

PREDICTION OF OFF-DESIGN PERFORMANCE

OF AXIAL-FLOW COMPRESSORS

by

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ABSTRACT

A finite-difference technique has been developed for the prediction of the off-design performance of multi-stage axial-flow compressors. The equations governing the turbulent and compressible flow were solved together with a consideration of the necessary cascade correction. Inclusion of the turbulent stress terms in the theoretical equations was effected by the introduction of a simple definition of effective viscosity.

Detailed hot-wire tests were conducted to obtain a further insight into the structure of the shear flow between the blade rows of a three stage axial-flow compressor. A special hot-wire probe for operation in a low speed compressor was designed and used. It was arranged to carry an inclined wire which could operate free from the prong wake-interference effects in three orientations relative to the direction of local resultant velocity. Some useful results for the radial distribution of turbulent shear and effective viscosity were obtained downstream of a rotor blade row. The accuracy of the shear measurement within the compressor was established by the ability of the same hot wire to measure shear stress within $\pm 5\%$ for a fully developed pipe-flow.

The computer programme developed to predict the flow distribution in axial-flow compressors was arranged to be quite general and the various steps in the solution procedure were executed in separate subroutines; this

arrangement was selected to allow further scope in developing the programme. Important features developed specifically for this computer programme were the subdivision of the flow field into a quasi-orthogonal network and an effective field scanning procedure.

The programme was tested and found satisfactory for the relatively simple cases of developing flow through circular pipes and parallel annuli. Further test runs with the data of the three-stage axial-flow compressor and a comparison with the experimental measurements of the development of axial velocity profile through the blade rows indicated reasonable success with the solution procedure.

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PREFACE

The references in the text are designated by the name of the first author mentioned in the title and the year of publication. The bibliography in alphabetical order is given after the main text.

A consistent nomenclature has been used throughout and a list of notation is appended following the bibliography.

The equations and sketches are numbered for each chapter whereas the tables and figures are placed after the appendices and numbered consecutively.

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CHAPTER 1

INTRODUCTION

The gradual improvement in the performance of turbomachines has generally come about as a result of improvements in the theoretical model of fluid flow in the machine. The flow through the blading of turbomachines is an extremely complicated three-dimensional phenomenon. The flow parameters in compressors have strong gradients in each of the three dimensions.

In view of this complexity it is necessary to make several simplifying assumptions to represent the flow by a theoretical model. In compressors as in other problems of fluid flow the major assumption is usually that the flow is inviscid.

The performance of both the axial-flow and centrifugal compressors is sensitive to the accuracy of blade settings; this sensitivity increasing with Mach number. The achievement of the accurate settings of the blades depends upon the ability to analyse the flow pattern in the bladed passage for a wide range of operating conditions. The analysis or the prediction of the fluid flow in a compressor is, in turn, based upon the solution of the most complete theoretical model of the flow.

In the past, the flow models in axial-flow compressors have been inadequate in the regions close to the walls. This weakness has always been reconciled

by employing empirical values of a "work done factor" and a "boundary layer blockage parameter" for prediction techniques using theoretical models with experimental results. Correction factors for stagnation-temperature rise and stagnation pressure rise have also been used in addition to the weight-flow blockage factor. The growth of the boundary layers close to the hub and casing walls of the compressor must be taken into account for greater accuracy in the prediction of the flow pattern.

More recently attempts have been made to apply boundary layer techniques to predict the growth of the boundary layers on the walls of axial-flow compressors. These analyses are discussed in the review of the literature and it is noted that values of the gross correction factors leading to realistic results can be determined.

The complexity of flow through compressors requires the development of a theoretical model which can account for the turbulent structure of the flow close to the compressor walls. It was expected that the development of such a theoretical model would provide a better understanding of the flow in axial-flow compressors and, at the same time, that a more accurate method for predicting the design and off-design performance of axial-flow compressors would be made available.

The literature is first reviewed, the theoretical model is presented and a computer programme to provide solutions of the theoretical model is described in the

thesis. Experimental data necessary to describe the turbulent structure of the flow were obtained and finally the results obtained from the computer programme are presented and analysed.

CHAPTER 2

SURVEY OF PREVIOUS WORK

Historical Survey

As has been pointed out in the Introduction (Chapter 1) the prediction of off-design performance is an essential phase of a successful design process; the history of the prediction techniques is as old as the design itself. Although one of the first axial-flow compressors was conceived and developed by Sir. C.A. Parsons towards the end of the 19th century, the basic aero-thermodynamic requirements were not established until shortly before and during World War II. Following the early studies undertaken at the Royal Aircraft Establishment in 1936, technical reports by Howell (1942, 1945), Constant (1945) and Carter (1948) presenting design information on blading appeared. In Germany, during World War II, similar research led to the development of turbojet engines (Jumo 004 and BMW 003). In the United States, a first multistage axial-flow compressor designed on the basis of aerofoil theory was reported by Sinnette (1943) of the then National Advisory Committee for Aeronautics.

Attempts were made in some of the early axial-flow compressors to satisfy the equations of motion by using the simple radial equilibrium condition. A more accurate representation was provided by the actuator disc theory first reported by Ruden (1944) and later

developed by Hawthorne (1952) and Horlock (1952).

Horlock (1958) provided a complete review of the literature on axial-flow compressors and in the supplement, Horlock (1966), the more recent developments were presented.

A new era of design and development of Turbo-machinery began with the advent of high speed digital computers. For axial-flow compressors advanced solution techniques have been published by Swan (1961), Serovy (1963), Rowe (1965), Marsh (1966), Jansen (1967) and Novak (1967). Computerised methods have been established at the National Gas Turbine Establishment in Britain, at the National Aeronautics and Space Administration in the United States and at leading gas-turbine companies throughout the world.

Calculation of blade losses and fluid deflections which form an essential phase of the present day analysis techniques has been studied by various authors since the earliest correlations suggested by Howell (1942) and Carter (1950). An analytical approach put forward by Schlichting (1955) was based upon the main assumptions of two-dimensionality and incompressibility but the results did not compare well with experiment. Lieblein (1959, 1963) correlated experimental data and provided loss and deviation curves which could be used at both design and off-design incidence.

Classification of the Prediction Techniques

More recent procedures for solution of the flow are achieved in two phases; determination of flow in the meridional plane and a blade-to-blade solution. A theoretical solution for flow between blade rows, referred to by blade-to-blade solution, is not available at present and available cascade correlations are usually used instead. In addition, assumption of axisymmetry simplifies the problem. Determination of the flow pattern in the meridional plane is, therefore, a major analysis problem. Three practical ways of achieving this have been in use: The first of these is a 'Simple Radial Equilibrium' method where a simplified form of radial momentum equation with only the pressure and whirl velocity terms as non zero governs the flow. This method was later extended to include more terms in the radial equilibrium equation (3.27) and two new solution procedures have become commonly accepted, namely the 'Streamline Curvature' procedure and 'Finite-Difference' technique. The basis and success of these two methods is outlined in the following sections.

Streamline Curvature Method

This method for determining the radial distribution of flow properties from hub to tip ~~by~~ employs the radial equilibrium equation, energy equation and continuity condition for a number of axial stations. It is necessary, however, that these distributions, obtained

separately at each station should be consistent from station to station and the radial acceleration of fluid particles be accounted for. This is achieved by predicting shapes for the meridional streamlines consistent with the continuity condition at each station and expressing the radial acceleration in terms of streamline slope and curvature. A successive improvement of the streamline pattern is obtained. This general scheme was first proposed by Traupel (1942) but later independently conceived by many other analysts.

An approach by Novak (1967) specifically developed for axial compressors for use in jet engines employed a typical streamline curvature method. In his method, the radial equilibrium equations was rearranged by replacing the static-pressure gradient with its equivalent in terms of stagnation enthalpy and entropy. The radius of curvature, r_m , of the meridional projection of a streamlines was defined by

$$\frac{1}{r_m} = - \partial \phi / \partial m \quad \dots(2.1)$$

where $\phi = \tan^{-1} C_r / C_x$ and m is the meridional direction. The rearranged radial equilibrium equation was expressed as

$$\partial C_m^2 / \partial r + P(r) C_m^2 = T(r) \quad \dots(2.2)$$

where

$$P(r) = 2 \left[- \frac{\sin \phi}{C_m} \frac{\partial C_m}{\partial m} + \frac{\cos \phi}{r_m} + \frac{1}{2} \left(\frac{1}{Q} \cdot \frac{\partial Q}{\partial r} \right) \right] \quad \dots(2.3)$$

$$T(r) = 2 \left[\frac{1}{Q} \frac{\partial (H Q)}{\partial r} - \frac{C_\theta}{r} \frac{\partial (r C_\theta)}{\partial r} - \frac{C_\theta^2}{2} \left(\frac{1}{Q} \frac{\partial Q}{\partial r} \right) \right] \quad \dots(2.4)$$

For mechanised computational procedure equation (2.2) was expressed in a form,

$$\frac{\partial C_m}{\partial z} = \left[\frac{1}{2} T(z) \right] / C_m - \left[\frac{1}{2} P(z) \right] C_m \quad \dots(2.5)$$

This form is useful both for the design and off-design problems. However, for the off-design case, an alternative form of relations (2.3) and (2.4) are preferred for convenience in using cascade correlations. The alternative form is

$$T(z) = 2 \sin^2 \beta_m \left[\frac{1}{Q} \frac{\partial (I \cdot Q)}{\partial z} + \frac{\omega^2 z^2}{2} \left(\frac{1}{Q} \frac{\partial Q}{\partial z} \right) \right] \quad \dots(2.6)$$

$$P(z) = 2 \sin^2 \beta_m \left[- \frac{\sin \phi}{C_m} \frac{\partial C_m}{\partial m} + \frac{\cos \phi}{z_m} + \frac{\csc^2 \beta_m}{2} \left(\frac{1}{Q} \frac{\partial Q}{\partial z} \right) + \frac{1}{2} \frac{\partial \cot^2 \beta_m}{\partial z} + \frac{\cot^2 \beta_m}{z} + 2 \frac{\omega}{C_m} \cot \beta_m \right] \quad \dots(2.7)$$

where $\beta_m = \tan^{-1} C_m / W_\theta$.

Equations (2.6) and (2.7) reduce to simpler form (with $I = H$, $\omega = 0$, $\beta_m = \tan^{-1} C_m / C_\theta$) for stationary blade rows. The solution of equation (2.2) with subsidiary relations (2.6) and (2.7) was iterative in nature, starting with assumed streamlines and using correlated cascade information.

In the initial stages of development, the streamline curvature method suffered from the problems of non-convergence and instability but the problems have since been overcome and the method has become widely accepted in industry. Further investigation for handling cascade characteristics for use with this method was made by Jansen (1965). A study of the

streamline curvature method for supersonic flows is being conducted by Bindon (1968).

Finite Difference Approach

This scheme of design and off-design analysis employs the basic laws of fluid flow and thermodynamics in a more direct manner. The general approach consists of writing the equations governing the flow in terms of the stream function in each of the three zones: upstream of the blades, within the bladed passage and downstream of the blades. The equations are then arranged in an appropriate finite-difference form. The field is subdivided into a net work or grid system and a distribution of the principal variables is assumed at the grid points; the appropriate boundary conditions are specified. A numerical solution to the set of equations is then sought.

This procedure is essentially directed towards obtaining a meridional solution which must be supplemented by a blade-to-blade solution or a reliable cascade correlation.

For subsonic flow, the principle partial differential equation (considered in Chapter 3) appears in an elliptic form, solvable as a boundary value problem, but for supersonic flows the principal equation is hyperbolic and requires a different solution procedure.

Marsh (1966) developed a computer programme to calculate the "through flow solution" for a turbomachine using finite-difference method. The equations provided by Wu (1952) for inviscid flow were written in a form that could be used for mixed-flow and axial-flow turbomachines, both within and between the blade rows. A finite-difference

form of these equations was solved by a standard matrix inversion procedure.

Although the programme demonstrated the possibility of a finite-difference solution by the matrix method, a great deal remains to be done. The use of inviscid flow equations and neglect of cascade data are considered to be responsible for the discrepancy observed between the calculated and experimental velocity profiles especially in the hub and tip regions.

The programme did not lead to a converged solution when either the absolute or the relative Mach number approached unity. This difficulty was mainly due to the misleading results offered by the matrix inversion method of solving the set of algebraic equations.

Another difficulty inherent with the procedure was that the curved boundaries were dealt with by special unequal-grid formulae by using higher degree interpolations. In order to achieve more accurate solutions, the network may be made finer but the computer time for matrix inversion would increase seriously. In such cases where curved boundaries are unavoidable and the time factor and convergence are important considerations, alternative solution techniques must be explored. In most cases where the diagonal terms in the matrix predominate and a large number of null terms appear, some iterative techniques may be usefully employed.

Comparison of Streamline Curvature and Finite-Difference Techniques

The finite difference approach is, in principle, more general than the streamline curvature method described earlier. The approach has greater scope for more exact solutions attainable by reducing the number of assumptions in the equations and by improving the solution technique. Accuracy of the finite-difference form of the equations may be improved by using a large number of grid points in the flow field, the upper limit being provided by the consideration of round-off error and the memory locations in the digital computer.

At Mach numbers approaching unity, the streamline curvature method often encounters convergence problems which are difficult to overcome. In the finite-difference method, the convergence problems are not serious since these can be overcome by employing alternative numerical techniques and by exercising more control over the iterative solution.

These considerations led to the selection of the finite-difference approach for developing a general programme for the analysis of flow through axial-flow compressors. Alternative numerical methods associated with this approach are discussed in the following section.

Solution of Simultaneous Algebraic Equations

The finite difference approach described earlier provides one linear algebraic equation for each grid point, thus reducing the problems to that of finding a solution for a set of simultaneous linear algebraic equations.

Two methods are of interest; the matrix method and the iterative method.

Matrix Method.

The set of difference equations may be solved by an exact, matrix inversion method. Writing these equations as:

$$[A][X] = [R] \quad \dots(2.8)$$

where

$[A]$ is a square matrix of the order of the number of equations n and $[X]$ and $[R]$ are the column matrices with n elements in each.

If $[A]^{-1}$, the inverse of matrix $[A]$ is known, then on multiplying each term in Equation (2.8) by $[A]^{-1}$,

$$[X] = [A]^{-1} [R] \quad \dots(2.9)$$

which represents a solution of the set of equations.

Standard matrix-inversion techniques are available in the computer library.

The main limitation of the matrix method is the fact that the computer time can be excessive for a large number of grid points. In addition there is a possibility of a singular or a near singular matrix during the iteration procedure and this can lead to erroneous results.

Iterative Techniques

An alternative approach to solving a set of linear

algebraic equations which does not suffer from the drawbacks of the matrix inversion method is to follow a suitable iterative technique. A variety of procedures are available such as the Gauss-Reduction, Crout's, Jacobi's, extrapolated Liebmann's or Gauss-Seidel's approach. The procedures may also be classified as the stationary gradient methods and linearly accelerated ones. Selection of a technique depends primarily upon the system of equations and their convergence criteria. The Gauss-Seidel method is generally considered to be superior to any other in the solution of stress analysis, heat flow and other potential flow problems. The distinctive feature lies in the successive improvement of the value of each of the sought quantities. For example, given $x_1^1, x_2^1, \dots, x_n^1$ one may compute:

$$x_1^2 = (b_1 - a_{12}x_2^1 - \dots - a_{1n}x_n^1)/a_{11}$$

$$x_2^2 = (b_2 - a_{21}x_1^2 - \dots - a_{2n}x_n^1)/a_{22}$$

$$x_n^2 = (b_n - a_{n1}x_1^2 - \dots - a_{n,n-1}x_{n-1}^2)/a_{nn} \quad \dots(2.9)$$

The technique outlined above may be expressed in matrix equation for further consideration. As before, writing the set of difference equations as

$$[A] \cdot [X] = [R] \quad \dots(2.10)$$

and splitting $[A]$ into $[B]$ and $-[C]$,

$$[B] \cdot [X]^{K+1} = [C] \cdot [X]^K + [R] \quad \dots(2.11)$$

which represents a 'stationary process'.

Alternatively, let

$$[A] = [D] - [L] - [U] \quad \dots(2.12)$$

in terms of diagonal, lower and upper triangular

matrices. The Gauss-Seidel method is a particular case when

$$[B] = [D] \text{ and } [C] = [L] + [U] \quad \dots(2.13)$$

$$\text{If instead, } [B] = [D]/\omega - [L] \quad \dots(2.14)$$

$$\text{then, } [C] = [U] + (1 - \omega)/\omega [D], \omega > 0 \quad \dots(2.15)$$

This case represents a successive overrelaxation process as an improvement over the basic Gauss-Seidel method.

Convergence Problem in Iterative Techniques

The iterative techniques yield a converged solution for a boundary value problem provided certain convergence criteria are satisfied. From equation (2.11) it follows that

$$[B] ([X]^{K+1} - [X]) = [C] ([X]^K - [X]) \text{ and } \dots(2.16)$$

$$[X]^{K+1} - [X] = [B]^{-1} [C] ([X]^K - [X]) = ([B]^{-1} [C])^K ([X]^0 - [X]) \quad \dots(2.17)$$

$$\text{If } [X]^0 - [X] \text{ is expanded as } \alpha_1 u_1 + \alpha_2 u_2 \dots \alpha_n u_n \quad \dots(2.18)$$

where $u_1, u_2 \dots u_n$ are the eigenvectors and principal vector of $[B]^{-1} [C]$, corresponding to eigenvalues $\lambda_1, \lambda_2 \dots \lambda_n$, then

$$[X]^K - [X] = \alpha_1 \lambda_1^K u_1 + \alpha_2 \lambda_2^K u_2 + \dots \alpha_n \lambda_n^K u_n \quad \dots(2.19)$$

The progress of iteration depends therefore upon $\rho([B]^{-1} [C])$, the spectral radius of $[B]^{-1} [C]$. The spectral radius is the name given to the modulus of the largest eigenvalue. It has been established by Forsyth (1960) that for convergence

$\rho < 1$. The smaller the value of ρ , the quicker the

convergence process. Splitting $[A]$ into $[B]$ and $-[C]$ is thus an important task in achieving a rapidly converging process. This problem of spectral radius and of criteria of convergence has been the subject of Young (1960), Varga (1962) and others.

The relaxation method is an effective mode of solution particularly suited to digital computation. A 'molecular representation' of the partial differential equation is used in sequence at each grid point and the residual is minimized by successively improving the field values. Expressing $[B]$ and $[C]$ in terms of $[D]$, $[L]$, $[U]$ and ω as in Equations (2.14) and (2.15) for an accelerated Gauss-Seidel process and substituting in Equation (2.17) Fox (1962) obtained the relation

$$\det. \{ (\lambda + \omega - 1) [I] - \lambda \omega [E] - \omega [F] \} = 0 \quad \dots(2.20)$$

for the eigenvalues λ of iteration matrix M which is given by

$$[M] = ([I] - \omega [D]^{-1} [L]^{-1}) \{ (1-\omega) [I] + \omega [D]^{-1} [U] \} , \dots(2.21)$$

$$[E] = [D]^{-1} [L] , \quad [F] = [D]^{-1} [U] \quad \dots(2.22)$$

and $[I]$ is an identity matrix.

Using the "property A" as first defined by Young (1954), it is shown that,

$$\det. \{ (\lambda + \omega - 1) [I] - \lambda \omega [E] - \lambda \omega [F] \} = 0 \quad \dots(2.23)$$

Further, if μ are the eigenvalues of $([E] + [F])$

$$\det. (\mu [I] - [E] - [F]) = 0 \quad \dots(2.24)$$

Comparing these two sets of determinants

$$\lambda \omega^2 \mu^2 = (\lambda + \omega - 1)^2 : \quad \omega \neq 0, \quad \lambda \neq 0 \quad \dots(2.25)$$

It may be stated that 'property A' used above is an essential

feature of a matrix of linear equations but does not impose serious limitations on the type of problem in hand.

The choice of optimum value of ω , the extrapolation factor depends upon a knowledge of μ , the eigenvalues. For matrix $[A]$ symmetric and diagonally dominant, the eigenvalues are all real and less than unity in modulus, optimum ω may be expressed in terms of $\bar{\mu}$, the eigenvalue of largest modulus as

$$\omega = 2/(1 + \sqrt{1 - \bar{\mu}^2}) \quad \dots(2.26)$$

This is, however, the simplest case. Studies have been made by Forsyth (1960) and Fox (1962) for cases of matrix

A without "property A" and for over relaxation by lines.

The convergence criteria, in general, have not yet been established and some trial and error is required for the particular type of equations being considered. The availability of high speed digital computers permits the over relaxation factor to be established for a class of problems by analysing the effect of different choices of ω for a simpler case.

The relaxation procedure for solving the finite-difference equations was considered superior to the matrix inversion method and to alternative matrix methods and was selected as a numerical procedure in the computer solution to predict the off-design performance of the axial-flow compressors.

Assumption of Axisymmetry

As has been noted, the assumption of axisymmetry is common to most of the analyses reported. The significance of this assumption is discussed below.

Axisymmetry is equivalent to assuming that the three dimensional flow through a blade row can be represented, fairly accurately by a circumferential average of the flow conditions at each radial point. A major mathematical simplification is achieved by considering that $\partial/\partial\theta \equiv 0$. In the first instance this assumption enables a streamfunction to be defined from the continuity equation and secondly it reduces the equations governing the flow to a simpler form readily amenable to a solution. An axially symmetric flow may be considered as the one restricted to hypothetical surfaces formed when the number of blades tend to infinity such that the blade thickness to spacing ratio remains the same. The total circulation associated with the blades thus remains constant. The blades may still be assumed to retain their turning effect in accordance with the normal blade-element theory. For stations within the blade rows, the action of the blades is computed by representing the blades by a distribution of forces in the flow equations.

Two Dimensional Cascade Correlations

Use of cascade information for design and off-design prediction problems was recognised in the earlier stages of development of the compressor.

N.G.T.E. Correlations

One of the earliest correlations was made by Howell (1942) using an arbitrary definition of 'Stall' as the

condition where loss becomes twice the minimum loss and of 'Nominal' condition where deflection = 0.8* stall-deflection. The deviation, defined by Constant's rule, was expressed as

$$\delta = m \theta \sqrt{(s/c)} \quad \dots(2.27)$$

where m was a function of the maximum camber ($\approx .26$) and θ = camber angle.

Howell produced a correlation for nominal deflection with nominal outlet angle, presented Reynolds-number and space/chord correction factors and also suggested a method of calculating deflection for a given incidence.

Carter (1950) prepared a correlation for 'optimum' conditions where lift/drag ratio was highest. The optimum deviation was expressed as

$$\delta = m \theta \sqrt{(s/c)}$$

where m was a function of stagger.

Optimum incidence was plotted against θ at different values of S/C and outlet angle β_2 .

Carter's suggestion for an off-design correlation was similar to that of Howell.

N.A.C.A. Correlations

Another series of correlated cascade data were reported by Lieblein (1959) at N.A.C.A. These correlations were based upon a large number of cascade tests with wall-suction to eliminate boundary layer effects. Correlations for design conditions and off-design prediction were prepared in a set of curves. Further studies in stagnation pressure loss evaluation were made and correlations using 'Equivalent

Diffusion Factor' were proposed by Lieblein (1965). More details of Lieblein's correlations are given in Chapter 3.

Serovy (1957, 1963) used the Lieblein correlation in his computer programme based upon simple radial equilibrium equations. Swan (1961) extended the Lieblein's correlations for transonic axial-flow compressors.

Recent Studies in Turbulent Boundary Layers

Some advances have been made, in recent years, in the prediction of compressible turbulent boundary layers.

Stratford (1967) has presented a method of calculating the growth of the annulus boundary layers and estimating the blockage factor which has not, until recently, been calculable. This was achieved by considering an axial momentum balance in the mainstream and another axial momentum balance in the boundary layer. With the assumption that the static pressure and the blade force were constant in the boundary layer and neglecting the effects of tip clearance, the growth in axial momentum thickness was given by

$$\delta(\rho C_x^2 \theta) = \tau_x \delta x - \rho C_x (\delta C_x) \quad \dots(2.28)$$

Numerical integration of this equation along the length of the compressor was proposed so as to include it within the overall iterative solution procedure. Local values of ρ , C_x and μ were used and the axial skin friction component τ_x was approximately determined from a consideration of flow on a flat plate.

Although this simplified blockage calculation technique appears to be a preliminary representation of the complex annulus boundary layers, it is reported to yield results comparable with experiment and with some correlations previously proposed by Stratford (1961).

Another report describing the mechanics of boundary layers associated with blades and side walls in turbomachinery was published by Schlichting (1959). The importance of intimate synthesis of theory and experiment to analyse the real flow in cascades has been emphasized, and some of the results for lift, drag and loss coefficient have been achieved. Further developments in this direction may provide more useful results.

Some basic studies in turbulent flows have been in progress at Imperial College, London. Spalding's (1967-68) recent monograph on turbulent boundary layers have revealed a possibility of analysing complex flows. Formulation of the primary and secondary conservation laws and consequent simplification by introducing convenient and justifiable concepts of effective exchange coefficients have been published. The initial progress in confined turbulent flows such as developing flows in straight circular pipes and diffusers was based upon the boundary layer solution procedure developed earlier and with an assumption for the axial pressure gradient dp/dx . Further work in turbulent flows suggested that a more exact model of turbulence from a basic theoretical concept is necessary in order to achieve reliable predictions and such work is now in progress.

A finite-difference integration procedure for the prediction of flow and mixing patterns in a confined steady,

turbulent, axisymmetric diffusion flame in combustion chambers has been published by Spalding and Pun (1967). The equations governing the flow were considered in turbulent form and were linked together by auxiliary relations from the thermodynamics of flow and by intuitive guesswork about the effective transport properties of the turbulent flow. In general, it was suggested that the effective viscosity should be expressible in terms of local fluid variables, such as,

$$\mu_{eff} = \ell^2 \rho \left[\left(\frac{\partial C_x}{\partial z} \right)^2 + \left(\frac{\partial C_1}{\partial x} \right)^2 + \left(\frac{\partial C_2}{\partial x} \right)^2 + \left(\frac{\partial C_3}{\partial z} \right)^2 \right]^{0.5} \quad \dots(2.29)$$

but this relation was not used in the analysis. However, for simplicity it was assumed that,

$$\mu_{eff} = K.(D^2 \rho^2 / (W \dot{m} C^2))^{1/3} \quad \dots(2.30)$$

where D and W are chamber dimensions and $\dot{m}$ = mass flow rate of gas.

The flowfield was divided into a rectangular network and iterative solutions for some practical cases were obtained. The predictions were plausible only in their qualitative features since the distribution of effective viscosity was obtained by trial and error in selecting a reasonable value of K in Equation (2.30). A theoretical basis for the effective-viscosity hypothesis must, therefore, be established before the solution procedure is extended to deal with more complicated flows as in axial flow compressors.

Furthermore, an extension of this theoretical approach to the compressor problem would also require a complete idealisation of the blades, secondary flows and losses which, as pointed out earlier, is difficult at present. Spalding's approach has, however, scope for future application to such more complex flowfields.

Hot Wire Measurements

Although hot wire anemometry has gained wide popularity in the field of aerodynamics and in the studies of boundary layers, its use has been very restricted as far as the analysis of the structure of flow within a turbomachine is concerned. Hot-wire transducers have been used for time indication during stall propagation in compressors and cascade wind tunnels by Carmichael (1958). The trace from this probe indicated the presence of a stall cell by a large amplitude trace of random fluctuations due to the turbulence during separation. The probe was considered effective since the beginning and end of separation were well defined.

Measurement of the turbulence intensity in three turbomachinery wind tunnels and in a single stage axial flow compressor have been reported by Shaw (1964). A DISA constant temperature anemometer, with external potentiometer and galvanometer for accurate measurement of wire voltage, was used. Level of turbulence in the wind tunnel was recorded between 0.3 and 0.5% in all cases whereas the level at spaces between the blade rows of the compressor was reported as

4 to 5%, at diffuser exit 10 to 11%, and outside the compressor approximately 50%. Higher turbulence intensity at diffuser exit and outside the compressor is mainly due to lower mean velocity which enter the definition of the intensity as

$$\sqrt{\frac{1}{3}(\overline{C_x^2} + \overline{C_i^2} + \overline{C_e^2})} / U * 100 \%. \quad \text{Another reason for higher}$$

atmospheric turbulence was established as being due to the narrow return passage for air in the small room housing the compressor assembly. It was also reported that a fine gauze installed at inlet to the compressor reduced the intensity of turbulence by about 40% at locations upstream of the I.G.V. and by 20% at spaces between the blade rows.

The results on the compressor in the above measurement were not considered conclusive since the level of turbulence dropped with the flow through the blades and a check showed that the atmospheric turbulence near the entry was excessive due to the unsteady flow in the return air circuit in the small room. Confirmation was obtained by the authors by an analysis of the frequency spectrum, showing that very long frequency turbulence occurred in the return air circuit and very high frequency turbulence existed immediately downstream of the stator row.

Reports are also available about the use of a hot wire in supersonic flow. It may be remarked that no attempts have been reported elsewhere regarding the measurement of the apparent shear stresses and effective viscosity within a turbomachinery. However, a large amount of work on this technique in boundary layer problems is available, chiefly due to Laufer (1951), Newman (1951), Sandborn (1955) and Ferris (1966).

Although these authors were referred to, a summary of literature on hot wire anemometry is considered beyond the scope of this presentation. This is because little information of direct application to the present measurements was made available. Development of a special probe, calibration and the measurement technique are discussed in Chapter 5.

Optical Flow Studies

Optical techniques for the measurement of three dimensional flow patterns have met with certain success. The basic advantage lies in obtaining a qualitative and quantitative information without disturbing the fluid. This approach is particularly useful in visualising a flow relative to the rotor (by rotating the camera at the same speed) and for separated and circulating flows where conventional means fail to operate. The technique requires essentially a transparent inspection window and provisions to illuminate the tracer particles by a beam of light at right angles to the direction of observation. Arrangements must also be made to obtain a series of pictures at intervals of a fraction of a millisecond (usually 0.5 msec.)

The path lines show up as dotted lines on the film, and the sense of the velocity vector is guessed easily. The magnitude of the velocities may be worked out by the length of the dotted lines and the spaces between them. Colour

photography offers an interesting approach in the measurements of the three components of velocity. Here, parallel beams of different coloured light are projected into the passage and a particle would appear as a definite colour when approaching or receding away from the camera. The scheme can work very well if the velocity in line with the optical axis of the camera is small. The lengths of the streaks on the film yield information about the velocities in a plane perpendicular to the optical axis. Photographic studies have been reported by Young (1955) and Simpson and Cinnamond (1964).

Laser Flow Studies

Recent studies in utilizing the Doppler Shift of scattered laser radiation for turbulent flow measurements by Goldstein (1967) and Yeh (1964) have revealed that such a technique could provide an alternative to the hot wire method.

A typical apparatus would consist of a source of monochromatic light (He-Ne laser), a splitter plate to divide the beam into two and a set of mirrors to reunite the beams to focus at the same point in the test section. The reference beam is passed directly to the photo-cathode whereas a fraction of light from the other beam that is scattered in the direction of the photomultiplier combines with it. The resulting beat signal is assessed by a spectrum analyser and a heterodyne, to give the Doppler shift. The intensity of the beat signal at a given

frequency bears a proportionality with the fraction of time at which the fluid is at the velocity corresponding to that Doppler frequency. In a turbulent flow this would correspond to the probability function for the turbulent velocity. Assuming the Gaussian velocity probability-function to represent the line profile for turbulent flow, relations may be established to evaluate the r.m.s. values of turbulent intensities.

The successful experiments performed thus far have only considered the flow of water with the light scattering produced by 0.6μ or smaller polystyrene particles, 1:30000 by volume concentration. No success has yet been reported in the measurement of turbulent shear stress and other correlations. This method promises scope for future measurements, especially in separated and recirculating flows.

CHAPTER 3

EQUATIONS AND CASCADE CORRELATIONS FOR FLOW THROUGH TURBOMACHINES

The purpose of this chapter is two-fold;

- 1) to present the basic equations of fluid flow in terms of selected variables and in a form convenient for the present problem and
- 2) to discuss the information required to define the blade element performance.

The chapter has, therefore, been split into two sections, A - Consideration of the basic equations and B - Cascade correlations. The computer programme uses the equations derived in section A together with the correlations of section B to provide the basis for predicting the off-design performance of axial-flow compressors. The method, in principle, is also valid for studying the development of flow in a passage. An adaptation of the technique for analysing the flow through circular pipes and parallel annuli is also described.

SECTION A

CONSIDERATION OF THE BASIC EQUATIONS GOVERNING THE FLOW.

In general, the three-dimensional compressible, turbulent flow of a viscous fluid, is governed by the basic laws of aero-thermodynamics as follows:

1. Law of conservation of mass (Continuity equation)
2. Newton's second law of motion (Momentum equation)
3. First law of Thermodynamics (Energy equation).

The three independent equations obtained from the

basic laws define the flow of a gas through a turbomachine completely; that is they provide a sufficient number of relations for the evaluation of the velocity vector and two thermodynamic properties of the gas.

Since the basic equations do not relate the same variables in the original form, it is necessary to invoke more relations in order that more than five variables are introduced. The additional relations are as follows:

$$p = \rho g_s R T \quad (\text{perfect gas law}) \quad \dots(3.1)$$

$$u = C_v T \quad (\text{internal energy for perfect gas}) \quad \dots(3.2)$$

$$p/\rho^\gamma = C. \quad (\text{reversible adiabatic process}) \quad \dots(3.3)$$

$$H = u + p/\rho g_s J \quad (\text{definition of specific enthalpy}) \quad \dots(3.4)$$

$$T ds = dH - dp/\rho g_s J \quad (\text{definition of entropy}) \quad \dots(3.5)$$

$$H_0 = H + C^2/2g_s J \quad (\text{stagnation enthalpy}) \quad \dots(3.6)$$

$$R = (C_p - C_v) J \quad (\text{gas constant}) \quad \dots(3.7)$$

$$\gamma = C_p/C_v \quad (\text{index of adiabatic process}) \quad \dots(3.8)$$

The purpose of this section is to list the basic equations and to express them in a more useful form.

Selection of the Variables

In the following sections, the equations have been set up and rearranged in terms of Ψ , C_θ , H_0 and s as the basic variables. Other variables were linked together using the additional relations (3.1) to (3.8).

The use of Stoke's streamfunction Ψ in preference to two variables C_x and C_r resulted in the reduction of basic unknown variables and the governing equations by one. The equilibrium equation for inviscid flow is reduced to elliptic

form in terms of ψ . For a turbulent flow, the equations are more complicated but ellipticity has still been assumed to hold by expressing Equation (3.27) in a form (A.I.1) for a numerical solution. Introduction of ψ , however, increased the number of total unknowns by one and the number of total equations also increased by one since the continuity equation was replaced by two relations arising from the definition of ψ .

The thermodynamic variables H_0 and s were introduced in a conventional manner to eliminate the pressure terms from the momentum equations. The energy addition in a rotor blade row appears as H_0 while it is convenient to express the stagnature pressure losses in the form of entropy rise.

Continuity Equation

The law of conservation of mass applied to a compressible flow with axisymmetry under steady state leads to the continuity equation,

$$\frac{1}{r} \frac{\partial}{\partial r}(\rho \cdot C_r \cdot r) + \frac{\partial}{\partial x}(\rho C_x) = 0. \quad \dots(3.9)$$

For turbulent flows, the instantaneous quantities may be split into mean and fluctuating components. Thus,

$$C_x = \bar{C}_x + C'_x, \quad C_r = \bar{C}_r + C'_r, \quad \rho = \bar{\rho} + \rho' \quad \dots(3.10)$$

Time averaged continuity equation becomes,

$$\frac{1}{r} \frac{\partial}{\partial r}(\bar{\rho} \cdot \bar{C}_r \cdot r) + \frac{\partial}{\partial x}(\bar{\rho} \bar{C}_x) + \frac{1}{r} \frac{\partial}{\partial r}(\overline{\rho' C'_r \cdot r}) + \frac{\partial}{\partial x}(\overline{\rho' C'_x}) \quad \dots(3.11)$$

This equation shows the the mean velocity and density do not satisfy the simple continuity equation. However, since the density and velocity fluctuations are small compared to the average values, the last two terms, following convention, are neglected.

A stream function Ψ associated with the meridional velocity components may be defined by

$$\frac{\partial \Psi}{\partial z} = \rho r C_x f \quad \text{and} \quad \frac{\partial \Psi}{\partial x} = -\rho r C_z f \quad \dots(3.12)$$

such that the equation of continuity is identically satisfied and replaced by these relations. The introduction of an arbitrary factor f makes it possible to specify $\Psi = 0$ and $\Psi = 100$ at the inner and outer boundaries respectively.

Although the streamfunction has been defined in terms of instantaneous velocities, it was assumed to hold good for the temporal mean velocity components in a turbulent flow.

From the definition of Ψ (3.12), it follows that

$$\frac{1}{\rho r f C_x} \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{z} + \frac{1}{\rho} \frac{\partial \rho}{\partial z} + \rho z f \frac{\partial \Psi}{\partial z} \cdot \frac{\partial C_x}{\partial z}$$

or

$$\rho z f \frac{\partial C_x}{\partial z} = \frac{\partial^2 \Psi}{\partial z^2} - \frac{\partial \Psi}{\partial z} \left(\frac{1}{z} + \frac{\partial \ln \rho}{\partial z} \right) \quad \dots(3.13)$$

and similarly,

$$-\rho z f \frac{\partial C_z}{\partial x} = \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial \Psi}{\partial x} \left(\frac{\partial \ln \rho}{\partial x} \right) \quad \dots(3.14)$$

These relations have been used in the reorganisation of the momentum equations at a later stage and elsewhere. It may, however, be noted that for the incompressible case, relations

$$(3.13) \text{ and } (3.14) \text{ reduce to } \rho z f \frac{\partial C_x}{\partial z} = \frac{\partial^2 \Psi}{\partial z^2}, \quad -\rho z f \frac{\partial C_z}{\partial x} = \frac{\partial^2 \Psi}{\partial x^2} \dots(3.15)$$

and for the special case of irrotationality,

$$\text{when, } \frac{1}{z} \frac{\partial}{\partial z} (z C_x) - \partial C_z / \partial x = 0 \quad \dots(3.16)$$

relations (3.15) may be combined into a single equation,

$$\frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{z} \frac{\partial \Psi}{\partial z} = 0 \quad \dots(3.17)$$

The flow rate through the compressor annulus was estimated as follows, by referring to an axial position with uniform axial velocity:

$$\psi = \rho f C_x \int r dr = \rho f C_x r^2/2 + K$$

Specifying $\psi = 0$ at the rotor wall, $r = R_{in}$ and $\psi = 100$ at the casing, $r = R_{out}$,

$$K = -R_{in}^2/2, \quad f = 200/(\rho C_x (R_{out}^2 - R_{in}^2)) \quad \dots(3.18)$$

and the weight flow rate is given by

$$\rho g C_x (R_{out}^2 - R_{in}^2) \pi = \frac{200}{f} \cdot \pi \cdot g \quad \dots(3.19)$$

Momentum Equations

Newton's second law of motion when applied to the equilibrium of an elementary fluid mass in axisymmetric flow of a compressible and variable-viscosity fluid results in the following three equations of motion:

$$\rho \left(\frac{DC_r}{Dt} - \frac{C_r^2}{r} \right) = \left\{ \frac{1}{r} \left(\frac{\partial}{\partial r} (r \tau_{rr}) \right) + \tau_{\theta\theta}/r + \frac{\partial \tau_{rx}}{\partial x} \right\} + F_r, \quad \dots(3.20)$$

$$\rho \left(\frac{DC_\theta}{Dt} + \frac{C_r C_\theta}{r} \right) = \left\{ \frac{1}{r} \left(\frac{\partial}{\partial r} (r^2 \tau_{r\theta}) \right) + \frac{\partial \tau_{\theta x}}{\partial x} \right\} + F_\theta \quad \text{and} \quad \dots(3.21)$$

$$\rho \frac{DC_x}{Dt} = \left(\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rx}) + \frac{\partial \tau_{xx}}{\partial x} \right) + F_x \quad \dots(3.22)$$

where the stress components are,

$$\tau_{rr} = \mu \left(2 \cdot \partial C_r / \partial r - \frac{2}{3} (\nabla \cdot \vec{C}) \right) - p$$

$$\tau_{\theta\theta} = \mu \left(2 \cdot C_r / r - \frac{2}{3} (\nabla \cdot \vec{C}) \right) - p$$

$$\tau_{xx} = \mu \left(2 \cdot \partial C_x / \partial x - \frac{2}{3} (\nabla \cdot \vec{C}) \right) - p$$

$$\tau_{r\theta} = \mu \left(\partial C_\theta / \partial r - C_\theta / r \right)$$

$$\tau_{rx} = \mu \left(\partial C_x / \partial r + \partial C_r / \partial x \right)$$

$$\tau_{x\theta} = \mu \cdot \partial C_\theta / \partial x \quad \dots(3.23)$$

and $\nabla \cdot \vec{C} = \frac{1}{r} \frac{\partial}{\partial r} (r C_r) + \partial/\partial x C_x$ for axisymmetric flow.

For turbulent flow, relations such as (3.10) may be invoked and substitution in the equations of motion and temporal averaging would lead to valid equations. This procedure would result in a large number of double and triple correlations among the velocity components, density and their fluctuations, as has been shown in Appendix III. An analysis in respect of the order of magnitude of those correlation terms is unlikely due to lack of experimental information and hence, this procedure is not used.

Alternatively, gross assumptions regarding the existence of an 'effective viscosity' and other exchange coefficients are necessary. Such assumptions are also needed in simplifying the energy and entropy equations and these have been treated separately (Equations 3.51) in this chapter.

With the introduction of exchange coefficients the stress terms in the radial equation, $\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial \tau_{rx}}{\partial x} - \frac{\tau_{\theta\theta}}{r}$ are replaced by $\frac{1}{r} \left(\frac{\partial}{\partial r} (-pr + 2\mu r \frac{\partial C_r}{\partial r} - \rho r \bar{C}_r^2) \right) + \frac{\partial}{\partial x} (\mu (\frac{\partial C_r}{\partial x} + \frac{\partial C_x}{\partial r}) - \rho \bar{C}_r' \bar{C}_x') + \frac{p}{r} - 2\mu \frac{C_r}{r^2} + \frac{\rho}{r} \bar{C}_\theta^2$
 $\approx -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (2\mu_{rz\text{eff}} r \cdot \frac{\partial C_r}{\partial r}) + \frac{\partial}{\partial x} (\mu_{rx\text{eff}} \cdot \frac{\partial C_x}{\partial r})$

In terms of the streamfunction, assuming that this term is unaffected by compressibility, it reduces to

$$\frac{1}{r^2} \left[\frac{2}{r} \frac{\partial}{\partial r} (\mu_{rz\text{eff}} r \left(\frac{\partial^2 \psi}{\partial r \partial x} \cdot \frac{1}{r} - \frac{\partial \psi}{\partial x} \frac{1}{r^2} \right)) + \frac{\partial}{\partial x} (\mu_{rx\text{eff}} \left(\frac{1}{r} \frac{\partial^2 \psi}{\partial r \partial x} - \frac{1}{r^2} \frac{\partial \psi}{\partial r} \right)) - \rho f \frac{\partial p}{\partial r} \right] \quad \dots (3.24a)$$

In a similar way, the stress terms in the axial and circumferential equilibrium equations appear as

$$\frac{\partial}{\partial x}(2\mu_{xx\text{eff}} \frac{\partial C_x}{\partial x}) + \frac{\partial}{\partial r}(\mu_{xr\text{eff}} \frac{\partial C_x}{\partial r}) \quad \dots(3.24b)$$

and,

$$\frac{\partial}{\partial x}(\mu_{xo\text{eff}} \frac{\partial C_o}{\partial x}) + \frac{\partial}{\partial r}(\mu_{or\text{eff}} \frac{\partial C_o}{\partial r}) \quad \dots(3.24c)$$

respectively.

It was considered more convenient from the point of view of a solution that the momentum equations be transformed in terms of Ψ , H_o and s as the basic variables. Some other variables, connected to these by algebraic relations were, however, retained. From equation (3.6),

$$\frac{\partial H_o}{\partial r} = \frac{\partial H}{\partial r} + (C_x \frac{\partial C_x}{\partial r} + C_o \frac{\partial C_o}{\partial r} + C_r \frac{\partial C_r}{\partial r})/g.J$$

Combining this with the definition of entropy (3.5),

$$\frac{\partial H_o}{\partial r} = T \frac{\partial s}{\partial r} + (1/\rho \frac{\partial p}{\partial r} + C_x \frac{\partial C_x}{\partial r} + C_o \frac{\partial C_o}{\partial r} + C_r \frac{\partial C_r}{\partial r})/g.J \quad \dots(3.25)$$

Eliminating $\partial p/\partial r$ between this and (3.20),

$$C_x(\frac{\partial C_r}{\partial x} - \frac{\partial C_x}{\partial r}) + g.J \frac{\partial H_o}{\partial r} - g.J T \frac{\partial s}{\partial r} - \frac{C_o}{r} \frac{\partial (r C_o)}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r}(\mu_{rr\text{eff}} r \cdot r \cdot \frac{\partial C_r}{\partial r}) + \frac{\partial}{\partial x}(\mu_{rx\text{eff}} \frac{\partial C_x}{\partial r}) \quad \dots(3.26)$$

Expressing it in terms of Ψ and rearranging,

$$\begin{aligned} \frac{\partial^2 \Psi}{\partial r^2} + \frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{r} \frac{\partial \Psi}{\partial r} = & -(e \cdot r \cdot f)^2 / \frac{\partial \Psi}{\partial r} \left[\frac{C_o}{r} \frac{\partial}{\partial r}(r C_o) - g.J \frac{\partial H_o}{\partial r} + \right. \\ & g.J T \frac{\partial s}{\partial r} \left. \right] - (e r f)^2 / \frac{\partial \Psi}{\partial r} \left[-\frac{r}{2} \frac{\partial}{\partial r}(\mu_{rr\text{eff}} r \cdot \frac{\partial}{\partial r}(1/(e \cdot r \cdot f) \frac{\partial \Psi}{\partial x})) \right. \\ & + \frac{\partial}{\partial x}(\mu_{rx\text{eff}} \frac{\partial}{\partial r}(1/(e \cdot r \cdot f) \frac{\partial \Psi}{\partial r})) \left. \right] + \\ & (\frac{\partial \Psi}{\partial r} \frac{1}{e} \frac{\partial e}{\partial r} + \frac{\partial \Psi}{\partial x} \frac{1}{e} \frac{\partial e}{\partial x}). \end{aligned} \quad \dots(3.27)$$

This constitutes the principal equation and has sometimes been referred to as the transformed radial-equilibrium equation. Solution of this equation requires a simultaneous consideration of the other equations governing the flow. It is the purpose of the following pages to establish other relations in a convenient form.

The axial equilibrium equation, on transformations as above, appears similar to (3.27) excepting that the stress term is

$$-\frac{(r_2 f)^2}{\partial \Psi / \partial r} \left[\frac{\partial}{\partial x} \left(2 \mu_{xx} \left(\frac{1}{r_2 f} \frac{\partial \Psi}{\partial r} \right) \right) + \frac{\partial}{\partial r} \left(\mu_{rr} \left(\frac{1}{r_2 f} \frac{\partial \Psi}{\partial r} \right) \right) \right] \quad \dots (3.28)$$

and in tangential direction,

$$\frac{\partial \Psi}{\partial r} \frac{\partial (r C_\theta)}{\partial r} + \frac{\partial \Psi}{\partial x} \frac{\partial (r C_\theta)}{\partial x} + \frac{\partial}{\partial x} (\mu_{x\theta} \frac{\partial C_\theta}{\partial x}) + \frac{\partial}{\partial r} (\mu_{r\theta} \frac{\partial C_\theta}{\partial r}) = 0 \dots (3.29)$$

Energy Equation

In a familiar form, the energy equation appears as,

$$\frac{DH_0}{Dt} = Q + \frac{\rho}{\rho} + \frac{1}{\rho} \frac{\partial p}{\partial t} + \vec{C} \cdot \left[\vec{F} + \frac{\mu}{\rho} \left\{ \nabla^2 \vec{C} + \frac{1}{3} \nabla (\nabla \cdot \vec{C}) \right\} + \frac{1}{\rho} \left\{ 2 [(\nabla \mu) \cdot \nabla] \vec{C} + (\nabla \mu) \times (\nabla \times \vec{C}) - \frac{2}{3} (\nabla \cdot \vec{C}) (\nabla \mu) \right\} \right] \quad \dots (3.30)$$

for compressible, time unsteady, viscous flow in three dimensions.

The equation reduces with proper units to,

$$g_0 J \frac{DH_0}{Dt} = g_0 J \left(C_x \frac{\partial H_0}{\partial x} + C_r \frac{\partial H_0}{\partial r} \right) = \vec{F} \cdot \vec{C} + Q \quad \dots (3.31)$$

for the special case of steady, axially symmetric flow of a non-viscous fluid. Further, in an inviscid flow, the blade force must be normal to the surface of the blade, and

consequently to the relative velocity of the gas; that is

$$\vec{F} \cdot (\vec{C} - \vec{U}) = 0. \quad \dots(3.32)$$

$$\text{or } \vec{F} \cdot \vec{C} = \vec{F} \cdot \vec{U} = F_x C_x + F_r C_r + F_\theta C_\theta = F_\theta \cdot r \omega.$$

Rewriting the circumferential equilibrium equation (3.21)

$$F_\theta = \left[C_r \frac{\partial(r C_\theta)}{\partial r} + C_x \frac{\partial(r C_\theta)}{\partial x} \right] / r = \frac{1}{r} \frac{D(r C_\theta)}{Dt}$$

With these relations, the energy equation becomes:

$$g \cdot J \frac{DH_\theta}{Dt} = Q + \omega \frac{D(r C_\theta)}{Dt} \quad \dots(3.33)$$

Assuming the flow of gas as an adiabatic process,

$$Q = 0.$$

$$g \cdot J \frac{DH_\theta}{Dt} = \omega \frac{D(r C_\theta)}{Dt} \quad \text{or,} \quad \dots(3.34)$$

$$g \cdot J \left(C_x \frac{\partial H_\theta}{\partial x} + C_r \frac{\partial H_\theta}{\partial r} \right) = \omega \left\{ C_x \frac{\partial(r C_\theta)}{\partial x} + C_r \frac{\partial(r C_\theta)}{\partial r} \right\}$$

If $Cr \approx 0$, then it reduces to

$$g \cdot J \frac{\partial H_\theta}{\partial x} = \omega \frac{\partial(r C_\theta)}{\partial x} \quad \dots(3.35)$$

which is, as stated across a blade row,

$$g \cdot J (H_{\theta 2} - H_{\theta 1}) = \omega (r_2 C_{\theta 2} - r_1 C_{\theta 1}) \quad \dots(3.36)$$

It has been shown by Wu (1950) that the energy relation $g \cdot J \frac{DH_\theta}{Dt} = \omega \cdot \frac{D(r C_\theta)}{Dt}$ can also be derived for a viscous fluid under the same flow conditions as before.

This however, involves an additional assumption that the heat generated by the frictional work (as the gas particles do work against the viscous forces exerted by the surrounding particles) is added back to the same gas stream, thus

nullifying the negative work in overcoming viscous forces. More general forms of energy equations are also obtainable but the success of a form lies in its usefulness in dealing with turbulent flows. Hence, with the simplified energy equation obtained above, it was thought satisfactory to proceed as follows:

Assuming that an effective viscosity μ_{eff} and effective Prandtl number Pr_{eff} can be defined by (3.51) to (3.53) as to represent the heat transfer by molecular and turbulent transport processes and assuming further that the effective exchange coefficients for turbulent flow replace the molecular counterparts in laminar flow, the radial and axial components of the effective heat flux vector are given by $-\mu_{\text{eff}}/Pr_{\text{eff}} \cdot g_o T/\rho \frac{\partial H_o}{\partial r}$ and $-\mu_{\text{eff}}/Pr_{\text{eff}} \cdot \frac{g_o T}{\rho} \frac{\partial H_o}{\partial x}$ respectively. Expressing the heat flux vector analogous to the conduction terms in laminar flow, the energy equation may be written in a form

$$g_o T \left(C_x \frac{\partial H_o}{\partial x} + C_r \frac{\partial H_o}{\partial r} \right) = \omega \left(C_x \frac{\partial (r C_\theta)}{\partial x} + C_r \frac{\partial (r C_\theta)}{\partial r} \right) + g_o T \left\{ \frac{\partial}{\partial x} \left(\frac{\mu_{\text{eff}}}{Pr_{\text{eff}}} \frac{\partial H_o}{\partial x} \right) + \frac{\partial}{\partial r} \left(\frac{\mu_{\text{eff}}}{Pr_{\text{eff}}} \frac{\partial H_o}{\partial r} \right) \right\} \quad \dots(3.37)$$

Simplification of this equation for flow through axial flow compressors can be achieved by assuming that $Cr \approx 0$ and

$\partial H_o / \partial r = 0$. Under such conditions, the energy equation reduces to

$$g_o T \frac{\partial H_o}{\partial x} = \omega \frac{\partial (r C_\theta)}{\partial x} + \frac{g_o T}{\rho C_x} \frac{\partial}{\partial x} \left(\frac{\mu_{\text{eff}}}{Pr_{\text{eff}}} \frac{\partial H_o}{\partial x} \right) \quad \dots(3.38)$$

The last term in equation (3.38) is only one approximate form of the contribution by turbulence depending mainly upon the assumptions made and on the definition of effective exchange coefficients.

Entropy Equation

Choice of entropy as a thermodynamic variable necessitates the formulation of an entropy equation. This has been obtained from the statement of the first law of thermodynamics and equilibrium considerations as follows:

According to the first law,

$$\frac{\partial}{\partial t} \left[\left(u + \frac{C^2}{2} + \varepsilon \right) \rho \right] + \text{Div} \left[\left(H + \frac{C^2}{2} + \varepsilon \right) \vec{G} \right] = H_0 \quad \dots(3.39)$$

where u = specific internal energy = $H - p/\rho g_T$

and ε = specific potential energy of the fluid, the other variables being with usual notation.

For time-steady process in the absence of body forces, it

$$\text{reduces to } \text{Div} (H \cdot \vec{G}) = H_0 \quad \dots(3.40)$$

Considering the net heat transfer per unit volume due only to conduction, that is $-\text{Div} \vec{q}$ where, $\vec{q}$ = heat conduction vector

$$= k \frac{\partial T}{\partial x} + k \frac{\partial T}{\partial r} \quad \text{for axisymmetric case, the external work}$$

done per unit volume as a consequence of the motion of the

points of application of the shear stresses is $\sum \text{Div} (C_i \tau_i)$.

The inflow of enthalpy per unit volume resulting from the diffusive transfer may be expressed as $\text{Div} (H \cdot \vec{G}_d)$, hence

$$H_0 = -\text{Div} (\vec{q} + \sum \text{Div} (C_i \tau_i) + \text{Div} (H \cdot \vec{G}_d))$$

For turbulent flow, invoking relations as

$$H = \bar{H} + H', \quad H_0 = \bar{H}_0 + H'_0 = \bar{H} + H' + \bar{k} + k' + \frac{(\bar{C} + C')^2}{2}$$

and on temporal averaging, the first law becomes

$$\text{Div}(\vec{H}_0 \vec{G}) = -\text{Div}(\vec{q} + \overline{H'_0 \vec{G}'} + \overline{H G_d} - \sum (\overline{C_i \tau_i}))$$

If Q_0 is defined as $\vec{q} + \overline{H'_0 \vec{G}'} + \overline{H G_d}$, then the energy equation is written as

$$\text{Div}(\vec{H}_0 \vec{G}) = -\text{Div}(\vec{Q}_0 - \sum (\overline{C_i \tau_i})) \quad \dots (3.43)$$

Using the exchange coefficients (3.51 a to d),

$$\begin{aligned} \text{Div}(\sum \overline{C_i \tau_i}) &= \text{Div}(C_x \tau_{x2} + C_\theta \tau_{\theta 2}) \\ &\approx \frac{1}{2} \frac{\partial}{\partial z} \left[\left(\rho / \mu_{x2\text{eff}} \cdot C_x \frac{\partial G_x}{\partial z} \right) + \left(\rho / \mu_{\theta 2\text{eff}} \cdot C_\theta \frac{\partial (C_\theta / 2)}{\partial z} \right) \right] + \text{small terms.} \\ &\approx \frac{\partial}{\partial z} \left(\mu_{x2\text{eff}} \frac{\partial}{\partial z} (C_x^2 / 2) \right) + \frac{1}{2} \mu_{\theta 2\text{eff}} \frac{\partial}{\partial z} (C_\theta^2 / 2) + \frac{1}{2} \mu_{x2\text{eff}} \frac{\partial}{\partial z} (C_x^2 / 2) + \\ &\quad \frac{\partial}{\partial z} \left(\mu_{\theta 2\text{eff}} \frac{\partial}{\partial z} (C_\theta^2 / 2) \right) - \frac{1}{2} \frac{\partial}{\partial z} (C_\theta^2 \mu_{\theta 2\text{eff}}). \end{aligned} \quad \dots (3.44)$$

$$\text{and } Q_2 = -\mu_{x2\text{eff}} / \rho_{\text{eff}} \cdot \frac{\partial H}{\partial z} - \mu_{\theta 2\text{eff}} / \rho_{\text{eff}} \cdot \frac{\partial k}{\partial z} \quad \dots (3.45)$$

With these terms and writing $g_0 J H = g_0 J H_0 - k - G_x^2 / 2 - C_\theta^2 / 2$, the first law, applied to axisymmetric flow is rearranged as

$$\begin{aligned} g_0 J \left(\rho C_x \frac{\partial H_0}{\partial x} + \rho C_\theta \frac{\partial H_0}{\partial z} \right) &= \frac{\partial}{\partial z} \left(\mu_{x2\text{eff}} g_0 J \frac{\partial H_0}{\partial z} \right) - \\ &\quad \frac{1}{2} \frac{\partial}{\partial z} (C_\theta^2 \mu_{\theta 2\text{eff}}) + \frac{1}{2} \left(\mu_{x2\text{eff}} \frac{\partial (C_x^2 / 2)}{\partial z} + \mu_{\theta 2\text{eff}} \frac{\partial (C_\theta^2 / 2)}{\partial z} \right) \end{aligned} \quad \dots (3.46)$$

Assuming that the contribution by turbulence in the principal equation (3.27), other than through the entropy term, is small and that the contributions in the axial and circumferential equations (3.28, 3.29) are only through $\frac{\partial}{\partial z} (\mu_{x2\text{eff}} \frac{\partial G_x}{\partial z})$ and $\frac{\partial}{\partial z} (\mu_{\theta 2\text{eff}} \frac{\partial C_\theta}{\partial z})$ respectively, the axial and circumferential equations may be combined to result in,

$$\begin{aligned} -C_x \frac{\partial C_x}{\partial x} + C_x \frac{\partial G_x}{\partial z} + g_0 J \frac{\partial H_0}{\partial x} - g_0 J T \frac{\partial s}{\partial x} + \frac{C_\theta}{2} \frac{C_x}{G_x} \frac{\partial (2 C_\theta)}{\partial z} \\ + \frac{C_\theta}{C_x} \frac{\partial}{\partial z} \left(\mu_{\theta 2\text{eff}} \frac{\partial C_\theta}{\partial z} \right) + \frac{\partial}{\partial z} \left(\mu_{x2\text{eff}} \frac{\partial G_x}{\partial z} \right) \end{aligned}$$

Multiplying each term of this equation by C_x and equation (3.27) by C_r , adding and rearranging,

$$\begin{aligned} g_o J \left(C_x \frac{\partial H_o}{\partial x} + C_r \frac{\partial H_o}{\partial r} \right) &= g_o J T \left(C_x \frac{\partial s}{\partial x} + C_r \frac{\partial s}{\partial r} \right) \\ - C_o \frac{\partial}{\partial r} \left(\mu_{or,eff} \frac{\partial C_o}{\partial r} \right) - C_x \frac{\partial}{\partial r} \left(\mu_{xr,eff} \frac{\partial C_x}{\partial r} \right) &= 0. \end{aligned} \quad \dots(3.47)$$

From equations (3.46) and (3.47)

$$\begin{aligned} \rho g_o J T \left(C_x \frac{\partial s}{\partial x} + C_r \frac{\partial s}{\partial r} \right) &= C_o \frac{\partial}{\partial r} \left(\mu_{or,eff} \frac{\partial C_o}{\partial r} \right) + C_x \frac{\partial}{\partial r} \left(\mu_{xr,eff} \frac{\partial C_x}{\partial r} \right) + \\ g_o J \frac{\partial}{\partial r} \left(\mu_{xr,eff} \frac{\partial H_o}{\partial r} \right) - \frac{1}{2} \frac{\partial}{\partial r} \left(C_o^2 \mu_{or,eff} \right) &+ \\ \frac{1}{2} \left[\mu_{xr,eff} \frac{\partial}{\partial r} (C_x^2/2) + \mu_{ro,eff} \frac{\partial}{\partial r} (C_o^2/2) \right] & \end{aligned} \quad \dots(3.48)$$

This is an entropy equation for axi-symmetric, turbulent and steady flow. Validity of this equation to compressible flow depends largely on the appropriate definition of the effective exchange coefficients. Even in this form, number of terms on r.h.s. is rather large and following additional assumptions have been made at this stage.

