

Integrating Operational Decisions into the Planning of One-Way Vehicle-Sharing Systems under Uncertainty

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Abstract— This study proposes a systematic approach for planning and operating a one-way vehicle-sharing system (VSS) under demand uncertainty. It investigates the distribution of parking spaces and vehicles considering stochastic demand and interactions with the major operational decisions, namely vehicle redistribution activities. An optimization model is formulated that aims to determine the best deployment strategy for minimizing overall cost while achieving a certain level of service (LoS). Then, a simulation-based solution approach based on a discrete-event simulator (DES), Particle Swarm Optimization (PSO), and Optimal Computing Budget Allocation (OCBA) is devised to solve the mathematical model. The methodology is then applied to a car-sharing system in Singapore. Results demonstrate that considering rebalancing activities is imperative in making deployment strategies. The case study also provides managerial insights regarding designing and operating one-way VSS.

Keywords— *vehicle-sharing system; discrete-event simulation; particle swarm optimization; optimal computing budget allocation*

1 INTRODUCTION

Rapidly growing populations, intensified land scarcity, and increasing environmental concerns pose unprecedented challenges to urban transportation systems. As a response to these ongoing challenges, vehicle-sharing systems (VSSs) have emerged in recent years. Vehicle systems are perceived as a promising alternative for urban transportation because of their ease of use and potentially large overall social benefits. For example, for car-sharing systems, users enjoy the convenience and comfort of private automobiles without the associated high cost, insurance requirements, need to refuel, service and repair demands, or parking problems (2008b). Substantial evidence suggests that encouraging people to use VSS is effective to alleviate people's reliance on private vehicles (Katzev, 2003). Therefore, more parking areas will be freed up for other public use and road congestion will be reduced.

A VSS involves a fleet of vehicles strategically located at stations across the transportation network. Vehicle fleets can comprise bicycles, low emission cars, or electric vehicles (Nair and Miller-Hooks, 2011). Stations in VSSs mainly consist of parking areas for cars or bikes, and charging facilities if electric vehicles are used. Existing VSSs can be categorized into two types, one-way or two-way. For two-way sharing systems, users are required to return the used vehicle to the station where it was picked up. Car-sharing systems are mostly two-way. Companies like Zipcar (<http://www.zipcar.com>) and Hertz (<https://www.hertz.com>) currently operate such systems. Compared with the two-way sharing systems, one-way VSSs are more flexible: users can walk to a nearby station to pick up a vehicle, and then drop off the vehicle at any station near their final destination. Initially, most one-way VSSs were bike-sharing systems that gradually became a popular choice for urban mobility, such as Vélib in Paris (<http://www.velib.paris.fr>) and Hubway in Boston (<http://www.thehubway.com>). Recently, more and more one-way car-sharing systems have been implemented, with the success of Car2go (<https://www.car2go.com>) in North America indicating the potential of such systems. Because of the usage mode, one-way VSSs are more likely to be utilized as a connection to existing public transport modes or as their substitute. The one-way VSSs can be further subdivided into station-based ones, where users need to return the vehicle to any parking areas rented by the company, and free-floating ones, where users have the freedom to drop off the vehicle anywhere within a certain area.

However, successfully deploying and operating a one-way VSS is even more difficult than two-way systems. In addition to the demand being difficult to predict and highly fluctuating, which calls for dynamic decision-making and prompt actions, the inherent imbalance of urban traffic flows frequently leads to an inefficient use of the system: areas with higher return rates may become overstocked with a large number of idle vehicles that could

be better used if relocated to places in need of vehicles (Jorge and Correia, 2013). Moreover, the performance of a one-way VSS is largely influenced by the intricate interactions between different levels of decisions, among which the trade-offs may not be very explicit.

This paper targets the planning and operating issues of one-way station-based VSS. As compared with free-floating systems, more managerial and operational difficulties may be involved for operators of such systems. More specifically, it aims to assist stakeholders to determine the distribution of vehicles and parking spaces when faced with stochastic and imbalanced demand. More importantly, it investigates how such strategic decisions interact with operational decisions, namely vehicle redistribution activities. Take the problem illustrated in Figure 1 as an example. The geographical region of interest is divided into eight zones with dots representing their centers. The zones represent clusters of adjacent parking spots, namely the maximum straight-line distance, namely the diameter of the circle, between parking spots located in the same subarea is within walking distance range. Therefore, within each zone, there is no difference for customers to pick up or drop off vehicles at any specific parking spot, but such difference does exist between zones. There are other studies utilizing complex models, e.g. clustering analysis Vogel et al. (2011), to understand how these zones can be more accurately defined. This study focuses more on how to plan and operate the whole system, however, and after the zones are already defined. Under such assumptions, this study intends to devise an approach to determine the following parameters for each zone: 1) the number of parking spots to be rented and 2) the number of vehicles to be placed at the beginning of the day.

Figure 1 Illustration of the target problem

ABOUT HERE

The rest of the paper is organized as follows. The next section reviews the relevant literatures, identifies the research opportunity and explains the motivations for this study. Following this, the paper elaborates on the proposed methodology, including the models and the computational algorithm. Then, the methodology is further illustrated by a case study that demonstrates the effectiveness of the proposed analysis. Finally, the paper concludes by discussing the major findings and opportunities for future work.

2 LITERATURE REVIEW AND MOTIVATION

The literature on the quantitative planning and detailed operation of VSS was virtually non-existent until a few years ago, but has been growing rapidly in recent years. Existing studies can be roughly categorized into

three sub-streams: data analytics, strategic planning, and rebalancing operations. Some of these studies address bike-sharing systems, while others pertain to car-sharing systems.

Authors in the area of data analytics focus on understanding and characterizing the usage patterns of VSSs. For example, Vogel et al. (2011) apply clustering analysis to riding data from Vienna's bike-sharing system, "Citybike Wien", identifying five distinct clusters based on pickup and return patterns over time. Borgnat et al. (2011) rely on non-stationary statistical modeling and data mining to describe the evolution of the dynamics of movements within the Vélib system in Paris. The spatial and temporary demand patterns are also described, and the social behavior of the users is explained. Using demographics and travel survey data, Ciari et al. (2010) develop an agent-based simulation (ABS) model to estimate and predict demand for a car-sharing system during its planning phase. In another study, (Martínez et al., 2017) developed an agent-based model to simulation relocation operations in car sharing systems. The simulation-based approach by Barth and Todd (1999) evaluated the performance of vehicle-sharing systems given a particular set of decisions. The study showed that vehicle-to-trip ration is critical to the system performance. Becker et al. (2017) used spatial regression and conditional logit of original data of free-floating car sharing systems and determined that such systems are mainly used for discretionary trips, for which only inferior public transport alternatives are available.

The second category of papers presents models that deal with strategic planning issues. Lin et al. (2013) formulate a mixed integer programming model for optimizing the design of bike-sharing systems. Although the model encompasses a wide range of strategic decisions, their analysis is deterministic and does not capture dynamic behavior, such as the daily or even weekly fluctuations of demand that these systems experience. Considering demand variations within a day, Raviv and Kolka (2013) explore the optimal number of vehicles to be made available at each station at the beginning of each day. They formulate a penalty cost function and further utilize it in a simulation-based optimization model in the search for the best solution. Shu et al. (2010) slightly simplify this same problem and adopt a linear approximation optimization model. Their analysis captures broader considerations by incorporating the day-to-day demand changes within a week. They also demonstrate the benefits of redistributing vehicles at the beginning of each day. Jorge et al. (2014) similarly apply a mixed integer programming model developed by Correia and Antunes (2012) to solve a similar problem in a deterministic environment, and further evaluate the solution via an ABS model. Levin (2017) proposes a linear programming model for shared autonomous vehicle optimal dynamic traffic assignment, considered congestion constraints. The study showed that peak hour travels significantly increase congestion and vehicle miles traveled as compared to off-peak conditions.

The above studies are helpful for determining an initial configuration for a VSS system. However, an essential part of daily operations, namely rebalancing of vehicle supplies, is missing from the formulations. In practice, imbalanced traffic flows frequently leave some stations empty of vehicles and others in need of additional parking space to accommodate an excess of vehicles. This necessitates redistributing vehicles and rebalancing of supplies as part of the daily operations of a VSS.

Pavone et al. (2012) use a fluid-model approximation to develop quasi-optimal rebalancing policies. They first identify the conditions under which a VSS reaches equilibrium, and then propose a simple optimization model to compute fixed rebalancing rates in such an equilibrium state. Schuijbroek et al. (2013) model vehicle flows at each station through a M/M/1/K queuing system that is then used to obtain an equation for computing bounds on the inventory of vehicles needed at each station. In addition, a cluster-first route-second heuristic model is proposed to obtain optimal routings of the rebalancing vehicles. Nair et al. (2013) and Nair and Miller-Hooks (2011) apply chance constraint programming to model the problem. An advanced algorithm is also developed to transform the stochastic problem to a set of mixed integer programming problems. Kek et al. (2009) devise a decision support tool to determine the optimal rebalancing strategy. They apply the model to data from a Singapore-based car-sharing system and demonstrate the significant cost reduction that can be achieved through the introduction of vehicle relocation operations.

