MODELLING THREE-DIMENSIONAL GEOLOGICAL DISCONTINUITIES FOR MINERAL EVALUATION

by

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July 1992

A Thesis submitted for
the Degree of Doctor of Philosophy
of the University of London

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ABSTRACT

Three-dimensional geological modelling plays an important part in the generation of reserve models for use in planning the exploration, evaluation and exploitation of most mineral deposits. Computerised techniques are widely used for geological modelling, as well as for three-dimensional geometric modelling in other fields.

The importance of using geological modelling to impose geological controls on computerised reserve/resource estimation has been widely recognised. Examples from the Cerro Colorado copper deposit (Riotinto, southwest Spain) and the Neves-Corvo copper-tin mine (southern Portugal), both in the Iberian Pyrite Belt; and the Bama-Brandelos copper deposits (Santiago, northwest Spain), provide good illustrations of this approach.

The main techniques used for geological modelling in commercially available systems suffer from several limitations. A system for modelling three-dimensional geological discontinuities, with a model structure based on storing sets of points, lines and triangles, provides several advantages. Implementation of this approach allows close links to be maintained between geological databases and the reserve models used for mine planning. Development of this modelling technique has concentrated on the following aspects:

- Links with drillhole and mapping databases
- Graphical presentation
- Interpretative hierarchies
- Interactive editing
- 3-D visualisation

- Conversion between 2-D and 3-D discontinuity models
- Geological analysis
- Conversion to volumetric models
- Geological controls on grade estimation
- Prediction and quantification of errors
- Reconciliation studies
- Areas requiring further study

This technique offers significant advantages over the approaches used in most existing computerised reserve estimation systems. In particular it facilitates greater geological control during the generation of reserve models, and allows an increased amount of geological detail to be presented to planning engineers. Where properly applied such advantages should lead to improved accuracy and precision in the resultant reserve models, thus facilitating more effective deposit evaluation, mine design, and production control.

DEDICATION

TO MY PARENTS

Who provided for my early education,
and have given moral support and encouragement
throughout the duration of my doctoral studies.

ACKNOWLEDGEMENTS

Whilst the work presented in this thesis is entirely the writers responsibility, it would have been impossible to complete the studies presented without the support and assistance of several individuals and organisations. In this regard I wish to acknowledge the following contributions:

- Dr Jeff O'Leary for introducing me to the field of computerised reserve estimation, and for promoting the funding of such research by the RTZ Group.

- Mr Colin Dixon, my internal supervisor, for suggesting the use of the University of London's PRI/IRL scheme, and for several reviews of the progress of my work which helped to keep me on the right track.

- Dr John Forkes, my external supervisor, for numerous fruitful discussions on my research work and other aspects of computerised reserve estimation, and for his persistence in prompting me to keep making progress towards my ultimate objective.

- All three of the above, for reviewing several early drafts of this thesis, and for helpful suggestions on ways of improving the final presentation.

- RTZ Technical Services Limited (TSL), Bristol, and RTZ Limited, London, for financial support, plus the freedom to carry out research work whilst maintaining my job as a geological consultant within the RTZ Group.

- Somincor (Sociedade Mineira de Neves-Corvo) for permission to use data from the Neves-Corvo deposit, as well as some of the work done for them in a consulting capacity.

- Rio Tinto Minera (RTM) for permission to use data from the Cerro Colorado and Bama-Brandelos copper deposits in Spain.

- Mr David Richards (Somincor) and Dr. Felix Garcia Palomero (RTM), who facilitated the use of data from their respective companies, as well as reviewing final drafts of relevant sections of this thesis.

- The geologists, and geological field crews, in Somincor and RTM, who provided the geological data and interpretations used to develop and test the modelling techniques presented here. In the final analysis a model can never be any better than the data and interpretations on which it is based.

- Fellow system developers and programmers at RTZ TSL who developed many of the library routines necessary for implementing the system in an X-Windows environment on Sun workstations. In particular Mark Howson and John Hibberd who were most closely associated with this aspect of the system's development.

- RTZ TSL for permission to use proprietary software to prepare many of the figures and plates presented in this thesis.

- Sheila Billard for drafting the large number of figures which I did not have the time, or inclination, to generate using a computer.

- Numerous colleagues within RTZ TSL, Riofinex, RTM and Somincor, as well as other friends and relations, who provided encouragement at various stages during the lengthy process of completing my research work and preparing this thesis. Thank you all for having had more faith in my ability to complete my work than I myself had on several occasions during the last 7 years.

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1. INTRODUCTION

1.1 Overview

The collection and interpretation of geological data has always played a fundamental part in mineral resource exploration, evaluation and exploitation. Technological developments in recent years have led to important changes in the type and amount of data collected, as well as the techniques available for interpretation purposes.

For instance, improved drilling technology has led to lower drilling costs per metre on many deposits, resulting in increased amounts of drilling data being available for reserve estimation purposes. In addition lower analytical costs, and the availability of low cost multi-element analytical methods (such as X-ray fluorescence and Induced Coil Plasma), mean that more samples tend to be taken and analysed for a wider range of elements.

On the interpretative side, advances in computer hardware and software mean that the data collected can be processed and interpreted using techniques which could not be applied manually. Such changes are reflected in the almost ubiquitous usage of personal computers at several stages during data collection and processing. In addition the widespread use of mechanical pen plotters, digitising tablets, screen graphics applications, and the storage of large volumes of information on magnetic disk or tape, reflect technological developments and price changes over recent decades.

The availability of increased amounts of raw data together with computerised processing techniques allow a much wider range of questions to be addressed during the evaluation of a mineral deposit. Such aspects include investigation of the effects of minor element contents on technical and economic viability, study of alternative extraction

strategies, use of different grade estimation techniques, and other detailed technical studies. In order to take advantage of such possibilities efficient strategies and techniques need to be applied in the administration and processing of geological data.

The 1980's saw the commercialisation of several software packages for geological data handling, reserve evaluation and mine planning. Nevertheless development of software for geological modelling has tended to lag behind developments on the mine planning side. This is illustrated by the fact that most existing packages model geology using techniques developed for modelling mine openings or grade variations. This thesis tackles this aspect from another angle, by considering the nature of the geological data available during reserve estimation, and developing a modelling technique which uses this as its starting basis.

The modelling approach developed represents a significant advance, which enables the cycle of data collection - interpretation - evaluation - decision making to be speeded up. It is also shown that the methodology adopted results in improvements in the accuracy and precision of reserve models. These advantages facilitate improved decision making and tighter planning control on existing mining operations. Such considerations are of increasing importance in an era of data explosion, and high financial costs, where mining operations need to make appropriate decisions on mineral extraction strategies in as short a timeframe as possible.

1.2 Research objectives

The general objective of the research work on which this thesis is based, was to investigate methods by which the geological input to computerised ore reserve estimates could be improved. Following an initial review of existing techniques, and work done previously on the deposits studied, attention was focused on the incorporation of geological discontinuities into orebody models. Aspects studied included the following:

- a review of methods currently used for the incorporation of geological information into computerised reserve models

- investigation of the use of interactive graphics as an aid to geological modelling

- consideration of alternatives to the use of regular block models as a means of representing orebody shapes

- closer linking of the geological interpretation and grade estimation stages in orebody evaluation

- approaches available for orebody modelling and reserve estimation in folded and faulted orebodies

This work led to the development of a technique for storing and manipulating geological discontinuities in the form of triangulated surface models. Software for generating and manipulating such models was developed and implemented as part of an integrated reserve estimation system. This allowed the geological models to be linked to geological data bases, grade estimation methods, and volumetric models used for reserve estimation and mine planning purposes.

The implementation of this modelling technique in conjunction with existing reserve estimation techniques was an important objective of this work. This allowed the advantages over conventional approaches to be demonstrated practically by carrying out comparative studies with existing reserve models.

1.3 Thesis layout

The contents of each Chapter in this thesis are summarised below:

1. Introduction: Overview of the field of research together with a summary of the research objectives.

2. Computer Aided Orebody Modelling: A summary of the current status of computer usage in orebody modelling.

3. Deposits Studied: A brief description of the geology, mining history, investigation data, and reserve estimation methods for the three deposits studied.

4. Geological Modelling for Reserve Estimation Purposes: A discussion of the ways in which geological contacts and structures are modelled for reserve estimation purposes.

5. A Geological Discontinuity Modelling System: A description of the modelling system developed during the course of this research project.

6. Modelling Geological Discontinuities: Examples of the usage of the modelling system for geological modelling purposes.

7. Modelling Geological Volumes: Examples of the usage of the modelling system for modelling and estimating volumes directly, or by interfacing with conventional volumetric modelling techniques.

8. Geological Controls on the Estimation of Internal Properties: Examples of ways in which the modelling system can be used to apply geological controls to grade estimation.

9. Errors in Discontinuity Modelling: A discussion of the errors associated with discontinuity models, and methods by which they can be measured and predicted.

10. Reconciliation Studies: Examples illustrating the comparison of results obtained using different modelling methods, and the estimation of model accuracy and precision, are presented.

11. Conclusions: A summary of the results of the research work, and discussion of areas meriting further study.

12. Bibliography: Bibliography and references.

Appendix I: Statistical summary of some of the discontinuity models used at Neves-Corvo.

Appendix II: Glossary of technical terms used, some of which may have different usages in other fields.

2. COMPUTER AIDED OREBODY MODELLING

2.1 General

Orebody modelling is the process of collecting geological and other information from a mineral deposit, and using it to build up three dimensional representations of the deposit characteristics. The resultant models allow the properties of specified volumes within the deposit to be predicted. Such predictions are used when planning the evaluation, extraction, beneficiation, and marketing of the contained metal or mineral.

Several categories of orebody modelling can be recognised, as indicated in Table 2.1.

TYPE	DESCRIPTION	EXAMPLES
CONCEPTUAL	Based on general concepts such as genetic ideas, metallogenetic provinces, and tectonic history.	Volcanogenic massive-sulphide deposits
GEOMETRIC	Definition of geometric features such as geological discontinuities and limits of different geological units	Geological maps & cross-sections
VOLUMETRIC	Definition of internal variations within defined geological volumes	Reserve grade models
PROCESS	The study of spatial models which change with time	Process simulation

Table 2.1. Orebody modelling categories

This thesis concentrates on the interpretation and representation of geological discontinuities, and the areas or volumes which they define. Such geometric models form a framework within which subsequent volumetric modelling and grade estimation are done when evaluating resource inventories and mining reserves.

The data collection and interpretation activities carried out during the investigation of a typical mineral deposit are summarised in Figure 2.1. Such work may be done during feasibility studies on new deposits, or as routine geological work on existing mines.

Given that all computerised aspects of such studies are based on the storage and manipulation of numeric data, it is important to recognise the existence of two fundamentally different types of numeric data, as illustrated in Figure 2.2.

These categories represent two distinct types of orebody property, namely;

- **Discontinuous properties**, such as lithology or oretype, which are usually modelled by interpreting the discontinuities which separate them. Such properties are normally recorded as integer or numeric codes in computer databases and handled as integer or character variables during computer modelling.

- **Internal variations**, such as metal grades, which are part of a continuum within given geological units. Such properties are generally recorded as analytical measurements, such as laboratory grade assays, and handled as real numbers during computer modelling.

Selection of an appropriate modelling technique is strongly dependent on the type of property being modelled. In the discussion below, and in subsequent Chapters, modelling techniques are divided into two main categories corresponding to these two numeric types, namely;

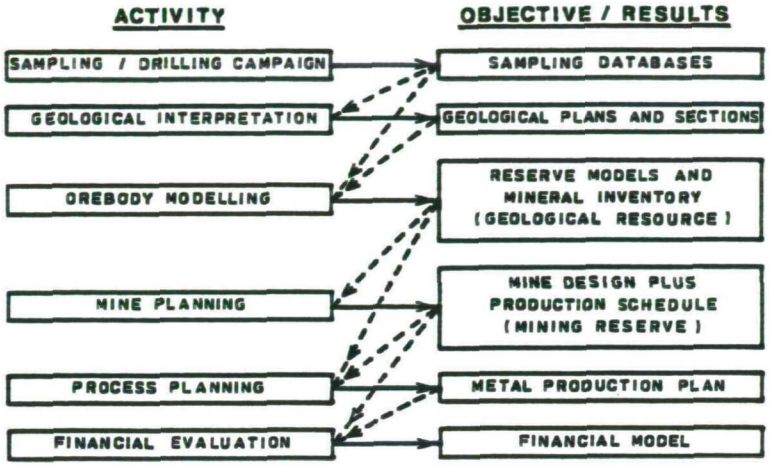


Figure 2.1 Simplified flowchart showing work-phases involved in the evaluation of mineral deposits

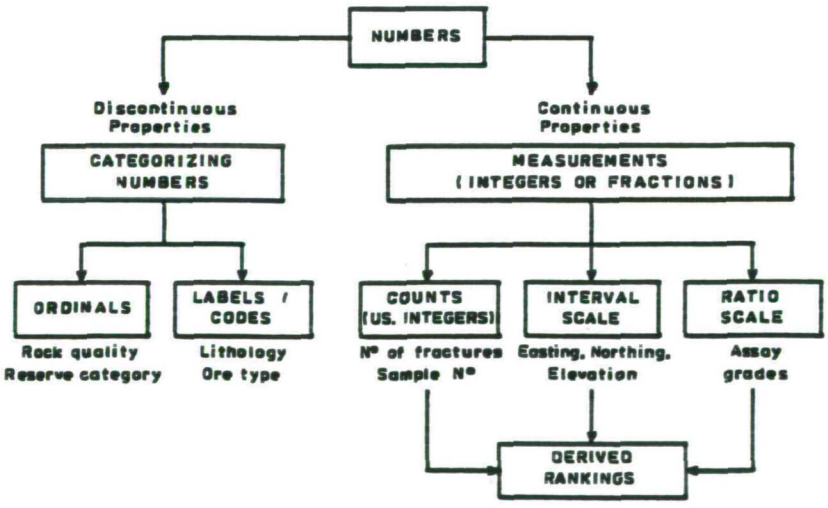


Figure 2.2 Numeric Data Types (modified after Sprent, 1981)

- **Geometric modelling**: the analysis, representation and display of variations in properties which are best represented by sharp boundaries (normally interpreted directly by a geologist)

- **Volumetric modelling**: the analysis, representation and display of properties which vary continuously within volumes of common geological characteristics (ie within boundaries defined by geological modelling). Such properties are largely modelled using geomathematical or geostatistical techniques.

This Chapter gives a brief review of the way in which the application of computer technology to orebody modelling has evolved, as well as a general overview of current usage.

2.2 Evolution of Computer Applications

The rapid advances in computer hardware and software technology in recent years have inevitably had a major impact in the field of orebody modelling, as well as in many other technical and scientific applications. Initial usage was by small specialist technical groups, mostly within major mining companies, running complex programs on mainframe computers. This has since evolved to the present situation where several integrated reserve modelling and mine planning packages are commercially marketed, and where much of the data used during reserve estimation resides in one or more computer databases.

Some of the broad trends seen in the evolution of computer applications in this field are summarised in Figure 2.3.

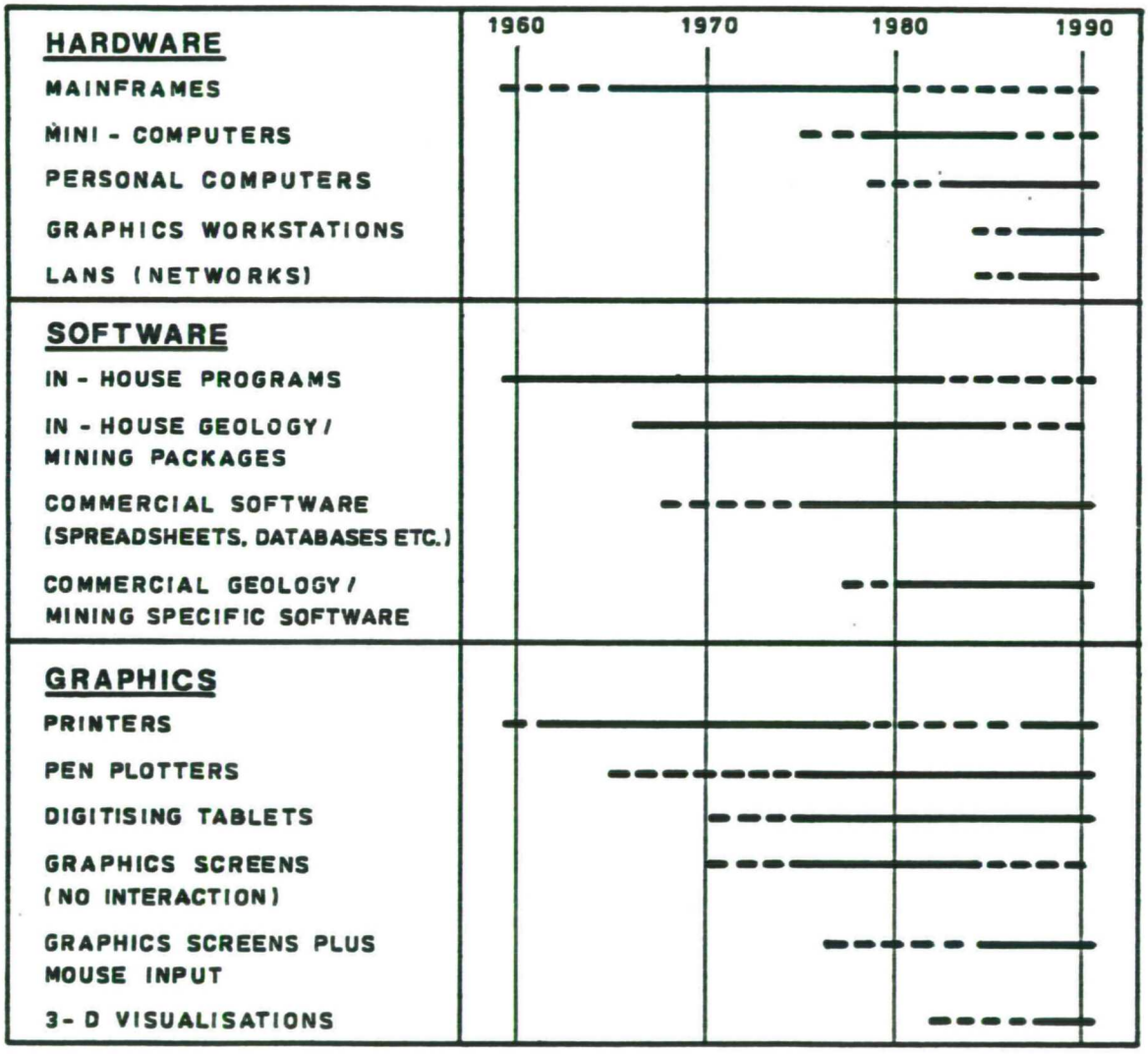


Figure 2.3 Evolution of Computer Applications in Orebody Modelling

The development of such applications is illustrated by papers presented in the proceedings of the APCOM (Applications of Computers and Operations Research in the Minerals Industries) conferences, 23 of which have been held over the last three decades (APCOM, 1961-1992).

One of the earliest, and most widespread, uses of computers in mining is in the maintenance of information databases. In many large companies this was initially done on centralised mainframe computers whose primary role was commercial. Nowadays most such information is maintained on independent databases within the departments where it is generated (eg. survey, laboratory, geology, mine production). Such applications have paralleled the evolution of database systems from flatfile, through relational, to object oriented database structures. Currently there is increasing interest in the development of databases containing a mixture of numeric, textual and graphic information.

Another important application of computers is in the graphical display of original data and derived models. Initially this was restricted to fairly crude representations generated on line printers. Subsequently, the advent of mechanical pen plotters as a cost-effective tool led to the widespread adoption of computer aided drafting techniques. Currently the trend is towards the increasing use of interactive graphic displays, which can be linked directly to different databases (eg drillhole information or surveyed mine workings), and reserve models. Current price performance trends in graphics workstations suggest that within a few years routine use will be made of programs which allow the user to interact with visualisations of three-dimensional models in real-time.

The processing capabilities of computers were initially used to carry out calculations previously done by hand, as in cases where systems were set up to computerise earlier manually generated polygonal reserve models.

It was soon realised that computers facilitated the use of other more computationally intensive methods of orebody modelling. Early work in this area concentrated on the use of conventional mathematical approaches such as trend surfaces, polynomial curve fitting, and spline surfaces as illustrated in the Computer Contributions of the Kansas Geology Survey published in the late 60's (eg James (1966), Merriam and Cocke (1967), Whitten (1968)).

The development and widespread application of one particular estimation technique, namely kriging, is closely associated with the adoption of computer aided orebody modelling techniques in the mining industry (Matheron (1963), Krige (1966), David(1977)). The number and complexity of calculations that have to be carried out during kriging means that the widespread application of this technique would have been impossible without the advent of comparatively cheap and widely available computing facilities. The evolution of orebody modelling from manual polygon and section based methods, to the current emphasis on geostatistically estimated regular block models, is illustrated by numerous published case studies (eg APCOM (1961-1992), South African Institute of Mining and Metallurgy (1966), Weiss (1979), Australian Institute of Mining and Metallurgy (1980, 1983)).

In recent years several techniques have been borrowed from other fields, such as the analysis of satellite imagery, computer aided design or manufacturing (CAD/CAM), and medical tomography, which deal with similar problems of trying to build up and represent three dimensional models on the computer. Several companies have also been set up specifically to design and market software packages for the mining industry. This has led to the development of integrated packages which allow the transfer of geological data directly onto the computer, and the ability to interactively create and edit three dimensional orebody models (eg. Wheeler & Stokes (1988), Houlding(1987)).

2.3 Current Practice

2.3.1 General

This section presents a brief overview of the current usage of computers in orebody modelling, in order to outline the overall context in which the research work was carried out. Functional aspects of computerised reserve estimation systems are stressed, since they have an important bearing on the development of new modelling techniques. Compatibility with existing systems is also an important consideration in the adoption of new techniques in this area.

The aspects discussed in this Section are described in more detail in many of the references given in the bibliography presented in Chapter 12.

2.3.2 Geological Databases

One of the most widespread uses of computers in the field of reserve estimation is to maintain databases of geological and analytical information. Such data can be very varied in character and may come from several different sources, such as drillhole sampling or logging, sampling and mapping of exposed mineralisation, geophysical logging, etc..

Three basic types of information must be available for such data to be used in reserve modelling, namely;

Identification:- hole and sample number, details of sampling procedure, date sampled, etc.

Location:- spatial co-ordinates which allow data to be correctly positioned with respect to one another and mine excavations

Characterisation:- analytical values or other measured properties.

Such information allows the selection of data according to the basis on which they have been collected, and their spatial location and display. Sample characteristics are used to define patterns of variation of the volume of ground being studied in order to generate models for use in planning work.

2.3.3 Graphic Displays

Graphic display of raw data and derived models is one of the most effective methods of representing the results of geological and reserve estimation studies. The graphics capabilities provided by modern computing hardware and software make this an essential part of any modern orebody modelling system. As well as eliminating the requirement for much manual drafting, such options can facilitate use of the same information by different applications programs.

One of the most important developments in recent years is the increasing use of interactive graphics systems. Most commercially available mine planning programs allow display of reserve models, and other geological data, on the computer screen as a guide to the design of mine excavations. This approach is also used in geological modelling whereby data stored in geological databases is accessed and displayed whilst doing interpretative work.

Graphic displays used for planning geological field programmes, and mining excavations, involve the following five elements;

Reference grid:- to allow location of geological data and interpretations with respect to other objects such as the mine infrastructure and surface topography.

Uniform scale:- to allow determination of the separation distance between different objects directly from the display.

Symbolic Representations:- the display of point symbols, lines, coloured or crosshatched areas, and rendered surfaces representing physical features in the real world should be possible, along with an appropriate legend for interpreting the displayed features.

Annotation:- labelling of features shown on the display so as to provide additional information on their characteristics. Since such annotation does not represent actual physical features, it is not scaled.

Interactive Capability:- In the case of interactive graphics applications on a computer, the ability to query the display, or input new information directly is necessary. Such interaction can be done from previously drawn scaled plans using a digitising tablet, or by indicating points on screen views using a graphics mouse.

The first four elements are generally taken into account when planning any graphic display, whether it is to be prepared manually or by a computer program. The fifth aspect is crucial to the successful implementation of interactive graphics applications.

The use of computer graphics to generate three dimensional visualisations of complex geological or mining shapes is an area of increasing applicability (eg Bak and Mill (1989), Van Driel (1989), Smith and Paradis (1989)). At present this capability is mainly used in the final presentation of reserve models (eg Plate 2.1), since the ability to interactively modify such three dimensional models is still somewhat limited. Such limitations stem from the fact that these displays do not offer clear solutions to the five elements described above. Nevertheless, current trends in the price and performance of graphics workstations, and developments in the software which can be used on them, make it likely that in a few years time such applications will be used routinely for geological modelling and mine planning.

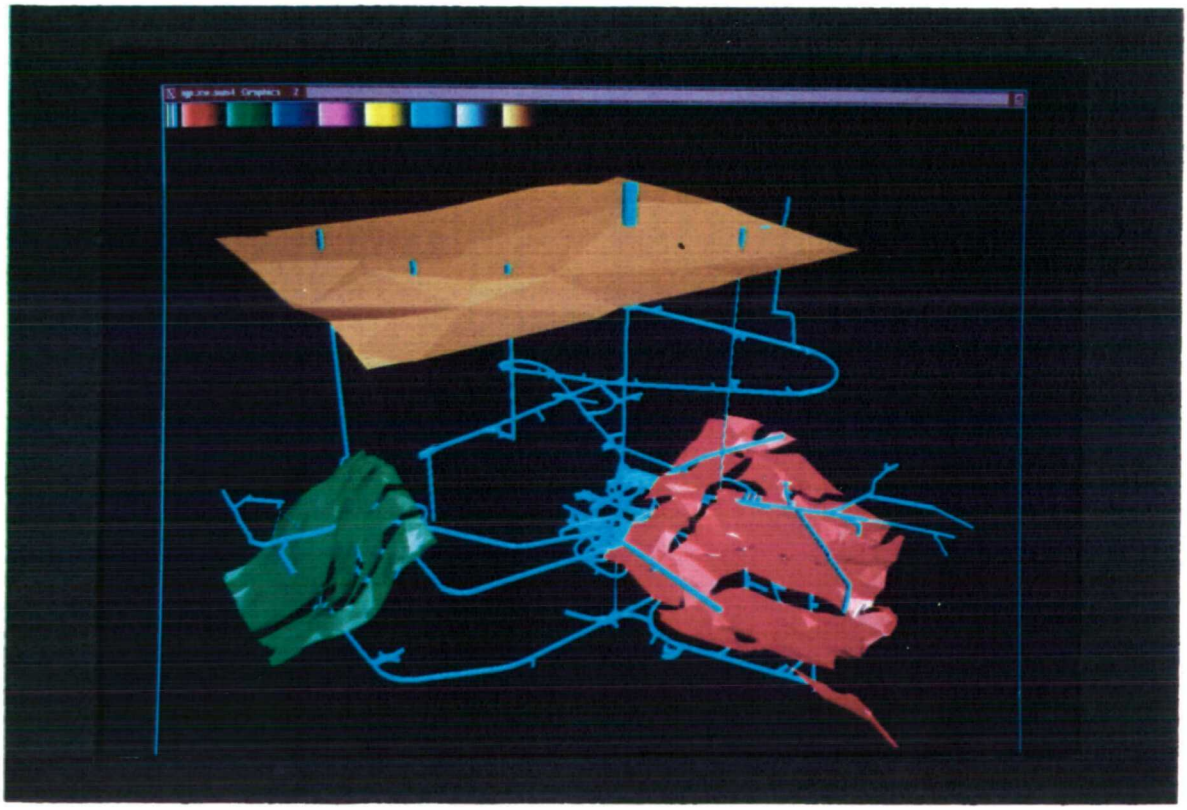


Plate 2.1 **Three-dimensional visualisation of the Neves-Corvo mine infrastructure, and footwall of the main orebodies.** Main infrastructure shown in blue, surface topography in brown, footwall of Graca orebody in green, and footwall of Corvo orebody in red.

2.3.4 Geometric Modelling

Most mineral deposits have geological controls on the disposition and morphology of mineralised zones. Geological controls on grade variations are also frequent since most mineralising processes are related to the geological history of a deposit. Incorporation of such controls into computer-based orebody models depends on the geometric modelling of discontinuities representing the boundaries between different geological units. Such models are used to define the methods and pattern of sampling as well as being used to impose geological controls on the grade estimation process.

Geometric modelling for reserve estimation purposes has several important functions, including the following;

- **Interpretation of geological discontinuities**:- The identification and representation of discontinuities forms the starting basis for most geological interpretations. In this instance, the term discontinuity is used in a broad sense to include features such as contacts between different lithologies, alteration zones, or mineralisation types, as well as structural features such as major faults. Less well defined features such as interpreted grade zone boundaries (Forkes (1982)) may also be of interest.

- **Definition of geological shapes and volumes**:- Geological units of common lithology, alteration or mineralisation type are defined by the areas and volumes bounded by the different discontinuities in a deposit. Modelling of these volumes is an essential prerequisite to volume and tonnage estimation.

- **"Zonal" control on volumetric modelling**:- Zonal control on the estimation of internal properties is applied by treating samples which fall within different geological volumes separately. This is essential where sharp grade changes are present across such boundaries, since the absence of such a control leads to the blurring of ore boundaries by smoothing out grade variations across them.

- **"Structural" controls on volumetric modelling**:- In many deposits stratigraphic or tectonic controls result in anisotropic internal variations. In such cases additional geological controls are often imposed during the estimation of internal properties. For instance, the relative stratigraphic and/or structural position of samples and reserve blocks may be used during such estimation.

The research work presented here concentrates on the geometric aspects of modelling geological discontinuities and shapes, as described in more detail in Chapter 4.

2.3.5 Volumetric Modelling

Volumetric modelling is defined as the study of properties, such as metal grades, which vary more or less continuously throughout a deposit on the scale at which they are being studied. These include physical properties, such as density and geomechanical characteristics, as well as grades of the main metals or minerals of interest.

In the context of reserve estimation, volumetric models are used for two purposes, namely;

- **Representation**; the representation, storage, and display of variations in the physical properties of interest throughout the deposit. The results of such work provide a three dimensional framework in which planning studies, such as design of sampling programmes, selection of mining methods, mine design and production scheduling, are carried out. The shape of the ore/waste boundary at different cutoffs may also be important.

- **Estimation**; the estimation of average values, and/or the statistical distribution of the modelled physical and chemical properties, within defined volumes. This function is essential in order to arrive at tonnage and grade estimates for geological mineral inventories or mining reserves. The ability to estimate average densities and grades within designed mining volumes is one of the main requirements of mine planning programs.

An understanding of the fundamental difference between these two requirements is essential in order to correctly apply modern computerised reserve estimation techniques.

An example of the application of different approaches to achieve these objectives would be the use of a block model to represent grade variations in a deposit with very narrow high grade ore shoots. A small block size would be required to accurately represent the grade variations within a regular block model. The uncertainties associated with grade estimates for individual blocks within such a model could be quite high. Ideally the grades of mining volumes would be independently estimated after the design had been completed using the block model data as a guide to the variation of grades in the deposit. Alternatively the uncertainties associated with individual block estimates would be reduced by ensuring that mining volumes used for production scheduling were much larger than the individual reserve blocks.

2.3.6 Integrated Planning Systems

The use of computer applications in most other technical fields in the mining industry mean that geological modelling and reserve estimation systems cannot be considered in isolation. An essential function of such systems is that they should generate reserve models in a form which can be used as input for other programs. The most important application in this context is mine planning. Nowadays such work is often done using interactive graphics programs which require the ability to access, display and interrogate the geological data and reserve models generated by other systems.

A fundamental basis for the interaction between different systems is the definition of common file formats so as to facilitate transfer of information between the different programs used. For interactive graphics applications the use of common graphics interface modules is essential.

In many modern mines integrated hardware and software solutions result in close physical links between geological and mine planning systems. Figure 2.4 shows the computer network used for these applications at Neves-Corvo, where much of the work presented in this thesis was done. This configuration allows current reserve models to reside at a single location on the network, whilst remaining accessible to all users. This facilitates the use of displays of different types of geological information whilst planning mine layouts (eg Plate 2.2).

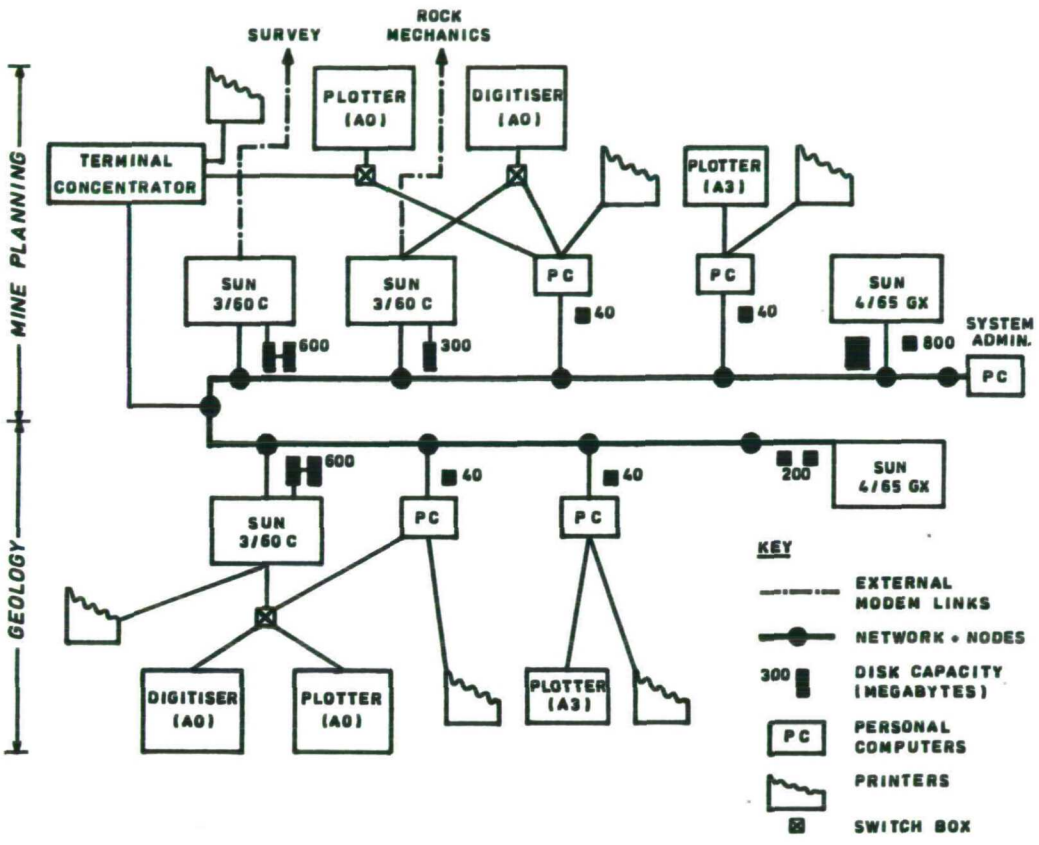


Figure 2.4 Technical computer network used by the geology and mine planning departments at Neves-Corvo mine (May 1991).

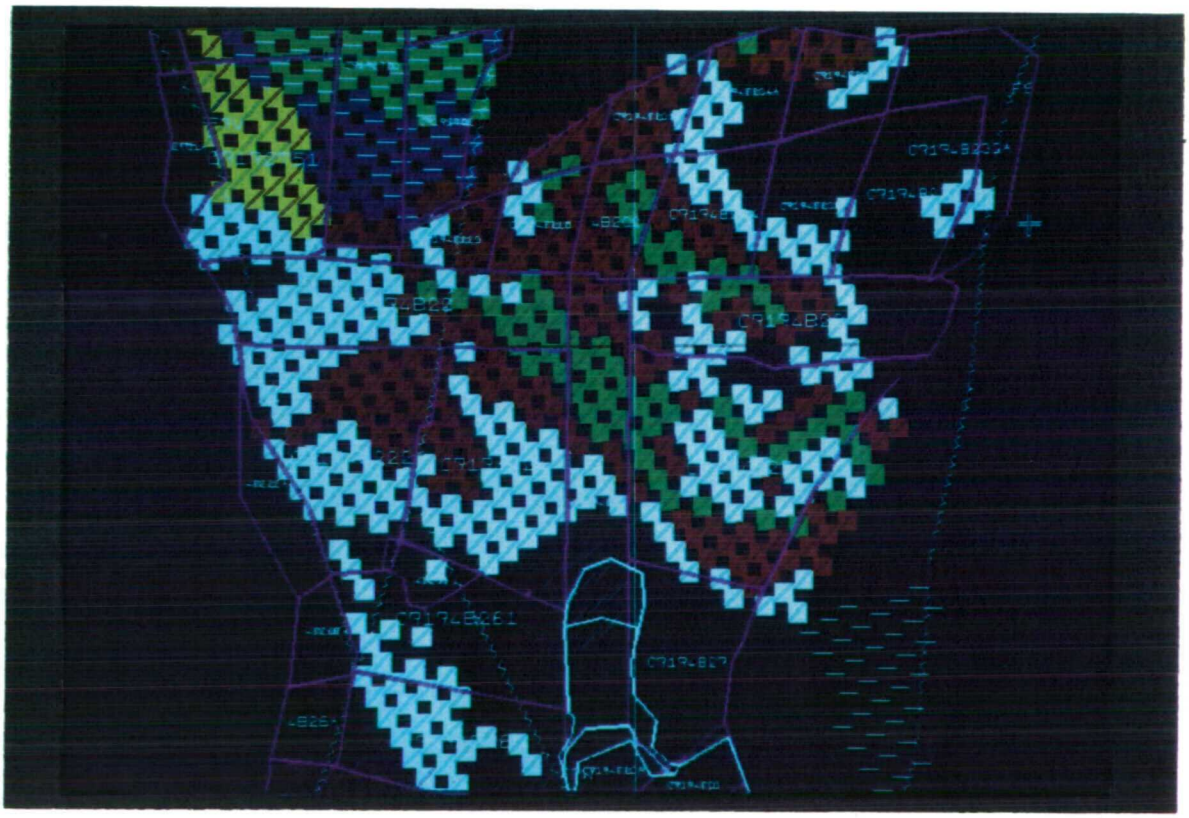


Plate 2.2 **Typical screen display of information used when planning mine layouts at Neves-Corvo.** Display shows surveyed mine openings (white, bottom centre), planned stoping layout (lilac), copper grades (coloured squares), and tin grades (diagonal symbols), and main faults.

2.4 Discussion

Many of the limitations of present reserve modelling techniques are due to the fact that identical modelling techniques are used for both continuous and discontinuous numeric data types (see Figure 2.2). A common example is the use of regular block models for handling both grade estimates, and interpreted rock types. This approach limits the usefulness of new techniques developed in other fields of computing. For instance, geometric modelling techniques developed for CAD/CAM applications can be used fairly readily to model geological contacts, but not for modelling internal grade variations.

The approach adopted in this study has been to treat the modelling techniques for these two numeric data types separately. An initial review of current reserve estimation procedures, and work done previously on the deposits studied, identified the modelling of geological discontinuities as the key area of interest.

The next two Chapters provide a more detailed outline of the context in which this research work was done. Chapter 3 gives a brief review of the geology and reserve estimation procedures used at the three deposits studied. Chapter 4 presents a review of the techniques normally used to represent geological discontinuities in computerised reserve models.

3. DEPOSITS STUDIED

3.1 Introduction

Information from three different mines was used to develop and test the techniques presented in this thesis. The deposits studied were the Cerro Colorado deposit at Riotinto in southwest Spain, the Bama-Brandelos copper deposits in northwest Spain, and the Neves-Corvo deposits in southern Portugal (Figure 3.1).

The writer was employed in routine ore reserve modelling on these deposits, at different times since 1983. This led to a detailed understanding of the geological data and interpretations used, as well as the limitations of existing computerised reserve estimation techniques. Consequently research work was able to focus on development of new techniques to overcome some of these difficulties.

A brief description of the geological setting, mining history, investigation methods, deposit geology and reserve estimation methods for each set of deposits is given below.

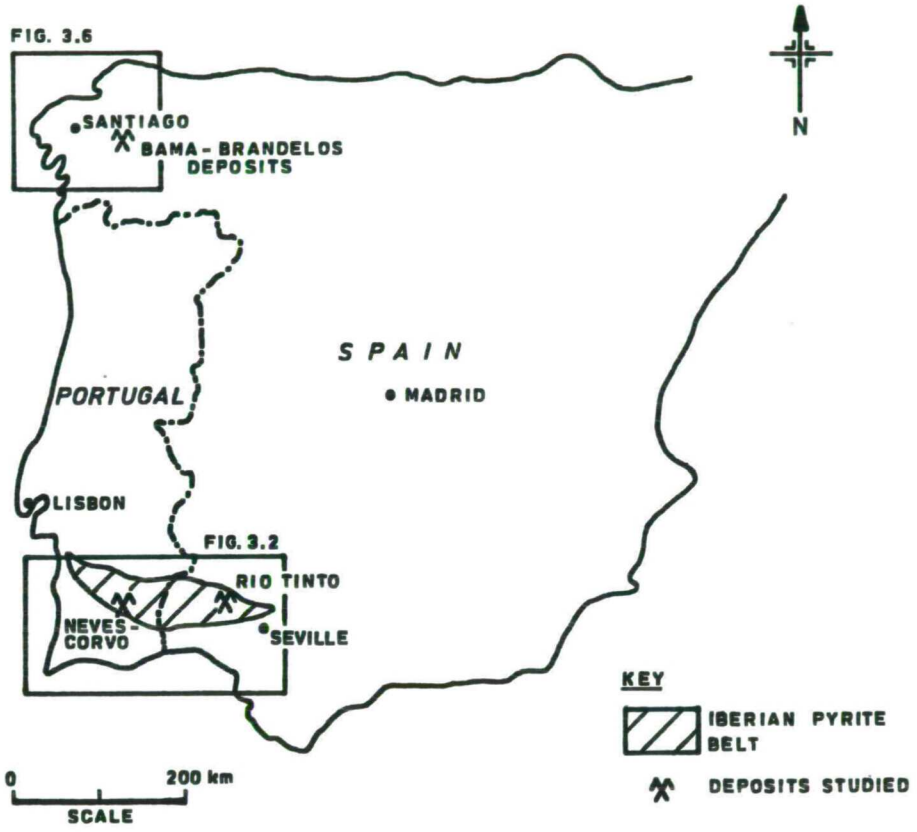


Figure 3.1 Map of Iberia showing location of the deposits studied

3.2 Cerro Colorado Deposit, Spain

3.2.1 Location

The Cerro Colorado deposit is a low grade copper orebody which forms one of a variety of different types of mineralisation in the Riotinto area. The location of this, and other major deposits in the Iberian Pyrite Belt, are shown in Figure 3.2.

3.2.2 Mining History

Mining activity in the Riotinto area dates from pre-Roman times, with significant activity between the third and first centuries BC, as reflected by up to 6 million tonnes of slag derived from treatment of gold and silver ore (Rothenberg et al (1989)). Intermittent mining of high grade copper ores from zones of secondary enrichment occurred through medieval times up until the middle of the last century.

The modern mining history of the area dates from the formation of the Rio Tinto Mining Company in 1873, when activity was concentrated on the exploitation of primary massive sulphides for sulphur production (Avery (1974)). Under different economic and technological conditions, production of copper from the Cerro Colorado open pit mine commenced in mid 1969 based on low grade mineralisation in sulphide stockworks (enriched near surface by supergene activity). At the same time a plant was constructed for production of gold and silver from a surfical iron oxide cap (Gossan). Underground production of stockwork copper ore was also carried out in the Alfredo mine up until 1987.

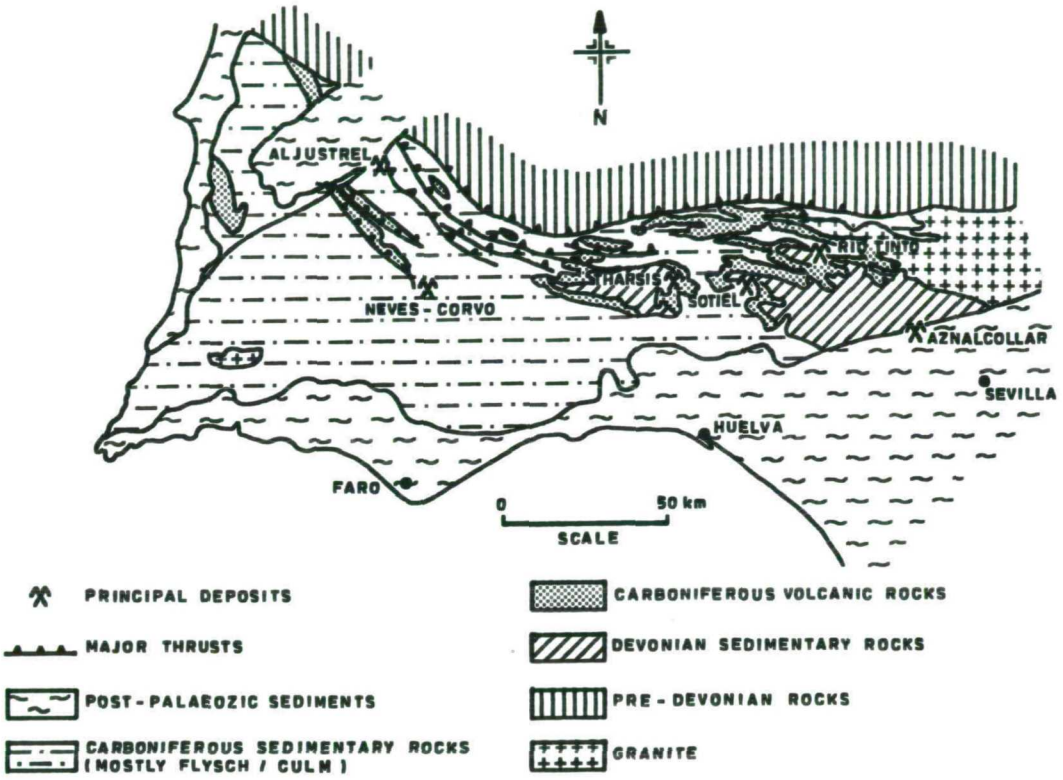


Figure 3.2 Geology of the Iberian Pyrite Belt (redrawn after Garcia Palomero, 1990)

Expansion of the Cerro Colorado open pit production from 3Mt to 6Mt of ore per annum was initiated in 1980. Subsequent changes in economic conditions in recent years have led to the expansion of Gossan production from 2 to 5 million tonnes per annum in 1985, and the cessation of operation on the Cerro Colorado copper operation during 1989. Pyrite has been produced more or less continuously from the Atalaya open pit since the end of the last century.

3.2.3 Geological Setting

The Riotinto deposits lie in the Iberian Pyrite Belt, a metallogenic zone of upper Palaeozoic age (Figure 3.2). This is characterised by the occurrence of abundant massive sulphide deposits spatially, and genetically, associated with a volcano-sedimentary sequence of upper Devonian to lower Carboniferous age (Williams (1934), Rambaud (1969), Garcia Palomero (1980, 1990)).

The different types of mineralisation occuring in the Riotinto area are spatially associated with the Riotinto anticline. This is a periclinal fold with volcanic rocks (exposed in the anticlinal core) overlain by a sedimentary sequence termed the Culm (Figures 3.3 and 3.4). The volcanics are underlain by a sedimentary succession (slates with lesser quartzites), dated as Famennian near the top (Schermerhorn (1971)).

The volcanic unit is divided into a lower Basic complex (andesitic volcanics and pyroclastics), and an upper Acid complex (dacitic-rhyolitic pyroclastics and volcanics). At the top of the volcanic sequence a heterogeneous pyroclastic level, locally termed the Transition Series, hosts the massive sulphide mineralisation, as well as laterally associated jasper and manganese occurrences. The overlying Culm, consists of a greywacke-shale sequence of turbiditic origin.

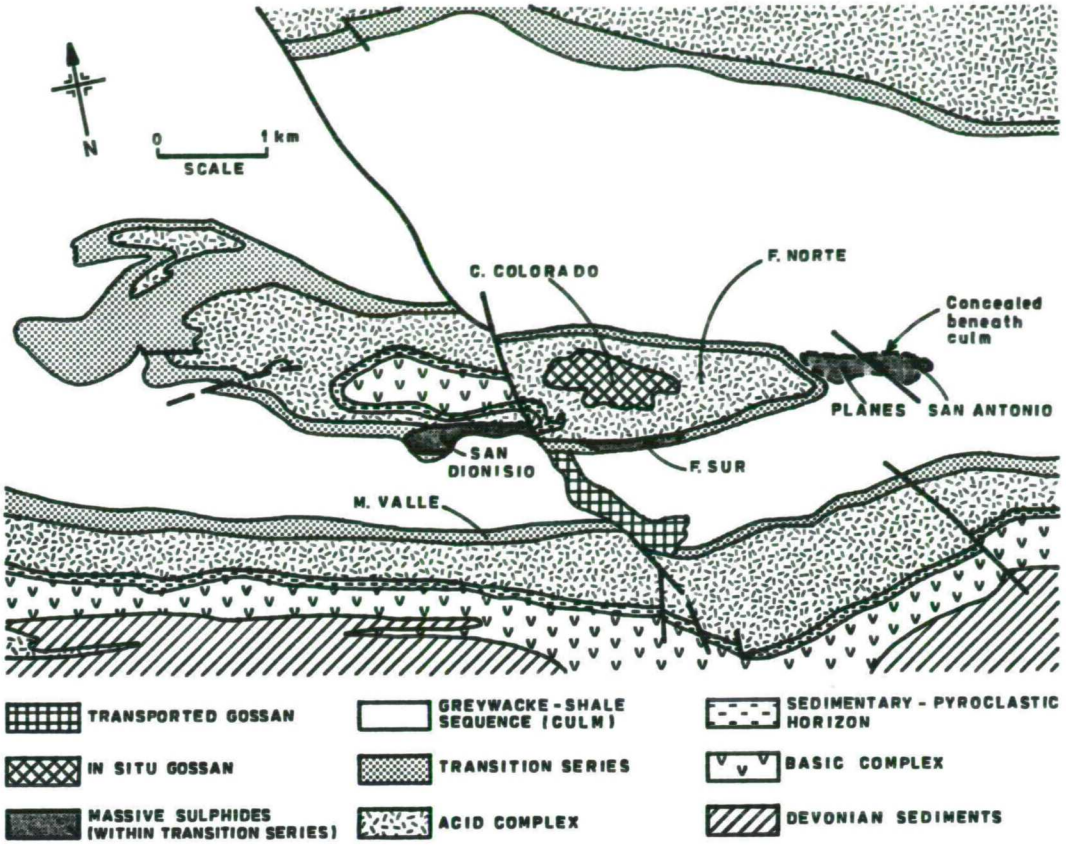


Figure 3.3 Geology of the Riotinto area (redrawn after Garcia Palomero, 1990)

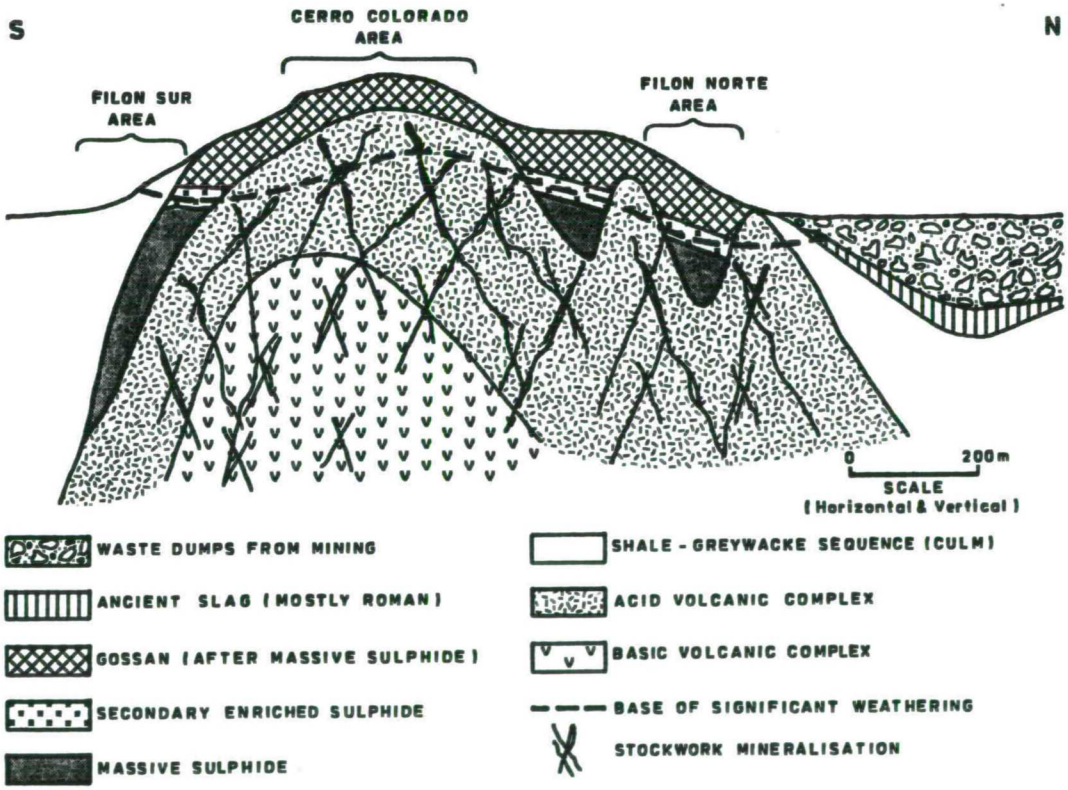


Figure 3.4 Simplified geological cross-section of the Cerro Colorado deposit (redrawn after Garcia Palomero, 1990)

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Three main types of mineralisation occur in the area, namely;

- **Stratiform massive sulphide lenses** which occur at the upper contact of the volcanic unit,
- **Sulphide stockworks** in the underlying altered volcanics,
- **Gossans** which represent near surface iron oxide caps formed by weathering of the sulphide deposits.