(a) $\mu_{xr,eff} = \mu_{or,eff} = \mu_{eff}$ = effective viscosity along the direction of resultant velocity = $\mu_{Cr,eff}$.

(b) The R.H.S. of Equation (3.48) expands to:

$$\begin{aligned} \mu_{eff} \left[C_o \frac{\partial^2 C_o}{\partial r^2} + C_x \frac{\partial^2 C_x}{\partial r^2} + g_o J \frac{\partial^2 H_o}{\partial r^2} - \frac{C_o}{r} \frac{\partial C_o}{\partial r} + \frac{C_x}{r} \frac{\partial C_x}{\partial r} \right] \\ + \frac{\partial}{\partial r} \left[\mu_{eff} \left[C_o \frac{\partial C_o}{\partial r} + C_x \frac{\partial C_x}{\partial r} + g_o J \frac{\partial H_o}{\partial r} - \frac{C_o^2}{r} \right] \right] \end{aligned}$$

It is assumed that this may be approximated by:

$$\frac{\partial}{\partial r} \left(\mu_{eff} \frac{\partial C_o}{\partial r} \right) + \frac{\partial}{\partial r} \left(\mu_{eff} g_o J / C_o \frac{\partial H_o}{\partial r} \right) \quad \dots(3.49)$$

This appears to be adequate since it still allows the entropy gradient to be related to the momentum transport, energy

production and the effective viscosity along the direction of the velocity vector.

(c) The axial gradient of entropy at a location outside the blade rows is due only to the irreversibilities along the flow in non-bladed passage. This is negligible compared to the radial gradient of entropy at the same place which is due to the cumulative frictional effects of the blade-passages and secondary flows, etc. Hence the entropy equation, in its simplified form may be stated as

$$gJT \frac{\partial s}{\partial r} = \frac{zf}{\partial \psi / \partial x} \left[\frac{\partial}{\partial r} (\mu_{eff} \cdot \frac{\partial C}{\partial r}) + \frac{\partial}{\partial r} (\mu_{eff} / C_c \cdot \frac{\partial H_o}{\partial r}) \right] \quad \dots(3.50)$$

This entropy relation (3.50) and the more general form (3.48) are derived by the author for dealing with flow within an axial-flow compressor. The form of these relations is, however, not unique since the definition of effective viscosity and other simplifying assumptions could be made in alternative ways. For inviscid flow, equation (3.48) reduces to

$$C_x \frac{\partial s}{\partial x} + C_r \frac{\partial s}{\partial r} = 0. \quad \dots(3.50a)$$

Equation (3.50a) has been used by Marsh (1966) in his computer-prediction technique.

Effective Exchange Coefficients

It is well known that the shear stresses due to turbulence in a flow can be expressed as being analogous to the viscous shear in a laminar flow, that is $(-\bar{\rho} \overline{C_i' C_j'})$

may be grouped together with (τ_{ij}) in the momentum equation.

Similarly it is also well established that the turbulent energy-transfer vector $(-\bar{\rho} \overline{C_j H_e})$ can be considered as the turbulent counterpart of $(k/c_p \frac{\partial \bar{H}_e}{\partial x_j})$.

An extension of the analogy leads one to an assumption that the fluxes in exchange processes in turbulent flow are proportional to the gradients of the relevant fluid property in the same way as they are in laminar flow. Such an assumption would reduce the turbulent flow equations to a form similar to that for laminar case. In general, the fluxes of an exchange process vary in different directions, and hence the effective exchange coefficients have to be defined appropriately. The effective viscosities in axial and circumferential directions may be defined as:

$$\mu_{xr\text{eff}} = \tau_{xr} / \left(\frac{\partial C_x}{\partial r} \right) \quad \dots(3.51a)$$

and,

$$\mu_{\theta r\text{eff}} = \tau_{\theta r} / \left(r \cdot \frac{\partial (r C_\theta)}{\partial r} \right) \quad \dots(3.51b)$$

Similarly, the effective exchange coefficient for energy flux is defined as

$$T_{he\text{eff}} = -Q_r / \left(\frac{\partial H}{\partial r} \right) \quad \dots(3.52)$$

and by analogy, for turbulent kinetic energy exchange,

$$T_{ke\text{eff}} = -Q_{kr} / \left(\frac{\partial k}{\partial r} \right) \quad \dots(3.53)$$

In the absence of any detailed information to the contrary, a further simplifying assumption that the ratio of some of the effective exchange coefficients is unity.

Assumed here in particular, is that $\mu_{xr\text{ eff}} = \mu_{\theta r\text{ eff}} = \mu_{\text{eff}}$ and that $\mu_{\text{eff}}/T_{h\text{ eff}} = \text{turbulent Prandtl number}$ also equals unity. Such assumptions have been made by other authors e.g. Spalding (1967) and are felt justifiable at this stage.

There is no published information on the distribution of effective viscosity in axial-flow compressors or indeed in other types of turbomachinery. It was, therefore, imperative to determine experimentally the distribution of effective viscosity in a representative compressor. This has been carried out and is described in Chapters 5 and 6.

Losses in Axial-Flow Compressors

It is well known that the loss mechanisms at the ends of compressor blades is very complex. Many empirical relations have been developed to predict these losses and these have been reviewed by Horlock (1963) and by Hansen (1965). In the present study only two forms of losses have been considered, profile loss and secondary losses. The secondary losses are defined here as the total loss over blade row less the profile loss. The profile loss has been accounted for by correlation of blade element loss and appears in Equation (3.57) as entropy rise while the secondary losses have been associated with the distribution of effective viscosity the measurement of which is described in Chapters 5 and 6. Correlation of profile loss is presented later in this chapter.

Limitation of the Equations

The equations of motion neglecting the body forces describe the flow only at the stations intermediate between the adjacent blade rows. Hence the grid points were fixed at these intermediate locations. From a theoretical viewpoint, this restriction also imposes a limitation on the validity of terms involving axial gradients. These are determined by numerical techniques using the values at grid points beyond adjacent blade rows upstream and downstream. In practice, such terms are usually small compared with the others, when dealing with the transformed radial-equilibrium equation and no serious inaccuracies would occur in the present procedure. This is an assumption also made by many other authors.

A second limitation is associated with the inability to express the exact nature of turbulent flow by using simplified equations. The basic equations for viscous flow have been transformed by replacing the instantaneous quantities by a sum of mean and fluctuating components and by the introduction of arbitrarily defined effective exchange coefficients.

SECTION B

The finite difference approach for analysing the flow in axial-flow compressors provided earlier in this chapter requires blade information in the form of fluid exit angle and profile loss as a function of incidence. This information has been derived from Lieblein's correlations (1959, 1965).

Alternative correlations are briefly surveyed in Chapter 2, but Lieblein's technique has been considered superior since his correlations were based upon considerable data including information from annular and rotating cascades.

Exit angle correlations and the profile loss correlations are discussed separately in the following pages.

Lieblein's Correlation for Deviation

Lieblein's (1959) basic deviation and basic profile loss are determined at the minimum-loss incidence. Correction factors are applied to account for differences between the actual incidence and the minimum loss incidence. The determination of minimum-loss incidence is therefore an important step towards finding the deviation and profile loss used in the prediction computer-programme.

Lieblein (1959) provided a detailed description of his method of determining minimum-loss incidence and deviation. In a later report, Lieblein (1965), a method for determining the profile losses was given. A brief description of Lieblein's correlation is given below.

Minimum-Loss Incidence and Deviation

The methods for determining both the minimum incidence, $i_{ref.}$ and the deviation at minimum loss, $\delta_{ref.}$ are very similar. A zero camber incidence or deviation is first determined from curves given by Lieblein and corrections for blade profile, shape, thickness, camber and other geometrical features are applied.

At values of incidence other than that for minimum loss, Lieblein provided a method for determining the deviation. The expression for this deviation is

$$\delta = \delta_{ref.} + (i - i_{ref.}) \cdot \left(\frac{d\delta}{di} \right)_{ref.} \quad \dots(3.54)$$

where $\left(\frac{d\delta}{di} \right)_{ref.}$ is a function of inlet angle, β_1 , and solidity, σ provided in graphical form by Lieblein.

Correlation for Profile Loss Coefficient

The procedure consisted of calculation of an equivalent diffusion factor Deq. due to Lieblein (1959), an estimation of the wake momentum thickness Θ and the evaluation of the stagnation pressure loss coefficient for an assumed form factor H_2 . The equivalent diffusion factor associated with the extent of diffusion of the velocity on the blade surfaces is given by

$$\begin{aligned} \text{Deq.} = \cos\beta_1 / \cos\beta_2 \left[1.12 + a(i - i_{ref})^{1.43} \right. \\ \left. + 0.61 \cos^2\beta_1 / \sigma (\tan\beta_1 - \tan\beta_2) \right] \quad \dots(3.55) \end{aligned}$$

where $a = 0.007$ for British C-4 section blades.

$a = 0.0117$ for NACA-65 10% blades.

β_1, β_2 = Relative inlet and exit air angles for the blade element.

The stagnation pressure loss coefficient $\bar{\omega}$ depends upon the boundary layer growth and breakaway on the blade surfaces. Hence, $\bar{\omega}$ has been related to the wake momentum thickness to chord ratio, Θ/C which is a direct consequence of boundary layer growth, separation and mixing. Blade

solidity σ has a pronounced effect of $\bar{\omega}$; $\bar{\omega}$ is very high for low and high solidity but it has an optimum value at a medium solidity. Lieblein (1953) provided the following expression which related the profile loss coefficient $\bar{\omega}$, to θ/C and blade conditions:

$$\begin{aligned}\bar{\omega} &= z\left(\frac{\theta}{c}\right) \frac{\sigma}{\cos\beta_2} \left(\frac{\beta_1}{\beta_2}\right)^2 \left[1 - \theta/c \frac{\sigma H_2}{\cos\beta_2}\right]^3 \left(\frac{2H_2}{3H_2-1}\right) \\ &= -\Delta P_o / \left(\frac{1}{2} \rho C^2\right)\end{aligned}\quad \dots(3.56)$$

The value of shape factor H_2 suggested by Lieblein was 1.6 and this was used in the computer programme.

Relationship between the profile loss and entropy rise can be determined from the thermodynamic relations. Lieblein (1953) gave the following expression for the change of entropy.

$$\begin{aligned}(S_n - S) J/R &= L_n \left(\left(\frac{T_{0n}}{T_o} \right)^{\gamma/\gamma-1} / \left(\frac{P_{0n}}{P_o} \right) \right) \\ &= \frac{1}{1 - \frac{\bar{\omega}}{(P_{0n}'/P_o)_{ideal}} \left[1 - \left(\frac{1}{1 + (\gamma-1)/2 M'^2} \right)^{\gamma/\gamma-1} \right]}\end{aligned}\quad \dots(3.57)$$

where $(P_{0n}'/P_o)_{ideal}$ = Ideal total pressure ratio across the blade row.

$$= \left[1 + \frac{\gamma-1}{2} \frac{U_n^2}{\gamma g_o R T} \left(1 - \frac{z^2}{R_{zn}^2} \right) \right]^{\gamma/\gamma-1}\quad \dots(3.58)$$

U being the circumferential velocity of the blade element = $r.\omega$.

Also along the assumed stream surface, the density across a blade row is given by

$$\begin{aligned}\rho_n &= \rho \left(\frac{T_n}{T} \right)^{1/\gamma-1} e^{-J/R(S_n-S)} \\ &= \rho \left[\frac{H_{0n} - C_n^2/2g_o J}{H_o - C^2/2g_o J} \right] e^{-J/R(S_n-S)}\end{aligned}\quad \dots(3.59)$$

Equations (3.57), (3.58) and (3.59) have been used in computer programme.

ADAPTATION OF THE PRESENT METHOD TO DEAL WITH THE PROBLEMS OF DEVELOPING PIPE AND ANNULUS FLOWS.

Consideration of a developing flow through an annulus or a pipe is a special case of the general compressor programme. An adaptation of the general procedure to deal with such cases was desired with the viewpoint of testing the computer programme and investigating the effect of a downstream boundary condition.

For the sake of completeness, it is shown in the following sections how the problem of a fully developed pipe or annulus flow can be handled and how a developing flow may be solved based upon the present technique.

Fully Developed Flow

An inspection of the transformed equilibrium equations (3.27), (3.28) and (3.29) shows that each term in the axial and circumferential direction equations vanishes identically for a steady, axisymmetric, potential flow with $C_r = C_\theta = 0$. From the transformed radial equilibrium equation (3.27)

$$\frac{1}{g \cdot J} C_x \frac{\partial C_x}{\partial r} = T \frac{\partial s}{\partial r} \quad \dots(3.60)$$

It is also assumed, in writing Equation (3.60), that the viscous term in the radial equilibrium equation (3.27) is negligible. However, it is not negligible in the axial momentum equation. It follows from this analysis that Equation (3.60) is the governing equation in the present

technique corresponding to the familiar equation obtained from axial equilibrium,

$$\frac{\partial p}{\partial x} = \mu \left(\frac{\partial^2 C_x}{\partial z^2} + \frac{1}{2} \frac{\partial C_x}{\partial z} \right) \quad \dots(3.61)$$

In equation (3.60) the radial gradient of entropy is different from zero even though the radial variations in static pressure do not exist. The most trivial case of zero entropy gradient would correspond with zero radial gradient of axial velocity, that is plug flow. Entropy may now be expressed in terms of stagnation pressure and temperature conditions for adiabatic flow referred to an assumed value of entropy at a reference state.

$$S - S_{ref} = \frac{R}{J} \ln \left\{ \left(\frac{T_o}{T_{o,ref}} \right)^{1/\gamma-1} / \left(\frac{P_o}{P_{o,ref}} \right) \right\} \quad \dots(3.62)$$

For a reference point at the inlet or within the passage, the stagnation temperature ratio is unity and hence

$$\frac{\partial S}{\partial z} = - \frac{R}{J} (L_n P_o) \quad \dots(3.63)$$

which is a valid relationship for viscous and inviscid fluids.

The variations of static temperature are given by

$$T = T_o - C_x^2 / (2g J C_p) \quad \dots(3.64)$$

With these subsidiary relations (3.63) and (3.64), Equation (3.60) becomes

$$\begin{aligned} \frac{1}{g J} C_x \frac{\partial C_x}{\partial z} &= \frac{R}{J} \left(T_o - C_x^2 / (2g J C_p) \right) \cdot \frac{\partial}{\partial z} (L_n P_o) \\ \text{or,} \quad \frac{-\frac{\partial}{\partial z} (-C_x^2/2)}{(T_o g J C_p - C_x^2/2)} &= \frac{\partial}{\partial z} (L_n P_o^{(\gamma-1)/\gamma}) \end{aligned} \quad \dots(3.65)$$

On integration,

$$L_n\left(T_0 C_p g J - \frac{C_x^2}{2}\right) = L_n(\rho_0^{(1-\gamma)/\gamma}) + a \text{ constant.} \quad \dots(3.66)$$

At $r = r_{\text{ref}}$, $C_x = C_{x_{\text{ref}}}$ and $P_0 = P_{0_{\text{ref}}}$, hence evaluating the above constant and rearranging,

$$\left(H_0 - \frac{C_x^2}{2gJ}\right) = \left(H_0 - \frac{C_{x_{\text{ref}}}^2}{2gJ}\right) \cdot \left(\frac{\rho_0}{\rho_{0_{\text{ref}}}}\right)^{(1-\gamma)/\gamma} \quad \dots(3.67)$$

This equation may be interpreted as a relation connecting the axial velocity with the stagnation pressure which, in turn, depends upon the value of P_0 at entry on the same streamline minus the stagnation pressure loss over the distance from entry. The loss is necessarily a cumulative effect of viscosity, turbulence and surface roughness. It is evident that, at any axial station, P_0 is less nearer the wall than in the core region.

For a pipe flow, a convenient reference point is at the centreline where $C_{x_{\text{ref}}} = C_{x_{\text{max}}}$, and $P_{0_{\text{ref}}} = P_{0_{\text{max}}}$.

Hence,

$$\left(H_0 - \frac{C_x^2}{2gJ}\right) = \left(H_0 - \frac{C_{x_{\text{max}}}^2}{2gJ}\right)^{\frac{\gamma}{\gamma-1}} * \left(\frac{\rho_0}{\rho_{0_{\text{max}}}}\right)^{\frac{\gamma-1}{\gamma}} \quad \dots(3.68)$$

It may, however, be noted that the wall, $C_x = 0$ and

$$P_0 = \rho_{0_{\text{max}}} * \left[\frac{H_0 - \frac{C_{x_{\text{max}}}^2}{2gJ}}{H_0} \right]^{\frac{\gamma}{\gamma-1}} \quad \dots(3.69)$$

In order to obtain the velocity distribution using equation (3.68), it is necessary to evaluate P_0 at each point.

This is achieved by an iterative process using Equations (3.61) and (3.68) together with the continuity equation implied by the definition of Ψ , Equation (3.12) starting with an assumed P_0 distribution.

It may be stated that, in this procedure, the radial distribution of axial velocity at an axial station is governed by the principal radial-equilibrium equation (3.68) and the distribution of stagnation pressure is obtained from loss considerations.

Developing Flow

In this case $Cr \neq 0$ and $\partial/\partial x \neq 0$ and the radial momentum equation appears as

$$C_x \left(\frac{\partial C_r}{\partial x} - \frac{\partial C_x}{\partial r} \right) - g_s T \frac{\partial s}{\partial r} = 0. \quad \dots(3.70)$$

viscosity terms being negligible and the axial equilibrium equation reduces to

$$C_x \frac{\partial G}{\partial x} + C_r \frac{\partial G}{\partial r} = -\frac{1}{r} \frac{dp}{dx} + \frac{\mu}{r} \left(\frac{\partial^2 G}{\partial r^2} + \frac{1}{r} \frac{\partial G}{\partial r} \right) \quad \dots(3.71)$$

The former (3.70), in terms of Ψ is written as

$$\frac{\partial^3 \Psi}{\partial x^3} + \frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{r} \frac{\partial \Psi}{\partial r} = \frac{R g_s (P r f)^2}{C_p} \left(H_0 - \frac{G^2}{2 g T} \right) \frac{1}{P_0} \frac{\partial P_0}{\partial r} \quad \dots(3.72)$$

and the latter (3.71) can be expressed as

$$\begin{aligned} \frac{\partial p}{\partial x} = & \frac{1}{r(rf)^2} \left(-\frac{\partial \Psi}{\partial r} \cdot \frac{\partial^2 \Psi}{\partial r \partial x} + \frac{\partial \Psi}{\partial x} \left(\frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{r^2} \frac{\partial \Psi}{\partial r} \right) \right) \\ & + \frac{\mu}{r r f} \left(\frac{\partial^3 \Psi}{\partial r^3} + \frac{1}{r} \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial \Psi}{\partial r} \right) = \frac{\partial}{\partial x} \left(P_0 - G^2/2 \right) \quad \dots(3.73) \end{aligned}$$

With these two equations established, a procedure similar to that used for fully developed flow is used to relax for ψ in equation (3.72), obtaining P_0 iteratively from equation (3.73). The procedure is the same for the pipe and annulus problems.

Summary

The basic equations for axisymmetric compressible flow in axial-flow turbomachines have been developed including terms expressing the effects of turbulent shear stresses. The equations are

1. the definition of stream function, Equation (3.12),
2. the principal radial equilibrium equation, (3.27),
3. the energy equation, (3.36),
4. the entropy equation (3.50), and
5. various thermodynamic relations.

It was observed that the distribution of eddy-viscosity in axial-flow compressors necessary to solve the above equations was not available in the published literature. Experiments described in Chapters 5 and 6 were designed to provide some basic results.

The cascade performance data required in the basic equations were obtained from the correlations due to Lieblein for deviation and profile loss. The entropy change due to profile loss is given in Equation (3.57).

A procedure for analysing developing and fully developed flow in pipes and annuli is presented. The basic equations for fully developed flow are (3.12), (3.61) and (3.68) and for the developing flow are (3.12), (3.72) and (3.73).

CHAPTER 4

THE DEVELOPMENT OF THE COMPUTER PROGRAMME

In the previous chapter, the equations applicable to the flow in axial-flow compressors have been developed and correlations of cascade data have been discussed. The next step was to arrange these equations and correlations in a suitable form for the computer solution and to develop the optimum solution procedure which are described in this chapter.

The problems associated with the boundaries of the flow field are first discussed, the convergence problems are next considered and the solution procedure is then described. Finally, test runs for simple developing flows are presented.

Mathematical details and block diagrams for the computer programme are given in Appendix II.

The field-scanning formula used in the computer programme is presented in Appendix I.

Problems Associated with the Boundaries of the Flow-Field

There are, in general, three types of problem in this context:

1. The specification of the boundary conditions.
 2. The occurrence of large gradients in various field values.
 3. Curved and sharp edged boundaries, which present additional problems in the solution of the equations.
- Each of these aspects is discussed in the following pages.

Boundary Conditions

The specification of the boundary conditions requires a physical knowledge of the flow at the boundaries of the domain. The number of boundary conditions for each dependent variable depends upon the terms of the highest order for the variable. When the highest order is $\partial^n \psi / \partial y_1^n$, n boundary conditions are required at the limiting values of y_1 . If another highest term $\partial^n \psi / \partial y_2^n$ occurs in the same equation, then n further boundary conditions at the bounds of y_2 must also be specified. Now, for the solution of Equation (A.I.1), that is

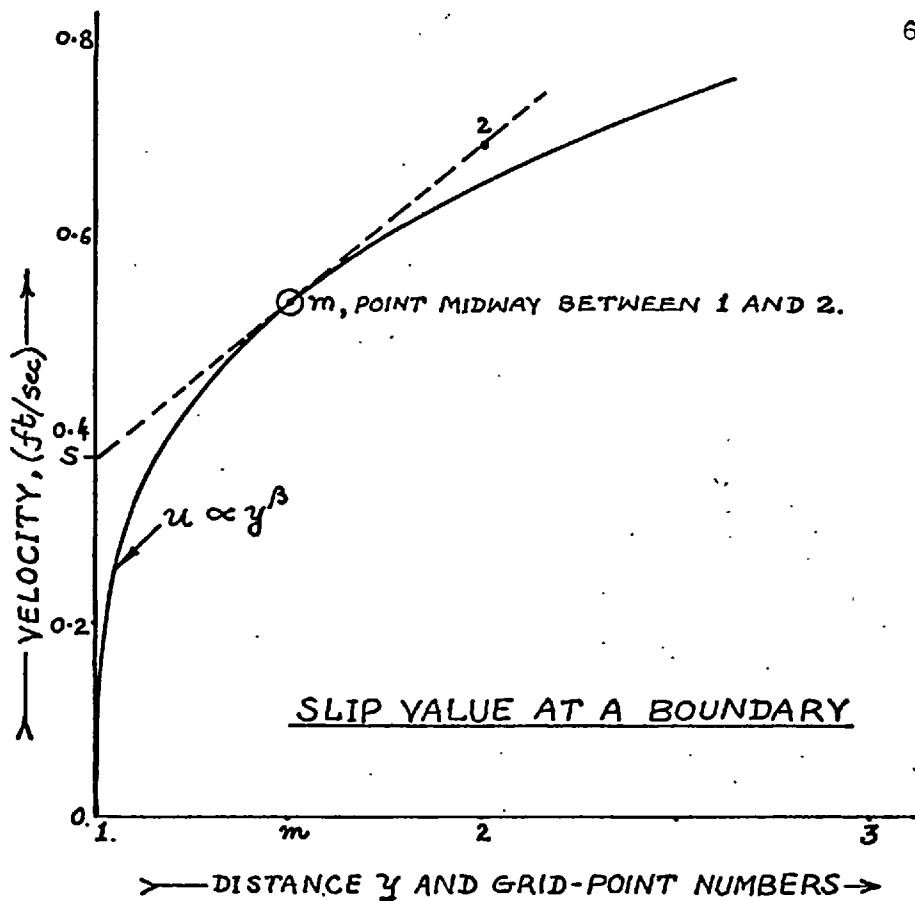
$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{z} \frac{\partial \psi}{\partial z} = B(x, z, \psi) \quad \dots(4.1)$$

for $B(x, r)$ predetermined at each grid point, one boundary condition at each of $r = r_1$, $r = r_0$, $x = x_1$, and $x = x_0$ is required. Specification of $\psi = \text{const.} = 0$ at the inner boundary r_1 , $\psi = \text{const.} = 100$ at the outer boundary r_0 , would be correct if large gradients did not occur near these boundaries; this aspect is discussed in the next paragraph. At inlet to the compressor, $x = x_1$, all flow conditions are given but at the exit, $x = x_0$, it was assumed that the axial gradients in ψ , H_0 , C_0 , ... vanish. The boundary condition at the exit from the compressor is not unduly weak but it has a marked influence on the convergence problem and is discussed later.

Large Gradients

When large gradients in the field function occur close to a boundary, this leads to gross inaccuracies if a coarse network is employed. The problem is further aggravated if the grid points do not lie on the boundary. An obvious solution is to construct a fine mesh in the flow field. A larger number of mesh points will require more computer time for relaxation or matrix inversion and the maximum size of the array is limited by the computer storage. One possible solution is to devise a coarser grid in the core region of flow and a finer mesh near the boundary, provided the steep gradients do not occur in the flow of the core region. However, the number of grid points would still be increased resulting in computer storage problem and excessive computer time.

The programme adapted here was to keep the grid size uniform in the radial direction and to overcome the problem of large gradients close to the walls by the introduction of false or 'slip' values at the boundaries. A slip value of velocity at the wall, u_s , is defined as the one which, when used in conjunction with the velocity at the grid point next to the wall, results in the correct value of the velocity and gradient of velocity at a point midway between the boundary and the next grid point. This procedure is illustrated in Sketch (S.4.1).



SKETCH (S.4.1)

The correct slope and value of the velocity at the point midway between the boundary and the next grid point were obtained from a form of the universal wall-function.

For the present case, it was required to establish the slip values for the stream function at the hub and tip of the compressor. These were obtained by assuming a velocity distribution, $u = Ky^\beta$, near a wall. From the definition of slip values,

$$ky_m^\beta = k/\beta y_m^{\beta-1} \cdot y_m + u_s \quad \dots(4.2)$$

$$\text{whence, } u_s = (1-\beta)/(1+\beta) u_2, \quad u_m = \frac{1}{1+\beta} u_2$$

$$\text{and } \left(\frac{\partial \psi}{\partial z} \right)_m = \frac{1}{1+\beta} \left(\frac{\partial \psi}{\partial z} \right)_2 \cdot \left(\frac{r_m}{r_2} \right) \quad \dots(4.3)$$

Replacing Equation (4.3) by a finite difference form results in

$$\psi_s = (r_m \psi_3 - 2 \cdot r_2 \psi_2 (1+\beta)) / (r_m - 2 \cdot r_2 (1+\beta)) \quad \dots(4.4)$$

Value of β used in Equation (4.2) is obtained from a suitable drag law; the form used here due to van Driest (1956) is as follows:

$$S_* = R_*^{-1} - 0.1561 R_*^{-4.5} + 0.08723 R_*^{-3} + 0.0371 R_*^{-1.8} \dots (4.5)$$

where

$$S_* = S/k^2 = \tau_s / (\rho U^2 k^2) \dots (4.6)$$

$$k = \text{constant in } \mu_{eff} = \rho k^2 y^2 \left| \frac{\partial U}{\partial y} \right|$$

$$\text{and } R_* = R k^2 = \rho U y k^2 / \mu \dots (4.7)$$

In using Equation (4.5), the pressure gradient factor has been neglected following convention and for the sake of simplicity.

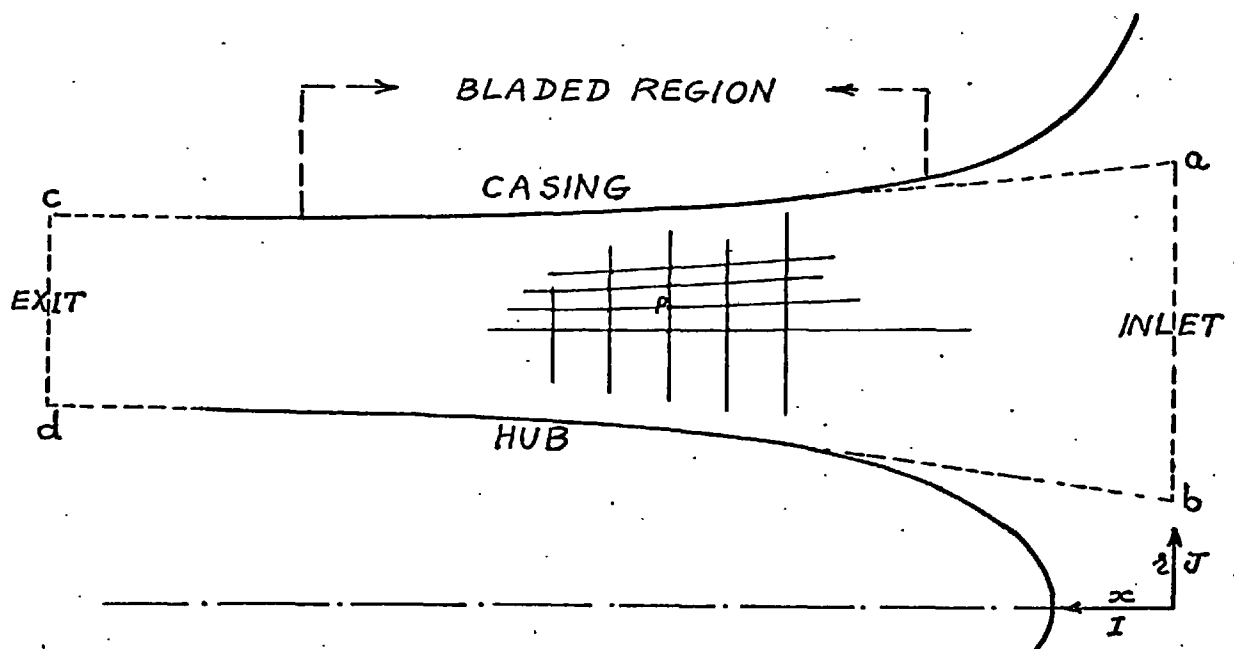
$$\beta = \left\{ (S+F)_m \right\}^{1/2} / k \dots (4.8)$$

Curved Boundaries

The presence of curved boundaries poses some further problems. For a rectangular network used in the field, the functions at the grid points adjacent to the boundary have to be evaluated separately using an amended relaxation routine, or by a process of suitable extrapolation from the known values at other points. In addition, a rectangular grid system is particularly unsuitable where the number of points in the interior of a region at a certain cross section becomes too few, this is likely to occur towards the exit of a high pressure-ratio axial-flow compressor. Further subdivision of the rectangular grid in such zones may provide a solution but a network of variable size along the flow makes the computer programme more cumbersome and discontinuities may

result in the solution at the section where the grid is changed, due to a change in the relative accuracy of the finite difference form of the equations.

Alternatively, a curved grid system may be preferred to a rectangular one. One way of achieving this is by the transformation of equations with a change of co-ordinate system. Another method, which was used in the present study, consists of dividing the field into a quasi-orthogonal net. The grid points along the flow were arranged to follow approximate streamlines while the grid points across the flow were on true radial lines, as illustrated in Sketch (S.4.2).



FLOW-FIELD, THE BOUNDARIES AND
THE GRID POINTS

SKETCH (S.4.2)

This is particularly suitable for regions with inner and outer curved boundaries. The radial lines across the flow were drawn at locations intermediate between the blade rows, each line was divided into equal number of parts, and the corresponding points were joined to construct the quasi-orthogonal mesh. For each point P in the region, values of HL and HR, the distances of transverse lines on its left and right and G, the transverse interval were noted. A finite difference form of the field equation was arranged to form a 'molecule' to apply at each point in the field. The details are given in Appendix I. The molecule appears as

$$\psi_{new} = (1-\omega) \psi_P + \frac{\omega}{M_P} (M_A \psi_A + M_B \psi_B + M_U \psi_U + M_D \psi_D - B_P)$$

with the symbols also explained in the appendix. $M_A, M_B \dots$ are, in general, functions of the grid size and shape and ω is a relaxation parameter governing the rate of convergence.

Convergence Considerations

A molecule representing the finite-difference form of the principal partial differential equation (3.27) is applied at each point in the flow field in order to obtain an improved value of stream function. An arbitrary initial distribution of the streamfunction, ψ , may be selected although the number of iterations could be reduced by selecting a realistic distribution. The main problems associated with the iteration technique were those of instability and convergence. Problems of instability may occur when the computer programme is operated with high Mach-number flows but in the development of the computer programme for low speed flows no problems associated with instability were encountered.

The simplified programme was that for a potential flow i.e. when B in Equation (A.I.1) was zero. The diagonal of the matrix of the algebraic equations so set up was predominant, a condition that often leads to the stability of the solution by an iterative technique of successive over-relaxation using the Gauss-Seidel approach. This technique is most promising since it uses an implicit iterative routine assuming the successively improved values of the unknown at each point and it makes use of the fact that most of the terms in the matrix are zero. With the simplified computer programme the following investigations were made.

- (1) Effect of values of O.R.'s (over-relaxation parameter ω) and of varying O.R. in successive iterations.
- (2) the influence of different field scanning techniques.

Over-Relaxation

Optimum O.R. was obtained for the annulus geometry of the three stage compressor described in Chapter 5 by plotting the maximum field residual against the number of iterations for difference percentages of over-relaxation as shown in Fig. 1. It was noted that the relaxation to achieve a particular convergence with O.R. = 1 may be as much as ten times that with a 60% over-relaxation. It was also observed from the curves on Fig. 1 that a higher over-relaxation is more effective in the initial iterations and less useful thereafter. This fact led to a consideration of differential

O.R., that is a high O.R. used in the initial stages, gradually decreasing to a lower value. Use of O.R. equal to 1.9 for the initial iteration and gradually reducing to 1.4 achieved a 10% reduction in convergence time compared with a constant over-relaxation of 1.6. It was considered that the complexity associated with differential over-relaxation was not warranted in view of the comparatively small reduction in computer time. Hence a constant value of O.R. = 1.6 was selected for the axial-compressor computer programme.

Field-Scanning

The convergence of the solution results from updating the field values to satisfy the governing equations. This is, in turn, achieved by sweeping the errors at field points towards the boundary where the error is reduced to a minimum. It was thus considered that a sequence of scanning so as to communicate with the boundaries frequently would be the quickest. In fact, Fox (1962) suggested the use of vertical and horizontal oscillations.

Various scanning procedures were considered but it was found that the most suitable scheme for the axial-flow compressor geometry was to scan the field from inlet to exit, from hub to tip and from tip to hub in successive iterations.

Description of the Computational Process

The computer programme was arranged to carry out the following operations.

1. From the input geometry of the compressor and the blades, the flow field, the boundaries and the net work were determined.
2. A starting distribution of ψ was assumed and the boundary conditions were specified.
3. Initial values of the velocity components were determined consistent with the assumed ψ distribution.
4. The value of B for Equation (A.I.1) was determined for each grid point starting from the compressor inlet. After each stator blade row, the cascade correlation was used to find the outlet angle and the entropy terms. After each rotor, the velocity triangle relationship together with the cascade correlation were used to obtain the values of outlet angles and entropy terms. The radial gradients of H_0 , e and rC_0 and the constituents of the B-term downstream the blade row were evaluated. The terms due to turbulence in the equilibrium equation and in the entropy equation were calculated for the specified distribution of effective viscosity.
5. The values of B were incorporated in the finite-difference molecule at each grid point and the scanning procedure to relax for ψ was carried out. This scanning procedure was continued until the residuals at each grid point were below the tolerance.
7. This cycle, from 4 to 5, was repeated until the maximum change in velocity at any point in the field over one iteration was less than a specified percentage of the maximum axial velocity.
8. When the converged solution was obtained, the final values of all fluid angles, velocity components and stream

function were printed out and the computation was stopped.

Iteration Control

The overall solution procedure, as outlined in the preceding section was iterative in nature. Starting with an arbitrary distribution of the field variables, one complete cycle comprised calculations for newer estimation of the constituent terms in $B(x,r)$ and a relaxation of the field for an improved ψ -distribution. Controls were, therefore necessary on the overall cycles of calculation as well as on the field scanning for ψ -distribution.

The overall cycles of calculation were repeated until the axial velocity at any point changed by less than 0.5% of the maximum axial velocity in successive cycles. A termination limit such as this was arbitrarily fixed but was observed to be effective since other variables like C_θ and ψ also converged to nearly the same extent as C_x . An upper limit to the number of cycles of overall iteration was fixed at 10.

During the relaxation procedure, the greatest residual in the field was determined and the procedure was repeated until the greatest residual anywhere was below the tolerance which was set at 0.005. If the desired convergence was not achieved within a specified number of cycles, the procedure was terminated. The tolerance limit referred to above was compared with the numerical difference between the present and updated values of ψ rather than on any percentage basis which has been used by some other authors. This was important from the point of view of achieving same absolute accuracy in difference values of ψ and hence resulting in a more

accurate determination of the velocity field.

Simple Test Cases

A simplified version of the general computer programme was written, on the basis of relations (3.60) to (3.73), to solve the problems of flow through an annulus and through a pipe. The simplification was achieved by the removal of the subroutines and statements associated with cascade correlations and calculations across the blade rows.

Tests were conducted to establish the validity of the general computing technique for simpler problems and to investigate the effect of a downstream boundary condition on the convergence with a view to incorporating suitable boundary conditions both downstream of the blade rows and at the exit plane of the axial-flow compressor.

Pipe Flow

The computer programme was run for fully developed flow at entry assuming laminar viscosity only. It was found that the solution converged rapidly to the parabolic velocity profile at every axial station in the pipe.

The computer programme was run with developing flow again assuming laminar viscous effects only. The results are shown in Fig. 2 where it can be seen that the programme yields realistic results.

However, it was felt that the comparatively coarse grid used in the computer programme for the relatively thin, growing boundary layer at the entrance of the pipe would not produce accurate results.

Parallel Annulus Flow

Test runs for fully developed annulus-flows were conducted assuming laminar viscosity and a plausible distribution of effective viscosity. For the laminar viscosity case, the solution converged to give the fully developed velocity profile at every axial station. For the turbulent eddy-viscosity example, the experimental fully developed velocity-profile was put as an entry condition and the converged solution was only slightly different at exit compared with the entry profile, Fig. 3. The slight discrepancy between the inlet and exit velocity perhaps arose from the eddy viscosity distribution used in the computer programme which has been based on the compressor results given in Chapter 6.

The problem of developing turbulent flow through an annulus was analysed. The effective viscosity variation was assumed to be the same as that in the axial-flow compressor. No experimental results were available to compare with the computed solution but the results appear credible, as shown in Fig. 4. However, as in the case of developing pipe flow the grid was relatively coarse compared with the boundary layer thickness so that accurate results were not expected.

Conclusions from the Simple Tests Cases

The general computing technique was found to be perfectly adequate since accurate results were obtained for fully developed flows and plausible results for the developing flows.

There were no computing difficulties associated with the specification of the exit boundary condition ($\partial\psi/\partial x=0$) both for the developing and fully developed flow cases. Moreover, the number of iterations was not excessive in obtaining convergence to reasonable limits.

CHAPTER 5

DEVELOPMENT OF EXPERIMENTAL APPARATUS AND TECHNIQUE

Objective of the Experimental Work

It has been shown in Chapter 3. that there is a need to conduct detailed tests on the flow through an axial-flow compressor in order to obtain a realistic picture of the structure of turbulence and the distribution of effective viscosity. The model of flow for the computerised prediction of the off-design performance of the compressor may be then improved to include the effect of turbulent shear flows in addition to the effect of blade losses. However, there are many experimental difficulties associated with the determination of the distribution of effective viscosity in an axial-flow compressor.

In two dimensional boundary layer flows, the constant stress region derives its supply of turbulent energy almost exclusively from the interaction of the turbulent shear stress and the mean velocity gradient within itself. The energy transfer from the turbulent flow outside the region and due to mean pressure gradients in the axial direction is small and hence various simplifying assumptions can be justified. The boundary layers within turbomachinery are more complex because there is an alternating energising and de-energising of the boundary layer by work input from the rotor blades and frictional resistance offered by the passage. In addition, there are centrifugal forces and radial pressure gradients causing radial shift of flow.

In view of the complexity of the problem and the amount of information needed on the one hand and the relatively undeveloped state of the transducer (hot wire) for such measurements on the other hand, it was not expected that all the desired information would be obtained from the experimental studies described here but the intention can be summed up as follows:-

1. To explore the use of the suitable transducers,
2. To develop a practical technique,
3. To procure necessary information,
4. To interpret results, and
5. To express the results in a form useful for the general prediction computer-programme.

The measurements within the compressor necessary to determine the distribution of effective viscosity are the turbulent shear stresses and the radial velocity gradients. The development of the test technique is described in the following pages.

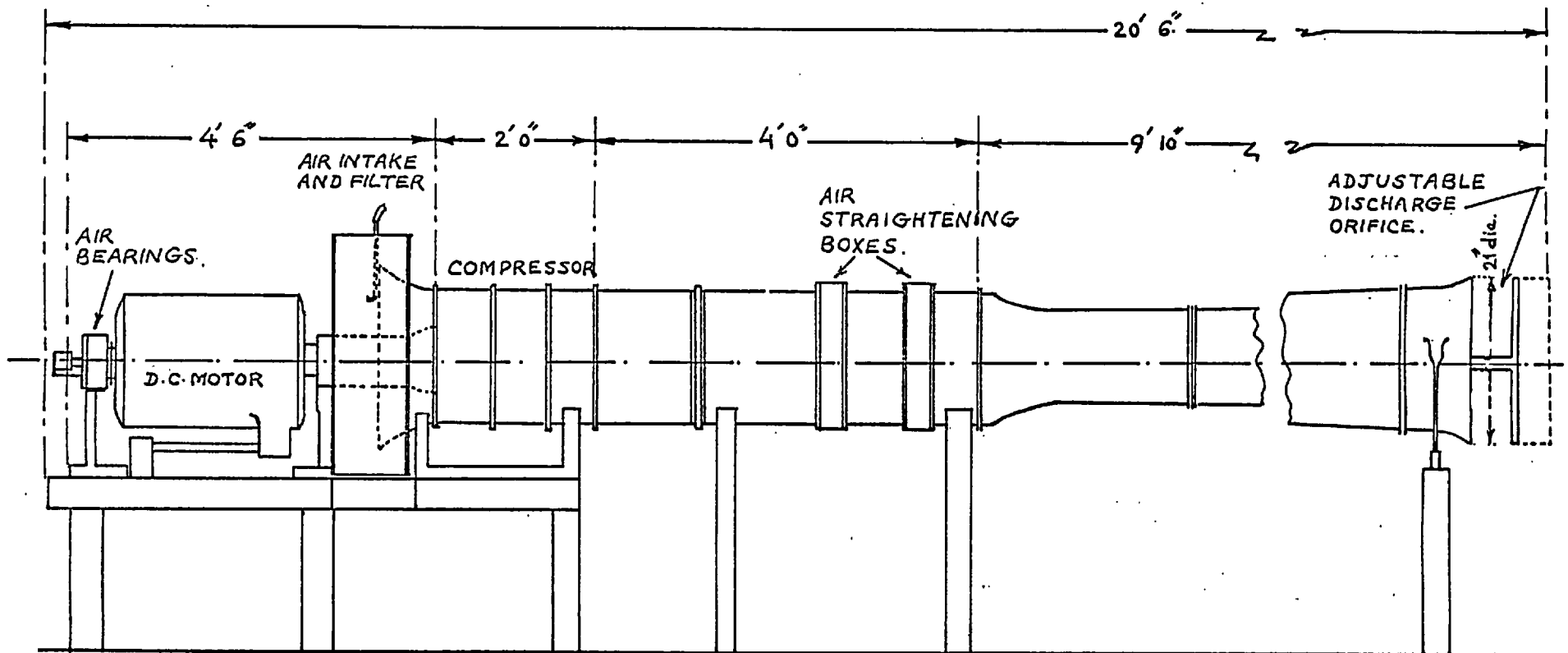
The Laboratory Compressor Test Facility

A compressor with three identical stages preceded by a set of inlet guide vanes was used for the tests. The blades were of free vortex design with 50% reaction at the mean diameter and had C-4 circular-arc camber-line aerofoils with a constant hub tip ratio of 0.7. More details of the blading and the compressor are given in Table A.

The compressor rotor was of drum construction and the extension shafts at either end were supported in ball and roller bearings fixed in the inlet and outlet stator casings through six webs which also served to straighten the flow at entry to the guide vanes and at exit from the last stator row as shown in the accompanying sketch (S5.1). The stator blades were held in half-rings located in position by adjustable spacers.

The compressor was coupled to a 200 Volt D.C. compound motor developing 15 h.p. at 3000 r.p.m. The speed was controlled in coarse steps of about 100 r.p.m. by a rheostat in series with the armature. Fine regulation of speed was achieved through the adjustment of smaller rheostates in the armature field circuits at lower (less than 2000 r.p.m.) and higher speeds respectively. A tacho-generator was used to measure the speed. The torque measurements could be made through the reaction on the rotor casing designed to swing freely in air bearings.

On discharge from the last stator, the air was passed through two lengths of straighteners, a converging passage with a venturimeter at the throat followed by a 5° diffuser with a throttle plate at the exit end. The mass flow was varied by adjusting the throttle plate and was referred to by the number of revolutions of the throttle plate from an arbitrary zero at the minimum possible opening.

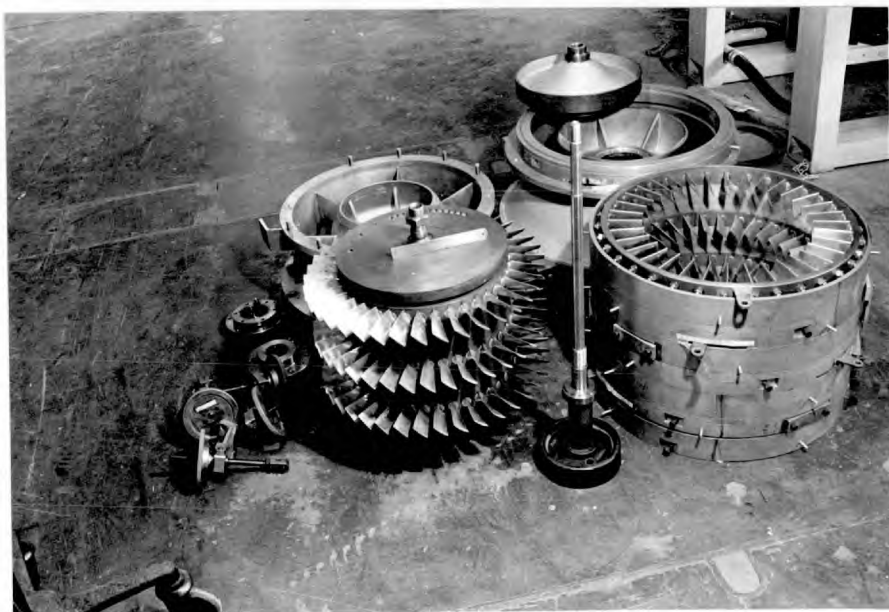


THREE-STAGE AXIAL FLOW COMPRESSOR TEST FACILITY

Sketch S.5.1.



Components of the Compressor with the Motor on the Foundations



Close-Up of the Components

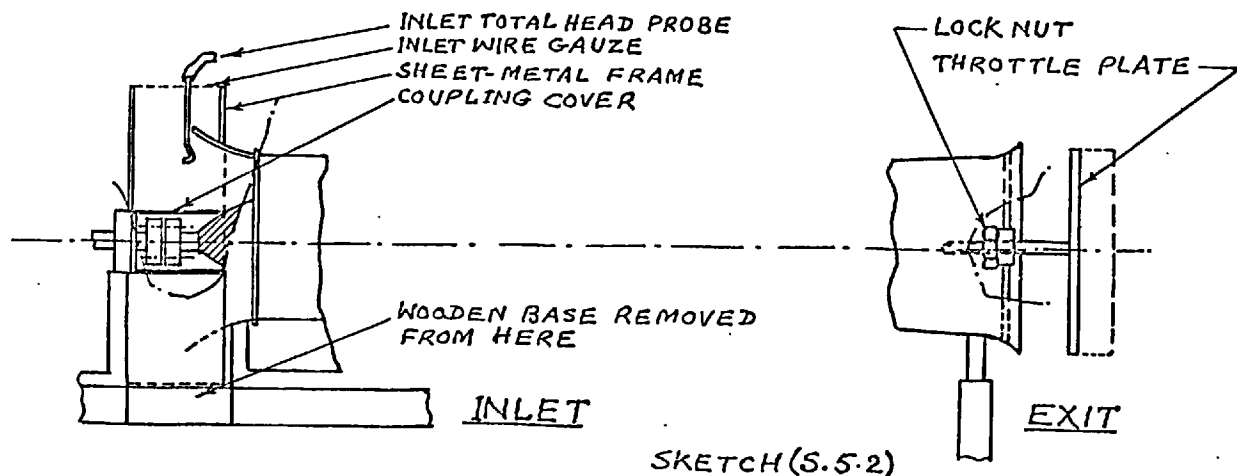
Measuring stations were provided midway between the blade rows, at inlet to the I.G.V. and ^{at} exit from the last stator. The circumferential positions of each of the stator rows and of the I.G.V. could be adjusted through $1\frac{1}{2}$ blade pitches relative to the probe so that the whole area between the pitches could be explored. There were static tapings at four circumferential places between each row of blades so as to provide an average static pressure for each station. All pressures were measured on a 24-tube sloping manometer.

Compressor Inlet and Throttle Modifications

The inlet of the compressor needed some modifications for three reasons:

1. To make the air enter axisymmetrically and to eliminate the disturbing influence due to the motor casing, coupling and the supporting table etc.
2. To filter the air in order to protect the hot wire transducer.
3. To facilitate velocity measurements.

The changes made to achieve these objectives are shown in Sketch (S.5.2). Also shown in the sketch is a



means of securing the throttle plate at exit end by introducing a lock nut, and a special spanner was made to tighten the lock nut.

The calibration of the inlet total head tube is shown in Fig. 5.

Hot Wire Equipment

It was considered that a constant temperature hot wire transducer would offer the most satisfactory approach to the measurement of the mean velocity together with the r.m.s. values of the fluctuating components and the correlation terms and hence a standard set of DISA equipment was procured. A brief description of the circuit has been given in Appendix VI.

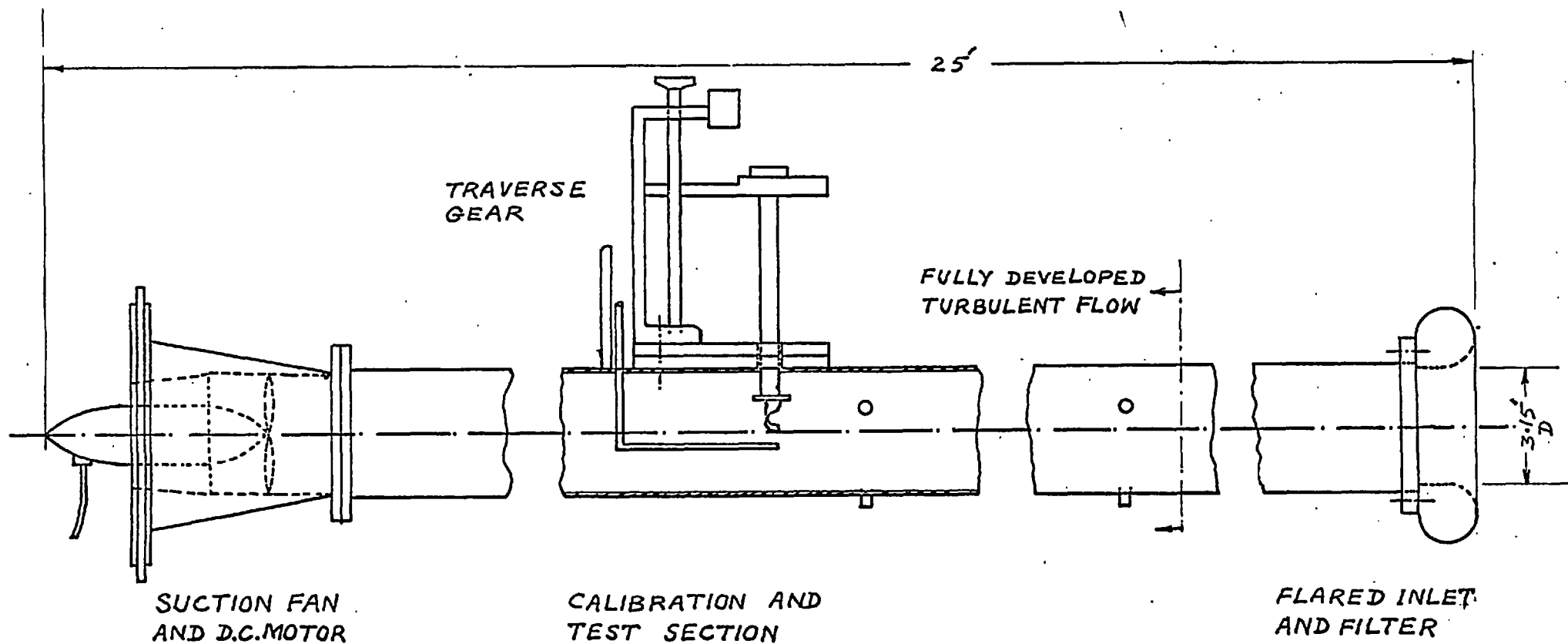
The selection of the shape of hot wire probe was governed by the limitations imposed by the compressor. The short gap between the blade rows did not permit the use of a standard hot wire with the stem facing upstream as is used in the wind tunnel and boundary layer measurements. Only a straight probe with a wire attached to the tips of the prongs could be used. The wire, however,, could be placed at inclination with the probe axis. In order to compute the components of the mean velocity as well as the structure of turbulence, it was necessary to obtain the D.C. and A.C. voltage response of the same or similar wires in different orientations. Cross

wires (X-probes) were ruled out because parts of one or both the wires operate in the wake of the prongs and cause mutual interference effects. Repeated traverses with one or more single wire probes was the obvious method to use. However, this introduced additional problems of running the compressor at a constant speed and relocating the probe at the same place in different runs. The problem of maintaining a constant speed was overcome by measuring the inlet velocity head by a micromanometer to maintain a constant Reynold's number. Accuracy in probe locations was achieved by designing a suitable traverse gear.

The wire material used in the hot wire anemometer was the usual platinum plated tungsten because it was necessary to have adequate mechanical strength, high temperature coefficient of resistance, high specific resistance and its ability to function at elevated temperatures. The specifications of the hot wire probe are listed in Table B and those of the anemometer in Table C.

The Pipe Apparatus

In order to become familiar with the technique of hot wire measurement and with the intentions of developing a suitable probe, a pipe apparatus was constructed. This consisted of a cast acrylic pipe of sufficient length to ensure fully developed turbulent flow at the test section. The total length of 18 ft was made up of three equal pieces flanged together, the ends being matched as to minimize the size of the steps between sections. The



THE PIPE APPARATUS

SKETCH (S.5.3)

pipes were circular within a maximum discrepancy of 0.004 inch in 3.150 inches nominal diameter. The steps between the lengths were less than 0.005 inch each.

At the inlet of the pipe, a smooth contraction in aluminium was followed by a fine wire mesh to serve both as a dust filter and to minimize disturbances due to draughts. A suction fan driven by a D.C. motor (with an accompanying transformer and rectifier) was suitably joined by a flange to the downstream end of the pipe.

The test station for the hot wire calibration and shear traverse was fixed at 60 diameters from the entrance. Provision was made to insert a total head pitot tube from a location 6" downstream of the hot wire station, the pitot head lying in the transverse plane containing the wire. For calibration purposes the wire and the pitot end were placed in close proximity at the centre of the pipe. Two horizontal static tappings were used to measure the static pressure.

The calibration of the hot wire for velocity measurement was achieved by a direct comparison with the pitot tube. The measured velocity profile was also compared with a standard fully developed turbulent profile. Shear stress at the wall was estimated by extrapolation of the apparent turbulent shear measurements at various radial locations. Comparisons were made with the alternative methods of wall shear determination by momentum balance, by reference to standard friction diagrams, Clauser Plots and by certain empirical relations.

Hot-Wire Calibration Tunnel

A calibration tunnel was used to calibrate the probes to be inserted into the compressor because the range of velocities in the compressor was greater than that in the pipe-apparatus.

The tunnel consisted of a centrifugal blower passing air through a ducting turned twice at right angles and a receiving chamber which discharged it through a sharp edged orifice. The centrifugal fan was mechanically isolated from the rest of the equipment to prevent the whole system from vibrating.

