

**Stability design of steel structures by
second order inelastic analysis**

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Statement of Originality

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Abstract

The stability design of steel structures by second order inelastic analysis with strain limits is addressed in this thesis. In the proposed design approach, a geometrically and materially nonlinear analysis with imperfections (GMNIA) of members and frames is carried out and the ultimate strength of the structure is signified by reaching either the strain limit defined according to the Continuous Strength Method (simulating cross-section failure) or the peak load factor, whichever occurs first. The method has been established for the in-plane design of prismatic structural components. In the present thesis, its scope is extended to the in-plane design of members with three-flanged cross-sections (representing the haunch and apex regions of portal frames) and web-tapered members, subjected to combined loading conditions ranging from pure compression to pure uniform and non-uniform bending. Explicit formulae, which have been developed previously to predict the elastic local buckling stress of single I-sections, are first extended to cover the case of three-flanged cross-sections, subjected to combined loading conditions ranging from pure compression to pure major axis bending. The predictions are shown to be more accurate than the calculations performed on an element-by-element basis – the way in which the local buckling of cross-sections is typically treated in current structural steel design specifications.

Attention then turns to the development of the proposed design approach for scenarios in which out-of-plane stability effects, with a focus on lateral-torsional buckling, govern. First, equivalent imperfections for the out-of-stability design of steel and stainless steel members by second order inelastic analysis are established. Then, the existing strain limits for in-plane design are extended to allow for the influence of shear, torsion and warping, to enable their general applicability to three-dimensional buckling problems. It is shown that the proposed design method provides more accurate and consistent ultimate strength predictions than traditional design methods and also streamlines the design process by eliminating the need for cross-section classification and individual member design checks.

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List of Publications

The peer-reviewed journal articles and conference proceeding papers that have originated from the work presented in this thesis are listed below:

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Notation

A	Cross-sectional area
A_{d+s}	Area of three-flanged cross-section
A_v	Shear area
B	Warping bimoment
b	Width of section; width of plate; mean value correction factor
B_{el} and B_{pl}	Elastic and plastic warping bimoment resistances, respectively
CoV	Coefficient of variation
C_1 and C_2	Coefficients of quad-linear material model
E	Young's modulus
E_{sh}	Strain hardening modulus
E_y	Tangent modulus at 0.2% proof stress
e_0	Amplitude of equivalent imperfection for flexural buckling design; equivalent imperfection amplitude
$e_{0,bow}$	Amplitude of equivalent imperfection applied through direct modelling as lateral bow imperfection
$e_{0,bow,1}$	Amplitude of half-sine wave imperfection
$e_{0,bow,2}$	Amplitude of full-sine wave imperfection
$e_{0,LT}$	Amplitude of equivalent imperfection for lateral-torsional buckling design
$e_{0,mod}$	Amplitude of equivalent imperfection applied through scaling of the first lateral-torsional buckling eigenmode
$e_{0,req}$	Required equivalent imperfection amplitude
f_M	Factor to account for influence of bending moment diagram
f_u	Ultimate strength
f_y	Yield strength

$f_{y,\text{mean}}$	Mean yield strength
$f_{y,\text{nom}}$	Nominal yield strength
G	Shear modulus
h	Depth of section
h_d	Depth of deeper I-section; depth of the section at the deep end of tapered member
h_s	Depth of shallower I-section (deducting a flange thickness); depth of the section at the shallow end of tapered member
h_w	Clear depth of web
I_{d+s}	Major axis second moment of area of three-flanged cross-section
I_t	St. Venant torsion constant
j_{req}	Required value of non-dimensional equivalent imperfection
k	Plate buckling coefficient
k_{dw}	Buckling coefficient of web within deeper I-section
k_{dw+sw}	Buckling coefficient of web within three-flanged cross-section
L	Member length
L_{cr}	Local buckling half-wavelength
$L_{b,cs}$	Local buckling half-wavelength of full cross-section
M	Bending moment
M_{bow}	Ultimate bending moment capacity obtained using equivalent imperfection $e_{0,\text{bow}}$
$M_{b,Rd}$	Design buckling resistance moment
M_{cr}	Elastic critical buckling moment
$M_{c,Rk}$	Characteristic cross-section bending moment resistance
M_d and M_s	Bending moments applied at deep end and at shallow end of tapered member, respectively
M_{EC3}	prEN 1993-1-1 member buckling resistance
M_{eff}	Effective moment capacity
M_{el}	Elastic moment capacity

