

Maximum Likelihood Frequency Estimation of Unbalanced Three-Phase Power Signals

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Abstract

Accurate detection of the system frequency in unbalanced three-phase power systems is a prerequisite for the operation of future smart grid. To this end, we introduce the Cramer-Rao Lower Bounds (CRLBs) for frequency estimation, based on the $\alpha\beta$ -transformed unbalanced voltage contaminated with noise. Next, the maximum likelihood estimation (MLE) method for frequency estimation is introduced as a maximiser of an “augmented periodogram”. The use of the augmented complex statistics caters for all the available second order information, including the noncircularity associated with unbalanced systems. To find the ML solution, Newton’s iterative method is employed and its initialisation is implemented by a discrete Fourier transform (DFT) based dichotomous search technique. We show that the MLE of phases and amplitudes of both the positive and negative phase-sequence components within the $\alpha\beta$ -transformed voltage can be generically derived based on the ML frequency estimates. In this way, a unified framework is provided to accurately detect voltage characteristics of the positive and negative phase-sequence components within unbalanced three-phase power system when its frequency experiences off-nominal conditions. Simulations verify that the proposed MLE approaches theoretical Cramer-Rao Lower Bounds (CRLBs) for all parameters under consideration.

Index Terms

Unbalanced three-phase power signals, frequency estimation, phase-sequence estimation, maximum likelihood estimation (MLE), augmented periodogram

I. INTRODUCTION

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POWER system frequency is a key parameter used in the monitoring, protection and control of power grid, as its deviation from the nominal value reflects a mismatch between the demand and generation, generator trips, or load shedding. Power system frequency estimation from single-phase measurements is well established, including zero-crossing techniques [1], [2], least squares methods [3], Kalman filters [4], [5], recursive estimators [6], [7], as well as discrete Fourier transform (DFT) based approaches [8]–[11]. Recent methods are also robust against noise and harmonic contamination, however, a single phase signal does not sufficiently reflect system frequency, since six such single-phase voltages exist in a three-phase system (if line-to-line voltages are also considered). To characterise power system frequency from multi-phase measurements, it is convenient to exploit the complex-valued $\alpha\beta$ -transformed voltage, which allows for a simultaneous modelling of all the available information from the three-phase voltages [12], [13].

Most complex-valued frequency estimation algorithms applied to the $\alpha\beta$ -transformed voltage are designed for nominal, balanced, power system conditions, and also operate well in noisy and harmonically contaminated environments [14]–[16]. However, unbalanced systems cause the complex-valued $\alpha\beta$ -transformed voltage to no longer represent a single complex exponential (positive phase-sequence component) that rotates at the fundamental frequency anticlockwise in the complex plane, but a sum of the positive phase-sequence component and the negative one, that is, a complex exponential rotating clockwise at the system frequency. Standard phase angle calculation techniques employed by current frequency estimators are designed only for the positive phase-sequence component, but it is the negative phase-sequence component, associated with unbalanced systems, that introduces bias and oscillations at twice the system frequency, resulting in suboptimal performance of frequency estimation methods [17].

Power quality parameter estimation under unbalanced system conditions is therefore a long standing problem in distributed power systems [18]–[20], and many algorithms have been proposed for this purpose. One solution to mitigate the effect of the negative phase-sequence component is to employ phase-locked loop (PLL) systems enhanced by dq theory [21], [22] or double synchronous reference frame PLL (DSRF-PLL) [23] to extract the positive phase-sequence component from the complex-valued $\alpha\beta$ -transformed voltage, so that the standard phase angle detection algorithms (designed for balanced power systems) can be directly applied. For a direct and online determination of the positive and negative phase-sequence components of unbalanced three-phase power systems, frequency-domain methods based on recursive DFT were proposed [24]–[27]. Especially, in [28], a combination of the SRF-PLL and recursive DFT schemes is proposed. The SRF-PLL is first used for computing the positive phase-sequence, which serves as the input of the recursive DFT block to produce the estimated fundamental-frequency components of the three-phase power signals, and the Fortescue transformation is applied for separating the positive and negative phase-sequence components. Nevertheless, this method is sensitive to higher-order harmonics in

the grid voltages, since they affect the SRF-PLL performance. To allow for harmonic sequence component separation for unbalanced and distorted three-phase power signals, space vector based DFT approaches have been employed more recently [29], [30]. Other time-domain approaches include proportional-integral-derivative (PID) controllers which iteratively correct the phase angle of the $\alpha\beta$ -transformed voltage under unbalanced power conditions [31], generalized delayed signal cancellation [32], [33], parallel finite impulse response (FIR) and notch filtering [34]–[38]. Based on the finding that under unbalanced conditions, the $\alpha\beta$ -transformed voltage and its complex conjugate obey a first-order widely linear (WL) autoregressive model [39]–[41], augmented complex-valued and widely linear (WL) modeling have been recently used to exploit the second-order noncircular statistical nature of the unbalanced $\alpha\beta$ -transformed voltage. This makes it possible to extend standard phase angle estimation algorithms so as to account naturally for the information contained in the negative phase-sequence component, in order to produce unbiased frequency estimates [42]–[47]. Other attempts along this direction employ a pre-linear combination of the the $\alpha\beta$ -transformed voltage and its complex conjugate, so that strictly linear frequency estimators are adequate to achieve statistically unbiased frequency estimate of unbalanced three-phase power signals [48], [49].

In most frequency detection methods for unbalanced power systems, for mathematical simplification, a fundamental noise model has been explicitly or implicitly ignored, thus posing an open question regarding the bounds on the accuracy of the estimation of system frequency in real-world scenarios. We here address this issue through statistical investigation of the $\alpha\beta$ -transformed voltage of unbalanced three-phase power signals polluted by white Gaussian noise. Such insight into the joint probability density functions of the phase voltages and measurement noise allows us to introduce both the Cramer-Rao lower bounds (CRLBs) and novel unbiased maximum likelihood estimation (MLE) of all the system parameters.

In this paper, we first demonstrate that in theory, the MLE of frequency of unbalanced power systems is the maximiser of an “augmented periodogram”, a sum of the standard periodogram of the $\alpha\beta$ -transformed voltage observations and their complex conjugates, referred to as the “conjugate periodogram”. This finding is of particular interest, since in a broad sense it conforms with our previous findings that in order to achieve unbiased frequency estimation, both the information contained in the complex-valued $\alpha\beta$ -transformed voltage and its complex conjugate should be jointly considered. To find the maximum of the proposed augmented periodogram efficiently, Newton’s iterative method is employed and its initialisation is implemented using the discrete Fourier transform (DFT) based dichotomous search technique; this makes it possible to derive the MLE of the phases and amplitudes of both the positive and negative phase-sequence components. This not only answers the question, from the statistical perspective, of why system frequency is of particular importance among all the voltage characteristics, but also facilitates an accurate synthesis of both the phase-sequence components of unbalanced power signals in the time-domain. This

may be beneficial for grid synchronisation and sequence detection in unbalanced power systems when their frequency experiences off-nominal conditions [17], [32], [37]. As desired, the proposed MLE method approaches the theoretical Cramer-Rao lower bounds (CRLBs) for parameter estimation of unbalanced three-phase power signals; this finding is supported by comprehensive simulations.