In the existing VSS literature, it is common that the problems of planning/designing a VSS and operating it are decomposed into several parts and then solved independently. Relocation/rebalancing decisions are made under an assumed system configuration and, conversely, the configuration of the system is determined without consideration of the impact of the relocation/rebalancing policies. In reality, however, ignoring the close interactions between these two sets of decisions may lead to significant problems and to poorly designed and poorly operating VSS. For example, stations may be designed with a larger capacity than would have been necessary if rebalancing policies were taken into account. As demonstrated in a case study by Lin et al. (2013) involving a bicycle-sharing system, bicycle stocks at stations can be overestimated by thousands if rebalancing policies are not considered. In fact, two studies have further investigated the interactions between planning and operational decision-making for VSSs. Cepolina and Farina (2012) apply a simulation-based approach to determine the optimal distribution of vehicles for a VSS consisting of Personal Intelligent City Accessible Vehicles (PICAVs) to solve the mobility problem in a pedestrian area. Their analysis considers the variability in customers' travel patterns as well as a relocation policy reliant on a system supervisor to instruct customers to complete the relocation task. However, such rebalancing practices may not be easily executed in reality,

especially for car-sharing systems where switching destination stations may lead to a relatively long walking distance, in which case customers may not be willing to cooperate. In contrast, in the paper by Boyacı et al. (2015), the system operators assume the responsibility of rebalancing. Although their study provides interesting results on integrating the location and sizing issue of VSS stations with operational decisions, the analysis is deterministic and thus daily fluctuations in demand are not fully considered. In fact, because of this deterministic approach, rebalancing decisions are made at the same time as planning decisions, even though rebalancing decisions are supposed to react in real-time to the realization of demand. In addition, the paper mainly deals with a reservation-based system in which operators have the power to choose among customers; however, for some VSSs such as Car2go, reservations are not compulsory. Furthermore, some VSSs are purely on-demand, like the UCR IntelliShare system (Barth et al., 2000). Such systems allow greater flexibility for customers, but are more difficult in terms of system management.

In summary, the existing literature does not provide an integrated approach that is helpful for determining the initial set-up configuration for VSS by considering the combined influence of operational decisions and fluctuations in demand. Without such an approach, VSS may suffer from inefficient use of resources, and operational difficulties. Motivated by this gap in the literature, this study aims to develop a decision support tool comprising a model and a computational approach that investigates the aforementioned factors. More specifically, the tool will address the following research issue: how to formulate a model, as well as propose a computational approach, to determine the optimal distribution of stations and vehicles for a one-way station-based VSS adopting a first-come-first-served operating policy under the conditions of stochastic demand, and frequent rebalancing activities.

3 METHODOLOGY

In order to deal with the stochasticity of demand and model the rebalancing operations, this study relies on a simulation-based methodology, as it is a widely used approach to the analysis of transportation systems (Barth and Todd, 1999, Chevallier and Leclercq, 2009). A constrained optimization model is first formulated and later solved using a simulation-based optimization approach. This solution approach consists of a discrete event simulator (DES), which is applied to estimate the objective function of each candidate solution considering the stochastic demand and hourly rebalancing operations, and a computational algorithm based on Particle Swarm Optimization (PSO) and Optimal Computation Budget Allocation (OCBA), which is to generate solutions and provide guidance on how to allocate simulation budget wisely across the candidate solutions. The framework of the computation approach is inspired by some past studies on VSSs (Cepolina and Farina, 2012) and other urban

transportation systems (Sidi et al., 2008, Borne et al., 2003), which mainly consists of a decision support tool to evaluate different decisions and an optimization algorithm to identify the optimal decision.

3.1 Integrated optimization model

$$\min C_i \sum_{i \in N} x_i + w * v + R(\mathbf{x}, v) + Q(\mathbf{x}, v) \quad (1)$$

$$s.t. \quad g(\mathbf{x}, v) \leq \alpha \quad (2)$$

$$x_i \geq v_i \quad \forall i \in N \quad (3)$$

$$\sum_{i=1}^N v_i = v \quad (4)$$

$$x_i, v_i \in Z_+ \quad (5)$$

In the optimization model, C_i is the rental cost of one parking spot at zone i ; $\mathbf{x}$ is a vector whose dimension is equal to the number of zones and the value of its component is the number of parking spots at each zone (also referred as capacity of each zone); w is the depreciated daily cost of one vehicle; v is the total number of vehicles in the system; v_i is the number of vehicles placed at zone i at the beginning of the day; $R(\mathbf{x}, v)$ represents the rebalancing cost, which is a function of $\mathbf{x}$ and v ; $Q(\mathbf{x}, v)$, which is also a function of $\mathbf{x}$ and v , represents the extra parking cost incurred when customers arrive at the destination zone but cannot find any available parking spots, hence having to park somewhere nearby that is not rented in advance; $g(\mathbf{x}, v)$ is the customer loss rate due to the lack of available vehicles upon arrival is proportional to the total number of customers approaching the system; α is the requirement on the maximum customer loss rate.

As indicated in (1), the objective of the optimization model is to minimize the overall daily cost, including rental cost for parking spots, depreciation cost of vehicles, and rebalancing cost. The practice of adopting daily cost has been seen in other studies, e.g., (Jorge et al., 2012, Cepolina and Farina, 2012). Equation (2) defines the main constraint of this optimization model, which sets an upper bound on the rate of lost customers. This constraint can be regarded as placing a requirement on the LoS that the system must provide. Here LoS is defined as the No. of satisfied customer divided by the total realized demand, which is related to customer loss rate. (3)-(5) define the search space.

For easier computation, similar to the work by Cepolina and Farina (2012), the aforementioned model was transformed into the following relaxed model. In this transformed optimization model, violation of constraint (2) is incorporated into the objective function as a penalty cost. μ is considered a large number since LoS is an important performance metric in the analysis. Note that in the problem formulation in Equation (6) transportation

cost is accounted for indirectly, but not modeled explicitly. Given that a required LoS is imposed through constraint (2), accounting for transportation cost would penalize the objective function twice due to customer loss, which would not be correct. Rebalancing cost, however, occurs independently of the penalty incurred from customer loss, and is modeled explicitly in Equation (6).

$$\min f(x, v) = C_i \sum_{i \in N} x_i + w * v + R(x, v) + Q(x, v) + \mu * (g(x, v) - \alpha)^2 \quad (6)$$

$x, v \in D$, D is the decision space

Due to the stochastic nature of the demand, accounting for the effects of rebalancing operations into the formulation is very difficult computationally, as rebalancing operations are responsive to real-time realization of the demand. Therefore, instead of taking an analytical approach to solve the above problem, this study favors a simulation-based approach. A discrete event simulator (DES) is developed to estimate the value of $f(x, v)$ given x and v . As indicated in (7), $\hat{R}(x, v)$, $\hat{Q}(x, v)$, and $\hat{g}(x, v)$ are the estimators of the function $R(x, v)$, $Q(x, v)$, and $g(x, v)$, respectively. Provided with the decision variables, parameters, and the number of simulations (for example, K), the simulator can generate the required number of sample points containing the rebalancing cost (r_k), the extra parking cost (q_k), and the rate of lost customers (g_k) under each realization of demand. In the simulator, each sample point is calculated using the equations shown from (8) – (10). γ_{ij} is the unit rebalancing cost from zone i to zone j ; r_{ijt} is the number of vehicles redistributed from zone i to zone j at time step t ; p_i is the unit temporary parking cost at zone i ; s_{it} is the number of vehicles at zone i at time step t ; d_{ijt} is the total vehicle pick up demand from zone i to zone j at time step t . The following section provides more details about the DES.

$$f(x, v) \sim C_i \sum_{i \in N} x_i + w * v + \hat{R}(x, v) + \hat{Q}(x, v) + \mu * (\hat{g}(x, v) - \alpha)^2 \quad (7)$$

$$\hat{R}(x, v) = \frac{\sum r_k}{K}, \quad r_k = \sum_{t \in T} \sum_{j \in N, \neq i} \sum_{i \in N} \gamma_{ij} * r_{ijt} \quad (8)$$

$$\hat{Q}(x, v) = \frac{\sum q_k}{K}, \quad q_k = \sum_{t \in T} \sum_{i \in N} p_i * \max(s_{it} - x_i, 0) \quad (9)$$

$$\hat{g}(x, v) = \frac{\sum g_k}{K}, \quad g_k = \frac{\sum_{t \in T} \sum_{i \in N} \max(\sum_{j \in N, \neq i} d_{ijt} - s_{it}, 0)}{\sum_{t \in T} \sum_{j \in N, \neq i} \sum_{i \in N} d_{ijt}} \quad (10)$$

3.2 Discrete Event Simulator

The discrete event simulator (DES) was developed in MATLAB. The randomness of the demand and a sub-optimization model to calculate rebalancing operations are coded into the simulator. The “passing of time” in the simulator is based on a fixed time increment.

3.2.1 Assumptions

This study makes several assumptions to define the behavior of customers and operators in the VSS being addressed. These assumptions are coded into the simulator and applied to guide its progress.

