

**COLLAPSE OF LONGITUDINALLY STIFFENED CYLINDERS
SUBJECT TO AXIAL AND PRESSURE LOADING**

by

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**A thesis submitted for the degree of Doctor of Philosophy
in the Faculty of Engineering of the University of London.**

**Imperial College of Science
and Technology , London .**

November 1984

To my parents.

"But, look, the morn, in russet mantle clad
Walks o'er the dew of yon high eastern hill."

Hamlet I.i., W. Shakespeare.

ABSTRACT

A new method for the fabrication of small scale axially stiffened cylinders is discussed. The technique is a development of existing methods used for the fabrication of small scale box-girders. 320 mm diameter models are produced with clamped end conditions, which are free of in-built stresses before loading.

6 models have been loaded, 3 in uniaxial compression, and 3 by a combination of axial load and end moment. Experimental results suggest that failure of the most heavily loaded panel is a useful criterion for failure of the cylinder.

Initial lateral surface imperfections and lateral deflections under load have been measured and analysed by regression into fourier coefficients.

Eigenvalues of the models were obtained using finite element idealisations and also from classical orthotropic shell theory. The panel buckling mode is critical for the geometries examined.

Theoretical behaviour of stiffened panels, modelled by elastic and elasto-plastic finite elements, is compared with the experimental response. An elastically supported beam-column is also discussed.

A general collapse mechanism theory, applied to panels, is established using rigid panel elements. An established constitutive relationship is used to obtain stress resultants. The principle of virtual work, coupled with an iterative procedure, is used to obtain collapse loads of a panel at a given lateral deflection.

The theory is applied to initially flat panels, and the parameters affecting results are discussed. The concept of redundant yield lines is introduced. Results are given for panels subject to inplane and lateral load, with modifications to the theory allowing for elastic strain energy.

The theory is applied to cylindrical panels. Results for panels subject to lateral and inplane load are given in terms of the panel aspect ratio, A/B , and the panel curvature parameter, A^2/Rt . Ring and stringer stiffeners are discussed.

Recommendations are made for the design of panels subject to inplane load.

Acknowledgements

Many people have assisted the completion of this work. It is not possible to name all; however, it is appropriate to mention those whose help has been particularly appreciated.

The support and guidance of Professor P.J. Dowling of the Department of Civil Engineering at Imperial College was essential to the orderly completion of the theoretical and experimental work. The experimental research was sponsored by the Department of Energy (UK) and the Science and Engineering Research Council. The Department of Civil Engineering at Imperial College supported the theoretical research and the completion of this thesis.

The author is grateful to Research Models and Equipment Ltd. and in particular to Mr. N. Scott for their assistance in the development of a satisfactory model manufacturing technique. Many of the staff of the Steel Structures Laboratory under the direction of Mr. J. Neale were involved in the experimental work. Mr. R. Philpott, Mr. G. Scopes, and Mr. S. Blackaller, in particular, gave their time with enthusiasm. Mr. C. Hargreaves developed the software which controlled the data logging and processing during experiments.

Photographs of the experiments were taken by Mr. A. Chipling, Miss J. Gurr and Mr. R. Packer, who kindly carried out the final processing of photographs in chapter 2 and the microfilm throughout the remainder of the thesis. Microfilm was mostly generated by computer using a software package written by Dr. C. Burgoyne. Hand drawings and other graphical work were expertly completed by Mrs. A. Langford.

A finite element package developed by Dr. U. Treub and Mr. D. Bates was used. Both developers made their time freely available in helping overcome problems encountered during the work. The author is grateful to Dr. E.J. Fahy of Dupont, Dr. S. Byrne and Dr. D. Bogle, both formerly of Chemical Engineering Department at Imperial College, for their advice in the choice of satisfactory non-linear equation solution routines. The staff of the Lyon-Playfair and Civil Engineering Libraries were generous with their time in tracing references during the literature survey.

Typing of the manuscript was started by Ms. G. Vickary and ably completed by Mrs. S. Wright.

Finally, the author remains indebted to his family and to colleagues and friends at Imperial College for their continuous support and advice throughout the project and the proof reading of this text.

Notation

- A = width of panel
 A_{pl} = cross-sectional area of a plate under inplane compression.
 A_R = cross-sectional area of a ring stiffener.
 A_{st} = cross-sectional area of a stringer stiffener.
 B = length of panel in the direction of uni-axial inplane load.
 D_{st} = depth of stringer stiffener
 $D'U_{ij}^l$ = transformation matrix representing the relationship between global degrees of freedom and local displacements at a point l . (§7.4.1.)
 E = Young's modulus ($=205000\text{N/mm}^2$).
 F_i = stress resultant vector at a point (§7.5.3).
 H_i = operator on a local velocity vector which generates stress resultant vector (§7.5.3).
 I_s = 2nd moment of area of a stringer stiffener, about its neutral axis.
 M = moment component of the stress resultant vector at a point on a yield line.
 M_O = full plastic moment at a point $= \frac{1}{2} t^2 \sigma_y$.
 N = (used as a force) normal component of the stress resultant vector at a point on a yield line.
 N = (used with a vector) number of degrees of freedom of a system.
 N_O = full plastic normal stress resultant at a point on a yield line $= t \sigma_y$.
 N_{st} = number of stringer stiffeners in a cylinder.
 P = axial load capacity of a panel under inplane load.
 P_O = maximum axial load capacity of a panel under inplane load $= 2At \sigma_y$ (virtual work) (§7.16.1).
 R = radius of a cylinder.
 S = shear stress component of the stress resultant vector at a point on a yield line.
 S_O = full plastic shear stress resultant at a point on a yield line $= \frac{1}{\sqrt{3}} t \sigma_y$.
 W_k = total internal virtual work vector.
 a = half width of a panel $= A/2$.

- b = half length of a panel = $B/2$.
 l_{st} = eccentricity of the centre of area of a stringer stiffener from the mid-surface of a panel (§8.18.1).
 f = redundant yield line stress factor, (§7.14.3).
 i, j, k, l = subscripts, superscripts used for vectors (also used to denote summation - see conventions below).
 m = non-dimensional moment component of stress resultant at a point on a yield line (Table 7.1).
 m = (used as a subscript, superscript) denotes a particular component of a vector
 n = non-dimensional normal stress component of the stress resultant at a point on a yield line (Table 7.1).
 r_s = $1/\sqrt{3}$ (Table 7.1).
 s = non-dimensional shear stress component of the stress resultant at a point on a yield line (Table 7.1).
 t = thickness of a yield line = thickness of a panel.
 t_{st} = thickness of a stringer stiffener.
 $\bar{u}, \bar{v}, \bar{w}$ = vectors
 $\hat{u}, \hat{v}, \hat{w}$ = unit vectors } see conventions
 Δ_j = vector representing the general degrees of freedom of a structure.
 $\dot{\Delta}_j$ = a set of relative velocities (displacement rates) at collapse of a structure, expressed in terms of the general degrees of freedom of the structure. (§ 7.8.1.)
 λ = load factor
 Γ_k = residual work vector, the difference between internal and external virtual work (§7.9.1.).
 ϕ = ridge angle of yield line 1. (figure 7A1.1(b)).
 α = spiral angle of a sloping ridge line (figure 8.2(a)).
 δ = central lateral deflection of a panel.
 δ_0 = initial central lateral deflection of a panel.
 $\delta' u_1^l$ = displacements at a point on a yield line expressed in terms of the local axes at the point (§7.4.1).
 ϵ_0 = an externally set error tolerance (§7.13.1.)
 ϵ_j = a vector of small values to increment Δ_j . (§7.9.1.)
 ν = Poisson's ratio (= 0.3).
 ρ = uniformly distributed lateral pressure on a panel.
 ρ_0 = reference uniform lateral pressure. (§7.6.1.)

σ_y = yield stress of material, § appendix 2A1. (taken as 355 N/mm² in figures 7.39 - 49)

Conventions

Two conventions are used in Chapters 7 and 8 to simplify the vector algebra presented.

A single bar over a letter denotes a vector of arbitrary length e.g. $\bar{v}$.

A vector will be represented for algebraic purposes by a letter with a single subscript of i, j, k, l. Thus $\bar{v} \equiv v_i \equiv v_j$ etc.

A superscript will be used as an index to denote a specific location on the structure. Thus for example dw_j^l denotes the particular values of the vector dw_j , at the point l. In addition the superscript index will be used to denote a specific term of a vector. e.g. ϵ^m is a specific term of the vector ϵ_j , §7.9.1.

An inflection over a letter denotes a vector of unit length.

Thus $\hat{v}$ where $\hat{v} \cdot \hat{v} = 1.0$

The particular unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ refer to the unit vectors along the (x, y, z) axes, respectively, in a right-handed cartesian coordinate system.

A single row vector, e.g. v_i is a one dimensional tensor.

A rectangular matrix, e.g. a_{ij} , is a two dimensional tensor.

An accepted convention in tensor algebra is used in this text.

The subscripts (and superscripts) i, j, k, l, are used without discrimination.

For example $u_{ij} \equiv u_{kl}$.

The order of subscripts (and superscripts) is important. Thus

$u_{ij} \equiv u_{kl}$ or $u_{ij} \equiv u_{lk}$ but $u_{ij} \not\equiv u_{ji}$ or $u_{kl} \not\equiv u_{lk}$.

A repeated subscript denotes a summation carried out over the range of values denoted by the repeated letter.

Thus

$$a_{ij}.b_j \equiv \sum_{j=1,N} a_{ij}.b_j$$

where N = no. of rows of b .

A repeated superscript denotes a summation carried out at all the specified points in the structure. Thus:

$$\text{or} \quad w t^{\ell} d w_K^{\ell} \equiv \sum_{\ell=1,N} w t^{\ell} . d w_K^{\ell}$$

The convention is limited in this text as follows:

A subscript and superscript of the same letter does not denote summation. Thus $a^{\ell}.b_{\ell} \neq a^{\ell}.b^{\ell}$. The convention is also limited to usage outside and inside function operators.

Thus $H\{ \}$ denotes a function (§7.5.3)

Then $F_i = H_i\{\delta'u_i\}$.

These conventions do not correspond exactly with standard covariant and contravariant notation systems of tensor algebra (241). Nonetheless the conventions are consistent and result in simple and concise equations.

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CHAPTER 1.

The Behaviour of Unstiffened and Stringer Stiffened Cylinders:
A Literature Review.

1.1 Introduction

1.1.1 The use of stiffened cylinders

The cylinder is a standard structural component in many types of construction from space vehicles to submarines. Its geometry enables it to resist external pressure effectively and when appropriately stiffened it can facilitate a highly efficient use of material. It is also an obvious structural form for minimising drag forces caused by fluid motion and it is for this reason that most fixed offshore structures are composed of tubular members.

These tubulars are usually stocky in the standard "jacket" structure. Recently, however, there has been a move towards the use of lighter stiffened cylinders which make more efficient use of materials. This type of structure, while new to offshore construction, is reminiscent of the geometry of aircraft and space vehicles. The offshore structures are designed to resist stresses close to material yield stress, but nevertheless they share many of the stability problems of aircraft shells. As a consequence the design strength of these stiffened cylinders, though lying between two relatively well understood extremes, is not well known and until recently has been the subject of little research.

Many of the doubts surrounding the elastic behaviour of shells have been resolved in recent years. In addition the increased complexity and speed of digital computers has enabled the solution of many elasto-plastic problems of flat plated structures. When research funds became available for the investigation of the elasto-plastic behaviour of cylinders, many researchers took on the challenge of extending existing techniques and the subject is currently an area of extensive research effort.

1.1.2 Scope of literature review

The theoretical and experimental research into the behaviour of shell structures is approximately a century old, and as time has progressed the number of papers issued annually has risen exponentially. Sechler (1) estimated that in 1960 the annual rate of issue of papers, N , obeyed a

rule $N = e^{0.074n} - 1$, where n = no. of years since 1886. There are probably about 10,000 papers now published under the general heading of shell buckling. It is not the intention of the present author to give a general and complete survey of the work available. Rather it is the intention to concentrate on a number of areas specifically relevant to this thesis, after a general review of the subject. A historical perspective is essential since much of our current knowledge has been acquired through a bias forced on the subject by outside circumstances. The state of the art in respect of unstiffened cylinders will be examined as it provides the basis for the fundamental equations governing stiffened cylinder behaviour and an insight into the problems which occur to a reduced extent in stiffened cylinders. The literature available on the behaviour of longitudinally (stringer) stiffened cylinders will be reviewed and a later section on the collapse of panels will also be included as much of the theoretical work of the thesis concerns the behaviour of curved panels, which form part of a stiffened cylinder.

1.2 Early research on unstiffened cylinders

1.2.1 Governing equations

Prior to 1900 the equilibrium of shells was considered by Kirchhoff, Lamé and Love. However, the first notable contribution to the stability of cylinders was made by Lorenz who studied the behaviour of cylinders under axial load and also under pressure (2,3). Timoshenko published his first papers on cylinders under axial compression in 1910 (4,5), and Von Mises solution to the buckling of cylinders under pressure appeared in 1914 (6). Southwell was also developing buckling theories for cylinders at this time (7,8).

Von Mises solution for lateral pressure buckling

$$\rho_c = \frac{Et}{R} \left\{ \frac{1}{12(1-\nu^2)} \left[\frac{t}{R} \right]^2 \left[n^2 - 1 + \frac{\lambda^2(2n^2 - 1 - \nu)}{(n^2 - \lambda^2)} \right] + \frac{\lambda^4}{(n^2 - 1)(n^2 + \lambda^2)^2} \right\} \quad \text{where } \lambda = \frac{\pi R}{L} \quad \text{----- (1)}$$

is reasonably accurate for long cylinders.

Lorenz solution for buckling under axial load

$$\sigma_{CR} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R} \approx 0.605 \frac{Et}{R} \quad \text{-----} \quad (2)$$

remains the standard "classical" solution for axial load buckling.

Flügge (9) suggested governing equations for the behaviour of cylinders under general load. These coupled equations have become the standard reference equations and solutions obtained through them are the basis for comparison for more approximate theories (10, Chap. 6).

The coupled equations are very cumbersome for use. Donnell (11) achieved a break-through in the study of shell stability by proposing uncoupled equations for a cylinder

$$\left. \begin{aligned} \frac{Et^3}{12(1-\nu^2)} D^4 w + \frac{Eh}{R^2} \frac{\partial^4 w}{\partial x^4} &= D^4 \left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \\ R \cdot D^4 u &= -\nu \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2} \\ R \cdot D^4 v &= -(2+\nu) \frac{\partial^3 w}{\partial x^2 \partial y} - \frac{\partial^3 w}{\partial y^3} \end{aligned} \right\} \quad (3)$$

where $D^4 = (D^2)^2$ etc

$$\text{and} \quad D^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Solutions to these equations may be obtained by solving the lateral displacement function first for w , instead of solving simultaneously for u , v and w , as is required when the equations are coupled. Equations 3 were developed independently by Mushtari in the USSR (246) and have become known as the Donnell-Mushtari equations. They will be known as the Donnell equations, for simplicity, in this text.

The Donnell equations are simpler and more symmetric in their structure than most others (13). The uncoupled nature of the equations is principally responsible for their widespread use. The equations are accurate when the number of circumferential waves is greater than 3 (14). Thus they are valid for axially loaded cylinders and relatively short cylinders under lateral load.

Various modifications have been proposed by subsequent authors, especially Loo (15) whose modifications enable the equations to be used

for long cylinders under lateral pressure. (Loo's modifications have been criticised by Armenakas and Herrmann as inconsistent (16). Nevertheless their accuracy recommends them for use). Donnell (10, Chap. 6) discusses the equation 3 above very fully and proposes extra terms when $n \leq 3$. The left-hand-side of the lateral displacement equation then becomes

$$\frac{Et^3}{12(1-\nu^2)} \left\{ D^4 w + \frac{1}{R^2} D^2 \left(5 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{2 \partial^4 w}{\partial y^4} \right) + \frac{1}{R^4} \left(3 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) \right\} + \frac{Et}{R^2} \frac{\partial^4 w}{\partial x^4}$$

The amended version of equation (3) is recommended when the number of complete circumferential waves is less than 4. Sanders (17) has proposed a very accurate set of governing equations. Unfortunately a closed form solution is not possible with them, so that solutions can really only be obtained if a computer is used (13). Vlasov's equations of equilibrium of a shell (12) are also very accurate. A thorough review and appraisal of the various proposed governing equations of shells has been given in reference (243).

1.2.2 Lateral loading

It can be shown (11) that the Donnell equations produce buckling solutions for lateral pressure loading and axial loading using a lateral displacement function

$$w = w_{mn} \sin \frac{m\pi x}{l} \cdot \sin \frac{ny}{R}$$

The solutions for lateral pressure loading when minimised with respect to n , are seen to be a function of $Z = L^2/Rt\sqrt{1-\nu^2}$, the so-called Batdorf factor (18).

Equation 3 produces results for lateral load which are approx. 25% too high for $n = 2$. In the limit as Z becomes very large, the buckling stress of a cylinder subject to lateral pressure alone approaches the buckling stress of a ring. This solution is given by the Von Mises formula (equation 1) or the modified version of equation 3.

1.2.3 Axial loading

Equation (3) may be used to determine the buckling stress of a cylinder under axial compression (19). The result is

$$\sigma_{CR} = \frac{Et}{\sqrt{12(1-\nu^2)}R} \left(A + \frac{1}{A} \right) \quad (4)$$

$$\text{where } A = \frac{Rt}{\sqrt{12(1-\nu^2)}} \frac{\left[\left(\frac{m\pi}{l} \right)^2 + \left(\frac{n}{R} \right)^2 \right]^2}{\left(\frac{m\pi}{l} \right)^2}$$

When minimised with respect to A, the value of A is 1.0 and the buckling stress is the same as given in equation 2, as proposed by Lorenz (3). Donnell's modification to the equation 3, to include minor terms, does not alter the theoretical buckling stress, although it does specify a relationship between m and n, which is not given in equation 4 (10, Chap. 7).

1.2.4 Combinations of axial and bending loads

A cylinder subject to a combination of axial and bending loads may generally be treated in the same way as cylinders subject to pure axial load, if the cylinder is fairly short. When the cylinder is long the curvature of the cylinder generators and the variation of stress around the cross-section causes a flattening of the cross-section (20). The ovaling of the cross-section was studied in detail by Brazier (21) and has become known as the Brazier effect. The distortion of the cross section causes a rotation of the cylinder ends, hence reducing the effective stiffness. The load at which the stiffness is reduced to zero is independent of R/t. The result is, however, only 55% of the actual theoretical buckling stress for low R/t.

1.2.5 Correlation of theory with experiment

In 1932 Flügge (22) carried out a series of tests on celluloid cylinders under axial compression and bending and found that the experimental values fell far below the theoretically computed values. A similar set of tests by Lundquist (23) on duralumin and by Donnell (19) on steel and brass cylinders confirmed Flügge's findings. In many cases the buckling

strengths obtained were only 10-15% of the theoretical value. Not only were the recorded values much lower than predicted, but a wide scatter of experimental results seemed also to occur with apparently identical specimens.

These findings caused investigators to realise that the buckling phenomenon in cylinders is very different to the same problem in plates and struts. Three different approaches were taken to the problem in the short term.

Some authors attempted to find more accurate governing equations than had been proposed by Donnell. More accurate governing equations are appropriate (as indicated above) when there are few circumferential waves. However, in the case of axial compression, where the greatest disparity exists between theory and experiment, there are many circumferential waves and the equations published by Donnell in 1933 (11) are quite accurate.

It was claimed that boundary conditions were important. Subsequent workers (Yamaki (24) for lateral load, Almroth (26) for axial load and Sobel (25) for axial and lateral loads) used the coupled form of the Donnell equations to obtain the buckling loads of cylinders with various boundary conditions. These investigations showed that boundary conditions were important for some circumstances, but that in themselves they were not sufficient to explain the disparity between theory and experiment.

In 1934 Donnell (19) proposed that initial imperfections were important and attempted to explain the disparity for axially loaded cylinders using a combination of two "sympathetic" initial imperfections which interact under load. A yield stress failure criterion was used to determine the maximum load of the cylinder. This explanation, though not complete in itself, provides a basis for strength predictions for cylinders under different loads (27).

In the longer term a fourth approach to the cylinder buckling problem was adopted. Koiter published his general postbuckling theory in 1945 (28) and, subsequent to its more widespread availability in the early 1960's, the theory has been enthusiastically taken up by other investigators (Thompson at University College London, and Hutchinson at Harvard

University) (13). The theory calculates the critical buckling load of an initially perfect, or imperfect, structure and also the slope of the postbuckling curve at the point of bifurcation. It is, however, an infinitesimal theory and although the qualitative results derived from it are important, the theory does not provide a complete understanding of the actual deformations under load.

1.2.6 Direction of research

The understanding of imperfection sensitivity and postbuckling behaviour has taken place largely since 1960. Following the discoveries of Flügge, Lundquist and Donnell there followed a period of intense activity in the research of cylindrical shells. This was caused principally by the growth in usage of metal-skin fuselage for aircraft and was hastened by the approach of war (1). The absence of satisfactory theoretical solutions in the behaviour of shell structures caused a growth of empirical design formulae, based solely on experimental results. (Similar methods were being formulated for flat panels (29), but in this case it was to use the considerable postbuckling capacity that the methods were proposed.)

These empirically derived formulations, whilst necessary at the time, have had a strong impact on subsequent research and development. The effect has not been entirely beneficial.

Many of the experiments were poorly documented and were carried out on material with widely differing strength properties, eg. celluloid, mylar, duralumin, brass and steel, and with differing forms of construction eg. rolled, spun, with glued or welded joints. The imperfections resulting from the different construction forms are different and enhance the scatter of results. The use of materials with different σ_y/E values strongly affects results. For example, duralumin and steel cylinders tested by Donnell (19) showed different properties. Length of cylinder and practical boundary conditions can affect the critical loads of cylinders - boundary conditions by as much as 20% (26).

In the absence of a reliable theoretical background these results were accepted without discrimination. Attempts are now being made (30) to correlate the data obtained from these early (and later) experiments, but information is frequently inadequate.

The legacy of empirical formulae, derived in isolation from theory, is also to be found in current design codes (31). The disparity is particularly wide in the $R/t = 50-200$ range, which is the relevant range for offshore structures. The pursuit of empirical formulae in the absence of a full theoretical understanding has led to some false concepts reaching a wide level of acceptance, as noted by, for example, Seide and Weingarten (32).

In the last 25 years the theoretical approach has been followed more enthusiastically and an essential understanding of the behaviour of cylinders is now generally accepted. The inherent scatter in buckling loads causes practical buckling values to be given by empirical formulae. However, these formulae can now be put on a sounder basis. The use of accurate numerical models will, in the very near future, produce a wide range of theoretical results which can be expected to agree with experiments with the same parametric values. These results will enable empirical strength formulae to be given with a greater degree of confidence. As a result it may be possible to ignore many of the older experimental results.

1.3 The Buckling of Unstiffened Cylinders

1.3.1 Influence of boundary conditions on the classical buckling load of a cylinder in axial compression

Flügge (22) pointed out that axisymmetric buckles are propagated from the ends of a pinned cylinder in axial compression. This is a result of the containing effect at the ends of a cylinder where the end conditions prevent the Poisson expansion. Flügge proposed this effect in an attempt to explain why the celluloid cylinders which he tested had failed at $0.52-0.65 \sigma_{Cl}$. His cylinders were, however, quite long ($l/R = 1.76-5.0$) and it is improbable that the effect was a major contribution to the decrease in buckling load. When cylinders of large R/t , as tested by Donnell and by Lundquist (19,23), failed at stresses as low as $0.15 \sigma_{Cl}$, the axisymmetric propagation of buckles could not be accepted as the cause.

Work was carried out in the early 1960's by Stein (33,34), Fisher (35),

and later by Hoff and his associates (36-39) and by Ohira (40,41) on the effect of various boundary conditions on the buckling load of a semi-infinite cylinder. The coupled Donnell equations were used and solutions obtained in closed form and also by computer. An axisymmetric prebuckling deformation of the form $w = B \sinh \lambda_1 x \sin \lambda_2 x + B_2 \cosh \lambda_1 x \cos \lambda_2 x + w_p(x)$ was assumed and a buckling deformation of the form $w = e^{px} \sin(n\phi)$ was used. Hoff found solutions for 4 simply supported boundary conditions for a semi-infinite cylinder and concluded that shells of all lengths, whose edges are free to rotate (w, x not restricted) and are free to displace in the circumferential direction (v not restricted), buckle at one half the classical critical value of the axial compression stress. When the edges are completely free the critical stress is $0.5 \sigma_{cl}$ for axisymmetric buckling, and $0.38 \sigma_{cl}$ for multilobed buckling (41). Almroth (26) solved the nonlinear Donnell equations resulting from non-membrane prebuckling stresses using a finite difference technique. Four simply supported and clamped boundary conditions were considered. No prebuckling radial movement of the cylinder ends (finite length) was allowed. The effect of length was plotted for $R/t = 100$ and it was found that, for $l/R < 1.0$ (simply supported), and $l/R < 2.0$ (clamped), the effect on buckling load is strong. A sharp minimum in buckling load is obtained for $l/R \approx 0.3$ (simply supported) and $l/R \approx 0.7$ (clamped). However, for very short cylinders the effect of decreasing length is an increase in capacity. It is important to note, however, that for practical boundary conditions and medium and long lengths, the critical stress is reduced by as much as 20% for simply supported ends and by 7-10% for clamped ends. A figure of 7% reduction for clamped ends is widely accepted (42).

1.3.2 Other loading conditions

A prebuckling membrane stress state is generally assumed for the loadings. Sobel (25) considered a hydrostatic loading condition and presented results for $R/t = 100$. He showed that the results of Von Mises (simply supported ends) (6) and of Nash (clamped ends) (135) are contained within a more general set of results. Sobel's results show that the effect of clamping is significant for $l/R < 1.0$, and the axial restraint is important for $0.5 < l/R < 2.0$. For $l/R \approx 1.0$ restraining inplane displacements ($u = 0$) as opposed to inplane forces ($N_x = 0$) results in a 25% increase of strength.

Yamaki (24,43) considered cylinders under lateral load for various boundary conditions. It was found for practical simply supported ends that the prebuckling deformations had a slightly detrimental effect for $8 < Z < 30$; but for longer cylinders the prebuckling deformations had no effect. For shorter cylinders, $Z < 8$, the effect of the prebuckling deformations was to enhance the buckling pressure greatly. The prebuckling deformations had a positive effect on buckling capacity for clamped edges, $Z < 20$. Serious errors in computed buckling strength are obtained for $Z < 10$, if prebuckling deformations are ignored. It was suggested that cylinders of $Z < 10$, would fail by axisymmetric yielding.

Rehfield and Hallauer (44) have considered the effect of boundary conditions on a longitudinally loaded panel, with a membrane prebuckling stress state. Their results show that the various boundary conditions affect the Z value at which panel buckling stress approaches that of a full cylinder.

1.3.3 The effect of length on buckling stresses

Batdorf (18) showed that the parameters $Z = L^2/Rt \sqrt{1-\nu^2}$ for a cylinder, and $Z = b^2/Rt\sqrt{1-\nu^2}$ for a curved panel, emerge from the Donnell shallow shell theory (11) as the relevant non-dimensional parameters for cylindrical structures under pressure and longitudinal axial load. Donnell's equations are inaccurate for long cylinders under pressure and as a result the R/t parameter of a cylinder enters the buckling formulae for large Z values. The effect of boundary conditions becomes important for short cylinders (45) as mentioned above. Almroth's analysis (26) also includes the prebuckling axis-symmetric displacements due to the radial constraints imposed at the ends. His analysis shows that the prebuckling deformations can reduce the critical buckling stress for some lengths. Yamaki's work (24,43) is of importance for pressure and axial loading.

Reference (46) by Tennyson *et al.* is of note when studying the behaviour of short cylinders. The authors obtained elastic buckling interaction capacities for short cylinders of $R/t = 100$. A convex interaction was found to be inapplicable for clamped ends and $Z \leq 20$. For $Z \leq 10$ the commonly used (and supposedly conservative) linear interaction between

pressure and axial load was shown to be unconservative. The theoretical study predicted two circumferential waves for most of the buckling range at low Z . However, at $P \approx 0.18 P_{\text{crit}}$, there was a sudden change to multi-mode buckling. The sudden change in behaviour was related to the critical axi-symmetric buckling length, which for clamped ends is $\ell = 2\pi (Rt)^{1/2} / (12(1-\nu^2))^{0.25}$. (For $R/t = 100$ this length corresponds to $Z \approx 12$). The concave interaction for elastic buckling was verified using almost perfect spun-cast epoxy cylinders.

The points raised by reference (46) are important for elastic buckling, but may, however, be of lesser importance for steel structures (47). Bushnell (48) states that cylinders of $R/t < 90$ in axial compression are insensitive to imperfections since they fail by the formation of elasto-plastic bulges near the ends. Ostapenko *et al.* (49,50) have published experimental results which show that axially loaded cylinders of $R/t < 100$ fail by forming an elasto-plastic axisymmetric bulge close to the ends. Reference (51) also contains similar results. In such cases the elastic critical stresses are less important.

1.3.4 Imperfection sensitivity of axially loaded cylinders

Donnell (19) suggested that the discrepancy between theoretical buckling loads and experimentally recorded values was due to initial imperfections. He suggested an initial imperfection of the form

$$w = A_{11} \cos \frac{\pi x}{\ell_x} \cdot \cos \frac{\pi y}{\ell_y} + A_{20} \cos \frac{2\pi x}{\ell_x}$$

The second term was introduced to eliminate the average circumferential stretching of the surface due to the first imperfection term. A first yield criterion for failure was used.

This attempt to explain the discrepancy of theory and experiment was unsuccessful, but nonetheless paved the way for Von Karmen and Tsien (52) to assess the postbuckling behaviour of a cylinder with an initial imperfection of the form

$$w = A_{00} + A_{11} \cos \frac{m\pi x}{a} \cos \frac{\pi y}{R} + A_{20} \cos \frac{2m\pi x}{a} + A_{02} \cos \frac{2\pi y}{R}$$

The important contribution of this behaviour was that it established a

postbuckling path for an axially loaded cylinder.

With the advent of the computer it became possible to consider an increased number of initial imperfection terms (54-57). At one stage it was thought that minimum postbuckling loads could be deduced (53,58), but this proved false. In any case the minimum postbuckling loads suggested were often quite conservative, but sometimes unconservative.

Madsen and Hoff (59) established limits to the minimisation process by fixing a minimum relationship between the number of circumferential waves and the cylinder R/t ratio. The main thrust of the research effort, to find the postbuckling path and thereby propose a minimum design strength, proved ultimately unsatisfactory.

Donnell and Wan (27) followed a slightly different line of research. An initial imperfection, similar to that used by Von Karmen and Tsien (52) was used:

$$w_0 = a_0 t \left(\cos \frac{\lambda x}{R} \cos \frac{ny}{R} + b \cos \frac{2\lambda x}{R} + c \cos \frac{2ny}{R} + d \right)$$

The potential energy was minimised with respect to n , λ , b , c . The value of the governing initial imperfection a_0 was judiciously chosen as

$$a_0 = \frac{U}{\pi^2 t^2} (l_x)^{1.5} (l_y)^{0.5}$$

where l_x and l_y are the wavelengths in the longitudinal and radial directions respectively. U is a nondimensional imperfection parameter and is closely allied to the similar imperfection parameter defining the strength of struts.

By choosing different values of U the authors (27) were able to produce stress-end shortening curves. In addition, they produced values of the maximum cylinder loads where either first yield or elastic peak load determined the strength. The results are particularly impressive when one realises that only hand calculations were possible. (Donnell (60) also applied this method to cylinders under lateral pressure, but the results appear to be ironically less satisfactory than for axial load).

Donnell and Wan's analysis has been criticised as being theoretically inconsistent because the potential energy function was minimised with respect to variable initial imperfections, and also because the circumferential wavenumber is discrete, not continuous (13). The criticisms are mathematically justified; however, more rigorous derivations can and have been given which do not alter the final conclusions reached in Reference 27 (59).

An interesting result of (27) is that very reasonable values of U can explain the wide scatter of results for differing materials. Since quite different test values on different materials are explained by the same value of U , much greater caution should be exercised in grouping results from tests on different materials together. Possibly the greatest obstacle to the wide use of Donnell and Wan's work is that the parameter U has not been measured satisfactorily for a wide range of cylinder constructions.

1.3.5 Koiter's postbuckling theory

Koiter's Ph.d. thesis of 1945 (28) is a significant contribution to postbuckling theory. The work did not reach a wide audience until 1963 when a translation became available in English (28). The postbuckling theory is general and it is not the intention to assess it in this survey. Excellent introductions to the subject have been given by Hoff (13) and by Brush and Almroth (61) and to the more general theory in a paper by Budiansky (62). Comment will be restricted to the result for a cylinder under axial load. It can be shown that an imperfection causes a reduction of the theoretical bifurcation load below that given by the classical buckling theory (equation 2). For an axisymmetric imperfection, the ratio ρ of theoretical buckling stress of an imperfect cylinder to the classical critical stress of a perfect cylinder is given by

$$\rho\phi = \left[\frac{4}{27}(1-\nu^2) \right]^{\frac{1}{3}}(1-\rho^2)$$

where $\phi = k.R/t$ is some imperfection (63).

Experimental verification of Koiter's theory has been obtained for some structures (64, 65) and the theory qualitatively explains the difference between theory and experiment for cylinders under standard loadings (67).

The theory also allows an assessment of imperfection sensitivity through a postbuckling coefficient. The postbuckling coefficient has been calculated for both unstiffened and stiffened cylinders. The work of Budiansky and Amazigo (66) in particular has been incorporated into current design recommendations (68) for design of unstiffened cylinders under hydrostatic and lateral pressure loading.

As noted above, some of the published theories are not entirely rigorous. These difficulties can often be overcome by more rigorous mathematical analyses. Some theories have, however, been proposed which are unsound from the engineering point of view. Amongst these are the snap-through theories proposed by some authors (55,69,70). The term "snap-through" is perhaps unfortunate in that it suggests that some discontinuity exists in the deformation behaviour of a cylinder. Such a discontinuity is central to some theories. Donnell (10) dismisses such a possibility and affirms that the loading behaviour of a structure is always continuous.

1.3.6 Experimental results for axially loaded cylinders

Timoshenko (71) traces the first experimental assessment of the strength of tubes to Fairburn in 1845 (72). As noted above the early work of Flügge, Lundquist and Donnell showed that the strength of cylinders under axial load was uncertain. This gave rise to empirical design rules as a result of tests in aircraft building companies (73,74,18). Many of these rules are still used today. The buckling curves for axially loaded cylindrical panels in the Det norske Veritas design recommendations for offshore structures (68) are essentially the same as those given by a NACA report published in 1947 for aircraft structures (74). Amongst the results on which reference (74) was dependent were results by Lundquist on duralumin cylinders (23) and by Wilson and Newmark (75,76) on steel cylinders, some of which had an L/R ratio as large as 84.

Schultz (30) attempted to gather together as many of these results as possible, and it is hoped that with increased knowledge of the buckling phenomenon it will be possible to treat these results more judiciously (31).

Efforts are now being made to measure initial imperfections more widely. Arbocz has been an advocate of large scale imperfection measurements on both small-scale and prototype models (78-82). In addition a data imperfection bank has been set up so that data from various sources can be assembled together (82). The intention is that, with initial imperfection information available from widely differing sources, realistic less conservative empirical design rules will be possible for various different forms of construction.

Of particular note are experimental results of Tennyson on near perfect cylinders. The manufacturing technique allows consistently high quality cylinders to be produced (83,46,84,85) and tests on these indicate that buckling loads close to the theoretical value can be obtained for a variety of loadings (46).

Another aspect of buckling theory which has been clarified by careful experimentation is the disparity between the number of theoretically predicted circumferential waves in buckling and the observations made in experiments. Buckling theory of isotropic cylinders predicts a greater number of buckles than are observed in practice. Esslinger has shown that the circumferential wave number n decreases along the minimum load path in the unstable buckling phase (70). Photographs taken during experiments have tended to confirm this. High speed photographs have been taken during the buckling phase and have shown that initially small buckles form and later join to form large buckles (83,86).

1.3.7 Elastic buckling of cylinders in bending

Von Karmen noted in 1911 that the stresses around the cross section were such as to cause ovaling of the cross-section (20). Brazier studied this effect in more detail but his solution is quite conservative (21). Seide and Weingarten used the Galerkin method to obtain the elastic critical buckling load for a cylinder in bending (32). Their results show that critical stresses in bending only very slightly exceed the uniaxial critical stresses. Yao (87) considered the load capacity by using the large deflection equations and an assumed ovaling failure pattern.

The consideration of a cylinder loaded in bending is an interesting example of the misconceptions which arise in the absence of rigorous

experimental-theoretical considerations. In 1932 Flügge's experimental results (22) indicated that a cylinder buckled in pure bending at 1.3 times the value of a cylinder in uniaxial compression. Donnell's experiments in 1934 (19) indicated that a factor of 1.4 might be appropriate, although he felt that the value should be 1.0 according to theory. For many years a value of 1.3 was accepted and observers attempted to correlate their results with this figure! The matter was settled rigorously by Seide and Weingarten in 1961 (32). Nevertheless as late as 1970 reference (10) was advocating a factor of 1.4, and design codes (68,88) still appear to allow a factor of 1.3 on maximum stress in bending relative to the maximum uniaxial stress. Bushnell (48) accepts the Seide and Weingarten solution provided linear theory can be assumed to apply.

1.4 Elasto-Plastic Behaviour

1.4.1 Plasticity theories

The plastic buckling of columns was a subject of research and controversy in the early years of this century (89-91). The study of plastic buckling of plates was researched during the early post war period (92,93). A theoretical problem was encountered at that time. It was found that bifurcation loads calculated using the simplest incremental, or flow, theories of plasticity consistently overestimated the buckling loads of plates in tests. Calculations based on the less respectable deformation theories of plasticity gave reasonably good agreement with test results (94). This theoretical inconsistency is still to be resolved. Hutchinson (95) and Onat *et al.* (96) have shown, however, that the discrepancy between flow theory and deformation theory largely disappears in the presence of imperfections.

1.4.2 Bifurcation theory

Bifurcation theory in the plastic range does not present any major theoretical obstacles in itself, apart from the choice of flow or deformation theories. A number of extensive literature reviews exist on the subject, (94,97). Postbuckling behaviour and imperfection sensitivity theories have followed the Koiter elastic postbuckling approach. No general theory, analagous to Koiter's elastic theory is, however,

applicable. This results from the complications which accompany the introduction of elasto-plastic behaviour. Nevertheless Hutchinson has discussed, in considerable detail, a model which is applicable to cylinders under axial compression and to spherical caps under lateral load (95).

His conclusions are that elasto-plastic models are more imperfection sensitive than elastic models for small imperfections, where the buckling stress is close to the first yield stress. This conclusion supports a judgement based on the fact that even columns, which have a stable postbuckling behaviour, exhibit an imperfection sensitivity where $\sigma_{CR} \approx \sigma_y$. Reference (95) is illustrated by an example of a spherical cap under pressure. The conclusions reached are supported by those of Leckie (98).

Despite the imperfection sensitivity exhibited by theoretical models Hutchinson concludes that the disagreement between theory and experiment is likely to be less in the elasto-plastic region than in the elastic region for shell structures. This is because the imperfections are likely to be smaller for the range of slendernesses where plasticity becomes important. (Most authors assume that the initial deviations from the perfect shape of a cylinder are proportional to the shell R/t ratio). Bushnell (99) supports the idea that shell structures subject to elasto-plastic buckling are not very imperfection sensitive in practice.

Gerard (101) has also produced a theoretical model which predicts the strength of axially loaded cylinders in the elasto-plastic region. Results are given in terms of the elastic critical load and a single plasticity factor which is calculable from the known material characteristics. Gerard claims a good correlation between theory and experiment for aluminium cylinders of $R/t \approx 30-50$.

1.4.3 Experimental results

There are not many experimental results available for unstiffened steel cylinders in the elasto-plastic and plastic range. Many of the tests in the $R/t = 100$ range have been on mylar or aluminium cylinders. It is not entirely satisfactory to extrapolate the results from these materials to structural steel. Some tests have recently been conducted on cylinders

with closely spaced ring stiffeners (100). The scope of the present study excludes a fuller consideration of this work.

Sobel and Newman (51) have published results of 4 tests for $R/t \approx 18.5$ in axial compression. They found that the results could be accurately predicted by STAGS finite difference program and by a plastic buckling analysis due to Gerard (101). Predictions based on a model by Batterman (102) were too high.

A series of tests have been carried out at Lehigh University on axially loaded cylinders with $R/t = 42-124$ (49,50). The conclusions of the test series are

1. End conditions and initial imperfections are not very significant in affecting the failure load.
2. Residual stresses are not very important (Odland (103) found differently in tests on ring stiffened cylinders. Miller *et al* (104) found the strength of ring stiffened cylinders subject to pressure load to be sensitive to residual stresses).
3. The DnV (1974) (68) design recommendations best predict failure load. Other predictions, eg. API (88) are very conservative.
4. The governing geometric parameter for strength is

$$\sqrt{\frac{E}{\sigma_y}} \frac{t}{R}, \text{ not } \frac{E}{\sigma_y} \frac{t}{R} \text{ as commonly used.}$$

(Tests by Sherman (105) on cylinders in bending support this conclusion).

5. The type of initial buckling was influenced by the parameter

$$\alpha = \frac{1}{2} \frac{E t}{\sigma_y R}$$

A combination of ring and lobular buckles developed for $2.4 < \alpha < 3.6$, whilst rings only appeared for $\alpha > 3.6$ and lobular buckles only for $\alpha < 2.4$.

The discrepancy between the results of the Lehigh tests, which fell on a smooth curve and previous tests which showed a considerable scatter was attributed to the uncertainty of material yield strengths and also to the length effect. The Lehigh tests were for $l/R = 3.2-6.0$.

A series of three tests carried out at Imperial College for $R/t = 250$ and 150 on cylinders with a construction representative of that used in actual construction of offshore structures appears to corroborate the results of the Lehigh tests (107).

Axial tests have also been carried out by Batterman on cylinders of $9.7 \leq R/t \leq 116.6$ (102). Axisymmetric failure was recorded in all cases. Samuelson tested three axially loaded cylinders of $R/t \approx 117$, $l/R \approx 1.07$ and obtained creep buckling failure (106). Creep buckling has been looked at by various authors including Hoff (108,109) and Rickards *et al* (110). A series of tests results are given by Walker *et al*. (111) for ring stiffened cylinders where the ring spacing is $l/R \approx 1.0$ and $R/t = 150$ or 250. The models of $R/t = 100$ all failed by axisymmetric failure close to a ring stiffener, but although the geometry was nominally identical for different models there was a considerable scatter in the failure loads.

Miller (121) correlated results obtained by many investigators on the buckling of axially compressed cylinders (unstiffened and ring stiffened). The results of the various tests are grouped according to material type, and a failure criterion based on Donnell and Wan theory (27) is found to be applicable in both the elastic and inelastic range. In the inelastic range Miller includes an extra plasticity reduction factor, which should not be required as the Donnell and Wan theory is based on either yield or maximum load. Miller also compares results of the effect of ring stiffener spacing with the theoretical predictions based on (71). Reference (71) may, however, be outdated for such a comparison. More up to date theoretical predictions are perhaps more appropriate. Of particular note is the absence of satisfactory test results for fabricated cylinders.

1.4.4 Combined pressure and axial loading

A recent paper by Galletly and Pemsing (47) discussed the various test data available on buckling of cylinders subject to lateral pressure and axial load. In addition the paper discusses the formulae available at present for interactive buckling, and compares them to results obtained using the BOSOR5 package. The authors conclude that a linear interaction for pressure and axial loading is appropriate. It should be noted, however, that the BOSOR5 package predicted strengths for axially

loaded cylinders gave results which were up to 40% unconservative when compared with test results obtained by Batterman, for $R/t \approx 115$ (102).

The study of cylinders subject to pressure and axial load forms an important part of current research (112,113). Various forms of interaction formulae are allowed for unstiffened cylinders by various codes (112). The most recent papers indicate that all codes are quite conservative in dealing with pressure especially where ring stiffener spacing is small (100).

Current research programmes in both Europe and the United States are progressing rapidly to provide clearer insight into the behaviour of unstiffened and stiffened structures of particular use to the offshore construction industry. Recent work by Chicago Bridge and Iron Co (112), sponsored by a group of oil companies, is of note. The present research programme in Britain, of which the experimental work in this thesis forms a part, is the major European experimental contribution. Most of the experimental work at present is, however, directed towards ring stiffened cylinders, reflecting the current needs of the industry (100,113-120).

1.4.5 Plastic bending of cylinders

The plastic collapse of unstiffened cylinders in bending for low R/t has been researched in recent years (105, 122, 123). Investigations in the United States have been carried out to determine the largest R/t for which a full plastic hinge will develop at collapse. The AISC code (124) accepts a value of $R/t = 1650/F_y$ (where $F_y = \text{k.s.i}$) from results of tests by Shilling (122). Sherman (105) tested long tubes in the $R/t = 18-51$ range ($\sigma_y \approx 300 \text{ N/mm}^2$) and questioned the AISC value. In a later paper the results are reassessed and the AISC value is affirmed (123).

Reddy (125) and Gellin (126) have looked in greater detail at the process of collapse of tubes in bending. A number of observations have been at variance with the accepted plastic bifurcation theory. Calladine (127) re-examined these observations and concluded that collapse is likely to result from the propagation of end effect ripples along the cylinder length. These end effects are very similar to the axisymmetric buckles, first discussed by Flügge (9), which propagate from the ends of

an uniaxially loaded cylinder. The axisymmetric buckles of an axially loaded cylinder are subject to exponential decay. Calladine was unable to explain why the decay is not very rapid for buckles induced by bending.

Bushnell (48) summarised the state of the art in respect to plastic bending failure of tubes used in pressure vessel design. The effects of pressure were included. The predicted behaviour from the BOSOR5 package is not completely in agreement with observed behaviour from Sherman's tests, but this may be due to material modelling within the program and the effects of residual stresses and imperfections on experimental behaviour. The program does, however, predict bifurcation with the formation of ripples at the intrados of bending at a load below the maximum load.

It is concluded that although general results are possible for plastic bending the detailed phenomena still require further research.

1.4.6 Codes of practice

Practical methods for defining the buckling resistance of unstiffened shells are in general empirical or semi-empirical, despite the recent research in the plastic range. Odland and Faulkner (128) have reviewed the approaches taken by the various codes of practice in establishing an elasto-plastic buckling load. The approach is invariably to take some knocked down value for the initial elastic buckling stress and to this apply a plasticity reduction factor.

Bornschauser (129) has shown that the safety factors, plasticity reduction factors and knockdown factors employed by the various design codes result in widely varying design loads for axially loaded cylinders for $R/t \leq 300$. The ASME code (130) is the most conservative in its treatment, the Austrian ÖNORM B-4650 (131) is the least conservative. A later paper by Bornschauser discusses the various factors contributing to the disparity of load capacity predicted by the various codes in further detail (31).

Recent trends in specification of design loads for axially unstiffened cylinders are the abandonment of the elastic knockdown factor. This approach leads to considerable simplification of load assessment for

axially loaded cylinders. A single equation can be used to relate the characteristic strength, σ_u/σ_y , to the non-dimensional slenderness $\sqrt{\sigma_y/\sigma_{CR}}$ in a manner similar to that employed for columns (132).

1.5 Orthotropic Cylinders

1.5.1 Governing equations

Early work on the buckling of orthotropically stiffened cylinders was carried out by Flügge in 1932, by Dshon, and by Taylor (see Lakshmikantham *et al.* (133)). Van der Neut (134), Nash (135), Kendrick (136) and Bijlaard (137-139) extended the theory. Van der Neut's work, in particular, showed that use of outside ring stiffeners led to greater buckling loads under hydrostatic pressure than inside ring stiffeners.

The most commonly quoted equations covering the buckling of orthotropic cylinders are those derived by Baruch and Singer (140). The governing equations are proposed in a simple form by considering the reference axes to lie at the centre of the cylindrical shell. The stiffener eccentricity is considered relative to these axes. Strains are expressed in linear terms only and Donnell's uncoupled equations for isotropic cylinders are generalised for orthotropic behaviour. A uniform membrane stress state is considered to exist in the prebuckling state. Individual sinusoidal buckling displacements are considered, in a form which satisfies simply supported end conditions. A determinant is evaluated which is linearly dependent on the load. The buckling or bifurcation load is that load at which the determinant of the displacement function becomes zero (61). The minimum bifurcation load may be evaluated by successively computing for different buckling wavelengths.

Baruch and Singer (140) correlated these calculations qualitatively with known results for both ring stiffened cylinders and stringer stiffened cylinders. Theoretical calculations from the paper suggested that external stiffening results in significantly higher buckling loads than internal stiffening. This result agrees with other theoretical and experimental results (134).

The majority of authors considering the buckling of stiffened cylinders

have subsequently followed the approach of reference (140) because of its simplicity (141). Current European codes of practice use the approach to establish the orthotropic buckling load of stiffened cylinders (68, 142). Miller (143) proposed an effective width of panel to account for panel local buckling when considering the cross-sectional properties within the orthotropic buckling approach.

Singer *et al* (144) proposed solutions for the buckling of cylinders with clamped ends.

A more rigorous development of orthotropic buckling theory was advanced by Geier (145) with results for simply supported ends given in reference (146). An arbitrary reference plane was considered for the axes. The resulting equations are considerably more complicated than those of reference (140).

The results of reference (140) are accurate asymptotically to the linear buckling solution for an isotropic shell (as the stiffening decreases to zero). This author has compared the results of the two approaches, §4.2.3, and found that a considerable error results from the use of the simpler equation of reference (140), when the area of the stiffening is large relative to the shell area and the eccentricity of the stiffener centre of area is large relative to the shell thickness. This is particularly relevant where tee stiffeners are used as rings or stringers. In such cases reference (140) overestimates the buckling load.

1.5.2 The effects of external stiffening

Van der Neut (134) pointed to the beneficial effects of external ring stiffening. Eccentrically placed stiffeners (rings or stringers) have two effects on the stiffness of the structure (144,147). The bending stiffness is directly increased by virtue of the stiffener. This is largely independent of the side of the shell on which the stiffening is placed. The second effect is the coupling of axial forces in the shell and stiffener with bending deformations. Singer *et al* (144,147) explain the beneficial effects of external stiffening in detail. Thieleman also examines the effect of eccentricity (148). The primary effect is that the axial bending stiffness for outside stiffeners is larger than that for inside stiffeners, by virtue of the coupling of the axial force in the bending

deformations. A secondary effect, directly dependent on the Poisson ratio, is the increase of extentional stiffness in the circumferential direction for inside stiffeners relative to outside stiffeners.

The primary and secondary effects are opposing. The secondary effect prevails for larger loads. Thus for short cylinders (low Z), internal stringer stiffening is superior to external stiffening. For longer cylinders external stiffeners result in higher buckling loads. The eccentricity effect depends very strongly on the shell geometry, whilst the stiffener geometry only influences the magnitude of the effect.

Stringer stiffeners are only of benefit in opposing axial loads and are not of great benefit in the presence of larger pressure loads.

Singer concluded that external stiffening is in general superior to internal stiffening (141, 148-150). The ring stiffener spacing in offshore structures generally causes buckling to occur between rings. Stringer stiffened shells used in that context are often classified as short and the conclusions for short cylinders apply. As a result internal stiffening may be superior for many offshore structures.

1.5.3 The effect of boundary conditions

The tangential restraint, $v = 0$, has a strong effect on the buckling of axially loaded isotropic cylinders. Singer and Rosen (151-153) have indicated that the effect is important for ring stiffened cylinders; the effect is unimportant for stringer stiffening (154,155). The axial restraint, $u = 0$ or $N_x = 0$, has a significant effect on the buckling load of simply supported stringer stiffened cylinders. The effects are strongly dependent on the shell Z value and on stiffener area and eccentricity. The effect is greater for internal stiffening than for external stiffening. The axial restraint has little effect for clamped end conditions.

Theoretical studies show that radial restraints can have as large an effect on the buckling of stiffened cylinders as on unstiffened cylinders (151-153). This effect need not be of great concern for practical cases, since adequate lateral restraint is usually ensured. The only exception is where local radial restraint is not continuous - eg. where there is intermittent tack welding (99).

1.5.4 Simply supported vs clamped boundary conditions

The effect of clamping the ends of stringer stiffened cylinders is very much greater than for isotropic cylinders (156). Seggelke and Geier (146) have shown that the effect is large for $Z < 1000$. Soong (155) found similar theoretical effects. Practical results lie somewhere between simply supported and clamped theoretical values (157). An extensive series of tests (158) using a variety of boundary conditions has shown the significant effect of clamping.

It is always conservative to assume simply supported supports. It is necessary to obtain an accurate idea of the end conditions if it is intended to use the beneficial effects of clamping or if a true correlation between theoretical and experimental buckling loads is required (141,159,160). Vibration techniques have been useful in assessing the degree of rotational restraint (161-164). Singer concludes that it is almost impossible to achieve total end fixity (159). The author was familiar with Singer's conclusions when the design of end-conditions of the experiments contained in this thesis was considered. A very dense epoxy-sand mix was chosen as a bedding material to ensure fixity. Given, however, the very rapidly varying forces and shell lateral displacements in the vicinity of the clamped ends, the author is generally in agreement with Singer's conclusion.

1.5.5 Choice of end detail

Eccentricity of axial load with respect to the shell middle surface has a significant effect on the buckling load of a stringer stiffened cylinder (162,158). The end detail also has a large effect on the linearity of results. It is almost impossible to achieve a totally satisfactory simply supported end condition, because the end rotates under load. A relatively consistent end detail is more likely to be obtained with a clamped support.

1.5.6 Correlation between theory and experiment

A logical result of the expected reduction in imperfection sensitivity for stringer stiffened cylinders in axial compression is a better correlation

between classical linear buckling theory (using Donnell's equations) and experimental results (165). Singer coined the term "linearity", when referring to this correlation (166). The conclusions of various comparisons is given in references 167,141,156,158. The linearity of stringer stiffened cylinders under axial load is at best as high as that of unstiffened cylinders under pressure load. In general the linearity factor $\rho = P_{exp}/P_{Cl}$ is greater than 0.65 for $A_{st}/Bt > 0.2$. The linearity increases towards unity as Z (shell) approaches 1×10^3 . There is a considerable scatter ($\pm 20\%$) throughout the entire range of Z (158). An average linearity of $\rho \approx 0.8$ (± 0.2) for $A_{st}/Bt > 0.2$ seems possible from a wide range of experiments. A much better "weighted linearity", $\rho/(1 + A_{st}/Bt)$, has been obtained which is largely independent of Z . The average weighted linearity is ≈ 0.6 .

The rotational boundary conditions have a marked effect on the theoretical buckling load (158). Consequently, linearity of experimental results will vary greatly depending on which theoretical buckling load is used for the correlation. It has been found that when theoretical buckling loads were computed from accurately defined boundary conditions the linearity was significantly improved and the scatter decreased.

Unstable panel postbuckling behaviour ($\theta > 0.64$) causes a reduction in the linearity of overall buckling (158,168). Nonlinear material effects may affect linearity even when buckling occurs in the elastic stress range (169).

Doubt has been cast on the applicability of linear theory because of suggested imperfection sensitivity (170-172). Postbuckling theory suggests a decreased sensitivity with increased Z . Reference (171) suggests a sensitivity for externally stiffened cylinders, but not for internal stiffening. It has been pointed out (141), however, that there is a very marked decrease in theoretical imperfection sensitivity when nonlinear prebuckling stresses are considered. Reference (141) failed to establish a correlation between linearity and the initial postbuckling coefficient, b .

Tennyson (173) conducted experiments on stringer stiffened epoxy resin models under both axial load and combinations of axial and pressure load. The linearity of the results was approximately uniform. (Tennyson

concluded that an unstiffened cylinder is structurally more efficient than a stringer stiffened cylinder. This is refuted by Weller *et al.* (158). A direct comparison of buckling loads, without reference to the knockdown factors required for unstiffened cylinders, is inappropriate).

Relatively few results are available for stringer stiffened cylinders in bending. Results available suggest a linearity of ≈ 0.95 is obtained (174). The critical stresses in bending of the models tested were 12% greater than those predicted for similar shells in axial compression.

1.5.7 Discreteness effect

Smeared stiffener theory is valid only if the discreteness of the stiffeners can be neglected. The discreteness effect has been studied by a linear discrete stiffener theory. Stiffeners are represented by linear discontinuities through the Dirac delta function (175,176). Singer (156) has found the discreteness effect to be small, but this conclusion is more applicable to aeronautical structures than to offshore structures. Reference (176) indicates that the discreteness effect is greater than 10% for axial loading for $Z < 100$. The effect is greater for inside stiffeners than for outside stiffeners.

The linearity of experimental results was significantly affected when the total panel curvature parameter θ exceeded 0.64 (177). Koiter (178) gives θ as

$$\theta = \frac{(12(1-\nu^2))^{1/4}}{2\pi} [B^2/Rt]^{1/2}$$

$\theta < 0.64$ has been recommended for design purposes (158, 179). Boros (180) tested a series of spun mylar stringer stiffened cylinders under axial load. Local buckling effects became significant and affected the overall load for $\theta \geq 0.95$.

Reference (158) suggests that the overall buckling load should be less than the local buckling load. Reference (177) suggests that panels may be considered to have clamped edges, since stiffeners will have considerable torsional rigidity in practical cases. Another condition is required to ensure the applicability of orthotropic buckling theory. Reference (114) suggests that the number of stringer stiffeners, N_{st} ,

should satisfy the inequality $N_{st} \geq 4 n_w$, where n_w is the number of complete circumferential waves. It may be suggested that a minimum of 4 stiffeners are required per half-wave to ensure orthotropic behaviour. In this case $N_{st} \geq 8n_w$ would be more appropriate.

1.5.8 Effective width of panel

Various authors have suggested an effective width concept may be used to accommodate the effects of panel buckling (142,143,165,181). However, caution may be appropriate in this approach. The role of stringer stiffeners is usually to enforce lines of symmetry on an overall buckling mode involving the shell area only, when few stringers are used (182). Many theoretical buckling loads are clustered around the lowest buckling load (10). Thus unless stringer stiffeners fully participate in the lateral buckling process the actual buckling load may not be much greater than that of the unstiffened cylinder. Typically 9-15 circumferential waves form around the circumference of an axially loaded unstiffened cylinder (70).

Reference (182) gives the experimental results of a thin mylar cylinder ($R/t \approx 520$) with 8 stringer stiffeners. The model was subject to axial and hydrostatic loading. The stringer stiffening was not found to increase the initial axial or hydrostatic buckling loads, above those of an unstiffened cylinder. The additional presence of ring-stiffeners (which of themselves did not increase initial buckling capacities) resulted in very much higher post buckling loads. In addition stringer stiffeners prevented a torsional mechanism from forming in the advanced postbuckling region. This torsional mode had been observed in ring stiffened and unstiffened cylinders under hydrostatic load.

The influence of the stringer torsional stiffness on local panel buckling load was investigated in reference (176). Stringers had no effect on panel buckling for $\theta > 1.9$, and very little effect for $\theta > 1.0$.

1.6 Imperfect stringer stiffened cylinders

1.6.1 The use of initial imperfections in design

The role of initial imperfections in determining the strength of cylinders

is generally accepted and most experimental programmes now include the measurement of imperfections before model testing (160). There exists an initial imperfection data bank for cylinders of various constructions at Delft University of Technology (81,82). Imperfections are generally analysed by double fourier analysis, with coefficients of the fourier analysis presented rather than raw imperfection data. This approach was first advocated by Arbocz (79,80,183).

In the case of unstiffened and ring stiffened cylinders these initial imperfections are used to establish empirical knockdown factors through computer programs which establish the strength from known geometry. When an understanding of the range of imperfections, obtained in practice, is known it will be possible to predict strengths on a statistical basis.

A number of recent papers have suggested the use of initial imperfections to determine the strength of longitudinally stiffened cylinders subject to combinations of axial and pressure loading (184,114,185). The basis of the method is that the initial imperfections of the stiffened cylinder grow under load in a manner similar to that observed in columns. Failure of the cylinder is assessed on a first yield basis. The basic column unit is a typical element of shell plate and attached stiffener.

The method may be applied unquestionably to cylinders with closely spaced stiffeners which fail by means of a single half-wave along the length and with no circumferential waves. Such cylinders have neither theoretical nor observed susceptibility to imperfections. A first yield failure criterion for these cylinders will, however, be very conservative.

The method may be subject to qualifications when applied to cylinders with few stiffeners. Much of the experimental work at the Technion has been concentrated on measuring the buckling linearity of closely stiffened cylinders which buckle orthotropically. Cylinder geometries for which the linearity is close to unity can be taken as imperfection insensitive, in the sense that the presence of reasonable imperfections does not lead to bifurcation loads lower than those calculated by linear theory. Singer believes most stringer stiffened cylinders which fail orthotropically are covered by this statement (141).

Thus stiffened cylinders which fail orthotropically by means of a single half-wave along the length and with several circumferential waves can be treated in a manner suggested by reference (185).

The buckling modes of a stringer stiffened cylinder are essentially orthogonal, although several bifurcation loads exist at approximately the same stress. Hence each imperfection will grow independently according to the Southwell rule and no mode interaction will occur. This statement cannot be made in general for cylinders which can buckle by means of more than one longitudinal wave. The author is not aware of theoretical results on the degree of mode interaction for stringer stiffened cylinders in the overall modes. In practical structures, ring spacing is likely to dictate the length of buckles in the longitudinal direction and as a result one half wave along the length is likely to be the preferred mode of buckling.

Singer has noted that the buckling linearity is noticeably affected when the panel curvature $\theta > 0.64$. Reference (185) suggests, however, that the first yield of column approach is applicable when panel curvature is as large as $\theta \approx 1.45$.

1.6.2 The effect of imperfections on buckling interaction

A theoretical study has been published by Byskov and Hutchinson on the interaction between panel buckling and overall buckling (186). The authors use a development of Koiter's postbuckling theory to predict buckling stresses for individual modes of buckling, considering interaction terms for both perfect shells and imperfect shells. The degree of interaction between buckling modes, with consequent reduction in buckling stress is maximum when overall and local (panel) buckling stresses coincide. As a consequence a theoretical optimisation procedure which seeks to maximise the buckling stresses by finding the coincident overall and local buckling stress is seen to be of reduced value. The torsional stiffness of the stiffeners has, however, been ignored. This assumption may be justified for the panel buckling mode if the panels are wide. The torsional stiffness is significant for the overall buckling mode. The paper also calculates the overall buckling stress using orthotropic buckling

theory when the circumferential halfwave extends over very few stiffeners.

1.6.3 Non-destructive testing

Westergaard (187) established that the maximum lateral deflection of a column or flat plate with a single sinusoidal initial imperfection is given by

$$w_p = w_o / (1 - \sigma / \sigma_{CR}) \text{ ----- (5)}$$

where w_o is the amplitude of the sinusoidal component at zero load. The relationship is valid for any system with a single imperfection whose behaviour is governed by linear differential equilibrium equations, and equation (5) follows directly if the differential equation solutions are written in determinant form. Equation 5 was popularised by Southwell (188) who used it in slightly modified form to predict the buckling loads of imperfect columns.

A general initial imperfection may be represented by a fourier series with coefficients w_{io} and the above equation may be written more generally for a column or for a flat plate subject to small lateral displacements as

$$w_p = \sum w_{pij} = \sum w_{ijo} / (1 - \sigma / \sigma_{CR}(ij)) \sin \frac{i\pi x}{a} \cdot \sin \frac{j\pi y}{b} \text{ ----- (6)}$$

Equation 5 is valid for an unstiffened cylinder with a single sinusoidal imperfection. Small deflection theory is, however, not valid for cylinders. Where several fourier modes of imperfection exist in conjunction, an interaction will occur which will make equation 6 invalid.

Donnell (189) chose an initial imperfection

$$w_o \sin \frac{\pi x}{l} \cdot \sin \frac{\pi y}{R} + w_o^2 \frac{\pi^2 R}{4 l^2} \cos \frac{2\pi x}{l}$$

and showed that the lateral deflection will be

$$w \sin \frac{\pi x}{l} \cdot \sin \frac{\pi y}{R} + w(w + 2w_o) \frac{\pi^2 R}{8 l^2} \cos \frac{2\pi x}{l}$$

at a load P given by

$$P = \frac{w_0}{w_0 + w} P_C \left[\frac{\frac{\pi^4 R^2}{1 + \frac{4l^4}{\pi^4 R^2} (w + 2w_0)(w + w_0)}}{1 + \frac{\pi^4 R^2}{8l^4} (w + 2w_0)^2 \left(\frac{2w_0 + w}{w + w_0} \right)} \right]$$

The term in parenthesis is a modification of the classical critical buckling load P_C , and depends on w_0 . It was shown, for example, that if $w_0 = 7.5t$ the value of P_C was modified by the expression in parenthesis to give $0.78 P_C$ for $R/t = 1000$.

Koiter (28) proved that bifurcation loads exist at loads much lower than the classical critical buckling load for very small imperfections. Donnell and Wan (27) developed the work of Donnell (189) and Von Karman and Tsien (52) by considering more elaborate initial imperfections and showed that buckling will occur at loads below the elastic classical bifurcation load. The simplified method advocated by Southwell is no longer appropriate for determining the buckling load.

As a result of the above points very little non-destructive examination of the capacity of isotropic cylinders has been carried out. Flugge (190), however, suggested that the Southwell technique could be usefully applied, provided an individual displacement was judiciously chosen. As a result of his suggestion a number of papers have been published by Horton and his associates at Georgia Institute of Technology (191-195). These investigations showed that individual deflections grow in a manner given by Southwell's formula, but a buckling load is initially predicted equal to or greater than the classical bifurcation value. The choice of initial deflection to be studied had to be made very carefully and in fact it was only after buckling had occurred that the critical location was known. However, if a modified Southwell technique was used and if loads in excess of 60% of the actual buckling load were considered a good estimate of the correct buckling load was obtained (193).

The technique was not found to be very satisfactory because of the high compressive loads required to accurately predict the buckling load. A more promising technique involves the determination of the local shell stiffness to lateral point loads. The lateral stiffness decreases with increasing load and extrapolation of the stiffness from low axial loads indicated a buckling load within 4% of the observed value (194,195).

Little published work exists on non-destructive testing of orthogonally stiffened cylinders. Singer found that strain gauge readings grew in orthotropic cylinders according to Southwell's rule (196). The technique has not been followed up. Horton and Craig (192) re-analysed published work of Arbocz and Babcock (197) and found that a very good estimate of the actual buckling load could be obtained by Southwell plots of the fourier imperfection coefficients at various loads. Rosen and Singer (161-163) have used vibration techniques to predict the buckling loads of orthotropically stiffened shells.

1.7 Curved Panels

1.7.1 Elastic critical stresses

A significant paper on the elastic buckling of curved panels under axial load was published by Koiter in 1956 (178). The buckling stress of an axially loaded panel with simply supported edges is given by

$$\sigma_{CR} = \frac{E\pi^2}{3(1-\nu^2)} \left[\frac{t}{B} \right]^2 (1 + \theta^4) \text{ ----- (7)}$$

$$\text{where } \theta = \frac{1}{2\pi} (12(1-\nu^2))^{1/4} \left[\frac{B}{Rt} \right]^{1/2} = (0.2893 \sqrt{\frac{B^2}{Rt}} \text{ for } \nu=0.3)$$

Possibly the most important contribution of the paper was that the assessment of the imperfection sensitivity was given in terms of the parameter θ . The slope of the elastic postbuckling curve was found to be positive for $\theta < 0.64$; whilst the slope decreases rapidly for $\theta > 0.64$. Koiter noted that for clamped boundaries stable postbuckling behaviour will be possible for some $\theta > 0.64$.

Koiter's threshold of negative initial postbuckling slope ($\theta = 0.64$) has become widely accepted as the boundary between "wide" and "narrow" panels. Wide panels are taken to be imperfection sensitive and the postbuckling reserve of a flat panel is not usually considered applicable. It is interesting to note, however, that Koiter suggested that curved panels for which θ is at least as large as unity will in practice display a postbuckling behaviour very similar to a flat panel.

Rehfield and Hallauer (44) considered the effect of edge restraints on the elastic buckling stress of panels for $0 \leq \theta \leq \sqrt{5}$. The effect of clamping the edges decreases with increasing θ and for $\theta > 2$ the clamped panel has a buckling stress only 12.5% greater than a simply supported panel. The most significant changes in buckling loads occur in the range $1 \leq \theta \leq 1.5$ for various forms of inplane edge restraint.

Syngellakis and Walker (198-200) used an advanced form of Koiter's buckling theory to calculate the elastic bifurcation loads of imperfect curved panels bounded by longitudinal stiffeners and simply supported ends. Computations were made numerically simple by assuming that no lateral deflection of the stiffeners occurs along their length. The torsional stiffness of the stiffeners was found to be significant for the geometries considered. In reference (198) where the stiffener slenderness was $D_{st}/t_{st} = 18.8$ the recorded buckling load was found to lie between the torsional buckling load of the stiffener and the classical buckling load of an unstiffened cylinder. The observed mode of buckling of the araldite model was initial buckling with a long wavelength in the longitudinal direction, followed by snapping to a shorter axial wavelength with a single inward buckle between each stiffener. The theoretical development of the papers is commendable in considering elastic bifurcation of imperfect panels. Results have, however, not yet been given in a generally useful form.

The same comment may also be made about other theoretical applications of Koiter's theory as extended by Hutchinson (201-203). The imperfection sensitivity of panels in the elastic and elastoplastic regime are considered by means of the initial postbuckling coefficients. The initial postbuckling behaviour fails to give an adequate indication of panel behaviour at advanced postbuckling states.

1.7.2 Elasto-plastic behaviour

Dowling and Ho (204) considered the elasto-plastic behaviour of curved panels by solving the differential equations of equilibrium using an 8×8 finite difference grid. Their results show very clearly that curved panels with simply supported edges ($\theta > 1$) fail in an essentially similar fashion to initially flat panels. It was found that inward initial imperfections have a considerably graver effect on maximum strength than outward

imperfections of the same magnitude. The number of half waves in the initial imperfection along the generators was significant for outward deflections; however, for inward imperfections the single sinusoidal half wave in the axial and circumferential directions resulted in a reasonably good estimate of the minimum panel strength. The level of inward initial imperfection was found to have a strong effect on ultimate strength for the panel geometry most extensively studied ($B/t = 60$, $R/t = 200$, $A/B = 1.0$ i.e. $A^2/Rt = 18$ or $\theta = 1.225$). This paper also showed that the panel strength for $B/t = 60$, as given by reference (68) corresponded closely with an inward initial imperfection of $w_0/t = 0.374$.

A subsequent paper (107) reaffirmed the above results and showed that the behaviour of multiple panels, separated by longitudinal stiffeners, corresponds quite closely to that of a single panel with constrained edges. Single sinusoidal imperfections (inward) were considered.

1.7.3 Rigid-plastic mechanism theory of flat panels

The strength of two-dimensional panels subject to lateral and/or inplane load has been considered by many authors, by an assessment of panel behaviour through rigid-plastic formulations (205-210). Panels subject to lateral load have been treated as parabolically dished membranes by Jaeger (205) using the Tresca yield criterion or as a series of encastre transverse strips when the panel is long.

The majority of the work on the subject has followed one of two approaches to the problem. The more accurate analyses have satisfied the yield surface flow rules, and are restricted to panels in inplane compression only. In another method, a panel at collapse is divided into parallel independent strips, each of which is assumed to be at a state of collapse and each of which makes an independent contribution to panel strength.

Important contributions to the parallel strip theory have been made by Sherbourne (211), Korol and Sherbourne (212), Kato (213), Murray (214) and Davies *et al.* (215). The most advanced of these papers is reference (215), where shear, axial force and bending moment are considered to exist simultaneously at the hinges which develop on the strips. The various stress resultants satisfy a relationship

$$\frac{M}{M_0} + \left[\frac{N}{N_0} \right]^2 + \left[\frac{S}{S_0} \right]^2 = 1$$

where $M_0 = \sigma_y \cdot \frac{1}{2} \cdot t^2$, $N_0 = \sigma_y t$, $S_0 = \sigma_y t/2$. The results are integrated along the yield lines and as a consequence a closed form solution is obtained. This final solution can be minimised with respect to the ridge angle which characterises the yield line geometry.

The use of the normality relationship to relate the forces at a hinge to the relative strains was proposed by Haythornthwaite (216) to determine the difference in strength between encasté and flexurally clamped beams. The method was used by Jones (217) to determine the lateral load capacity of a laterally deflected square panel with four diagonal yield lines.

A contribution to the collapse mechanism theory was made by Dean (218) when he developed unique equations relating the non-dimensional stresses at a point on a yield line to the relative displacements of the rigid plate segments forming the yield line. The equations satisfy the Prandtl-Reuss flow rule and the Von Mises yield criterion. The force-displacement equations were expressed in terms of the local axes at the yield lines, and Dean was able to show how the global kinematic variables of a panel (assuming rigid plate movements) could act as fundamental variables in satisfying the equilibrium conditions of the panel. The method was applied to panels under inplane load and lateral load and to diaphragms (using two different types of mechanism). Results were initially presented for constrained panels (218) and later extended to panels with unrestrained edges (219).

The approach adopted by Dean (218) may be seen to have further reaching consequences. In an attempt to reduce the global kinematic variables to a fundamental form Dean, however, presents the method in a somewhat complicated fashion. This form is indeed useful when a closed form solution is possible (220). In the more general case, however, a closed form solution is not practically possible. The complicated form of the virtual work integrals forces solutions to be calculated numerically. A major part of this thesis is concerned with a presentation of the collapse mechanism method in a form where it has a general application.

1.7.4 Collapse mechanism approach applied to cylinder collapse

Pugsley and MacAuley (221), and Alexander (222) studied the application of collapse mechanism theory to unstiffened or ring stiffened cylinders. More recently the subject has been studied by Andronicou and Walker (223) and by Tsang and Harding (224). The strip approach is ideally applied. The biaxial stress state complicates the stress state at the hinges, but it is nonetheless possible to satisfy the Prandtl Reuss flow rule and also to obtain a closed form solution to the load capacity. Upper bound solutions have been obtained for the strength of cylinders subject to both axial and lateral load.

1.8 Layout of thesis

1.8.1 Scope of Thesis

Several areas of research readily suggest themselves in the literature survey. There is a need for carefully documented experiments where the parameters affecting experimental scatter are carefully measured and documented. In particular the shortage of results for steel cylinders in the $100 \leq R/t \leq 300$ range is noticeable. Numerical techniques have advanced the knowledge of flat plates in recent years. These techniques can now be applied to stiffened cylinders. Numerical analyses can model in detail the material behaviour and geometric imperfections and as a result should be capable of reproducing experimental results accurately. There is, however, a clear need to identify the parameters which affect the elasto-plastic strength of curved panels. Without this knowledge parametric studies may be conducted less effectively than would otherwise be the case.

The research described in this thesis was conducted as part of an overall research programme and consequently the choice of direction was limited somewhat by the overall requirements of the programme. In particular, model parameters were set within the overall programme and the author was free to work on the details of the experiments. An accurate method of manufacture of small scale cylinders was required and this will be discussed in chapter 2. The boundary conditions affect the strength of stringer stiffened cylinders and a solution is required to the problem of providing consistent boundary conditions throughout the entire model

loading cycle. This will be resolved within chapter 2.

Some development is required to adapt the numerical techniques, which have been applied successfully to flat panel analysis, to curved panels. It was not possible to guarantee the availability of numerical problem solving packages, e.g. the finite element elasto-plastic package or the collapse mechanism technique until some time into the research efforts. Neither could the general problem of relevant geometric parameters be addressed until quite late in the programme of research. As a result a number of other studies were carried out in considerable detail in an effort to gain an insight into model behaviour. These analyses have been retained within the thesis to provide a complete picture of the stiffened cylinder behaviour. Some of the chapters in this work can stand separately as pieces of work, and the reader may pass them over when following the main thread of research. Thus chapters 3, 4 and 6 may be read separately and the principal direction of the thesis may be seen to stretch through chapter 2 and chapter 5 and then through chapters 7 and 8.

1.8.2 Contents of following chapters

Chapter 2 contains the description of an experimental programme which was carried out as the first of a new overall programme of research. The details of the experimental rig and description of model manufacture are given in detail because these formed the basis of an extensive programme subsequent to the initial experiments described in this work.

Chapter 3 contains an analysis of the raw data collected during tests. An attempt is made to use the surface imperfection data to predict buckling loads.

Chapter 4 contains the results of various theoretical elastic buckling predictions for the geometries tested in chapter 2. The buckling predictions are based on classical orthotropic theory as well as on eigenvalue analyses of finite element representations.

Elastic and elasto-plastic finite element analyses of the model geometries are discussed in chapter 5. The elastic analyses are compared in detail with experimental responses. The experimental results obtained in

chapter 2 are also compared with the results of experiments carried out at other locations.

The results of a beam on elastic foundation model analysis are presented in chapter 6. The beam consists of a typical repeated stiffened panel unit. Axisymmetric behaviour is considered.

A general collapse mechanism theory is advanced in chapter 7 with results given specifically for a rectangular two dimensional panel.

The collapse mechanism analysis is extended in chapter 8 to curved panels. Simple and combined mechanisms are discussed and some results are compared with results derived from finite element analyses.

The results of the experimental programme and much of the analytical research programme are used to suggest a procedure for design in chapter 9.

CHAPTER 2.

The Experimental Response of Six Stringer Stiffened Cylinders under Uniaxial and Eccentric Axial Load.

2.1 Description of experimental rig

2.1.1 Experimental rig

The principal criterion in the design of the loading rig was that it should have a load capacity of at least 50 tonne and that load should be displacement controlled. No standard loading machine with these characteristics existed in the laboratory at the time. It was however, possible to re-design a rig which had been previously used in another laboratory. This meant that one was not completely free in the choice of components; the result was nevertheless satisfactory for the series of tests.

2.1.2 Particulars of rig

Figure 2.1 shows a general view of the rig used for the tests. A vertical screw jack, centre, is driven by a variable speed motor, left. The model is situated between the uppermost circular platens and is surrounded by a deflection measuring frame. A computerised data acquisition system is located on the right hand side. The screw jack and drive motor unit is seen in detail in figure 2.2.

The motor is a Carter Hydraulic Variable Speed F13 unit with a 2HP English Electric 3 phase motor. The output shaft runs at 0-1400 rpm with a minimum recommended continuous speed of 50 rpm. The electric control box with on, off and trip switches is mounted in front of the motor, where the chain linked variable control dial is also positioned. A 151/1 double worm drive reduction gear supplied by Wiseman of Grantham reduces the output speed to the screw jack. The screw jack is a 100 ton capacity worm gear Duff Norton Jackmotor supplied by Consolidated Pneumatic Tool Co. Ltd., London. It has 150 mm travel. Couplings are Crown Pin Flexible Couplings which allow for small inaccuracies in alignment of the drive shafts. Trip switches attached to the screw jack prevent the jack overrunning.

The central section of the rig as seen in figure 2.1 consists of 4,

1 m diameter, 125 mm thick platens. Top and bottom platens are fixed relative to one another by means of 6 columns of 50 mm diameter studding, which thread freely through the intermediate platens. Lateral stability for the intermediate platens is provided by 3, 75 mm diameter columns which are fixed at the central support and free to move vertically, top and bottom. The screw jack, located between the bottom platens acts against the model through the two intermediate platens between which are located 3, 50 ton load cells. The load cells are 50 ton spherical capped cells type Datasense 200, supplied by Transducer Ltd (CEL) of Reading. They have an error range of 2% over the range used. Symmetrically positioned about the vertical axis they can measure any eccentricity of load through the model.

A typical model is seen positioned in the rig in figure 2.3. 80 mm thick plates at either end of the model distribute the load into the model. Concentric or eccentric axial load can then be applied to the model through spherical bearings fitting between the model end plates and the loading platens. The spherical bearings, figure 2.4, of 50 ton capacity have a steel-ptfe sliding surface giving a very low coefficient of friction. With the load applied through these bearings it is reasonable to conclude that the centre of load corresponds to the geometric centre of the bearing.

A model is accurately positioned at the centre of the rig by means of a theodolite which is also used to position the spherical bearings. Rotation of the model ends could be measured by means of dial gauges positioned around the circumference of the end plates, figure 2.3.

The entire rig was designed to be self-reacting. However, very substantial extra stiffness was given to the rig by means of a 200 ton capacity beam bearing onto the top platen and secured to the laboratory strong-floor.

The computerised data logging equipment, figure 2.5, consists of a Solatron System 35 Central unit with two floppy disc drives for program and data storage. Peripheral units are an Adds visual display unit for controlling the system, a Digital decprinter by which processed readings are printed, a central master scanner for controlling the choice of channel numbers scanned at any one time,

and two "slave" units which act as terminals for the transducers and strain gauges. This computing system was used to carry out both initial and final processing of much of the data acquired during tests. As data were stored on floppy disc as well as being produced in hard copy format, they could also be transferred to a more powerful computing system in the college for final processing and some graph plotting.

2.2 Description of models

2.2.1 Models. General

The models tested in this series of experiments were part of an ongoing programme of work sponsored by the Department of Energy (116). The programme was chiefly designed to provide experimental results for various cylinder types, as a correlation for design strength predictions and to identify possible areas of inadequacy in these predictions. The models tested were longitudinally stiffened rigid cylinders of nominal diameter 320 mm. Models had either 20 or 40 longitudinal stiffeners.

Model dimensions are given in Table 2.1. Figures 2.6, 7 show the typical stiffened panel units of each model. The manufacturing technique involved initial fabrication of the models 40 mm oversize in length. The extra 20 mm at each end of a model allowed appropriate end fixity to be added in the form of steel rings.

2.2.2 Manufacture of models

The manufacture of small scale models in steel can cause considerable problems, particularly for the manufacture of models with a significant proportion of stiffening. The Structures Laboratories at Imperial College have many years of experience in the testing of 1/4 and 1/12 scale box girder models (244). The chief difficulties arise in obtaining thin steel plate with similar structural properties to thicker plate used in full scale construction and in the control of geometric distortions due to welding to a level representative of the full scale prototype.

The problem of obtaining steels with representative properties was solved in the past by obtaining a special order of steel plate. An adequate supply of this material existed at the beginning of this project. The control of welding distortions was overcome by the use of extensive jigging, and careful welding procedures, followed by stress relieving.

Walker and Davies (245) used a tee-butt welding procedure (a butt weld incorporating adjacent plate panels and tee-stiffener, laid from the back of the tee-joint), to produce 1/20 scale cylinder models. The cylinders were, as a result, produced very economically but resulted in a level of imperfection which may not satisfy the scaled geometric tolerances specified in the codes, e.g. (68).

An alternative technique exists in the aerospace industry for the production of stiffened cylinders. This entails machining plate area to a reduced thickness to leave raised stiffness. Although this technique has proved satisfactory with aluminium structures (156), and tests have been carried out by Weller and Singer (177) on integrally machined steel models, the proposed geometry of stiffeners for this series of experiments made this method of production unsatisfactory. After considerable consultation and tests the method described here was adopted, and resulted in the initial imperfection tolerances (68), being satisfied.

2.2.3 Manufacture of a Model

A sheet of 0.84 mm thick steel plate was accurately cut to a length equal to the circumferential length of the proposed cylinder. Stiffeners were cut and machined. A jig, figures 2.8,9, was made up so that stiffeners could be positioned at the required spacing, figure 2.10. The spacing jig, figure 2.9, was designed such that a flow of inert gas could be directed at the back of the stiffener-plate junction to be welded. The welding procedure was a single fillet weld pass along one side only of the stiffened-plate junction. A tungsten-inert gas procedure, with no filler wire, was used and the welding torch was hand held. The problem of burning through the thin 0.84 mm plate - caused mainly by air and impurities in the weld jet was overcome by

directing a backing flow of inert gas through the rig and by carrying out the entire longitudinal weld in one pass - without intermediate starts.

As soon as one weld was complete the plate could be moved and reset, so that the stiffener last welded acted in the jig as a reference for the next stiffener. The process was repeated until all the stiffeners were in position. Figure 2.11 shows the partially completed process and figure 2.12 shows the plate after all the stiffeners had been attached. The curvature of the plate is a result of welding induced shrinkage and is an aid to the next step of bending the plate to form a cylinder.

The tig-welding process (without filler wire) produced a weld, figure 2.13 which accurately represents the scaled size of a full-scale fillet weld. A slight undercutting of the stiffeners occurred with the melting of the stiffener metal to form the welds. This did not, however, affect the structural integrity of the stiffeners. Minute cracks in the fillet welds were noticed along their lengths. It was decided to ignore these flaws after tests had indicated that the welds remained intact when typical stiffened plate units were loaded to collapse. Figure 2.13 is a detail of the stiffened plate, showing the high quality of weld finish and the low level of weld induced plate distortion. This is a mark of the high level of workmanship of the welder and the effectiveness of the jiggling procedure.

The thinness of the plate allowed it to be wrapped elastically around a circular mandrel, figure 2.14. Accurately machined spacers helped the process by preventing the localised formation of folds at any section of the plate and these spacers also ensured that distortion of the stiffeners during the later heat relieving process was prevented. A wide strap held the stiffened plate in position around the mandrel while the final tee butt weld formed the plate into a cylinder.

The outer case of the mandrel, figure 2.14, was secured and the entire assembly was sent for heat treatment. It was decided to totally stress relieve the models. In addition to the jiggled models a number of coupons for tensile testing were also included in the heat treatment process. The strength characteristics of the steel after stress

relieving could be determined from the tensile coupons and strengths are summarised in Appendix 2A1.

A typical model after heat treatment is seen in figure 2.15. The closing tee-butt weld, figures 2.16, 2.17, produced a slight overspill of weld liquid around the stiffener-plate junction in some models. This was small enough to be ignored and a slight redesign of the rig for later models eliminated the overspill entirely.

2.2.4 End conditions for models

A considerable effort has been directed in the past into assessing the importance of boundary conditions of cylindrical shells and into the relative effects of boundary conditions and initial imperfections on the failure of axially loaded cylindrical shells. Vibration studies by Singer have assessed the experimental boundary conditions and attempted to correlate actual boundary conditions with buckling loads. These studies were relevant for structures with a far higher Z geometric parameter than the models of this series. It was felt, however, that boundary conditions could still be of major importance and the detail, figure 2.18, was decided upon as an improvement on the detail used by Rosen and Singer (162) or by Sridharan and Walker (117). This detail provides full rotational and inplane restraint at the model ends, and ensures uniformity of loading in the model.

A typical model after heat treatment, figure 2.15, was very flexible, and susceptible to oval distortions. A model was placed in a 2 mm deep groove on a flat platen, figure 2.19, so that the model was held in a circular form. The machined steel end rings were clamped in position, figure 2.19, and a layer of silicon rubber a few millimetres thick was cast around the model base. When the silicon rubber had solidified a layer of dense araldite-sand mix was packed around the model base to the height of the end rings. When this had set the fixing clamps were removed, the silicon rubber layer peeled off, and the gap left by the rubber layer carefully filled with sand-epoxy mix. A little scraping clean of the plate and material surface had to be carried out to remove excess araldite adhering to the inside of the models.

Similarly the other end of the model had restrained rings attached. When the end rings were in position the loading surfaces of the model were ground flat and later lapped into contact with the end plates. Figure 2.20 shows two of the completed models.

2.2.5 Strain gauging

The intention to obtain a complete picture of the growth of initial imperfections under load meant that the number of strain gauges on the outer surface of the models had to be severely limited. In order to obtain an accurate indication of the strain level at a point on the model surface, it was necessary to use very small gauges. Gauges used were 2 mm long linear gauges, type PL2, 60 Ω and 3 mm square cross gauges, type PC3 120 Ω , manufactured by Tokyo Sokki Kenkujo Ltd. The installation of these gauges, especially on the inside surface of a model required patience and skill. As a result of the time involved and also for financial reasons, the number of gauges per model had to be economised. Figures 2.21-26 show the strain gauge layouts adopted. No strain gauges were applied to model no. 2 and only on model no. 1 were strain gauges applied to the outside and inside surface.

2.3 Deflection measurement rig

2.3.1 Description

The internal diameter of the loading rig was 710 mm. A movable assembly, to measure surface displacements during load, was designed to fit within this space. This movable rig was also used to measure initial imperfections.

The rig, figure 2.29, consisted of four vertical banks of transducers, rigidly mounted on a circular frame. Five transducers were set at 13 mm spacing for model nos 1 and 2, nine at 18 mm spacing for model nos 3 and 6, while a staggered arrangement of nine transducers for model nos 4 and 5 allowed readings to be taken at 6.5 mm spacing.

Individual transducer blocks could be moved over ± 5 mm while setting

up was taking place. Once a satisfactory arrangement had been determined, the blocks were clamped and were located on the rig by means of dowel pins. This allowed the transducer blocks to be removed, to allow a model to be positioned, and returned to position without altering the original datum.

The outer ring of the transducer frame was free to move on ball-bearings, set in a groove, figure 2.30, in the lower fixed plate of the rig. This arrangement was considered to provide sufficient restraint against lateral movement of the upper movable ring. The rotational positions of the transducer rig were determined by lining up a series of equally spaced scribed lines on the movable ring, with the cross-hair of a low power microscope set on the main experimental rig. Thus it was possible to obtain readings of surface deflections around a model, at a spacing of 6.28 mm, i.e. giving a total of 160 circumferential positions around a model.

Individual transducers were 1K Ω and 10K Ω 8FLP 10A linear motion potentiometers supplied by Techni Measure Ltd, with a stroke of 11 mm. Each transducer was individually calibrated before use, and subsequent readings had an accuracy of ± 0.022 mm, or $\pm 2.5\%$ of the thickness of the plate of a model.

A zero datum was established as follows. A perfect right circular cylinder had been fabricated and ground to a polished surface. This cylinder was located in the rig, figure 2.31, and the transducer frame was rotated into each position around the circumference. A set of readings was taken at each position and was later used as a zero datum when a model was in position. A new set of datum values was obtained before each model was tested.

Initial imperfections for each model were determined with the model inside the loading rig. In this way the deflection measurement apparatus was not disturbed when load was applied, and deflections measured under load could be readily compared to the initial imperfections. The utmost care was used, by means of dowels and continuous support in the bearing race, to ensure that individual transducer blocks and the transducer frame did not move from the initially preset positions. Analysis of later readings indicate that some error may have occurred in repositioning transducer blocks.

This shows as a slight step in the circumferential scans between transducer blocks, and in the failure of extreme points on a circumferential scan to match. In general these mis-matchings were small and do not affect the incremental displacements.

2.4 Testing

2.4.1 Setting up procedure

A common procedure was adopted for each test.

- a) Strain gauges on the model were checked to ensure that all were functioning.
- b) Model ends were lapped into contact with their respective loading surfaces.
- c) A zero datum was obtained for the radial transducers.
- d) The model was carefully positioned in the rig and accurately located by theodolite.
- e) Top loading plates, bearing, and top platen of the rig were positioned, and top bolts of the rig secured.
- f) Strain gauges and internal transducers for the model were connected up to the logging equipment.

At this stage a number of the delicate electrical contacts to the strain gauges were found to be broken. In some instances assembly was stopped and these electrical contacts repaired. In other instances it was decided to continue with the test and to do without the relevant strain gauges.

- g) When model nos 4, 5, 6 were being tested a heavy top cross frame was placed across the top of the rig and lightly prestressed into position.

- h) Initial imperfections for the model were determined at zero load.
- i) A number of elastic loading cycles were carried out and strain gauge readings checked to ensure that load was being applied as appropriate (concentric load for model nos 1-3, and eccentric load for model nos 4-6). Some adjustments of the top nuts of the loading rig were generally required before an acceptable load distribution was obtained.
- j) Where the model was acceptably set up, load was applied slowly. The loading cycle was stopped at various load levels to allow a complete scan of strain gauges and to obtain a scan of surface movements. In general two sets of surface displacements were obtained below the first sign of distress to the model and three sets of surface displacements after the first buckles had appeared on the surface. Load was applied to a model up to a high elastic level in the first day of loading. Load was statically maintained overnight and on the second day of loading the model was loaded to collapse.

2.4.2 Presentation of surface deflection data

The detailing of the model end rings, figure 2.18, made it impossible to obtain surface measurements along the lines of support. It was decided to extrapolate data from the actual transducer positions to obtain an estimate of the end deflection value. A second order polynomial was used for this purpose. Thus the deflection values for initial imperfection surface are only estimated values at the model ends.

In the case of model no. 2, the end rings were a distance of one half a transducer spacing from the final transducer. The end value obtained in this case was linearly interpolated between the last real transducer value and a fictitious transducer value, at a distance of one transducer spacing, obtained by the above extrapolation technique.

Retrospectively, a more appropriate method of extrapolation would

have been by a best fit line through the three transducer values used in the second order polynomial method.

End conditions of the models assumed total restraint against lateral movement. In the absence of measurements of real deflections at the end rings it would be inconsistent to adopt an extrapolating estimate which, under load, assumed deflections to occur at this position. Thus in the analysis of surface deflections under load, it has been assumed that the movement of the model ends was zero.

2.4.3 Analysis of deviations from best-fit perfect cylinder

The most concise method of presentation of the surface scans of the model has been to carry out a fourier analysis of the data obtained. Each set of data was initially analysed by a "best-fit cylinder" routine. This routine fitted a perfect cylinder through the original data and obtained the deviations of the data points from this best-fit surface.

The deviations from the best fit surface were analysed by fourier analysis techniques. Two separate techniques were used. The deviated surface was analysed as a cosine half-series expansion

$$\frac{w(x,y)}{t} \approx \sum_{k=0}^5 \sum_{l=0}^{40} \cos \frac{k\pi x}{h} \left\{ A_{kl} \cos \frac{l\pi y}{b} + B_{kl} \sin \frac{l\pi y}{b} \right\}$$

or as a sine half-series expansion

$$\frac{w(x,y)}{t} \approx \sum_{k=1}^5 \sum_{l=0}^{40} \sin \frac{k\pi x}{h} \left\{ C_{kl} \cos \frac{l\pi y}{b} + D_{kl} \sin \frac{l\pi y}{b} \right\}$$

where h = height of model, t = thickness of surface,

$$b = \pi \times \text{Radius}(R)$$

This format expresses the surface continuously in the circumferential direction, and as an even function over the height for the cosine half-series expansion, or as an odd function over the height for the sine half-series expansion.

The cosine half-series is a better method of presenting the initial imperfections, although other investigators (117,230) have tended to

use the sine half-series. The latter series assumes that the imperfect surface has a zero value at the end rings. Any further use of this series to represent the surface thus introduces a false curvature at the model ends. The cosine expansion is not ideal, since it uses the fictitious end values described above. It does, however, generally produce a better fitting surface.

The deflected surface under load has, it is assumed, zero movement at the end rings. Thus it becomes logical to represent the small growth of surface deflections in terms of sine half-series. The procedure adopted in this thesis is to present initial imperfections in terms of both cosine half-series and sine half-series. Subsequent analysis of the surface under load is presented in terms of sine half-series expansions. Mean coefficient values of the form

$$\sqrt{A_{Kl}^2 + B_{Kl}^2}, \text{ or } \sqrt{C_{Kl}^2 + D_{Kl}^2},$$

have been presented in table and graph form, so that the relative values of mode shapes in the expansions can be assessed.

2.5 Model no. 1

2.5.1 Behaviour

As indicated in table 2.1, model no. 1 had a test length of 65 mm with forty 6.72 mm deep longitudinal stiffeners. Strain gauges, figure 2.21, were located on three panels. Panel gauges for stiffener no. 1 were located at the centre and ends of the panel, while on the remaining two panels gauges were located at the base of the panel only. These two panels thus provided a check on the uniformity of load and supported the stress distribution observed at the ends of panel no. 1. Due to the presence of strain gauges and associated wiring on the outer surface of the model only 50% of the initial surface imperfection readings could be satisfactorily obtained. As a result it was decided not to present these data for further processing.

Figure 2.32 shows the overall behaviour of the model under load, while figures 2.33,34 show the behaviour of stiffener no. 1. At

midheight the stiffener outstand is seen to be more heavily loaded than the plate. At the base of the model the plate area was the more heavily loaded. This is a consequence of the pinching effect of the base restraints, which prevent Poisson expansion at the base, and induces outward bending of the section. Strain gauge readings from gauges 22 and 25 indicate that eccentricity of load was low across the stiffener outstand at midheight, with the larger strain being 12-14% greater than the average strain at elastic loads close to yield. In addition, the strain distribution at the panel centre (gauges 1, 2 and 4, 5) indicates only negligible bending strains in the axial direction, at large elastic strain levels.

Transverse strains at midheight vary from 9% (tensile) of the average (axial plate strain at midheight) at the stiffener position to 24% (tensile) of the average at mid-panel. Transverse bending strains at midheight resulted in outer surface strains being 30-35% greater than the average. This is consistent with the outward movement of the panel centre relative to the stiffener. A comparison of strains in the plate area at the base shows that, even at high elastic strains individual panels show no more than 3% variation in strain. On the outer surface of the panel, strains are approximately 20% higher at mid-panel (base of) by comparison with the stiffener position. This is understood as resulting from the presence of the stiffener preventing outward bending of the plate surface. Large bending strains are seen to occur in the plate area.

Figures 2.33 and 2.34 show that a small amount of nonlinearity occurs in the most stressed regions even at moderate elastic strains. However, the overall stress-strain curve, figure 2.32, indicates that this effect is minimal when the model is considered as a whole. First yield was reached at the end of the base of the model at a stress of 281 N/mm^2 ($0.81\sigma_y$). The stiffener outstand at midheight reached yield at 309 N/mm^2 ($0.89\sigma_y$). However, although the plate area at the base continued to take load after first yield, the stiffener outstand was unable to sustain load after first yield and strains grew rapidly. This was not due to eccentric loading across the stiffener thickness, as even after strains had grown substantially greater than yield the bending strains across the stiffener were not large. Rather the reason for the plate continuing to take load after first yield is that

the bending stresses at the base were very localised. Further, the average strain in the stiffener at midheight was greater than the average strain in the plate base.

At the final stage of positive loading load was shed at midheight from the stiffener to the plate area. When the average strain in the plate at midheight reached 335 N/mm^2 ($0.96\sigma_y$), the ultimate capacity of the model was reached. No increase in load could be taken and load remained roughly steady until a panel mechanism in the plate was advanced. After this a gradual fall-off in the load occurred with increasing large strain. Figure 2.36 shows the model after collapse. The ridge-type mechanism can be clearly seen along the entire model at midheight. It is worth noting that the stiffener outstands at the base of the model unloaded without reaching yield after maximum load had been reached.

2.6 Model no. 2

2.6.1 Behaviour

Model no. 2 had a test length of 65 mm with twenty, 6.72 mm longitudinal stiffeners. Due to time constraints on the installation of strain gauges this model was tested without any strain gauges. As a result the elastic behaviour is not known in precise detail. Care was taken to ensure a uniform distribution of loading by lapping loading surfaces into contact, and by ensuring that load distribution through the load cells was central to the model axis. Full surface scans of the surface displacements were taken at zero load, 120 N/mm^2 (load case 1), 249 N/mm^2 (load case 2), 220 N/mm^2 (load case 3) - after buckling and 200 N/mm^2 , in addition to other intermediate loads not presented here.

Figure 2.36, shows the computed mean of the A and B coefficients. A fourier sine analysis is also presented, figure 2.37. Vertical sections through mid-panel and stiffener positions are presented in figures 2.38 and 2.39.

Figures 2.36 and 2.37 indicate that two different types of initial imperfection are present. The largest modes are associated with

circumferential waves 0-4. The remainder of the terms are very much smaller but show distinct peaks in both types of analysis, around circumferential modes 20 and 40. Figures 2.38 and 2.39 show that in general vertical sections are almost straight. In the region of stiffeners 5-10, however, profiles are distorted by what on careful examination was shown to be an incorrectly set transducer. Wiring of the transducer set the positive direction incorrectly. Thus the apparent dimple on the surface in this region is not real. Figure 2.40, a circumferential profile, shows the two trends of initial imperfections very clearly. The overall tendency is a sinusoidal wave around the entire surface, with small but noticeable waves of the "hungry horse" type centred on the stiffeners. These result from the original welding induced distortions and are approximately 0.2 mm deep. Figure 2.41 shows the overall stress-strain curve for the model. Strain was measured by means of three internal vertical transducers measuring the overall shortening of the model. The curve is seen to be almost perfectly linear up to a stress of 300 N/mm^2 ($0.86\sigma_y$). At 319 N/mm^2 ($0.92\sigma_y$) a small drift in transducer readings was noticed when the load was retained at a steady level. At 335 N/mm^2 ($0.96\sigma_y$), load was held for a few minutes after application, but dropped rapidly as buckles were seen to form around the entire model surface. Load dropped to 220 N/mm^2 . The buckles were contained entirely between stiffeners and formed adjacent to the restrained ends. Increased end shortening resulted in a gradual drop in load, and a growth of the initially formed buckles at the ends. Figure 2.42 shows the model after collapse.

Figures 2.43 and 2.44 show circumferential section no. 3 at different loads. Surface movements before buckles formed were very small ($\leq 0.06 \text{ mm}$). Buckles formed with an initial amplitude of 0.5-1.0 mm. Figure 2.44 shows very clearly the trend that can be seen both in figure 2.42 and from figures 2.47,48 - of an alternating inward and outward buckle formation. Figure 2.45 shows the incremented mean coefficient values. By subtracting the initial mean coefficients, we are left with an analysis of the shape developing under load. A clear waveform can be seen corresponding to the wavelength of one panel width and a half panel width. The peak in the curve corresponding to the circumferential wave, $l = 1$, seems to indicate a lack of full convergence of the "best fit cylinder" routine.

Figure 2.46 shows a very similar type of plot. It seems remarkable, however, that apart from the pure barrelling mode $k = 1$, $l = 0$, which doubled in value, the main terms at $l = 20$ and 40 have not increased very much in value. (One would expect a doubling of values if purely elastic theory applied).

Figure 2.47 shows that at buckling a wave of circumferential number $l = 10$ suddenly appeared. Although it is difficult to show in these graphs, plots of selected mid-panels show that deflections were approximately linear up to buckling. After buckling it is noted that the original waves remain and at, in general, not greatly increased values. Thus the buckled wave is directly superimposed on existing elastic waves. It is of note that prior to the formation of the full buckled shape, no sign of these buckles was detected, even at 250 N/mm^2 ($0.72\sigma_y$). Figure 2.48 shows the final buckled shape, with the original elastic mode remaining constant and circumferential mode, $l = 10$, developing, while lower circumferential modes, $l = 0-4$, show the original oval shape terms to be increasing again at an advanced state of failure.

2.7 Model no. 3

2.7.1 Behaviour

Model no. 3 was 180 mm long with forty, 13.44 mm longitudinal stiffeners, and because of its length it was expected to behave in a substantially different way from model nos. 1 and 2. One stiffened panel unit was strain gauged internally, figure 2.22. The usual care was taken to position the model in the rig and to obtain a uniform stress distribution under load around the model. During loading full surface scans of the model were obtained at 148 N/mm^2 (load case 1), 309 N/mm^2 (load case 2), 330 N/mm^2 (load case 3) - just below maximum load, 222 N/mm^2 (load case 4) and 215 N/mm^2 (load case 5) - both after a collapse mechanism had developed.

Tables 2.8-2.13 give the fourier analysed initial imperfections, and figures 2.49,50 show the mean coefficients plotted for the cosine and sine half-series respectively. Figures 2.51-54 give vertical sections of the models at mid-panel and stiffener positions. From figures

2.49,50 it can be seen that the most prominent imperfection is the mode $(k,l) = 0,2$ for the cosine analysis and $(k,l) = 1,2$ for the sine analysis. This shape is that of an oval cylinder and figure 2.55, a circumferential plot at midheight, shows this clearly. Mode shape $(k,l) = 0,40$ (cosine analysis) or $(k,l) = 1,40$ (sine analysis) shows up the effect of the "hungry horse" mode of initial imperfection due to welding distortions. Figure 2.55 again shows up this mode very clearly, with the surface being seen to cave inwards from the perfect shape, and from the stiffener positions. Figures 2.51-55 which give vertical profiles through the model show that these profiles are less straight than those of model no. 2. This is to be expected as the model has a test length approximately 2.8 times the former model. Profile imperfections are exaggerated by the method of extrapolation to the ends of the model. The effect of out-of-roundness can also be seen in these figures. The effect of the end rings constrains the surface at the ends of the profiles, causing the profile in some places to be displaced relatively uniformly along its length, and in other places causing one end to be displaced considerably more than the other - thus resulting in non-vertical, but almost straight, profiles.

A plot of the overall stress-strain behaviour is seen in figure 2.56. The model behaved linearly up to a stress of 267 N/mm^2 ($0.77\sigma_y$). Although it cannot be confirmed by strain gauge readings, it is thought that at this point yield occurred at the base of the model on its outer plate surface. A small decrease in model stiffness is noted at this point. At 308 N/mm^2 ($0.88\sigma_y$) yield strain was recorded on one side of the strain gauged stiffener at midheight, figure 2.57. In addition, the base of the model reached yield strain throughout the plate area, figure 2.58. At this load a hinge is seen to have developed at the base, and with increased displacement plate strains increase rapidly and the stiffeners unload. Simultaneous with the local hinge at the base of the model, stiffener strains at midheight are seen to drop with increased load. It is suggested that this is a result of lateral tripping of the stiffeners on reaching yield. (After the test the strain gauges on the stiffener were seen to be at some distance from where the lateral hinge in the stiffener developed). Maximum stress was reached at 332 N/mm^2 ($0.95\sigma_y$) as buckles began to appear on the model surface at midheight. Because of the narrow panel mechanism the drop in load was gradual and could be controlled with relative ease.

Figure 2.59 shows the model after collapse, with alternate inward and outward ridge mechanisms at midheight.

Figures 2.60-64 show change of the fourier coefficients (sine analysis) with load. The most obvious component of growth was a pure barrelling mode $(k, l) = 1, 0$. This term grows nonlinearly, reaching a value of 0.117 times the thickness just before failure. Although not shown in figures 2.63, 64, this term continues to be (by far) the largest component after failure. Other components seen to grow in the pre-buckling range are centred around the term $(k, l) = 1, 11$. These indicate the growth of an overall buckling mode of approximately 4-panels' width. Figures 2.65-68 show midheight circumferential sections. No particular trend can be seen in load case 2. At load case 3, however, a mode shape corresponding to 11 circumferential waves is clearly seen. This mode, with an amplitude of 0.1 mm was not visible to the eye, but could be assumed from the fourier analysis. Figure 2.67 does not reveal any precise information, but it is apparent from the section that some overall mode is linking in with the visible panel buckling. Figure 2.63 shows that 8 and 12 circumferential waves are prominent. In the elastic range of load there is almost no sign of the development of interpanel waves. It has been noted, however, in an alternative method of plotting imperfection growth, §3.2.1, that the mode shape $(k, l) = 4, 20$ grows in the elastic range and is still present after buckling. This mode corresponds to a localised panel buckle, alternating inward and outward in adjacent panels.

2.8 Model no. 4

2.8.1 Behaviour

This model was similar to model no. 1 and was eccentrically loaded, the eccentricity being 0.25R (40mm). Strain gauging of the model is shown in figure 2.23. This layout of gauges was designed to check the distribution of load around the model and to look in detail at the most heavily loaded area - around stiffener no. 1. Initial elastic tests led to some concern about the stress distribution in panel no. 1. Additional gauges were temporarily located on the external surface of this panel, but these failed to give any conclusive results.

Figures 2.69,70 show the initial imperfections. As with earlier models the most prominent imperfection shape is the oval mode, figure 2.71, but mode shapes $(k,l) = 1,10$ and $0,40$ in figure 2.69 show that the "hungry horse" mode is also present. Tables 2.14-19 summarise the initial imperfection analysis and figures 2.72-75 give profiles of the initial shape. It can be seen that, end points excluded, the vertical profiles are almost straight and the main imperfection is caused by the displaced positions of the end rings. Local deviations of individual transducers are less than 0.15 mm and between extreme transducers the maximum bowing is less than 0.25 mm.

A full scan of surface displacements was obtained at an average stress of 90 N/mm² (load case 1), 175 N/mm² (load case 2), 254 N/mm² (load case 3) - after yielding had occurred at the critical section but before deflections had become visible to the eye, and on the unloading curve at 258 N/mm² (load case 4), 202 N/mm² (load case 5) and at 156 N/mm² (load case 6) - both these latter scans being taken at large overall strains.

Figure 2.76 shows the overall behaviour of the model under load. The position of the centre of load was critical in determining the maximum stress in the model. The spherical bearings should locate the position of the load centre accurately. It was found, however, in this and in the remaining tests, that the position of the load indicated by the load cells varied by as much as 6 mm when stresses were redistributed on failure of the most critical section. In addition, some lateral movement of the load centre was observed - not greatly increasing the eccentricity of load, but moving maximum load to stiffeners adjacent to that intended. Figure 2.77 and 2.78 show the mid-height and base strains for stiffeners no. 6 and 1 respectively.

Load was increased from 258 N/mm² to 369 N/mm² on the most critically loaded section in one step. At the higher stress the stiffener is seen to have yielded. It is suggested that strains continued linearly until first yield occurred and then grew rapidly. Thus in figure 2.76 first yield in the stiffener is suggested at 297 N/mm² ($0.85\sigma_y$). Figure 2.78 shows that strains grew rapidly after yield had occurred at the base of the plate and at mid-height of the

stiffeners. The stiffener base is seen to unload, as expected from the outward movement of the surface at mid-height. After initial failure buckles were seen to form around a portion of the circumference. Figure 2.79 shows the circumferential section 6 at load case 3. Deflections at the most highly stressed area are seen to be in the form of inward-outward buckles over individual panels, but are less than 0.1 mm in depth. In addition, in figure 2.79 a wave of larger wavelength is seen to form. This corresponds to a mode of approximately $l = 11$ in figure 2.82. The amplitude of this wave is about $0.05t$, but is significant in that it incorporates the stiffener in lateral movement.

Strain gauge readings around stiffener no. 1 have been found difficult to explain. Panels 1, 6 and 36 all show the same pattern of strain development. Stiffener no. 6 is found to be the most highly stressed stiffener at mid-height. After failure has occurred at mid-height in the stiffener and buckles observed to grow, mid-height plate strains were observed to drop, and in fact never reached yield throughout the test. Strains at the base of panel no. 1 developed as expected (compare with figure 2.34). Thus panel strains are at variance with all other observations of the test.

Loading was continued to large overall strains. As plastic strains developed around stiffener no. 1 buckles spread to eventually encompass about 50% of the circumference. On the remainder of the model strains decreased; towards the end of the test the least loaded area of the model lifted clear of the loading platens.

Maximum stress in the most critically loaded area was 371 N/mm^2 ($1.07\sigma_y$). This was calculated using the formula:

$$\sigma_{\max} = \frac{P}{A} + \frac{P.e.R}{I}$$

where A = area of model

I = second moment of area of the entire model section

e = eccentricity of load.

Although no truly accurate stress distribution around the model can be

given after first yield, the above stress indicates that some redistribution (10%) of load is possible at the most heavily loaded area.

Figure 2.85 shows the model after collapse, showing that with very large strains the individual panel buckles merge to give an almost continuous wave around the failed region. This wave only develops in the later stages of unloading. Figures 2.81-84 show the development of fourier coefficients with load. It is difficult to determine any clear trend in the elastic range, figure 2.79 being a clearer indicator that a mode of $l = 8-11$ is significant. In load cases 4 and 5, however, the alternating inward-outward mode is shown up in the size of the terms around $l = 20$. The diffuse nature of the fourier spectra may be attributed to the eccentric development of buckles over the model surface.

2.9 Model no. 5

2.9.1 Behaviour

This model was similar to model no. 2 with a test length of 65 mm and twenty internal longitudinal stiffeners. Load was applied at an eccentricity of $0.4R$ (64 mm). Strain gauges, figure 2.24, monitored the growth of strain in the most heavily loaded section, panel no. 1 and in sections close by. Gauges were omitted from the base of the stiffeners.

Figures 2.86,87 show the mean imperfection coefficients, summarising tables 2.20-25. It is apparent, once again, that the oval mode of imperfection is the most important - the corresponding coefficient values being somewhat larger than for model no. 2. The smaller coefficient terms, indicating the "hungry horse" mode are close in value to those of model no. 2, approximately $0.02t$ in magnitude. Profiles through panels and stiffeners, figures 2.89,90, show that within the measured length of the model straightness is satisfactory. The distortion due to end rings is once again apparent.

Figure 2.91 summarises the behaviour of the model under load. The overall strain rate was somewhat greater than expected at low elastic

loads. However, at stresses greater than 70 N/mm^2 the slope of the stress-strain graph is parallel to that of model no. 2. At 268 N/mm^2 ($0.77\sigma_y$) a drift in strain was noted when end shortening displacement was held constant. It is suggested that this coincides with first yield at the base of the model. First yield was recorded at one side of stiffener no. 1 outstand at 304 N/mm^2 ($0.87\sigma_y$). The strain response becomes increasingly non-linear up to maximum stress of 326 N/mm^2 ($0.94\sigma_y$) and strain was found to drift considerably on the next application of displacement - accompanied by a drop in load. Compared with model no. 2, however, the drift of strain was far smaller. Two reasons account for this. Firstly the rig had been substantially stiffened by the use of a strong cross frame. Secondly the load was shed from the most heavily loaded panel to adjacent panels on first panel failure. Thus as failure advanced it is noted that the axial strain in panels 4 and 18 increased to the same level as panel 1, before following the same trend of that panel after failure.

Figure 2.92 shows the individual strain gauge response for panel 1. It has been noted that other panels behaved similarly. When the average strain across the stiffener outstand reached yield (at $\sigma = 320 \text{ N/mm}^2$) all strains in the axial direction decreased. A slight increase in load was possible as load was redistributed (the eccentricity of load measured by the load cells was also noted to drop slightly). As overall straining proceeded the strain in the panel and stiffener dropped while the hoop strains increased markedly. This latter increase is a clear indication of the development of a panel buckle. A drop in plate axial strains could be expected to occur at mid-height on the formation of a buckle - on the basis of classical flat plate post buckling theory. However, no satisfactory explanation can be given for the drop in axial strain throughout the stiffener depth at mid-height. Stiffener tripping was not suggested from the strain response of the stiffener outstand until stresses had dropped to 270 N/mm^2 and the overall strain in the critical panel had exceeded $4000 \mu\epsilon$. It is possible that alternate inward-outward panel buckling forced the entire stiffener section to rotate close to the base, thereby shedding load from the entire section. This would not become apparent at mid-height until displacements had advanced considerably.

Loading was continued until buckles had spread to around 50% of the

panel and the unloaded side of the model had begun to lift off the loading platens. Figure 2.93 shows the model after collapse. The inward-outward buckles are clearly visible at the base - similar to model no. 2.

Deflections were measured during load at 176 N/mm^2 (load case 1), 234 N/mm^2 (load case 2), 314 N/mm^2 (load case 3) - before the first buckle had become visible, 252 N/mm^2 (load case 4), 188 N/mm^2 (load case 5) and 177 N/mm^2 (load case 6). Figures 2.94 - 97 show the growth of sine coefficients. At elastic loads the most significant terms are those associated with the growth of the panel waves, while the overall loading effect is indicated in the term $(k, l) = 1, 0$. Symmetry of behaviour along the length is indicated by the small size of even modes $k = 2, 4$. It is noted that at load case 3 initial failure had occurred in the most critical panel but from figure 2.95 there is no noticeable presence of buckling modes which later become important, figures 2.96, 97. The buckling terms $(k, l) = (1, 10), (2, 10)$ are the dominant post buckling terms. Minor harmonics at $l = 20, 30$, and 40 are evident but small. The $(2, 10)$ term results from the axially non-symmetric nature of the buckle. It seems noteworthy that although the model is eccentrically loaded like model nos 4 and 6 these other models do not show the same presence of one strong buckling mode. Model nos 2 and 5 show certain differences. Because of the eccentric loading in model no. 5 the least heavily loaded section was loaded to only $0.16\sigma_y$ when the most heavily loaded section was at failure load. As a result the coefficients at $l = 20$ are not very large before failure of the critical section. A comparison of circumferential sections close to mid-height for both models shows that at 250 N/mm^2 the mid-panel displacements were similar at approximately 0.06 mm . On formation of buckles both models show very similar buckling coefficients, the harmonic terms in model no. 5 being smaller than those of model no. 2. Figures 2.98, 100 show circumferential sections at mid-height before and after buckling developed, figure 2.99 showing that even just prior to collapse there is no evidence of the buckled shape, thus supporting figure 2.95.

2.10 Model no. 6

2.10.1 Behaviour

The dimensions of this eccentrically loaded model were similar to those of model no. 3. Strain gauges figures 2.27 and 28 were located on the most heavily loaded areas. Initial imperfection scans were obtained. The results of these are presented in tables 2.26-31, and figures 2.101, 102 summarise the outputs. The oval mode of imperfection, figure 2.103, is once again prominent, while the "hungry horse" mode is seen in the $l = 40$ terms. The not insignificant value of the terms $(k, l) = (1, 7)$ and $(1, 8)$ in the sine analysis indicate initially occurring terms which are found to develop under load. Vertical sections, figures 2.104-107, show that the level of imperfection along the generators is very small.

The model was loaded at an eccentricity of $0.4R$ and the response to load is summarised in figure 2.108. Detailed response of the critical section is seen in figures 2.109, 110 where mid-height and base strains are plotted. The overall stress-strain response was linear until first yield was recorded on one side of the stiffener outstand at 312 N/mm^2 ($0.89 \sigma_y$). Some additional load was carried by the stiffener before lateral tripping became apparent with the sudden growth of average strain in the outstand. A very small amount of additional load was carried by panel 1 and as soon as this failed load was redistributed into adjoining panels. Maximum load was reached at 365 N/mm^2 ($1.02 \sigma_y$) and loading was continued until the unloaded edge was visibly clear of the loading platen.

The response of model no. 6 was very similar to that of model no. 3. Although the ultimate load of model no. 6 is considerably higher than that of model no. 3 the response of panel 1 was very close to the earlier model, and failure stresses were almost identical. Figure 2.111 shows the ridge mechanism mode of collapse, while figure 2.112 shows the lateral collapse of the stiffeners.

Scans of the surface imperfections were obtained at 205 N/mm^2 (load case 1), 300 N/mm^2 (load case 2), 330 N/mm^2 (load case 3) - just before failure, and at 243 N/mm^2 (load case 4). Figures 2.113-115

show the incremented coefficients and circumferential sections are shown in figures 2.116-119. It is difficult to attach significance to apparent trends at load cases 1 and 2. Fourier coefficients are small, apart from the pure barrelling mode. At load case 3, however, along with the barrelling, mode terms $(k,l) = (1,3) - (1,8)$ have grown significantly. A wave form is clearly seen in figure 2.118, corresponding to 8 circumferential waves. Although only 0.1 mm in amplitude it is significant as this wave number is predicted as a buckling mode by orthotropic shell buckling theory. The final buckled shape, however, does not show this mode shape. There, the only clearly identifiable mode is that corresponding to the panel collapse mode as seen in figure 2.111.

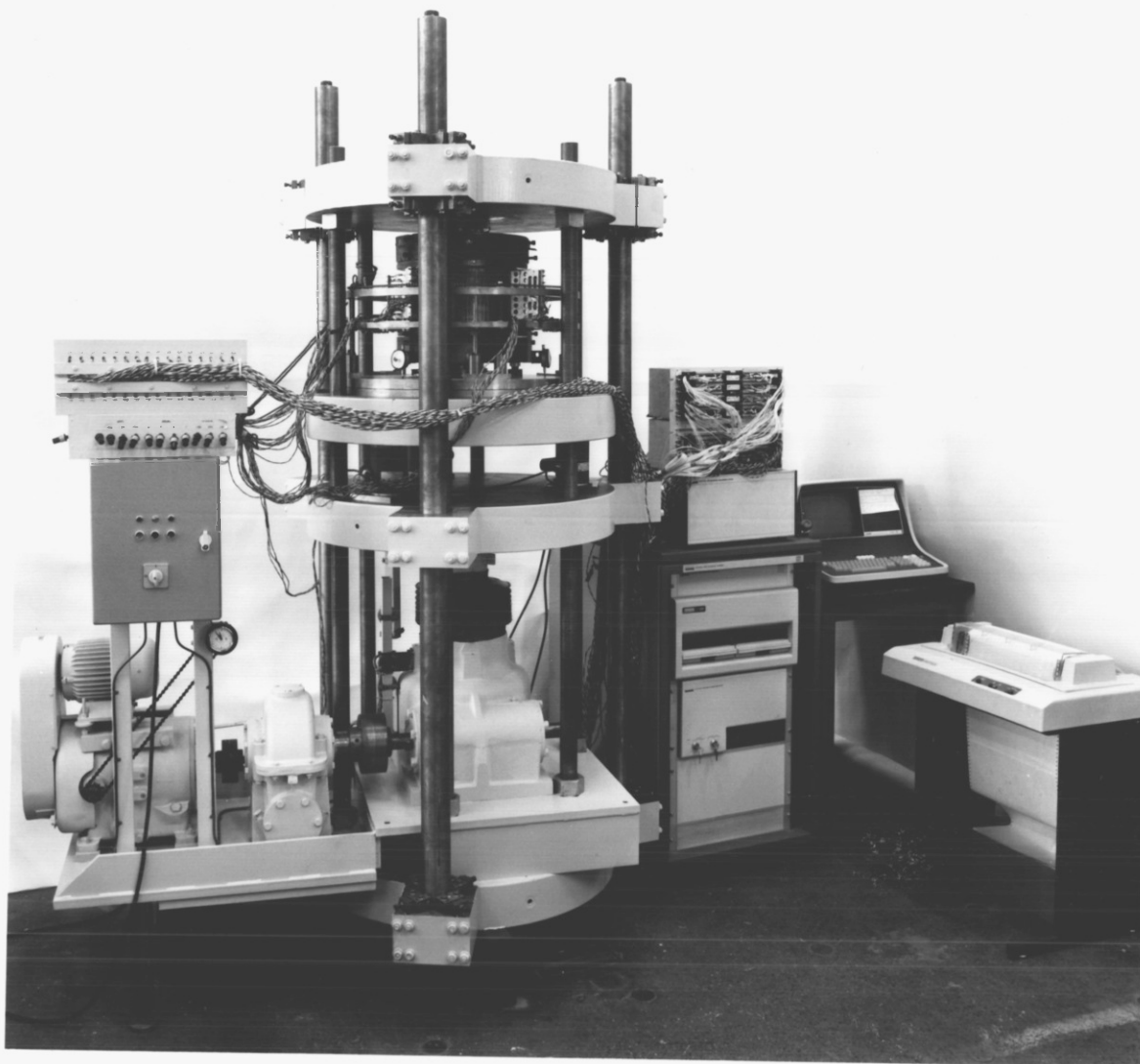
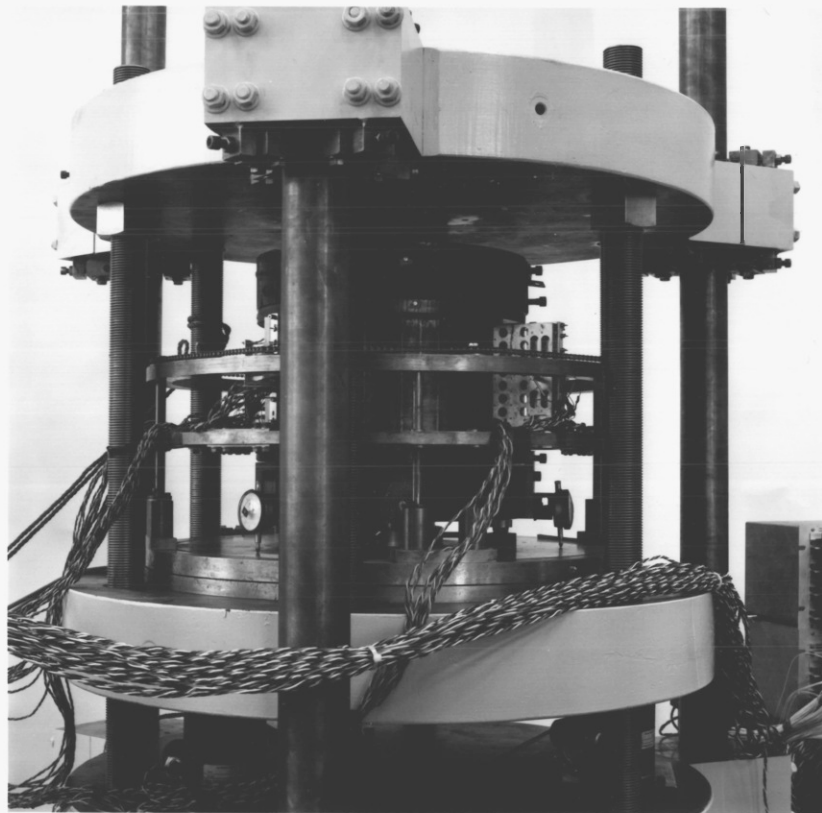
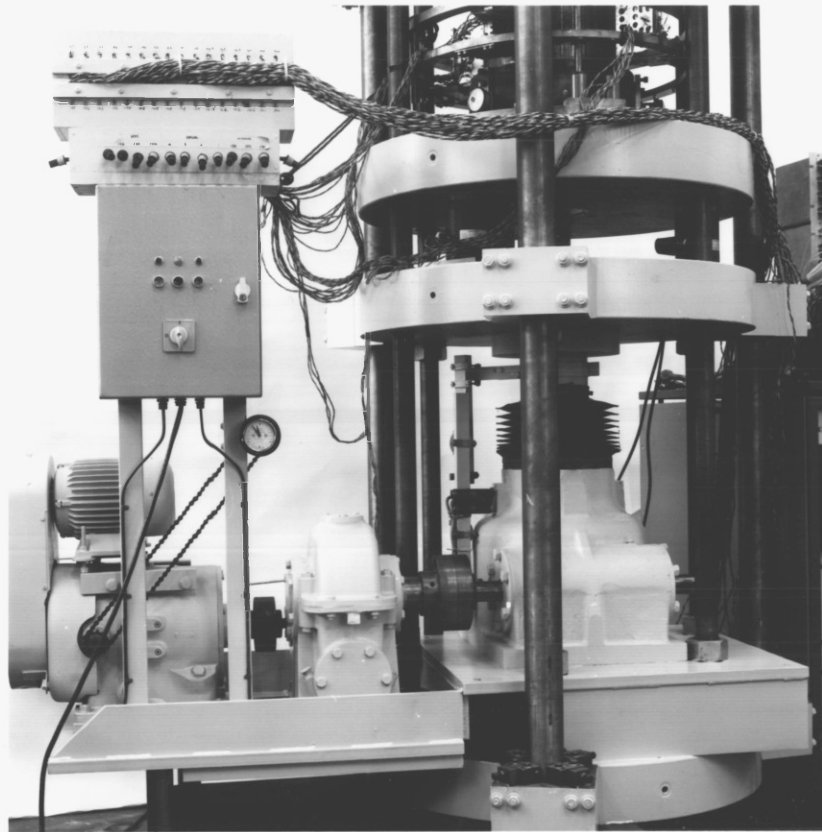


figure 2.1. General view of rig.



upper figure 2.2. Detail of screw jack and drive unit.

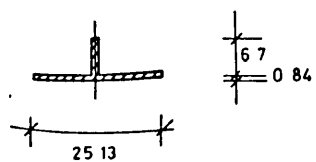
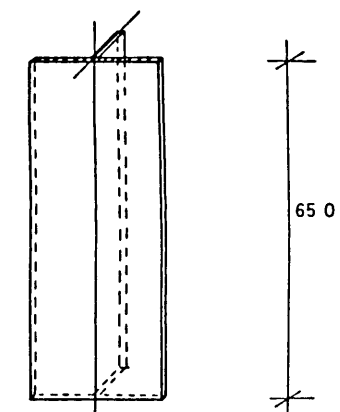
lower figure 2.3. Detail of model within rig.



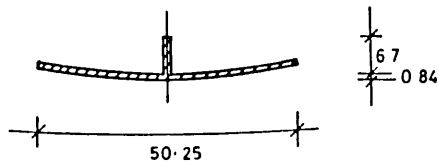
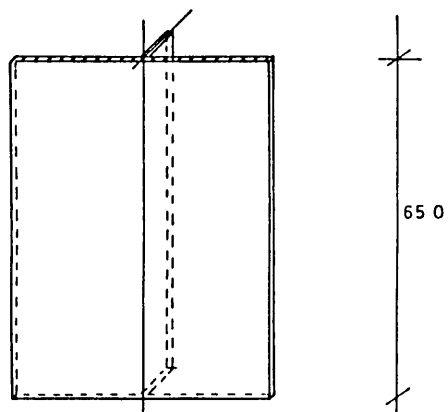
upper figure 2.4. Spherical bearing.

lower figure 2.5. Computerised data logging equipment.

MODELS No 1 & 4
PLATE-STIFFENER UNIT



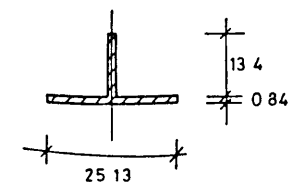
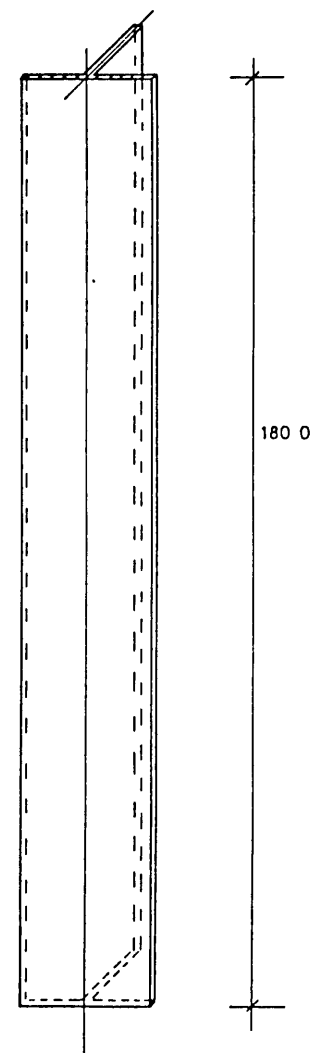
MODELS No 2 & 5
PLATE-STIFFENER UNIT



Scale 1:1
Dimensions in mm

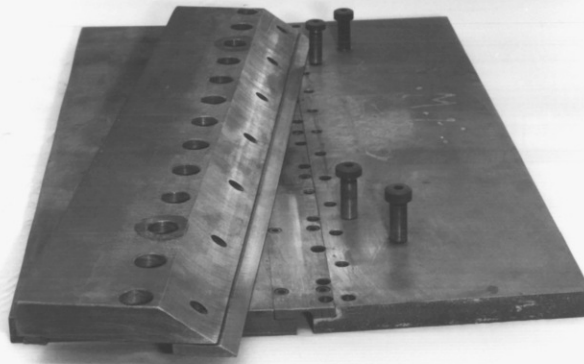
figure 2.6. Section and plan of short models.

MODELS No 3 & 6
PLATE-STIFFENER UNIT



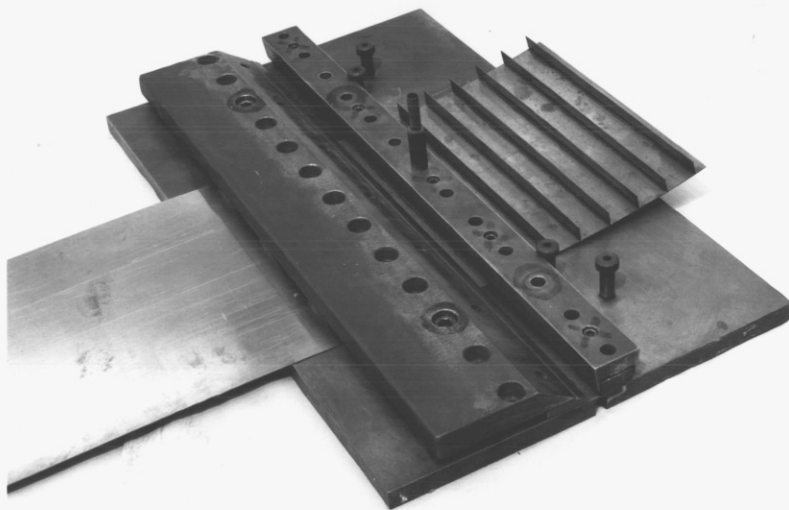
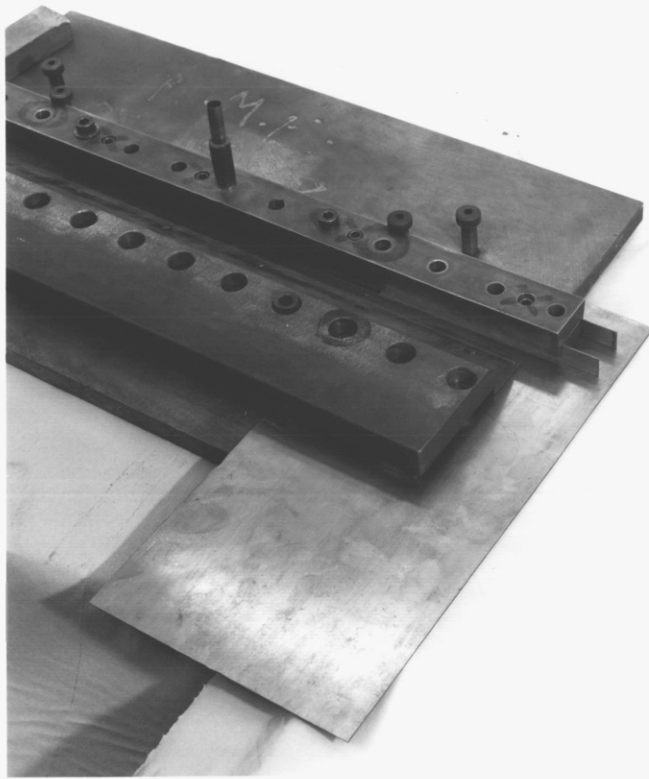
Scale 1:1
Dimensions in mm

figure 2.7. Section and plan of long model.



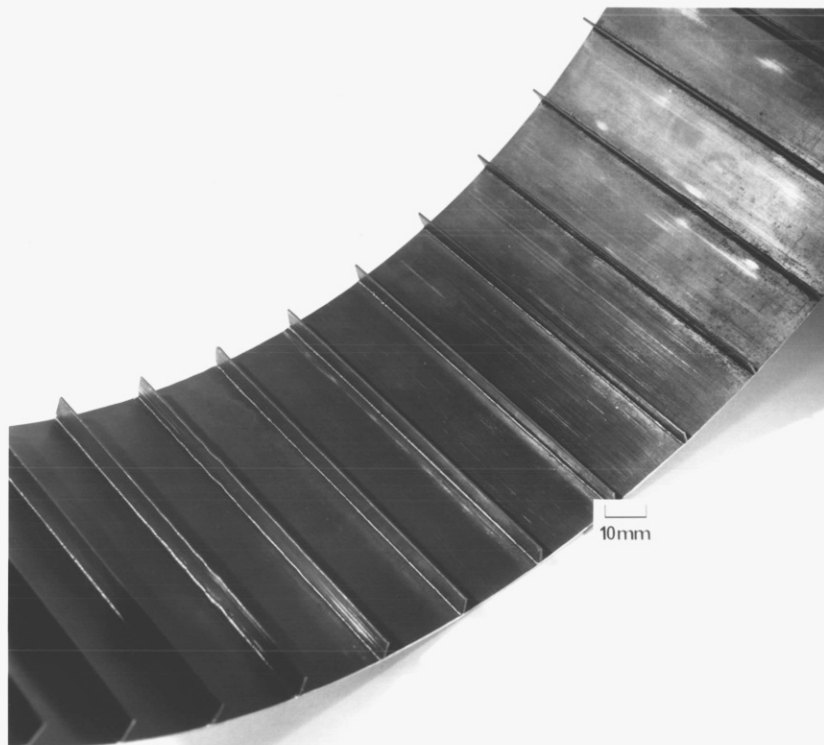
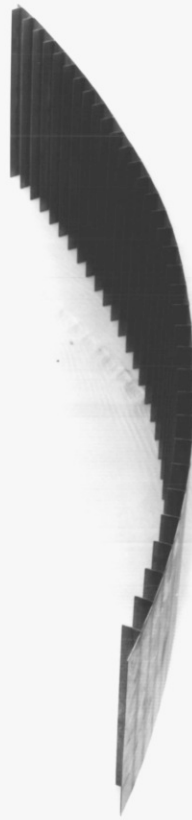
upper figure 2.8. Base plate and heat sink of jig.

lower figure 2.9. Jig component for locating stiffener in position.



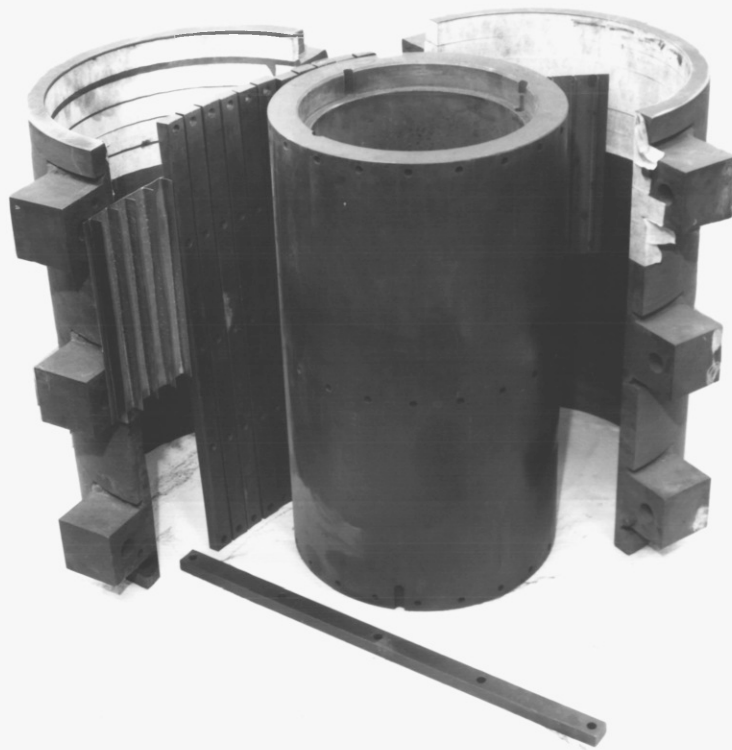
upper figure 2.10. Stiffeners located in jig.

lower figure 2.11. Partially completed stiffened plate.



upper figure 2.12. Completed plate before final seam weld.

lower figure 2.13. Detail of fillet weld of stiffened plate.



50mm

upper figure 2.14. Mandrel for jigging model and forming closing weld.
lower figure 2.15. Typical model after heat treatment.



upper figure 2.16. Closing tee-butt weld viewed from inside.

lower figure 2.17. Closing tee-butt weld viewed from outside.

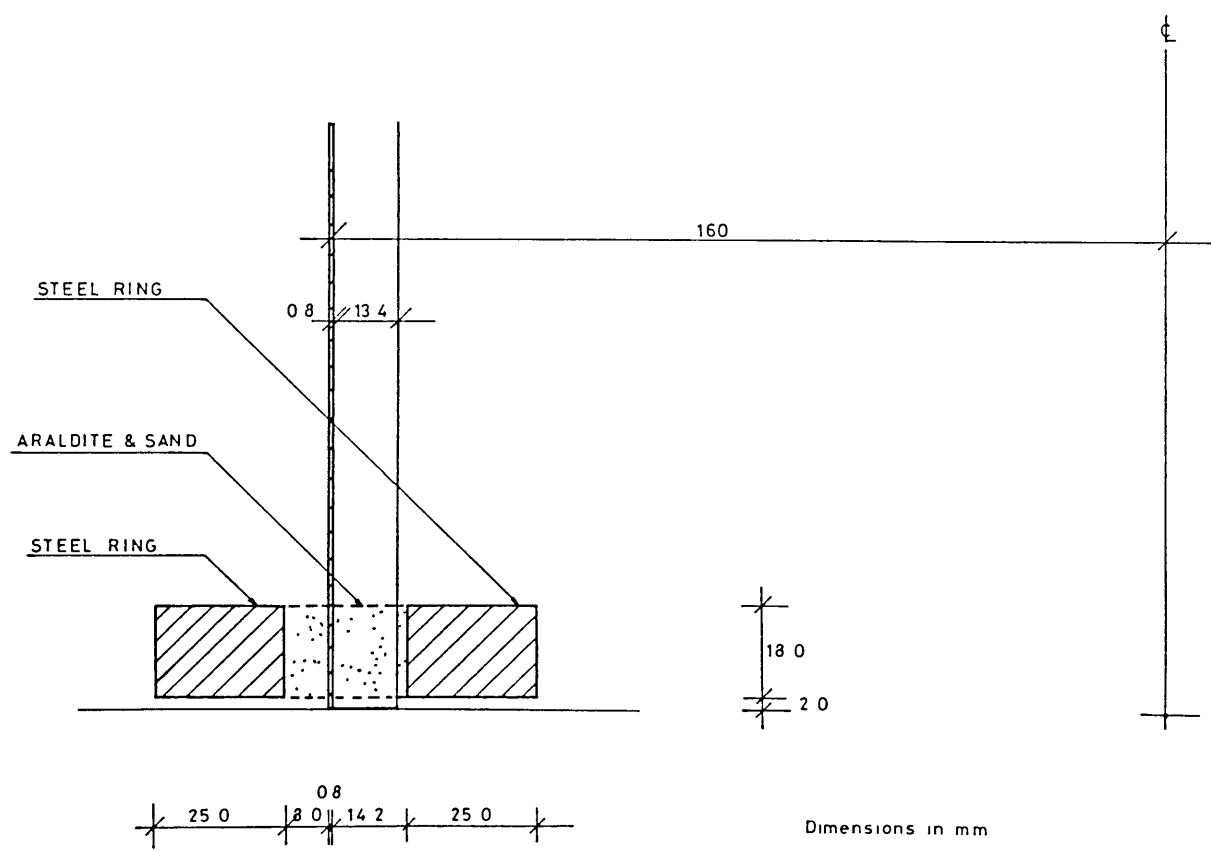


figure 2.18 Detail of end conditions.



upper figure 2.19. Procedure for attaching end ring.

lower figure 2.20. Models with end rings attached.

