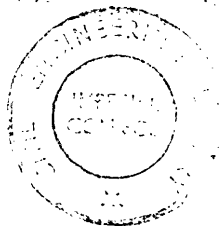


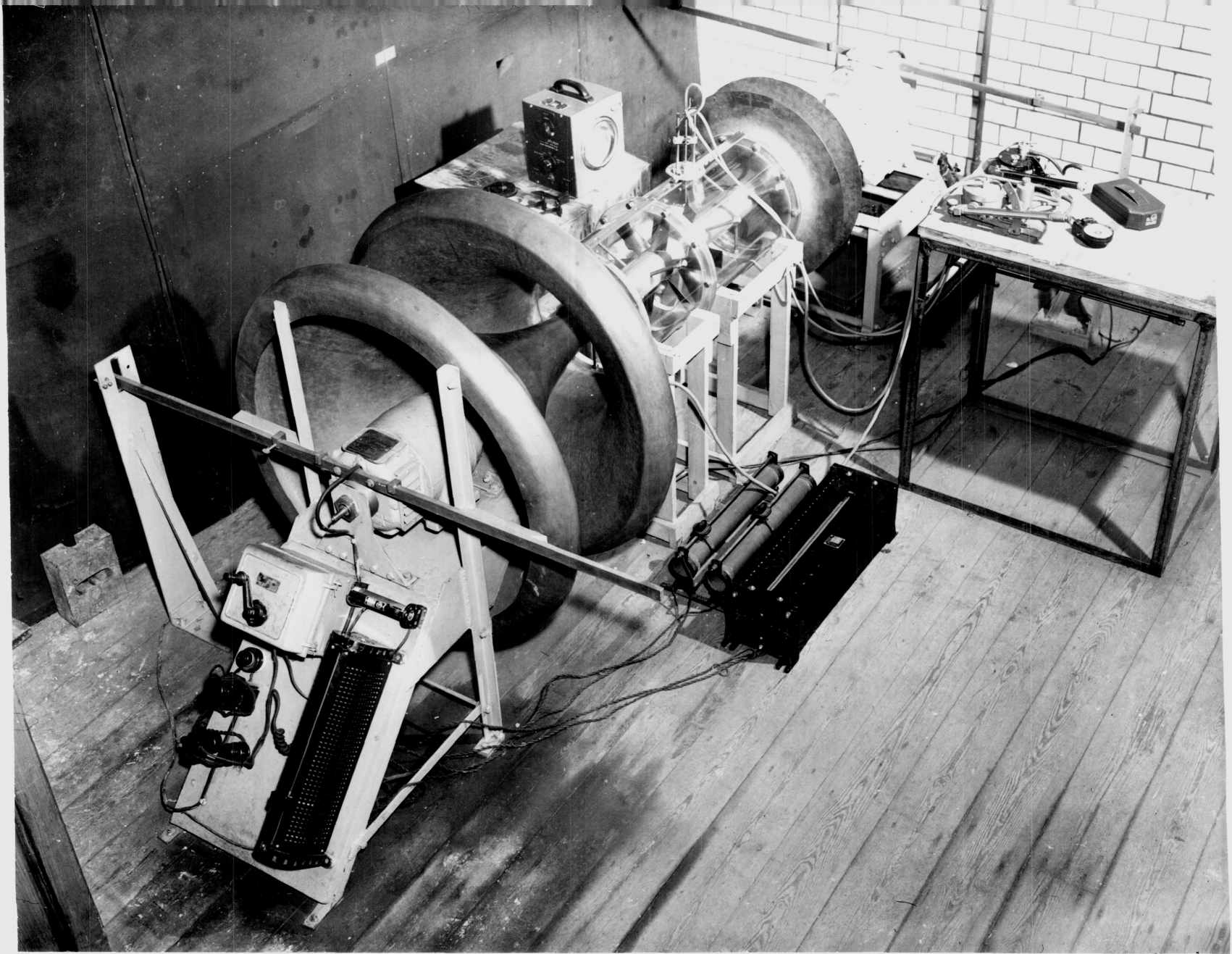
London University

THE PROPELLER AS APPLIED TO PUMPS AND FANS

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NOTATION

c	=	chord length	ft.
C_D	=	drag coefficient	
C_L	=	lift coefficient	
C_{Di}	=	coefficient of induced drag	
D	=	drag force per unit span	lb./ft.
D_i	=	induced drag per unit span	lb./ft.
h_c	=	manometric head coefficient	
	=	$H_m / \frac{1}{2} \rho (\Omega R_o)^2$	
H_m	=	manometric head	lb./ft. ²
L	=	lift force per unit span	lb./ft.
N	=	rotor speed	r.p.m.
n	=	rotor speed	r.p.s.
Q	=	flow rate	cusecs.
q_c	=	flow coefficient	
	=	Q / nD^3	
R_o	=	rotor tip radius	ft.
R_H	=	hub radius	ft.
S	=	blade spacing	ft.
S/c	=	solidity ratio	
T_c	=	torque coefficient	
	=	$T / \frac{1}{2} \rho n^2 D^5$	

NOTATION

T	=	torque ft.lb.
$\bar{U}$	=	mean axial velocity ft./sec.
α	=	angle between flow and axial direction
θ	=	camber angle
ν	=	kinematic viscosity
	=	1.59×10^{-4} sq.ft./sec.
ρ	=	fluid density
	=	0.002378 slug/cu.ft.
σ	=	stagger angle, defined as the angle between axial direction and the blade chord
Ω	=	rotor speed radians/sec.

Suffixes

1	=	upstream of the rotor, or the upstream rotor in case of the contra-rotators
2	=	downstream of the rotor, or the downstream rotor in case of the contra-rotators.

Part I

- i - Summary of the present Aerofoil and Cascade theories.
- ii - Secondary motions that render the above theories inadequate.

There is no doubt that much information about the details of motion in axial flow machines can be obtained from the so called aerofoil theory based on the concept of the blades of the machine functioning rather as deflecting bodies than as the walls of curved passages through which the fluid flows as in a pipe. Before proceeding to discuss this theory, certain conventional quantities describing the aerofoil need definition. . Two lines, the chord and the camber line, are used as a basis. The chord is usually defined as the line joining the mid-points of the leading and trailing edges. For simplicity, the tangent to the concave surface of the profile is sometimes used in place of, and described as the chord. The curved centre line of the aerofoil is known as the camber line. Usually it is of simple form, either an arc of a circle or a parabola, though other more complicated curves are also used and have their advantages. Measurements normal to the camber line and on either side of it serve to define the actual shape of the profile, and a single set of such measurements can, if scaled up or down, serve to define a family of aerofoils which are then described as all having the same "base profile".

Fig. 1.1, illustrates an aerofoil and defines the following items:-

- c = chord length ,
- t = maximum thickness ,
- b = maximum camber ,
- a = distance from the leading edge to the point of maximum camber ,
- r_1 = the radius of curvature at the leading edge ,
- r_2 = the radius of curvature at the trailing edge ,

all of which are usually expressed as a percentage of the chord length.

χ_1 and χ_2 = the angles between the chord and the tangents to the camber line at the leading and trailing edges respectively ,

$$\begin{aligned}\theta &= \text{the camber angle} \\ &= \chi_1 + \chi_2 .\end{aligned}$$

The conventional nomenclature specifying a certain aerofoil takes the following form:- 12 R.A.F. 6 / 25 P 40, where (12) is the maximum thickness expressed as a percent of the chord, R.A.F.6 denotes the selected base profile, (25) is the camber angle in degrees, P denotes a parabolic camber line, and (40) is the distance from the leading edge to the point of maximum camber as a percent of the chord.

The position of the aerofoil with respect to the

direction of the flow is specified by the angle between the flow direction and the chord. This angle is usually called the geometrical angle of incidence " α ".

An aerofoil of an infinite span, placed in an infinite stream moving with a uniform velocity "V", will exercise a resultant force "R". This force can be resolved into two components, along and normal to the direction of the relative motion. The normal component is usually known as the lift, while the other component as the drag. The magnitude for these two components can be put in the form:

$$L = \frac{1}{2} \rho V^2 \cdot c \cdot C_L ,$$

and,

$$D = \frac{1}{2} \rho V^2 \cdot c \cdot C_D ,$$

where:

L = Lift force per unit length of span,

D = Drag force per unit length of span,

C_L = Lift coefficient,

C_D = Drag coefficient,

c = Chord Length,

V = The undisturbed fluid velocity,

ρ = Fluid density.

The lift coefficient is a function of the angle of attack " α ". It increases with increasing values of the angle of attack, up to a certain maximum value corresponding to the usually called critical angle of incidence.

Beyond this value the aerofoil is stalled. For values of " α " below the critical value the function is approximately linear, and in most cases does not become zero until " α " is considerably negative.

The Circulatory Flow Around an Aerofoil:

The force experienced by the aerofoil is due mainly to the difference in the pressures acting over the lower and the upper surfaces of the profile, which is accompanied by velocity differences. It becomes apparent that there is a certain asymmetry in the distribution of the motion along both sides.

In order to obtain a measure of this asymmetry, use is made of the conception of circulation, defined as the line integral of the velocity along a closed curve encircling the aerofoil, or in other words:

$$\Gamma = \oint V_s \cdot ds .$$

The same flow pattern can be obtained in the neighbourhood of the aerofoil, if it is replaced by a vortex, usually known as the bound vortex, the strength of which is the same as the circulation around the aerofoil.

For this case of plane motion, the Kutta-Joukowski theorem gives the mathematical relation between the circulation around the aerofoil and the lifting force

experienced by it. This theorem states that:

$$L = \rho \cdot V \cdot \Gamma \quad ,$$

or substituting the lift in terms of the lift coefficient, we get:

$$C_L = 2 \Gamma / V \cdot c \quad .$$

As consequence of the theorem that vortices cannot end or begin within a frictionless fluid, it becomes evident that for an aerofoil with finite span, in an unlimited fluid, the "bound vortex" must extend out into the flow to form two trailing vortices, and that all together must form one continuous vortex ring. The trailing vortices usually have their axes normal to that of the bound vortex and lie in a direction parallel to the relative motion. Each of these trailing vortices is of a strength equivalent to that of the bound vortex. Fig. 1.2, illustrates diagrammatically the bound and trailing vortices in relation to the aerofoil. F.W. Lanchester was the first to suggest the existence of the trailing vortices.

It is necessary to mention here that the vortices represented in Fig.1.2, correspond to an aerofoil with constant circulation all along its span. In general, when, as may happen, the circulation varies along the span, there will be other trailing vortices. If, for instance, the circulation at sections a-a and b-b of the aerofoil were Γ_a and Γ_b respectively, then there would exist trailing vortices between these two sections of a total strength

$(\Gamma_a - \Gamma_b)$. If, however, the circulation varies gradually along the span, there will be a whole sheet of trailing vortices extending downstream from the aerofoil. The strength of a vortex sheet is measured by the circulation around a strip of the sheet having a unit length in the direction normal to the axis of the vortices. A vortex sheet, as that shown in Fig. 1.3, will consist of an infinite number of vortices, uniformly distributed across the whole width of the sheet. Hence the strength of such a sheet will be:

$$\bar{\gamma} = v_{x_2} - v_{x_1}$$

where v_{x_1} , and v_{x_2} , are the velocities along both sides of the sheet.

Fig. 1.4 shows a vortex sheet of a strength $\bar{\gamma}$, with its axes parallel to the z-axis. The fluid velocities along both sides of this sheet are v_1 and v_2 , which for simplicity reasons will be assumed to lie in the plane of the sheet. It is clear that the components of these velocities along the Z-axis must be the same. The strength of this vortex sheet will be equal to the vectorial difference between the two velocities:

$$\vec{\gamma} = \vec{v}_2 - \vec{v}_1$$

Now if $\vec{V}_m$, is the vectorial mean of these two velocities and which makes an angle θ with the Z-direction, then the following equations can be reached:

$$\begin{aligned} v_1^2 &= (v_m + \frac{1}{2} \bar{\gamma} \sin \theta)^2 + (\frac{1}{2} \bar{\gamma} \cos \theta)^2 \\ v_2^2 &= (v_m - \frac{1}{2} \bar{\gamma} \sin \theta)^2 + (\frac{1}{2} \bar{\gamma} \cos \theta)^2 \end{aligned}$$

Applying Bernouilli's equation the following is obtained:

$$p_2 - p_1 = \frac{1}{2} \rho (v_1^2 - v_2^2) ,$$

where p_1 and p_2 are the pressures on both sides of the sheet.

$$\begin{aligned} \text{Hence, } p_2 - p_1 &= \frac{1}{2} \rho (v_1^2 - v_2^2) \\ &= \rho \bar{\gamma} v_m \sin \theta . \end{aligned}$$

The force exerted by the sheet upon the fluid, is of the same magnitude as the pressure difference and in the opposite direction. Hence:

$$f = \rho |\bar{\gamma}| v_m \sin \theta . \quad (1.1)$$

Generally speaking the fluid motion is in a direction normal to the axis of the vorticity, as in the case of the bound vortex; hence the expression for the experienced force will become:

$$f = \rho \bar{\gamma} v_m . \quad (1.2)$$

The resemblance between the above expression and Kutta-Joukowski theorem, is worth mentioning. In fact it is a special form of this theorem, with the exception that it applies only to an element of the aerofoil. For the case of plane two dimensional motion, in summing up the forces on all elements of the aerofoil, the terms due to additional velocities will cancel. The resulting expression for the lift force per unit span will only include the velocity of the undisturbed stream "V". This is due to the fact that for this case, there will be no trailing vortices. The only existing vortices are the bound vortices, which are of equal strength and have their

axes normal to the main flow.

Regarding the effect of the trailing vortices, Fig. 1.5, these will cause additional downward velocities downstream of the aerofoil. Hence, according to equation (1.2), there will be an additional force acting from the aerofoil upon the fluid, and against the relative motion. This force evidently, must be the reaction of that resistance, experienced by the aerofoil, usually referred to as the induced drag. If it is assumed that this downward velocity " w' " is uniform along the span, the expression for this additional resistance per unit span will be:

$$D_i = \rho \Gamma w'.$$

The following expression for the induced drag is due to a reasoning given by Prandtl. Instead of considering velocities in the wake of the aerofoil, it will be assumed that the action of the aerofoil upon the fluid is confined to an area " A ", which is normal to the direction of the relative motion.

In passing through this area the fluid experiences the reaction of the lift force, hence a downward momentum will be communicated to it. Hence the lift force will be equivalent to this acquired downward momentum, and can be expressed thus:

$$L = \rho A V w',$$

or,

$$w' = L / \rho A V,$$

the kinetic energy imparted to the fluid will be:

$$E = \frac{1}{2} \rho V A w^2$$

and the work expended in generating this kinetic energy will be:

$$D_i \cdot V,$$

hence the expression for the induced drag will be:

$$D_i = E/V = L^2/2 \rho V^2 A.$$

This equation does not give a definite value for the induced drag, unless the value of the area "A" is known.

It is found that a good approximation for the value of this area, is given by the area of the circle described by the span of the aerofoil. In which case the expression for the induced drag will be:

$$D_i = L^2/2 \rho V^2 \pi b^2,$$

where,

$$2b = \text{the span of the aerofoil.}$$

It is feasible to regard half this downward velocity as being imparted to the fluid before reaching the aerofoil, the other half being generated after passing the aerofoil. Hence at the aerofoil there will exist a downward velocity "w",

$$w = w'/2 = L/2 \rho V \pi b^2.$$

When this velocity "w" is combined with the original fluid velocity "V", it will give a new resultant velocity: $(V^2 + w^2)$, making an angle " ϕ " with the velocity V, where

$$\tan \phi = w' / 2V ,$$

which can be put approximately as:

$$\phi \doteq w' / 2V .$$

The fact that lift is necessarily accompanied by the induced drag, was first pointed out by F.W. Lanchester.

Denoting the profile drag by D_o , the following expression for total drag can be obtained:

$$D = D_o + D_i ,$$

or in the coefficient form,

$$C_D = C_{D_o} + C_{D_i} .$$

It can be shown that the coefficient of induced drag has the form:

$$C_{D_i} = (C_L^2 / \pi) \cdot (S / (2b)^2) ,$$

where,

S = The projected area of the aerofoil,

and,

$$\lambda = \text{Aspect ratio} = (2b)^2 / S .$$

Hence,

$$C_{D_i} = C_L^2 / \pi \lambda . \quad (1.4)$$

If the angle of attack is corrected to take into account this downward inclination " ϕ ", the system of trailing vortices can be assumed not to exist. Hence an effective angle of attack " i ", is given by:

$$i = \alpha - \phi = \alpha - C_L / \pi \lambda . \quad (1.5)$$

The corrected angle of attack, is shown diagrammatically in Fig. 1.6.

Assuming that the aerofoil can be replaced by a "loaded line" of a length equal to the span, i.e. "2b", and lying along the Y-axis, Fig. 1.7, then the loading of this line per unit length will be:

$$\Lambda = \rho V \Gamma .$$

If the circulation varies along the span, there will be, as previously mentioned, other trailing vortices other than the two corresponding to the constant circulation case. The strength of these secondary vortices will be equivalent to:

$$\gamma_x = \left(\frac{1}{\rho} \cdot V \right) \times \left(d\Lambda/dy \right) ,$$

which can be put in the form,

$$\gamma_x = d\Gamma/dy .$$

Before proceeding to discuss cascades of aerofoils, it is convenient to consider thin plate aerofoils of simple geometrical forms. Though such aerofoils are of no practical importance, yet their aerodynamical characteristics are of great use. These characteristics apply with good approximation to cases of actual profiles.

First the case of thin flat plate, Fig. 1.8, will be discussed. For this case the mathematical solution, for instance by conformal transformation, gives the following relation between the angle of attack " α ", and the lift coefficient:

$$C_L = 2 \pi \sin \alpha ,$$

which for small values of " α " becomes:

$$C_L = 2\pi \sin \alpha \doteq 2\pi \alpha$$

It is evident that for this case the lift-angle of attack relation is linear, and has a slope " 2π ".

For the case of circular arc profile, Fig. 1.9, the theoretical solution gives:

$$C_L = 2\pi \sin(\alpha + \theta/4) / \cos \theta/4 ,$$

where,

θ = the camber angle.

For small values of camber, the above relation becomes,

$$C_L = 2\pi \sin(\alpha + \theta/4) = 2\pi \sin(\alpha + 2f/c) ,$$

as it can be easily proved that:

$$\theta = 8f/c .$$

Also, it is apparent that lift vanishes at an angle of attack:

$$\alpha_0 = -\theta/4 = -2f/c .$$

The flow direction corresponding to this angle, is usually referred to as the line of zero lift. Hence this line can be obtained by joining the trailing edge and the point of maximum camber. Practical evidence shows that this rule holds with good approximation in the case of actual profiles.

Birnbaum, introduced a general method for calculating the aerodynamic characteristics of any given profile. This method is based on assuming that the profile can be replaced by the camber line, which in effect, represents the mean for the upper and lower surfaces of the profile. Further by

assuming that the vorticity is located on the chord, it can be shown that the lift coefficient takes the following form:

$$C_L = 2\pi (\alpha + \epsilon_0),$$

where ϵ_0 , can be evaluated once the geometrical form of the camber line is known.

Theoretical solution for a family of Joukowski aerofoils gives the following expression for the lift coefficient,

$$C_L = 2\pi \sin\alpha (1 + 0.77 t/c),$$

where,

t = maximum thickness,

c = chord.

This means that with increasing thickness, steeper lift-angle of attack curves can be obtained; which is not the actual case. The divergence between theory and practice is due to neglecting viscous action near the surface in the theoretical approach.

An explanation due to Penkerton attributes this divergence to the unequal growth of the boundary layers on both sides of the profile. Hence the "effective profile" about which the non-viscous theoretical flow would realise, is changed by changing the angle of attack. Thus one could say that the actual lift curve is a composite of a great number of theoretical aerofoils each developing the theoretical lift curve slope, but having decreasing camber.

In the previous discussion little attention was paid to forces of an adverse nature, which are usually referred to as drag. One form of it was previously mentioned as the induced drag, which as a matter of fact, is an unavoidable bye-product of the lift. As mentioned before the coefficient for the induced drag takes the form:

$$C_{D_i} = k \cdot C_L^2 \quad (1.6)$$

where "k" is a constant depending on the geometrical particulars of the aerofoil used. Apart from the induced drag, there are two other forms of drag; one due to skin friction, and the other is what is usually known as "form", or "shape" drag. It is rather difficult to express the latter two explicitly, and hence their combined effect is usually summed up, and referred to simply by the "drag". It is possible to evaluate the drag experienced by the aerofoil if the velocity along its surfaces outside the boundary layer zones, were known. The method for such evaluation will be discussed briefly hereafter.

The expression for the drag can be put in the following form:

$$\begin{aligned} D &= C_D \cdot \frac{1}{2} U^2 \cdot c \\ &= \rho \int_{-\infty}^{\infty} u (U - u) dy , \end{aligned}$$

where,

u = velocity in the wake,

U = velocity of the main stream.

The integration is to be taken in the wake at a point

sufficiently far downstream to justify the assumption that the static pressure in the wake, at that section, is the same as that of the main stream. Hence the coefficient of drag will be:

$$C_D = \frac{2}{c} \int_{-\infty}^{\infty} \frac{u}{U_o} \left(1 - \frac{u}{U_o}\right) dy$$

$$= \frac{2\delta^{**}}{c},$$

where,

$$\delta^{**} = \int_{-\infty}^{\infty} \frac{u}{U_o} \left(1 - \frac{u}{U_o}\right) dy$$

$$= \text{the momentum thickness,}$$

and, U_o = the stream velocity outside the wake.

It is assumed that the momentum thickness of the wake, is equal to that of the boundary layers of the upper and lower surfaces of the aerofoil. For a laminar boundary layer Karman's solution can be used. This gives the skin friction at any point on the surface as:

$$\tau_o = \mu \delta u / \delta y = \mu (\lambda + 12) U / 6 \delta,$$

and the coefficient for skin friction as

$$C_f = 2 \tau_o / \rho U_o^2 = (\lambda + 12) \nu U / 3 \delta U_o^2$$

where,

U = velocity at the edge of the boundary layer,

δ = boundary layer thickness,

$\lambda = \delta^2 / \nu$,

U_o = velocity outside the wake.

Howarth gave the following expression for the momentum thickness in the case of laminar boundary layer, based on Karman-Pohlhausen's method:

$$\delta^{**}/\delta = (5328 - 48 \lambda - 5 \lambda^2)/45360.$$

By this method it is possible to evaluate the drag, provided that the boundary layer remained laminar all along the aerofoil surfaces. Experimental evidence, however, shows that the laminar boundary layer changes over to a turbulent one shortly downstream the leading edge, after passing a transition point not far from the leading edge. Therefore, Pohlhausen's method can only apply over the short region between the leading edge and the transition point. Beyond this point equations for the turbulent boundary layers should be applied. At the transition point it can be assumed that the momentum thickness is continuous and that the skin friction undergoes a rapid increase over a short region near the transition point.

Karman's momentum equation for boundary layers, can be put in the form:

$$\tau_o = dM/dx + \rho \delta^* U dU/dx$$

where

$$M = \rho \int U^2 \delta^{**}$$

$$\delta^* = \int_0^{\delta} (1 - u/U) dy ,$$

$$y = \text{distance measured normal to the surface.}$$

Putting,

$$H = \delta^*/\delta^{**},$$

the equation of momentum becomes:

$$\tau_o / \rho U^2 = d \delta^{**} / dx + (U' / U) (H+2) \delta^{**} . \quad (1.7)$$

Young showed that by assuming that Karman's solution for turbulent boundary layer along flat plates, serves as an approximation for the existing case, and making use of the results of Nikuradse and Buri's tests on diverging and converging channels, which indicate that it is sufficiently accurate to assume a constant value of (1.4), for "H", equation (1.7) can be written as:

$$d \zeta / dx + 6.13 U' / U = U / \nu \cdot F(\zeta) ,$$

where,

$$F(\zeta) = 10.411 \zeta^{-2} e^{-0.3914 \zeta}$$

and $\zeta^2 = \rho U^2 / \tau_o$

For solving this equation, it is necessary to know the value of "U" as a function of "x" and " ζ " at the transition point. For obtaining the momentum thickness at any point, the relation for turbulent boundary layer along flat plate can be used. This relation is as follows:

$$U \delta^{**} / \nu = 0.2454 e^{0.3914 \zeta}$$

At a traverse in the wake, equation (1.7) becomes,

$$d \delta^{**} / dx + U' / U \cdot (H+2) \delta^{**} = 0,$$

where "x" is measured downstream, along the centre line of the wake. This equation can be put in the following form:

$$d/dx (\log_e \delta^{**}) = -(H+2) d/dx (\log_e U / U_o) . \quad (1.8)$$

Integrating equation (1.8), from the trailing edge

to infinity, and making suitable assumptions, Young gave the following expression for the drag:

$$C_D = 2(\delta_1^{xx}/c)(U_1/U_\infty)^{3/2} \quad (1.9)$$

where,

δ_1^{xx} = the momentum thickness of the wake at the trailing edge, which is assumed to be equal to the sum of the momentum thickness of the two boundary layers corresponding to both surfaces of the aerofoil at the trailing edge,

U_1 = velocity at the trailing edge,

U_∞ = velocity of the main stream at infinity.

Young gave a comparison between the values of drag coefficients, as obtained by the above method and by experimental measurements. This showed close agreement between both. One point worth mentioning, is that the value of the drag coefficient thus computed, depends to a great extent on where the transition point is assumed to be. In axial flow machines, however, it is expected that the boundary layer is more likely to become turbulent at a point much closer to the leading edge, than in the case of an aerofoil in a non-turbulent stream. Hence a transition point within 10% of the chord from the leading edge can be assumed feasible. With a transition point within such a percentage, Young showed that the

computed drag coefficient becomes less sensitive to where the transition point is assumed to be.

Recently the flow problem through blade cascades was the subject for intensive study by several investigators, both from the practical and theoretical points of view.

Generally speaking, a cascade can be formed of thin flat plates, thin circular arcs, or aerofoils. Cascades formed of thin plates are of no practical importance, nevertheless, they serve as an approximation for simplifying the theoretical approach.

Fig. (1.10)(a, b, and c), illustrates cascades formed of thin flat plates, circular arcs, and aerofoils, and define the following items:

Suffixes 1 and 2 denote the inlet and outlet sides of the cascade.

S = blade spacing or pitch,

α = the angle between the flow and axial directions,

σ = the stagger, defined as the angle between the axial direction and the blade chord,

ϵ = the turning angle, defined as the angle through which the fluid is turned

$$= \alpha_1 - \alpha_2,$$

i = the angle of incidence, defined as the angle between the flow direction at inlet, and the tangent to the camber line at the leading edge

of the blade,

V_r = the resultant fluid velocity,

w_r = the "whirl", or the tangential component of the fluid velocity,

U = the axial flow component,

δ = the deviation, defined as the angle between the flow direction at outlet, and the tangent to the camber line at the trailing edge of the aerofoil,

θ = the camber angle, which is the angle between the tangents to the camber line at the trailing and leading edges,

s/c = solidity ratio, defined as the ratio between the blade spacing and the chord.

The flow through a cascade of thin flat plates, was theoretically solved by F. Weinig. His solution gives a correction factor for the theoretical value of the lift coefficient for a single flat plate. The expression for the corrected lift coefficient is:

$$C_{Lc} = f \cdot 2\pi \sin(\alpha_m - \sigma)$$

where,

f = the correction factor,

α_m = the angle between the vectorial mean of the inlet and outlet velocities, and the axial direction, or in other words,

$$\tan \alpha_m = \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2)$$

A. Collar gave the value for this correction factor as:

$$f = \sin a / (1 - \sin^2 a \cdot \sin^2 \sigma)^{1/2} \{ \cos \sigma \sinh^{-1} (\tan a \cdot \cos \sigma) + \sin \sigma \sinh^{-1} (\sin a \sin \sigma) \},$$

where "a" is a function of solidity ratio, and the stagger, and is given as:

$$s/c = \pi/2 \cdot \{ \cos \sigma \sinh^{-1} (\cos \sigma \tan a) + \sin \sigma \sinh^{-1} (\sin \sigma \sin a) \}.$$

Several investigators solved the flow problem through cascades of arbitrary aerofoils by successive transformation. These methods being too elaborate and laborious, beside the three-dimensional nature of the flow, limits the usefulness of such methods to their academic value.

The following discussion is based on the assumption that two dimensional flow prevails, and that the complications due to the existing viscous forces can be disregarded for the time being.

Fig. 1.11 illustrates a cascade of aerofoils and the corresponding inlet and outlet velocity diagrams. For simplicity, the aerofoils are replaced by their chords and camber lines.

It is obvious that the circulation along a closed contour encircling any of the aerofoils is:

$$\begin{aligned} \Gamma &= \oint \mathbf{v}_s \cdot d\mathbf{s} = (w_{r_1} - w_{r_2}) \cdot S \\ &= \Delta w_r \cdot S \end{aligned} \quad (1.10)$$

The tangential force experienced by the aerofoil can

be evaluated by applying Euler's equation, thus giving:

$$\begin{aligned} f_t &= \rho U S (w_{r_1} - w_{r_2}) \\ &= \rho U \Gamma, \end{aligned}$$

where,

f_t = the tangential force per unit depth of span.

The force experienced in the axial direction will be only due to the static pressure rise, as it is assumed that the momentum along that direction does not change. Hence for ideal fluid this force will be:

$$\begin{aligned} f_a &= S(p_2 - p_1) \\ &= S \cdot \rho / 2 (w_{r_1}^2 - w_{r_2}^2) \\ &= \rho (w_{r_1} + w_{r_2}) / 2 \cdot \Gamma \\ &= \rho \cdot \overline{w_r} \cdot \Gamma, \end{aligned} \tag{1.12}$$

where,

$\overline{w_r}$ = the arithmetical mean for the tangential components upstream and downstream of the cascade.

The resemblance between the above equations and the Kutta-Joukowski theorem is obvious. The expression for the resultant force experienced by the aerofoil can be put in the form:

$$F = \rho \cdot V_m \cdot \Gamma \tag{1.13}$$

where,

V_m = the vectorial mean flow velocity.

The angle between this velocity and the axial direction will be:

$$\alpha_m = \tan^{-1} \overline{w_r} / U \tag{1.14}$$

The experienced force can be put in terms of a corrected lift coefficient, by using the following relation:

$$C_{L_c} = \frac{2\Gamma}{V_m \cdot c} \quad , \quad (1.15)$$

Hence substituting in equation (1.13), we get the following expression:

$$F = \rho \cdot \frac{V_m}{2} c \cdot C_{L_c} \quad , \quad (1.16)$$

where,

C_{L_c} = the lift coefficient corrected for cascade effect.

The above outlines the simple two dimensional theory for ideal flow through cascades. It shows that a cascade of aerofoils can be replaced by one single aerofoil, in a hypothetical infinite and uniform stream, moving with a velocity equivalent to the vectorial mean of actual upstream and downstream velocities.

For a real fluid flow, however, it is necessary to make some adjustments to take into account viscosity effects. For a single aerofoil, viscosity manifests its existence by the losses usually referred to as drag. In the case of a cascade of blades, which in effect is a cascade of diffusers, the losses will take the form of a drop in total energy. Though the losses in both cases are akin, yet they are of a more complicated nature in the later.

Denoting the mean drop in total head through the cascade by: $\overline{\delta P_L}$, equation (1.12) will become:

$$\begin{aligned} f_a &= S \{ (P_2 - P_1) - \overline{\delta P_L} \} \\ &= S \left\{ \rho/2 V^2 (\tan^2 \alpha_1 - \tan^2 \alpha) - \overline{\delta P_L} \right\} . \quad (1.17) \end{aligned}$$

The expression for the tangential component will remain unaltered. Resolving these two components along, and normal to the vectorial mean flow velocity, fig. (1.11), the following expressions for the hypothetical lift and drag will be obtained:

$$\begin{aligned} L &= c \cdot \rho/2 \cdot V_m^2 \cdot C_L \\ &= f_t \cos \alpha_m + f_a \sin \alpha_m , \end{aligned} \quad (1.18)$$

and,

$$\begin{aligned} D &= c \cdot \rho/2 V_m^2 \cdot C_D \\ &= f_t \sin \alpha_m - f_a \cos \alpha_m . \end{aligned} \quad (1.19)$$

Hence, it can be shown that:

$$\begin{aligned} C_L &= 2 \cdot \frac{S}{c} (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m + \\ &\quad + C_D \tan \alpha_m , \end{aligned} \quad (1.20)$$

and,

$$C_D = \frac{S}{c} \cdot \frac{\cos^3 \alpha_m}{\rho/2 V_1^2 \cos^2 \alpha_1} \cdot \overline{\delta P_L} . \quad (1.21)$$

For the case of axial flow machines, a change in axial velocity is usually encountered. Tests on two dimensional cascades with varying axial velocity, are not easy to carry out, which beside the absence of Coriolis forces, will reduce their practical value for studying the flow through a rotating cascade, where under such operating condition, these forces will exist.

Nevertheless, it is usually assumed that the vectorial mean flow method can be applied for such cases as an approximation. The error due to this assumption is, however, less serious in case of two dimensional cascades, and axial rotors with large hub to outer diameter ratio, and relatively small blade height in comparison with the outer diameter, e.g. axial flow compressors.

Fig. 1.11, illustrates velocity diagrams for constant as well as varying axial flow velocity. For varying axial velocity, the experienced tangential force can be put as:

$$f_t = \rho \cdot S \cdot \frac{U_1 + U_2}{2} (w_{r1} - w_{r2}),$$

while the axial force will be partly due to the static pressure rise, and partly to the change in the axial momentum, hence,

$$\begin{aligned} f_a &= S(P_2 - P_1) + \frac{U_1 + U_2}{2} \cdot S \cdot \rho (U_1 - U_2) \\ &= S \cdot (H_1 - H_2), \end{aligned}$$

where,

H = the total energy.

It can be shown that the corresponding lift and drag coefficients will be:

$$C_L = 2 \cdot \frac{S}{c} \cdot \bar{C} \cdot \cos^3 \alpha_m + \frac{S}{c} C_H \cdot \sin \alpha_m \quad (1.22)$$

where,

$$\bar{C} = \frac{w_{r1} - w_{r2}}{\bar{U}},$$

and,

C_H = the coefficient of the change in the total energy

$$= \frac{H_1 - H_2}{\frac{1}{2} \rho V_m^2}$$

and,

$$C_D = 2 \cdot \frac{S}{c} \cdot \bar{E} \cdot \cos^2 \alpha_m \cdot \sin \alpha_m - \frac{S}{c} \cdot C_H \cdot \cos \alpha_m, \quad (1.23)$$

putting:

$$\begin{aligned} C_T &= \text{coefficient for the tangential force} \\ &= 2 \frac{S}{c} \cdot \bar{E} \cdot \cos^2 \alpha_m, \end{aligned}$$

and,

$$\begin{aligned} C_S &= \text{coefficient for the axial force} \\ &= \frac{S}{c} \cdot C_H, \end{aligned}$$

equations (1.22) and (1.23) can be reduced to:

$$C_L = C_T \cos \alpha_m + C_S \sin \alpha_m, \quad (1.24)$$

and,

$$C_D = C_T \sin \alpha_m - C_S \cos \alpha_m. \quad (1.25)$$

In the previous discussion, two dimensional flow was assumed to prevail. This, to say the least, represents nothing but a rough approximation to what is actually taking place. Deviation from this simplified flow picture is due to secondary motions brought about by the existing viscous forces.

As mentioned before, a cascade of aerofoils, can be looked upon as a series of curved channels. Hence, it is justified to expect a secondary flow similar to that known

to take place in curved conduits, fig. 1.12. The explanation for this type of secondary motion is based on the need for a certain adjustment in the flow pattern to attain a balance between centrifugal forces and the prevailing pressure gradients. White and Adler, both found this secondary motion experimentally for laminar flow. The former observed that it was by no means as evident once the flow became turbulent. This, he ascribed to the fact that in turbulent flow the velocity distribution is much more nearly uniform than in laminar flow, and correspondingly the pressure gradients producing the secondary motion are less. However for the case of cascades, where curvature is usually small, and the flow is more likely to be turbulent, such type of secondary motion will not exist, or at least not in an appreciable strength.

Another type of secondary motion is that caused by trailing vortices shed downstream of the cascade, fig.1.13(a). Though the circulation " Γ ", is assumed to be constant along the aerofoil span, yet it is inevitable that at the boundary walls circulation must become zero. Hence, within the boundary layers its magnitude will gradually change from its normal value " Γ " down to the zero value at the walls, 1.13(b). Consequently, within the boundary layers, a large number of trailing vortices will be developed, the strength of which can be put in the form:

$$\bar{\gamma} = \frac{d\Gamma}{dy} \cdot \delta_y ,$$

these being of an unstable nature, will finally roll up in two main trailing vortices, each of a total strength Γ . Experimental evidence by several investigators emphasised the existence of such vortices.

Replacing the cascade by these trailing vortices, and solving for the induced velocities, Carter and Cohen deduced the following expression for the coefficient of induced drag:

$$C_{D_i} = \frac{1/4 C_L^2}{S/c} \left(1 - \frac{h'}{h} \right) \frac{h'}{h}, \quad (1.26)$$

where,

$2h$ = the span of the blades,

$2h'$ = the distance between the cores of the trailing vortices, which by measurement is found to be slightly less than $2h$.

The resemblance between this equation and equation (1.4) is worth noting.

Howell gave the following empirical rule for the induced drag:

$$C_{D_i} = 0.018 C_L^2 \quad (1.27)$$

Furthermore, as the circulation is closely related to the velocity upstream of the cascade, it becomes obvious that any irregularity in the approach velocity will give rise to further secondary losses, as a consequence of the resulting non uniformity in the dis-

tribution of circulation. Moreover, it is plausible to expect that the upstream velocity will give a qualitative representation for the distribution of circulation.

As previously mentioned, the effect of these secondary losses is taken account of, by using a modified drag coefficient, as given by the following expression:

$$C_D = C_{D_0} + C_{D_i} + C_{D_a} ,$$

where,

C_D = the effective drag coefficient,

C_{D_0} = the drag coefficient corresponding to one isolated aerofoil of an infinite span,

C_{D_i} = coefficient of induced drag,

C_{D_a} = the drag coefficient giving the equivalent loss due to skin friction along the boundary walls. The effect of the adverse pressure gradient must be taken into account as it is apt to increase the skin friction.

Now the axial flow propeller will be considered. Any imaginary coaxial cylindrical surface will intersect the blades in what will form, when developed, a cascade of profiles, as that shown in Fig. 1.10. If the flow is assumed to be of a two dimensional nature, i.e. proceeding along coaxial and cylindrical stream surfaces, it becomes obvious that the flow pattern in one case can be transformed

to the other. In other words, assuming two dimensional flow, it is possible to say that the flow characteristics in one case can be used as a basis for predicting those of the other.

Though this, as shown experimentally (fig.1.20), is often far from being the case, yet it will help in forming an approximation to reality.

Before proceeding further, the general equations of motion will be discussed. The cylindrical polar coordinates are chosen as they match better the fluid field in this case. Fig. 1.14 shows these coordinates in relation to the rotor axis, which is chosen as the z-axis. The Cartesian coordinates will thus have the following relations:

$$x = r \cos \phi,$$

and,

$$y = r \sin \phi,$$

The axial, radial and tangential velocity components will be denoted by w , u , and v respectively.

The three dimensional equations for steady flow will be:

(a) - Equation of Continuity:

$$\frac{1}{r} \frac{\partial}{\partial r} (r u) + \frac{1}{r} \frac{\partial v}{\partial \phi} + \frac{\partial w}{\partial z} = 0 \quad (1.26)$$

(b) - Vorticity Equations:

$$\left. \begin{aligned} \xi &= \frac{1}{r} \frac{\partial w}{\partial \phi} - \frac{\partial v}{\partial z}, & (\text{radial}) \\ \eta &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}, & (\text{tangential}) \\ \zeta &= \frac{1}{r} \frac{\partial}{\partial r}(rv) - \frac{1}{r} \frac{\partial u}{\partial \phi}, & (\text{axial}) \end{aligned} \right\} (1.27)$$

(c) - Equations of motion:

$$\left. \begin{aligned} u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \phi} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \phi} \right), \\ u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \phi} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \\ &= -\frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \phi} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \phi} - \frac{v}{r^2} \right), \\ u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \phi} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w, \end{aligned} \right\} (1.28)$$

where,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}.$$

If non viscous and axially symmetrical flow is assumed, equations 1.26, 1.27 and 1.28 can be reduced to:

(a) - Equation of Continuity:

$$\frac{1}{r} \frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \quad (1.29)$$

(b) - Vorticity equations:

$$\left. \begin{aligned} \xi &= -\frac{\partial v}{\partial z} && \text{(radial)} \\ \eta &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} && \text{(tangential)} \\ \zeta &= \frac{1}{r} \frac{\partial}{\partial r} (r v) && \text{(axial)} \end{aligned} \right\} (1.30)$$

(c) - Equations of motion:

$$\left. \begin{aligned} u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} \\ u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + u \frac{v}{r} &= 0 \\ u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} \end{aligned} \right\} (1.31)$$

If now as an approximation, it is assumed that Bernouilli's equation can be applied, or in other words at any point in the given fluid field the following relation is valid as it is when the three vortices (1.30) are zero,

$$\begin{aligned} H &= p + \frac{\rho}{2} (u^2 + v^2 + w^2) \\ &= p + \frac{\rho}{2} V^2 \end{aligned}$$

or,

$$p = H - \frac{\rho}{2} V^2$$

where,

H = the total energy at that point,

p = the static pressure,

V = the resultant fluid velocity.

Then by eliminating the pressure gradients in equations (1.31), and substituting from equations (1.30), equation (1.31) can be put in terms of vorticity thus:

$$\left. \begin{aligned} w.\zeta - v.\xi &= -\frac{1}{r} \frac{\partial H}{\partial r} , \\ w.\xi - u.\zeta &= 0 , \\ \text{and} \\ v.\xi - u.\eta &= -\frac{1}{r} \frac{\partial H}{\partial z} . \end{aligned} \right\} \quad (1.32)$$

For two dimensional irrotational flow, where the stream surfaces are moving with uniform axial velocity along coaxial cylindrical surfaces, the following relations can be obtained from equations 1.30, 1.31 and 1.32:

$$\frac{\partial v}{\partial z} = 0 , \quad \frac{1}{r} \frac{\partial}{\partial r} (r v) = 0 , \quad \frac{v^2}{r} = \frac{1}{r} \frac{\partial p}{\partial r} \quad \text{and,}$$

$$\frac{\partial p}{\partial z} = \frac{\partial H}{\partial r} = \frac{\partial H}{\partial z} = 0 .$$

These imply the following facts relating to such flow:

- a - Angular momentum, and consequently the circulation are constant along any radius, and are independent of the z value.
- b - The total energy is also constant along any radius irrespective of z .

c - the centrifugal forces are balanced by a radial pressure gradient, which is expressed by:

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{v^2}{r} .$$

This type of flow is usually referred to as the free-vortex régime.

The design requirements for a propeller to yield this two dimensional flow will be discussed next. As mentioned before, such a propeller will consist of a large number of cascades, having the profiles of their blades lying on surfaces of coaxial cylinders. The geometrical particulars for each of these cascades, will be dictated by both the desired type of flow, as well as the learned facts from two dimensional cascade testing. The following notations will be used throughout the coming discussion, as well as those mentioned before for cascades:

R_o = tip radius,

Ω = impeller rotational speed in radians per second,

r_{rh} = the ratio between the hub radius to the tip radius,

r_r = the radius ratio at any point, expressed as the ratio of the radius r at that point, to that of the tip R_o ,

U = mean axial velocity,

u = axial velocity at any point of a radius
ratio r_r ,

v = tangential or 'whirl' velocity,

z = number of rotor blades.

Suffixes 1 and 2 will be used to designate points upstream and downstream of the impeller respectively.

Fig. 1.15 illustrates the blade element at radial station r and the corresponding velocity diagrams. The tangential and axial forces per unit depth per blade can be found by applying either Kutta-Joukowski theorem or Euler's and Bernouilli's equations. The expression for these will be

$$\left. \begin{aligned} f_t &= \gamma \cdot \frac{2\pi r}{z} \cdot v \cdot U \\ &= \gamma \cdot U \cdot \Gamma \end{aligned} \right\} \quad (1.33)$$

and,

$$f_a = \gamma \cdot \frac{2\pi r}{z} \cdot v \cdot \left(\Omega r - \frac{v}{2} \right) \quad (1.34)$$

where,

f_t , and f_a : are the tangential and axial forces
per blade per unit depth.

The condition for constant circulation can be used to help in selecting the profiles for the blade and their geometrical setting. The following relation can be used for that purpose:

$$\Gamma = c \cdot \frac{V_{r_m}}{2} \cdot C_L \quad (1.35)$$

However, such a two dimensional flow cannot be realized precisely; not even under the most favourable working conditions.

Viscosity as well as the existence of trailing vortices cause the actual pattern to differ from that ideal one.

Furthermore, even if such a flow pattern could be realised, then it will be only for one specific flow rate, usually referred to as the design or normal flow. Once the flow is shifted either beyond or below the design point, the balance between centrifugal forces, and the radial pressure gradient will be upset, thus giving rise to further secondary flow, that will restore the balance again. This type of secondary flow will be discussed next. For simplicity the one dimensional flow theory will be assumed. This implies that the relative outlet velocity will have, at all rates of flow, a constant inclination, corresponding to that of the outlet blade angle. In other words, the flow will behave as if it were guided by an infinite number of blades. Fig. 1.16 illustrates a blade element of such an imaginary cascade and the corresponding velocity diagrams for part, normal and over loads. If such a rotor is designed to give a free vortex régime at design point, then the tangential velocity at any radius will be given by:

$$v = K/r,$$

where,

$$K = \text{constant.}$$

Furthermore this tangential velocity can be expressed as:

$$v = \Omega r - U \cot \alpha_2 ,$$

where,

$$\cot \alpha_2 = \frac{U_n}{\Omega r - K/r} ,$$

U_n designates the axial flow velocity corresponding to normal or design condition.

Putting λ as the ratio between any specific flow rate to that corresponding to design point, the following expression for the whirl velocity can be obtained:

$$v = (1 - \lambda) \Omega r + \lambda \frac{K}{r} \quad (1.36)$$

Assuming a 100% hydraulic efficiency, then Euler's equation will give the rise in total energy at any radius r as:

$$E_r = \rho \cdot v \cdot \Omega r ,$$

and the corresponding pressure rise as:

$$p_r = E_r - \frac{\rho v^2}{2} ,$$

substituting from equation (1.36) this can be written as:

$$p_r = \frac{\rho}{2} \Omega^2 r^2 (1 - \lambda^2) + \rho \Omega K \lambda^2 - \frac{\rho \lambda^2 K^2}{2 r^2} \quad (1.37)$$

The inward radial pressure acting upon the annulus enclosed between the two coaxial cylindrical surfaces of radii r_1 and r_2 , will be:

$$p_2 - p_1 = \frac{\rho}{2} \Omega^2 (1 - \lambda^2) (r_2^2 - r_1^2) + \frac{\rho \lambda^2 K^2}{2} \left(\frac{r_2^2 - r_1^2}{r_1^2 r_2^2} \right) \quad (1.38)$$

This inward radial pressure will be counteracted by the centrifugal forces set up by the tangential velocities. The difference between these pressures will be the factor deciding the nature and strength of the resulting secondary flow.

The centrifugal forces acting upon the annulus can be expressed thus:

$$(p_2 - p_1)_{cent.} = \int_{r_1}^{r_2} dp = \rho \int_{r_1}^{r_2} v^2/r \, dr,$$

which when evaluated will give:

$$\begin{aligned} (p_2 - p_1)_{cent.} &= \rho \Omega^2 (1 - \lambda)^2 \left(\frac{r_2^2 - r_1^2}{2} \right) + \rho \lambda^2 K^2 \left(\frac{r_2^2 - r_1^2}{2 r_1^2 r_2^2} \right) \\ &\quad + 2 \rho \Omega K \lambda (1 - \lambda) \log_e \frac{r_2}{r_1}, \end{aligned} \quad (1.39)$$

Subtracting equation (1.38) from (1.39) the following expression for the unbalanced pressure force can be obtained:

$$P = \rho \lambda (1 - \lambda) \left[\Omega^2 (r_2^2 - r_1^2) + 2 \Omega K \log_e \frac{r_2}{r_1} \right] \quad (1.40)$$

This pressure force will give rise to secondary flow which will be some function of it. The important conclusion

that can be drawn from equation (1.40) is that for overloads, the resulting secondary flow will be radially inward, while for reduced loading, the reverse will take place.

The above calculation was made on the assumption that the secondary adjusting flow does not take place till the fluid has acquired its total energy and has passed downstream through the rotor, and that till then the fluid has been moving with a uniform axial velocity. This as will be shown later, is not the case; on the contrary it is believed that this adjusting flow comes into action well before the fluid has left the rotor. Nevertheless, it is more likely that this will not affect the conclusion reached about the direction of the secondary flow.

The unavoidable existence of secondary flows makes the two dimensional theory, not to mention the one dimensional approach, inadequate. It needs some adjustments before it can be claimed to predict the performance of a certain propeller. Hence it is justified to expect that such adjustment will yield better results if it were based on an accurate knowledge about the texture of these secondary flows. Therefore, the next step will be to present some experimental evidence that is hoped to throw light upon the nature of the secondary flows encountered in axial flow propellers. Before doing so, however, a brief description of the apparatus used for the present investigation will

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be given.

Plates (1 and 2) show front and inlet side views of the apparatus. Fig. 4.1, is a diagrammatic sketch showing the inlet bell mouth, the cylindrical casing and the outlet control gate. This apparatus was designed to provide for testing one, as well as two contra-rotating impellers. For that purpose two D.C. motors were incorporated to drive each rotor independently, in the later case. Both motors were carried in ball bearings and equipped with balance arms for torque measurements. The downstream motor was fitted with a "Newton-Derby" make and break speed governor. It was mounted to the motor frame to avoid its losses being included in torque measurements. Furthermore, special deflectors were fitted to direct the motor's cooling air in an axial direction, thus safeguarding against the possibility of unequal cooling air stream reactions affecting torque readings. Reohstats and potentiometers were included in each motor's circuit, thus providing for independent speed control over a range from 300 to 2600 r.p.m.

The casing surrounding the rotors was made of Perspex, convenient for visual observation. A stroboscope was used for speed measurement, as well as an aid for maintaining it constant.

Commercial propellers were used, as specially designed rotos were difficult to make. Nevertheless, it may be

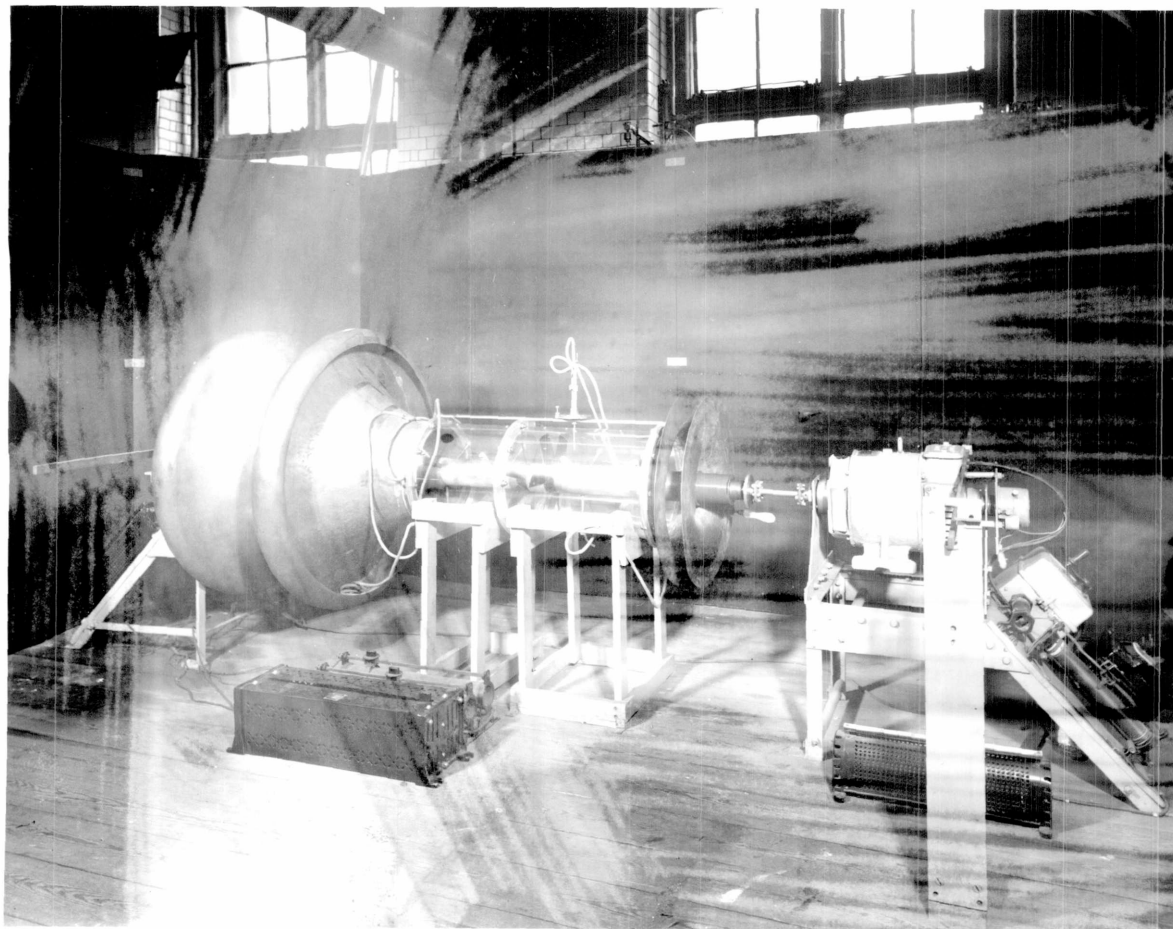


Plate 1

Plate 1 - Front view of the test apparatus.

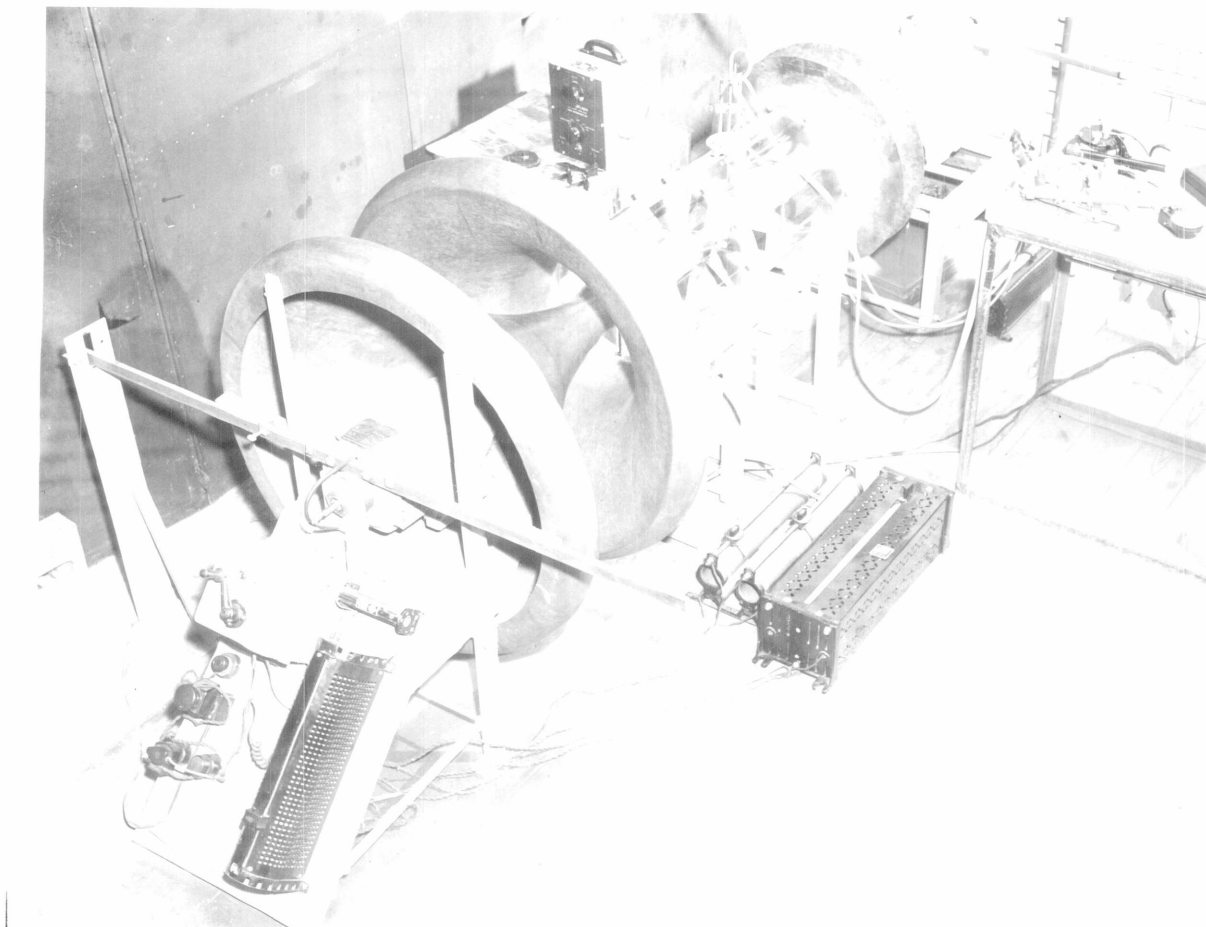


Plate 2 - View of the apparatus from the inlet side.

remarked that suitable plastic materials of Bakelite type are now available, and if a simple way can be found to construct a mould cheaply, the process of casting presents no further difficulty. A mould made of plaster of Paris was used to cast one circular back untwisted blade. The result was very satisfactory with regard to surface finish and the sharp leading and trailing edges.

The first type of secondary flow to be discussed is that due to operating at reduced or over loads. The experimental data, given hereafter, were obtained from the following tests:

(a) - A single rotor operating at a constant speed of 2200 R.P.M., and at varying flow rates. The layout of the testing rig is as shown in fig. 4.1 . Velocity traverses were taken upstream and downstream of the rotor at varying discharges.

(b) - A single rotor, followed 28 inches further downstream by a boosting propeller, fig. 4.1 . Similar velocity traverses were taken upstream and downstream of the first rotor at varying flow rates and at speeds ranging from 800 up to 2000 R.P.M.

The obvious criterion for secondary flow of this type is the ratio between the axial velocities upstream and downstream of the rotor. Taking this ratio $\xi_s = U_2/U_1$ as a factor expressing the strength of the secondary flow,

and plotting it at various upstream flow coefficients:

$$\varphi_1 = U_1 / \Omega r$$

where,

Ω = the rotor's speed expressed in radians per second,

and,

r = the radius at the point considered,

the curves shown in Figs. 1.17, 1.18 and 1.19 were obtained for the radii:

$$\frac{r}{R_0} = 0.76, 0.50 \text{ and } 0.37.$$

The one dimensional theory, as previously shown, lead to the conclusion that at any radial station, the secondary flow factor can be expressed as:

$$\xi = f [\lambda (1 - \lambda)] ,$$

where,

$$\lambda = \frac{U}{U_n} \gtrless 1.0 ,$$

which would yield a continuous function over the whole operating range, including over and part loads.

The noticable asymmetry of this function at part and over load, as experimentally found, figs. 1.17, 1.18, and 1.19, can, however, be explained by the fact that at these loads the blade setting in relation to the flow distinctly differs from each other. A symmetrical profile

with no camber, will probably give a secondary flow relation nearer to that reached by assuming the one dimensional theory.

The radial secondary flow at any radius r , can be put as:

$$q_r = \int_{r_h}^r (U_1 - U_2) \cdot 2\pi r dr ,$$

where,

r_h = the hub radius,

r = the radius at which the secondary flow is to be evaluated.

This can be expressed as a ratio to the upstream flow in the following way:

$$\frac{q_r}{q_a} = \frac{\int_{r_h}^r (U_1 - U_2) \cdot r dr}{\int_{r_h}^r U_1 \cdot r \cdot dr} , \quad (1.41)$$

using this expression, the following tables were obtained, where the radial secondary flow is given as a percentage of the upstream flow, at different radial stations.

TABLE I

One single rotor operating at 2200 r.p.m.

$q_c = Q_n D^3$	0.64	0.61	0.55	0.45	0.43	0.35
r/R_o	% of upstream flow					
0.37	22.1	26.8	32.5	67.1	83.3	117.0 *
0.50	10.1	17.0	25.6	49.5	58.5	84.5
0.63	3.4	9.0	13.7	29.5	36.7	50.0
0.76	0.9	3.2	5.6	12.3	17.7	22.6
0.89	- 2.9	1.2	2.4	3.4	4.0	6.1

* Back flow

TABLE II

One rotor followed 28 inches further downstream
by a boosting propeller

N r.p.m.	800	1000	1200	1300	2000	2000
$\eta_c = \frac{Q}{nD^3}$	1.28	1.05	0.93	0.83	0.54	0.43
r/R_0	% of upstream flow					
0.37	1.9	-4.1	-6.8	-4.4	38.8	84.7
0.50	5.1	0.4	-1.3	-0.6	29.7	52.5
0.63	8.6	5.3	1.8	-0.5	15.9	31.4
0.76	9.3	6.9	2.9	0.2	5.7	13.5
0.89	5.0	3.9	2.3	0.0	1.6	3.2

Note: The negative sign designates inward radial flow.

Fig. 1.20 shows these values plotted against the flow coefficient.

If it is assumed the design point corresponding to the best working conditions is that accompanied by the minimum secondary flow, then accordingly, the design flow coefficient for this rotor would be somewhere near:

$\eta_c = 0.8$. Though it must be mentioned that this is not necessarily the point of peak efficiency. The parabolic nature of these curves at reduced rates of flow, as well as their changed characteristics beyond that point is noticeable. Obviously the previous explanation given in connection with figs. 1.17, 1.18 and 1.19 will also apply here.

For radii nearer to the outer radius, however, a clearer picture of the nature of the secondary flow near the outer wall is obtained by taking the limits for the integrals in equation (1.41) as the tip radius and that corresponding to the point at which the secondary flow is to be evaluated. The following tables give the secondary flow thus calculated:

TABLE III

One single rotor operating at 2200 R.P.M.

$q_c = Q/nD^3$	0.64	0.61	0.55	0.45	0.43	0.35
r/R_0	% of upstream flow					
0.95	21.4	-8.9	-16.5	-9.8	-22.0	-43.4
0.89	11.1	-4.4	-8.8	-3.8	-15.2	-25.2
0.82	4.7	-3.5	-7.0	-13.3	-18.4	-26.2

TABLE IV

One rotor followed 28 inches further downstream by a
boosting propeller

N r.p.m.	800	1000	1200	1300	2000	2000
$q_c = Q/nD^3$	1.28	1.05	0.93	0.83	0.54	0.43
r/R_0	% of upstream flow					
0.95	-20	-25	-24.8	-1.5	-11.2	-20.5
0.89	-20	-14.7	-8.6	0.1	-5.8	-12.4
0.82	-14.8	-11.3	-5.0	0.0	-6.6	-13.4

Note: The negative sign indicates outward radial flow.

Fig. 1.21 shows these values plotted for varying flow coefficients.

The total percentage of useful flow which undergoes secondary motions can be put as:

$$\frac{|\sum q_r|}{Q} = \frac{\int_{r_h}^{r_o} |U_1 - U_2| \cdot r \cdot dr}{\int_{r_h}^{r_o} U_1 \cdot r \cdot dr} \times 100 \quad (1.42)$$

where,

r_h & r_o : are the radii corresponding to the hub
and the outer casing.

Using equation (1.42), the following tables were prepared:

TABLE V

One single rotor operating at 2200 R.P.M.

$q_c = \frac{Q}{nD^3}$	0.64	0.61	0.55	0.45	0.43	0.35
$\frac{ \sum q_r }{Q} \%$	4.1	6.7	10.1	21.0	33.2	36.6

TABLE VI

One rotor followed 28 inches further downstream by a
boosting propeller

N r.p.m.	2000	2000	1800	1500	1500	1500
q_c	0.43	0.54	0.67	0.71	0.74	0.80
$\frac{ \sum q_r }{Q} \%$	22.4	11.7	4.9	2.2	2.2	3.6

(continued)

TABLE VI (Continued)

N.r.p.m.	1300	1300	1200	1000	800
q_c	0.83	0.86	0.93	1.05	1.28
$\Sigma q_r/q$ %	1.5	3.0	4.6	8.2	10.4

Fig. 1.22 shows the percentage of useful flow undergoing secondary motions, due to operating at loads other than those corresponding to design conditions. As the flow is reduced further a stage will be reached, when back flow starts downstream of the rotor, close to the hub. These back flow currents will increase in radial depth by further throttling. In this type of back flow, which incidentally is distinct from that due to separation or to eddy formation, it starts when the secondary or "radial pumping" at a particular radius becomes of such strength that it consumes all the incoming flow and a region of low pressure develops downstream of the rotor in the neighbourhood of the hub, with the result that the already pumped fluid flows backwards into the rotor, and is discharged again at a greater radius. On further reduction of flow, this secondary pumping ultimately exceeds the useful flow, with the result that part of the secondary flow is discharged upstream of the rotor close to the outer casing, and against the main flow direction. In one experiment with extreme over-throttling, the back flow was observed to pass well into the inlet bell mouth. This back flow current

will envelope the fresh inflowing stream and by transferring angular momentum to it, considerable pre-rotation is thus acquired by the latter.

Fig. 1.23 shows the depth of back flow upstream and downstream of the rotor at varying flow coefficients. It is noticed that back flow downstream of the rotor starts at a flow rate corresponding to 46% of the design rate, while it starts upstream of the rotor at 32% of the normal flow and downstream back flow is then occupying about 20% of the total flow depth.

Fig. 1.24 shows the pre-rotation, deviation from the axial direction, and the depth of the back flow stream at a distance of $3\frac{1}{4}$ inches upstream of the rotor for flow rate corresponding to 22.5% of the normal rating.

Another type of secondary flow to be discussed, is that associated with the boundary layers close to the blade. For the flow close to the blade, the equations of motion, equations (1.28), although apparently neglecting viscosity, can be rewritten as:

$$\left. \begin{aligned} u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \Omega^2 r &= -\frac{1}{\rho} \frac{\partial p}{\partial r}, \quad (a) \\ \frac{\partial p}{\partial \phi} &= -2 \rho r \Omega u, \quad (b) \end{aligned} \right\} (1.43)$$

in which the viscous action has to some extent been included by taking the tangential velocity close to the blade to be the same as that of the blade, and also $\frac{\partial v}{\partial \phi} = 0$,

so that this tangential component does not vary within a fluid depth of the same order as the boundary layer thickness, and that $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial \phi} = 0$, so implying that the axial velocity is independent on r and ϕ .

These assumptions, however, are far from being true, therefore equation (1.43) can hardly be relied upon quantitatively, though it is of interest in giving some idea of the Coriolis forces resulting from the radial motion. As shown later, Plate 3, these radial secondary flows, and no doubt the resulting forces also, attain their maximum close to the trailing edge.

Equation 1.43(b) shows clearly that this local radial flow creates a pressure drop across the B.L., which counteracts the adverse pressure gradient arising from the main stream. This obviously will help in keeping down the boundary layer thickness, and may delay separation. The resulting pressure distribution along the low pressure side of the aerofoil is diagrammatically sketched in fig. 1.25. This, however, does not mean that the pressure distribution in the main stream has also assumed this modified pressure curve. Outside the boundary layer zones this secondary flow will die out, and the pressure distribution curve will be of the normal shape, shown by the solid line in fig. 1.25.

Though this type of secondary flow sounds advantageous, it must be remembered, however, that it is maintained at the

cost of an increase in the applied torque, as can be easily seen from equation 1.43(b) and fig. 1.25.

A better approximation can be reached by assuming that the tangential velocity is a function of the angular position " ϕ ", i.e.:

$$v = f(\phi) ,$$

with the following boundary conditions:

$$v_{\phi_0} = \Omega r ,$$

where,

ϕ_0 = the angular position corresponding to the point on the blade surface,

and ,

$$v_{\phi_s} = w_r ,$$

where,

ϕ_s = the angular position corresponding to the edge of the boundary layer,

and,

w_r = the tangential velocity component of the main stream,

hence the equations of motion can be put as:

$$\left. \begin{aligned} u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{(f\phi)^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} \\ u \frac{\partial f(\phi)}{\partial r} + \frac{f(\phi)}{r} \cdot f'(\phi) + \frac{u}{r} f(\phi) &= -\frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \phi} \end{aligned} \right\} \quad (1.44)$$

Weske by assuming that,

$$\frac{\partial P}{\partial r} = \frac{\partial u}{\partial z} = 0,$$

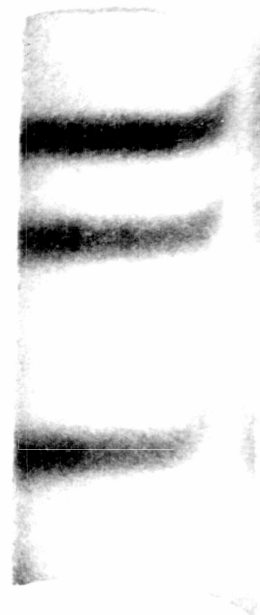
managed to get an expression for the radial secondary velocity. These assumptions, however, are not justified, since the radial pressure gradient is considerably as determined by the vortex régime in the main stream. As for the second part of the assumption this will not be so except at a point close to the trailing edge, where the radial velocity has attained its maximum.

It would seem that from the moment the fluid particle enters the zone of the boundary layer at the leading edge, it is acted upon by centrifugal force which gradually increases its radial velocity as it proceeds along the blade. Furthermore, its axial velocity is retarded by the adverse pressure forces. A point will be reached where the radial velocity becomes so prominent that the resultant relative velocity becomes almost radial. It is obvious that the centrifugal accelerating forces are increasing all the time as the particle moves along the blade.

Plate 3 shows ammonial traces on the low pressure side of the aerofoil. These traces show clearly the secondary motions within the boundary layer. These were obtained by sticking "Ozalid" paper on to the low pressure side of the blade, and injecting anhydrous Ammonia gas about two chords upstream of the rotor. The injector was made of a hypodermic



A



B

Plate 3 - Ammonia traces on the low pressure side of the rotor's blade, at radii r/R_0 . A: 0.29, 0.6, 0.89 and B: 0.44, 0.6, 0.80.

stainless steel tube, bent in the manner of a Prandtl's static pressure tube, thus enabling the gas to be released in the same direction as that of the main stream. Traces obtained for the high pressure side of the aerofoil did not show any radial displacement. Unfortunately, the difficulty of sticking the "Ozalid" paper on to the high pressure side of the blade, made the samples obtained for that side unsatisfactory.