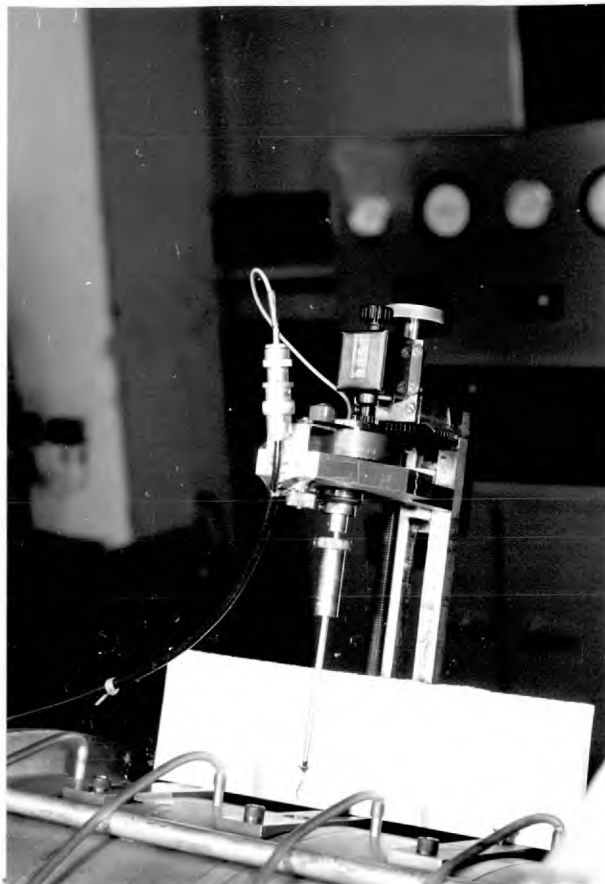
The fan speed was controlled by a regulator but it was not adequate to cover the whole range of velocities required. Throttling at the inlet of the fan by covering it in parts was used as an additional means to regulate the velocity.

The probe was held in the traverse gear which was in turn held in a clamp stand in front of the orifice. A pitot tube and an inclined manometer were used for the velocity measurement.

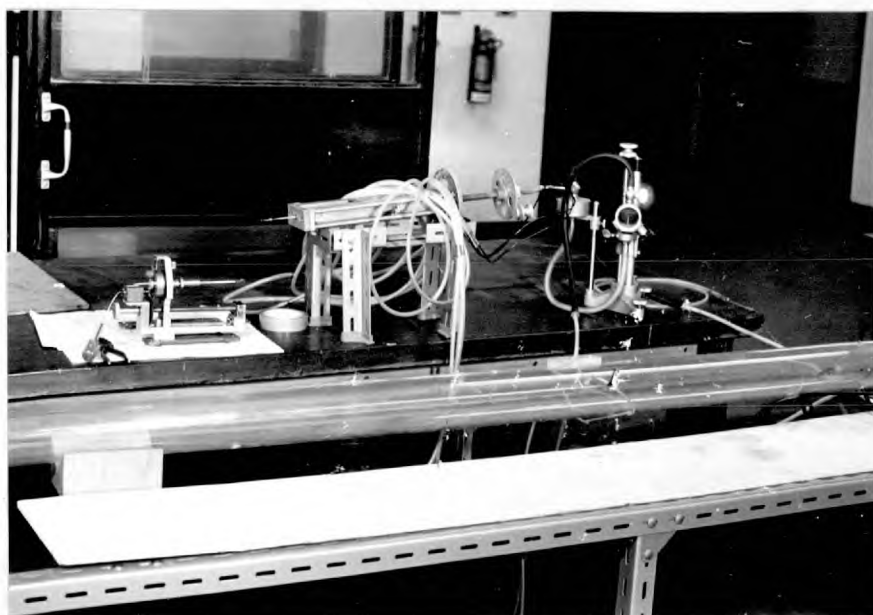
The Probe Traverse Gear

A traverse gear was manufactured with the intended functions of accurate radial and angular locations and quick 45°-settings. A front view is given in Sketch (S.5.4).

The assembly is capable of being mounted on the compressor body or the Pipe-apparatus by two bolts AA. The probe support is held by a tap screw in a bush which



Traverse Gear for the Hot Wire Probe



Hot-Wire Traverse Gear, Yaw-Probe Traverse Gear and the Micromanometer with the Pipe-Apparatus in the Foreground

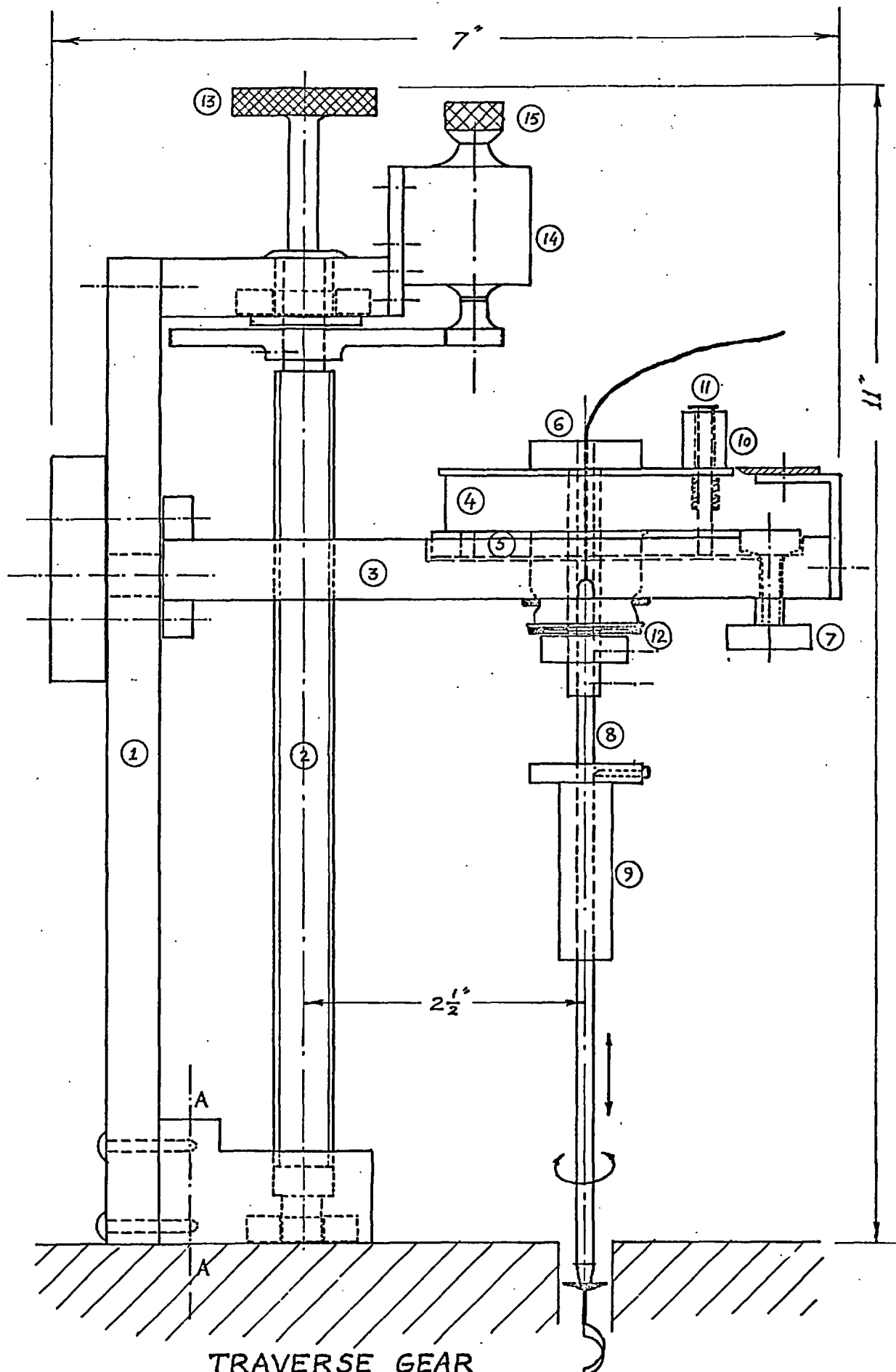
forms a part of the device for rotation and linear movement. The linear traverse was measured by a counter correct to 0.001 inch and the angular settings could be secured within one sixth of a degree.

A distinctive feature of the gear is its capability to reset the probe at intervals of 45° from any arbitrary reference position. This is achieved by locking the knob (7) in the reference position, unscrewing (10) which results in pulling out of pin (11) from the gear, and turning the brass plate assembly (4) until the pin drops into the next hole. The knob (10) is then screwed with the pin pushed in the new position. The main gear (5) has eight accurately drilled and reamed holes at 45° intervals. This feature was incorporated to save time in reorienting the wire with short leg facing and long leg facing positions once the normal wire position and yaw angle are determined.

A sliding brass bush (9) was lowered to block the hole when the probe was lowered into the compressor. It also helped in reducing the probe "wobble" to a minimum. In order to eliminate the vertical play, unidirection traverses were used.

List of the Main Components of the Traverse Gear

1. Supporting frame with a guide slot in it.
2. Threaded bar $3/8$ " B.S.F.
3. Movable plate carrying the probe holder



TRAVERSE GEAR
FOR HOT-WIRE PROBE

SKETCH (S. 5.4)

4. Brass plate with angular scale on top of it and vernier on the right.
5. Main gear attached to the probe.
6. Brush with the probe support clamped in.
7. Knurled knob to change the angular setting.
8. Probe holder with probe at the bottom.
9. Sliding bush to seal the compressor-wall hole.
10. Knob for rapid 45° settings.
11. Steel pin running through the brass piece and gear.
12. Circlips to keep the assembly intact.
13. Knurled knob to raise or lower the probe.
14. Cyclometer reading in 0.001 inch.
15. Zero setting knob for the cyclometer.

Problems Encountered with the Probe

Probe End Effects

A difficulty with many types of instruments when the probe axis is perpendicular to the direction of velocity is associated with the wake behind the probe being blown into the measuring device. This problem was encountered with the hot wire tests when it was observed that the r.m.s. voltage increased with time. The increase in the reading occurred gradually at first and then increased suddenly. Over the same period of time the D.C. voltage would not vary by a measurable amount. The phenomenon was observed to occur for all orientations of the wire to a greater or lesser degree. Examination of the output on an oscilloscope showed that there exists,

over and above the fluctuating component of velocity, a source of disturbance of the order of 150 cycles/second. The initial gradual rise in A.C. r.m.s. reading was due to the development of the source of disturbance and the latter jump may be attributed to a condition of resonance so created.

Attempts were then made to determine the source of trouble and some possible remedies. It was suspected that the induced vibrations of the wire, or probe wake interference sometimes referred to as the "Telegraph Pole" effect could have caused the erroneous response of the instrument. The wire was checked under an optical projector with a high (x 100) magnification and it did not seem likely that the wire or prongs could vibrate in the low-speed tests.

In order to establish the cause and as a trial for a possible remedy, a plane cardboard disc with its diameter three times that of the stem of the probe was fitted on the probe immediately above the prongs holding the wire. Tests on the pipe under similar conditions as before showed that the time unsteady effects and the jump in r.m.s. reading now vanished. The diagnosis was thus confirmed by the remedy and it was adopted as a standard practice to fix a thin machined brass disc on every probe made thereafter.

Wire Orientations

The flow at the pipe test section was one dimensional and hence output from two wires or from the same wire in x-r plane at $+\theta$ and $-\theta$ to the direction of velocity could be used to determine the shear stress. A straight probe with a single inclined wire could be used to operate free from prong-wake effects only in one of the above orientations. Use of a 90° adapter was impracticable from the point of view of later measurements with the compressor. Two alternative means of overcoming the problem were suggested:

1. To select a number of wake-free orientations which may, in general, contain the contribution by all the six components of the apparent turbulent shear tensor.
2. To modify the probe prongs in such a way as to allow wake free operation of the wire in the desired orientations.

Although the second alternative turned out to be more promising, possibilities were explored of analysing the six sets of wire traverse data necessary to obtain the required shear-stress information. Based upon the assumption of low level of turbulence, a set of six equations were set up in terms of the six unknowns and a computer program was developed to evaluate the mean velocity components, the coefficients for the six equations and to solve the set of equations. The solution frequently resulted in absurd conclusions such as 'negative $\overline{u^2}$ ' and 'random change in sign for the shear stress'. The

serious discrepancies were attributed to a number of possible sources of error. The most serious of which were associated with the fact that the important velocity components and correlating factors were obtained by processes involving small differences between large numbers.

The second alternative calling for a modification of the probe-design was considered. The prongs of a DISA straight probe 55-A-22 were bent with one prong cut short to make a 45° inclined wire to operate in clear stream of air. A disc was necessary to isolate the wire from the stem-wake effects. Tests for the velocity and shear distribution in the pipe confirmed that the wire behaved reasonably well and that such a wire could be relied upon for measurements within the compressor.

A first series of tests were conducted on the compressor and the analysis of the results showed that the velocity and turbulence distribution were measured inaccurately. This was explained by the fact that the wire on the crooked prongs was too close to the disc and the stem, and the disturbance created by the disc affected the wire response appreciably. The adverse effects were larger at higher velocities within the compressor than at low velocities in the pipe. A straight-forward solution was to procure a probe with extra long prongs and to redesign it. The special probe was constructed as explained in the next section.

The Special Probe

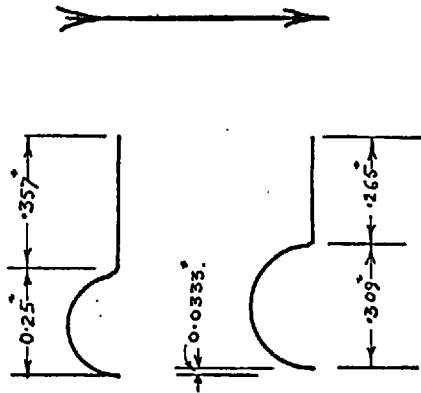
A straight long-legged hot wire probe was adapted for a wake-free operation in the three desired orientations. It was decided to bend the prongs in semicircular arcs of such diameters as to result in the line joining the tips to be at 45° with the probe axis and the distance apart was adjusted to keep the length of the wire close to 1.2 mm. This limit to the wire length was necessary since a longer wire would have a higher resistance and the corresponding operating resistance for an overheat ratio of 1.8 could fall outside the range of anemometer bridge.

A jig was constructed to facilitate bending the probes to the required diameters. Sketch (S.5.5) shows the long-pronged probe as it was delivered and the final probe. An examination of the probe under the optical projector with a high magnification showed that a desired wake-free operation was possible. A circular brass disc was attached with epoxy cement "Araldite" on the stem of the probe to minimize the effects described earlier.

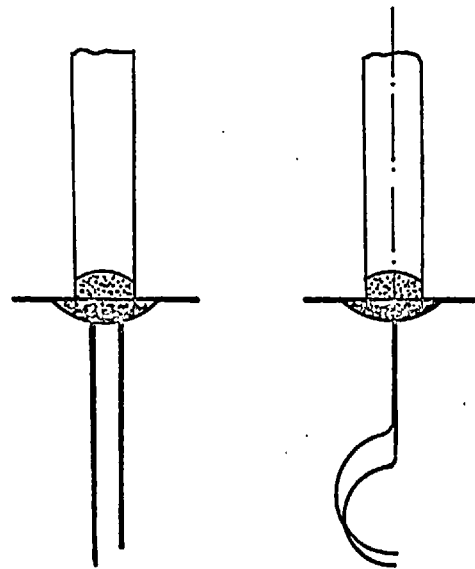
The calibration curves for the probe were satisfactory. The angular response, although it was systematic and better than before, it was not the same for the wire placed at $+9$ and -9 to the mainstream. The prongs did not have a direct interference but there could be boundary layers descending the prongs and sweeping over the wire in one or both of these positions. There was a possibility of such a phenomenon in view of the bent prongs. In order to improve the response, the prongs were slightly re-adjusted



ORIGINAL PROBE



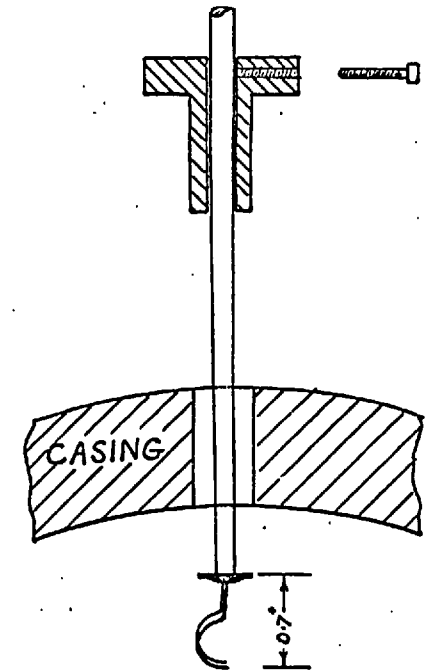
THE BENT PRONGS



MODIFIED PROBE

THE SPECIAL PROBE

SKETCH (S.5.5)



THE PROBE IN USE

SKETCH (S.5.6)

near the tips and a new wire was welded and tested. The discrepancy in angular response still persisted.

A check was made on the turbulence response of the wire in a near-potential homogeneous isotropic flow available near the inlet end of the pipe-apparatus when the wire gauze was removed with the flared entry piece in place. A constant low r.m.s. reading (4mV) in each angular setting of the wire showed the absence of the interference encountered with earlier probes.

The discrepancy in angular response described above was allowed for by considering the probe as three different probes for its use in three orientations. This consideration introduced the necessity of different partial response-factors (f_1 and f_2) in the hot-wire equations (A.IV.23a and b) and the data analysis technique was accordingly adjusted.

The special probe was now used for a set of measurements within the axial-flow compressor.

Calibration of the Hot-Wire

In principle, a hot wire probe is a thin finite electrically heated cylinder with end supports placed in a stream of fluid. Radiation and free convection effects being negligible it loses heat mainly to the flow ^{by} forced convection and to the supports by conduction. With the additional assumptions of steady, uniform and isothermal flow, a dimensional analysis leads to $N=f(\text{Pr}, \text{Re}, \frac{\ell}{\alpha}, \frac{T_m - T_a}{T_a}, \alpha)$ where N, Pr and Re are the Nusselt, Prandtl and the

Reynolds numbers respectively. For constant-temperature anemometry, $N = f(\text{Pr}, \text{Re}_{\text{eff}})$ where the effective Reynolds number Re_{eff} includes the effect of α and ℓ/d . The functional form may be expressed by a semi-empirical equation and the rate of heat lost by the wire may be equated to the rate of electrical heating for the equilibrium state. The feedback bridge voltage E may be expressed as

$$E^2 = A + BU^p \quad \dots(5.1)$$

This equation is slightly more general than the King's Law (1914) where $p = 0.5$. Although some more complicated relations have also been proposed by some authors, Equation(5.1) was assumed to be adequate for the present analysis.

The calibration of the wire, thus required the evaluation of the constants A , B and p . In the first instance, it may appear to consist of a simple process of recording E and U for a range of flow conditions but in practice, it is complicated by the considerations described below:

1. A least square fit to the data may result in a wrong set of constants if one or two points fall outside the smooth trend formed by the rest of the points. This is chiefly due to the relative insensitivity of the residual with the index p . Inaccuracies in the index may severely effect the reliability of shear measurements. A programme was written (details are given in Appendix VII) to perform the least squares fit and to indicate the standard

deviation and the residual in the solution of indicial equation. As shown in Fig. 10, the standard deviation was minimum at $p = 0.44$ but the residual in the indicial equation was not the minimum. Such a discrepancy was observed with the data of different wires and it was later noted that such a fit gives misleading results for shear stress in a fully developed pipe flow. In view of this problem, it was considered to be better to select a value of p consistent with the expected results in the pipe flow and use the same value for compressor measurements. The constants A and B were again determined by a least square fit.

2. The measurements in the compressor requires the wire response to be found in three orientations:

The wire 'normal' to the flow, the wire with short leg facing the flow (S.L.F.) making $+\theta$ with the velocity and the wire with long leg facing (L.L.F.), making $-\theta$ with the velocity. On theoretical grounds only one calibration with the wire 'normal' to the flow should be adequate but experimental results described earlier showed that the cosine cooling law was inadequate and that the additive or deficit cooling for short leg forward (S.L.F.) and long leg forward (L.L.F.) positions could be approximated by the introduction of partial response factors f_1 and f_2 respectively, as shown in Equations (A.IV.23a and b). It was, therefore, considered necessary to calibrate the wire in each of the three orientations required for subsequent measurements.

3. The calibration constants are effected by the level of turbulence at the calibration station. It was verified that the turbulence was low both at the centre of the pipe and at the exit of the orifice of the calibration tunnel.

4. It has been observed that for a tentative value of p , a relationship of the type $E^2 = A + BU^p$ does not hold good for the entire range of velocities. In general, A and B showed dependence on the magnitude of velocity but linear fits between E^2 and U^p are quite valid for a short range of velocities. At higher velocities, A is higher and B is less than the corresponding values at lower velocities. A typical response corresponding to a set of calibration curves shown in Fig. 6 suggests:

A	B		p
34.5	6.4	for 5-25 ft/sec	0.5
37.5	6.0	for 25-90 ft/sec	0.5

This behaviour of the wire conforms with a still lower zero-velocity voltage squared of 31.5 in this case.

5. Because of the variations of the wire response with time due to dust collection, ageing of the wire and the change in ambient conditions, it is necessary to calibrate the wire frequently. Efforts were made to establish a calibration station before the inlet guide vanes but unfortunately the direction of velocity was

found to deviate from the axial direction due to the interference of the six webs supporting the casing.

For practical reasons, it was decided to conduct a series of tests in a consecutive order without loss of time and to recalibrate the wire only towards the end of the series. Average values of the constants were used for the analysis of the data.

Yaw Sensitivity and Stem Inclination Effects

It was realised that a single wire does not possess good angular response especially when placed normal to the flow.

Considering a wire whose normal makes an angle θ with the direction of uniform velocity U . The effective cooling velocity is made up of the normal component and a small fraction of the component parallel to the wire.

$$\text{Hence, } U_{\text{eff}}^2 = U^2 \cos^2 \theta + f^2 U^2 \sin^2 \theta \quad \dots(5.2)$$

From the relation, $E^2 = A + B \sqrt{U_{\text{eff}}}$, differentiating

$$\text{w.r.t } \theta, \quad 2E \frac{dE}{d\theta} = B \frac{d}{d\theta} (U_{\text{eff}}) \quad \dots(5.3)$$

Substituting U_{eff} from above, and simplifying,

$$\frac{dE}{d\theta} = \frac{B\sqrt{U}}{2E} \left(-\frac{1}{2} \frac{\sin \theta}{\sqrt{\cos \theta}} + f^2 \sin \theta \frac{4 - \sin^2 \theta}{8 \cos^{5/2} \theta} \right) \quad \dots(5.4)$$

For a typical case, $A = 30$, $B = 5$, $f = .2$,

$$U = 60 \text{ ft/sec, } E = 7.8, \quad \frac{B\sqrt{U}}{2E} \approx 2.5 \text{ and } \frac{dE}{d\theta} = 1.25 \frac{\sin \theta}{\sqrt{\cos \theta}} +$$

$$.1 \sin \theta \frac{4 - \sin^2 \theta}{8 \cos^{5/2} \theta} \quad \dots(5.5)$$

Fig. 8 shows that $\frac{dE}{d\theta}$ is very small as $\theta \rightarrow 0$ for a normal wire.

The alternative of using a parallel wire for the indication of the direction of velocity by a sudden rise in the turbulence level due to prong interference was explored but it was not found to be successful.

In a publication by Delleur (1966) a technique has been presented which doubles the angular response by using a cross probe. Additional electronic equipment is necessary to render the probe insensitive to the variations in mean velocity and extra-sensitive to the directional variations. This method cannot be used in the compressor since the use of cross wires is ruled out.

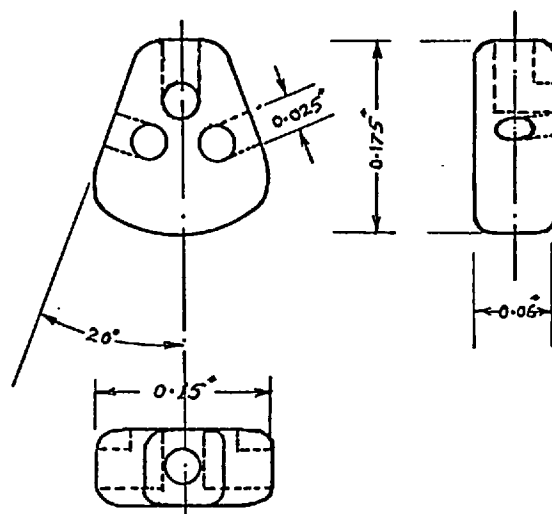
More tests were conducted in order to assess the effect of the inclination β of the stem axis with the direction of velocity. With the wire maintained normal to the flow (by observing maximum D.C. reading), in the exit plane of the free jet of the calibration tunnel, it was observed that the D.C. voltage was highest when the probe axis was normal to the flow and least when the probe axis was in line with the flow. The test was then extended to observe the variations in the voltmeter reading with β varied in steps of 15° . It was thus established that the response of the wire under such conditions could differ by as much as 15% in the measurement of velocity.

The above test was repeated with the wire now kept at $+45^\circ$ and -45° with the direction of velocity and a similar variation in the response was seen to occur with varying β .

The conclusion of this test, therefore, was that the probe should be calibrated in the same orientation relative to the velocity as it is intended to be used. In view of these results the probes for the axial-flow compressor tests were always calibrated with the stem axis perpendicular to the direction of flow.

Yawmeter Probe and Traversing Mechanism

A combination yaw, static and total pressure probe primarily intended to measure the inclination of the velocity with the axial direction was made in the laboratory. It was not a standard type but was designed to be small enough to be used in the gaps between the blade rows and still retain the qualities of a good probe. As shown in Sketch (S.5.7) it was a small wedge shaped element with a hole drilled in the front and one perpendicular to each of the two side surfaces.



COMBINATION YAW, STATIC
AND TOTAL PRESSURE PROBE

SKETCH (S.5.7)

Extreme care was taken in drilling the holes. The tappings were brought up by drilling three holes from the upper surface. Half inch lengths of hypodermic tube (outside diameter 0.025", inside diameter 0.012") with their edges cut at 45° were fixed in the top holes (each 0.026" dia.). Two feet lengths of hypodermic tube (outside dia. 0.05", inside 0.03") were joined to the smaller tubes by silver soldering. The larger diameter tubes were used for greater lengths to ensure quick response time and to impart rigidity to the set of three tubes which were considerably long.

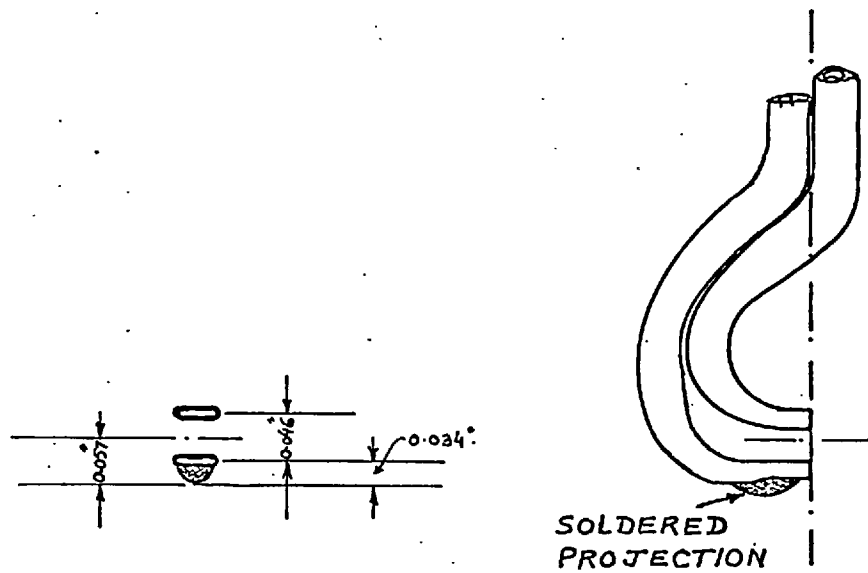
The traversing mechanism was capable of recording linear distances with an accuracy of 0.005" on the sliding vernier scale and to within half a degree in the angular setting.

In order to use the probe for pressure and velocity measurements, it was calibrated in a uniform free stream for these in addition to the angular scale calibration which was, of course, necessary. The calibration proved very useful since a single traverse in the compressor resulted in a great deal of information. The calibrations is given in Fig. 9.

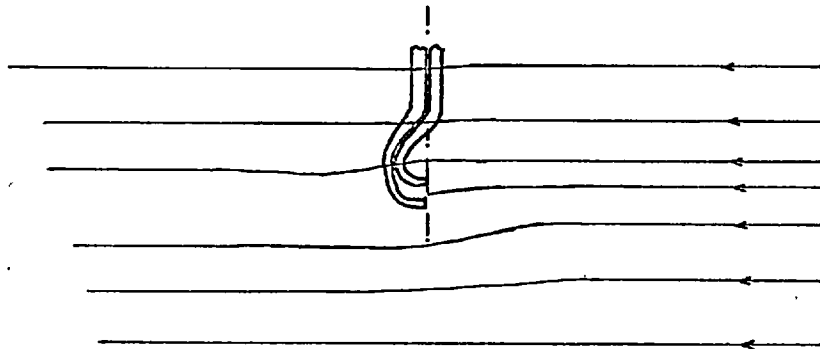
Velocity Gradient Probe

Although sufficient confidence was placed in the observations by hot wire probe for the determination

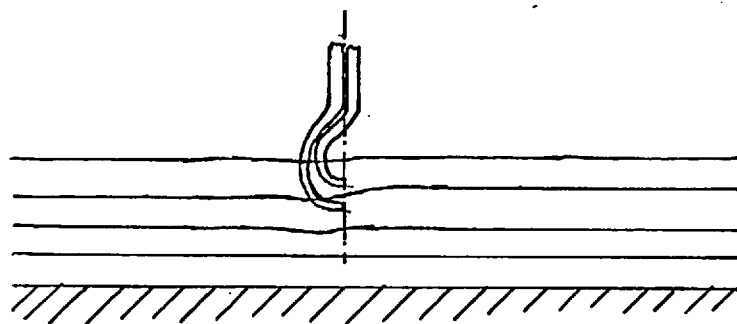
of mean velocity and turbulence, it did not provide any direct means of measuring the velocity gradient. Data smoothing and 'best curve fit' techniques were used to obtain the velocity gradient. For the purpose of comparison, techniques offering direct measurement of the gradient were explored. The probe selected for this purpose consisted of a pair of flattened-end total head pitot tubes placed close to each other. The overall shape of the probe and its dimensions were governed by the gap between the blade rows. The size of the tubes were so chosen to be small enough as not to create undue disturbance and large enough to have a reasonably low response time and free from blocking troubles. Although the tubes were flattened at the end, the opening was adjusted to be slightly wider in front than the rest of it. If the tubes were placed too close together or touching each other, the pair might act as a stagnation point and no velocity gradient would be observed; on the other hand, increased distance between the tubes would result in errors in the velocity gradient. In view of these contradictory requirements, it was decided to keep the tubes 2 to 4 external diameters apart. The pair of tubes was placed together with an epoxy cement "Araldite" and connected to larger size tubes. The completed instrument is shown in Sketch (S 5.8a).



(a) CONSTRUCTIONAL DETAILS



(b) PROBE IN NEAR-UNIFORM FLOW.



(c) PROBE NEAR A WALL

VELOCITY-GRADIENT PROBE

SKETCH (S.5.8)

Performance of the Velocity Gradient Probe

For the case of fully developed pipe flow, the mean velocities computed from the readings the upper tube were close to the velocity obtained by hot wire traverse. The dimensions of the probe limit the minimum distance from the wall to 0.070". The measurements of the velocity gradient from the differential pitot reading were observed to be erroneous and, in general, much higher than those obtained graphically from the velocity distribution or computed by short-range polynomial-fit to the velocities indicated by the hot wire.

The velocity sensed by the upper tube was generally 5-7% less in comparison with the hot wire results and the velocity gradients were too high by a factor of 3 or so.

An important step in the use of this instrument was to calibrate it in a uniform flow. The differential reading did not vanish for reasons of slight inaccuracies in the fabrication and the development of a flow-pattern in the vicinity of the probe. An adjustment of the probe was made to indicate a minimum differential reading for zero velocity gradient, and it was observed that the differential total head was proportional to the velocity squared. The upper pitot reading was calibrated against a true pitot placed close to the probe in order to facilitate the measurement of the velocity gradient and the velocity in a single traverse. The calibration curves and the results obtained with the instrument are given in Figs. 10 and 11.

Explanation of its Erroneous Behaviour

The need to calibrate the probe arises from the fact that a uniform flow intended to be measured is distorted by the presence of the probe. When the probe is placed in a stream of fluid with a certain velocity gradient, the flow pattern near the probe is further complicated by a radial shift of the streamlines as sketched in (S.5.8'b,c). This shift depends upon the velocity gradient, magnitude of velocity and the size and shape of the probe. The shift is really a consequence of the vortex lines in a viscous flow getting stretched round the obstruction. The stretching of the vortex lines and changes in area are accompanied by differential increase in vorticity by Kelvin's Theorem. This, in turn, results in a cross flow in the direction of decreasing velocity.

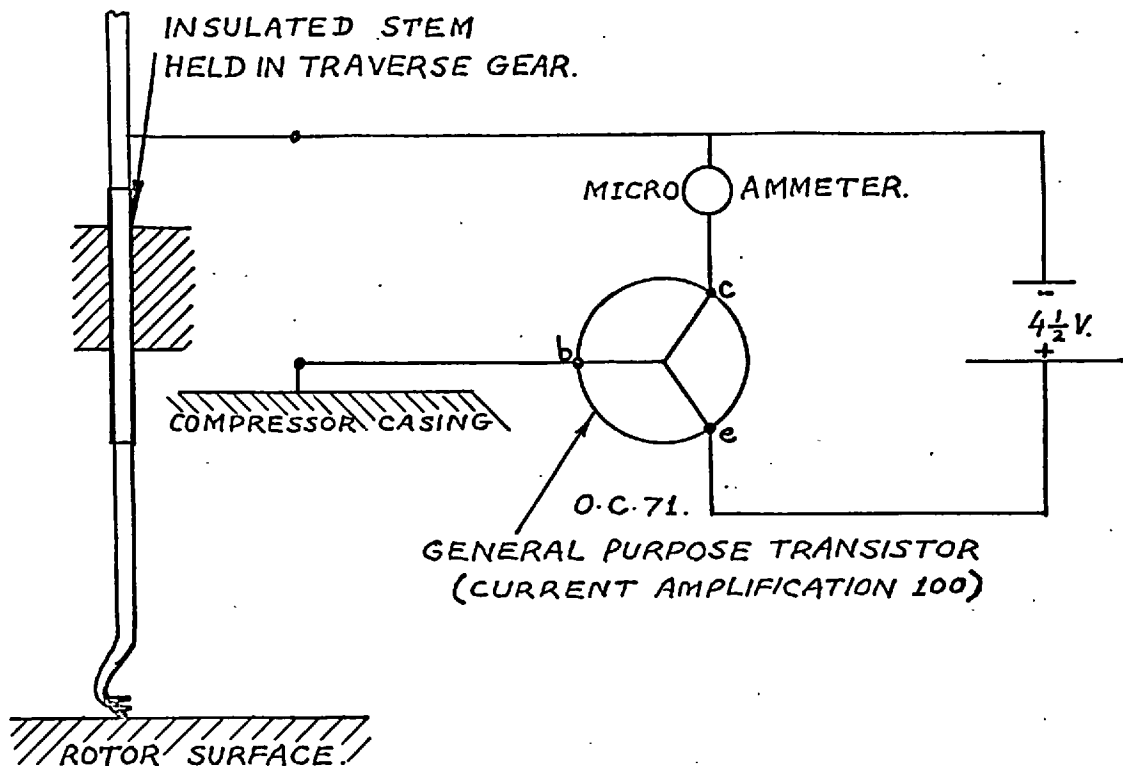
Experimental studies by Young and Moss (1937) and other investigators for different sizes and shapes of the pitot tubes showed that $\delta/d \approx 0.16$ to 0.18 . Theoretical investigations by Lighthill (1957) revealed that δ/d should increase with a parameter $(d \cdot dC_x/dr / 2C_x)$. No such information is available for a pair of pitot tubes placed a short distance apart. The resulting flow pattern when the probe approaches a wall is still more complex. Therefore, the probe cannot be relied upon to measure the velocity gradient of an oncoming flow. In brief, it suffers from the defect of 'creating its own flow pattern and measuring it'.

Another problem associated with the velocity gradient probe was the measurement of very small differential readings. A micromanometer was employed to do this but an error of only 0.002" in 0.04" corresponds to an error in velocity gradient of 10%, the results could not be expected to be very accurate. Development of this device was, therefore, terminated.

Contact Indication

Although the cyclometer on the probe gear functioned satisfactorily it was thought advisable to make a check on the measurement of the distances from the rotor. A simple electrical contact circuit was devised to indicate when the yawmeter or velocity gradient probe just touched the rotor. Due to a layer of insulating dust adhering to the rotor surface, the circuit indicated a very high resistance of 20 M Ω and with a 4.5 volt battery the current was 0.25 μ amps. This was too small to show up on the microammeter and hence an O.C. 71 general purpose transistor was employed to boost the meter current. This increased the meter current to 20 μ amps and the meter deflection on contact was quite noticable. The measurements subsequently confirmed that the cyclometer was accurate to 0.001".

The circuit is shown in Sketch (S.5.9).



TRANSISTORISED CIRCUIT
TO INDICATE CONTACT WITH THE ROTOR

SKETCH (S.5.9)

Spot Welding of Wire on Probes

In view of the odd shape of the probe prongs the task of welding the wire on it called for special care. It was decided to repair the wires using a DISA precision micromanipulator and sport welding unit. This process also resulted in saving time and expenditure in the event of a wire breakage during a series of tests on the compressor.

The special probe could not be held in any of the standard holders and an alternative holding technique was developed. Care was taken to clean the prongs, the electrode and the wire before each weld. The contact

pressure between the probe and the electrode with the wire held in between was optimized. The capacitance for the power-supply-capacitors was kept between 4000 μF and 6000 μF with 5-7 volts across the open circuit for clean and point welding. Extra wire on either side was burnt off with a high capacitance setting (8000 μF) and a high voltage across them.

As soon as a wire was welded, the probe was re-examined under a high powered optical projector (magnification x 50) for any defects in welding and the inclination of the wire with the probe, θ was measured.

CHAPTER 6

THE COMPRESSOR TESTS AND THE ANALYSIS OF DATA

The compressor inlet modification and the traverse gear manufacture have been described in the previous chapter. The problems associated with the response of hot wire probe and the development of a special probe have also been outlined. Having overcome the initial difficulties, three series of tests were conducted on the compressor and with the help of relevant hot wire relations derived in Appendix IV, the data were analyzed. Although only limited success was achieved by the first two series, a brief description has been included and the sources of error are discussed. The third series of tests which resulted in more useful information have been described in greater detail and the implications of the results are discussed.

Selection of the Operating Conditions for the Test

The operating conditions were defined by the Reynolds Number and the flow function C_x/U (sometimes referred to by the number of turns of the throttle opening). The Reynolds Number Re for an axial flow compressor is usually denoted in terms of the blade chord as a characteristic length and the mid-annulus inlet relative velocity at design conditions as a characteristic speed. Thus, $Re = W_1 \cdot c \cdot \rho / \mu$(6.1)

From the velocity diagram, $W_1 = U/(\sin \beta_1 + \cos \beta_1 \tan \alpha_2) \dots (6.1a)$

and for conditions of reference minimum loss incidence

for the three stage compressor it was found that

$\beta_1 = 53.2^\circ$, $\alpha_2 = 32^\circ$, $c = .1$ ft. Then $Re = 0.00505 \text{ } N/\mu$.

Considering the slight variation of viscosity with

temperature

$$\mu = (28 + 0.18T) \times 10^{-7} \text{ lbm/ft. sec.}$$

and from the perfect gas law $\rho = p/(R \times T)$.

Then,

$$Re = 37200 \times H \times N / (460 + t) / (616 + t) \dots (6.2)$$

where N = compressor speed in r.p.m.

H = Barometer reading in inches of mercury.

t = Ambient temperature in $^\circ\text{F}$, T in $^\circ\text{F}$ Absolute.

A 'carpet plot' depicting the selection of the compressor speed N for a desired Reynolds Number and the recorded ambient conditions (H and t) was prepared as shown in Fig. 12 and kept near the compressor control-panel for ready reference.

The maximum attainable Reynolds number was between 85000 and 90000 at 3000 r.p.m. compressor speed. The highest operating Re was fixed at 80000 so as not to require an over-speed for the range of anticipated ambient conditions. The minimum operating Reynolds number was fixed at 50,000.

Control of the number of turns of the exit throttle referred to an arbitrary zero (maximum closure) controlled the mass flow. For the three stage compressor, 8 turns

refer to the design flow, 6 for near-stall and 14 for a high mass flow.

First Series of Tests

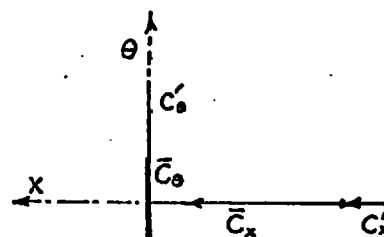
To Determine the Complete Stress Tensor.

The first series of tests was undertaken using a straight wire, an inclined wire and a yawmeter. In order to obtain the necessary data at each station, the traverses were repeated with each of the above instruments. The response of the straight wire in three orientations and of the inclined wire in another three angular positions shown in Sketch (S.6.1) was recorded at each point.

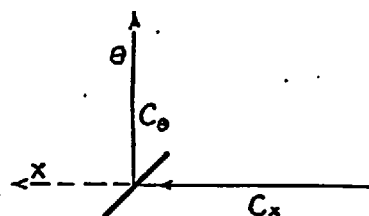
The three components of the velocity may be estimated by an analysis of the D.C. voltages recorded for three different inclinations of the hot wire, preferably in one traverse. The evaluation of $\sqrt{\overline{C_x'^2}}$, $\sqrt{\overline{C_y'^2}}$, $\sqrt{\overline{C_z'^2}}$, $\overline{C_x' C_y'}$, $\overline{C_y' C_z'}$ and $\overline{C_z' C_x'}$, the six-independent components of the turbulent-stress tensor may similarly be evaluated from an analysis of the r.m.s. response of the wire in six independent positions. Three of these six orientations may preferably be the same as used for finding the velocity components.

With this approach in mind, a set of equations were derived from consideration of the effective cooling velocities and the electrical response relations. A computer programme was developed to solve a set of three simultaneous equations for the velocity components and

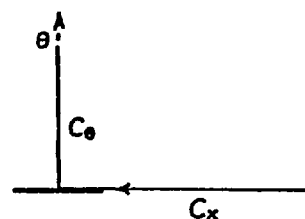
1. STRAIGHT WIRE NORMAL TO X-AXIS.



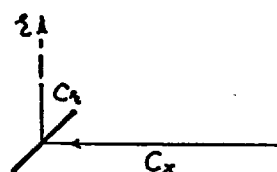
2. STRAIGHT WIRE AT 45° TO X-AXIS (S.L.F)



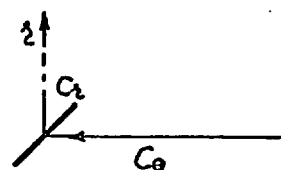
3. STRAIGHT WIRE ALONG X-AXIS.



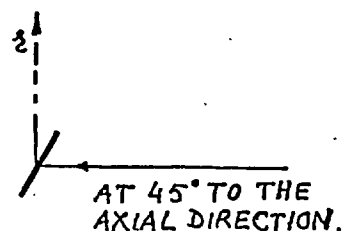
4. INCLINED WIRE NORMAL TO θ -AXIS.



5. INCLINED WIRE NORMAL TO X-AXIS.



6. INCLINED WIRE TURNED 45° ON THE PROBE AXIS FROM POSITION 4.



HOT-WIRE ORIENTATIONS
FOR THE FIRST SERIES OF TESTS

SKETCH (S.6.1)

the six more-complicated equations for the stresses.

The latter set was more involved as the velocity components formerly obtained and stored formed a part of the coefficients of the stress terms in the equations.

The calculated mean axial and tangential velocities were within 5% of the measurements with a pitot probe, but the radial velocity measurements were incorrect. A manual check on the wire-response revealed that an error in the second decimal place in recording the D.C. voltage ($\approx 0.2\%$) could cause as much as 5% error in the resultant axial and tangential velocities and many times more in the radial component. Measurement of the voltages by a more accurate digital voltmeter was the obvious step but unfortunately fluctuations in readings occurred and no marked improvement in accuracy was noted. More details of the errors involved in the measurements have been given in Appendix V.

The results for the turbulent quantities were obviously incorrect since the solution of the simultaneous equations resulted in a negative value of one or more of the terms $\overline{C_x^2}$, $\overline{C_u^2}$ and $\overline{C_v^2}$.

The following factors account for the misleading results in the above procedure.

1. The data was obtained from two separate traverses by two different hot wire probes and grouped together to solve a set of equations. This is unsatisfactory since the location of wires at coincident points in space is very difficult, and the running of the compressor at exactly the same operating conditions presented a problem.

2. A relatively small error in only one reading in a set could result in an absurd solution of the equations.
3. As it has been mentioned earlier, the probe response was subject to an error due to the closeness of the disc and the wire.
4. The hot wire analysis relations were based upon the assumption of low level of turbulence and complete wake-free operation. The equations were re-examined and it was noticed that the solution of the equations was sensitive to a slight change in the constant B and index p in addition to the random errors in the bridge D.C. and r.m.s. voltages.

An attempt to improve the above procedure could be made by setting up more than six equations and solving them for the six unknowns by an optimization technique. This, however, would involve the use of more than two wires which would tend to introduce further problems.

The procedure adopted in the first series of measurements was abandoned and the idea of using more than one wire was given up.

Second Series of Tests.

To Determine the Effective Viscosity Distribution

The second series of tests on the compressor were performed with the special probe and the yawmeter. One traverse of the special probe in three orientations preceded by a yawmeter traverse was sufficient to obtain the required data. Although the flow is in general

three dimensional, the radial velocity was assumed small compared with the resultant velocity. The direction of the resultant velocity was established with a yawmeter and the wire, placed in the X-R plane, was oriented in three positions relative to the mean flow, that is normal, at $+45^\circ$ and at -45° to it. These positions are referred to as normal, short leg facing (S.L.F.) and long leg facing (L.L.F.).

The traverses were conducted at the exit plane of each of the three rotors for six operating conditions obtained by varying the Reynolds number (60,000 and 80,000) and by adjusting the exit throttle plate to give maximum, designed and near-stall flow (14, 8 and 6 turns open, referred to as an arbitrary closing). The set of data included the reading on the traverse gear, three D.C. voltages, three root mean square of the fluctuating hot wire voltages and the yaw angle.

The analysis of the data was made with the help of a computer programme described in Appendix VII. Variation of shear stress across the annulus was incorrect since the sign of evaluated shear stress changed randomly. The velocity profile was, however, realistic. It was also observed that the same hot wire, with the same angular settings, yielded satisfactory results in the pipe test. It was, therefore, thought that the hot wire might have been subjected to adverse conditions of operation and that the flow did not possess an established pattern. The rotor wakes passing downstream were

considered as a likely source of error since the probe placed downstream close to the rotor could thus be subjected to a flow where the structure of turbulence was not yet established.

The problem, therefore, was reduced to eliminate the undesirable random wake-effects and still be able to measure the turbulent shear stress distribution.

One of the alternatives was to obtain a frequency analysis of the signal, to distinguish those frequencies contributed by the rotor wake effects from those arising from the turbulent motion of the fluid and to filter out the former. This procedure seems sound in theory but it would pose many practical problems even if a frequency analyser and other related equipment were available.

The value of the results so obtained would be questionable to the same extent as the frequency analysis and filtering process. A second alternative described below was therefore adopted to overcome the problem.

Third Series of Tests.

For Effective Viscosity Distribution with a Single Rotor

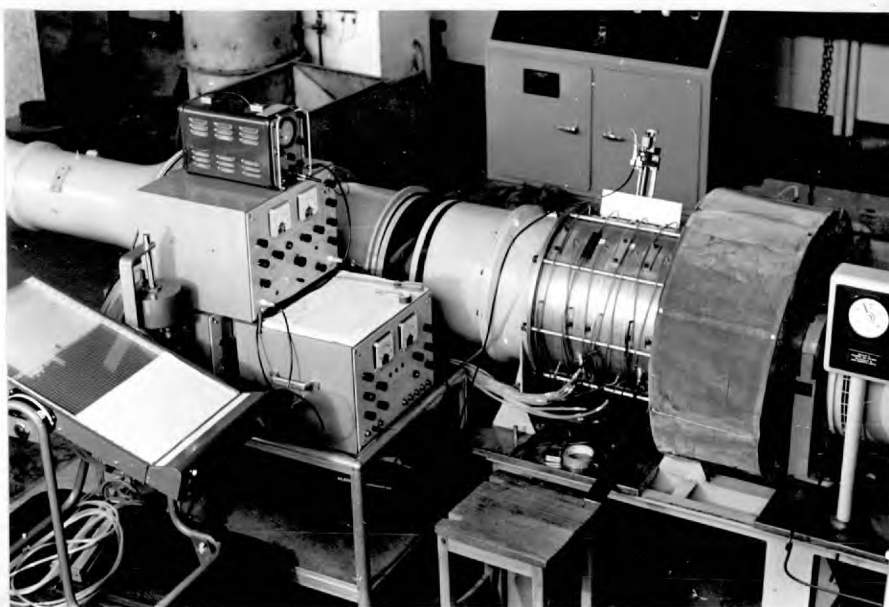
Since the major cause of discrepancy was considered to be the closeness of the measuring station to the rotor, it was thought that if tests were conducted some distance downstream of the rotor, better results may be obtained. The experimental compressor had the provision to vary the distance between the blade rows from $3/8$ " to $7/8$ " only.

The only way to be able to set up measuring stations at reasonable distances from the rotor row was to dismantle the compressor and remove the blade rows after the first rotor.

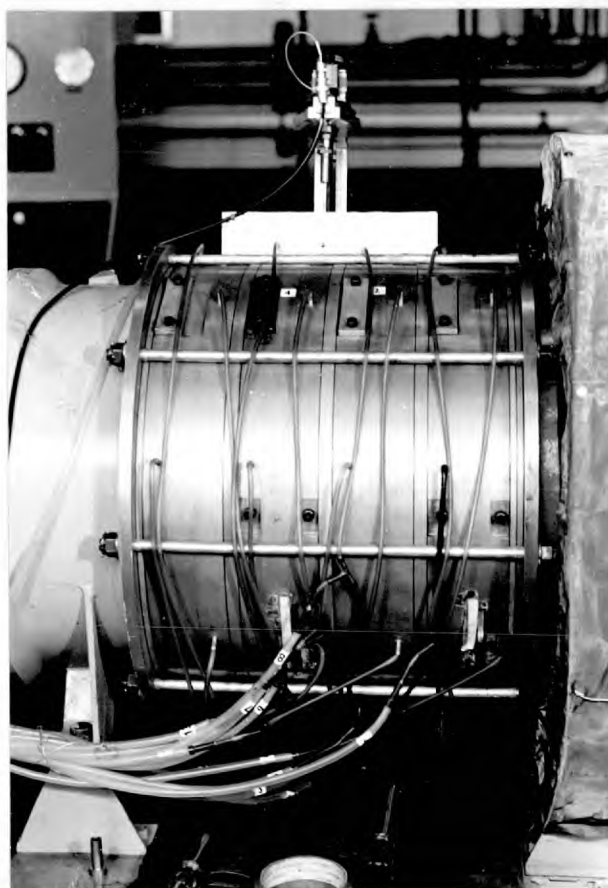
The compressor was subsequently dismantled and all the blade rows except the I.G.V., first rotor blade-row and the last stator blade-row, were removed. Care was taken to cover the holes left by the removal of the blades. This was achieved by cleaning the surfaces and wrapping adhesive tape on them, special care being taken to secure the ends properly with a thin layer of epoxy cement "Araldite".

Stations 1 to 5 could now be used to test at varying distances from the rotor. A single tests at station 5 at an arbitrary running condition revealed that the results were realistic. The effective viscosity was seen to be positive at each radial point and the distribution in the radial direction appeared satisfactory.

A new wire was spot-welded to the probe in order to achieve the maximum accuracy with the results and examined under a high-magnification optical projector. In accordance with the procedure followed in the previous tests, the wire probe was calibrated both on the calibration tunnel and in the pipe-experiment. The calibration curves are shown in Fig. 6. The apparent turbulent shear stress distribution at the test station in the pipe apparatus was measured with the hot wire and



The Axial-Flow Compressor Under Operating Conditions



Close-up of the Compressor

extrapolated to obtain the total wall shear as shown in Fig. 13. The calibration constants A, B and p for equation (5.1) were then selected as outlined in Chapter 5. The tests on the pipe apparatus were concluded with the determination of effective viscosity distribution as plotted in Figs. 14 and 15.

Before starting the third series of tests it was intended to investigate the effect of dispersion of the blade-wakes on the response of the hot wire. The probe was placed in the Station 1 and the compressor was run at low speed (760 r.p.m.). The number of blades being 38, the frequency of the blade wakes was $760 \times 38/60 \approx 480$ c.p.s. It was observed on the oscilloscope that on a scale of 1 millisec/cm the wavelength was 2.1 cm., which corresponds to a frequency of $1000/2.1 \approx 480$ c.p.s. The probe was then mounted at stations 3 and 5 and for the compressor under the same conditions, no overall waveform of the type observed at station 1 was detected. A repetition of the test at higher rotational speed confirmed the above feature.

The preliminary tests had been successful and, therefore a third series of tests were undertaken. It was realised, however, that the near-stall, design and the high mass flow conditions previously obtainable at 6, 8 and 14 turns throttle open were now possible at 11.5, 16 and 28 turns respectively.

The data obtained in the final series of tests was analyzed with the help of a computer programme, details

of which are given in Appendix VII.

A Note on the Reliability of the Measurement

The results computed for turbulence, shear stress and velocity distribution from the data of hot wire traverses can be deemed as reasonably reliable for the following reasons:

1. The calibration of the wire was satisfactory in the beginning of the test and a recalibration of the wire towards the end of the test did not differ appreciably from the first. An error of 5% could result from the change in the calibration constants due to change in its cold resistance by dust accumulation and ageing.

2. The distribution of velocity, shear and effective viscosity in the fully developed pipe flow was measured within $\pm 5\%$ of the expected results. The same wire being used to measure the same quantities in the compressor is liable to an additional error due only to:

(a) slightly higher level of turbulence at stations 3 and 5, though appreciably higher level of turbulence exists at station 1 immediately downstream the rotor blades.

(b) three dimensional flow effects which could be high for high radial velocities.

3. The wire was adequately protected against the "telegraph pole effect" and the wake interference from the probes. Furthermore, the wire was used in the same orientations as it was calibrated.

4. The distances from the rotor wall were measured accurately. Checks with the depth micrometer and careful initialisation of the wire at entry into the compressor ensured an accuracy within ± 0.002 inch. The unidirectional traverses with a cyclometer-type revolution counter enable the relative distances to be measured within ± 0.001 inch and thus the velocity gradients could be relied upon.

It was considered adequate to adjust the angular position of the hot wire exactly only to a degree on the mainscale since the wire sensitivity to yaw did not demand finer adjustment.

5. The results for the tests conducted at various operating conditions possess even better comparative value since the same wire transducer was used in the whole set of observations. On theoretical grounds, different wires, properly calibrated, should yield the same results and it has indeed been experimentally verified that the different wires indicate the same shear and velocity distribution in the pipe flow and the same order of effective viscosity in the compressor. However, practical difficulties of rewelding the wire at precisely the same position on the tips of the prongs, chopping off the extra wire exactly, calibrating and orienting the probe, the comparative value of the results could have been lessened or lost if more than one wire was used for the complete set.

It may be mentioned that the wire was lost after 15 runs when the compressor was being run at near-stalled conditions at 11.5 throttle turns open. This did not

jeopardise the results since the major part of the experiment was already performed.

6. Inaccuracies in recording A.C. and D.C. voltmeter readings result in random error. An error analysis has been included in Appendix V.

Implication and Discussion of the Results

Almost all the useful results were obtained from the analysis of the data of the third series of tests performed on the reassembled compressor with only the I.G.V., first rotor blade row and the last stator blade row. The results are given in Figs. 16 to 23. The following statements summarise the inferences made from a critical study and comparison of the results for the series of tests.

1. Just downstream of the rotor, negative values of effective viscosity were observed due to both the incorrect shear measurements and the arbitrary basis of the effective viscosity hypothesis. The D.C. voltages were not as much in error as the r.m.s. voltage output. This was mainly due to the cyclic interference of the wakes from the rotor blades and only partly due to a higher level of turbulence. The presence of the wakes did not permit the measurement of the turbulent shear even with the special hot wire probe.

2. The magnitude and the direction of velocity did not change by more than 5% at stations 2 to 5 downstream of the rotor. The level of turbulence dropped with distance from the rotor. The level at station 1 was 40% higher and that at station 3 was 20% higher than that at station 5. These figures suggest a possible

explanation of the discrepancies in measurements close to the rotor.

3. Frequency analysis of the signal showed that the wake effects died out as the flow progressed from station 1 to 5. The distribution of the effective viscosity and the shear distribution indicated more systematic and interpretable results at station 5 than at station 3, where more scatter was observed. It is inferred that the structure of turbulence was better established and the wakes lost their identity in the course of fluid flow from the rotor exit to station 5. It is assumed that the measurements at station 5 were an indication of those as 'at the rotor exit' since the turbulent shear is a consequence of the microstructure of turbulence which did not change significantly over the distance.

4. Examination of the results at station 5 showed that:

(a) Whatever the Reynolds' number and whatever the flow rate, the effective viscosity is of the same order as the molecular viscosity near the rotor wall, it rises steeply to a maxima at 15% blade height, drops towards the centre of the annulus with a defined minimum in some cases and increases again. The trend could not be established beyond 1.7" from the rotor due to the casing hole effects and the geometry of the special probe.

(b) The resultant velocity C_c was always observed to be the highest near the rotor drum and least at the casing while the velocity gradient, always negative, were highest near the wall and decreased towards the core region.

(c) At high Reynolds' numbers of operation, the peak values of μ_{ratio} and shear stress were higher than those at lower Reynolds' number.

(d) At a fixed Reynolds' number, the peak and minimum values of the shear and μ_{ratio} were the least at medium throttle opening (16 turns) rising slowly with further opening and rising sharply as the compressor ran under partial or fully stalled conditions.

(e) The minima of shear and μ_{eff} distributions occurred near the centre of the annulus for medium throttle opening but they were shifted towards the rotor wall at other throttle settings. This was a consequence of the character of the velocity profile and the level of turbulence. The possibility of three dimensional flow effects to a varying degree to have caused the shift should not be ruled out. It was also noted that the location of the shear and μ_{ratio} maxima was unaltered with either the Reynolds' number or the throttle opening.

(f) The observation that the 'least overall values' of shear and μ_{ratio} occurred at a certain Reynolds' number and for a certain throttle opening relates itself to the fact that the phenomena of wake effects, cross-flows, secondary effects etc. are least significant near the compressor design conditions. At off-design running conditions, the shear stress and the effective viscosity were more unevenly distributed in that the curves were more peaky.

5. A comparison of the results at station 3 with those at station 5 revealed certain similarities but the fact that turbulence level and scatter is greater at station 3 rules out any further interpretation of the data.