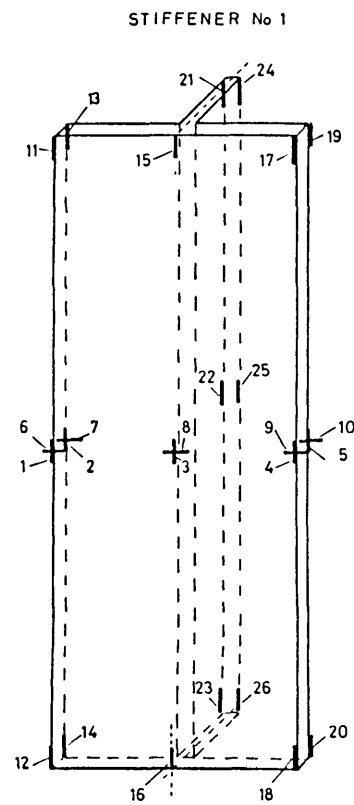


figure 2.21. Strain gauging of model 1.

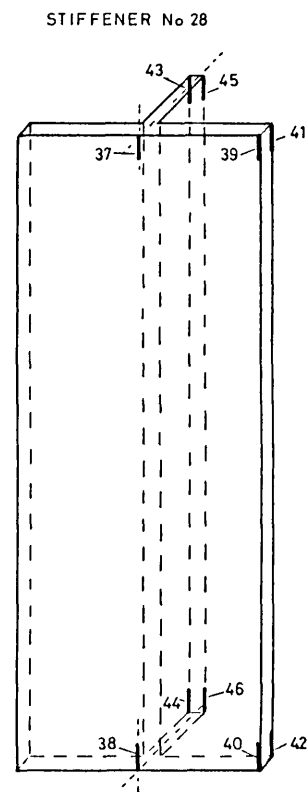
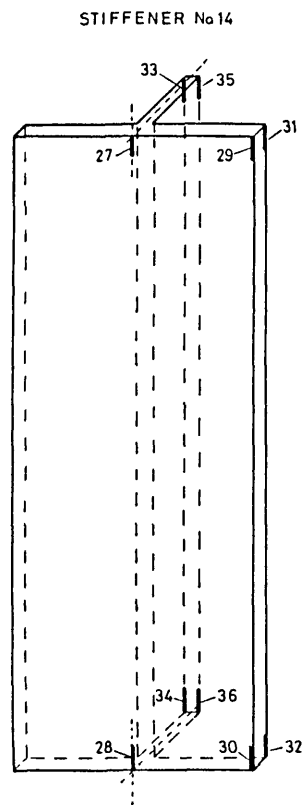
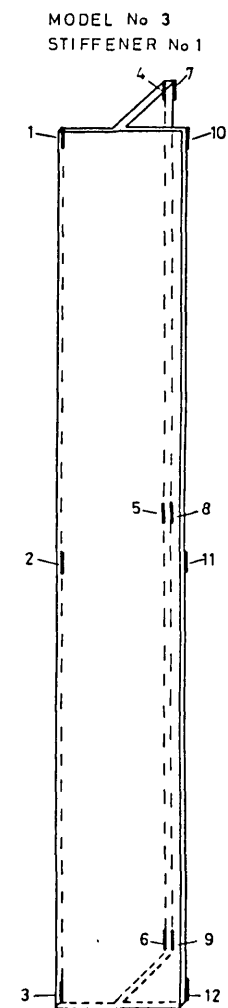
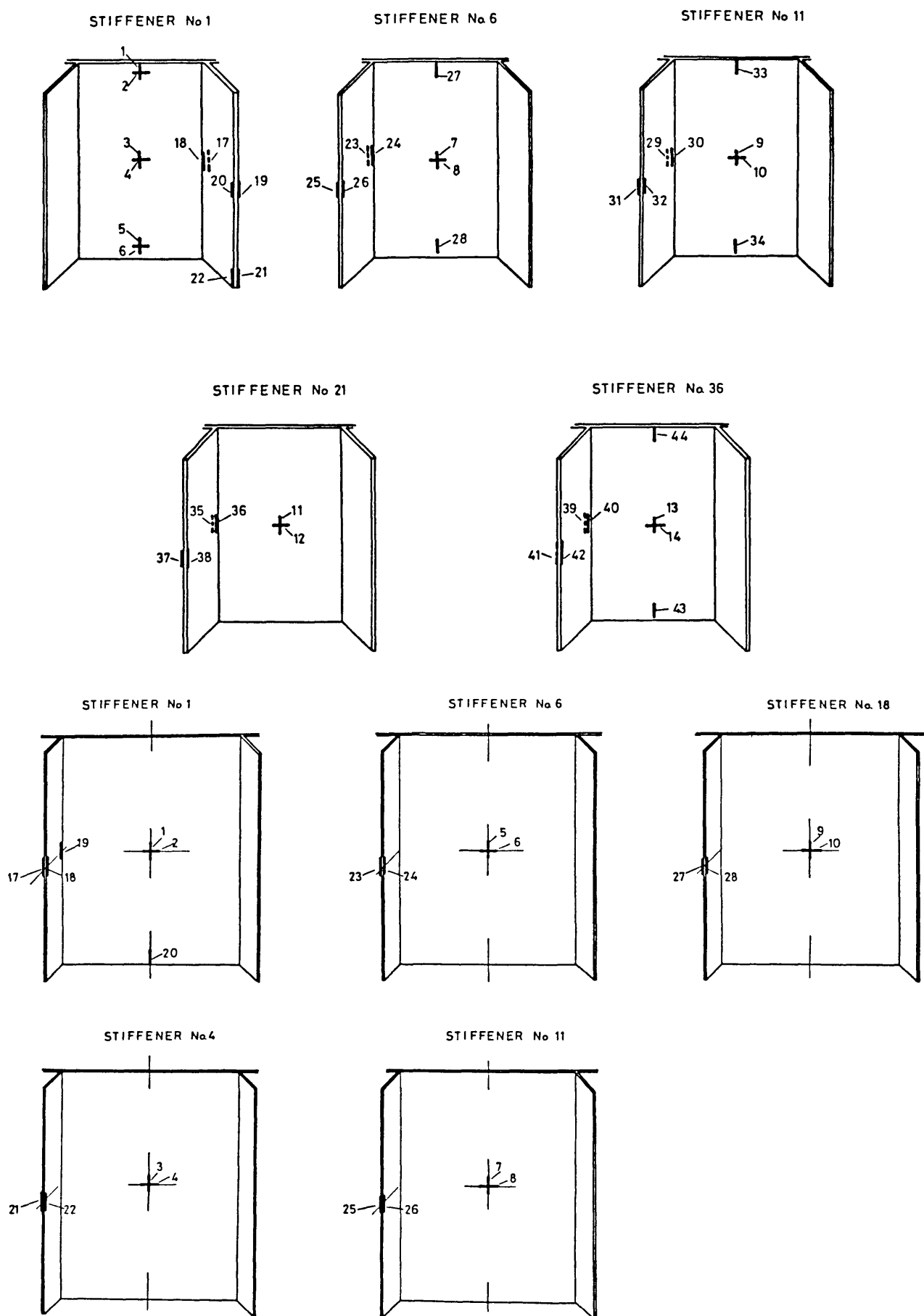


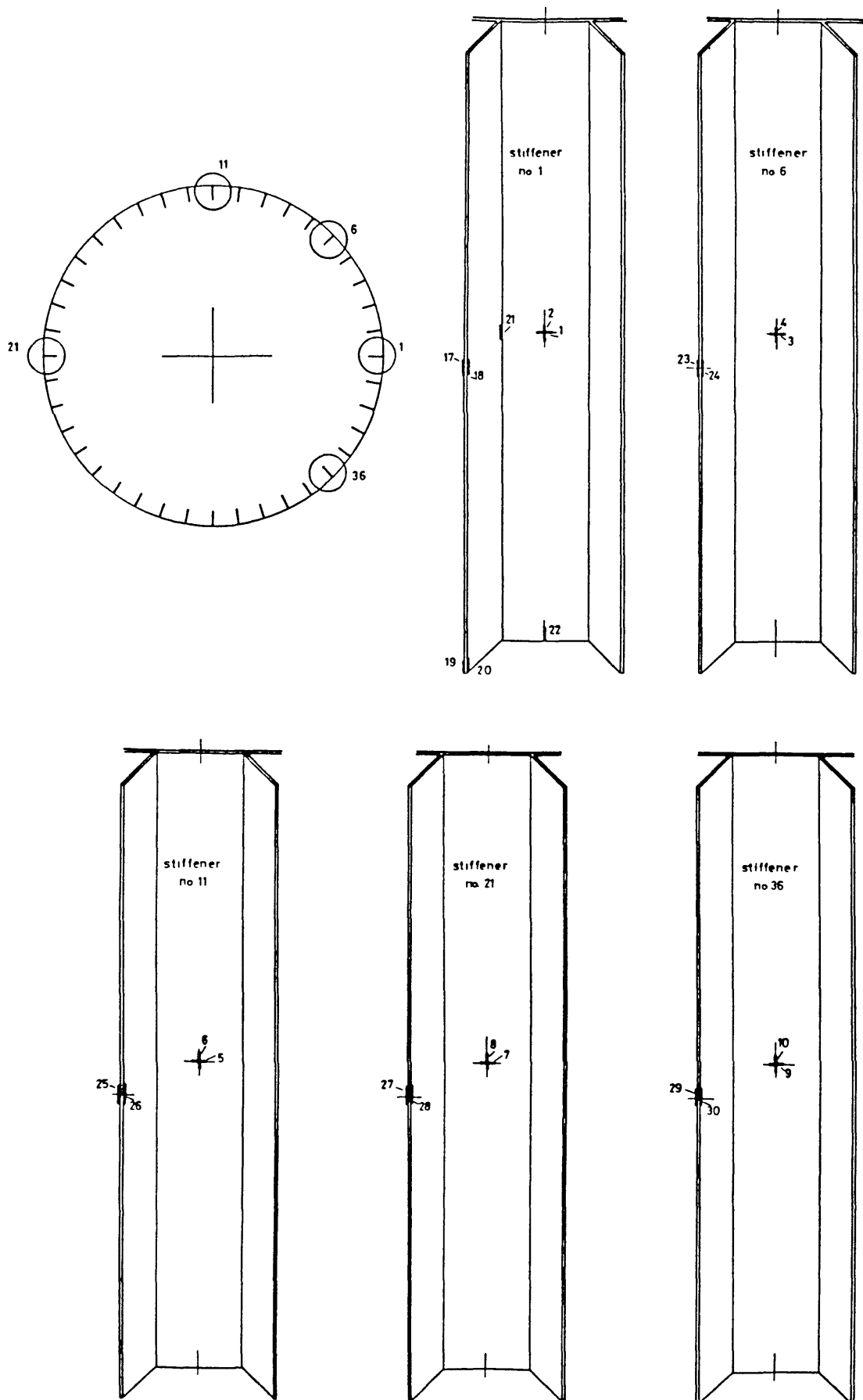
figure 2.22. Strain gauging of model 3.





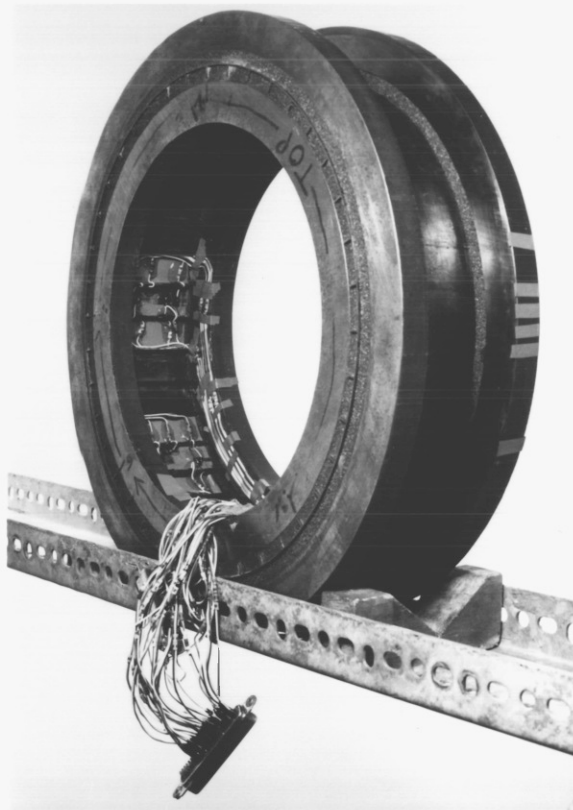
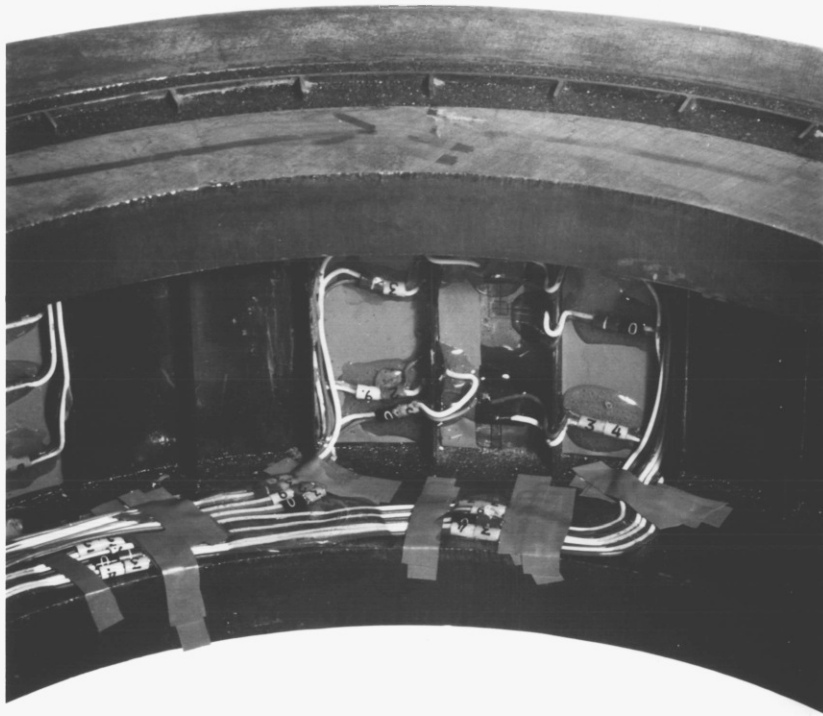
upper figure 2.23. Strain gauging of model no. 4.

lower figure 2.24. Strain gauging of model no. 5.



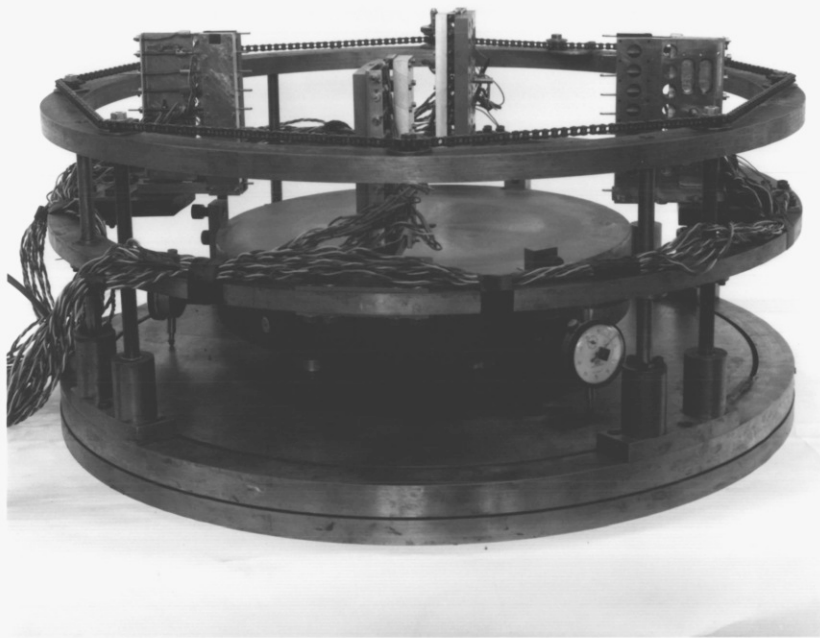
upper figure 2.25. Strain gauging of model no. 6.

lower figure 2.26. Strain gauging of model no. 6.



upper figure 2.27. Strain gauge layout on a typical stiffened panel.

lower figure 2.28. Model ready for testing.



upper figure 2.29. Deflection measuring rig.

lower figure 2.30. Base of deflection measuring rig.



figure 2.31. Perfect cylinder in position.

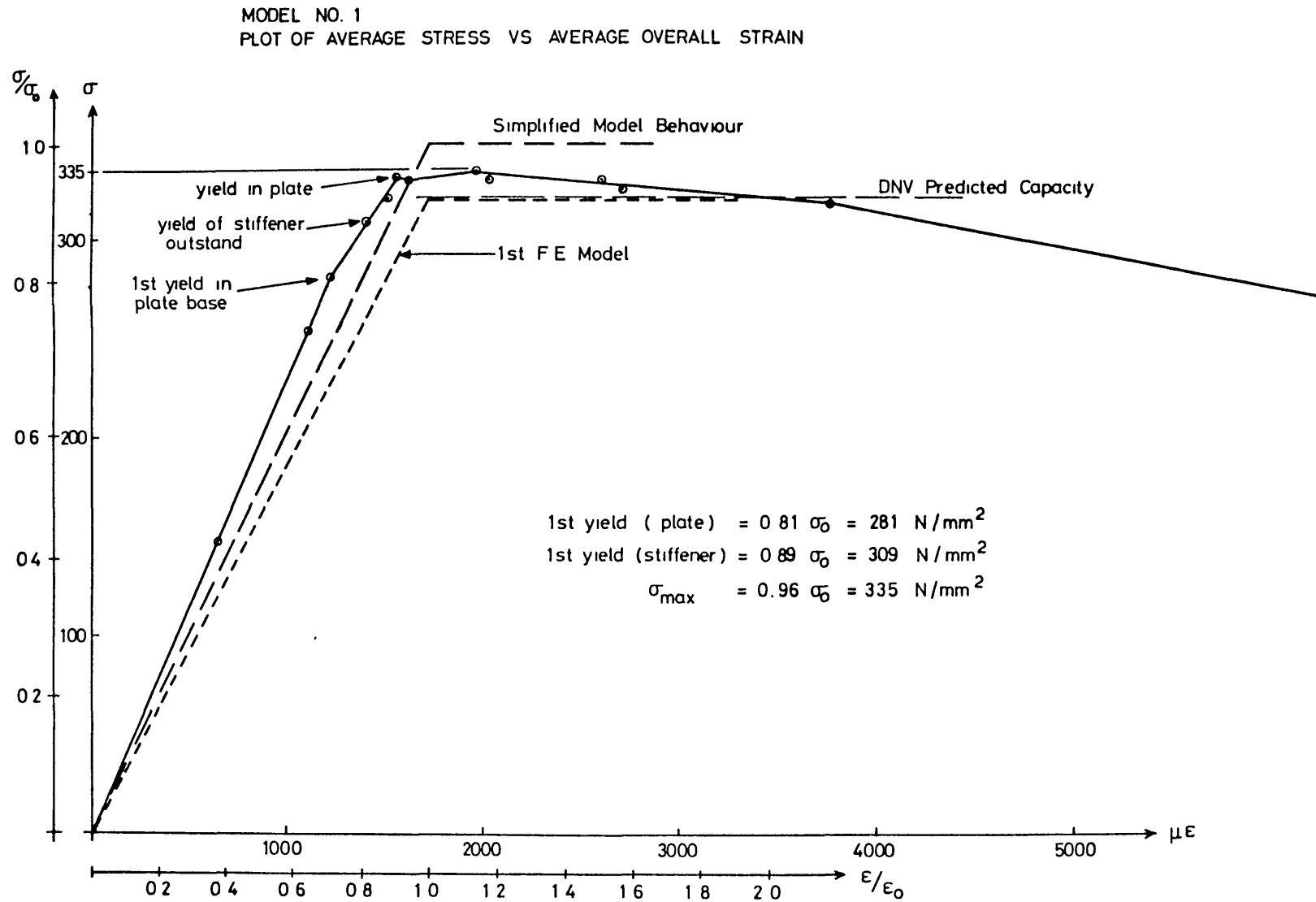


figure 2.32. Average stress-strain response of model no. 1.

Model no 1 Mid-height Strains vs Stress

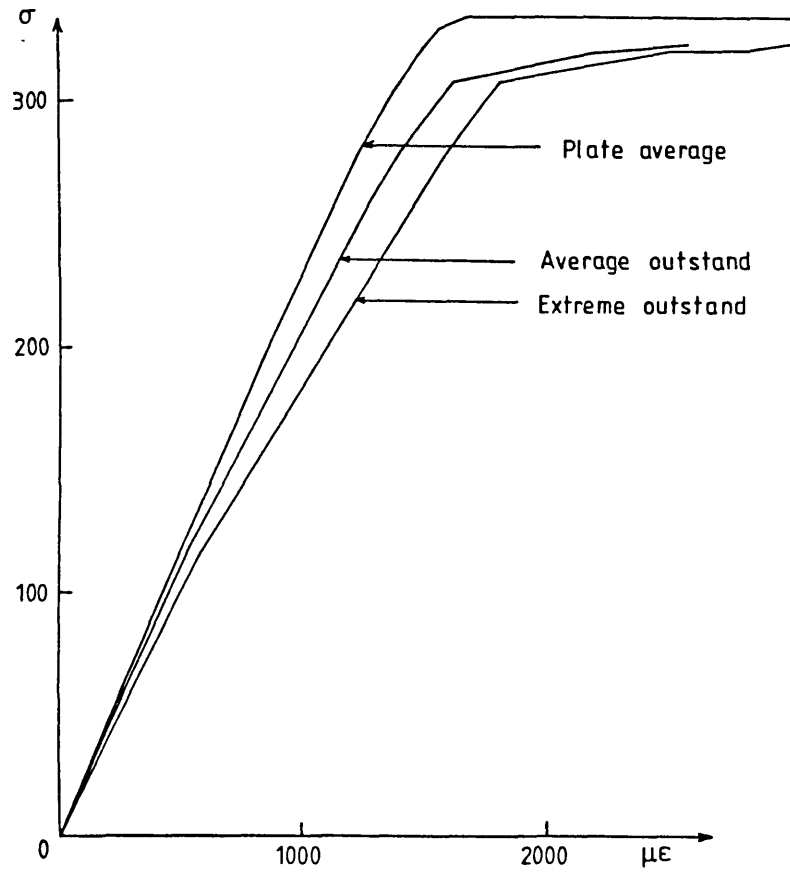


figure 2.33. Mid-height strain gauge response of model no. 1.

Model no 1 Strains at base vs stress

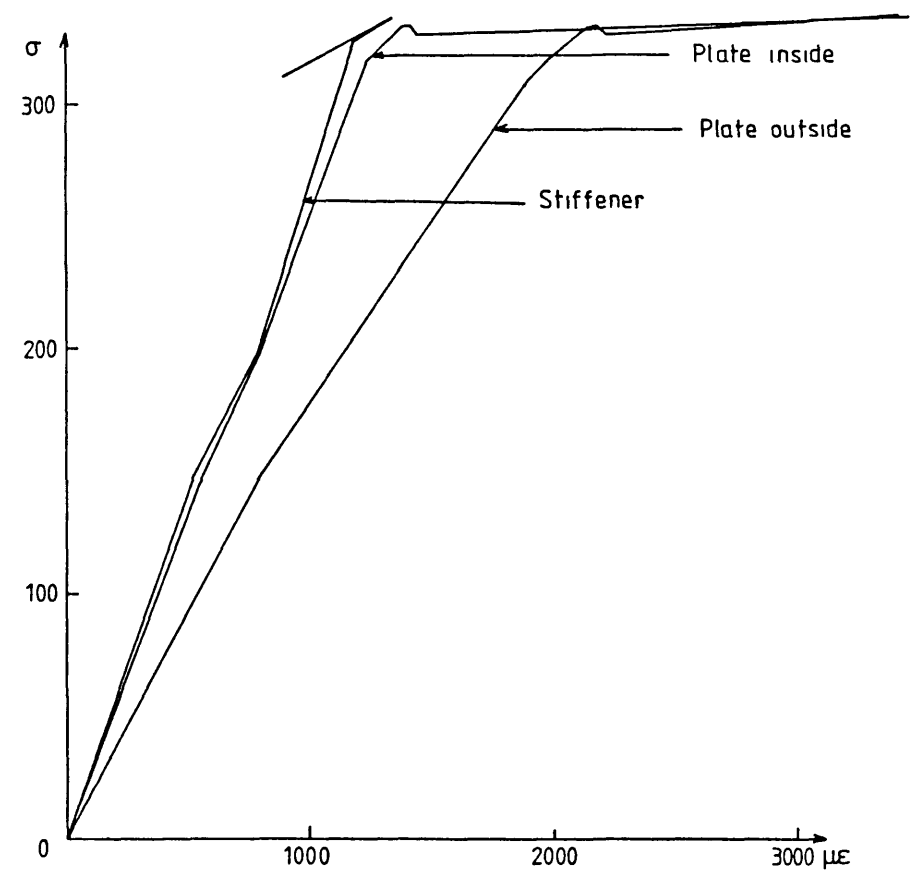


figure 2.34. Base strain gauge response of model no. 1.

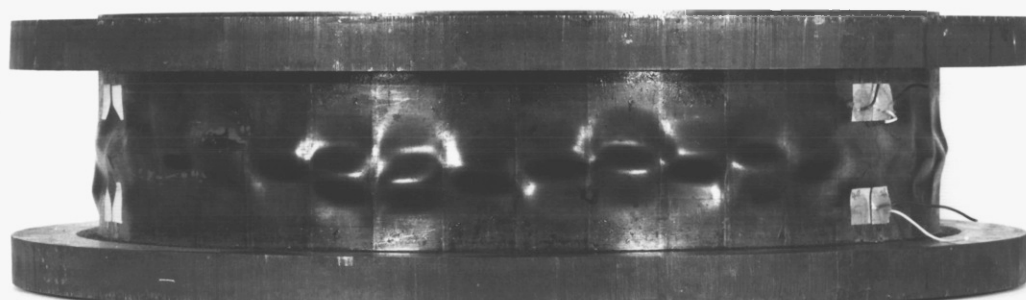
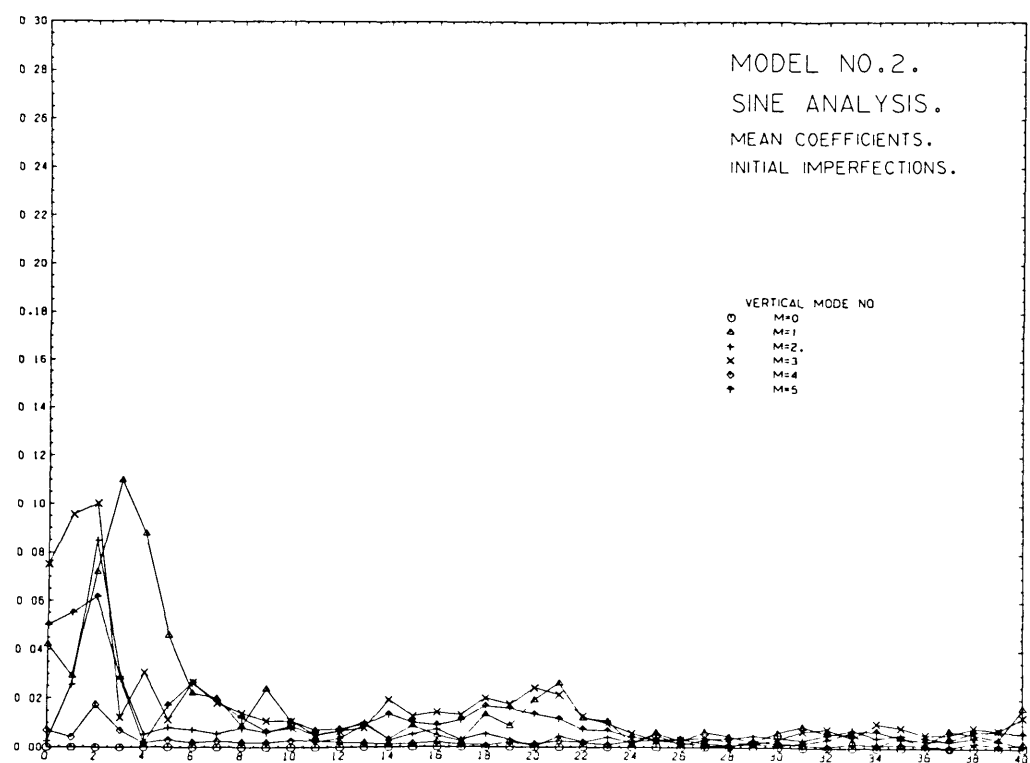
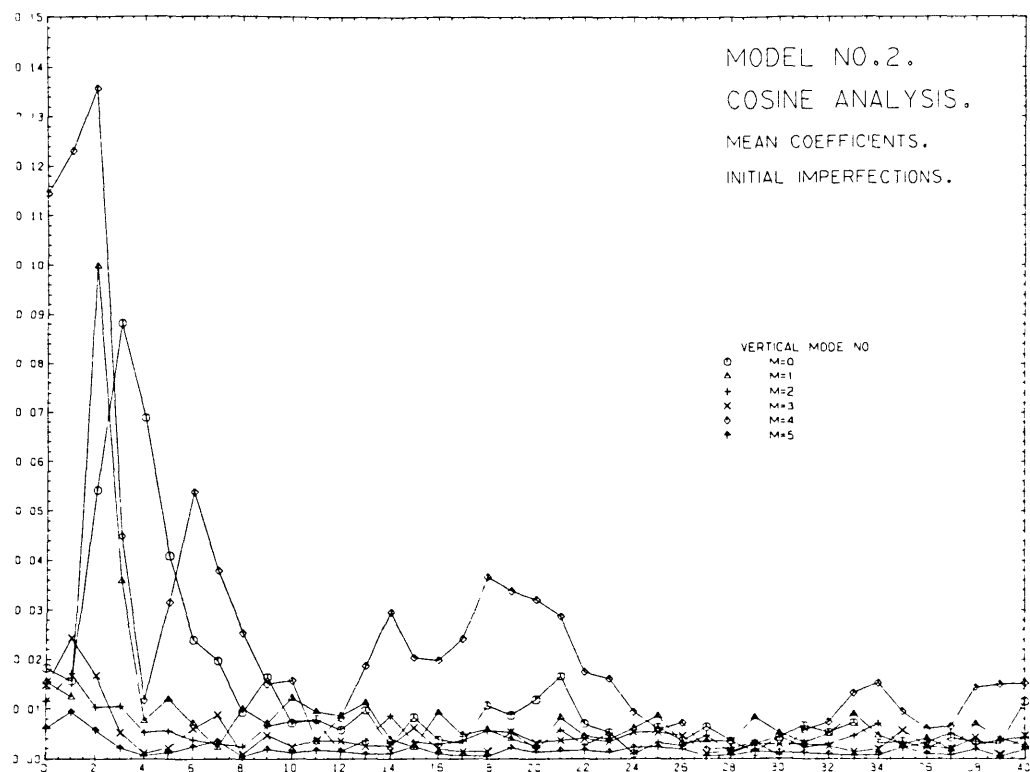
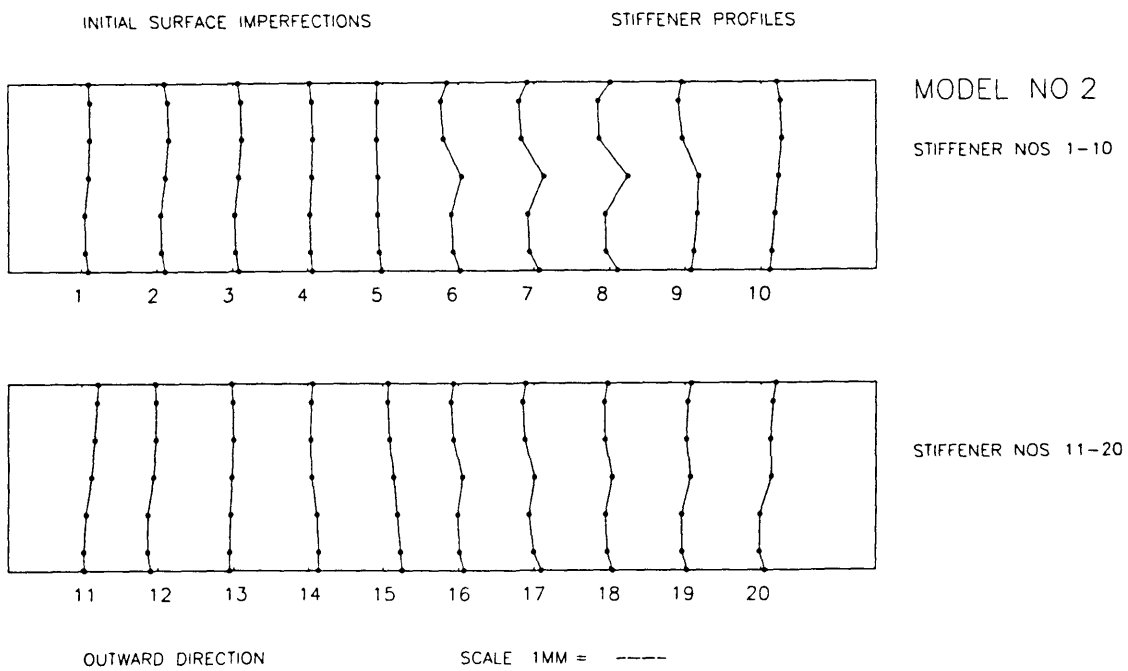
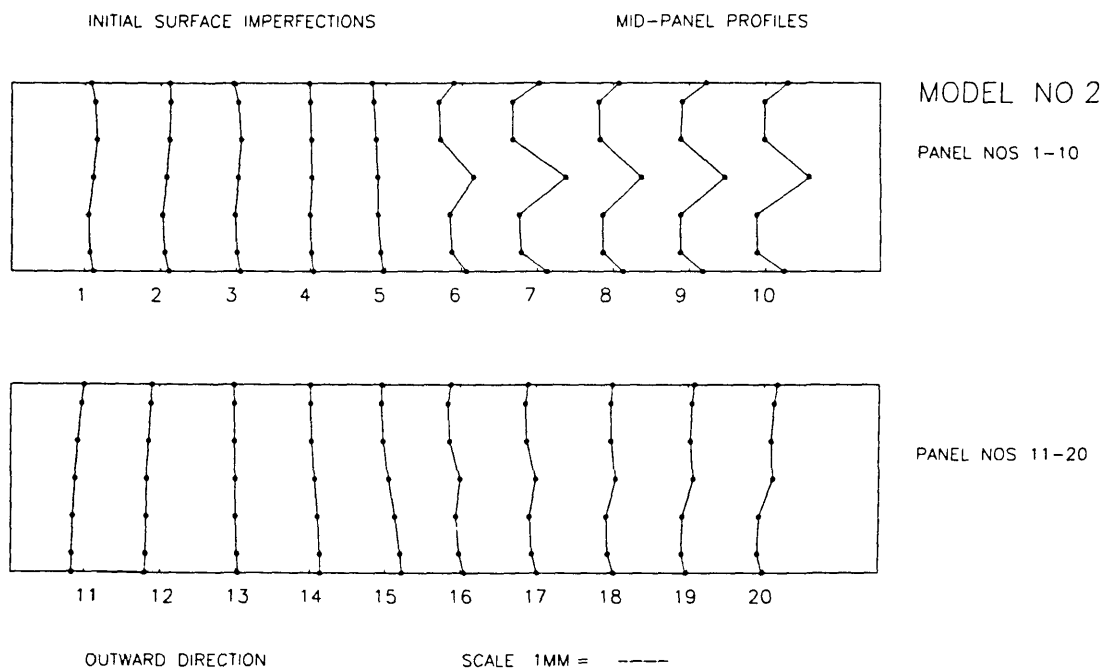


figure 2.35. Model no. 1 after collapse.



upper figure 2.36. Initial imperfection analysis.

lower figure 2.37. Initial imperfection analysis.



upper figure 2.38. Vertical profiles of initial imperfections.

lower figure 2.39. Vertical profile of initial imperfections.

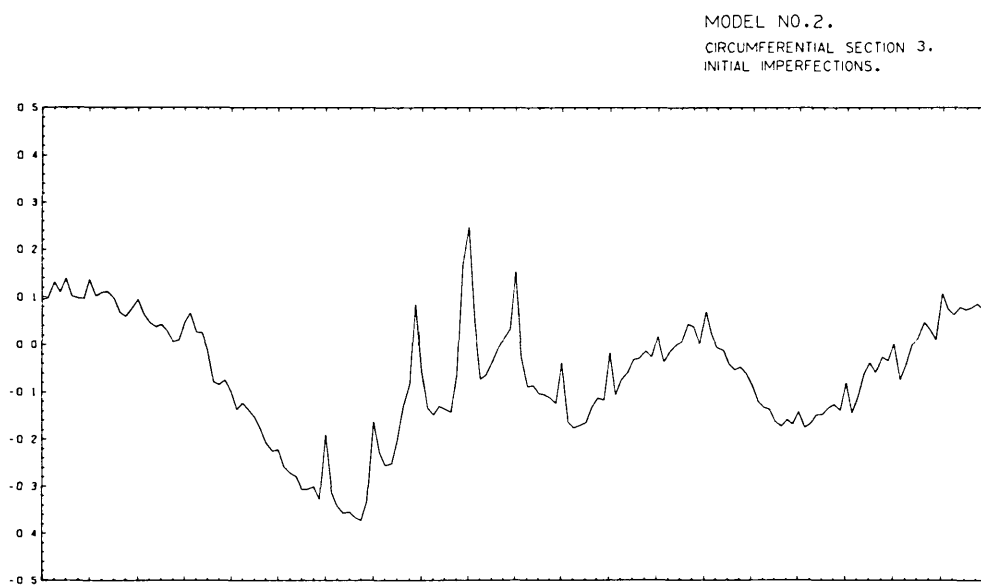


figure 2.40. Circumferential section 3.

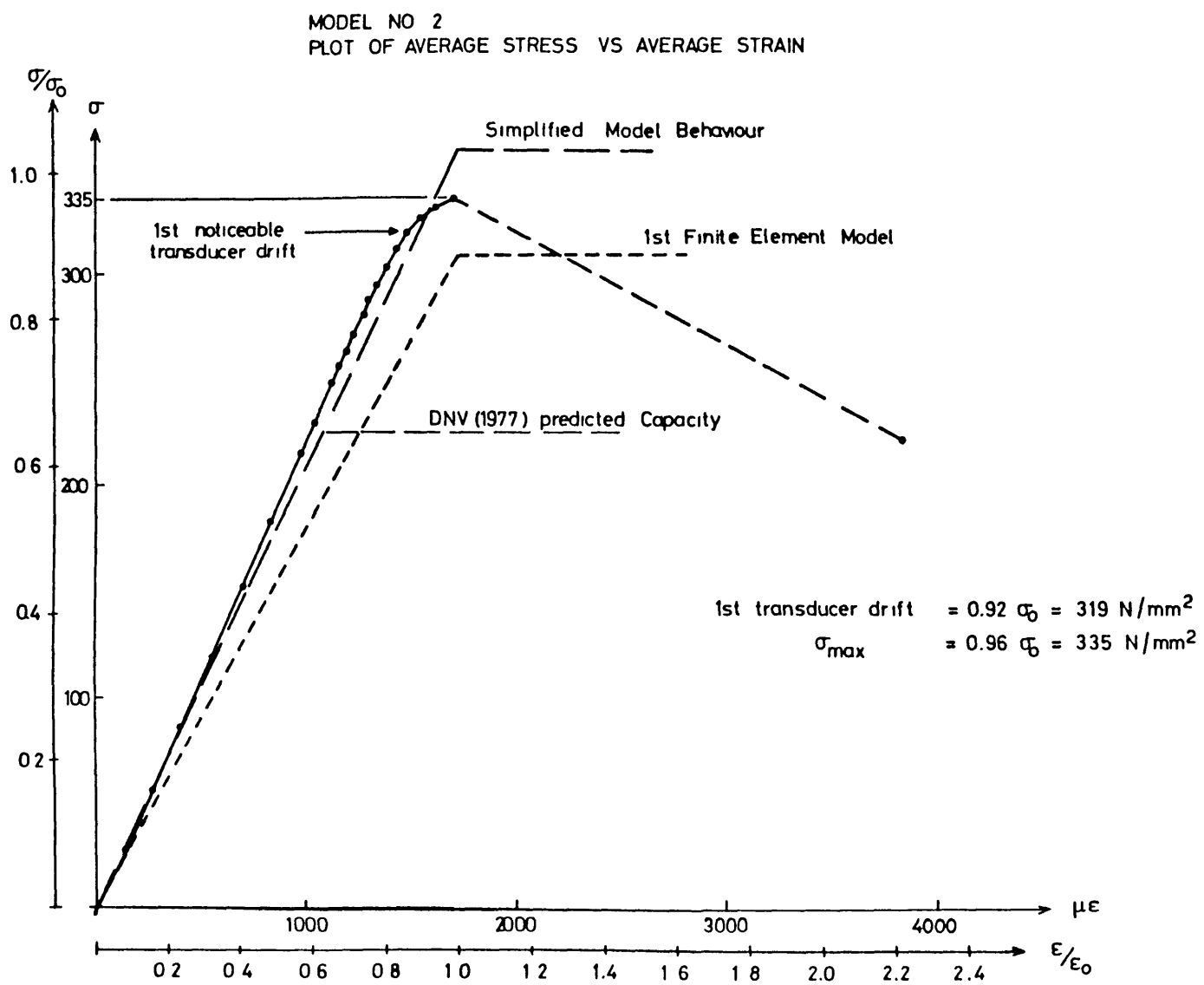


figure 2.41. Overall stress-strain response of model no. 2.

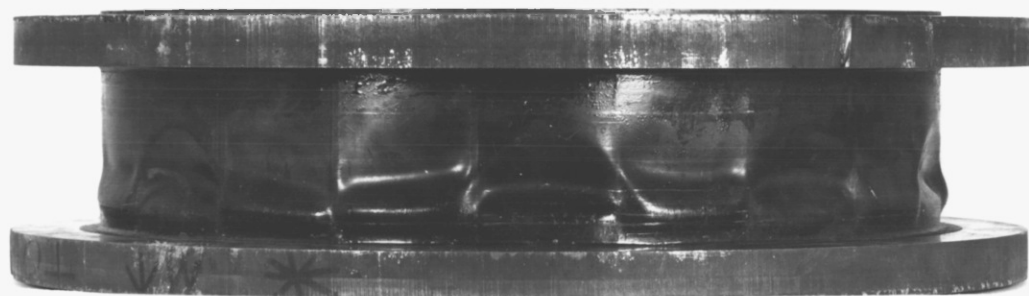
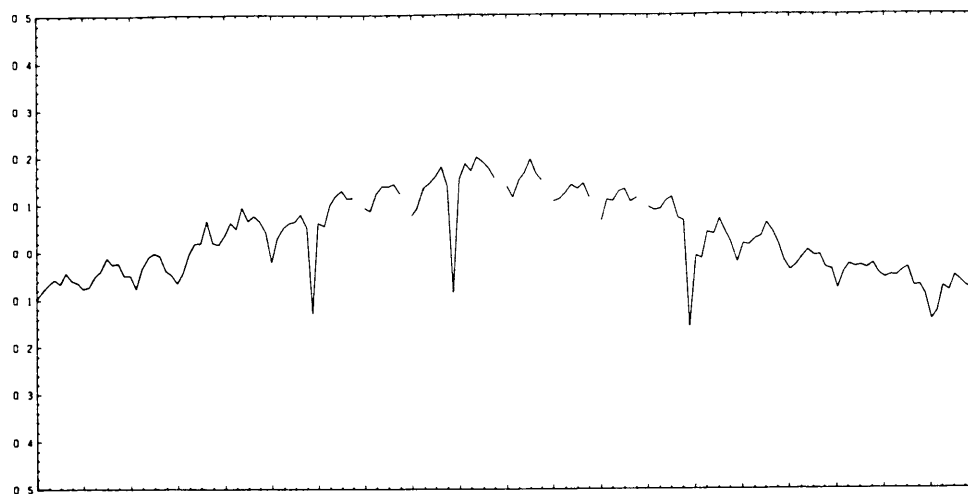
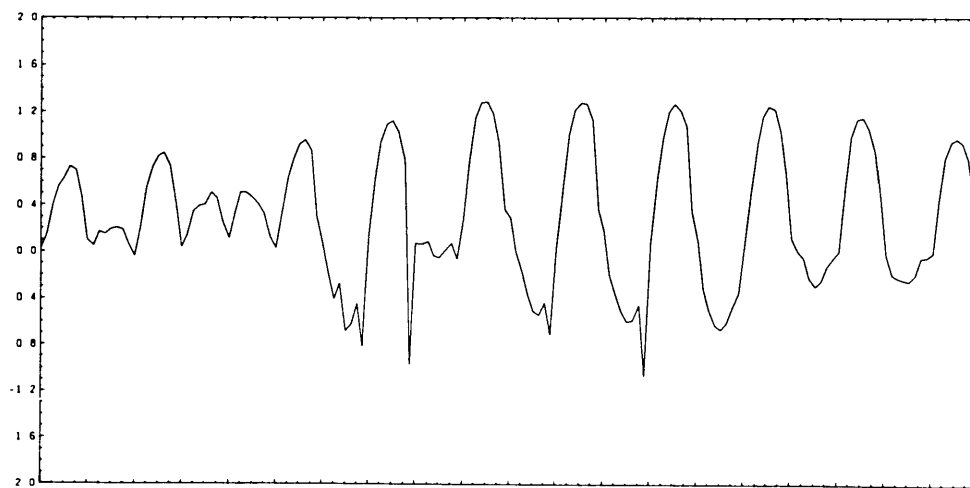


figure 2.42. Model no. 2 after collapse.

MODEL NO.2.
CIRCUMFERENTIAL SECTION 3
LOAD CASE 2.

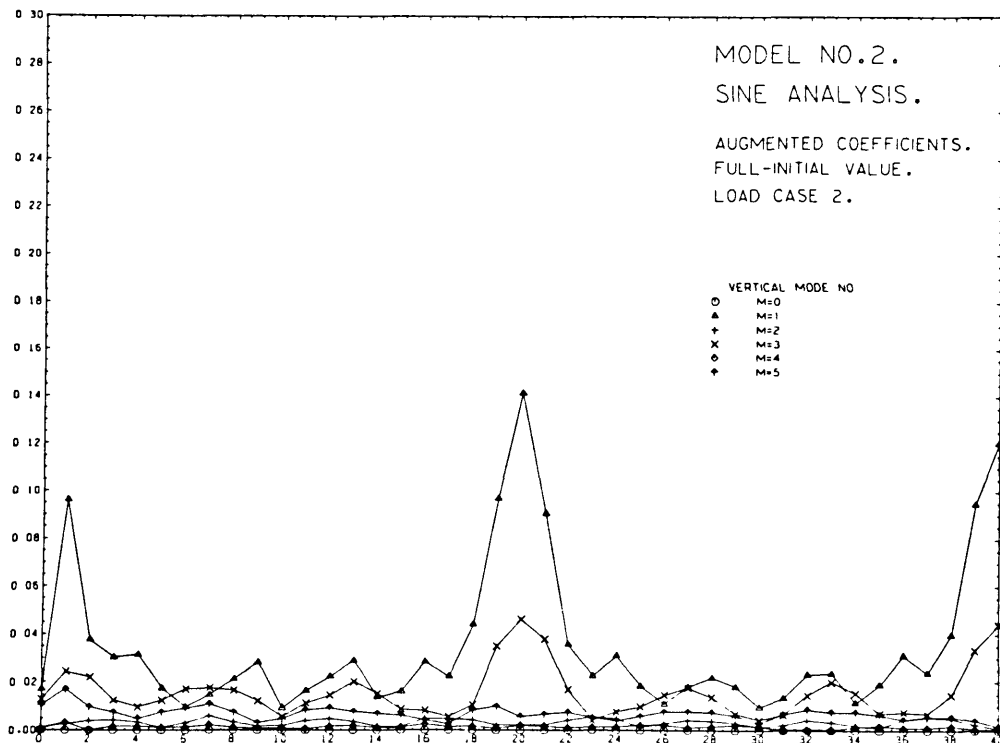
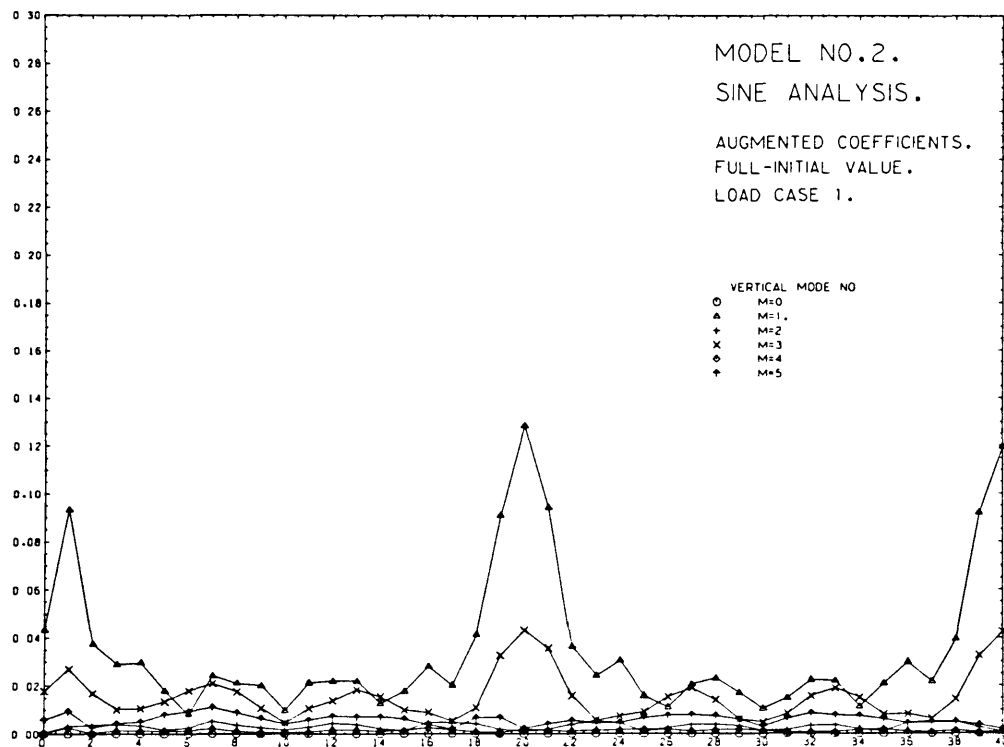


MODEL NO.2.
CIRCUMFERENTIAL SECTION 3
LOAD CASE 3.



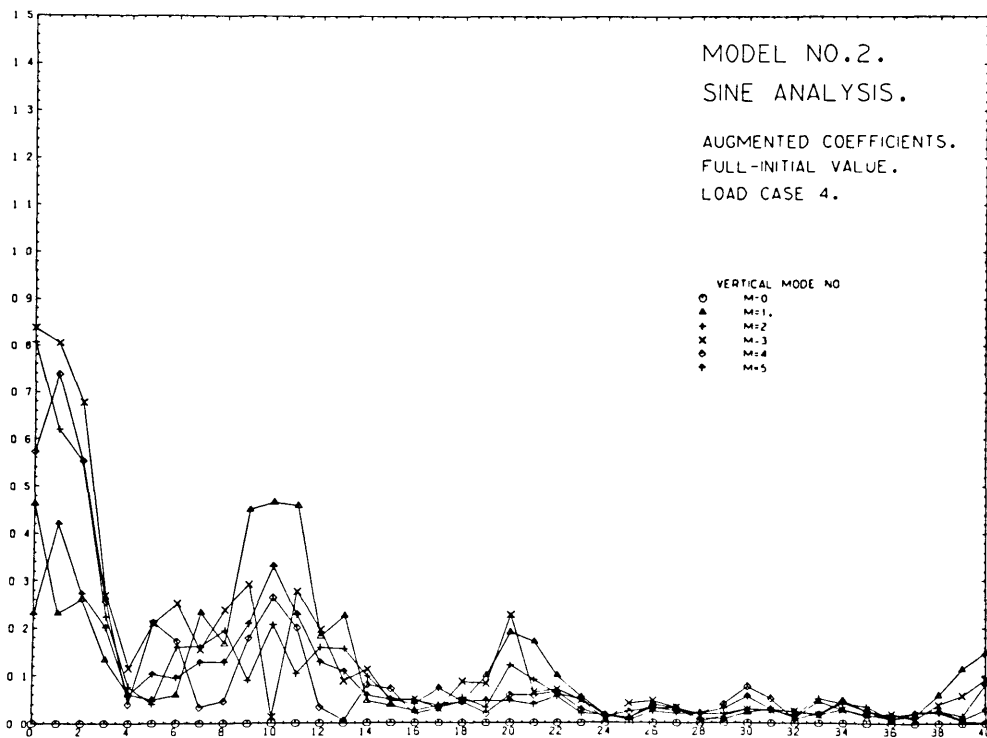
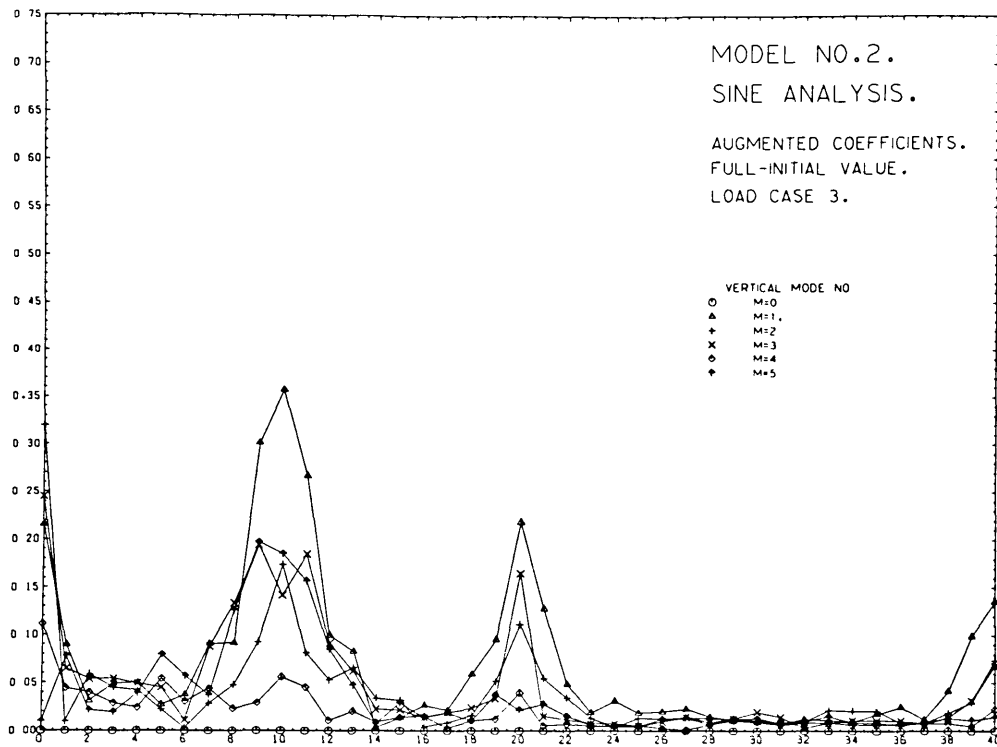
upper figure 2.43. Circumferential profile 3 at load case 2.

lower figure 2.44. Circumferential profile 3 at load case 3.



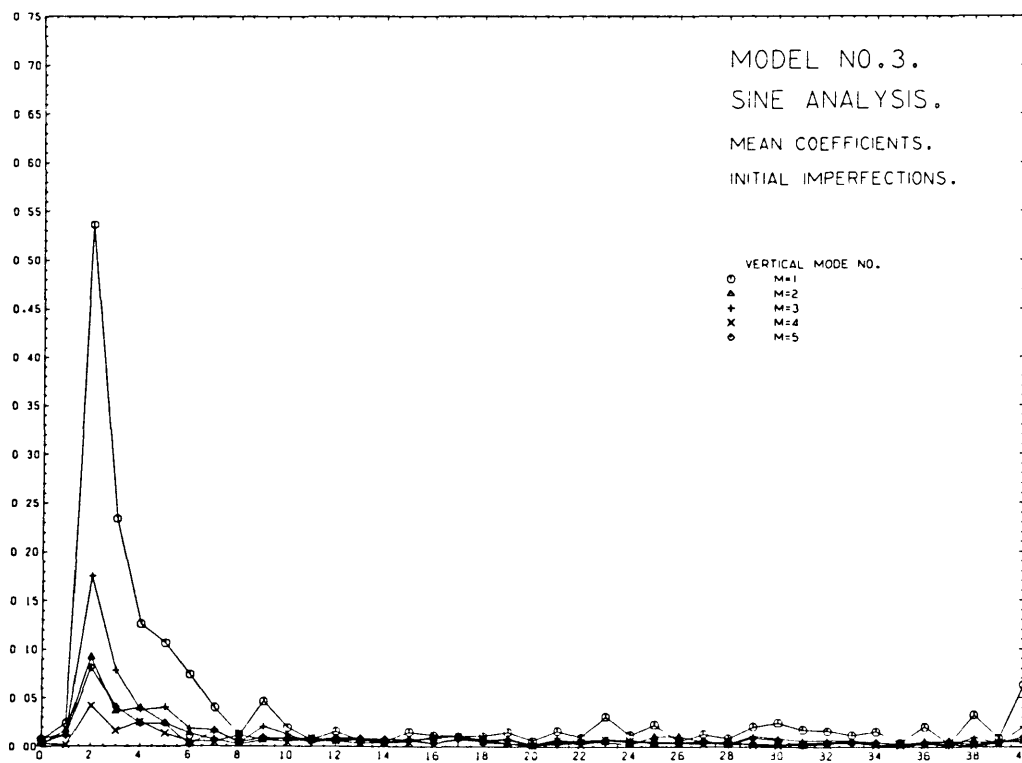
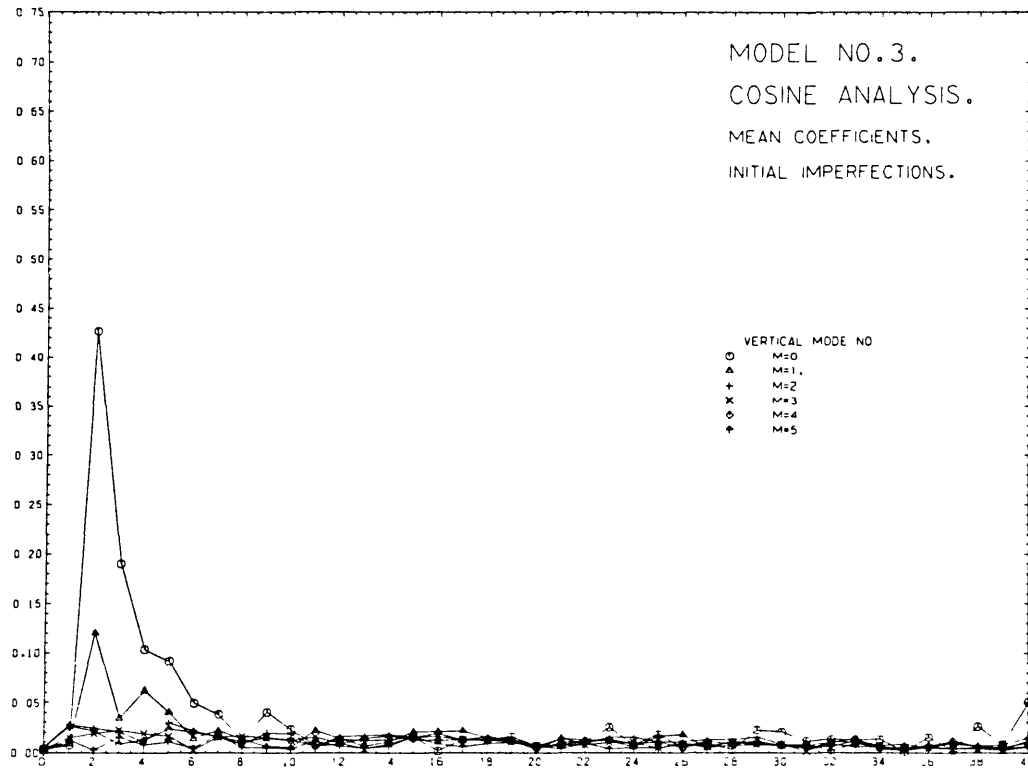
upper figure 2.45. Imperfection analysis, load case 1.

lower figure 2.46. Imperfection analysis, load case 2.



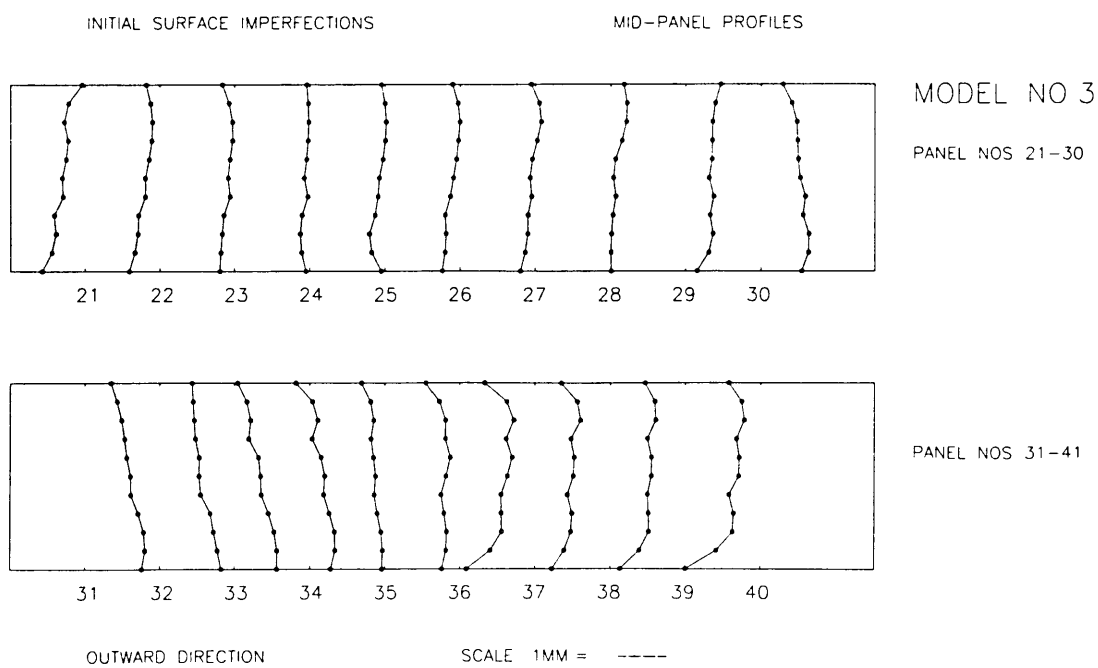
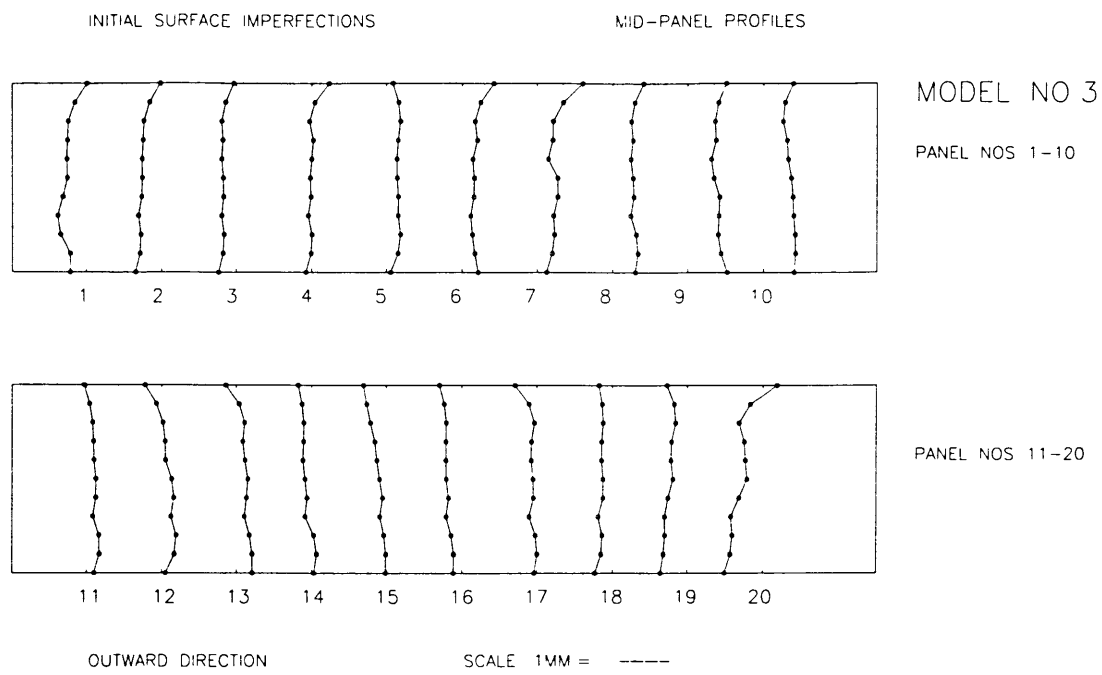
upper figure 2.47. Imperfection analysis, load case 3.

lower figure 2.48. Imperfection analysis, load case 4.



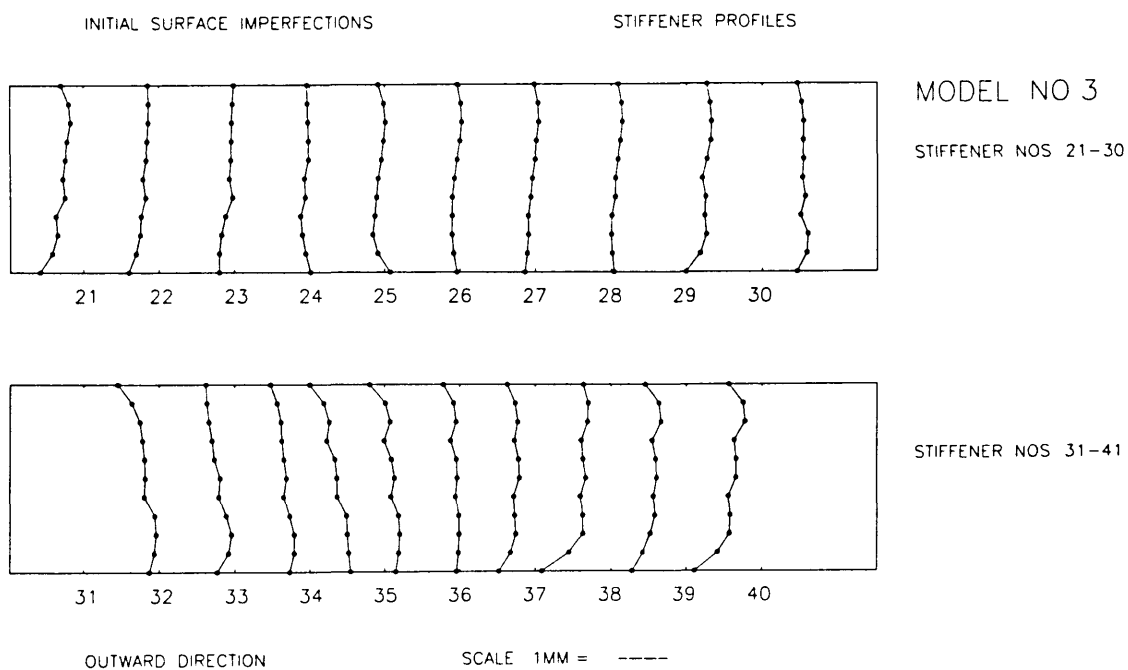
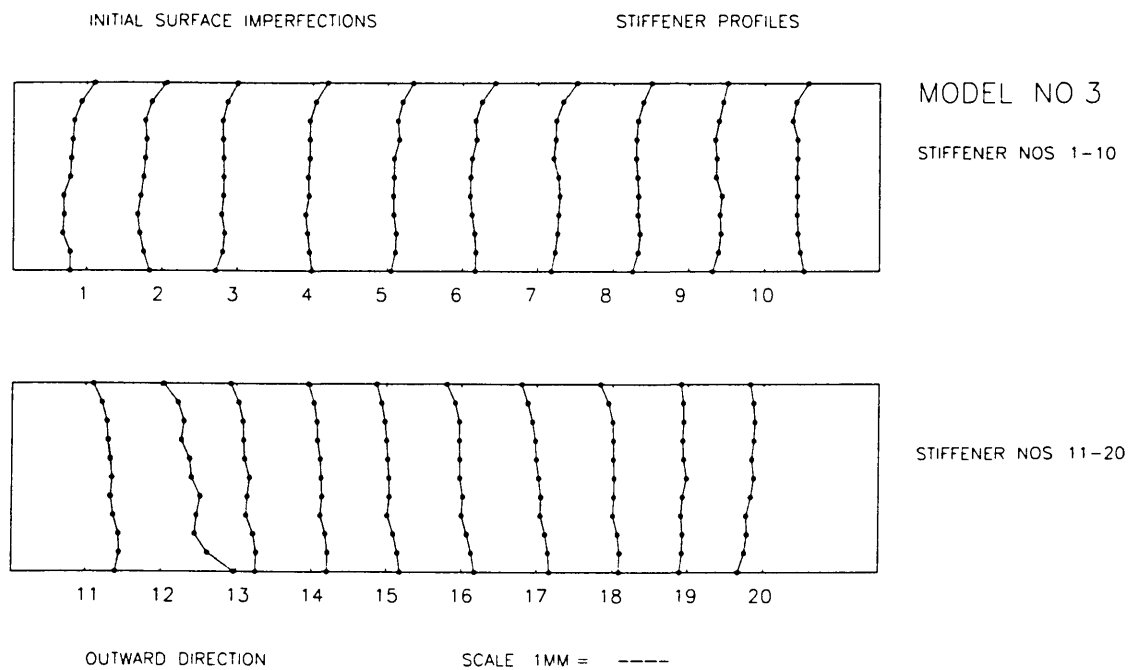
upper figure 2.49. Initial imperfection analysis.

lower figure 2.50. Initial imperfection analysis.



upper figure 2.51. Vertical profiles, panels 1-20.

lower figure 2.52. Vertical profiles, panels 21-40.



upper figure 2.53. Vertical profiles, stiffeners 1-20.

lower figure 2.54. Vertical profiles, stiffeners 21-40.

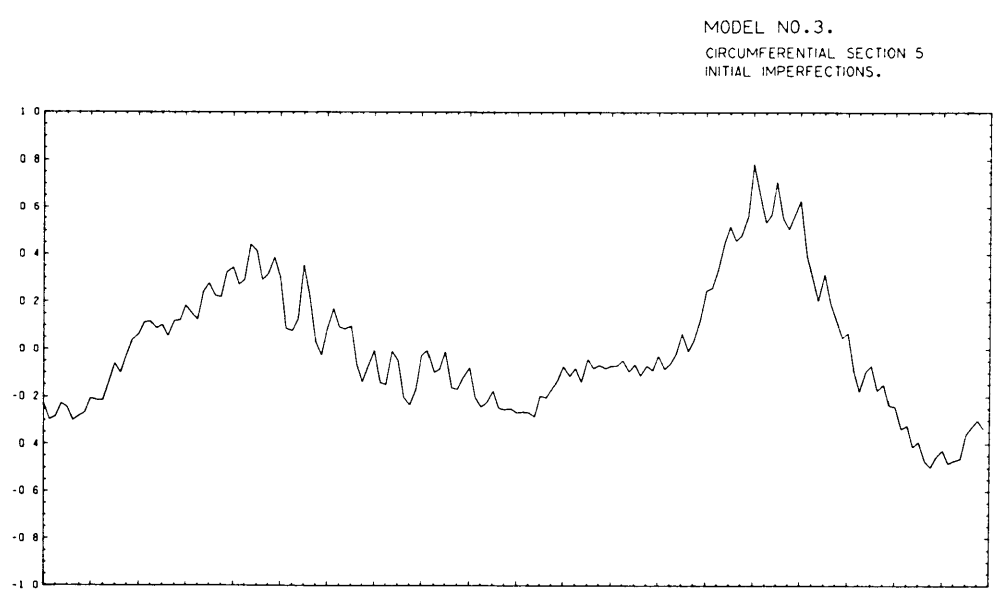


figure 2.55. Circumferential section no. 5.

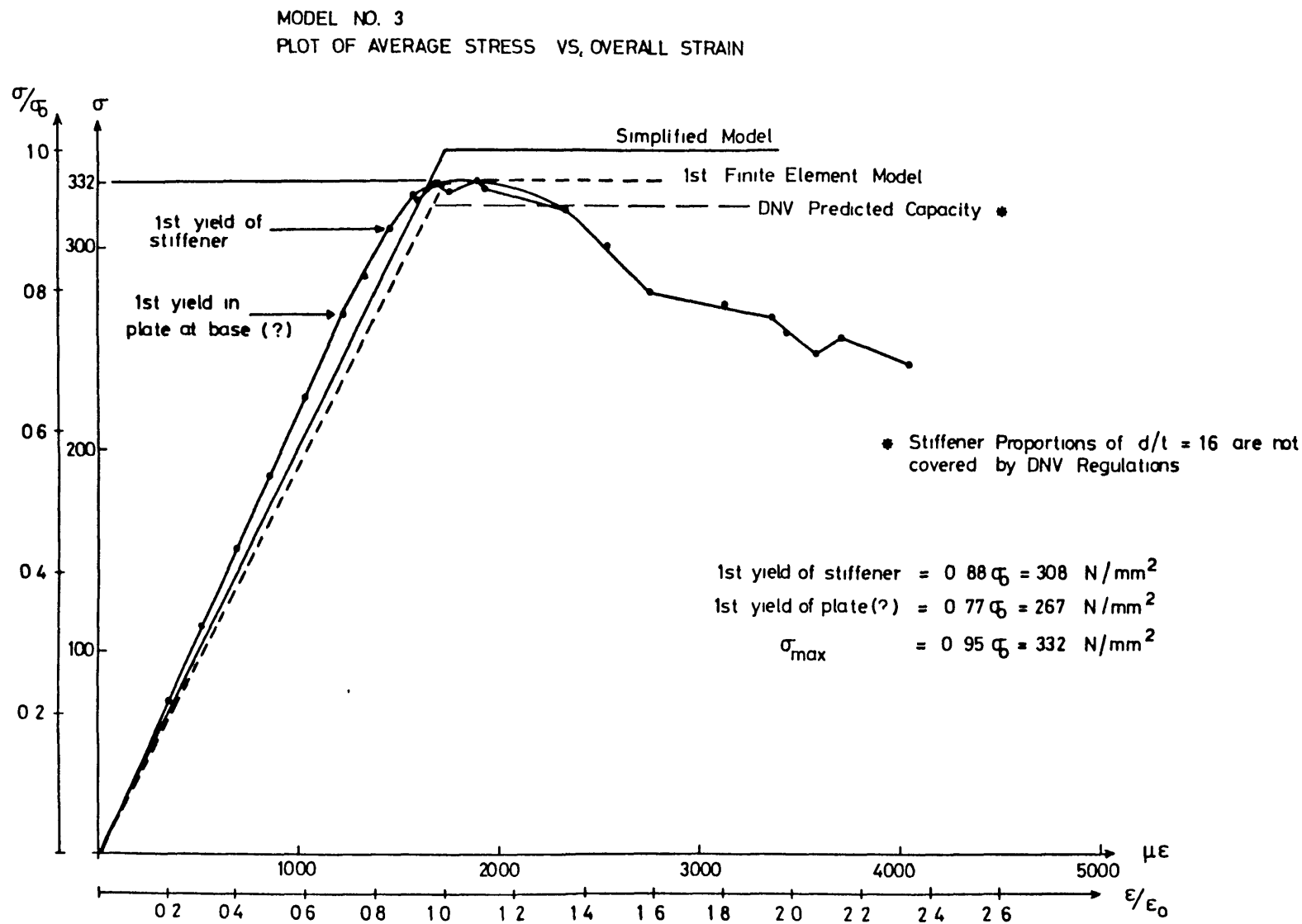


figure 2.56. Overall stress-strain response of model no. 3.

Model no 3 Mid-height strains vs stress

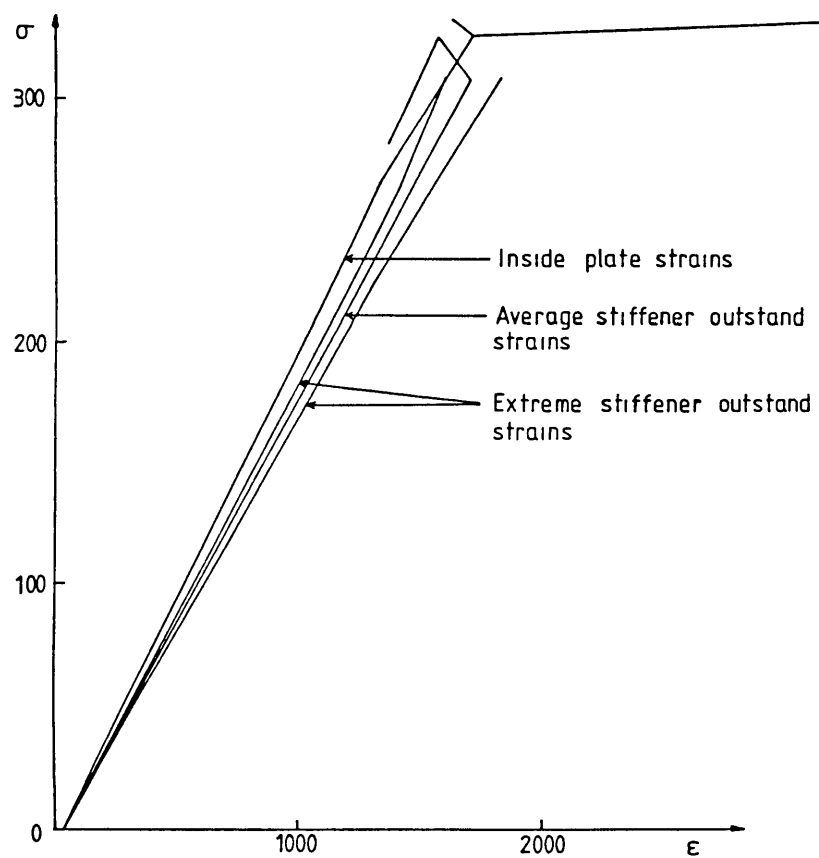


figure 2.57. Strain gauge response at midheight of model no. 3.

Model no 3 Base strains vs. stress

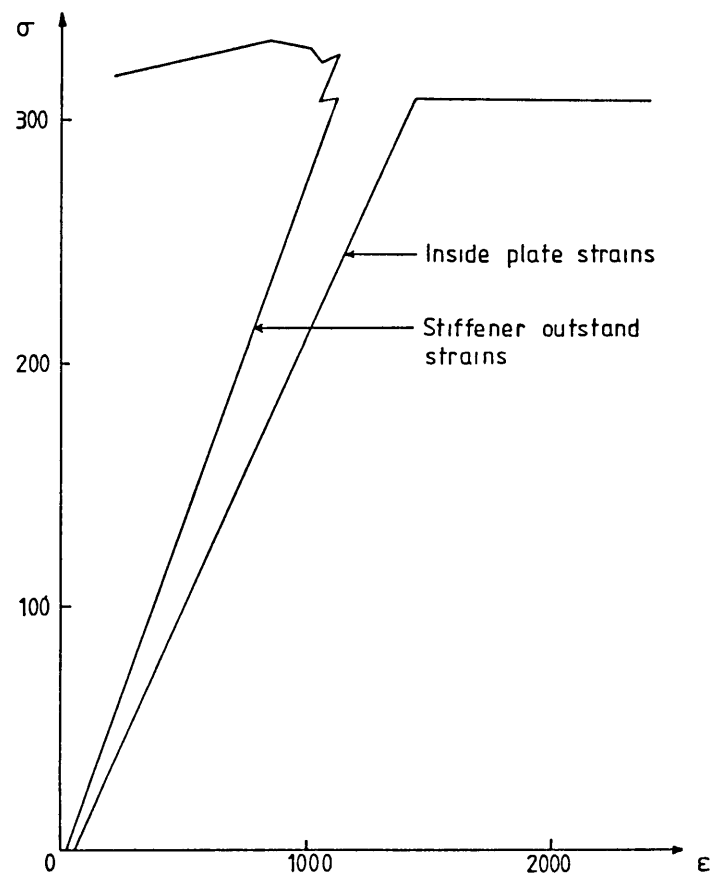


figure 2.58. Strain gauge response at base of model no. 3.

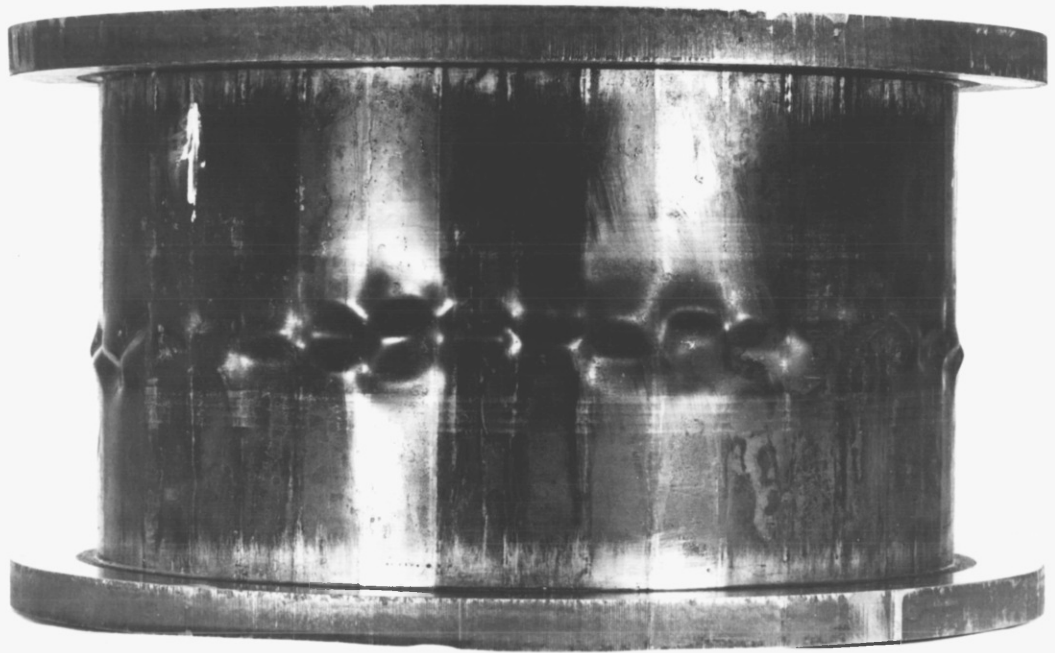
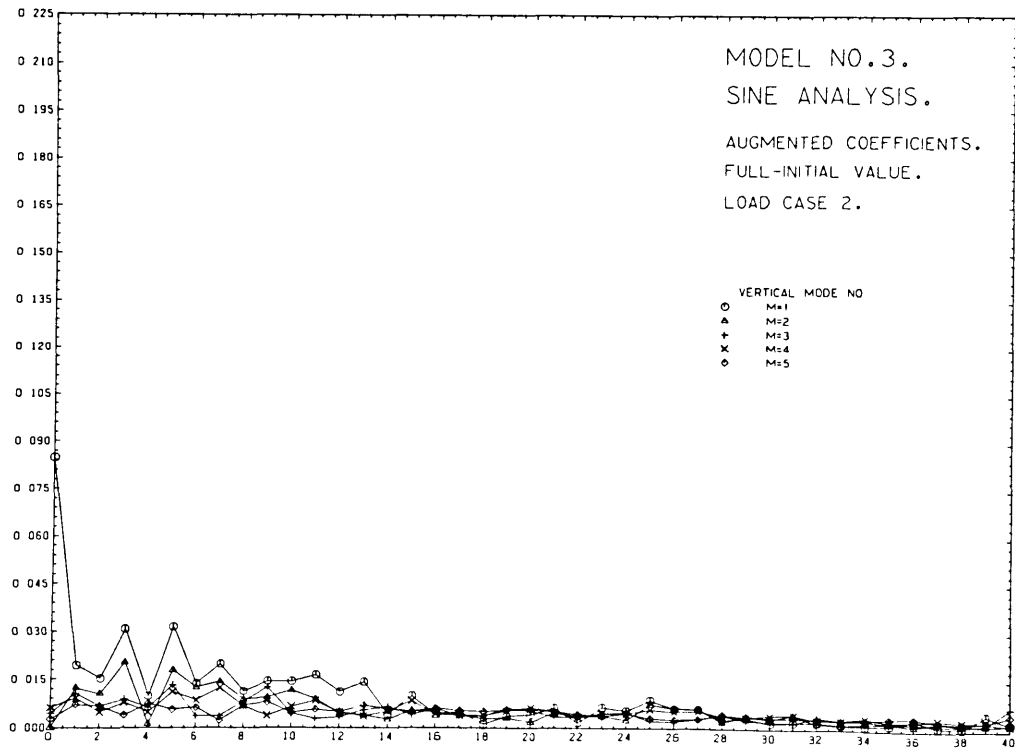
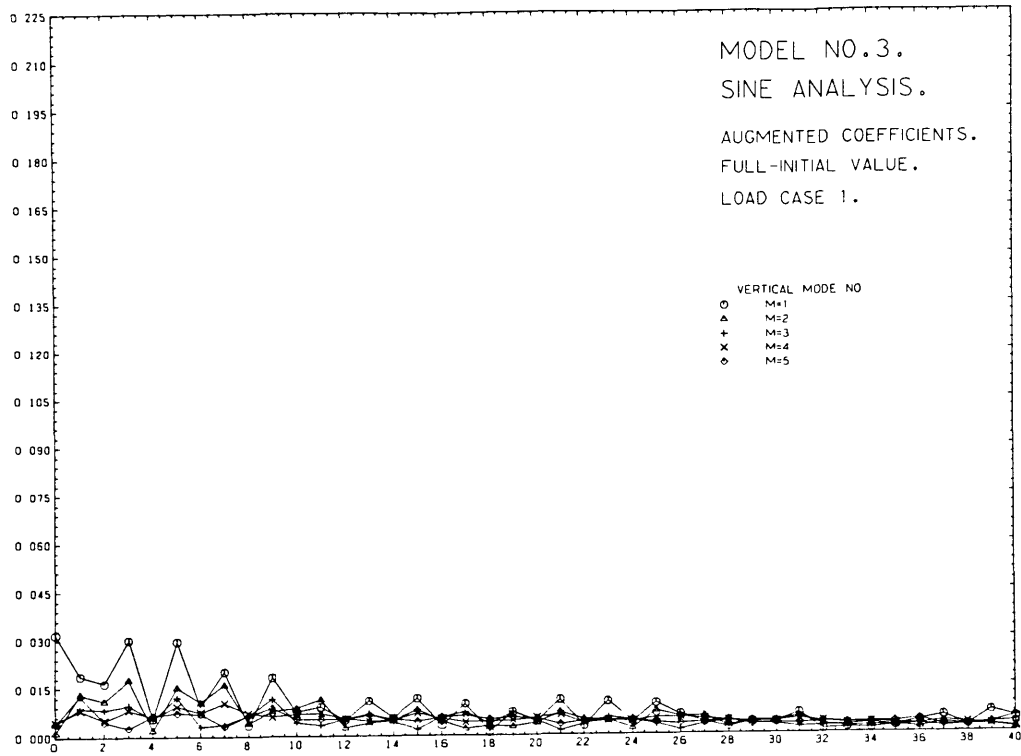
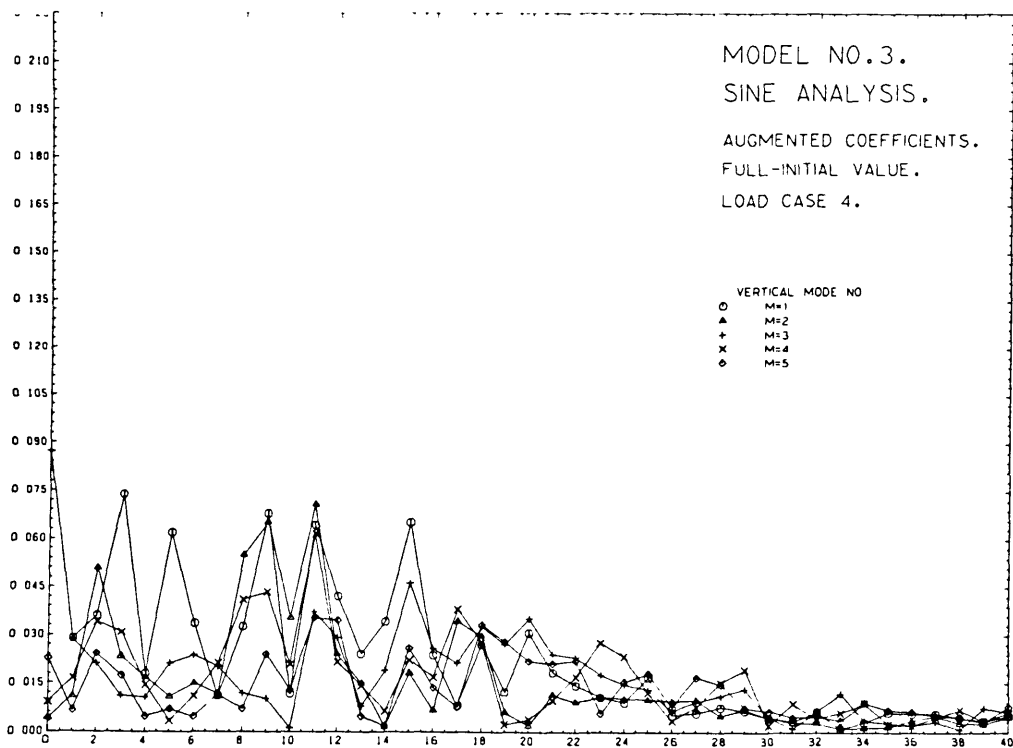
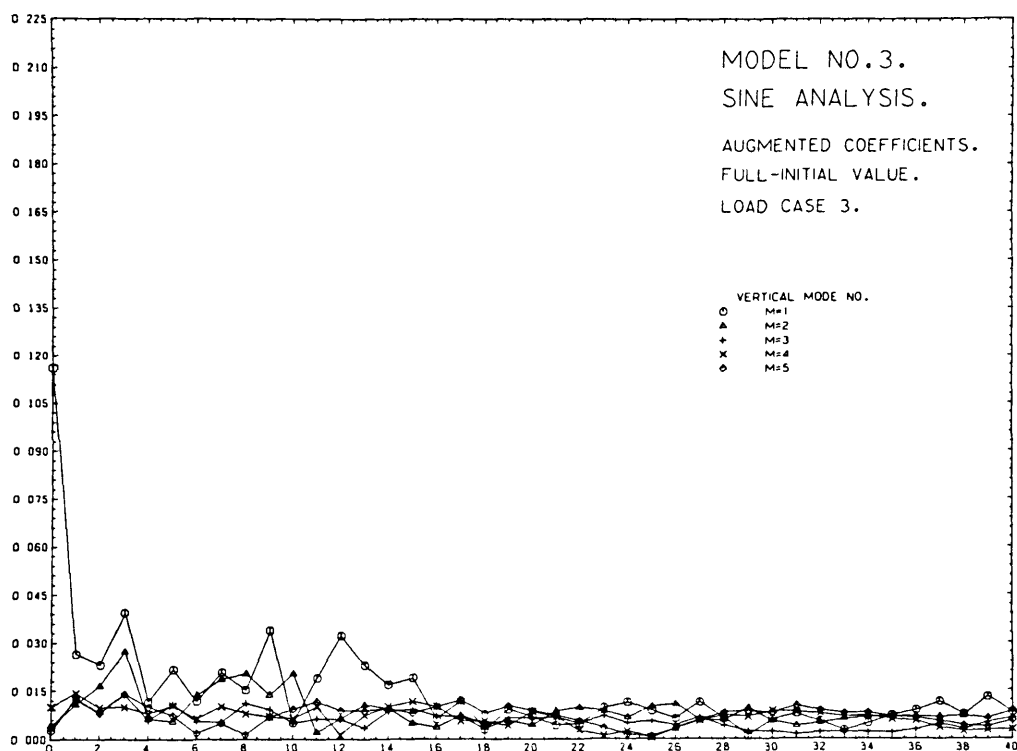


figure 2.59. Model no. 3 after collapse



upper figure 2.60. Imperfection analysis, load case 1.

lower figure 2.61. Imperfection analysis, load case 2.



upper figure 2.62. Imperfection analysis, load case 3.

lower figure 2.63. Imperfection analysis, load case 4.

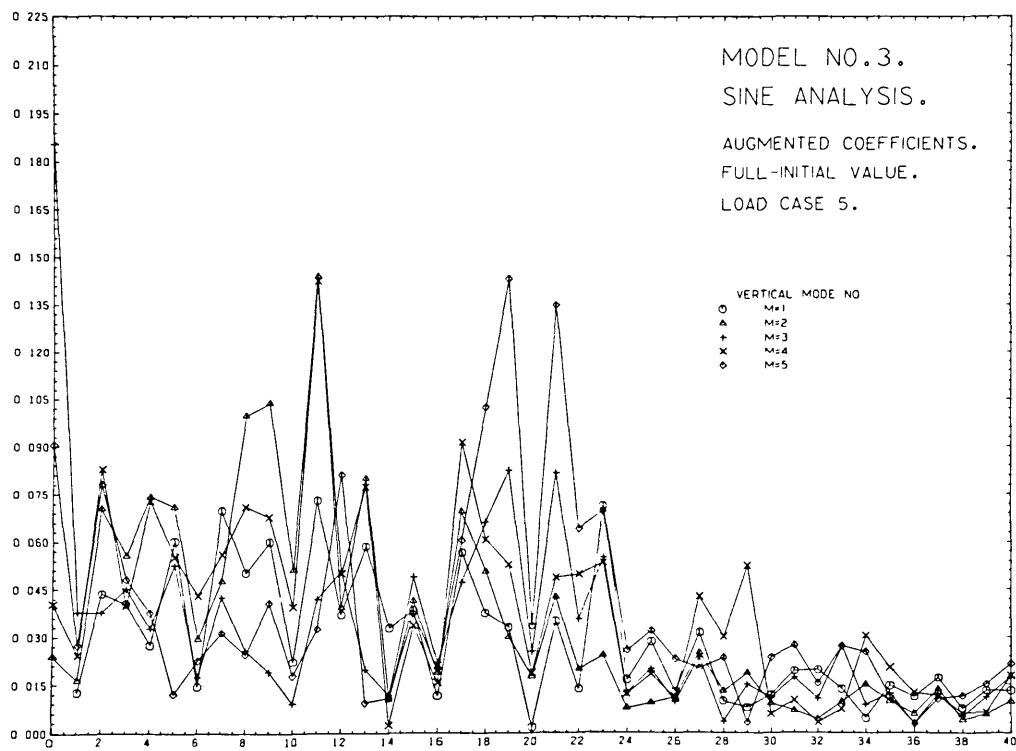
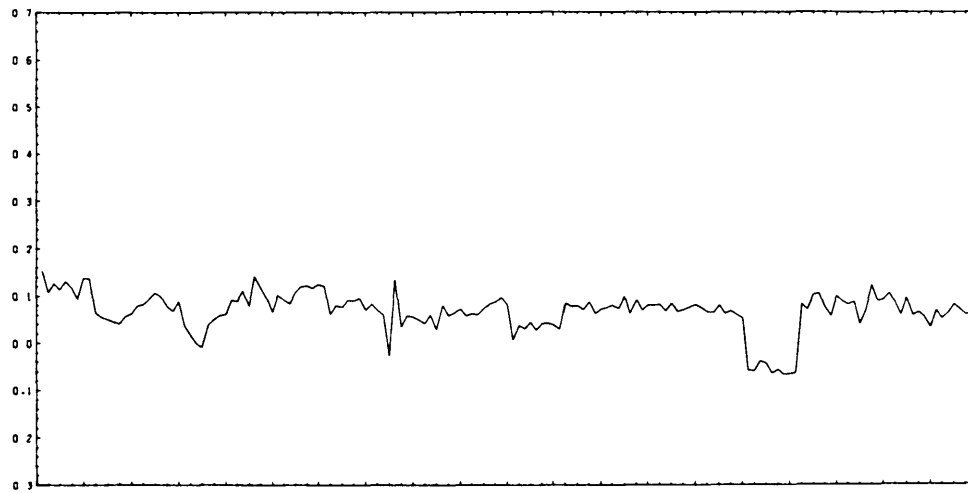
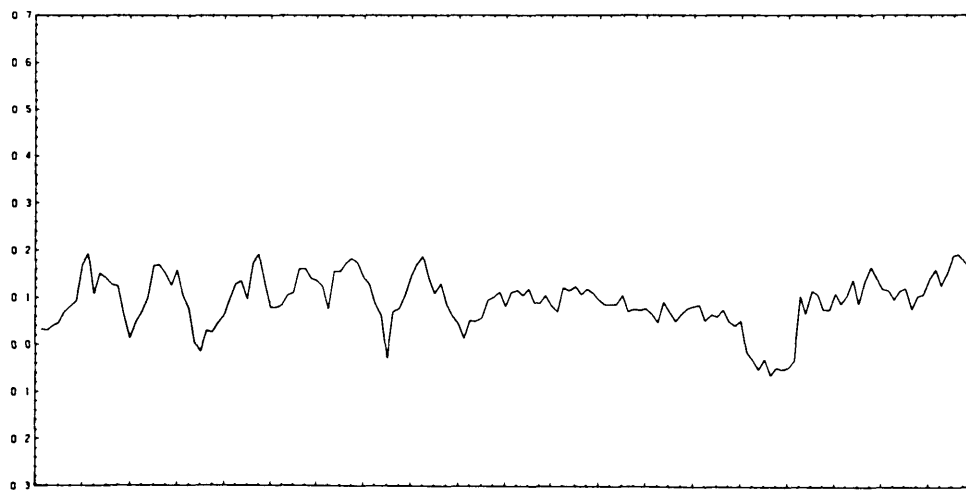


figure 2.64. Imperfection analysis, load case 5.

MODEL NO.3.
CIRCUMFERENTIAL SECTION 5.
LOAD CASE 2.



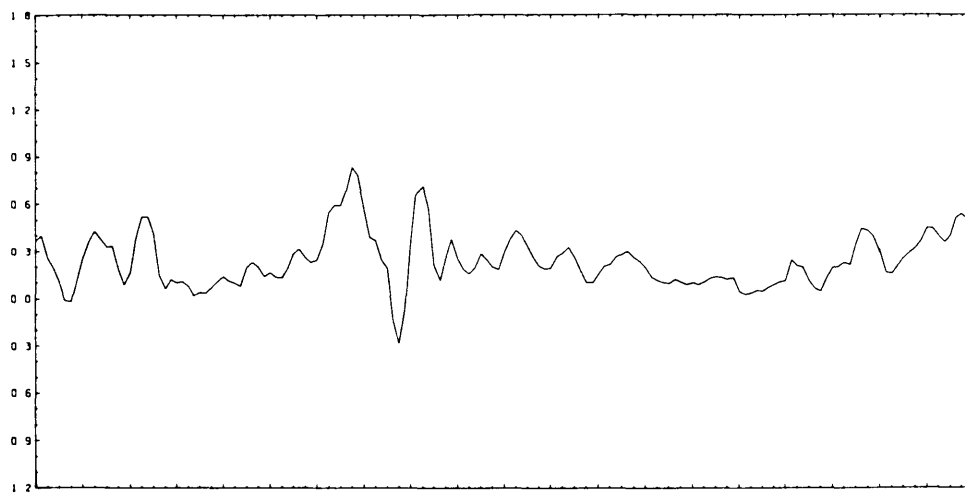
MODEL NO.3.
CIRCUMFERENTIAL SECTION 5
LOAD CASE 3.



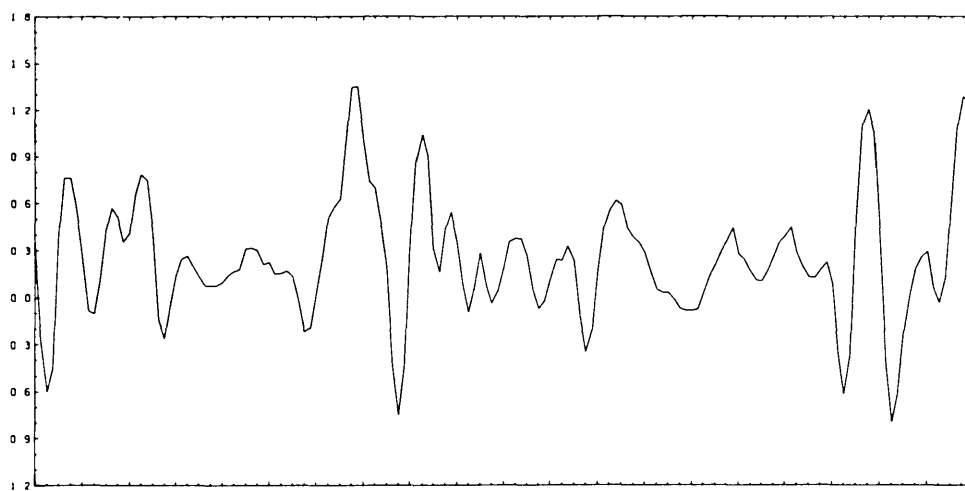
upper figure 2.65. Circumferential section 5, load case 2.

lower figure 2.66. Circumferential section 5, load case 3.

MODEL NO.3.
CIRCUMFERENTIAL SECTION 5
LOAD CASE 4.

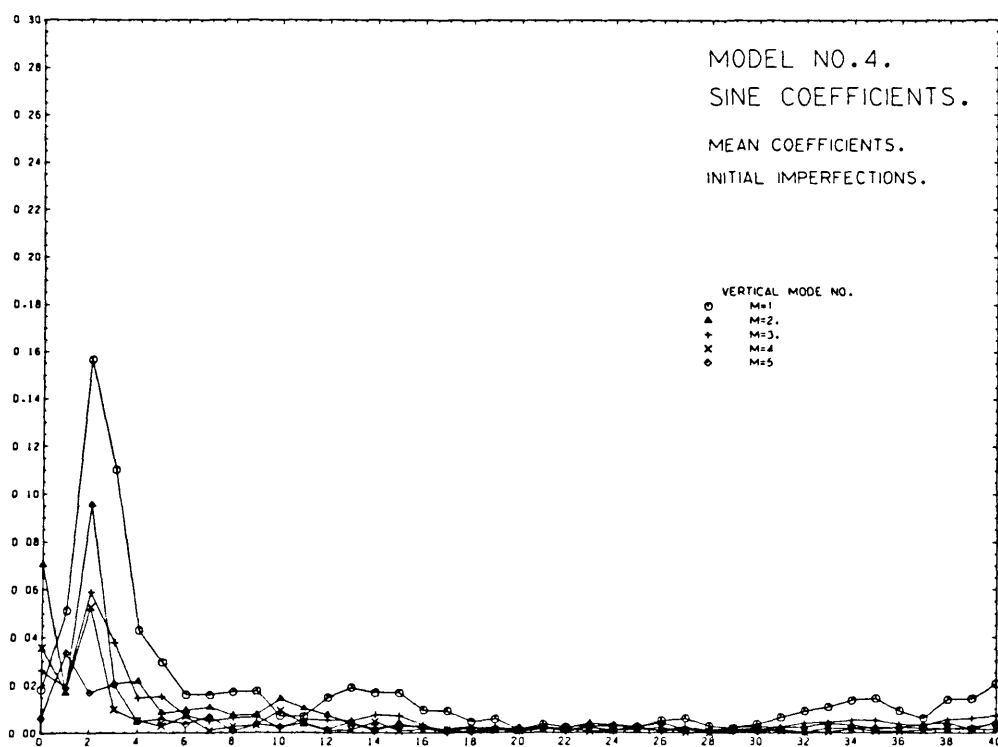
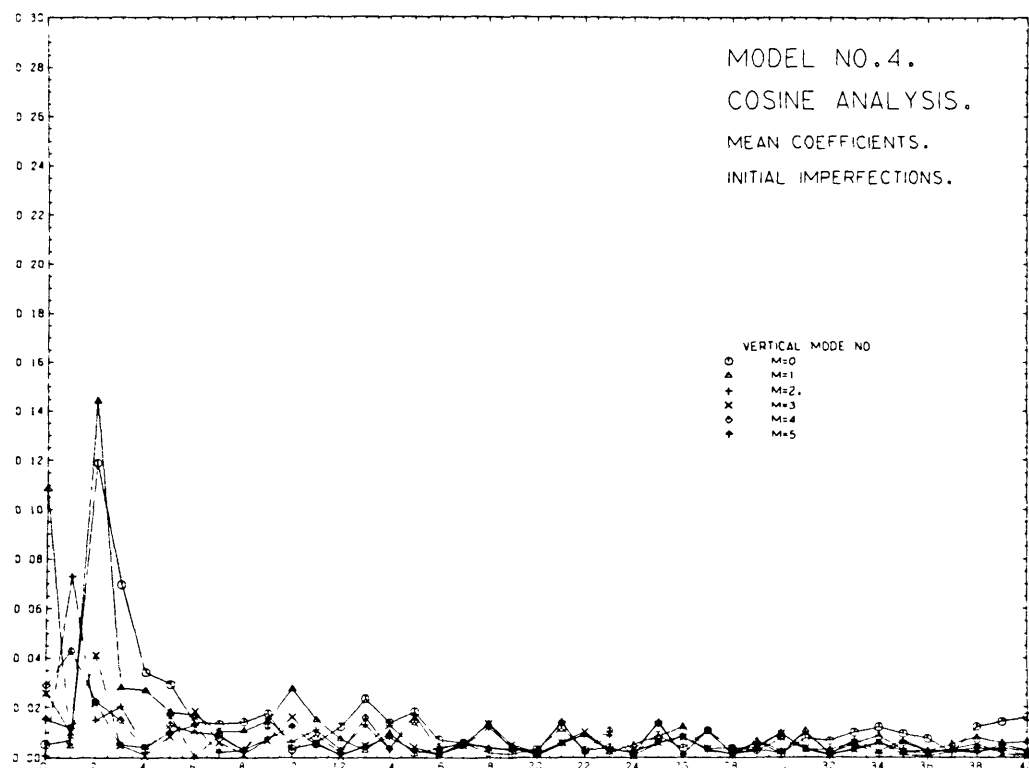


MODEL NO.3.
CIRCUMFERENTIAL SECTION 5
LOAD CASE 5.



upper figure 2.67. Circumferential section 5, load case 4.

lower figure 2.68. Circumferential section 5, load case 5.



upper figure 2.69. Initial imperfection analysis.

lower figure 2.70. Initial imperfection analysis.

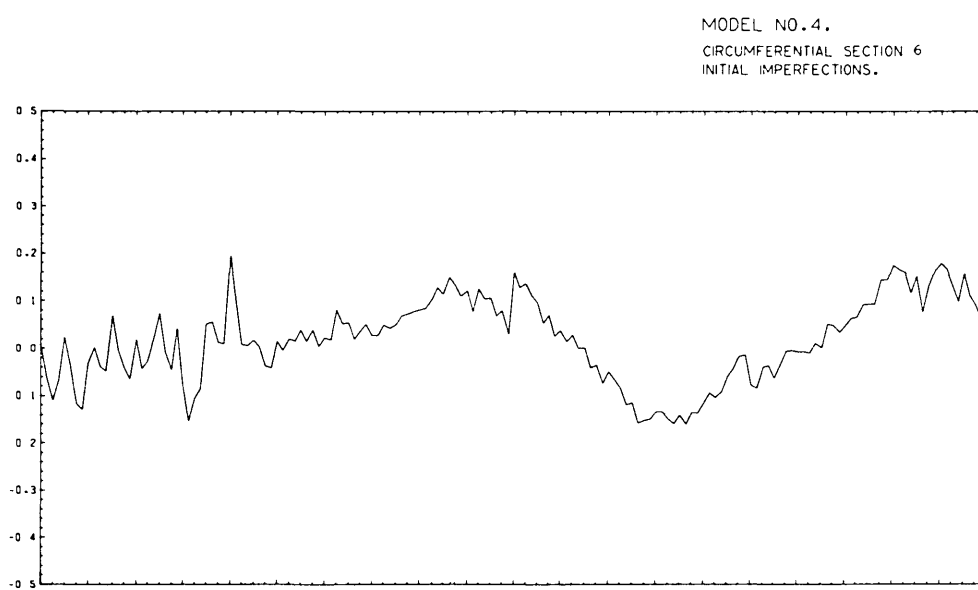
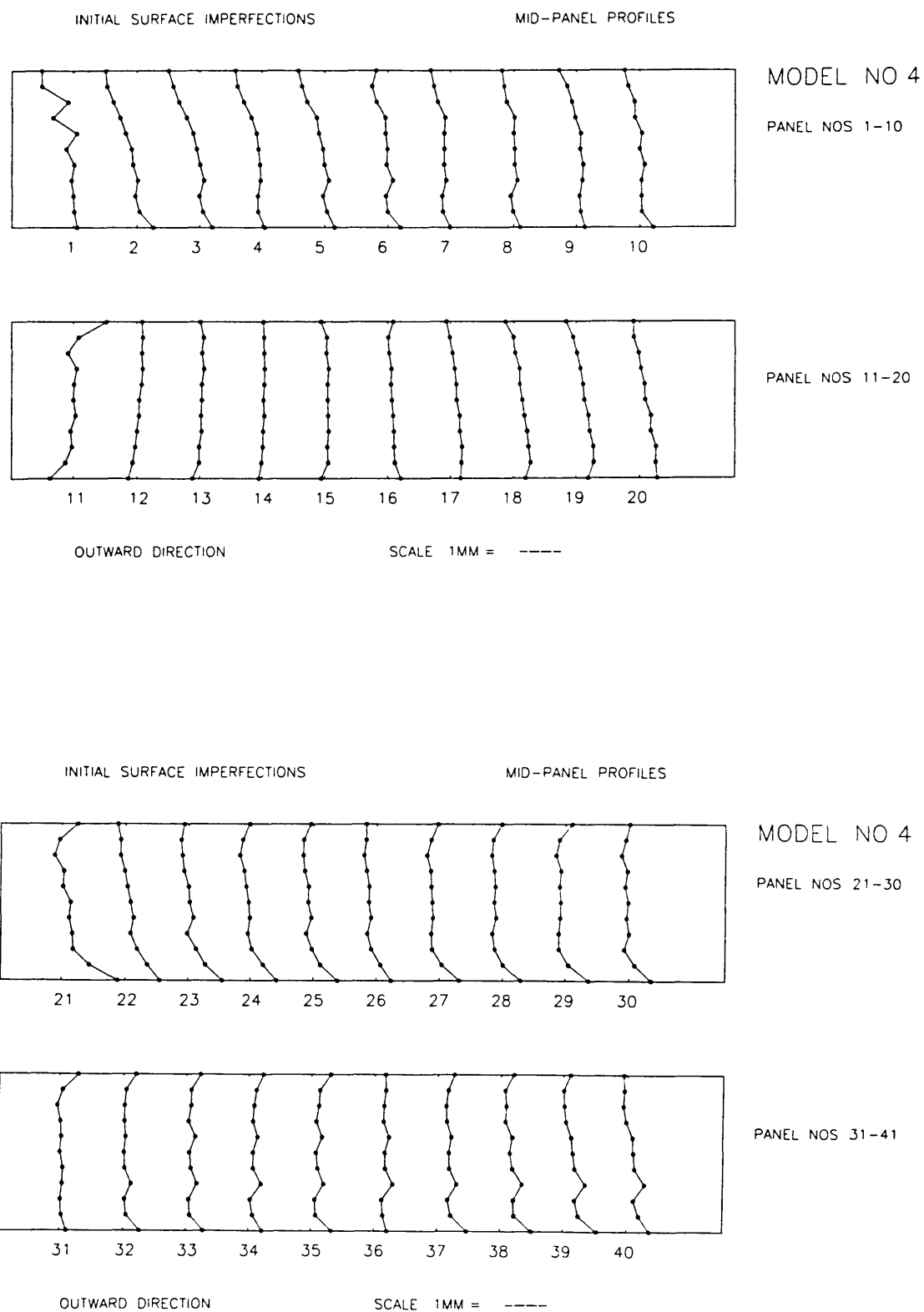
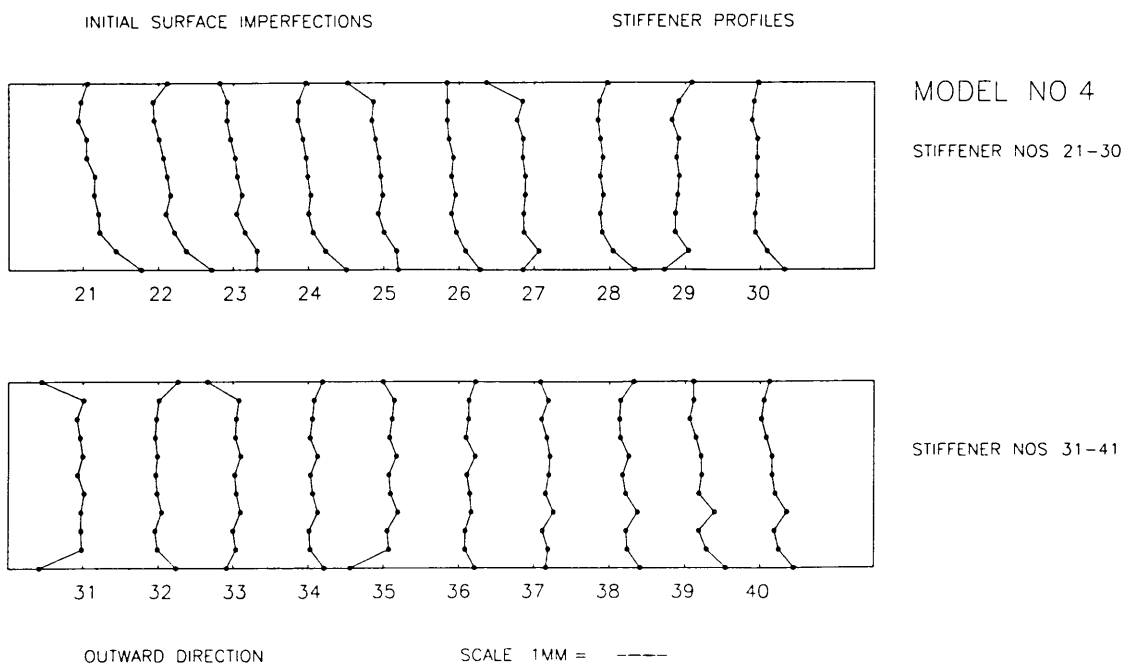
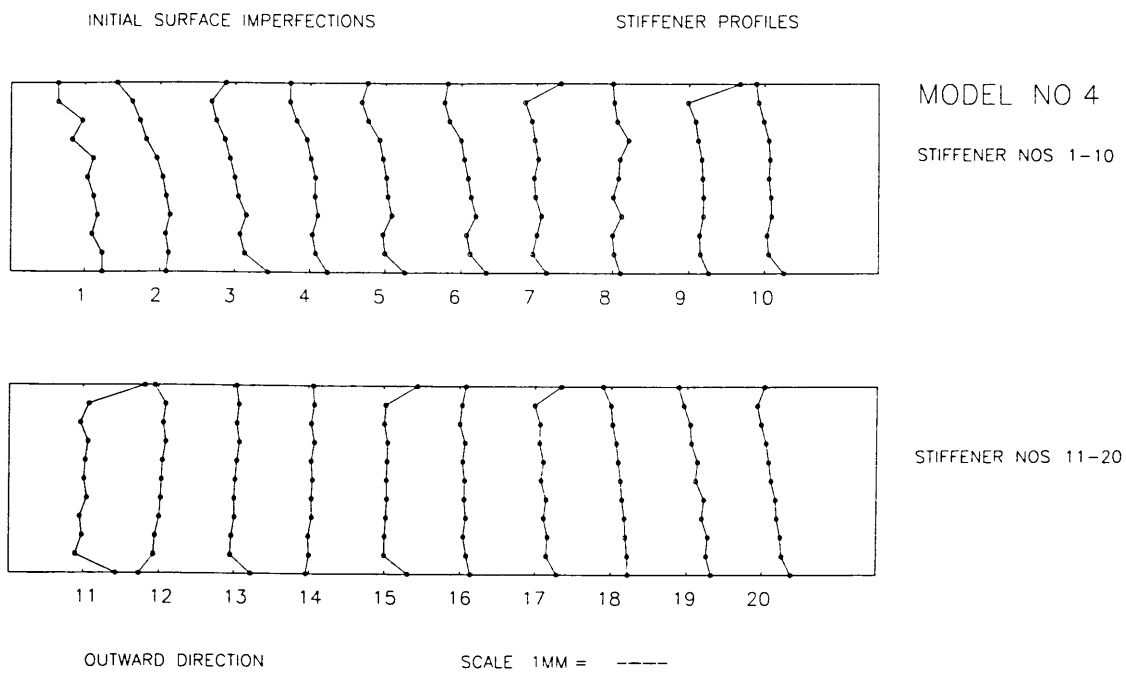


figure 2.71. Circumferential section 6, zero load.



upper figure 2.72. Vertical profiles, panels 1-20.

lower figure 2.73. Vertical profiles, panels 21-40.



upper figure 2.74. Vertical profiles, stiffeners 1-20.

lower figure 2.75. Vertical profiles, stiffeners 21-40.

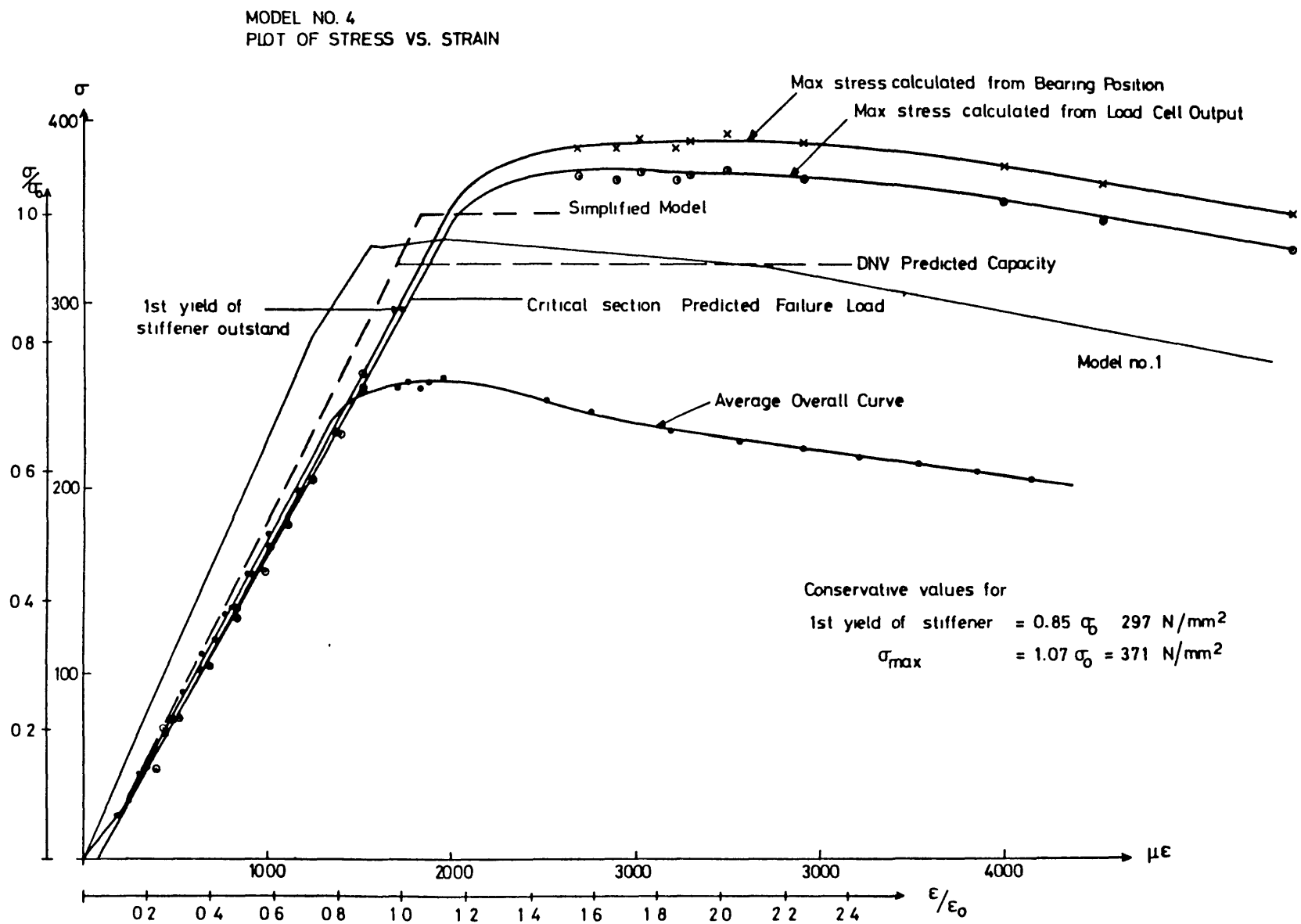


figure 2.76. Overall stress-strain response of model no. 4.

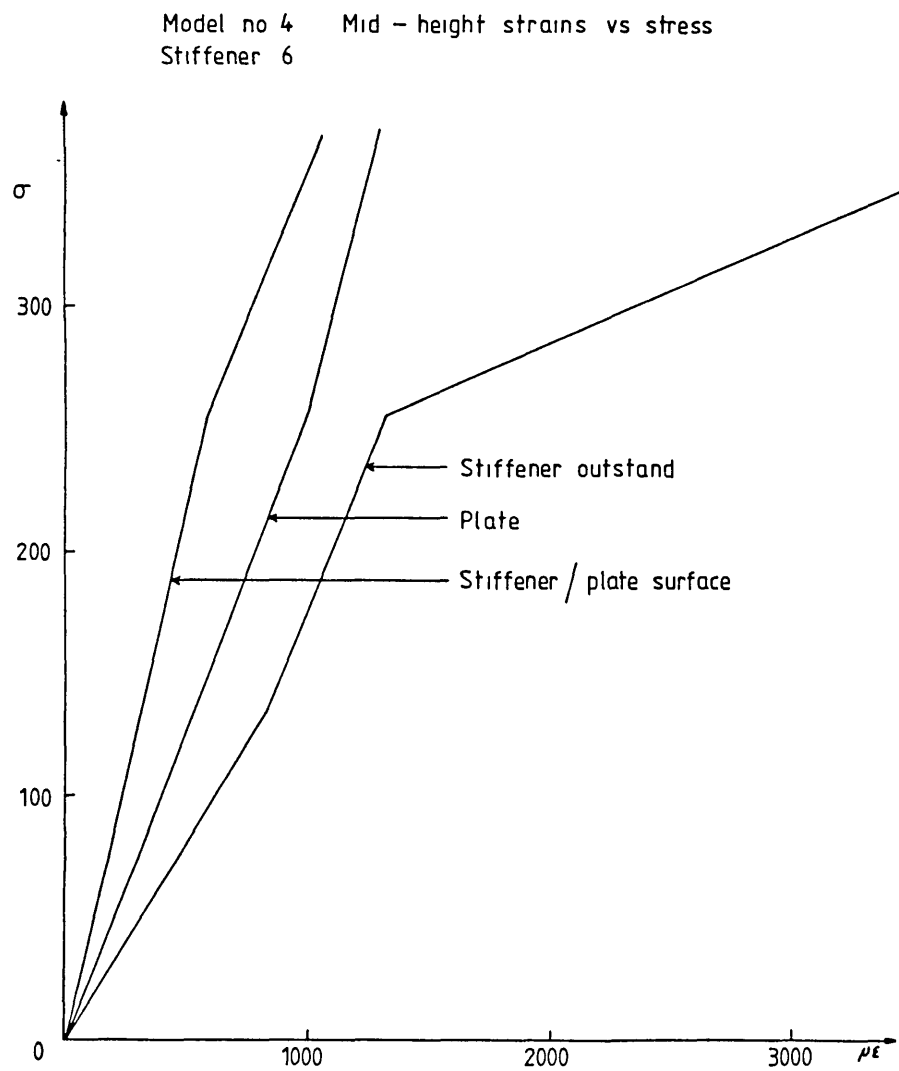


figure 2.77. Strain gauge response at mid-height, stiffener no. 6.

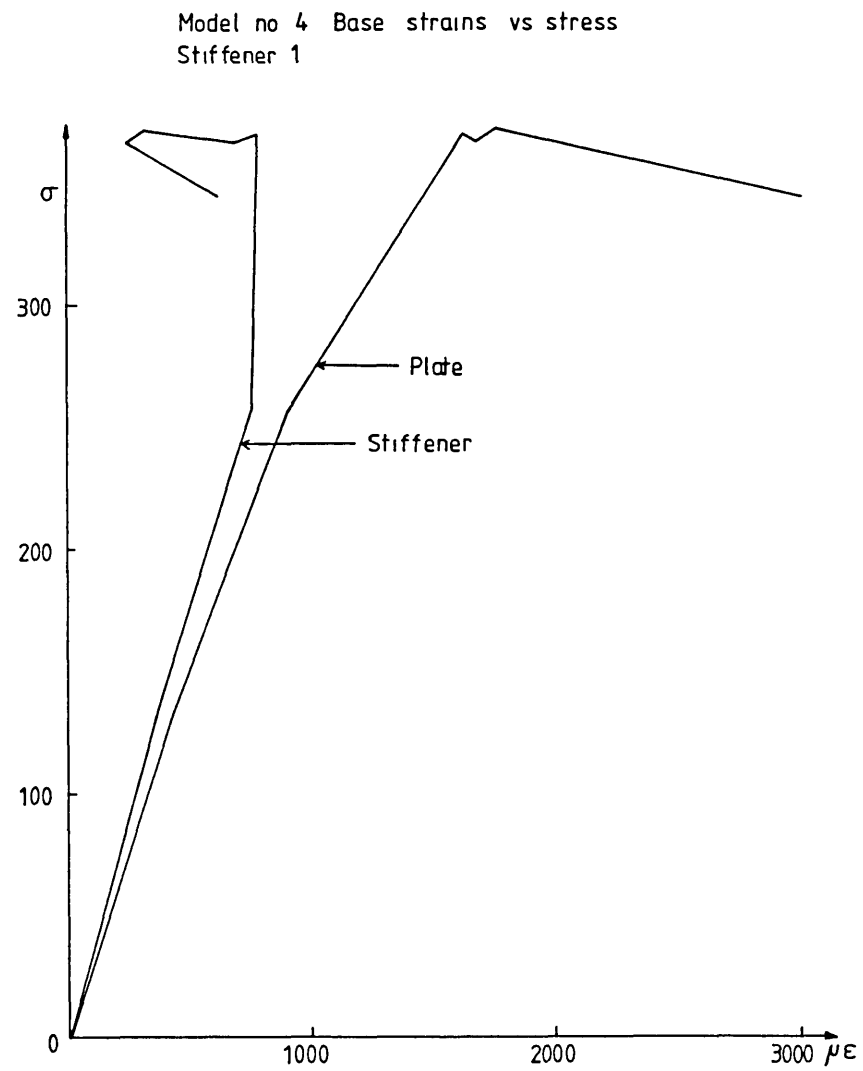
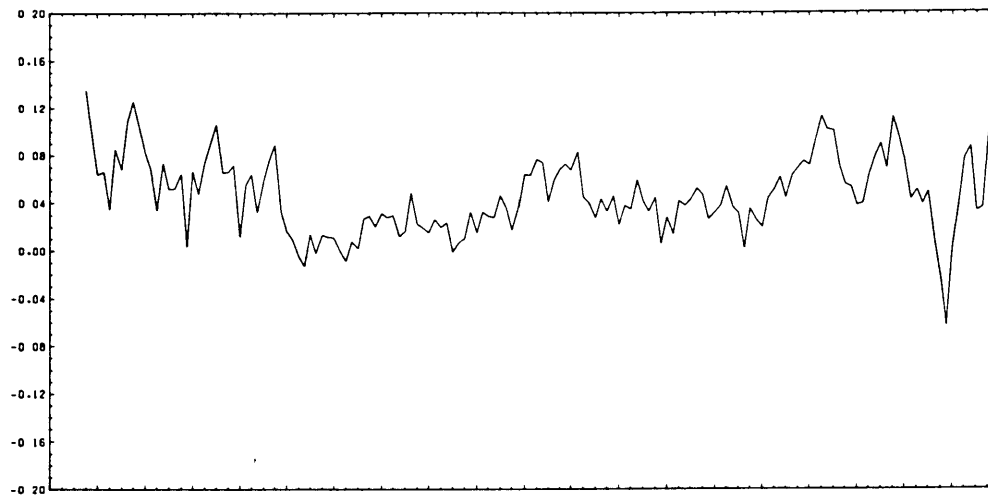
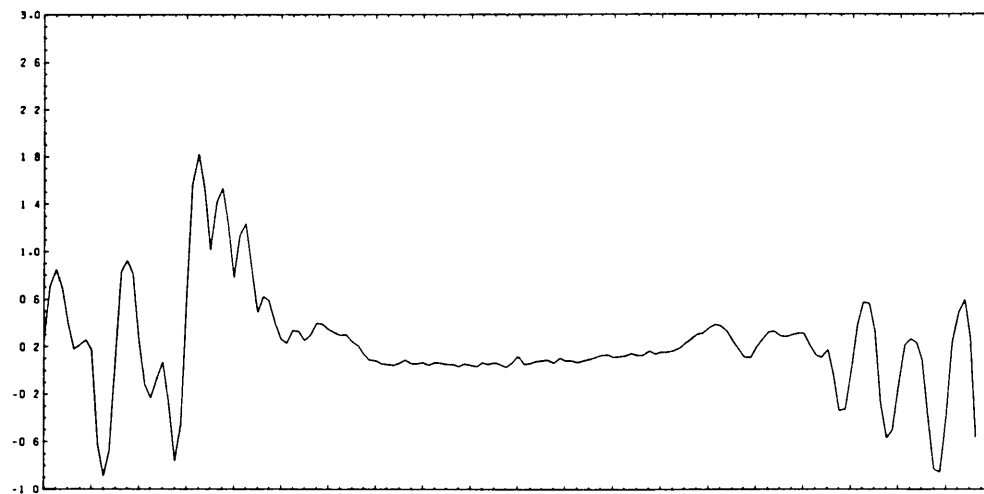


figure 2.78. Strain gauge response at base, stiffener no. 1.

MODEL NO. 4.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 3.

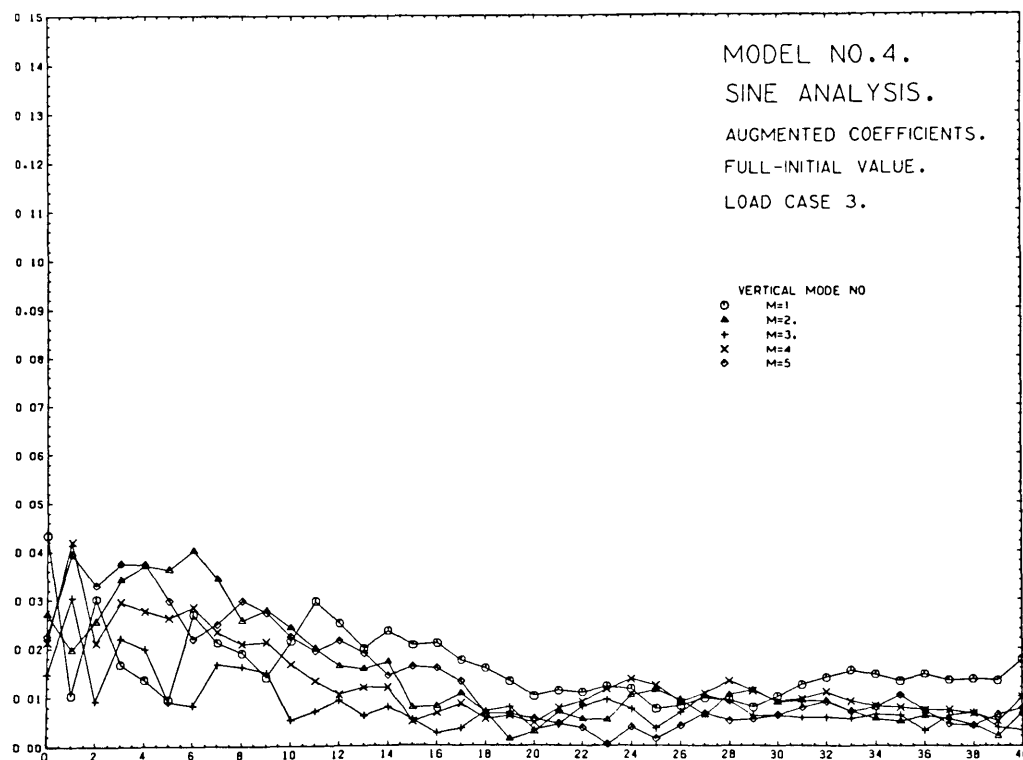
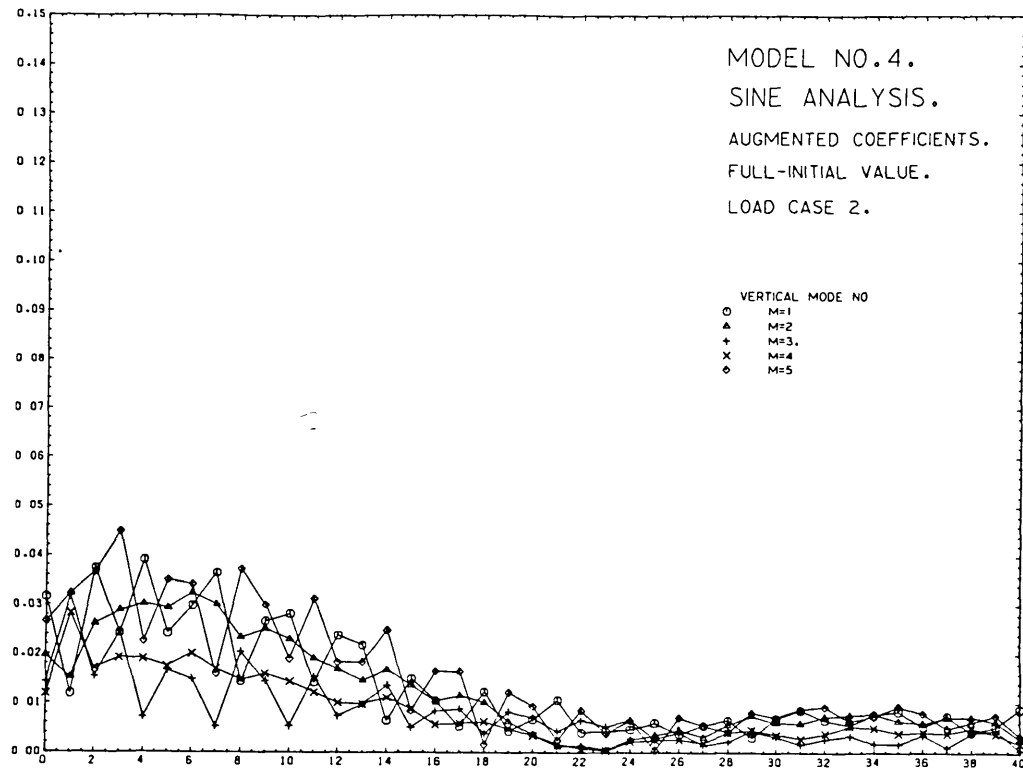


MODEL NO. 4.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 4.



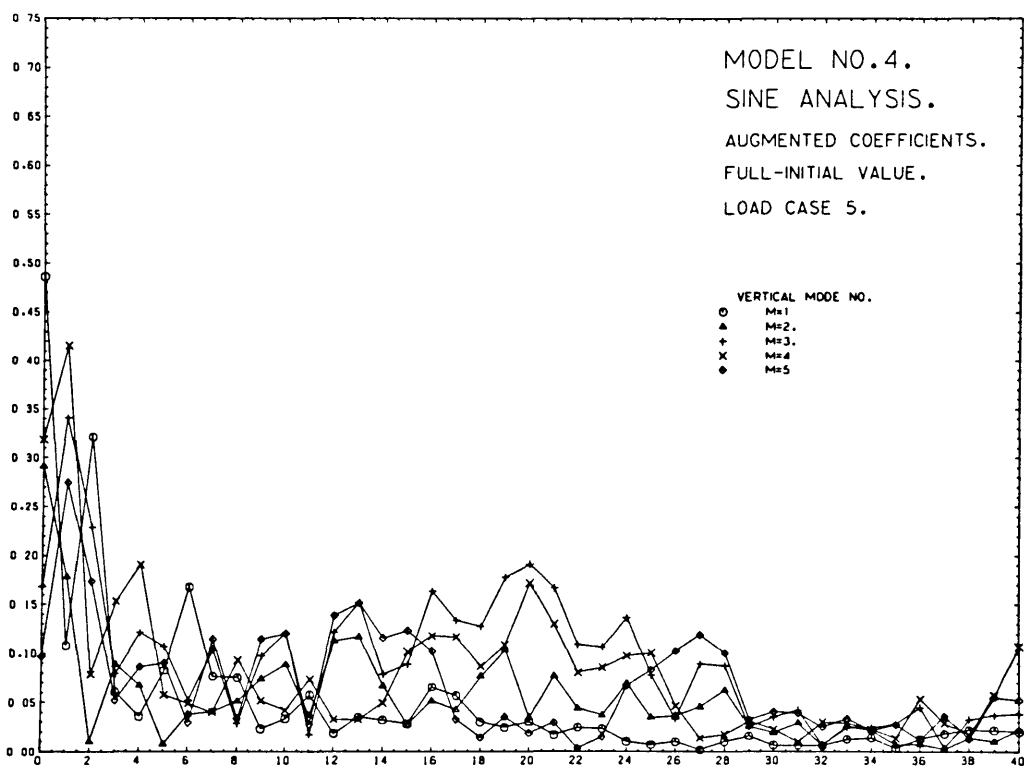
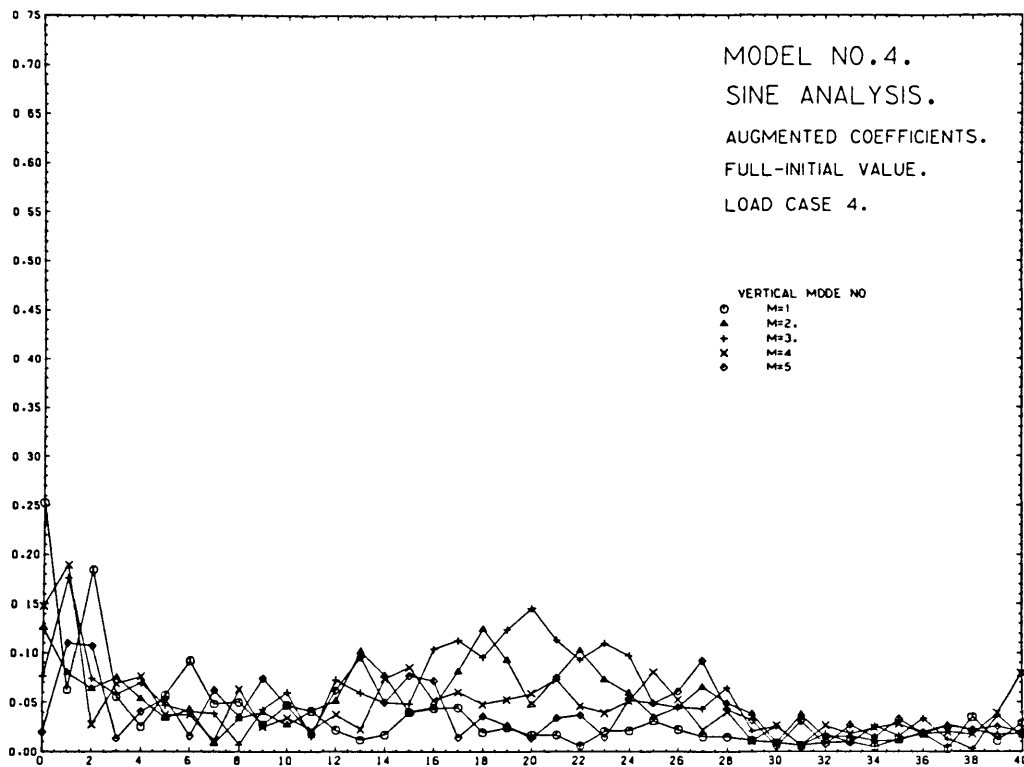
upper figure 2.79. Circumferential section 6, load case 3.

lower figure 2.80. Circumferential section 6, load case 4.



upper figure 2.81. Imperfection analysis, load case 2.

lower figure 2.82. Imperfection analysis, load case 3.



upper figure 2.83. Imperfection analysis, load case 4.

lower figure 2.84. Imperfection analysis, load case 5.