Plate 3, also shows clearly that part of the Ammonia gas entrained in the wake, was pumped back and left its traces near the trailing edge. It is plausible to think that such a secondary motion as that sketched in fig. 1.26 takes place close to the trailing edge of each blade.

Hence it may be concluded that part of the fluid in the wake travelling with each blade, is pumped radially outwards near the trailing edge of the blade, and is discharged again into the wake at the blade tip. Part of its energy is transferred to the boundary layer at the tip clearance and the rest is dissipated during its downward journey in the wake, at the end of which the cycle repeats itself.

A point worth mentioning is that after long testing periods, faint traces of dust, on lines similar to those shown in Plate 3, were noticed. Hence an attempt was made to accelerate this process by injecting through the inlet

bell mouth french chalk by using an ordinary powder puff. The results of these tests were unsuccessful, regarding the original aim for which they were carried out. Probably this is due to the relatively larger size of the powder particles, in comparison with that of the dust particles. However, they gave results very much in agreement with those published by Davidson in connection with axial flow compressor blade fouling. The explanation he gave was based on the fact that these solid particles shoot through the boundary layer, and rest on the blade surface where there are no forces sufficiently large to move them. The sketch shown in fig. 1.27 was reproduced from his paper, showing the fouling regions on a compressor blade.

PART II

A Simplified Three-Dimensional Approach

In the previous section, the two dimensional theory for axial flow propellers was presented, and secondary motions that make such theory inadequate, were discussed. Generally speaking, these secondary flows can be put under two main categories:-

(i) Radial secondary flow: this is of an adjusting nature, and is mainly brought about to establish a balanced state of affairs between the different forces acting within the fluid.

(ii) Secondary motions associated with boundary layers.

The first type of secondary motion, occurs in the case of rotors designed for a free vortex régime when operating under conditions other than those corresponding to the design point, as well as in the case of rotors designed for a vortex régime other than the irrotational type. This, however, is not wholly detrimental, for it combines the motions of radial and axial pumping and produces in effect a sort of mixed flow rotor. Rotors designed for a vortex régime other than the free vortex gave efficiencies comparable with those designed for irrotational flow.

The free vortex is clearly the ideal when the load does not vary, but further experiments are needed to find whether or not other régimes are more flexible.

The present experiments confirm that all three components of the motion may be large, and the theoretical approach can

hardly proceed far except on a three dimensional basis, though many terms in the equations may be dropped, or simplified without unduly invalidating the results.

Before considering three dimensional flow in axial machines however, it is interesting to study first the less complicated case of an aerofoil with finite span. As previously mentioned, pp. 5 to 12, the forces experienced by such an aerofoil will be associated with bound and trailing vortices, hence the inevitable result that in some parts of the fluid field rotational motion will exist. Therefore, the influence of such rotational motion on the equations of hydrodynamics in general and on the application of Bernouilli's equation in particular, will be considered. The following is based on an explanation given by Th. von Karman and J.M. Burgers.

Assuming an aerofoil of a finite span, in an infinite fluid moving with a constant velocity V parallel to the x -axis, and that the disturbances caused by the action of the forces are relatively small, the equations of hydrodynamics can be put as:

$$\left. \begin{aligned} V \frac{\partial w_x}{\partial x} &= - \frac{1}{\rho} \frac{\partial H}{\partial x} + \frac{k_x}{\rho} \\ V \frac{\partial w_y}{\partial x} &= - \frac{1}{\rho} \frac{\partial H}{\partial y} + \frac{k_y}{\rho} \\ V \frac{\partial w_z}{\partial x} &= - \frac{1}{\rho} \frac{\partial H}{\partial z} + \frac{k_z}{\rho} \end{aligned} \right\} \quad (2.1)$$

where,

w_x, w_y, w_z : denote the components of additional velocity,

H is defined by:

$$H = p + \frac{1}{2} \rho w^2 - p_0 \quad (2.2)$$

where,

$$w = \sqrt{w_x^2 + w_y^2 + w_z^2} ,$$

p_0 = the constant pressure at infinity,

and,

$$K_x = f_x + g_x ,$$

$$K_y = f_y + g_y ,$$

$$K_z = f_z + g_z ,$$

where,

f_x, f_y, f_z : are the actual external forces,

$$g_x = \rho (w_y \gamma_z - w_z \gamma_y) ,$$

$$g_y = \rho (w_z \gamma_x - w_x \gamma_z) ,$$

$$g_z = \rho (w_x \gamma_y - w_y \gamma_x) ,$$

$\gamma_x, \gamma_y, \gamma_z$: are the components of the **vorticity vector** $\bar{\gamma}$.

Equations(2.1) are the exact equations of fluid motion, provided the values of K_x, K_y & K_z are put in. As a first approximation, however, the forces of the second order, i.e. g_x, g_y & g_z can be neglected. Moreover, these forces are zero in every part of the fluid where the vorticity is zero.

This approximation is equivalent to the assumption that the proper motion of the vortices under their mutual influence can be neglected in comparison with the general velocity V .

Next, integrating with respect to x the first equation of (2.1), the following can be obtained:

$$\rho V w_x = -H + \int_{-\infty}^x k_x dx, \quad .$$

The constant of integration can be omitted, as both w_x and H vanish for large negative values of x .

Putting:

$$k_x = \frac{\partial Q_x}{\partial x}, \quad u_x = V + w_x, \quad v_y = w_y \quad \text{and}$$

$$u_z = w_z,$$

and substituting from equation (2.2), the following can be obtained:

$$p + \frac{\rho}{2} (u_x^2 + v_y^2 + u_z^2) = p_0 + \frac{\rho}{2} V^2 + Q_x,$$

The value of Q_x can differ from zero, only within a definite region of the fluid, where the external forces exist, as well as in the wake of the profile. Hence, wherever Q_x is zero, Bernouilli's equation holds, otherwise a definite quantity has to be included, which can only be dropped as an approximation when the external forces are relatively small.

Considering now the case of an axial flow machine, then

with the notations of fig. 1.14, the complete equations of motion are:-

$$\left. \begin{aligned} u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \phi} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \\ = -F_r - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial u}{\partial \phi} \right), \\ u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \phi} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \\ = -F_\phi - \frac{1}{\rho r} \frac{\partial p}{\partial \phi} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \phi} - \frac{v^2}{r} \right), \\ u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \phi} + w \frac{\partial w}{\partial z} \\ = -F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w, \end{aligned} \right\} (2.3)$$

where,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2},$$

w = axial velocity component,

v = tangential velocity component,

u = radial velocity component,

F_r , F_ϕ and F_z are the components of extraneous forces in the radial, tangential and axial directions.

In the case of axial flow propellers the fluid field can conveniently be divided into three zones:-

- a - upstream of the rotor,
- b - within the rotor,
- c - downstream of the rotor.

The extraneous forces only exist within zone "b", where they are the forces exerted by the rotor blades on the fluid.

Therefore, it will be easier to consider first the fluid field upstream and downstream of the rotor. If as an approximation, Bernouilli's equation is assumed to hold downstream of the rotor,^{*} and that the motion is steady and axially symmetrical, then as previously shown, the equation of continuity can be reduced to:

$$\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0, \quad (2.4)$$

Defining the three vorticity components by:

$$\left. \begin{aligned} \xi &= -\frac{\partial v}{\partial z}, & \text{radial (a)} \\ \gamma &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}, & \text{tangential (b)} \\ \zeta &= \frac{1}{r} \frac{\partial}{\partial r} (rv), & \text{axial (c)} \end{aligned} \right\} (2.5)$$

the equations of motion can be put as:

$$\left. \begin{aligned} w\gamma - v\zeta &= -\frac{1}{g} \frac{\partial H}{\partial r}^* + v(\nabla^2 u - \frac{u}{r^2}), & (a) \\ w\xi - u\zeta &= 0, & (b) \\ v\xi - u\gamma &= -\frac{1}{g} \frac{\partial H}{\partial z}^* + v\nabla^2 w, & (c) \end{aligned} \right\} (2.6)$$

where,

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2},$$

H = total energy.

* except for a small correcting term, δH , which is sufficiently small not seriously to invalidate the use of Bernouilli's equation for computing pressure difference from changes in velocity (pp. 57).

Furthermore, it will be assumed that any necessary adjusting radial secondary flow, takes place within the rotor, well before the fluid being discharged. Consequently it is feasible to expect for a particular blade speed, that the tangential velocity component will be solely a function of the outlet flow velocity, and of the blade outlet angle, irrespective of the strength and direction of the secondary flow. As will be shown later by experimental evidence, pp.74, both assumptions are justified.

With the above assumptions, equations (2.4, 2.5 and 2.6) simplify to:

a - Equation of Continuity:

$$\partial w / \partial z = 0 \quad , \quad (2.7)$$

b - Equation of motion:

$$w \cdot \gamma - v \cdot \zeta = - \frac{1}{g} \frac{\partial H}{\partial r} \quad , \quad (2.8)$$

where,

$\gamma = - \frac{\partial w}{\partial r}$, and $\zeta = \frac{1}{r} \frac{\partial}{\partial r} (r v)$ the tangential and axial vorticities, the third vorticity component $\xi_z = - \frac{\partial v}{\partial z}$ being zero.

The differential equation (2.8) gives the relation between axial velocity, angular momentum, and total energy

* See page 57 and footnote page 61 .

at any point within the fluid field down-stream of the rotor. Both the angular momentum and the total head can be put as functions of the shape of the rotor blade and its speed, thus making it possible to reduce equation (2.8) to a linear differential equation of the first order, which can then, for particular cases, be solved for the axial flow velocity, w .

For that purpose an arbitrary propeller of the following dimensions will be assumed:

- R_h = radius of the rotor hub,
- R_o = radius of the blade tip,
- r = radius at any particular section,
- Ω = propeller rotational speed in radians per second,
- $\bar{U}$ = mean axial flow velocity,
- α'_2 = the angle between axial direction, and the tangent to the blade camber line at the trailing edge,
- δ^* = the deviation defined as the angle between the flow at outlet, and the tangent to the camber line at the trailing edge.

For simplicity, let:

$$\delta^* = 0.26 \theta \sqrt{S/c} \quad (2.9)^*$$

where,

$$\delta^* = \text{deviation at design conditions,}$$

* An empirical rule derived by H. Constant, based on two dimensional cascades.

θ = camber angle,

S = blade spacing ,

c = blade chord. (see fig. 1.10)

As a further simplification, let the deviation angle remain constant for working conditions near the design point. Then the angle between the axial and the relative flow directions at outlet will be a function of the radius r , and of the form of the blade which itself is a function of r , so:

$$\tan (\alpha'_2 + \delta^*) = \phi(r) \quad . \quad (2.10)$$

The tangential velocity component, or "whirl" at any point downstream of the propeller, and at a radius r , can thus be written as:

$$v = \Omega r - w \tan (\alpha'_2 + \delta^*) \quad . \quad (2.11)$$

From Euler's equation the total energy at the selected point, measured from a datum corresponding to the total energy upstream of the rotor, is:

$$\begin{aligned} H &= \eta \cdot v \cdot \Omega r \cdot \eta_h \\ &= \eta \cdot \eta_h \cdot \Omega r [\Omega r - w \tan (\alpha'_2 + \delta^*)] , \end{aligned} \quad (2.12)$$

where,

η_h = the hydraulic efficiency at the particular radial station considered.

$\doteq 1.0$,

hence the expressions for the radial total energy gradient, and the axial vorticity component can be written as:

$$\frac{1}{\rho} \frac{\partial H}{\partial r} = 2r\Omega^2 - \Omega w\Phi - \Omega r\Phi \frac{\partial w}{\partial r} - \Omega r w\Phi', \quad (2.13)$$

and,

$$\zeta = 2\Omega - \frac{w}{r}\Phi - w\Phi' - \Phi \frac{\partial w}{\partial r}. \quad (2.14)$$

Substituting these expressions in equation (2.8), the following differential equation can be obtained:

$$\frac{\partial w}{\partial r} + w \left[\frac{\Phi^2 + r\Phi\Phi'}{r(1+\Phi^2)} \right] = \frac{2\Omega\Phi}{1+\Phi^2}, \quad (2.15)$$

putting:

$$F(r) = \frac{\Phi^2 + r\Phi\Phi'}{r(1+\Phi^2)}, \quad (2.16)$$

and,

$$G(r) = \frac{2\Omega\Phi}{1+\Phi^2}, \quad (2.17)$$

the differential equation (2.15) can be written as:

$$\frac{\partial w}{\partial r} + w.F(r) = G(r). \quad (2.18)$$

Equation (2.18) is a linear differential equation of the first order which when solved gives the following expression for the axial velocity component at any radial station downstream of the arbitrary propeller:

$$w = e^{-\int F(r) dr} \cdot \int e^{\int F(r) dr} \cdot G(r) dr + c e^{-\int F(r) dr}, \quad (2.19)$$

the constant c is determined by the equation of continuity, which can be written as:-

$$\frac{1}{U(R_o^2 - R_H^2)} \int_{R_H}^{R_o} 2 w \cdot r dr = 1.0 \quad (2.20)$$

Once the axial velocity distribution is determined, the tangential velocity, total energy, and any other entity can be evaluated by substituting in the appropriate equation.

F. Marble, by assuming that vorticity components travel with the corresponding fluid velocity component, reduced the equations of motion to a similar form to that expressed by equations (2.6). Then by making a series of further assumptions, he solved the resulting linearised differential equations. His assumptions, however, were based solely on mathematical grounds, which made his solution more complicated than the one given above. He gave expressions for the local radial secondary velocities, under different operating conditions. But his solution was based on assuming a rotor of infinite number of blades of zero chords, i.e. his rotor was an infinitesimally thin disc at which the fluid underwent a discontinuity; an

unrealistic assumption not very helpful in connection with the design of the rotor blades.

A. Kahane introduced an iterative solution for rotors designed for three dimensional flow. This solution was based on the assumption that the tangential velocity distribution is known.

J. Bowen, R. Sabersky, and W. Rannie by making similar assumptions to those used in connection with the present solution, as well as further simplifications, deduced an expression for the axial velocity distribution. Their expression, however, is not in a direct form.

Returning to the present solution 2.8 and 2.19, the interesting question is whether it confirms with experiments. As previously mentioned, the propeller was a six bladed, commercial, 12-inch rotor made by Messrs. Vent-Axia. Its nominal dimensions according to the working drawing are:-

$$R_o = \text{Outer radius} = 5 \frac{31}{32} \text{ in.},$$

$$R_h = \text{Hub radius} = 1.75 \text{ in.},$$

$$\text{Hub axial length} = 1 \frac{5}{16} \text{ in.}$$

The blades are detachable. The working drawing gives the particulars of the blade profiles at five radial stations. These profiles, however, represent the intersections of a series of planes parallel to the rotor axis, and passing by the leading and trailing edges of the corresponding radial station. This is diagrammatically presented in fig. 2.1,

which is reproduced from the working drawing. Table I of the appendix gives the nominal measurements as obtained or estimated from the working drawing.

To supplement the above nominal dimensions, certain direct measurements of the blade profiles, at different radial stations, were carried out for a right-hand, as well as for a left-hand propeller. For measurement, the propeller was mounted on the dividing head of a universal milling machine. A 5/16 in. thick steel strip was locked against the propeller hub, and served as a datum from which measurements were taken. A dial gauge, reading up to 0.0001" was supported from the tool holder, with its axis parallel to that of the propeller, fig.2.2.(a). The spherical end of the dial gauge spindle was replaced by a sharp pointed needle, and the spindle was also modified so that the needle could be set to a particular reading. This was necessary to avoid pressure between needle and blade with possible deflection of the blade. These modifications are shown diagrammatically in fig. 2.2(b). It was noticed that the pressure of the needle could cause an error at the blade tip of the order of 0.015". The contact between needle and blade was checked visually with the aid of a concentrated light. With this arrangement, it was possible quickly to measure the profile at the various radial stations. The profile so obtained was the intersection of the blade with a fictitious cylindrical

surface concentric with the propeller.

With the aid of the dividing head the readings were taken in steps of one or two degrees except near the leading and trailing edges, where the intervals were ten minutes.

The profiles were then drawn to the following enlarged scales:

1 inch to the degree ,

1 inch to 0.100 inch,

from which the data given in Table II of the appendix were obtained. Using this table, figs. 2.3(a, b, c and d) were prepared. These show the different angles measured from the axial direction, defining the shape of the blade profiles at the different radial stations.

The Experiments:

In connection with formulae (2.8 and 2.19) experiments were made under the following conditions:-

- (a) with one rotor operating at a constant speed of 2200 R.P.M. The layout of the apparatus for these tests was as shown in fig. 3.1.,
- (b) with the boosting arrangement. The boosting rotor was 28 in. downstream of the tested propeller. These tests were carried out under varying speed. The layout for these tests is as shown by the chain dotted lines, fig. 3.1.,

(c) with the contra-rotating set up. The two rotors were $2\frac{1}{4}$ in. apart in this case - see Plates 1 and 2,

(d) with one rotor having an altered hub of 8" diameter operating at 2000 R.P.M.

Regarding series (d) with the large hub, the rotor and the upstream and downstream hub continuations, are shown in fig. 2.4 and Plate 4. While the original and modified rotors are compared in Plate 5. The stationary parts of the hub-fairing, upstream and downstream of the rotor were turned out of a specially hardened paraffin wax. As for the rotor, two aluminium discs were bolted to the propeller. The space in between them, was filled with Alabastine, which after setting was scraped off to form the new 8" hub surface. "Durofix" diluted with acetone was used to apply a plastic coat enveloping the alabastine, thus preventing flaking-off.

This modified propeller allowed an annulus of the rotor to be tested to find how the secondary flow influenced the main flow, including the effect on the distribution downstream. In comparing the annulus, i.e., the fan with the 8 inch hub, with the complete fan, 3.5 inch hub, there are geometrical differences incidental to the change in the hub diameter. For example, the spacing of the blades with the large hub is nearly twice the average spacing of the whole fan, and, being near the tip, the blades are flatter. Also

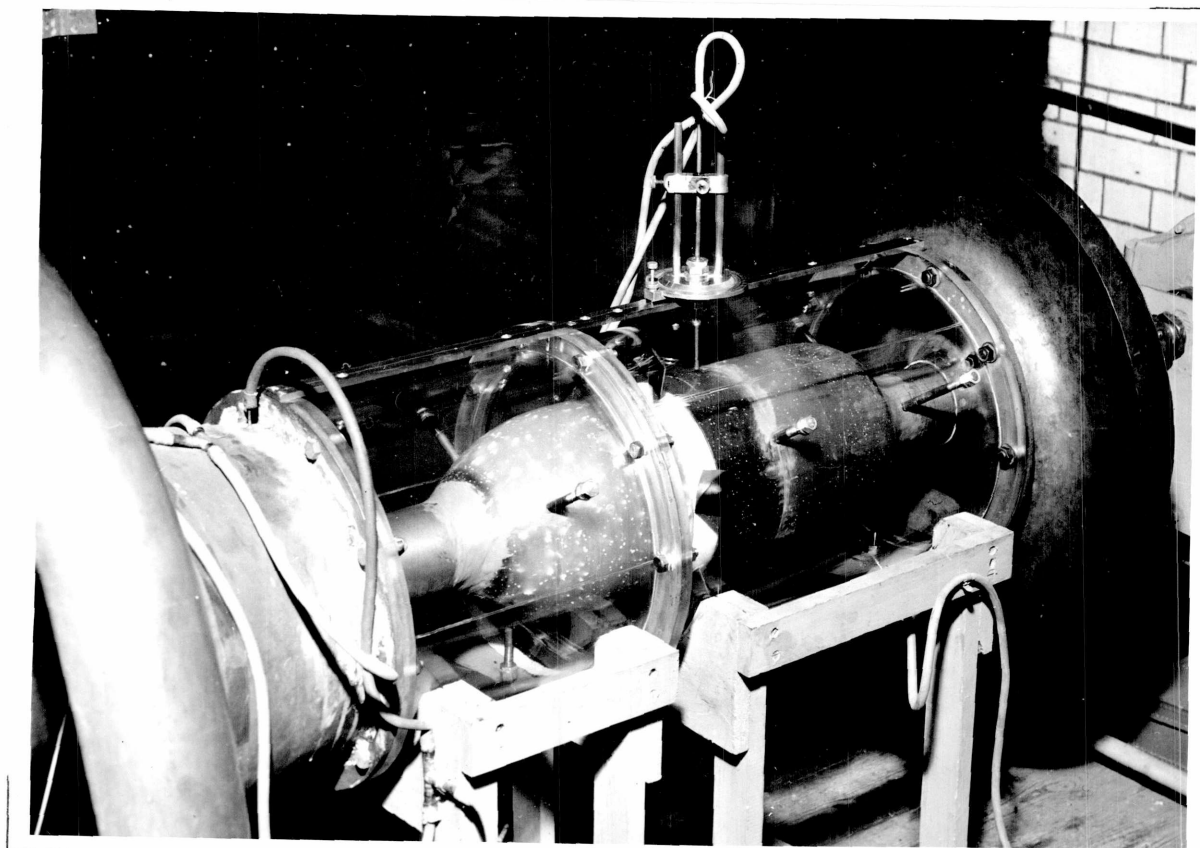


Plate 4 - Set-up for testing the modified 8" hub rotor.

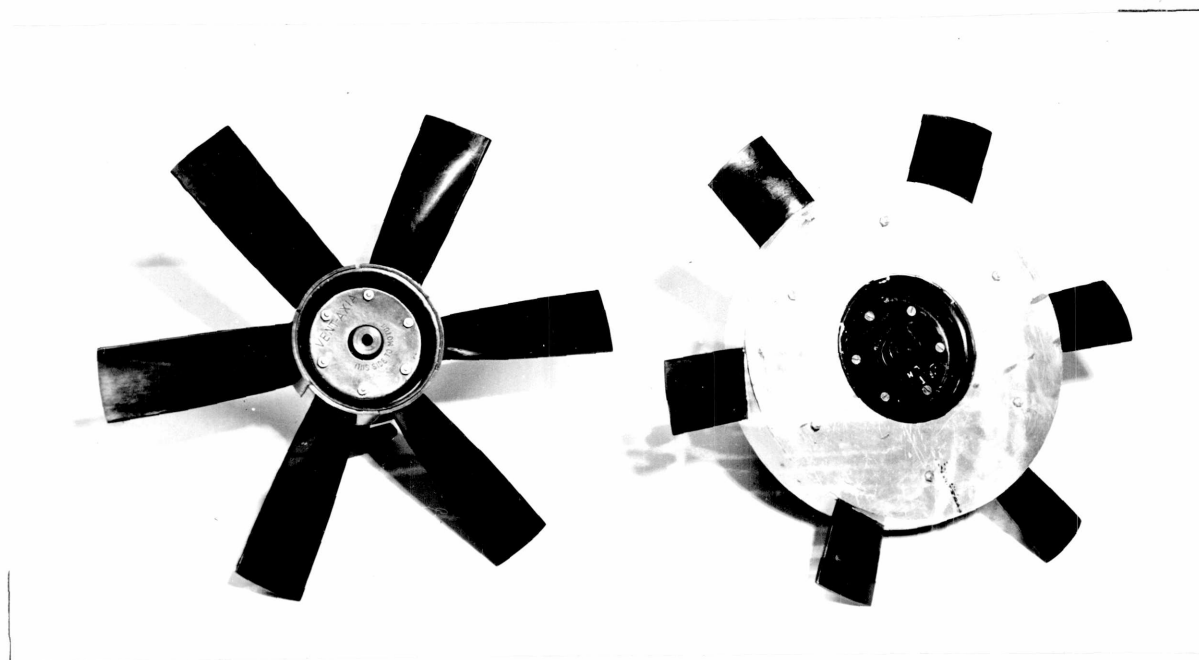


Plate 5 - The original propeller, and the modified 8" hub rotor.

the flow coefficient defined as:

$$q_c = \frac{Q}{nD^3} ,$$

where,

Q = the flow through the propeller in cusecs.,

n = the rotor speed in radians per second,

D = Rotor outer diameter,

is no longer a suitable basis for comparison between the two.

Some new definition is necessary and the simplest seems to be:

$$U_c = \frac{2\bar{U}}{nD} \quad (2.21)$$

where,

$\bar{U}$ = the mean flow velocity through the rotor,

D = the propeller outer diameter,

Two sets of results are given in figs. 2.5 and 2.6.

Figs. 2.5 (a, b, c, d and e), for the original rotor give the velocity distribution downstream and upstream of the rotor for flow coefficients ranging from about 0.15 up to 0.35 (new basis). Traverses shown in fig. 2.5 (a, b, c and d) were obtained from tests at 2200 R.P.M., while those in fig. 2.5 (e) were obtained with the boosting arrangement at a speed of 1500 R.P.M. Figs. 2.6(a, b, c, d and e) for the rotor with 8 inch hub show velocity traverses from tests at 2000 R.P.M. for a flow coefficient varying from 0.16 to 0.30 approximately.

In all these traverses, the downstream curve crosses the upstream curves. In the original rotor the downstream curve is rising steeply, and so the radial part of the secondary flow is clearly strong and outwards, except in fig. 2.5(e) where the secondary hardly exists; with the large hub, the reverse is the case, the radial is inwards, and is so over the whole operating-range tested.

The change in the direction of radial secondary flow, throws light on the location of the secondary motion, and on whether it has any direct influence on the flow direction downstream of the propeller. To clarify this question, convenient variables are the tangential or "whirl" component, and the direction of the flow relative to the rotor blade, both downstream of the rotor. It should be mentioned that upstream of the propeller the flow was axial and uniform. Also the following coefficients are convenient:-^{*}

$$\begin{aligned} \phi &= \text{the local flow coefficient} \\ &= \frac{w}{\Omega r} , \\ \xi &= \text{the factor expressing the strength of the} \\ &\quad \text{radial secondary flow} \\ &= \frac{w_2}{w_1} , \\ \alpha_2 &= \tan^{-1} \frac{\Omega r - u_2}{w_2} \\ &= \text{the angle between the axial direction and the} \\ &\quad \text{flow relative to the blade} , \end{aligned}$$

* As previously, subscripts 1 and 2, denote conditions upstream and downstream of the rotor, respectively.

$$\epsilon = v_2 / w_2 ,$$

where,

w = the axial flow velocity at a particular radius r ,

v = the tangential or "whirl" component at that radius,

Ωr = peripheral speed of the rotor at radius r .

Successive attempts to plot the tangential velocity component, as well as the angular deviation of the relative flow as functions of rotor peripheral speed and the flow coefficient upstream of the rotor, showed great scatter of the points. Clearly these variables are not directly related, or at least there is no simple connection between them. However, when referred to the axial flow velocity downstream of the rotor, all the experimental points for large and for small hub, for single and for contra-rotating propellers followed one single curve.

Fig. 2.7(a) shows the secondary flow factor ξ , plotted against the downstream flow coefficient ϑ_2 both for tests on the original rotor, as well as for those with the 8 inch hub, at radius $r = 0.69 R_0$. It should be mentioned that the secondary flow with the original rotor was radially outward, except for a narrow range corresponding to $\vartheta_2 \div 0.5$ at this radial station. On the other hand, with the 8 inch hub, the secondary flow was invariably radially inward.

Figs. 2.7 (b and c), show values of ε and $\cot \alpha_2$, plotted against the downstream flow coefficient ϑ_2 , at the same radial station $r = 0.69 R_0$. Further results, though for different radii, are given in figs. 2.8 to 2.10. The relevant numerical data are given in Table III of the appendix.

The wide variety in the shapes of the curves seen in figs. 2.7 to 2.10 clearly show that the tangential velocity, as well as the direction of the flow relative to the blade downstream of the propeller, when operating with uniform axial upstream velocity, are functions of the axial velocity on the downstream side of the rotor, and obviously of the blade shape at the radial station considered. This was so whether the secondary motion was weak or strong, and whether it was outward as with the $3\frac{1}{2}$ " hub, or inward as with the 8" hub. In other words, neither the strength nor the direction of the secondary motion affects in a direct way, the whirl component or the flow direction downstream of the rotor. These secondary motions, however, do alter the axial velocity downstream of the propeller, which will have definite flow direction and tangential velocity components. This also leads to the conclusion that the secondary flow takes place within the rotor disc, well before the fluid is discharged. It is interesting to note also that this was true at radial station: $r = 0.95 R_0$, in spite of the fact that

the blade at that radius in the case of the original rotor, was operating well within the boundary layer.

Algebraically, the relations are not clear, except that α_2 , w_2 , and α'_2 are connected, i.e.:

$$\cot \alpha_2 = \frac{w_2}{\Omega r - u_2} = f(w_2, \tan \alpha'_2), \quad (2.22)$$

where,

α_2 = the angle between the axial direction, and the flow relative to the rotor,

w_2 = the axial flow velocity downstream of the rotor,

α'_2 = the angle between axial direction and the tangent to the camber line at the trailing edge.

Regarding the nature of the relation (2.22.) the preceding results together with those given in Table IV of the appendix, cover the following range of:-

$$r/R_0: 0.32, 0.43, 0.50, 0.69, 0.89 \text{ and } 0.95,$$

and the following conditions:

- (i) tests with one single rotor,
- (ii) tests with the boosting arrangement,
- (iii) tests with the contra-rotators. For this case the data relate to the upstream rotor.

The relevant results are shown in Figs. 2.11 (a to g), where the angle between relative flow and the axial direction, and also the secondary flow factor, both plotted against downstream flow coefficients. It is interesting to note how

much the experimental results deviate from the one dimensional theory, which requires that $\cot \alpha_2$ should remain constant, fig. 2.11(a), where the experimental curve is seen to be anything but a horizontal line at $\cot^{-1} 30.8^\circ$ as required by the one dimensional theory, in which the fluid would move with the blade when the axial velocity is zero.

For comparing the tested propeller with the equivalent two dimensional cascade, Constant's rule (2.9) was used to estimate the relative flow directions shown by the chain dotted lines drawn parallel to φ_2 -axis, figs. 2.11 (a to g). The relevant numerical values are given in Table V. It is interesting to note that these lines intersect the curves at points where the secondary motion is weak, or in other words when two dimensional flow prevails.

TABLE V

r/R_0	δ^* deg. Nominal Deviation.	α'_2 deg. Camber line angle at trailing edge.	Flow direction D.S. of the cascade.	
			$\alpha_2 = \alpha'_2 + \delta^*$	$\cot \alpha_2$
0.32	10.2	30.8	41.0	1.15
0.43	10.4	40.8	51.2	0.80
0.50	10.4	45.0	55.4	0.69
0.69	10.3	53.0	63.5	0.50
0.89	10.5	56.0	66.5	0.43
0.95	10.5	57.4	67.9	0.41

N.B. All angles are measured from the axial direction.

Limiting the present discussion to the useful operating range of flow coefficient, it becomes obvious that the relative angular deviation of the fluid downstream of the propeller is very much in agreement with that corresponding to two dimensional cascades, provided that the radial secondary motion in the former is weak. With appreciable secondary motion, however, the $\cot\alpha_2 - \psi_2$ relation deviates from that corresponding to the two dimensional values, and approaches the origin as the secondary motion becomes stronger.

The above experimental evidence clearly shows that the flow direction relative to the blade on the discharge side, is by no means independent of the loading. This fact limits the applicability of the solution given by equation (2.19), to a narrow range of the "working part" where the angular deviation of the flow relative to the blade can be assumed to remain constant.

This, however, does not invalidate equation (2.8). This equation is:

$$w \cdot \eta - v \cdot \xi = - \frac{1}{g} \frac{\partial H}{\partial r} ,$$

where,

w = the axial velocity component,

v = the tangential velocity component,

H = the total energy at the radial station considered,

$$\zeta = - \frac{\partial w}{\partial r}$$

= the tangential vorticity component ,

$$\zeta = \frac{1}{r} \frac{\partial}{\partial r} (r \cdot v)$$

= the axial vorticity component,

and each of these terms can be obtained from experimentally measured quantities. Readings in steps of $\Delta r = 0.065 R$, were taken. An approximation was used for computing vorticities and the radial gradient of energy as follows:

denoting by (a), the radial station to which the computation refers, then:

the radius corresponding to this radial station = r_a ,

$$2 dr = 0.065 \text{ ft.} ,$$

$$\zeta = - \frac{\partial w}{\partial r} = \frac{w_{r_a + dr} - w_{r_a - dr}}{2 dr} ,$$

$$\zeta = \frac{1}{r_a} \cdot \frac{\partial (r \cdot v)}{\partial r} = \frac{|(v \cdot r)_{r_a + dr}| - |(v \cdot r)_{r_a - dr}|}{2 dr} ,$$

and,

$$\frac{1}{\rho} \frac{\partial H}{\partial r} = \frac{1}{\rho} \frac{H_{r_a + dr} - H_{r_a - dr}}{2 dr} .$$

Hence the left hand side of equation (2.8) can be compared directly with the radial gradient of total energy. The difference between the two quantities, i.e. the vorticity term and the radial gradient of total energy, as will be shown later, is so small that it may be neglected altogether.

The data for the computation were chosen from tests with one rotor, operating both alone and with the boosting arrangement. The numerical values are given in Table VI of

the appendix, which clearly shows that apart from few points, the agreement between the gradient of total energy computed by equation (2.8) and that obtained by direct measurement is fairly good. Even under extreme operating conditions, e.g. at $q_c = 1.05$ and 0.35 , the agreement was still satisfactory.

As previously shown, pp. 64 , the differential equation (2.8), was linearised by assuming a 100% hydraulic efficiency for the whole operating range. Obviously such an assumption cannot be realised in practice, as is apparent in Table VII of the appendix giving the actual hydraulic efficiency for the radius $r = 0.69 R_o$. Fig. 2.12 shows these values plotted together with the corresponding efficiency values based on actual static pressure rise for the same radial station.

Before concluding this part, the aerodynamical characteristics computed for the propeller, as obtained by the method described in Part I, p.24 will be compared with those relating to a 10% circular back aerofoil as given in the paper R. & M. 2301, p.16, and Table VIII of the appendix. This circular back aerofoil was found to be the nearest available profile to that of the rotor blade. However, as the mean operating Reynold's number for the present propeller was about 65,000, as compared with 280,000 at which the 10% circular back aerofoil was tested, no conclusive deductions based on such comparison should be made.

The lift coefficient and the percentage of radial

secondary flow, plotted against the effective angle of attack, measured from the line of zero lift, for the rotor's profile at $r = 0.63 R_0$, are shown in figs. 2.13 and 2.14. The discontinuity of the lift curve, fig. 2.14, at an angle of attack -2.5° , can be explained by the fact that once radial secondary flow started, fig. 2.13, more energy is added by the associated Coriolis forces, hence the apparent increase in the value of the calculated lift coefficient. These forces are by no means accounted for in the aerofoil theory.

The numerical values used for plotting the two curves shown in figs. 2.13 and 2.14, are given in Table IX of the appendix.

The conclusions reached in the previous discussion are summarised hereafter:-

1. With no appreciable secondary flow, the fluid direction downstream of the rotor can be determined with good approximation either by Constant's rule, or by knowledge learned from two dimensional cascade tests.
2. Over a wide range of operating conditions, even with back flow established downstream of the rotor, the simplified equation of motion given by equation (2.8) holds.
3. A radial secondary flow corresponding to about 45% change in axial velocity, has no direct influence

on the flow direction downstream of the propeller.

4. The direction of the flow downstream of the propeller, at loadings other than the two dimensional design conditions is a function of the flow coefficient downstream of the rotor, the shape of the blade, and the corresponding peripheral speed. Algebraically, the relations are not clear, except that α_2, α'_2 and ϑ_2 are connected, i.e.

$$\cot \alpha_2 = f(\vartheta_2, \alpha'_2),$$

where,

α_2 = the angle between axial direction and the flow direction relative to the blade,

$$\vartheta_2 = \frac{w_2}{\Omega r}$$

= the flow coefficient downstream of the rotor,

α'_2 = the angle between axial direction and the tangent to the camber line at the trailing edge.

5. The conventional method used for applying the aerofoil theory in the case of a propeller operating at conditions, other than two dimensional flow, by replacing the actual fluid field by an imaginary flow moving with a velocity equivalent to the vectorial mean of both upstream and downstream

velocities, is by no means more than a rough approximation. The stronger the radial secondary motions, the larger the error caused by such an approximation, as there is nothing in the aerofoil theory to account for the Coriolis forces associated with radial secondary motion. This error, however, need not be so serious in the case of axial flow compressors, where larger hub to outer diameter ratio, and smaller blade height in comparison with rotor diameter, are more likely to be used.

PART III

THE CONTRA-ROTATORS

For efficient pumping the whirl velocity cannot be changed by more than a fraction of blade speed without abrupt changes in direction, and these necessarily cause much dissipation; to restrict losses it is therefore necessary to restrict the changes in whirl and so to restrict the work done per stage.

When however two adjacent rotors are rotated in opposite directions the difference in their speeds is twice their individual speeds, and apparently four times the work could be done by the combination, while averting the need for fixed guide blades.

Such a principle was long ago successfully used in the Ljungström contr-rotation radial flow steam turbine, in several fields of aeronautical and marine engineering, and within recent years in fans, e.g. Woods of Colchester, and also in certain pumps for paper mills.

The drive for contra-rotating propellers presents certain mechanical difficulties, though advances in electrical motor designs seem likely to overcome these, for example by rotating both stator and rotor of the motor as has been done by the De Bothezat Fans Division of American Machine and Metals, Inc.

If guide vanes are to be avoided, the two propellers must be of different form, or alternatively must rotate at different speeds. That this is so is apparent when the

symmetrical case is considered, for then by symmetry the fluid between the two propellers would rotate neither clockwise nor anticlockwise, and the fluid both before and after would have to be whirling. Thus guides before and after would be necessary.

The practical alternatives seem to be:-

- a. Two propellers rotating at equal but opposite speeds, the second propeller being differently designed to operate in the whirl generated by the first, and so requiring greater torque to drive it, but of the same pitch as the first.
- b. Two propellers rotating at different speeds but exerting equal torques. The pitch of the second should be greater and its camber less.

The present experiments, due to practical difficulties, relate to case (b) in which different speeds are used, though both propellers have the same pitch, and therefore neither could operate perfectly. The layout of the apparatus is shown in fig. 3.1 and plates 1 and 2. For the purpose of testing the unit with different axial spacing between the rotors, the housing for the shaft bearing was so designed to allow for changing the axial gap between the limits $5/16$ and $2\frac{1}{4}$ inches. Fig. 3.2 shows diagrammatically the arrangement used for that purpose. With the $2\frac{1}{4}$ inches axial gap, a $2\frac{1}{4}$ in. long cylindrical spinner, turned out of perspex was

fitted to the upstream rotor's hub, to form a continuation of the hub surface. The tests fall into two groups:-

(i) a measurement of the overall performance of the pair of propellers at various speeds and with various discharges without regard to the inside story of the flow within the machine.

(ii) A more detailed study of the motion upstream and downstream of the rotors, as well as in between them.

In measuring the performance as a whole the speed of the propellers ranged from 800 R.P.M. to 2400 R.P.M., in various combinations in which the speed of the second propeller ranged from 0.4 to 2.0 times the speed of the first. The flow was measured by regarding Section A, fig. 4. 1, as the throat of a venturimeter, the necessary pressure holes being drilled through the wall, and serving also to record the pressure at inlet to the fan. As for the delivery head, a pressure tapping was made in the cylindrical casing downstream of the rotors. As this reading, when the flow is spinning, by no means represents the mean delivery pressure, it is recorded as the "nominal" delivery head.

The speed of both rotors was varied and adjusted by suitably connected reohstats and potentiometers. For speed measurements, a mechanical tachometer as well as a variable frequency stroboscope were used. In order to keep the two

motors operating at the selected speeds, the frequency of stroboscopic light was so chosen to serve both rotors simultaneously. This was done by choosing the frequency to be equal to or a multiple of the common factor between the two rotational speeds multiplied by the number of the blades.

The torques were measured by sliding weights on arms attached to the frames of the two motors both of which were slung in ball bearings. The torques were corrected for shaft friction as measured with the propellers removed, and it was assumed the friction would not increase appreciably when the propellers in operation exerted their thrust on the shafts.

For examining the details of the flow in the second group of tests, the unit was operated at four rotational speed ratios: 0.75, 1.0, 1.25, and 1.5. Velocity and static pressure surveys were made upstream and downstream of each rotor by using the pilot cylinder and Prandtl's static tube. These tests were restricted to the case in which the rotors were $2\frac{1}{4}$ inches apart.

Preliminary experiments with the rotors $5/16$ in. apart, were made to confirm that the contra-rotating system was insensitive to scale effect, in regard to overall performance.

These tests were carried out with the rotors operating

at equal speeds, i.e. at $N_1/N_2 = 1.0$, where N_1 and N_2 are the rotational speeds of the upstream and downstream rotors respectively. Tests were carried out at speeds ranging from 800 R.P.M. up to 2400 R.P.M. At speeds such as 400 R.P.M. the measured quantities became so small that the magnitude of the possible experimental error were prohibitive. The characteristic curves obtained by these tests were reduced to a speed of 1000 R.P.M. on the usual simple similitude basis. Then for four specific discharges, the corresponding specific torque and manometric head were measured off the different characteristic curves (Table I)*. If the laws of similitude hold, then these measured values should be independent of the speed at which the test was carried out.

Fig. 3.3 (a, b and c) shows the values for the reduced head, total torque and torque ratios plotted against the actual test speed, and it is seen the test points do tend to drift away somewhat from the mean values to which they should adhere were scale effects entirely absent. However, experimental error at low speeds could account for the whole of the differences, and on the whole the results may be regarded as satisfactory evidence that interpolation can safely be done on the simple similarity basis.

Proceeding to the main series of performance tests the unit, with the rotors $2\frac{1}{4}$ in. apart, was run at different

* See appendix

rotational speed ratios ranging from $N_1/N_2 = 0.5$ up to 3.0, and the results are given in Table II of the appendix. It should be noted that the mechanical efficiencies in the Table are uncorrected for spin at exit which reduces the mean pressure at exit to a value somewhat less than that measured at the wall of the casing, and a correction as given in pp.92, is necessary.

Figs. 3, 4, 5, 6 and 7 show the head coefficient, torque ratio, torque coefficient and mechanical efficiency plotted against flow coefficient. From the efficiency curves, the optimum rotational speed is in the neighbourhood of $N_1/N_2 = 1.5$. It also shows that there is no point in operating the unit at speed ratios beyond $N_1/N_2 = 2.0$, as it will only result in a drop in the peak efficiency obtainable.

The curves of head against discharge seen in Fig. 3.4 form a regular and simple family, which by logarithmic plotting or by trail can be shown when $N_1 > N_2$ to reduce to:

$$H / (N_1 + N_2)^{1.85} = \Phi \left\{ Q / (N_1 + N_2)^{0.92} \right\},$$

where,

N_1 and N_2 : are the rotational speeds of the upstream and downstream rotors respectively,
while when $N_2 > N_1$, a single curve as seen in fig. 3.8 is

obtained by using the arithmetical mean of the two speeds. Similarly, regarding efficiency, this tends to reach its maximum when the discharge per unit area is a particular fraction of the arithmetical mean of the two blade speeds, though as seen in fig. 3.9, the ratio of the two speeds is not without a residual influence.

Regarding the influence of the axial spacing between the two rotors, tests carried out with the rotors $5/16$ in. and $2\frac{1}{4}$ inches apart when compared, as in figs. 3.10 (a, b, c and d) show remarkably little difference, except perhaps a rather irregular influence on the efficiency noticeable in fig. 3.10(d).

The efficiency as may be seen in Table III of the appendix differs by $\pm 4\%$ as the spacing between the propellers is increased from $5/16"$ to $2\frac{1}{4}"$, improving when N_1 is twice, but deteriorating when N_1 equals N_2 . Intermediately when $N_1 = 1.50 N_2$, there was also improvement, the efficiency being 79% with $5/16"$ spacing and 82% with $2\frac{1}{4}"$ spacing. When N_1 was less than N_2 , there was no measurable difference.

Why this should be is a little puzzling, though perhaps it may be explained in terms of the velocity of whirl in the space between the rotors. The values of the whirl reached at $r = 0.63 R_0$, were: -

N_1/N_2	1.50	1.00	0.75
$\frac{\text{Whirl}}{\text{Axial vel.}} = \frac{U_2}{W_2}$	0.35 to 0.60	0.21 to 0.36	0.09 to 0.28

Two factors seem involved, (a) friction between casing and whirling fluid, (b) interference between the two propellers. The first seems to increase with the spacing, while the second decreases. However, when N_1/N_2 is small there is little circulation about the first set of blades and presumably the wakes would be correspondingly narrow. When N_1/N_2 is large, the first rotor has to operate with an excessively large angle of attack, the wakes are large and also the distortion of the pattern, so space between the rotors is needed to reduce interference.

This view is supported by the absence of noise when the two rotors were $2\frac{1}{4}$ " apart, for they were then apparently no more noisy than a single rotor, whereas with $5/16$ " spacing they emitted a loud noise, especially when run at 2400 R.P.M.

Regarding the magnitude of the correction needed to convert the exit pressure from its casing value, as measured, to the true mean across the section, the latter, duly "weighed" for the different discharge in the different parts is:

$$P_{dm} = \frac{\int_{R_H}^{R_o} P_s \cdot W \cdot 2\pi r dr}{Q},$$

where,

p_{dm} = the mean or "weighed" static pressure,

p_s = the local static pressure at any radius r ,
in the chosen traverse downstream of the
rotor,

w = the local axial velocity at the same radial
station,

Q = the flow rate at the particular loading, at
which the mean delivery pressure is to be
computed.

The following table gives the ratio of the static pressure at radial station $r/R_o = 0.89$ to that at $r/R_o = 0.37$, for rotational speed ratios $N_1/N_2 = 1.5, 1.0$ and 0.75 , when operating at different flow coefficients, the ratio of axial velocity at the outer radial station to that nearer to the hub is also included.

TABLE IV

N_1/N_2	q_c	$p_{s,r}$	w_r	N_1/N_2	q_c	$p_{s,r}$	w_r
1.50	0.575	1.048	1.09	1.00	0.603	1.07	1.34
1.50	0.543	1.032	1.25	1.00	0.532	1.11	3.64
1.50	0.513	1.038	1.56	0.75	0.767	1.40	1.34
1.50	0.490	1.044	2.41	0.75	0.730	1.30	1.45
1.50	0.455	1.089	2.41	0.75	0.685	1.22	1.76
1.00	0.645	1.018	1.11	0.75	0.657	1.23	2.06
1.00	0.630	1.043	1.19	0.75	0.625	1.24	5.15

where,

N_1 and N_2 : rotational speeds of the upstream and downstream rotors respectively

$$q_c = \frac{60 Q}{N_1 D^3} ,$$

$p_{s,r}$ = ratio of the static pressure at radial station $r/R_o = 0.89$, to that at $r/R_o = 0.37$,

w_r = ratio of the axial velocity component at radial station $r/R_o = 0.89$ to that at $r/R_o = 0.37$.

The above table shows that static pressure variations, are more pronounced when the unit is operating at rotational speed ratio $N_1/N_2 = 0.75$. Therefore, the error resulting from taking the static pressure at the outer wall, in place of the true mean value, will be computed for this speed ratio as it is liable to give the maximum value for such error.

The above equation giving p_{dm} was used for calculating the true mean value for the static pressure. The following table gives the local pressure at radial station $r/R_o = 0.37$, and 0.89 expressed as a ratio to the mean or "weighed" static pressure:

TABLE V

$$N_1/N_2 = 0.75$$

$q_c = 60 Q / N_1 D^3$	0.767	0.730	0.685	0.657	0.625
$(p_s/p_{dm})_{r/R_o = 0.37}$	0.740	0.799	0.840	0.835	0.823
$(p_s/p_{dm})_{r/R_o = 0.89}$	1.038	1.034	1.027	1.028	1.020

This table clearly shows that with pressure variations across the flow section of the order 40%, the error resulting from taking the pressure at the outer wall, in place of the true mean or weighed value, will not be more than 4%, in the worst case. This result can be easily explained by the fact that generally speaking, static pressure variations across the flow section is associated with axial flow velocity changes, which when combined with the larger flow areas available at outer radial stations, will make the pressure values at these stations have more weight in deciding the value of mean static pressure.

A further point of importance concerns the kinetic energy of the fluid downstream of the rotors. Generally speaking, where energy is added by dynamic forces due to velocity changes, the process involves two main steps:

- a) imparting the energy by augmenting the kinetic energy,
- b) recuperating or converting the major part of this added kinetic energy into useful pressure energy. For efficient pumping, the fluid has to leave with the least possible kinetic energy.

Therefore it is interesting to estimate the excess kinetic energy downstream of the contra-rotating unit. Two cases are considered, one with $N_1/N_2 = 1.50$, approximating to the optimum value, and another with

$N_1/N_2 = 0.75$, representing an extreme condition with appreciable excess kinetic energy downstream of the rotors. The excess kinetic energy, expressed as a percentage of the total energy rise across the contra-rotating unit was computed by using the following equation:

$$\text{Excess kinetic energy} = \frac{100}{Q} \int_{R_H}^{R_o} \frac{\frac{\rho}{2}(W^2 - \bar{U}^2)}{T_3 - T_1} \cdot 2\pi r \cdot w dr,$$

where,

- W = the absolute velocity at a radius r , in the selected traverse downstream of the rotors,
- $\bar{U}$ = the mean flow velocity,
- w = the local axial flow velocity at the radial station considered,
- $T_3 - T_1$ = the rise in total energy across the two rotors, at the same radial station,
- Q = the flow rate at which the excess kinetic energy is to be evaluated.

The following table gives the excess kinetic energy as a percent of the total energy imparted to fluid, together with observed efficiencies, at different flow coefficients, for rotational speed ratios $N_1/N_2 = 1.5$ and 0.75 . Also, a modified efficiency value based on the assumption that all the excess kinetic energy has been successfully converted into useful pressure energy is included.

TABLE VI

a - $N_1/N_2 = 1.50$

$q_c = 60 Q / N_1 D^3$	0.575	0.543	0.513	0.490	0.455
Excess K.E. %	2.0	2.5	1.8	2.6	4.1
Nominal Effy. %	68.5	78.5	82.5	82.5	79.9
Modified Effy. %	70.0	80.5	84.0	84.8	83.4

b - $N_1/N_2 = 0.75$

$q_c = 60 Q / N_1 D^3$	0.767	0.730	0.685	0.657	0.625
Excess K.E. %	11.8	9.6	9.3	9.2	13.5
Nominal Effy. %	68.0	73.0	76.0	77.0	75.5
Modified Effy. %	77.0	80.9	84.0	85.0	87.3

Fig. 3.11(a and b) shows excess kinetic energy as a percentage of total energy imparted, nominal and modified efficiencies plotted against flow coefficient for both rotational speed ratios: $N_1/N_2 = 1.50$ and 0.75 .

It is interesting to note that though the variations of the axial flow velocity across the flow section downstream of the rotors, for the rotational speed ratio: $N_1/N_2 = 1.50$ was appreciable, e.g. W_r which is the ratio of axial velocity component at $r/R_o = 0.89$ to that at $r/R_o = 0.37$, was about 1.56 at a flow coefficient $q_c = 0.513$ (Table IV), yet the excess kinetic energy as a percentage

of the total energy imparted to the fluid was only 1.8% and never exceeded 5% for the range of flow coefficients embraced by the experiments. Before concluding this section which is more or less dealing with the overall performance of the contra-rotating unit, it is convenient to consider some axial velocity traverses taken downstream of the rotors. They concern the speed ratio: $N_1/N_2 = 1.50$, and flow coefficients $q_c = 0.575$, 0.513 and 0.455 . The following table and fig. 3.12 give the axial flow velocity expressed as a ratio to the mean flow velocity:

TABLE VII

r/R_o	$\frac{w}{U} = \frac{\text{Local axial vel.}}{\text{Mean flow vel.}}$		
	$q_c = 0.575$	$q_c = 0.513$	$q_c = 0.455$
0.32	0.85	0.58	0.38
0.37	0.93	0.68	0.51
0.43	1.00	0.86	0.62
0.56	1.07	1.09	1.08
0.69	1.04	1.12	1.32
0.76	1.05	1.11	1.30
0.89	1.01	1.07	1.22
0.99	0.74	0.72	0.88

It is noticable that the axial velocity distribution downstream of the rotors is far from being uniform, even when the efficiency is a maximum as it is when $q_c = 0.513$, there is great inequality. However, the values in the table perhaps over-emphasise this, since when expressed as a percentage of the total energy imparted, the excess K.E. is actually not more than 2%. Nevertheless, the velocity distribution does suggest that a conical hub as in fig. 3.13, would be an advantage for such contra-rotating units.

It is interesting to compare the overall results with those for a single rotor fitted with an imaginary 100% efficient recuperator or guide vanes, but first a few words will be said about torque measurements. As previously mentioned, torque for both rotors was mechanically measured. It is possible also to compute torques by evaluating the total change in angular momentum from the velocity measurements upstream and downstream of the rotor, and this has been done for the case $N_1 = N_2$. The total change in angular momentum was calculated from:

$$\text{Torque } T = \rho \int_{R_H}^{R_o} 2 \pi r \cdot w \cdot v \cdot dr \quad \pm \quad \rho \int_{R_H}^{R_o} 2 \pi r w \cdot v \cdot dr ,$$

where the positive and negative signs are used when the angular momentum upstream and downstream of the rotor are in the opposite or the same directions respectively. The above equation can be reduced to the following form:

$$\text{Torque} = \rho \pi \left\{ \left(\int_{R_H}^{R_o} r^2 W \sin 2\alpha \, dr \right)_{D.S} \pm \left(\int_{R_H}^{R_o} r^2 W^2 \sin 2\alpha \, dr \right)_{U.S} \right\},$$

where,

w = the axial velocity component,

v = the tangential velocity component,

W = the absolute velocity,

α = the angle between the axial and flow directions.

The following table gives the resulting values of T_1' , and T_2' , which denote the computed torques applied to the upstream and downstream rotors respectively, together with the mechanically measured values T_1 and T_2 .

Table VIII

$$N_1 = N_2 = 2000 \text{ R.P.M.}$$

Mechanically measured			Computed		
$q_c = \frac{60Q}{ND^3}$	T_1 ft.lb.	T_2 ft.lb.	$q_c = \frac{60Q}{ND^3}$	T_1' ft.lb.	T_2' ft.lb.
0.592	0.155	0.228	0.594	0.151	0.222
0.618	0.138	0.211	0.624	0.139	0.226
0.642	0.125	0.193	0.635	0.127	0.207
0.666	0.114	0.175	0.680	0.109	0.158
0.680	0.110	0.163			
0.690	0.108	0.154			
0.700	0.107	0.143			
0.712	0.106	0.133			

Fig. 3.14 (a and b) shows the torques obtained by both methods plotted against flow coefficients, and the agreement is very good.

To compare the performance of the contra-rotators with that of a single propeller and guide vanes, it is necessary to know the performance of the guide vanes, and as that is an indefinite quantity, it is assumed for simplicity that they are perfect, that they convert all excess.K.E., whether spin or radial inequality, into useful pressure. The efficiencies and manometric head so obtained are ideal values, which by no means can be realised in practice. The corresponding delivery pressure is computed from:

$$P_{th} = \frac{1}{Q} \int_{R_H}^{R_o} \left(T_2 - \frac{\rho}{2} \bar{U}^2 \right) 2\pi r \cdot w_2 \, dr,$$

where,

P_{th} = the theoretical mean static pressure downstream of the rotor, when provided with a perfect recuperator,

Q = the rate of flow at which the static pressure is computed,

T = the total energy at any radial station r , downstream of the rotor,

$\bar{U}$ = the mean axial flow velocity,

and w_2 = the local axial velocity at the radial station considered.

Then using the computed mean static pressure-rise, the ideal manometric head is obtained. Such computations were carried out for one rotor operating alone, for one rotor operating with the boosting set-up as well as for when it was operating in tandem with a contra-rotating propeller, $2\frac{1}{4}$ inches downstream of it.

The following tables give values of the manometric head coefficients at the different flow coefficients for the three conditions:

TABLE IX

Single rotor N = 2200 R.P.M.		Single rotor with boosting arrangement		
$q_c = \frac{60 Q}{N D^3}$	$h_c = \frac{H_m}{\frac{1}{2} \rho (\Omega R)^2}$	R.P.M.	q_c	h_c
0.609	0.109	1500	0.800	-0.031
0.553	0.145	1500	0.710	0.032
0.450	0.180	2000	0.667	0.058
0.425	0.188	2000	0.535	0.149
0.352	0.210			

TABLE X

The upstream propeller of two contra-rotators, spaced at $2\frac{1}{4}$ inches apart				
N_1/N_2	N_1 R.P.M.	N_2 R.P.M.	$q_c = \frac{60 Q}{N D^3}$	$h_c = \frac{H_m}{\frac{1}{2} \rho (\Omega R_o)^2}$
1.0	2000	2000	0.685	0.050
1.0	2000	2000	0.635	0.067
1.0	2000	2000	0.594	0.095
1.0	2000	2000	0.590	0.139
1.5	2100	1400	0.543	0.137
1.5	2100	1400	0.513	0.145
1.5	2100	1400	0.490	0.153
1.5	2100	1400	0.455	0.164

Fig. 3.15 shows these head coefficients plotted against varying flow coefficients, for the three testing arrangements assuming a perfect diffuser has been added. The plotted points are seen to lie on a single curve and so confirm the conclusion reached in Part II, pp. 73, that the performance of the upstream rotor, is completely independent on whether it is operating alone or in tandem with another propeller downstream of it.

The following table gives the torque coefficients as obtained for one rotor when operating at 2200 R.P.M.

TABLE XI

$q_c = \frac{60 Q}{ND^3}$	$T_c = \frac{T}{\eta (N/60)^2 D^5}$	q_c	T_c
0.705	0.0338	0.611	0.0522
0.690	0.0374	0.591	0.0550
0.675	0.0406	0.575	0.0577
0.660	0.0435	0.556	0.0605
0.644	0.0465	0.537	0.0626
0.627	0.0492	0.518	0.0650

Fig. 3.16 shows torque coefficients plotted against flow coefficient. Using Tables IX and X a theoretical value for the efficiency can be obtained from the following expression:

$$\eta = \frac{\pi \cdot h_c \cdot q_c}{4 T_c} \times 100 \quad \%$$

Fig. 3.17 shows the resulting efficiency and relates to a single rotor followed by a perfect recuperator. The corresponding efficiency curves for the contra-rotators unit, at various rotational speed ratios, are also included from which it may seem the contra-rotating unit has lower efficiency, though this perhaps is misleading since the single rotor is assumed to have a perfect recuperator.

Actually with an inefficient recuperator the single fan would be inferior. Both would have equal peak efficiency,

when the recuperator was 80 per cent efficient. At best, the single fan, even with 90% recovery, would show no more than 5 per cent advantage over the contra-rotators. The latter, however, by varying the speed ratio, would be better able to cope with a wide range of flows.

A further comparison is shown in fig. 3.18 (a and b) where the torque and head coefficients are plotted against flow coefficients for the contra-rotators when operating at speed ratio, $N_1/N_2 = 1.0$, as well as those corresponding to an equivalent two stage set, each of which comprises one propeller fitted with a perfect recuperator.

It is noticeable that the head coefficient for the contra-rotators, as obtained by direct measurements is on the average 7% higher than that for two single fans in series each with perfect recuperators. This can be explained since the second fan of the contra-rotators meets air already rotating in the opposite sense and so the velocities relative to the downstream rotor blades are greater, and generally speaking, will be associated with larger "lift" forces and larger torques.

Regarding the interaction of the two propellers, it was shown in Part II, that when the upstream flow was axial and uniform, the direction of the flow relative to the blade on the downstream side, is a function of both the geometrical form of the blade, and of the downstream

velocity of flow. Furthermore, it was confirmed experimentally that the flow pattern within the propeller is, generally speaking, independent of the conditions prevailing immediately downstream, except for their effect on the rate of discharge. In other words, with the contra-rotating arrangement the downstream rotor has no direct influence on the flow picture through the upstream propeller.

On the other hand, the flow pattern through the downstream rotor is greatly influenced by the upstream propeller. Not only is the flow rotating as it reaches the second rotor but the axial component will generally be far from uniform. Further, the field as a whole will deviate considerably from the ideal potential flow upstream of the first rotor.

Therefore it is interesting to study the performance of the second rotor, and to compare it with a single rotor. For this comparison the downstream flow direction relative to the blade, at two radial stations: $r/R_0 = 0.43$ and 0.69 are convenient. The cotangent of the angle formed by the axial and the relative flow direction, and the downstream flow coefficient are given in Table XII. The spin upstream of the rotor, expressed as a ratio to the blade peripheral speed at the radial station considered is also included. The table gives these values at both radial stations, and shows their variation with change of speed ratio.

TABLE XII

a. Radial station $r/R_o=0.43$

N_1/N_2	N_2	ϑ_2	$\cot \alpha_2$	$v_r/\Omega r$	N_1/N_2	N_2	ϑ_2	$\cot \alpha_2$	$v_r/\Omega r$
1.5	1400	0.945	0.788	-0.396	1.25	1600	0.836	0.806	-0.316
1.5	1400	0.864	0.759	-0.498	1.25	1600	0.833	0.833	-0.379
1.5	1400	0.825	0.730	-0.549	1.25	1600	0.732	0.765	-0.412
1.5	1400	0.664	0.633	-0.527	1.25	1600	0.622	0.688	-0.447
1.5	1400	0.510	0.527	-0.556	1.25	1600	0.522	0.607	-0.472
1.0	2000	0.730	0.803	-0.180	0.75	2000	0.596	0.787	-0.040
1.0	2000	0.693	0.798	-0.222	0.75	2000	0.543	0.762	-0.099
1.0	2000	0.655	0.772	-0.244	0.75	2000	0.486	0.678	-0.123
1.0	2000	0.576	0.691	-0.280	0.75	2000	0.398	0.569	-0.169
					0.75	2000	0.389	0.565	-0.187

TABLE XII (Continued)

b. Radial station $r/R_0=0.69$

N_1/N_2	N_2	ϑ_2	$\cot \alpha_2$	$v_1/\Omega r$	N_1/N_2	N_2	ϑ_2	$\cot \alpha_2$	$v_1/\Omega r$
1.5	1400	0.595	0.505	-0.208	1.25	1600	0.520	0.481	-0.146
1.5	1400	0.565	0.481	-0.245	1.25	1600	0.503	0.471	-0.175
1.5	1400	0.554	0.475	-0.257	1.25	1600	0.489	0.464	-0.209
1.5	1400	0.541	0.472	-0.308	1.25	1600	0.486	0.462	-0.232
1.5	1400	0.545	0.476	-0.320	0.75	2000	0.435	0.486	-0.086
1.0	2000	0.463	0.470	-0.095	0.75	2000	0.419	0.480	-0.157
1.0	2000	0.450	0.467	-0.125	0.75	2000	0.404	0.458	-0.185
1.0	2000	0.442	0.459	-0.130	0.75	2000	0.395	0.454	-0.234
1.0	2000	0.423	0.442	-0.137	0.75	2000	0.388	0.448	-0.279

Where,

ϑ_2 = the flow coefficient downstream of the second propeller

$$= \frac{w_2}{\Omega r} ,$$

$\cot \alpha_2$ = the cotangent of the angle formed between the axial and the relative flow direction

$$= \frac{w_2}{\Omega r - u_1}$$

Ωr = the peripheral speed of the rotor blade at the same radial station,

u_1 = the tangential velocity component upstream of the second rotor at the radial station considered.

In Fig. 3.19 (a and b) $\cot \alpha_2$ is shown plotted against the downstream flow coefficient for both stations: $r/R_0 = 0.43$, and 0.69 . The corresponding values for a single propeller with potential flow upstream, i.e. uniform and axial, is also included. The curves are remarkably similar in appearance and the effect of the whirl and other irregularities on the second rotor is to diminish $\cot \alpha_2$, and the effect is progressively more pronounced as the ratio of the rotational speeds increases. However, $\cot \alpha_2$ is primarily a function of the magnitude of the relative pre-spin rather than of the speed ratio, and one may write:

$$\cot \alpha_2 = f(\alpha'_2, \vartheta_2, \chi) ,$$

where,

α_2 = the angle between the axial and flow direction
relative to the blade,

α'_2 = the angle between the tangent to the trailing
edge of the camber line, and the axial direction,

ϑ_2 = the downstream flow coefficient,

χ = the relative upstream pre-spin
= $v_t / \Omega r$,

v_t = the tangential velocity component upstream of
the rotor.

However, the contra-rotators were tested only over a limited range of flow coefficients, in the region of the optimum loading and the experimental data are hardly sufficient to fully establish the function, particularly as they do not include the ranges in which either of the rotors is excessively stalled, nor when back flow develops downstream of the second propeller. Nevertheless, there is enough information on the effect of pre-spin on the second rotor to give a general idea.

According to the results obtain^{ed} _{χ} at station $r/R_0 = 0.43$, the value of the function at one particular downstream flow coefficient $\vartheta_2 = 0.50$, depends on the relative pre-spin, in the manner shown in the following table:-

TABLE XIII

N_1/N_2	$\cot \alpha_2$	$\chi = \frac{U_1}{\Omega r}$	$C_{\cot \alpha_2} = \frac{\cot \alpha_2}{(\cot \alpha_2)_{\chi=0}}$
1.50	0.51	-0.57	0.68
1.25	0.58	-0.48	0.77
1.00	0.63	-0.33	0.84
0.75	0.70	-0.12	0.94
single rotor	0.75	0.00	1.00

The negative sign for the relative pre-spin indicates that the direction of the pre-spin was in a direction opposite to the propeller rotation. Fig. 3.20 gives the factor $C_{\cot \alpha_2} - \chi$ plotted against the corresponding values of the relative pre-spin.

Unfortunately, with the present contra-rotators, it was not possible to investigate the effect of pre-spin of the same direction as that of the propeller rotation. However, the $C_{\cot \alpha_2} - \chi$ relation can be expected to assume a skew-symmetrical form, as shown in Fig. 3.20 by extrapolated dotted line.

The remaining item to be considered is the application of the equation of motion pp.62 , to the fluid downstream of the second propeller. This equation is:

$$w \cdot \gamma - v \cdot \zeta = - \frac{1}{\rho} \frac{\partial H}{\partial r}$$

where

$$\begin{aligned} \zeta &= \text{the tangential vorticity component,} \\ &= -\frac{\partial w}{\partial r}, \\ \gamma &= \text{the axial vorticity component} \\ &= \frac{1}{r} \frac{\partial(r.v)}{\partial r}. \end{aligned}$$

When the axial vorticity component is much smaller than the tangential vorticity component, there is no need to verify this equation; the term $v.\gamma$ is negligible and the result is simply Bernouilli's equation expressed in differential form.

With the contra-rotators, however, over the whole range of loading there were few tests in which the axial vorticity downstream of the second rotor was **relatively large** compared with the tangential component of vorticity.

The procedure, pp.77, originally used for verifying the equation of motion downstream of the single propeller, was again followed on preparing Table XIV in which the radial total energy gradient as obtained both by using the simplified equation of motion and by direct measurements, is given for the radial station $r/R_0 = 0.43$.

According to Table XIV, it is clear from columns 7 and 8, that the simplified equation of motion holds for the fluid on the downstream side of second propeller, despite the considerable upstream vorticity seen in column 4.

TABLE XIV

N_1 / N_2	N_2	ϑ_2	$(\frac{1}{\rho} \frac{\partial H}{\partial r})_{US}$	w. ζ	v. ζ	Radial gradient of total energy		Difference
						Measured $\frac{1}{\rho} \frac{\partial H}{\partial r}$	Calculated* $\frac{1}{\rho} \frac{\partial H}{\partial r}$	%
1.25	1600	0.522	2440	-2378	-330	2120	2048	3.4
1.112	1800	0.509	2340	-3440	-296	3185	3144	1.3
0.75	2000	0.596	-255	-1410	333	1785	1743	2.4
0.75	2000	0.543	127	-1770	454	2230	2224	0.3
0.75	2000	0.486	615	-1770	386	2280	2156	5.5

* Calculated from the simplified equation of motion

Summary of findings regarding Contra-rotators

- 1) For any particular rotational speed ratio, the experiments confirmed the following relations:

$$Q \propto N$$

$$H \propto N^2$$

$$T \propto N^2, \dots \text{etc.},$$

where,

Q = the rate of flow ,

H = the manometric head,

T = the applied torque.

Thus the simple laws of similarity are confirmed.

- 2) For two identical rotors, the efficiency reached its maximum in the neighbourhood $N_1/N_2 = 1.5$. Beyond $N_1/N_2 = 2.0$, the efficiency diminished appreciably.
- 3) With constant specific upstream rotor speed, the head-discharge characteristic can be deduced from that at $N_1/N_2 = 1.0$, by the following approximate relations:

$$Q_x / Q_{1.0} = 0.50 (1 + 1/N_R) ,$$

and

$$H_x / H_{1.0} = 0.25 (1 + 1/N_R)^2 ,$$

where,

x : denotes values relating to any arbitrary speed ratio N_R ; and (1) denotes values

corresponding to $N_r = 1.0$

- 4) Regarding the axial spacing of the rotors, curves were substantially the same at axial spacings 0.18 and 1.3 chords. However, when $N_1/N_2 = 1.5$ and 2.0, the efficiency was slightly lower with the smaller axial spacing, for the reasons given on pp.91. The unit was noisy with the small axial gap, but as quiet as a single fan, when operating with the larger axial spacing, thus showing how rapidly the disturbances from the first rotor fade away.
- 5) The pressure at the outer wall was found to be a reliable estimate of the mean exit pressure. The error under the extreme operating conditions at $N_1/N_2 = 0.75$, was no more than 4%.
- 6) The excess kinetic energy downstream of the contra-rotators, as a percentage of the total energy imparted to fluid was about 2.0% on the average, and never exceeded 5%, for the optimum loading at $N_1/N_2 = 1.50$.
- 7) The behaviour of the upstream rotor of a contra-rotating unit is uninfluenced by the downstream rotor and is identical with that of a single rotor operating alone.

- 8) A contra-rotator unit yields an efficiency within 5-10% of that possible with a single rotor fitted with a perfect recuperator, and equal to one with 80% recovery.
- 9) The contra-rotating unit was more flexible from the point of view of maintaining high efficiency over a wider range of flow coefficients.
- 10) The manometric head generated by a contra-rotating unit operating at $N_1/N_2 = 1.0$ is about equal to that with a two stages of single rotors each with a perfect recuperator.
- 11) The flow direction relative to the blade downstream of the second rotor, is a function of the relative pre-spin, as well as of the downstream flow coefficient and of the geometrical features of the blade.
- 12) The simplified equation of motion holds for the fluid downstream of the second rotor.

