

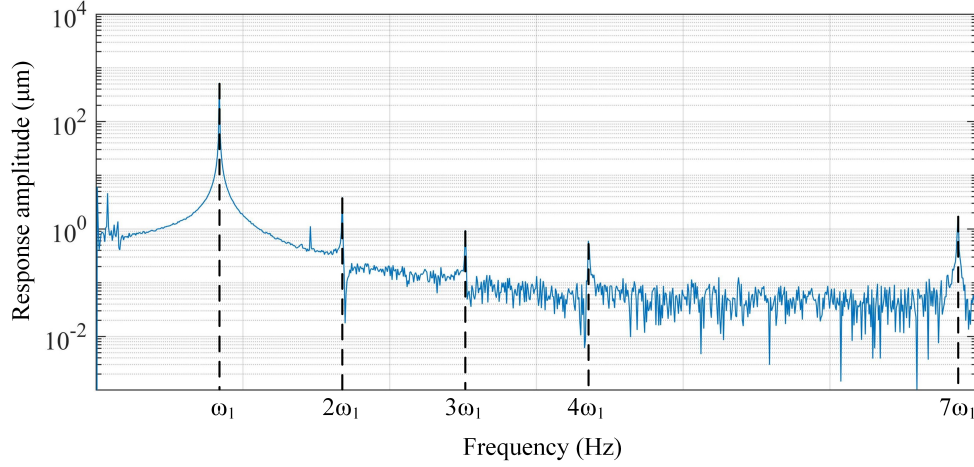


Figure 14: Response spectrum of response-amplitude-controlled fixed sine test at 10 g.

cameras [48]. Although several harmonics of the first mode are present, their amplitude is several orders of magnitude lower than the fundamental harmonic. In order to smooth the results, the data are filtered by a third-order bandpass butterworth filter centered at the first natural frequency with a bandwidth of 40% of the first natural frequency. The results of the mode shape estimation are shown in **Fig. 15** for each amplitude level, both for the upper and lower lines. Two additional plots shed further light on the amplitude-dependent properties of the mode shape. The first one, **Fig. 16** (a), shows the upper line mode shape of a central portion of the beam, normalized with respect to the left tip displacement. From the lowest to the highest excitation amplitude, the displacements of the central part of the

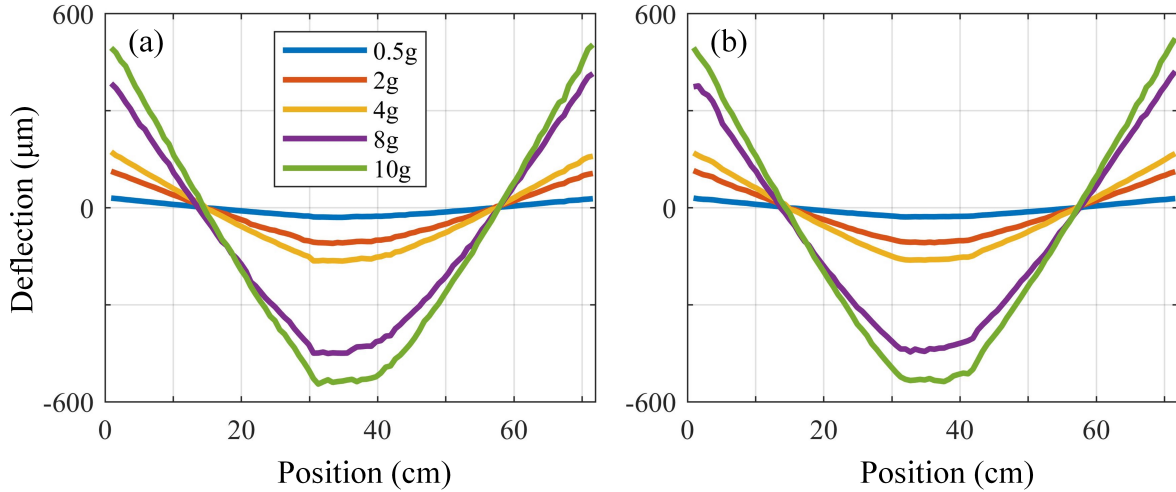


Figure 15: Amplitude-dependent mode shape of the 1st mode of the HBRB from response-amplitude-controlled testing, showing (a) DIC lower line and (b) DIC upper line. Each line is the mode shape calculated from the maximum displacement for an excitation level.