The paper is organised as follows. Section II introduces the mathematical model of the unbalanced three-phase signals in noisy environments while the Cramer-Rao Lower Bound (CRLB) for unbalanced three-phase power signals is derived in Section III. The maximum likelihood (ML) parameter estimation problem is discussed in Section IV and an implementation of the MLE based on Newton's iterative method is given in Section V. Comprehensive simulation results are presented in Section VI to validate the proposed MLE method and evaluate its performance. Finally, the conclusions are given in Section VII.

II. MATHEMATICAL MODELLING OF UNBALANCED POWER SYSTEMS

The measured voltages in the three-phase power system can be represented in a discrete time form as

$$\begin{aligned} v_a(k) &= V_a \cos(2\pi f k \Delta T + \phi_a) + n_a(k) \\ v_b(k) &= V_b \cos(2\pi f k \Delta T + \phi_b) + n_b(k) \\ v_c(k) &= V_c \cos(2\pi f k \Delta T + \phi_c) + n_c(k) \end{aligned} \quad (1)$$

where V_a, V_b, V_c and ϕ_a, ϕ_b, ϕ_c are respectively the peak values and initial phases of each fundamental voltage component at a time instant k , $\Delta T = \frac{1}{f_s}$ is the sampling interval, where f_s is the sampling frequency, ϕ the initial phase, f the system frequency, and n_a, n_b and n_c phase measurement noises. Although the system frequency can be estimated directly from any of the three-phases in (1), the use of information from all three phases gives more robust frequency estimates [12], [13], [31]. To achieve this, the dimensionality of the signal is first reduced from $\mathbb{R}^3$ in (1) to $\mathbb{C}$ via the (fixed-frame) $\alpha\beta$ -transformed coordinates using the following Clarke transformation [50]

$$\begin{bmatrix} v_r(k) \\ v_i(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(k) \\ v_b(k) \\ v_c(k) \end{bmatrix} \quad (2)$$

The inphase, $v_r(k)$, and the quadrature, $v_i(k)$, components then form the complex-valued $\alpha\beta$ -transformed voltage $v(k)$, in the form

$$v(k) = v_r(k) + jv_i(k) \quad (3)$$

When the three-phase power system deviates from its normal operation, the accurate form of the complex $\alpha\beta$ -transformed voltage $v(k)$ is given by [42]–[47]

$$v(k) = V_+ e^{j(2\pi f k \Delta T + \phi_+)} + V_- e^{-j(2\pi f k \Delta T + \phi_-)} + n(k) \quad (4)$$

where V_+ and V_- are respectively the amplitudes of the positive and negative phase-sequence components within the $\alpha\beta$ -transformed voltage, for which the initial phases are ϕ_+ and ϕ_- , while the system noise $n(k)$ in (4) is a complex-valued doubly white circular Gaussian process $n \sim \mathcal{N}(0, \sigma_n^2)$, with $n = n_r + jn_i$, $\sigma_{n_r}^2 = \sigma_{n_i}^2 = \frac{\sigma_n^2}{2}$, and $n_r \perp n_i$. The statistical independence between n_r and n_i and their equal powers follow from the usual assumption that the three phases are contaminated by independent and identically distributed zero-mean Gaussian noises with variances $\frac{\sigma_n^2}{2}$. By comparing (3) and (4), we then have

$$v_r(k) = V_+ \cos(2\pi f k \Delta T + \phi_+) + V_- \cos(2\pi f k \Delta T + \phi_-) + n_r(k) \quad (5)$$

$$v_i(k) = V_+ \sin(2\pi f k \Delta T + \phi_+) - V_- \sin(2\pi f k \Delta T + \phi_-) + n_i(k) \quad (6)$$

so that the joint probability density function of the $\alpha\beta$ -transformed voltage vector, $\mathbf{v} = [v(0), v(1), \dots, v(K-1)]^T$ with K observations, for an unknown parameter vector $\boldsymbol{\alpha}$, defined as

$$\boldsymbol{\alpha} = [f, V_+, V_-, \phi_+, \phi_-]^T \quad (7)$$

is given by

$$p(\mathbf{v}; \boldsymbol{\alpha}) = \left(\frac{1}{2\pi\sigma_n^2} \right)^K \exp \left(-\frac{1}{2\sigma_n^2} \sum_{k=0}^{K-1} [(v_r(k) - \mu(k))^2 + (v_i(k) - \eta(k))^2] \right) \quad (8)$$

From (4), we can now obtain the variables $\mu(k)$ and $\eta(k)$ in (8) as

$$\mu(k) = V_+ \cos(2\pi f k \Delta T + \phi_+) + V_- \cos(2\pi f k \Delta T + \phi_-) \quad (9)$$

$$\eta(k) = V_+ \sin(2\pi f k \Delta T + \phi_+) - V_- \sin(2\pi f k \Delta T + \phi_-) \quad (10)$$

For this multivariate estimation problem we shall consider the three main aspects: (i) theoretical performance bounds via the Cramer-Rao lower bounds (CRLBs), (ii) maximum-likelihood (ML) estimators of the vector signal parameters and their analysis, and (iii) practical implementation of ML estimators and simulation case studies.

III. CRAMER-RAO LOWER BOUNDS FOR PARAMETER ESTIMATION IN UNBALANCED POWER SYSTEMS

It is well known that based on the probability density function of the observations $\mathbf{v}$, that is $p(\mathbf{v}; \boldsymbol{\alpha})$ parametrised by the vector of unknown parameters $\boldsymbol{\alpha}$, as shown in (8), the variance of any unbiased estimator for any of the parameters, α_i within $\boldsymbol{\alpha}$ in (7), must satisfy [51], [52]

$$\text{var}(\hat{\alpha}_i) \geq \mathbf{I}_{ii}^{-1}(\boldsymbol{\alpha}), \quad i = 1, \dots, 5 \quad (11)$$

where $\mathbf{I}(\boldsymbol{\alpha})$ is the 5×5 Fisher information matrix, defined as

$$\mathbf{I}_{ij}(\boldsymbol{\alpha}) = -E \left[\frac{\partial^2 \ln(p(\mathbf{v}; \boldsymbol{\alpha}))}{\partial \alpha_i \partial \alpha_j} \right] \quad i, j = 1, \dots, 5 \quad (12)$$

and $\mathbf{I}_{ii}^{-1}(\boldsymbol{\alpha})$ is the $[i, i]$ element of the inverse of the Fisher information matrix $\mathbf{I}(\boldsymbol{\alpha})$. Expression (12) is evaluated based on the true values of the unknown vector parameter $\boldsymbol{\alpha}$. When $p(\mathbf{v}; \boldsymbol{\alpha})$ are given by (8), the elements of the Fisher information matrix $\mathbf{I}(\boldsymbol{\alpha})$ are calculated based on

$$\mathbf{I}_{ij}(\boldsymbol{\alpha}) = \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} \left(\frac{\partial \mu(k)}{\partial \alpha_i} \frac{\partial \mu(k)}{\partial \alpha_j} + \frac{\partial \eta(k)}{\partial \alpha_i} \frac{\partial \eta(k)}{\partial \alpha_j} \right) \quad (13)$$

The detailed expressions of all the elements $\mathbf{I}_{ij}(\boldsymbol{\alpha})$ in (13) are provided in Appendix A, based on which, the performance bounds of any unbiased estimator of an unknown parameter α_i within $\boldsymbol{\alpha}$ in (7) are given by CRLB in (11). Note that we here consider the most general situation where all the five system parameters of the unbalanced three-phase power signals $\mathbf{v}$ in $\boldsymbol{\alpha}$ are unknown. When some elements in $\boldsymbol{\alpha}$ are known *a priori*, the subscripts i and j through (11)-(13) should refer to only the other unknown ones. For instance, if the phases of the positive and negative-sequence components, that is, ϕ_+ and ϕ_- , are known in hand, then $\mathbf{I}(\boldsymbol{\alpha})$ becomes a 3-by-3 matrix obtained by deleting the fourth and fifth rows and columns from (13).