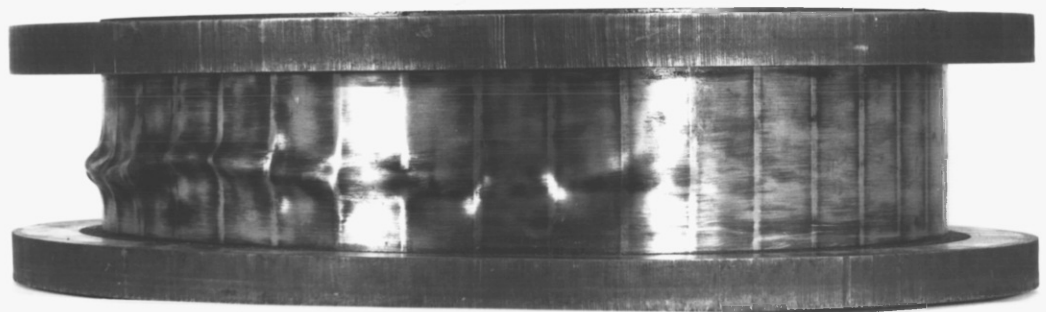
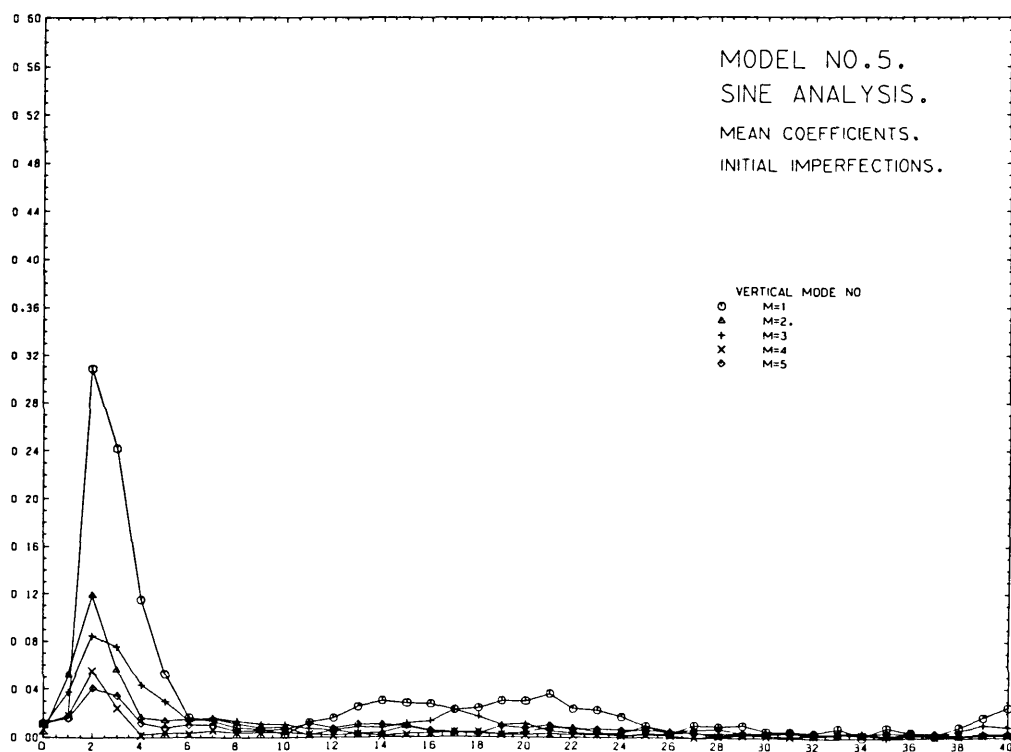
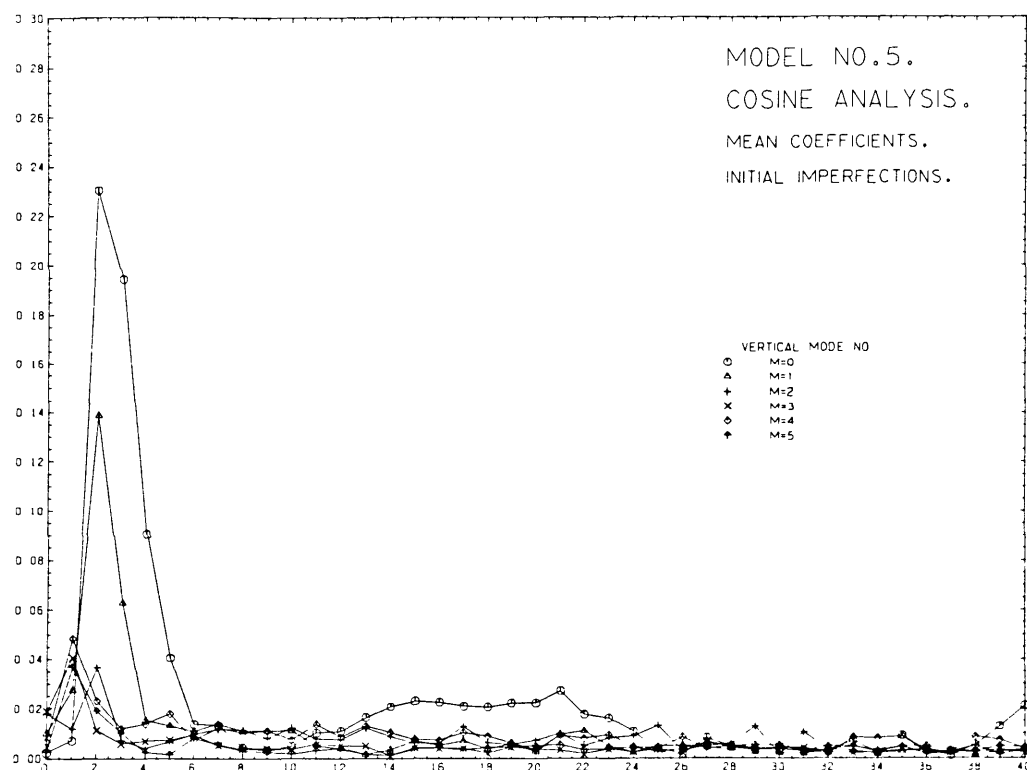


figure 2.85. Model no. 4 after collapse.



upper figure 2.86. Initial imperfection analysis.

lower figure 2.87. Initial imperfection analysis.

MODEL NO.5.
CIRCUMFERENTIAL SECTION 5.
INITIAL IMPERFECTIONS.

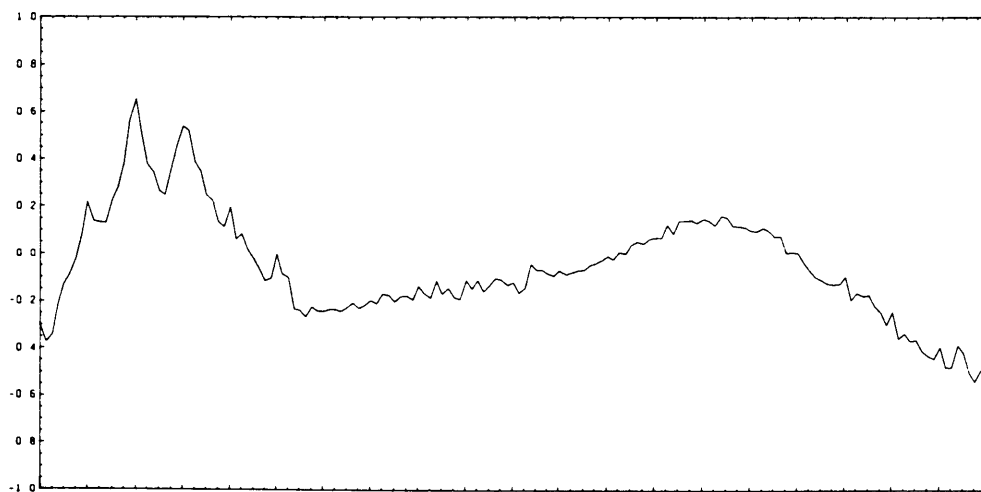
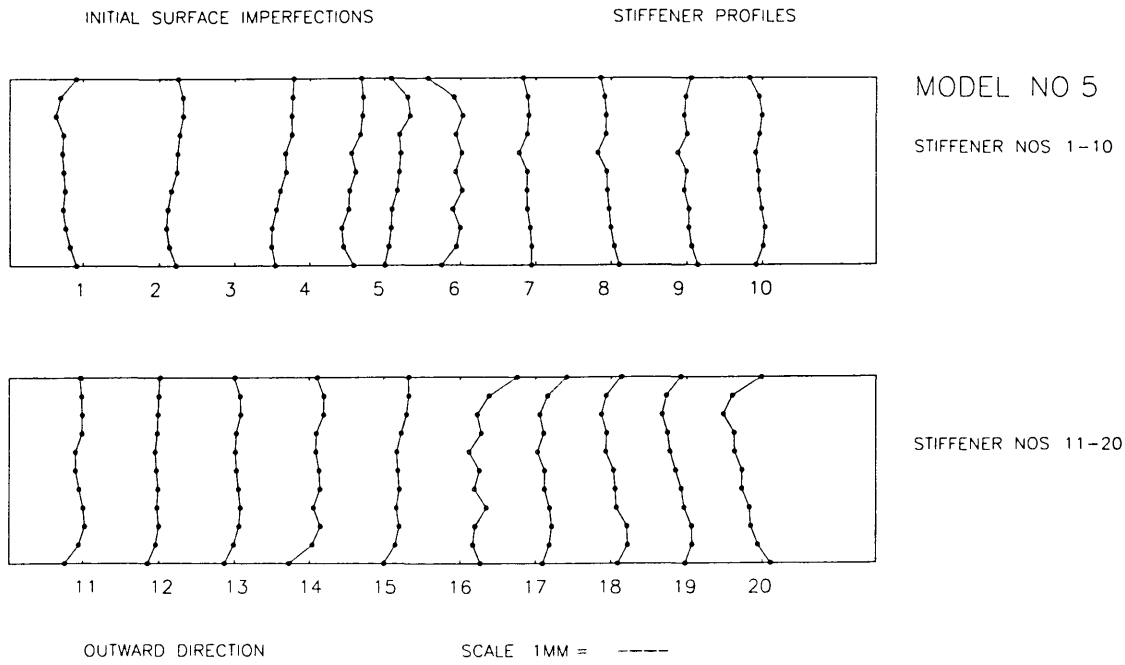
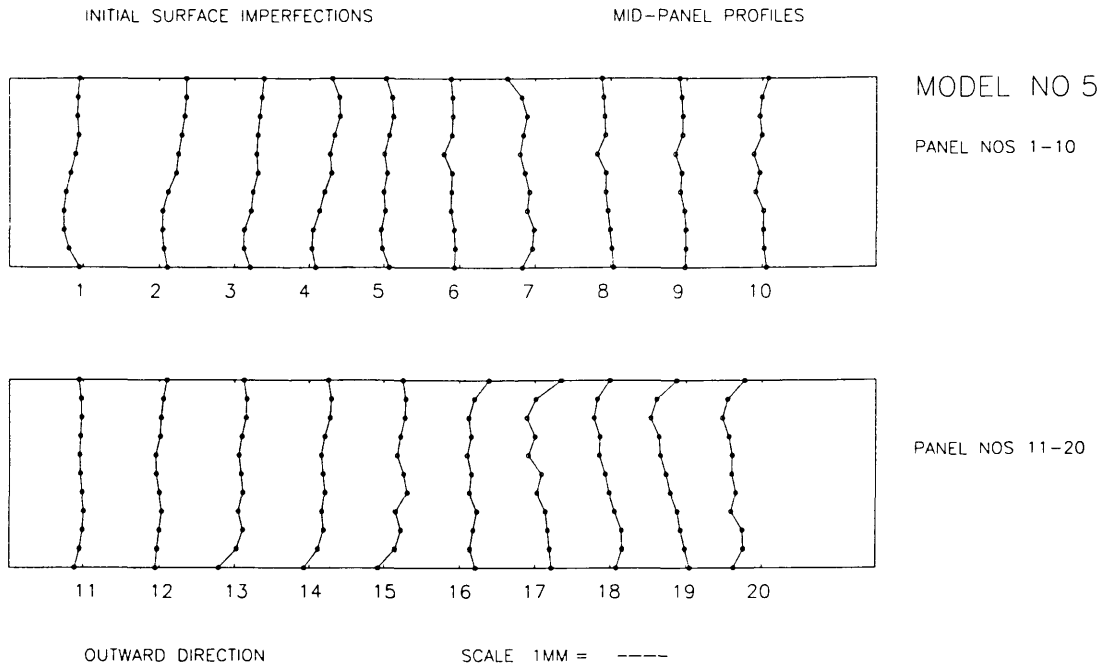


figure 2.88. Circumferential section 5, zero load.



upper figure 2.89. Vertical profile of panels.

lower figure 2.90. Vertical profile of stiffeners.

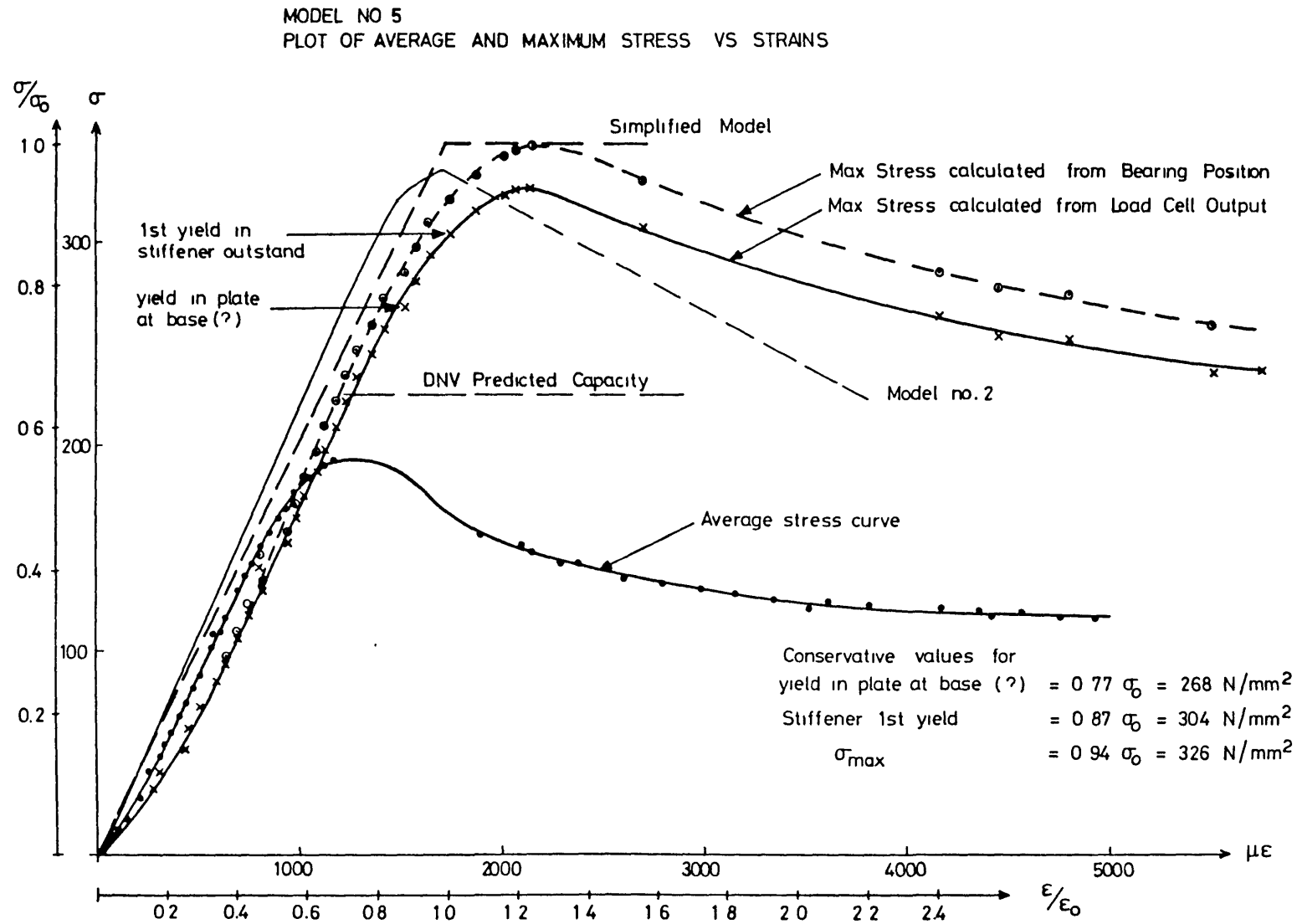


figure 2.91. Overall stress-strain response of model no. 5.

Model no 5 Plot of Panel 1 Strains vs Max.Stress

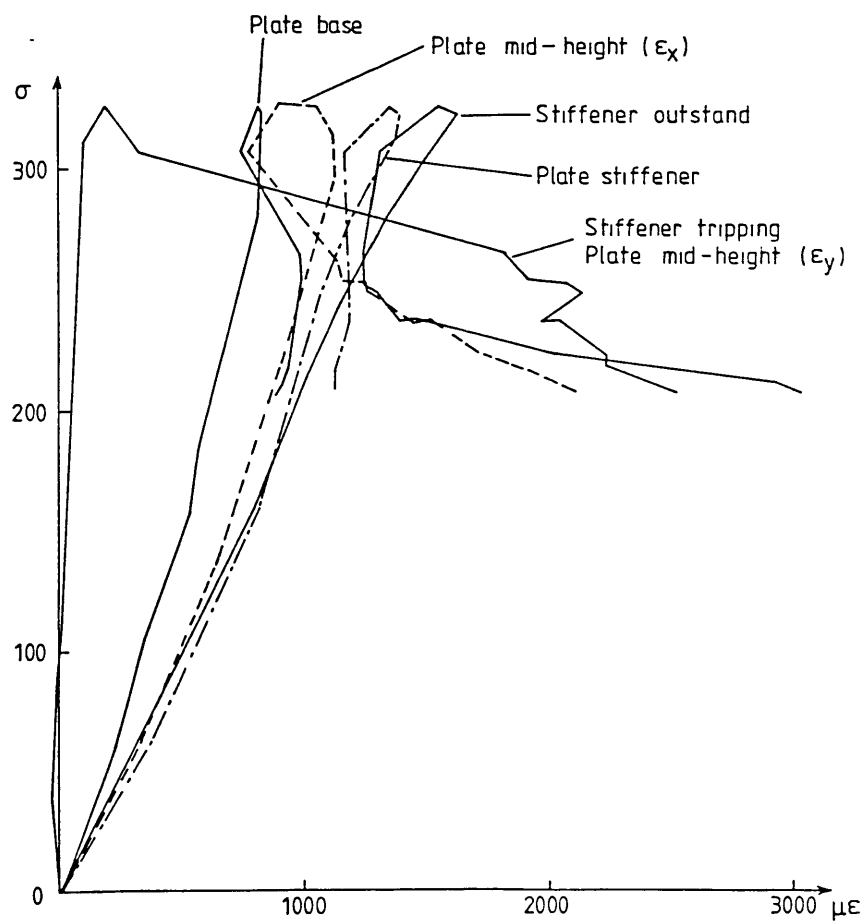


figure 2.92. Strain gauge response in panel no. 1.

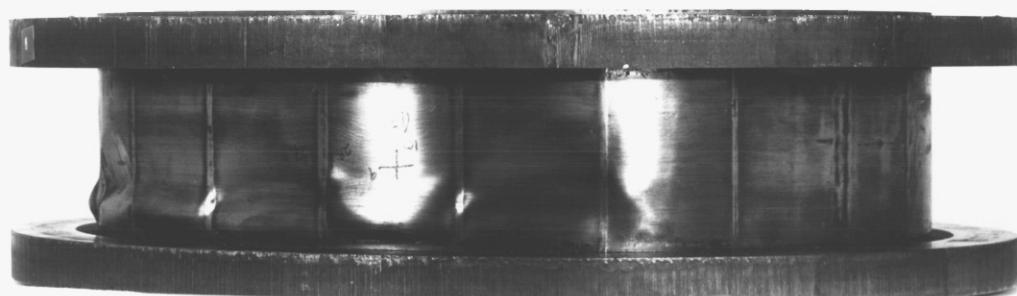
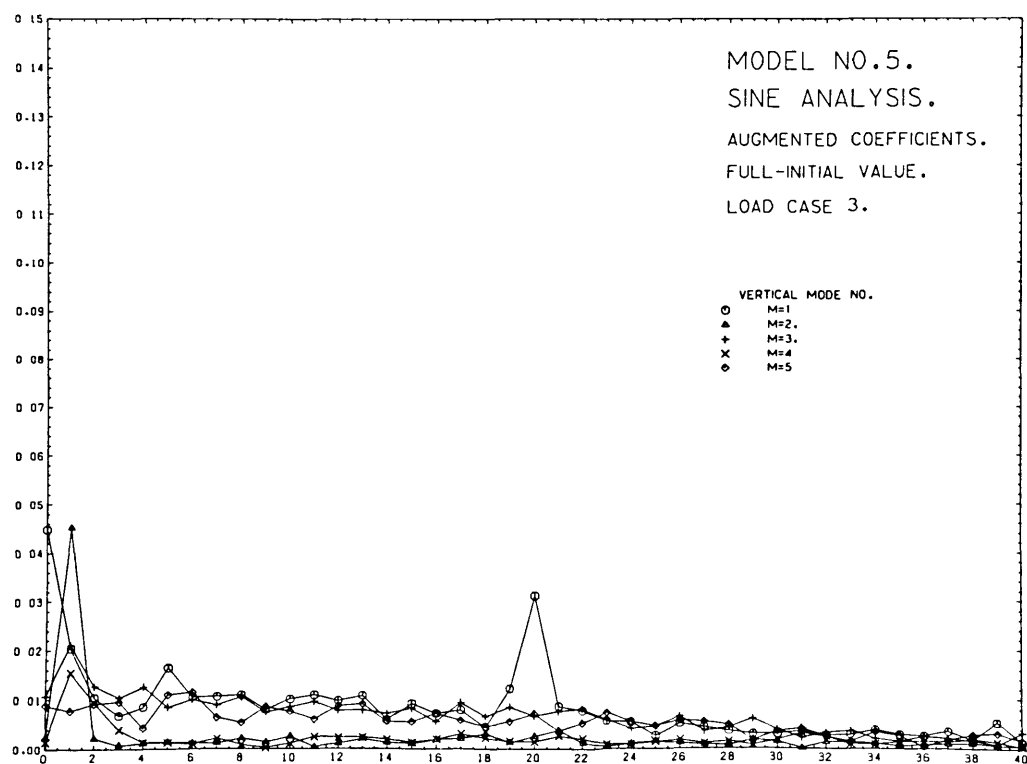
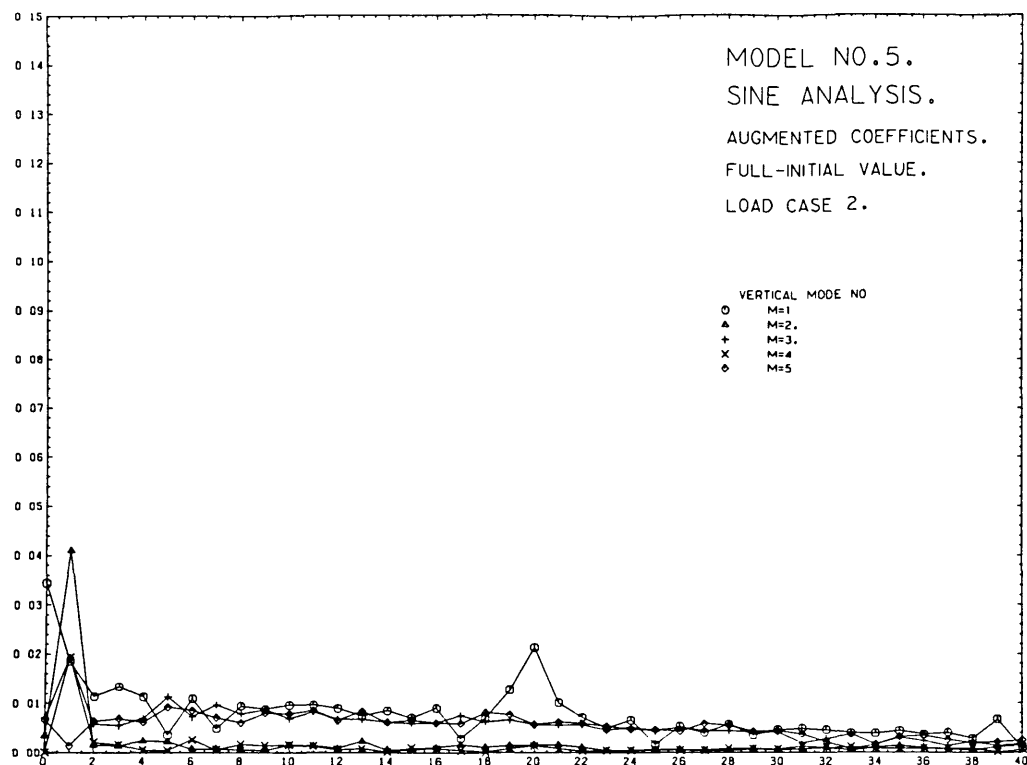
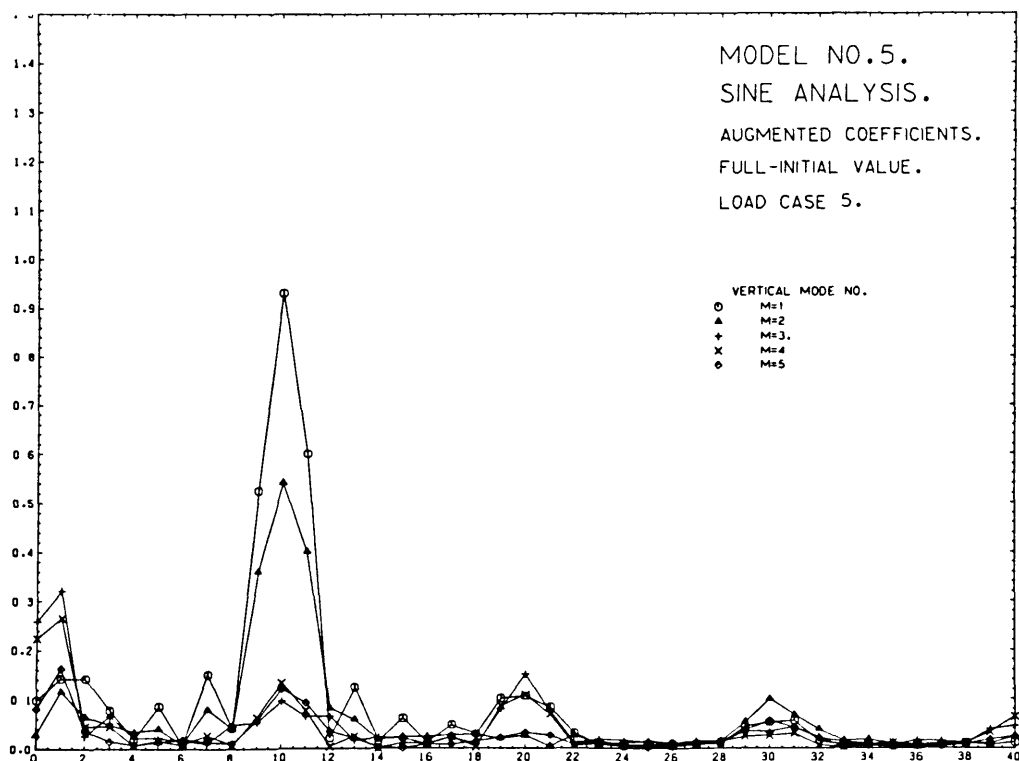
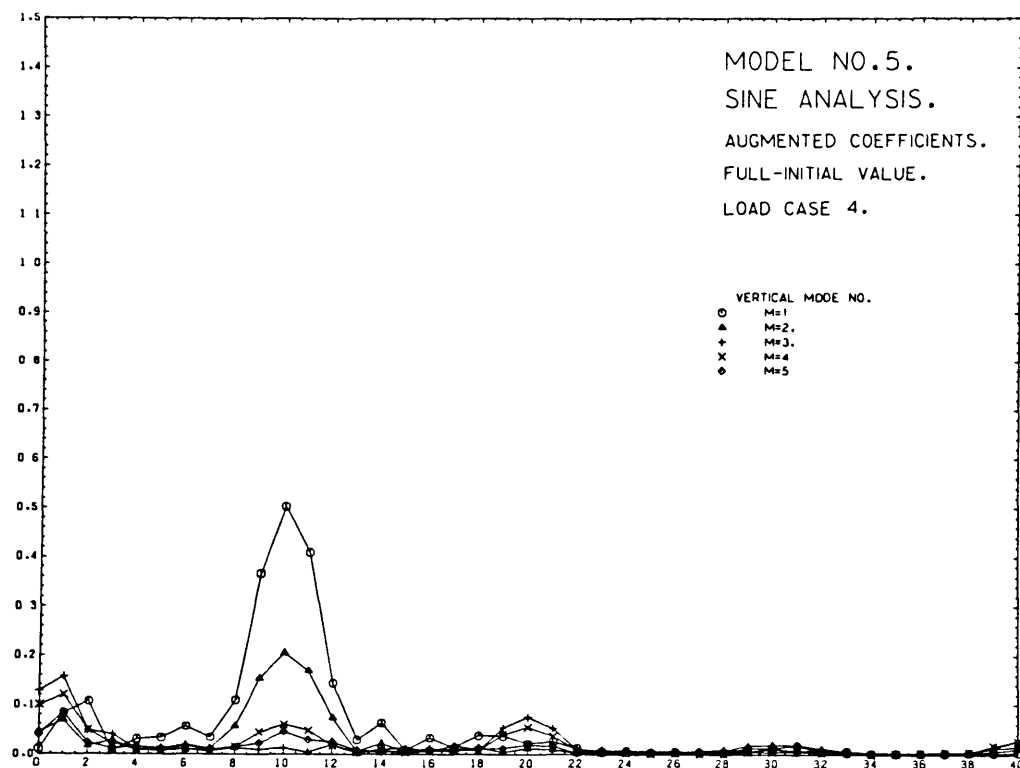


figure 2.93. Model no. 5 after collapse.



upper figure 2.94. Imperfection analysis, load case 2.

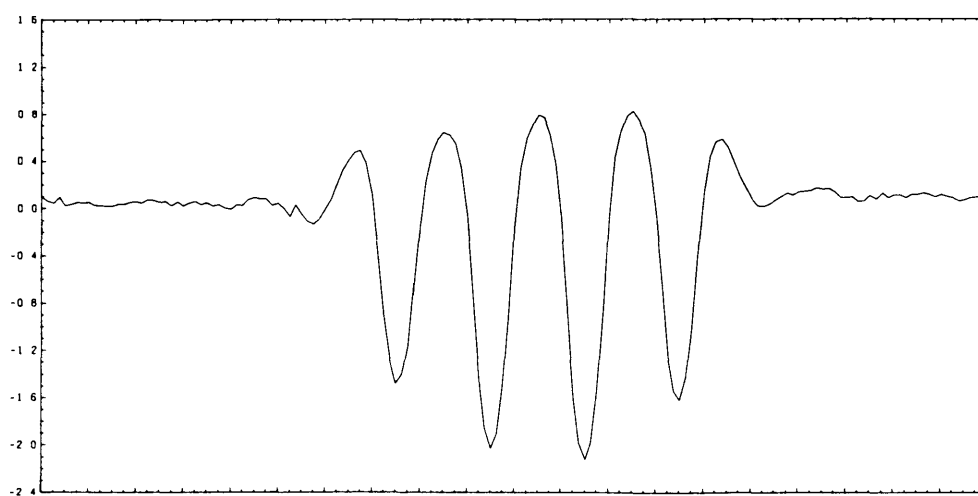
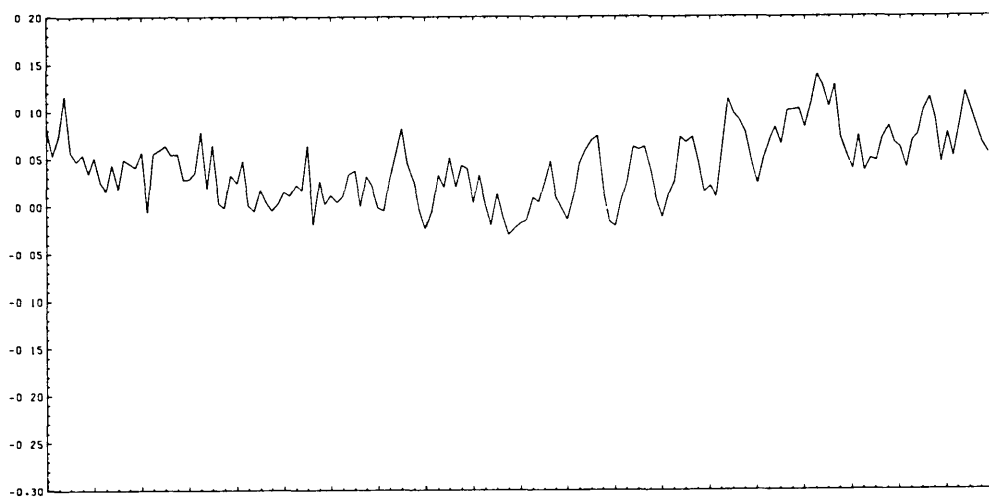
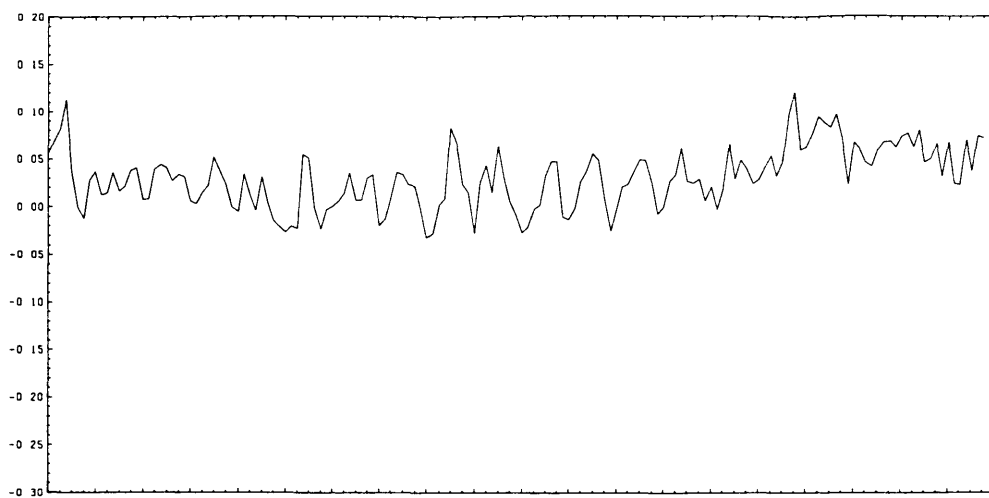
lower figure 2.95. Imperfection analysis, load case 3.



upper figure 2.96. Imperfection analysis, load case 4.

lower figure 2.97. Imperfection analysis, load case 5.

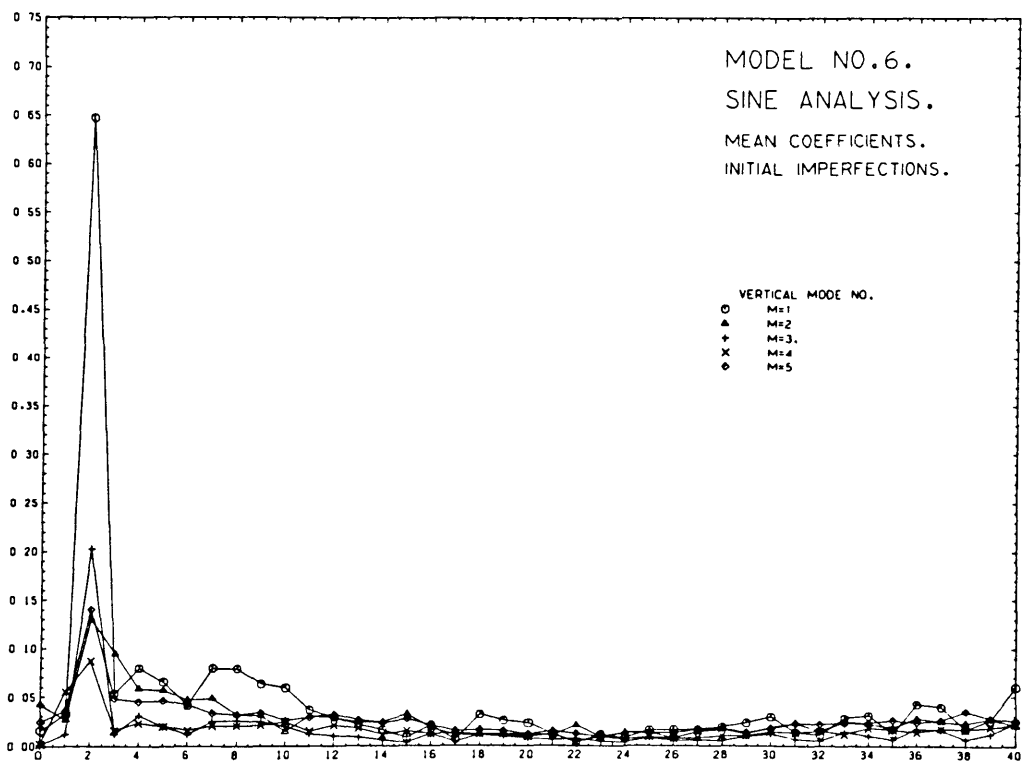
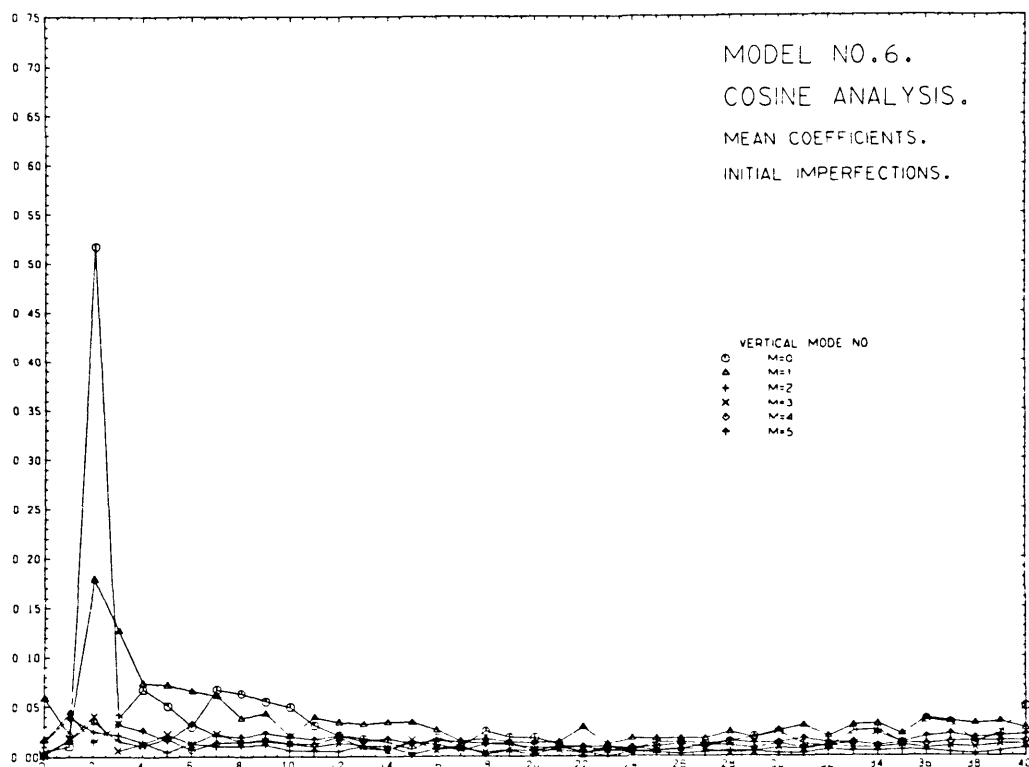
MODEL NO.5.
CIRCUMFERENTIAL SECTION 5.
LOAD CASE 2.



upper figure 2.98. Circumferential section 5, load case 2.

centre figure 2.99. Circumferential section 5, load case 3.

lower figure 2.100. Circumferential section 5, load case 4.



upper figure 2.101. Initial imperfection analysis.

lower figure 2.102. Initial imperfection analysis.

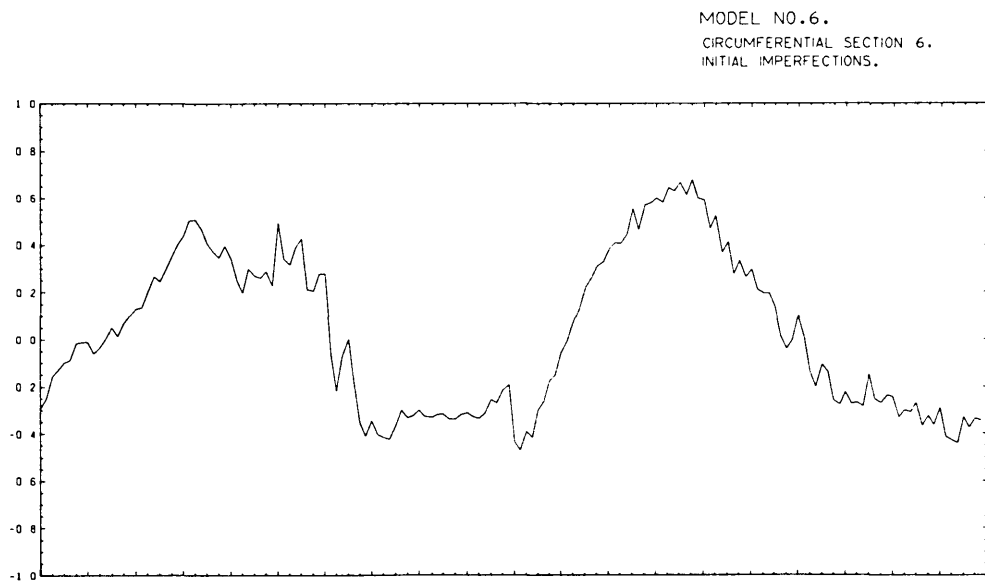
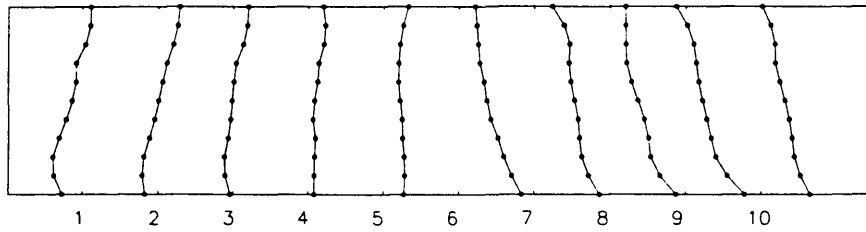


figure 2.103. Circumferential section 6, zero load.

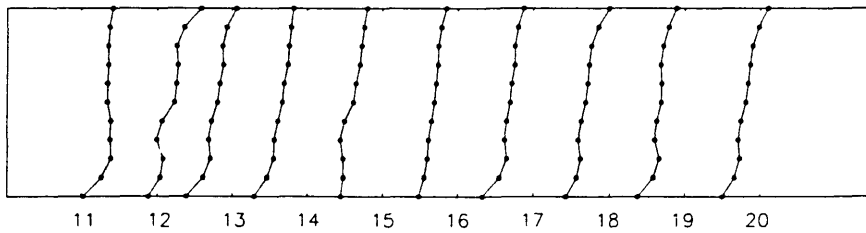
INITIAL SURFACE IMPERFECTIONS

MID-PANEL PROFILES



MODEL NO 6

PANEL NOS 1-10



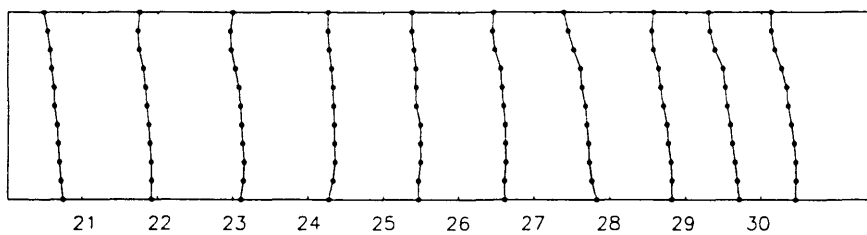
PANEL NOS 11-20

OUTWARD DIRECTION

SCALE 1MM = ----

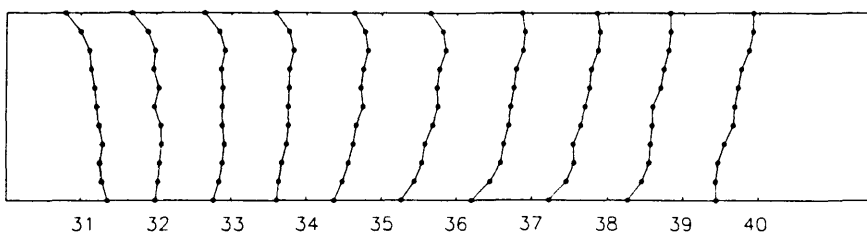
INITIAL SURFACE IMPERFECTIONS

MID-PANEL PROFILES



MODEL NO 6

PANEL NOS 21-30



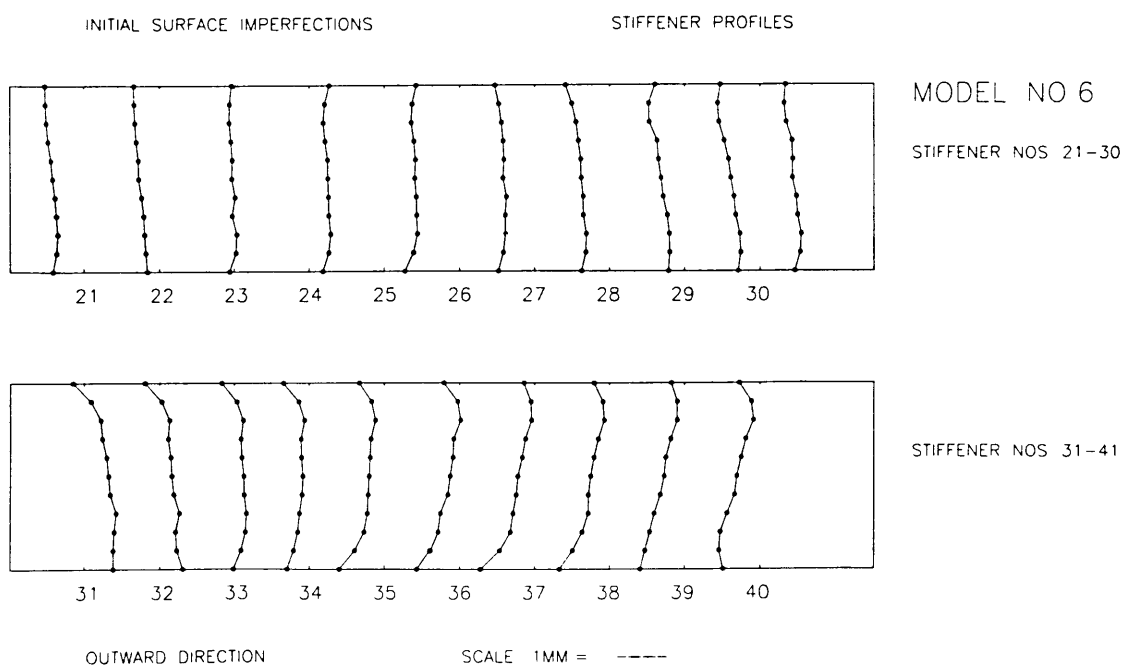
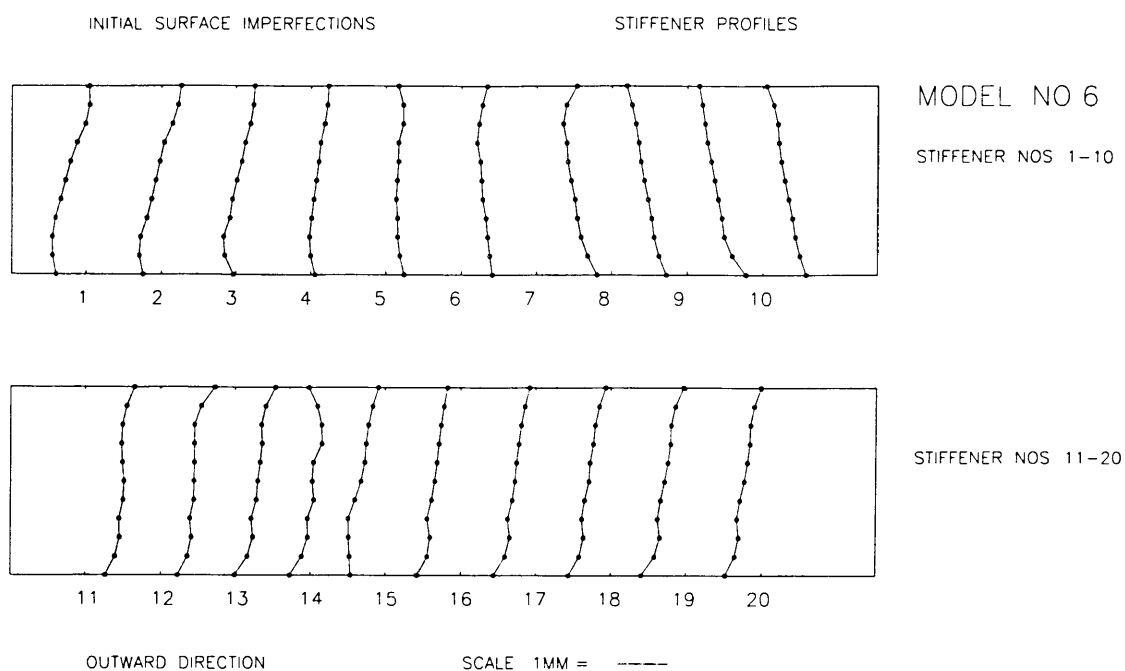
PANEL NOS 31-41

OUTWARD DIRECTION

SCALE 1MM = ----

upper figure 2.104. Vertical profiles, panels 1-20.

lower figure 2.105. Vertical profiles, panels 21-40.



upper figure 2.106. Vertical profiles, stiffeners 1-20.

lower figure 2.107. Vertical profiles, stiffeners 21-40.

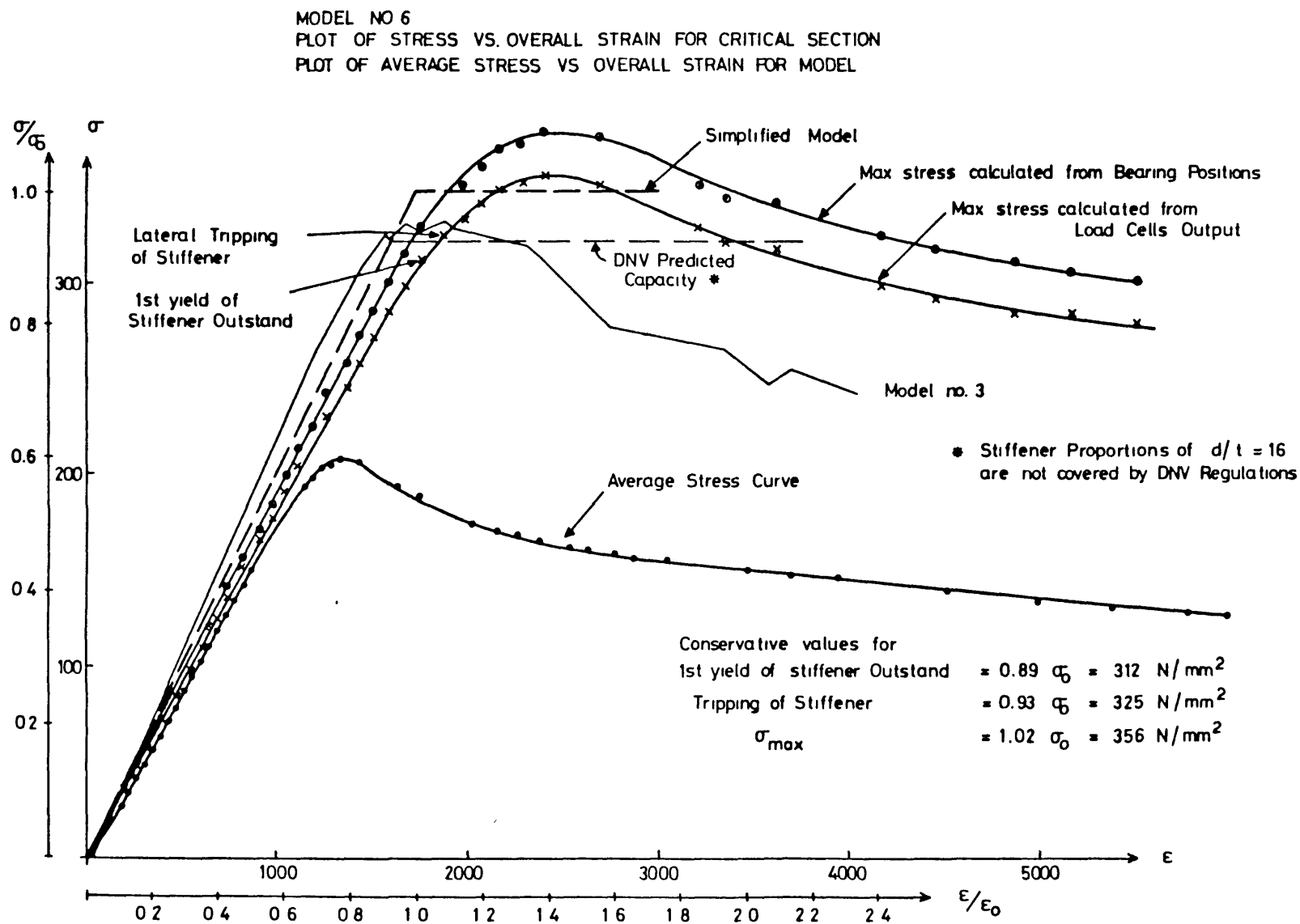


figure 2.108. Overall stress-strain response of model no. 6.

Model no 6 Mid - height strains vs. maximum stress
Stiffener no 1

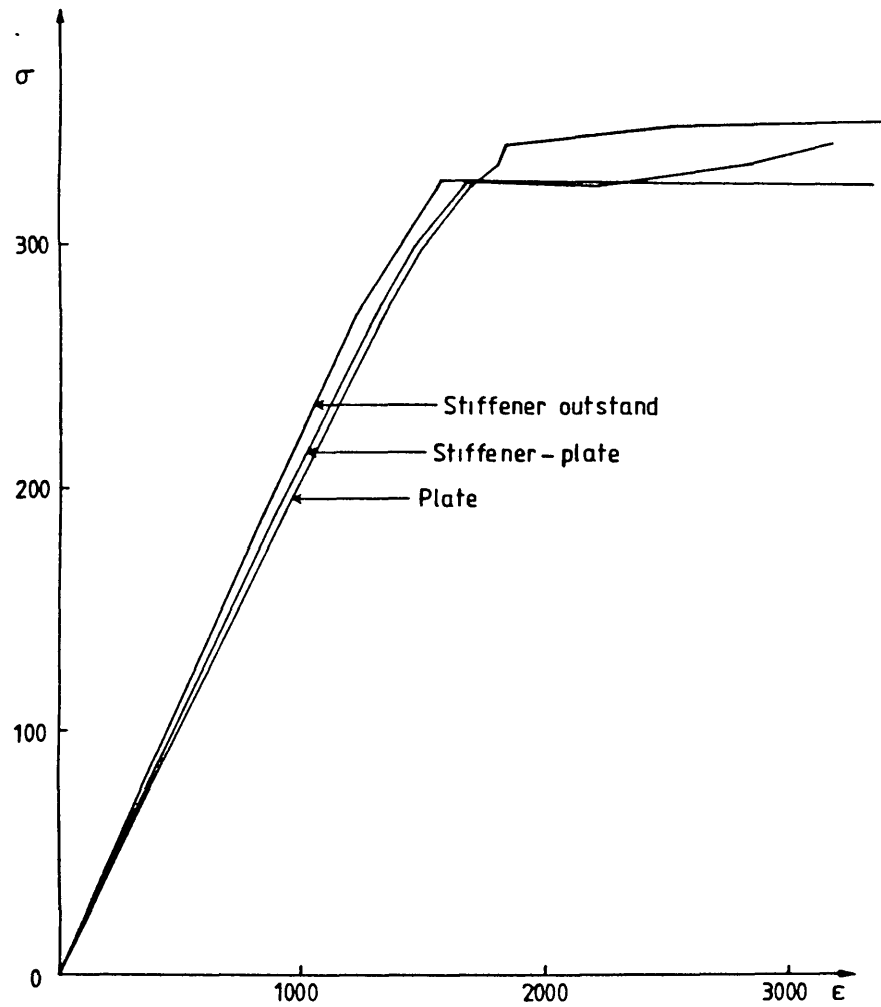


figure 2.109. Strain gauge response at mid-height.

Model no 6 Base strains vs. maximum stress
Stiffener no 1

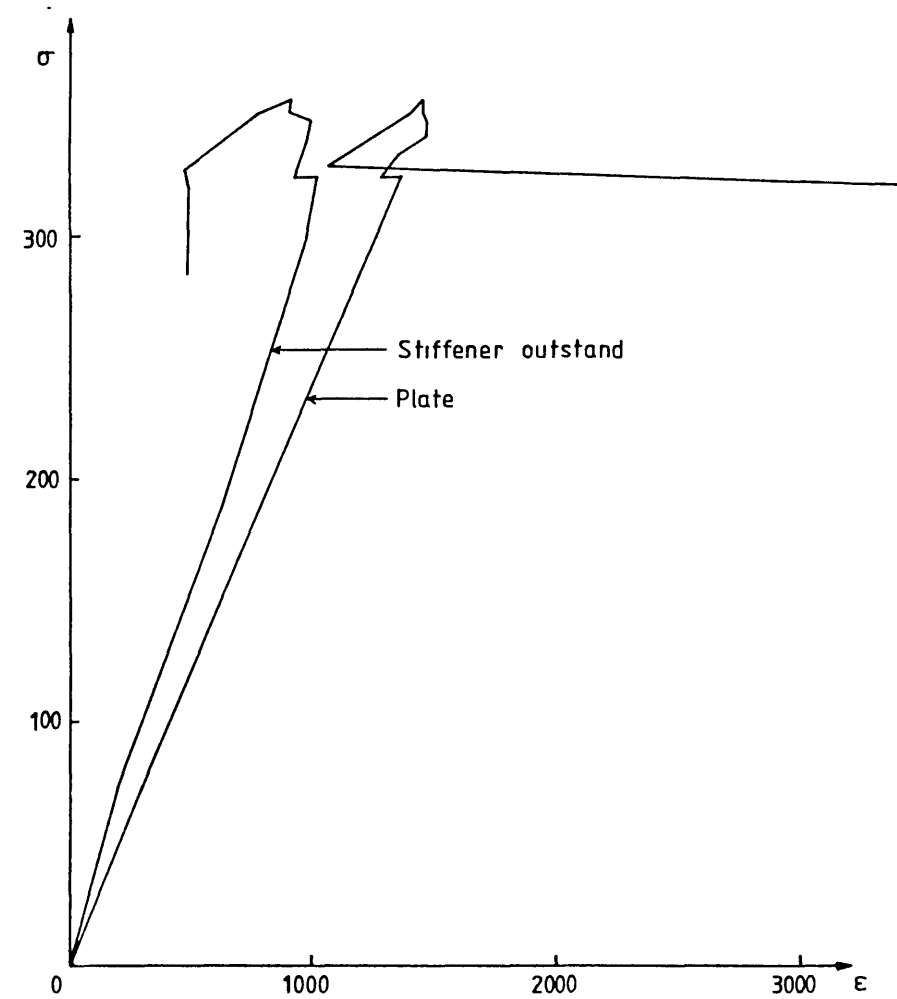
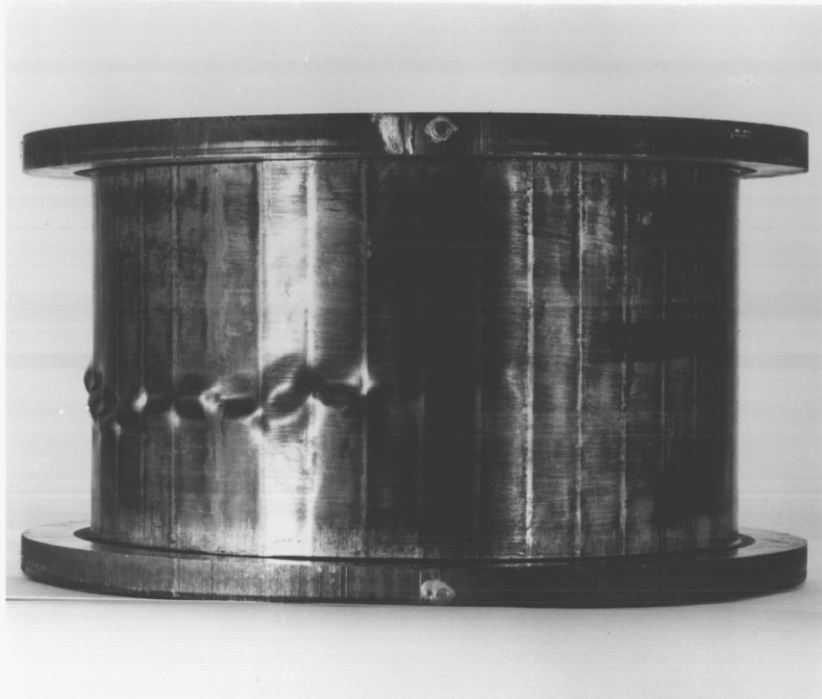
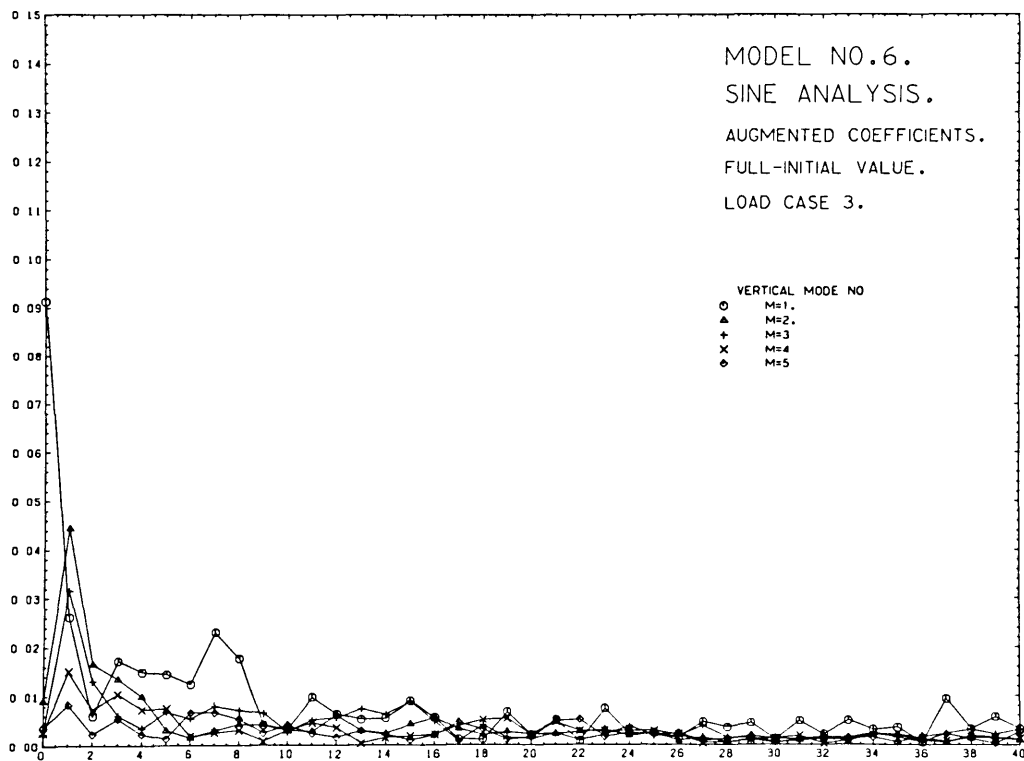
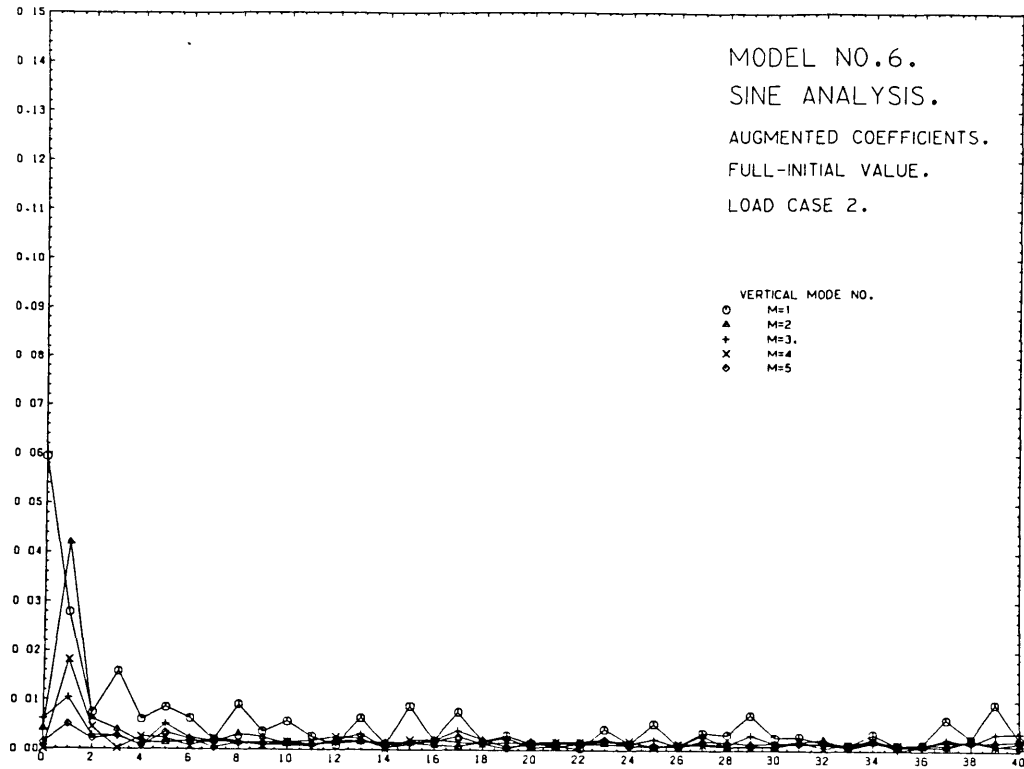


figure 2.110. Strain gauge response at base.



upper figure 2.111. Model no. 6 after collapse.

lower figure 2.112. Internal view of model no. 6 after collapse.



upper figure 2.113. Imperfection analysis, load case 2.

lower figure 2.114. Imperfection analysis, load case 3.

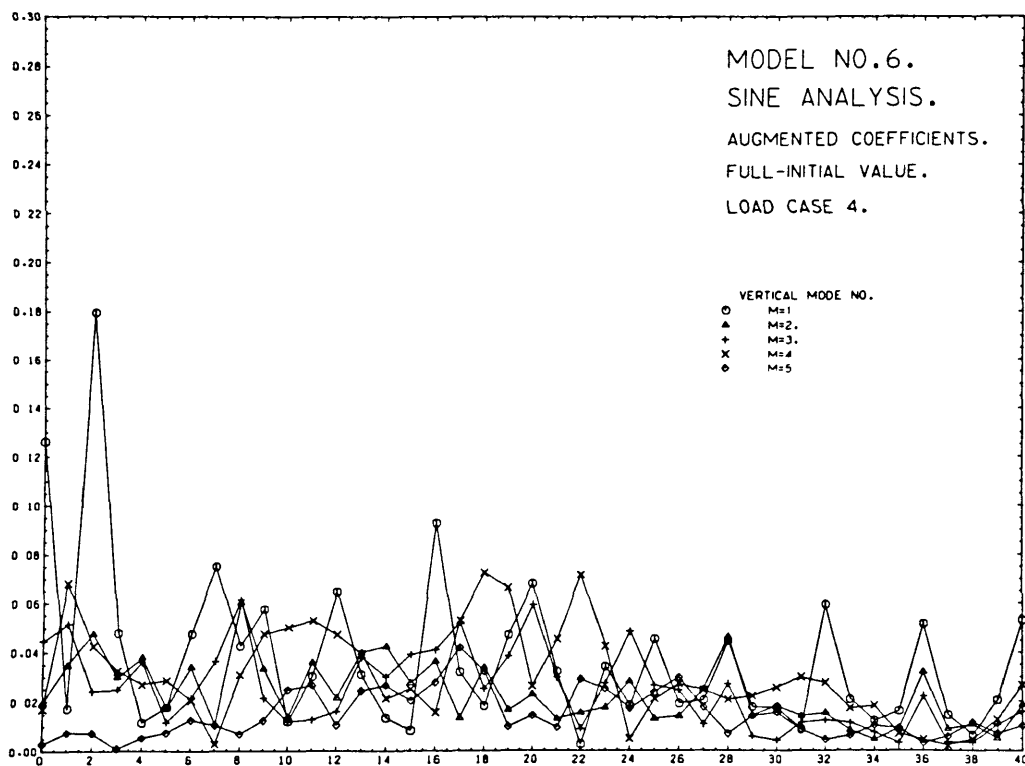
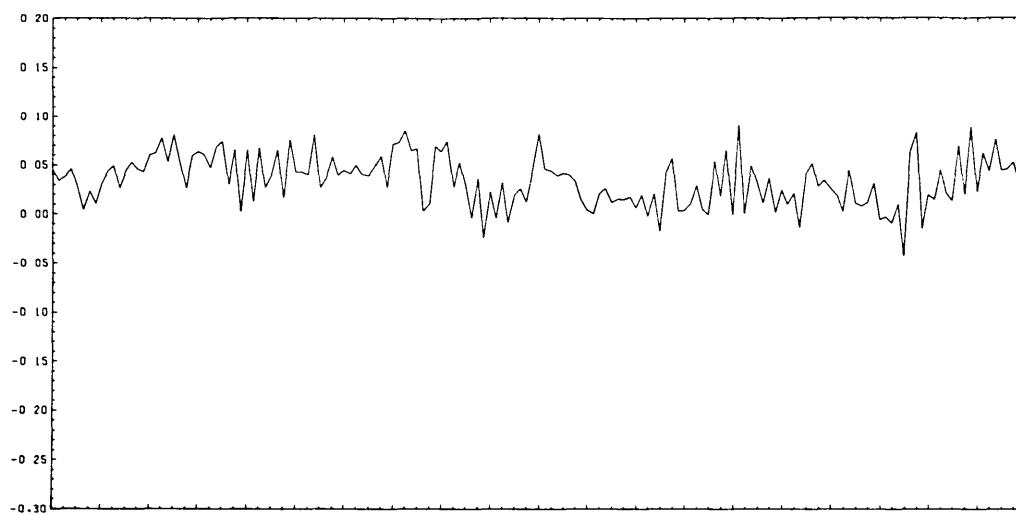
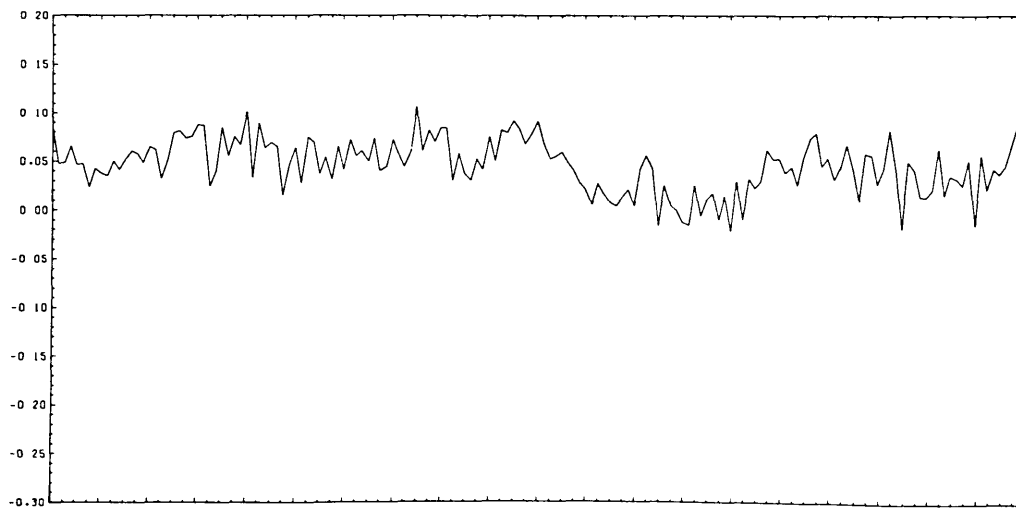


figure 2.115. Imperfection analysis, load case 4.

MODEL NO.6.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 1.



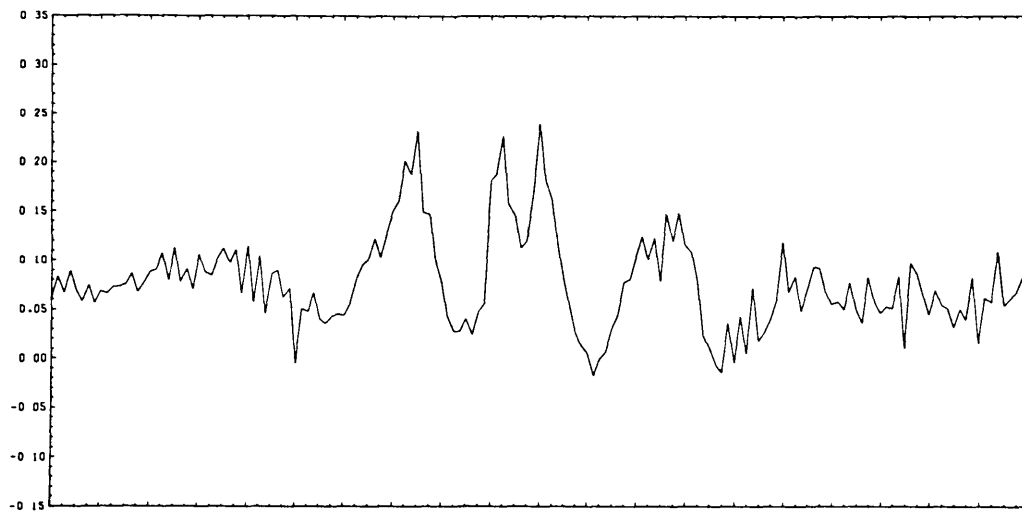
MODEL NO.6.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 2.



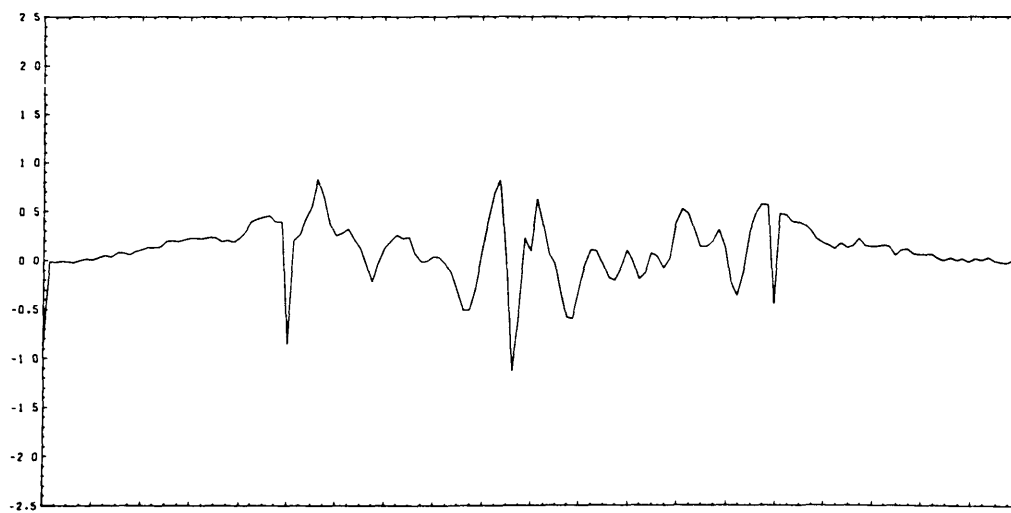
upper figure 2.116. Circumferential section 6, load case 1.

lower figure 2.117. Circumferential section 6, load case 2.

MODEL NO.6.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 3.



MODEL NO.6.
CIRCUMFERENTIAL SECTION 6.
LOAD CASE 4.



upper figure 2.118. Circumferential section 6, load case 3.

lower figure 2.119. Circumferential section 6, load case 4.

MODEL NO	RADIUS = R (mm)	PLATE THICKNESS = t (mm)	LENGTH = A = L (mm)	R/t	L/R	NO OF STIFFENERS	D _{st} /B _{st}	B _{st} /t _{pl}	A/B (PANEL)	B/t (PANEL)
1	160	0.84	65	190	0.41	40	8	1	2.59	29.9
2	159.91	0.84	65	190	0.41	20	8	1	1.29	59.8
3	159.93	0.84	180	190	1.125	40	16	1	7.16	29.9
4	159.90	0.84	65	190	0.41	40	8	1	2.59	29.9
5	159.89	0.84	65	190	0.41	20	8	1	1.29	59.8
6	159.98	0.84	180	190	1.125	40	16	1	7.16	29.9

Table 2.1

MODEL NO. 2 FILE NO. 0

COSINE ANALYSIS

COSINE COEFFICIENTS

L	K=	0	1	2	3	4	5
0		-.01821	-.01551	.01155	-.01477	.11450	-.00609
1		.01238	.01236	.00158	-.02430	-.08863	-.00930
2		.05390	.09979	-.00375	-.00972	-.00301	-.00189
3		.03191	-.02357	.00070	.00445	.03469	.00129
4		.00372	-.00187	.00506	-.00024	-.00325	-.00013
5		-.02785	.00280	-.00537	.00056	-.03119	.06027
6		.01837	-.00208	.00266	-.00544	.02524	-.00217
7		.00460	-.00020	-.00273	.00798	-.00330	.00312
8		.00424	-.00209	.00054	.00076	-.00974	.00026
9		-.01378	.00332	.00513	-.00376	.01114	-.00141
10		.00103	-.00438	-.00481	.00144	-.00748	.00048
11		.00771	-.00685	.00665	-.00211	.00328	-.00096
12		-.00361	-.00547	-.00064	.00229	.00705	.00079
13		-.00153	.00603	-.00304	-.00229	-.01307	-.00078
14		.00259	-.00333	.00845	-.00205	.00797	-.00087
15		-.00289	-.00139	-.00223	.00611	-.00019	.00236
16		.00361	.00518	.00270	-.00123	-.00153	-.00038
17		-.00120	-.00464	.00009	-.00073	.00617	-.00037
18		.00647	-.00048	-.00428	-.00053	-.01904	-.00022
19		.00168	-.00246	.00488	.00137	.02977	.00049
20		.01081	.00095	-.00142	.00149	-.03181	.00060
21		-.01461	.00387	.00203	.00002	.02088	.00008
22		.00019	-.00380	.00066	-.00129	-.00685	-.00058
23		-.00100	-.00382	.00146	-.00002	-.00505	-.00008
24		.00069	.00164	.00090	-.00062	.00816	-.00021
25		-.00144	-.00780	-.00160	-.00466	-.00513	-.00198
26		.00068	-.00009	.00080	.00114	-.00245	.00044
27		.00185	-.00315	-.00161	.00074	.00078	.00022
28		.00013	-.00175	.00091	.00109	.00150	.00039
29		-.00120	.00816	.00047	.00262	-.00210	.00119
30		-.00306	-.00175	-.00269	.00207	.00023	.00078
31		.00658	-.00235	.00104	-.00219	-.00569	-.00091
32		-.00483	-.00012	-.00293	.00122	.00593	.00048
33		-.00450	.00171	-.00238	.00115	-.00486	.00048
34		.00291	-.00053	.00704	-.00179	.00249	-.00071
35		-.00329	-.00122	-.00175	.00554	.00424	.00215
36		.00174	.00240	.00301	-.00235	.00045	-.00087
37		-.00188	-.00177	.00236	.00197	.00348	.00074
38		.00244	-.00072	-.00275	-.00334	-.00804	-.00132
39		.00038	-.00375	.00349	-.00039	.01280	-.00022
40		.01013	-.00258	-.00240	-.00413	-.01519	-.00167

MODEL NO. 2 FILE NO. 0

COSINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1		-.00944	.00010	-.01704	-.00017	.08545	-.00007
2		.00589	-.00269	.00967	-.01356	-.13570	-.00537
3		.08224	-.02690	-.01048	-.00266	.02848	-.00156
4		-.05878	-.00733	.00102	.00101	.01133	.00057
5		.02981	-.01165	.00125	-.00203	.00464	-.00102
6		-.01534	-.00677	-.00262	.00241	-.04752	.00081
7		.01911	-.00211	.00106	-.00357	.03763	-.00144
8		-.00822	-.00979	-.00234	-.00010	-.02333	-.00023
9		.00887	-.00626	-.00404	-.00270	.01001	-.00118
10		-.00699	-.01134	.00566	-.00179	-.01374	-.00092
11		.00170	-.00646	-.00350	-.00291	.00142	-.00126
12		-.00441	.00660	.00116	.00263	-.00395	.00116
13		.00952	-.00940	.00188	.00080	.01329	.00013
14		-.00033	.00182	.00075	-.00147	-.02837	-.00054
15		.00771	-.00175	-.00243	.00002	.02025	-.00003
16		.00076	-.00761	.00037	-.00134	-.01969	-.00067
17		.00277	-.00102	.00356	-.00118	.02327	-.00048
18		-.00846	-.00581	-.00343	-.00132	-.03120	.00040
19		.00847	-.00304	.00200	-.00513	.01588	-.00207
20		-.00474	.00236	.00143	.00279	.00341	.00114
21		.00774	-.00722	-.00532	-.00362	-.01962	-.00156
22		-.00691	-.00243	.00258	-.00375	.01600	-.00152
23		.00513	-.00070	-.00484	.00333	-.01521	.00129
24		-.00056	-.00585	-.00009	-.00520	.00453	-.00215
25		.00600	-.00337	.00278	.00236	.00263	.00086
26		-.00332	.00310	-.00234	.00440	-.00668	.00178
27		.00603	.00153	.00429	.00064	.00158	.00028
28		-.00344	-.00275	-.00020	.00090	.00130	.00030
29		.00002	.00121	-.00286	.00138	-.00233	.00056
30		-.00293	-.00474	.00331	-.00191	-.00033	-.00084
31		-.00044	-.00209	-.00195	.00197	-.00028	.00073
32		-.00199	.00490	-.00035	-.00215	-.00433	-.00075
33		.00557	-.00872	.00380	.00061	.01228	.00007
34		-.00085	.00463	-.00068	-.00045	-.01502	-.00009
35		-.00066	-.00249	-.00101	.00081	.00838	.00027
36		.00070	-.00321	.00107	-.00028	-.00597	-.00017
37		.00374	.00047	-.00458	-.00007	.00541	-.00002
38		-.00227	-.00695	-.00005	-.00247	-.01185	-.00111
39		.00287	.00086	.00060	-.00063	.00769	-.00023
40		-.00535	.00038	-.00356	.00193	.00016	.00076

upper table 2.2

lower table 2.3

-----MODEL NO. 2 FILE NO. 0 -----

-----COSINE ANALYSIS-----

MEAN IMPERFECTION AMPLITUDES

L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
0		.01821	.01551	.01155	.01477	.11450	.00609
1		.01557	.01236	.01712	.02430	.12311	.00930
2		.05423	.09983	.01037	.01668	.13574	.00570
3		.08822	.03577	.01050	.00519	.04488	.00203
4		.06898	.00756	.00517	.00103	.01169	.00055
5		.04079	.01198	.00551	.00211	.03153	.00106
6		.02393	.00708	.00373	.00595	.05381	.00232
7		.01966	.00212	.00293	.00874	.03777	.00344
8		.00925	.01001	.00241	.00077	.02528	.00035
9		.01639	.00709	.00652	.00463	.01498	.00184
10		.00706	.01215	.00743	.00230	.01565	.00104
11		.00790	.00942	.00751	.00359	.00357	.00159
12		.00570	.00857	.00133	.00349	.00808	.00141
13		.00964	.01117	.00358	.00242	.01864	.00080
14		.00261	.00380	.00848	.00252	.02947	.00102
15		.00824	.00223	.00329	.00611	.02025	.00235
16		.00368	.00920	.00272	.00182	.01975	.00077
17		.00302	.00475	.00356	.00138	.02407	.00061
18		.01085	.00583	.00548	.00142	.03655	.00046
19		.00864	.00391	.00528	.00531	.03374	.00213
20		.01181	.00254	.00201	.00316	.03199	.00129
21		.01653	.00819	.00570	.00362	.02865	.00156
22		.00692	.00451	.00267	.00396	.01741	.00162
23		.00523	.00388	.00515	.00333	.01603	.00129
24		.00089	.00607	.00080	.00523	.00933	.00216
25		.00617	.00850	.00321	.00522	.00577	.00216
26		.00339	.00310	.00247	.00454	.00711	.00184
27		.00631	.00350	.00458	.00097	.00176	.00036
28		.00344	.00326	.00094	.00141	.00198	.00050
29		.00120	.00825	.00289	.00296	.00314	.00131
30		.00424	.00505	.00427	.00282	.00041	.00115
31		.00660	.00315	.00221	.00294	.00570	.00116
32		.00523	.00491	.00295	.00247	.00734	.00089
33		.00716	.00889	.00448	.00130	.01321	.00049
34		.00304	.00466	.00707	.00185	.01523	.00072
35		.00335	.00277	.00202	.00560	.00939	.00217
36		.00188	.00401	.00319	.00236	.00599	.00089
37		.00418	.00183	.00515	.00197	.00643	.00074
38		.00333	.00699	.00275	.00415	.01432	.00173
39		.00289	.00385	.00354	.00074	.01493	.00032
40		.01146	.00261	.00429	.00456	.01519	.00183

-----MODEL NO. 2 FILE NO. 0 -----

-----SINE ANALYSIS-----

COSINE COEFFICIENTS

L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
0	0	-.04202	-.00242	-.07519	-.00700	.05012	
1	0	.02548	.02574	.06420	-.00409	-.03930	
2	0	.06918	.08471	.01763	.01697	.00357	
3	0	.03514	-.02143	-.01205	-.00343	.01632	
4	0	.00267	-.00131	.00657	-.00043	-.00131	
5	0	-.02834	.00183	.00815	.00070	-.01651	
6	0	.01853	.00196	-.00904	-.00187	.01304	
7	0	.00740	-.00543	.00203	.00210	-.00100	
8	0	.00621	-.00214	.00824	-.00021	-.00407	
9	0	-.02089	.00509	-.00882	-.00036	.00367	
10	0	.00444	-.00439	.00232	-.00047	-.00315	
11	0	.00610	-.00398	.00454	-.00191	.00196	
12	0	-.00506	-.00580	-.00630	-.00046	.00291	
13	0	.00111	.00624	.00630	.00057	-.00559	
14	0	-.00169	-.00126	.00083	-.00120	.00358	
15	0	-.00254	-.00512	-.00223	.00137	-.00027	
16	0	.00341	.00487	.00391	.00069	-.00046	
17	0	-.00229	-.00316	-.00446	-.00111	.00270	
18	0	.01236	-.00003	.01217	-.00024	-.00795	
19	0	-.00382	-.00283	-.01624	-.00012	.01354	
20	0	.01799	-.00024	.02393	.00059	-.01347	
21	0	-.02167	.00302	-.01752	.00076	.00813	
22	0	.00076	-.00214	.00505	-.00109	-.00312	
23	0	-.00131	-.00298	.00394	-.00075	-.00244	
24	0	-.00051	.00170	-.00471	.00016	.00375	
25	0	-.00041	-.00304	.00194	-.00278	-.00241	
26	0	.00078	-.00083	.00236	.00029	-.00108	
27	0	.00296	-.00296	-.00091	-.00042	.00058	
28	0	-.00044	-.00209	-.00038	-.00005	.00066	
29	0	-.00146	.00467	.00129	.00230	-.00108	
30	0	-.00258	-.00274	-.00286	.00021	-.00008	
31	0	.00840	-.00040	.00664	-.00105	-.00204	
32	0	-.00537	-.00090	-.00740	.00030	.00237	
33	0	-.00390	.00058	.00025	.00064	-.00253	
34	0	.00004	.00077	.00371	-.00059	.00116	
35	0	-.00379	-.00462	-.00501	.00125	.00170	
36	0	.00071	.00343	.00216	-.00016	.00026	
37	0	-.00386	-.00269	-.00147	.00018	.00134	
38	0	.00530	.00165	.00445	-.00104	-.00335	
39	0	-.00271	-.00269	-.00623	-.00084	.00575	
40	0	.01558	.00070	.01200	-.00161	-.00593	

upper table 2.4

lower table 2.5

MODEL NO. 2 FILE NO. 0

SINE ANALYSIS

SINE COEFFICIENTS						
L	K	0	1	2	3	4
1	0	0	-.01421	.00020	-.07077	-.00003
2	0	0	.01937	.00684	.09847	-.00416
3	0	0	.10372	-.01936	.00188	-.00599
4	0	0	-.08740	-.00509	-.02977	-.00171
5	0	0	.03593	-.00780	.00762	-.00283
6	0	0	-.01206	-.00690	.02493	-.00068
7	0	0	.01869	.00070	-.01805	-.00137
8	0	0	-.00628	-.00761	.01135	-.00195
9	0	0	.01169	-.00313	-.00624	-.00195
10	0	0	-.00966	-.00771	.01037	-.00271
11	0	0	.00357	-.00315	-.00257	-.00205
12	0	0	-.00555	.00344	.00189	.00200
13	0	0	.00935	-.00790	-.00451	-.00163
14	0	0	.00270	.00240	.01927	-.00004
15	0	0	.00826	-.00139	-.01244	-.00034
16	0	0	.00317	-.00508	.01361	-.00185
17	0	0	-.00105	-.00002	-.01237	-.00051
18	0	0	-.00512	-.00542	.01584	-.00079
19	0	0	.00767	.00100	-.00651	-.00197
20	0	0	-.00698	.00001	-.00296	.00121
21	0	0	.01451	-.00327	.01233	-.00239
22	0	0	-.01176	.00057	-.01136	-.00148
23	0	0	.01054	-.00275	.00876	.00075
24	0	0	-.00121	-.00116	-.00326	-.00254
25	0	0	.00584	-.00420	.00199	-.00003
26	0	0	-.00223	-.00047	.00188	.00179
27	0	0	.00531	.00078	.00364	.00047
28	0	0	-.00435	-.00275	-.00214	-.00030
29	0	0	.00165	.00003	-.00022	.00061
30	0	0	-.00515	-.00245	.00132	-.00144
31	0	0	.00039	-.00294	-.00118	.00012
32	0	0	-.00178	.00527	.00200	.00039
33	0	0	.00366	-.00724	-.00396	-.00155
34	0	0	.00110	.00393	.00931	.00079
35	0	0	-.00137	-.00249	-.00644	-.00027
36	0	0	.00110	-.00233	.00488	-.00071
37	0	0	.00613	.00042	-.00522	.00007
38	0	0	-.00135	-.00383	.00711	-.00202
39	0	0	.00235	.00109	-.00380	.00000
40	0	0	-.00502	-.00098	-.00412	.00060

MODEL NO 2 FILE NO 0

SINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES						
L	K	0	1	2	3	4
0	0	0	.04202	.00242	.07519	.00700
1	0	0	.02918	.02574	.09555	.00409
2	0	0	.07184	.08499	.10004	.01747
3	0	0	.10052	.02888	.01220	.02792
4	0	0	.08745	.00525	.03049	.00176
5	0	0	.04576	.00801	.01116	.00291
6	0	0	.02211	.00718	.02652	.00199
7	0	0	.02010	.00547	.01816	.00251
8	0	0	.00883	.00791	.01403	.00196
9	0	0	.02394	.00597	.01080	.00199
10	0	0	.01063	.00887	.01063	.00275
11	0	0	.00707	.00508	.00522	.00280
12	0	0	.00751	.00674	.00658	.00205
13	0	0	.00941	.01007	.00775	.00173
14	0	0	.00319	.00271	.01929	.00120
15	0	0	.00864	.00530	.01264	.00141
16	0	0	.00466	.00704	.01416	.00198
17	0	0	.00252	.00316	.01314	.00122
18	0	0	.01336	.00542	.01998	.00082
19	0	0	.00857	.00300	.01749	.00197
20	0	0	.01930	.00024	.02411	.00134
21	0	0	.02608	.00445	.02143	.00251
22	0	0	.01179	.00221	.01243	.00184
23	0	0	.01062	.00406	.00961	.00107
24	0	0	.00132	.00206	.00573	.00255
25	0	0	.00586	.00519	.00278	.00278
26	0	0	.00236	.00095	.00302	.00181
27	0	0	.00608	.00306	.00375	.00063
28	0	0	.00437	.00345	.00217	.00030
29	0	0	.00220	.00467	.00131	.00238
30	0	0	.00576	.00368	.00315	.00146
31	0	0	.00841	.00297	.00674	.00105
32	0	0	.00566	.00534	.00766	.00049
33	0	0	.00534	.00727	.00397	.00168
34	0	0	.00110	.00401	.01002	.00098
35	0	0	.00403	.00524	.00816	.00128
36	0	0	.00131	.00415	.00534	.00072
37	0	0	.00724	.00272	.00543	.00019
38	0	0	.00547	.00416	.00839	.00227
39	0	0	.00359	.00290	.00729	.00084
40	0	0	.01636	.00120	.01269	.00172

upper table 2.6

lower table 2.7

-----MODEL NO. 3 FILE NO. 0-----

-----COSINE ANALYSIS-----

COSINE COEFFICIENTS

--L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
0		.00289	-.00464	-.00489	-.00310	-.00395	-.00061
1		-.00258	-.00625	.01777	.01304	.00452	.01203
2		-.41880	.09199	-.00185	.00163	.00955	.00073
3		-.12127	.00462	.00458	-.02210	-.00111	-.01418
4		.08428	-.00312	.00249	.00022	.00771	.00099
5		.08647	-.03953	.02600	-.01204	.01866	-.00920
6		-.04900	.01338	.01890	-.00221	.01742	-.00414
7		.03167	-.02188	.01332	-.01495	.00873	-.01728
8		.01062	-.01241	.00758	-.01605	.00954	-.00505
9		.03884	-.00492	.01551	-.01443	.00784	-.00472
10		.01863	.00377	.01673	-.00986	.01136	-.00220
11		.00358	-.01307	-.00023	-.01136	.00418	-.00616
12		.01206	-.01038	.00964	-.00683	.00738	-.00352
13		.00857	.00352	.00976	-.00192	.00754	-.00338
14		.00846	-.00576	.01314	-.00235	.01142	-.00306
15		.00176	-.01071	.01157	-.01183	.00494	-.01153
16		-.00120	-.01144	.01230	-.01268	.00778	-.00505
17		.01163	-.02170	.00868	-.01121	.00550	-.00525
18		.01256	-.01091	.01152	-.00628	.01016	-.00759
19		.01491	-.01138	.00807	-.01022	.00470	-.00903
20		.00201	-.00235	.00189	-.00485	.00107	-.00438
21		.00095	-.01208	.00898	-.00590	.00558	-.00220
22		.00597	-.00435	.00713	-.00525	.01078	-.00453
23		-.00997	.00142	.01244	-.00260	.00753	-.00080
24		.00683	-.00317	.00701	-.00250	.00184	-.00378
25		.01754	-.01472	.00112	-.00259	.00582	-.00431
26		-.00044	-.00580	.00830	-.00741	.00273	-.00678
27		-.00803	-.00427	.00436	.00112	.00379	.00293
28		.00231	-.00806	.00577	-.00669	.00290	-.00628
29		.02200	-.00595	.01373	-.00184	.01015	-.00402
30		-.00799	.00340	.00569	-.00225	.00281	-.00199
31		-.01111	-.00136	.00429	-.00110	.00677	.00149
32		.01037	-.00955	.00414	-.00522	.00029	-.00603
33		.00245	-.00728	.00966	-.00464	.00745	-.00039
34		-.01239	-.00345	.00074	-.00343	-.00263	-.00463
35		.00066	-.00191	-.00302	-.00224	-.00172	-.00108
36		.01492	-.00183	-.00318	-.00425	-.00123	-.00607
37		.00148	-.00345	.00530	-.00318	.00680	-.00070
38		-.01618	-.00074	.00440	-.00150	.00293	-.00073
39		-.00761	.00736	-.00132	.00102	-.00109	-.00177
40		.05044	-.00501	.00259	-.00139	.00062	-.00119

-----MODEL NO. 3 FILE NO. 0-----

-----COSINE ANALYSIS-----

SINE COEFFICIENTS

--L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
1		-.00643	-.00517	.02091	.00586	.02547	.00175
2		.08131	.07752	.02380	.01873	.01930	.00175
3		.14622	-.03389	.01981	.00041	.00851	.00610
4		.05990	-.06199	.01000	-.01885	.00967	-.00736
5		-.03084	-.00820	.01377	-.01113	.01390	.00410
6		-.00536	.00295	.01067	.00038	.01006	-.00017
7		.02129	.00154	.00772	-.00296	.01357	.00160
8		-.00482	-.00355	.00352	-.00245	.00424	-.00045
9		-.01115	.00334	.01172	-.00184	.01222	-.00011
10		.01327	-.00290	.00786	-.00625	.00419	-.00275
11		.00618	-.01739	.00463	-.01053	.00417	-.00918
12		-.00416	-.00869	.01237	-.00602	.00438	-.00504
13		.00834	.00549	.01375	-.00285	.01134	.00232
14		.00912	-.00872	.01169	-.00823	.01197	-.00553
15		.01599	-.01775	.00931	-.00998	.01240	-.01102
16		-.00060	-.01759	.01374	-.00980	.00877	-.00634
17		.00439	-.00025	.00917	-.00550	.01133	.00302
18		.00048	-.00910	.01067	-.00996	.00864	-.00418
19		.00523	-.00620	.00852	-.00353	.01043	-.00337
20		.00560	-.00572	.00199	-.00396	.00332	-.00534
21		-.00753	-.00709	.01172	-.00362	.00819	-.00246
22		.00883	-.01125	.00428	-.00531	.00189	-.00784
23		.02288	-.01184	.00818	-.00963	.01068	-.00325
24		-.00280	-.00431	.01352	-.00964	.00815	-.00306
25		-.00150	-.00658	.01020	-.00365	.00813	-.00033
26		.00511	-.01699	.00573	-.00468	.00420	-.00606
27		.00409	-.00085	.01157	-.00511	.01025	-.00349
28		-.00195	-.00718	.01097	-.00801	.00754	-.00278
29		.00326	-.00671	.00709	-.00744	.00500	-.00669
30		.01885	-.00616	.00613	-.00774	.00555	-.00742
31		-.00136	-.00688	.00435	-.00107	.00484	-.00404
32		-.00771	-.00272	.00395	-.00877	.00142	-.00529
33		.01226	-.01137	.00667	-.00924	.00518	-.00552
34		.00428	-.00147	.00657	-.00500	.00302	-.00309
35		.00048	-.00053	.00779	-.00206	.00440	-.00108
36		.00282	-.00723	.00307	-.00170	.00446	-.00169
37		.00160	-.00770	.00919	-.00011	.00928	-.00348
38		.02026	-.00599	.00406	-.00348	.00373	-.00489
39		.00143	-.00077	.00464	-.00226	.00309	-.00116
40		-.00363	-.01376	.00917	-.01139	.00602	-.00560

upper table 2.8

lower table 2.9

MODEL NO. 3 FILE NO. 0

COSINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES

L	K=	0	1	2	3	4	5
0		.00289	.00464	.00489	.00310	.00395	.00061
1		.00693	.00811	.02744	.01430	.02587	.01216
2		.42662	.12030	.02387	.01880	.02153	.00189
3		.18997	.03421	.02033	.02211	.00858	.01544
4		.10339	.06207	.01030	.01885	.01236	.00743
5		.09181	.04037	.02942	.01640	.02327	.01008
6		.04929	.01370	.02170	.00224	.02011	.00414
7		.03816	.02193	.01539	.01524	.01614	.01736
8		.01166	.01290	.00836	.01624	.01044	.00507
9		.04040	.00594	.01944	.01455	.01451	.00472
10		.02287	.00476	.01648	.01168	.01211	.00352
11		.00714	.02176	.00463	.01548	.00590	.01106
12		.01276	.01354	.01568	.00910	.00859	.00615
13		.01196	.00652	.01686	.00344	.01361	.00410
14		.01244	.01045	.01759	.00856	.01655	.00632
15		.01609	.02073	.01485	.01548	.01335	.01595
16		.00135	.02098	.01845	.01603	.01172	.00810
17		.01243	.02170	.01262	.01249	.01260	.00525
18		.01257	.01421	.01570	.01178	.01334	.00866
19		.01580	.01296	.01173	.01081	.01144	.00963
20		.00595	.00618	.00274	.00626	.00348	.00691
21		.00759	.01400	.01476	.00692	.00991	.00330
22		.01066	.01206	.00831	.00746	.01094	.00905
23		.02496	.01203	.01489	.00997	.01307	.00331
24		.00738	.00535	.01523	.00996	.00836	.00486
25		.01761	.01613	.01026	.00448	.01000	.00433
26		.00513	.01796	.01009	.00876	.00501	.00909
27		.00901	.00436	.01237	.00523	.01093	.00456
28		.00303	.01080	.01239	.01044	.00808	.00686
29		.02224	.00897	.01545	.00766	.01132	.00780
30		.02048	.00517	.00836	.00806	.00622	.00768
31		.01119	.00702	.00611	.00153	.00832	.00430
32		.01292	.00993	.00572	.01021	.00145	.00802
33		.01251	.01350	.01174	.01034	.00907	.00553
34		.01311	.00375	.00661	.00607	.00400	.00557
35		.00081	.00198	.00835	.00304	.00472	.00153
36		.01518	.00746	.00442	.00458	.00463	.00630
37		.00218	.00843	.01061	.00318	.01150	.00355
38		.02593	.00604	.00599	.00379	.00474	.00494
39		.00775	.00740	.00482	.00248	.00328	.00211
40		.05057	.01464	.00953	.01147	.00605	.00572

MODEL NO. 3 FILE NO. 0

SINE ANALYSIS

COSINE COEFFICIENTS

L	K=	0	1	2	3	4	5
0	0	.00718	-.00226	.00355	-.00353	.00893	
1	0	-.01135	-.01367	.00969	.00020	.00857	
2	0	-.52914	.07522	-.17195	.02808	-.07496	
3	0	-.15496	.01861	-.04339	-.00261	-.02325	
4	0	.10432	-.00313	.02948	-.00209	.01967	
5	0	.09496	-.02393	.03815	-.01167	.02352	
6	0	-.07239	.01312	-.01770	.00566	-.00033	
7	0	.03308	-.00658	.01615	-.00345	.01246	
8	0	.00862	-.00052	.00215	-.01034	.00393	
9	0	.04085	.00490	.01961	-.00688	.00904	
10	0	.01437	.00897	.01016	-.00356	.00624	
11	0	.00421	-.00334	-.00121	-.00649	.00335	
12	0	.01020	-.00424	.00706	-.00505	.00857	
13	0	.00532	.00485	.00424	.00287	.00328	
14	0	.00326	-.00301	.00417	-.00130	.00563	
15	0	-.00349	-.00010	.00556	-.00219	.00416	
16	0	-.00808	-.00145	.00238	-.00761	.00274	
17	0	.01014	-.01061	.00684	-.00929	.00541	
18	0	.00930	-.00401	.00545	-.00143	.00567	
19	0	.01466	-.00210	.00833	-.00339	.00594	
20	0	.00157	.00152	.00141	-.00070	.00119	
21	0	-.00358	-.00637	.00253	-.00584	.00240	
22	0	.00300	.00016	-.00014	-.00132	.00546	
23	0	-.01916	.00261	-.00072	-.00108	.00120	
24	0	.00528	-.00049	.00636	.00029	.00308	
25	0	.02079	-.00985	.00322	-.00265	.00403	
26	0	-.00450	.00062	.00405	-.00138	.00302	
27	0	-.01262	-.00474	-.00294	-.00276	-.00053	
28	0	-.00014	-.00177	.00249	-.00204	.00075	
29	0	.02017	-.00318	.01044	-.00009	.00699	
30	0	-.01300	.00185	-.00116	-.00013	-.00011	
31	0	-.01679	-.00090	-.00602	-.00243	.00063	
32	0	.01127	-.00370	.00692	-.00132	.00327	
33	0	-.00238	-.00327	.00173	-.00449	.00142	
34	0	-.01565	-.00001	-.00258	.00033	-.00344	
35	0	.00243	-.00017	-.00054	-.00126	-.00043	
36	0	.02041	.00199	.00452	.00118	.00224	
37	0	-.00135	-.00101	-.00039	-.00264	.00332	
38	0	-.02279	.00029	-.00542	-.00080	-.00213	
39	0	-.00888	.00582	-.00313	.00399	-.00194	
40	0	.06236	-.00295	.02083	-.00097	.00951	

upper table 2.10

lower table 2.11

MODEL NO. 3 FILE NO. 0

SINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1	0	0	-.02085	-.00887	-.00621	-.00170	.00745
2	0	0	.08866	.05251	.03170	.03103	.01385
3	0	0	.17463	-.03009	.06493	-.01586	.03336
4	0	0	.06971	-.03954	.02290	-.02506	.01328
5	0	0	-.04695	.00026	-.01251	-.00617	-.00118
6	0	0	-.01318	.00171	-.00303	-.00018	-.00154
7	0	0	.02177	.00234	.00455	-.00324	.01035
8	0	0	-.00827	-.00171	-.00267	-.00281	-.00010
9	0	0	-.02104	.00370	-.00534	-.00035	.00095
10	0	0	.01266	.00151	.00755	-.00298	.00417
11	0	0	.00532	-.00693	.00315	-.00539	.00446
12	0	0	-.01142	-.00286	.00358	-.00259	.00093
13	0	0	.00300	.00551	.00518	-.00229	.00765
14	0	0	.00466	-.00172	.00299	-.00404	.00557
15	0	0	.01420	-.00723	.00324	-.00393	.00523
16	0	0	-.00814	-.00801	.00273	-.00698	.00261
17	0	0	-.00001	.00272	.00021	-.00383	.00549
18	0	0	-.00527	-.00144	.00157	-.00648	.00398
19	0	0	.00135	-.00254	.00030	-.00163	.00382
20	0	0	.00572	-.00150	.00113	-.00025	.00189
21	0	0	-.01572	-.00354	-.00025	-.00295	.00336
22	0	0	.00898	-.00498	.00503	-.00124	.00249
23	0	0	.02371	-.00399	.00664	-.00736	.00683
24	0	0	-.01060	.00213	.00264	-.00569	.00375
25	0	0	-.00754	-.00351	.00068	-.00437	.00299
26	0	0	.00328	-.01040	.00275	-.00378	.00189
27	0	0	-.00126	.00276	.00273	-.00099	.00614
28	0	0	-.00841	-.00107	.00130	-.00537	.00222
29	0	0	.00048	-.00013	.00332	-.00173	.00505
30	0	0	.02044	.00063	.00806	-.00122	.00700
31	0	0	-.00430	-.00430	-.00094	.00032	.00152
32	0	0	-.01166	.00353	-.00120	-.00266	-.00042
33	0	0	.01179	-.00323	.00565	-.00513	.00418
34	0	0	.00212	.00205	.00426	-.00155	.00303
35	0	0	-.00339	.00169	.00270	-.00061	.00296
36	0	0	.00161	-.00479	.00008	-.00215	.00209
37	0	0	-.00325	-.00567	.00064	.00058	.00454
38	0	0	.02322	-.00217	.00792	-.00060	.00481
39	0	0	-.00056	.00203	.00199	-.00056	.00311
40	0	0	-.00933	-.00414	.00119	-.00748	.00371

MODEL NO. 3 FILE NO. 0

SINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES

L	K=	0	1	2	3	4	5
0	0	0	.00718	.00226	.00355	.00353	.00893
1	0	0	.02374	.01629	.01151	.00171	.01136
2	0	0	.53652	.09173	.17485	.04185	.08115
3	0	0	.23347	.03538	.07809	.01607	.04066
4	0	0	.12547	.03967	.03733	.02515	.02373
5	0	0	.10593	.02393	.04015	.01320	.02355
6	0	0	.07358	.01323	.01796	.00567	.00158
7	0	0	.03960	.00699	.01678	.00473	.01619
8	0	0	.01195	.00179	.00342	.01072	.00393
9	0	0	.04595	.00614	.02032	.00689	.00909
10	0	0	.01915	.00910	.01266	.00464	.00751
11	0	0	.00679	.00770	.00337	.00844	.00558
12	0	0	.01531	.00511	.00791	.00567	.00862
13	0	0	.00611	.00734	.00669	.00367	.00832
14	0	0	.00569	.00347	.00513	.00424	.00792
15	0	0	.01463	.00723	.00643	.00450	.00669
16	0	0	.01147	.00814	.00362	.01032	.00378
17	0	0	.01014	.01095	.00685	.01005	.00771
18	0	0	.01069	.00426	.00567	.00664	.00693
19	0	0	.01473	.00329	.00834	.00376	.00706
20	0	0	.00593	.00213	.00181	.00074	.00223
21	0	0	.01612	.00728	.00254	.00654	.00413
22	0	0	.00947	.00498	.00504	.00182	.00600
23	0	0	.03048	.00476	.00668	.00744	.00694
24	0	0	.01185	.00218	.00688	.00569	.00485
25	0	0	.02212	.01045	.00329	.00511	.00502
26	0	0	.00557	.01042	.00490	.00403	.00356
27	0	0	.01268	.00549	.00402	.00293	.00616
28	0	0	.00841	.00207	.00280	.00575	.00234
29	0	0	.02018	.00318	.01095	.00173	.00862
30	0	0	.02422	.00196	.00815	.00123	.00700
31	0	0	.01733	.00439	.00609	.00245	.00165
32	0	0	.01621	.00512	.00702	.00297	.00330
33	0	0	.01203	.00459	.00591	.00682	.00441
34	0	0	.01580	.00205	.00498	.00159	.00458
35	0	0	.00417	.00169	.00275	.00140	.00299
36	0	0	.02048	.00518	.00452	.00245	.00307
37	0	0	.00352	.00575	.00075	.00270	.00563
38	0	0	.03254	.00219	.00960	.00100	.00526
39	0	0	.00890	.00616	.00371	.00403	.00366
40	0	0	.06306	.00508	.02086	.00754	.01021

upper table 2.12

lower table 2.13

MODEL NO. 4 FILE NO. 0

COSINE ANALYSIS

COSINE COEFFICIENTS

L	K=	0	1	2	3	4	5
0		-.00767	-.10619	.00432	-.02431	.02899	-.01549
1		.00591	.00716	-.03789	.00338	-.02303	-.00592
2		.04780	-.11792	-.01584	-.02620	-.00427	-.00964
3		-.07172	.00410	-.00316	-.00667	.00389	-.01067
4		.00639	-.03207	.00083	-.00096	.00186	-.00811
5		.00405	-.02100	-.00881	-.00985	-.01132	-.00183
6		-.00862	.00399	-.00184	-.00305	-.00447	-.01231
7		-.00230	.01439	.00230	-.00005	-.00430	-.00503
8		-.00952	-.00567	-.00246	-.00027	-.00173	-.00031
9		-.00283	-.01778	-.01853	-.00861	-.00731	-.00830
10		.00192	-.00836	-.00363	-.01707	.00356	-.00435
11		-.00531	.00411	-.01379	-.00561	-.00747	-.00317
12		-.00993	.00336	-.00113	-.00172	-.00344	-.00198
13		-.00666	.00587	.00015	.00136	-.00843	-.00650
14		.00528	.00154	.00041	-.00007	-.00224	-.01001
15		.01023	-.00678	-.00379	-.00527	-.00384	.00555
16		.00451	-.00802	.00171	-.00208	.00132	-.00398
17		-.00103	-.00075	-.00777	-.00613	-.00225	-.01005
18		-.00016	-.00451	-.00283	-.01337	-.00164	-.00348
19		-.00497	-.00121	.00299	-.00376	-.00312	.00062
20		-.00277	.00133	-.00062	.00230	-.00400	-.00089
21		.00461	-.00340	.01277	-.00510	.00596	-.00154
22		.00149	-.00026	-.00245	-.01131	-.00100	-.00116
23		.00581	.00614	.00296	-.00244	.00754	-.00698
24		-.00425	-.00635	.00251	-.00193	.00350	-.00461
25		-.00309	-.01300	-.00356	-.00655	-.00188	.00312
26		-.00042	-.00066	.00030	.00365	.00015	-.00765
27		-.00097	.00773	.00758	.00575	.00168	-.00138
28		.00081	-.00356	-.00024	-.00084	-.00423	.00148
29		.00099	-.00622	-.00401	-.00457	.00228	-.00300
30		-.00233	-.00919	.00129	-.00927	.00561	.00169
31		-.00254	-.00433	.00250	-.00065	.00897	-.00105
32		-.00490	-.00704	-.00084	-.00068	-.00202	.00057
33		-.00268	.00053	-.00329	.00455	-.00647	-.00436
34		.01337	.00349	.00092	.00752	.00149	-.00427
35		.01257	-.00478	.00261	-.00259	.00114	.00693
36		-.00108	-.00472	.00418	-.00118	.00392	-.00404
37		-.00462	-.00143	-.00702	-.00216	.00027	-.00829
38		.00628	-.00083	.00066	-.00597	.00370	.00288
39		.01073	.00254	.00915	.00127	.00147	.00366
40		.01412	.00532	-.00004	.00345	-.00487	-.00151

MODEL NO. 4 FILE NO. 0

COSINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1		.01077	-.01022	-.05236	.02027	-.03608	.00306
2		-.11108	-.07726	.00223	-.03646	.01919	-.01683
3		-.05154	-.02163	-.01225	-.00345	-.01094	.00114
4		-.02972	-.00098	-.00761	.00618	.00441	-.00492
5		-.02816	.00593	-.02075	.00169	-.00717	.00301
6		-.00576	-.01791	.00901	-.01034	.00327	-.00359
7		-.01692	-.00024	-.01186	-.00668	-.01351	-.00289
8		-.01116	.00989	-.01011	.00131	.00475	.00132
9		-.01725	-.00693	-.00234	.01521	-.00282	.00060
10		.00083	-.02614	-.00211	-.01128	.00169	-.00728
11		-.01084	-.00646	-.01846	-.01896	-.01482	-.00192
12		-.00627	.00452	.00845	-.00040	-.00690	.00094
13		-.01836	.00490	-.00559	.00395	-.00767	.00169
14		-.01496	-.00930	-.01262	-.01221	.00348	-.01134
15		-.01358	-.00198	-.01061	-.00522	-.01382	-.00480
16		-.00242	.00979	.00250	-.00362	-.00171	.00743
17		.00517	-.00597	-.00441	.00446	.00191	-.00446
18		.00346	-.01256	-.00568	-.00996	-.00057	.00915
19		-.07295	-.00313	-.00134	-.01447	-.00190	.00106
20		-.00040	-.00058	-.00150	.00024	-.00085	-.00046
21		.06191	-.00589	-.01036	.00753	-.00922	-.00731
22		-.00310	-.00665	.00130	-.00751	-.00332	-.00677
23		-.00110	.00588	-.00673	-.00824	-.01295	.00166
24		.00317	-.00151	-.00790	.00461	-.00005	-.00436
25		-.00451	.00064	-.01390	-.00080	-.00804	.00018
26		-.00145	-.00927	.00788	-.00491	-.00539	-.00209
27		-.00592	-.00724	-.00709	-.00686	-.00577	-.00537
28		-.00331	.00100	-.01071	-.00008	.00660	.00061
29		-.00179	-.00367	.00274	.01162	.00089	-.00292
30		.00211	-.00711	-.00129	-.00725	-.00146	-.00643
31		-.00397	.00269	-.01214	-.01745	-.01134	-.00096
32		-.00372	.00055	.01153	-.00127	-.00527	-.00106
33		-.01039	-.00026	-.00119	.00193	-.00073	-.00076
34		-.00697	-.01019	-.01242	-.00839	.00275	-.01159
35		.00092	-.00209	.00247	-.00281	-.00375	-.00285
36		.00394	.00866	.00673	-.00435	-.00204	.00568
37		-.00299	-.00600	-.00530	.00278	-.00091	-.00538
38		-.00944	-.00967	-.00389	-.00507	.00211	-.00507
39		-.07684	-.00014	-.00471	-.01203	-.00282	.00444
40		.00391	.00208	-.00013	-.00057	.00021	-.00050

upper table 2.14

lower table 2.15

-----MODEL NO. 4 FILE NO. 0 -----

-----COSINE ANALYSIS-----

MEAN IMPERFECTION AMPLITUDES

--L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
0		.00767	.10619	.00432	.02431	.02899	.01549
1		.01229	.01248	.06463	.02055	.04280	.00667
2		.12093	.14098	.01600	.04490	.01966	.01940
3		.08832	.02201	.01265	.01136	.01160	.01073
4		.03040	.03209	.00766	.00625	.00479	.00948
5		.02845	.02182	.02255	.01000	.01340	.00353
6		.01037	.01834	.00919	.01078	.00554	.01282
7		.01708	.01439	.01208	.00668	.01418	.00580
8		.01467	.01140	.01041	.00134	.00506	.00136
9		.01748	.01908	.01867	.01748	.00784	.00832
10		.00209	.02745	.00420	.02046	.00394	.00848
11		.01207	.00765	.02304	.01978	.01660	.00371
12		.01174	.00564	.00853	.00176	.00771	.00219
13		.01953	.00765	.00559	.00418	.01140	.00672
14		.01587	.00942	.01263	.01221	.00414	.01513
15		.01700	.00707	.01127	.00742	.01434	.00734
16		.00511	.01266	.00303	.00417	.00216	.00843
17		.00527	.00602	.00893	.00759	.00295	.01100
18		.00346	.01335	.00635	.01667	.00173	.00979
19		.07312	.00335	.00328	.01495	.00365	.00123
20		.00280	.00145	.00163	.00231	.00409	.00100
21		.06208	.00680	.01644	.00909	.01098	.00747
22		.00344	.00666	.00278	.01358	.00346	.00687
23		.00592	.00850	.00735	.00860	.01489	.00718
24		.00530	.00653	.00829	.00500	.00350	.00634
25		.00547	.01302	.01435	.00660	.00825	.00312
26		.00151	.00929	.00789	.00611	.00540	.00793
27		.00600	.01059	.01038	.00896	.00601	.00555
28		.00341	.00370	.01072	.00085	.00784	.00160
29		.00205	.00723	.00486	.01248	.00245	.00418
30		.00314	.01085	.00182	.01177	.00580	.00664
31		.00471	.00510	.01239	.01746	.01446	.00143
32		.00615	.00707	.01156	.00144	.00565	.00120
33		.01073	.00059	.00350	.00494	.00651	.00443
34		.01508	.01077	.01245	.01127	.00313	.01235
35		.01260	.00521	.00360	.00382	.00392	.00749
36		.00409	.00986	.00793	.00451	.00442	.00697
37		.00550	.00617	.00880	.00352	.00095	.00988
38		.01133	.00970	.00394	.00783	.00426	.00584
39		.07758	.00255	.01029	.01209	.00318	.00576
40		.01466	.00571	.02013	.00349	.00488	.00159

-----MODEL NO. 4 FILE NO. 0 -----

-----SINE ANALYSIS-----

COSINE COEFFICIENTS

--L--	K=	--0--	--1--	--2--	--3--	--4--	--5--
0		0	-.01800	-.07041	-.02607	-.03571	-.00609
1		0	.02887	.00833	-.01724	.01687	-.02728
2		0	.06691	-.08549	.01325	-.03729	.00958
3		0	-.09249	.00417	-.02715	-.00050	-.00639
4		0	.01194	-.02138	.00381	-.00495	.00469
5		0	.01047	-.00567	.00443	-.00124	.00070
6		0	-.01347	.00615	-.00181	-.00021	-.00211
7		0	-.00197	.01017	.00124	.00106	.00062
8		0	-.01149	-.00135	-.00601	.00049	-.00025
9		0	.00554	-.00680	-.00326	-.00268	.00015
10		0	.00676	.00288	-.00060	-.00148	.00140
11		0	.00015	.00857	-.00144	.00168	-.00214
12		0	-.01234	.00659	-.00390	.00083	-.00009
13		0	-.00595	.00275	.00103	.00085	-.00132
14		0	.00272	.00183	.00343	-.00126	.00079
15		0	.01484	.00170	.00433	-.00065	.00236
16		0	.00894	-.00155	.00299	-.00127	.00065
17		0	-.00085	-.00006	-.00006	-.00155	-.00035
18		0	.00003	-.00029	.00079	-.00108	-.00030
19		0	-.00383	.00192	-.00137	-.00099	.00024
20		0	-.00124	.00106	-.00018	-.00074	.00177
21		0	.00222	.00149	.00103	-.00031	.00045
22		0	.00179	.00168	.00100	-.00013	-.00057
23		0	.00135	.00289	.00186	.00026	.00098
24		0	-.00266	-.00027	-.00109	-.00060	.00079
25		0	-.00228	-.00222	-.00191	-.00137	.00099
26		0	-.00477	-.00250	-.00070	-.00127	-.00008
27		0	-.00368	.00066	.00033	-.00013	.00048
28		0	.00073	.00054	.00079	-.00011	-.00033
29		0	.00217	-.00071	-.00008	-.00114	-.00010
30		0	-.00121	-.00268	-.00083	-.00148	-.00007
31		0	-.00618	-.00173	-.00209	.00036	.00057
32		0	-.00641	-.00226	-.00311	-.00041	.00004
33		0	.00030	-.00387	-.00136	-.00037	-.00120
34		0	.01204	-.00188	.00367	-.00013	.00139
35		0	.01403	.00159	.00527	.00058	.00220
36		0	.00044	.00061	.00071	.00008	.00080
37		0	-.00618	-.00324	-.00317	-.00074	-.00165
38		0	.00575	-.00266	.00145	-.00023	.00115
39		0	.01234	.00165	.00479	.00104	.00220
40		0	.01981	.00373	.00696	.00134	.00338

upper table 2.16

lower table 2.17

MODEL NO. 4 FILE NO. 0

SINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1	0	0	.04215	-.01444	-.00776	-.00390	-.01940
2	0	0	-.14135	-.04279	-.05733	-.03728	-.01381
3	0	0	-.05960	-.02015	-.02641	-.00955	-.01917
4	0	0	-.04111	.00269	-.01407	.00160	-.00073
5	0	0	-.02757	-.00610	-.01460	-.00253	-.00570
6	0	0	-.00847	-.00719	-.00702	-.00698	.00277
7	0	0	-.01577	.00349	-.00477	-.00026	-.00673
8	0	0	-.01250	.00715	-.00100	.00251	-.00054
9	0	0	-.01645	-.00343	-.00573	-.00190	-.00371
10	0	0	.00191	-.01387	-.00184	-.00920	-.00194
11	0	0	-.00689	-.00533	-.00540	-.00349	-.00331
12	0	0	-.00797	.00352	-.00365	-.00012	.00131
13	0	0	-.01786	.00208	-.00497	.00073	-.00426
14	0	0	-.01653	0	-.00675	-.00441	-.00012
15	0	0	-.00759	.00200	-.00585	-.00037	-.00318
16	0	0	.00309	.00222	-.00169	-.00045	.00191
17	0	0	.00912	.00113	.00011	-.00021	-.00038
18	0	0	.00468	-.00197	.00039	-.00241	-.00015
19	0	0	-.00430	-.00082	-.00138	-.00062	-.00111
20	0	0	.00040	-.00056	-.00017	.00020	.00088
21	0	0	-.00267	-.00139	-.00217	-.00134	-.00230
22	0	0	-.00165	-.00082	-.00011	-.00169	.00010
23	0	0	.00255	.00262	.00093	.00034	-.00198
24	0	0	.00077	.00323	.00042	.00061	.00011
25	0	0	-.00084	.00131	-.00066	.00006	-.00266
26	0	0	-.00142	-.00316	-.00019	-.00165	.00053
27	0	0	-.00499	-.00153	-.00250	-.00016	-.00145
28	0	0	-.00277	.00088	-.00034	-.00002	.00017
29	0	0	.00013	.00207	.00060	-.00025	-.00108
30	0	0	.00339	-.00041	.00083	-.00139	-.00051
31	0	0	-.00158	.00120	-.00107	-.00059	-.00118
32	0	0	-.00658	.00123	-.00255	-.00005	-.00032
33	0	0	-.01079	-.00054	-.00411	-.00010	-.00165
34	0	0	-.00665	-.00262	-.00375	-.00162	-.00133
35	0	0	.00323	.00012	.00061	-.00050	.00023
36	0	0	.00914	.00220	.00349	.00049	.00200
37	0	0	-.00024	.00119	-.00011	-.00064	-.00110
38	0	0	-.01272	-.00316	-.00519	-.00169	-.00137
39	0	0	-.00736	-.00053	-.00380	-.00111	-.00010
40	0	0	.00509	.00214	.00174	.00066	.00222

MODEL NO. 4 FILE NO. 0

SINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES

L	K=	0	1	2	3	4	5
0	0	0	.01800	.07041	.02607	.03571	.00609
1	0	0	.05109	.01667	.01891	.01731	.03347
2	0	0	.15639	.09560	.05884	.05277	.01681
3	0	0	.11003	.02058	.03788	.00966	.02021
4	0	0	.04281	.02155	.01458	.00520	.00475
5	0	0	.02949	.00833	.01526	.00282	.00574
6	0	0	.01591	.00946	.00725	.00698	.00348
7	0	0	.01589	.01075	.00493	.00109	.00676
8	0	0	.01698	.00728	.00609	.00256	.00060
9	0	0	.01736	.00762	.00659	.00329	.00371
10	0	0	.00702	.01417	.00194	.00932	.00239
11	0	0	.00689	.01009	.00559	.00387	.00394
12	0	0	.01469	.00747	.00534	.00084	.00131
13	0	0	.01883	.00345	.00508	.00112	.00446
14	0	0	.01675	.00183	.00757	.00459	.00080
15	0	0	.01667	.00262	.00728	.00075	.00396
16	0	0	.00946	.00271	.00343	.00135	.00202
17	0	0	.00916	.00113	.00013	.00156	.00052
18	0	0	.00468	.00199	.00088	.00264	.00034
19	0	0	.00576	.00209	.00194	.00117	.00114
20	0	0	.00130	.00120	.00025	.00077	.00198
21	0	0	.00347	.00204	.00240	.00138	.00234
22	0	0	.00243	.00187	.00101	.00169	.00058
23	0	0	.00289	.00390	.00208	.00043	.00221
24	0	0	.00277	.00324	.00117	.00086	.00080
25	0	0	.00243	.00258	.00202	.00137	.00284
26	0	0	.00498	.00403	.00073	.00208	.00054
27	0	0	.00620	.00167	.00252	.00021	.00153
28	0	0	.00286	.00103	.00086	.00011	.00037
29	0	0	.00217	.00219	.00061	.00117	.00108
30	0	0	.00360	.00271	.00117	.00203	.00051
31	0	0	.00638	.00211	.00235	.00069	.00131
32	0	0	.00919	.00257	.00402	.00041	.00032
33	0	0	.01079	.00391	.00433	.00038	.00204
34	0	0	.01375	.00322	.00525	.00163	.00192
35	0	0	.01440	.00159	.00531	.00077	.00221
36	0	0	.00915	.00228	.00356	.00050	.00215
37	0	0	.00618	.00345	.00317	.00098	.00198
38	0	0	.01396	.00413	.00539	.00171	.00179
39	0	0	.01437	.00173	.00611	.00152	.00220
40	0	0	.02045	.00430	.00717	.00149	.00404

upper table 2.18

lower table 2.19

-----MODEL NO. 5 FILE NO. 0-----

-----COSINE ANALYSIS-----

COSINE COEFFICIENTS						
--L--	K=	--0--	--1--	--2--	--3--	--4--
0		-.00212	.00992	.01798	.01876	.00673
1		-.00639	-.00093	-.00179	-.00854	.04839
2		-.12878	-.02142	.00433	-.01098	.02180
3		-.11156	-.00613	-.00638	-.00402	.00706
4		-.08917	.01136	-.00068	-.00539	.01268
5		-.04033	.01019	-.00110	-.00104	.01787
6		-.01235	.00436	-.00805	.00343	.00722
7		-.00631	-.00259	-.01100	.00191	.01050
8		-.00003	-.00683	-.01053	.00330	.00792
9		.00166	-.00705	-.00449	.00121	.01027
10		.00269	.00428	.00054	.00345	.00686
11		.00487	.00504	-.00699	.00524	.00746
12		.00476	.00213	-.00664	.00270	.00384
13		.00725	.00121	-.00290	.00041	.01210
14		-.00468	-.00269	.00098	.00087	.00941
15		-.01360	-.00682	-.00571	.00289	.00436
16		-.02237	-.00132	-.00228	.00304	-.00016
17		-.01221	.00001	-.00271	-.00040	.00516
18		.00061	.00328	-.00150	.00062	.00400
19		.01989	.00379	-.00493	.00175	-.00022
20		.02166	.00039	-.00106	.00338	-.00282
21		.01859	-.00802	.00175	.00075	.00553
22		.00253	-.00873	.00041	-.00108	.00565
23		.00077	-.00435	.00649	-.00233	.00813
24		-.00468	.00219	-.00022	.00209	-.00360
25		-.00052	-.00063	-.00016	.00245	-.00164
26		-.00103	-.00124	.00292	-.00144	.00041
27		.00442	-.00362	.00393	-.00547	.00072
28		.00042	.00008	-.00052	-.00032	-.00475
29		-.00212	.00057	-.00224	.00196	-.00359
30		-.00357	.00109	.00137	-.00136	-.00227
31		-.00252	.00155	-.00053	-.00402	-.00231
32		.00280	.00121	.00010	-.00024	-.00013
33		.00165	-.00108	-.00323	.00180	-.00629
34		-.00107	-.00155	-.00047	-.00023	-.00517
35		.00020	-.00200	-.00143	-.00309	-.00035
36		-.00001	-.00105	-.00121	-.00136	.00195
37		-.00077	-.00150	-.00144	.00068	-.00180
38		.00162	-.00077	-.00496	.00120	-.00737
39		.01217	-.00090	.00076	-.00256	-.00302
40		.02052	-.00266	.00241	-.00180	.00130

-----MODEL NO. 5 FILE NO. 0-----

-----COSINE ANALYSIS-----

SINE COEFFICIENTS						
--L--	K=	--0--	--1--	--2--	--3--	--4--
1		.00312	.02741	-.01170	-.03978	-.00025
2		.19122	.13701	-.03637	-.00237	-.00742
3		.15927	.06231	.00312	-.00411	.00950
4		.01479	.01010	.00389	-.00412	.00510
5		.00360	.00812	.00666	-.00720	.00073
6		.00598	.00987	-.00528	-.00876	.00023
7		.01156	.01347	.00325	-.00470	.00614
8		.00428	.00827	-.00444	-.00135	.00628
9		.00256	-.00833	-.00663	-.00364	.00376
10		-.00390	-.01016	-.01220	-.00075	.00252
11		.00887	-.00032	.00301	.00169	.01131
12		.00977	.00300	-.00358	-.00421	.00759
13		.01466	-.00032	-.01166	-.00479	.00426
14		.02013	-.00211	-.00862	.00034	.00401
15		.01846	-.00063	-.00195	.00278	.00607
16		-.00154	-.00491	-.00528	-.00228	.00703
17		-.01697	-.00693	-.01227	-.00378	.00891
18		-.02046	.00122	-.00592	.00170	.00778
19		-.00973	.00426	-.00228	.00400	.00608
20		.00477	.00235	-.00698	-.00114	.00178
21		.01962	-.00495	-.00971	-.00304	.00706
22		.01736	-.00664	-.00488	-.00112	.00562
23		.01607	-.00158	-.00480	.00267	.00453
24		.00967	.00031	-.00883	.00085	.00273
25		.00245	-.00456	-.01302	-.00265	.00347
26		.00104	-.00491	.00044	-.00120	.00831
27		.00681	-.00209	-.00260	.00368	.00392
28		.00431	.00339	-.00577	.00411	-.00045
29		.00267	-.00208	-.01239	-.00222	-.00376
30		.00056	-.00147	-.00020	-.00398	.00447
31		-.00123	-.00032	-.01036	.00030	-.00209
32		-.00077	.00304	-.00352	.00229	-.00169
33		.00676	.00239	.00295	.00019	.00543
34		.00174	-.00048	.00306	-.00213	.00620
35		.00894	-.00142	.00431	-.00056	.00866
36		.00286	.00329	-.00071	.00244	-.00005
37		.00072	.00029	-.00153	.00238	.00133
38		-.00472	-.00068	.00014	-.00146	.00415
39		-.00346	-.00226	.00060	-.00207	.00643
40		-.00435	.00191	-.00283	-.00013	-.00327

upper table 2.20

lower table 2.21

MODEL NO. 5 FILE NO. 0

COSINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES

L	K=	0	1	2	3	4	5
0		.00212	.00992	.01798	.01876	.00673	.00260
1		.00712	.02742	.01184	.04069	.04839	.03694
2		.23055	.13868	.03653	.01124	.02303	.01806
3		.19445	.06261	.00710	.00575	.01184	.01028
4		.09039	.01520	.00395	.00679	.01367	.00214
5		.04049	.01302	.00675	.00727	.01789	.00145
6		.01372	.01079	.00962	.00940	.00723	.00806
7		.01317	.01372	.01147	.00508	.01216	.00544
8		.00428	.01073	.01143	.00357	.01011	.00264
9		.00305	.01091	.00801	.00383	.01093	.00238
10		.00474	.01102	.01221	.00353	.00731	.00174
11		.01012	.00505	.00761	.00551	.01355	.00290
12		.01087	.00368	.00754	.00500	.00851	.00384
13		.01636	.00125	.01202	.00480	.01283	.00146
14		.02067	.00342	.00868	.00093	.01023	.00108
15		.02292	.00685	.00604	.00401	.00747	.00367
16		.02243	.00508	.00575	.00380	.00703	.00421
17		.02091	.00693	.01257	.00380	.01030	.00325
18		.02047	.00350	.00611	.00181	.00875	.00406
19		.02214	.00570	.00543	.00437	.00609	.00448
20		.02218	.00238	.00705	.00357	.00333	.00495
21		.02703	.00942	.00986	.00313	.00897	.00526
22		.01754	.01096	.00490	.00155	.00787	.00336
23		.01609	.00463	.00808	.00354	.00930	.00371
24		.01074	.00221	.00883	.00225	.00452	.00421
25		.00251	.00460	.01302	.00361	.00384	.00260
26		.00146	.00506	.00295	.00187	.00832	.00366
27		.00812	.00418	.00471	.00659	.00398	.00583
28		.00434	.00339	.00580	.00412	.00477	.00498
29		.00341	.00216	.01259	.00296	.00519	.00384
30		.00361	.00183	.00139	.00421	.00502	.00356
31		.00281	.00158	.01037	.00403	.00312	.00281
32		.00291	.00327	.00352	.00231	.00170	.00468
33		.00696	.00262	.00437	.00181	.00831	.00482
34		.00204	.00163	.00309	.00214	.00808	.00261
35		.00894	.00245	.00454	.00314	.00867	.00440
36		.00286	.00345	.00140	.00279	.00195	.00476
37		.00106	.00153	.00210	.00248	.00224	.00297
38		.00499	.00103	.00496	.00189	.00845	.00255
39		.01266	.00244	.00097	.00329	.00710	.00461
40		.02097	.00327	.00372	.00181	.00352	.00406

MODEL NO. 5 FILE NO. 0

SINE ANALYSIS

COSINE COEFFICIENTS

L	K=	0	1	2	3	4	5
0	0	-.01100	-.00362	.00907	.01039	.01208	
1	0	-.01341	.00139	-.03696	-.01550	.01513	
2	0	-.16738	-.01436	-.06272	-.02358	-.01805	
3	0	.12772	-.00323	-.05577	-.00497	-.00624	
4	0	-.11362	.01212	-.04311	-.00113	-.00960	
5	0	-.05267	.00894	-.02874	.00193	-.00100	
6	0	-.01239	.00073	-.01343	-.00006	.00621	
7	0	-.00396	-.00369	-.01570	-.00092	.00685	
8	0	.00383	-.00783	-.01212	-.00156	.00392	
9	0	.00277	-.00592	-.00956	.00064	.00358	
10	0	.00240	.00122	-.00283	.00202	.00498	
11	0	.00884	.00029	-.00618	.00158	.00935	
12	0	.00869	-.00016	-.00462	.00068	.00429	
13	0	.00897	.00073	-.00725	.00042	.00638	
14	0	-.00738	-.00265	-.00699	.00015	.00479	
15	0	-.01479	-.00778	-.01086	-.00205	.00299	
16	0	-.02708	-.00356	-.00974	-.00150	-.00290	
17	0	-.01471	.00077	-.00966	.00168	.00104	
18	0	.00098	.00303	-.00336	.00401	.00207	
19	0	.02764	.00201	.00562	.00168	.00688	
20	0	.02836	-.00257	.01036	-.00155	.00549	
21	0	.02181	-.00732	.00406	-.00283	.00382	
22	0	.00209	-.00611	-.00335	-.00133	.00007	
23	0	-.00333	-.00183	-.00213	-.00110	-.00072	
24	0	-.00538	.00006	.00039	-.00041	-.00248	
25	0	-.00038	-.00217	.00079	.00021	-.00069	
26	0	-.00269	.00039	.00121	.00108	-.00016	
27	0	.00344	.00120	.00288	-.00022	-.00207	
28	0	.00122	.00005	.00255	-.00086	-.00335	
29	0	-.00116	-.00145	.00033	-.00161	-.00108	
30	0	-.00492	.00167	.00079	-.00052	-.00250	
31	0	-.00269	.00419	.00006	.00032	-.00222	
32	0	.00348	.00130	.00115	.00085	.00009	
33	0	.00454	-.00264	.00351	-.00190	.00028	
34	0	-.00034	-.00185	.00333	-.00181	-.00040	
35	0	.00104	.00067	-.00032	-.00031	.00072	
36	0	.00008	.00077	-.00288	.00209	-.00194	
37	0	-.00016	-.00168	-.00028	-.00026	-.00168	
38	0	.00534	-.00179	.00279	-.00124	-.00112	
39	0	.01551	.00098	.00772	-.00062	.00260	
40	0	.02448	-.00034	.00815	.00124	.00254	

upper table 2.22

lower table 2.23

MODEL NO. 5 FILE NO. 0

SINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1	0	.00943	.05149	-.00583	.00866	.00237	
2	0	.25913	.11776	.05676	.04990	.03686	
3	0	.19831	.05583	.05745	.02338	.03372	
4	0	.01633	.01069	.00516	.00081	.00598	
5	0	.00193	.01051	.00718	-.00282	.00781	
6	0	.01044	.01436	.00059	.00249	.00761	
7	0	.01249	.01483	.00303	.00508	.00684	
8	0	.00663	.00801	-.00552	.00308	.00343	
9	0	.00591	-.00454	-.00543	-.00357	.00392	
10	0	.00063	-.00793	-.01011	-.00307	.00437	
11	0	.00831	-.00110	-.00255	.00136	.00554	
12	0	.01307	.00551	-.00348	.00075	.00576	
13	0	.02360	.00249	-.00368	-.00247	.00852	
14	0	.02907	-.00194	.00034	-.00034	.00877	
15	0	.02355	-.00266	.00227	-.00071	.00810	
16	0	-.00030	-.00262	-.00839	-.00229	.00424	
17	0	-.01669	-.00328	-.01884	-.00294	.00328	
18	0	-.02388	-.00007	-.01632	.00098	.00213	
19	0	-.01164	.00037	-.00810	.00032	.00531	
20	0	.00923	.00252	-.00303	-.00021	.00486	
21	0	.02851	-.00138	-.00277	.00051	.00913	
22	0	.02346	-.00439	-.00012	-.00091	.00625	
23	0	.02200	-.00354	.00042	-.00152	.00653	
24	0	.01605	-.00078	-.00326	-.00137	.00529	
25	0	.00889	-.00200	-.00889	-.00174	.00538	
26	0	.00012	-.00300	-.00476	-.00076	.00422	
27	0	.00948	-.00420	-.00099	.00006	.00555	
28	0	.00852	-.00066	-.00039	-.00062	.00567	
29	0	.00982	-.00038	-.00335	-.00143	.00321	
30	0	.00019	.00150	-.00312	-.00077	.00138	
31	0	.00371	-.00064	-.00484	-.00058	.00251	
32	0	.00113	.00020	-.00045	-.00142	.00277	
33	0	.00646	.00108	.00071	-.00202	.00299	
34	0	.00001	.00127	-.00159	.00027	.00257	
35	0	.00813	-.00017	.00006	.00182	.00387	
36	0	.00408	.00084	.00122	.00061	.00240	
37	0	.00161	-.00173	-.00103	-.00079	.00274	
38	0	-.00659	.00048	-.00477	-.00024	.00005	
39	0	-.00558	.00026	-.00578	.00155	.00029	
40	0	-.00373	.00172	-.00110	.00071	-.00118	