6. An average value of μ_{ratio} was seen to be of the order of 50-70 and the peak values were as high as 120. Since no experimental or theoretical results for such flows are available in literature, no comparisons of the magnitude were possible. However, reference may be made to the measurements of μ_{ratio} by Jonsson and Sparrow (1966) in turbulent flows through concentric and eccentric annuli. The radial distribution of shear stress was determined by the measurement of axial pressure gradient and using the momentum equation, that is $\tau = -\frac{1}{2} \frac{dp}{dx} \cdot |r - r_c|$. The velocity gradient was obtained by the pitot traverse. It was seen that for the diameter ratio of 0.7 and $Re = 80,000$, average value of μ_{ratio} was around 50 and a maximum of 70. The corresponding results from the compressor were,

however, 30 to 50% higher. Although the flow within an axial-flow compressor is more complicated than that in a simple annulus, a comparison showed that the values of eddy viscosity were not very different. Conclusions from such a comparison are limited by the difference in the definitions of effective viscosity in the two cases.

Conclusion from the Experimental Studies

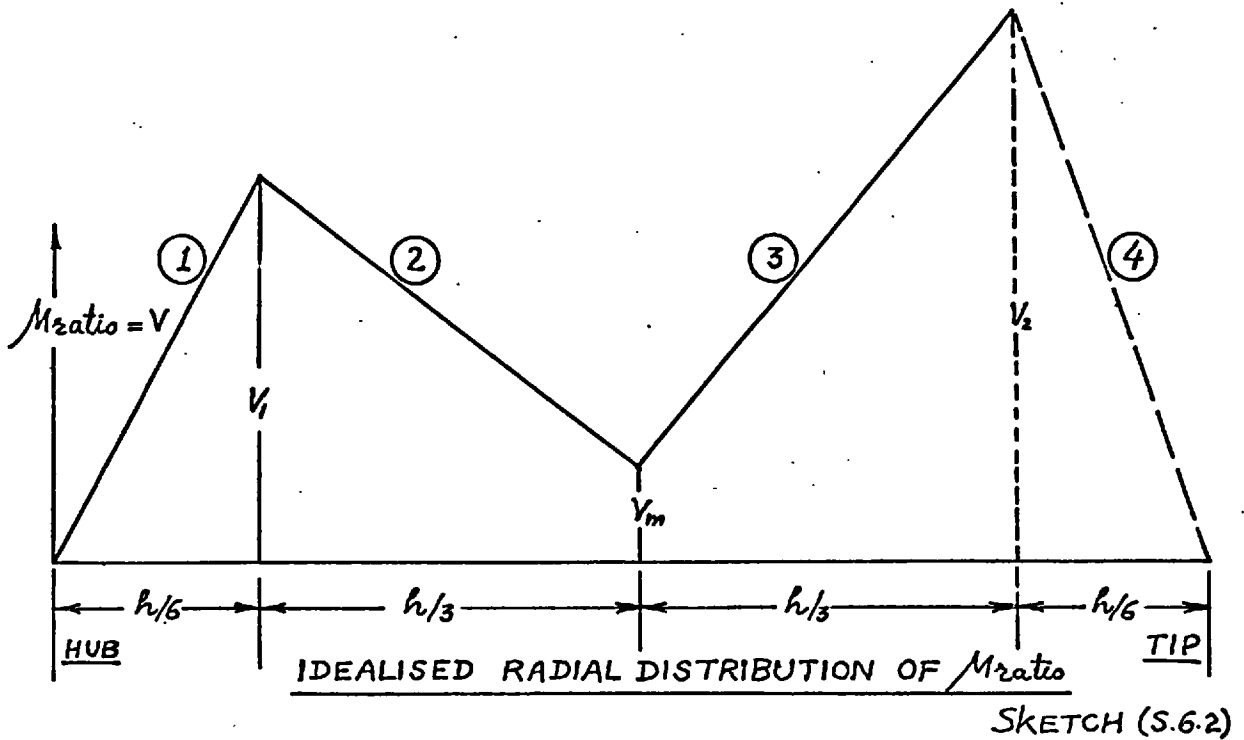
The distribution of μ_{ratio} within the axial-flow compressor varied with the speed, the ambient conditions and the mass flow through it. The typical values so obtained indicated trends regarding the radial distribution and an idealisation have been made to simplify matters for input to a programme designed to use $\mu_{\text{effective}}$. Assuming that the variations were linear and the μ_{ratio} diagram was as shown in Sketch (S.6.2), it is tentatively suggested that the relations for μ_{ratio} in the four regions of the annulus could be expressed as

$$1. \quad \mu_{\text{ratio}} = \frac{6V_1}{h} r \quad \dots(6.1a)$$

$$2. \quad \mu_{\text{ratio}} = V_1 - \frac{3}{h} (V_1 - V_m)(r-h/6) \quad \dots(6.1b)$$

$$3. \quad \mu_{\text{ratio}} = V_m + \frac{3}{h} (V_2 - V_m)(r-h/2) \quad \dots(6.1c)$$

$$4. \quad \mu_{\text{ratio}} = V_2 - \frac{6V_2}{h} \left(r - \frac{5h}{6} \right) \quad \dots(6.1d)$$



Further, it is also tentatively suggested that V_1 , V_m and V_2 may be considered as double linear-functions of Reynolds' number Re and flow function $\bar{C}_x/U$, as follows:

$$V_1 \approx 55 + A(\bar{C}_x/U - 0.55) + B(Re - 60,000) \quad \dots(6.2)$$

$$V_m \approx 15 + C(\bar{C}_x/U - 0.55) + D(Re - 60,000) \quad \dots(6.3)$$

$$V_2 \approx 1.5 V_1 \quad \dots(6.4)$$

where,

$$A = 70, B = 0.00135, C = 25, D = 0.00115$$

for flow functions higher than the design value ($= 0.55$).

For flow functions less than the design value, the data was not adequate to establish these constants, but from a knowledge of the trends, it is suggested that double

the above listed values of A, B, C and D should be used close to stall.

The experimental results obtained and arranged in the form of equations (6.1a to 6.4) were valid only downstream of the first rotor blade row. Since no useful results for the distribution of effective viscosity downstream of other blade rows were obtained, it was assumed, for the purpose of input to the Prediction-programme, that the radial distribution of μ_{eff} did not change appreciably across successive blade rows.

Furthermore, until experiments are conducted on other compressors, it can only be assumed that the magnitude and distribution of effective viscosity for any axial-flow compressor are given by the correlating equations given above.

CHAPTER 7

THE PREDICTION OF COMPRESSOR PERFORMANCE

The theoretical equations and cascade correlations describing the flow of fluid in axial-flow compressors have been established. Development of the computer programme to analyse the fluid flow in the compressors has been described and test runs for simple flows through pipes and annuli have showed that the programme yields converged and plausible results. In addition, the distribution of effective viscosity in axial-flow compressors has been measured from the series of tests on a compressor. The computer programme was now available to predict the off-design performance of axial-flow compressors.

Predicted Results for the Three Stage Compressor

The prediction computer-programme was tested for several operating conditions for the data of the three stage compressor. In each case, the ambient temperature and pressure, the compressor speed in r.p.m., the average flow function $\bar{C}_x/U$ and the profile of axial velocity at entry to the compressor were supplied as input to the computer programme. The inlet passage of the laboratory compressor had been designed for accelerating flow and hence it was considered reasonable to assume a uniform velocity profile upstream of the inlet guide vanes. The maximum running speed of the compressor was 3000 r.p.m. but in order not to exceed the speed under any ambient conditions, the experimental tests were conducted at a maximum Reynolds

number of 80,000 which corresponded to a speed between 2500 and 2800 r.p.m. The computer programme was therefore tested for the same value of Reynolds number. The design flow function was 0.55 and an adjustment of the throttle at exit of the compressor had enabled the compressor to run at off-design flow rates giving flow functions of 0.45 for stable operation close to stall and 0.63 for a high mass-flow condition.

The results of the prediction computer programme showing the axial velocity profiles after the various blade rows are presented in Figs. 24 to 27. The predicted distributions of axial velocity exhibit the trend observed in axial-flow compressors with a growth of wall boundary layer in successive stages. The predicted boundary-layer growth on the outside wall is greater than on the hub.

Comparison of the Prediction with Experiment

The experimental results from the laboratory tests obtained by the author and also by Elsey (1966) were plotted to show the comparison with predicted results. The non-dimensional velocity $C_x/\bar{C}_x$ were obtained with reference to the average axial velocity $\bar{C}_x$ at each blade row.

The comparison between the computer-predicted and experimental results shown in Figs. 24, 25 and 26 shows quite good agreement at all conditions. The axial velocity distribution close to the compressor walls was not determined experimentally because of the difficulty and uncertainty of measurements in the regions of the walls.

Test Runs to Determine the Effect of Inlet Velocity Profile

The computer programme was also used to examine the effect of a non-uniform inlet velocity profile; the results for the design mass flow are shown in Fig. 27. It is observed that the predicted axial velocity profile downstream of the first stage is more uniform than the inlet profile. There are growths of boundary layers on the walls in subsequent stages. There was no experimental confirmation of these results.

A comparison of the development of the axial-velocity profile through a compressor for a uniform inlet profile and the non-uniform entry profile is also shown in Fig. 27. It can be seen that the influence of the non-uniform inlet profile remains after three compressor stages but the difference between the two axial-velocity profiles after the third stage is smaller than the initial difference in axial velocity distributions.

Discussion

It appears that the computer programme developed for the off-design prediction of the performance of the multistage axial-flow compressors provides realistic results for the test cases. The predictions are superior to those from other methods, which do not include the effects of turbulent shear in the flow.

Compilation and Computation-Time

The programme, written in FORTRAN IV, was run on an I.B.M. 7094 Computer. The central processor of the computer took 2.8, 5.6 and 8.4 microseconds for floating point addition, multiplication and division operations respectively. The card reader was capable of reading 1200 cards/minute and the line printer could print 1100 lines/minute.

The compilation time taken by the peripheral units was 1.2 minutes for the cards punched in FORTRAN IV but the time dropped to 1.0 minute for binary decks.

The computational time taken to achieve a solution to a tolerance in axial velocity set at 0.5% of the maximum axial velocity over successive cycles was 2.0 minutes. The solution of the flow within the three stage axial-flow compressor referred to above was obtained by sub-dividing the flow field into a 16 x 17 network.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

Conclusions

A model of the axisymmetric flow in axial-flow compressors taking account of a form of the turbulent shear terms was developed. The model is valid for the entire range of subsonic flow.

It was found that the simplest model for turbulent flow required a knowledge of the variation in effective viscosity within an axial-flow compressor. The distribution of effective viscosity was determined by a special hot-wire technique. This technique was developed in order to measure the structure of turbulence between the blade rows of a multistage compressor.

The problem of wake interference in the measurements immediately downstream of the blade rows was overcome by removing the intermediate rows. By doing so it was possible to set up a measuring station far downstream of the first rotor row. Plausible results were obtained from a series of measurements conducted on the compressor operating at different conditions.

The distributions of mean velocities downstream of each blade row of the three-stage compressor predicted from the computer programme, based upon the model of flow, were in good agreement with the measured results. The present investigation was, however, limited to low speed subsonic flows. At Mach numbers approaching unity, it is possible that achievement

of rapid convergence may require a revision of the over-relaxation parameter used in the present case.

Recommendations for Further Work

The results of the author's measurements have provided an insight into the structure of turbulence within an axial-flow compressor. The conclusions and the idealisation of the distribution of μ_{ratio} presented in Chapter 6 are based upon a limited number of tests. A further generalisation would require an elaborate series of tests conducted on different compressors. The tests should be directed to determine the variation of the μ_{ratio} with Reynolds number, stage flow-function, Mach number, hub-tip ratio, the type of blading and other parameters.

Since the meaningful experimental results were obtained with only one rotor in the compressor, the change in μ_{ratio} over successive blade rows could not be found. An attempt to overcome the difficulties associated with the measurements close to and downstream of each blade row in a multistage compressor would be a useful step. An axial-flow compressor with large axial gaps between the blade rows, designed specially for this purpose, may facilitate such measurements. An alternative approach can be to obtain a spectral analysis of the shear stress at each point, to recognise the undesirable frequency range and to filter it out. Additional electronic equipment is needed for this approach.

It is suggested that turbulence measurements within the blade rows may also be made. The information so obtained may

enable further refinement of the model of flow.

The trends in μ_{eff} based upon the shear $\overline{C_\theta C_r}$ and the radial gradient of the resultant velocity C_θ do not provide a complete description of the turbulent flow. Further studies to develop hot-wire techniques and to measure the shear stresses $\overline{C_x C_z}$ and $\overline{C_\theta C_r}$ both between and within the blades should be undertaken. When a knowledge of all the turbulent correlations is available then improvements to the model of flow and prediction method will undoubtedly come about.

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NOMENCLATURE

The Symbols hold the following meanings unless otherwise stated in the text.

<u>Symbol</u>	<u>Meaning</u>	<u>Computer-Notation</u>
A	A calibration constant for the hot wire, equation (5.1)	A
a	A constant for the blade-form, equation (3.5.5)	
B, B(x, r)	Field variable on the r.h.s. of equation (A.I.1)	B(I, J)
B	A calibration constant for the hot wire, equation (5.1)	B
c	Blade chord	BC(J)
C, C _i	Velocity vector and components, ft/sec.	C(I, J)
C _x	Axial velocity	CX(I, J)
C _r	Radial velocity	CR(I, J)
C _θ	Circumferential velocity	CU(I, J)
C _c	Resultant of C _x and C _θ	CC(I, J)
C _{xen} , $\bar{C}_x$	Uniform velocity at entry to the compressor	CXEN
C _{eff}	Effective cooling velocity for the hot wire	
$\bar{C}_x, \bar{C}_r, \bar{C}_\theta$	Temporal mean axial, radial and circumferential velocities	
C' _x , C' _r , C' _θ	Instantaneous fluctuations in axial, radial and circumferential velocities	
C _p	Specific heat at constant pressure (BTU/°F lbm)	CP
D	Diameter	D
D	Diffusion factor	D

Deq.	Equivalent Diffusion Factor, Equation (3.55)	DEQ
Dr	Degree of reaction	
E	D.C. Bridge voltage for the hot wire probe	ED(J)
e	A.C. r.m.s. reading	EA(J)
e'	Tip clearance	
$\vec{F}$	Blade Force vector	
F_x, F_r, F_θ	Axial, radial and circumferen- tial components of $\vec{F}$	
f	A constant in the definition of ψ , equation (3.12)	FC
f	An arbitrary function, equation (A.I.2) etc.	
f	Partial response factor for the fraction of cooling of hot wire caused by tangential component of velocity, equation (A.IV.23)	
G	Grid spacing in the radial direction	G(I)
$\vec{G}$	Mass velocity vector ($=\rho\vec{C}$)	
g	Acceleration due to gravity (ft lbf/lbf sec ²)	AG
GU, GD, G	Radial grid spacings above and below the grid point	GU, GD, G(I)
H	Form factor, equation (3.56)	H2
H, H_0	Static and stagnation enthalpy	H(I, J), HO(I, J)
HL, HR	Axial grid spacings on left and right of the grid point	HL(I), HR(I)
h	Blade height	BH(I)
I_0	Relative stagnation enthalpy ($=H_0 - \omega.r.C_\theta$)	
i	Incidence angle	AINC(J)
i_0	Zero-camber incidence angle	

I, J	Index for field point in axial and radial directions	I, J
J	Joule's constant (ft lbf/BTU)	AJ
k	Instantaneous kinetic energy of turbulence	
$\bar{K}, K'$	Mean and fluctuating components of kinetic energy of turbulence	
K	A constant, equation (2.33)	
K_{sh}, K_t	Correlation factors in Lieblein's correlation	PKSH, PKT
l	Mixing length, proportional to the size of passage	
m	Meridional direction	
M	Number of field points in radial direction	M
M	Mach number	AM(I, J)
m, n	Slope factors in Lieblein's correlation, equations (3.52, 3.53)	ASFN, ASFN
N	Rotational speed of the compressor in r.p.m.	RPM
N	Number of grid points in the axial direction	N
n	Number of simultaneous difference equations	N
	Number of grid points with the field	
p, P	Static pressure	P(I, J)
p	Percentage over-relaxation, equation (A.I.13a)	
p	Index in the hot wire calibration, equation (5.1)	P
P_o	Stagnation pressure	PO(I, J)

P_{eff}	Effective Prandtl number for turbulent flows	
P_o	Stagnation pressure loss across a blade row	
Q	Entropy function = e^{-s/C_p}	
Q	Heat exchange between the fluid and surroundings, equation (3.30)	
$\vec{q}$	Conductive heat flux vector	
R	Gas constant (ft lbf/°F lbm)	AR
Re	Reynolds' number of Operation	REY
r	Radial distance from the axis of rotation	
$r_{\text{in}}, r_{\text{out}}$	Radius of the rotor drum and compressor casing	RIN, ROUT
r	Radial direction, radius	R(I,J)
s	Spacing between blades	S
S	Specific entropy (BTU/°F lbm)	S(I,J)
S	Residual at a grid point, equation (A.I.12)	BIGD
S	Strength of a vortex	
t/c	Blade thickness to chord ratio	TBYC (J)
t	Time	
U	Blade peripheral velocity, equation (6.1a)	CBU(I,J)
U	Uniform stream velocity	CXEN
U_{eff}	Effective cooling velocity for the hot-wire probe	

u Specific internal energy
W Relative velocity

Additional Notation for the Computer Programme

Control Flags KEQ, KVEL, KCOMP, KWA etc. as defined in the listing (Appendix VIII)

Number of iteration in progress ITN

Maximum number of iterations MAXIT

$\partial\psi/\partial r$, $\partial\psi/\partial x$ DSIDR(I,J), DSIDX(I,J)

Number of points for POLYFT NP

Order of Polynomial KOR

Coefficients of the polynomial CO,C(K)

Reference incidence and deviation REFI(J), REFD(J)

Number of blades on I.G.V., rotor and stator NBG, NBR and NBS

Inner and outer radii at entry RINI, ROUTI

Inner and outer radii at exit RINO, ROUTO

Greek Symbols

Meaning

α, α' Absolute air and blade angle, respectively ALFA, ALFD

β, β' Relative air and blade angle, respectively BETA, BETD

γ Stagger angle GAMA(J)

All blade angles measured with x-axis, as shown in Sketch (A).

γ Ratio of specific heats, (C_p/C_v) GAMA

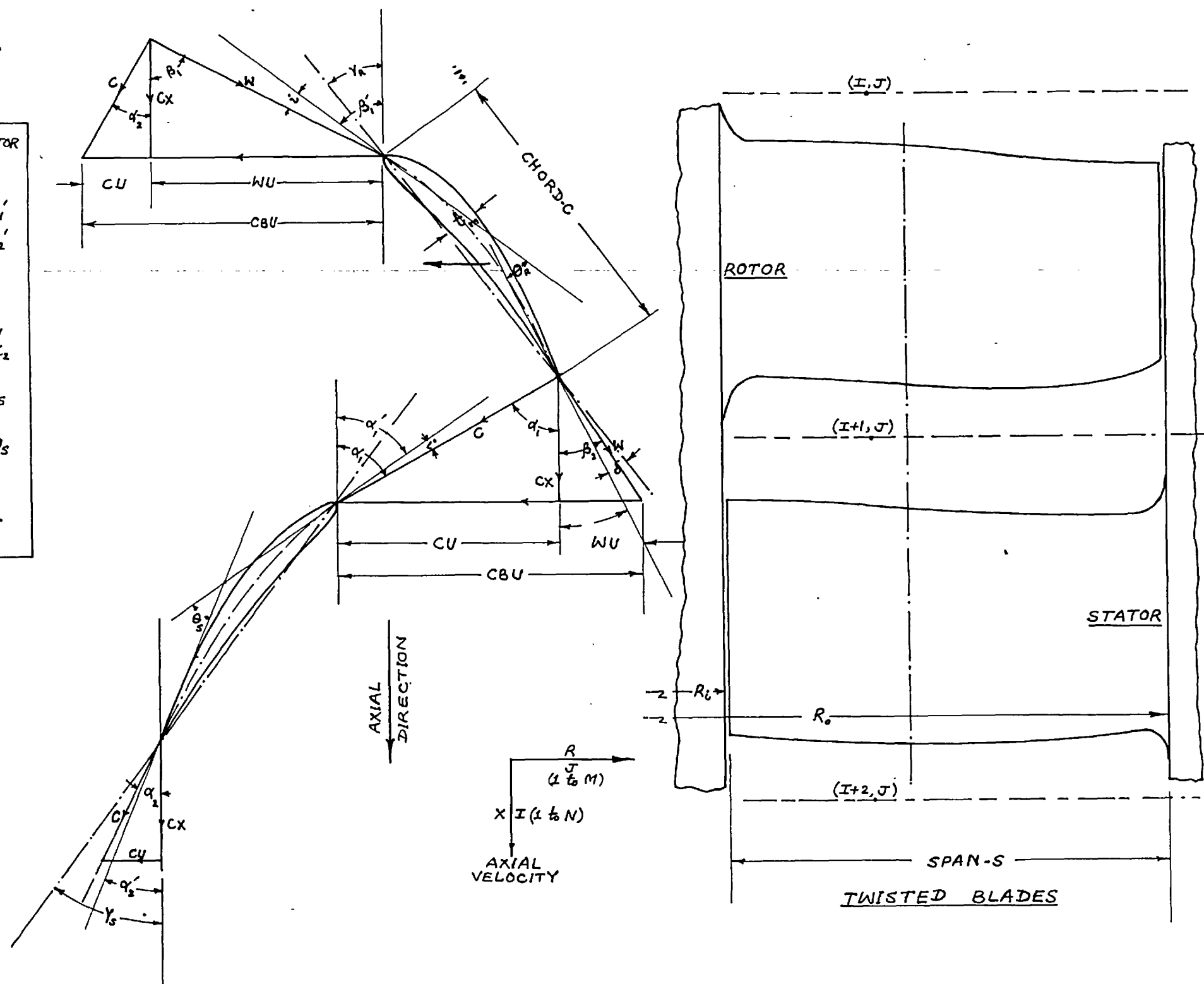
Γ Effective Exchange Coefficient, equation (3.51)

δ Deviation angle ADEV(J)

δ Boundary layer thickness

δ^* Displacement thickness

	ROTOR	STATOR
BLADE ANGLES		
INLET	β'_1	α'_1
EXIT	β'_2	α'_2
AIR ANGLES		
INLET	β_1	α_1
EXIT	β_2	α_2
STAGGER ANGLE	γ_R	γ_S
CAMBER ANGLE	θ_R	θ_S
INCIDENCE	i	i
DEVIATION	δ	δ



BLADES NOMENCLATURE

SKETCH (A)

ϵ	Potential energy of the fluid/ unit mass, equation (3.41)	
ϵ	Maximum permissible residual in relaxation	EPS
ζ, η	Substitutions as defined in Appendix IV	
θ	Axial momentum thickness, equation (2.28)	
θ	Wake momentum thickness, equation (3.56)	
λ	Eigen value of a matrix, equation (2.19)	
μ	Dynamic viscosity tbf.sec/ft ²	AMEW
μ_{eff}	Effective (apparent + dynamic) viscosity in turbulent flow	EMEWR(I,J)
μ_{ratio}	Effective/Dynamic viscosity ratio	
ρ	Density of the fluid	ROW(I,J)
σ	Blade solidity (chord/ spacing ratio)	SIGIB(I,J)
τ	Shear stress, lbf/ft ²	SHEAR(J)
τ_w	Wall shear-stress	
ϕ	$\tan^{-1} C_r/C_x$	
ϕ	Flow function $\bar{C}_x/U$	FL/UF
ψ	Stoke's Stream function, equation (3.12)	U(I,J)
ω	Over-relaxation factor $1 +$ $p/100$, equation (A.I.13a)	OMEGA
ω	Compressor speed, radians per second	
ω	Stagnation pressure loss- coefficient, equation (3.56)	OMEGAD

Miscellaneous Notation Used

$\bar{A}$	Time averaged value of A
$\rightarrow$	Vectorial addition
$\vec{C}$	Vector $C = C_x \rightarrow C_r \rightarrow C_\theta$
$\partial/\partial x ()$	Partial Differentiation w.r.t. x only
$\partial^2/\partial x \partial y, \partial^2/\partial x^2$	Second Partial Derivatives
$D/Dt (S)$	Total Differentiation of S
$\det.A$	Determinant of matrix A
$\nabla \cdot \vec{C},$ $\text{Div } \vec{C}$	Divergence of $\vec{C}, \frac{1}{r} \frac{\partial}{\partial r} (r \cdot C_r) + \frac{\partial C_\theta}{\partial \theta}$ for axisymmetric flow
$\sum C_i \tau_i$	$C_x \tau_{xr} + C_\theta \tau_{\theta r}$ for axisymmetric flow
$-\bar{\rho} \overline{C'_i C'_j}$	Apparent Turbulent Shear Stresses
L_n	Natural logarithm
$[A]$	Matrix A, $A(I, J)$

SubscriptsMeaning

1,2	at inlet/ ^{to} and exit from the blade, for velocities and angles.
1,2	in inclined positions equally inclined to the normal position on either side of it, for voltages and partial response factors.
0	zero camber value, equations (3.52),(3.53)
av	averaged magnitude
b	for the blades
c	resultant of axial and tangential components
A, B, P, U or D	at points A, B, P, U or D in Sketch (S.A.I.1)
eff	effective value
h	energy flux, equation (3.51c)
i	components in the coordinate directions

I, J	location in x and r directions
k	kinetic energy of turbulence, equation (3.51d)
m	in meridional plane
mean	at mean diameter
max	maximum value
min	minimum value
n	at next downstream station, equations (3.57 to 3.59)
new	newer calculated value, equation (A.I.13)
o	stagnation value
p	at point (I, J)
r	along radial direction
ref	reference minimum stagnation loss value
ref	reference condition, equation (3.62)
x	along axial direction
x, r	in a plane perpendicular to x and in r-direction, equation (3.51a)
θ	along circumferential direction
θ	for an orientation when the flow makes θ with the normal wire, equation (A.IV.24b)

SuperscriptsMeaning

*	for design condition
(k)	Kth estimate
'	instantaneous fluctuation of
'	relative to rotor

APPENDIX I

DETAILS OF THE QUASI-ORTHOGONAL COMPUTATIONAL

MOLECULE

From the considerations of curved boundaries discussed in Chapter 4, it was decided to adopt a quasi-orthogonal grid system in the flow field. A finite-difference representation and the consequent numerical model of the equation termed as 'computational molecule' were then considered. A general molecule shown in sketch (S.A.I.1) was made for the principal equation.

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{z} \frac{\partial \psi}{\partial z} = B(x, z, \psi) \quad \dots(A.I.1)$$

where $B(x, z, \psi)$ is a group of non-linear terms as seen in Equation (3.27), also involving unknowns other than ψ which were evaluated by using other equations governing the flow.

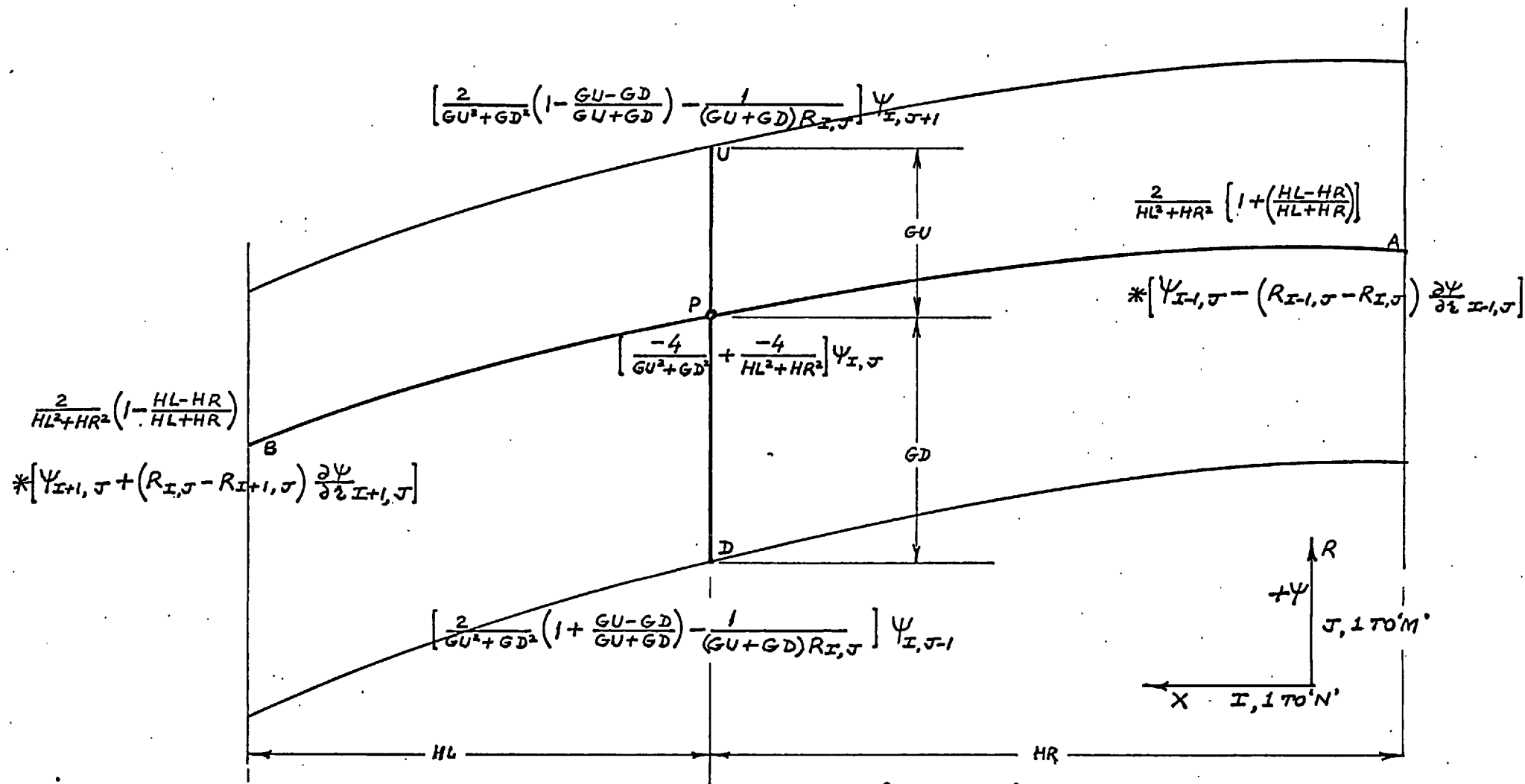
The grid spacing in either direction could be non-uniform as shown in Sketch (S.A.I.1). By Taylor's Expansion Theorem applied to a function $f(x, r)$ in this field,

$$f(A) = f(P) - HR \frac{\partial f(P)}{\partial x} + HR^2/2 \frac{\partial^2 f(P)}{\partial x^2} \quad \dots(A.I.2)$$

$$f(B) = f(P) + HL \frac{\partial f(P)}{\partial x} + HL^2/2 \frac{\partial^2 f(P)}{\partial x^2} \quad \dots(A.I.3)$$

$$f(U) = f(P) + GU \frac{\partial f(P)}{\partial z} + GU^2/2 \frac{\partial^2 f(P)}{\partial z^2} \quad \dots(A.I.4)$$

$$f(D) = f(P) - GD \frac{\partial f(P)}{\partial z} + GD^2/2 \frac{\partial^2 f(P)}{\partial z^2} \quad \dots(A.I.5)$$



COMPUTATIONAL MOLECULE FOR $\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{2} \frac{\partial \Psi}{\partial z} = B(x, z, \Psi)$
IN QUASI-ORTHOGONAL NET-WORK

SKETCH (S.A.I.1)

From Equations (A.I.2) and (A.I.3),

$$\frac{\partial f(P)}{\partial x} = \frac{f(B) - f(A)}{HR + HL} \text{ neglecting } \left(\frac{HR^2 - HL^2}{2} \right) \frac{\partial^2 f(P)}{\partial x^2} \quad \dots (A.I.6a)$$

and smaller terms. Similarly,

$$\frac{\partial f(P)}{\partial z} = \frac{f(U) - f(D)}{GU + GD} \quad \dots (A.I.6b)$$

Where $HR = HL$ and $GU = GD$, the above representations assume better accuracy since the coefficients of second order error terms vanish.

Also from (A.I.2) and (A.I.3), on addition.

$$f(A) + f(B) = 2f(P) + (HL - HR) \frac{\partial f(P)}{\partial x} + (HR^2 - HL^2)/2! \frac{\partial^2 f(P)}{\partial x^2}$$

and substituting for $\frac{\partial f(P)}{\partial x}$ from (A.I.6a) and on transposing terms,

$$\frac{\partial^2 f(P)}{\partial x^2} = f(A) \cdot HM(1 + HF) + f(B) \cdot HM(1 - HF) - 2 \cdot HM \cdot f(P) \quad \dots (A.I.7)$$

where

$$HM = \frac{2}{HR^2 + HL^2} \quad \text{and} \quad HF = \frac{HL - HR}{HL + HR} \quad \dots (A.I.7a)$$

Similarly,

$$\frac{\partial^2 f(P)}{\partial z^2} = f(D) \cdot GM(1 + GF) + f(U) \cdot GM(1 - GF) - 2 \cdot GM \cdot f(P) \quad \dots (A.I.8)$$

with

$$GM = \frac{2}{GU^2 + GD^2} \quad \text{and} \quad GF = \frac{GU - GD}{GU + GD} \quad \dots (A.I.8a)$$

For equal axial and radial intervals, the relations (A.I.7) and (A.I.8) assume a more conventional simpler form.

Equation (A.I.1) was thus replaced by a finite difference form,

$$\begin{aligned} & \Psi_A(1+HF)HM + \Psi_B(1-HF)HM + \Psi_U \left[(1-GF)GM - \frac{1}{R \cdot GS} \right] \\ & + \Psi_D \left[(1+GF)GM + \frac{1}{R \cdot GS} \right] - (2 \cdot HM + 2 \cdot GM) \Psi_P = B_P \end{aligned}$$

...(A.I.9)

where $GS = GU + GL$

For the grid system adopted here, points U and D must be the upper and lower grid points for P whereas A and B may or may not coincide with the grid points on the right and left of P. An extra step of calculating Ψ_A and Ψ_B from an interpolation was thus necessary. This, however, did not entail excessive work since the distribution of $\partial\Psi/\partial x$ and $\partial\Psi/\partial z$ had to be calculated and stored for other purposes.

The interpolates appear as,

$$\Psi_A = \Psi_{I-1,J} - \left(\frac{\partial\Psi}{\partial z} \right)_{I-1,J} * (R_{I-1,J} - R_{I,J}) \quad \dots(A.I.10)$$

and,

$$\Psi_B = \Psi_{I+1,J} + \left(\frac{\partial\Psi}{\partial z} \right)_{I+1,J} * (R_{I,J} - R_{I+1,J}) \quad \dots(A.I.11)$$

It was thus possible to replace the principle partial -differential equation (A.I.1) by a finite difference form at each grid point,

$$M_A \Psi_A + M_B \Psi_B + M_U \Psi_U + M_D \Psi_D - M_P \Psi_P - B_P = 0 \quad \dots(A.I.12)$$

where $M_A, M_B, \dots$ were appropriate multipliers.

Starting with an arbitrary distribution of Ψ , the terms on the left hand side of this equation add up to make a

residual S . The purpose of the relaxation technique is to minimise the residual at each point. For a straight relaxation, S was set equal to zero and ψ was updated as

$$\psi_{pnew} = \frac{1}{M_p} (M_A \psi_A + M_B \psi_B + M_U \psi_U + M_D \psi_D - B_p) \quad \dots(A.I.13)$$

For faster convergence, it was required to over-relax, that is reduce and reverse the sign of the residual. For $p\%$ over relaxation, the over-relaxation ω is said to be $(1 + p/100)$, and the over-relaxed residual = $-s(\omega - 1)$... (A.I.13a)

Thus,

$$\begin{aligned} -S(\omega-1) &= -(M_A \psi_A + M_B \psi_B + M_U \psi_U + M_D \psi_D - M_p \psi_p - B_p) \\ &= (M_A \psi_A + M_B \psi_B + M_U \psi_U + M_D \psi_D - M_p \psi_{pnew} - B_p) \quad \dots(A.I.13b) \end{aligned}$$

whence,

$$\psi_{pnew} = (1-\omega) \psi_p + \frac{\omega}{M_p} (M_A \psi_A + M_B \psi_B + M_U \psi_U + M_D \psi_D - B_p) \quad \dots(A.I.14)$$

For the present case of uniform radial grid spacing $G(I)$, this expression, written for computer use, appears as:

$$\begin{aligned} \psi_{I,J}^{new} &= (1-\omega) \psi_{I,J} + \omega / (2.HM + 2/G(I)^2) \\ &\quad * \left[(1+HF).HM \left\{ \psi_{I-1,J} - \left(\frac{\partial \psi}{\partial r} \right)_{I-1,J} * (R_{I-1,J} - R_{I,J}) \right\} \right. \\ &\quad \left. + (1-HF).HM \left\{ \psi_{I+1,J} + \left(\frac{\partial \psi}{\partial r} \right)_{I+1,J} * (R_{I,J} - R_{I+1,J}) \right\} \right. \\ &\quad \left. + \psi_{I,J+1} \cdot \left(\frac{1}{G(I)^2} - \frac{1}{2.R.G(I)} \right) + \psi_{I,J-1} \left(\frac{1}{G(I)^2} + \frac{1}{2.R.G(I)} \right) - B_{I,J} \right] \quad \dots(A.I.15) \end{aligned}$$

This was incorporated in the computer programme to relax for ψ .

APPENDIX II

OUTLINES OF THE COMPUTER PROGRAMME FOR THE OFF-DESIGN

PERFORMANCE PREDICTION

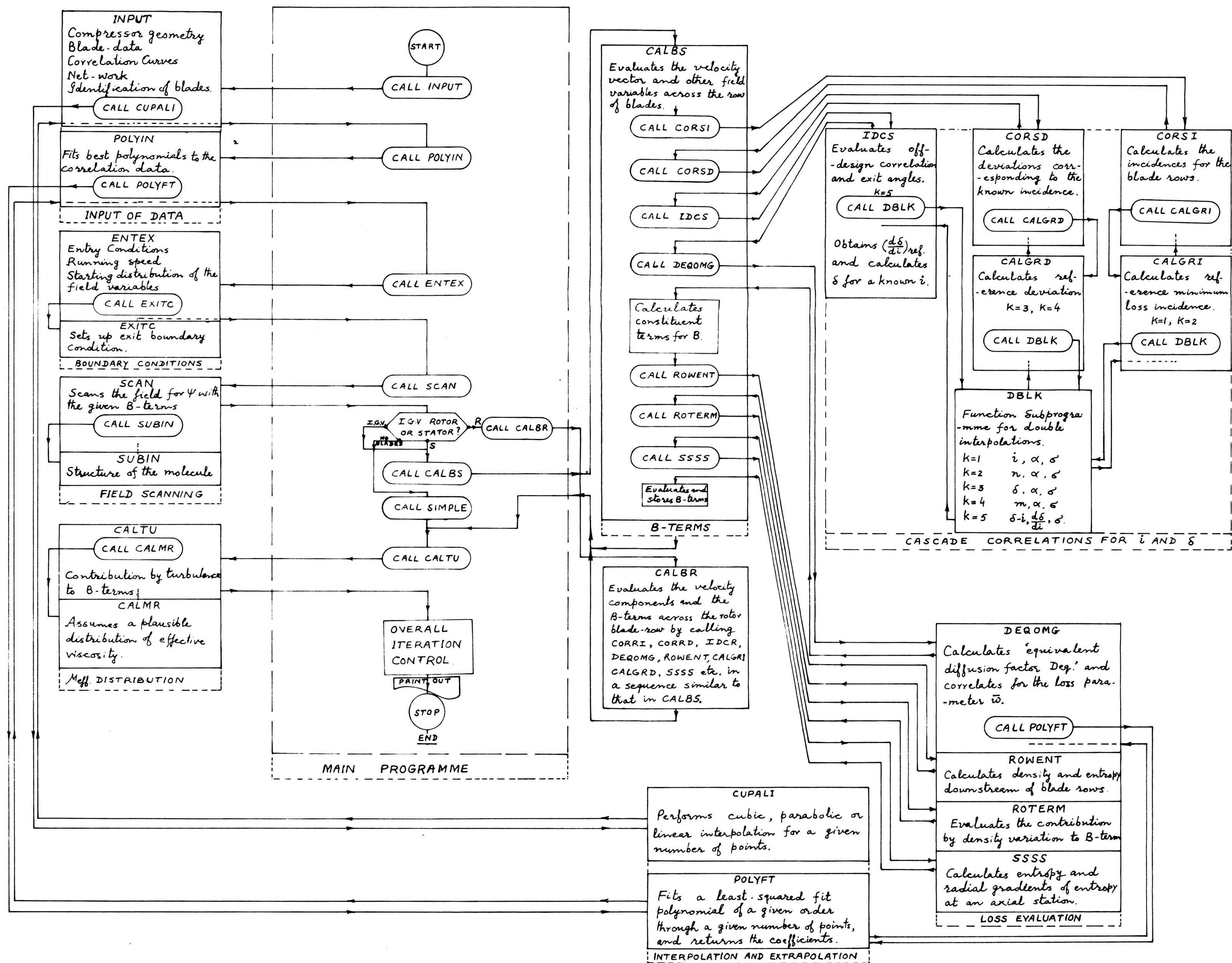
The overall flow diagram of the programme is shown in Sketch (S.A.II.1). This indicates the order of execution of the main programme and the subroutines, and the purpose of each of them. A brief description of each subprogramme is given below.

MAIN

The main programme starts the computation and controls the sequence of operations. It specifies an iteration control, convergence parameter and termination limits. It calls in appropriate subroutines to deal with the flow across the I.G.V., rotor and stator blades rows. It also decides the extent of printout required from each subroutine. The execution is stopped if the convergence criterion is satisfied or a specified number of iterations have been made. A block diagram is given in Sketch (S.A.II.2).

INPUT

This is to read data of the compressor geometry and blades. In addition, input of the blade correlation curves and their mathematical representation are provided here with the help of subroutines POLYIN, POLYFT, and CUPALI. From the specification of grid spacings, a quasi-orthogonal network is set up and the location of rotor and stator blades is identified.



OVERALL BLOCK DIAGRAM FOR THE PREDICTION COMPUTER PROGRAMME.

ENTEX

The subroutine ENTEx is intended to supply the entry conditions for the fluid and the compressor running conditions. It also sets up an initial distribution of various quantities in the flow field and at the exit end.

EXITC

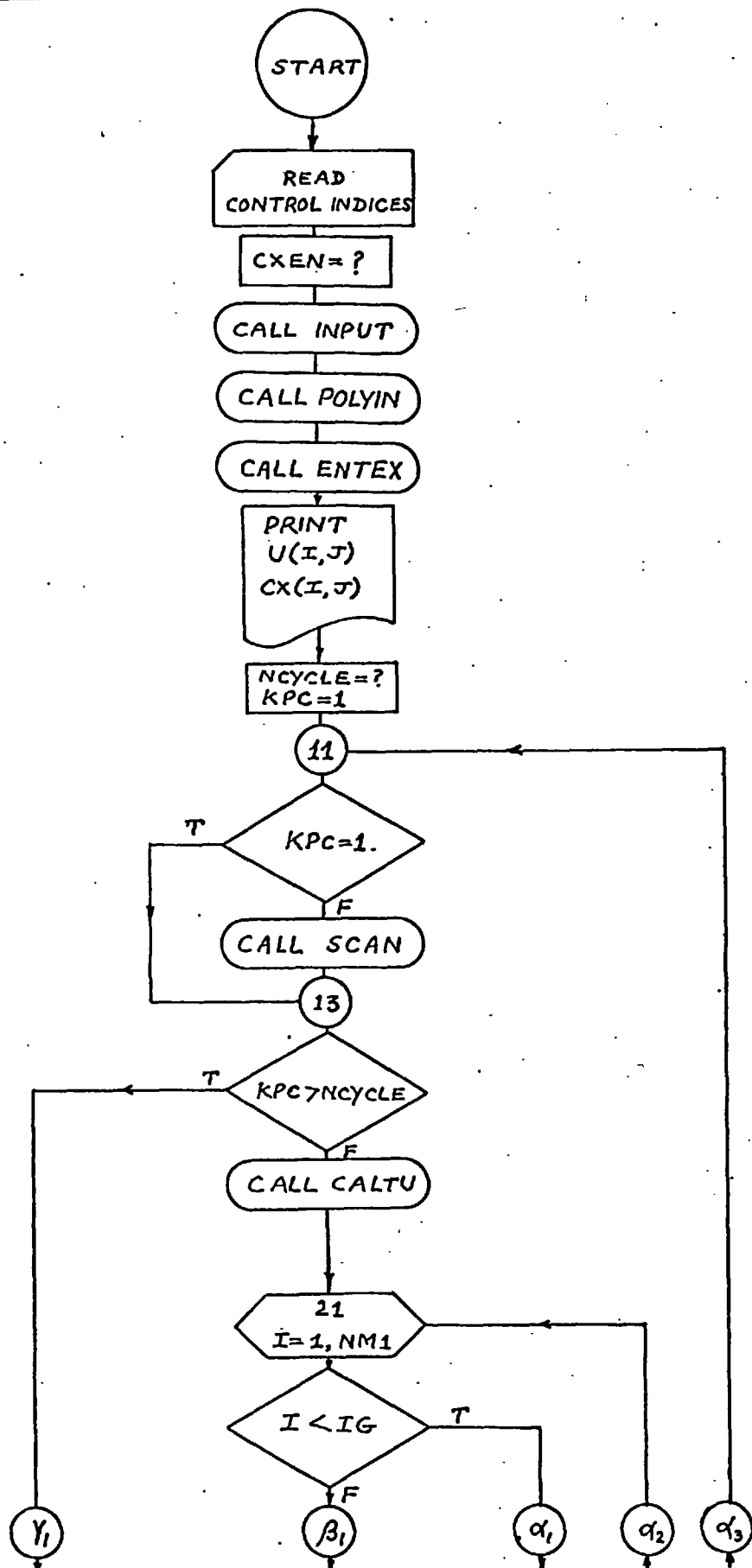
The purpose of the subroutine EXITC is to estimate the distribution of Ψ and C_x at the exit station for a fully developed turbulent flow or a fully developed laminar flow according as an index KVEL is specified = 1 or 2 respectively. 'Modified' trapezoidal rule is used in the integration process.

SCAN

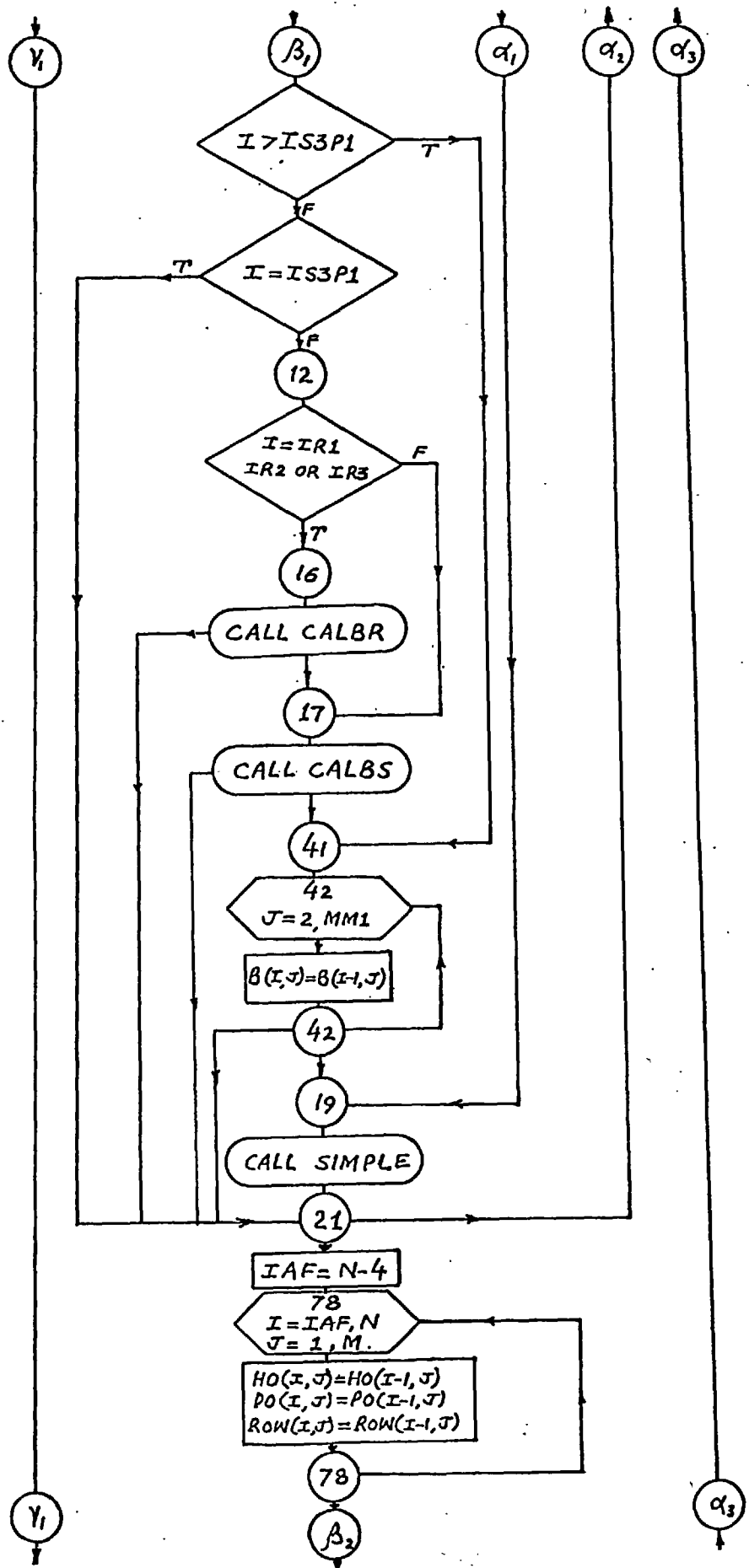
The task of scanning or relaxing the field by a computational molecule is performed in the subroutine SCAN. The scanning procedure consisting of marching from inlet end to exit, from hub to tip and tip to hub is organised in this subroutine. The exit boundary conditions are also implemented during the scanning operation. The subroutine incorporates an iteration control procedure and when the convergence criterion is satisfied, evaluation of $\partial\Psi/\partial r$, $\partial\Psi/\partial x$, C_x and C_r is also carried out.

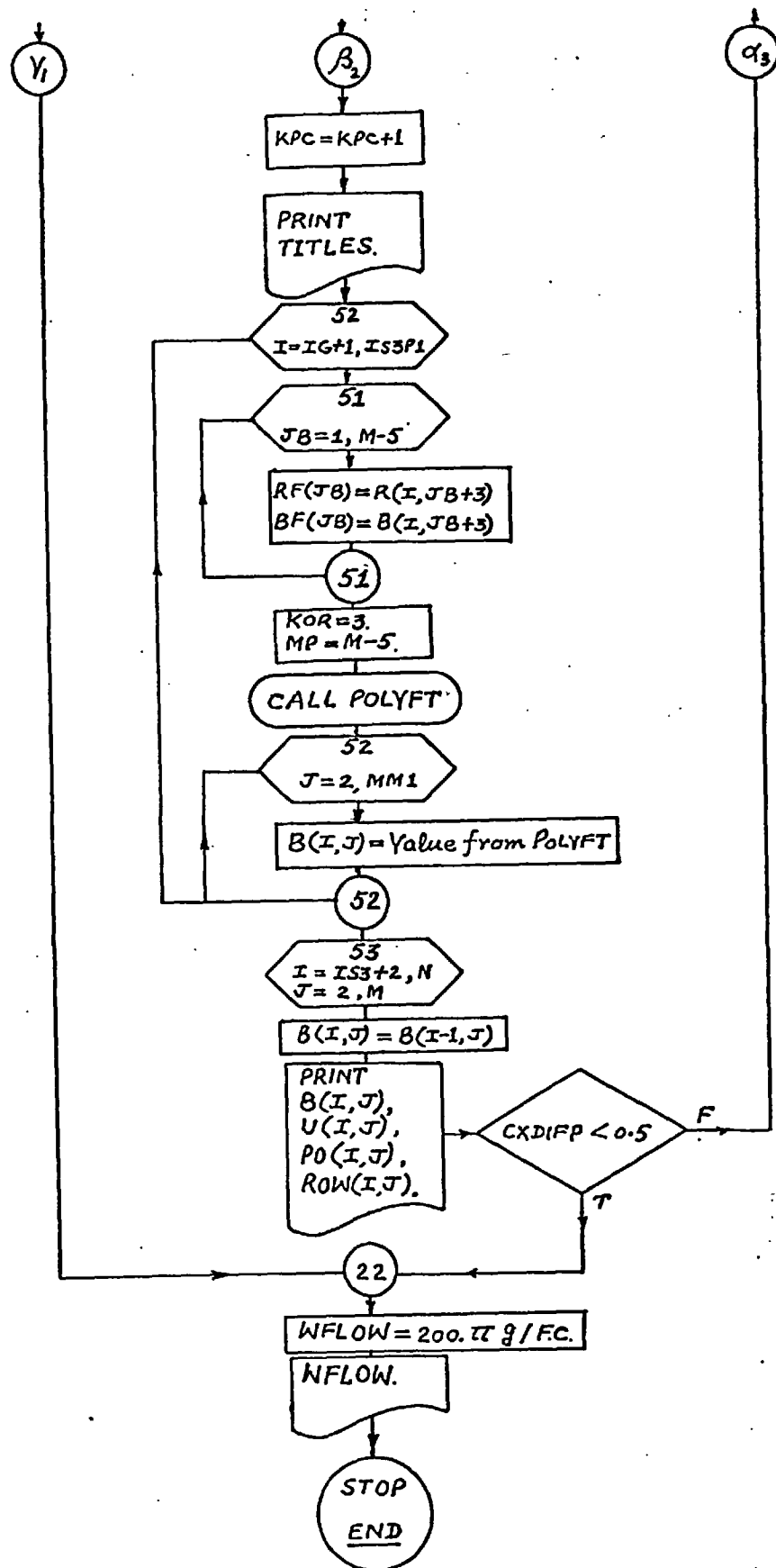
SUBIN

The subroutine SUBIN supplies a sequence of operations to update the value of Ψ at a grid point according to Equation (A.I.15). This subroutine represents the computational

BLOCK DIAGRAM—MAIN PROGRAMME FOR THE PREDICTION TECHNIQUE—

SKETCH (S.A.II.2) contd....





SKETCH (S.A.II.2). (concluded).

molecule for the partial differential equation governing the flow. It is called only in subroutine SCAN.

SIMPLE

This subroutine is called by the main programme when it is required to deal with the bladeless passage before the guide vanes. Its purpose is to calculate the distribution of $B(I,J)$ for the assumed entry conditions specified by ENTEX. Effect of updating the quantities in the bladed region is propagated to the bladeless entry region through the execution of this subroutine.

CALBS

The subroutine CALBS is designed to determine the flow across a stator row and to calculate the value of B at points downstream of the row. It deals with I.G.V's instead when required. Air angle, temperature, velocity vector and other conditions at inlet to the row are assumed known and the calculation of reference incidence, reference deviation and the air outlet angle are made by calling the subroutines CORSI, CORSD and IDCS. Having established the flow conditions at the exit of the row,

$\partial H_0/\partial r$, $\partial \psi/\partial r$, C_0/r , $\partial/\partial r(r C_0)$ are evaluated.

The loss parameter ω is obtained from DEQQMG subroutine and used in ROWENT subroutine to determine the density and entropy rise across the blade row. An estimate of the radial gradient in entropy is made in subroutine SSSS by assuming a plausible distribution of effective viscosity in subroutine CALMR.

Contribution to B by the turbulent terms is made by the subroutine CALTU. Finally an addition of constituent terms to make the field function B, given by R.H.S. of Equation (3.27) is achieved and the value of B at each grid point is stored.

CORSI, CALCRI

These subroutines work out the reference minimum loss incidence and the corresponding air angle for the given blades and solidity.

CROSD, CALGRD

The reference deviation corresponding to the reference minimum loss incidence is correlated by these two subroutines.

IDCS

Corresponding to an incidence i on the blade, the deviation angle δ is obtained from the relation $\delta = \int_{i_{ref}}^{(i-i_{ref})} \frac{d\delta}{di_{ref}}$ and the exit air angle is then determined.

CALBR

Calculations concerned with the flow across the rotor blade row and evaluation of B term downstream the row are made in the subroutine CALBR in conjunction with the subroutines CORRI, CORRD, CALCRI, CALGRD and IDCR similar to those for flow through a row of stators.

CAIMR

The purpose of the subroutine CAIMR is to calculate a realistic distribution of effective viscosity corresponding to the running conditions of the compressor. The information supplied to this subroutine is based upon the limited experimental findings on the laboratory compressor.

CALTU

The terms associated with the effective viscosity in the calculation of B at each point are calculated and supplied by this subroutine.

DEQOMG

From the knowledge of inlet and exit air angles for a blade row, the calculation of equivalent diffusion factor -Deq. and the stagnation pressure loss coefficient ω is performed in the DEQOMG subroutine. Corrections are made in ω for Mach number from the published corrections.

ROWENT

The function of this subroutine is to work out, from a knowledge of the thermodynamic relations, the variations in density and entropy over a step along the flow. The values so obtained are stored for further use in all other subroutines.

ROWTERB

This subroutine is devised to calculate the contribution of compressibility term, that is the last term

in Equation (3.27). This contribution is, however, small at low subsonic speeds.

SSSS

This subroutine calculates the value of entropy and the radial gradient of entropy at each radial point for every axial position. The calculations are based upon a simplified form of the entropy equation (3.50) after the magnitude of entropy at the centre of the annulus across a blade-row is obtained by ROWENT subroutine.

POLYIN

The correlation data read in INPUT subroutine is passed on to subroutine POLYIN where the data is rearranged and expressed in terms of polynomials by calling subroutine DBLK. The coefficients of these polynomials are then returned to the calling subroutine for cascade correlations.

DUMP

This subroutine, called in case of unrecognizable execution errors, dumps out the information in the core of the computer, which can be helpful for diagnosis.

CUPALI

The subroutine CUPALI derives its name from its capability to perform a cubic, parabolic or linear interpolation as required. This subroutine is used to calculate the blade angles, camber, etc. at a larger number of radial positions when the specifications are available only at three or four radial points.

POLYFT

The subroutine POLYFT is capable of performing a 'least-squares' polynomial fit of a specified order through a given number of points. The subroutine returns the values of CO and C(I) such that

$$Y = CO + C(1)*X + C(2)*X^2 + \dots$$

This subroutine is called, for data smoothing, for fitting best curves and for determining the gradient dY/dX by various subroutines.

KPWALL

The function subprogramme KPWALL extrapolates linearly or parabolically the value of a function f at a wall from a known distribution close to it.