IV. MAXIMUM LIKELIHOOD PARAMETER ESTIMATION FOR UNBALANCED THREE-PHASE POWER SIGNALS

The MLE for the unknown parameter vector $\boldsymbol{\alpha}$ is denoted by $\hat{\boldsymbol{\alpha}}$, and maximises $p(\mathbf{v}; \boldsymbol{\alpha})$ when $\mathbf{v}$ is the observed sample vector. To find maximum of the log-likelihood function, $\log(p(\mathbf{v}; \boldsymbol{\alpha}))$, it is sufficient to examine the exponent in (8), given by [52]

$$L_o(\boldsymbol{\alpha}) = -\frac{1}{K} \sum_{k=0}^{K-1} ((v_r(k) - \mu(k))^2 + (v_i(k) - \eta(k))^2) \quad (14)$$

Since the terms $\sum_{k=0}^{K-1} v_r^2(k)$ and $\sum_{k=0}^{K-1} v_i^2(k)$ are constants **once the voltage observations $v(k)$ have been collected, and they** do not depend on the unknown variables in $\boldsymbol{\alpha}$, **we can drop them from $L_o(\boldsymbol{\alpha})$ and maximise the expression**

$$L(\boldsymbol{\alpha}) = \frac{2}{K} \sum_{k=0}^{K-1} (v_r(k)\mu(k) + v_i(k)\eta(k)) - \frac{1}{K} \sum_{k=0}^{K-1} (\mu^2(k) + \eta^2(k)) \quad (15)$$

The substitution of $\mu(k)$ and $\eta(k)$ in (9) and (10) into (15) yields

$$\begin{aligned}
L(\boldsymbol{\alpha}) = & \frac{2V_+}{K} \sum_{k=0}^{K-1} (v_r(k)\cos(2\pi f k \Delta T + \phi_+) + v_i(k)\sin(2\pi f k \Delta T + \phi_+)) \\
& + \frac{2V_-}{K} \sum_{k=0}^{K-1} (v_r(k)\cos(2\pi f k \Delta T + \phi_-) - v_i(k)\sin(2\pi f k \Delta T + \phi_-)) - \frac{1}{K} \sum_{k=0}^{K-1} (V_+^2 + V_-^2) \\
& - \frac{2V_+V_-}{K} \sum_{k=0}^{K-1} (\cos(2\pi f k \Delta T + \phi_+)\cos(2\pi f k \Delta T + \phi_-) - \sin(2\pi f k \Delta T + \phi_+)\sin(2\pi f k \Delta T + \phi_-)) \quad (16)
\end{aligned}$$

This can be further simplified through a complex exponential representation and a trigonometric identity, to give

$$\begin{aligned}
L(\boldsymbol{\alpha}) = & 2V_+ \Re[\exp(-j\phi_+)V_1(f)] + 2V_- \Re[\exp(-j\phi_-)V_2(f)] \\
& - (V_+^2 + V_-^2) - \frac{2V_+V_-}{K} \sum_{k=0}^{K-1} \cos(4\pi f k \Delta T + \phi_+ + \phi_-) \quad (17)
\end{aligned}$$

where $\Re[\cdot]$ denotes the real part operator, and

$$V_1(f) = \frac{1}{K} \sum_{k=0}^{K-1} v(k) \exp(-j2\pi f k \Delta T) \quad (18)$$

$$V_2(f) = \frac{1}{K} \sum_{k=0}^{K-1} v^*(k) \exp(-j2\pi f k \Delta T) \quad (19)$$

When the number of the observations K is large enough, or when it is approximately equal to a multiple of the period of twice the fundamental system frequency, the magnitude of the last term on the right hand side (RHS) of (17) is negligible; this results in a simplified log-likelihood function, given by

$$L(\boldsymbol{\alpha}) = 2V_+ \Re[\exp(-j\phi_+)V_1(f)] + 2V_- \Re[\exp(-j\phi_-)V_2(f)] - (V_+^2 + V_-^2) \quad (20)$$

Suppose that all five elements of $\boldsymbol{\alpha}$ are unknown, and the amplitudes of the positive and negative phase-sequence components, that is, V_+ and V_- are positive. The MLE for the phase angles ϕ_+ and ϕ_- can then be performed by setting the partial derivatives of L in (20) with respect to ϕ_+ and ϕ_- equal to zero, to yield

$$\begin{aligned}
\frac{\partial L}{\partial \phi_+} = & 2V_+ \Re[-j \exp(-j\phi_+)V_1(f)] \\
= & -2V_+ \Im[\exp(-j\phi_+)V_1(f)] \\
= & 0 \quad (21)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial \phi_-} = & 2V_- \Re[-j \exp(-j\phi_-)V_2(f)] \\
= & -2V_- \Im[\exp(-j\phi_-)V_2(f)] \\
= & 0 \quad (22)
\end{aligned}$$

where $\Im[\cdot]$ is the imaginary part operator. From (21) and (22), and after some simple calculus, we arrive at

$$\hat{\phi}_+ = \angle(V_1(f)) \text{ and } \hat{\phi}_- = \angle(V_2(f)) \quad (23)$$

where $\angle(\cdot)$ is the phase angle operator, taken $\text{mod } 2\pi$ for convenience. The log-likelihood function L within the MLE of $\hat{\phi}_+$ and $\hat{\phi}_-$ now becomes

$$L = 2V_+|V_1(f)| + 2V_-|V_2(f)| - (V_+^2 + V_-^2) \quad (24)$$

and can be further maximised over V_+ and V_- by setting

$$\frac{\partial L}{\partial V_+} = 0 \text{ and } \frac{\partial L}{\partial V_-} = 0 \quad (25)$$

to give

$$\hat{V}_+ = |V_1(f)| \text{ and } \hat{V}_- = |V_2(f)| \quad (26)$$

Upon substituting the estimates $\hat{V}_+$ and $\hat{V}_-$ in place of V_+ and V_- in (24), the ML estimate, $\hat{f}$, for the system frequency, f , becomes the argument that maximises

$$\hat{f} = \arg \max_f (Y(f)) \quad (27)$$

where

$$\begin{aligned} Y(f) &= |V_1(f)|^2 + |V_2(f)|^2 \\ &= V_1(f)V_1^*(f) + V_2(f)V_2^*(f) \\ &= \underbrace{\frac{1}{K^2} \left| \sum_{k=0}^{K-1} v(k) \exp(-j2\pi f k \Delta T) \right|^2}_{\text{standard periodogram}} + \underbrace{\frac{1}{K^2} \left| \sum_{k=0}^{K-1} v^*(k) \exp(-j2\pi f k \Delta T) \right|^2}_{\text{conjugate periodogram}} \end{aligned} \quad (28)$$

where $|V_1(f)|^2$ is the standard, widely used, periodogram. In analogy to widely linear modelling [40], we refer to $|V_2(f)|^2$ as the *conjugate periodogram* and to $Y(f)$ as the *augmented periodogram* of $\mathbf{v}$, since it is the sum of the standard and conjugate periodograms. Note that $\hat{f}$ does not necessarily maximise the standard and conjugate periodogram individually, but their sum under unbalanced grid voltage conditions.