1. 17 hours (07:00-24:00) are simulated and analyzed every day. Each time step in the simulation model represents 20 minutes in reality.
2. The travel demands between zones are modeled as non-stationary Poisson processes. For each time step, there is an O-D matrix representing the arrival rates of the Poisson distributions that calibrate the travel demand between zones.
3. Two concepts related to demand modeling are defined in this study: time segment and demand pattern. The time segment is defined as a period in a day in which the same O-D matrix applies for each time step. The demand pattern characterizes the demand during a whole day. More specifically, one particular demand pattern contains the O-D matrixes for all the time segments in a day.
4. Within each zones, it makes no difference at which parking spot customers pick up or return vehicles. However, if a zone runs out of vehicles, customers who demand vehicles there will be lost. Similarly, if a zone runs out of parking spots, customers will not travel to another zone and will have to drop off the vehicle at a nearby location.
5. In order to simplify the computational process, instead of directly optimizing each v_i , we optimize the total number of vehicles and apply the distribution rule specified by (11). The heuristic rule allocates vehicles to each zone at the beginning of the day in proportion to the customer arrival rates during the first time segment, while being constrained by the capacity of the zone. This study will test the efficiency of the heuristic allocation rule later by comparing the results obtained under such a rule to the results of an optimization analysis where the initial vehicle distribution is individually optimized. λ_{i1} is the arrival rate of customers at zone i during the first time segment of an operating day.

$$v_i = v * \frac{\lambda_{i1}}{\sum_{i=1}^N \lambda_{i1}} \quad (11)$$

6. The DES incorporates an option to conduct rebalancing activities. At the beginning of every hour, an integer-programming model is applied to compute the rebalancing decisions between zones. The following section presents the formulation of the optimization model for calculating the rebalancing operations.

3.2.2 Integer-programming model for hourly rebalancing

$$\min \sum_{j \in N, \neq i} \sum_{i \in N} r_{ijt} \gamma_{ij} \quad (12)$$

$$L_{it} \leq s_{it} - \sum_{j \in N, \neq i} r_{ijt} + \sum_{j \in N, \neq i} r_{jit} \leq U_{it} \quad (13)$$

$$\sum_{j \in N, \neq i} r_{ijt} \leq s_{it} \quad (14)$$

$$r \in \mathbb{Z}^+ \quad (15)$$

As indicated in the model, the objective is to find the optimal rebalancing operation that minimizes cost and guarantees each zone can meet a specified LoS in the next hour in a demand responsive manner. Despite the LoS on satisfying arriving customers, a requirement is also set for the rate of successfully returning vehicles to the existing parking lots. Inspired by Schuijbroek et al. (2013), this model calculates the upper (U_{it}) and lower bounds (L_{it}) of vehicles that can be stocked at each zone to meet the required LoS on vehicle returns and pickups, respectively. This enables considering explicitly the stochastic nature of the demand and its variations (i.e. changing pick-up and drop-off rates) across the whole day. This formulation is indicated in (13). Constraint (14) limits the rebalanced-out vehicles to no more than the total number of existing vehicles in the zone, while (15) constrains the solution to non-negative integers.

The following equations indicate how to calculate the bounds under different situations. To simplify the computation, instead of formulating vehicle returns and pick-ups as an M/M/1/K queue, as done by Schuijbroek et al. (2013), this paper identifies three situations and calibrates the arrival and departure of vehicles for each situation by a probability distribution. Here, λ_{it} and μ_{it} are the customer arrival rate and vehicle return rate, respectively, for the next hour after time t .

$$a) \quad \lambda_{it} = 0, \mu_{it} \neq 0$$

$$U_{it} = x_i \quad (16)$$

$$L_{it} = \arg \min_k \sum_{x=0}^k \frac{(\mu_{it})^x e^{-\mu_{it}}}{x!} \geq 1 - \alpha \quad (17)$$

$$b) \quad \mu_{it} = 0, \lambda_{it} \neq 0$$

$$L_{it} = 0 \quad (18)$$

$$U_{it} = x_i - \arg \min_k \sum_{x=0}^k \frac{(\lambda_{it})^x e^{-\lambda_{it}}}{x!} \geq 1 - \alpha \quad (19)$$

$$c) \quad \mu_{it} \neq 0, \lambda_{it} \neq 0$$

$$L_{it} = \arg \min_k \sum_{x=-\infty}^0 P(x, \mu_{it}, \lambda_{it}) + \sum_{x=0}^k P(x, \mu_{it}, \lambda_{it}) \geq 1 - \alpha \quad (20)$$

$$U_{it} = x_i - \arg \min_k \sum_{x=-\infty}^0 P(x, \lambda_{it}, \mu_{it}) + \sum_{x=0}^k P(x, \lambda_{it}, \mu_{it}) \geq 1 - \alpha \quad (21)$$

* P is the Skellam distribution

(16)-(17) show the calculation when there are only vehicle pickups in the next hour. In this case, the upper limit of stocked vehicles is simply the capacity of that zone, as indicated in (16), although there must also be a minimum number of vehicles so that a certain number of arriving customers can be served, which is the $(1 - \alpha)$ -th quantile of the Poisson distribution calculated by (17). Similarly, (18) and (19) show the case when there are only vehicle returns. With no customers requiring vehicles, the lower bound of the vehicle stock is zero, while the upper bound is the total capacity minus the $(1 - \alpha)$ -th quantile of the Poisson distribution for vehicle returns since a sufficient number of parking spaces must be reserved for the coming vehicles in the next hour. When both returns and pickups exist, as shown in (20) and (21), the $(1 - \alpha)$ -th quantile of the Skellam distribution used to calibrate the difference between two Poisson distributions is applied to calculate the upper and lower bounds.

Infeasibility may occur when calculating solutions for the rebalancing optimization model, e.g., when the total number of vehicles is not sufficient to satisfy the required LoS or when extra parking spaces are needed to accommodate excessive vehicles. In the case of infeasibility, no rebalancing will be executed in the next hour.

3.2.3 Simulation procedure

Figure 2 illustrates the process of the simulator starting with the first time step of the day. At the beginning of each time step, it calculates the number of temporary parking spots needed by comparing the number of vehicles at each zone to the total number of parking spots rented beforehand. Next, the simulator generates demands for each zone (d_{ijt}). For each demand outcome, the simulator generates a random number between 0 and 1, and then uses a O-D probability matrix to determine the destination of the demand. In this way, the procedure guarantees that for each pick-up, there will be a drop-off. Then the DES calculates the lost demand, the satisfied demand, and the arrival of vehicles ($u_{ij(t+t_{ij})}$), based on the travel time between zones (t_{ij}). If this time step is the beginning of every hour, then the integer-programming model introduced earlier is applied to calculate the rebalancing operations. Following that, the number of arriving vehicles at each zone in the subsequent time steps ($u_{ij(t+t_{ij})}$) is further updated based on the optimization results of the integer-programming model. The simulator then moves to next time step and updates the number of vehicles at each zone. This process is repeated until the simulation reaches the end of the day.

Figure 2 Procedure of simulating a day

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3.3 Computational procedure

This section addresses the computation issues arising in connection with the simulation-based optimization model. There are two main questions to be answered: 1) how do we identify potentially good solutions within the decision space and 2) how do we efficiently estimate their performance so that the computational effort is minimized. To answer these questions, this study modifies and implements a hybrid computation procedure combining PSO and OCBA. PSO aims to achieve an efficient search, while OCBA reduces the computational budget for the simulation.

3.3.1 Particle swarm optimization

Particle Swarm Optimization (PSO) is a population-based optimization technique developed by Eberhart and Kennedy (1995). The algorithm works similarly to the way individual members (or “particles”) in a group of birds or fish collaborate in search of the location of food (the optimal solution in the context of optimization). In the algorithm, the solutions are represented by the location of a swarm of particles that moves within the decision space to seek the best location. The goodness of each location is determined by the objective function value. Initially, the locations of all particles are randomly generated within the decision space. In later iterations, each particle moves towards its personal best location (*pbest*) and the global best location across the whole swarm (*gbest*). The search stops when a specified criterion is satisfied, e.g., after a maximum number of iterations or after only limited improvement can be achieved. Figure 3 shows how one particle in a swarm moves from one iteration to another.

Figure 3 Illustration of PSO

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Because PSO can be easily implemented and adopted for a vast array of problems, it has been applied to many fields including communication networks, control, robotics, and even entertainment (Poli, 2008). The successful

implementation of PSO on transportation systems was also demonstrated in many studies, e.g. Gong et al. (2012). The convergence of this algorithm was theoretically studied and explained by Clerc and Kennedy (2002).

However, the efficiency of PSO may be affected when stochasticity is introduced into the search process, because information, namely the ranking of the goodness of locations, may be diluted by noise. In such cases, the particles may move towards locations where their performance is not necessarily better. Therefore, many studies have introduced optimal computing budget allocation (OCBA) into PSO, which aims to obtain a more accurate ranking of solutions using a limited simulation budget. Such practice leads to higher efficiency in terms of updating the solutions and, hence, better performance of the search algorithm (Pan et al., 2006, Zhang et al., 2011).

3.3.2 Optimal computing budget allocation

Optimal computing budget allocation (OCBA) derives from the concept of ordinal optimization that emphasizes the relative order of candidate solutions rather than their exact performance. The underlying principle is that, instead of spending great effort estimating the exact performance of every solution by running a large number of simulations, a ranking of solutions with sufficient accuracy can be obtained using a smaller computational budget. Studies on OCBA aim to generalize certain rules to determine the optimal allocation of the budget.