DBLK

This function subprogramme evaluates the ordinate DBLK for a given set of values XX and YY from a chart of Z(ordinate) versus X for varying y by a double interpolation procedure. A second order 'best' polynomial fit through each curve, $y_n = \text{constant}$ results in a set of equations,

$$Z = a_{1n} + a_{2n}x + a_{3n}x^2 \quad \text{with } n = 1, 2, 3, \dots$$

Assuming again that each of a_1 , a_{2n} , a_{3n} can be approximated by second order polynomials such that

$$a_1 = b_1 + b_{2y} + b_{3y}^2$$

$$a_2 = b_4 + b_{5y} + b_{6y}^2$$

and
$$a_3 = b_7 + b_{8y} + b_{9y}^2$$

Then, DBLK, the ordinate for a given XX and YY is expressed as

$$\begin{aligned} \text{DBLK} = & b_1 + b_2 \text{YY} + b_3 \text{YY}^2 + (b_4 + b_5 \text{YY} + b_6 \text{YY}^2) * \text{XX} \\ & + (b_7 + b_8 \text{YY} + b_9 \text{YY}^2) * \text{XX}^2. \end{aligned}$$

APPENDIX III

EQUATIONS FOR TURBULENT FLOW

A transformation of the r.h.s. of Equation (3.20) for turbulent flow is demonstrated below.

$$\rho \left(\frac{DC_z}{Dt} - \frac{C_\theta^2}{r} \right) = \rho \left(C_x \frac{\partial C_z}{\partial x} + C_r \frac{\partial C_z}{\partial r} + \frac{\partial C_z}{\partial t} - \frac{C_\theta^2}{r} \right) \quad \dots (A.III.1)$$

$$\text{By continuity, } \frac{1}{r} \frac{\partial}{\partial r} (\rho r C_r) = - \frac{\partial}{\partial x} (\rho C_x) \quad \dots (A.III.2)$$

and by replacing the instantaneous quantities by a sum of mean values and fluctuating components,

$$\rho = \bar{\rho} + \rho', \quad C_x = \bar{C}_x + C'_x, \quad C_r = \bar{C}_r + C'_r, \quad C_\theta = \bar{C}_\theta + C'_\theta \quad \dots (3.10)$$

it is possible to carry out the transformation.

Expression (A.III.1) reduces to

$$\begin{aligned} & \bar{\rho} \left(\frac{D\bar{C}_z}{Dt} - \frac{\bar{C}_\theta^2}{r} \right) + \bar{\rho} \left[\overline{C'_x \frac{\partial C_z}{\partial x}} + \overline{C'_r \frac{\partial C_z}{\partial r}} - \overline{\frac{C_\theta'^2}{r}} \right] + \\ & \overline{\rho' \frac{\partial C_z}{\partial t}} + \overline{\rho' \frac{\partial C_z}{\partial x}} \bar{C}_x + \overline{\rho' \frac{\partial C_z}{\partial r}} \bar{C}_r - \overline{\rho' \frac{C_\theta'^2}{r}} + \\ & \overline{\rho' C'_x \frac{\partial \bar{C}_z}{\partial x}} + \overline{\rho' C'_r \frac{\partial \bar{C}_z}{\partial r}} + \text{triple correlation terms.} \end{aligned} \quad \dots (A.III.3)$$

Equation (A.III.2) yields

$$\overline{\rho' \frac{\partial C_z}{\partial r}} + \bar{C}_r \frac{\partial \bar{\rho}'}{\partial r} + \frac{\bar{\rho} C'_r}{r} + \overline{\rho' \frac{\partial C_z}{\partial x}} + \bar{C}_x \frac{\partial \bar{\rho}'}{\partial x} = 0. \quad \dots (A.III.4)$$

Combining (A.III.3) and (A.III.4) and using the continuity condition,

$$\bar{\rho} \left(\frac{D\bar{C}_z}{Dt} - \frac{\bar{C}_\theta^2}{r} \right) = \bar{\rho} \left(\bar{C}_x \frac{\partial \bar{C}_z}{\partial x} + \bar{C}_r \frac{\partial \bar{C}_z}{\partial r} - \frac{\bar{C}_\theta'^2}{r} \right)$$

$$\begin{aligned}
& + \overline{\rho' \frac{\partial G'}{\partial t}} + \overline{\rho' C_x} \frac{\partial \bar{G}}{\partial x} + \overline{\rho' C_z} \frac{\partial \bar{G}}{\partial z} + 2 \cdot \overline{\rho' C_\theta} \frac{\bar{G}_\theta}{z} \\
& + \overline{\rho' \frac{\partial G'}{\partial x}} \bar{C}_x + \overline{\rho' \frac{\partial G'}{\partial z}} \bar{C}_z + \overline{\rho' C_x} \frac{\partial \bar{G}_x'}{\partial x} + \text{similar} \\
& \dots (\text{A.III.5})
\end{aligned}$$

triple correlation terms.

For incompressible flow, (A.III.3) reduces to

$$\rho \left(\frac{D \bar{G}_z}{Dt} - \frac{\bar{G}_\theta^2}{z} \right) + \rho \left(\overline{G'_x \frac{\partial G'_x}{\partial x}} + \overline{G'_z \frac{\partial G'_z}{\partial z}} - \frac{\bar{G}_\theta^2}{z} \right) \dots (\text{A.III.6})$$

and this is further simplified by the continuity condition for incompressible flow, that is

$$\frac{\partial C_z}{\partial z} + \frac{C_z}{z} + \frac{\partial C_x}{\partial x} = 0.$$

Whence by multiplying throughout by C_r and using relation (3.10)

$$\overline{G'_x \frac{\partial G'_x}{\partial z}} + \frac{\bar{G}_\theta^2}{z} + \overline{G'_z \frac{\partial G'_z}{\partial x}} = 0.$$

Adding it to equation (A.III.6).

$$\rho \left[\left(\frac{D \bar{G}_z}{Dt} - \frac{\bar{G}_\theta^2}{z} \right) + \frac{1}{z} \frac{\partial}{\partial z} (z \bar{G}_\theta^2) + \frac{\partial}{\partial x} \overline{G'_x C'_x} - \frac{\bar{G}_\theta^2}{z} \right] \dots (\text{A.III.7})$$

Equation (A.III.5) has been referred to in the text as the one with large number of correlations and thus further analysis has been made in simplified terms, introducing the assumption of incompressibility.

APPENDIX IV

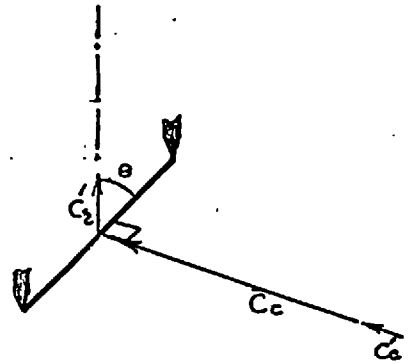
ANALYSIS OF THE HOT WIRE RESPONSE

For an inclined wire placed at an angle θ with the probe axis, designed to operate free from disturbing effects in three positions shown in Sketch (S.IV.1), relations have been established to estimate the shear stress $\overline{C_c C_r'}$, the level of turbulence and other turbulence parameters. The main assumptions are the low level of turbulence, negligible radial velocity and the consideration of axisymmetry.

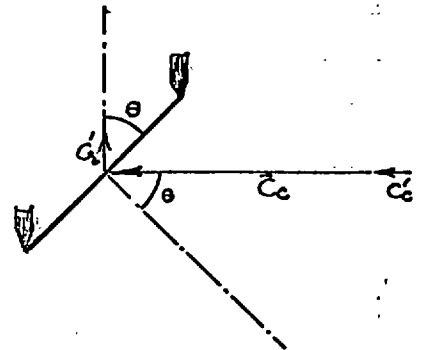
From the experiment, the cosine law was considered inadequate and tangential-velocity cooling correction factors, f_1 and f_2 , were used for angular response of the hot wire on either side of the 'normal' position. The analysis is also general as far as the angle θ between the wire and the probe axis is concerned. There is no restriction on the choice of the index p of the velocity in the calibration equation $E^2 = A + BU^p$. This is an important consideration as it has been shown elsewhere that a small change in p does not appreciably change the velocity calibration and yet the shear distribution may be severely in error. The selection of p was therefore governed by the correct measurements of velocity and shear distribution in the pipe test and used for the compressor data analysis. The value of index p was close to 0.5 for most cases.

Assuming at the first instance that all the velocity components and the fluctuations exist, then from the knowledge

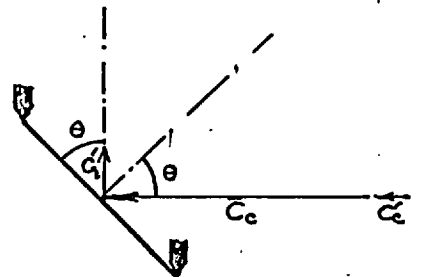
1. WIRE NORMAL TO THE RESULTANT VELOCITY C_c AND AT AN INCLINATION OF ANGLE θ WITH THE RADIAL DIRECTION.



2. WIRE WITH THE SHORT SUPPORTING LEG FACING THE FLOW (S.L.F.)



3. WIRE WITH THE LONG SUPPORTING LEG FACING THE FLOW (L.L.F.)



WIRE ORIENTATIONS FOR THE
SECOND AND THIRD SERIES OF TESTS

SKETCH (S.IV.1)

of the wire placed in relation to the flow field expressions for the effective cooling velocity and its fluctuations may be set up as follows:

For position 1 as shown in Sketch (S.A.IV.1),

$$C_{eff,1}^2 = C_c^2 + C_r^2 (\sin^2 \theta + f^2 \cos^2 \theta) \quad \dots (A.IV.1)$$

and

$$C'_{eff,1} = [C'_c \cdot 2 \cdot \bar{C}_c + C'_r \cdot 2 \cdot \bar{C}_r (\sin^2 \theta + \cos^2 \theta \cdot f^2)] / 2 \cdot \bar{C}_{eff,1}$$

$$= C'_c \cdot \eta + C'_r \cdot \zeta$$

where

$$\eta = \bar{C}_c / \bar{C}_{eff,1}$$

and

$$\zeta = \bar{C}_r (\sin^2 \theta + f^2 \cos^2 \theta) / \bar{C}_{eff,1}$$

and hence

$$\overline{C'^2_{eff,1}} = \overline{C'^2_c} \eta^2 + \overline{C'^2_r} \zeta^2 + 2 \cdot \overline{C'_c C'_r} \eta \zeta \quad \dots (A.IV.2)$$

For position 2

$$C_{eff,2}^2 = (C_c \cos \theta + C_r \sin \theta)^2 + f_i^2 (C_c \sin \theta - C_r \cos \theta)^2$$

$$= C_c^2 (\cos^2 \theta + f_i^2 \sin^2 \theta) + C_r^2 (\sin^2 \theta + f_i^2 \cos^2 \theta)$$

$$+ 2 \cdot C_c \cdot C_r \sin \theta \cos \theta (1 - f_i^2) \quad \dots (A.IV.3)$$

whence

$$C'_{eff,2} = [C'_c \cdot 2 \cdot \bar{C}_c (\cos^2 \theta + f_i^2 \sin^2 \theta) + C'_r \cdot 2 \cdot \bar{C}_r (\sin^2 \theta + f_i^2 \cos^2 \theta)$$

$$+ C'_c \cdot 2 \cdot \bar{C}_r (1 - f_i^2) \sin \theta \cos \theta + C'_r \cdot 2 \cdot \bar{C}_c \sin \theta \cos \theta$$

$$\cdot (1 - f_i^2)] / 2 \cdot \bar{C}_{eff,2} = C'_c \eta_1 + C'_r \zeta_1$$

where

$$\eta_1 = \{ \bar{C}_c (\cos^2 \theta + f_i^2 \sin^2 \theta) + \bar{C}_r \sin \theta \cos \theta [1 - f_i^2] \} / \bar{C}_{eff,2}$$

and

$$\zeta_1 = \{ \bar{C}_r (\cos^2 \theta \cdot f_i^2 + \sin^2 \theta) + \bar{C}_c \sin \theta \cos \theta [1 - f_i^2] \} / \bar{C}_{eff,2}$$

and hence

$$\overline{C'^2_{eff,2}} = \overline{C'^2_c} \eta_1^2 + \overline{C'^2_r} \zeta_1^2 + 2 \cdot \overline{C'_c C'_r} \eta_1 \zeta_1 \quad \dots (A.IV.4)$$

Similarly for position 3,

$$\overline{C_{eff3}^2} = \overline{C_c'^2} \eta_2^2 + \overline{C_r'^2} \zeta_2^2 + 2 \cdot \overline{C_c' C_r'} \eta_2 \zeta_2 \quad \dots(A.IV.5)$$

where

$$\eta_2 = \left\{ \overline{C_c} (\cos^2 \theta + f_2^2 \sin^2 \theta) - \overline{C_r} \sin \theta \cos \theta (1 - f_1^2) \right\} / \overline{C_{eff3}}$$

$$\text{and } \zeta_2 = \left\{ \overline{C_r} (\sin^2 \theta + f_2^2 \cos^2 \theta) - \overline{C_c} \sin \theta \cos \theta (1 - f_1^2) \right\} / \overline{C_{eff3}}$$

For $Cr = 0$ as is nearly the case in a compressor, the following set of equations is applicable.

$$\overline{C_{eff1}^2} = \overline{C_c^2}, \quad \overline{C_{eff1}^{'2}} = \overline{C_c'^2} \quad \dots(A.IV.6)$$

$$\overline{C_{eff2}^2} = \overline{C_c^2} (\cos^2 \theta + f_1^2 \sin^2 \theta)$$

$$\begin{aligned} \overline{C_{eff2}^{'2}} &= \overline{C_c'^2} (\cos^2 \theta + f_1^2 \sin^2 \theta) + \overline{C_r'^2} \frac{\sin^2 \theta \cos^2 \theta (1 - f_1^2)^2}{\cos^2 \theta + f_1^2 \sin^2 \theta} \\ &\quad + \overline{C_c' C_r'} \cdot 2 \cdot \sin \theta \cos \theta (1 - f_1^2) \end{aligned} \quad \dots(A.IV.7)$$

$$\overline{C_{eff3}^2} = \overline{C_c^2} (\cos^2 \theta + f_2^2 \sin^2 \theta)$$

$$\begin{aligned} \overline{C_{eff3}^{'2}} &= \overline{C_c'^2} (\cos^2 \theta + f_2^2 \sin^2 \theta) + \overline{C_r'^2} \frac{\sin^2 \theta \cos^2 \theta (1 - f_2^2)^2}{\cos^2 \theta + f_2^2 \sin^2 \theta} \\ &\quad - \overline{C_c' C_r'} \cdot 2 \cdot \sin \theta \cos \theta (1 - f_2^2) \end{aligned} \quad \dots(A.IV.8)$$

Now, from the hot wire relation

$$E^2 = A + B C_{eff}^p, \quad C_{eff} = \left(\frac{E^2 - A}{B} \right)^{1/p} \quad \dots(A.IV.9)$$

$$\text{and } e' = \frac{pBC_{\text{eff}}^{p-1}}{2E} C'_{\text{eff}} \quad \dots(\text{A.IV.10})$$

$$\approx \frac{pB}{2E} C_{\text{eff}}^p \frac{C'_{\text{eff}}}{C_{\text{eff}}}$$

$$\text{or } \frac{C'_{\text{eff}}}{C_{\text{eff}}} = \frac{2E}{p(E^2-A)} e' \quad \dots(\text{A.IV.11})$$

On squaring each side and rearranging,

$$\overline{C_{\text{eff}}'^2} \cdot \overline{C_{\text{eff}}^2} = \left[\frac{2E(E^2-A)^{\frac{2-p}{p}}}{p B^{2/p}} \right]^2 \overline{e'^2}$$

$$\text{or, } \overline{C_{\text{eff}}'^2} = \left[\frac{2E(E^2-A)^{\frac{1-p}{p}}}{p B^{1/p}} \right]^2 \overline{e'^2} \quad \dots(\text{A.IV.12})$$

provides a useful expression to serve as the right-hand side of each of the equations for the wire response.

Thus, in terms of the electrical outputs, the equations may be written as follows:

$$\overline{C_c'^2} = \left[\frac{2E_1(E_1^2-A)^{\frac{1-p}{p}}}{p B^{1/p}} \right]^2 \overline{e_1'^2},$$

$$\begin{aligned} & (\cos^2\theta + f_1^2 \sin^2\theta) \overline{C_c'^2} + 2 \sin\theta \cos\theta (1-f_1^2) \overline{C_c' C_2'} \\ & + \frac{\sin^2\theta \cos^2\theta (1-f_1^2)^2}{\cos^2\theta + f_1^2 \sin^2\theta} \overline{C_c'^2} = \left[\frac{2 \cdot E_2 \cdot (E_2^2-A)^{\frac{1-p}{p}}}{p B^{1/p}} \right]^2 \overline{e_2'^2} \end{aligned}$$

...(\text{A.IV.13})

$$\begin{aligned} \text{and, } & (\cos^2\theta + f_2^2 \sin^2\theta) \overline{C_c'^2} - 2 \sin\theta \cos\theta (1-f_2^2) \overline{C_c' C_2'} \\ & + \frac{\sin^2\theta \cos^2\theta (1-f_2^2)^2}{\cos^2\theta + f_2^2 \sin^2\theta} \overline{C_c'^2} = \left[\frac{2 \cdot E_3 \cdot (E_3^2-A)^{\frac{1-p}{p}}}{p B^{1/p}} \right]^2 \overline{e_3'^2} \end{aligned}$$

...(\text{A.IV.14})

These relations constitute a set of linear algebraic equations which were solved to analyse the data of the second and the third series of tests.

If the radial component of velocity Cr was not negligible, the set would then consist of linear algebraic equations on terms of more complicated coefficients,

$$\eta_n^2 \overline{C_c'^2} + 2 \eta_n \zeta_n \overline{C_c' C_i'} + \zeta_n^2 \overline{C_i'^2} = \left[\frac{2 \cdot E_1 (E_1^2 - A)^{\frac{2-p}{p}}}{p B^{2/p}} \right]^2 \overline{e_1'^2} \dots (A.IV.15)$$

$$\eta_{1n}^2 \overline{C_c'^2} + 2 \eta_{1n} \zeta_{1n} \overline{C_c' C_i'} + \zeta_{1n}^2 \overline{C_i'^2} = \left[\frac{2 \cdot E_2 (E_2^2 - A)^{\frac{2-p}{p}}}{p B^{2/p}} \right]^2 \overline{e_2'^2} \dots (A.IV.16)$$

$$\eta_{2n}^2 \overline{C_c'^2} + 2 \eta_{2n} \zeta_{2n} \overline{C_c' C_i'} + \zeta_{2n}^2 \overline{C_i'^2} = \left[\frac{2 \cdot E_3 (E_3^2 - A)^{\frac{2-p}{p}}}{p B^{2/p}} \right]^2 \overline{e_3'^2} \dots (A.IV.17)$$

where

$$\eta_{1n} = \eta_1 \cdot \overline{C_{eff1}}, \quad \eta_n = \eta \cdot \overline{C_{eff}}$$

and $\eta_{2n}, \zeta_{1n}, \zeta_n, \zeta_{2n}$ being similarly defined.

On the other hand, if $Cr = 0$ and $f_1 = f_2$ for symmetrical angular response of the wire, it is obvious that

$$-\overline{C_c' C_i'} = \frac{(\overline{C_{eff3}^2} - \overline{C_{eff2}^2})}{4 \cdot \sin \theta \cos \theta (1-f^2)} \dots (A.IV.18)$$

and the shear stress is ρ times this value. Introducing the electrical output relations as mentioned earlier,

$$-\rho \overline{C_c' C_i'} = \frac{\rho}{p^2 B^{2/p}} \frac{[2 \cdot E_3 (E_3^2 - A)^{\frac{1-p}{p}}]^2 \overline{e_3'^2} - [2 \cdot E_2 (E_2^2 - A)^{\frac{1-p}{p}}]^2 \overline{e_2'^2}}{4 \cdot \sin \theta \cos \theta (1-f^2)} \dots (A.IV.19)$$

If now, in addition, $p = .5$ and $\theta = 45^\circ$

$$E_1 = E_2 = E$$

then,

$$-\rho \overline{C_c' C_i'} = \frac{\rho}{B^4 \cdot 2 \cdot (1-f^2)} * [4 E (E^2 - A)]^2 (\overline{e_3'^2} - \overline{e_2'^2}) \dots (A.IV.20)$$

For calibration index $p = .5$, $Cr = 0$, $f_1 = f_2 = f$ and

$$E_1 = E_2 = E,$$

$$-p \overline{C_c' C_i'} = p \frac{[2 E (E^2 - A)]^2}{B^2 \sin \theta \cos \theta (1 - f^2)} (\overline{e_3'^2} - \overline{e_2'^2}) \quad \dots (A.IV.21)$$

For the simplest case of $f_1 = f_2 = f$, $p = .5$, $Cr = 0$, $\theta = 45^\circ$,

$E_1 = E_2 = E$ and $C_\theta = 0$, the relation reduces to a commonly used form

$$p \overline{C_x' C_i'} = \frac{8 E (E^2 - A)^2}{B^2} (\overline{e_3'^2} - \overline{e_2'^2}) \quad \dots (A.IV.22)$$

f-Factors and Their Evaluation

The procedure to calculate the f-factors for a wire is outlined here.

Assuming that the cross components of flow did not exist at the calibration station, the effective cooling velocity for a normal wire is given by $C_{\text{eff}_n}^2 = C_x^2$ and for a wire whose normal makes θ with the direction of velocity C_x ,

$$C_{\text{eff}}^2 = C_x^2 (\cos^2 \theta + f^2 \sin^2 \theta) \quad \dots (A.IV.23)$$

From the electrical output relations,

$$E_n^2 = A + B_n C_{\text{eff}_n}^p \quad \text{for the 'normal' wire and}$$

$$E_\theta^2 = A + B_\theta C_{\text{eff}_\theta}^p = A + B_\theta' C_{\text{eff}_n}^p$$

$$= A + B_n (\cos^2 \theta + f^2 \sin^2 \theta)^{p/2} C_{\text{eff}_n}^p$$

$$\frac{E_\theta^2 - A}{E_n^2 - A} = (\cos^2 \theta + f^2 \sin^2 \theta)^{p/2} = \frac{B_\theta'}{B_n}$$

$$\text{and } f = \left[\frac{\left(\frac{B'_0}{B_n} \right)^{2/p} - \cos^2 \theta}{\sin^2 \theta} \right]^{\frac{1}{2}} \quad \dots (\text{A.IV.24})$$

where B'_0 and B_n were the slopes of the linear plots of E_0^2 and E_n^2 respectively $V/S C_{\text{eff}_n}^p$.

$$\text{For } p = 0.5, \quad f^2 = \frac{(B_0/B_n)^4 - \cos^2 \theta}{\sin^2 \theta} \quad \dots (\text{A.IV.24a})$$

and for a special case of the wire placed at 45° to the probe axis

$$f^2 = 2 \cdot (B_0/B_n)^2 - 1. \quad \dots (\text{A.IV.24b})$$

For a wire possessing unequal angular response on either side of the normal setting, it is assumed that the introduction of f_1 and f_2 factors as follows overcomes the problem:

$$C_{\text{eff}_{\theta+}}^2 = C_x^2 (\cos^2 \theta + f_1^2 \sin^2 \theta)_+ \quad \dots (\text{A.IV.23a})$$

and,

$$C_{\text{eff}_{\theta-}}^2 = C_x^2 (\cos^2 \theta + f_2^2 \sin^2 \theta)_- \quad \dots (\text{A.IV.23b})$$

The evaluation of f_1 and f_2 is similar to that shown in Equation (A.IV.24) above.

APPENDIX V

ERROR ANALYSIS FOR HOT WIRE MEASUREMENTS

Estimates of error in the measurement of mean velocity due to an error in recording the bridge D.C. voltage, in orienting the wire at an angle with the flow and due to high turbulence level are presented here. Extent of errors in the measurement of turbulence due to a faulty record of D.C. voltage and due to the assumption of 'low turbulence' are also included.

$$(1) \quad E^2 = A + B\sqrt{U_{\text{eff}}} \quad \text{and for small changes in } E \text{ and } U_{\text{eff}},$$

$$2E\delta E = \frac{1}{2} \cdot B \cdot (U_{\text{eff}})^{-\frac{1}{2}} \delta U_{\text{eff}} \quad \dots(\text{A.V.1})$$

$$\text{For a wire normal to flow, } \frac{\delta U_{\text{eff}}}{U_{\text{eff}}} = \frac{\frac{4E^2}{E^2-A}}{\frac{E^2}{E}} \frac{e'}{E}$$

$$\text{or } u'/U_{\text{eff}} = 4Ee'/(E^2-A) \quad \dots(\text{A.V.2})$$

Differentiating each side w.r.t. E and rearranging,

$$\frac{d(u'/U_{\text{eff}})}{u'/U_{\text{eff}}} = - \frac{E^2 + A}{E^2 - A} \frac{\delta E}{E} \quad \dots(\text{A.V.3})$$

If it is intended to evaluate the error in u'/U_G , then writing $u'/U_G = u'/U_{\text{eff}} * U_{\text{eff}}/U_G$, it is evident that

$$\delta\left(\frac{u'}{U_G}\right) = \frac{U_{\text{eff}}}{U_G} \frac{\partial(u'/U_{\text{eff}})}{\partial E} \delta E + u'/U_{\text{eff}} \cdot \frac{\partial(U_{\text{eff}}/U_G)}{\partial E} \delta E \quad \dots(\text{A.V.4})$$

$$\text{and hence, } \delta\left(\frac{u'}{U_G}\right)/\left(\frac{u'}{U_G}\right) = - \frac{E^2+A}{E^2-A} \frac{\delta E}{E} + \frac{4E^2}{E^2-A} \frac{\delta E}{E}$$

$$= [3E^2-A]/(E^2-A) \cdot \frac{\delta E}{E} \quad \dots(\text{A.V.5})$$

For an example, considering a typical set of values:

$$A = 35, \quad B = 6, \quad E = 9.5 \text{ volts}$$

$$U_{\text{eff}} \approx 85 \text{ ft/sec.}$$

$$\frac{\delta U_{\text{eff}}}{U_{\text{eff}}} = 6.55 \frac{\delta E}{E},$$

$$\frac{\delta (\bar{u}'/U_{\text{eff}})}{\bar{u}'/U_{\text{eff}}} = -2.28 \frac{\delta E}{E} \quad \text{and}$$

$$\frac{\delta (\bar{u}'/U_0)}{\bar{u}'/U_0} = 4.27 \frac{\delta E}{E}$$

Thus, under such circumstances 1% error in E would result in 6 to 7% error in the mean velocity and 4 to 4.5% error in the turbulent fluctuations of the velocity.

(2) It has been shown in the consideration of yaw-sensitivity of the hot wire in Chapter 5 that the angular response is not very good especially when the wire is normal to the flow. A small deviation of the wire from its intended inclination with the velocity will, therefore, cause no noticeable change in the readings and the errors in the measurement of shear stresses are kept to a minimum.

(3) For a wire placed normal to C_0 , the resultant of C_x and C_0 , assuming the radial velocity as negligible, the effective cooling velocity is given by $C_{\text{eff}}^2 = (\bar{C}_c + C'_c) + C'_i{}^2$ and

$$\sqrt{C_{\text{eff}}} = \sqrt{\bar{C}_c} \left(1 + \frac{1}{2} \frac{C'_c}{\bar{C}_c} - \frac{1}{8} \frac{C'^2_c}{\bar{C}_c^2} + \frac{1}{4} \frac{C'^2_i}{\bar{C}_c^2} \dots \right),$$

$$\sqrt{C_{\text{eff}}} = \sqrt{\bar{C}_c} \left(1 - \frac{1}{8} \frac{\bar{C}_c^2}{\bar{C}_c^2} + \frac{1}{4} \frac{\bar{C}_c^2}{\bar{C}_c^2} \dots \right)$$

(A.V.6)

It is necessary to assume, at this stage, what $\bar{E}$ really corresponds to:

$$\bar{E}^2 = A + B\sqrt{C_c} \quad \text{or} \quad \bar{E}^2 = A + B\sqrt{C_c}$$

There is hardly any justification in favour of the former or latter. However, the second relation appears to be better since it implies that the wire senses the instantaneous velocity. Then, rewriting it,

$$\bar{E}^2 = A + B\sqrt{C_c} \left(1 - \frac{1}{8} \frac{\bar{C}_c'^2}{C_c^2} + \frac{1}{4} \frac{\bar{C}_c'^2}{C_c^2} + \dots \right) \quad (\text{A.V.7})$$

Now, from $E^2 = A + B\sqrt{C_c}$,

$$\bar{E}^2 + e^2 + 2\bar{E}e = A + B\sqrt{C_c} = R, \text{ say.} \quad (\text{A.V.8})$$

$$\begin{aligned} e &= -\bar{E} + \sqrt{R} \\ &= -\bar{E} + (A + B\sqrt{C_c})^{1/2} \left[1 + \frac{B\sqrt{C_c}}{2(A + B\sqrt{C_c})} \left\{ \frac{1}{2} \frac{C_c'}{C_c} \right. \right. \\ &\quad \left. \left. - \frac{1}{8} \frac{C_c'^2}{C_c^2} + \frac{1}{4} \frac{C_c'^2}{C_c^2} + \dots \right\} \right] \end{aligned} \quad (\text{A.V.9})$$

Substituting for $(A + B\sqrt{C_c})$ from the assumption made above, and taking r.m.s. of each side should provide an expression for the r.m.s. measured voltage. As a first approximation, neglecting the terms of higher orders,

$$\begin{aligned} \sqrt{e'^2} &= \frac{E^2 - A}{4E} \left[\sqrt{C_c'/C_c} \left(1 + \frac{1}{4} \frac{\bar{C}_c'^2}{C_c^2} - \frac{1}{2} \frac{\bar{C}_c'^2}{C_c^2} \right) \right. \\ &\quad \left. + \frac{1}{8} \frac{\bar{C}_c'^2}{C_c^2} - \frac{1}{4} \frac{\bar{C}_c'^2}{C_c^2} \right] \end{aligned} \quad (\text{A.V.10})$$

On the other hand, assumption of 'low level of turbulence' implies that

$$\sqrt{\bar{C}_{eff}} = \sqrt{\bar{C}_c} \quad \text{and} \quad (A.V.11a)$$

$$\sqrt{\bar{C}^2} = \frac{E^2 - A}{4E} \frac{\sqrt{\bar{C}_c^2}}{\bar{C}_c} \quad (A.V.11b)$$

which are associated with errors of

$$-\left(\frac{\bar{C}_c^2 - 2\bar{C}_i^2}{4\bar{C}_c^2}\right) * 100 \% \quad \text{and} \quad (A.V.12a)$$

$$\left[\frac{1}{4} \frac{\bar{C}_c^2}{\bar{C}_c^2} - \frac{1}{2} \frac{\bar{C}_i^2}{\bar{C}_c^2} + \frac{1}{8} \sqrt{\bar{C}_c^2} / \bar{C}_c - \frac{1}{4} \frac{\bar{C}_c^2}{\sqrt{\bar{C}_c^2}} / \bar{C}_c\right] * 100 \% \quad (A.V.12b)$$

respectively.

The error in the velocity measurement should be zero if $\bar{C}_c^2 = 2 \cdot \bar{C}_i^2$. For isotropic turbulence, there is a small error and for a region close to the wall $\bar{C}_c^2 = 2$ to 3 times $\bar{C}_i^2$ and the error is again small.

(4) Another error that arises from the finite length of the wire is concerned chiefly with the determination of the gradient of velocity. When the wire is placed in a flow with steep velocity gradients, it is subjected to differential cooling along its length, and the boundary layer flow along the prongs may tend to assume importance. Under these conditions, the wire may only measure a weighted average velocity and the superimposed prong-disturbances. Numerical estimates of such errors are difficult to obtain.

APPENDIX VI

DETAILS OF THE ELECTRONIC EQUIPMENT

The electronic equipment used in conjunction with the hot wire transducer consisted of two constant temperature anemometers (DISA 55-A-01), a random signal indicator and correlator (DISA 55-A-06), a digital voltmeter (SOLARTRON IM 902.2) and a single beam oscilloscope (METROVICK - 300). A brief description of each of these components is given below:

The anemometer embodied a bridge circuit, one arm of which was the transducer and the other variable arm consisted of three resistance decades. The bridge was fed by an amplifier whose input voltage was controlled by the bridge unbalance. The resulting loop system constituted a null-seeking device.

The resistance decades were adjusted to an operating resistance corresponding to a pre-determined operating temperature of the hot wire. For equilibrium, the transducer resistance must be equal to the preset adjustable bridge arm-resistance. If the two are unequal, a voltage will be present across the output terminals of the bridge and hence across the amplifier input. This voltage undergoes a large amplification in the D.C. preamplifier and in D.C. output amplifiers and is fed back to the bridge as supply voltage.

As the effective cooling velocity increases, it results in additional cooling or a temperature drop and hence the transducer resistance drops. The amplifiers were so wired as to increase the bridge voltage, causing the bridge current to increase resulting in heating of the probe, The

increase in temperature and resistance of the wire reduced the original amount of bridge unbalance.

Because of a high loop gain, the electronic feed back operated to obtain a condition of balance corresponding to the the alterations in the effective cooling velocity. The average flow velocity was thus related to the indicated bridge D.C. voltage.

A calibrated r.m.s. indicator consisting of an A.C. amplifier and a thermocouple was also connected across the bridge terminals via a set of adjustable low-pass and high-pass filters and two cathode followers. The instrument provided the r.m.s. value indications of the bridge voltage fluctuations reduced by approximately 4% by the cathode followers. The r.m.s. millivoltmeter reading formed an indication of a correlation of turbulent velocities depending upon the placement of the wire relative to the direction of velocity.

The technical data of the anemometer has been summarised in Table D.

The signal indicator and correlator was designed to measure the r.m.s. value of two electrical input signals $\bar{A}$ and $\bar{B}$ and their correlations such as $(\overline{A + B})$, $(\overline{A - B})$, $\tau \frac{dA}{dt}$, $(\overline{A/B})$ and $(\overline{A + B})/(\overline{A - B})$. The correlator is a useful equipment if a cross wire or two or more single wire probes are used simultaneously. This was used when a cross-wire was employed in the early stages but the unsuitability of an X-wire eliminated its use.

In principle, it is only an analogue computer with limited function to perform.

Its main specifications are given below:

Frequency response 3 c/s - 200,000 c/s.

Voltage ranges 0-10, 0-31.5, 0-100, 0-315 mV.
 0-1, 0-3.15, 0-10, 0-31.5, 0-100 V.

Noise level less than 50 μ V r.m.s.

Two damping positions, and meter selector switches.

The oscilloscope used was a single beam general purpose version. It was included in the circuit to make a visual check for the operation of the wire and to examine the existence of the undesirable frequencies such as those of the stem vortices in the pipe apparatus and the blade wakes in the compressor.

On X scale, the range was from 3 microseconds/cm to 10 milliseconds/cm and on the Y scale from 1 volt/cm to 100 volts/cm.

A Solartron Digital Voltmeter was employed at times to obtain a more accurate reading of the D.C. voltage. Although it could read accurately up to the third decimal place, the reading was not steady enough to be recorded. The dead zone units of 2 and 4 were switched in so as to get a reasonably steady reading. A mental averaging process and later, recorded averaging method were used to make a record. This, therefore, did not result in improved accuracy but provided a check on the anemometer readings, and offered itself as an alternative digital device free from parrallax errors and capable of being read from longer distances.

APPENDIX VII

DETAILS OF THE COMPUTER PROGRAMME FOR THE HOT WIRE

DATA ANALYSIS

(a) Description

The computer programme for the analysis of the data has been organised in three sections. Section A deals with the calibration of the wire, including the determination of f_1 and f_2 factors. The calibration constants are worked out by the least-square-fit considerations. If, instead, it is preferred to determine the constants graphically, these may be supplied as input.

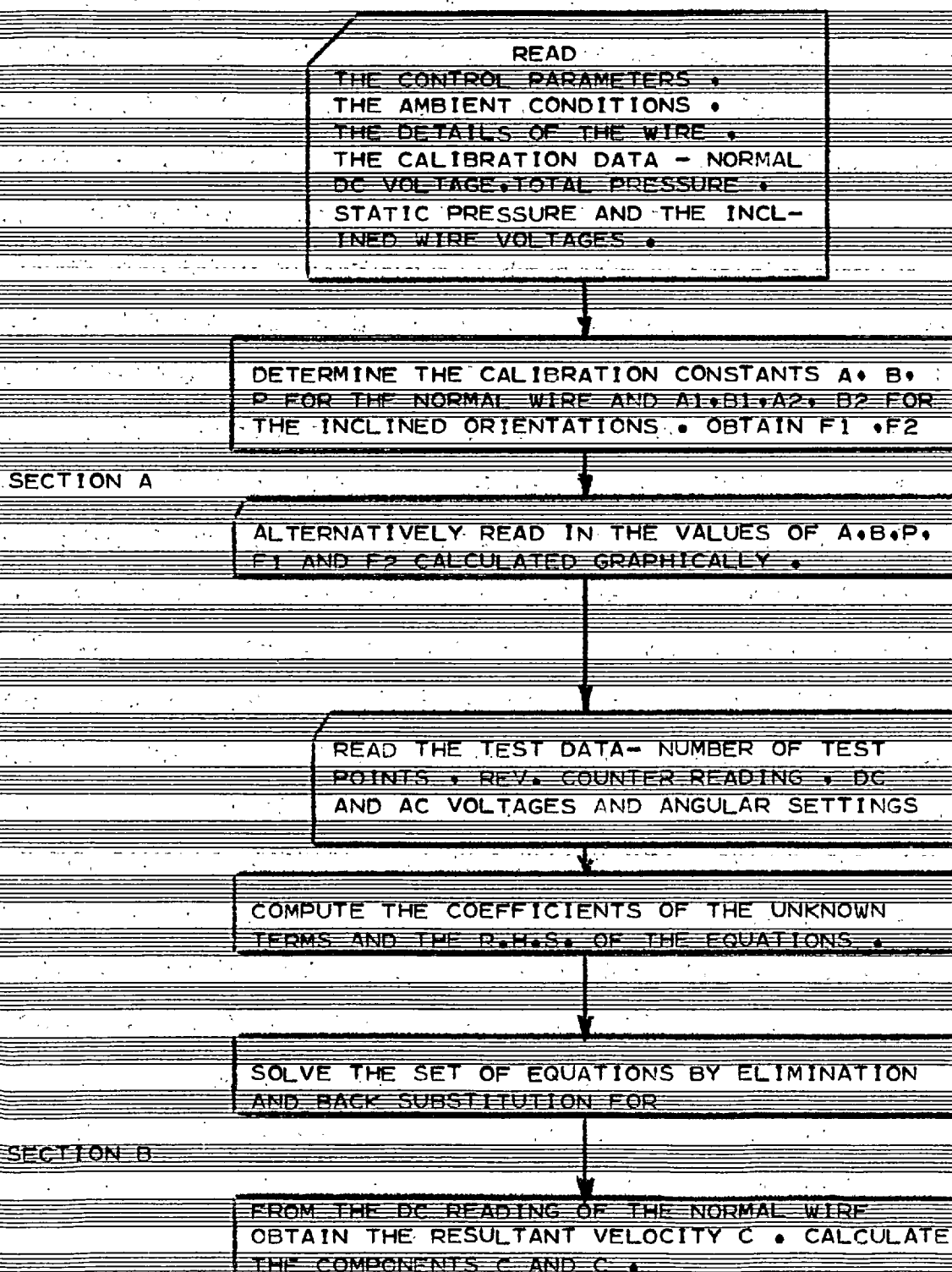
Section B covers the setting up and the solution of three equations in $\overline{C_c^2}$, $\overline{C_v^2}$ and $\overline{C_c C_v}$. It provides the distribution of shear stress, the turbulent kinetic energy, level of turbulence and the shear to K.E. ratio. The calculations of the velocity profile, velocity gradient, effective viscosity and the ratio of effective to molecular viscosity are performed in Section C.

Provision was made to permit the programme to be used for any of the following tasks:

1. To calculate the wire constants only.
2. To determine the shear stress and effective viscosity distributions only.
3. To calibrate the wire as well as to compute the distribution of shear stress, and effective viscosity.

This was achieved by a suitable indicator in the programme input. Further, the calibration section can be

(b) BLOCK DIAGRAM FOR THE MAIN PROGRAMME
FOR HOT-WIRE DATA ANALYSIS



α

CALCULATE THE LEVEL OF TURBULENCE , THE
NON-DIMENSIONAL SHEAR AND THE RATIO OF
SHEAR TO TURBULENT KINETIC ENERGY .

CLCULATE THE RADIAL GRADIENTS OF RESULTANT
VELOCITY BY FITTING BEST THIRD ORDER POLY-
NOMIALS THROUGH THE POINT AND TWO MORE
POINTS ON EITHER SIDE OF IT .

SECTION C

CALCULATE THE RADIAL DISTRIBUTION OF
EFFECTIVE VISCOSITY , AS A RATIO OF
SHEAR TO VELOCITY GRADIENT , CALCULATE THE
RATIO OF APPARENT TO MOLECULAR VICOSITY .

used for calibrating the wire either in only the normal position, or in inclined positions or in all the three positions. For index KCOMP set equal to unity, the controls are transferred to deal with measurements on the compressor, otherwise the data is assumed to refer to the pipe measurements.

(c) Determination of the Calibration Constant and Optimisation of the Fit.

For a set of data and other relevant information, values of velocity corresponding to the D.C. voltages are first computed. In order to fit a straight line $E^2 = A + B C^p$, by the least-square-method

$$\partial S / \partial A = 0, \quad \partial S / \partial B = 0. \quad (\text{A.VII.1})$$

where

$$S = \sum_i^N \left(E^2 - [A + B C^p] \right)^2 \quad (\text{A.VII.2})$$

On simplification, the conditions require that

$$N \cdot A - \sum E^2 + B C^p = 0. \quad \text{and}$$

$$B \sum C^{2p} - \sum E^2 C^p + A \sum C^p = 0.$$

From these two equations,

$$A = \frac{\sum E^2 \sum C^{2p} - \sum C^p \sum (E^2 C^p)}{N \sum C^{2p} - (\sum C^p)^2} \quad (\text{A.VII.3})$$

and,

$$B = \frac{N \sum (E^2 C^p) - \sum E^2 \sum C^p}{N \sum C^{2p} - (\sum C^p)^2} \quad (\text{A.VII.4})$$

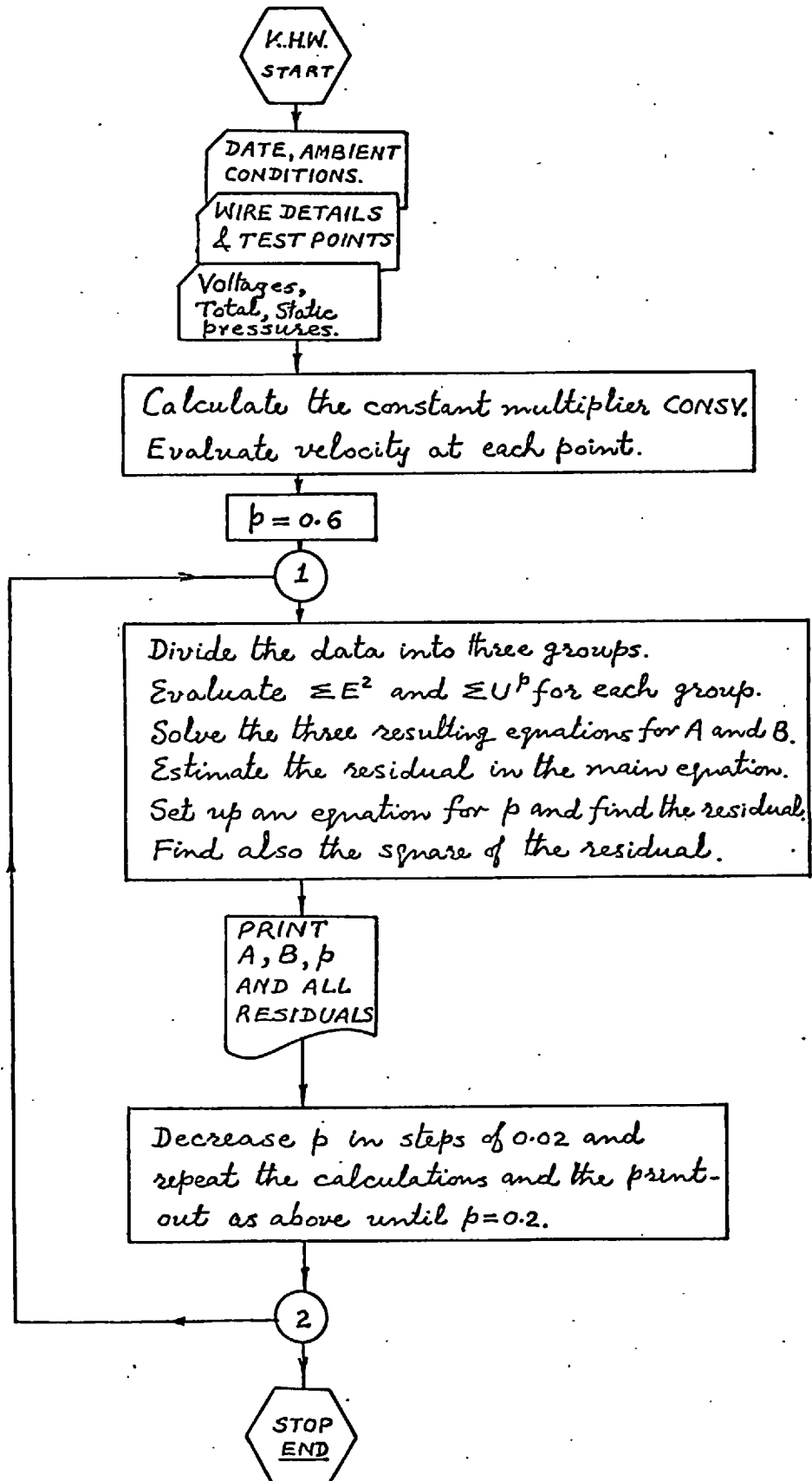
Determination of A and B in this manner for the Normal, SLF and LLF wire has been incorporated in the first section of the Data-Analysis Programme. Theoretically all positions of a wire must have a common constant A due only to conduction loss which is independent of the velocity. In most practical cases, an averaged value of A was assumed unless A's differed appreciably where a manual check was called for.

The calibration described above was adequate to determine A and B constants, p being an assumed index close to 0.5, based upon theoretical grounds. This calibration technique formed section 'A' of the Hot Wire Programme. For the present case, index p was selected to result in satisfactory wall shear stress in the pipe flow and this was generally close to 0.5.

Alternatively, if another equation is set up by using the condition $\partial S / \partial p = 0$ and the set of three equations may be solved uniquely for A, B and p. Unfortunately, this condition leads to a non-linear equation with p both in the coefficient and in the index. An alternative approach with the following steps was preferred.

1. The data was divided into three groups.
2. Residuals in the solution of p-equations in each group were evaluated.
3. For a given value of p, the three equations constituted by the groups were solved to get A and B.
4. Step 3 was repeated for different values of p, and overall residuals were calculated.

(d) OPERATION-DIAGRAM FOR HOT-WIRE PROGRAMME
TO OPTIMISE THE CALIBRATION CONSTANTS



5. The data was regrouped into different groups and steps 1 to 4 were repeated.
6. The results were plotted and A, B and p for minimum overall residual were considered as a 'best solution'.

The procedure outlined above was not adopted as a standard practice as an estimate of a 'best' set of constants from an examination of the plots of residuals was not always possible. Moreover, the earlier adopted practice of estimating A and B from the calibration curves with a value of p to yield correct wall shear stress in the pipe tests was considered more realistic and hence better.

```

C MAIN PROGRAMME
C
C THE MAIN PROGRAMME CONTROLS THE SEQUENCE OF OPERATIONS .
COMMON/BLKA/XM,YM,XM1,YM1,D2YM1,XM2,YM2,D2YM2
COMMON/BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKIT/MAXIT,OMEGA,EPS,BIGD,BIGDM,ITN
COMMON/BLKGR5/16,IR1,IS1,IR2,IS2,IR3,IS3,IS3P1
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KCC,KVEL
COMMON/BLKRT/RPM,TURNS,CXEN
COMMON/WORKA/ A(36),C(6),MF(17),NF(17),EXTRAA(468)
COMMON/BLK1/THETG(17),GAMG(17),CRSETG,CAMBG(17),ALFDG(17),
1ALFD1(17)
1 /BLK2/THETR(17),GAMR(17),CRSETR,CAMBR(17),BETID(17),
1BET2D(17)
1 /BLK3/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALFID(17),
1ALF2D(17)
1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1 /BLK5/BC(17),BH(17),BASPR(17),BLER,BTER,BFR,TIPCLR,TBYC(17)
1 /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON/BLKM2/ CX(16,17),CRU(16,17),B(16,17),CU(16,17),CR(16,17)
1,U(16,17),CV(16,17)
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
COMMON/BLKT/TERM(16,17),EMEWR(16,17),EMEWRX(16,17),ROTERM(16,17)
COMMON/BLKSIG/SIGMB(16,17)
DIMENSION RE(17),BF(17)

C
READ(5,123) KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KVEL
123 FORMAT(12I3)
C KCOMP=1 COMPRESSOR PROBLEM IN HAND .
C KANNU=1 FOR FLOW THROUGH AN ANNULUS .
C KPIPE=1 TO DEAL WITH THE PIPE FLOW ONLY .
C KEQ=1 OMIT THE DOWNSTREAM BOUNDARY CONDITION DSIDX=0.
C KWA OMIT WRITING ROWENT,IDCR/S,CALBR/S EXCEPT B IF KPC=KWA
C KWB=1 OMIT WRITING THE CONSTITUENTS OF THE B TERMS .
C KWA,KWB EACH AS UNITY TO OMIT ALL CORRELATION WRITE STATEMENTS .
C KPA=1 ESCAPE GIVEN BLADE ANGLE INTERPOLATIONS IN INPUT .
C KPB= NUMBER OF ITERATIONS FOR REFERENCE EVALUATION .
C KVEL 1 OR 2 FOR THE ASSUMED EXIT VELOCITY PROFILE .
CXEN=120.
CALL INPUT
CALL ENTEX
C INITIALISED FIELD PRINTOUT .
WRITE(6,6) ((U(1,J),I=1,N),J=1,M)
WRITE(6,6) ((CX(1,J),I=1,N),J=1,M)
6 FORMAT(1H0,16F7.2)
C CONTROL TO ITERATE NCYCLE TIMES COMPLETE CYCLES .
NCYCLE=8
KCC=1
CXP=0.
11 CONTINUE
C BYPASS THE CALL SCAN STATEMENT THE FIRST TIME .
IF (KCC.EQ.1) GO TO 13
JM=(M+1)/2
CXP=CX(153P1,JM)
CALL SCAN

```

```

13  CONTINUE
    IF(KCC.GT.NCYCLE) GO TO 22
    CALL      CALTU
C   DEPENDING UPON THE SITUATION OF I CALL THE APPROPRIATE BLADE
C   ROW SUBROUTINE TO CALCULATE THE B-TERMS .
    DO 21 I=1,MM1
    IF(I.LT.IG) GO TO 19
    IF(I.GT.IS3P1) GO TO 41
    IF(I.EQ.IS3P1) GO TO 21
12  CONTINUE
    IF(I.EQ.IR1.OR.I.EQ.IR2.OR.I.EQ.IR3) GO TO 16
    GO TO 17
16  CALL      CALBR
    GO TO 21
17  CALL      CALBS
    GO TO 21
41  DO 42 J=2,MM1
42  B(I,J)=B(I-1,J)
    GO TO 21
19  CALL SIMPLE
21  CONTINUE
C   AFTER THE LAST STAGE ASSUME AS FOLLOWS .
    IAF=N-4
    DO 78 I=1AF,N
    DO 78 J=1,M
    H0(I,J)=H0(I-1,J)
    P0(I,J)=P0(I-1,J)
    ROW(I,J)=ROW(I-1,J)
78  CONTINUE
    KCC=KCC+1
    WRITE (6,66)
66  FORMAT(1H0,5X,34HTHE FOLLOWING IS WRITTEN FROM MIKC)
    WRITE(6,23) ((B(I,J),I=2,MM1),J=2,MM1)
23  FORMAT(3X,14F8.1)
    IGP1=IG+1
    DO 52 I=IGP1,IS3P1
    DO 51 JB=1,12
    RF(JB)=R(I,JB+3)
51  BF(JB)=B(I,JB+3)
    KOR=3
    NP=12
    CALL      POLYFT(RF,BF,NP,KOR,C,C0,A)
    DO 52 J=2,MM1
    B(I,J)=C0+C(1)*R(I,J)+C(2)*R(I,J)*R(I,J)+C(3)*R(I,J)
    I*R(I,J)*R(I,J)
52  CONTINUE
    IS3P2=IS3P1+1
    DO 53 I=IS3P2,N
    DO 53 J=2,M
53  B(I,J)=B(I-1,J)
    DO 54 I=2,MM1
    IF(B(I,2).LT.B(I,3)) B(I,2)=B(I,3)
54  CONTINUE
    DO 55 J=2,MM1
55  B(5,J)=B(5,J)*2.
    WRITE(6,23) ((B(I,J),I=2,MM1),J=2,MM1)
    CXDIF=ABS(CX(IS3P1,JM)-CXP)
    IF(CXDIF.LE.0.2) GO TO 22
    GO TO 11

```

```

22  CONTINUE
    WFLOW=2.*3.14159*100.*32.17/PC
    FLOWF=CXEN/CBU(3,9)
    WRITE(6,112) WFLOW, FLOWF
112  FORMAT(3X,30HWEIGHT FLOW IN LBS PER SECOND=F6.3,9H   FLOWF=F4.2)
    STOP
    END

```

SUBROUTINE INPUT

```

C -----
C  SUBROUTINE FOR INPUT OF COMPRESSOR GEOMETRY AND BLADE DATA.
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON /BLK2/ OMEGAD,ANG2,STGMK,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON /BLK3/ BI(9),BD(9),BN(16),BM(9),BR(16)
COMMON /BLK4/ XM,YN,XM1,YM1,D2YM1,XM2,YM2,D2YM2
COMMON /BLK5/ IG,IR1,IS1,IR2,IS2,IR3,IS3,IS3P1
COMMON /BLK6/ KPIPE,KCOMP,KANNU,KEG,KI,KWA,KWB,KWC,KPA,KPB,KPC
COMMON /WORKA/ A(36),C(6),EXTRAC(442),
1      RD(4),THETAG(4),GAMAG(4),CAMBRG(4),ALFADG(4),
1      THETAR(4),GAMAR(4),CAMBRR(4),BETAID(4),BETA2D(4),
1      THETAS(4),GAMAS(4),CAMBRS(4),ALFAID(4),ALFA2D(4)
COMMON /BLK7/ ANG(8),STGMC(10),TBYCG(8)
COMMON /BLK8/ MAXIT,OMEGA,EPS,BIGD,BIGDM,ITN
COMMON /BLK9/ A1010(8,9),ASFN(8,9),FKT(8)
COMMON /BLK10/ A010(8,9),ASF0(8,9),FKTD(8)
COMMON /BLK11/ ADDIR(9,6),STGMF(9),ANGF(6)
COMMON /BLK12/ THETG(17),GAMG(17),CRSETG,CAMBG(17),ALFDG(17),
1ALFDI(17)
1      /BLK13/ THETR(17),GAMR(17),CRSETR,CAMBR(17),BETID(17),
1BET2D(17)
1      /BLK14/ THETS(17),GAMS(17),CRSETS,CAMBS(17),ALFID(17),
1ALF2D(17)
1      /BLK15/ RINI(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1      /BLK16/ BC(17),BH(17),BASPR(17),BLER,BTER,BFR,TIPCLR,TBYC(17)
1      /BLK17/ ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON /BLK18/ STIG/STIGB(16,17)
C -----
    WRITE(6,65)
65  FORMAT(1H1)
    WRITE(6,66)
66  FORMAT(1H0,5X,41HTHE FOLLOWING IS WRITTEN FROM SUB. KINPT )
    READ(5,31) N,M,MAXIT,OMEGA,EPS
31  FORMAT(3I3,2F5,2)
    NM1=N-1
    NP1=N+1
    MM1=M-1
    MP1=M+1
    FMM1=FLOAT(MM1)
C  COMPRESSOR GEOMETRY AND CONFIGURATION DATA
C  ALL LENGTH DIMENSIONS IN INCHES
    READ(5,1) ROUT1,RINI,RMEAN1,RATIO1,AL
    READ(5,1) ROUT0,RINO,RMEAN0,RATIO0
C  READ SOME CONSTANTS WHICH WILL BE USED LATER.
    READ(5,48) CP,AG,AJ,AR,GAMA,H2
48  FORMAT(6F12,3)
    GJ=AG*AJ
1  FORMAT(5F8,3)
    ROUT1=ROUT1/12,