MODEL NO. 5 FILE NO. 0

SINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES

L	K=	0	1	2	3	4	5
0	0	.01100	.00362	.00907	.01039	.01208	
1	0	.01639	.05151	.03742	.01775	.01532	
2	0	.30849	.11863	.08459	.05519	.04104	
3	0	.24145	.05593	.07536	.02391	.03430	
4	0	.11479	.01616	.04342	.00139	.01131	
5	0	.05271	.01380	.02962	.00341	.00787	
6	0	.01621	.01437	.01344	.00249	.00982	
7	0	.01311	.01528	.01599	.00516	.00968	
8	0	.00766	.01127	.01331	.00345	.00521	
9	0	.00653	.00746	.01099	.00362	.00531	
10	0	.00248	.00802	.01049	.00368	.00663	
11	0	.01213	.00114	.00668	.00208	.01086	
12	0	.01569	.00551	.00578	.00101	.00718	
13	0	.02525	.00260	.00813	.00251	.01065	
14	0	.02999	.00328	.00700	.00037	.00999	
15	0	.02780	.00822	.01110	.00217	.00863	
16	0	.02708	.00442	.01285	.00273	.00513	
17	0	.02225	.00337	.02207	.00339	.00344	
18	0	.02390	.00303	.01666	.00413	.00297	
19	0	.02999	.00204	.00986	.00171	.00869	
20	0	.02982	.00360	.01080	.00156	.00733	
21	0	.03589	.00745	.00492	.00287	.00990	
22	0	.02355	.00752	.00335	.00161	.00625	
23	0	.02225	.00399	.00217	.00188	.00657	
24	0	.01693	.00078	.00328	.00143	.00584	
25	0	.00890	.00295	.00892	.00176	.00543	
26	0	.00270	.00302	.00491	.00132	.00422	
27	0	.01009	.00437	.00305	.00023	.00592	
28	0	.00861	.00066	.00258	.00106	.00659	
29	0	.00989	.00150	.00336	.00215	.00339	
30	0	.00492	.00224	.00322	.00093	.00286	
31	0	.00458	.00424	.00484	.00067	.00335	
32	0	.00366	.00132	.00123	.00166	.00277	
33	0	.00790	.00285	.00358	.00278	.00300	
34	0	.00034	.00224	.00369	.00183	.00261	
35	0	.00819	.00069	.00033	.00185	.00394	
36	0	.00408	.00115	.00313	.00218	.00308	
37	0	.00162	.00241	.00106	.00083	.00322	
38	0	.00848	.00185	.00552	.00127	.00112	
39	0	.01649	.00102	.00965	.00167	.00261	
40	0	.02476	.00175	.00823	.00143	.00280	

upper table 2.24

lower table 2.25

MODEL NO. 6 FILE NO. 0

COSINE ANALYSIS

COSINE COEFFICIENTS

L	K=	0	1	2	3	4	5
0		.00568	.05848	-.01427	.00203	.00112	.01545
1		-.00325	.02240	.00853	.00657	-.01405	-.04205
2		-.32901	.13836	.00928	.01798	.00595	.00911
3		.03788	.12106	-.00270	.00104	.01491	.02726
4		.00502	-.05966	.00967	.01219	-.00963	-.02569
5		-.03116	.00753	-.00080	-.02226	.01398	.00154
6		-.02641	-.01056	.00016	-.01051	-.00083	-.01131
7		.06697	-.00477	.00735	.02181	-.00044	-.00072
8		-.04013	.03643	-.00753	-.01346	.01328	.01797
9		-.00649	-.04208	-.00308	.00220	-.01601	-.02201
10		.00235	.01208	.00477	.00386	.00698	.00799
11		-.01551	.03137	.00120	-.00354	.00875	.01187
12		.01817	-.03137	-.00434	.01297	-.01817	-.01963
13		-.01665	.01482	-.00445	-.01056	.00782	.01252
14		-.00483	-.00763	.00810	-.00390	.00159	-.00572
15		.00614	-.01391	-.00046	.00832	-.00326	-.00112
16		-.01754	.01558	-.00674	-.00549	.00758	.00977
17		.00694	-.01466	.00534	.00256	-.00174	-.01355
18		-.00732	.01577	.00004	.00040	.00748	.00819
19		-.00045	.00395	-.00371	-.00346	-.00543	-.00182
20		.00547	-.00946	.00037	.00438	-.00255	-.00319
21		-.00014	.00838	.00605	-.00459	.00947	.00777
22		.00121	-.02041	.00421	.00158	-.00865	-.00699
23		-.00837	.01025	-.00019	-.00074	.00274	.00746
24		.00298	-.00156	-.00419	-.00202	-.00353	-.00211
25		.01277	.00212	.00233	.00209	-.00292	-.00301
26		-.00932	.00969	-.00118	.00004	.00460	.00902
27		.01137	-.01698	-.00310	.00412	-.01078	-.00812
28		-.01330	.01162	.00403	-.00155	.00749	.01135
29		-.00628	-.00103	.00374	-.00342	-.00032	-.00142
30		.01271	-.02420	-.00142	.00022	-.00942	-.00923
31		-.00150	.03008	-.00087	-.00618	.00749	.01675
32		.00025	-.01259	.00537	.00990	-.00856	-.00835
33		-.00694	.00415	.00090	.00034	.00025	.00359
34		-.00817	.00631	-.00437	-.00729	.00336	.00765
35		.01177	-.01937	.00376	.01061	-.00948	-.01272
36		-.01449	.03244	-.00236	-.00426	.00980	.01660
37		.01351	-.01835	.00063	-.00355	-.00608	-.01081
38		-.01246	-.02369	-.00007	-.00098	-.00488	-.01006
39		-.02179	.03225	-.00179	-.01072	.01326	.01799
40		.04916	-.02201	.00341	.01061	-.01411	-.01835

MODEL NO. 6 FILE NO. 0

COSINE ANALYSIS

SINE COEFFICIENTS

L	K=	0	1	2	3	4	5
1		-.00906	.00378	.03771	.01545	.00733	-.00783
2		.39906	-.11285	.02296	-.03613	.03486	.01080
3		-.00759	.03555	.02133	.00372	-.00356	-.01650
4		-.06703	.04333	-.00976	.00104	-.00267	.00024
5		.03998	.07129	-.00384	-.00222	.01068	.01570
6		-.01263	-.06445	.01239	.00552	-.00563	-.02964
7		.00056	.06035	-.00613	-.00635	.01301	.02049
8		.04840	-.00970	-.00586	.00124	-.00421	-.00600
9		-.05458	.00609	.01071	.01436	-.00214	-.00580
10		-.04901	.01589	-.00330	-.01183	.01063	.01790
11		.02665	-.02295	.00434	.00804	-.00624	-.01136
12		-.01018	.01261	.00183	-.00063	.00597	.00578
13		.00437	.02770	-.00853	.00081	-.00116	.00780
14		.01265	-.03283	.00584	.00380	-.00580	-.01621
15		-.00190	.03061	.00104	-.01362	.01263	.01250
16		.00578	-.02105	-.00253	-.00220	-.00902	-.01136
17		.00528	-.00106	.00821	.00714	.00457	.00270
18		-.02335	.00727	.00146	-.00368	.00909	.00960
19		.01838	-.01344	-.00361	.00494	-.01211	-.01069
20		-.01714	.00951	.00040	-.00170	.00440	.00790
21		.01113	.01085	.00535	.00301	.00307	-.00153
22		-.00174	-.01984	.00091	-.00212	-.00127	-.00569
23		-.00328	.00615	.00004	-.00565	.00413	.00581
24		.00417	-.01693	.00087	.00375	-.00712	-.00764
25		.00410	.01655	.00179	-.00391	.00367	.00858
26		-.00944	-.01390	-.00047	-.00718	-.00379	-.00775
27		.00250	.00104	-.00096	.00365	-.00076	.00024
28		.00316	.02045	.00003	-.00216	.00608	.00857
29		.01786	-.01764	.00238	.00202	-.00725	-.01253
30		-.01955	.00646	-.00116	-.00782	.00737	.00922
31		.01138	-.00284	.00162	.00058	-.00183	-.00368
32		.00837	-.01391	-.00108	.00114	-.00467	-.00963
33		-.02313	.02996	-.00430	-.00758	.01272	.01292
34		.02412	-.03073	-.00042	.00307	-.00893	-.02032
35		.00376	.00979	.00438	-.00053	.00754	.00427
36		-.03280	.01809	-.00427	-.00580	.00574	.00776
37		.03011	-.02877	.00296	-.00822	-.01250	-.01868
38		.00268	.02151	.00156	-.00792	.01388	.01453
39		.00169	-.01056	.00355	.00279	-.00588	-.00684
40		.00387	-.01563	.00570	.00136	-.00250	-.00924

upper table 2.26

lower table 2.27

-----MODEL NO. 6 FILE NO. 0 -----

-----COSINE ANALYSIS-----

MEAN IMPERFECTION AMPLITUDES

L--	K=	0--	1--	2--	3--	4--	5--
0		.00568	.05848	.01427	.00203	.00112	.01545
1		.00962	.02272	.03867	.01679	.01585	.04278
2		.51720	.17854	.02477	.04035	.03537	.01413
3		.03863	.12620	.02151	.00582	.01533	.03186
4		.06722	.07373	.01374	.01224	.01000	.02569
5		.05069	.07168	.00392	.02237	.01759	.01578
6		.02928	.06531	.01239	.01187	.00569	.03173
7		.06698	.06054	.00957	.02272	.01301	.02051
8		.06287	.03770	.00955	.01352	.01394	.01894
9		.05497	.04252	.01114	.01453	.01615	.02276
10		.04907	.01996	.00581	.01245	.01271	.01960
11		.03083	.03886	.00451	.00879	.01157	.01643
12		.02083	.03381	.00471	.01298	.01913	.02046
13		.01722	.03141	.00862	.01059	.00791	.01475
14		.01354	.03371	.00999	.00544	.00601	.01719
15		.00643	.03362	.00114	.01596	.01304	.01255
16		.01847	.02619	.00720	.00591	.01179	.01498
17		.00872	.01470	.00979	.00759	.00489	.01381
18		.02447	.01736	.00146	.00370	.01177	.01262
19		.01838	.01401	.00518	.00603	.01327	.01084
20		.01800	.01341	.00055	.00470	.00509	.00852
21		.01113	.01371	.00808	.00549	.00996	.00792
22		.00212	.02846	.00431	.00264	.00874	.00901
23		.00899	.01195	.00020	.00570	.00496	.00945
24		.00512	.01700	.00428	.00426	.00795	.00792
25		.01341	.01668	.00294	.00444	.00469	.00910
26		.01326	.01694	.00127	.00718	.00596	.01190
27		.01165	.01702	.00324	.00550	.01080	.00812
28		.01366	.02352	.00403	.00266	.00965	.01422
29		.01893	.01767	.00444	.00397	.00726	.01261
30		.02332	.02504	.00183	.00782	.01196	.01305
31		.01148	.03021	.00184	.00621	.00771	.01715
32		.00837	.01876	.00547	.00997	.00975	.01255
33		.02415	.03024	.00439	.00759	.01272	.01341
34		.02547	.03137	.00439	.00791	.00954	.02171
35		.01236	.02170	.00578	.01062	.01211	.01342
36		.03586	.03715	.00488	.00720	.01136	.01833
37		.03300	.03413	.00303	.00895	.01390	.02158
38		.01275	.03200	.00156	.00798	.01471	.01768
39		.02186	.03393	.00397	.01108	.01451	.01925
40		.04931	.02699	.00664	.01069	.01433	.02054

-----MODEL NO. 6 FILE NO. 0 -----

-----SINE ANALYSIS-----

COSINE COEFFICIENTS

L--	K=	0--	1--	2--	3--	4--	5--
0	0		.01548	.04219	-.00091	.00193	.02420
1	0		-.00847	.02668	.00587	.05085	-.03150
2	0		-.41994	.10282	-.12537	.04569	-.05799
3	0		.04941	.09114	.01087	.00837	.03845
4	0		-.00032	-.04707	.00248	.01921	-.04274
5	0		-.03836	.01557	-.01382	-.01973	.02598
6	0		-.03354	-.00056	-.01047	-.00099	-.00806
7	0		.07937	-.01382	.02459	.01806	-.00887
8	0		-.04594	.03067	-.01910	-.01964	.03109
9	0		-.00699	-.02904	-.00156	.01203	-.03245
10	0		.00033	.00576	.00079	-.00129	.00768
11	0		-.01964	.02385	-.00581	-.00615	.02018
12	0		.02476	-.02648	.00791	.02052	-.03106
13	0		-.01808	.01330	-.00794	-.01750	.02158
14	0		-.01031	-.00228	.00120	.00143	-.00400
15	0		.00744	-.01496	.00095	.00434	-.01092
16	0		-.01833	.01208	-.01040	-.01098	.01890
17	0		.00527	-.00909	.00283	.01188	-.01395
18	0		-.00954	.01012	-.00561	-.00394	.00859
19	0		.00215	.00525	.00234	-.00039	.00209
20	0		.00622	-.00871	.00111	.00464	-.00963
21	0		-.00315	.00683	.00093	-.00900	.01462
22	0		.00001	-.01532	.00671	.00286	-.01049
23	0		-.01046	.00686	-.00397	-.00439	.00340
24	0		.00602	.00023	.00048	-.00020	-.00179
25	0		.01494	.00174	.00678	.00557	-.00410
26	0		-.01091	.00512	-.00435	-.00661	.01012
27	0		.01588	-.01343	.00533	.00692	-.01594
28	0		-.01873	.00723	-.00469	-.00851	.00969
29	0		-.00953	.00126	.00043	-.00153	-.00064
30	0		.01680	-.01710	.00662	.00296	-.01326
31	0		-.00069	.02300	-.00001	-.01253	.02261
32	0		-.00236	-.01246	.00477	.01319	-.01812
33	0		-.00885	.00203	-.00100	-.00252	.00285
34	0		-.00767	.00635	-.00453	-.01195	.01195
35	0		.01232	-.01691	.00635	.01669	-.02419
36	0		-.01659	.02352	-.00723	-.01154	.02226
37	0		.01697	-.01025	.00787	.00251	-.00583
38	0		-.01631	-.01573	-.00599	.00329	-.01836
39	0		-.02609	.02625	-.01061	-.01775	.02666
40	0		.06003	-.01780	.02304	.02056	-.02139

upper table 2.28

lower table 2.29

MODEL NO. 6 FILE NO. 0
SINE ANALYSIS

SINE COEFFICIENTS							
L	K=	0	1	2	3	4	5
1	0	0	-.03120	-.00137	.01035	.02303	-.01786
2	0	0	-.00219	-.07953	.15908	-.07432	.12780
3	0	0	-.02119	-.02632	.00514	.01319	-.02958
4	0	0	-.07987	.03521	-.03095	.01324	-.01461
5	0	0	-.05318	.05468	.01409	.00183	.03837
6	0	0	-.02408	-.04696	-.00439	.01553	-.04136
7	0	0	.00423	.04653	-.00420	-.00859	.03212
8	0	0	.06396	-.00704	.01673	.00361	.00401
9	0	0	.06274	-.00974	.02429	.01688	-.00802
10	0	0	-.05910	.01297	-.01927	-.02418	.02423
11	0	0	.03040	-.01858	.01022	.01337	-.02048
12	0	0	-.01398	.00922	-.00522	-.00204	.00533
13	0	0	.01077	.01957	.00129	-.00055	.01467
14	0	0	.01205	-.02373	.00566	.01067	-.02242
15	0	0	-.00267	.02833	-.00225	-.01406	.02492
16	0	0	.00901	-.01310	.00471	.00303	-.00863
17	0	0	.00175	-.00482	.00217	.00356	-.00445
18	0	0	-.02998	.00491	-.00974	-.01045	.01357
19	0	0	.02513	-.01018	.00867	.01105	-.01487
20	0	0	-.02171	.00633	-.00706	-.00646	.00593
21	0	0	.01106	.00790	.00518	.00696	.00131
22	0	0	-.00284	-.01266	-.00062	-.00181	-.00538
23	0	0	-.00402	.00622	-.00170	-.00820	.00842
24	0	0	.00479	-.01334	.00358	.00618	-.01296
25	0	0	.00476	.01320	.00335	-.00639	.01210
26	0	0	-.01157	-.00541	-.00264	-.00135	-.00707
27	0	0	.00337	-.00080	-.00030	.00347	-.00360
28	0	0	.00421	.01511	.00070	-.00503	.01430
29	0	0	.02112	-.01159	.00904	.00955	-.01332
30	0	0	-.02402	.00643	-.00967	-.01349	.01088
31	0	0	.01376	-.00178	.00641	.00269	.00043
32	0	0	.01056	-.00899	.00188	.00694	-.01252
33	0	0	-.02685	.02418	-.01344	-.01126	.02297
34	0	0	.02988	-.02104	.00853	.01324	-.02016
35	0	0	.00240	.00694	.00137	-.00216	.01046
36	0	0	-.03910	.01542	-.01569	-.00719	.00811
37	0	0	.03590	-.02212	.01391	.01704	-.02555
38	0	0	.00305	.01674	.00021	-.01598	.02994
39	0	0	.00065	-.00793	.00486	.00625	-.00850
40	0	0	.00138	-.01053	.00259	.00649	-.01435

MODEL NO. 6 FILE NO. 0
SINE ANALYSIS

MEAN IMPERFECTION AMPLITUDES							
L	K=	0	1	2	3	4	5
0	0	0	.01548	.04219	.00091	.00193	.02420
1	0	0	.03232	.02671	.01190	.05582	.03621
2	0	0	.04699	.12999	.02054	.08724	.14034
3	0	0	.05376	.09486	.01207	.01552	.04001
4	0	0	.07987	.05878	.03105	.02333	.04517
5	0	0	.06558	.05686	.01974	.01982	.04633
6	0	0	.04129	.04696	.01135	.01556	.04214
7	0	0	.07948	.04854	.02494	.02004	.03332
8	0	0	.07874	.03146	.02539	.01997	.03135
9	0	0	.06313	.03063	.02434	.02073	.03343
10	0	0	.05910	.01419	.01929	.02422	.02542
11	0	0	.03619	.03023	.01176	.01472	.02875
12	0	0	.02843	.02804	.00948	.02062	.03151
13	0	0	.02103	.02367	.00804	.01751	.02610
14	0	0	.01586	.02384	.00578	.01077	.02277
15	0	0	.00790	.03203	.00244	.01471	.02721
16	0	0	.02043	.01782	.01141	.01139	.02078
17	0	0	.00555	.01029	.00356	.01240	.01464
18	0	0	.03146	.01125	.01124	.01117	.01606
19	0	0	.02522	.01146	.00898	.01105	.01501
20	0	0	.02259	.01077	.00714	.00795	.01131
21	0	0	.01150	.01044	.00526	.01138	.01468
22	0	0	.00284	.02053	.00674	.00338	.01179
23	0	0	.00769	.00926	.00432	.00930	.00908
24	0	0	.01568	.01334	.00361	.00619	.01308
25	0	0	.01590	.01332	.00756	.00848	.01277
26	0	0	.01624	.00744	.00509	.00675	.01234
27	0	0	.01920	.01345	.00533	.00774	.01634
28	0	0	.02317	.01675	.00474	.00989	.01727
29	0	0	.02931	.01166	.00905	.00967	.01334
30	0	0	.01082	.01826	.01172	.01381	.01715
31	0	0	.01378	.02307	.00641	.01282	.02261
32	0	0	.01082	.01536	.00513	.01491	.02203
33	0	0	.02827	.02426	.01348	.01153	.02314
34	0	0	.03085	.02197	.00965	.01783	.02344
35	0	0	.01256	.01828	.00649	.01683	.02635
36	0	0	.04247	.02812	.01728	.01360	.02369
37	0	0	.03971	.02438	.01598	.01723	.02621
38	0	0	.01660	.02297	.00599	.01631	.03512
39	0	0	.02610	.02742	.01167	.01882	.02798
40	0	0	.06004	.02068	.02319	.02156	.02576

upper table 2.30

lower table 2.31

MODEL NO	1	2	3	4	5	6
Total area (mm ²)	1070	925	1295	1070	925	1295
Length (mm)	65	65	180	65	65	180
Max outward imperfection (·80)	—	·56	·94	·41	·74	·80
Max inward imperfection (—80)	—	—40	—67	—52	—65	—65
Out of roundness (0·02 R)	—	0·005	0·011	0·004	0·009	0·013
Out of straightness (·39)	—	·12	·19	·17	·23	·15
Deviation from circle E	—	·44 (·44)	·46 (·59)	·29 (·44)	·52 (·44)	·58 (·58)
Load eccentricity	0	0	0	·25 R	·4 R	·4 R
Calc max stress at 1st yield	0·81 σ_y	0·92 σ_y	0·88 σ_y	0·85 σ_y	0·87 σ_y	0·89 σ_y
Max average stress	0·96 σ_y	0·96 σ_y	0·95 σ_y	0·78 σ_y	0·54 σ_y	0·51 σ_y
Max stress at crit section $\sigma_{max} = P(1/A + eR/I)$				1·07 σ_y	0·94 σ_y	1·02 σ_y
Posn of 1st yield from strain gauges	PLATE OUTER SURFACE		STIFFENER OUTSTAND MID HT *	STIFFENER OUTSTAND MID HT	STIFFENER OUTSTAND MID HT *	STIFFENER OUTSTAND MID HT

$$\sigma_y = 348 \text{ N/mm}^2$$

* = 1st yield suspected at external base of panel

table 2.32

Appendix 2A1

A total of twenty tensile specimens were tested to determine the material properties of the steel used for the models. These specimens were cut in the longitudinal and transverse directions from the sheets used in the model manufacture. Ten of the specimens were subjected to the same heat cycle (stress relieving) as the models and the remainder were untreated. All specimens were machine finished to eliminate stresses due to flame cutting. Specimen dimensions are given in figure 2A1.1. Testing was carried out in an Amsler 35 ton testing machine, at an approximately constant rate of elongation. Each loading cycle took about 5 minutes. Specimens were loaded to the limit of travel of the transducers, giving a total extension of 10mm in a gauge length of 100mm. Figure 2A1.2 shows typical load vs. extension for ex-stock and heat treated specimens. Table 2A1.1 summarises the results. Two load levels have been noted,

1. at 1st yield or 0.2% permanent elongation,
2. at limit of transducer travel.

All the specimens from each group were found to behave very similarly.

It is evident from figure 2A1.2 that specimens tested ex-stock do not show the very marked 1st yield point shown by heat treated specimens. On the other hand the average ultimate stress was found to be very similar for the two groups. This can be attributed to the presence of very considerable in-plane residual stresses due to rolling of the material to the 0.84mm thickness. These stresses are much smaller in normal construction material.

It is suggested that it could be difficult to model normal plate material behaviour without the stress relieving process adopted for these models.

Ex-stock specimens, tested parallel to the direction of rolling, showed a proof stress 10-15 N/mm² higher than those tested transverse to the direction of rolling. This difference is small, however, by comparison with the difference between the treated and ex stock specimens. After heat treatment there is no apparent difference between first yield stress for longitudinal and transverse specimens.

Tests were carried out in compression on a number of made-up specimens. Each specimen consisted of a 75mm long laminated section of 4 x 0.85mm thick plates glued together with araldite. The thickness of araldite between each layer was less than $25 \times 10^{-3}\text{mm}$ ($1 \times 10^{-3}\text{ins}$). The plate material had been stress relieved and loaded surfaces were machine finished after gluing.

Loading response was linear and stable up to a nominal stress of approx. 338 N/mm^2 . At this point, however, specimens became laterally unstable and the glue bond failed, making further loading impossible.

It can be concluded that the first yield value obtained in tension is also applicable in compression. It has, however, been accepted that for this study a compression yield value 10 N/mm^2 greater than tension yield value would be assumed. Thus throughout the study

$$\sigma_{yc} = \sigma_{yt} + 10 = 348 \text{ N/mm}^2$$

$$\epsilon_{yc} = 1730\mu\epsilon$$

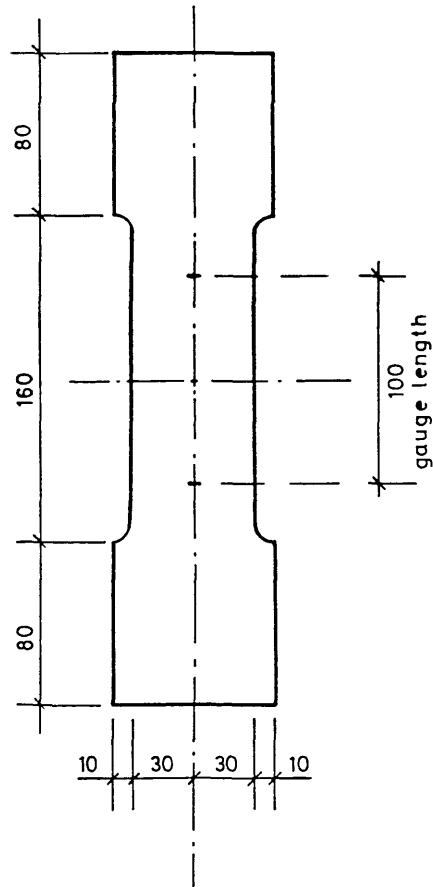


Figure 2A1.1 Tensile test coupon

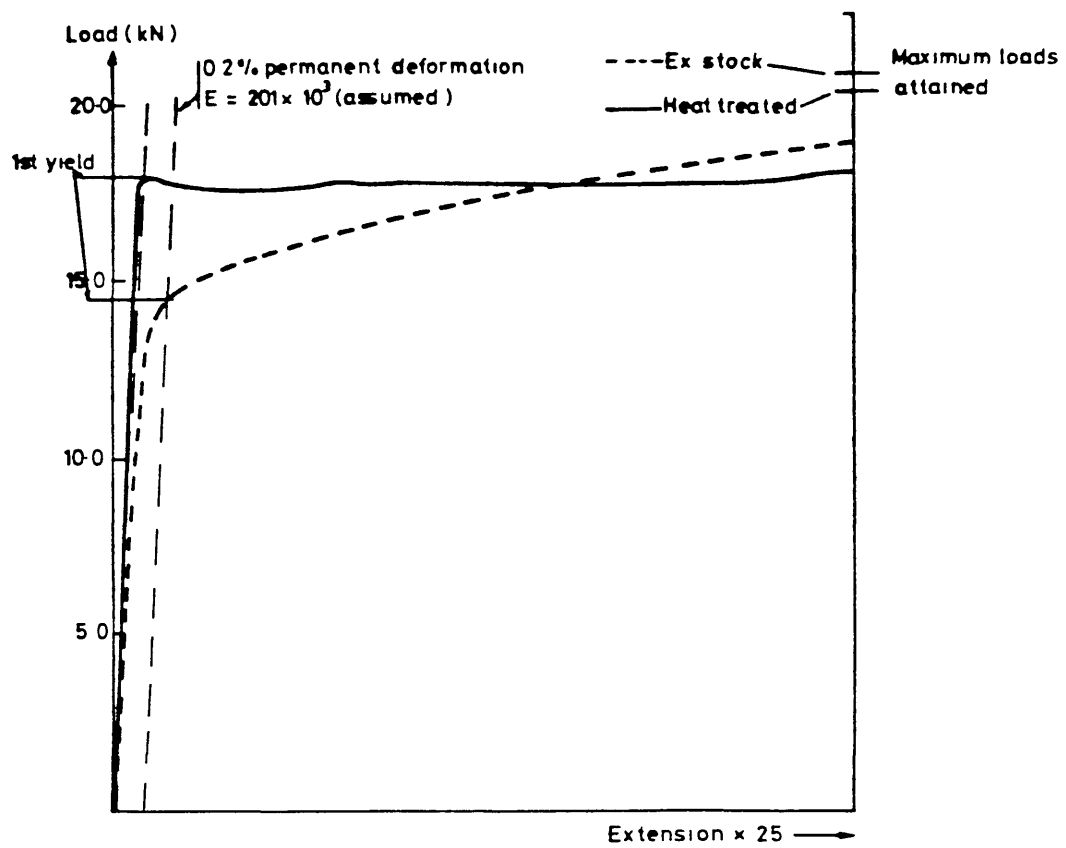


Figure 2A1.2

TABLE 2A1.1 SUMMARY OF TENSILE TEST DATA			
	1st yield * (N/mm ²)	Ult yield (N/mm ²)	Young's modulus (N/mm ²)
EX STOCK	271	401	201×10^3 (assumed)
HEAT TREATED	338	396	201×10^3
Strain at 1st yield (heat treated) $1670 \mu\epsilon$			

CHAPTER 3.

The Analysis of Surface Deflections Recorded During Experiments.

3.1 Introduction

3.1.1 Analysis of experimental data

Lateral surface displacements formed an important component of the experimental data recorded during the loading of the six cylinders in chapter 2. A total of 800 or 1440 readings were taken at each load step and it is impractical to present these data in any manner other than in an abbreviated form.

Longitudinal profiles and a single circumferential profile were presented for the unloaded models in chapter 2. The most systematic method of presenting the recorded data for uniaxially loaded models, however, is by means of the coefficients of the fourier analysis of the surface. The total value of the deflection coefficient of a fourier mode is more useful than the individual sine or cosine components. Thus the coefficient $\sqrt{A_{mn}^2 + B_{mn}^2}$ or $\sqrt{C_{mn}^2 + D_{mn}^2}$ has been used in the graphs in chapter 2.

A method of presentation of fourier coefficients for models under load is by means of the ratio of the coefficient value at any load to the initial coefficient value (at zero load). The results for model nos. 2 and 3 are presented in this manner in this chapter. Some underlying trends in behaviour will be clear when results are thus presented.

Non-destructive methods of testing have been discussed in §1.6.3. It has been noted that buckling loads have been predicted using the Southwell plot technique. Fourier coefficients for model nos. 2, 3, 5, 6 will be analysed in this manner in this chapter and the results will be compared with theoretically computed buckling loads.

3.1.2 Advantages of the fourier analysis method

The fourier analysis technique is particularly effective in eliminating "noise" from readings and in pointing out the overall trends in the deflection pattern which are repeated over the entire surface. (It is not a good technique, however, for analysing localised patterns which are not repeated over the whole cylinder surface.)

The deflection of adjacent panels can be expected to be fairly close to one another, given the consistently uniform fabrication technique. Hence, the fourier coefficients corresponding to circumferential wavelengths equal to one panel in width and length will express very accurately the behaviour of typical repeated panel units.

An idealised orthotropic buckling analysis for a cylinder with simply supported ends considers buckling modes based on sinusoidal displacement functions. The individual imperfections from a fourier analysis of initial imperfections can be used in the buckling analysis as the fourier analysis technique highlights the imperfection modes of an experimental model which are relevant to a theoretical buckling analysis. The major initial imperfections are not necessarily relevant to the critical behaviour of a model. The models of this series had large initial imperfections in modes $(m,n) = (1,2)-(1,4)$. These modes are of little importance for axial loading (although the terms are important for pressure loading). This can be seen quite clearly in the coefficient values computed in chapter 2 for models under load. The prebuckling panel deflections can be clearly seen when the initial imperfection values are subtracted from the total values recorded under load.

3.2 Analysis of surface imperfection data under load

3.2.1 Coefficient values from fourier analysis under load relative to initial values.

Figures 3.1-5 show the growth of fourier coefficients of model no. 2 under load, expressed relative to the initial coefficients. The effects to be seen most strongly are the growth of the $(m,n) = (1,10)$, $(2,20)$, $(5,10)$ and $(5,30)$ terms. The prebuckling values associated with all these terms are small. In the postbuckling regime, however, their relative values are very much larger than any other terms. In contrast the $(m,n) = (1,20)$ term grows moderately large in the prebuckling regime but in the postbuckling regime the values are almost unchanged.

The large terms correspond to the physical buckling of the panels into the panel-only buckling mode. The dominant buckling shape is a single

halfwave along the height and alternate inward and outward buckles around the circumference. The physical buckle is not a pure single sinusoidal wave in the $(m,n) = (1,10)$ mode. Lesser harmonics in $m = 5$ and to a lesser extent $m = 3$, combine with the $m = 1$ wave to describe the major buckling shape along the height of the model.

The end conditions of the cylinder have a strong effect on the local deflected shape of the panels, § chapter 6. The ends tend to cause a localised barrelling failure, and one end will generally fail first. The stringer stiffeners tend to restrain the axisymmetric barrelling effect and the result is a failure which tends to be symmetric with respect to the stringer stiffeners. The barrelling effect is seen in the prebuckling regime for model no. 2 in the large growth of the terms $(m, n) = (1,20)$. On buckling, however, the failure is at one end and only the term $(m,n) = (2,20)$ grows very large by comparison with the prebuckled value.

Figure 2.59 shows that the failure mode of model no. 3 is a single short wavelength buckle at about midheight of each panel. The length of the buckle is short relative to the surface measurement grid of the deflection scanning system and, coupled with the fact that the buckle is not repeated in the axial direction, the buckling shape does not show up very clearly in the fourier analysis. The fourier analysis data do not indicate very clear trends in figures 3.6-10. There is not so sharp a difference between prebuckling and postbuckling terms as there was for model no. 2. There is some indication of the panel failure in the terms $(m,n) = (3,20)$ and $(5,20)$ but the indication is not very clear. The most noticeable buckling term is $(m,n) = (2,8)$. This term is from an overall mode of buckling which did not grow numerically large because of its small initial value. Hence in figures 2.60 - 64 the term is not dominant. It can be inferred from figure 3.7, however, that there is a latent overall mode of buckling for model no. 3 corresponding with the fourier mode $(m,n) = (2,8)$. This particular model of buckling may have a localised low overall buckling value because of the discrete nature of the stiffeners.

3.3 Experimental imperfection studies

3.3.1 Predicted growth of surface imperfections

Let the initial surface deflection of a cylinder be given by

$$w_0(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{l} \left(C_{0mn} \cos \frac{ny}{R} + D_{0mn} \sin \frac{ny}{R} \right)$$

The amplitude of a typical imperfection, w_{0mn} , is given by

$$w_{0mn} = \sqrt{C_{0mn}^2 + D_{0mn}^2}$$

If the separate imperfection modes are orthogonal, then any initial imperfection amplitude will grow under load and its value will be given by

$$w_{mn} = w_{0mn} / (1 - \sigma / \sigma_{mn}) \text{ ----- (1)}$$

w_{mn} expresses the absolute magnitude of the imperfection at a load σ .
 σ_{mn} is the orthotropic buckling stress of the imperfection.

Equation (1) was popularised by Southwell (188) for axially loaded columns in the form

$$\frac{w_{1,mn}}{\sigma} \sigma_{mn} - w_{0mn} = w_{1,mn} \text{ ----- (2)}$$

where $w_{1,mn} - w_{0mn} = w_{0mn}$ is the change of coefficient value under stress σ .

Equation (1) may alternatively be written in the form

$$\sigma_{mn} = \frac{w_{mn}}{w_{mn} - w_{0mn}} \sigma \text{ ----- (3)}$$

3.3.2 First method of calculating buckling loads

The value of σ_{mn} obtained from equation (3) has been calculated for the vertical mode $m = 1$ for model nos. 2, 3, 5, 6. The equation was solved at various stress levels. The results are presented in figures 3.11-14. The orthotropic buckling stress curves of Geier and of Miller (145, 143) are presented for comparison. The comparison is

directly relevant for model nos. 2 and 3 but for model nos. 5 and 6, which were eccentrically loaded, the theoretical curves are not directly relevant.

A number of points are illustrated in figure 3.11 for model no. 2.

1. Buckling stresses approximately equal to yield (350 N/mm^2) are predicted for almost all imperfection modes.
2. A wide scatter exists between the buckling loads predicted from the various readings for a particular mode.
3. Predicted buckling stresses are generally less than 0.3 times the theoretical value.
4. The experimental results do not follow the trend of the theoretical results.

The trend of experimental results for model no. 3, figure 3.12 appears to be partly in line with the theoretical predictions. However, figure 3.12 is divided into two regions by the predicted values. The first region spans modes $n = 4 - 18$, the second spans modes $n = 18 - 34$. High buckling stresses are predicted around $n = 4, 18, 34$. The predictions lie close to yield stress around $n = 11$ and 26 . Although buckling stresses in the first region roughly coincide with the theoretical curves of Miller and Geier, no theoretical reason exists for the second region. It is noted that $n = 34$ is an approximate harmonic of $n = 18$, suggesting that the division into two regions may be the result of high experimental buckling stresses associated with the mode $n = 18$. The experimental values correspond well with theoretical values for modes $n = 4-6$. The scatter of results, however, and the generally low values predicted for the other modes - in particular those thought critical from the theoretical analyses, $n = 7 - 11$, make it only possible to suggest that a general trend is observable.

As noted above a comparison between the theoretical buckling values, as predicted by references 145 and 143, and equation (3) is not directly relevant. Nevertheless for model no. 5, figure 3.13, the trend of the experimentally predicted buckling values follows that of the theoretically predicted curves for uniform loading. The experimental values are, however, low for the critical modes $n = 7 -$

11.

The experimental scatter for model no. 6 is too large to make any conclusions possible.

3.3.3 Second method of calculating buckling loads

The value of σ_m was calculated from eq. (2) by means of a best fit straight line through the four prebuckling data points available for each mode shape. The results are shown in figures 3.15 and 16 for model nos. 2 and 3. Five vertical modes are presented for each model.

Curve fitting through the available data points was difficult as expected from the scatter of results observed in figures 3.11 and 12. The critical values obtained show the erratic nature of the result. In some cases negative (i.e. tensile) buckling stresses were predicted; these values are not plotted in figures 3.15 and 16. All the critical values are scattered around a value of 300 N/mm², although computed from the prefailure imperfections.

The predicted buckling values of all vertical modes for $n = 10$, figure 3.15, are close to 350 N/mm². These modes correspond to panel buckling, which had occurred before the highest load case considered. (The buckles were not visible to the eye during the experiment). In contrast, the buckling stresses for $n \approx 20$ show the same amount of scatter as many of the other circumferential modes. It is noted, however, from figures 2.45 - 47, that the most dominant elastic mode is $(m,n) - 1,20$. It is concluded that the coefficient values for load case 3 strongly weighted the results on the best fit lines and caused the convergence of predicted buckling stresses values for $n = 10$.

3.3.4 Assessment of Results

The theoretical maximum deflections of model no. 2 at 300 N/mm² are 0.10 mm for the panel and 0.03 mm for the stiffener, §5.3.2. The maximum theoretical deflection for model no. 3 is 0.08 mm. In Chapter 2 the tolerance of transducer readings was given as ± 0.02 mm.

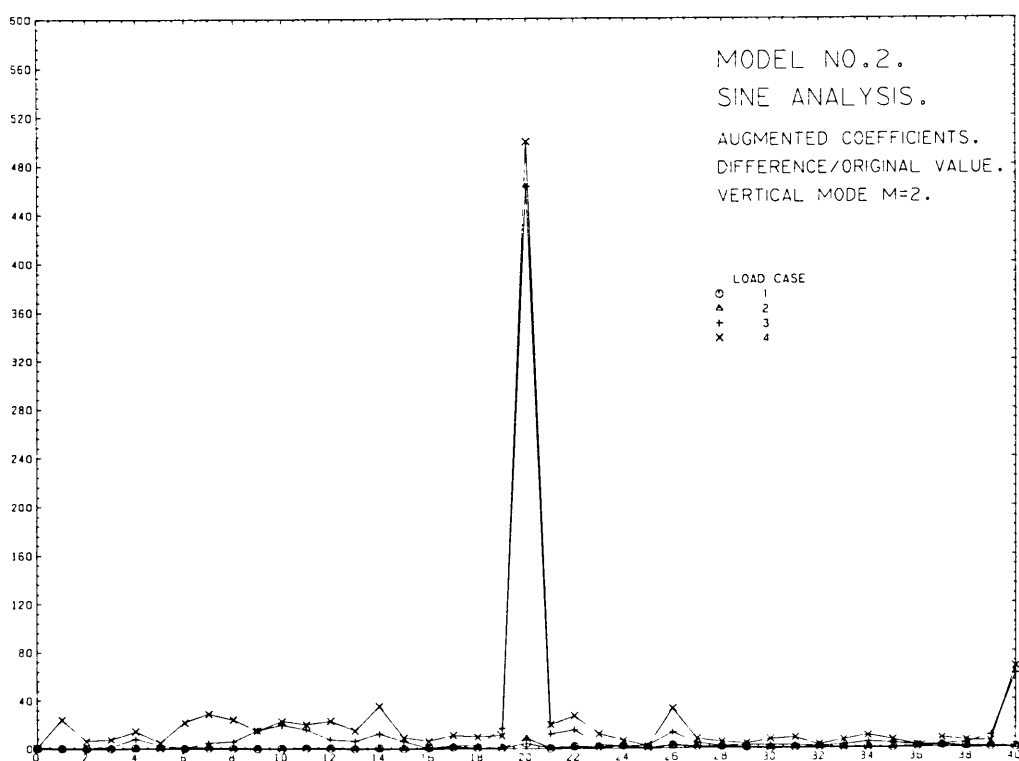
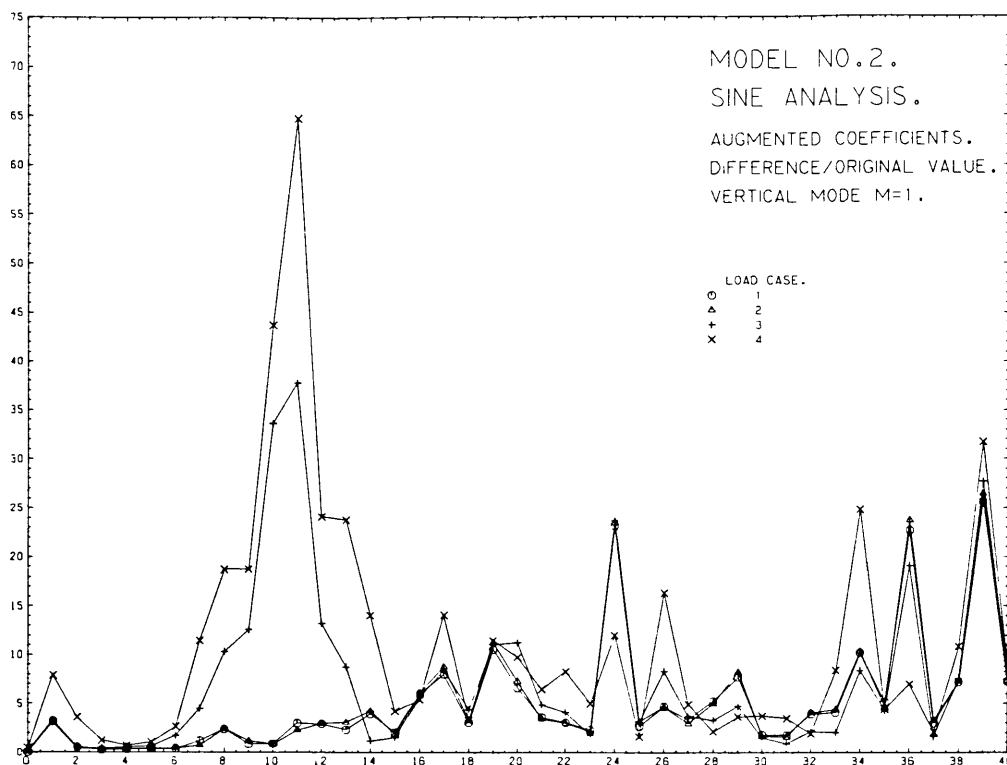
The displacement transducers were chosen to provide not only readings of the elastic displacements, but also were required to measure the deflections at failure. Deflections of ± 2 mm (minimum) were anticipated at failure. In addition, the design of the measuring frame necessitated its rotation through 90° . The model tolerance on out-of-roundness was $0.01 \times$ radius. Thus the average distance between the unloaded cylinder and a transducer could be expected to vary by 1 mm between extreme measuring positions.

A minimum range of transducer movement of $\pm 2.5 - 3$ mm was expected before the experiments were carried out. The transducers chosen had a range of ± 5 mm. Measurements accurate to 1% of the maximum deflection are required in order to carry out an effective imperfection survey under load. An accuracy of 0.001 mm was required for model nos. 2 and 3. It is clear at this stage of the study that this measurement accuracy was not consistent with either the design of measuring rig or the total expected range of measurement. An accuracy of 1 part in 10^4 was required for detailed measurement; this was not possible within the budget of the experiment.

It must be concluded that an increased number of readings, obtained at both a closer load and spacing interval, could do no more than partially smooth a very coarse measurement system.

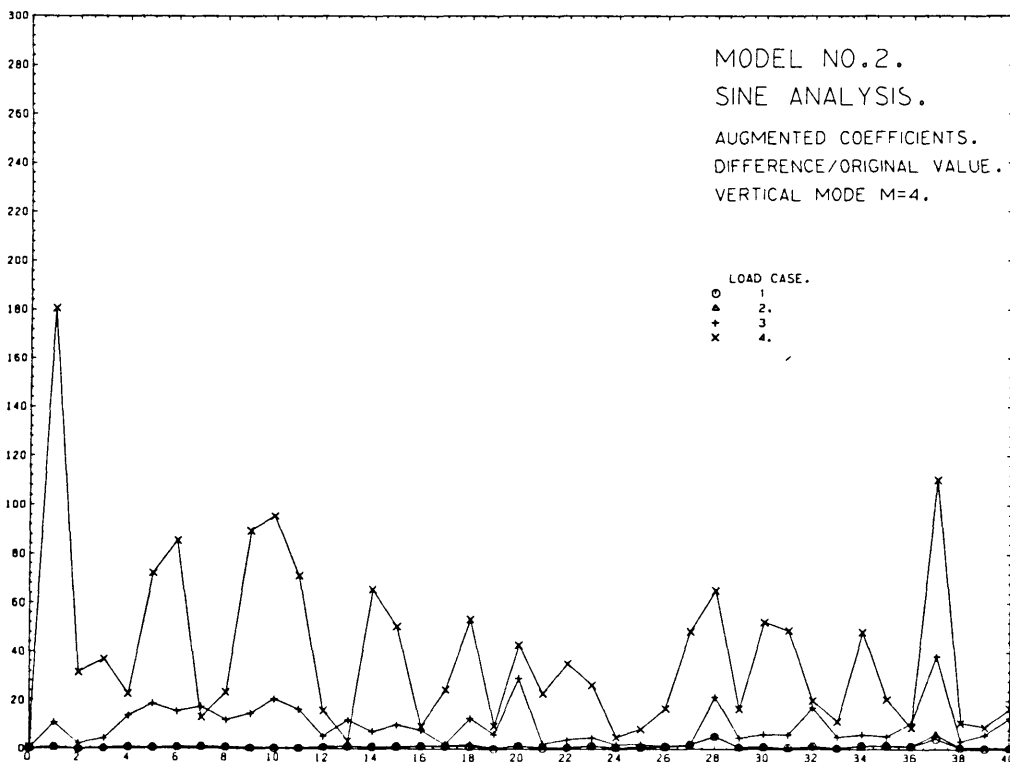
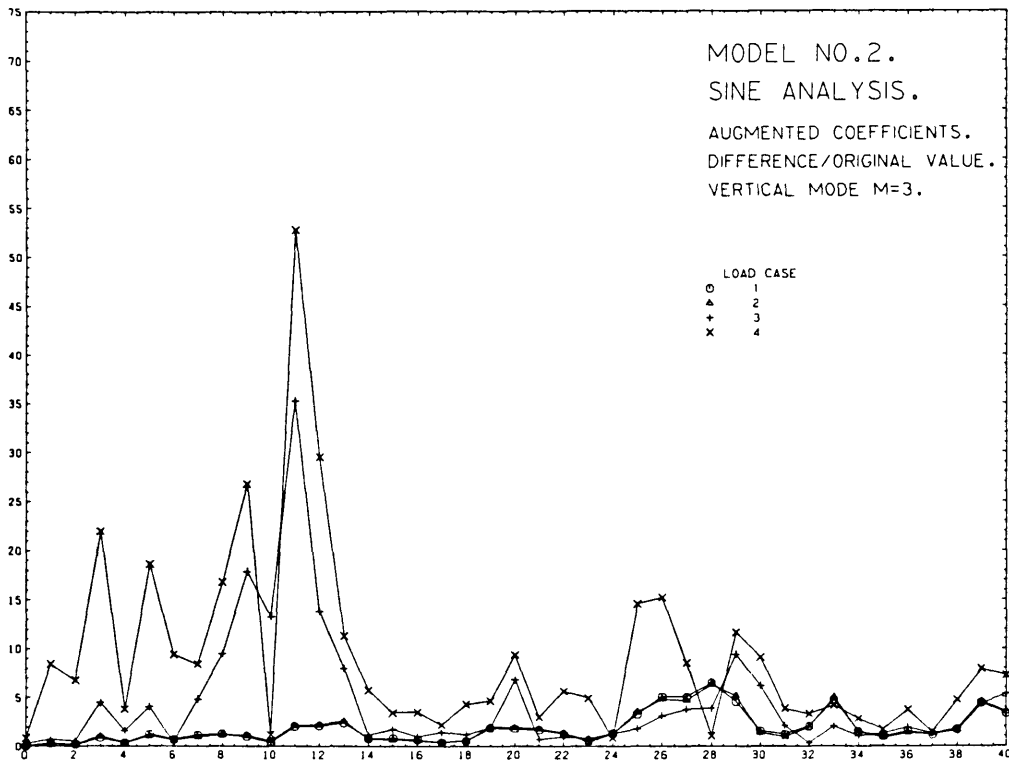
The following conclusions may be drawn from the study:

1. A large scatter of buckling stresses was predicted.
2. Experimentally predicted buckling stresses are much lower than theoretical values.
3. Measurements were not accurate enough to study the behaviour of models in the detail required by the method.
4. No quantitative results could be derived from the use of the Southwell plot method.



upper figure 3.1: Relative growth of fourier coefficients, $m = 1$.

lower figure 3.2: Relative growth of fourier coefficients, $m = 2$.



upper figure 3.3: Relative growth of fourier coefficients, $m = 3$.

lower figure 3.4: Relative growth of fourier coefficients, $m = 4$.

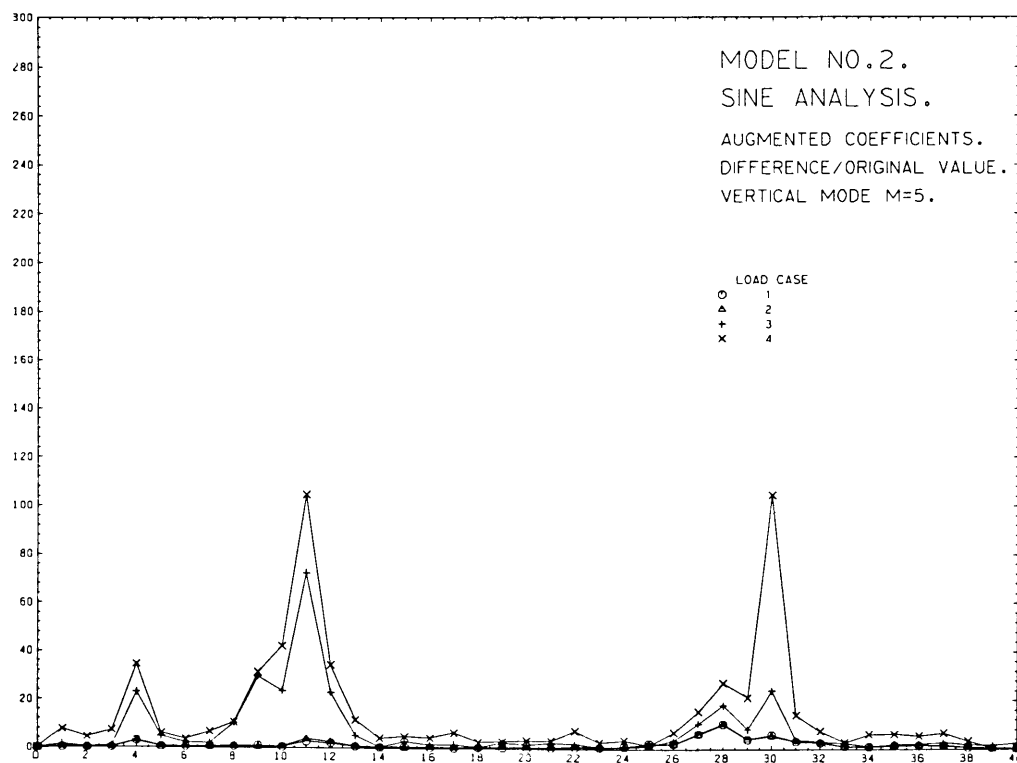
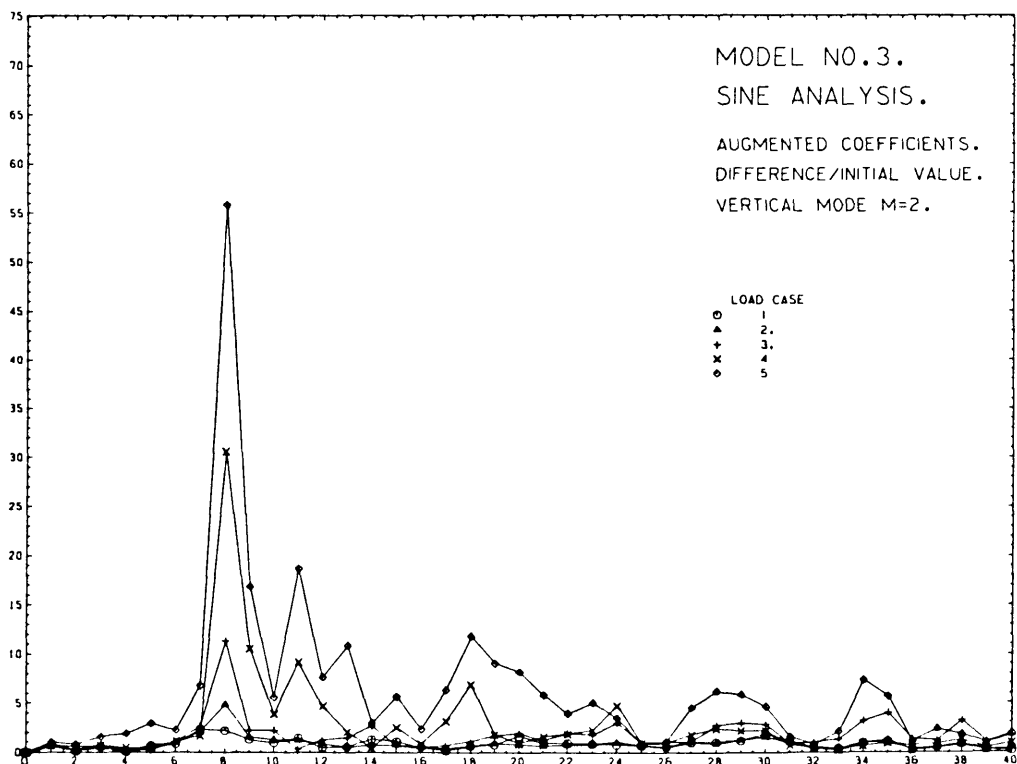
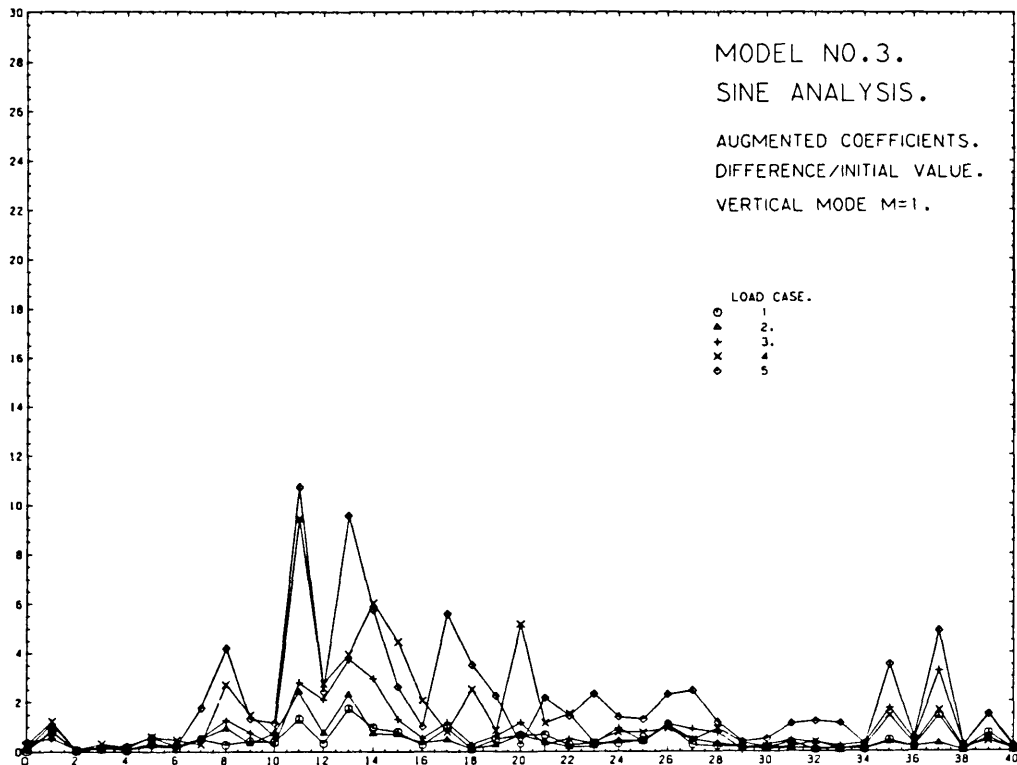
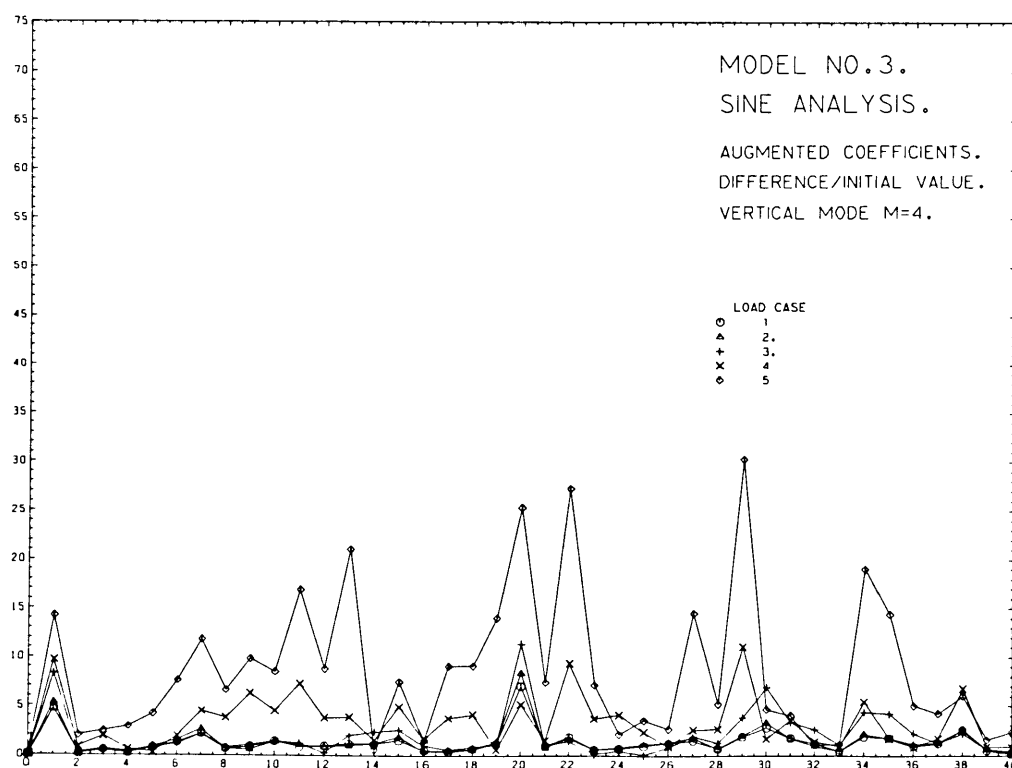
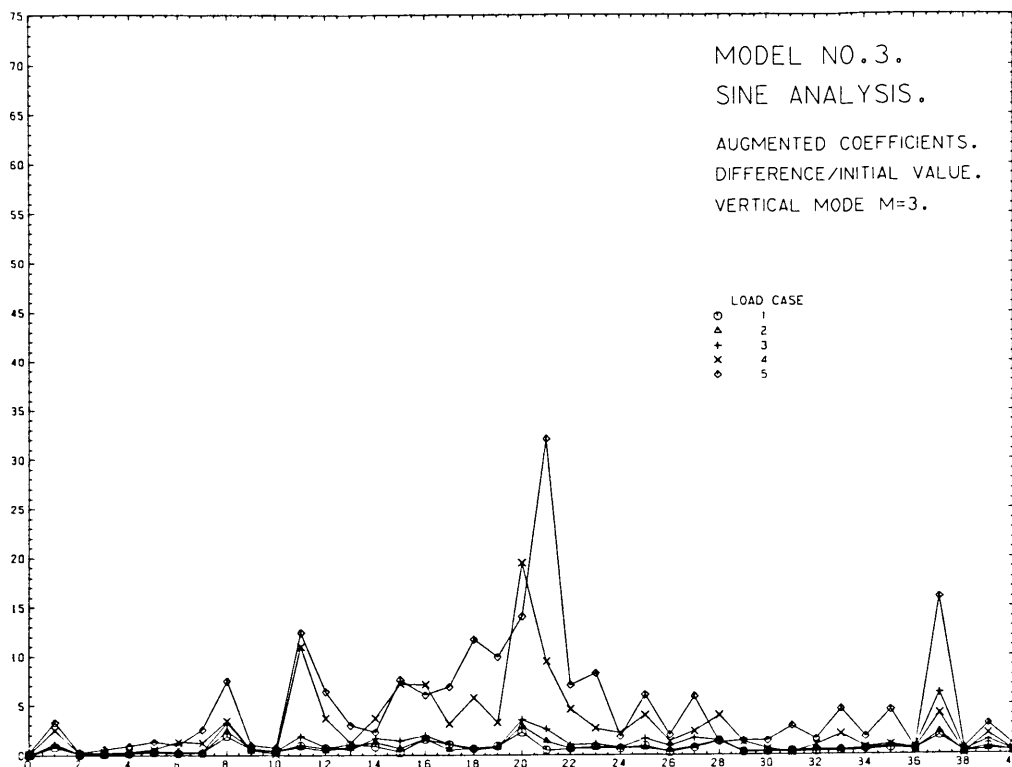


figure 3.5: Relative growth of fourier coefficients, $m = 5$.



upper figure 3.6: Relative growth of fourier coefficients, $m = 1$.

lower figure 3.7: Relative growth of fourier coefficients, $m = 2$.



upper figure 3.8: Relative growth of fourier coefficients, $m = 3$.

lower figure 3.9: Relative growth of fourier coefficients, $m = 4$.

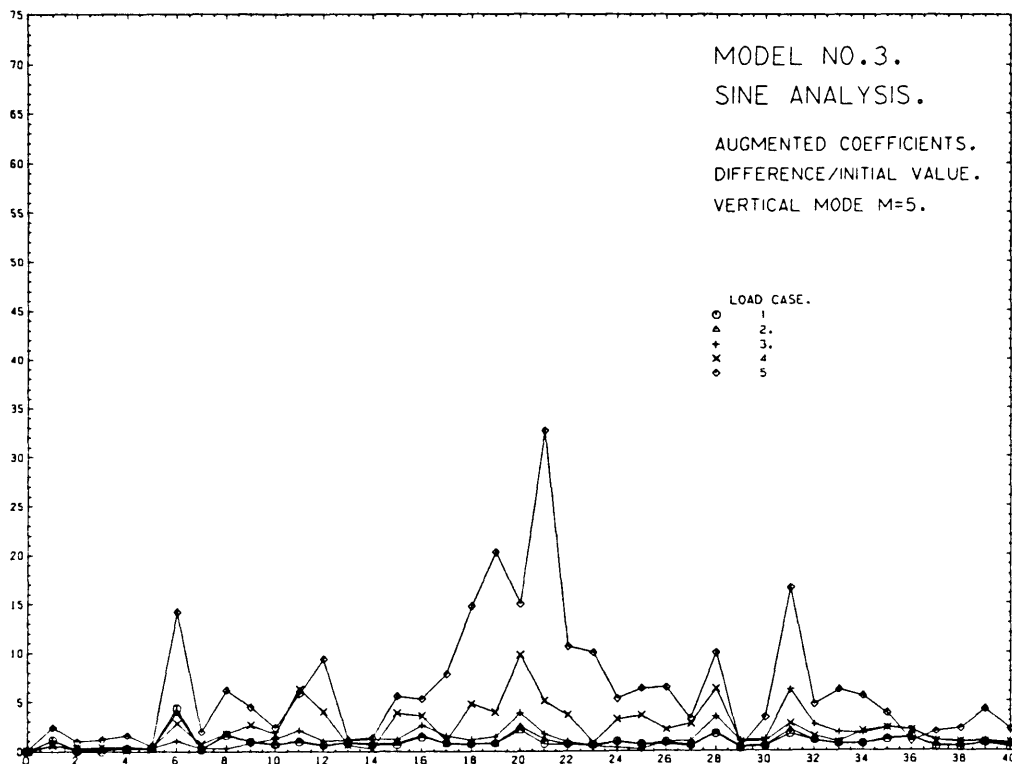
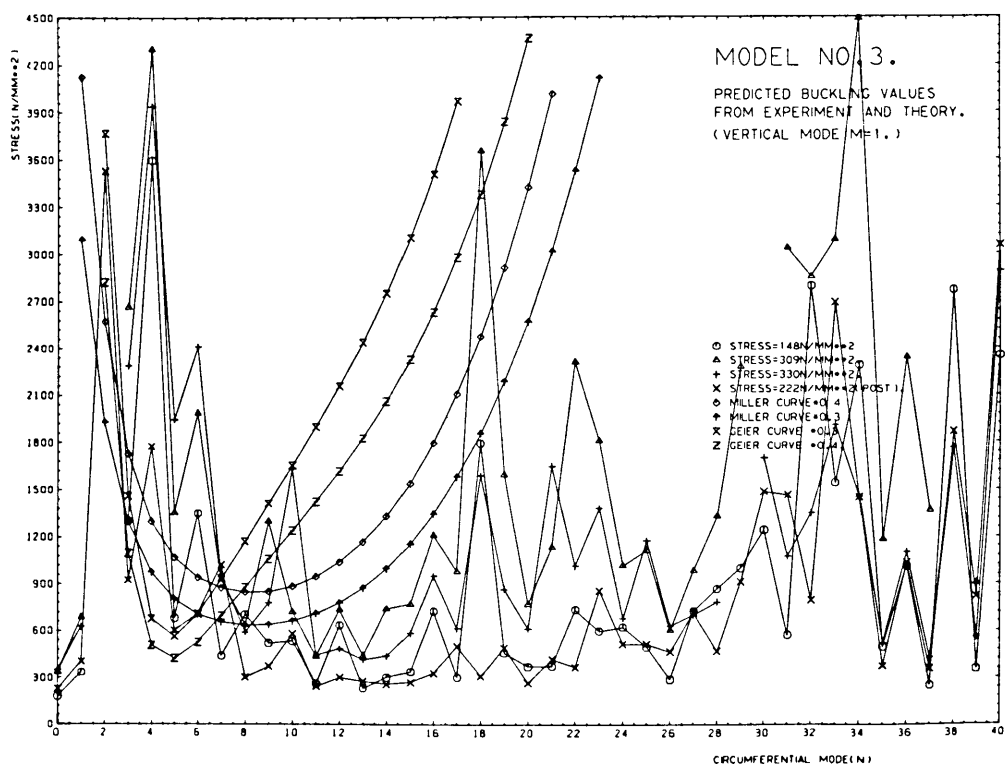
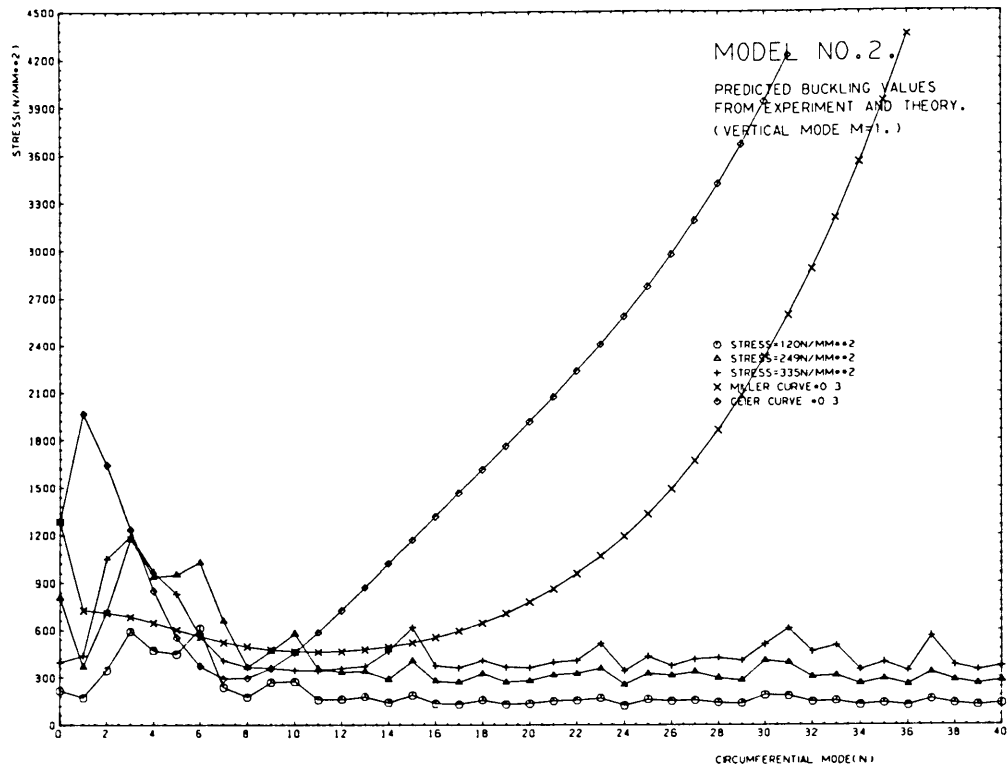
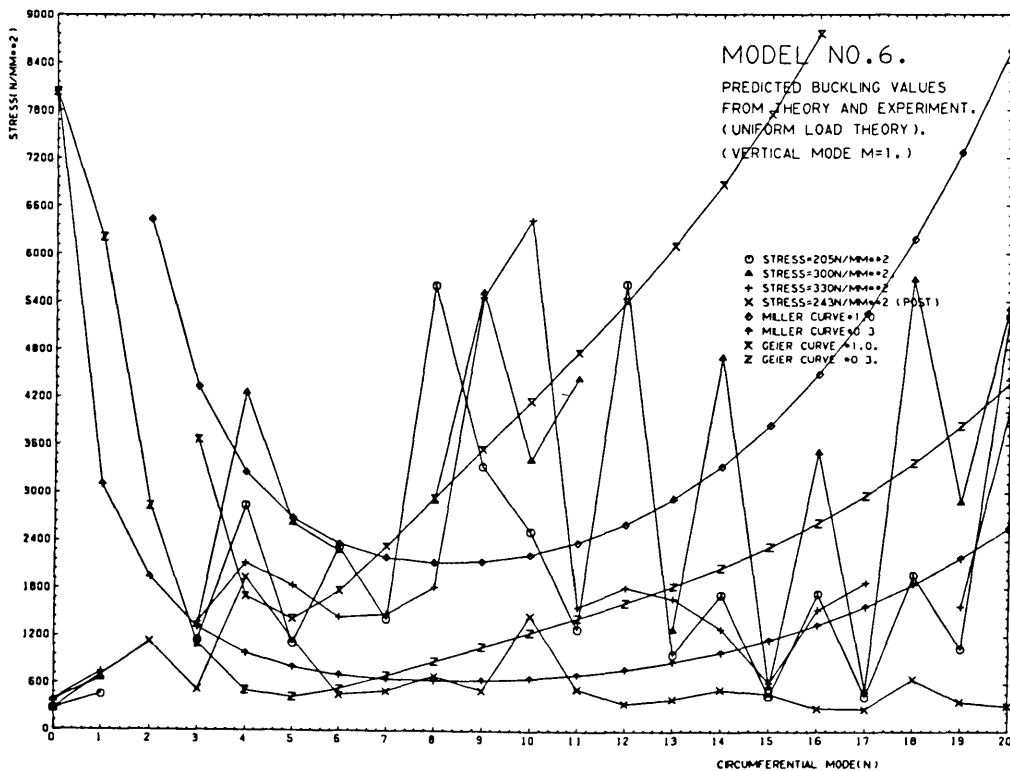
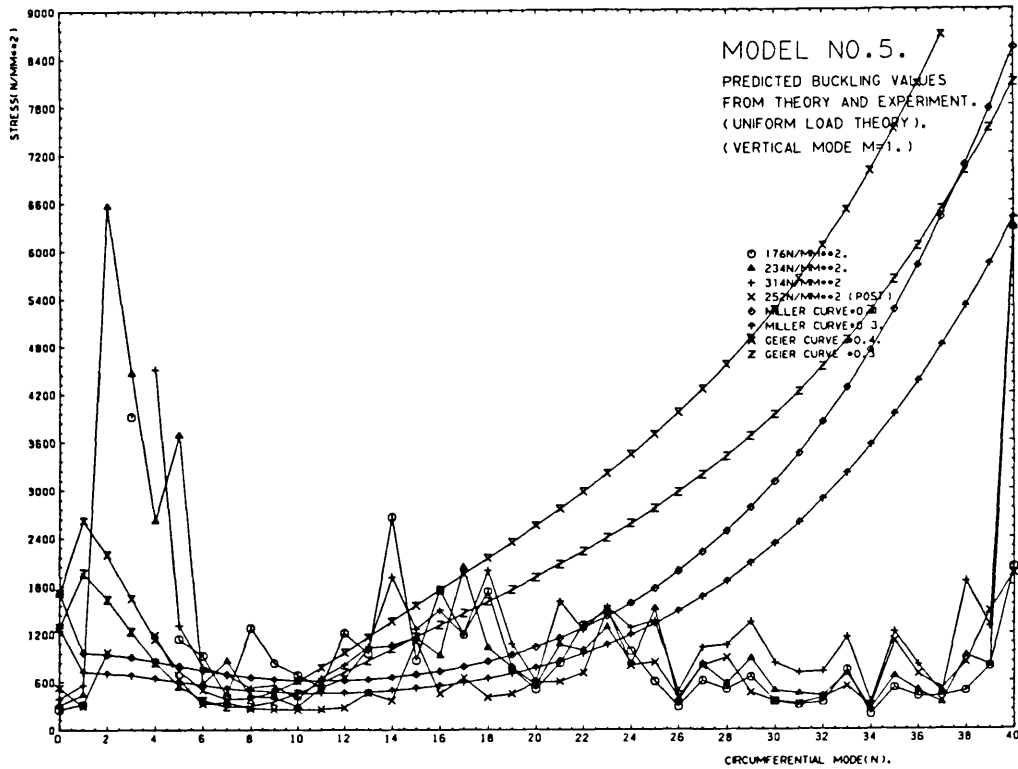


figure 3.10: Relative growth of fourier coefficients, $m = 5$.



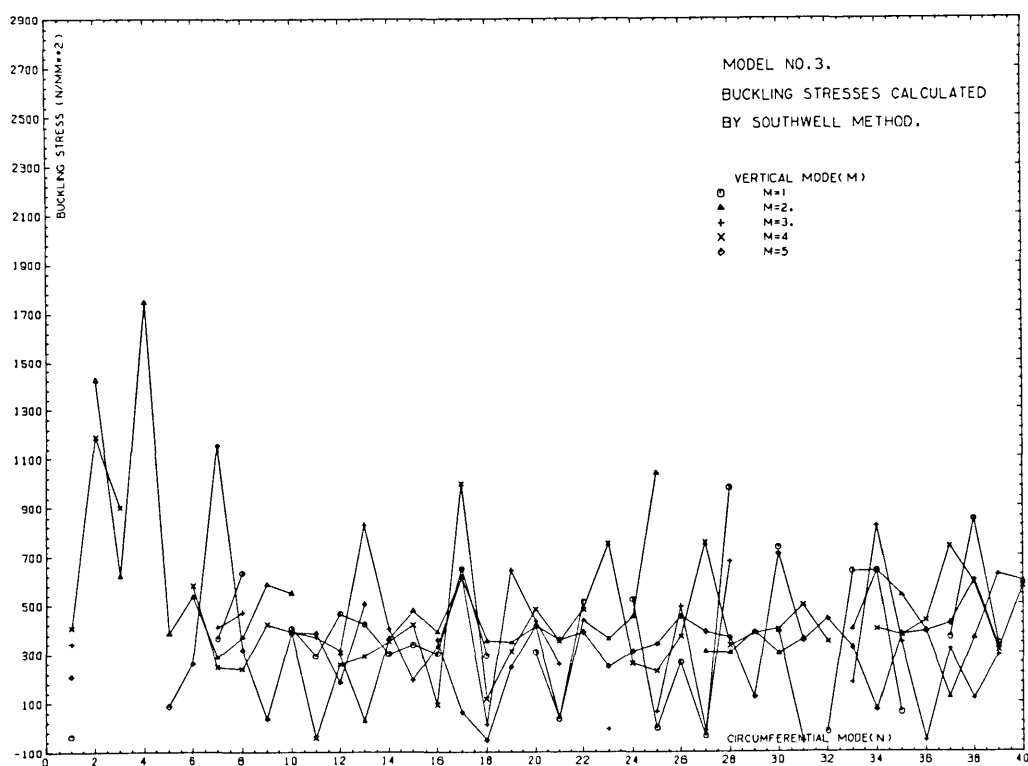
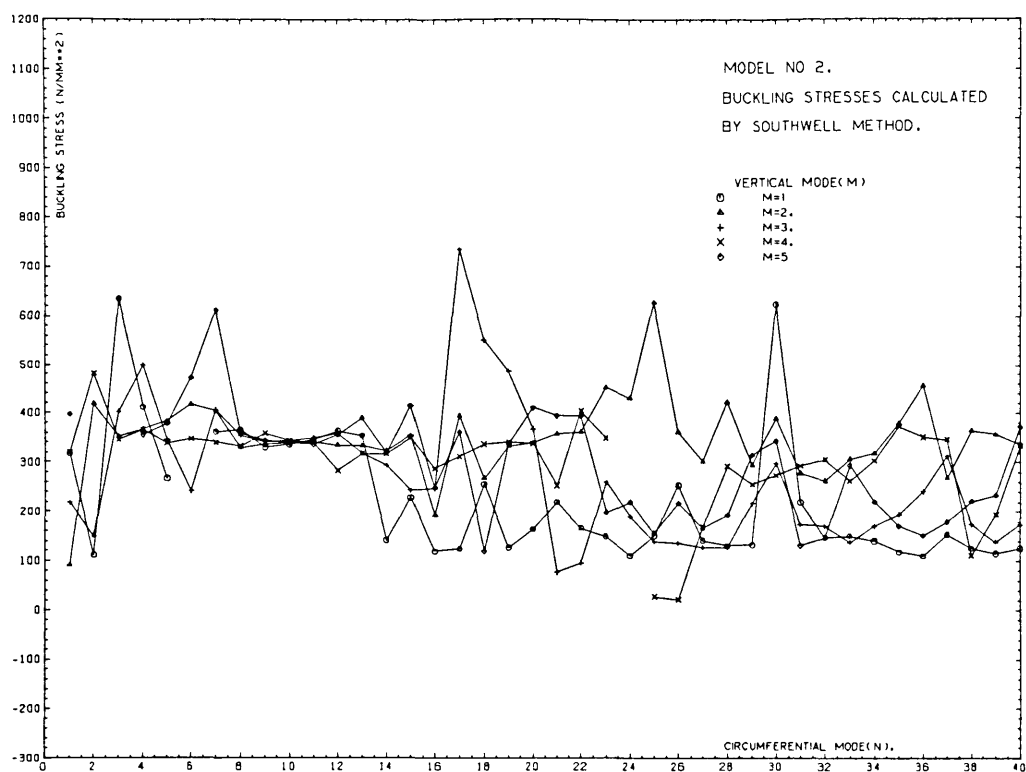
upper figure 3.11: Predicted buckling stresses, method 1, model no. 2.

lower figure 3.12: Predicted buckling stresses, method 1, model no. 3.



upper figure 3.13: Predicted buckling stresses, method 1, model no. 5.

lower figure 3.14: Predicted buckling stresses, method 1, model no. 6.



upper figure 3.15: Predicted buckling stresses, method 2, model no. 2.

lower figure 3.16: Predicted buckling stresses, method 2, model no. 3.

CHAPTER 4.

Theoretical Elastic Buckling Loads of the Experimental Models.

4.1 Introduction

4.1.1 Scope of Chapter

A perfect elastic structure which is subject to compressive stresses due to the presence of external loads will be capable of existing at equilibrium in two neighbouring displacement states at some level of loading. The structure is said to be unstable at this load and the load is termed the elastic buckling or elastic critical load of the structure. The elastic buckling load of a column is an important indication of the strength of a column. The elastic buckling load of a flat plate in axial compression is, however, of less value because a very considerable postbuckling reserve of strength exists and the material yield strength is often the important parameter affecting strength.

It has been pointed out in the literature survey in chapter 1 that the elastic buckling load of a perfect unstiffened cylinder in axial compression is not necessarily a good indication of the experimental buckling load. The magnitude of initial imperfections reduces the elastic buckling load. Research into the behaviour of stiffened cylinders indicates that these structures are not imperfection sensitive and hence the elastic buckling load is a useful parameter in determining strength.

The buckling stresses of the uniaxially loaded cylinders tested in the experimental section of this work will be calculated in this chapter. The cylinders will be considered to have simply supported ends and the stiffening will be considered uniformly smeared over the surface. Two approaches will be considered and the results will be compared.

Buckling stresses of a typical entire cylinder will also be calculated using an eigenvalue analysis in conjunction with a finite element package. The individual stiffened panels will be modelled. It will be shown that the panel mode of buckling governs for the three cylinders whose overall geometry is modelled. The eigenvalue analysis

of a typical repeated stiffened panel will also be considered in isolation. A mesh refinement study will be carried out for the stiffened panel analysis.

A semi-empirical method will be suggested for the buckling stress calculation of a stiffened panel unit where the panels are relatively narrow. It will be shown that this model is useful in determining the elastic critical stress when slender stringer stiffeners are used.

4.2 Orthotropic buckling analysis

4.2.1 Orthotropic buckling

The linear differential equations describing the equilibrium of an orthotropic cylinder were used by Rosen and Singer (140) to calculate the buckling stress of a stiffened cylinder subject to axial load. The buckling stress of a perfect cylinder is the load at which the determinant, used to obtain displacements from the differential equations of equilibrium, becomes zero. The determinant is a linear function of axial stress and the buckling stress can be easily computed given any displacement set which satisfies the boundary conditions. The minimum buckling stress is obtained by trying, successively, different sinusoidal displacement functions.

Miller (143) suggested the use of an effective width concept to account for local panel buckling. In the following analysis, however, the effective width of a flat panel, as advocated by von Karmen (29) will be used. The effective width is given by

$$B_c = 1.9t \sqrt{\frac{E}{\sigma_y}} \leq B \quad \dots \dots \dots (1)$$

where σ_y is the perfect curved panel buckling stress.

As pointed out in the literature survey, §1.5.1, an alternative buckling stress formula was suggested by Geier (145).

Results will be presented using reference 145, and the modified version of reference 143.

4.2.2 Orthotropic buckling stress results

Figure 4.1 shows the plot of theoretical buckling stresses corresponding to various lateral displacement functions using reference 145. The lateral displacement function is

$$w = w_a \sin \frac{m\pi x}{a} \cdot \sin \frac{n y}{R}$$

where w_a is a buckling displacement of arbitrary value.

Results are given as curves for successive values of m , the number of longitudinal half-waves. The number of circumferential waves, n , is plotted along the horizontal axis, $3 \leq n \leq 20$. The lowest buckling curve is for $m = 1$. Figure 4.2 shows the variation of buckling stress for $m = 1$, for the three uniaxially loaded models of the experimental series. Figure 4.3 shows the buckling stresses for the three geometries using the formulae proposed by Miller, and the effective width given in equation 1. Table 4.1 gives the lowest buckling stresses from figures 4.2 and 4.3 and also the stresses obtained by direct use of reference 143 and from reference 225 (on which reference 143 is based).

4.2.3 Comparison of results

Table 4.1 shows that Geier's formula predicts the lowest minimum buckling stress for all three geometries. The results derived from reference 143 using the effective width approach and also from reference 145 are also compared in figure 4.4 for $m = 1$. The greatest disparity exists between the results for model no. 3, where the stiffener eccentricity is greatest. The disparity may be explained by observing how the axial strains are given in the different references. In reference 143 the plane of reference is taken as the shell mid-surface and hence the shell axial strain is

$$\epsilon_x = \partial u / \partial x$$

The stringer stiffener axial strain is

$$\epsilon_x = \partial u / \partial x - z_{st} \partial^2 w / \partial x^2$$

where z_{st} is the distance of the stiffener centre of area from the

mid-surface of the shell.

By contrast reference 145 takes an arbitrary reference axis system but a neutral axis for the cross-section in each direction is found. As a result the buckling formulae become much more complicated, but more accurate.

The error inherent in the approach of reference 143 is clearly seen in the limit as the thickness of shell is reduced to zero. The eccentricity of the stiffener, z_{st} , remains constant. The equilibrium equation in the circumferential direction may be assumed to vanish and the resulting buckling load is

$$P_X = B.N_X = (EA z_{st}^2) \left[\frac{m\pi}{a} \right]^2 + EI_S \left[\frac{m\pi}{a} \right]^2 \dots \dots \dots (2)$$

The second term on the right hand side of equation 2 is recognised as the Euler buckling load of a strut. The first term on the right hand side of the equation should become zero, since, if properly chosen, $z_{st} \rightarrow 0$ as $t_s \rightarrow 0$. z_{st} is the eccentricity of the stiffener centroid from the reference axis. However, if z_{st} is chosen as the distance of the stiffener centroid to the shell mid-surface, the buckling stress formula calculated from (143) becomes increasingly inaccurate as the stiffener eccentricity increases. It is obvious, for example, that the error incurred when tee-stiffeners are used is considerable.

4.3 Finite element eigenvalue analysis

4.3.1 Modelling for overall buckling analysis

A finite element package has been built up at Imperial College which facilitates the solution of the classical eigenvalue problem (226). The actual stress distribution throughout the model and the non-linear geometric stiffness of the structure at the current stress level is used in the solution technique adopted by the package for this problem. The stiffness of the shell continuum is modelled by an 8-node shell finite element. The stiffeners of the models tested, §2.2.1, may be modelled by a single shell element through the depth of the stiffener. Alternatively a beam finite element, with a single line of eccentrically

positioned nodes, may be used. This element is specific to the package used.

A 2 x 2 gauss point grid is used in the shell element and the shape functions are exact for membrane and bending stresses. The 2 x 2 grid prevents shear locking. 2 gauss points are used in the beam element. The eccentric positioning of the beam nodes allows the shell nodes of a structure to double as the beam nodes. As a result the structural matrix bandwidth of the stiffened shell is identical to that of an unstiffened shell.

An exact and complete representation of any of the geometries tested in the experimental part of this work would require the solution of a very large problem. It was decided to model one quarter of the circumference and one half the height of each model and to simulate the remainder of the structure by means of symmetry. Figure 4.5 shows a schematic representation of the finite element simulation of a typical model for the overall buckling analysis.

Model nos. 1 and 3 had 10 stiffeners over the quarter circumference. A typical panel between stiffeners was represented by a 5 x 2 grid of shell elements. The stiffeners were modelled over the half-height by 5 beam elements. Figure 4.5 also indicates the boundary conditions used to simulate symmetry of behaviour with adjacent areas of the cylinder. Side 3 is totally fixed in displacement and rotation. Side 1 represents the cylinder mid-height and displacements are free, but rotations are restrained. The structure is cut at the mid side of a panel along sides 2 and 4, in order to simulate symmetry along the longitudinal edges of the panel. (The alternative cut, coinciding with a stiffener position, does not create a symmetry condition). Both longitudinal edges are restrained against tangential displacement and against rotation about the generator (longitudinal) axis. The edges are also restrained against rotation about the radial line from the cylinder centre.

Figure 4.5 represents model nos. 1 and 3 exactly. Model no. 2 was similarly generated with 5 stringer stiffeners in the quarter circumference and each panel was represented by a 5 x 4 shell grid. The boundary conditions of model no. 2 were the same as for the

other models.

4.3.2 Overall buckling analysis results.

Axial load was applied to edge 1 of the models, so that all nodes displaced equally in the longitudinal direction of the cylinder. The lowest eigenvalue was found for model nos. 1 and 2. The 4 lowest eigenvalues of model no. 3 were found. As will be apparent from the results below, the lowest eigenvalue corresponded to a panel mode of buckling for all three models and the analysis was extended for model no. 3 to ascertain if an overall buckling mode could be obtained.

The eigenvectors for the overall buckling analysis are shown in figures 4.6-11. The stiffeners are not shown in the figures because the plotting technique used only the displacements of the nodes. The nodes of the stiffener correspond to the shell nodes. The location of the stiffeners is clear when figure 4.5 is compared with the deflected shapes. Figures 4.6-9 are plan representations of the buckled models. Thus the stiffener positions correspond throughout with the zero point of the lateral displacements. The stiffeners rotate (in a torsional mode) about the stiffener-shell junction for compatibility.

Figures 4.6 and 7 show the buckling shape of model nos. 1 and 2. The lowest eigenvalue is a sinusoidal panel buckle. One buckling wave occurs along the model height. Figure 4.8 is the eigenvector corresponding to the lowest eigenvalue for model no. 3. This figure appears to depict a complicated buckling function because of the view of the buckling profile. It will be seen much more clearly below, figure 4.16, that figure 4.8 depicts a panel buckling pattern with a number of buckling waves along the model height. Figure 4.10 is a view of the overall buckling shape from another angle. The buckling shape is best seen by looking at the right hand longitudinal edge of the model. Very small lateral displacements occur in the lowest shell element, because the bottom edge of the model is clamped. The second and third elements from the base undergo large lateral displacements whilst the fourth and top elements forming the longitudinal edge again displace by only a small amount. It had been expected that the largest lateral displacements would occur at mid-height, as with model nos. 1 and 2. It will be seen below in the

detailed analysis that the largest displacements of the panels of model no. 3 do in fact occur at midheight. The explanation for the inaccurate buckling shape, seen in figures 4.8 and 10 lies in the coarseness of the finite element grid over the panel.

The finite element grid of model no. 3 was not detailed enough to accurately depict the local buckling phenomenon. It was quite accurate enough to depict the overall buckling patterns. The next three lowest eigenvalues were obtained for model nos. 3 using the finite element representation described above.

The second eigenvalue corresponds to an eigenvector, figure 4.9, in which the panel buckling mode described above is coupled with an overall buckling mode. The wave number of the overall buckling pattern corresponds to $n = 2$. It is not clear why this overall buckling mode is physically possible. It seems more likely that it is a spurious mathematical solution peculiar to this finite element idealisation.

The third eigenvector was a panel buckling mode similar to that shown in figure 4.8. In this case, however, the lateral displacements occur predominantly in the first and second elements along the height. The buckling wave is gradually damped out along the generator in elements 3-5, figure 4.11.

The lateral displacements around the circumference of the fourth eigenvector were similar to those of the second eigenvector, i.e. a combination of an overall and panel buckling mode around the circumference. The lateral displacements of the fourth eigenvector along the generator were similar to those of the third vector.

The four lowest eigenvalues obtained for model no. 3 all lay within 7.5% of the lowest value. The computational effort was very large, as the eigenvalue solution strategy involves the successive solution for each eigenvalue. A great many iterations were required because of the proximity of the solutions to one another. It was not a practical strategy to attempt to jump in the solution process beyond the lowest eigenvalue cluster, in an effort to find overall buckling eigenvalues. The cost of finding more than 4 eigenvalues appeared prohibitive and

as a result the effort to find overall buckling eigenvectors was not further pursued.

The overall buckling analyses indicated that the preferred mode of buckling of all three models was a lateral displacement mode of the panels coupled with a torsional mode of the stiffeners for compatibility. The lowest overall eigenvalues are given in figure 4.13. A more detailed analysis was carried out on a simulated single repeated panel-stiffener unit, as described below.

4.3.3 Finite element idealisation of a single stiffened panel.

Figure 4.12 shows a finite element idealisation of a single stiffened panel, as used in the eigenvalue study of a panel. The panel is represented in the figure by a 5 x 4 shell grid. A number of different grid sizes were considered in the study. The upper table included in figure 4.12 details the element layouts. 5 x 2 elements were used to represent the panel of model nos. 1 and 3 in the coarsest analysis. The finest grid was of 15 x 6 elements, or 90 elements per half panel.

The stiffener is positioned at mid-panel. A single eccentric beam element was used to represent the stiffener through its depth. Alternatively a single shell element was used through the depth, to check the accuracy of the beam element. The number of stiffener elements equalled the number of shell elements used in the longitudinal direction of the panel.

The boundary conditions for the four sides of the panel are shown in the lower table of figure 4.12. These boundary conditions ensure symmetry of behaviour about the panel mid-height and about the longitudinal edges of the panel.

4.3.4 Results of stiffened-panel eigenvalue analysis

Figure 4.13 shows the results of the eigenvalue analysis of the perfect panels for various grid sizes. The maximum number of elements used was 90. It is clear, however, that 20 finite elements per panel give the buckling stress accurately. Indeed the panel buckling stress of

model nos. 1 and 2 can be obtained with considerable accuracy by a 5 x 2 grid. It is clear from the eigenvectors of model no. 3, figure 4.16, that a greater number of elements was required for the representation of model no. 3 because of the greater number of buckling waves along the length of the panel.

The eigenvectors are shown for the three models in figures 4.14-16, with the eigenvalue, λ , given above each panel representation. The stiffener is represented in figures 4.14-16 by shell elements; hence the displaced shape of the stiffener is clearly seen.

The eigenvectors are similar for model nos. 1 and 2. The fixed end of the panel causes the panel displacements to be largest at midheight. It is also clear from figures 4.14 and 15 that the torsional displacements of the stiffener of model no. 1 are much larger than for model no. 2, for similar magnitudes of panel deflection.

Figure 4.16 shows the eigenvector of model no. 3 for various panel grid sizes. It is clear that one and one half deep halfwaves form along the length of the panel as shown, whilst very much smaller lateral displacements occur close to the model end. The boundary conditions along side 3 force the lateral displacement buckling waves to zero amplitude at the model end. Figure 4.17 is another view of the buckling shape of model no. 3. The stiffener has been drawn separate from the panel in this figure. The view of the stiffener is along the y-axis, as given in figure 4.12. The stiffener is seen to be clamped at the fixed end of the panel, whilst buckling along the length is in a torsional mode. The shell stiffener junction remains straight.

The panel eigenvalue analyses were carried out with both shell and beam elements representing the stiffener. Very little difference in eigenvalues was observed as a result of the differing stiffener representations. The eigenvalues obtained from the quarter cylinder representation are given in figure 4.13. In all cases the eigenvalues obtained correspond with those obtained from the same grid size from the stiffened panel representation given in figure 4.12.

4.4 Prediction of stiffened panel buckling stress

4.4.1 An adaption of flat panel analysis to curved panels

The elastic buckling of stiffened panels, as given in §4.3.4 may be seen as the buckling of panels which are elastically restrained by stiffeners at their longitudinal edges. Bleich (228) has summarised the buckling of elastically restrained flat panels in a manner which allows the buckling stress of a panel with longitudinal stiffeners to be calculated. A modification of this method is suggested to make it applicable to calculations with cylindrical panels.

The buckling stress of a curved panel under longitudinal compression has been given by reference (68) as

$$\sigma_{CR} = C_{Fz} \frac{\pi^2 E}{(1-\nu^2)} (t/B)^2$$

where the coefficient, C_C , is given in (68), based on experimental results. A curved panel which displays a stable postbuckling behaviour behaves quite similarly to a flat panel. It is suggested that a curved panel, of width B , may be replaced by a flat panel, of width B_{el} , for the purposes of calculating the buckling stress of a combined panel-stiffener unit.

$$\text{If } B_{el} = B \sqrt{\frac{4}{C_C}} \dots \dots \dots (1)$$

the flat panel alone has the same buckling strength as the curved panel (given the conservative nature of experimentally derived values of C_C in (68)). The elastic buckling load of the stiffened panel of width B_{el} , is obtained from reference (228).

This hypothesis was tested on the geometry of model nos. 1 - 3. The buckling stresses are given in table 4.2.

The finite element eigenvalue analysis of model no. 1 gives a buckling stress 11% lower than that calculated using the present hypothesis. The difference in predicted values is largely accounted for by the prebuckling hoop stresses in the panel. When a calculation is made

taking these hoop stresses into account, the difference is reduced to 2.5%.

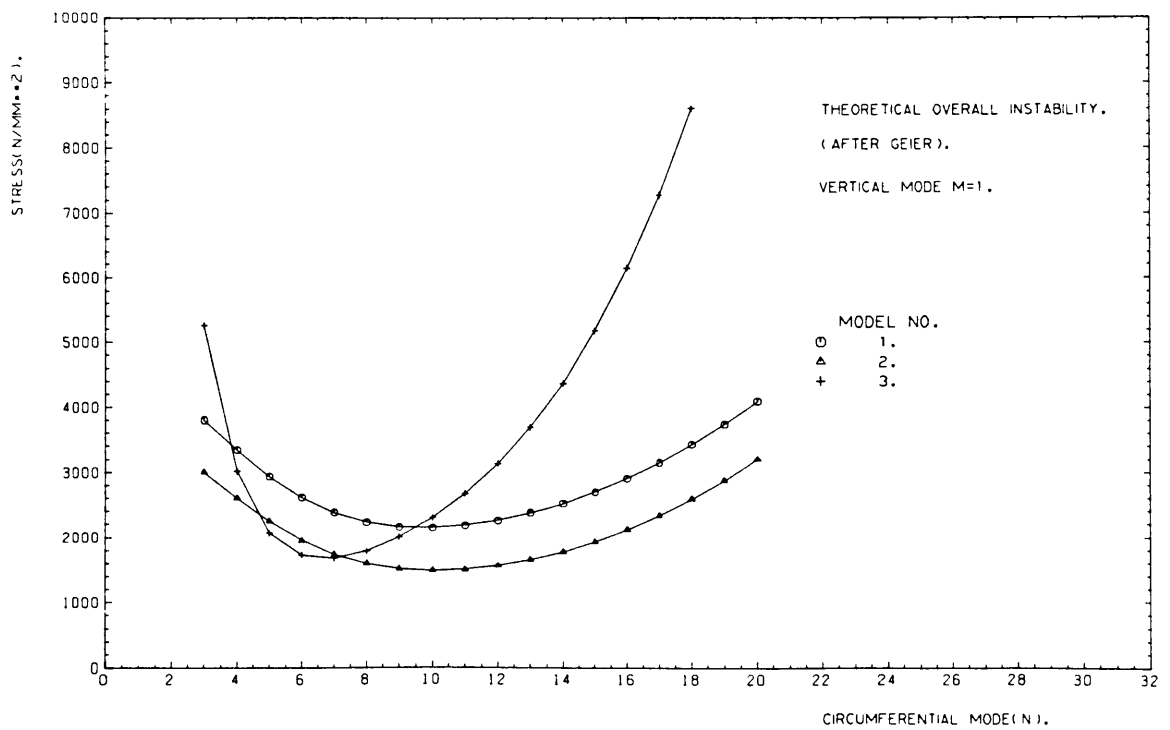
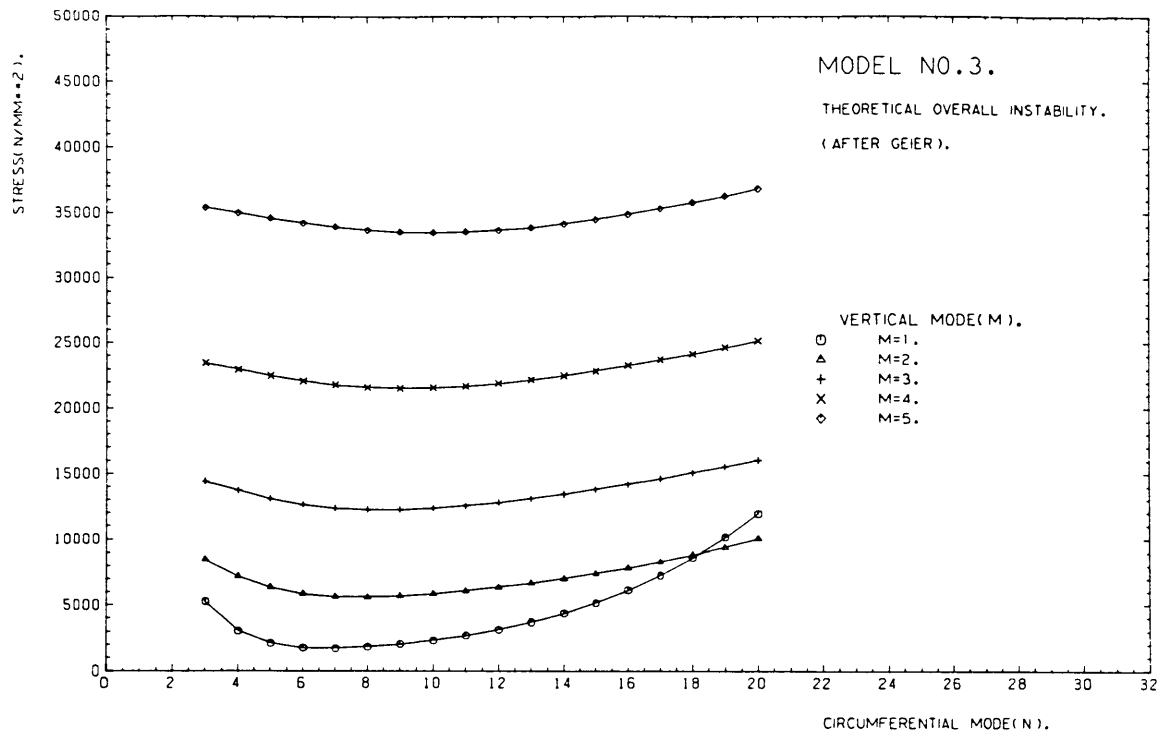
The curved panels of model no. 2 have a value of $\theta = 1.23$, §1.7.1, and are moderately imperfection sensitive based on reference (178) predictions. Hence the coefficient C_c (68), based on experimental results, will predict rather conservatively the buckling stress of a curved panel of these proportions. This may explain the difference in predicted buckling values in table 4.2 for model no. 2. The stiffened panel buckling stress, as predicted by the hypothesis (320 N/mm^2), is very much lower than that predicted by the eigenvalue analysis (545 N/mm^2). It is noted that the classical buckling stress of the equivalent simply supported cylindrical panel, as given in reference (178) or (71) is 705 N/mm^2 . Thus the finite element result lies approximately mid-way between the classical buckling stress of the panel and the empirically derived buckling stress of the stiffened panel. The discrepancy is, however, large and as this study has not been continued to accurately ascertain the imperfect elastic panel buckling behaviour, it must be concluded that the semi-empirical method suggested above is not applicable to panels where the curvature is as large as $\theta = 1.23$.

The suggested method predicts that the stiffened panels of model no. 3 will buckle at 560 N/mm^2 . In addition the method predicts that the panel will act as a restraint for the stiffener. (In model nos. 1 and 2, the stiffener acts as a restraint for the panel). The finite element eigenvalue analysis predicts a buckling stress of 509 N/mm^2 , or approximately 10% lower than the value given by the suggested method. The elastic stress distribution in the stiffener of the uniaxially loaded model no. 3 is given in figure 6.12, and it is noted that the stiffener stress is somewhat greater than the average stress throughout most of the model length. If the unequal stress distribution is taken into account in the suggested method, and the stiffener is considered to be the critical member, the difference between the finite element result and the result of the semi-empirical analysis above is approximately 6%.

4.4.2 Limits of proposed method

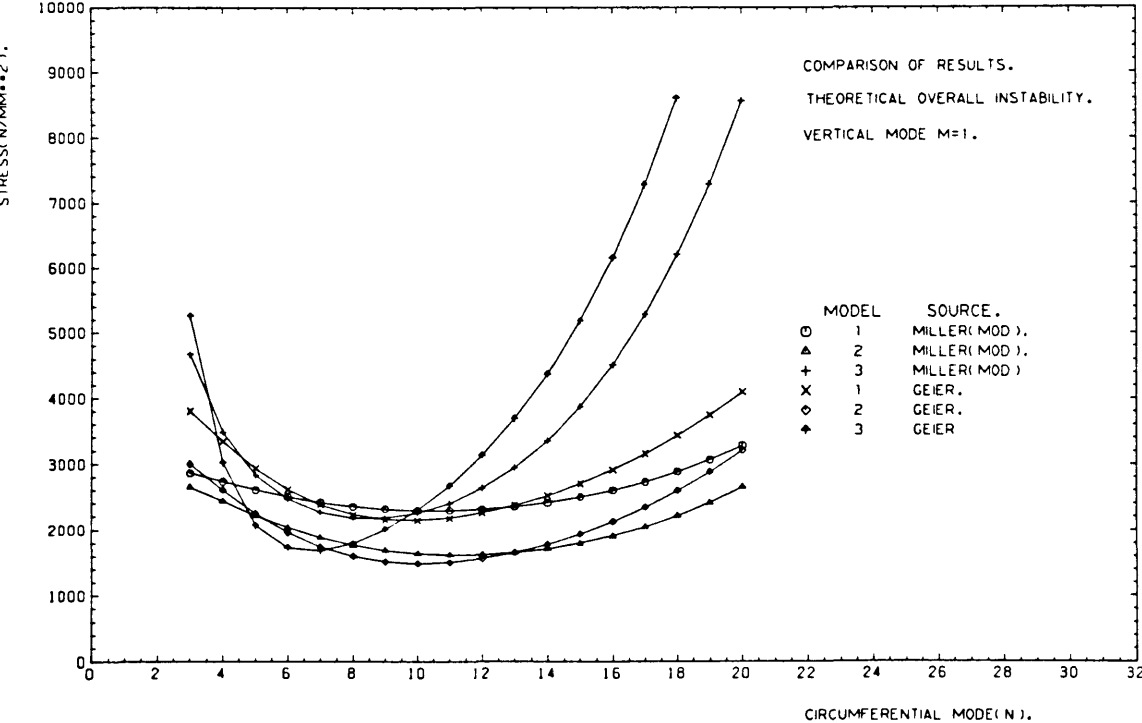
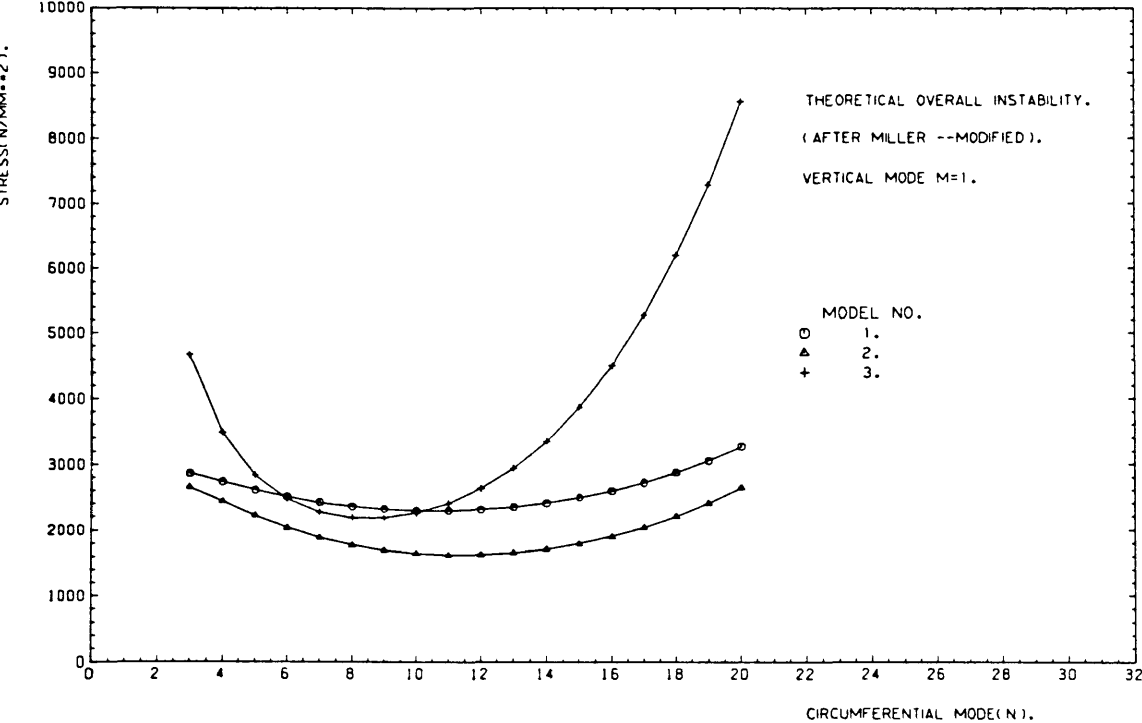
It is difficult to give accurate limits of applicability to the proposed semi-empirical method. The stiffener proportions are not constrained, provided panel buckling governs overall strength. The panel dimensions must, however, be limited as the buckling load of panels of large θ values is not accurately predictable. It is over conservative to limit the method to very narrow panels, i.e. panels of $\theta \leq 0.64$. Most research indicates that panels of much greater curvature have a stable post-buckling behaviour. In any case the semi-empirical method predicts the buckling stress conservatively for perfect (or near perfect) panels. (The experimental results of model no. 2 of this series of experiments indicates that buckling occurs at 320 N/mm^2 (approx.). There was, however, a considerable amount of plastic strain at this stress and hence the experimental results cannot be relied upon for indications of elastic panel behaviour.)

It is prudent to limit the use of the proposed semi-empirical analysis to panels of $\theta \leq 1.0$.



upper figure 4.1 : Orthotropic buckling stresses, model no. 3

lower figure 4.2 : Lowest orthotropic buckling stresses (Ref. 145).



upper figure 4.3 : Lowest orthotropic buckling stresses, (Ref. 143).

lower figure 4.4 : Comparison of lowest buckling stresses.

Finite element layout for overall Eigen value analysis

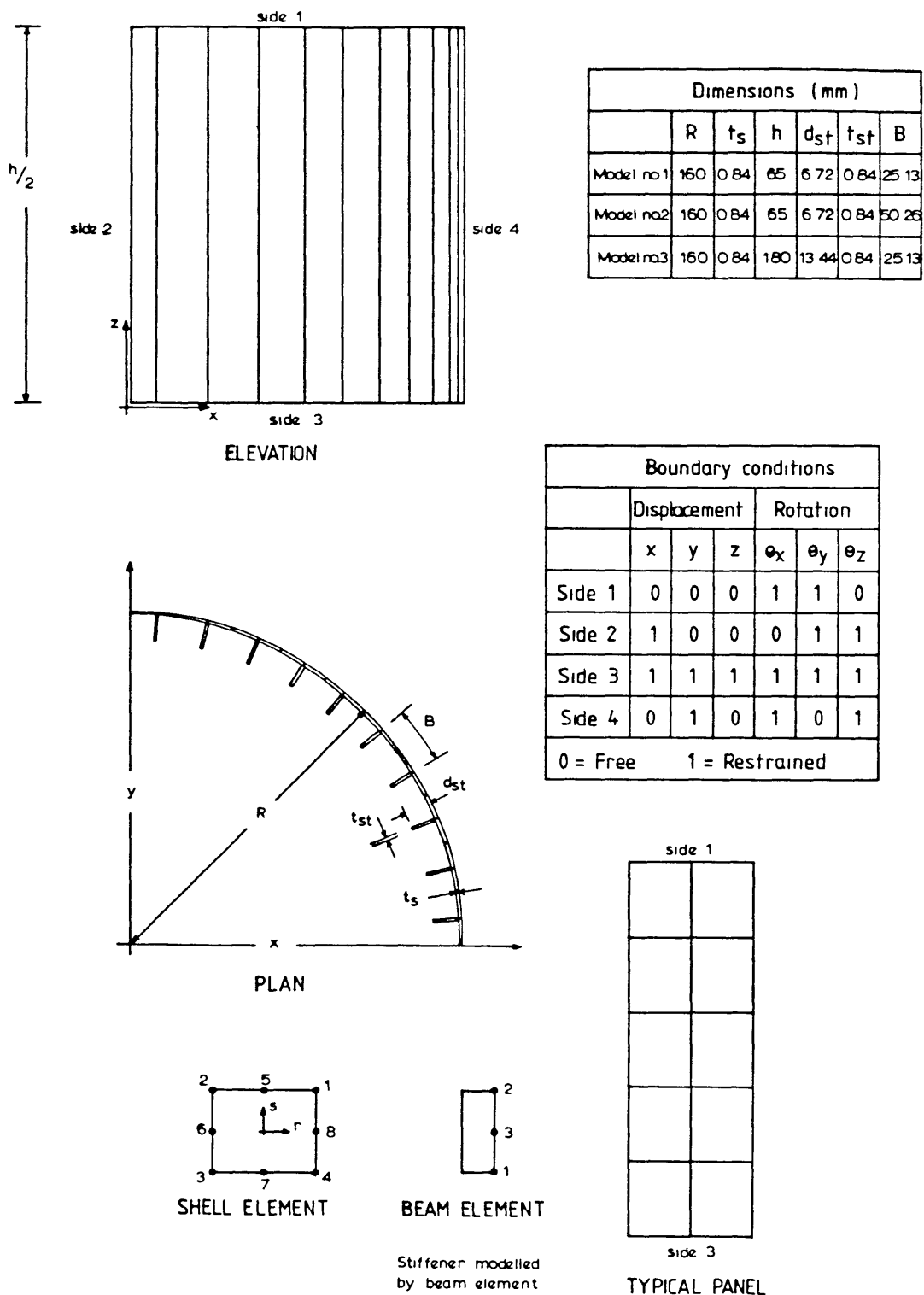
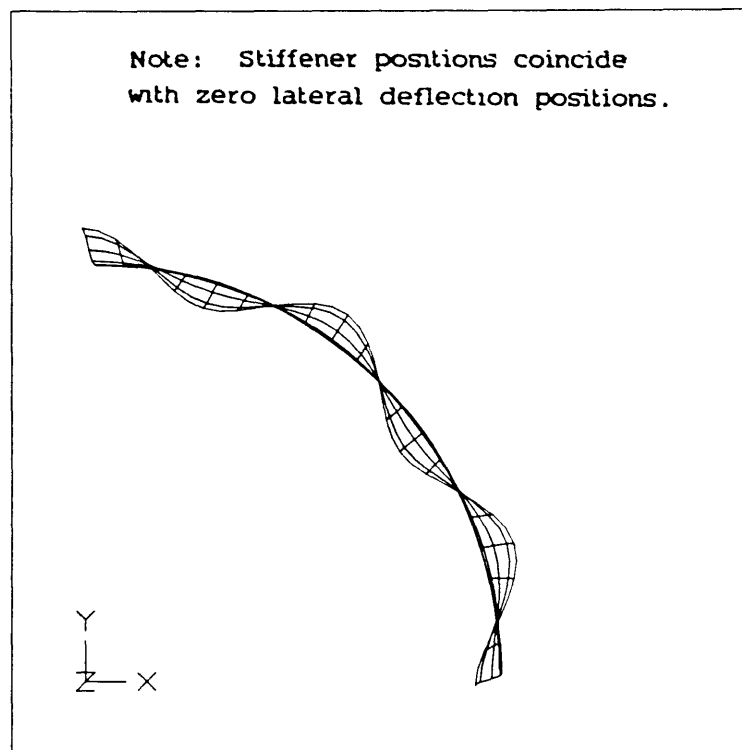
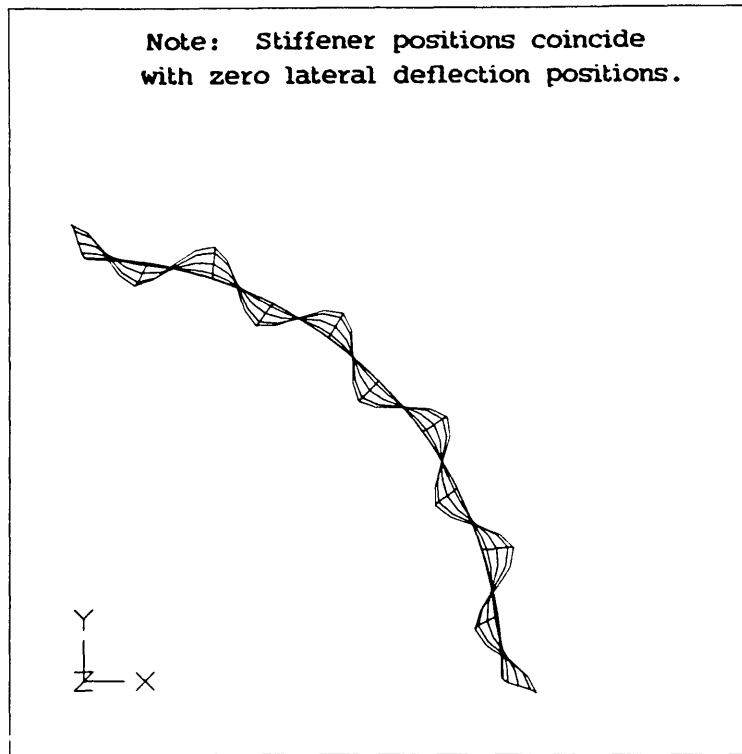
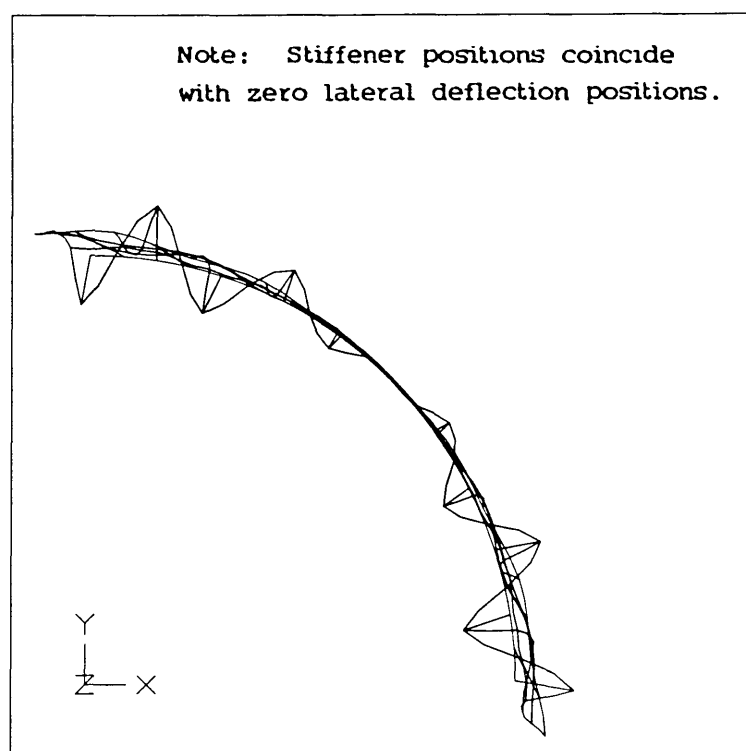
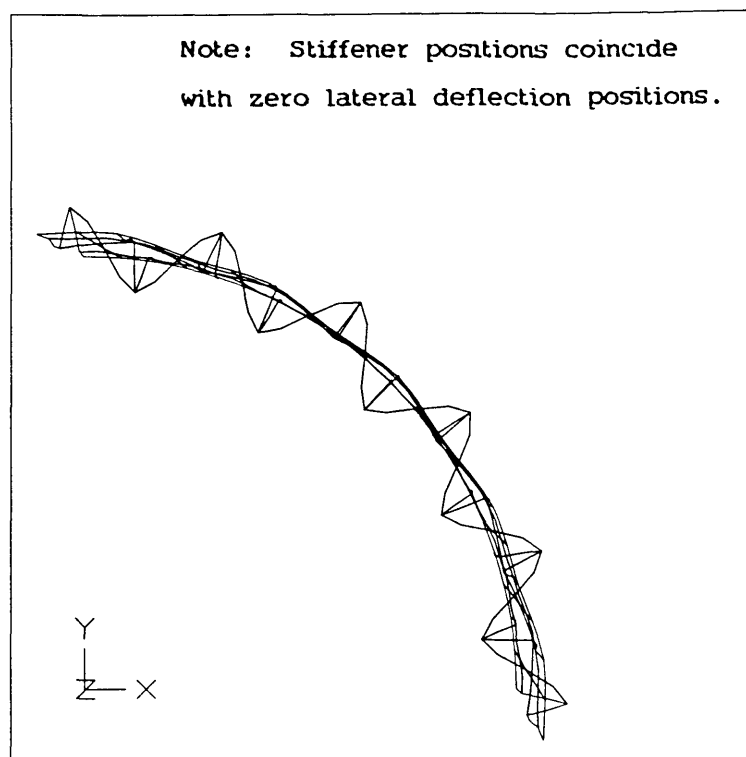


figure 4.5 : Finite element idealisation for overall eigenvalue analysis.



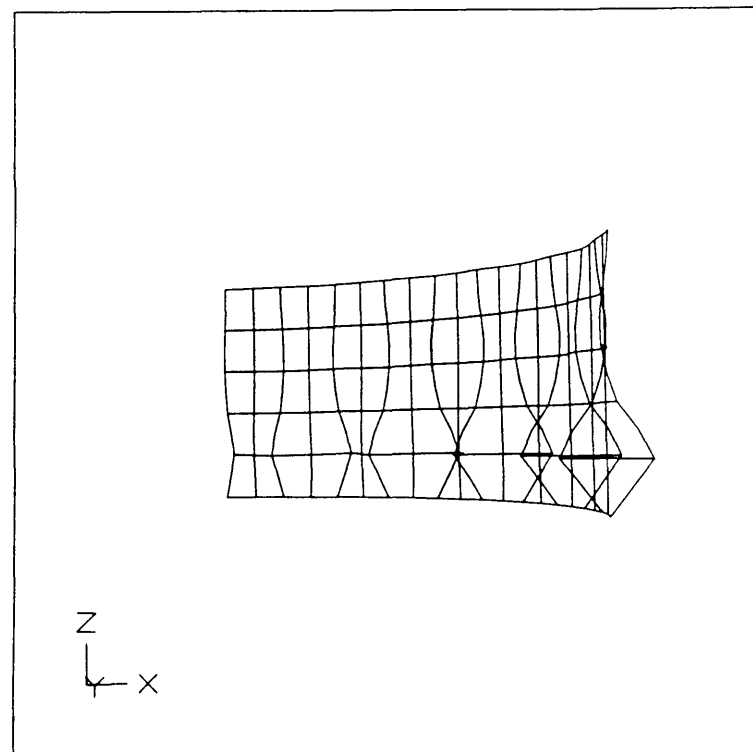
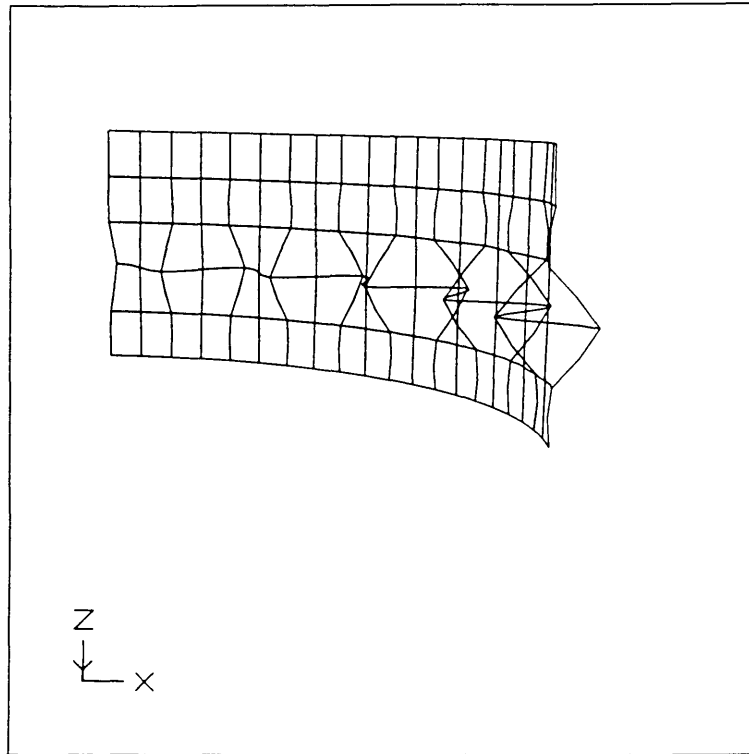
upper figure 4.6 : Lowest eigenvector for model no. 1, plan view.

lower figure 4.7 : Lowest eigenvector for model no. 2., plan view.



upper figure 4.8 : Lowest eigenvector for model no. 3, plan view.

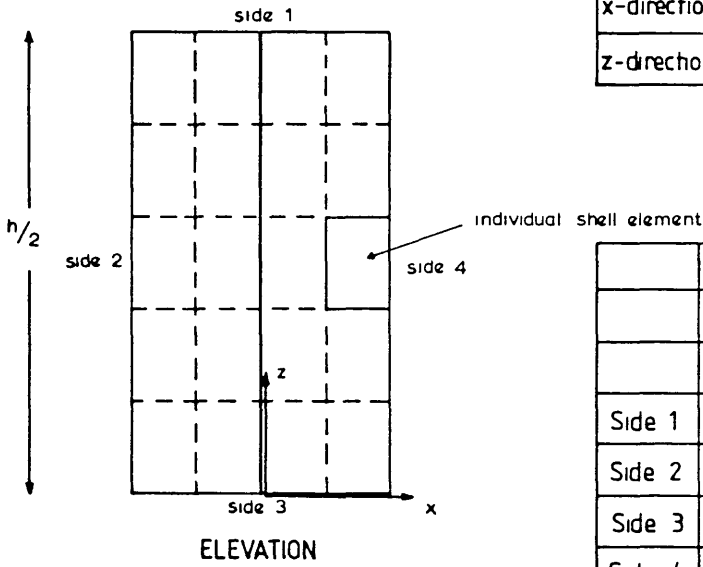
lower figure 4.9 : Second eigenvector for model no. 3 , plan view.



upper figure 4.10: Lowest eigenvector for model no. 3, side elevation.

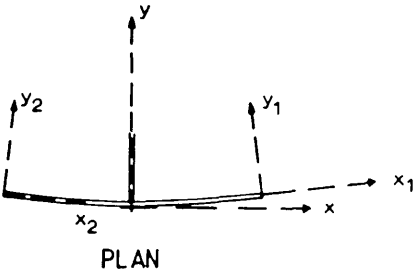
lower figure 4.11: Third eigenvector for model no. 3, side elevation.

Finite element layout for panel
Eigen value analysis model nos 1-3



No of panel elements used				
Case no	1	2	3	4
x-direction	2	4	4	6
z-direction	5	5	10	15

	Boundary conditions					
	Displacement			Rotation		
	x	y	z	θ_x	θ_y	θ_z
Side 1	0	0	0	1	1	0
Side 2	1_2	0	0	0	0	1_2
Side 3	1	1	1	1	1	1
Side 4	1_1	0	0	0	0	1_1
0 = Free 1 = Restrained Subscripts refer to local axes						



Stiffener modelled by beam element
or by single shell element through
depth

figure 4.12: Finite element idealisation of a single stiffened panel.

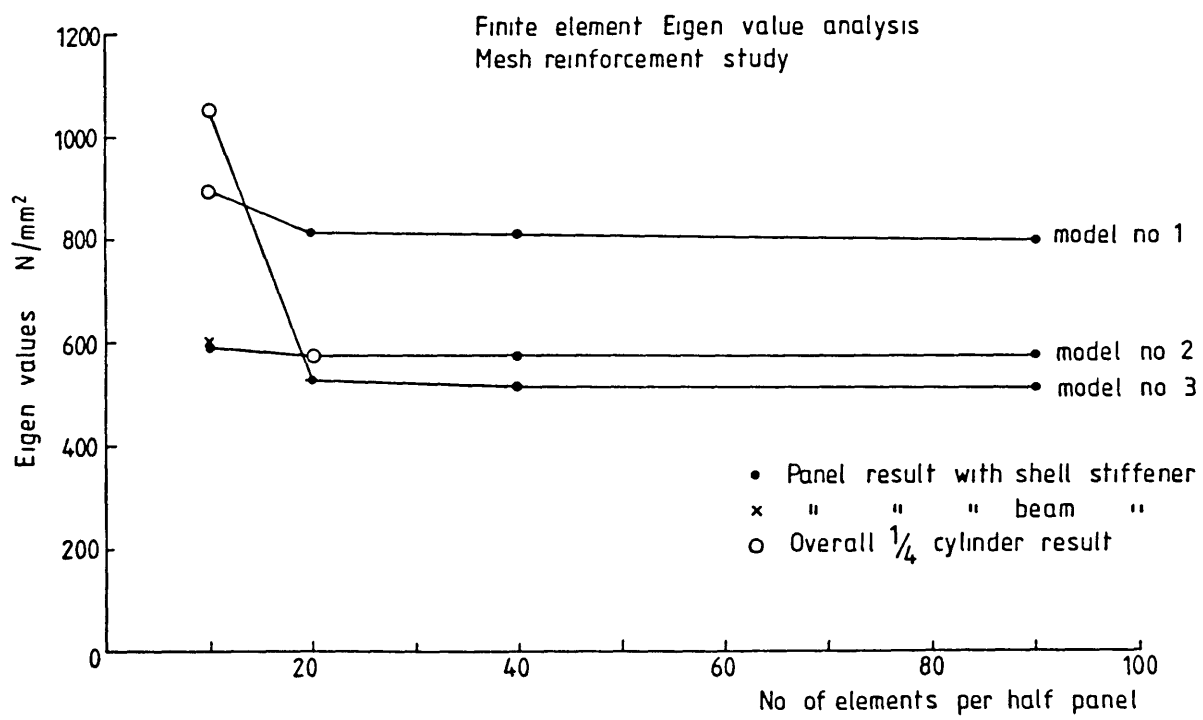
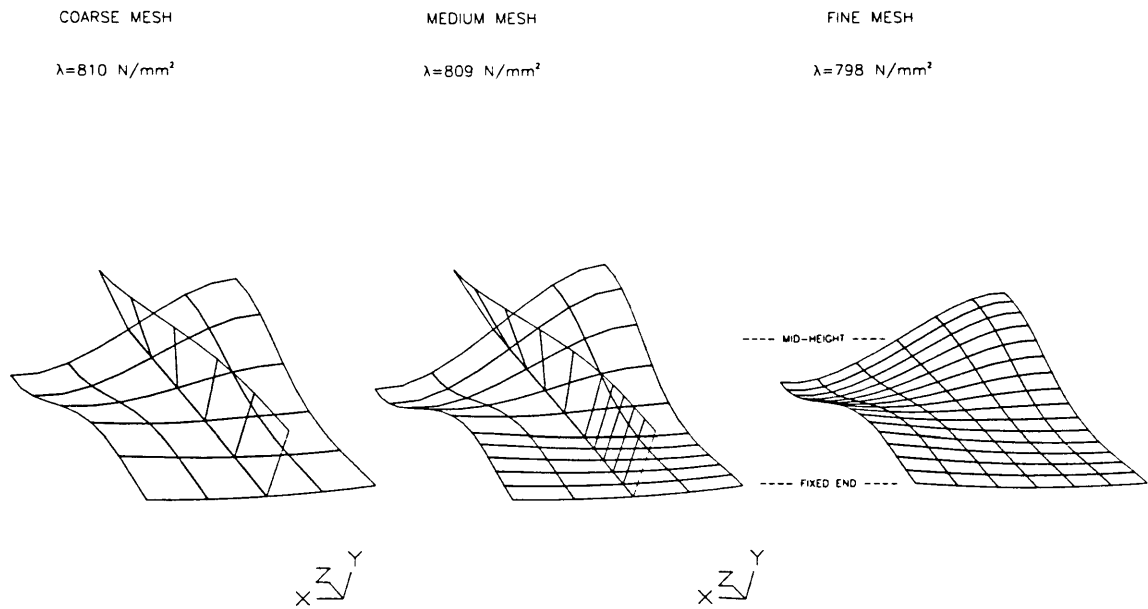
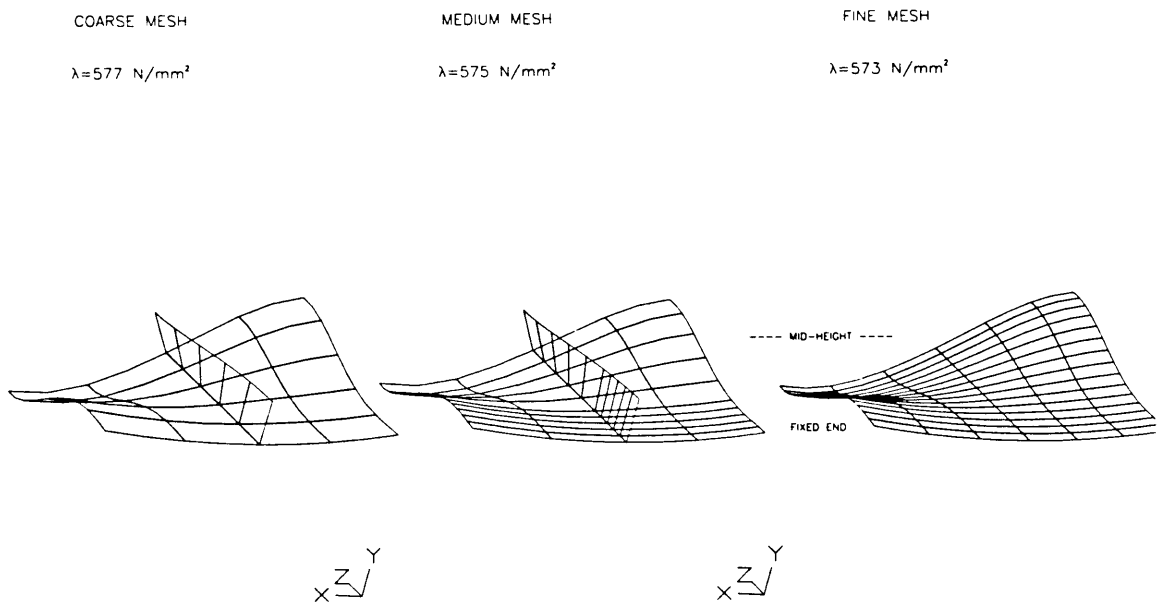


figure 4.13: Results of eigenvalue analysis.

FIRST EIGENVECTOR FOR PANEL SEGMENT OF MODEL NO 1



FIRST EIGENVECTOR FOR PANEL SEGMENT OF MODEL NO 2



upper figure 4.14: Stiffened panel eigenvectors, model no. 1.

lower figure 4.15: Stiffened panel eigenvectors, model no. 2.

FIRST EIGENVECTOR FOR PANEL SEGMENT OF MODEL NO 3

COARSE MESH

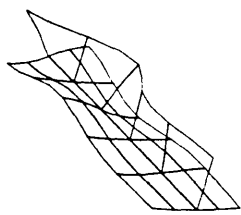
$$\lambda = 527 \text{ N/mm}^2$$

MEDIUM MESH

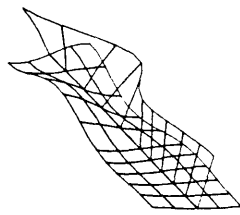
$$\lambda = 516 \text{ N/mm}^2$$

FINE MESH

$$\lambda = 509 \text{ N/mm}^2$$

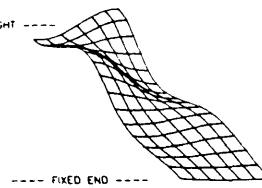


X
Z
Y

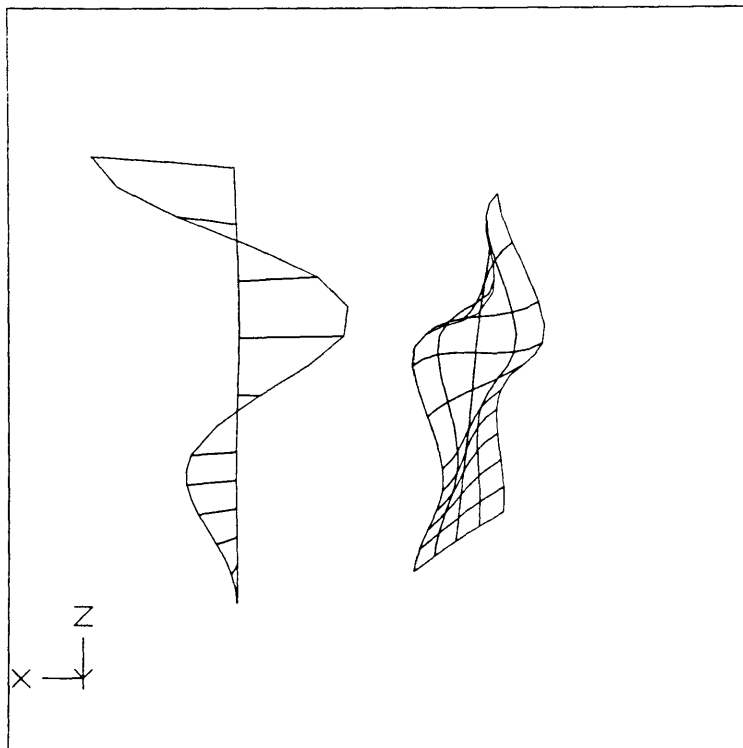


X
Z
Y

--- MID-HEIGHT ---



--- FIXED END ---



upper figure 4.16: Stiffened panel eigenvectors, model no. 3.

lower figure 4.17: View of stiffener and panel buckling shape, model no. 3.

Source	Overall elastic buckling stress (N/mm ²) (no of circumferential waves)		
	Model no 1	Model no. 2	Model no 3
Geier (145)	2144 (10)	1490 (10)	1686 (7)
Miller (143)	2294 (11)	1619 (11)	2184 (9)
Block et al (227)	2427 (11)	1709 (12)	2254 (9)
Proposed (using modified eff width)	2404 (11)	1624 (11)	2240 (9)

Theoretical buckling stresses of stiffened panel (N/mm²)

Method	Model no 1	Model no 2	Model no. 3
1	935	702	935
2	795	573	510
3	810	220	810
4	888	320	560

Method 1 Curved panel stress (ref 71)

Method 2 Finite element idealisation

Method 3 Restrained flat panel stress (ref 228)

Method 4 Restrained equivalent flat panel stress
(proposed { 4 4 1 })

upper table 4.1 : Theoretical orthotropic buckling stresses.

lower table 4.2 : Stiffened panel buckling stresses.