The accuracy of the ranking of solutions is measured by the probability of correct selection - or $P(CS)$ - which is defined as the probability that the selected best design is indeed the true best design (Dai, 1996). According to Shi (2000), $P(CS)$ is approximated by (22) and (23). The derivation of the equations is not presented here and readers can refer to Shi (2000) if interested. Several budget allocation rules have been derived to maximize $P(CS)$. One such rule comes from Chen et al. (2000), whose numerical experiments showed that this allocation rule generally led to better results than other rules. Therefore, Chen et al. (2000)'s allocation rule is adopted in this study. (24)-(26) provide the details of this allocation rule. In brief, in each iteration, this rule picks the best solution and allocates a simulation budget according to its relative variance; for the rest of the solutions, the rule attempts to allocate more of the simulation budget to ones with a larger sample variance or a better sample mean. The reader can find more details about how this rule is deduced in Chen et al. (2000).

$$P_{cs(between\ i\ and\ j)} = P(\bar{X}_i < \bar{X}_j) = \phi\left(\frac{\bar{X}_j - \bar{X}_i}{\sqrt{\frac{s_j^2}{N_j} + \frac{s_i^2}{N_i}}}\right) \quad (22)$$

$$P_{cs(i\ is\ best\ design)} \approx \prod_{i \neq j}^n P_{cs(between\ i\ and\ j)} \quad (23)$$

$$\frac{N_i}{N_j} = \left(\frac{s_i/\delta_{b,i}}{s_j/\delta_{b,j}} \right), \quad i, j \in N, i \neq j \neq b \quad (24)$$

$$N_b = s_b \sqrt{\sum_{i=1, i \neq b}^k \frac{N_i^2}{s_i^2}} \quad (25)$$

$$\delta_{b,i} = \bar{X}_i - \bar{X}_b \quad (26)$$

n is the number of solutions

ϕ is the standard normal distribution

$\bar{X}_i$ is the posterior distribution of the unknown mean X_i , $\bar{X}_i \sim N(\bar{X}_i, \frac{s_i^2}{N_i})$

$\bar{X}_i$ and s_i^2 are the sample mean and the sample variance of solution i

N_i is the number of simulations allocated to solution i

b is the best solution based on sample mean

3.3.3 Computation procedure in this study

This study develops a new version of PSO+OCBA to solve the optimization problem. The motivation is to guarantee a certain level of accuracy of the ranking of solutions in each iteration, specifically to satisfy a pre-defined $P(CS)$. In such a case, PSO is expected to update the solutions more effectively. The target of achieving a certain level of $P(CS)$ was not emphasized in past studies. Most studies related to OCBA focus more on the theoretical aspect of the algorithm, where more efforts were dedicated to discuss the computational efficiency rather than the optimality of the final solutions (Shi, 2000, Dai, 1996, Chen et al., 2000). The modified computation procedure is as follows:

1. Randomly generate the initial locations of all particles in the feasible design space.
2. Apply OCBA to estimate the performance of all particles at their current locations:
 - i. Perform an equal number of simulations for all particles.
 - ii. Calculate $P(CS)$ using (22) and (23).
 - iii. If $P(CS)$ is satisfied or the maximum budget allowed for a single solution (MAS) is reached, go to step 3; if not, increase the total value of the simulation budget, use (24)-(26) to allocate the extra budget, run simulations accordingly, and go back to step ii.

3. Based on the results obtained from step 2, update $pbest$ and $gbest$, and the velocity and location of each particle.
4. For each particle, use OCBA to determine its $pbest$ between the new location and the existing $pbest$:
 - i. Perform an initial number of simulations for the new location.
 - ii. Use (22) to estimate the $P(CS)$ between the new location and the existing $pbest$.
 - iii. If $P(CS)$ is satisfied or MAS is reached, update $pbest$ and go to step 5; if not, use (24)-(26) to allocate the extra budget, run simulations accordingly, and go back to step ii.
5. For the current set of $pbests$, use OCBA to select the $gbest$:
 - i. Use (22) and (23) to estimate $P(CS)$.
 - ii. If $P(CS)$ is satisfied or MAS is reached, update $gbest$ and go to step 6; if not, use (24)-(26) to allocate the extra budget, run simulations accordingly, and go back to step i.

Check if either the maximum number of iterations is reached, or if $gbest$ does not change for a number of consecutive iterations. If either is true, stop; if not, go to step 3.

4 APPLICATION

This section presents a case study on a car-sharing system under planning in Singapore. It illustrates the use of the proposed methodology and tests the efficiency of the methodology with respect to finding an optimal VSS design. A sensitivity analysis is conducted to see how different factors may influence the final decision.

The analysis is implemented in MATLAB on a desktop computer (Intel 3.30 GHz Core i5 and 8 GB of memory) with a Microsoft Windows 7 operating system. At every three time steps, the IBM CPLEX optimizer for MATLAB is called to solve the hourly rebalancing optimization model.

4.1 Problem setting

The prototype problem is assumed to be a planned VSS based in Singapore that includes 20 zones. The whole geographic region is shown in Figure 4. The picture is provided by Map data ©OpenStreetMap contributors. The 20 zones are centered at Raffles Place Mass Rapid Transit (MRT) (S1), One-North MRT (S2), City Hall MRT (S3), Tanjong Pagar MRT (S4), Buona Vista MRT (S5), Expo MRT (S6), Clementi MRT (S7), Tiong Bahru MRT (S8), Pasir Ris MRT (S9), Bishan MRT (S10), Ang Mo Kio MRT (S11), Lavender MRT (S12),

Commonwealth MRT (S13), Jurong East (S14), Orchard MRT (S15), Somerset MRT (S16), VivoCity MRT (S17), Clarke Quay MRT (S18), Bugis MRT (S19), and Chinatown MRT (S20).

Figure 4 Geographical setting
ABOUT HERE

As the system is currently under planning considerations, historical usage data is not available. Therefore, this study adopts the following procedure to make reasonable assumptions on the demand profile. First, the average daily demand is proportioned to that of Car2Go (2014) in Minneapolis, accounting for the difference between the populations of Singapore and Minneapolis. The reason to refer to Minneapolis' usage data is that it is published on the government website, and therefore, it is a reliable data source. As this study only covers a partial area of Singapore, the volume is reduced. In the end, this study assumes the average daily trips in the system to be around 1,650. The day is partitioned into four time segments: morning (T1) (0700-0900) and evening peaks (T3) (1800-2100), one segment between these two peaks (T2) (0900-1800), and another after the evening peak (T4) (2100-2400). Different proportions of total daily demand are then assigned to these four time segments, with 0.125 to T1, 0.55 to T2, 0.2 to T3, and 0.125 to T4. The aforementioned zones are divided into three categories: working places (S1-S6), residential areas (S7-S13), and entertainment areas (S14-S20). On the basis of temporal and spacial categorization as well as geographical and demographics information, with reference to the demand characteristics summarized in Vogel et al. (2011), this study makes assumptions about the travel flows between different geographical zones at different times of a day. Figure 5 and Figure 6 display the hourly vehicle pickup and return rates for different zones throughout the day. The vehicle pickup rates added by the probability matrices that are used to generate the return rates calibrate the Poisson distributions used to model the travel demand.

Figure 5 Hourly vehicle pickup rates
ABOUT HERE

Figure 6 Hourly vehicle return rates
ABOUT HERE

Major assumptions of the daily costs are shown in Table 1. Meanwhile, the depreciated daily cost of one vehicle is assumed to be €17, and the rebalancing cost per vehicle per 20 minutes is €4. Except for temporary parking

costs based on market rates in Singapore (2016), all other costs are adopted from Jorge et al. (2014). For convenience of presentation, all costs are displayed in Euros, as in the study by Jorge et al. (2014).

Table 1 Cost parameters
ABOUT HERE

4.2 Benchmark calculation and evaluation

To demonstrate the necessity of incorporating rebalancing operations into the decision-making process of the planning decisions, as well as to examine the effectiveness of the proposed methodology, this study first applies an approach inspired from (Shu et al.) to obtain a benchmark solution. Similar to this study, Shu et al. (2010)'s approach also formulates travel demand into non-stationary Poisson processes. They derived and solved an approximation model in order to obtain an optimal distribution of vehicles and parking spaces in the case whereby the system reaches the equilibrium state. Besides the difference in the choice of modeling and computation method, the other major differences between their study and this paper are: 1) the objective of their optimization model is to maximize the expected number of customers being served, and 2) their analysis does not incorporate rebalancing operations. Since this study and the one presented here rely on the same dataset, it is considered an appropriate benchmark for the purpose of comparing the results obtained.

Figure 7 displays the benchmark solution obtained by applying Shu et al. (2010)'s approach to on our case study, which is solved using IBM ILOG CPLEX Optimization Studio. The solution is then evaluated using the DES developed in this study - but with hourly rebalancing operations being taken out - as this is the original assumption of (Shu et al., 2010). 500 demand scenarios are generated for the performance estimation. The results suggest that 1027 parking spaces and 547 vehicles are needed to satisfy 0.94 of the daily demand, which is perceived as a relatively large system. Dividing the total cost against the average number of trips that can be satisfied, without considering other costs and expenses (e.g. fuel cost), it suggests that at least €8.35 should be charged for each trip to make a breakeven case. This price charge is relatively high in this case study, especially considering that the average distance between zones is only 11km and the rental cost of a parking space is assumed to be quite low.

Figure 7 The benchmark solution
ABOUT HERE

Table 2 Performance of the benchmark solution
ABOUT HERE

4.3 Optimization result

In this section, the methodology proposed in this study is applied to identify the optimal system configuration for the aforementioned problem. The optimal solutions identified under this circumstance are then compared with the benchmark solution summarized in Figure 7 and Table 2. In Figure 7, for each decision variable, the six bars from left to right correspond to solutions obtained under different runs with the last bar being the average. For example, in the first category, the number of parking spaces in the zone of Raffles Place, the first five bars indicate the solutions from Run 1 to Run 5, and the last bar is the average of the five solutions. The optimization algorithm is run five times to allow fair comparisons, as well as to determine the robustness of the computational procedure. The parameters of PSO are set according to Eberhart and Shi (2000) with inertial weights of 0.729 and constriction factors of 1.494. The number of particles in a swarm is set to 15, which is a relatively small number, to reduce the computational effort demanded in each iteration. The study sets MAS as 250, since under this condition the simulation is already well converged. The parameter μ is assumed to be 2,500,000. This choice is tuned by running several rounds of optimization under different values, and the value of μ resulting in a relatively stable performance is selected. To adjust for the disparity in objective function between this study and the benchmark study, the required level of customer loss rate (α) in our model is set to 0.06. The intent is to see if, with equal customer satisfaction, the solution obtained using our approach can outperform the benchmark solution in terms of cost savings. If it can, how much cost can be saved and what contributes to the improvement? Figure 8 displays the optimal solutions of the five runs and the mean values of the solutions. The performances of the optimal solutions obtained from the five runs are summarized in Table 3. The average computation time for each optimization run is 6,228 seconds.