```

```

RINI=RINI/12.
RMEANI=RMEANI/12.
AL=AL/12.
ROUTO=ROUTO/12.
RINO=RINO/12.
RMEANO=RMEANO/12.
C IDENTIFICATION OF THE BLADE ROWS .
READ(5,333) IG,IR1,IS1,IR2,IS2,IR3,IS3
IS3P1=IS3+1
333 FORMAT(7I6)
DO 32 I=IG,IS3P1
ROUT(I)=ROUTI+(ROUTO-ROUTI)/FLOAT(IS3P1-IG) *FLOAT(I-IG)
32 RIN(I)=RINI
DO 36 I=1,4
RIN(I)=RINI
ROUT(I)=ROUTI
36 CONTINUE
DO 37 I=IS3P1,N
RIN(I)=RINO
ROUT(I)=ROUTO
37 CONTINUE
WRITE(6,8) ROUT(1),ROUT(2),ROUT(7)
WRITE(6,333) N,M,IG,IS3P1
RMEAN =RMEANI
RATIO =RATIOI
C READ FOR AXIAL DIRECTION GRID SPACING
READ(5,74)HR(2),HR(6),HR(8)
74 FORMAT (4F9,4)
HR(2)=HR(2)/12.
HR(6)=HR(6)/12.
HR(8)=HR(8)/12.
DO 4 I=2,5
HR(I) =HR(2)
4 HR(I) =HR(I+1)
DO 5 I=7,16
5 HR(I) =HR(8)
DO 6 I=1,15
6 HL(I) =HR(I+1)
C CALCULATE RADIAL DIRECTION GRID SPACINGS
DO 7 I=1,N
G(I)=(ROUT(I)-RIN(I))/FLOAT(M-I)
DO 7 J=1,M
7 R(I,J)=RIN(I)+G(I)*FLOAT(J-I)
C COMPRESSOR BLADES DATA
C C-4 SECTION 10/CHORD MAXTHICKNESS BLADES ON CIRCULAR CAMBRELINE
C TWISTED BLADES ON FREE VORTEX DESIGN
C THE BLADES HAVE CONSTANT CHORD AND CONSTANT HEIGHT
READ(5,8) BC1,BH1,BASPR1,TM
READ(5,9) BLER,BTER,BFR,TIPCLR
8 FORMAT(4F8,3)
9 FORMAT(4F9,4)
BC1=BC1/12.
BH1=BH1/12.
TM=TM/12.
BLER=BLER/12.
BTER=BTER/12.
BFR=BFR/12.
TIPCLR=TIPCLR/12.
READ BLADE SETTINGS AND ANGLES AT FOUR RADIAL PLACES AS MEASURED

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```

C      AND APPLY THE THICKNESS CORRECTIONS.
      DO 10 J=1,M
      BC(J)=BCI
      BH(J)=BHI
      BASPR(J)=BASPRI
10     TBYC(J)=TM/BC(J)
      READ (5,3) (RD(JD),JD=1,4)
      DO 35 JD=1,4
      RD(JD)=RD(JD)/12.
35     CONTINUE
      READ(5,3) (THETAG(JD),JD=1,4) , (THETAR(JD),JD=1,4) ,
1      (THETAS(JD),JD=1,4)
      READ(5,3) ( GAMAG(JD),JD=1,4) , ( GAMAR(JD),JD=1,4) ,
1      ( GAMAS(JD),JD=1,4)
      READ(5,3) (CAMBRG(JD),JD=1,4) , (CAMBRR(JD),JD=1,4) ,
1      (CAMBRS(JD),JD=1,4)
      READ(5,3) CRSETG,CRSETR,CRSETS
      DO 13 JD=1,4
      GAMAG(JD)=GAMAG(JD)+CRSETG
      GAMAR(JD)=GAMAR(JD)+CRSETR
      GAMAS(JD)=GAMAS(JD)+CRSETS
13     CONTINUE
C      NBG,NBR,NBS NUMBER OF BLADES ON GUIDEVANE, ROTOR AND STATOR RESP.
      READ(5,11) NBG,NBR,NBS
11     FORMAT(3I5)
      DO 12 J=1,M
19     CONTINUE
      DO 12 I=1G,1S3
      TPIR=2.*3.14159*R(I,J)
      IF(I.EQ.1G) GO TO 52
      GO TO 53
52     SIGMB(I,J)=FLOAT(NBG)*BC(J)/TPIR
      GO TO 55
53     IF(I.EQ.1R1.OR.I.EQ.1R2.OR.I.EQ.1R3) GO TO 54
      SIGMB(I,J)=FLOAT(NBS)*BC(J)/TPIR
      GO TO 55
54     SIGMB(I,J)=FLOAT(NBR)*BC(J)/TPIR
55     CONTINUE
12     CONTINUE
C      READ THE BLADE ANGLES AT ALL RADIAL LOCATIONS .
      IF(KPA.EQ.1) GO TO 75
C      CALCULATE THE BLADE ANGLES . IF NOT KNOWN .
75     CONTINUE
      READ(5,72) (THETG(J),J=2,MM1)
      READ(5,72) ( GAMG(J),J=2,MM1)
      READ(5,72) (THETR(J),J=2,MM1)
      READ(5,72) ( GAMR(J),J=2,MM1)
      READ(5,72) (THETS(J),J=2,MM1)
      READ(5,72) ( GAMS(J),J=2,MM1)
      READ(5,72) (ALFD1(J),J=2,MM1)
      READ(5,72) (ALFDG(J),J=2,MM1)
      READ(5,72) (BET1D(J),J=2,MM1)
      READ(5,72) (ALF1D(J),J=2,MM1)
      READ(5,72) (BET2D(J),J=2,MM1)
      READ(5,72) (ALF2D(J),J=2,MM1)
72     FORMAT (6F12,6)
      READ(5,38) (BI(K),K=1,9)
      READ(5,38) (BD(K),K=1,9)
      READ(5,34) (BN(K),K=1,16)

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      READ(5,38) (BM(K),K=1,9)
      READ(5,34) (BR(K),K=1,16)
34  FORMAT(4F18,8)
38  FORMAT(3F18,8)
      READ(5,2) (ANGC(IC),IC=1,8)
      READ(5,2) ((AI010(IC,JC),IC=1,8),JC=1,9)
2   FORMAT(8F8,3)
      READ(5,33) (SIGMC(IC),IC=1,9)
33  FORMAT(9F8,3)
3   FORMAT(4F8,3)
      READ(5,2) ((ASFN(IC,JC),IC=1,8),JC=1,9)
      READ(5,2) (FKT(IC),IC=1,7)
      READ(5,2) ((AD010(IC,JC),IC=1,8),JC=1,9)
      READ(5,2) ((ASFM(IC,JC),IC=1,8),JC=1,9)
      DO 51 IC=1,8
      DO 51 JC=1,9
51  ASFN(IC,JC)=-ASFN(IC,JC)
      READ(5,2) (TBYCG(IC),IC=1,7)
      READ(5,2) (FKTD(IC),IC=1,7)
      READ(5,33) (SIGMF(IC),IC=1,9)
      READ(5,33) ((ADD1R(IC,JC),IC=1,9),JC=1,6)
      READ(5,2) (ANGF(IC),IC=1,6)
      RETURN
      END

```

SUBROUTINE ENTEX

```

C
C
C THIS SUBROUTINE DEFINES THE ENTRY AND STARTING CONDITIONS .
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKGRS/IG,IR1,IS1,IR2,IS2,IR3,IS3,IS3P1
COMMON/BLKQ/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON/BLKRT/RPM,TURNS,CXEN
1   /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEG,KT,KWA,KWB,KWC,KPA,KPB,KPC,KVEL
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1   ,U(16,17),CV(16,17)
1   /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
1   /BLKS/S(16,17),DSDR(16,17),CM(16,17)
C
C
      READ(5,1) TOEN,POEN,RPM,TURNS
      WRITE(6,8) TOEN,POEN,RPM,TURNS
1   FORMAT(6F12,5)
      DO36 J=1,M
      RI=RIN(1)*RIN(1)
      RIO=ROUT(1)*ROUT(1)-RIN(1)*RIN(1)
      U(1,J)=100*(R(1,J)*R(1,J)-RI)/RIO
36  CONTINUE
C
C TO TOTAL TEMPRATURE IN DEGREES F .
C
C PO TOTAL PRESSURE IN INCHES OF MERCURY .
      PRESEN=POEN*13.59*62.4/12
      TEMPEN=TOEN+459.7
      HOEN=CP*TEMPEN
      RHOEN=PRESEN/AR/TEMPEN/AG
      DO 3 J=1,M
      PO(1,J)=PRESEN
      HO(1,J)=HOEN
      ROW(1,J)=RHOEN

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```

3  CONTINUE
   FC=200./((ROW(1,2)*CXEN*(ROUT(1)*ROUT(1)-RIN(1)*RIN(1)))
   CXEN= (U(1,3)-U(1,1))/(2.*G(1)*R(1,2)*ROW(1,2)*FC)
   WRITE(6,8) TEMPN,PRESN,CXEN,RHOEN,H0EN,FC
8  FORMAT(2X,7F15.5)
   DO 16 J=1,M
     DO 16 I=1,N
       ROW(I,J)=ROW(I,J)
16  CX(I,J)=CXEN
C   KXA AN INDEX FOR THE INLET VELOCITY PROFILE .
C   KXA=0.1 AND 2 UNIFORM,DEVELOPED TURBULENT AND DEVELOPED LAMINAR .
   KXA=0
   IF(KXA.EQ.0) GO TO 18
   KVEL=KXA
   CALL EXITC
   DO 17 J=1,M
     CX(I,J)=CX(N,J)
17  U(I,J)=U(N,J)
18  CONTINUE
   KVEL=1
   CALL EXITC
   DO 99 J=1,M
     DO 99 I=2,N
       CX(I,J)=CX(I,J)+(CX(N,J)-CX(I,J))*FLOAT(I-1)/FLOAT(N-1)
       U(I,J)= U(I,J)+( U(N,J)- U(I,J))*FLOAT(I-1)/FLOAT(N-1)
99  CONTINUE
   DO 98 J=1,M
     DO 98 I=2,IG
       CX(I,J)=CX(I,J)
98  U(I,J)=U(I,J)
   DO 23 I=1,N
     U(I,1)=0.
     U(I,M)=100.
   DO 23 J=2,M
     DSIDR(I,J)=100./((ROUT(1)-RIN(1))
23  CONTINUE
   DO 24 J=2,MM1
24  DSIDR(1,J)=(U(1,J+1)-U(1,J-1))/(2.*G(1))
     DSIDR(1,1)=DSIDR(1,2)*R(1,1)/R(1,2)
     DSIDR(1,M)=DSIDR(1,MM1)*R(1,M)/R(1,MM1)
     DO 4 I=2,NM1
       DSIDR(I,1)=0.
       DSIDR(I,M)=0.
4  CONTINUE
C   DETERMINE CBU . THE PERIPHERAL VELOCITY .
   DO 5 I=1,N
     DO 5 J=1,M
       B(I,J)=0.
       CBU(I,J)=2.*3.14159*R(I,J)*RPM/60.
5  CONTINUE
C   STARTING DISTRIBUTION OF THE VELOCITY COMPONENTS,DENSITY,ENTROPY
C   ETC. AT ALL GRID POINTS .
   DO 6 I=1,N
     DO 6 J=1,M
       B(I,J)=0.
       IF(I.EQ.1.OR.J.EQ.1) GO TO 6
       IF(I.EQ.N.OR.J.EQ.M) GO TO 6
       A1=R(I-1,J)-R(I,J)
       A2=G(I-1)-A1

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```

      B1=R(I,J)-R(I+1,J)
      B2=G(I+1)-B1
6     CONTINUE
      DO 9 I=2,NM1
      DO 9 J=2,MM1
      DSDR(I,J)=(U(I,J+1)-U(I,J-1))/(2.*G(I))
      UA=(U(I-1,J)*A2+U(I-1,J-1)*A1)/G(I-1)
      UB=(U(I+1,J)*B2+U(I+1,J+1)*B1)/G(I+1)
      DSDX(I,J)=(UB-UA)/(HL(I)+HR(I))
9     CONTINUE
      DO 7 I=1,N
      DO 7 J=1,M
      DSDX(I,1)=0.
      DSDX(I,M)=0.
      DSDX(1,J)=0.
      DSDX(N,J)=0.
      CU(I,J)=0.
      S(I,J)=0.5963
      DSDR(1,J)=0.
      PO(1,J)=PO(1,J)
      HO(1,J)=HO(1,J)
      IF(I.EQ.1.OR.J.EQ.1) GO TO 7
      IF(I.EQ.N.OR.J.EQ.M) GO TO 7
      ROWRFC=ROW(I,J)*R(I,J)*FC
      CX(I,J)=DSDR(I,J)/ROWRFC
      CR(I,J)=-DSDX(I,J)/ROWRFC
7     CONTINUE
      IA=IG+1
      DO 10 I=IA,IS3P1.2
      CU(I,M)=CBU(I,M)*0.15
10    CU(I,1)=CBU(I,1)*0.30
      IB=IR1+1
      IC=IR3+1
      DO 11 I=IB,IC.2
      CU(I,M)=CBU(I,1)*.75
11    CU(I,1)=CBU(I,1)*0.96
C     ENFORCE THE CONDITION THAT THE AXIAL AND THE TANGENTIAL VELOCITIES
C     AT THE WALLS ARE EACH ZERO .
      DO 22 I=1,N
      CX(I,1)=0.
      CX(I,M)=0.
      CR(I,1)=0.
      CR(I,M)=0.
22    CONTINUE
      RETURN
      END

```

SUBROUTINE EXITC

```

C     ----
C     THIS SUBROUTINE SETS UP AND IMPLEMENTS THE NECESSARY CONDITIONS
C     AT THE EXIT END OF THE COMPRESSOR .
      COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
      COMMON /FLAGS/ KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC,KVEL
1     /BLK3/ THETS(17),GAMS(17),CRSETS,CAMBS(17),ALFID(17),
1     ALF2D(17)
1     /BLK4/ RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
      COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1     ,U(16,17),CV(16,17)

```

```

COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
1 /BLKS/S(16,17),DSDR(16,17),CM(16,17)
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKRT/RPM,URNS,CXEN
COMMON/BLKQ/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON/WORKA/A(36),G(6),FDX(15),RFIT(15),FFIT(15),EXX(42)
C -----
IF(KVEL.EQ.2) GO TO 35
C KVEL=1 FULLY DEVELOPED TURBULENT FLOW .
CXMAX=CXEN/0.82
ALAMB=SQRT((1.-.49)/(2.*ALOG(1./0.7)))
RLAMB=0.6667*ALAMB
CX(N,1)=0.
CX(N,M)=0.
DO 15 J=2,MM1
IF(R(N,J).GT.RLAMB) GO TO 13
TAU=0.1*0.6667/(2.*2.)
CX(N,J)=CXMAX-2.5*SQRT(TAU/ROW(N,J))*SQRT((ALAMB*ALAMB-0.49)/.7)
1*ALOG((ALAMB-0.7)*0.6667/(R(N,J)-0.7*0.6667))
GO TO 14
13 CX(N,J)=CXMAX-2.5*SQRT(TAU/ROW(N,J))*SQRT(1.-ALAMB*ALAMB)
1*ALOG((R(N,M)-ALAMB*R(N,M))/(R(N,M)-R(N,J)))
14 CONTINUE
WRITE(6,25) R(N,J),ALAMB,CX(N,J),TAU,RLAMB
25 FORMAT(6F12.5)
15 CONTINUE
IF(KVEL.NE.2) GO TO 45
35 CONTINUE
C FULLY DEVELOPED LAMINAR VELOCITY PROFILE .
RMAX=SQRT((R(N,M)*R(N,M)-R(N,1)*R(N,1))/(2.*ALOG(R(N,M)/R(N,1))))
VISC=0.000012/AG
CXB=CXEN
DPDX=-8.*VISC*CXB/(AG*(R(N,M)*R(N,M)+R(N,1)*R(N,1)-2.*RMAX*RMAX))
DO 36 J=1,M
36 CX(N,J)=AG/(2.*VISC)*DPDX*((R(N,J)*R(N,J)-R(N,M)*R(N,M))/2.
1-RMAX*RMAX*ALOG(R(N,J)/R(N,M)))
45 CONTINUE
U(N,1)=0.
A=1.
IK=1
U(N,M)=99.
9 CONTINUE
U(N,M)=U(N,M)*A
C INTEGRATION USING THE MODIFIED TRAPEZOIDAL RULE .
DO 24 J=2,MM1
CXRM=CX(N,J-1)*R(N,J-1)
CXRP=CX(N,J+1)*R(N,J+1)
ROWFCM=ROW(N,J-1)*FC
ROWFCP=ROW(N,J+1)*FC
PM=CXRM*ROWFCM
PP=CXRP*ROWFCP
FDX(J)=(PP-PM)/(2.*G(1))
24 CONTINUE
MM2=M-2
DO 26 J=1,MM2
RFIT(J)=R(N,J+1)
26 FFIT(J)=FDX(J+1)
NP=MM2
KOR=2

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CALL POLYFT(RFIT,FFIT,NP,KOR,C,CO,A)
DO 27 J=1,M
27 FDX(J)=CO+C(1)*R(N,J)+C(2)*R(N,J)*R(N,J)
DO 21 J=1,MM1
  CXR=CX(N,J)*R(N,J)
  ROWFC=ROW(N,J)*FC
  CXRP=CX(N,J+1)*R(N,J+1)
  ROWFCP=ROW(N,J+1)*FC
  U(N,J+1)=U(N,J)+(CXR*ROWFC+CXRP*ROWFCP)/2.*G(N)
  I+G(N)*G(N)/12.*(FDX(J)-FDX(J+1))
21 CONTINUE
  IF((IFIX(U(N,M)).EQ.100) GO TO 7
  A=100./U(N,M)
  FC=FC+A
  CX(N,M)=0.
  IF(1K.EQ.3) GO TO 7
  1K=1K+1
  GO TO 9
7 WRITE(6,37) ( U(N,J),J=1,M)
  WRITE(6,37) (CX(N,J),J=1,M)
37 FORMAT(17F7,2)
  RETURN
  END

```

SUBROUTINE SCAN

```

C ----
C SUBROUTINE TO SCAN THE FIELD FOR RELAXATION OF PSI(U)
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON /BLK1T/ MAXIT,OMEGA,EPS,BIGD,BIGDM,ITN
COMMON /FLAGS/ KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC,KVEL
1 /BLK4/ RIN(16),ROUT(16),G(16),ML(16),HR(16),R(16,17)
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,U(16,17),CV(16,17)
1 /BLKM3/ HO(16,17),PO(16,17),ROW(16,17)
COMMON /BLKMOL/ DSIDR(16,17),DSIDX(16,17),FC,FMM1
COMMON /BLKGR5/ IG,IR1,IS1,IR2,IS2,IR3,IS3,IS3PI
COMMON /WORKA/ A(36),C(6),CUF(17),RFIT(17),BIG(4),EXN(464)
C -----
C MA AND MZ MARK THE RADIAL LIMITS OF THE SCANNING FIELD .
MA=2
MZ=M-1
ITN=1
C SLIP VALUES CALCULATIONS
C IT=1 FOR INNER BOUNDARY AND IT=2 FOR OUTER BOUNDARY .
VISC=0.0000123/32.2
AKK=0.4
AKKS=AKK*AKK
DO 68 IT=1,2
DO 68 I=2,N
  IF(IT.EQ.2) GO TO 65
  RM=(R(I,1)+R(I,2))*0.5
  RR=ROW(I,2)*CX(I,2)*(R(I,2)-R(I,1))/VISC
  GO TO 66
65 RM=(R(I,N)+R(I,N-1))*0.5
  RR=ROW(I,M-1)*CX(I,M-1)*(R(I,M)-R(I,M-1))/VISC
66 RST=RR*AKKS
  SST=1./RST-0.1561/(RST**0.45)+0.08723/(RST**0.3)+0.0371/(RST**0.18)
  SS=SST*AKKS

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```

FF=0.
BET=SQRT(SS+FF)/AKK
IF(I1.EQ.2) GO TO 67
UUS=1./(RM-2.*R(I,2)*(1.+BET))*((RM*U(I,3)-2.*R(I,2)*U(I,2)*
I*(1.+BET))
U(I,1)=UUS
VELS=(1.-BET)/(1.+BET)*CX(I,2)
VELM=1./(1.+BET)*CX(I,2)
CX(I,1)=VELS
DSIDR(I,1)=CX(I,1)*ROW(I,1)*R(I,1)*PC
GO TO 69
67 UUS=1./(RM-2.*R(I,M-1)*(1.+BET))*((RM*U(I,M-2)-2.*R(I,M-1)*U(I,M-1)
I*(1.+BET))
VELS=(1.-BET)/(1.+BET)*CX(I,MM1)
VELM=1./(1.+BET)*CX(I,MM1)
CX(I,M)=VELS
DSIDR(I,M)=CX(I,M)*ROW(I,M)*R(I,M)*PC
U(I,M)=UUS
69 CONTINUE
WRITE(6,78)RR,RM,RST,SS,BET,UUS,VELS,VELM
78 FORMAT(10F11.3)
68 CONTINUE
7 BIGD=0.
DO 31 KMANY=1,2
DO 31 I=2,NM1
DO 31 J=MA,MZ
CALL SUBIN
31 CONTINUE
IF(KEQ.EQ.1) GO TO 42
DO 41 J=1,M
41 U(N,J)=U(N-1,J)
42 CONTINUE
BIG(1)=BIGD
BIGD=0.
DO 32 J=MA,MZ
DO 32 I=2,NM1
CALL SUBIN
32 CONTINUE
IF(KEQ.EQ.1) GO TO 52
DO 51 J=1,M
51 U(N,J)=U(N-1,J)
52 CONTINUE
BIG(2)=BIGD
BIGD=0.
BIGDM=0.
DO 33 JB=MA,MZ
DO 33 I=2,NM1
J=MP1-JB
CALL SUBIN
33 CONTINUE
DO 45 J=MA,MZ
I=4
CALL SUBIN
45 CONTINUE
IF(KEQ.EQ.1) GO TO 62
DO 61 J=1,M
61 U(N,J)=U(N-1,J)
62 CONTINUE
BIG(3)=BIGD

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```

DO 34 K=1,3
  IF(BIG(K).LT.BIGDM) GO TO 34
  KMAX=K
  BIGDM=BIG(K)
34 CONTINUE
  DO 35 I=2,MM1
    DO 35 J=2,MM1
      A1=R(I-1,J)-R(I,J)
      A2=G(I-1)-A1
      B1=R(I,J)-R(I+1,J)
      B2=G(I+1)-B1
      UA=(U(I-1,J)*A2+U(I-1,J-1)*A1)/G(I-1)
      UB=(U(I+1,J)*B2+U(I+1,J+1)*B1)/G(I+1)
      DSIDR(I,J)=(U(I,J+1)-U(I,J-1))/(2.*G(I))
35 DSIDX(I,J)=(UB-UA)/(HL(I)+HR(I))
      DO 303 J=1,M
        DSIDR(I,J)=DSIDR(2,J)
        DSIDX(I,J)=DSIDX(2,J)
        DSIDX(N,J)=DSIDX(N-1,J)
303 DSIDR(N,J)=DSIDR(N-1,J)
      WRITE (6,27) BIGDM,KMAX
27 FORMAT(F9.3 ,14)
      IF(ITN=10) 25,20,25
20 WRITE(6,21)
21 FORMAT(1H1)
      WRITE(6,28) ((U(I,J),I=1,N),J=1,M)
      WRITE (6,29)
29 FORMAT(1H0)
28 FORMAT(1H0,16F7.2)
      WRITE (6,24) ITN,MAXIT,BIGDM,OMEGA,N,M
24 FORMAT(2I4,2F8.2,2I4)
25 ITN=ITN+1
      IF(EPS.GT.BIGDM) GO TO 26
      IF(ITN=MAXIT) 7,26,26
26 WRITE (6,21)
      WRITE(6,28) ((U(I,J),I=1,N),J=1,M)
      WRITE (6,29)
      WRITE (6,29)
      WRITE (6,24) ITN,MAXIT,BIGDM,OMEGA,N,M
      WRITE(6,56)
      WRITE(6,28)((DSIDR(I,J),I=1,N ),J=1,M)
      WRITE(6,56)
      WRITE(6,28)((DSIDX(I,J),I=1,N ),J=1,M)
      DO 9 I=1,N
        DO 9 J=1,M
          RRFC=ROW(I,J)*R(I,J)*FC
          CX(I,J)=DSIDR(I,J)/RRFC
          CR(I,J)=DSIDX(I,J)/RRFC
9 CONTINUE
      WRITE(6,56)
      WRITE(6,71)
71 FORMAT(10X,27HAXIAL VELOCITY DISTRIBUTION)
      WRITE(6,28)((CX(I,J),I=1,N ),J=1,M)
      WRITE(6,56)
      WRITE(6,72)
72 FORMAT(10X,28HRAIAL VELOCITY DISTRIBUTION)
      WRITE(6,28)((CR(I,J),I=1,N ),J=1,M)
      DO 82 I=1G,153P1
      DO 81 J=2,MM1

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      CUF(J)=CU(I,J)
81  RFIY(J)=R(I,J)
      KOR=3
      NP=M-2
      CALL POLYFT(RFIT,CUF,NP,KOR,C,CO,A)
      DO 82 J=1,M
      CUI(I,J)=CO+C(1)*R(I,J)+C(2)*R(I,J)*R(I,J)+C(3)*R(I,J)**3
82  CONTINUE
      DO 75 I=IS3P1,MM1
      DO 75 J=1,M
75  CUI(I+1,J)=CUI(I,J)
      WRITE(6,56)
      WRITE(6,28)((CUI(I,J),I=1,N),J=1,M)
56  FORMAT(1H0)
      RETURN
      END

```

SUBROUTINE SUBIN

```

C -----
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKIT/MAXIT,OMEGA,EPS,BIGD,BIGDM,ITN
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC
1  /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1  ,U(16,17),CV(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
C -----
      RGF=1./(2.*R(I,J)*G(I))
      HL2=HL(I)*HL(I)
      HR2=HR(I)*HR(I)
      HLR2=1./(HL2+HR2)
      HM=2.*HLR2
      HF=(HL(I)-HR(I))/(HL(I)+HR(I))
      GM=1./(G(I)*G(I))
      UNEW=(1.-OMEGA)*U(I,J)+(OMEGA/(2.*GM+4.*HLR2))*(HM*(1.+HF)*
1  (U(I-1,J)-(R(I-1,J)-R(I,J))*DSIDR(I-1,J))+HM*(1.-HF)*(U(I+1,J)
1  +(R(I,J)-R(I+1,J))*DSIDR(I+1,J)))+(GM-RGF)*U(I,J+1)+(GM+RGF)*
1  U(I,J-1)-B(I,J))
      DSIDR(I,J)=(U(I,J+1)-U(I,J-1))/(2.*G(I))
      RESID=ABS(UNEW-U(I,J))
      IF (RESID-BIGD)11,11,10
10  BIGD=RESID
11  U(I,J)=UNEW
      CONTINUE
      RETURN
      END

```

SUBROUTINE SIMPLE

```

C -----
C THIS SUBROUTINE IS MEANT TO PROPAGATE THE EFFECT OF UPDATING THE
C STREAMFUNCTION IN THE BLADED REGION TO THE BLADELESS REGIONS .
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKGRS/IG,IR1,IS1,IR2,IS2,IR3,IS3,IS3P1
COMMON/BLKG/OMEGA,ANG2,SIGMC,ATNC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1  /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1  ,U(16,17),CV(16,17)

```

```

1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
1 /BLKS/S(16,17),DSDR(16,17),CM(16,17)

```

```

C -----
4 WRITE(6,4) 1
  FORMAT(1H0,17HSIMPLE WORKING,1*,13)
  CV(1,J)=SQRT(CX(1,J)*CX(1,J)+CR(1,J)*CR(1,J))
  K=1+1
  DO 5 J=2,MM1
    ROWRFC=ROW(K,J)*R(K,J)*FC
    CX(K,J)=DSIDR(K,J)/ROWRFC
    CR(K,J)=-DSIDX(K,J)/ROWRFC
    CU(K,J)=CU(K-1,J)
    PO(K,J)=PO(1,J)
    HO(K,J)=HO(1,J)
    TTN=HO(K,J)/CP
    CV(K,J)=SQRT(CX(K,J)*CX(K,J)+CR(K,J)*CR(K,J)+CU(K,J)*CU(K,J))
    CM(K,J)=SQRT(CV(K,J)*CV(K,J)/(GAMA*AG*AR*TTN))
    RGF=1./(2.*R(K,J)*G(K))
    HL2=HL(K)*HL(K)
    HR2=HR(K)*HR(K)
    HLR2=1./(HL2+HR2)
    HM=2.*HLR2
    HF=(HL(K)-HR(K))/(HL(K)+HR(K))
    GM=1./(G(K)*G(K))
    B(1+1,J)=(-(2.*GM+4.*HLR2)*U(K,J)+HM*(U(K-1,J)+U(K+1,J))
    + (GM+RGF)*U(K,J-1)+(GM-RGF)*U(K,J+1))
5 CONTINUE
7 FORMAT(6F15,5)
  WRITE(6,7) PO(K,3),HO(K,3),CV(K,3),TTN,CM(K,3)
  WRITE(6,8) (B(1+1,J),J=2,MM1)
8 FORMAT (15F8,0)
  RETURN
END

```

SUBROUTINE CALBS

```

C -----
C CALCULATION OF B-TERM ACROSS A STATOR BLADE-ROW .
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKGRS/IG,IR1,IS1,IR2,IS2,IR3,IS3,IS3P1
COMMON/BLKG/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1 /BLKAS/ANG1,SIGMI,THET1
COMMON/FLAG/KPI,PF,KCOMP,KANNU,KEG,KT,KWA,KWB,KWC,KPA,KPB,KPC,KVEL
COMMON/BLK1/THETG(17),GAMG(17),CRSETG,CAMBG(17),ALFDG(17),
1ALFD1(17)
1 /BLK3/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALFID(17),
1ALF2D(17)
1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1 /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,U(16,17),CV(16,17)
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
COMMON/BLKT/TTN(16,17),EMEWR(16,17),EMEWRX(16,17),ROTERM(16,17)
COMMON/BLKSIG/SIGMB(16,17)
1 /BLKS/S(16,17),DSDR(16,17),CM(16,17)
1 /BLKDH/DHODR(16,17),CUBYR(16,17),DDGRU(16,17)
COMMON/BLKDEV/REF1(17),REFD(17),ADEV(17)

```

```

C -----
      IF (KPC.LT.KWA) WRITE(6,16)
16   FORMAT(1H1)
      WRITE(6,66) I
66   FORMAT(1H0,5X,45HTHE FOLLOWING IS WRITTEN FROM SUB. KCALBS .I=.13)
C   ASSUME THE VALUE I AS IN THE CALLING SUBROUTINE .
      DO 22 J=2,MMI
      IF(1.NE.16) GO TO 12
      CR(I,J)=0.
      CU(I,J)=0.
      CV(I,J)=CX(I,J)
      HO(I,J)=HO(I-1,J)
      TTP=HO(I,J)/CP
      CM(I,J)=SQRT(CV(I,J)*CV(I,J)/(AG*AR*GAMA*TTP))
      TEMP1=ALF1D(J)
      ALF1D(J)=ALFD1(J)
      TEMP2=ALF2D(J)
      ALF2D(J)=ALFDG(J)
      ALF1(J)=0.
      AINC=ALF1D(J)
      TEMP3=THETS(J)
      THETS(J)=THETG(J)
12   CONTINUE
      ALF1(J)=180./3.14159*ATAN(CU(I,J)/CX(I,J))
      TTP=HO(I,J)/CP
      CMSQ = (CX(I,J)*CX(I,J)+CU(I,J)*CU(I,J)+CR(I,J)*CR(I,J))/
      I(GAMA*AG * AR *TTP)
      CM(I,J)=SQRT(ABS(CMSQ))
      AINC=ALF1(J)-ALF1D(J)
      CV(I,J)=SQRT(CX(I,J)*CX(I,J)+CU(I,J)*CU(I,J)+CR(I,J)*CR(I,J))
      CALL      CORST
      CALL      CORSD
      CALL      IDCS
      ROWREC=ROW(I+1,J)*R(I+1,J)*FC
      CX(I+1,J)=DSIDR(I+1,J)/ROWREC
      CR(I+1,J)=-DSIDX(I+1,J)/ROWREC
      TEMP=3.14159/180.*ALF2(J)
      CU(I+1,J)=CX(I+1,J)*TAN(TEMP)
      K=I+1
      HO(K,J)=HO(I,J)
      PO(K,J)=PO(I,J)
      DSIDR(K,J)=(U(K,J+1)-U(K,J-1))/(2.*G(K))
      CV(K,J)=SQRT(CX(K,J)*CX(K,J)+CR(K,J)*CR(K,J)+CU(K,J)*CU(K,J))
      TTN=HO(I+1,J)/CP
      CMSQ = (CX(K,J)*CX(K,J)+CU(K,J)*CU(K,J)+CR(K,J)*CR(K,J))/
      I(GAMA*AG * AR *TTN)
      CM(K,J)=SQRT(ABS(CMSQ))
      IF (KPC.LT.KWA) WRITE(6,4) CM(I,J),CBU(I,J),CX(I,J),CX(I+1,J),
      ICU(I,J),CU(I+1,J),HO(I,J),HO(I+1,J),BBS
      ANG1=ALF1(J)
      ANG2=ALF2(J)
      SIGMC=SIGMB(I,J)
      IF(1.NE.16) GO TO 42
      ALF1D(J)=TEMP1
      ALF2D(J)=TEMP2
      THETS(J)=TEMP3
42   CONTINUE
      CUBYR(K,J)=CU(K,J)/R(K,J)
      DHODR(I+1,J)=(HO(I+1,J)-HO(I+1,J-1))/G(I+1)

```

```

DDCRU(I+1,J)=(R(I+1,J)*CU(I+1,J)-R(I+1,J-1)*CU(I+1,J-1))/G(I+1)
CALL      DEGOMG
CALL      ROWENT
CALL ROTERB
RORFCS=ROW(I+1,J)*ROW(I+1,J)*R(I+1,J)*R(I+1,J)*FC*FC
FF=RORFCS/DSIDR(I+1,J)
BBU=FF*CUBYR(I+1,J)*DDCRU(I+1,J)
BBH=-FF*AG*AJ*DHODR(I+1,J)
BBS=FF*AG*AJ*TSN*DSDR(I+1,J)
B(I+1,J)=BBU+BBH+TTERM(I+1,J)+ROTERM(I+1,J)
IF(KWB.EQ.1) GO TO 22
WRITE(6,5) B(I+1,J),FF,BBU,BBH,BBS,TTERM(I+1,J),ROTERM(I+1,J),
1 DSDR(I+1,J),CUBYR(I+1,J),DDCRU(I+1,J),DHODR(I+1,J)
4  FORMAT(1X,8F10.3,3F13.7)
5  FORMAT(1X,6F10.3,2F13.7,3F11.4)
22  CONTINUE
CALL      SSSS
DO 31 J=2,MM1
TSN=HO(K,J)/CP-CV(K,J)*CV(K,J)/(2.*AG*AJ*CP)
BBS=FF*AG*AJ*TSN*DSDR(I+1,J)
31  B(I+1,J)=B(I+1,J)+BBS
10  FORMAT(23F5.2)
TEST=ABS(B(K,3))*1.2
IF(ABS(B(K,2)).GT.TEST) B(K,2)=B(K,3)
RETURN
END

```

SUBROUTINE CORST

C

```

EXTERNAL DBLK
COMMON /BLK1/ I,J,N,M,NM1,MP1,MM1,MP1
COMMON/BLK2/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1  /BLK3/ANG1,SIGM1,THET1
1  /BLK4/YI0,IOC,YSFNC
COMMON/BLK5/KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC
1  /BLK6/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALFID(17),
1 ALF2D(17)
1  /BLK7/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1  /BLK8/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON/BLK9/SIGMB(16,17)
COMMON/BLK10/REFI(17),REFD(17),ADEV(17)

```

C

```

KCALL=1
20  REFI(J)=0.0
THET1=THETS(J)
11  ANGLE=ALFID(J)+REFI(J)
ANG1=ANGLE
SIGM1=SIGMB(1,J)
CALL      CALGR1(DBLK)
C  CONTROL THE REFERENCE INCIDENCE ITERATION TO ONE .
IF(KCALL.GT.KPB) GO TO 16
IF(ABS(REFI(J)-REFIN).GT.0.5) GO TO 12
KCALL=KCALL+1
16  REFI(J)=REFIN
ANGLE=ALFID(J)+REFI(J)
GO TO 19
12  REFI(J)=REFIN
GO TO 11

```

```

19  CONTINUE
C   APPLY LIEBLEIN'S SIMPLE 3DIM CORRECTION TO CORRELATION.
    REFI(J)=REFI(J)+1.10
    ANGLE=ANGLE+1.1
15  CONTINUE
    RETURN
    END

```

SUBROUTINE CALGRI(DBLK)

```

C   ----
EXTERNAL DBLK
COMMON/BLKC/ANGC(8),SIGMC(10),TBYCG(8)
COMMON/BLKQ/OMEGAD,ANG2,SIGMK,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON/BLKGI/A1010(8,9),ASFN(8,9),FKT(8)
1   /BLKAS/ANG1,SIGM1,THET1
1   /BLKGR1/Y1010C,YSFNC
COMMON/BLKY/YFK1,FKSH,YFKD
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
COMMON/WORKA/A(36),C(6),EXTRAH(502)

```

```

C   ----
    K=1
    Y1010C=DBLK(ANG1,SIGM1,K)
    K=2
    YSFNC=DBLK(ANG1,SIGM1,K)
    TBYCC=0.10
    FKSH=1.10
C   FOR SAVING TIME IN DEALING WITH BLADES OF CONSTANT THICKNESS
C   TO CHORD RATIO RADIALY, BYPASS THE ROUTE CALLING POLYFT .
    KPASS=1
    IF(KPASS.EQ.1.AND.J.GT.2) GO TO 9
C   LET THE ORDER OF FITTING POLYMONIAL BE KOR =2
    KOR =2
    NP=7
    CALL POLYFT(TBYCG,FKT,NP,KOR,C,CO,A)
    YFK1=CO +C(1)*TBYCC+C(2)*TBYCC*TBYCC
9   CONTINUE
    THETC=THET1
    REFIN=Y1010C*YFK1*FKSH+YSFNC*THETC
    RETURN
    END

```

SUBROUTINE CORSD

```

C   ----
EXTERNAL DBLK
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
1   /BLKGRD/YD010C,YSEMC,REFDN
1   /BLKAS/ANG1,SIGM1,THET1
1   /BLK3/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALF1D(17),
1ALF2D(17)
1   /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1   /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON/BLKSIG/SIGMB(16,17)
COMMON/BLKDEV/REFI(17),REFD(17),ADEV(17)
C   ----
    ANG1=ANGLE
    THET1=THETS(J)
    SIGM1=SIGMB(1,J)

```

```

CALL      CALGRD(DBLK)
REFD(J)=REFDN
C  APPLY LIEBLEIN'S SIMPLE 3DIM CORRECTION TO CORRELATION.
REFD(J)=REFD(J)+0.63
15 CONTINUE
RETURN
END

```

```

SUBROUTINE CALGRD(DBLK)
C  -----
EXTERNAL DBLK
COMMON/BLKC/ANGC(8),SIGMC(10),TBYCG(8)
COMMON/BLKGD/AD010(8,9),ASEM(8,9),FKTD(8)
1  /BLKAS/ANG1,SIGM1,THET1
2  /BLKGRD/YD010C,YSEMC,REFDN
COMMON/BLKY/YFK1,FKSH,YFKD
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
COMMON/WORKA/A(36),C(6),EXTRA1(502)
C  -----
K=3
YD010C=DBLK(ANG1,SIGM1,K)
K=4
YSEMC=DBLK(ANG1,SIGM1,K)
TBYCC=0.10
FKSH=1.10
C  FOR SAVING TIME IN DEALING WITH BLADES OF CONSTANT THICKNESS
C  TO CHORD RATIO RADIALY, BYPASS THE ROUTE CALLING POLYFT.
KPASS=1
IF(KPASS.EQ.1.AND.J.GT.2) GO TO 9
C  LET THE ORDER OF FITTING POLYMONIAL BE KOR.
KOR =2
NP=7
CALL      POLYFT(TBYCG,FKTD,NP,KOR,C,C0,A)
YFKD=C0+C(1)*TBYCC+C(2)*TBYCC*TBYCC
9  CONTINUE
THETC=THET1
REFDN=YD010C*YFKD*FKSH+YSEMC*THETC
RETURN
END

```

```

SUBROUTINE IDC5
C  -----
EXTERNAL DBLK
COMMON/BLKQ/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
1  /BLKPID/YDDIRC
1  /BLKAS/ANG1,SIGM1,THET1
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC
1  /BLK3/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALF1D(17),
1ALF2D(17)
1  /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON/BLKSIG/SIGMB(16,17)
COMMON/BLKDEV/REF1(17),REFD(17),ADEV(17)
C  -----
ANG1=ANGLE
SIGM1=SIGMB(1,J)
K=5

```

```

YDDIRC=DBLK(SIGMC,ANG1,K)
C PREDICT THE OFF-DESIGN INCIDENCE-DEVIATION RELATION.
IF(KPC.GE.KWA) GO TO 51
WRITE(6,8)
8 FORMAT(77H AINC ADEV ALF1 ALF2 ANGLE R
1EF1 REFD J)
ADEV(J)=REFD(J)+(AINC-REF1(J))*YDDIRC
51 CONTINUE
IF(I.GT.4) GO TO 52
C FOR THE GUIDE VANES - CALCULATE ALF2(J)
ALF2(J)=ALF2D(J)-ADEV(J)
GO TO 55
52 ALF2(J)=ALF2D(J)+ADEV(J)
55 CONTINUE
IF(KPC.GE.KWA) RETURN
WRITE(6,53)AINC,ADEV(J),ALF1(J),ALF2(J),ANGLE,REF1(J),REFD(J),J
53 FORMAT(7(F7.2,3X),17)
RETURN
END

```

SUBROUTINE CALBR

```

C ----
C CALCULATION OF B-TERM ACROSS A ROTOR BLADE ROW .
EXTERNAL PWALL
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLK2/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1 /BLK3/ANG1,SIGM1,THET1
COMMON/BLK4/KPIPE,KCOMP,KANNU,KEG,KT,KWA,KWB,KWC,KPA,KPB,KPC,KVEL
COMMON/BLK5/THETG(17),GAMG(17),CRSETG,CAMBG(17),ALFDG(17),
1ALFD(17)
1 /BLK6/THETR(17),GAMR(17),CRSETR,CAMBR(17),BET1D(17),
1BET2D(17)
1 /BLK7/THETS(17),GAMS(17),CRSETS,CAMBS(17),ALF1D(17),
1ALF2D(17)
1 /BLK8/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1 /BLK9/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
COMMON /BLK10/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,UI(16,17),CV(16,17)
1 /BLK11/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLK12/DSIDR(16,17),DSIDX(16,17),FC,FMM1
COMMON/BLK13/TTERM(16,17),FMEWR(16,17),FMEWRX(16,17),ROTERM(16,17)
COMMON/BLK14/SIGMB(16,17)
1 /BLK15/S(16,17),DSDR(16,17),CM(16,17)
1 /BLK16/DHODR(16,17),CUBYR(16,17),DDCRU(16,17)
COMMON/BLK17/REF1(17),REFD(17),ADEV(17)
C ----
IF(KPC.LT.KWA) WRITE(6,16)
16 FORMAT(1H1)
WRITE(6,66) I
66 FORMAT(1H0,5X,45H THE FOLLOWING IS WRITTEN FROM SUB. KCALBR ,I=,13)
K=I+1
DO 3 J=2,MM1
BET1(J)=180./3.14159*ATAN((CBU(I,J)-CU(I,J))/CX(I,J))
AINC=BET1(J)-BET1D(J)
CV(I,J)=SQRT(CX(I,J)*CX(I,J)+CU(I,J)*CU(I,J)+CR(I,J)*CR(I,J))
TTP=HO(I,J)/CP
CM(I,J)=SQRT(CV(I,J)*CV(I,J)/(GAMA*AG*AR*TTP))
CALL THE SUB THAT CALCULATE THE ROTOR OUTLET CONDITIONS .

```

```

CALL      CORRI
CALL      CORR0
CALL      IDCR
ROWREC=ROW(I+1,J)*R(I+1,J)*EC
CX(I+1,J)=DSIDR(I+1,J)/ROWREC
CR(I+1,J)=-DSIDX(I+1,J)/ROWREC
TEMP=BET2(J)*3.14159/180.
CU(I+1,J)=-CX(I+1,J)*TAN(TEMP)+CBU(I+1,J)
ALF1(J)=180./3.14159*ATAN(CU(I+1,J)/CX(I+1,J))
CV(I+1,J)=SQRT(CX(I+1,J)*CX(I+1,J)+CU(I+1,J)*CU(I+1,J)+CR(I+1,J)*
ICR(I+1,J))
HO(I+1,J)=HO(I,J)+(CBU(I+1,J)*CU(I+1,J)-CBU(I,J)*CU(I,J))
1/(AG*AJ)
TTN=HO(I+1,J)/CP
CM(I+1,J)=SQRT(CV(I+1,J)*CV(I+1,J)/(GAMA*AG*AR*TTN))
IF(KPG.LT.KWA) WRITE(6,4) CM(I,J),CBU(I,J),CX(I,J),CX(I+1,J),
ICU(I,J),CU(I+1,J),HO(I,J),HO(I+1,J),BBS
DHODR(I+1,J)=(HO(I+1,J)-HO(I+1,J-1))/G(I+1)
DSIDR(I+1,J)=(U(I+1,J+1)-U(I+1,J-1))/(2.*G(I+1))
CUBYR(I+1,J)=CU(I+1,J)/R(I+1,J)
DDCRU(I+1,J)=(R(I+1,J)*CU(I+1,J)-R(I+1,J-1)*CU(I+1,J-1))/G(I+1)
ANG1=BET1(J)
ANG2=BET2(J)
SIGMC=SIGMB(I,J)
CALL      DEQOMG
CALL      ROWENT
CALL ROTERB
RORFCS=ROW(I+1,J)*ROW(I+1,J)*R(I+1,J)*R(I+1,J)*EC*EC
FF=RORFCS/DSIDR(I+1,J)
BBU=FF*CUBYR(I+1,J)*DDCRU(I+1,J)
BBH=-FF*AG*AJ*DHODR(I+1,J)
BBS=FF*AG*AJ*TSN*DSDR(I+1,J)
B(I+1,J)=BBU+BBH*TTERM(I+1,J)+ROTERM(I+1,J)
IF(KWB.EQ.1) GO TO 3
4  FORMAT(1X,8F10.3,3F13.7)
   WRITE(6,5) B(I+1,J),FF,BBU,BBH,BBS,TTERM(I+1,J),ROTERM(I+1,J),
IDSDR(I+1,J),CUBYR(I+1,J),DDCRU(I+1,J),DHODR(I+1,J)
5  FORMAT(1X,6F10.3,2F13.7,3F11.4)
3  CONTINUE
CALL SSSS
DO 31 J=2,MM1
TSN=HO(I+1,J)/CP-CV(I+1,J)*GV(I+1,J)/(2.*AG*AJ*CP)
BBS=FF*AG*AJ*TSN*DSDR(I+1,J)
31 B(I+1,J)=B(I+1,J)+BBS
B(K,2)=PWALL(RA,PA,RB,PB,RC,PC)
TEST=ABS(B(K,3))*1.2
IF(ABS(B(K,2)).GT.TEST) B(K,2)=B(K,3)
MA=M-1
MB=M-2
MC=M-3
MD=M-4
RA=R(K,MA)-R(K,MB)
RB=R(K,MA)-R(K,MC)
RC=R(K,MA)-R(K,MD)
PA=B(K,MB)
PB=B(K,MC)
PC=B(K,MD)
B(K,MA)=PWALL(RA,PA,RB,PB,RC,PC)
RETURN

```

END

SUBROUTINE CORR1

EXTERNAL DBLK

COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1

COMMON/BLK0/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA

1 /BLK2/THETR(17),GAMR(17),CRSETR,CAMBR(17),BETID(17),

1BET2D(17)

1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)

1 /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE

COMMON/BLK5IG/SIGMB(16,17)

COMMON/BLKDEV/REF1(17),REED(17),ADEV(17)

1 /BLKAS/ANG1,SIGM1,THET1

1 /BLKGR1/Y10,OC,YSFNC

COMMON/FLAGS/KR1DE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC

KCALL=1

20 REF1(J)=0.0

11 ANGLE=BETID(J)+REF1(J)

THET1=THETR(J)

ANG1=ANGLE

SIGM1=SIGMB(1,J)

CALL CALGR1(DBLK)

C CONTROL THE REFERENCE INCIDENCE ITERATION TO ONTWO .

IF(KCALL.GT.KPB) GO TO 16

IF(ABS(REF1(J)-REFIN).GT.0.5) GO TO 12

KCALL=KCALL+1

16 REF1(J)=REFIN

ANGLE=BETID(J)+REF1(J)

GO TO 19

12 REF1(J)=REFIN

GO TO 11

19 CONTINUE

C APPLY LIEBLEIN'S SIMPLE 3DIM CORRECTION TO CORRELATION.

REF1(J)=REF1(J)+1.10

ANGLE=ANGLE+1.1

RETURN

END

SUBROUTINE CORR2

EXTERNAL DBLK

COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1

1 /BLKAS/ANG1,SIGM1,THET1

1 /BLKGRD/YD0,OC,YSFMC,REFDN

1 /BLK2/THETR(17),GAMR(17),CRSETR,CAMBR(17),BETID(17),

1BET2D(17)

1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)

1 /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE

COMMON/BLK5IG/SIGMB(16,17)

COMMON/BLKDEV/REF1(17),REED(17),ADEV(17)

ANG1=ANGLE

THET1=THETR(J)

SIGM1=SIGMB(1,J)

CALL CALGRD(DBLK)

```

      REFD(J)=REFDN
C     APPLY LIEBLEIN'S SIMPLE 3D IM CORRECTION TO CORRELATION.
      REFD(J)=REFD(J)+0.63
15    CONTINUE
      RETURN
      END

```

SUBROUTINE IDCR

```

C     -----
      EXTERNAL DBLK
      COMMON/BLK0/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
      COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
1     /BLKAS/ANG1,SIGM1,THET1
      COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEQ,KT,KWA,KWB,KWC,KPA,KPB,KPC
1     /BLK2/THETR(17),GAMR(17),CRSETR,GAMBR(17),BET1D(17),
      IBET2D(17)
1     /BLK6/ALF1(17),ALF2(17),BET1(17),BET2(17),ANGLE
      COMMON/BLK5IG/SIGMB(16,17)
      COMMON/BLKDEV/REFI(17),REFD(17),ADEV(17)

```

```

C     -----
      ANG1=ANGLE
      SIGM1=SIGMB(1,J)
      K=5
      YDDIRC=DBLK(SIGMC,ANG1,K)
C     PREDICT THE OFF-DESIGN INCIDENCE-DEVIATION RELATION.
      IF(KPC,GE,KWA) GO TO 51
      WRITE (6,8)
8     FORMAT (76H      AINC      ADEV      BET1      BET2      ANGLE      RE
1     IF1      REFD      J)
51    CONTINUE
      ADEV(J)=REFD(J)+(AINC-REFI(J))*YDDIRC
      BET2(J)=BET2D(J)+ADEV(J)
43    CONTINUE
      IF(KPC,GE,KWA) RETURN
      WRITE(6,53)AINC,ADEV(J),BET1(J),BET2(J),ANGLE,REFI(J),REFD(J),J
53    FORMAT (7(F7.2,3X),17)
      RETURN
      END

```

SUBROUTINE CALMR

```

C     -----
      COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
1     /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1     /BLK5/BC(17),BH(17),BASPR(17),BLER,BTER,BFR,TIPCLR,TBYC(17)
      COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1     ,U(16,17),CV(16,17)
1     /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
      COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
      COMMON/BLKT/TTERM(16,17),EMEWR(16,17),EMEWRX(16,17),ROTERM(16,17)
      COMMON/BLKRT/RPM,TURNS,CXEN
      COMMON/WORKA/ PML(16,17),EXTRAB(272)
C     -----
      WRITE (6,66)
66    FORMAT(1H0,5X,41HTHE FOLLOWING IS WRITTEN FROM SUB. KCALMR)
      BARO=30.
      TF=70.
      FLOWF=CXEN/CBU(3,9)

```

```

VISCO=0.00000038
JM=(M+1)/2
IF(FLOWF.LT.0.55) GO TO 11
A=70.
E=0.00135
C=25.
D=0.00115
TH=50.-(FLOWF-0.55)*30.
GO TO 12
11 A=140.
E=0.00270
C=50.
D=0.0023
TH=50.+(0.55-FLOWF)*30.
12 TH=TH*3.14159/180.
W1=CX(1,9)/COS(TH)
REY=W1*BC(JM)*ROW(2,JM)/VISCO
WRITE(6,54) B1,W1,REY,FLOWF,TH,CXEN
VIS1=55.+A*(FLOWF-0.55)+E*(REY-60000.)
VISM=15.+C*(FLOWF-0.55)+D*(REY-60000.)
IF(VISM.LT.0.) VISM=1.
VIS2=1.5*VIS1
DO 19 I=2,NM1
DO 19 J=2,MM1
DISIN=R(1,J)-R(1,1)
IF(DISIN.GT.BH(1)/6.) GO TO 13
EMEWR(1,J)=0.00000038*6.*VIS1/BH(1)*DISIN
GO TO 18
13 IF(DISIN.GT.BH(1)/2.) GO TO 14
EMEWR(1,J)=0.00000038*(VIS1-3./BH(1))*(VIS1-VISM)*(DISIN-BH(1)/6.)
GO TO 18
14 IF(DISIN.GT.BH(1)*5./6.) GO TO 15
EMEWR(1,J)=0.00000038*(VISM+3./BH(1))*(VIS2-VISM)*(DISIN-BH(1)/2.)
GO TO 18
15 EMEWR(1,J)=0.00000038*(VIS2-6.*VIS2/BH(1))*(DISIN-5.*BH(1)/6.)
18 CONTINUE
IF(1.EQ.3) WRITE(6,54) R(1,J),DISIN,BH(1)
54 FORMAT(9F13,5)
19 CONTINUE
WRITE(6,54) VIS1,VISM,VIS2,A,E,C,D,FLOWF,REY
WRITE(6,55)
55 FORMAT(1H0)
DO 10 J=2,MM1
EMEWR(5,J)=EMEWR(5,J)/2.
DO 7 I=2,4
7 EMEWR(1,J)=EMEWR(5,J)*FLOAT(I-1)/4.
DO 8 I=7,11
8 EMEWR(1,J)=EMEWR(1,J)*FLOAT(I-6)/FLOAT(11-6)*0.5+EMEWR(1,J)
DO 9 I=12,N
9 EMEWR(1,J)=EMEWR(11,J)
10 CONTINUE
WRITE(6,6) ((EMEWR(1,J),I=2,NM1),J=2,MM1)
6 FORMAT(7F15,8)
WRITE(6,55)
RETURN
END

```

```

C -----
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
COMMON /BLK2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,U(16,17),CV(16,17)
1 /BLK3/HO(16,17),PO(16,17),ROW(16,17)
COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
COMMON/BLKT/ITERM(16,17),EMEWR(16,17),EMEWRX(16,17),ROTERM(16,17)
COMMON/WORKA/ AIN(16,17),BIN(16,17)

```

```

C -----
C ASSUMING AT THIS STAGE THAT EMEWR=EMEWRX .
CALL CALMR
DO 20 I=2,NM1
DO 20 J=2,MM1
EMEWRX(I,J)=EMEWR(I,J)
D2URX=(DSIDX(I,J+1)-DSIDX(I,J-1))/(2.*G(I))
RSQI=1./(R(I,J)*R(I,J))
AIN(I,J)=(D2URX/R(I,J)-DSIDR(I,J)*RSQI)*EMEWR(I,J)*R(I,J)
BIN(I,J)=(D2URX/R(I,J)-DSIDR(I,J)*RSQI)*EMEWRX(I,J)
20 CONTINUE
WRITE(6,19) AIN(3,4),BIN(3,4),DSIDR(3,4),DSIDX(3,4)
19 FORMAT(4F15,4)
C FOLLOWING IS A SIMPLE APPROXIMATION .
DO 21 J=1,M
AIN(1,J)=AIN(2,J)
AIN(N,J)=AIN(NM1,J)
BIN(1,J)=BIN(2,J)
BIN(N,J)=BIN(NM1,J)
21 CONTINUE
DO 22 I=1,N
AIN(I,1)=AIN(I,2)
AIN(I,M)=AIN(I,MM1)
BIN(I,1)=BIN(I,2)
BIN(I,M)=BIN(I,MM1)
22 CONTINUE
DO 23 I=2,NM1
DO 23 J=2,MM1
OUT=ROW(I,J)*FC*R(I,J)*R(I,J)/DSIDR(I,J)
ITERM(I,J)=OUT*((AIN(I,J+1)-AIN(I,J-1))/G(I)/R(I,J)+(BIN(I+1,J)-
1BIN(I-1,J))/(HL(I)+HR(I)))
23 CONTINUE
WRITE(6,24) ((ITERM(I,J),I=2,NM1),J=2,MM1)
24 FORMAT(7F15,8)
WRITE(6,55)
55 FORMAT(1H0)
RETURN
END

```

SUBROUTINE DEQOMG

```

C -----
C CALCULATION OF THE EQUIVALENT DIFFUSION PARAMETER , MOMENTUM
C THICKNESS TO CHORD RATIO AND OMEGAD , THE LOSS PARAMETER .
COMMON/BLKG/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1 /BLKAS/ANG1,SIGM1,THET1
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEG,KT,KWA,KWB,KWC,KPA,KPB,KPC
1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1 /BLK5/S(16,17),DSDR(16,17),CM(16,17)

```

```
COMMON/BLKDEV/REFI(17),REFD(17),ADEV(17)
COMMON/WORKA/A(36),C(6),EXTRAJ(502)
DIMENSION RDEQ(7),RMTBYC(7),OMMM( 9),AMACH( 9)
```

```
C -----
C ALL THE CURVES AND THE CONSTANTS FROM LIEBLEIN'S PAPER RM E57A28 .
DATARDEQ/1.2,1.4,1.6,1.8,2.0,2.2,2.4/
DATARMTRYC/0.005,0.007,0.009,0.011,0.017,0.03,0.04/
DATAOMMM/1.,1.01,1.02,1.03,1.05,1.1,1.1,2.1,23,1.36/
DATAAMACH/0.0,0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6/
TEM1=ANG1*3.14159/180.
TEM2=ANG2*3.14159/180.
DIF=AINC-REFI(J)
IF(DIF.GE.0.) GO TO 7
DIF=-DIF
COEFF=-0.007
GO TO 8
7 COEFF=0.007
8 DEG=(COS(TEM2)/COS(TEM1))*(1.12+ COEFF*DIF **1.43+0.61*
ICOS(TEM1)*COS(TEM1)*(TAN(TEM1)-TAN(TEM2))/SIGMC)
NP=7
KOR=2
CALL POLYFT(RDEQ,RMTBYC,NP,KOR,C,CO,A)
AMTRYC=CO+C(1)*DEG+C(2)*DEG*DEG
OMEGAD=(2.*AMTRYC*SIGMC/COS(TEM2))*(COS(TEM1)*COS(TEM1)/
ICOS(TEM2)/COS(TEM2))*((2.*H2)/(3.*H2-1.))/(1.-AMTRYC*SIGMC*H2/
2COS(TEM2))*3
KOR=3
NP=9
CALL POLYFT(AMACH,OMMM,NP,KOR,C,CO,A)
YMOM=CO+C(1)*CM(1,J)+C(2)*CM(1,J)*CM(1,J)+
IC(3)*CM(1,J)*CM(1,J)*CM(1,J)
IF(KWA.EQ.1) GO TO 10
WRITE(6,9) TEM1,TEM2,DIF,DEG,SIGMC,AMTRYC,OMEGAD,YMOM
9 FORMAT(2X,8F14.5)
10 CONTINUE
IF(OMEGAD.GT.0.13) OMEGAD=0.13
RETURN
END
```

