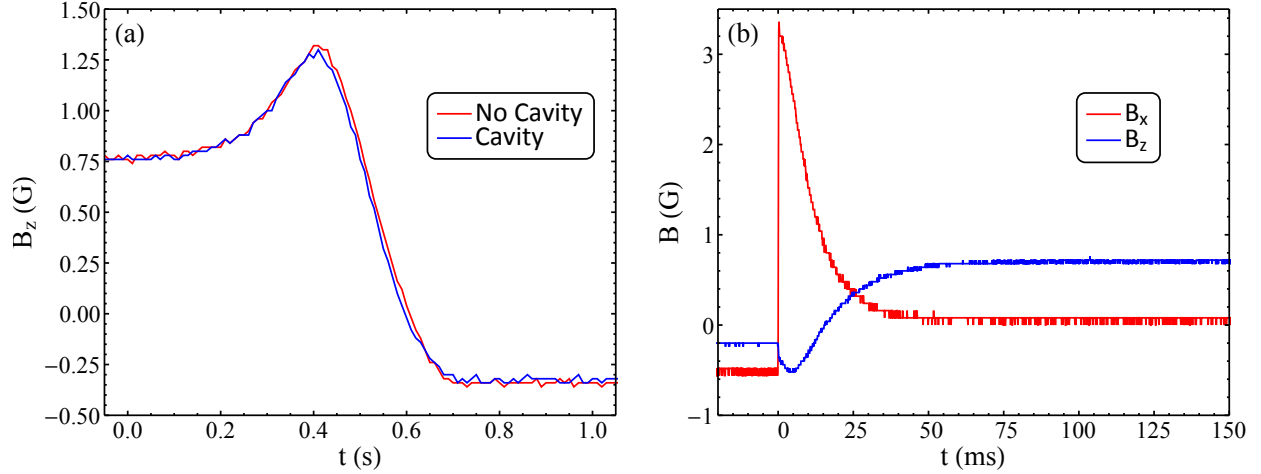


Figure 4.27: Magnetic induction measurements at the centre of the microwave trap. (a) Moving the external trap with $I = 10$ A per coil to the centre of the cavity. (b) Switching the trapping currents to zero from $I = 10$ A over the cavity centre.

chamber, and sets the limit for how fast we can release atoms into the microwave trap.

Lifetime in the microwave chamber

Once atoms had been transported to the microwave chamber, it was possible to make a lifetime measurement. Rather than transport atoms back to the MOT (which would introduce additional losses), absorption images were taken in the magnetic trap after various hold times at the cavity centre. Figure 4.28 shows the data obtained, from which the measured $1/e$ lifetime is $3.9(2)$ s. The pressure in the microwave trap chamber for these experiments was $1.3(1) \times 10^{-9}$ mbar, which is within about 10% of the pressure in the MOT chamber. The difference in the lifetime in each is a factor ~ 1.5 , which is likely explained by differing gauge calibrations, or vacuum conductance between the gauges and their respective pumps. A separate measurement of the lifetime with the microwave chamber pressure at $6(1) \times 10^{-10}$ mbar provided $8.4(1.0)$ s, which is consistent with background gas collisions dominating loss from the trap. However, this vacuum quality could not be attained without compromising the microwave power delivered to the experiment, as discussed in section 5.5.4.

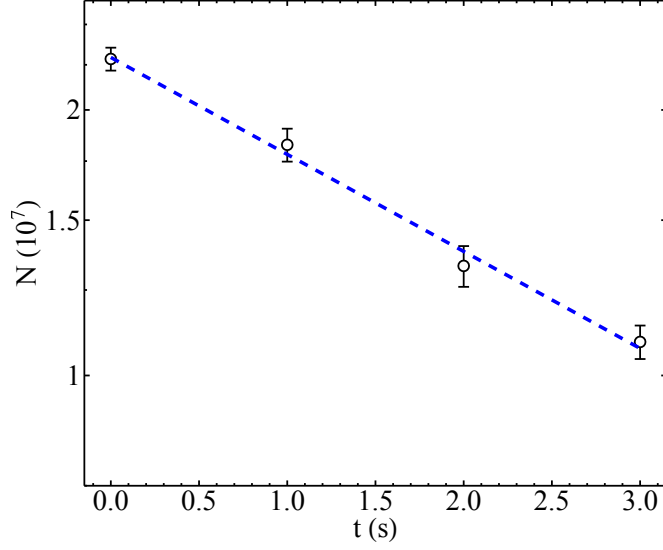


Figure 4.28: Magnetic trap lifetime in the microwave chamber.

Overall Transport Efficiency

Having measured the trap lifetime in the microwave chamber, it was then possible to assess the overall efficiency of the transport stage. To do this, absorption images were compared in the external trap before transport, and then 2 seconds later, after the cloud had arrived at the cavity centre. From these measurements, the fraction of atoms remaining at the microwave cavity was calculated as 56(2)%. If we assume the trap lifetime during transport is the same as measured in the MOT chamber, then we would expect to retain just 44%. Assuming the lifetime measured in the microwave chamber, we expect to retain 65% of the atoms. This provides a lower limit for the overall transport efficiency of 86%.

Estimation of the Phase-Space Density

Since large background magnetic fields due to eddy currents in the microwave chamber persist for several milliseconds, it was not possible to perform ballistic expansion measurements after transport. Instead, we can estimate the phase space density at this stage in the experiment using the results of section 4.6.2. We know that the dominant source of heating occurs when moving through the transport tube entrance, that this effect is less severe at the exit of the tube, and that the acceleration of the stage does not measurably heat the atoms. The optimised temperature after moving the cloud 60 mm along the transport tube and returning

to the MOT is $130(3) \mu\text{K}$, measured by ballistic expansion from the internal quadrupole trap with $b_i = 32 \text{ G cm}^{-1}$. The peak density measured 0.5 ms after release from the trap is $1.84(3) \times 10^9 \text{ cm}^{-3}$, assuming 2×10^7 atoms. This corresponds to a phase space density of $3.58(15) \times 10^{-7}$ at the centre of the trap. Although we actually transport the cloud at $b_e = 48 \text{ G cm}^{-1}$, the compression is adiabatic and so should conserve the phase space density.

4.7 Trajectory Simulations in the Quadrupole Trap

The assessment that our cloud is sufficiently cold for loading into the microwave trap assumes that any heating of the cloud along the transport direction will show up in the orthogonal velocity distributions. Motion in the quadrupole trap is complex, and therefore one would expect motion along each axis to couple and mix together. However, a previous experiment using magnetically transported Rb reported a clear increase in the velocity distribution along the transport direction, but not in the orthogonal directions (see figure 4 of [137]). To ascertain whether this might be a problem for our experiment, I carried out trajectory simulations in the quadrupole trap.

The equation of motion for an atom in the magnetic quadrupole trap is

$$\frac{d^2}{dt^2} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{\mu b}{m} \frac{1}{\sqrt{z^2 + \frac{x^2+y^2}{4}}} \begin{pmatrix} x/4 \\ y/4 \\ z \end{pmatrix} . \quad (4.28)$$

We can make equation 4.28 dimensionless by rescaling the variables. For convenience, we rescale the position coordinates by the typical axial displacement of a particle, $z_r = z/\lambda_z$ and rescale time by $t_r = \tau/2\pi$, where τ is the circular orbit time for a radial circular orbit at displacement $z_r = 1$, given by

$$m\left(\frac{2\pi}{\tau}\right)^2 \lambda_z = \mu b . \quad (4.29)$$

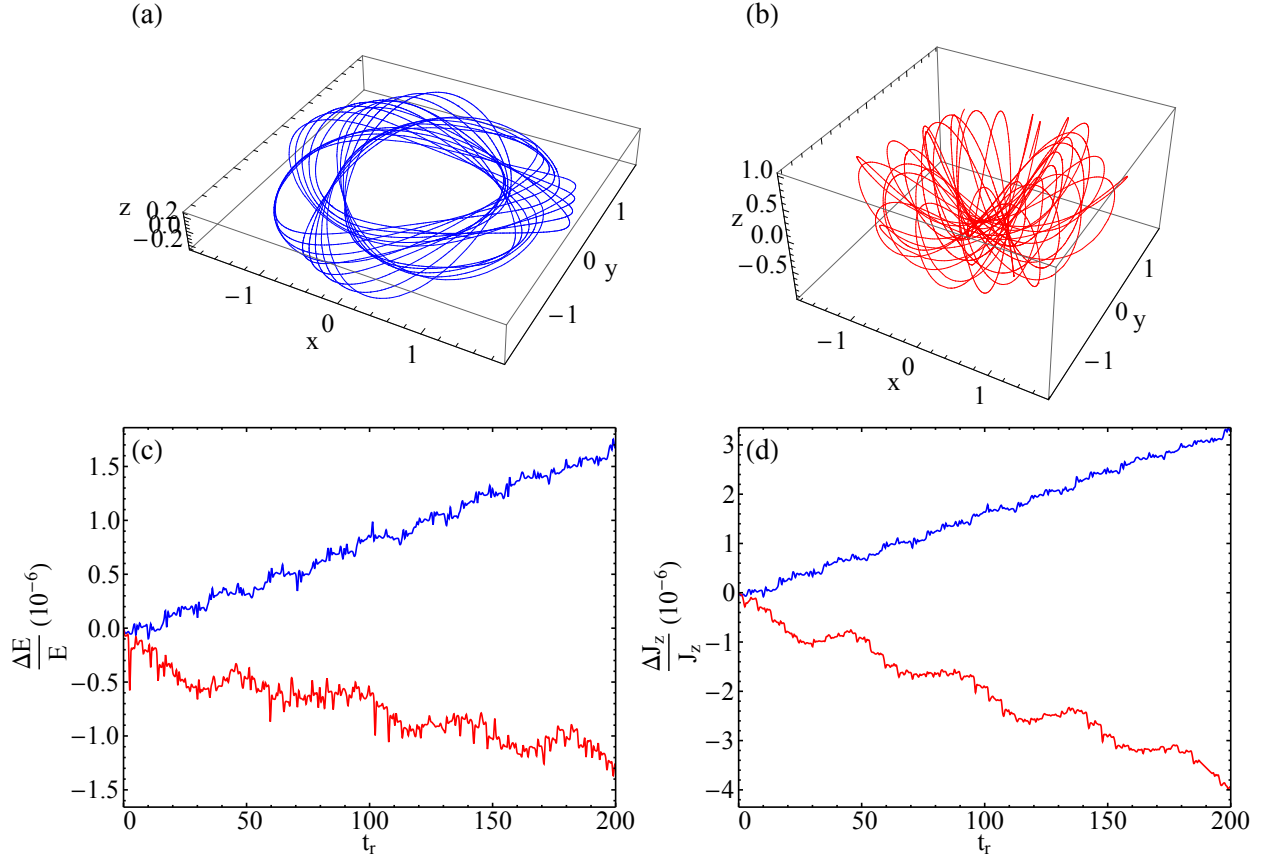


Figure 4.29: Single particle trajectories in the magnetic quadrupole trap. (a) and (b): trajectories $\mathbf{x}(t)$ followed by two particles for $0 < t_r < 200$. (c) and (d): relative change in the total energy E and axial angular momentum J_z along the trajectories, compared to $t_r = 0$.

After rescaling, equation 4.28 becomes

$$\frac{d^2}{dt_r^2} \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} = - \frac{1}{\sqrt{z_r^2 + \frac{x_r^2 + y_r^2}{4}}} \begin{pmatrix} x_r/4 \\ y_r/4 \\ z_r \end{pmatrix}. \quad (4.30)$$

Here, x_r, y_r, z_r, t_r refer to the rescaled coordinates.

Figure 4.29 shows the trajectories obtained by numerically solving equation 4.30, for $0 < t_r < 200$. To assess the validity of the simulation results, we can monitor the expected conservation laws for this system, in particular the total energy E , and the angular momentum along the trap axis J_z . These quantities are conserved in the simulation to better than 1 part in 10^5 . Monte Carlo simulations of equation 4.30 were completed to model atomic

ensembles in the trap. Initial coordinates (x_i, p_i) were drawn from Gaussian distributions centred on $(x_i, p_i) = (0, 0)$, and the trajectories of $N = 5000$ particles were calculated for $0 < t < 200t_r$. In the rescaled system of variables, an ensemble at temperature T has $\langle v_i^2 \rangle = 1$; for the position variables, the mean potential energy of the start distribution is determined by,

$$\langle PE \rangle = \int \mu b P(x, y, z) \sqrt{\frac{x^2 + y^2}{4} + z^2} dx dy dz . \quad (4.31)$$

With $N = 5000$, we can expect statistical uncertainties in thermodynamic variables $\sim 1.4\%$.

First, we studied the evolution of an ensemble initialised with the properties of our atomic cloud after Raman molasses cooling. Initial coordinates were generated according to the distribution function $\rho(x_i, p_i) = e^{\frac{-p_i^2}{2mk_B T}} e^{\frac{-x_i^2}{2\sigma^2}}$, where σ is the spatial width. Setting $\sigma = 2.724\lambda_z$ means that the initial conditions satisfy the virial theorem. For lithium atoms at $100\mu\text{K}$, and $b = 32\text{ G cm}^{-1}$, $\tau/2\pi \sim 2\text{ ms}$. Hence the simulation time is 200 ms , which is less than the expected collision time at the densities achievable in our experiment. This means it is reasonable to assume that particles move independently during the simulation.

Figure 4.30 shows the results of these simulations. After some initial breathing motion over $0 < t_r < 50$, the velocity distribution stabilises, and we see that the mean-square velocities are the same along the x - and y - axes, but that along the z -axis the mean-square velocity is larger. The partitioning of the total system energy closely follows the virial theorem result. Histograms of the particle energy and angular momentum show that they change by less than 1 part in 10^4 across the distribution. We studied the dependence of these results on the initial position and momentum distribution of the ensemble. The total energy per particle was fixed at $\frac{9}{2}k_B T$, and three different scenarios for the initial distributions were tested: a symmetric Gaussian distribution with $\frac{\langle PE \rangle}{\langle KE \rangle} = 3.5$ at $t = 0$; a symmetric Gaussian distribution with $\langle PE \rangle = \langle KE \rangle$ at $t = 0$; and a box distribution with $\langle PE \rangle = 2\langle KE \rangle$. Figure 4.31a shows the mean-squared velocity along z , which shows that the final velocity spread is only weakly affected by the start distribution. Figure 4.31b shows the ratio of potential and kinetic energies during the simulations, demonstrating that the virial theorem result is approached regardless of the start distribution. To simulate transport heating in the trap, we tried two different approaches. The first was to run simulations with

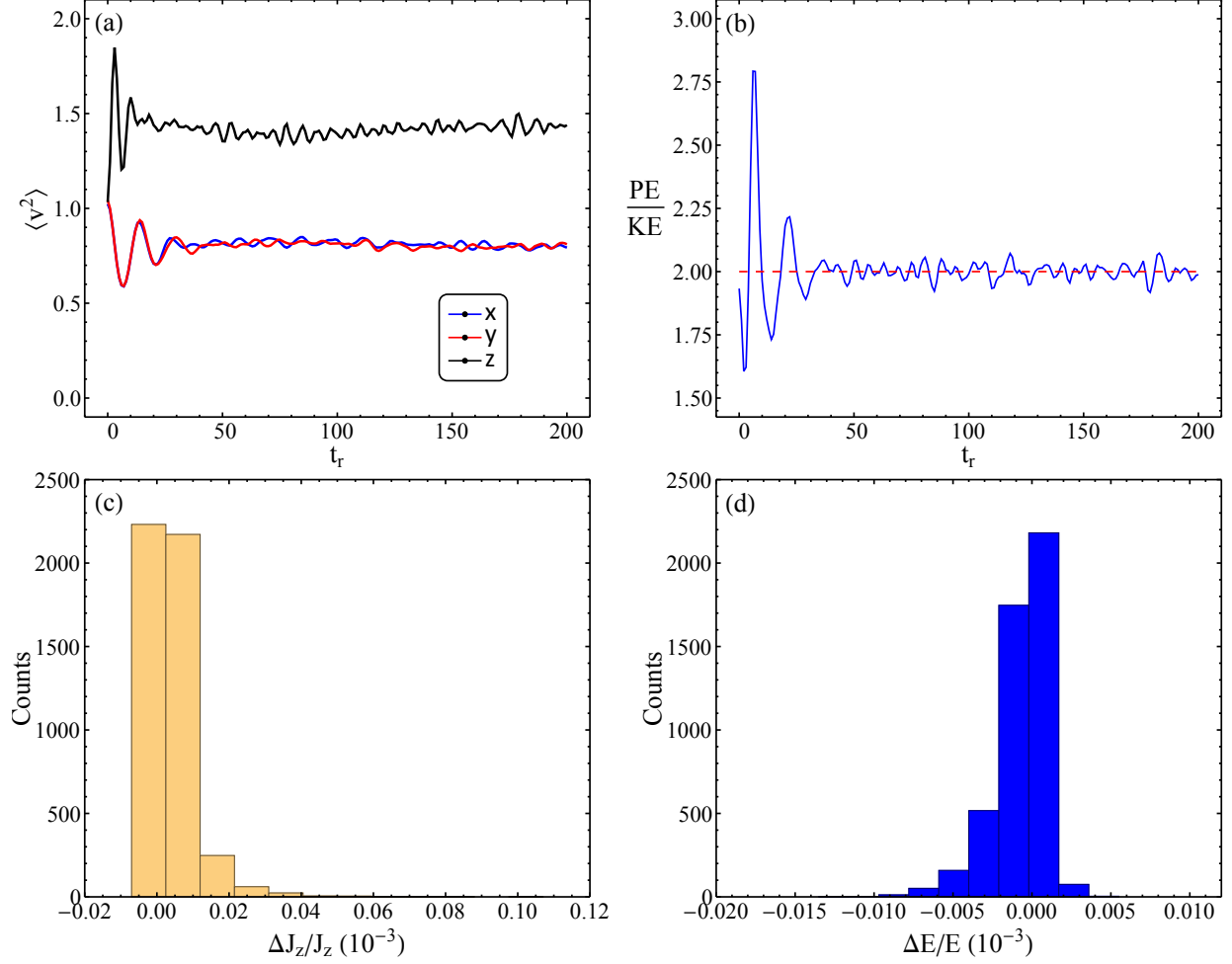


Figure 4.30: Results of a Monte Carlo simulation for particle trajectories in the magnetic trap. (a) axis-resolved mean square velocities and (b) partitioning of energy during the simulation. The dashed red line shows the equilibrium result expected from the virial theorem. (c) histogram of the fractional energy change over the trajectories at $t_r = 200$. (d) histogram of the relative change in angular momentum component J_z at $t_r = 200$.

the centre of mass offset from the trap centre by $\mathbf{r}_0$, equivalent to moving the trap in a time period much less than the orbit time period τ . The second was to include a time dependence of the trap centre in the equation of motion 4.30, which allows us to model experimental move profiles directly. These simulations were seeded with the final coordinates of the simulation in figure 4.30.

In studying a sudden shift in the trap centre, we can identify three cases of interest: a shift along the transport direction x ; a shift along the trap axis z ; and a combined shift along x and z . Figure 4.32 shows axis-resolved mean square velocities for $0 < t_r < 200$

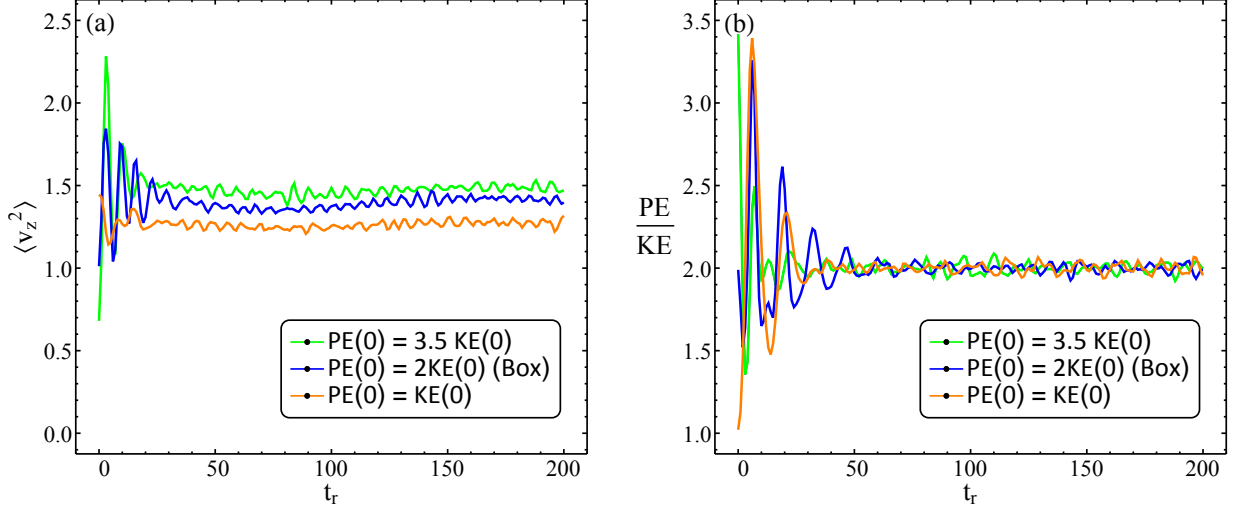


Figure 4.31: Trajectory simulations with different starting distributions for $0 < t_r < 200$. Left: mean-squared velocity along the z-axis versus t_r . Right: ratio of the total kinetic and potential energy in the system.

for each scenario. The z - and x -offsets used were $z_0 = 3$ units and $x_0 = 5.42$ units; the potential energy increase per particle at $t_r = 0$ was 1.15 units when either offset was applied separately, and 1.98 units when both were applied. The mean-square velocity plots show that when the cloud is offset along z , this additional energy is transferred only to the axial velocity distribution, and $\langle v_x^2 \rangle, \langle v_y^2 \rangle$ remain close to their values at $t_r = 0$. When the offset is applied in the x -direction, the additional potential energy transfers to both the x and y velocity distributions, with $\langle v_z^2 \rangle$ relatively unchanged. When offsets are applied in both directions at once, the system behaves at long times as the linear combination of applying the two offsets separately. Taken together, this would suggest that transport heating of the cloud would be observable in the ballistic expansion; a sudden offset along x (the transport direction) always results in heating along y , which is observable. We would expect to observe heating via the expansion along y provided we measure 50 ms after transport.

When including the motion of the trap directly in the simulations, we used $x(t_r) \rightarrow x(t_r) - x_{\text{trap}}(t_r)$, with $x_{\text{trap}}(t_r)$ a triangular move profile employed in the experiment. We focus on two cases: $a = 3 \text{ m s}^{-2}$, $x = 0.05 \text{ m}$, which we have tested experimentally, and $a = 20 \text{ m s}^{-2}$, $x = 0.3 \text{ m}$, to exaggerate the transport heating. Figures 4.33 and 4.34 show the simulation results for these two parameter sets. The total energy per particle, measured

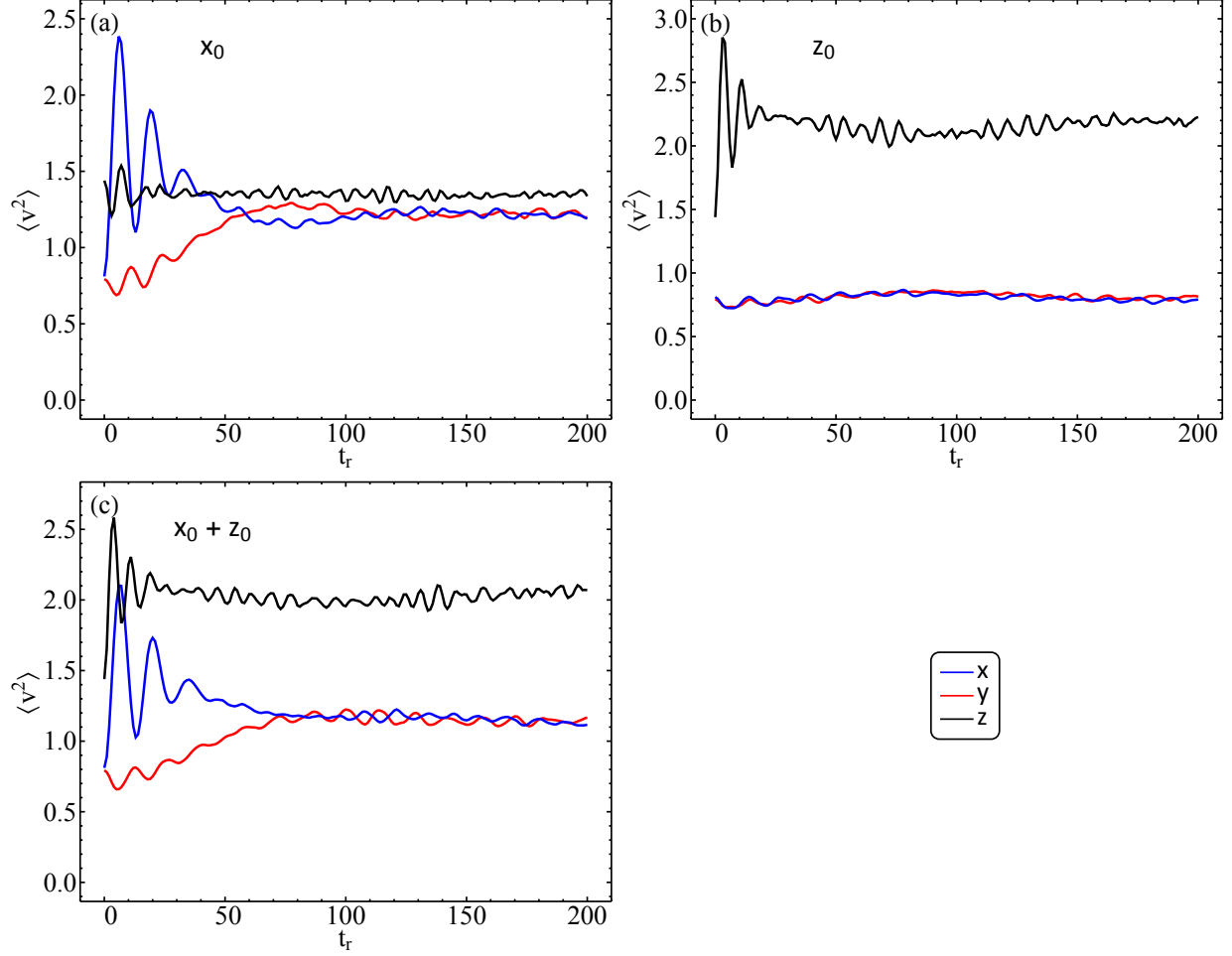


Figure 4.32: Axis resolved velocity components following a sudden offset applied to the trap centre at $t_r = 0$. (a) An offset along x of $x_0 = 5.42$ units. (b) An offset along z of $z_0 = 3$ units. (c) a combination of both x_0 and z_0 .

in the instantaneous frame of the moving trap, serves as a diagnostic for transport heating. In figure 4.33a, we see that energy is delivered to the ensemble when the trap centre jerks, and that it takes time for the system to stabilise during the constant acceleration periods. With $a = 3 \text{ m s}^{-2}$, the total energy at the end of the simulation has increased by around 0.1%, and the trap-frame velocity distributions in figure 4.33b are relatively unchanged. This supports our observations from figure 4.21. For the simulation using $a = 20 \text{ m s}^{-2}$ (figure 4.34), the total energy at the end of the transport stage corresponds to a temperature increase of 6%. At $t_r = 90$, when the jerk of the trap centre is largest, the velocity distributions vary erratically, but at $t_r = 250$, the x - and y - mean square velocities are converging. We can therefore conclude that we would observe any heating in the radial expansion of the cloud

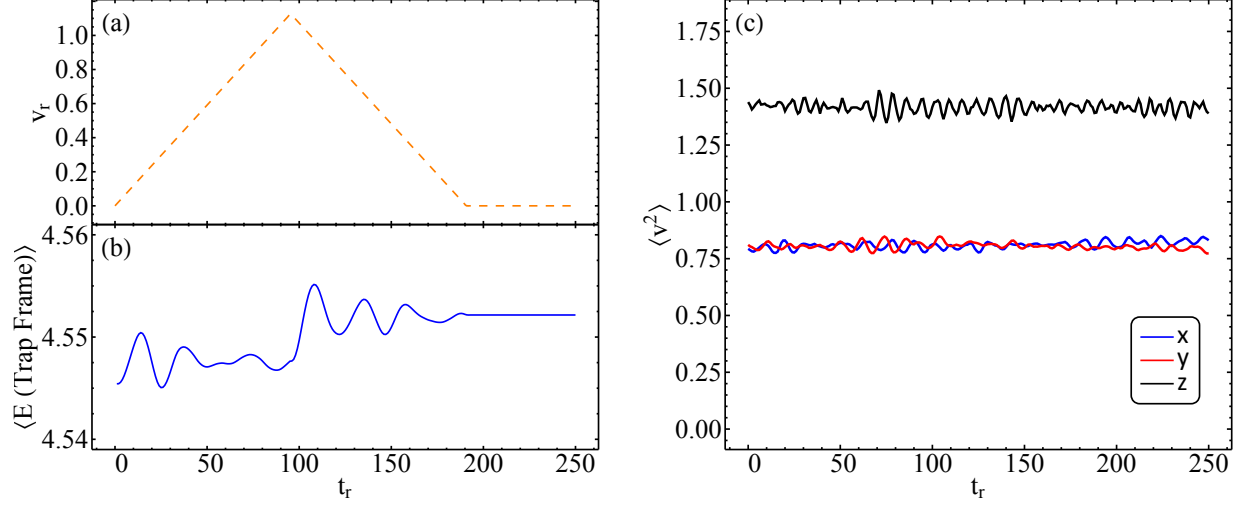


Figure 4.33: Transport simulations for a moving trap, using $a = 3 \text{ m s}^{-2}$ and $x = 50 \text{ mm}$. (a) Trap velocity profile in rescaled coordinates. (b) Energy per particle as measured in the frame of the trap center. (c) Axis-resolved mean-square velocities.