Figure 8 Optimal solutions of five runs
ABOUT HERE

The results suggest that the solutions obtained from each run of the algorithm are not identical, but their overall performances are nearly the same. This demonstrates that the computational procedure is relatively stable. Meanwhile, compared with the decisions on the number of parking spaces at each zone, the variation in the total number of vehicles purchased is relatively small from one run to another. This can be explained by the fact that the unit cost of one vehicle is assumed to be much higher than the unit cost of one park space. Therefore, the

fluctuations in the number of parking spaces at each zone only have a minor effect on the objective function value. In contrast, more consistency can be identified regarding the total number of parking spaces within the system. This study defines another indicator, the Vehicle-to-Capacity (VTC) ratio, which is calculated by dividing the total number of vehicles by the total number of parking spaces. It is found that across the five runs, VTC is maintained at a relatively stable level ranging from 0.31 to 0.39 with a mean of 0.35.

Table 3 Performance of optimal solutions
ABOUT HERE

The study also dives further into the operational level decisions, specifically to examine how rebalancing operations are conducted throughout a day. The study takes the solution of Run 2 and averages the number of rebalancing operations across all demand scenarios as an example. In other words, the planning decisions obtained in Run 2 is further evaluated in another round of simulation, and the rebalancing decisions calculated in every demand scenario of the simulation are averaged to obtain the number shown in Figure 9 and Figure 10. Figure 9 displays the number of vehicles rebalanced out from each zone throughout a day, while Figure 10 shows the number of vehicles rebalanced in. Two patterns with respect to rebalancing operations can be observed from these two figures. First, a sharp increase in rebalancing operations normally happens at transitions between time segments. For example, at the beginning of T2 (starting from 9am), a large number of vehicles are transported from working places to either residential areas or entertainment areas, because most activities now originate from these areas and they are the areas where the vehicles in need. Second, when there is more demand imbalance (i.e. the absolute difference between hourly vehicle return rates and vehicle pickup rates at each zone), more rebalancing activities are carried out. As shown in Figure 9 and Figure 10, the number of rebalancing operations stays much higher throughout T3 (6pm-21pm) compared with T2, as the average demand imbalance at one zone is only 0.95 in T2, while it goes up to 8.45 in T3. It can also be noticed that more rebalancing operations are needed at the beginning and end of days, due to morning and evening rush hours. In sum, the results here indicate that the rebalancing operations are helpful in terms of handling both change in demand as well as unbalanced demand.

Figure 9 Average number of vehicles rebalanced out of each zone at the beginning of each hour
ABOUT HERE

Figure 10 Average number of vehicles rebalanced into each zone at the beginning of each hour
ABOUT HERE

4.3.1 Comparison with benchmark solution

This subsection looks into the comparison between the benchmark solution and the solutions obtained using the methodology of this study. As seen in Figure 7 and Figure 8, the optimal scale of the system is significantly reduced when rebalancing operations are considered. In the optimal solutions of this study, a much smaller number of vehicles and parking spaces is required to achieve the same level of customer satisfaction (the same customer loss rate), namely an average of 207 vehicles and 590 parking spaces using our approach versus 547 vehicles and 1027 parking spaces in the benchmark solution. This cost saving in initial investment is also visualized in Table 2 and Table 3, as less than half of the setup cost of the benchmark solution is incurred for the solutions of this study. The study also calculates the VTC of the benchmark, which is 0.53, suggesting that more vehicles are required should the system possess the same number of parking spaces as in our solutions. Since this study assumes a much higher unit cost of vehicles, the VTC of the solutions obtained in this study seems to be more cost effective. Furthermore, Table 2 and Table 3 indicate that, in the operational level, conducting rebalancing operations is effective in terms of reducing temporary parking cost. Overall, the solutions obtained in this study achieve an average cost reduction of €6,436, roughly 50% of the benchmark solution, without compromising the LoS.

Another interesting observation can be made on the decisions with respect to the number of parking spaces at each zone. In the benchmark solution, as indicated in Figure 7, the number of parking spaces at each zone seem to coincide with the maximum hourly demand imbalance of the zone. In the optimal solutions obtained in this study, this set of decisions seems not only to relate to the demand imbalance but also to the unit rent cost of the parking spaces at each zone. To further examine this hypothesis, this study conducts a correlation analysis between the numbers of parking spaces with the maximum hourly demand imbalance as well the unit parking space cost for the two sets of solutions obtained. Table 4 summarizes the results of the correlations analysis: in the benchmark solution, the distribution of parking spaces is strongly positively correlated with demand imbalance, while the optimal solutions obtained in this study not only takes the demand profiles into account, but also give more attention to cost efficiency. This might be largely attributable to the fact that performing rebalancing operations provides more flexibility to the higher level planning decisions, thus the cost issue can also be accounted for.

Table 4 Correlation analysis between No. of parking spaces and possible relevant factors
ABOUT HERE

4.4 Further analysis

4.4.1 Test of the initial vehicle distribution rule

As mentioned earlier, a heuristic rule is applied to decide on the distribution of vehicles at the beginning of the day, after the total number of vehicles is determined. To investigate the efficiency of this rule, another round of optimization analysis is carried out where the number of vehicles at each subarea is optimized individually. Table 5 displays the estimated performance. A slight difference is observed between the solutions obtained using the more complex model here and the ones obtained using the heuristic rule (roughly a 7% decrease in total cost and a 3% increase in customer loss rate), though such difference is considered to be within an acceptable range. More importantly, the computational time increases significantly when the heuristic rule is not used, from 6,228 seconds to roughly 13,000 seconds, due to the increase in the dimensionality of the problem.

Table 5 Optimization results without using the heuristic rule for the initial vehicle distribution

ABOUT HERE

4.4.2 Sensitivity analysis

In the sensitivity analysis, three cost parameters—namely unit vehicle cost, unit parking space cost, and unit rebalancing cost—are doubled to see how the optimal solutions change in accordance with different assumptions. The study also conducts sensitivity analyses on the assumptions regarding the total demand and rebalancing frequency. Under each varied set of assumptions, five rounds of optimization are conducted. The averaged performance of each set of five solutions is then compared against the solutions under the original assumptions (displayed in Figure 8). The relative percentage changes are summarized in Figure 11.

Figure 11 Sensitivity analysis
ABOUT HERE

As seen in Figure 11, when the unit parking cost is doubled, the setup cost increases accordingly. There is little variation in the rebalancing cost and customer loss rate, but, as the parking spaces become more expensive,

the VTC increases by roughly 10%, indicating that less parking spaces will be utilized to accommodate an identically sized vehicle fleet, which consequently leads to an increase in the temporary parking cost. Overall, the increase in unit rent cost of parking spaces causes the total cost to be 25% higher than the original solutions.

The doubled unit vehicle cost has an even larger effect on the total cost, which increases by about 62% under the new assumption. The setup cost also increases by a larger percentage than when the unit parking cost was altered. Similar to the case for higher unit parking cost, a higher unit vehicle cost has almost no effect on the rebalancing cost and customer loss rate; however, the temporary parking cost and VTC change in opposite directions in this case, with the vehicles becoming even more expensive.

Meanwhile, a rise in the unit rebalancing cost is shown to have less effect on the total cost due to the lower percentage of the total cost it accounts for. It is also interesting to observe the trade-offs that exist between the rebalancing operations and planning decisions. When the unit rebalancing cost doubles, the total rebalancing cost does not directly increase by 100%. Instead, the optimal solutions slightly increase the scale of the system, particularly by renting more parking spaces to reduce the number of rebalancing operations needed, so there is a smaller increase in the total rebalancing cost. More parking spaces also contribute to a lower temporary parking cost and a lower VTC.

The figure also suggests that assumptions about demand play a critical role in the decision-making process. The doubled demand results in a significant increase for every cost item. More importantly, the observable increase in customer loss rate indicates that a larger penalty cost should be imposed to force the final solutions to conform to the required LoS.

Finally, this study widens the interval between two rebalancing operations from one hour to two hours. The results demonstrate the importance of choosing a proper rebalancing frequency. Under the varied assumption, although the rebalancing cost is reduced, the system has to prepare for demand in the upcoming two hours rather than upcoming one hour in the original solution. Therefore, more vehicles and parking spaces are required to accommodate the demand. It is also interesting to note that the reduction in rebalancing operations is accompanied by a decrease in temporary parking cost. One reason for this might be that fewer rebalancing operations result in less traffic flow, which means that less demand is satisfied. In turn, this reduces the number of occasions that temporary parking is needed.

In sum, the sensitivity analysis illustrates how changes to the assumptions might influence the optimal decisions, and further reveals the interactions and trade-offs within the system.