Remark 1: After obtaining the ML estimate for the frequency, $\hat{f}$, the ML estimates for the phases and the amplitudes of the positive- and negative-phase sequence components can be performed by using $\hat{f}$ in place of f in (23) and (26). This also indicates that the MLE of all the voltage characteristics within unbalanced $\alpha\beta$ -transformed voltage depends statistically on that of the system frequency.

A. MLE in the Cases When Some System Parameters Are Known

When some parameters within α are known *a priori*, the MLE framework for the unbalanced $\alpha\beta$ -transformed voltage can be derived in a similar manner as above, and the results are summarised as follows.

If the system frequency f is unknown, then its MLE $\hat{f}$ maximises:

- 1) $|V_1(f)|^2 + |V_2(f)|^2$, if the phases of both phase-sequence components, ϕ_+ and ϕ_- , and their amplitudes, V_+ and V_- , are all unknown;
- 2) $V_+|V_1(f)| + V_-|V_2(f)|$, if the phases of both phase-sequence components are unknown but their amplitudes are known;
- 3) $\Re^2[\exp(-j\phi_+)V_1(f)] + \Re^2[\exp(-j\phi_-)V_2(f)]$, if the phases are known but the amplitudes are unknown;
- 4) $V_+\Re[\exp(-j\phi_+)V_1(f)] + V_-\Re[\exp(-j\phi_-)V_2(f)]$, if the phases and amplitudes of both sequence components are all known;

If the phases ϕ_+ and ϕ_- are unknown, then the MLEs for $\hat{\phi}_+$ and $\hat{\phi}_-$ are equal to:

- 5) $\hat{\phi}_+ = \angle(V_1(\hat{f}))$ and $\hat{\phi}_- = \angle(V_2(\hat{f}))$, if the frequency f and amplitudes are unknown, where $\hat{f}$ is extracted from 1) above;
- 6) $\hat{\phi}_+ = \angle(V_1(\hat{f}))$ and $\hat{\phi}_- = \angle(V_2(\hat{f}))$, if the frequency is unknown, but amplitudes are known, where $\hat{f}$ is extracted from 2) above;
- 7) $\hat{\phi}_+ = \angle(V_1(f))$ and $\hat{\phi}_- = \angle(V_2(f))$, if the frequency is known, no matter whether the amplitudes are known or not;

If the amplitudes V_+ and V_- are unknown, then the MLEs of $\hat{V}_+$ and $\hat{V}_-$ are equal to:

- 8) $|V_1(\hat{f})|$ and $|V_2(\hat{f})|$, if the phases and frequency are unknown, where $\hat{f}$ is extracted from 1) above;
- 9) $|V_1(f)|$ and $|V_2(f)|$, if the phases are unknown but the frequency is known;
- 10) $\Re[\exp(-j\phi_+)V_1(\hat{f})]$ and $\Re[\exp(-j\phi_-)V_2(\hat{f})]$, if the phases are known but the frequency is unknown and is extracted from 3) above;
- 11) $\Re[\exp(-j\phi_+)V_1(f)]$ and $\Re[\exp(-j\phi_-)V_2(f)]$, if the phases and the frequency are known.

The discussion above provides a detailed guidance on how to achieve accurate synthesis of both the positive and negative phase-sequence components of unbalanced power signals in the time-domain when some parameters are known *a priori*. This may be beneficial for grid synchronisation and phase-sequence component detection in unbalanced power systems when the frequency experiences off-nominal conditions [17], [32], [37].

Remark 2: Observe that according to items 7) and 9), for the detection of phase-sequence of fundamental-frequency when the system frequency is known in advance, the MLEs of the amplitudes are independent of those of the phases for both the phase-sequence components of unbalanced power signals.

V. AN IMPLEMENTATION OF THE PROPOSED MLE SOLUTION FOR UNBALANCED THREE-PHASE POWER SIGNALS BASED ON THE DISCRETE FOURIER TRANSFORM (DFT) AND NEWTON'S ITERATIVE METHOD

For generality, we shall focus on the general MLE solution for the detection of phase-component sequences within the $\alpha\beta$ transformed voltages of unbalanced three-phase power signals, when all system parameters are unknown. As discussed in Section IV, the MLE solution for the system frequency is the prerequisite for the estimation of the amplitudes and phases of both the positive and negative phase-sequence components, since their MLE can be subsequently achieved by using (22) and (26) respectively. We now present a fast and efficient implementation of ML frequency estimates. The structure of the proposed algorithm consists of a coarse search and a fine search. The coarse search is similar to that presented in [52], based on the maximum bin search within the proposed augmented periodogram using the discrete Fourier transform (DFT) of the $\alpha\beta$ -transformed observation voltage vector $\mathbf{v}$ and its complex conjugate $\mathbf{v}^*$. The fine search is based on a simple dichotomous search of the peak of the augmented periodogram, followed by application of Newton's iterative method. The dichotomous search procedure is simple and is particularly suited for DSP implementation, since only real multiple-and-accumulate operations are involved [53], and its refined frequency resolution is instrumental to guarantee rapid convergence of the subsequent Newton's iterative method.

A. Coarse search stage

Consider the discrete Fourier transform (DFT) of the voltage observation vector $\mathbf{v}$ and its complex conjugate $\mathbf{v}^*$, given by

$$V_1(m) = \frac{1}{K} \sum_{k=0}^{K-1} v(k) \exp \left(-j \frac{2\pi km}{K} \right), \quad (29)$$

$$\begin{aligned} V_2(m) &= \frac{1}{K} \sum_{k=0}^{K-1} v^*(k) \exp \left(-j \frac{2\pi km}{K} \right) \\ &= V_1^*(K - m) \end{aligned} \quad (30)$$

where $m = 0, 1, \dots, K - 1$. Note that the last step in (30) is obtained using the complex conjugate property of DFT for complex-valued sequence for a more computationally efficient implementation [54]. Comparing the DFT coefficients of the voltage observation vector $\mathbf{v}$ and its complex conjugate in (29) and (30) with the definition of the augmented periodogram in (28), we obtain

$$\begin{aligned} Y(m) &= Y \left(\frac{m}{K\Delta T} \right) \\ &= |V_1(m)|^2 + |V_2(m)|^2 \\ &= |V_1(m)|^2 + |V_1(K - m)|^2 \end{aligned} \quad (31)$$

where $m = 0, 1, \dots, K - 1$. By selecting the $m_{\max}$ bin which corresponds to the largest $|V_1(m)|^2 + |V_1(K - m)|^2$, we then obtain a coarse approximation to the system frequency f_o as

$$\hat{f}_{\text{coarse}} = \frac{m_{\max}}{K \Delta T} \quad (32)$$

and for a sufficient high signal-to-noise ratio (SNR), it is very likely that the true frequency f_o lies in the interval [55]

$$\left[\frac{m_{\max} - 1/2}{K \Delta T}, \frac{m_{\max} + 1/2}{K \Delta T} \right]$$