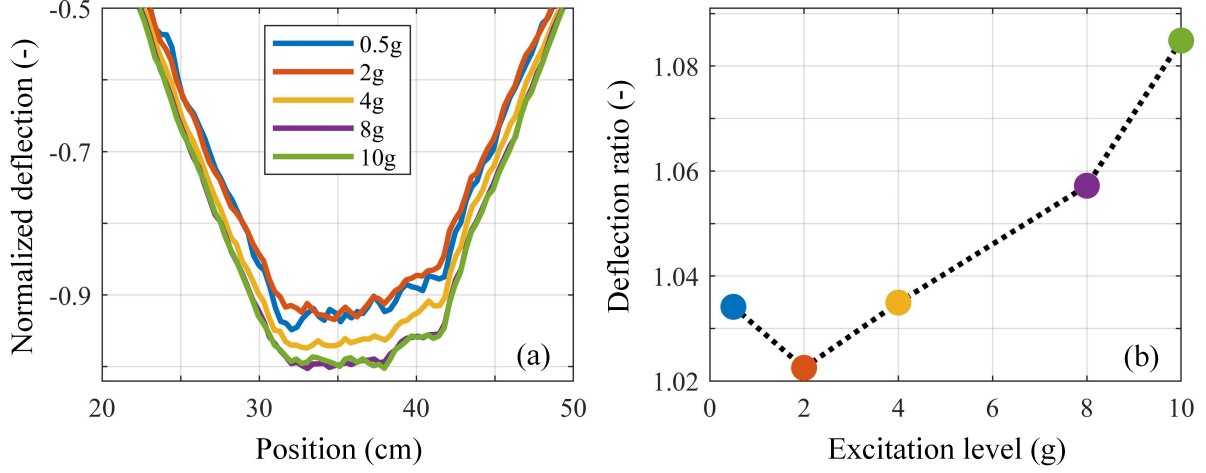


Figure 16: Amplitude-dependent mode shape of the 1st mode of the HBRB from response-amplitude-controlled testing of DIC upper line, showing (a) normalized deflection and (b) deflection ratio between the middle of the beam to the tip displacement.

beam increase approximately by 7%, with respect to the tip displacement, but decrease for the 2 g level. This suggests a softening behavior of the modal stiffness. The overall trend resembles that of the backbone curve, but the variation in this case is greater with respect to the frequency change in the backbone curve, cf. **Fig. 8**, since this is a local phenomenon while the natural frequency depends on global characteristics. This behavior is illustrated more compactly in **Fig. 16** (b) with the deflection ratio between the displacement of the middle of the beam to the tip displacement. Precisely, the tip displacement is computed as the average between the displacements of the ten leftmost points from DIC, and the middle displacement is the average among the ten displacements around the beam center. While the locations of the mode shape nodes remain unchanged, the principal variation of the mode shape is concentrated in the middle of the beam, suggesting that further studies should connect this macroscopic observation with microscopic phenomena of the contact patch.

3.4.3. Energy of the Beam

The total energy of a structure consists of kinetic energy and potential (strain) energy. As shown in the STFT analysis (**Fig. 5** (b)), the free decay response obtained from the shaker ringdown testing only contains the first mode of the HBRB. The potential energy is assumed to be zero at the equilibrium position, corresponding to the configuration when all displacements are zero. As verified previously, the vibration is synchronous and, during the decay, every point passes through zero at the same time. At these time instances, the total energy of the structure is therefore equal to the kinetic energy.