PART IV

Boundary Layers and Pre-rotation
in the Inlet Bellmouth

The inlet to the fan, designed to avoid irregularities in the stream as it approached the rotor, took the form of the bellmouth seen in fig. 4.1., and plate 6. It also served the subsidiary purpose of metering the flow, which was done by regarding section A , as the throat of a nozzle. There were three pressure tapings drilled through the outer wall, and spaced at 90° . They were $1\frac{1}{8}$ in. upstream of the cylindrical casing, and were manifolded to a single pressure lead.

Earlier, a definite though slight pre-rotation had been found in fluid as it approached the rotor, and curiously this pre-rotation was always anti-clockwise, that is it was rotating always with the rotor, which raises the question as to whether it could not have been caused by recirculation of still-spinning air which from the downstream exit might have found its way back to the inlet. Accordingly certain of the traverses were re-examined regarding the spin at exit. They included a wide range of conditions, and may conveniently be grouped as follows:-

A. A single rotor discharging through a lateral gate, as shown in fig. 4.1.

B. A pair of contra-rotators, spaced $2\frac{1}{4}$ in. apart, discharging through the lateral gate as in A.

Cases with clockwise and anticlockwise spin in the discharge and also some with no spin are included in this group.

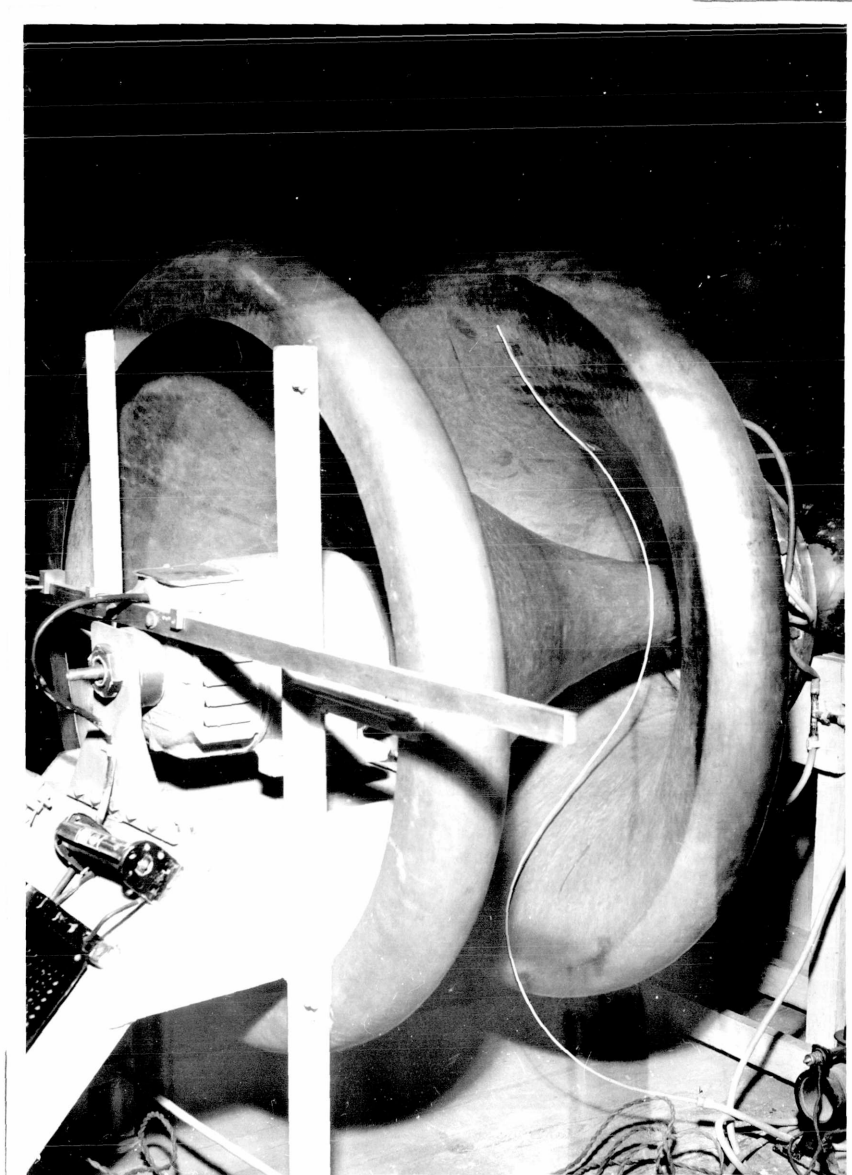


Plate 6 - The inlet bellmouth, and the method used for measuring the pressure along the curved boundary walls.

C. A single rotor, followed 28 inches further downstream by a booster fan rotating in the opposite sense, and discharging axially as shown by the chain dot lines in fig. 4.1. Some of the experiments in this group include upstream velocity traverses at section T₀ , fig. 4.1.

The following are the particulars for the throat section A, fig. 4.1:

Internal diameter of the outer wall $2R_o$	=	12.18"
External diameter of the hub $2R_h$	=	3.64"
Cross sectional area of throat	=	0.736 sq.ft.
Wetted perimeter	=	4.14 ft.
Hydraulic mean depth m	=	0.178 ft.
Equivalent hydraulic diameter d	=	0.712 ft.

Reynold's number for throat section A:

$$R_n = \frac{\bar{U}_{fl} \cdot d}{\nu} = 0.448 \times 10^4 \bar{U}_{fl} ,$$

where,

$\bar{U}_{fl}$ = the mean flow velocity through the throat area,

ν = the kinematic viscosity for air = 1.59×10^{-4} ft²/se

The coefficient of discharge will be:

$$C_d = \bar{U}_{fl} / \sqrt{2 \times (p_o - \bar{p}) / \rho} ,$$

where:

ρ = the air density = 0.002378 slug/cu.ft.,

p_o = the pressure outside the bell mouth inlet, where

the fluid velocity is practically zero,

$\bar{p}$ = the mean pressure at the throat section in lb./ft.²

Assuming for the moment the traverses are exact, then their integrated discharge inserted in the nozzle formula above, gives the discharge coefficient C_d . The resulting values are tabulated below together with the spin at section T_2 downstream of the rotor, fig.4.1., as measured by the tangent of the angle included between the flow and the axial direction at $r = 0.63 R_o$. The positive sign indicates a clockwise spin, when looking axially downstream the cylindrical casing, thus:

$$w = \tan \theta = u_2 / w_2 ,$$

where:

θ = the angle the fluid makes with the axial direction.

u_2 = the tangential velocity component,

w_2 = the axial velocity component.

Table IV

Group	$p_o - p$ lb./ft. ²	R_n	$\frac{u}{w} = \tan \theta$ at Sec. T_2	C_D Mean.
A - single rotor	Min. 0.640 Max. 1.280	1.01×10^5 1.43×10^5	-0.695 -0.308	0.971
B - contra-rotators	Min. 0.575 Max. 1.160	0.96×10^5 1.36×10^5	-0.287 +0.392	0.975
C - booster (contra-rotating)	Min. 0.587 Max. 0.998	0.97×10^5 1.28×10^5	- -	0.974

Details will be found in Tables I, II and III of the appendix.

The slightly smaller values of discharge coefficient for group A, may be due to a wooden cube, six inches high, used in those early tests to support the inlet. Later it was found to be unnecessary and was removed. No doubt the cube by creating eddies could have reduced the discharge coefficient by 0.4% as observed.

Clearly the discharge coefficient is independent of the discharge conditions, which is not surprising since the spin in the pre-rotation never exceeded the value: $\tan \theta = \frac{U}{W} = 0.09$, and on the average ranged from $\tan \theta = -0.03$ to -0.05 , and these variations could hardly alter the nozzle-coefficient.

Therefore some other quantity to serve as a measure, more sensitive to changes in the pre-rotation is needed. Considering a balanced vortex régime:

$$\frac{\partial P}{\partial r} = \rho \frac{U^2}{r} ,$$

and the radial drop in the static pressure is seen to be just that needed criterion. It is given in Table V of the appendix, as the pressure difference between two different radii, $r = 0.89 R_0$ and $0.37 R_0$ at section T₁, fig. 4.1, where the dimensions, differing only very slightly from those previously given in pp.115 for section A, are now:

Outer diameter $2R_o$ 12.078"

Hub diameter $2R_h$ 3.453"

Flow area 0.731 sq.ft.

Wetter perimeter 4.07 ft.

Hydraulic mean depth 0.180 ft.

Equivalent hydraulic diameter 0.72 ft.

Corresponding Reynold's number = $0.453 \times 10^4 \bar{U}_{f1}$,

where $\bar{U}_{f1}$ is the mean flow velocity at section T_1 .

Judged from the pressure difference ($\frac{P_2}{P_1} - 1$) the pre-rotation seemed not to vary in any systematic way. The average values in the three groups of experiments were:-

Table VI

Group	R_n	$\frac{P_2}{P_1} - 1$	Equivalent vorticity rad./sec. *
A - single rotor	Min. 1.03×10^5 Max. 1.46×10^5	0.050	6.9
B - contra-rotators	Min. 1.08×10^5 Max. 1.39×10^5	0.038	6.0
C - booster (contra-rotating)	Min. 0.976×10^5 Max. 1.24×10^5	0.041	6.3

* These correspond to $r = 0.89 R_o$. Free vortex, and 30 ft./sec. mean flow velocity were assumed.

p_2 and p_1 : are the static pressures at $r = 0.89 R_o$ and $0.37 R_o$ respectively.

At first sight these values seem consistent with the idea that pre-rotation did originate downstream, yet the following experiments in which traverses were taken both at inlet and outlet show conclusively it is not so.

TABLE VII

Comparison of whirl at inlet and outlet

Condition B (Contra-rotators).

$N_1/N_2 = 1.00$ $Q = 17.63$ cusecs. and $R_n = 1.09 \times 10^5$

r/R_0	Upstream of rotors		Downstream of rotors	
	θ , degs.	$\tan \theta_1 = \frac{U_1}{W_1}$	θ_2 degs.	$\tan \theta_2 = \frac{U_2}{W_2}$
0.37	+ 4.0	+ 0.070	-51.5	-1.26
0.50	+ 2.5	+ 0.040	-24.5	-0.45
0.63	+ 1.3	+ 0.020	-11.3	-0.20
0.76	+ 2.0	+ 0.035	-4.0	-0.07
0.89	+ 1.9	+ 0.033	-3.0	-0.05

Furthermore, a comparison of the torque obtained by mechanical measurements with that by angular momentum computations provides another method for estimating the pre-spin, and the relevant data are given in Table VIII, for the upstream rotor operating as follows:-

$N_1/N_2 = 1.0$, $N_1 = N_2 = 2000$ R.P.M.,

the axial gap between the rotors = $2\frac{1}{4}$ in.

Table VIII

Flow Q cusecs.	Torque by mechanical measurements ft.lb.	Flow Q cusecs.	Torque by Angular Momentum	
			Downstream ft.lb.	U.S. and D.S. ft.lb.
19.70	0.155	17.63	0.208	0.195
20.60	0.138	19.80	0.163	0.151
21.40	0.125	20.80	0.161	0.139
22.40	0.114	21.15	0.150	0.127
22.70	0.110	22.78	0.128	0.109
23.00	0.108			
23.30	0.107			
23.78	0.106			

and it is seen, both from Table VIII, and from fig. 4.2 prepared from the data of the Table, that the two methods of measurement are mutually consistent. Were the pre-spin ignored as if it was due to faulty measurement, then a discrepancy of 10 per cent would have to be accounted for. Such an error is most unlikely, and the existence of the secondary upstream spin, as well as its sense of rotation, seems confirmed.

From the above analysis the following conclusions are reached:-

1. There is a small, but quite marked secondary spin or pre-rotation upstream of the rotor, in the same

rotational direction as the propeller.

2. This pre-rotation was found under a variety of operating conditions.
3. The spin of the discharged stream had no apparent influence on this secondary spin. Cases with no spin on the discharge side, as well as with strong reversed spin confirmed this.

How it could possibly arise from the rotor, or from some inherent feature of the inlet bell mouth, is difficult to conceive. Certainly the axial velocity is greater near the hub as if the low pressure near the centre of the rotor were exerting an influence upstream, but that would be in the nature of the presence upstream of the weak outer parts of the blade vortices, fluctuating plus and minus about their mean and so reducing the pressure, but not transmitting spin to the fluid. Actually the speed in the pre-rotation was small, no more than 2 ft. per second, so perhaps for the time being it is best attributed to thermal movements of the air within the laboratory.

Next the boundary layers along the curved boundary walls will be considered. As the boundary layer computations are based on Blasius's and Karman's solutions, a summary for both will be given.

If y denotes distance measured normally to the surface considered, x distance measured along curves orthogonal to the

normals, and measured from the stagnation point, fig. 4.3., then the boundary layer equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} , \quad (4.1)$$

$$0 = - \frac{1}{\rho} \frac{\partial p}{\partial y} \quad (4.2)$$

and the equation of continuity is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.3)$$

where

u and v are the velocity components along the x and y directions,

ρ = fluid density,

ν = fluid kinematic viscosity,

If U denotes the velocity of the main stream at the edge of the boundary layer, then Bernouilli's equation gives:

$$\frac{p}{\rho} + \frac{1}{2} U^2 = \text{const.} , \quad (4.4)$$

hence:

$$\frac{1}{\rho} \frac{dp}{dx} = - U \frac{dU}{dx} . \quad (4.5)$$

Blasius's Method:

Let the velocity at the edge of the boundary layer be expressed by the polynomial:-

$$U = u_1 x + u_3 x^3 + u_5 x^5 + \dots , \quad (4.6)$$

where, $u_1, u_3, \dots$ are functions of y .

Bernoulli's equation gives:

$$-\frac{1}{\rho} \frac{dp}{dx} = u_1^2 x + 4 u_1 u_3 x^3 + (6 u_1 u_5 + 3 u_3^2) x^5 + \dots \quad (4.7)$$

To satisfy the equation of continuity, there must be a stream function ψ for the flow in the boundary layer where,

$$u = \frac{\partial \psi}{\partial y}, \quad \& \quad v = - \frac{\partial \psi}{\partial x},$$

Blasius's method is based on the assumption that this stream function for the fluid in the boundary layer can be expressed in the form:

$$\psi = F_1 x + F_3 x^3 + F_5 x^5 + \dots, \quad (4.8)$$

where,

$F_1, F_3, \dots$ etc. are functions of y .

This gives:

$$u = F_1' x + F_3' x^3 + F_5' x^5 + \dots, \quad (4.9)$$

$$-v = F_1 + 3 F_3 x^2 + 5 F_5 x^4 + \dots. \quad (4.10)$$

Substituting these values in the equation of motion (4.1) and equating coefficients of each power of x , the following can be obtained:

$$F_1''^2 - F_1 F_1''' = u_1^2 + \nu F_1''', \quad (4.11)$$

$$4 F_1' F_3' - 3 F_1'' F_3 - F_1 F_3'' = 4 u_1 u_3 + \nu F_3'', \quad (4.12)$$

$$\begin{aligned}
 6F'_1 F'_5 - 5F''_1 F_5 - F_1 F''_5 = \\
 6u_1 u_5 + 3u_3^2 - 3(F_3'^2 - F_3 F_3'') + \nu F_5''' , \quad (4.13) \\
 \dots\dots\dots \text{etc.}
 \end{aligned}$$

All the dashes denote differentiation with respect to y .

Now, considering equation (4.11),

put $\eta = \alpha y$ where α is to be determined.

then: $F_1(y) = A f_1(\eta)$,

hence,

$$A^2 \alpha^2 (f_1'^2 - f_1 f_1'') = u_1^2 + \nu A \alpha^3 f_1''' ,$$

equating the three factors: $A \alpha$, u and $A \alpha$,

A and α can be put as:

$$A = \sqrt{\nu u_1} \quad \& \quad \alpha = \sqrt{u_1 / \nu} ,$$

therefore putting,

$$\eta = \sqrt{\frac{u_1}{\nu}} y \quad . \quad \text{and} \quad F_1(y) = \sqrt{\nu u_1} f_1(\eta)$$

reduces equation (4.11) to the following form:

$$f_1'^2 - f_1 f_1'' = 1 + f_1''' . \quad (4.14)$$

In a similar way, it can be shown that by putting:

$$F_3 = 4f_3 u_3 (\nu / u_1)^{\frac{1}{2}} ,$$

$$F_5 = 6u_5 (\nu / u_1)^{\frac{1}{2}} (g_5 + \frac{u_3^2}{u_1 u_5} h_5) ,$$

reduces equations (4.12) and (4.13) into the following form:

$$4f_1'f_3' - 3f_1''f_3 - f_1f_3'' = 1 + f_3'' \quad , \quad (4.15)$$

$$6f_1'g_5' - 5f_1''g_5 - f_1g_5'' = 1 + g_5'' \quad , \quad (4.16)$$

.....etc.

Here dashes denote differentiation with respect to η .

Next assuming a power series for each of the above functions, i.e.:

$$f_1(\eta) = A_0 + A_1\eta + \frac{A_2}{2!} \eta^2 + \frac{A_3}{3!} \eta^3 + \dots ,$$

where the boundary conditions for $\eta = 0$, can be satisfied by putting: $A_0 = A_1 = 0$,

hence :

$$f_1 = \frac{A_2}{2!} \eta^2 + \frac{A_3}{3!} \eta^3 + \dots ,$$

$$f_1' = A_2\eta + \frac{1}{2} A_3\eta^2 + \dots ,$$

$$f_1'' = A_2 + A_3\eta + \frac{A_4}{2!} \eta^2 + \dots ,$$

$$f_1''' = A_3 + A_4\eta + \frac{A_5}{2!} \eta^2 + \dots ,$$

These values are then substituted in the corresponding differential equation, i.e. equation (4.14) for this case, and rearranged in powers of η . The differential equation has a solution only if the coefficients of every power of η vanishes by itself; since in this case the differential equation is satisfied for all values of η . Hence the

following can be obtained:

$$u = u_1 f_1' x + 4 u_3 f_3' x^3 + 6(u_5 g_5' + \frac{u_3^2}{u_1} h_5') x^5 + 8(u_7 g_7' + \frac{u_3 u_5}{u_1} h_7' + \frac{u_3^3}{u_1^2} k_7') x^7 + \dots \quad (4.17)$$

$$-v = \sqrt{\frac{y}{u_1}} \left[u_1 f_1 + 12 u_3 f_3 x^2 + 30(u_5 g_5 + \frac{u_3^2}{u_1} h_5) x^4 + 56(u_7 g_7 + \frac{u_3 u_5}{u_1} h_7 + \frac{u_3^3}{u_1^2} k_7) x^6 + \dots \right] \quad (4.18)$$

$$\frac{\partial u}{\partial y} = \sqrt{\frac{u_1}{y}} \left[u_1 f_1'' x + 4 u_3 f_3'' x^3 + 6(u_5 g_5'' + \frac{u_3^2}{u_1} h_5'') x^5 + \dots \right] \quad (4.19)$$

Howarth in his paper gives the value for, $f_1, f_1', f_1'', f_3, f_3', f_3'', \dots$ etc., as functions of $\eta = y (u_1/\nu)^{\frac{1}{2}}$. In this particular case of the inlet bell mouth, the flow being accelerated, there will be no danger of separation. Nevertheless, the error due to neglecting terms of higher order, becomes considerable with large values of x . Calculation of this error as suggested by Howarth, is done by estimating the value of the neglected seventh power term, which is:

$$8 \left(\frac{u_3 u_5}{u_1} h_7' + \frac{u_3^3}{u_1^2} k_7' \right) x^7 \quad \text{provided } u_7 = 0; \text{ this can be done if it is assumed that } h_5' > h_7' \text{ and using the tables provided in his paper. The value of this error will show how far along the boundary wall Blasius's method can be used without serious error.}$$

Karman's Method:

Integrating the equation of motion for the flow in the boundary layer, between the limits $y = 0$ and $y = \delta$, where δ = the boundary thickness, the following is obtained:-

$$\int_0^\delta u \frac{\partial u}{\partial x} dy + \int_0^\delta v \frac{\partial u}{\partial y} dy = -\frac{1}{\rho} \int_0^\delta \frac{\partial p}{\partial x} dy + \int_0^\delta \nu \frac{\partial^2 u}{\partial y^2} dy, \quad (4.20)$$

Integrating by parts the second term on the left hand side of equation (4.20) gives:

$$\int_0^\delta v \frac{\partial u}{\partial y} dy \equiv \int_0^\delta v du = [uv]_0^\delta - \int_0^\delta u dv \equiv U(v)_{y=\delta} - \int_0^\delta u \frac{\partial v}{\partial y} dy ,$$

where,

$u = U$ = the main stream velocity at $y = \delta$.

The equation of continuity, equation (4.3), gives:

$$\frac{\partial v}{\partial y} = - \frac{\partial u}{\partial x} ,$$

hence,

$$(v)_{y=\delta} = - \int_0^\delta \frac{\partial u}{\partial x} dy \quad (4.21)$$

The left hand side of equation (4.20) can now be written in the form:

$$\int_0^\delta u \frac{\partial u}{\partial x} dy - U \int_0^\delta \frac{\partial u}{\partial x} dy + \int_0^\delta u \frac{\partial u}{\partial x} dy = \frac{\partial}{\partial x} \int_0^\delta u^2 dy - U \frac{\partial}{\partial x} \int_0^\delta u dy .$$

Since the extra terms arising from the variability of δ with x cancel out, then equation (4.20) will take the form:

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dy - U \frac{\partial}{\partial x} \int_0^\delta u dy = - \frac{1}{\rho} \frac{\partial p}{\partial x} \int_0^\delta dy + \nu \left[\frac{\partial u}{\partial y} \right]_0^\delta \quad (4.22)$$

Bernouilli's equation gives:

$$- \frac{1}{\rho} \frac{\partial p}{\partial x} = U \frac{\partial U}{\partial x} ,$$

hence,

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dy - U \frac{\partial}{\partial x} \int_0^\delta u dy - U \frac{\partial U}{\partial x} \int_0^\delta dy = \nu \left[\frac{\partial u}{\partial y} \right]_0^\delta ,$$

or,

$$\frac{\partial}{\partial x} \int_0^\delta (u^2 - \frac{1}{2} U^2) dy - U \frac{\partial}{\partial x} \int_0^\delta u dy = \nu \left(\frac{\partial u}{\partial y} \right)_{y=0} , \quad (4.23)$$

as,

$$\frac{\partial u}{\partial y} = 0, \quad \text{at } y = \delta.$$

Equation (4.23) represents Karman's theorem of momentum for the flow in the boundary layer. At Karman's suggestion, Pohlhausen took a polynomial of the fourth degree as an approximation to the velocity profile in the boundary layer.

If u is the velocity at a distance y normal to the boundary wall, and x from the stagnation point, where $y < \delta$ then:

$$u = ay + by^2 + cy^3 + dy^4, \quad (4.24)$$

where a , b , c and d are functions of x .

Equation (4.1) gives for $y = 0$,

$$\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=0} = \frac{1}{\mu} \frac{\partial p}{\partial x} = -\frac{U}{y} \frac{\partial u}{\partial x} = -\frac{1}{y} UU'. \quad (4.25)$$

The boundary conditions are:

$$\left. \begin{aligned} u = U, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0 & \quad \text{at } y = \delta, \text{ \&} \\ u = 0, \quad y \frac{\partial u}{\partial y} = -UU' & \quad \text{at } y = 0. \end{aligned} \right\} \quad (4.26)$$

Pohlhausen introduced the parameter $\lambda = U' \frac{\delta^2}{y}$, and obtained the values of the coefficients of the above polynomial,

$$\left. \begin{aligned} a &= U \frac{(12 + \lambda)}{6 \delta} & b &= -U \frac{\lambda}{2 \delta^2} \\ c &= -U \frac{(4 - \lambda)}{3 \delta^3} & d &= U \frac{(6 - \lambda)}{6 \delta^4} \end{aligned} \right\} \quad (4.27)$$

then, using these values to get the integrals of equation (4.23), he obtained the differential equation

$$\frac{dz}{dx} = \frac{0.9 [-9072 + 1670.4U'Z - (47.4U'^2 + 4.8UU'')Z^2 - U'(U'^2 + UU'')Z^3]}{U(-213.12 + 5.76U'Z + U'^2Z^2)} \quad (4.28)$$

where,

$Z = \frac{\lambda}{U'} = \frac{\delta^2}{y}$ and U' , U'' are the first and second derivatives w.r.t.x, of the main stream velocity at the edge of boundary layer.

Equation (4.28) can be put in the following simplified form:

$$dz/dx = f(\lambda)/U + Z^2 U'' g(\lambda) \quad (4.29)$$

This differential equation can be solved numerically, or graphically.

Using equation (4.27) the velocity distribution in the boundary layer can be put in the form:

$$u/U = \frac{(12+\lambda)}{6} \frac{y}{\delta} - \frac{\lambda}{2} \left(\frac{y}{\delta}\right)^2 - \frac{(4-\lambda)}{2} \left(\frac{y}{\delta}\right)^3 + \frac{(6-\lambda)}{6} \left(\frac{y}{\delta}\right)^4, \quad (4.30)$$

or

$$u/U = F\left(\frac{y}{\delta}\right) + \lambda G\left(\frac{y}{\delta}\right) \quad (4.31)$$

Howarth, gave in his paper tables for $f(\lambda)$, and $g(\lambda)$. Using these tables, curves of $(dz/dx) = \text{constant}$, can be drawn in the Z-x plane. Marking the direction of the integral

curves on those of constant slopes, will help in drawing the required curve giving the solution for the particular problem. For the stagnation point, both the Z-value and the slope, can be determined in the following manner:

At $x = 0$, dZ/dx , becomes equal to infinity except for those values of λ , for which the numerator of equation (4.28) vanishes. This will give three possible values for λ_0 , but it can be shown that λ_0 must be positive and less than 12, which is the corresponding value at the separation point. Hence,

$$\lambda_0 = U' Z = 7.052 \quad (4.32)$$

From equation (4.29) the slope of the required curve at $x = 0$ is

$$\begin{aligned} (dZ/dx)_{x=0} &= \lim_{x \rightarrow 0} f(\lambda)/U = \lambda'/U' \cdot f'(\lambda) \\ &= f'(\lambda) \cdot (\ddot{U}Z + \dot{U}\dot{Z}) / U' , \end{aligned}$$

which when computed gives:

$$(dZ/dx)_{x=0} = - 5.391 U'' / U'^2, \quad (4.33)$$

Thus equations (4.32) and (4.33) define the value of Z, at $x = 0$, as well as the slope of the required curve at that point.

Boundary Layer Calculations:

The static pressure along the curved boundaries of the inlet bell mouth, was measured by a flexible "Polythylene"

tube 2.5 mms. outside diameter, in the manner shown in Plate 6. Fig. 4.4, gives the velocity along the curved boundary of the bell mouth, till it joins the $3\frac{1}{2}$ in. diameter hub, at $x = 32$ ". The peculiar shape of the velocity curve between $x = 0$ and 7 in., seems to be caused by separation of the main flow from the boundary wall, due to the manufacturing fault seen in fig. 4.5(b). There the two arcs, the 2" R. and the 30" R., forming the curved boundary wall, instead of making a continuous smooth curve, actually intersect rather abruptly.

The following polynomial expresses the chain dotted curve fig. 4.4., which as an approximation, is assumed to express the velocity just outside the boundary layer of the hub curved boundary:

$$U = 3.589 x + 2.05 x^3 - 0.1186 x^5, \quad (4.34)$$

$$\eta = \sqrt{u_1/\nu} \cdot y = 150.2 y,$$

where:

U = the velocity in ft./sec. just outside the boundary layer, and at a distance x ft. measured from the stagnation point,

$$u_1 = 3.589,$$

$$\nu = \text{the kinematic viscosity} = 1.59 \times 10^{-4} \text{ sq.ft./sec.}$$

y = the distance in feet measured normally to the boundary wall.

The first and second derivatives of the velocity at the edge of the boundary layer, and along the hub curved

boundary, are shown in fig. 4.6.

Blasius's method, was used for computing the velocity profile within the boundary layer, at $x = 0.5'$, $0.95'$ and $1.0'$, which are given in Table IX of the appendix. The error due to neglecting the seventh power term of equation (4.17) was within 2% for both $x = 0.5'$ and $0.95'$. This error, however, became of the order 13% at $x = 1.0'$. This clearly shows that for this bell mouth, Blasius's method fails to give satisfactory results beyond $x = 1.0'$. Hence Karman's method was used for the curved boundary walls of the inlet bellmouth. Again the polynomial, given by equation 4.34, is assumed to express the velocity outside the boundary layer of the curved boundary wall of the hub, and that the corresponding values along the outer curved boundary wall are expressed by:

$$U = 15.155 x + 1.057x^3 + 0.408 x^5 \quad (4.35)$$

where,

U = the velocity in ft./sec. at the edge of the
boundary layer, and at a distance x ft.,
measured from the stagnation point.

The actual velocity along the outer curved boundary is shown by the full line, fig. 4.7, while the chain dotted line represents the approximate velocity distribution as

given by equation 4.35. Here again the peculiar shape of the actual velocity curve between $x = 3"$ and $6"$, is due to a fault in manufacture. This fault is the comparatively sharp edge formed by the intersection of the $2"R$, and the $16"R$ arcs of the outer boundary wall, as is diagrammatically shown in fig. 4.5(a). The first and second derivatives of the approximate velocity distribution, equation 4.35, are shown in figs. 4.8 (a and b).

The graphical solutions of the differential equation 4.29, for both curved boundary walls of the inlet bell mouth are shown in fig. 4.9 (a and b), and the relevant numerical values are given in Table X of the appendix. The solution although extending further than Blasius's, ceases to be of much value for distances beyond $x = 2.5$ ft. along the hub wall, and $x = 1.5$ ft. along the outer wall, where the actual velocities depart from those expressed by equations 4.34 and 4.35, as shown in figs. 4.4 and 4.7.

The velocity profile within the boundary layer, at any particular point along the curved boundaries, can be obtained by using equation 4.31 and Table XI of the appendix. Velocity profiles were thus computed at $x = 0.5, 0.95$ and 2.5 ft. along the curved boundary of the hub, and at $x = 1.5$ ft. along the outer curved wall, and the corresponding numerical values are given in Table XII.

For comparison between the more accurate Blasius's

TABLE XII

Inner Wall (hub)						Outer Wall	
$x = 0.5$ ft.		$x = 0.95$ ft.		$x^{(1)} = 2.5$ ft.		$x^{(2)} = 1.5$ ft.	
y inches	$\frac{u}{\text{ft./sec.}}$	y inches	ft./sec.	y inches	$\frac{u}{U_o}$	y inches	$\frac{u}{U_o}$
0.0190	0.605	0.0155	1.50	0.0092	0.284	0.0084	0.320
0.0286	0.855	0.0232	2.14	0.0139	0.402	0.0126	0.448
0.0381	1.080	0.0310	2.70	0.0185	0.511	0.0168	0.562
0.0572	1.425	0.0465	3.54	0.0276	0.678	0.0252	0.729
0.7620	1.660	0.0620	4.11	0.0370	0.799	0.0336	0.842
0.1142	1.940	0.0930	4.76	0.0555	0.943	0.0505	0.960
0.1522	2.025	0.1240	4.96	0.0740	0.993	0.0670	0.995
0.1910	2.047	0.1550	4.99	0.0924	1.000	0.0840	1.000

1&2: Here the velocity is expressed as a ratio to the velocity outside the boundary layer, as this is more convenient for comparing with certain measured values obtained 4 inches further downstream at section T_o , fig. 4.1.

(1) : This traverse lies 2.0 inches upstream of the cylindrical casing.

(2) : This traverse lies 4.0 inches upstream of the cylindrical casing.

solution; and Karmans's approximate method; the velocity profiles corresponding to the distances $x = 0.5$, and 0.95 ft. along the curved boundary of the hub, as obtained by both methods, are shown in figs. 4.10(a and b).

Additional velocity measurements were taken by the pitot cylinder, at traverse T_0 , fig. 4.1, close to the hub and outer wall boundaries. These velocities are given in Table XIII of the appendix as a ratio to that of the main stream U_0 , at a point where it ceased to increase appreciably by moving any further from the boundary walls.

Figs. 4.11 (a and b) show the calculated and measured velocity profiles close to the hub and the outer curved walls. Velocities calculated by Karman's method relate to the points:

$x = 2.5$ ft., $4\frac{3}{8}$ inch upstream of traverse T_0

and,

$x = 1.5$ ft., $6\frac{3}{8}$ inch upstream of traverse

T_0 , fig. 4.1,

the distances being measured along the hub and the outer walls of the inlet cone respectively.

Fig.4.5(c) shows a manufacturing defect, causing a step-down at the section where the hub-side wall of the bell mouth joins the rotor hub. This, no doubt, is one of

the causes of the greater discrepancy between measurement and computation in fig. 4.11(a) as compared with fig.4.11(b).

The factors causing the deviation between the velocity profiles obtained by direct measurement and by Karman's method could be summarised in the following:-

1. The velocity profiles obtained by the two methods do not relate to the same section, and between the two sections the boundary layer experiences a retarding pressure gradient.
2. The surface discontinuity caused by the manufacturing fault especially at the hub wall, where it is more pronounced, fig.4.5(c).
3. Karman's boundary layer calculation does not take into account the existing secondary motion. This secondary motion is influential in shaping the velocity profiles at traverses in the cylindrical casing.
4. The manufacturing faults shown diagrammatically in figs. 4.5 (a, and b).

The manner in which the static pressure varies with the distance along the curved boundaries of the inlet cone is seen from the tables below. The pressure, measured by the method described in pp.130, is expressed as a ratio to the mean velocity head as reckoned at section T_0 , fig. 4.1. The distance x , which is the curved distance, is negative

as it is measured in a direction opposite to the flow, the origin arbitrarily being at the downstream flange of the cone.

TABLE XIV

Outer Wall		Inner Wall (hub side)			
x in.	$\frac{p_o - p}{\frac{1}{2} \rho \bar{U}_{T_o}^2}$	x in.	$\frac{p_o - p}{\frac{1}{2} \rho \bar{U}_{T_o}^2}$	x in.	$\frac{p_o - p}{\frac{1}{2} \rho \bar{U}_{T_o}^2}$
0	1.112	0	1.224	-22	0.015
-2	0.995	-2	0.867	-24	0.006
-4	0.870	-4	0.665	-26	0.004
-6	0.612	-6	0.546	-28	0.002
-8	0.418	-8	0.410	-30	0.0014
-10	0.278	-10	0.284	-32	0.0028
-12	0.178	-12	0.191		
-14	0.110	-14	0.131		
-16	0.067	-16	0.089		
-18	0.056	-18	0.050		
-19	0.061	-20	0.031		

According to a traverse, further downstream, across section T_o , ($x = 2.375$ "), the pressure close to hub, at radius $r = 0.32 R_o$, was $-1.004 (\frac{1}{2} \rho \bar{U}_{T_o}^2)$, and $-0.992 (\frac{1}{2} \rho \bar{U}_{T_o}^2)$ at radius $r = 0.99 R_o$ close to outer wall, so there is a considerable adverse gradient between $x = 0$ and $x = +2.375$ ". These data are presented graphically in figs. 4.12 and 4.13,

in which the peaks noticeable at $x = 0$, denote separation of the main flow from the boundary walls, due to the manufacturing faults, previously mentioned pp. 130.

They also explain why the boundary layers at section T_0 are apparently thicker than the calculated values, which had to be based on the smooth curves shown by chain dotted lines in Figs. 4.4 and 4.7.

Before bringing this discussion to an end the effect of the secondary motion on the fluid velocity patterns, as it proceeds through the inlet cone into the cylindrical casing, will be considered. From the pressure survey along the curved boundaries, it can be noticed that at a section two inches upstream of the inlet to the cylindrical casing, ($x = -2$ in.), the velocity at the outer wall was 31.5 ft./sec., while it was 29.4 ft./sec. at the corresponding point on the hub wall. In other words the ratio between the velocities at the outer wall to that at the hub wall was 1.11. This shows that at this section the difference is that created directly by the curvature of the inlet boundary walls.

A study of the velocity traverses further downstream at sections T_0 and T_1 as given in Table XV, and fig. 4.14 indicates that there the fluid is under the influence of a secondary vortex having the same axis as the rotor.

TABLE XV

Local axial velocities as a ratio to mean velocity = $\frac{w}{U}$.

r/R_0	Traverse T_1 (fig.4.1)	Traverse T_0 (fig.4.1)
	Group A: Single rotor, $Q = 22.3$ cusecs. and $R_n = 1.37 \times 10^5$.	Group C: Booster $Q = 21.00$ cusecs. $R_n = 1.28 \times 10^5$.
0.32	0.990	0.940
0.37	1.038	1.010
0.50	1.034	1.023
0.63	1.032	1.032
0.76	1.029	1.032
0.89	1.020	1.024
0.99	0.760	0.774

APPENDIX

Instruments:

For velocity measurement, a pitot cylinder^{*} was used. Calibration against a standard pitot tube, in the 8 in. wind tunnel, showed that though the instrument was satisfactorily accurate for determining flow direction and total energy, yet it was not accurate enough for measuring static pressure. Hence a Prandtl's static tube was used for measuring the latter.

The variable inclination manometers, shown in Plate 7, were used after being calibrated statically against a Chattock micromanometer.

* See references: 1, 5, 23 and 44.

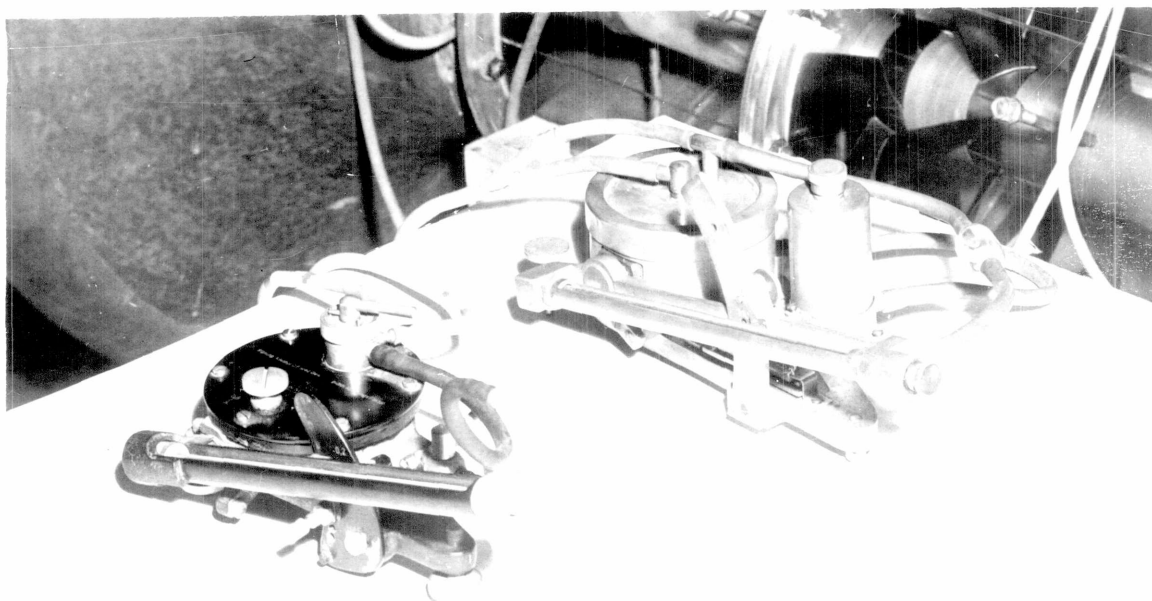


Plate 7 - Variable Inclination Manometers.

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A.C.A. = Australian Council for Aeronautics.

A.R.C. - R. & M. = Aeronautical Research Council -
Reports and Memoranda.

E.D. = The Engineer's Digest.

Jour.A.Sc. = Journal of the Aeronautical Sciences, U.S.A.

Jour.Inst.H.V.E. = Journal of the Institution of Heating
and Ventilating Engineers.

N.A.C.A. = National Advisory Committee for Aeronautics,
U.S.A.

N.C.I.H. = National Conference on Industrial Hydraulics,
U.S.A.

Proc.I.C.A.M. = Proceedings of the Seventh International
Congress for Applied Mechanics.

Proc.Inst.M.E. = Proceedings of the Institution of
Mechanical Engineers.

R.A.S. = The Royal Aeronautical Society.

Trans.Am.S.Mech.E. = Transactions of the American Society
of Mechanical Engineers.

The Engr. = The Engineer (London).

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TABLE I - Nominal Dimensions

Radial Station: r"	5.75	4.50	3.50	2.50	1.75
Chord: C inches	1.70	1.68	1.66	1.65	1.67
Solidity ratio: S/c	3.55	2.80	2.21	1.58	1.10
Maximum thickness % of chord	8.30	8.40	8.40	8.60	8.90
Maximum camber % of chord	6.0	7.0	7.6	7.8	8.9
σ deg*	74.00	68.50	63.00	54.50	43.75
$\tan \sigma$	3.487	2.539	1.963	1.402	0.957
α_1' deg.	80.5	77.0	74.5	64.5	55.5
$\tan \alpha_1'$	5.976	4.33	3.61	2.10	1.46
α_2' deg.	63.0	56.0	52.0	40.5	29.0
$\tan \alpha_2'$	1.96	1.48	1.28	0.85	0.55
$\sigma + \alpha_0$ deg.	67.0	62.0	55.0	45.5	34.0
$\tan \sigma + \alpha_0$	2.36	1.88	1.43	1.02	0.67

where:

σ = the stagger angle,

for the other angles see fig. 1.10

* In the working drawing the chord was given as the line tangential to the concave surface of the aerofoil.

TABLE II - Actual Dimensions

r/R	0.95	0.91	0.83	0.75	0.66	0.58	0.50	0.41	0.33
r in.	5.75	5.50	5.00	4.50	4.00	3.50	3.00	2.50	2.00
c in.	1.71	1.71	1.71	1.70	1.68	1.69	1.66	1.68	1.71
S/c	3.52	3.38	3.06	2.77	2.49	2.16	1.89	1.56	1.22
σ°	72.3	71.8	70.4	67.5	65.8	62.5	58.5	54.5	48.9
$\tan \sigma$	3.13	3.04	2.81	2.41	2.23	1.92	1.63	1.40	1.15
$\sigma^{*\circ}$	74.1	73.0	71.7	69.4	66.9	63.8	59.6	55.1	49.4
$\tan \sigma^*$	3.51	3.27	3.04	2.66	2.34	2.03	1.70	1.43	1.17
α_1°	79.0	79.0	78.3	78.2	77.1	75.6	74.4	71.2	67.4
$\tan \alpha_1'$	5.14	5.14	4.83	4.79	4.37	3.90	3.58	2.94	2.40
α_2°	57.4	58.0	55.9	53.0	52.7	49.1	42.9	40.0	32.5
$\tan \alpha_2'$	1.56	1.50	1.48	1.33	1.31	1.15	0.93	0.84	0.64
$\theta = \alpha_1' - \alpha_2'$	21.6	21.0	22.4	25.2	24.4	26.5	31.5	31.2	34.9
α_0°	-6.4	-6.2	-8.0	-8.0	-8.3	-8.5	-9.3	-9.5	-8.5
$\sigma + \alpha_0^\circ$	65.9	66.2	62.4	59.5	57.5	54.0	49.2	45.0	40.4

TABLE II

c: Chord in inches,

S/c: Solidity ratio,

σ : Stagger angle,

σ^* : Stagger angle based on nominal chord,
defined as the tangent to the concave
surface of the aerofoil. This is given
only for comparison with values in
Table I, from the working drawing.

For the other angles see figure 1.10.

Table 2.III

TABLE III

Radial Station $r = 0.69 R_0$

Group	N.R.P.M.	$\vartheta_2 = \frac{w}{\Omega r}$	$\epsilon = \frac{U_2}{W_2}$	$\cot \alpha_2 = \frac{W_2}{\Omega r - U_2}$	$\xi = \frac{W_2}{W_1}$
Single Rotor	N = 2200 R.P.M.	0.459	0.281	0.527	1.11
		0.432	0.340	0.505	1.10
		0.404	0.435	0.490	1.12
		0.350	0.608	0.444	1.19
		0.297	0.773	0.386	1.30
		0.302	0.702	0.383	1.15
		0.287 ⁽¹⁾	0.810	0.374	1.32
		0.218	1.090	0.285	-
		0.121	1.840	0.156	-
		0.053 ⁽²⁾	4.320	0.700	-
Booster Arrangement	1000	0.597	-0.096	0.564	0.87
	1200	0.560	-0.004	0.558	0.92
	1300	0.525	0.054	0.540	0.97
	1300	0.544	0.033	0.554	0.96
	1500	0.524	0.056	0.540	1.00
	1500	0.485	0.150	0.523	1.00
	1500	0.472	0.194	0.518	1.02
	2000	0.392	0.488	0.485	1.13
	2000	0.347	0.623	0.443	1.24

Table 2.III

TABLE III (cont.)
 Radial Station $r = 0.76 R_0$

Group	N.R.P.M.	φ_2	ε	$\cot \alpha_2$	ξ
Single Rotor	N = 2200 R.P.M.	0.398	0.268	0.446	1.05
		0.382	0.330	0.438	1.07
		0.367	0.412	0.433	1.12
		0.336	0.559	0.414	1.25
		0.306	0.630	0.379	1.27
		0.282 ⁽¹⁾	0.773	0.360	1.35
		0.276	0.778	0.351	1.39
		0.240	0.914	0.308	-
		0.187	1.150	0.238	-
		0.114 ⁽²⁾	1.960	0.146	-
Booster Arrangement	800	0.732	-0.04	0.748	0.972
	1000	0.589	0.00	0.590	0.940
	1200	0.525	0.021	0.532	0.950
	1300	0.510	0.037	0.520	0.990
	1500	0.492	0.079	0.510	1.020
	1300	0.480	0.080	0.500	0.970
	1500	0.451	0.141	0.481	1.030
	1500	0.421	0.215	0.462	1.000
	2000	0.361	0.445	0.431	1.140
	2000	0.330	0.600	0.411	1.290

Table 2.III

TABLE III (cont.)

Radial Station $r = 0.89 R_0$

Group	N.R.P.M.	v_2	ϵ	$\cot \alpha_2$	ξ
Single Rotor	N = 2200 R.P.M.	0.335	0.277	0.369	1.05
		0.314	0.337	0.352	1.04
		0.296	0.392	0.335	1.07
		0.241	0.625	0.283	1.07
		0.223	0.729	0.266	1.11
		0.209	0.894	0.257	1.21
		0.217 ⁽¹⁾	0.960	0.273	1.30
		0.080 ⁽²⁾	4.010	0.120	-
Boosting arrangement	1000	0.535	0.050	0.550	1.01
	1200	0.450	0.060	0.462	0.96
	1300	0.421	0.090	0.436	0.96
	1300	0.407	0.105	0.424	0.97
	1500	0.405	0.090	0.420	1.01
	1500	0.375	0.158	0.398	1.00
	1500	0.363	0.177	0.376	1.02
	2000	0.281	0.467	0.334	1.08
	2000	0.245	0.690	0.296	1.14

TABLE III (cont.)

Radial station $r = 0.95 R_0$

Group	N.r.p.m.	ϑ_2	ε	$\cot \alpha_2$	ξ
Single Rotor	N = 2200 r.p.m.	0.276	0.420	0.313	1.01
		0.256	0.445	0.289	0.99
		0.253	0.516	0.291	1.07
		0.208	0.745	0.246	1.09
		0.187	0.932	0.225	1.09
		0.175	1.134	0.218	1.20
		0.183 ⁽¹⁾	1.115	0.231	1.47
		0.130	2.360	0.188	-
Booster arrangement	1000	0.490	0.035	0.500	1.10
	1200	0.407	0.107	0.425	1.03
	1300	0.364	0.132	0.382	1.00
	1500	0.345	0.109	0.358	1.04
	1300	0.335	0.119	0.349	0.93
	1500	0.292	0.194	0.310	0.92
	1500	0.256	0.268	0.276	0.85
	1800	0.240	0.325	0.260	0.87
	2000	0.236	0.672	0.280	1.05
	2000	0.202	0.839	0.243	1.14

TABLE III (cont.)

Contra Rotators - Radial Station $r = 0.69 R_0$

N_1/N_2	N_1 r.p.m.	θ_2	ϵ	$\cot \alpha_2$
0.75	1500	0.510	0.092	0.535
0.75	1500	0.472	0.185	0.518
0.75	1500	0.472	0.210	0.524
0.75	1500	0.446	0.276	0.508
0.75	1500	0.441	0.305	0.515
1.112	2000	0.442	0.284	0.505
1.112	2000	0.432	0.290	0.508
1.112	2000	0.413	0.403	0.495
1.112	2000	0.388	0.468	0.473
1.25	2000	0.452	0.259	0.512
1.25	2000	0.427	0.306	0.497
1.25	2000	0.416	0.403	0.499
1.25	2000	0.398	0.467	0.489
1.50	2100	0.428	0.324	0.497
1.50	2100	0.404	0.403	0.483
1.50	2100	0.394	0.435	0.475
1.50	2100	0.370	0.556	0.466

where:

 N_1 = upstream rotor's speed, N_2 = downstream rotor's speed.

TABLE III (cont.)

Contra-Rotators - Radial Station $r = 0.76 R_0$

N_1/N_2	N_1 r.p.m.	φ_2	ϵ	$\cot \alpha_2$
0.75	1500	0.468	0.107	0.485
0.75	1500	0.435	0.148	0.464
0.75	1500	0.420	0.212	0.461
0.75	1500	0.410	0.278	0.462
1.112	2000	0.403	0.259	0.456
1.112	2000	0.394	0.306	0.447
1.112	2000	0.388	0.366	0.452
1.112	2000	0.357	0.446	0.425
1.250	2000	0.414	0.249	0.461
1.250	2000	0.388	0.304	0.441
1.250	2000	0.374	0.385	0.437
1.250	2000	0.366	0.424	0.434
1.500	2100	0.394	0.296	0.445
1.500	2100	0.366	0.365	0.423
1.500	2100	0.355	0.405	0.415
1.500	2100	0.334	0.533	0.407
1.500	2100	0.321	0.550	0.391

Table 2.III

TABLE III (Cont.)

Contra-Rotators - Radial station $r = 0.89 R_0$

N_1/N_2	N_1 r.p.m.	ϑ_2	ϵ	$\cot \alpha_2$
0.750	1500	0.398	0.107	0.417
0.750	1500	0.369	0.139	0.389
0.750	1500	0.364	0.193	0.392
0.750	1500	0.344	0.270	0.379
1.112	2000	0.337	0.259	0.370
1.112	2000	0.322	0.287	0.355
1.112	2000	0.316	0.327	0.353
1.112	2000	0.303	0.420	0.348
1.250	2000	0.344	0.249	0.376
1.250	2000	0.325	0.306	0.361
1.250	2000	0.310	0.401	0.354
1.250	2000	0.298	0.470	0.347
1.500	2100	0.320	0.296	0.354
1.500	2100	0.310	0.365	0.350
1.500	2100	0.292	0.467	0.338
1.500	2100	0.272	0.582	0.324
1.500	2100	0.274	0.550	0.306

TABLE III (Cont.)

Contra Rotators - Radial station $r = 0.95R_0$

N_1/N_2	N_1 r.p.m.	φ_2	ϵ	$\cot \alpha_2$
0.750	1500	0.330	0.143	0.346
0.750	1500	0.314	0.177	0.333
0.750	1500	0.280	0.255	0.302
0.750	1500	0.247	0.313	0.268
0.750	1500	0.226	0.456	0.252
1.112	2000	0.251	0.384	0.278
1.112	2000	0.231	0.484	0.260
1.112	2000	0.181	0.755	0.209
1.112	2000	0.183	0.895	0.218
1.250	2000	0.252	0.405	0.280
1.250	2000	0.202	0.629	0.232
1.250	2000	0.176	0.885	0.212
1.250	2000	0.172	1.000	0.207
1.500	2100	0.218	0.535	0.247
1.500	2100	0.181	0.783	0.211
1.500	2100	0.170	1.000	0.204
1.500	2100	0.155	1.188	0.189

(1) : Back flow downstream of the propeller.

(2) : Back flow both upstream and downstream
of the propeller.

Table III (Cont.)

8"-Hub Propeller operating at 2000 r.p.m.

Radial Station r	ϑ_2	ϵ	$\cot \alpha_2$	ξ
0.76 R_0	0.418	0.222	0.461	1.08
	0.405	0.286	0.458	1.09
	0.388	0.325	0.444	1.13
	0.354	0.446	0.420	1.09
	0.333	0.508	0.401	1.07
	0.290	0.635	0.355	1.11
	0.279	0.687	0.345	1.24
0.69 R_0	0.467	0.264	0.534	1.07
	0.455	0.307	0.528	1.08
	0.435	0.384	0.523	1.07
	0.420	0.424	0.512	1.10
	0.392	0.557	0.502	1.10
	0.337	0.673	0.437	0.99
	0.311	0.726	0.402	1.01
	0.305	0.812	0.403	1.07

Table 2.III

TABLE III (cont.)

8"-Hub Propeller operating at 2000 r.p.m.

Radial Station r	φ_2	ε	$\cot \alpha_2$	ξ
0.95 R_0	0.273	0.249	0.293	0.870
	0.270	0.268	0.292	0.880
	0.252	0.335	0.276	0.870
	0.234	0.456	0.261	0.864
	0.216	0.724	0.254	0.887
	0.212	0.650	0.245	0.845
	0.201	0.855	0.242	0.926
	0.187	0.985	0.229	0.910
	0.141	1.600	0.182	0.810
0.89 R_0	0.344	0.194	0.369	1.04
	0.326	0.262	0.356	1.03
	0.312	0.315	0.346	1.07
	0.291	0.414	0.330	1.07
	0.282	0.454	0.324	1.07
	0.251	0.602	0.295	1.06
	0.222	0.780	0.269	1.00
	0.176	1.190	0.222	0.93

TABLE V

Radial station $r = 0.32 R_o$

Group	N r.p.m.	φ_2	$\cot \alpha_2$	ξ
Single rotor	N = 2200 r.p.m.	0.70	1.04	0.79
		0.58	0.88	0.70
		0.45	0.75	0.59
		0.10	0.19	0.17
		0.02	0.03	0.03
Booster arrangement	800	1.64	1.37	0.98
	1000	1.43	1.14	1.01
	1200	1.24	1.10	1.01
	1300	1.20	1.17	1.07
	1300	1.14	1.17	1.04
	1500	1.10	1.16	1.05
	1500	1.00	1.21	1.06
	1500	0.92	1.17	1.00
	1800	0.82	1.12	0.92
	2000	0.44	0.74	0.61
	2000	0.10	0.16	0.18

TABLE V (cont.)

Radial station $r = 0.43 R_0$

Group	N r.p.m.	ϑ_2	$\cot \alpha_2$	ξ
Single rotor	N = 2200 r.p.m.	0.65	0.86	0.98
		0.57	0.78	0.91
		0.47	0.68	0.81
		0.28	0.47	0.57
		0.22	0.33	0.51
		0.09	0.14	0.25
		0.04	0.06	0.10
Booster arrangement	800	1.23	1.07	0.93
	1000	1.07	0.89	0.97
	1200	0.94	0.85	0.96
	1300	0.89	0.85	0.98
	1300	0.83	0.82	0.95
	1500	0.83	0.85	0.98
	1500	0.77	0.86	0.99
	1500	0.73	0.85	0.97
	1800	0.70	0.84	1.00
	2000	0.42	0.62	0.75
	2000	0.27	0.45	0.59

TABLE V (cont.)

Radial station $r = 0.50 R_0$

Group	N r.p.m.	ϑ_2	$\cot \alpha_2$	ξ
Single rotor	N = 2200 r.p.m.	0.59	0.75	1.02
		0.54	0.70	0.97
		0.46	0.62	0.91
		0.32	0.47	0.76
		0.25	0.38	0.68
		0.19	0.30	0.60
		0.12	0.17	0.38
Booster arrangement	800	1.03	0.93	0.89
	1000	0.89	0.74	0.91
	1200	0.80	0.74	0.94
	1300	0.74	0.72	0.94
	1300	0.71	0.71	0.95
	1500	0.73	0.75	0.99
	1500	0.67	0.75	0.99
	1500	0.63	0.72	0.97
	1800	0.62	0.74	1.02
	2000	0.47	0.64	0.96
	2000	0.30	0.41	0.77

TABLE V (Cont)

Contra Rotators - Radial station $r = 0.43 R_0$

N_1/N_2	N_1 r.p.m.	ϕ_2	$\cot \alpha_2$
0.75	1500	0.80	0.85
0.75	1500	0.77	0.89
0.75	1500	0.72	0.86
0.75	1500	0.69	0.88
1.112	2000	0.71	0.90
1.112	2000	0.63	0.87
1.112	2000	0.56	0.80
1.112	2000	0.54	0.82
1.25	2000	0.70	0.93
1.25	2000	0.62	0.89
1.25	2000	0.60	0.89
1.25	2000	0.53	0.83
1.50	2000	0.65	0.88
1.50	2000	0.56	0.86
1.50	2000	0.53	0.84
1.50	2000	0.46	0.71

TABLE V (Cont.)

Contra Rotators - Radial station $r = 0.50 R_0$

N_1/N_2	N_1 r.p.m.	φ_2	$\cot \alpha_2$
0.75	1500	0.68	0.73
0.75	1500	0.67	0.75
0.75	1500	0.63	0.74
0.75	1500	0.60	0.74
1.112	2000	0.60	0.74
1.112	2000	0.57	0.75
1.112	2000	0.53	0.71
1.112	2000	0.48	0.66
1.25	2000	0.62	0.78
1.25	2000	0.56	0.75
1.25	2000	0.54	0.75
1.25	2000	0.49	0.71
1.50	2100	0.58	0.76
1.50	2100	0.52	0.71
1.50	2100	0.48	0.67
1.50	2100	0.42	0.60

Table 2.VI

Table VI.

Radius r	q_c $= \frac{60Q}{ND^3}$	N r.p.m.	$w.\eta$	$v.\zeta$	$\frac{1}{\eta} \frac{\partial H}{\partial r}$ obtained from $w.\eta - v.\zeta$	$\frac{1}{\eta} \frac{\partial H}{\partial r}$ from observed head.	Difference %
0.43 R_0	1.05	1000	657	87	-570	-552	3.3
	0.80	1500	-253	5	258	212	22.0
	0.67	1800	-600	261	861	892	3.5
	0.61	2200	-2385	855	3240	3130	3.5
	0.55	2200	-2320	940	3260	3240	0.6
	0.45	2200	-2070	1097	3167	3080	2.8
	0.35	2200	-796	3200	3996	3820	4.6
0.69 R_0	1.05	1000	-741	-97	644	636	1.0
	0.80	1500	-441	18	459	467	1.7
	0.67	1800	804	74	-730	-560	30.0
	0.61	2200	320	93	-227	-247	8.1
	0.55	2200	-609	238	847	876	3.3
	0.45	2200	-1832	561	2393	2340	2.3
	0.35	2200	-1158	1050	2208	2260	2.3
0.89 R_0	1.05	1000	77	-7	-84	340	?
	0.80	1500	1380	41	-1339	-1271	5.4
	0.67	1800	2985	87	-2898	-2820	2.8
	0.61	2200	2535	637	-1898	-1960	3.2
	0.55	2200	1560	767	-793	-783	1.3
	0.45	2200	1885	880	-1005	-1132	11.2
	0.35	2200	2000	2060	-60	-65	8.3

TABLE VIIBlade Element at $r = 0.69 R_0$

$\varphi = \frac{w_2}{\Omega r}$	$\eta_H \% = \frac{\Delta H}{\rho \varphi^2 \Omega r} \times 100$	$\eta_{st.} \% = \frac{\Delta p_s}{\rho \varphi^2 \Omega r} \times 100$
0.49	19.0	13.9
0.47	44.2	35.2
0.46	66.5	54.0
0.43	89.7	71.5
0.40	91.6	73.4
0.35	94.8	73.6
0.30	89.8	70.5

where,

 w_2 = the flow velocity downstream of the propeller, ΔH = the rise in total energy at the radial
station considered, Δp_s = the rise in static pressure at the same
radial station, v = the tangential or whirl component downstream
of the rotor.

TABLE VIII

10% circular-back aerofoil. (R. & M. No. 2301)

 $R_n = 0.28 \times 10^6$ (no lower value available)

α (degs.)	$\alpha - \alpha_0$ (degs.)	C_L
-9.2	-2.9	-0.164
-7.9	-1.6	-0.100
-6.6	-0.3	-0.018
-5.3	1.0	0.077
-4.1	2.2	0.168
-2.8	3.5	0.253
-1.6	4.7	0.341
-0.3	6.0	0.428
2.1	8.4	0.599
3.4	9.7	0.625
4.6	10.9	0.674
5.9	12.2	0.725

$\alpha - \alpha_0$: the effective angle of attack measured
from the line of zero lift.

Table IX

Radial station $r = 0.63 R_0$

$q_c = \frac{60 Q}{ND^3}$	α (degs.)	q_r/q_a %	C_L
0.350	8.1	50.0	1.400
0.450	4.8	29.5	1.280
0.535	1.5	-	1.020
0.550	1.1	13.7	0.990
0.610	-0.7	9.0	0.800
0.640	-1.9	3.4	0.664
0.670	-2.0	-	0.170
0.710	-3.3	-	0.146
0.740	-3.9	-	0.100
0.800	-5.2	-	0.020
0.830	-4.8	-0.5	-0.010

The angle of attack " α " is measured between the chord line and the mean flow direction.

The secondary flow is expressed as a percentage of the total axial flow passing through the annular area enclosed between the hub, and the coaxial cylindrical surface passing by this radial station.

Table 3-I

TABLE I

Actual r.p.m.	$Q_s=9$ cusecs.			$Q_s=10$ cusecs.		
	Reduced to $N_s=1000$ r.p.m.			Reduced to $N_s=1000$ r.p.m.		
	H_{ms} lb./ft ²	$(T_1 + T_2)_s$ lb.ft	T_1 / T_2	H_{ms} lb./ft ²	$(T_1 + T_2)_s$ lb.ft	T_1 / T_2
2400	0.92	0.100	0.71	0.78	0.090	0.66
2200	0.94	0.098	0.72	0.78	0.091	0.64
2000	0.94	0.099	0.68	0.76	0.090	0.64
1800	0.93	0.099	0.69	0.78	0.093	0.68
1500	0.95	0.098	0.68	0.77	0.091	0.64
1200	1.01	0.093	0.72	0.77	0.092	0.60
1100	0.92	0.095	0.71	0.79	0.090	0.71
1000	1.00	0.098	0.72	0.83	0.091	0.69
900	0.97	0.095	0.68	0.81	0.092	0.68
800	0.97	0.091	0.67	0.81	0.086	0.67
Mean	0.96	0.097	0.70	0.79	0.091	0.66

Actual r.p.m.	$Q_s=10$ cusecs.			$Q_s=11.8$ cusecs.		
	Reduced to $N_s=1000$ r.p.m.			Reduced to $N_s=1000$ r.p.m.		
	H_{ms} lb./ft ²	$(T_1 + T_2)_s$ lb.ft	T_1 / T_2	H_{ms} lb./ft ²	$(T_1 + T_2)_s$ lb.ft	T_1 / T_2
2400	0.51	0.074	0.60	0.32	0.060	0.68
2200	0.54	0.075	0.61	0.32	0.059	0.65
2000	0.53	0.074	0.60	0.28	0.057	0.68
1800	0.52	0.074	0.64	0.30	0.058	0.74
1500	0.50	0.070	0.58	0.37	0.062	0.69
1200	0.54	0.074	0.66	0.30	0.068	0.71
1100	0.52	0.073	0.68	0.35	0.055	0.71
1000	0.56	0.073	0.72	0.34	0.056	0.72
900	0.57	0.074	0.73	0.29	0.047	0.78
800	0.60	0.071	0.68	0.40	0.053	0.69
Mean	0.54	0.073	0.65	0.33	0.058	0.71

Table 3.I

where,

$$Q_s = Q \times \frac{1000}{N_1}, \quad H_{ms} = H_m \left[\frac{1000}{N_1} \right]^2, \quad T_s = T \left[\frac{1000}{N_1} \right]^2,$$

and Q , H , T and T are the flow rate, manometric head and torque values corresponding to the rotational speed N at which the unit was tested. The rotors were 5/16 in. apart.

TABLE II

$N_1/N_2=2.5$ $N_1=2400$ r.p.m.
 $N_2=960$ r.p.m.

$N_1/N_2=2$ $N_1=2200$ r.p.m.
 $N_2=1100$ r.p.m.

q_c	h_c	T_c	T_1/T_2	$\beta_m \%$	q_c	h_c	T_c	T_1/T_2	$\beta_m \%$
0.560	0.064	0.056	-5.65	50.0	0.566	0.068	0.053	-6.55	57.0
0.544	0.082	0.061	-8.21	57.2	0.552	0.085	0.056	-10.02	64.4
0.524	0.095	0.062	-13.72	62.9	0.537	0.100	0.062	-24.6	68.0
0.513	0.109	0.066	39.20	66.2	0.525	0.115	0.064	31.6	74.5
0.504	0.117	0.067	18.00	69.5	0.517	0.124	0.067	23.5	75.8
0.496	0.127	0.070	13.10	71.0	0.508	0.134	0.068	9.95	78.3
0.485	0.137	0.071	9.10	73.2	0.496	0.144	0.071	7.70	78.7
0.470	0.147	0.072	6.80	75.2	0.483	0.156	0.073	6.10	80.6
0.455	0.158	0.073	5.23	77.2	0.466	0.168	0.076	4.29	80.9
0.433	0.169	0.075	4.61	76.3	0.445	0.178	0.076	3.96	81.9
0.406	0.176	0.076	4.10	76.3	0.415	0.185	0.075	3.95	80.4

TABLE II (Continued)

$N_1/N_2=1.5$ $N_1=2400$ r.p.m.
 $N_2=1600$ r.p.m. $N_1/N_2=1$ $N_1=N_2=2000$ r.p.m.

q_c	h_c	T_c	T_1/T_2	$\zeta_m \%$
0.599	0.077	0.063	5.00	57.2
0.588	0.095	0.066	3.54	66.4
0.574	0.113	0.074	3.08	68.5
0.560	0.130	0.077	2.58	74.0
0.554	0.141	0.080	2.15	76.4
0.549	0.153	0.084	2.24	78.2
0.535	0.164	0.086	1.96	79.8
0.519	0.177	0.089	1.83	80.9
0.504	0.191	0.092	1.71	82.2
0.480	0.203	0.093	1.74	81.9
0.450	0.210	0.094	1.74	78.7

q_c	h_c	T_c	T_1/T_2	$\zeta_m \%$
0.708	0.099	0.091	0.799	60.0
0.695	0.106	0.095	0.746	61.0
0.685	0.114	0.099	0.702	61.7
0.676	0.127	0.103	0.674	65.6
0.661	0.148	0.110	0.652	70.0
0.638	0.182	0.121	0.645	75.4
0.612	0.212	0.132	0.655	77.2
0.587	0.244	0.145	0.682	75.8
0.570	0.259	0.151	0.685	77.0
0.542	0.280	0.155	0.711	77.0
0.531	0.286	0.154	0.736	77.5

TABLE II (Continued)

$N_1/N_2=0.75$

$N_1=1800$ r.p.m.

$N_2=2400$ r.p.m.

$N_1/N_2=0.625$

$N_1=1500$ r.p.m.

$N_2=2400$ r.p.m.

q_c	h_c	T_c	T_1/T_2	$\xi_m \%$
0.800	0.147	0.152	0.071	60.2
0.780	0.178	0.169	0.106	64.7
0.764	0.213	0.185	0.149	69.2
0.740	0.246	0.199	0.182	72.2
0.726	0.265	0.210	0.202	72.2
0.723	0.262	0.201	0.200	74.6
0.706	0.282	0.209	0.206	74.9
0.690	0.308	0.214	0.258	77.9
0.669	0.334	0.230	0.298	76.1
0.645	0.357	0.239	0.304	75.6
0.609	0.383	0.237	0.326	77.4

q_c	h_c	T_c	T_1/T_2	$\xi_m \%$
0.889	0.182	0.229	-0.098	55.1
0.879	0.211	0.246	-0.062	58.8
0.856	0.248	0.263	-0.030	63.4
0.840	0.286	0.270	-0.021	68.8
0.819	0.302	0.272	0.019	71.2
0.808	0.330	0.285	0.022	69.2
0.796	0.355	0.295	0.062	74.8
0.770	0.378	0.305	0.950	74.5
0.740	0.407	0.310	0.106	76.5
0.715	0.441	0.324	0.140	76.4
0.684	0.478	0.331	0.190	77.4

Table (3 - II)

$N_1/N_2 = 0.50$
 $N_1 = 1200 \text{ r.p.m.}$
 $N_2 = 2400 \text{ r.p.m.}$

q_c	h_c	T_c	T_1/T_2^*	$\eta_m \%$
1.03	0.254	0.503	-	42.8
1.014	0.288	0.477	-	47.3
0.990	0.334	0.485	-	51.0
0.966	0.378	0.510	-	54.9
0.954	0.407	0.522	-	57.1
0.932	0.435	0.535	-	59.6
0.916	0.470	0.547	-	61.7
0.894	0.509	0.551	-	64.5
0.864	0.550	0.559	-0.018	65.9
0.83	0.595	0.563	0.008	68.9
0.79	0.643	0.594	0.023	68.5

* Measurements of the upstream rotor torque was not possible for flow coefficients between 1.03 and 0.894. For efficiency and torque coefficient calculations within that range, the upstream rotor torque was neglected.

Table II (Continued)

q_c = Flow coefficient

$$= 60 Q / N_1 D^3 ,$$

h_c = Nominal manometric head coefficient referred to the upstream rotor

$$= H_m / \frac{1}{2} (\Omega R_o)^2$$

T_c = Total torque coefficient referred to the upstream rotor

$$= 3600 (T_1 + N_2 T_2 / N_1) / \Omega N_1^2 D^5 ,$$

η_m = Nominal mechanical efficiency

$$= Q H_m / (2\pi / 60) \cdot (N_1 T_1 + N_2 T_2) ,$$

where,

Q = The through flow in cusecs.

H_m = Nominal manometric pressure head lb./ft.²

ΩR_o = Upstream rotor's tip speed ft./sec.

T_1 = Torque applied to the upstream rotor.

T_2 = Torque applied to the downstream rotor.

N_1 & N_2 are the upstream and downstream rotor speeds respectively.

The rotors were 2.25 in. apart.