$M_{el,y}$ and $M_{el,z}$	Elastic major and minor axis bending moment resistances, respectively
M_{FE}	Ultimate bending moment capacity obtained from benchmark beam FE models
M_{fl}	Cross-section major axis bending moment resistance considering flanges alone
M_{mod}	Ultimate bending moment capacity obtained using equivalent imperfection $e_{0,mod}$
M_{peak}	Peak bending moment
M_{pl}	Plastic bending moment capacity
$M_{pl,d}$ and $M_{pl,s}$	Plastic bending moment capacities at the deep end and at the shallow end of tapered member, respectively
$M_{pl,y}$ and $M_{pl,z}$	Plastic major axis and minor axis bending moment resistances
M_u	Ultimate bending moment capacity
$M_{u,EC3}$	Ultimate bending moment capacity obtained from traditional design method provided in Eurocode 3
$M_{u,d}$ and $M_{u,s}$	Ultimate bending moments at the deep end and at the shallow end of tapered member
$M_{u,prop}$	Ultimate bending moment capacity obtained from proposed design method
$M_{u,shell}$	Ultimate bending moment capacity obtained from shell FE models
$M_{u,test}$	Ultimate bending moment capacity obtained from tests
M_w	Cross-section minor axis bending moment resistance considering web alone
M_y and M_z	Major axis bending moment and minor axis bending moment
$M_{y,Ed}$	Applied design major axis bending moment
m and n	Ramberg-Osgood strain hardening exponents
N	Applied axial load
N_{cr}	Euler buckling load
N_{Ed}	Applied design axial load
N_{pl}	Cross-section axial yield load
$N_{pl,s}$	Axial yield load of the shallow end cross-section of tapered member
N_u	Ultimate compressive capacity

$N_{u,prop}$	Ultimate compressive capacity obtained from proposed design method
$N_{u,shell}$	Ultimate compressive capacity obtained from shell FE models
r_e	Benchmark resistance
r_t	Predicted resistance
T_{Ed}	Total internal torsion moment
T_{St}	St. Venant torsion
$T_{St,Ed}$	Internal St. Venant torsion
$T_{St,Rk}$	Cross-section St. Venant torsion resistance
T_w	Warping torsion
$T_{w,Ed}$	Internal warping torsion
$T_{w,el}$ and $T_{w,pl}$	Elastic and plastic warping torsion resistances, respectively
$T_{w,Rk}$	Cross-section warping torsion resistance
t	Plate thickness
t_f and t_w	Flange thickness and web thickness
V_E	CoV of Young's modulus
V_{fy}	CoV of yield strength
$V_{pl,y}$ and $V_{pl,z}$	Plastic cross-section vertical and lateral shear force capacities
$V_{pl,y,T}$	Reduced plastic vertical shear resistance considering influence of torsion
V_r	Combined CoV incorporating variability of model and basic variables
V_{rt}	Combined CoV of material and geometric basic variables
V_w	Flange shear force due to warping torsion
V_{Wpl}	CoV of geometric properties
$V_{y,Ed}$ and $V_{z,Ed}$	Design vertical shear force and design lateral shear force
$V_{y,Rk}$ and $V_{z,Rk}$	Vertical and lateral cross-section shear resistances
V_{δ}	CoV of results obtained using proposals relative to benchmark FE results
$W_{eff,y}$	Effective major axis section modulus
$W_{el,y}$ and $W_{el,z}$	Elastic major axis section modulus and minor axis section modulus
$W_{pl,y}$ and $W_{pl,z}$	Plastic major axis section modulus and minor axis section modulus