In order to mitigate the problems at low SNR, when the bin index may not be close to the augmented periodogram peak, we can next extend the observation vector $\mathbf{v}$ to N samples by appending $(N - K)$ zeros (zero padding), and this allows us to use the N -point DFT to find the correct bin. This technique is also referred to as the maximum bin search and has been widely applied in the literature [52], [53], [56].

B. Fine search stage

The first step of the fine search consists of a simple dichotomous search with refined frequency resolution in order to locate a more accurate augmented periodogram peak. Within $Y(m)$, the coefficients at both sides of the maximum, i.e., $Y(m_{\max} - 1)$ and $Y(m_{\max} + 1)$, obtained in the coarse search step, are compared, and the frequency estimate is then adjusted toward the larger coefficient so as to halve the estimation resolution, and the DFT coefficient midway between $Y(m_{\max})$ and the next highest coefficient is calculated [53]. The procedure is repeated for M iterations, as summarised in Table I. Since this scheme yields an estimate of f_o for which the variance is somewhat larger than the corresponding CRLB [56], we proceed to define a fast and iterative Newton's technique to solve the augmented periodogram problem starting at the estimated frequency obtained by the dichotomous search algorithm. The augmented periodogram maximisation problem is solved via Newton's method through the following iterative form [57]

$$\hat{f}(q + 1) = \hat{f}(q) - \frac{Y'(\hat{f}(q))}{Y''(\hat{f}(q))}, \quad q = 0, 1, \dots, Q \quad (33)$$

where

$$Y'(f) = \frac{\partial Y(f)}{\partial f} \quad \text{and} \quad Y''(f) = \frac{\partial^2 Y(f)}{\partial f^2} \quad (34)$$

Based on the augmented periodogram in (28), the complex-valued partial derivative $Y'(f)$ can be calculated based on $Y(f) = V_1(f)V_1^*(f) + V_2(f)V_2^*(f)$, and using the chain rule [40], we obtain that

$$\frac{\partial Y(f)}{\partial f} = 2\Re \left[\frac{\partial V_1^*(f)}{\partial f} V_1(f) \right] + 2\Re \left[\frac{\partial V_2^*(f)}{\partial f} V_2(f) \right] \quad (35)$$

Note that

$$\frac{\partial V_1^*(f)}{\partial f} = \frac{1}{K} \sum_{k=0}^{K-1} (j2\pi k \Delta T) v^*(k) \exp(j2\pi f k \Delta T) \quad (36)$$

$$\frac{\partial V_2^*(f)}{\partial f} = \frac{1}{K} \sum_{k=0}^{K-1} (j2\pi k \Delta T) v(k) \exp(j2\pi f k \Delta T) \quad (37)$$

which allows (35) to take the form

$$\begin{aligned} \frac{\partial Y(f)}{\partial f} &= \frac{2}{K^2} \Re \left[\sum_{k=0}^{K-1} (j2\pi k \Delta T) v^*(k) \exp(j2\pi f k \Delta T) \cdot \sum_{k=0}^{K-1} v(k) \exp(-j2\pi f k \Delta T) \right] \\ &\quad + \frac{2}{K^2} \Re \left[\sum_{k=0}^{K-1} (j2\pi k \Delta T) v(k) \exp(j2\pi f k \Delta T) \cdot \sum_{k=0}^{K-1} v^*(k) \exp(-j2\pi f k \Delta T) \right] \end{aligned} \quad (38)$$

In the same spirit, $\frac{\partial^2 Y(f)}{\partial f^2}$ can be shown to be

$$\begin{aligned} \frac{\partial^2 Y(f)}{\partial f^2} &= \frac{2}{K^2} \Re \left[\sum_{k=0}^{K-1} (j2\pi k \Delta T)^2 v^*(k) \exp(j2\pi f k \Delta T) \cdot \sum_{k=0}^{K-1} v(k) \exp(-j2\pi f k \Delta T) \right] \\ &\quad + \frac{2}{K^2} \Re \left[\sum_{k=0}^{K-1} (j2\pi k \Delta T)^2 v(k) \exp(j2\pi f k \Delta T) \cdot \sum_{k=0}^{K-1} v^*(k) \exp(-j2\pi f k \Delta T) \right] \\ &\quad + \frac{2}{K^2} \left| \sum_{k=0}^{K-1} (j2\pi k \Delta T) v^*(k) \exp(j2\pi f k \Delta T) \right|^2 + \frac{2}{K^2} \left| \sum_{k=0}^{K-1} (j2\pi k \Delta T) v(k) \exp(j2\pi f k \Delta T) \right|^2 \end{aligned} \quad (39)$$

Dividing (38) by (39) and using the q th iterated frequency estimate $\hat{f}(q)$ instead of f , we next obtain the expression for the update term within the iteration in (33), for which the initialisation $\hat{f}(0)$ is obtained from the simple dichotomous search procedure discussed in the previous section. It should be noted that if the initialisation value is close to the true parameter value (guaranteeing a convergence), Newton's method can achieve quadratic convergence [57], justifying its use for frequency estimation. With the so resolved MLE of frequency $\hat{f}$, we are now able to subsequently perform the corresponding MLEs of both the amplitudes and phases of the positive and negative phase-sequence components within the unbalanced three-phase power signals, using (26) and (22), respectively.

C. Implementation and computational complexity of the proposed ML frequency estimator

The implementation of the proposed ML frequency estimator involves both the coarse and fine search stages, as discussed above. The main computational burden of the coarse search stage lies in the DFT operation on the voltage observation vector $\mathbf{v}$ in (29), which requires K^2 complex multiplications, $K(K-1)$ complex adds and K^2 complex exponential evaluations to calculate $V_1(m)$, where $m = 0, 1, \dots, K$. For computational efficiency, the DFT operation on the complex conjugate of the voltage observations $\mathbf{v}^*$ to calculate each $V_2(m)$ in (30) can be achieved by using K complex conjugate operations. In a similar way,

TABLE I
DICHOTOMOUS SEARCH OF THE AUGMENTED PERIODOGRAM PEAK

Set $\Delta = 1$, $Y_{-1} = Y(m_{\max} - 1)$, $Y_0 = Y(m_{\max})$ and $Y_1 = Y(m_{\max} + 1)$
For M iterations do
$\Delta = \Delta/2$
if $Y_1 > Y_{-1}$
then $Y_{-1} = Y_0$ and $m_{\max} = m_{\max} + \Delta$
else $Y_1 = Y_0$ and $m_{\max} = m_{\max} - \Delta$
$Y_0 = \left \sum_{k=0}^{K-1} v(k) \exp(-j2\pi k m_{\max}/N) \right ^2 + \left \sum_{k=0}^{K-1} v^*(k) \exp(-j2\pi k m_{\max}/N) \right ^2$
Finally, $\hat{f} = \frac{m_{\max}}{N\Delta T}$

TABLE II
COMPUTATIONAL COMPLEXITY OF THE PROPOSED ML FREQUENCY ESTIMATOR.