CHAPTER 5.

Elastic and Elasto-plastic Finite Element Analysis of the Experimental Models.

5.1 Introduction

5.1.1 Range of comparisons

The results obtained in the experiments outlined in chapter 2 will be assessed in this chapter. The standard design code for offshore structures in the North Sea (68) gives guidance on the strength of cylinders and curved panels in longitudinal compression. Results will be compared to the predicted strengths as given by this code of practice.

The experimental results clearly indicate that the repeated stiffened panel unit is the basic unit of failure of the cylinders tested. The eigenvalue analyses of chapter 4 firmly support this hypothesis. The model response was also characterised by a high degree of linearity, almost to collapse load. The results of an elastic finite element analysis will be compared in detail with experimental readings.

The results of elasto-plastic analyses of the stiffened panel unit will also be presented. The analyses are for perfect and imperfect panel geometries.

Results will be compared with the results of a similar series of experiments carried out at University College London (117).

5.2 Predicted strengths

5.2.1 Det norske Veritas predictions

A predicted failure load as computed by reference (68) is indicated on the average stress-strain curves presented in chapter 2. The code requires consideration of various failure modes and predicts failure through panel buckling for each of the geometries tested.

Panel strengths are conservatively predicted for all models. The predicted failure stress is 97% of the experimentally recorded value for models nos. 1 and 3. In the case of model no. 3, however, the code

does not advise in favour of the slender stiffeners used. The maximum recommended slenderness is $D_{st}/t = 0.4 E/\sigma_y$ ($= 9.6$, for $\sigma_y = 348 \text{ N/mm}^2$). Failure of model no. 3 was by a combination of panel and stiffener collapse. Had stockier stiffeners been used panel failure alone would have governed.

The strength of model no. 2 was assessed at 67% of the recorded failure load. It will be shown below that the high strength recorded by this model was chiefly as a result of the initially near perfect shape. The strength of a wide panelled cylinder, such as model no. 2 is much closer to the predicted value when the imperfections are of the order permitted by the code (§5.4.2).

Load eccentricity is considered by evaluating the strength of a critical section where elastic stresses from axial and bending loads combine. Figures 2.76, 91, 108 show the stresses calculated by the simple elastic method for combining axial and bending forces.

On this basis model nos. 4 and 6 are able to carry loads 10% greater than the uniaxially loaded equivalent models, because load can be redistributed after the initiation of failure in the critical panel. Model no. 5 was not able to redistribute load from the critical region, and although the unloading phase of model no. 5 was more controlled than for model no. 2 this is primarily attributable to stiffening of the test rig.

The code does not allow for redistribution of load after the initiation of failure in the most highly stressed panels. It is very difficult to guarantee the point of application of an eccentric load. Even in these experiments, where low friction steel-ptfe bearings were used, there is evidence of a drift of load centre. Consequently it is not advisable to consider the 10% extra strength which is apparently available in the most heavily loaded section of a narrow panelled cylinder.

5.3 Elastic finite element analysis

5.3.1 Idealisation of elastic behaviour

An elastic finite element idealisation of a stiffened panel unit is shown in figure 5.1 for the two panel widths of the experimental series. The

panels were without lateral surface imperfections, and symmetry was not taken into account in the longitudinal direction. The boundary conditions between circumferentially adjacent panels assume full symmetry of behaviour. Two shell elements model the panel in the circumferential direction for model nos. 1 and 3. 4 elements are used circumferentially for model no. 2. 12 elements are used in the longitudinal direction, 3 elements being used to model the first 10mm. of length, so that the rapidly changing forces are accurately represented. A 3-node beam is used as a stiffener, the nodes of the beam corresponding with shell nodes.

Loading was applied at one end of the model so that uniform end displacements were obtained.

5.3.2 Elastic response in general

The lateral displacements of the surface and stress distributions along the model length are summarised in figures 5.2-4 at an average stress of 1 N/mm².

The surface deflection profiles of model nos. 1 and 2 show that the panel and stiffener respond with considerable independence of one another. This is especially evident for model no. 2 where the maximum plate deflection is 7 times the stiffener deflection. The plate deflection is 1.8 times the stiffener deflection for model no. 1.

The deflections of plate and stiffener of model no. 3 are $.226 \times 10^{-3}$ mm and $.215 \times 10^{-3}$ mm at unit axial stress. It will be shown in §6.2.3 that the lateral deflections of the surface are caused by a Poisson expansion effect and by end forces induced at the cylinder ends due to the clamping of the end rings. The expansion due to the Poisson effect is $.238 \times 10^{-3}$ mm at unit axial stress. Thus the lateral deflections of model no. 3 are only very slightly curtailed by the end rings.

The lateral deflections of the panel of model no. 1 are curtailed as a result of the interaction between panel and stiffener. It is noted, however, that the panel deflections of model no. 2 are approximately 1.6 times the deflections predicted by the Poisson effect.

5.3.3 Elastic stress distributions

The stresses in model nos. 1 and 2 are strongly influenced by the end effects. The bending moments induced by the end rings introduce stress concentrations of 1.6 - 1.9 times the average stress close to the end rings. The large stress concentration is limited to 5 - 7mm from the end rings.

The end rings affect the stresses in the stiffener over a considerably greater distance than at the panel centre line. The stiffener outstand stresses are magnified at model mid-height to 1.2 - 1.3 σ_{av} , for both model nos. 1 and 2.

The effect of the end rings on stresses in model no. 3 is considerably less than for the other models. The maximum stiffener stress at mid-height is approximately 1.15 σ_{av} . The stress concentrations in the panel are limited to 5 - 7mm from the base. This indicates that local panel response at the ends is not very strongly influenced by length or stiffener spacing.

It will be shown in §6.4.4, that local panel stresses at the base are influenced only by the R/t ratio of the cylinder, where the panel can be assumed to act independently of the stiffener.

5.3.4 Comparison of finite element and experimental elastic strains

It has been noted above that the strain response of the experimental models was linear. The experimental strains and strains computed from the elastic finite element results are compared in figures 5.5 - 10. Strains are compared at the base and at mid-height of the models. There is some difficulty in accurately comparing strains at the base of the models because of the rapidly varying forces in the region of the rings. The smallest strain gauges used were 2mm. long and they could not be guaranteed to be within 1 mm. of the end restraint. In addition, only model no. 1 was strain gauged externally, where the maximum stresses occur in the panel. Thus in figures 5.5 - 10 the finite element strains,

calculated from the axial and hoop stress output, are indicated for the two positions closest to the estimated position of the relevant strain gauges on the experimental panels. Some degree of averaging is appropriate also for stiffener strains, since the centre of the strain gauges in the experiments was approximately 2mm. from the stiffener outstand.

The correlation at the base of model no. 1 is close for both the panel (inside and outside) and the stiffener, figure 5.5. The experimental strains on the outer surface of the panel are within the range predicted by the two appropriate finite element readings. The strain in the stiffener is also close to the average strain in the stiffener predicted by the finite element analysis. Panel strains correlate satisfactorily at mid-height, but stiffener strains are slightly less than expected from the finite element analysis.

No strain gauges were used on model no. 2. It is, however, possible to compare the strains recorded at the critical section of model no. 5 with the finite element output for model no. 2. Panel strains at the base, albeit only for the inside surface, are reasonably in accordance with predicted values, figure 5.7. The experimental response at mid-height is characterised by a somewhat non-linear response in both stiffeners and panel. The initial panel imperfections were small. Nevertheless the non-linearity of strain response at stresses greater than 200 N/mm^2 may be a result of a non-linear growth of panel deflections. Strains in both the panel and the stiffener are somewhat less than predicted by the elastic analysis.

The predicted and recorded strains are in good agreement at the base of model no. 3, figure 5.9. There is also good agreement at mid height, although the variation of strains in panel and stiffener is not very large.

5.4 Elasto-plastic finite element analyses

5.4.1 Finite element models.

The elasto-plastic panel behaviour of the models was studied by a modified idealisation of a stiffened panel, figure 5.11. Symmetry of behaviour was assumed about the mid-height of the panel and about the longitudinal centre-line of the panel. A rigid beam along the base of the

panel ensured equal longitudinal displacements of the panel base.

The elasto-plastic response of a perfect panel was obtained for all 3 geometries. Each stiffened-panel model was also subject to a number of initial imperfections.

Barrelling imperfections and panel (only) imperfections were used for model nos. 1 and 3. Model no. 2 was subject to panel (only) imperfections.

Barrelling imperfections were of the form

$$w(x,y) = w_0 \sin \pi x/B$$

Panel imperfections were of the form

$$w(x,y) = w_0 \sin \pi x/B \sin \pi y/A$$

A and B were the panel dimensions in the circumferential and longitudinal directions, respectively. The origin is at the base of the stiffener; x and y axes are along the longitudinal direction and parallel to the circumferential tangent at the origin.

Barrelling imperfections were

$$w_0 = 0.211 \text{ mm. (model no. 1)}$$

$$w_0 = 2.08 \text{ mm. (model no. 3)}$$

These are approximately 50% of the permissible imperfections §6.2.5 (68).

Panel imperfections were

$$w_0 = 0.36 \text{ mm (model no. 1)}$$

$$w_0 = 1.45 \text{ mm (model no. 2)}$$

$$w_0 = 0.72 \text{ mm (model no. 3)}$$

and are the maximum permitted by reference (68) as deviations from a true circle.

5.4.2 Elasto-plastic model behaviour

Model response is summarised in the elasto-plastic average stress-strain curves, figures 5.12 - 14. Average stress was obtained from the load factor and the average strain obtained from overall panel shortening. The experimental response of the 3 models is given for comparison.

The experimental response for all three models is seen to be slightly stiffer than the finite element response during the elastic loading phase. This may result from the additional axial stiffness given to the experimental models by the end rings.

The finite element response of model no. 1 is linear until very close to the collapse load. The ultimate axial stress of the perfect model (357 N/mm^2) is greater than yield stress. This is caused by the compression hoop stresses in the panel which are induced at the model base and which remain significant over the entire length.

The imperfect model responses are very similar to that of the perfect model. The maximum stress recorded for the barrelling imperfection was 350 N/mm^2 and 349 N/mm^2 for the panel imperfection. Both loading cycles were discontinued before a drop in load was recorded.

The response of the imperfect panels shows model no. 1 to be very insensitive to imperfections.

The experimental response of model no. 2 corresponds very closely with that of the elasto-plastic perfect panel, figure 5.13. The maximum experimental stress was 335 N/mm^2 and 346.7 N/mm^2 for the idealised model. The unloading of the experimental model is sharper than the idealised model. Unloading responses cannot be accurately compared since the flexibility of the experimental rig strongly affects the experimental model.

The imperfect finite element response is quite different from that of the perfect model. The imperfect model shows much greater flexibility in the elastic loading phase and the model reaches a maximum stress at 263 N/mm^2 ($0.76 \sigma_y$). The imperfection chosen ($a/l = 1/35 B/t$) would just meet the DnV tolerance requirement (over a gauge length of 32 mm), but is considerably more than would normally be allowed in a flat panel (229).

The response of the elasto-plastic idealisations of model no. 3 have been given in figure 5.14. The perfect model has been loaded through to the unloading phase. The response is completely linear up to the full yield stress. The maximum recorded stress is 348.5 N/mm^2 .

The response of a panel with a single sinusoidal imperfection is indistinguishable from the perfect model response. It should be noted, however, that the eigenvector, figure 4.16, for this model is not a single sinusoidal wave. The insensitivity to panel imperfections in figure 5.14 suggests that the change would not be large for a panel imperfection in the shape of the eigenvector.

The maximum stress attained by the panel with a barrelling imperfection was 336 N/mm^2 , although loading was not continued into the unloading phase. The behaviour of this model is close to that of the experimental model. The measured imperfections in the test indicate, however, that maximum imperfections were very much less than for the theoretical model.

No initial imperfections of the stiffeners have been considered in these elasto-plastic analyses. The stiffener proportions used in models nos. 1 and 2 ($D_{st}/t = 8$) suggest that these models are insensitive to these imperfections and may be expected to take large plastic strains.

The stiffener proportions of model no. 3 ($D_{st}/t = 16$) render this model susceptible to stiffener collapse once yield has been reached in the stiffener outstand. The growth of sinusoidal panel imperfections causes a lateral displacement at the mid-height of the stiffener. The maximum lateral displacement of the stiffener outstand ($\approx 3.4 \times 10^{-2} \text{ mm}$) is about 75% of the maximum panel deflection when the average stress is close to full yield. This deflection however, does not affect the outstand stresses greatly and is insufficient to initiate lateral tripping of the stiffener.

The elasto-plastic analyses are considered sufficiently complete while design regulations prohibit the use of stiffeners which would be unable to carry full collapse load.

5.5 Comparison of results with other experimental studies

5.5.1 University College London tests

A series of experiments was carried out at University College, London, on a geometrically similar series of models to those tested uniaxially in this

experimental programme (117). There were, however, important constructional differences between the models tested at the two centres.

Tee-butt fusion welds were used in the UC series to form the stiffener-panel weld, whereas tig-arc fillet welds were used in this series. Elaborate jigging and stress relief procedures were carried out for the Imperial College tests, whilst residual stresses of $0.18-0.3\sigma_y$ were measured in the other models. The stress relieving in particular also affects the yield properties of the material, § Appendix 2A1.

The boundary conditions of the UC series with a loss of $1/3$ of the panel area to allow for bolting and the method of clamping the model ends is such as to indicate that boundary conditions were simply supported rather than clamped. The models tested at IC had clamped ends, §5.3.4.

The UC models showed a very flexible response to load. The failure stress of UC 16 (corresponding with model no. 1) was $0.94\sigma_y$ at an overall strain of $4.5 \epsilon_y$. Model no. 1 failed at $0.96 \sigma_y$ but the failure strain was $1.0 \epsilon_y$. The closeness of failure load is a result of the insensitivity to imperfections of this geometry, §5.4.2.

IC model no. 2 attained a maximum stress of $0.96 \sigma_y$ at $\epsilon = \epsilon_y$, while the corresponding UC1 failed at $0.82 \sigma_y$ and $\epsilon = 5.5 \epsilon_y$. UC1 develops a panel mechanism at mid-height while IC no. 2 forms a panel mechanism close to the base of the model. It will be shown (§6.4.4) that the clamping moment at the base of this model induces a mechanism to form at the model base.

IC model no. 3 failed by the formation of a panel-stiffener mechanism at $0.95 \sigma_y$ and $\epsilon = \epsilon_y$. UC5 failed at $1.04 \sigma_y$ and $\epsilon = 2.8 \epsilon_y$. Failure of UC5 was initiated by stiffener failure near the model base, suggesting a simply supported end condition. The collapse mode of the model was by an overall mechanism, involving a number of stiffened panels.

It is suggested that the similar failure stresses recorded by narrow panelled models tested at the two centres indicates an insensitivity of these geometries to initial imperfections and to boundary conditions. This is corroborated by the elasto-plastic finite element analyses, §5.4. The difference in failure stresses recorded by the wide panelled models

suggests a geometry susceptible to panel imperfections, also verifying the elasto-plastic analyses.

5.6 Conclusions

5.6.1 Conclusions of experiment and finite element comparisons

The elastic finite element analyses of a stiffened panel unit compare well with strain measurements made during the experiments and enable first yield stresses to be predicted accurately.

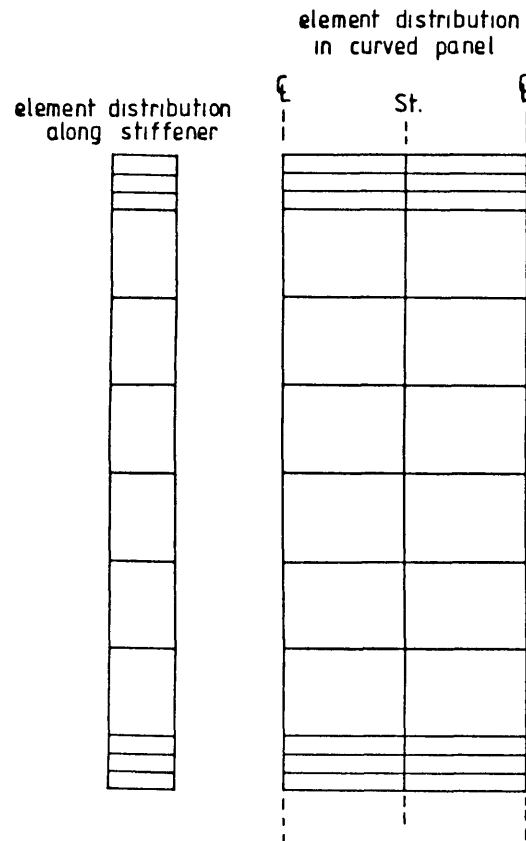
Elasto-plastic analyses of perfect models are in close agreement with the experimentally recorded behaviour. Imperfect model analyses indicate that model no. 1 geometry is totally insensitive to imperfections. Model no. 3 is insensitive to panel imperfections, but barrelling imperfections affect load capacity slightly. Model no. 2 is susceptible to panel imperfections.

Comparisons with predicted strengths indicate that the regulations are somewhat conservative, but not grossly.

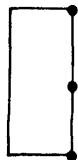
Comparisons with other experimental series corroborate the above conclusions.

Elastic Finite Element Models of Stiffened Curved Panel

Model no's 1 and 3



Stiffener
Element



Shell
Element

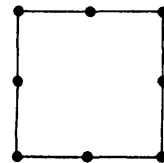
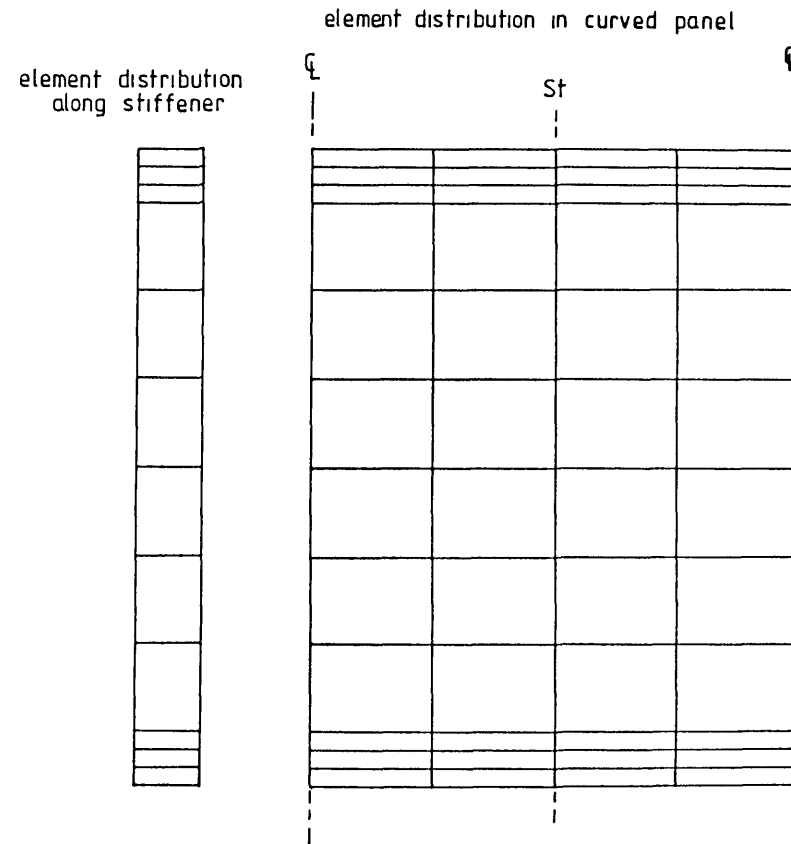
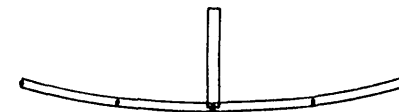


Figure 5.1

Model no 2



Plan of stiffened panel unit



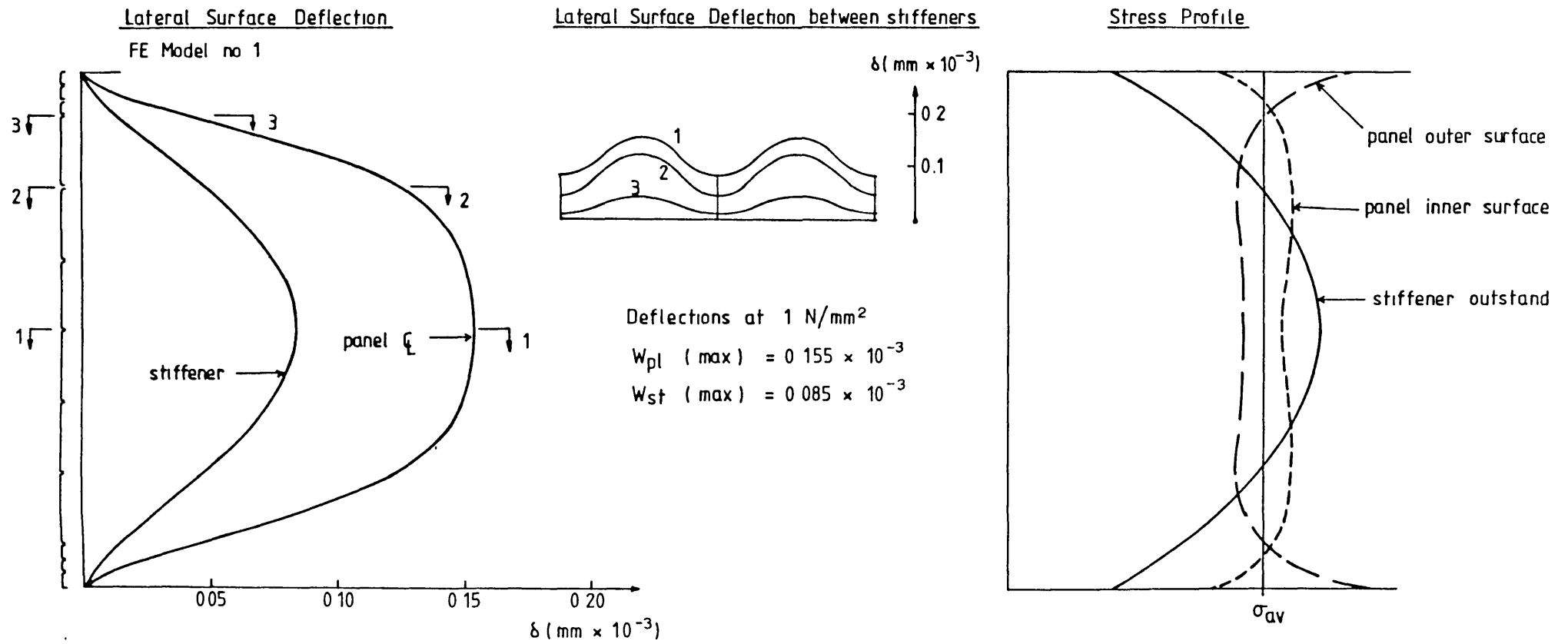
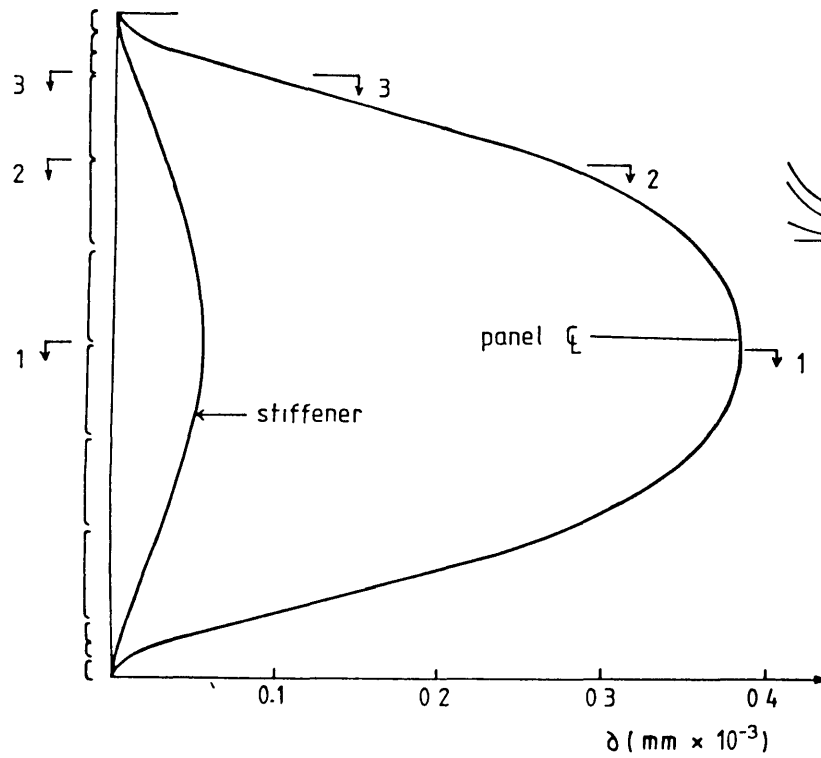


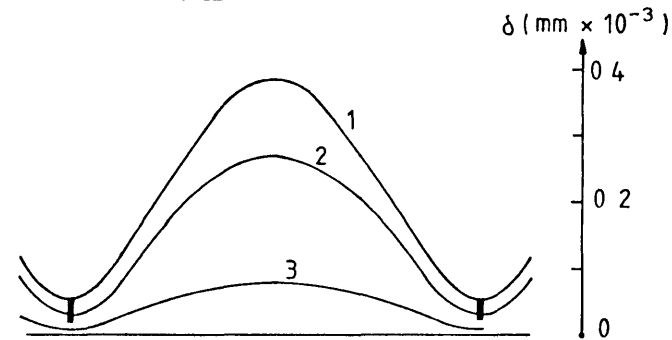
Figure 5.2

Elastic Finite Element Response for Model no 2 at unit axial stress

Lateral Surface Deflection



Lateral Surface Deflection between Stiffeners



Deflections at 1 N/mm^2

$$W_{pc} (\text{max}) = 0.384 \times 10^{-3} \text{ mm}$$

$$W_{st} (\text{max}) = 0.055 \times 10^{-3} \text{ mm}$$

Stress Profile

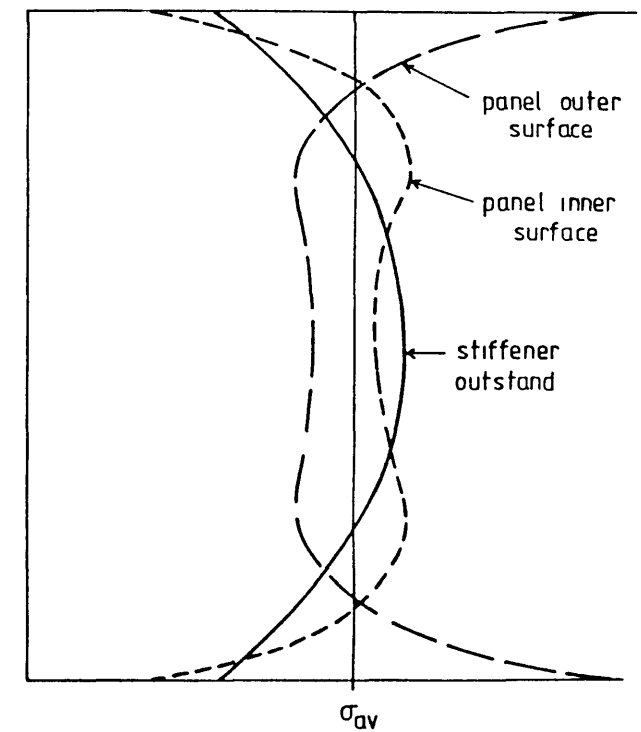


Figure 5.3

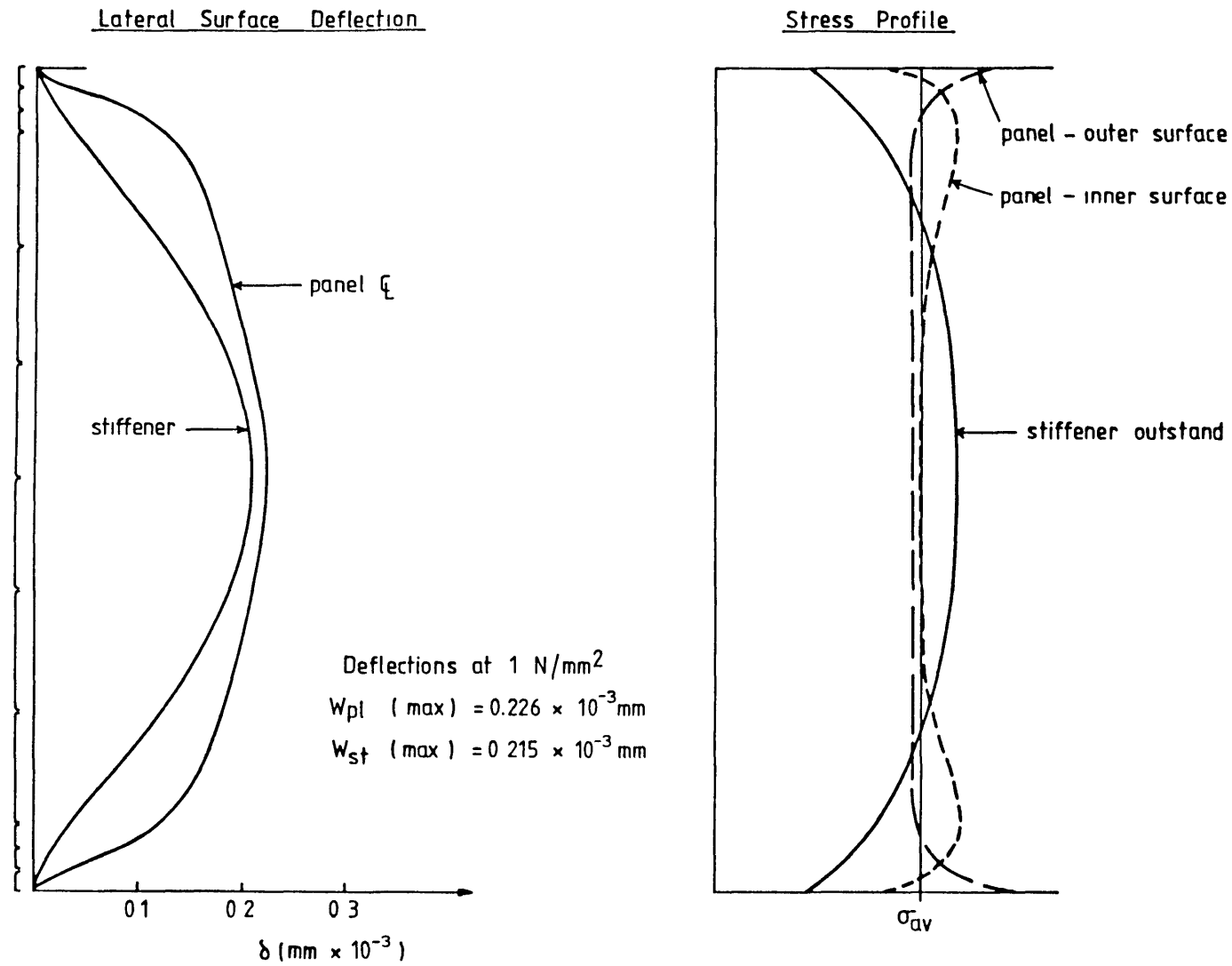


Figure 5.4

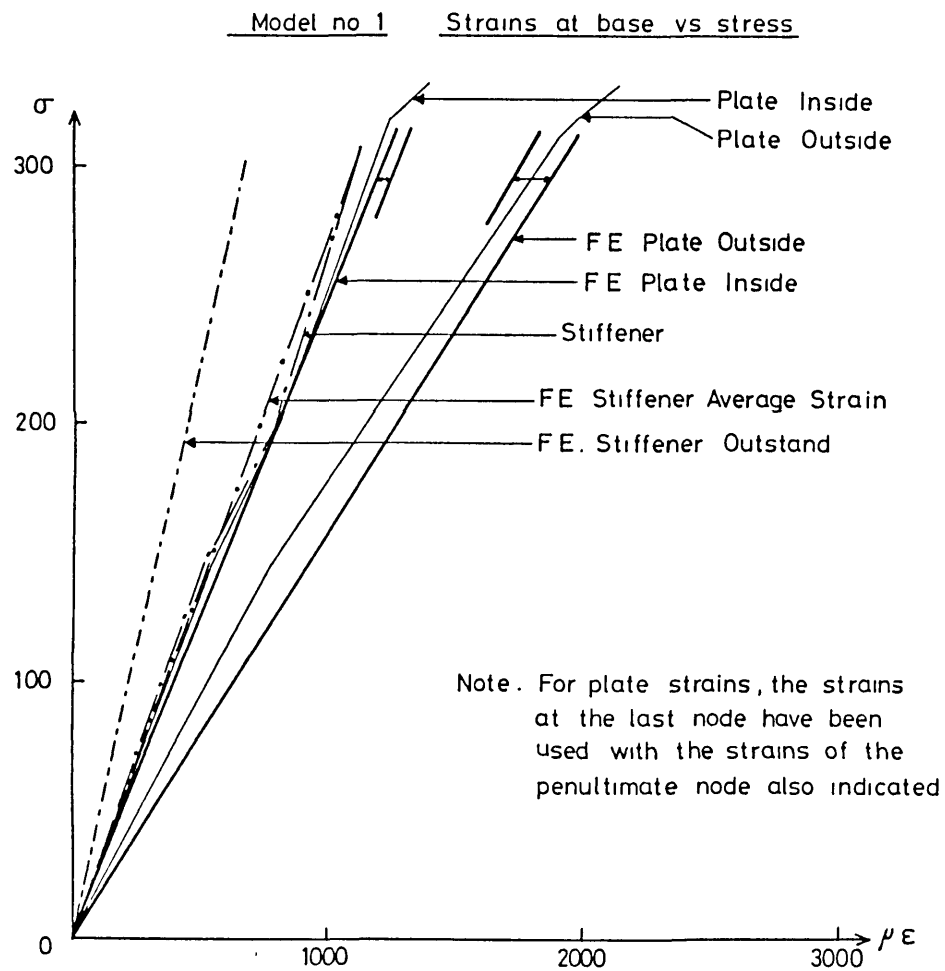


Figure 5.5

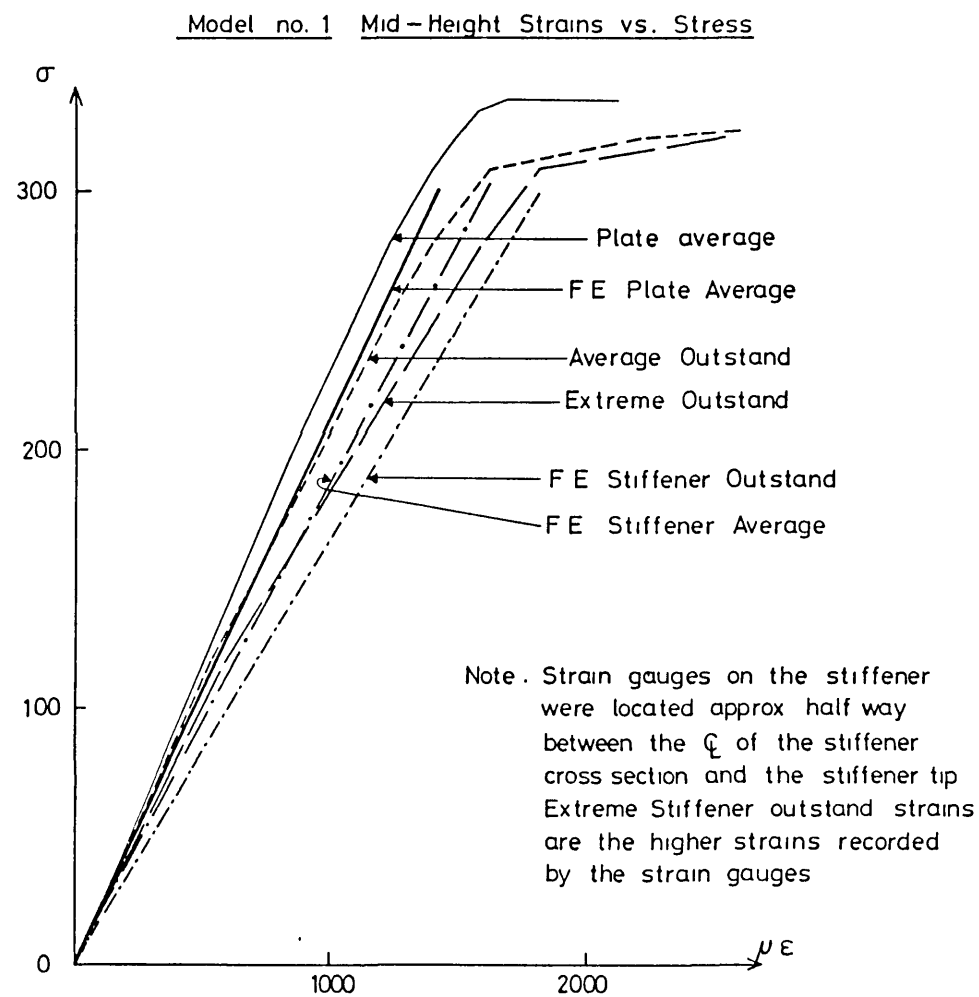


Figure 5.6

Model no 2 Strains at base vs stress

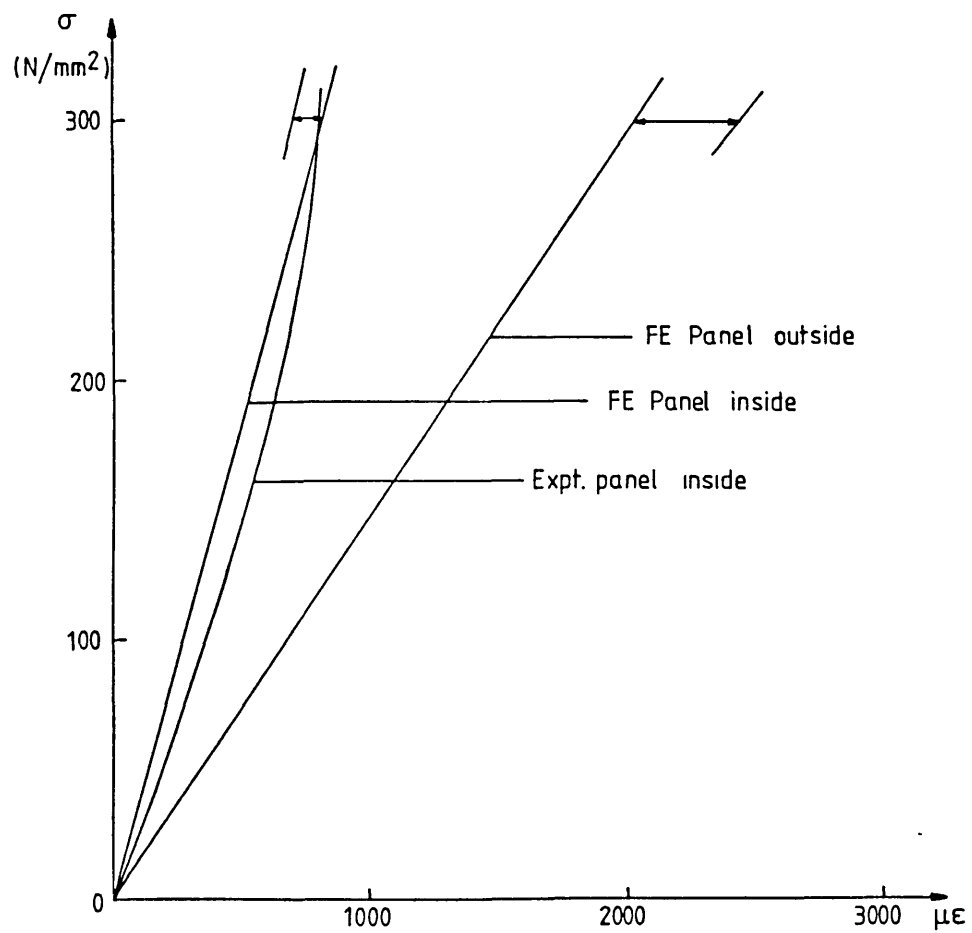


Figure 5.7

Model no.2 Strains at mid-height vs stress

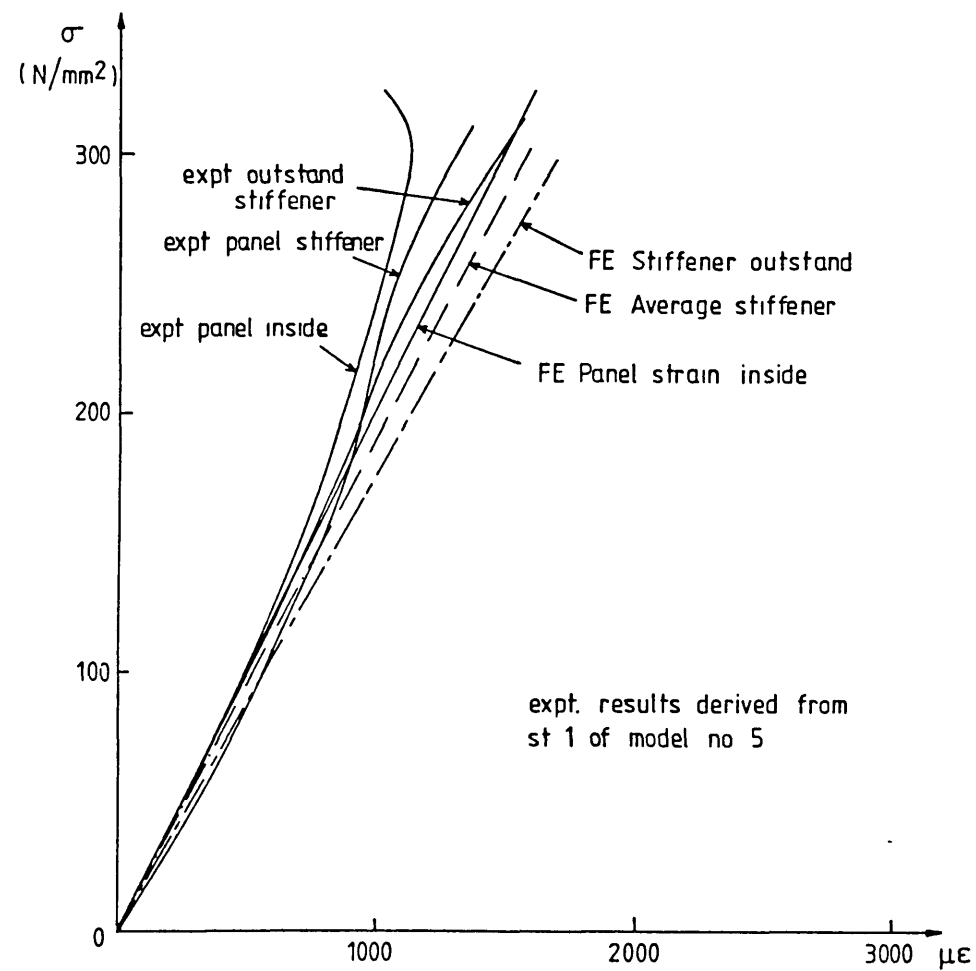


Figure 5.8

Model no 3 Strains at base vs stress

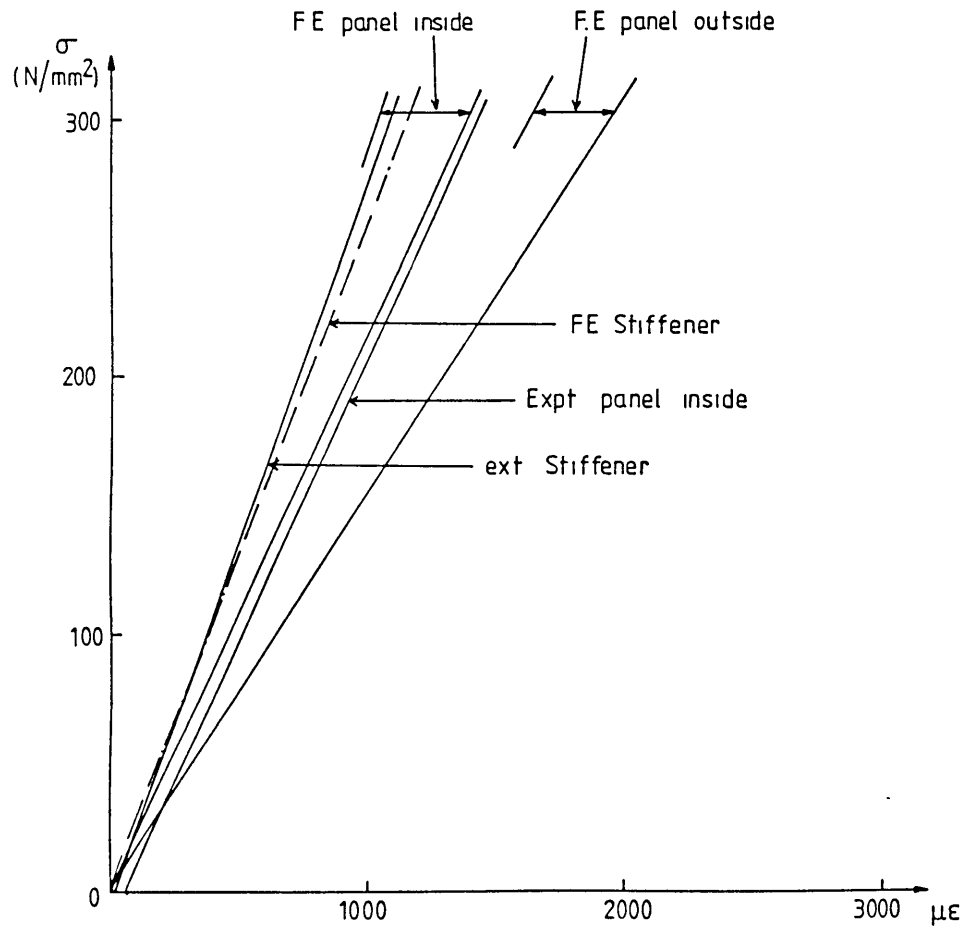


Figure 5 9

Model no 3 Mid - height strains vs. stress

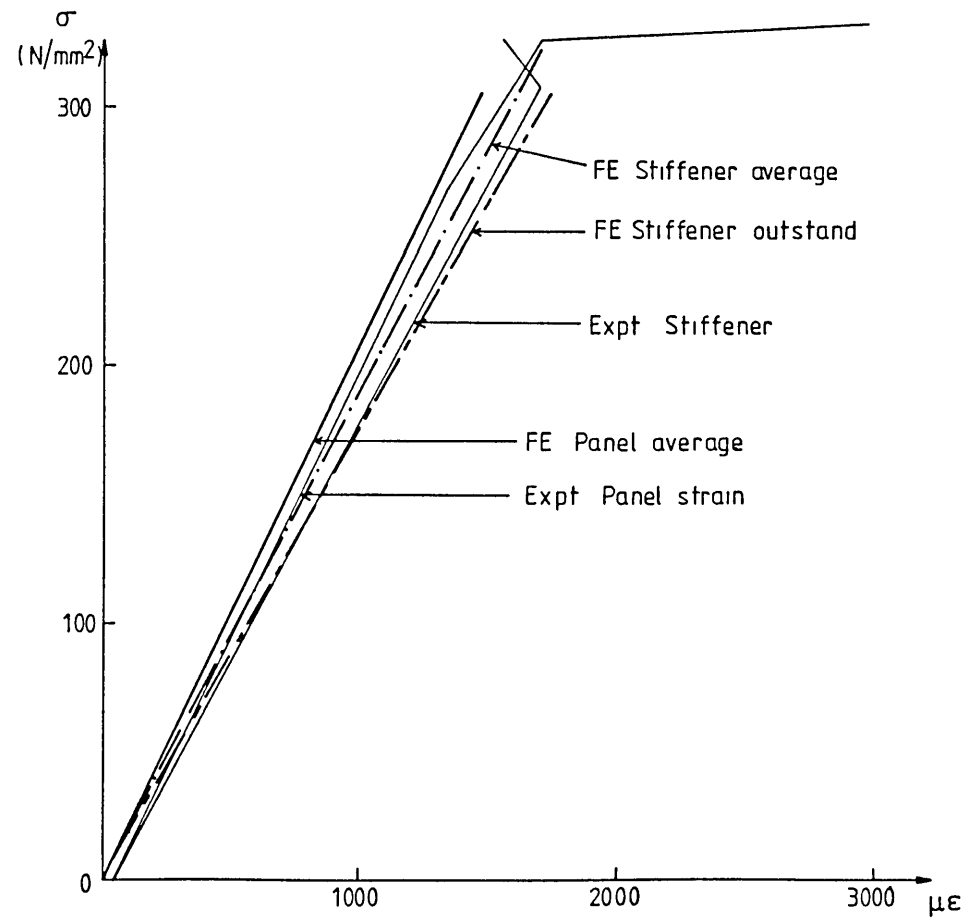


Figure 5 10

Finite Element Idealisation of Stiffened panel for Elasto-plastic Analysis

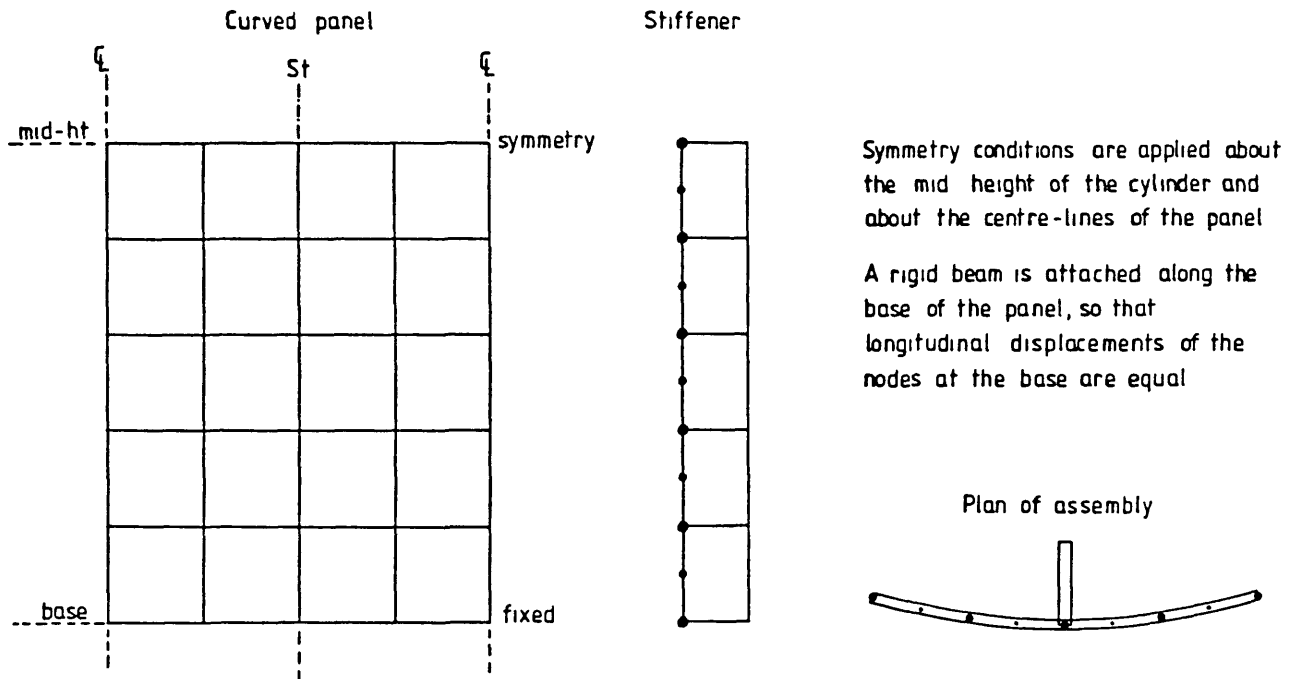


Figure 5.11

Model no.1 Plot of average stress-strain response for various finite element models

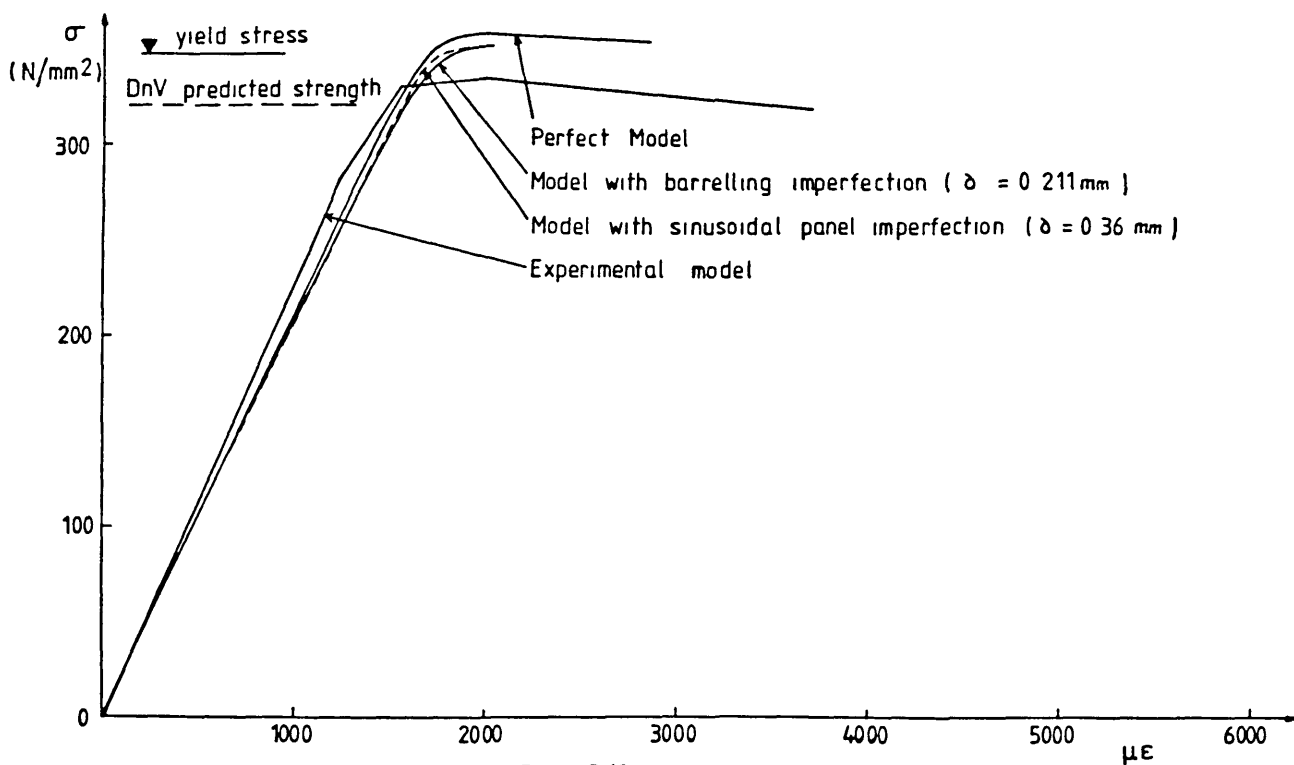


Figure 5.12

Model no 2 Plot of average stress-strain response for various finite element models

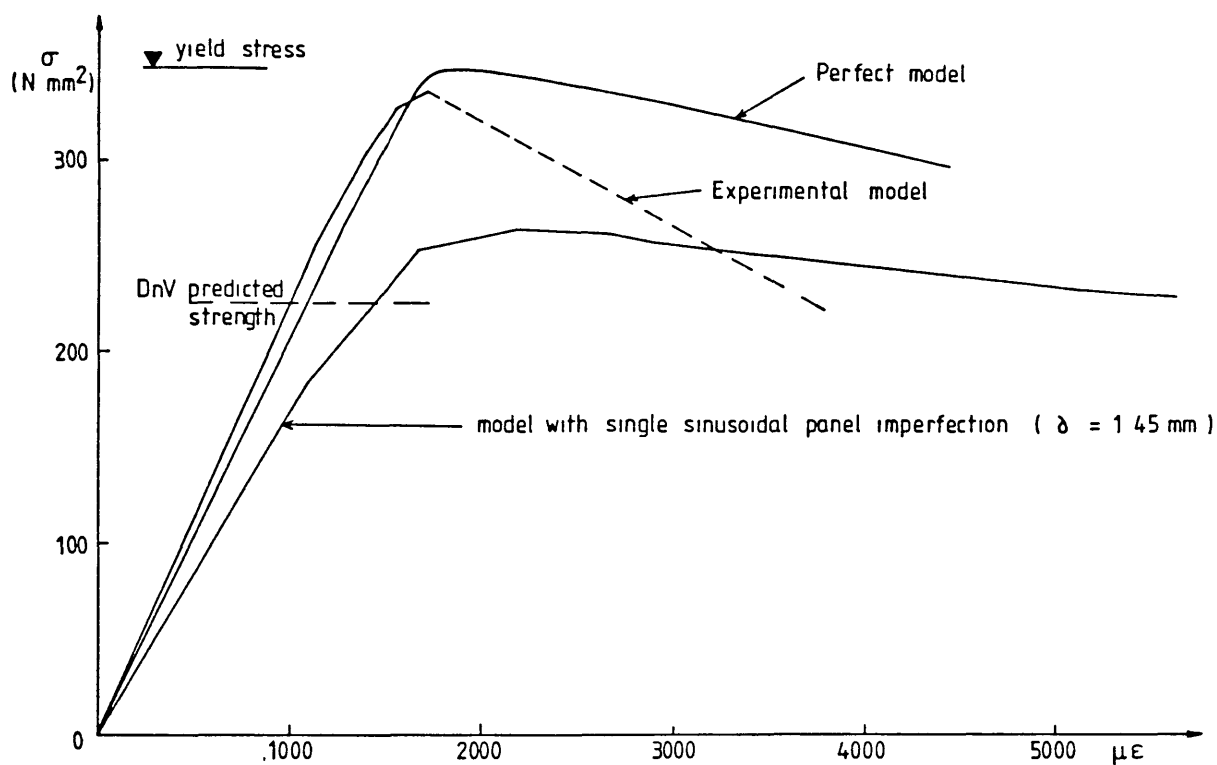


Figure 5 13

Model no 3 Plot of average stress-strain response for various finite element models

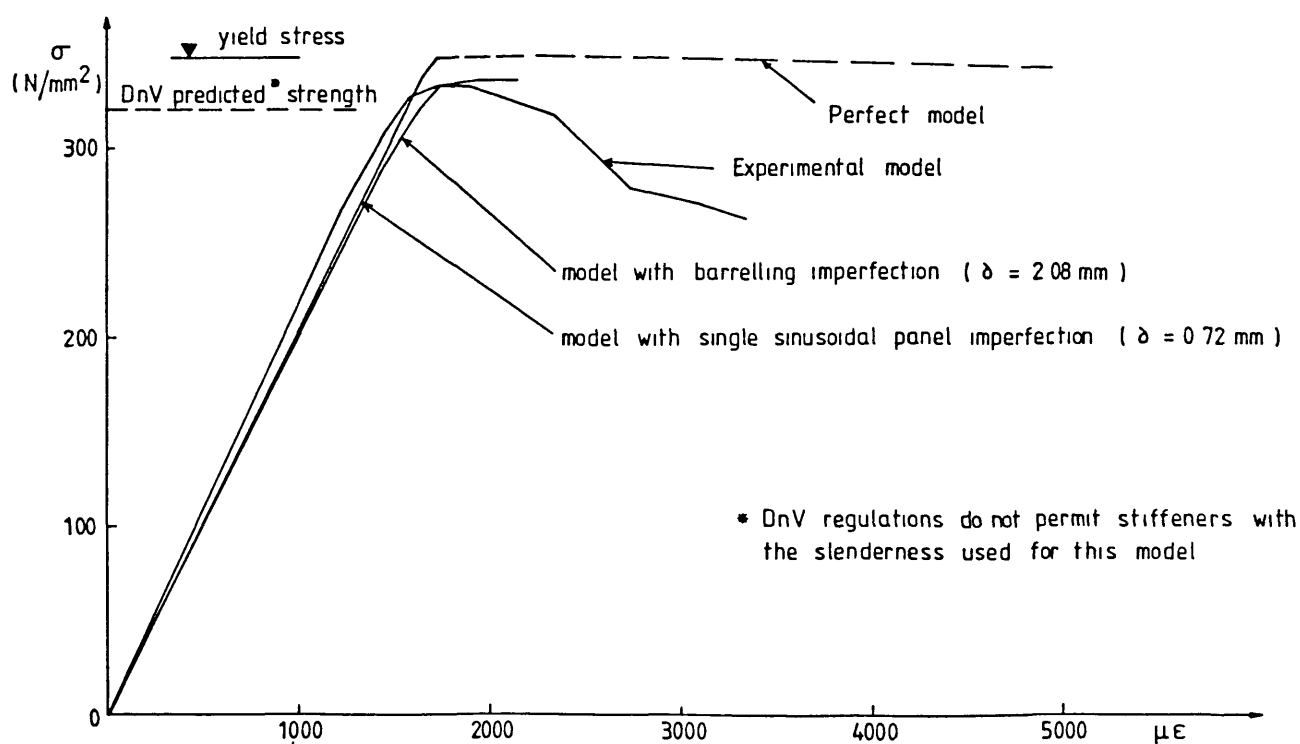


Figure 5 14

CHAPTER 6.

An Elastically Supported Beam-Column Analysis of Stiffened Cylinders with Narrow
Stiffener Spacing.

6.1 Introduction

6.1.1 Growth of sinusoidal imperfections under axial load.

The lateral deflection of the surface of a cylinder with any single sinusoidal imperfection can be shown to grow as

$$W_{\sigma} = W_0 / (1 - \sigma / \sigma_{CR}) \text{ ----- (1)}$$

where W_{σ} is the amplitude of the imperfection at the current stress, σ , W_0 is the initial amplitude of the imperfection and σ_{CR} is the critical stress for the imperfection mode (obtained by classical eigenvalue analysis).

It has been shown from the analysis of experimental observations of orthotropically stiffened cylinders with multiple initial imperfection modes that the imperfections associated with each mode grow separately, according to equation 1 (192).

6.1.2 Relevant critical stresses

Theoretical buckling analyses of the cylinders tested in the experimental part of this thesis have been discussed, §4.3. It has been shown that the panel buckling stress is critical. Accurate attempts to find the lowest buckling stress associated with the overall buckling mode were not successful, since the first four eigenvalues were associated with forms of the panel buckling mode.

Measurements of lateral deflections under load showed that the overall modes of buckling were not significant. Model behaviour was predominantly a combination of axisymmetric barrelling with local panel deflections superposed. Panel deflections were reasonably small for models with narrowly spaced stiffeners.

6.1.3 Elastically supported beam-column methods.

Simplified methods of analysis of cylinders with closely spaced

longitudinal stiffeners have been proposed by Valsgaard and Steen (184) and by Walker and Sridharan (114). Both papers apply equation 1 to establish a first yield criterion of failure for the cylinders considered, and are termed "elastically supported beam-column methods". The title may be better reserved for methods which consider the axisymmetric cylinder behaviour only.

It will be shown that standard elastically supported beam column methods may be applied to the analysis of some of the cylinders described in the experimental part of the thesis. The analysis is confined to cylinders with narrowly spaced longitudinal stiffeners. Standard results (231) may be used to establish a first yield failure criterion at mid-height of the cylinder.

6.2 Simple elastic model

6.2.1 Differential equations

The differential equation of equilibrium of an imperfect column on an elastic foundation is

$$EI \frac{d^4 w}{dx^4} - N \frac{d^2 w}{dx^2} + kw = N W_m \left[\frac{m\pi}{l} \right]^2 \sin \frac{m\pi x}{l} \quad \text{-----} \quad (2)$$

where w represents the displacement from the initial stress free position and

$$W = W_m \sin \frac{m\pi x}{l} \quad \text{-----} \quad (3)$$

is the initial stress free position.

Any particular solution of equation (2) may be combined with general solutions of

$$EI \frac{d^4 w}{dx^4} - N \frac{d^2 w}{dx^2} + kw = 0 \quad \text{-----} \quad (4)$$

to satisfy the boundary conditions of the column. The values of the coefficients in equations 2 and 4 for a longitudinally stiffened cylinder are

E = Young's modulus for the material

I = moment of inertia/unit width of a typical stiffened plate unit (figure 6.1).

N = axial force per unit width (positive in tension).

$k = Et/R^2$.

t = thickness of shell material.

R = radius of the shell.

A_e = cross sectional area per unit width.

6.2.2 General solutions to the differential equations.

Equation 2 is satisfied by a lateral displacement of the form

$$W_1 = W_m N/N_{CR(m)} / (1 - N/N_{CR(m)}) \sin \frac{m\pi x}{l} \text{ ----- (5)}$$

General solutions to equation 4 have been given in a form convenient for satisfying the boundary conditions of the column (231). These solutions have been given for reference in Table 6.1.

6.2.3 Boundary conditions

The boundary conditions at zero load are

$W = 0$ at $x = 0$ and $x = l$, and

$W' = W_0 \pi/l$ at $x = 0$ and $W' = -W_0 \pi/l$ at $x = l$

for an initial imperfection of the form

$$W = W_0 \sin \pi x/l \text{ ----- (6)}$$

These changes of slope must be counteracted by the end forces to maintain a zero change of overall slope at the ends.

An unrestrained cylinder under axial load will expand by an amount

$$\Delta R = - \frac{\nu R}{E} \frac{N}{A_e} \text{ ----- (7)}$$

where A_e = average thickness per unit width. The end expansion of a cylinder with heavy end rings may be taken as zero. This expansion of the cylinder may be accommodated within the differential equations by moving the origin of the reference axes of the cylinder by ΔR .

Forces P_1 and M_1 , figure 6.2, are chosen at the ends to satisfy

$$W_{x=0} = -\frac{\nu R}{E} \frac{N}{A_c} \quad \text{and}$$

$$W'_{x=0} = -W_0 \pi/l (N/N_{CR})/(1 - N/N_{CR})$$

The boundary conditions at $x = l$ are satisfied by symmetry. (Note the sign convention for forces and moments causes the inward radial direction to correspond with positive W).

$$\left. \begin{aligned} P_1 \cdot P_{11}(0, N) + M_1 \cdot P_{21}(0, N) &= -\frac{\nu R}{E} \frac{N}{A_c} \\ P_1 \cdot P_{12}(0, N) + M_1 \cdot P_{22}(0, N) &= -W_0 \pi/l (N/N_{CR})/(1 - N/N_{CR}) \end{aligned} \right\} \quad \text{--- (8)}$$

P_1 and M_1 are uniquely obtained from equation 8. The lateral deflection is

$$W = W_0 \cdot \frac{N/N_{CR}}{(1 - N/N_{CR})} \sin \frac{\pi x}{l} + P_1 \cdot P_{11}(x, N) + M_1 \cdot P_{21}(x, N) + \frac{\nu R}{E} \frac{N}{A_c} \quad \text{--- (9)}$$

6.2.4 Critical stresses

The analysis in this chapter is confined to the model nos. 1 and 3 referred to in previous chapters. The critical stress resultants of a model unit (figure 6.1) is

$$-N_{CR} = \frac{\pi^2 EI}{l^2} + \frac{k l^2}{EI} \quad \text{--- (10)}$$

for simply supported end conditions, and

$$-N_{CR} = \frac{4\pi^2 EI}{l^2} + 2\sqrt{kEI} \quad \text{--- (11)}$$

for clamped end conditions.

The initial imperfection assumed for each model is a single sine half-wave. Thus the relevant critical load for use in the above equations is obtained from equation 10. The critical compression stresses for model nos. 1 and 3 are 1500 N/mm² and 1025 N/mm² respectively.

6.2.5 Initial imperfections

The initial imperfections allowable in practical structures may be obtained from the DnV recommendations (68). These are

$$\delta g/t \leq \frac{1}{30} \sqrt{\frac{R}{t}} \quad \text{-----} \quad (12)$$

For a gauge length of $L_g = 4\sqrt{Rt}$

In general the beam length is longer than the gauge length. A sinusoidal imperfection as given in equation 3 will satisfy equation 12 if

$$W_0 = \delta g / \left[1 - \cos \left[\frac{\pi L_g}{2 L} \right] \right] \quad \text{-----} \quad (13)$$

The maximum imperfections for models 1 and 3 are $W_0 = .68$ mm and 4.78 mm respectively. These initial imperfections grossly overestimate the initial imperfections measured during experiments.

Initial imperfections of $W_0 = 0.125$ mm and .10 mm are relevant for models 1 and 3 respectively for comparison with the experimental results.

6.3 Linearised response to axial load

6.3.1 Linear equations

The critical stresses for model nos. 1 and 3 far exceed the working stresses and the material yield stress. As a result it is possible to suggest simpler linearised equations which ignore the non-linear effects of load on the boundary forces P_1 and M_1 . Table 6.2 gives the relevant deflections, slopes and moments for unit shear forces and moments applied symmetrically at the ends of a typical beam.

The lateral deflection of a beam-column may be given as

$$W(x) = \sigma(P_1.F11(x) + M_1.F21(x) - \frac{\nu R}{E}) + W_0 \cdot \frac{N/N_{CR}}{(1-N/N_{CR})} \sin \frac{\pi x}{l} \quad \text{-----} \quad (14)$$

Slopes and moments are obtained by differentiation of equation 14. (The derivatives of F_{11} and F_{21} are given in Table 6.2). P_1 and M_1 are calculated using equation 8 with $N = A_0$.

Figures 6.3-6 show the calculated stress profiles for model nos. 1 and 3 using both the non-linear and linear forces. There is very little error at 300 N/mm^2 for either model. Equation 14 and its derivatives will be used for the remaining discussion in this chapter.

6.3.2 Stress distribution at unit axial load

Equation 14 indicates that the beam-column response is an approximately linear function of load. It is thus satisfactory to study the behaviour at unit axial stress in compression, in order to obtain an understanding of model behaviour.

It has been shown in §5.3 that a linear finite element model accurately predicts the stresses in the experimental model when the initial imperfection along the generator is taken as zero. This linear finite element model will now be used as a standard for comparison with the elastic beam-column model. The panel and stiffener stresses at unit load are compared in figures 6.7,8 for model nos. 1 and 3. The panel and stiffener are assumed fully effective.

The beam-column model overestimates the stress in the stiffener at the end of the beam and underestimates the stress in the panel very considerably.

6.3.3 Effective panel width

Figure 6.9 shows the distribution of moment along the height of model nos. 1 and 3, due to unit axial stress in compression. A stiffened plate is seen to be subject to a strong clamping moment at the ends. The moment is zero at some point along the length.

It is suggested that a beam-column may be divided into three separate units (figure 6.10(a)):

- a) a central beam with zero end moments
- b) 2 cantilever beams symmetrically positioned at the ends of the beam column.

The central section may be treated as a simply supported beam under uniformly distributed load. The cantilevers are subject to a uniformly distributed load and a point load at the free end.

Stiffened beams which are subject to rapidly varying shear forces have been studied by Moffatt and Dowling (232). Stresses may be calculated using an effective flange width. The effective width of flange of the central and end beams differ and as a result differing moments of inertia must be used. Elastic forces and moments P_{X1} , M_{X1} , P_{X2} , M_{X2} due to a unit point load P_1 and moment M_1 are calculated, figure 6.10(b). Intermediate forces are obtained by imposing continuity of displacement and slope at P_2 for each of the unit forces at the beam-column ends.

A small computer program was written to carry out the calculation of stresses along the beam-column of variable inertia. Figures 6.11,12 show the stresses obtained, and are compared with stress distribution of the finite element model. The comparison is very satisfactory for panel stresses at the ends of both models.

6.3.4 A failure criterion

The highest stress at unit load is predicted in the plate at the ends of the model. The response is assumed linear; thus it is suggested that yield stress is first reached at the end of the model. Very little extra moment capacity is possible at the base once yield has been reached in the plate. The beam can, however, continue to carry load and it is suggested that a beam-column with plastic hinges at the ends, figure 6.13, describes the behaviour of the model.

The attainment of first yield in the stiffener outstand at mid-height is suggested as a failure criterion. This is a conservative estimate of failure but for slender stiffeners, e.g. model 3, it is appropriate. It is suggested that failure may extend through to 50% of the stiffener if the stiffener proportions are stocky.

6.3.5 Effective width of panel at collapse

The effective widths computed from (232) are relevant for elastic stress distributions only. Shear lag theory has not been developed for

elasto-plastic behaviour. At large compressive stresses the panel area between stiffeners may be treated as a clamped panel on rigid supports. The effective width approach has been used successfully when assessing the strength of such panels. It is suggested that the effective width of panel to be used in figure 6.13 is given to a good approximation by BS 5400 part 3 (229).

The effective width may be taken as uniform along the length of the model. There is not a very significant change of predicted load capacity when the effective flange width at the end of the beam is taken as the elastic effective width at initial failure.

A single linearised equation may be used to predict a failure stress.

$$\sigma_f = \frac{\sigma_y - M_p(F_{23}(l/2) + P_{12}.F_{13}(l/2))/Z}{(P_{11}.F_{13}(l/2) + (W_0 EI/\sigma_{CR}).(\pi/l)^2)/Z - 1} \quad (15)$$

where $P_{11} = \nu R/E \cdot 1/F_{11}(0)$

$P_{12} = -F_{11}(0)/F_{21}(0)$

W_0 = initial imperfection amplitude

Z = elastic modulus of the critical section.

$F_{11}(x)$ etc are evaluated using Table 6.2.

The plastic moment, M_p , at the end of the beam-column is dependent on the axial load. Thus equation 15 must be solved iteratively. A satisfactory solution may be obtained after very few iterations.

The distribution of moments at failure of model nos. 1 and 3 is given in figure 6.14. The stresses at failure are plotted in figure 6.15,16. It is assumed that model no. 1 can carry load until the elastic stress distribution in the stiffener indicates that yield has spread through 50% of the stiffener depth. First yield of the stiffener outstand is taken as the yield criterion of model no. 3. The elastic stress calculated in the plate at the ends of the model exceeds yield because a plastic moment has been assumed in this position.

Both figures 6.15 and 16 are based on zero initial imperfection. Table 6.3 gives the calculated average stress capacity at collapse for both models for 3 levels of initial imperfection: zero imperfection, the

measured imperfection, and the maximum imperfection allowed by reference (68).

6.4 Extension of analysis

6.4.1 The effect of ring stiffeners

The end effects on the beam-column are strongly influenced by the radial stiffness of the end rings. If the level of initial imperfection is small the end forces, P_1 and M_1 , are linearly dependent on the ring restriction of the Poisson expansion. Under these conditions P_1 and M_1 , are multiplied by a factor, f , where $f = k_2/(k_1 + k_2)$ and $k_1 = 1/F_{11}(0)$ and $k_2 = A_R E/R^2$.

P_1 and M_1 approach zero linearly as the ring area, A_R , approaches zero.

If the initial imperfection, W_0 , is not small, P_1 and M_1 must be obtained by solution of the following linear equations (for unit axial stress in compression).

$$\begin{aligned} (F_{11}(0) + R^2/A_R \cdot E) \cdot P_1 + F_{21}(0) \cdot M_1 &= \nu R/E \\ F_{12}(0) \cdot P_1 + F_{22}(0) \cdot M_1 &= -W_0 \pi / l \quad 1/\sigma_{CR} \end{aligned}$$

6.4.2 Exponential decay of end effects

The forces P_1 and M_1 are subject to exponential decay along the length of the beam. The decay factor is

$$e^{-\lambda x} \text{ where } \lambda = \left[\frac{t}{4R^2 I} \right]^{0.25} \text{-----} (16)$$

The effects of P_1 and M_1 have decayed to less than 10% their maximum value when $\lambda x = 0.75\pi$.

Thus the end effects are confined to a distance

$$l_1 \approx 3.5R (I/R^2t)^{0.25} \text{ ----- (17)}$$

at the ends of the model. The effects of the end forces are less than 8% the maximum values at distances greater than l_1 from the ends. Equation 15 is no longer a satisfactory failure criterion when

$$l/R > 7R (I/R^2t)^{0.25} \text{ ----- (18)}$$

6.4.3 The collapse of long models

Elastic beam-columns which satisfy equation 18 may be classed as long, and the effects of the end forces at each end are independent of one another. The maximum moment in the stiffener assembly is reached at

$$l_2 \approx 2.2R (I/R^2t)^{0.25} \text{ ----- (19)}$$

A plastic hinge will form in the stiffener at a distance l_2 from the end of the model. Two modes of collapse are possible for long beam-columns. A panel mechanism may form at a distance of l_2 from the ends of the cylinder when the stiffener sheds load.

Alternatively the stiffened panel assembly may be stable at the ends of the beam and failure will occur in the centre of the model through the growth of initial imperfections. It is suggested that an appropriate length of model for this form of collapse is $L_3 = (L - 2.l_2)$. Plastic hinges are considered at the ends of such a model.

6.4.4 Wide panelled models

A wide panelled model may be defined as a stiffened cylinder where the elastic lateral deflections of the panel are largely independent of the stiffener. Model no. 2 is clearly such a model, (figure 5.13). It is not possible to use the beam-column theory as stated above in the analysis of such models.

The elastically supported beam-column theory may be used to predict a collapse wavelength for the panel area.

Since the panel behaviour is assumed to be independent of the stiffener the beam-column theory is applied to the panel alone. The distribution of

moments due to the end effects is such as to cause a peak in moments at the model end and at a distance

$$\ell_4 \approx 1.2t\sqrt{\frac{R}{t}} \quad \text{from the end.}$$

These moment peaks will encourage the development of a panel collapse mechanism with an overall length of $2\ell_4$. The theory of curved panel collapse mechanisms is developed in chapter 8. It is suggested that one form of collapse of a wide panelled model is by the development of a panel mechanism at the model end. The stiffener may be included as a load carrying component in the mechanism if the stiffener can develop a full plastic hinge, i.e. for flat stiffeners $d_{st}/t \leq 9.6$. Where the stiffener proportions are such as to render it liable to tripping failure, the collapse load of the cylinder should be taken as that of the panel alone.

It should be emphasised that another mode of failure of all stiffened cylinders is through the growth of initial imperfections in accordance with equation 1.

$\ell_4 = 13.8$ mm for model no. 2. The measured distance of the hinge line in model no. 2 after collapse is, ≈ 13 mm, figure 2.42.

6.5 Conclusions

6.5.1 Narrow and wide panelled cylinders

The axisymmetric behaviour of a stiffened cylinder subject to axial load is considered. Non-linear and linear solutions are compared and the linear solution is found to be quite accurate. The behaviour of narrow-panelled cylinders, tested in the experimental part of the thesis, is shown to be strongly governed by the forces induced at the cylinder ends. The induced forces are caused by the prevention of Poisson expansion.

A satisfactory correlation of stresses at unit axial stress is obtained between a beam-column of variable moment of inertia and the linear finite element analysis presented in chapter 5. The variable moment of inertia results from different effective widths acting with a stiffener, due to the effect of shear-lag. Published results are used to calculate effective

widths.

A model for failure is suggested and the failure criterion is expressible in a single formula.

The effect of ring stiffeners on end forces is discussed and formulae given for modification of forces. A maximum length of cylinder for application of the theory is suggested.

End forces are shown to affect panel deformations of wide panelled cylinders. The end moments enforce a length over which a plastic mechanism may form. The predicted and observed lengths of panel mechanism correspond closely for model no. 2.

MODELS No 3 & 6
PLATE - STIFFENER UNIT

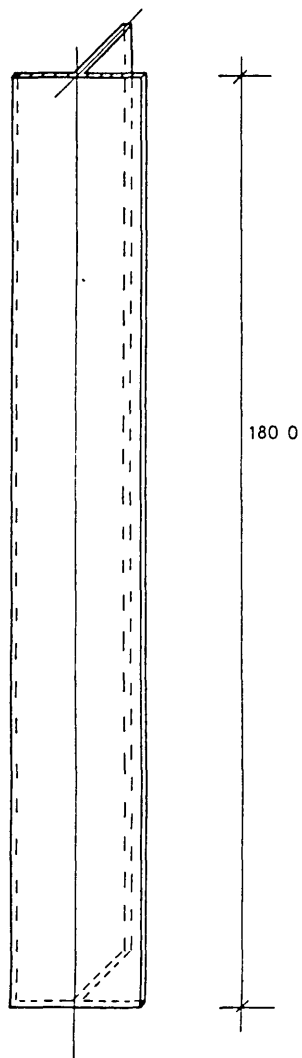


figure 6.1

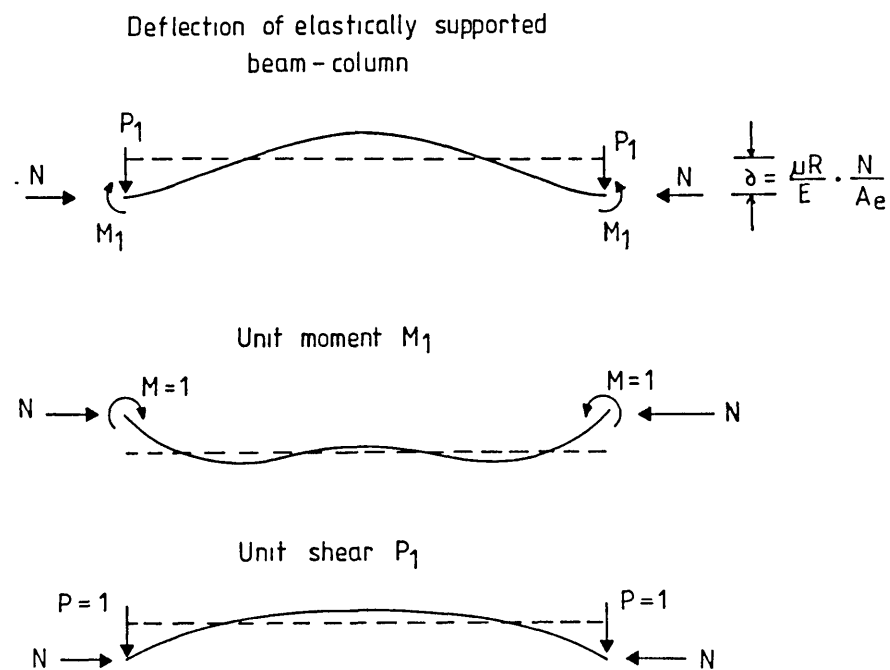
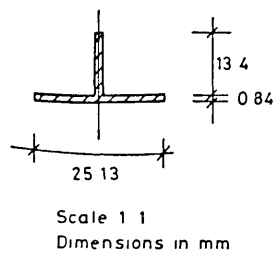
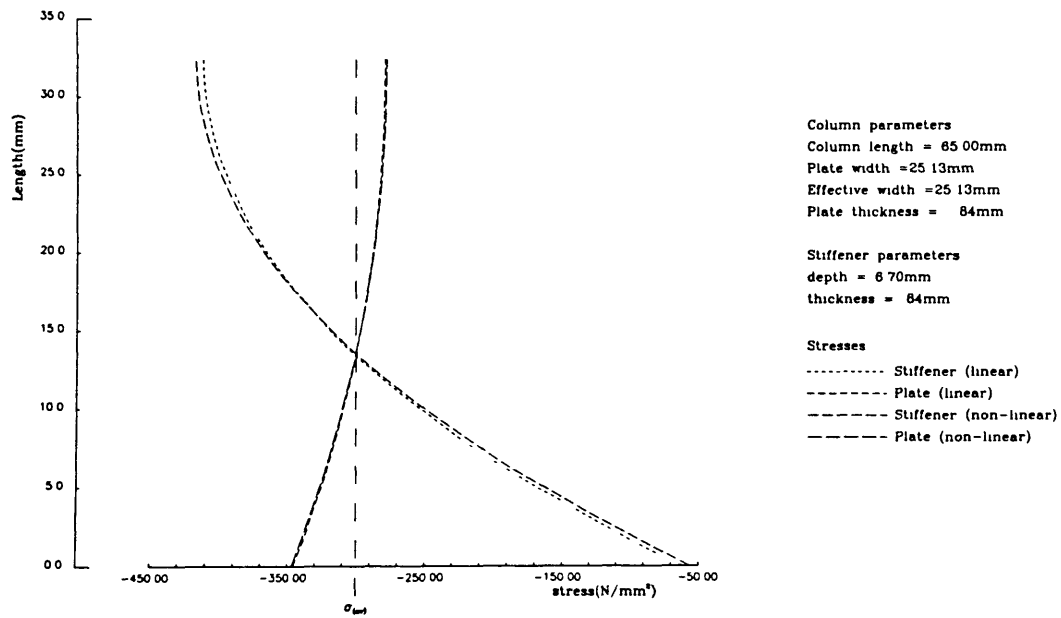
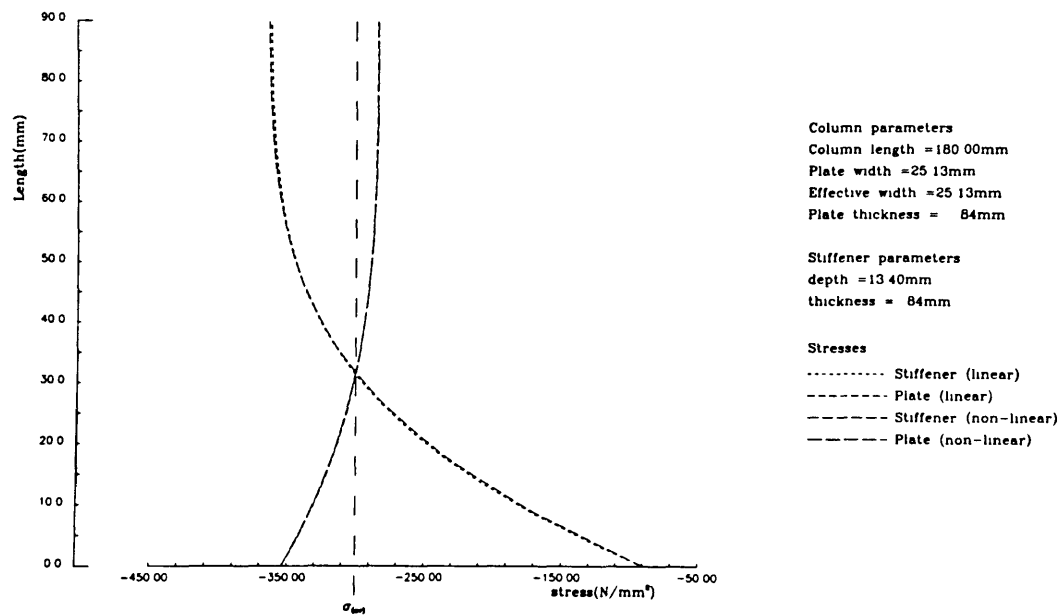


figure 6.2

Elastically supported beam-column under axial load
Column of constant inertia along length
A comparison of linear and non-linear behaviour



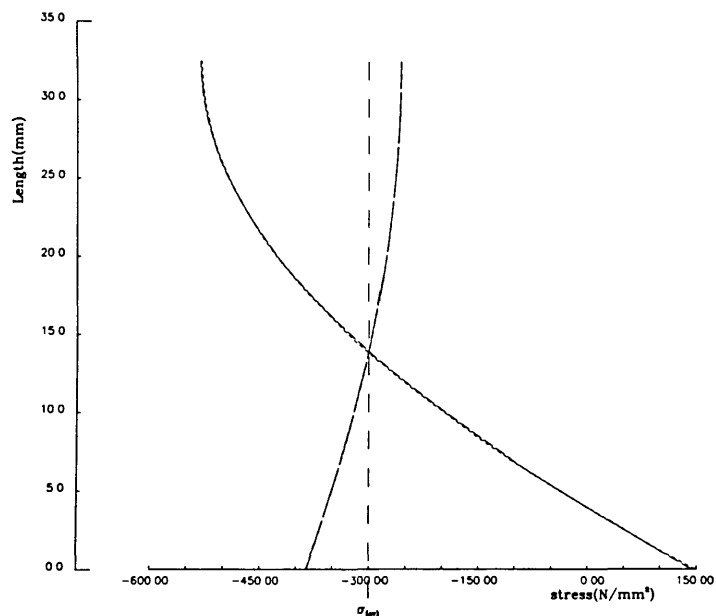
Elastically supported beam-column under axial load
Column of constant inertia along length
A comparison of linear and non-linear behaviour



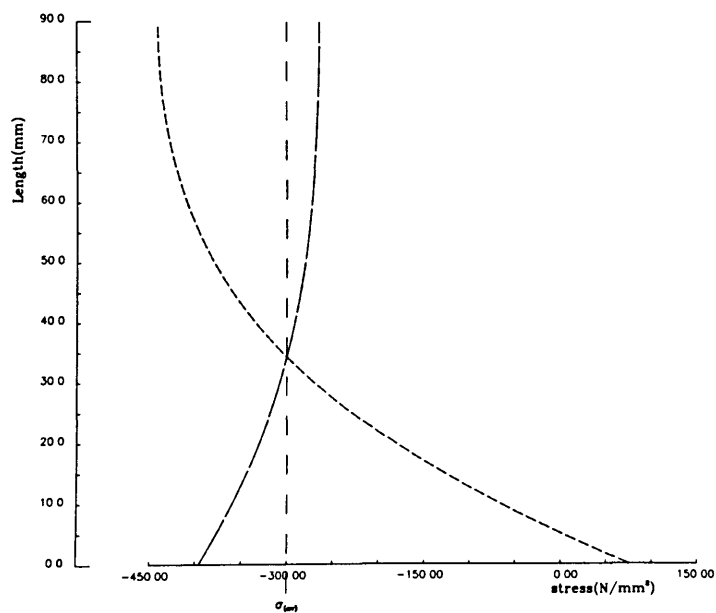
upper figure 6.3

lower figure 6.4

Elastically supported beam-column under axial load
 Column of constant inertia along length
 A comparison of linear and non-linear behaviour
 Maximum permitted initial imperfection



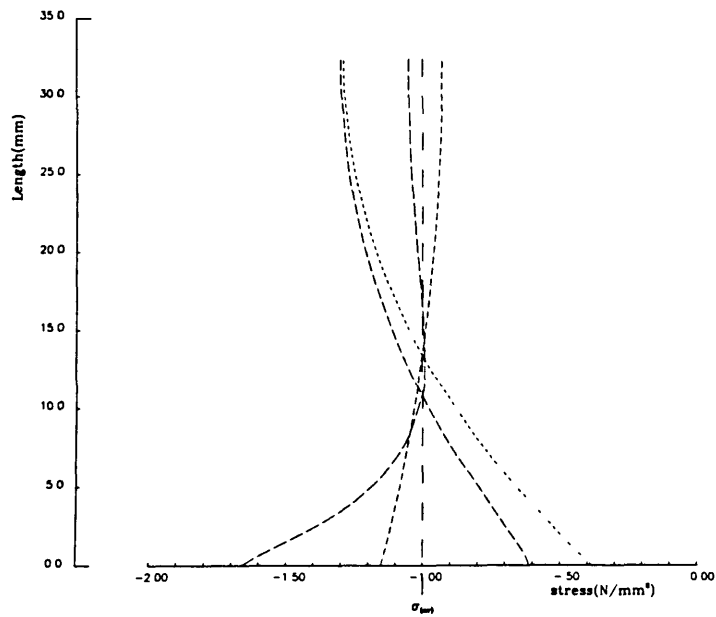
Elastically supported beam-column under axial load
 Column of constant inertia along length
 A comparison of linear and non-linear behaviour
 Maximum permitted initial imperfection



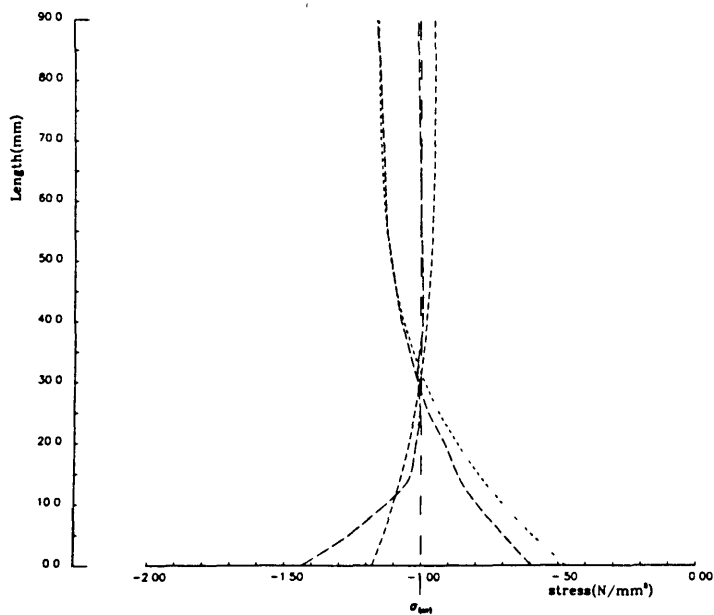
upper figure 6.5

lower figure 6.6

Elastically supported beam-column under axial load
 Plot of calculated stress against length
 A comparison of linear and finite element behaviour



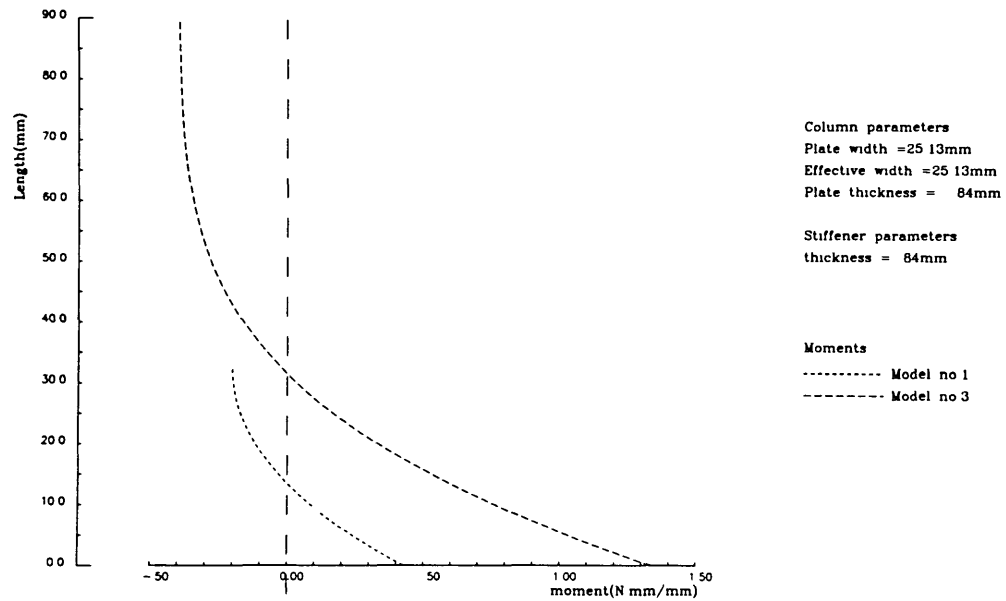
Elastically supported beam-column under axial load
 Plot of calculated stress against length
 A comparison of linear and finite element behaviour



upper figure 6.7

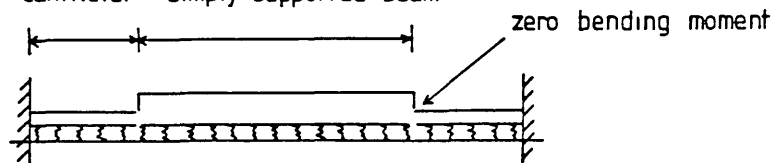
lower figure 6.8

Elastically supported beam-column under axial load
 Column of constant inertia along length
 Plot of calculated moment against length

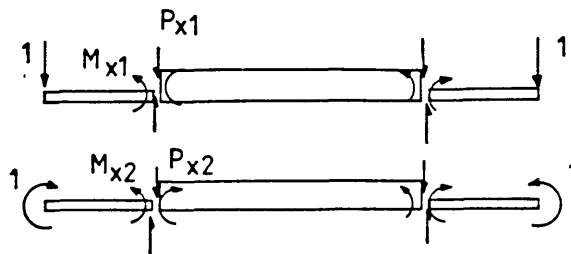


Beam of variable inertia

cantilever simply supported beam



6.10 (a)

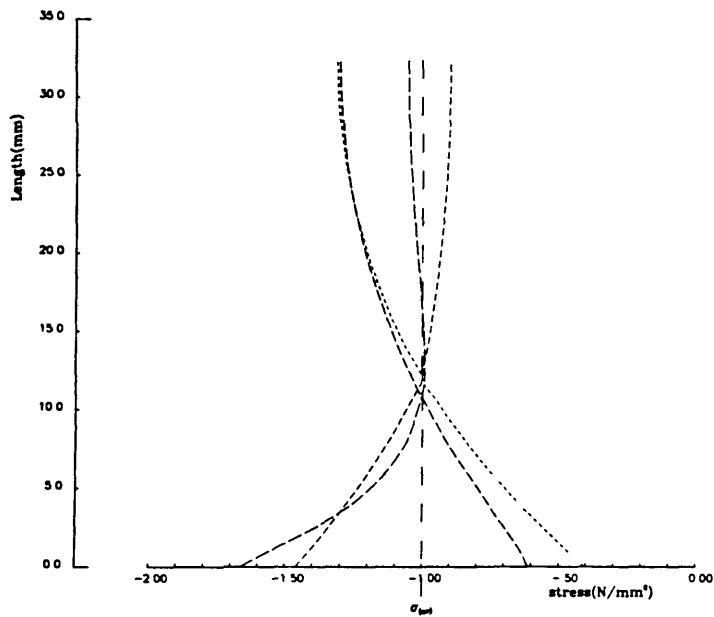


6.10 (b)

upper figure 6.9

lower figure 6.10

Elastically supported beam-column under axial load
 Plot of calculated stress against length
 A comparison of linear and finite element behaviour
 (Effective width calculated by shear-lag theory)

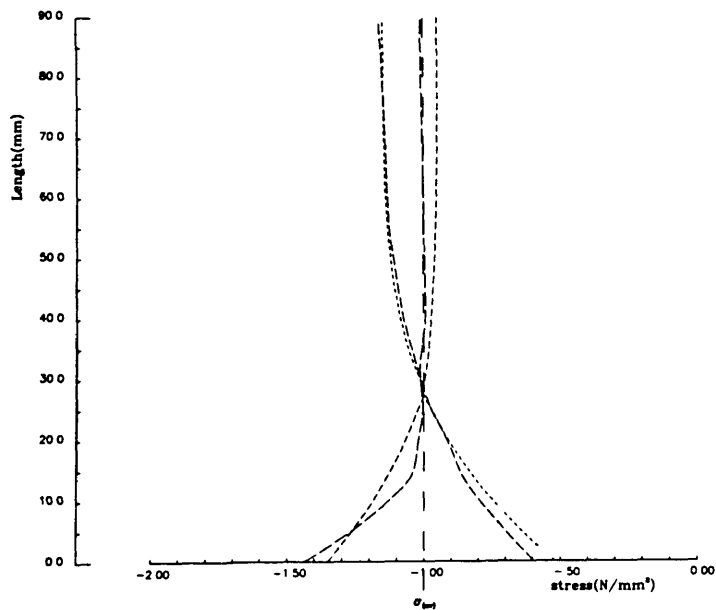


Column parameters
 Column length = 65 00mm
 Plate width = 25 13mm
 Plate thickness = 84mm

Stiffener parameters
 depth = 6 70mm
 thickness = 84mm

Stresses
 Stiffener (beam-col)
 ----- Plate (beam-col)
 -.-.-.- Stiffener (fin-elm)
 _____ Plate (fin-elm)

Elastically supported beam-column under axial load
 Plot of calculated stress against length
 A comparison of linear and finite element behaviour
 (Effective width calculated by shear-lag theory)



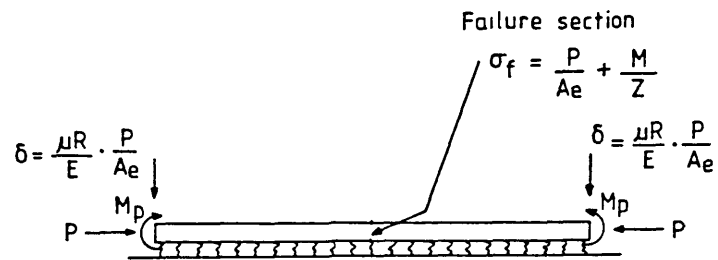
Column parameters
 Column length = 180 00mm
 Plate width = 25 13mm
 Plate thickness = 84mm

Stiffener parameters
 depth = 13 40mm
 thickness = 84mm

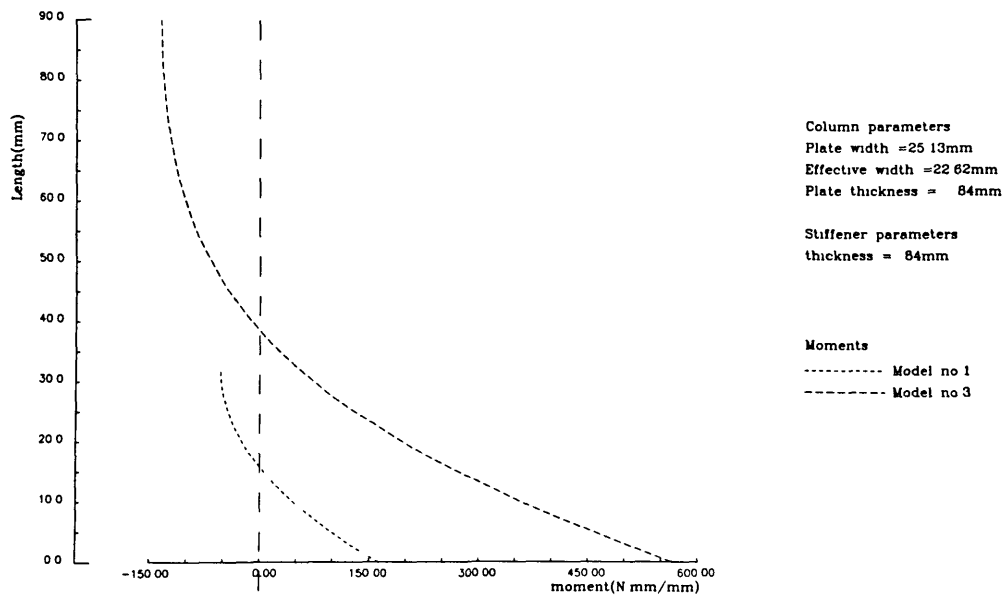
Stresses
 Stiffener (beam-col)
 ----- Plate (beam-col)
 -.-.-.- Stiffener (fin-elm)
 _____ Plate (fin-elm)

upper figure 6.11

lower figure 6.12



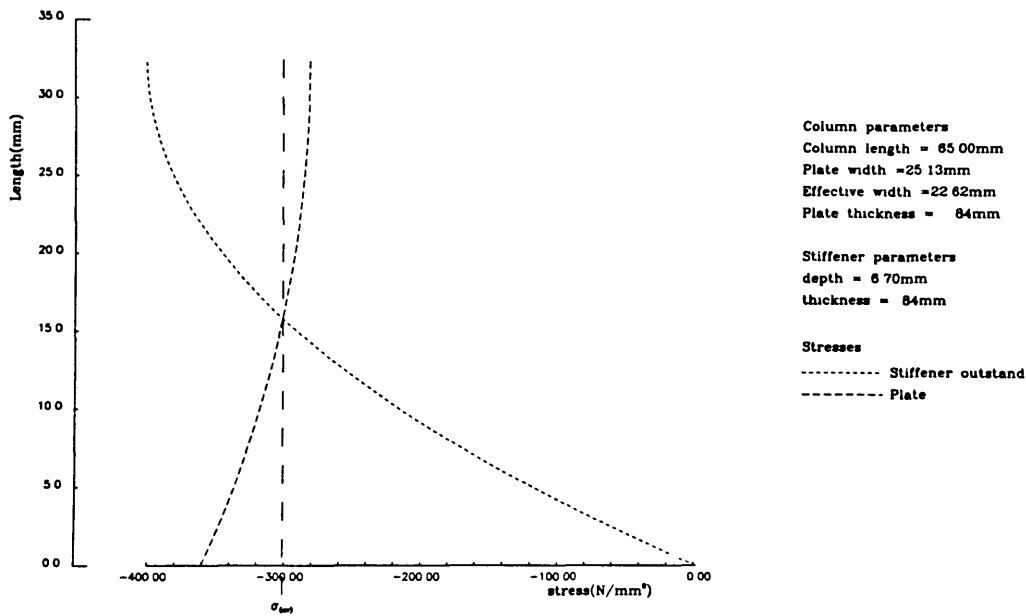
Elastically supported beam-column under axial load
 Column of constant inertia along length
 Plot of moment distribution at predicted failure load



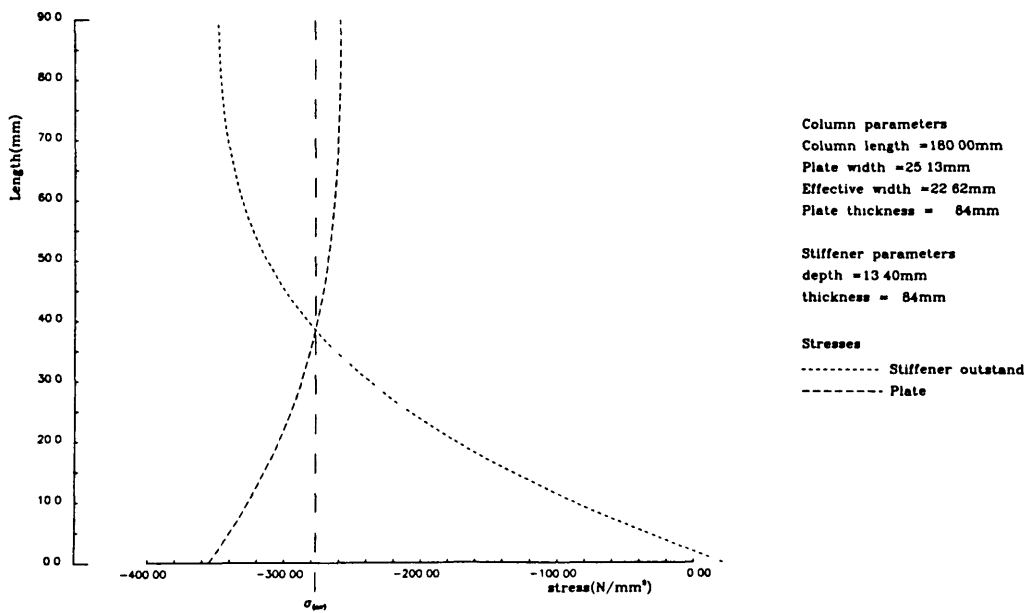
upper figure 6.13

lower figure 6.14

Elastically supported beam-column under axial load
Column of constant inertia along length
Plot of stress distribution at predicted failure load
Failure criterion Stiffener 50% plastic



Elastically supported beam-column under axial load
Column of constant inertia along length
Plot of stress distribution at predicted failure load
Failure criterion First yield at mid-height



upper figure 6.15

lower figure 6.16

$$D_1 = \beta(3\alpha^2 - \beta^2) \sinh\alpha l + \alpha(3\beta^2 - \alpha^2) \sin\beta l$$

$$f_2 = 1/(2\lambda^2 EI.D_1) \quad ; \quad f_3 = 1/(EI.D_1)$$

$$\alpha^2 = \lambda^2 + N/4EI \quad ; \quad \beta^2 = \lambda^2 - N/4EI$$

$$X' = l - X$$

Equal unit shear forces at ends of beam-column.

$$P11(X,N) = f_2 [2\alpha\beta(\cosh\alpha X.\cos\beta X' + \cos\beta X.\cosh\alpha X')]$$

$$+(\alpha^2 - \beta^2)(\sinh\alpha X.\sin\beta X' + \sin\beta X.\sinh\alpha X')]$$

$$P12(X,N) = (\alpha^2 + \beta^2)f_2[\beta.\sinh\alpha X.\cos\beta X' + \alpha.\cosh\alpha X.\sin\beta X'$$

$$-\alpha \sin\beta X.\cosh\alpha X' - \beta.\cos\beta X.\sinh\alpha X']$$

$$P13(X,N) = (\alpha^2 + \beta^2)^2 f_2.[\sinh\alpha X.\sin\beta X' + \sin\beta X.\sinh\alpha X']$$

Equal unit moments at ends of beam-column.

$$P21(X,N) = f_3[\alpha(\cosh\alpha X.\sin\beta X' + \cosh\alpha X'.\sin\beta X)$$

$$-\beta(\sinh\alpha X.\cos\beta X' + \sinh\alpha X'.\cos\beta X)]$$

$$P22(X,N) = f_3[(\alpha^2 - \beta^2)(\sinh\alpha X.\sin\beta X' - \sinh\alpha X'.\sin\beta X)$$

$$+2\alpha\beta(\cosh\alpha X'.\cos\beta X - \cosh\alpha X.\cos\beta X')]$$

$$P23(X,N) = f_3[(\alpha^3 - 3\alpha\beta^2)(\cosh\alpha X.\sin\beta X' + \cosh\alpha X'.\sin\beta X)$$

$$+(\beta^3 - 3\alpha^2\beta)(\sinh\alpha X'.\cos\beta X + \sinh\alpha X.\cos\beta X')]$$

Table 6.1 (Reference (231))

$$f_1 = 1/(\sinh \lambda l + \sin \lambda l)$$

$$f_2 = 2\lambda/k.1/(\sinh \lambda l + \sin \lambda l)$$

$$f_3 = -2\lambda^2/k.1/(\sinh \lambda l + \sin \lambda l)$$

$$X' = l - X$$

Equal unit shear forces at ends of beam

$$F_{11}(X) = f_2(\cosh \lambda X \cdot \cos \lambda X' + \cosh \lambda X' \cdot \cos \lambda X)$$

$$F_{12}(X) = \lambda \cdot f_2(\sinh \lambda X \cdot \cos \lambda X' + \cosh \lambda X \cdot \sin \lambda X' \\ - \sinh \lambda X' \cdot \cos \lambda X - \cosh \lambda X' \cdot \sin \lambda X)$$

$$F_{13}(X) = -1/\lambda \cdot f_1(\sinh \lambda X \cdot \sin \lambda X' + \sinh \lambda X' \cdot \sin \lambda X)$$

Equal unit moments at ends of beam.

$$F_{21}(X) = f_3(\sinh \lambda X \cdot \cos \lambda X' - \cosh \lambda X \cdot \sin \lambda X' \\ + \sinh \lambda X' \cdot \cos \lambda X - \cosh \lambda X' \cdot \sin \lambda X)$$

$$F_{22}(X) = 2\lambda f_3(\cosh \lambda X \cdot \cos \lambda X' - \cosh \lambda X' \cdot \cos \lambda X)$$

$$F_{23}(X) = f_1(\sinh \lambda X \cdot \cos \lambda X' + \cosh \lambda X \cdot \sin \lambda X' \\ + \sinh \lambda X' \cdot \cos \lambda X + \cosh \lambda X' \cdot \sin \lambda X)$$

Table 6.2 (Reference (231))

Predicted Overall Failure Stresses (N/mm²)

	Imperfection level		
	zero	measured	maximum
model no. 1			
first yield	271 (279)	261 (269)	220 (227)
50% st plast	292 (301)	285 (294)	256 (264)
model no 3			
first yield	269 (277)	267 (275)	213 (219)

Axial stress in beam is given in brackets (based on effective width = 0.9 × plate width)

Table 6 3

CHAPTER 7.

Collapse Mechanism Theory: A General Development and Application to Initially
Flat Panels.

7.1 Introduction

7.1.1 Discretisation of a structure

Engineers are familiar with the division of a continuous structure into a finite number of discrete areas or elements in the application of the finite element method. In essence this method

a) Attempts to represent accurately the infinite number of degrees of freedom of a continuum by a finite number of degrees of freedom of well chosen elements. The accuracy of the representation depends on both the number and sophistication of the elements.

b) Each finite element has specific limited freedoms which are uniquely related to the strain throughout its volume.

c) A constitutive law relates strains to stresses.

d) The principle of virtual work is used to compute the internal strain energy of the element in response to external forces. The internal strain energy is summed at representative points throughout the element.

The ideal finite element combines accuracy in structural representation with speed (and hence cost) of computation. A few elements may suffice when the overall response of a structure is required. Many elements may be necessary where the detailed local response to overall loading of a structure is required.

7.1.2 Localised nature of collapse

It is generally sufficient to know the overall response of a structure at collapse load. In addition, structures tend to collapse by forming localised areas of extreme deformation, suggesting that most of the internal strain energy absorption of the structure occurs at relatively few places in the structure.

For example, a single portal frame, subject to a wide range of loads, will collapse by forming hinges at some combination of the five positions in figure 7.1(a)(233). An obvious discretisation of the structure at collapse is the division of the frame into four elements, figure 7.1(b). The elements may be regarded as rigid away from the nodes which are located at the element ends. Strain energy at collapse need only be computed at the nodes.

7.1.3 Collapse of a panel

An unreinforced concrete panel under lateral load will collapse by forming yield lines, figure 7.2(a)(234). A yield line is defined as an area where gross deformations occur. The area is of considerable extent in one direction, but of negligible width perpendicular to that direction. A yield line is formed by the relative movement of two rigid adjoining areas, or elements.

An unreinforced panel has only one degree of freedom, namely the central lateral deflection, δ_c , figure 7.2(b), and the simplified stress state along a yield line is $M = M_0$.

A two-dimensional rectangular steel panel exhibits a similar yield line pattern of failure when subject to combinations of lateral and inplane loading. The stress state at a point on a yield line of a steel panel is considerably more complicated than that of an unreinforced concrete panel. The shear and overall axial stresses must be considered, in addition to the moments. Thus inplane displacements must be considered, in addition to the central lateral displacement.

7.1.4 Scope of chapter

The treatment of panel collapse by means of collapse mechanisms is not entirely new and the contributions of various authors have been discussed in §1.7.3. The work of Dean (218-220) in particular has been noted and will be used in the development of this chapter. The particular manner in which Dean's work is presented does not make it very suitable for more general applications.

Collapse mechanism theory may be more easily understood and more widely used if it is seen as a particular application of finite element theory. This approach will be used in this chapter and is a substantially new contribution to the collapse mechanism method. Displacements and rotations are treated directly through standard kinematic relationships and the basic degrees of freedom of the structure are retained intact throughout. As a result Dean's fundamental kinematic relationships are not used.

The collapse mechanism method differs substantially from the elastic (and elasto-plastic) finite element method in so far as a first solution cannot be obtained through the inversion of a stiffness matrix. The degree of freedom variables are highly non-linear, however, the solution method is similar to linear programming problems and techniques. (These have been used to establish the minimum collapse loads of large rectangular frameworks).

The numerical problems encountered in establishing acceptable collapse mechanism results are considerable and major emphasis will be placed in this chapter on understanding the problems and in defining their solutions. Numerical accuracy will also be discussed.

A particular form of numerical problem which required a new approach is that of a so-called redundant yield line or lines. Redundant yield lines will be defined in a general form and one particular application, that of an inplane yield line, will be fully discussed in this chapter. (The more general form of redundant yield line will be discussed in §8.10.). The concept of a redundant yield line is introduced as new in this thesis; the author is, however, aware of a particular use of an inplane yield line by Dean (219) in establishing the collapse load of an unrestrained two-dimensional panel under inplane load. The particular example given by Dean did not require the special treatment required for inplane yield lines in general.

A new first order of magnitude correction to the virtual work equations will be introduced through consideration of the elastic strain energy release of the structure at collapse. This will be shown to have a significant effect on the calculated collapse loads.

The theory and results presented in this chapter will be developed for two-dimensional rectangular panels (i.e. panels which in their "perfect" state are two dimensional). Results for aspect ratios of 1.0 and 3.0 will be given and both uniaxial and lateral loading conditions considered. A number of comparisons with elasto-plastic finite element solutions will be presented.

The collapse mechanism results for uniaxial loading will be used in combination with known elastic panel results to predict the strength of panels with arbitrary initial imperfections of a sinusoidal shape. These results in particular may be used for panel design.

Substantial numerical work not immediately relevant to the main continuity of ideas in this chapter (and chapter 8) have been placed for convenience in appendices. The results derived in these appendices are nonetheless important to the development of the thesis.

7.2 Discretisation of yield lines

7.2.1 Gauss points

The stresses along a yield line will not, in general, be constant. The stresses and virtual work are computed at discrete points along a yield line, and summed to give the virtual work of the line. Points are generally chosen according to the Gaussian Quadrature rule and are called gauss points (240).

The relative displacement between adjacent elements is the displacement of a gauss point. For convenience the gauss points are located on one side of a yield line, figure 7.3(a). The gauss point displacement is

$$\bar{\delta}_g = \bar{\delta}_B - \bar{\delta}_A \text{ ----- (1)}$$

7.2.2 Local reference axes for gauss points

The displacement, δ_g , may be resolved into components in a right-handed local reference frame at a gauss point, figure 7.3(b).

The frame of reference is always chosen, so that w is normal to the surface and v is parallel to the yield line. u is normal to the yield line.

7.3 Kinematics of a point on a yield line

7.3.1 A linear relationship between global degrees of freedom and local displacements

Let the point G be a typical point on a yield line formed by elements A and B , figure 7.4. The ordinates of G , relative to an axis set on the element A , are $\bar{x}_a = (x_{ga}, y_{ga}, z_{ga})$. The origin of the local axes in the element, relative to the global axes, is given by $\bar{x}_A = (X_A, Y_A, Z_A)$. The matrix $[B_A]$ expresses the local axes system in terms of the global axes.

Movement of G due to the movement of the element A is the sum of the displacement of G relative to the local reference axes in the element A , and the displacement of the local origin relative to the fixed global axes.

The element is rigid, hence there is no translation relative to the local origin of the element. Rotations may occur.

If $\bar{\theta}_A$ is the rotation of the element in the global reference axes, the displacement of G due to the rotation is

$$\bar{\theta}_A \times ([B_A^T][\bar{x}_a] + \bar{x}_A)$$

where $\times$ denotes the standard vector cross product.

The translation of the element in the global axes system is $\bar{\delta}_A$.

The relative displacement of G , in the global axes is

$$\bar{\delta}_{g1} = -\bar{\delta}_A - \bar{\theta}_A \times ([B_A^T][\bar{x}_a] + [\bar{x}_A]) \text{ ----- (2)}$$

Similarly the relative displacement of G , due to the movement of element B is

$$\bar{\delta}_{g2} = \bar{\delta}_B + \bar{\theta}_B \times ([B_B^T][\bar{x}_b] + [\bar{X}_B])$$

and the total relative displacement is

$$\begin{aligned} \bar{\delta}_g = \bar{\delta}_B - \bar{\delta}_A + \bar{\theta}_B \times ([B_B^T][\bar{x}_b]) - \bar{\theta}_A \times ([B_A^T][\bar{x}_a]) \\ + \bar{\theta}_B \times [\bar{X}_B] - \bar{\theta}_A \times [\bar{X}_A] \text{ ----- (3)} \end{aligned}$$

The relative rotation at G is

$$\bar{\alpha}_G = \bar{\theta}_B - \bar{\theta}_A \text{ ----- (4)}$$

Let $[B_G]$ express the local direction cosines of the yield line at G in terms of the global axes. Then the relative displacement and rotation at G is given in terms of the local axes as

$$\begin{aligned} \bar{\delta}_{gl} = [B_g][\bar{\delta}_g] = [B_g][\bar{\delta}_B] - [B_g][\bar{\delta}_A] \\ + ([B_g][\bar{\theta}_B]) \times ([B_g][B_B^T][\bar{x}_b]) \\ - ([B_g][\bar{\theta}_A]) \times ([B_g][B_A^T][\bar{x}_a]) \\ + ([B_g][\bar{\theta}_B]) \times ([B_g][\bar{X}_B]) \\ - ([B_g][\bar{\theta}_A]) \times ([B_g][\bar{X}_A]) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{ ----- (5)}$$

and $\alpha_{gl} = [B_g][\bar{\theta}_B] - [B_g][\bar{\theta}_A]$

Equation (5) establishes the linear relationship between the global degrees of freedom $\{X_A, X_B, \theta_A, \theta_B\}$ and the local displacements $[\delta_{gl}, \alpha_{gl}]$ at a point on a yield line.

7.4 The general D'U_{ij} matrix

7.4.1 Definition of D'U_{ij} matrix

Each of the general degrees of freedom of a structure has an effect on the relative displacements at the gauss points on the yield lines. A unit infinitesimal displacement, composed of translations and rotations may be linearly related to the local translations and rotations at a point, as shown in equation 5. The effect of all the general degrees of freedom of the structure, Δ_j , on the local displacements at the gauss points, l , may be written in simple tensor form

$$\delta'u_l^l = D'U_{lj}^l \cdot \Delta_j \text{ ----- (6)}$$

where i refers to the i^{th} component of displacement at the point, ℓ .

7.4.2 Relevant terms of $D'U_{ij}$

$\delta'u_i$ = has six components in general

$$\delta'u_i = (\delta_u, \delta_v, \delta_w, \alpha_u, \alpha_v, \alpha_w).$$

Only the components $(\delta_u, \delta_v, \alpha_v)$ are important when all the elements of the structure are planar. The remaining terms are either zero, (δ_w, α_u) , or are not relevant for determining the stresses at a point (α_w) .

The compatibility of movement is not perfect along the entire length of a curved yield line. In such problems the relative displacement component, δ_w , is retained. Curved yield lines arise when considering the collapse of shells.

Thus for plates

$$\delta'u_i = [\delta_u, \delta_v, \alpha_v] \text{ ----- } 7(a)$$

and for shells

$$\delta'u_i = [\delta_u, \delta_v, \delta_w, \alpha_v] \text{ ----- } 7(b)$$

7.5 Constitutive relationships

7.5.1 Dean's constitutive relationships

Dean (218) has given two satisfactory constitutive relationships for the stress state at a point on a yield line, Table 7.1. Both relationships express the stress state (n, s, m) in terms of the relative displacements of the adjacent elements (or strain rates) at the collapse of the structure. The reference axes for stresses and displacements are the local axes at the yield line, figure 7.3(b).

Table 7.1(a) gives the stress state at a point, based on an assumed

Table 7.1(a) gives the stress state at a point, based on an assumed rectangular stress block distribution, figure 7.5. Shear stress is allowed for by a reduction in the direct yield stress to $\sigma_y' = \sigma_y \sqrt{1-s^2}$.

The expressions in Table 7.1(b) are derived using an assumed linear strain rate distribution. Stresses and strain rates are related by means of the Prandtl-Reuss flow rule.

7.5.2 Numerical and physical meaning of the strain rate

The absolute magnitudes of the displacements at a yield line, and hence also the strain rates at a yield line, are unknown. Only the relative magnitudes of the strain rates are of importance in determining the stresses.

In physical terms, the displacements of a structure at collapse are time dependent. Thus the relevant displacement rates are described as relative velocities. The expression is useful as an analogy, but should not be taken as being entirely physically relevant.

7.5.3 Strain Rate - stress resultant relationship

The relative strain rates, or relative velocities, are related to the stress resultant at a point by a direct unique relationship

$$\text{Thus } F_i^l = H_i \left\{ \delta' u_i^l \right\} \text{-----} \quad (8)$$

$$\text{where } F_i = \left[\sigma_y t. n, \frac{1}{\sqrt{3}} \sigma_y t s, \frac{1}{4} \sigma_y t^2 m \right]$$

and (n, s, m) are obtained through Table 7.1.

7.5.4 Convention for shear stress resultants

The shear displacements, (δ_v, δ_w) for a curved yield line are added vectorily for the purposes of stress calculation.

$$\text{Thus } \delta_v(eq) = \sqrt{\delta_v^2 + \delta_w^2}$$

Stresses are then computed from the velocity vector

$(\delta_u, \delta_v(eq), \alpha_v)$, and the stress resultants are taken as

$$F_i = [\sigma_{yt}.n, \frac{1}{\sqrt{3}} \sigma_{yt}.s. \delta_v/\delta_v(eq), \frac{1}{\sqrt{3}} \sigma_{yt}.s. \delta_w/\delta_v(eq), \frac{1}{2}\sigma_{yt}^2.m]$$

The component $1/\sqrt{3} \sigma_{yt}.s. \delta_w/\delta_v(eq)$ is found to be small in practice. The displacement δ_w will in fact result in a combination of shear and locally increased curvature over a narrow band width on either side of the yield line. Within the limits of the theory, where the zone of plasticity is assumed to be of negligible width, only shear deformations can be accommodated.

7.6 Internal virtual work

7.6.1 Use of $D'U_{ij}$ matrices to obtain plastic internal virtual work

Equation 8 is a non-linear relationship between displacement rates and the stress state at a point. Let the general velocities of the global degrees of freedom be $\dot{\Delta}_j$ at the collapse state.

Equation 8 may be re-written as

$$F_i^l = H_i \{ D'U_{ij}^l . \dot{\Delta}_j \} \text{ ----- (9)}$$

The displacements at l , due to the unit displacements of the general degrees of freedom, are $D'U_{ij}^l$. Hence the internal virtual work at the point l is expressible in tensor form as

$$dw_K^l = H_i \{ D'U_{ij}^l . \dot{\Delta}_j \} . D'U_{ik}^l \text{ ----- (10)}$$

following the usual convention of repeated subscripts and superscripts to denote summation, inside and outside the $\{ \}$ brackets.

The total internal virtual work may be expressed as

$$w_K = wt^l H_i \{ D'U_{ij}^l . \dot{\Delta}_j \} . D'U_{ik}^l \text{ ----- (11)}$$

where wt^l is the weighting of the point l .

7.7 External virtual work

7.7.1 Convention adopted for external virtual work

Virtual work is done by the external forces on the structure due to a set of unit displacements along the general degrees of freedom of the structure.

The external virtual work is

$$R_k + \lambda \cdot p_k \text{ ----- (12)}$$

where R_k is the work done by some constant force system and p_k is the work done by a force system of variable magnitude, λ .

"Equilibrium" of the structure at collapse is satisfied when

$$R_k + \lambda p_k = wt \sum H_i \{ D' U_{ij}^L \cdot \dot{\Delta}_j \} \cdot D' U_{ik}^L \text{ ----- (13)}$$

7.8 The Choice of velocity set

7.8.1 Convention adopted for velocity set

The solution vector, $\dot{\Delta}_j$, must satisfy equation 13. The forces, equation 9, depend only on the relative magnitudes of the components of $\dot{\Delta}_j$.

A convention is adopted for $\dot{\Delta}_j$.

$$\dot{\Delta}_j \cdot \dot{\Delta}_j = 1.0 \text{ ----- (14)}$$

In practice this normalisation of $\dot{\Delta}_j$ proved successful. Another convention, which is equally acceptable, in theory, is

$$|\dot{\Delta}_{j(\max)}| = 1.0 \text{ ----- (15)}$$

The solution velocity set Δ_j was found by an iterative process, the success of which depended on the initial guess of $\dot{\Delta}_j$. The initial guess for a deformed state of a structure was found by extrapolation

of the known solution velocities of adjacent states of the structure. The use of equation 15 resulted in unnecessary discontinuities in the velocity solution sets for the loading of panels. These discontinuities made extrapolation difficult. Figure 7.6 shows the velocity set satisfying equation 15 plotted against central lateral deflection for a two-dimensional panel under inplane loading.

7.9 Solution of the virtual work equations

7.9.1 Numerical procedure to set up Jacobian

The solution velocity set, $\dot{\Delta}_j$ is dependent on the geometry of the structure, the boundary conditions and the state of deformation, for any given external loading system. $\dot{\Delta}_j$ is, however, independent of any previous strain state of the structure.

The internal virtual work of the system has N degrees of freedom. Equation 13 is not satisfied in general.

A residual vector Γ_k may be introduced.

$$\Gamma_k = W_k - R_k - \lambda P_k \text{ ----- (16)}$$

where $\lambda = (W_k - R_k) \cdot P_k$

and $P_k \cdot P_k = 1.0$

The internal virtual work, W_k , may be varied by variation of $\dot{\Delta}_j$.

Let $\dot{\Delta}_j^l = \dot{\Delta}_j^1 + \epsilon_j^l$ be any velocity vector,

where $\dot{\Delta}_j^1 =$ some initial guess at a satisfactory $\dot{\Delta}_j$

and ϵ_j^l is an n-dimensional vector.

If the terms of ϵ_j^l are small, the velocity vector $\dot{\Delta}_j^l$ may be used to compute a Jacobian of the residuals, Γ_k , for the initial vector $\dot{\Delta}_j^1$.

It has been noted above that only the relative magnitudes of $\dot{\Delta}_j$ are of importance.

$$\text{Thus } H_i \left\{ D'U_{ij} \cdot \dot{\Delta}_j \right\} = H_i \left\{ D'U_{ij} \cdot (\alpha \dot{\Delta}_j^1) \right\} \text{ ----- (17)}$$

for any $\alpha > 0$

Thus a variation $\epsilon_j = \alpha \dot{\Delta}_j$ will produce no change in the residual vectors Γ_k .

Thus ϵ_j can only produce a Jacobian of rank $n-1$ while there are n degrees of freedom. This problem can be dealt with in one of two ways.

a) One term of ϵ_j , say ϵ^m , is set to zero. $n-1$ columns of the Jacobian matrix are then found from the $n-1$ degrees of freedom of ϵ_j .

The term ϵ^m is reintroduced as a variable which produces a residual $\Gamma_k^m = \epsilon^m \cdot p_k$. The residual matrix Γ_{kl} , composed of the $n-1$ real variations in residual work and the imposed residual Γ_k^m , spans the vector space of W_k and is used as a Jacobian of rank n for estimating a better value of $\dot{\Delta}_j$.

b) All the terms of ϵ_j are allowed to vary. A variable ϵ^{n+1} is introduced and a residual $^*\Gamma_k^l = \Gamma_k^l + \epsilon^{n+1} \cdot p_k$ is computed for any $\dot{\Delta}_j^l = \dot{\Delta}_j + \epsilon_j^l$. A value $S^l = \dot{\Delta}_j \cdot \dot{\Delta}_j - 1$ is calculated. S^l is taken as the $n+1$ term of the residual associated with $\dot{\Delta}_j^l$ i.e. $^{**}\Gamma_k^l = \Gamma_k^l, S^l$.

There are now $n+1$ independent terms of the variable ϵ_j and the residual $^{**}\Gamma_k^l$ produces a Jacobian of rank $n+1$. It is thus possible to find a new guess of Δ_j which reduces the error Γ_k .

The second technique was adopted because the necessity of specifying externally the unchanging term, ϵ^m , could lead to additional poor conditioning of the Jacobian matrix.

The second method was easy to apply and because the artificial variable, ϵ^{n+1} , introduces linear terms in the Jacobian, the extra degree of freedom did not greatly increase the computational effort.

7.9.2 Solution algorithms

Many algorithms exist in library form for the solution of non-linear simultaneous equations. The author used three different library routines (235,236,237) on different occasions. The in-house routine

(237) proved to be the most successful and in some loading cases for panels was the only routine which could obtain a satisfactory solution for $\dot{\Delta}_j$. The iterative techniques for obtaining a solution rely heavily on some form of the Newton-Raphson technique for the solution of a non-linear equation. The technique computes a Jacobian matrix for the current "best" velocity set, $\dot{\Delta}_j$. The Jacobian matrix is prone to ill-conditioning for certain stress states, and the presence of cut-off points may also cause difficulties. The solution methods adopted to overcome these difficulties are discussed more fully, with specific reference to two-dimensional panels, in §7.15. The in-house routine (237) does not work directly with the Jacobian, and so does not fail when the Jacobian is ill-conditioned.

7.10 In-plane and lateral loading of an initially flat rectangular panel

7.10.1 Definition of flat panel mechanism

The matrix $D'U_{ij}^0$ has been evaluated for a two-dimensional, i.e. initially flat, panel in appendix 7A1. The symmetry of external inplane and lateral loading allows the structure to be represented by four elements, figure 7.7. A maximum of ten degrees of freedom and six yield lines are possible in the structure.

External virtual work is possible for any of the degrees of freedom. In practice, the freedoms dy_1 , dy_4 , $d\theta_3$ and $d\theta_4$ are necessary only to satisfy the equilibrium of yield line 4 and no external work occurs for these freedoms.

Yield lines 1 and 2 are required for every problem and lines 3 and 5 are required where the edges of the panel are restrained. Yield line 4 is not necessary for failure modes involving no lateral deflection of the supports. It may be described as a redundant yield line for such problems. The yield line does, however, become active at certain loading states. Redundant yield lines are discussed more fully in §7.14.

The numerical techniques, results and conclusions are discussed with reference to the geometry and collapse mechanisms shown in figure 7.7.

7.11 Solution techniques

7.11.1 Computerisation of solution strategy

A computer program was written to evaluate the matrices $D'U_{ij}^0$ in Table 7A1.3. The matrices were evaluated at chosen gauss-points on the yield lines. The program computed the internal virtual work, equation 11, and the residual work Γ_k , equation 16, for any velocity set $\dot{\Delta}_j$. A library algorithm was used to obtain a solution velocity $\dot{\Delta}_j$ to equation 13.

7.11.2 Limiting the range of problem considered

The terms of the matrices $D'U_{ij}^0$ are dependent on the lateral deflections h_0 and h_1 . $h_0 = 0.0$ has only been considered in this thesis, although the program accommodated general values of h_0 . Successive values of h_1 were used by the program to obtain continuous results for a panel over the range of central lateral deflection, $0 \leq h_1 \leq 5.0t$.

7.11.3 Initial guesses at velocity set

The solution velocity $\dot{\Delta}_j$ satisfied equation 14. The initial guess for any lateral deflection, h_1 , was found by extrapolation of the three previous solutions for adjacent values of h_1 . Extrapolation was by means of a best-fit straight line. If fewer than three previous solutions were known, the available solutions were used for extrapolation.

The technique was ideal for closely spaced values of h_1 . The slopes of the solution velocities satisfying equation 14 were usually continuous and thus the extrapolation technique usually gave a very good initial guess for the required solution.

Extrapolation was only possible when at least one previous solution was known. Another technique was required in order to find a solution when the first guess was very inaccurate. An algorithm was written whereby the external load was gradually changed from that external

load in equilibrium with the current best velocity estimate, to the actual external load on the panel. The steps chosen could be subdivided at two inner levels if a solution could not be obtained at any stage. It was thus always theoretically possible to move through the solution vector space of $\dot{\Delta}_j$ in a controlled manner until a solution was obtained to the correct loading at the current deformed state.

In practice, this technique worked well. It could not, however, be guaranteed to find a solution. It was necessary on occasions, to manually change the external loading vector from a well conditioned loading system, to which a solution velocity vector could readily be found, to the desired loading.

7.12 Choice of stress function for results

7.12.1 Square stress block and Prandtl-Reuss flow based stress functions

A square-block stress function or a yield function based on the Prandtl-Reuss flow rule may be used to relate stresses to velocities (Table 7.1).

The Prandtl-Reuss based function assumed zero change of strain rates along the yield line (218). Thus $\partial \epsilon_v / \partial v = 0.0$. This "plane strain" state is often not satisfied. Allowance is made in Table 7.1(b) for a variation in yield line strains by setting the constant, f_p , to unity. This is equivalent to a reduction in the maximum direct stress from $2/\sqrt{3}\sigma_y$ to $1.0\sigma_y$.

When f_p is reduced to unity there is a relative error between the shear and direct stresses. The effect of similarly reducing the maximum shear stress from $1/\sqrt{3}$ to 0.5, to eliminate the relative error, is small. Figure 7.8 shows the effect of the reduction of shear yield stress for a constrained panel under lateral load.

The effect of varying the stress function from a "plane strain" function at lines of constant stress to a "plane stress" function at lines of varying stress was found to be small. Lines of varying stress (line 1, figure 7.7) made the greatest contribution to the virtual work of the panel. In general Dean's formulation, with $f_p = 1.0$ and no allowance

for the discrepancy between shear stress and direct stress, was used for the results presented in this thesis.

7.12.2 Comparison of two stress functions

A comparison is made between strengths obtained from the two stress functions, Table 7.1(a) and Table 7.1(b) for a constrained panel under lateral load and under uniaxial inplane loading, figures 7.9 and 10. The results compare quite well.

7.12.3 Stress function adopted

The stress state derived from the Prandtl-Reuss flow rule is a better theoretical stress state than that derived from the square-block stress state. Unless stated to the contrary, the results presented in this paper have been obtained using the former method, and with $f_p = 1.0$.

7.13 Choice of convergence criteria

7.13.1 Nag, Harwell and inhouse routines

Δ_j is accepted as a solution of equation 13 when the residual work terms Γ_k , equation 16, have become sufficiently small. The non-linear equation solution algorithms have an inbuilt form of total error evaluation.

Nag (235) and Harwell (236) routines estimate the total error as Γ_i . $\Gamma_i = (\epsilon)^2$

The current estimate is accepted as the correct solution when $\epsilon < \epsilon_1$.

The inhouse routine (237) uses a criterion $|\Gamma_i| < \epsilon_2$ for all values of i , for acceptance of the current estimate as the correct solution. ϵ_1 and ϵ_2 are supplied by the user in every case.

The results presented in this study were obtained using the inhouse solution routine, where $\epsilon_2 = \epsilon_0^*/N$ and N = total number of degrees of freedom. $\epsilon_0^* = \epsilon_0 \cdot W_i \cdot p_i$ and ϵ_0 is set externally. W_i and p_i are defined in equations 11 and 12 respectively. Thus the total out of

balance virtual work at convergence is less than ϵ_0 times the external virtual work due to proportional loading.

The value of ϵ_0 chosen for the inhouse routine is approximately equivalent to a value $\epsilon_0/\sqrt{N}$ for the Nag or Harwell routines

where for the latter two routines

$$\{\Gamma_i, \Gamma_i\}^{\frac{1}{2}} < \epsilon_1 = \epsilon_0 \cdot w_i \cdot p_i \text{ ----- (18)}$$

The results in this thesis are for $\epsilon_0 = 1 \times 10^{-8}$. The effect of looser tolerances on strength results is discussed in § 7.16.

7.14 Redundant yield lines

7.14.1 Definition of redundant yield lines

The yield lines, which must exist to form a kinematically acceptable collapse mechanism, are called the minimum set of yield lines for that mechanism. A redundant yield line is any line, formed by the boundaries of two adjacent elements, which is not in the minimum set of yield lines of a particular collapse mechanism. Two types of redundant yield lines exist.

a) In the simplest cases of collapse, by central deflection of a panel, redundant yield lines are lines not necessary for the development of hinges. They exist by virtue of the large inplane forces within an element bound by hinge lines. These redundant yield lines will involve inplane movement only and are called inplane yield lines. Lines 4 and 6, figure 7.7(a) are two such lines.

b) In more general cases two independent collapse mechanisms may combine. For example, the collapse of a longitudinally stiffened panel under inplane and/or lateral load may involve the lateral displacement of longitudinal stiffeners in addition to the independent individual panel mechanism. The yield lines of the former mechanism will be redundant lines for the latter collapse mechanism. The general case of two independent mechanisms interacting will not be considered more fully in this chapter.

In-plane yield lines provide a useful cut-off point for the collapse load of a panel under inplane load. For such a loading condition a valid mechanism is lines 2 and 4, figure 7.7(a). No lateral displacements of the panel occur and the mechanism gives, by observation, the collapse virtual work as $P = 2At.\sigma_y = 1.0 P_0$. This value of $1.0 P_0$ is an upper limit for collapse load when considering collapse of the panel by lateral deflection.

7.14.2 Limiting the range of consideration of redundant yield lines.

Dean(219) showed that an inplane redundant yield line may be included in the set of yield lines for collapse of a panel, in order to reduce the collapse load of the panel under inplane load.

This present study limits itself to the inclusion of redundant yield lines adjacent to unrestrained edges. This is not a true constraint on redundant lines. In general, redundant lines will exist both within a structure and at edges where some degree of restraint exists. The treatment of such lines necessitates a more systematic development of yield line theory. A general planar element, without the simplifying assumptions on degrees of freedom in appendix 7A1, would be required. Such a development would follow the methods of the finite element theory very closely.

7.14.3 Numerical treatment of redundant yield lines

The stress state along any redundant yield line is defined by 4 parameters (u, v, α, f). The velocities (u, v, α) are used in Table 7.1 to give non-dimensional stress resultants (n, s, m) at a point on the line. The forces at the point are then ($f.n.N_0, f.s.S_0, f.m.M_0$). The value of f allows a stress state along a yield line to be evaluated using yield line theory, while the line may be in an elastic or plastic stress state.

$0 \leq f < 1.0$ defines a redundant yield line in an inactive (or elastic) state.

$f = 1.0$ defines an active yield line

$f > 1.0$ defines a situation where a correct equilibrium state has not

yet been evaluated within the assumed yield line pattern.

A redundant yield line is kinematically simple if it forms from P_O , the apex of the set of minimum yield lines, figure 7.11(a). The lines $P_O P_4$ and $P_O P_6$ are two minimum length redundant lines. Under certain loading conditions these minimum length lines represent the critical redundant lines.