Secondary enrichment of the sulphide deposits occurs beneath the leached cap represented by the gossan deposits. The economic geology of these deposits has recently been described by Garcia Palomero (1990), and different aspects are covered in many earlier publications (Garcia Palomero, (1980), Garcia Palomero et al (1986, 1988), Rambaud (1969), Williams (1934)).

The sulphide deposits at Riotinto are considered to be related to submarine hydrothermal activity in the waning phases of volcanic activity. The massive sulphide lenses are thought to have formed on the sea floor, the stockwork sulphides representing mineralisation formed by ascending hydrothermal fluids within the volcanic pile. This genetic model is supported by recent geological, isotopic and geochemical studies (Garcia Palomero (1990), Eastoe et al (1986), Halsall (1990)).

The Cerro Colorado copper operation was based on low grade sulphide stockworks occuring in altered volcanics in the core of the Riotinto anticline east of the Eduardo fault (Figures 3.3 and 3.4). Subsequent discussion in this thesis is restricted to the evaluation and reserve estimation methods applied at Cerro Colorado, rather than any of the other Riotinto deposits.

3.2.4 Investigation Methods

Initial interest in the Cerro Colorado stockwork mineralisation was based on mineralisation intersected by rail tunnels, cutting through the core of the anticline, developed as part of the mine infrastructure early this century. During the mid 1960's systematic studies of this mineralisation including exploration, reserve evaluation and mine planning, were started. This led to the initiation of the Cerro Colorado project based on the exploitation of the near surface secondary enriched part of the stockwork mineralisation (Pryor et al (1972)).

During the 1970's changed geological thinking led to implementation of an extensive drilling campaign in order to better define open-pitable reserves of primary stockwork sulphide mineralisation at depth.

Investigation of the Cerro Colorado reserves was done chiefly using vertical surface diamond drillholes, although results of some inclined drilling from underground workings were also utilised. Initial drilling was on a 100m by 100m grid infilled to 50m by 50m in areas of interest. In some cases holes on the wider grid were drilled to greater depths to provide information on reserves at depth. Holes were sampled on regular 2m intervals, with sampled core being split and routinely assayed for copper, sulphur, lead and zinc. Gold and silver were analysed for selected 10-12m bench composites in areas of interest.

3.2.5 Reserve Estimation

Initial reserve estimates for the Cerro Colorado project were derived from manual section-based geological calculations. Reserve models used for subsequent mine planning were based on a modified polygonal method, with regular blocks of 50m by 50m centred on the drillhole grid. Estimation of grades was done separately for each bench using composited bench grades.

During 1982 and 1983 a re-assessment of the methods of reserve estimation led to the development of a computerised block model. This model incorporated the use of geostatistics to estimate block copper grades, within an overall geological framework corresponding to a revised interpretation of the stockwork mineralisation (Garcia Palomero et al (1986), Garcia Palomero (1990)). The new geological interpretation recognised stratabound horizons of stockwork mineralisation with enhanced copper grades.

Statistical analysis of the assay data led to the definition of three separate copper grade zones defined by limits of 0.10% and 0.38% copper. An interpretation of these grade zones was used to control the geostatistical estimation of copper block grades (Figure 3.5). Sulphur and zinc grades were estimated by inverse distance weighting. Ranges of sulphur content were used to assign block densities for planning purposes.

To provide more detailed information for short term planning additional reverse circulation percussion drillholes (20-50 metres in length) were drilled within the operating pit, on a grid of 20m by 25m. Samples from these holes provided analytical data on minor metal variations, as well as samples for metallurgical testwork. This information was used to generate a separate reserve model for use in short-medium term planning.

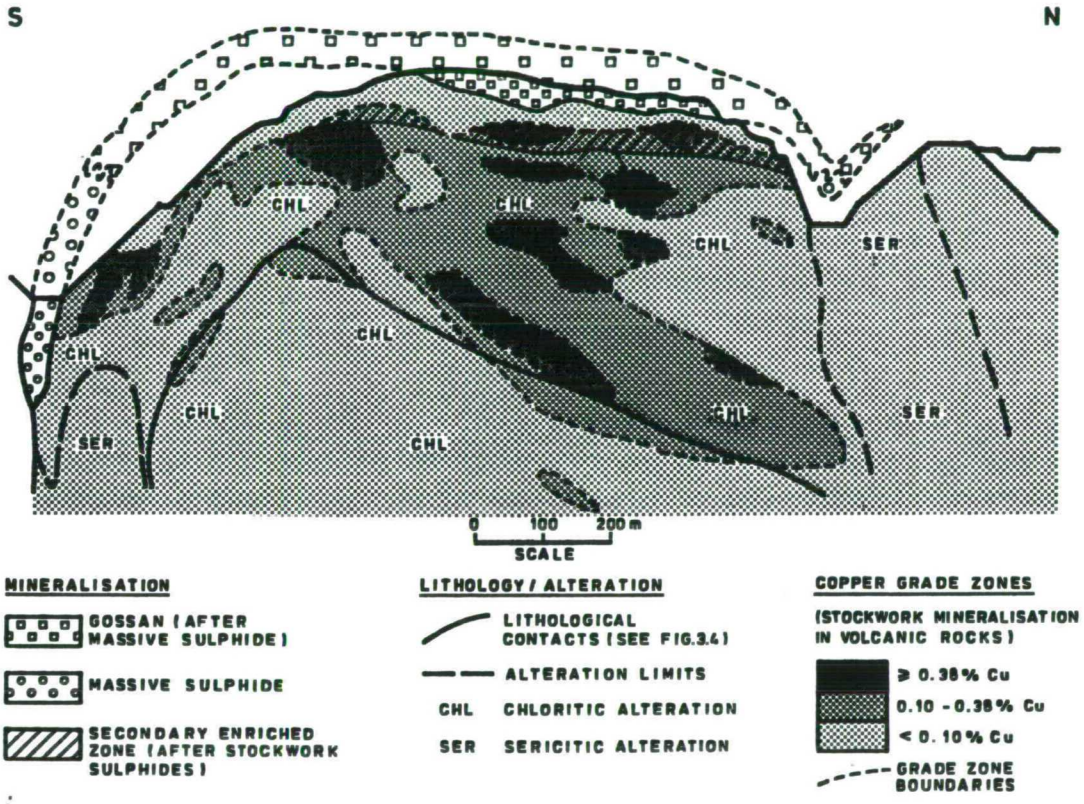


Figure 3.5 Cross-section of the Cerro Colorado deposit showing grade zone interpretation used for reserve estimation (redrawn after Garcia Palomero, 1990)

3.3 Bama-Brandelos Deposits, Spain

3.3.1 Location

The Bama-Brandelos copper deposits are located in the Galicia province of northwest Spain, 25km east of Santiago de Compostela (Figure 3.6).

3.3.2 Mining History

Copper mining in the area was started by Sociedade Riotinto Patino, SA in 1976 on the adjacent Arinteiro deposit (12Mt averaging 0.7% Cu) following a successful exploration programme.

The Bama-Brandelos deposits were discovered subsequently and were mined from about 1980 onwards when the reserves in the higher grade Arinteiro deposit were exhausted. Mining on these deposits reached a peak of 2Mt per annum averaging around 0.5% copper, but ceased in 1987 due to declining head grades and adverse economic conditions. All the deposits were mined by open pit methods with copper concentrate being recovered by flotation methods.

3.3.3 Geological Setting

The ore horizon in the deposits consists of an apparently stratabound level of disseminated pyrrhotite-chalcopyrite mineralisation hosted by a highly deformed metabasite succession belonging to the Ordenes complex (Figures 3.6 and 3.7). This unit has been interpreted as an ophiolite complex emplaced during the Hercynian orogeny (Badham and Williams (1981), Ries and Shackleton (1971), Williams (1982, 1983)).

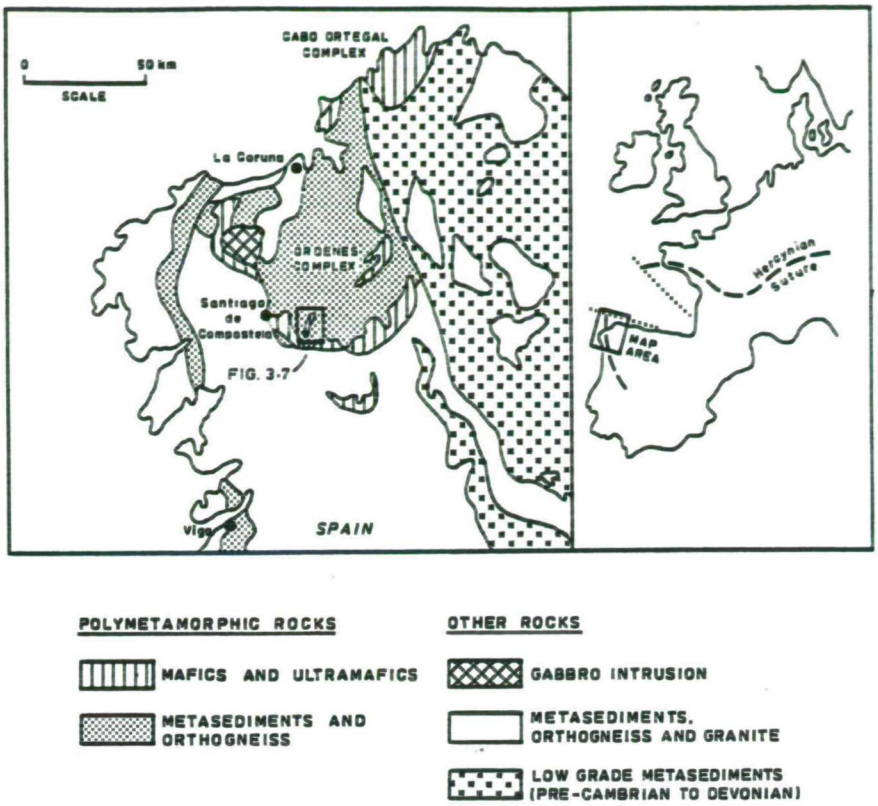


Figure 3.6 Geology of northwest Iberia showing the location of the Bama-Brandelos copper deposits (redrawn after Sides, 1987)

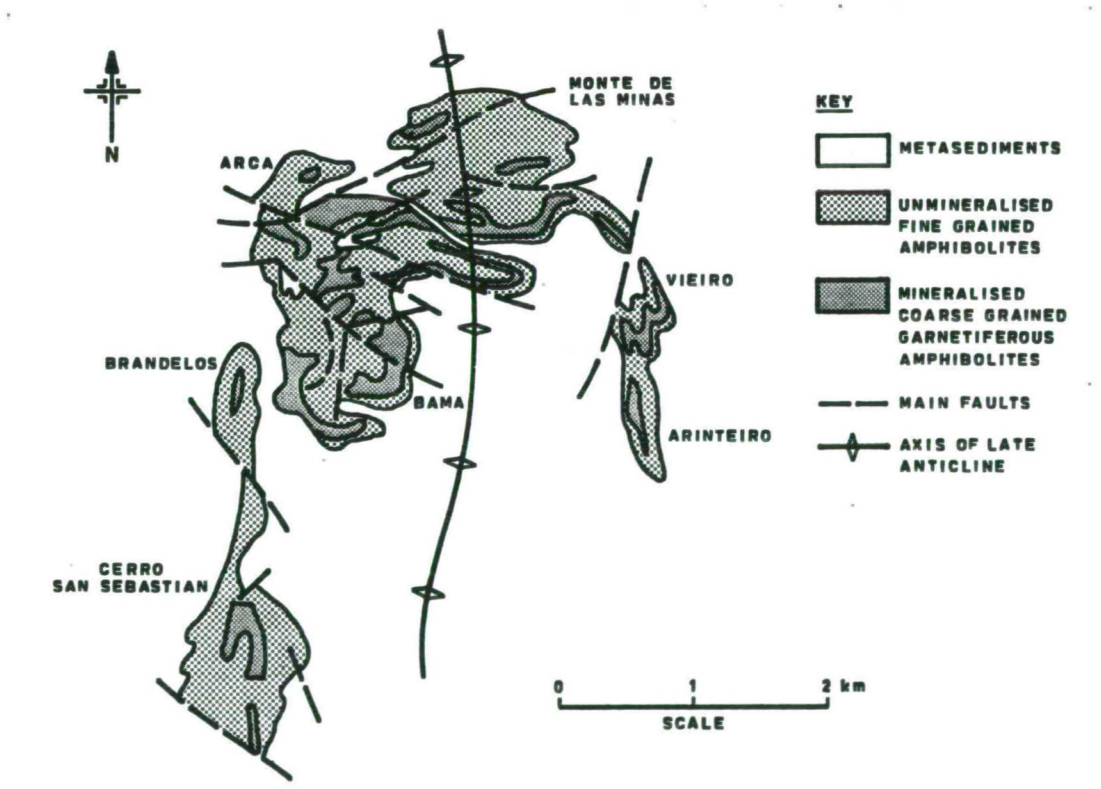


Figure 3.7 Surface geology of the Bama-Brandelos deposits (redrawn after Sides, 1987)

Ore mineralisation is closely associated with a coarse garnetiferous amphibolite which is generally surrounded by finely crystalline non-garnetiferous amphibolite and unmineralised schists (Figure 3.7).

The amphibolite units appear to represent a single deformed metabasite level, whose outcrop pattern in the mine area reflects the presence of a major late, gently northward plunging, open fold. Earlier isoclinal fold structures present within the metabasite horizon have been highly flattened by subsequent deformation, so that for evaluation purposes the ore horizon can be treated as a single, fairly openly folded, unit (Figure 3.8).

The mineralogy and metamorphic characteristics of the deposits were studied in detail by Williams (1982) who concluded that the ore horizon probably represents a stockwork system of intensely altered basaltic rocks veined with quartz and sulphides. Hydrothermal alteration and mineralisation are postulated to have occurred adjacent to a submarine fault zone (Williams, 1983).

3.3.4 Investigation Methods

Following initial targeting of favourable areas within the metabasite unit, by geological mapping, the deposits were evaluated by diamond drilling. An initial grid spacing of 200m by 200m was closed down to 100m by 100m, and then 50m by 50m in areas of interest. Most of the drillholes were vertical, except in structurally complex areas where several inclined holes were necessary. All mineralised core was split and sampled in regular one-metre lengths and analysed for copper. Separate analyses for soluble copper were done in weathered near surface zones.

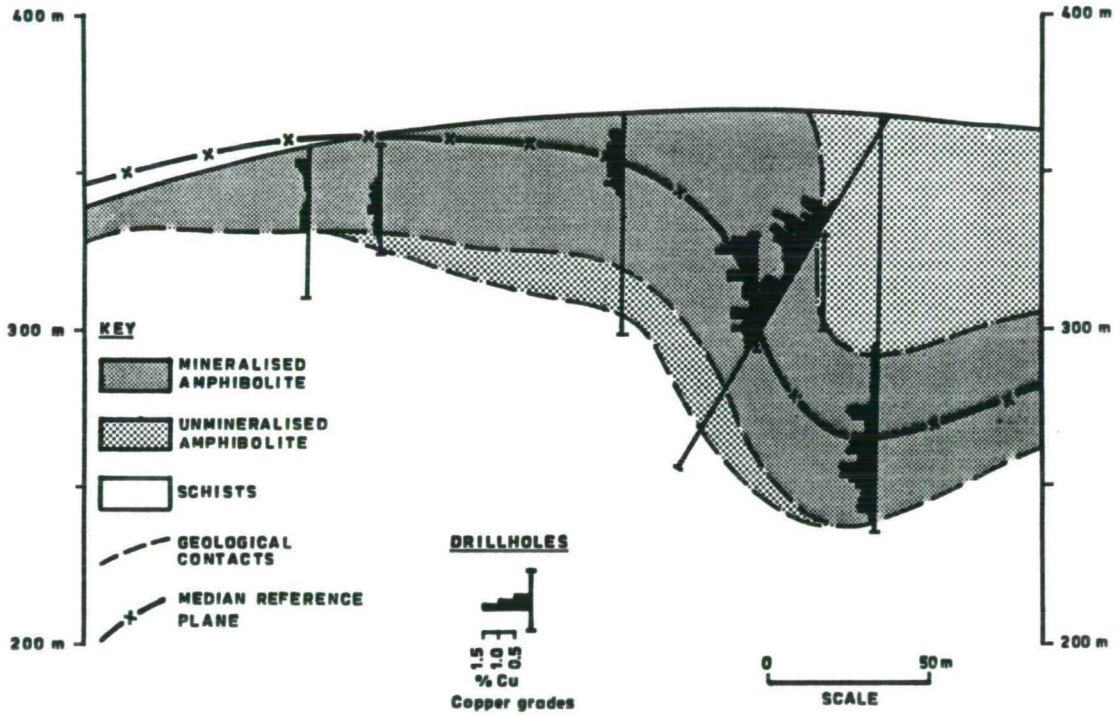


Figure 3.8 Typical geological cross-section of the Bama-Brandelos deposits (redrawn after Sides, 1987)

3.3.5 Reserve Estimation

Four rock types form the basis for the geological interpretation used in orebody modelling, namely:

- Coarse garnetiferous amphibolite
- Finely crystalline amphibolite
- Schists
- Crystalline garnetiferous amphibolite

Economic mineralisation is almost entirely restricted to the coarse garnetiferous amphibolite. The fourth type represents localised zones transitional between the first two types.

Using the results of surface geological mapping, and geological logging of drillcore, these units were interpreted on two sets of vertical sections (East-West and North-South). The interpretation was then transferred to horizontal plans, at 10m intervals, corresponding to the benches used in the open pit. A block model representation of the deposit geology was generated from these plans, and used to control the estimation of block copper grades by kriging. A "structural" control was imposed on the grade estimation process, in order to take account of the main fold structures, as discussed in Chapter 8.

3.4 Neves-Corvo Deposits, Portugal

3.4.1 Location

The Neves-Corvo deposits are located in the Baixo Alentejo province of southern Portugal, 220km south of Lisbon, and about 125km west of Riotinto (Figures 3.1 and 3.2).

3.4.2 Mining History

The Neves-Corvo deposits consist of a series of massive sulphide lenses which are currently mined for copper and tin by underground methods. They were discovered in 1977 as a result of an exploration programme based on geological mapping and gravimetric surveying followed up by surface diamond drilling (Leca (1990)). The drilling programme led to the definition of four massive sulphide lenses with associated copper and lead-zinc mineralisation, at depths of 250m to 650m below surface. (Leca et al (1985), Fernandez Rubio et al (1988), Richards and Sides (1991)). Continuing exploration work led to the discovery of a fifth sulphide lens in 1988 (Figure 3.9).

Detailed evaluation of the deposits started in 1980 with the development of a decline ramp and vertical shaft to access the orebody underground. A major underground evaluation drilling programme led to the discovery of associated tin mineralisation in 1986; and completion of feasibility studies and project financing for the copper project by mid 1987. The copper plant was commissioned in October 1988, and subsequently a project for a separate tin plant was approved. This was constructed during 1989 and commissioned in early 1990.

The Neves-Corvo mine currently produces and treats around 1.3Mt of copper ore, and 0.3Mt of copper-tin and tin ore per annum. The project history, evaluation methods, and evolution of reserve estimation methodology are discussed in detail by Richards and Sides (1991).

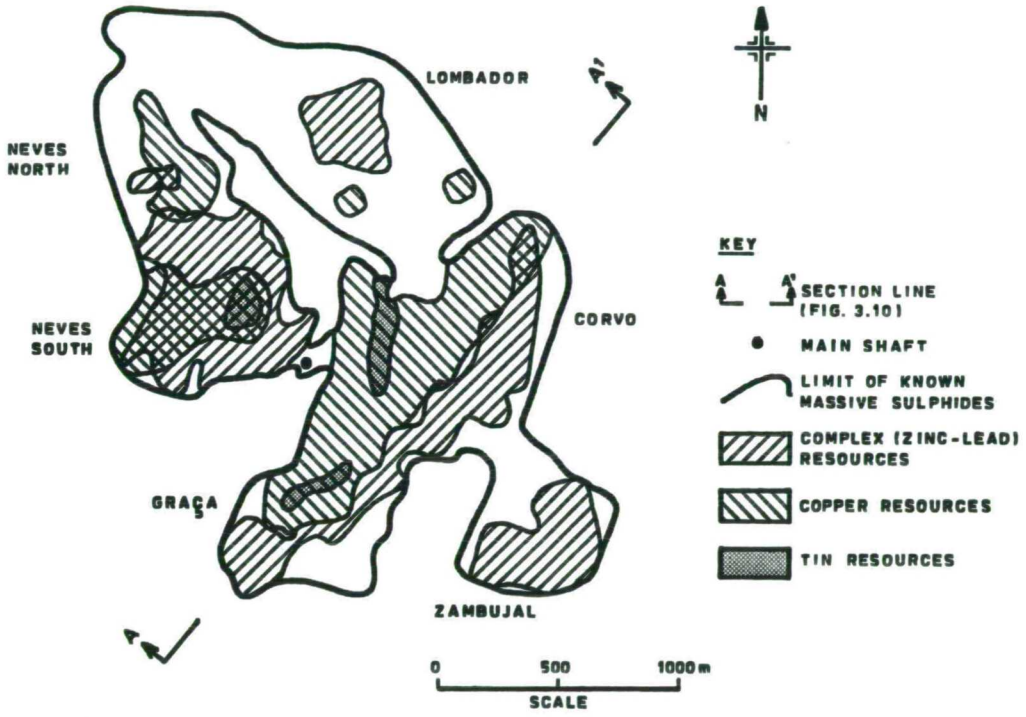


Figure 3.9 Plan of the Neves-Corvo deposits showing metal zonation patterns (redrawn after Richards and Sides, 1991)

3.4.3 Geological Setting

The Neves-Corvo deposits are located in the Iberian Pyrite Belt (Figure 3.2), and show many characteristics in common with other deposits in the Belt, such as Riotinto. Nevertheless, Neves-Corvo is atypical in several ways, particularly in the very high grades of copper and tin mineralisation within the sulphide lenses. Typical pyrite belt occurrences contain only limited zones with copper grades over 2% Cu, and grades over 5% Cu (less than the average for the Neves-Corvo copper reserves) are considered exceptional. Tin, in the form of cassiterite and stannite, is also normally only encountered in trace amounts in the other deposits and grades such as those seen at Neves-Corvo have not been reported elsewhere.

In general terms, the regional stratigraphy of the Pyrite Belt around Neves-Corvo is similar to that seen at Riotinto, but there are differences in detail. The volcanic unit below the orebodies (regionally called the Volcano-Sedimentary unit (VS)) is more sedimentary in character at Neves-Corvo, consisting mostly of very fine grained acid pyroclastics. The Neves-Corvo deposits are capped by a thin chert-carbonate unit, and overlain by an upper volcano-sedimentary sequence dominated by sedimentary rocks. This in turn is overlain by a greywacke-shale sequence similar to that seen at Riotinto. Stockwork mineralisation, and alteration, is encountered locally in footwall shales and volcanics. As yet no distinctive alteration zoning patterns, or potential feeder zones, have been identified.

Five separate massive sulphide lenses have been identified; Corvo and Graca, which contain most of the copper and tin ores, Neves, Zambujal and Lombador which are predominantly polymetallic (Cu-Pb-Zn). These lenses are locally linked by thin discontinuous sulphide horizons. Their main characteristics are summarised in Table 3.1.

OREBODY	GENERAL DESCRIPTION
CORVO	Elongated NE-SW, 1200m by 700m Maximum massive sulphide thickness of 90m Average dip 25 degrees to the northeast Located on NE flank of main anticline, at depths of 250-700m below surface About 60% barren pyrite Major copper and tin, minor zinc-lead Linked with Graca orebody by the 'Graca Bridge'
GRACA	Slight east-west elongation, 600m by 500m Maximum massive sulphide thickness of 80m Dips 50 degrees to the south in updip part, sub-horizontal at depth Located on SW flank of main anticline at depths of 250-425m below surface About 70% barren pyrite Major copper, minor tin and zinc-lead
NEVES	Elongated NNW-SSE, 1500m by 700m Maximum massive sulphide thickness of 50m Dips mostly sub-horizontal, 20-25 degrees to the northeast in the northern part Located on the crest of the main anticline, extending onto the northeast dipping flank About 30% barren pyrite Divided into two separate 'basins': North Neves - Major copper, minor zinc-lead South Neves - Major zinc-lead, minor copper-tin
ZAMBUJAL	Slight E-W elongation, 600m by 500m Maximum massive sulphide thickness of 50m Dips sub-horizontal to 20 degrees to north Located near the crest of the main anticline at depths of 300-450m below surface About 60% barren pyrite Major zinc-lead, minor copper Appears to be geologically and/or structurally complex
LOMBADOR	Elongated NW-SE, 700m by 500m Maximum massive sulphide thickness 50m (to date) Dips about 25 degrees to the northeast Located on NE flank of the main anticline at depths of 500-750m below surface About 85% barren pyrite Major zinc-lead Recent discovery, and geologically the least well know at present. Investigation continuing

Table 3.1 Summary of the main characteristics of the Neves-Corvo massive sulphide lenses (after Richards and Sides, 1991)

The distribution of the main mineralisation types is shown in Figure 3.9. Base metals, particularly copper, tend to form continuous zones along the footwall of the massive sulphide lenses, but also occur as separate horizons at higher levels within the massive sulphides.

The generalised cross-section, shown in Figure 3.10, illustrates the occurrence of four mineralisation types, namely; massive sulphide, banded shale-sulphide (rubane), fissural, and sulphide breccias. The massive sulphides have a distinctive stratabound character, as do the rubane ores which occur locally above them, although the latter are often strongly affected by tectonism. The morphology of the other two types is less well known, although some of the fissural mineralisation, which occurs in the footwall volcanics, may represent feeder zones.

The entire area has been gently folded by Hercynian tectonism, with NW-SE fold axes, and a parallel regional cleavage. Northeast dipping thrusts are present in the sequence overlying the ore horizon, and at least three sets of late steep-dipping faults cut the succession. Given the very high grade character of the economic copper and tin mineralisation, these late faults (see Figure 3.10) are very significant from the point of view of reserve estimation, and production control.

3.4.4 Investigation Methods

Initial evaluation of the deposits was done entirely by cored diamond drilling from surface, with many of the holes being pre-collared to 200m depth using a rotary drill. Most holes were collared vertically but significant deviations occurred such that the majority were 30-50 degrees off vertical where they cut the orebody at depths of 250 to 600 metres. For this reason all holes were surveyed at 40m intervals using downhole survey instruments.

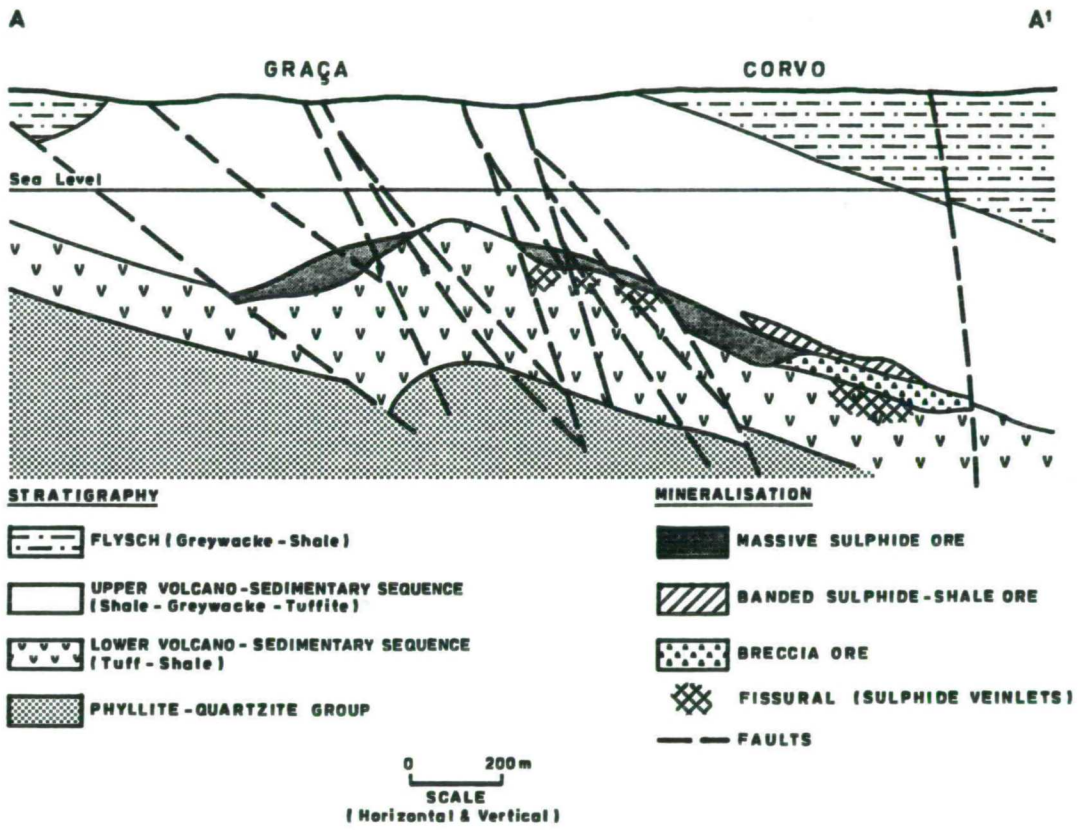


Figure 3.10 Generalised geological cross-section through the Corvo and Graca orebodies (redrawn after Fernandez-Rubio et al, 1988)

Surface drillholes were drilled on an approximate 100m by 100m grid, filled in to closer spacings in areas of interest. This did not result in a regular grid of ore intersections because of drillhole deviation.

Investigation by underground drilling commenced after the orebody had been accessed by underground workings in 1984. A major drilling campaign carried out between 1986 and 1988 formed the basis for detailed investigation of the copper and tin reserves of the Graca and Upper Corvo orebodies (Richards & Sides (1991)). Drillhole fans were drilled on transverse sections at 35m (Corvo), and 25m (Graca) spacings to obtain footwall intersections at approximately 35m intervals in the downdip direction. Holes were surveyed at 20m intervals, but deviation problems were much less significant than with the surface holes.

All core and open-hole cuttings were logged geologically, with the ore intersections of the later surface holes, and all underground holes, also being logged geotechnically.

Most of the underground core was photographed before being sampled. Mineralised intersections were sampled on one-metre intervals, sub-divided at the contacts of the main mineralisation types.

Analytical procedures changed slightly with the evolution of the project, but most of the ore intersections used for reserve evaluation were analysed routinely for Cu, Pb, Zn, S and Fe. In addition all samples with over 2% copper were also analysed for Ag, Hg, Sn, As, Sb, and Bi. Density measurements were made on most ore intercepts. All this information was entered into a computer database for use in subsequent reserve calculations.

Since the start of production in 1988, routine stope face sampling and underground geological mapping for production control, has been carried out. This information is also entered onto computer databases for use in reserve evaluation. Underground face samples are analysed for the same metals as the drillhole samples.

3.4.5 Reserve Estimation

Reserve estimation methods at Neves-Corvo have evolved with the development of the mine, in response to changing planning requirements, available databases, and overall project decisions. This meant that different reserve estimation techniques were, and continue to be, used at different stages between the initial discovery of mineralisation, and when it actually becomes available for mining. Given the very sharp cutoffs at the ore contacts, and the presence of several important faults which disrupt the orebody (see Figure 3.10), the accurate modelling of interpreted geological shapes has always been the dominant factor in the selection of orebody modelling techniques.

The evolution of the geological modelling and reserve estimation techniques used are described by Richards and Sides (1991), as summarised in Table 3.2. This illustrates the evolution from the use of mainly manual methods in the early phases of the project, to a computerised system based on interactive graphics workstations which is used for routine planning control on the operating mine.

The techniques developed in the course of the research work presented in this thesis form part of the orebody modelling system currently in use at Neves-Corvo, as illustrated by examples presented in subsequent Chapters.

	Phase and duration Definition (1980–85)	Evaluation (1985–87)	Preparation (1988)	Production (1989–90)
Objectives	Pre-feasibility study	Feasibility study for project financing. Choice of mining method and process design	Construction and commissioning of plant. Preparation of mine for full production	Production targets
Project development	Shaft and decline ramp down to orebody	Mine infrastructure, crusher station, haulage level and trial stope; some surface installations	Ore accesses and production level infrastructure. Construction of surface facilities and off-site infrastructure	Stoping, backfill and preparation of new ore accesses. Geotechnical monitoring, process control and optimization
Investigative work	Mostly surface drilling	Extensive underground drilling plus underground mapping	Further underground drilling plus underground mapping and sampling	Routine stope mapping and sampling plus definition drilling in stopes
Geological interpretation	Isobath plans and sections (1:2000, 1:1000)	Regular sections and isobath plans (1:500). Sections at 25-m (Graça) and 35-m (Corvo) spacing	Detailed sections (1:500)	Stope maps (1:250, 1:500) plus working sections. Detailed (1:500) level plans at 2-m intervals
Modelling of orebody shape	Trend-surface fitting, mostly no faults	Trend surface within individual fault blocks	Main faults and ore-lens contacts modelled as three-dimensional surfaces	Mapping database for minor faults and folds used to guide geological reinterpretation
Volumetric estimation	Polygonal (manual); columnar seam model, 10 m × 10 m (computer)	Multiple-gridded seam model (8 m × 8 m); split vertically on 4-m levels and ore contacts	Blocks derived from surface models (4 m × 4 m); split on 4-m levels and ore contacts	Mini-blocks, 2 m × 2 m × 2 m (single rock code for each)
Grade estimation	Polygonal (manual); geostatistical where possible (computer)	Geostatistical using unfolding method to account for stratigraphic position	Geostatistical, anisotropic models using relative stratigraphic position. Grades of barren pyrite wallrock also estimated	Anisotropic inverse-distance weighting, using average orebody orientation every 20-m interval. Wallrock pyrite grades estimated
Mine planning	Manual scheduling using overall reserve totals	Manual methods using level plans and longitudinal sections for design	Interactive graphics using surface models and block grades. Design based on regular levels	Interactive graphics system with enhancements to allow design of irregular stopes
Reserve categories	Inferred I (plus Inferred II)	Indicated and Inferred I (plus Inferred II)	Indicated (plus Inferred I/II)	Measured (plus Indicated and Inferred I/II)

Table 3.2 Evolution of reserve estimation methodology at Neves-Corvo (from Richards & Sides, 1991)

4. GEOLOGICAL MODELLING FOR RESERVE ESTIMATION PURPOSES

4.1 General

One of the fundamental requirements of orebody modelling is the identification and modelling of geological discontinuities within a mineral deposit. Such features may represent mineralisation limits, contacts between lithologies with different physical and mineralogical characteristics, or tectonic features such as faults or fold axes. Modelling such discontinuities provides information on the limits of materials with differing mining characteristics, as well as forming the basis for subsequent volumetric modelling of properties such as metal grades and density.

Methods used to model geological discontinuities are summarised in Table 4.1. This Chapter describes the main methods currently used in computerised geological modelling, with examples from the deposits studied. The geological framework and investigation data, on which these examples are based, have been described in Chapter 3.

The different methods presented in Table 4.1. are discussed separately below. The selection of appropriate modelling methods depends on both the geological characteristics of the deposit being studied, and the level of precision and accuracy required for planning purposes. These aspects are dealt with below, together with consideration of the limitations which exist with the most commonly used modelling techniques. This leads on to the development of a geological discontinuity modelling system, which is described separately in Chapter 5.

METHOD	DESCRIPTION
Serial Slice Models	Geological interpretation of polygonal outlines on a series of regularly-spaced parallel sections or plans. Volumetric modelling is based on polygonal outlines assigned a prismatic volume of influence halfway to adjacent slices, precision being dependent on slice spacing and polygon size. Model precision can be improved by linking polygons on adjacent slices in 3-D. This is the most common method used for manually based geological modelling.
Gridded Seam Models	Multiple sub-parallel surfaces represented by grids of elevation points. Volumetric modelling is based on rectangular prisms between adjoining interfaces, precision being dependent on grid dimensions, and number of interfaces. Limited to use in tabular bodies (effectively 2.5-D).
Block Models	Volume being modelled is divided into a series of regular rectangular blocks. Each block is normally assigned uniform geological and grade characteristics, precision being dependent on the minimum block size. Blocks are commonly equidimensional, but storage and handling efficiency can be improved through the use of blocks of different sizes (eg octree encoding). Regular block models are one of the most widely used methods in computerised reserve estimation.
Mathematical Models	Geological discontinuities are represented as global or piecewise mathematical functions, such as polynomial trend surfaces, cubic B-splines, etc. Such models have a very high inherent precision, however much depends on the goodness of fit against original data. Problems frequently arise with edge effects. Widely used in 2-D contouring packages, and for stratiform deposits that can be represented by sets of 2-D surfaces. More complex surfaces can be represented by mathematical techniques such as those used in CAD/CAM packages for automobile and aircraft design.
Solid Models	Based on modelling groups of geometric primitives defining volumes of common properties. Precision depends on model complexity, but can be very good for geological modelling. Such techniques are usually based on existing CAD/CAM software, so that difficulties may be encountered in establishing links with pre-existing geological databases and mine planning systems. This is a relatively new approach in reserve estimation the full potential of which has still to be exploited.
Boundary Representations	Based on modelling discontinuity surfaces within a given volume of ground. Precision depends on model complexity, but can be very good for geological modelling. Techniques are mostly adapted from applications in other fields, such as medical tomography. Commonly used for 3-D visualisation of geological and mining volumes, although handling multiple intersecting boundaries can be difficult. A lot of recent development work has concentrated on this type of model.

Table 4.1 Methods used for geological modelling in computer-based reserve estimation systems.

4.2 Serial Slice Models

Modelling of geology by interpretation of a set of parallel, usually regularly spaced, sections or plans, as illustrated by Plate 4.1, is one of the most widespread methods used for the evaluation of mineral deposits (King et al (1982)). This approach was used before the widespread introduction of computerisation (Popoff (1966), Hazen (1968)), and continues to be an essential adjunct to many computerised reserve systems. The use of a series of planar slices to create 3-dimensional models of solid objects has analogies in other fields such as medical tomography and reflection seismics.

In mineral deposit evaluation the choice of section spacing and orientation usually depends on an initial idea of the overall geological structure of the deposit. This determines the choice of drilling grid at an early stage of evaluation. In the case of tabular stratiform orebodies, the drill grid and sections used for geological interpretation are usually oriented normal to the strike of the orebody. In deposits with more irregular shapes in three dimensions, such as porphyry copper deposits, interpretation is often done on two orthogonal sets of sections.

Interpretation is normally facilitated by having section lines parallel to the drilling grid so that most of the information used lies in the plane of interpretation. Problems in the use of information projected onto the plane of interpretation are highlighted by the example shown in Figure 4.1. In this case the discontinuity model of the fault shows an apparent displacement from the locations of the intercepts in holes RC187 and RC183 on which it has been based. The use of 3-dimensional computer graphics linked to geological databases greatly facilitates the handling and interpretation of irregularly spaced drilling data.

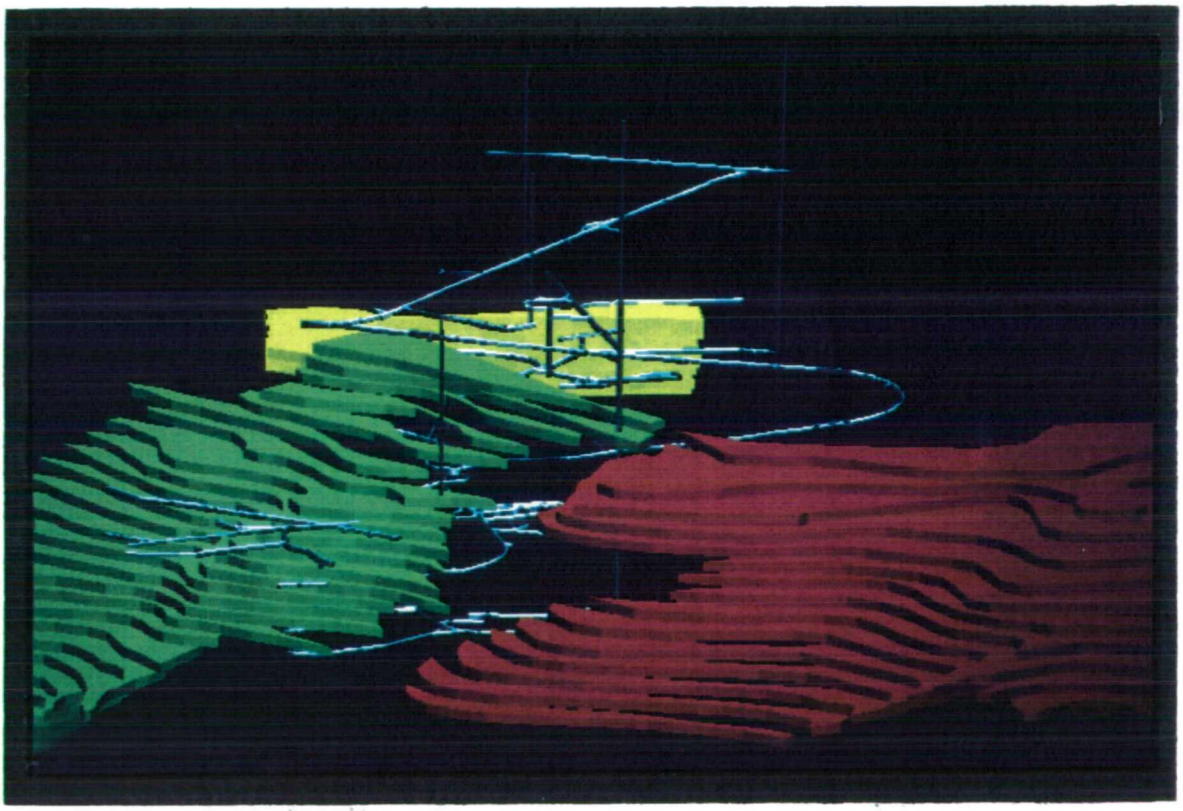


Plate 4.1 Serial-slice type models of the main massive sulphide lenses at Neves-Corvo. Based on interpretation of massive sulphide limits in horizontal plan, for Graca (yellow), Corvo (green) and Neves (red).

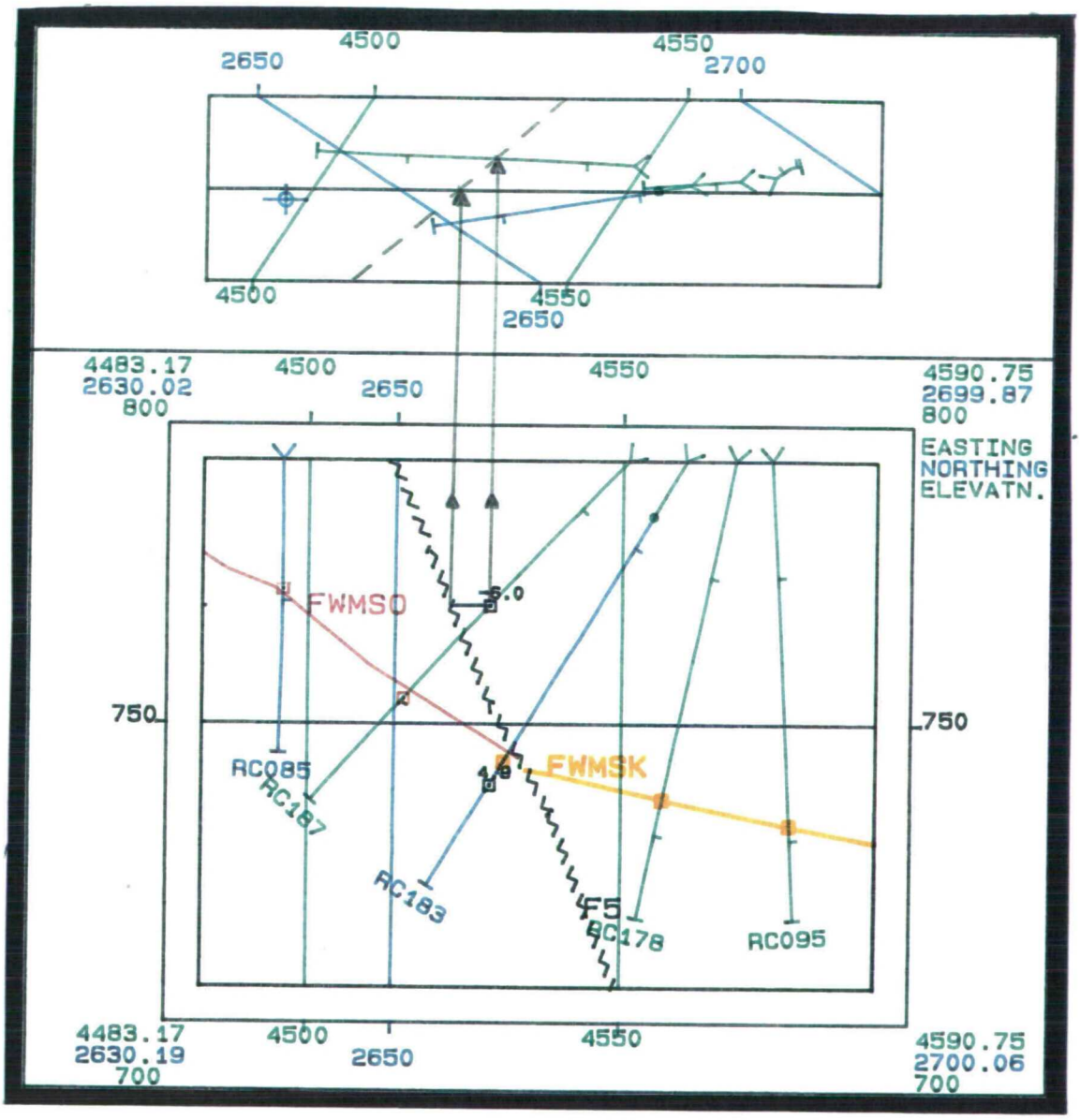


Figure 4.1 Drillhole section showing apparent displacement of contact and fault positions when using off-section data projected onto the section plane. Arrows indicate the method by which the strike of the fault plane (dashed line in upper part of plot) can be derived from the information presented.

Serial sections can form the basis for volumetric calculations where ore type or lithological units are defined as polygons on each section. The volume of each unit is calculated by measuring the polygon areas and assigning them a zone of influence half way to adjacent sections on either side. In some cases the polygons are also assigned grades based on the samples falling inside them, and can then be used to estimate geological reserve tonnages and grades on a section by section basis (King et al (1982)).

Geological interpretations on sections and plans are readily digitised and stored using computer aided drafting packages, and most currently available reserve evaluation software packages include this facility. When storing such interpretations on the computer there are two commonly used approaches.

1) **Geologically coded polygons**: Each section or plan is represented by a set of polygons defining areas of different geological types. Problems are encountered in ensuring that adjacent polygons match exactly without overlaps or gaps, and special procedures are also required for handling nested polygons. One way around these problems is to store polygons in a set sequence and ensure that those stored later in the sequence always overlap earlier ones along common boundaries. The geology for any particular point is that of the last polygon plotted over the area in which it lies. If this approach is followed then the boundaries of the stored polygons are often artifical, and do not match the interpreted geological features, so that if one fault or contact is changed several polygons may have to be redigitised.

2) **Polylines representing geological discontinuities:** In this instance the individual line segments making up each geological feature are stored only once for each section or plan. Polygons are then established by using pointers to the points in the individual line segments (Wray (1973), Burrough (1986)). This method presents problems in ensuring that all areas are properly closed, and that line segments abut properly against one another. This can be overcome by using special algorithms for handling the line intersection nodes.

One problem of using geological models based on serial slices for reserve evaluation, is that the slice spacing and orientation selected for initial geological interpretation may not provide sufficient information for detail reserve evaluation and mine planning. For this reason transfer of interpretations from section to plan, or from one set of sections to another, is often necessary. For instance, mine planning in most open pit mines is based on level plan interpretations corresponding to mining benches, however the initial geological interpretation of these deposits is generally based on vertical drillholes and sections.

Transfer to another set of sections is a fairly simple, though laborious procedure. The approach used is illustrated in Figure 4.2, where the intersections of a primary set of vertical sections have been drawn on a horizontal plan. The interpreted positions of the geological contacts on the section, at the required plan elevation, are then transferred to the plan, along with the codes for the intervals that they separate. This is repeated for all sections on each plan in turn, and the contact positions on the plan are then interpreted by joining up the intercept positions on adjacent sections.

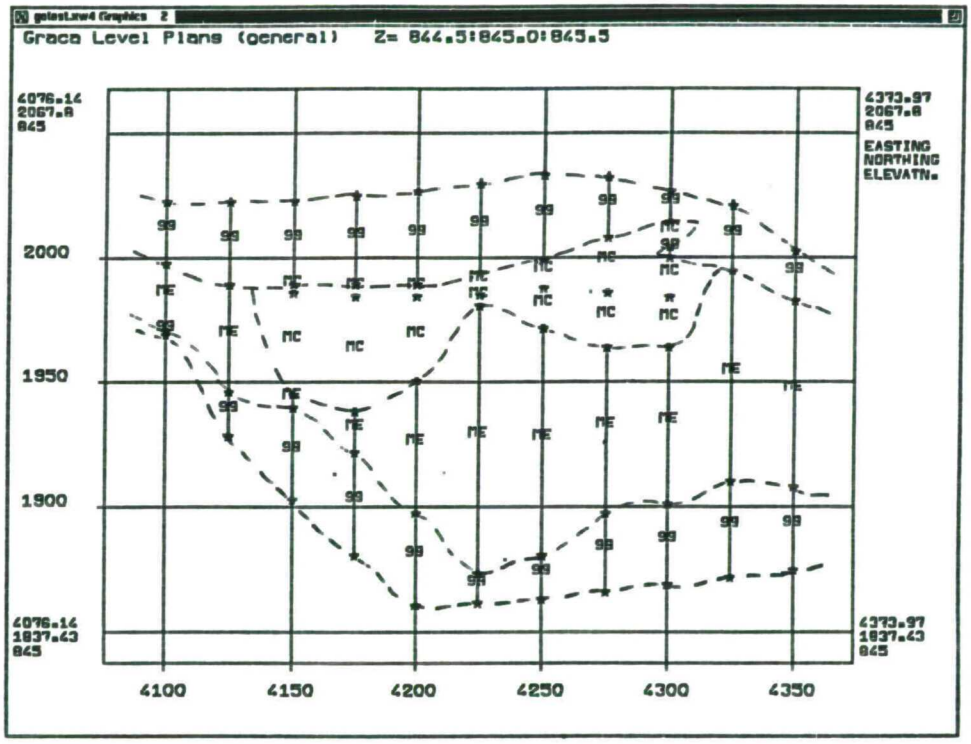


Figure 4.2 Level plan illustrating the approach adopted when transferring a serial-slice type interpretation from section to plan. Vertical lines with character codes indicate section line intersections; the dashed lines represent the interpretation of contacts derived from this information.

The example shown in Figure 4.2, was generated using software developed during work on the Neves-Corvo deposit. This is based on handling all stored 2-d discontinuities (digitised from plan or section interpretations) as 3-dimensional polylines. Display options allow contact position, name, and codes on either side, to be plotted on plans or sections at different orientations to those from which they were originally entered.

A modification to the conventional serial slice approach has been adopted recently by several workers in the field of geological modelling and reserve evaluation (Karonen (1985), Houlding (1987), Howson (1989)). This is termed the Linked-Slice Model which is similar to the Serial-Slice Model except that the polygonal outlines defined on each slice are linked together between sections by a set of linklines, defining rectangles or triangles, rather than giving each polygon a prismatic volume of influence halfway to adjacent slices. This approach has been widely used in the modelling of medical tomography images. Several different algorithms have been developed for automatically generating the most accurate links between adjacent slices (eg Keppel (1975), Fuchs et al (1977)).

A similar algorithm was implemented in the geological modelling system used for this research work. This technique allows geological discontinuities to be linked from slice to slice by a set of triangles which can then be used to establish three-dimensional models as described in the next Chapter. An example of this approach is shown in Plate 4.2.

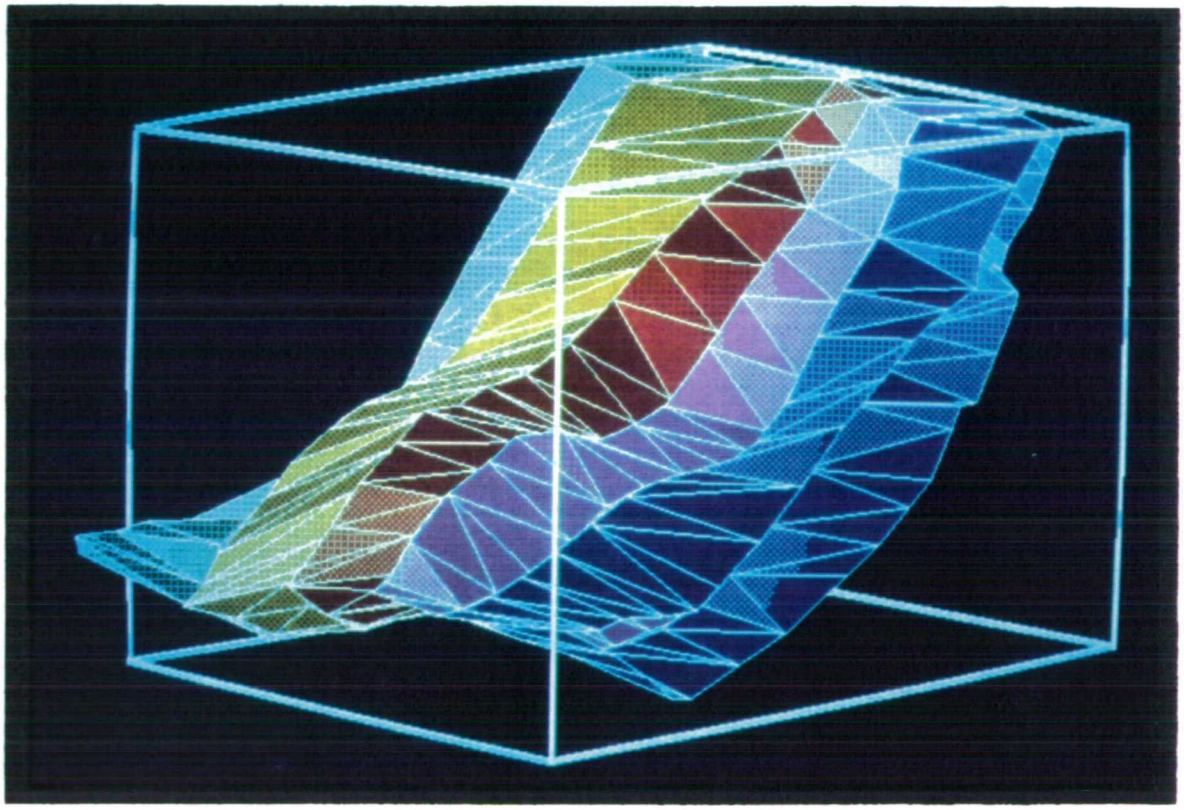


Plate 4.2 Example of a linked-slice type geological model (courtesy of RTZ Technical Services Ltd.)