5 DISCUSSION AND CONCLUSION

This study has developed a decision-support tool for the design and management of VSS operating under uncertainty. It assists with determining the optimal configuration of a VSS by accounting for stochastic demand and the effect of conducting vehicle redistribution as part of daily operations. An optimization model is formulated that aims to identify the solution with the minimal cost but an adequate LoS. A simulation-based approach is devised to solve the model. The computational approach consists of a discrete-event simulator that assesses the performance of different system configurations, as well as a hybrid algorithm combining PSO and OCBA to efficiently search the decision space. The paper presents a case study of a Singapore-based car-sharing system to illustrate the application of the methodology, leading to several interesting observations and findings.

First, rebalancing is demonstrated as an essential consideration when determining the configuration of a VSS system. As reported in the case study, the optimal solutions obtained under the condition of no rebalancing result in significantly more parking spots being rented and more vehicles being purchased, which does not lead to higher customer satisfaction (namely the same customer loss rate). As the system designer can identify a more realistic system plan (No. of parking spaces and size of the vehicle fleet) for the geographical area of interest, incorporating rebalancing into the decision making model constitutes a fairer way of evaluating the value of a potential VSS. The case study also shows that rebalancing operations are most helpful for dealing with abrupt changes in demand flow as well as large demand imbalance. Second, by conducting optimization under varied assumptions in a case study, this study explicitly illustrates the trade-offs and interactions within a VSS. Third, regarding the solution approach, as demonstrated by several runs of the numerical experiments in the case study, the computational procedure is shown to have a stable performance. The heuristic rule devised in this study that helps to allocate vehicles among zones at the beginning of the day is also shown to be effective in terms of reducing computational effort. This finding provides an opportunity to properly simplify the decision-making process.

As for future work, there are several directions that would strengthen this study and address its limitations. First, the model could be applied under different assumptions and parameters. One opportunity is to incorporate revenue. Currently, the objective of the optimization model is to find a system design that incurs minimal cost while satisfying a pre-defined LoS, and a large penalty cost is incurred every time the constraint on LoS is not met. The consideration here is that LoS is critical for the survival of a VSS. As previously demonstrated in Singapore, poor service quality can lead to a car-sharing system's demise (2008a). However, it may take some effort to determine the penalty cost, as several warm-up runs of optimization are needed to see how the

computational procedure performs under different choices. The other opportunity is to apply the methodology to a different case study rather than Singapore. Singapore is a relatively compact area, which makes frequent rebalancing a viable solution in terms of both time and cost, while results may be different under other geographical areas. Modifying and applying the methodology to a bike-sharing system is also another future direction. This study investigates the planning and operating a car-sharing system, but the methodology could also accommodate analysis of a bike-sharing system. A different formulation of demand and another set of cost parameters might be required, but these assumptions can be easily modified in the current model. The larger usage volume of a bike-sharing system might cause some difficulty as it would require higher computational effort. Therefore, along with the applications to bike-sharing systems, further investigation on a solution approach is another research opportunity.

Second, different formulations of the rebalancing optimization model could be incorporated in the simulator to explore how they influence the overall performance, as well as the optimal strategic design of the system. In this study, an integer-programming model was adopted to calculate the hourly rebalancing scheme, which required at least 17 runs of the optimization model to obtain the rebalancing cost for one scenario. Therefore, most of the computation time in the simulation process was spent calculating the rebalancing decisions. Future studies could work on finding a heuristic rebalancing rule or investigating interactions between the frequency of rebalancing and the optimal system configuration, which may lead to a reduction in the computational complexity.

Thirdly, this study takes the PSO+OCBA approach to find the solution to the problem. Because of the stochastic nature of the problem, in addition to the fact that a heuristic search algorithm is used, the global optimality of the final solution obtained cannot be guaranteed. In this case, it remains an opportunity for future research to use and evaluate different algorithms that may be applicable to the problem of interest.

Finally, the case study shows that changes to the demand assumptions can severely affect the optimal configuration of the system. As the system is operated and deployed over a long time scale (e.g., months and years) and faces regular variation in usage patterns, future work should investigate the ability the ability to adapt to changing conditions. Flexibility in engineering design, akin to embedding real options in the system to “provide the right, but not the obligation, to change the system as uncertainty unfolds”, provides a worthwhile paradigm to further improve the lifecycle performance of the system for more tactical and strategic level capacity decisions (Cardin, 2014, Trigeorgis, 1996, de Neufville and Scholtes, 2011). Except for the study by Cardin et al. (2017), very few studies have considered flexibility and real options in the context of mobility on-

demand transportation systems. Looking into the problem of optimizing the site location itself is also an opportunity for future work, which would benefit greatly from the flexibility analysis paradigm introduced above.

ACKNOWLEDGEMENTS

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Table 1

Zones	Fixed daily parking fee (€)	Temporary parking fee (per 20mins) (€)
Raffles Place	4	1
One-North	3	0.75
City hall	4	1
Tanjong Pagar	4	1
Buona Vista	3	0.75
Expo	3	0.75
Clementi	2	0.5
Tiong Bahru	2	0.5
Pasir Ris	2	0.5
Bishan	2	0.5
Ang Mo Kio	2	0.5
Lavender	2	0.5
Commonwealth	2	0.5
Jurong east	3	0.75
Orchard	4	1
Somerset	4	1
Vivo City	4	1
Clarke quay	4	1
Bugis	4	1
Chinatown	4	1

Table 2

Setup cost (€)	Temporary parking cost (€)	Total cost (€)	Standard error	Customer loss rate	Standard error
12,360	585	12,945	13.10	0.06	0.00

Table 3

	Setup cost (€)	Rebalancing cost (€)	Temporary parking cost (€)	Total Cost (€)	Standard error	Customer loss	Standard error
Run1	5,451	1,164	50	6,666	4.47	0.059	0.000
Run2	5,276	1,141	110	6,527	4.23	0.061	0.000
Run3	5,256	1,120	55	6,432	4.22	0.062	0.000
Run4	5,104	1,173	39	6,316	4.33	0.061	0.000
Run5	5,351	1,151	106	6,607	4.28	0.062	0.000
Average	5,288	1,150	72	6,509	4.31	0.061	0.000

Table 4

	Correlation with hourly maximum demand imbalance / P-value	Correlation with unit parking space cost/ P- value
Benchmark solution	0.79 / 0.00	-0.25 / 0.28
Optimal solutions of this study	0.65 / 0.00	-0.42 / 0.06

Table 5

Setup cost (€)	Rebalancing cost (€)	Temporary parking cost (€)	Total Cost (€)	Standard error	Customer loss	Standard error
4,866	1,162	38	6,066	4.00	0.063	0.000