TABLE III

$N_1/N_2 = .75$ $N_1 = 1500 \text{ r.p.m.}$ $N_1/N_2 = 1.0$ $N_1 = N_2 = 2000 \text{ r.p.m.}$
 $N_2 = 2000 \text{ r.p.m.}$

q_c	h_c	T_c	T_1/T_2	$\lambda_m \%$
0.596	0.407	0.260	0.421	73.3
0.632	0.372	0.243	0.360	76.0
0.666	0.332	0.228	0.310	76.9
0.700	0.291	0.211	0.269	76.1
0.731	0.249	0.193	0.224	73.5
0.760	0.206	0.178	0.176	69.2
0.775	0.184	0.169	0.068	66.3
0.788	0.163	0.159	0.117	63.8
0.809	0.130	0.142	0.063	59.4

q_c	h_c	T_c	T_1/T_2	$\lambda_m \%$
0.525	0.300	0.152	0.687	81.3
0.547	0.282	0.149	0.670	81.4
0.570	0.264	0.144	0.656	81.9
0.591	0.243	0.139	0.645	81.1
0.612	0.222	0.132	0.634	81.0
0.632	0.200	0.124	0.623	79.8
0.651	0.176	0.116	0.610	77.5
0.670	0.147	0.106	0.603	72.8
0.690	0.118	0.097	0.625	66.2
0.709	0.087	0.086	0.678	56.2

TABLE III (Continued)

$N_1/N_2=1.50$

$N_1=2100$ r.p.m.

$N_2=1400$ r.p.m.

$N_1/N_2=2.0$

$N_1=2000$ r.p.m.

$N_2=1000$ r.p.m.

q_c	h_c	T_c	T_1/T_2	$\beta_m\%$
0.445	0.221	0.102	1.60	76.8
0.463	0.212	0.099	1.68	78.3
0.475	0.207	0.980	1.73	79.0
0.503	0.190	0.095	1.87	79.3
0.531	0.168	0.090	2.04	78.0
0.557	0.143	0.084	2.33	75.1
0.572	0.127	0.078	2.83	73.3
0.582	0.116	0.076	2.82	69.7
0.591	0.104	0.072	3.11	67.1
0.605	0.087	0.067	3.98	62.0
0.620	0.069	0.060	6.05	56.2

q_c	h_c	T_c	T_1/T_2	$\beta_m\%$
0.418	0.188	0.080	2.86	77.0
0.433	0.183	0.081	3.08	77.2
0.447	0.178	0.080	3.33	78.0
0.461	0.171	0.079	3.69	78.3
0.474	0.164	0.078	4.18	77.8
0.487	0.155	0.077	4.79	77.0
0.500	0.146	0.075	5.8	75.7
0.512	0.135	0.073	7.32	74.4
0.525	0.123	0.070	10.38	71.9
0.535	0.110	0.087	18.21	68.5
0.547	0.096	0.064	88.7	64.4
0.559	0.082	0.062	-	58.6
0.570	0.067	0.060	-	50.5

Table (3 - III)

TABLE I
Single rotor

$P_o - p$ lb/ft ²	Q cusecs	R_n	$W = \tan \theta$ at $r/R_o = 0.63$	C_d
1.280	23.48	1.43×10^5	-0.308	0.972
1.152	22.30	1.35×10^5	-0.374	0.972
0.960	20.25	1.23×10^5	-0.477	0.966
0.640	16.68	1.01×10^5	-0.695	0.973

TABLE II
Group B - Contra-rotators

$P_o - p$ lb/ft ²	Q cusecs	R_n	N_1 / N_2	$W = \tan \theta$ at $r/R_o = 0.63$	C_d
1.160	22.40	1.36×10^5	1.112	+0.086	0.973
0.925	20.00	1.22×10^5	1.500	-0.287	0.972
0.912	19.90	1.21×10^5	1.112	-0.003	0.974
0.846	19.30	1.17×10^5	1.250	-0.096	0.982
0.722	17.63	1.07×10^5	1.000	+0.200	0.975
0.702	17.41	1.06×10^5	1.250	-0.114	0.972
0.627	16.51	1.01×10^5	0.750	+0.364	0.975
0.590	16.00	0.97×10^5	1.500	-0.213	0.973
0.575	15.85	0.96×10^5	0.750	+0.392	0.980

TABLE IIIGroup C - Booster arrangement

$P_o - P$ lb/ft. ²	Q cusecs	R_n	C_d
0.919	20.00	1.22×10^5	0.975
0.801	18.69	1.14×10^5	0.975
0.751	18.04	1.10×10^5	0.975
0.670	17.10	1.04×10^5	0.976
0.663	17.00	1.03×10^5	0.976
0.998	21.00	1.28×10^5	0.980*
0.885	19.55	1.19×10^5	0.971*
0.715	17.60	1.07×10^5	0.972*
0.587	15.94	0.97×10^5	0.970*

* These values are based on velocity
traverses taken at section T_o .

TABLE V

Q cusecs	R _n	W=tan θ * at r/R _o =0.63	$\frac{N_1}{N_2}$	$\frac{P_2}{P_1} - 1$	
23.48	1.46x10 ⁵	-0.308	Only one rotor	0.049	
22.30	1.37x10 ⁵	-0.374		0.048	
20.25	1.26x10 ⁵	-0.477		0.052	
16.68	1.03x10 ⁵	-0.695		0.053	
22.40	1.39x10 ⁵	+0.086		1.112	0.033
20.00	1.24x10 ⁵	-0.287		1.500	0.044
19.90	1.23x10 ⁵	-0.003		1.112	0.032
19.30	1.20x10 ⁵	-0.096		1.250	0.034
17.63	1.09x10 ⁵	+0.200		1.00	0.050
17.41	1.08x10 ⁵	-0.114		1.250	0.036
16.51	1.02x10 ⁵	+0.364	0.750	0.037	
16.00	0.985x10 ⁵	-0.213	1.500	0.038	
15.85	0.976x10 ⁵	+0.392	0.750	0.038	
20.00	1.24x10 ⁵	Not observed	Only one rotor	0.057	
18.69	1.16x10 ⁵			0.048	
18.04	1.12x10 ⁵			0.028	
17.10	1.06x10 ⁵			0.048	
17.00	1.05x10 ⁵			0.038	

* On the down stream side

TABLE IX
Inner Wall(Hub).

x=0.5 ft		x=0.95 ft	
y	u	y	u
inches	ft/sec	inches	ft/sec
0.016	0.535	0.008	0.902
0.032	0.960	0.016	1.646
0.048	1.286	0.024	2.354
0.064	1.529	0.032	2.865
0.080	1.707	0.048	3.698
0.112	1.908	0.064	4.208
0.128	1.960	0.080	4.550
0.143	2.000	0.112	4.830
0.160	2.016	0.144	4.900
0.176	2.030	0.160	4.950

The velocities at the edge of the boundary layer were 2.05 and 4.99 ft/sec., at x=0.5 and 0.95 ft. respectively.

TABLE X

Graphical solution of equation 4.29, for the curved boundary walls of the inlet bell mouth.

Inner wall(hub)		Outer wall	
x ft.	$Z = \delta^2/\nu$	x ft.	$Z = \delta^2/\nu$
0.00	1.970	0.00	0.466
0.10	1.948	0.10	0.465
0.20	1.890	0.20	0.463
0.50	1.582	0.60	0.445
0.95	1.050	0.80	0.424
1.00	0.994	1.00	0.398
1.50	0.609	1.20	0.365
2.00	0.440	1.50	0.309
2.50	0.373	1.80	0.237
2.75	0.370		
3.00	0.397		

Where, x = the distance in ft. measured from the stagnation point.

δ = A parameter of the nature of the boundary layer thickness, though it has no physical meaning.

Table XI

Values of the functions $F(y/\delta)$, & $G(y/\delta)$ of
the equation $u/U = F(y/\delta) + \lambda G(y/\delta)$

y/δ	$F(y/\delta)$	$G(y/\delta)$	y/δ	$F(y/\delta)$	$G(y/\delta)$
0.10	0.1981	0.0122	0.60	0.8976	0.00640
0.15	0.2938	0.0154	0.70	0.9541	0.00315
0.20	0.3910	0.0171	0.80	0.9856	0.00106
0.30	0.5577	0.0172	0.90	0.9981	0.00015
0.40	0.6976	0.0144	1.00	1.000	0.00000
0.50	0.8125	0.0104			

Table 4.XIII

Table XIII

Inner wall (hub)		Outer wall			
y in.	u/U_0	y in.	u/U_0	y in.	u/U_0
0.18	0.935	0.06	0.794	0.26	0.980
0.31	0.960	0.10	0.905	0.50	0.985
0.51	0.975	0.14	0.940	0.71	0.988
0.90	0.994	0.18	0.957	1.05	1.000
1.11	1.000	0.22	0.965		

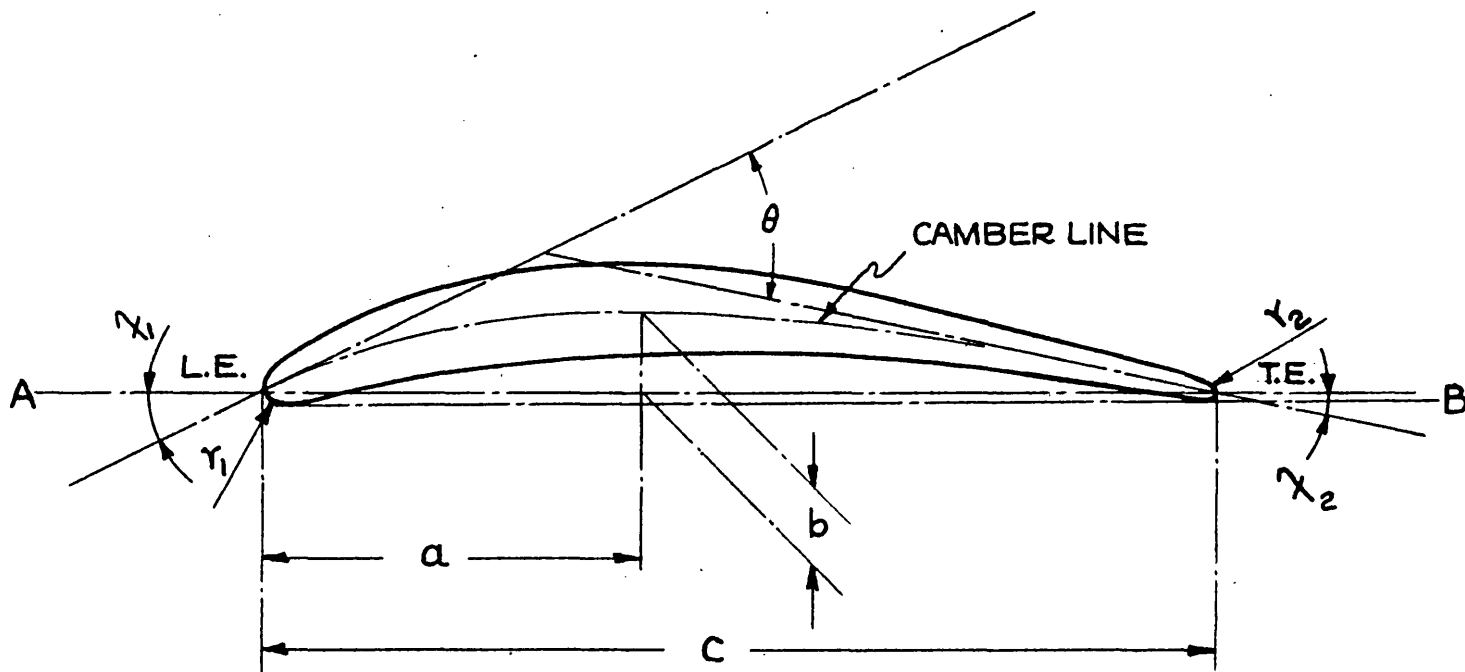


Fig. 1.1 SKETCH OF AN AEROFOIL, SHOWING
THE CONVENTIONAL MEASUREMENTS

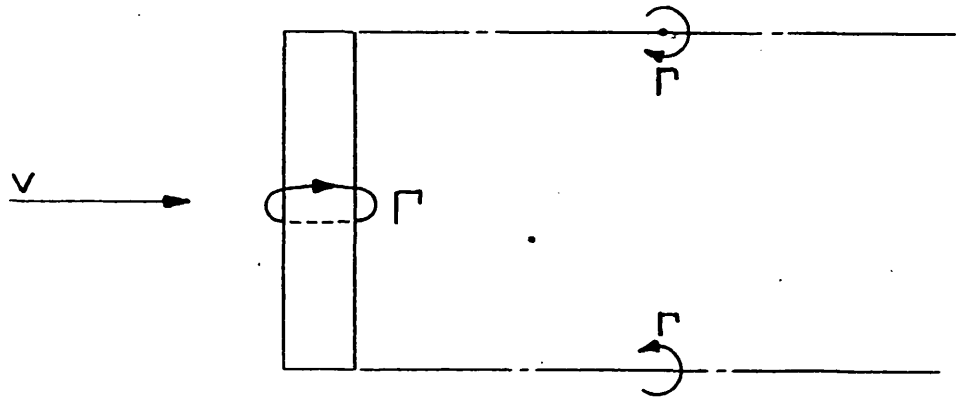


Fig. 1·2 VORTICES ASSOCIATED WITH A FINITE SPAN AEROFOIL

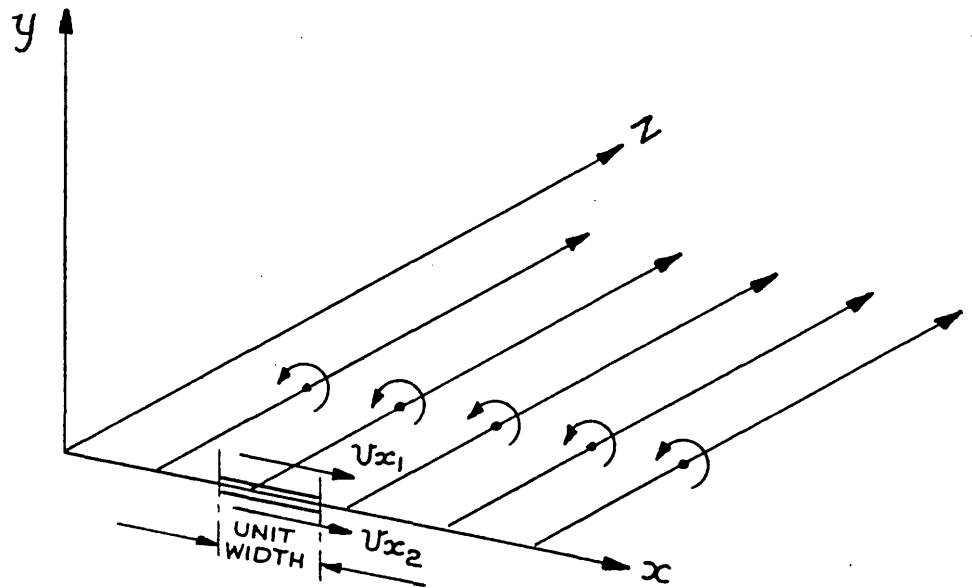


Fig. 1·3 A VORTEX SHEET

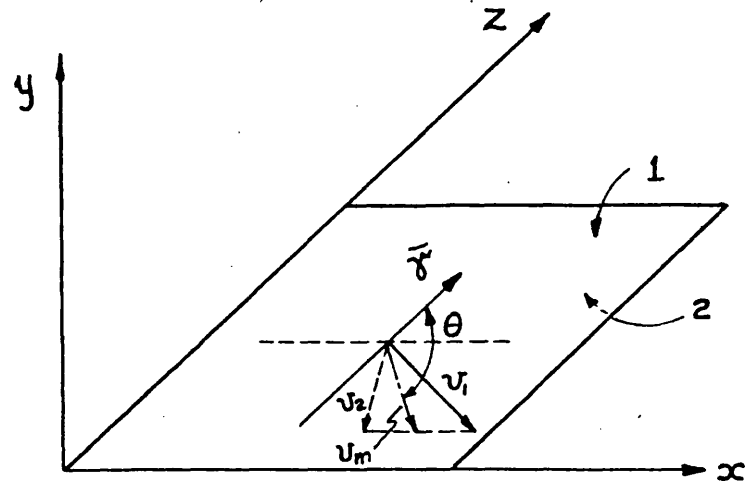


Fig. 1.4 A VORTEX SHEET WITH FLOW ON BOTH ITS SIDES

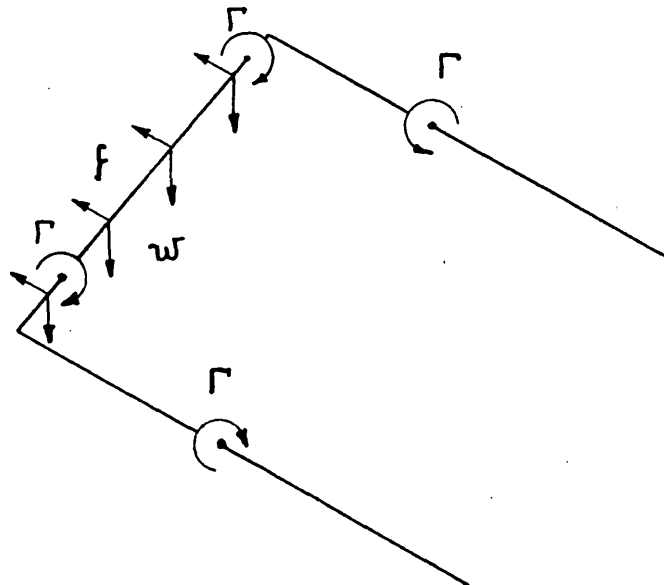


Fig. 1.5 BOUND AND TRAILING VORTICES, AND THE INDUCED VELOCITIES FOR A FINITE SPAN AEROFOIL.

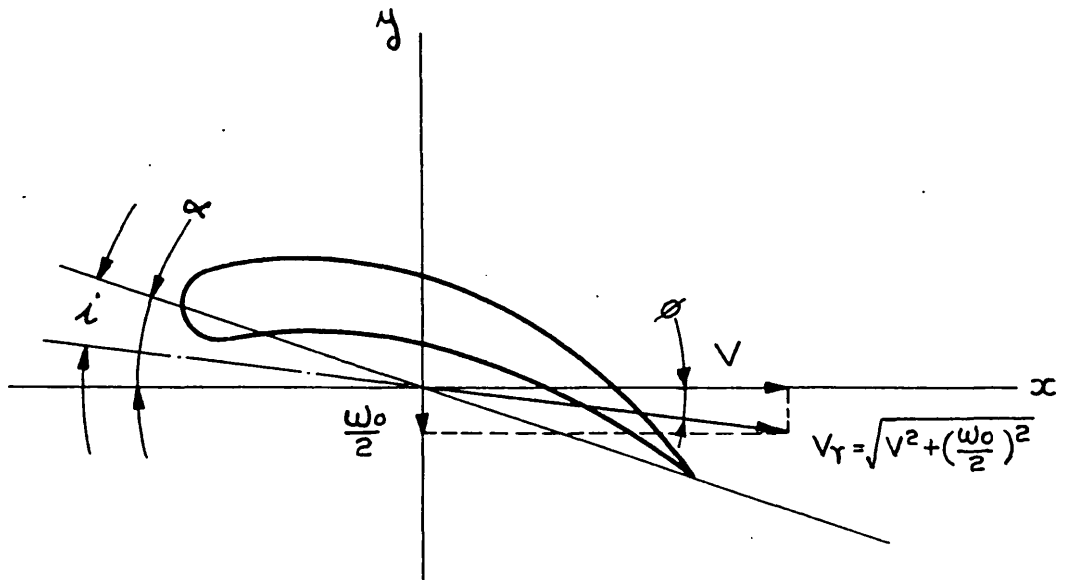


Fig. 1.6 THE EFFECTIVE ANGLE OF ATTACK FOR A FINITE AEROFOIL

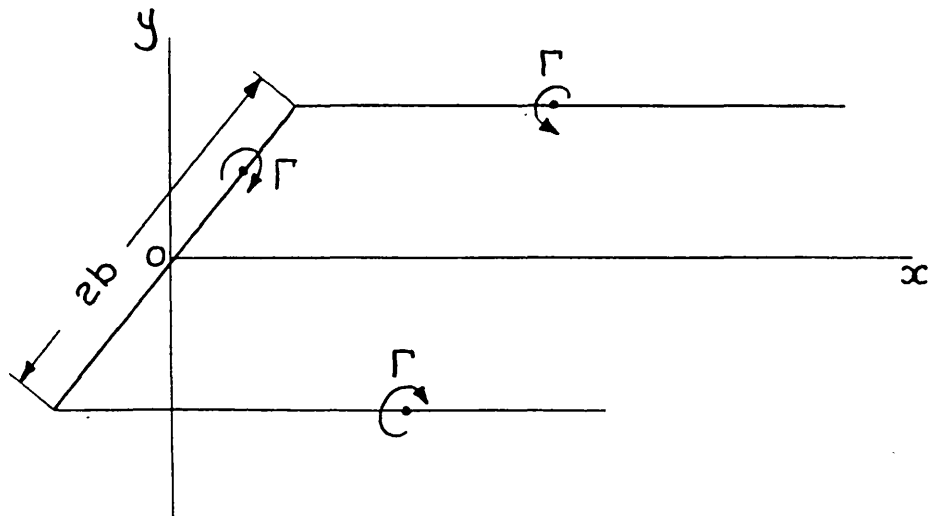


Fig. 1.7 LOADED LINE WITH BOUND AND TRAILING VORTICES, REPLACING A FINITE AEROFOIL

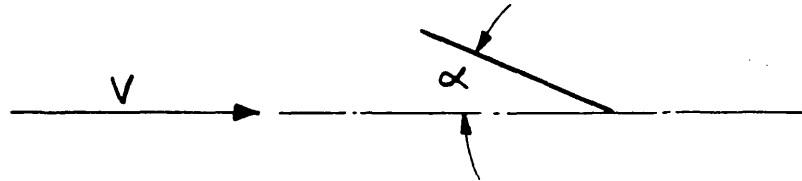


Fig. 1·8 A FLAT PLATE AS AN AEROFOIL

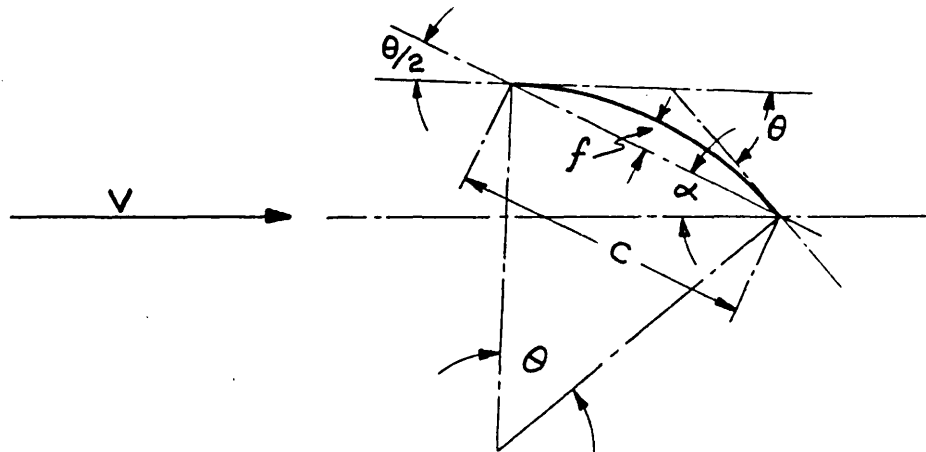


Fig. 1·9 THIN PLATE, CIRCULAR ARC AEROFOIL.

Fig. 1-10 (a,b&c)

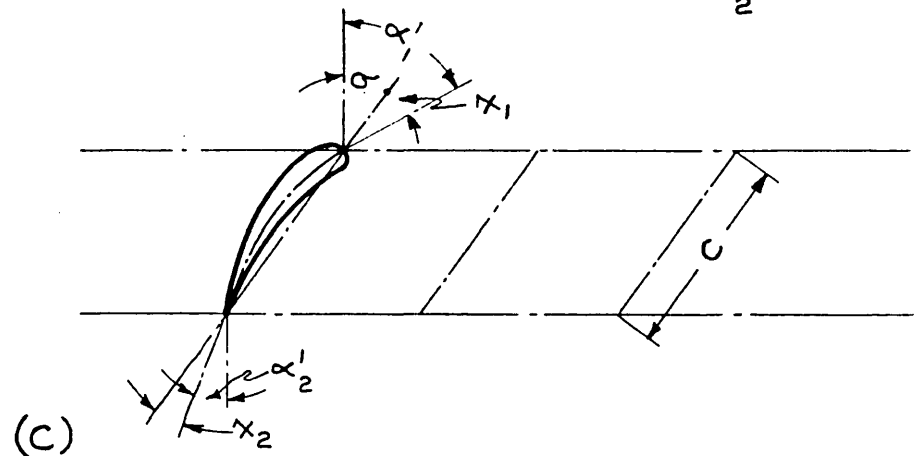
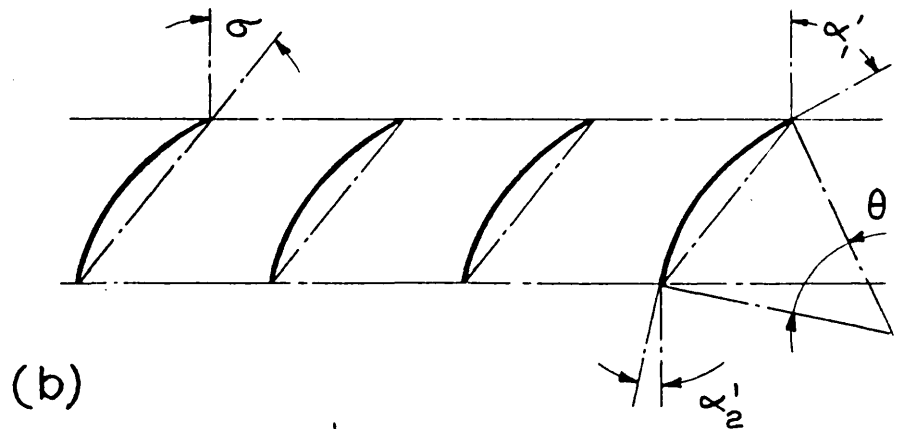
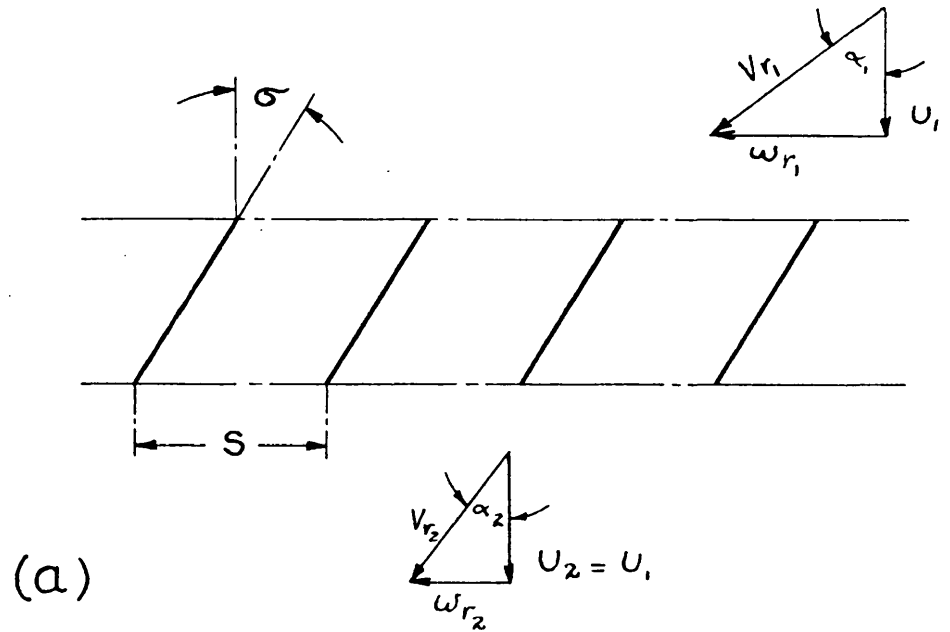


Fig. 1-10 (a,b&c) CASCADE OF BLADES

[illegible]

2

Fig. 1·12 & 13

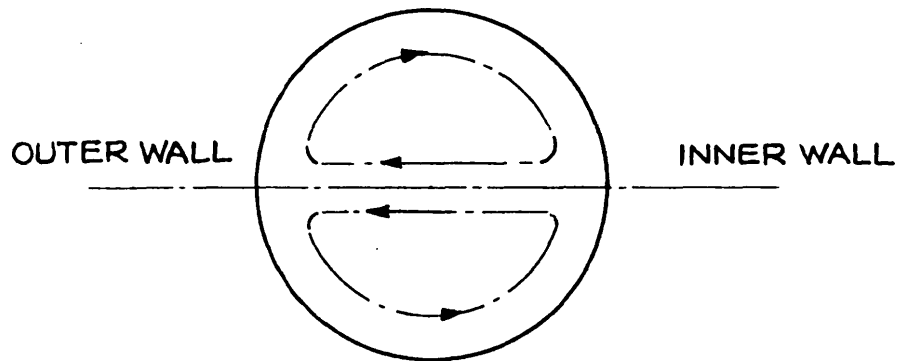


Fig. 1·12 SECONDARY FLOW IN A PIPE BEND

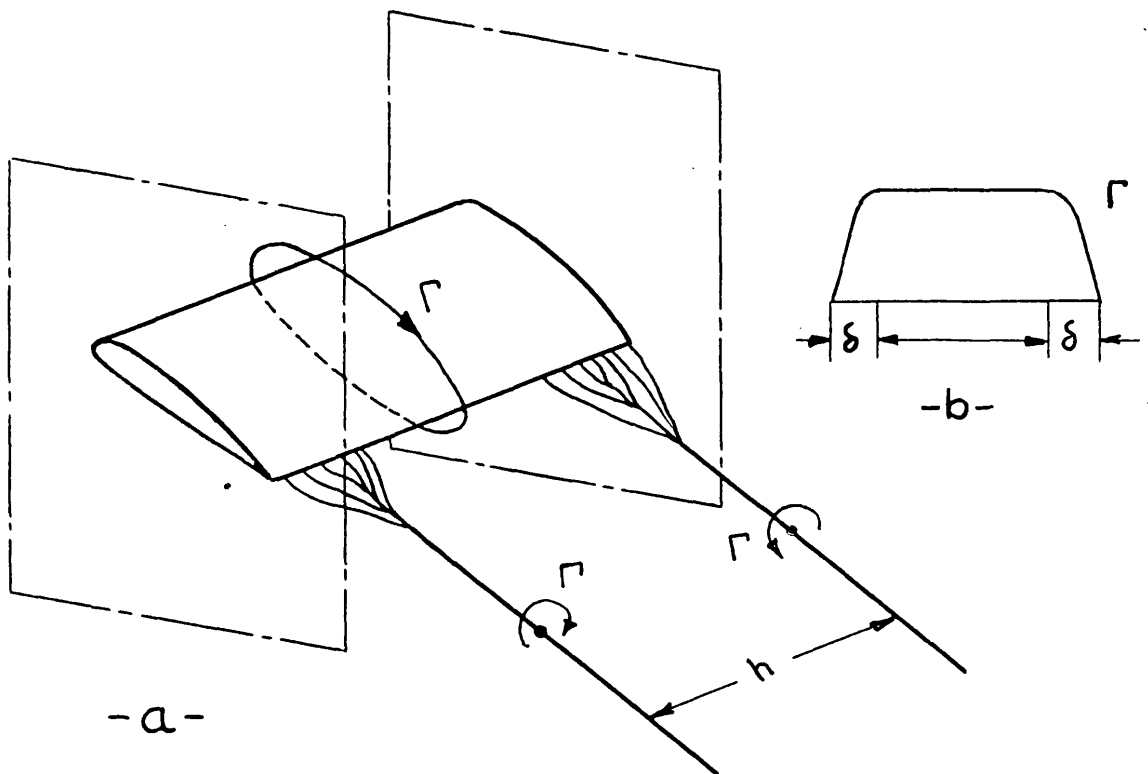


Fig. 1·13

CIRCULATION AND TRAILING VORTICES
FOR A FINITE AEROFOIL.

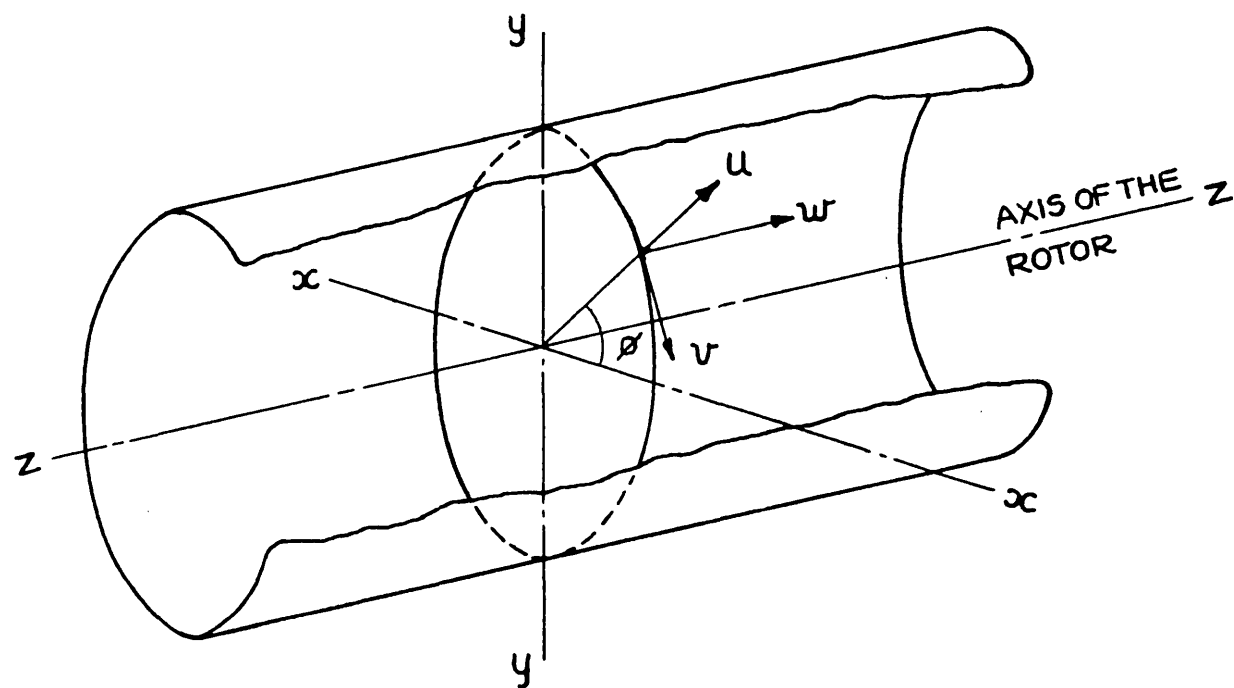


Fig. 1-14 SKETCH SHOWING THE CARTESIAN AND POLAR AXES IN RELATION TO THE ROTOR AXIS.

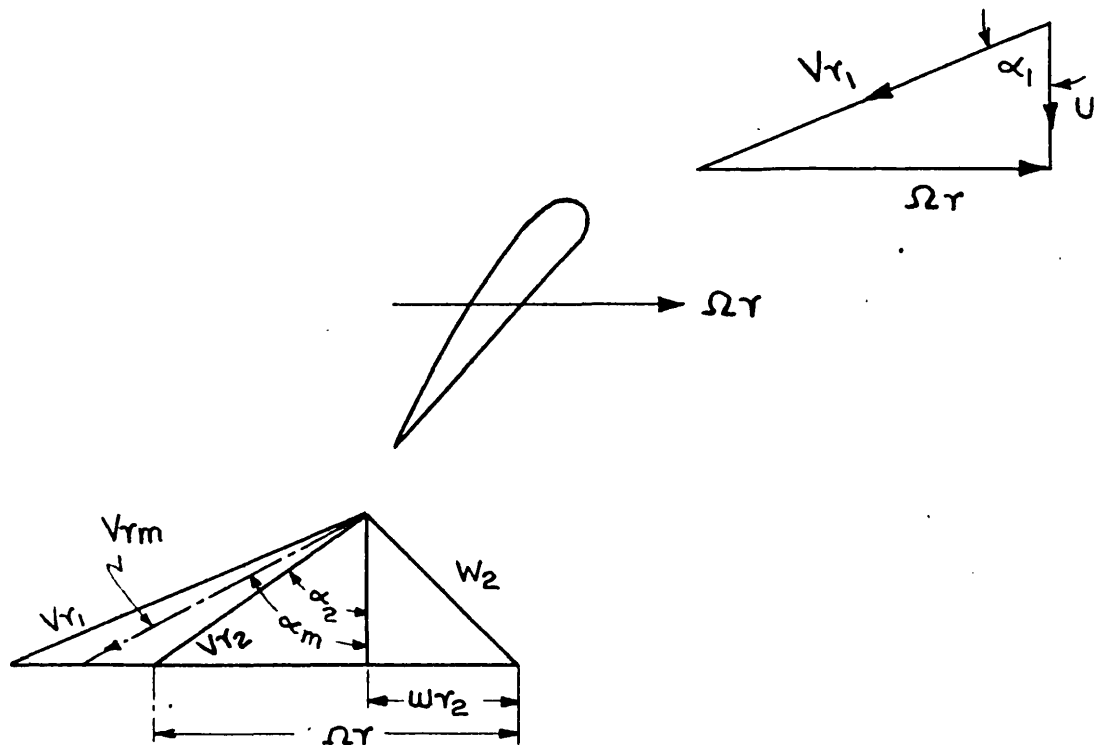


Fig. 1·15 VELOCITY DIAGRAMS FOR A BLADE ELEMENT AT RADIUS r .

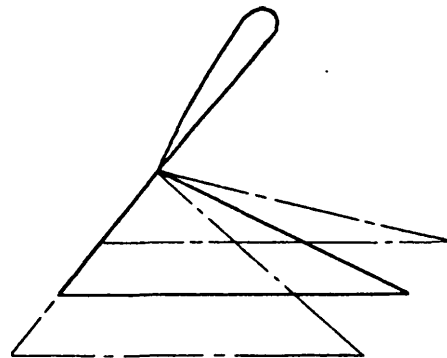


Fig. 1·16 VELOCITY DIAGRAMS FOR PART, NORMAL AND OVERLOADS, ACCORDING TO THE ONE DIMENSIONAL THEORY.

Fig. 1.17

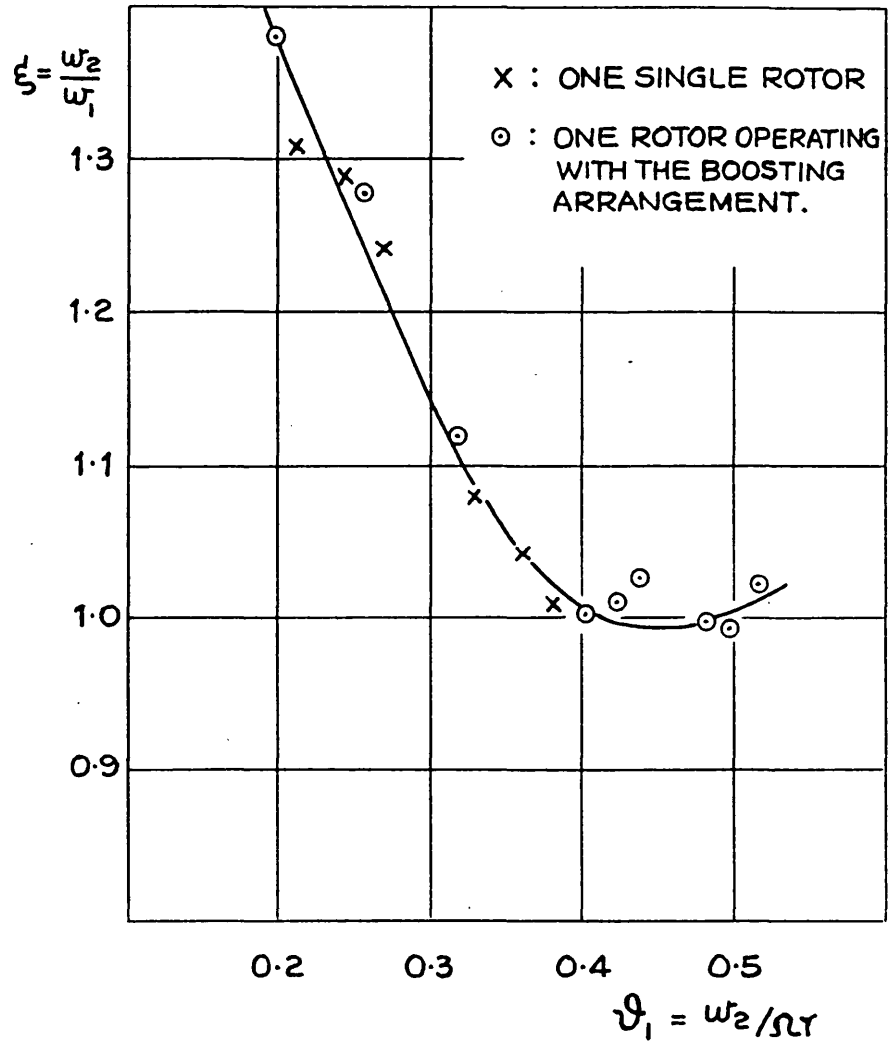


Fig. 1.17 SECONDARY FLOW FACTOR AT VARYING U.S. FLOW COEFFICIENTS FOR RADIAL STATION $\gamma_{R0} = 0.76$.

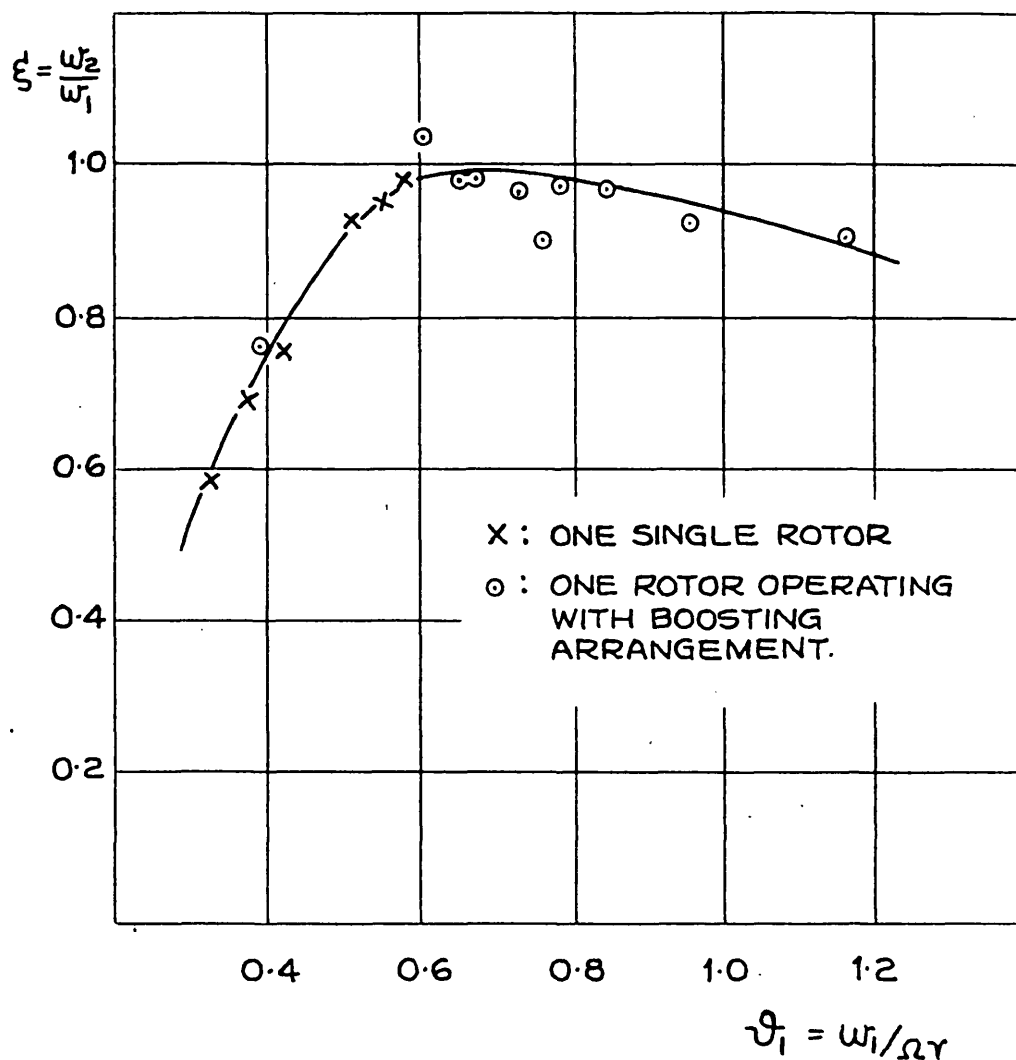


Fig. 1-18 SECONDARY FLOW FACTOR AT VARYING FLOW COEFFICIENT FOR RADIAL STATION $r/R_0 = 0.50$.

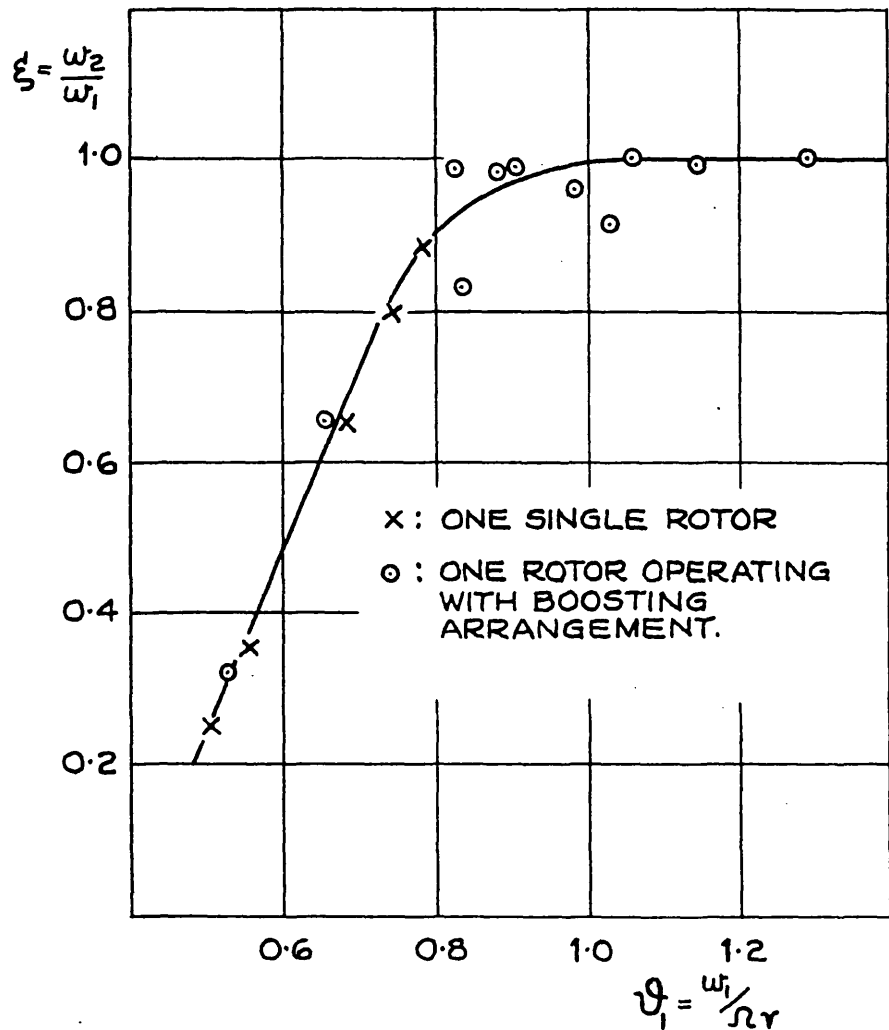


Fig. 1.19 SECONDARY FLOW FACTOR AT VARYING FLOW COEFFICIENT FOR RADIAL STATION $\gamma/R_0 = 0.37$.

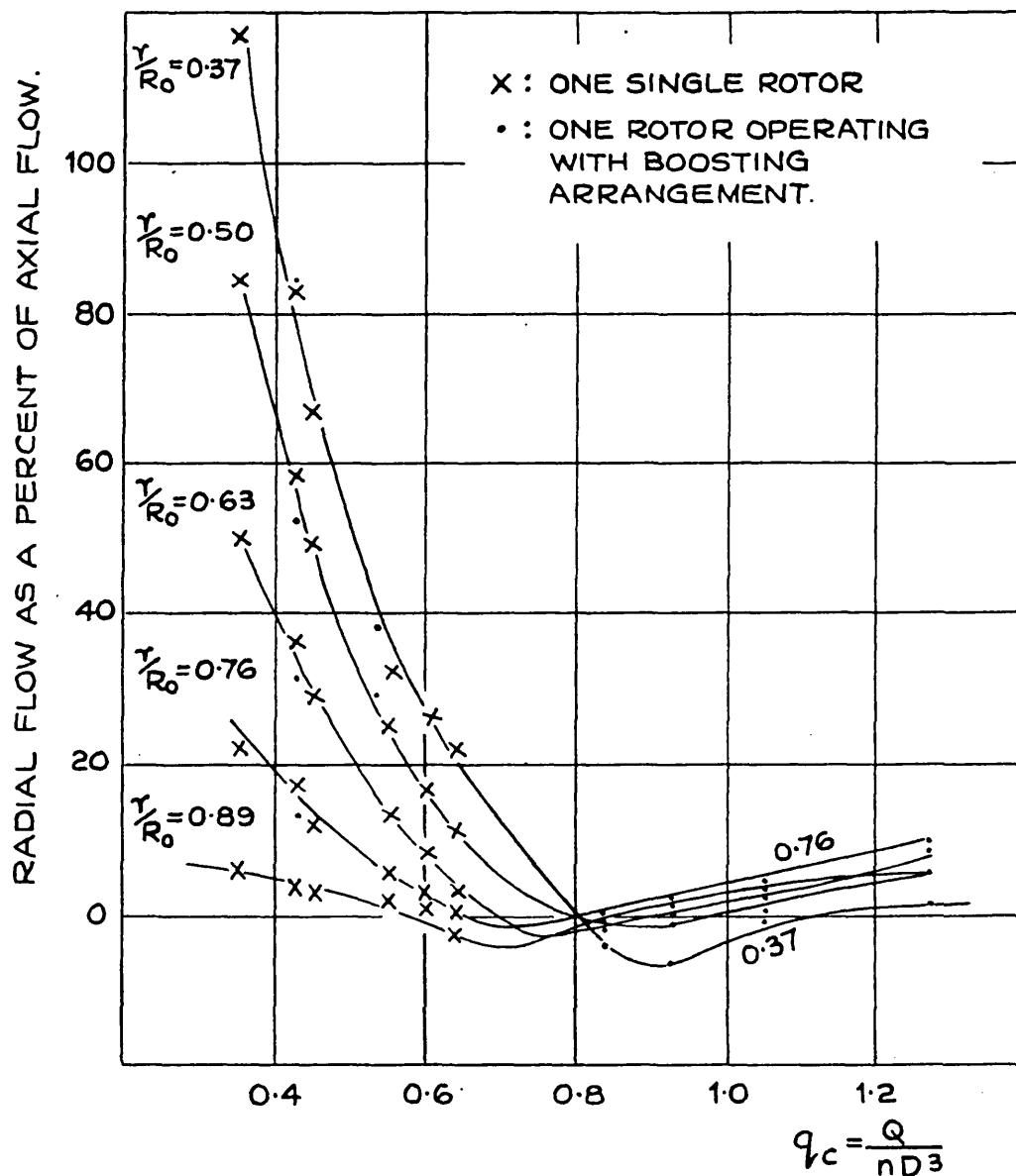


Fig. 1-20 SECONDARY FLOW AS A PERCENTAGE OF AXIAL FLOW FOR DIFFERENT RADIAL STATIONS, AT VARYING FLOW RATES.

Fig. 1.21

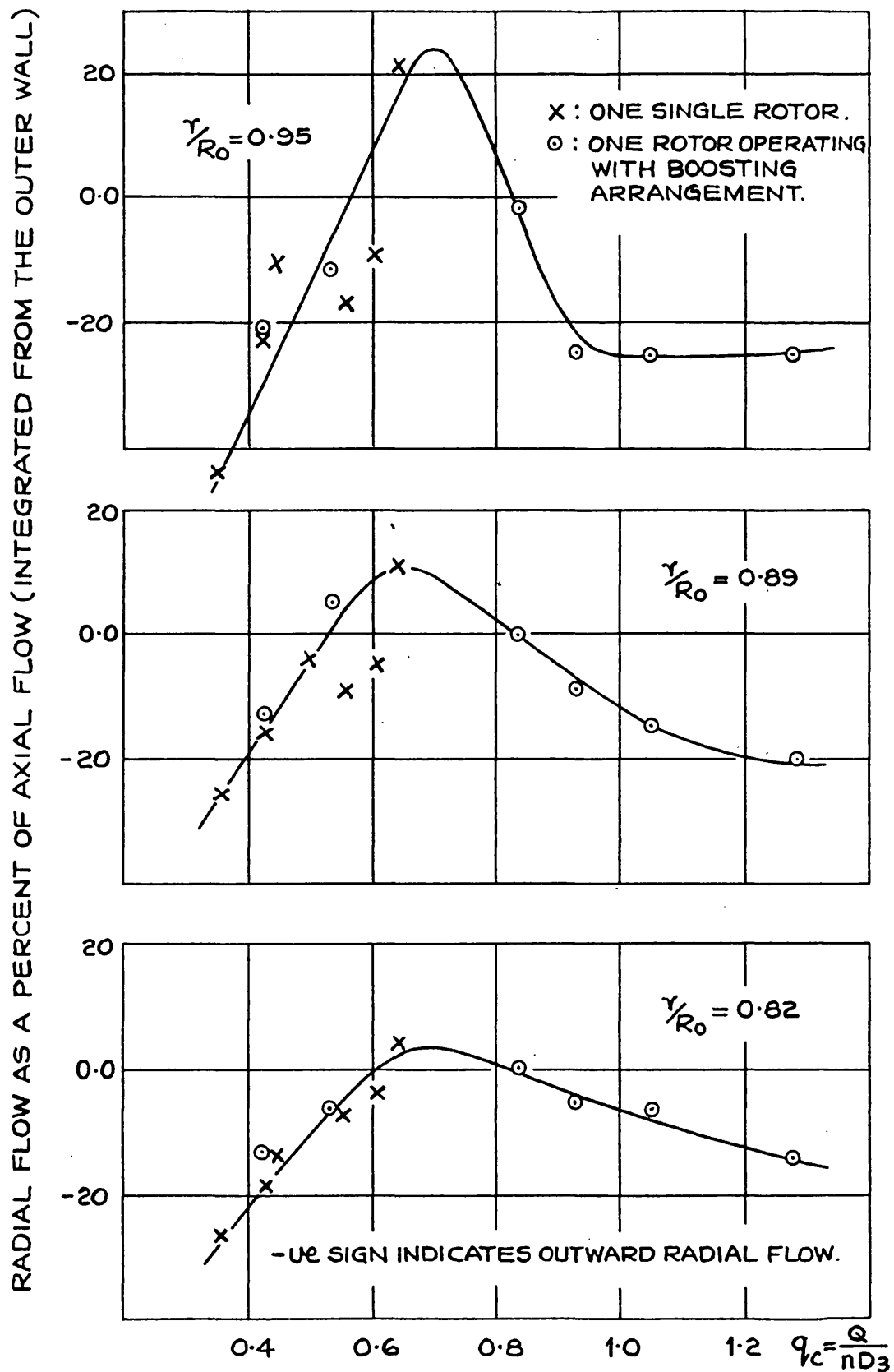


Fig. 1.21 SECONDARY FLOW AS A PERCENT OF AXIAL FLOW, BOTH INTEGRATED FROM THE OUTER WALL.

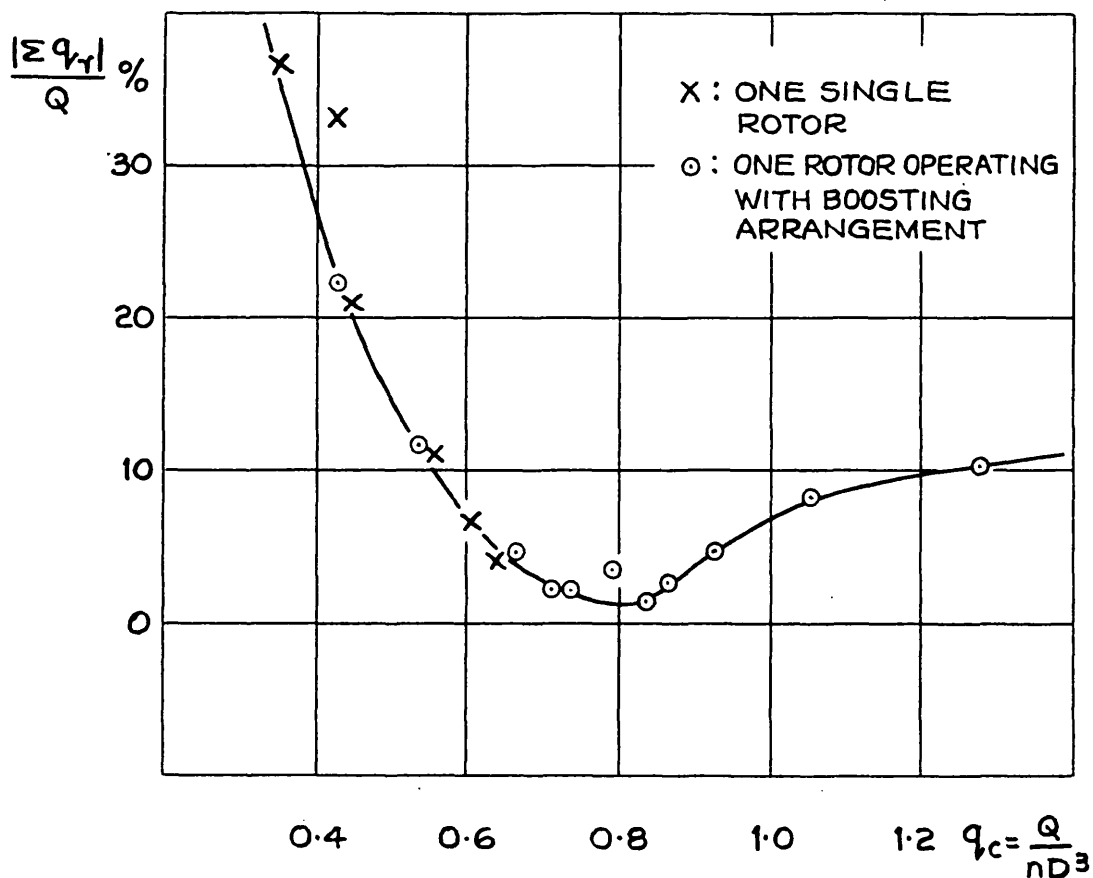


Fig. 1-22 TOTAL RADIAL FLOW AS A PERCENT OF AXIAL FLOW, AT VARYING RATES OF FLOW.

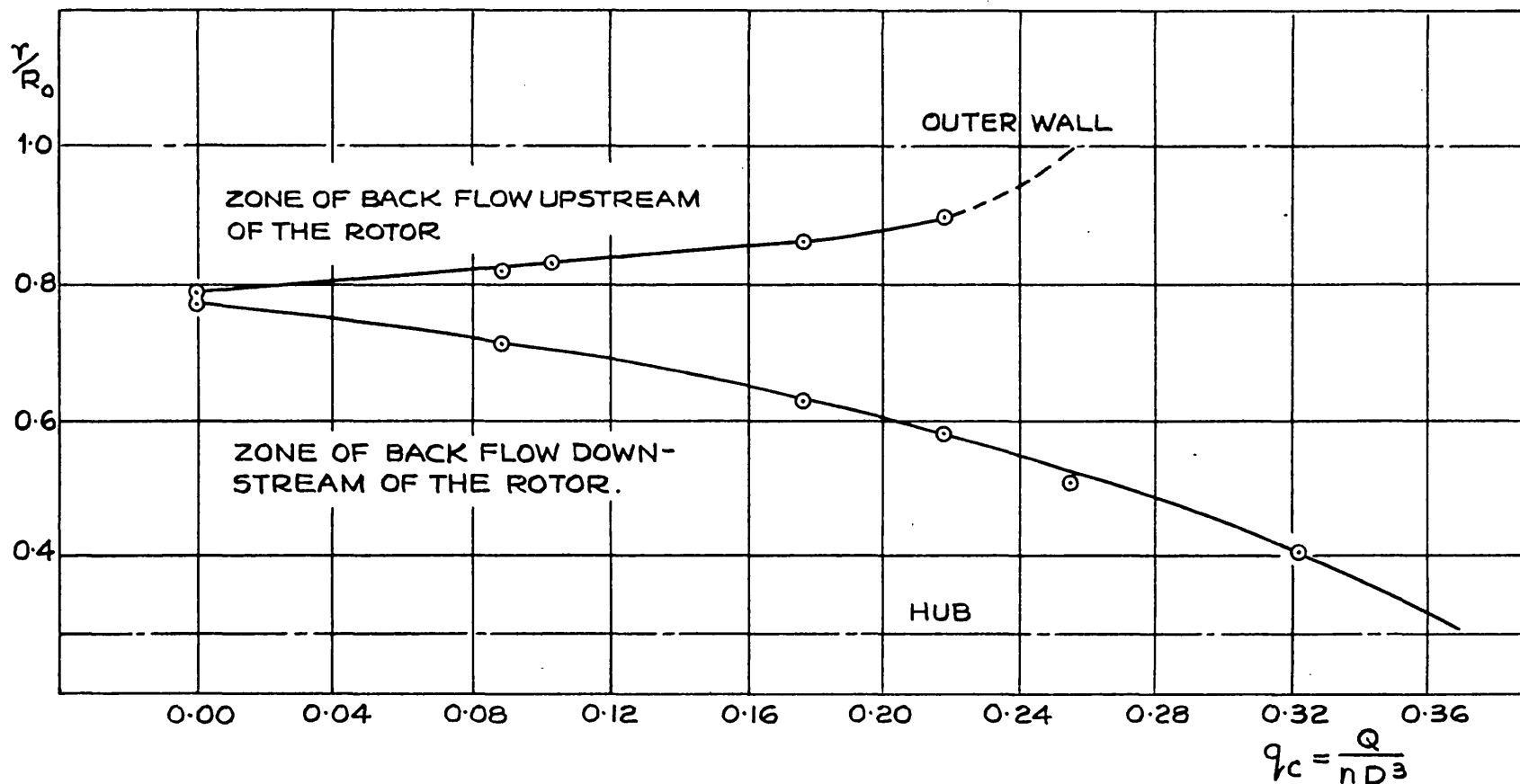


Fig. 1.23 DEPTH OF BACKFLOW STREAMS UPSTREAM AND DOWNSTREAM OF THE PROPELLER, AT THROTTLED FLOW RATES.

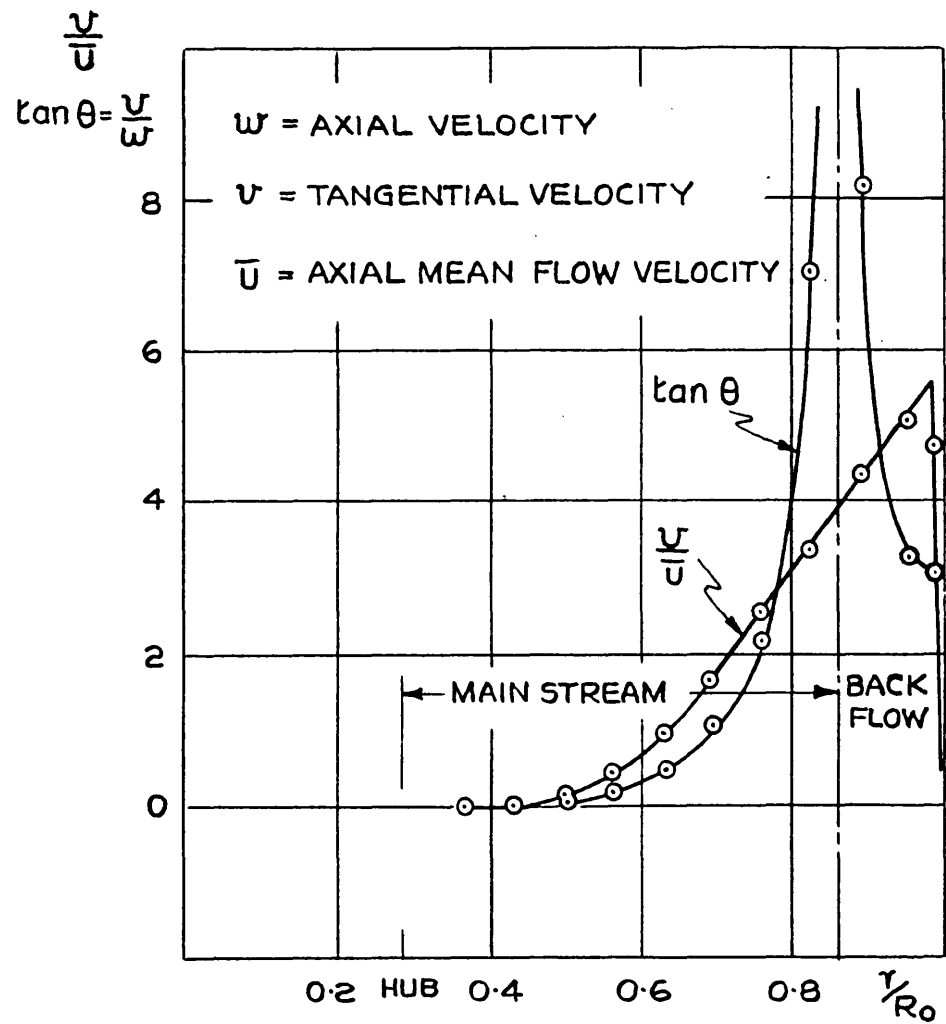


Fig. 1·24 PRE-ROTATION CAUSED BY BACK FLOW UPSTREAM OF THE PROPELLER, AT FLOW COEFFICIENT $q_c = 0.18$.

Fig. 1-25

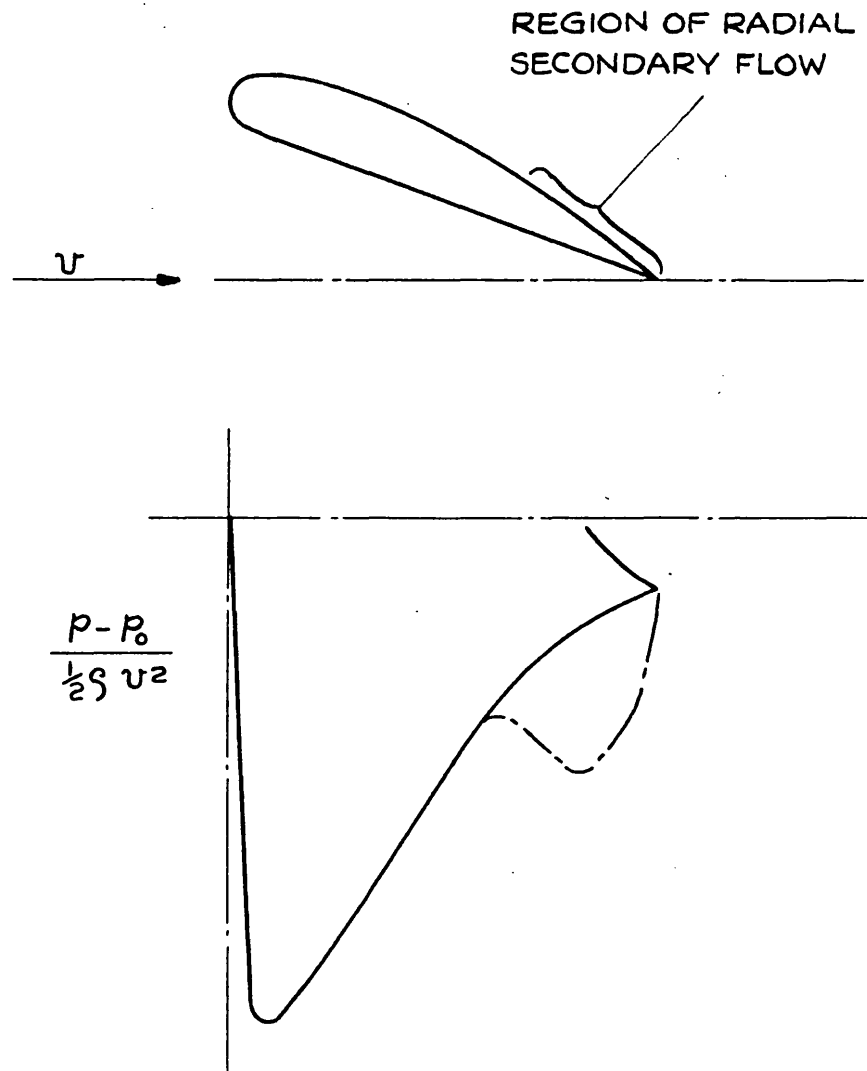


Fig. 1-25 PROBABLE MODIFICATION TO THE PRESSURE DISTRIBUTION ON THE LOW PRESSURE SIDE OF THE BLADE CAUSED BY THE RADIAL SECONDARY FLOW WITHIN THE BOUNDARY LAYER.