4.3 Mathematical Models

A very efficient way of representing irregular surfaces in a computer is through the use of mathematical functions. Two main types are commonly used, namely;

global: where a single function is used to represent the entire surface of interest
piecewise: where each surface is represented by a series of separate patches, each of which is described by a different mathematical function.

These type of models are really a subset of the boundary representation techniques described in Section 4.6.4, however they are discussed separately here because of their importance in the early use of computers for geological modelling.

Many of the earliest applications of computers to geological modelling centred on the use of contouring packages to represent fairly simple geological discontinuities. Most such applications were based on the use of polynomial trend surfaces, as illustrated by several of the early University of Kansas Computer Contributions, such as O'Leary et al (1966), James (1966), Merriam & Cocke (1966), and later work by other authors such as Whitten (1975) and Pflatz (1975).

Examples of the continued development of new methods in contouring packages (based on mathematical modelling), are given by several authors. Sibson & Thompson (1981) describe the use of piecewise quadratic functions, which have been used in the CONICON contouring package developed at Bath University. Likewise Smith & Paradis (1989) mention the use of a minimum curvature method, developed by Briggs (1974), for the representation of 3-dimensional contoured surfaces.

Similar modelling techniques have been widely used in CAD/CAM packages developed for aircraft and automobile design. Such applications make use of a wide range of functions, such as several types of B-splines, Bezier surfaces and Coons surfaces, as described by Rogers and Adams (1976). Several workers have used such approaches for geological modelling; a recent example being the use of Intergraph's Engineering CAD software for geological modelling, as described by Fisher & Wales (1990, 1992).

Given the wide range of mathematical modelling techniques available, and incorporated into existing contouring packages, it is surprising that such techniques have not been used extensively for geological modelling in reserve estimation. One exception is their application in the oil and coal industries, as described by authors such as Jones et al (1986).

The limited application of such techniques to geological modelling for reserve estimation is probably due to factors such as the following;

- models such as trend surfaces often give rise to very large errors near the margins of the area being studied (edge effects)
- whilst models of 2-dimensional surfaces can be readily visualised and altered by users this becomes much more difficult for truly 3-dimensional surfaces
- functions such as cubic splines often require extra control points, in addition to the raw data points, in order to define the form of the surface. Where the surface shape is known in advance (as in aircraft or automobile design) this does not present a problem, but can be a significant disadvantage when trying to model an unknown shape from a limited number of sample points.
- volumetric modelling requires that the models of different geological surfaces in a deposit be inter-dependent, which can be quite difficult where mathematical models require the addition of artifical control points.

4.4 Gridded Seam Models

Gridded seam models were developed to overcome some of the limitations of the mathematical models described in Section 4.3, and are a widely used technique in the case of stratiform or stratabound deposits, such as oil or coal deposits (eg Jones et al (1986), Eli (1983)). This approach is based on defining a vertical sequence of geological interfaces, usually stratigraphic contacts, which are modelled over the entire area of interest. Each interface is used to define the geological unit immediately above or below; absence of units being indicated by assigning the same position to adjacent interfaces.

Computer representations of such models are normally generated by superimposing a regular grid of points, and determining the elevation of each interface at each grid node. These elevations can either be derived directly from geological interpretations, or estimated by mathematical interpolation from known data. Use of a regular grid of points, and the imposition of a common vertical sequence for the interfaces modelled, facilitates the generation of an unambiguous model without any overlap between different units.

Triangulated irregular networks (TINs), can also be used for modelling the interfaces in a seam type model, in a similar way to their application in digital terrain modelling and other areas (eg Eli (1983)). Crossovers between adjacent seam interfaces are more difficult to avoid in such models, unless the same set of vertices is used for all the interfaces (Jones et al (1986)). So long as the absence of crossovers is guaranteed, the differences in volumes between specified interfaces in a seam type model can readily be used for volumetric calculations.

A gridded seam type model was selected for initial geological modelling work on the Neves-Corvo deposit (Richards and Sides (1991)). This suited the stratiform nature of the deposits and their wide lateral extent when compared to the thickness of the ore horizons. In addition, the irregular nature of the initial grid of surface drillhole intersections made interpretation of a regular set of sections difficult. The interfaces used corresponded to the footwall and hangingwall of the massive sulphide, and rubane, orebodies respectively, as well as the main internal ore lens contacts (see Figure 3.10). Interface elevations were estimated on a regular grid using local trend surface fitting of digitised, manually interpreted, isobath interpretations. Surface fitting parameters were adjusted so as to obtain the best possible match between the original manual interpretation and isobath contours derived from the modelled interface elevations.

Subsequent underground evaluation work revealed the presence of several major faults cutting the deposit, which could not be modelled accurately using this approach. This problem was overcome through the use of separate gridded-seam models for each structural block, based on discontinuity models of the main faults and contacts, as discussed in Section 7.2.

The file structure used in the gridded seam models at Neves-Corvo is given in Table 4.2. This structure allows grade information to be stored in the same files as information about the geological shape of the orebodies. Such files can then be used directly for the calculation of reserve tonnages and grades. Where significant grade variations occur vertically within individual lenses, or where the lens thickness is large in relation to the height of mine openings, this approach may prove unsatisfactory. In such instances the columnar blocks can be subdivided in the vertical sense, in order to allow a maximum block height to be defined for grade estimation purposes.

Position	Description
1	Header data (co-ordinates of model origin, orientation of X and Y axes, block size, etc.)
2	Limit of Interpretation (CHARACTER*4 array defining limits of interpreted area; plus fault block codes where more than one fault block is stored in a single model)
3	Model base (array of elevations defining the lowest valid elevation for each block)
4	Model top (array of elevations defining the highest valid elevation for each block)
5	Internal lens contacts 1 to N-1, in order from lowest to highest elevation (N-1 arrays of elevations defining the elevation of the lens contacts for each block; N-1 internal contacts are used to define N separate lenses)
6	Grade 1 for each lens (N arrays of grade values defining the value of grade 1 for the columnar blocks, of varying height,for each lens in turn)
7	Grade 2 for each lens (N arrays of grade values defining the value of grade 2 for the columnar blocks, of varying height,for each lens in turn)
	(continued for any number of different grade values)

Table 4.2 File structure used for gridded-seam type models at Neves-Corvo.

4.5 Block Models

Block models are widely used for reserve estimation and mine planning purposes. Several types of block model are used, as summarised in Table 4.3. Such techniques could be classed with the group of solid modelling techniques discussed in Section 4.6. They are discussed separately here, however, because of their dominant role in the field of computerised orebody modelling over most of the past two decades.

TYPE	DESCRIPTION
Regular	Volume of interest is divided into a three dimensional matrix of identical rectangular blocks each of which is given a unique geological code.
Seam-type	Volume of interest is divided into a regular rectangular grid in plan, and each geological unit assigned a base elevation and thickness within each cell. (also called a columnar lattice model).
Variable	Similar to a regular model, except that each block may be divided into sub-blocks where necessary for local detail (eg. cuboid sub-cell model).
Proportional	Model structure identical to the regular block model, except that each block may be assigned more than one geological code, along with the percentage of the block which it occupies.

Table 4.3 Types of block model used for computerised orebody modelling

The widespread usage of block models reflects the ease with which information stored in this format can be manipulated within computer programs. For instance, the characteristics of a point in a given block can be accessed very rapidly, and arithmetic operations between different reserve properties are facilitated. Model coherency is easily checked to avoid inconsistencies such as undefined spaces, or conflicts within the model.

Such models can be used to store both geological code type data, and block grade estimates, thus facilitating the transfer of information between geological modelling, grade estimation and mine planning programs. The remainder of this section discusses ways in which they are used to represent deposit geology, rather than for grade estimation.

Regular block models, as illustrated by the example shown in Plate 4.3, are the most widely used approach, and have the following advantages:

- Each block has the same dimensions and volume

- Data can be handled as arrays for each level or slice in the deposit, thus facilitating links between different data types (eg. grade estimates and rock codes)

- Grade estimation is facilitated since volume-variance relationships are constant for all blocks from any particular mineralised unit, and since each block is assumed to belong to only a single unit

- Each level in the deposit has the same number of blocks, thus facilitating disk storage and access

- The reserve data for any point in a deposit can be located very quickly because of the regular structure of the model

Initial input of geology to such models usually relies on the generation of a geological interpretation in the form of one of the other model types discussed in this Chapter. The approach used for the deposits studied was to prepare geological interpretations on a set of level plans corresponding to the horizontal slices in the block models (ie a serial-slice type interpretation). In the case of both Cerro Colorado and Bama-Brandelos, these plans corresponded to actual open-pit mining benches.

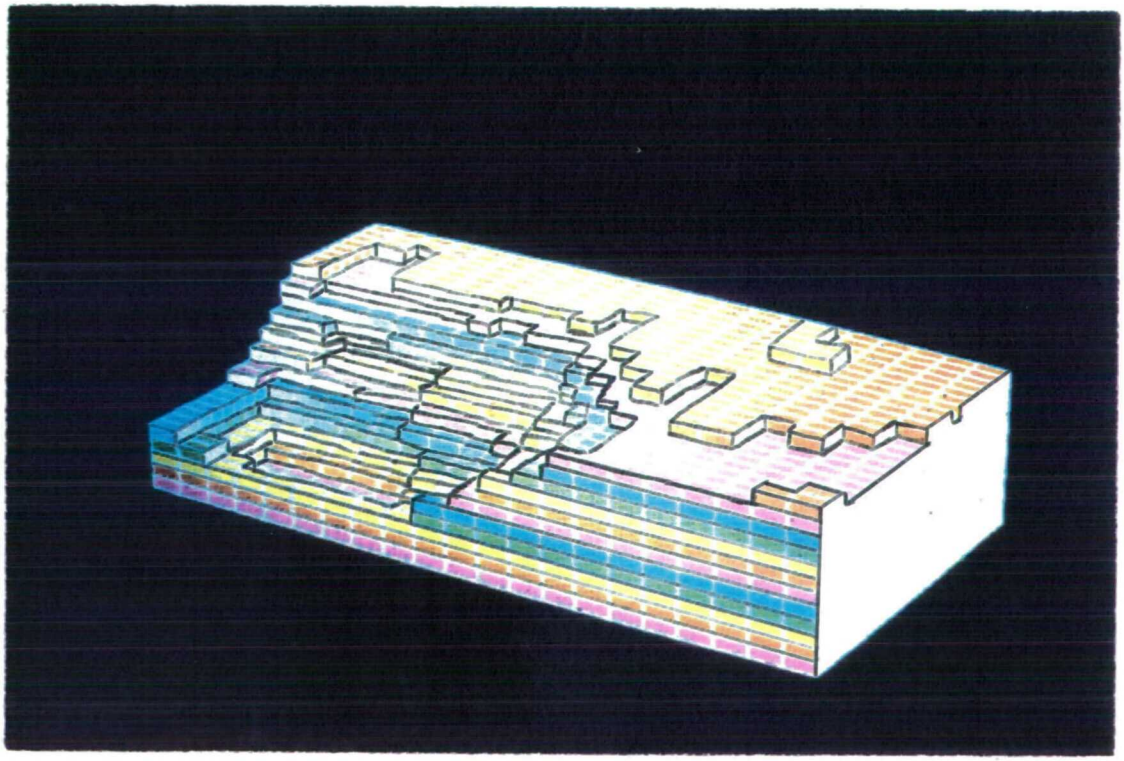


Plate 4.3 Three dimensional visualisation of a regular block model (courtesy of RTZ Technical Services Ltd.)

Block models were generated by digitising sets of polygonal areas on plans corresponding to the block model slices, and assigning an associated code to any blocks whose centres fell inside the polygon (Plate 4.4). Problems of overlaps and gaps were overcome by initialising all blocks on a level to a pre-defined default value, and then digitising a series of overlapping polygons. This approach also allowed for direct interfacing with stored geological serial-slice interpretations, so as to avoid having to digitise the same interpretations twice. Subsequent checking and editing of the resultant block models was done in an interactive mode using a graphics screen and screen mouse.

Where 3-dimensional solid models that allow point-in-volume tests are available, initialisation or modification of block models can be done simultaneously for a series of slices, rather than separately for each slice. Seam-type models can also be converted into regular block models, although this usually leads to the loss of some geological detail.

Access to information stored in block models, for display or evaluation purposes, is most efficient in planes parallel to the principal axes of the block model. Display of block model data in other orientations usually involves reading in and checking all the slices which may cut the desired view, and may consequently be slower than desirable in an interactive environment.

When regular block models are used, the entire volume being modelled has to be split into blocks. This often results in the storage and manipulation of a lot of redundant information in peripheral areas. The size of the files and memory arrays used in such instances can be reduced by employing special algorithms using run length encoding. The size of files archived on disk can also be reduced by using system compression utilities.

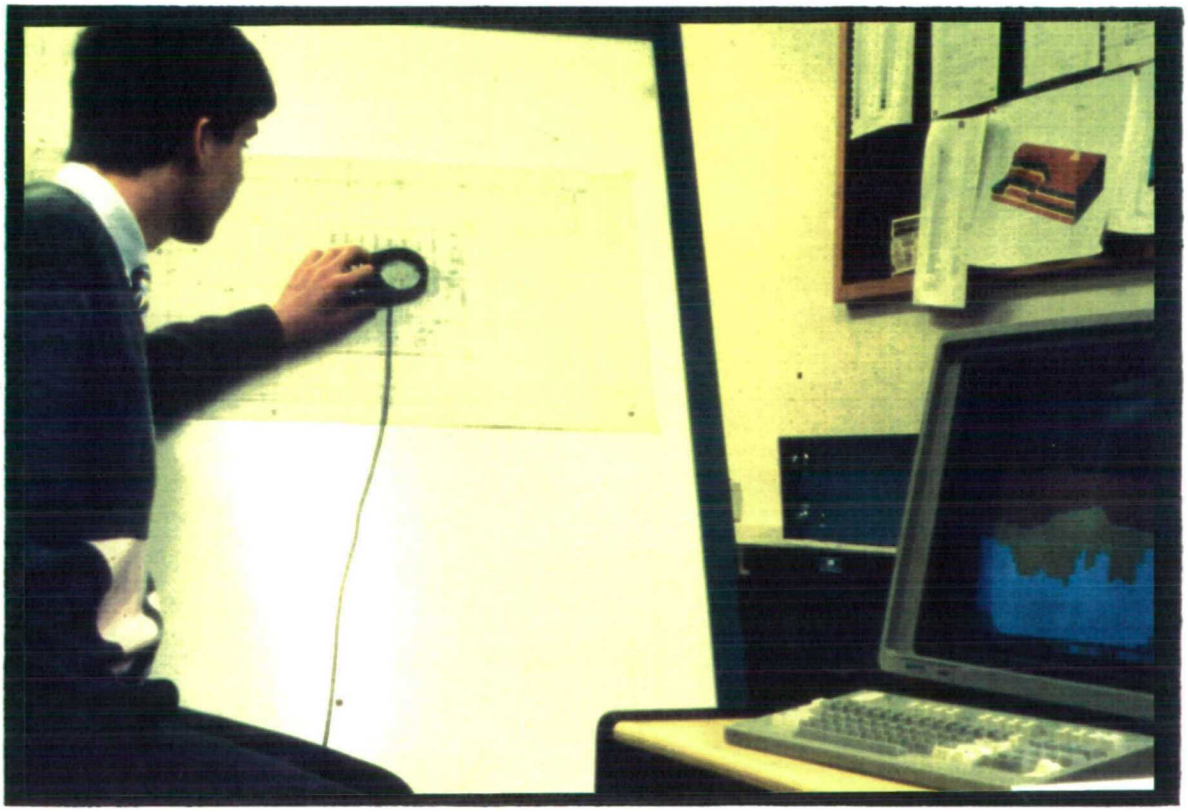


Plate 4.4 Use of an interactive graphics workstation for editing a geological block model (courtesy of RTZ Technical Services Ltd.)

4.6 Solid Modelling Techniques

4.6.1 General

Over the past decade an increasing amount of attention has been given to the application of solid modelling techniques (developed for computer graphics, CAD/CAM and medical applications) to geological modelling. The development of the study of geometric modelling in such applications is reviewed by Mortensen (1985). Such work has led to the definition of a mathematical/geometrical framework for use in the representation of rigid solid bodies by computer modelling, as discussed by Requicha (1980). The main types of solid model defined by Requicha are summarised in Table 4.4.

MODEL TYPE	DESCRIPTION
Spatial Occupancy Enumeration	Based on repeated sub-division of an initial rectangular prism enclosing the entire area to be modelled. Cells are assigned a code, if occupied by a single material type, or if at the lowest level of sub-division. (eg Octree models).
Constructive Solid Geometry (CSG)	Solids are modelled as a series of geometric primitives, either a series of different geometric shapes, or a set of half spaces which can be combined to represent the total solid.
Boundary Representations (BR)	Solids are modelled in terms of their bounding surfaces, as a collection of vertices, edges and facets
Other Schemes	Other methods include wireframe models, linked sections, and Cell Decompositions.

Table 4.4 Summary of the main model types used in computer-based solid modelling (after Requicha, 1980)

The use of these model types in other fields provides parallels with the problems encountered in trying to represent geological boundaries and volumes on the computer. A review of the available literature suggests that, whilst the theoretical background is more developed in the field of CAD/CAM, medical applications are more directly comparable to geological modelling. A lot of research and development work in the medical field has concentrated on the construction of three dimensional models using sets of serial scans through irregular and heterogeneous bodies such as the human brain (eg Drebin et al (1988), Merickel (1988), Herman et al (1992)).

The solid modelling techniques listed in Table 4.4 are only some of those described by Requicha (1980), however they include most of the techniques that have been applied in geological modelling and mine design over recent years. Examples of such applications are provided by authors such as Karonen (1985), Houlding (1987), Wheeler & Stokes (1988), Howson (1989) and Mill (1989) amongst others.

Some of the general principles defined by Requicha are already implicit in current approaches to orebody modelling. For instance, the calculation of mining reserves by combining geological models with models of the mine openings would correspond to the performance of Boolean operations in solid modelling systems. Likewise boundary determinism corresponds to the requirement that it should be clear from any reserve model what ore type occurs on either side of a contact.

Requicha also discusses disjointness, whereby various integral properties of the solids may be computed as the sum of the contributions of the separate components of a solid model. This property is usually taken for granted in the use of ore reserve models by mine planning programs, when tonnage and grade values are accumulated as weighted averages of individual block values. Spatial decomposition models, Cell Decompositions, and certain types of CSG model, possess the property of disjointness.

Two data types are recognised in solid modelling, namely geometric and topological data. Geometric data are used to define the actual or relative positions of a body in space (eg. the co-ordinates of a block model origin, the orientation of its axes, and the codes of individual blocks). Topological data describe the spatial connectivity between different geometric components (eg row, column and slice numbers may be used to define the location of individual values within a block model).

The solid modelling techniques listed in Table 4.4 are described briefly below.

4.6.2 Spatial Occupancy Enumeration

This type of model is defined by Requicha (1980) as being essentially a list of the spatial cells occupied by the solid. Such cells are generally regarded as cubes of a fixed size and lie in a fixed spatial grid. These cells are termed voxels (volume elements), by analogy with pixels (picture elements), in recent computer graphics literature (eg. Drebin et al, 1988). This technique has the advantage of being able to describe sub-units of differing characteristics within a single model, and is directly analogous to the regular block models described in Section 4.5.

Development of techniques for the storage and manipulation of raster graphic displays, have resulted in the definition of recursive spatial sub-division schemes, as described by Meagher (1982a, 1982b), Samet & Tamminen (1985), and many others. The most widely used of these schemes are those known as quadtree and octree models where the spatial units are divided twice in each dimension, at each stage in the subdivision, as illustrated in Figure 4.3.

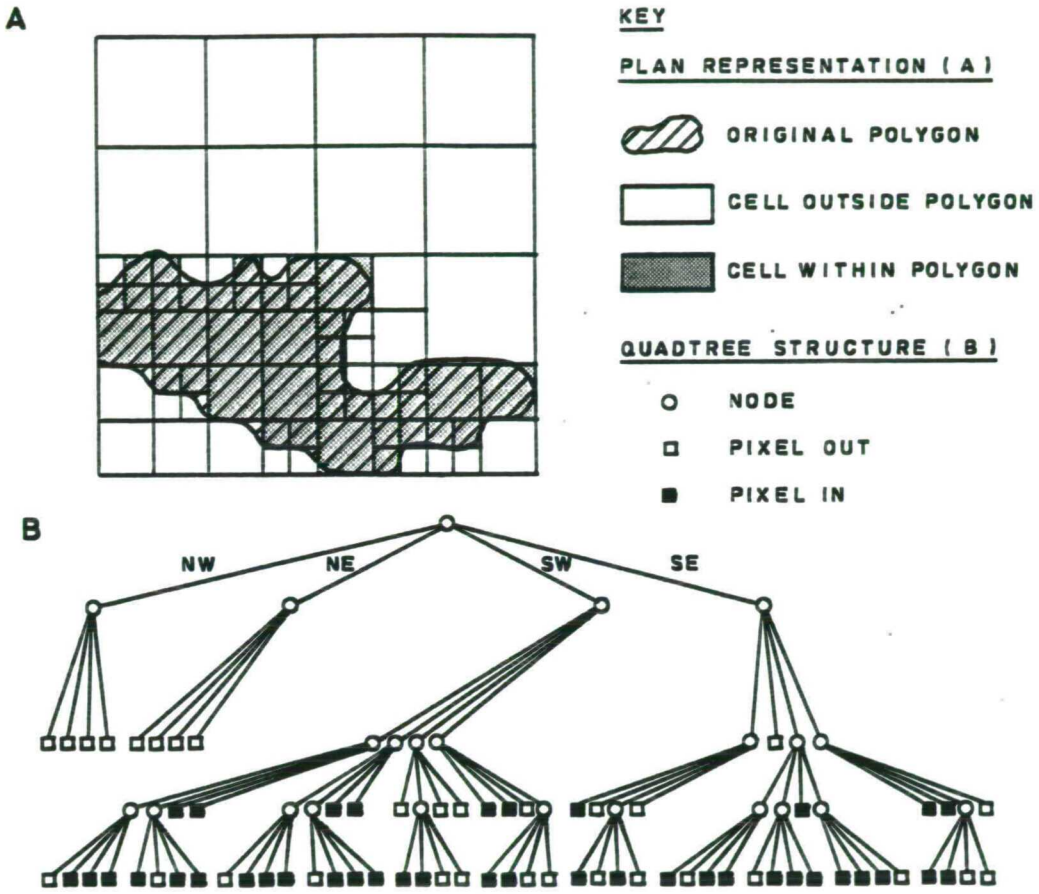


Figure 4.3 Diagram illustrating the modelling of a polygon using a quadtree representation scheme (modified after Burrough, 1986)

Some work has been done on the use of such techniques in mining applications (eg. Walker & Grant (1986), Samaddar (1988), Mill (1989), Dunstan and Mill (1989), Prissang and Skala (1990), and Bak (1991)), but their use in orebody modelling is still at an early stage. As well as helping to reduce file storage, and allowing variable precision within a single model, the fact that such recursive spatial subdivisions could also be applied to further dimensions such as time and reserve category, suggests other potential applications in reserve modelling.

4.6.3 Constructive Solid Geometry

Constructive Solid Geometry (CSG) defines a solid in terms of a collection of elementary geometric units, and a set of operations by which they must be combined to produce the desired object. Depending on the type of basic unit used, CSG models can be further subdivided into those based on;

Elementary half spaces:
This type is based on the fact that the mathematical definition of most planar and curved surfaces divides space into two portions. For instance, in the case of the equation for a plane of the form:

$$a_0 + a_1x + a_2y + a_3z = 0,$$

a point would be defined as being in one half space if its co-ordinates gave a positive value when substituted in this equation, and in the other if they gave a negative value. A value of 0 would indicate that the point lay on the surface.

Bounded primitives:
In this type a set of primitive solids (eg, cube, sphere, regular prism, etc), are combined together to represent the solid being modelled. Such combinations may involve addition, subtraction, rotation, etc..

CSG type models are widely used in the field of computer aided manufacturing (CAM) where the objects being modelled have to be described exactly in order to allow their replication by automated machinery. This is not the case in geological modelling where the solid objects being modelled are always imprecisely known. Nevertheless, the rigorous theoretical approaches adopted in solid modelling have a lot to offer since models are more readily passed between different applications (eg reserve modelling and mine planning), if no ambiguities are present in the model structures used.

4.6.4. Boundary Representation Models

In this case solid bodies are represented in terms of a collection of faces, edges and vertices which define their spatial boundaries. These include models such as the mathematical models discussed in Section 4.3. Such models are commonly applied in automobile and aircraft design, where the surface faces are defined by mathematical descriptions such as bicubic splines, Bezier curves, Coons patches, etc.. In such cases criteria must be established for;

a) defining the inside and outside of the solid;
b) ensuring that surfaces mesh together correctly at the edges; and
c) checking that no ambiguities exist in the model.

The special case where all the faces of the model are planar facets is known as a polyhedral or tesselated model. In such cases all the facet edges are straight lines, and curved or irregular surface edges are approximated by planes and lines. The geological discontinuity model structure described in Chapter 5 falls into this category.

4.6.5 Other Schemes

Some of the other solid modelling schemes discussed by Requicha (1980) have potential applications in the field of orebody modelling. This includes wire-frame models, where an object is described by a collection of edges. This method has been used by several authors to display three dimensional representations of models of orebodies or mine openings (eg Wheeler & Stokes, 1988). Requicha points out however that this method is ambiguous when used as the sole means of describing a solid.

Another technique mentioned is Cell Decomposition, whereby a solid triangulation of a rectilinear polyhedron results in its decomposition into tetrahedra. Such tetrahedra must be non overlapping, although they may share common faces, edges or vertices. Such a modelling approach is a generalisation of triangulations such as those discussed by Bowyer (1981), Watson (1981), and Watson & Philip (1984a, 1984b). Although not widely applied in the CAD/CAM field, some of the properties of such models (such as the existence of algorithms for determining whether a solid is one-piece or contains voids), suggests that their application in the field of orebody modelling warrants investigation.

4.7 Limitations to Conventional Approaches

The serial-slice type model was the most commonly used approach before the introduction of computerised modelling techniques (Popoff, 1966). This reflects the fact that it is really a graphic representation of how most geologists set about interpreting a given geological situation. Mental visualisation and interpretation of a given geological situation is usually simplified by reducing 3-dimensional interpretation problems to a series of interpretations in 2-dimensions.

Despite its popularity in geological modelling the serial-slice approach has been less popular in computerised applications, because of difficulties in developing suitable techniques for handling such data. Available models have tended to be rather laborious to construct and update. This is particularly true where a lot of the drilling or sampling data used in the interpretation does not fall in the plane of the interpretative sections.

Additional difficulties are presented in deciding how to represent the geology between interpreted sections. The traditional approach of assuming that the interpretation presented on each slice has a volume of influence halfway to the next slice is unrealistic in many situations.

The linked-slice approach, discussed in Section 4.2, overcomes some of the problems associated with the conventional serial-slice approach. This allows more accurate representation of the geology between adjacent slices by linking the interpreted lines or polygons on each slice with a series of triangles. Where this approach is used to model a series of closed volumes, problems of model coherency may arise through the introduction of overlaps or gaps between adjacent volumes; especially in the spaces between the planes of interpretation.

One of the main limitations of seam-type models is the fact that they cannot readily be used to model interfingering or overfolded tabular orebodies. For this reason their application is restricted to simple stratiform deposits (such as coal seams and oil reservoirs), as well as tabular vein deposits. Some of the more complicated situations, such as lensing out of particular horizons, can be handled by introducing artifical interfaces in the surrounding units.

Despite their advantages, several limitations exist in the use of regular block models, including the following:

- The resolution of such models is controlled by the block dimensions, often resulting in an imprecise match to well defined contacts such as surface topography.

- Such models are deterministic in nature in that they require the association of a single defined material type code with each block

- Geological boundaries in block models can only be represented where material with different codes is present on either side, resulting in a loss of geological information.

- Updating such models with new geological information can be a laborious procedure because the interpretation of geological contacts is not an integral part of them

- Such models present a uniform amount of detail throughout, even though data density may vary considerably

- The step-like representation of geological contacts is rigid and unrealistic, and can lead to false impressions of the true nature of the geology in the minds of planning engineers

- It is difficult to deal with blocks which contain more than one material type

The variants of the regular block model listed in Table 4.3 have been developed in order to overcome some of the disadvantages listed above. The seam-type model (see Section 4.4), improves the precision of boundary representation for contacts that are semi-parallel to the block model slices. The cuboid sub-cell technique (Henley & Stokes, 1984) is another approach which allows variable information density to be stored. The proportional block approach is another alternative which introduces a probabilistic element for dealing with the presence of more than one material type in a given block.

The relationship between different modelling techniques, and choice of model parameters (such as block model origin, block size, and orientation) on the accuracy and precision of the resultant geological models is discussed in more detail in Chapters 9 and 10.

4.8 Discussion

The work described in this Chapter has illustrated the application of different geological modelling techniques to a variety of different mineral deposits. The selection of an appropriate modelling technique to use in any particular situation depends on various factors, including the following;

- the objectives of the study (eg solely to guide future evaluation work, or for detailed reserve estimation)

- requirement for, and frequency of, future updates of the model

- the nature of the deposit being studied (eg high grade tabular stratiform deposits versus bulk disseminated deposits)

- the geological data available (ie nature and size of databases)

- available modelling techniques (eg limits of model precision and accuracy)

- available computing facilities (eg disk storage or memory capacity limitations on model size)

- ease of interface with grade estimation and mine planning programs

The conventional techniques described in this Chapter show significant limitations to allowable precision, loss of detail in some areas, and in some cases potential inaccuracies in the representation of the original geological interpretation. In bulk disseminated deposits with more irregular 3-dimensional shapes (generally mined by open-pit methods), these problems are less critical and regular block models are usually sufficient for geological modelling and reserve estimation. This is also the case with deposits where the geological contacts being modelled are gradational or 'fuzzy', as in the case of the Cerro Colorado grade zone contacts. Regular block models should be adequate in such cases unless the exact location of such contacts is crucial to reserve estimation or mine planning.

In high grade deposits with sharp ore contacts, as at Neves-Corvo, maximum precision and accuracy are required in the modelling of geological shapes in order to facilitate subsequent reserve estimation and mine planning studies. Using block models for this purpose would require small block sizes resulting in large models. Such models present problems in data storage and handling, speed of file access, model generation and editing. Other approaches such as the use of gridded seam models overcome some, but not all, of these difficulties.

The problems encountered during geological modelling at Neves-Corvo led to the definition of a set of objectives for an ideal geological modelling system, including the following:

- direct links with drillhole and mapping databases to facilitate model generation and checking

- ability to identify geological features such as faults, as well as boundaries between different geological units, in the models

- fully 3-dimensional representations of geological features so that they could be displayed on sections or plans at any orientation

- facilities for incorporating interpretations done in 2-dimensions, on sections or plans, into 3-dimensional models

- the ability to interface directly with the volumetric models used for grade estimation in order to facilitate more rapid updating of reserve models

- interfaces with other model types so as to allow integration with systems based on conventional model types

After consideration of available modelling options it was decided that a boundary representation scheme would be the most appropriate means of achieving these objectives. Available modelling software did not provide all the options required, so that a new geological discontinuity modelling system was developed. The structure on which this system is based, and the ways in which it can be used to generate, edit, display and analyse geological discontinuity models are discussed in Chapter 5. Subsequent Chapters illustrate the application of the discontinuity modelling technique in the context of computerised reserve estimation.

5. A GEOLOGICAL DISCONTINUITY MODELLING SYSTEM

5.1 General

The review of techniques available for computer-based geological modelling, given in Chapter 4, highlighted several limitations and led to the definition of the need for a more geologically based system. One of the main requirements was for a system that would facilitate the use of information stored in existing geological databases, as well as allowing links with the types of volumetric models used in conventional reserve estimation and mine planning systems.

A system for handling this type of model was developed, implemented, and tested during reserve estimation work done at Neves-Corvo. The model structure is based on the use of triangulated surfaces to represent the different geological discontinuities (faults, contacts, etc.) within a deposit. The system developed is termed a **geological discontinuity modelling system**, and is analogous to boundary representation models used in other fields. A preliminary description of the approach used was presented in Sides (1992a).

Figure 5.1 illustrates some of the requirements specified for the 3-dimensional geological modelling system. The need to incorporate raw data such as drillhole and mapping data, as well as lines representing manual interpretations done in two dimensions, is indicated. During initial interpretation a geologist may wish to leave ambiguous situations from a volumetric/solid modelling viewpoint, such as incompletely defined contacts at the edge of a deposit. Extrapolated contacts, such as pre-erosion geology, may also be modelled.

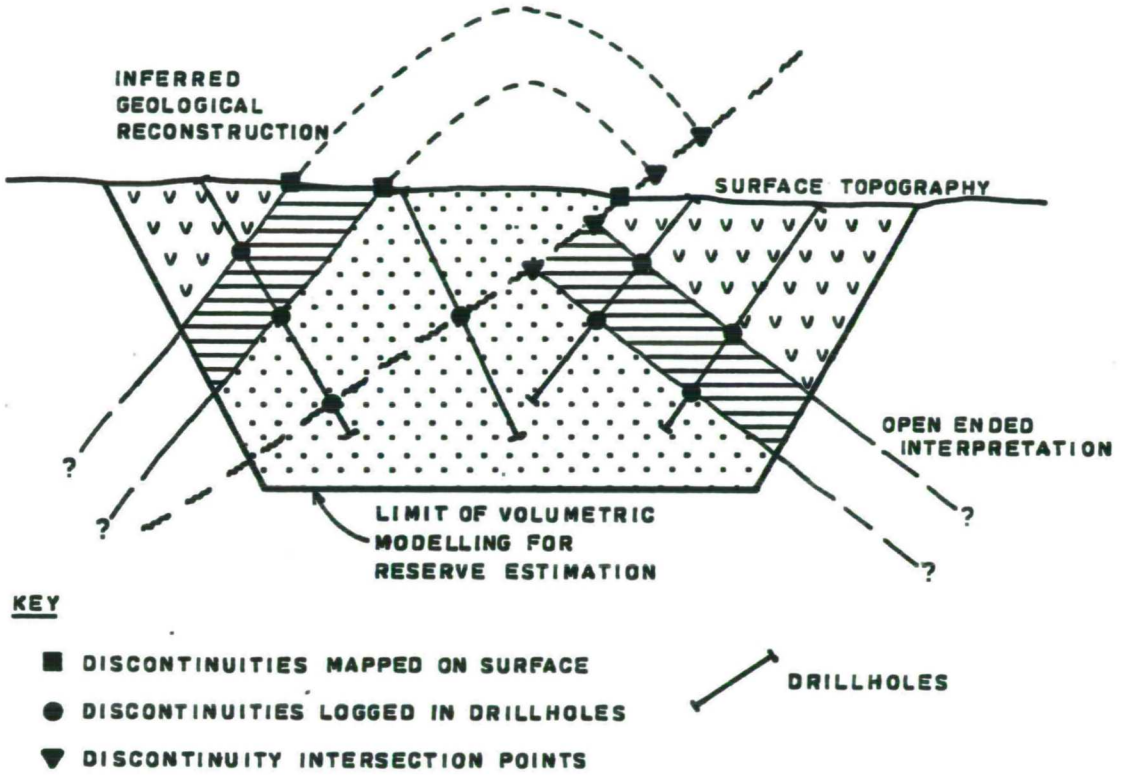


Figure 5.1 **Schematic cross-section illustrating some of the data types and interpretative requirements of a geological modelling system (redrawn after Sides, 1992a)**

Another typical feature of geological modelling is wide variability in the density of available information, and consequently in the level of detail that can be accurately modelled in an individual deposit. A further common requirement is the need to maintain different models for the same area, and to frequently modify and update models.

The technique used here is not entirely new, a somewhat similar approach being adopted by Wray (1973). More recently workers in other geological fields have applied similar approaches (eg Klein et al (1989), Pflug et al (1992), Ruber (1988), Schaeben (1988), Siehl (1990)). In addition triangulated irregular networks (TINs) are widely used for modelling 2-D surfaces in applications such as geographic information systems (Burrough, 1986).

Most of the commercially available reserve estimation and mine planning software systems developed over the past decade (eg DATAMINE, LYNX, MEDS, VULCAN, etc.) now include options for geometric modelling of geological information. In the majority of cases such options have been derived from solid modelling algorithms used for designing mine openings, or borrowed from other CAD/CAM applications. In such cases geological data and interpretation methods have to be organised around the modelling system, rather than the system being designed on the basis of existing databases and interpretation methods, as done in this instance. The fundamental difference between these two approaches is discussed in more detail below.

The software used for the work presented here was developed and implemented on Sun 3/60 and Sparc 1+ colour graphics workstations. The programs used were written in Fortran 77; software libraries developed by RTZ Consultants Limited being used for several lower level operations such as the graphics user interface, screen graphics output and digitiser input.

5.2 Criteria Used

The model structure adopted is based on several general principles, including the following:

- All co-ordinate data are stored as mine co-ordinate values to facilitate links with other databases

- All data items included in the model are reduced to geometric primitives so as to facilitate handling and analysis

- General attributes can be assigned to an entire model or to the individual primitives within it

- Links to geological databases are maintained by establishing groups of special point and line types

- The system is designed around open ended discontinuities, those bounding closed volumes being regarded as special cases

- All discontinuities are regarded as having unique positive and negative facing directions, with the possibility of assigning geological codes to the material types on either side

- The Z co-ordinate axis (elevation) is treated as a special case to facilitate the establishment of conventions for facing directions

- Definition of volumes is done in a geological fashion by establishing a hierarchy of surfaces for intersection purposes

- Model precision is controlled by the use of minimum tolerance distances when testing for point-point and point-line coincidence

5.3 Model Structure

Each individual discontinuity is modelled and stored as a separate entity; several separate blocks of data being stored for each, as summarised in Table 5.1.

Data type	Description
Header information	Name; version number; general description; positive and negative side codes; number of points, triangles, intersection lines, special points and edges; geometric limits; date generated and date last altered.
Point co-ordinates	X, Y, Z co-ordinates, plus up to 3 associated integer codes for all points in the model
Triangle pointers	Sets of three vertex pointers into the point array
Special points	Extra information about points derived from drillhole and mapping databases; such as drillhole name, depth, mapping location, orientation, etc.
Special edges	Triangle edges fixed by geological interpretation, to facilitate agreement with 2-D models
Intersection lines	Intersection lines with other surfaces, used for special display features, and to control generation of volumetric models

Table 5.1 Geological Discontinuity Model File Structure.

The header information facilitates rapid checking of the characteristics of a given model. The file structure has evolved considerably since its initial development, and consequently one of the first header parameters is a variable to indicate the version of the software with which the file was written. This facilitates future modifications, since programs use the version number to determine the way in which a file is to be read in, whilst always writing it out according to the latest version. This prevents incompatibilities between files generated with different program versions.

Other header information (such as surface name, associated codes, and geometric limits), allows rapid checks to be made, during surface selection for display and editing, without having to read in an entire file.

The header information is followed by the main structure of the discontinuity model which consists of a list of point co-ordinates and a list of triangle vertices (specified as pointers to the arrays of points). Point co-ordinates are stored using the local co-ordinate system so as to facilitate links with other databases. Each point has three associated integer codes; the first of which is used to indicate the nature of the point (eg drillhole intersection, digitised point, etc.), and the other two to control aspects of surface display.

To facilitate the use of algorithms for determining surface edges and hinge lines, a simple rule is used to control the order of the vertices stored for each triangle. The convention adopted is that a line connecting two points, to form a triangle edge, can only be used once as a vector in each sense. Consider, for instance, the two points P1 and P2 shown in Figure 5.2. Once the vector P1-P2 is used in one triangle, and P2-P1 in another, a line joining these two points cannot be used in another triangle. This convention facilitates determination of surface edges, and prevents the generation of models with Mobius strip type characteristics.

This rule does not have to be followed during initial editing of a surface, but is normally necessary for display and interfacing with volumetric models. Algorithms have been developed for determining whether the rule has been obeyed, and if not then correcting the surface by reversing the sense of offending triangles. This aspect is less critical for simple surfaces, where all triangles face in the same direction, but is essential for a structure capable of handling overfolds, as discussed below.

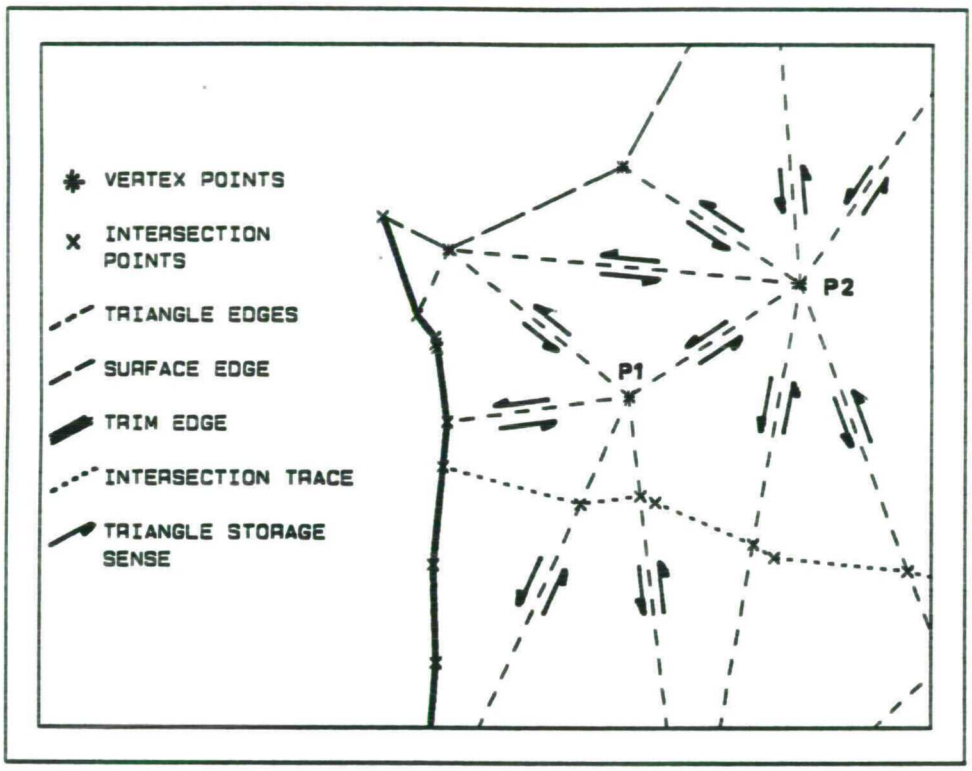


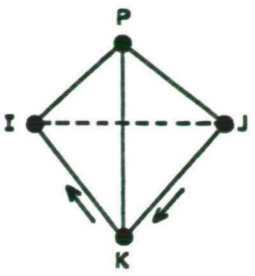
Figure 5.2 Portion of a geological discontinuity model illustrating the model structure (after Sides, 1992a)

All discontinuities have two associated codes to allow definition of material types on either side. These are defined as positive and negative, by taking a point on one side of the discontinuity and forming a tetrahedron using it as the first vertex. The other three vertices used are the triangle vertices taken in the order in which they are stored. The side of the discontinuity on which the point lies is then said to be positive or negative depending on whether the tetrahedron volume is positive or negative when calculated using the method given in Figure 5.3.

The first non-vertical triangle in any surface, or sub-segment of a surface, is set so as to be positive when viewed from above (as defined by the positive Z co-ordinate axis direction). By following this convention problems with overturned or contorted surfaces are avoided as illustrated schematically in Figure 5.4. This also facilitates the determination of hinge lines such as fold axes, or the limits of lenticular bodies.

In addition to the individual point codes discussed above, two sets of special named points are allowed. These possess additional information which enables links to be maintained with drillhole and mapping databases. For drillhole points, hole name, intersection depth, and core angle are stored. For mapping points, location name, dip direction and dip angle are stored.

To allow lines (defined during 2-dimensional geological interpretation) to be retained in the 3-dimensional discontinuity models a set of fixed links are defined which are preserved during editing. These links are defined as point pairs with reference to the array of points stored at the start of the file. Such links represent geological discontinuities interpreted in 2-dimensions on sections or plans, and consequently become triangle edges in the 3-dimensional models.



P = test point
IJK = triangle in the discontinuity model which is closest to point P
X_I = X co-ordinate of Point I
Y_I = Y co-ordinate of Point I
Z_I = Z co-ordinate of Point I
X_J = X co-ordinate of Point J
etc.

The volume of the tetrahedron is calculated as follows:

$XIP = X_I - X_P$ $\quad YIP = Y_I - Y_P$ $\quad ZIP = Z_I - Z_P$
$XJP = X_J - X_P$ $\quad YJP = Y_J - Y_P$ $\quad ZJP = Z_J - Z_P$
$XKP = X_K - X_P$ $\quad YKP = Y_K - Y_P$ $\quad ZKP = Z_K - Z_P$

$$DIJ = YIP*ZJP - ZIP*YJP$$
$$DKJ = YKP*ZJP - ZKP*YJP$$
$$DKI = YKP*ZIP - ZKP*YIP$$

$$VOL = (1/6) * (XKP*DIJ - XIP*DKJ + XJP*DKI)$$

Volume of tetrahedron PIJK = ABS (VOL)

P is defined as being on the positive or negative side of the discontinuity model containing the triangle IJK, according to the sign of the calculated value of VOL.

Figure 5.3 Calculation of tetrahedron volume for determination of discontinuity facing directions

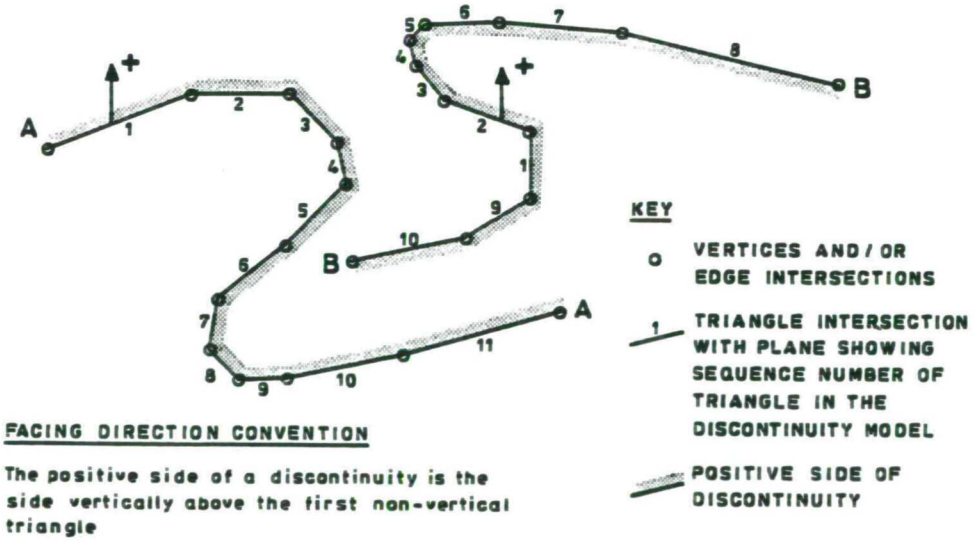


Figure 5.4 Schematic section illustrating the convention used for surface facing direction

Intersection lines consist of a series of point-pairs, derived from intersection with another surface, which are stored with the surface being studied. Such lines allow intersecting surfaces to be truncated according to a defined hierarchy, and also allow the traces of surface intersections on irregular surfaces to be displayed. Information as to which triangle each intersection line is associated with, the intersecting surface to which it belongs, and the difference in hierarchy between the two surfaces, is also stored. The generation and use of this information is described in Section 5.10.

When a surface is read into memory a set of pointers are set up for each point to indicate which triangles it belongs to. This pointer array allows rapid checking of surface integrity, determination of surface edges, unattached points, and other properties.

The size of individual files is governed by software limits on the maximum number of points, triangles, and the other data stored (as listed in Table 5.1). Typical amounts of information used are indicated in Appendix I where statistics for some of the discontinuities used to model the Neves-Corvo orebodies are given.

5.4 File Indexing

Work on an individual deposit or project is controlled by a set of surface indices which allow grouping of surfaces at different levels. One level of association is normally defined on the basis of common geological type, such as surface topography, reserve category limits, late faults, early faults, major contacts, internal contacts, etc. Such groupings are usually arranged such that they correspond to the hierarchy of intersection priority used to control surface-surface intersections. A second level of association may be used to separate models by deposit, user, interpretation date, or reserve category.

In most cases knowledge gained during the exploration phase can be used to define the interpretation sequence, and relative priorities, of the different discontinuities present within a deposit. Unless consistent criteria are used when carrying out a given geological interpretation it is impossible to arrive at coherent three-dimensional models. For instance, a situation where FAULT1 truncates and displaces FAULT2 on section 1, and FAULT2 truncates and displaces FAULT1 on section 2, could not be modelled geometrically, except by invoking the geological equivalent of a Mobius strip.

File names used during modelling are derived from the discontinuity names, with different groups of discontinuities being stored in separate sub-directories. Consequently modelling of any deposit should follow a fairly strict naming convention in order to facilitate file backups, the use of intersection hierarchies, selection for displays, etc.. For instance conventions may be adopted such that model limits are named with the first character as L, faults with F as the first character, etc..

5.5 Model Creation

Once aspects such as discontinuity nomenclature, groupings, and hierarchy, have been decided a surface can be selected for initialisation and editing. Several options are available for setting up surfaces in the initial stages of work, including the following:

- initialise a blank file
- import ASCII file of point co-ordinates
- import ASCII file of triangulated points
- import digitised serial-slice data
- import digitised linked-slice data
- digitise isobath plans of a surface
- select contact positions from a drillhole database
- select contact positions from a mapping database

Stored parameters are set to default values when initialising a new model, and are then modified according to the input data. Special data points and lines are flagged during the addition of new data, so as to preserve information about the source of the data incorporated into the model.

5.6 File Handling

During editing two separate discontinuity models can be stored in memory in order to facilitate checking of surface-surface intersections, and temporary saving of changes. Certain basic operations for handling these two structures and for file input/output are available for use during initialisation and editing, as summarised below:

Read - read in an existing file
Write - write out one of the channels in memory to file
Copy - copy an existing file to a new file (this requires a change in the name of the model, or the directory in which it is stored)
Clear - empty one of the memory channels
Swap - change the current working channel
Delete - delete a file from disk (normally done from the operating system rather than within the program)
Rename - to change the name of a surface, and hence the file to which it will be written

Delete, rename and copy options can also be done directly from the operating system. Protection of files between different users and projects is done outside the application software using facilities provided by the operating system.

Newer versions of the software have modified the options listed above in order to simplify the user interface. In these versions the user no longer has direct access to the two surfaces stored in memory, although they are still important from the programming point of view.

5.7 Global Editing

Global editing options perform functions which affect the whole model simultaneously, including the following;

- Automatic re-triangulation (Delaunay)
- Check triangle storage sense integrity
- Remove all points of selected type
- Remove all intersection lines
- Co-ordinate transformation
- Add another surface
- Delete part of a surface
- Extract part of a surface to form a new one
- Import additional points or triangles
- Surface-surface intersection options

Automatic re-triangulation, based on the Delaunay triangulation algorithm described by Watson (1982), is often the main editing technique used in the initial setting up of surfaces. For modelling topographic surfaces, or lithological contacts in simple stratiform-stratabound deposits, it may be the main editing technique used. Normally intersection lines are removed before re-triangulation otherwise problems may arise in subsequent surface intersections. Preservation of fixed links is possible using constrained Delaunay triangulation based on the approach described by Auerbach & Schaeben (1990a).

Triangle integrity checking can be useful, since in some cases interactive editing may result in impossible situations, such as crossing triangles or spurious surface edges. The integrity checking option checks the point-triangle pointer array, and tries to sort out any inconsistencies.

Co-ordinate transformations are used to shift or rotate a surface. This can be used for manipulating other information such as geochemical or geophysical data stored in the same format.

Removal of all intersection lines is commonly done when re-editing surfaces which have previously been incorporated into a model. Storage of intersection lines is normally done as the final stage of editing, although they can be generated and displayed at any time during editing.

Adding another surface creates one or more separate surface segments in the same file, since triangles which cut the existing surface are not added. The different segments can be merged using interactive or global editing facilities, although a single surface can contain more than one segment if desired.