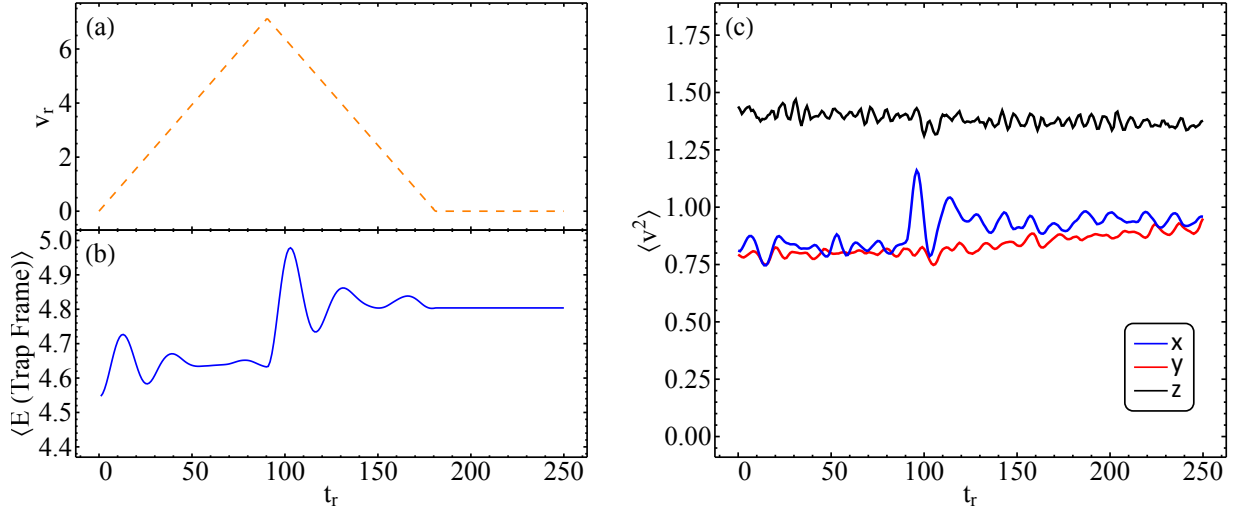


Figure 4.34: Simulation results for $a = 20 \text{ m s}^{-2}$, $x = 0.3 \text{ m}$. (a) Trap velocity profile in rescaled coordinates. (b) Average energy per particle measured in the instantaneous frame of the trap. (c) Axis-resolved mean-square velocities.

after transport, and that very large accelerations (and changes in those accelerations) are needed for there to be significant heating of the atoms. Such large accelerations cannot be produced by the movement of the translation stage alone, but can be produced when the atoms are moved at high speed through a sudden change in magnetic field.

4.8 Conclusions

This chapter has detailed our method for delivering our ultracold lithium samples to the microwave chamber, by loading and transporting a magnetic quadrupole trap. We capture up to 85(2)% of the atoms in a quadrupole trap formed by the MOT coils, by optical pumping to the $|2, 2\rangle$ state. Atoms are then transferred, without loss, to a second trap formed by the externally mounted coils, which is transported to the microwave chamber by the translation stage. The motion profile uses an acceleration of 0.75 m s^{-2} and a velocity of 0.5 m s^{-1} , moving the atoms a distance of 600 mm in 1.87 s.

When transferring between the magnetic traps, it was found that eddy currents induced by the external coils heated the cloud, and that we can minimise this problem by choosing current ramps $I(t)$ such that dI/dt is continuous during the transfer. A large defect magnetic field was discovered at the entrance of the transport tube, which abruptly shifts the cloud position during motion, resulting in heating and atom loss. To minimise the impact of the defect field, the magnetic field gradient of the external trap was increased to 50 G cm^{-1} and the trapping currents varied during motion, in order to keep the atoms centred in the transport tube.

The $1/e$ lifetime in the internal quadrupole trap was measured as 1.77(9) s, and outgassing by the coils limited the trapping gradient used for the internal trap. Once transferred to the external magnetic trap, the lifetime in the MOT chamber was measured as 2.47(15) s, which is limited by the background gas pressure in this part of the experiment. In the microwave chamber, the lifetime increased to 3.9(2) s at a background gas pressure of $1.3 \times 10^{-9} \text{ mbar}$. The overall efficiency of the transport process is better than 86% when accounting for losses already present due to background gas. We transport typically 2×10^7 atoms to the microwave cavity at a final phase space density of $3.58(15) \times 10^{-7}$. Trajectory simulations indicate that heating of the velocity distribution along the transport direction should be visible in the ballistic expansion of the cloud along the x -direction, and so we believe our estimate is a fair representation of the conditions at the point of loading the microwave trap.

Chapter 5

The Microwave Trap

This chapter describes the design, testing and operation of the microwave trap. I begin by summarising past research with microwave induced forces and cold matter, and the work on our trap which preceded my own efforts. I then explain our cavity design and derive the trap properties for lithium atoms. Next, I detail simulation work on thermal and pressure loading of the cavity, and our system for safely delivering the required power to the cavity. I study the behaviour of the cavity at an input power up to 1kW, verifying its trapping capability. Finally, I demonstrate loading of our ^7Li cloud into the trap and study its basic properties.

5.1 Microwaves and Cold Matter

In this section, I summarise the work previously undertaken to exert forces on atoms and molecules with microwave fields.

5.1.1 Deflection and Deceleration of Molecules with the Microwave Electric Dipole Force

The first attempt to exert forces on molecules using microwaves was an experiment in 1975, carried out by Gallagher and Hill [139]. They passed an effusive beam of CsF molecules through a box-like cavity ($Q \sim 2000$), which excited the TE_{111} mode close to the $J = 0 \rightarrow J = 1$ rotational transition at 11.019 GHz. Up to 24 W was delivered to the cavity,

leading to a peak electric field of $\sim 0.7 \text{ kV cm}^{-1}$. Despite the source temperature being around 1000 K, they were able to isolate deflection of the $J = 0$ rotational ground state using an electrostatic state selector. However, they were not able to fully understand their results and did not pursue this idea further.

The next significant advance was published in 2010, demonstrating microwave lensing of a beam of ammonia molecules [140]. This experiment used a pulsed supersonic source and Stark decelerator which slowed molecules in the rotationally excited $J = 1, K = 1$ (antisymmetric) inversion state to a forward speed of 20 m s^{-1} . After exiting the Stark decelerator, molecules passed through a 120 mm long cylindrical cavity ($Q \sim 5000$), with a TE_{11} resonance frequency close to the inversion transition¹ at 23.695 GHz. Molecules were detected after leaving the cavity by state-selective laser ionisation. The authors clearly demonstrated transverse focussing of the molecules and were able to overfocus the molecular pulse with an input power of 3.0 W (an equivalent peak electric field of 1.52 kV cm^{-1}). However the observed signal enhancement was an order of magnitude smaller than expected; the suggested cause of this discrepancy was the occurrence of non-adiabatic transitions to unfocussed rotational states, which are degenerate at the electric field nodes. This idea was investigated by comparing the behaviour with different TE_{11m} modes, where the number of nodes along the cavity axis increases with m .

The same experiment was later extended to include longitudinal focussing by operating as a decelerator [142], in a scheme first suggested by Mamose [143]. This concept is illustrated in figure 5.1; by modulating the electric field in the cavity as a molecular pulse moved through the cavity, it was possible to decelerate from 20 m s^{-1} to 16.9 m s^{-1} .

Transverse microwave lensing was also applied by Mamose and co-workers to a molecule in the rotational ground state, using the $K = 0, J = 0 \rightarrow 1$ transition in acetonitrile (CH_3CN), at 18.4 GHz [144]. In this experiment, molecules from a counter-rotating nozzle source, traveling at 120 m s^{-1} , were directed through a 400 mm long cylindrical cavity of radius 6.3 mm ($Q \sim 25,000$). The authors chose to use a transverse magnetic mode (TM_{014}), which unlike the TE modes has azimuthal symmetry in the electric field amplitude. This has the

¹Incidentally, this was the same transition used more than 50 years earlier by Townes and co-workers, demonstrating a MASER [141].

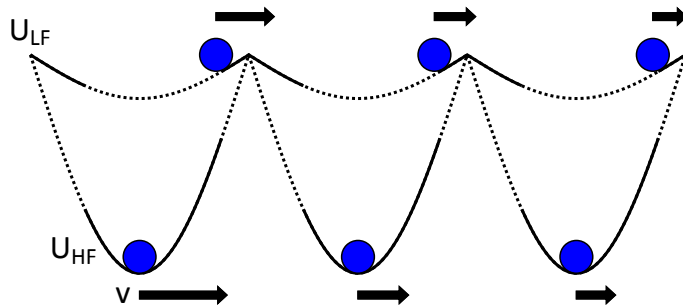


Figure 5.1: Microwave Stark deceleration. By modulating the amplitude of the microwave field, the potential experienced by a molecule switches between the two curves. A so-called synchronous molecule with initial velocity v experiences the potential given by the solid line, and is decelerated as a result.

advantage of providing a uniform focusing force around the cavity axis. The two molecular rotations of acetonitrile are well separated, which can be understood from the diagram in figure 5.2. The K quantum number refers to angular momentum about the molecular axis of symmetry, in which only the hydrogen atoms move. Consequently the rotational constant of the K rotations is 158.4 GHz, much larger than for the J rotations, which is 9.2 GHz. This means the K rotations played no role in the experiment. As a demonstration of focusing, the authors observed an increase in the number of molecules leaving through the cavity exit hole when the microwave field was applied. However, the analysis of their results was complicated by their detection system, which was based on a residual gas analyser and therefore not state-selective. Since the $J = 0$ and $J = 1$ states experience opposite forces in the cavity, the observed gain in molecules in the focussed state was offset by a loss of molecules in the unfocused state. Given the rotational temperature of the molecular source, the two states both constituted around 5% of the total population. As a result the increase in the number of detected molecules, whilst clearly observable, was small.

Recently, the acetonitrile experiment described above was further developed and transverse focussing was applied to a PbO molecular beam [145]. This molecule has a similar rotational constant to acetonitrile but its molecular mass is 5.5 times larger. To achieve focussing, the surfaces of the microwave cavity were coated with a superconducting Pb-Sn alloy, which has a critical temperature of 7-8 K. By cooling the cavity with liquid helium,

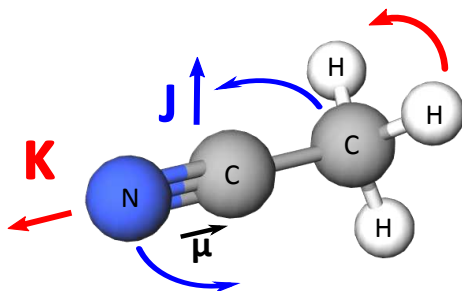


Figure 5.2: Rotations of the acetonitrile molecule (CH_3CN). The quantum numbers J and K refer to rotations parallel and perpendicular to the molecular symmetry axis.

the unloaded quality factor of the cavity increased to $\sim 1 \times 10^6$, resulting in a potential depth of 0.11 K for 1.9 W of microwave power. The molecules were detected by laser induced fluorescence, allowing state-selectivity of molecules in the $J = 0$ state. The flux density of these molecules at the detection region was increased by up to a factor 9 when focussing was applied. In practice, the reduced cooling power at liquid helium temperatures limits the microwave power to a few watts. If the critical temperature is exceeded, the mirrors are no longer superconducting and the enhancement of the quality factor disappears. The peak electric field in the cavity is determined by the product of its input power and quality factor, and this can be similar or even greater with a room temperature resonator.

5.1.2 Trapping Atoms with the Microwave Magnetic Dipole Force

In the experiments described in section 5.1.1, the microwave field strength was too weak for three-dimensional confinement, in part because of the temperature and longitudinal velocities of the molecular sources. However, the advances of atomic laser cooling and trapping experiments opened up the possibility of confining atoms in a cavity. Although ground state atoms have no permanent electric dipole moment, most have a permanent magnetic dipole moment, which can interact with the magnetic component of a microwave field. The interaction is strongest when the microwave field is resonant with a hyperfine interval, in which case it has a strength of order $\mu_B B = \mu_B \mathcal{E}/c$ where $\mathcal{E}$ and B are the electric and magnetic fields of the light. For a field strength of 20 kV/cm, which (as I will show) is feasible inside a good resonator, this interaction strength is 4.5 mK, deep enough to confine laser-cooled atoms.

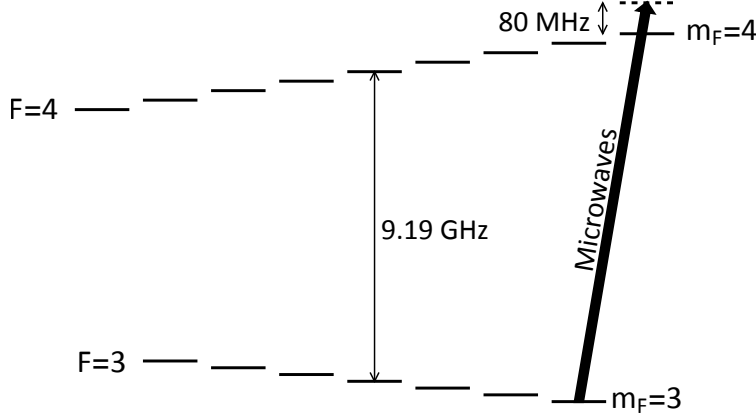


Figure 5.3: Level scheme used for microwave trapping of cesium. The arrow shows the main transition addressed by the microwave field at the trap centre.

Such a trap was designed and demonstrated with laser-cooled Cesium as part of the search for BEC in the 1990s [146]. A spherical cavity ($Q \sim 5500$) was operated at the TE_{110} resonance, tuned slightly above the 9.19 GHz $F = 3 \rightarrow 4$ hyperfine transition in Cs. This cavity mode has a magnetic field maximum at its centre, which for positive detuning attracts atoms in the $F = 4$ sublevels. A static magnetic field of 70 G was applied to separate the Zeeman sublevels, so that the $|F, m_F\rangle = |3, 3\rangle \rightarrow |4, 4\rangle$ approximates a two level system as illustrated in figure 5.3. Atoms were prepared from a MOT in the $|4, 4\rangle$ state at a temperature of 5 μK , then loaded directly into the cavity. By coupling up to 83 W into the cavity, it was possible to generate a trap depth of 100 μK . However, the trap diameter of 1 cm meant that this potential could not support the atoms against gravity. In addition to the static magnetic field, a levitating gradient had to be applied to form a trap in three dimensions. However, this resulted in an effective detuning variation of the microwaves over the trap volume, with a knock-on effect on the trapping potential.

Whilst this trap successfully confined ground state atoms, the microwave frequency is highly dependent on the hyperfine structure of the electronic ground state. In addition to this, there is an electric field minimum at the centre of the cavity, which restricts trapping for polar molecules in the rotational ground state. Specifically, the microwave frequency must be tuned above the $J = 0 \rightarrow 1$ rotational transition to ensure the ground state is trapped at the cavity centre. This introduces the possibility of coupling to higher excited levels, and this coupling will always oppose the intended trapping force. Therefore this kind of trap has

severely limited use for our purposes.

5.2 Design of our Trap

The design of our microwave trap evolved through several iterations before reaching the version described in this thesis. The reader is referred to [147] for the details of this process. I shall describe the theory of operation, starting with a description of the ideal modes in a closed cavity. I shall present a heuristic treatment of how power is coupled in through the hole and discuss the quality factor, the crucial parameter in determining the trap depth.

5.2.1 The Cavity Assembly

Figure 5.4 shows an exploded view of the cavity assembly. The cavity itself is formed by two spherical copper mirrors in near-confocal arrangement, mounted onto copper coolant baths. The cavity length is maintained by copper legs connecting the coolant baths, two of which are hollow to connect the reservoirs. The upper coolant bath contains a central pillar, through which a waveguide cross-section is cut to feed power to the cavity. The upper bath is then attached to a DN160CF rotatable flange, which forms a vacuum seal with the 8 inch spherical octagon microwave chamber. Unlike the cylindrical resonators described in section 5.1.1, our design provides an open geometry to allow as much access as possible for loading and detecting samples. The minimum distance between the mirror surfaces is 8 mm, and in the radial plane the access is only limited by the connecting legs of the coolant baths.

Figure 5.5 shows a cross-section through the centre of the cavity in the $y-z$ plane, which illustrates the details more clearly. The most challenging part of the design to fabricate is the thermal break between the coolant baths and the laboratory environment, incorporated so that liquid nitrogen can be used to cool the mirrors. The predecessor to our current design used Invar for this part, which has a thermal conductivity $\kappa = 13 \text{ W m}^{-1} \text{ K}^{-1}$, about 40 times lower than copper. However, it also has an anomalously low thermal expansion coefficient of around $1 \times 10^{-6} \text{ K}^{-1}$, about a factor twenty below copper. When the coolant baths were loaded with liquid nitrogen, large vacuum leaks appeared, most likely caused by thermally induced strain.

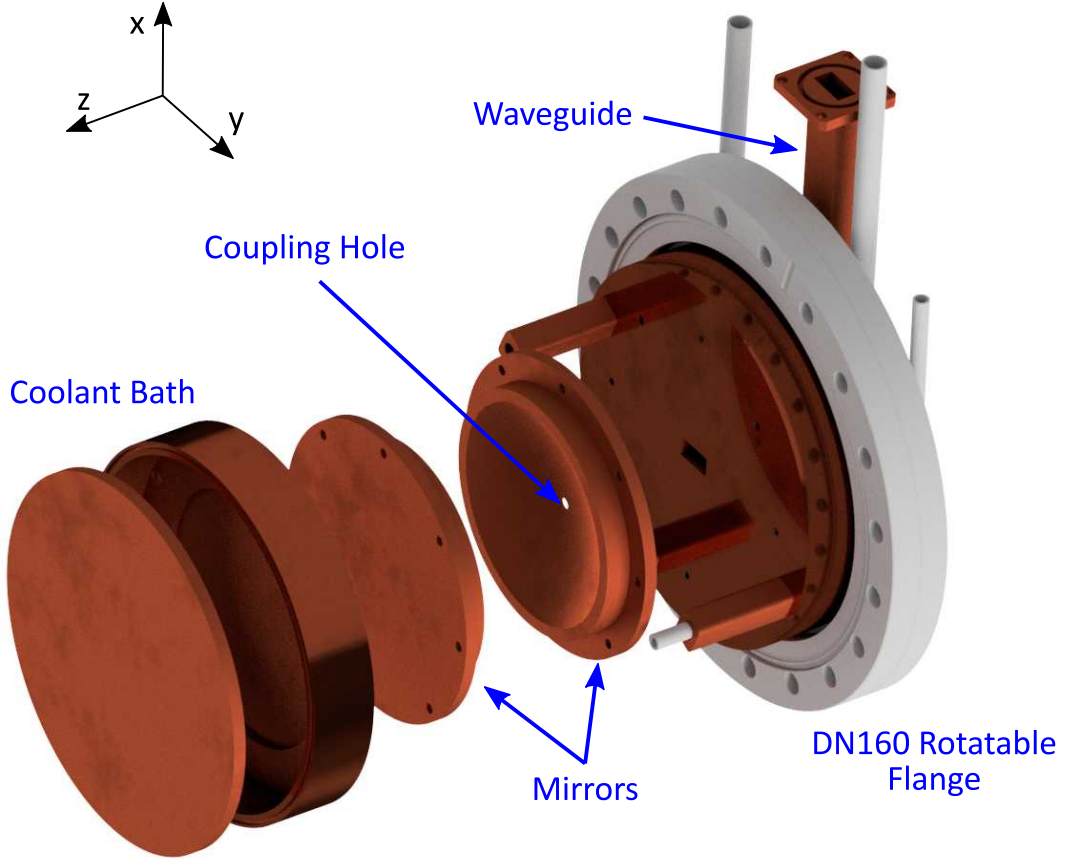


Figure 5.4: Exploded view of the microwave cavity assembly.

In the current version, the thermal break section is made of stainless steel. This sacrifices a factor of about three in thermal isolation, whilst ensuring a good thermal expansion match with the copper and surrounding vacuum chamber. The cross-sectional area at the waveguide section of this part is $A = 2.0 \text{ cm}^2$ and depth $\Delta z = 3.87 \text{ mm}$. Assuming that the coolant-laboratory temperature differential is entirely across this boundary, then the heating power from the environment to the coolant baths is $P = \kappa A \frac{\Delta T}{\Delta z}$. With $\kappa = 20 \text{ W m}^{-1} \text{ K}^{-1}$, $P[\text{W}] = 1.3 \Delta T[\text{K}]$; therefore with a liquid nitrogen coolant, we can expect an upper bound of 260 W of heating from the environment, about 10% of the maximum intended microwave power. The joint between the thermal break and the input waveguide is soft soldered (Alroy Microwave & Electronics Ltd.); for both parts, the microwave-facing surfaces were silver coated to minimise microwave loss.

The copper body of the coolant baths is a single piece whose component parts were ad-

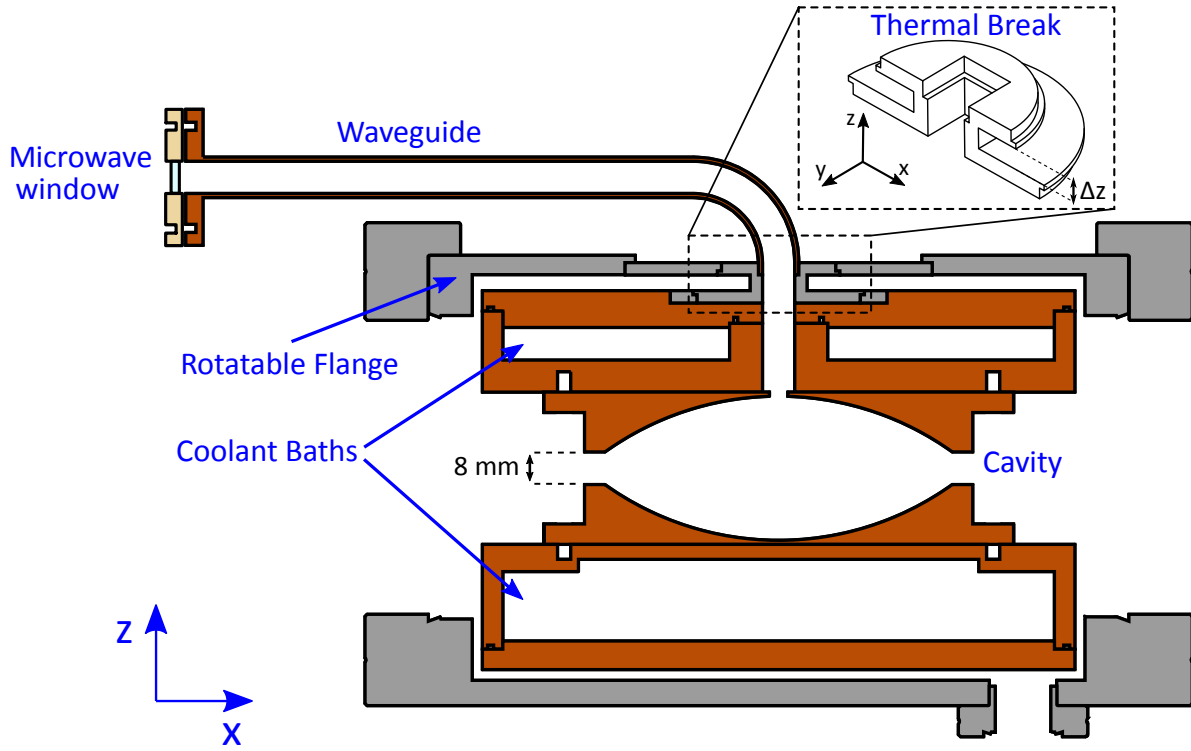


Figure 5.5: A cross section through the cavity assembly in the $x - z$ plane. The inset shows the thermal break section, and the thermal resistance between the coolant baths and the rest of the experiment is dominated by the blue shaded region.

joined by vacuum brazing (Special Techniques Group, Culham Laboratory). In this process, the joints are formed by heating to 780°C in a vacuum with a silver eutectic forming the bond. The top bath mounts into the thermal break section using four M4 screws. These mounting screws can be accessed by removing the DN16CF (previously DN10CF) flanges mounted onto the top flange, visible in figure 5.6. In this way, good electrical continuity is assured along the waveguide whilst still allowing the copper body to be separated from the flange.

Even with the design alteration to the waveguide flange, we could not be sure that the cavity would remain vacuum-tight at cryogenic temperatures without filling the coolant baths with liquid nitrogen. As shown in section 5.5.1, water cooling turns out to be a viable option when operating at high power for few seconds, and so I decided to proceed with this method in the experiments. This reduces the trap depths possible for lithium by about a factor two.

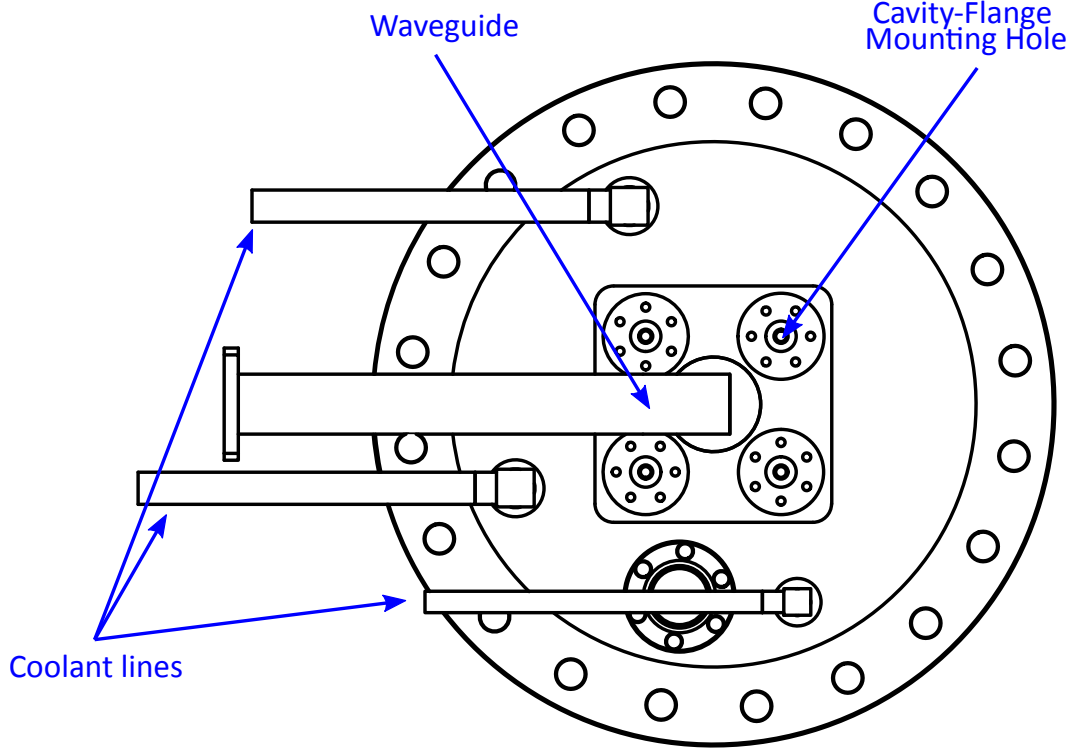


Figure 5.6: A top-down view of the cavity, showing the waveguide, coolant lines and cavity flange mounting.

5.2.2 Fabry-Perot Cavity Modes

To find the modes of a closed, perfectly reflecting cavity, we start with the paraxial wave equation,

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + 2ik \frac{\partial A}{\partial z} = 0. \quad (5.1)$$

Here, $A(x, y, z)$ is the envelope function describing the amplitude of the field vector, $\mathcal{E} = \mathcal{E}_0 A e^{i(kz - \omega t)}$. This equation relies on the paraxial approximation, $|\frac{\partial^2 A}{\partial z^2}| \ll |k \frac{\partial A}{\partial z}|$, which is to say that the variation of A with z is small over one wavelength. It is a *scalar* equation, and tells us nothing about the direction of the electric field vector. However, Maxwell's equations in free space dictate that $\mathcal{E}$, $\mathcal{B}$ and the local wavevector (the gradient of the field phase) must be mutually perpendicular. Therefore it is sufficient to say that $\mathcal{E}$ and $\mathcal{B}$ are perpendicular and lie in the plane of the wavefronts. In fact, the electric field polarisation in our trap is set by the input mode of the waveguide, and is linearly polarised along x .

Solutions of equation 5.1 can be expressed as the Hermite-Gaussian modes:

$$A_{nm}(x, y, z) = \frac{1}{\sqrt{1 + \frac{z^2}{z_0^2}}} H_n\left(\frac{\sqrt{2}x}{w}\right) H_m\left(\frac{\sqrt{2}y}{w}\right) \times e^{-\frac{x^2+y^2}{w^2}} e^{\frac{ik(x^2+y^2)}{2R}} e^{-i(n+m+1)\phi} . \quad (5.2)$$

The variables w , R and ϕ are the local beam width, radius of curvature and the Gouy phase respectively. All depend explicitly on z :

$$w(z) = w_0 \sqrt{1 + \frac{z^2}{z_0^2}} , \quad (5.3a)$$

$$R(z) = z + \frac{z_0^2}{z} , \quad (5.3b)$$

$$\phi(z) = \arctan\left(\frac{z}{z_0}\right) . \quad (5.3c)$$

The Hermite polynomials H_n can be generated according to

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} . \quad (5.4)$$

Cavity modes are determined by the boundary condition that the tangential component of the electric field must be zero at the mirror surfaces. This imposes two conditions on the solutions given by (5.2). First, the radius of curvature of the mode at each mirror surface must equal that of the mirror, which fixes the Rayleigh range z_0 from equation (5.3b). Second, the phase accumulated on a round trip of the cavity must be an integer multiple of 2π . From these conditions, it can be shown that the frequencies of the cavity modes, $\nu_{N,n,m}$, satisfy

$$\nu_{N,n,m} = \frac{c}{2L} \left[N + \frac{n+m+1}{\pi} \arccos(\sqrt{g_1 g_2}) \right] . \quad (5.5)$$

Here, $g_{1,2} = (1 - \frac{L}{R_{1,2}})$ are called the cavity g-factors, where $R_{1,2}$ refers to the radius of curvature of each mirror. We require $-1 < g_1 g_2 < 1$ for physical solutions. The set of

	Value	Unit
R_m	73	mm
L	35	mm
z_0	31.2	mm
w_0	14.7	mm

Table 5.1: Cavity properties for our microwave trap, assuming a perfectly closed cavity.

solutions given by equation 5.5 are the Transverse Electromagnetic Modes, or the TEM_{Nmn} modes. The case $g_1 g_2 = 0$ occurs when the mirrors are in the confocal arrangement, and under this condition all ν_{Nnm} are degenerate for a given $(N + \frac{1}{2}(n + m))$. We want to avoid degeneracies in order to control the power and field distribution in the cavity.

To make an effective microwave trap, we couple power into the TEM_{300} mode of our cavity, with $\nu_{300} \approx 14.3$ GHz. The electric field amplitude in this mode is

$$\mathcal{E} = \mathcal{E}_{\text{peak}} \frac{e^{-\frac{x^2+y^2}{w^2}}}{\sqrt{1 + \frac{z^2}{z_0^2}}} \cos(\Theta(x, y, z)) \cos(2\pi\nu_{300}t), \quad (5.6)$$

with $\Theta(x, y, z) = kz + \frac{k(x^2+y^2)}{2R(z)} - \phi(z)$ and $k = 2\pi\nu_{300}/c$.