	Coarse Search Stage	Fine Search Stage (per iteration)	
		Dichotomous Search	Newton's Iteration
Complex Multiplications	$K^2 + K + 1$	$2K + 3$	$6K + 4$
Complex Adds	K^2	$2K$	$6K - 2$
Complex Exponential Evaluations	K^2	K	K
Complex Conjugate Oeprations	K	2	$2K + 2$
Complex Divisions	0	0	1
Real Part Operations	0	0	2

to construct the augmented perodogram $Y(m)$ in (31), where $m = 0, 1, \dots, K$, we futher need K complex multpilcations and K complex adds. For the last step of the coarse search stage, one more multipilcation is required to obtain the coarse frequency estimate $\hat{f}_{\text{coarse}}$ in (32). Note that we here consider only the compulsory operations, as the constant $\frac{1}{K\Delta T}$ in (32) can be predefined in the initialisation. The total computation burden of the corase search stage is listed II, where by contrast, those of the dichotomous search and the Newton's iterative method required in the fine search stage of the proposed frequency estimator is also provided. By considering that the number of observations used in the DFT operation is usually much larger than the numbers of iterations inlvolved in the dichotomous search and Newton's iterative method, that is, $K \gg M, Q$, the computational burden of the proposed ML frequency estimator still mainly lies in the DFT operation. After obtaining the MLE of frequency, $\hat{f}$, by using (33), the ML estimates for the phases and the amplitudes of the positive- and negative-phase sequence components can be performed by using $\hat{f}$ in place of f in (23) and (26).

D. Discussion of the proposed ML frequency estimator with existing methods

Compared with some existing time-domain methods with simple implementations, such as delayed signal cancellation method and its generalised version [32], [33], and widely linear model based frequency estimators [42]–[45], [48], [49], the proposed ML frequency estimator generic considers noise measurements in its mathematical voltage modelling, and its derivation, which originates from the joint probability density functions of the unbalanced $\alpha\beta$ -transformed voltages and measurement noises, enables enhanced robustness against noisy pollution in the sense that it approaches the theoretical CRLBs for all the system parameters by design. However, this is achieved at an increased computational complexity due to the involvement of the full DFT operation. On the other hand, the coarse search stage of the proposed ML frequency estimator has a very similar implementation as compared with the space-vector DFT based methods [29], [30], [33], but the implicit coherent sampling assumption that the voltage cycle covers integer periods at the fundamental frequency, i.e., 50/60 Hz, results in their sub-optimality in the asynchronous sampling situations, for which an additional SRF-PLL structure is further equipped for phase angle adaptation. While, within the proposed MLE method, the frequency off-nominals are inherently addressed by its fine search stage. However, we should note that the space-vector DFT methods further allow for higher-order harmonic sequence component separation when the unbalanced power systems experience harmonic distortions, for which the proposed MLE method is not optimal due to its modelling insufficiency.

VI. SIMULATIONS

To verify the effectiveness of the proposed MLE method, numerical simulations were carried out in the MATLAB programming environment for various types of unbalanced power systems. For comparison, statistical performance of the conventional block based least squares (LS) frequency estimation algorithm was also considered [16]. To achieve unbiased frequency estimates for unbalanced power systems, the underlying LS observation model was modified by employing the widely linear (WL) framework to give the WL-LS model; we refer to [42], [43] for more detail. In the implementation of the considered algorithms, the sampling frequency f_s of the simulated power systems was fixed at $f_s = 1000$ Hz, $K = 100$ consecutive voltage samples were used as the available observations for the coarse search of the augmented periodogram peak by the DFT operation. $M = 3$ iterations were performed in the dichotomous search to improve the frequency domain resolution, and the initial frequency of the iterative Newton's method was set to the nominal 50 Hz.

In the first stage, we investigated the effects of the number of iterations Q used in the Newton's iterative method within the proposed ML frequency estimator. The system parameters of the unbalanced three-phase power system considered here were those obtained by an experimental setup of a small-scale power

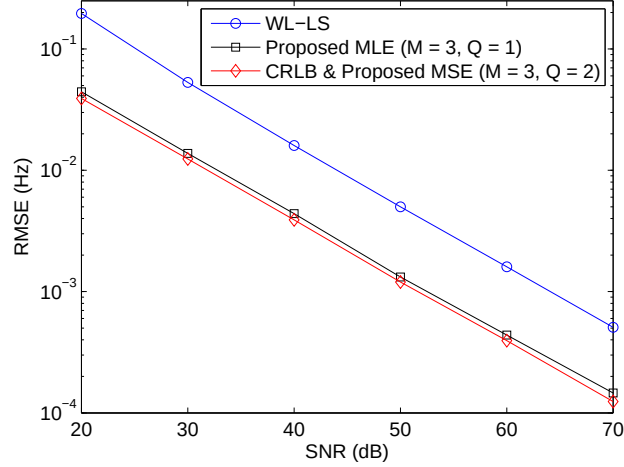


Fig. 1. Frequency estimation performances against the CRLB, in terms of RMSE, of all the considered algorithms for a simulated unbalanced power system and at different levels of noise. The results were obtained by averaging 10000 independent trials.

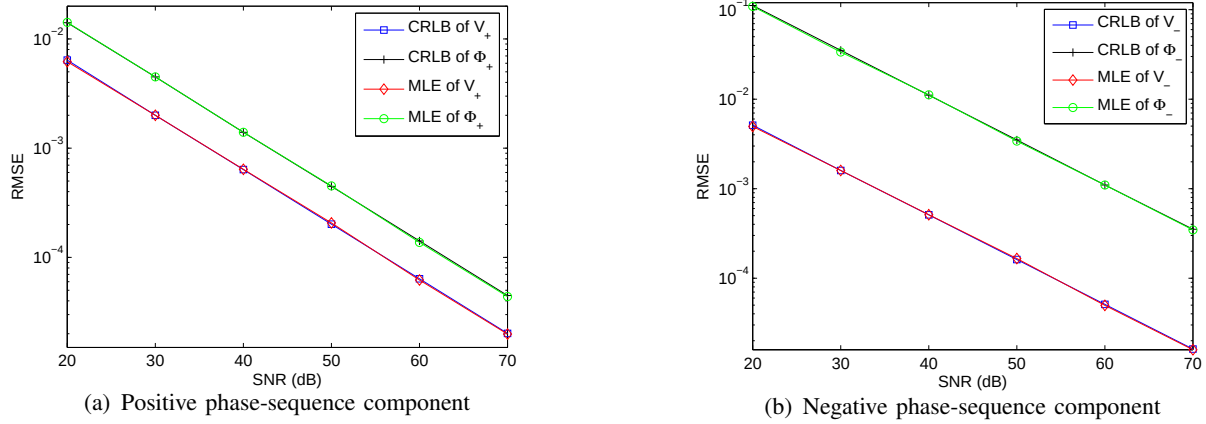


Fig. 2. Estimation performances, in terms of RMSE, of the proposed MLE method with $Q = 2$ of the amplitudes and phases of the positive and negative phase-components of the unbalanced power system in different levels of noise, evaluated against the CRLB. The results were obtained by averaging 10000 independent trials.