As DIC provides responses of a large number of points, the HBRB can be discretized into lumped mass elements. The filtered displacement responses (**Fig. 9** (b)) can be transferred into velocity responses by numerical differentiation. Thus, with the full-field information of

the structure and the mass of the whole beam ($M = 1.796$ kg as measured), the energy E at time instant t can be approximated by

$$E(t) = \sum_{i=1}^N \frac{1}{2} m v_i^2(t), \quad (5)$$

where N is the number of the measurement points, v_i is the velocity of the i th point, and $m = M/N$ is the mass of each measurement point.

Here, the total number of the measurement points is $N = 206$, which is determined during the processing of the DIC. The energy trend, computed using all measurement points, over the shaker ringdown testing is shown in **Fig. 17** (a) with a black line, and the red dots depict energy peaks. In order to study the influence of the number of the measurement points on the calculation of E , N is varied from 5 to 206 over an equally spaced grid. **Figure 17** (b) shows the energy of the beam, at the time instant of the first peak, as a function of N , which converges for $N > 100$. In other words, the number of points N needed to have an accurate estimation of the energy is higher than 100. This spatial resolution is not achievable with contact sensors such as accelerometers or strain gauges, but only with a non-contact measurement approach, such as videography and DIC.

The relationships between energy and displacement/acceleration amplitudes for the accelerometer located on the shaker (see Accel 3 in **Fig. 2** (a)) is shown in **Fig. 17** (b). A clear, quadratic trend, as defined in Eq. (5), is observed. The accuracy of the energy calculation by DIC is validated in **Fig. 17** (d) by means of the dissipated energy. The red and green dots are the work done by the forcing for the response-control (red) and force-control (green) testing procedures for different amplitudes, cf. **Fig. 7** (b). These quantities are computed using the measurement of the shaker force and the acceleration of the driven point. As argued in **Sect. 3.2**, this work is equal to that done by the non-conservative forces over one oscillation cycle. For the shaker ringdown, the peaks in the kinetic energy correspond to the total energy of the system, since the motion is synchronous. Hence, the difference between the energy of the k -th peak and that of the $(k+2)$ -th peak (see the inset in **Fig. 17** (a)) is the work done by the non-conservative forces in one oscillation cycle. The dissipative work computed from forced testing and from shaker ringdown is expected to be comparable from theoretical speculations. Indeed, **Fig. 17** (d) shows that there is a very good agreement with the two. Here, the blue line shows the dissipative work, computed using the kinetic energy decay obtained from full-field data, plotted against the acceleration amplitude level. It agrees well with the energy measurements obtained in forced testing using a single accelerometer.

The energy computed here could serve for model validation or tuning, but it is also useful to construct frequency-energy plots, typically adopted in the NNM literature [9] to characterize the dynamical feature of nonlinear responses. Frequency-energy plots are particularly meaningful for conservative systems, where each NNM orbit lies on a specific energy level [29]. However, they are relevant also for weakly forced, damped structures as their resonant periodic motions typically show small deviations from their conservative counterpart, which