Extraction of a portion of an existing surface to form a new surface is another approach which could be used to establish a detailed model of an area for which a general model already exists. This would be done by digitising a limit which enclosed all the points and triangles required. Any points falling outside this limit would be excluded, as would any triangles linked to them. By deleting part of a model and then adding in a smaller more detailed model, a general model could be updated using detailed local interpretations.

The option to import additional data allows for the same choices available during file initialisation (see Section 5.5). This allows files containing new information to be added to an existing surface. Normally triangles are not added if they cut existing triangles, and points which coincide with existing ones are ignored. An exception to the latter is made in cases where an existing digitised or interpreted point is to be altered to a named drillhole or mapping point.

Surface-surface intersections, which are fundamental to certain aspects of the display and generation of volumetric models, are discussed separately in Section 5.10.

5.8 Graphic Displays

The intersection of stored models with a plane in any orientation is fundamental to most display options. This problem is reduced to the calculation of the intersection between a triangle and a plane, which is repeated for all triangles in a given surface. This is based on simple geometric principles which are described in most texts on computer graphics and geometric modelling (eg Angell and Griffith (1987), Mortensen (1985), Bowyer and Woodwark (1983)). The main difficulty in implementation is the establishment of rules for dealing with the special cases which occur when one or more of the triangle vertices are co-planar with the plane of interest. These aspects are discussed further in Section 9.3.

In general terms two types of graphic display are utilised, namely;

- **2-D Graphic Displays:** Generation of such displays requires the selection of a set of points and lines to be displayed, along with appropriate display symbols, colours and labelling. Such displays are always generated with reference to a particular viewplane, which can be altered in scale, and moved forwards, backwards, sideways, up or down. The data displayed is controlled by specified minimum and maximum projection distances onto the viewplane. For editing purposes a narrow depth of field is normally used, however an unrestricted depth of field may be used for generating isobath plots.

  Display parameters can be associated with different model features, such as;

  - Points stored in the model
  - Lines stored in the model
  - Lines generated by surface-plane intersection

  Examples of 2-D graphic displays are provided by several of the Figures and Plates presented in Chapters 6 to 8.

- **3-D visualisations:** These include 3-D graphics representations or isometric views of contoured surfaces. Since many sophisticated 3-D display packages are available commercially, the approach adopted has been to provide interfaces to existing display systems, rather than to develop such options from scratch. This requires the system to output the stored surfaces in a form which can be used by other systems, such as a set of polygons with associated colours.

  The 3-D displays shown in several Plates (eg Plate 5.1) were generated using a visualisation program developed by RTZ Technical Services Limited.

5.9 Interactive Editing

Interactive editing provides the main method by which a discontinuity model can be adjusted to incorporate field knowledge of the geological characteristics of a deposit.

In the simplest case, this involves editing of general characteristics, such as surface name, description and associated codes. This is normally done by displaying the current values and asking the user to confirm or modify them.

Plate 5.2 illustrates the use of some of the interactive editing options. Such editing is controlled by selection of one or more points using either the screen mouse or, less commonly, a digitising tablet. For editing purposes a surface can be displayed at any orientation within a specified depth of view. Points can be shifted about in the plane of the display using the mouse or digitiser. The distance at which points lie vertically above or below the viewplane can also be altered.

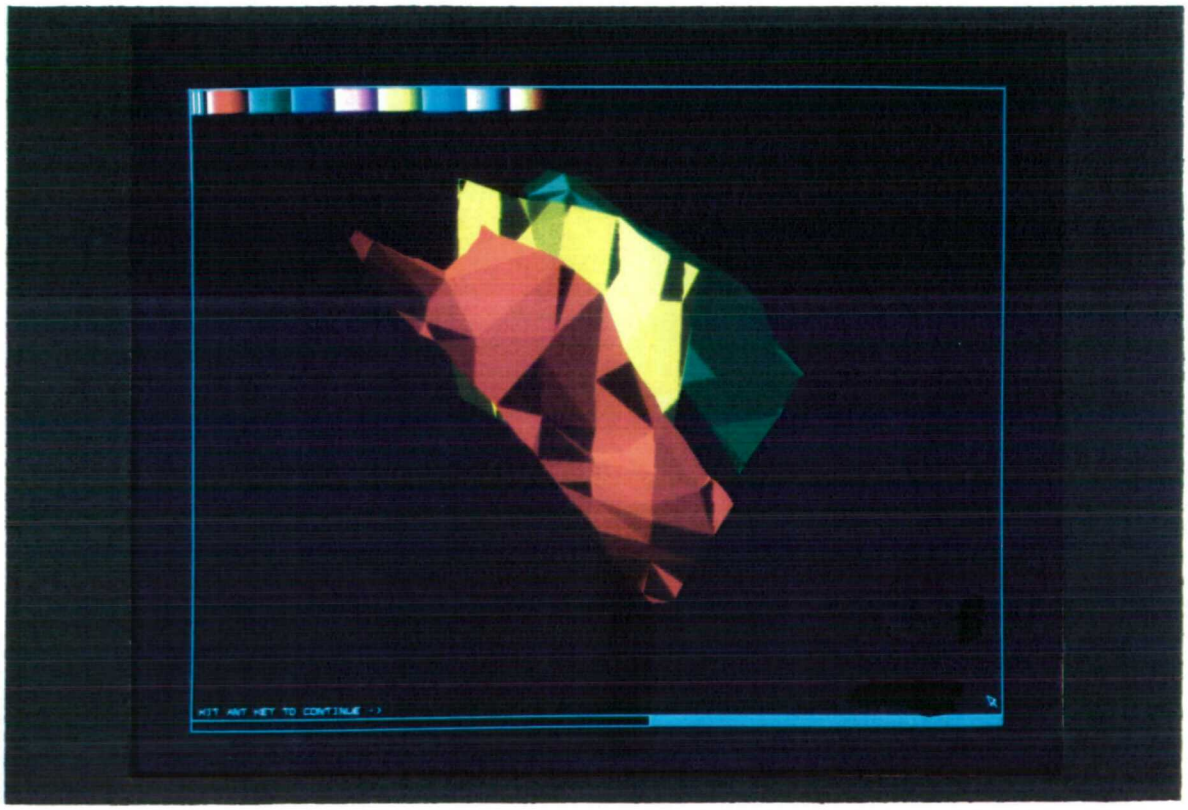


Plate 5.1 Example of a 3-D visualisation of a geological discontinuity model. Display shows displacement of orebody footwall by a major fault (yellow) in the Corvo orebody. Note faint intersection trace of footwall (green) with the fault indicating an apparent vertical displacement down towards the viewer.

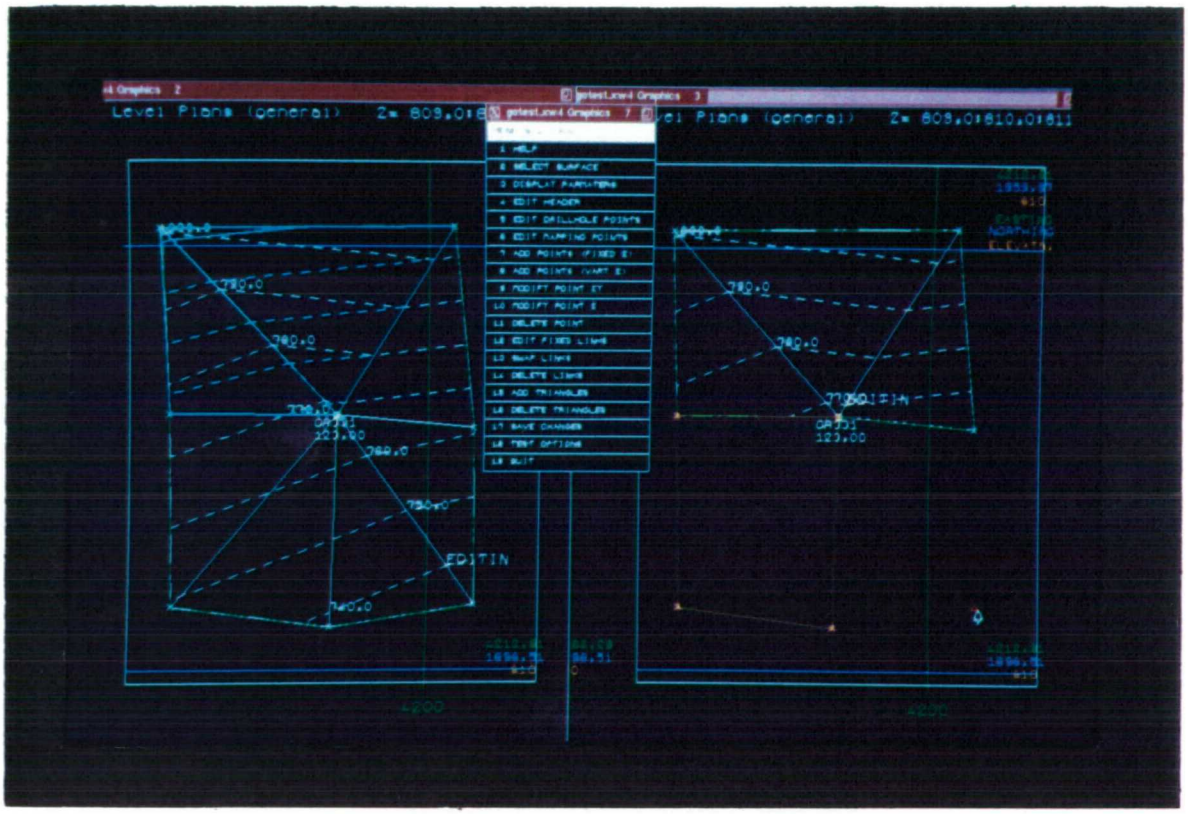


Plate 5.2 Examples of interactive editing of a triangulated discontinuity model. Editing options are indicated in the central pop-up menu. Addition of new points and triangles to an existing model is shown in the bottom right. The effect of changing a point position is shown in the top left (blue lines represent the new contours).

Addition or deletion of a triangle normally requires all three vertices to be selected on the screen using the graphics mouse. When triangles are modified by adding or deleting points or triangles, checks are made to ensure that the triangle storage sense rule is obeyed. Swapping triangle links can sometimes lead to problems if a new link is generated which cuts across other pre-existing triangles. For this reason it is difficult to edit overhanging surfaces, other than in the same plane in which they were originally interpreted.

If a point selected for editing is flagged as a special point (ie drillhole intersection, or mapped location), the user is prompted for different information during editing. In the case of a drillhole point, a drillhole index file containing the information used for calculating downhole co-ordinates (collar co-ordinates, and co-efficients derived from downhole surveys; as described by Howson and Sides (1986)) is checked. On locating the hole a new depth can be entered, and the new co-ordinates are then automatically calculated. In this manner editing of drillhole points is confined along a curved line representing the drillhole trace. Strike and dip data associated with mapping points can also be entered and edited.

Intersection lines cannot be edited directly by the user and are usually eliminated, prior to interactive editing, in order to avoid the creation of invalid intersection lines. This can occur if triangles containing intersection lines are altered such that they are no longer co-planar with the plane in which the intersection lines lie.

5.10 Surface-Surface Intersection

The generation of a coherent set of geological discontinuity models for a given deposit, that can be used to establish volumetric models, is fundamental to the use of this approach in reserve estimation. This is achieved by modelling cross-cutting surfaces in order to ensure proper closure of the volumes defined by intersecting discontinuities. This aspect is less important when doing such modelling for exploration purposes, or when modelling simple deposits where the main geological boundaries do not intersect (eg thick continuous coal seams). In order to set up a system of general applicability, however, procedures for generating and handling surface-surface intersections are essential.

The calculation of a surface-surface intersection line is fairly simple in geometric terms, being based on repeated triangle-triangle intersections. This again reduces to the problem of calculating a triangle-plane intersection, as discussed in Section 5.8. As was the case for calculating surface-plane intersections for display purposes, most of the programming effort is concerned with establishing rules for handling special cases where one or more triangle vertices are coincident with the plane of interest (discussed further in Section 9.3).

To generate a complete set of intersections for a pair of surfaces, each possible pair of triangles is checked, and valid intersections are calculated and stored in memory. The resultant point-pairs are then sorted into polyline strings before further processing is done.

Definition of volumes for display and evaluation purposes is done by defining a hierarchy of intersection priority. On the basis of the relative ranking of pairs of surfaces two categories of surface-surface intersection are recognised, namely;

- **trim lines**, where the intersected surface truncates the current surface. Such lines are used to define visible and invisible parts of a surface by **surface trimming** as illustrated in Figure 5.5.

- **intersection traces**, where the current surface truncates the intersected one, and the intersection is used for display purposes only. This can be used, for instance, to generate a representation of surface geology by intersecting all modelled geological discontinuities with a discontinuity model representing the surface topography.

The intersection hierarchy adopted is based on assigning lower relative rankings to the truncating surfaces. In this manner surfaces such as topography or model limits, will generally have the lowest rankings, followed in ascending order by late faults, early faults, main mineralisation type boundaries, and internal ore contacts. In this fashion the intersection hierarchy is set up so as to mimic the interpreted geological sequence of events or – more accurately speaking – the sequence of interpretation of different features.

To speed up intersection generation, groups of files can be assigned a common ranking, whilst maintaining individual rankings within each group. In practice this means that a stratigraphic contact would automatically be checked against all faults, without worrying about the relative order of precedence of the different faults. The usage of the ranking hierarchy for modelling of the Neves-Corvo deposit is illustrated in Appendix I.

The procedure used for surface trimming is illustrated in Figure 5.6. In order for this to function the storage sense of intersection lines, and of the triangles in which they lie, is critical. Consequently, the user is required to verify the sense in which the intersection lines are stored, during the generation of trim lines.

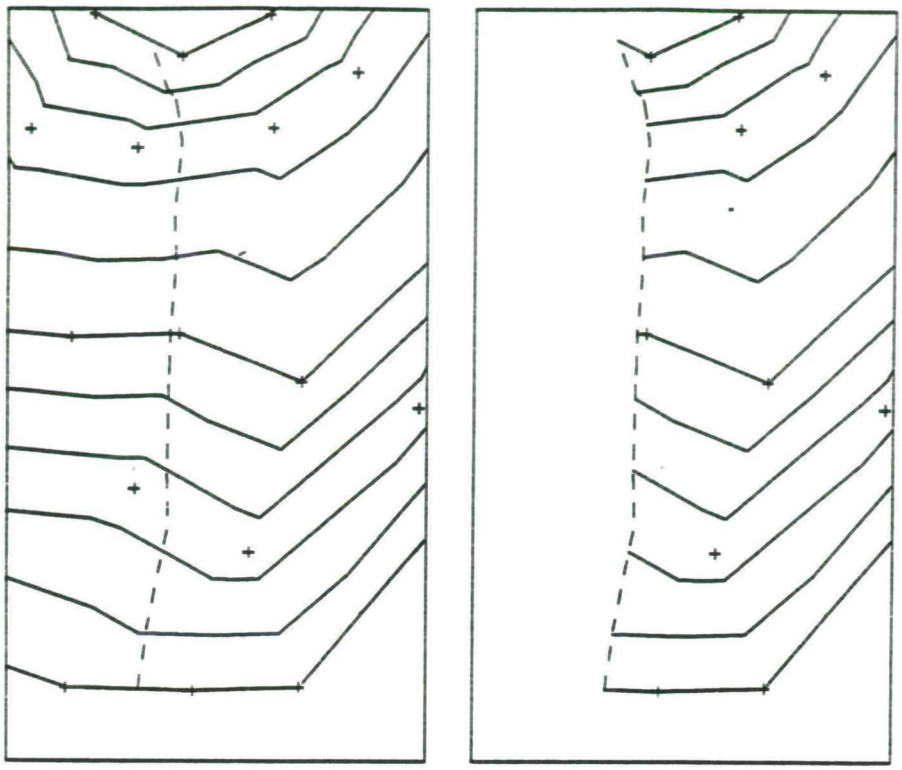


Figure 5.5 Untrimmed (left) and trimmed (right) displays of part of a contoured discontinuity model (after Sides 1992a)

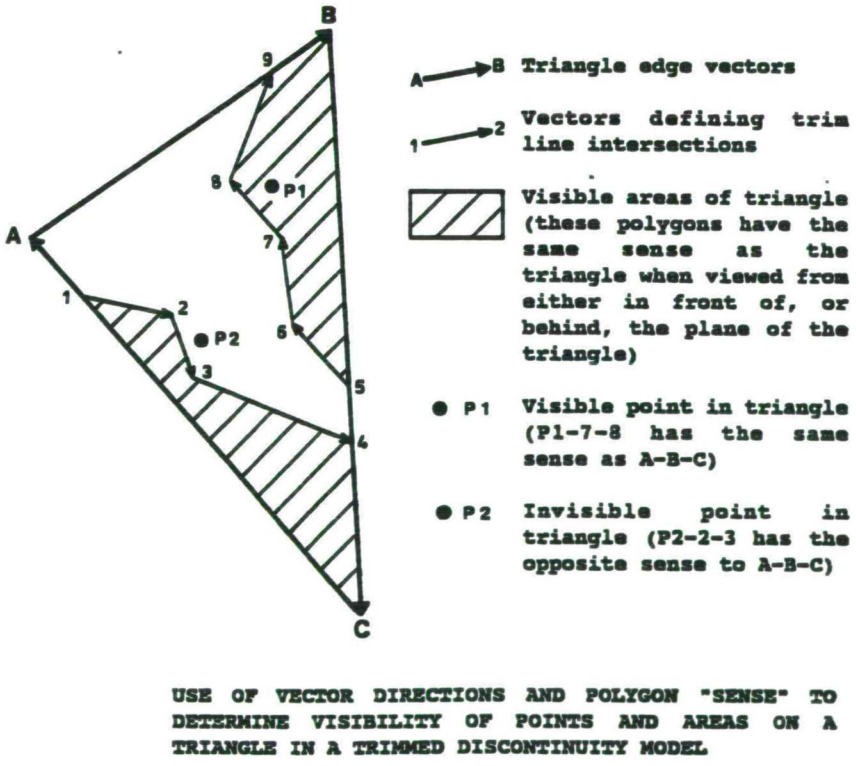


Figure 5.6 Application of surface trimming procedure to a single triangle

Trimming of an entire surface is done using the following sequence of steps;

- review and confirm surface intersection hierarchy (ie rank numbers of individual files and groups of files)

- generate intersections with all surfaces having rank numbers less than the current surface

- for each intersection line specify the sense in which it should be stored (by indicating the visible part of the surface closest to it). Some lines can be rejected from further consideration at this point

- when all intersection polylines are generated check them to eliminate trailing edges (ie portions of the intersection lines that fall within the invisible part of the surface)

- run a software check to flag all "invisible" triangle vertices (the second integer code stored with each point, mentioned in Section 5.3, is used for this purpose)

At present this process requires a considerable degree of user interaction, but it is considered that continued development of the system should allow several aspects to be made semi-automatic in the future.

5.11 Data Output

An important aspect of any modelling system is the ability to output model data for reporting purposes, use in other applications, further analysis, etc.. In addition to the graphic display options discussed in Section 5.8, the following output options have been implemented;

- ASCII file of all model data
- ASCII file of point co-ordinates +/- triangle vertices
- Printer listing of file contents
- 2-D graphic displays as plot files
- Interface files for 3-D visualisation software
- Gridded seam file output (see Chapter 7)
- Serial slice type file output (see Chapter 7)

Output options such as those listed above are essential for the integration of this modelling system with other applications. Other options such as conversion to formats used by commercial modelling packages, such as AutoCAD, could also be implemented.

5.12 Other Features

The discontinuity model structure enables the derivation of several summary statistics, based on simple arithmetic and geometric calculations, including the following:

- Number of vertices and triangles
- Vertex/triangle ratio
- Number of intersection lines
- Total surface area (sum of individual triangle areas)
- Length of discontinuity margin
- Enclosed volume (closed structures only)
- Centre of gravity
- Average orientation (weighted by triangle area)
- Surface roughness (based on measures such as that of Watson & Philip (1989))
- Best fit plane

Such statistics have not been studied in detail, but it is considered that they could provide a basis for comparative tests between different models of the same discontinuity, or between discontinuity models from different deposits.

Continued development of the modelling programs should allow for several additional display options, such as;

- gradient variations
- hinge lines
- variations in discontinuity roughness
- enhanced three-dimensional display options
- use of variable opacities
- use of different colours on either side of a surface

5.13 Discussion

The development of the geological discontinuity model structure described in this Chapter was based on the objectives outlined in Section 4.8. The structure established achieves most of these aims and has several advantages over other techniques of geological modelling. This owes much to the fact that the structure has been set up specifically for 3-dimensional geological modelling, rather than principally for use in grade estimation or mine planning, or in CAD/CAM type applications.

Some of the main advantages over other systems include the following:

- contact positions logged in drillholes, or recorded during geological mapping, can be incorporated directly in the models

- links with drillhole databases facilitate initial interpretation and checking of models

- links can be established with 2-dimensional geological interpretations, done on sections or plans, thus facilitating integration of such models with conventional manual interpretation methods

- variable amounts of detail can be stored in the same model

- surfaces do not have to define closed volumes; those that do are treated as special cases

- a single surface can be made up of more than one disjoint segment

- storage of surface-surface intersections allows the generation of contact traces on faults, display of modelled geology on irregular surfaces, etc.

- the trimming mechanism applied for display purposes allows extrapolated geological contacts to be modelled; more than one model to be stored and used for a given area; and facilitates proper closure of contacts in order to generate volumetric models

- general geological characteristics can be associated with each surface (such as codes above and below, etc.)

- the definition of a hierarchical order of surfaces allows the model to reflect the interpreted geological history of the deposit

- discontinuity models can be displayed on sections or plans drawn at any orientation

- the model structure allows several checks of surface integrity to be done in order to test the validity of overall interpretations

- statistical measures of discontinuity characteristics can be derived

- isobath and isopachyte plans can be generated using discontinuity models of stratigraphic contacts

- automatic (Delaunay) triangulation of non-overhanging surfaces is possible

- input from linked-slice type models allows features such as overhangs and lensing out to be modelled

Although several software packages exist which a geologist can use for geological modelling and reserve estimation, their use often involves compromises and loss of important geological information in the final reserve models. The approach described here allows geological interpretations to be done in a conventional manner, whilst taking advantage of the database handling, and interactive graphics, capabilities offered by computerised systems.

It is considered that a sound basic structure for handling the data used in interpreting geological discontinuities, such as that established here, can eliminate many of the problems encountered with existing orebody modelling techniques. In order to encourage wider usage such a structure needs to allow for future modification and enhancement as geological ideas and modelling techniques evolve. This aspect is facilitated by the modular structure adopted here which would allow extra types of geological data to be added relatively easily.

From the viewpoint of reserve estimation and mine planning these advantages are of little interest unless the resultant models can be used to estimate reserve tonnages and grades for planned mining volumes. The discontinuity modelling approach cannot be used for modelling grade variations directly, and is not intended as a replacement to existing methods of volumetric modelling. Consequently, it is essential that discontinuity models can be interfaced with the volumetric models used for reserve estimation and mine planning purposes.

Chapters 6 to 8 illustrate the application of the discontinuity modelling technique in the context of reserve estimation on the three deposits studied. Chapters 9 and 10 then discuss the problems of measuring and predicting errors associated with such models.

6. MODELLING OF GEOLOGICAL DISCONTINUITIES

6.1 General

The modelling of geological discontinuities is one of the first steps in the evaluation of any mineral deposit, as discussed in Chapter 2. Selection of drilling and sampling methods, as well as sampling density and patterns, is usually based on geological models established during initial exploration. Subsequent investigation work concentrates on the interpretation and modelling of the main geological discontinuities present. Such models are used to define geologically-based volumes which control subsequent grade estimation and mineral inventory calculations. Consequently, the modelling of geological discontinuities fulfils an important role both during initial exploration, and in subsequent detailed evaluation work.

This Chapter describes the application of the discontinuity modelling system for purely geological purposes. In most deposits such work leads to the definition of several groups of discontinuities which are then used to define geological volumes for grade estimation and mine planning purposes, as discussed in subsequent Chapters.

6.2 Drillhole and Mapping databases

The data on which geological interpretations, of most mineral deposits, are based are derived mainly from field geological observations of surface or underground exposures, coupled with geological logging of drillhole samples. This information is generally treated as point data, thus providing the geologist with a network of points on which an interpreted framework of geological discontinuities can be built. The ability of a computerised geological modelling system to incorporate such data directly into a model is thus an important advantage from the geologists point of view.

Three approaches are commonly used when setting up databases of geological mapping and drillhole logging information, namely;

a) File structures based on linear segments of common geological type, with data fields containing descriptive information for each type (eg hole name, depth from, depth to, assay grades, etc.)

b) File structures based on defining the position of boundaries between different geological types, with data fields containing information on the attitude and characteristics of each boundary (eg hole name, contact depth, intersection angle, unit above, etc.)

c) A combination of the above two methods where the characteristics of linear segments of common geological type are defined, along with a description of their start or end boundary.

Most drillhole databases are of type (a) since the geological information is usually associated with lengths of core sampled for analytical purposes. Mapping databases tend to be of type (b), since geological mapping usually depends on the recognition and recording of important boundaries. Several of the limitations of these two different approaches can be overcome by using the third approach which allows recording and storage of the widest range of relevant information at the same time.

The establishment of such databases is often complicated by the lack of standardisation of nomenclature and data recording practices, leading to considerable variations between information collected by different geologists working on the same project. Nevertheless, it is usually possible to establish major subdivisions (on a lithological, mineralogical or other geological basis) that allow different units, and their boundaries, to be coded for identification purposes.

Coding of boundaries between different units is complicated by the fact that whereas some discontinuities are readily identified individually during mapping or logging, others are only identified by type. For instance, contacts between different sedimentary units or major intrusive contacts automatically define the geological material present on one side. On the other hand the full significance of features such as major faults is usually only defined at the interpretation stage.

Links between such databases and the geological modelling system are provided in three ways, namely;

a) **Display of database information**: This is illustrated by an example from Neves-Corvo where both drillhole and mapping databases are maintained. Plate 6.1 shows a display of drillhole data (derived from drillhole collar co-ordinates, downhole survey measurements, and sample oretype codes), together with mapping data illustrating the recorded attitude of faults and contacts between different oretypes. Such information can be displayed on the screen, or output to a plotter, to provide working plans or sections for a geologist interpreting a given area.

b) **Flagging of discontinuity model points**: This is illustrated in Figure 6.1 which shows an interpretation of the massive sulphide footwall contact for part of the Neves-Corvo orebody. Hole numbers and downhole depths of points incorporated in the discontinuity model are indicated, using information stored in the model. The interpretation procedure used in this instance is to set up initial discontinuity models based solely on drillhole intercepts. These are then plotted on sections or plans for revision by the geologist carrying out the interpretation. This facilitates the correct incorporation of off-section data without necessitating complicated calculations on the part of the geologist.

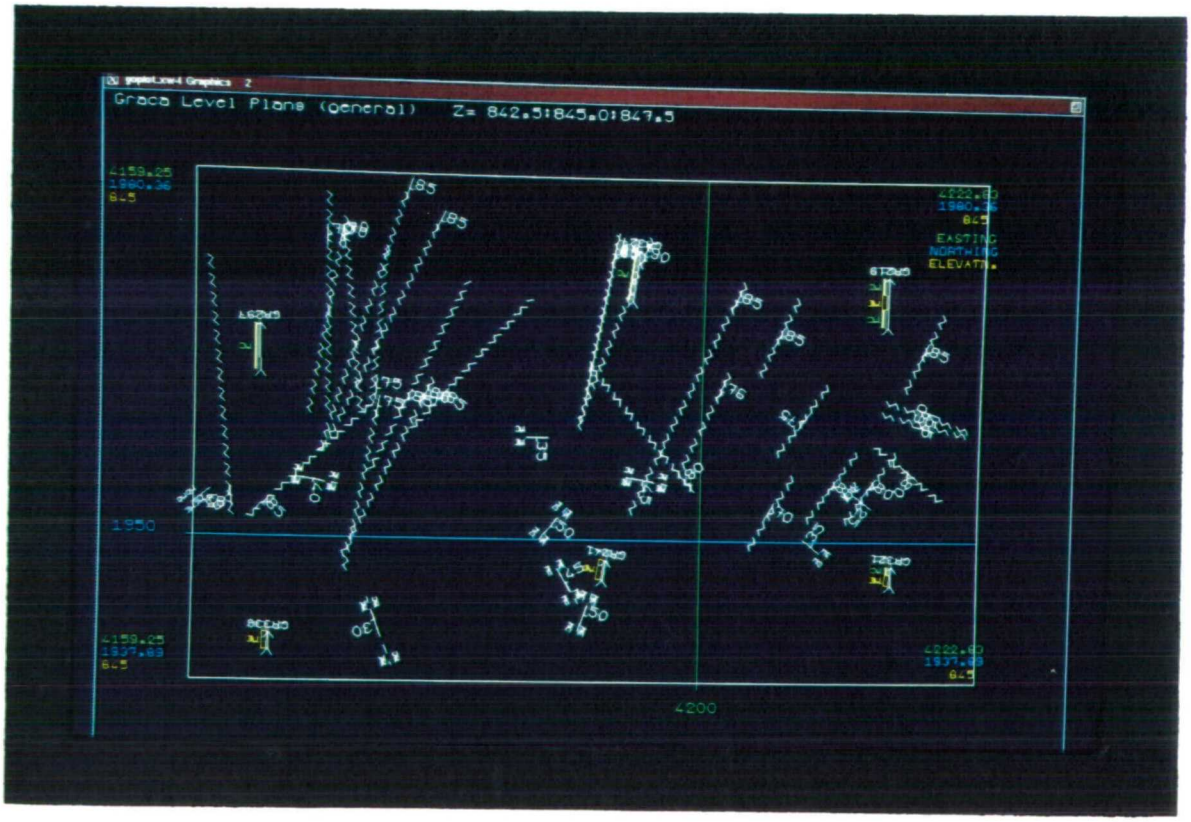


Plate 6.1 Display of mapping and drillhole data used as a guide for geological interpretation. White lines and symbols represent data from a stope mapping database. Green and yellow coloured letters represent the geological codes of ore intercepts for drillholes intersecting the volume displayed.

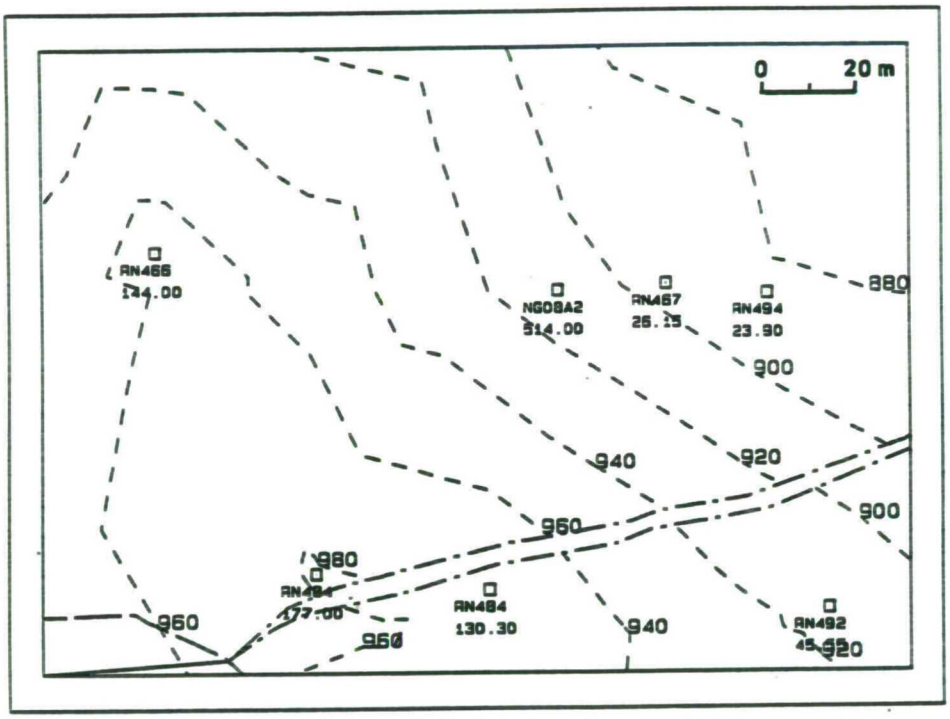


Figure 6.1 Isobath plan for the massive sulphide footwall in the Neves orebody showing drillhole data points incorporated in the geological discontinuity model (after Sides, 1992a)

c) **Checking database information against the final models**: Graphic displays of both original data, and derived models, are used to check that the points flagged in the models correspond with the original data. Such displays may reveal differences where errors in collar co-ordinates or downhole surveys have been corrected subsequent to the initial interpretation. Whilst, theoretically, it may be desirable to maintain "live links" with the database (which would change every time an alteration was made in the database), this is not possible in practice, since a significant change in a single discontinuity may require re-interpretation of several discontinuities. In this sense the information flagged in the discontinuity models corresponds to a database "frozen" at the time of interpretation.

The discontinuity modelling system also facilitates the establishment of procedures for automatically intersecting existing or planned drillholes with established discontinuity models. This allows further verification of models against original data, as well as enabling predictions to be made for new holes.

6.3 2-D geological interpretation

Most interpretative data handled during geological modelling is derived from interpretations of the intersection traces of given geological discontinuities on a series of planes (eg geological section sets, topographic contour maps, and isobath plans). In many cases the information used as the basis for interpretation is deliberately collected along particular planes, to facilitate subsequent interpretation, as reflected in the use of regular drilling grids on many deposits. In some cases, such as coal or evaporite deposits, the information used as the basis for geological interpretation may consist of seismic or other geophysical profiles from which discontinuities are interpreted on vertical planes.

The widespread use of this approach reflects the ease with which interpretative information prepared in this fashion can be checked for consistency, and communicated to other individuals. Considerable attention was therefore given to ways of facilitating the incorporation, and generation, of 2-dimensional line data when modelling geological discontinuities.

The first step taken was to establish databases of 2-dimensional geological data for use during initial interpretation. Such databases are common in most commercially available geological modelling systems (Gibbs Associates (1990)), as well as packages such as AutoCAD which have been used in several instances as an aid to geological interpretation (eg Gibbs and Krajewski (1992), Marschallinger (1991)). Such information also forms the basis for geographic information systems (GIS) which have achieved widespread acceptance in most applications concerned with the recording, storage, display and analysis of geographical data. A lot of attention has been paid in this field to the problems involved in setting up structures for dealing with such data (eg Burrough (1986)). The data handled by GIS systems has a lot in common with information generated during geological interpetation, and the approach described in this Section has many parallels with such systems.

To facilitate the use of 2-d discontinuity data in conjunction with 3-d discontinuity models, three co-ordinate values are stored for all the points in the 2-d database used here. This facilitates transfer of data between files, as well as the display of 2-d data on planes at different orientations to those on which they have been generated. A hierarchy of data types are used for handling such data, namely;

- **Point**: defined by a set of three co-ordinate values; two co-ordinate values might be sufficient where data are stored in a file structure containing a header defining the plane in which they lie;

- **Line**: a pair of points defining a simple line segment; the sense of the vector may be significant in which case the order of the points is important;

- **Polyline**: a set of points defining a continuous complex discontinuity made up of several lines. Some of the points on a polyline may represent special nodes, which are used for subdividing it into different **line segments** for intersection or polygon generation purposes;

- **Polygon**: a special type of polyline where the first and last point, in the string of points, are identical; this defines a closed area;

Some of the problems encountered in handling such data (such as dealing with nested polygons, the intersection of different sets of polygons, etc.) are reviewed in detail by Burrough (1986).

In the context of the work done here it was found that an important aspect in setting up databases of such information is the way in which different sets of polylines are grouped together. In general three approaches were recognised, namely;

a) **Project based**: all the discontinuities, used in a particular interpretation, derived from a set of serial-slice sections or plans;

b) **Interpretation plane based**: all the discontinuities interpreted on a particular plane;

c) **Discontinuity based**: all the polylines pertaining to a particular discontinuity, interpreted on several different sections or plans.

The third approach is used for extracting data for incorporation into 3-D discontinuity files, and is also useful for handling individual discontinuities with a large amount of data (such as surface topography).

For use during display and interpretation the second approach is often preferred, since interpretative work normally proceeds from plane to plane with information on adjoining planes being compared in order to arrive at a consistent overall interpretation. The 2-d discontinuity files used contain groups of polylines, identified by discontinuity name and section reference value. This allows flexibility in the way in which individual discontinuities are grouped. Facilities for transferring data from one file to another allow information to be grouped in different ways for different studies.

An important preliminary step to incorporating such data into 3-D discontinuity models is the provision of a facility for linking between sections (Linked-Slice type modelling). An example of the use of this modelling technique is given in Plate 6.2. Since totally automatic linking between slices often gives rise to spurious effects the user is allowed to impose fixed ties, between slices, which control subsequent generation of links. The algorithm used in this instance was initially developed for use in modelling of mining volumes (Howson (1989)), and is similar to existing algorithms used in a lot of medical applications (eg Keppel (1975), Fuchs et al (1977), Boissonnat (1988)).

The point and line data, and triangles in the case of linked-slice interpretations, stored in such databases can be incorporated directly into 3-d discontinuity models using an import option. Unless controlled in some manner subsequent editing of the 3-d models may result in the loss of the line information generated during the original interpretation. For this reason a special set of lines, termed **fixed links**, are defined in the 3-d geological discontinuity model structure. In this manner the lines interpreted on section and plan, together with any fixed ties defined between adjoining sections, can be preserved. Special checks are carried out during editing to ensure that such lines are dealt with separately.

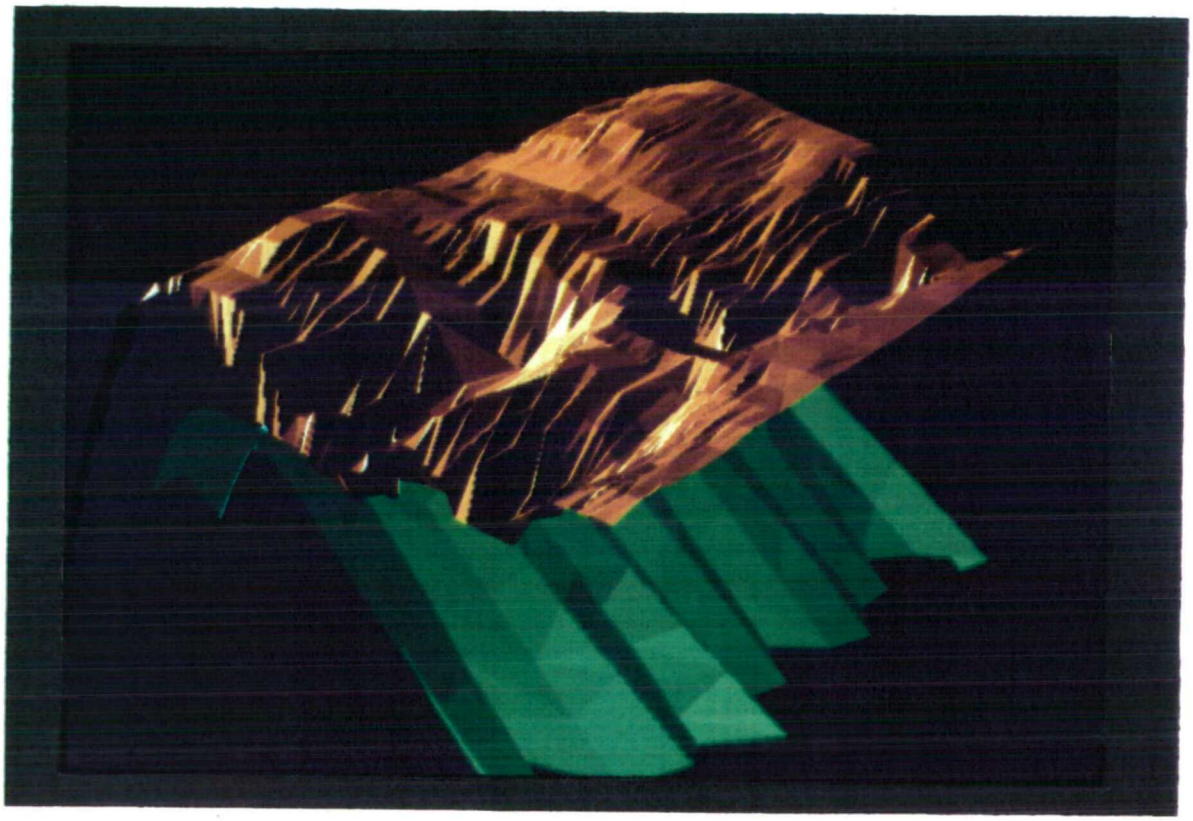


Plate 6.2 Examples of 3-d discontinuity models generated using the linked-slice method; the original interpretation was done in 2-d on a series of parallel sections. Model shows surface topography (brown), and fold structure (green), from the Cerro Colorado deposit. Note the lines of the interpreted sections (diagonals from bottom right to top left), and the triangles linking them.

In the case of automatic triangulation, a constrained Delaunay triangulation option, similar to that described to Auerbach and Schaeben (1990a) can be used. Plate 6.3 shows an example of a topographic surface modelled in this fashion where the original digitised contours, from which the 3-d model is derived, are separately distinguished from contours derived directly from the 3-d model.

An important aspect of discontinuity modelling is the ability to generate 2-d databases from the final 3-d models. This is often necessary in order to pass information to other applications, and can also be used as a preliminary step in the definition of geological volumes, as described in Section 7.3. A facility to generate 2-d discontinuity files in the same format as the original 2-d interpretative databases is one of the export options from the 3-d modelling system.

6.4 Discontinuity hierarchies

Any geological interpretation must take into account the geological sequence of events in an area. This is usually reflected in the interpretation of geological discontinuities sequentially from youngest to oldest. This is necessary because the most recent events in the geological history of an area often displace or disrupt earlier features. For instance, the present-day exposure pattern in an orogenic belt may reflect batholith intrusion into older sedimentary rocks, subsequent folding and faulting, followed by uplift and erosion.

To take advantage of this aspect, the principle of discontinuity hierarchies was established (see Chapter 5). This allows the grouping of discontinuities, of similar geological age or characteristics, in separate index files to facilitate access for display and/or processing operations such as intersection. This approach is particularly important for trimming discontinuities for display purposes.

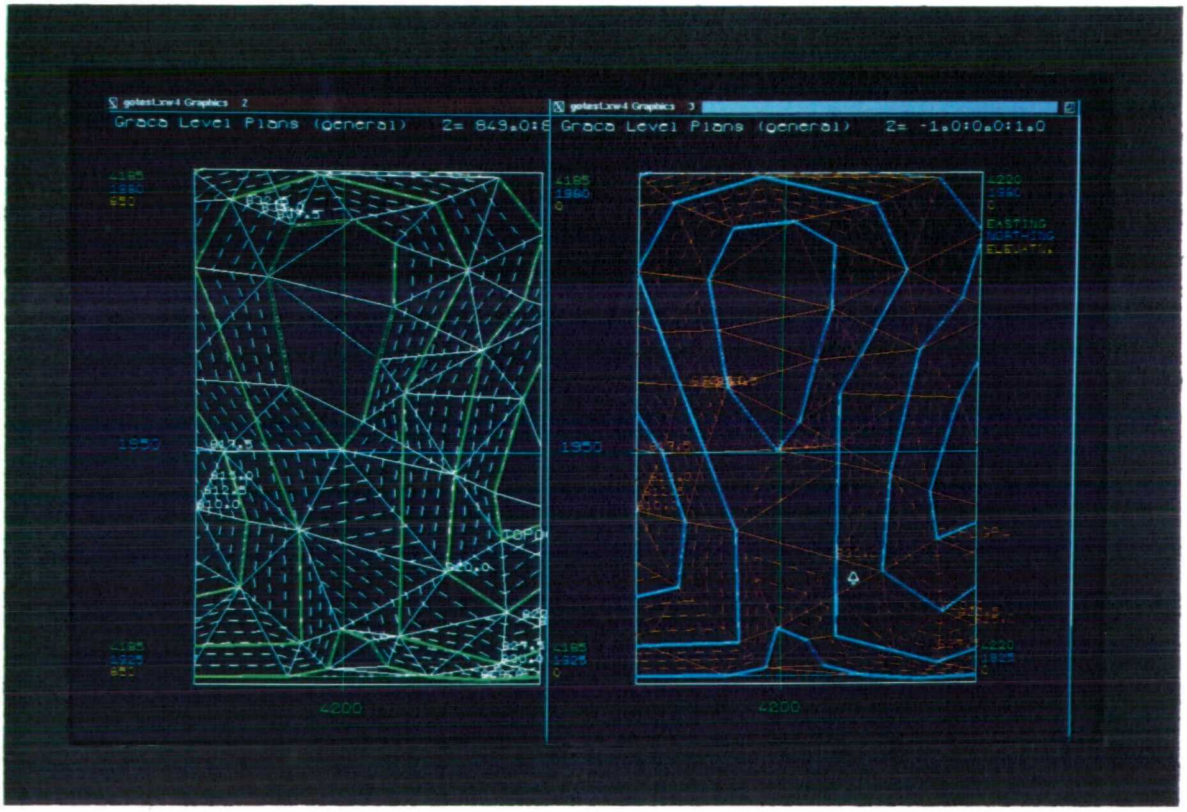


Plate 6.3 **Topographic model generated with (right), and without (left), the preservation of digitised contours as fixed links in a 3-D discontinuity model.** Solid lines represent digitised contours which are stored as fixed links; dashed lines represent contours derived from the discontinuity models. The dashed lines cross the solid lines at several points in the left hand display (eg centre of bottom half of plot), indicating that the geometry of the original interpretation is not preserved during re-triangulation.

This is done using a procedure whereby discontinuities representing early events (such as stratigraphic contacts) are truncated by those representing later events (eg faults or surface topography), as discussed in Section 5.10.

Discontinuities modelled in a particular interpretation phase may include both real geological features, and artifical boundaries such as limits of interpretation. In such cases an interpretative sequence is established in order to avoid arriving at ambiguous interpretations. Figure 6.2 illustrates an interpretation for part of the Corvo orebody, where discontinuities were divided into four groups, interpreted in the following order;

1- **limits of interpretation**: defining the limits of the area to be used during subsequent reserve estimation
2- **major faults**: cutting and displacing all earlier contacts
3- **major stratigraphic contacts**: defined by lithological contacts recorded during underground mapping and logging of drillcore
4- **internal lens boundaries**: defined on the basis of analyses of drillcore samples, and interpreted within envelopes defined by the major stratigraphic contacts

Another important aspect in the context of geological interpretation on operating mines is the knowledge that a hierarchy of interpretations, corresponding to different reserve categories, is often necessary. As areas come closer to being mined much more detailed data becomes available, allowing more detailed interpretations to be made. This is illustrated by a series of sections presented by Richards & Sides (1991), showing interpretation changes on a particular geological section during the evolution of the Neves-Corvo project. An initial interpretation based on widely spaced surface drillholes, was subsequently superceded by more detailed interpretations based on underground mapping and drilling.

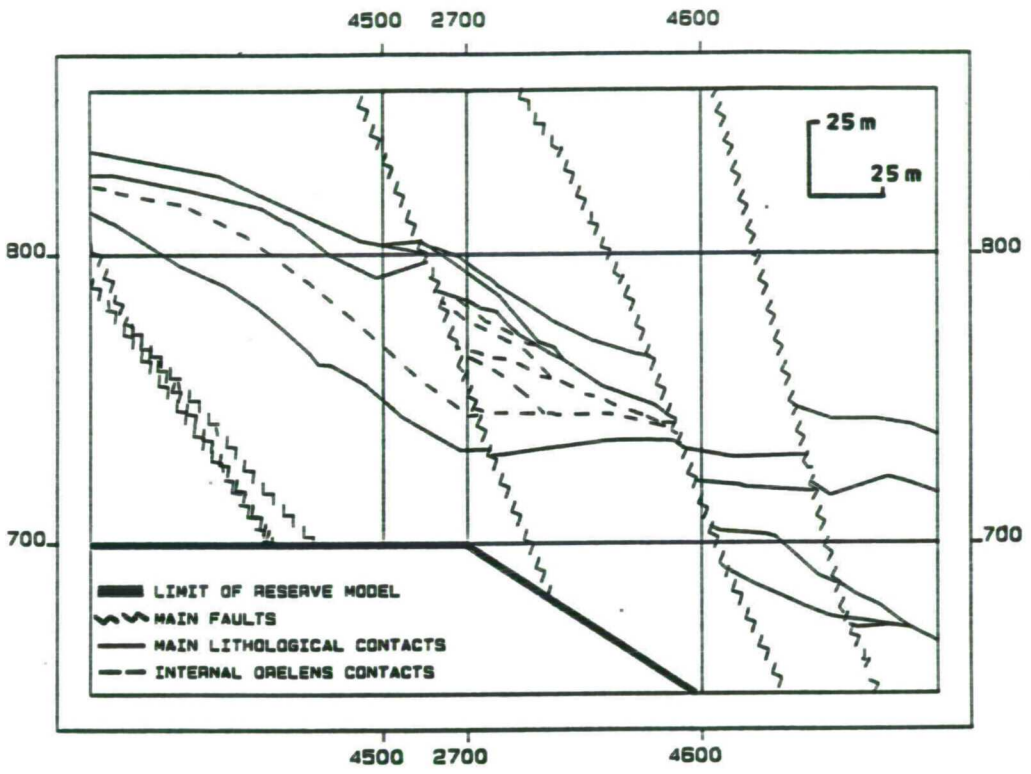


Figure 6.2 Geological cross-section of part of the Corvo orebody based on the use of a four-fold discontinuity hierarchy (after Sides, 1992a)

The discontinuity hierarchies established in the modelling system allow different 3-d discontinuity models to be maintained for the same area, rather than having to delete parts of earlier interpretations and replace them with subsequent more detailed interpretations. Using the trimming procedure in conjunction with appropriate indices and discontinuity hierarchies, the parts of interpretations duplicated by later information can be hidden for display purposes. This is illustrated in Plate 6.4 which shows models used for the Inferred and Indicated reserve categories in part of the Graca orebody. Parts of the Inferred model which would normally be completely hidden are illustrated in a different colour to allow comparison with the later more detailed interpretation. This facility allows direct measurement of the differences between different reserve interpretations; as discussed in Chapter 10.

6.5 Geological Analysis

An important aspect of geological modelling is the analysis of the characteristics and inter-relationships of modelled discontinuities. This approach is used both to check the validity of interpretations from a geological point-of-view, and also for predictive purposes. These aspects are illustrated below with an example from the work done.

Plate 6.5 shows a display of the intersections of stratigraphic contacts on either side of a steeply dipping north-south fault plane viewed from the hangingwall side (dashed lines in front of fault plane, solid lines behind it). The shape of the massive sulphide lens on either side of the fault plane is clearly defined. The absence of vertical displacement at the extremities of the area shown suggests a lateral (dextral) movement, of about 40m, on the fault plane.

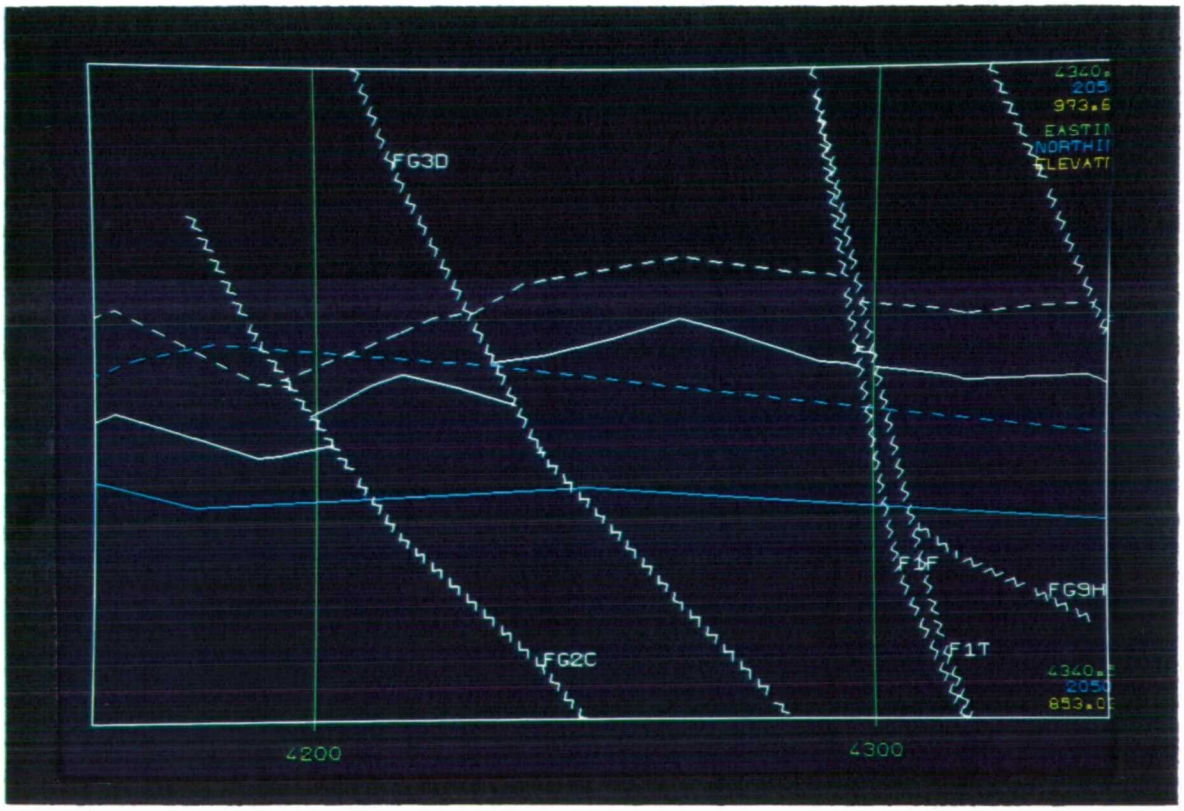


Plate 6.4 Longitudinal section of part of the Graca orebody illustrating two different geological models for the same area, corresponding to interpretations used for different reserve estimates. Blue line represents early interpretation based on surface holes only, white lines represented more detailed interpretation based on results of underground drilling. Solid lines represent massive sulphide footwall, dashed lines represent orebody hangingwall.