Figure 1

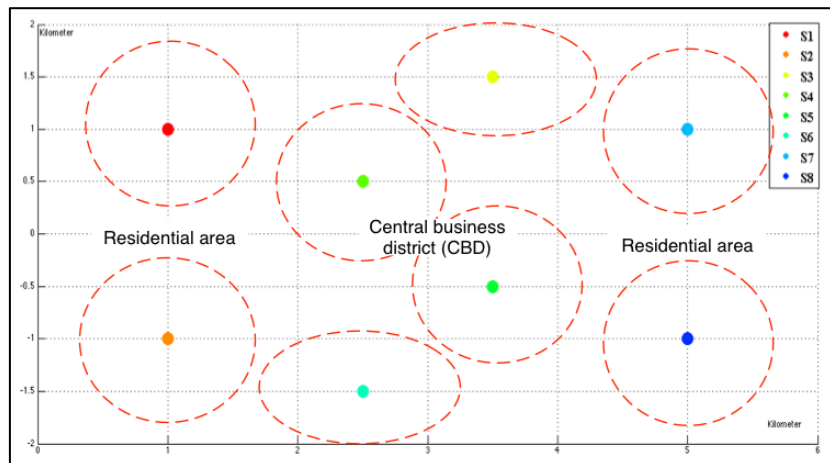


Figure 2

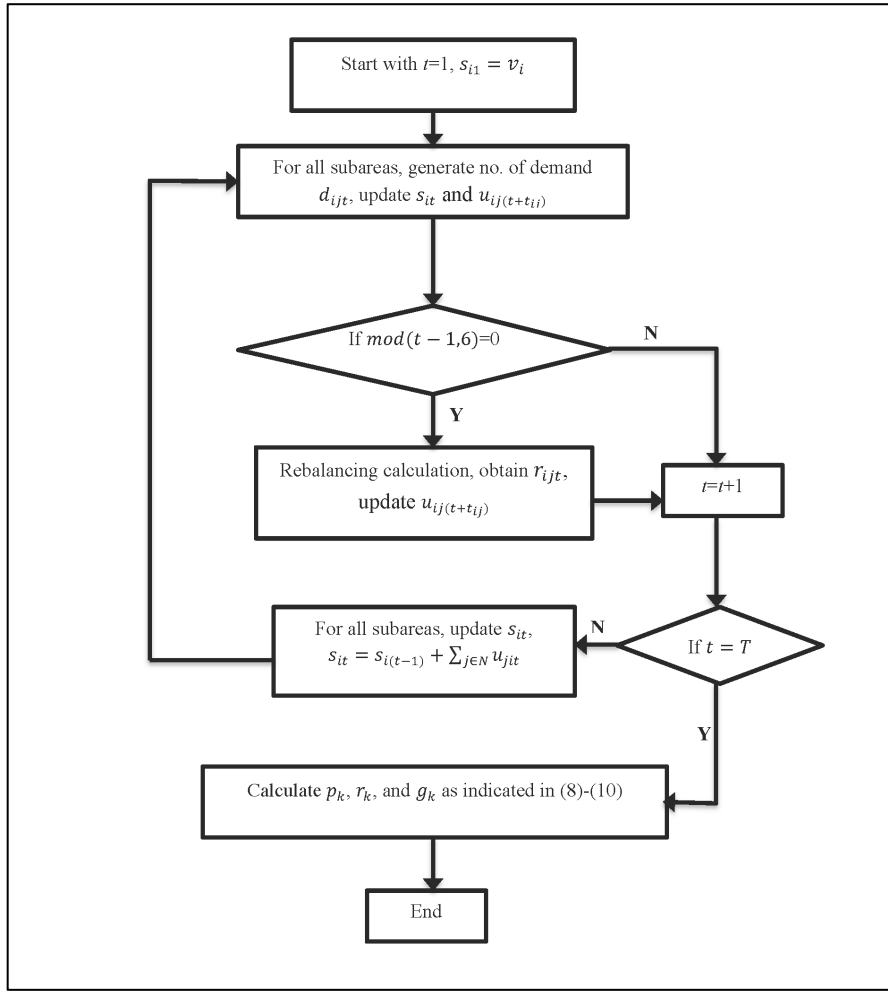


Figure 3

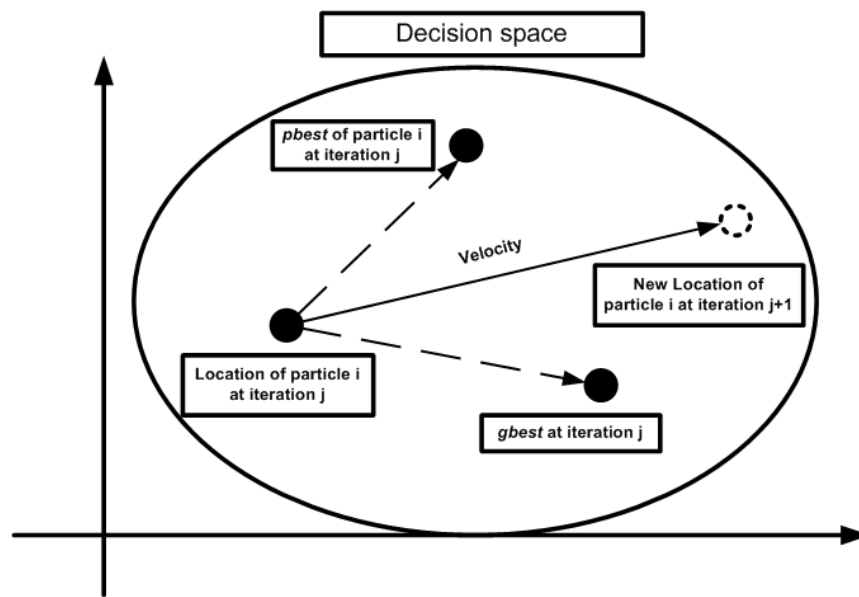


Figure 4

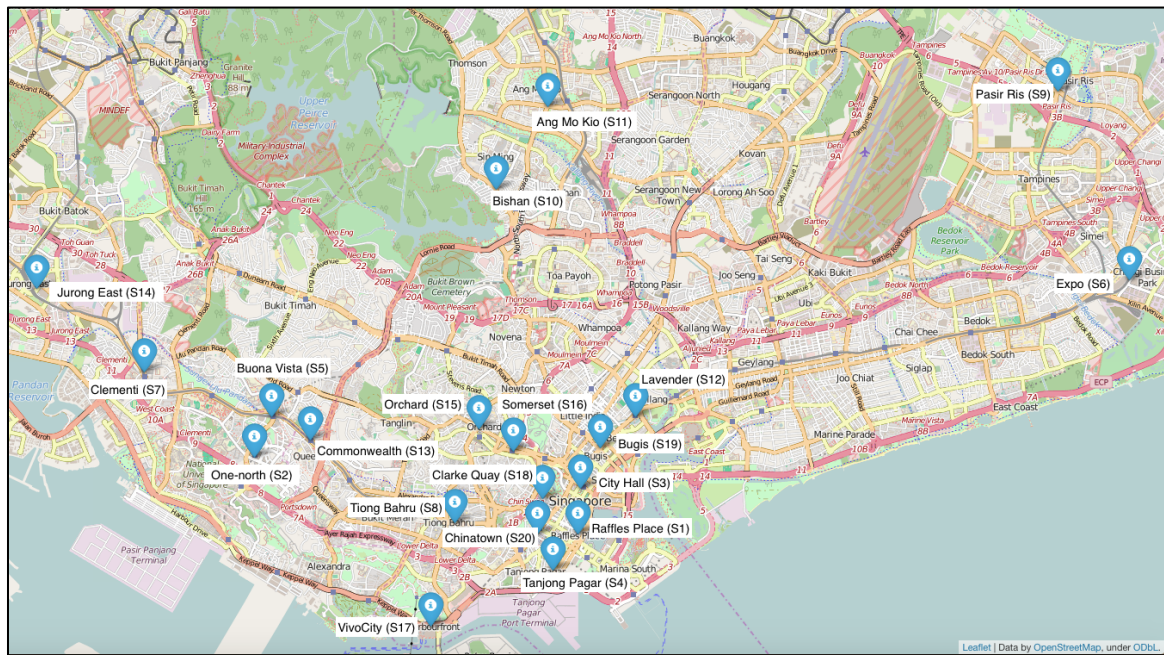


Figure 5

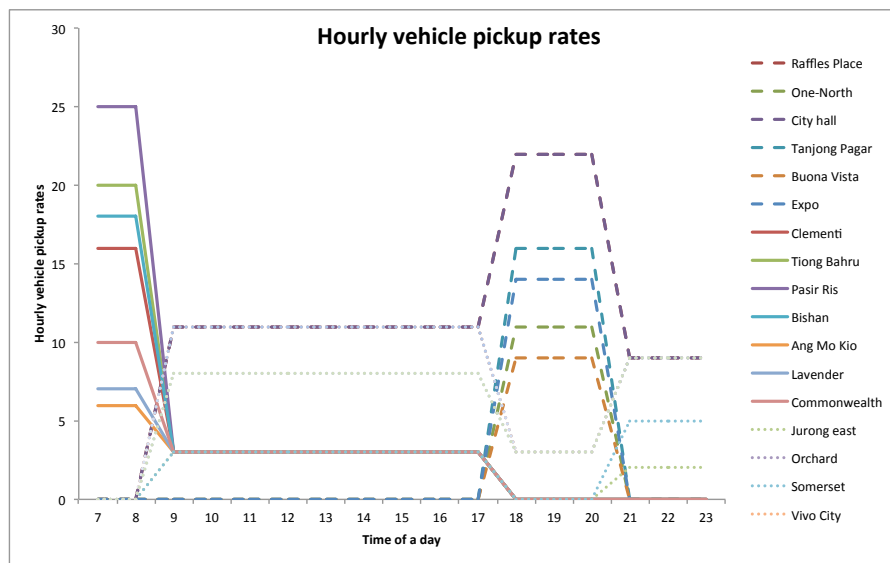


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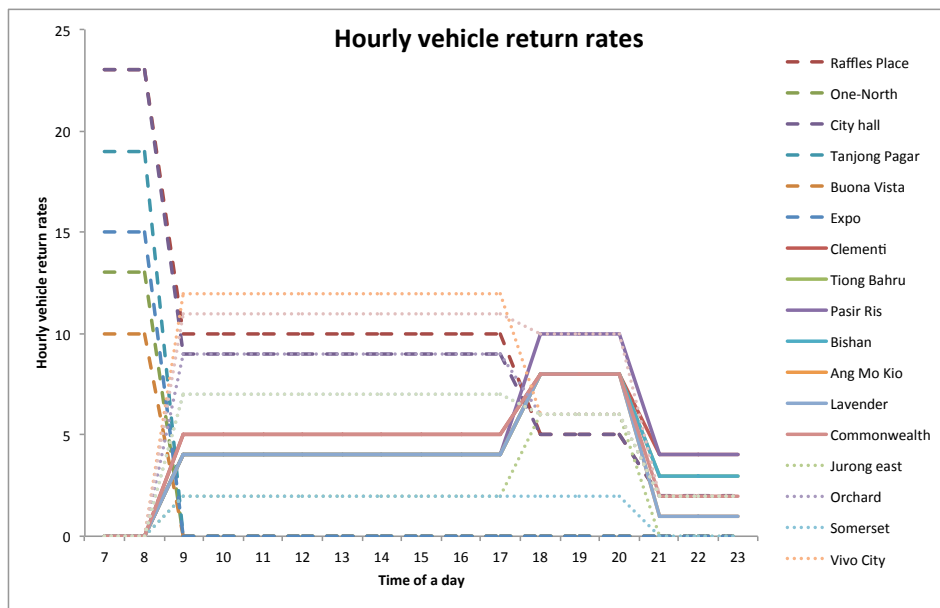