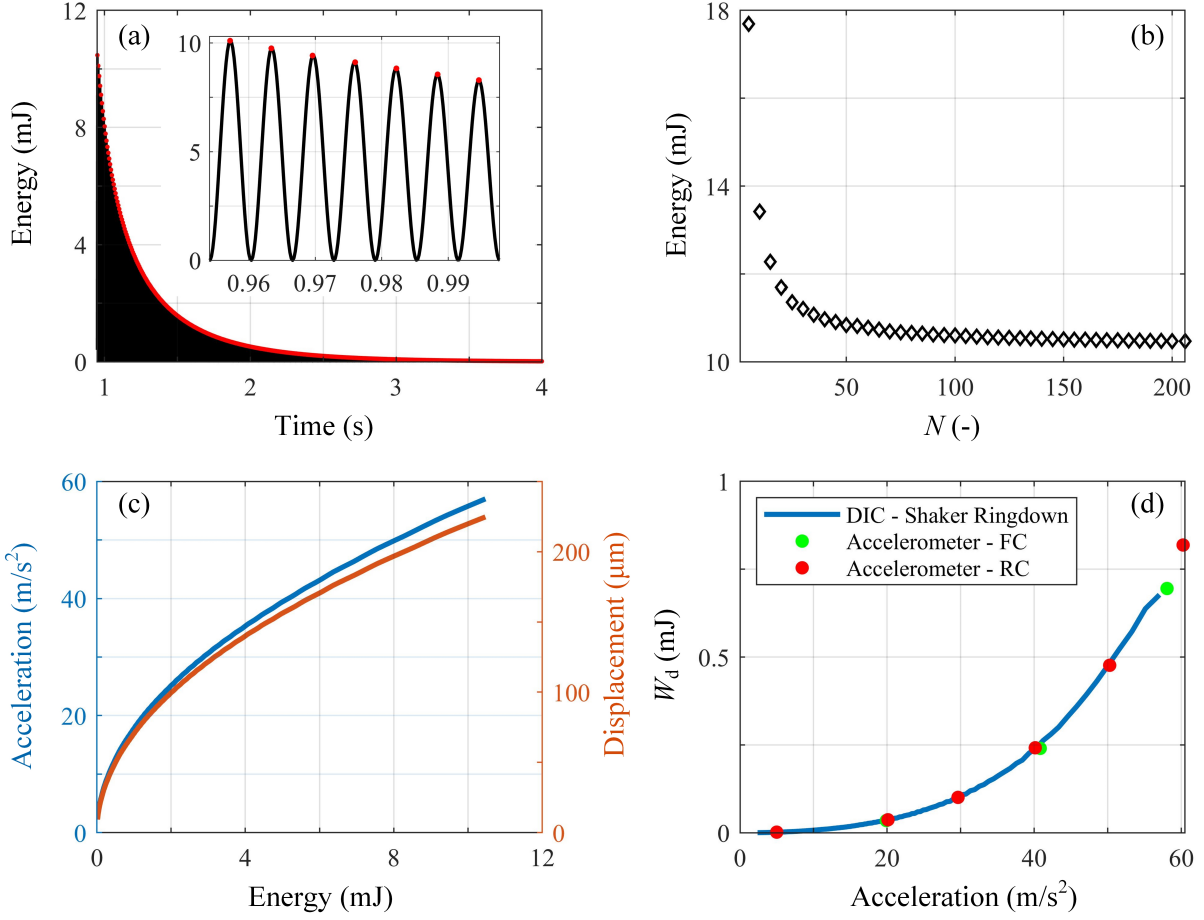


Figure 17: The energy of the HBRB where (a) shows the trend of the kinetic energy over time (with the inset showing the first seven peaks) and (b) the dependence of the energy on the number of the measurement points. Plot (c) shows the relation between energy and displacement/acceleration amplitudes at the location of Accel 3 in **Fig. 2**. The last plot in (d) shows the dissipated energy over one oscillation cycle computed via accelerometer measurements from forced testing (red and green dots, cf. **Sect. 3.2**) and the same quantity extracted using full-field data from shaker ringdown.

can be neglected [46, 47, 61]. Experimentally, the construction of frequency-energy plots is not practical since measuring the energy of a structure accurately is impractical with accelerometers, and thus the frequency-amplitude (typically acceleration amplitude) backbone plots are often reported instead, which is a flawed metric [19, 20]. In this study, however, the calculation of energy is feasible due to the full-field information from DIC.

Figure 18 shows the nonlinear characteristics of the system using the full-field DIC data. Here, all of the 206 DIC points are considered to ensure a sufficiently converged result, since the use of $N = 206$ instead of a smaller value, such as 100, comes at no significant computational cost increase. As an alternative to the displacement or acceleration amplitude used previously (see **Figs. 8** or **11**), the damping and frequency are now combined with the energy calculations, as shown in **Fig. 18** (a)-(d). Specifically, **Fig. 18** (c) and (d)