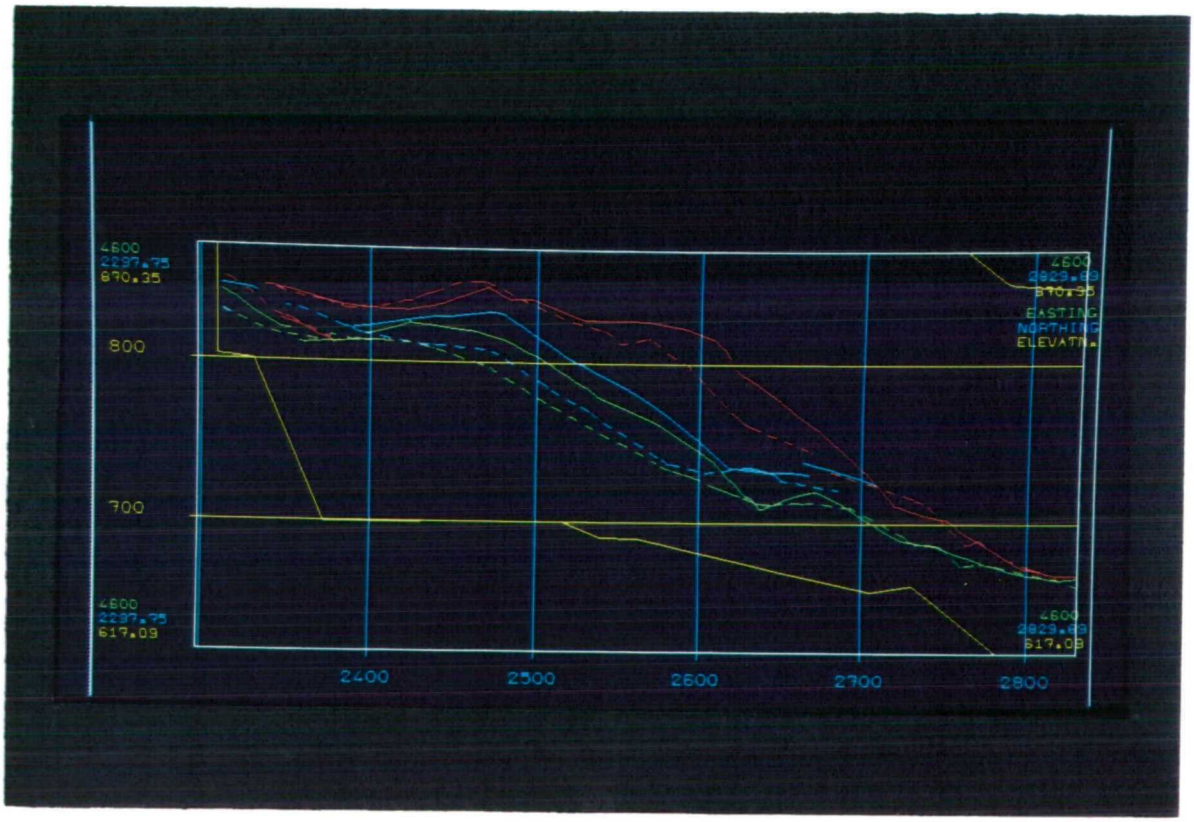


Plate 6.5 Geology on either side of a fault plane, as derived from the intersections with a set of discontinuity models. Dashed lines represent contact positions on front of fault plane, solid lines those behind the fault plane. Contacts shown are the massive sulphide footwall (green), top of basal cupriferous unit (blue), and hangingwall of massive sulphide (red). A lateral (dextral) movement of about 40m is suggested.

This form of analysis provides a useful tool for checking modelled discontinuities against the geological field data. Such displays may also reveal inconsistencies in the interpretation which are not obvious when considered purely in section or plan.

A similar procedure, to that used in generating Plate 6.5, could be used to create a display of the predicted geology on an irregular topographic surface, such as a planned open pit outline or underground stope. This would be done by intersecting all the modelled discontinuities with the surface of interest, and then displaying them in the same manner as on a normal section or plan.

Predictions of the expected intersection points and intersection angles, for planned drillholes, could also be derived using the 3-D models. This would facilitate their use for predictive purposes during the exploration or evaluation phases of a project.

Another aspect of geological analysis which may be applied in certain cases is the extraction of statistical measures (eg. average orientation, irregularity, area, etc.) of the modelled discontinuities. The model structure used facilitates extraction of such statistics, provided they can be calculated simply for individual planar triangles or polygons. Global statistics can be derived by weighting such values by triangle or polygon area.

6.6 Discussion

Examples presented in this Chapter have illustrated specific ways in which the geological discontinuity model structure can assist in the interpretation of geological contacts, faults and other boundaries. This approach offers several advantages over other methods used for geological modelling (as described in Chapter 4). The ability to link the discontinuity models to geological databases (which form the foundation for all geological interpretations of a deposit), is of particular importance. The incorporation, into the models, of information about the data from which they have been derived greatly assists model verification and updating. This approach helps to minimise the introduction of spurious errors during modelling.

The psychology of geological visualisation and interpretation undoubtedly plays a major part in the acceptance of certain approaches, such as serial-slice interpretations. For this reason special attention has been paid to allowing the 3-d discontinuity modelling system to be used in conjunction with conventional interpretative techniques.

This Chapter has been restricted to a discussion of the modelling of geological discontinuities, since this forms the starting basis for the majority of geological interpretations used during reserve interpetations. At the exploration stage, this is often as far as geological modelling needs to proceed, and the analytical techniques provided make this type of modelling a powerful tool in its own right.

Where a project proceeds past the exploration phase, geological modelling must be linked to the generation of volumetric models for use in grade estimation and mine planning studies. The next Chapter discusses how discontinuity models can be used to define the volumes of different material types which make up a deposit.

7. MODELLING GEOLOGICAL VOLUMES

7.1 General

The examples presented in Chapter 6 illustrated how comprehensive geological models of mineral deposits can be created using sets of discontinuity models. In the context of reserve estimation and mine planning it is essential to be able to define the volumes of different geological material types present. Such models allow variations in internal properties, such as density and metal grades, to be studied separately within each geological unit. They can also be used to impose geological controls on the estimation of internal properties by mathematical or geostatistical methods, as well as allowing the categorisation of material to be mined according to geological criteria. Consequently the ability to interface with volumetric models is an essential pre-requisite for using geological discontinuity models directly during reserve estimation and mine planning.

The fundamental requirement in this context is the ability to assign a geological code at any point within a deposit. Where volumetric models are to be used in interactive graphics programs this must be done as rapidly as possible. Properties such as lack of ambiguity, the ability to carry out Boolean operations, completeness and disjointness (see Section 4.6) are also essential. These, and other requirements, have led to the adoption of several common modelling techniques as described in Chapter 4. Methods by which such models can be generated directly from 3-d geological discontinuity models (such as those illustrated in Chapter 6) are discussed in this Chapter.

7.2 Gridded-Seam Models

Gridded-seam models are based on regular grids of points for which elevations are assigned to a series of discontinuities arranged in a unique vertical sequence, as described in Section 4.4. The conversion of a series of discontinuity models into multiple gridded seam models is illustrated here using an example from Neves-Corvo.

Three basic procedures are involved in this conversion procedure, namely;

a) **Gridding**: in order to generate a gridded-seam model it is necessary to superimpose a regular grid of points onto a 3-d discontinuity, and automatically determine the elevation of the discontinuity at each grid node. This is done using an algorithm based on calculating the intersections of a series of vertical lines, one for each grid node, with the triangulated surfaces. The problem is thus reduced to checking the intersection of a series of lines with a series of triangles. Where an intersection occurs within a triangle the elevation of the intersection point can be determined directly from the triangle data, thus providing the elevation for the grid point being tested. Preliminary checks on the overall limits of a particular surface, and of individual triangles, are used to minimise the number of calculations required. The results of such gridding are illustrated in Figure 7.1 which shows the grid points and elevations established on a contoured discontinuity surface.

b) **Determination of model limits**: In addition to any limits of interpretation specified during geological interpretation, the overall structure of a gridded-seam model is normally controlled by an overall base and an overall top.

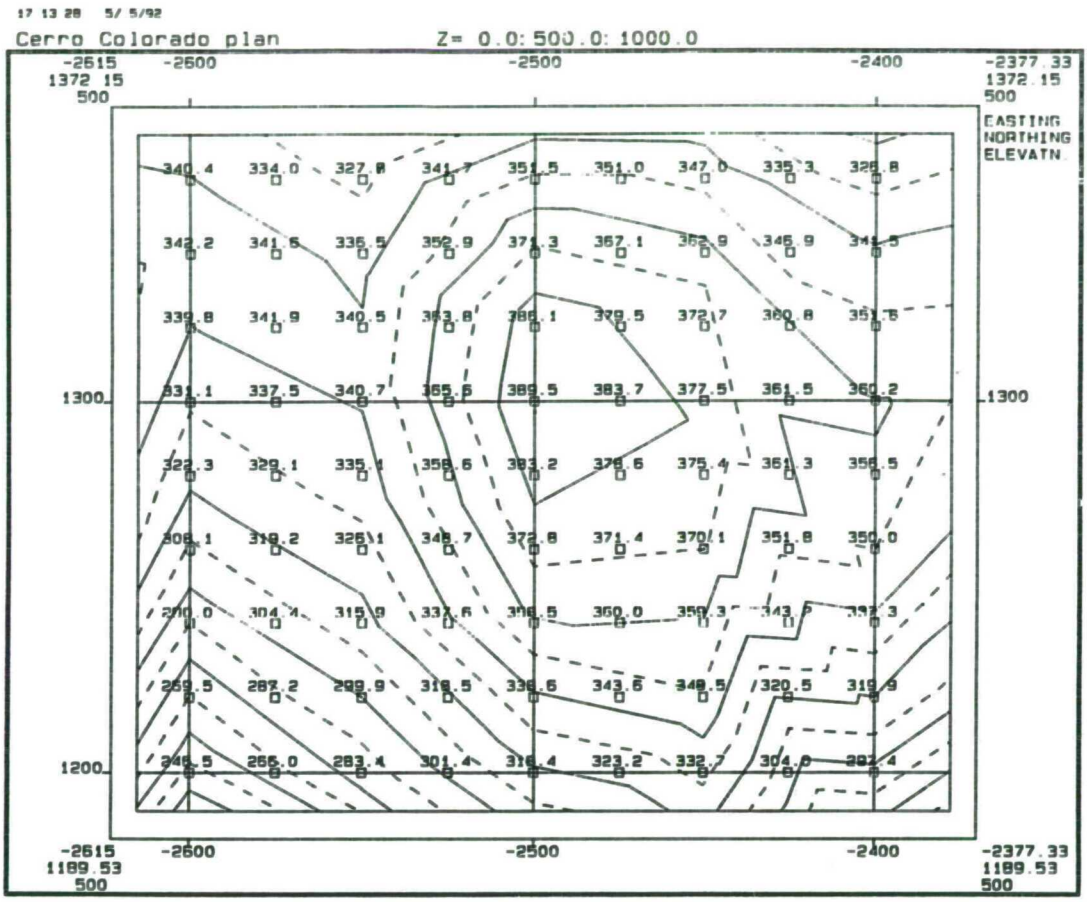


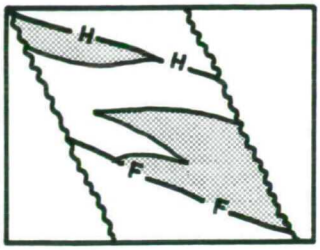
Figure 7.1 Isobath plan of a discontinuity model showing the grid points and elevations derived from it using a gridding algorithm

The program used for conversion of 3-d discontinuities specifies one principal basal surface, but allows any number of additional surfaces to be used as well. All grid points are first initialised, and those falling outside the limits of interpretation are flagged in order to avoid considering them further.

A regular grid is then superimposed on the "visible" portion of the principal basal surface and used to reset some of the grid points. The additional basal bounding surfaces are then checked in turn and used to assign elevations to other grid points. Where these points fall below previously assigned grid elevations they are ignored. If, however, they lie above the previously assigned values these are reset upwards. This procedure is illustrated schematically in Figure 7.2.

A similar approach is used for defining the elevation of the top surface. In this case all grid points are first checked against the overall base, and if they fall below it they are reset equal to the basal elevations. When using additional upper bounding surfaces elevations are reset downwards rather than upwards.

c) **Vertical sequence and coding**: When the overall top and base elevations have been correctly determined, other internal interfaces may be established in order to subdivide the overall model. This procedure is governed by a pre-defined vertical sequence of discontinuities, which should also be used when modelling the interfaces as discontinuities, in order to avoid ambiguities.



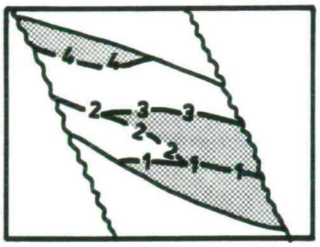
A. MODEL 3-D DISCONTINUITIES DEFINING GEOLOGICAL LIMITS OF STRUCTURAL COMPARTMENT

F Footwall contact

H Hangingwall contact

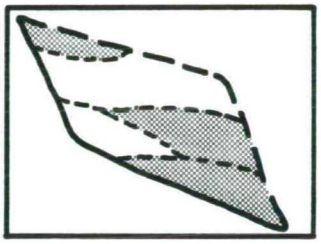
Major faults

Mineralisation



B. DEFINE INTERNAL SEQUENCE OF LENS CONTACTS FROM BASE UPWARDS, AND MODEL THEM AS 3-D DISCONTINUITIES

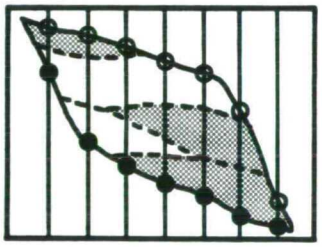
1 Internal lens contacts



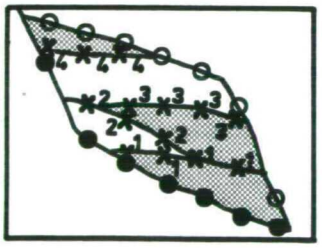
C. DEFINE PRINCIPAL, AND SUBSIDIARY, DISCONTINUITIES DEFINING:

OVERALL BASE, AND

OVERALL TOP, OF THE STRUCTURAL COMPARTMENT



D. DEFINE GRID CELL POSITIONS AND THEN GENERATE GRID ELEVATIONS FOR BASE (●) AND TOP (o) OF THE STRUCTURAL COMPARTMENT



E. GENERATE GRID ELEVATIONS FOR THE INTERNAL BOUNDARIES (X1) WORKING FROM THE BASE UP. VALUES ARE CONSTRAINED WITHIN THE LIMITS DEFINED BY THE LAST UNDERLYING VALUE, AND THE VALUE FOR THE OVERALL TOP.

Figure 7.2 Schematic cross-section illustrating the procedure used to control generation of gridded-seam models from 3-d geological discontinuity models

Grid elevations are assigned to each interface in turn, working upwards from the base. In this instance each grid elevation is checked first against the underlying interface, and then against the overall top. Where elevations of the surface being added fall below that of the underlying surface, or above the overall top, they are reset upwards or downwards, respectively. Since some of the intermediate interfaces may not cover the entire model area, it is necessary to automatically extend them over the whole area as a preliminary step.

This automatic procedure can sometimes generate errors, so that it may be necessary to edit the original discontinuity models after checking the results obtained.

The generation of such models facilitates the derivation of isopachytes in complex geological situations such as those modelled at Neves-Corvo. Figure 7.3 shows an example of a cross-section through a set of 3-d geological discontinuities, and the gridded-seam model derived from them.

7.3 Serial-Slice Models

Serial-slice type models, based on interpreting geology on a series of parallel sections or plans, are one of the most widespread methods used to build up 3-dimensional geological interpretations of a deposit. As mentioned in Chapter 4, this approach can also be used for modelling geological volumes. This is done by defining a series of polygons on each interpretation plane, and assigning them a volume of influence half way to adjacent planes. Generation of such models depends on the ability to define a set of mutually exclusive polygons on each interpretative plane.

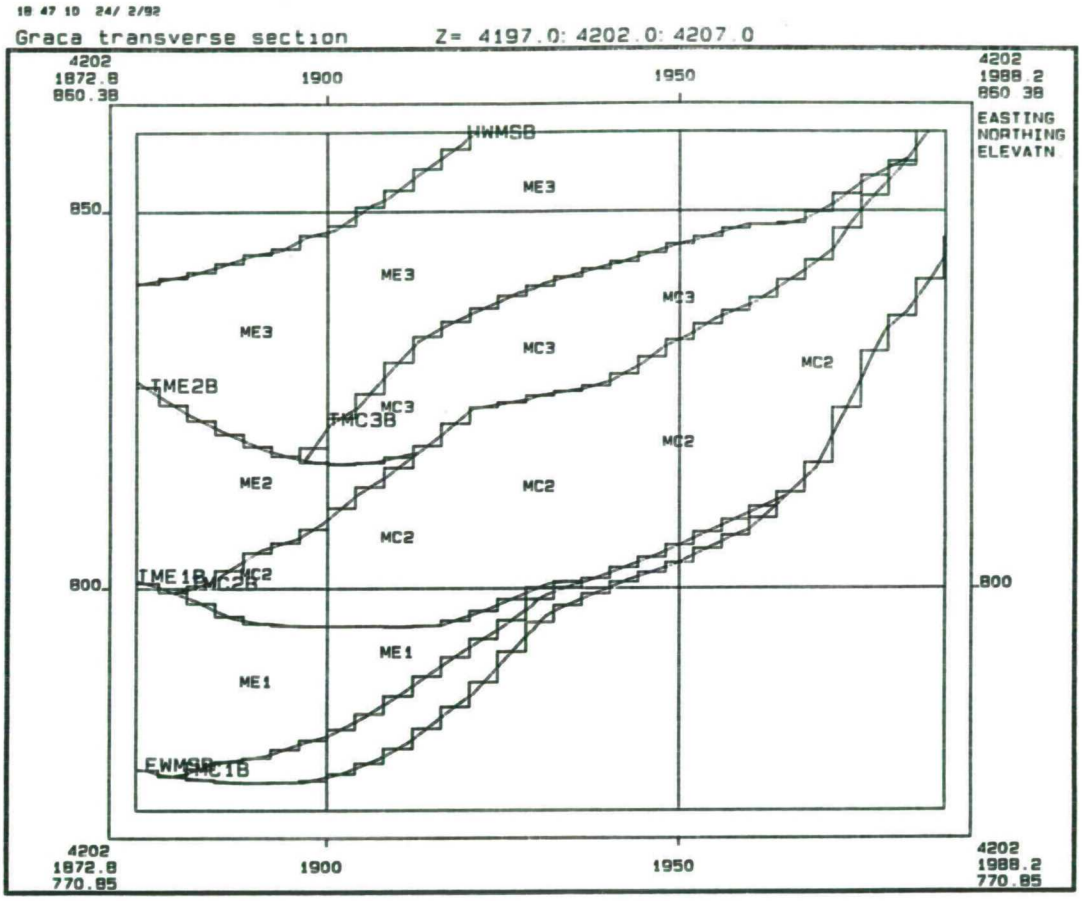


Figure 7.3 Cross-section showing geological discontinuities and an equivalent gridded seam model

The 3-d discontinuity modelling system allows rapid calculation and display of the intersection of all discontinuities in a model with any given plane. Consequently it is a fairly simple matter to calculate such lines and store them in separate files. Generation of the required set of polygons for each plane is somewhat more complicated, since each discontinuity may have to be split into several segments prior to merging them to form the polygons defining the entire plane. It may also be necessary to identify situations where nested polygons occur.

Conversion of the 3-d models to serial-slice models is done in four steps (illustrated schematically in Figure 7.4), namely;

a) **Polyline generation**: Calculation and storage of the intersection traces of the discontinuity models with each plane in the serial-slice set. Calculation of these polylines is done in the manner described in Section 5.8, and they are stored in a similar format to that used for inputting 2-d geological interpretations (as described in Section 6.3).

b) **Node generation**: The resultant polylines are then checked against one another and special nodes generated to mark the ends of each line, as well as points where they intersect or touch. At the same time checks are made to ensure that duplicated line segments, and other problematical data, are eliminated.

c) **Polygon generation**: Using the nodes created in the last step, polygons are generated by following each polyline from node to node in a clockwise direction, until all possible paths have been checked. The resultant polygons are then sorted by area, with the largest first, in order to be able to test for nested polygon structures.

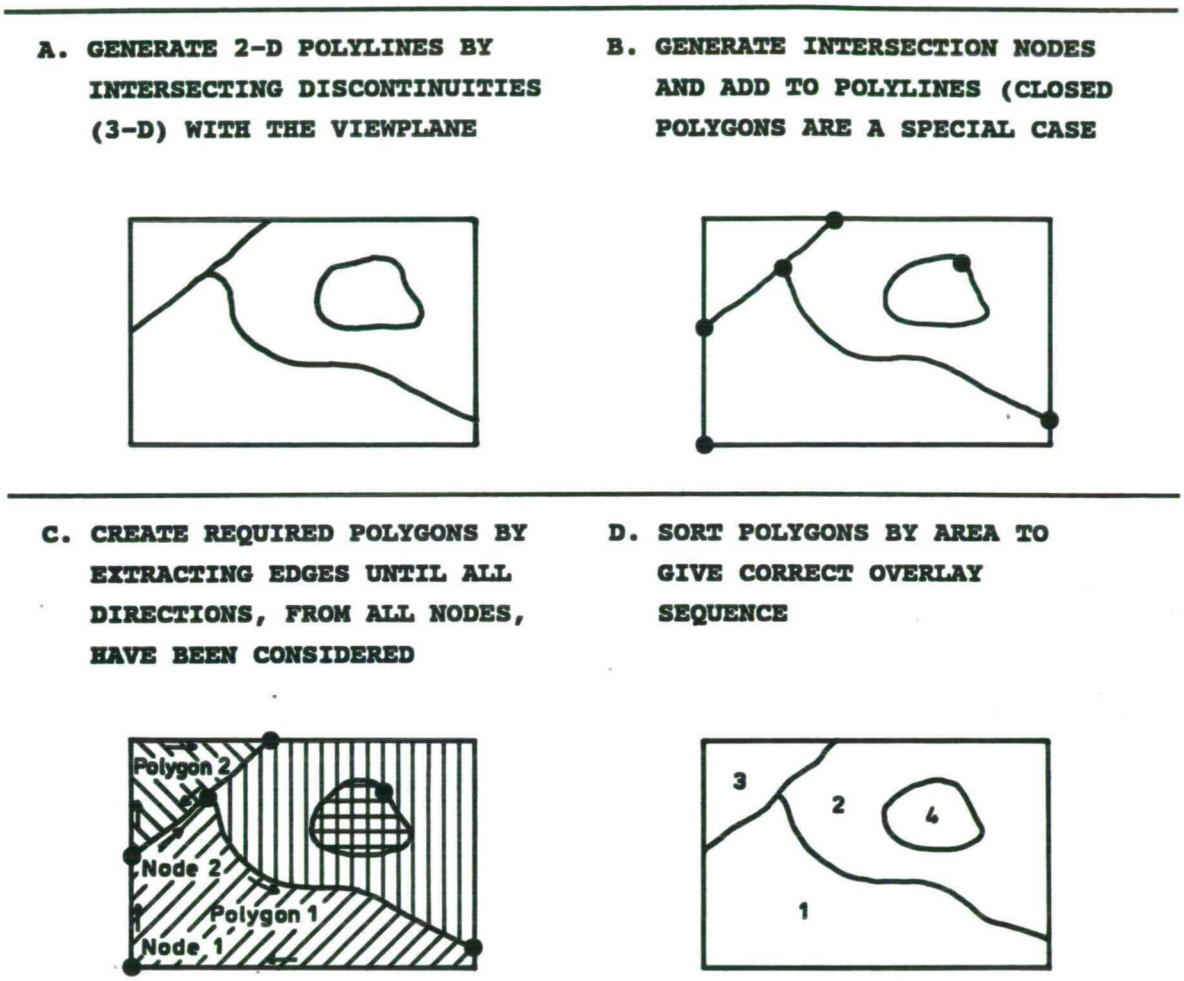


Figure 7.4 Schematic section illustrating the approach used to generate 2-d polygons (for use in a serial-slice model), from a set of 3-d discontinuity models

Trailing edges which do not link up with any other line on the plane are eliminated at this stage. (Such lines may represent incorrectly trimmed discontinuities which have allowed gaps to remain in the overall interpretation).

d) **Polygon coding**: Geological codes are then assigned to each polygon. In some cases this can be done totally automatically, especially where most of the discontinuities used are interpretation independent (see Section 6.2). For instance, most of the polygons shown in Plate 7.1 can be correctly coded geologically using the same vertical sequence of interfaces defined during derivation of seam-type models. Where this is not the case manual intervention may be necessary in order to assign codes using an interactive graphics program.

The algorithms used for polygon generation are based on the development of polygon merging subroutines for mine planning applications, as mentioned by Howson (1989). The main difficulty encountered, in implementing this application, related to problems in the generation of the polyline data from the discontinuity models in cases where a lot of co-planar edges existed (eg where the model had been derived from digitised data on the same plane). Such problems have been reduced by implementing further checks in the display routines to ensure that duplicated display lines were not generated.

The disadvantage of not being able to automatically assign codes to all the polygons generated stems from the fact that some discontinuities (eg major faults or surface topography) may have several geological material types on either side. Consequently whilst many of the lines used to build the polygons can automatically be assigned codes on either side (derived from the positive and negative side codes stored with each discontinuity model), this is not true in all cases.

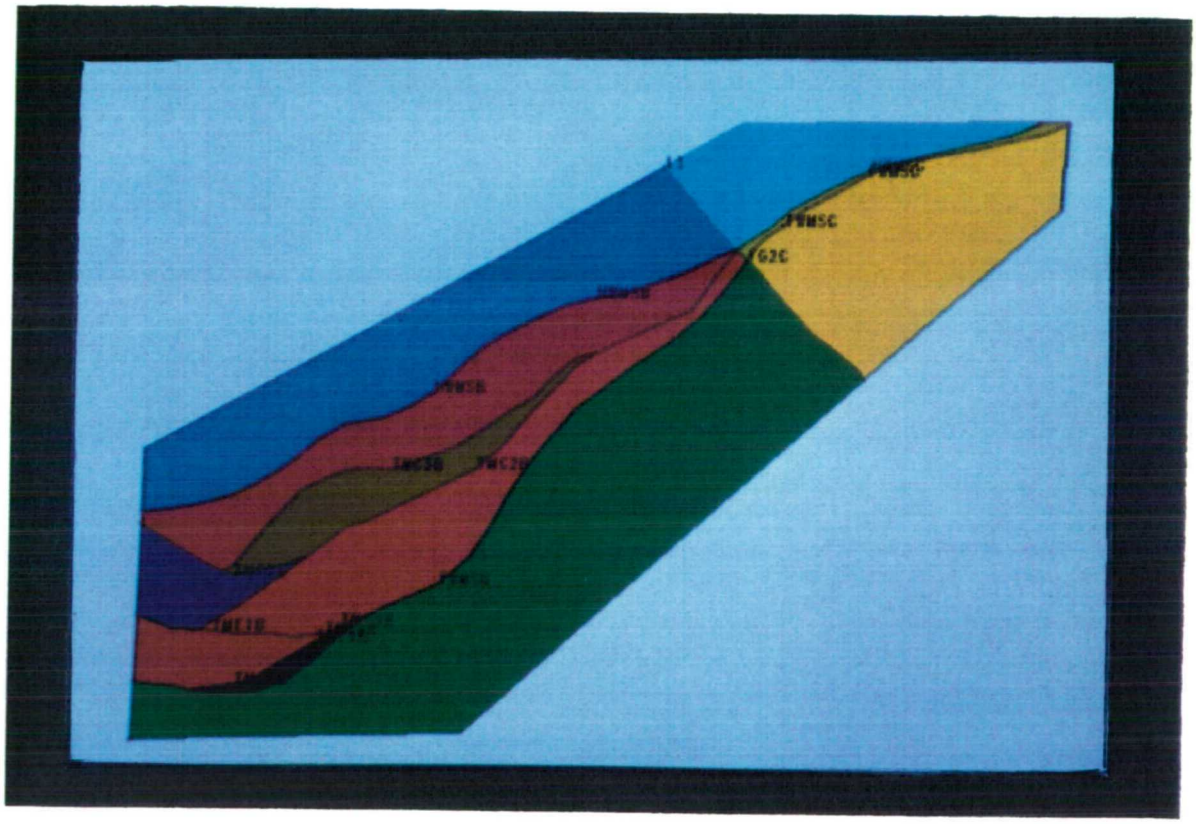


Plate 7.1 Polygon mosaic (for a transverse section in the Graca orebody) generated using stored 3-d discontinuity models. Polygon colours are assigned according to polygon size rather than geological code.

Future development of the system may allow unambiguous determination of codes at any point on either side of a discontinuity model. Should this be successful, totally automatic generation of serial-slice type volume models, from geological discontinuity models, should be possible.

7.4 Closed Discontinuity Models

The discontinuity model structure described in Chapter 5 can be used to store models defining closed volumes directly. So long as the rules pertaining to triangle storage sense are obeyed (see Figure 5.2), a discontinuity model with no edges defines a closed volume. The fact that each triangle edge can only be shared by two triangles, and that it must be used in opposite senses in each, means that ambiguous structures such as a Klein bottle (a higher dimension equivalent of a Mobius strip) cannot be generated.

The algorithm developed for determining on which side of a discontinuity a point lies can thus be used for in/out tests, since all points inside a closed discontinuity model are on the same side of it. Since the positive and negative sides of a discontinuity are defined by the location of the first non-vertical triangle, the interior may be either negative or positive depending whether the triangle selected is at the top or base of the closed discontinuity model. To avoid errors when using such models, an initial check is done so as to ensure that the positive side of a closed volume is outside. Where this is not the case triangles, and trim lines, are reversed before using the model for volume calculations.

For simple shapes, defined by a single discontinuity, such models can be generated rapidly using the linked-slice method. This results in a model defining an irregular cylinder with a hole through the middle. This is then closed off by interactive editing, with additional points and triangles being added at either end, in order to fill in the polygons defined on the first and last interpreted planes. The application of this approach is illustrated schematically in Figure 7.5.

The triangulated discontinuity models described here have also been used to model irregular mine openings for mine planning purposes, using an approach termed flexible level stope design (Howson (1989)). This design method is based on modelling the floor and roof of irregular stopes as two separate discontinuity models using the techniques developed for geological discontinuity modelling by the writer. During subsequent design work, closed polygons are digitised on horizontal planes and superimposed on the two discontinuity models. The areas falling within these polygons (on each surface) are then combined with the irregular vertical prism (linking the polygon between the roof and floor models) in order to define the triangles representing the stope volume. This allows direct calculation of the geometric volume of the designed opening, as well as estimation of density and grades by checking against reserve block models. A photograph showing a stope designed in this manner is presented in Plate 7.2.

The discontinuity model structure is thus perfectly adequate for defining closed volumes, as illustrated by the two examples given above. The model structure was in fact designed with this in mind, the original intention being to automatically combine the individual geological discontinuity models in a given deposit to produce models of all the geological volumes present. The reason that this was not achieved deserves discussion since it highlights problem areas that need more study.

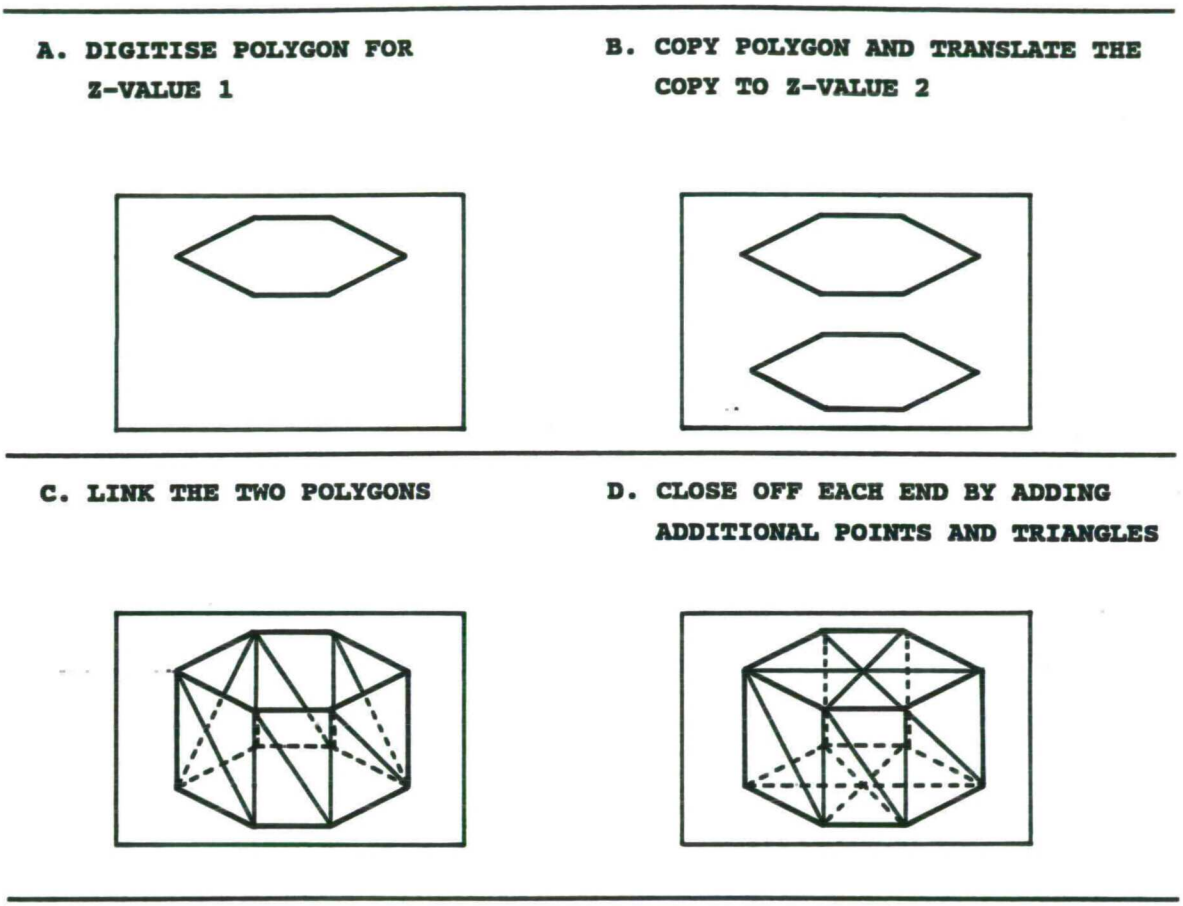


Figure 7.5 Diagram illustrating the procedure used to generate a closed volume discontinuity model using a linked-slice interpretation

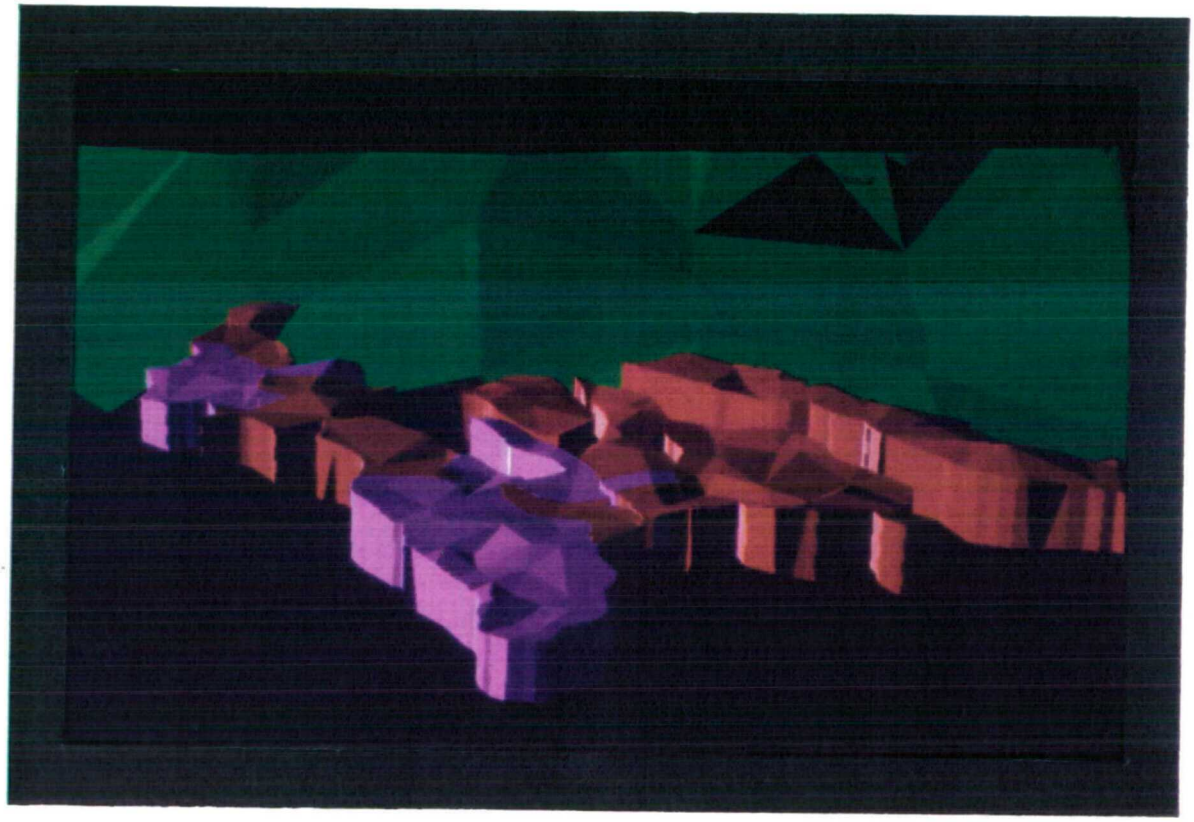


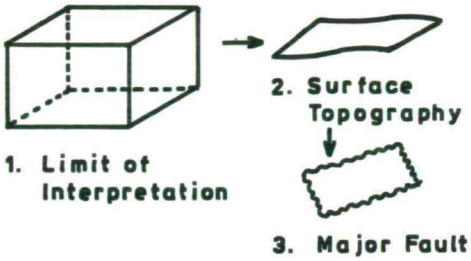
Plate 7.2 **3-d visualisation of a cut-and-fill stope, in the Neves-Corvo mine, modelled using the flexible level design method described in the text.** Red and magenta colours indicate stopes in two different ore types. The massive sulphide footwall is shown in green.

The procedure originally envisaged for generating a geological volume, defined by portions of different geological discontinuities, is illustrated schematically in Figure 7.6. The steps involved are as follows;

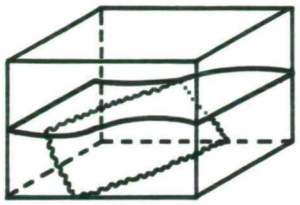
a) Define the intersection hierarchy to be used in modelling the discontinuities, ensuring that the model at the top of the hierarchy defines an overall volume of interest which is to be split into smaller geological sub-volumes
b) Create and edit the individual discontinuity models ensuring proper closure of intersecting discontinuities
c) Starting with the first two discontinuities at the top of the hierarchy (eg limit of interpretation and surface topography) generate their mutual intersection
d) Save the intersection points in the lower rank discontinuity model, and use them to trim off the "trailing edges". Retriangulate the remaining surface delimited by the intersection line.
e) Store the intersection line in the higher rank model, and generate triangles incorporating the intersection points, ensuring that none of them cross the intersection trace, or alter the planarity of the triangle they lie in.
f) Split the higher rank model into two separate patches as defined by the intersection line now stored with it.
g) Merge the patches of the higher rank model with the lower rank one so as to form two sub-volumes.
h) Using each of these sub-volumes in turn return to step (c) and repeat the procedure if an intersection occurs with the next discontinuity in the hierarchy

Despite considerable effort on the part of the writer and associated workers this approach proved impracticable during initial development of the modelling system. The main reason for this failure was that the model structure rapidly degenerated when this approach was attempted with fairly simple test cases. This was reflected in the generation of a lot of long thin triangles, and closely separated points, as illustrated in Plate 7.3.

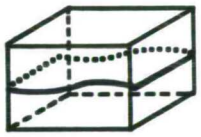
A. DEFINE INTERSECTION HIERARCY WITH CLOSED VOLUME AT TOP



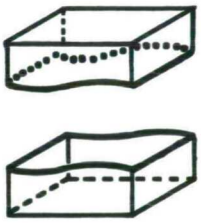
B. MODEL DISCONTINUITIES AND TRIM TO ENSURE PROPER CLOSURE (USING HIERARCHY DEFINED IN STEP A)



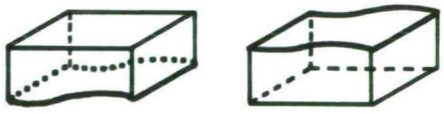
C. ADD THE INTERSECTION LINE OF THE TWO DISCONTINUITIES AT THE TOP OF THE HIERARCHY INTO THE ONE WITH THE HIGHER RANKING



D. SPLIT THE HIGHEST RANKED DISCONTINUITY INTO TWO SEPARATE DISCONTINUITIES



E. ADD THE LOWER RANKED DISCONTINUITY (TRIMMED) TO BOTH DISCONTINUITIES TO FORM TWO CLOSED VOLUMES



F. SELECT ONE OF THE TWO NEW VOLUMES TO FORM THE TOP OF A NEW HIERARCHY (STARTING AT NUMBER 3 IN THE ORIGINAL HIERARCHY) AND RETURN TO STEP C.

Figure 7.6 Diagram illustrating the procedure used to generate a closed volume discontinuity model from two or more individual discontinuities

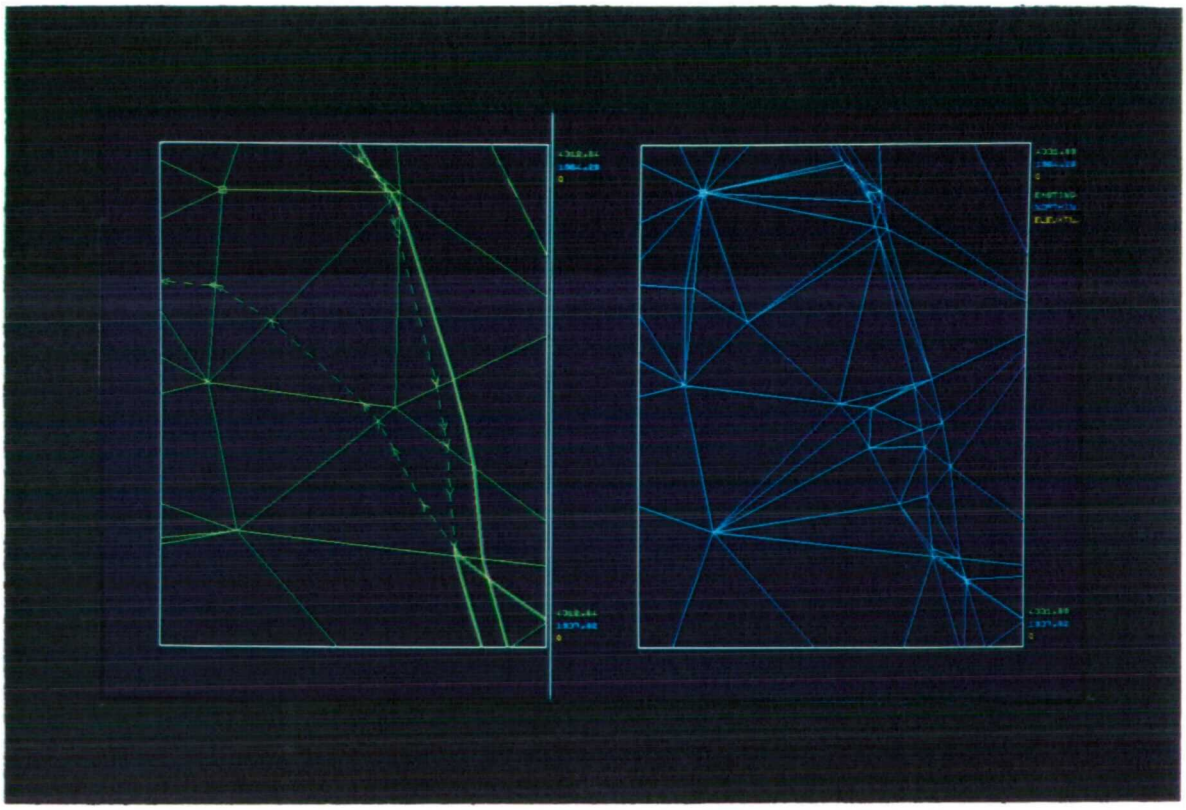


Plate 7.3 **Example showing the effects, on triangle size and shape, of incorporating a fairly simple intersection line into a discontinuity model.** Left hand display shows the intersection line (dashed line) and the triangles in the original discontinuity model; right hand display shows the new discontinuity model which incorporates all the intersection points and lines as triangle vertices and edges, respectively.

In such situations the problem of identity (ie when to consider two points as identical, or a line as coplanar with a given triangle) becomes critical, and this resulted in an excessive number of special cases causing errors in the low level geometric subroutines used. Consequently, this approach was abandoned in favour of using the trimming procedure (see Section 5.10), coupled with the methods for interfacing with other types of volumetric models that are described in this Chapter.

7.5 Block Models

The use of discontinuity models to generate block models usually involves the creation of another type of volumetric model as intermediate step. Consequently most of the aspects involved in such conversions have already been discussed in the preceeding sections of this Chapter. In work done to date, seam-type models have been used extensively as a method of generating block models for grade estimation purposes at Neves-Corvo. As this approach is restricted in applicability, use of serial-slice type models has been investigated as a method of more general applicability to a wide variety of deposits.

An example showing a comparison of contacts as represented in a regular block model, and the discontinuities from which they are derived is shown in Figure 7.7. The block model illustrated was generated using a seam-type model as an intermediate step (see Section 7.2). In this instance the grid used for generating the seam-type model corresponded exactly with the horizontal dimensions of the blocks in the block model. Codes for individual blocks were generated by checking the vertical sequence of interfaces at each grid node. The elevations of the block centres corresponding to each slice in the block model were checked against the elevations assigned to the different seam interfaces to determine the code at that point.

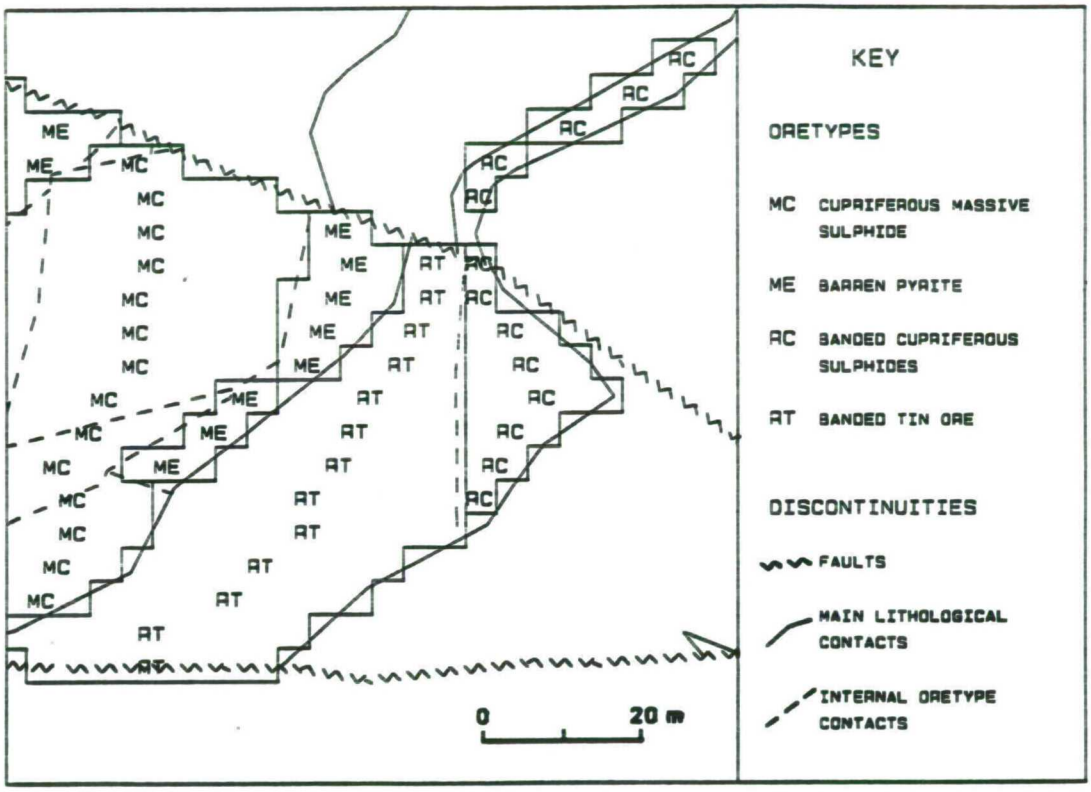


Figure 7.7 Level plan showing comparison of a regular block model and the discontinuity models from which it was derived (after Sides 1992a)

If desired, further checks could be carried out to determine the dominant geological material type present along the vertical axis cutting each block, rather than merely assigning the code at the centre. In fact, for grade estimation purposes, a sequential file of block centres and heights was generated, with separate partial blocks where a contact occurred within an individual block. The grades estimated for each of these partial blocks were recombined into a regular block model structure prior to their use in mine planning work.

The serial-slice type models discussed in Section 7.3 can also be used to generate regular block models. This is done by generating serial-slice models for a set of planes corresponding to one of the three axial planes of the block model. Block codes can then be assigned by doing in/out checks of the blocks on a particular plane against all the polygons stored for that plane. Because of the difficulties of automatically assigning codes to the serial-slice polygons this procedure often requires a considerable amount of manual intervention when assigning codes to the polygons.

Closed discontinuity models can also be used to generate or modify block models, by doing in/out checks on all blocks within the geometric limits affected by such models. This approach has been used at Neves-Corvo for the modelling of mined-out volumes. In this instance it is necessary to flag blocks in the reserve models as being mined-out, in order to discount these volumes from overall mineral inventory totals. This is done using closed discontinuity models (generated using the flexible level stope design method described in Section 7.4) to model the mined-out stopes. Blocks falling within these volumes are then flagged in the geological block models used. Plate 7.4 shows the outline (on a horizontal plan) of a mined-out stoping area, as defined in a geological block model, together with the flexible level designs from which the block codes were derived.

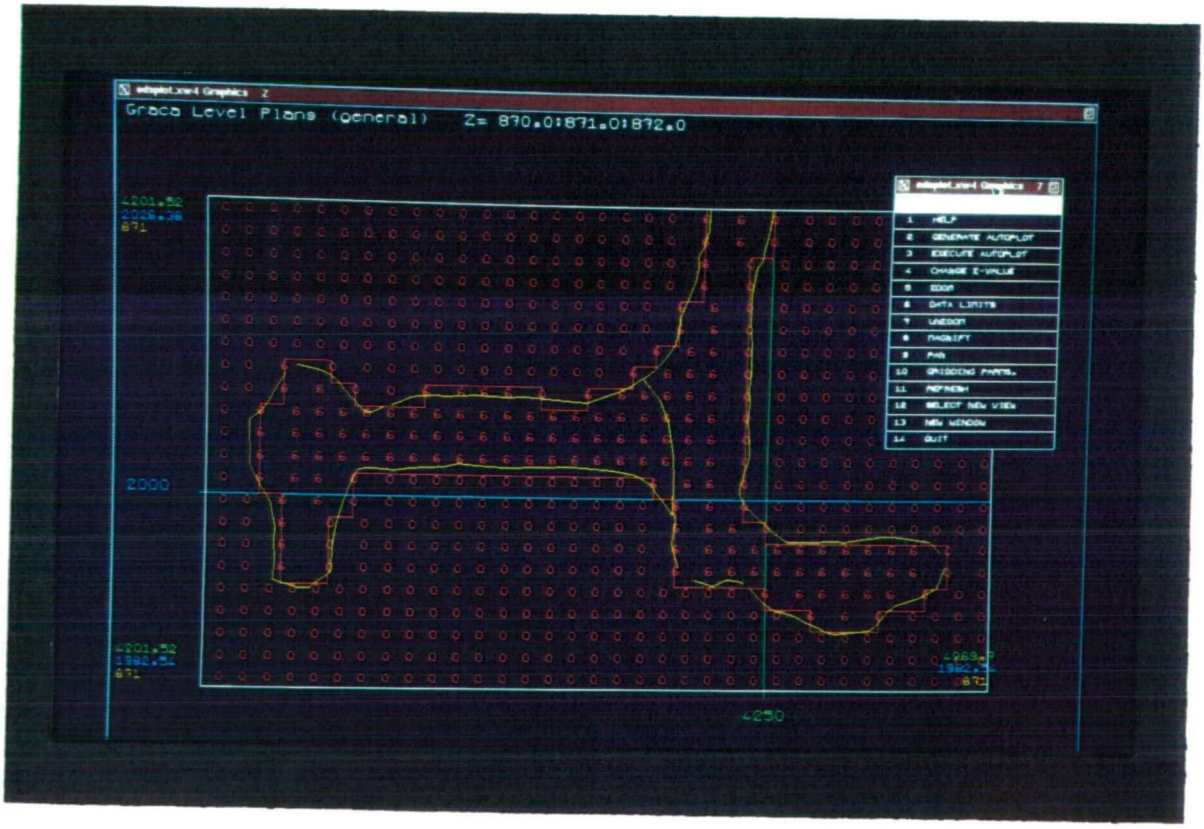


Plate 7.4 **Plan showing mined-out volumes for a given six month period, as represented in a geological block model, and the stope designs from which the block codes were derived.** Yellow lines represent intersection of stope designs (flexible design method) with the plane of interest. Code 6 indicates mined out blocks; code 0 indicates material remaining in situ.