W_y and W_z	Major axis section modulus and minor axis section modulus
$\alpha_{cr,d}$	Critical load factor of deeper section
$\alpha_{cr,s}$	Critical load factor of shallower section
$\alpha_{cr,d+s}$	Critical load factor of three-flanged cross-section
α_{LT}	Imperfection factor for lateral-torsional buckling
α_{csm}	Load factor corresponding to attainment of CSM strain limit
α_{peak}	Peak load factor
α_{Rd}	Design resistance factor
α_{Rk}	Characteristic resistance factor
α_u	Ultimate load factor
$\alpha_{u,EC3}$	Ultimate load carrying capacity obtained from traditional design method provided in Eurocode 3
$\alpha_{u,FE}$	Ultimate load carrying capacity obtained from FE models
$\alpha_{u,prop}$	Ultimate load carrying capacity obtained from proposed design method
$\alpha_{u,shell}$	Ultimate load carrying capacity obtained from shell FE models
$\alpha_{u,test}$	Ultimate load carrying capacity obtained from tests
α_z	Imperfection factor for minor axis buckling
β	Reference relative bow imperfection for flexural buckling
β_{LT}	Reference relative imperfection for lateral-torsional buckling
β_{req}	Required value of the reference relative imperfection
γ	Ratio of critical load factor of the shallower section to that of the deeper section; taper ratio
γ_{M1}	Partial safety factor for member buckling resistances
γ_{M1}^*	Required partial safety factor for member buckling resistances
ε	Strain; material parameter
ε_{csm}	Strain limit determined from Continuous Strength Method (CSM)
$\varepsilon_{csm,V}$	Reduced CSM strain limit to take account of interaction between bending, shear and torsion
ε_{Ed}	Design value of compressive strain

$\varepsilon_{Ed,av}$	Average design value of compressive strain
ε_{sh}	Strain hardening strain
ε_y	Yield strain
ε_u	Ultimate strain
ε_w	Warping torsion normal strain
$\varepsilon_{0.2}$	Total strain at yield strength (0.2% proof stress)
ζ	Element interaction coefficient
ζ_d and ζ_s	Element interaction coefficients within deeper and shallower I-sections
ζ_{d+s}	Compound element interaction coefficient of three-flanged cross-section
ζ_d^*	Coefficient to represent influence of deeper section on critical shallower section
ζ_s^*	Coefficient to represent influence of shallower section on critical deeper section
ζ^{Eq}	Element interaction coefficient obtained from equations
ζ^{FSA}	Element interaction coefficient obtained from finite strip analysis
η_y and η_z	Coefficients to present utilisation of a cross-section under combined shear and torsion for shear forces acting through cross-section depth and cross-section width, respectively
$\bar{\lambda}_{LT}$	Member slenderness for lateral-torsional buckling
$\bar{\lambda}_p$	Cross-section slenderness
$\bar{\lambda}_{p,cr}$	Cross-section slenderness of critical cross-section
$\bar{\lambda}_y$ and $\bar{\lambda}_z$	Member slendernesses for major axis and minor axis flexural buckling
$\mu_{y,fl}$	Ratio of cross-section major axis bending moment resistance considering flanges alone to elastic major axis bending resistance of full cross-section
$\mu_{z,w}$	Ratio of cross-section minor axis bending moment resistance considering web alone to elastic minor axis bending resistance of full cross-section
ν	Poisson's ratio
ρ	Shear reduction factor
ρ_{csm}	Reduction factor applied to CSM strain limit to consider effect from high shear force