Figure 7

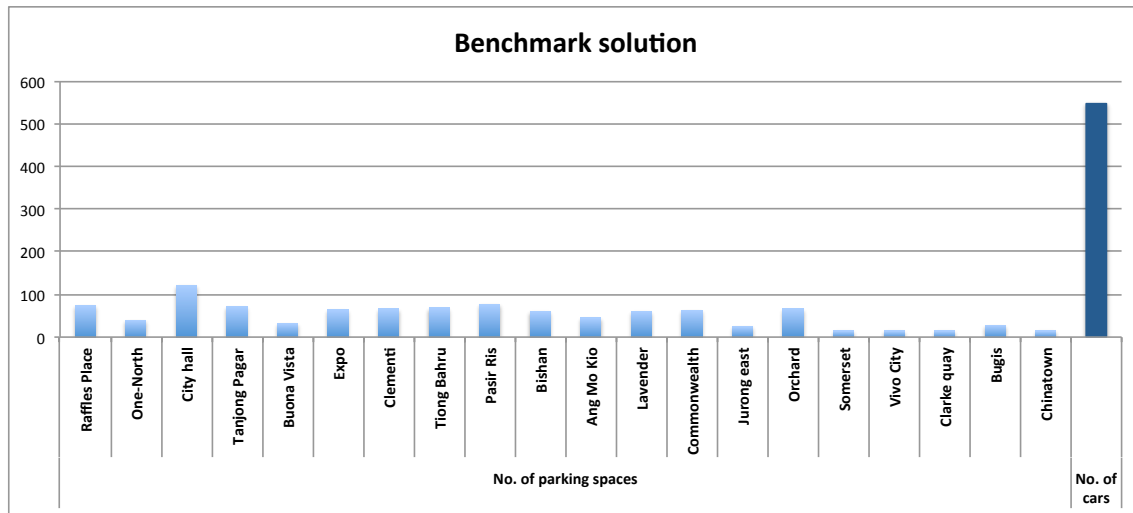


Figure 8

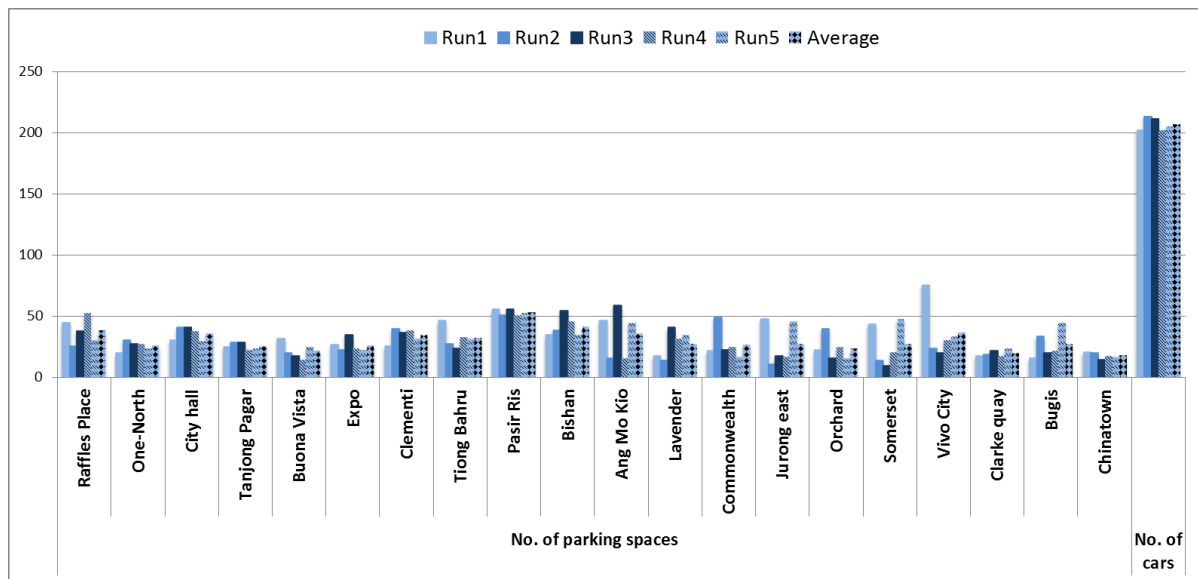


Figure 9

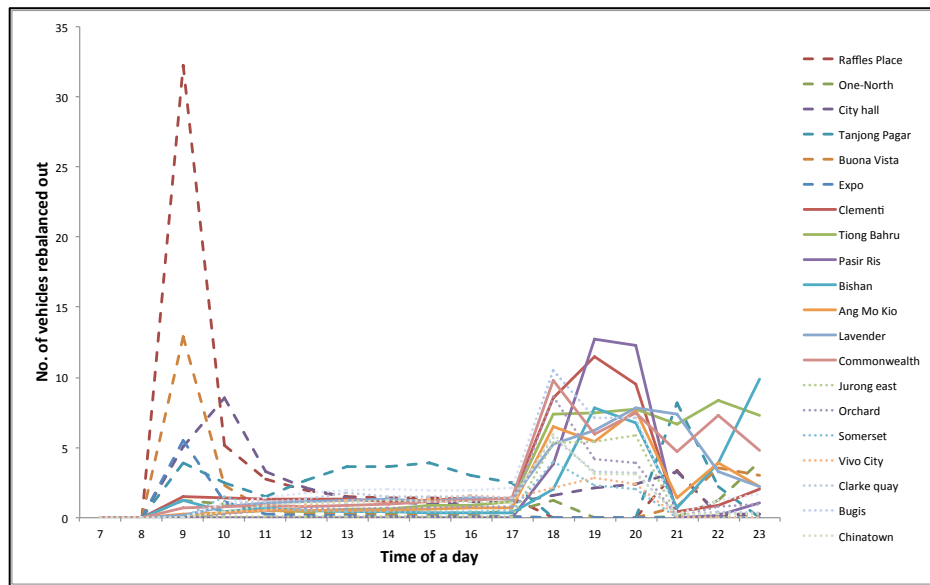


Figure 10

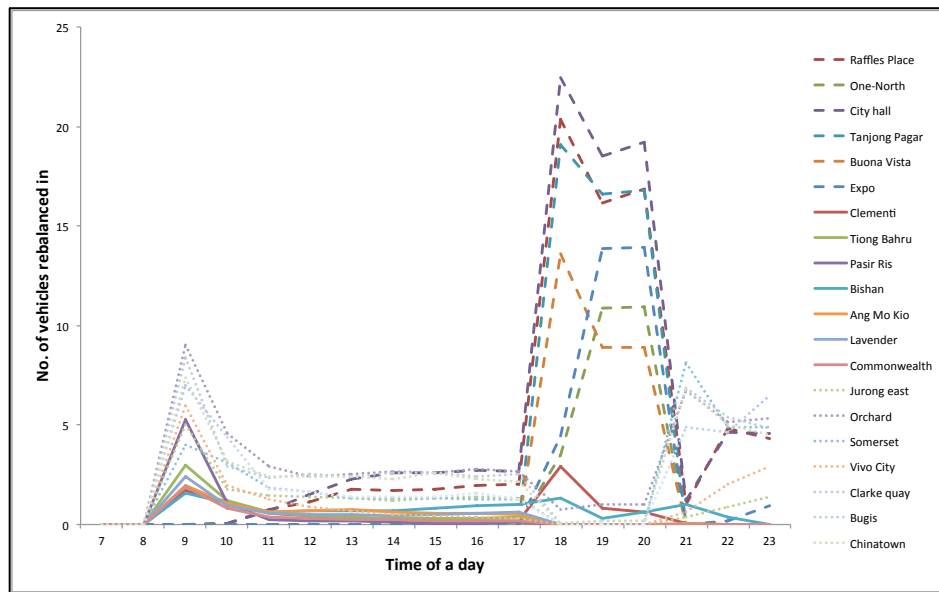
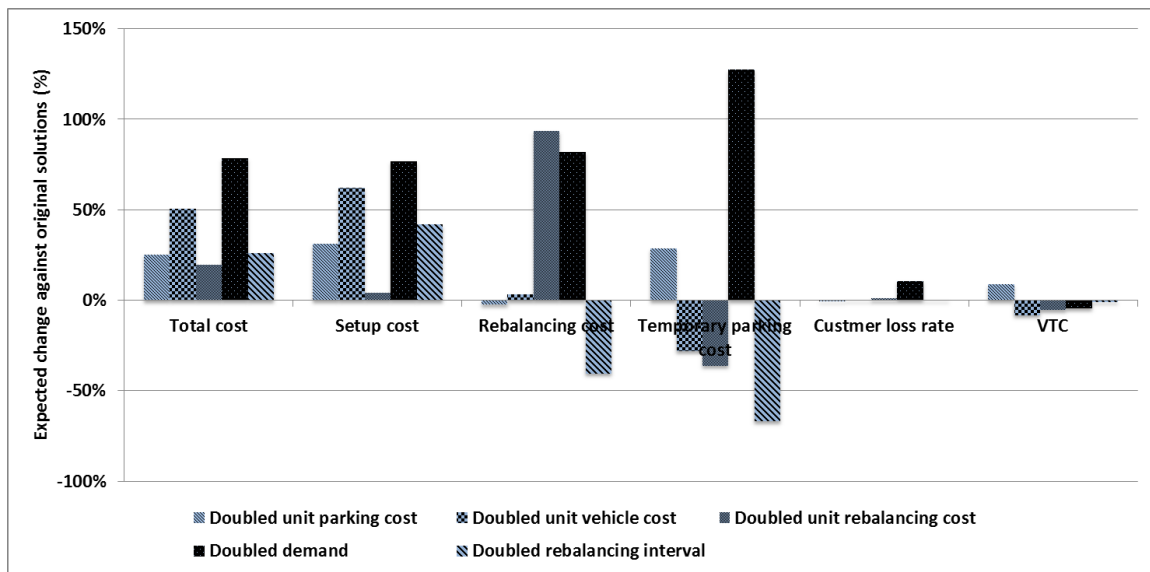


Figure 11



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