An inplane yield line B is considered at an angle α to a yield line A. In figure 7.11(b) yield line A is at some known state of inplane collapse. The equilibrium state of line B is considered in appendix 7A2.

f is the yield line factor for line B

For $0 \leq f \leq 1.0$ line A is the critical yield line.

For $f > 1.0$ line A is no longer sufficient to fully describe the inplane collapse state, since the stress state on line B exceeds yield.

The yield line factor of line B has been plotted for various values of α and various loading states of yield line A, figures 7.12 - 17. Yield line A is usually critical for low values of shear stress, S , along line A, figures 7.16,17. Yield may be exceeded along line B where shear stress exists in conjunction with overall inplane load along line A, figures 7.13,14,15.

7.14.4 Problems considered with redundant yield lines

Redundant yield lines will be considered for two loading conditions

a) In-plane uniaxial loading

Yield line 4, figure 7.7, will be assumed to exist for inplane loading. It is assumed that the inplane load is applied so that the loading edge, line 5, is constrained to remain straight. Yield line 6 is excluded.

Symmetry of displacement of element A, figure 7.7(b), results in no inplane shear along line 4. From figures 7.16 and 17, it is concluded that yield line 4 is the critical inplane yield line, except for values of overall load, P/P_O , close to unity, when the inplane

force, N/N_0 , on line 4 is also close to unity.

b) Lateral pressure loading

Yield lines 4 and 6 are possible when an unrestrained panel is subject to lateral load. Symmetry of movement of element A precludes the existence of shear forces along line 4. Hence the net force F_{X1} , figure 7.11, on yield line 1 is zero, and as a result there is no overall inplane axial force on line 6.

It is noted in figure 7.15, that yield is exceeded on inclined yield lines for such a loading state (on line 6). The maximum values of f are $f = 1.098$ for $\tan(\alpha) = 0.4$ and $f = 1.082$ for $\tan \alpha = 0.2$, when $N/N_0 = 0.01$ along line 6.

In general, yield line 6 does not define the appropriate inplane yield line for lateral loading of a panel. Yield will occur over a fan shaped area, figure 7.11(c). However, the collapse load obtained using yield line 6 will be at most 10% greater than the minimum possible from a more extensive yield line pattern.

In the specific case of a panel with an aspect ratio, $A/B=1.0$, yield lines 4 and 6 define the minimum collapse load. In this case symmetry of the geometry of the panel results in the forces on lines 4 and 6 being equal. As a result $N_4 = 0.0$ and $S_6 = 0.0$. (The value of f , for inclined yield lines, for $N = S = 0$, decreases from unity as α increases, figures 7.16, 17).

7.14.5 Requirement for the solution of problems involving redundant yield lines.

The factor, f , must be included for redundant yield lines of a panel. An attempt to find an equilibrium state for the panel when $f < 1.0$, without including the value of f (i.e. by assuming the redundant line is active), will not lead to a state of equilibrium.

A procedure was introduced into the equilibrium calculations whereby the state of redundant lines is assessed. Only when the value of f , for a redundant line reached unity was the line and the independent velocities associated with it included in the plastic work equations.

The inclusion of the degrees of freedom dy_1 and $d\theta_3$, for yield line 4, figure 7.11 resulted in poorly conditioned Jacobian matrices of the system. An accurate solution to the velocity set and equilibrium was possible only when the inhouse nonlinear equation solution routine (237) as used. Use of Harwell or Nag routines (235,236), resulted in divergence from an equilibrium state.

Results of the analysis of a flat panel under inplane and lateral loading, with redundant lines included, are discussed more fully in § 7.16.

7.15 Numerical stability

7.15.1 The limitation of techniques

No universally valid non-linear equation solution routine exists. The methods used in this thesis have relied considerably on a variation of the Newton-Raphson technique (237) for finding a better guess $\dot{\Delta}^{n+1}$ to the velocity vector $\dot{\Delta}_j$, given a known guess $\dot{\Delta}_j^n$.

A series of effective procedures have been outlined which have made it possible to find a solution velocity vector $\dot{\Delta}_j$ in most cases. The numerical stability of these techniques is discussed in this section.

7.15.2 Factors affecting numerical stability

a) The geometry of the panel was not found to greatly affect the stability of the method. Answers were most easily found for square panels.

b) The type of loading had a significant effect on numerical stability. Almost any initial velocity guess led to a satisfactory solution where lateral load was the most important loading on the panel. A reasonably judicious guess at the solution velocity had to be made when inplane loading was considered.

Total symmetry of inplane loading on a square panel, with the ridge angle $\phi = 45^\circ$, led to the same ill-conditioning as described in (g) below.

c) Lateral deflection of the panel had a large effect on the numerical stability for inplane loading. The Jacobian matrix is stress dependent. At an inplane load $P = 1.0P_0$, any change of the inplane velocity component leads to either no change in the inplane stress, or to a negative change in stress along the yield lines. Thus in the neighbourhood of $P = 1.0P_0$, the Jacobian is poorly conditioned and unless the initial guess at the velocity is fairly accurate, a satisfactory solution will not be found.

Where inplane loading was applied and a "good" initial guess was not known, the range of satisfactory initial guesses was considerably widened by first finding the solution velocity for a large lateral deflection. Failing this, a large component of lateral pressure could be added. The lateral pressure could be reduced rapidly once a solution was obtained.

d) The choice of nonlinear equation solution routine had a large effect on the speed of solution and on the range of acceptable initial first guesses. The inhouse routine (237) proved to be very much more successful and faster than the other routines (235,236).

The Nag and Harwell routines use a combination of the Newton-Raphson technique and the "Steepest Descent" technique for convergence to a solution. The Steepest Descent method takes a guess $\dot{\Delta}\eta^{j+1}$ along the gradient of the solution surface at the current guess $\dot{\Delta}\eta^j$. This is unsatisfactory where the Jacobian matrix is poorly conditioned or where almost uncoupled velocity vectors exist as solutions. The inhouse routine does not use the "Steepest Descent" technique, and does not rely directly on using the inverted Jacobian of the system. In addition, the Nag and Harwell routines use the the least squares sum of the residuals for assessing the significance of a new guess. The least square sum has an averaging effect on the effect of the residuals, making it more difficult to identify the improvements in the individual residual components.

e) The type of yield function did not affect the stability of the solution technique.

f) The number of gauss-points did not affect the stability of the method, although the solution velocities were significantly affected by the number of gauss points.

g) An important condition for obtaining a satisfactory Jacobian matrix is that the yield behaviour at the yield lines should be continuous. If there is no shear force on a yield line and there is a variation of inplane strain along the line, the stress behaviour along the line may not be continuous. For example, a redundant yield line, line 4, figure 7.7, has no shear strain. The stress state at any gauss point along the yield line is $n = \pm 1.0$. As a result, the contribution of the line to the virtual work terms, $\dot{y}_1$ and $\dot{\theta}_3$, will vary in steps, as shown in figure 7.18. An attempt to decrease the size of the steps, by increasing the number of gauss-points, was not found to improve greatly the numerical stability, since the Jacobian is computed from very small changes in the basic velocities.

A satisfactory solution to this problem may be obtained by treating a yield line where no shear occurs in a different manner. The line is taken as a beam of width t and depth d^* , where t = thickness of the panel, and d^* = length of the yield line. Yield line 4, figure 7.7, has been treated in such a manner in appendix 7A1. A continuous response is then obtained from the yield line, figure 7.18.

h) Redundant yield lines have been considered in §7.14. Two problems are associated with these lines, as noted earlier.

1) If a yield line is not active, any attempt to find a solution velocity, without introducing an "elastic" parameter, f , for the redundant yield line, will not be successful.

2) The Jacobian, created by including an active redundant yield line, is not well conditioned. The Harwell and Nag routines were unable to find a solution velocity for a laterally loaded or inplane loaded panel which included redundant yield lines, even when the initial guess was very close to the correct answer.

j) When a panel was totally clamped along its edges, the inplane load, P , reached a value of $1.0P_0$ before the lateral deflection

became zero. Attempts to find a solution velocity for smaller lateral deflections than that giving $P = 1.0P_0$, were unsuccessful.

7.15.3 Accuracy

The accuracy of any answer is affected by three factors, within the bounds of the theory.

a) Round-off errors and cut-off points.

Round-off errors were generally not significant because a CDC mainframe computer was used for computation. This machine uses a 60-bit word length, which is approximately equivalent to double accuracy on an IBM machine.

Cut-off points for some variables had to be introduced as the constitutive relationship involved the possibility of division by a small number, or the use of the logarithm of a small number. A cut-off value of 10^{-12} was chosen for any variable which was used as a denominator in Table 7.1. This cut-off value did not affect results, although larger cut-off values of 10^{-6} had a noticeable effect on some results.

b) An acceptable value of residual work must be defined (§7.13). An error value, $\epsilon_0 = 10^{-6}$, has been used to produce results. Figures 7.19 and 20 show the effect of varying ϵ_0 . It can be seen that a value of $\epsilon_0 = 10^{-2}$ is required in order to produce continuous load-deflection curves. For both loading cases considered the number of gauss points used on line 1 did not affect the continuity of the results.

The extra convergence was sought because very little extra computation was required for stable problems and because the looser tolerances did not, on occasions, reveal the problems associated with redundant yield lines (and thus did not reveal the true response of the model).

c) The number of gauss points along a yield line had a considerable effect on the solution velocities, figure 7.23. The effect on the load-deflection behaviour was less pronounced. The effect of the variation in the number of gauss points on load capacity along line 1

is seen in figures 7.21 and 22, for a panel of aspect ratio, $A/B=3.0$, under lateral and inplane load. The short sides of the panel are constrained, i.e. forced to remain straight, but free to pull in. Three gauss points are probably adequate for most loading conditions. Five gauss points were used to obtain the results produced in the following sections of this chapter.

7.16 Panel strengths

7.16.1 Non-dimensionalisation of results

Collapse mechanism load capacities have been obtained for panels of aspect ratio $A/B = 1.0$ and 3.0 , when subject to uniaxial inplane loading and lateral pressure. Results have been non-dimensionalised as follows:

The lateral displacement parameter, δ/t , (the ratio of lateral deflection to the panel thickness) is the only geometric parameter to affect panel strength for a particular aspect ratio (218).

Inplane load capacities have been expressed relative to the maximum inplane external virtual work along line 5, $P_0 = 2A.t.\sigma_y$. The results presented in figures 7.8, 9, 19, 21 have been non-dimensionalised with respect to the lateral load capacity ρ_0 , at zero lateral displacement, for the particular ridge angle, ϕ , in question. Both the internal and external virtual work vary with the angle ϕ and a common datum must be chosen for comparing the results obtained for different angles. The inplane velocity components are zero at zero lateral displacement and the panel capacity for lateral load is given (for simply supported edges) as

$$\rho = 2 \sigma_y \left[\frac{t}{B} \right]^2 \left\{ \frac{A/B + \tan\phi}{A/B - 1/(3\tan\phi)} \right\} \text{-----} (19)$$

ρ is obtained when The minimum value of

$$\tan\phi = \frac{1 + \sqrt{1 + 3A^2/B^2}}{3A/B} \text{-----} (20)$$

The lateral load capacity, when ϕ satisfies this equation, is taken as

ρ_0 , for the remaining figures of this chapter.

Equation 20 is satisfied by $\phi = 34.96^\circ$ when $A/B = 3.0$. Symmetry governs when $A/B = 1.0$ and $\phi = 45^\circ$ was used.

Results will be presented for 3 sets of boundary conditions: unrestrained, constrained and clamped/fixed edges.

7.16.2 Lateral loading results

The effect of varying the ridge angle, ϕ , is different for a panel under lateral pressure, for the three boundary conditions considered ($A/B = 3.0$). The minimum strength at zero lateral deflection is given by $\phi = 35^\circ$ for the three boundary conditions. The minimum strength for an unrestrained panel at large lateral deflections is given by $\phi = \phi_{\min} = 18.43^\circ$. At intermediate deflections the ridge angle decreases from $\phi = 35^\circ$ and the lowest strength is given over much of the range $0 \leq \delta/t \leq 5.0$ by $\phi = 22.5^\circ$ - 25° , figure 7.24.

The strength of a constrained panel at large lateral deflections is obtained for $\phi = 55^\circ$, figure 7.25. The ridge angle increases from $\phi = 35^\circ$ at intermediate deflections. The minimum strength is defined by $\phi = 50^\circ$ or 55° for $\delta/t > 1.0$.

Figure 7.26 shows the strength of a panel with fixed sides and clamped ends (free to pull in) as given by $\phi = 18.43^\circ$ for most lateral deflections. There is very little difference in strength for $30^\circ \leq \phi \leq 40^\circ$, for $\delta/t < 0.7$.

As mentioned above, there is no need to vary the ridge angle for a panel of aspect ratio, $A/B = 1.0$.

Figures 7.27 and 28 present the minimum strength curves for the 2 panel ratios. Comparatively little increase in strength is obtained at large lateral deflections for an unrestrained panel ($A/B = 3.0$). The strength of a panel of aspect ratio, $A/B=1.0$ with unrestrained edges, increases considerably more than for $A/B=3.0$. The unrestrained panel has a much lower strength than a constrained panel at large deflections.

The increase in load capacity for a constrained panel occurs because large inplane forces can develop at the panel centre. These tensile forces are balanced by compressive forces close to the edges. The inplane forces cannot develop effectively when the panel is unrestrained.

Inplane yield lines 4 and 6 may develop for unrestrained boundaries, figure 7.7. Yield line 6 becomes active when the lateral deflection $\delta/t \geq 0.2$ for most angles of ϕ ($A/B = 3.0$). Yield line 4 remains inactive with a yield line factor of $f = 0.2 - 0.3$ for the entire range of deflections covered. Symmetry of displacements dictate that lines 4 and 6 become active simultaneously for an aspect ratio of unity. It was found however, that by setting line 6 as active and line 4 as inactive, the yield line factor for both lines remained at unity. A lack of symmetry between the velocities results. The equilibrium condition along line 1 is, however, that of zero shear, i.e. $d\dot{X}_1 \cos 45 = d\dot{X}_4 \cos 45 + d\dot{Y}_2 \cos 45$ or $d\dot{X}_1 = d\dot{X}_4 + d\dot{Y}_2$

This condition is met, figure 7.29.

It has been noted above, §7.14, that the redundant yield line 6 does not completely represent the minimum yield mechanism since yield may be exceeded by about 10% at neighbouring yield line positions. In the special case of $\phi = 18.43^\circ$ symmetry of displacements governs along line 6. From figure 7.17, this condition of no shear causes yield line 6 to be the critical inplane yield line. Hence the results for $\phi = 18.43^\circ$ are the minimum obtainable from the theory. It must be concluded, however, that for much of the range of lateral deflections covered, $1.0 < \delta/t < 4.0$, the lowest strength given in figure 7.27, for an unrestrained panel, may be unconservative by as much as 10% of the load value. For $\delta/t < 1.0$ and $\delta/t > 4.0$ the unrestrained panel strength given is quite accurate within the bounds of the theory.

The minimum strengths given otherwise in figures 7.27 and 28 represent accurate lower bounds to strength as predicted by the yield line theory.

7.16.3 Inplane loading results

Inplane loading results are given for 2 aspect ratios, figures 7.30 - 35, and are summarised as minimum strength curves in figures 7.36 and 37. In figures 7.30 and 33, the strength of panels with unrestrained edges decreases with increased ridge angle, ϕ , at larger lateral deflections. In fact, $\phi = 70^\circ$ was found to give the lowest strength for δ/t 5.0, for both aspect ratios considered. The increase of ridge angle, whose result forms the lowest strength at large lateral deflections, may be taken to correspond physically with a flattening of the peak lateral deflection as the overall deflection increases, as has been observed in tests (238).

Inplane yield lines 4 are active at zero lateral displacement, for aspect ratio, $A/B = 1.0$. These yield lines remain active for all lateral displacements and result in a large reduction of strength of the panel, over that of a constrained panel, figure 7.36.

The lateral displacement at which yield lines 4 become active for an aspect ratio of 3.0 varies with the ridge angle, ϕ . The variation of yield line factor is shown in figure 7.38 for $\phi = 18.43^\circ$. The figure clearly shows that the unrestrained panel strength is less than the constrained panel strength when the yield line factor exceeds unity, i.e. for $\delta/t > 1.75$ (approx).

It has been noted above that an attempt to include an inactive redundant yield line does not lead to convergence of a result. This explains why no curve has been plotted in figure 7.33 for $\phi = 18.43^\circ$ when $\delta/t < 1.75$. Results have only been given in that figure for displacements at which a solution was possible with yield lines 4 active. The constrained panel results are appropriate where no solution is given in figures 7.30 and 33.

7.17 Comparison of results

7.17.1 Comparison with Dean's results

Collapse mechanism theory has been used by Dean to predict the strength of panels under various loadings and boundary conditions

(218, 219, 220). Dean obtained closed form solutions where symmetry reduced the number of fundamental variables to one (220). Otherwise an iterative procedure was used to obtain results.

This author used Dean's constitutive relationships, but developed a solution technique using the independent kinematic variables directly related to the geometry rather than the fundamental variables used by Dean. Dean's fundamental variables and the independent kinematic variables are related by a quotient relationship $K\dot{\alpha}_1(\text{Dean}) = 4/t \cdot \dot{\gamma}_z / \dot{\theta}_z(\text{Fahy})$. Results of the two applications should agree.

The author found himself able to reproduce Dean's closed form results exactly. (Only some of these results are presented here). However, he was unable to agree entirely with Dean's plotted results, where the iterative technique was used by Dean. In cases where inplane loading is considered the differences are often small, although not insignificant. In the case of laterally loaded panels, however, the differences can be large.

The author repeated his calculations using Dean's variables and equations (in so far as they are structurally correct) for a number of cases. These calculations agreed exactly with the results presented in this thesis.

7.18 A first order elastic strain energy compensation

7.18.1 Limitation of direct plastic work equation

The fundamental work equation, equation 13, assumes that the internal plastic work exactly equals the external work done on a panel. The theory assumes that the panel remains rigid away from the yield lines, and so the elastic strain energy of the panel has been ignored. A first order correction may be made to the theory to allow for the effect of inplane strain energy in panels subject to uniaxial inplane load.

7.18.2 Elastic strain energy release.

It is assumed that changes of elastic deformations are small in comparison with the plastic deformations at the yield lines.

The yield lines are assumed of negligible extent in the inplane direction perpendicular to the yield line.

Let the inplane displacement be s and the average stress σ_0 .

Let an adjacent inplane displacement be $s + \Delta s$ and the average stress is then σ .

The change of elastic inplane strain energy is

$$dE = \frac{4abt}{2E} (\sigma^2 - \sigma_0^2) = \frac{4abt}{E} \sigma_{av} (\sigma - \sigma_0)$$

σ is known in terms of the lateral displacement δ

$$\sigma - \sigma_0 \approx \frac{d\sigma}{d\delta} \cdot \frac{d\delta}{ds} \cdot \Delta s$$

$$\text{then } dE \approx \frac{Pb}{E} \frac{d\sigma}{d\delta} \cdot \frac{d\delta}{ds} \cdot \Delta s$$

where $P = 4 \cdot at \cdot \sigma_{av}$.

The virtual work equilibrium equation may be rewritten more exactly as

$$P \cdot \Delta s \approx W \cdot \Delta s + \frac{Pb}{E} \frac{d\sigma}{d\delta} \cdot \frac{d\delta}{ds} \cdot \Delta s$$

$$\text{or } P \approx W / \left\{ 1 - \frac{b}{E} \frac{d\sigma}{d\delta} \cdot \frac{d\delta}{ds} \right\}$$

$$\approx W / \left\{ 1 - \frac{\sigma_y}{E} \left[\frac{b}{t} \right]^2 \cdot \frac{d\sigma_1}{d(\delta/t)} \cdot \left[\frac{t\dot{\theta}}{\dot{Y}} \right] \right\} \text{----- (21)}$$

The ratio $\frac{d\sigma_1}{d(\delta/t)}$ is the slope of the non-dimensional stress σ_1 , with

respect to the non-dimensional lateral displacement. The ratio $t\dot{\theta}/\dot{Y}$ is a fundamental kinematic parameter and is independent of the geometric parameter, b/t .

Load capacity has been expressed non-dimensionally as P/P_0 . Then $P/P_0 = \sigma_1$.

Previously we have written $P/P_0 = W_k/P_0 = \sigma_A$. Then the correct equation may be written as

$$\sigma_1 = \sigma_A \left[1 - \frac{d\sigma_1}{d(\delta/t)} \cdot \left[\frac{b}{t} \right]^2 \frac{\sigma_y}{E} \frac{t\dot{\theta}}{\dot{Y}} \right] \text{-----} (22)$$

$$\text{If } \frac{d\sigma_1}{d(\delta/t)} \approx \frac{\sigma_1(\text{old}) - \sigma_1}{\Delta(\delta/t)} \text{-----} (23)$$

and $\sigma_1(\text{old})$ is the known value of σ_1 at an adjacent value of δ/t , equation (22) may be solved as a quadratic equation in σ_1 . $\dot{\theta} \equiv \dot{\theta}_2$ and $\dot{Y} \equiv \dot{Y}_2$ are the relevant velocities.

The correction is clearly dependent on the panel half-length to thickness ratio, b/t , and increases with increased b/t . The correction has a major effect on the calculated load capacity. Figure 7.39 shows the effect on a 3:1 panel (unrestrained) for $\phi = 45^\circ$. (B = full panel length).

The slope of the uncorrected curve decreases with increased δ/t , but is never less than zero. Thus the corrected curves will never lie below the minimum solution computed at a large value of δ/t .

The corrected curve becomes increasingly parallel to the displacement axis, δ/t , with increasing values of b/t .

New lower bound curves are given for unrestrained, constrained, and clamped panels of aspect ratios 1.0 and 3.0, figures 7.40-45. The small step which occurs in figure 7.44 at $\delta/t = 0.7$ is a result of using the corrected results for $\delta/t < 0.7$ from those predicted by one ridge angle, and the results for $\delta/t > 0.7$ from results obtained using another ridge angle. The criterion for moving from one set of results to another was the uncorrected ($B/t = 0$.) result. This causes a small disparity at large B/t ratios. The rapid change of slope for $\delta/t < 0.1$ results from the rapid change change of $(\dot{\theta}_2/\dot{Y}_2)$ close to $\delta/t = 0.0$.

The clamped panel strengths in figure 7.42 are slightly conservative, since they are based on an assumption that $\dot{\theta}/\dot{Y} = \text{constant}$, for $\delta/t < 0.8$. This assumption was required since no result was possible for small lateral deflections, $\delta/t < 0.8$, (figure 7.32). This figure is conservative in particular for $\delta/t < 0.1$ where the velocity ratio $\dot{\theta}_2/\dot{Y}_3$ tends towards zero for the other boundary conditions. Figure 7.45 is similarly conservative for $\delta/t < 0.1$.

7.19 A comparison of axial load results with finite element results

7.19.1 Method for predicting the strength of a panel under axial load.

The strength of a panel subject to inplane load varies with the lateral deflection of the panel. It is not adequate to consider the initial imperfection (at zero load) to determine the panel strength, since imperfections increase with load.

Lateral deflections of the panel will grow elastically until a load is reached where, for any increase of the lateral deflection, the elastic load exceeds the load given by the collapse mechanism theory. The collapse mechanism method is an upper bound approach and thus for increased lateral deflections, the panel will follow the unloading behaviour predicted by the mechanism theory, rather than the elastically derived loading behaviour.

Williams and Walker (239) have considered the elastic behaviour of rectangular two-dimensional panels subject to inplane load. They give formulae for the growth of lateral deflections under load for arbitrary initial imperfections. These formulae may be used to determine the collapse load of a panel by obtaining the intersection between the elastic loading curve and the plastic unloading curve.

Figure 7.47 gives the suggested behaviour of unrestrained panels of varying slenderness and with an initial imperfection $\delta/t = 1/165 B/t$. (The elastic energy effect has not been considered in this figure).

This suggested model of panel behaviour is an upper bound approach which may be expected to be most accurate when the transition from

elastic to plastic behaviour is fairly short.

7.19.2 Comparison with finite element results.

The elastic-plastic panel behaviour is compared with an elasto-plastic finite element analysis for a simply supported unrestrained panel of the two aspect ratios $A/B = 1.0$ and 3.0 , as given above.

The finite element model used a 6×6 grid division of the panel. Symmetry of panel behaviour allowed a total of 9 elements to be used in the analysis to represent the panel. The loading edge of the panel was constrained to remain straight by a rigid shell which was attached to the panel only. Thus uniform inplane movements of the entire loading edge were permitted but the shear rigidity of the attached shell curtailed relative inplane displacements of the panel edge. The unloaded edges were free to move inplane, and were simply supported against other movements.

An 8-node shell finite element which has been developed at Imperial College was used in the comparison study. This element is theoretically accurate and in practice has proved capable of giving accurate results to theoretically closed form problems. It compares very favourably with the best shell elements available, for a wide range of problems.

A single half-wave sinusoidal imperfection has been used for the finite element analyses. The amplitude of the sine wave imperfection is taken as the lateral deflection of the panel.

Results are compared for the two methods for $B/t = 30, 60, 90$.

7.19.3 Results for aspect ratio $A/B = 1.0$

The correlation for $B/t = 30, 60$ is very close for the plastic unloading state. Reference (239) predicts the elastic loading behaviour satisfactorily. The error between maximum strength predictions is 3.5% for $B/t = 30$ and 5% for $B/t = 60$. For $B/t = 90$ there is less correlation between the unloading behaviour predicted by the two methods. This is to be expected, since the onset of plasticity is more gradual for such a slender panel. Nevertheless the difference

between maximum strength predicted by the collapse mechanism theory and the finite element result is 14% of the finite element result. It is not anticipated that the collapse mechanism theory could be satisfactorily applied for $B/t > 90$, because the elastic flexure effects not considered by the theory are expected to be excessively large.

Figure 7.47 shows also that the first order correction for elastic strain energy of the panel is an appropriate and accurate correction for panels subject to uniaxial loading and $B/t \leq 60$.

7.19.4 Results for aspect ratio $A/B = 3.0$

Figure 7.48 shows the correlation of panel behaviour for $A/B = 3.0$. Collapse mechanism results are given for the unrestrained panel.

The correlation between the two methods is not as good as for the aspect ratio $A/B=1.0$. The collapse mechanism results lie below the finite element results for $B/t=30$ and 60 . It is surprising that an upper bound technique gives lower results than the finite element approach.

There are two possible reasons for the lack of agreement.

a) The finite element model required an attached beam to keep the loading edge straight. This beam had an axial rigidity in addition to a shear and bending rigidity. Thus free contraction of the loading edge was prevented, preventing also free contraction of the panel side at the junction with the loading edge. The length of the panel, B , is small relative to the panel width, A , and this constraint on the panel loading edge may considerably affect results. A comparison of the finite element results with the mechanism results for a constrained panel show a closer level of agreement, figure 7.49.

It has not been possible to impose the straight loading edge condition in the finite element analysis in any other way. An attempt was made to use a stiffener finite element which had no axial rigidity, but had a large bending and shear rigidity. It was not possible to obtain satisfactory numerical convergence using this stiffener element.

A 3 x 9 element idealisation of the panel for $B/t = 60$ was also attempted, but no improvement of the correlation was obtained. It is of note that this finite element idealisation of the panel showed the same flattening of the panel imperfection at large displacements ($\delta/t > 1.0$), as is indicated from the change of ridge angle, ϕ , in the constrained mechanism results. The ridge angle suggested in the finite element analysis is approximately the same as that obtained in the collapse mechanism analysis (constrained panel) for the same central lateral deflection.

b) Yield line 4 may have a stress of $\pm \sigma_y$ at any point along its length. A square-block stress distribution only may occur on the line. This simplification of the stress state along the line may underestimate the inplane capacity of the yield line for a given moment on the line. This would have a considerable effect on the panel capacity.

Time restraints have limited the number of finite element comparisons possible. A more complete comparison will allow the accuracy of the collapse mechanism results to be more fully assessed. In addition this would enable the inaccuracies resulting from the present use of redundant yield lines to be separated from the inaccuracies of the finite element representation.

7.20 Conclusions

7.20.1 Collapse mechanism theory and application to two-dimensional panels

A theory has been proposed for the discretisation of a structure into rigid interconnecting units, or elements, in order to determine the behaviour of the structure at collapse. The theory has been advanced in a generalised manner and may be applied to a variety of structural forms. It is illustrated in this chapter as applied to rectangular two-dimensional panels. The theory produces a number of equilibrium equations expressed non-linearly in terms of partially independent kinematic variables. The restrictions on the values of the kinematic variables have been outlined. Acceptable solutions to the equilibrium equations have been shown to exist under most conditions.

It has been shown that some variables are redundant and a systematic

method was evolved for determining when the redundant variables are "active" and when they are "inactive". Redundant variables have a major effect on the feasibility of finding an acceptable solution, and on the computed strength of a panel. Specialised solution routines were required to obtain solutions to equations with redundant variables.

Results were obtained for a number of geometries, boundary conditions and loadings, and are compared with published work. The author was unable to find the cause of disagreement where comparisons have failed to match.

A correction has been made for changes of elastic strain energy of a uniaxially loaded panel. The correction is shown to have a significant effect on the predicted panel behaviour. A method has been proposed whereby the strength of a panel under uniaxial compression may be predicted.

Comparisons with an accurate finite element elasto-plastic analysis proved very favourable for an aspect ratio $A/B = 1.0$. Differences occurring between the collapse mechanism results and the finite element results for $A/B = 3.0$ have not been satisfactorily resolved, and require further study.

Single bay portal frame

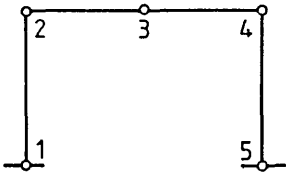


Figure 7 1 (a)

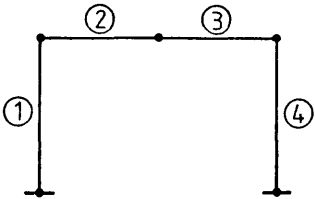


Figure 7 1 (b)

Unreinforced concrete panel under lateral load

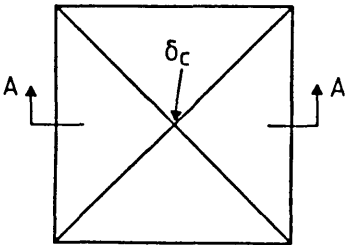


Figure 7 2 (a)

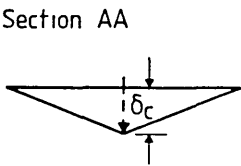


Figure 7 2 (b)

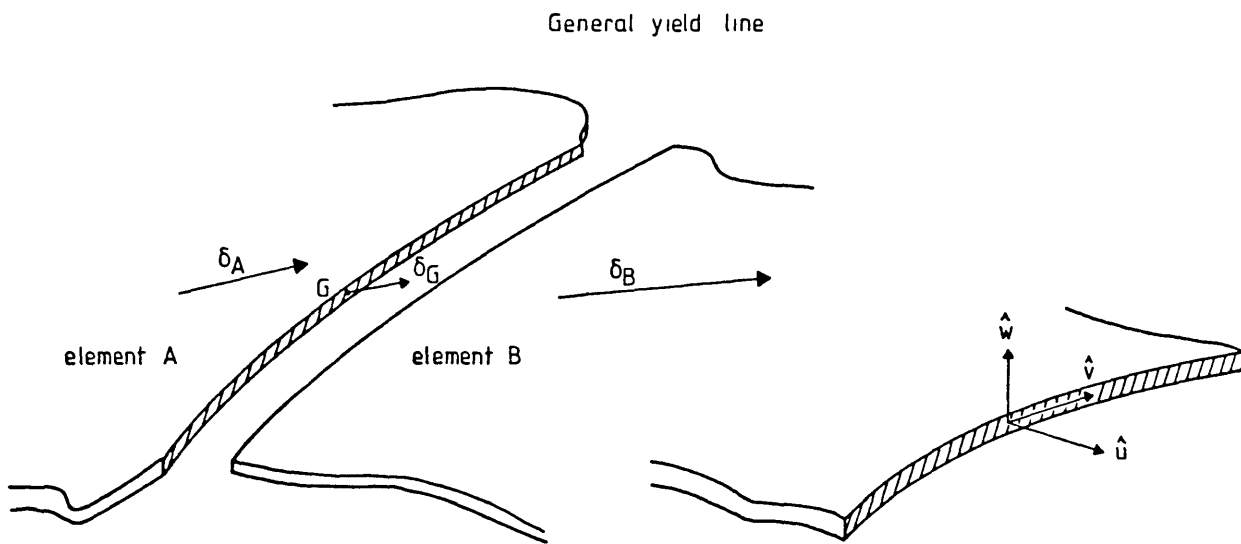


Figure 7.3(a)

Figure 7.3(b)

Adjacent planar elements

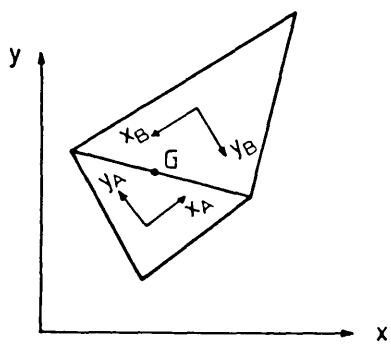


Figure 7.4

Point on a yield line

deformation state assumed stress state

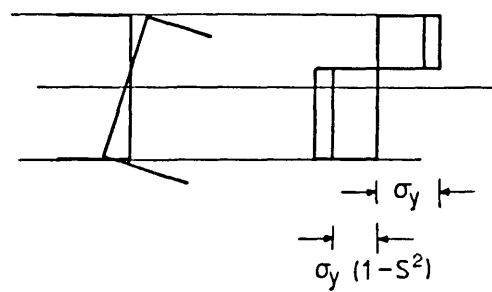


Figure 7.5

Initially flat panel under inplane load
 Plot of central lateral deflection against relative velocity
 The effect of varying the relative velocity function

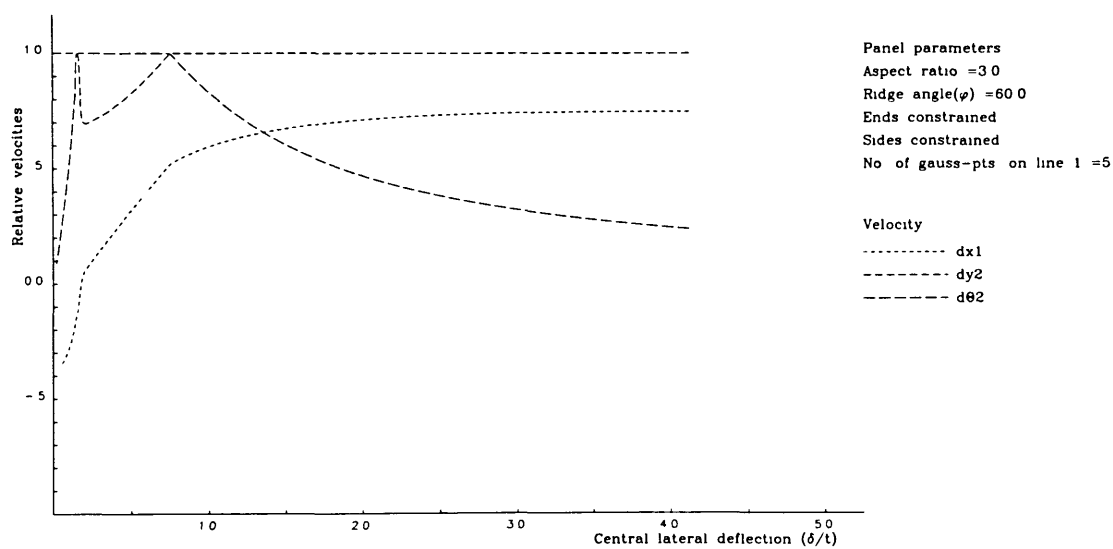
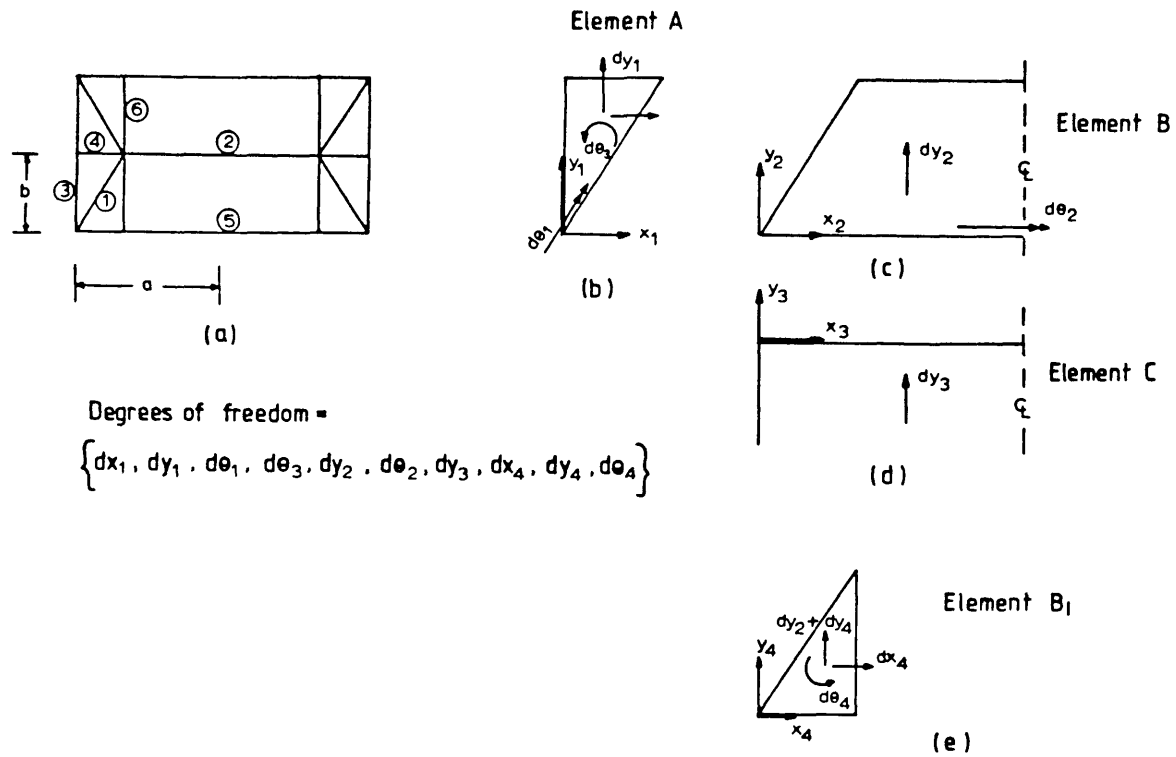
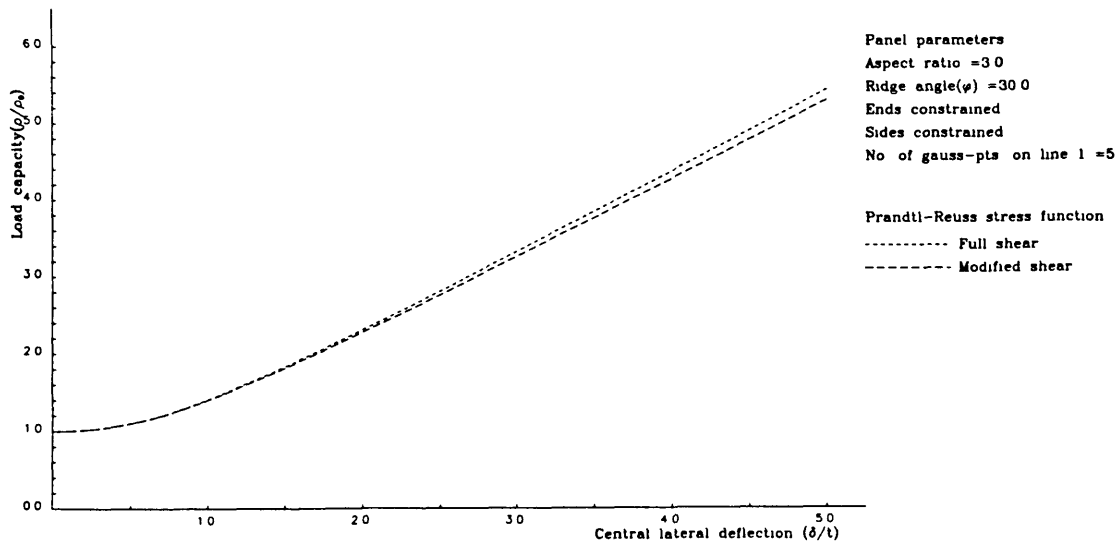


figure 7.6

The elements of a rectangular planar panel



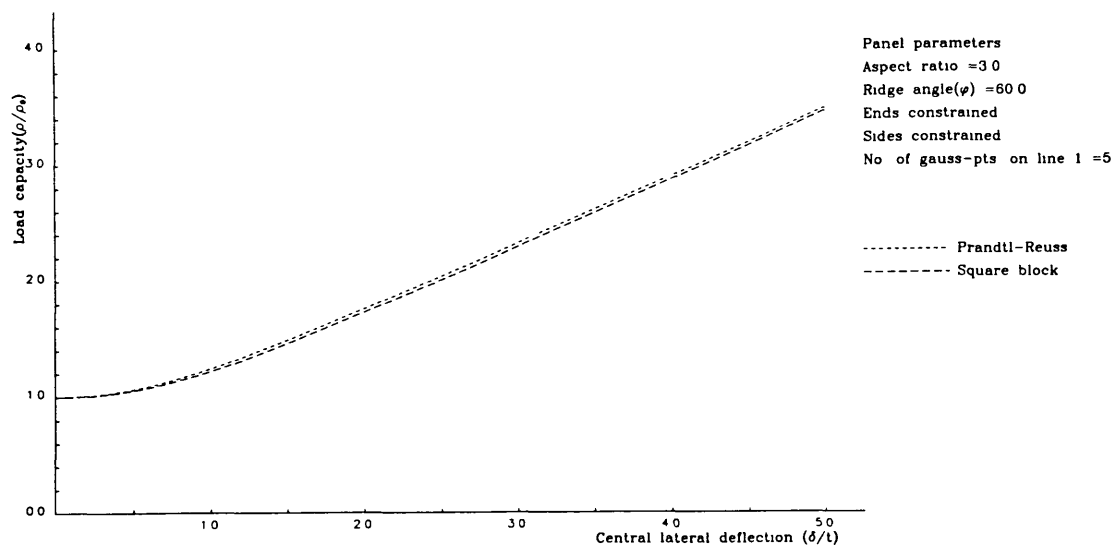
Initially flat panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of modifying the shear stress function



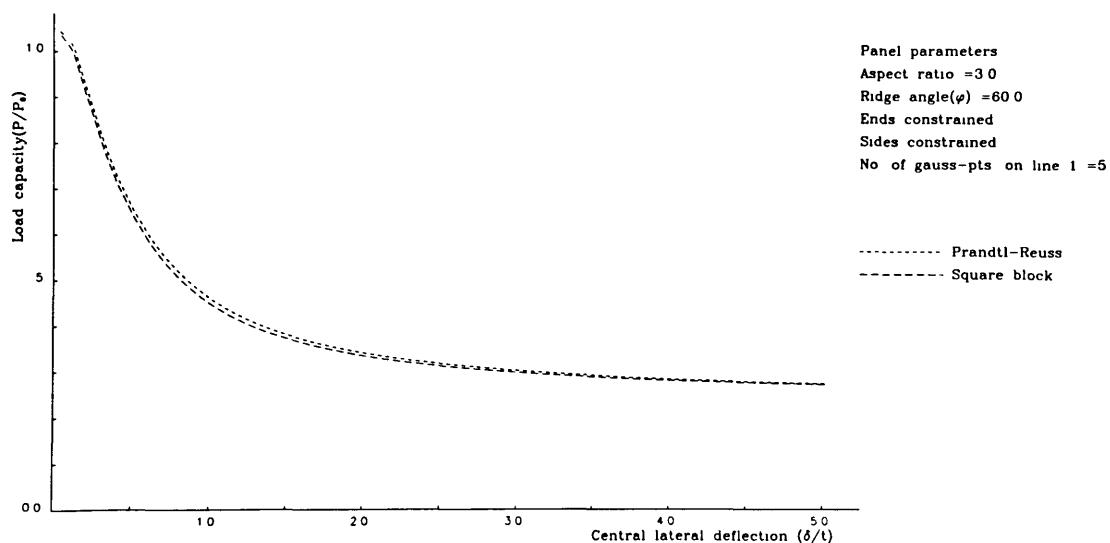
upper figure 7.7

lower figure 7.8

Initially flat panel under lateral load
 Plot of central lateral deflection against load capacity
 A comparison of Prandtl-Reuss derived and square-block stress functions

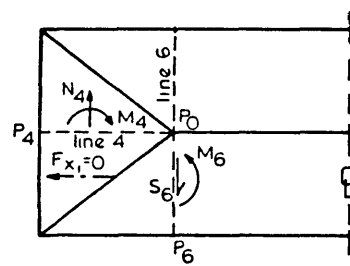


Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 A comparison of Prandtl-Reuss derived and square-block stress functions

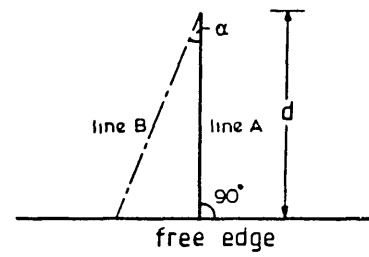


upper figure 7.9

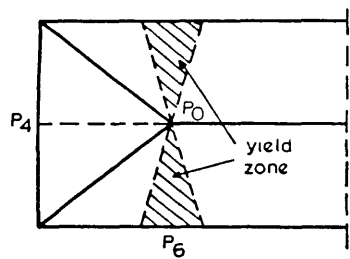
lower figure 7.10



(a)



(b)

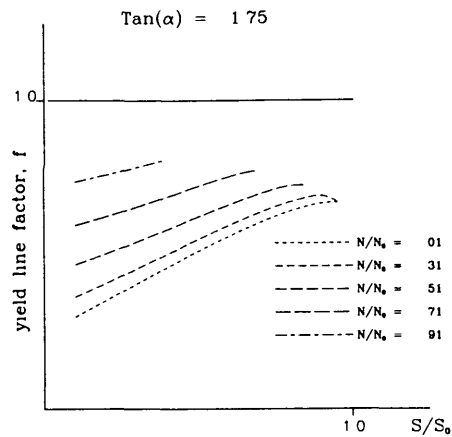
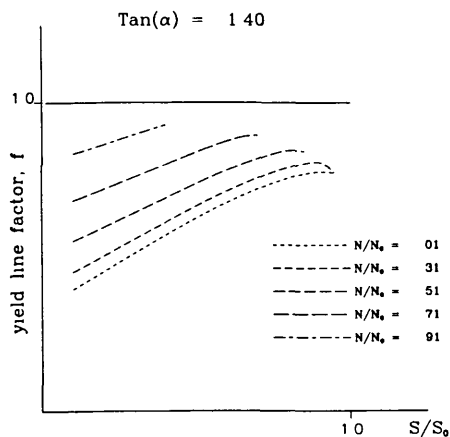
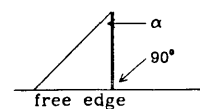


(c)

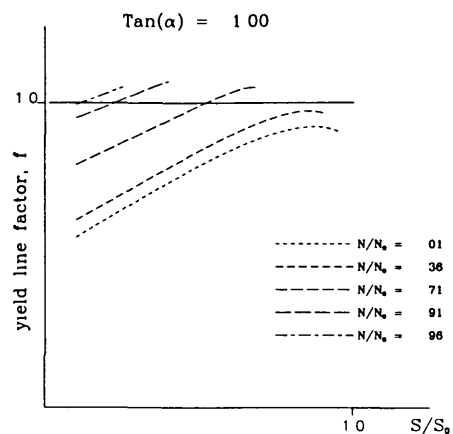
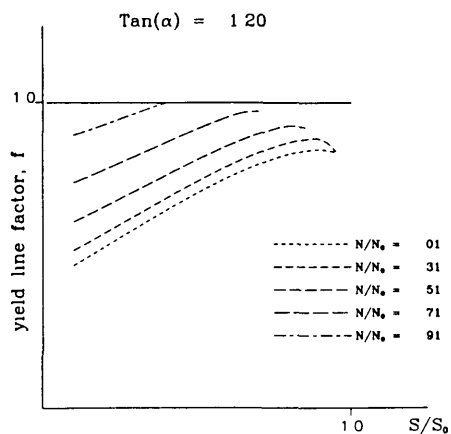
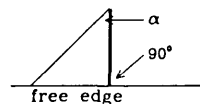
Redundant yield lines

figure 7.11

Inplane yield lines

Plot of shear stress against yield line factor, f ,for a line at angle α to a known yield line(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)

Inplane yield lines

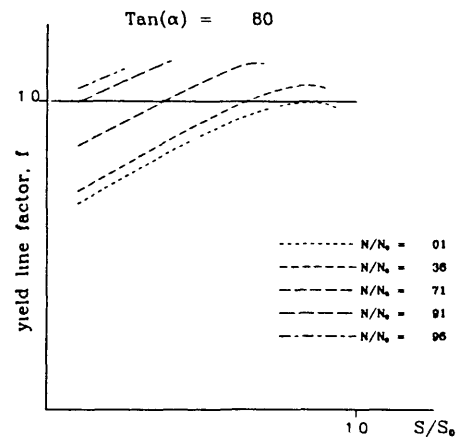
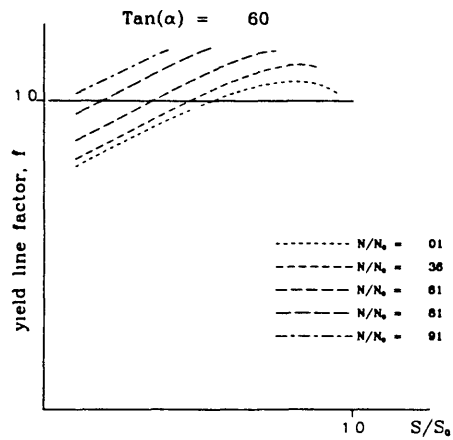
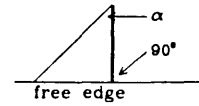
Plot of shear stress against yield line factor, f ,for a line at angle α to a known yield line(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)

upper figure 7.12

lower figure 7.13

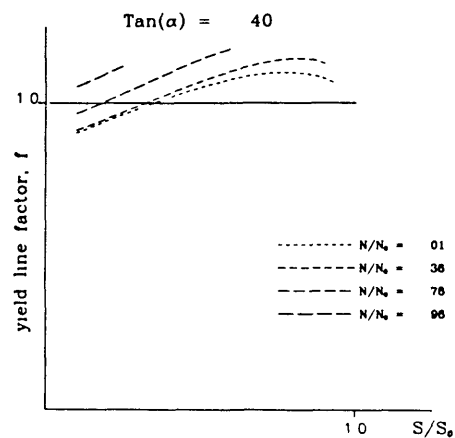
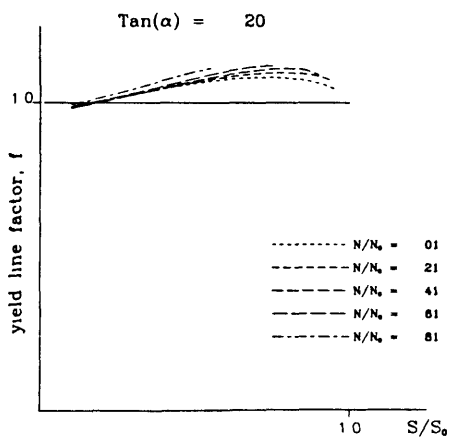
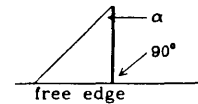
Inplane yield lines

Plot of shear stress against yield line factor, f ,
for a line at angle α to a known yield line
(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)



Inplane yield lines

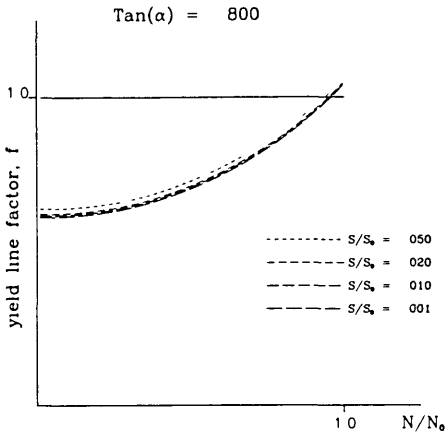
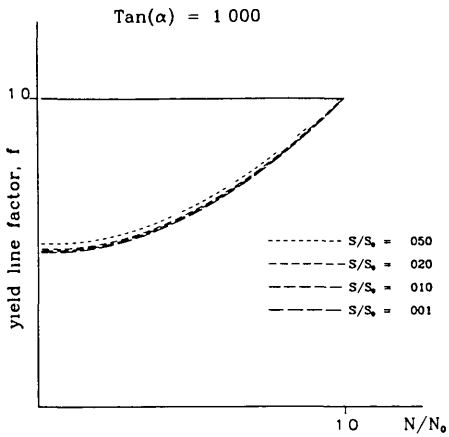
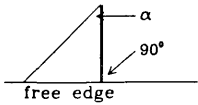
Plot of shear stress against yield line factor, f ,
for a line at angle α to a known yield line
(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)



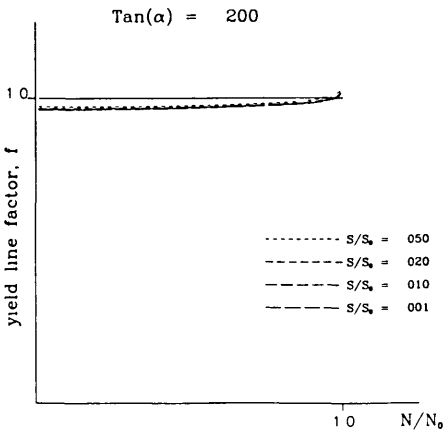
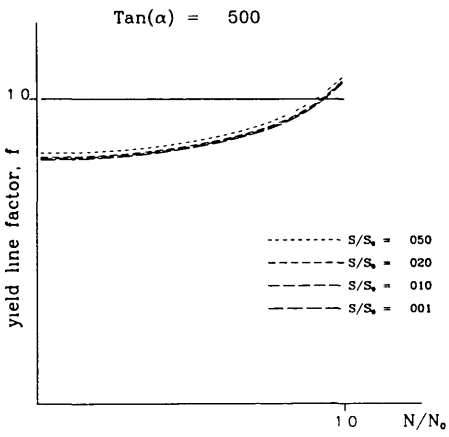
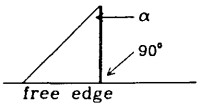
upper figure 7.14

lower figure 7.15

Inplane yield lines
Plot of axial stress against yield line factor, f ,
for a line at angle α to a known yield line
(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)



Inplane yield lines
Plot of axial stress against yield line factor, f ,
for a line at angle α to a known yield line
(line is elastic for $f < 1.0$, yield is exceeded for $f > 1.0$)



upper figure 7.16

lower figure 7.17

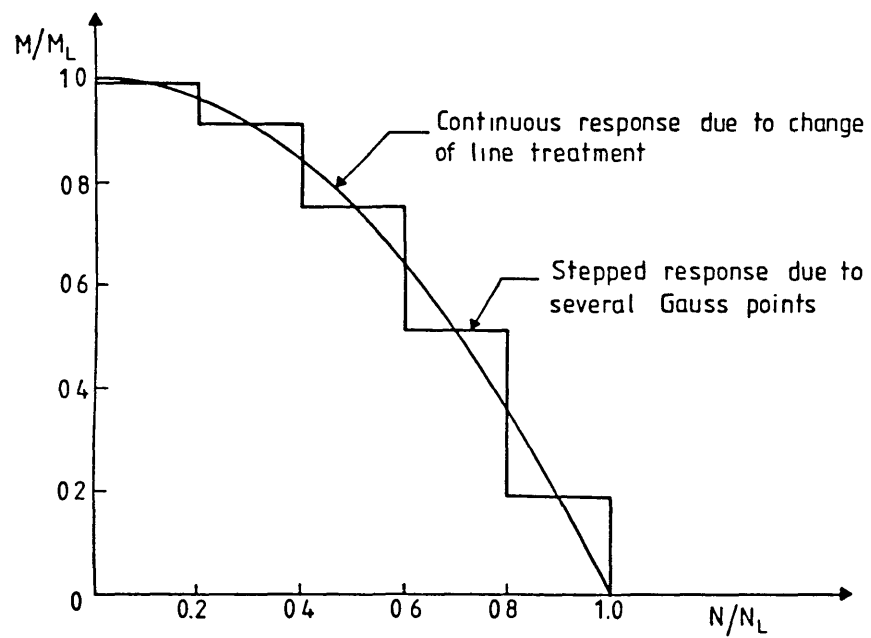
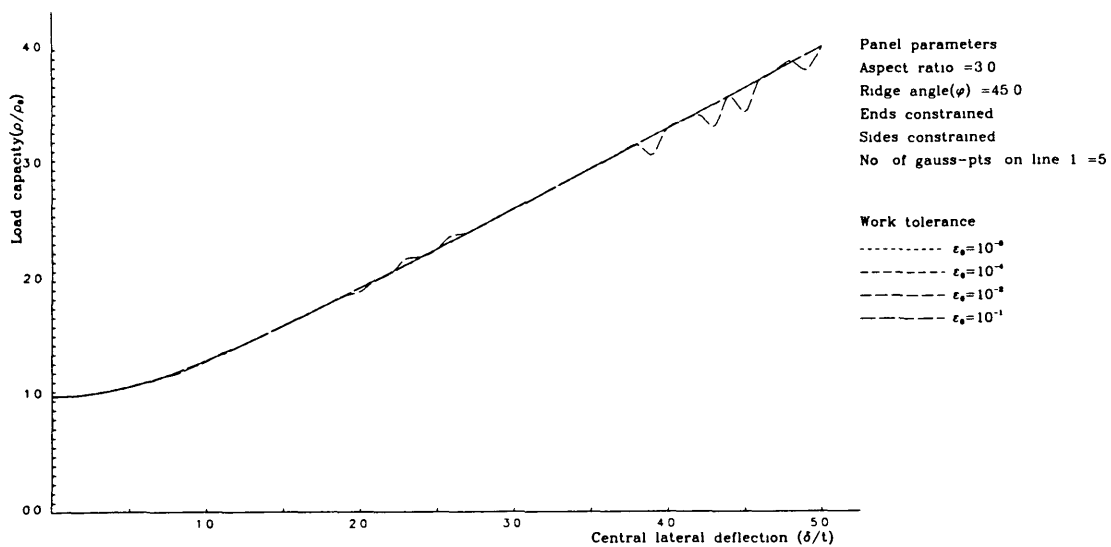
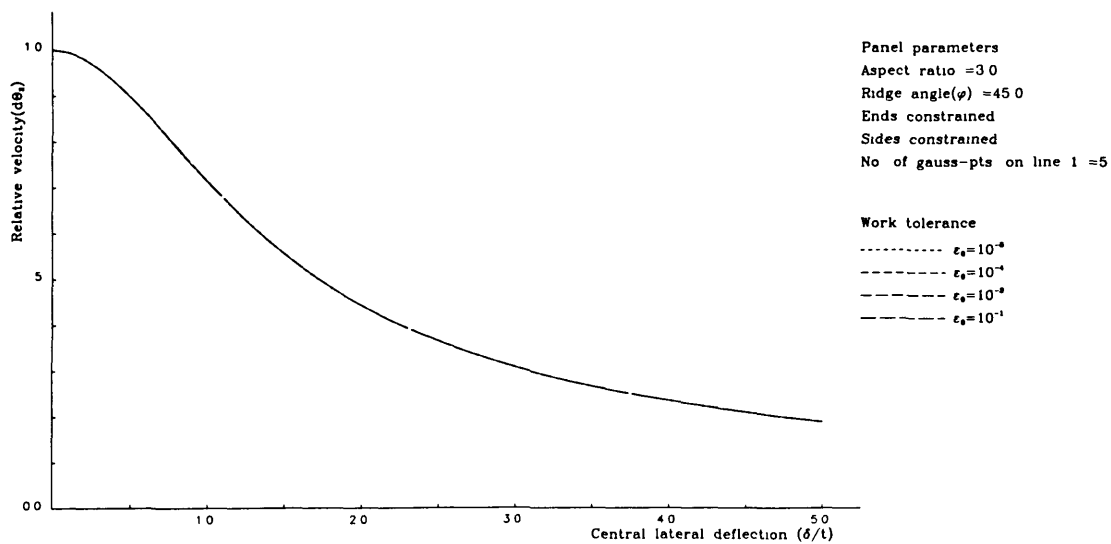


figure 7.18

Initially flat panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of varying the residual work tolerance (ϵ_0)



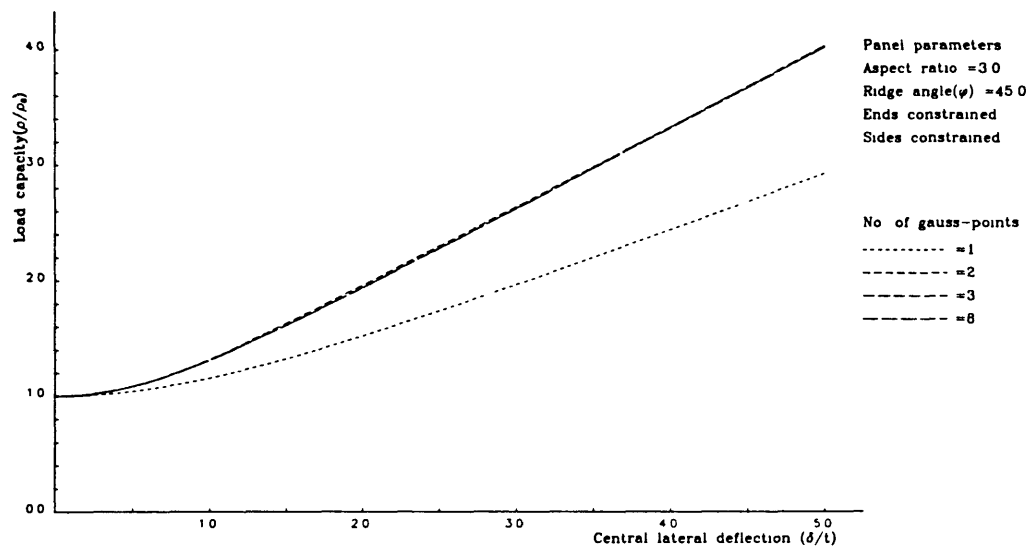
Initially flat panel under lateral load
 Plot of central lateral deflection against relative velocity
 The effect of varying the residual work tolerance (ϵ_0)



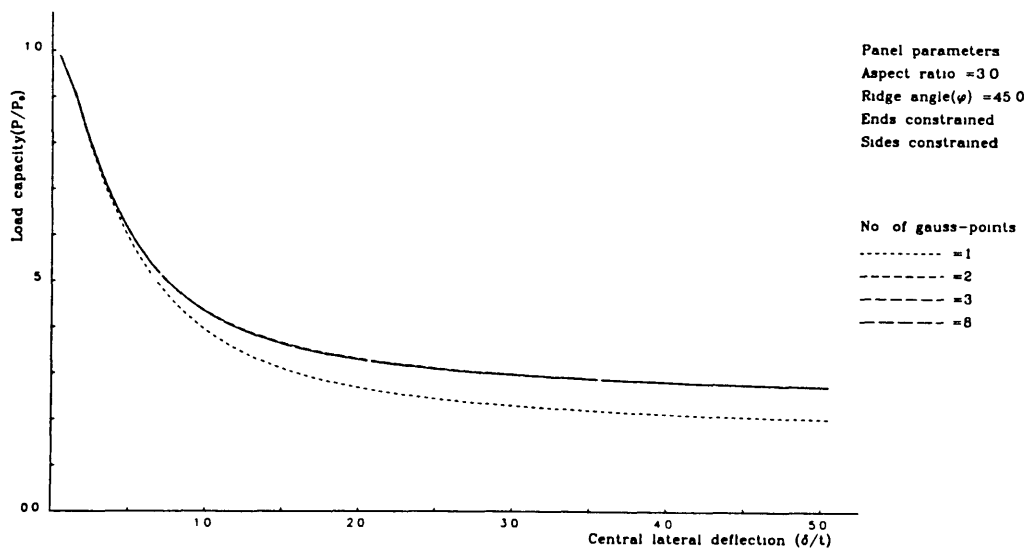
upper figure 7.19

lower figure 7.20

Initially flat panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of varying the number of gauss-points on yield line 1



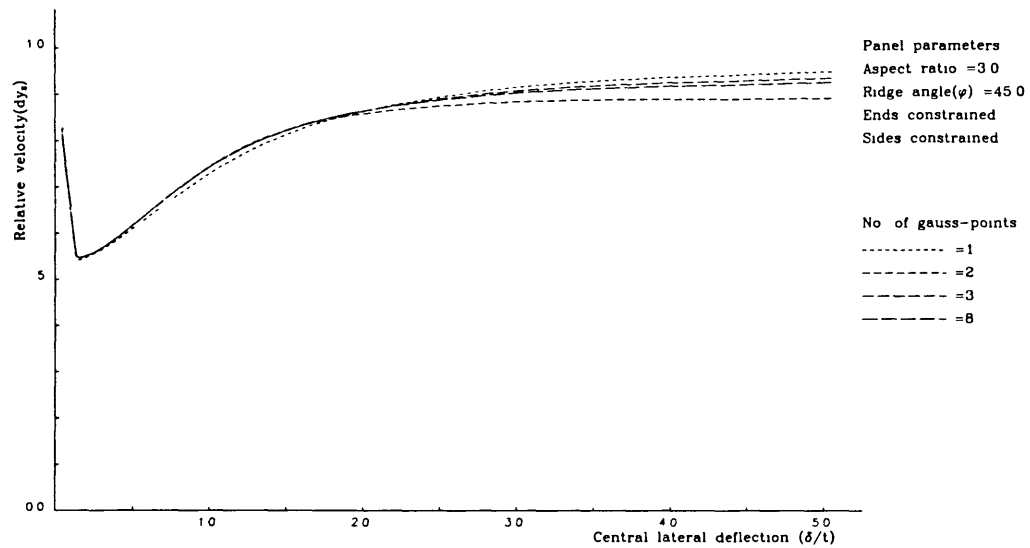
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the number of gauss-points on yield line 1



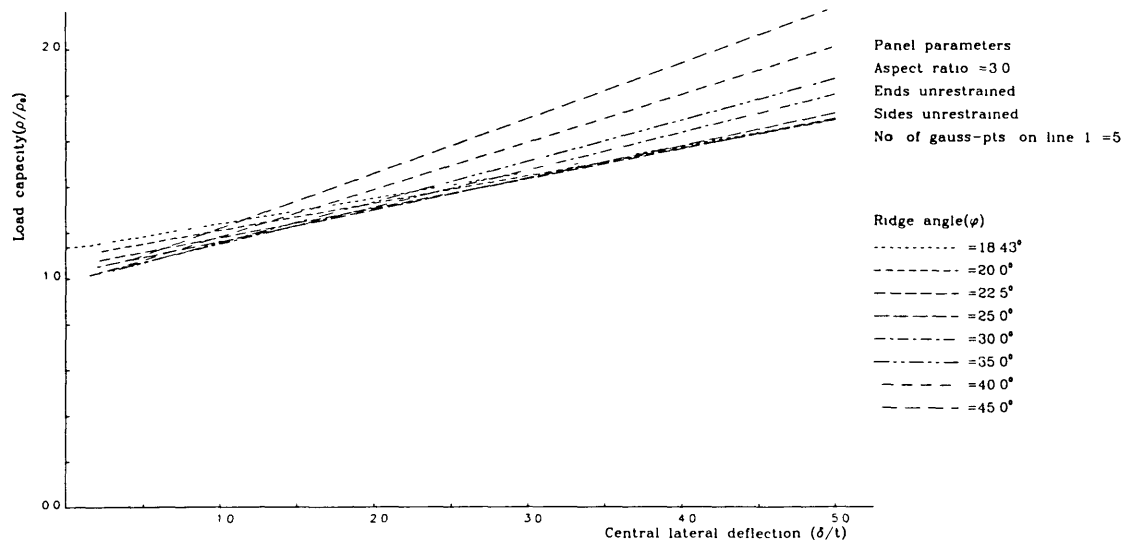
upper figure 7.21

lower figure 7.22

Initially flat panel under inplane load
 Plot of central lateral deflection against relative velocity
 The effect of varying the number of gauss-points on yield line 1



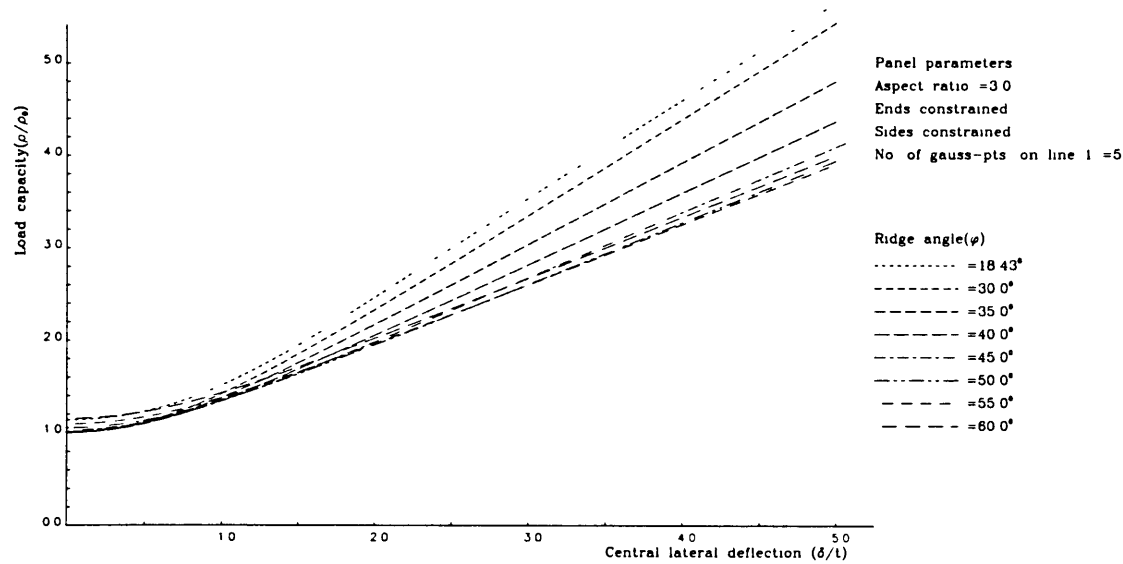
Initially flat panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(ϕ)



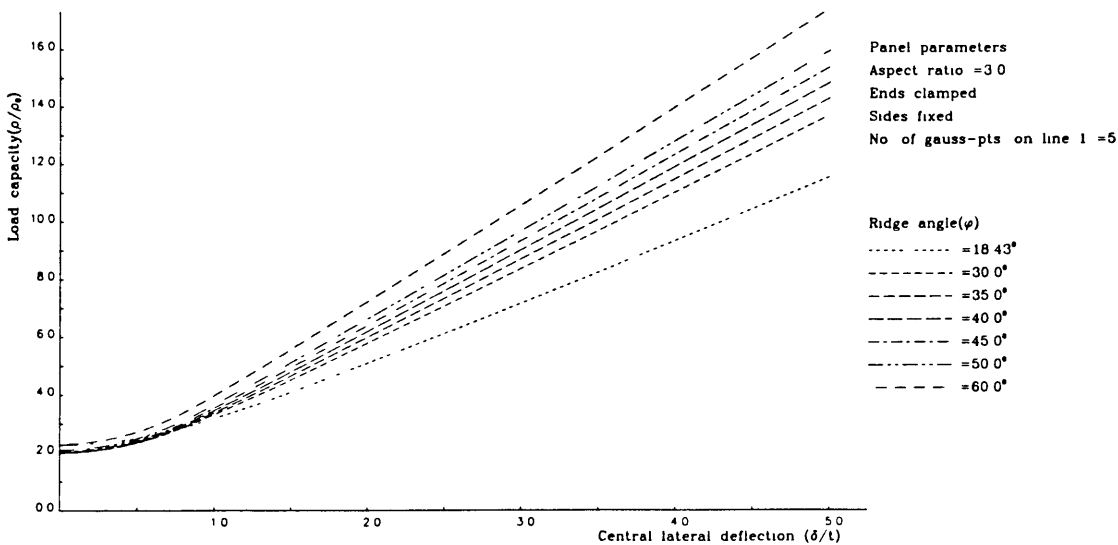
upper figure 7.23

lower figure 7.24

Initially flat panel under lateral load
Plot of central lateral deflection against load capacity
The effect of varying the ridge angle(φ)

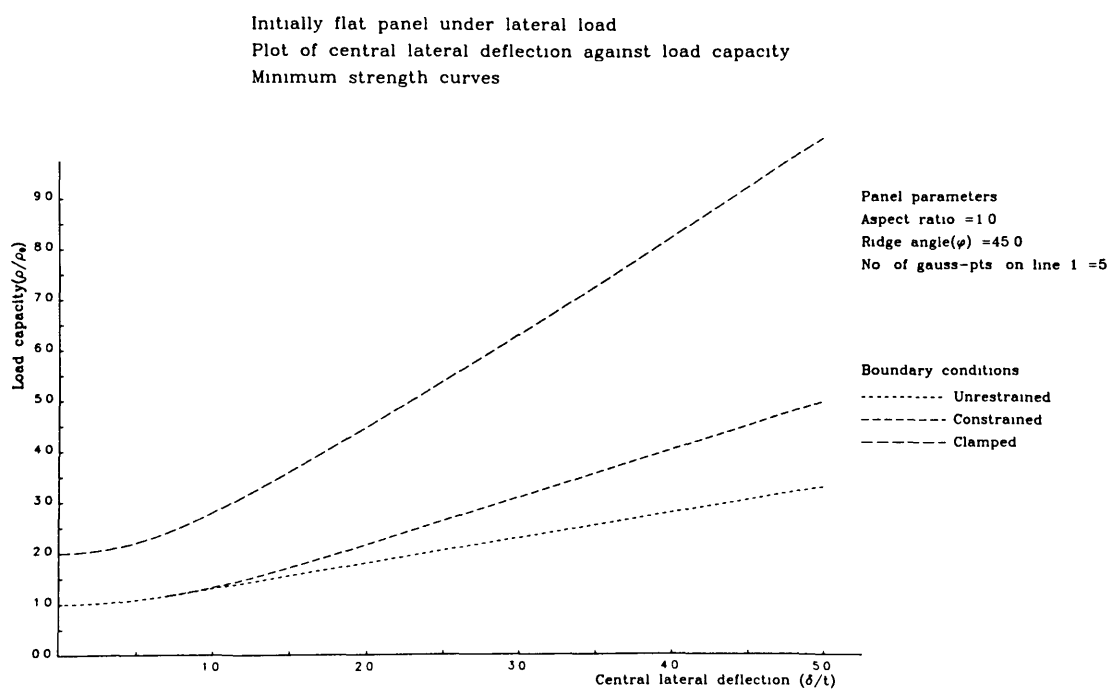
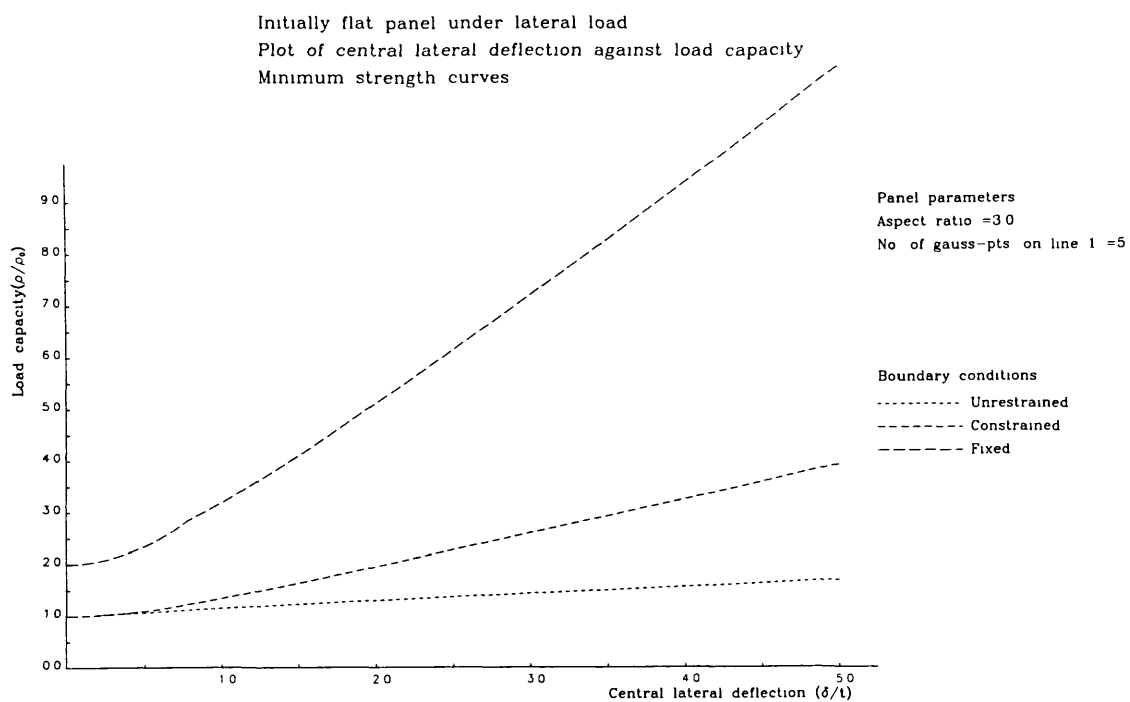


Initially flat panel under lateral load
Plot of central lateral deflection against load capacity
The effect of varying the ridge angle(φ)



upper figure 7.25

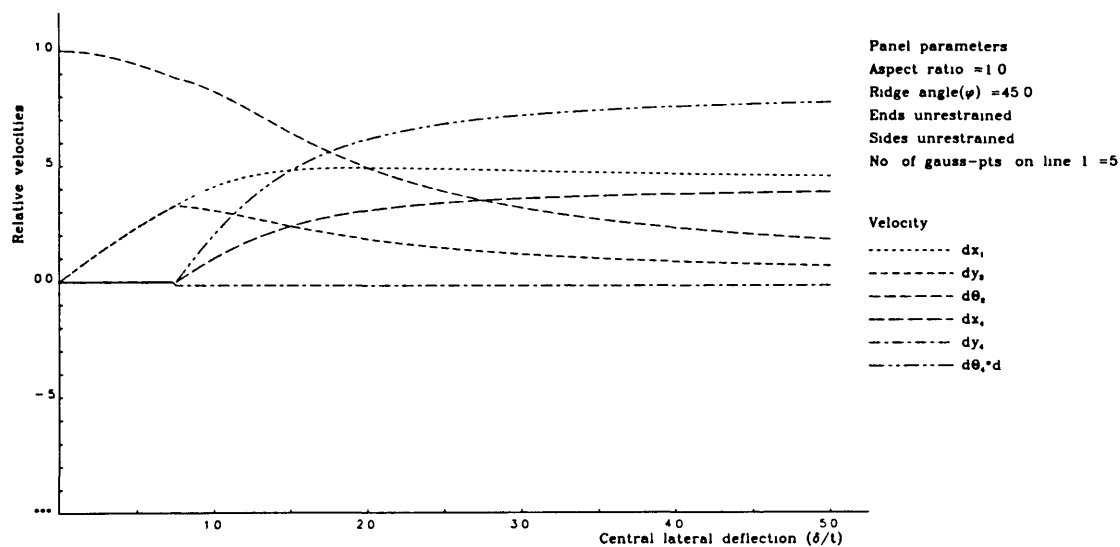
lower figure 7.26



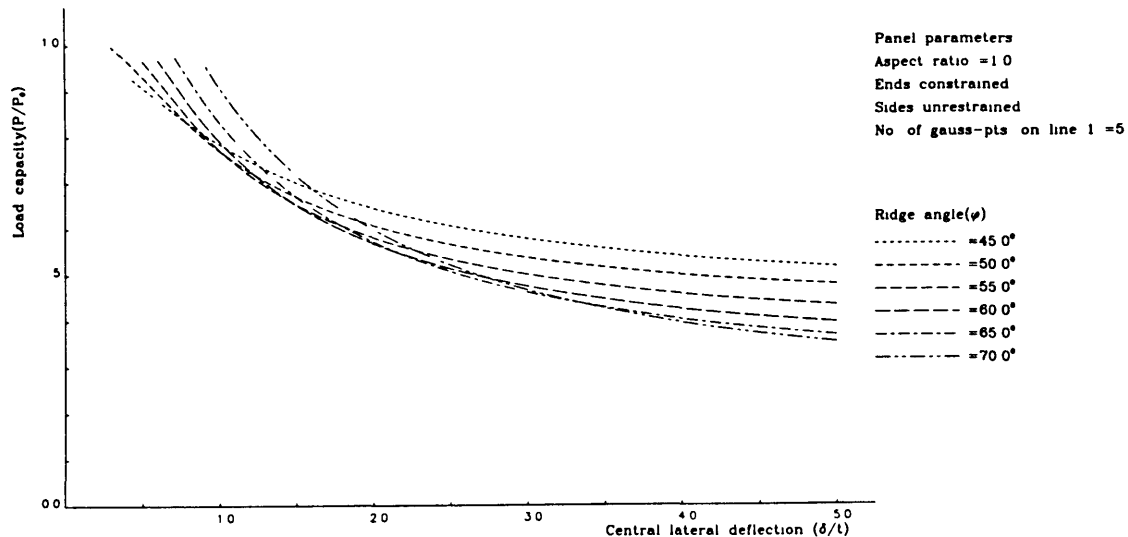
upper figure 7.27

lower figure 7.28

Initially flat panel under lateral load
Plot of central lateral deflection against relative velocity



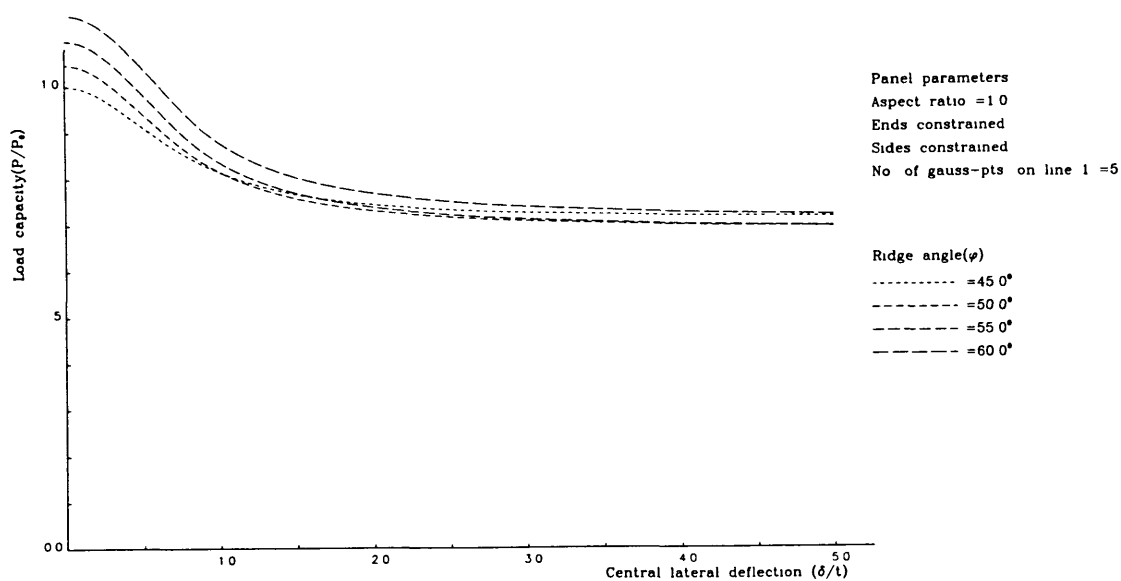
Initially flat panel under inplane load
Plot of central lateral deflection against load capacity
The effect of varying the ridge angle(φ)



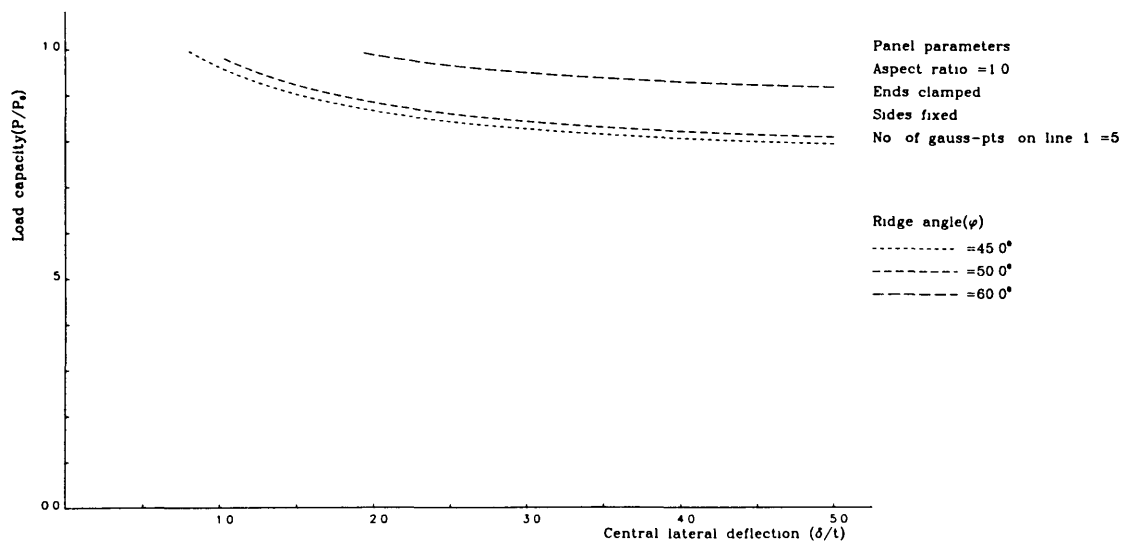
upper figure 7.29

lower figure 7.30

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(φ)



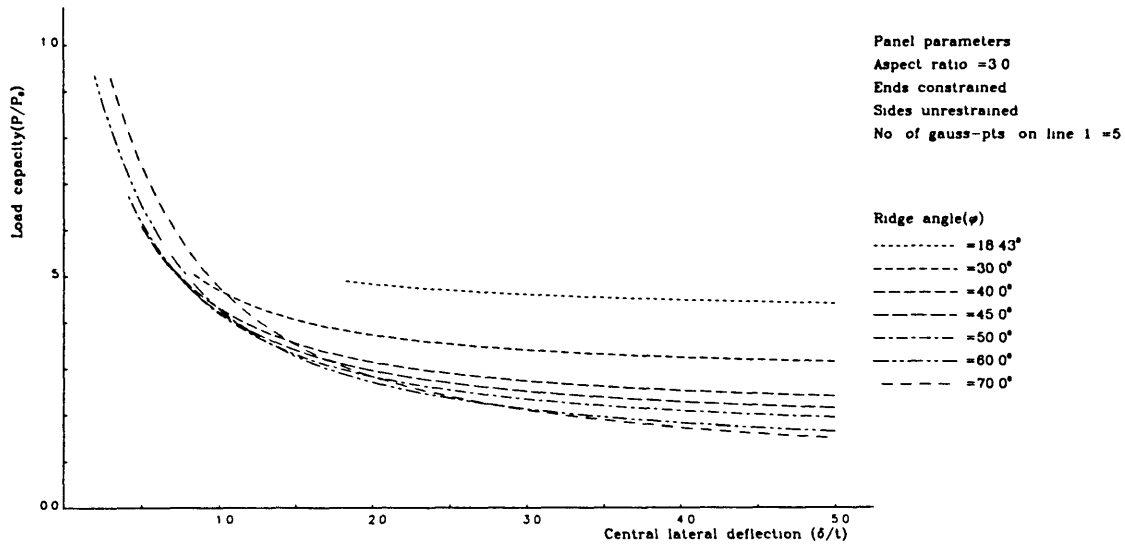
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(φ)



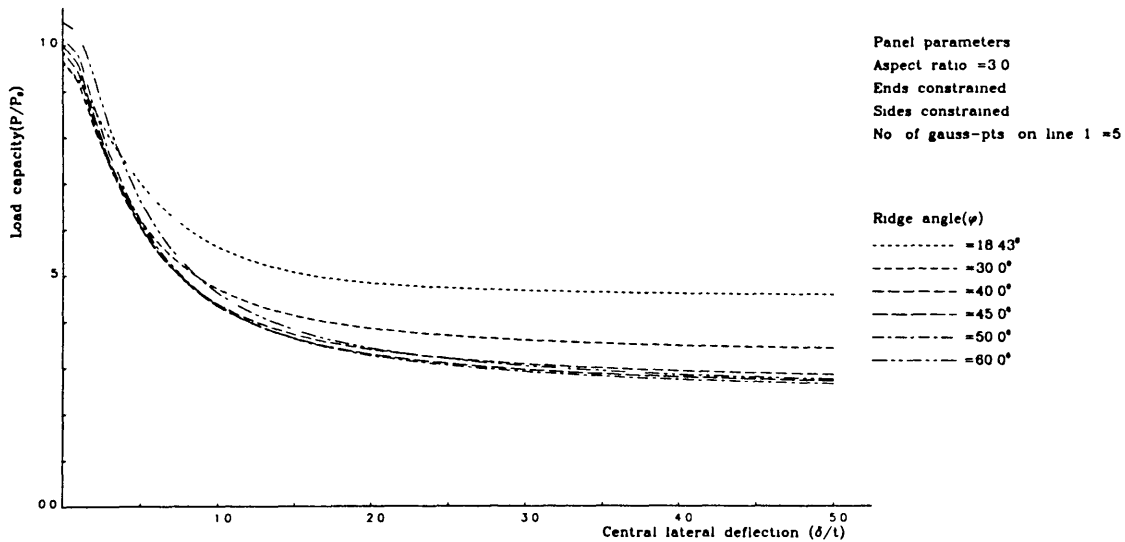
upper figure 7.31

lower figure 7.32

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(φ)



Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(φ)



upper figure 7.33

lower figure 7.34

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the ridge angle(φ)

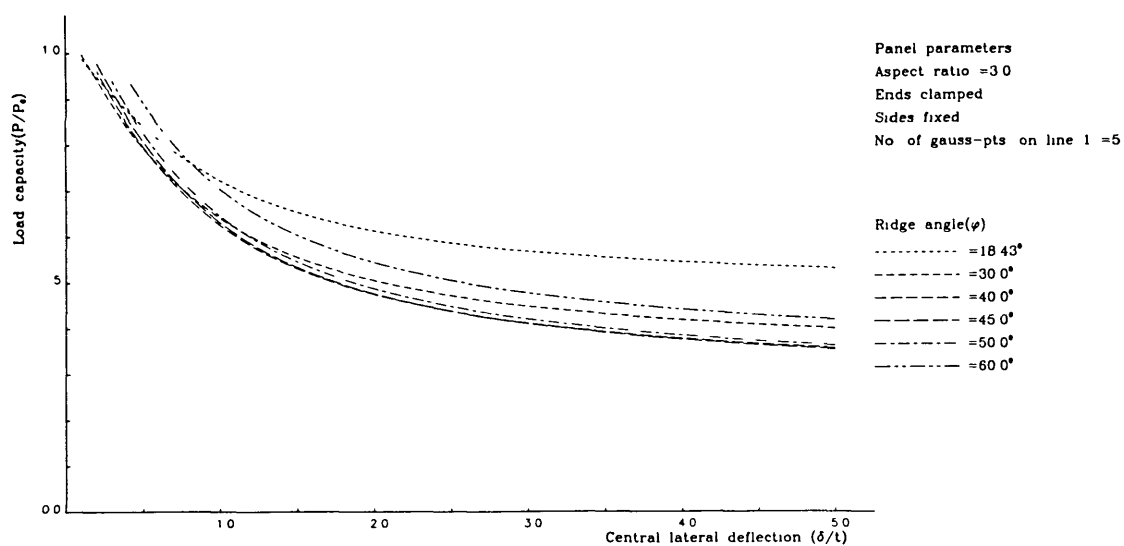
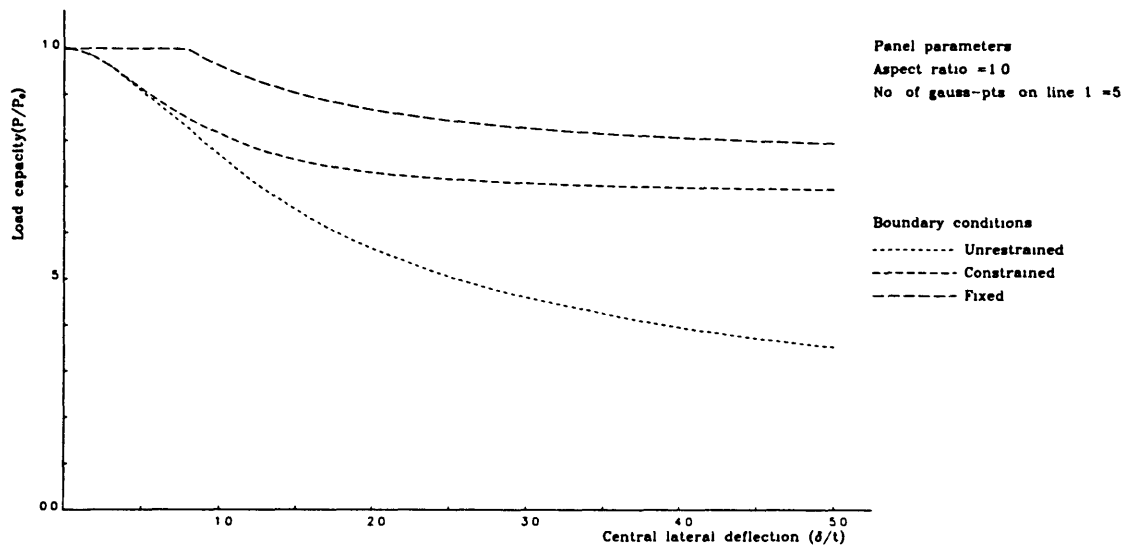
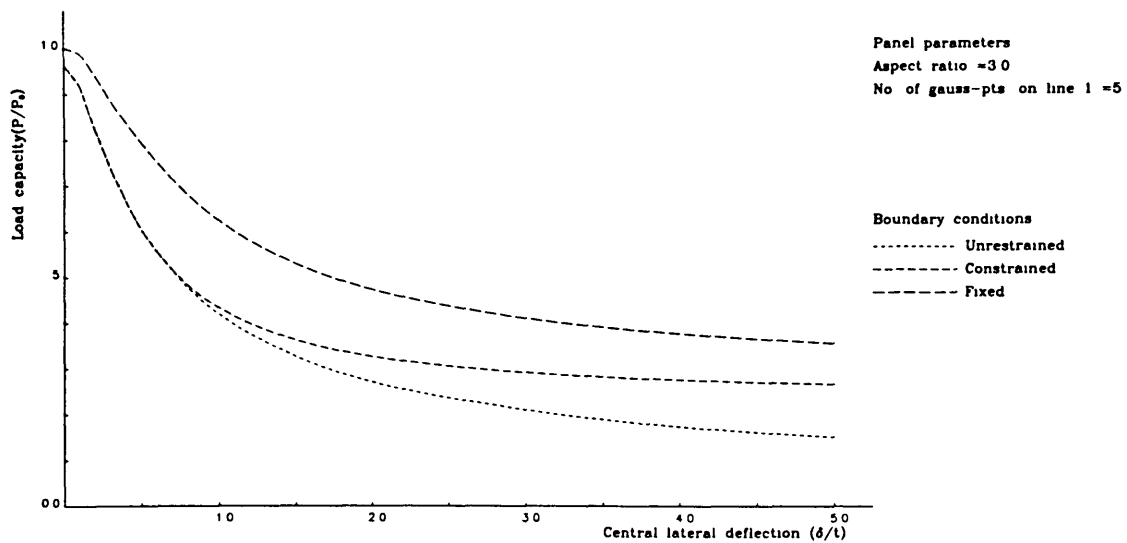


figure 7.35

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves



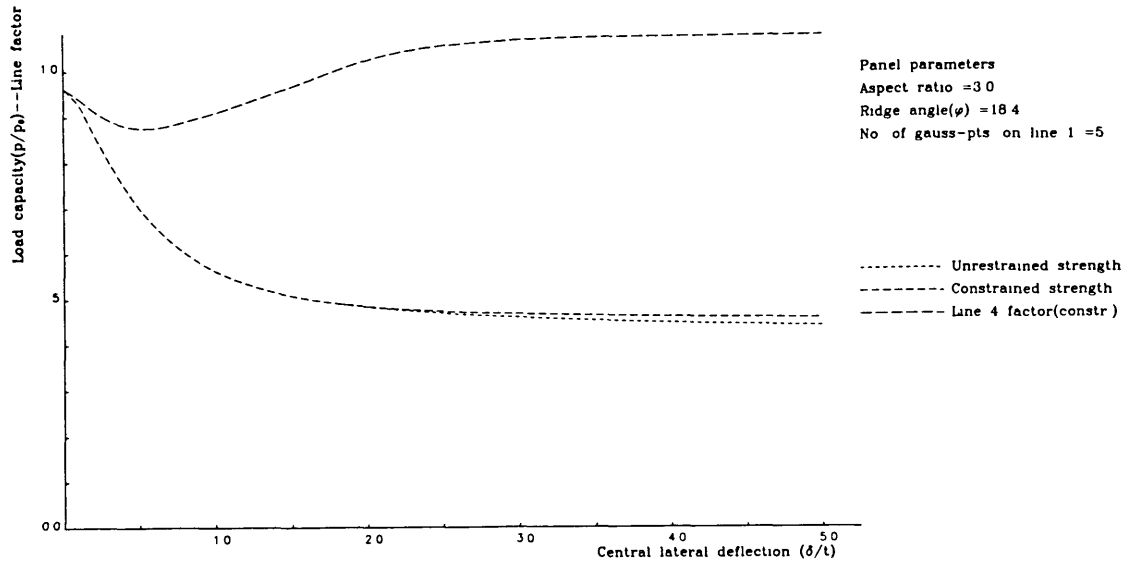
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves



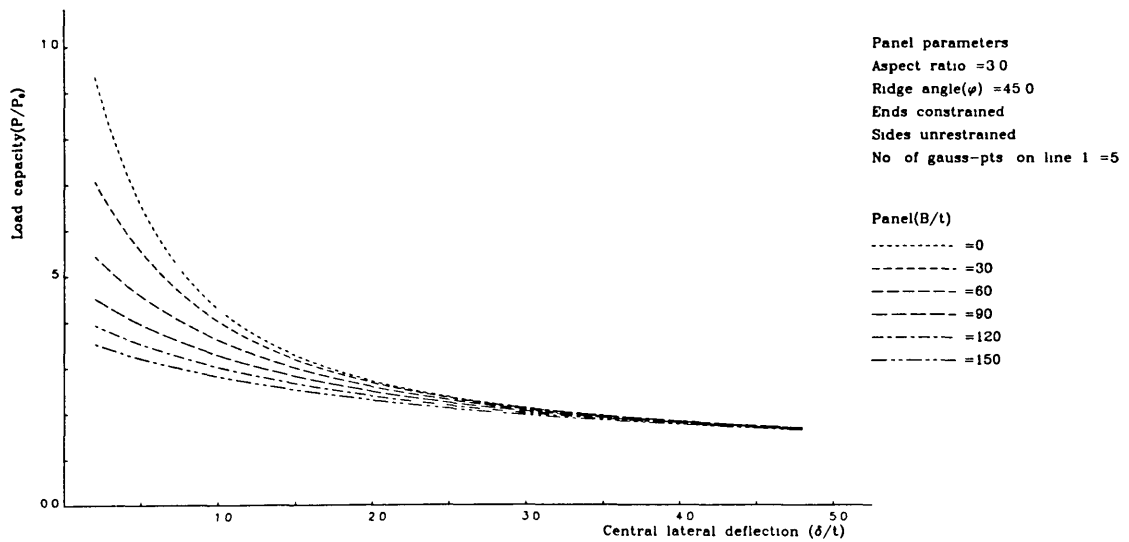
upper figure 7.36

lower figure 7.37

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Plot of central lateral deflection against yield line factor



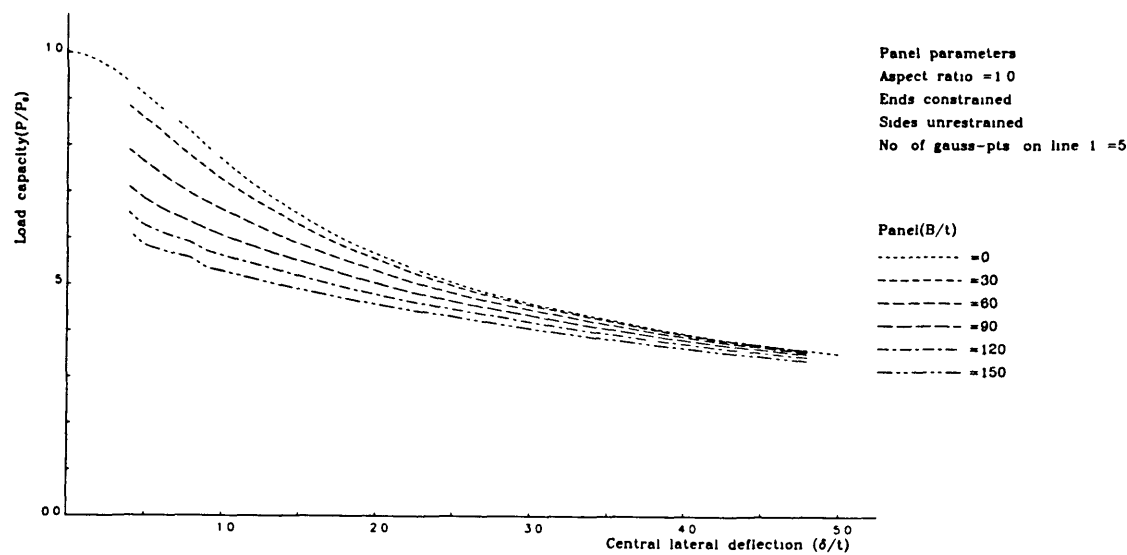
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of elastic energy compensation



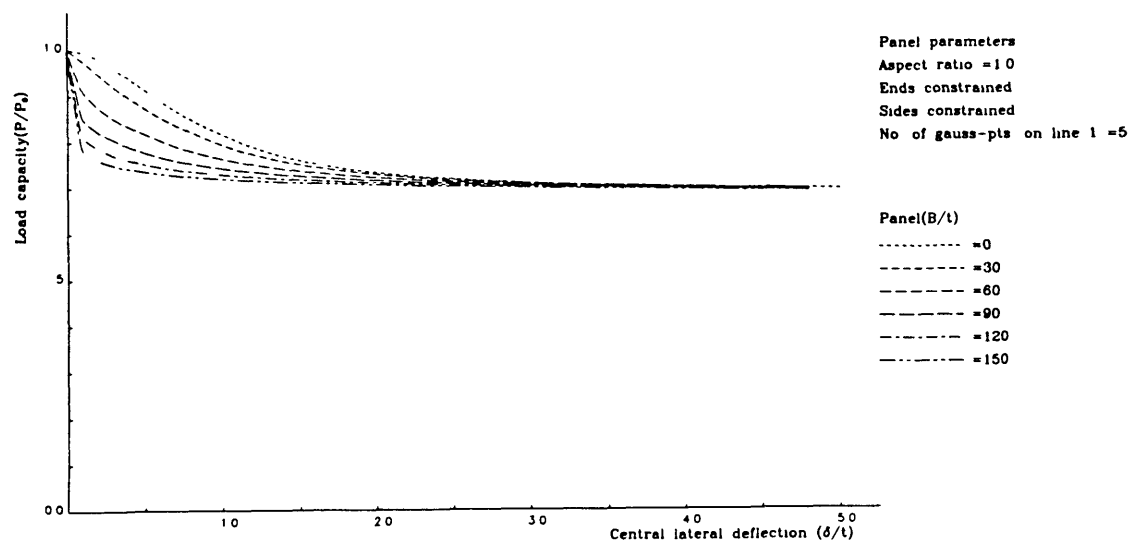
upper figure 7.38

lower figure 7.39

Initially flat panel under inplane load
Plot of central lateral deflection against load capacity
Minimum strength curves, with elastic energy compensation



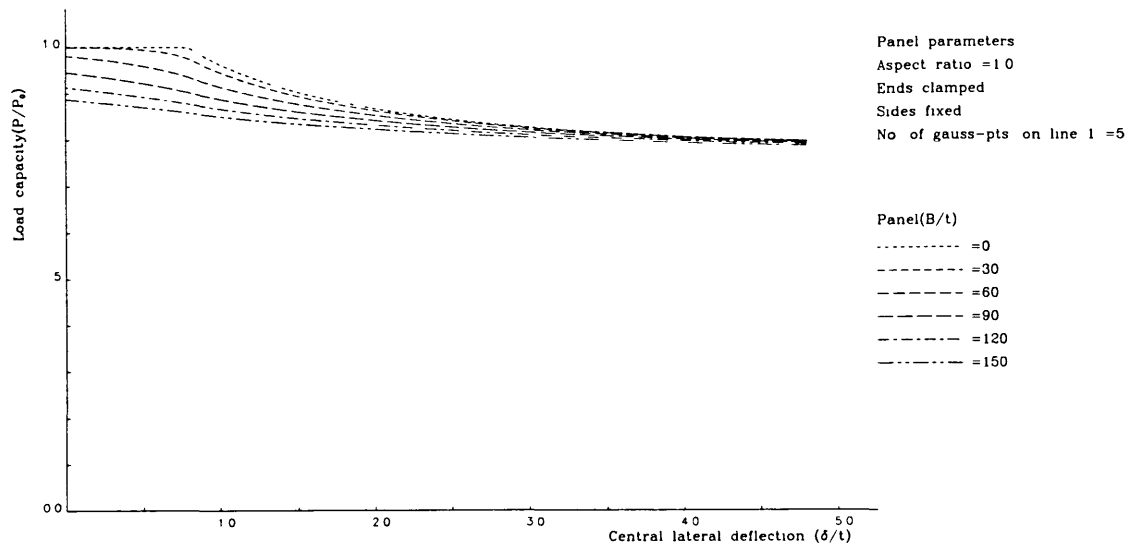
Initially flat panel under inplane load
Plot of central lateral deflection against load capacity
Minimum strength curves, with elastic energy compensation



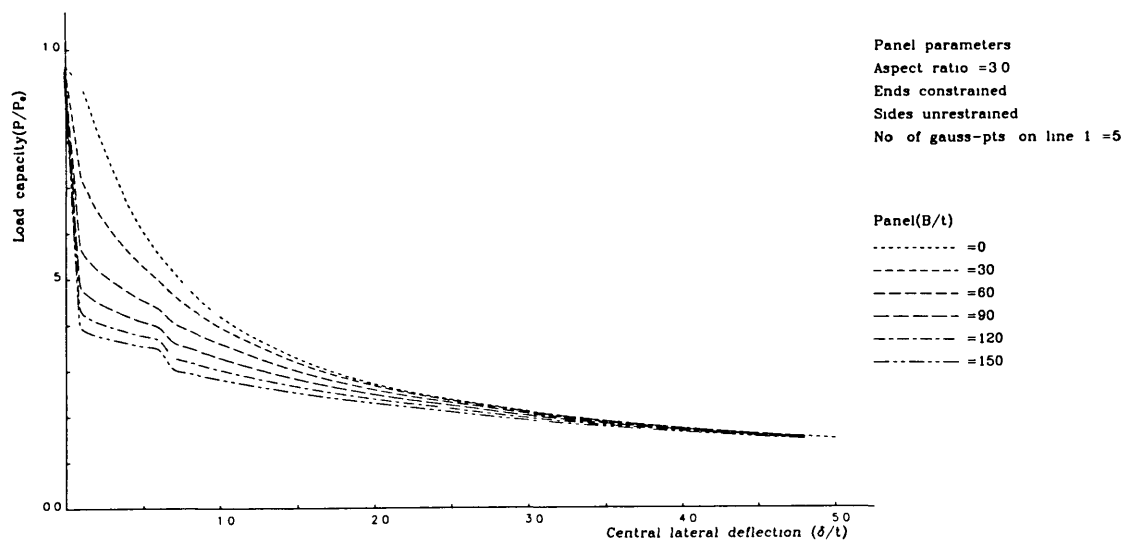
upper figure 7.40

lower figure 7.41

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves, with elastic energy compensation



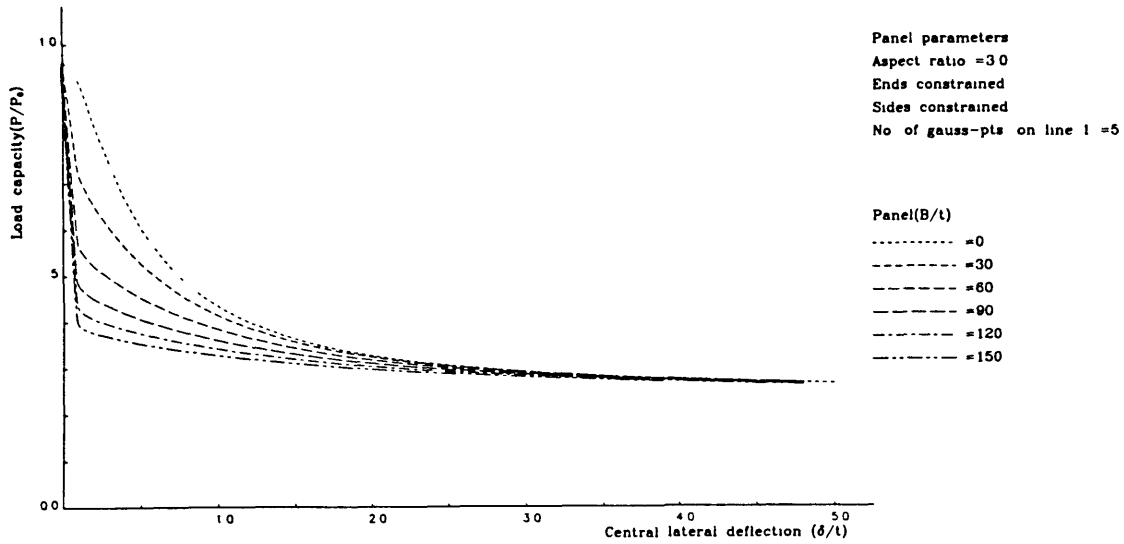
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves, with elastic energy compensation



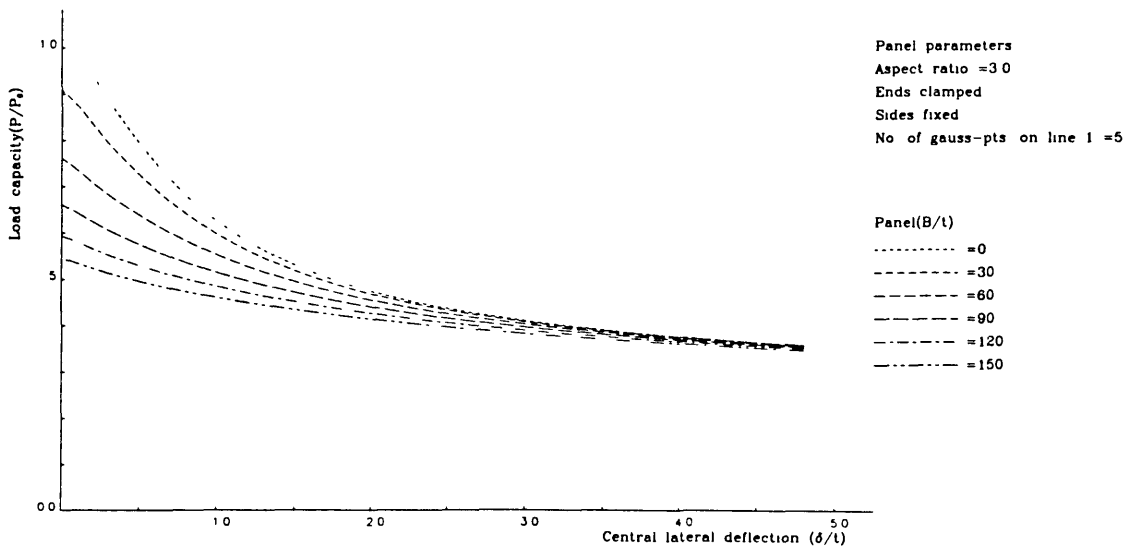
upper figure 7.42

lower figure 7.43

Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves, with elastic energy compensation



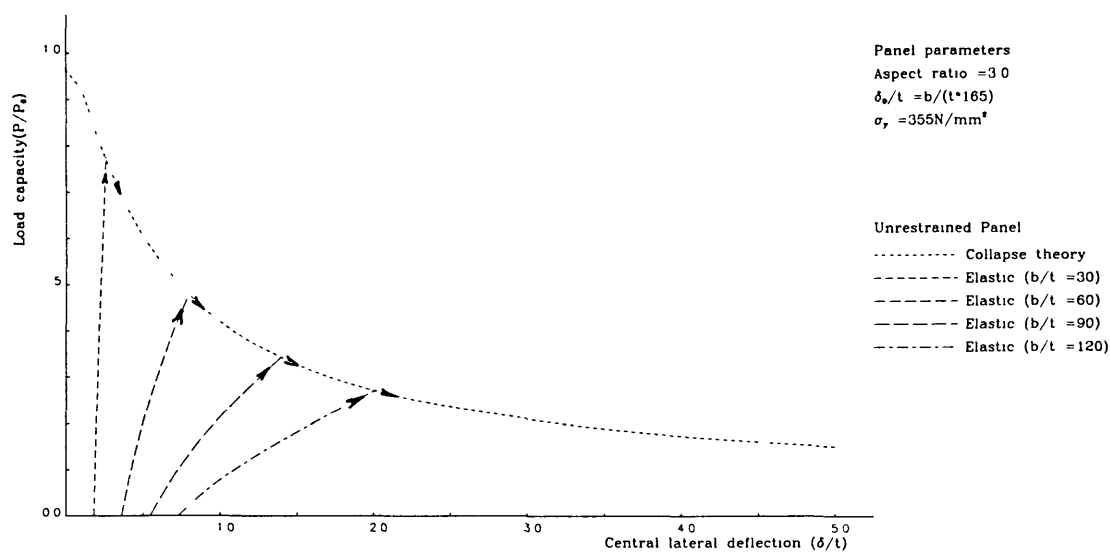
Initially flat panel under inplane load
 Plot of central lateral deflection against load capacity
 Minimum strength curves with elastic energy compensation



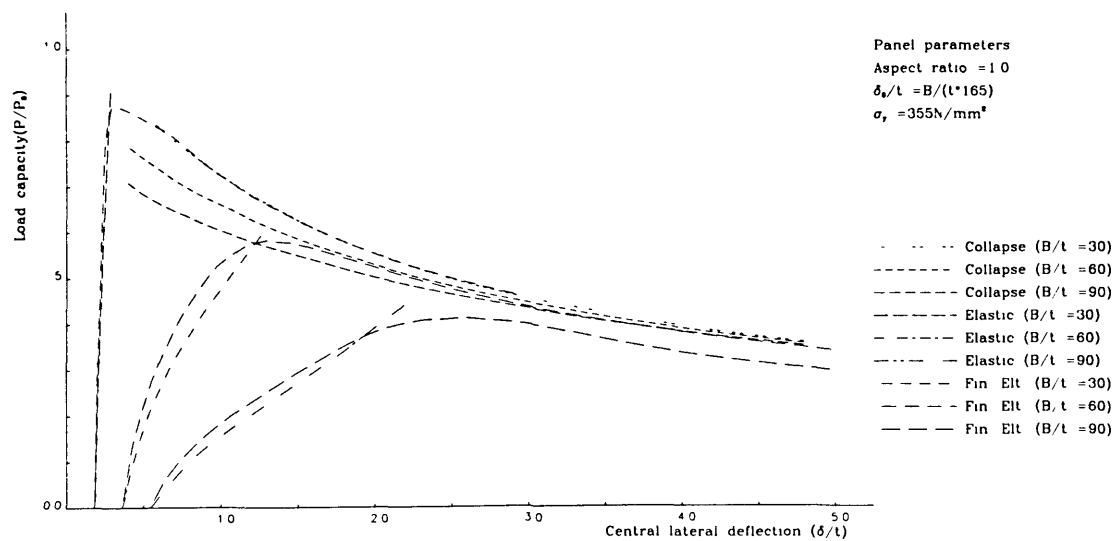
upper figure 7.44

lower figure 7.45

Imperfect flat panel under inplane load
Predicted Elastic-Plastic behaviour



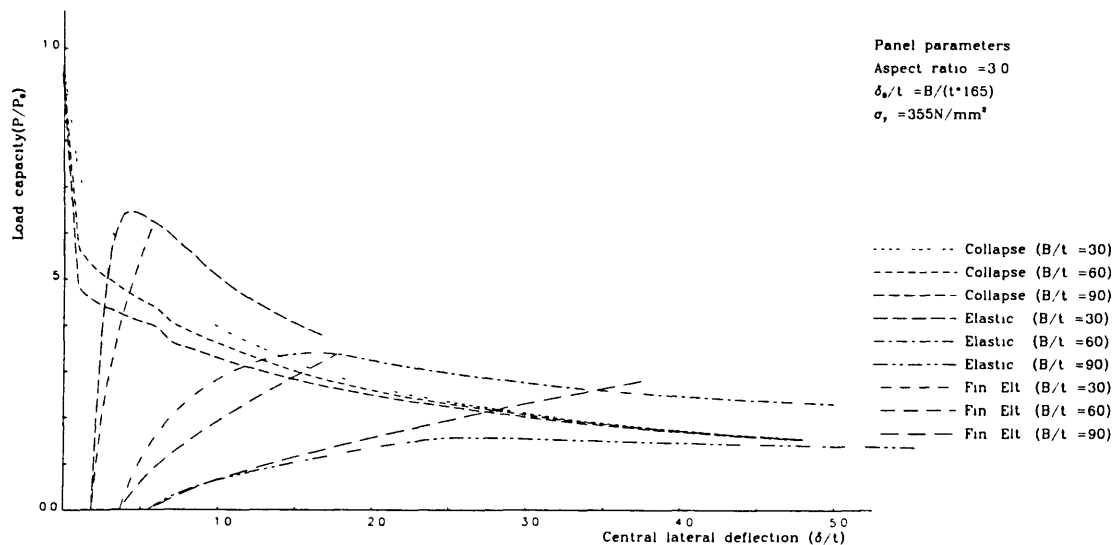
Imperfect unrestrained flat panel under inplane load
A comparison of panel behaviour, predicted by
collapse mechanism theory and finite element elasto-plastic analysis



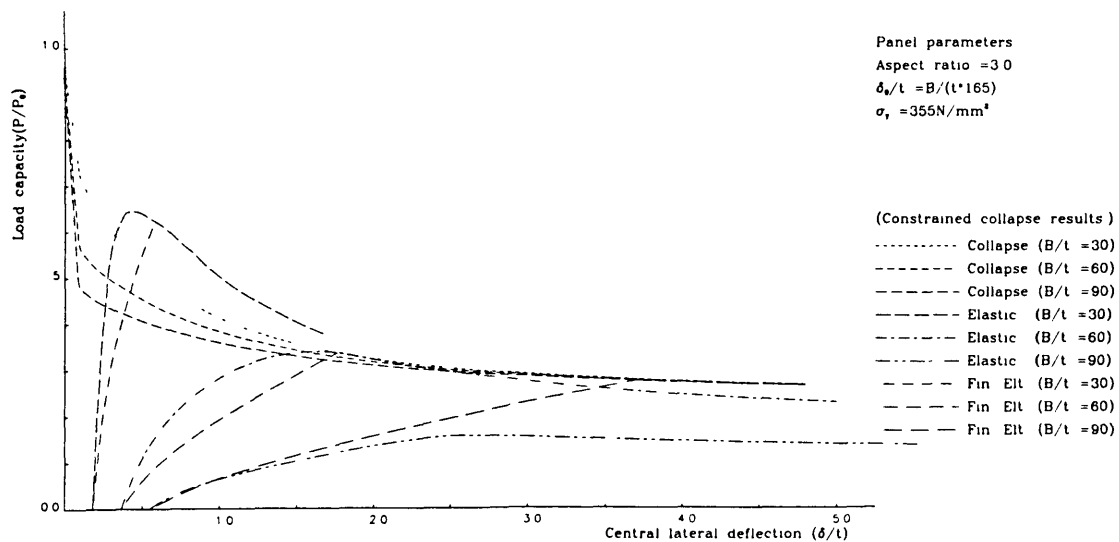
upper figure 7.46

lower figure 7.47

Imperfect unrestrained flat panel under inplane load
 A comparison of panel behaviour predicted by
 collapse mechanism theory and finite element elasto-plastic analysis



Imperfect unrestrained flat panel under inplane load
 A comparison of panel behaviour predicted by
 collapse mechanism theory and finite element elasto-plastic analysis



upper figure 7.48

lower figure 7.49

Assumed Square-Block Stress Distribution			Prandtl-Reuss Derived Stress Distribution	
	$ U \leq 2$	$ U > 2$		All values (for $V=0$, Table 1 (a) is used)
n	$\frac{U (1 + U^2/4)}{2 \left\{ r_s^2 V^2 + (1 + U^2/4)^2 \right\}^{1/2}}$	$\frac{1}{\left\{ 1 + r_s^2 V^2/U^2 \right\}^{1/2}}$	n	$\frac{f_p}{4} \left\{ \left(\frac{1}{4} V^2 + (2+U)^2 \right)^{1/2} - \left(\frac{1}{4} V^2 + (2-U)^2 \right)^{1/2} \right\}$
s	$\frac{r_s V_s}{\left\{ r_s^2 V^2 + (1 + U^2/4)^2 \right\}^{1/2}}$	$\frac{V/U}{\left\{ 1 + r_s^2 V^2/U^2 \right\}^{1/2}}$	s	$\frac{V}{8} \ln \left\{ \frac{(2-U)^2 + V^2/4}{(2+U)^2 + V^2/4} \right\}$
m	$\frac{1 - U^2/16}{\left\{ r_s^2 V^2 + (1 + U^2/4)^2 \right\}^{1/2}}$	00	m	$\frac{f_p}{8} \left[(2+U) \left(\frac{V^2}{4} + (2-U)^2 \right)^{1/2} + (2-U) \left(\frac{V^2}{4} + (2+U)^2 \right)^{1/2} \right]$ $+\frac{f_p}{64} V^2 \ln \left\{ \frac{[(V^2/4 + (2-U)^2)^{1/2} + 2 - U][(V^2/4 + (2+U)^2)^{1/2} + 2 + U]}{[(V^2/4 + (2-U)^2)^{1/2} + 2 + U][(V^2/4 + (2+U)^2)^{1/2} - 2 - U]} \right\}$
$r_s = 1/\sqrt{3}$				
$U = \frac{4}{t} \frac{\dot{u}}{\dot{\alpha}} \quad V = \frac{4}{t} \frac{\dot{v}}{\dot{\alpha}}$ $N = \sigma_y t n$ $S = 1/\sqrt{3} \sigma_y t s$ $M = 1/4 \sigma_y t^2 m$			$f_p = 10 \text{ (} \epsilon_y \neq 0 \text{ along yield line)}$ or $f_p = 2/\sqrt{3} \text{ (} \epsilon_y = 0 \text{ along yield line)}$ $U = \frac{4}{t} \frac{\dot{u}}{\dot{\alpha}} \quad V = \frac{4}{t} \frac{\dot{v}}{\dot{\alpha}}$ $N = \sigma_y t n$ $S = 1/\sqrt{3} \sigma_y t s$ $M = 1/4 \sigma_y t^2 m$	

Table 7 1 (a)

(after Dean)

Table 7 1 (b)

APPENDIX 7A1

D'U_{ij} matrices for a rectangular panel.

Figure 7A1.1(a) describes the geometry of an initially flat panel and its division into 4 discrete elements on collapse. The panel is subject to uniform inplane and lateral loads.

The basic yield line pattern, lines 1 and 2, describes the ridge-type mechanism which occurs with lateral deflection of the panel. These lines are described by elements A and B. Yield line 1 is at an angle ϕ to the global x-axis. Inplane yield lines, lines 4 and 6 are redundant in the basic ridge mechanism. These lines may, however, reach yield under certain loading conditions, and must be considered. They are described by elements A and B1.

Yield lines 3 and 5 must be considered if there are inplane external restraints on the panel at collapse. Yield line 3 may be considered to form the boundary between the panel under consideration and an identical adjacent panel. Displacements along this line have been considered to be mirrored symmetrically by the adjacent panel. Anti-symmetric panel behaviour may be considered by ignoring the virtual work contributions of line 3 in addition to proscribing the displacements dX_1 , $d\theta_1$, $d\theta_3$.

Yield line 5 is formed by elements B and C. Element C is necessary only for consideration of the rotation and inplane displacements of line 5. It is assumed that the degrees of freedom dX_4 , dY_4 , $d\theta_4$ are considered only when line 5 is unrestrained and hence line 5 makes no contribution to the virtual work associated with these freedoms.

Figure 7A1.1(b) shows the degrees of freedom of the panel elements. Figure 7A1.2 shows these freedoms more clearly. As the forces along the yield lines are not constant, representative points are chosen on the yield lines. Virtual work is calculated at these points and summed to estimate the virtual work of the yield lines. Points are chosen according to the Gaussian Quadrature rule. Table 7A1.1 summarises the locations and weightings of the points on the 6 yield lines. The gauss points on any yield line are associated, for convenience,

with a particular side of the yield line, so that the relative movement of the adjoining elements can be described in the global axes of the panel. The local direction cosines of the gauss points may be calculated from the local axes systems shown in figure 7A1.1(b).

There are 10 degrees of freedom in the system. Inplane displacements are not restrained by compatibility. Element A is forced, for compatibility, to rotate through $-1/\tan\phi \cdot d\theta_2$ about the y-axis when element B rotates through $d\theta_2$.

$d\theta_1$ is a rotation of element A which does not change the lateral deflection of P_0 or P_1 . The inplane rotations, $d\theta_3$ and $d\theta_4$, have been given the same dimensions as the inplane translations by using $b \cdot d\theta_3$ and $b \cdot d\theta_4$ for $d\theta_3$ and $d\theta_4$, respectively. In considering the effect of $d\theta_1$, it has been assumed for simplicity that lateral deflections are small, and that little error results from taking the component of $d\theta_1$ in the z-global axis to be zero. It is also assumed that the half-length of the panel, b , is unchanged by lateral displacement of the panel. No account has been taken of the effect of lateral displacement on the direction cosines of points on the yield lines.

The procedure for evaluating the matrix $D'U_{ij}$ for a point on a yield line is relatively simple and follows two steps.

- a) The relative displacement and rotation at a point, due to the 10 degrees of freedom, is evaluated in the global reference axes.
- b) The matrix so obtained is pre-multiplied by the direction cosines matrix of the local axes along the yield line.

This procedure is illustrated in Table 7A1.2 for points on yield line 1. Table 7A1.2(a) contains the relative displacement effects in global axes. Table 7A1.2(b) contains the direction cosines matrix of the line. Table 7A1.2(c) contains the final $D'U_{ij}$ for any point on line 1. The position of any point on a line is defined in terms of its y-ordinate (for lines 1, 2, 3, 6) or its x-ordinate (for line 4).

Only the local displacement and rotation terms du , dv , $d\alpha_v$ need be

retained in the $D'U_{ij}$ matrix. The $D'U_{ij}$ matrices for all lines are given in Table 7A1.3.

Several gauss points must be chosen on yield line 4 when lateral displacement of P_4 is allowed in the system. When a panel is rigidly supported on all sides from lateral displacement it is possible to treat yield line 4 in a more efficient manner. Thus, when $d\theta_1$ is zero, yield line 4 may be accommodated by a single point on a yield line with a thickness equal to $b/\tan\phi$. $d\theta_3$ then becomes the out-of-plane rotation and the inplane displacement is $dY_1 + b/2\tan\phi \cdot d\theta_3$. The single gauss point is in the middle of the yield line. The $D'U_{ij}$ matrix for yield line 4 under these circumstances is given in Table 7A1.4.

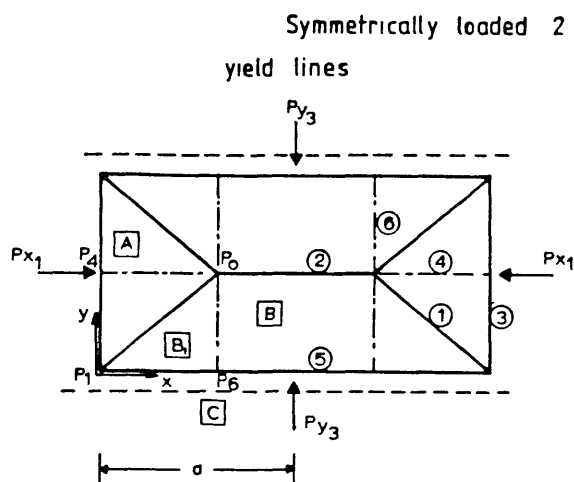


Figure 7A1.1 (a)

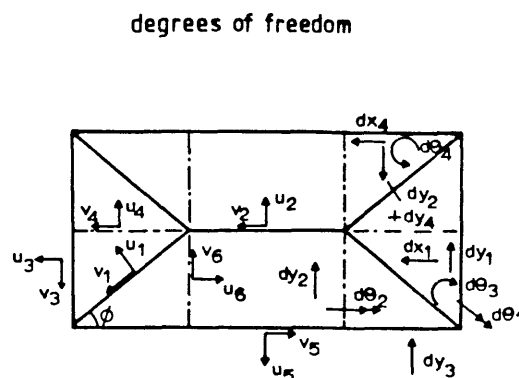


Figure 7A1.1 (b)

Panel has 4 elements A, B, B1, C

10 degrees of freedom [$dx_1, dy_1, d\theta_1, d\theta_3, b, dy_2, d\theta_2, dy_3, dx_4, dy_4, d\theta_4$]

6 yield lines

Degrees of freedom of a 2 dimensional panel
(part of a stiffened panel assembly)

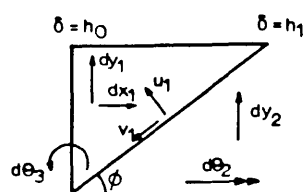


Figure 7A1.2 (a)

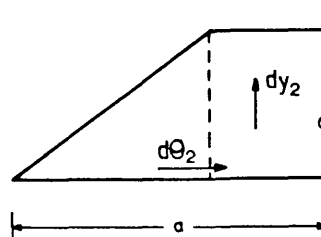


Figure 7A1.2 (b)

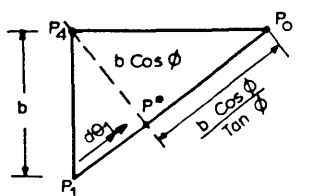


Figure 7A1.2 (c)

Note : the rotation $d\theta_1$ does not change the lateral deflection of $P_0 P_1 P_4$ moves along $P_4 P^*$ a distance $(h_0 - h_1 \sin^2 \phi) d\theta_1$. This inplane displacement affects lines 3 and 4

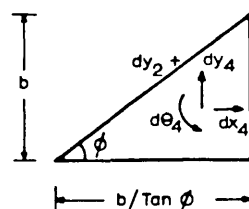


Figure 7A1.2 (d)

Line number	Length	Number of lines	Gaussian Quadrature		Number of points
			ordinate	weighting	
1	$b / \sin \phi$	4	$y = \frac{b}{2} (1 + \xi_g)$	$\frac{b}{2} \sin \phi \cdot \eta_g$	variable
2	$a - \frac{b}{\tan \phi}$	2	$y = b$	$a - \frac{b}{\tan \phi}$	1
3	b	4	$y = \frac{b}{2} (1 + \xi_g)$	$\frac{b}{4} \cdot \eta_g$	variable
4	$b / \tan \phi$	2	$x = \frac{b}{\tan \phi} (1 + \xi_g)$	$\frac{b}{2 \tan \phi} \cdot \eta_g$	variable
5	a	4	$y = 0.0$	a	1
6	b	4	$y = \frac{b}{2} (1 + \xi_g)$	$\frac{b}{2} \cdot \eta_g$	variable

η_g = Gaussian Quadrature weighting
 $-1 \leq \xi_g \leq 1 =$ " " position
 t = Thickness of lines

Table 7A1.1

Line 1

freedom	dx_1	dy_1	$d\theta_1$	$d\theta_3 b$	dy_2	$d\theta_2$	dy_3	dx_4	dy_4	$d\theta_4 b$
u_x	+1 0	0 0	0 0	$-y/b$	0 0	$-\frac{y}{b} h_1 \tan \phi$	0 0	-1 0	0 0	y/b
u_y	0 0	+1 0	0 0	$+\frac{y}{b} \frac{1}{\tan \phi}$	-1 0	$\frac{y}{b} h_1$	0 0	0 0	-1 0	$-\frac{y}{b} \frac{1}{\tan \phi}$
α_x	0 0	0 0	$\cos \phi$	0 0	0 0	-1 0	0 0	0 0	0 0	0 0
α_y	0 0	0 0	$\sin \phi$	0 0	0 0	$-\tan \phi$	0 0	0 0	0 0	0 0

(a)

(b)

$-\sin \phi$	$\cos \phi$	0
$-\cos \phi$	$-\sin \phi$	
0		$-\sin \phi \quad \cos \phi$
		$-\cos \phi \quad -\sin \phi$

$D'U_{ij} =$

$-\sin \phi$	$+\cos \phi$	0 0	$\frac{y}{b} \frac{1}{\sin \phi}$	$-\cos \phi$	$\frac{y h_1}{b} \frac{1}{\cos \phi}$	0 0	$\sin \phi$	$-\cos \phi$	$\frac{y}{b} \frac{1}{\sin \phi}$
$-\cos \phi$	$-\sin \phi$	0 0	0 0	$\sin \phi$	0 0	0 0	$\cos \phi$	$\sin \phi$	0 0
0 0	0 0	-1 0	0 0	0 0	$\frac{1}{\cos \phi}$	0 0	0 0	0 0	0 0

(c)

Table 7A1 2

$\dot{D}U_{ij}$ matrices for an initially flat rectangular panel

Degrees of freedom

dx_1	dy_1	$d\theta_1$	$d\theta_3 b$	dy_2	$d\theta_2$	dy_3	dx_4	dy_4	$d\theta_4 b$
--------	--------	-------------	---------------	--------	-------------	--------	--------	--------	---------------

Line 1

$-\sin \phi$	$\cos \phi$	00	$\frac{y}{b} \frac{1}{\sin \phi}$	$-\cos \phi$	$\frac{yh_1}{b} \frac{1}{\cos \phi}$	00	$\sin \phi$	$-\cos \phi$	$-\frac{y}{b} \frac{1}{\sin \phi}$
$-\cos \phi$	$-\sin \phi$	00	00	$\sin \phi$	00	00	$\cos \phi$	$\sin \phi$	00
00	00	-1.0	00	00	$\frac{1}{\cos \phi}$	00	00	00	00

Line 2

00	00	00	00	-20	$2h_1$	00	00	00	00
00	00	00	00	00	00	00	00	00	00
00	00	00	00	00	20	00	00	00	00

Line 3

20	00	$2\xi_3 \sin \phi$	$-2 \frac{y}{b}$	00	00	00	00	00	00
00	00	00	00	00	00	00	00	00	00
00	00	$2 \sin \phi$	00	00	$-2 \tan \phi$	00	00	00	00

$$\xi_3 = (h_0 - h_1 \sin^2 \phi) y/b$$

Line 4

00	-20	$2\xi_4 \cos \phi$	$-2 \frac{x}{b}$	00	00	00	00	00	00
00	00	00	00	00	00	00	00	00	00
00	00	$2 \cos \phi$	00	00	00	00	00	00	00

$$\xi_4 = (h_0 - h_1 \sin^2 \phi) (1 - \tan \phi \cdot x/b)$$

Line 5

00	00	00	00	10	00	-10	00	00	00
00	00	00	00	00	00	00	00	00	00
00	00	00	00	00	-10	00	00	00	00

Line 6

00	00	00	00	00	00	00	-10	00	y/b
00	00	00	00	00	00	00	00	-10	$-1/\tan \phi$
00	00	00	00	00	00	00	00	00	00

Table 7A1.3

D'_{ij} matrix of line 4 for $d\theta_1 = 0$

freedom	dx_1	dy_1	$d\theta_{3.b}$	dy_2	$d\theta_2$	dy_3	dx_4	dy_4	$d\theta_{4.b}$
---------	--------	--------	-----------------	--------	-------------	--------	--------	--------	-----------------

$D'_{ij} =$	0 0	-2 0	$-1 0 / \tan \phi$	0 0	0 0	0 0	0 0	0 0	0 0
	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
	0.0	0 0	$-2 0 / b$	0 0	0 0	0 0	0 0	0 0	0 0

Thickness of line = $b / \tan \phi$
 no of Gauss points = 1
 weighting of line = t
 no of lines = 2

Table 7 A14

APPENDIX 7A2

Redundant yield lines

It is considered that known inplane yield lines exist at 90° to an unrestrained (free) edge of a panel, figure 7A2.1(a).

The yield state of a postulated yield line, B, is required. Line B is at an angle α to a known yield line A, figure 7A2.1(b).

There are 3 equilibrium conditions for the element formed by yield line A and B and the unrestrained edge. The net forces on the element along directions (X_1, Y_1, θ_1) are zero.

Infinitesimal unit displacements $(dX_1, dY_1, d\theta_1)$ are considered. The effects of these displacements on gauss points on lines A and B are given in figure 7A2.1(c).

If the inplane direct and shear stress resultants at any point on a line are N, S respectively, the three equilibrium equations are given by the principle of virtual work.

The summation convention, e.g. $N_{1i} \eta_i$, is the usual tensor convention for repeated subscripts. For convenience i will refer to line A, and j will refer to line B.

Equilibrium along X_1 :

$$-N_{1i} \cdot \eta_i \cdot d + N_{2j} \cdot \cos \alpha \cdot \eta_j \cdot d / \cos \alpha + S_{2j} \cdot \sin \alpha \cdot \eta_j \cdot d / \cos \alpha = 0$$

i.e.

$$-N_{1i} \eta_i + N_{2j} \eta_j + S_{2j} \eta_j \tan \alpha = 0 \text{ ----- (1)}$$

Equilibrium along Y_1 :

$$-S_{1i} \cdot \eta_i \cdot d - N_{2j} \cdot \sin \alpha \cdot \eta_j \cdot d / \cos \alpha + S_{2j} \cdot \cos \alpha \cdot \eta_j \cdot d / \cos \alpha = 0$$

i.e.

$$S_{1i} \eta_i - N_{2j} \eta_j \tan \alpha + S_{2j} \eta_j = 0 \text{ ----- (2)}$$

Equilibrium along θ_1 :

$$N_{1i} \cdot \frac{y_i}{d} \eta_i \cdot d - N_{2j} \cdot \frac{y_j}{d} \cdot \frac{1}{\cos \alpha} \cdot \eta_j \cdot \frac{d}{\cos \alpha} = 0.0$$

i.e.

$$N_{1i} y_i \eta_i - N_{2j} y_j \eta_j / \cos^2 \alpha = 0.0 \text{ ----- (3)}$$

The summation in equations 1, 2, 3 may also be represented as:

$$N_{1i} \eta_i = N_1 \text{ etc}$$

and

$$N_{1i} y_i \eta_i = M_1$$

Then N_2 and S_2 are evaluated from equations 1 and 2.

M_2 is obtained from equation 3.

$$N_2 = (N_1 - S_1 \tan \alpha) / (1 + \tan^2 \alpha) \text{ ----- (4)}$$

$$S_2 = (S_1 + N_1 \tan \alpha) / (1 + \tan^2 \alpha) \text{ ----- (5)}$$

$$M_2 = M_1 / (1 + \tan^2 \alpha) \text{ ----- (6)}$$

Dean (218) gives the constitutive relationship for a yield line as

$$n = \frac{1}{\{1 + r_s^2 v^2 / u^2\}^{1/2}} = \frac{u/v}{\{(u/v)^2 + r_s^2\}^{1/2}}$$

$$s = \frac{r_s v/u}{\{1 + r_s^2 v^2 / u^2\}^{1/2}} = \frac{r_s}{\{(u/v)^2 + r_s^2\}^{1/2}}$$

For an inplane yield line $u = k'_0 + k'_1 X$, for $0 \leq X \leq 1.0$

and $v = k_2$

Then $u/v = U/V = k_0 + k_1 X$

$$\text{Thus } n = \frac{k_0 + k_1 X}{\{(k_0 + k_1 X)^2 + r_s^2\}^{1/2}}$$

$$s = \frac{r_s}{\{(k_0 + k_1 X)^2 + r_s^2\}^{1/2}}$$

$$N = \sigma_{yt} \cdot n$$

$$S = r_s \sigma_{yt} \cdot s$$

As σ_{yt} is common to all expressions we will write $N = n$ and $S = r_s \cdot s$

The summation of equations 1, 2, 3 may be represented as integrals.