ρ_y and ρ_z	Reduction factors to allow for influence of high vertical shear and high lateral shear on ultimate cross-section resistance, respectively
σ	Stress
$\sigma_{cr,cs}$	Elastic local buckling stress of cross-section
$\sigma_{cr,d}$ and $\sigma_{cr,s}$	Elastic local buckling stresses of deeper and shallower I-sections
$\sigma_{cr,f}$ and $\sigma_{cr,w}$	Elastic local buckling stresses of flange and web, respectively
$\sigma_{cr,d+s}$	Elastic local buckling stress of three-flanged cross-section
$\sigma_{cr,p}$	Elastic local buckling stress of plate
$\sigma_{Ed,max}$	Maximum stress in cross-section
σ_w	Warping torsion normal stress
σ_1 and σ_2	Maximum compressive stress and minimum compressive stress
$\sigma_{cr,cs}^{Eq}$	Elastic local buckling stress obtained from equations
$\sigma_{cr,cs}^{FSA}$	Elastic local buckling stress obtained from finite strip analysis
$\sigma_{cr,f}^F$ and $\sigma_{cr,f}^{SS}$	Elastic local buckling stresses of flange with fixed and simply-supported boundary conditions along the adjoined edges, respectively
$\sigma_{cr,p}^F$ and $\sigma_{cr,p}^{SS}$	Elastic local buckling stresses of isolated plate with fixed and simply-supported boundary conditions along the adjoined edges, respectively
$\sigma_{cr,w}^F$ and $\sigma_{cr,w}^{SS}$	Elastic local buckling stresses of web with fixed and simply-supported boundary conditions along the adjoined edges, respectively
τ_{St} and τ_w	St. Venant torsion shear stress and warping torsion shear stress
$\tau_{St,Ed}$ and $\tau_{w,Ed}$	Shear stresses due to St. Venant torsion and due to warping torsion
τ_y	Shear yield strength
$\tau_{y,Ed}$ and $\tau_{z,Ed}$	Shear stresses due to vertical shear force and due to lateral shear force
ϕ	Ratio of elastic local buckling stress of isolated and simply-supported flange to that of web
χ_{LT}	Lateral-torsional buckling reduction factor
ψ	Stress distribution ratio of plate or cross-section; ratio of bending moment applied at two ends of member
ψ_f and ψ_w	Stress distribution ratio of flange and stress distribution ratio of web
ψ_{dw+sw}	Stress distribution ratio over depth of three-flanged cross-section

Chapter 1

Introduction

1.1 Background

In current structural design specifications, such as EN 1993-1-1 (2005), the stability of members is typically assessed through buckling curves. Alternatively, the stability design of members can be performed more directly using either elastic or inelastic second order analysis, also referred to as geometrically nonlinear or advanced analysis, through finite element (FE) modelling.

With improvements in computational efficiency, second order inelastic analysis, also referred to as geometrically and materially nonlinear analysis with imperfections (GMNIA) or advanced inelastic analysis, typically implemented using beam finite elements, is being increasingly employed in the design of steel structures. This approach brings a number of advantages, including (i) the need for carrying out individual member design checks can be avoided, (ii) global buckling behaviour e.g. flexural buckling and lateral-torsional buckling (LTB) can be directly and accurately captured and (iii) the structural failure modes can be explicitly visualised. Beam elements are highly computationally efficient, but are not able to capture the influence of local buckling on the cross-section resistance and rotation capacity of steel members. In current structural steel design codes, the influence of local buckling on the response of steel members is typically considered through the concept of cross-section classification, which restricts the cross-section resistance (e.g. plastic, elastic or effective moment capacity in bending) and the structural analysis type (i.e. plastic or elastic) on the basis of the classes of the cross-sections of the members making up the structure. However, this approach is not only overly-simplistic but also generates artificial steps in the resulting resistance predictions.

Recently, a new structural steel design approach utilising advanced inelastic analysis, together with the adoption of strain limits from the Continuous Strength Method (CSM) (Gardner, 2008) to capture the influence of local buckling on the resistance of the cross-sections, has been put forward (Gardner et al., 2019a). According to the proposed design approach, a GMNIA of the structure is performed using beam finite elements, considering explicitly buckling of the members and frame, the influence of imperfections and the spread of plasticity throughout the structure. The ultimate load carrying capacity of the structure is taken as either (i) the peak load factor obtained from the analysis or (ii) the load factor at which the CSM strain limit is attained at any cross-section in the considered member or structure, whichever occurs first. The accuracy and reliability of the proposed approach of design by GMNIA with strain limits have been verified for carbon steel (Fieber et al., 2019a; Fieber et al., 2020) and stainless steel structures (Walport et al., 2020a). However, the approach is currently limited to structures that feature prismatic members and are not susceptible to out-of-plane instability effects, thus limiting its scope of applicability.