It will be useful for section 5.3 to expand the electric field about the centre of the cavity. To second order in the position coordinates,

$$\mathcal{E}(x, y, z) \approx \mathcal{E}_{\text{peak}} \left[1 - \frac{x^2 + y^2}{w_0^2} - \frac{k^2}{2} \left(1 - \frac{2}{kz_0} + \frac{2}{k^2 z_0^2} \right) z^2 \right] \cos(2\pi\nu_{300}t) . \quad (5.7)$$

Likewise, the square of the electric field amplitude, can be found as

$$\mathcal{E}(x, y, z)^2 \approx \mathcal{E}_{\text{peak}}^2 \left[1 - 2\frac{x^2 + y^2}{w_0^2} - k^2 \left(1 - \frac{2}{kz_0} + \frac{2}{k^2 z_0^2} \right) z^2 \right] \cos^2(2\pi\nu_{300}t) . \quad (5.8)$$

The magnetic field can be derived directly from Maxwell's equation $\nabla \times \mathcal{E} = -\frac{\partial \mathbf{B}}{\partial t} = i\omega \mathbf{B}$. The full form is rather protracted, but importantly the magnetic field must be zero exactly at the cavity centre, because there is an electric field antinode here.

Table 5.1 lists the properties of the cavity, neglecting the effect of the coupling hole. For these dimensions, the frequency spacing of the TEM modes is ~ 1.3 GHz (transverse) and 4.3 GHz (longitudinal). The TEM modes closest to our chosen TEM_{300} mode at 14.3 GHz are the TEM_{2nm} modes with $n + m = 6$, at around 14.1 GHz.

5.2.3 Cavity Properties

Energy Stored in the Cavity

The electromagnetic energy stored in the cavity, U , is given by the integral over the cavity volume,

$$U = \int_V \frac{1}{2} \epsilon_0 |\mathcal{E}(t)|^2 + \frac{1}{2\mu_0} |B(t)|^2 dV . \quad (5.9)$$

The average over a cycle of the magnetic and electric terms in 5.9 are equal for a TEM mode, and so we have

$$\langle U \rangle = \int_V \epsilon_0 \langle |\mathcal{E}|^2 \rangle dV . \quad (5.10)$$

Next, we substitute equation 5.6 for the electric field, giving

$$U = \epsilon_0 |\mathcal{E}_{\text{peak}}|^2 \langle \cos^2(\omega t) \rangle \int_V \frac{e^{-2\frac{x^2+y^2}{w^2}}}{1 + \frac{z^2}{z_0^2}} \cos^2(\Theta(x, y, z)) dx dy dz . \quad (5.11)$$

To perform the integral, we approximate the wavefronts as plane waves, so that $\Theta = kz$. This is a reasonable approximation since $w_0 \ll R(z)$. We first carry out the integral in the (x, y) plane, noting the radial symmetry. This cancels the denominator in the integrand because $w^2 = w_0^2(1 + \frac{z^2}{z_0^2})$. The integral along z then simplifies to $\frac{L}{2}$. Putting all this together, we arrive at,

$$\langle U \rangle = \frac{w_0^2 L \pi \epsilon_0}{8} |\mathcal{E}_{\text{peak}}|^2 . \quad (5.12)$$

The Quality Factor

The important figure of merit for a resonator is its quality factor Q_0 , which measures the rate at which energy is dissipated relative to the resonant frequency, ω . If a resonator is not driven by an external source, the stored energy $U(t)$ decays as

$$\frac{dU}{dt} = -\frac{\omega U}{Q_0} \quad \rightarrow \quad U(t) = U(0) \exp\left(-\frac{\omega}{Q_0} t\right) . \quad (5.13)$$

If a cavity mode is excited with incident power P at $t = 0$, then $U(t) = \frac{PQ_0}{\omega}(1 - \exp(-\frac{\omega t}{Q_0}))$.

For a microwave resonator, there are three main sources of energy loss: resistive losses in the walls of the resonator, diffraction loss from the spill-over of the mode at the mirror edges, and dielectric losses in the medium filling the cavity. In our experiment the mirrors are under high vacuum, meaning there are no dielectric losses. If the mirrors have diameter D and mode width at the mirror faces is w_m , then a fraction $2e^{\frac{-D^2}{2w_m^2}}$ of the power is lost per round trip, the so-called diffraction loss. For our cavity dimensions, this is 6×10^{-6} per round trip for an ideal Gaussian mode. This is orders of magnitude below the resistive losses. If the cavity round trip time is $\tau = \frac{2L}{c}$ and the power transmission coefficient into the mirrors is $\mathcal{T}$, it follows that

$$Q_0 = \frac{\omega\tau}{2\mathcal{T}} = \frac{L}{c} \frac{\omega}{\mathcal{T}} . \quad (5.14)$$

To calculate $\mathcal{T}$, we consider the reflection of microwaves from a metal surface with conductivity σ . We assume normal incidence, since this is a condition for the Gaussian modes described by 5.5. Starting from the fourth of Maxwell's equations:

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t} \mathbf{D} + \mathbf{J} , \quad (5.15)$$

Applying the time derivative to both sides,

$$\nabla \times \left(\frac{\partial}{\partial t} \mathbf{H} \right) = \frac{\partial^2}{\partial t^2} \mathbf{D} + \frac{\partial \mathbf{J}}{\partial t} . \quad (5.16)$$

Using the third of Maxwell's equations,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} = -\mu_0\mu_r \frac{\partial}{\partial t} \mathbf{H} , \quad (5.17)$$

we can substitute for $\frac{\partial \mathbf{H}}{\partial t}$ in equation 5.16 to obtain

$$\nabla^2 \mathbf{E} = \frac{\partial^2}{\partial t^2} \mathbf{D} + \frac{\partial \mathbf{J}}{\partial t} , \quad (5.18)$$

where we have assumed that there are no free charges in the system, such that $\nabla \times \nabla \times \mathbf{E} =$

$-\nabla^2 \mathcal{E}$. We now introduce the properties of our metal, with relative permittivity ϵ_r such that $\mathbf{D} = \epsilon_0 \epsilon_r \mathcal{E}$, and conductivity σ such that $\mathbf{J} = \sigma \mathcal{E}$. If we make these substitutions in 5.18 and search for plane wave solutions of the form $\mathcal{E} = \mathcal{E}_0 e^{i(\mathbf{k}_m \cdot \mathbf{r} - \omega t)}$, we find the dispersion relation²,

$$k_m^2 = \mu_0 \mu_r (\omega^2 \epsilon_r \epsilon_0 + i \sigma \omega) \approx i \mu_0 \mu_r \sigma \omega \quad \rightarrow \quad k_m = \frac{1+i}{\sqrt{2}} (\sigma \omega \mu_0 \mu_r)^{\frac{1}{2}} . \quad (5.19)$$

The imaginary part of k_m leads to the decaying field amplitude inside the conductor. It is common to define the skin depth, δ :

$$\delta = \sqrt{\frac{2}{\sigma \omega \mu_0 \mu_r}} , \quad (5.20)$$

such that the field decays as $e^{-x/\delta}$ inside the conductor surface. For a typical metal, $\sigma \sim 2 \times 10^6 \text{ S/m}$, meaning the skin depth is less than $10 \mu\text{m}$ at microwave frequencies. Noting that $|k_m| = \sqrt{2}/\delta$, we can see that $|k_m| \gg |k|$, where $k = \omega/c$ is the free space wavevector. The power reflection from the conductor, $|\mathcal{R}|^2$ can be found as

$$\mathcal{R} = \left| \frac{k - k_m}{k + k_m} \right|^2 . \quad (5.21)$$

Therefore, the power transmission coefficient $\mathcal{T}$ is

$$\mathcal{T} = 1 - \mathcal{R} \approx k \sqrt{\frac{8}{\sigma \omega \mu_0 \mu_r}} . \quad (5.22)$$

Combining equations 5.22 and 5.14, we find

$$Q_0 = L \sqrt{\frac{\sigma \omega \mu_0 \mu_r}{8}} = L \sqrt{\frac{\omega \mu_0 \mu_r}{8 \rho}} . \quad (5.23)$$

where $\rho = 1/\sigma$ is the resistivity of the conductor. Equation 5.23 tells us that a high conductivity mirror material will give us the best Q . However, we must also consider the need for good thermal conduction to transport heat away from the mirror surfaces. Copper offers a combination of high electrical and thermal conductivity, and is therefore a good choice for our purpose. At $T = 298 \text{ K}$, we expect $\mathcal{T} = 3.3 \times 10^{-4}$ and $Q = 3.2 \times 10^4$.

²The approximation here relies on the fact that, for a good conductor, $\sigma \gg \omega \epsilon_0 \epsilon_r$ at microwave frequencies.

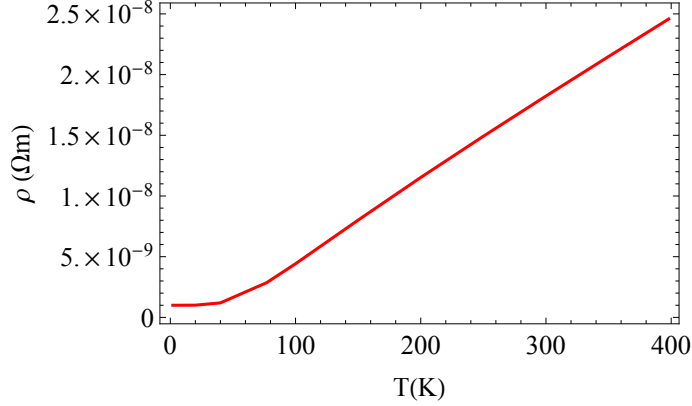


Figure 5.7: Resistivity of copper versus temperature as calculated using the Bloch–Grüneisen formula.

To consider the effect of temperature on the mirrors, we can use the Bloch Grüneisen formula [148]:

$$\rho(T, \Theta) = \rho_0 + \frac{C}{m\Theta} \left(\frac{T}{\Theta}\right)^5 \int_0^{\Theta/T} \frac{z^5 e^z}{(e^z - 1)^2} dz. \quad (5.24)$$

Here, ρ_0 is the residual resistivity as $T \rightarrow 0$ K, which is determined by defects in the metal; Θ is the Debye temperature, m is the atomic number, and C is a normalisation constant chosen to match an experimentally measured value. Whilst only a basic model, equation 5.24 captures the essential physics of electron scattering by phonons: at low temperature the scattering scales as T^5 , and as T/Θ increases it begins to scale linearly with T .

Figure 5.7 plots $\rho(T)$ for copper using $C = 1.85 \times 10^{-3} \text{ } \Omega \text{ m K}^{-1}$, $\Theta = 343.5 \text{ K}$, and $m = 63.5$, as in [85]. The resistivity is essentially linear with temperature for $100 < T < 400 \text{ K}$. Therefore the quality factor should scale with $1/\sqrt{T}$; for example, we would expect $\frac{Q(398)}{Q(298)} = 0.85$.

Measuring the quality factor of a cavity can be done either in the time or frequency domain. In the time domain, one observes the decay of $U(t)$, known as the “ring-down”, and extracts the decay time τ . In the frequency domain, one measures the linewidth of the resonance and relates this back to the quality factor. For our Q_0 , the ring down time is less than $1 \text{ } \mu\text{s}$ and so a frequency domain measurement is more straightforward.

Property	Value	Unit
a	15.9	mm
b	7.8	mm
Cutoff frequency (TE ₀₁)	9.49	GHz
Cutoff frequency (TE ₁₀)	18.98	GHz
k_x at 14.3 GHz	0.35	cm ⁻¹
Group velocity at 14.3 GHz	0.75	c
Wave impedance at 14.3 GHz	513	Ω

Table 5.2: WG18 waveguide properties.

Calculation of the Peak Electric Field Strength

The crucial factor in determining our trap depth is the peak electric field strength in the cavity. This is difficult to measure directly, and is instead calculated from knowledge of the input power and quality factor. In the steady state, energy loss must balance energy input, which requires $U = P_{\text{in}}Q_0/\omega$. It therefore follows, from rearranging equation 5.12, that

$$\mathcal{E}_{\text{peak}} = \left(\frac{8P_{\text{in}}Q_0}{\pi w_0^2 L \omega} \right)^{\frac{1}{2}} \quad (5.25)$$

5.2.4 Coupling Power into the Cavity

Waveguide modes

At high powers, waveguide becomes the safest and most practical way to build microwave circuits. I shall discuss below some relevant formulae for our experiment; for detailed descriptions and derivations of these results, the reader is referred to (for example) [149].

We use the TE₁₀ mode of WG18 waveguide (also referred to as WR62/R140 waveguide) to deliver power to the cavity. The properties of this waveguide are listed in table 5.2. The cross-section of the waveguide is rectangular with side lengths $a = 2b = 15.9$ mm. For propagation along the x -axis, the electric field in the TE₀₁ mode is

$$\mathcal{E} = A \sin(\pi y/a) \cos(k_x x - \omega t) \mathbf{z} , \quad (5.26)$$

where A is the field amplitude and the guided wavevector $k_x = \sqrt{(\frac{\omega}{c})^2 - (\frac{\pi}{a})^2}$. The magnetic field can be found using $\nabla \times \mathcal{E} = -\frac{\partial \mathbf{B}}{\partial t}$, as

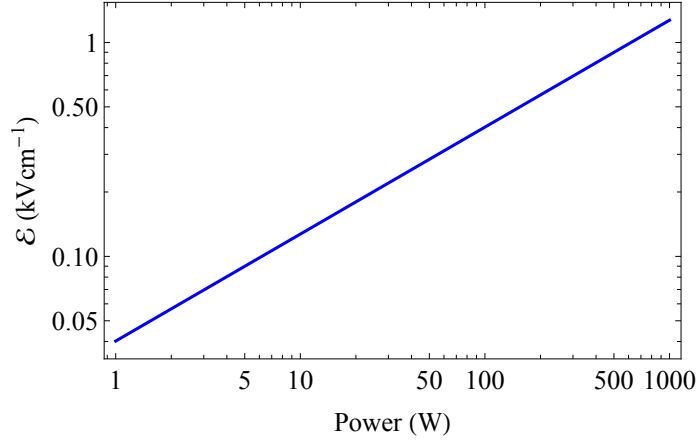


Figure 5.8: Electric field amplitude versus power transmitted in WG18 waveguide, at 14.3 GHz.

$$\mathbf{B} = A \left[\frac{k_x}{\omega} \sin\left(\frac{\pi y}{a}\right) \cos(k_x x - \omega t) \mathbf{y} - \frac{\pi}{a\omega} \cos(\pi y/a) \sin(k_x x - \omega t) \mathbf{x} \right] . \quad (5.27)$$

We can calculate the direction of energy flow along the waveguide from the time average of the Poynting vector, $\langle \mathbf{E} \times \mathbf{H} \rangle$:

$$\langle \mathbf{E} \times \mathbf{H} \rangle = |A|^2 \left(\frac{k_x}{\mu_0 \omega} \right) \sin^2\left(\frac{\pi y}{a}\right) \langle \cos^2(k_x x - \omega t) \rangle \mathbf{x} . \quad (5.28)$$

Integrating equation 5.28 across the waveguide cross-section allows us to relate the peak electric field to the power flow:

$$P = |A|^2 \left(\frac{k_x}{\mu_0 \omega} \right) \frac{ab}{4} . \quad (5.29)$$

Figure 5.8 shows the electric field amplitude in the waveguide versus power. At 1 kW, the field amplitude is 1.3 kV cm^{-1} .

Power loss also occurs along the waveguide due the finite conductivity of the walls. This attenuates power reaching the cavity by a factor $e^{-2\beta z}$. A formula for β is given by equation 8.63 of Jackson's book [150], and for our parameters we expect to dissipate about 5% of the output power from the klystron in the waveguide.

It is also worth stating results for the fundamental mode of a cylindrical waveguide of radius a , with its axis oriented along z . The lowest frequency mode which is supported is the TE_{11} mode, which has the electric field profile:

$$\mathcal{E}(\rho, \phi, z) = A \begin{pmatrix} \frac{-1}{k_c \rho} \cos(\phi) J_1(k_c \rho) \\ \sin(\phi) J'_1(k_c \rho) \\ 0 \end{pmatrix} e^{i(k_z z - \omega t)} . \quad (5.30)$$

Here, $J_1(x)$ is a Bessel function of the first kind, $J'_n(x) = \frac{d}{dx} J_n(x)$, and $k_z = \sqrt{(\frac{\omega}{c})^2 - k_c^2}$. The cutoff wavevector k_c is determined by the condition that $\mathcal{E}_\phi = 0$ at $\rho = a$, and given approximately by

$$k_c = \frac{1.8412}{a} . \quad (5.31)$$

These results are referred to in section 5.4.2 in the calculation of attenuation factors for microwave power leaking from the experiment.

Aperture Coupling

Power is delivered to the cavity through a small hole, of radius $r_h = 2.31$ mm and thickness $t = 0.7$ mm, in the centre of the upper mirror. Aperture coupling was originally given a full vectorial treatment by Bethe [151], and for an heuristic interpretation the reader is referred to chapter 7 of [149]. We can understand the effect of the hole by considering the two scenarios illustrated in figure 5.9, which represent a short-circuited waveguide with and without a hole. The electric and magnetic fields must satisfy

$$\mathbf{n} \times \mathcal{E} = 0 , \quad (5.32a)$$

$$\mathbf{n} \cdot \mathbf{H} = 0 , \quad (5.32b)$$

where $\mathbf{n}$ denotes the normal to the conductor surface. These boundary conditions specify that the tangential electric field and normal magnetic field must vanish at the conductor surface. An equivalent statement is that the electric (magnetic) field must be normal (parallel) to the conductor at its surface. When we introduce the hole, it follows that the lines of magnetic field must follow the edges of the hole in its vicinity. This leads to the magnetic

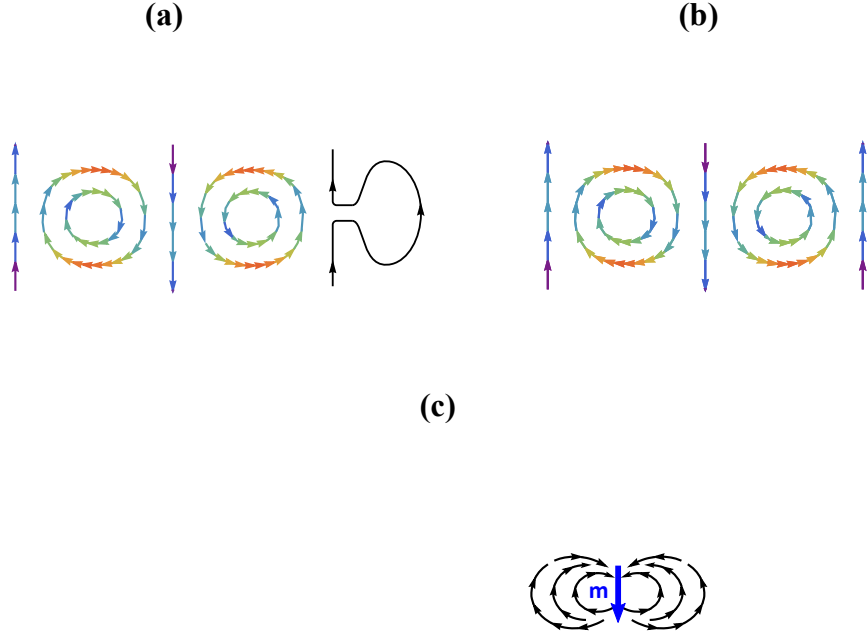


Figure 5.9: Coupling microwave power through the mirror hole. (a) A schematic of the magnetic field profile close to the hole, satisfying boundary conditions imposed by the conductor surface. (b) Magnetic field of a short circuited waveguide. (c) Magnetic field produced by a magnetic dipole in the plane of the hole at its centre.

field protruding out from the hole by some distance, as shown in figure 5.9a. Now, if the hole is small compared with the microwave wavelength, we can neglect the phase change in the fields across the hole area. The protruding magnetic field outwards into the hole looks similar in form to the combination of the original field profile (figure 5.9b), plus a small magnetic moment $\mathbf{m}$ in the plane of the hole at its centre (figure 5.9c). To a first approximation, we can treat the resultant field on the right hand side of the waveguide as the field of the short-circuited waveguide (i.e. zero) plus the oscillating magnetic moment in place of the hole. The size of the moment must grow in proportion to the magnetic field strength, which leads to the idea of a “hole” magnetization α_m such that $\mathbf{m} = \alpha_m \mathbf{H}$. As the hole thickness approaches zero, Bethe’s theory predicts $\alpha_m = \frac{4}{3}r_h^3$; for a hole of finite thickness t , a correction factor f_t must be included³ such that $\alpha_m = \frac{4}{3}r_h^3 f_t$. For the dimensions of our

³For further details on this correction factor, the reader is referred to [85] and the references therein.

coupling hole, $f_t = 0.52$.

When the microwave frequency is far from a cavity resonance, the magnetic moment is small and oscillates in phase with the incident field. The resultant field in the waveguide is a standing wave of the TE_{10} mode, with a small correction accounting for the dipole radiation emitted from the hole. As the frequency approaches resonance, the magnetic moment presented by the hole increases, and a phase difference develops relative to the incident field. Exactly on resonance with the cavity mode, the dipole lags behind the incident field by $\pi/2$, and the magnetic moment reaches a maximum. This situation is analogous to a driven mechanical oscillator, in which maximum power is delivered to the system when there is a $\pi/2$ phase difference between the driving force and the oscillator.

The full expression for the cavity reflectance, $\mathcal{R}$, is [85]

$$\mathcal{R} = 1 - \frac{4\kappa\nu_n^2/(2Q_0)^2}{(\nu - \nu_n + \delta)^2 + (1 + \kappa)^2\nu_n^2/(2Q_0)^2} \quad . \quad (5.33)$$

Here, κ is the coupling coefficient, which we can think of as the impedance ratio of the cavity to the waveguide, and δ defines the shift in the resonance frequency from the ideal value in equation 5.5. As defined, the power transmission on resonance, $T_{\text{res}} = \frac{4\kappa}{(1+\kappa)^2}$. For convenience, we can rewrite equation 5.33 as

$$\mathcal{R} = 1 - \frac{T_{\text{res}}}{1 + \frac{4\Delta^2}{\Gamma^2}} \quad , \quad (5.34)$$

Here, $\Delta = \nu - \nu_n + \delta$ measures the detuning of the source from resonance, and $\Gamma = \nu_n(1 + \kappa)/Q_0$. It is common to refer to the “loaded quality factor”, $Q_L = Q_0/(1 + \kappa)$ so that $\Gamma = \nu_n/Q_L$. The ring-down of a coupled cavity will then decay with a time constant $\tau = \frac{Q_L}{\omega}$. For a strongly undercoupled cavity ($\kappa \ll 1$), we recover the ideal closed cavity linewidth ν_n/Q_0 , whereas for a critically coupled cavity ($\kappa = 1$), we measure $2\nu_n/Q_0$. However, it is important to note that equation 5.25 for the peak electric field depends on Q_0 , *not* Q_L . If we were to assume Q_L instead, then we would be implying that a critically coupled cavity would dissipate only half of the input power, which cannot be true if $\mathcal{R} = 0$.

The ideal cavity theory discussed thus far is modified by the finite mirror conductivity and the presence of the coupling hole. Firstly, the impedance presented by the cavity is

a combination of the impedance of the hole and the impedance represented by the quality factor of the mode. As a result, the hole size has to be carefully chosen to maximise power transmission on resonance. Secondly, an effect of the hole is to broaden the mode perpendicular to the cavity axis. This reduces the quality factor by increasing the diffraction spill-over at the mirror edges, an effect which grows with the hole size. Using the data presented in [85] (figures 3a and 3b), we can extrapolate to estimate w_0 with our coupling hole of radius 2.31 mm. Along the long (x) axis of the waveguide, $w_0 = 17.25$ mm, and along the short (y) axis $w_0 = 19.2$ mm. Assuming these values as a lower bound for w_m , then the ratio of the diffraction loss to resistive loss is about 5% at room temperature.

Together, these effects shift the resonance frequency from that of an ideal cavity, ν_n , by

$$\delta = \left(\frac{1}{2Q_0} + \frac{8r_h^3 f_t}{3\pi w_m^2 L} \right) \nu_n = \delta_Q + \delta_g \quad . \quad (5.35)$$

We identify δ_Q as being due to the imperfect reflectivity of the copper mirrors, and δ_g as being caused by the coupling hole geometry. These shifts will hereon be referred to as the Q -shift and the geometric shift respectively. At room temperature, $\delta_Q \approx 0.3$ MHz and $\delta_g = 13$ MHz.

5.3 Trap Potential for ^7Li

From the description of the cavity mode, we can derive the trap potential for lithium. Because the microwave frequency is so small compared to the frequencies of all electric dipole transitions from the ground state, the interaction with the microwave field is almost identical to the DC Stark shift. In general the energy shift, ΔE , for an atom is determined by its scalar and tensor polarisabilities, but the spherical symmetry of the electronic ground state in alkali atoms means that the tensor part is negligible compared to the scalar part⁴. Therefore for lithium atoms in the microwave trap we may write,

$$\Delta E = -\frac{1}{2} \alpha_{\text{Li}} |\mathcal{E}(x, y, z, t)|^2 \quad , \quad (5.36)$$

⁴There is a non-zero tensor part when hyperfine structure is included, but it is tiny compared to the scalar part.

with α_{Li} the (scalar) ground state polarisability. The trap potential is determined by the time average of the Stark shift combined with the gravitational potential, which gives:

$$U(x, y, z) = -\frac{1}{4}\alpha_{\text{Li}}\mathcal{E}^2(x, y, z) + mgz \quad . \quad (5.37)$$

We should also consider the effect of the magnetic field in the cavity on the trap potential. This interacts with the electron magnetic moment μ_{m} via the magnetic dipole interaction, $\boldsymbol{\mu}_{\text{m}} \cdot \mathbf{B}$, which couples the $F = 1$ and $F = 2$ hyperfine levels. Since the microwave field is tuned above the hyperfine transition, the $F = 2$ levels are strong-*magnetic* field seeking; because the magnetic field is zero at the cavity centre, the trap depth for these states is lowered by the hyperfine coupling. The transition matrix element is at most $\sqrt{3/2}\mu_B B$ ⁵, and we can calculate the energy shift, U_{m} , using second-order perturbation theory:

$$U_{\text{m}} \approx \left(\frac{\mu_{\text{m}} B^2}{2} \right) \left[\frac{1}{h(f_{\text{MW}} - f_{\text{HFS}})} - \frac{1}{h(f_{\text{MW}} + f_{\text{HFS}})} \right] \quad .$$

Here, f_{HFS} is the 803.5 MHz ground state hyperfine splitting and f_{MW} is the microwave frequency; the prefactor of one half arises from the decomposition of the magnetic field into its two counter-rotating components. For lithium in our trap, U_{m} provides a correction of less than 1.5% to the total trap depth and we ignore this contribution.

Figure 5.10a plots equation 5.37 for the TEM₃₀₀ cavity mode, assuming an input power of 700 W and $Q_0 = 2 \times 10^4$. The influence of gravity results in an asymmetry in the axial potential, shifting the trap centre and lowering the overall depth. Note that, unlike the microwave trap in [146], applying a magnetic field gradient to compensate for gravity would not affect the trapping mechanism. Figure 5.10b plots the trap depth against power; the threshold power for trapping is 110 W, and above this input power the depth increases at $0.28 \mu\text{K W}^{-1}$. The shift in the axial trap centre relative to the centre of the cavity mode is plotted in figure 5.10c.

⁵This value is obtained with a circularly polarised field driving stretched $|F = 1, m_F = \pm 1\rangle \rightarrow |F = 2, m_F = 2\rangle$ transitions.

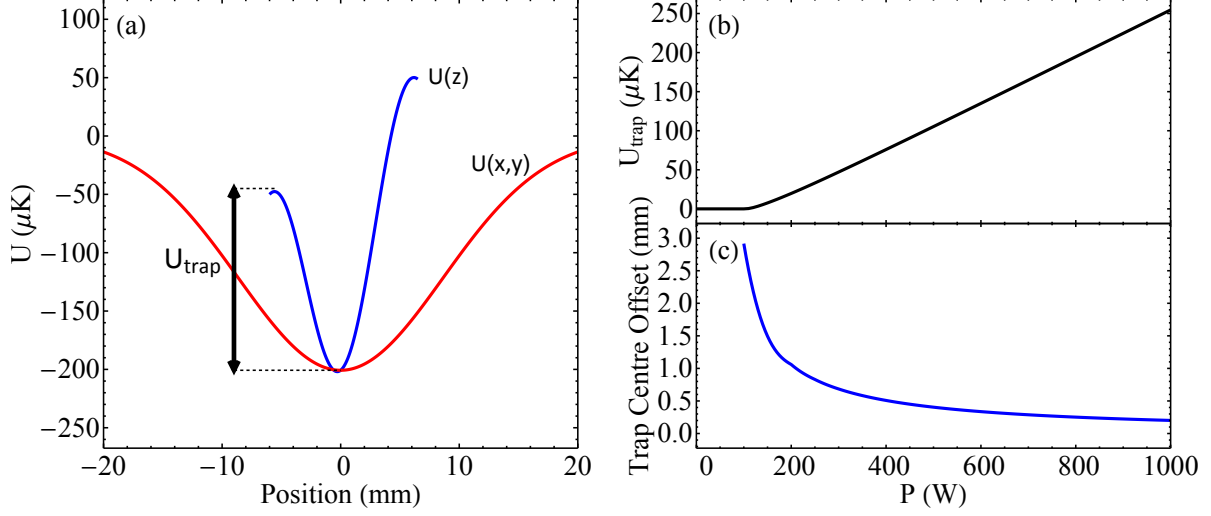


Figure 5.10: Microwave trap potentials for ^7Li ($Q_0 = 2 \times 10^4$). (a) Trap potentials $U(z)$ and $U(x, y)$, for $P_{\text{in}} = 700 \text{ W}$. (b) Trap depth versus cavity input power. (c) Axial offset between the trap and cavity mode centres versus power.

Trap Oscillation Frequencies

To find the trap oscillation frequencies, we use equation 5.8 for the second order expansion of the electric field close to the cavity centre:

$$U(x, y, z) \approx \frac{\alpha \mathcal{E}_{\text{peak}}^2}{4} \left[\left(1 - \frac{2}{kz_0} + \frac{2}{k^2 z_0^2}\right) k^2 z^2 + 2 \frac{x^2 + y^2}{w_0^2} - 1 \right] . \quad (5.38)$$

Equation 5.38 represents an anisotropic harmonic potential. The motional frequencies for atoms near the trap centre, $\Omega_{x,y,z}$, are given by:

$$\Omega_x = \Omega_y = \sqrt{\frac{\alpha_{\text{Li}} \mathcal{E}_{\text{peak}}^2}{m_{\text{Li}} w_0^2}} , \quad (5.39a)$$

$$\Omega_z = \sqrt{\frac{\alpha_{\text{Li}} \mathcal{E}_{\text{peak}}^2 k^2 \left(1 - \frac{2}{kz_0} + \frac{2}{k^2 z_0^2}\right)}{2m_{\text{Li}}}} . \quad (5.39b)$$

In thermal equilibrium the density profile of a sample, $n(x, y, z)$, will follow a Gaussian distribution:

$$n(x, y, z) = n_0 e^{-\frac{m_{\text{Li}}}{2k_B T} (\Omega_x^2 x^2 + \Omega_y^2 y^2 + \Omega_z^2 z^2)} . \quad (5.40)$$

We can apply the equipartition theorem for a cloud at temperature T to find the mean-square radii of a trapped sample:

$$\langle x^2 \rangle = \langle y^2 \rangle = \frac{k_B T}{m_{\text{Li}} \Omega_x^2}, \quad \langle z^2 \rangle = \frac{k_B T}{m_{\text{Li}} \Omega_z^2}. \quad (5.41)$$

The trap aspect ratio, $\sqrt{\frac{\langle y^2 \rangle}{\langle z^2 \rangle}}$ is the ratio of the trapping frequencies, given by

$$\Omega_z / \Omega_y = w_0 k \sqrt{\frac{1}{2} \left(1 - \frac{2}{k z_0} + \frac{2}{k^2 z_0^2} \right)}. \quad (5.42)$$

For our cavity parameters this is 3.28, so that in equilibrium a cloud of atoms would take on a “pancake” formation.