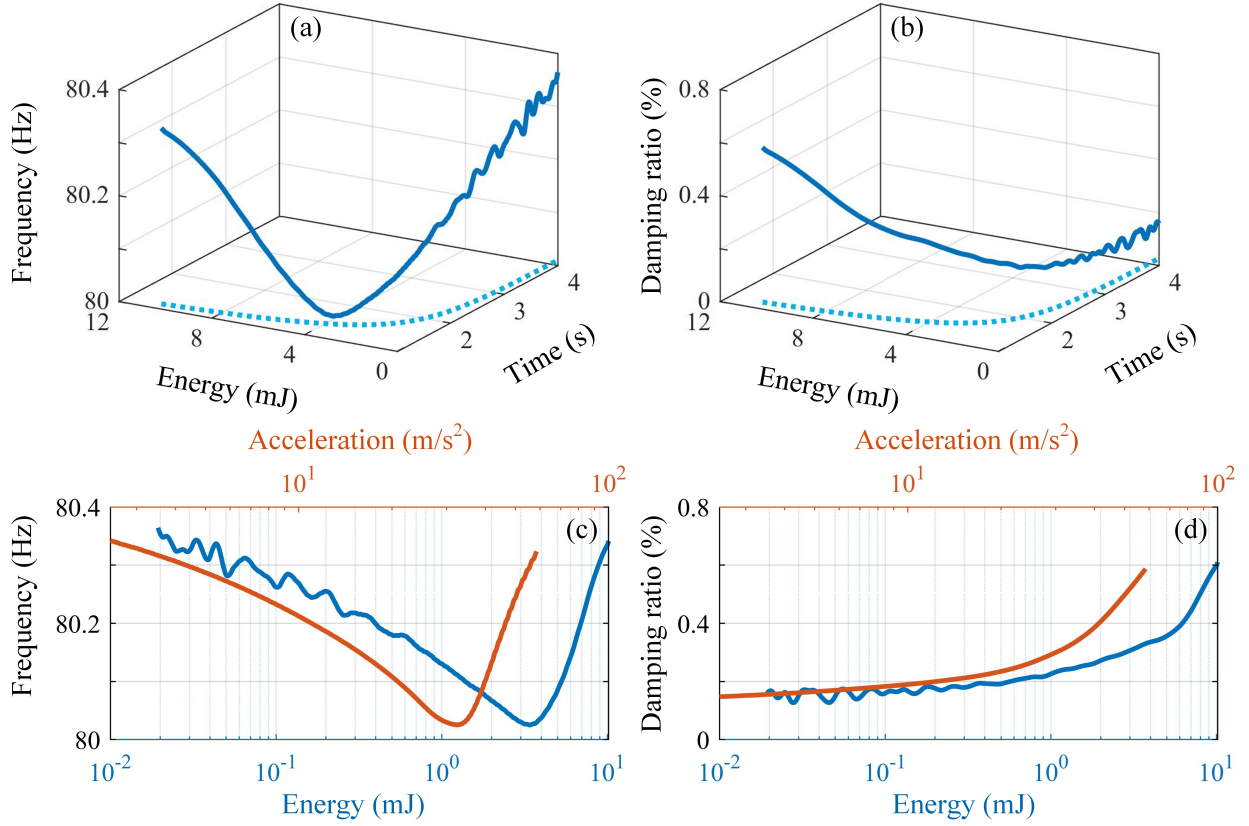


Figure 18: The relationship between the time, energy, amplitude, and nonlinear characteristics of the HBRB, including (a) relationship between time, energy, and frequency, (b) relationship between time, energy, and damping ratio, (c) energy-dependent and amplitude-dependent frequencies, (d) energy-dependent and amplitude-dependent damping ratios. The latter amplitudes refers to the acceleration amplitudes measured by Accel 3 in **Fig. 2**.

superimpose amplitude-dependence and energy-dependence. Clearly, the two properties show consistent values of frequency and damping whose amplitudes are related by a scaling factor and a quadratic relationship. Amplitude-dependent property plots act as proxies for the energy-dependent property plots in this case, but the latter may provide a more global perspective on the system under analysis. However, the energy-dependent plots in **Fig. 18** (c) and (d) show an increased variability towards small amplitudes, due to the degradation of the signal-to-noise ratio in DIC measurements. By contrast, amplitude-dependent curve shows a smoother behavior since the accelerometer data, at the same amplitudes, is not subject to these noise issues. It is important to note that if the system was not vibrating synchronously, the amplitude dependent plots may have a less clear relationship with the energy plots depending on which point is used to measure the amplitude.