grid [32], where $V_+ = 0.896$ p.u., $V_- = 0.058$, $\Phi_+ = 0 \pi$ and $\Phi_- = 0.5156 \pi$, giving a degree of unbalance $V_-/V_+ = 6.49\%$. The power system frequency f deviated from the nominal 50 Hz, and was fixed at $f = 50.5$ Hz. As shown in Fig. 1, due to fact that in general the power grid frequency does not deviate too much from its nominal value and the fast convergence of the Newton's iterative method, $Q = 2$ iterations were enough to enable the performance of the proposed ML frequency estimator, measured in root mean square error (RMSE), to match well the theoretical CRLB calculated using (11) over all the SNR regions considered, and the performance advantages of the proposed ML framework over the WL-LS frequency estimator can be observed. With the so achieved MLE of system frequency, we were able to subsequently achieve MLEs of amplitudes and phases of both the positive and negative phase-sequence components within the unbalanced $\alpha\beta$ transformed voltage by using (23) and (26), which were also close

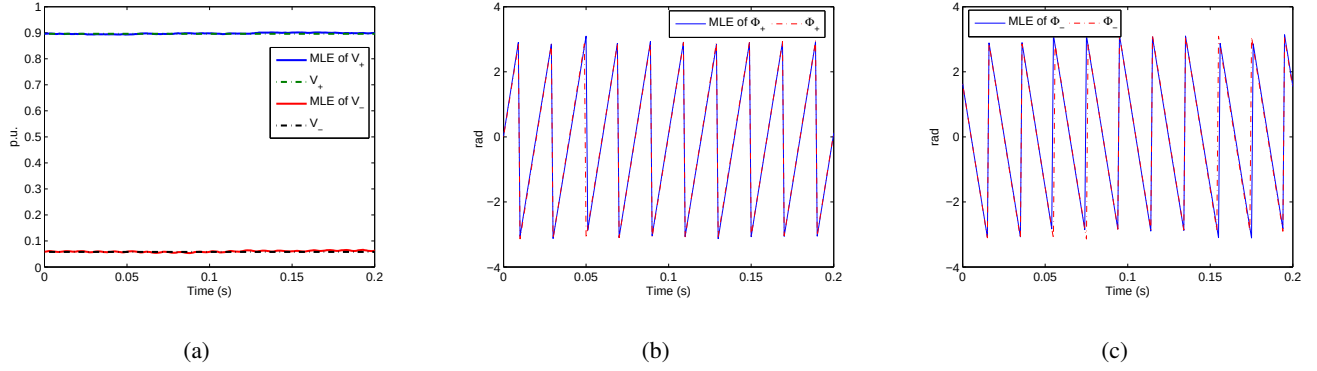


Fig. 3. A single trial illustration of the proposed MLE framework for a simulated unbalanced episode, lasting for 0.2 s, with a SNR at 40 dB. (a) MLE of amplitudes of both the positive and negative phase-components. (b) MLE of phase of the positive phase-component. (c) MLE of phase of the negative phase-component.

TABLE III

PERFORMANCE COMPARISON, IN TERMS OF BOTH THE ESTIMATION BIAS AND RMSE, OF THE CONSIDERED FREQUENCY ESTIMATORS FOR DIFFERENT OFF-NOMINAL FREQUENCY CONDITIONS.

Frequency (Hz)	Bias ($\times 10^{-3}$ Hz)		RMSE ($\times 10^{-3}$ Hz)		
	WL-LS	MLE	WL-LS	MLE	CRLB
49.5	1.887	0.292	51.434	12.729	12.426
49.6	1.698	0.231	50.699	12.722	12.422
49.7	1.942	0.131	51.490	12.644	12.418
49.8	1.592	0.175	53.061	12.600	12.414
49.9	1.422	0.213	52.374	12.484	12.411
50.0	1.990	0.124	52.279	12.486	12.409
50.1	1.687	0.142	53.922	12.532	12.407
50.2	1.356	0.289	51.439	12.622	12.406
50.3	1.179	0.179	53.685	12.506	12.405
50.4	1.413	0.143	52.898	12.495	12.405
50.5	1.188	0.248	51.746	12.440	12.405

to their corresponding CRLBs as shown respectively in Fig. 2 (a) and (b). In this way, by using a simple sliding window technique, the proposed MLE method facilitated accurate synthesis of both the positive and negative phase-sequence components within unbalanced power signals in the time-domain, as shown in Fig. 3.

In the next case study, the performance comparison of the considered methods was evaluated when the simulated unbalanced three-phase power system experienced different off-nominal frequency conditions. White noise at 30 dB SNR was added to the simulated power signals. As illustrated in Table III, in all off-nominal frequency conditions, the RMSE of the proposed MLE closely followed the CRLB, and its estimation bias was much smaller than that of the WL-LS approach.

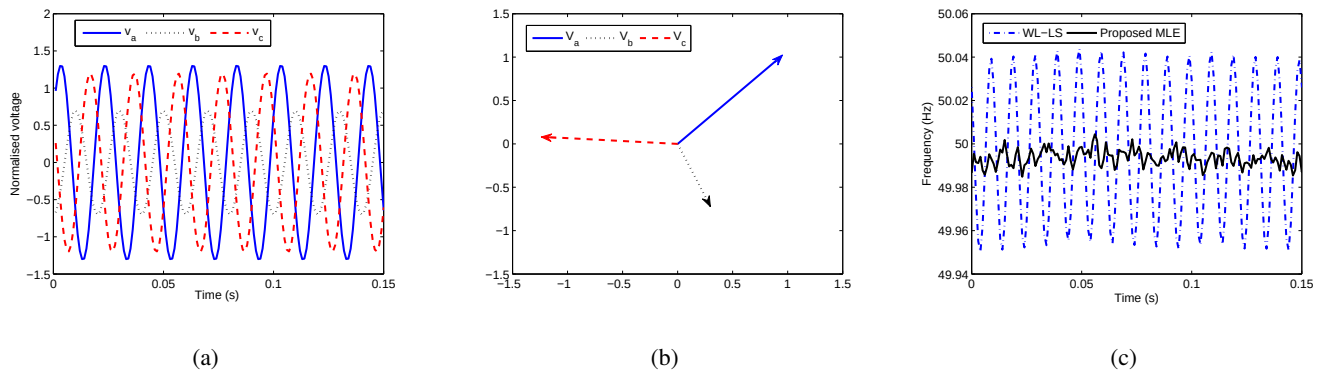


Fig. 4. Frequency estimation results of both the considered approaches for a real-world unbalanced three-phase power system. (a) Time domain waveforms of the three-phase voltages. (b) Phasor representation of the three-phase voltages. (c) Frequency estimation results.