5.4 Microwave System

Figure 5.11 shows a schematic of the microwave system. The microwave source is a Yttrium iron garnet (YIG) sweep oscillator (HP 8620C, with RF plugin 86260B), which has a variable output power of up to 20 mW. The signal from the sweep oscillator is amplified by a klystron amplifier, which delivers up to 1 kW to the experiment via WG18 waveguide. Directional couplers pick off a small portion (−40 dB) of the forward and reflected powers, which are measured using microwave diode detectors (Pasternack PE8013). Each detector outputs a DC voltage, and by analog division of the two signals we can deduce a measure of the cavity reflectance $\mathcal{R}$. Additional 10 dB attenuators are included to ensure that the rated power of the detectors is never exceeded.

In the original characterisation of the cavity, measurements were carried out using a Vector Network Analyser (VNA). This enabled accurate measurements of the intensity and phase of reflected power by calibrating for the various reflections along the chain of microwave components, and consequently the quality factor and reflected power could be measured with high accuracy. Unfortunately our VNA developed an (unserviceable) fault before I was able to use it. Instead, to test the cavity at low power, we bypass the amplifier, switch to −20 dB couplers, and remove the protective attenuators. We then monitor the ratio of the reflected

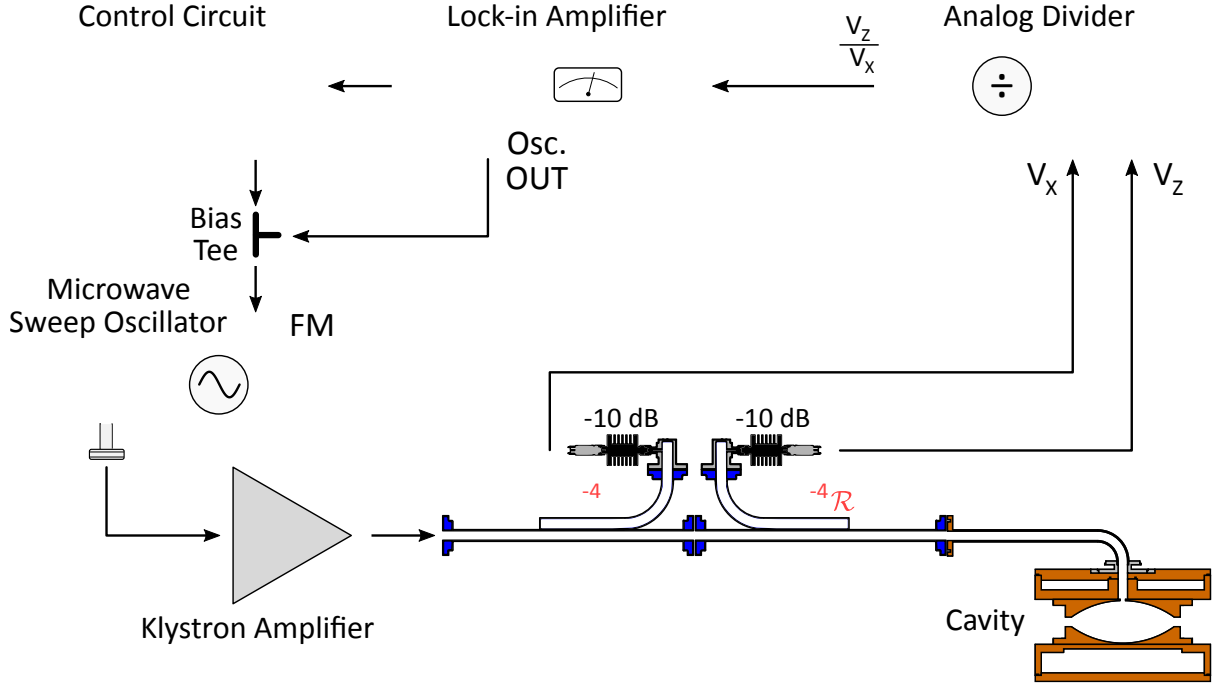


Figure 5.11: Schematic of the microwave system.

power to the forward power as the microwave frequency is swept across the cavity resonance.

Figure 5.12a shows an early measurement where the frequency is scanned over the TEM_{300} resonance, with a hole radius $r_h = 2.31$ mm and an input power of 10 mW. The data is calibrated assuming $\mathcal{R} \rightarrow 1$ when the frequency is far from the cavity resonance ($\Delta \gtrsim 10$ MHz), and $\mathcal{R} \rightarrow 0$ when replacing the cavity with a matched waveguide terminator. The dashed line shows a fit to equation 5.34, which is used to extract the loaded quality factor from the full-width at half maximum. This provides $Q_L = 1.515(16) \times 10^4$. To calculate the unloaded quality factor Q_0 , we use $Q_0 = Q_L(1 + \kappa)$. The coupling coefficient κ is found by the on-resonance power transmission T_{res} ; figure 5.12b shows that small changes in the transmission cause large changes in κ , which makes it difficult to know Q_0 accurately. I found that the presence of offset ground noise on the detectors through the waveguide, and difficulty in knowing the exact detector-waveguide coupling, made it difficult to measure T_{res} to within about 1%. From the VNA measurements in figure 5(a) of [85], we expect $T_{\text{res}} \approx 95\%$ at our hole size, but the data in figure 5.12a provides 98.8(5)%. Using the latter value results in $Q_0 = 2.74(6) \times 10^4$.

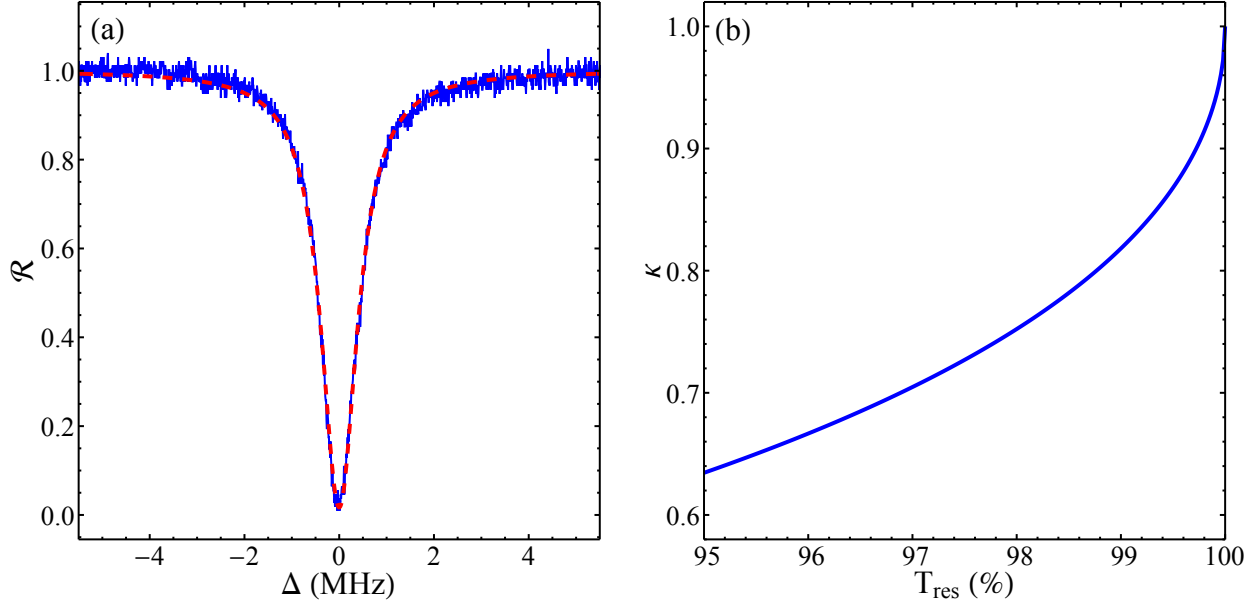


Figure 5.12: (a) A frequency scan over the TEM_{300} cavity resonance. Dashed red line: fit to equation 5.33. (b) Dependence of the coupling coefficient κ on the power transmission into the cavity on resonance.

5.4.1 Frequency Locking

Building a robust and reliable frequency lock for the microwave trap is essential for our experiment for a number of reasons. First and foremost, the trap depth is proportional to the microwave energy in the cavity, meaning maximum trap depth is achieved precisely at the resonant frequency. Secondly, the klystron amplifier is designed with the assumption that most of its output is transmitted and dissipated across the output load. Reflected power from the cavity caused by a deviation from the resonant frequency could permanently damage the amplifier, and the manufacturer suggests an absolute maximum reflected power of 200 W. Ensuring a reasonable trap depth for our lithium gas necessitates transmitting at least 500 W into the cavity, and therefore a failure to frequency-stabilise the system could be potentially catastrophic.

Since the precise frequency of the cavity resonance is unimportant, we choose to lock the generator to the cavity resonance rather than attempting to stabilise the cavity length. The resonance will remain inside the gain width of a given klystron band provided it does not drift by more than 80 MHz. This requires that the cavity length is stable to within 0.5%, and assuming copper has a thermal expansivity of $1.8 \times 10^{-5} \text{ K}^{-1}$, places an upper limit on

the cavity temperature increase of around 280 °C. This is significantly larger than acceptable with water cooling, and therefore not a limiting factor in the experiments.

To introduce feedback into the microwave system, we apply a small modulation, $\alpha \sin(\omega t)$, to the frequency of the sweep oscillator. The amplitude of this modulation is chosen to be much less than the resonance width, and its frequency is chosen to be fast compared with laboratory noise. With the frequency modulation off, sweeping the generator across the resonance simply maps out the reflectance given by equation 5.34. We wish to remain at the minimum of the curve, which requires information about the gradient; modulating the reflectance signal with a lock-in amplifier allows us to measure this. The demodulated output of the lock-in amplifier, S_ω , is given by

$$S_\omega(\alpha, \Delta) \approx \langle [\mathcal{R}(\Delta) + \frac{d\mathcal{R}}{d\Delta} \alpha \sin(\omega t) + \frac{1}{2} \frac{d^2\mathcal{R}}{d\Delta^2} \alpha^2 \sin^2(\omega t) + \frac{1}{6} \frac{d^3\mathcal{R}}{d\Delta^3} \alpha^3 \sin^3(\omega t) + \dots] G \sin(\omega t) \rangle . \quad (5.43)$$

Here, $\mathcal{R}(\Delta)$ is the reflectance of the cavity as given by equation 5.34, G is the combined gain of the analog divider and lock-in, and the angle brackets denote time averaging. Terms in odd powers of $\sin(\omega t)$ average to zero, meaning

$$S_\omega(\alpha, \Delta) = G \left[\frac{1}{2} \frac{d\mathcal{R}}{d\Delta} \alpha + \frac{3}{48} \frac{d^3\mathcal{R}}{d\Delta^3} \alpha^3 + \dots \right] . \quad (5.44)$$

The numerical factors derive from the time averages of $\sin^2(\omega t)$ and $\sin^4(\omega t)$.

Figure 5.13a shows the reflectance signal as we scan over the cavity resonance at around 20 MHz/s. The output of the lock-in amplifier, S_ω is shown in figure 5.13b, using $\alpha = 40$ kHz and an averaging time of 1 ms. This provides the error signal which we then feed back to a control circuit in order to lock the sweep oscillator. By keeping the frequency modulation fast compared to laboratory noise, and small compared to the resonance width, we can minimise the resultant reflected power to acceptable levels.

The control circuit drives $S_\omega(\Delta)$ to zero, where all odd derivatives of $\mathcal{R}(\Delta)$ are zero by symmetry. Therefore in the ideal case the lock-in will drive the generator to the frequency of minimum reflectance from the cavity, as required. We can estimate the residual reflected power due to the frequency modulation as,

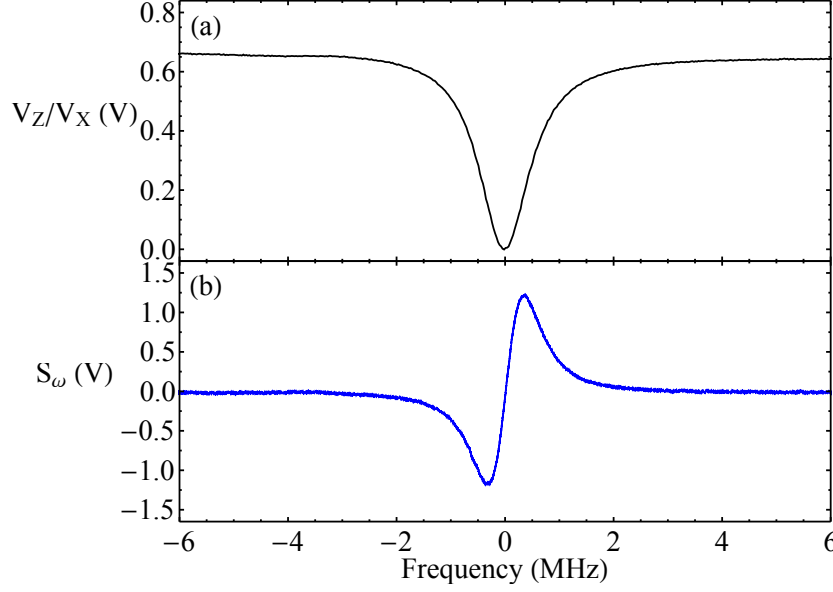


Figure 5.13: Locking to the cavity resonance. (a) Reflectance signal as the generator frequency is swept over the resonance. (b) Lock-in amplifier output, S_ω .

$$\langle \mathcal{R}_{locked} \rangle = \mathcal{R}(0) + \frac{1}{2} \frac{d^2 \mathcal{R}}{d\Delta^2} \bigg|_{\Delta=0} \alpha^2 \langle \sin^2(\omega t) \rangle = \mathcal{R}(0) + \frac{T_{res}}{2} \left[\frac{8}{(1 + \frac{4\Delta^2}{\Gamma^2})^2} - \frac{128\Delta^2/\Gamma^2}{(1 + \frac{4\Delta^2}{\Gamma^2})^3} \right] \bigg|_{\Delta=0} \frac{\alpha^2}{2} \quad (5.45)$$

$$\approx \mathcal{R}(0) + \frac{2\alpha^2}{\Gamma^2} . \quad (5.46)$$

Therefore we expect $2\alpha^2/\Gamma^2$ additional reflectance when locked exactly on resonance. We use a modulation amplitude of 40 kHz and the resonance width Γ is about 1 MHz, so $\alpha = 0.04$, and at on resonance we expect 0.32% additional reflectance from the cavity. At the maximum intended operating power of 2 kW this amounts to 6.4 W, and therefore a small contribution.

Klystron Amplification

To achieve the required microwave input power to the cavity for loading ultracold atoms, we use a CPI Klystron High Power Amplifier (KHPA) to amplify the generator input. This device was an invention credited to the Varian brothers, who published their work in 1939 [152], and has an interesting history. The operating principle is illustrated in figure 5.14:

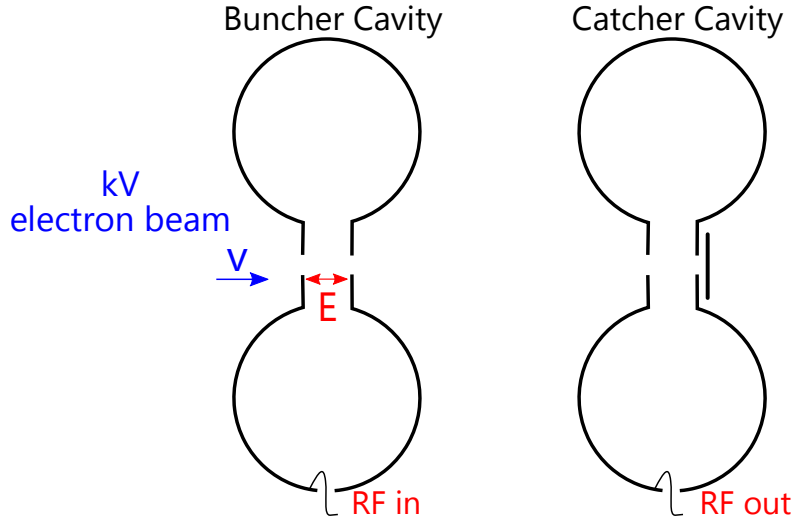


Figure 5.14: The klystron operating principle.

a high energy electron beam passes through a cavity, where it experiences a time-varying electric field from the input signal. The resultant velocity modulation causes “bunching” of the electron beam to develop as it propagates through space. A second “catcher” cavity is used to extract energy from the beam, which by this point is effectively a pulse train of charges at the microwave input frequency. This picture inspired the name, which derives from the Greek word, “klyzo”, meaning the breaking of waves on a beach. It is possible to operate klystrons as oscillators by feeding back a portion of the output signal to the input buncher cavity (under appropriate phase conditions) and this allowed the device to extend microwave frequency communication further into the gigahertz regime than was possible with contemporary magnetron devices. The klystron opened the possibility of airborne radar during the second world war, and is credited with being pivotal to the success of the allied forces.

Our KHPA consists of an initial semiconductor amplifier which provides 38 dB gain, followed by a klystron which provides a further 45 dB gain. The overall specified gain is 80 dB when losses are accounted for, at a maximum power output of 2 kW. There are 12 frequency bands covering 13.75 – 14.5 GHz. In the amplification band used in this work, the gain is approximately uniform in a ± 10 MHz range around 14.265 GHz, and the 3 dB bandwidth is roughly 80 MHz; changing band requires a power down of the machine. When operating in the experiment, we fix the gain of the KHPA and vary the input power to

increase the output power. To prevent the klystron from producing excessive output power, we attenuate by 26 dB at the input to ensure that the power here cannot exceed -17 dBm.

5.4.2 Safety

An important concern for the experiment was ensuring that the microwave power leaving the vacuum chamber did not exceed safe levels. There are two key exit points from the experiment: waveguide flanges and glass viewports of the vacuum system.

To our knowledge there has not been detailed studies of the effects of microwave radiation in our frequency range, and so we followed guidance from the International Commission on Non-Ionizing Radiation Protection (ICNIRP), who published a report in 1998 covering time-varying electromagnetic fields up to 300 GHz [153]. The report states that at frequencies from 10 MHz to 300 GHz, “heating is the major effect of absorption of electromagnetic energy”, and that “At very high frequencies (10–300 GHz), absorption of electromagnetic energy is confined largely to the epidermal layers of the skin, subcutaneous tissues, and the outer part of the eye”. The authors comment that there is only limited data available on the exposure risk, and much of this is poorly reported. A review of the current guidance is underway and expected to be published in 2018, although in December 2017 the ICNIRP website published a statement confirming the 1998 guidance remains protective.

The recommended safe limit based on the available data is 5 mW cm^{-2} for occupational exposure, and 1 mW cm^{-2} for the general public. The former was adopted into UK legislation directly in 2016 [154]. We opted to use the latter restriction as an absolute maximum given the presence of other people in the laboratory.

Cutoff Attenuators

Although most of the microwave power is dissipated in the mirrors, a small portion is diffracted by the hole and couples into the surrounding environment. The viewports on the vacuum chamber are transparent to microwaves, leading to dangerous levels in the laboratory. For example, with 1 W input power, I measured 2 mW with the receiver horn positioned 5 cm from a viewport and directed towards the cavity centre. A few layers of alu-

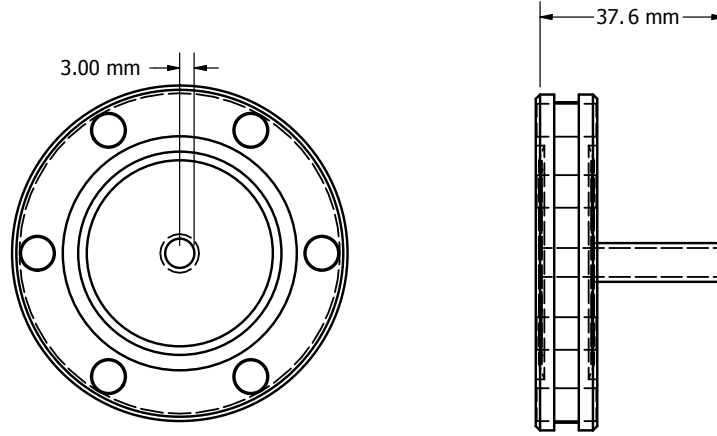


Figure 5.15: Microwave cutoff attenuators used to minimise power leakage from the experiment. The protruding tube extends into the vacuum chamber.

minium foil was sufficient to reduce the leakage by 25 dB, but this obviously means optical access is not possible. Furthermore, previous work on the cavity encountered problems with the microwave leakage interfering with the pressure gauge, leading to erratic and unreliable readings.

Our solution to both of these problems is to use hollow cylindrical tubes as cut-off attenuators, meaning the radius r is sufficiently small that the lowest order TE_{11} mode does not propagate. The power attenuation of a tube of length L is $e^{-2\beta L}$, with

$$\beta = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{1.8412}{r}\right)^2}, \quad (5.47)$$

where the numerical value 1.8412 derives from Bessel function properties of the cylindrical modes as described in section 5.2.4.

For the viewport attenuators, we use a single tube of inner diameter 6 mm and length 37.6 mm, shown in figure 5.15. The attenuator is formed by welding a tube of sufficient length into a double sided DN40 flange. This flange is mounted between the viewport and the vacuum chamber with the tube directed towards the cavity, minimising external protrusion to avoid blocking the transport coils. The power attenuation is approximately 175 dB.

To protect the vacuum gauge, we drilled an array of 97 tubes into a double sided DN40



Figure 5.16: Protective DN40 flange used to shield the vacuum gauge from microwave power.

flange, as shown in figure 5.16. With $r = 1$ mm, and $L = 10$ mm, the expected attenuation is ~ 160 dB, but we maintain about 25% of the vacuum conductance relative to the bored hole in the vacuum gauge. This attenuator is mounted directly onto the vacuum gauge.

An additional consideration is leakage from the microwave chamber to the MOT chamber via the transport tube. The inner diameter of the transport tube is 9 mm, meaning that the $1/e$ decay length for the microwave power is 1.8 mm, and the attenuation along the tube is far stronger than for the viewport attenuators described above.

Monitoring Microwave Power

To monitor the microwave power leaking from the cavity, we used a standard gain horn antenna (Pasternack PE9854/SF-20). The power received by the horn when directed towards the cavity, at a distance r from its centre is

$$P_{\text{horn}} = I(r, \theta, \phi) A_{\text{eff}} G_{\text{horn}} , \quad (5.48)$$

where $I(r, \theta, \phi)$ is the radiated intensity at the position of the horn, A_{eff} is the effective area of an isotropic antenna and G_{horn} measures the gain of the horn along its direction (relative to an isotropic antenna).

It can be shown from thermodynamic arguments that $A_{\text{eff}} = \frac{\lambda^2}{4\pi}$, which is 0.35 cm^2 for our wavelength, and the nominal gain for the horn used was $G_{\text{horn}} = 20$ dB. Therefore the

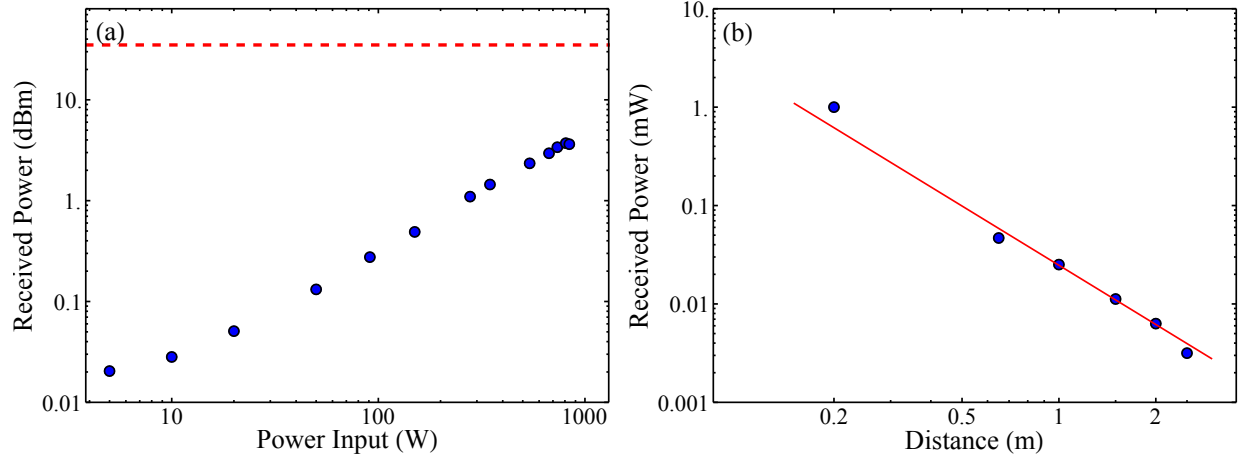


Figure 5.17: Monitoring power leakage from the cavity. (a) Received power versus input power at a distance of 1m from the cavity. Dashed red line: power received at the intensity limit for CW exposure [153]. (b) Received power versus distance from the cavity centre. Solid red line: Inverse square relationship to guide the eye.

intensity at the horn location is

$$I(r, \theta, \phi) [\text{mWcm}^{-2}] = \frac{P_{\text{horn}}}{A_{\text{eff}} G_{\text{horn}}} = \frac{P_{\text{horn}} [\text{mW}]}{35} . \quad (5.49)$$

Figure 5.17 shows the maximum power received 1 m away from the cavity centre, with the horn perpendicular to the cavity axis and directed towards its centre. The dashed line indicates our conservative safety limit, and we evidently do not breach this limit.

5.5 Cavity Performance

5.5.1 FEM Simulations

We modelled the thermal and mechanical behaviour of the cavity under the intended experimental operating conditions using finite element software COMSOL Multiphysics 5.2. The geometry of the mirrors and their respective coolant baths was partitioned with a tetrahedral mesh of $\sim 30,000$ elements. The material properties were chosen to be representative of copper at room temperature, and are summarised in table 5.3.

Material Property	Value	Unit
Young's Modulus	110	GPa
Thermal Conductivity	400	$\text{W m}^{-1} \text{K}^{-1}$
Heat Capacity	400	$\text{J kg}^{-1} \text{K}^{-1}$
Density	8960	kg m^{-3}

Table 5.3: Material properties used in the FEM simulations of the cavity assembly.

Thermal Modelling

To study the thermal behaviour, a heat load was distributed across the mirror faces according to the intensity of the cavity mode (which may be derived from equation 5.6). To study the response to input power P_{in} , $\frac{P_{\text{in}}}{2}$ was dissipated across the bottom mirror, and $\frac{1.037P_{\text{in}}}{2}$ was dissipated across the in-coupling mirror. The numerical factor corrects for the effect of the hole size under the assumption that the mirrors dissipate equal power. Constant temperature boundary conditions were imposed across all surfaces of the cooling baths. Perfect thermal contact was assumed between the mirrors and the coolant baths, and the cavity was thermally insulated from the surroundings.

Figure 5.18 shows steady state temperature profiles for $P_{\text{in}} = 2 \text{ kW}$. The region around the coupling shows a temperature increase of 59°C , and the outline of the hollow waveguide cross-section is clearly visible. By contrast, the temperature increase at the centre of the bottom mirror is 21°C , a factor ~ 3 smaller. This difference is evidently the result of the small effective area for heat conduction at the interface between the waveguide and the top mirror. In an experiment, we can only monitor the temperature at one of the mirror mounting holes with a thermocouple, as shown in the figure. Importantly, the temperature increase here is less than 0.1°C , and tells us nothing about what happens at the centre of the mirror. Figure 5.19a shows the temperature at the coupling hole for a time-dependent study using a 4 s microwave pulse at $P_{\text{in}} = 2 \text{ kW}$, and figure 5.19b shows the steady state temperature increase versus P_{in} . These simulations predict that the steady state temperature is approached with a thermal time constant of 0.12 s, and that this temperature increases at 29.5°C for each kilowatt of input power.

The assumption of fixed temperature at the coolant baths is motivated by the original cryogenic cooling method, where evaporation of liquid nitrogen at atmospheric pressure

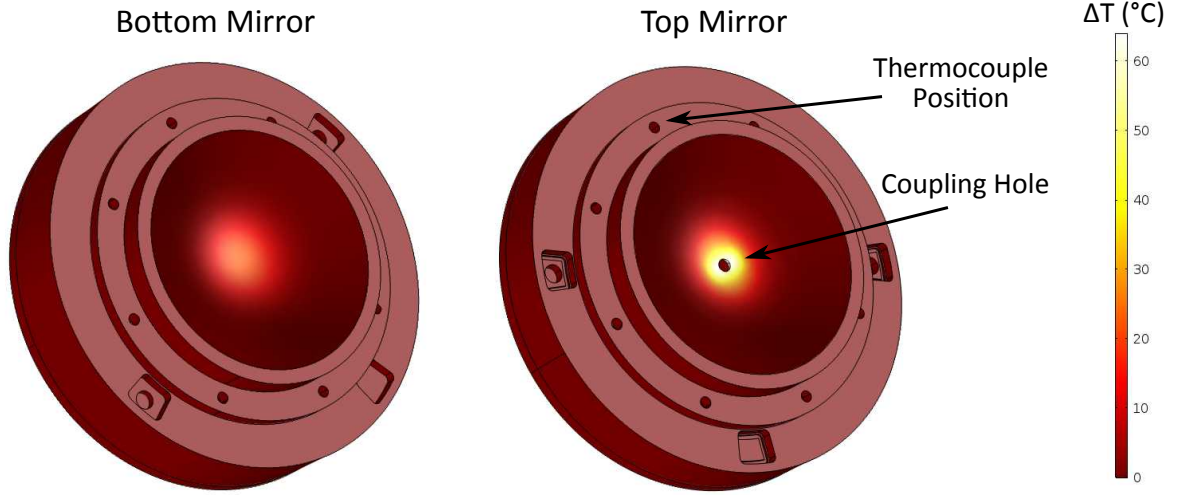


Figure 5.18: Steady state thermal profiles of the cavity mirrors when subjected to a 2 kW heat load distributed according to intensity of the $n = 3$ mode. Colour bar: temperature increase ($^{\circ}\text{C}$) relative to the fixed coolant bath temperature.

cools the cavity. For water cooling, a more realistic approach is to introduce a heat transfer coefficient at the walls of the coolant baths. With this model, the heat flux across the boundary, q is determined by

$$q = h\Delta T_{\text{bath}} \quad . \quad (5.50)$$

Here, ΔT_{bath} is the temperature difference across the boundary, and h is the heat transfer coefficient. For a water-copper boundary, h is typically in the range $340\text{--}455 \text{ W m}^{-2} \text{ K}^{-1}$. Figure 5.20 shows results of a time dependent study for $P_{\text{in}} = 2 \text{ kW}$, applied for 4 s. There are a number of important differences to the fixed boundary temperature simulations. First, the system does not reach thermal equilibrium when the heat load is applied, which we can interpret as the interfacial heat transfer limiting the conduction away from the mirrors. The temperature increase at the mirror hole is 91°C at $t = 4 \text{ s}$, which is 32°C larger than for fixed temperature boundaries. There is a delayed response at the thermocouple position, peaking at 5.7 K when $t = 8.5 \text{ s}$. After the heat load is removed, the temperature at the mirror hole equalises with the temperature at the thermocouple, and both then return slowly to the initial coolant temperature.