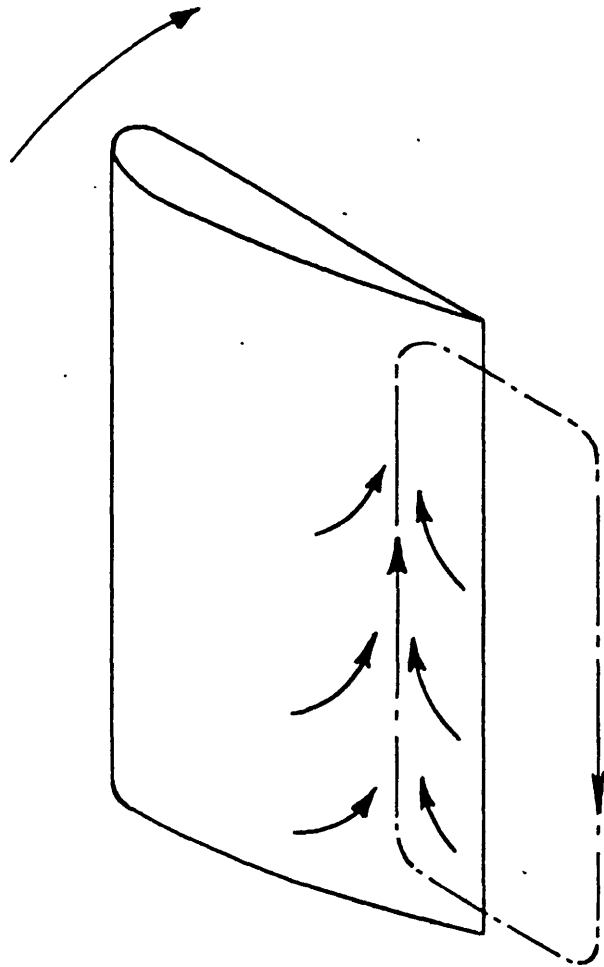


Fig. 1-26 SECONDARY FLOW WITHIN
THE BOUNDARY LAYER OF
LOW-PRESSURE SIDE OF
THE ROTOR BLADE

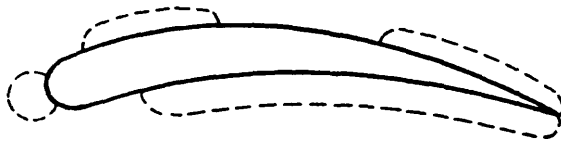


Fig. 1-27 FOULING REGIONS ON A COMPRESSOR
BLADE. (REPRODUCED FROM A
PAPER BY DAVIDSON)

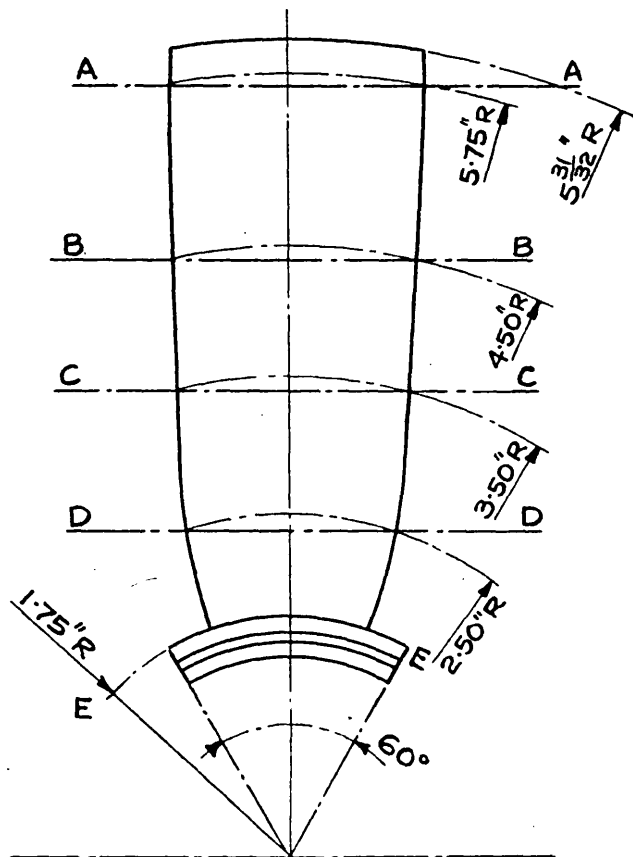


Fig.2-1 DIAGRAMATIC SKETCH OF THE ROTOR BLADE

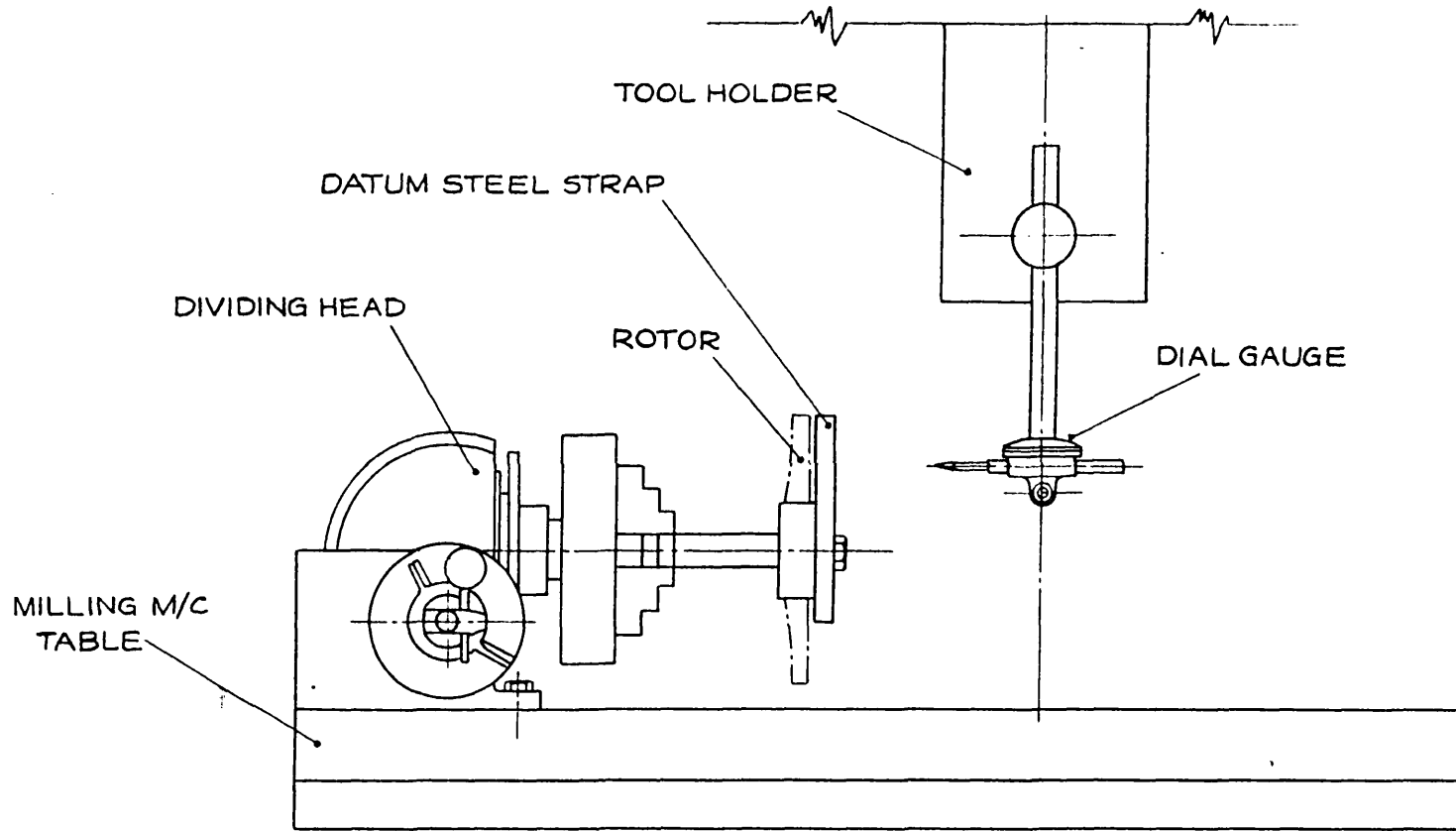


Fig. 2.2(a) DIAGRAMATIC SKETCH SHOWING THE METHOD USED FOR BLADE PROFILE MEASUREMENT.

Fig. 2-2 (b)

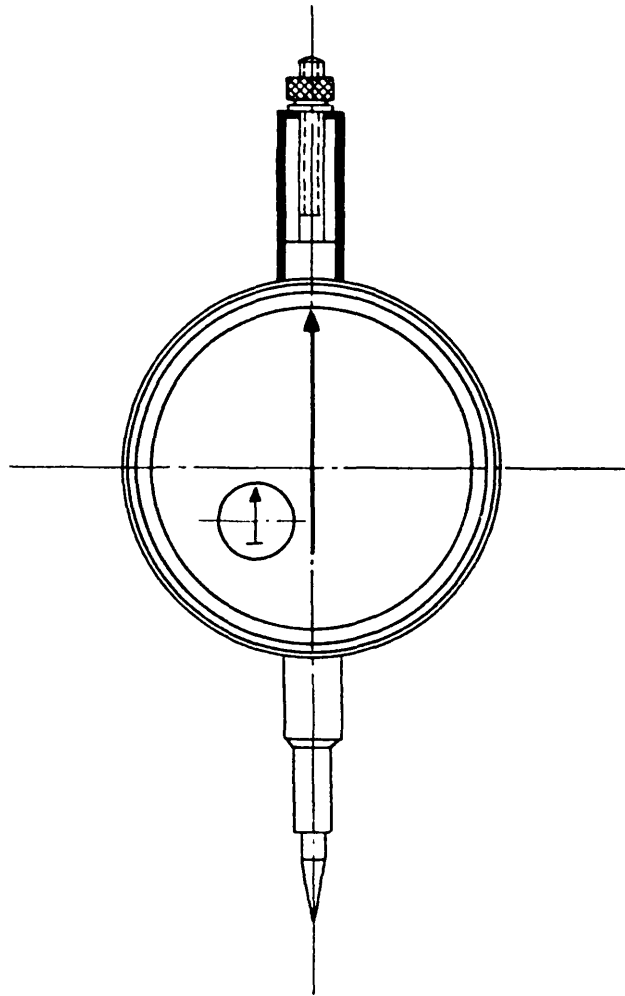


Fig. 2-2 (b) DIAGRAMATIC SKETCH OF THE
MODIFIED DIAL GAUGE USED
FOR BLADE MEASUREMENT.

Fig. 2.3 (a & b)

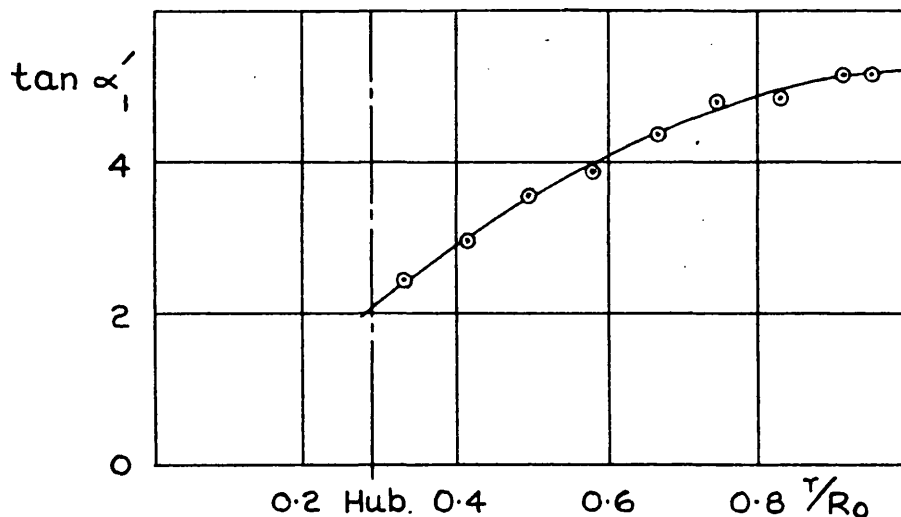


Fig. 2.3 (a) ANGLE BETWEEN THE TANGENT TO THE LEADING EDGE OF THE CAMBER LINE, AND AXIAL DIRECTION.

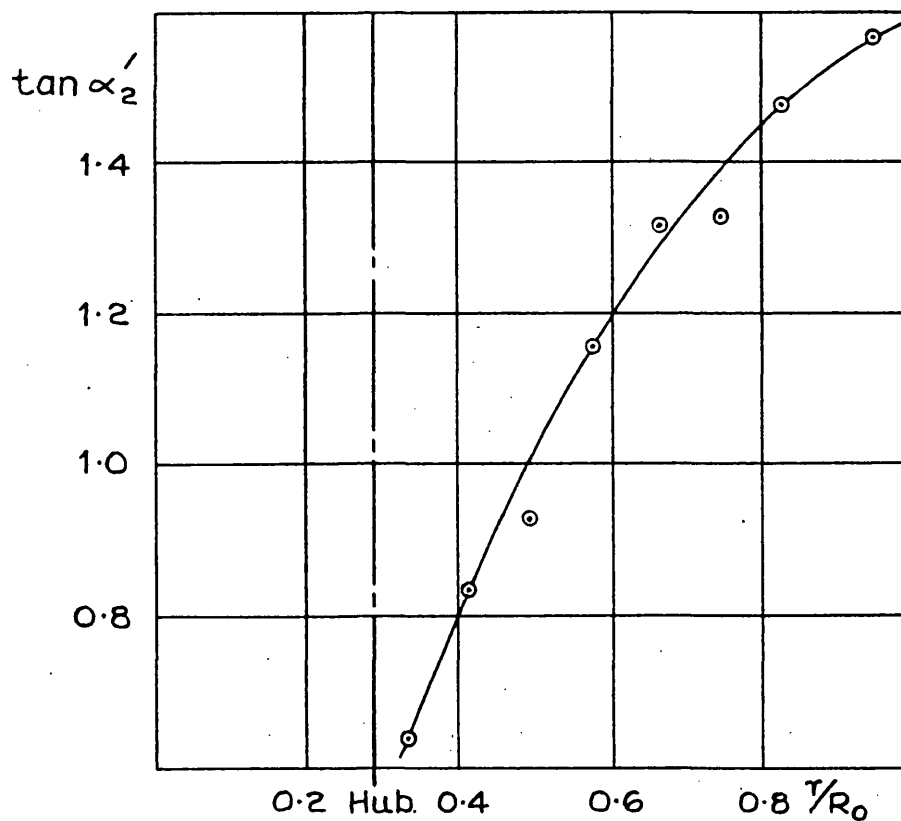


Fig. 2.3 (b) ANGLE BETWEEN THE TANGENT TO THE TRAILING EDGE OF THE CAMBER LINE AND AXIAL DIRECTION. (TABLE II)

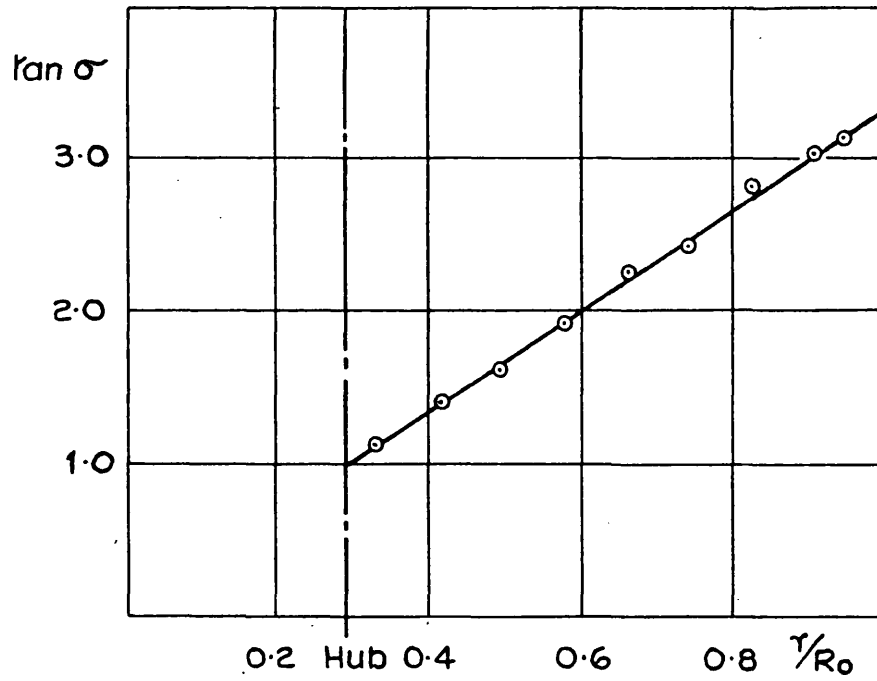


Fig. 2·3(C) CHORD ANGLE MEASURED FROM AXIAL DIRECTION.

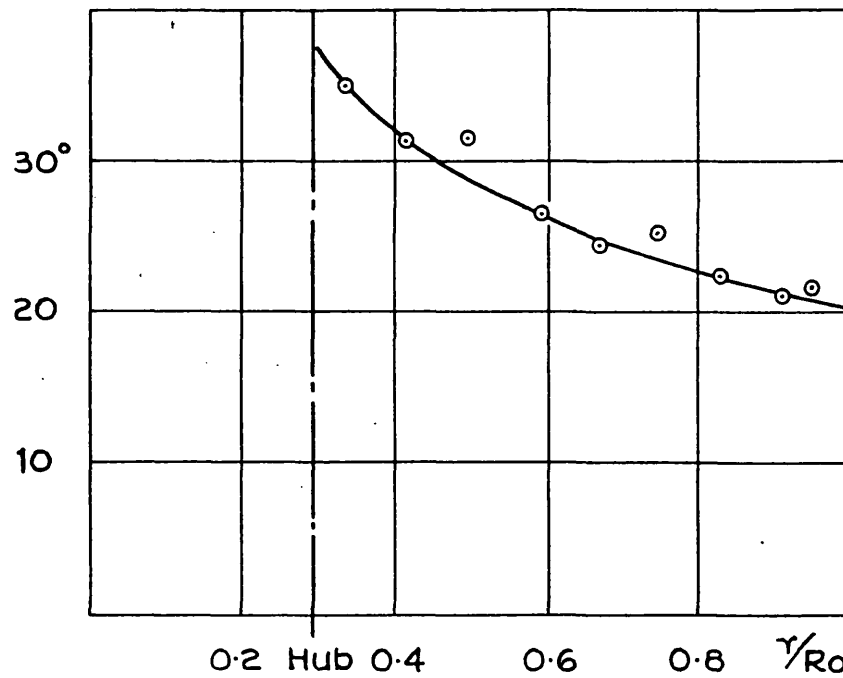


Fig. 2·3(d) BLADE CAMBER ANGLE AT DIFFERENT RADIAL STATIONS. (TABLE II).

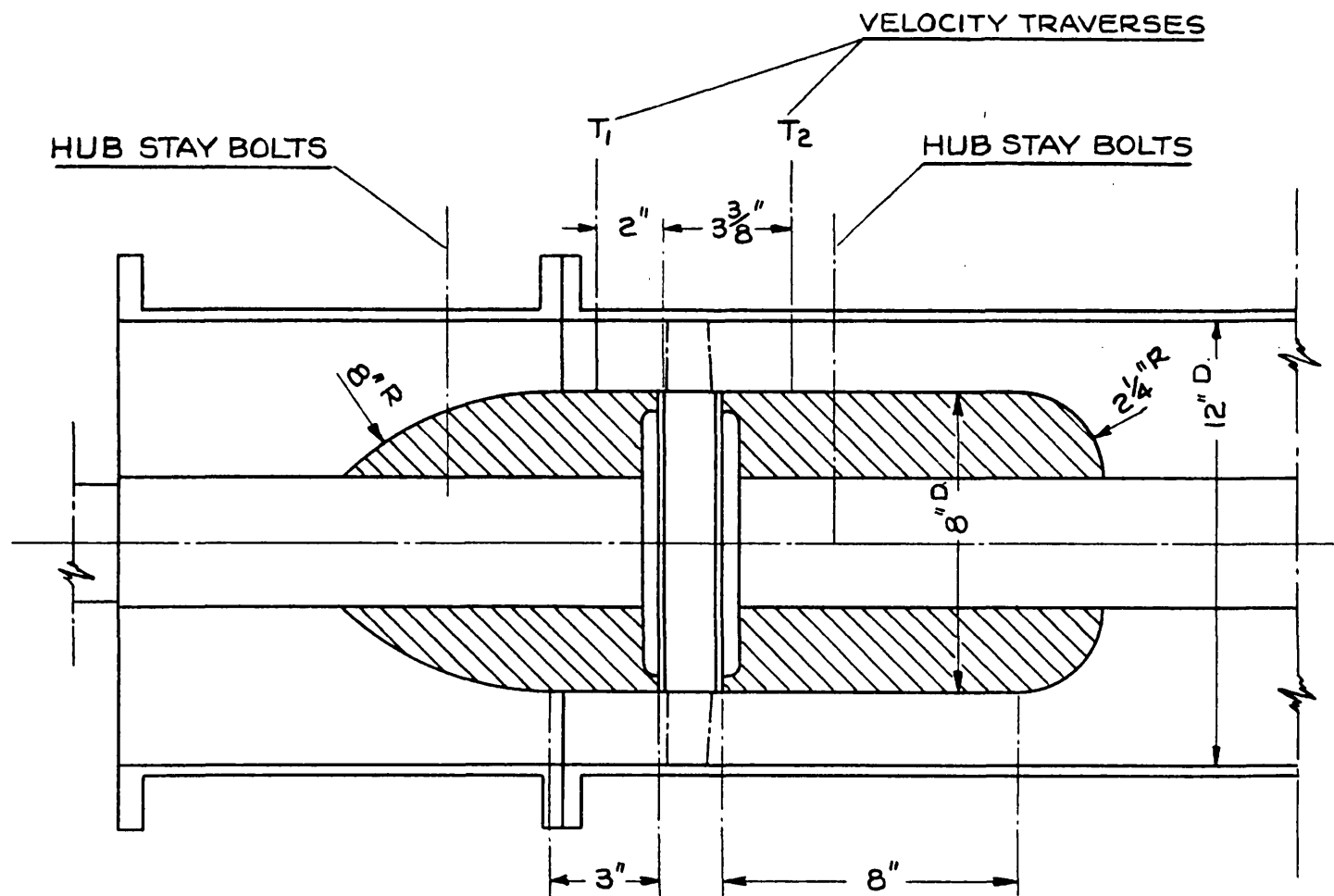


Fig. 2.4 SET-UP FOR TESTING 8" MODIFIED PROPELLER

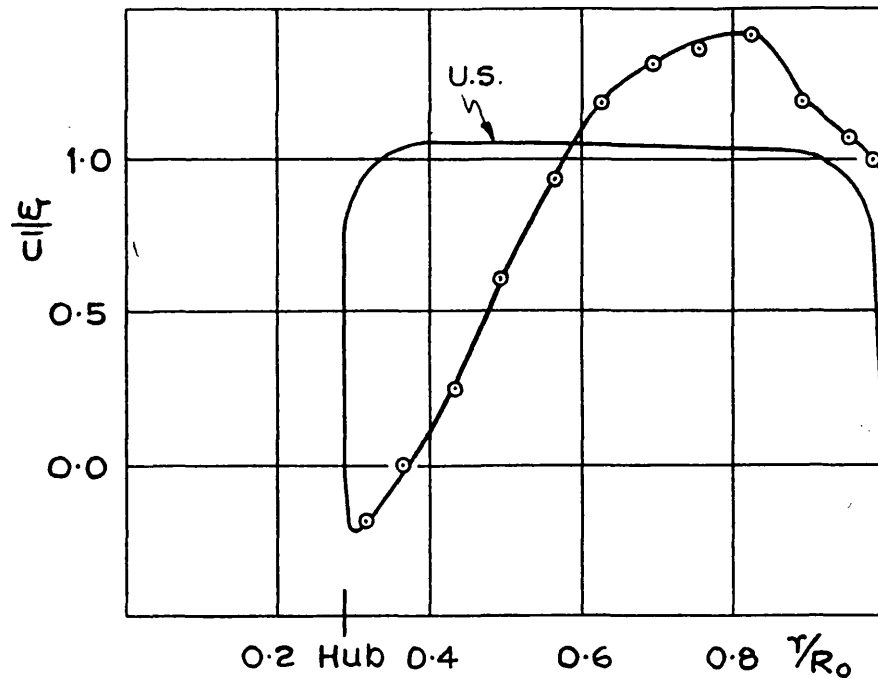


Fig. 2.5(a) UPSTREAM AND DOWNSTREAM AXIAL VELOCITY DISTRIBUTION AT $U_c = 0.15$ ($q_c = 0.35$)

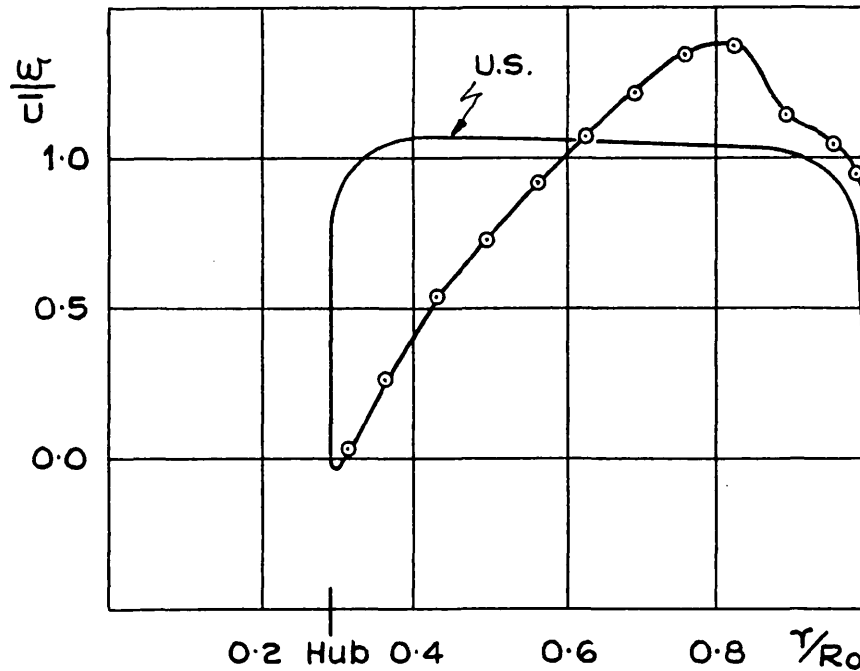


Fig. 2.5(b) UPSTREAM AND DOWNSTREAM AXIAL VELOCITY DISTRIBUTION AT $U_c = 0.17$ ($q_c = 0.43$)

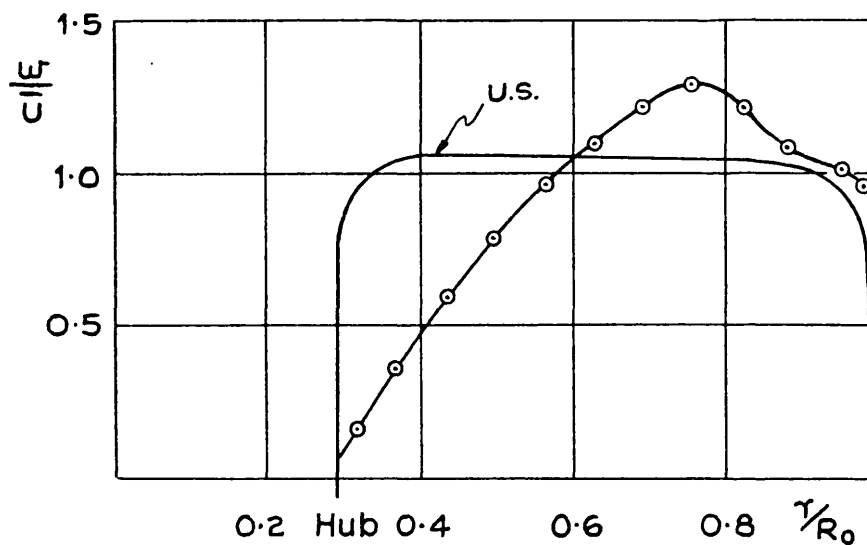


Fig. 2.5(c) UPSTREAM AND DOWNSTREAM AXIAL VELOCITY DISTRIBUTION AT $U_c = 0.20$ ($q_c = 0.45$)

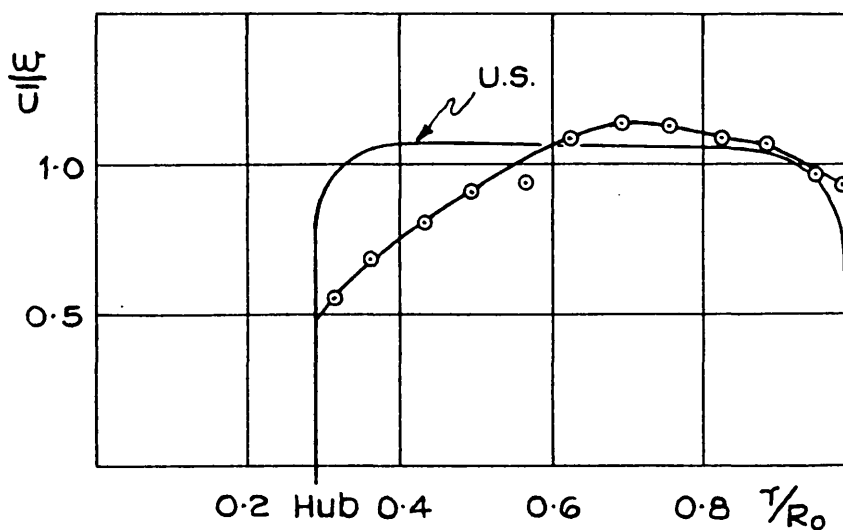


Fig. 2.5 (d) UPSTREAM AND DOWNSTREAM AXIAL VELOCITY DISTRIBUTION AT $U_c = 0.24$ ($q_c = 0.55$)

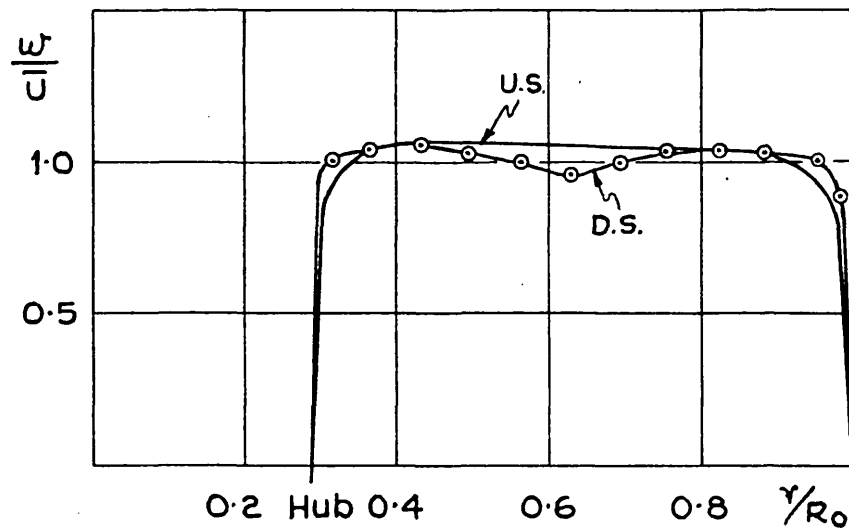


Fig. 2.5(e) UPSTREAM AND DOWNSTREAM VELOCITY DISTRIBUTION AT $U_c = 0.35$ ($q_c = 0.80$)

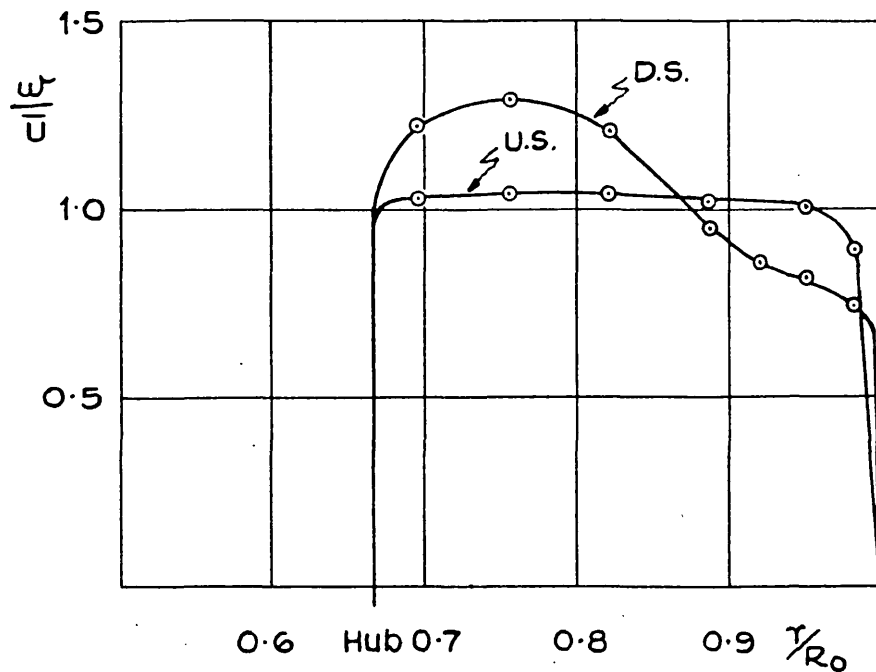


Fig. 2.6(a) UPSTREAM AND DOWNSTREAM VELOCITY DISTRIBUTION FOR THE 8"-HUB PROPELLER AT $U_c = 0.165$

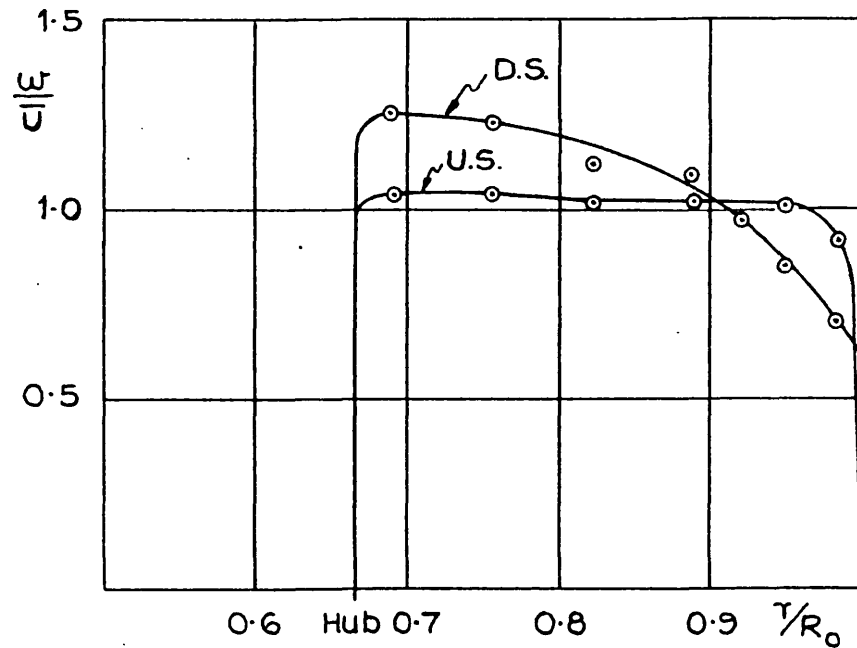


Fig. 2-6 (b) UPSTREAM AND DOWNSTREAM VELOCITY DISTRIBUTION FOR THE 8"-HUB MODIFIED PROPELLER AT $U_c = 0.238$.

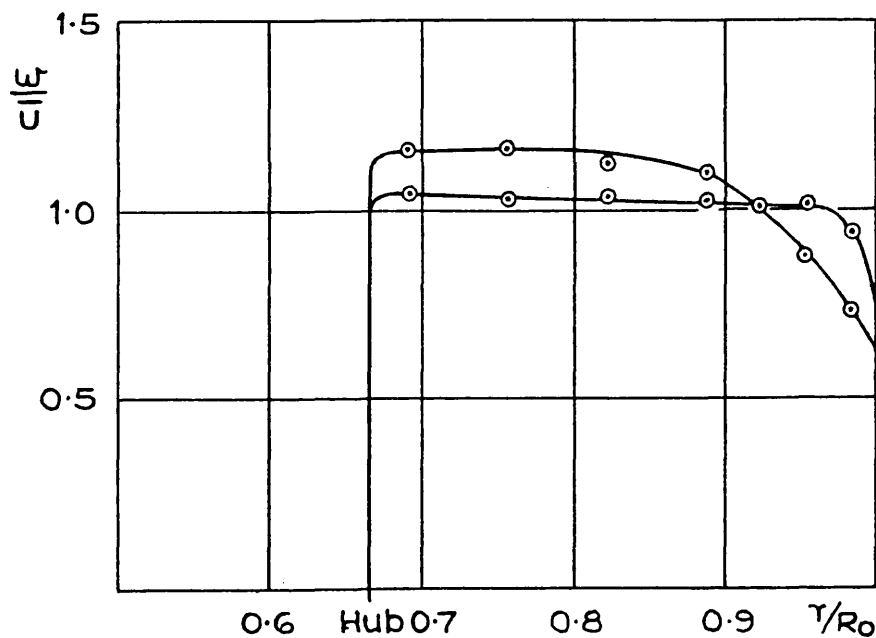


Fig. 2-6 (c) U.S., & D.S. VELOCITY DISTRIBUTION FOR THE 8"-HUB MODIFIED PROPELLER AT $U_c = 0.256$.

Fig. 2·6 (d & e)

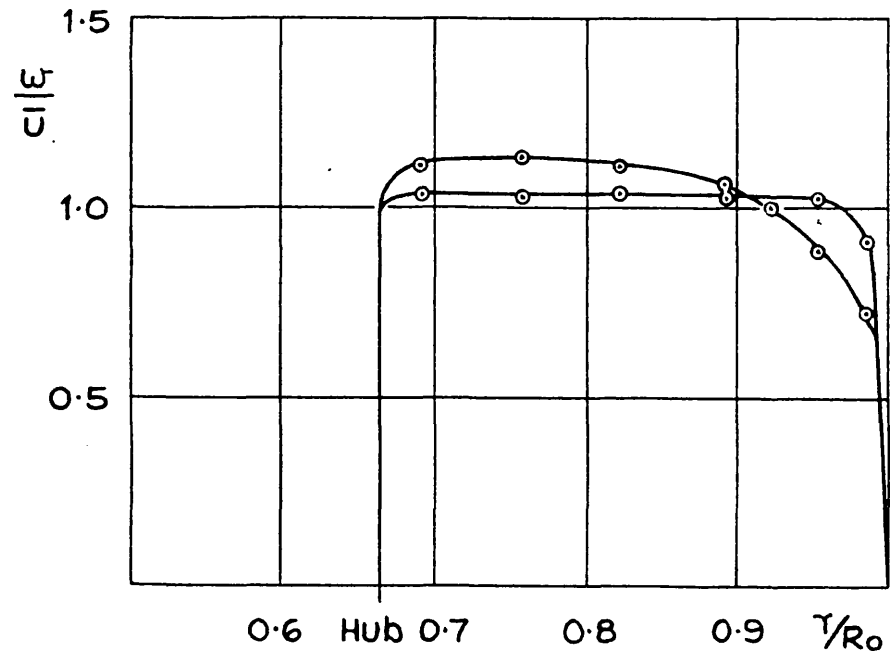


Fig 2·6 (d) U.S., & D.S. VELOCITY DISTRIBUTION FOR THE 8" HUB PROPELLER AT $U_c = 0.274$

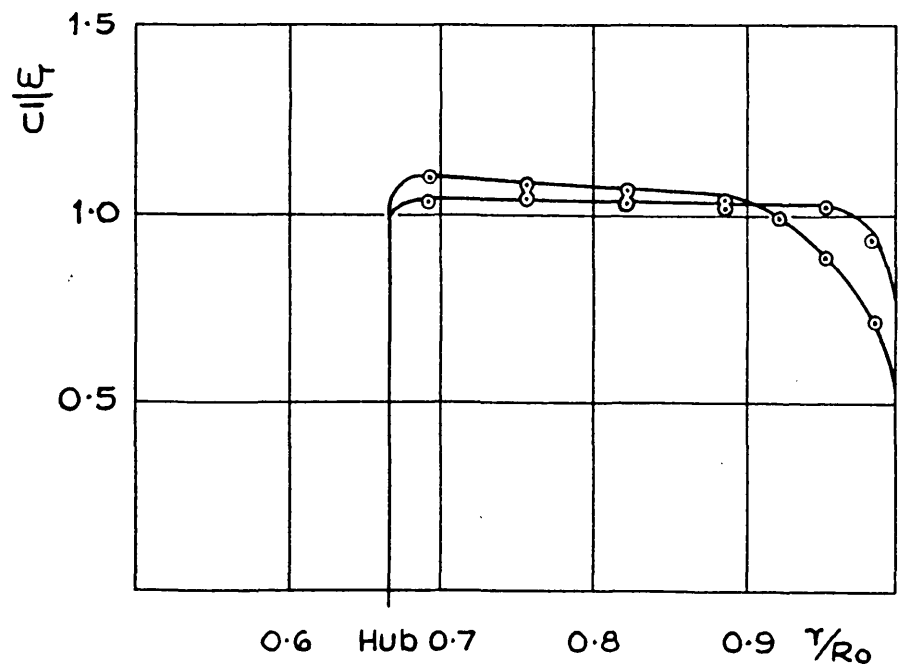


Fig 2·6 (e) U.S., & D.S. VELOCITY DISTRIBUTION FOR THE 8" HUB PROPELLER AT $U_c = 0.296$

Fig. 2.7 (a)

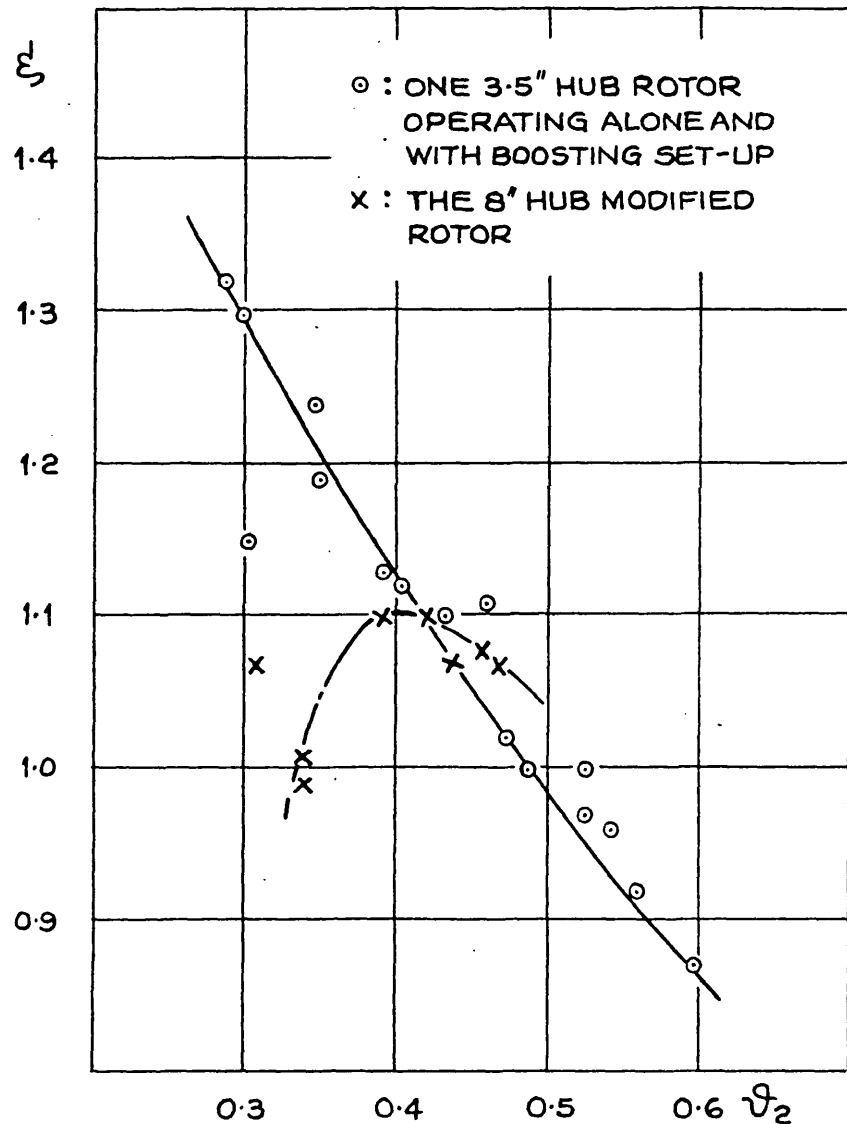


Fig. 2.7 (a) SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $r/R_0 = 0.69$ SECONDARY FLOW WAS RADIALLY OUTWARDS FOR THE 3½"-HUB ROTOR, AND INWARDS FOR THE 8"-HUB PROPELLER (TABLE III)

Fig 2-7 (b)

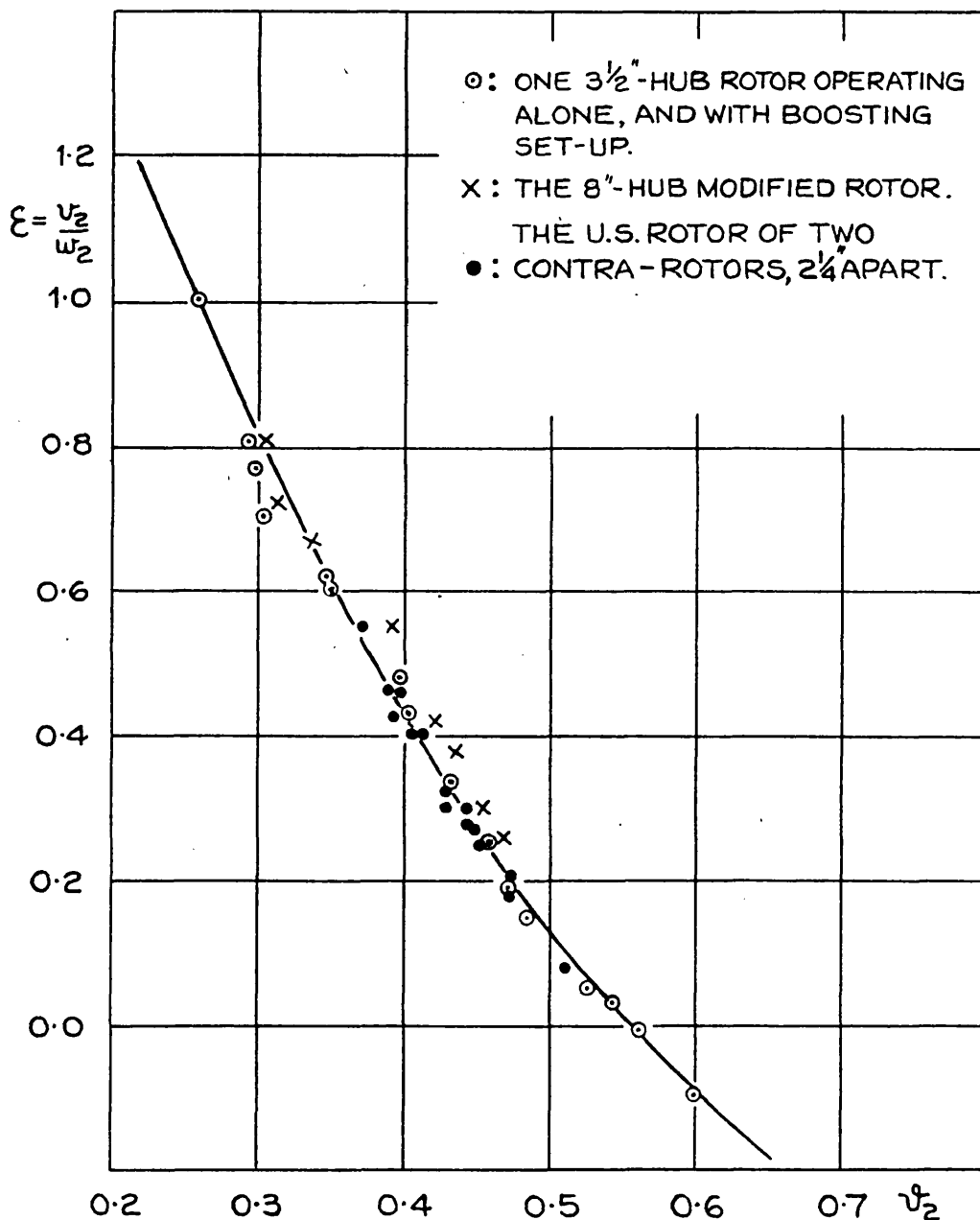


Fig. 2-7 (b) TANGENTIAL VELOCITY COMPONENT, AS A RATIO TO AXIAL VELOCITY, DOWNSTREAM OF THE PROPELLER, V.S. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $\frac{r}{R_0} = 0.69$ (TABLE III)

Fig. 2·7 (C)

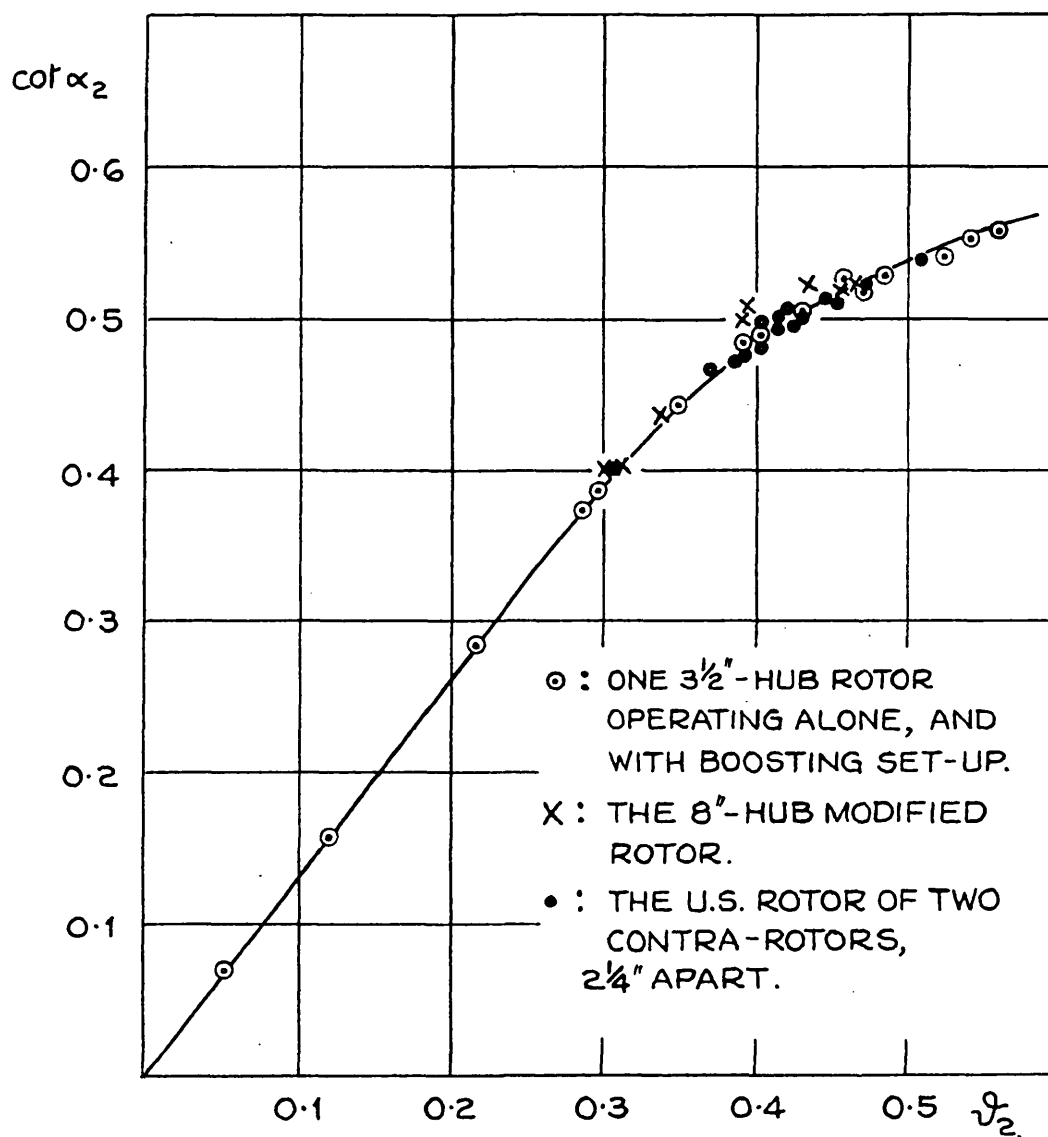


Fig. 2·7 (C) DOWNSTREAM FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, V.S. D.S. FLOW COEFFICIENT, FOR $\gamma/R_0 = 0.69$. (TABLE III)

Fig. 2·8 (a)

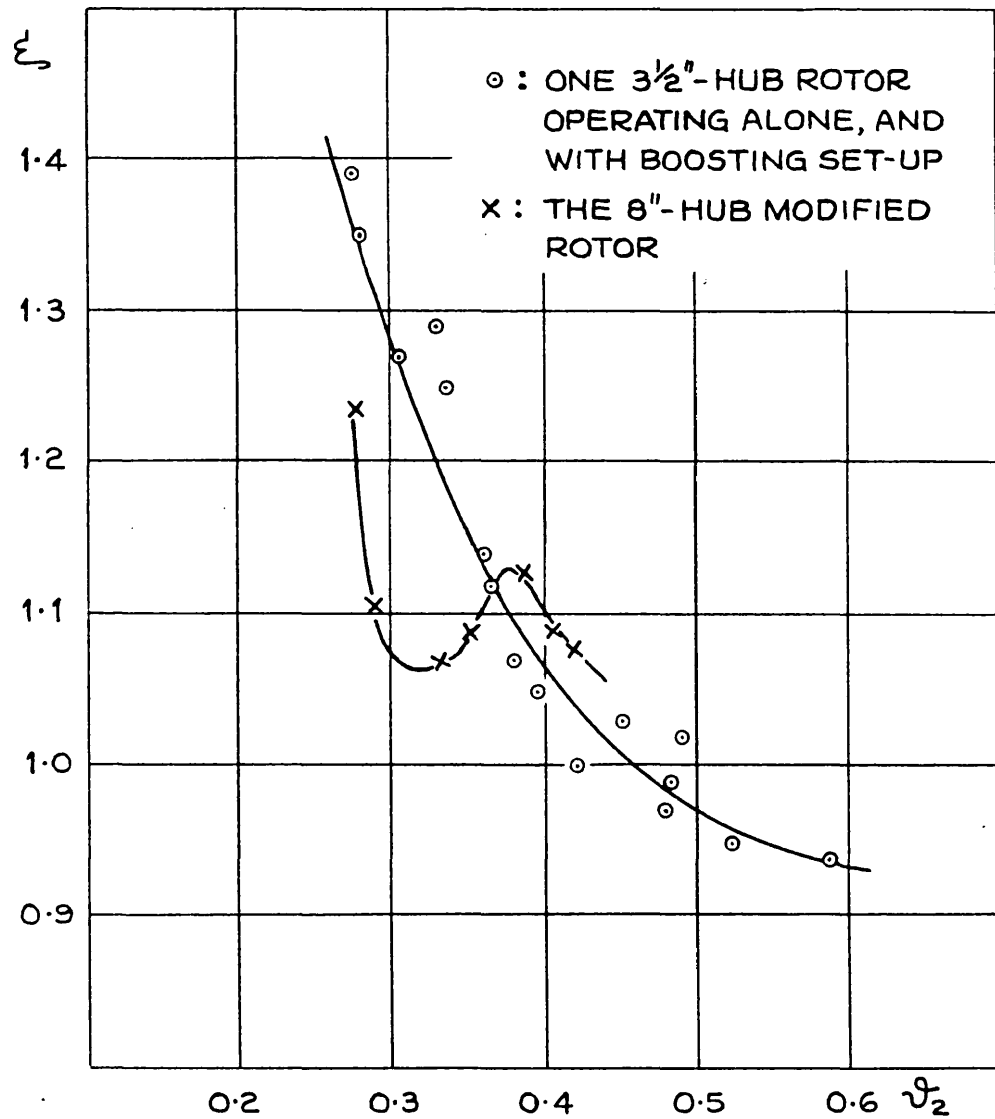


Fig. 2·8 (a) SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $\gamma_{R_0} = 0.76$ SECONDARY FLOW WAS RADIAL OUTWARDS FOR THE $3\frac{1}{2}$ " HUB ROTOR AND INWARDS FOR THE 8"-HUB PROPELLER. (TABLE III)

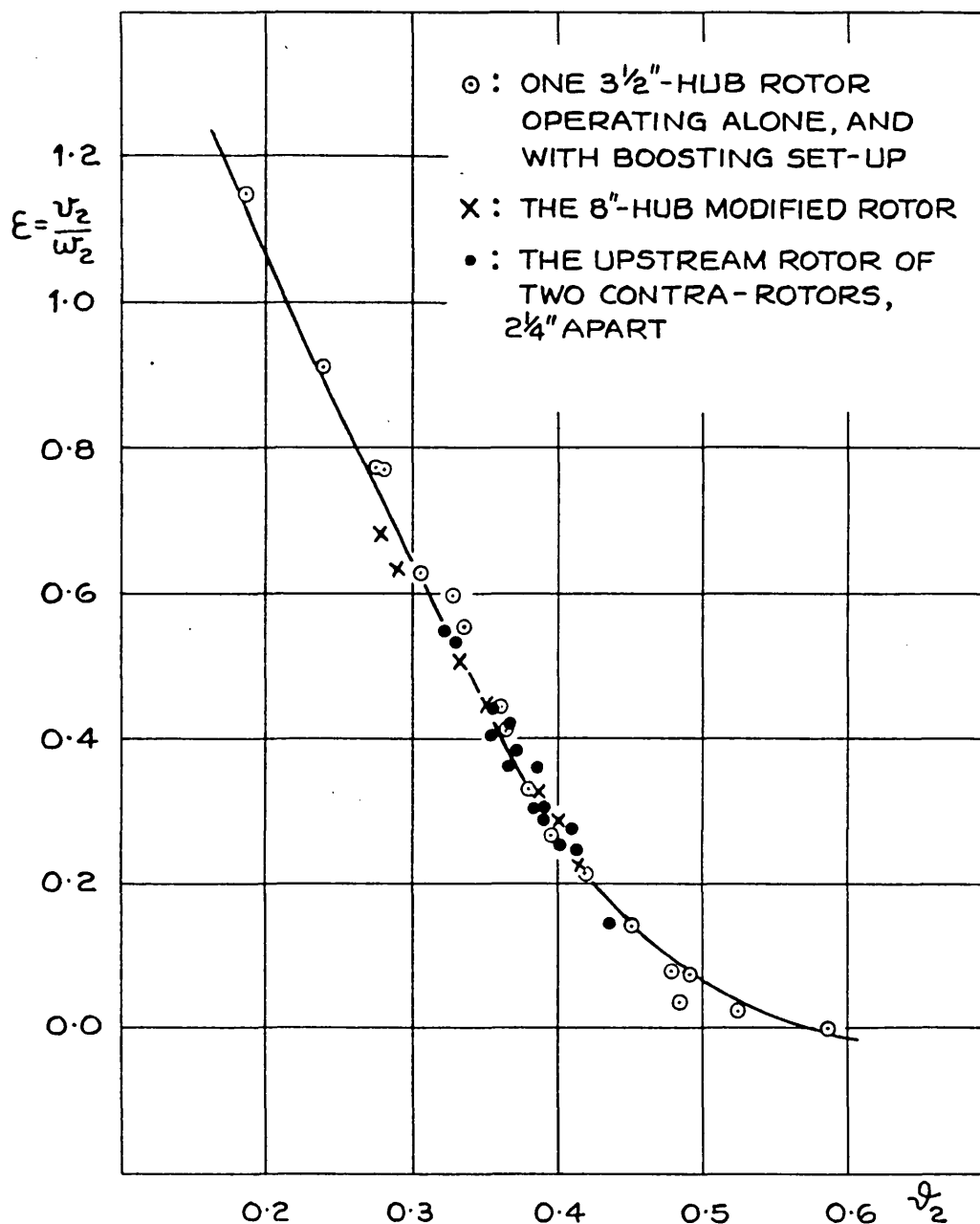


Fig. 2·8 (b) TANGENTIAL VELOCITY COMPONENT AS A RATIO TO AXIAL VELOCITY D.S. OF THE PROPELLER, VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $r/R_0 = 0.76$ (TABLE III)

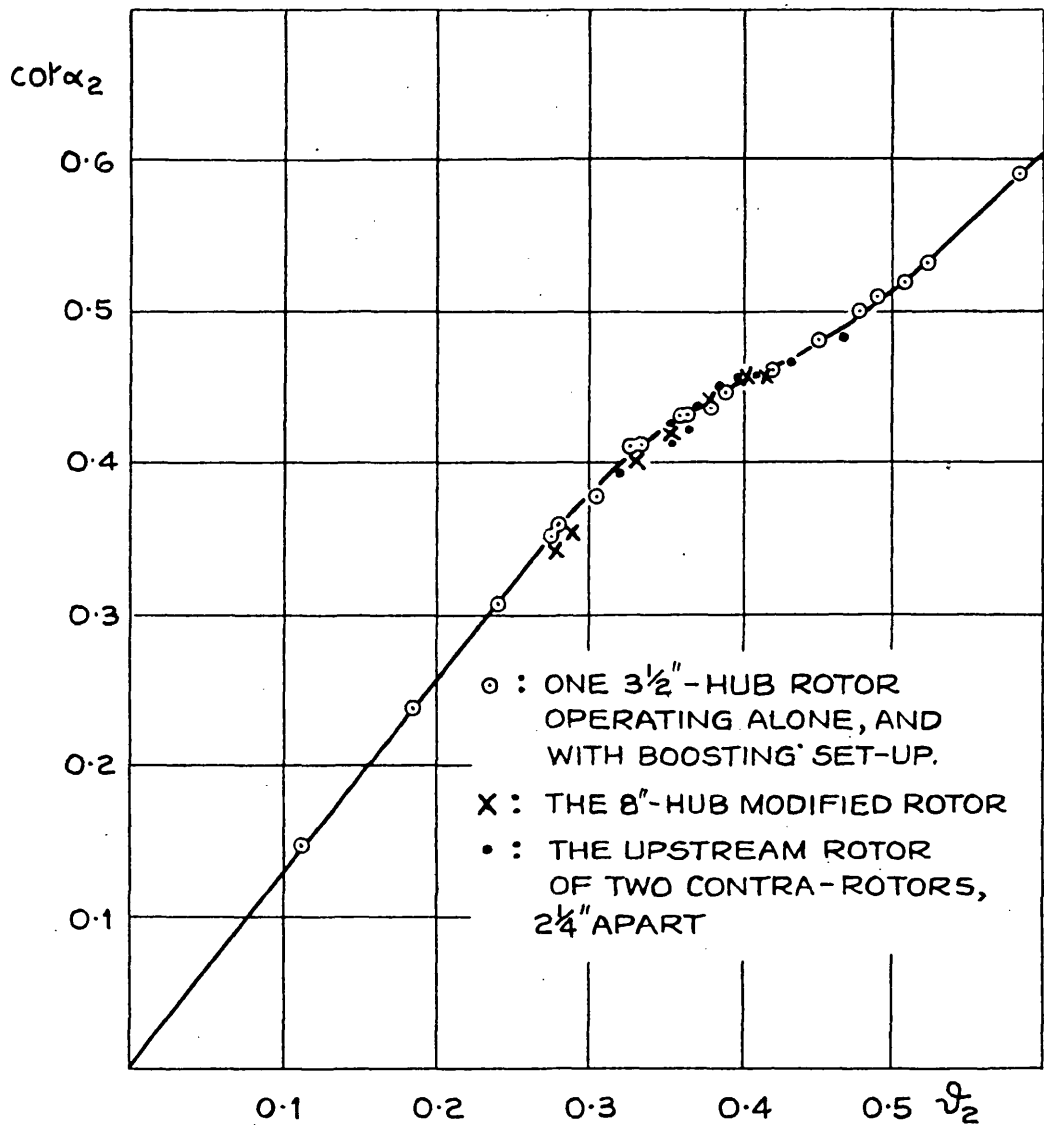


Fig. 2-8 (c) DOWNSTREAM FLOW DIRECTION RELATIVE TO THE ROTOR BLADE VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $r/R_0 = 0.76$ (TABLE III)

Fig. 2·9 (a)

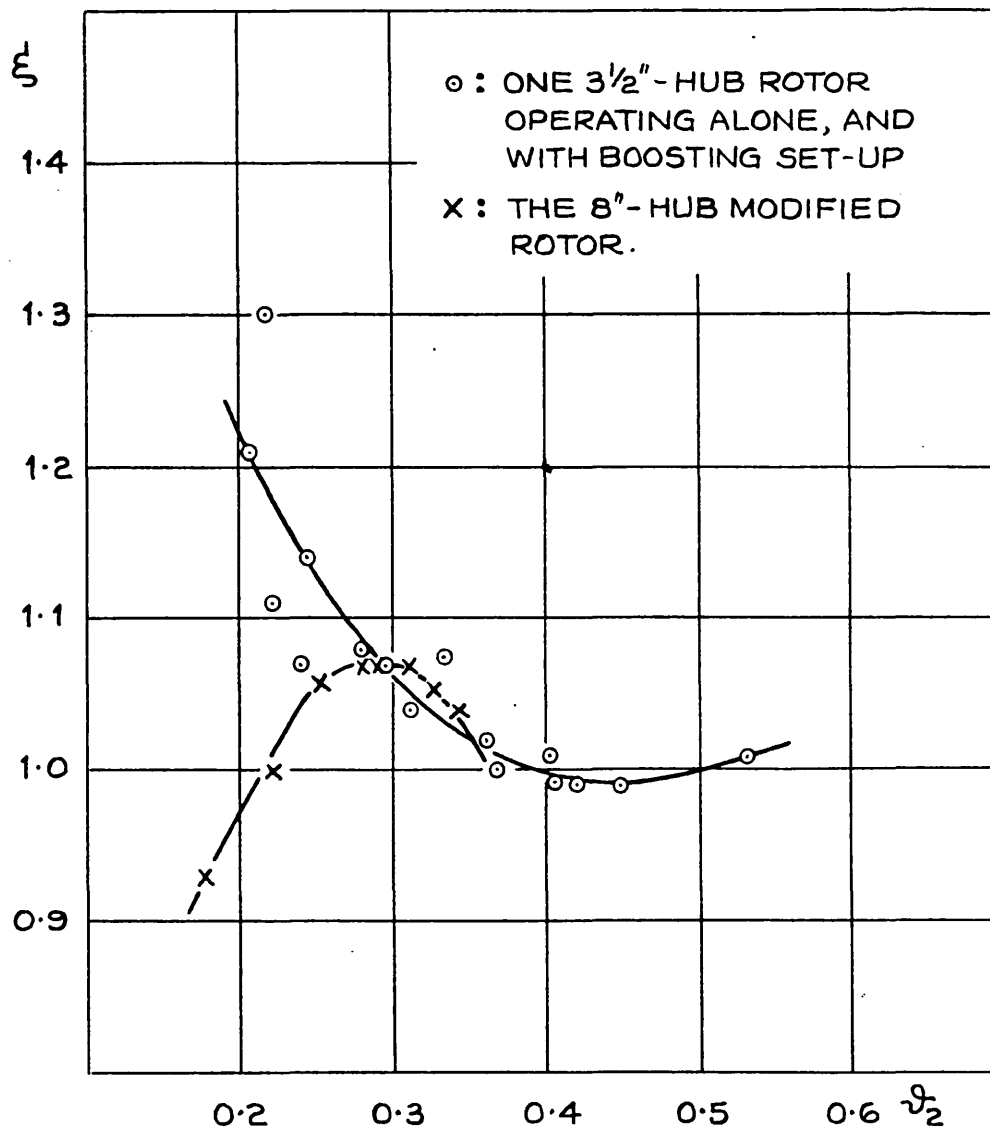


Fig. 2·9 (a) SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $\gamma_{R_0} = 0.89$. SECONDARY FLOW WAS RADIALY OUTWARDS FOR THE 3½" HUB ROTOR AND INWARDS FOR THE 8"-HUB PROPELLER (TABLE III)

Fig. 2·9 (b)

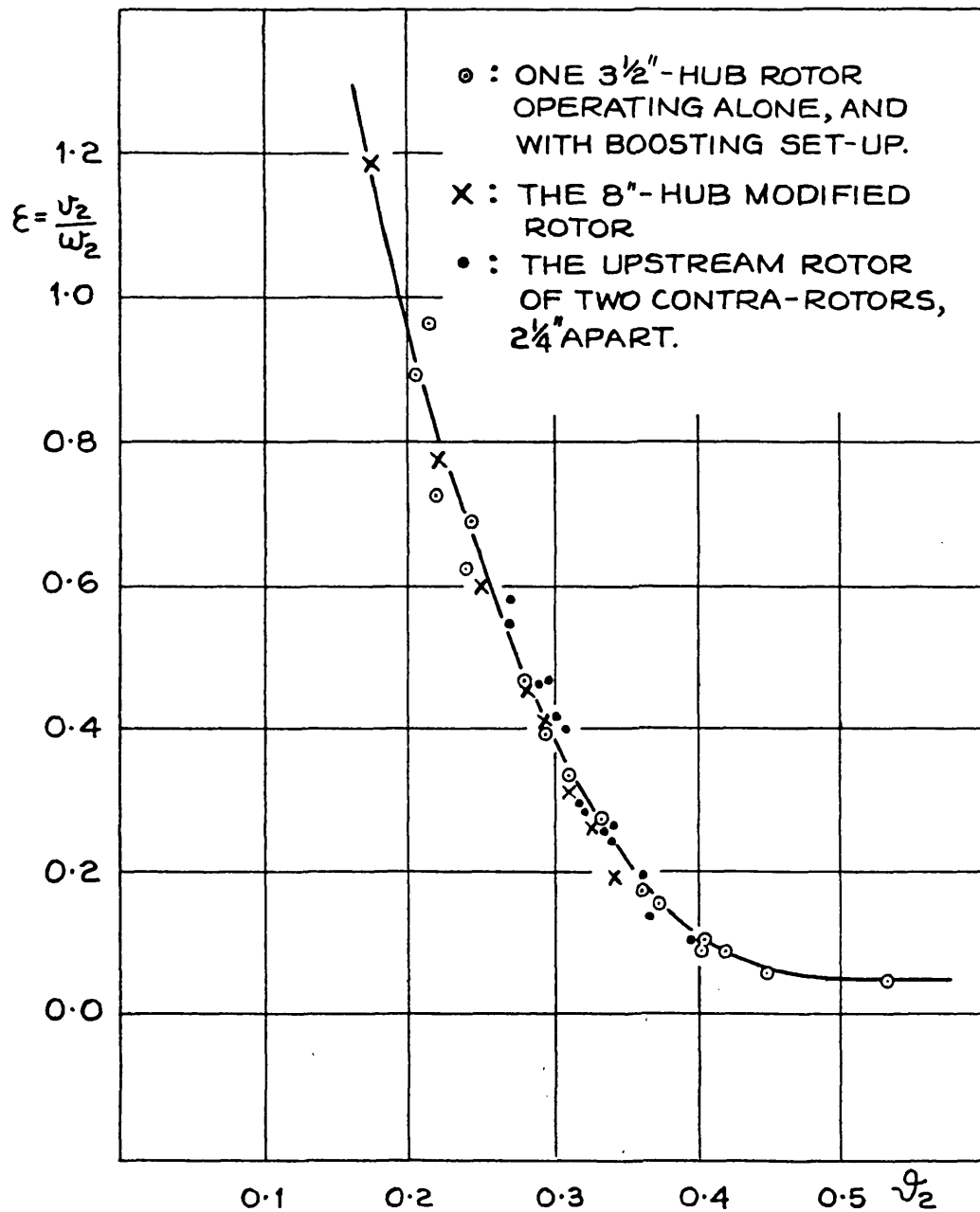


Fig. 2·9 (b) TANGENTIAL VELOCITY COMPONENT AS A RATIO TO AXIAL VELOCITY D.S. OF THE PROPELLER, VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION, $r/R_0 = 0.89$. (TABLE III)