In the present thesis, the scope of the method is extended to the in-plane design of members with three-flanged cross-sections (representing the haunch and apex regions of portal frames) and web-tapered members. Explicit formulae for predicting the elastic local buckling stress of three-flanged cross-sections are first developed. Attention then turns to the extension of the scope of the proposed design method to the scenarios in which out-of-plane stability effects, with a focus on LTB, govern. Equivalent imperfections for the out-of-stability design of steel and stainless steel members by GMNIA are also established.

1.2 Research scope and objectives

Overall, the aim of this research study is to develop safe and efficient methods for stability design of steel structures by second order inelastic analysis. More specifically, the objectives of this research include:

- To review traditional methods of analysis and stability design of steel structures, as given in current design specifications, and identify potential areas of improvement;
- To extend the existing formulae for calculating the elastic local buckling stress of standard I-sections under combined compression and major axis bending to a broader range of dimensions and stress distribution ratios;

- To investigate the elastic local buckling behaviour of three-flanged cross-sections and the interaction within and between the two I-sections composing the three-flanged cross-sections;
- To develop formulae for calculating the elastic local buckling stress of three-flanged cross-sections subjected to combined loading conditions ranging from pure compression to pure major axis bending, allowing for the effects of element interaction, and to assess the accuracy and safety of the developed formulae;
- To develop a method of in-plane design performed by second order inelastic analysis with CSM strain limits for web-tapered steel members subjected to combined loading conditions ranging from pure compression to uniform and non-uniform bending;
- To assess the accuracy of the proposed design method for tapered members against the results from shell finite element modelling as well as a number of experiments and the traditional design method provided in EN 1993-1-1, covering various taper ratios, geometrical properties, member slenderness values and loading conditions;
- To develop equivalent imperfections, which account for both geometric and material (residual stress) imperfections, for use in LTB design by second order inelastic analysis of steel and stainless steel members;
- To assess the accuracy of the proposed equivalent imperfections for use in LTB design against benchmark finite element results and the calculations of the traditional design method provided in prEN 1993-1-1, covering various material grades, geometrical properties, member slenderness values and loading conditions;
- To develop an approach to allow for the influence of shear, torsion and warping on the out-of-plane stability design of steel members utilising second order inelastic analysis with CSM strain limits;
- To assess the accuracy of the proposed method for LTB design against benchmark shell finite element results and the calculations of the traditional design method provided in prEN 1993-1-1;
- To provide recommendations for further research.

1.3 Thesis overview

This thesis comprises seven chapters. This chapter provides the general background information and research aims of this thesis.

In Chapter 2, a comprehensive review of the literature related to this research is presented, covering the essential knowledge of traditional methods provided in current structural design specifications and the proposed method for the stability design of steel structures.

In Chapter 3, existing formulae for calculating the elastic local buckling stress of single I-sections subjected to combined compression and major axis bending are extended to three-flanged sections. The predicted elastic local buckling stresses are compared with results obtained from finite strip analysis and with elastic buckling stresses calculated on an element-by-element basis (i.e. ignoring the influence of element interaction).

In Chapter 4, the proposed method of design by second order inelastic analysis with strain limits is extended to web-tapered steel members subjected to compression, bending and combined compression plus bending. The predictions obtained from the proposed design approach are compared against the results from nonlinear shell finite element modelling as well as a number of experiments, and also compared with the calculations obtained from the traditional design methods provided in design standards.

In Chapter 5, equivalent imperfections for the out-of-plane stability design of steel and stainless steel beams by second order inelastic analysis are proposed. The proposals are calibrated against benchmark finite element results and also compared with current design provisions.

In Chapter 6, the proposed method of design by second order inelastic analysis with strain limits is extended to scenarios in which out-of-plane stability effects, with a focus on LTB, govern. The existing strain limits for in-plane design are extended to allow for the influence of shear, torsion and warping. The accuracy and safety of the proposed method are assessed against the results of nonlinear shell finite element modelling and also compared against the calculations with reference to current design guidance.