From figure 5.20b, we see that the thermal response at the mirror hole depends very

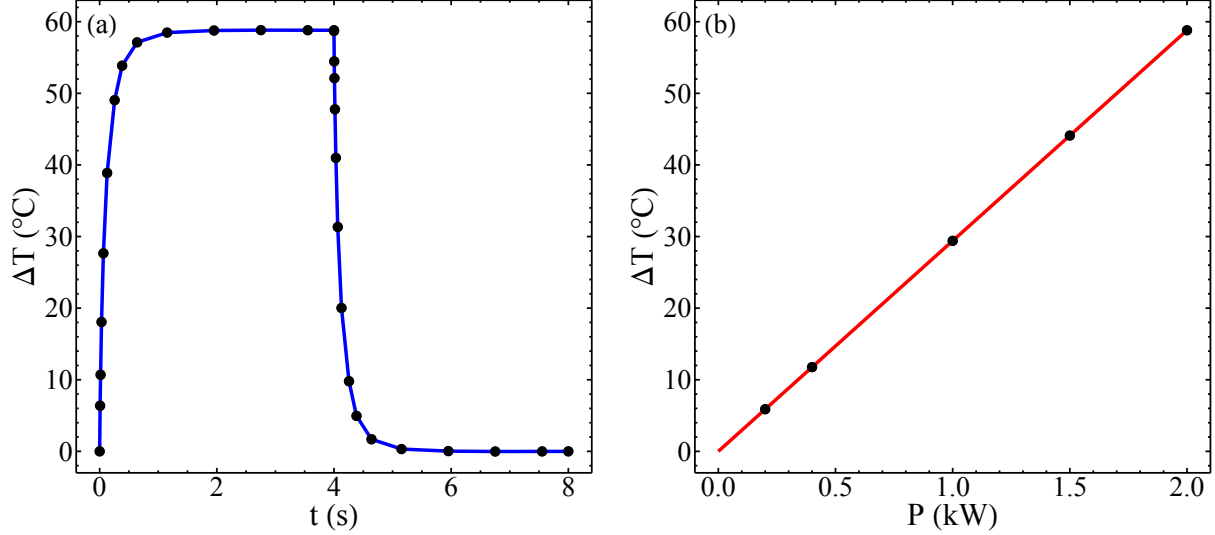


Figure 5.19: FEM simulation of the mirror hole temperature for fixed temperature boundary conditions at the coolant baths. (a) Time dependence of the temperature when applying a 2 kW heat load for 4 s. (b) Temperature increase versus total input power.

weakly on the heat transfer coefficient for $h < 1000 \text{ W m}^{-2} \text{ K}^{-1}$. This indicates that the water cooling is not effective at short time scales, and that most of the heat is stored in the cavity assembly over the first few seconds of operation. Figure 5.21 shows the steady state temperature against the heat transfer coefficient, for $P_{\text{in}} = 1 \text{ kW}$. The coupling hole is consistently around 50°C hotter than the mounting mounting hole. Assuming $h = 340 \text{ W m}^{-1} \text{ K}^{-1}$, we would expect a steady state temperature increase of 100°C at the coupling hole, and 50°C at the thermocouple.

Comparison with Experiment

Figure 5.22 shows measured temperature data at the thermocouple position. The time dependent data in figure 5.22 (a) was taken with a constant input power of 300 W, applied for 10 minutes from $t = 0$. We see that the temperature takes several minutes to stabilise, much longer than a typical run of a cold atom experiment. Figure 5.22 (b) shows the steady state data obtained for $0 < P_{\text{in}} < 500 \text{ W}$. The shaded region shows the expected range from the simulations for $340 < h < 455 \text{ W m}^{-2} \text{ K}^{-1}$. The simulations clearly underestimate the actual temperature increase, which is most likely explained by our assumption of perfect thermal contact between the mirror and the coolant bath. A linear fit to the experimental

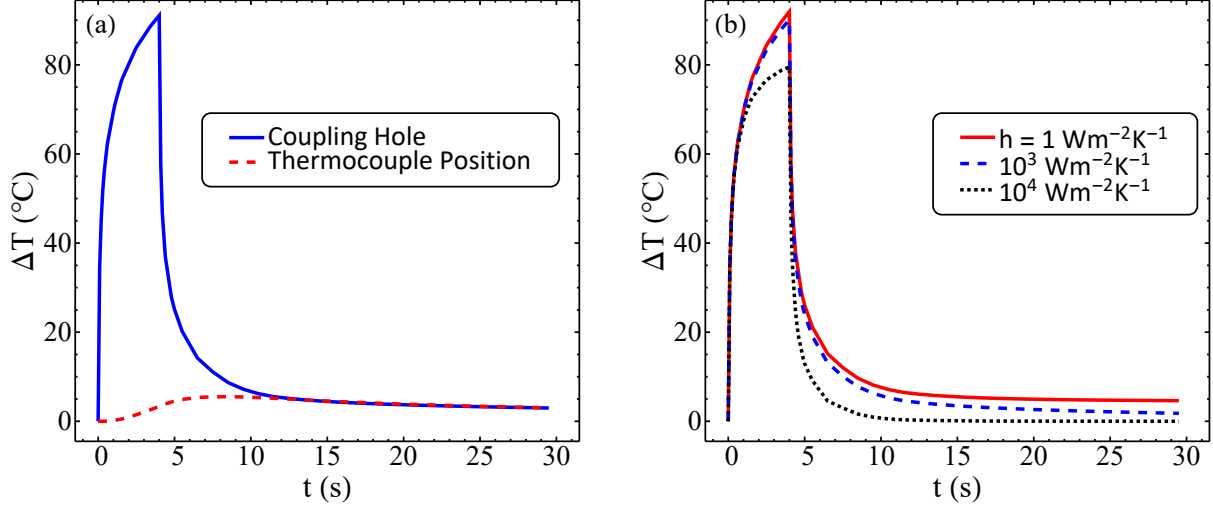


Figure 5.20: FEM temperature simulation for the microwave cavity when a 2 kW microwave heat load is applied for $0 < t < 4$ s; a heat transfer coefficient h is assumed at the coolant bath boundaries. (a) Temperature at the coupling hole and thermocouple position, for $h = 340 \text{ W m}^{-2} \text{ K}^{-1}$. (b) Temperature at the mirror hole for different of values of h .

data provides $\Delta T = 70^\circ \text{C kW}^{-1}$, from which we can infer $h = 247 \text{ W m}^{-2} \text{ K}^{-1}$.

Using this empirically observed heat transfer coefficient, the simulations were repeated and compared directly with time-dependent measurements at the thermocouple position. Figure 5.23a shows a comparison with $P_{\text{in}} = 750 \text{ W}$ applied from $0 < t < 1.5 \text{ s}$. The measured temperature increases faster than the simulations predict when the microwave power is on, then decreases for a short time after. This is likely the result of power leaking from the cavity into the surrounding vacuum chamber; the simulations assume heat only reaches the thermocouple by conduction from the spherical surfaces of the mirrors. For $t > 10 \text{ s}$, there is close agreement with the simulation. Figure 5.23b shows the simulation results at the coupling hole for the same experiment. The temperature peaks at 53°C , an increase of 30°C relative to the coolant baths; at $t = 14 \text{ s}$ the coupling hole remains 0.75°C hotter than at $t = 0$. We can therefore conclude that water cooling the cavity is a viable option for high power pulses of a few seconds, provided the duty cycle is around 10%. Given that it takes about 10 s to cool and transport cold atom samples to the microwave chamber, this is not a severe limitation for our experiment.

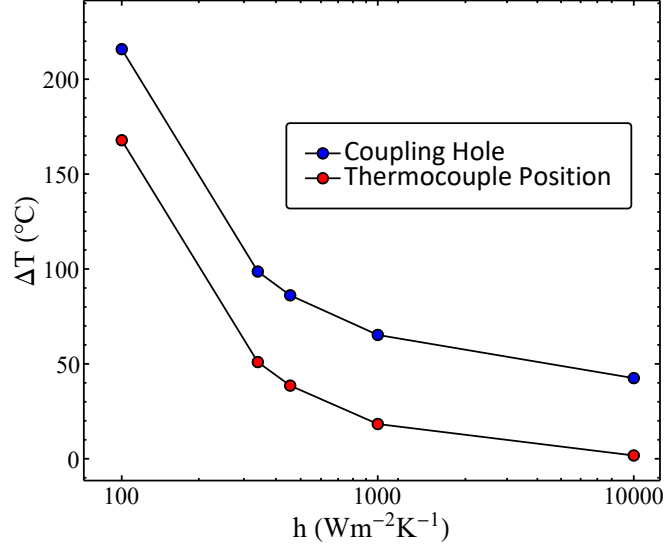


Figure 5.21: Simulated steady state temperature increases for different heat transfer coefficients, assuming a 0.5 kW microwave heat load per mirror.

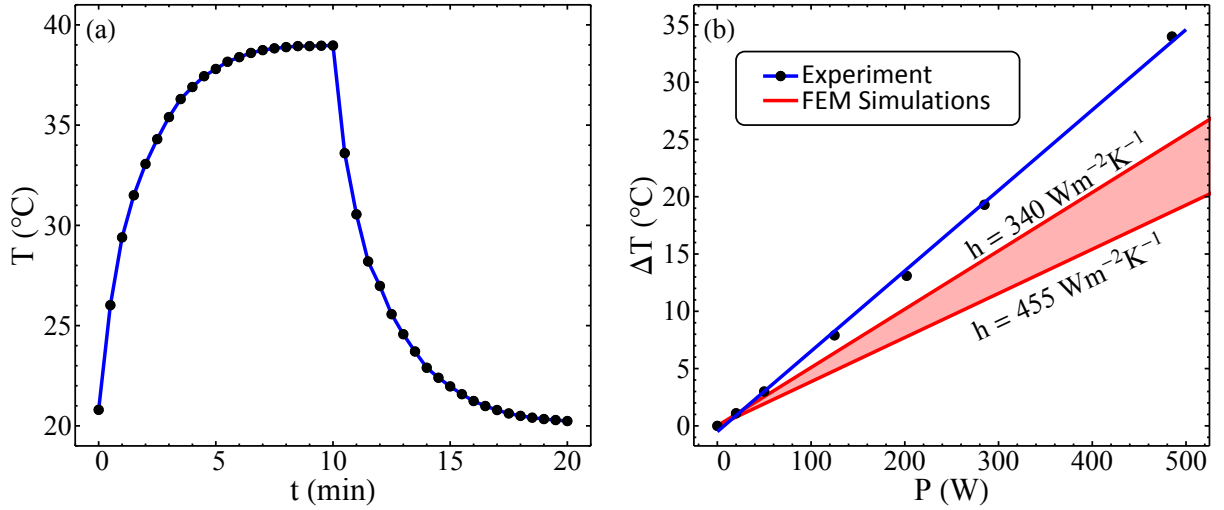


Figure 5.22: Experimental data at the thermocouple position under high power microwave testing. (a) Time dependence of the temperature with $P_{\text{in}} = 300$ W, applied for $0 < t < 10$ minutes. (b) Steady state temperature versus P_{in} . The shaded region shows the FEM predictions for $340 < h < 455$ $\text{W m}^{-2} \text{K}^{-1}$.

Pressure Loading

We studied the mechanical response of the cavity to a fixed pressure load of 100 kPa applied to the cooling bath walls, to determine the effect of evacuating the vacuum chamber whilst having a coolant at atmospheric pressure in the baths. Figure 5.24 shows the deformation profiles obtained; the deformation of the top (bottom) mirror was approximately 1 (5) μm

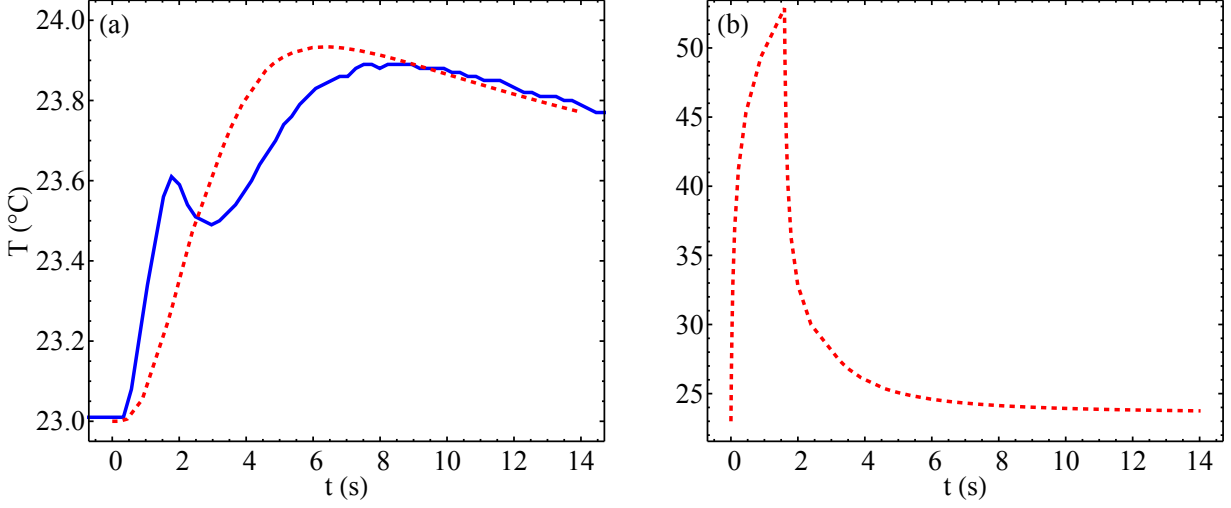


Figure 5.23: Comparison of simulations using the measured heat transfer coefficient, $h = 247 \text{ W/m}^2/\text{K}$. (a) Simulated (red, dashed line) and measured (solid, blue line) temperatures at the thermocouple position. (b) Simulated temperature at the coupling hole under the same conditions.

under these conditions, with the top mirror deforming least due to the central support provided by the waveguide. Assuming a maximum deformation of $6 \mu\text{m}$, we have an upper bound on the trap frequency shift under vacuum of around 2 parts in 10^4 , or 3 MHz. In addition to this, we want to ensure that the baths do not deform plastically in vacuum; an accepted criterion is to compare the von Mises stress to the yield stress of the material⁶. Figure 5.25 shows the von Mises stress profile for the pressure simulation. The yield stress for copper is a highly process dependent property, but typically lies in the 40–80 MPa range. However, annealing copper can reduce the yield stress by an order of magnitude, and the high temperatures used in the vacuum brazing process probably had this effect.

Unfortunately, when water was supplied from our laboratory cooling system, the coolant baths plastically deformed, with the bottom bath bowing by around 1 mm at its centre. Thankfully, this did not seem to significantly degrade the performance of the trap, and the quality factor remained around 2×10^4 . Therefore, we decided to proceed regardless, and installed a pressure reducing valve at the water cooling inlet. This ensures the pressure at the inlet is less than about 1 bar. When the cooling supply is switched on, the cavity resonance frequency shifts by +1.5 MHz when the water cooling pressure inlet was switched to 1 bar.

⁶This collapses the stress tensor to a scalar, removing the influence of the loading scenario on the yield point. For a description, see [155].

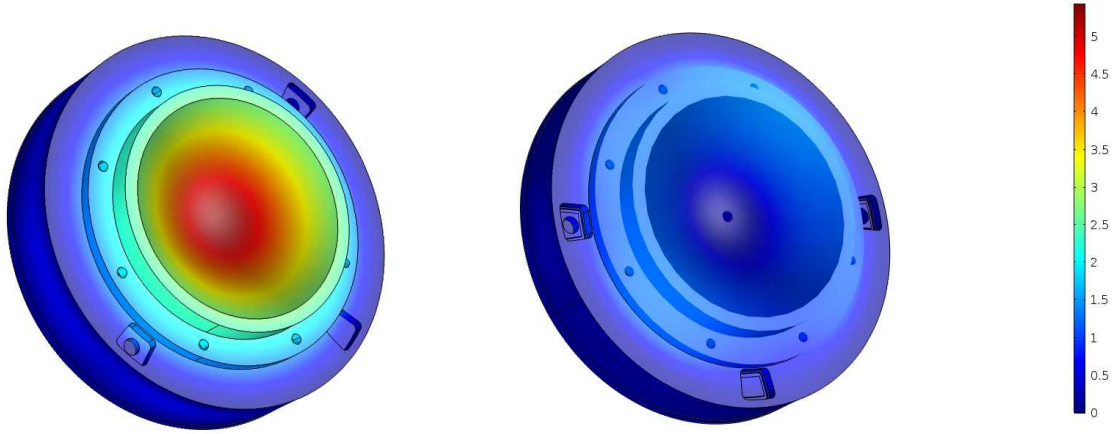


Figure 5.24: FEM simulation results for 1 bar pressure applied at the walls of the coolant baths. Colour bar: deformation (μm).

If we assume this is caused by simple extension of the cavity we would predict $\Delta L = 3.5 \mu\text{m}$, similar to the deformations predicted by the pressure simulations.

Conclusions

In conclusion, the simulation work indicates that resistive heating of the cavity mirrors is most significant at the coupling mirror hole. For fixed temperature boundary conditions, we expect the mirror temperature to stabilise in under a second, and predict that temperature increases at the coupling hole by 29.5°C per kilowatt of input power. With a fixed heat transfer coefficient $h = 247 \text{ W m}^{-1} \text{ K}^{-2}$ at the mirror-coolant bath interface, the temperature measured at the thermocouple position is in reasonable agreement with the simulations. Using this value of h , we predict that the steady-state temperature increase at the coupling hole when $P_{\text{in}} = 1 \text{ kW}$ is around 100°C . When operating with $P_{\text{in}} = 750 \text{ W}$ for 1.5 s , the temperature increase predicted at the coupling hole is 30°C . This shows that water cooling of the mirrors is a viable option when operating the cavity for short periods.

Elastic deformation of the cavity assembly occurs when the mirrors are in vacuum, with the coolant baths are at atmospheric pressure. The expected resonance frequency shift is less than 3 MHz . The von Mises stress is close to that for annealed copper, and was probably exceeded when we initially flowed coolant under pressure through the baths.

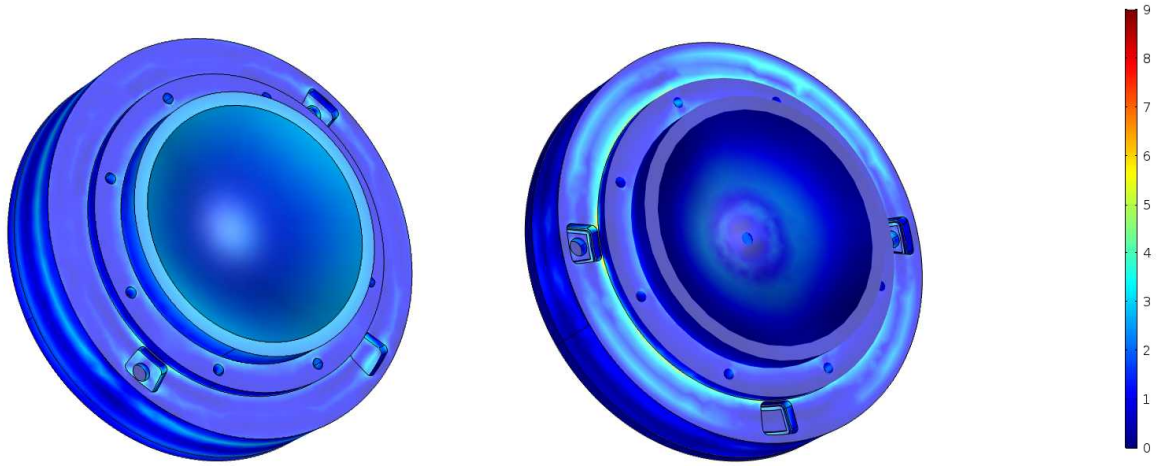


Figure 5.25: FEM simulation results for the von Mises stress with 1 bar pressure applied at the walls of the coolant baths. Colour bar: von Mises stress (MPa).

5.5.2 Frequency Shift at High Power

Since we frequency lock the generator to the cavity resonance, we expect some shift in the microwave source frequency at high input power due to thermal expansion. This frequency shift needs to be tracked faithfully by the feedback loop to prevent the generator unlocking. To investigate this effect, the generator was locked using about 10 W of microwave power, and the power was subsequently ramped up over 10 ms, held constant for 4 s, and then returned to 10 W over 10 ms. During each test, the frequency of the resonance was measured by monitoring the servo loop control voltage and converting this to a frequency shift. Figure 5.26 shows this frequency shift for various input powers. The time constant for stabilisation when switching from low to high power is around 0.5 s, and the behaviour is reversed when the power is switched from high to low. Compared to the FEM simulations, the system takes a factor 2 longer than expected to stabilise; this can be attributed to the idealised assumption of fixed temperature at the coolant walls.

Figure 5.26b shows the steady state resonance frequency for high power testing up to 800 W, measured using an EIP 575 microwave frequency counter. The frequency shifts downwards by 17 MHz, a fractional shift of 1.2 parts per thousand. We were able to pulse the power on for 4 s at up to 1 kW, at which point the frequency shifted outside the uniform gain region of the klystron's amplification band. At this point, it was not possible to maintain

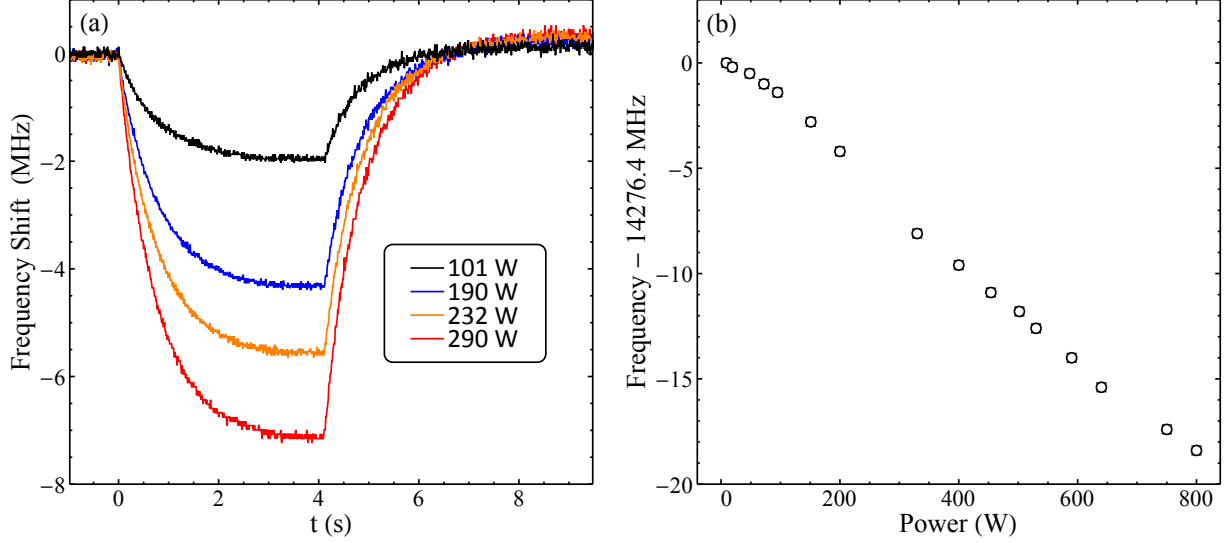


Figure 5.26: The effect of input power on the cavity resonance frequency. (a) Time dependence of the cavity resonance frequency during a 4 s high-power microwave pulse. (b) Steady state frequency shift versus input power.

constant input power to the cavity, and typically there would be $\sim 10\%$ drift over the course of a test. For these tests, the cavity reflectance was 2-3% when measured using the output and reflected power monitors of the klystron, and about 3 – 4% when measured by our own detectors.

The resonance frequency depends on R_m, L, Q_0 and r_h , so to understand the frequency shift we should consider how these parameters might change at high power. Losses in the cavity shift the resonance from the ideal cavity frequency by

$$\delta_Q = \frac{1}{2Q_0} \nu_n \quad . \quad (5.51)$$

At low power we can use the measured Q_0 to calculate $\delta \approx \frac{\nu}{40000} = 0.36$ MHz. This means that in order to explain the observed frequency shift, the quality factor would have to decrease by a factor ~ 50 . Such a large change in Q_0 would significantly alter the impedance of the cavity and we would expect to see a large increase in the reflected power, which we do not see.

A change in the cavity dimensions will affect the resonance both by changing ν_n , and by the geometric frequency shift,

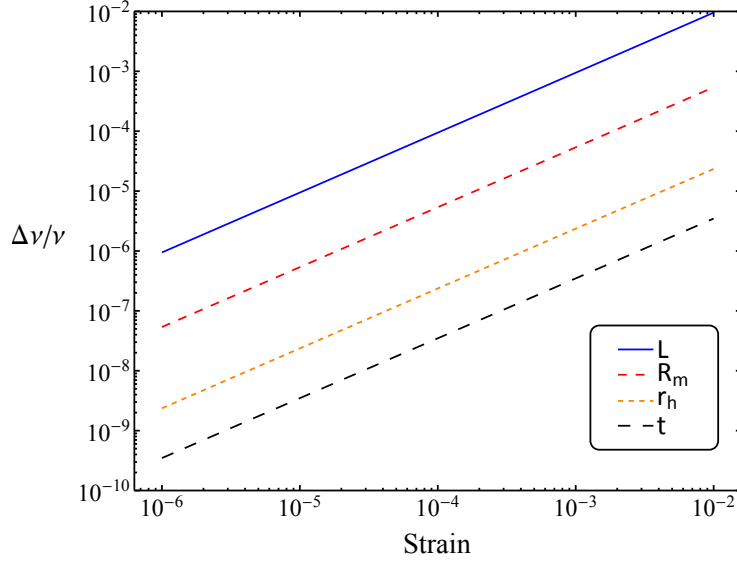


Figure 5.27: Fractional frequency shift, $\frac{\Delta\nu}{\nu}$, against linear strain for the geometric parameters of our cavity. Blue, thick line: cavity length L ; red, dashed line: Mirror radius R_m ; orange, thin line: hole radius, r_h ; thin, black line: hole thickness, t .

$$\delta_g = \frac{8r_h^3 f_t}{3\pi w_m^2 L} \nu_n \quad . \quad (5.52)$$

This frequency shift can change with temperature, because ν_n , L and r_h may all change. We investigate this below.

Figure 5.27 shows the calculated fractional frequency shift against linear strain for the geometric parameters of our cavity. The largest effect comes from the cavity length L , by an order of magnitude. The influence of L is predominantly through changing the ideal cavity frequency ν_n , rather than through the geometric correction factor. To explain our observed frequency shift of 1.2 parts per thousand, we would require a strain in the cavity length of 10^{-3} at 800 W. Taking $18 \times 10^{-6} \text{K}^{-1}$ as the thermal expansivity of copper, we infer a temperature increase of 66 K in the cavity pillars when stabilised. This is far hotter than the temperatures recorded at the thermocouple, or we could expect to be possible given the water cooling parameters.

Another effect is the radiation pressure on the mirrors due the circulating microwave power, which exerts a force $\sim \frac{Q_0 P_{\text{in}}}{c}$. For $P_{\text{in}} = 800 \text{ W}$, the force is 0.05 N, and the stress when loaded on the cavity pillars is 170 Pa. Taking the Young's modulus of copper to be

129.5 GPa, this results in a frequency shift at the 10 Hz level, six orders of magnitude below our range of interest.

Considering all this, we cannot currently explain the large shift in the cavity resonance frequency observed at high microwave power. Whilst the frequency shift is problematic in that it restricts our input power to the cavity, this could be solved by using a klystron with a higher bandwidth. Of greater importance is maintaining a large Q_0 , whose high-power dependence was unknown.

5.5.3 Measurement of the Quality Factor at High Power

Our predictions of the microwave trap potential depth are based on the assumption that the cavity Q_0 at high input power is the same as at low power. We expect the cavity losses to increase at high power due to heating of the mirrors, and so there will be some consequent reduction in the actual trap depth. Measurement of the quality factor by sweeping over the resonance is impractical for two reasons. Firstly, we cannot exceed the 200 W back reflection limit for the klystron amplifier, which restricts us to measuring close to the resonant frequency. Secondly, the resonance position takes a few seconds to stabilise at high power, making it difficult to measure the steady state Q_0 by a frequency scan.

What we would like to have access to is the curvature of the cavity reflectance, $\frac{d^2\mathcal{R}}{d\nu^2}$, since this is both $\sim Q_L^2$ and maximised at resonance. The frequency dithering used for locking effectively Taylor expands $\mathcal{R}$ in powers of $\alpha \sin(\omega t)$; to measure $d^2\mathcal{R}/d\Delta^2$, we just need to select the $\alpha^2 \sin^2(\omega t)$ term, which oscillates at 2ω . This can be done straightforwardly with the lock-in amplifier by demodulating at twice the drive frequency.

If our microwave detectors and the divider circuit were perfectly linear, then a change in $S_{2\omega}$ at fixed modulation amplitude would indicate a change in $\frac{d^2\mathcal{R}}{d\nu^2}$. However, the incident power meter begins to saturate at high input powers, so we must account for this effect. To do this, we replace $\mathcal{R}$ with

$$F(P_0, \mathcal{R}) = \frac{V_z(P_{\mathcal{R}})}{V_x(P_0)} \quad , \quad (5.53)$$

where V_x and V_z are the incident and reflected power detector voltage responses to input

power P_0 and reflected power $P_{\mathcal{R}} = \mathcal{R}P_0$.