SUBROUTINE ROWENT

```
C -----
COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
COMMON/BLKQ/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
1 /BLKAS/ANG1,SIGM1,THET1
COMMON/FLAGS/KPIPE,KCOMP,KANNU,KEG,KT,KWA,KWB,KWC,KPA,KPB,KPC
1 /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,U(16,17),CV(16,17)
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
1 /BLKS/S(16,17),DSDR(16,17),CM(16,17)
C -----
TTP=HO(I,J)/CP
CMSQ = (CX(I,J)*CX(I,J)+CU(I,J)*CU(I,J)+CR(I,J)*CR(I,J))/
1(GAMA*AG * AR *TTP)
CM(I,J)=SQRT(ABS(CMSQ))
TSP=TTP/( 1.+(GAMA-1.)*0.5 *CM(I,J)*CM(I,J))
TTN=HO(I+1,J)/CP
TSN=TTN/( 1.+(GAMA-1.)*0.5 *CM(I+1,J)*CM(I+1,J))
```

```

C      CBUSQ=CBU(I,J)*CBU(I,J)
C      PRRI IS THE IDEAL RELATIVE TOTAL PRESSURE RATIO ACROSS THE ROTOR .
      PRRI=(1.+(GAMA-1.)/2.)*(CBUSQ/(GAMA*AG*AR*
1      TTP ))*(1.-R(I,J)*R(I,J)/(R(I+1,J)*R(I+1,J)))**((GAMA/
2(GAMA-1.))
      DENOM=1.-(OMEGAD/PRRI)*(1.-(1./(1.+(GAMA-1.)*CM(I,J)*CM(I,J)/2.))
1**((GAMA/(GAMA-1.)))
      ENTDF=ALOG(1./DENOM)*AR/AJ
      S(I+1,J)=S(I,J)+ENTDF
      DSDR(I,J)=(S(I,J)-S(I,J-1))/G(I)
      DSDR(I+1,J)=(S(I+1,J)-S(I+1,J-1))/G(I+1)
C      DISTRIBUTION OF DENSITY AT THE NEXT STATION.
      TRATIO=TSN/TSP
      POWER=-AJ/AR*ENTDF
      IF(POWER.GT.0.) GO TO 17
      POWER=ABS(POWER)
      EP=1./EXP(POWER)
      GO TO 18
17     EP=EXP(POWER)
18     ROW(I+1,J)=ROW(I,J)*TRATIO**((1./(GAMA-1.))*EP
3     CONTINUE
      TRATIO=ABS(TRATIO)
      IF(TRATIO.GT.1.1) TRATIO=1.1
      IF(KPC.GE.KWA) RETURN
      WRITE(6,6) TSP,TTP,TSN,TTN,PRRI,CBUSQ,DENOM,TRATIO
6     FORMAT(8F15,7)
      WRITE(6,6) ROW(I,J),ROW(I+1,J),CMSQ,S(I,J),S(I+1,J),DSDR(I+1,J)
1.ENTDF,POWER
      RETURN
      END

```

SUBROUTINE ROTERB

```

C      ----
C      CALCULATION OF THE CONTRIBUTION BY RO-TERM TO THE B-TERM .
      COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
      COMMON/BLK2/TTERM(16,17),EMEW(16,17),EMEWX(16,17),ROTERM(16,17)
1      /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)
      COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
C      ----
      K=I+1
      DRODR=(ROW(K,J+1)-ROW(K,J-1))/(2.*G(K))
      DRODX=(ROW(K+1,J)-ROW(K-1,J))/(HL(K)+HR(K))
      TERMA=DSIDR(K,J)*DRODR/ROW(K,J)
      TERMB=DSIDX(K,J)*DRODX/ROW(K,J)
      ROTERM(K,J)=TERMA+TERMB
      RETURN
      END

```

SUBROUTINE SSSS

```

      COMMON /BLK1/ I,J,N,M,NM1,NP1,MM1,MP1
1      /BLK4/RIN(16),ROUT(16),G(16),HL(16),HR(16),R(16,17)
      COMMON/BLK2/TTERM(16,17),EMEW(16,17),EMEWX(16,17),ROTERM(16,17)
      COMMON /BLKM2/ CX(16,17),CBU(16,17),B(16,17),CU(16,17),CR(16,17)
1 ,UI(16,17),CV(16,17)
      COMMON/BLKMOL/DSIDR(16,17),DSIDX(16,17),FC,FMM1
1 /BLKM3/HO(16,17),PO(16,17),ROW(16,17)

```

```

1      /BLKS/S(16,17),DSDR(16,17),CM(16,17)
COMMON/BLKG/OMEGAD,ANG2,SIGMC,AINC,REFIN,H2,CP,AG,AJ,AR,GJ,GAMA
COMMON/WORKA/DCVR(16,17),DHOR(16,17)
DO 4 J=2,MM1
  K=1+1
  DCVR(K,J)=(CV(K,J)-CV(K,J-1))/G(K)
  DHOR(K,J)=(HO(K,J)-HO(K,J-1))/G(K)
4  CONTINUE
DO 5 J=2,MM1
  TS=-CV(K,J)*CV(K,J)/(2.*AG*AJ*CP)+HO(K,J)/CP
  RA=1./((ROW(K,J)*AG*AJ*TS)*(EMEWR(K,J)*DCVR(K,J)-EMEWR(K,J-1)*DCVR
1(K,J-1))/G(K)
  RB=1./((ROW(K,J)*TS)*(EMEWR(K,J)*DHOR(K,J)/CV(K,J)-EMEWR(K,J-1)
1*DHOR(K,J-1)/CV(K,J-1))/G(K)
  RHS=RA+RB
  DIF=RHS*G(K)
  JM=(M+1)/2
  IF(J.GE.JM) S(K,J+1)=S(K,J)+ABS(DIF)
  IF(J.LE.JM) JB=JM-J+1
  IF(J.LE.JM) S(K,JB)=S(K,JB+1)+ABS(DIF)
  IF(J.EQ.6.OR.J.EQ.7) WRITE(6,8) RA,RB,TS,ROW(K,J),EMEWR(K,J),
1CV(K,J),DHOR(K,J),DCVR(K,J)
5  CONTINUE
DO 6 J=2,MM1
  DSDR(K,J)=(S(K,J)-S(K,J-1))/G(K)
6  CONTINUE
WRITE(6,8) (S(K,J),J=1,M)
8  FORMAT(8F15,8)
WRITE(6,8) (DSDR(K,J),J=1,M)
RETURN
END

```

\$IBMAP *XDUMP

```

C -----
XDUMP  SAVE      1,2
       CALL      PDUMP(12288,16384,=3)
       RETURN    XDUMP
       END

```

SUBROUTINE CUPALI(X1,X2,X3,X4,X,Y1,Y2,Y3,Y4,Y)

```

C -----
COMMON /BLK1/ 1,J,N,M,NM1,NP1,MM1,MP1
COMMON /BLKA/XM,YM,XM1,YM1,D2YM1,XM2,YM2,D2YM2
DIMENSION Y(17),X(16,17)
C -----
WRITE (6,66)
66  FORMAT(1H0,5X,41HTHE FOLLOWING IS WRITTEN FROM SUB. KCPL1 )
NM1=N-1
MM1=M-1
C -----
IT PERFORMS PARABOLIC INTERPOLATION IF X3=X4 AND Y3=Y4 .
X12=X1-X2
X13=X1-X3
X14=X1-X4
X11=X1*X1
X22=X2*X2
X33=X3*X3
X44=X4*X4

```

```

X111=X11*X1
X222=X22*X2
X333=X33*X3
X444=X44*X4
X123=((X11-X22)/X12 - (X11-X33)/X13)
X124=((X11-X22)/X12 - (X11-X44)/X14)
X123A=((X111-X222)/X12 - (X111-X333)/X13)
X124A=((X111-X222)/X12 - (X111-X444)/X14)
Y12=Y1-Y2
Y13=Y1-Y3
Y14=Y1-Y4
Y123=Y12/X12-Y13/X13
Y124=Y12/X12-Y14/X14
IF(X2.EQ.X3) GO TO 7
IF(X3.EQ.X4) GO TO 7
D=(Y123/X123-Y124/X124)/(X123A/X123-X124A/X124)
GO TO 8
7 D=0.
8 CONTINUE
IF(X2.EQ.X3) GO TO 9
C=Y123/X123-D*X123A/X123
GO TO 10
9 C=0.
10 CONTINUE
B=(Y12-C*(X11-X22)-D*(X111-X222))/X12
A=Y1-B*X1-C*X11-D*X111
DO 101 J=2,MM1
Y(J) = A+B*X(I,J)+C*X(I,J)*X(I,J)+D*X(I,J)*X(I,J)*X(I,J)
101 CONTINUE
WRITE(6,71)(Y(J),J=2,MM1)
71 FORMAT(1HP,6F12.8)
11 FORMAT(F7.3,5X,F7.3)
IF(X2.EQ.X3) GO TO 12
GO TO 14
12 WRITE(6,13)
13 FORMAT(1H0,32H THIS IS BY LINEAR INTERPOLATION)
14 CONTINUE
IF(X3.EQ.X4) GO TO 15
GO TO 17
15 WRITE(6,16)
16 FORMAT(1H0,35H THIS IS BY PARABOLIC INTERPOLATION)
GO TO 19
17 CONTINUE
WRITE(6,18)
18 FORMAT(1H0,35H THIS IS BY CUBICURVE INTERPOLATION)
19 CONTINUE
IF(X3.EQ.X4) GO TO 20
GO TO 23
20 XM=-B/2./C
YM=A+B*XM+C*XM*XM+D*XM*XM*XM
WRITE(6,21)
21 FORMAT(1H0,38H THE DETAILS OF P MAXIMA OR MINIMA ARE:)
D2YDX2=2.*C
WRITE(6,11)XM,YM ,D2YDX2
GO TO 25
23 CONTINUE
IF(4.*C*C-12.*B*D) 26,27,27
27 XM1=(-2.*C-SQRT(4.*C*C-12.*B*D))/(6.*D)
XM2=(-2.*C+SQRT(4.*C*C-12.*B*D))/(6.*D)

```

```

YMI=A+B*XM1+C*XM1*XM1+D*XM1*XM1*XM1
YM2=A+B*XM2+C*XM2*XM2+D*XM2*XM2*XM2
D2YM1=2.*C+6.*D*XM1
D2YM2=2.*C+6.*D*XM2
WRITE(6,21)
WRITE(6,24)XM1,YM1,D2YM1,XM2,YM2,D2YM2
24 FORMAT(2F10.2,F10.3,2F10.2,F10.3)
GO TO 25
25 WRITE(6,25)
28 FORMAT(1H0,32HXMAXIMA AND MINIMA ARE IMAGINARY)
25 CONTINUE
RETURN
END

```

SUBROUTINE POLYFT (X, Y, NP,KOR,C,CO,A)

```

C -----
COMMON/WORKA/A(36),C(6),SOME(50),SUMX(200),SMYX(100),AMEANX(100),
1EXTRAF(52)
DIMENSION X(30),Y(30)
C -----
NOUT=6
N=NP
PTS= N
NT = NOUT
KTOR= 2*KOR
C INITIALIZATION
DO 1 I=1,KOR
SMYX(I)= 0.0
1 AMEANX(I)= 0.0
DO 2 I=1,KTOR
2 SUMX(I)= 0.0
SUMY= 0.0
C NORMALIZATION WITH RESPECT TO XMAX
XMAX= X(1)
DO 100 I=2,N
IF( X(I) -XMAX) 100,101,101
101 XMAX= X(I)
100 CONTINUE
DO 102 I=1,N
102 X(I)= X(I)/XMAX
C FORMULATION OF NORMAL EQUATIONS
DO 3 J=1,KTOR
DO 3 I=1,N
3 SUMX(J)= SUMX(J) +X(I)**J
DO 4 I=1,N
4 SUMY= SUMY+ Y(I)
AMEANY= SUMY/PTS
DO 6 J=1,KOR
AMEANX(J)= SUMX (J)/PTS
DO 6 I=1,N
6 SMYX(J)= SMYX(J) +Y(I)*X(I)**J
DO 8 I=1,KOR
C(I)= SMYX(I) -PTS*AMEANX(I)*AMEANY
DO 8 J=1,KOR
K= I+J
IJ =(J-1)*KOR +I
8 A(I,J) =SUMX(K) -PTS* AMEANX(I)*AMEANX(J)
CROUT=5 REDUCTION METHOD

```

```

DO 11 I=2,KOR
  I1I=(I-1)*KOR + 1
11 A(I1I) = A(I1I)/A(I)
DO 12 J=2,KOR
  KM= J-1
DO 14 I=J,KOR
  API= 0.0
DO 14 K=1,KM
  IK =(K-1)* KOR+ 1
  KJ =(J-1)* KOR+ K
114 API = API + A(IK) *A(KJ)
  IJ =(J-1)* KOR+ 1
14 A(IJ) = A(IJ) -API
  JP= J+1
  IF (JP= KOR) 444, 444, 445
444 DO 16 I=JP,KOR
  API= 0.0
DO 16 K=1,KM
  JK =(K-1)* KOR + J
  KI =(I-1)* KOR + K
116 API =API +A(IK) *A(KI)
  JI =(I-1)* KOR + J
  JJ =(J-1)* KOR + J
16 A(JI) = (A(JI) -API)/A(JJ)
445 DUMMY= 0.0
12 CONTINUE
C(I) =C(I)/A(I)
DO 18 I=2,KOR
  API= 0.0
  IM=I-1
DO 118 K=1,IM
  IK =(K-1)* KOR + 1
118 API=API +A(IK) *C(K)
  II =(I-1)* KOR + 1
18 C(II) =(C(II) - API) / A(II)
  KORM= KOR-1
  IF (KORM) 122, 123, 122
122 DO 21 I=1,KORM
  API= 0.0
  M= KOR-1
  MP= M+1
DO 121 K=MP,KOR
  MK =(K-1)* KOR +M
121 API =API + A(MK)* C(K)
21 C(M) =C(M) -API
123 API= 0.0
DO 24 I=1,KOR
24 API =API +AMEANX(I) *C(I)
  CO = AMEANY -API
  IF (NT) 777,778,778
C DENORMALIZATION WITH RESPECT TO XMAX
777 DO 779 I = 1,KOR
779 C(I) = C(I)/XMAX**I
  RETURN
778 SRES = 0.0
DO 77 I=1,N
  SMOOTH= CO
DO 27 J=1,KOR
27 SMOOTH =SMOOTH + C(J)*X(I)**J

```

```

RES= Y(I)-SMOOTH
SRES= SRES +RES**2
X(I)= X(I)*XMAX
77 CONTINUE
STDV=SQRT(ABS(SRES/PTS))
DO 28 I=1,KOR
28 C(I) =C(I) /XMAX**I
RETURN
END

```

FUNCTION PWALL(RA,PA,RB,PB,RC,PC)

```

C -----
C FUNCTION PWALL RETURNS THE PARABOLICALLY EXTRAPOLATED VALUE
C OF A PROPERTY P FROM THE SUPPLIED VALUES PA,PB,PC AT RA,RB,RC
C DISTANCES FROM THE WALL RESPECTIVELY .
C IT PERFORMS LINEAR EXTRAPOLATION IF RB IS SET EQUAL TO RC .
C -----
IF(RB.EQ.RC) GO TO 2
RAC=RA-RC
RBC=RB-RC
RAB=RA-RB
DENOM=RAB*RAC*RBC
PWALL=((PA*RB-PB*RA)*RBC*RC-(PB*RC-PC*RB)*RAB*RA)/DENOM
GO TO 3
2 PWALL=(RA*PB-RB*PA)/(RA-RB)
3 CONTINUE
WRITE(6,1) RA,RB,RC,PA,PB,PC,RAB,RAC,RBC,PWALL
1 FORMAT (14F8.3)
RETURN
END

```

FUNCTION DBLK(XX,YY,K)

```

C -----
C FUNCTION PROGRAM TO EVALUATE DBLK FROM GIVEN XX ,YY BY DOUBLE
C INTERPOLATION TECHNIQUE .
COMMON/BLKP/B1(9),BD(9),BN(16),BM( 9),BR(16)
COMMON/WORKA/ BK(16),EXTRAD(528)
C -----
X2=XX*XX
X3=X2*XX
Y2=YY*YY
Y3=Y2*YY
IF(K.GT.1) GO TO 3
DO 2 IN=1,9
2 BK(IN)=B1(IN)
GO TO 15
3 IF(K.GT.2) GO TO 5
DO 4 IN=1,16
4 BK(IN)=BN(IN)
GO TO 16
5 IF(K.GT.3) GO TO 7
DO 6 IN=1,9
6 BK(IN)=BD(IN)
GO TO 15
7 IF(K.GT.4) GO TO 9
DO 8 IN=1,9
8 BK(IN)=BM(IN)

```

```
GO TO 15
9 DO 10 IN=1,16
10 BK(IN)=BR(IN)
GO TO 16
15 DBLK=(BK( 1)+BK( 2)*YY+BK( 3)*Y2)+
1(BK( 4)+BK(5)*YY+BK( 6)*Y2)*XX+
2 (BK( 7)+BK( 8)*YY+BK( 9)*Y2)*X2
GO TO 17
16 DBLK=(BK( 1)+BK( 2)*YY+BK( 3)*Y2+BK( 4)*Y3)+
1 (BK( 5)+BK( 6)*YY+BK( 7)*Y2+BK( 8)*Y3)*XX+
1 (BK( 9)+BK(10)*YY+BK(11)*Y2+BK(12)*Y3)*X2+
1 (BK(13)+BK(14)*YY+BK(15)*Y2+BK(16)*Y3)*X3
IF(K.EQ.2) GO TO 17
DBLK=1./DBLK
17 CONTINUE
RETURN
END
```

DESIGN DIAMETER	13.6
HUB DIAMETER	11.2
TIP DIAMETER	16.0
HUB-TIP RATIO	0.7
BLADE HEIGHT	2.4
BLADE CHORD	1.2
ASPECT RATIO	2.0
NOMINAL SPACING BETWEEN BLADE ROWS	0.75

IGV Rotor Stator				
NUMBER OF BLADES	34	38	37	
CIRCUMFERENTIAL PITCH	1.202	1.076	1.105	DIAMETER
STAGGER	20.25	24.75	39.50	
CAMBER	40.50	41.25	28.00	11.75
CAMBER RADIUS	1.730	1.700	2.410	
STAGGER	18.45	36.30	36.00	
CAMBER	36.90	27.30	27.50	13.2
CAMBER RADIUS	1.895	2.540	2.530	
STAGGER	17.00	44.00	33.60	
CAMBER	34.00	18.00	27.20	14.5
CAMBER RADIUS	2.050	3.850	2.550	
STAGGER	15.75	49.25	31.70	
CAMBER	31.00	12.25	26.75	15.75
CAMBER RADIUS	2.210	5.500	2.600	
BASE AEROFOIL C-4 CIRCULAR ARC CAMBER LINE				
L.E. RADIUS 0.0144 • T.E. RADIUS 0.0072				
MAXIMUM THICKNESS TO CHORD RATIO 0.1				

DETAILS OF THE COMPRESSOR AND THE BLADE ROWS

TABLE A

DIAMETER OF THE WIRE	0.005 MM
LENGTH OF THE WIRE (APPROXIMATELY)	1.2 MM
COLD RESISTANCE	3.5 0.5
TEMPERATURE COEFFICIENT OF RESISTANCE	4000. /C
MAXIMUM OPERATING TEMPERATURE	300. C
OPERATING RESISTANCE / COLD RESISTANCE	1.8 - 2.0
UPPER FREQUENCY LIMIT AT 10 M/SEC VELOCITY	15 KC/SEC
UPPER FREQUENCY LIMIT AT 100 M/SEC VELOCITY	50 KC/SEC
MAXIMUM PERMISSIBLE AIR VELOCITY	150 M/SEC
TYPICAL WIRE SELF-TIME CONSTANT	1 MILLISEC

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SPECIFICATIONS OF THE HOT-WIRE PROBE

TABLE B

FREQUENCY RESPONSE	0.60 KC/SEC FOR 5M PROBE CABLE
PROBE OPERATING RANGE ON PANEL	1 TO 10 OHMS
MAXIMUM AVAILABLE PEAK CURRENT	250 MILLIAMPS
OPERATING RESISTANCE VARIATIONS	0.1 /
DC VOLTMETER RANGES	2.5, 10 AND 20 VOLTS
RMS INDICATOR RANGES	5, 10, 20, 50, 100, 200, 500, 1000 MV
LOW PASS FILTERS	100, 50, 20, 10, 5, 2 AND 1 KC/SEC
HIGH PASS FILTERS	5, 20, 50, 100, 200, 500, 1000 C/SEC
NOISE LEVEL - FILTERS OUT	3.7 MILLIVOLTS

TECHNICAL DATA OF THE HOT-WIRE ANEMOMETER

TABLE C

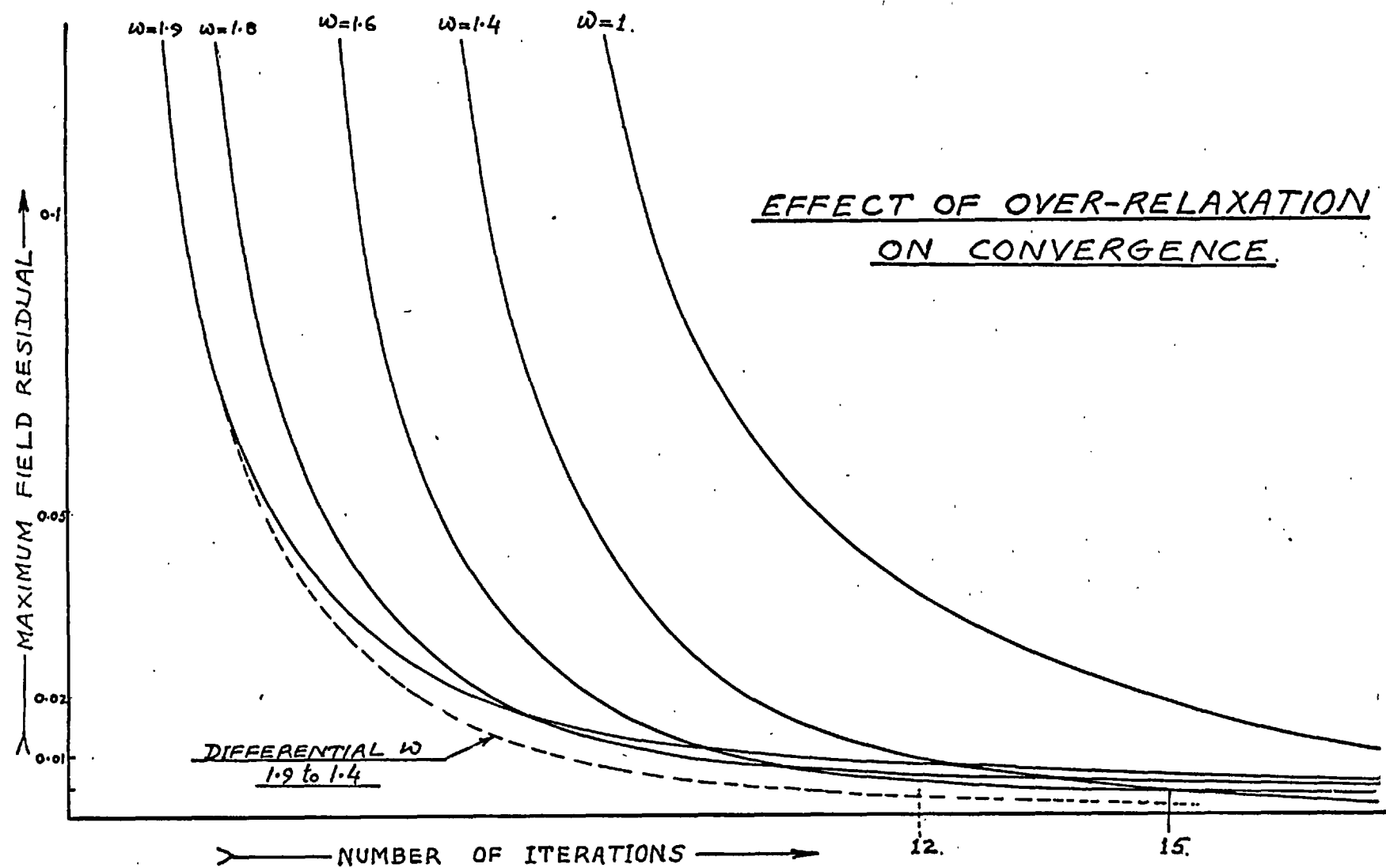
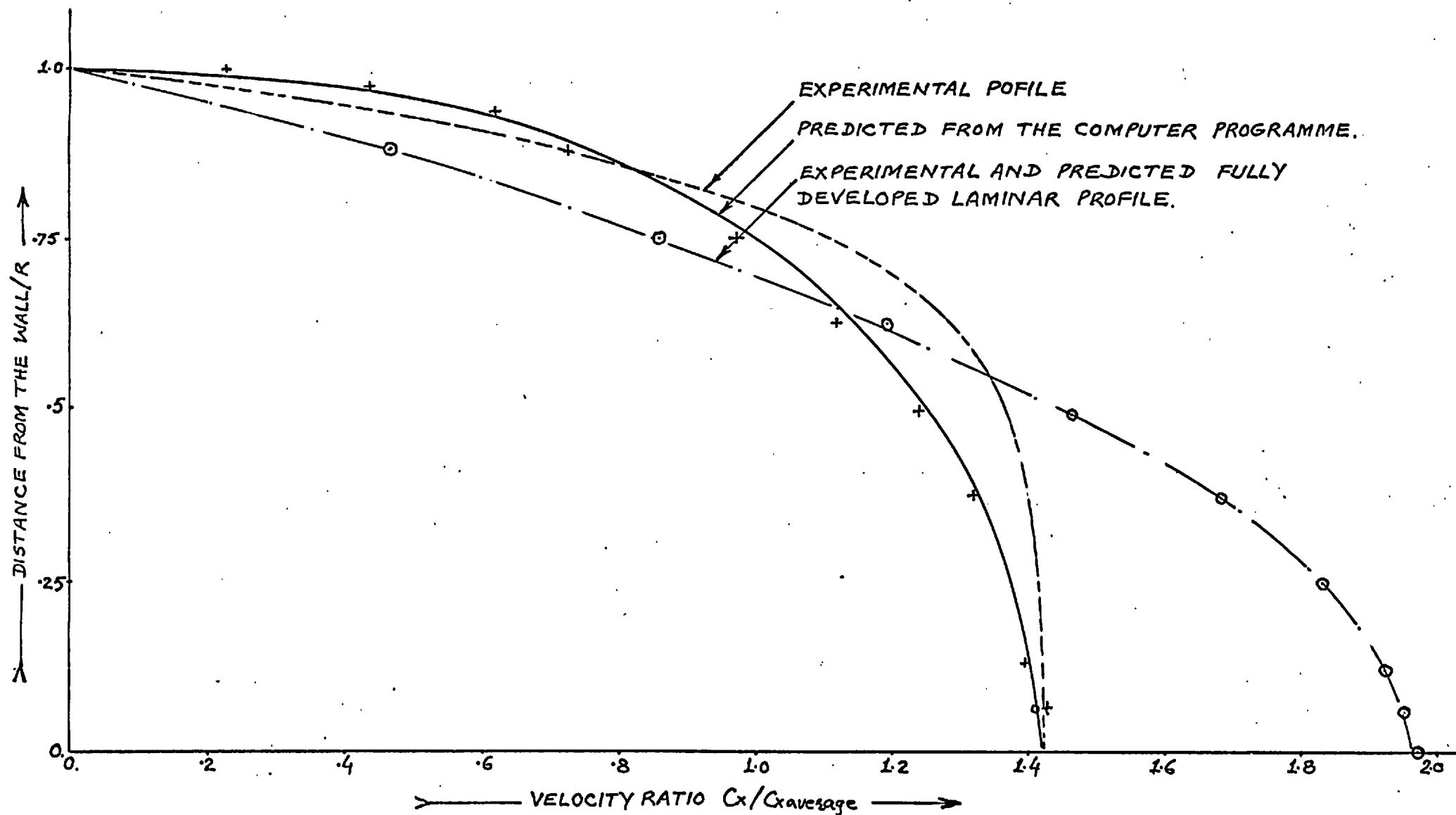


FIG. 1.



VELOCITY DEVELOPMENT IN CIRCULAR PIPES

FIG. 2

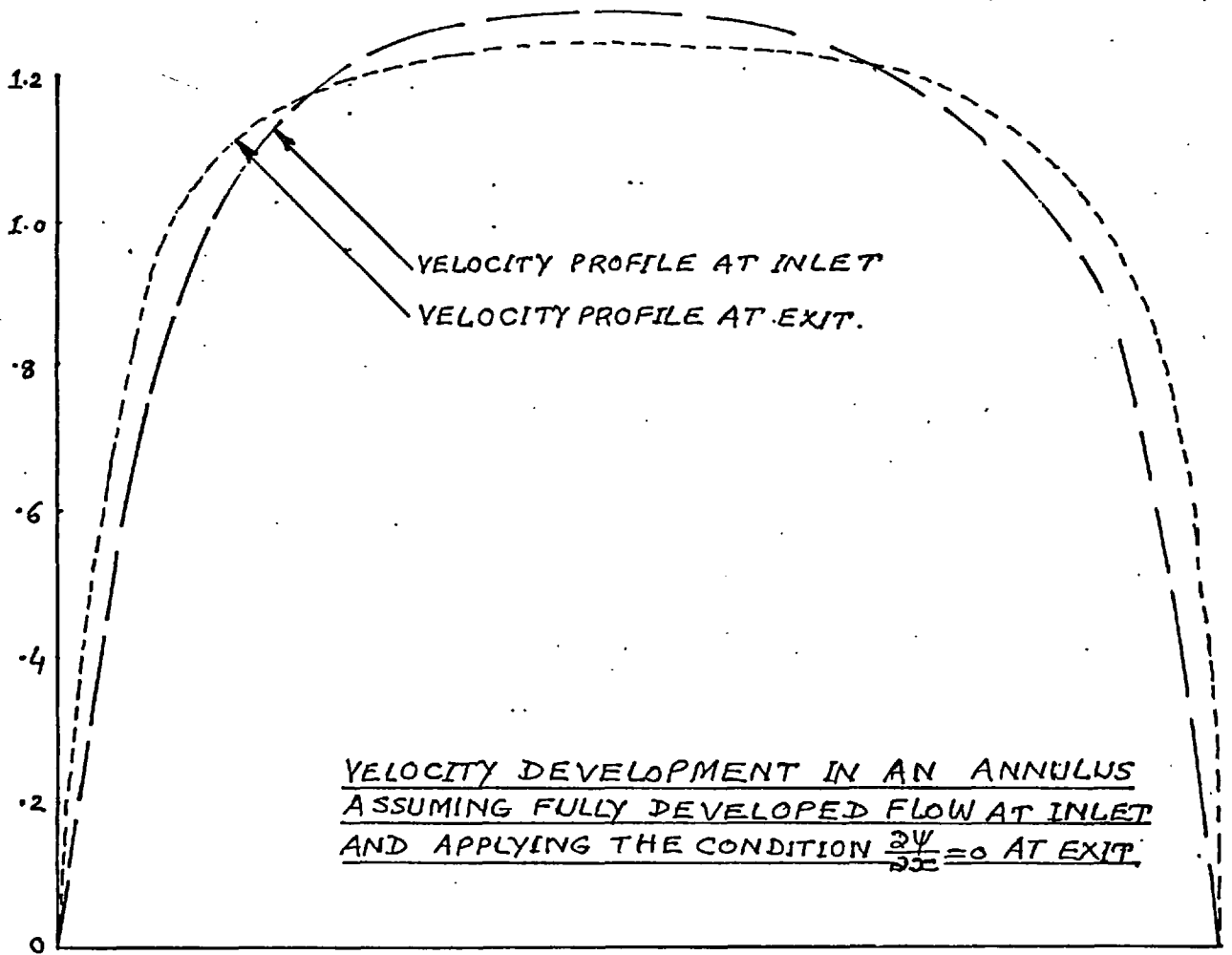


FIG. 3.

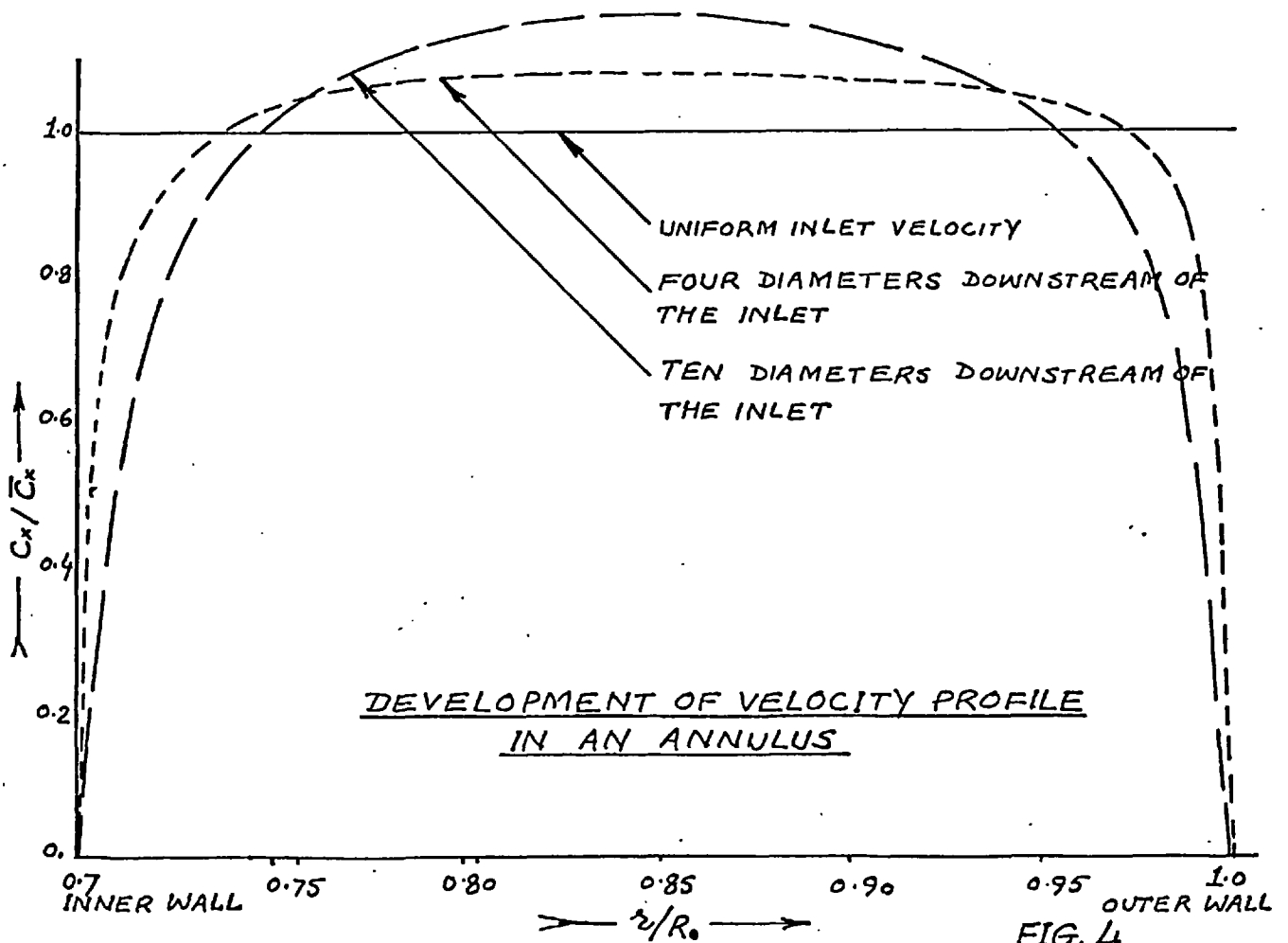


FIG. 4

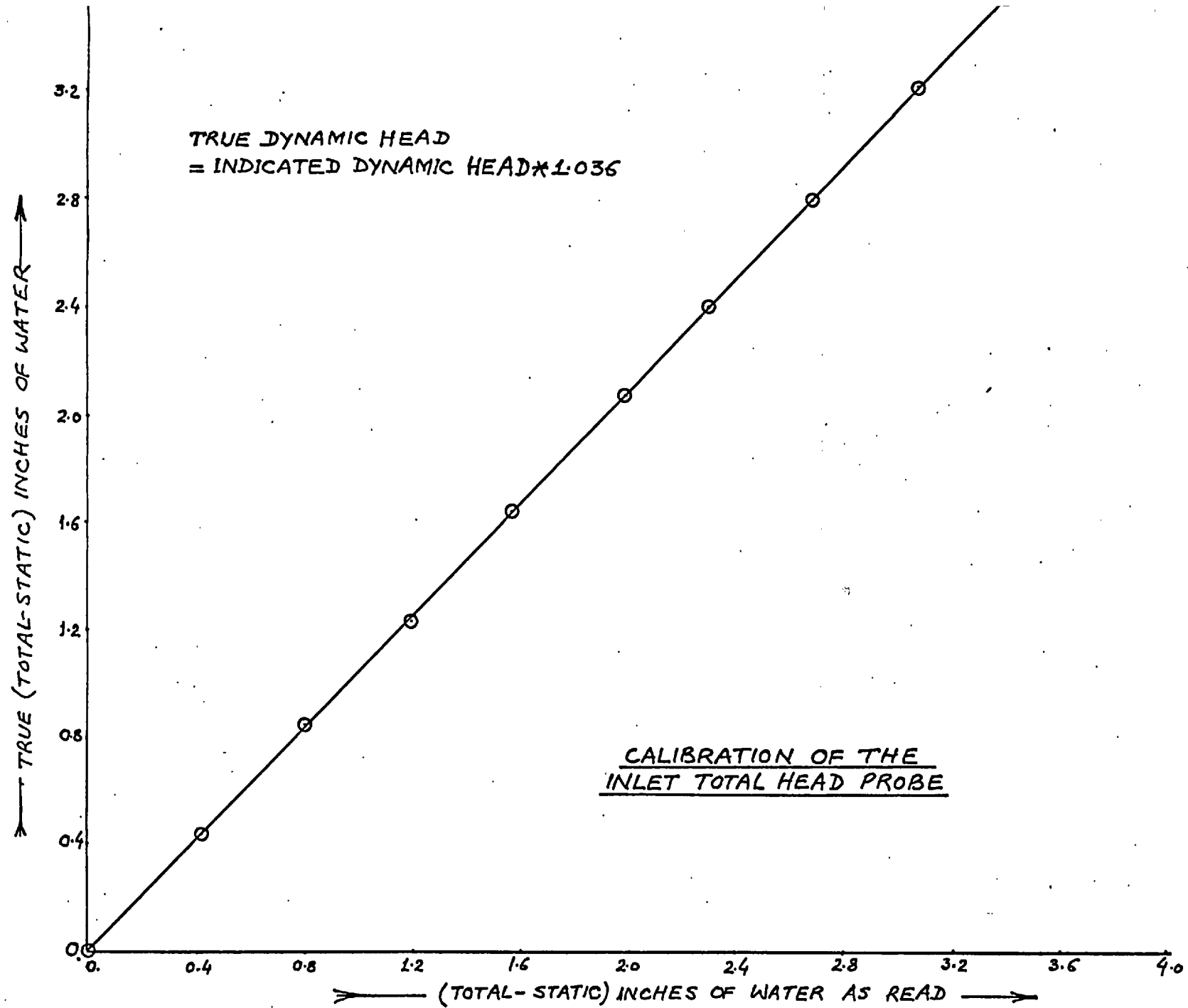


FIG. 5

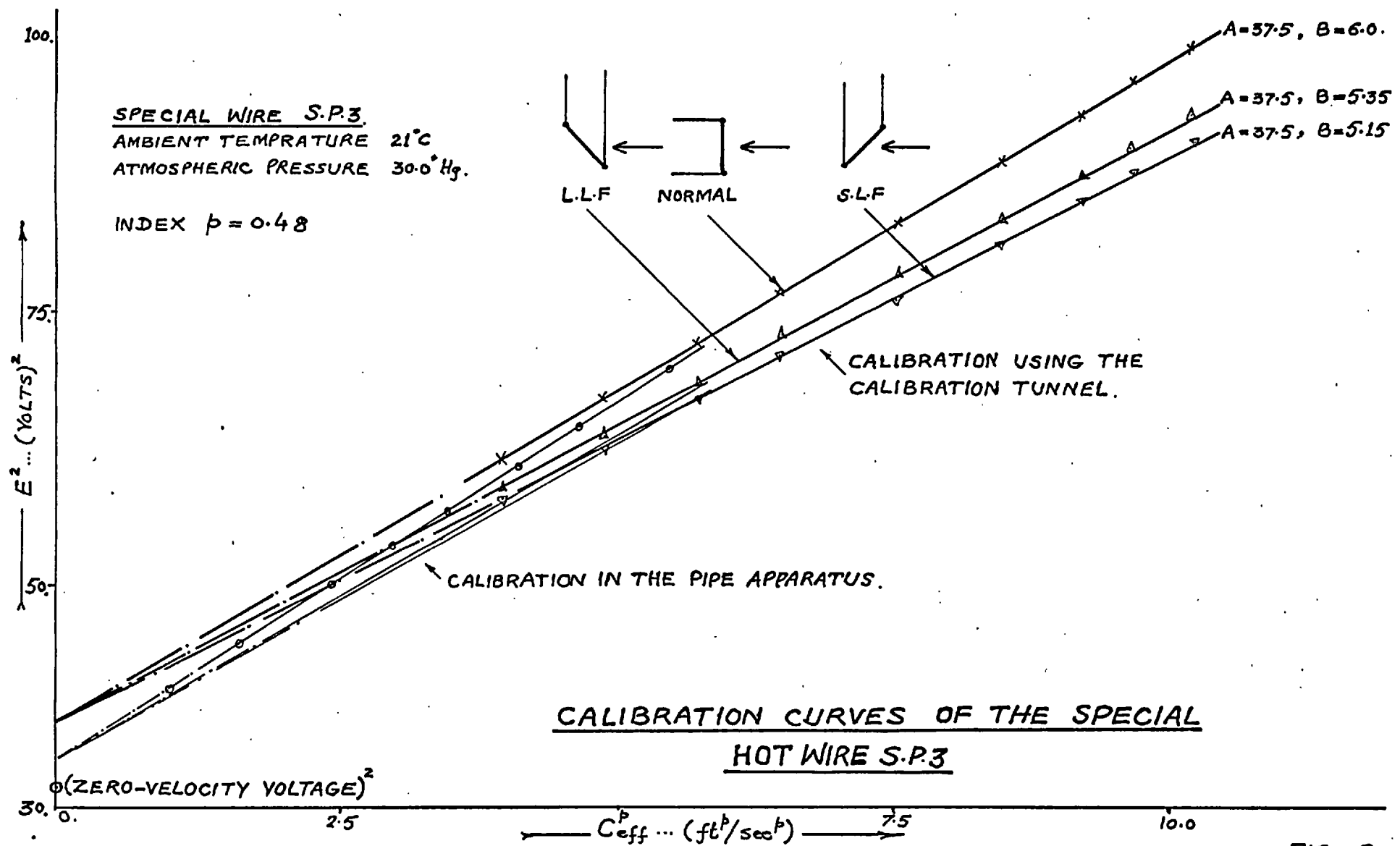


FIG. 6.

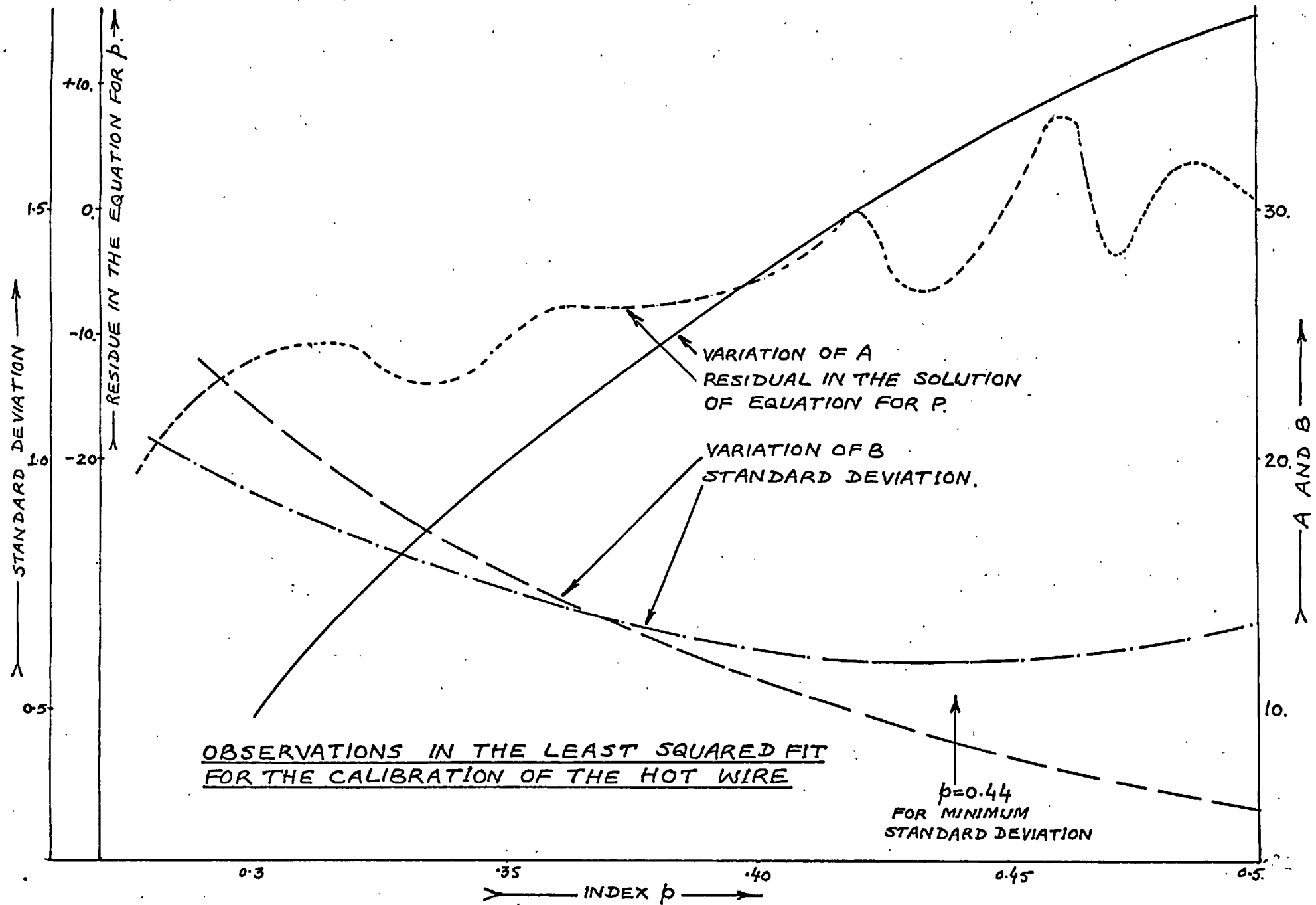
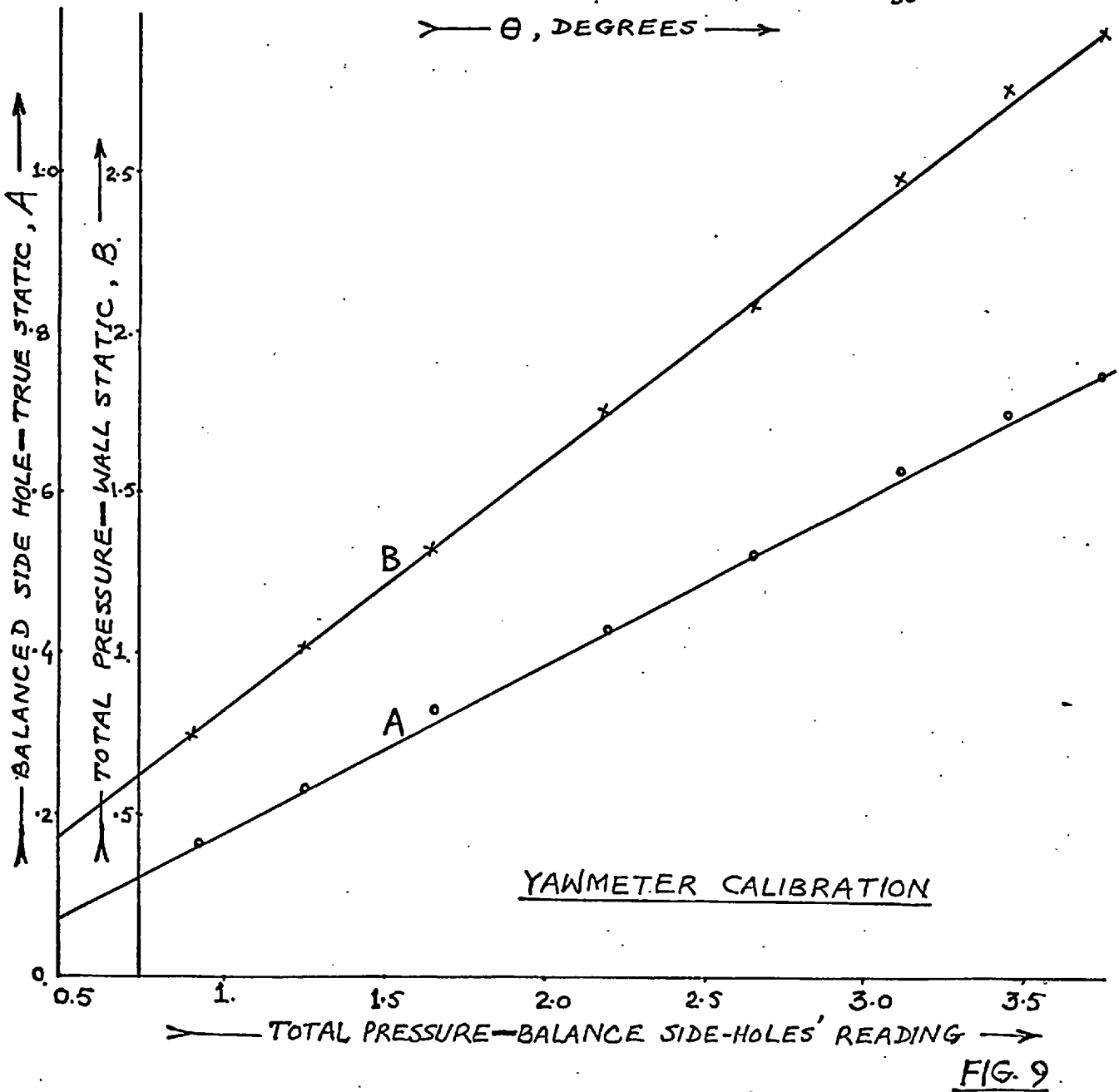
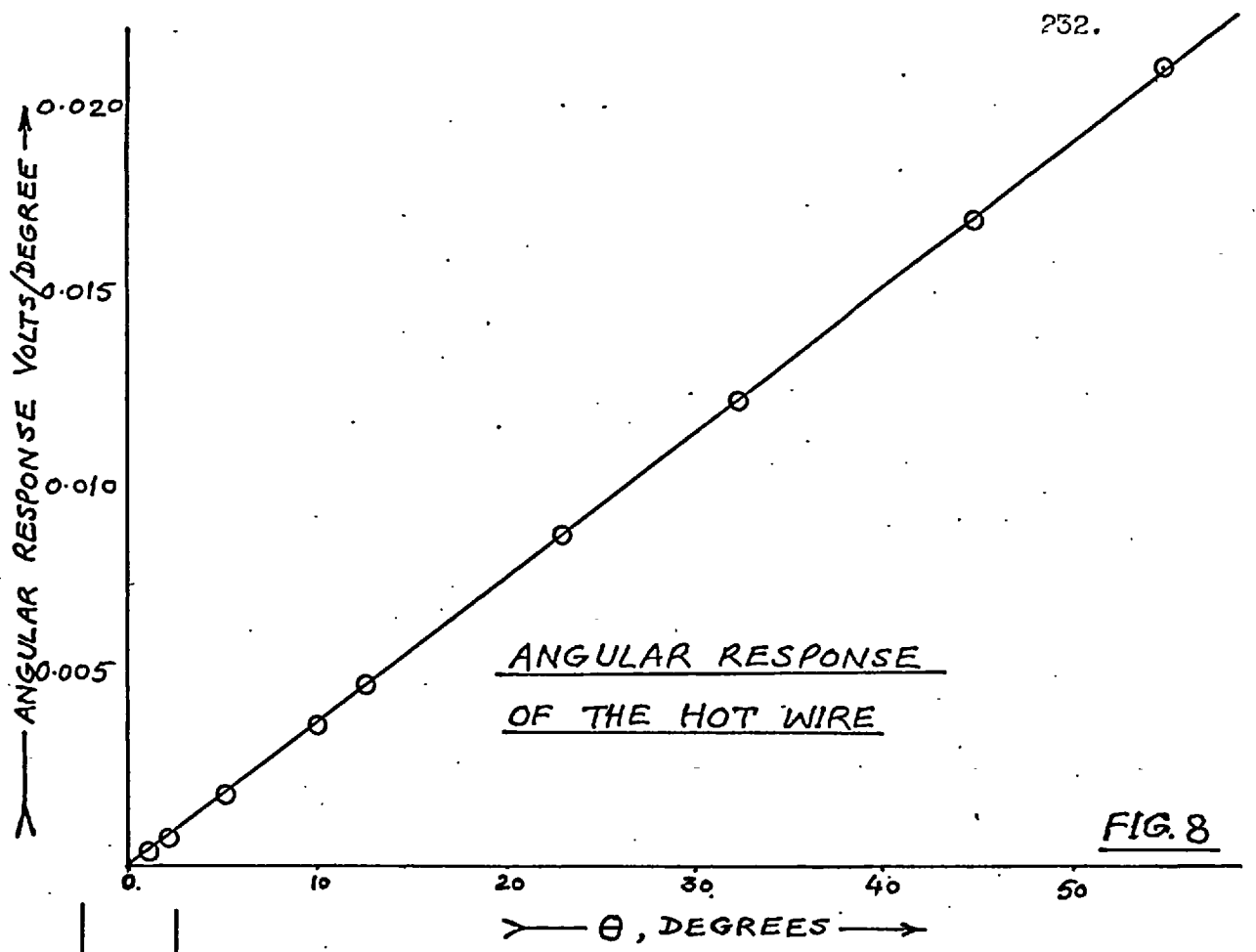


FIG.7



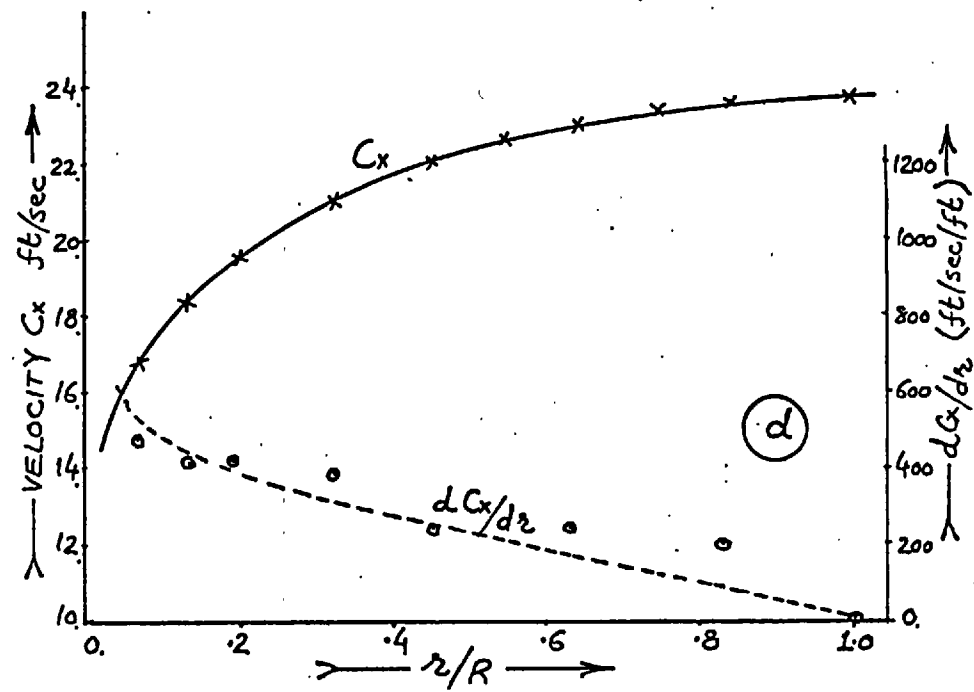
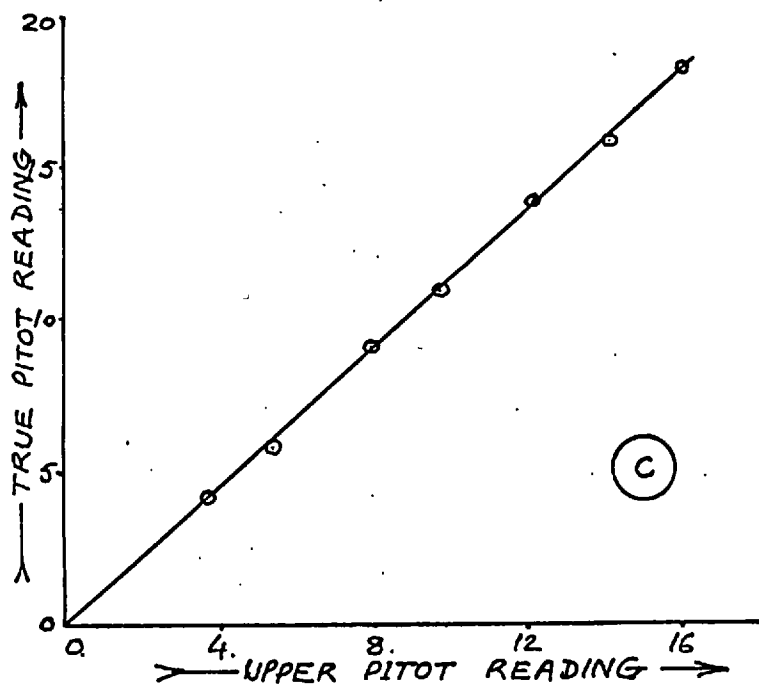
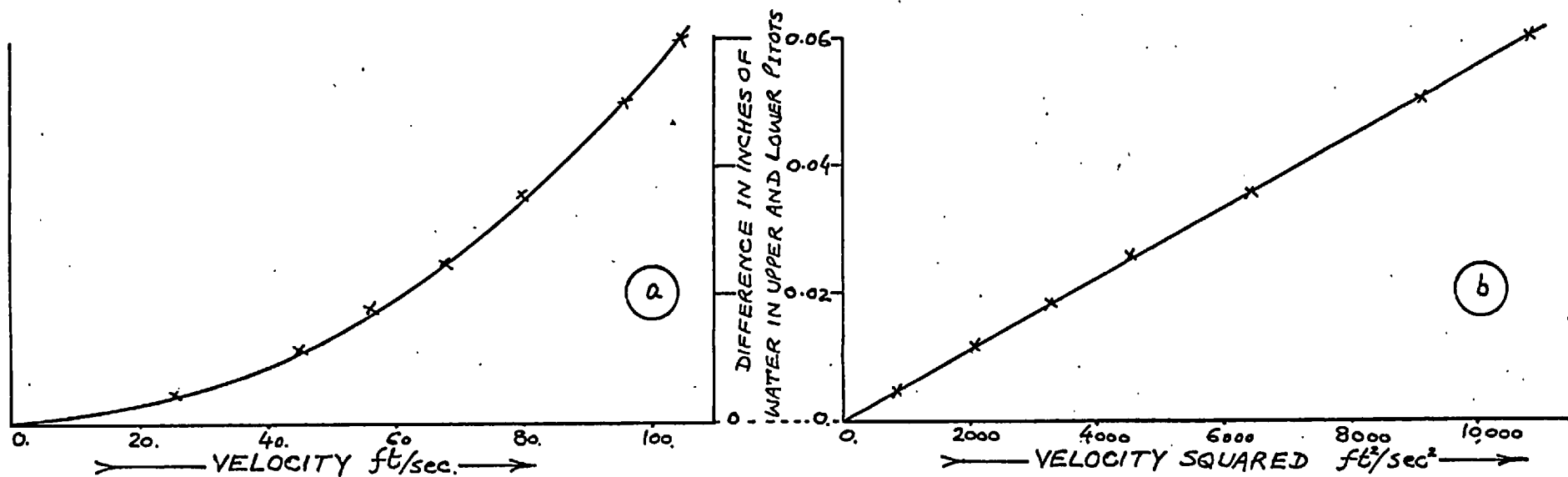


FIG 10.