Finally, in Chapter 7, the key findings and conclusions of this research and suggestions for future work are summarised.

Chapter 2

Literature review

2.1 Introduction

In this chapter, a review of the background literature and previous work related to this thesis is presented. The traditional design methods of steel structures are first discussed, at both cross-section and member levels. Then, the key concepts and components of the design method by second order inelastic analysis with strain limits developed in this study are introduced. Further discussion on the literature is later elaborated within the relevant chapters.

2.2 Traditional steel design approaches

2.2.1 Cross-section resistance and cross-section classification concept

In current design specifications, such as EN 1993-1-1 (2005) and AISC 360-16 (2016), the cross-section resistances of steel members are determined with reference to the cross-section classification concept. Based on the susceptibility to local buckling, a cross-section can be classified into one of four classes, namely Class 1 (plastic), Class 2 (compact), Class 3 (semi-compact) and Class 4 (slender). With reference to the typical moment-curvature relationships shown in Figure 2.1, the following definitions are used in EN 1993-1-1 (2005): Class 1 cross-sections can attain their full plastic moment resistance M_{pl} and have sufficient rotation capacity ($R = \theta_{rot}/\theta_{pl} - 1 > 3$) for plastic design, where θ_{rot} is the limiting rotation at which the moment resistance falls back below M_{pl} and θ_{pl} is the elastic component of rotation upon reaching M_{pl} . Class 2 cross-sections can also reach their full plastic moment capacity M_{pl} but, because of their limited rotation capacity, may only be used in elastic design (i.e. no allowance is made for plastic redistribution). For Class 3 cross-sections, the elastic moment capacity M_{el} may be used as the bending resistance, while for Class 4 cross-sections, only the effective bending moment

resistance M_{eff} may be used, which is calculated according to the effective width method given in EN 1993-1-5 (2005). Although it is straightforward to apply, there are a number of shortcomings with the cross-section classification concept.

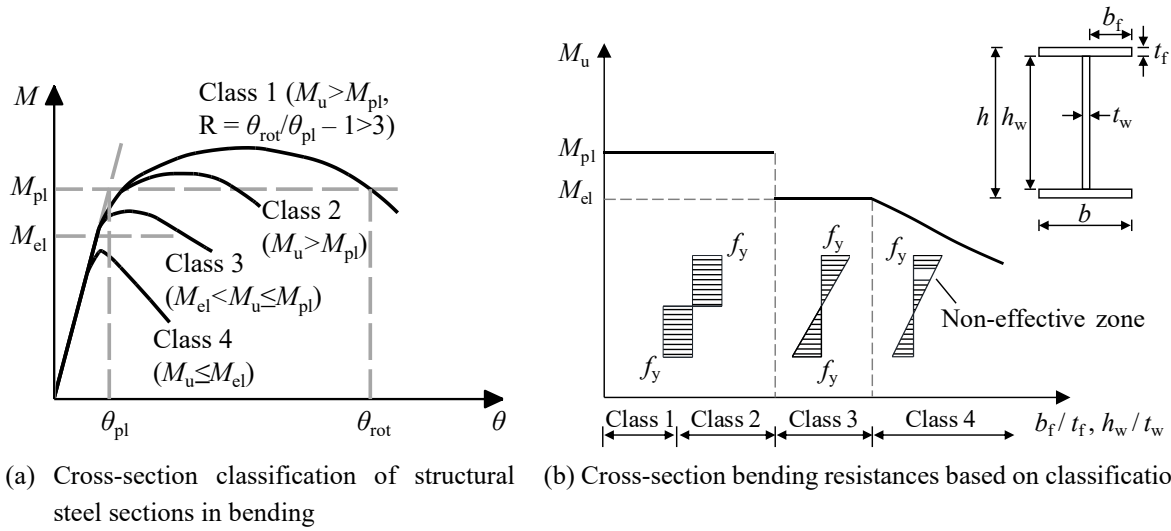


Figure 2.1 Cross-section classification and corresponding idealised bending resistances according to EN 1993-1-1 (2005)