The output of the lock-in when in second harmonic mode, $S_{2\omega}$, is then

$$S_{2\omega}(t, \alpha, \Delta) \approx \langle [F(\Delta) + \frac{dF}{d\Delta} \alpha \sin(\omega t) + \frac{1}{2} \frac{d^2 F}{d\Delta^2} \alpha^2 \sin^2(\omega t) + \frac{1}{6} \frac{d^3 F}{d\Delta^3} \alpha^3 \sin^3(\omega t) + \dots] G_{2\omega} \sin(2\omega t + \phi_{2\omega}) \rangle \quad . \quad (5.54)$$

Time averaging now removes all odd derivatives of the expansion, leaving

$$S_{2\omega}(\alpha, \Delta) = \frac{G_{2\omega}}{2} \frac{d^2 F}{d\Delta^2} \alpha^2 \langle \sin^2(\omega t) \sin(2\omega t + \phi_{2\omega}) \rangle + \mathcal{O}(\alpha^4) \quad . \quad (5.55)$$

The optimal value of the phase $\phi_{2\omega}$ is $\pm \frac{\pi}{2}$, resulting in

$$S_{2\omega}(\alpha, \Delta) = \frac{G_{2\omega}}{2} \frac{d^2 F}{d\Delta^2} \alpha^2 \left[\frac{-1}{4} \right] \quad (5.56)$$

Next, we calculate $\frac{d^2 F}{d\Delta^2}$:

$$\frac{d^2 F}{d\Delta^2} = \frac{1}{V_x(P_0)} \frac{d^2 V_z(P_{\mathcal{R}})}{d\Delta^2} \quad , \quad (5.57)$$

where we have made use of the fact that the incident power, and therefore $V_x(P_0)$, is unaffected by the frequency modulation. Using $\frac{dV_z}{d\Delta} = \frac{dP_{\mathcal{R}}}{d\Delta} \frac{dV_z}{dP_{\mathcal{R}}}$, we find

$$\frac{d^2 F}{d\Delta^2} = \frac{1}{V_x(P_0)} \frac{dV_z}{dP_{\mathcal{R}}} \frac{d^2 P_{\mathcal{R}}}{d\Delta^2} = \frac{P_0}{V_x(P_0)} \frac{dV_z}{dP_{\mathcal{R}}} \frac{d^2 \mathcal{R}}{d\Delta^2} \quad , \quad (5.58)$$

where in the last step we used $P_{\mathcal{R}} = \mathcal{R}P_0$, and $\frac{dP_0}{d\Delta} = 0$. The microwave detectors were calibrated against an Agilent U2000A USB power meter, which showed that $\frac{dV_z}{dP_{\mathcal{R}}}$ is constant over our range of input powers, whereas $\frac{P_0}{V_x(P_0)}$ increases with P_0 due to the detector saturation.

From equation 5.34 for the cavity reflectance, $\frac{d^2 \mathcal{R}}{d\Delta^2}$ at resonance is $8Q_L^2/\nu_n^2$. Combining this with equation 5.58 and substituting into 5.56, we find:

$$S_{2\omega} = \frac{P_0}{V_x(P_0)} \frac{dV_z}{dP_{\mathcal{R}}} \frac{G_{2\omega} Q_L^2 T_{\text{res}} \alpha^2}{\nu_n^2} \quad . \quad (5.59)$$

This allows us to measure Q_L with the generator locked to the cavity resonance, without disturbing the locking system. The signal is suppressed by an additional factor α relative

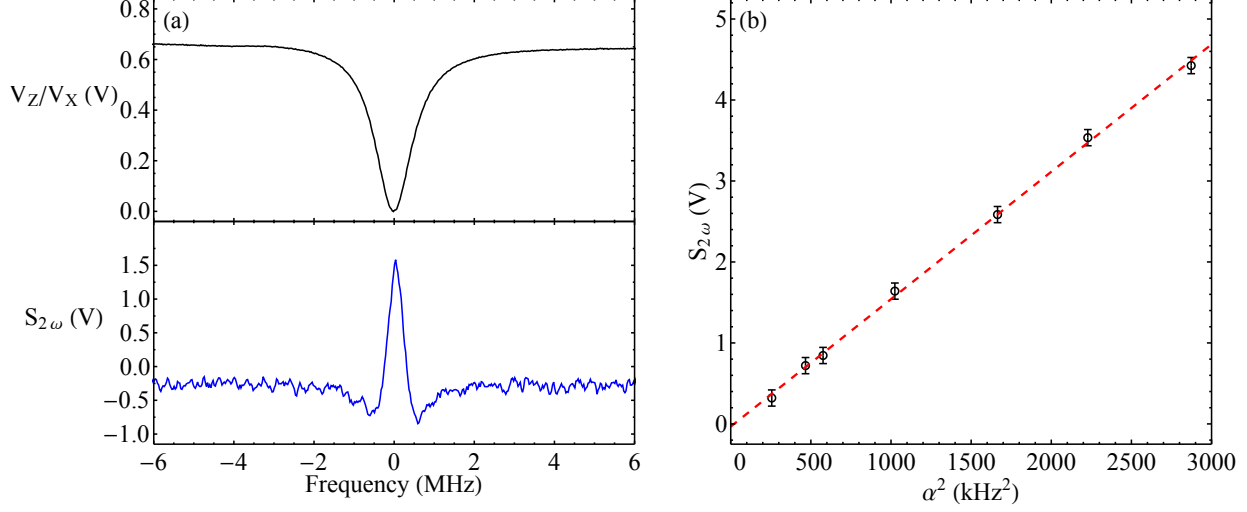


Figure 5.28: Measurement of the cavity quality factor by lock-in detection. (a) Reflected power signal (upper panel) and lock-in amplifier output (lower panel) when scanning over the cavity resonance. (b) $S_{2\omega}$ with the generator locked at the cavity resonance, against the squared modulation amplitude. Dashed line: linear fit to the data.

to the first harmonic, but visible after time-averaging to remove noise. Figure 5.28a shows a scan over the cavity resonance (upper panel) with the second harmonic from the lock-in (lower panel). For these data, the scan rate was 15 MHz/s and the filter time constant on the demodulation output was 1 ms, and the modulation amplitude was $\alpha = 32$ kHz; 60 averages were taken. Figure 5.28b shows $S_{2\omega}$ against the square of the modulation amplitude, using an input power of 67 W, showing the expected linear relationship.

Figure 5.29a shows measurements of $S_{2\omega}$ during 4.5 s long high power pulses, switched from 10 W up to various test powers over 50 ms. The signal increases with input power due to the forward power detector saturating, but remains stable within about 10% during each test. The cavity reflectance is shown as a function of input power in figure 5.29b; $\mathcal{R}$ remains roughly constant, at around 5%, and these values are roughly twice those reported by the in-built reflection monitor of the klystron. Figure 5.29c plots $S_{2\omega}$ against power. For these data, the second harmonic gain $G_{2\omega}$ was measured as 3.16 V/mV, by calibration with a sinusoidal input; the dithering frequency was 22 kHz ($2\omega = 2\pi \times 44 \times 10^3 \text{ s}^{-1}$), and the dithering amplitude applied to the generator was 34.4 kHz. The filter time constant at demodulator output was 30 ms, and the demodulation phase $\phi_{2\omega}$ was optimised for maximum signal. The solid line shows the prediction of equation 5.59 after calibrating the analog divider circuit

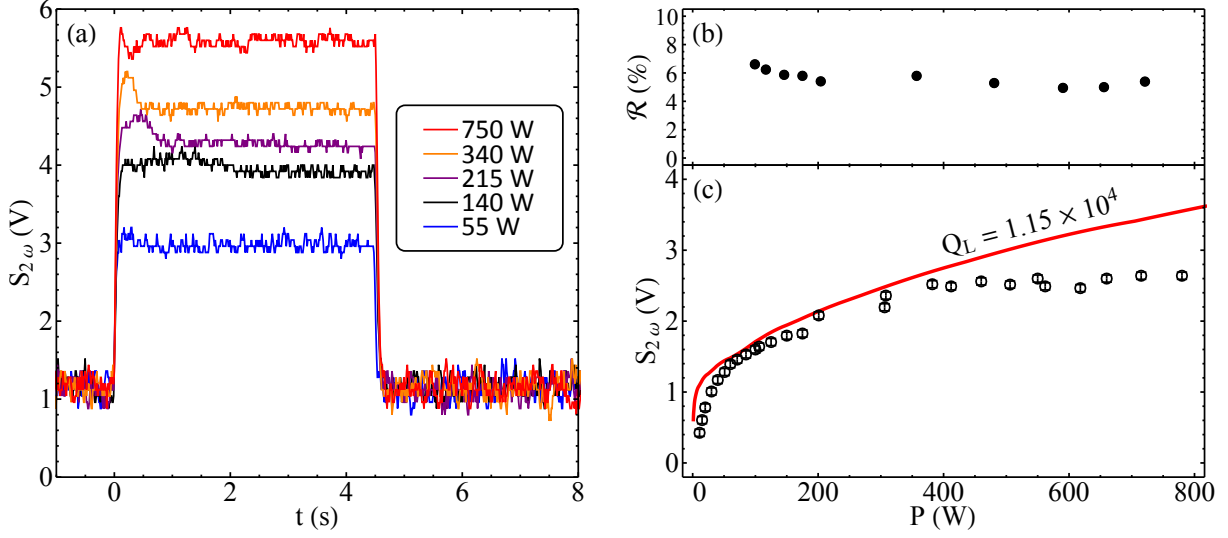


Figure 5.29: (a) $S_{2\omega}$ versus time for high power microwave tests, using 4.5 s pulses applied at $t = 0$. (b) Cavity reflectance during high power tests. (c) Average $S_{2\omega}$ between $t = 2$ s and $t = 4$ s against input power. The line shows the predicted curve from equation 5.59 assuming $Q_L = 1.15 \times 10^4$ and $T_{\text{res}} = 0.95$.

and lock-in amplifier, with $T_{\text{res}} = 0.95$ and $Q_L = 1.15 \times 10^4$, the values measured at low power. It is clear the experimental data are consistently below the predicted values, but at high power the disagreement is larger which suggests the cavity properties are changing. However, it is difficult to use the data at low power to infer the quantitative change because the data clearly disagrees with the model here as well. After unsuccessfully searching for a source of calibration error in the measurements, I decided on a different approach. Instead, I decided to measure $Q(P_0)$ and $Q(P_0/10)$, and for the latter measurement remove the 10 dB microwave attenuators protecting the detectors. This allows us to change the cavity input power by a factor 10 whilst still receiving the same power at the detection system. In this way we can isolate the change in $S_{2\omega}$ caused by power dependence of Q_L .

Figure 5.30 plots $S_{2\omega}$ against the forward detector power for the two cases above, over the range $10 < P_0 < 1000$ W. We can see that at low power, the two curves overlap, but they begin to deviate at $P_X = 2$ mW, equivalent to $P_0 = 200$ W. From this, we can conclude that the quality factor of the cavity is constant with power for $P_0 < 200$ W. Therefore we can compare the data at $P_{\text{max}} = 1$ kW and 100 W to estimate the relative change in the Q -factor. Since $S_{2\omega} \sim Q_0^2$, we find that $\frac{Q_0(1 \text{ kW})}{Q_0(100 \text{ W})} = 90(2)\%$, which means that the trap depth

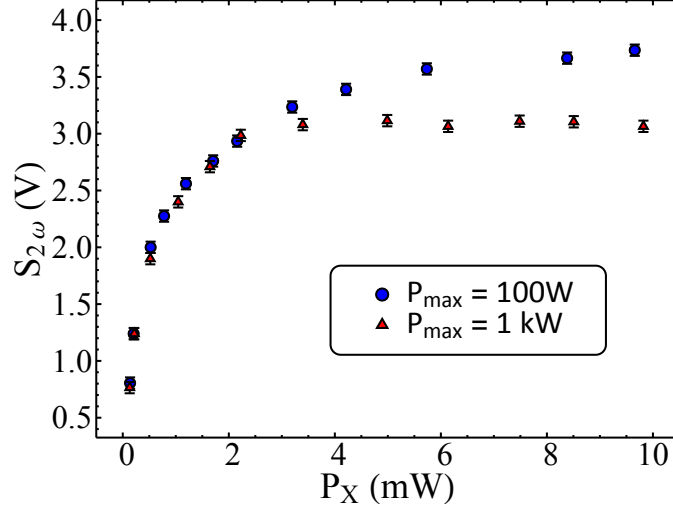


Figure 5.30: $S_{2\omega}$ versus the detector forward power, with and without 10 dB attenuation applied to the detection system.

is reduced by 10% at high power.

5.5.4 Attaining Ultra-high Vacuum

The nature of our experiment places high requirement on the vacuum quality, as we expect sympathetic/evaporative cooling to require several seconds of trapping time. In terms of attaining UHV, there are three clear weakpoints in the design. First, a thin, epoxy-sealed, quartz window, seals the vacuum at the end of the microwave waveguide. The window can tolerate up to 2 kW of CW input power; the microwave reflectance quoted by the manufacturer is less than 0.25%, and I measured around a factor ten smaller. However, it was found to be slightly porous, and, without some remedy, this limits the attainable vacuum. To prevent this from degrading the vacuum quality, a short (10 cm) waveguide section was added directly before the cavity input. The pressure in this section was maintained at $\sim 3 \times 10^{-3}$ mbar by a separate pump, suppressing leakage through window at the UHV side to safe levels. Secondly, the microwave window is sealed to the waveguide by an o-ring. It is the only o-ring in the entire vacuum system, so potentially limits the attainable vacuum. Finally, the welded connection between the waveguide and the top flange of the vacuum chamber is potentially a weak point where leaks can open when the system is heated or cooled. These weakpoints, along with the waveguide section under rough vacuum, are illustrated in figure 5.31.

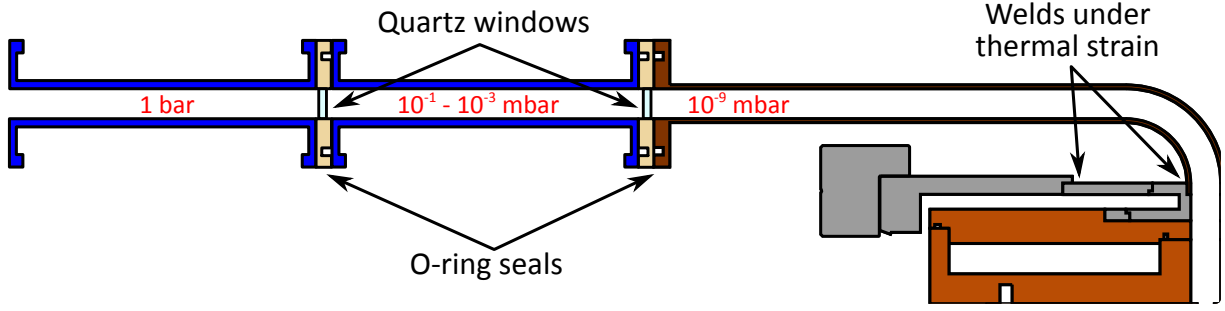


Figure 5.31: Diagram showing the weakpoints for attaining UHV, and the waveguide assembly used suppress gas leakage through the quartz windows.

Attaining sufficient vacuum quality requires that the pumping speed s of our system at our intended pressure p_{exp} is sufficient to balance the combined rate of leaks through the various seals and any outgassing from materials inside the chamber, Φ :

$$p_{\text{exp}}s = \Phi \quad . \quad (5.60)$$

We first proceeded with a non-evaporable getter pump (SAES CapactiTorrr Z200) to maintain vacuum in this section of the experiment, which has an (initial) pumping speed for nitrogen of $s = 70 \text{ l s}^{-1}$. The choice of a passive pump was made with the aim of minimising any effects of the microwave power on the vacuum system; previous transport experiments had already demonstrated a lifetime of several seconds in the magnetic trap, using a similar pump [84]. We opted to use a Viton o-ring for the waveguide window seal, and chose the BS119 standard size (ID 23.47 mm, circular cross-section thickness 2.62 mm). Studies of the outgassing rate completed by the manufacturer (Polymax) suggested we should expect an outgassing rate from the o-ring of around $2.5 \times 10^{-6} \text{ mbar l s}^{-1}$ prior to bakeout [156].

The cavity assembly was sonicated with Decon 90 detergent (1vol%) at 60°C for 40 minutes, and subsequently in deionised water for a further 40 minutes. After drying in an oven overnight at 100°C , we cleaned all accessible areas thoroughly by hand with acetone, methanol and isopropanol sequentially. The assembly was left to dry overnight at 80°C .

After fully assembling the vacuum chamber, we measured the leak rate with a helium leak detector and found a maximum of $\sim 1 \times 10^{-10} \text{ mbar l s}^{-1}$ around the waveguide window flange. This leak rate is comfortably within our pumping capabilities. Prior to bakeout, we typically measured a pressure of $2.5 \times 10^{-7} \text{ mbar}$ when using a turbomolecular pump with a pumping

speed of 70 l s^{-1} . Following activation of the getter pump, this fell to $7.5 \times 10^{-8}\text{ mbar}$. This is roughly consistent with the expected limit due to the Viton outgassing. We intended to try other sealing options (Kalrez 8900 polymer o-rings, or Helicoflex metallic spring o-rings) to ascertain which was best, but found Viton was sufficient for our purposes and decided it was not worth testing further due to time constraints.

When baking the chamber, we ensured the temperature was nowhere greater than 200°C , the maximum tolerable by the quartz viewports and microwave windows; the o-ring was kept below 180°C , and the vacuum gauge was held below 150°C . We initially baked for 1 week at $120\text{--}130^\circ\text{C}$, and found the pressure after cooling down reached $3.5 \times 10^{-8}\text{ mbar}$. I found that it was never possible to maintain a pressure less than 10^{-6} mbar with the getter pump alone, whereas we were able to reach $5 \times 10^{-9}\text{ mbar}$ with both the getter and turbo pumps. The cause of this is most likely permeability of the o-ring to non-reactive gases (e.g. Argon) which are not pumped effectively by the getter pump. This permeability is not present with CF seals and therefore was not a problem encountered with the previous transport experiments in [84]. To effectively pump all species leaking into the system, I replaced the getter pump with a combined ion and getter unit (SAES NexTorr100-5). I found it was possible to maintain $3 \times 10^{-7}\text{ mbar}$ with both pumping elements active prior to bakeout, similar to that attained with the turbo pump alone. With the ion pump installed, the pressure after bakeout was typically $1 \times 10^{-9}\text{ mbar}$.

When making the microwave window seal, we had to balance the need for good electrical contact between the waveguide flanges with compression of the o-ring into the seal groove. To rule out permeability of the o-rings as a limit on the pressure, we baked the system with even thicker o-rings, to maximise compression of the o-ring. I found that with sufficiently thick o-rings it was possible to reach $6 \times 10^{-10}\text{ mbar}$ following bakeout, unfortunately at the expense of efficient coupling into the cavity, which relies on good electrical contact between the waveguide flanges. This was ultimately what limited the pressure attained, rather than leakage through the microwave window. In the end, we returned to the original BS119 Viton o-rings and operated with $1 \times 10^{-9}\text{ mbar}$.

Conditioning the Cavity with Microwave Power

Once the chamber had reached the UHV regime, the effects of microwave power became far more pronounced. I found that the pressure was prone to spiking rapidly (over seconds) by orders of magnitude when coupling in greater than 50 W, but that over time these bursts reduced significantly. Therefore we continued to pump on the chamber with the turbomolecular pump in an attempt to remove these contaminants from the experiment. There are two likely explanations for this behaviour: microwave induced electrical breakdown, or microwave activated outgassing from absorbing materials in the chamber. Given that we expect only a small percentage of the circulating power to leak out from the cavity, it seems most likely that any outgassing is caused by localised heating of the region around the coupling hole, where the heat load is concentrated. The only non-metallic components near the microwave cavity are some ceramic beads used to electrically isolate the wires of the thermocouple, and some silicon carbide plates at the edges of the cavity which are designed to safely absorb microwave power that leaks out of the cavity. There was no sign of any damage to either, and therefore it does not seem likely that anything else in the chamber was being heated excessively.

I could not find any studies related to microwave breakdown at UHV in the literature. A study by Platzman and Solt [157] observed breakdown in 8.85 GHz rectangular box cavities for pressures in the range 1 mbar to 1 bar. They observed minimum breakdown fields of around 0.1 kV cm^{-1} , identifying the breakdown threshold by a sudden increase in the power reflection. I did not observe these increases in our experiment. In [158], breakdown was studied theoretically for a 10 GHz spherical cavity, and the authors predict a minimum breakdown threshold of about 0.7 kV cm^{-1} at a pressure $\sim 10 \text{ mbar}$. Similar breakdown fields are calculated for the TE_{10} mode of rectangular waveguide at 10 GHz in [159]. I verified that breakdown was not present in the waveguide roughing section, by deliberately increasing the pressure in this section to $1 \times 10^{-1} \text{ mbar}$ with a less effective pump. This had no significant effect on the observed behaviour.

Initially I operated the klystron with continuous power output, and found that with 140 W the pressure rose to around $2 \times 10^{-7} \text{ mbar}$ over around 20 minutes, before falling. Whilst

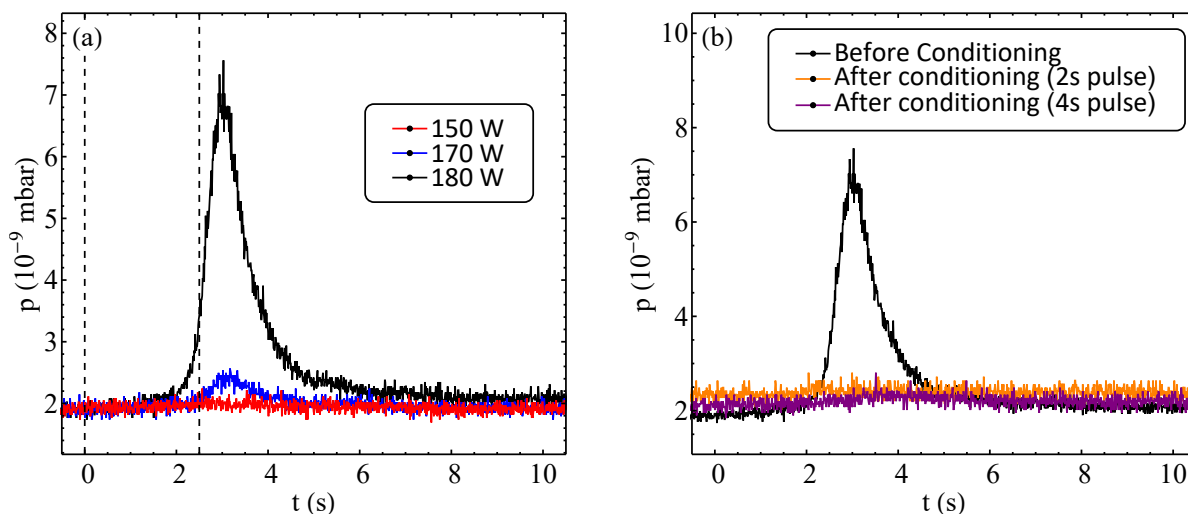


Figure 5.32: (a) Pressure in the microwave chamber before conditioning, using 2.5 s pulses applied from $t = 0$. Dashed lines indicate the duration of the pulses. (b) Comparison of the pressure in the microwave chamber with $P = 180$ W, before and after the conditioning procedure.

the temperature at the thermocouple increased linearly with power, the pressure grew more rapidly, and soon it became inoperable to run the microwaves continuously. Both required around 20 minutes to stabilise, which is much longer than the duration of an experiment. Therefore it made sense to determine the limits when pulsing the power for a few seconds.

Figure 5.32a demonstrates the surprising sensitivity of the pressure to the power input, using 2.5 s pulses of microwave power separated by 20 s: increasing the power from 170 W to 180 W causes the pressure spike to increase by about an order of magnitude. I found that this behaviour was independent of the ion pump operation, and so it appears to be unrelated to its inclusion in the setup.

The behaviour of the pressure under pulsed testing also showed improvement over time, and so with care I was able to increase the pulse power and duration. I found a safe approach was to fix the pulse power and increase the pulse duration in 0.1 s increments, ensuring that the pressure transients were below 10^{-7} mbar. Once the pulse length reached around 4 s, the power was increased by around 5% and the process repeated. After conditioning with 400 W, 4 s pulses, the response at 180 W was retested. Figure 5.32b compares the pressure recorded before and after the pulsed conditioning, demonstrating that even with a 4 s pulse, the pressure transient is considerably reduced.

I found that the improvements gained by pulsed conditioning started to drop as $P >$

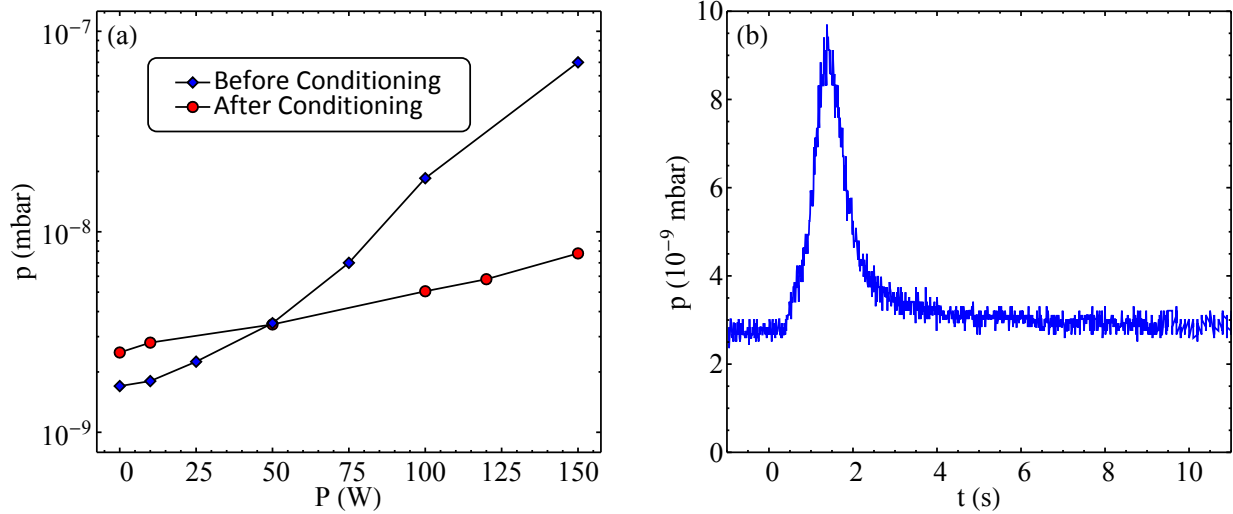


Figure 5.33: (a) Comparison of the steady state pressure with continuous microwave power fed to the cavity. (b) Pressure during a 850 W 2 s pulse applied from $t = 0$.

700 W. Figure 5.33a shows a comparison of the steady state pressure measured before the pulsed conditioning, for $0 < P < 150$ W. The steady state pressure at $P = 150$ W is improved by an order magnitude following the conditioning phase; the increased pressure as $P \rightarrow 0$ after conditioning was the result of a campus power outage which slightly degraded our vacuum quality.

In summary, it seems that the microwave power activates outgassing which is not addressed by the bakeout procedure, but that this outgassing can be improved by carefully conditioning the cavity with pulses. It would be useful in future to analyse the gas products released by the microwaves, determine the source and consider how to improve this issue.

External Cavity Coupling

With the microwave chamber at UHV, an unexpected observation appeared after opening the gate valve to the main experiment. I found that slight adjustment of the gate valve screw measurably affected the coupling into the cavity, and the on-resonance reflection could change as much as 2%. With the sweep oscillator locked to the resonance and the klystron output $P = 130$ W, I was able to shift the cavity resonance by 0.3 MHz by adjusting the screw. The only plausible explanation for this effect is that the cavity mode couples to the surrounding environment, and that its effective impedance is “tuned” by rotating the valve

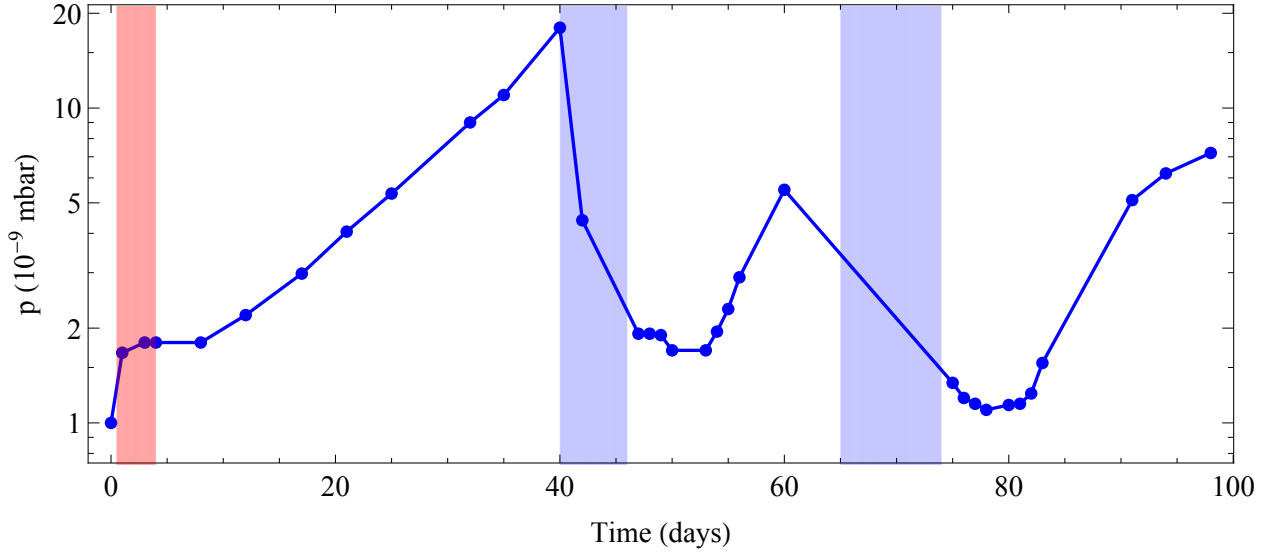


Figure 5.34: Long term pressure behaviour of the microwave chamber. The red shaded region indicates the pulsed conditioning tests, and the blue shading shows bakeout periods.

screw. This suggests that power leaking from the cavity may be larger than a diffraction loss estimate would suggest, and may explain the observed outgassing behaviour. About 0.05 MHz of the observed frequency shift can be accounted by the power-dependence of the resonance frequency discussed in section 5.5.2, since if the cavity coupling changes by 2% then so does the circulating power. This leaves 0.25 MHz unaccounted for. In practice, I found that it was possible to minimise reflection from the cavity without blocking the atomic cloud during the transport stage, and so proceeded with this strategy when loading the trap.

Long Term Pressure Behaviour

The steady state pressure in the cavity chamber after bakeout, whilst initially stable following the bakeout, consistently began to increase around a week later. This limited the available time in which to run experiments. Figure 5.34 shows the pressure readings over a period of three months, in which a series of bakeouts were completed, each time returning the cavity to a similar pressure. Reducing the pressure in the waveguide section a rough vacuum by a factor 10 did not affect this behaviour, meaning it is more likely to be the result of a small leak on the o-ring seal.

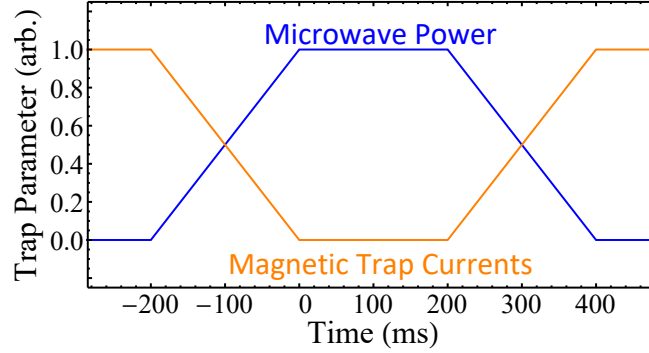


Figure 5.35: Procedure for loading the microwave trap from the magnetic trap.