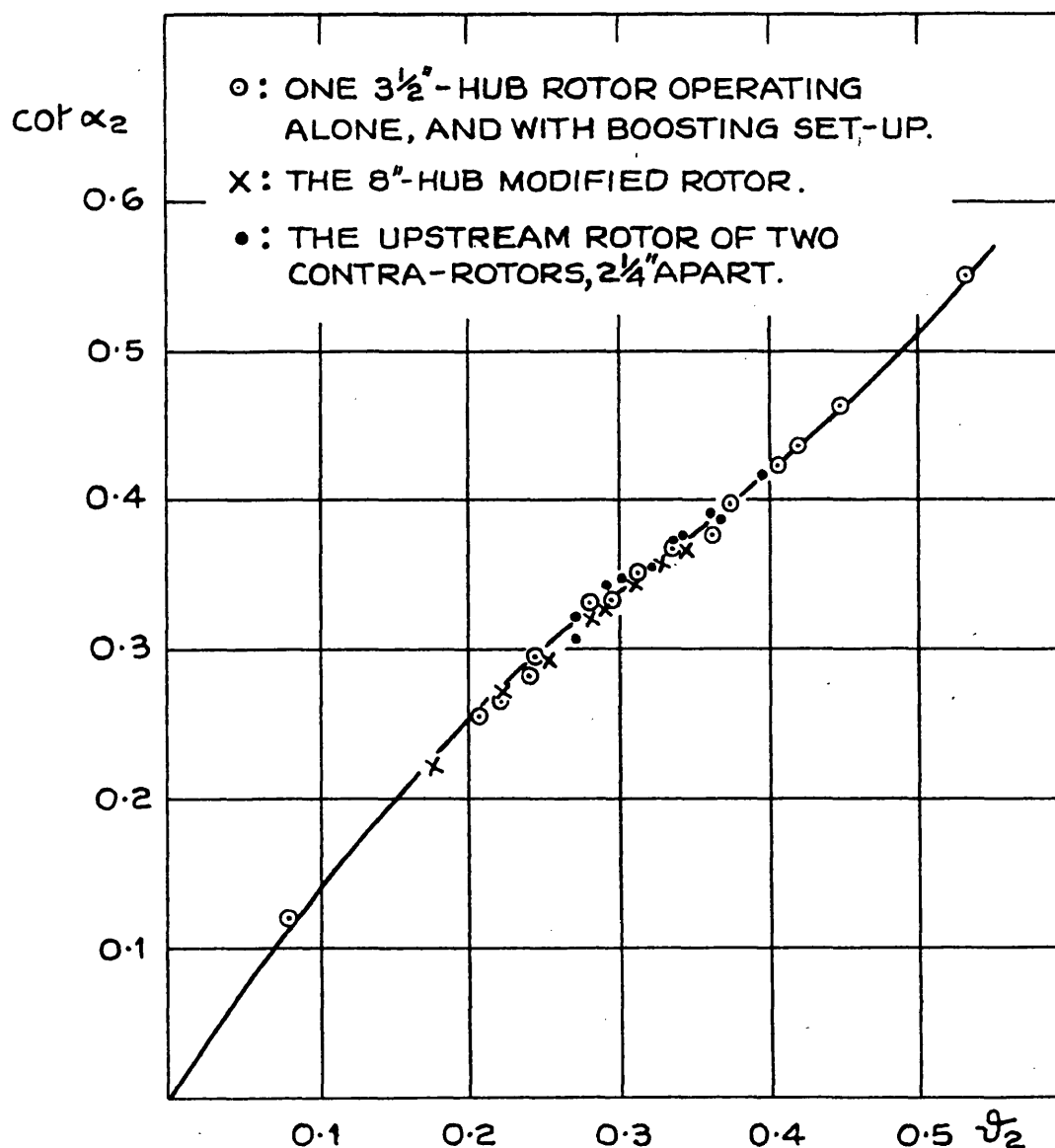


Fig. 2.9 (c) DOWNSTREAM FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $r/R_0 = 0.89$. (TABLE III)

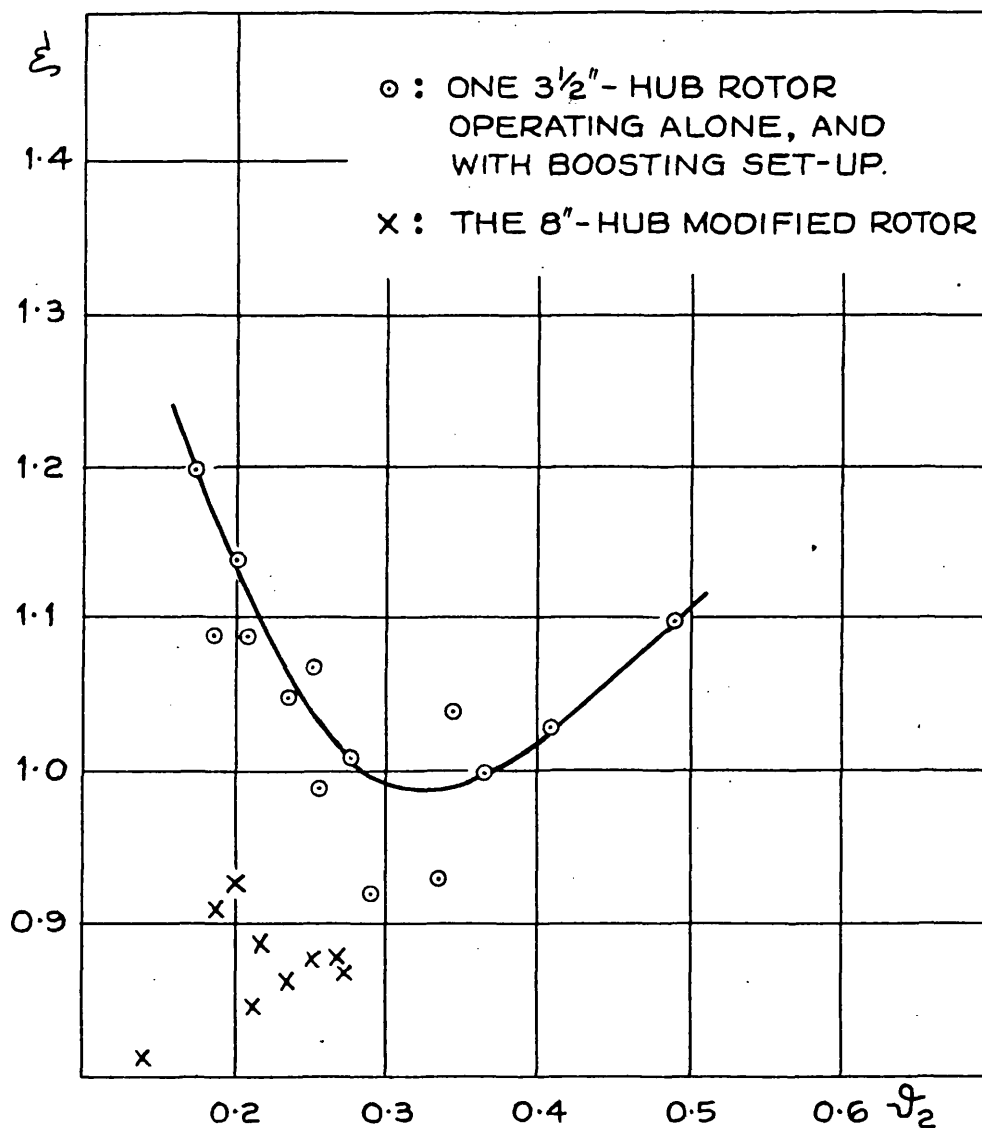


Fig. 2.10(a) SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $\gamma/R_0 = 0.95$. SECONDARY FLOW WAS RADIALLY OUTWARDS FOR THE 3½" HUB ROTOR AND INWARDS FOR THE 8"-HUB PROPELLER. (TABLE III)

Fig. 2·10 (b)

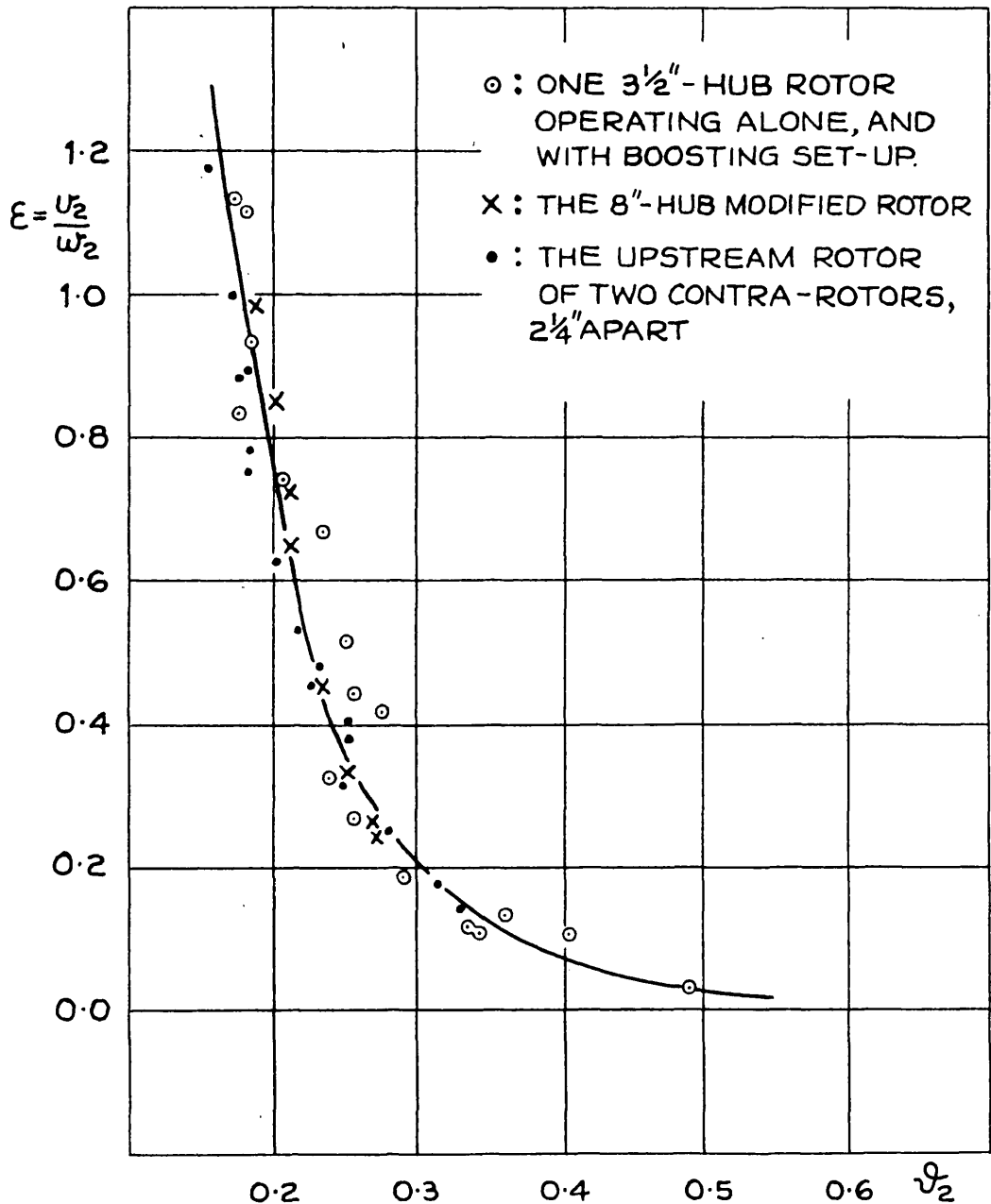


Fig. 2·10(b) TANGENTIAL VELOCITY COMPONENT AS A RATIO TO AXIAL VELOCITY D.S. OF THE PROPELLER, VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $r/R_0 = 0.95$ (TABLE III)

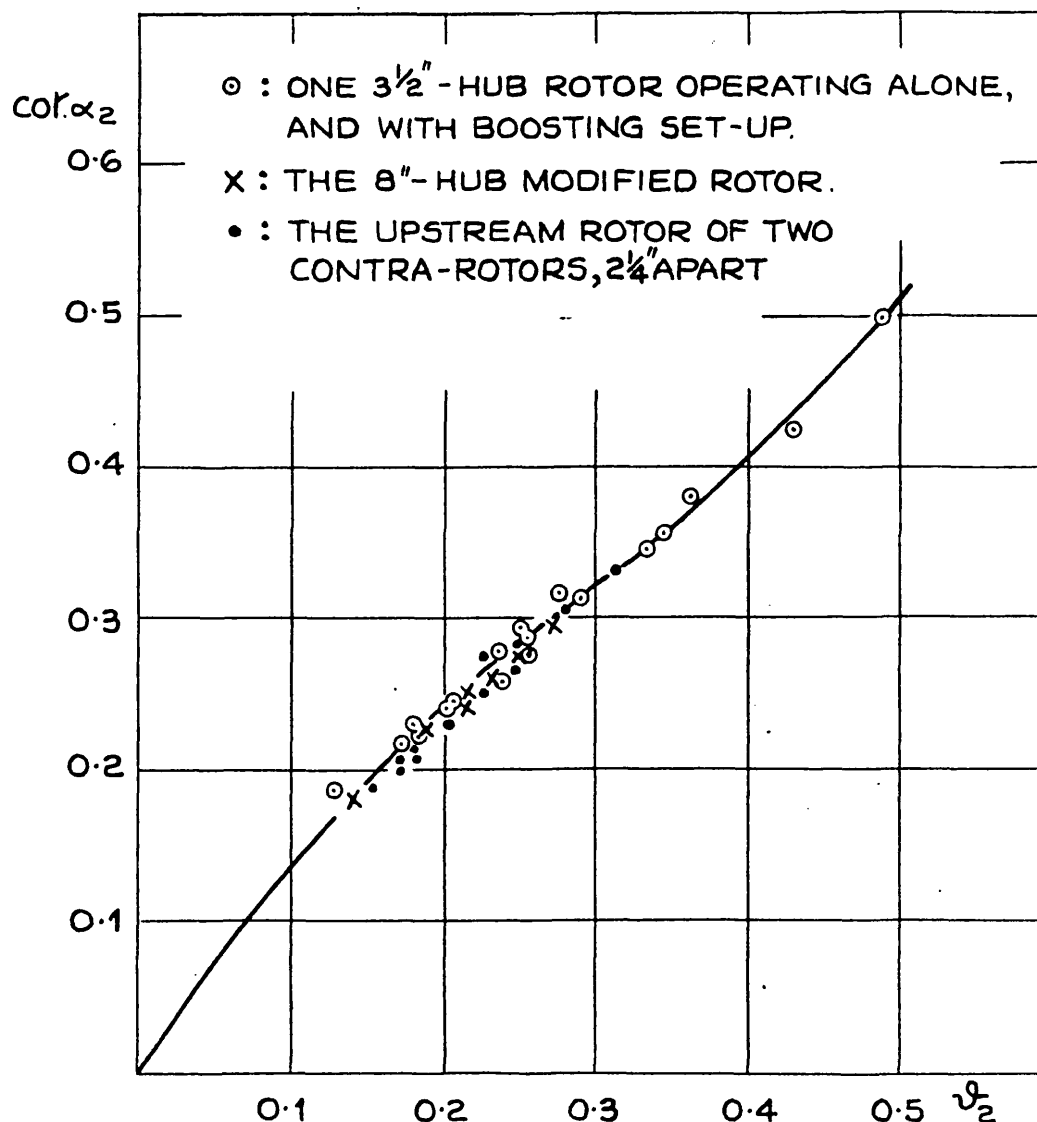


Fig. 2·10 (C) DOWNSTREAM FLOW DIRECTION RELATIVE TO THE ROTOR BLADE VS. D.S. FLOW COEFFICIENT, FOR RADIAL STATION $r/R_0 = 0.95$ (TABLE III)

Fig. 2·11 (a)

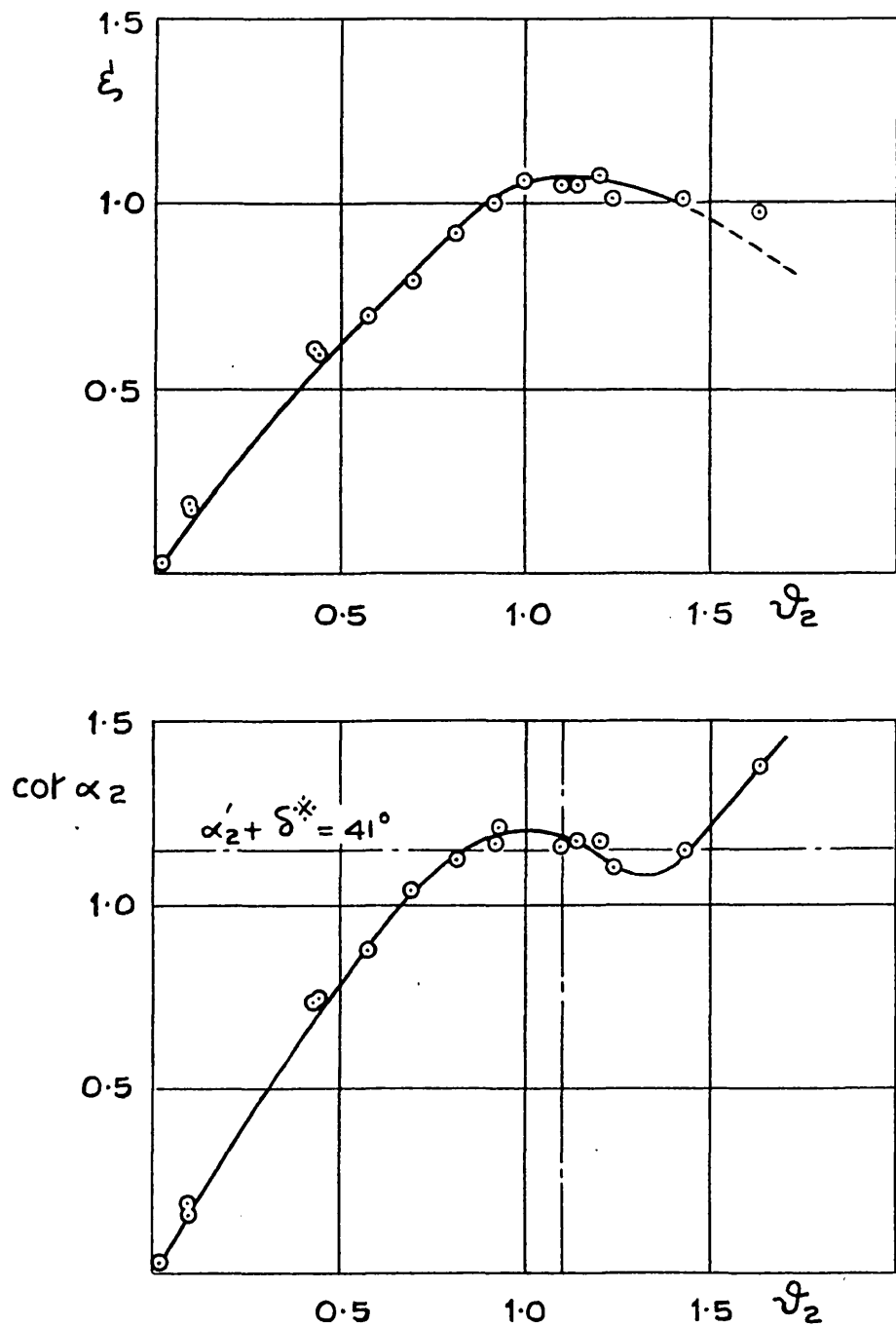


Fig. 2·11 (a) DOWNSTREAM FLOW DIRECTION
RELATIVE TO THE ROTOR BLADE
AND THE SECONDARY FLOW FACTOR
VS. D.S. FLOW COEFFICIENT FOR
RADIAL STATION $r/R_0 = 0.32$
(TABLE VI)

Fig. 2.11 (b)

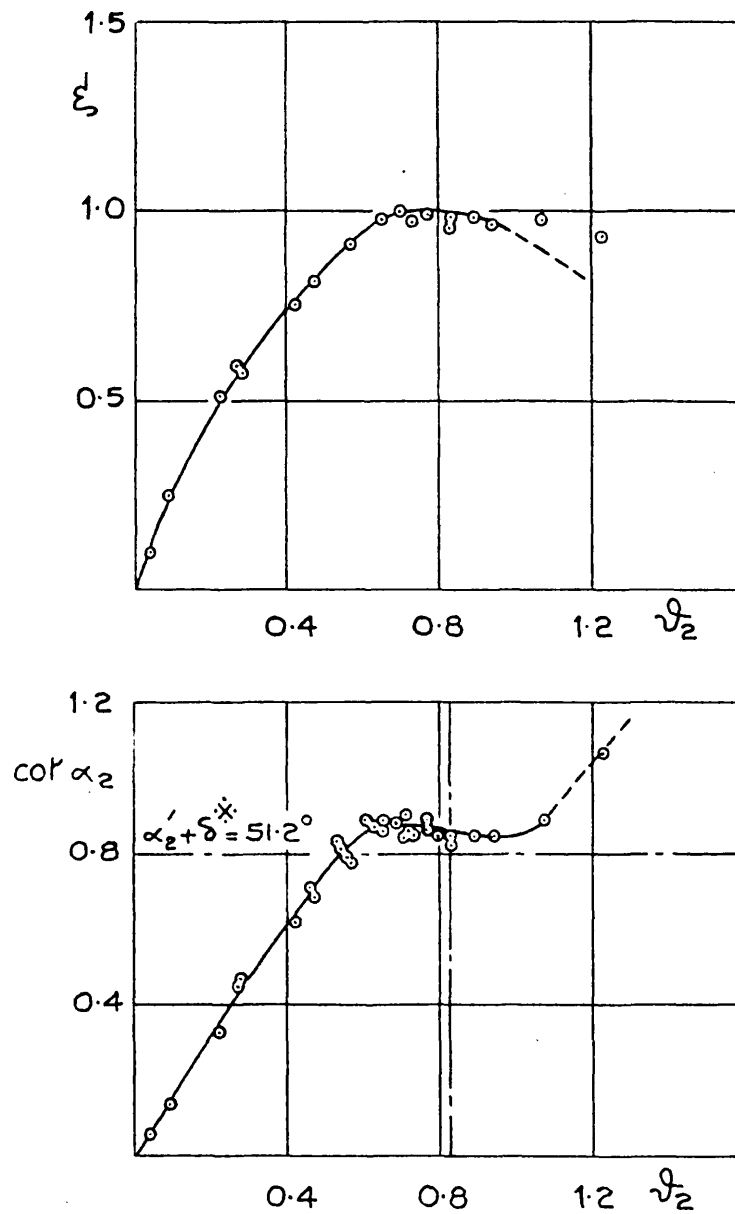


Fig. 2.11 (b) DOWNSTREAM FLOW DIRECTION
RELATIVE TO THE ROTOR BLADE
AND THE SECONDARY FLOW FACTOR
VS. DS. FLOW COEFFICIENT FOR
RADIAL STATION $\gamma_{R_0} = 0.43$
(TABLE VI)

Fig. 2.11 (C)

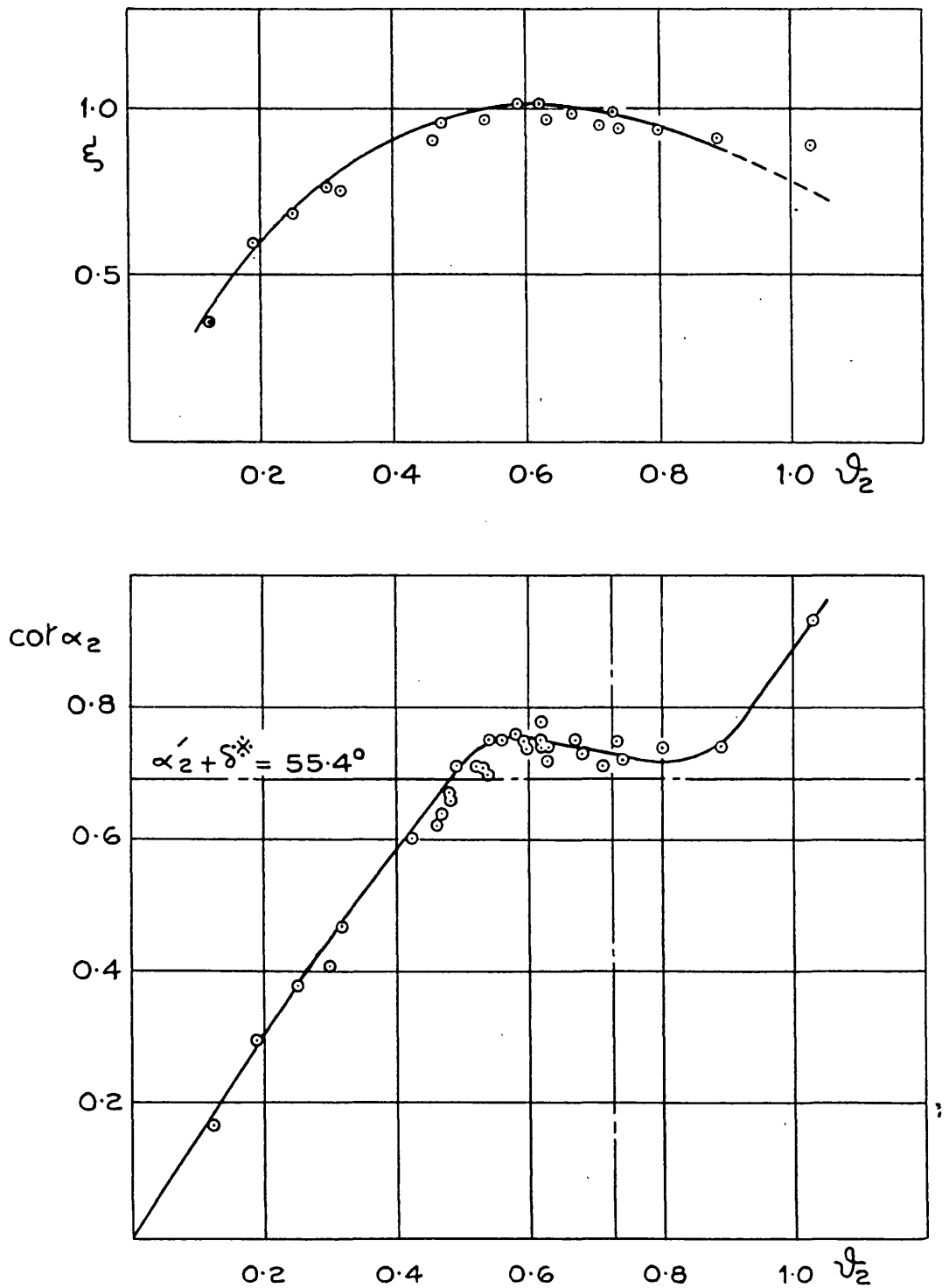


Fig. 2.11 (C) D.S. FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, AND SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $r/R_0 = 0.50$ (TABLE VI)

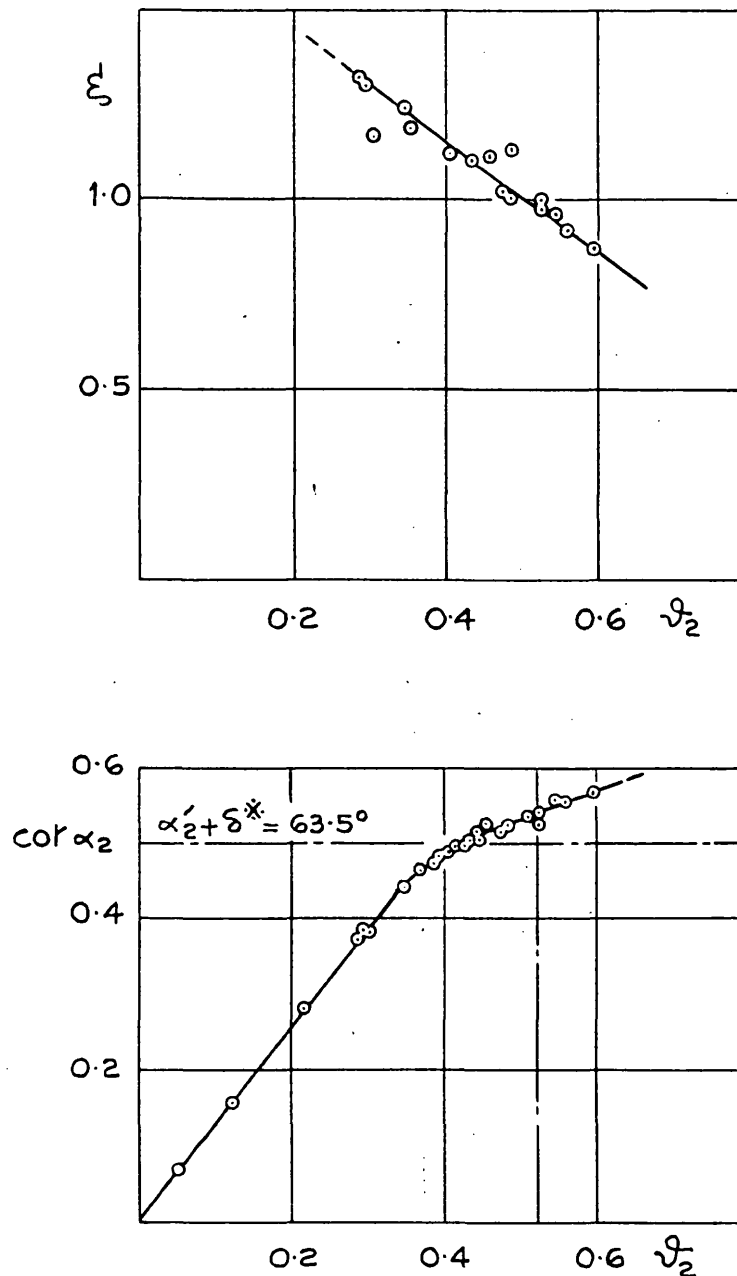


Fig. 2.11 (d) D.S. FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, AND SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $\gamma_{R_0} = 0.69$ (TABLE III)

Fig. 2.11 (e)

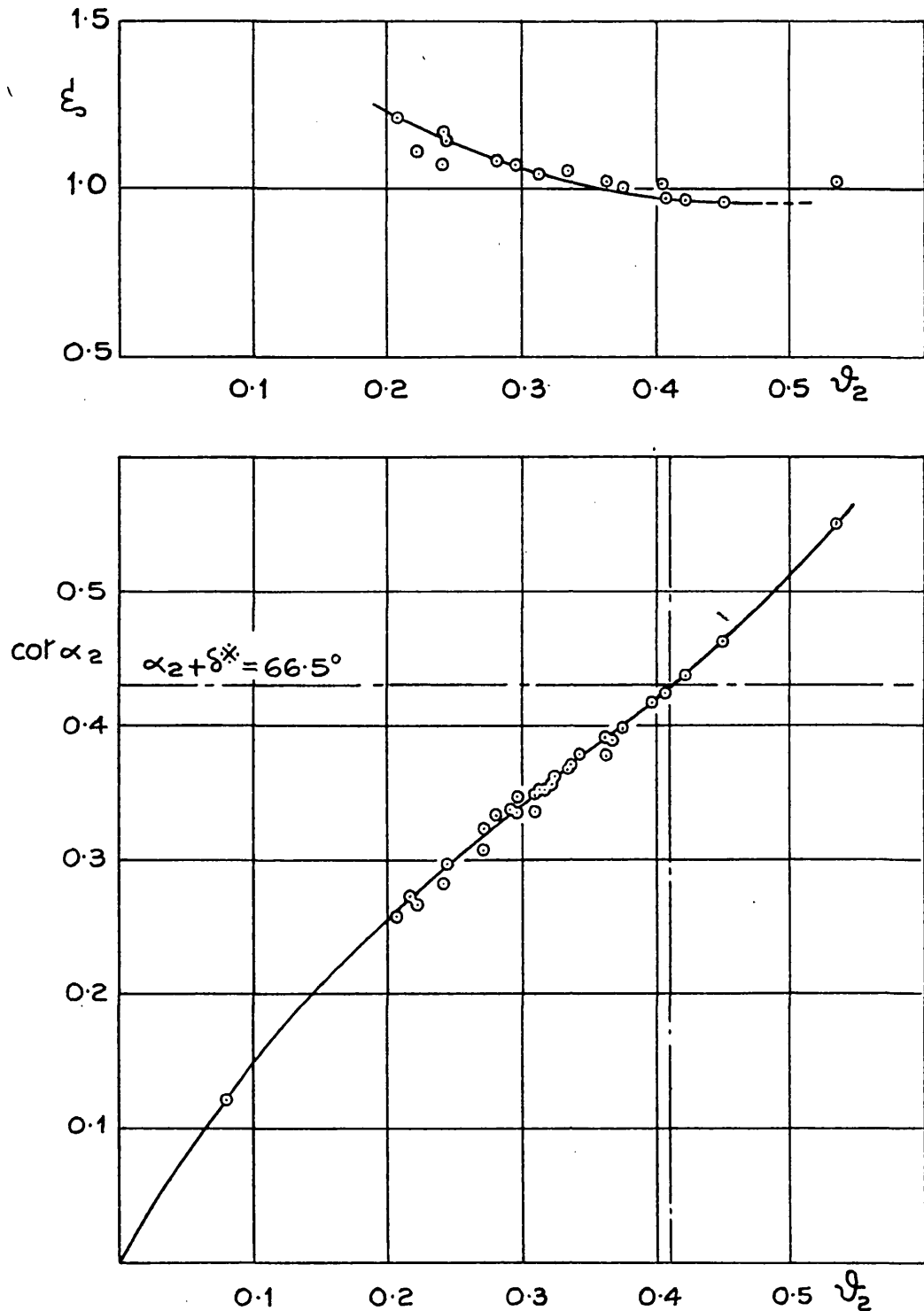


Fig. 2.11 (e) D.S. FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, AND SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $\gamma/R_0 = 0.89$. (TABLE III)

Fig. 2.11 (f)

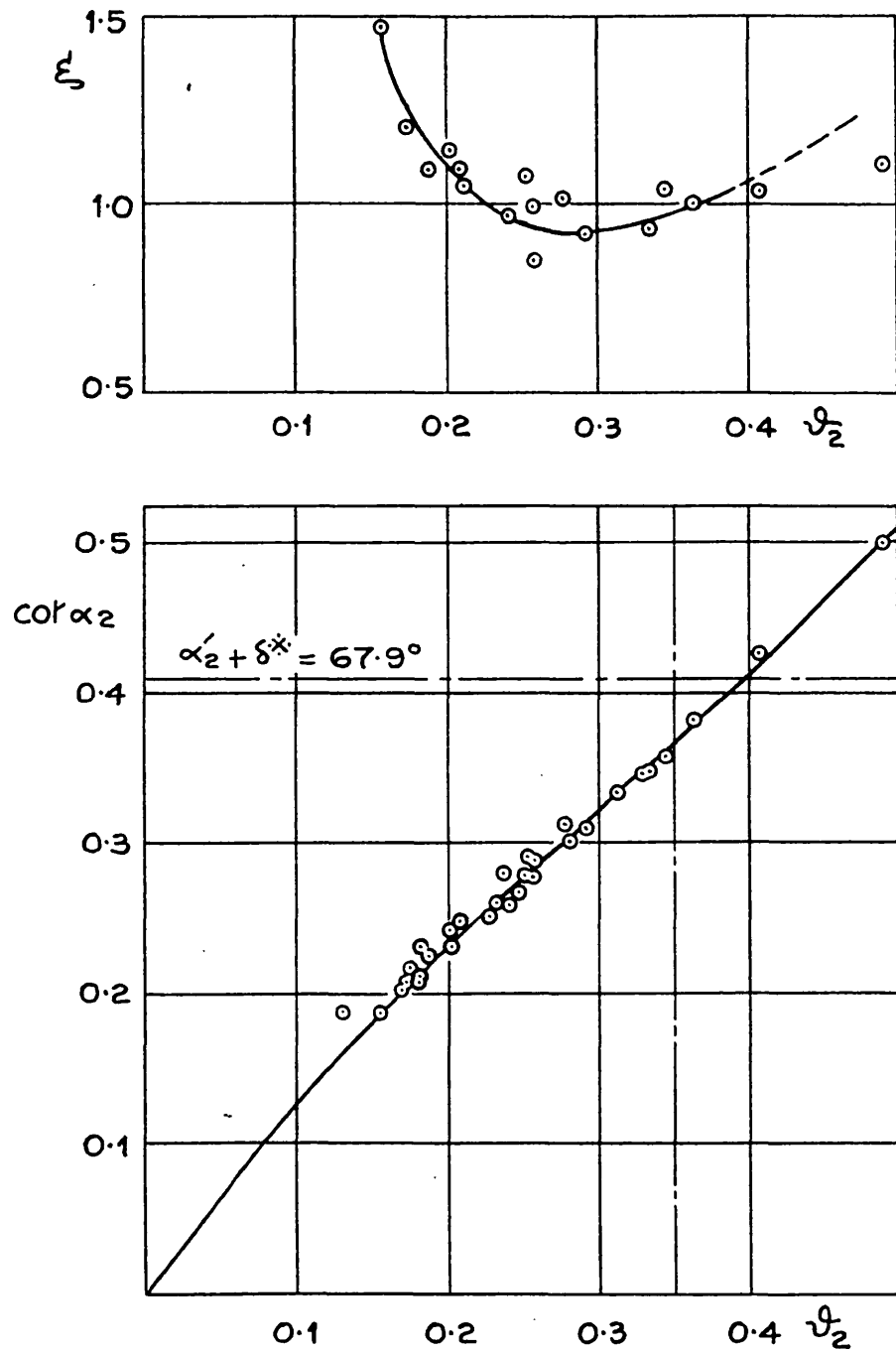


Fig. 2.11 (f) D.S. FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, AND SECONDARY FLOW FACTOR VS. D.S. FLOW COEFFICIENT FOR RADIAL STATION $r/R_0 = 0.95$ (TABLE III)

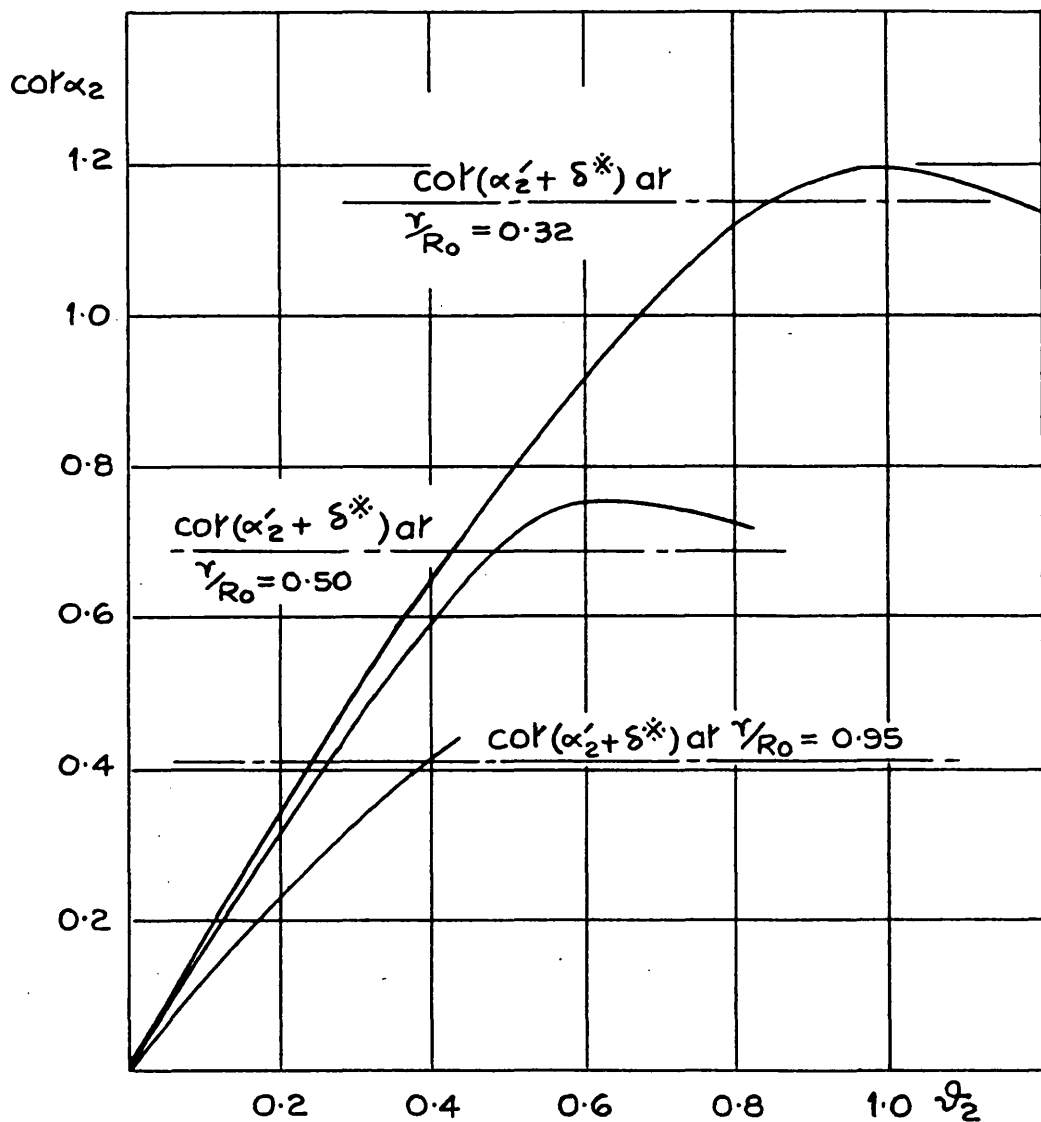


Fig. 2.11 (g) D.S. FLOW DIRECTION RELATIVE TO THE ROTOR BLADE, VS. FLOW COEFFICIENT FOR RADIAL STATIONS $r/R_0 = 0.32$, 0.50 , AND 0.95 .

Fig. 2.12

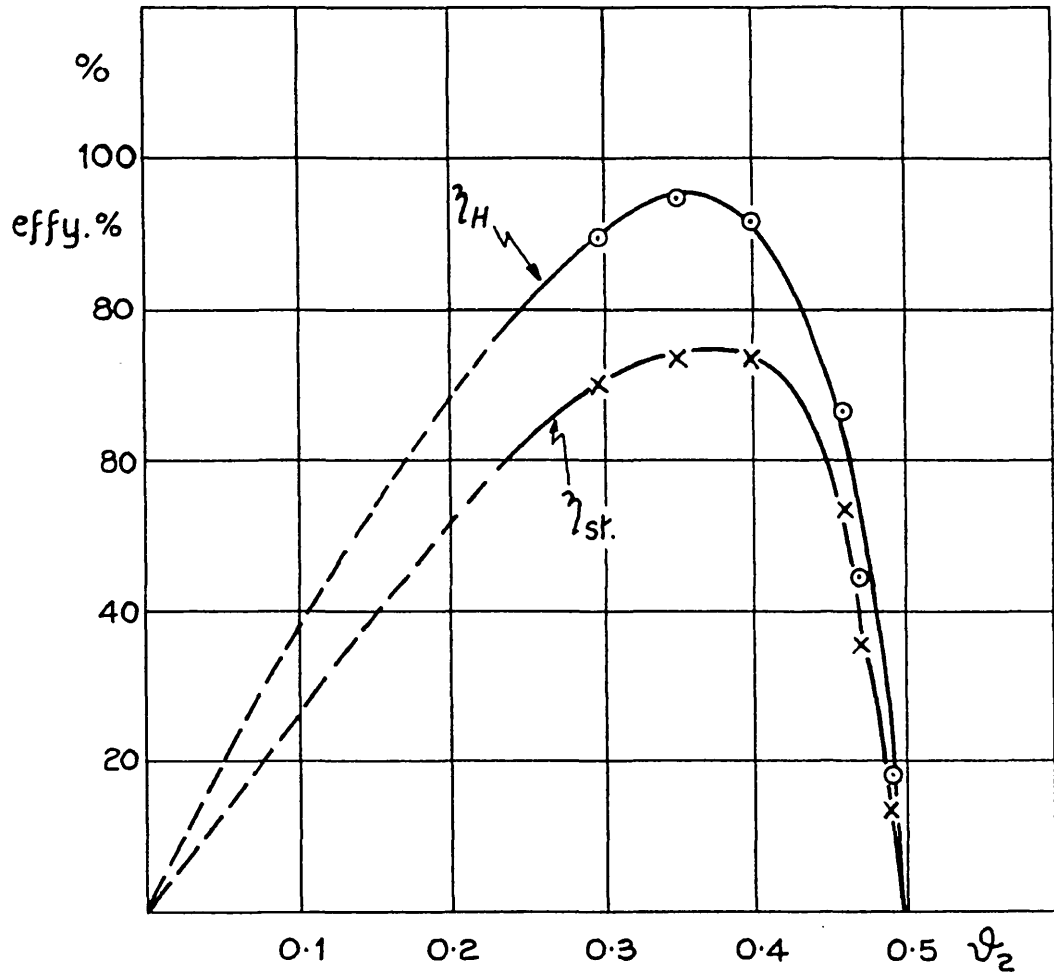


Fig. 2.12 HYDRAULIC EFFICIENCIES, BASED ON THE RISE IN TOTAL ENERGY AS WELL AS STATIC PRESSURE, FOR RADIAL STATION $r/R_0 = 0.69$ (TABLE VII)

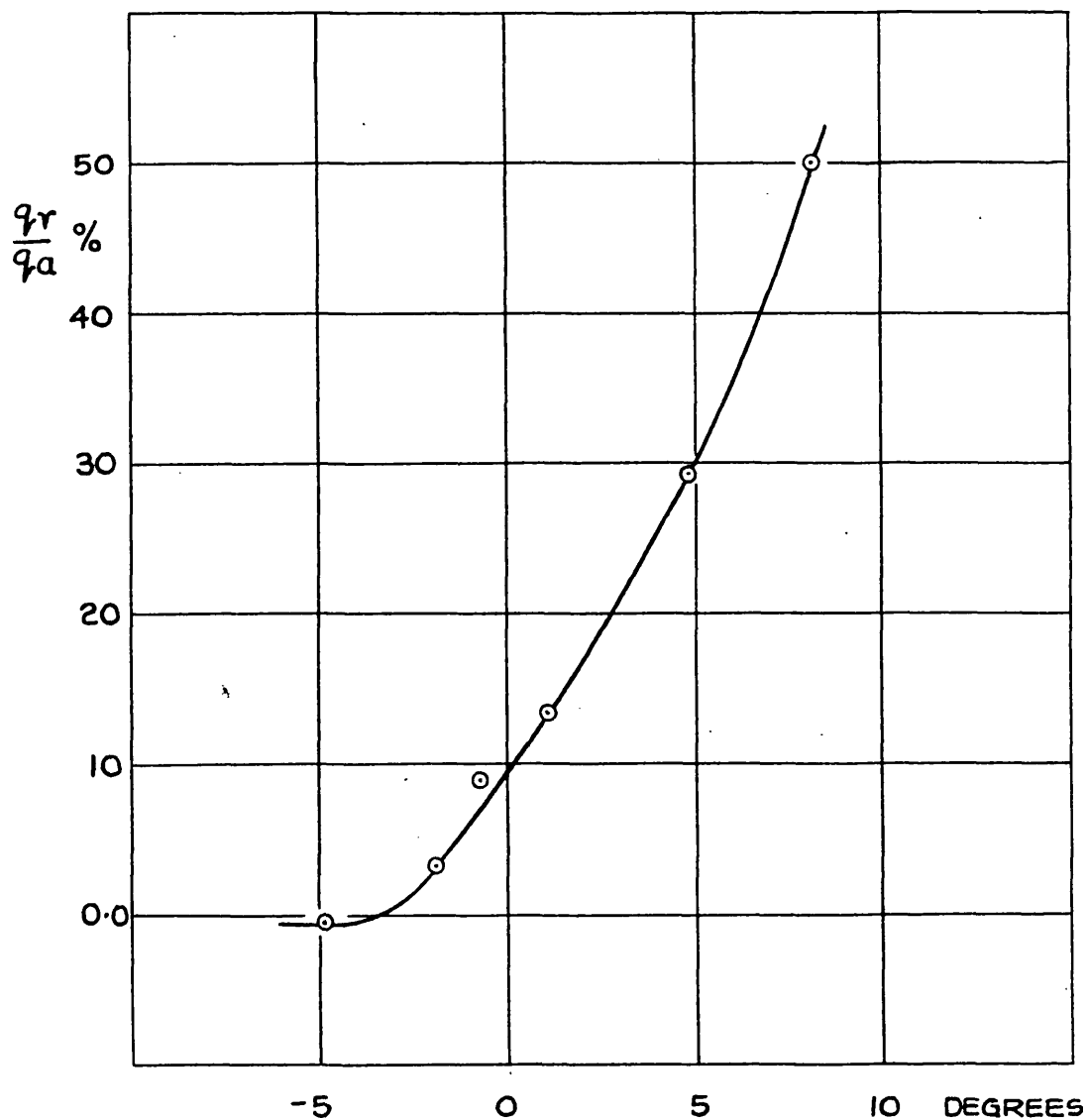


Fig. 2·13 TOTAL RADIAL SECONDARY FLOW AT RADIAL STATION $r/R_0 = 0.63$, PLOTTED AGAINST THE CORRESPONDING ANGLE OF ATTACK. (TABLE IX)

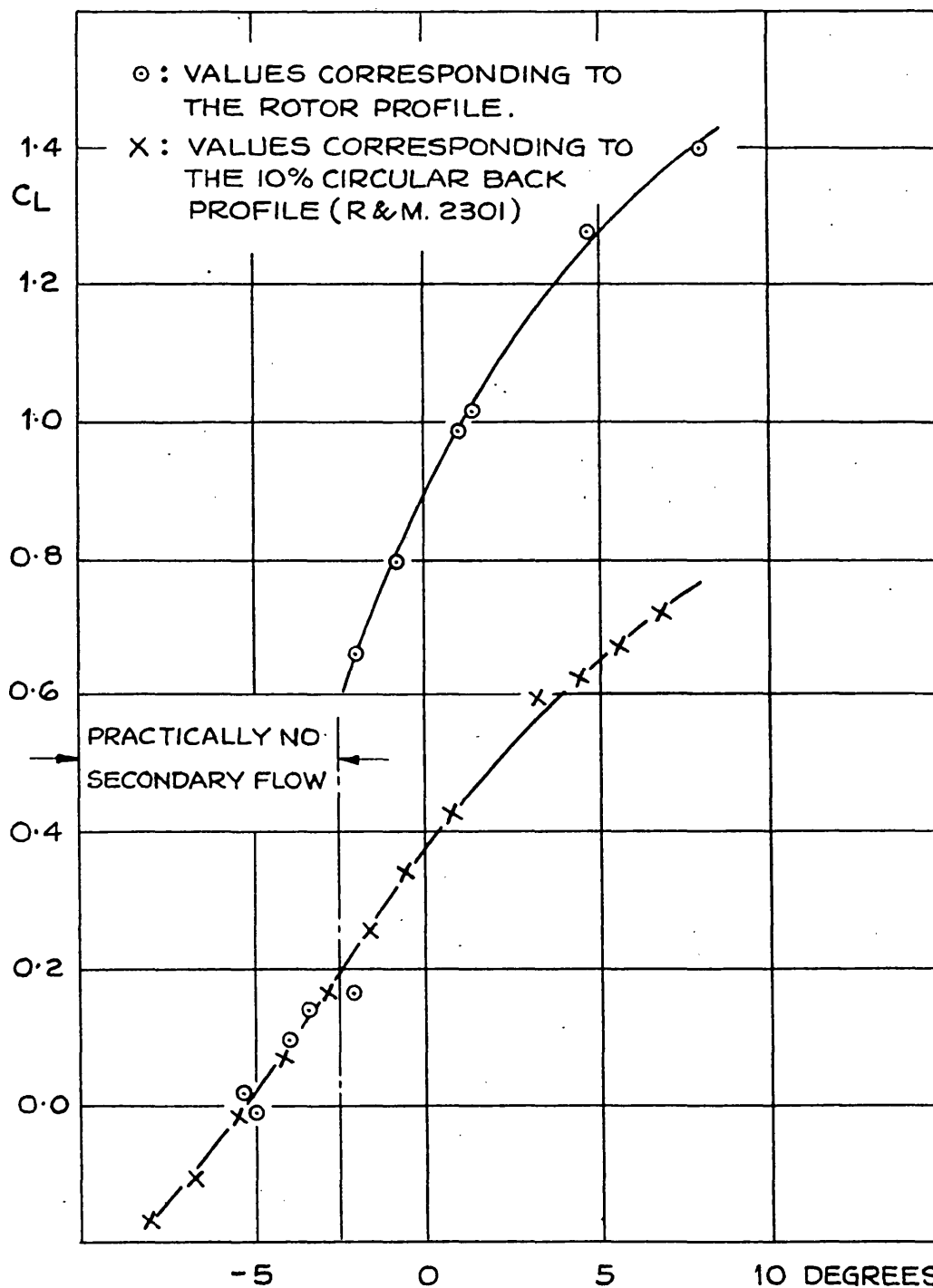
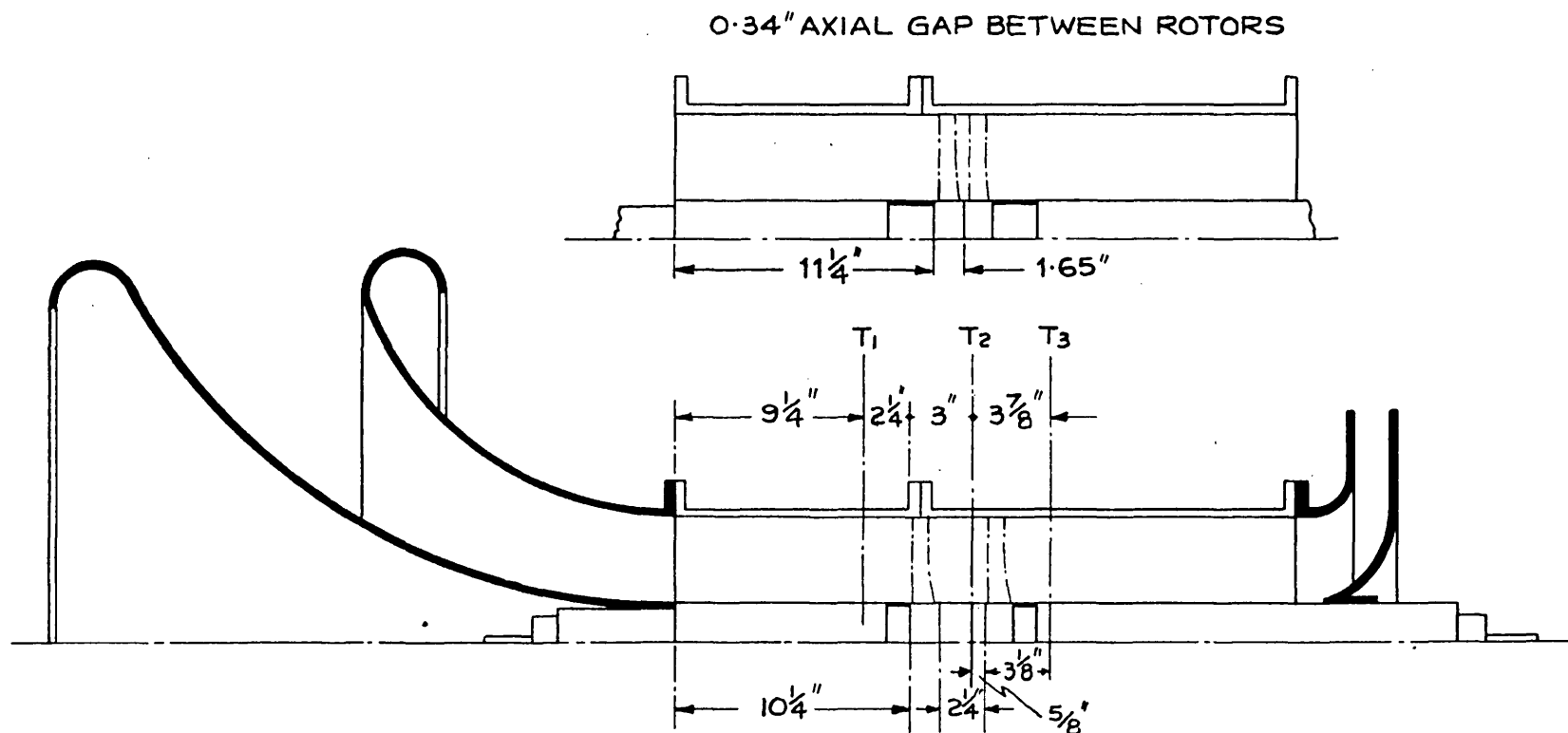


Fig. 2-14 LIFT COEFFICIENTS VS. EFFECTIVE ANGLES OF ATTACK FOR THE ROTOR PROFILE AT $\gamma/R_0 = 0.63$, AND THE 10% CIRCULAR BACK PROFILE (R. & M. 2301) (TABLES VIII & IX)



T₁, T₂ & T₃ : VELOCITY TRAVERSES

PROPELLER HUB WIDTH = 1 5/16"

Fig. 3.1

SET UP FOR TESTING TWO CONTRA-PROPELLERS SPACED
AT 2 1/4" & 5/16"

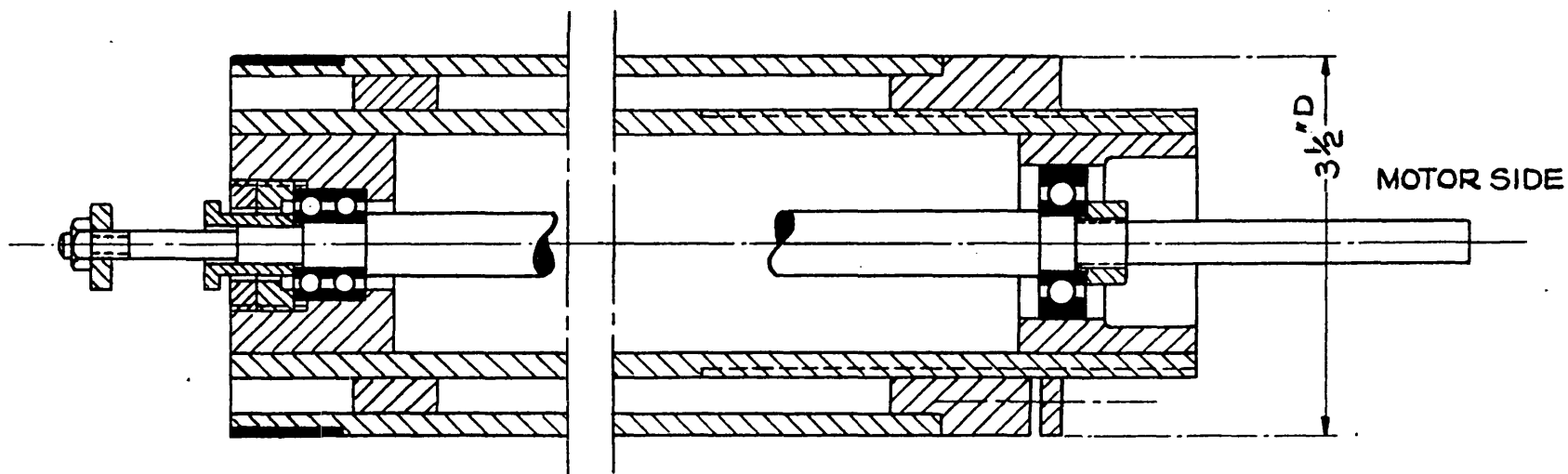


Fig. 3-2 LAYOUT OF ROTOR SHAFT BEARINGS,
ARRANGED FOR AXIAL GAP VARIATION.

Fig. 3.3 (a&b)

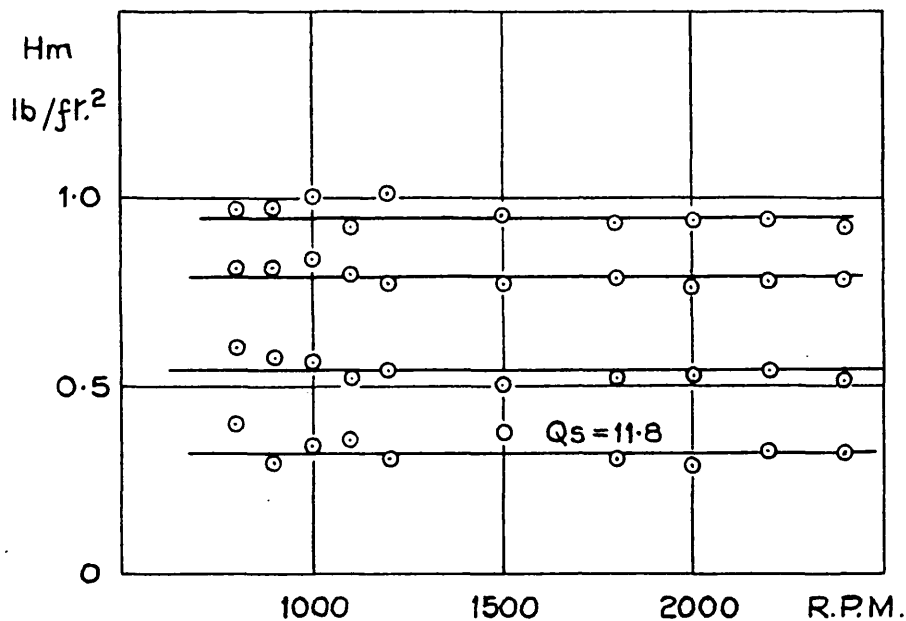


Fig. 3.3 (a) SPECIFIC MANOMETRIC HEAD VS. ROTOR SPEED AT SPECIFIC FLOWS : 9, 10, 11 & 11.8 CUSECS.

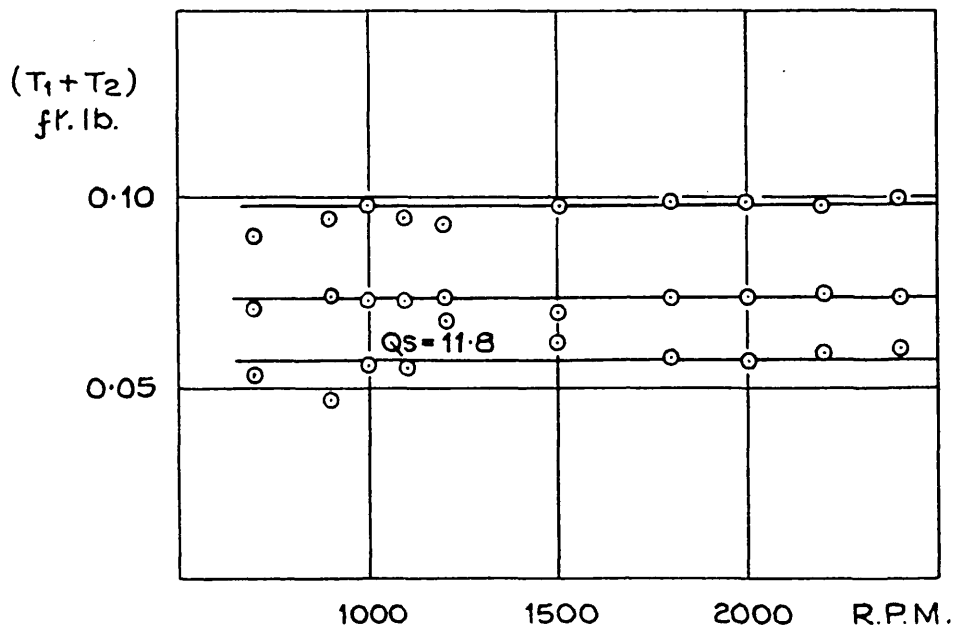


Fig. 3.3 (b) TOTAL SPECIFIC TORQUE VS. ROTOR SPEED AT SPECIFIC FLOWS : 9, 11 & 11.8 CUSECS.

Fig. 3.3 (c)

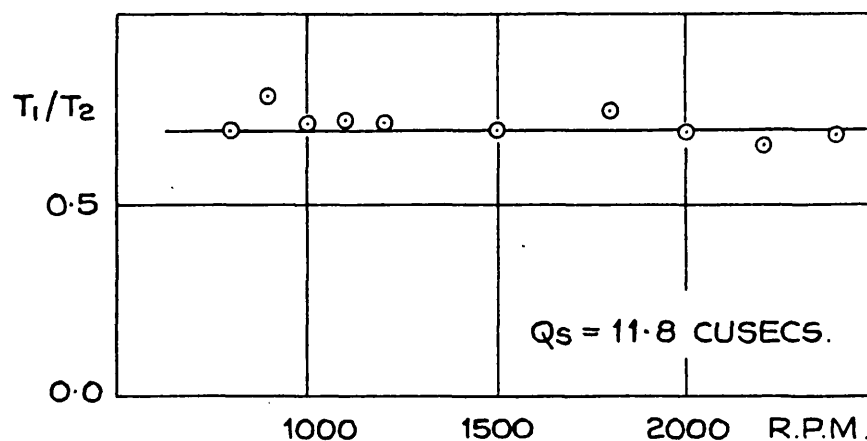
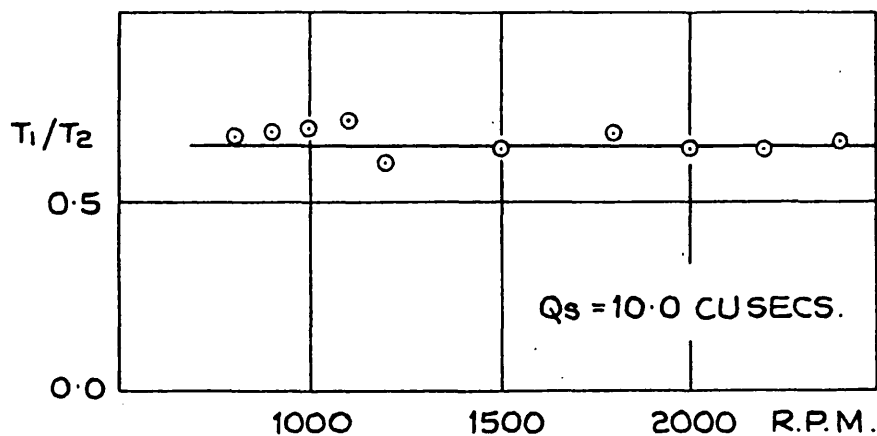
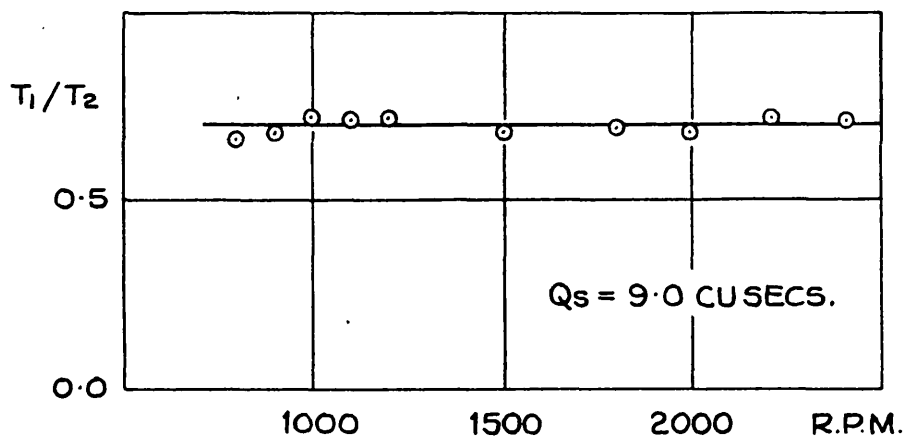


Fig. 3.3 (c) TORQUE RATIOS VS. ROTOR SPEED AT SPECIFIC FLOWS: 9, 10 & 11.8 CU SECS.

Fig. 3.4

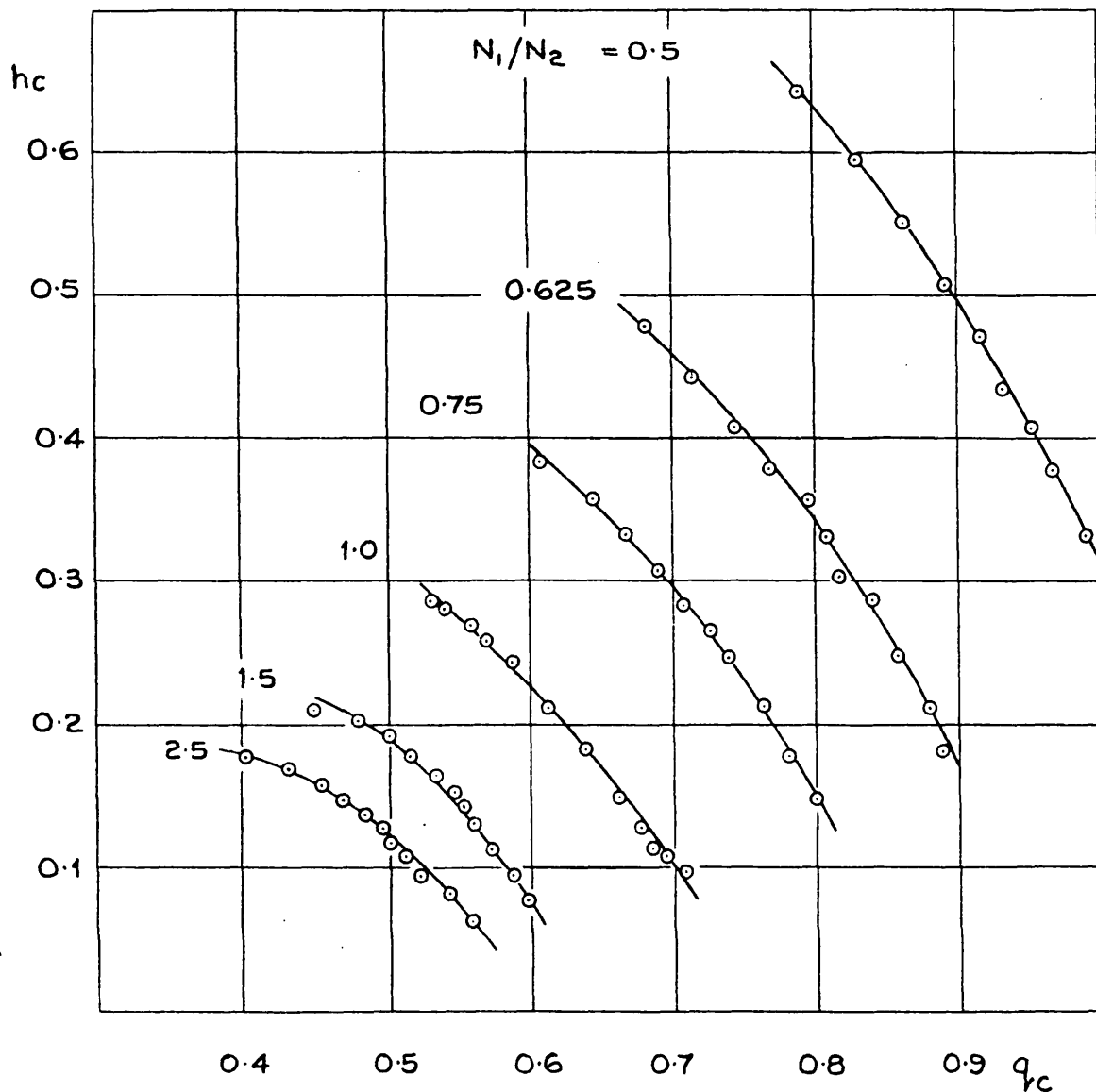


Fig. 3.4 FLOW - HEAD CURVES FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLE II).