7.6 Discussion

The examples given in this Chapter illustrate ways in which discontinuity models, created using the modelling approaches discussed in Chapter 6, can be used to generate different forms of volumetric models. Despite the fact that the modelling system has been used successfully in this context at Neves-Corvo, the system is still fairly limited in its ability to cope with geologically complicated deposits. The main reason behind this limitation is the difficulty of automatically determining the geological code pertaining to a point, at any position, on either side of a discontinuity. This results in a fairly complicated procedure being necessary for generating seam-type models, and a considerable amount of manual checking being required in the case of serial-slice models. It also means that geological codes at any position in space cannot be determined directly from the nearest discontinuity model.

To overcome this limitation, the possibility of storing all possible intersection traces, with each discontinuity model, is being investigated. These traces would then represent a network of 3-dimensional polylines, which could be merged to form sets of polygons (lying in many different planes) defining the geological codes for different patches on either side of the discontinuities. The algorithms used for generating 2-dimensional polygons during the conversion of discontinuity models to serial-slice type models are being adapted for this purpose.

The approach being followed is analogous to constructing a house in which the interior walls, floors, ceilings and internal doors, are painted in a different colour within each room. Provided there are no windows or open doors, ("proper discontinuity model closure") one could then determine which room one was in from the colour seen when looking in any direction.

In this instance the overall limit of interest would correspond to the external walls of the house, and the internal dividing walls and floors to individual discontinuities within a deposit.

This approach would allow parameters such as the contained volumes of different geological materials to be calculated directly from the discontinuity "patches" flagged with particular codes. Where closed volumetric models are required by other applications this approach could be used to select the required bounding surface patches.

Despite these possibilities alternative forms of volumetric model will continue to be used, for grade estimation and mine planning purposes, for the foreseeable future. The particular type of volumetric model used in such instances depends on several factors, such as;

- orebody morphology;
- mining method;
- spatial and geological controls on grade variation;
- accuracy and precision required;
- software used in other applications;
- time, and hence cost, involved in model generation and updates;
- frequency with which the models have to be updated;

The ability of the discontinuity modelling system to interface with different types of volumetric model is a distinct advantage under such situations, since it allows a single geological model to be used for a wide variety of applications.

This is illustrated by the application of the discontinuity models in the generation of block models, as described in Section 7.5. Depending on the intermediate model type used several block model parameters can be varied, without necessitating any additional interpretative work, as follows;

- **seam-type model interface:** block heights and starting elevation can be varied (ie 1-dimension of variation)

- **serial-slice type model interface:** block dimensions, origin and orientation of the block grid, can be varied within the planes corresponding to the serial slices (ie 2-dimensions of variation).

- **closed volume discontinuity model interface:** block dimensions, origin and orientation of the block grid can be varied in all three dimensions

One of the main advantages of this is that the effects of using different block dimensions, or block grid origin and orientation, can be studied before establishing the block models to be used as the basis for grade estimation and mine planning (see Section 10.2). Conventional approaches would involve a considerable amount of additional geological modelling work in such instances. Consequently whilst such studies can be done in a matter of hours using the procedures described here, they are usually regarded as impracticable when using conventional approaches because of the days or weeks of additional work involved.

An example would be the case of a deposit which has to be evaluated for both open-pit and underground mining scenarios. In most cases this requires the reserve models used for mine planning to be generated on a different basis for each planning option. The fact that a single geological model could readily provide geological block models based on different block sizes, orientations, and locations, would allow much more thorough analysis of alternative mining strategies.

Generation of volumetric models representing the different geological units present in a deposit are an important preliminary step to the estimation of internal properties such as density and grades. Such estimates are essential for the calculation of in-situ and extractable mineral inventories. The methods available for converting geological discontinuity models, as described in this Chapter, require a significant amount of manual intervention and control, except in the case of fairly simple deposit geometries. Nevertheless, they are no less time consuming than alternative procedures used to generate such volumetric models, and offer several distinct advantages, as discussed above.

Continued development work on the processing of geological discontinuity models should make it possible to automatically determine the geological code at any point on either side of a discontinuity. This would greatly reduce the need for manual intervention, and would consequently enhance the applicability of the discontinuity modelling system.

This Chapter has shown how the geological discontinuity modelling approach provides some distinct advantages in the generation of volumetric models. The next Chapter illustrates some of the ways in which discontinuity modelling can also provide improved geological control during the estimation of internal properties within such volumes.

8. GEOLOGICAL CONTROLS ON THE ESTIMATION OF INTERNAL PROPERTIES

8.1 General

Planning the evaluation and exploitation of any mineral deposit is dependent on a detailed knowledge of variations in the physical properties of the contained rocks and minerals. For instance, estimates of bulk density are required in order to calculate tonnages of material to be extracted from planned mining volumes. Likewise variations in metal grades have to be modelled so as to predict the amount of mineral product or metal, that will be produced from given mining volumes, in order to be able to optimise production schedules.

Internal properties may vary widely within given geological units, as defined by modelling procedures such as those described in the last two Chapters. For this reason the ability to predict variations in internal properties (such as density and metal grades) on a smaller scale, is an essential part of any computerised orebody modelling system. This aspect constitutes one of the fundamental differences between such systems, and the solid modelling approaches used in other fields such as CAD/CAM. In such applications the main emphasis is on correctly designing and representing the overall structure of a designed object, such as a car or a house. Such designs are normally assembled from different individual components which are assumed to have uniform physical properties. This factor has meant that CAD-type software used in other fields can rarely be applied directly to solve all reserve estimation and mine planning needs.

Representation of internal property variations is usually done using one of the block model types described in Section 4.5. A wide variety of geostatistical and mathematical modelling techniques can be used to estimate the values of different properties within such models.

Such aspects are discussed in detail by a large number of authors, such as Matheron (1963, 1965, 1971), Davis (1973), David (1977, 1988), Koch & Link (1971) and Isaaks & Srivastava (1989), amongst others. Numerous case histories illustrating such work have been presented in the proceedings of the APCOM conferences (APCOM (1961-1992)).

In the early days of the application of mathematical and geostatistical techniques, to the estimation of internal properties in mineral deposits, little attention was given to the use of geological controls. It was gradually realised that it was not possible to model all variation on an entirely mathematical or geostatistical basis. The use of the geological characteristics of a deposit to provide a framework for controlling the estimation process was recognised as a means of improving the accuracy of the final reserve models. Such applications are illustrated by several case studies, such as Barnes (1982), Dowd (1989), Forkes (1982), Rendu (1985) and Sides (1987).

The choice and implementation of the estimation algorithms used in such applications are not discussed in detail here, this being an aspect which is considered to fall outside the scope of the thesis. Instead this Chapter illustrates ways in which geological discontinuity models can be linked to the process of estimating internal properties in order to allow improved geological control on the estimation procedure. Such controls may depend on the recognition of geologically controlled anisotropies related to the mineralising processes which formed the deposit. In addition the effects of post-mineralisation folding and faulting may have to be taken into account.

In the work done, on the three deposits studied, three types of geological control were used, namely;

- **"Zonal"**: whereby the properties of different geological units are estimated independently; properties within a given unit being estimated using only sample values taken from within the same unit.

- **"Stratigraphic"**: whereby the position of a sample or block vertically within a given stratigraphic unit is used to control the estimation.

- **"Structural"**: whereby the position of samples with reference to structural features such as fold axes, or folded reference planes, are used to control the estimation.

These three aspects are discussed separately below.

8.2 Zonal Control

In many deposits there are sharp changes in the internal properties of interest at the boundaries between different geological units. In such cases, if sample data from different units is combined for estimation purposes artifical smoothing of variations across geological boundaries is likely to occur in the resultant reserve models. To avoid this effect zonal controls are applied, such that when estimating properties within any particular unit, only samples taken from that particular unit are used. A fundamental requirement in such instances is that individual samples must be correctly coded with geological codes corresponding to the interpreted units within which they fall.

Frequently, one of the most time consuming aspects of orebody modelling in such instances is assigning and checking sample codes. This is required in order to verify that sample codes correspond correctly to the interpretation currently in use. Although automatic procedures (for assigning and checking sample codes) can be applied, these are not always sufficient.

One approach is to assign sample codes on the basis of associated analyses. Plate 8.1 shows underground samples on a level plan representing a 2m horizontal slice through part of the Graca orebody. The sample symbols reflect the code assigned in the database, and their size reflects the associated copper grade. This illustrates some of the problems that may occur when codes are assigned purely on the basis of analytical results, since occasional isolated low grade samples fall inside high grade zones (and vice versa), and have to be recoded following completion of the overall interpretation.

Another possibility is to assign codes using point-in-volume checks between the samples and the volumetric model of the geology which is to be used to control the estimation of internal properties. Although this method can be successfully applied in many cases, it is very dependent on the precision of the volumetric modelling technique used. This is illustrated in Plate 8.2 where there is a very sharp grade break at the footwall contact of the orebody. When such a model is converted into a regular block model, some of the samples on either side of such a contact will fall within blocks coded as lying on the other side. Additional manual intervention may be necessary in such instances.

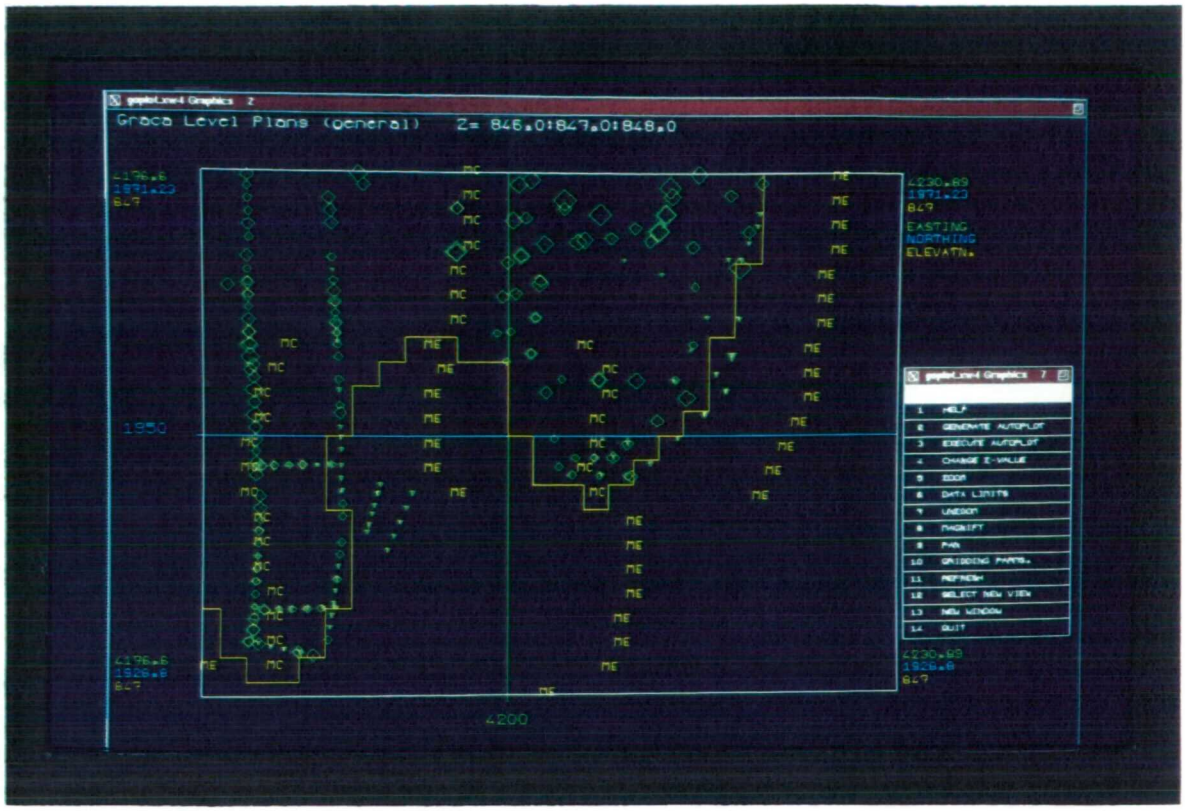


Plate 8.1 Plan from part of the Graca orebody showing underground samples and orelens contacts

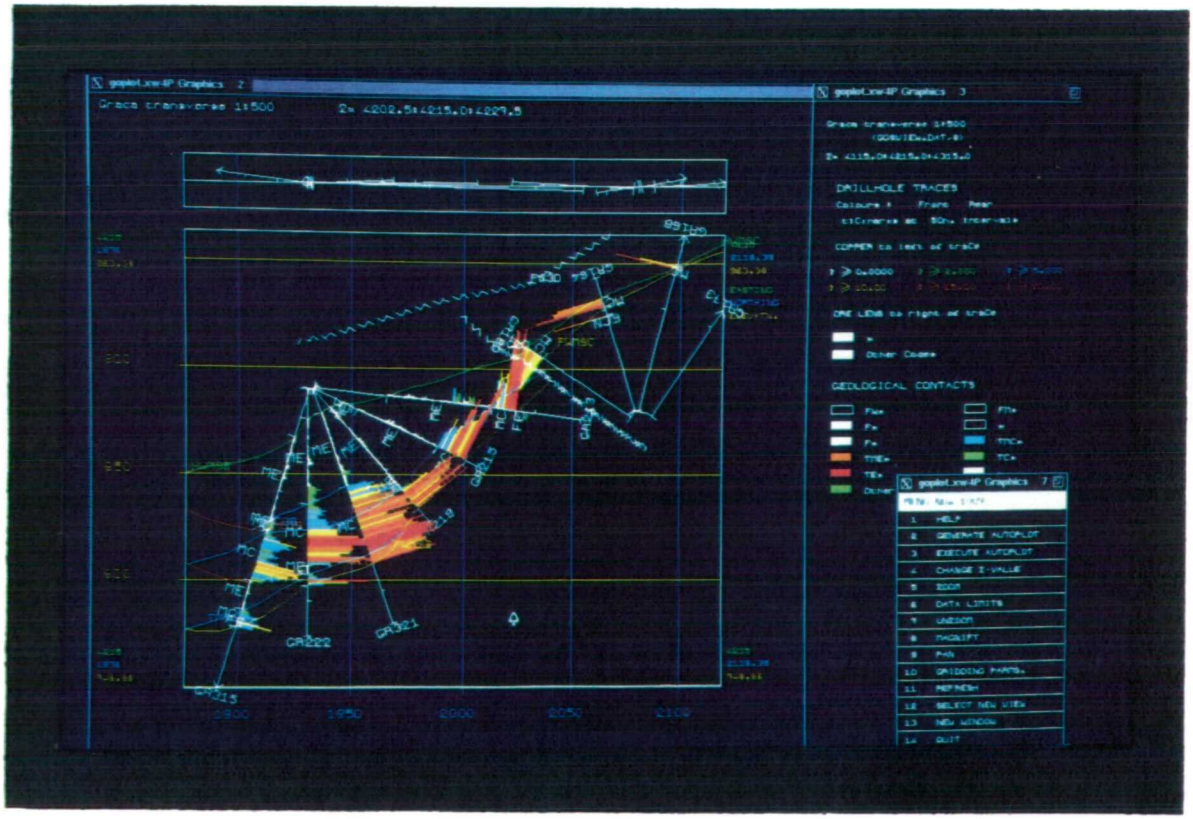


Plate 8.2 Cross-section of the Graca orebody showing drillhole copper grades and ore-lens contacts

The discontinuity models illustrated in Plate 8.2 were in fact converted into seam type models (see Section 7.2). This allowed point-in-volume checks to be carried out in order to check the sample coding, as discussed in more detail in Section 8.3 below. Although the precision (with which sub-horizontal contacts were defined) was better using this model type, it was still dependent on the dimensions of the horizontal grid used in setting up the seam-type models.

The main advantage of the discontinuity modelling system for use in zonal control is the ability to view the sample data, and the geological interpretation used, in plans or sections at any orientation. Displays such as those presented in Plates 8.1 and 8.2 can be viewed interactively during a final stage of model verification, or to investigate inconsistencies revealed during other checks between sample codes and the volumetric model.

Plate 8.2 also illustrates how the drillhole sample assay database is biased towards sampling mineralised material, with long intervals of footwall and hangingwall rocks remaining unsampled. This presents further difficulties when checking the correspondence of sample codes against the volumetric models being used.

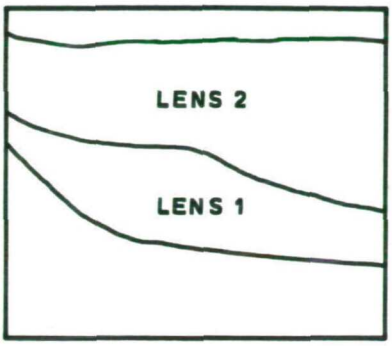
8.3 Stratigraphic control

Many stratiform and stratabound deposits show strong anisotropies in the variation of internal properties. Such anisotropies are usually reflected by much more rapid variations in such properties across the stratigraphic sense, as opposed to within the plane of stratification. In order to quantify such anisotropies using geostatistical approaches, and to apply the resultant models during estimation of internal properties, it is necessary to assign stratigraphic co-ordinates to the samples and reserve blocks used.

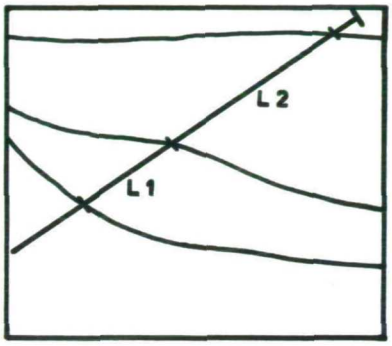
In the case of simple tabular deposits this effect can be taken into account by applying a linear transformation to the co-ordinate system used so that the XY plane is adjusted to coincide approximately with the plane of stratification. In more irregular deposits, as well as those where folding or faulting occurs, this method is not sufficient particularly with regard to estimating the "vertical" stratigraphic co-ordinate. The approach normally used in such instances is to assign a particular stratigraphic plane as reference, and to determine the stratigraphic Z co-ordinate of samples by calculating their position above or below the reference horizon, as illustrated by David (1988). Stratigraphic XY co-ordinates can be applied by reference to an arbitary grid superimposed on the reference plane.

This approach was applied at Neves-Corvo using discontinuity models as reference stratigraphic planes, in order to improve the estimation of grades in proximity to faults. The procedures adopted for conversion of the discontinuity models into gridded-seam type models, as described in Section 7.2, were utilised.

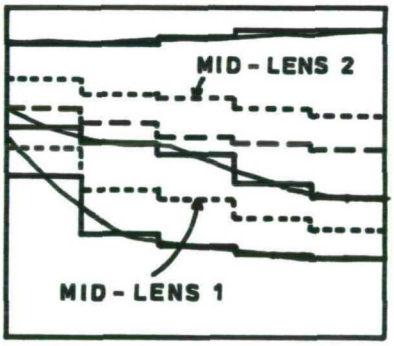
The gridded seam model block in which each sample fell was first determined. The sample elevation was then checked against the elevations of the bounding surfaces of the lens which it was coded as belonging to. Where code mismatches occurred these were flagged for checking, and such samples were temporarily eliminated from the output file used for subsequent estimation of internal properties. Correctly coded samples were assigned a stratigraphic Z co-ordinate by calculating their vertical position above or below the centre of the ore lens within which they fell. A second Z co-ordinate was assigned on the basis of their position within the overall group of lenses to which they belonged. This procedure is illustrated schematically in Figure 8.1.



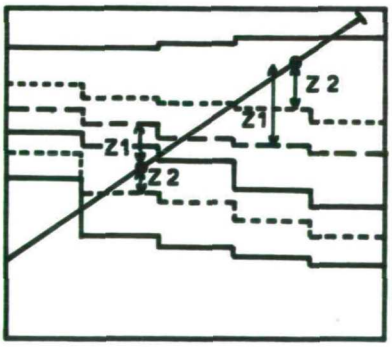
A. ESTABLISH ORE LENS STRATIGRAPHY AND MODEL BOUNDING DISCONTINUITIES



B. CODE DRILLHOLE AND OTHER SAMPLES TO CORRESPOND WITH THE GEOLOGICAL INTERPRETATION



C. CONVERT DISCONTINUITIES INTO A GRIDDED SEAM MODEL, AND ESTABLISH THE MID-ELEVATIONS OF THE MINERALISED SEQUENCE AND FOR EACH OF THE INDIVIDUAL LENSES



D. FOR EACH SAMPLE; DETERMINE THE BLOCK IN WHICH IT LIES AND THEN CALCULATE THE ELEVATION DIFFERENCES RELATIVE TO THE MIDDLE OF THE OVERALL SEQUENCE (Z1), AND RELATIVE TO THE MIDDLE OF THE LENS TO WHICH IT BELONGS (Z2)

Figure 8.1 Schematic cross-section illustrating the procedure used for calculating Z stratigraphic co-ordinates

Subsequent geostatistical studies were carried out using original (untransformed) X and Y co-ordinates together with the stratigraphic Z co-ordinate. Although this approach did not show dramatic differences in overall reserve totals, or the experimental variograms obtained, it was often significant in terms of local estimation. This is illustrated in Figure 8.2 which shows a comparison of grade estimates in the vicinity of a major fault.

8.4 Structural control

In stratiform and stratabound deposits which have suffered post mineralisation folding, it becomes more difficult to apply the type of control described in the previous Section. In such instances it may be necessary to carry out separate modelling of the main fold structures, in addition to normal modelling of lithological or ore-type contacts. Following preparation of structural models, algorithms are established for calculating the structural co-ordinates of samples and ore blocks for use in subsequent estimation. Barnes (1982) and Dowd (1986, 1989) give examples of the application of this type of approach on other deposits.

The development of the discontinuity modelling system described in this thesis owes much to early work done on the Bama-Brandelos deposits, as described by Sides (1987). To take account of the folding of this stratabound type orebody (see Figure 3.8), a structural median reference plane was modelled by digitising points from two sets of sections. The plane used was defined by a set of triangles, although the manner in which it was stored and handled pre-dates the discontinuity models described here. The manner in which the structural Z co-ordinate was calculated is illustrated in Figure 8.3. Structural X and Y co-ordinates were assigned by calculating the distance along the plane from specified north-south and east-west reference co-ordinate lines.

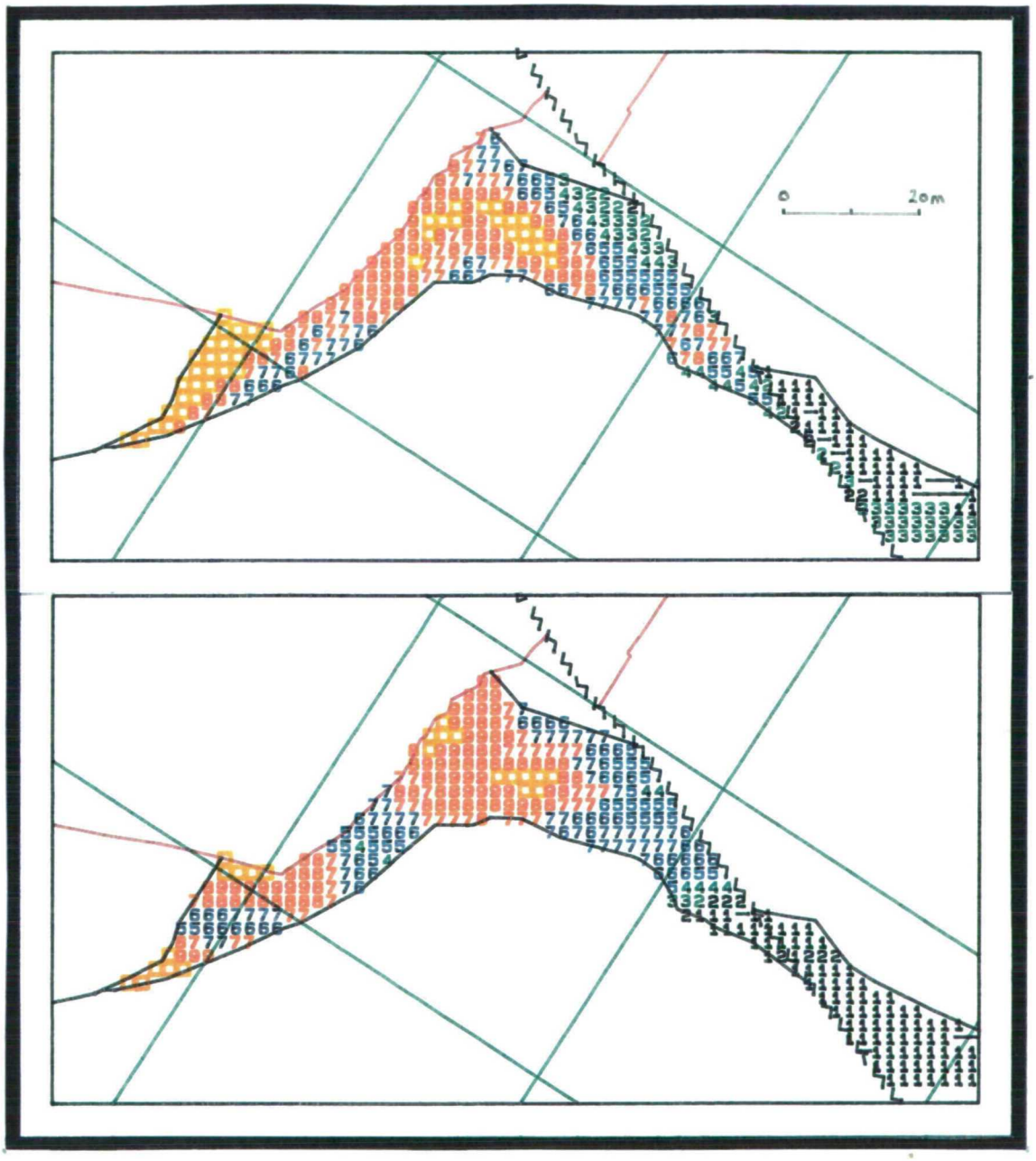
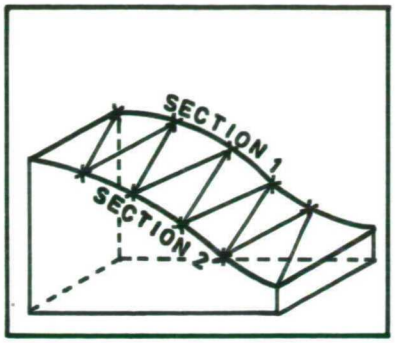
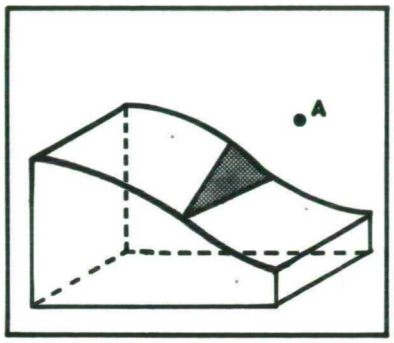


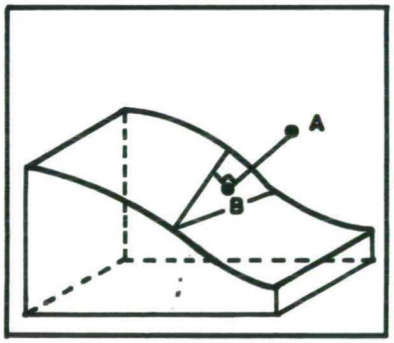
Figure 8.2 Plan comparison of grade estimates with (upper), and without (lower), the use of stratigraphic control. Note more coherent grade trends (centre left part of area), and better representation of grade changes across fault (bottom right part of area), in the upper plot.



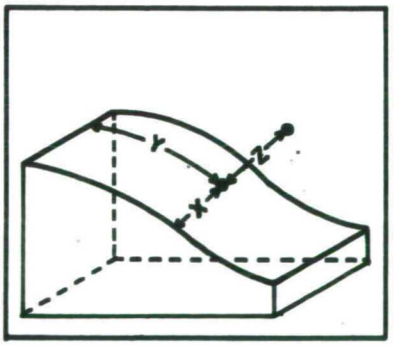
A. MODEL STRUCTURAL REFERENCE PLANE USING THE LINKED-SLICE APPROACH



B. SELECT THE "CLOSEST" TRIANGLE TO A (THE SAMPLE POINT BEING TESTED)



C. DETERMINE THE POSITION OF B, THE ORTHOGONAL PROJECTION OF A ONTO THE SELECTED TRIANGLE. THE DISTANCE AB PROVIDES THE STRUCTURAL Z CO-ORDINATE (NEGATIVE IF A IS ON THE NEGATIVE SIDE OF THE DISCONTINUITY MODEL)



D. DETERMINE STRUCTURAL X AND Y CO-ORDINATES BY MEASURING DISTANCE ALONG THE REFERENCE PLANE FROM POINT B TO THE "X" AND "Y" BASEPLANES (eg FOLD AXES)

Figure 8.3 Diagram illustrating the use of a structural reference plane to assign structural co-ordinates to samples or reserve blocks (modified after Sides, 1987)

The application of this unfolding technique resulted in a significant improvement in the variogram models used, as illustrated in Figure 8.4. An overall reduction in nugget variance was noted, when using the structural ("unfolded") co-ordinates. This effect is interpreted as being due to improved selection of sample pairs (within the same stratigraphic level), for experimental variogram calculation. This is particularly important for samples from fans of inclined drillholes close to the axis of one of the major folds in the deposit (see Figure 3.8). The lower nugget variance of the variogram model (based on the "unfolded" data) resulted in a lower block grade estimation variance than would otherwise have been the case.

Following implementation of the discontinuity modelling system the same approach was applied to the Cerro Colorado deposit, in order to determine whether a comparable improvement in reserve estimation could be attained. In this instance the structural co-ordinates of samples were assigned using a reference plane represented by a discontinuity model. The results of this study are discussed in Section 10.4.

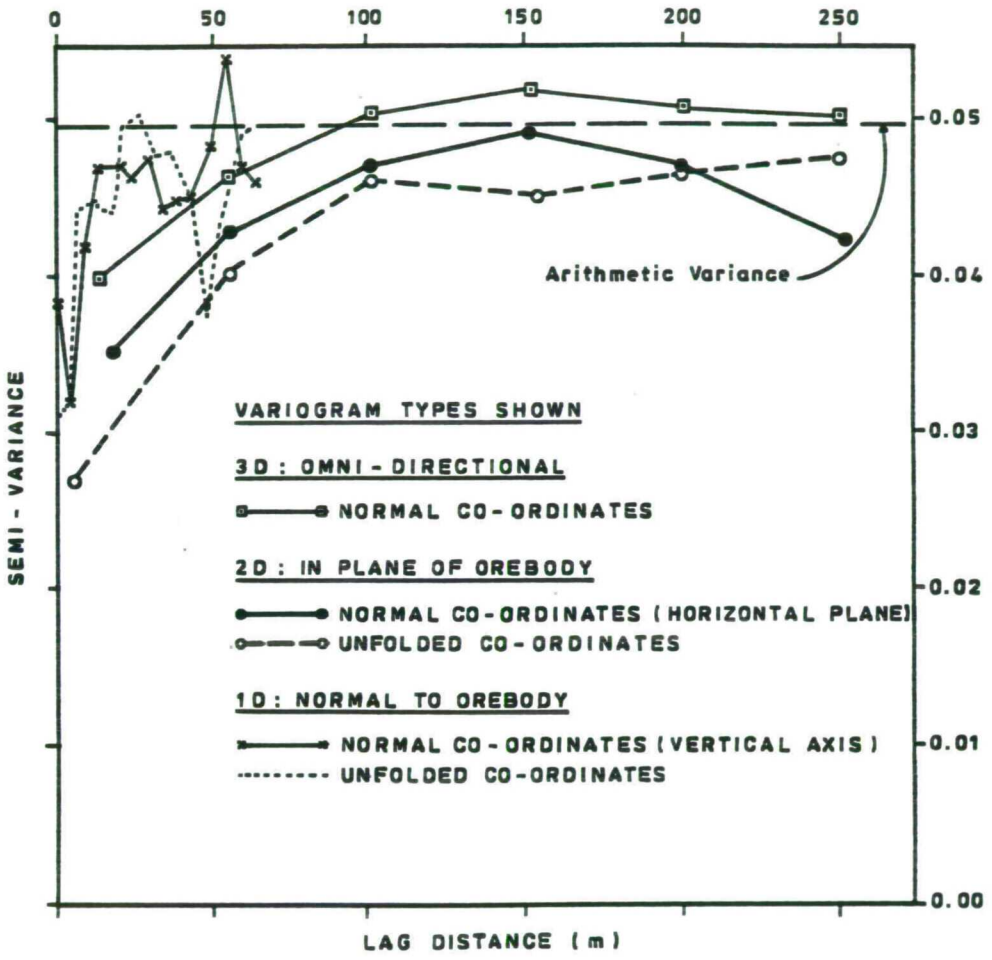


Figure 8.4 Comparison of experimental copper semi-variograms from the Bama-Brandelos deposits, with and without the use of structural co-ordinates (modified after Sides 1987)

8.5 Discussion

The examples presented above have illustrated some of the ways in which discontinuity models were used to apply geological controls to the estimation of internal properties on the deposits studied. Whilst some of these approaches were applied using conventional volumetric models, derived from the discontinuity models, others take advantage of the properties of the discontinuity model structure.

The main limitation to the use of discontinuity models for zonal control is the inability to do point-in-volume tests. Continued development work (described in Section 7.6), is being directed at overcoming this limitation. Successful completion of this work would allow sample codes to be checked against geological models which are more precise than those represented in conventional volumetric models. This would facilitate correct assignment of codes in the presence of sharp contacts such as those illustrated in Plate 8.2.

To date, the stratigraphic control discussed in Section 8.3 has only been applied by assigning a stratigraphic Z-co-ordinate relative to the intersection of a vertical ray with the discontinuities bounding the different seams in a deposit. Using rotated co-ordinate systems this approach could also be applied to sub-vertical tabular vein type deposits. Greater control could also be obtained by estimating the local orientation, of the seam or vein being studied, from the orientation of the closest triangles in the bounding discontinuities. The ability to derive average orientations within a defined search prism, weighted by triangle area, could be applied in such cases.

The application of structural controls, as discussed in Section 8.4, could be enhanced in order to take into account the style of deformation present in a deposit. The method used to calculate the structural Z-cordinate, illustrated in Figure 8.3, uses a normal projection onto the folded reference plane. This would tend to assume a **parallel fold** style (see Ramsay and Huber (1987) for definition of fold types) formed mainly by bending of the component layers. If the deformation was related more to cleavage parallel shear, resulting in a **similar fold** style, it would be more appropriate to calculate the stratigraphic Z co-ordinate by projection parallel to the fold axes. In such cases discontinuity models of the fold axial planes could be used in conjunction with folded reference planes to assign structural co-ordinates.

The previous Chapter showed how geological discontinuity models could be used in the generation of volumetric models of the geological units present in a deposit. The way in which such models are used to provide a zonal control on the estimation of internal properties has been illustrated above. The other examples presented in this Chapter have shown how the ability to distinguish individual discontinuities within a deposit, allows more specific geological controls to be applied during the estimation of internal properties. Such controls can take advantage of geological knowledge of the mineralising processes involved, as well as post-mineralisation processes such as folding and faulting, which have resulted in the current patterns of internal variation.

In addition to the procedures described in this and preceeding Chapters, an important aspect of orebody modelling is the identification and quantification of errors associated with the modelling and estimation process. This aspect is discussed in the next two Chapters.

9. ERRORS IN DISCONTINUITY MODELLING

9.1 General

Inevitably, errors are introduced in any modelling process; particularly so in orebody modelling where only a very small proportion of the volume being investigated is sampled. General aspects of errors associated with reserve estimation are discussed by King et al (1982), and Sides (1992b). Both papers stress the need to identify, quantify and control the errors that arise.

A correct understanding of the terms error, accuracy, and precision, is essential in order to follow the arguments presented in this Chapter. If in doubt, readers should refer to the definitions given in the Glossary presented in Appendix II.

The main types of error that arise in orebody modelling and reserve estimation are summarised in Table 9.1. It is important to realise that errors can be introduced at several stages in the sampling and modelling process, and that many such errors are unavoidable.

The study of errors associated with sampling for reserve estimation purposes led to the formulation of a comprehensive theoretical background for controlling the errors associated with sampling preparation procedures (Gy, 1989). Likewise the development of geostatistics as an estimation technique (Matheron (1963, 1965, 1971), Krige (1966), Serra (1967)) provided a theoretical basis for predicting the errors associated with the estimation of regionalised variables such as reserve grades (eg estimation variance). Work done in these two areas provides a firm basis for studying the inherent uncertainties associated with sampling, modelling and estimating the internal properties of mineralised bodies.

RECONCILIATION STUDIES	ACTIVITY	AVOIDABLE ERRORS (MISTAKES)	UNAVOIDABLE ERRORS (INHERENT UNCERTAINITIES)
DATA VALIDATION	SAMPLE SELECTION	Survey mistakes Driller interval errors	Survey precision Downhole survey precision
	MAPPING/LOGGING	Map location errors Recording or coding errors	Locational precision (map scale) Precision of downhole interval measurement
	SAMPLING	Sample mis-numbering Non-representative samples	Sampling precision (sample weight, size reduction, etc.)
	ANALYSIS	Sample contamination Calibration errors Sampling Handling Mixups	Sample preparation variances Precision of analytical method Differences between analytical methods
	DATABASE ENTRY	Transcription errors Mismatching of results and original sampling records	Precision of data fields Digitising precision
MODEL VALIDATION	MODELLING TECHNIQUES	Geological interpretation errors Use of inappropriate techniques	Valid alternative interpretations Maximum resolution of model (eg block size)
CHECKING OF RESULTS	RESERVE MODELS	Errors in control parameters Errors in grade estimation algorithms Errors in mine planning programs	Repeatability of estimation process Predicted estimation variance Differences between alternative models
	PRODUCTION RESULTS	Bias between estimates and production results Gross errors in the data compared	Empirical measures of predictive variances over different time periods

Table 9.1 Examples of errors that arise during reserve estimation (after Sides, 1992b)

An area which has received less attention is the measurement and prediction of errors associated with the modelling of shapes and estimation of volumes. This aspect is fundamental to the work presented here which has centred on the geometric modelling of geological discontinuities within an orebody. In this context two types of orebody model can be recognised, namely;

Probabilistic: where the values predicted are an approximation of the average value for a volume (eg block grades). Strictly speaking, such values should be expressed as a statistical distribution of values within a given volume, specified by a predicted mean and variance.

Deterministic: where the model indicates an exact value for any given point (eg in a geological block model a unique code is usually given to each block). This may still have an associated error related to the degree of confidence on which the model is built.

These two types reflect the two different types of numeric data handled, as discussed in Section 2.1. The discontinuity model structure is essentially deterministic (a point lies on either one side or the other of the discontinuity, if not on it). Most of the subsequent discussion in this Chapter is devoted to the identification, analysis and prediction of errors associated with deterministic models.

Most present-day evaluation campaigns on mineral deposits, for feasibility studies and routine production control purposes, include the use of standard procedures for measuring sampling and analytical errors. Reserve estimation work done as part of such evaluation programmes often includes the use of geostatistics to predict the magnitude of errors associated with the estimation of reserve grades. These approaches provide a means of identifying and controlling many of the errors associated with orebody modelling and reserve estimation.

In addition to controls such as those outlined above, orebody modelling systems should also include controls that help to exclude avoidable errors (ie mistakes). It is important to try to quantify the expected magnitude of unavoidable errors (ie inherent uncertainties) and to minimise them where possible within the time and cost constraints imposed by project requirements. In this context it is also important to study the sensitivity of project planning to errors in different reserve parameters and to ensure that effort is not wasted trying to reduce errors which have relatively little effect on the overall end result.

9.2 Sources of Error

Taking into account Burrough's (1986) discussion of sources of error associated with geographic information systems, one can recognise five potential sources of error in connection with the generation of discontinuity models, namely;

a) **Inaccuracies associated with original data:** these can include gross errors in topographic surveys; differences between samples collected on different dates or using different methods; biases due to sample clustering effects, or incorrect orientation of drill grids. Identification of such errors is facilitated by storing as much information as possible about data sources in the relevant databases. Such information facilitates the use of graphical displays during validation of information stored in such databases.

b) **Sampling and analytical errors:** these include the effects of the accuracy and precision of different sampling and analytical methods, as well as the possibility of gross inaccuracies due to incorrect calibration of equipment.

In relation to the recording of geological information used to interpret discontinuities, locational accuracy and precision are determined by the surveying methods and scale of mapping used. Logging and mapping of geological information is often highly subjective such that there may be significant differences in mapping or core logging data related solely to the fact that it has been recorded by different geologists.

Such errors can only be identified and controlled by carrying out periodic check studies, such as duplicate mapping and interpretation of the same area by different geologists. The application of standardised criteria for data recording, such as agreed coding conventions and descriptive terminologies, also helps to reduce such errors.

c) **Errors due to natural variation:** regardless of the precision and accuracy with which measurements are made, the presence of natural variations makes exact reproducibility of measurements impossible.

With respect to the identification and recording of discontinuities there is a clear difference between sharp clearly recognisable features (such as surface topography), and other interpretation dependent features (such as internal contacts based on metal grade zoning).

Some geostatistical techniques such as indicator kriging have been used in the analysis of natural variation associated with boundary type data (eg Journel (1982), Soares (1990)). Fractal geometry also provides a means of studying the natural variability of boundaries at different scales (Mandlebrot (1983) and Goodchild (1980)). These aspects are discussed further in Section 9.6 (see Appendix II for definitions).

d) **Errors in data capture:** unless rigorous data input and verification procedures are applied, errors can often be introduced when data are input to computerised systems. This is a crucial area, with respect to the application of such systems, that often receives relatively little attention.

Such errors can include obvious ones such as data transcription errors, errors in file transfer between different systems, or sample number mix-ups (eg associating analytical results with the wrong sample identification data). In addition less obvious effects such as hardware or software limitations often play a part in the introduction of errors during the capture of discontinuity information by digitising, or scanning, from interpreted plans or sections. Control of such errors requires the establishment of systematic data verification procedures.

e) **Computer processing errors:** errors may be introduced at the processing stage due to hardware and software limitations, inherent limitations of model structures, or errors associated with the interpretation of geological data. Burrough (1986) defines these as including "the mistakes, errors and misapprehensions that can arise as a result of carrying out certain kinds of processing", and notes that they are often the "most difficult to spot because they require an intimate knowledge of not only the data, but also of the data structures and the algorithms used".

One such error, highlighted by Burrough (op cit), is the finite limit to numeric precision on different computer systems. Although not normally of concern when using FORTRAN compilers on 32-bit workstations, such factors can cause gross errors in certain instances. The writer has personally experienced problems with algorithms used for drillhole trace calculation (using the method described by Howson and Sides (1986)), and automatic triangulation, when using large co-ordinate values (see Section 9.3).

9.3 Discontinuity Model Precision

The discontinuity modelling system described in this thesis contains several inherent limitations to modelling precision (see Appendix II for definitions). These are listed below, together with an indication of their relative importance.

Large-Small uncertainty:

- locational uncertainties associated with included mapping data points (due to precision of survey plans and sections, and the precision with which a geologist records the field location of contacts)

- precision with which the discontinuity can be identified (ie do all geologists record it at exactly the same position in a hole or outcrop)

Medium-Small uncertainty:

- locational uncertainties associated with included drillhole contact positions (ie precision of collar survey; downhole survey instruments; downhole trace calculation method; depth measurement down hole; etc.)

- precision associated with the inherent irregularity (roughness) of the discontinuity being modelled

Small uncertainty:

- scale used for mapping and interpretation

- precision of digitising or other data input method used

- precision to which the point co-ordinates are stored (this affects the determination of point-point, and point-line equality)

- numerical precision limits when processing data for section plotting, intersection, etc.

- precision associated with approximating a curved irregular surface by a set of planar triangular facets (this has opposite effects on concave and convex boundaries, tending to truncate peaks and troughs)

The cumulative effect of these limitations is that there is always a spatial location error associated with discontinuity models, as illustrated schematically in Figure 9.1. Such errors vary for different parts of a given model being at a minimum near the most precisely located points. They tend to a maximum in those parts of the model which are at a distance from known data points, or close to points whose true location is less well known.

There is no simple way of measuring such locational imprecisions, particularly since they are often anisotropic, and may be propogated from raw data through to interpreted data. For instance the distance down a drillhole is usually determined with a greater certainty than the location of the three-dimensional co-ordinates of the end of the hole. Consequently the prediction of the true location of a drillhole trace, at a given level of confidence, is defined by a conical volume, as illustrated in Figure 9.2. Any interpretative data generated subsequently would include such locational uncertainties, plus additional ones associated with the generation and entry of the geological interpretation.

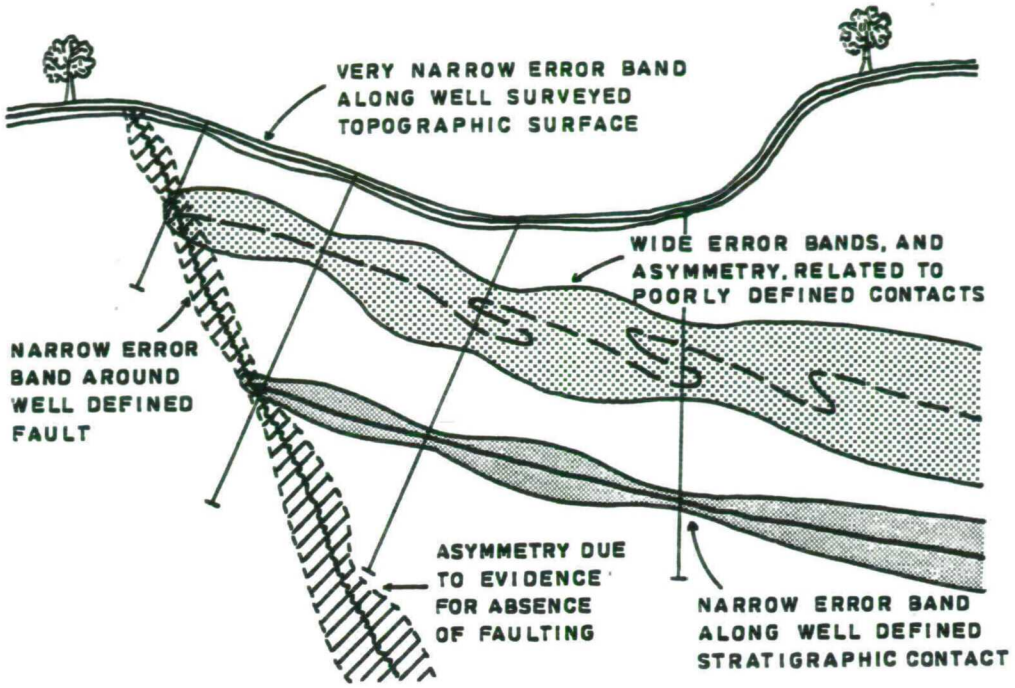


Figure 9.1 Schematic cross-section showing locational error bands associated with different discontinuities in a hypothetical deposit

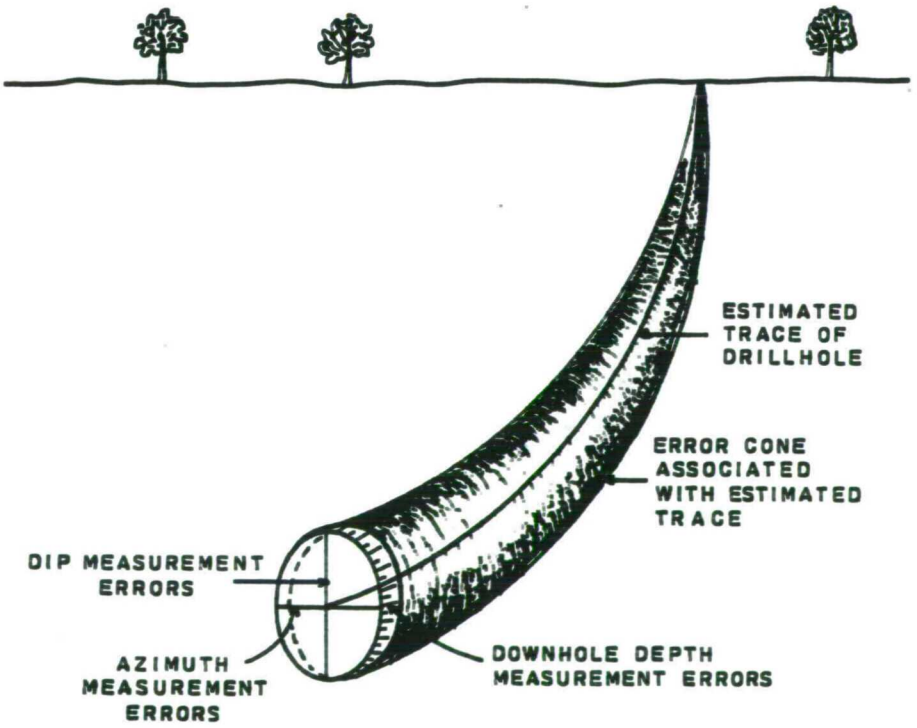


Figure 9.2 Locational error cone associated with the trace of an inclined borehole

With regard to the quantification of the precision errors described above, procedures already exist for quantifying survey instrument errors. In addition some attention has been paid to errors associated with the input of mapped, or interpreted, boundaries in the GIS literature (eg Burrough (1986)). Other aspects, such as the uncertainties associated with geological data recording and interpretation, are more difficult to quantify. These could, however, be measured by carrying out duplicate mapping and interpretation exercises and quantifying the differences between the resultant models.

Another important aspect of the inherent precision limits of discontinuity models is associated with determining whether different points, lines, or areas, coincide. In the computer programs established as part of this work these aspects were controlled by a minimum tolerance distance. Points less than this distance apart (5mm in this instance) cannot be discriminated in the models used. In developing programs to handle discontinuity modelling, a coherent set of rules had to be established for the determination of point-point, point-line, line-line, line-area, and area-area, identity. Special care had to be taken to follow such rules in order to avoid problems when special cases are encountered.

Processing limitations, associated with the magnitude of the co-ordinate values used, were noted in two instances. Firstly associated with the calculation of downhole survey locations, and secondly in the automatic triangulation of points. In both cases the use of UTM co-ordinates with up to 10 significant figures, including 2 decimal places, caused some of the calculation routines to become unstable. These problems were eliminated by using smaller maximum co-ordinate values, in the first instance by adding 250000 to reduce the absolute magnitude of the maximum value from -235000 to 15000, and in the second case by dividing by 1000. Time did not permit a detailed analysis of these problems, however they serve to highlight some of the limitations inherent in the system.

As yet relatively little work has been done on the quantification of the errors associated with the prediction of discontinuity locations. Approaches adopted for measuring such errors are discussed in the next Section. It is essential that such methods be established in order to allow the quantification, and control, of the errors associated with discontinuity modelling.

9.4 Measuring Discontinuity Errors

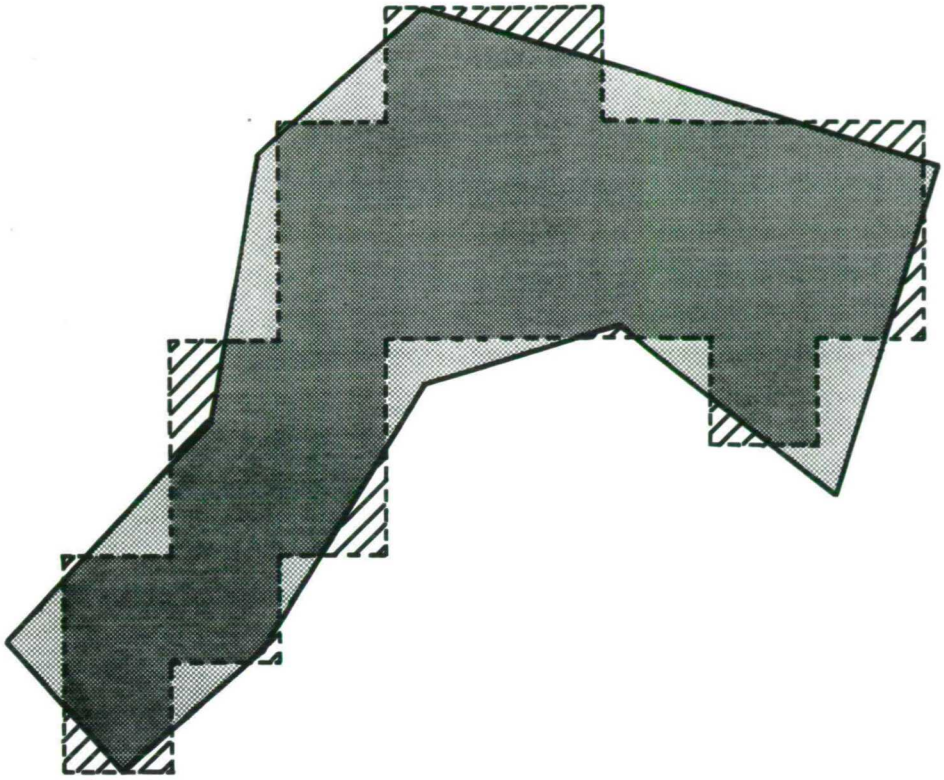
There is little point in making predictions which cannot be tested by subsequent studies. Consequently, the empirical measurement of errors associated with discontinuity models is an essential precursor to any attempt at their prediction. Little has been published about the measurement of errors associated with modelling geological discontinuities during the application of computerised reserve estimation and mine planning systems; a rare exception being a recent paper by Houlding (1991a). In contrast a considerable amount of work has addressed this problem in the GIS field, as summarised in Burrough (1986).