5.6 Loading the Trap with ^7Li

5.6.1 First Loading Attempts

To load the microwave trap, we followed the scheme illustrated in figure 5.35. During the MOT, molasses and transport stages, the microwave power to the cavity was fixed at 10 W. The resulting trap potential is too weak to support the cloud against gravity, and therefore the trap is effectively switched off. A 100 ms hold time followed the transport phase, in order to provide some tolerance on the timing of the stage motion. Next, the microwave power was linearly ramped up to the trapping power over 200 ms, and the current in the transport coil was ramped to zero. After a variable hold time in the microwave trap, the transfer ramp was applied in reverse and the remaining atoms were then returned to the magnetic trap.

Initially the field of view was so restricted by the cutoff attenuators that it was not possible to observe the cloud directly in the microwave trap, due to a slight misalignment between the cavity centre and the transport tube. However, recovery of the atomic cloud to the magnetic trap after several hundred milliseconds in the microwave trap made us confident that the trap was indeed working. To extend the field of view in the absorption images, I removed the relevant attenuator from inside the microwave chamber, and mounted a similar attenuator outside the chamber. This could be adjusted whilst the system was under vacuum, and allowed direct observation of the atomic cloud in the trap.

Figure 5.36a shows a sequence of images obtained with a 200 ms hold time in the microwave trap, using a trapping power of 609 W. The sharp edge at the top of each image

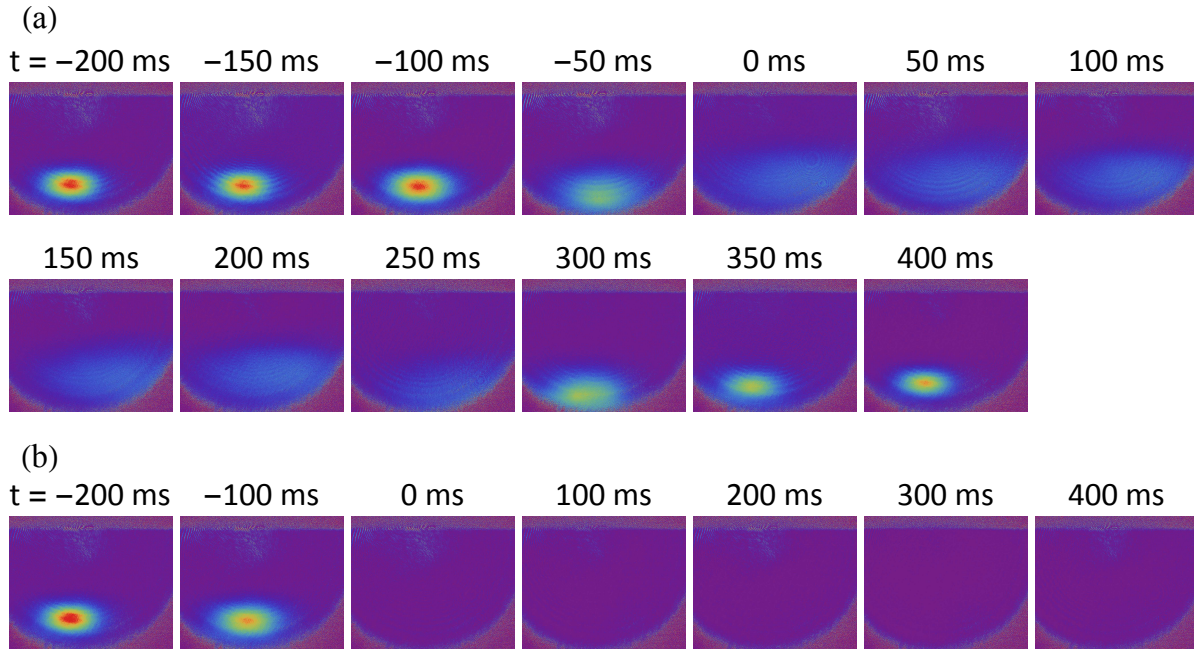


Figure 5.36: Loading the microwave trap. (a) Releasing from the magnetic trap into the microwave trap ($-200 < t < 0$ ms), and transfer back to the magnetic trap ($200 < t < 400$ ms). (b) Repeating the experiment with the microwave power off.

marks the edge of the top cavity mirror, and the sharp curve at the bottom of the image is due to the edge of the transport tube. The cloud expands as the magnetic trap is lowered, and its edges move slightly out of the observation region. However, the central part of the cloud remains visible, and it was possible to fit the line density profiles to find the cloud centre coordinates and its width. We recover a clear signal on reloading the magnetic trap from the microwave trap. For comparison, figure 5.36b shows the images obtained with the microwave power off for the duration of the experiment. It is clear that we do not recover a signal in the magnetic trap in this case, verifying confinement by the microwave trap.

Figure 5.37 shows the percentage of atoms recaptured into the magnetic trap versus trapping power. The recapture fraction saturates at an input power around 500 W, reaching 80%. If the atomic spins depolarize amongst the $F = 2$ sublevels when released into the microwave trap, then we would expect to recover at most 40%. From this we can conclude that the spin polarisation is reasonably well preserved during the transfer process, most likely due to the presence of a background magnetic field. To fit to the data, we assume that at

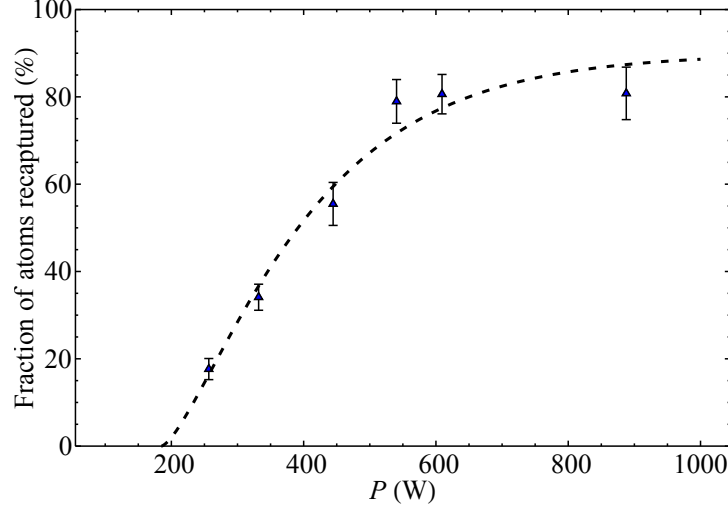


Figure 5.37: Percentage of atoms recaptured to the magnetic trap versus trapping power. The dashed line is a fit to equation 5.62 with $\beta = 0.28 \mu\text{K W}^{-1}$.

$t = 0$ the trap is loaded with an initial energy distribution, $f(E)$, given by

$$f(E) = \frac{2\sqrt{E}}{\sqrt{\pi}(k_B T)^{\frac{3}{2}}} \exp\left(-\frac{E}{k_B T}\right) . \quad (5.61)$$

The fraction of atoms loaded into the microwave trap is modelled as the integral of this energy distribution up to the effective trap depth $U_{\text{eff}}(P)$:

$$\eta = \eta_{\text{max}} \int_0^{U_{\text{eff}}(P)} f(E) dE . \quad (5.62)$$

Here, η_{max} is a fit parameter to account for any unknown loss during the loading process. The value of U_{eff} depends both on the gravitational sag and the offset between the centres of the magnetic and microwave traps, as illustrated in figure 5.38. Nevertheless, I find that U_{eff} is well-approximated by $\beta(P - P_0)$, where $\beta = 0.28 \mu\text{K W}^{-1}$ is calculated from equation 5.37. The threshold power for trapping, P_0 , depends on the offset between the trap centres. The dashed line in figure 5.37 shows a fit of the recapture data to equation 5.62, from which the fitted values of T and P_0 are $44(12) \mu\text{K}$ and $180(20) \text{ W}$ respectively. The value of P_0 is larger than our prediction of 110 W . This could be explained by an underestimate of the quality factor, but this explanation is not supported by our trap frequency measurements presented later. At low power, gravity shifts the trap centre shifts further from centre of the cavity, reducing U_{eff} . This could explain the higher than expected threshold for trapping.

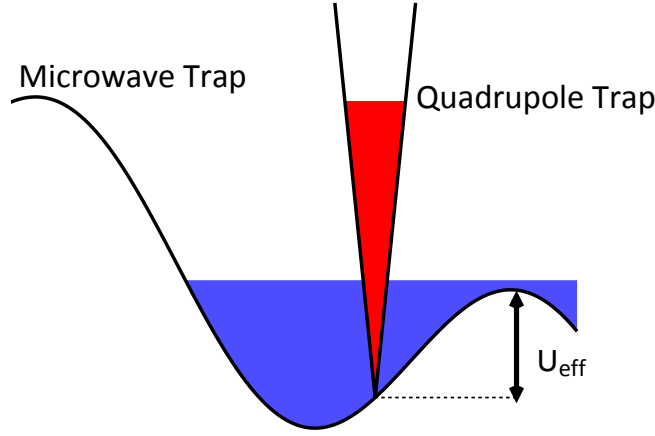


Figure 5.38: The microwave trap and quadrupole trap axial potentials.

The measured threshold power is consistent with loading the trap about 1 mm away from the centre along z .

The recapture experiments were repeated for different initial positions of the magnetic trap, in order to optimise the loading process. The location of the cloud along the transport direction was adjusted by varying the distance travelled by the translation stage. To vary the trap centre position along the axis of the cavity, the trapping coil currents were adjusted prior to release. Figure 5.39 shows the data obtained; the loading efficiency is clearly more sensitive to the axial cloud position, which is expected because the trap is tighter along this direction. It was not possible to scan further along the transport direction without relocating the translation stage, and we decided against this step as there was a risk of reducing the transport efficiency.

5.6.2 Trap Oscillations

If the trap transfer is fast in comparison to the periods of motion in the microwave trap, we violate the condition of adiabaticity and the density profile of the cloud evolves in time. The shift in the trap centre causes motion of the cloud's centre-of-mass, and the change in the effective trap volume excites oscillations in its width. These are commonly referred to as the centre-of-mass and breathing modes respectively.

To excite these motions, the magnetic trap was switched off as quickly as possible, and

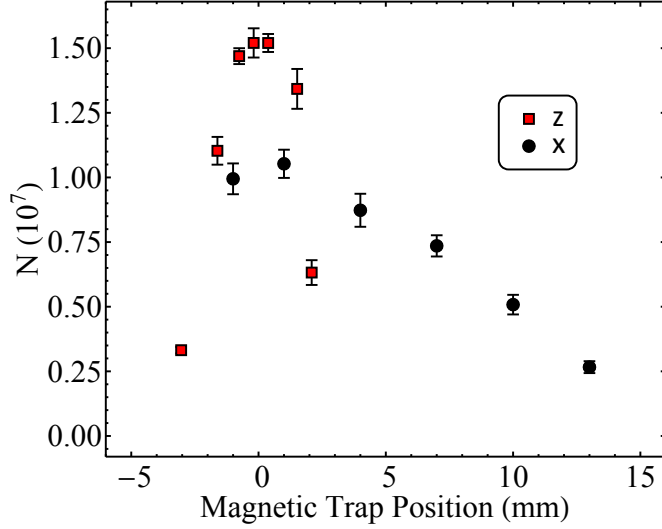


Figure 5.39: Number of recaptured atoms versus position of the magnetic trap.

the cloud imaged directly in the microwave trap. Eddy currents induced in the cavity by the external coils provide a lower limit to the switching time of about 40 ms. The current ramp was chosen to coincide with the end of the 200 ms switch on ramp, and a further 50 ms was allowed to be sure Eddy currents had dissipated. Absorption images were taken every 5 ms for $50 < t < 200$ ms, and the centre of mass and width of the cloud extracted.

Figure 5.40 shows the centre of mass oscillations for three different trapping powers. The axial motion settles to slowly decaying oscillations, and several oscillations are visible within the imaging time. The radial motion is slower as expected, and the cloud executes a little more than one oscillation. To estimate the oscillation frequencies, the function $A \sin(\Omega t + \phi)$ was used to fit the data from $t = 50$ ms onwards. Apart from a slow decay in the oscillation amplitudes, the data fit well to this model.

The trap frequencies should be proportional to the electric field at the mode centre, so that we expect $\Omega_{x,y,z} \sim \mathcal{E}_{\text{peak}} \sim \sqrt{P_{\text{in}} Q_0}$. Figure 5.41a shows the data obtained, along with the model predictions. The error bars on the electric field estimates reflect the systematic uncertainty in the absolute power circulating in the cavity, and we have assumed 5% power loss along the waveguide, $\mathcal{R} = 5(3)\%$ and $Q_0 = 2.0(1) \times 10^4$. The measured frequencies are in agreement with the theory, but systematically a little higher, implying the electric field is slightly greater than the estimated values.

A more convenient test of the theory is the ratio of the trapping frequencies Ω_z/Ω_y ,

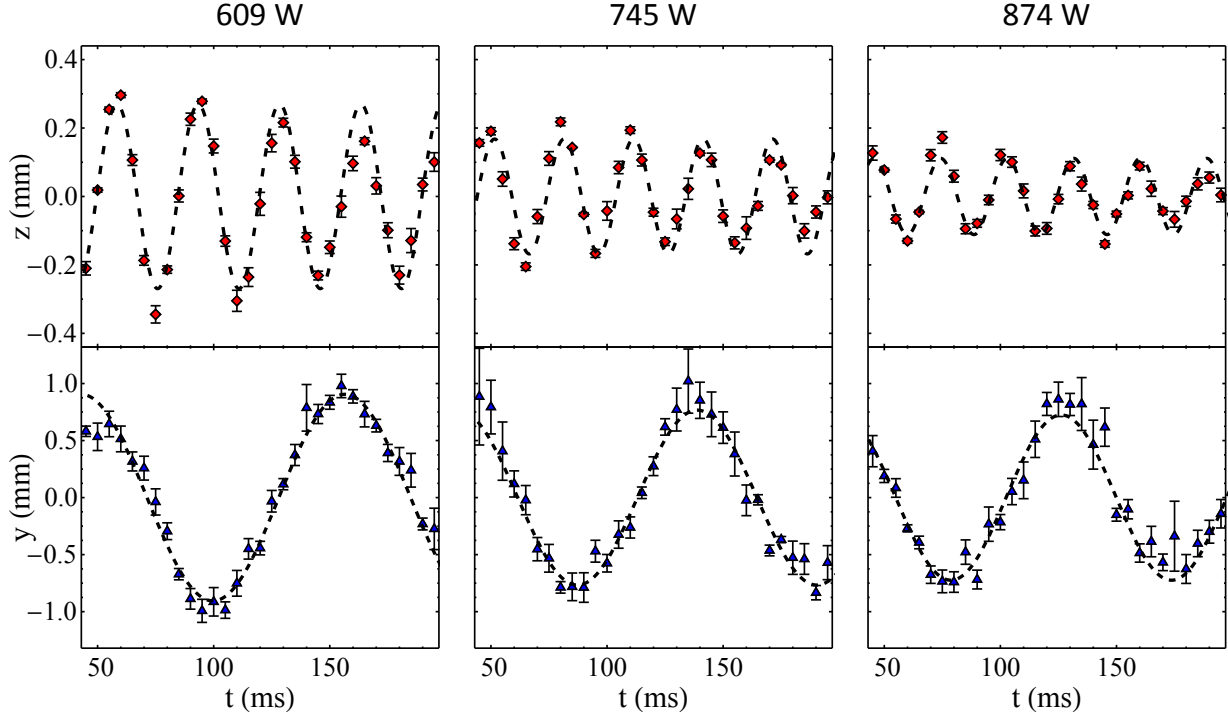


Figure 5.40: Centre-of-mass oscillations in the microwave trap, at three different input powers. Top row: axial oscillations. Bottom row: radial oscillations. Error bars are the uncertainties obtained from fitting Gaussian profiles to five images. Dashed lines are fits to $A + B \sin(\Omega t + \phi)$.

which depends on the shape of the mode and not on the absolute electric field strength. Figure 5.41b shows the ratio at each input power, alongside the prediction from equation 5.42. There is reasonable agreement between data and theory, and the average of our three measurements gives $\frac{\Omega_z}{\Omega_y} = 3.33(6)$, consistent with the predicted value of 3.28.

5.6.3 Temperature and Phase-Space Density

The measured trap frequencies allow us to predict the equilibrium size of our atomic cloud, from equation 5.41. If we assume an initial temperature $T_i = 130 \mu\text{K}$, we would expect $\sigma_x = 7.1 \text{ mm}$ and $\sigma_z = 2.2 \text{ mm}$. Even when accounting for the breathing motion, the measured widths are more than a factor 2 smaller, $\sigma_x = 2.7(3) \text{ mm}$ and $\sigma_z = 1.06(8) \text{ mm}$. The geometric mean temperature from these values is $22(3) \mu\text{K}$ and the phase-space density with 1.8×10^7 atoms is $4(1) \times 10^{-7}$. We see that the temperature falls as the cloud expands into the microwave trap, which is what we expect if the phase-space density is conserved. The phase-space density we measure here is indeed consistent with the one in the magnetic trap,

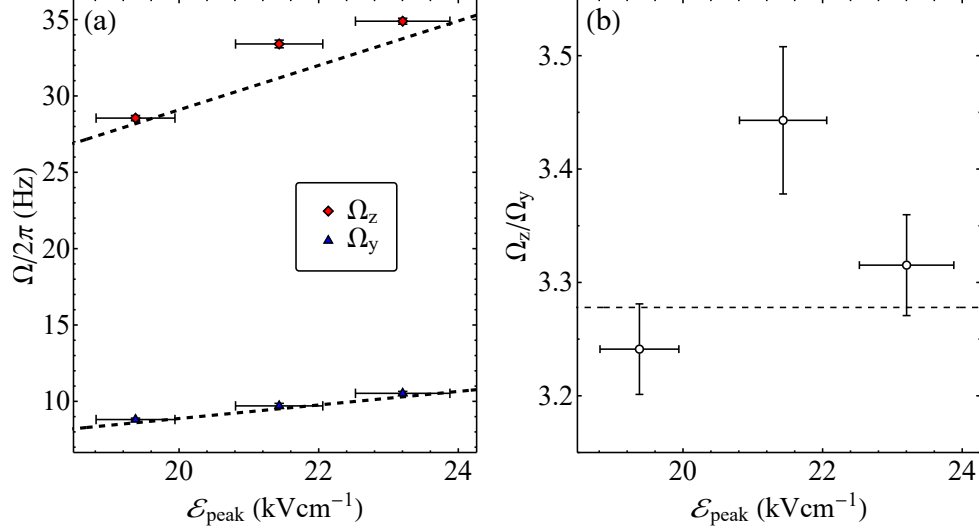


Figure 5.41: Trap oscillation frequencies versus the peak electric field. (a) Measured frequencies along the z - and y - directions. Dashed lines show the predictions of equations 5.39a and 5.39b. (b) Ratios of the trap frequencies in (a). The dashed line is the prediction from equation 5.41.

which we estimated in section 4.6.3 to be $3.58(15) \times 10^{-7}$.

5.6.4 Lifetime in the Trap

Figure 5.42 shows the number of atoms returned to the magnetic trap against hold time in the magnetic trap. For $0 < t < 1.3$ s, the decrease in signal follows an exponential decay with $1/e$ lifetime 1.76(12) s. After correcting for the higher pressure in the microwave trap experiments, the loss rate is consistent with the measurements in the magnetic trap, implying that there are no extra loss mechanisms beyond collisions with residual background gas. From $t = 1.5$ s onwards the pressure in the chamber rises sharply, as shown by the inset to figure 5.42. This coincides with an abrupt increase in the loss rate. At present, this behaviour of the pressure is the main limitation to the lifetime in the trap.

5.7 Conclusions

This chapter has explained the motivation for building our microwave trap, and demonstrated its operation with ultracold lithium. There were a number of technical challenges to overcome to achieve this, in particular verifying the cavity properties at high power, and maintaining

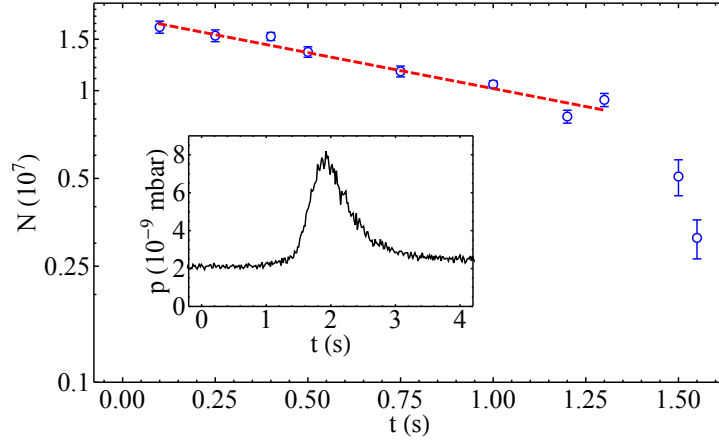


Figure 5.42: Number of atoms recaptured into the magnetic trap versus hold time in the microwave trap. Inset: typical pressure trace for an experiment using a 1.55 s hold time.

sufficient vacuum quality to test loading of the trap. The frequency of the cavity resonance drifts by 29 MHz/kW for input powers greater than 50 W, which we cannot account for in terms of the expected thermal strain. However, the quality factor at 1 kW is 90(2)% of the value at low power, and the potential depths we can generate are sufficient to load lithium directly from the magnetic trap.

Simulations of the cavity under thermal loading indicated that the temperature increase is concentrated at the mirror hole, and that water cooling the mirrors is sufficient to run the trap at high power for short periods < 10 s. This allowed us to sidestep the risks of cryogenically cooling the assembly. To achieve ultra-high vacuum, it was necessary to condition the cavity with microwave pulses after bakeout, and we can operate the trap with the pressure below 3×10^{-9} mbar for 1.5 s. This relatively short operation time is the most important issue to resolve for future use of the trap, and if liquid nitrogen cooling proves viable then the strong cryo-pumping effect should provide a significant gain here.

The $1/e$ lifetime in the microwave trap was measured as 1.76(12) s, consistent with the loss rate being dominated by the background gas pressure. The motional oscillation frequencies in the trap are within 10% of the theory predictions, and the axial to radial frequency ratio was measured as 3.33(6). This is consistent with the theoretical value of 3.28. From the width of the cloud and its oscillations, we can deduce the temperature and phase-space density. The phase-space density in the trap is $4(1) \times 10^{-7}$, and consistent with the estimated value from the transport experiments.

Chapter 6

Summary and Outlook

In this chapter, I summarize the work of this thesis thus far, and consider trapping molecules in our microwave cavity.

6.1 Outlook: Trapping Molecules

6.1.1 The AC Stark Shift for Polar Molecules

To illustrate the confinement possible for a polar molecule in the microwave trap, I shall follow the method of DeMille [81]. I apply this analysis to the low lying rotational levels of CaF in its vibrational ground state. Table 6.1 below gives the relevant molecular data.

We consider the molecule as a rigid rotor with electric dipole moment μ , and apply the dressed state formalism. The basis of states $\{|\psi\rangle\}$ are labelled as $|J, m_J, N\rangle$, which are the eigenstates of the system when there is no dipole interaction. The labels J , m_J are the quantum numbers for the rotational angular momentum of the molecule. The cavity photon number, N , is measured relative to the mean photon number without the molecules present. The Hamiltonian for our system is

$$\mathcal{H} = B_{\text{rot}} \hat{J}^2 + \hbar\omega \hat{N} - \mu \boldsymbol{\mathcal{E}} \cdot \boldsymbol{n} \quad . \quad (6.1)$$

Here, B_{rot} is the rotational constant for the molecule, ω is the photon angular frequency, μ is the molecular dipole moment, $\boldsymbol{\mathcal{E}}$ is the electric field vector, and $\boldsymbol{n}$ is a unit vector along

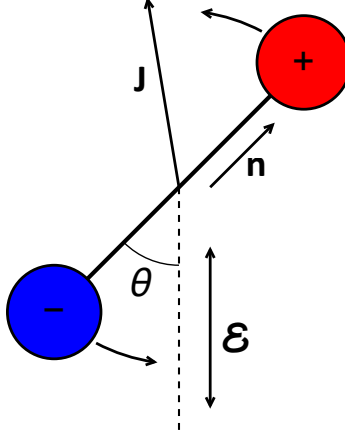


Figure 6.1: A polar molecule in a microwave field.

Property	Value	Source
Internuclear Distance	1.967 Å	[160]
Molecular Dipole Moment	3.07 D (1.01×10^{-29} Cm ² /V)	[161]
Rotational Constant	0.338 cm ⁻¹ (10.1 GHz)	[160]
Vibrational Constant	581.1cm ⁻¹ (17.4 THz)	[160]
Reduced Mass	12.9 amu	

Table 6.1: Properties of CaF relevant to the microwave trap

the molecular axis. This scenario is illustrated by figure 6.1. We ignore effects of electronic and nuclear spin, which interact with the magnetic component of the microwave field. This coupling is a small fraction of the electric dipole term, at most around $\sim \frac{\mu_D \mathcal{E}}{\mu_B B}$, which with $\mu_D = 1$ D is a suppression of at least a factor 100. One might question whether the molecular bond would extend at strong microwave fields, changing μ and B_{rot} . To estimate this effect, we can treat the molecule as a classical spring with natural frequency ω_{vib} , and mass m_r . The extension from the equilibrium bond length x_0 is $\delta x = \frac{q\mathcal{E}}{\omega_{\text{vib}}^2 m_r}$, with q defined such that $qx_0 = \mu$. For a field strength of 30 kV cm⁻¹, $\frac{\delta x}{x_0} \sim 10^{-5}$ and therefore not significant.

It is convenient to rewrite equation 6.1 as,

$$\mathcal{H} = B_{\text{rot}} \left(\mathcal{H}_0 + \lambda \mathcal{H}_{\text{Int}} \right) , \quad (6.2)$$

$$\mathcal{H}_0 = \hat{J}^2 + \frac{\hbar\omega}{B_{\text{rot}}} \hat{N} , \quad \mathcal{H}_{\text{Int}} = \frac{\mathbf{\mathcal{E}} \cdot \mathbf{n}}{|\mathcal{E}|} , \quad \lambda = \frac{\mu |\mathcal{E}|}{B_{\text{rot}}} .$$

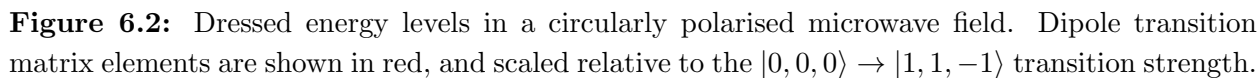
The first term in equation 6.2 is diagonal in our chosen basis, and its matrix elements are straightforward to find. The second term is the dipole interaction term, and determined by the selection rules; the parameter λ scales the dipole interaction to the rotational constant of the molecule. The dipole interaction matrix element between two states $|J_1, m_1, N_1\rangle$ and $|J_2, m_2, N_2\rangle$ evaluates to,

$$\langle J_1, m_1, N_1 | \mathcal{H}_{\text{Int}} | J_2, m_2, N_2 \rangle = \frac{\mu}{2} \sum_q \mathcal{E}_q (\delta_{N_2, N_1-1} C_{J_2, m_2 1, q; J_1, m_1} + \delta_{N_2, N_1+1} C_{J_2, m_2 1, -q; J_1, m_1}) \quad . \quad (6.3)$$

The symbols $C_{J_2, m_2 J_3, m_3; J_1, m_1}$ are the Clebsch-Gordon coefficients projecting individual angular momentum states $|J_2, m_2\rangle$ and $|J_3, m_3\rangle$ into total angular momentum $|J_1, m_1\rangle$, and the summation over q runs over the electric field components in the spherical basis. We can understand the first term in equation 6.3 as arising from absorption of a photon with angular momentum projection q and the second term describing emission of a photon with angular momentum projection q . When dealing with a near-resonance trap, one normally makes the rotating wave approximation, in which we only include matrix elements where absorption (emission) of a photon increases (decreases) J . In the dressed state picture, these terms dominate because they couple states whose energy difference is small, whereas the reverse processes only couple states with a large energy difference. However, in a far-detuned trap all terms are off-resonant and so we include both terms in equation 6.3.

It is instructive to consider how the dipole interaction term changes with the polarisation of the microwave field. Figure 6.2 shows the low-lying rotational levels with $-2 < N < 2, 0 < J < 3$ and $\hbar\omega = 1.5B_{\text{rot}}$, for a σ^+ polarised field $\mathcal{E} = |\mathcal{E}|(1, 0, 0)$. States which are coupled by the field are linked by red arrows, which are labelled by their relative transition strengths. Because of the angular momentum selection rules, the microwave field divides the full basis into distinct subsets. For the subset of linked states in figure 6.2, the $|0, 0, 0\rangle$ state is the true ground state; as J increases, the total number of states scales as $\frac{1}{2}(J+1)(J+2)$.

Figure 6.3 shows the same system of energy levels coupled by a linearly polarised microwave field, $\mathcal{E} = |\mathcal{E}|(0, 1, 0)$. In this case, the $\Delta m_J = 0$ selection rule means that any state $|J, m_J, N\rangle$ indirectly couples to state $|J, m_J, N-2\rangle$, and therefore this state is no longer



To simulate the effect of the microwave trap, we must calculate the shift in energy levels as the field is adiabatically switched on. This requires diagonalising $\mathcal{H}$ and increasing λ in small increments $\delta\lambda$, where $\delta\lambda \ll 1$. At each value of λ , the set of eigenstates $\{\psi(\lambda)^{(i)}\}$ are returned as

After each increment $\delta\lambda$, the new set of states are matched with the previous by computing their projections $\langle\psi^{(i)}(\lambda)|\psi^{(j)}(\lambda+\delta\lambda)\rangle$. The state j with maximum projection onto i is taken as its adiabatic continuation.