Then

$$N_1 = \frac{d}{k_1} \int_0^1 \frac{k_1(k_0 + k_1 X) dX}{\{(k_0 + k_1 X)^2 + r_S^2\}^{\frac{1}{2}}}$$

$$S_1 = d \cdot r_S^2 \int_0^1 \frac{dX}{\{(k_0 + k_1 X)^2 + r_S^2\}^{\frac{1}{2}}}$$

$$M_1 = d^2 \int_0^1 \frac{(k_0 + k_1 X) X dX}{\{(k_0 + k_1 X)^2 + r_S^2\}^{\frac{1}{2}}}$$

These integrals give

$$\frac{N_1}{d} = \left\{ \left[\left[\frac{k_0}{k_1} + 1 \right]^2 + \left[\frac{r_S^2}{k_1} \right]^{\frac{1}{2}} \right] - \left[\left[\frac{k_0}{k_1} \right]^2 + \left[\frac{r_S^2}{k_1} \right]^{\frac{1}{2}} \right] \right\} \dots \dots \dots (7)$$

$$\frac{S_1}{d} = \frac{r_S^2}{k_1} \ln \left\{ \frac{\frac{k_0}{k_1} + 1 + \left[\left[\frac{k_0}{k_1} + 1 \right]^2 + \left[\frac{r_S^2}{k_1} \right]^{\frac{1}{2}} \right]}{\frac{k_0}{k_1} + \left[\left[\frac{k_0}{k_1} \right]^2 + \left[\frac{r_S^2}{k_1} \right]^{\frac{1}{2}} \right]} \right\} \dots \dots \dots (8)$$

$$\frac{2M_1}{d^2} = \left[\left[\frac{k_0}{k_1} + 1 \right]^2 + \left[\frac{r_S^2}{k_1} \right]^{\frac{1}{2}} \right] - \frac{1}{k_1} \left[\frac{S_1}{d} + \frac{N_1}{d} \cdot k_0 \right] \dots \dots \dots (9)$$

Equations 7, 8, 9 express the relationship between the fundamental velocity variables k_0 , k_1 and the forces N_1/d , S_1/d , M_1/d^2 on the yield line. These equations are valid for a line where the yield state is identically satisfied, i.e. on yield line A. The yield state of line B is unknown. Yield may not have been reached, or it may have been exceeded.

The forces in line B may be expressed as:-

$$\frac{N_2}{d} = f \cdot \left\{ \left[\left[\frac{k_3}{k_4} + 1 \right]^2 + \left[\frac{r_S^2}{k_4} \right]^{\frac{1}{2}} \right] - \left[\left[\frac{k_3}{k_4} \right]^2 + \left[\frac{r_S^2}{k_4} \right]^{\frac{1}{2}} \right] \right\} \dots \dots \dots (10)$$

$$\frac{S_2}{d} = f \cdot \frac{r_S^2}{k_4} \ln \left\{ \frac{\frac{k_3}{k_4} + 1 + \left[\left[\frac{k_3}{k_4} + 1 \right]^2 + \left[\frac{r_S^2}{k_4} \right]^{\frac{1}{2}} \right]}{\frac{k_3}{k_4} + \left[\left[\frac{k_3}{k_4} \right]^2 + \left[\frac{r_S^2}{k_4} \right]^{\frac{1}{2}} \right]} \right\} \dots \dots \dots (11)$$

$$\frac{2M_2}{d^2} = f \cdot \left[\left[\frac{k_3}{k_4} + 1 \right]^2 + \left[\frac{r_S}{k_4} \right]^2 \right]^{\frac{1}{2}} - \frac{1}{k_4} \left[\frac{S_2}{d} + \frac{N_2}{d} \cdot k_3 \right] \text{ ----- (12)}$$

where $f \geq 0$

For any set (k_0, k_1) for line A, N_1, S_1, M_1 can be evaluated from equations 7, 8, 9. Equations 4, 5, 6 are used to calculate N_2, S_2, M_2 and by a trial approach equations 10, 11, 12 are solved for k_3, k_4, f . (In fact, N_1 and S_1 can be chosen initially. Equations 7, 8, 9 can be used to evaluate k_0, k_1, M_1 .)

Plots of f have been obtained for various values of α and are plotted in figures 7.12 - 17. For $0 \leq f \leq 1.0$ yield line B is elastic or line B meets the yield criterion. For $f > 1.0$ the yield criterion is violated along line B.

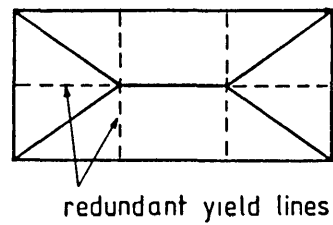


Figure 7A2.1 (a)

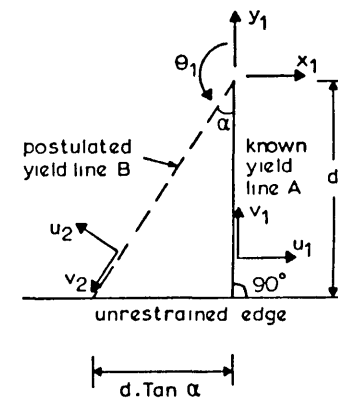


Figure 7 A2.1 (b)

Line A

$$\begin{bmatrix} u_{1i} \\ v_{1i} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & y_i/d \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dy_1 \\ d\theta_1 \end{bmatrix}$$

$$w_{ti} = d \cdot \eta_i$$

Figure 7 A2.1 (c)

Line B

$$\begin{bmatrix} u_{2j} \\ v_{2j} \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha & -y_j/d \cos \alpha \\ \sin \alpha & \cos \alpha & 0 \end{bmatrix} \begin{bmatrix} dx_1 \\ dy_1 \\ d\theta_1 \end{bmatrix}$$

$$w_{tj} = \frac{d}{\cos \alpha} \cdot \eta_j$$

CHAPTER 8.

The Application of Collapse Mechanism Theory to Cylindrically Curved Panel
Assemblies under Combinations of Axial and Pressure Loading.

8.1 Introduction

8.1.1 Scope of chapter

The collapse of longitudinally stiffened cylinders under longitudinal and lateral load will be considered in this chapter. In many cases the proportioning of cylinder components is such that the preferred mode of failure is by a combination of panel and stiffener collapse. The longitudinal stiffeners enforce a symmetry on the failure mode, so that a repeated collapse mechanism forms around the circumference. The basic unit of such a mechanism will incorporate a number of stiffeners and a segment of panel.

The collapse mechanism theory as developed can model the collapse of stiffened panels and reference will be made to such a mechanism. The main consideration of the chapter will, however, be confined to the specific case of a stiffened panel where a single unstiffened curved panel, bound on its longitudinal edges by stiffeners, forms the basic unit of collapse, figure 8.1.

Results will be presented for the collapse of a single curved panel, under both lateral pressure and longitudinal compression. The presence or absence of ring stiffeners must be considered since the ability of the overall cylinder to contract radially has an important influence on panel strength. The treatment of ring stiffeners will be confined to the simplest possible mechanisms.

The method which is adopted will accommodate the collapse of the longitudinal stiffeners at the edge of the curved panel. It will be shown how the adequacy of stringer stiffeners may be assessed. The eccentricity of the stiffener from the shell mid-surface and the stiffener proportions will be shown to influence the adequacy of the stiffeners.

8.1.2 Assumptions

It was shown in chapter 7 that the computation of $D'U_{ij}$ matrices for points on the yield line was an essential part of the solution process for establishing the collapse behaviour of a panel. The panels considered in chapter 7 were 2-dimensional and it was assumed that the orientation of the local axes at the points on the yield lines did not change with the out-of-plane rotation of the panel. This enabled $D'U_{ij}$ matrices to be evaluated by inspection.

The yield lines of a curved panel are curved in 3-dimensional space, so that it is no longer possible to compute $D'U_{ij}$ matrices by inspection. A more complete consideration of the panel geometry is required.

The assumptions made for curved panels are:

- a) Panels remain rigid inplane and are only free to bend laterally at the yield lines.
- b) The collapse displacements of the panel are adequately described by a small number of rigid-body movements of the rigid panel elements.
- c) There is no distortion of the panel other than by rigid body movements. Thus, changes of yield line shape are excluded.
- d) Yield lines do not change in length.

There is no assumption that the angles of rotation of the panel elements are small. The orientation of the local axes at points on the yield lines is allowed to vary to accommodate real rotations of the panel elements. The effects of such re-orientation will, however, be shown to be minimal.

8.2 Definition of curved panel

8.2.1 Idealisation of panel by 3 elements

Let the curved panel be as shown in plan in figure 8.2(a), and in section in figure 8.2(b). The panel is considered to consist of two typical elements A and B, figure 8.2(c). A third element, C, facilitates a clamped connection for the ends of the panel. A symmetry condition about the mid-height of the panel implies that elements A and B are repeated in the longitudinal direction. Symmetry also exists about the mid generator of the panel.

One quarter of the panel may be considered as the basic panel unit. An origin is chosen for a global axis system, u_i , at P_1 . The global axis system is shown in figure 8.2(b).

8.2.2 Definition of points on the yield lines

Five possible yield lines exist, defined by the points P_1 - P_5 , figure 8.2(c). Yield lines 1, 2, 4 and 5 are easily defined since they are either parallel or perpendicular to the cylinder generator. Yield line 3 is not uniquely defined by points P_1 and P_3 . It will be assumed that line 3 is part of a helix.

A helix is characterised by constant rates of change of height and angular position on a cylinder. A point on a helix may be defined in terms of (H, θ, α) . The height and angular displacements are H and θ . The angle α is the angle of the helix with the cylinder generators.

The points on the five yield lines, figure 8.2, may be described in this manner. Thus the gauss points for the yield lines may be described very simply as

$$\left. \begin{aligned} H_i &= \frac{1}{2}(1-\xi_i)H_1 + \frac{1}{2}(1+\xi_i)H_2 \\ \theta_i &= \frac{1}{2}(1-\xi_i)\theta_1 + \frac{1}{2}(1+\xi_i)\theta_2 \\ \alpha_i &= \alpha_j \end{aligned} \right\} \text{----- (1)}$$

where (H_i, θ_i) are the height and angular positions of the point i ,

and (H_1, θ_1) and (H_2, θ_2) are the height and angular positions of the end points of the relevant yield line. α_j is the angle of the helix of the yield line. ξ_i is the gaussian quadrature weighting of the point. Then, for the point i , the cartesian co-ordinates in the global axis system are

$$(R(\cos\theta_i - 1), R \sin\theta_i, H_i) \text{ ----- (2)}$$

The normal to the undisturbed surface of the cylinder at i is

$$w = \cos\theta_i \hat{i} + \sin\theta_i \hat{j} + 0 \hat{k} \text{ ----- (3)}$$

The tangent to the surface, parallel to the helix is

$$v = -\sin\theta_i \sin\alpha \hat{i} + \cos\theta_i \sin\alpha \hat{j} + \cos\alpha \hat{k} \text{ ----- (4)}$$

The unit vectors, $\hat{w}$ and $\hat{v}$, are used to obtain $\hat{u}$

$$\hat{u} = \hat{v} \times \hat{w} \text{ ----- (5)}$$

$\hat{u}$, $\hat{v}$, $\hat{w}$ form the basis for the transformation matrix from the global axis framework to the local axis framework at the point i . The local framework at the yield line has been defined in §7.2.2.

The global axes - local axes transformation is written as

$$u'_i = A_{ij} u_j \text{ ----- (6)}$$

Thus by means of (H_i, θ_i, α) the co-ordinates and local direction co-sines of all points are defined.

8.3 Definition of reference planes for the panel

8.3.1. Relationship between reference axes

The global axis system u_i could suffice for the problem being considered. Two more convenient axis systems, corresponding with the elements A and B, are chosen to enable a better visualisation of the problem. These axis systems have the same origin as the global

axes. Thus transformations may be made very easily between the 3 systems.

P_1, P_2, P_3 define a plane. The y-axis of the plane is chosen parallel to the generator of the panel. (This is the z-axis of the global framework.) The z-axis of the plane is defined as the outward normal to the plane, as defined by $\overline{P_1P_3} \times \overline{P_1P_2}$. The x-axis is defined in a right handed system as $\hat{u}^1 = \hat{v}^1 \times \hat{w}^1$. The transformation

$$B1_{ij} u_j = u_i^1 \text{ ----- (7)}$$

expresses the connection between the global axis and the axis system of element A.

The points P_1, P_3, P_A , figure 8.3(a), define a plane for the second element, the x-axis of which is along P_1P_A . The z-axis is the outward normal to the plane and the y-axis is given by $\hat{v} = \hat{w} \times \hat{u}$. The transformation

$$B2_{ij} u_j = u_i^2 \text{ ----- (8)}$$

gives the connection between the global axes and the axis system of element B.

$$\text{Also } u_i^2 = CC_{ij} u_j^1 \text{ ----- (9)}$$

where $CC_{ij} = B2_{ik} B1_{jk}$

The local direction cosines at the points on the yield lines are given as

$$u_i' = A_{ij} B1_{kj} u_k^1 = A_{ij} B2_{kj} u_k^2 \text{ ----- (10)}$$

In a thoroughly consistent treatment of the problem of collapse there would exist several elements and hence several planes of reference, like planes 1 and 2. In that type of problem all displacements would ultimately be referred to a global reference axis system, such as shown in figure 8.2(a). The problem under consideration in this

ultimately be referred to a global reference axis system, such as shown in figure 8.2(a). The problem under consideration in this chapter is simplified in visual terms by referring displacements to the plane of element A. In particular, the degrees of freedom of the system are most easily visualised when referred to the plane of element A.

8.4 Degrees of freedom

8.4.1 Panel degrees of freedom

Seven degrees of freedom will be considered initially, figure 7.2(c) and figure 8A1.1. They form a set $\{\theta_x, \theta_y, \theta_z, X_1, Y_1, Y_2, Y_3\}$ and are expressed in terms of the axes of plane 1. Table 8.1 summarises the components of these freedoms. Table 8.1 also gives the expressions for freedoms Y_2 and Y_3 in terms of the axes of plane 2.

8.4.2 Change of cylinder radius

The seven degrees of freedom are sufficient when there is no overall contraction of the cylinder radius, i.e. when ring stiffeners restrain the cylinder at collapse. When ring stiffeners are absent, or are themselves subject to collapse, an eighth degree of freedom is required. This freedom is the change of radius, dR . However, the freedom is more conveniently considered as an overall inplane shortening, dX_3 , when panel failure is under consideration.

$$dX_3 = \frac{\pi}{N_{St}} \cdot dR \text{ ----- (11)}$$

8.5 Compatibility of displacements

8.5.1 Lateral displacements of end points of yield lines

Two compatibility conditions ensure that the relative lateral displacement of P_1 and P_3 is zero. Symmetry of displacements ensures that no relative lateral movement occurs at P_2 , P_4 and P_5 .

Compatibility of displacement is ensured at P_1 when the inplane displacement X_1 is written as

$$X_1 = \begin{bmatrix} k_{10} \\ 0 \\ k_{20} \end{bmatrix} X_1 \text{ ----- (12)}$$

k_{10} and k_{20} are defined in appendix 8A1.

Compatibility of displacement is ensured at P_3 by rotation of element B about the x-axis of plane 2.

The freedoms θ_x , θ_z , θ_y , X_1 produce lateral displacements of P_3 . These displacements are resolved along the normal to the surface at P_3 . The rotation $\Gamma_2 = \Gamma_2 u_1^2 = \Gamma_2 i_2$ of the element B is computed in appendix 8A1.

The rotation may be expressed as

$$\Gamma_2 = \xi_1 X_1 + \xi_2 \theta_x + \xi_3 \theta_y + \xi_4 \theta_z \text{ ----- (13)}$$

The values of $\xi_1 - \xi_4$ are given in appendix 8A1. The rotation $\Gamma_2 i_2$ becomes

$$\Gamma_2 = CC_{11} \Gamma_2 \hat{i}_1 + 0 \hat{j}_1 + CC_{13} \Gamma_2 \hat{k}_1 \text{ ----- (14)}$$

in the plane of element A. ($CC_{12} = 0$, by inspection, since the x-axis of plane 2 has no component along the y-axis of plane 1.)

8.5.2 Lateral displacements of points on line 3

Compatibility of lateral displacements is ensured only at the end points of line 3. Symmetry of displacement ensures zero relative lateral displacement of lines 1, 2, 4, 5.

A small component of relative lateral movement will occur at points along line 3 due to the freedoms X_1 , θ_x , θ_z . These components are largest close to the mid-point of the line.

In the particular case of X_1 , for a panel where P_3 and P_5 coincide, the relative lateral displacement is

$$w^1 = \frac{k_{20}^3}{k_{10}^2 - k_{20}^2} X_1$$

at the mid-point of the line.

$$w^1 = .004 X_1 \text{ for } N_{st} = 10$$

$$\text{and } w^1 = .0005 X_1 \text{ for } N_{st} = 20$$

These values are very small in comparison to the inplane components which are

$$u^1 \approx -\cos\alpha X_1 \text{ and } v^1 \approx -\sin\alpha X_1$$

The very small lateral displacements have been included, for completeness, in the DU'_{ij} matrices of the points on all lines. The forces consequent to the relative lateral displacements are evaluated in the manner indicated in §7.5.4.

8.6 Symmetry of displacements

8.6.1 Reflective symmetry of movement for lines 1, 2 and 5.

The total relative displacements of lines 3 and 4 are obtained by adding the relative displacements of elements A and B for line 3 and of elements B and C for line 4. The total relative displacements of lines 1, 2, 5 are best obtained by considering the symmetry of movements of elements A and B with respect to the appropriate yield line. The elements displace with reflective symmetry with respect to the yield line.

The reflective symmetry is slightly complicated by the plane of reflection being at an angle to the reference axes for lines 1 and 5. A matrix multiplication is required for both displacement and rotational movements. These matrices are given in appendix 8A1.

8.7 Changes due to permanent lateral deformation of the panel

8.7.1 Changes of co-ordinates at ends of yield lines.

The collapse of a panel will involve gross lateral displacements, which may be considered permanent, figure 8.4. These permanent lateral deformations require a re-evaluation of the geometry of the panel. In a complete analysis the co-ordinates and direction cosines of the points are evaluated. The results in this study have been obtained by a re-evaluation of the point co-ordinates only. The improved accuracy resulting from a complete and accurate re-evaluation of co-ordinates and direction cosines is discussed in §8.11.5.

Permanent deformations are considered to be caused by finite rotations θ_x and θ_y of the element A. The effects of θ_x and θ_y have been evaluated for infinitesimal rotations. It is considered satisfactory to consider the finite rotations as composed of a succession of small rotations of magnitudes (approx) one hundredth of a radian.

P_1 undergoes no change of co-ordinates.

P_2 displaces along $-k_{20} \hat{i}_1 + k_{10} \hat{k}_1$ due to θ_x . P_2 moves laterally and inplane due to θ_y , figure 8.4(b).

The change of co-ordinates of P_2 is

$$dP_2 = \begin{bmatrix} dx_1 \\ dy_1 \\ dz_1 \end{bmatrix}_{P_2} = \begin{bmatrix} -k_{20} & k_{10} & P_y(z) \\ 0 & & \\ k_{10}^2 & & P_y(z) \end{bmatrix} \theta_x + \begin{bmatrix} k_{20}^2 & P_z(z) + k_{10} k_{20} P_x(z) \\ 0 & & \\ -k_{10} k_{20} P_z(z) - k_{10}^2 P_x(z) \end{bmatrix} \theta_y$$

P_2 becomes $P_2 + dP_2$

P_3 is constrained to move along $k_{20} \hat{i}_1 + k_{10} \hat{k}_1$, figure 8.4(c).

Then

$$dP_3 = \begin{bmatrix} dx_1 \\ dy_1 \\ dz_1 \end{bmatrix}_{P_3} = \begin{bmatrix} k_{10} & k_{20} & P_y(z) \\ 0 & & \\ k_{10}^2 & & P_y(z) \end{bmatrix} \theta_x + \begin{bmatrix} k_{20}^2 & P_z(z) - k_{10} k_{20} P_x(z) \\ 0 & & \\ k_{10} k_{20} P_z(z) - k_{10}^2 P_x(z) \end{bmatrix} \theta_y$$

P_4 does not change its position.

P_5 moves laterally due to the permanent rotation

$$R_z = \xi_z \theta_x + \xi_3 \theta_y$$

The resulting displacement is resolved along the vector $\beta_z \hat{j}_z + \beta_1 \hat{k}_z$ where $\beta_1 = \cos\phi^*$ and $\beta_z = \sin\phi^*$ and

$$\phi^* = \tan^{-1} \left\{ \left[\frac{R}{b} \left[\frac{\theta_z}{2} - \theta_1 \right] - \cos \left[\frac{\theta_z}{2} \right] \right\}$$

θ_1 and θ_2 are defined in figure 8.2(b).

8.7.2 Changes of co-ordinates of gauss points.

Gauss points on the yield lines undergo a permanent change of co-ordinates in accordance with their distance from the line end points, figure 8.4. Thus for, for example, the change of co-ordinates of points on line 1 is

$$dP_1 = \frac{1}{2}(1-\xi_1)dP_1 + \frac{1}{2}(1+\xi_1)dP_2 = \frac{1}{2}(1+\xi_1)dP_2$$

Points on line 4 do not undergo any change, since both P_1 and P_4 do not change coordinates.

8.7.3 Changes of direction cosines at gauss points

Permanent displacements affect the normal to the surface at P_3 , figure 8.4(a). θ_x is assumed to cause no change in the normal to the surface at P_3 . This assumption is consistent if points on lines 2 and 5 displace radially due to permanent geometry changes. This assumption is only approximately correct.

Line 2 rotates through θ_y due to θ_y , while line 5 does not rotate in the plane (i_1, k_1) due to θ_y .

The average rotation of the normal to the surface at P_3 is $\theta_y/2$. Thus after θ_y , the values of k_1 and k_2 are

$$\frac{k_1 - k_2/2 \theta_y}{\sqrt{1+k_1^2 \theta_y^2}} \quad \text{and} \quad \frac{k_2 + k_1/2 \theta_y}{\sqrt{1+k_1^2 \theta_y^2}} \quad \text{respectively.}$$

k_1 and k_2 are the current values of k_{10} and k_{20} respectively.

8.7.4 Assumptions implicit in assumed geometry changes.

The permanent change of ordinate of the gauss points is inconsistent with the basic assumption that the elements do not bend. Moreover, the permanent geometry changes assume a stretching of the yield lines.

The change in x and y ordinates results in gauss points remaining at identical relative positions on the displaced yield lines. Stretching of the yield lines is not real in the sense that virtual work calculations should be evaluated for the movement.

The inconsistencies produce, in fact, only very minor errors in geometry (see §8.11.5). These errors are inconsequential by comparison with the basic assumption that the panel elements do not bend.

8.8 Procedure for computation

8.8.1 Evaluation of $D'U_{ij}$ matrices

The above sections describe how the ordinates and local direction cosines of points on yield lines may be evaluated relative to two reference axes. The rotation and displacement effects at these gauss points due to specified rigid body movements are given in general terms.

The unique effect of each degree of freedom at each gauss point may be computed and stored in a matrix $D'U_{ij}^k$, for each point k . A computer program was written to evaluate the $D'U_{ij}$ matrices.

The steps followed by this program are summarised as follows.

1. The ordinates of the points $P_1 - P_5$ are evaluated from the parameters B , N_{st} , R , α .

2. The matrices $B1$, $B2$, which define the coordinate transformations

to the planes, are evaluated (equations 7 and 8). CC is evaluated (equation 9).

3. For each point, as defined by a helix, the ordinates, P_i (equation 1), and the local direction cosines, A_{ij} , (equation 6), at the yield line are evaluated. These are transformed to the reference axes of the relevant planes. The resulting information P_i (1 or 2) and A_{ij} (1 or 2) is stored for longterm re-use.

4. A set of permanent deformations (θ_x , θ_y) is chosen.

5. The ordinates of all gauss points are re-evaluated, and stored.

6. For each gauss point, P , whose position relative to the origin is given by P_i , the relative displacement dp_{ij} due to an infinitesimal unit rotation $d\theta_j$ is computed.

$$dp_{ij} = d\theta_j \times P_i \text{ ----- (15)}$$

$d\theta_j$ is the relative rotation of the yield line resulting from degree of freedom j . In the case of points on line 3, $d\theta_j$ is the relative rotation of the element A and B. For points on lines 1, 2, 5, $d\theta_j$ is the rotation of element A or B due to freedom j .

The relative inplane displacements of the point are

$$du_{ij} = dx_j + dp_{ij} \text{ ----- (16)}$$

where dx_j is the relative translation of the element due to the degree of freedom j .

7. A matrix

$$\begin{bmatrix} du_{ij} \\ d\theta_j \end{bmatrix}$$

is temporarily composed.

The symmetry conditions are imposed if the point is on lines 1, 2 or 5. The resulting matrix DV_{ij} contains, for each degree of freedom j ,

the relative displacements and rotations of the point in the framework of the planes 1 or 2 (as appropriate to the yield line).

8. The matrix dV_{ij} is transferred to the local axes of the point by pre-multiplication

$$dV'_{ij} = AA'_{ik} dV_{kj} \text{ ----- (17)}$$

where

$$AA'_{ik} = \begin{bmatrix} A_{ik} & 0 \\ 0 & A_{ik} \end{bmatrix}$$

and A_{ik} is derived from equation 6.

The terms $[du, dv, dw, d\alpha_v]$ are taken from the columns of dV'_{ij} for each degree of freedom j , and stored for later re-use as the matrix $D'U_{ij}$. (The general derivation of the $D'U_{ij}$ matrices of the points is discussed in §7.4).

9. Steps 6-8 are executed for each gauss point of the structure.

8.8.2 Use of $D'U_{ij}^l$ matrices

$D'U_{ij}^l$ contains the relevant displacement and rotation, i , at a point, l , on the yield lines, due to the degrees of freedom j . It has been shown in §7.6 and 7.7 how $D'U_{ij}$ may be used to compute a set of collapse velocities in "equilibrium" with a set of external forces on a panel.

$D'U_{ij}$ matrices are specific for a given state of permanent deformation. Thus steps 4-9 of the computer program are repeated for each successive state of permanent lateral displacement of the panel when a continuous understanding of the panel behaviour at collapse is required. Only the current set of $D'U_{ij}$ matrices are in store at any stage of the program execution.

8.9 Stringer and ring stiffeners. The range of application of the present theory.

8.9.1 Stringer stiffeners

The 8 degrees of freedom presented for the curved panel, §8.4, may also be used to consider the collapse of stringer and ring stiffeners, combined with panel collapse.

$D'U_{ij}$ matrices are given in appendix 8A1 for stringer stiffeners along line 1. The stiffeners are discretely occurring and can be considered exactly within the bounds of collapse mechanism theory. Compatibility of displacements at the stiffener-plate boundary is ensured.

Intermediately positioned stringer stiffeners on the panel may be considered in this manner also. Additional named points, which correspond to the intermediate stiffener positions at a yield line, are taken. The internal virtual work computation is extended over these points. The computing program can accommodate these stiffeners easily, although they are not discussed further in this work.

The inclusion of the virtual work for stringer stiffeners, derived from the 8 degrees of freedom, above, is satisfactory when the cross sectional area of an individual stiffener is relatively small by comparison with the total cross-sectional area of the panel. This assumption has been made for the examples considered in this chapter.

Two changes in the $D'U_{ij}$ matrices of the stiffener have been considered in appendix 8A1 for cases where the stiffener area is large and either symmetric or anti-symmetric panel mechanisms develop. Additional degrees of freedom are required. The importance of these degrees of freedom has not been assessed.

8.9.2 Ring stiffeners

The effect of ring stiffeners will be considered in this chapter. $D'U_{ij}$ matrices for the rings are given in appendix 8A1. The treatment is not sophisticated. The ordinates of points on yield line 4 have been

assumed to remain unchanged during collapse. Since the ring stiffener position corresponds with line 4 the ring must remain flexurally and torsionally rigid during panel collapse. If the ring stiffener itself collapses the present theory can accommodate only full inplane yield of the stiffener. This yield is assumed to occur at P_1 .

8.9.3 Limitations of rigid panel mechanisms

The fundamental assumptions on the behaviour of the panel elements limit the range of application in this chapter. The degrees of freedom chosen are consistent with the assumption of rigid panel elements. This assumption is unjustified for cylinders of large R/t , or where significant flexural displacements occur. A set of freedoms, allowing for limited panel flexure, is appropriate for such cases.

8.10 Kinematically acceptable mechanisms

8.10.1 Independent mechanisms

The eight degrees of freedom, §8.4, constitute a redundant system when describing the collapse of a curved panel. Three possible mechanisms exist.

1. Panel mechanism

degrees of freedom $\{\theta_y, X_1, Y_2, Y_3\}$
active yield lines 3, 4, 5, (1)

This is the simplest mechanism and the only one possible when the stringer stiffeners and ring stiffeners are guaranteed adequate.

2. Stringer stiffener collapse mechanism

degrees of freedom $\{\theta_x, \theta_z, Y_1\}$
active yield lines 1, 2, 3, stringer stiffeners.

This is not normally a realistic independent mechanism since a panel mechanism will develop at a lower load factor.

3. Ring stiffener collapse mechanism

degrees of freedom $\{X_1\}$
 active yield lines 1 and ring stiffeners
 or lines 3 and ring stiffeners.

This mechanism may occur independently in a very stocky system under lateral pressure, when a hoop stress equal to material yield will develop throughout the ring stiffener and the panel. The mechanism is also possible if closely spaced heavy stringer stiffeners allow the panel to develop a hoop stress equal to material yield.

The collapse load associated with each of these mechanisms may be evaluated on the assumption that they are truly independent mechanisms. Such an assumption requires that, while one mechanism develops, the equilibrium forces from the remaining inactive mechanisms are taken as zero. Alternatively elastic redistribution of forces, away from the active mechanism, occurs so as to render invalid the stresses calculated in association with the kinematics of the remaining inactive mechanisms.

The independent mechanisms are clearly upper-bound mechanisms, and alternative compound mechanisms may result in lower load factors.

8.10.2 Assessment of redundant mechanisms

An upper bound approach, which considers the collapse forces developed along the yield lines of one mechanism, while assessing the forces for another redundant mechanism, is discussed in §7.14. This procedure should be used in general when considering the strength of a curved panel as part of a stiffened cylinder. The panel mechanism is taken as the main mechanism and proportional factors (f_1 and f_2) for the redundant stringer collapse and ring collapse mechanisms are computed subsequently. The procedure is as follows.

1. The entire external work, W_{ext} , is computed due to a proportional loading system. (The constant of proportionality, λ , is set to unity). The external work components, W_{ext}^1 , due to θ_y , X_1 , Y_2 , Y_3 are entered as the external work terms of the panel mechanism. (This

mechanism may be symmetric or anti-symmetric. Virtual work terms for line 1 are included for a symmetric panel mechanism).

2. Internal work terms due to lines 3, 4, 5 (and 1, if appropriate) are included in the internal virtual work computation. A velocity system is computed from which the internal virtual work, W_{int} , along the eight degrees of freedom is calculated. The virtual work "equilibrium" equation

$$W_{int}^1 = \lambda_1 W_{ext}^1 \text{ ----- (18)}$$

will be satisfied for the internal work W_{int}^1 associated with θ_y , (X_1) , Y_2 , Y_3 . λ_1 is the load factor for the panel mechanism.

3. The external virtual work difference

$$W_{ext}^A = W_{ext} - W_{int} \text{ ----- (19)}$$

is evaluated

4. The terms of W_{ext}^A associated with $(\theta_x, \theta_z, Y_1)$, equal to W_{ext}^z , are entered as the external virtual work for a mechanism involving a lateral displacement of P_2 .

The stringer stiffener collapse mechanism is considered subject to an external force system W_{ext}^z . Internal work is evaluated for line 2 (and 1 if appropriate) and the stringer stiffeners. (Line 3 is already in collapse, so no work may be computed for it). A set of velocities $(\theta_x, \theta_z, Y_1)$ are computed so that the internal work along the present degrees of freedom satisfies

$$W_{int}^z = \lambda_2 W_{ext}^z \text{ ----- (20)}$$

5. The yield line factor $f_1 = W_{ext}^z / W_{int}^z$ is computed. (This may be computed from the work terms of any individual degree of freedom). If $f_1 \leq 1.0$, the stringer stiffeners are adequate and panel failure governs.

6. The ring stiffener collapse mechanism is considered. The term of W_{ext}^A , equation 19, for X_3 is entered as the external work, W_{ext}^3 .

Internal work, W_{Int}^3 , is evaluated for the ring stiffener only. (Line 1, (or 3) is already in collapse, so may not be included in work terms). The velocity set $X_3 = 1.0$ is unique, so that $f_2 = W_{\text{Ext}}^3/W_{\text{Int}}^3$ is evaluated without iteration.

7. If $f_2 \leq 1.0$ the ring stiffener is adequate and the panel mechanism governs.

8.10.3 Determination of critical mechanism

If either $f_1, f_2 > 1.0$ a new compound mechanism should be assessed. If only $f_1 > 1.0$ the new mechanism to be tried involves the collapse of the stringer stiffener, in addition to the panel mechanism.

If only $f_2 > 1.0$ the compound mechanism involves the simultaneous collapse of ring stiffeners and the panel. The stringers are assumed to remain intact.

When f_1 and $f_2 > 1.0$ one compound mechanism may be evaluated and the state of the remaining redundant mechanism assessed, as above.

For cases where a compound mechanism is assessed a new value of $\lambda < \lambda_1$ will be found. Only when λ has been found and the associated f_i , for the redundant mechanisms, satisfy $f_i \leq 1.0$, can the answer be accepted as satisfactory.

8.10.4 Limitation of mechanisms to a single panel unit

A compound stringer stiffener cum panel mechanism exists with line 1 inactive. This mechanism involves a rotation

$$\theta^1 = k_{10} \theta_x + k_{20} \theta_z \text{ ----- (21)}$$

The degrees of freedom of this mechanism are $(\theta^1, \theta_y, Y_1, Y_2, Y_3)$ or $(\theta^1, Y_1, Y_2, Y_3)$. The external work term associated with θ^1 is

$$W_{\theta^1} = k_{10} W_{\theta x} + k_{20} W_{\theta z} \text{ ----- (22)}$$

and the internal work $k_{10} W_{\theta x}(\text{int}) + k_{20} W_{\theta z}(\text{int})$ is computed using internal velocities

$$\dot{\theta}_x = k_{10} \dot{\theta}^1 \text{ and } \dot{\theta}_z = k_{20} \dot{\theta}^1$$

(and $\dot{\theta}_y, \dot{Y}_1, \dot{Y}_2, \dot{Y}_3$ as set in the iterative process)

When $\dot{\theta}_y$ is zero this mechanism may be assessed using a single panel/stiffener unit. When $\dot{\theta}_y \neq 0$, however, the mechanism incorporates a panel mechanism with alternate inward/outward lateral displacements of adjacent panels. This particular compound mechanism will not be considered further since two panels are required for full consideration of the mechanism.

The simple alternate panel mechanism will be considered although the above comment governs in this case also. A lower bound approach will be used in the case of the simple alternate panel mechanism.

8.11 The parameters affecting results

8.11.1 The eighth degree of freedom

The single major parameter affecting panel loading is the presence or absence of the eighth degree of freedom, X_3 . As mentioned above this degree of freedom is related to the ability of the cylinder to expand or contract radially. X_3 is a permissible degree of freedom when no ring stiffeners or weak ring stiffeners exist. X_3 is excluded when the ring stiffeners are considered adequate to prevent overall radial movements of the curved panel. A criterion for adequacy of ring stiffeners will be established in § 8.15. Results will be presented below, in graph form, for the two extremes of ring stiffening. Weak and strong ring stiffener cases will be plotted in separate graphs.

8.11.2 Stringer stiffening

Stringer stiffeners have an influence on results, for both axial and lateral loading conditions. When stringer stiffeners are adequate, however, the degrees of freedom (θ_x, θ_z, Y_1) are excluded, as is yield line 2. The panel results given here will in general assume that

the stringer stiffeners are adequate. §8.16 will consider briefly a number of particular cases of stringer stiffening.

8.11.3 Fundamental geometric parameters of a curved panel.

A number of geometric variables influence the panel strengths under various loading conditions. Dean (218) has shown that only the panel aspect ratio initially influences the collapse mechanism results for a flat panel. Thus the results are independent of the panel slenderness, B/t . It has, however, been shown in §7.18 that the panel slenderness affects results when the elastic energy due to the inplane stress state is considered. This correction may be made when the initial results have been obtained.

A first order of magnitude assessment of the panel geometric parameters is considered in appendix 8A2. It is concluded that the non-dimensional panel curvature parameter, A^2/Rt , is the unique parameter affecting the results of a cylindrical panel of given aspect ratio. Results are otherwise related to the flat panel results.

Since appendix 8A2 claims only a first order of magnitude accuracy, the hypothesis must also be tested numerically. Figures 8.5-6 compare the panel strengths of two constrained panels under inplane and lateral load. The panels have widely differing A/t and R/t ratios, but identical A^2/Rt ratio. The results are very close, and justify quoting all results in terms of the curvature parameter A^2/Rt . Similarly, plots of velocities, figures 8.7-8 show that they are functions of A^2/Rt , to a first order of magnitude, as contended in appendix 8A2.

8.11.4 Scope of results of panel analysis

The non-dimensionalisation of results in terms of the curvature parameter reduces the number of panel variables immensely. Nevertheless a wide range of variations must be covered in order to provide a comprehensive set of results. It was decided to limit the aspect ratios covered to the single case of $A/B = 1.0$. In addition results are usually presented for the single spiral angle, $\alpha = 45^\circ$. The effect of the spiral angle α on results is illustrated in figures

8.9-10 for a panel subject to inplane (axial) loading and another subject to lateral loading. The effect is significant in both cases. A considerably more complete study is required if the minimum strength curves are to be obtained.

8.11.5 The effect of re-orientation of local axes

It has been stated in §8.7.1 that direction cosines of the gauss points are not re-evaluated when permanent changes of geometry take place. A numerical routine was written to carry out a more accurate and consistent re-evaluation of co-ordinates and of direction cosines at the gauss points.

New co-ordinates of the gauss points were evaluated by considering a no change of curvature condition along the yield lines. The effect of changes of yield line length was considered by moving gauss points tangentially along yield lines. A change of value weighting was not considered for the gauss points, on the assumption that changes of line length are balanced by a reduction of panel thickness in the region of the yield line.

The direction cosines of the gauss points were re-evaluated allowing for the displaced position of gauss points on the yield lines, the rotation of the yield lines and the discontinuity in curvature across the yield lines. The new z-axis bisected the angle formed between the normal to the adjacent elements at the gauss points.

The effect of these elaborate changes of ordinates and direction cosines was found to be almost negligible for two test loading conditions, figures 8.11-12. The results presented elsewhere rely on the changes outlined in §8.7.

8.11.6 Loading cases considered for panel results

The computed collapse strength results will be presented for three types of loading: inplane load along the generator direction only, lateral pressure only, and hydrostatic loading. Hydrostatic loading consists of lateral pressure, ρ , on a panel and an inplane load ρRA along the generator direction.

Panels will be considered part of a cylinder with longitudinal and ring stiffeners, so that the support condition of the edges is always a clamped condition. Ring stiffeners are considered torsionally rigid, so that no anti-symmetry is possible with respect to line 4. Anti-symmetric mechanisms are possible about the stringer stiffeners. It should be noted that when an anti-symmetric mechanism develops, in a weak ring stiffener mechanism, no internal virtual work is done along line 1, despite the clamping condition.

Table 8.2, contains a summary of the degrees of freedom included to simulate the various types of loading and boundary conditions.

Five gauss points are used along yield lines of variable inplane displacement for all results. One gauss point is used for yield lines of constant inplane displacements.

The stress function derived from the Prandtl-Reuss flow rule, Table 7.1, will be used for all results. A convergence tolerance, §7.16.1, of $\epsilon_0 = 10^{-8}$ will be used.

8.12 Panels subject to longitudinal load

8.12.1 The effect of ring stiffening

The effect of strong ring stiffeners for this loading condition is to allow inplane stresses to develop along yield line 1. The overall inplane stress for line 1 is zero for the weak ring stiffener case. Thus only a moment M_p can develop along yield line 1 when weak ring stiffeners are considered, in the symmetric panel mechanism. No virtual work occurs for line 1 for an anti-symmetric mechanism in conjunction with weak ring stiffeners.

8.12.2 Panels supported by strong ring stiffeners

Figures 8.13-16 summarise the results obtained for curved panels supported by strong ring stiffeners. Inward displacements produce considerably lower load capacities than outward displacements. Consequently panels may be expected to fail by inward collapse when a symmetric mechanism occurs. (A symmetric mechanism may occur when

stringer stiffeners of large torsional rigidity separate adjacent panels, or where deep stringer stiffeners are firmly attached to closely spaced ring stiffeners - thereby preventing stiffener rotation).

It is of note that for the symmetric mechanism cases minimum strength values exist. These values are reached at fairly large lateral deflections. The minimum strength is a function of A^2/Rt , decreasing with increased A^2/Rt . Thus the maximum value of minimum strength is for a flat panel ($A^2/Rt = 0$).

The inward displacement mode for an anti-symmetric panel mechanism results in lower strength than the corresponding symmetric mechanism. Minimum strengths exist for panels in the anti-symmetric mode also. Inward displacements in one panel are accompanied by equal outward displacements in adjacent panels. The load capacity of the equivalent outward displacement is considerably greater than the equivalent inward displacement. It is inconsistent for adjacent panels to carry differing loads in the presence of strong ring stiffeners, which may be assumed to re-distribute unequal loads. It is justifiable to take an average value of strength derived from the inward and outward displacement curves; nevertheless it is conservatively assumed that the inward displacement curve, figure 8.13, defines the strength curve.

8.12.3 Panels supported by weak ring stiffeners

The behaviour of panels with weak ring stiffeners subject to inplane (axial) load is qualitatively similar to the equivalent panel with strong rings, figures 8.17-20. The minimum strength values for the symmetric mechanism are slightly decreased in the absence of strong rings and for outward displacements both sets of graphs correspond closely for a wide range of displacements. However, for small and intermediate inward displacements there are significant differences in strength predicted between panels with strong and weak ring stiffeners. These differences are large for $A^2/Rt > 13$, increasing with increased A^2/Rt . These differences can be expected to affect stocky panels (small B/t). It was noted in §7.18 that the elastic energy correction reduced panel strengths at intermediate displacements. Thus the disparity between predicted strengths may be much less for slender panels (large B/t) than is indicated by the direct comparison of

figures 8.15 and 19.

Lowest panel strengths are understandably predicted for panels with weak ring stiffeners in the anti-symmetric mode of displacement. Load capacities predicted in figure 8.17 may be realistically used to predict the maximum load of a panel which has weak ring stiffeners. Redistribution of load between adjacent panels may not be possible in this case. The strength of panels in cylinders with widely spaced ring stiffeners, but closely spaced stringer stiffeners may be derived from figure 8.17, provided failure occurs by panel collapse.

An important qualitative conclusion is drawn from figure 8.17. Panels with the same level of imperfection and panel slenderness (B/t), but with differing R/t values will have significantly different initial failure loads, the higher failure loads being associated with lower values of R/t . The reserve strength of panels of low R/t is, however, much less than that of the flatter panels.

8.12.4 The particular case of a flat panel

It is apparent from the above figures that the behaviour of a flat panel is but a particular case of the curved panel problem. Curved panel results become asymptotic to the flat panel results where $A^2/Rt = 0$. The flat panel results in §7.16 are obtained from a different computer program, and are mostly quoted for simply supported panels. A comparison of the results obtained from the separate programs is given in figure 8.21 for simply supported panels in uniaxial compression. The gradual change of strength with A^2/Rt is clear as is the asymptotic character of the change. There is only a very small difference between the result obtained for $A^2/Rt = 0.16$ and the flat panel result, obtained from the earlier program.

8.12.5 Inplane elastic strain energy

The release of elastic strain energy in unloading was shown to have a significant effect on panel strength in §7.18. Equation 19 (chapter 7) is dependent on the assumed length of panel over which unloading takes place. Failure can be expected to occur in each longitudinal segment of a cylinder with closely spaced ring stiffeners. In such a

case unloading may be similar to the unloading of a flat panel. However, if elastic energy is released in a long stiffened cylinder, while panel failure occurs locally in one region only, the effect of elastic energy release will be greatly magnified. It is proposed that equation 19 (chapter 7) be rewritten for a curved panel as

$$P \approx W \left[1 - \frac{\sigma_y}{4E} \cdot \left[\frac{B_1 B_2}{t^2} \right] \cdot \frac{d\sigma_1}{d(\delta/t)} \cdot \frac{t\theta}{Y} \right] \text{-----} (23)$$

where B_1 = length of the panel in the collapse buckle and B_2 = distance between ring stiffeners.

Thus an equivalent panel slenderness may be used for strength calculations, namely

$$\frac{B_{eq}}{t} = \frac{1}{t} \sqrt{B_1 B_2} \text{-----} (24)$$

8.13 Panels subject to lateral load

8.13.1 Range of problems considered

A curved panel subject to lateral load will be subject also to load in the hoop direction of $2\rho Rb$, in the elastic range of loading. The external virtual work due to x_3 is $-4\rho Rb$ (or $1/3 \rho a^2/t^2 \{-12Rt/a^2\}$ non-dimensionally).

A panel with strong ring stiffeners will shed the hoop load of the panel onto the ring stiffeners at panel collapse - since no overall change in radius can occur. Such a panel will be subject to the lateral load only at collapse. A panel with weak ring stiffeners will be subject to both the inplane hoop load and the lateral load at collapse.

The lateral load virtual work components are derived in appendix 8A1. The external virtual work due to unit rotation θ_y is $-1/3 \rho a^2 b \{2(3k-k^2)\}$ where $k = \tan \alpha$, and α = spiral angle of the line 3. (The pressure sign convention is positive along the positive z-axis of the panel elements, i.e. positive outwards).

This section will consider the strength of panels only. The adequacy of the stringer stiffeners is assumed. The panel degrees of freedom are thus $\{\theta_y, X_1, Y_2, Y_3, X_3\}$. Symmetric or anti-symmetric panel mechanisms are possible in theory for panels with strong or weak rings. However, the external virtual work for the anti-symmetric mechanism is zero along θ_y and X_1 , since the virtual work components in adjacent panels cancel. Thus no virtual work is done by the loading on a panel with strong ring stiffeners in the anti-symmetric collapse mode. This collapse mode is unrealistic for such a panel. Only the inward collapse mode of the symmetric panel mechanism need be investigated for a panel with strong rings. A panel with weak rings must be checked for the anti-symmetric mode (inward and outward displacements) as well as the symmetric collapse mode of failure.

The hinge at yield line 1 vanishes in the anti-symmetric mechanism, and the degrees of freedom X_1 and X_3 become one. Thus the external virtual work along the direction X_1 , is $1/3 \rho a^2 b \{-12R/a^2\}$. If the loss of internal virtual work along line 1 is greater than the loss of external virtual work along θ_y (and the component $1/3 \rho a^2 b \{2(3k-k^2)\}1/R$ along X_1) the panel will fail in the anti-symmetric mode.

Results will be presented for panels with strong and weak ring stiffening. A criterion for ring stiffeners will be established in §8.15.2.

8.13.2 Panels supported by strong ring stiffeners

The behaviour of curved panels with strong rings is summarised in figure 8.22 for the symmetric panel mechanism of collapse and spiral angle $\alpha = 45^\circ$. The strength of panels at zero lateral deflection increases with A^2/Rt . The lowest initial capacity is $2\rho_0$, for a flat panel. (ρ_0 is the capacity of a simply supported flat panel at zero lateral deflection. $\rho_0 = 3/2 \sigma_y a^2/t^2$ for $a=b$). The increase in panel strength at zero lateral deflection corresponds very closely with the increase in strength of an initially flat panel due to a lateral displacement $\delta/t = 1/8 A^2/Rt$. This is the "rise" in the curved panel at P_5 (figure 8.3).

The strength of a panel reaches a minimum value at some lateral deflection for all A^2/Rt . Figure 8.22 suggests an envelope of lower bounds to panel strength for all A^2/Rt . It must be noted, however, that because sides 1 and 4 are different for a curved panel, it can no longer be assumed that $\alpha = 45^\circ$ defines the minimum strength for all A^2/Rt . In fact, figure 8.10 shows that for $A^2/Rt = 26.18$ the minimum panel strength is obtained when $\alpha = 35^\circ$. This study has not been extended to allow the minimum strength envelope to be plotted for all A^2/Rt .

8.13.3 Panels supported by weak ring stiffeners

It has been pointed out, above that the essential difference between panels with weak instead of strong ring stiffeners is the presence of the inplane hoop load. The hoop load becomes more important with decreasing A^2/Rt . Results will be plotted relative to the lateral pressure ρ_0 and in some cases relative to the full inplane yield load P_0 (along X_3 or X_1). Solutions are possible for inplane loading only when the inplane load is less than full inplane yield load. In practice this means solutions are possible when the rotation velocity, $\dot{\theta}_y$, is greater than zero. Convergence problems were experienced at low lateral deflections. The solution procedure adopted was to start by solving for a large displacement and solutions were obtained by decreasing the deflection stepwise towards lower displacements, until the solution technique failed. It will be seen that in some cases the solution technique failed at quite large lateral displacements. These convergence failures always occurred at large inplane loads.

The solutions obtained for the symmetric collapse mode (inwards) are plotted in figures 8.23,24; pressure load is plotted on the y-ordinate axis in figure 8.23, hoop load is plotted in figure 8.24. The maximum load attained is full inplane yield for all A^2/Rt , and the capacity appears to reach a constant level at large displacements.

The external loading on a panel in the anti-symmetric collapse mode is identical to that on a panel under inplane load only along X_1 . Results have been plotted in figure 8.25-27, ρ/ρ_0 as the y-ordinate in figure 8.25-26, and P/P_0 as the y-ordinate in figure 8.27. The outward mode of deflection gives lower load capacities than the inward mode of

deflection for similar lateral deflections. This difference decreases with A^2/Rt ; there is very little difference for $A^2/Rt = 3.27$. (At zero A^2/Rt there is theoretically no difference between inward and outward displacements, and hence no difference in load capacity). Figure 8.27, shows that panels have a load capacity plateau at approx. $0.8P_0 = P(x_1)$. This capacity is slightly dependent on A^2/Rt .

The load capacities attained at large lateral deflections are slightly lower for the symmetric mechanism than for the anti-symmetric mechanism. (It is assumed that the lower of the inward and outward deflection strengths governs for the anti-symmetric mechanism). Nevertheless, at lower lateral deflections the anti-symmetric mechanism strength is significantly lower than the symmetric mechanism strength. This difference in strength increases with A^2/Rt . It is relatively small for low values of A^2/Rt . It is concluded that panels with weak ring stiffeners will fail initially by the formation of an anti-symmetric panel mechanism. As deflections grow, however, the symmetric mechanism will prevail and outward deflections will cease to grow - they may indeed be reversed.

Figures 8.24 and 27 indicate clearly that lower bounds to panel strength can be given for panels under lateral load. This lower bound to strength exists at large lateral deflections ($\delta/t > 5.0$), and is best expressed in terms of the inplane hoop load on the panel. The present study is incomplete for setting minimum strength levels. The spiral angle, α , must be varied and solutions must be obtained at larger lateral deflections.

8.14 Panels subject to hydrostatic load

8.14.1 The effect of axial load.

Panels subject to a combination of axial and lateral load are encountered in many design situations. The particular combination of axial and lateral loading known as hydrostatic loading, where the axial load is $1/3 \rho a^2 b \{-6R/ab\}$ (or $-12 \rho/\rho_0 \gamma/(A^2/Rt)$ non-dimensionally), occurs frequently and is of especial note.

The effect of the axial load is to decrease the lower bounds to strength obtained from that obtained for lateral load only. At small lateral deflections an increase in panel strength has been obtained for some mechanisms. The basic trend in behaviour, however, remains similar to the laterally loaded panel behaviour.

8.14.2 Panels supported by strong ring stiffeners

A set of lower bounds to strength may be deduced from figure 8.28 for panels with strong ring stiffeners. The effect of the axial inplane load increases with decreasing A^2/Rt . The minimum strength at $A^2/Rt = 26.18$ is reduced to 65% of the lateral load strength; the reduction is to 35% for $A^2/Rt = 3.27$.

It is clear from figure 8.29 that at low A^2/Rt the strength of the panel is closely related to the axially loaded strength. A comparison of the anti-symmetric failure mode for the axially loaded panel, figure 8.17, with the hydrostatic loading result in figure 8.29 shows that for $A^2/Rt = 3.27$ the strengths are very similar.

8.14.3 Panels supported by weak ring stiffeners

Two collapse mechanisms were considered for panels with weak ring stiffeners, figures 8.30-35. Convergence problems were experienced for the anti-symmetric mechanism, and results were obtained for only 3 values of A^2/Rt in the outward deflection mode, figure 8.32. A comparison of the results of the outward and inward deflection modes reveals once again that the outward mode is the more critical at low lateral deflections. The difference decreases with increasing displacement, and with decreasing A^2/Rt .

The results for both inward and outward displacements are plotted in figures 8.31, 33 with load normalised with respect to the full inplane load, P_0 . Figure 8.31 shows that the maximum load for inward deflections is recorded as $P=2/3 P_0$. This is not incompatible with the results in figure 8.33, where the slope of the load-lateral deflection curves is negative at $\delta/t = 0$. Figure 8.33 suggests that, for some inward deflections, load capacities considerably higher than values at $\delta/t = 0$ are possible.

Results for the symmetric panel mechanism are given in figures 8.34, 35. Comparison with the anti-symmetric mechanism results (outwards), figure 8.33, shows that the anti-symmetric mechanism produces a lower load capacity at low lateral deflections, but at large lateral deflections the symmetric mechanism governs. This phenomenon decreases with decreasing A^2/Rt .

Figures 8.31, 33, 35 show that the most suitable method of presentation of results is in non-dimensionalised form relative to the full inplane virtual work capacity, $P_0 = 2Bt \sigma_y$. It is also noted from figures 8.33, 35 that no minimum strength has been reached for $\delta/t = 5.0$. Rather the curves show behaviour asymptotic to a straight line of slightly negative slope. An extension of this study to larger deflections is required in order to establish a minimum bound to panel strength.

8.15 The effect of ring stiffening

8.15.1 Non-dimensionalisation of external virtual work

The external virtual work on any panel may be expressed in non-dimensional form. Loads due to inplane and lateral load may be non-dimensionalised both with respect to the full inplane load, P_0 , for inplane loads, and with respect to the collapse load for a flat simply supported panel for lateral load. The external virtual work is written as

$$W_{ext} = \lambda \left\{ k_1, 6k_2 \left[\frac{Rt}{A^2} \right], 3k_3 \left[\frac{Rt}{A^2} \right] \right\} \dots\dots\dots (25)$$

along $\{\theta_y, X_{1or3}, Y_3, \}$ for the combination of lateral and hydrostatic load discussed in the above sections.

For strong ring stiffeners

$$\begin{aligned} (k_1, k_2, k_3) &= (1, 0, 0) \text{ for lateral load, and} \\ &= (1, 0, 1) \text{ for hydrostatic load.} \end{aligned}$$

For weak ring stiffeners

$(k_1, k_2, k_3) = (1, 1, 0)$ for lateral load, and
 $= (1, 1, 1)$ for hydrostatic load.

The terms $\lambda 6k_2(Rt/A^2)$, $\lambda 3k_3(Rt/A^2)$ indicate the importance of the inplane loads. Neither may exceed unity if the yield function is satisfied throughout the panel.

The effect of strong ring stiffeners is to remove the loading term $6\lambda/(Rt/A^2)$, and this effect is clearly an inverse function of A^2/Rt . Thus the effect of strong stiffeners, for lateral load only, is to raise the minimum load factor λ from approx. 0.38 to 2.05 for $A^2/Rt = 3.05$ whilst for $A^2/Rt = 34.9$ the introduction of strong rings raises λ from approx 4.15 to 4.3.

The effect of ring stiffeners on the hydrostatically loaded panel is less dramatic, since an inplane load exists along Y_3 even with strong stiffeners. For $A^2/Rt = 3.27$, $\lambda = 0.8$ (approximately), for strong ring stiffening, and $\lambda = 0.3$ for weak stiffeners. For $A^2/Rt = 34.92$, $\lambda = 2.9$ for strong ring stiffeners, and $\lambda = 2.8$ for weak ring stiffeners.

8.15.2 Variable ring stiffening

Results have been quoted for the two extremes of ring stiffening. Variable ring stiffening may be accommodated within the present collapse mechanism theory. It is noted, however, that the theory assumes ring stiffeners of stocky proportions, not subject to flexural or torsional failure.

The effect of the ring stiffener is to remove some of the hoop forces from the panel at collapse. The internal virtual work due to the ring stiffener is $-2A_R \sigma_y$ along X_3 . The total external virtual work due to a lateral load p is $-4pbR$. Thus the external virtual work on the panel, in the presence of the ring stiffener is

$$W_{X3} = -4pbR + 2A_R \sigma_y$$

which may be written non-dimensionally as

$$\frac{W_{x3}}{\sigma_y b t} = -4 \frac{\rho}{\sigma_y} \frac{R}{t} + \frac{2A_R}{b t} \text{-----} (26)$$

The ring stiffener removes all the virtual work from the panel when $W_{x3} = 0$.

If ρ/σ_y is expressed in terms of the non-dimensional pressure $\rho_0/\sigma_y = 3/2 t^2/a^2$ and $\rho = \lambda \rho_0$, then the ring stiffener area may be written as

$$\frac{A'_R}{B t} = \lambda \left[\frac{\rho_0}{\sigma_y} \frac{a^2}{t^2} \right] \frac{t^2}{a^2} \frac{R}{t} = 6 \lambda_S \frac{R t}{A^2} \text{-----} (27)$$

Equation (27) gives the conditions that the ring stiffener remove the hoop forces from the panel at collapse. It may be termed the sufficiency condition for stocky ring stiffeners. In general the ring stiffener will have a sufficiency k_R . Then

$$k_R = \frac{A_R}{B t} \cdot \frac{1}{6 \lambda_S} \frac{A^2}{R t} \text{-----} (28)$$

λ_S is the load factor for a panel with strong ring stiffeners, and in general is a function of A^2/Rt and δ/t .

The range of k_R is $0 \leq k_R \leq 1.0$. $k_R = 0$ defines a panel with weak rings, and $k_R = 1.0$ defines a panel with strong rings.

The minimum value of λ_S defines the maximum value of k_R .

For any value of k_R , $0 \leq k_R \leq 1$, the collapse load factor $\lambda = \rho/\rho_0$ will lie between the weak ring and strong ring stiffener strength. As noted above this strength difference is a function of A^2/Rt and δ/t . Figure 8.36 was obtained for $A^2/Rt = 6.54$ and $\delta/t = 1.0, 2.0, 4.0$, by varying the value of k_R for the ring stiffener. It is noted that a good approximation to the panel strength is given by

$$\lambda_R = \frac{\rho}{\rho_0} = k_R \lambda_S + (1-k_R) \lambda_W \text{-----} (29)$$

where λ_W = strength of a panel with weak ring stiffeners.

By substitution from equation 28

$$\lambda_R = \frac{A_R}{6Bt} \cdot \frac{A^2}{Rt} \left[\frac{\lambda_S - \lambda_W}{\lambda_S} \right] + \lambda_W \leq \lambda_S \text{ ----- (30)}$$

This equation defines the collapse load for a panel with arbitrary stocky ring stiffeners. It may be conservatively assumed that λ_W and λ_S refer to the minimum panel strength for weakly stiffened and strongly stiffened panels respectively. Equation 30 also expresses very concisely the value of providing some level of ring stiffening A_R/Bt . The parameter

$$\frac{A^2}{6Rt} \left[\frac{\lambda_S - \lambda_W}{\lambda_S} \right]$$

is the measure of the value of the ring stiffening.

8.16 The study of stringer stiffeners

8.16.1 Range of problems considered

The results presented above for the strength of curved panels under both inplane and/or lateral load have assumed that the stringer stiffeners, and line 2 are adequately strong to remain rigid at panel collapse. The collapse of stocky stringer stiffeners is represented in the degrees of freedom $\{\theta_x, \theta_z, Y_1\}$. Additional degrees of freedom are required for slender stiffeners. $D'U_{ij}$ matrices are given in Appendix 8A1 for stocky stringer stiffeners along line 1. Terms suitable for use with symmetric or anti-symmetric mechanisms are given. The need to accommodate large stringer stiffener forces is discussed in §8.9.1.

A detailed study of the effects of stringer stiffeners has not been possible within this work. A number of specific examples are given to demonstrate the ability of the solution technique to find solutions for compound mechanisms. The results will be used to discuss a number of parameters affecting stiffener adequacy. The examples given are for laterally loaded panels. The simplest mechanism for the stiffener is used, i.e. a symmetric panel mechanism, ignoring the degree of freedom, Y_4 . Strong ring stiffeners are assumed, and the solution procedure outlined in §8.10.2 and 8.10.3 is used.

8.16.2 Numerical ill-conditioning possible

The external virtual work along θ_x and θ_y is given in appendix 8A1 as

$$W_{\theta x} = 2ab^2k_{10}\rho \quad \text{and} \quad W_{\theta y} = -2/3a^2b(3k-k^2)\rho$$

$k_{10} \approx 1.0$ and $k = 1.0$ for a square panel of relatively small A^2/Rt . Then $w_{\theta x} = 2b^3\rho$ and $w_{\theta y} = -4/3.b^3\rho$.

Two independent mechanisms are possible for a stringer stiffened panel, as noted in §8.10.1. In some instances these mechanisms may remain independent, even in regions of lateral displacement, adjacent to where both mechanisms have the same load ρ .

Consider the extreme case of a simply supported rectangular panel at zero lateral displacement. The internal virtual work is

$$W_{\theta x} = 4bM_p + 4M_{st} \quad ; \quad W_{\theta y} = 8M_pb$$

Hence $\rho_{\theta x} = 2/b^2(M_p + M_{st}/b)$ and $\rho_{\theta y} = 6 M_p/b^2$.

If $\rho_{\theta x} \geq \rho_{\theta y}$ a panel mechanism will form and

$$M_{st}/b \geq 2M_p. \text{-----} (31)$$

For a stiffener of zero eccentricity

$$D_{st}^2 t_{st} \geq 2bt^2 \quad \text{i.e.} \quad D_{st}^2 t_{st} \geq Bt^2 \text{-----} (32)$$

The collapse mechanism involving θ_x , but excluding θ_y , will form at some $\rho_{\theta x}$, when the central lateral deflection of the panel is δ/t . This mechanism will have a constant load capacity, since no inplane forces may form. Thus an increase of the lateral displacement of the stringer stiffener will occur, but there will be no change of load capacity $\rho_{\theta x}$.

The independent panel mechanism is known to have a load capacity $\rho_{\theta y}$ which increases with panel deflection. It is concluded that if equation

(31) is satisfied a panel mechanism will form until some central lateral deflection, δ/t , is reached, where $\rho_{\theta x} = \rho_{\theta y}$. At this point the panel mechanism will cease and a beam-type mechanism, involving the stiffener will ensue at constant load. The mathematical result at δ/t will be a discontinuity in the velocities as one mechanism stops and another starts.

A cylindrical panel will escape this discontinuity if the coupling of the two mechanisms at line 3 is sufficiently strong. The problems attempted in this section bypass the mathematical discontinuity mentioned above by clamping the boundaries, so that internal virtual work occurs along line 4 due to θ_x and θ_y .

Numerical illconditioning resulted from an attempt to include the degree of freedom θ_z . The problem, whose solution was attempted, had a value of $A^2/Rt = 6.54$; thus corresponding to a moderately flat panel. θ_z corresponds to an inplane rotation of the element A, figure 8A1.1. Element A is surrounded on 2 sides by fully developed yield lines, where the inplane axial forces across the yield line are large. The response of line 2 to θ_z is very close to that of an inplane yield line subject to inplane rotation. This is the condition at which it is most difficult to find a satisfactory solution. Various well tried techniques were used in an attempt to find a solution, but without success. It was decided to proceed, excluding θ_z from consideration.

8.16.3 Solution of compound mechanism

A stiffened panel subject to lateral load was assumed to form a symmetric panel mechanism, involving degrees of freedom $\{\theta_y, X_1, Y_2, Y_3\}$. The adequacy of the stringer stiffener was assessed using degrees of freedom $\{\theta_x, Y_1\}$. A solution was obtained for continuously varying, δ/t . When the yield line factor of the redundant yield lines (line 2 and the stiffener) reached 1.0 the compound mechanism was attempted. A successful solution was found and lateral displacements of the panel were increased. The velocities $\dot{\theta}_x, \dot{\theta}_y, \dot{Y}_1$, have been plotted in figure 8.37 for the compound mechanism, for 2 values of stiffener eccentricity.

The effect on the load capacity of the stiffener in the combined

mechanism is seen in figure 8.38. It should be noted, however, that the strength given for the combined mechanism is not totally accurate since the increase in central lateral deflection, δ/t , was obtained by the rotation θ_y only. The value of $\dot{\theta}_x$ in figure 8.37, though small relative to $\dot{\theta}_y$, implies that the lateral deflection of P_2 should increase simultaneously as the point P_3 . It is suggested that the change in lateral displacements of θ_x and θ_y be in proportion to the instantaneous velocities $\dot{\theta}_x$ and $\dot{\theta}_y$. The change in lateral displacements of P_2 and P_3 are then approximately

$$\Delta P_{Z(2)} = -b\dot{\theta}_x \Delta\theta \text{ ----- (33)}$$

$$\text{and } \Delta P_{Z(3)} = (-b\dot{\theta}_x + ka\dot{\theta}_y)\Delta\theta \text{ ----- (34)}$$

where $\Delta\theta$ is some arbitrarily chosen small rotation.

8.16.4 Adequacy of stringer stiffeners

It is satisfying to be able to solve mathematically the problem of the compound stiffened panel mechanism. It is not, however, necessary within the present work. The information most relevant to the stiffener is its adequacy during the formation of a panel mechanism. Thus it is sufficient to obtain the line factor and to ensure it is less than unity. It is not possible to separate the line factors for line 2 and the stringer stiffener.

Yield line factors are plotted in figure 8.39 for stiffener proportions $D_{st}/t_{st} = 7.5$ and 3.3 , and for three eccentricities; zero eccentricity, internal stiffening (negative eccentricity) and external stiffening (positive eccentricity). A value of $A_{st}/A_{pl} = 0.6$ was used for all stiffeners.

The yield line factors, figure 8.39, are seen to be an approximately linear function of displacement, δ/t . It is noted, however, that panel capacity is also an approximately linear function of displacement, over the plotted range of displacements.

The proportions of the stiffener have a significant effect on the yield line factor. The proportion $D_{st}/t_{st} = 3.3$ leads to a much higher factor than $D_{st}/t_{st} = 7.5$. Line factors are approximately parallel for

each level of eccentricity chosen, in the case of both stiffener proportions chosen. Line factors for internally placed stiffeners are complicated by the presence of negative values of line factor. Only the absolute value of the line factor is relevant. Thus the stiffener proportion $D_{st}/t_{st} = 7.5$ results in a greater line factor than $D_{st}/t_{st} = 3.3$, for internal stiffening.

The most important result obtained from figure 8.39 is that internal stiffening leads to a significant decrease in stiffener line factor. For $\delta/t = 1.3$ an internal stiffener, $D_{st}/t_{st} = 3.3$ is adequate to withstand loading, while an external stiffener is overloaded. A stiffener with zero eccentricity just adequate. For the stiffener proportion $D_{st}/t_{st} = 7.5$, an internal stiffener and a stiffener of zero eccentricity, are equally adequate, whilst the external stiffener is overloaded.

It is inconvenient to set out a closed form formula, indicating the necessary increase in stiffener proportions when the yield line factor exceeds unity. The formula depends on the relative velocities θ_x and Y_1 , and on the panel lateral deflection, δ/t . The simplest procedure seems to be a trial and correct method to find an adequate level of stiffening.

The results obtained in this section are for a very limited range of parameters, and for a very limited type of mechanism. Generalised conclusions are not possible from so few results. This caution is particularly appropriate since the degree of freedom, Y_4 , has been ignored for the stiffener. A condition of excluding Y_4 is that the area of the stiffener is small relative to the panel area. This condition has not been satisfied here. Nevertheless, it is the intention of the section to demonstrate that a stiffener mechanism can be assessed. This has been demonstrated.

8.17. Correlation of collapse mechanism results with finite element predictions for axial loading

8.17.1 Choice of panel parameters for comparison

A wide variety of preliminary results have been presented above.

These results are consistent within themselves but their general value can only be assessed when correlated with a reliable standard.

The results obtained using the finite element analysis have been used as the standard in chapter 7. It has been shown that a very satisfactory degree of correlation exists between the collapse mechanism results and the finite element results for a two-dimensional rectangular panel of aspect ratio $A/B = 1.0$.

It has been shown in this chapter that results can be expressed non-dimensionally with respect to the panel curvature parameter A^2/Rt . Koiter(178) as defined the "total panel curvature" as

$$\theta = \left[12(1-\nu^2) \right]^{\frac{1}{4}} \left[\frac{A}{2\pi} (Rt)^{-\frac{1}{2}} \right] = .2893 \sqrt{\frac{A^2}{Rt}} \text{ ----- (35)}$$

When $\theta \leq 0.64$ Koiter predicts stable postbuckling behaviour for a simply supported panel under axial load; when $\theta > 0.64$, unstable elastic postbuckling behaviour is predicted.

Thus a simply supported panel for which $A^2/Rt > 4.89$ will have an unstable postbuckling state.

The amount of correlation possible within this thesis is limited. A suitably large value of panel curvature should be chosen to test the validity of the collapse mechanism method as applied to cylindrical panels. The curvature $A^2/Rt = 13.09$ was chosen since it corresponds to the middle of the range of results quoted in the above sections. In addition some degree of postbuckling instability may be detected in the finite element results. This might be expected to decrease the level of correlation between the finite element and collapse mechanism analyses.

The choice of boundary conditions for the comparison study was dictated by the finite element study. The curved loading edges were clamped to ensure a uniform inplane displacement of the loading edge. The longitudinal edges of the panel were simply supported for rotations and lateral deflections, and unrestrained for movements tangential to the hoop direction of the panel.

The clamped loading edges unavoidably stabilise the panel postbuckling behaviour. Nonetheless the correlation check will give a good indication of the accuracy of the collapse mechanism results.

8.17.2 Collapse mechanism analysis

Results have not been given previously for the boundary conditions of the correlation study. A number of special considerations are required. The inplane degrees of freedom θ'_z and X_1 simulate the unrestrained edge condition of the panel.

θ'_z is a rotation of the element A about the axis $-k_{20} \hat{i}_1 + 0\hat{j}_1 + k_{10} \hat{k}_1$, figure 8.3. This rotation ensures that no lateral displacement of the yield line 1 occurs, whilst allowing tangential displacements of line 1 in the hoop direction. The degrees of freedom of the panel are

$$\left\{ k_{20} \theta'_z, \theta_y, k_{10} \theta'_z, X_1, Y_1, Y_2, Y_3, 0 \right\}$$

The degree of freedom θ'_z is considered more fully in appendix 8A1, along with the treatment of yield line 2 in the presence of the degrees of freedom $\{\theta'_z, Y_1\}$.

Figures 8.40, 41 show the basic mechanism results. The load capacity is given for spiral angles $20^\circ \leq \alpha \leq 45^\circ$ at 2.5° intervals for $0 < \partial/t \leq 5.0$. As with the flat panel behaviour, the minimum strength at $\partial/t \approx 0$ is given by $\alpha = 45^\circ$ and at $\partial/t = 5.0$ the minimum strength is given by $\alpha = 20^\circ$.

Results are discussed with reference to $\alpha = 45^\circ$. It is known that convergence of a compound mechanism is possible only when the redundant lines are active. Convergence problems are encountered at $\partial/t \approx 2.1$. Figure 8.42 shows that at this deflection the line factor for line 2 is close to unity.

It was, however, possible to obtain stable solutions for the unrestrained mechanism for $\partial/t \leq 1.2$. It was decided to use a simple linear interpolation for the variables (including load capacity) for the

interval $1.2 < \theta/t < 2.1$. A comparison with figure 8.17 shows this to be satisfactory. Strength is overestimated by a maximum of $0.015P_0$ when compared with the constrained panel strength.

Figure 8.42 shows that the yield line factor for line 2 varies erratically for $\theta/t < 2.1$ (constrained panel mechanism). The unstable behaviour for $\theta/t < 2.1$ is also to be seen in figure 8.43 where the velocities $\dot{\theta}_2$ and $\dot{Y}_1$ are plotted for the separate and compound (combined) mechanisms. The instability is somewhat surprising since tolerance on the out-of-balance work terms was $\epsilon_0 = 10^{-5}$, §7.13.

The instability results from the fact that the degree of freedom, θ_2 , produces an inplane displacement $25\theta_2$ and a rotation $2\theta_2$ (for the geometry of the panel used in the analysis). This results in equilibrium equations for line 2 which are very sensitive to variations of $\dot{\theta}_2$. The ideal velocities would be a $\dot{\theta}_2$ which produces rotations only and inplane displacements produced by $\dot{Y}_1$.

The observations, above, refer to the panel behaviour for $\alpha = 45^\circ$. They are applicable for the entire range of α for which results are quoted. The range of θ/t for which the interpolation procedure was adopted was largest for $\alpha = 45^\circ$. Interpolation was required for only a few values of θ/t (interval of 0.1) for the lower values of α .

The instability of the solution process at P/P_0 close to unity is reflected in the discontinuity of calculated load response for $\alpha = 20^\circ$ and 22.5° . However, the load values obtained for these spiral angles are not critical in this region of θ/t , and the discontinuity of response has not been investigated more thoroughly.

The basic mechanism results were processed to include the effect of the release of elastic inplane strain energy. In this type of mechanism the relevant velocities for use in equation 22 (chapter 7) are

$$\begin{aligned}\dot{Y} &= \dot{Y}_3 \text{ and} \\ \dot{\theta} &= \dot{\Gamma}_2 = \epsilon_1 \dot{X}_1 + \epsilon_2 \dot{\theta}_x + \epsilon_3 \dot{\theta}_y + \epsilon_4 \dot{\theta}_z\end{aligned}$$

(The error produced by taking $\dot{\theta} = \tan\alpha \cdot \dot{\theta}_y$ is, however, undetectable

on the plotted output.)

B/t ratios of 0, 30, 45, 60, 90, 120 were used in the calculations for $20^\circ \leq \alpha \leq 45^\circ$. Minimum strength curves were chosen for each B/t, based on the range of α , $20^\circ \leq \alpha \leq 45^\circ$. Minimum strength curves are plotted in figure 8.44.

8.17.3 Finite element analyses

The finite element analyses of panel behaviour were based on a cylindrical panel with a single half-wave sinusoidal imperfection imposed on the perfect panel geometry. The imperfection was inward with an initial magnitude of $\delta_0/t = 1/165$ B/t.

Two axes of symmetry allowed the analysis to be confined to a $\frac{1}{4}$ panel. A 3 x 3 finite element grid was used to represent the panel itself, and a rigid beam of 3 x 1 shell elements compelled the panel loading edge to move uniformly along the generator direction, figure 8.45. The loading edge was clamped against rotations. This choice of finite element model was made after attempts to use a stiffener element along the loading edge failed to produce results which converged to an equilibrium state.

A flexible solution strategy could be adopted by the finite element package. The iteration procedure to a state of equilibrium could use the stiffness matrix determined at the beginning of a load step throughout the process. Alternatively the stiffness matrix could be reformed up to a maximum rate of one reformation after each iteration step, i.e. full Newton-Raphson strategy. Increments of displacement were applied parallel to the generator, with the maximum displacement acting as a control. This was usually the lateral displacement of the node at the panel centre. Displacements were in increments of 0.1-0.2 t. The finite element results for B/t = 30,45,60,90 are plotted with the collapse mechanism results in figure 8.46.

8.17.4 Comparison of collapse mechanism and finite element results.

The results for the two analyses are presented, for 4 values of B/t,

separately in figures 8.47-50. The two analyses may be compared for panel unloading and figures 8.47-50 indicate a very acceptable level of comparison.

The maximum panel strength predicted by the finite element analysis for $B/t = 30$, is 1.4% greater than the strength predicted by the mechanism analysis, figure 8.47. The constrained panel response may be taken from figure 8.17. (The constrained longitudinal edge corresponds to the clamped edge condition for this mechanism). The minimum strength response is given by $\alpha = 45^\circ$ at low values of lateral deflection, and calculations show that the elastic energy correction for $B/t = 30$ and $\delta/t < 0.35$ is very small. The maximum strength predicted by the finite element analysis corresponds to the strength predicted by the constrained collapse mechanism for the same lateral deflection ($\delta/t \approx 0.24$).

The difference between predicted strengths over most of the unloading range is less than 2.5% for $B/t = 30, 45, 60$, figures 8.47-49. The difference is less than 4% for $B/t = 90$, figure 8.50.

The finite element result slightly exceeds the mechanism result for $0.5 < \delta/t < 2.1$ for $B/t = 45$ and 60. It was noted earlier that the mechanism result showed signs of instability in this range. Nevertheless the load capacity predicted by the mechanism analysis is sufficiently stable to be acceptable. The underestimation of panel strength by the collapse mechanism analysis for $B/t = 30, 45, 60$, over some of the unloading range, must be attributed to the inaccuracy inherent in the simplified treatment of the inplane yield line (line 2).

In general the close correlation between finite element and collapse mechanism results indicates that the collapse mechanism model is very accurate.

8.17.5 Prediction of panel strengths.

The work of Williams and Walker (239) on the elastic behaviour of two dimensional panels facilitated a method of prediction of panel strengths in §7.19. No such elastic data exists for cylindrical panels.

There are problems which impede such a treatment. The classically derived buckling load for a perfect cylindrical panel is inaccurate for imperfect panels. It is generally agreed that the buckling load is highly non-linear and is dependent on both boundary conditions and on the level of initial imperfection. In addition, the postbuckling behaviour of cylindrical panels is not well understood for deflections significantly away from the point of buckling.

The comprehensive investigation of the elastic response of curved panels is outside the scope of this thesis. The absence of accurate published guidance on the subject means that the mechanism results for curved panels cannot as yet be used to predict the collapse load of cylindrical panels, as was possible for two dimensional rectangular panels. It will, however, be possible to predict minimum strengths by a comprehensive extension of the work presented above to cover a range of spiral angles.

8.18 Conclusions

8.18.1 Conclusions of study of cylindrical panels

The results presented above, whilst not complete, allow a number of conclusions to be drawn about the behaviour of a panel under inplane and lateral load.

The relevant geometric parameter of a curved panel is the non-dimensional curvature, A^2/Rt .

An essential difference exists between the behaviour of panels with strong ring stiffeners and panels with weak ring stiffeners under lateral load. This difference results from the ability of a panel to shed hoop forces onto strong ring stiffeners. A minor improvement in strength results from the use of strong ring stiffeners when panels are subject to axial load.

Minimum panel strengths may be deduced from the extension of the results presented, for inplane (axial) and pressure loading. The use of such minimum strengths may treat the axial capacity very conservatively. Lower bounds to strength are influenced by the

parameter A^2/Rt .

Hydrostatic loading causes axial forces to act on the panel in addition to the lateral loading. The effect of the axial load is not uniform since the relative size of the non-dimensionalised load varies with A^2/Rt , for panels with strong ring stiffeners. Hydrostatic loading on a panel with weak ring stiffeners produces inplane loads in a fixed ratio to one another and a lateral loading component of lesser importance. Minimum strengths have not been obtained for this loading, but are possible by an extension of lateral displacements beyond $\delta/t = 5.0$.

A criterion for the adequacy of stocky ring stiffeners has been produced, and the panel strength given for any level of ring stiffening.

The technique of checking the adequacy of stringer stiffeners has been demonstrated. Stiffener proportions D_{st}/t_{st} and e_{st}/t have been shown to have a significant effect on the adequacy of the stringer stiffener for the area of stringer chosen.

The correlation study carried out for a particular set of boundary conditions and a single value of A^2/Rt indicates that the collapse mechanism analysis is an accurate method of assessing the unloading behaviour of cylindrical panels.

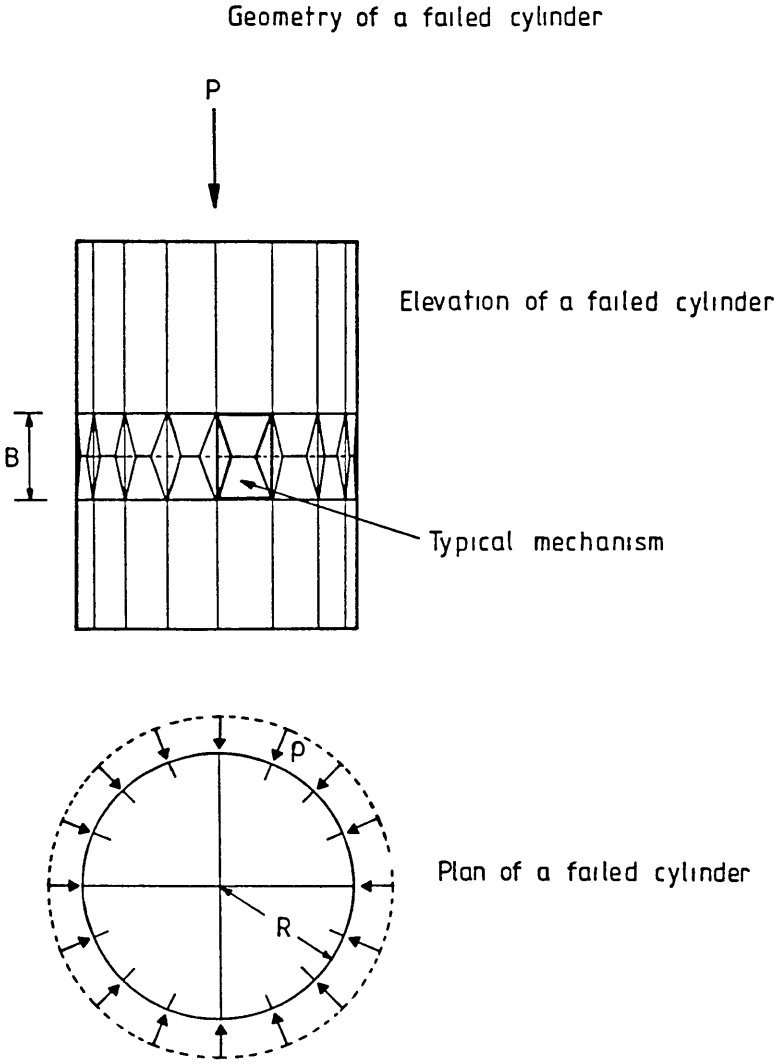


Figure 8 1

Basic geometry of yield mechanism

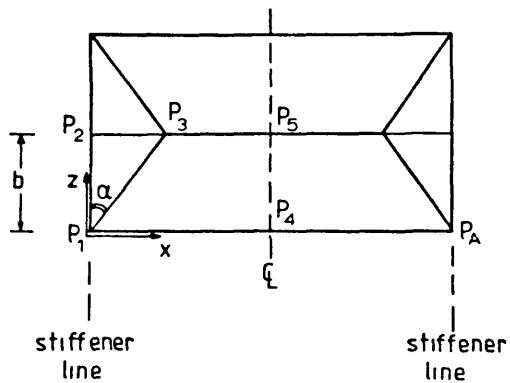
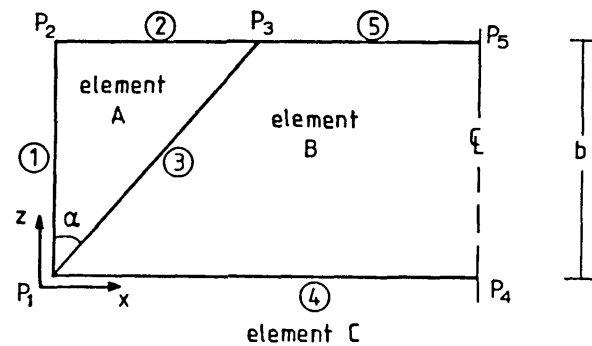


Figure 8 2 (a)

Idealisation of panel



Element A	bounded by points	P_1, P_2, P_3
Element B	" " "	P_1, P_4, P_3, P_5
Element C	" " "	P_1, P_4 and the remainder of the cylinder

Figure 8 2 (c)

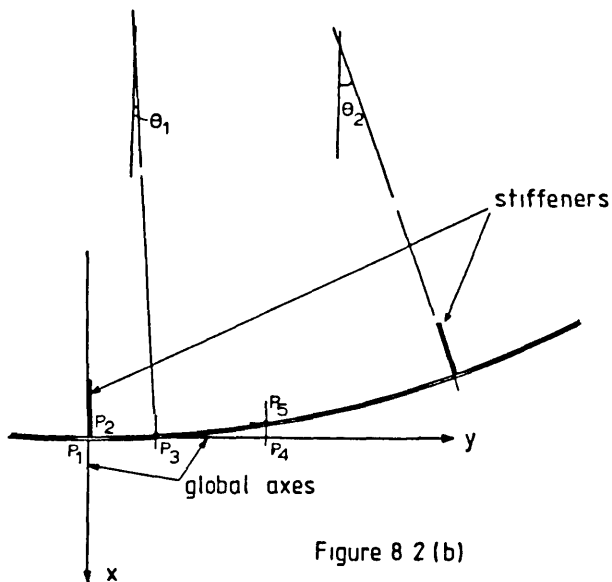


Figure 8 2 (b)

note z_2 axis does not lie completely in the plane of the cross section

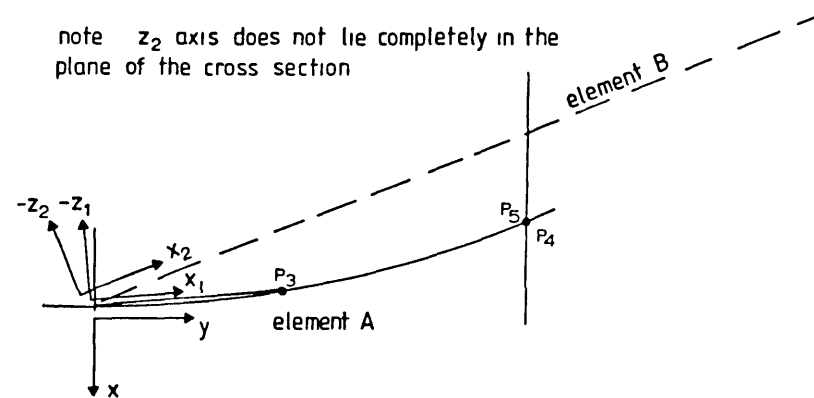
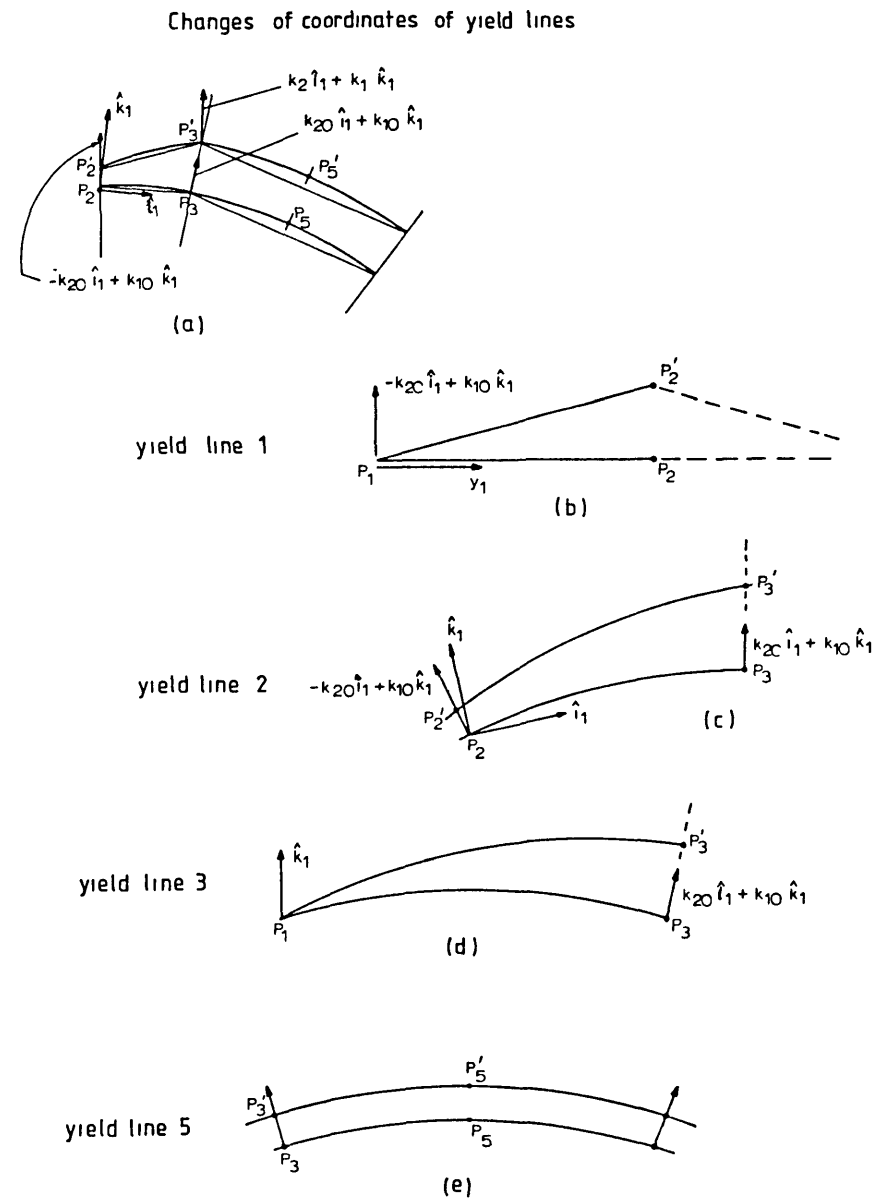
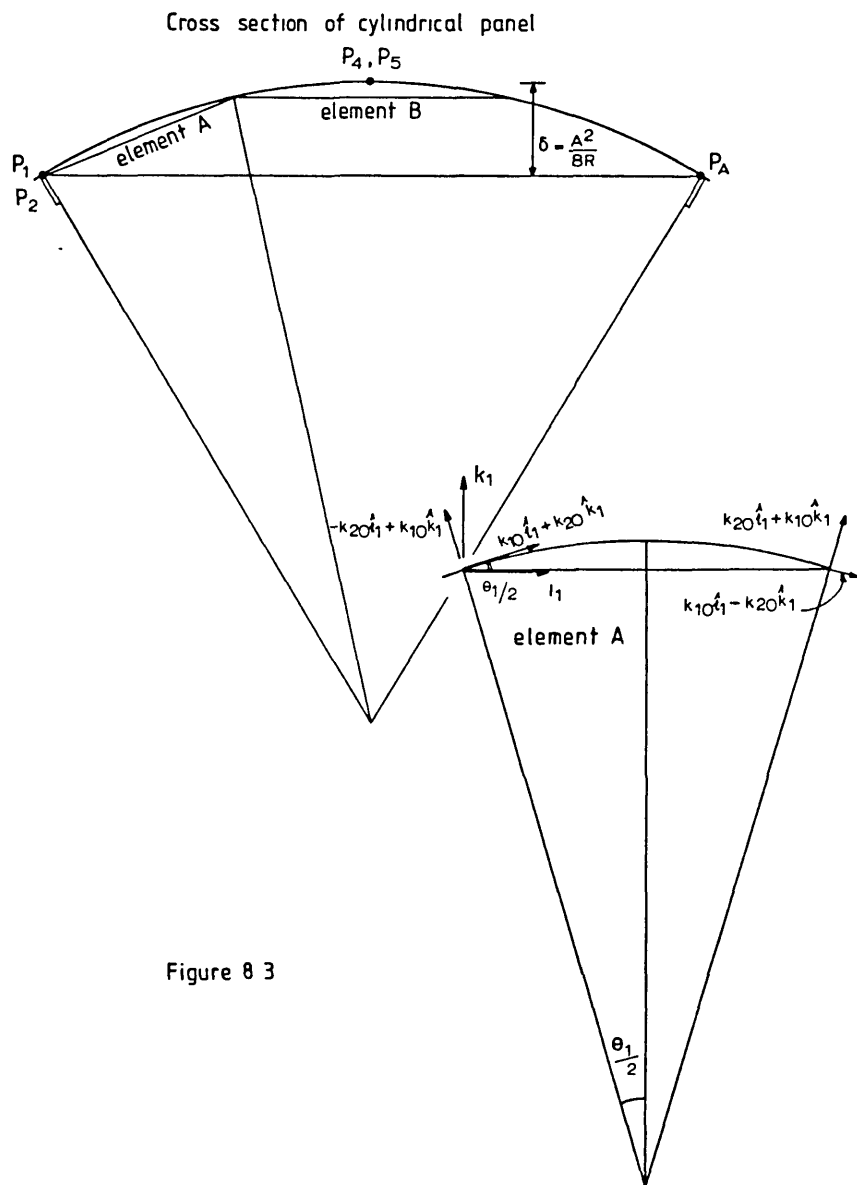
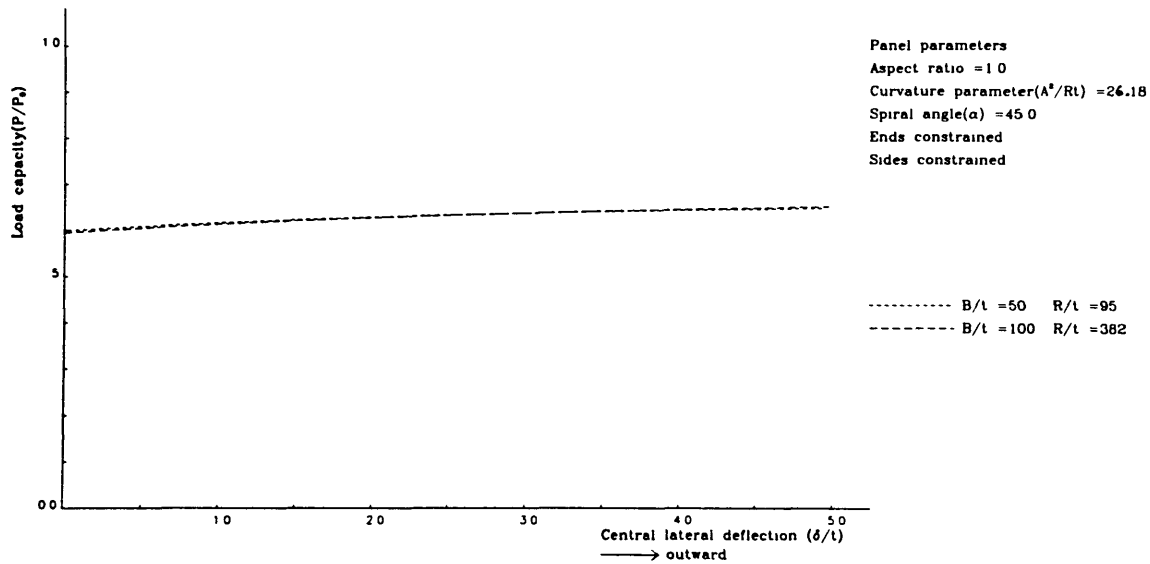


Figure 8 2 (d)



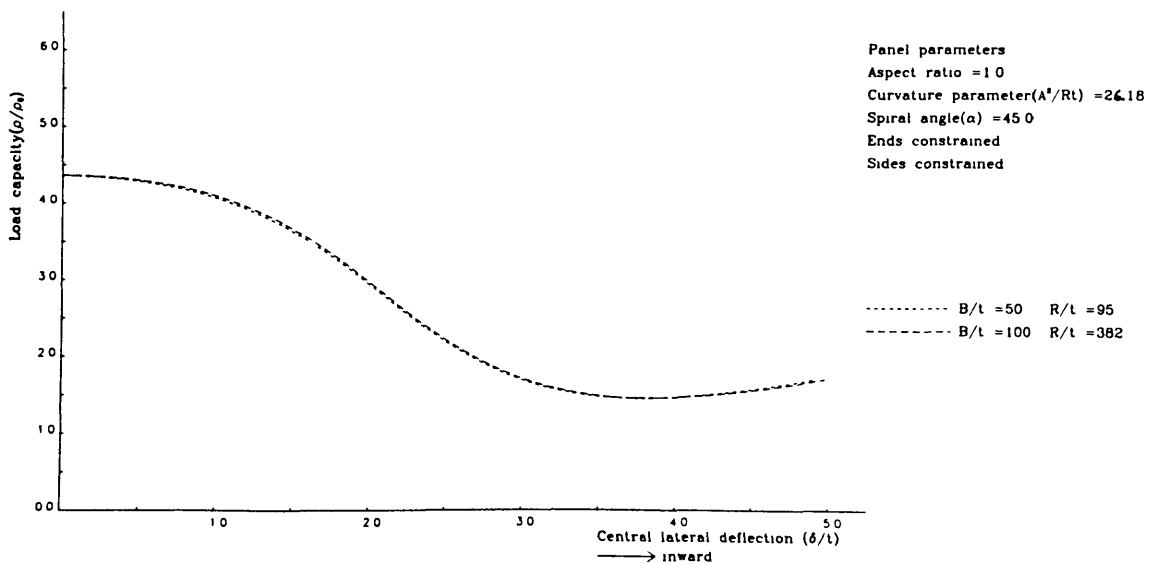
Cylindrical panel under inplane load

Plot of central lateral deflection against load capacity

The effect of varying R/t and A/R , but keeping A^2/Rt constantDegrees of freedom $\{0, \theta_y, 0, X_1, 0, Y_2, 0, 0\}$ Loading $\{0, 0, 0, 0, 0, P_x, 0, 0\}$ 

Cylindrical panel under lateral load

Plot of central lateral deflection against load capacity

The effect of varying R/t and A/R , but keeping A^2/Rt constantDegrees of freedom $\{0, \theta_y, 0, X_1, 0, Y_2, 0, 0\}$ Loading $1/3 \rho a^3 b \{0, -4, 0, 4/R - 12R/a^2, 0, 0, 0, 0\}$ 

upper figure 8.5

lower figure 8.6

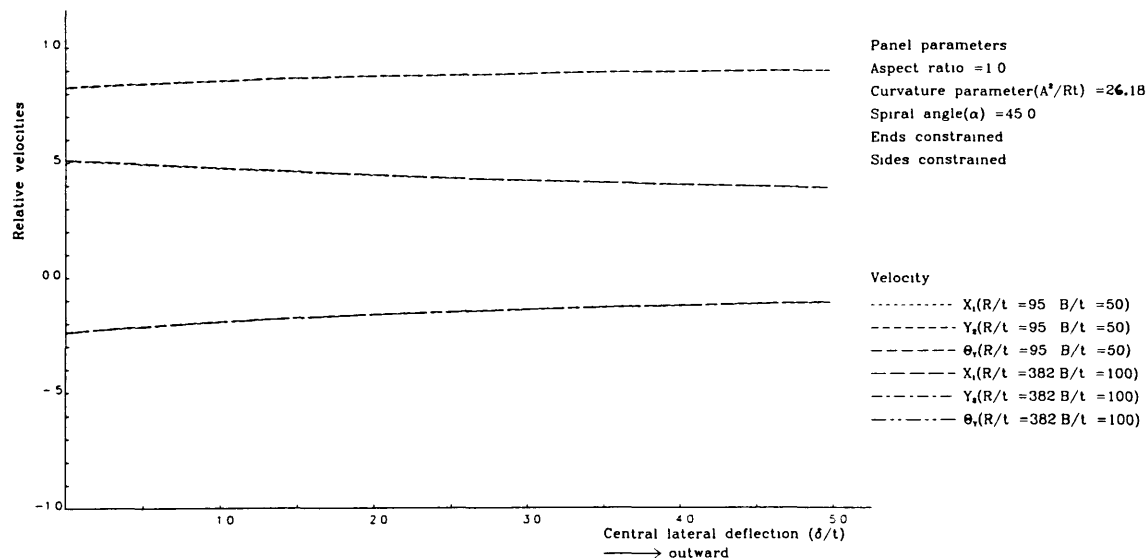
Cylindrical panel under inplane load

Plot of central lateral deflection against relative velocity

The effect of varying R/t and A/R , but keeping A^2/Rt constant

Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, 0, 0\}$

Loading $\{0, 0, 0, 0, 0, P_{rz}, 0, 0\}$



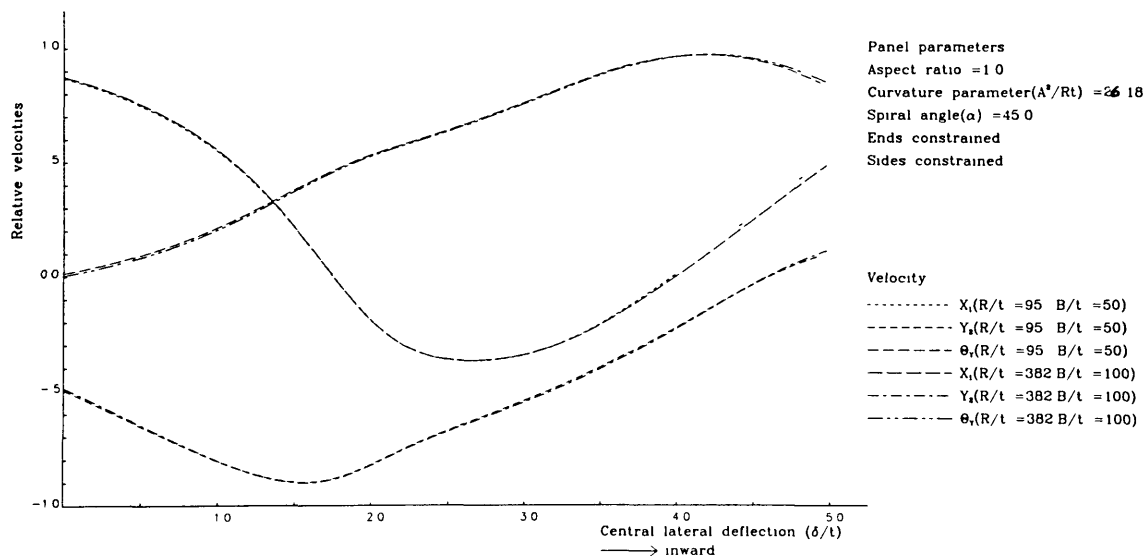
Cylindrical panel under lateral load

Plot of central lateral deflection against relative velocity

The effect of varying R/t and A/R , but keeping A^2/Rt constant

Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, 0, 0\}$

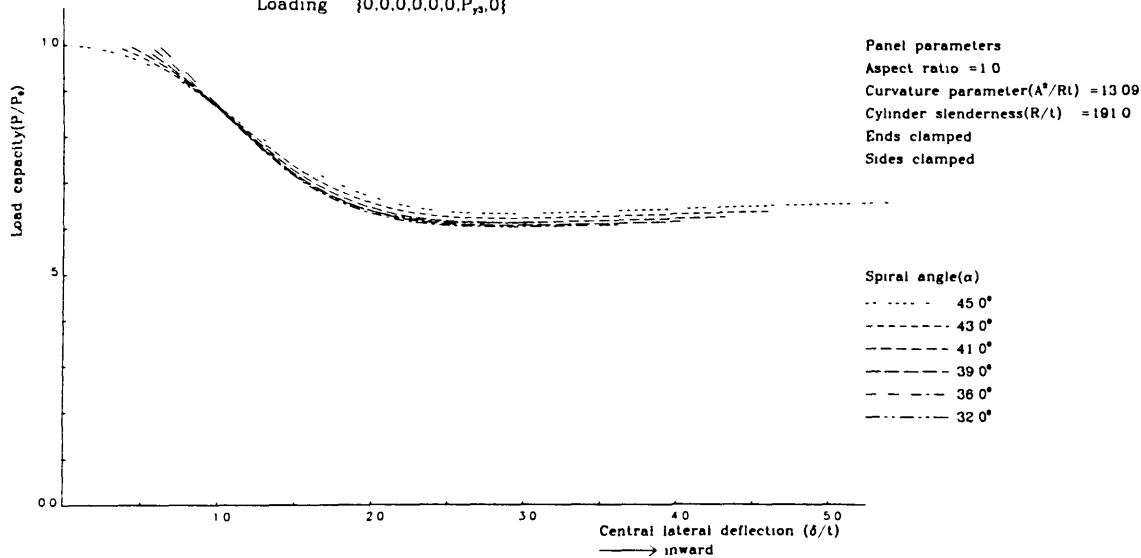
Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R - 12R/a^2, 0, 0, 0, 0\}$



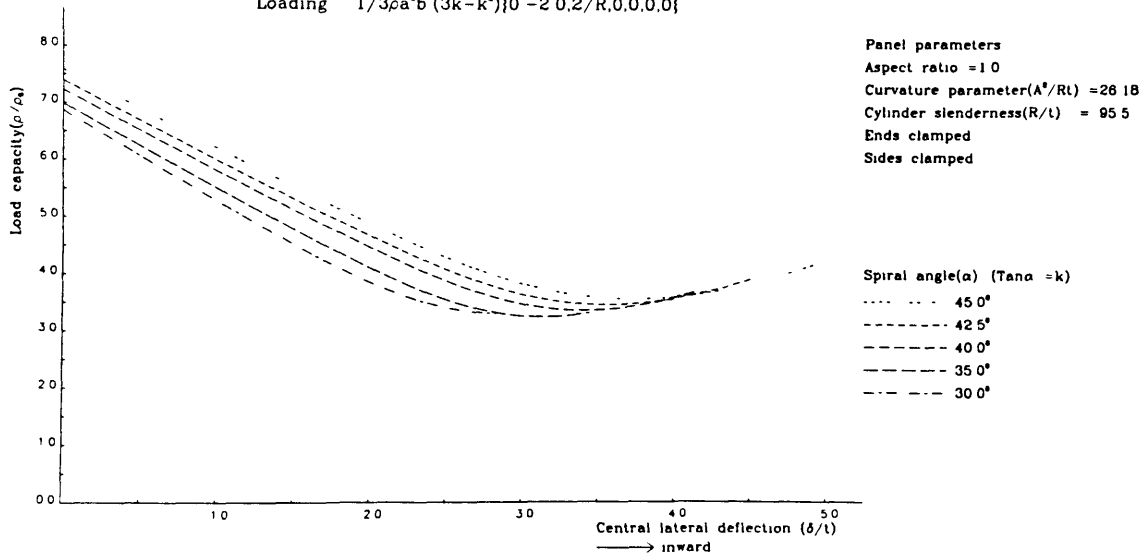
upper figure 8.7

lower figure 8.8

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of varying the spiral angle(α)
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_3, 0\}$



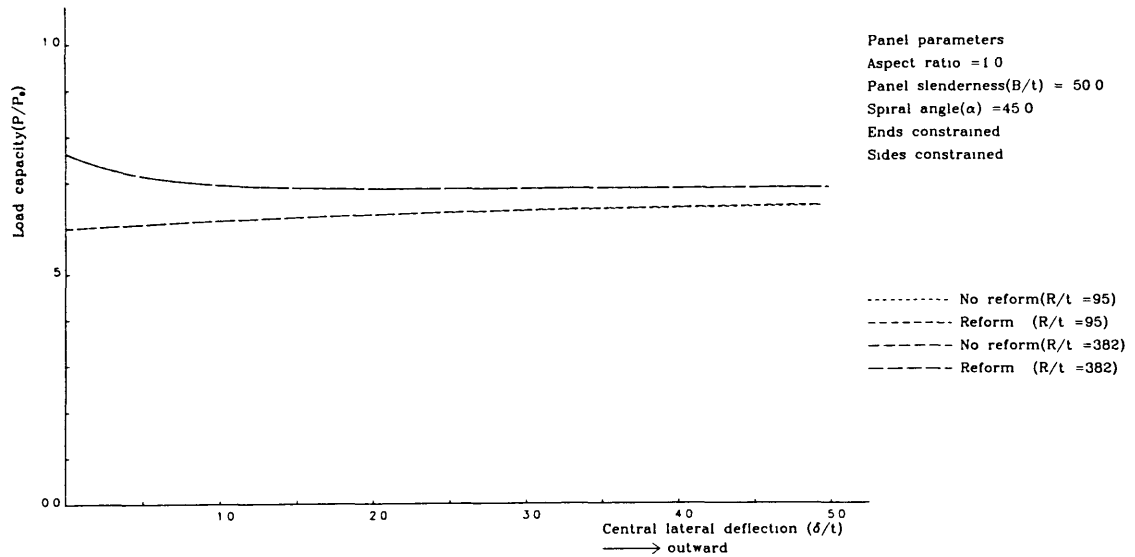
Cylindrical panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of varying the spiral angle(α)
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3 \rho a^2 b (3k - k^2) \{0 - 2, 0, 2/R, 0, 0, 0, 0\}$



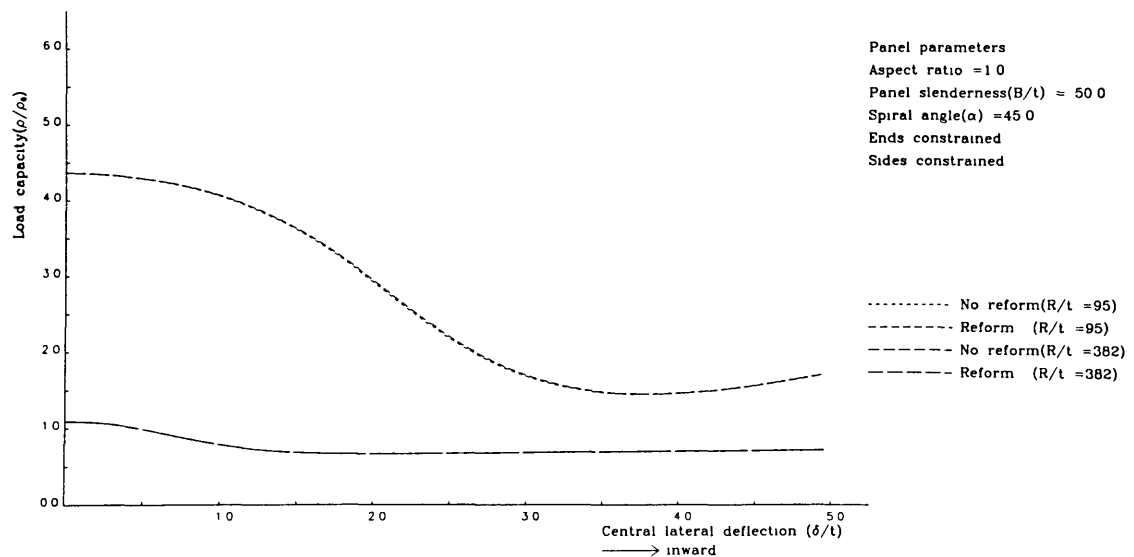
upper figure 8.9

lower figure 8.10

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 The effect of consistent reformation of the direction cosines of points
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_z, 0, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_{rz}, 0, 0\}$



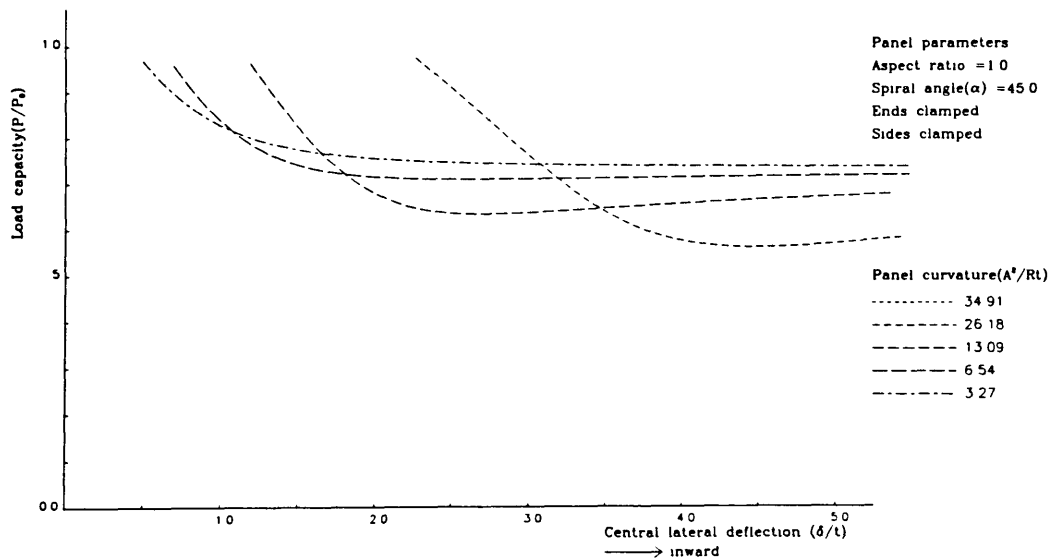
Cylindrical panel under lateral load
 Plot of central lateral deflection against load capacity
 The effect of consistent reformation of the direction cosines of points
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_z, 0, 0\}$
 Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R - 12R/a^2, 0, 0, 0, 0\}$



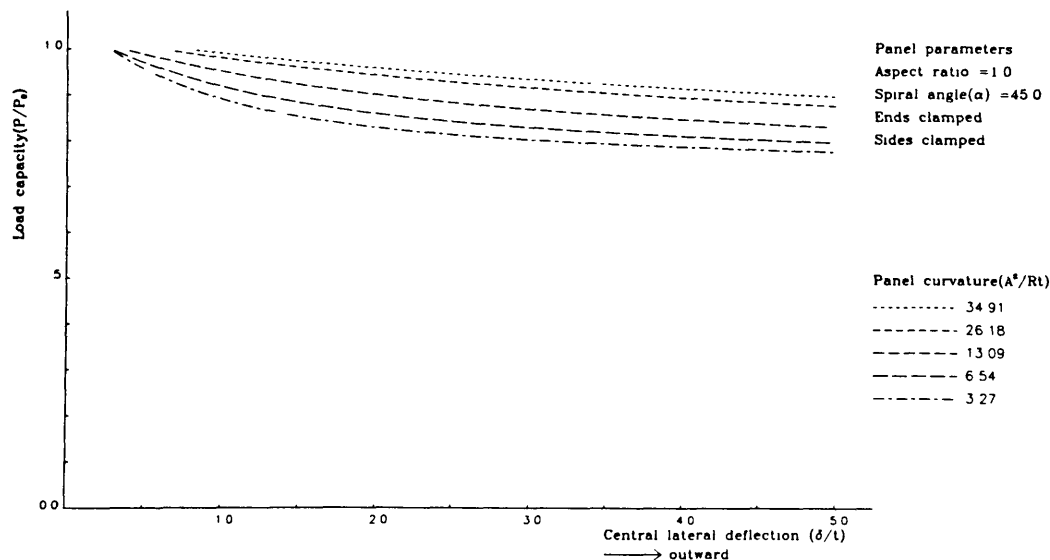
upper figure 8.11

lower figure 8.12

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, 0, 0, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_r, 0\}$



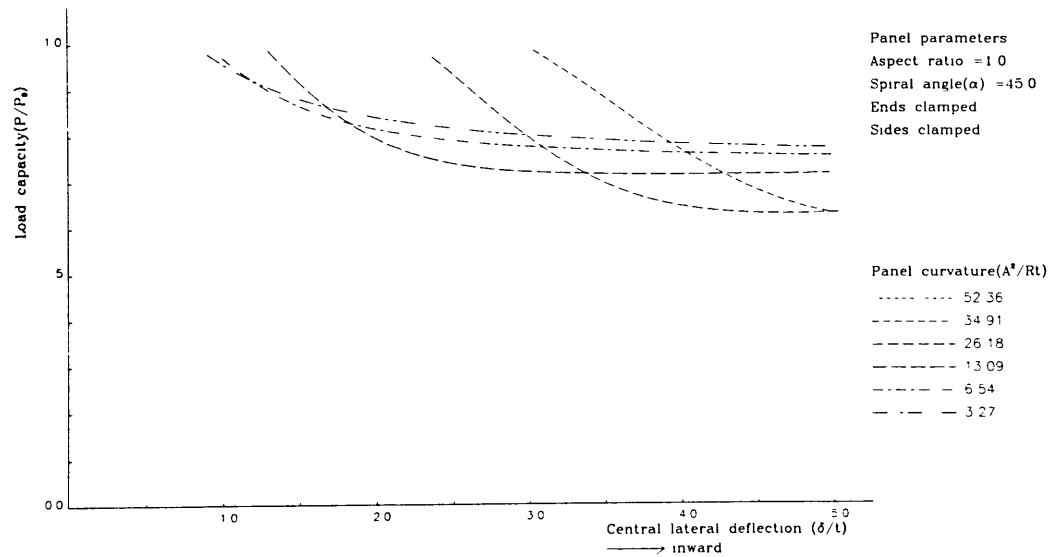
Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, 0, 0, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_r, 0\}$



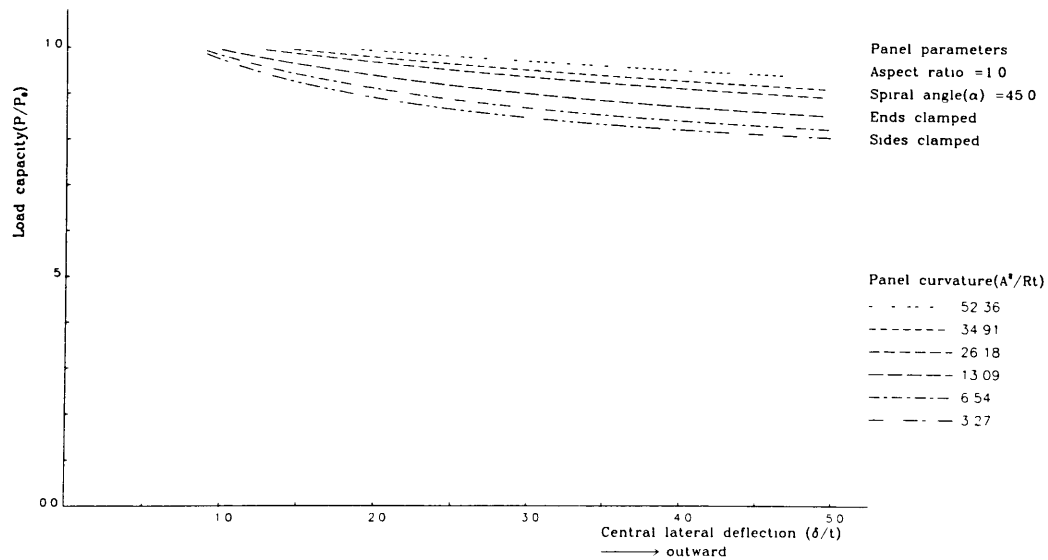
upper figure 8.13

lower figure 8.14

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_{13}, 0\}$



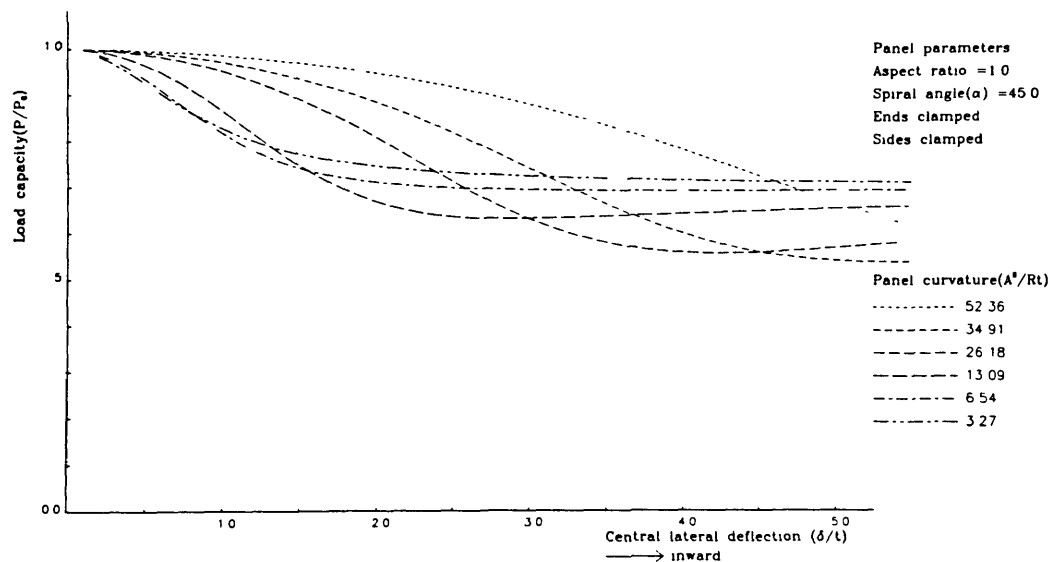
Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_{13}, 0\}$



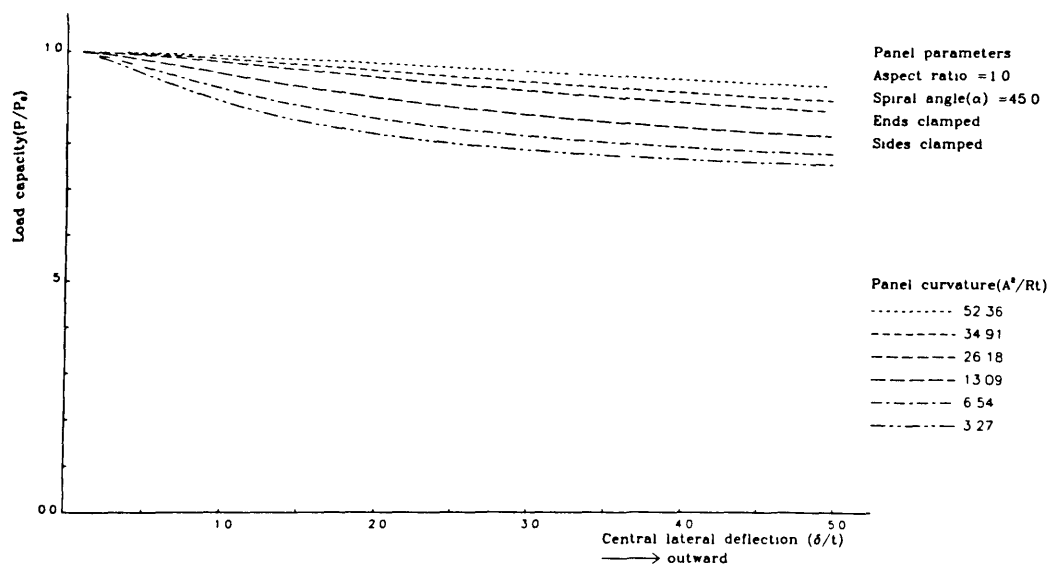
upper figure 8.15

lower figure 8.16

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_{r3}, 0\}$

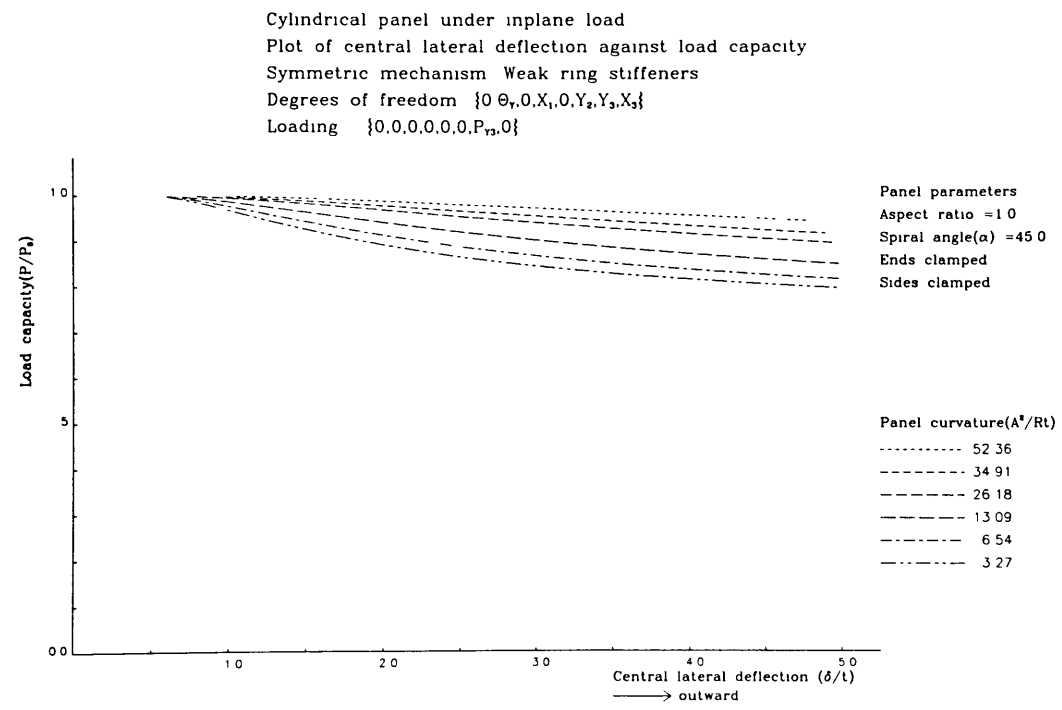
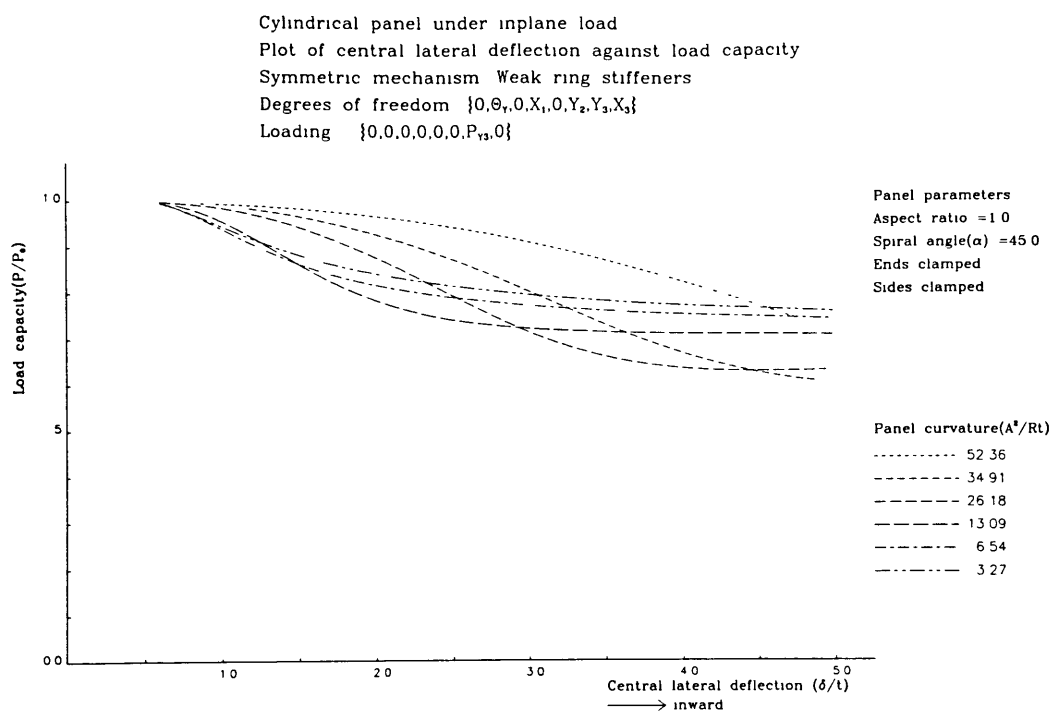


Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_{r3}, 0\}$



upper figure 8.17

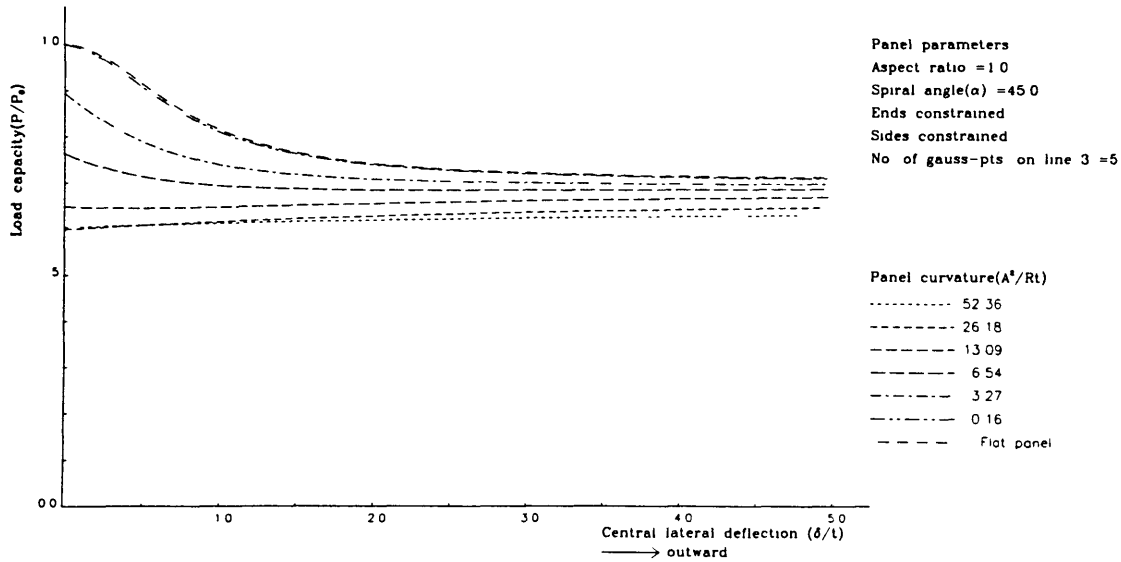
lower figure 8.18



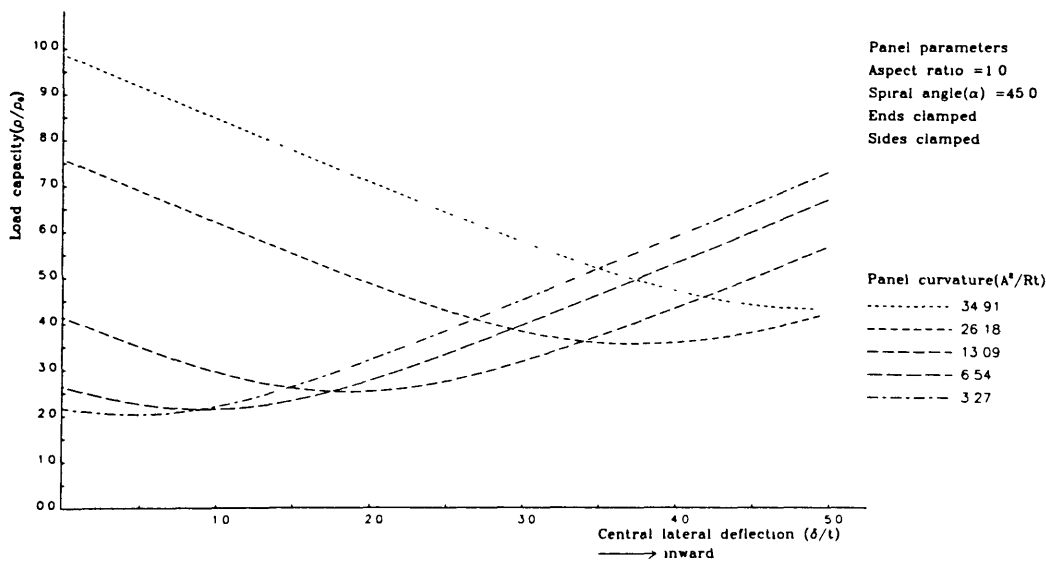
upper figure 8.19

lower figure 8.20

Cylindrical panel under inplane load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, 0, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_{cr}, 0, 0\}$



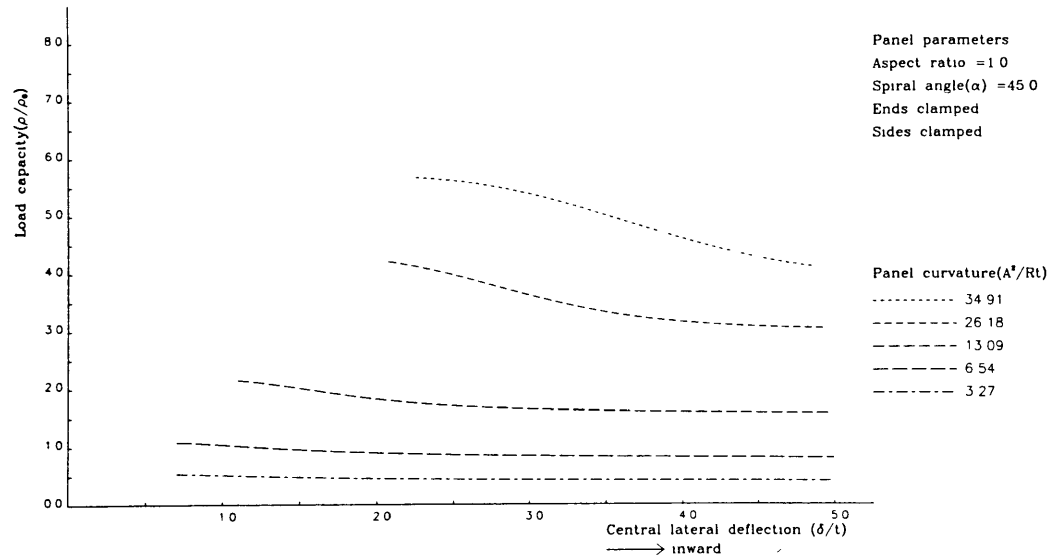
Cylindrical panel under lateral load
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R, 0, 0, 0, 0\}$



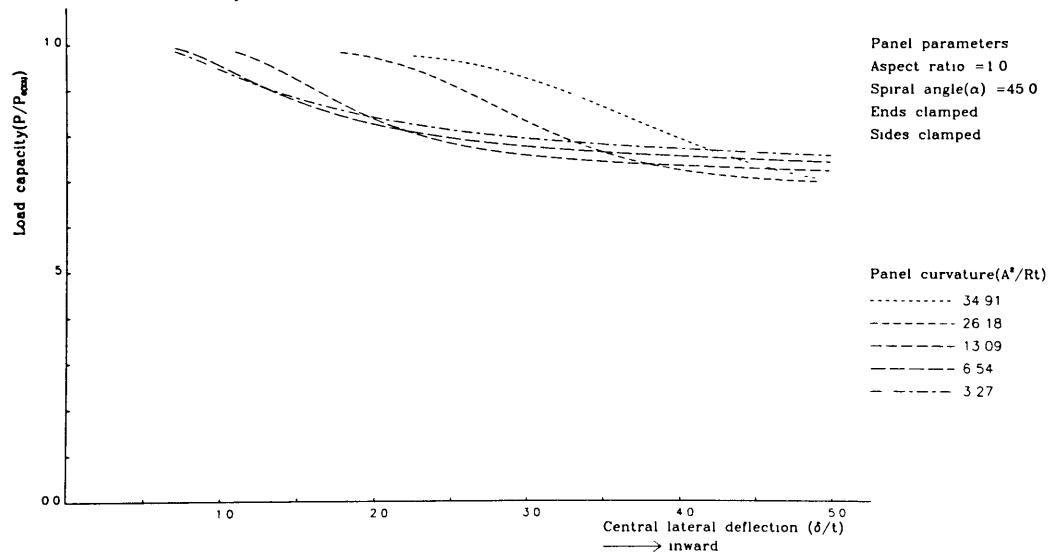
upper figure 8.21

lower figure 8.22

Cylindrical panel under lateral load
Plot of central lateral deflection against load capacity
Symmetric mechanism Weak ring stiffeners
Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, X_3\}$
Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R, 0, 0, 0, -12R/a^2\}$



Cylindrical panel under lateral load
Plot of central lateral deflection against load capacity
Symmetric mechanism Weak ring stiffeners
Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, X_3\}$
Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R, 0, 0, 0, -12R/a^2\}$



upper figure 8.23

lower figure 8.24

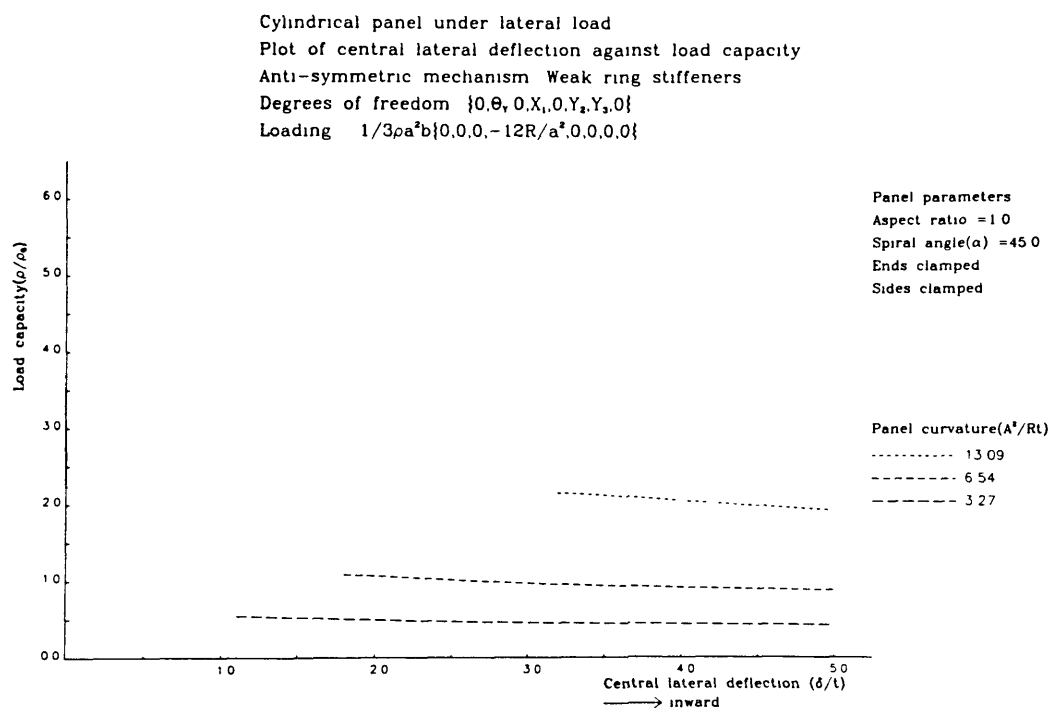
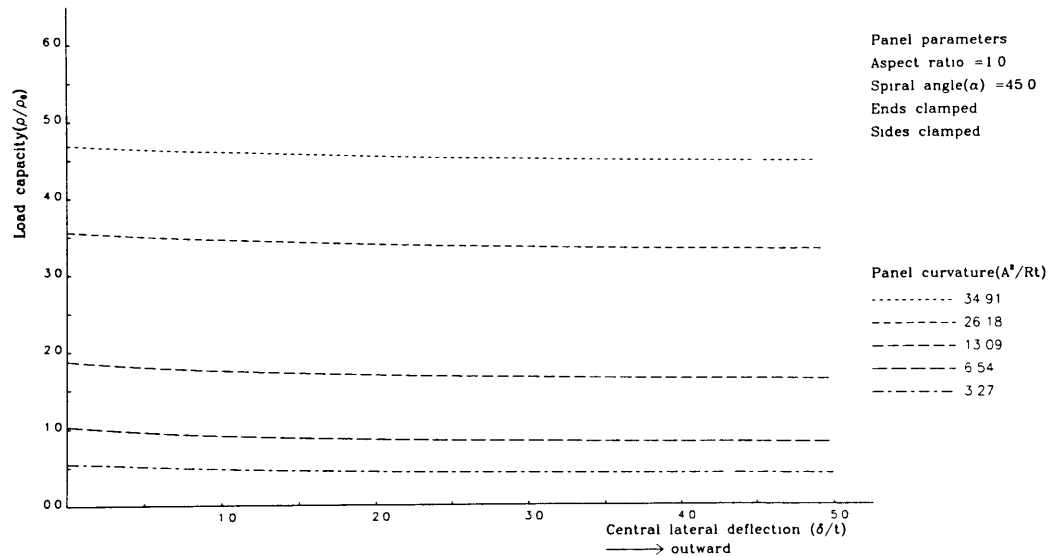
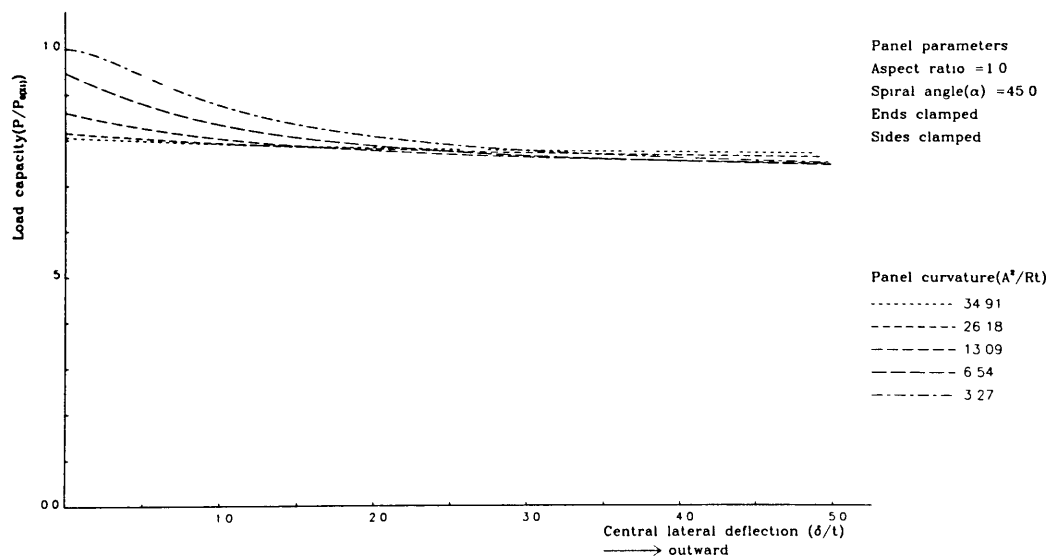


figure 8.25

Cylindrical panel under lateral load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3 \rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, 0, 0\}$