Houlding (1991a) describes two types of error which can be measured in the study of the approximation of irregular polygons by rectangular blocks, namely;

a) **Boundary error:** Half the total misclassified area (or volume) as a proportion of the total true area (or volume). This can be regarded as the precision of the area (or volume) estimate.

b) **Volume error:** the difference in the two area (or volume) estimates as a proportion of the true total area (or volume). This can be regarded as a measure of the accuracy of the area (or volume) estimate.

These two types of error are illustrated schematically in Figure 9.3.



ORIGINAL POLYGON

POLYGON AS REPRESENTED BY A RECTANGULAR APPROXIMATION

A: AREA INSIDE BOTH POLYGONS

B: INCLUDED AREAS WHICH SHOULD BE EXCLUDED

C: EXCLUDED AREAS WHICH SHOULD BE INCLUDED

TRUE POLYGON AREA: P = A - B + C

AREA OF RECTANGULAR APPROXIMATION: R = A + B

VOLUME ERROR: VE = ((B - C) /P) * 100.0

BOUNDARY ERROR: BE = (((B + C) * 0.5) /P) * 100.0

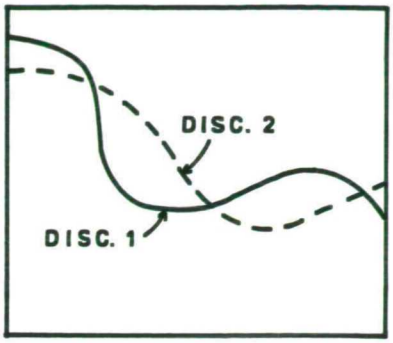
Figure 9.3 Schematic diagram illustrating the calculation of boundary and volume error (modified after Houlding, 1991a)

Despite being essential to the assignment of confidence to area or volume estimates; neither of these measures provides a means of measuring the locational uncertainty associated with different discontinuities in a deposit. For instance the ore envelope shown in Figure 9.3 may be defined by different types of discontinuity, such as surface topography, a major fault, a sharp stratigraphic contact, etc., each of which would have different associated locational uncertainties.

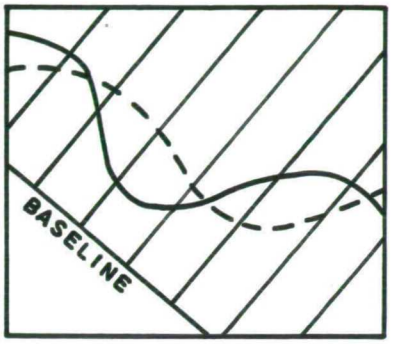
For this reason a method is proposed for measuring the locational error associated with the prediction of discontinuities, on a linear or areal basis. This procedure is based on the accumulation of the linear differences, between two discontinuities, along a set of equally spaced lines (see Figure 9.4). The lines along which differences are measured can be arranged to lie in a single plane (linear comparison), or to form a regular grid (areal comparison). Statistics of the differences, along such a set of lines, allow the average difference between the two discontinuities to be derived, as well as the calculation of variances which allow confidence intervals to be defined.

The differences measured may be dependent on the exact location of the test lines, and statistical studies should be based on measurements obtained from a series of similar grids the origins of which are selected at random. Geometrical anisotropy of locational errors would be expected, and can be verified by comparing error statistics obtained from sample line sets oriented in different planes.

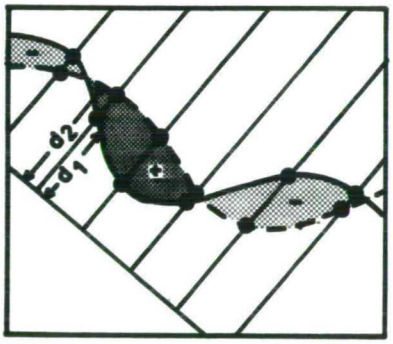
It should be noted that the accuracy of a particular discontinuity model is a function of the resolution at which it is measured; a concept which forms the basis of fractal geometry. In the context of reserve estimation the resolution of interest will depend on the scale, and selectivity, of the mining method used.



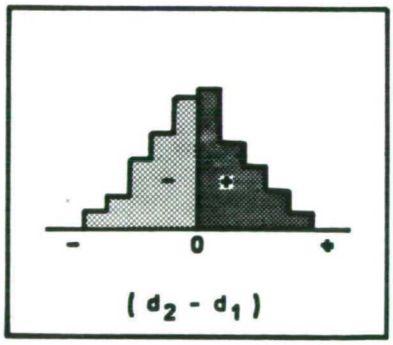
A. SELECT THE DISCONTINUITIES TO BE COMPARED



B. SELECT TEST LINE ORIENTATION AND SPACING, AND ESTABLISH A SET OF TEST LINES WHICH INTERSECT BOTH DISCONTINUITIES



C. CALCULATE CO-ORDINATES OF THE PAIRS OF INTERSECTION POINTS WHICH THE TEST LINES MAKE WITH THE DISCONTINUITIES, AND OUTPUT TO FILE



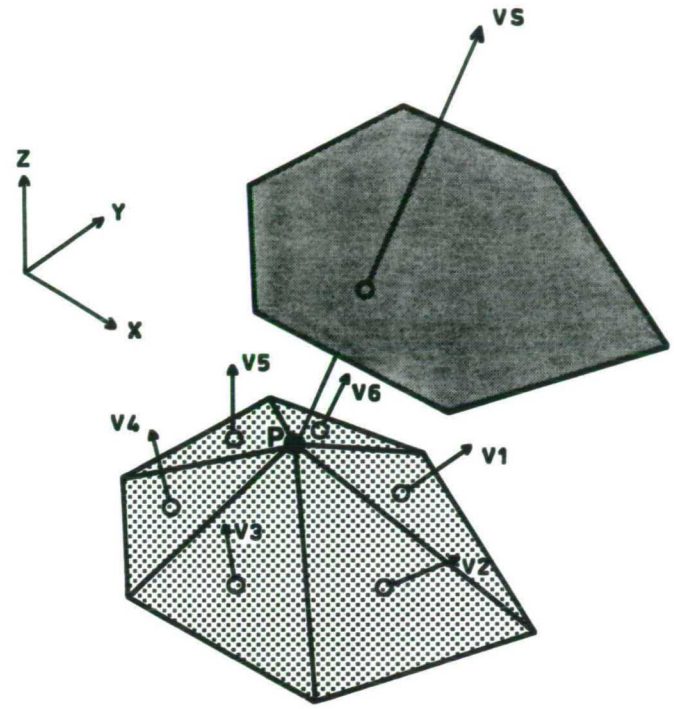
D. PERFORM STATISTICAL ANALYSIS OF DIFFERENCES BETWEEN THE DISCONTINUITIES, AS MEASURED ALONG THE TEST LINES

Figure 9.4 Schematic diagram illustrating the method adopted for measuring locational error of discontinuities

The precision with which the location of a given discontinuity can be predicted will depend not only on the sampling density used, but also on the inherent shape variability. An important aspect in this regard is the ability to apply measures of "surface roughness". In the context of the discontinuity model structure, the roughness index proposed by Philip and Watson (1986) has considerable potential. Their index is based on the use of triangles formed between each data point and its natural neighbours, as illustrated in Figure 9.5. A "best-fit plane" based on the cross product of the vector normals to each triangle, weighted by triangle area, is then calculated. The roughness index is defined such that it is 0.0 for a perfectly planar surface, and approaches 1.0 as variation becomes infinitely large.

Measures such as the roughness index provide a means of assigning relative uncertainties to points at different locations on a discontinuity. Locational errors would be expected to be higher where discontinuities are rougher (ie more irregular), and consequently the roughness index could provide a relative measure of locational error.

Errors inherent in the data points used to define a discontinuity must also be incorporated when predicting the locational error. The concept of "fuzzy" data points was introduced by Mallett (1988, 1989) to take account of differences in the quality of the data points used. For instance, a drillhole or mapped point is generally regarded as much less fuzzy than one interpreted from a geophysical survey. Since such errors arise during data collection and interpretation, they are best quantified by means of duplicate sampling and interpretation studies, as discussed in Section 9.3. The results of such studies could provide a more rigorous basis for quantifying the "fuzziness" of points used in discontinuity modelling. Such measures have been applied as a constraint during the modelling of discontinuities by Mallett (1989) using a technique termed Discrete Smooth Interpolation.



ROUGHNESS INDEX: R = 1.0 - (A2/A1)

where

A1 = Sum of the areas of the triangles making up the convex hull surrounding the data point (P)

and

A2 = Area of convex hull projected onto the plane normal to the vector sum (VS) of the triangle cross products (V1, V2, etc.)

NOTE: To facilitate comparisons, data values are standardised so as to lie within a unit cube (ie. values of Z, and either X or Y, occupy the full range 0.0 to 1.0)

Figure 9.5 Determination of the roughness index associated with a point on a triangulated surface (modified after Philip & Watson, 1986)

The discussion given in this Section has established a basis for the measurement of locational uncertainties associated with discontinuity models, as well as the accuracy and precision of area and volume estimates. The next Section investigates ways in which the magnitude of such errors can be estimated.

9.5 Estimating Discontinuity Errors

As discussed in the previous Section, it is important to have a clear idea of exactly which sort of error one is dealing with before trying to predict its magnitude. The data collection and estimation methods used are normally selected so as to minimise the liklihood of inaccuracies, since these usually reflect the application of incorrect methods or erroneous assumptions. Consequently, in the context of discontinuity modelling, one is mainly interested in trying to predict locational error bands associated with each of the discontinuities modelled, as illustrated in Figure 9.1. Such error bands should then enable the determination of the errors associated with area, or volume, estimation.

One approach to this problem is through the use of geostatistics (eg David (1977, 1988), Dowd (1992), Isaaks and Srivastava (1989), Journel (1982), Royle(1979a), Soares (1989)). Consider, for instance, the estimation of a topographic surface based on a set of spot heights. Geostatistical approaches could be applied to estimate the locational errors in two ways, namely;

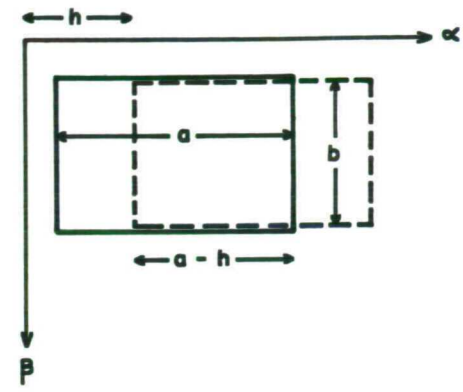
- treating the projection distance from the discontinuity onto a reference plane as the spatial variable of interest (eg elevation)

- using an indicator function (Journel (1982)) defined by which side of the discontinuity each sample or block lies (eg 0=above, in air; 1=below, in rock)

The first approach would enable the determination of locational uncertainties, for estimated points on the surface, in terms of the estimation variance associated with their estimated elevation. In the second instance one would be able to derive the probability that a given point or reserve block was correctly classified as rock or air.

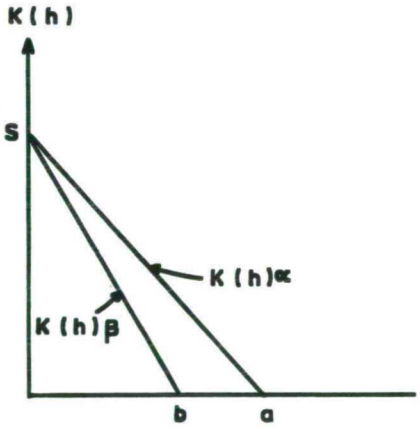
In the context of determining errors associated with the estimation of areas, the application of the geometric co-variogram (termed the transitive co-variogram by some workers), has also been used (Royle (1979a)). This method, illustrated in Figure 9.6, relies on taking a given shape and moving it different distances from its true location. An analysis of spatial correlation is then carried out by measuring the area of overlap of the two shapes at different lags. Such an analysis should obviously take into account inherent anisotropies in the shape being studied. Royle (1979a) illustrates how this approach can be used to measure the uncertainty associated with the estimation of irregular areas through rectangular approximation.

An alternative set of procedures has been established by work done in the field of cartography and GIS, as discussed in detail by Burrough (1986). One group of methods is based on the use of measures of the complexity of rasterised maps to determine the errors associated with their derivation from vector maps. The theory behind this approach is based on the fact that such errors will be proportional to the number of cells crossed by the vector boundaries. Estimation of these errors thus depends on calculation of the number of cells containing different materials on either side. Burrough (1986) summarises the work done by various authors in this regard, in particular Frolov and Maling (1969), Switzer (1975) and Goodchild (1980).



Geometric co-variance K(h), for a given shape, is determined from the area of overlap at different displacement distances.

Displacing a rectangle (top left) in directions α and β results in the geometric variograms shown (bottom left). These are defined as follows:



$$K(h)_{\alpha} = b*(a-h) = S - b|h|$$

$$K(h)_{\beta} = a*(b-h) = S - a|h|$$

where S = rectangle area

Irregular shapes will give non-linear geometric co-variograms.

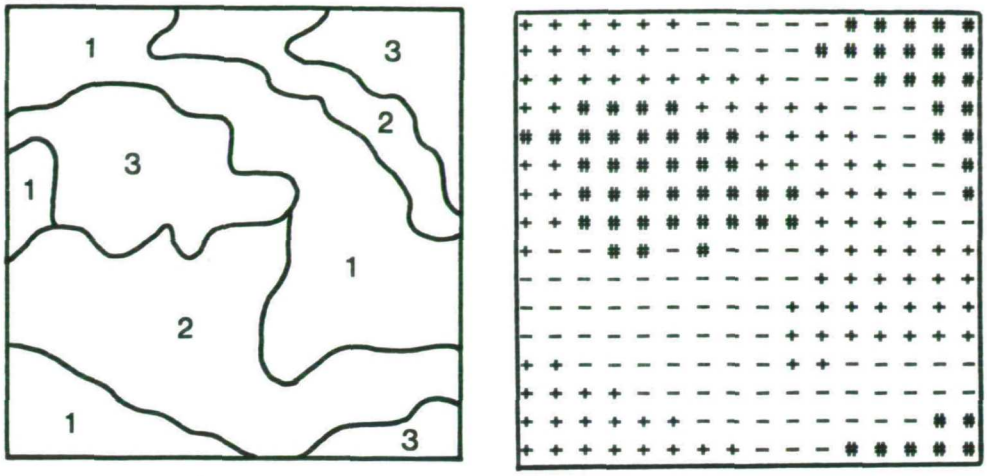
Figure 9.6 Example of the calculation of geometric co-variograms for a shape which shows geometrical anisotropy (modified after Royle, 1979a)

Because of the obvious applicability of this approach to the prediction of errors associated with rectangular approximation in regular blocks (as defined by Houlding, 1991a) an example of the calculation method is given in Figure 9.7 and Table 9.2. Use of this approach could be used a guide to the selection of the optimum block size to apply when using regular block models for geological modelling. Where such models are derived from 3-d discontinuity models, these estimates of uncertainty could be calculated in preliminary tests without actually needing to generate a large number of block models.

The method illustrated above may give misleading results where the grid cell size used is comparable to the size of the polygons in the original vector maps. This problem is highlighted by the results presented in Section 10.2.

An important limitation to this approach is that it assumes that the rectangular representation has been derived from a "true" vector representation of the area, or volume, modelled. Due to locational uncertainties associated with the data and interpretations used (see Sections 9.2 and 9.3), this is obviously untrue. Consequently, additional methods must be used to predict the locational uncertainties associated with the individual discontinuities which make up a vector model.

Burrough (1986) summarises some approaches to this problem, based on the work of Perkal (1966), Blakemore (1984) and MacDougal (1975). These approaches follow the same principles outlined at the start of this Section, namely of trying to define an error band associated with each discontinuity, and using these to predict the proportion of points that might be misclassified during point-in-polygon tests.



A. Original map based on polygons bounded by irregular discontinuities

B. Rasterised equivalent (16 by 16 square grid)

PROCEDURE USED TO CALCULATE PRECISION OF RASTERISED MAP:

1. Estimate total number of cell pairs within the map area at separation d=1 cell width;
   NPAIRS = 4*NCOL*NROW - 2*d*(NCOL+NROW)

2. For each pair of cells at separation d=1 (along rows and up columns) count the number of pairs that lie in different geological units i and j ($TALLY_{ij}$)

3. Compute probabilities for each pair of geological units: P_{ij} $(n^{-1/2})$ = $TALLY_{ij}$ / NPAIRS

4. Repeat steps 1 to 3 for d=2 to estimate P_{ij} $(2n^{-1/2})$

5. Calculate mismatch between units i and j:
   $L_{ij} = 0.60*P_{ij}(n^{-1/2}) - 0.11*P_{ij}(2n^{-1/2})$

6. Calculate total mismatch for each unit:
   L_i = sum of L_{ij}s for j=1,n (Note: $L_{ij} = L_{ji}$)

7. Calculate total mismatch: L = sum of L_is for i = 1,n

Figure 9.7 Calculation of misclassification errors associated with the rasterisation (rectangular approximation) of vector maps (modified after Burrough, 1986)

Probability estimates:

Polygon pairs	16 by 16 grid (as shown in Figure 9.7)		32 by 32 grid (not illustrated)	
	P_{ij} $(n^{-1/d})$	P_{ij} $(2n^{-1/d})$	P_{ij} $(n^{-1/d})$	P_{ij} $(2n^{-1/d})$
1 2	0.05104	0.09821	0.02545	0.05130
2 3	0.02188	0.04464	0.01184	0.02422
3 2	0.03438	0.05804	0.01689	0.03333

Mismatch per polygon pair:

L12 0.020	L21 0.020	L31 0.008	L12 0.010	L21 0.010	L31 0.004
L13 0.008	L23 0.014	L32 0.014	L13 0.004	L23 0.006	L32 0.006

Mismatch per polygon:

L1 0.028	L2 0.034	L3 0.022	L1 0.014	L2 0.016	L3 0.011

Total mismatch:

8.46%	4.11%

Table 9.2 Calculation of precision associated with the rasterisation of polygonal map outlines for the example illustrated in Figure 9.7 (after Burrough, 1986)

9.6 Discussion

The discontinuity models described in this thesis are all representations of some unknown "reality" derived from a relatively small sample of measurements of the features being studied. Uncertainties will always be introduced during the modelling process, due to the limitations of the sampling method, the inherent variability of the features under study, and the limitations of the modelling method used. It is therefore crucial that methods for quantifying the errors associated with individual models should form part of any discontinuity modelling system. Such measures provide an empirical basis for the analysis of such errors, without which a theoretical basis for their prediction could not be established.

These two aspects, namely; **Error measurement**, and **Error prediction**, are essential in order to control the accuracy and precision of the models generated. Where avoidable errors (mistakes) are detected (see examples in Table 9.1), remedial action should be taken to avoid their recurrence. Errors inherent in the sampling or modelling procedures used have to be taken into account when selecting appropriate methods to apply during discontinuity modelling. Such errors need to be monitored in order to keep them within acceptable limits. Examples of both types of error have been given in Sections 9.2 and 9.3.

As discussed in Section 9.4, there are as yet no widely accepted procedures for the measurement of errors associated with discontinuity modelling in the context of ore reserve estimation. Previous work has mainly concentrated on calculating the errors associated with area or volume estimates (Houlding, 1991a), rather than trying to determine the locational certainty with which given discontinuities can be predicted. The method of quantifying discontinuity locational errors proposed in Section 9.4 may allow a more rigorous approach to this problem in future studies.

Section 9.5 has illustrated some methods by which the errors associated with discontinuity models can be predicted, although time has not permitted a detailed comparison of the different approaches available. One aspect that does not appear to have been taken into account in any previous work is the fact that the discontinuities bounding a given area, or volume, may have totally different characteristics. For instance, the outline of a stratiform orebody may be defined by the intersection of surface topography, sharp fault contacts, a sharp footwall contact defined by lithological differences, and a gradational hangingwall contact defined by sample assays. Such discontinuities should be treated separately when predicting the errors associated with the estimation of the orebody outline. In this regard it is considered that the methods used in the cartographic and GIS fields may be more applicable than geostatistical approaches.

An interesting overview of some of the approaches used in geographical applications is given by Goodchild (1980). He points out that methods used by different workers to predict cartographic errors can be linked through the concept of fractal dimensionality. His main conclusion is that the errors associated with area estimation, and point-in-polygon classification (by means of grid representation), are related to the number of grid cells cut by the discontinuities bounding the area under investigation. This in turn depends on the fractal dimensionality of the surface being studied. In the context of discontinuity modelling for reserve estimation this may not be critical, given the range of scales of interest (eg 1:250 to 1:5000). Nevertheless, it suggests that investigation of the fractal dimensionality of the different discontinuities used to define orebody limits merits further study. Parallel studies of the application of the roughness index concept, proposed by Philip and Watson (1986), would also be worthwhile.

In summary, this Chapter has outlined some general principles for the recognition, measurement, analysis and prediction of errors associated with discontinuity modelling. This has highlighted the applicability of techniques developed in the cartographic and geographic fields to the analysis of discontinuity model errors. There may also be potential for using techniques from other fields such as image analysis, photogrammetry, pattern recognition, and robotics in this regard. A comprehensive study of such aspects falls outside the scope of this thesis, however the review presented above provides a framework within which the results of the reconciliation studies presented in Chapter 10 can be interpreted.

10. RECONCILIATION STUDIES

10.1 General

Reconciliation studies can be used to compare different sets of information in order to determine their degree of correspondence, and to analyse differences between them (ie errors) in terms of accuracy and precision. Such studies form an essential part of computerised reserve modelling systems, as discussed by Sides (1992b). In this context, they are used to validate the data handling procedures and modelling techniques used, as well as providing a means of checking the quality of the resultant models.

This Chapter illustrates examples of some of the approaches which can be adopted in reconciliation studies aimed at determining model accuracy and precision. The examples given illustrate different features of the application of geological discontinuity modelling in the context of mineral evaluation. Three particular aspects are highlighted, namely;

- comparisons of volumes estimated using different model structures (with examples from the Bama-Brandelos and Lombador deposits)

- the measurement of locational errors associated with discontinuity models (with an example from the Neves-Corvo deposit)

- comparison of block grades estimated using different geological controls (with an example from the Cerro Colorado deposit)

Due to the lack of accepted standard procedures for carrying out reconciliation studies of this type, the examples given are partly exploratory in nature. The use of some of the techniques discussed in Chapter 9 is illustrated.

10.2 Comparison of different model structures

To illustrate the effects of using different types of model structure, for the modelling of geological volumes, comparisons of the use of regular block models and gridded seam models on stratabound deposits are presented. The deposits selected for this study were the Lombador polymetallic massive sulphide lens at Neves-Corvo (see Section 3.4), and the mineralised amphibolite unit in the Brandelos deposits (see Section 3.3). Both deposits consist of fairly gently dipping tabular mineralised units, where it was considered that there might be significant differences between the use of different model structures for representing geological volumes.

In both cases three dimensional geological discontinuity models were established for the base and top of the mineralised sequences, using the modelling technique described in Chapter 5. A regular grid was then plotted on a horizontal plan and overlaid on these discontinuity models. Elevations for the top and base of the mineralised units, at each grid point, were then derived using the technique discussed in Section 7.2. These data provided the reference points for the gridded seam model, the difference between the two elevations being treated as the most reliable measure of the true thickness of the unit being modelled. These elevations were then used to estimate the volumes corresponding to regular block models, with different block heights and start elevations, based on the same horizontal grid.

Results for the two model types were output to a series of ASCII files containing the gridded seam estimates, and corresponding regular block model estimates, for the same set of base and top elevations and thicknesses. Differences between the two sets of results were then studied in order to try to quantify differences in precision between the two approaches. The total volumes obtained using the different approaches were also compared in order to determine differences in the overall accuracy of the two modelling methods.

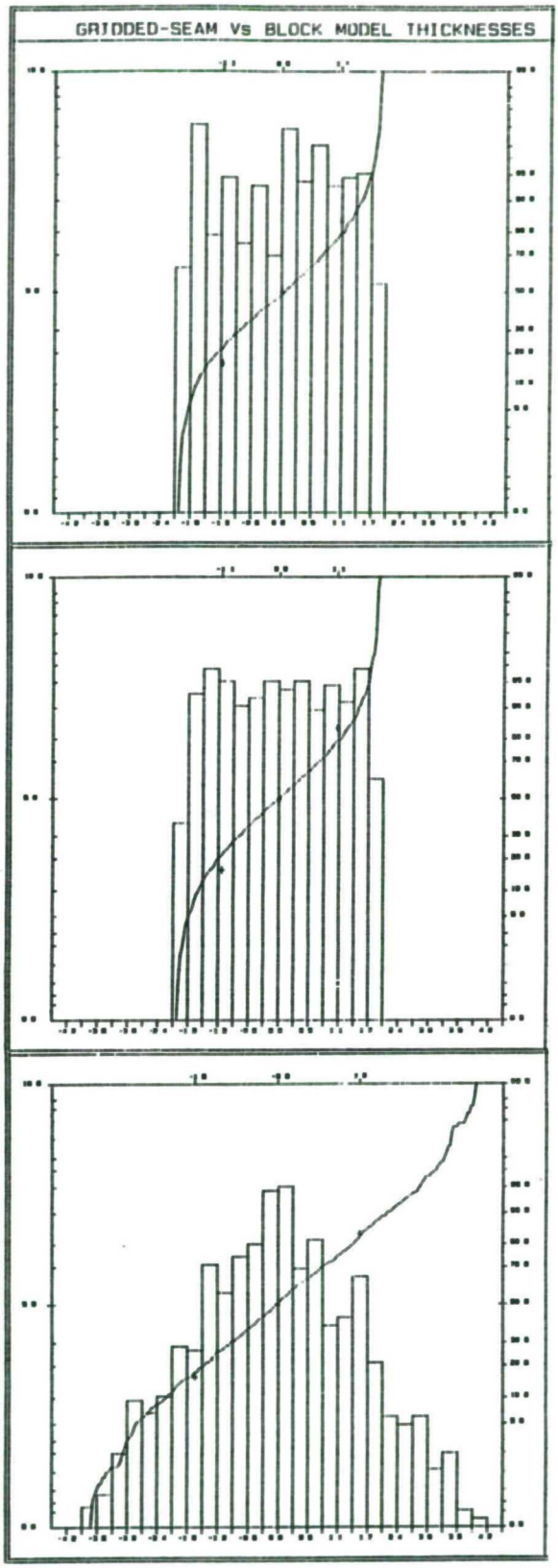
Figures 10.1 and 10.2 illustrate the results obtained for the Lombador deposit, using block heights of 4m and 24m respectively. The graphs presented in these Figures show the distribution of differences between estimated base, top and thickness, for each grid cell in the model. If the gridded seam model is assumed to be inherently a more accurate representation (although this is not necessarily the case), then these differences can be regarded as distributions of errors for individual cells on a fixed grid spacing.

The distributions of errors for the estimated base and top elevations show a similar box-shaped form (ie rectangular distributions) for both examples. This reflects the fact that such errors are independent of the elevation value being estimated. The limits of the distribution are equal to half the block height under consideration. In all cases the distribution of errors for the base and top are centred on a mean of zero, which also corresponds to the median of the distributions. This indicates that there is no bias between the different methods on a block by block basis. The variance of the distributions is higher in both cases for the larger block height; 1.3 versus 47.3 for the base, and 1.4 versus 45.6 for the top, as would be expected.

Difference_top

No. of values = 1080

Mean = 0.013
Variance = 1.38
Std. deviation = 1.18



Difference_base

Mean = 0.017
Variance = 1.33
Std. deviation = 1.15

Difference_thickness

Mean = -0.0045
Variance = 2.71
Std. deviation = 1.65

Figure 10.1 Statistical distribution of differences between a gridded seam model and a regular block model (4m block height) for the Lombador polymetallic massive sulphide deposit

Difference_top

No. of values = 1080

Mean = 0.057
Variance = 45.6
Std. deviation = 6.75

Difference_base

Mean = -0.068
Variance = 47.3
Std. deviation = 6.88

Difference_thickness

Mean = 0.125
Variance = 92.3
Std. deviation = 9.61

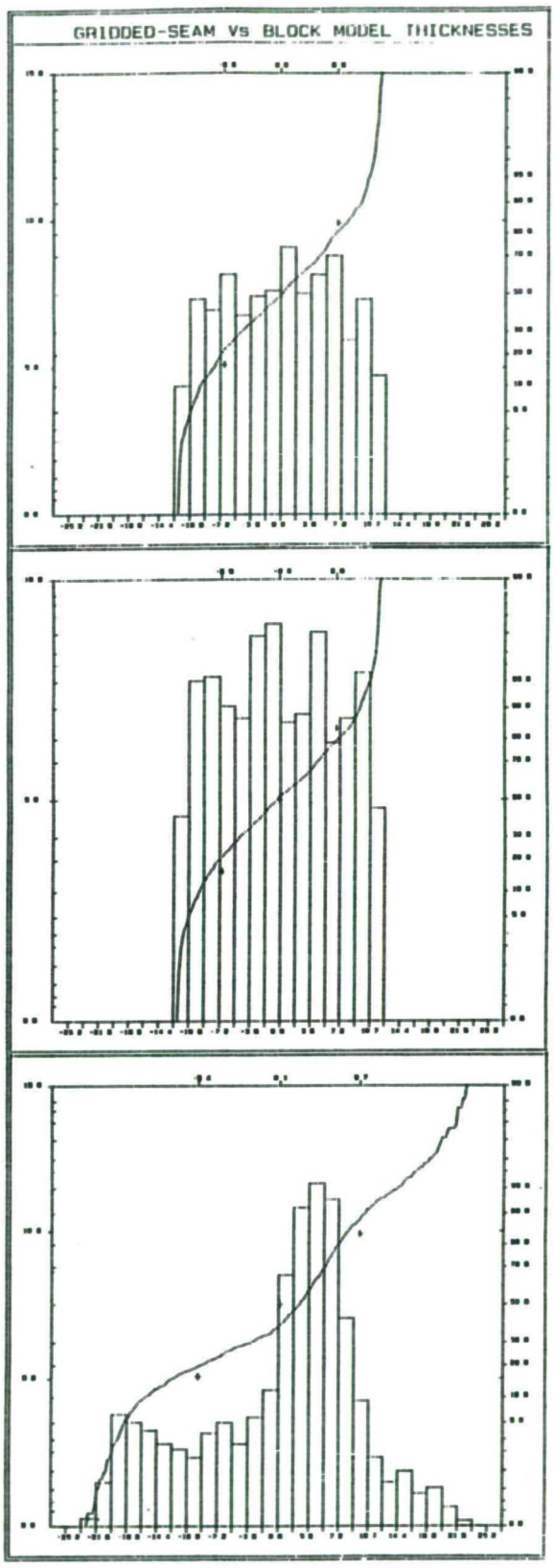


Figure 10.2 Statistical distribution of differences between a gridded seam model and a regular block model (24m block height) for the Lombador polymetallic massive sulphide deposit

The third graph in Figures 10.1 and 10.2 illustrates the effect of changes in block height on the thickness estimation errors. Although both these distributions are still centred on a mean with a very small value (0.004m and 0.125m) the shape of the distributions are quite different. For the 24m block height this results in a median difference which is significantly different from the mean (about 2.5m as against 0.125m). This example highlights the fact that the combination of apparently regular error distributions will not necessarily result in a similarly shaped error distribution for the combined value. It also suggests that the use of different block heights may result in significant differences in the overall volume estimated, despite the fact that the mean block difference is very small in both cases.

The potential introduction of biases in the estimation of total orebody volume, using different block heights, was followed up by a study done on the Brandelos deposit. In this case the volume of ground between the surface topography and the top of the unmineralised schists in the orebody footwall was modelled within a limit of interpretation corresponding to the Brandelos openpit (see Section 3.3). A grid size of 20m by 20m, corresponding to the block size used for reserve modelling when the mine was operational, was applied. The effects of changing block height, and start elevation (in a regular block model), on the overall volume estimate were then studied.

The results obtained are summarised in Table 10.1. In all cases the vertical range of blocks in the regular block model included the entire deposit, so that all volume estimates correspond to an identical volume of ground. The results presented show that in general the total volumes estimated using the regular block model are within 1% (relative) of the gridded-seam volume. Nevertheless, some very significant differences occur when the block height is increased beyond a certain limit (eg -11.6% for block height of 50m).

Block height (m)	Start elevation (m)	Total thickness (m)	Absolute difference (m)	Relative difference (%)
5	0.0	11475	-2.5	-0.0%
10	0.0	11550	72.5	+0.6%
15	0.0	11520	42.5	+0.4%
20	0.0	11540	62.5	+0.5%
30	0.0	11370	-107.5	-0.9%
40	0.0	11400	-77.5	-0.7%
50	0.0	10150	-1327.5	-11.6%
60	0.0	12240	762.5	+6.6%
40	0.0	11400	-77.5	-0.7%
40	5.0	11640	162.5	+1.4%
40	10.0	11640	162.5	+1.4%
40	15.0	11520	42.5	+0.4%
40	20.0	11480	2.5	+0.0%
40	25.0	11680	202.5	+1.8%
40	30.0	11440	-37.5	-0.3%
40	35.0	11360	-117.5	-1.0%

Note:
Total accumulated gridded seam thickness = 11477.5 m
Absolute difference = Total block thickness - 11477.5
Relative difference = (Difference * 100.0) /11477.5

Table 10.1 Effects of changing block height and start elevation on overall volume estimates (mineralised unit plus overburden) at the Brandelos deposit.

Likewise the relative errors vary if the start elevation of the regular block model is changed (ranging from -1.0% to +1.8% in the case of a 40m block height). Such effects will be most pronounced where the block height is comparable to the thickness of the geological unit being modelled.

In order to try to quantify the modelling precision of different models, another type of graphical presentation of the results discussed above was prepared. This is illustrated in Figure 10.3, where the thicknesses obtained using 10m and 40m block heights are compared against the gridded seam thicknesses using a Thompson-Howarth type precision plot (Thompson & Howarth (1973)). The use of this type of plot for graphically estimating precision associated with duplicate sample results has been discussed by Sides (1992b).

The results presented in the upper graph in Figure 10.3 imply that the precision (for modelling overall thickness) using a 10m block height model is about 28% at a 95% level of confidence, or 20% at a 90% confidence level. This suggests that one could be fairly certain that estimates of overall thickness (derived from a regular block model) will be within 20% of their true value (based on the gridded seam model) when a 10m block height is used. The lower graph in Figure 10.3 shows that for a 40m block height there is a total loss of precision with more than 10% of the estimated values (regular block model) being 100% different from the "true" value (gridded seam model). Most of the poor precision values are for mean thicknesses of less than half the block height (implying that the thickness derived from the regular block model is zero). This reflects the fact that regular blocks cannot accurately represent thin features. The tuning fork shaped pattern, centred on a thickness value of 40m, reflects a close agreement between the two sets of models where the seam thickness more or less corresponds to the block height. The errors gradually increase as the seam thickness diverges upwards and downwards from this value.

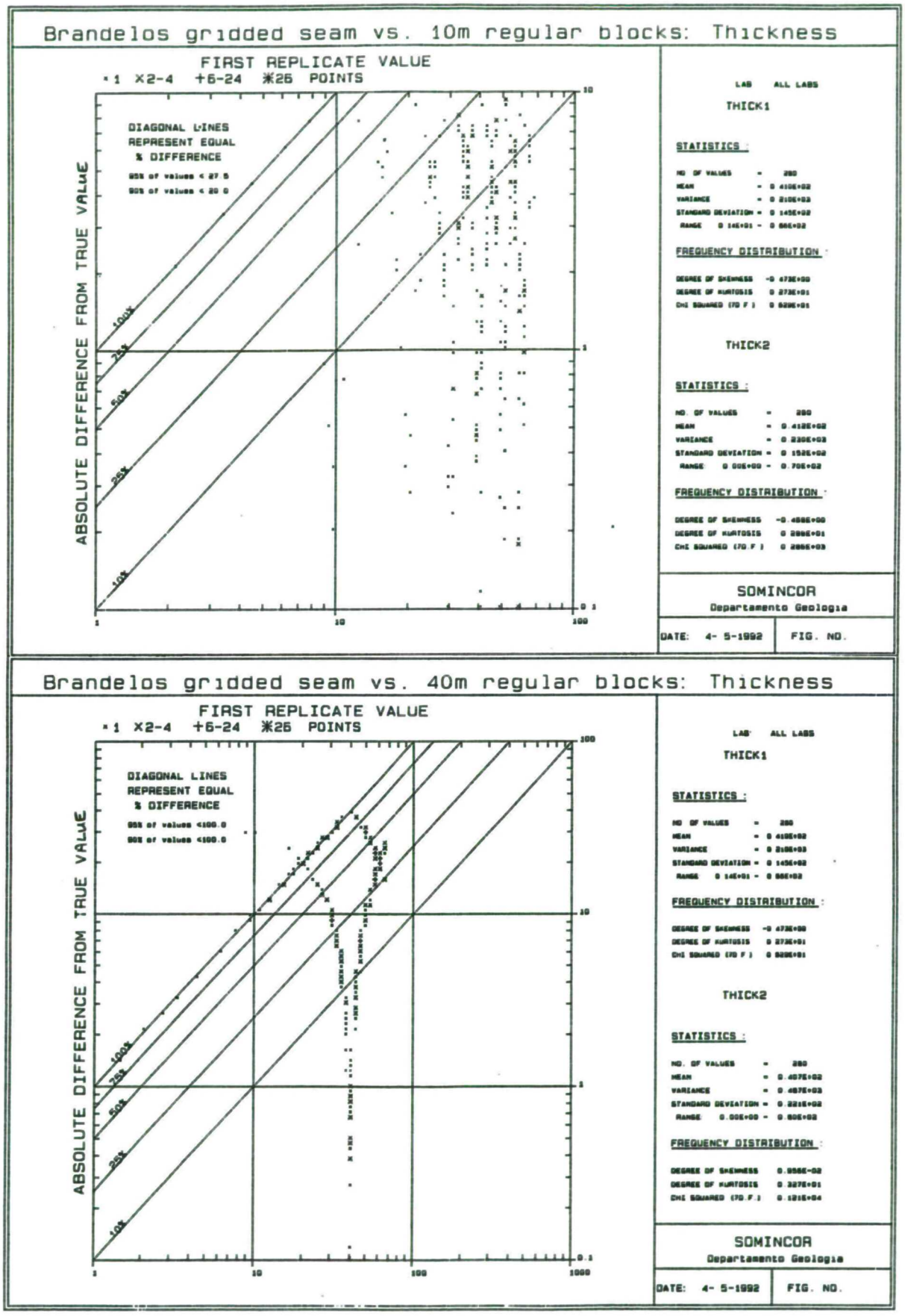


Figure 10.3 Thompson-Howarth precision plot comparing thickness estimates obtained from a gridded seam model against those obtained using 10m high and 40m high regular blocks, for the Brandelos deposit

10.3 Measuring locational accuracy and precision

In addition to estimating the overall volumes of different geological types, an important factor in reserve modelling is predicting the location of the geological discontinuities bounding such volumes. The accurate spatial location of volumes of ore to be mined is critical for successful mine design and the control of production sequences. Predicting the location of the boundaries of such volumes to a required degree of locational precision is consequently an important aspect of the modelling of geological discontinuities.

A method for measuring the locational accuracy and precision of such boundaries has been proposed in Section 9.4. In order to test this method interpretations of the ore limits for two different categories of reserve interpretation in the Graca orebody at Neves-Corvo (see Section 3.4) were selected. The first interpretation was that based on fans of underground drillholes, on vertical sections at 25m spacing, which was used for Indicated Reserve modelling in 1988. The second interpretation was based on re-interpretation of the geology, on level plans at 2m vertical intervals, using additional geological mapping information obtained whilst mining the orebody in the area of interest. The latter interpretation was used for Measured Reserve modelling in 1991, after a large proportion of the ore volume being modelled had been mined out. Polygons representing the ore outlines for the two different interpretations were digitised from level plans at 2m vertical intervals between the 840 and 860 levels, as illustrated by the example presented in Figure 10.4.

Using the method described in Section 9.4, and illustrated in Figure 9.4, sets of points were generated along north-south lines at 2.5m spacings between 4150E and 4300E. This resulted in a total of 600 points for each interpretation (ie. 60 points per level from 841 to 859 at 2m intervals). Statistical comparisons were then carried out on the differences between these two sets of data.

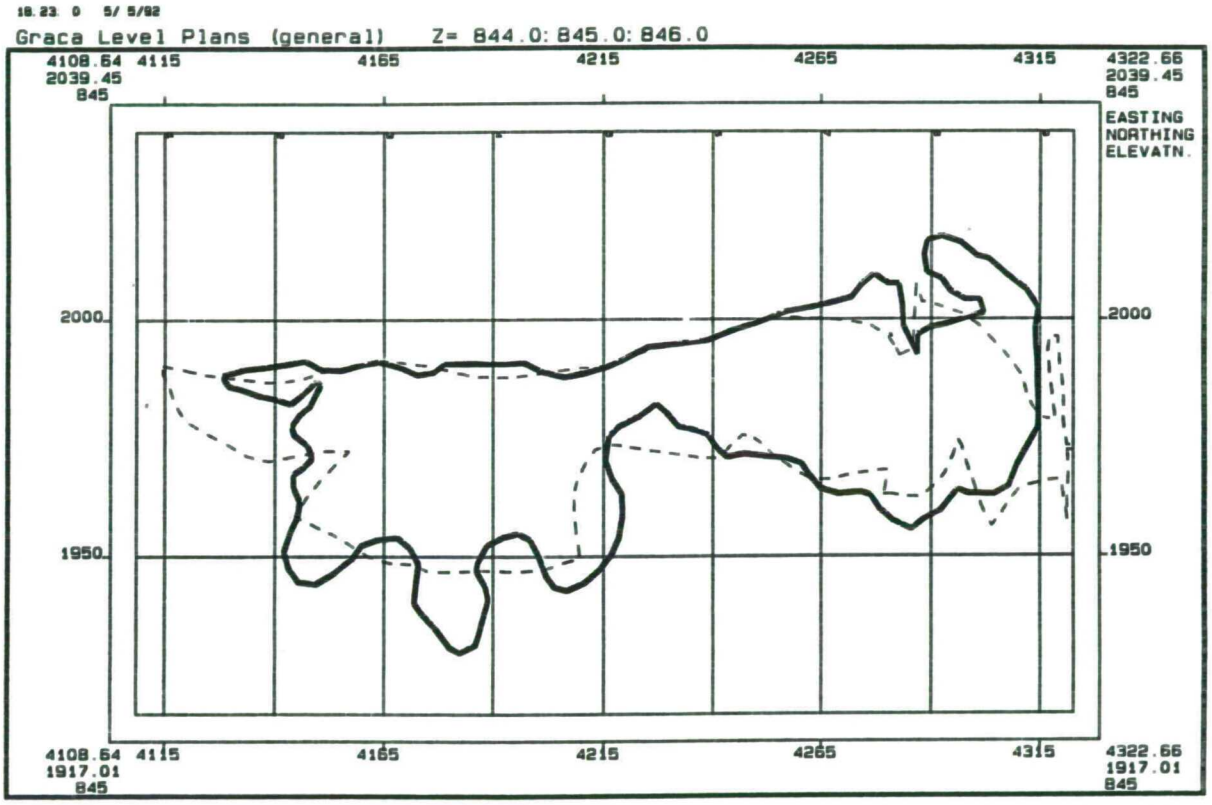


Figure 10.4 Level plan of part of the Graca orebody illustrating the two different reserve interpretations used for measuring locational errors in discontinuity modelling. Dashed line indicates interpretation based on underground drilling only (Indicated reserves, 1988); solid line indicates final interpretation based on in-stope mapping (Measured reserves, 1991).

The test line orientation and spacing was chosen so as to reflect the effect of differences in the prediction of the orebody location on the design of transverse accesses which constitute the initial mining development in this orebody. Results were analysed for errors in the prediction of footwall and hangingwall positions, as well as the lateral extent of the orebody in a north-south sense. The predictions of the footwall and hangingwall positions were given in terms of distance (in metres) from the centre of the orebody (as defined in the Measured reserve interpretation). Results were only analysed along test lines where the orebody was present in both interpretations. Where interfingering contacts occurred, the extreme positions of the footwall and hangingwall were used, and segments of waste between these limits were excluded from the thicknesses used in the comparisons.

The data points used were split into sets, corresponding to points close to the line of a drill section (projection distance <6.25m), and those falling between sections (projection distance >=6.25m). These two sets of points were analysed separately in order to quantify differences in the reliability of interpretations made close to the line of the drill sections, as against the interpretations corresponding to zones falling between sections.

Statistical summaries of the errors obtained in prediction of orebody footwall and hangingwall position, and horizontal thickness (along N-S horizontal lines), are presented in Figures 10.5 to 10.7, respectively. The quantitative differences between the two interpretations presented in these Figures demonstrates a greater accuracy and precision along drill sections, as opposed to between them, as would be expected. This is illustrated by the summary presented in Table 10.2.

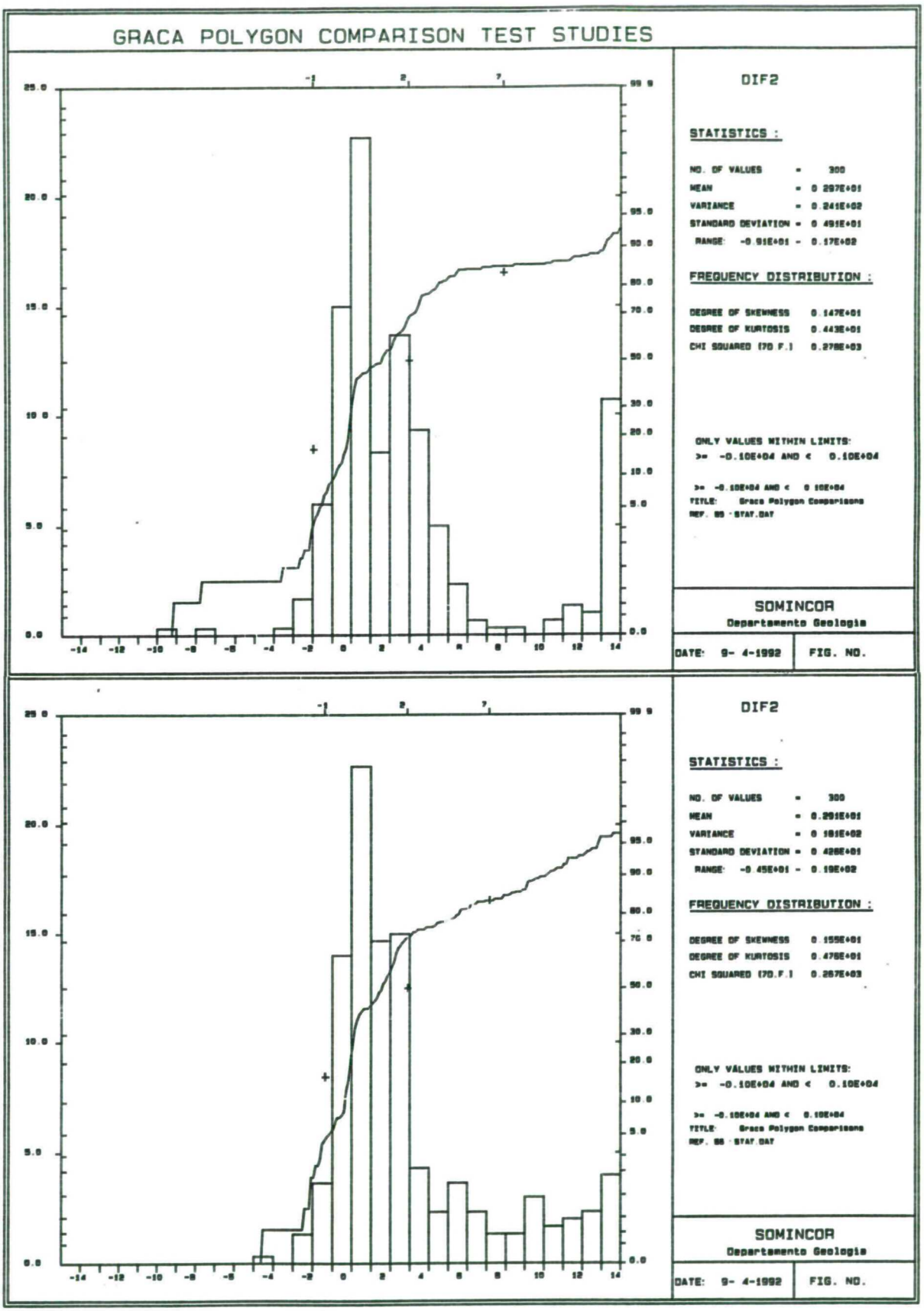


Figure 10.5 Errors in prediction of orebody footwall location along section lines (upper), and between section lines (lower). Graca orebody 4150-4300E, 840-860 elevation.

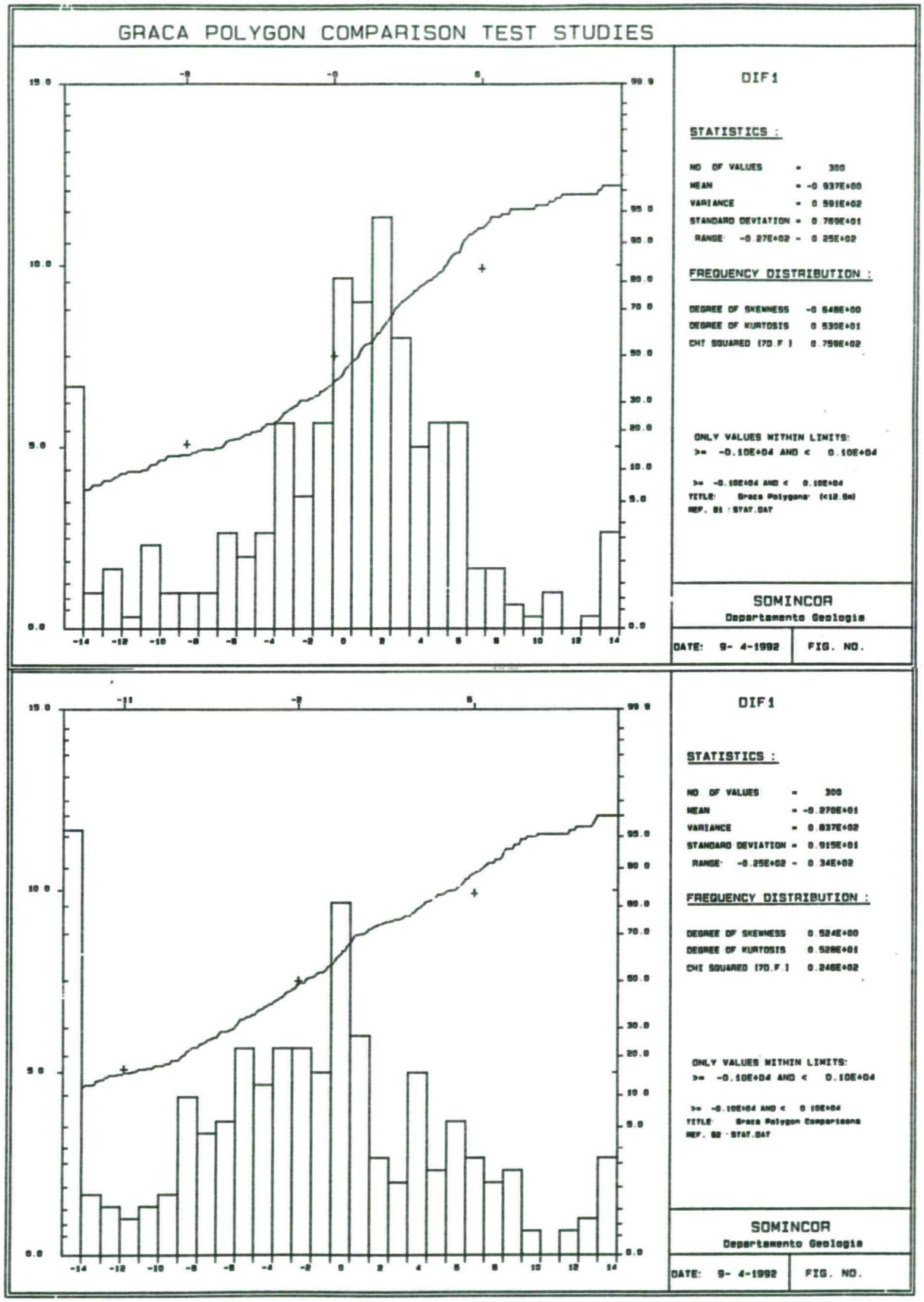


Figure 10.6 Errors in prediction of orebody hangingwall location along section lines (upper), and between section lines (lower). Graca orebody 4150-4300E, 840-860 elevation.