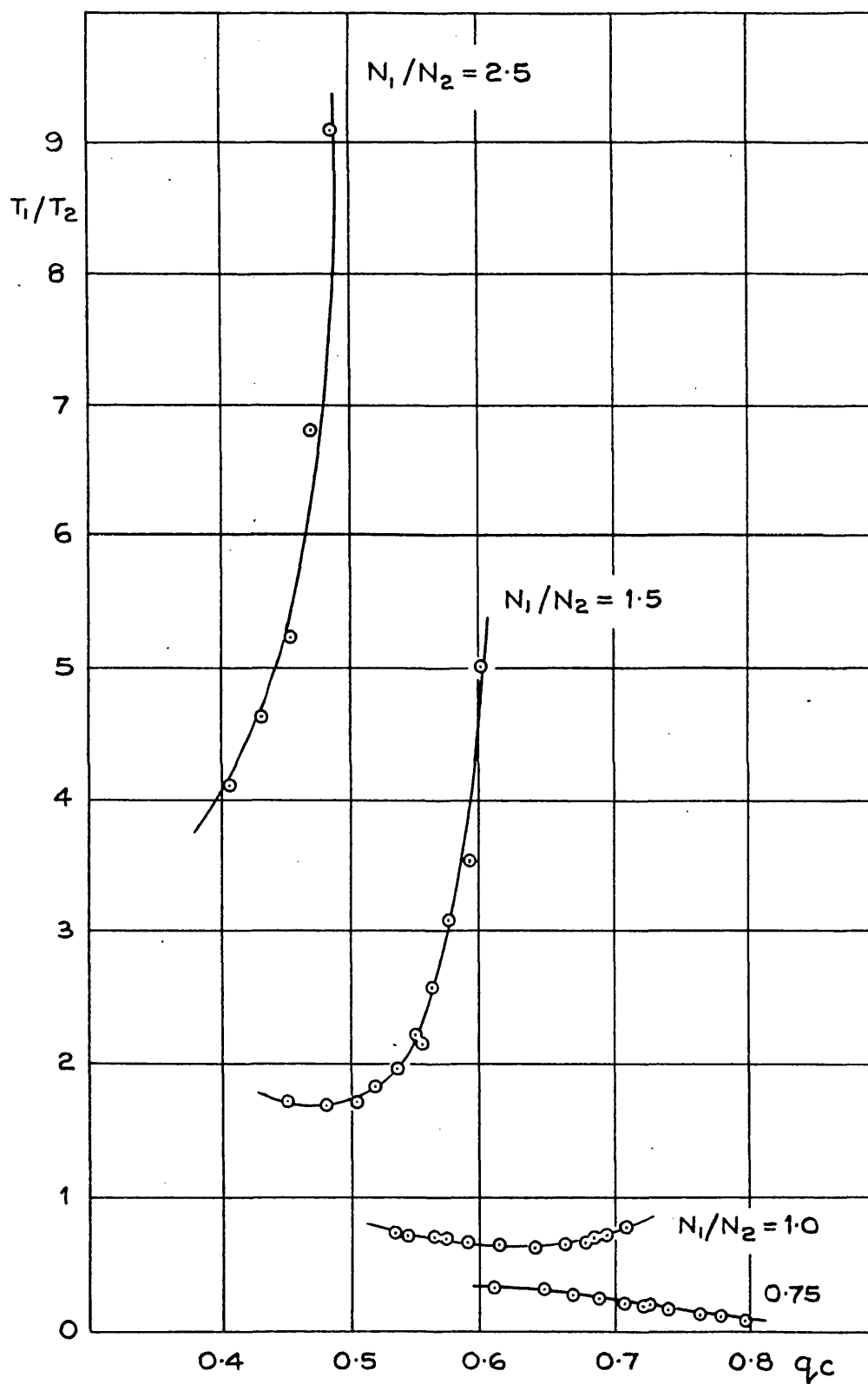


Fig. 3.5 TORQUE RATIO VS. FLOW COEFFICIENT FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLE II)

Fig. 3.6

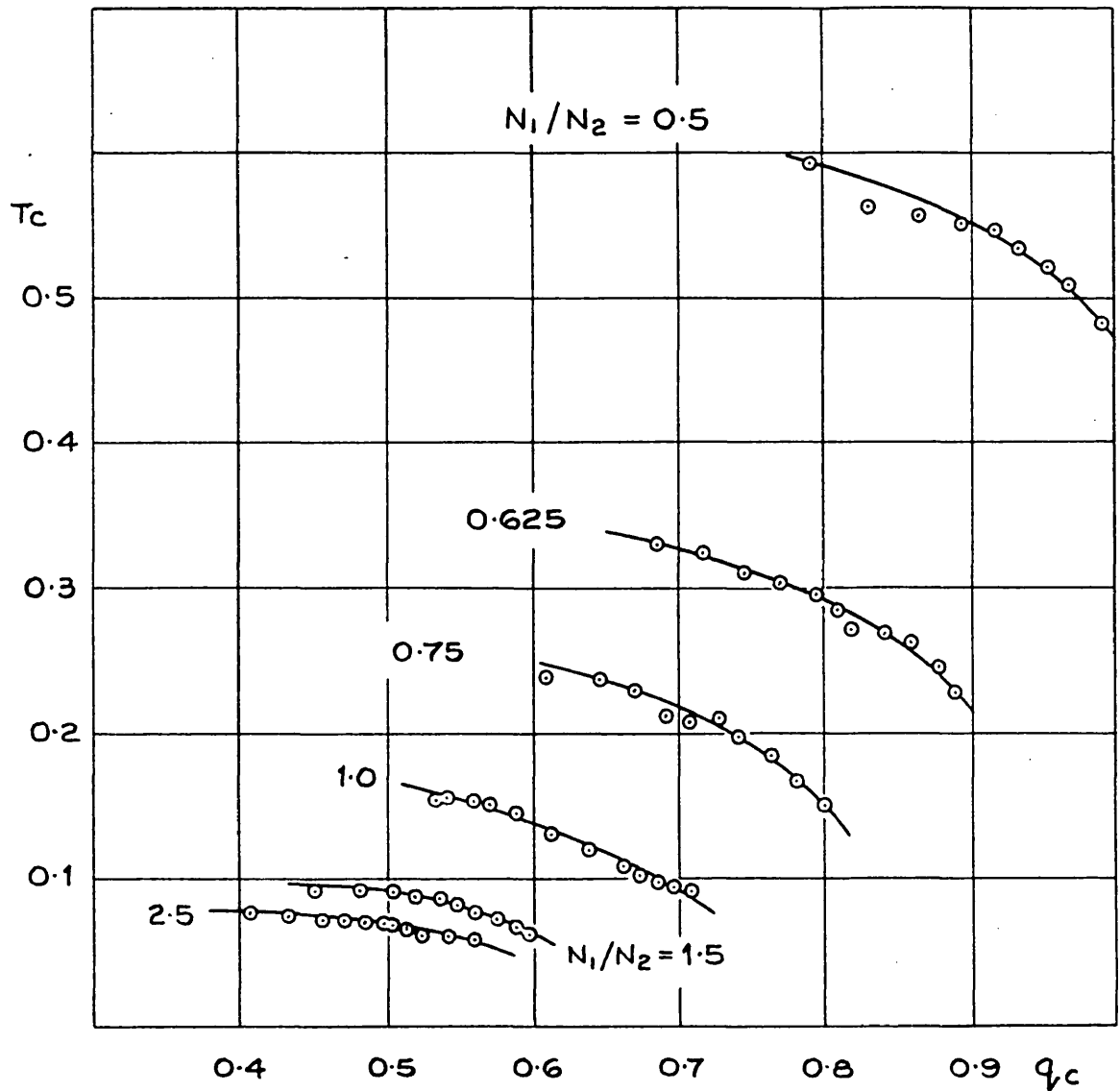


Fig. 3.6 TOTAL TORQUE COEFFICIENT, REFERRED TO THE U.S. ROTOR, VS. FLOW COEFFICIENT FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLE II).

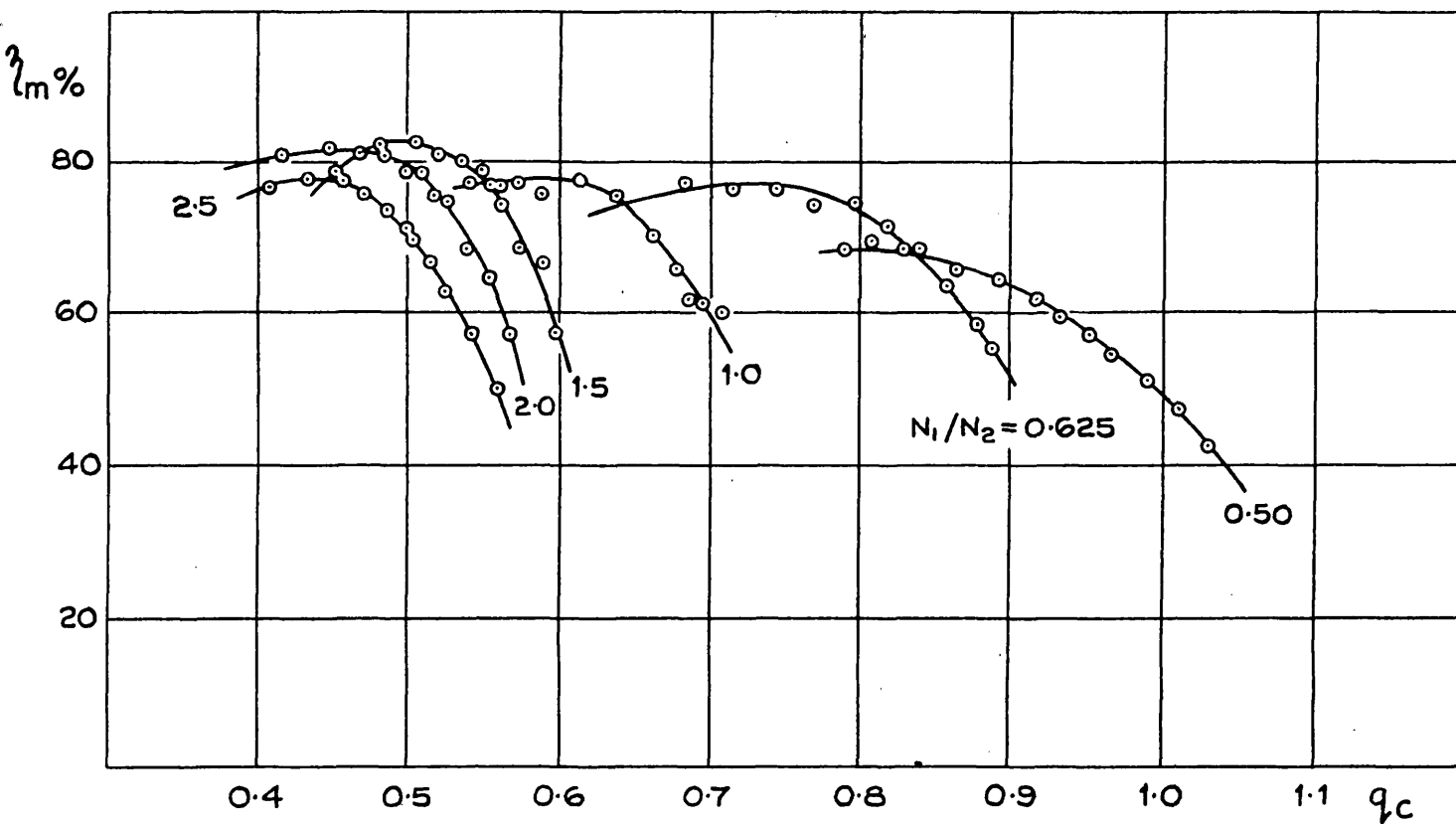


Fig. 3.7 MECHANICAL EFFICIENCY VS. FLOW COEFFICIENT FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLE II).

Fig. 3.8

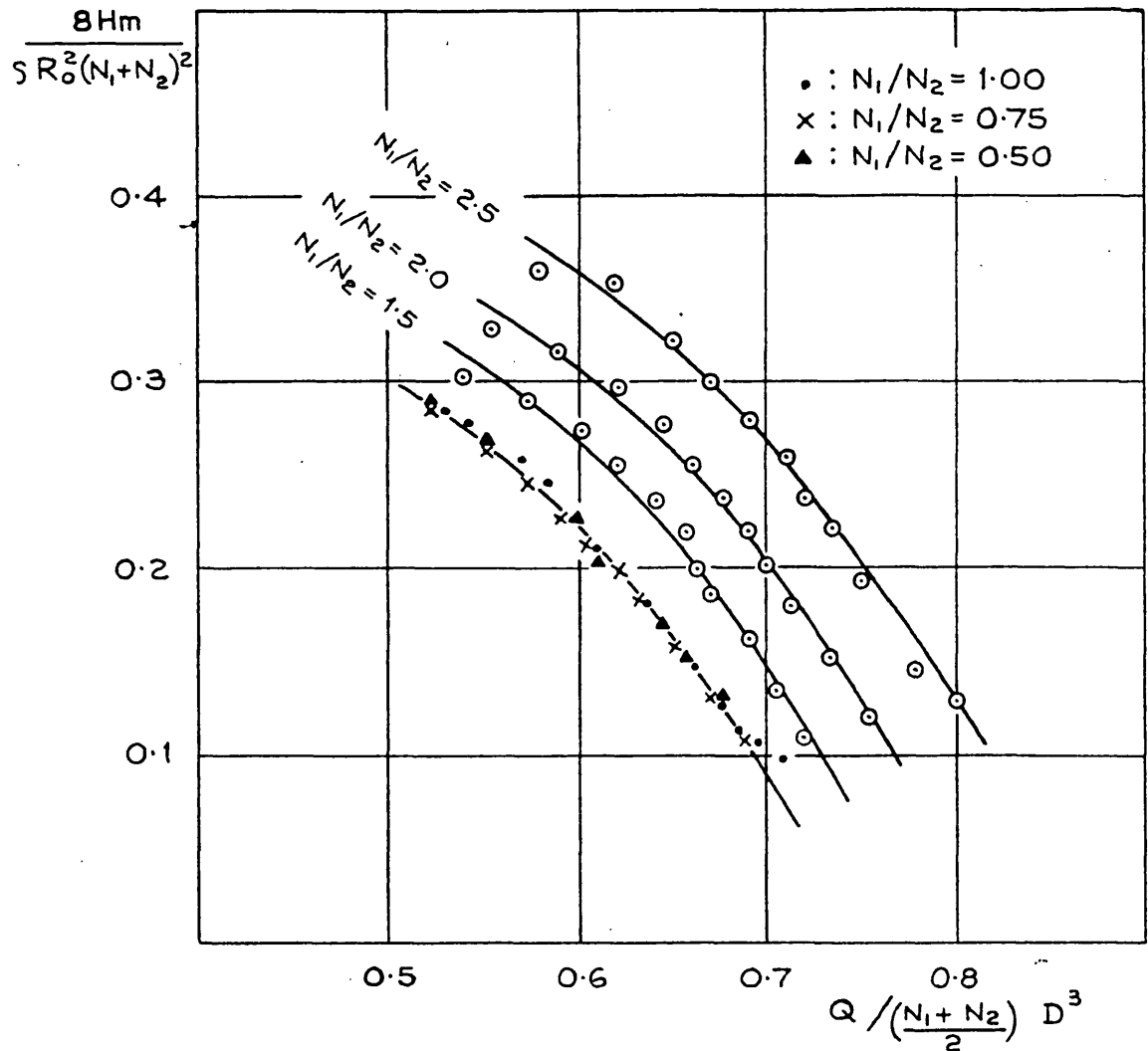


Fig. 3.8 MANOMETRIC HEAD AND DISCHARGE COEFFICIENTS IN TERMS OF MEAN ROTATIONAL SPEED, FOR CONTRA-ROTATORS OPERATING AT DIFFERENT ROTATIONAL SPEED RATIOS.

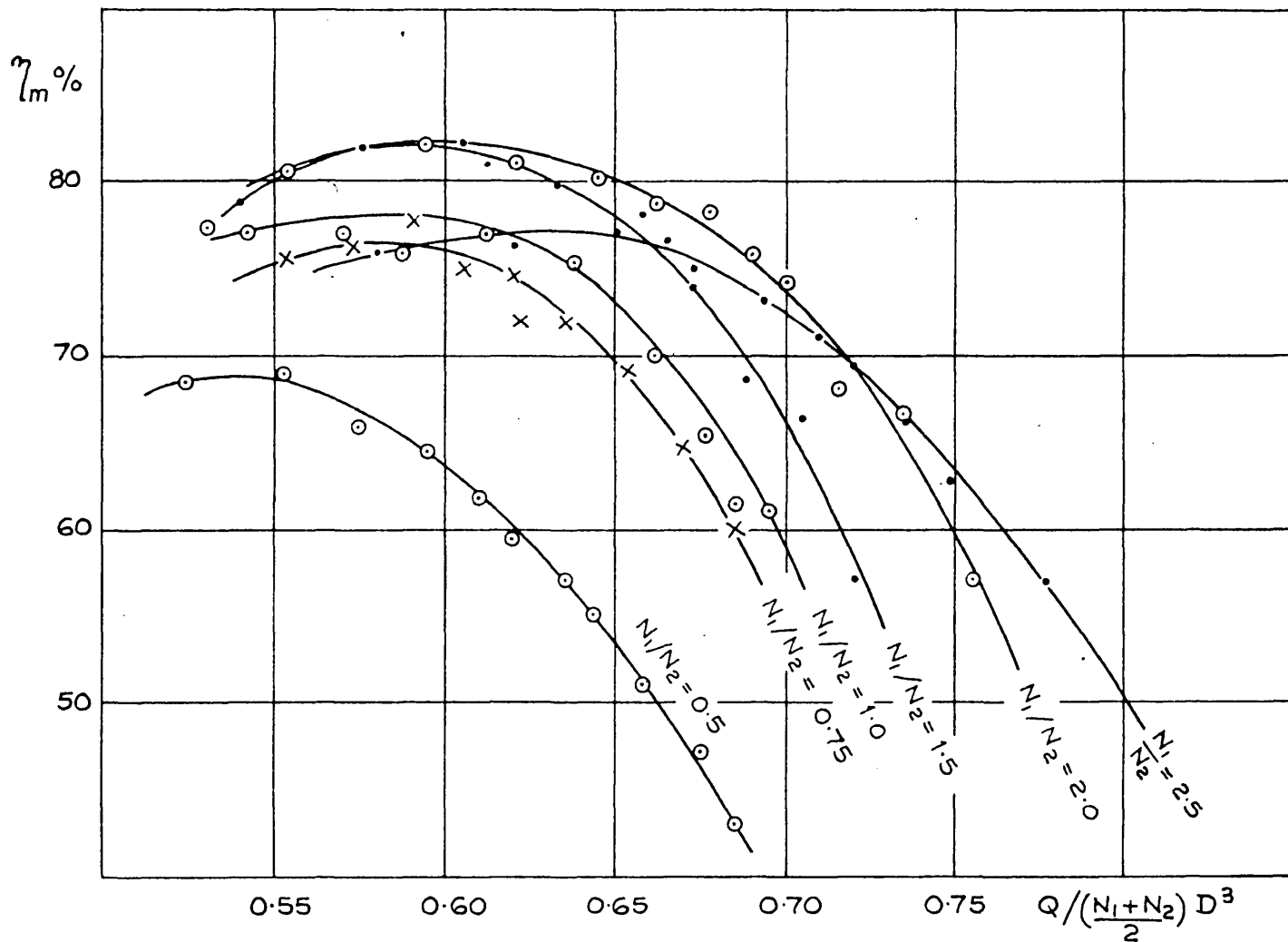


Fig. 3.9 EFFICIENCY CURVES FOR THE CONTRA-ROTATORS OPERATING AT VARIOUS ROTATIONAL SPEED RATIOS.

Fig. 3-10 (a)

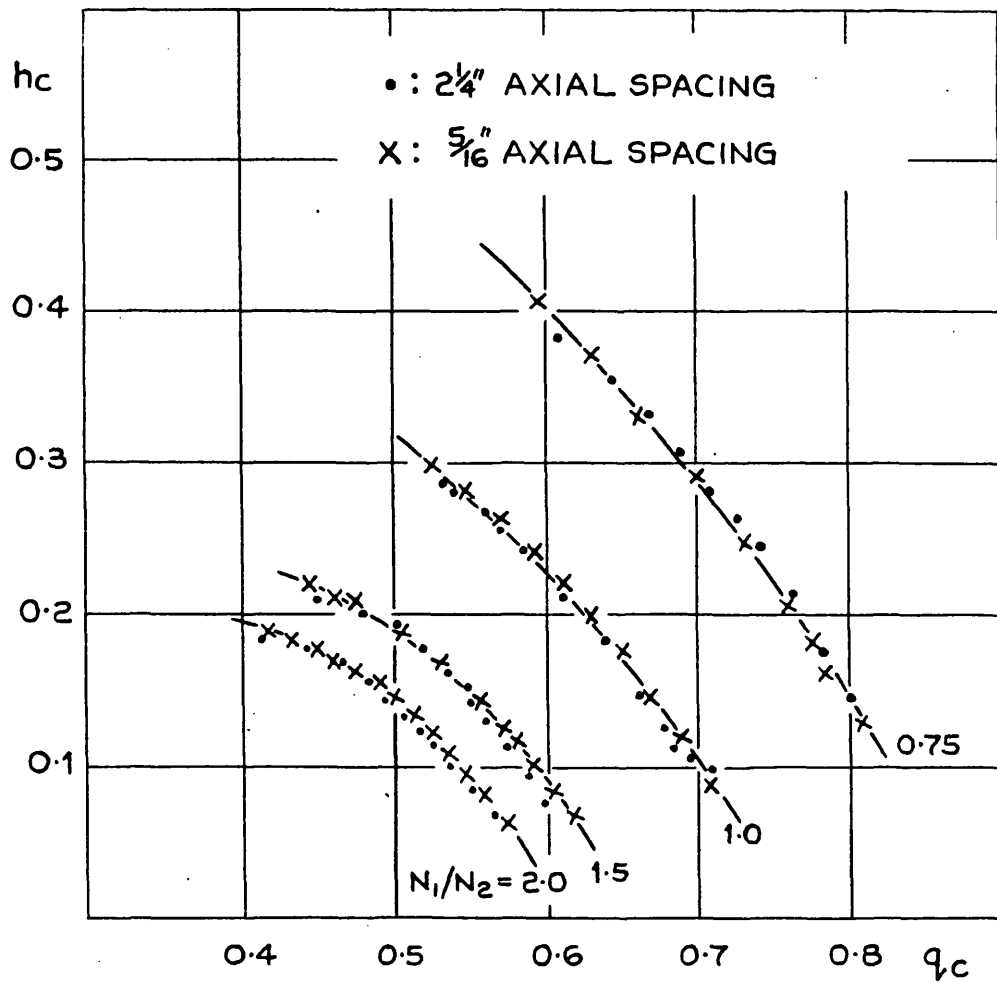


Fig.3-10(a) FLOW-HEAD CHARACTERISTIC CURVES WITH $2\frac{1}{4}"$ & $\frac{5}{16}"$ AXIAL SPACING BETWEEN THE ROTORS, FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLES II, & III).

Fig. 3·10(b)

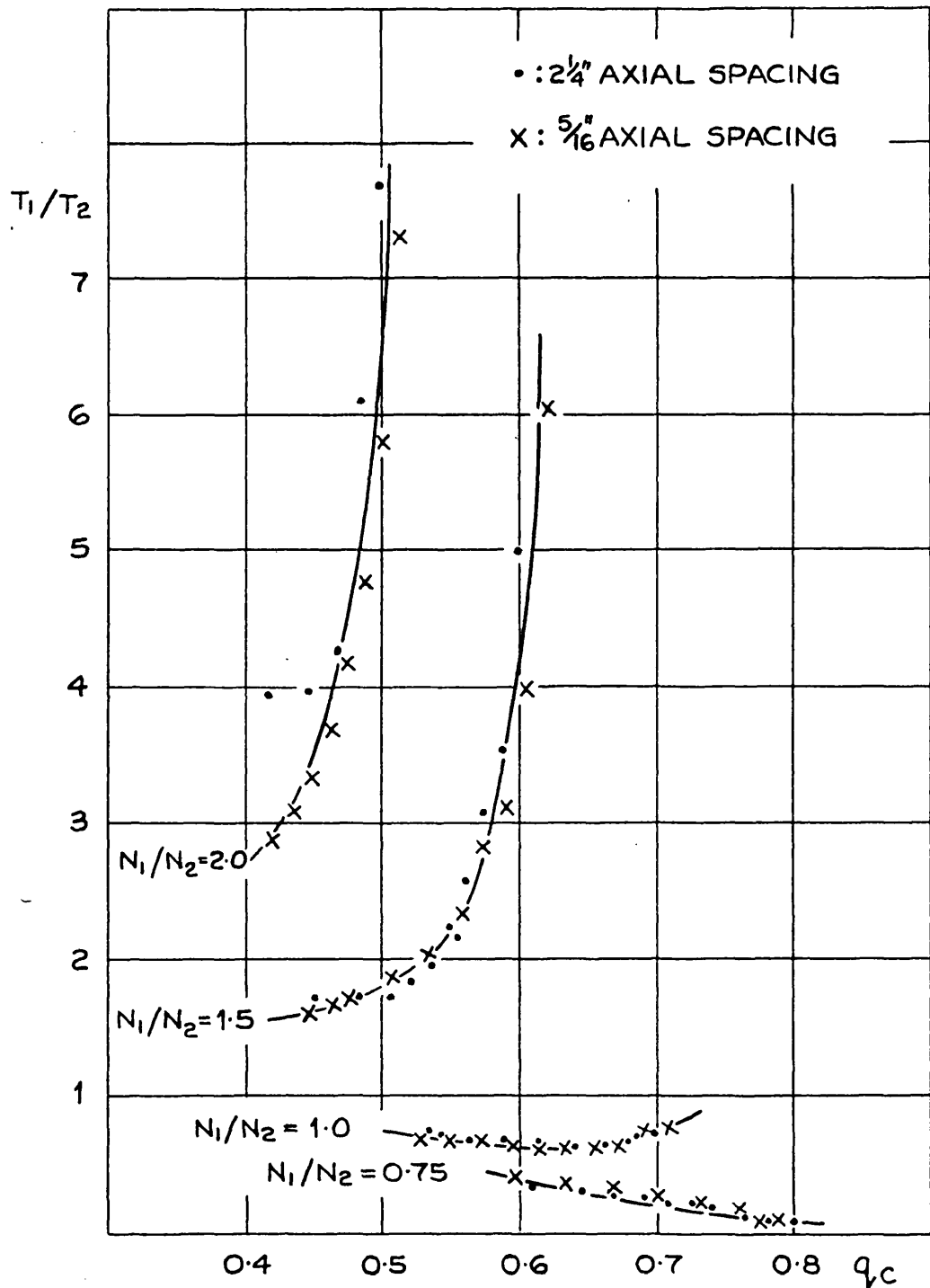


Fig. 3·10(b) TORQUE - RATIO VS. FLOW COEFFICIENT WITH $2\frac{1}{4}$ " & $\frac{5}{16}$ " AXIAL SPACING BETWEEN THE ROTORS, FOR DIFFERENT ROTATIONAL SPEED RATIOS. (TABLES II, & III).

Fig. 3·10 (c & d)

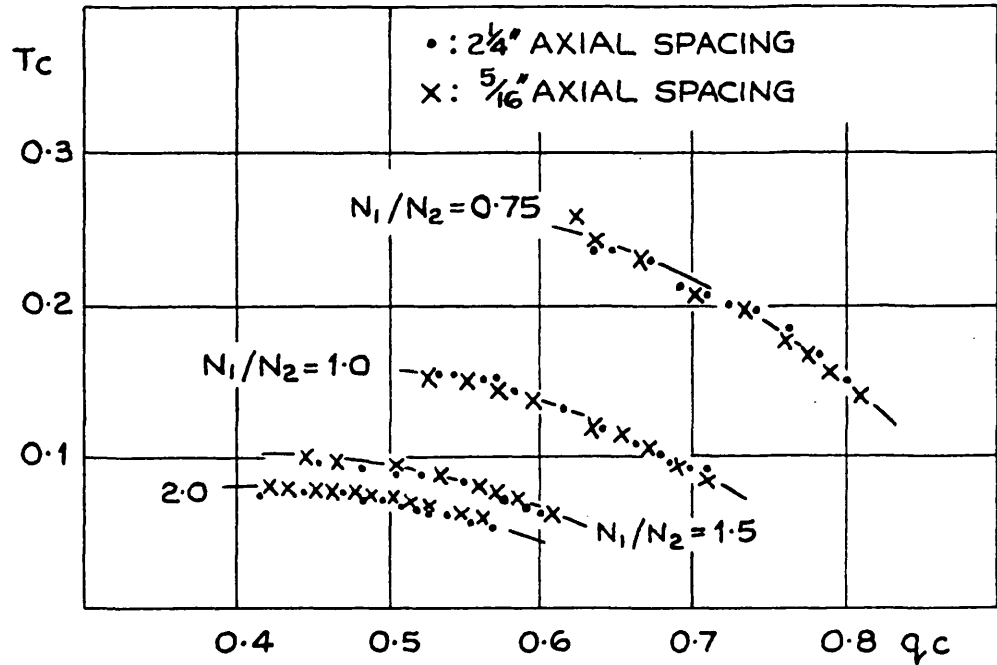


Fig. 3·10(c) TORQUE COEFFICIENT V.S. FLOW COEFFICIENT WITH $2\frac{1}{4}"$ & $\frac{5}{16}"$ AXIAL SPACING

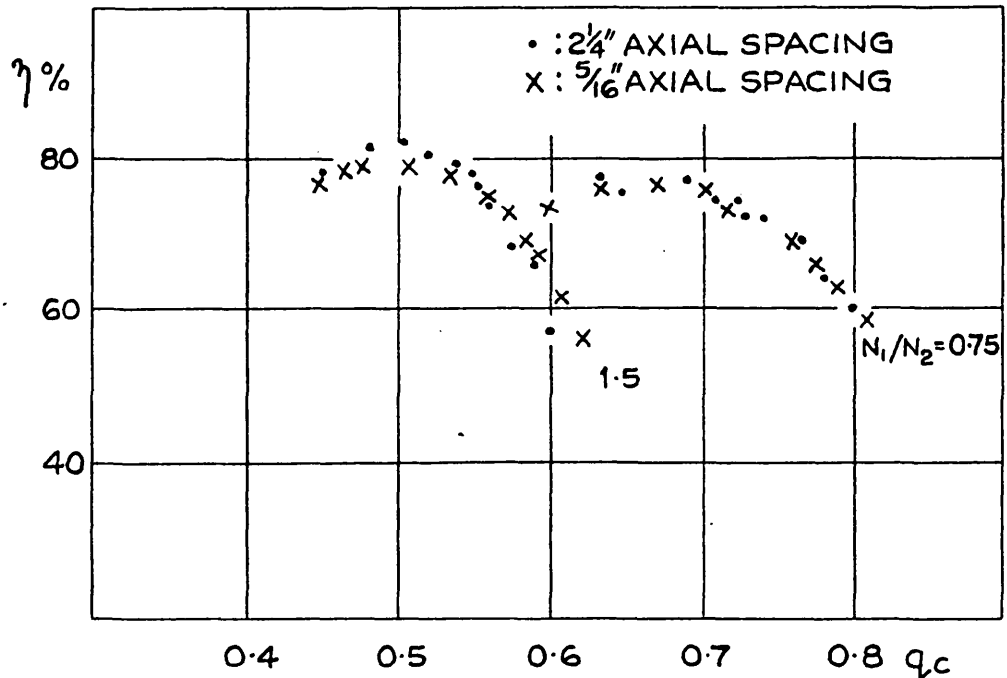


Fig. 3·10(d) EFFICIENCY-FLOW COEFFICIENT CURVES WITH $2\frac{1}{4}"$ & $\frac{5}{16}"$ AXIAL SPACING. (TABLES II, & III.)

Fig. 3.10 (d)

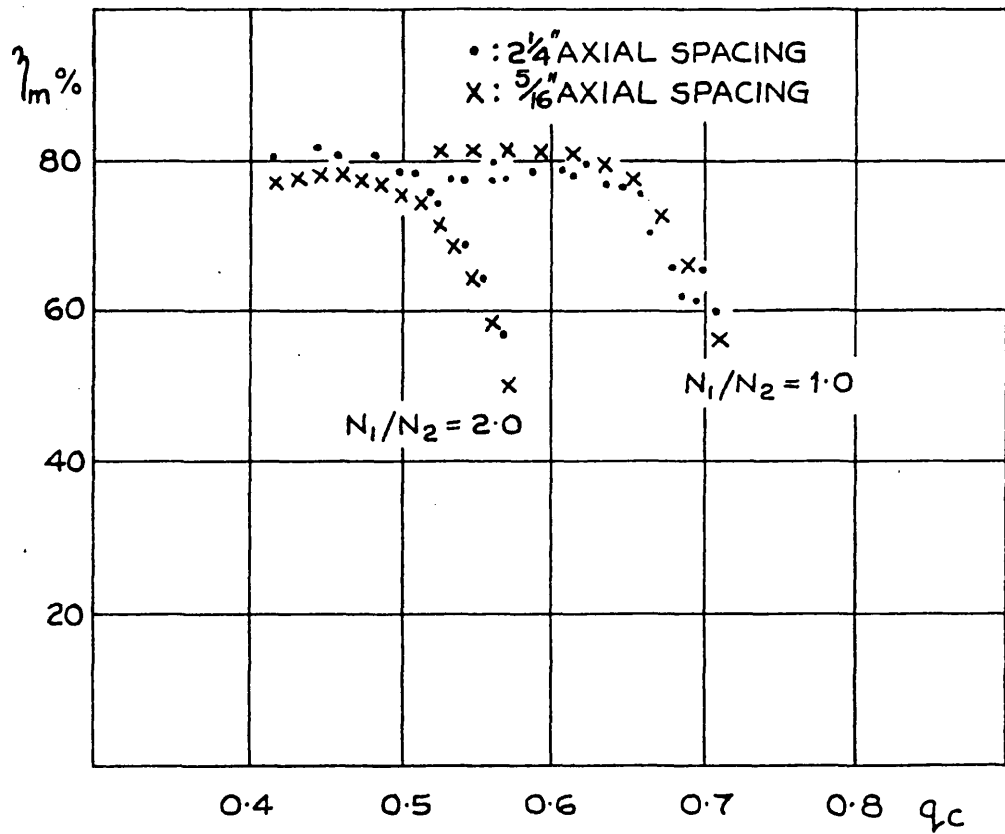


Fig. 3.10(d) EFFICIENCY-FLOW COEFFICIENT CURVES WITH $2\frac{1}{4}"$ & $\frac{5}{16}"$ AXIAL SPACING. (TABLES II, & III).

Fig. 3.11 (a)

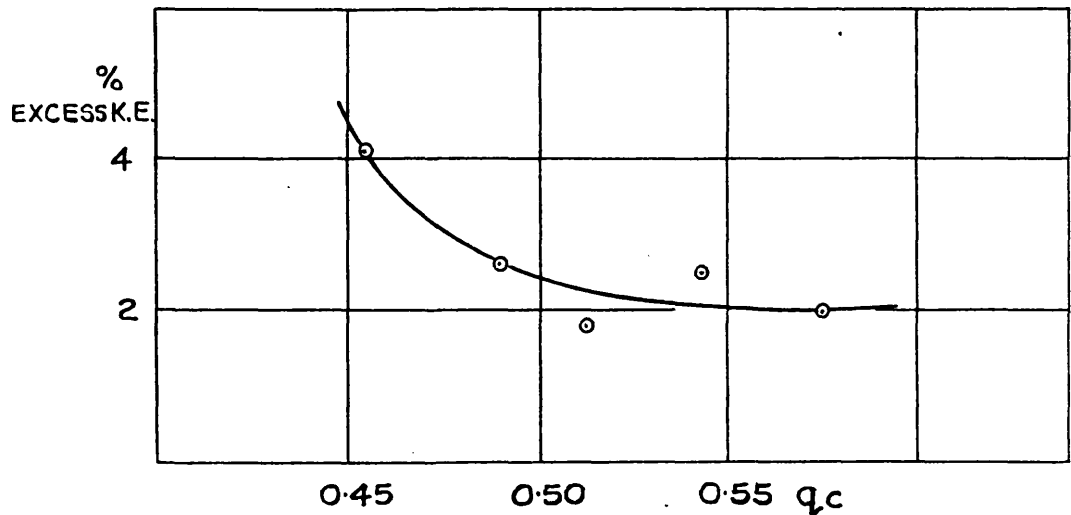


Fig. 3.11(a) EXCESS K.E., AS A PERCENT OF TOTAL ENERGY IMPARTED, FOR VARYING FLOW COEFFICIENTS AT $N_1/N_2 = 1.50$

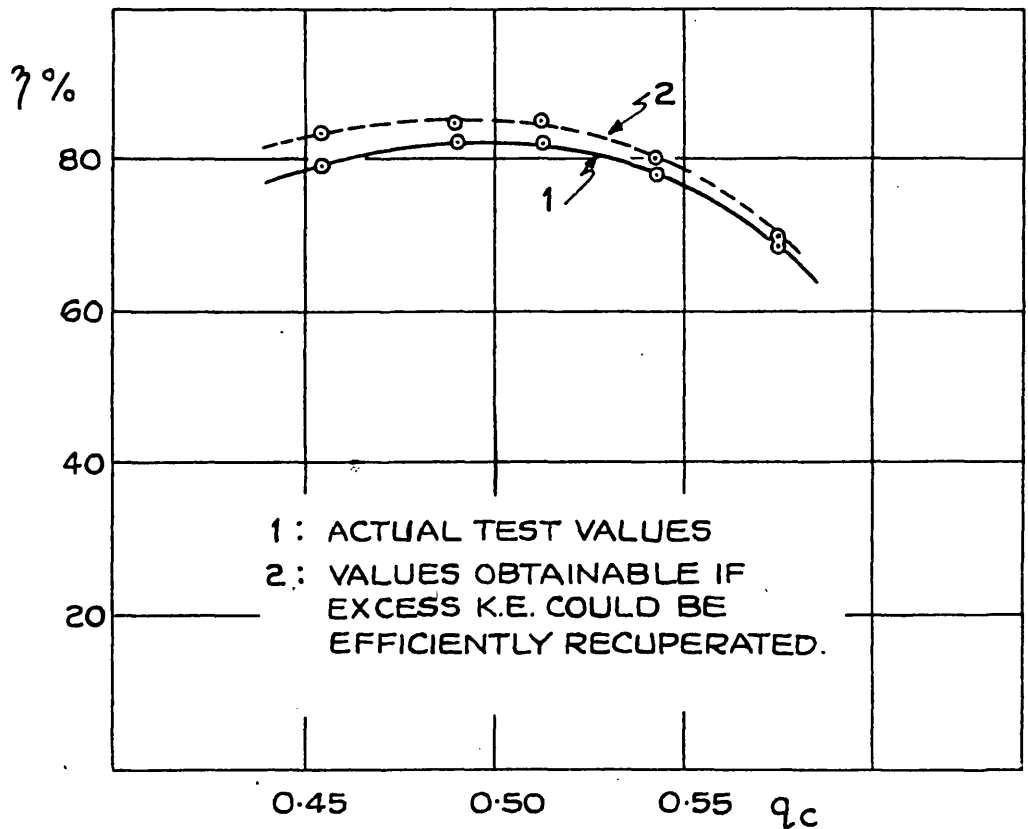


Fig. 3.11 (a) ACTUAL AND MODIFIED MECHANICAL EFFICIENCIES FOR $N_1/N_2 = 1.50$

Fig. 3.11 (b)

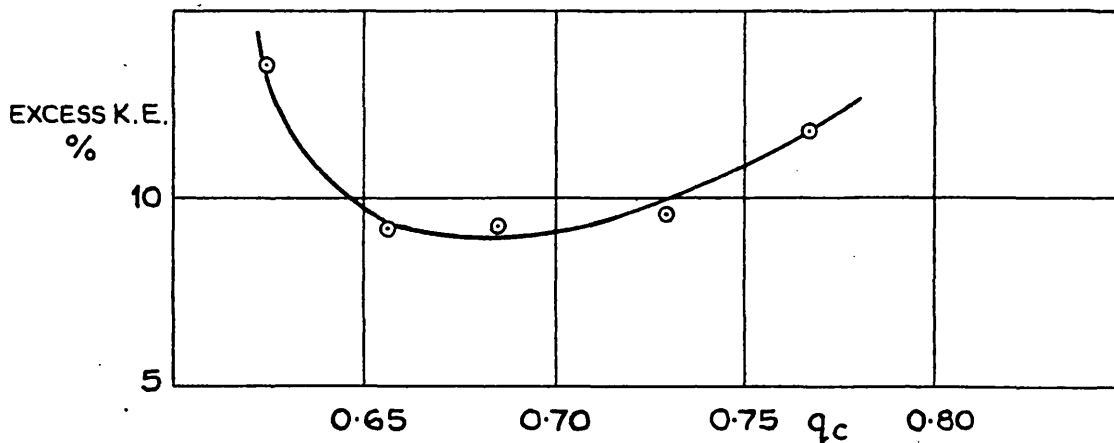


Fig. 3.11(b) EXCESS K.E., AS A PERCENT OF TOTAL ENERGY IMPARTED, FOR VARYING FLOW COEFFICIENTS, AT $N_1/N_2 = 0.75$

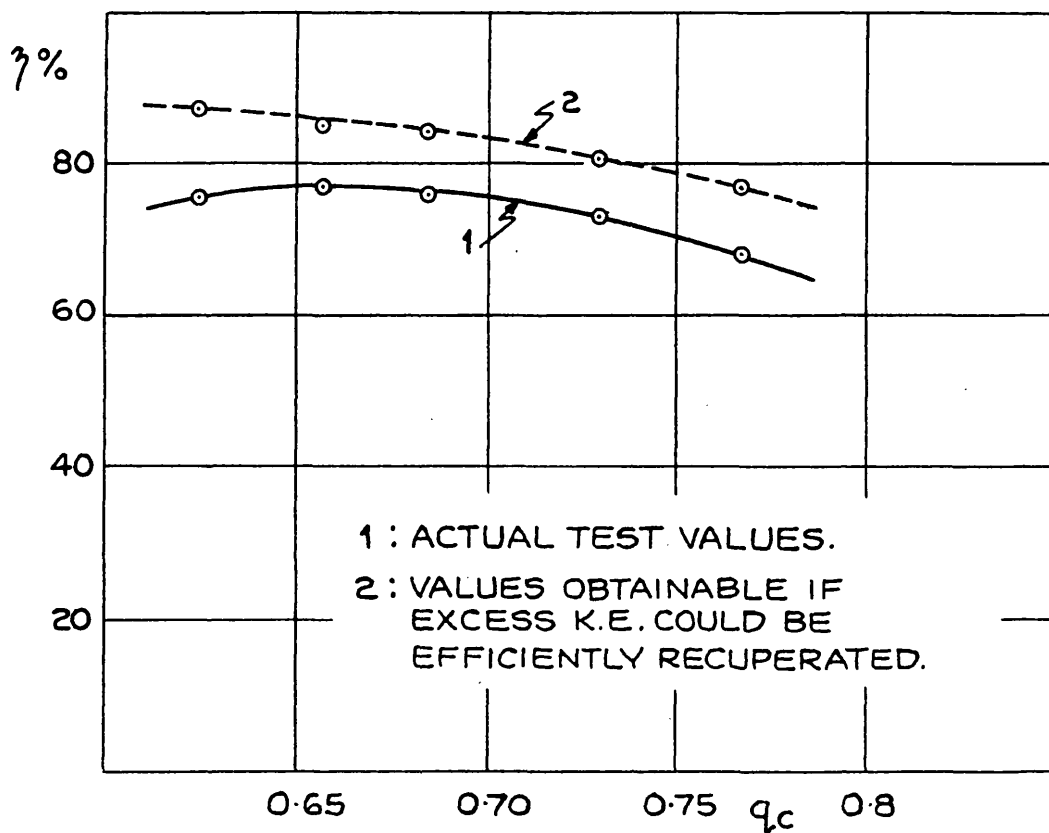


Fig. 3.11(b) ACTUAL AND MODIFIED MECHANICAL EFFICIENCIES FOR $N_1/N_2 = 0.75$

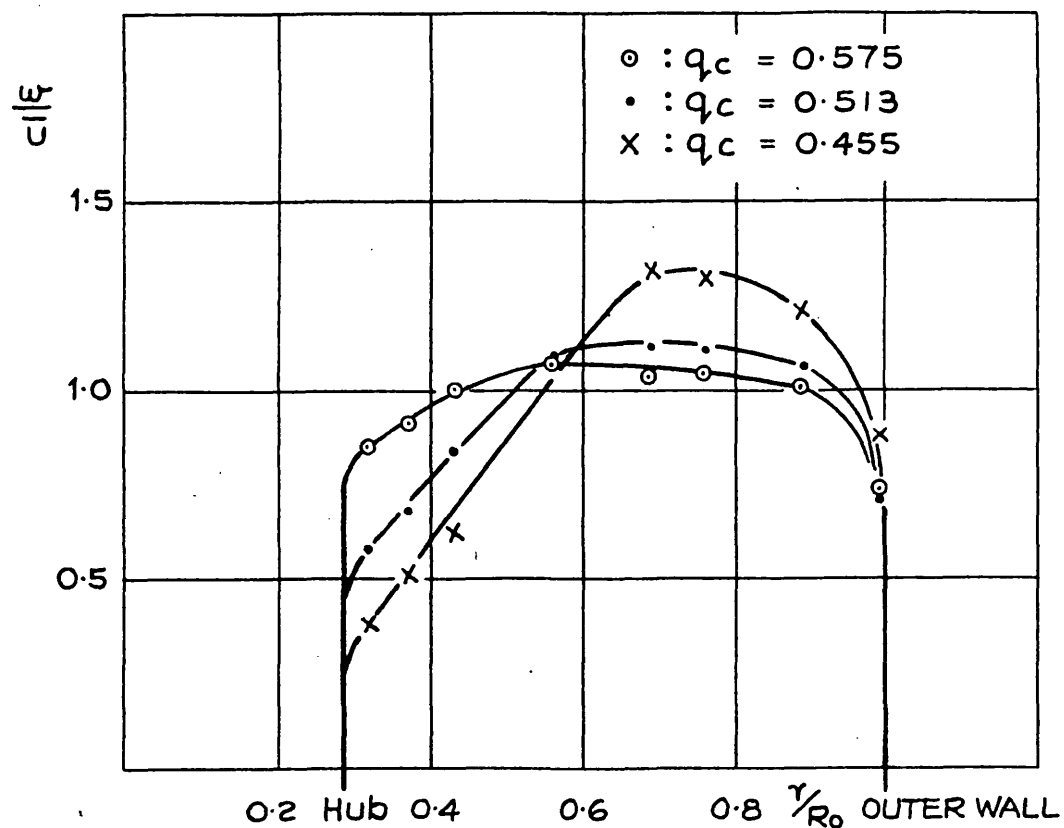


Fig. 3.12 AXIAL VELOCITY DISTRIBUTION DOWNSTREAM OF THE 2ND ROTOR OF A CONTRA ROTATING UNIT OPERATING AT $N_1/N_2 = 1.50$

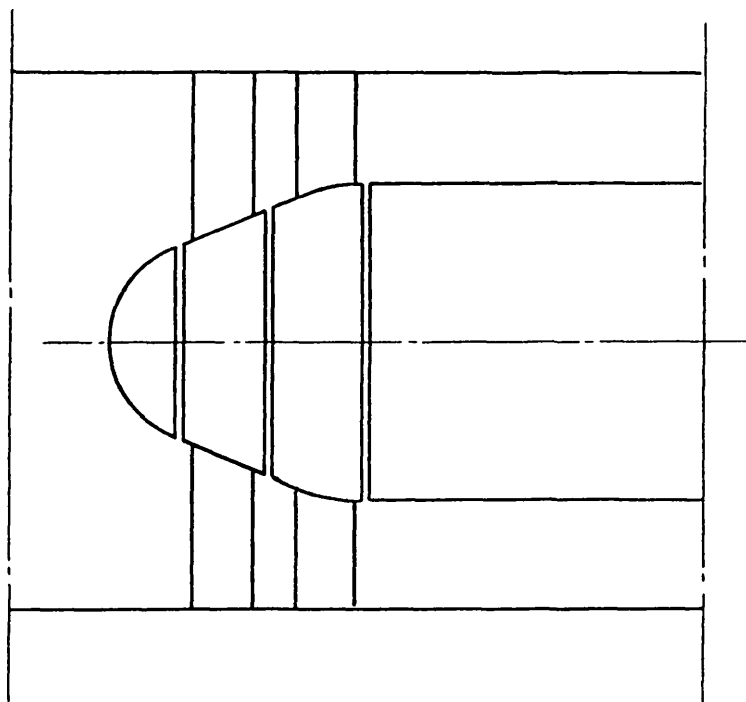


Fig. 3·13 PROPOSED ARRANGEMENT FOR
A CONTRA ROTATING UNIT.

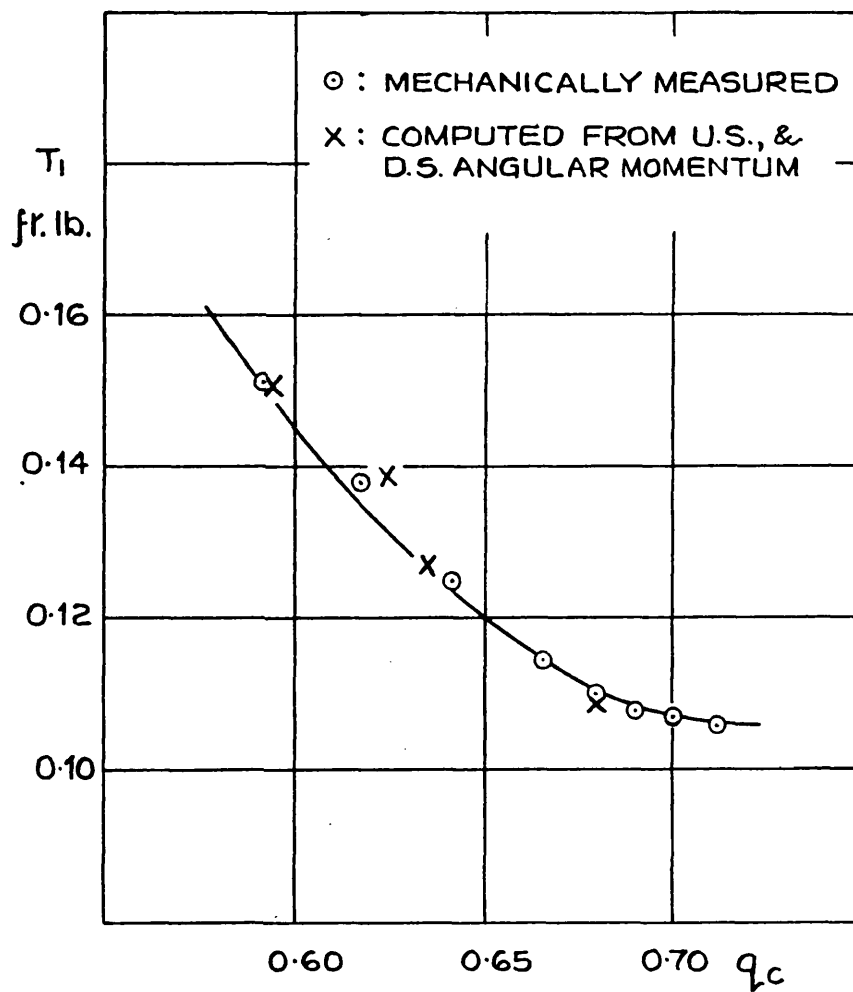


Fig. 3.14(a) UPSTREAM ROTOR TORQUE
AGAINST FLOW COEFFICIENT.

Fig. 3·14 (b)

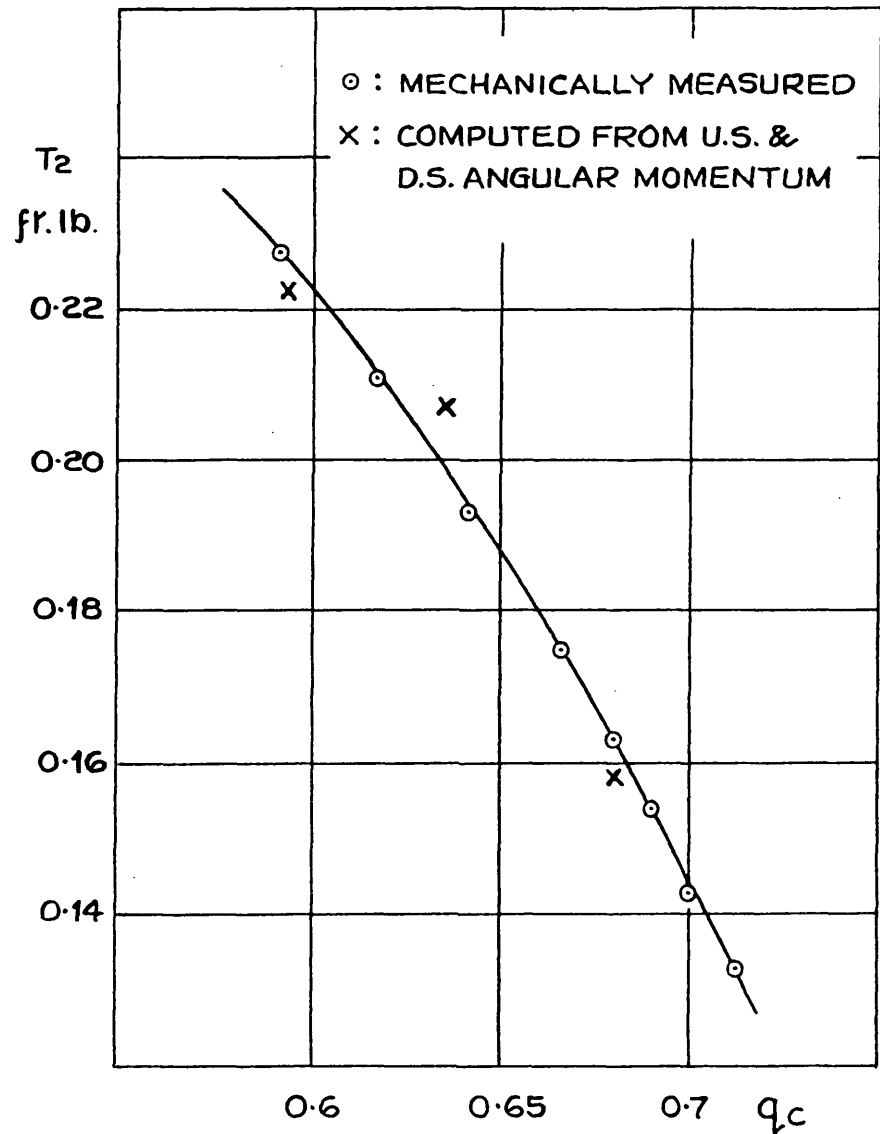


Fig. 3·14 (b) DOWNSTREAM ROTOR TORQUE
AGAINST FLOW COEFFICIENT

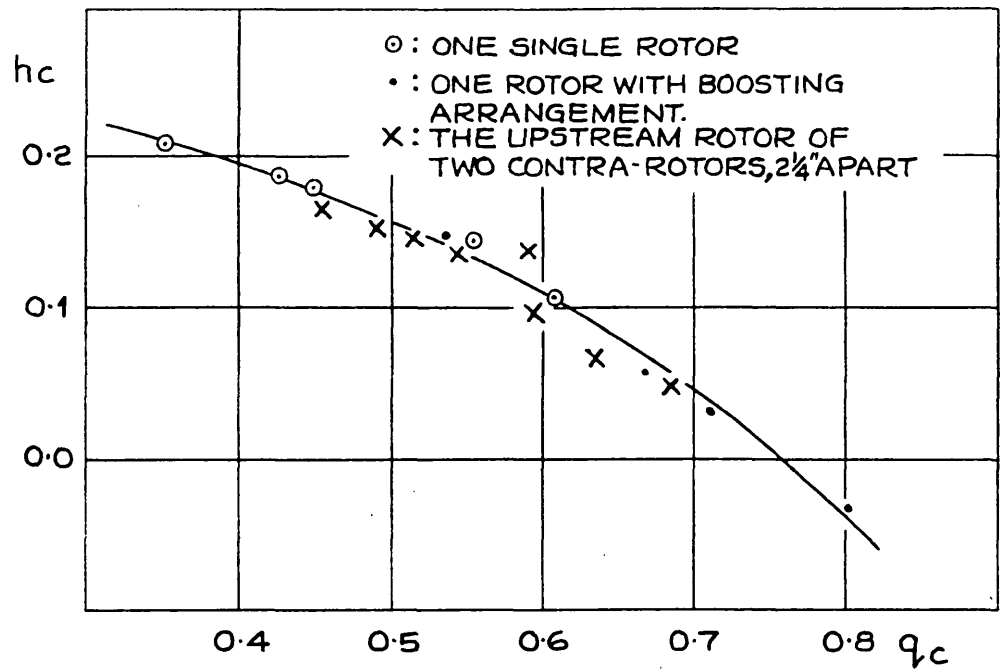


Fig. 3.15 HEAD-FLOW COEFFICIENTS CURVE, FOR ONE ROTOR FITTED WITH A 100% EFFICIENT RECUPERATOR.

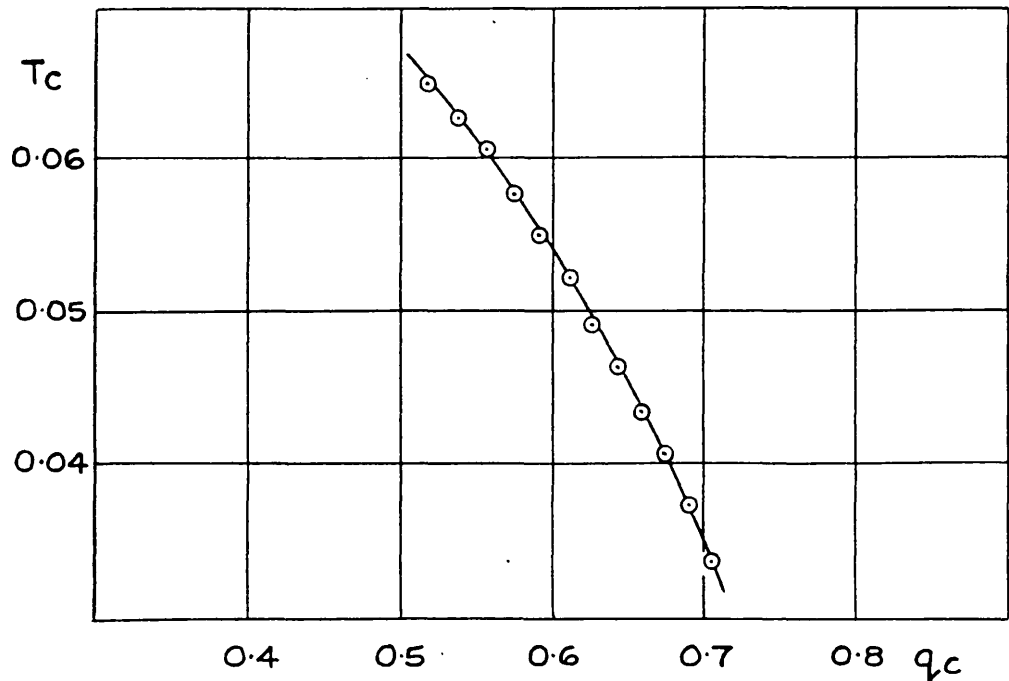


Fig. 3.16 TORQUE COEFFICIENT AT VARYING FLOW COEFFICIENTS FOR A SINGLE ROTOR. (MECHANICALLY MEASURED)

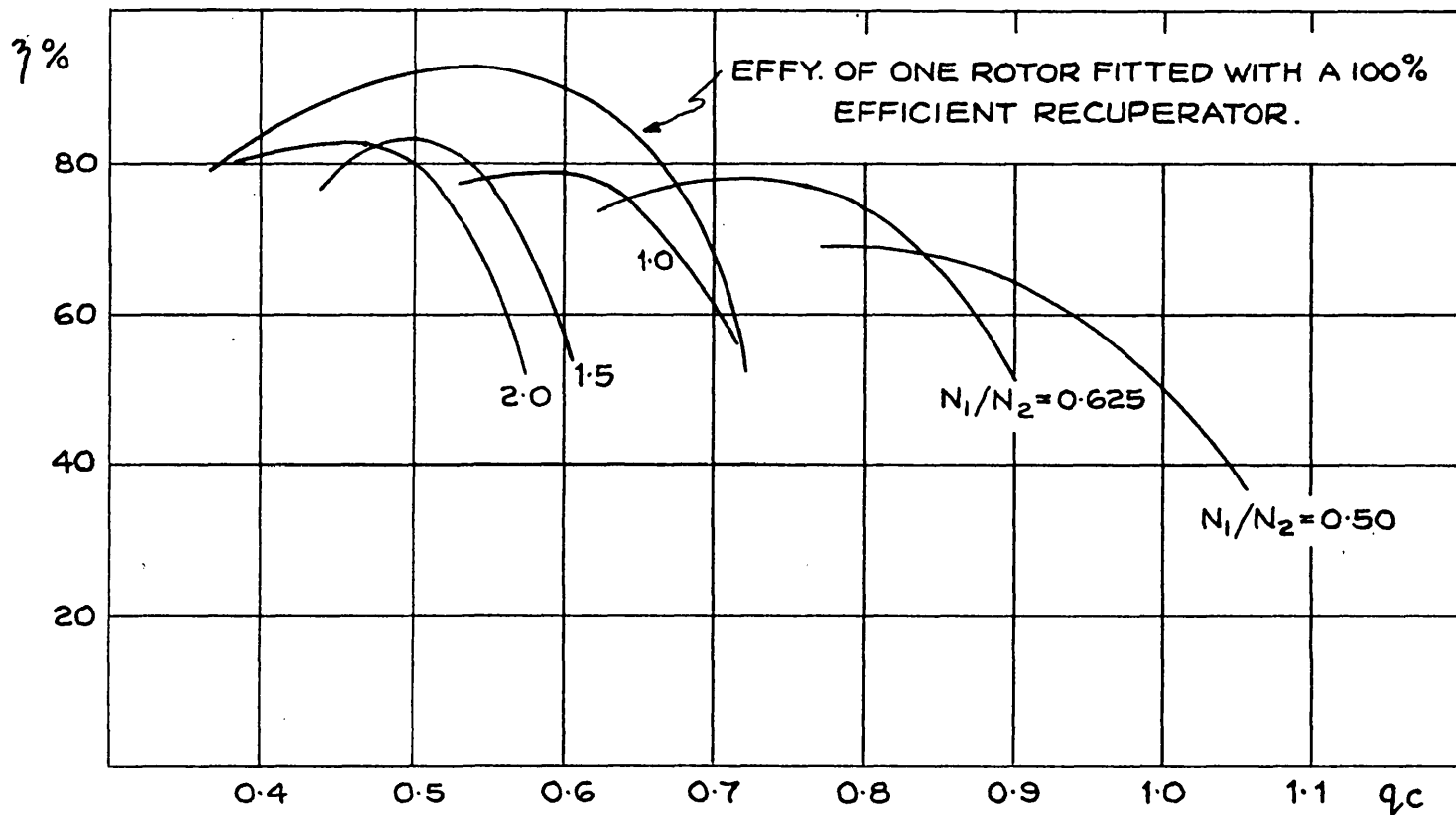


Fig. 3.17 EFFICIENCY CURVE FOR A SINGLE PROPELLER FITTED WITH A 100% EFFICIENT RECUPERATOR, AND THOSE FOR A CONTRA-ROTATING UNIT OPERATING AT DIFFERENT ROTATIONAL SPEED RATIOS.

Fig. 3.18 (a & b)

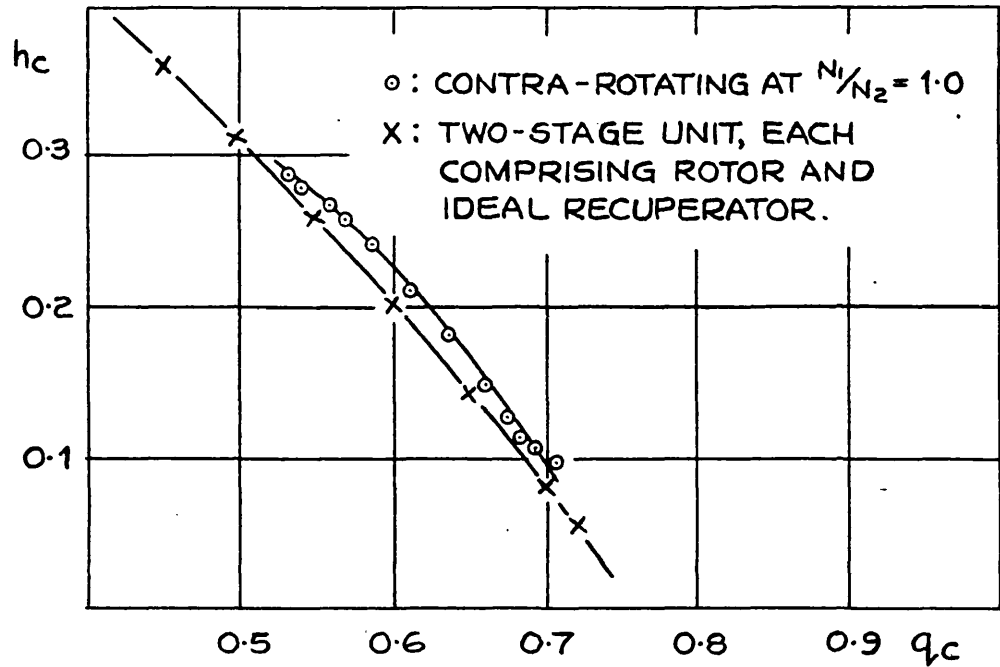


Fig. 3.18 (a) HEAD-FLOW COEFFICIENTS FOR A CONTRA-ROTATING AND TWO STAGE ROTOR-IDEAL RECUPERATOR UNITS.