In the last set of simulations, a real-world power system was considered. The three-phase voltage was recorded at a 110/20/10 kV transformer station. The REL 531 numerical line distant protection terminal, produced by ABB Ltd., was installed in the station and was used to monitor three phase-ground voltages on the 20 kV busbars (neutral earthed by 40 ohms resistor). The measured three phase-ground voltages with a system frequency around 50 Hz were sampled at 1000 Hz by the equipment and were further normalised with respect to their respective normal peak voltage values, as shown in Fig. 4(a). The three-phase power system was in a unbalanced condition, where phase v_b experienced an earth fault, causing a 30.3% voltage drop, and 29.9% and 19.6% voltage swells in phases v_a and v_c , respectively. The phasor representations of the three-phase voltages also indicated that besides of the unbalanced voltage magnitudes, the phase differences between v_a and v_b , between v_b and v_c , and between v_c and v_a were respectively 114.5° , 116.2° and 129.3° , and all deviated from the nominal 120° . The frequency tracking capabilities for both the approaches are shown in Fig. 4(c), from which, the enhanced robustness in the presence of measurement noise of the proposed ML frequency estimator over the WL-LS one can be clearly observed, evidenced by a much smaller estimation disturbance.

VII. CONCLUSION

We have introduced a unified framework for accurate parameter estimation in unbalanced power systems based on the maximum likelihood principle. By investigating the log-likelihood function of the three-phase voltages through their $\alpha\beta$ -transform contaminated with white Gaussian noise, it has been shown that the proposed ML frequency estimator maximises the augmented periodogram, a sum of the periodograms of the $\alpha\beta$ -transformed voltage observations and their complex conjugates. To achieve a computationally sufficient solution for the augmented periodogram maximiser, Newton's iterative method has been employed, the initialisation of which has been implemented using the discrete Fourier transform (DFT) based dichotomous search technique. It has also been found that the MLE of the phases and amplitudes of

both the positive and negative phase-sequence components within the unbalanced $\alpha\beta$ transformed voltage can be generically derived based on the proposed ML frequency estimates. For rigour, the introduced MLE method has been shown to attain the theoretical Cramer-Rao lower bounds (CRLBs) for parameter estimation in various unbalanced power conditions. The proposed methodology has also been shown to exhibit higher immunity to noise pollution as compared with the recently developed widely linear least squares (WL-LS) frequency estimator [in both simulated and real-world unbalanced three-phase power systems](#).

APPENDIX A. DETAILED EXPRESSION FOR THE ELEMENTS IN THE FISHER INFORMATION MATRIX

$\mathbf{I}(\boldsymbol{\alpha})$ IN (13)

[According to \(9\) and \(10\)](#), the first order partial derivatives in (13) are given by

$$\begin{aligned} \frac{\partial \mu(k)}{\partial \boldsymbol{\alpha}} &= \left[\frac{\partial \mu(k)}{\partial f}, \frac{\partial \mu(k)}{\partial V_+}, \frac{\partial \mu(k)}{\partial V_-}, \frac{\partial \mu(k)}{\partial \phi_+}, \frac{\partial \mu(k)}{\partial \phi_-} \right]^T \\ &= \begin{bmatrix} -2\pi k \Delta T [V_+ \sin(2\pi f k \Delta T + \phi_+) \\ + V_- \sin(2\pi f k \Delta T + \phi_-)] \\ \cos(2\pi f k \Delta T + \phi_+) \\ \cos(2\pi f k \Delta T + \phi_-) \\ -V_+ \sin(2\pi f k \Delta T + \phi_+) \\ -V_- \sin(2\pi f k \Delta T + \phi_-) \end{bmatrix} \end{aligned} \quad (40)$$

and

$$\begin{aligned} \frac{\partial \eta(k)}{\partial \boldsymbol{\alpha}} &= \left[\frac{\partial \eta(k)}{\partial f}, \frac{\partial \eta(k)}{\partial V_+}, \frac{\partial \eta(k)}{\partial V_-}, \frac{\partial \eta(k)}{\partial \phi_+}, \frac{\partial \eta(k)}{\partial \phi_-} \right]^T \\ &= \begin{bmatrix} 2\pi k \Delta T [V_+ \cos(2\pi f k \Delta T + \phi_+) \\ - V_- \cos(2\pi f k \Delta T + \phi_-)] \\ \sin(2\pi f k \Delta T + \phi_+) \\ -\sin(2\pi f k \Delta T + \phi_-) \\ V_+ \cos(2\pi f k \Delta T + \phi_+) \\ -V_- \cos(2\pi f k \Delta T + \phi_-) \end{bmatrix} \end{aligned} \quad (41)$$

which, when combined with (13), give the elements within the Fisher information matrix as follows

$$\begin{aligned} \mathbf{I}_{11} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} (2\pi k \Delta T)^2 (V_+^2 + V_-^2 - 2V_+ V_- \cos(4\pi f k \Delta T + \phi_+ + \phi_-)) \\ \mathbf{I}_{12} &= \mathbf{I}_{21} = \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -2\pi k \Delta T V_- \sin(4\pi f k \Delta T + \phi_+ + \phi_-) \end{aligned}$$

$$\begin{aligned}
\mathbf{I}_{13} = \mathbf{I}_{31} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -2\pi k \Delta T V_+ \sin(4\pi f k \Delta T + \phi_+ + \phi_-) \\
\mathbf{I}_{14} = \mathbf{I}_{41} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} 2\pi k \Delta T (V_+^2 - V_+ V_- \cos(4\pi f k \Delta T + \phi_+ + \phi_-)) \\
\mathbf{I}_{15} = \mathbf{I}_{51} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -2\pi k \Delta T (V_+ V_- \cos(4\pi f k \Delta T + \phi_+ + \phi_-) - V_-^2) \\
\mathbf{I}_{22} = \mathbf{I}_{33} &= 2K / \sigma_n^2 \\
\mathbf{I}_{23} = \mathbf{I}_{32} &= \frac{2}{\sigma_n^2} \sum_{k=1}^K \cos(4\pi f k \Delta T + \phi_+ + \phi_-) \\
\mathbf{I}_{24} = \mathbf{I}_{42} = \mathbf{I}_{35} = \mathbf{I}_{53} &= 0 \\
\mathbf{I}_{25} = \mathbf{I}_{52} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -V_- \sin(4\pi f k \Delta T + \phi_+ + \phi_-) \\
\mathbf{I}_{34} = \mathbf{I}_{43} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -V_+ \sin(4\pi f k \Delta T + \phi_+ + \phi_-) \\
\mathbf{I}_{44} &= 2K V_+^2 / \sigma_n^2 \\
\mathbf{I}_{45} = \mathbf{I}_{54} &= \frac{2}{\sigma_n^2} \sum_{k=0}^{K-1} -V_+ V_- \cos(4\pi f k \Delta T + \phi_+ + \phi_-) \\
\mathbf{I}_{55} &= 2K V_-^2 / \sigma_n^2
\end{aligned} \tag{42}$$

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