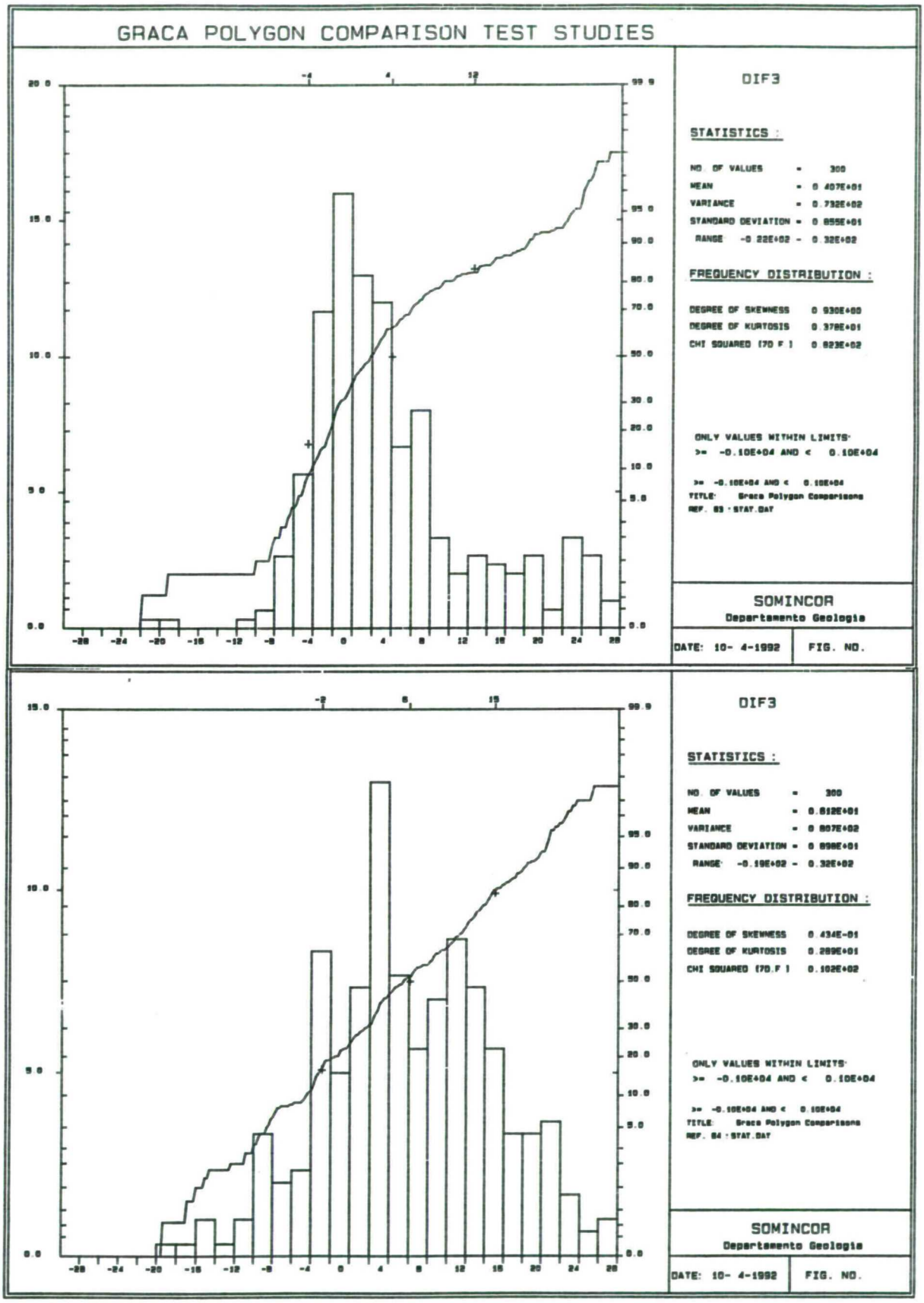


Figure 10.7 Errors in prediction of orebody thickness along section lines (upper), and between section lines (lower). Graca orebody 4150-4300E, 840-860 elevation.

	Along section lines Projection <6.25m	Between section lines Projection >=6.25m
Footwall position:		
Accuracy	1.7m	2.9m
Precision	7.7m	8.4m
Hangingwall position:		
Accuracy	-2.2m	-2.7m
Precision	5.2m	5.6m
Horizontal thickness:		
Accuracy	-3.0m	-5.6m
Precision	9.0m	9.3m

Note:

In this instance accuracy and precision are with respect to measurements along horizontal north-south lines at 2.5m spacing. The accuracy is derived from the mean difference of the set of test comparisons, and the precision from the standard deviation of the values about this mean.

Table 10.2 Summary of differences in accuracy and precision of Indicated orebody shape interpretations, when compared against Measured orebody shapes, along and between drillhole section lines. (Graca orebody 4150-4300E, 840-860 elevation).

The values given as accuracy measures in Table 10.2 are derived from the differences obtained by subtracting the Measured values from the corresponding Indicated values. In the case of the footwall and hangingwall positions, this is in terms of distance from the centre of the Measured orebody shape. Consequently the positive difference in footwall positions shows that the Indicated footwall was on average further away from this line (ie further north) than the Measured footwall. The opposite is true in the case of the hangingwall position, which is on average further south in the case of the Measured orebody shapes. The overall effect of these differences in terms of the difference in horizontal width of the orebody is negative, with the Measured shapes being wider than the Indicated shapes.

The values given for precision are the standard deviation of the differences about the mean value. These values are somewhat larger than might be expected giving a very wide range of expected variation if applied using 90% or 95% confidence limits. Nevertheless they serve as a useful relative measure which again demonstrates a slightly greater variability in the differences between sections as opposed to along them. It should be noted that there is a much larger difference in accuracy for the two sets of values, particularly for the horizontal width of the orebody.

10.4 Effects of geological control on grade estimation

In order to test the effects of geological control on grade estimation, studies were carried out using a set of data from the Cerro Colorado copper deposit (as described in Section 3.2). Specific objectives of these studies were to try to establish methods of quantifying the accuracy and precision of block model grade estimates, and to use such measures as a means of choosing the most appropriate geological controls to apply during grade estimation.

Production control in the Cerro Colorado open-pit (which ceased operation in 1989), was based on analyses of blasthole cuttings, as sampled from the debris cone created during drilling. The results of such analyses were used to define ore and waste composites during production, as illustrated by the example presented in Figure 10.8. Although there were several doubts about the quality, and representativity, of the blasthole samples, these results were used as the basis for reconciliation studies during mining. In particular a blasthole block model was generated (by averaging all blasthole values falling within a given reserve block), for use in comparisons against different reserve grade estimates.

The data used for the comparisons presented here was derived from the blasthole block model for Bench 388, which is based on a block size of 25m east-west, 20m north-south, with a 12m height. A condition used in generating the blasthole block values was that a block had to have at least 6 blastholes within it before a block value would be generated. This process ensured that partly mined blocks at the edge of the bench were excluded from the comparisons.

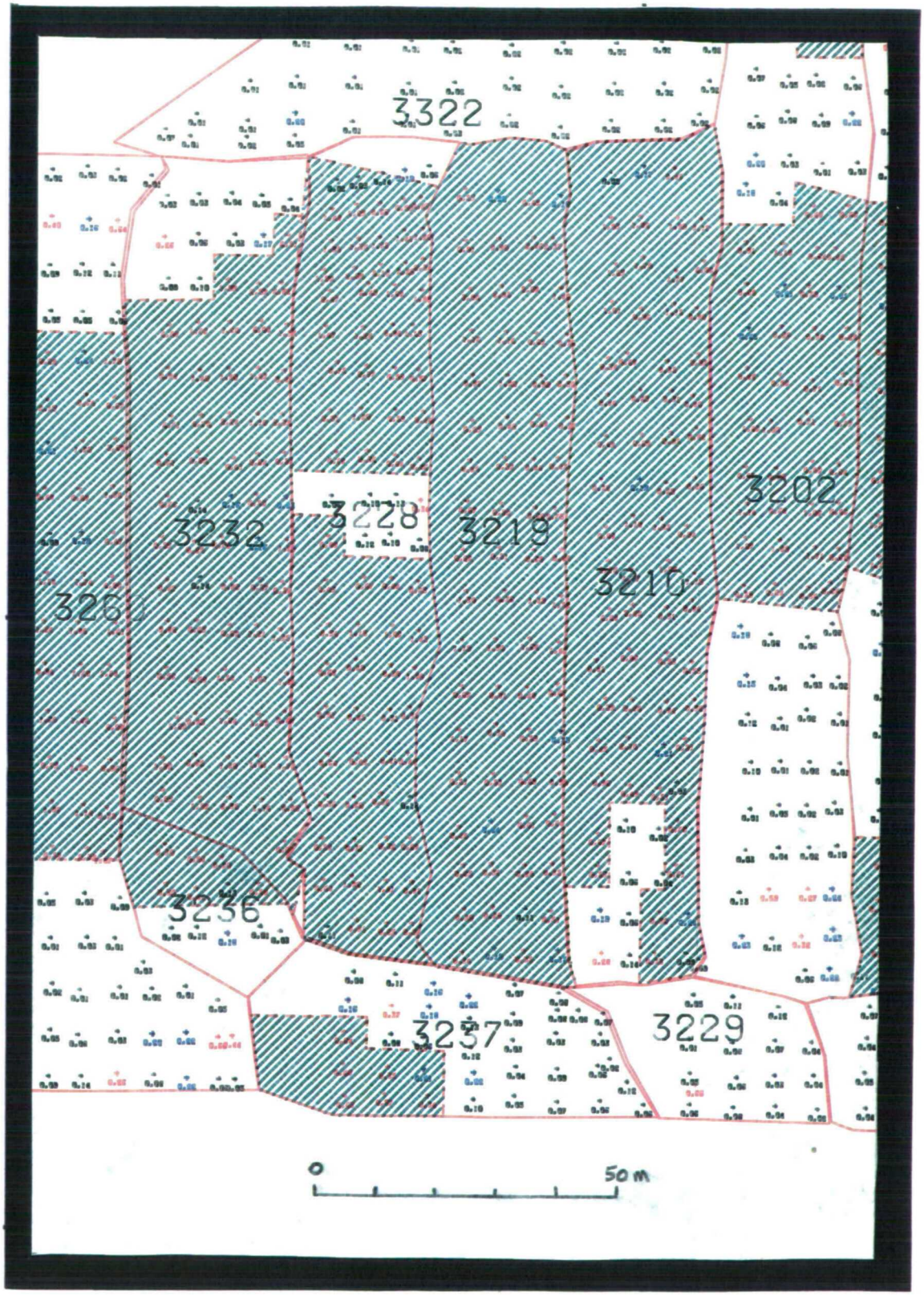


Figure 10.8 **Plot showing typical distribution of blasthole samples and ore composites in the Cerro Colorado open-pit (courtesy of RTM).** Ore composites are indicated by cross hatching.

The statistical distribution of the blasthole block values used are summarised in Figure 10.9. This illustrates the highly skewed nature of the distribution which suggests that the use of arithmetic averages (ie Gaussian dependent statistics) to arrive at the block values may be questionable.

The geological evaluation of the Cerro Colorado copper deposit has been discussed in Section 3.2.4. The reserve grade estimates presented here are based on using drillhole assays composited on a bench basis. In cases where a grade zone control was applied such composites were split along grade zone boundaries. The statistical distribution of the drillhole composite values used show similar characteristics to those for the blasthole block values presented in Figure 10.9.

Three different types of geological control were studied, namely;

a) **Normal Grade Zoning:** using a similar basis to that applied for the reserve update done in 1983, as described in Section 3.2.5. In this instance two groups of grade zones were dealt with completely separately during geostatistical analysis and grade estimation; zone 1 (generally <0.10% Cu), and zones 2+3 (generally >0.10% Cu) (see Figure 3.5).

b) **Structural Control:** using a median structural plane, interpreted on a similar basis to that described by Sides (1987), and modelled as a three-dimensional discontinuity (see Plate 6.2). The Z co-ordinate used for grade estimation purposes was derived by projecting sample points, and block centres, vertically onto this discontinuity.

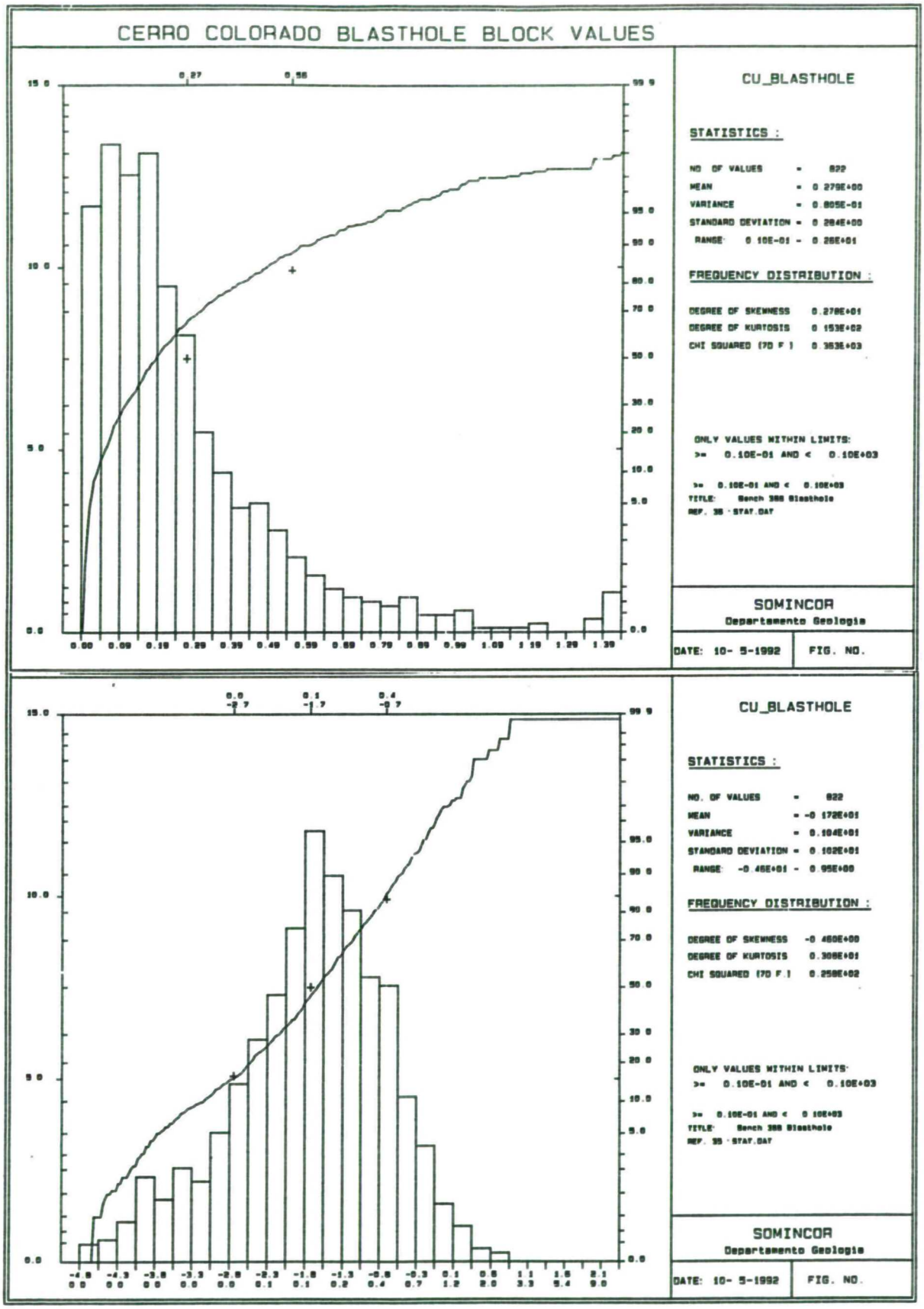


Figure 10.9 Statistical summaries of the blasthole block values used for reconciliation studies. Graphs show normal copper assay values (upper), and the log transform of the same values (lower)

c) **Modified Grade Zoning:** this used a similar approach to the normal grade zone control described above, with the introduction of a modification in order to smooth out the abrupt break in block grade estimates (around 0.10% Cu) introduced by the standard approach. This modification involved the use of composites from zones 1+2+3 when estimating blocks in zones 2+3, although such estimates were based on variogram models derived from studies of zones 2+3 on their own.

Separate estimates were generated for each of the blocks on bench 388 that had an associated blasthole average value using these different approaches. This provided six sets of estimates which were then compared statistically against the blasthole average values, as described below.

In order to test the accuracy of the estimates with respect to the blasthole block values, scatterplots with linear regression lines were generated. Because of the highly skewed nature of the distributions, both normal and log scaled plots were produced, as illustrated by the examples given in Figures 10.10 and 10.11. The correlation co-efficient given on such plots can be used as a measure of the overall error associated with the estimate. The linear regression of Y on X indicates the presence, and nature, of any biases in the estimation. Where the slope of this regression line is not equal to 1.0 a relative error is indicated, whereas if the regression constant is not equal to 0.0 then an absolute error is suggested.

To test the precision of the estimates Thompson-Howarth type plots (as described in Section 10.2) of the pairs of block values were generated, as illustrated by the examples presented in Figure 10.12. This type of plot illustrates variations in the differences between the estimates and the blasthole block values over the range of grades being studied. Numerical measures of precision are also presented for 95% and 90% confidence levels.

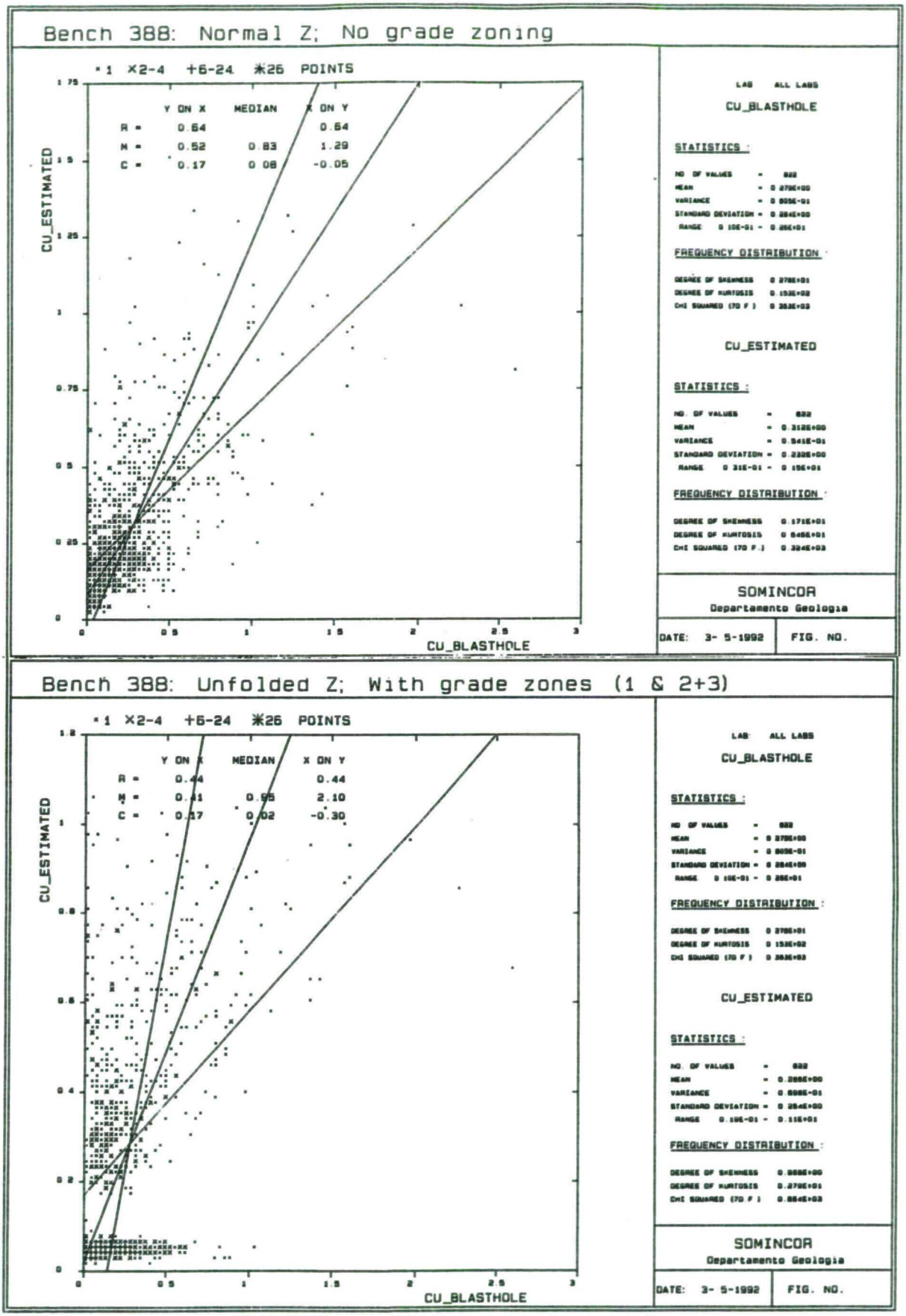


Figure 10.10 Scatterplots showing comparisons of blasthole block values against block grade estimates generated without any geological control (upper), and using normal grade zoning with structural control (lower)

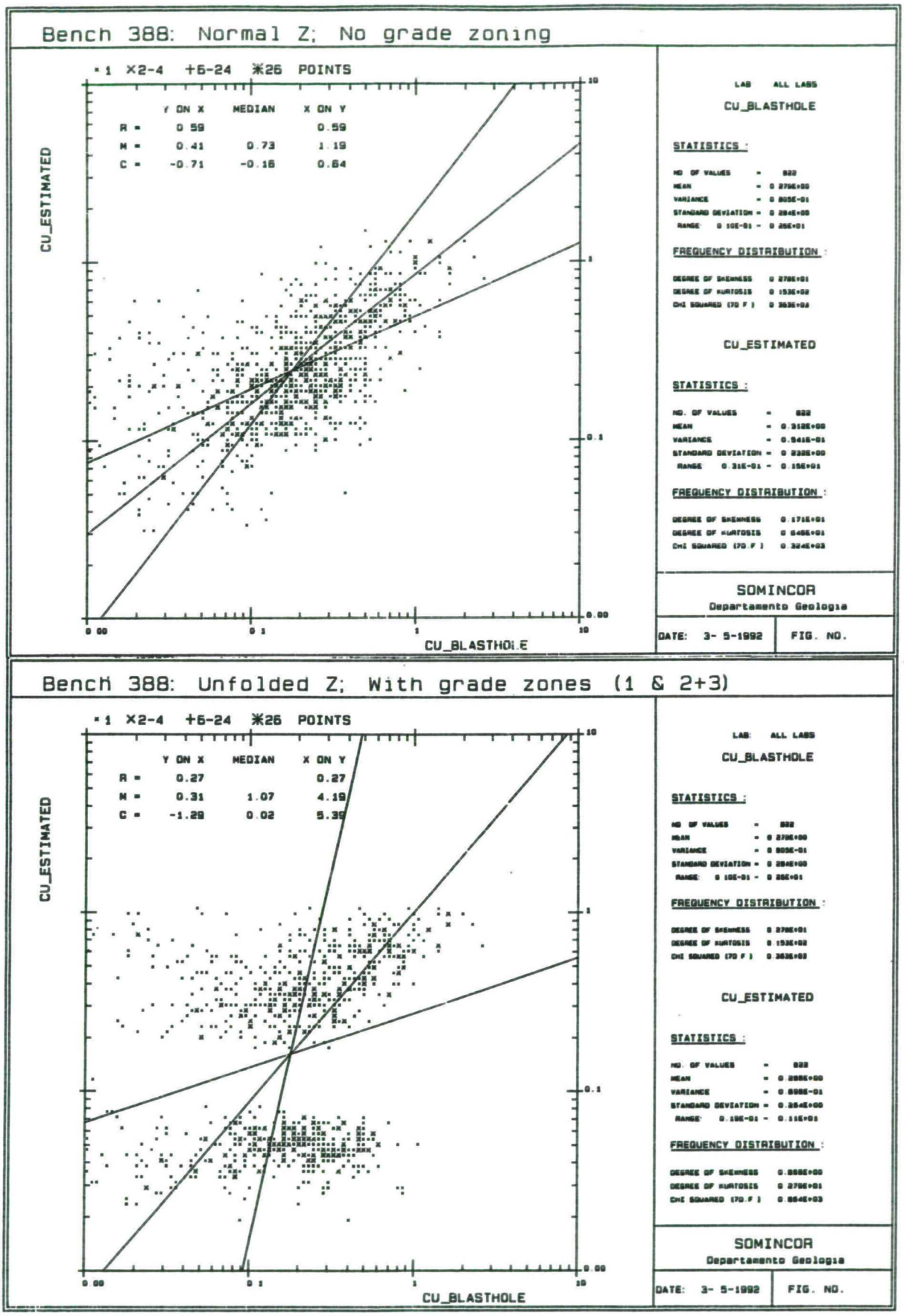


Figure 10.11 Log-scaled scatterplots showing comparisons of blasthole block values against block grade estimates generated without any geological control (upper), and using normal grade zoning with structural control (lower)

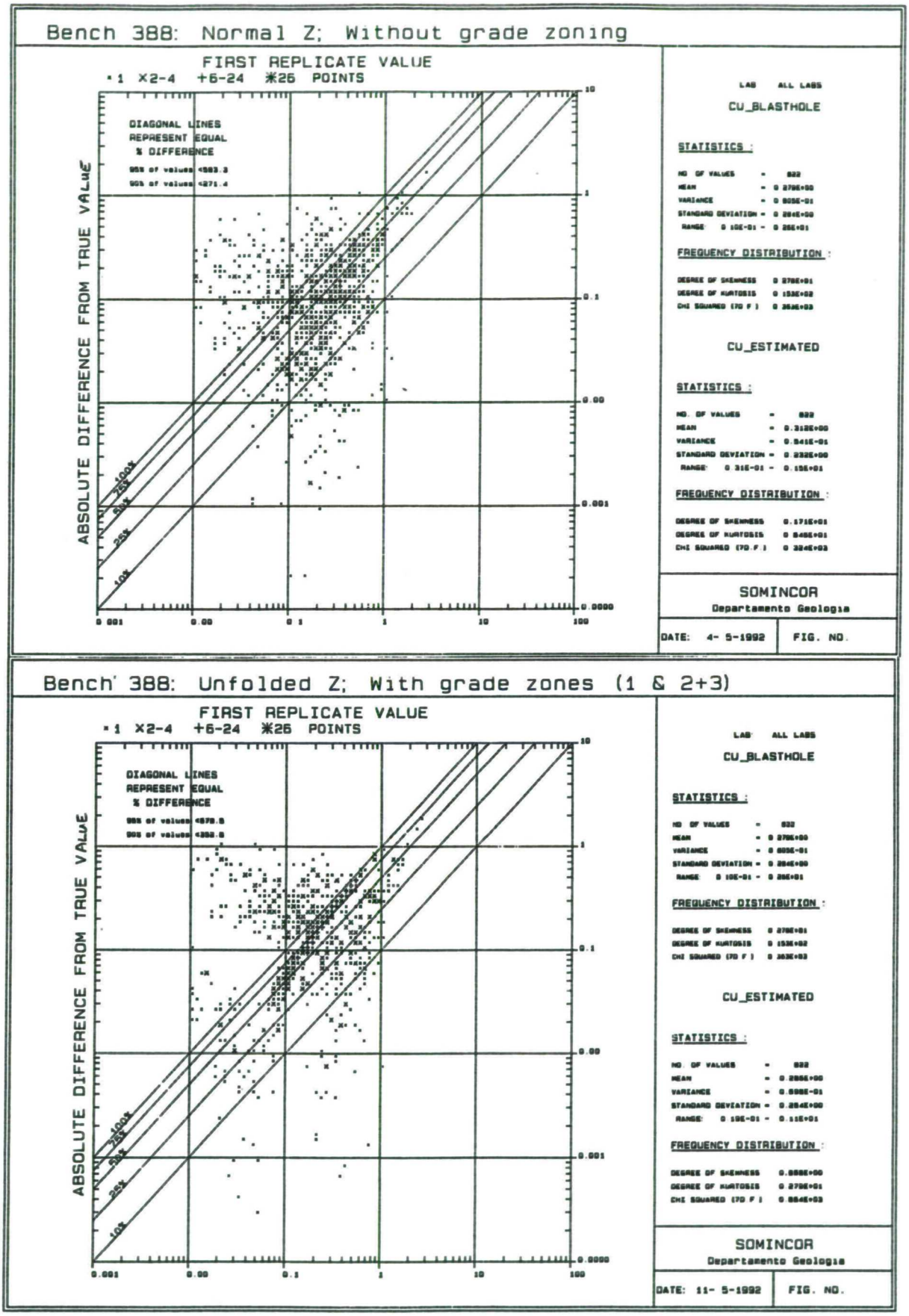


Figure 10.12 Thompson-Howarth precision plots comparing blasthole block values against block grade estimates generated without any geological control (upper), and using normal grade zoning with structural control (lower)

The results of the statistical comparisons for the six sets of estimates studied are summarised in Table 10.3. This table shows a set of summary statistics to indicate differences in the accuracy and precision with which the reserve estimates match the blasthole block values. Separate sets of statistics are given for all block values, and for those with average blasthole grades >0.25% Cu (cut-off grade used for mine planning). The first three statistics shown are derived from a linear regression of the reserve estimates on the blasthole values (eg Figure 10.10), and the last two from plots such as that shown in Figure 10.12.

In all cases the comparisons show a very poor match, with suggestions of a strong bias between the two sets of results, as well as a very high degree of imprecision in the reserve predictions. All the geological controls on grade estimation that were tested show a worsening of the match between the two sets of values. These poor results, which are much worse than was expected, deserve some clarification.

The most likely explanation for the large differences is that factors other than those studied in these reconciliation studies need to be taken into account. Such factors might include the following:

a) A bias may exist between the blasthole cutting samples with respect to samples of solid drillcore, due to the preferential concentration of mineralised particles within the cone of cuttings produced by the blasthole drills.

b) The non-Gaussian nature of the distribution of values (as illustrated in Figure 10.9) suggests that averaging of blasthole grades, and the use of simple linear kriging for reserve grade estimation, may be inappropriate estimation techniques in this instance.

		Stratigraphic control			
		Normal Z		Unfolded Z	
Zonal control		All	>0.25%Cu	All	>0.25%Cu
No grade zones	R	0.64	0.53	0.58	0.49
	M	0.52	0.46	0.47	0.41
	C	0.17	0.21	0.19	0.24
	P_{95}	583%	89%	788%	104%
	P_{90}	271%	74%	322%	77%
Grade zones 1 & 2+3	R	0.45	0.50	0.44	0.49
	M	0.41	0.45	0.41	0.46
	C	0.17	0.14	0.17	0.14
	P_{95}	656%	94%	680%	96%
	P_{90}	329%	90%	353%	92%
Modified grade zone method	R	0.58	0.49	0.52	0.46
	M	0.43	0.40	0.41	0.39
	C	0.09	0.11	0.11	0.12
	P_{95}	265%	92%	362%	94%
	P_{90}	146%	90%	194%	90%

Note:

R = linear correlation co-efficient
M = slope of regression line (Y on X)
C = constant for regression line (Y on X)
P_{95} = 95% precision level derived from Thompson-Howarth type plot
P_{90} = 90% precision level

Table 10.3 **Summary of differences between blasthole block copper grades and reserve block grade estimates obtained using different types of geological control, for Bench 388 in the Cerro Colorado open pit.**

c) There may be mismatches between the predicted and realised grade zone shapes. Strictly speaking a new grade zone interpretation, based on the realised production results, should be done prior to carrying out such studies. Drillhole composites would then be recoded to correspond to the new shapes, prior to grade estimation for comparison purposes.

The main conclusion of these particular studies is that these three aspects should be looked at in detail before any further attention is paid to improved geological control on grade estimation. Reconciliation studies of the differences between mine production results (based on surveyed volumes and composited blasthole grades), against mill production results could also be used to confirm whether a significant bias existed in the blasthole sample grades.

10.5 Discussion

This Chapter has illustrated the use of reconciliation studies to try to quantify the predictive accuracy and precision of different methods of reserve modelling. The principles behind the work done were discussed in Chapter 9. The results presented here have illustrated the use of several methods to analyse differences between different modelling methods; between models generated at different dates; and between reserve predictions and production results.

In Section 10.2 it has been demonstrated that in certain circumstances the use of regular block models may introduce significant biases into reserve models. The symmetrical pattern of errors illustrated in Figure 10.3 suggests that rules based on simple geometric relationships could be developed to assist in the selection of appropriate block model orientations and block sizes for use in modelling tabular orebodies.

In Section 10.3 the application of a technique for quantifying differences between different interpretations of orebody shape has been illustrated. This has demonstrated, in a quantified manner, that initial interpretations along drillhole section lines tend to be slightly more accurate, and more precise, than interpretations in the zones falling between sections. One implication of this for mine planning is that, where possible, it would be preferable to develop initial ore accesses along the lines of existing drillhole sections in order to reduce the chance of errors in the prediction of orebody contacts. Routine studies of this type could also be used to assist in the choice of drillhole grid spacings.

Further work on this aspect needs to be undertaken in order to determine the best way of applying the difference statistics that are produced. The large magnitude of the precision values given in Table 10.2 suggests that it may not be correct to apply these directly in predicting locational uncertainty. This may be due to the fact that such differences are spatially correlated, and that some interlinking with geostatistical methods of estimating uncertainties may be necessary.

The third set of studies, presented in Section 10.4, have illustrated approaches that can be used to measure differences between different sets of block grade estimates. In this instance the results obtained were disappointing in that they failed to indicate any improvement in reserve prediction with the introduction of geological controls. Since only a small area of the Cerro Colorado openpit was studied, this is not necessarily representative of the entire deposit. In addition it is suggested that other factors may influence the end result to a much greater extent than the different reserve estimation approaches studied.

The main emphasis of the work presented in this Chapter has been the development of improved methods for quantifying errors between different reserve models, and between reserve models and production results. Such approaches fulfil several functions, including the following;

a) Supporting preliminary studies aimed at selecting the most appropriate model structure(s) to use on a new deposit.

b) Highlighting areas of uncertainty within reserve models in order to target further evaluation, and to provide extra information to guide mine planning studies.

c) Assisting in the selection of new estimation techniques, or modifications to existing techniques. Such changes should only be introduced if they can be shown to give a measurable improvement in reconciliations against production results.

The use of reconciliation studies is an essential part of any computerised reserve estimation system, as discussed by Sides (1992b). Continued development and application of techniques such as those illustrated in this Chapter are fundamental to the improvement of existing computerised reserve modelling systems.

11. CONCLUSIONS

Technological advances in drilling and analytical techniques made in the last two decades, and reductions in associated costs, have led to an explosion of the amount of data generated during the evaluation of mineral deposits. Parallel advances in computer hardware and software have provided many new tools for handling this information. The use of such technologies is particularly important in an era of high financial costs, where successful mining companies require more and more detailed planning of production strategies and schedules. Such criteria lead to the need for continued development of new approaches to reserve modelling in order to make the most efficient use of existing data and computer handling methods.

The work done in the studies presented here identified the modelling of geological discontinuities as a critical area of computerised reserve modelling requiring further attention. A review of methods used for computer modelling of three-dimensional geological discontinuities highlighted several limitations associated with the approaches used in most commercially available reserve estimation systems. Such limitations often stem from the fact that such methods have been adapted from techniques used for solid modelling in other areas; particularly the geometric modelling of mine openings, and CAD/CAM approaches used in fields such as automobile design and architecture. It is noted that techniques used in the medical field, for modelling scanning tomography results, may be more appropriate to the modelling of poorly sampled irregular three-dimensional geological shapes. As yet, such techniques have received relatively little attention in the field of geological modelling.

The conclusions arrived at above led to the definition of a series of criteria which a geologically based three-dimensional geometric modelling system should satisfy. This resulted in the development of a system for modelling three-dimensional geological discontinuities using a boundary representation type file structure in which all data are reduced to geometric primitives (ie points, lines and triangles). This approach was implemented as part of an integrated geological database and reserve estimation system, at the Neves-Corvo copper-tin mine in Portugal, during the course of this research project.

This approach to geological modelling is not entirely new, as discussed in Chapters 4 and 5. Its implementation in the context of an integrated reserve estimation system is, however, believed to be unique and represents a significant advance in terms of allowing greater geological control on reserve estimation. Particularly important features of the system include the following:

- links with drillhole and mapping databases
- the ability to maintain 2-d interpretative data within a 3-d model structure
- the use of a variety of graphic displays of discontinuity models, in conjunction with displays of information from other geological databases and reserve models, facilitates a better understanding of the geometry of geological shapes, as well as assisting the verification of interpretations
- open-ended discontinuities can be handled easily, since the boundaries of closed volumes are treated as special cases
- the use of surface trimming to ensure proper closure of intersecting discontinuities
- the use of interpretative hierarchies to control surface trimming and volume generation
- the ability to convert the discontinuity models into several different types of volumetric model

Separate Chapters in this thesis have illustrated the use of this modelling system in three separate areas, in the context of deposit modelling and reserve estimation, namely;

- geological modelling
- volumetric modelling
- geological controls on the estimation of internal properties

Examples presented have shown how this approach enables greater geological control on the reserve estimation process.

A basis for measuring and interpreting errors associated with discontinuity modelling is developed in Chapter 9. This is a particularly important area which merits further study. It is considered that the treatment of geometric errors in orebody modelling has been neglected in the past, with most attention being given to the study of grade estimation errors using geostatistics.

The application of techniques for measuring errors associated with reserve models has been illustrated by the reconciliation studies presented in Chapter 10. To a large extent these studies are exploratory in nature, since this aspect warrants a full research project in its own right. The results presented here have illustrated methodologies that can be applied to the measurement of the accuracy and precision of different reserve models with respect to one another or to production results.

The most significant conclusions of the work done are as follows:

- there is a need for continual refinement of the methods used for orebody modelling, in order to take advantage of technological advances, and to meet the requirements of increasingly detailed planning

- it is essential to develop geological modelling techniques in close association with the geological databases and interpretation methods used. A geological discontinuity modelling system developed on these lines provides several advantages over conventional approaches.

- maximising the amount of geological input into orebody modelling ensures that a greater amount of geological information is taken into consideration during subsequent planning work.

- it is essential to be able to interface geometric and volumetric models in order to allow geological controls to be imposed during the estimation of internal properties using geomathematical or geostatistical techniques.

- when performing volumetric modelling and estimation for mine planning purposes, there is no universal solution, and compromises are often made to allow a single reserve model to fulfil both functions. Under such circumstances, the use of geometric models which can interface to different types of volumetric model can provide much greater overall planning flexibility.

- it is important to be able to measure differences between different geometric models in order to allow the control of errors introduced during the modelling process. The work presented has demonstrated that:

    a) regular block models can give large errors and potentially misleading results under certain circumstances, and

    b) the accuracy and precision of shape models are better close to the planes of drillhole sections, than for the zones falling between sections

Several aspects requiring further study have been mentioned in Chapters 5 to 10. The following aspects are regarded as being of particular importance in this regard;

- the development of methods for automatically assigning codes to any point in space using a set of "trimmed", open-ended, discontinuities
- the implementation of methods for deriving orientation statistics from the discontinuity models
- the investigation of the use of surface roughness measures as a means of predicting the uncertainties associated with different discontinuity models
- the development of interfaces with commercially available CAD packages
- the use of new developments in other fields, such as 3-dimensional visualisation and virtual reality systems, in conjunction with the modelling approaches developed here
- the development of a more rigorous basis for measuring and predicting the errors associated with discontinuity models
- the development of standard procedures for carrying out reconciliation studies between different types of reserve models, and between reserve models and production results

Further work on these aspects should lead to increased geological input into the resultant reserve models, and hopefully result in improvements in their accuracy and precision. Nevertheless, it should always be borne in mind that computerised techniques are only a tool which, if applied incorrectly, can cause problems rather than solve them. Whilst such tools can greatly enhance a geologist's ability to model the three dimensional form of mineral deposits, they can never replace the role of the geologist in data collection and interpretation.

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APPENDIX I

STATISTICAL SUMMARY OF DISCONTINUITY MODELS USED FOR INDICATED RESERVE MODELLING AT NEVES-CORVO IN 1988

GRACA:- 1988 Indicated geological interpretation

No.	Name	Rank	D	M	V	T	V/T	IL	IP	NV	NP
1	LI	0:	0	0	114	224	0.51:	0	0	0	114
2	F1F	5:	4	0	48	77	0.62:	125	124	124	172
3	F1G	1:	6	0	106	199	0.53:	82	82	106	212
4	F4J	1:	13	0	115	212	0.54:	84	84	92	207
5	FG1A	7:	3	0	70	123	0.57:	110	109	109	179
6	FG2C	5:	4	0	109	199	0.55:	133	133	135	244
7	FG2D	5:	2	0	51	87	0.59:	114	125	114	165
8	FG2G	5:	5	0	50	85	0.59:	117	117	117	167
9	FG3D	1:	29	0	183	347	0.53:	108	111	109	292
10	FG9H	5:	3	0	58	101	0.57:	115	115	115	173
11	FWMSA	10:	10	0	76	135	0.56:	99	100	100	176
12	FWMSB	10:	57	0	295	563	0.52:	219	219	226	521
13	FWMSC	10:	8	0	85	143	0.59:	166	167	166	251
14	FWMSD	10:	18	0	134	243	0.55:	268	266	266	400
15	FWMSE	10:	3	0	29	44	0.66:	131	130	130	159
16	FWMSF	10:	1	0	25	30	0.83:	71	72	71	96
17	FWMSG	10:	4	0	46	73	0.63:	129	129	129	175
18	FWMSH	10:	10	0	78	134	0.58:	146	146	146	224
19	FWMSI	10:	4	0	57	97	0.59:	115	115	114	171
20	FWMSJ	10:	25	0	175	329	0.53:	162	162	175	350
21	HWMSA	20:	4	0	77	135	0.57:	161	158	161	238
22	HWMSB	20:	8	0	290	551	0.53:	200	401	206	496
23	HWMSC	20:	7	0	58	99	0.59:	107	107	107	165
24	HWMSD	20:	28	0	161	299	0.54:	320	315	315	476
25	HWMSE	20:	6	0	32	50	0.64:	100	100	100	132
26	HWMSF	20:	1	0	33	46	0.72:	92	98	92	125
27	HWMSG	20:	8	0	53	90	0.59:	134	134	134	187
28	HWMSH	20:	7	0	84	143	0.59:	142	142	141	225
29	HWMSI	20:	0	0	4	2	2.00:	49	49	49	53
30	HWMSJ	20:	29	0	218	408	0.53:	223	221	221	439
31	TMC1B	30:	7	0	70	126	0.56:	94	94	94	164
32	TMC2B	40:	44	0	295	557	0.53:	420	409	412	707
33	TMC2C	40:	1	0	29	43	0.67:	72	76	72	101
34	TMC2D	40:	5	0	99	169	0.59:	198	194	194	293
35	TMC2E	40:	1	0	67	114	0.59:	137	150	137	204
36	TMC2F	40:	2	0	16	21	0.76:	62	62	62	78
37	TMC2G	40:	2	0	33	50	0.66:	121	121	121	154
38	TMC2I	40:	2	0	39	61	0.64:	85	85	84	123
39	TMC2J	40:	11	0	81	138	0.59:	330	330	330	411
40	TMC3B	50:	21	0	116	207	0.56:	177	176	175	291
41	TMC4B	60:	7	0	49	84	0.58:	269	264	260	309
42	TME1B	35:	20	0	184	342	0.54:	282	280	283	467
43	TME1G	35:	1	0	11	12	0.92:	66	66	66	77
44	TME2B	45:	18	0	153	273	0.56:	338	338	338	491
45	TME3B	55:	2	0	61	105	0.58:	217	215	215	276

D = number of drillhole points M = number of mapped points
V = number of vertices T = number of triangles
V/T = ratio of number of vertices to number of triangles
IL = number of intersection trim lines
IP = number of intersection points
NV = number of non-vertex points
NP = total number of points stored

CORVO:- 1988 Indicated geological interpretation

No.	Name	Rank	D	M	V	T	V/T	IL	IP	NV	NP
1	LIM	0:	0	0	120	236	0.51:	0	1	0	120
2	F1	1:	1	8	40	68	0.59:	78	78	78	118
3	F1A	5:	1	4	35	55	0.64:	83	82	82	117
4	F2	1:	0	7	23	30	0.77:	70	76	70	93
5	F20K	7:	2	0	21	26	0.81:	213	211	211	232
6	F20X	7:	4	0	38	56	0.68:	153	160	152	190
7	F3K	5:	4	3	31	45	0.69:	112	112	112	143
8	F3O	5:	3	0	73	127	0.57:	171	200	174	247
9	F3R	5:	11	0	36	56	0.64:	143	140	140	176
10	F4	1:	29	16	154	278	0.55:	150	154	150	304
11	F4A	2:	2	2	37	56	0.66:	119	119	119	156
12	F5	1:	35	11	136	246	0.55:	161	163	160	296
13	F7U	5:	15	2	50	83	0.60:	111	112	111	161
14	F7V	5:	18	6	71	120	0.59:	140	140	140	211
15	F7W	5:	8	1	28	42	0.67:	113	113	113	141
16	F8	1:	23	14	132	234	0.56:	143	147	143	275
17	F9	1:	23	9	95	170	0.56:	112	112	112	207
18	FMSAA	10:	0	0	10	11	0.91:	73	78	71	81
19	FMSAB	10:	0	0	12	12	1.00:	56	61	56	68
20	FMSAD	10:	0	0	11	12	0.92:	35	35	35	46
21	FMSAE	10:	0	0	15	20	0.75:	70	80	70	85
22	FMSK1	10:	2	0	11	13	0.85:	34	34	34	45
23	FMSX1	10:	7	0	23	35	0.66:	80	79	79	102
24	FWFTV	15:	1	0	14	18	0.78:	57	57	64	78
25	FWFTW	15:	2	0	34	55	0.62:	105	111	105	139
26	FWMSK	10:	39	0	85	141	0.60:	239	241	238	323
27	FWMSO	10:	35	0	125	219	0.57:	236	256	236	361
28	FWMSP	10:	0	0	31	50	0.62:	181	189	179	210
29	FWMSR	10:	2	0	47	72	0.65:	127	139	127	174
30	FWMSS	10:	12	0	46	76	0.61:	159	166	159	205
31	FWMST	10:	3	0	18	23	0.78:	70	72	70	88
32	FWMSU	10:	22	0	72	120	0.60:	156	157	156	228
33	FWMSV	10:	5	0	42	62	0.68:	113	113	119	161
34	FWMSW	10:	0	0	42	67	0.63:	118	137	126	168
35	FWMSX	10:	21	0	61	103	0.59:	185	181	181	242
36	FWMSY	10:	4	0	20	25	0.80:	90	90	90	110
37	FWRUK	80:	23	0	124	217	0.57:	378	398	368	492
38	FWRUO	80:	25	0	115	199	0.58:	373	377	367	482
39	FWRUS	80:	2	0	31	48	0.65:	203	198	198	229
40	FWRUU	80:	18	0	89	154	0.58:	280	276	276	365
41	FWRUV	80:	4	0	41	62	0.66:	123	136	122	163
42	FWRUW	80:	11	0	46	75	0.61:	220	214	214	260
43	FWRUX	80:	8	0	48	82	0.59:	258	247	247	295
44	FWRUY	80:	1	0	20	28	0.71:	121	117	117	137
45	HMSAA	20:	0	0	11	13	0.85:	67	76	67	78
46	HMSK1	20:	2	0	11	13	0.85:	45	45	45	56
47	HMSX1	20:	6	0	21	33	0.64:	58	62	58	79
48	HWMSK	20:	17	0	97	167	0.58:	251	297	250	347
49	HWMSO	20:	65	0	149	265	0.56:	252	255	255	404

continued on next page

No.	Name	Rank	D	M	V	T	V/T	IL	IP	NV	NP
50	HWMSP	20:	4	0	40	67	0.60:	273	261	260	300
51	HWMSR	20:	2	0	45	70	0.64:	170	184	168	213
52	HWMSS	20:	6	0	40	65	0.62:	151	152	151	191
53	HWMST	20:	2	0	16	20	0.80:	62	61	61	77
54	HWMSU	20:	21	0	93	157	0.59:	149	183	149	242
55	HWMSV	20:	7	0	35	54	0.65:	186	186	183	218
56	HWMSW	20:	12	0	43	70	0.61:	152	153	157	200
57	HWMSX	20:	19	0	68	117	0.58:	188	194	187	255
58	HWMSY	20:	8	0	30	43	0.70:	104	106	104	134
59	HWRUK	90:	11	0	121	211	0.57:	306	352	299	420
60	HWRUO	90:	1	0	110	187	0.59:	406	466	394	504
61	HWRUS	90:	1	0	46	77	0.60:	183	187	174	220
62	HWRUU	90:	24	0	94	169	0.56:	296	302	284	378
63	HWRUV	90:	8	0	43	68	0.63:	124	132	122	165
64	HWRUW	90:	5	0	53	88	0.60:	174	196	172	225
65	HWRUX	90:	10	0	43	73	0.59:	292	278	277	320
66	HWRUY	90:	5	0	26	38	0.68:	109	109	106	132
67	TC5X1	30:	3	0	16	21	0.76:	69	69	69	85
68	TMC5K	30:	32	0	56	89	0.63:	230	228	227	283
69	TMC5O	30:	42	0	116	200	0.58:	345	338	338	454
70	TMC5R	30:	3	0	45	70	0.64:	277	261	261	306
71	TMC5S	30:	1	0	34	49	0.69:	175	174	171	205
72	TMC5T	30:	3	0	14	16	0.88:	73	74	73	87
73	TMC5U	30:	24	0	70	119	0.59:	160	159	158	228
74	TMC5V	30:	1	0	24	31	0.77:	143	142	141	165
75	TMC5W	30:	0	0	16	21	0.76:	93	100	93	109
76	TMC5X	30:	17	0	73	125	0.58:	289	295	284	357
77	TMC5Y	30:	3	0	23	31	0.74:	148	144	143	166
78	TMC6K	40:	4	0	48	83	0.58:	137	142	137	185
79	TMC6U	40:	7	0	56	92	0.61:	228	236	225	281
80	TMC6Y	40:	2	0	17	21	0.81:	91	89	88	105
81	TMC7K	50:	0	0	22	33	0.67:	24	32	24	46
82	TME5K	35:	0	0	40	61	0.66:	112	145	114	154
83	TME5U	35:	9	0	47	76	0.62:	216	212	210	257
84	TME5V	35:	1	0	19	24	0.79:	55	55	76	95
85	TME5X	35:	1	0	16	23	0.70:	78	75	75	91
86	TME5Y	35:	2	0	20	24	0.83:	79	82	79	99
87	TME6K	45:	2	0	26	36	0.72:	99	103	98	124
88	TMTK	32:	1	0	17	22	0.77:	32	32	32	49
89	TMTV	32:	1	0	21	28	0.75:	87	94	104	125
90	TMTW	32:	0	0	13	18	0.72:	85	82	82	95
91	TRTK	95:	2	0	33	54	0.61:	140	155	139	172
92	TRTV	95:	5	0	25	33	0.76:	115	113	113	138
93	TRTW	95:	1	0	38	60	0.63:	122	138	118	156

D = number of drillhole points M = number of mapped points
V = number of vertices T = number of triangles
V/T = ratio of number of vertices to number of triangles
IL = number of intersection trim lines
IP = number of intersection points
NV = number of non-vertex points
NP = total number of points stored

APPENDIX II

GLOSSARY

GLOSSARY

Some of the terms used in this thesis have strict, and sometimes slightly different, usages in other fields. A glossary of definitions of such terms is given below in order to clarify the sense in which they are used here. Definitions are based mainly on Merks (1985), Kirkpatrick (1983) and Pickover(1990).

Accuracy: the reliability of a measurement in terms of the closeness of agreement between this estimate and the unknown true value.

Delaunay triangulation: the process of generating the most equiangular set of triangles linking a set of points in a two-dimensional plane. The procedure used relies on the fact that one of the properties of this triangulation is that no point falls inside the circumcircle of any of the triangles (the three vertices will of course lie on the circumcircle). The term **Dirichlet tesselation** is also used for this type of triangulation. The polygons formed by connecting the bisectors of the edges of the Delaunay triangles define a nearest neighbour network. These are often termed **Thiessen** or **Voronoi polygons.**

Discontinuity: interval, break or interruption in a property which is otherwise connected throughout in space or time in a continuous fashion. Hence the term **geological discontinuity** is used for a geological boundary or surface across which the properties of materials on either side vary sharply.

Error: a difference between a measurement, or the mean of a set of measurements, and the most reliable estimate of its unknown true value (ie reality). Such differences may be **absolute** (constant over a range of values), or **relative** (proportional to the magnitude of the value being measured).

Estimate: an approximate judgement of number, amount, quality, character, etc.

Estimation: the process of calculating an estimate.

Fractals: objects (or sets of points, curves, or patterns) which exhibit increasing detail ("bumpiness") with increasing magnification (after Pickover (1990)).

Grade zones: geologically interpreted zones of similar grades, whose boundaries are defined by sharp changes in grade gradient. Interpretation of such zones is often used to separate geostatistically different populations of assay values during reserve estimation. An important criterium is that they should form coherent three-dimensional shapes, and consequently their boundaries do not coincide exactly with isograde lines, since they may include isolated samples which fall outside the nominal bounding grade limits.

Indicator function: a function used to separate all samples into two populations by reference to a defined grade cut-off or other geological criteria. All samples are thus assigned a value of 0 (below cut-off), or 1 (above cut-off). This function can be used during geostatistical studies to estimate the probability of a given ore block falling above or below the defined cut-off, using a procedure known as **indicator kriging.**

Klein bottle: a one-sided four-dimensional surface, which in three dimensions can be represented as a surface obtained by pulling the narrow end of a tapering cylinder through the wall of the cylinder and then stretching the narrow end and joining it to the larger end.

Line: a pair of points defining a simple line segment.

Mobius strip: the one-sided surface formed by joining together the two ends of a long rectangular strip, one end being twisted through 180 degrees before the join is made.

Model: a representation of reality based on recorded measurements which can be used as a predictive tool to forecast other measurements from the same location.

Modelling: the process of creating or modifying a model.

Polyline: a set of points defining a continuous complex discontinuity (normally in a single two-dimensional plane) made up of several lines.

Polygon: a special type of polyline where the first and last point are the same.

Precision: the magnitude of random variation between different sets of measurements, or estimates, of the same true value.

Reconciliation studies: comparative studies between different sets of information aimed at examining the consistency between them, and analysing differences in terms of accuracy and precision.

Serial-slice model: a geometric model based on sampling or interpreting the shape of a complex solid on a set of regularly spaced parallel planes. Sets of sections or plans used for geological interpretations during mineral evaluation are good examples of this type of model.