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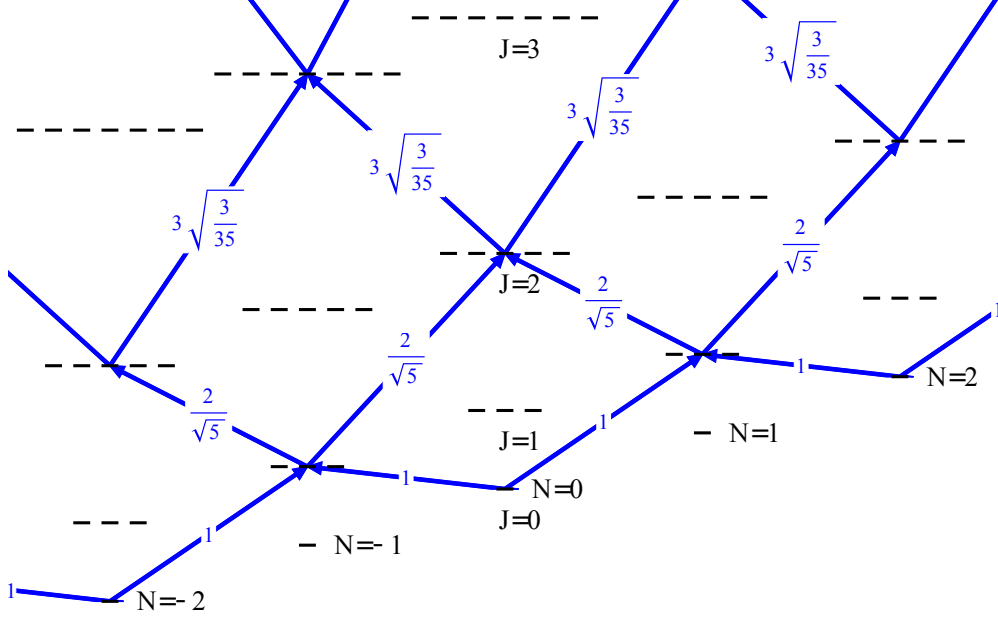


Figure 6.3: Dressed energy levels in a linearly polarised microwave field. Dipole transition matrix elements are shown in blue, and scaled relative to the $|0, 0, 0\rangle \rightarrow |1, 0, -1\rangle$ transition strength.

dipole interaction couples states separated by a single photon, it is practical to truncate the basis used at some maximum $J_{\max}$ and $\pm N_{\max}$. From hereon, states are labelled as $|\psi_{J,M,N}\rangle$ according to their initial composition at $\lambda = 0$, although the reader should remember that they are in general an admixture of zero-field eigenstates.

Figure 6.4a shows the results of our calculation for $J_{\max} = 10$, $N_{\max} = 10$, $\omega = 1.57B_{\text{rot}}$ (a red-detuned trap), with a circularly polarised electric field; for comparison, these are identical parameters to those in figure 2(a) of DeMille’s paper [81]. The energy of the $|0, 0, 0\rangle$ state shifts monotonically downwards as the field strength increases, and this state remains the absolute ground state of our basis. Figure 6.4b shows the composition of this state in terms of the field free eigenstates. When $\lambda < 1$, mixing occurs primarily with $|1, 1, -1\rangle$, which is the closest in energy and directly coupled via the dipole interaction. As λ increases further, increasing contributions from $|1, -1, 1\rangle$ and $|2, 2, -2\rangle$ appear, reflecting the increasing importance of higher order terms in the interaction. Figure 6.4c compares the energy of $|\psi_{0,0,0}\rangle$ when the basis is truncated at $N_{\max}, J_{\max} = 10$ to that with $N_{\max}, J_{\max} = 11$; from this, we conclude that the solution has converged to within 1 part in 10^{13} .

We now apply the calculation to CaF in a linearly polarised field with $\omega = 2\pi \times 14.3 \text{ GHz}$, which is the frequency of our trap. Figure 6.5a shows the calculation results for $0 < \lambda < 8$,

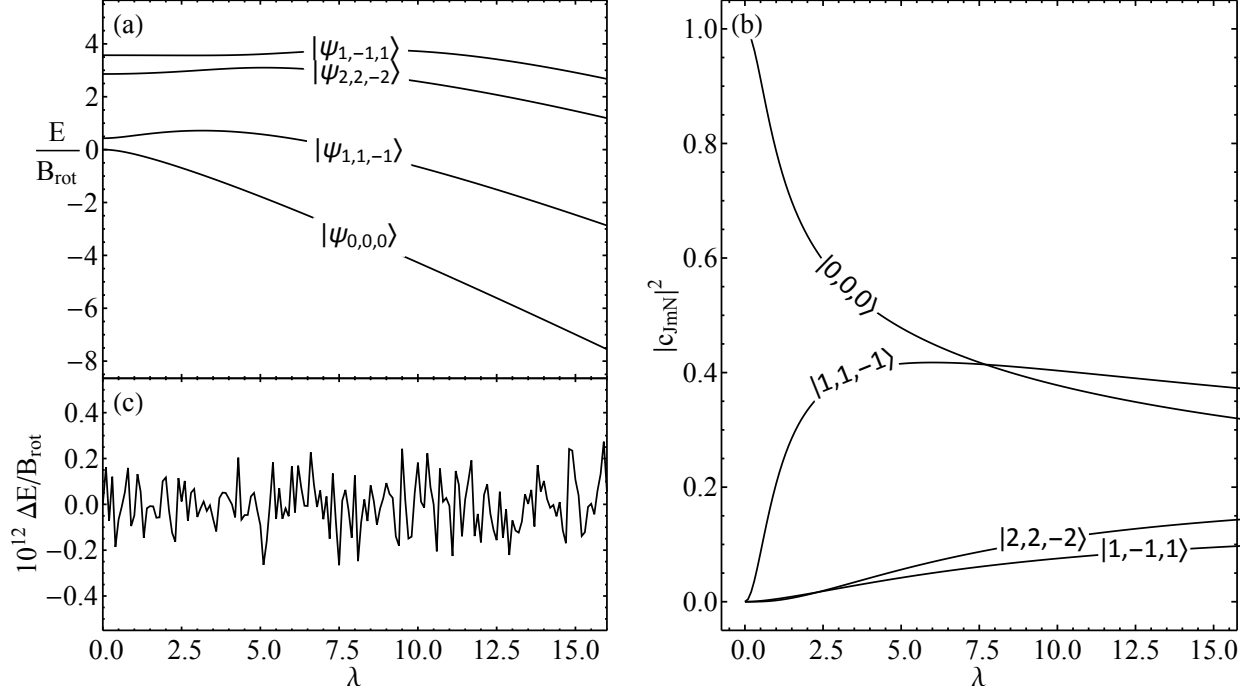


Figure 6.4: AC Stark shifts for the low lying rotational levels of a polar molecule in a circularly polarised microwave field. For these calculations, the basis was restricted to $J_{\text{max}} = 10$, $\Delta N_{\text{max}} = 10$, with $\omega = 1.57 B_{\text{rot}}$, as for figure 2(a) in [81]. (a) Energies of the four lowest dressed levels. (b) Composition of the state $|\psi_{0,0,0}\rangle$ state as the field strength increases. (c) energy change of $|\psi_{0,0,0}\rangle$ state when the basis is expanded to $(J_{\text{max}}, N_{\text{max}}) = (11, 11)$.

which corresponds to a peak electric field of 50 kV cm^{-1} . The spectrum of energy levels is more complex than for a circularly polarised field. Because there is no longer a true ground state in the basis set, the energy of any given state will meet a multitude of others as the field is increased. Whenever two states come close to meeting, an avoided crossing appears between them. We can understand this as the microwave field driving multiphoton transitions between the various levels in the basis. This concept is illustrated by figure 6.6, in which the $|0,0,0\rangle$ state is shown linked to the $|2,0,-6\rangle$ state by a 6 photon transition. The coupling leads to the broad avoided crossing around $\lambda = 7$ in figure 6.5a; the splitting is extremely large here and so a molecule will tend to follow this crossing adiabatically.

Figure 6.5b shows the composition of the state labelled $|\psi_{0,0,0}\rangle$ for $0 < \lambda < 5$. Compared with circular polarisation, the mixing with the near-resonant $|1,0,-1\rangle$ level is weaker, peaking around $\lambda = 2.5$, and the mixing with the other states is more significant. Figure 6.5c shows the difference in energy between the states $|\psi_{0,0,0}\rangle$ and $|\psi_{0,0,2}\rangle$ as the field strength

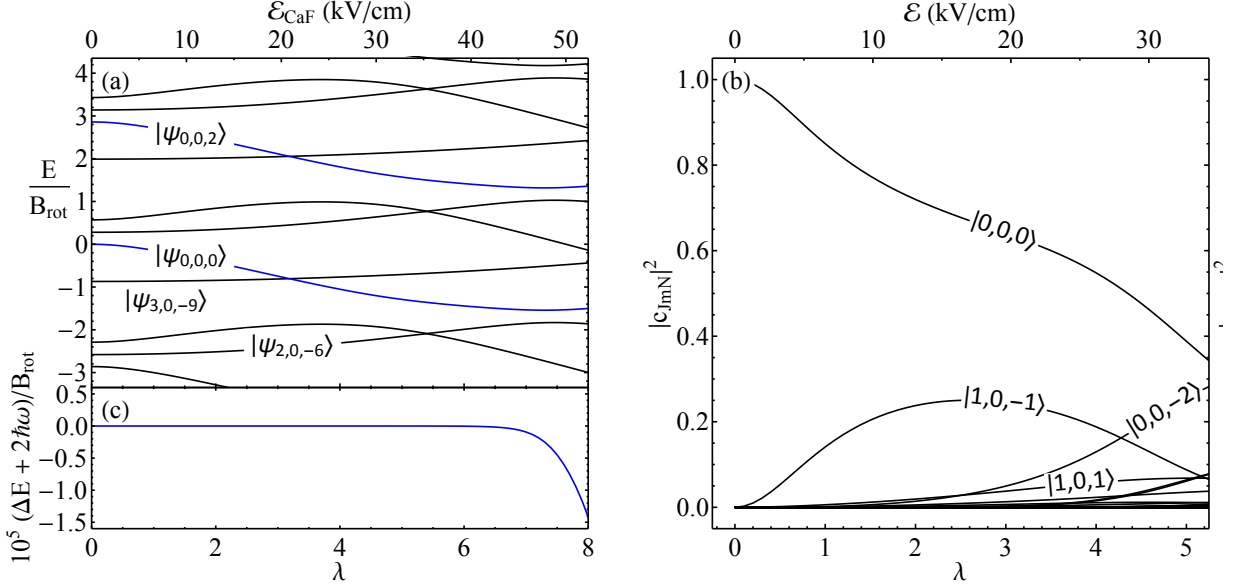


Figure 6.5: AC Stark shifts for CaF in a linearly polarised microwave field with $\hbar\omega = 1.43B_{\text{rot}}$. (a) Low lying energy levels for $0 < \lambda < 8$ (b) Composition of the $|0,0,0\rangle$ state for $0 < \lambda < 5$. (c) Energy difference between the states $|\psi_{0,0,0}\rangle$ and $|\psi_{0,0,2}\rangle$, after subtracting the energy of two photons.

increases. These two states should follow identical curves, because of the symmetry between $|J, m_J\rangle$ states with N and $N + 2$ photons in linear polarisation. The convergence is weaker compared with circularly polarised calculation, because there are many more states nearby in energy, and the coupling to these states becomes increasingly important as $\lambda > 1$.

Figure 6.7a focusses on the behaviour of the $|\psi_{0,0,0}\rangle$ and $|\psi_{3,0,-9}\rangle$ states, which cross near $\mathcal{E} = 20 \text{ kV cm}^{-1}$. The two states in fact avoid each other as they approach, opening up a crossing with a minimum separation of around 0.1 MHz. A molecule moving in this region of the trap will tend to follow an energy curve adiabatically if it traverses the crossing sufficiently *slowly*, and will tend to transfer between states if it moves quickly through the crossing. It follows that there must be some velocity at which point either outcome is possible. To determine whether this velocity is problematic for experiments, we need to know the shape of the trap potential.

Trap Potential for CaF

The calculation results of figure 6.5 allow us to find the trap potential for CaF in our cavity, U . In the following, I assume a peak electric field $\mathcal{E}_{\text{peak}} = 25 \text{ kV cm}^{-1}$, representative of

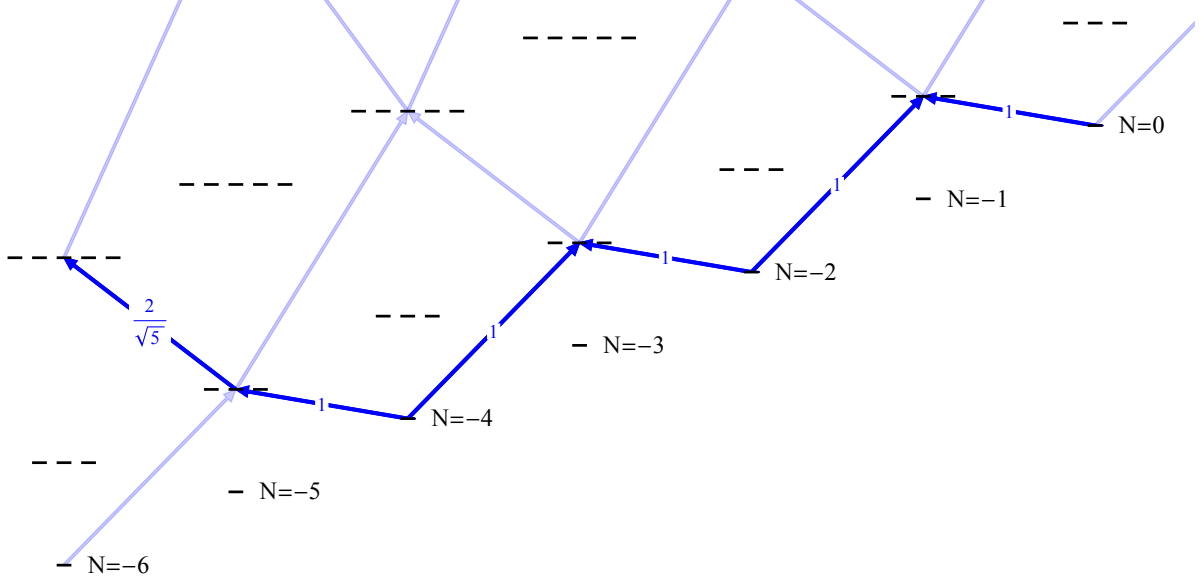


Figure 6.6: Multiphoton couplings in a linearly polarised microwave field.

our current experiments. The difference in gravitational potential across the trap is less than 1 mK, which is below 1% of the depth of the microwave potential; in the interests of simplicity we neglect this term. Close to the cavity centre, we can expand the potential as follows:

$$U(x, y, z) = U(0, 0, 0) + \left. \frac{dE}{d\mathcal{E}} \right|_{\mathcal{E}_{\text{peak}}} (\mathcal{E}_{\text{peak}} - \mathcal{E}(x, y, z)) \quad . \quad (6.5)$$

By substituting equation 5.7 for the electric field close to the cavity centre, we find,

$$U(x, y, z) = U(0, 0, 0) + \left. \frac{dE}{d\mathcal{E}} \right|_{\mathcal{E}_{\text{peak}}} \mathcal{E}_{\text{peak}} \left[\frac{x^2 + y^2}{w_0^2} + \frac{k^2}{2} \left(1 - \frac{2}{kz_0} + \frac{2}{k^2 z_0^2} \right) z^2 \right] \quad . \quad (6.6)$$

Therefore the trap potential is harmonic close to its centre. The trap frequencies are,

$$\begin{aligned} \Omega_x = \Omega_y &= \sqrt{\frac{2\beta\mathcal{E}_{\text{peak}}}{m_{\text{CaF}}w_0^2}} \quad , \\ \Omega_z &= \sqrt{\frac{\beta\mathcal{E}_{\text{peak}}k^2(1 - \frac{2}{kz_0} + \frac{2}{k^2z_0^2})}{m_{\text{CaF}}}} \quad , \end{aligned} \quad (6.7)$$

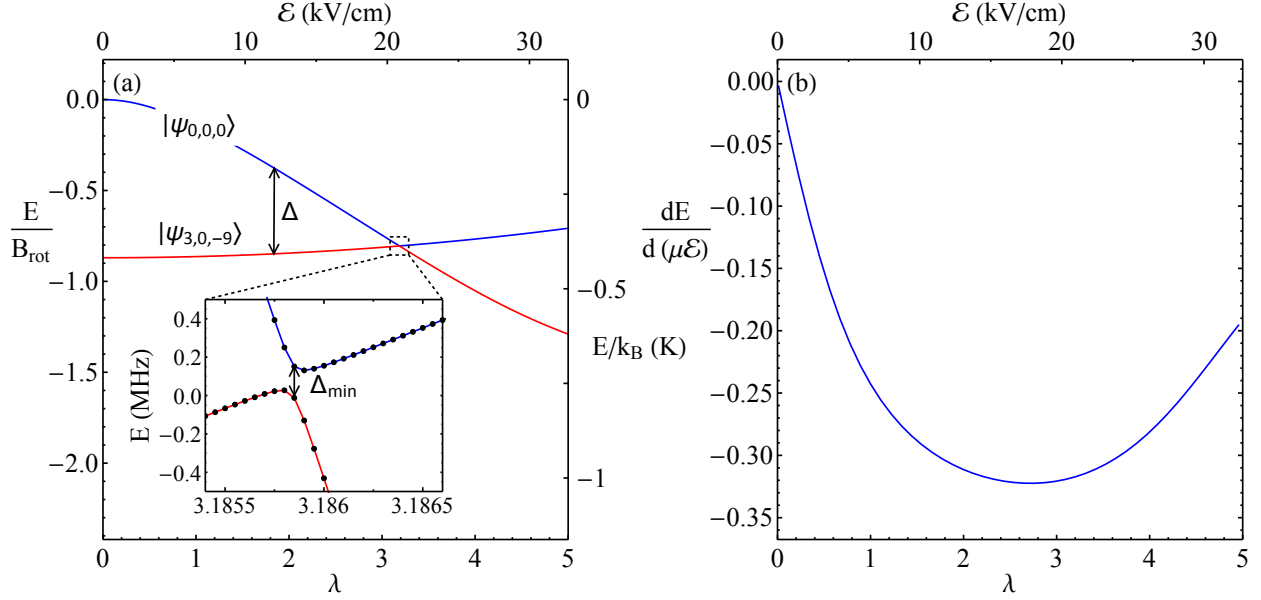


Figure 6.7: AC Shifts of the rotational ground state of CaF in a linearly polarised microwave field, with $\hbar\omega = 1.43B_{\text{rot}}$. (a) Energies of the $|\psi_{0,0,0}\rangle$ and $|\psi_{3,0,-9}\rangle$ states as a function of field. The inset is a close-up showing the narrow avoided crossing at around $\lambda = 3.186$ (b) The gradient of energy with respect to field, $dE/d(\mu\mathcal{E})$ for a molecule in the $|\psi_{0,0,0}\rangle$ state which diabatically transfers through the crossing.

with $\beta = \frac{dE}{d\mathcal{E}}|_{\mathcal{E}_{\text{peak}}}$. At $\mathcal{E}_{\text{peak}} = 25 \text{ kV cm}^{-1}$, $\beta = 0.23\mu$, and we find $\Omega_z = 360 \text{ Hz}$, $\Omega_x = \Omega_y = 130 \text{ Hz}$. The ratio $\frac{\Omega_z}{\Omega_x}$ is the same as for the atomic potential, and so we would expect the same aspect ratio in the trap for atoms and molecules.

We can now return to the problematic multiphoton transition at $\mathcal{E} = 20 \text{ kV cm}^{-1}$ discussed above. The avoided crossing represents a Landau-Zener problem, for which the adiabatic formula provides the probability of diabatic transfer, P_{transfer} :

$$P_{\text{transfer}} \approx \exp\left(-\frac{\Delta_{\text{min}}^2}{\hbar d\Delta/dt}\right) . \quad (6.8)$$

Here, $d\Delta/dt$ is the rate of change of the energy separation of the states away from the crossing, and Δ_{min} is the minimum separation, as shown in the inset of figure 6.7a. Substituting $\frac{d\Delta}{dt} = v \frac{d\Delta}{dx}$, we can define the “lossy” velocity, v_{loss} , as

$$v_{\text{loss}} \frac{d\Delta}{dx} = \frac{\Delta_{\text{min}}^2}{\hbar} . \quad (6.9)$$

Molecules with speeds close to v_{loss} will sometimes traverse the avoided crossing diabatically,

and sometimes adiabatically. This is likely to result in a change of state after traversing the crossing several times, and thus to loss of molecules. Writing $\frac{d\Delta}{dx} = \mu \frac{d\Delta}{d(\mu\mathcal{E})} \frac{d(\mathcal{E})}{dx}$ allows us to relate v_{loss} to the shape of the trap potential. From equation 5.8 it can be shown that $\frac{d\mathcal{E}}{dx} \approx w_0 \sqrt{2 \frac{\mathcal{E}}{\mathcal{E}_{\text{peak}}} (1 - \frac{\mathcal{E}}{\mathcal{E}_{\text{peak}}})}$. Putting all this together, we find

$$v_{\text{loss}} = \frac{\Delta_{\text{min}}^2 w_0}{\hbar \mu \mathcal{E}_{\text{peak}} \frac{d\Delta_{\text{min}}}{d(\mu\mathcal{E})} \sqrt{2 \frac{\mathcal{E}}{\mathcal{E}_{\text{peak}}} (1 - \frac{\mathcal{E}}{\mathcal{E}_{\text{peak}}})}} \quad . \quad (6.10)$$

For our chosen $\mathcal{E}_{\text{peak}}$, the multiphoton transition occurs at $\frac{\mathcal{E}}{\mathcal{E}_{\text{peak}}} = 0.85$, where $\frac{d\Delta}{d(\mu\mathcal{E})} \approx 0.2B_{\text{rot}}$. Given the splitting at the crossing Δ_{min} is 0.1 MHz, we find that $v_{\text{loss}} = 0.15 \text{ m s}^{-1}$. This has to be compared to the typical velocity for a molecule in the trap. For a cloud of molecules oscillating in the trap with a displacement 1 mm, the peak axial velocity is 2.3 m s^{-1} . This is a similar order of magnitude to v_{loss} , and therefore likely to be problematic. If the field strength in the cavity is restricted to remove the risk of such transitions, then the trap depth is limited to 380 mK.

Elliptical Polarisation

We have seen that the behaviour of a circularly polarised microwave trap is simpler and more effective than with linear polarisation. It is possible in principle to make a circularly polarised cavity trap, and it is then interesting to ask how pure the polarisation state should be. To answer this question, we can repeat the eigenstate calculation for a near-circular polarisation. In the spherical basis, this can be represented by $\mathcal{E} = |\mathcal{E}|(1, 0, \alpha)$, where $\alpha \ll 1$. Figure 6.8a shows a calculation of the eigenstates with $\alpha = 0.09$; the increase in the size of the basis states leads to an avoided crossing between the $|0, 0, 0\rangle$ and $|1, 1, -3\rangle$ states at $\lambda = 0.7$. The closest approach of these states is plotted for different values of α in figure 6.8b, and follows a quadratic relationship, $\Delta \sim \alpha^2$. With $\alpha = 0.01$ the crossing is still 1 MHz wide, which means a molecule is likely to follow adiabatically through the crossing. To achieve this degree of circularity in a cavity is already quite challenging, and so it appears that non-adiabatic transitions will provide a limit to the trap depth even when the field is intended to be circularly polarised.

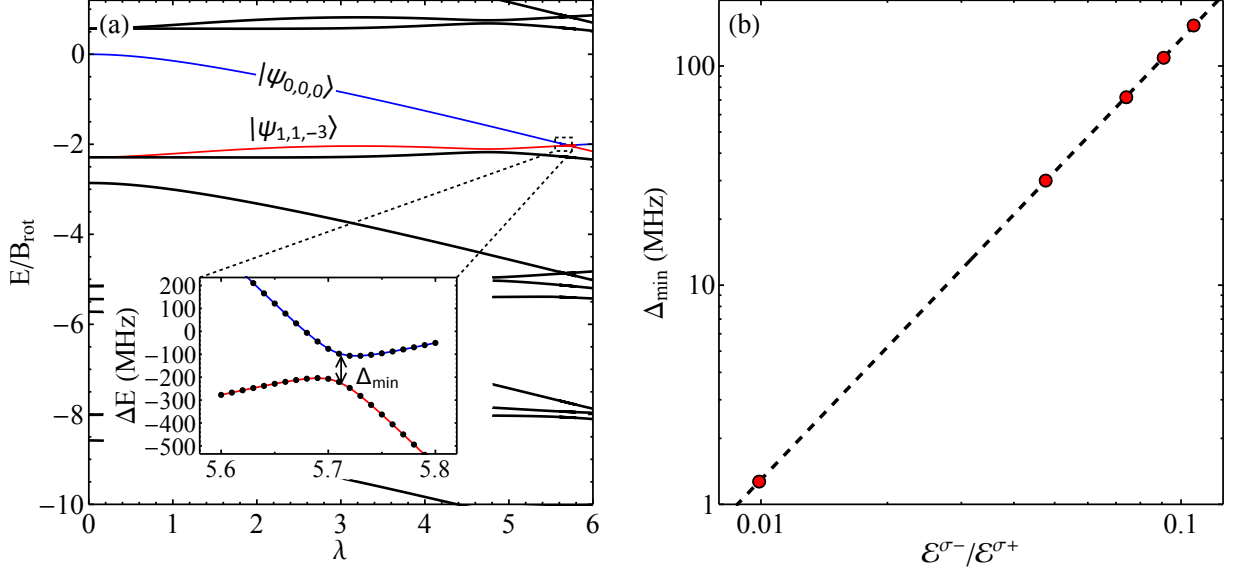


Figure 6.8: Dressed levels in an elliptically polarised field. (a) AC Stark shifts with ellipticity parameter $\alpha = 0.09$. Inset: close-up of the avoided crossing between the $|\psi_{0,0,0}\rangle$ and $|\psi_{1,1,-3}\rangle$ states. (b) Avoided crossing splitting, Δ_{min} , between the $|\psi_{0,0,0}\rangle$ and $|\psi_{1,1,-3}\rangle$ states against the ellipticity parameter α . Dashed line: quadratic relationship.

6.2 Thesis Summary

The aim of this project was to demonstrate that a microwave trap, a new trap with a number of advantages over current methods, works. This thesis has detailed the steps taken to load the trap with ultracold ^7Li , first by laser cooling to ultracold temperatures, and then transporting the atoms into the cavity with sufficient phase-space density. The microwave trap operates at a frequency near 14.3 GHz, and has been tested with up to 1 kW of input power for periods of a few seconds. Despite the complications associated with delivering such high microwave power to an ultrahigh vacuum system, of thermal changes to the cavity properties, of microwave leakage from the cavity, the trap behaves more or less as expected. It was possible to directly observe transfer from the magnetic trap to the microwave trap, and recover up to 81(6)% of the atoms to the magnetic trap. Centre-of-mass oscillations in the the microwave trap were observed, and the oscillation frequencies compared with theory. Whilst it is difficult to know the electric field in the cavity precisely, our measurements are close to the expected values, and the ratio of the axial and radial oscillation frequencies agrees well with predictions based on the geometry of the cavity mode.

One of the most significant challenges in the experiment was magnetic induction by the externally mounted trapping coils. This limits the switching time for magnetic fields at the location of the MOT to several milliseconds. To overcome this difficulty, in-vacuum coils were designed and installed, reducing the switching time to a few hundred microseconds. These coils were essential for the sub-Doppler Raman grey molasses to become effective, and also for the subsequent magnetic trapping stage. Atoms were then delivered to the microwave chamber by transferring to the externally mounted coils, and translating these coils a distance 600 mm to the external chamber. When transferring between magnetic traps, induction in the MOT chamber was found to jerk the cloud and heat the atoms. In addition, a large magnetic defect around the transport tube entrance resulted in both atom loss and heating. To minimise these issues, careful control of the trapping currents proved effective, and in the end we were able to deliver 2×10^7 atoms to the microwave chamber at a phase space density of $3.58(15) \times 10^{-7}$. The transport retains 56% of the atoms transferred into the magnetic trap; when losses due to background gas collisions is considered, the overall transport efficiency is better than 86%.

6.3 Future Work and Improvements

The most obvious next step for the experiment is to definitively show that ground state polar molecules can be trapped in the cavity. This is where the microwave trap offers the strongest benefits compared to other traps. Whilst the phase-space acceptance of the trap is sufficient to attempt loading from a decelerated molecular beam, it would be sensible to first try a laser cooled species with a high phase-space density. Calcium monofluoride molecules are already available in such conditions in our group, and loading this species could follow a similar sequence of steps to those used in this work. Many of the issues encountered in the transport experiments in chapter 4 provide some useful guidance for such an endeavour. Calculations outlined in this chapter indicate that multiphoton transitions may limit the trap depth for CaF to about 380 mK in our cavity, but there are many other unexplored issues at present. It is not known whether inelastic molecule-molecule collisions become detrimental in a strong microwave field, and this could pose significant issues to the viability of our trap.

Before advancing forward, there are a number of relatively simple improvements which can be made to the current experiment. Firstly, although water cooling was sufficient to demonstrate the trap, it should be tested with cryogenic coolant in the copper baths. Cooling with liquid nitrogen is known to improve the quality factor by around a factor two, but caused vacuum leaks in earlier designs. This problem should be solved in the current design, and if this proves true then we would expect a significant improvement in the chamber pressure by a cryo-pumping effect. If the pressure continues to spike after seconds of trap operation, then investigations using residual gas analysis could be used to gain further insight here.

Second, the microwave power delivery and detection system could be improved. For instance, using biased diode detectors would reduce non-linearities in the incident and reflected power monitors, which altered the effective gain of the frequency stabilisation system. This would improve the reliability of the frequency lock and simplify measurements of the quality factor. A feedback loop could also be installed to servo the cavity input power, ensuring that the trap potential remains stable in time.

Finally, optical access was somewhat restricted in the experiment by the need to ensure microwave safety in the laboratory. It may be possible to circumvent this issue by utilising optically transparent microwave absorbers, which have recently been demonstrated using a Salisbury screen configuration [162, 163]. Devices presented thus far can provide better than 80% optical transparency over the entire visible range, and broadband microwave transmission loss of around 20 dB. These could form externally mounted attenuators for the viewports on the microwave chamber, improving optical access compared to the cutoff attenuators in this work.

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Appendix A

Relating the spontaneous decay rate to the reduced matrix element

Consider the decay of an atomic state $|L', m'_L\rangle$ to the states $|L, m_L\rangle$. The spontaneous emission rate is given by

$$\Gamma_{L' \rightarrow L} = \frac{8\pi^2}{3\epsilon_0 \hbar \lambda^3} \sum_{m_L, q} |\langle L', m'_L | dC_q^{(1)} | L, m_L \rangle|^2 \quad , \quad (\text{A.1})$$

where the sum runs over all final states with angular momentum L , and the pre-factor derives from the vacuum energy density at the frequency of the transition. By applying the Wigner Eckart theorem, we have

$$\Gamma = \frac{8\pi^2}{3\epsilon_0 \hbar \lambda^3} |\langle L' || dC^{(1)} || L \rangle|^2 \sum_{m_L, q} \begin{pmatrix} L & 1 & L' \\ -m_L & q & m'_L \end{pmatrix} \begin{pmatrix} L & 1 & L' \\ -m_L & q & m'_L \end{pmatrix} \quad . \quad (\text{A.2})$$

Because of the symmetry of the Wigner 3j symbols, the sum evaluates to $\frac{1}{2L'+1}$. Therefore, we find

$$|\langle L' || dC^{(1)} || L \rangle| = \sqrt{\frac{3\epsilon_0 \hbar \lambda^3 \Gamma}{8\pi^2}} \sqrt{2L' + 1} \quad (\text{A.3})$$

This is then used to produce equation 3.25.