4. Comparison of the Methods from a Practitioner Viewpoint

Nonlinear system identification, as shown in this paper, requires a toolbox approach in which different techniques have to be deployed depending on the system under analysis and the testing procedure. Force-response testing tends to provide robust information since steady states can be properly isolated, but the shaker unavoidably influences the system dynamics. Hammer excitation is easy to perform, but it generates a multi-modal component. This is less than ideal if the identification targets a particular mode of the structure, as much of the energy is distributed across modes that are not of interest. In this context, shaker ringdown should be the preferred test method as it combines the advantages from both shaker testing (i.e., single frequency and high level of excitation at that frequency) and hammer excitation (i.e., transient free decay data). However, decoupling the shaker from the structure is often not a straightforward task [48].

Excluding SS-SSM, all of the methods used in this paper are signal processing techniques suited for time-frequency analyses. The extraction of frequency patterns in a time series varies according to the theoretical foundations of the method, but a common requirement of time-frequency methods is the need to filter the transient data before extracting the nonlinear characteristics. By contrast, for SS-SSM, the frequency-amplitude relation is intrinsic in the identified nonlinear model of the system. Encouragingly, all of the techniques showed good agreement in the results for shaker ringdown and hammer testing.

Among all of the methods, DMD is the only one that was originally born for the analysis of multi-point measurements of the dynamical system. The other techniques are more suited for single-point measurements, and hence required suitable rearrangements in order to be applied to multiple measurements simultaneously. Moreover, the HT, PFF, SS-SSM, and WBEMD methods have been developed and extensively used for studying transient regimes in mechanical systems. By contrast, DMD, broadly known in the fluid dynamics community for unfolding the dynamics of ergodic attractors, has presented challenges for tackling transient phenomena [68]. However, the results of DMD show a very good agreement with the other techniques in the context of this research.

Every technique deployed in this paper presents either computational or implementation challenges. From the complexity viewpoint, all of the techniques have drawbacks. The HT and PFF methods require filtering that needs to be manually tuned for the best results [58] in order to retain the core harmonic information of the time series [25]. Moreover, the HT method suffers from the issue of end effect in the estimation of the damping curve, even though appropriate signal pre-processing is performed, cf. **Fig. 4** (b) and see [25]. For the SS-SSM method, model fitting is fundamental for obtaining adequate results [34], which requires a computational routine to determine the coefficients that describe the SSM and its dynamics [31]. For DMD, the results depend on the selection of time scales (e.g., sampling, windowing) and of the number of dynamic modes. Finally, WBEMD needs an optimization in order to identify the intrinsic mode functions, and hence it requires some application dependent parameters [57]. Overall, DMD, HT, and PFF may seem to be less complex in terms of implementation for the context of the paper. However, SS-SSM may also be

715 exploited to construct data-driven reduced models, and WBEMD can efficiently uncover
716 multi-modal transient phenomena.

717 5. Conclusions

718 This paper compared the performance of multiple nonlinear identification methods based
719 on the experimental data from a jointed structure. The backbone curve and amplitude-
720 damping curve were extracted from the signals using Hilbert Transform (HT), Peak Finding
721 and Fitting (PFF), Dynamic Mode Decomposition (DMD), State-Space Spectral Submani-
722 fold (SS-SSM), and Wavelet-Bounded Empirical Mode Decomposition (WBEMD). The re-
723 sponses were measured by accelerometers in the experiments including hammer testing,
724 shaker ringdown testing, and response/force-control stepped sine testing. During the shaker
725 ringdown testing, apart from the accelerometers, two high-speed cameras recorded the im-
726 ages of the jointed beam. Then, Digital Image Correlation (DIC) was adopted to extract
727 the displacement responses from the images to reconstruct the mode shapes of the whole
728 beam. The main conclusions of this paper are summarized as follows:

- 729 • Backbone and damping curves obtained from hammer testing and shaker ringdown
730 testing showed a similar trend. However, the amplitude range in the shaker ringdown
731 testing was wider than that in the hammer testing despite the peak amplitude of the
732 acceleration of the tip of the beam being smaller in the shaker tests. This happens
733 because the hammer testing excites multiple modes, while the shaker ringdown testing
734 is able to excite one single mode of the structure. Especially at high amplitudes,
735 the amplitude-dependent properties obtained via hammer testing deviated from those
736 obtained via shaker ringdown testing, although proper filtering was applied. This
737 observation proves that other modes can indeed influence the amplitude-dependent
738 properties of a specific mode. Therefore, hammer testing should be carefully handled
739 when aiming to extract single-mode nonlinear information.
- 740 • Amplitude-dependent properties computed from shaker ringdown testing and from
741 forced-response testing presented good agreement, as expected from theoretical re-
742 sults. In this paper, an energy criterion was used to extract backbone and damping
743 curves from response/forced-control testing. Although it requires precise tuning and an
744 effective decoupling mechanism, shaker ringdown testing is more efficient with respect
745 to forced testing, where the acquisition of dense datasets can necessitate extensive
746 experimental campaigns.
- 747 • Other than amplitude-dependent properties, which show good agreement with those
748 coming from accelerometer data, full-field data from two high-speed cameras are able
749 to reconstruct the mode shape of the beam. The deformation induced by the lap joint
750 is visible and the variation of the mode shape with the amplitude of motion can be
751 assessed as well as motion synchronism. Moreover, full-field data allows the definition
752 of the kinetic energy of the beam, which is able to determine amplitude-dependent

properties. The value of the kinetic energy also showed good convergence properties as the number of measurement points was increased. Moreover, the dissipated energy computed from the decay of kinetic energy during a shaker ringdown test is consistent with the dissipated energy computed in forced testing. The metric of energy, which cannot be achieved with traditional accelerometers alone due to the need for a spatially dense data set for adequate convergence, can be useful for practices such as model validation and model updating. However, especially when dealing with long structures in free vibrations, the cameras are limited in memory, and thus they are only able to record a short period of time.

- All of the methods used in the paper agreed well in the extraction of amplitude-dependent properties from free vibrations. Other than showing performances, this paper also critically compared these methods, pointing out benefits and potential challenges in order to enrich the toolbox of practitioners in structural dynamics.
- In terms of the frequency-amplitude and damping-amplitude curves, the beam under investigation showed limited frequency change, i.e., 0.5 Hz, while the damping ratio rose from about 0.1% at low amplitude to more than 0.6% at high amplitudes. Additionally, the mode shape presented substantial amplitude dependence, especially in the area of the contact patch. The deflection ratio, which tries to quantify this variation, exhibited a similar trend to that of the backbone curve, but with a much wider range amounting to 7%. Thus, the strongest nonlinearities for the jointed beam studied were observed in the damping behavior and the variation of the mode shape. Further studies can help to uncover the connection of these macroscopic phenomena with interactions occurring in the contact patch.
- The relationship between the energy and nonlinear frequency and damping ratio of the beam have been obtained due to the full-field DIC data. The energy-dependent characteristics provide the information of the whole beam instead of a few points on the structure. Accelerometers, as a traditional way of measurement, make the measurement of the beam's energy prohibitively difficult since it is not practical to place a large number (i.e., 100s) of accelerometers on this specific structure, let alone the effect of the added mass. By contrast, high-speed cameras provide a convenient way of measuring the structure without contact. Additionally, responses of the structure can be obtained with a high spatial resolution, making it possible to accomplish the identification of the whole structure.

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Data Access

The data used in the paper is available at <https://github.com/mattiacenedese/BRBtesting>.

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