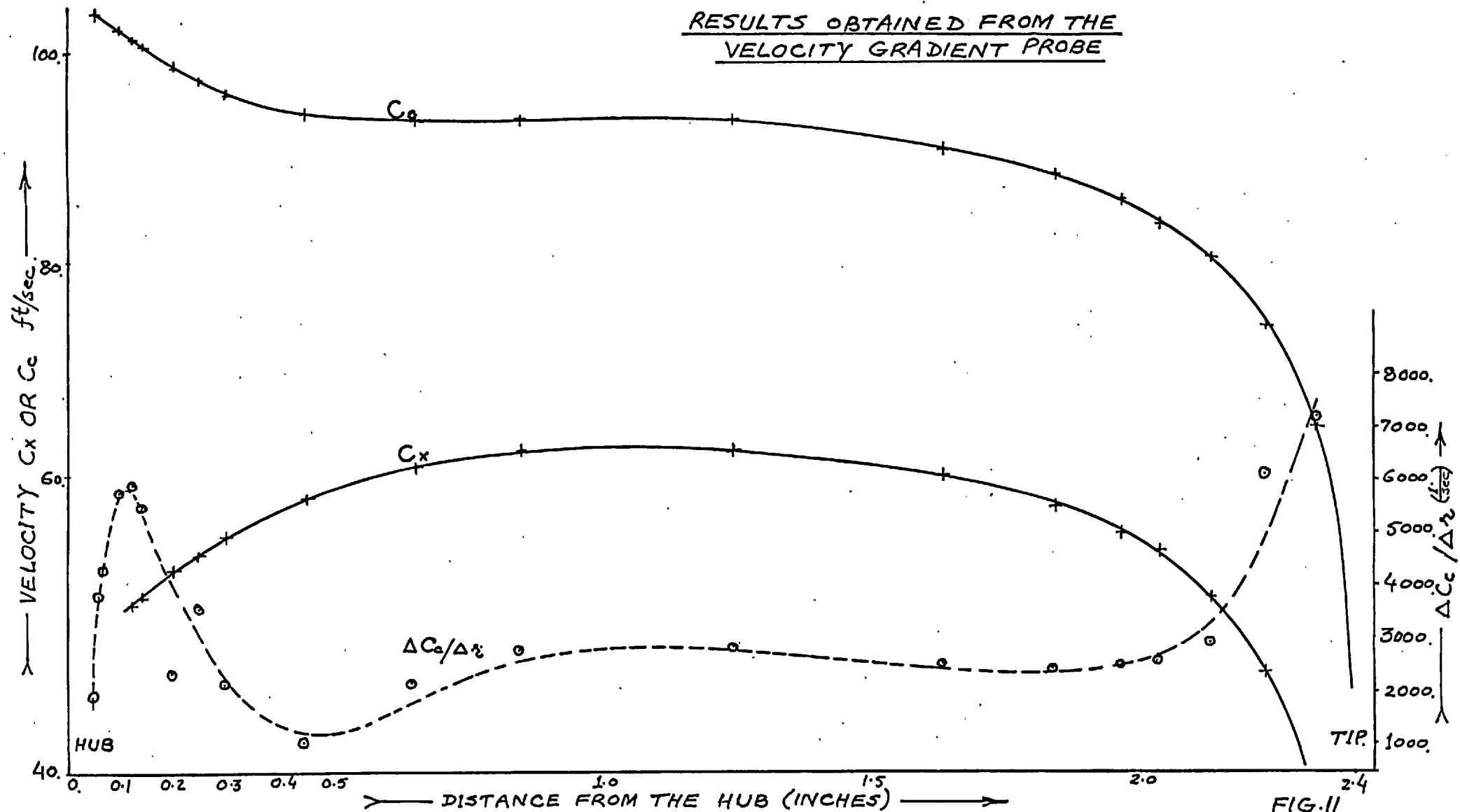
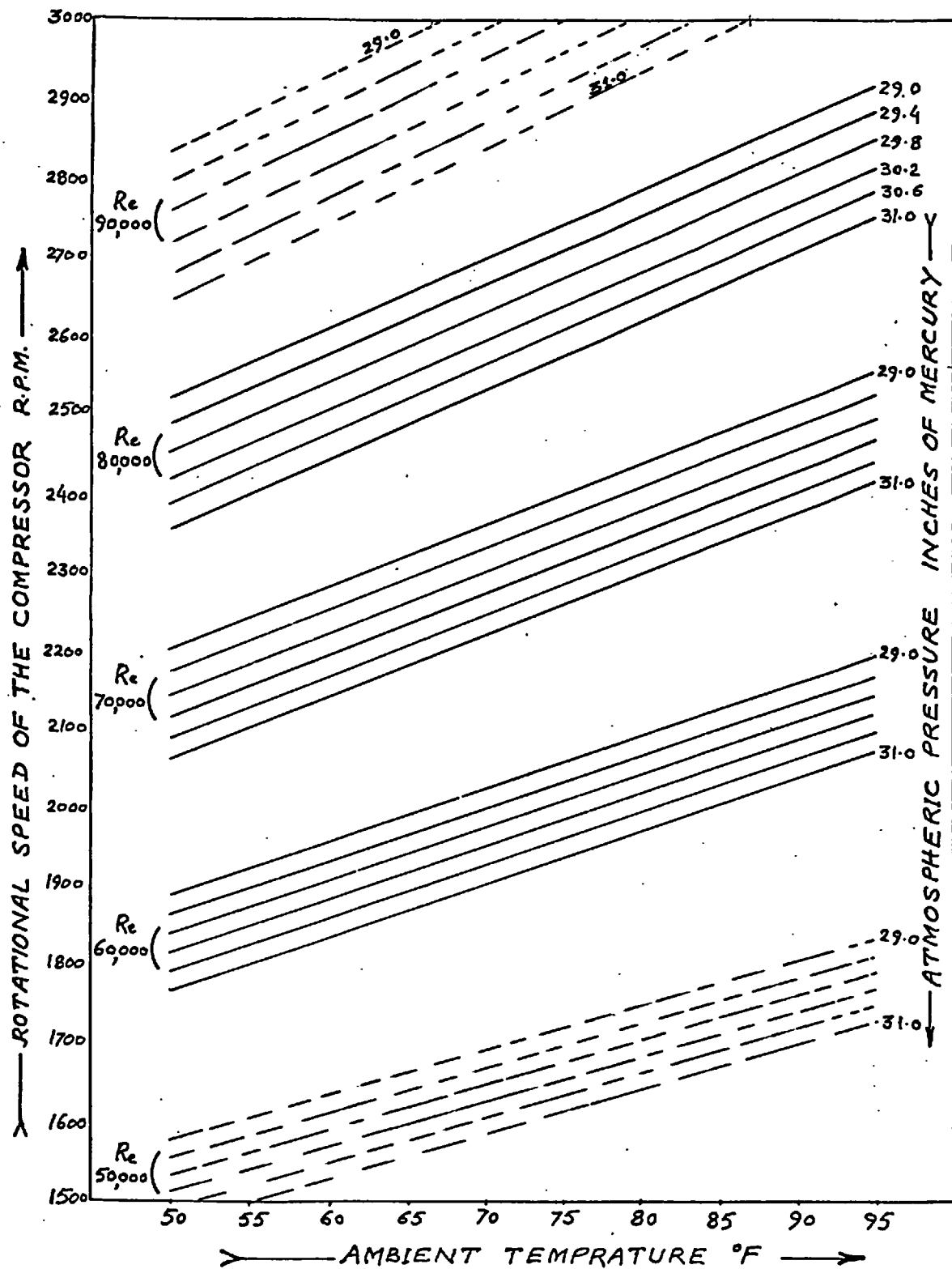


FIG. 11



CARPET PLOT TO DETERMINE THE RUNNING SPEED

FIG. 12.

SHEAR STRESS DISTRIBUTION
IN FULLY DEVELOPED PIPE FLOW

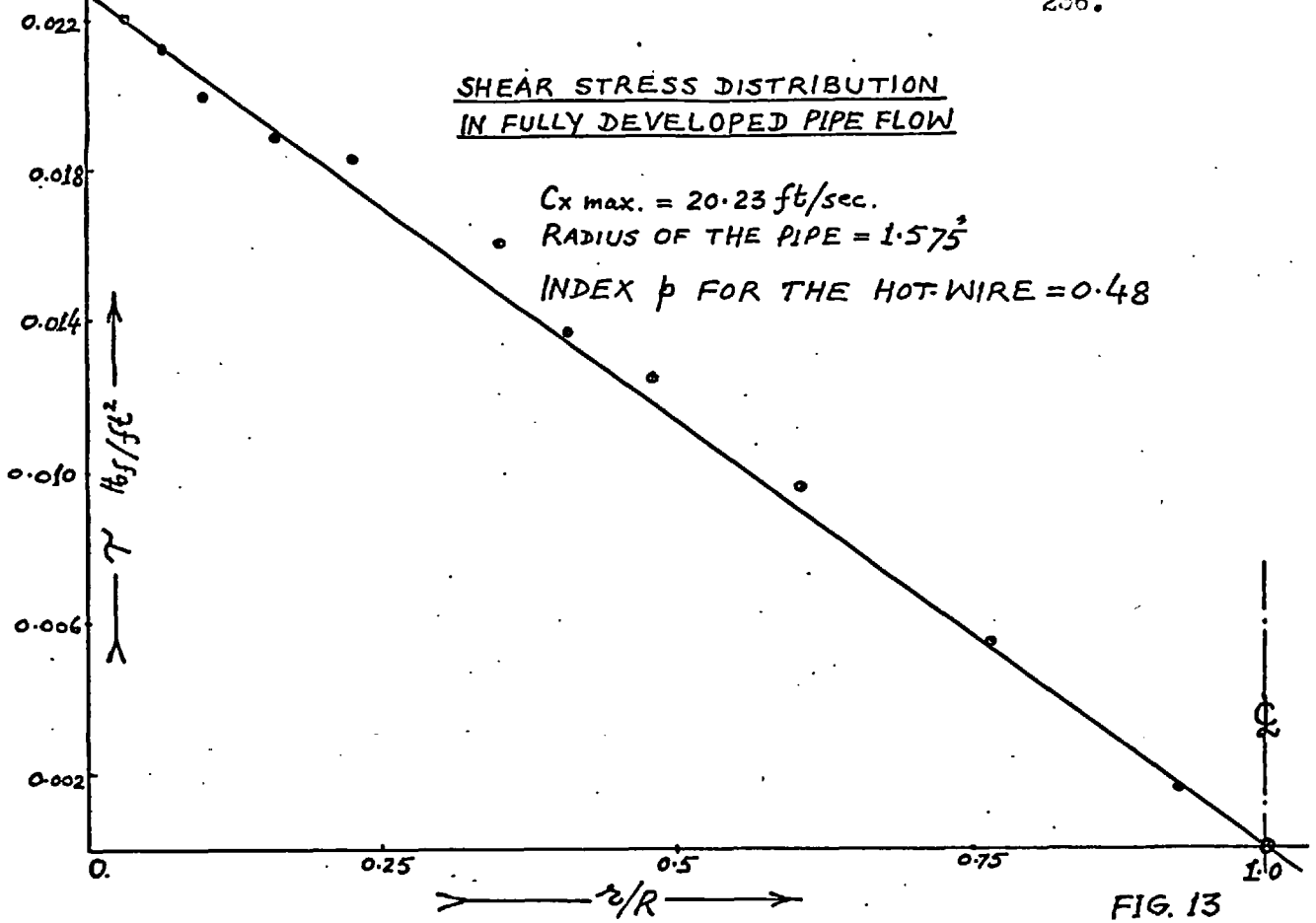
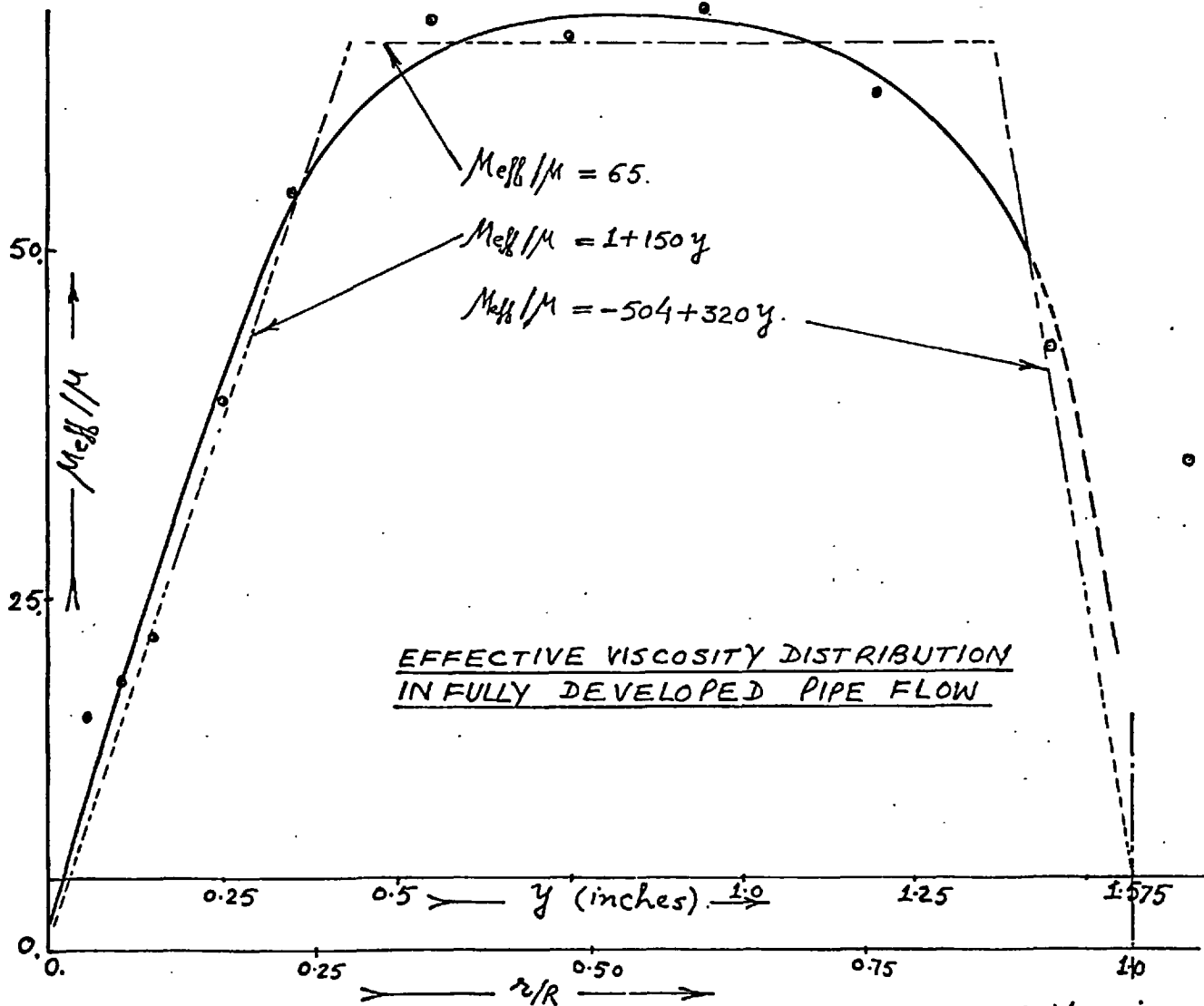
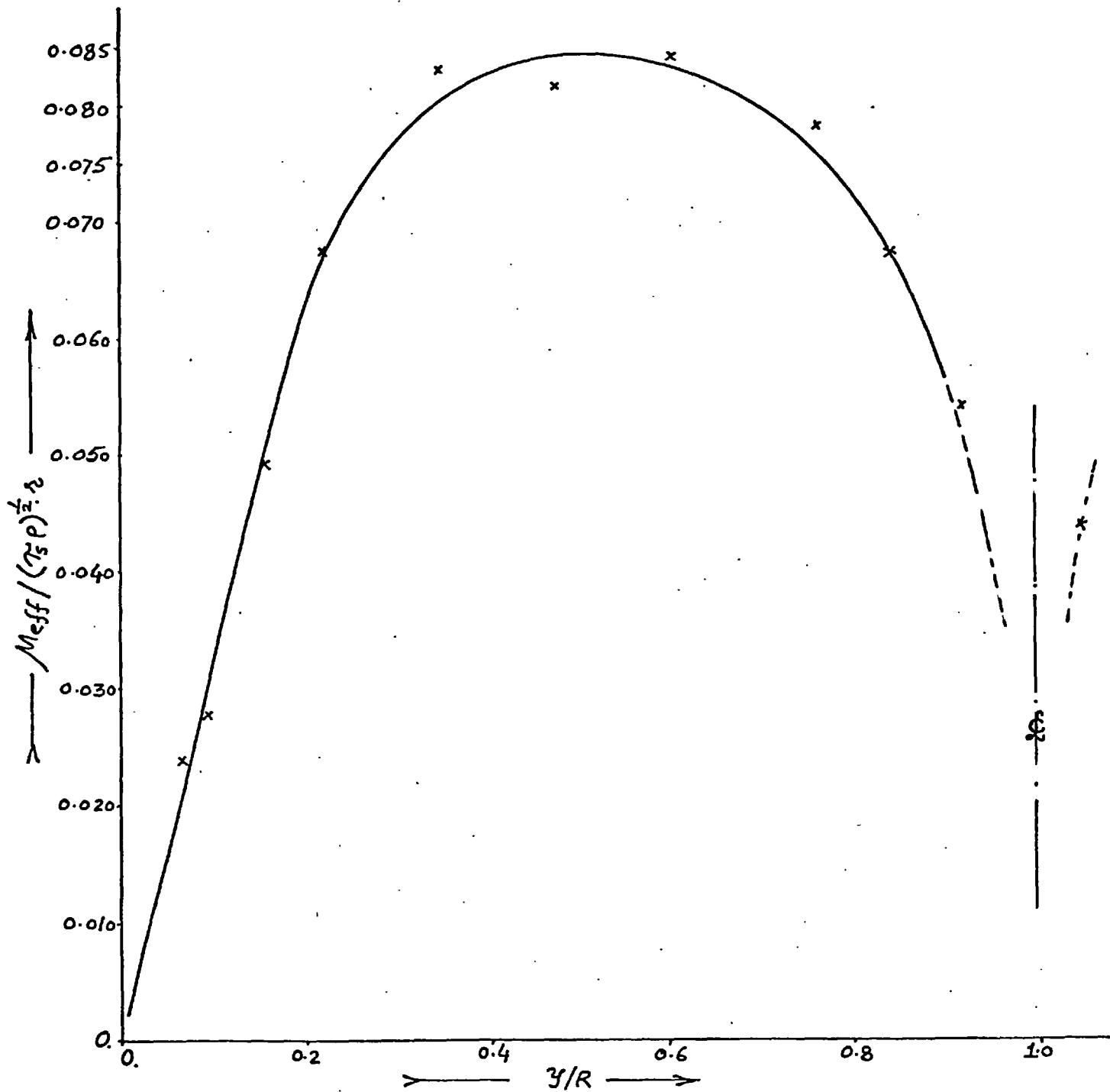


FIG. 13



EFFECTIVE VISCOSITY DISTRIBUTION
IN FULLY DEVELOPED PIPE FLOW

FIG. 14



EFFECTIVE VISCOSITY DISTRIBUTION IN A
FULLY DEVELOPED PIPE FLOW (MEASURED)

FIG. 15.

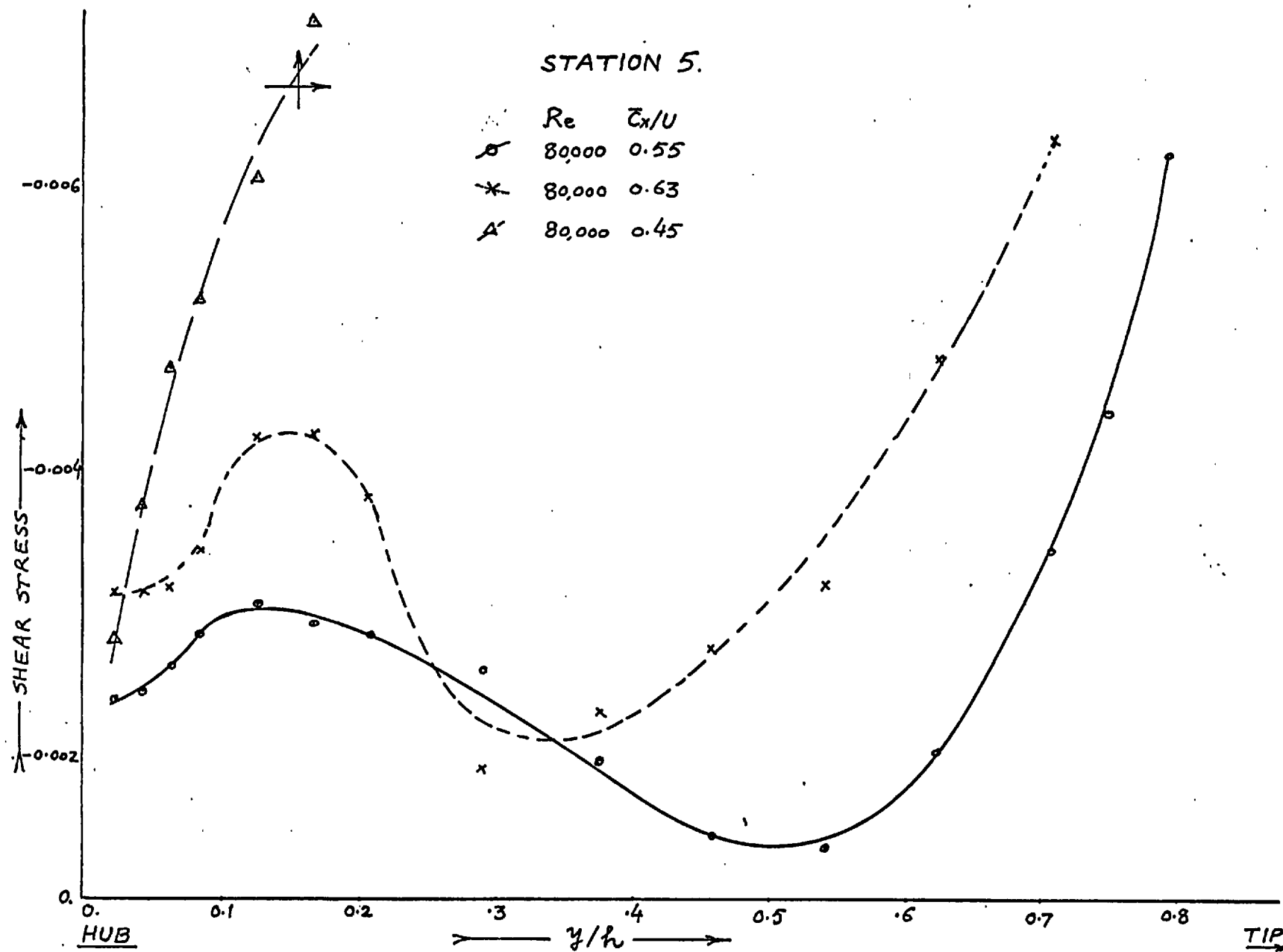


FIG. 16.

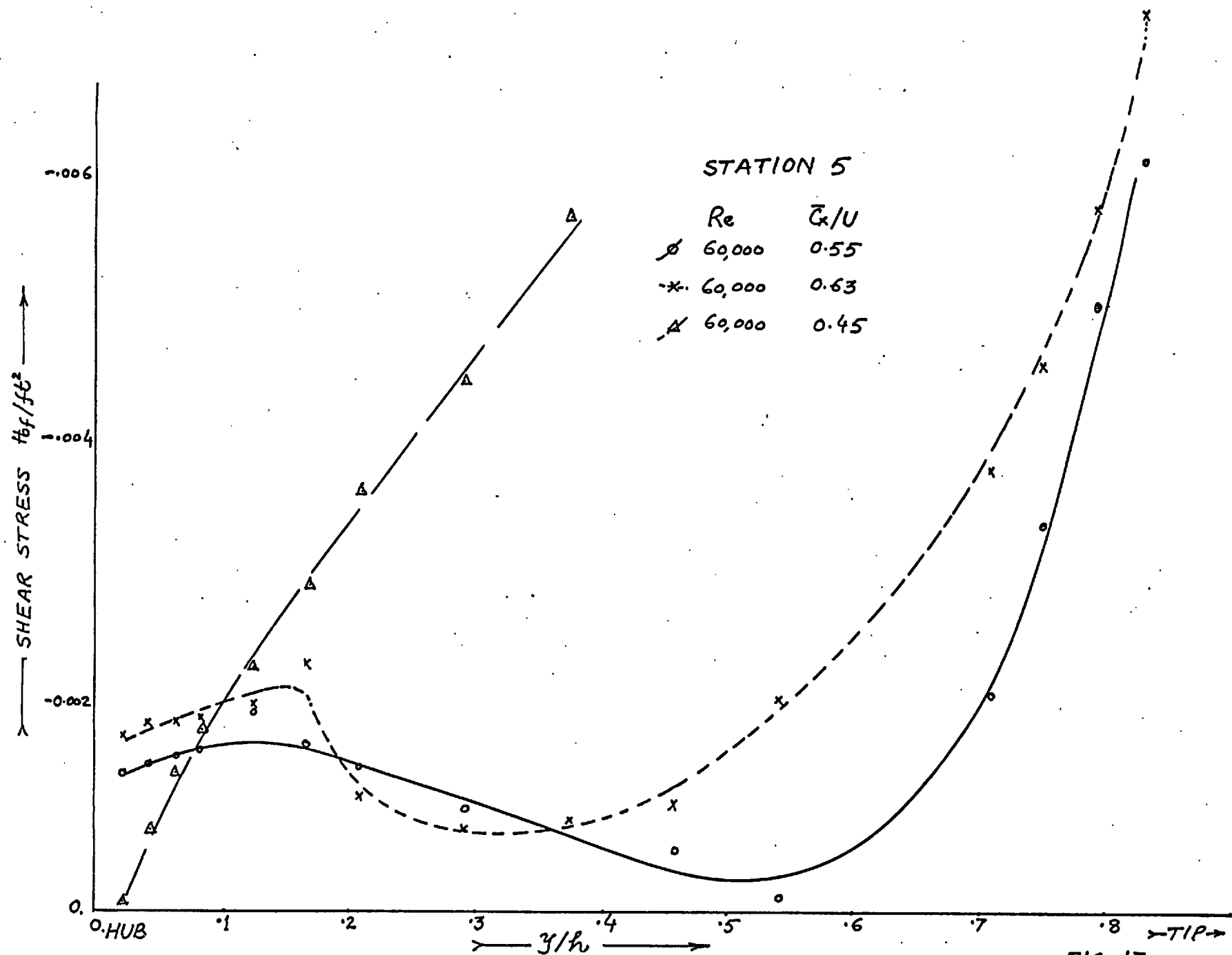
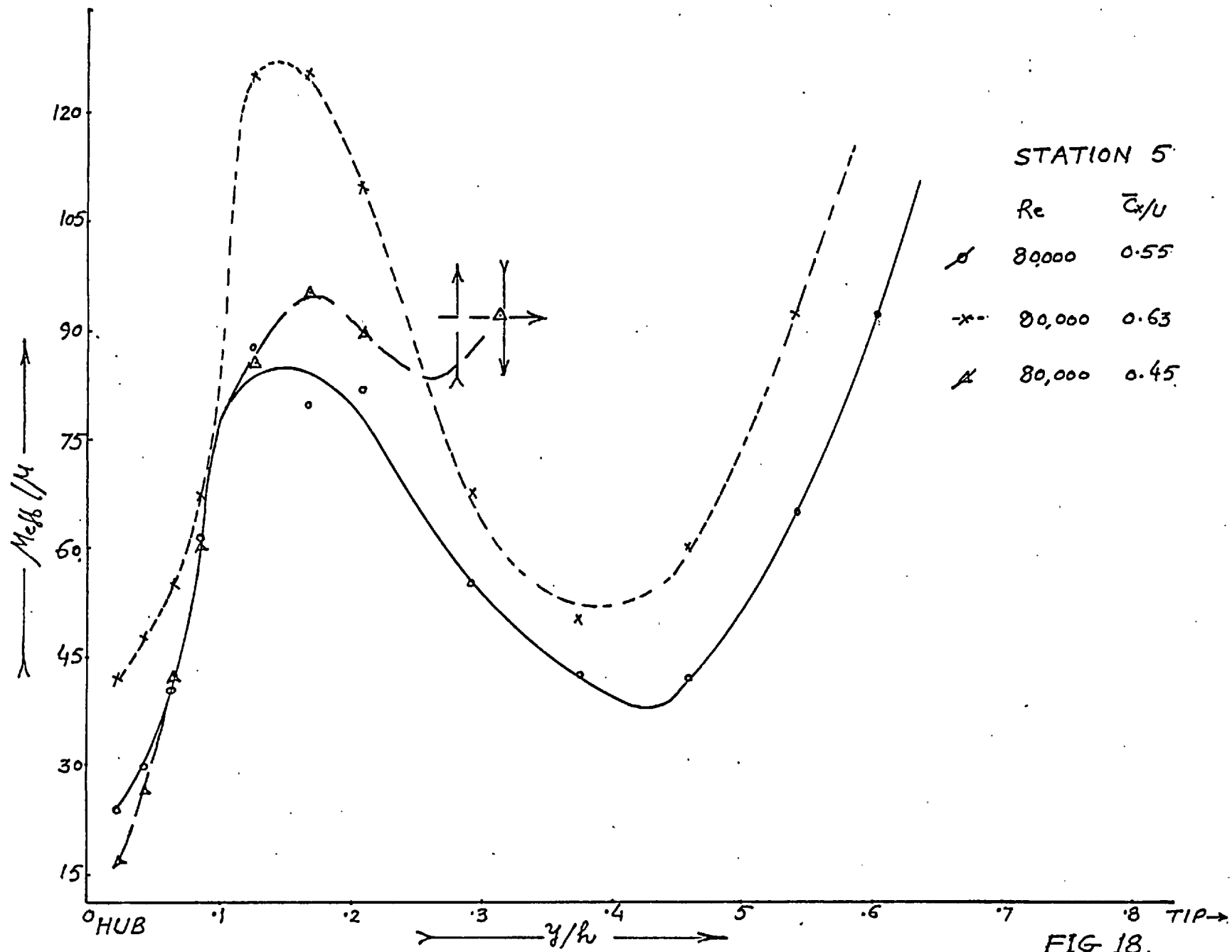


FIG. 17



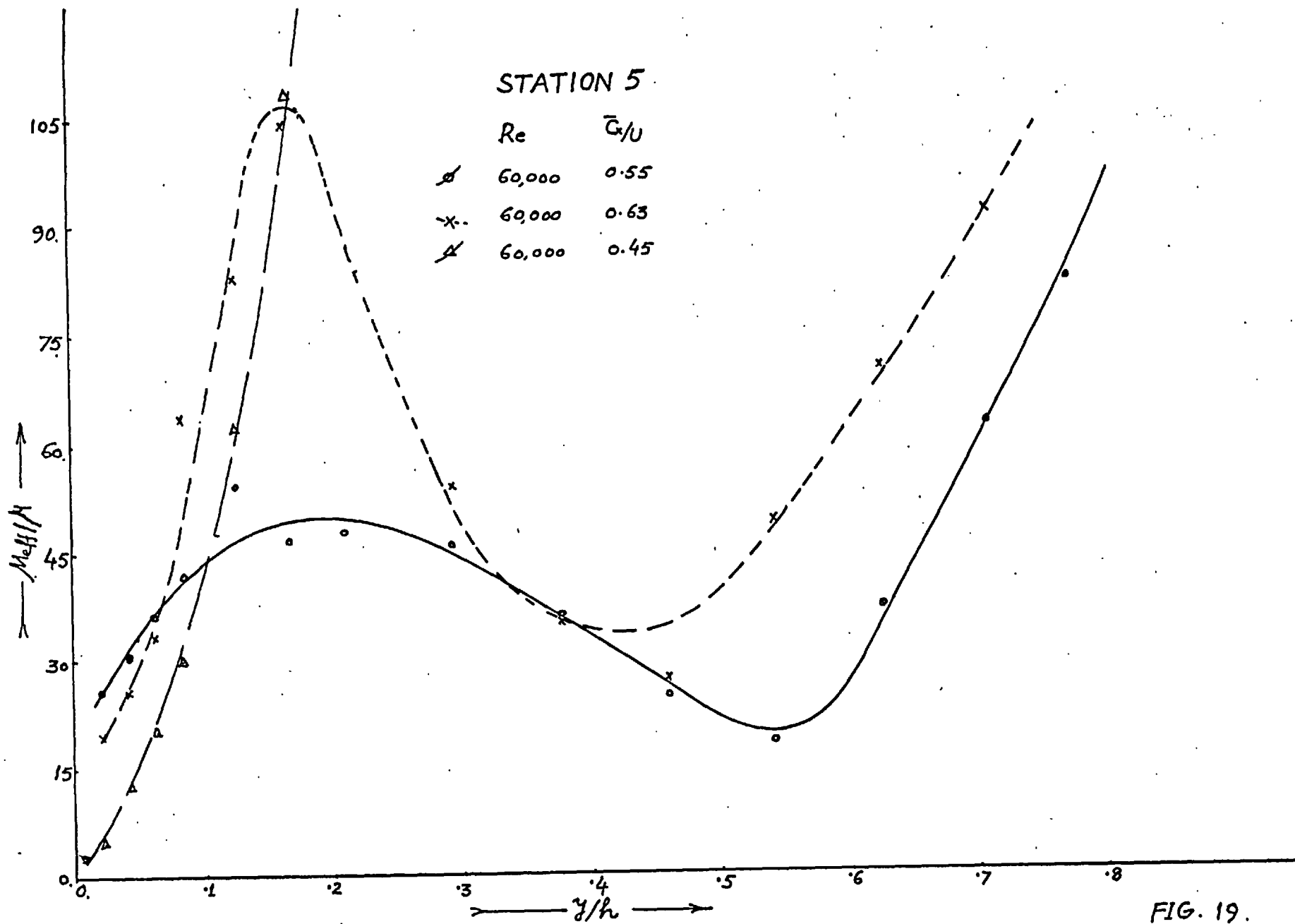


FIG. 19.

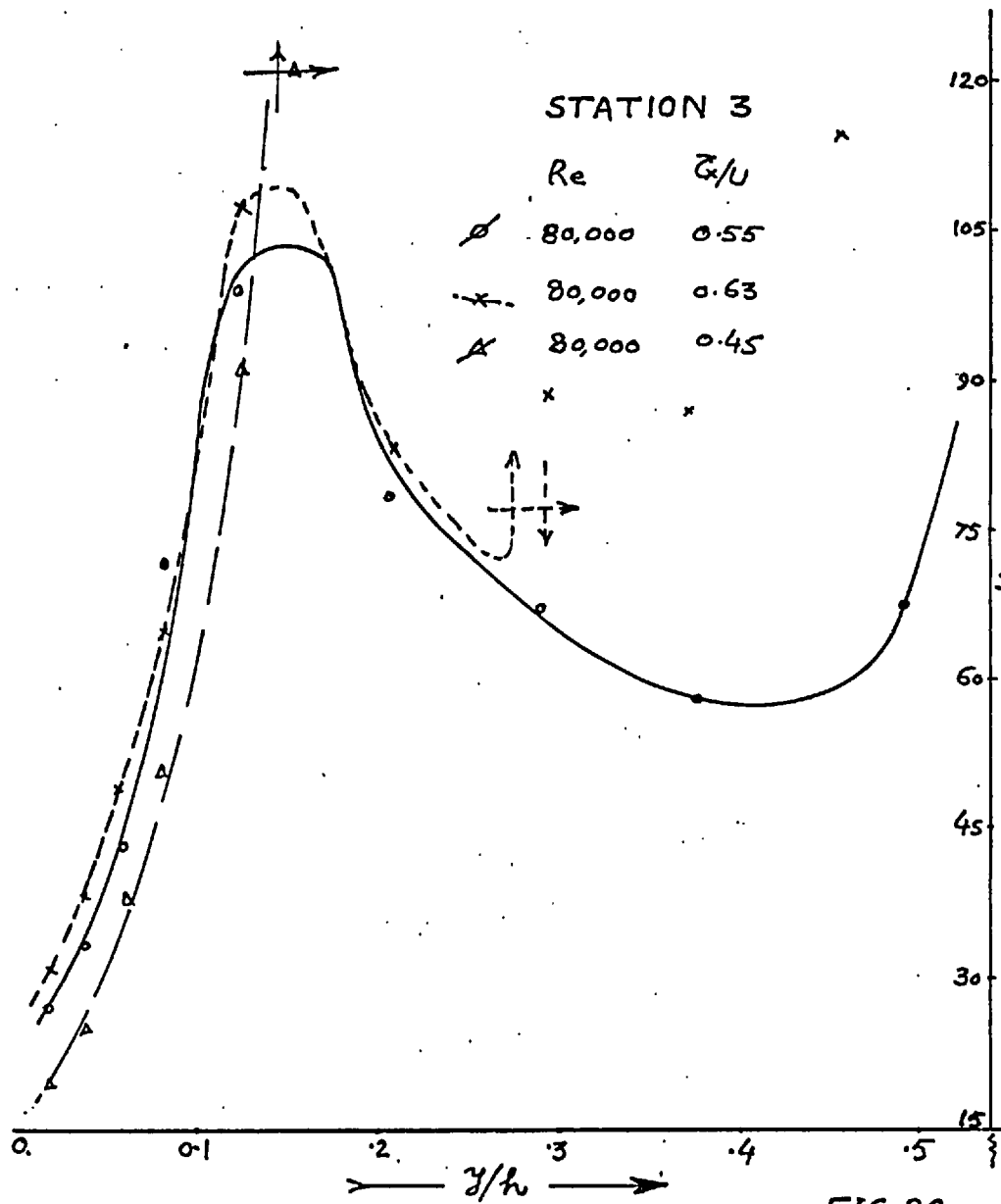


FIG. 20

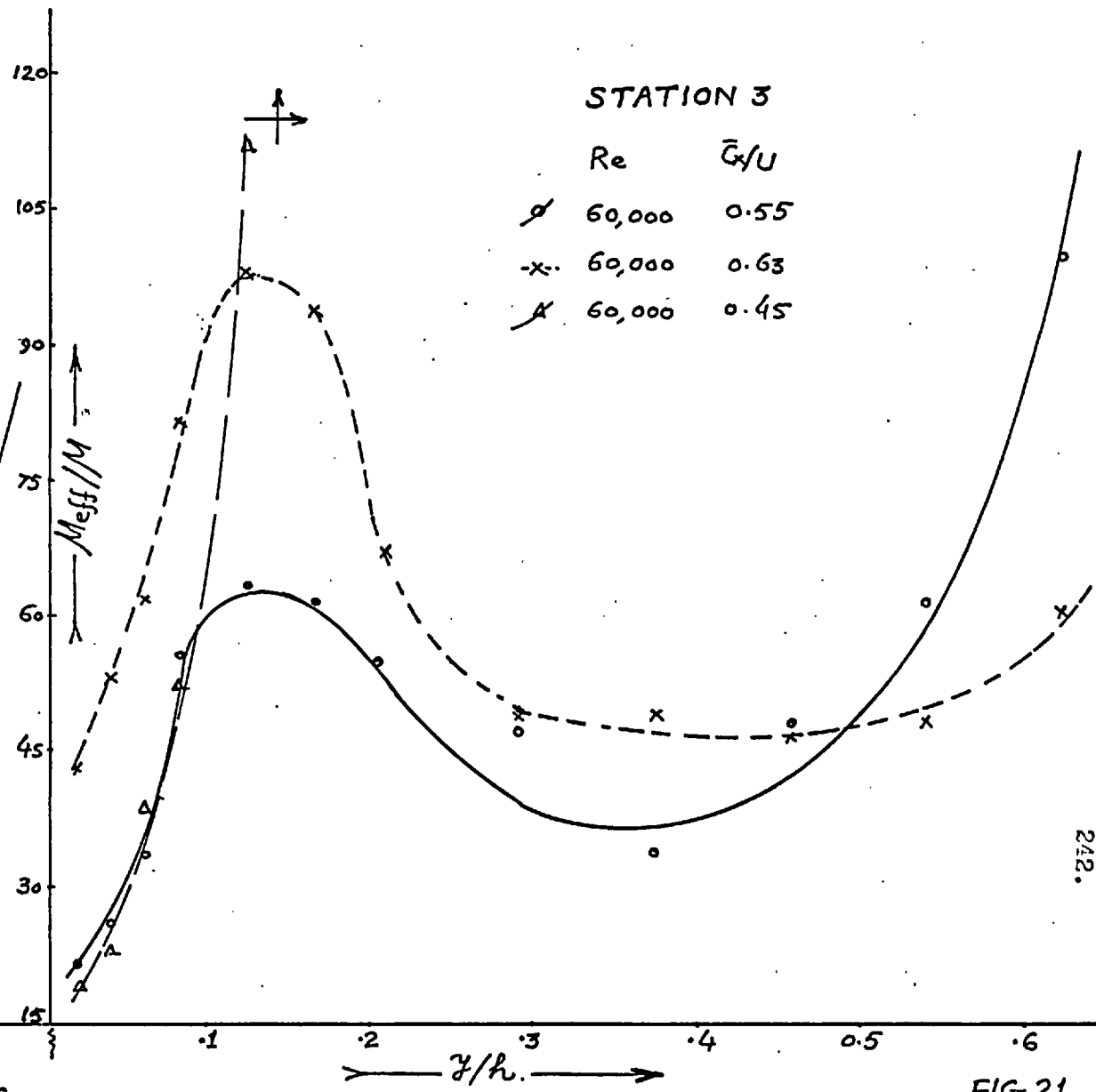


FIG. 21.

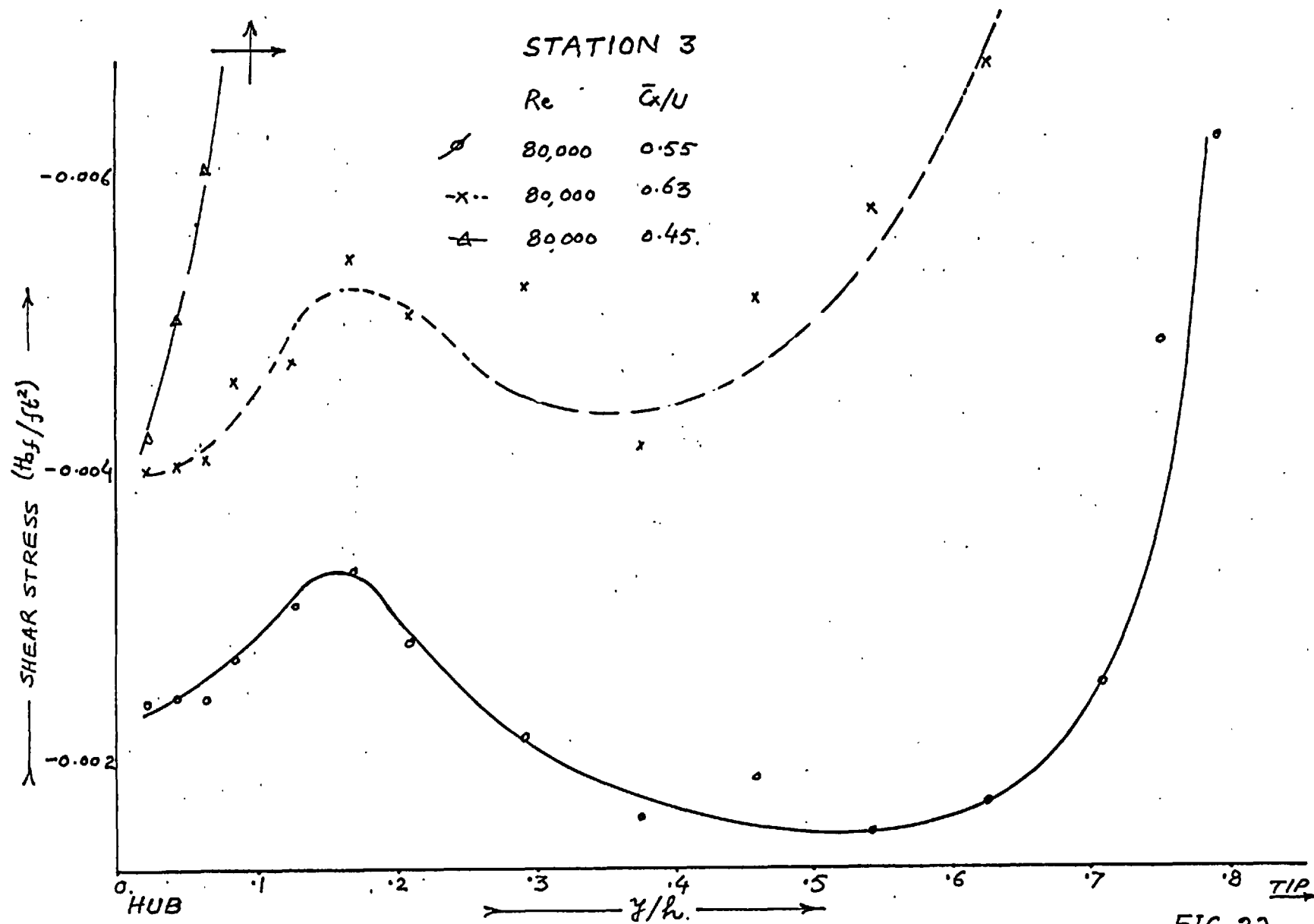


FIG. 22.

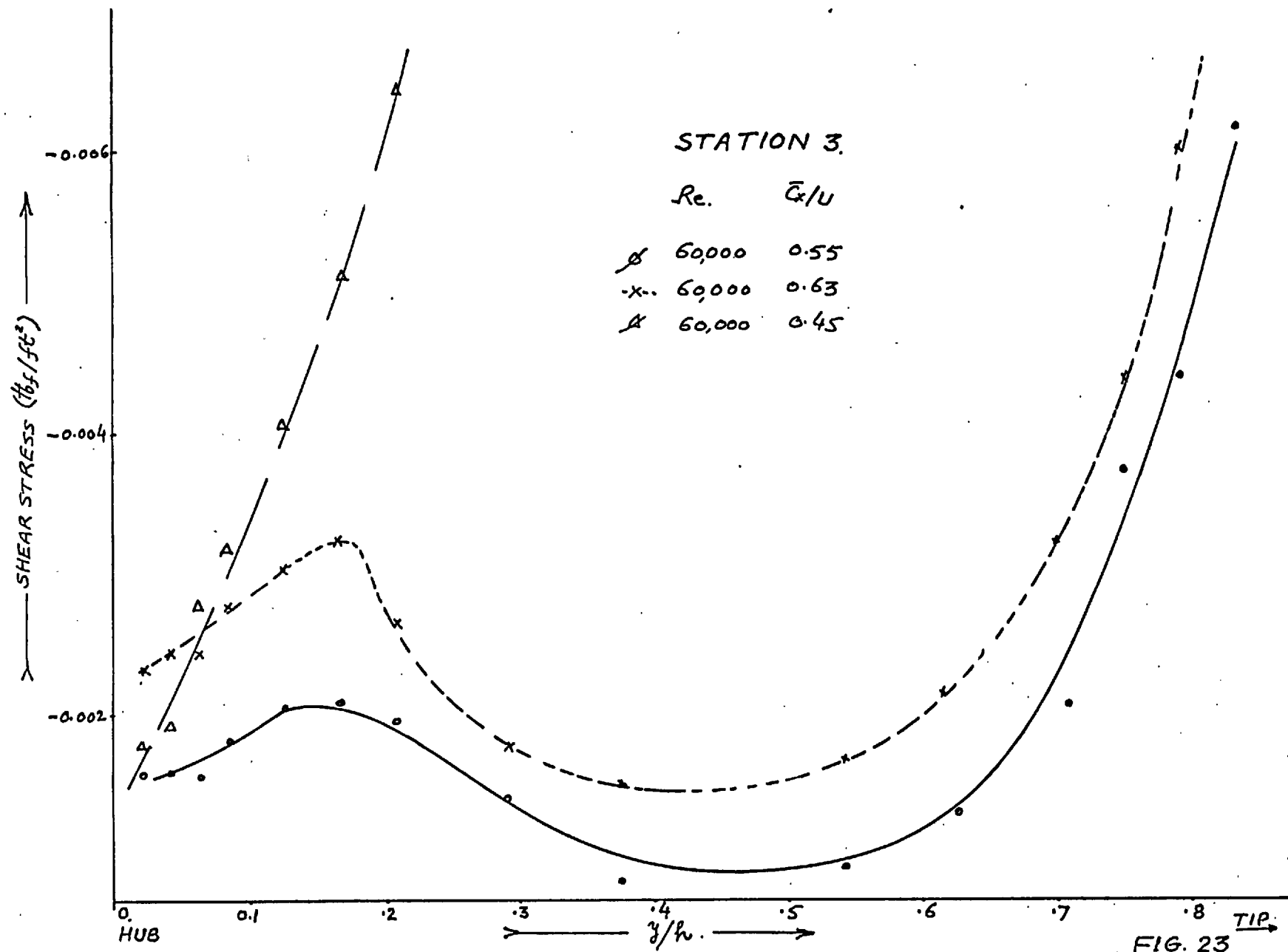
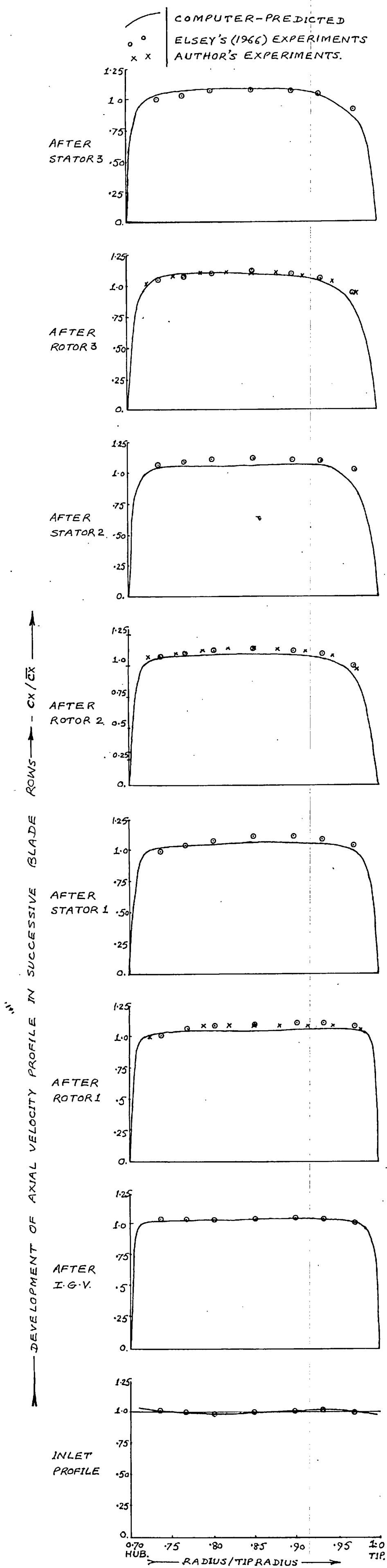


FIG. 23



OPERATION CLOSE TO DESIGN

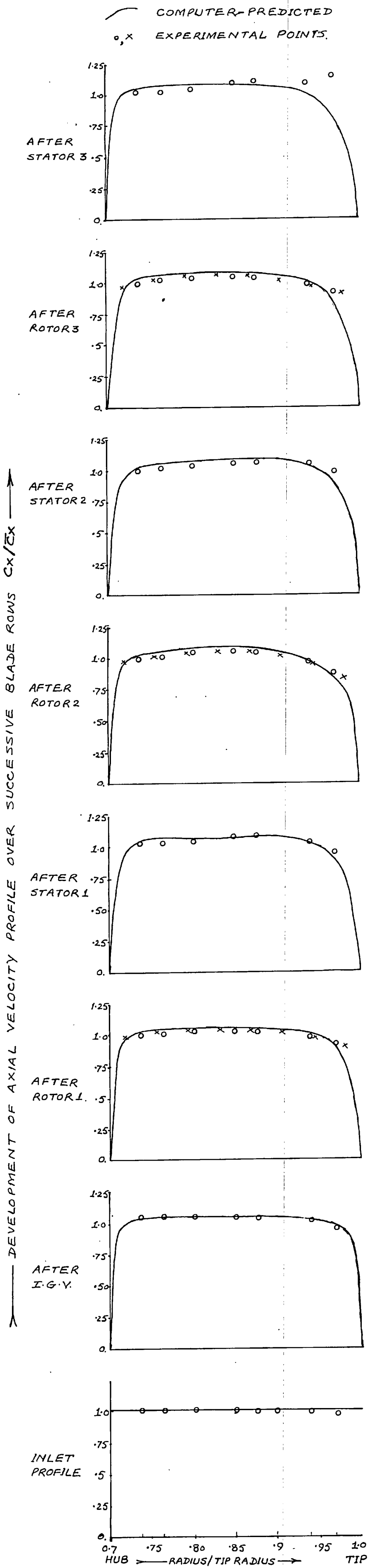
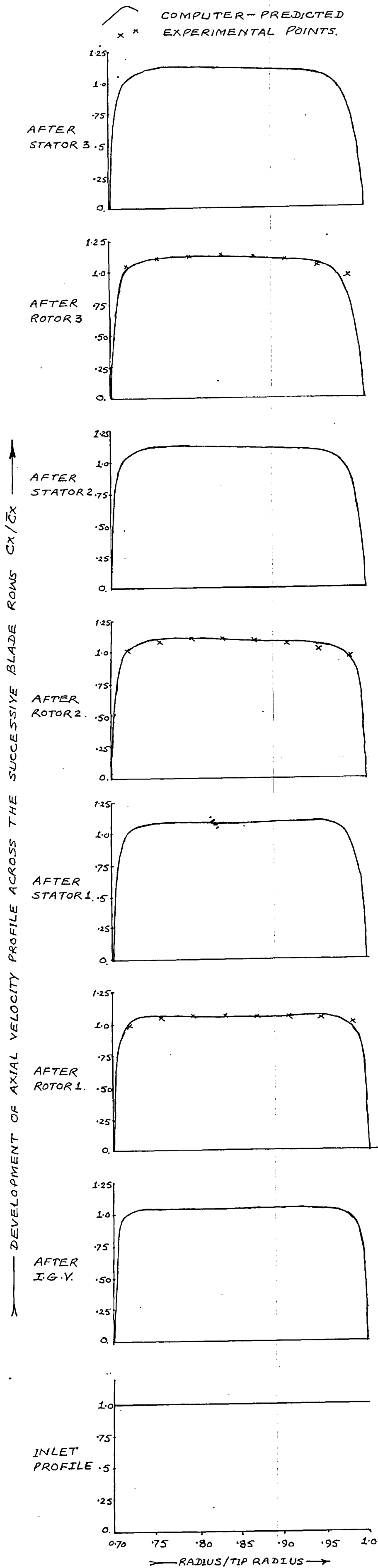
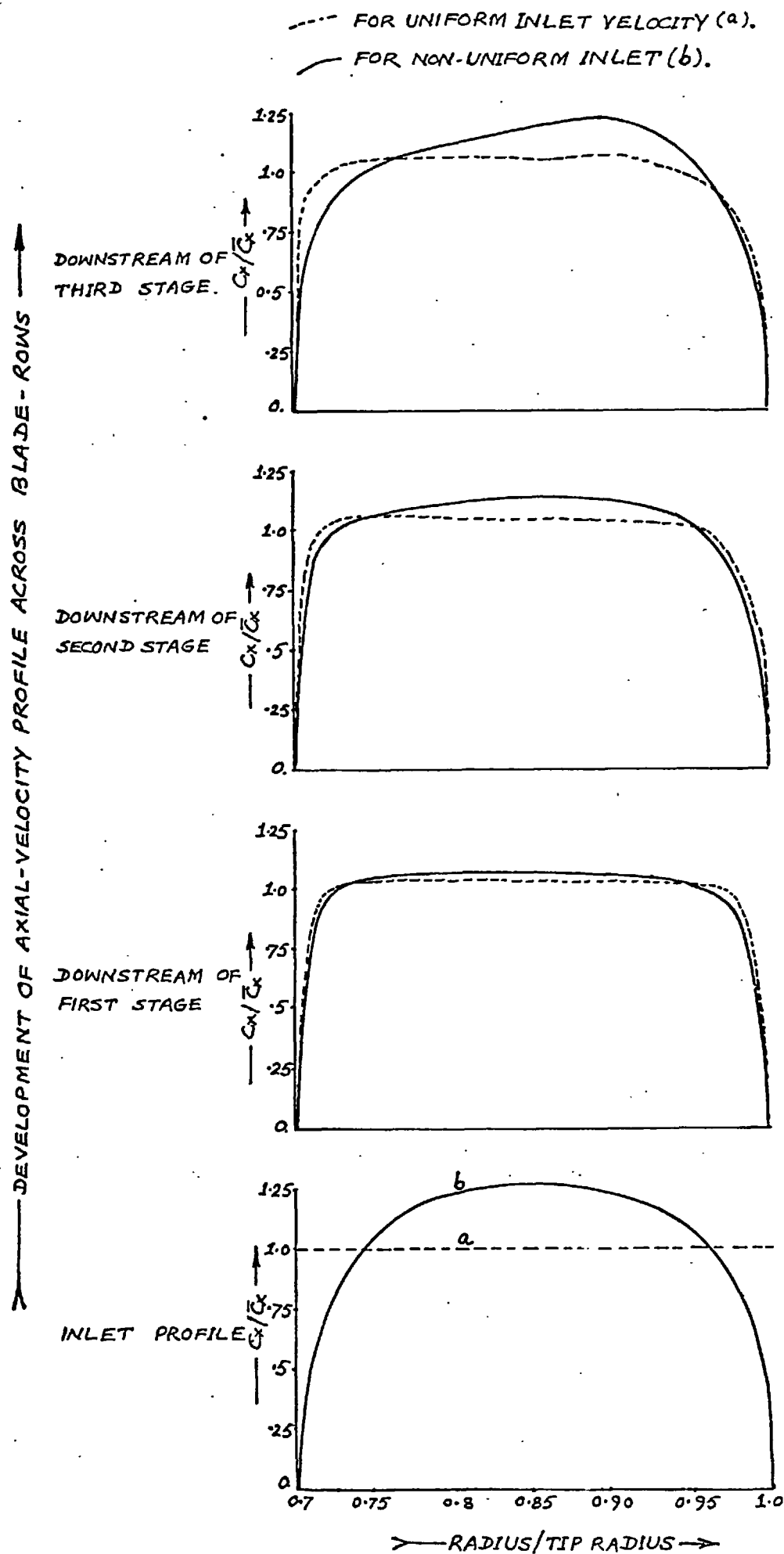


FIG. 25.



OPERATION AT HIGH MASS FLOW



EFFECT OF NON-UNIFORM INLET VELOCITY PROFILE
ON THE PERFORMANCE OF THE BLADE ROWS

FIG 27.