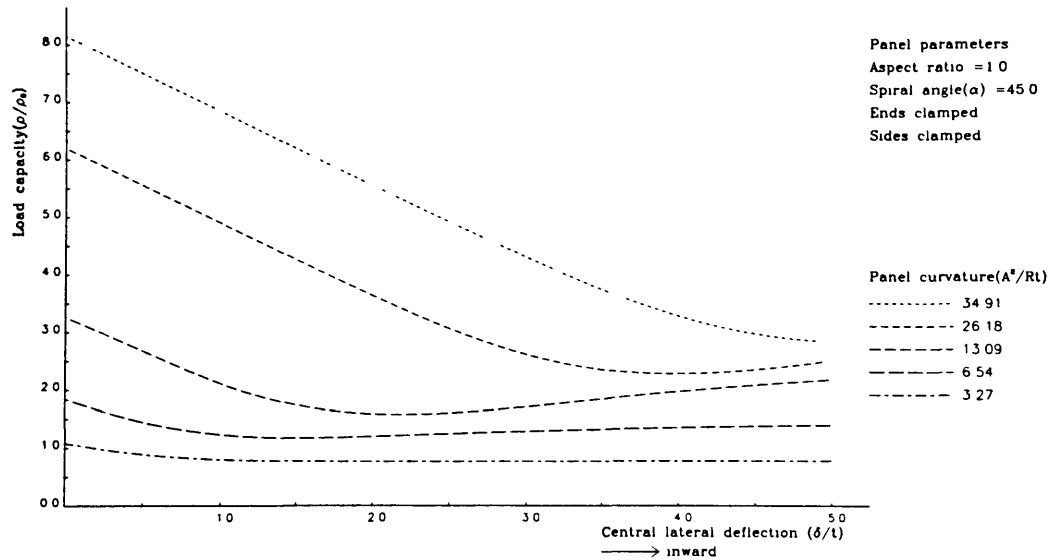
Cylindrical panel under lateral load
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3 \rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, 0, 0\}$



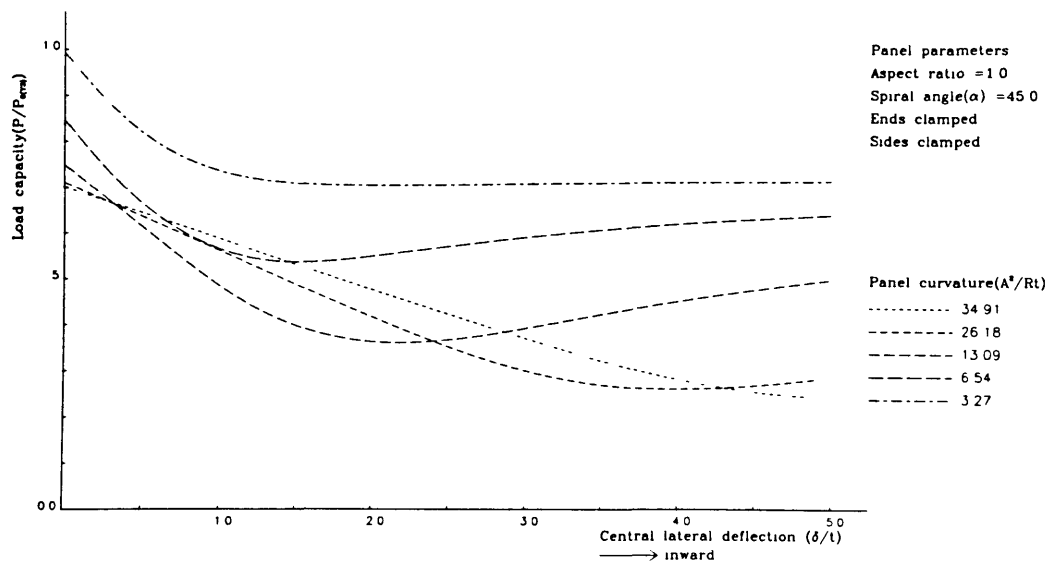
upper figure 8.26

lower figure 8.27

Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_1, Y_3, 0\}$
 Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R, 0, 0, -6R/ab, 0\}$



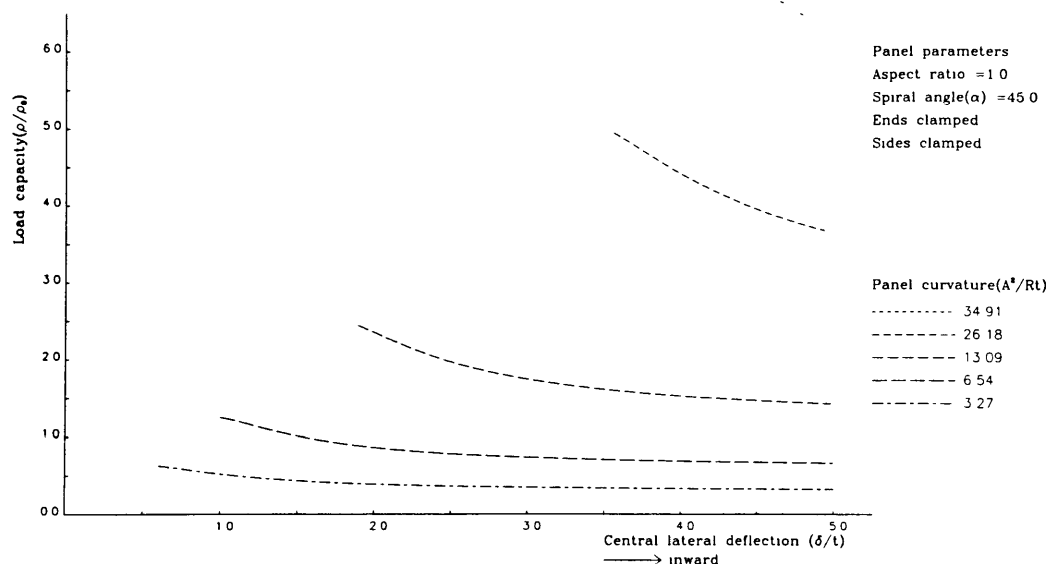
Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Strong ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_1, Y_3, 0\}$
 Loading $1/3 \rho a^2 b \{0, -4, 0, 4/R, 0, 0, -6R/ab, 0\}$



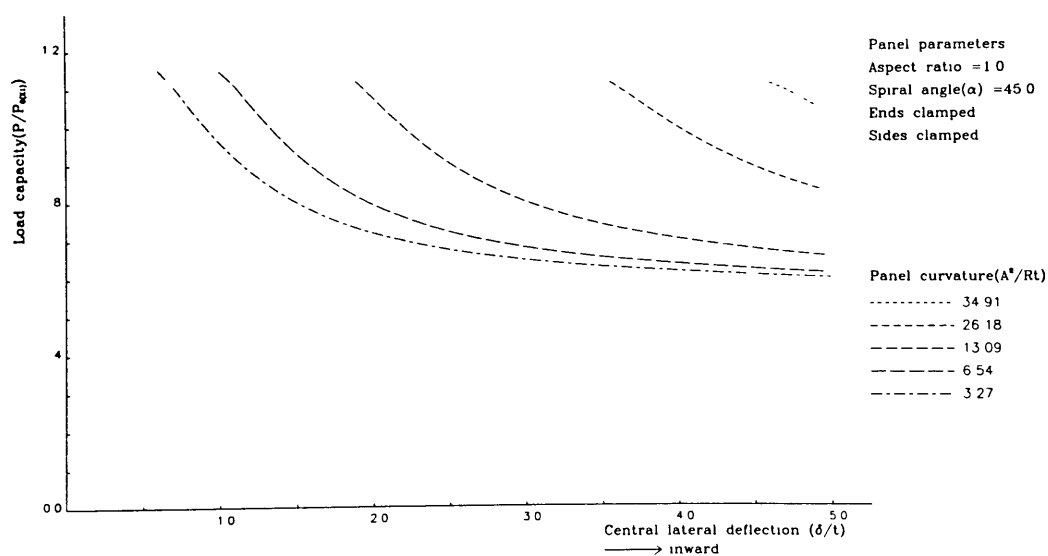
upper figure 8.28

lower figure 8.29

Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3\rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, -6R/ab, 0\}$



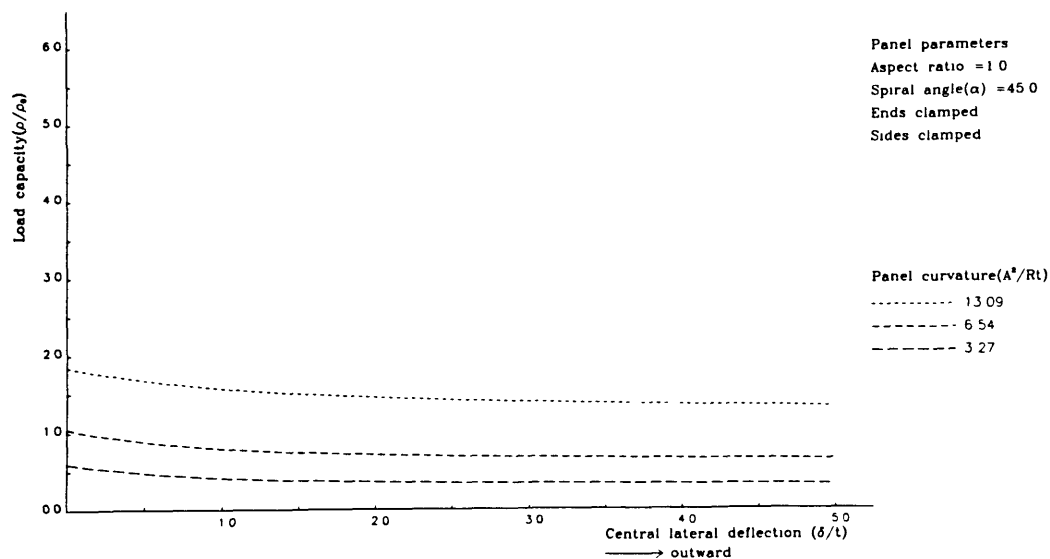
Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3\rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, -6R/ab, 0\}$



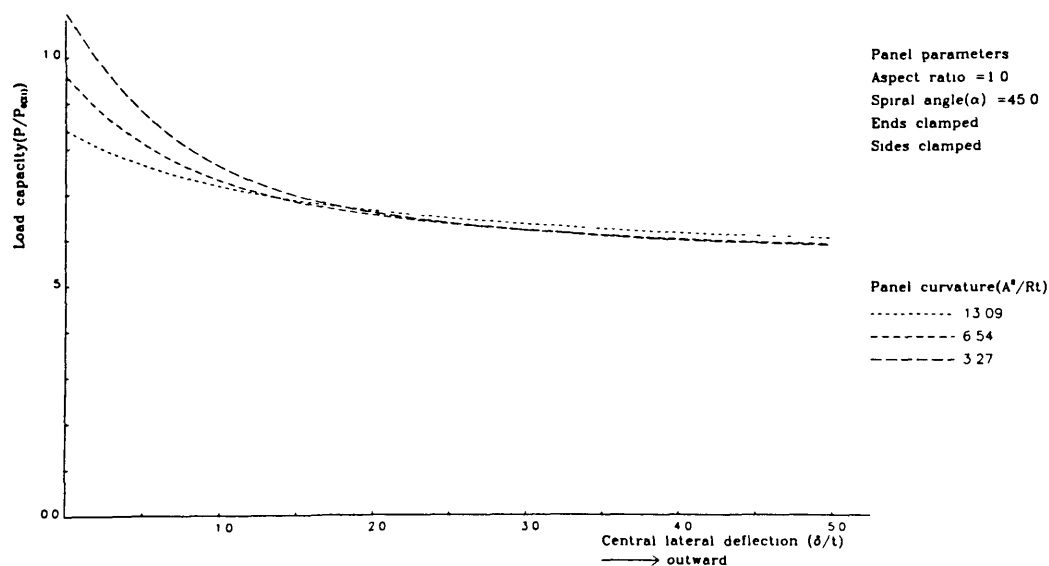
upper figure 8.30

lower figure 8.31

Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3\rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, -6R/ab, 0\}$



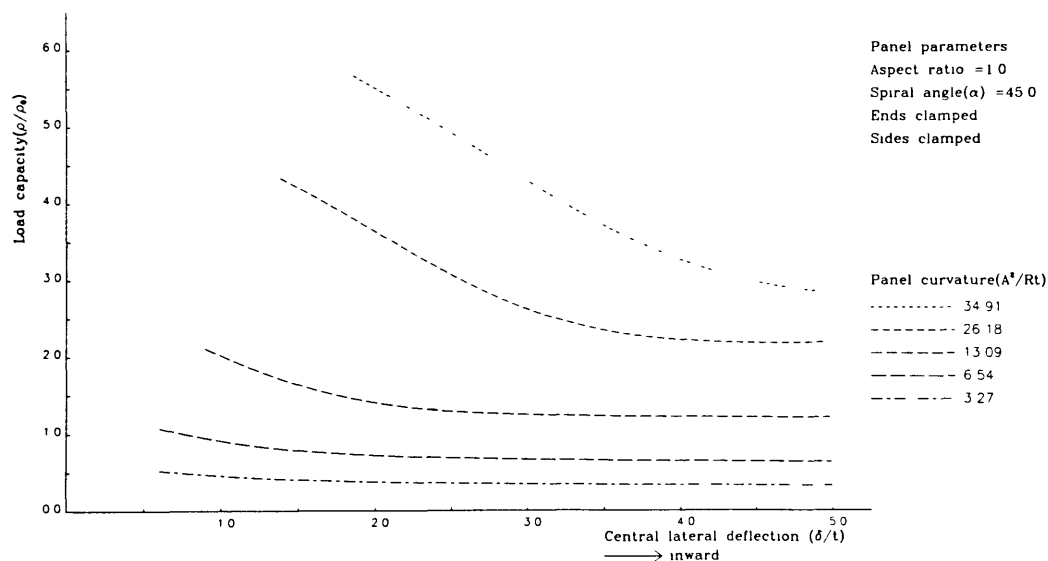
Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Loading $1/3\rho a^2 b \{0, 0, 0, -12R/a^2, 0, 0, -6R/ab, 0\}$



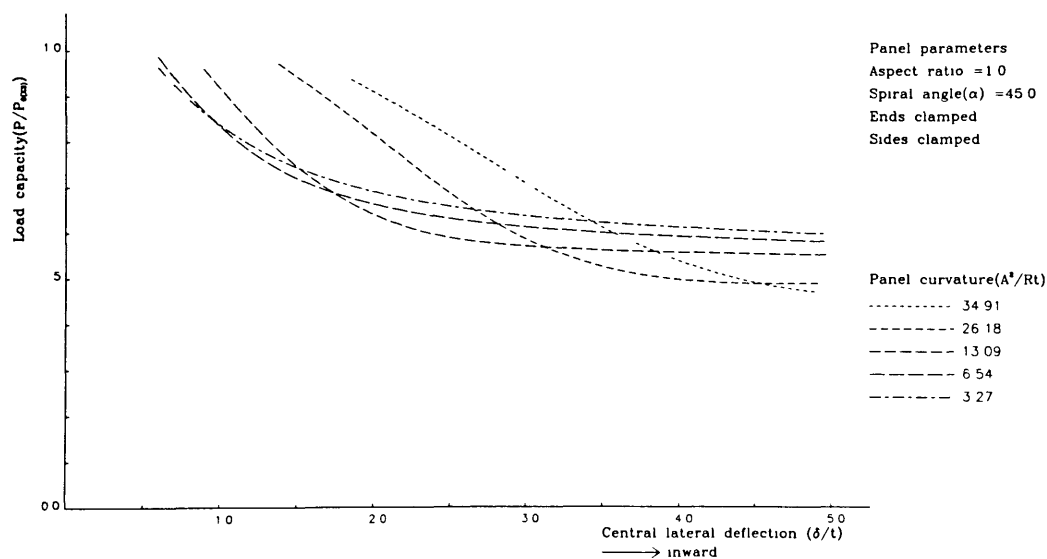
upper figure 8.32

lower figure 8.33

Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, X_3\}$
 Loading $1/3\rho a^2 b \{0, -4.0, 4/R, 0, 0, -6R/ab, -12R/a^2\}$



Cylindrical panel under hydrostatic pressure
 Plot of central lateral deflection against load capacity
 Symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, X_3\}$
 Loading $1/3\rho a^2 b \{0, -4.0, 4/R, 0, 0, -6R/ab, -12R/a^2\}$



upper figure 8.34

lower figure 8.35

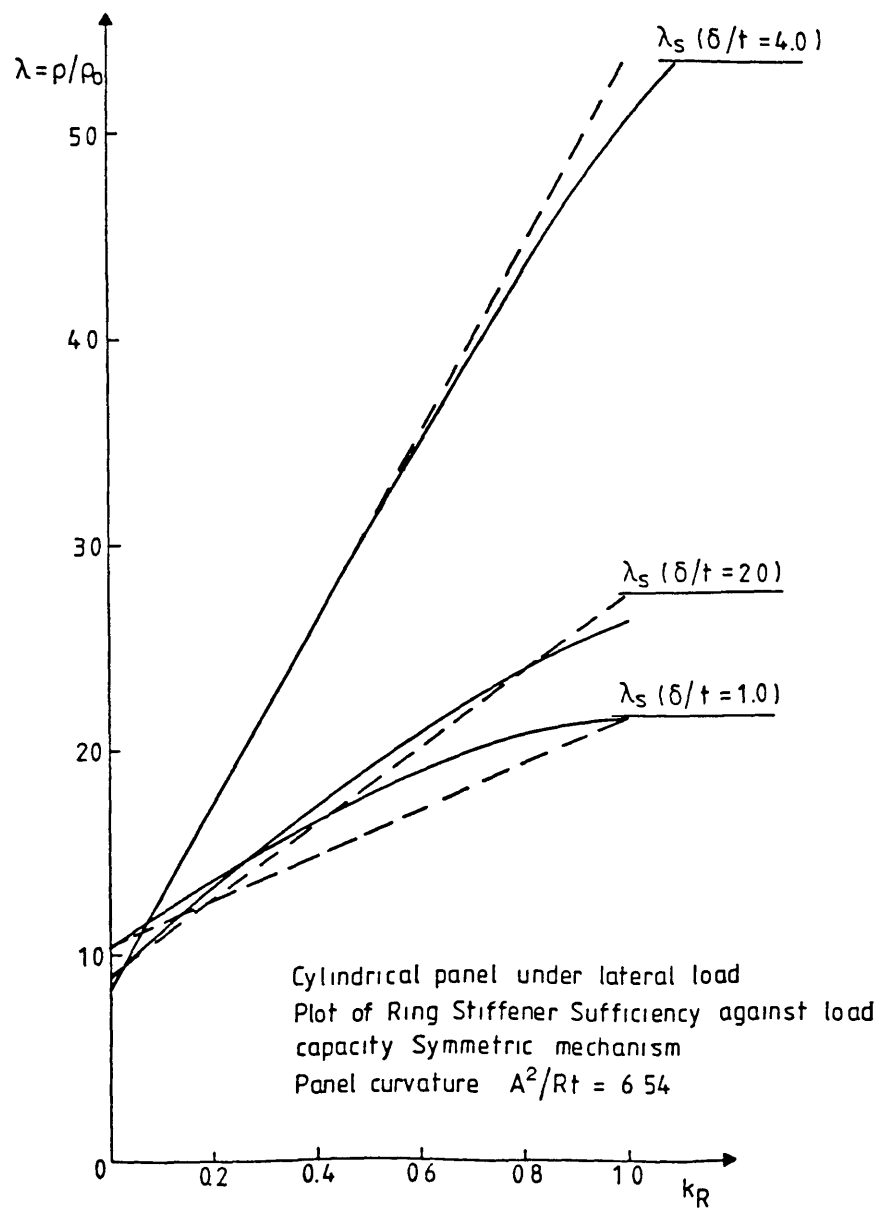
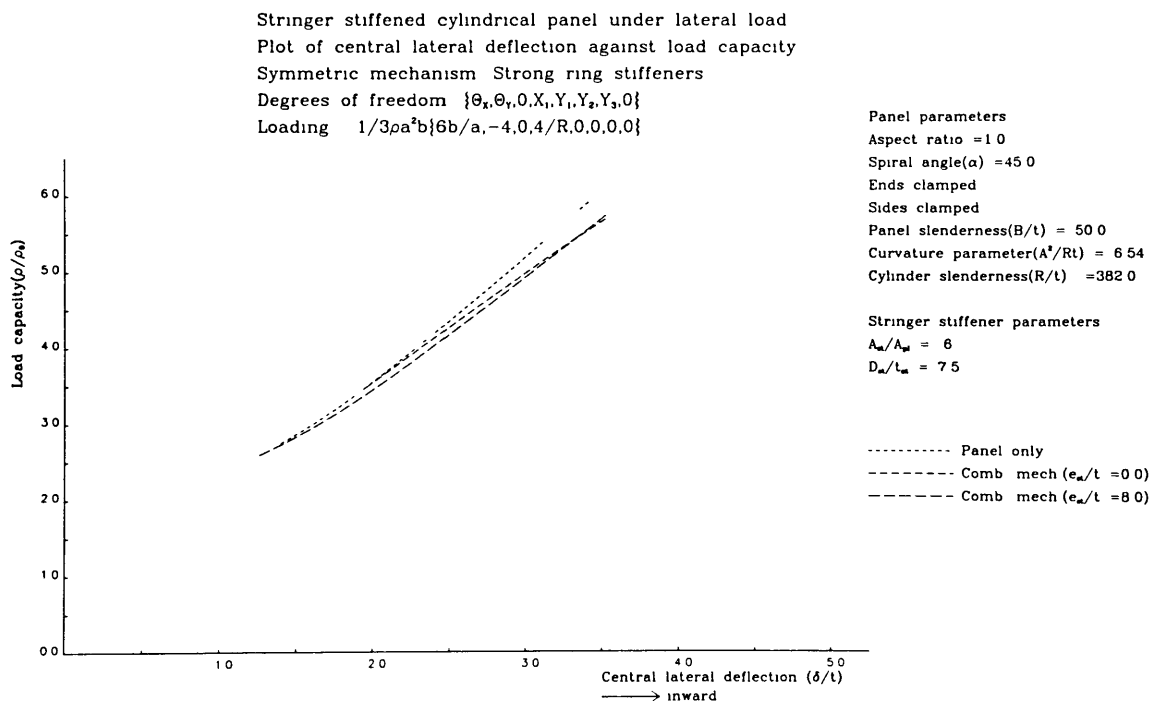
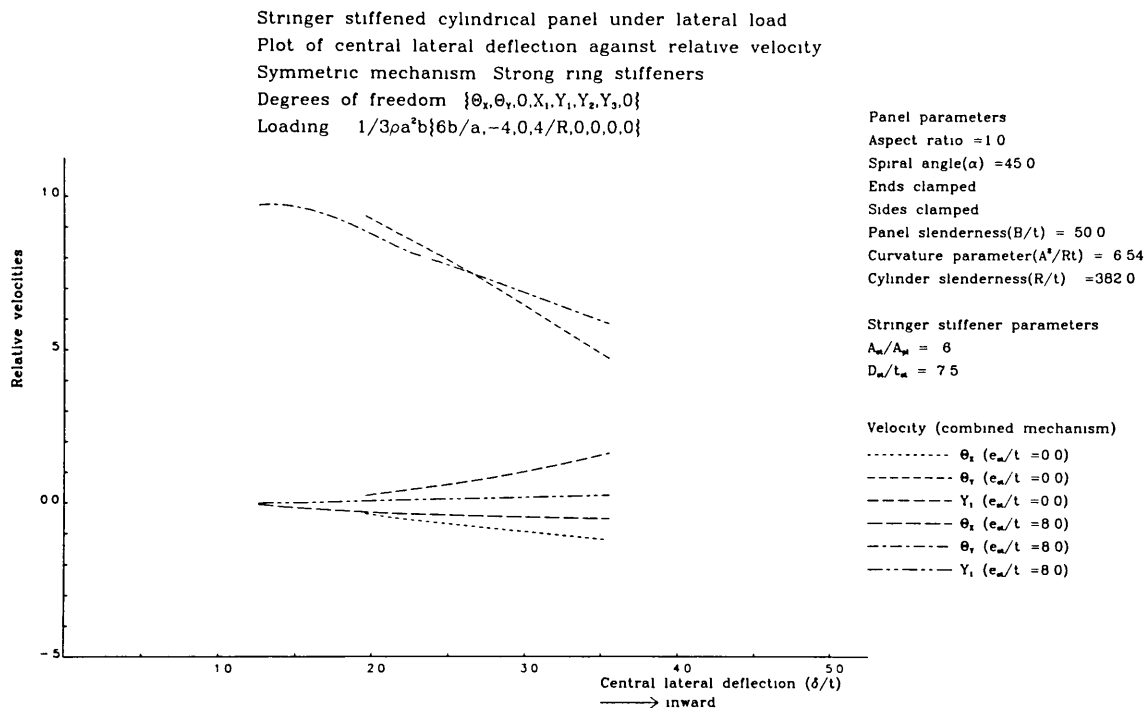


figure 8.36



upper figure 8.37

lower figure 8.38

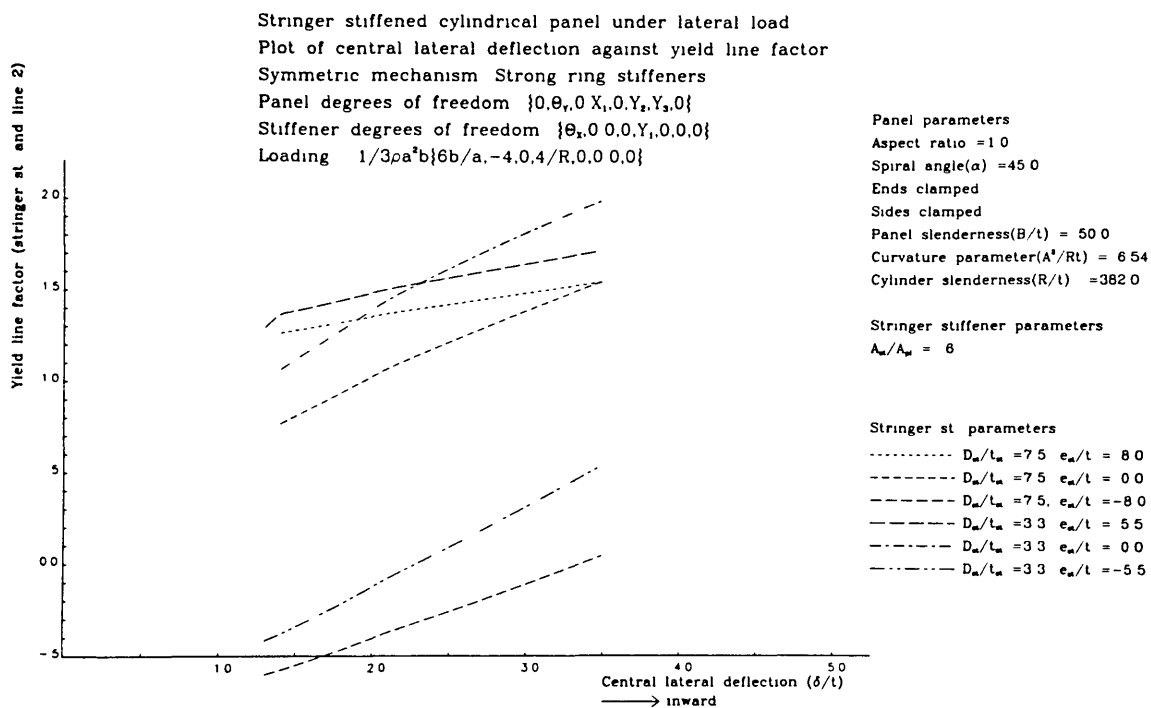
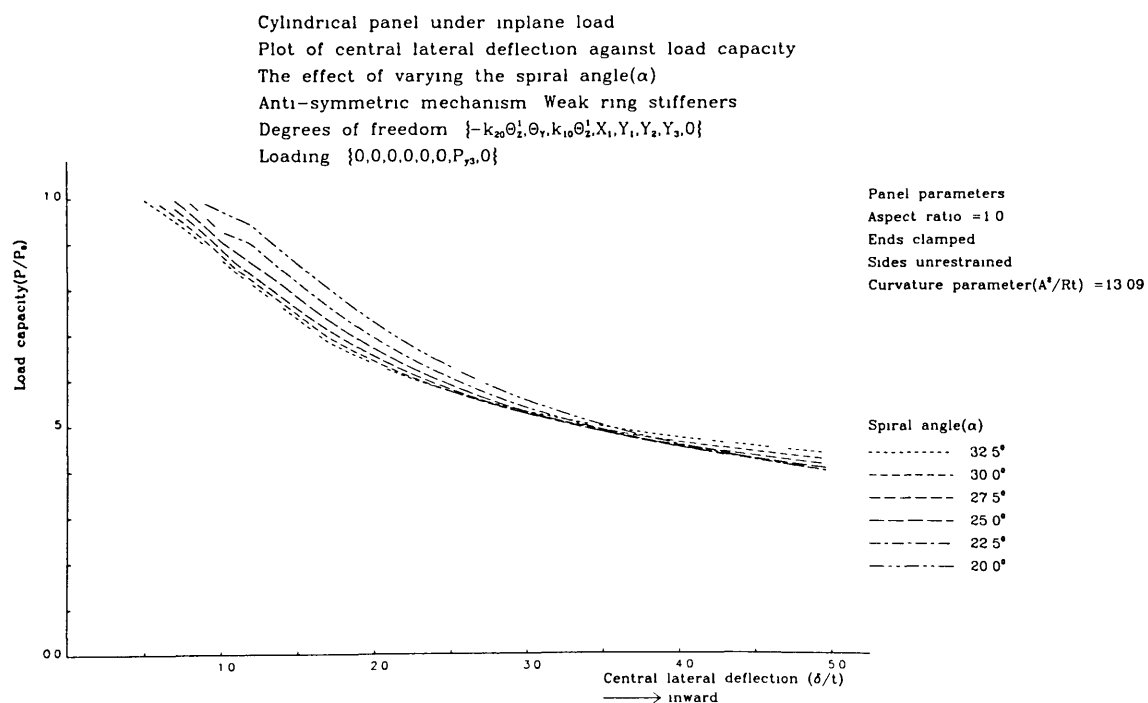
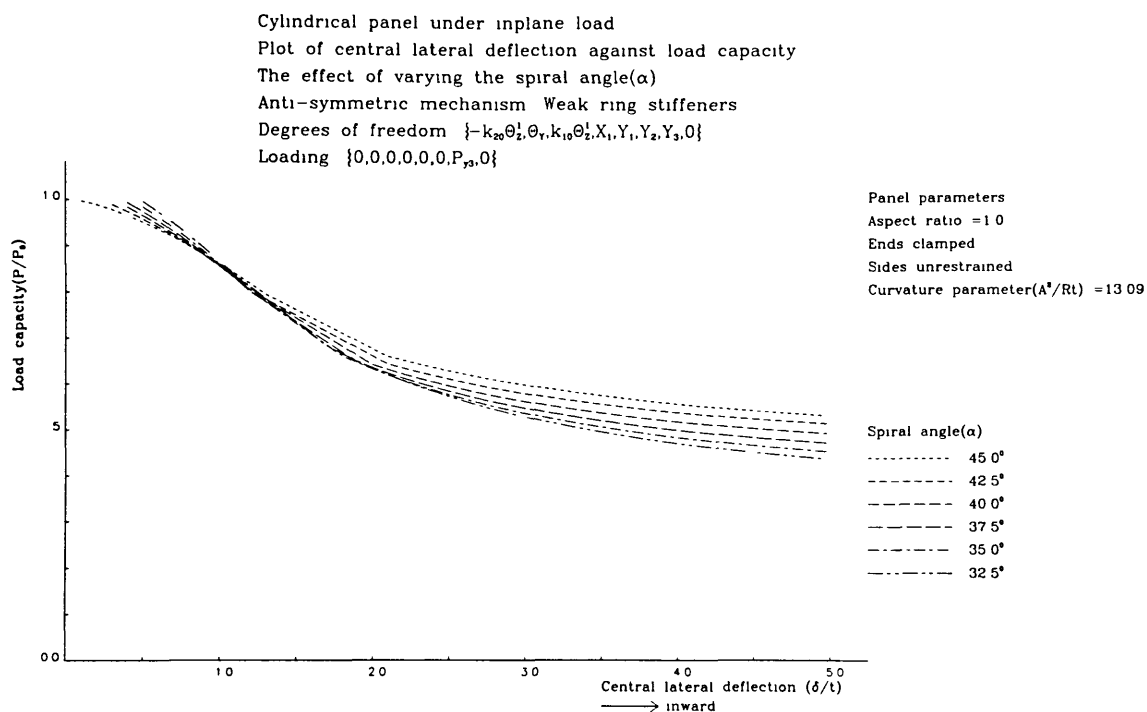


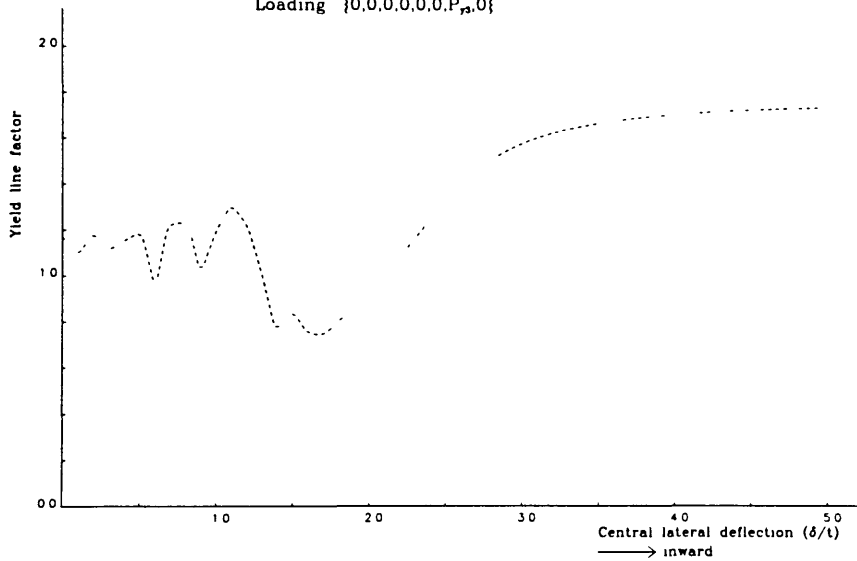
figure 8.39



upper figure 8.40

lower figure 8.41

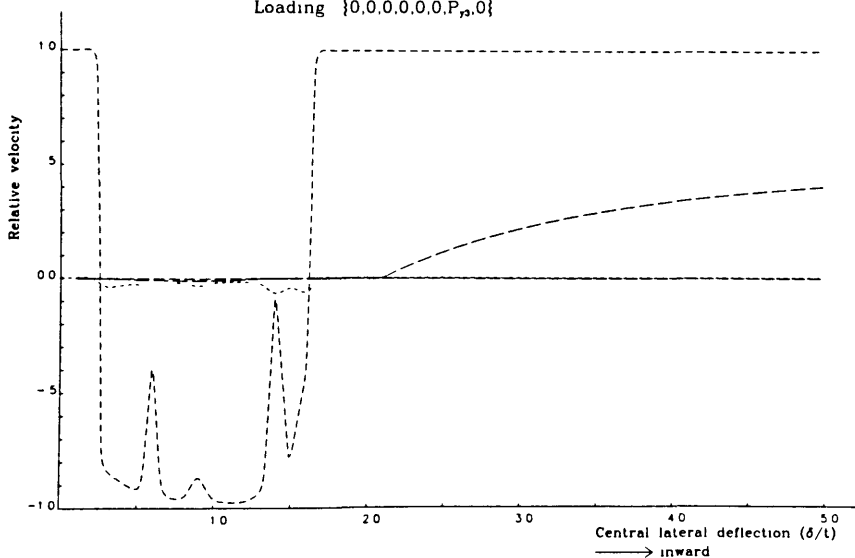
Cylindrical panel under inplane load
 Plot of central lateral deflection against yield line factor
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Degrees of freedom $\{-k_{20}\theta_2^1, 0, k_{10}\theta_2^1, 0, Y_1, 0, 0, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_r, 0\}$



Panel parameters
 Aspect ratio = 1.0
 Spiral angle(α) = 45.0
 Ends clamped
 Sides unrestrained
 Curvature parameter(A^2/Rt) = 13.09

----- Line 2 factor

Cylindrical panel under inplane load
 Plot of central lateral deflection against relative velocity
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{0, \theta_r, 0, X_1, 0, Y_2, Y_3, 0\}$
 Degrees of freedom $\{-k_{20}\theta_2^1, 0, k_{10}\theta_2^1, 0, Y_1, 0, 0, 0\}$
 Loading $\{0, 0, 0, 0, 0, P_r, 0\}$



Panel parameters
 Aspect ratio = 1.0
 Spiral angle(α) = 45.0
 Ends clamped
 Sides unrestrained
 Curvature parameter(A^2/Rt) = 13.09

Relative velocities

----- θ_2 (separate mech)
 -.-.-.- Y_1 (separate mech)
 ----- θ_2 (combined mech)
 -.-.-.- Y_1 (combined mech)

upper figure 8.42

lower figure 8.43

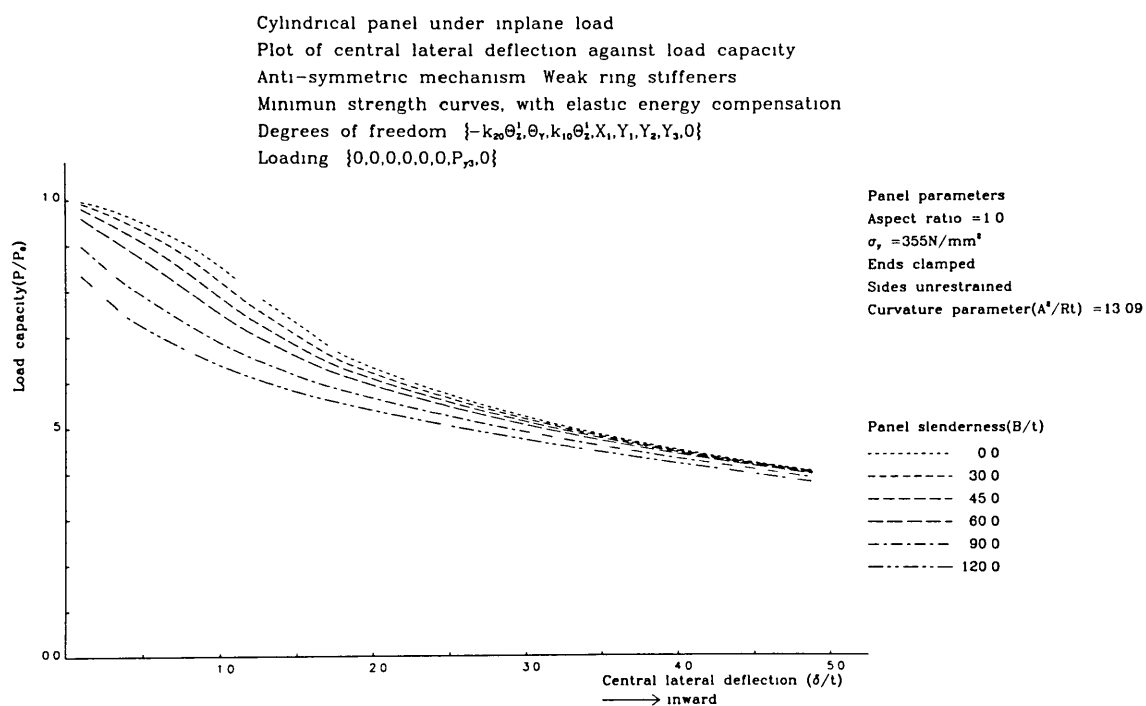


figure 8.44

Finite element idealisation of curved panel

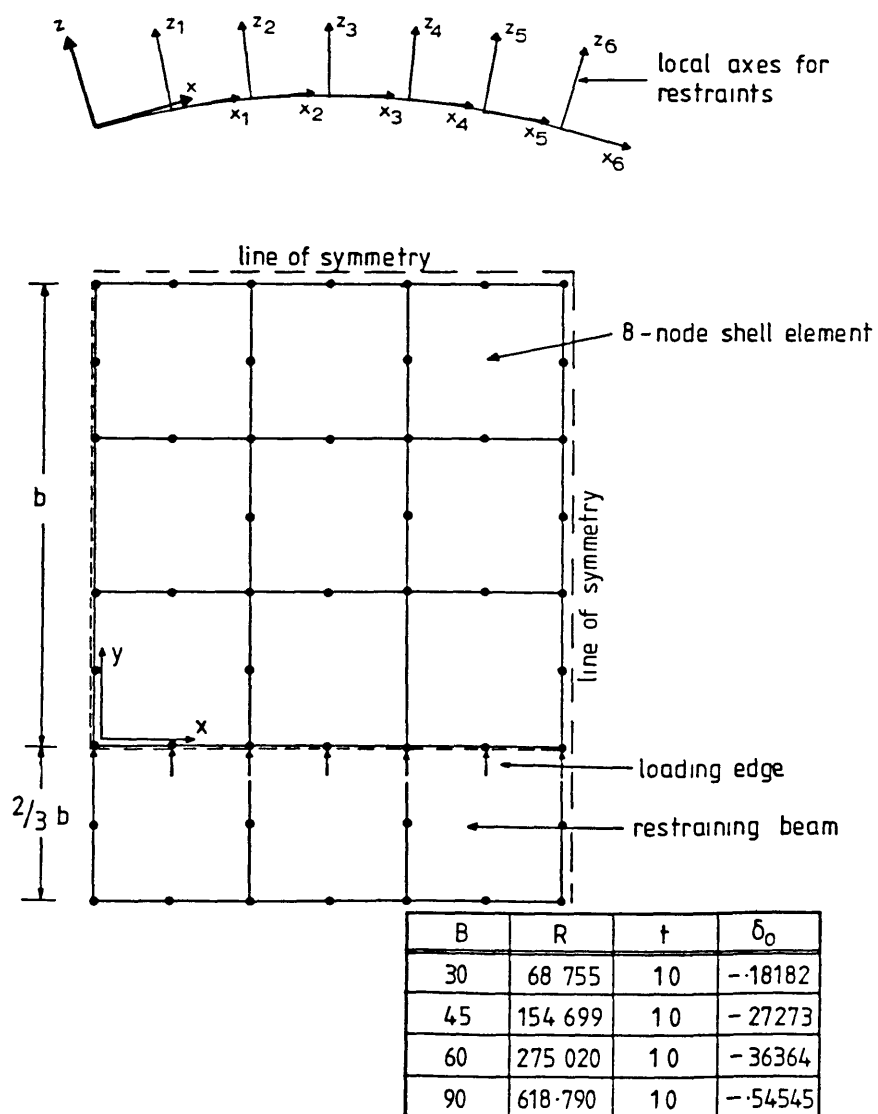


figure 8.45

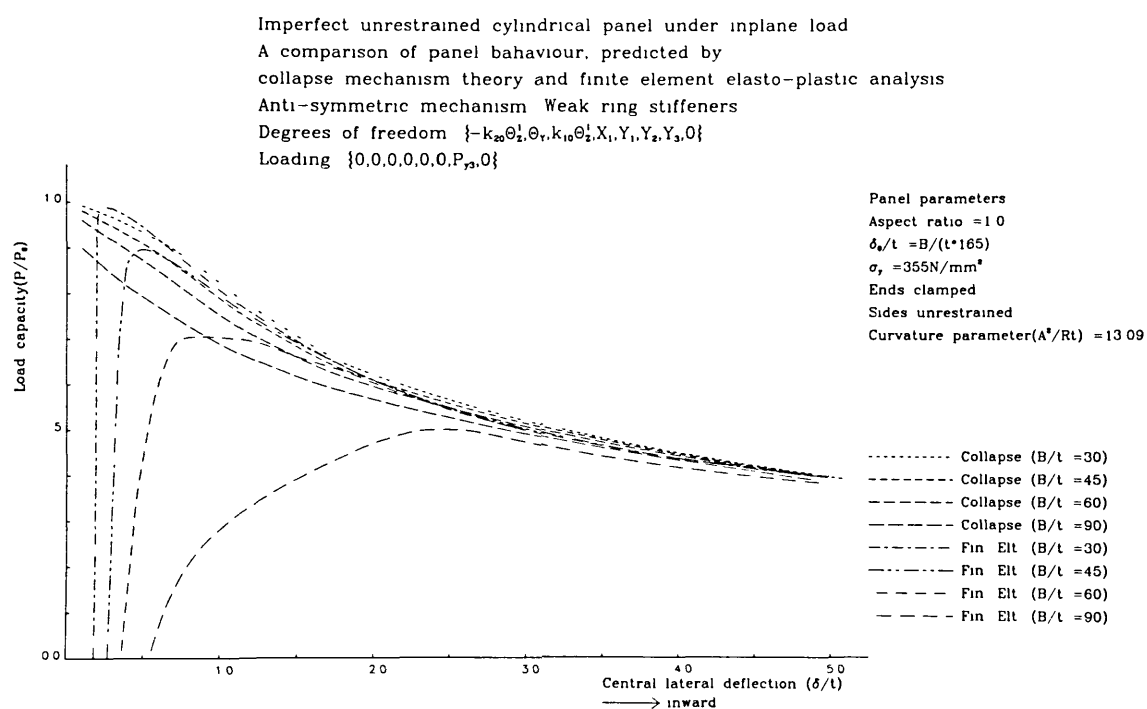
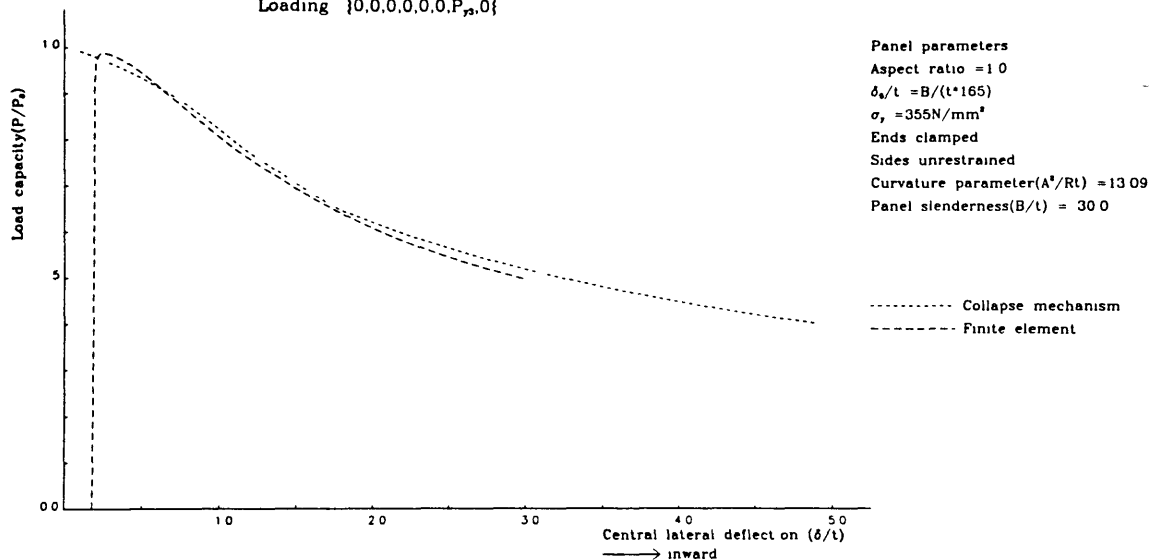
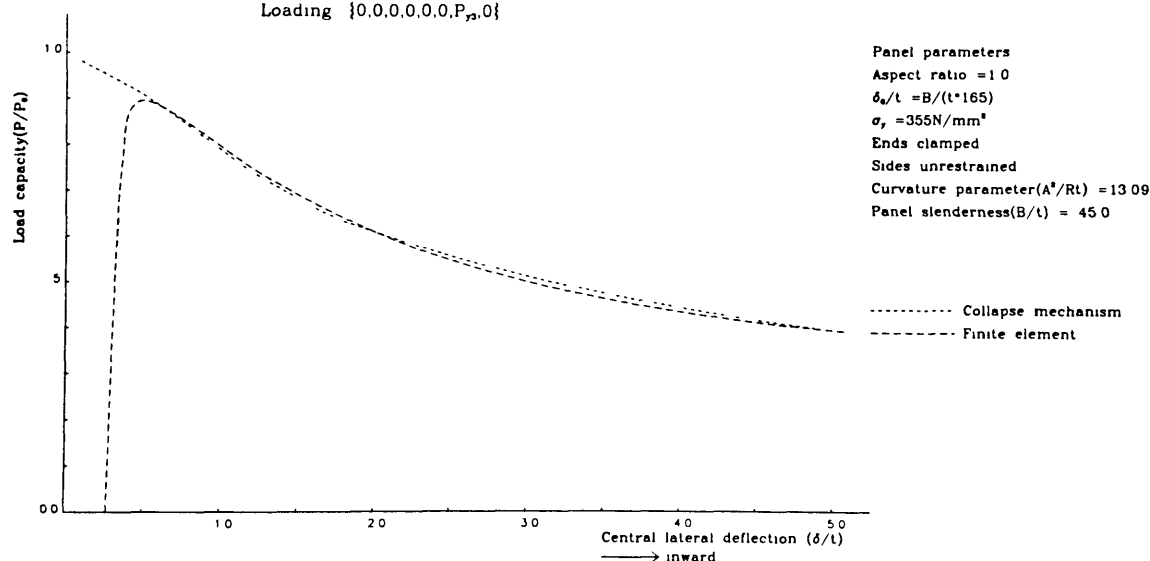


figure 8.46

Imperfect unrestrained cylindrical panel under inplane load
 A comparison of panel behaviour, predicted by
 collapse mechanism theory and finite element elasto-plastic analysis
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{-k_{\theta}\theta_1^i, \theta_7, k_{10}\theta_2^i, X_1, Y_1, Y_2, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_3, 0\}$



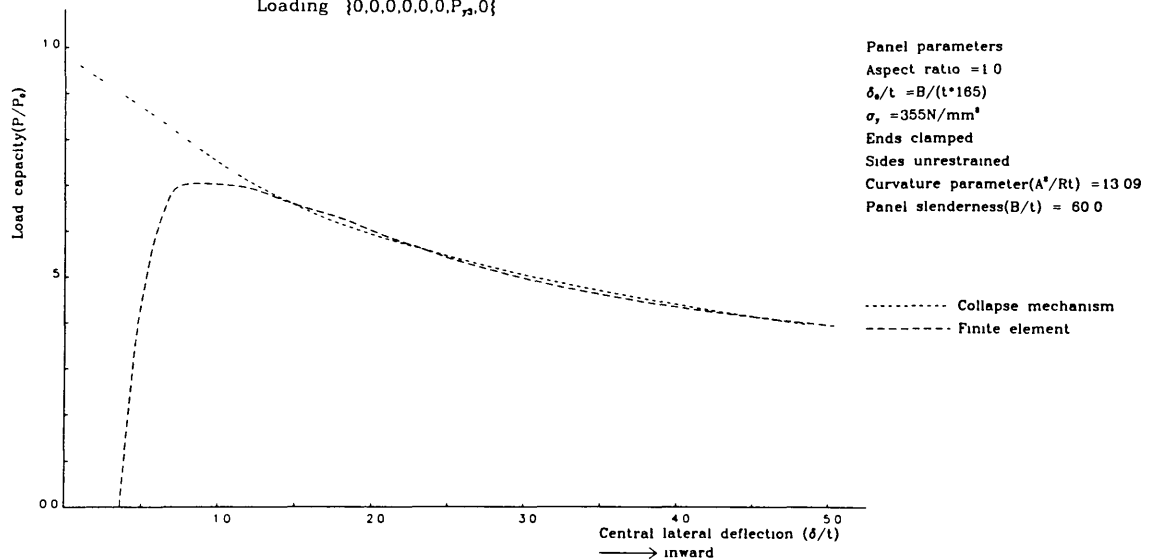
Imperfect unrestrained cylindrical panel under inplane load
 A comparison of panel behaviour, predicted by
 collapse mechanism theory and finite element elasto-plastic analysis
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{-k_{\theta}\theta_1^i, \theta_7, k_{10}\theta_2^i, X_1, Y_1, Y_2, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_3, 0\}$



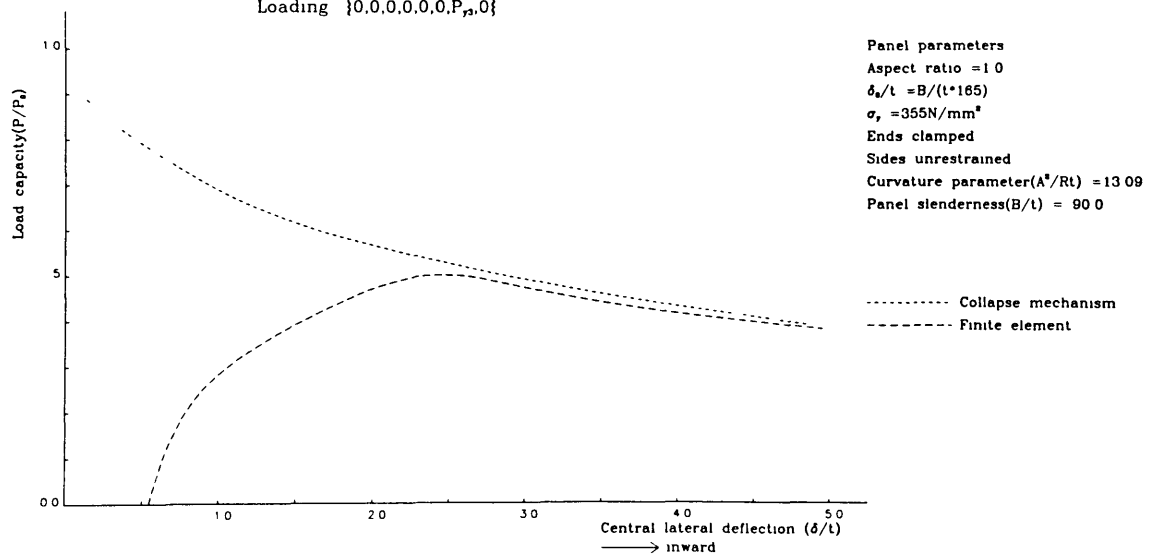
upper figure 8.47

lower figure 8.48

Imperfect unrestrained cylindrical panel under inplane load
 A comparison of panel behaviour, predicted by
 collapse mechanism theory and finite element elasto-plastic analysis
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{-k_{20}\theta_z^1, \theta_r, k_{10}\theta_z^1, X_1, Y_1, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_{21}, 0\}$



Imperfect unrestrained cylindrical panel under inplane load
 A comparison of panel behaviour, predicted by
 collapse mechanism theory and finite element elasto-plastic analysis
 Anti-symmetric mechanism Weak ring stiffeners
 Degrees of freedom $\{-k_{20}\theta_z^1, \theta_r, k_{10}\theta_z^1, X_1, Y_1, Y_2, Y_3, 0\}$
 Loading $\{0, 0, 0, 0, 0, 0, P_{21}, 0\}$



upper figure 8.49

lower figure 8.50

Degrees of Freedom						
translations				rotations		
freedom	u_i	u_j	u_k	α_i	α_j	α_k
Reference axes - Plane 1						
θ_x	formed by vector cross product $\bar{\theta} \times \bar{P}$			1	0	0
θ_y				0	1	0
θ_z				0	0	1
X_1	k_{10}	0	k_{20}	0	0	0
Y_1	0	1	0	0	0	0
Y_2	0	0	0	0	0	0
Y_3	0	0	0	0	0	0
movement of element A						
Reference axes - Plane 1						
θ_x	formed by vector cross product $\bar{\theta} \times \bar{P}$			$CC_{11}\xi_2$	0	$CC_{13}\xi_2$
θ_y				$CC_{11}\xi_3$	0	$CC_{13}\xi_3$
θ_z				$CC_{11}\xi_4$	0	$CC_{13}\xi_4$
X_1				$CC_{11}\xi_1$	0	$CC_{13}\xi_1$
Y_1	0	0	0	0	0	0
Y_2	0	1	0	0	0	0
Y_3	0	0	0	0	0	0
movement of element B						
Reference axes - Plane 2						
θ_x	formed by vector cross product $\bar{\theta} \times \bar{P}$			ξ_2	0	0
θ_y				ξ_3	0	0
θ_z				ξ_4	0	0
X_1				ξ_1	0	0
Y_1	0	0	0	0	0	0
Y_2	0	C_{22}	C_{32}	0	0	0
Y_3	0	0	0	0	0	0
movement of element B						
Reference axes - Plane 2						
y_3	0	C_{22}	C_{32}	0	0	0
movement of element C						
Reference axes - Plane 1						
x_3	k_{10}	0	k_{20}	0	0	0
movement of fictitious element						

Table 8.1

Mechanisms

Mechanism	Ring stiffener	active degrees of freedom							yield lines excluded
-----------	----------------	---------------------------	--	--	--	--	--	--	----------------------

panel only mechanisms

symmetric	strong	0	θ_Y	0	X_1	0	Y_2	Y_3	0	2
symmetric	weak	0	θ_Y	0	X_1	0	Y_2	Y_3	X_3	2
Anti-symmetric	strong	0	θ_Y	0	0	0	Y_2	Y_3	0	1,2
Anti-symmetric	weak	0	θ_Y	0	X_1	0	Y_2	Y_3	0	1,2

simple version of panel and stringer stiffener compound mechanism

compound symmetric	strong	$k_{10} \theta'_X$	θ_Y	$k_{20} \theta'_X$	X_1	Y_1	Y_2	Y_3	0	
approximation (when $k_{20} \ll 10$)		θ_X	θ_Y	0	X_1	Y_1	Y_2	Y_3	0	
reduces to (simple mechanism)		0	θ_Y	0	X_1	0	Y_2	Y_3	0	2, stiffener (2)

unrestrained isolated panel

compound		$-k_{20} \theta'_Z$	θ_Y	$k_{10} \theta'_Z$	X_1	Y_1	Y_2	Y_3	0	1
reduces to (simple mechanism)		0	θ_Y	0	X_1	0	Y_2	Y_3	0	1,2

Table 8 2 (a)

External Virtual Work

external virtual work components	external pressure			hydrostatic loading ***	longitudinal loading	
		symmetric mechanism	anti symmetric mechanism	extra component (clamped ends)	clamped ends	constrained ends
W_{θ_X}	$\rho a^2 b$	$2 k_{10} b/a$	$2 k_{10}^2 b/a$	0	0	0
W_{θ_Y}		$-\frac{2}{3} (3k - k^2)$	0	0	0	0
W_{θ_Z}		$-b/9R (9k - 8k^2)$	$2 k_{10} k_{20} b/a$	0	0	0
W_{X_1}		$\frac{1}{3R} (6k - 2k^2)$	$-4R/a^2 **$	0	0	0
W_{Y_1}		0	0	0	0	0
W_{Y_2}		0	0	0	0	$4at\sigma$
W_{Y_3}		0	0	$-2R/ab$	$4at\sigma$	0
W_{X_3}		$-4R/a^2 *$	0	0	0	0

* enters only for weak ring stiffeners

** enters only for weak ring stiffeners (radial expansion work component)

*** hydrostatic loading consists of lateral pressure plus work terms in this column

Note: variable ring stiffening is accommodated by taking a constant external work component of $-\lambda_{st} W_{X_3}$ (λ_{st} = load factor for strong ring stiffening for a particular lateral deflection)

Table 8 2 (b)

APPENDIX 8A1

Degrees of freedom of elements

The eight degrees of freedom, figure 8A1.1, of the panels are chosen to represent the rigid body movements of elements A, B, C in the simplest physical manner. These degrees of freedom define the system at collapse.

θ_x , θ_y , θ_z are rotations of the element A about the x, y, z axes of plane 1 respectively. θ_x and θ_z are mobilised when yield line 2 is active. θ_y is the main rotation associated with lateral deflection of the panel. X_1 is a translation of element A. X_1 has no component of movement parallel to $-k_{20} \hat{i}_1 + k_{10} \hat{k}_1$ since P_1 does not move laterally. Thus the simplest representation of X_1 is as $k_{10} X_1 \hat{i}_1 + k_{20} X_1 \hat{k}_1$

$$X_1 = X_1 \begin{bmatrix} k_{10} \\ 0 \\ k_{20} \end{bmatrix} \text{-----} (1)$$

Y_1 is a translation of element A, and is parallel to the j_1 axes, i.e. parallel to the cylinder generator. Y_2 and Y_3 are translations of elements B and C, respectively. They are simplest chosen parallel to the generator, i.e. parallel to the j_1 axis.

From equation 9, chapter 8

$$\left. \begin{aligned} Y_3(\text{plane 2}) &= (CC_{12} \hat{i}_2 + CC_{22} \hat{j}_2 + CC_{32} \hat{k}_2) Y_2 \\ Y_3(\text{plane 2}) &= (CC_{12} \hat{i}_2 + CC_{22} \hat{j}_2 + CC_{32} \hat{k}_2) Y_3 \end{aligned} \right\} \text{-----} (2)$$

and

$$\left. \begin{aligned} Y_2(\text{plane 1}) &= Y_2 \hat{j}_1 \\ Y_3(\text{plane 1}) &= Y_3 \hat{j}_1 \end{aligned} \right\} \text{-----} (3)$$

The degree of freedom, X_3 , represents the inplane component of the change of overall radius. It may be taken as the movement, parallel to $k_{10} \hat{i}_1 + k_{20} \hat{k}_1$, of a fictitious neighbouring element at line 1, when line 1 is active. When line 1 is not active, the inplane movement due to change of cylinder radius, may be represented by the freedom X_1 . In this case, however, no virtual work is evaluated along line 1.

Compatibility of lateral displacements

When X_1 is chosen as described above (equation 1), compatibility of inplane displacements, parallel to $k_{10} \hat{i}_1 + k_{20} \hat{k}_1$ is automatically satisfied at P_1 .

Compatibility of displacements at P_3 must be ensured for each of the 4 degrees of freedom $[X_1, \theta_X, \theta_Y, \theta_Z]$. If $\Gamma_2 = \Gamma_2 \hat{i}_2$, is the rotation (about the x-axis of plane 2) of element B, Γ_2 may be expressed as

$$\theta = \Gamma_2 \begin{bmatrix} CC_{11} \\ CC_{12} \\ CC_{13} \end{bmatrix} = \Gamma_2 \begin{bmatrix} CC_{11} \\ 0 \\ CC_{13} \end{bmatrix} \text{-----} (4)$$

($CC_{12} = 0$ by inspection)

The lateral displacement of P_3 on element B, due to $\theta = \Gamma_2$ is

$$\Gamma_2 \begin{bmatrix} CC_{11} & P_X(3) & i \\ 0 & P_Y(3) & j \\ CC_{13} & P_Z(3) & k \end{bmatrix} \det = \Gamma_2 \begin{bmatrix} -CC_{13} P_Y(3) \\ CC_{13} P_X(3) - CC_{11} P_Z(3) \\ C_{11} P_Y(3) \end{bmatrix}$$

The component of this displacement parallel to $k_2 \hat{i}_1 + k_1 \hat{k}_1$ is $\delta = \Gamma_2 P_Y(3) (CC_{11} k_1 - CC_{13} k_2)$.

The displacement of P_3 on element A is

$$\begin{bmatrix} \theta_Y P_Z(3) - \theta_Z P_Y(3) \\ \theta_Z P_X(3) - \theta_X P_Z(3) \\ \theta_X P_Y(3) - \theta_Y P_X(3) \end{bmatrix} + \begin{bmatrix} k_{10} X_1 \\ 0 \\ k_{20} X_1 \end{bmatrix}$$

The components Y_1 and Y_2 need not be considered as they do not lead to lateral displacements of P_3 . The component along $k_2 \hat{i}_1 + k_1 \hat{k}_1$ is

$$k_2 (k_{10} X_1 + P_Z(3) \theta_Y - P_Y(3) \theta_Z) + k_1 (k_{20} X_1 + P_Y(3) \theta_X + P_X(3) \theta_Y).$$

The lateral displacements of P_3 on elements A and B are equated.

Then if

$$\Gamma_2 = \xi_1 X_1 + \xi_2 \theta_X + \xi_3 \theta_Y + \xi_4 \theta_Z \text{-----} (5)$$

$$\left. \begin{aligned} \xi_1 &= (k_{10} k_2 + k_{20} k_1) / P_y(3) [CC_{11} k_1 - CC_{13} k_2] \\ \xi_2 &= k_1 / [CC_{11} k_1 - CC_{13} k_2] \\ \xi_3 &= (k_2 P_z(3) - k_1 P_x(3)) / P_y(3) [CC_{11} k_1 - CC_{13} k_2] \\ \xi_4 &= -k_2 / [CC_{11} k_1 - CC_{13} k_2] \end{aligned} \right\} \text{----- (6)}$$

Symmetry of displacements

A symmetry condition is used at lines 1, 2, 5 to determine the relative displacements at the gauss points along these lines. In all cases the total displacements of the gauss points must be evaluated, before the symmetry condition is imposed.

A local framework, t , is set up at line 1, for symmetry at that line. The x-axis of the t framework is parallel to $k_{10} \hat{i}_1 + k_{20} \hat{k}_1$

Then

$$t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} k_{10} & 0 & k_{20} \\ 0 & 1 & 0 \\ -k_{20} & 0 & k_{10} \end{bmatrix} \begin{bmatrix} i_1 \\ j_1 \\ k_1 \end{bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} i_2 \end{bmatrix}$$

Displacements and rotations are transposed to the t framework. The symmetry of displacements condition is that

$$t_1 \rightarrow -2t_1 \quad \text{and} \quad t_2, t_3 \rightarrow 0.$$

The rotational symmetry condition is that

$$t_1 \rightarrow 0, \quad t_2 \rightarrow -2t_2 \quad \text{and} \quad t_3 \rightarrow -2t_3.$$

(Displacements and rotations are referred relative to the points on yield line 1, thus the multiplier is -2). After symmetry has been applied, the components of displacement and rotation are referred to the i_1 framework the resulting symmetry conditions are expressed as

$$\bar{u}(\text{relative}) = -2 \begin{bmatrix} k_{10}k_{10} & 0 & k_{10}k_{20} \\ 0 & 0 & 0 \\ k_{10}k_{20} & 0 & k_{20}k_{20} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}_1 \text{----- (7)}$$

$$\bar{\alpha}(\text{relative}) = -2 \begin{bmatrix} k_{20}k_{20} & 0 & -k_{10}k_{20} \\ 0 & 1 & 0 \\ -k_{10}k_{20} & 0 & k_{10}k_{20} \end{bmatrix} \begin{bmatrix} \alpha_i \\ \alpha_j \\ \alpha_k \end{bmatrix}_1 \text{----- (8)}$$

where u_i is the translation of a point on line 1 and α_i is the rotation of element A (in the axes of plane 1).

The reflective symmetry for line 2 is expressed, by inspection, as

$$\bar{u}(\text{relative}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}_1 \text{-----} \quad (9)$$

$$\bar{\alpha}(\text{relative}) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_i \\ \alpha_j \\ \alpha_k \end{bmatrix}_1 \text{-----} \quad (10)$$

where u_i is the translation of a point on line 2 and α_i is the rotation of element A (in the axes of plane 1).

All translations and rotations are transferred to plane 1 to express the symmetry of line 5. (Rotational symmetry may be obtained by inspection).

Displacements are

$$u^1_i = CC_{ji} u^2_j$$

The symmetry conditions is that only $i = 2$ remains. The relative displacement is then

$$\delta = -2u^1_2 = -2CC_{j2} u^2_j$$

On conversion to the second plane

$$\delta = CC_{k2} (-2u^1_2) = -2CC_{k2} CC_{j2} u^2_j$$

Thus the relative displacements are ($CC_{12} = 0.0$)

$$\bar{u}(\text{relative}) = -2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & CC_{22}CC_{22} & CC_{22}CC_{32} \\ 0 & CC_{32}CC_{22} & CC_{33}CC_{33} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}_2 \text{--} \quad (11)$$

$$\alpha(\text{relative}) = -2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_j \\ \alpha_k \end{bmatrix} \frac{1}{2} \text{-----} (12)$$

where u_i is the translation of points on line 5 and α_i is the rotation of element B (in the axes of plane 2).

Weighting of gauss points

Gauss point positions are easily obtained on a spiral, since all rates of change are constant.

The length of a spiral, hence the length of a yield line is

$$L = R \left\{ 1 + \left[1/\tan\alpha \right]^2 \right\}^{\frac{1}{2}} \left\{ \theta_2 - \theta_1 \right\} \text{-----} (13)$$

where θ_2 and θ_1 define the angular positions of the end points of the spiral. α is the angle of the tangent to the spiral with the cylinder generator.

D'U_{ij} matrices of stiffeners

The method of obtaining D'U_{ij} matrices for points on curved yield lines has been outlined in §8.8. The relevant matrices for points on the stiffeners may be obtained by inspection. There are 2 points on each stiffener, corresponding with P₁ and P₂ respectively, figure 8A1.1. Let the stiffener be a single flat-bar plate stiffener. It may be distributed in any proportion on both sides of the shell mid-surface. Let the stiffener be treated as a discretely occurring beam, with an eccentricity, e_{st} , of the centre of stiffener area from the shell mid-surface, figure 8A1.2. The relative displacements at P₁ are

$$\begin{bmatrix} 0 \\ -\theta_z PS_{x(1)} + \theta_x PS_{z(1)} - Y_1 \\ 0 \end{bmatrix} = \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}$$

These are resolved along the local axes at P₁ to give

$$\begin{bmatrix} u \\ v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -PS_{z(1)} & 0 & PS_{x(1)} & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{10} & 0 & -k_{20} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ X_1 \\ Y_1 \\ Y_2 \\ Y_3 \\ X_3 \end{bmatrix}$$

$$PS_{(1)} = \begin{bmatrix} PS_{x(1)} \\ PS_{y(1)} \\ PS_{z(1)} \end{bmatrix} = e_{st} \begin{bmatrix} -k_{20} \\ 0 \\ k_{10} \end{bmatrix} \text{----- (14)}$$

where e_{st} is positive valued if the centre of area of the stiffener is external to the cylinder mid-surface.

Similarly

$$PS_{(2)} = \begin{bmatrix} PS_{x(2)} \\ PS_{y(2)} \\ PS_{z(2)} \end{bmatrix} = \begin{bmatrix} P_{x(2)} \\ P_{y(2)} \\ P_{z(2)} \end{bmatrix} + e_{st} \begin{bmatrix} -k_{20} \\ 0 \\ k_{10} \end{bmatrix} \text{----- (15)}$$

The relative displacement of the stiffener at P_2 is

$$\begin{bmatrix} u \\ v \\ \alpha_v \end{bmatrix} = 2 \begin{bmatrix} PS_{z(2)} & 0 & -PS_{x(2)} & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{10} & 0 & k_{20} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ X_1 \\ Y_1 \\ Y_2 \\ Y_3 \\ X_3 \end{bmatrix}$$

The thickness used for stress formulae (Table 7.1) is $t_1 = D_{st}$ (total depth of stiffener). The value weighting given to the stiffener positions is $A_{st} = D_{st} \cdot t_{st}$ (cross-sectional area of the stiffener).

The $D'U_{ij}$ matrices of a stringer stiffener as given above are the least elaborate treatment for stringer stiffeners at the edges of a panel at collapse. The stringers have been given no independent degrees of freedom. This may be considered satisfactory when the stiffener area, and hence the forces in the stiffener, are small by comparison with the panel area and forces.

The $D'U_{ij}$ matrices of the stiffener are exact when the stiffener can be assumed to act integrally with the panel. Variable stresses are accommodated through the stiffener depth, whilst ensuring compatibility of displacements, and hence strain, at the panel-stiffener junction.

Intermediate panel stiffeners may be treated in this way, by considering the incidence of the intermediate stiffener at a yield line as another gauss point. The thickness, eccentricity and weighting are particular to the stiffener and the relative displacements at the point are derived from the relative element displacements.

When the discretely occurring stiffener has an area which is a significant proportion of the total panel area, the above treatment of the stiffener may be inadequate. Independent degrees of freedom may be required for the stiffener. These independent degrees of freedom have not been implemented into the computer program, but will be considered for 2 specific cases.

Consider a symmetric panel mechanism. A hinge is formed at yield line 1 coincident with the stiffener position. The forces at the ends of the stiffener may differ significantly. Full inplane yield may occur in the stiffener at P_1 . However, the moment interaction at P_2 may result in a significantly different inplane force at P_2 .

If the area of the stiffener is significant, the difference in axial force along the stiffener cannot be ignored. The forces may be treated by means of a degree of freedom, Y_4 . An arbitrarily narrow band of panel acts integrally with the stiffener. The degrees of freedom of this stiffener are $\{\theta'_x, X_3, Y_4\}$, figure 8A1.3. $\theta'_x = k_{10} \theta_x + k_{20} \theta_z$. Yield line 1 is slightly displaced from the stiffener. The degrees of freedom of the panel are as before

$$\{\theta_x, \theta_y, \theta_z, X_1, Y_1\}$$

The displacements at line 1 are a result of the relative velocities

$$\{-\theta_y, -X_1, X_3, -Y_1, Y_4\}$$

Thus shear forces may develop between the stiffener and element A, and a redistribution of forces between panel and stiffener is possible

within the collapse mechanism theory.

The $D'U_{ij}$ matrices along line 1 have an additional term

$$\begin{bmatrix} u \\ v \\ w \\ \alpha_v \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 0 \end{bmatrix} Y_4$$

($v = -2Y_4$ to remain consistent with the symmetry condition imposed on the $D'U_{ij}$ matrices for line 1). The $D'U_{ij}$ matrices for the stiffener positions are altered by substituting the effect of Y_1 for Y_4 .

A twisting displacement occurs at P_1 in the stiffener when an anti-symmetric mechanism has developed. When significant rotations, θ_y , occur in the panel, a single yield line in the stiffener at P_1 is not adequate. Two yield lines, at an angle β to the z -axis of the panel, are assumed to form, figure 8A1.4. New gauss points must be considered on these lines. An extra degree of freedom, Z_4 , must be introduced for the stiffener. Y_4 is redundant, thus the inplane displacement of the stiffener is Y_1 . A number of symmetries may be utilised to express the $D'U_{ij}$ matrices for gauss points on both the stiffener yield lines at P_1 . The displacements of points on both lines are the same, so that only one sloping yield line need, in fact, be included in the summations for the internal virtual work.

The angle β , figure 8A1.4 is likely to vary with both stiffener proportions and with relative magnitudes of the velocities as well as with the angle of twist, θ_y . It is not convenient to vary β for each problem and since this mechanism has not been implemented no further guidance can be given. A short parametric study will reveal suitable values of β to be considered for problems. The $D'U_{ij}$ matrices for points on the stiffener yield line are

$$\begin{bmatrix} u \\ v \\ w \\ \alpha_v \end{bmatrix} = \begin{bmatrix} -z/(2\cos\beta) & \delta/(4\sin\beta) & \frac{1}{2}\cos\beta & -\frac{1}{2}\cos\beta & -\sin\beta \\ 0 & 0 & -\frac{1}{2}\sin\beta & \frac{1}{2}\sin\beta & -\cos\beta \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1/(2\sin\beta) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x' \\ \theta_y \\ Y_1 \\ Y_3 \\ Z_4 \end{bmatrix}$$

$$\theta_x' = k_{10} \theta_x + k_{20} \theta_z$$

where $\delta = \text{ABS } (z, \theta_y)$

and $z = z$ -ordinate of gauss point relative to the central plane of the panel

$$z = K_p \cdot \frac{1}{2} (t + (\xi_g + 1)D_{st})$$

ξ_g = gauss quadrature position weight

$K_p = +1$ for outside stiffener

$K_p = -1$ for inside stiffener

The thickness of the point (for force formulae) is $t_1 = t_{st}$

Total quadrature weighting for a point (taking 2 lines into account) is

$$\frac{D_{st}}{\cos \beta} \eta_g$$

η_g = gauss quadrature value weighting of the point.

Ring stiffeners

A ring stiffener is accommodated at line 4 with the following $D'U_{ij}$ matrix

$$\begin{bmatrix} u \\ v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ X_1 \\ Y_1 \\ Y_2 \\ Y_3 \\ X_3 \end{bmatrix}$$

The value weighting of the ring stiffener is A_R = cross-sectional area of the ring.

It has been noted above when an anti-symmetric panel mechanism develops that X_3 is absent and that X_1 represents the inplane component of the change of radius of the cylinder in the panel mechanism. The work terms associated with X_3 above, for the ring, will be associated with X_1 for an anti-symmetric panel mechanism.

External virtual work due to lateral pressure

A force is exerted due to pressure on element A.

force = $\frac{1}{2} \rho k a b$ acting at $k a / 6 R$ to the normal to plane 1.

The work done on element A due to θ_x is $1/3\rho kab^2\theta_x$

Total work done on triangles due to θ_x is

$$W_{\theta_x(1)} = 4/3 \rho kab^2 \cdot \theta_x \text{ ----- (16)}$$

Total work done on trapezoids due to unit rotation $\Gamma_z =$

$$W_{\Gamma_z} = 2/3 \rho ab^2(3 - 2k) \text{ ----- (17)}$$

The component of θ_x along 2nd x-axis is $\approx k_{10} \cdot \theta_x$

Thus total work is $W_{\theta_x} = 2/3\rho ab^2(3k_{10} + 2k(k_{10} - 1))$

To a good approximation $3k_{10} + 2k(k_{10}-1) \approx 3k_{10} - k \cdot k_{20}^2 \approx 3k_{10}$

$$\text{Thus } W_{\theta_x(\text{external})} = 2\rho ab^2 \cdot k_{10} \cdot \theta_x \text{ ----- (18)}$$

$$\text{The work done on element A due to } \theta_y \text{ is } -1/6\rho(ka)^2b \theta_y \text{ ---- (19)}$$

Total work done is $-2/3\rho(ka)^2b\theta_y$ on the triangles

The component of θ_y along 2nd x-axis is $-k_{10} P_{x(3)}/P_{y(3)} = -k_{10}\tan\alpha$

$$\text{Work} = -2/3\rho ab^2(3-2k) \cdot k_{10} \cdot ka/b = -2/3\rho ka^2b(3-2k) \text{ ----- (20)}$$

$$\text{Total work} = -2/3\rho^2ab\{3k-k^2\} \theta_y = W_{\theta_y(\text{external})} \text{ ----- (21)}$$

Similarly the external virtual work done to θ_z is

$$W_{\theta_z(\text{external})} = - \frac{\rho a^2 b^2}{R} \left\{ \frac{9k - 8k^2}{9} \right\} \cdot \theta_z \text{ ----- (22)}$$

and the virtual work due to X_1 is

$$\begin{aligned} W_{X_1(\text{external})} &= + \frac{\rho a^2 b}{R} \left\{ \frac{(6k - 4k^2)k_{10} - k^2k_{10} + 3k^2}{3} \right\} \\ &\approx \frac{\rho a^2 b}{R} \left\{ \frac{6k - 2k^2}{3} \right\} X_1 \text{ ----- (23)} \end{aligned}$$

The external virtual work due to X_3 is

$$W_{X_3}(\text{external}) = -4bR\rho = -\frac{\rho ab^2}{R} \cdot \left\{ \frac{4R^2}{a^2} \right\} X_3 \text{ ----- (24)}$$

The total external work due to pressure is

$$\rho a^2 b \begin{bmatrix} 2 k_{10} b/a \\ -2/3(3k-k^2) \\ -\frac{(9k-8k^2)}{9} \frac{b}{R} \\ (6k-2k^2)/3R \\ 0 \\ 0 \\ 0 \\ -4R/a^2 \end{bmatrix} = \begin{bmatrix} W_{\theta x} \\ W_{\theta y} \\ W_{\theta z} \\ W_{X_1} \\ W_{Y_1} \\ W_{Y_2} \\ W_{Y_3} \\ W_{X_3} \end{bmatrix}$$

This represents the forces along each of the degrees of freedom.

The external virtual work due to inplane compressive forces along Y_2 and Y_3 are computed, by inspection, as $4a\sigma_{Y_2}$, and $4a\sigma_{Y_3}$ respectively.

External virtual work along Y_1 is zero, since element A is not subject to external forces along the y-axis.

The treatment of an unrestrained panel excluding stiffener collapse

The lateral displacement of P_2 in the collapse of an unrestrained panel is zero, if stringer stiffeners are adequately strong. Thus there is no displacement of element A along $-k_{20} \hat{i}_1 + 0 \hat{j}_1 + k_{10} \hat{k}_1$. The inplane displacement $X_1 = k_{10} X_1 \hat{i}_1 + k_{20} X_1 \hat{k}_1$ satisfies this criterion. The rotations θ_x and θ_z must be in the proportion $\theta_x/\theta_z = -k_{20}/k_{10}$. Thus the rotational degree of freedom may be taken as $\theta' = (-k_{20} \hat{i}_1 + k_{10} \hat{k}_1) \theta'_z$. The velocity vector of an unrestrained panel is $\{-k_{20} \theta'_z, \theta_y, k_{10} \theta'_z, X_1, Y_1, Y_2, Y_3, 0\}$. When the internal virtual work is calculated the moment about $-k_{20} \hat{i}_1 + 0 \hat{j}_1 + k_{10} \hat{k}_1$ must be zero.

$W_{\theta x}$, $W_{\theta y}$, $W_{\theta z}$ are the moment terms of the virtual work. Thus $-k_{20} W_{\theta x} + k_{10} W_{\theta z}$ must equal zero at equilibrium.

The degree of freedom θ'_z is accommodated in practice by setting θ_z free and always setting $\theta_x = -k_{20}/k_{10} \theta_z$. The internal virtual work is calculated for θ_x , θ_y , θ_z separately. When the internal virtual work has been calculated at all points, set $W_{\theta z} = -k_{20} W_{\theta x} + k_{10} W_{\theta z}$ and $W_{\theta x} = 0$ for further considerations.

The absence of shear forces along line 2 causes ill-conditioning of the Jacobian §7.15.2. Line 2 must be considered as a beam with a single gauss point. Cross-sectional area of line 2 is $2Rt \sin^{-1}(k_{20})$.

The projected length of line 2 onto $k_{10} \hat{i}_1 + 0\hat{j}_1 + k_{20} \hat{k}_1$ is $k_{10} P_X(3)$. The thickness of the "beam" representing line 2 is $k_{10} P_X(3)$. The total weighting of line 2 (there are 2 such lines in the panel) is

$$Wt = \frac{4Rt \sin^{-1} k_{20}}{k_{10} P_X(3)} \approx \frac{2t}{k_{10}}$$

The gauss point of line 2 is situated at $\frac{1}{2} k_{10} P_X(3)$ so that a unit rotation θ'_z causes a relative displacement of $\frac{1}{2} k_{10} P_X(3) \theta'_z$. By symmetry

$$u = -k_{10} P_X(3) \left\{ -k_{20} \theta_x + k_{10} \theta_z \right\}$$

The $D'U_{ij}$ matrix for line 2 may be written as

$$\begin{bmatrix} k_{10} & k_{20} & P_X(3) & 0 & -k_{10}^2 & P_X(3) & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 k_{20} & 0 & -2 k_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with

$$t_1 = k_{10} P_X(3) \text{ and } Wt = \frac{4Rt \sin^{-1} k_{20}}{k_{10} P_X(3)}$$

The collapse of a stiffened cylinder by formation of an anti-symmetric panel mechanism

No hinge occurs at line 1.

The rotation degrees of freedom are

$$\theta' = (k_{10} \hat{i}_1 + 0 \hat{j}_1 + k_{20} \hat{k}_1) \theta'_x + \theta_y \hat{j}$$

The velocity set is

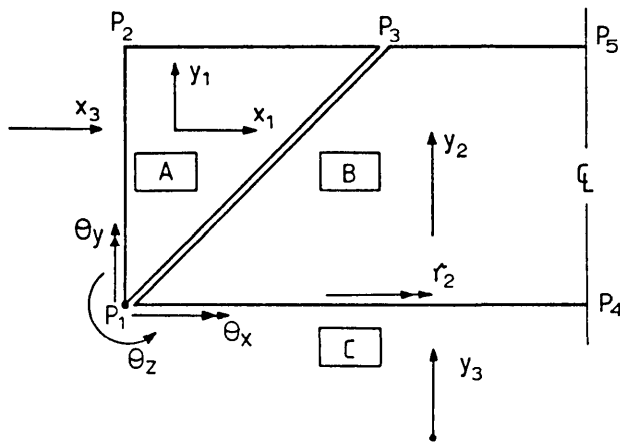
$$\left\{ k_{10} \dot{\theta}'_x, \dot{\theta}_y, k_{20} \dot{\theta}'_x, \dot{x}_1, \dot{y}_1, \dot{y}_2, \dot{y}_3, 0 \right\}$$

The virtual work for θ'_x is accommodated into the work equations by evaluating all the work terms independently and then setting

$$W_{\theta x} = W'_{\theta x} = k_{10} W_{\theta x} + k_{20} W_{\theta z} \quad \text{and} \quad W_{\theta z} = 0$$

for the iteration strategies.

The degrees of freedom of a curved panel at collapse



$\theta_x, \theta_y, \theta_z$ are rotations about the point P_1 of element A parallel to the x, y, z axes of element no 1 respectively

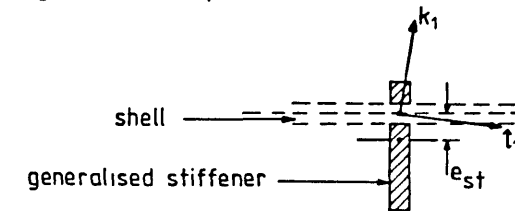
x_1 is parallel to the tangent to the circumference at P_1

y_1 is parallel to the y -axis of element A

y_2, y_3 are parallel to the y -axis of element A but represent translations of elements B and C respectively

Figure 8A1 1

Stringer stiffener positions



Normal stringer stiffener
area = $D_{st} \cdot t_{st} = A_{st}$
eccentricity = e_{st}

$$\text{Position} = e_{st} \begin{bmatrix} -k_{20} \\ 0 \\ k_{10} \end{bmatrix}$$

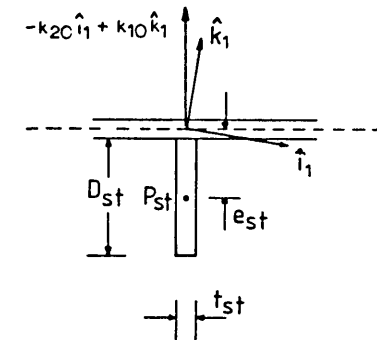


Figure 8A1 2

Independent degree of freedom of stiffener for
symmetric mechanism

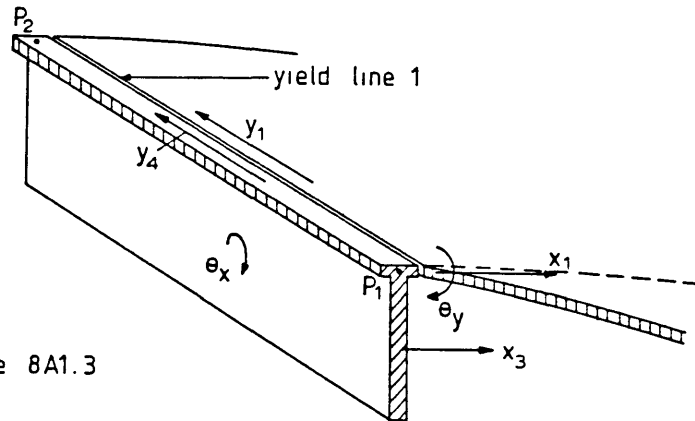
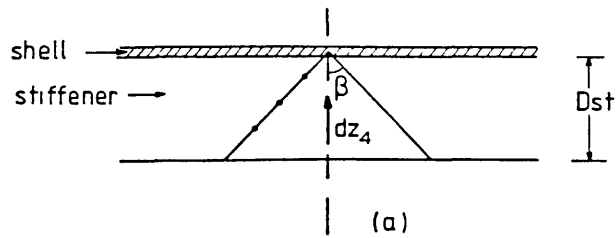
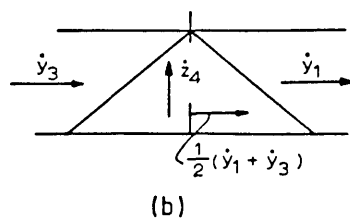


Figure 8A1.3

Independent degree of freedom of stiffener for
anti-symmetric mechanism



in plane displacements



rotations

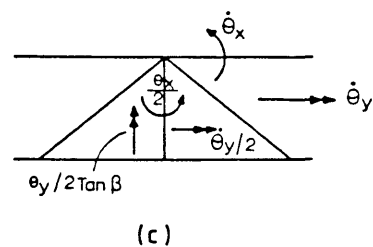


Figure 8A1.4

APPENDIX 8A2

This appendix is a first order assessment of the internal virtual work of a curved panel at collapse. The development is intended to identify the important work terms and the parameters influencing these terms.

It will be shown that the most important internal virtual work expressions are those of a flat panel and terms which are dependent on the non-dimensional curvature parameter, A^2/Rt . Other expressions, which are functions of t/R and $(A/R)^2$ also arise, but these will be shown to be very much smaller in magnitude.

The results of the virtual work expressions suggest a non-dimensional format for the presentation of numerical results in chapter 8.

The simplest degrees of freedom are considered for the panel, which is assumed supported on 4 sides and constrained or clamped at the supports. The panel is subject to inplane and/or lateral load. Figure 8A2.1 shows the plan of the panel. The degrees of freedom are $\{\theta_y, X_1, Y_2, Y_3\}$. Degrees of freedom $\{\theta_x, \theta_z, Y_1\}$ are suppressed. Yield line 2 does not exist.

P_3 has ordinates $P_3 = (ka, b, P_{Z(3)})$ (approximately) in the plane of element A (plane 1). $P_{Z(3)} = 0$, in the undeformed state. From figure 8A2.1(b)

$$\sin\beta = k_{20} \approx \frac{ka}{2R}$$

$$\text{and } \cos\beta = k_{10} \approx \left\{1 - \left[\frac{ka}{2R}\right]^2\right\}^{\frac{1}{2}}$$

For typical problems: -

$$2\pi R = 2aN_{st} \quad (N_{st} = \text{no. of stringer stiffeners of the cylinder})$$

$$a/2R = \pi/2N_{st} = k_{20}$$

A reasonable minimum number of stiffeners is 10.

$$\text{Then } k_{20} = .152$$

$$k_{10} = .987$$

This is an extreme case.

The following approximations will be used

$$k_{10} \approx 1.0 \text{ and } (k_{20})^2 \approx 0.0$$

$$k_1 = k_{10} \text{ and } k_2 = k_{20}$$

No reformation of the direction cosines of the gauss-point occurs.

The direction cosines of the gauss points on line 3 are (approximately)

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The z displacement of points on lines 2 and 3 are equal for points of the same x-ordinate.

$$\text{For line 2 } z_2 \approx \frac{x}{2R} (ka - x) ; \left[z_{\max} = \frac{(ka)^2}{8R} \right]$$

$$\text{For line 3 } z_3 = \frac{(ka)^2}{2R} (\epsilon - \epsilon^2)$$

where $\epsilon = x/ka$ and $0 \leq \epsilon \leq 1.0$

Line 1

The relative displacements for points on line 1 are

$$\begin{bmatrix} u \\ v \\ \alpha_y \end{bmatrix} = [D'U_{1j}] \begin{bmatrix} \theta_y \\ X_1 \\ Y_2 \\ Y_3 \end{bmatrix} \quad D'U_{1j} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix}$$

Line 3

The relative displacements of line 3 are not calculated by inspection.

The underformed state is considered initially.

θ_y and X_1 are degrees of freedom of element A which require rotations of element B for compatibility. The rotation of element B is

$$\Gamma_2 = \xi_1 X_1 + \xi_3 \theta_y$$

$$\xi_1 = \frac{k_{10} k_z + k_{z0} k_1}{P_{y(3)}(CC_{11}k_1 - CC_{13}k_z)} = \frac{2 k_{10} \cdot k_{z0}}{P_{y(3)}}$$

$$\xi_3 = \frac{k_z P_{z(3)} - k_1 P_{x(3)}}{P_{y(3)}(CC_{11}k_1 - CC_{13}k_z)} = \frac{k_{z0} P_{z(3)} - k_{10} P_{x(3)}}{P_{y(3)}}$$

(By the above assumptions $CC_{11}k_1 - CC_{13}k_z \approx k_{10}^2 + k_{z0}^2 = 1.0$)

Γ_z , transformed into the axes of element A, is

$$\Gamma_z = \begin{bmatrix} k_{10} \Gamma_z \\ 0 \\ -k_{z0} \Gamma_z \end{bmatrix} = \begin{bmatrix} 2k_{10}^2 k_{z0} / P_{y(3)} \\ 0 \\ -2k_{10} k_{z0}^2 / P_{y(3)} \end{bmatrix} X_1 + \begin{bmatrix} -k_{10}^2 P_{x(3)} / P_{y(3)} \\ 0 \\ k_{10} k_{z0} P_{x(3)} / P_{y(3)} \end{bmatrix} \theta_y$$

$$+ \begin{bmatrix} k_{10} k_{z0} P_{z(3)} / P_{y(3)} \\ 0 \\ -k_{z0}^2 P_{z(3)} / P_{y(3)} \end{bmatrix} \theta_y$$

Ignoring terms in k_{z0}^2 , and letting $k_{10} \approx 1.0$ and $\frac{P_{x(3)}}{P_{y(3)}} = \tan \alpha$

$$\Gamma_z \approx \begin{bmatrix} 2k_{z0} / P_{y(3)} \\ 0 \\ 0 \end{bmatrix} X_1 + \begin{bmatrix} -\tan \alpha \\ 0 \\ k_{z0} \tan \alpha \end{bmatrix} \theta_y + \begin{bmatrix} k_{z0} P_{z(3)} / P_{y(3)} \\ 0 \\ 0 \end{bmatrix} \theta_y$$

The third component of rotation is of order k_{z0}^2 and is small by comparison with the main component of θ_y . It will thus be disregarded.

Consider

$$\theta = \begin{bmatrix} 2k_{z0} / P_{y(3)} \\ 0 \\ 0 \end{bmatrix} X_1 + \begin{bmatrix} -\tan \alpha \\ -1.0 \\ k_{z0} \tan \alpha \end{bmatrix} \theta_y$$

as the relative rotation of the two elements to one another. This relative rotation occurs at line 3. If the ordinates of a point on line 3 are (x, y, z) the relative rotation causes a relative displacement

$$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} z \frac{2k_{z0}}{P_{y(3)}} X_1 + \begin{bmatrix} -z - k_{z0} y \tan \alpha \\ -z \tan \alpha + k_{z0} x \tan \alpha \\ x - y \tan \alpha \end{bmatrix} \theta_y$$

Setting $x/y \approx P_{x(3)} / P_{y(3)}$

$$\text{and } \eta = z + k_{z0} x = \frac{(ka)^2}{2R} (2\epsilon - \epsilon^2)$$

The inplane displacement is

$$\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} z \frac{2k_{z0}}{P_{y(3)}} X_1 + \begin{bmatrix} -P_{y(3)} \cdot \eta \\ P_{x(3)} \cdot \eta \\ 0 \end{bmatrix} \frac{1}{P_{y(3)}} \theta_y$$

These displacements, in terms of the local axes of line 3, are

$$\begin{bmatrix} \sin \alpha \\ -\cos \alpha \\ 0 \end{bmatrix} \cdot \frac{2z k_{z0}}{P_{y(3)}} X_1 + \begin{bmatrix} 1/\cos \alpha \cdot \eta \\ 0 \\ 0 \end{bmatrix} \theta_y$$

(A displacement along the z-axis of $(2y k_{z0}/P_{y(3)})X_1$ has been omitted, because this is cancelled when added to the direct lateral displacement due to X_1 . The relative lateral displacement due to X_1 is dealt with in §8.5.2.) Inplane displacements due to X_1 and Y_2 are

$$\begin{bmatrix} -\cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_2 \end{bmatrix}$$

The effects of a deformed state are summarised by considering the z-ordinate of all points on line 3 changed by $\epsilon P_{z(3)}$.

Then for line 3

$$\begin{bmatrix} u \\ v \\ \alpha_y \end{bmatrix} = \left\{ \frac{(ka)^2}{2R} (\epsilon - \epsilon^2) + \epsilon P_z \right\} \frac{2k_{z0}}{P_{y(3)}} \begin{bmatrix} 0 & \sin \alpha & 0 \\ 0 & -\cos \alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_y \\ X_1 \\ Y_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \\ -1/\cos \alpha & \sin \alpha & 0 \end{bmatrix} \frac{2k_{z0}}{P_{y(3)}} \begin{bmatrix} \theta_y \\ X_1 \\ Y_2 \end{bmatrix}$$

$$+ (\eta + \epsilon P_{z(3)}) \begin{bmatrix} -1/\cos \alpha & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_y \\ X_1 \\ Y_2 \end{bmatrix}$$

The first matrix, the inplane contribution due to the rotation caused by X_1 is small, because

$$\frac{2k_{z0}}{P_y(3)} \approx \frac{1}{2R}$$

The second matrix has the small rotation due to X_1 included, but is mainly the contribution due to an undisturbed flat panel.

The third matrix is the contribution due to lack of planarity of the system. The equivalent lateral displacement of points on line 3 is

$$\delta_z = \eta + \epsilon P_z(3) = \frac{(ka)^2}{2R} (2\epsilon - \epsilon^2) + \epsilon P_z(3)$$

Line 4

The relative rotation of line 4 in the plane of element B is

$$\begin{bmatrix} -2k_{10}k_{z0}/P_y(3) \\ 0 \\ 0 \end{bmatrix} X_1 + \begin{bmatrix} k_{10} \tan \alpha \\ 0 \\ 0 \end{bmatrix} \theta_y + \begin{bmatrix} -k_{z0} P_z(3)/P_y(3) \\ 0 \\ 0 \end{bmatrix} \theta_y$$

The third term is ignored because of size. Setting $k_{10} \approx 1.0$ gives a rotation of

$$\begin{bmatrix} -2k_{z0}/P_y(3) \\ 0 \\ 0 \end{bmatrix} X_1 + \begin{bmatrix} \tan \alpha \\ 0 \\ 0 \end{bmatrix} \theta_y$$

This results in an inplane displacement of

$$\frac{2k_{z0}}{P_y(3)} \begin{bmatrix} 0 \\ z \\ 0 \end{bmatrix} X_1 + \tan \alpha \begin{bmatrix} 0 \\ -z \\ 0 \end{bmatrix} \theta_y$$

(The y-ordinate of points on line 4 is very small)

The displacements of line 4 in terms of the local axes at the line are

$$\begin{bmatrix} u \\ v \\ \alpha_v \end{bmatrix} = \begin{bmatrix} z_4 \tan \alpha & -2k_{z0} \cdot z_4 / P_y(3) + 1.0 & -1.0 \\ 0 & 0 & 0 \\ \tan \alpha & -2k_{z0} / P_y(3) & 0 \end{bmatrix} \begin{bmatrix} \theta_y \\ X_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

We note that $-2k_{z0}/P_y(3)$ represents a small rotation by comparison with

the main component $\tan\alpha$. The rotation of line 4 is taken as $\tan\alpha \theta_y$.

The rotation

$$\frac{-2k_{20}}{P_y(z)} X_1 = -\frac{ka}{R} \frac{X_1}{P_y(z)} \approx -\frac{\tan\alpha}{R} X_1$$

will, however, be considered briefly at a later stage.

$$z_4 = \frac{2a^2}{R} \epsilon_4 (1 - \epsilon_4) \quad 0 \leq \epsilon_4 \leq 1.0$$

Line 5

The relative displacements along line 5 are

$$\begin{bmatrix} u \\ v \\ \alpha_v \end{bmatrix} = 2 \begin{bmatrix} -z_5 \tan\alpha & -2k_{20} \cdot z_5 / P_y(z) & -1.0 & 0 \\ 0 & 0 & 0 & 0 \\ -\tan\alpha & -2k_{20} / P_y(z) & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x \\ X_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

$$z_5 = \frac{a^2}{2R} (2 - \epsilon_5) + P_z(z); \quad 0 \leq \epsilon_5 \leq 1.0$$

The $D'U_{ij}$ matrices developed above give the local displacements and rotations of points on the yield lines to a first order of accuracy. It will be shown that the forces generated by these displacements correspond to those of a flat panel, modified by a non-dimensional curvature parameter, A^2/Rt .

Dean (218) has shown that the fundamental local displacement variables for stress computation are

$$U = \frac{4}{t} \cdot \frac{\dot{u}}{\alpha} \quad \text{and} \quad V = \frac{4}{t} \cdot \frac{\dot{v}}{\alpha}$$

Line 1

$$U = -4 \frac{\dot{X}_1}{t\dot{\theta}_y}; \quad V = 0.0$$

These variables are as occur in a flat panel

Line 3

$$\begin{aligned} \dot{u} &\approx -\delta_z / \cos\alpha \cdot \dot{\theta}_y - \cos\alpha \dot{X}_1 - \sin\alpha k \cdot \dot{Y}_2 \\ \dot{v} &\approx -\sin\alpha \dot{X}_1 + \cos\alpha \dot{Y}_2 \\ \dot{\alpha}_v &\approx -1/\cos\alpha \cdot \dot{\theta}_y \end{aligned}$$

$$\begin{aligned}\frac{4}{t} \cdot \frac{\dot{u}}{\dot{\alpha}} &= U \approx \frac{2(ka)^2}{Rt} (2\epsilon - \epsilon^2) + 4\epsilon \frac{P_z(z)}{t} + 4 \cos^2 \alpha \frac{\dot{X}_1}{t\dot{\theta}_y} \\ &\quad + 4 \sin \alpha \cos \alpha \frac{\dot{Y}_2}{t\dot{\theta}_y} \\ \frac{4}{t} \cdot \frac{\dot{v}}{\dot{\alpha}} &= 4 \sin \alpha \cos \alpha \frac{\dot{X}_1}{t\dot{\theta}_y} - 4 \cos^2 \alpha \frac{\dot{Y}_2}{t\dot{\theta}_y}\end{aligned}$$

Of these components

$$U_1 = 4\epsilon \frac{P_z(z)}{t} + 4 \cos^2 \alpha \frac{\dot{X}_1}{t\dot{\theta}_y} + 4 \sin \alpha \cos \alpha \frac{\dot{Y}_2}{t\dot{\theta}_y}$$

and

$$V = 4 \sin \alpha \cos \alpha \frac{\dot{X}_1}{t\dot{\theta}_y} - 4 \cos^2 \alpha \frac{\dot{Y}_2}{t\dot{\theta}_y}$$

are terms for a deformed flat panel.

The term

$$U_2 = \frac{2(ka)^2}{Rt} (2\epsilon - \epsilon^2) \text{ is a function of the non-dimensional}$$

geometric parameter A^2/Rt .

Line 4

The total rotation for line 4 is

$$\alpha = \tan \alpha \theta_y - 2k_{20}/P_y(z) X_1$$

Then $t\alpha = \tan \alpha (t\theta_y) - t/R \cdot \tan \alpha X_1$

The component, t/R , is not of the form A^2/Rt and so affects the results.

As noted above, however, its effect will be small by comparison with $\tan \alpha$.

$$U \approx \frac{4z_4}{t} - 4 \frac{z_4}{R} \frac{\dot{X}_1}{t\dot{\theta}_y} + \frac{4}{\tan \alpha} \frac{\dot{Y}_2}{t\dot{\theta}_y} - \frac{4}{\tan \alpha} \frac{\dot{Y}_3}{t\dot{\theta}_y}$$

$$\text{The term } U_1 = \frac{4}{\tan \alpha} \frac{\dot{Y}_2}{t\dot{\theta}_y} - \frac{4}{\tan \alpha} \frac{\dot{Y}_3}{t\dot{\theta}_y}$$

is that of a flat panel.

$$U_2 = \frac{4z_4}{t} \left[1 - \frac{t}{R} \frac{\dot{X}_1}{t\dot{\theta}_y} \right] \approx 4z_4/t$$

$$4z_4/t \approx \frac{8a^2}{Rt} \cdot \epsilon_4 (1 - \epsilon_4)$$

is a function of A^2/Rt .

Thus the approximation is correct when

$$\frac{t}{R} \frac{\dot{x}_1}{t\dot{\theta}_y} \ll 1.0$$

and the stresses are clearly a function of the flat panel parameters modified by the non-dimensional geometric parameter A^2/Rt .

Line 5

$$U_1 = \frac{4}{\tan \alpha} \frac{\dot{y}_2}{t\dot{\theta}_y} - P_z(s)$$

is the term for a deformed flat panel

$$U_2 = - \frac{2a^2}{Rt} (2 - \epsilon_s)$$

is a function of A^2/Rt .

In the first order of approximation only the non-dimensional parameter A^2/Rt alters the fundamental displacement parameter, and hence the stresses from those of a flat panel.

The stresses may be written as

$$n = n_i(f, c) = \text{axial stress}$$

$$s = s_i(f, c) = \text{shear stress}$$

$$m = m_i(f, c) = \text{moment}$$

where i refers to the yield line

f indicates the function is dependent on flat panel parameters

c indicates the function is dependent on A^2/Rt .

The stress resultants are (divided by yield stress σ_y)

$$N_i = t n_i$$

$$S_i = 1/\sqrt{3} t s_i$$

$$M_i = \frac{1}{4} t^2 m_i$$

Let L_i represent the length of the line i and η_j = gaussian quadrature

weighting of a point on a line. Then

$L_1 n_{1j} \eta_j$ = the sum of the axial contributions of line 1

(This is in accordance with the repeated subscript convention).

The stress resultants are used in conjunction with the $D'U_{ij}$ matrix to evaluate the internal virtual work along $[\theta_y, X_1, Y_2, Y_3]$.

The work contributions are

Line 1

$$2L_1 \begin{bmatrix} -\frac{1}{2}t^2 & m_{1j} & \eta_j \\ t & n_{1j} & \eta_j \\ 0 & & \\ 0 & & \end{bmatrix} = 2L_1 \eta_j t \begin{bmatrix} -\frac{1}{2}t & m_{1j} \\ n_{1j} \\ 0 \\ 0 \end{bmatrix}$$

Line 3

First contribution:-

$$L_3 \begin{bmatrix} -\frac{1}{2} & 1/\cos\alpha & t^2 & m_{3j} & \eta_j \\ -\cos\alpha & t & n_{3j} & \eta_j & -\sin\alpha & t/\sqrt{3} & s_{3j} & \eta_j & +\sin\alpha & \frac{2k_{20}}{P_y(3)} & \frac{t^2}{4} & m_{3j} & \eta_j \\ -\sin\alpha & t & n_{3j} & \eta_j & +\cos\alpha & t/\sqrt{3} & s_{3j} & \eta_j \\ 0 & & & & & & & \end{bmatrix}$$

$$=L_3 \eta_j t \begin{bmatrix} -\frac{1}{2}t & m_{3j} & \eta_j & 1/\cos\alpha \\ -\cos\alpha & n_{3j} & -\sin\alpha & 1/\sqrt{3} & s_{3j} & +\frac{1}{2}t \tan\alpha & \sin\alpha & \frac{t}{R} & m_{3j} \\ -\sin\alpha & n_{3j} & +1/\sqrt{3} & \cos\alpha & s_{3j} \\ 0 & & & \end{bmatrix}$$

Second contribution:-

$$\frac{L_3 t \eta_j}{\cos\alpha} \cdot \begin{bmatrix} -\delta_3 & n_{3j} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Line 4

Contribution:-

$$L_4 \begin{bmatrix} \tan \alpha \ z_4 \ t \ n_{4j} \ \eta_j + \tan \alpha \ t^2/4 \ m_{4j} \ \eta_j \\ -\tan \alpha \cdot t/R \ z_4 \ n_{4j} \ \eta_j - \tan \alpha \ 1/R \ t^2/4 \ m_{4j} \ \eta_j \\ t \ n_{4j} \ \eta_j \\ -t \ n_{4j} \ \eta_j \end{bmatrix}$$

$$= L_4 \ t \ \eta_j \begin{bmatrix} \tan \alpha \ z_4 \ n_{4j} + \tan \alpha \ t/4 \ m_{4j} \\ -\tan \alpha \cdot 1/R (z_4 \ n_{4j} + t/4 \ m_{4j}) \\ n_{4j} \\ -n_{4j} \end{bmatrix}$$

Line 5

Contribution:-

$$2L_5 \ t \ \eta_j \begin{bmatrix} -\tan \alpha \cdot z_5 \cdot n_{5j} - \tan \alpha \ t/4 \ m_{5j} \\ -\tan \alpha \cdot 1/R (z_5 \ n_{5j} + t/4 \ m_{5j}) \\ -n_{5j} \\ 0 \end{bmatrix}$$

The contributions are added to give the internal virtual work. For convenience, the terms L_i and η_j will be omitted. Thus, for example

$$t \cdot \tan \alpha \ z_4 \ N_4 \text{ replaces } L_4 \cdot t \cdot \tan \alpha \cdot z_4 \ n_{4j} \ \eta_j$$

The internal virtual work Θ_y :-

$$W\Theta_2/t^2 = -\frac{1}{2} M_1 - \frac{M_3}{4} \cdot \frac{1}{\cos \alpha} - \delta_3/t \cdot \frac{1}{\cos \alpha} \cdot N_3$$

$$+ \tan \alpha \left[\frac{z_4}{t} \cdot N_4 - \frac{z_5}{t} \cdot 2N_5 + \frac{1}{4} M_4 - \frac{M_5}{2} \right]$$

 X_1 :-

$$\frac{Wx_1}{t} = 2N_1 - N_3 \cos \alpha - \frac{S_3}{\sqrt{3}} \sin \alpha$$

$$+ \tan \alpha \cdot \frac{t}{R} \left[\frac{M_3}{4} \sin \alpha - \frac{z_4}{t} N_4 - \frac{M_4}{4} - \frac{2z_5}{t} N_5 - \frac{M_5}{2} \right]$$

$$= Wx_{11}/t + Wx_{12}/t$$

 Y_2 :-

$$\frac{Wy_2}{t} = -\sin \alpha \ N_3 + 1/\sqrt{3} \cos \alpha \ S_3 + N_4 - 2N_5$$

Y₃:-

$$\frac{W_{y3}}{t} = -N_4$$

The terms N_i , S_i , M_i are functions of the stress resultants on the yield lines, of the aspect ratio and are linear functions of the panel length B .

This may be written as

$$N_i = B \cdot f_{1i} (\gamma, \sigma_1(f, c)) = B \cdot f_{1i}$$

$$S_i = B \cdot f_{2i} (\gamma, \sigma_2(f, c)) = B \cdot f_{2i}$$

$$M_i = B \cdot f_{3i} (\gamma, \sigma_3(f, c)) = B \cdot f_{3i}$$

where σ_k is some function (stress)

γ = aspect ratio of the panel

i = line number

and f, c are used as above, and denote the geometric parameters which influence the stresses.

It is possible to write

$$\frac{W_{y3}}{Bt} = -f_{14}$$

$$\text{and } \frac{W_{y2}}{Bt} = -\sin \alpha f_{13} + \frac{\cos \alpha}{\sqrt{3}} f_{23} + f_{14} - 2 f_{15}$$

It is clear that $\frac{W_{y2}}{Bt}$ and $\frac{W_{y3}}{Bt}$ are functions of the aspect ratio and the stress state only.

$$\frac{W_{x11}}{Bt} = 2f_{11} - \cos \alpha \cdot f_{13} - \frac{1}{\sqrt{3}} \sin \alpha f_{23}$$

is a function of the aspect ratio and the stress state only.

$$\frac{W_{x12}}{Bt} = \tan \alpha \cdot \frac{t}{R} \left\{ \frac{\sin \alpha}{4} \cdot f_{33} - \frac{z_4}{t} f_{14} - \frac{1}{4} f_{34} - \frac{2z_5}{t} f_{15} - \frac{1}{2} f_{35} \right\}$$

is a function of the aspect ratio, the stress state and the parameters t/R , z_4/t , z_5/t .

z_4/t and z_5/t are functions of A^2/Rt and the central lateral displacement.

Typically $t/R < 1/50$ and $N_{St} \geq 10$

$$\text{Then } \frac{t}{R} \frac{z_4}{t} = \left[\frac{\pi}{N_{St}} \right]^2 2\epsilon_4(1 - \epsilon_4) < 0.02 \cdot 2\epsilon_4(1 - \epsilon_4)$$

$$\text{and } \frac{t}{R} \frac{z_5}{t} = \left[\frac{\pi}{N_{St}} \right]^2 \left[\frac{2 - \epsilon_5}{2} \right] + \frac{P_{Z(3)}}{R} \approx \frac{P_{Z(3)}}{R}$$

(The terms for line 5 will be significantly reduced because the length of line 5 will, in general, be small.)

$\frac{W_{X12}}{Bt}$ is judged to be a second order quantity.

It can be shown that $W_{\theta 2}/Bt^2$ is a function of the aspect ratio and the stress state on the yield lines.

It is concluded that the internal virtual work expressions are dependent on the flat panel dimensions and the curvature parameter A^2/Rt to a first order of accuracy.

External Virtual Work

The external virtual work for Y_2 and Y_3 is

$$W_{Y2} = 2\sigma_{Y2} \sigma_Y At = 2\gamma\sigma_{Y2} (Bt\sigma_Y)$$

$$W_{Y3} = 2\gamma \sigma_{Y3} (Bt\sigma_Y)$$

$$\text{Hence } \frac{W_{Y2}}{Bt \sigma_Y} = 2\gamma \sigma_{Y2} \text{ and } \frac{W_{Y3}}{Bt \sigma_Y} = 2\gamma \sigma_{Y3}$$

It is noted that the internal work is of the same form as these equations. Hence the non-dimensional axial capacity of the panel (σ_{Y2} , σ_{Y3}) is a function of the flat panel capacity and the curvature parameter A^2/Rt only.

The external work for X_1 is zero or very small for the longitudinal and lateral loading considered in chapter 8.

The total external work due to unit rotation θ_Y is

$$2/3 \rho a^2 b (3k - k^2)$$

where ρ is the external pressure (ρ is negative for compressive pressure on the outer cylinder surface). Then

$$\begin{aligned} \frac{W_{\theta y}(\text{ext})}{Bt^2 \sigma_y} &= \frac{1}{12} \frac{\rho \gamma^2}{\sigma_y} \left[\frac{B}{t} \right]^2 (3k - k^2) \\ &= \frac{\rho}{\sigma_y} \left[\frac{B}{t} \right]^2 \left\{ g(\gamma, k) \right\} \end{aligned}$$

where $g(\gamma, k)$ is a function of the aspect ratio and the value k only.

It has been shown that the internal work expressions

$$\frac{W_{\theta y}}{Bt^2 \sigma_y}, \quad \frac{W_{x1}}{Bt \sigma_y}, \quad \frac{W_{yz}}{Bt \sigma_y}, \quad \frac{W_{y3}}{Bt \sigma_y}$$

are functions of the non-dimensional parameters γ , k , A^2/Rt , $P_{Z(3)}/t$. The corresponding external work expressions are functions of σ_i/σ_y , γ , in the case of inplane stresses on yield line i , and a function of

$$\frac{\rho}{\sigma_y} \cdot \left[\frac{B}{t} \right]^2, \quad \gamma, \quad k,$$

in the case of external pressure p .

These external virtual work expressions are of the same form as the corresponding expressions for a flat panel. It is thus possible to generalise results (to a first order accuracy) obtained for any set of $\{A, B, R, t\}$ in terms of parameters A^2/Rt , γ , $P_{Z(3)}/t$ for panels subject to inplane stresses. It is possible to generalise the results for the strength of a curved panel under lateral load,

$$\rho^* = \frac{\rho}{\sigma_y} \left[\frac{B}{t} \right]^2$$

in terms of the strength of a flat panel multiplied by some function of the curvature parameter A^2/Rt for any constant aspect ratio.

CHAPTER 9.

A Recommended Design Procedure for Axially Loaded Stringer-Stiffened Cylinders.

9.1 General requirements of design of axially loaded cylinders

9.1.1 Member proportions

Cylinders subject to axial load and bending moment shall be designed as beam-columns. The local panel strength and the strength of the longitudinally stiffened panel cross section shall be calculated as given below, §9.2 and 9.3.

Ring stiffeners shall be used. Ring stiffeners shall not be subject to local plate instability. The orthotropic buckling stress of the ring stiffened cylinder shall exceed the buckling stress of the stringer stiffened cylinder spanning between rings.

Stringer stiffeners may be fabricated from built up sections, e.g. tee stiffeners, but shall not be subject to local plate instability.

These proposals consider stocky and slender stringer stiffeners separately. The limiting slenderness for stocky flat-bar stiffeners is

$$\frac{D_{st}}{t_{st}} \leq 0.416 \sqrt{\frac{E}{\sigma_y}} \text{ -----(1)}$$

$$\frac{D_{st}}{t_{st}} < 10 \text{ for } \sigma_y = 355 \text{ N/mm}^2$$

The panel width shall not be greater than B_s if slender stiffeners are used

$$\left[\frac{B_s^2}{Rt} \right]^{\frac{1}{2}} = 3.46 \text{ -----(2)}$$

9.1.2 Maximum axial stress

The stiffened cross section shall be designed to carry the maximum inplane stress calculated by simple bending theory.

$$\sigma_{\max} = \frac{P}{A} + M \frac{R}{I_O} \text{ -----(3)}$$

where A = the total cross sectional area

and I_O = second moment of area of the stiffened cross section.

P = axial load on the cylinder cross section

M = moment on the cylinder at a point along its length.

No allowance shall be made for possible redistribution of stresses after failure of the most heavily loaded region of the cross section.

The effect of residual stresses shall be considered by reducing the material yield stress by σ_R . The material design stress shall be taken as

$$\sigma_F = \sigma_y / \gamma_f - \sigma_R \text{ -----(4)}$$

γ_f = partial safety factor for material strength

σ_y = manufacturer's first yield or proof stress

σ_R = average residual stress in compression.

9.2 Strength of stiffened cross section with stocky stringer stiffeners

9.2.1 Local panel buckling

The elastic buckling stress of a panel is

$$\sigma_{CR1} = C_C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{B} \right)^2 \text{ -----(5)}$$

C_C is given conservatively in reference (68).

The beneficial effect of stiffeners on panel buckling may be considered by following the method given in §9.3.1.

9.2.2 Local failure of stiffened panel

Failure of the cross section may occur through local failure of a typical repeated stiffened panel unit. An effective width of panel, B_{ef} , shall be considered

$$B_{ef}/B = (1 + \sigma_f/\sigma_{CR1})^{-1/2} \text{ ----- (6)}$$

The strength of the stiffener and effective panel width is the strength of a column of length B_{el}

$$B_{el}/B = (4/C_C)^{1/2} \text{ ----- (7)}$$

9.2.3 Overall buckling of the stiffened cross section

The overall elastic buckling stress, σ_{CR2} , of the stiffened cylinder, between ring stiffeners, shall be calculated. An effective width of panel B_{ef} (equation 6) shall be used. The ends of the cylinder shall be considered simply supported. σ_{CR} may be calculated from reference (143) or (145).

9.2.4 Failure of the overall stiffened cross section

First yield of the overall cross section shall govern failure. The failure stress, σ_{f2} , may be calculated as

$$\sigma_{f2}/\sigma_f = \frac{1}{2}(f_1 + \xi + 1) - \{[\frac{1}{2}(f_1 + \xi - 1)]^2 - f_1\}^{1/2} \text{ ---- (8)}$$

where $f_1 = \sigma_{CR2}/\sigma_f$ and $\xi = \sigma_{CR2}/\sigma_f \cdot A_{eff} \cdot \delta_{ef}/z$

A_{eff} = effective area of the cross section per unit width

δ_{ef} = an effective overall imperfection at zero load

z = section modulus of the plate or stiffener outstand

9.3 Strength of stiffened cross section with slender stringer stiffeners

9.3.1 Local stiffened panel buckling

The elastic buckling stress, σ_{CR3} , of a typical panel and attached stiffener shall be calculated using an equivalent flat panel width B_{el} (equation 7). σ_{CR3} may be calculated from reference (228, Chap. 9).

9.3.2 Local failure of stiffened panel

Local failure of the stiffened plate may occur subsequent to buckling. An effective width of panel, B_{ef} , may be considered

$$B_{ef}/B = \{1 + \sigma_f/\sigma_{CR3}\}^{-1/2} = \phi \text{ -----(9)}$$

The stiffener area may be included when calculating the collapse load. The effective depth of the stiffener, should be reduced in a similar fashion as the effective panel width. The strength of the stiffened panel is the strength of a column of length B_{ef} (equation 7).

9.3.3 Overall buckling of the stiffened cross section

The overall elastic buckling stress, σ_{CR4} , of the stiffened cylinder between ring stiffeners shall be calculated. Reduced stiffener properties shall be used in addition to a reduced panel effective width, B_{ef} , when calculating the buckling stress.

$$\left. \begin{array}{l} A_{st}^* = \phi \cdot A_{st} \\ I_{st}^* = \phi \cdot I_{st} \\ B_{ef} = \phi \cdot B \end{array} \right\} \text{----- (10)}$$

where ϕ is given by equation 9.

9.3.4 Failure of the overall stiffened cross-section.

First yield of the overall cross section shall govern failure. The failure stress, σ_{f4} , may be calculated using equation 8, with σ_{CR4} replacing σ_{CR2} .

9.4. Discussion of design proposals

9.4.1 Comparison of stiffened cylinder behaviour with stiffened box behaviour.

The buckling of a longitudinally stiffened cylinder may be treated in an analogous fashion to the buckling of a longitudinally stiffened box section. An important difference exists, however, between the two types of

structure. The buckling stress of a longitudinally stiffened panel unit (stiffener and attached width of panel), which is part of a flat plate, is a good yet conservative estimate of the orthotropic buckling stress of the stiffened plate. This is not the case for a stiffened cylinder. The buckling stress of a stiffened panel, as a column spanning between rings, is a very conservative estimate of the buckling stress of the stiffener. When a stiffened plate fails locally in a cylinder, the relevant length is often much shorter than the distance between rings.

9.4.2 Local failure of a stiffened panel

Reference (68) gives the buckling coefficients for curved panels, based on experimental results. The strength of a curved panel, computed using reference (68), has been shown to correspond closely with panel strengths obtained with reasonable construction tolerances (204). In addition the strength of a group of stiffened panels has been shown to be close to the strength of an individual panel (107). Hence if a curved stiffened panel assembly is to fail through localised panel failure the appropriate effective width may be computed through equation 6 or 9.

A ridge-type mechanism forms in a panel at collapse. The length of this mechanism may be taken as the length of the half wave buckle in the longitudinal direction. Once a panel failure has been initiated a second mechanism may form, consisting of the stiffener and some effective width of panel. The second mechanism is a column mode of failure. The column may be considered simply supported and spanning the same length as the panel mechanism, i.e. B_{el} (equation 7). A reasonable effective width of panel is B_{eff} (equation 6).

This assessment of the combined stiffened panel collapse follows from the manner in which the combined mechanism strength was assessed in §8.10. In many instances the combined stiffener and effective width of panel will fail at the squash load of the unit. However, if the inertia of the unit is sufficiently low, lateral buckling of the unit will occur. As a consequence the overall failure stress of the stiffened panel will be reduced below the failure stress of the panel alone. The method is approximate and does not evaluate the strength of panels with internal stiffeners differently from the strength of panels with external stiffeners.

When slender stiffeners are used, collapse will be initiated by the stiffener and restraint will be given by the panel. The reduced strength of the stiffener under such circumstances has been considered in §9.3.2. It should be noted that very slender stiffeners are liable to local plate instability and will collapse independently of the panel. The stiffener load will, as a consequence, be shed onto the panel. Local plate instability of the stiffener has been excluded in §9.1.1. The width of panel which may be used with slender stiffeners has been limited, §9.1.1, so that the combined stiffener cum panel unit will be stable in the postbuckling regime.

9.4.3 Orthotropic buckling

It has been pointed out in the literature survey, §1.5.7, that the linearity of orthotropic buckling values is affected when individual panels are wide ($\theta > 0.64$). In addition local panel and overall orthotropic buckling modes may interact when their buckling stresses are close (186). Quantitative results are not available for local-overall buckling, for panels of $\theta \approx 1.0 - 2.5$.

An empirical approach to the treatment of local panel buckling within overall buckling has become accepted. The approach is reasonable; and there is, as yet, no alternative. An effective panel width is recommended in §9.2.3 and 9.3.3, and is based directly on the local panel strength. This effective width is a good estimate of the postbuckling effective width of panel.

It is clear from references (186, 182) that the effective width approach is questionable when very wide panels are considered. Nevertheless reference (185) suggests panel proportions as large as $B/t = 5\sqrt{R/t}$.

The error in orthotropic buckling stress as obtained through the use of reference (143) has been discussed in §4.2.3. The use of the reference has nonetheless been permitted for buckling stress calculations because in many cases the error will be acceptable.

9.4.4 Overall failure of a stiffened cylinder

The growth of lateral surface imperfections under load has been discussed in §1.6.3 and §3.3, and as a result a first yield failure criterion has been advocated on the basis of the growth of these imperfections. A number of other authors have advocated the same approach (184, 185, 111).

It has been suggested that orthotropic buckling theory is reasonably accurate if the number of buckling waves is less than $n/4$, where n is the number of stringer stiffeners (111). A more conservative approach would be to limit the direct application of orthotropic buckling theory to cases where $n_w \leq n/8$. Both suggested values of n_w are empirical rather than based on theory. The error in application of orthotropic theory when $n/4 \geq n_w \geq n/8$ has not been substantiated by experiment.

An appropriate initial imperfection must be specified. Several overall eigenvalues of a stiffened cylinder cluster around the lowest value. The eigenvectors of the same longitudinal wave number are orthogonal. Thus the initial imperfections associated with these modes will grow independently, but the modes clustered around the mode with the lowest eigenvalue will grow at a similar rate. The displacements of these modes will be additive in some locations on the circumference. Thus a reasonable worst imperfection, δ_{ef} , may be suggested as some combination of the individual mode components.

$$\delta_{ef} = \left\{ \delta_{k-1}^2 + \delta_k^2 + \delta_{k+1}^2 \right\}^{1/2} \text{ is suggested, where}$$

δ_k is the initial imperfection associated with the lowest eigenvalue and δ_{k-1} and δ_{k+1} are the initial imperfections associated with the neighbouring modes.

There is little information available on the imperfections of prototype structures. Attempts are being made to redress this situation. It will be possible in time to suggest more appropriate values of δ_{ef} which relate directly to actual construction forms in use.

9.4.5 Use of the proposed design recommendations

The proposed design method has been used to assess the strength of three experimental series of axially loaded cylinders (116, 117, 230). The results are summarised in the first two columns of table 9.1. The average axial stress at failure has been given for a local panel mode of failure and for an overall mode of failure of the cylinders between nominally simply supported ends. Local panel collapse is predicted to occur for five models, whilst overall buckling failure is predicted for the remaining thirteen models. All the models which are predicted to fail by the development of a panel mechanism were reported in experiments to fail in this manner, with the exception of model UC6 (117). This model failed in the experiment by initial panel buckling, but subsequently collapsed through overall buckling. A number of the models which are predicted to fail by overall buckling, failed by local panel collapse in the experiments. With the exception of model UC4, these models were either very short (hence local end effects exert a strong influence, (§ 6.3)) or had only 20 longitudinal stiffeners. The applicability of smeared orthotropic theory to cylinders with so few stiffeners must be seriously in doubt. The model UC4 had 30 stiffeners and failed by the development of local panel mechanisms. The proposed method estimates the overall buckling failure stress is slightly lower than the local panel failure stress for model UC4. The proposed method conservatively predicts the strength of all the cylinders of the experimental series, with the exception of the model GU3. This model was believed to have failed prematurely due to an under-strength weld. The proposed method is very conservative in predicting the strength of some wide panelled models. Nevertheless the method is not overly conservative for other models with wide panels. The disparity in strength between the predictions and experimental results for wide panels may be attributed to the differences in model fabrication, which resulted in near perfect cylinders in some instances.

It has been suggested that the strength of curved panels may be conservatively estimated by considering the panels to be flat (111). The overall buckling failure stresses of the models were computed using this hypothesis. The results are given in column 4 of table 9.1. In most cases the overall buckling strength using flat panel strength is slightly more conservative than the strength obtained using the curved panel strength. A major underestimation occurs when the panel curvature is

large, as in the case of model GU2. Reference (68) predicts the curved panel strength with an adequate margin of safety. It is unnecessary to be additionally conservative by assuming the panel as flat.

Reference (68) recommends that the overall strength of a stiffened cylinder may be estimated in a similar manner as stiffened flat panels, i.e. as a column spanning between rings. The strength of the cylinders has been calculated in this manner and the results are given in column 5 of table 9.1. It is clear that the method is unnecessarily conservative in the case of the longer cylinders of the test series.

CYLINDER NAME	1	2	3	4	5	FAILURE MODE
	LOCAL FAILURE	OVERALL FAILURE	OVERALL FAILURE 2	COLUMN FAILURE	EXPT.	
ICS 1	931	915	90	80	96	LOCAL
ICS 2	697	668	62	54	96	LOCAL
ICS 3	850	926	91	76	95	LOCAL
ICS B	930	889	87	55	99	LOCAL
ICS C	930	883	87	33	10	LOCAL
ICL 4	777	780	749	60	87	LOCAL
UC 1	70	646	58	52	82	LOCAL
UC 2	933	853	82	34	105	OVERALL
UC 3	668	689	63	51	76	LOCAL
UC 4	839	804	77	65	96	LOCAL
UC 5	864	911	88	74	104	OVERALL
UC 6	822	608	59	16	65	OVERALL
UC 7	832	809	78	51	86	OVERALL
UC 8	367	382	40	20	51	LOCAL
UC 9	644	623	62	34	66	OVERALL
GU 1	634	552	585	23	57	LOCAL
GU 2	817	623	313	31	89	LOCAL
GU 3	784	723	688	599	69	LOCAL

* Residual stresses have not been considered

Column 1 - Proposed method , local failure } Chap 9
 Column 2 - Proposed method , overall failure } Chap 9
 Column 3 - Overall failure , curved panel replaced by flat panel
 Column 4 - Column strength using BS 5400 (229)
 Column 5 Experimental results

Table 9.1 Assessment of Design Proposals

CHAPTER 10.

Conclusions of Research and Suggested Areas for Further Research.

10.1 Conclusions and further research

10.1.1 Conclusions

The object of the experimental programme was to set up a test rig and a system of model manufacture which would accurately model prototype structures at one twentieth scale. The method of model manufacture described in §2.2 proved successful. Imperfections were considerably less than those tolerated in prototypes because of the jiggling and stress relieving procedures adopted. Welds were scale modelled. The end conditions adopted, with model ends embedded in a dense araldite-sand mix, gave a satisfactory rotational restraint. Uniformity of axial load was ensured by projecting model ends through the mix and grinding the surface flat. Strain gauge readings indicated that rotational fixity was given to the models and that load was uniformly distributed around the circumference.

The method of applying axial load eccentrically through the base of three models was successful, as indicated by strain gauge readings at low stresses. At large axial stresses there was some drift in the centre of load application.

Uniaxially loaded models failed at a high proportion of squash load in a controlled manner. Eccentrically loaded models showed first failure at the same local stresses as uniaxially loaded models. There was, however, some redistribution of load at collapse of the first panels of the closely stiffened models. Local panel collapse mechanisms formed between stiffeners in all cases.

The response of the models was linear until just prior to collapse. An elastic finite element model was capable of predicting the collapse load reasonably accurately. In addition the elastic strains predicted by the finite element analyses of a perfect single repeated panel unit corresponded closely with those recorded during the experiments. Thus it may be concluded that the models tested behaved as perfect clamped elastic models until just prior to collapse.

Elasto-plastic analyses of the repeated panel units of the three geometries tested revealed that model no. 1 was quite insensitive to imperfections of either a panel or barrelling shape. Model no. 3 showed itself to be similarly insensitive to panel imperfections but barrelling imperfections affected strength. Model no. 2 was found to be sensitive to panel imperfections and the level of imperfection tolerated by the DnV code (68) reduced the perfect panel strength from $1.0 \sigma_y$ to $0.76 \sigma_y$. The imperfect elasto-plastic finite element analyses provided a satisfactory explanation for the differences in strength recorded by another set of experiments on nominally similar geometries.

The collapse mechanism theory is approached in a new manner in chapter 7. The development of the method is seen to closely parallel the development of a rigid-plastic finite element. The development falls short of setting up a rigid plastic finite element, because symmetries of panel loading are utilised to reduce the degrees of freedom used. The virtual work "equilibrium" is formulated as a single tensor equation in terms of the geometric variables or degrees of freedom. A satisfactory solution technique is employed in the solution of the non-linear virtual work equations. Factors affecting numerical accuracy are discussed and it is shown that the results obtained are not affected by the particular choice adopted for cut-off points, number of gauss points, and the size of residual work terms.

The new concept of a redundant yield line is introduced and conditions are laid down for the existence of redundant yield lines within the kinematically admissible solution set. The particular case of the inplane redundant yield line is discussed in chapter 7.

A first order of magnitude correction is introduced to allow for the release of elastic energy within panels under uniaxial load. This correction produces results where the generally accepted panel slenderness parameter

$$\frac{\sigma_y}{E} \left[\frac{B}{t} \right]^2$$

is seen to be the salient parameter affecting collapse mechanism results for a specific A/B ratio.

The collapse mechanism theory is illustrated in chapter 7 with examples of a two dimensional panel under either axial or lateral load. Two aspect ratios are considered. Results are compared to those obtained in reference (218). In addition comparisons are made between strengths predicted using the panel mechanism results in conjunction with known elastic loading behaviour of panels and results obtained from elasto-plastic finite element analyses of a panel. There is close agreement for square panels under uniaxial load; agreement is satisfactory for panels of 3:1 aspect ratio. The panel strengths produced in chapter 7 show that it is possible to use the collapse mechanism theory to generate general panel strengths for any level of initial imperfection. The collapse strength results are obtained for individual aspect ratios initially and later corrected to accommodate different panel slendernesses. As a consequence accurate parametric results can be generated at a small fraction of the cost of finite element parametric results.

The collapse mechanism theory is further developed for cylindrical stiffened panels in chapter 8. It is shown that the curvature parameter A^2/Rt is the factor which governs strength in addition to the parameters which affect flat panels. Thus only one extra parameter need be considered for curved panels over and above the flat panel parameters.

The treatment of redundant yield lines is extended and illustrated for the case of combined mechanisms developing in stringer stiffened panels. Results are produced for individual panels under uniaxial load and lateral pressure loading. It is found that the inward collapse mode produces more rapid unloading than outward collapse for panels under uniaxial load. This corroborates results of reference (107) derived by finite difference analysis of panels.

Panels subject to lateral load with strong ring stiffeners were found to possess minimum strengths which could be used as minimum strengths for design.

The analysis of curved panels subject to inplane and lateral load was not extended to produce design strengths, although these could be produced by more extensive analysis using the collapse mechanism method.

Comparisons between the unloading behaviour of curved panels under axial load as predicted by collapse mechanism theory and elasto-plastic finite element analysis proved very satisfactory for a panel of unit aspect ratio.

Ring stiffeners were considered. Rings were shown to have little effect on the behaviour of uniaxially loaded panels. However, ring stiffeners had a major effect on the behaviour of stiffened panels under lateral pressure. A criterion of adequacy was derived for ring stiffeners.

General results could not be produced for stringer stiffened panels. However, it was shown that the area of the stiffeners and stiffener eccentricity had a considerable effect on stiffener adequacy.

The analyses carried out in chapter 3 showed that the measurements of the surface imperfections were too crude to be able to satisfactorily predict buckling loads from the growth of initial lateral deflections of the surface.

Orthotropic buckling analyses conducted in chapter 4 for axially loaded models could not be corroborated by eigenvalue analyses of a cylinder portion using finite element idealisations. The eigenvalue analyses showed the panel mode of buckling to be critical for all geometries tested in the experiments. A semi-empirical method was developed to predict the stiffened panel buckling stress of relatively narrow panels and slender stiffeners.

The beam on elastic foundation model discussed in chapter 6 successfully explained the elastic stress distributions within the narrow stiffened panels under axial load. In addition it was possible to use the model to define a failure criterion of first yield at midheight for narrow panelled cylinders. This criterion was conservative in comparison to the experimental results of two of the models tested, because these models failed only after considerable plastic strains had occurred. It was possible to use the model to predict the length of the collapse mechanism within a panel with widely spaced stiffeners, or with no stringer stiffeners.

It was possible to suggest a design method for axially loaded cylinders.

This design method conservatively predicted the failure strength of most of the models tested within the United Kingdom. The method, however, falls short of using the collapse mechanism results because these were not sufficiently developed within the thesis to be suitable for design.

10.1.2 Future research

The cylinder proportions adopted in this experimental series were stocky and failure occurred very close to full material yield stress. There is a need for further experiments on axially loaded stiffened cylinders where failure occurs at a lower proportion of the full yield stress. In particular panel failure should be studied. Two parameters should be varied. The panel curvature, A^2/Rt , has been identified as the relevant panel parameter for any specific panel slenderness, B/t . These results will serve to confirm the collapse mechanism results obtained through the application of the theory in chapter 8. In practice a number of tests may serve to confirm comparisons made between elasto-plastic finite element analyses and collapse mechanism results.

Two further parameters affect the collapse load of a cylinder under axial load. Stocky stiffeners have been used in the experiments described in this thesis, and in addition the proportion of stiffener area to plate area was large.

Experiments must be carried out to determine the level of stiffening required to prevent overall buckling. Clearly this will be dependent on the relative values of elastic local and overall buckling stresses. Some guidance has been provided in reference (114) but this should be further clarified. The effect of practical discrete stiffening on the elastic orthotropic buckling stress could also be investigated during such experiments.

It has been suggested (184), that long stringer stiffened cylinders may be imperfection sensitive. Work has already been carried out at the Technion on the elastic buckling of stringer stiffened cylinders into more than one halfwave along the axial direction. This work should be extended to elasto-plastic buckling, and a limit obtained to the spacing of ring stiffeners required to ensure single halfwave buckling between rings.

The collapse mechanism theory has been advanced to a stage where it can be used as an analytical tool for quite general plated structures problems. The computer program developed for this thesis can be used to solve a number of curved panel strength problems.

Little extra computational effort is required to generate panel strengths for panels subject to inplane loading. A range of panel curvatures and aspect ratios should be covered. (Before these results become fully useful, however, elastic panel behaviour should be defined for curved panels, along the lines published by Williams *et al.* (239) for flat panels. This work could be carried out by a short parametric study using elastic geometrically nonlinear finite elements.) Such inplane strength curves enable the assessment of panel strengths when fabrication tolerances fail to meet the criteria laid down in codes of practice. In addition the strength of panels with very large initial imperfections e.g. damaged panels, can be easily calculated.

The minimum strength of panels subject to lateral loading may also be generated. Such minimum loads are, however, of little value except as a means of preventing excessive lateral deflection.

Strengths of panels subject to combinations of inplane load may be calculated by means of the collapse mechanism program developed in this thesis.

The treatment of stringers has been brief in this thesis. There is, however, scope within the collapse mechanism package for a study of the integrity of stringer stiffeners in cylinders subject to combinations of lateral and axial load. Little development work is required to enable the package to handle local tripping of stiffeners.

The collapse mechanisms considered in this thesis were limited to panels with straight edges in plan. This need not be a limitation on the collapse mechanism method. Experiments show that when cylinders fail by overall collapse oval buckles develop initially (242). These oval buckles become, in the limit, the classical diamond pattern of unstiffened cylinders. A particular collapse element may be developed to consider such a collapse state. This element would function within the present collapse mechanism package.

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