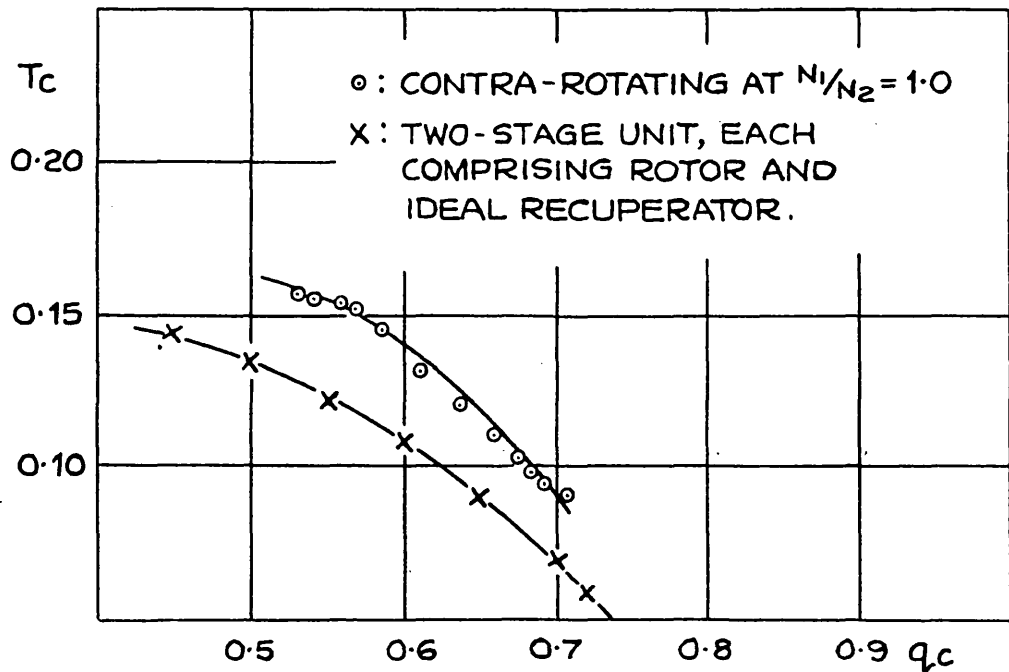


Fig. 3.18 (b) TORQUE-FLOW COEFFICIENTS FOR A CONTRA-ROTATING AND TWO STAGE ROTOR-IDEAL RECUPERATOR UNITS

Fig. 3.19 (a)

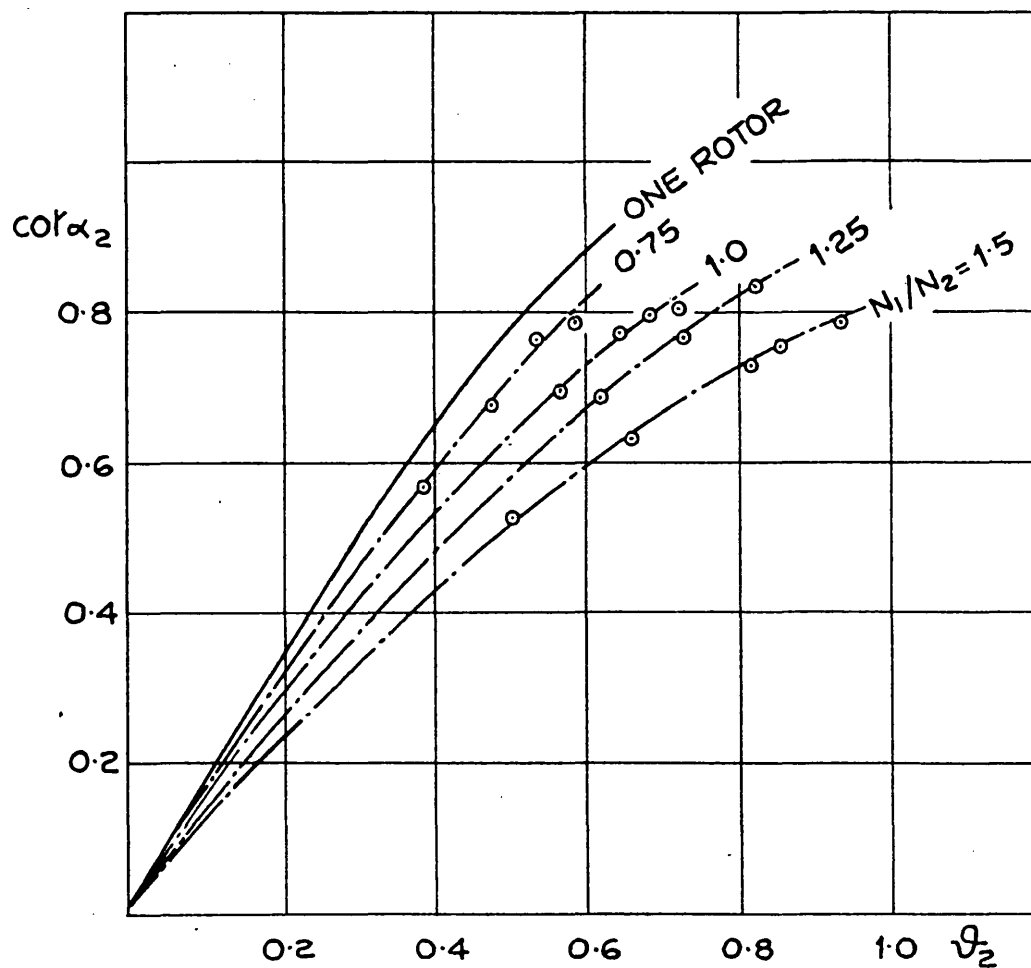


Fig. 3.19 (a) RELATIVE FLOW DIRECTION DOWNSTREAM OF THE 2ND ROTOR OF A CONTRA-ROTATING UNIT, OPERATING AT DIFFERENT ROTATIONAL SPEED RATIOS, AS WELL AS THAT CORRESPONDING TO A SINGLE PROPELLER, FOR RADIAL STATION $r/R_0 = 0.43$.

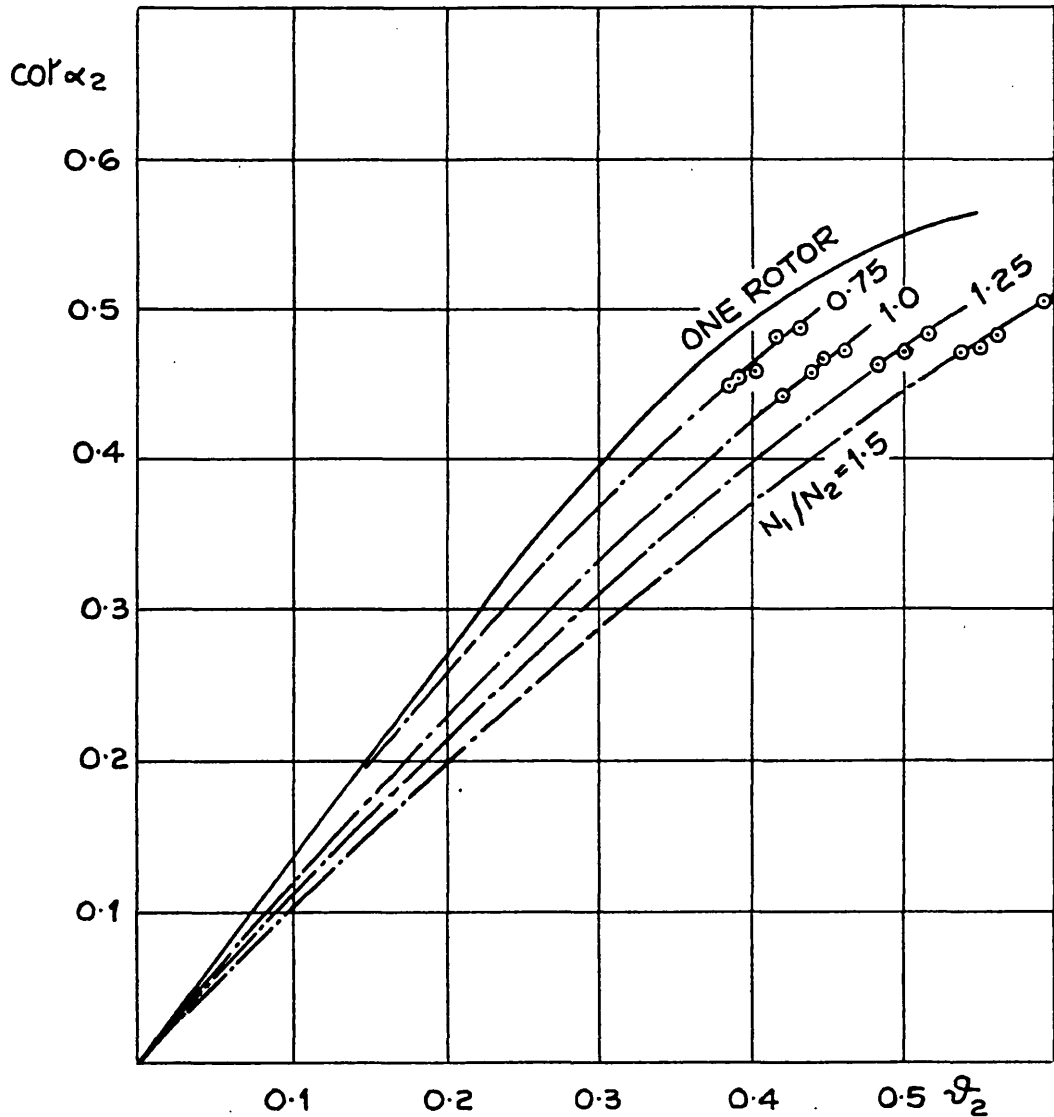


Fig. 3.19(b) RELATIVE FLOW DIRECTION DOWNSTREAM OF THE 2ND ROTOR OF A CONTRA-ROTATING UNIT, OPERATING AT DIFFERENT ROTATIONAL SPEED RATIOS, AS WELL AS THAT CORRESPONDING TO A SINGLE PROPELLER, FOR RADIAL STATION $r/R_0 = 0.69$

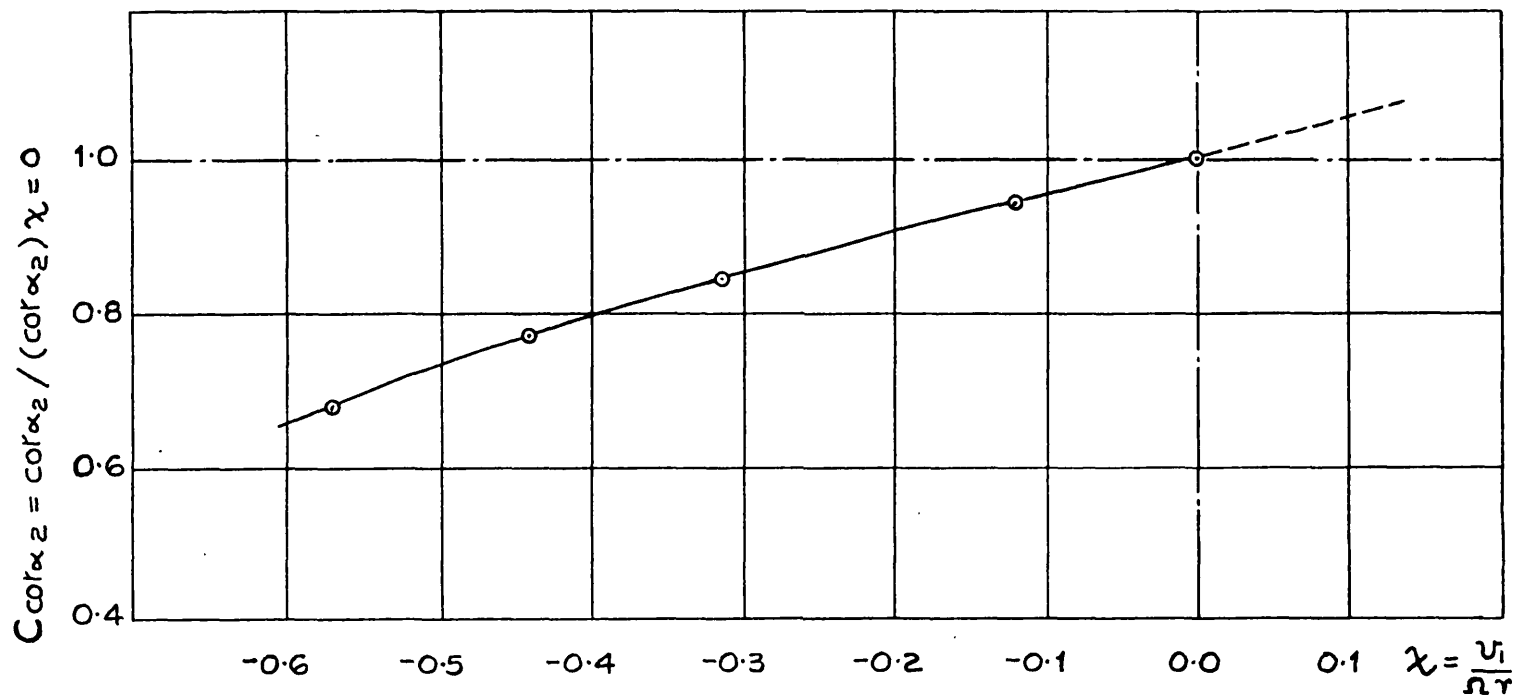


Fig. 3.20 CURVE SHOWING THE EFFECT OF PRE-ROTATION ON THE RELATIVE FLOW DIRECTION, FOR RADIAL STATION $\gamma_{R_0} = 0.43$

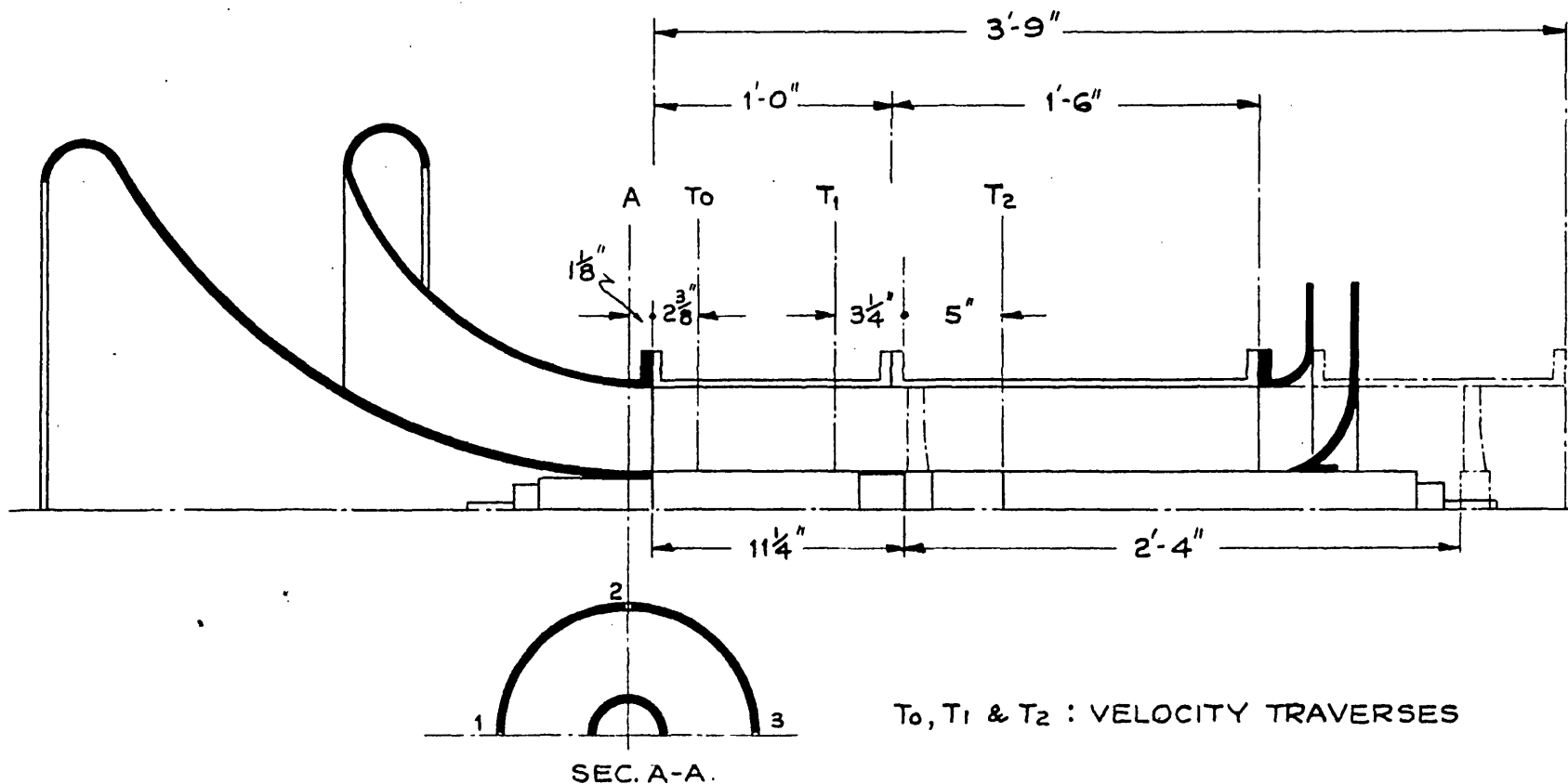


Fig. 4.1

SET UP FOR TESTING ONE PROPELLER ALONE, AND WITH BOOSTING ARRANGEMENT.

Fig. 4.2

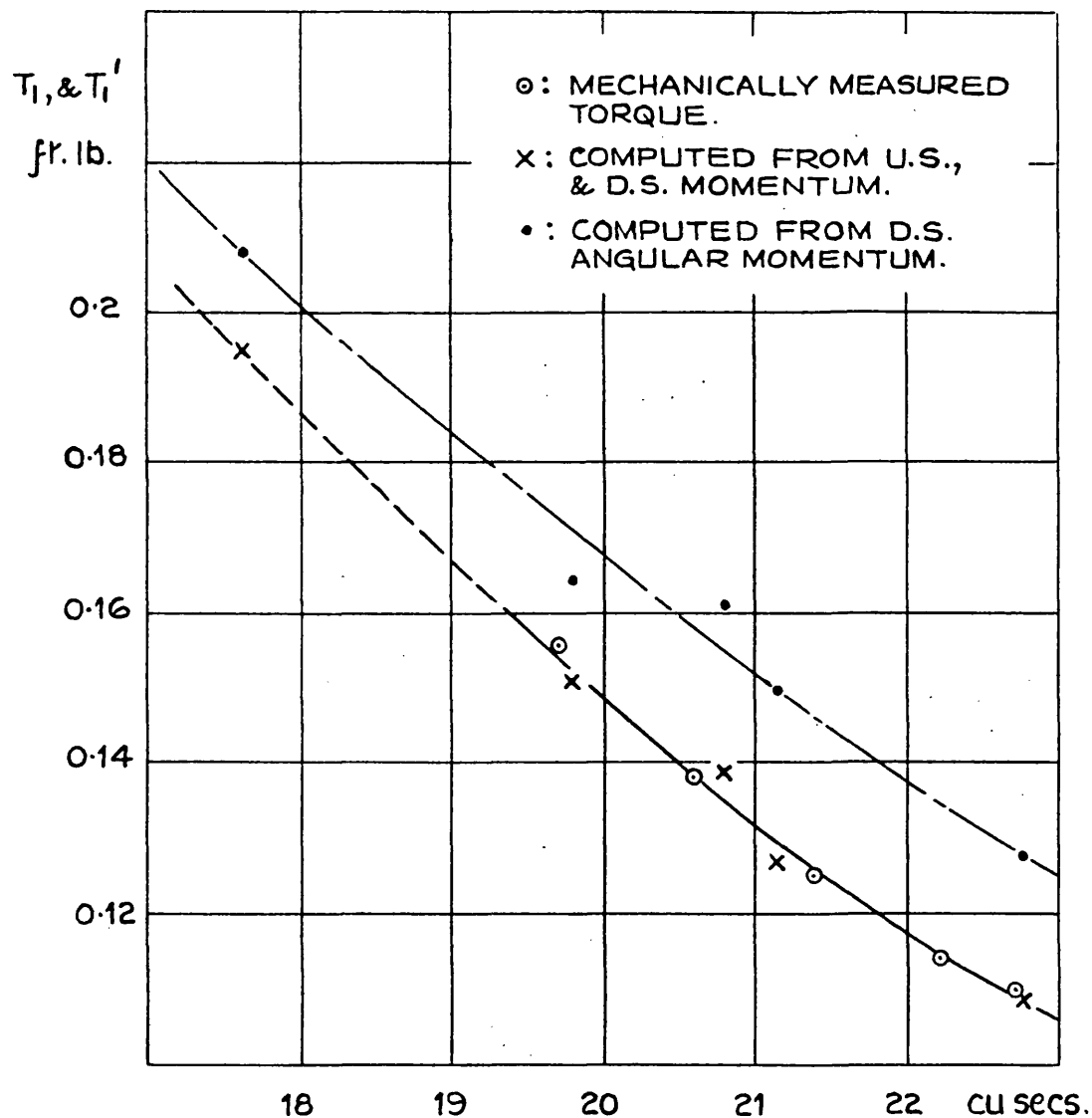


Fig. 4.2

APPLIED TORQUE VALUES, OBTAINED BY MECHANICAL MEASUREMENTS AND MOMENTUM COMPUTATIONS.

Fig. 4.3

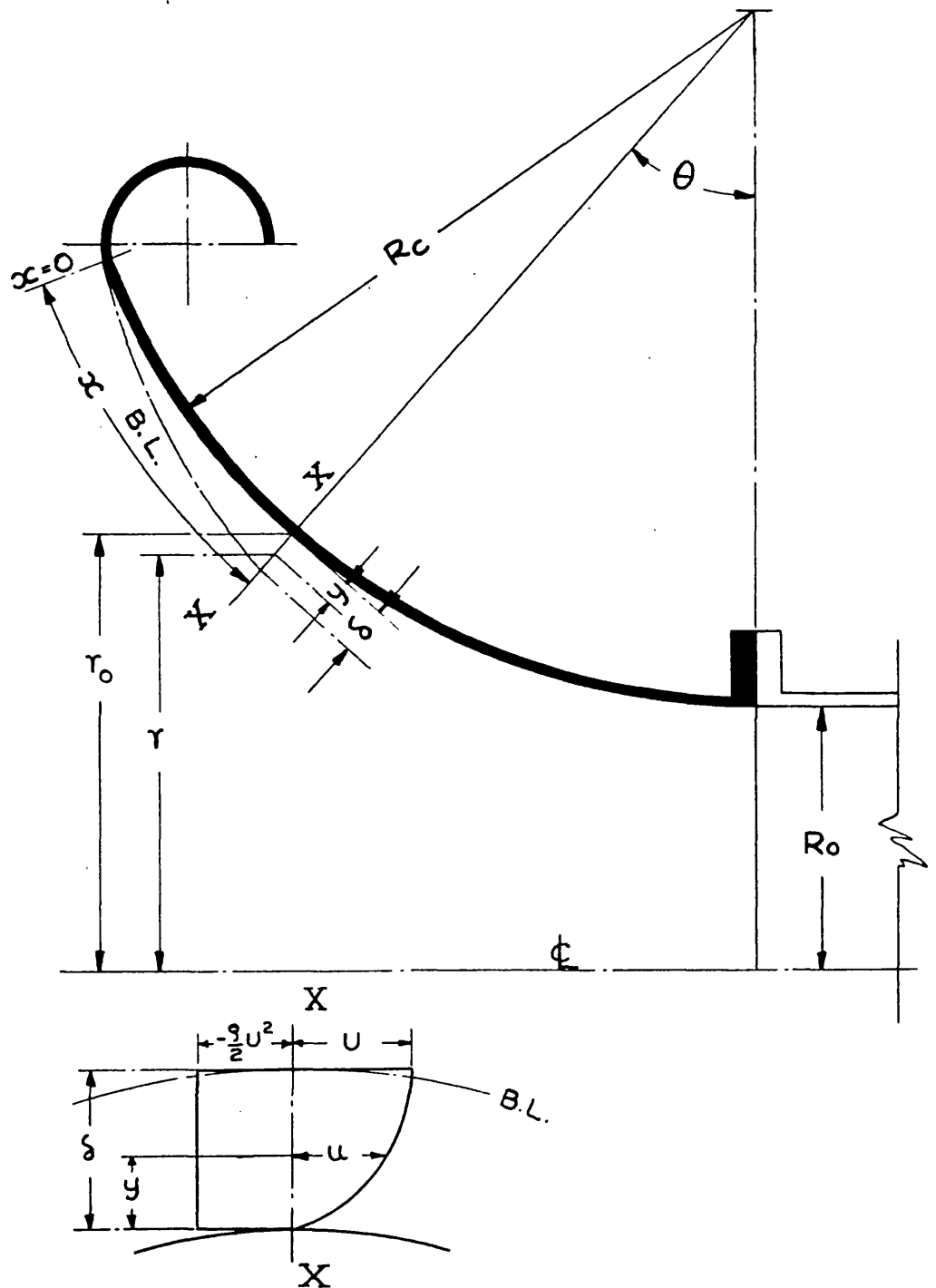


Fig. 4.3 VELOCITY AND PRESSURE DISTRIBUTION WITHIN THE BOUNDARY LAYER, AT A DISTANCE α , FROM THE HYPOTHETICAL STAGNATION POINT.

Fig. 4·4

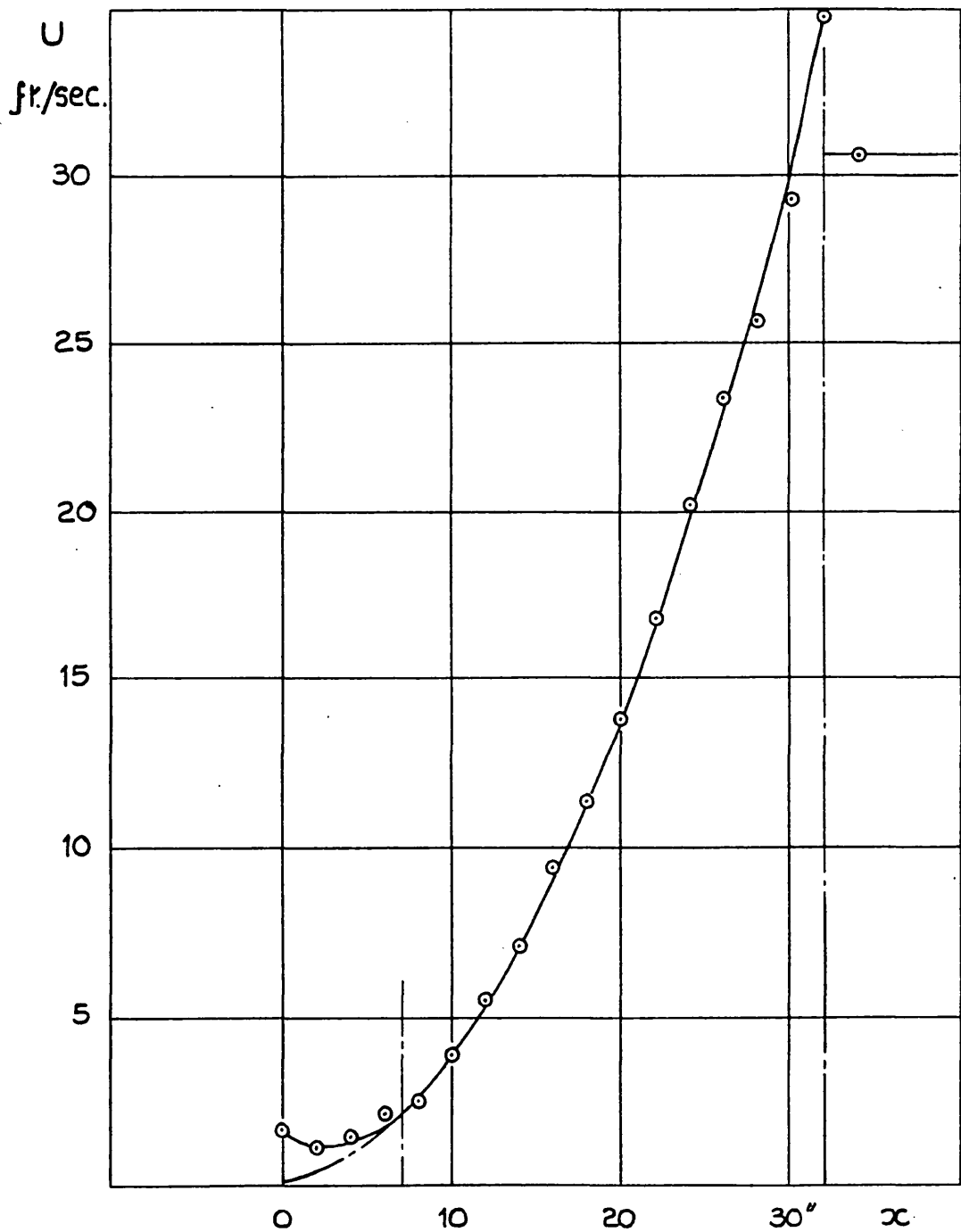
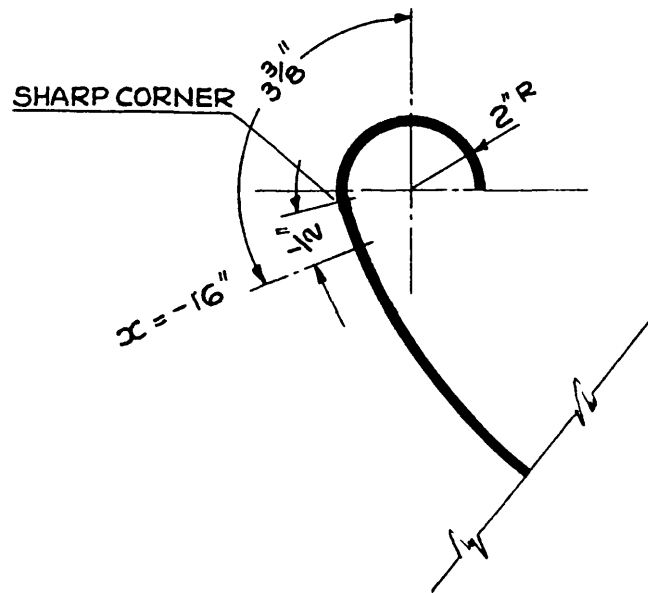
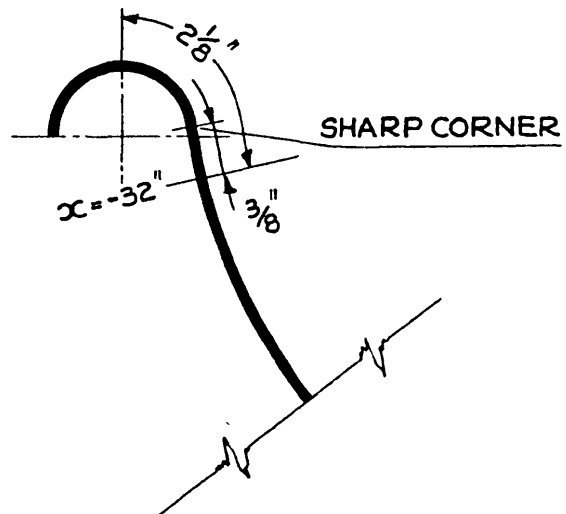


Fig. 4·4 VELOCITY ALONG THE HUB BOUNDARY WALL OF THE INLET CONE, AS COMPUTED FROM STATIC PRESSURE MEASUREMENTS.

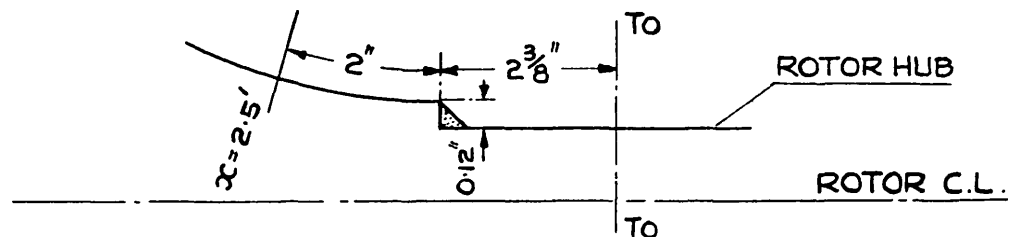
Fig. 4-5 (a,b&c)



a - OUTER BOUNDARY WALL OF THE INLET CONE



b - HUB-SIDE BOUNDARY WALL OF THE INLET CONE



c - THE STEP WHERE THE ROTOR HUB MEETS THE HUB-SIDE BOUNDARY WALL OF THE INLET CONE

Fig. 4-5(a,b&c) MANUFACTURING FAULTS IN THE INLET CONES

Fig. 4·6

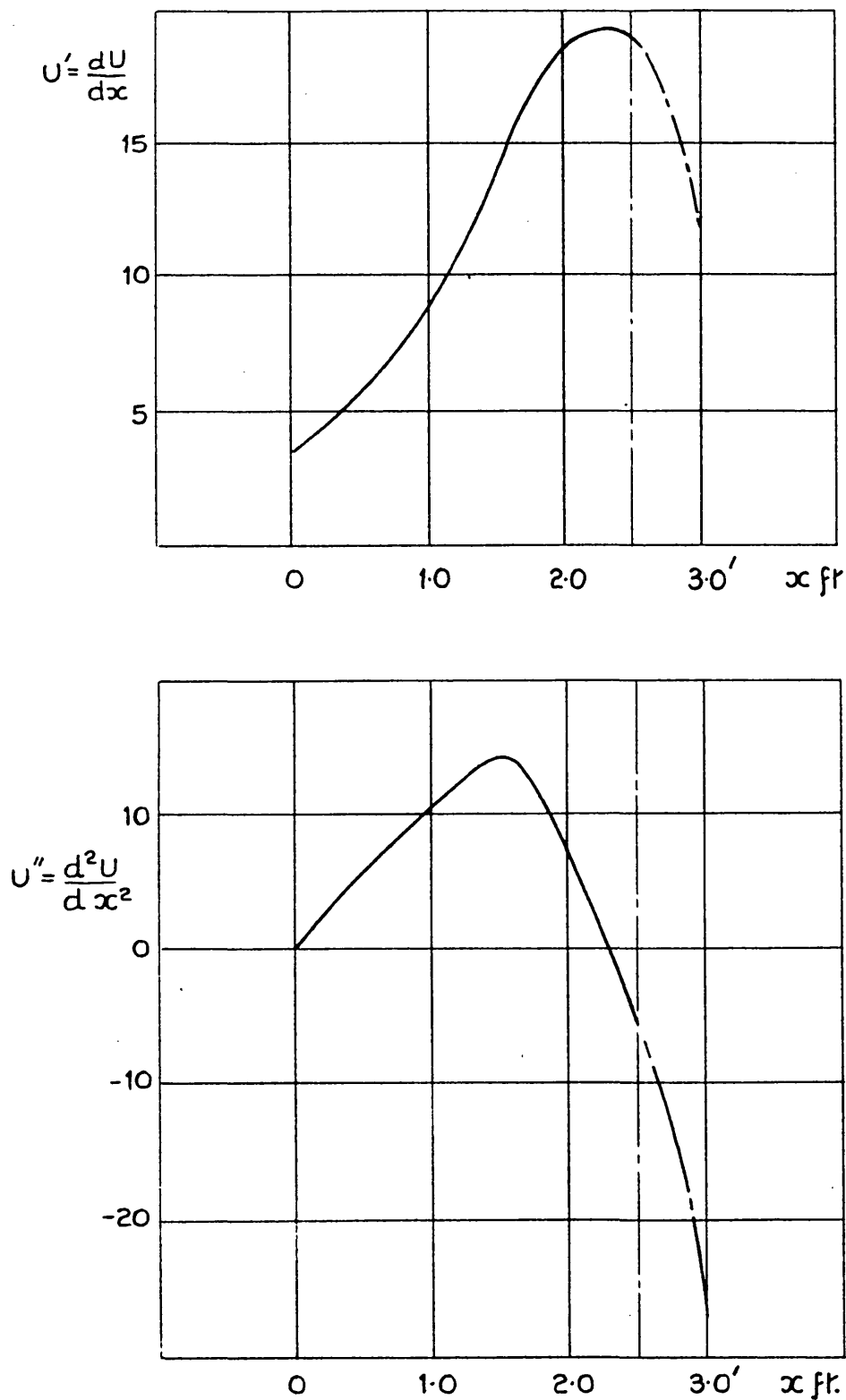


Fig. 4·6 FIRST AND SECOND DERIVATIVES OF VELOCITY ALONG HUB BOUNDARY WALL

Fig. 4.7

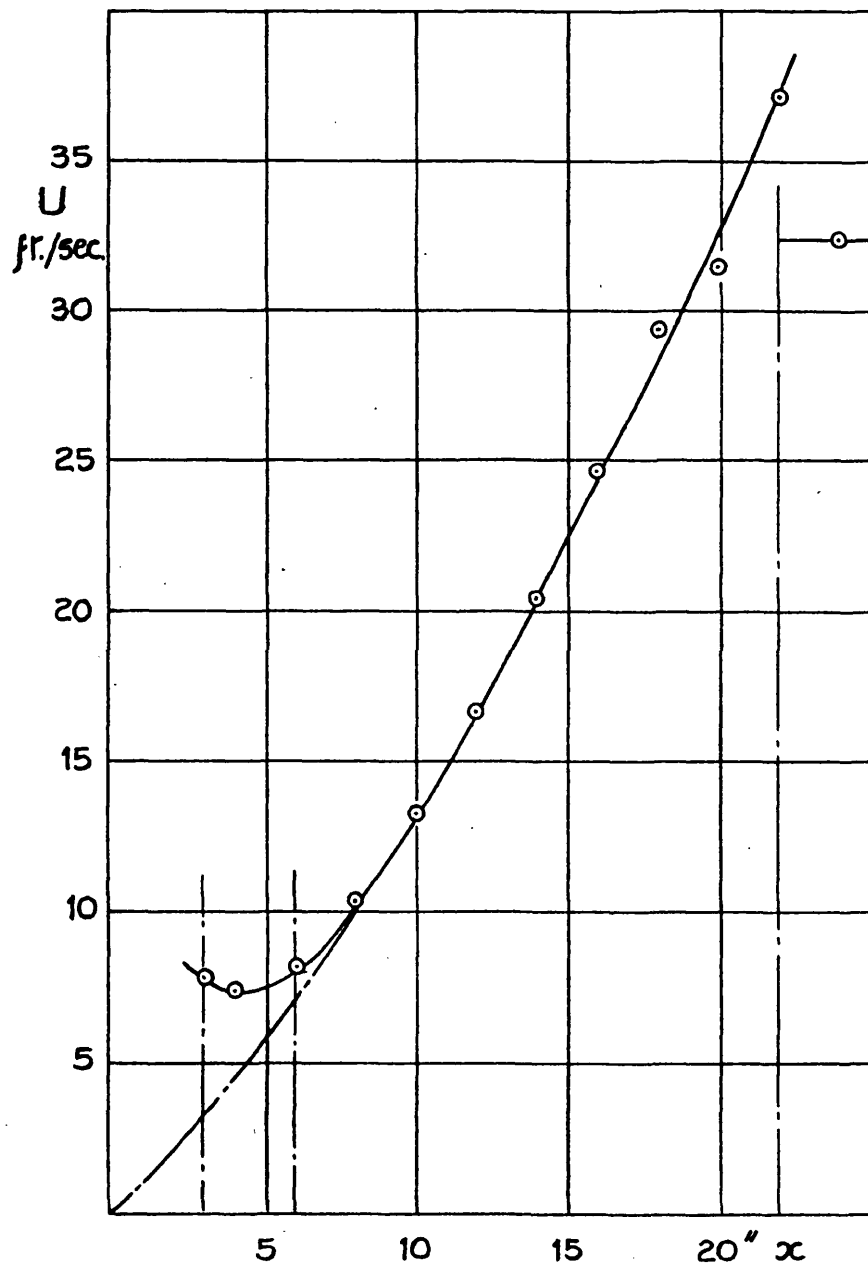


Fig. 4.7

VELOCITY ALONG THE OUTER
BOUNDARY WALL OF THE INLET
CONE, AS COMPUTED FROM
STATIC PRESSURE MEASUREMENTS.

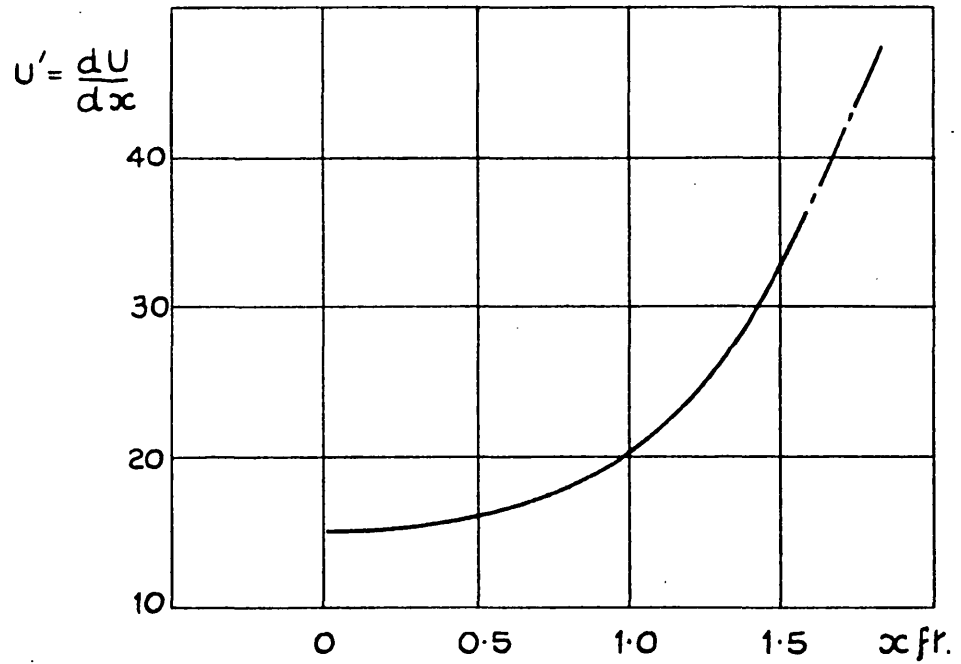


Fig. 4·8 (a) FIRST DERIVATIVE OF THE VELOCITY ALONG OUTER WALL.

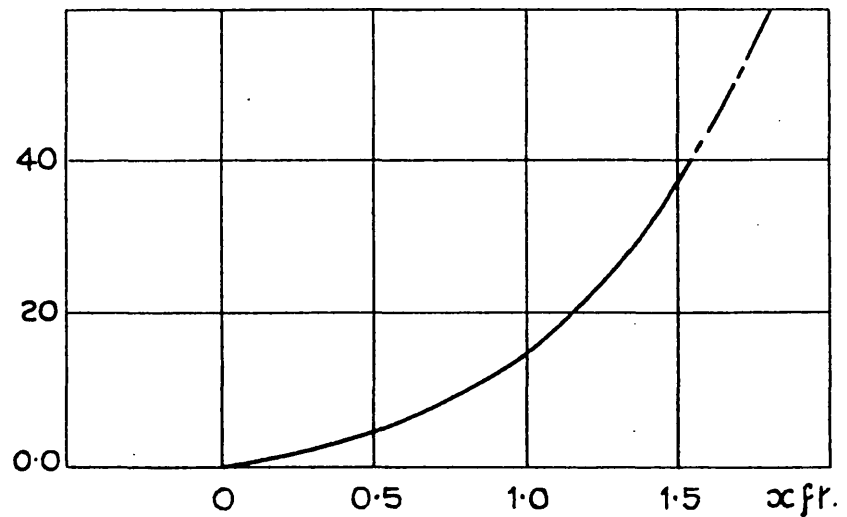


Fig. 4·8 (b) SECOND DERIVATIVE OF VELOCITY ALONG THE OUTER WALL.

Fig. 4.9(a)

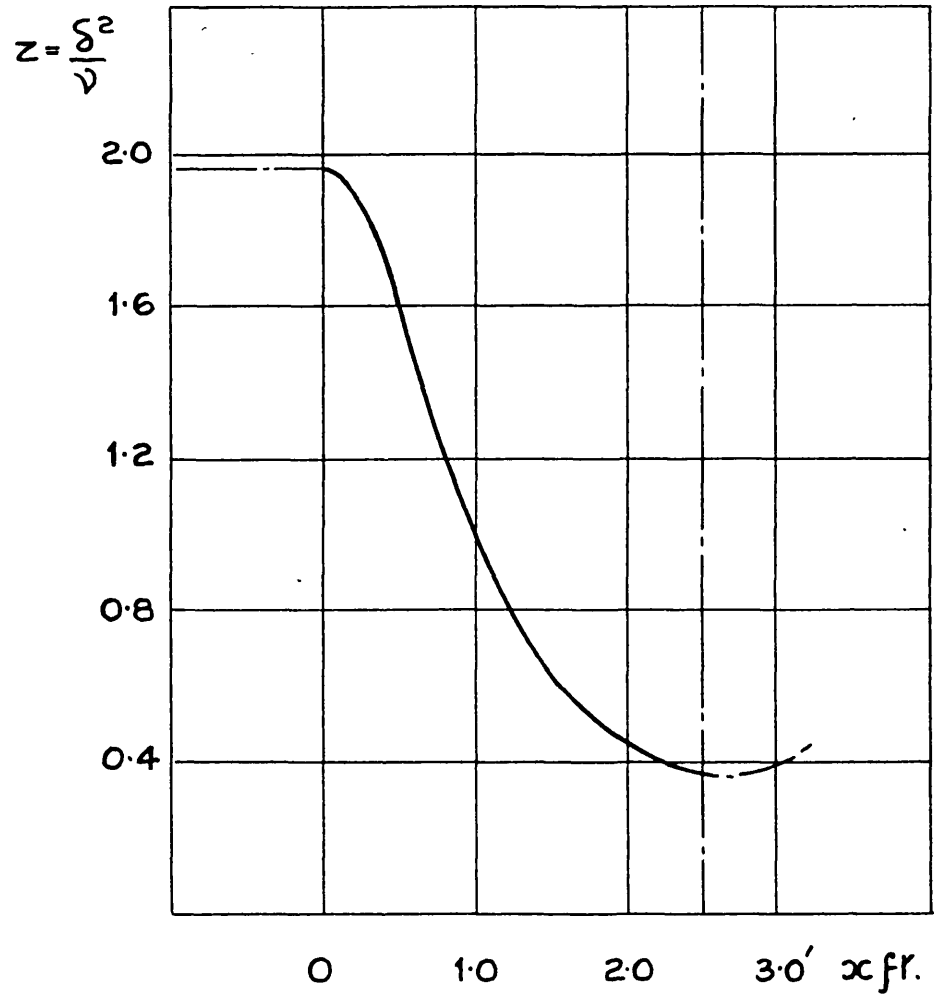


Fig. 4.9(a) SOLUTION REACHED BY KARMAN'S METHOD FOR THE OUTER WALL BOUNDARY LAYER OF THE INLET CONE. (TABLE X).

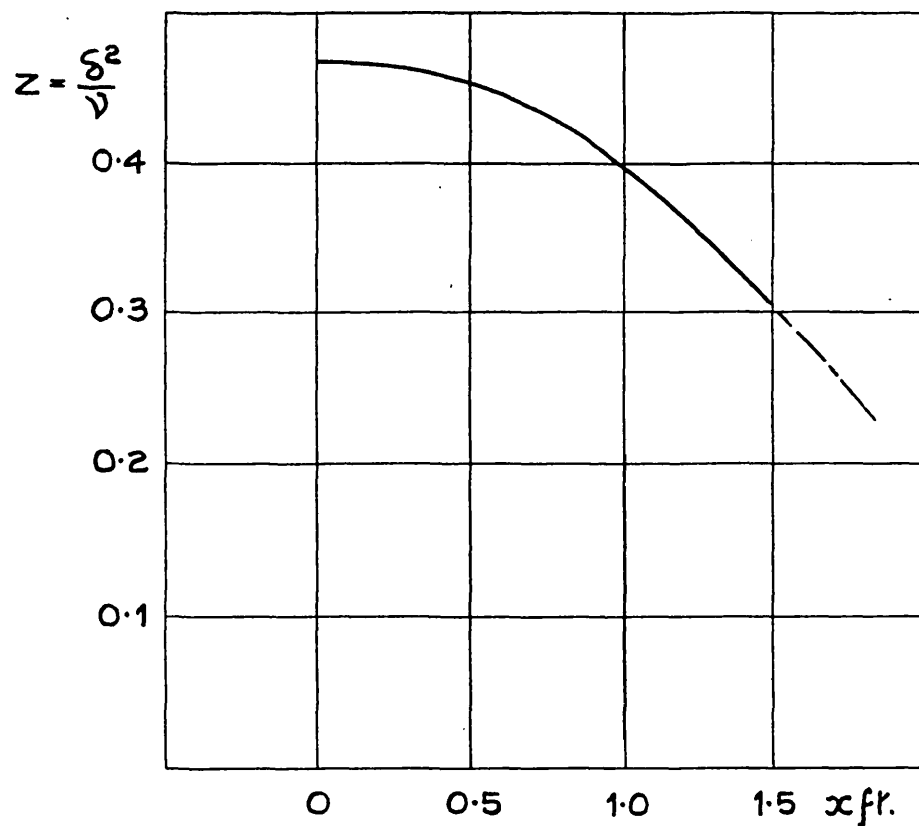


Fig.4.9(b) SOLUTION REACHED BY KARMAN'S METHOD FOR THE HUB WALL BOUNDARY LAYER OF THE INLET CONE. (TABLE X).

Fig. 4-10(a&b)

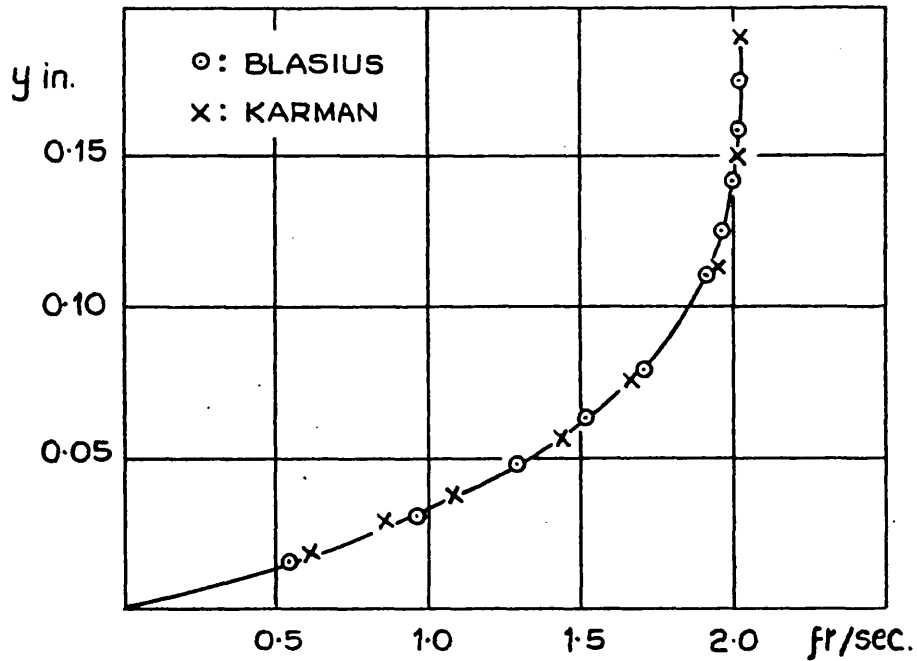


Fig. 4-10(a) VELOCITY PROFILE WITHIN THE HUB WALL BOUNDARY LAYER AT $x = 0.5$ ft., AS COMPUTED BY BLASIUS'S & KARMAN'S METHODS. (TABLES IX, & XII)

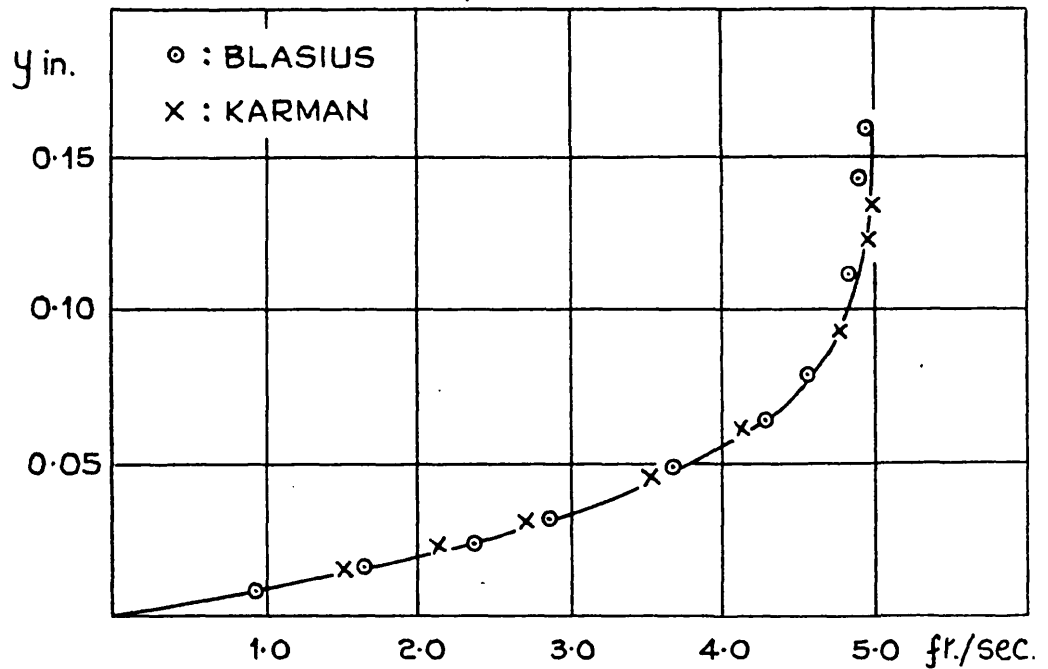


Fig. 4-10(b) VELOCITY PROFILE WITHIN THE HUB WALL BOUNDARY LAYER AT $x = 0.95$ ft., AS COMPUTED BY BLASIUS'S AND KARMAN'S METHODS. (TABLES IX, & XII.)

Fig. 4-11(a)

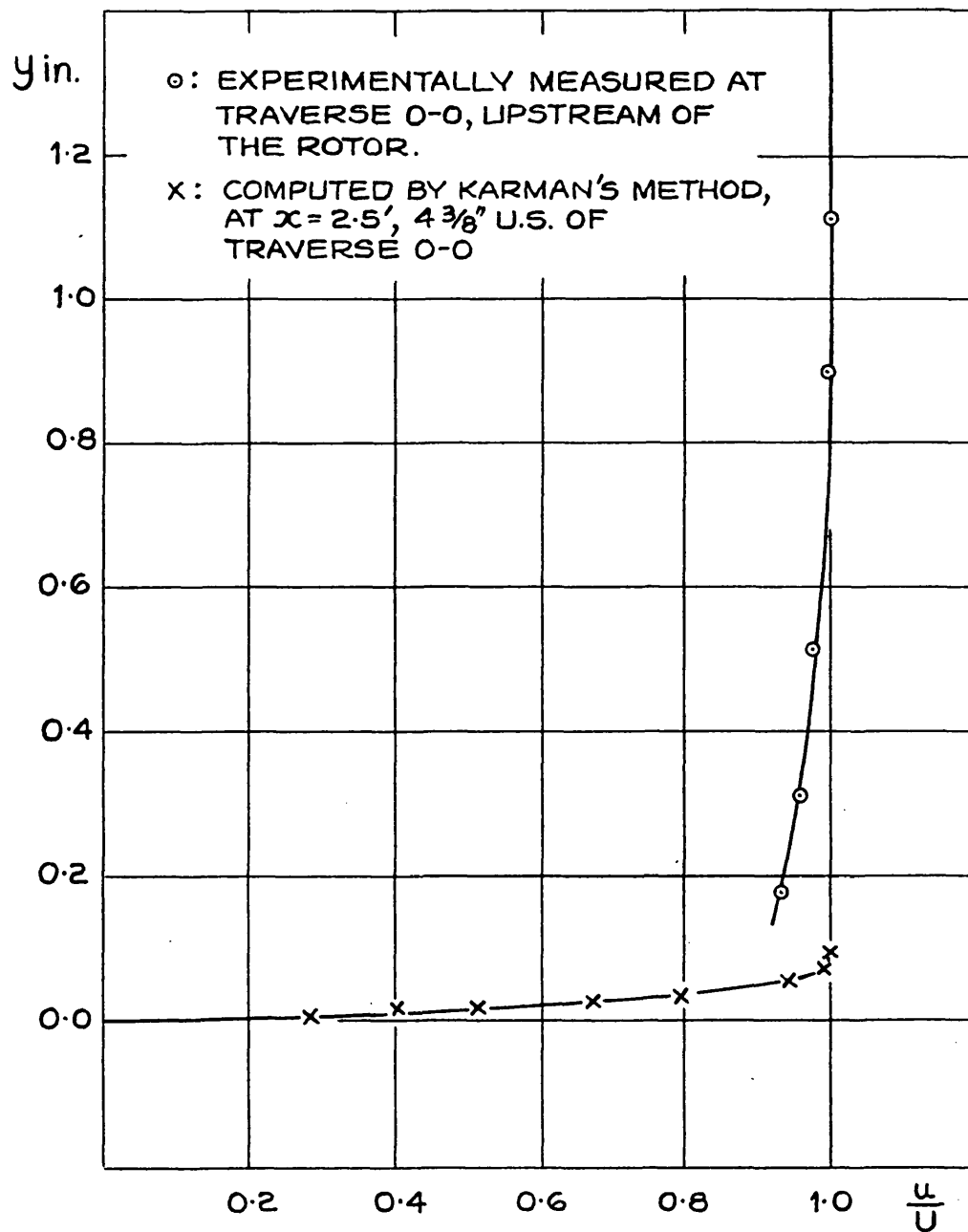


Fig. 4-11(a) VELOCITY PROFILE WITHIN THE BOUNDARY LAYER OF THE INLET CONE HUB WALL, AS COMPUTED BY KARMAN'S METHOD AND EXPERIMENTALLY MEASURED.

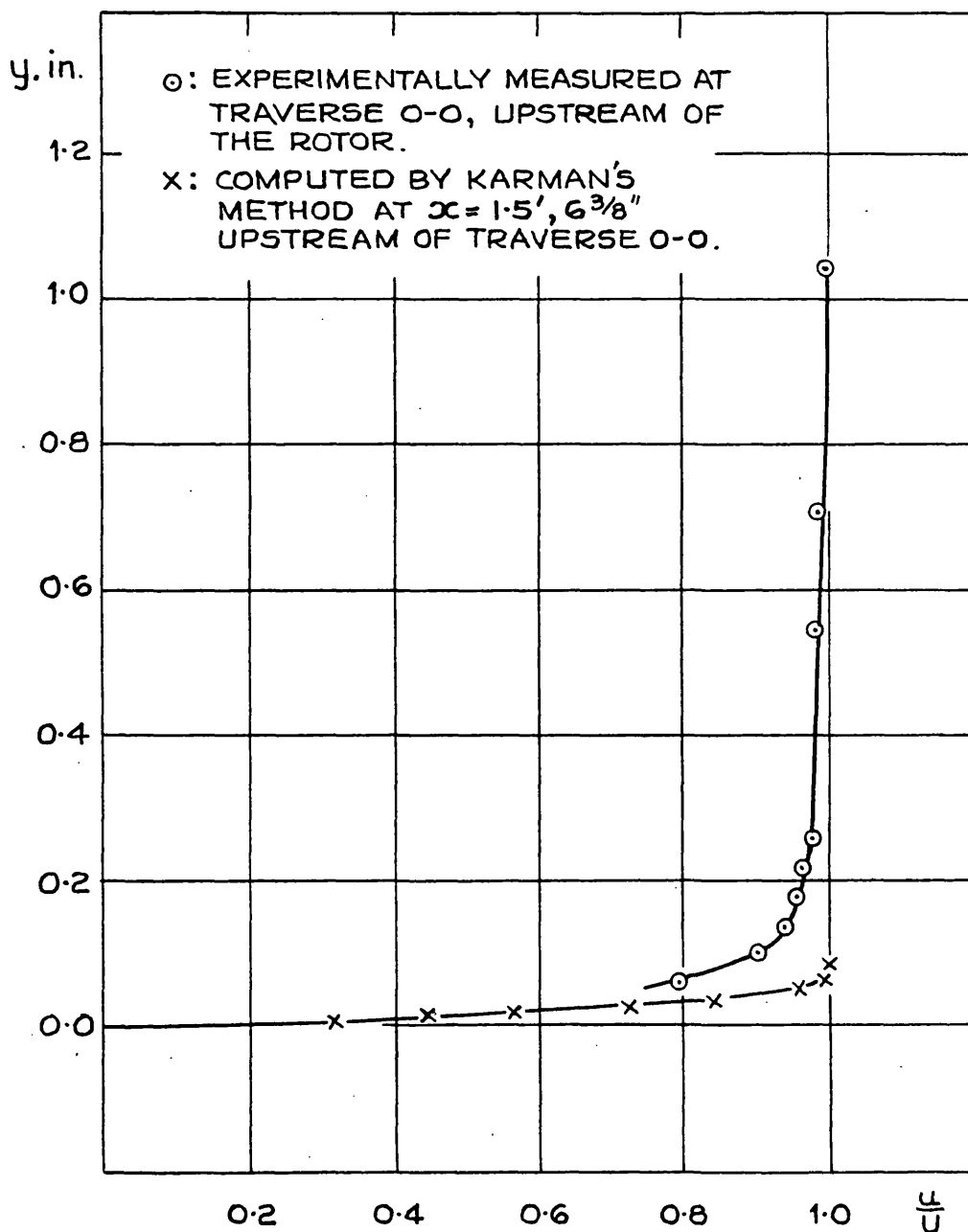


Fig.4-11(b) VELOCITY PROFILE WITHIN THE BOUNDARY LAYER OF THE INLET CONE OUTER WALL, AS COMPUTED BY KARMAN'S METHOD AND EXPERIMENTALLY MEASURED.

Fig. 4·12

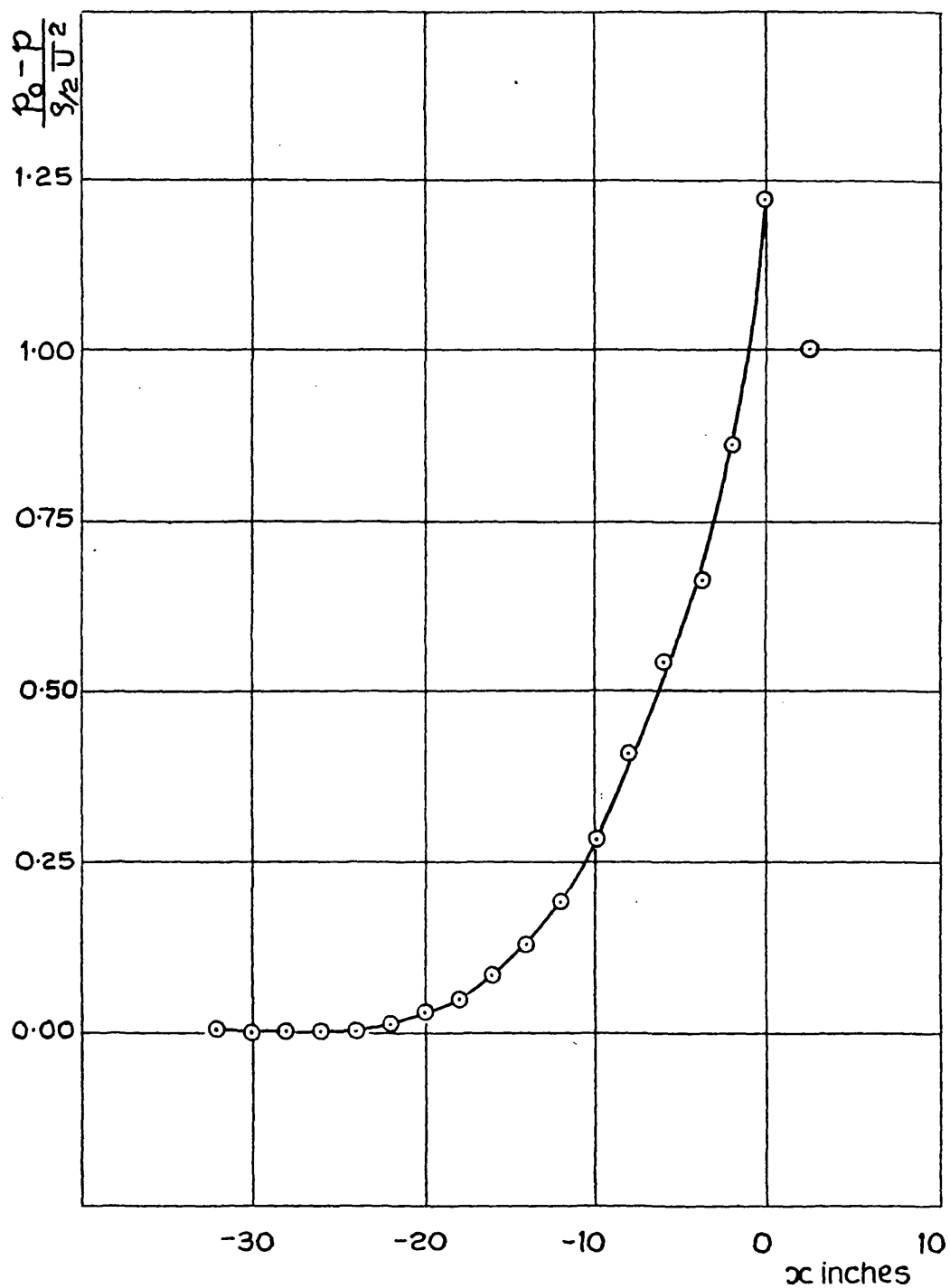


Fig. 4·12 STATIC PRESSURE ALONG THE HUB
BOUNDARY WALL OF THE INLET CONE

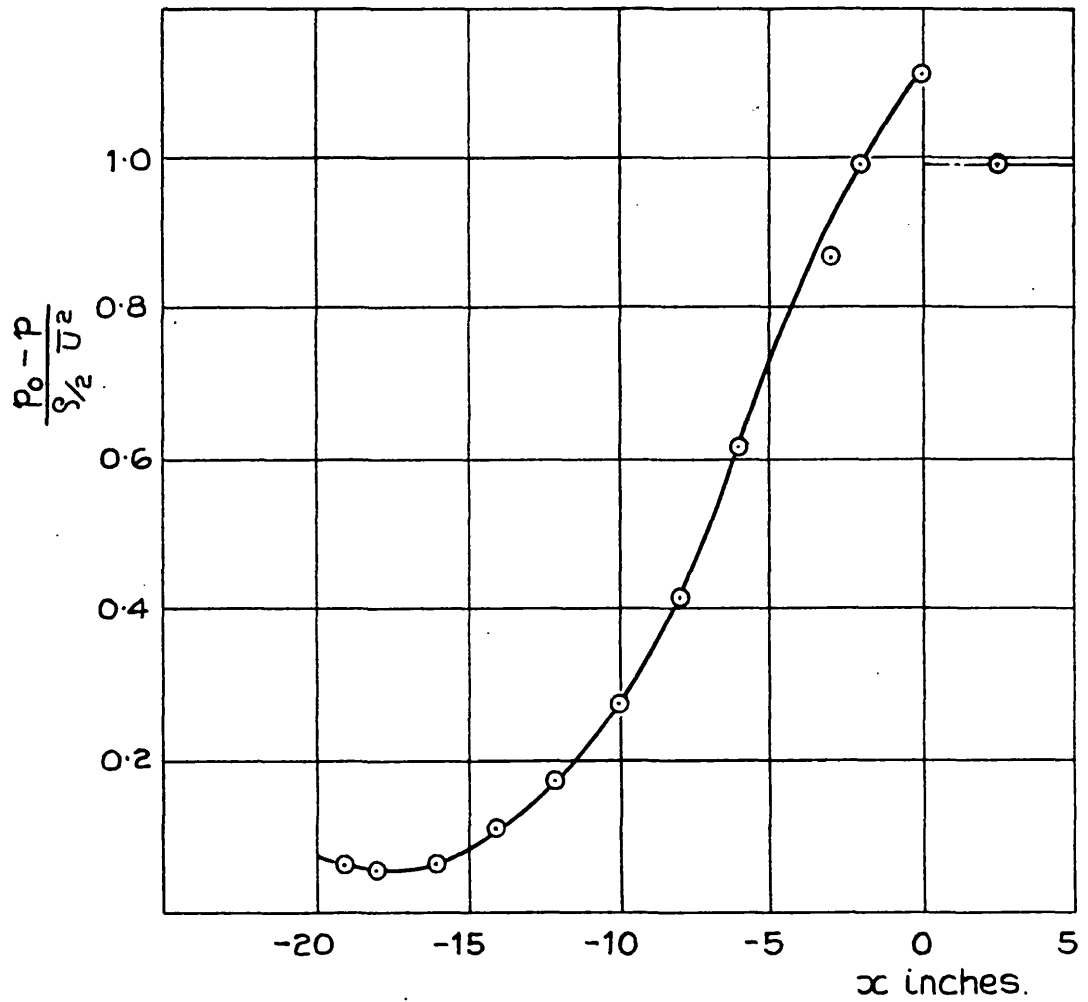


Fig. 4·13

STATIC PRESSURE ALONG THE
OUTER BOUNDARY WALL OF THE
INLET CONE.

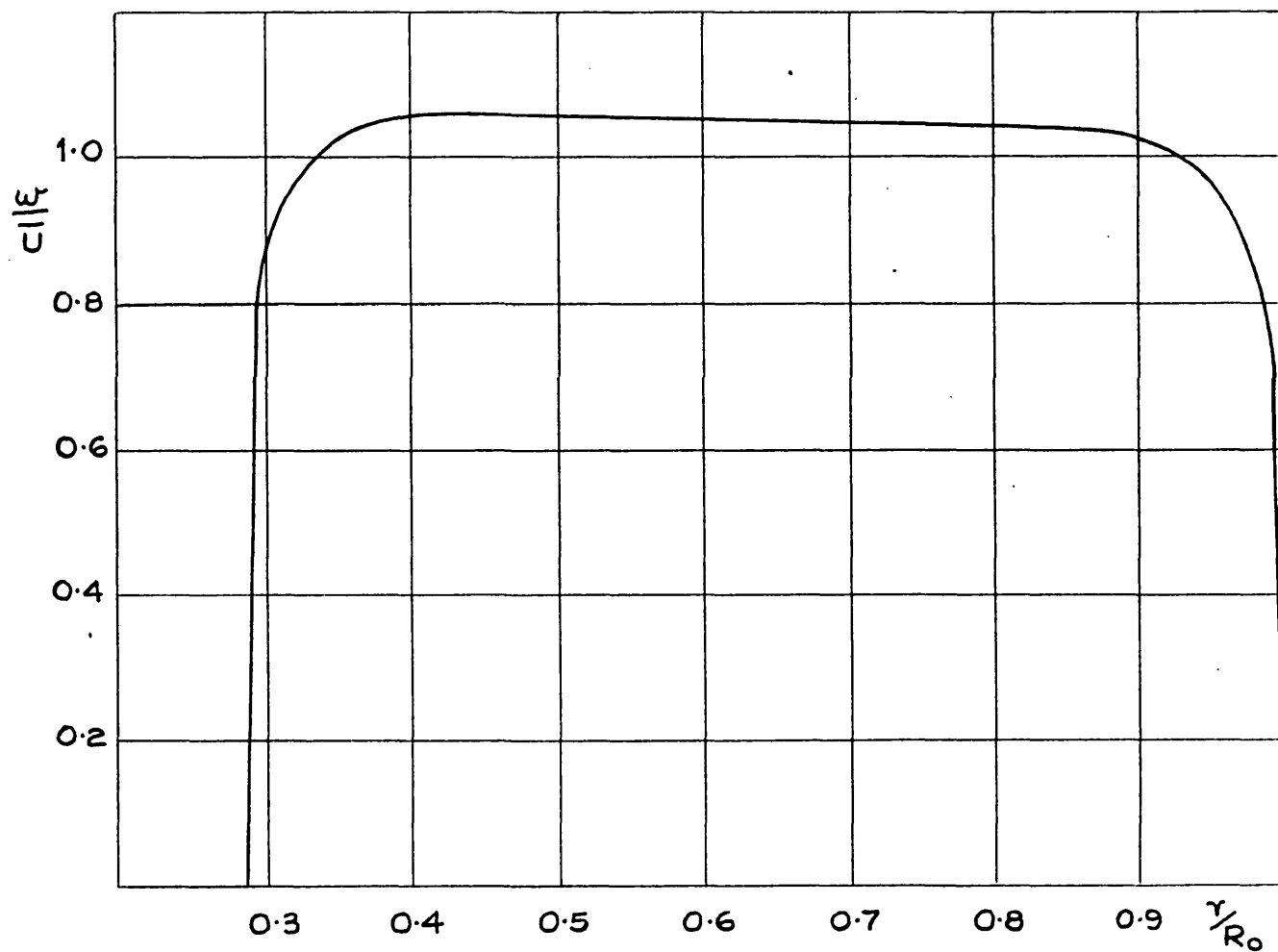


Fig. 4.14 VELOCITY PROFILE AT SECTION T₁
UPSTREAM OF THE PROPELLER