- The first shortcoming is that classification is carried out only by considering the width-to-thickness ratio of the most critical plate element within the cross-section by conservatively assuming the web-to-flange junctions to provide simply-supported boundary conditions to the individual plates, thus ignoring the beneficial effect of element interaction (Gardner et al., 2019b).
- The second shortcoming relates to the lack of consideration for partial plasticity in Class 3 (semi-compact) cross-sections, though following recent research (SEMI-COMP, 2009; Meng & Gardner, 2020c; Meng et al., 2020), the resulting artificial step in cross-section resistance has been overcome through the definition of an elasto-plastic bending resistance. This is due to be incorporated into the next revision to EN 1993-1-1 (2005).
- The third shortcoming is the lack of consideration given to material strain hardening in the determination of the resistance of stocky cross-sections, resulting in overly-conservative capacity predictions (Gardner et al., 2011).
- The fourth shortcoming relates to the inconsistent allowance for the beneficial effect of moment gradients on element stability. This beneficial effect is accounted for when considering member stability, but is disregarded when evaluating local cross-sectional stability. This is despite numerous studies (Driscoll Jr & Beedle, 1957; Lay & Galambos,

1964; Gioncu & Petcu, 1997; Gardner et al., 2010; Wang et al., 2016; Meng & Gardner, 2020b) showing improved bending resistances in beams tested in three-point bending (i.e. with a moment gradient) relative to those tested in four-point bending (i.e. with uniform moment in the central region).

2.2.2 Design of web-tapered members

Structural steel design requires any limit state that is not captured in the structural analysis to be assessed separately through strength and stability checks. It suffices to state that in current practice, the most widely used structural analysis types are first order (with or without amplification to simulate the effects of the deformed geometry at the overall frame level) or second order elastic analysis. Since only the influence of overall geometric nonlinearity is considered in such an approach, subsequent individual member design checks, against the corresponding internal forces, are required. The ultimate resistances of the members are determined using member design equations that incorporate the effects of geometric imperfections and residual stresses. For regular, prismatic members, member design equations given in steel design specifications are generally well established, accurate and straightforward to apply. However, for tapered steel elements, member design equations provided in steel design specifications (EN 1993-1-1, 2005; AISC 360-16, 2016) are typically less well established and less accurate, as shown in Marques et al. (2012), Marques et al. (2013), Kucukler et al. (2014), Kucukler & Gardner, (2018), Tankova et al. (2018a) and Tankova et al. (2018b). Other design methods, developed specifically for tapered steel members (Ermopoulos, 1997; Raftoyiannis & Ermopoulos, 2005) and yielding accurate resistance predictions also exist, but these methods are rather complex to apply, requiring a series of indirect steps, as outlined in Kaehler et al. (2011), Marques et al. (2012), Marques et al. (2013), Tankova (2018) and Simões da Silva et al. (2019), for the consideration of the tapered geometry and loading conditions on the structural response.

Design by second order elastic analysis, i.e. geometrically nonlinear analysis with imperfections (GNIA), of tapered steel members using beam finite elements with equivalent imperfections is also permitted in EN 1993-1-1 (2005). In this design approach, member out-of-straightness is explicitly modelled in the structural analysis using equivalent imperfection amplitudes that are greater than the maximum out-of-straightness tolerances provided in the European execution standard EN 1090-2 (2008) to allow for the influence of plasticity, residual

stresses and geometric imperfections on member strengths. Thus, this approach removes the need to apply member design equations and only requires cross-section strength checks, resulting in a very direct, though rather conservative (Kucukler & Gardner, 2018), design approach for tapered steel members.

2.2.3 Design of beams susceptible to lateral-torsional buckling (LTB)

Steel structures and their components are often susceptible to out-of-plane instability effects, such as LTB – see Figure 2.2. The influence of LTB on the resistance of steel beams is generally accounted for in design standards by either: (i) the application of a buckling reduction factor to the cross-section bending resistance (ECCS, 1976; Nethercot & Trahair, 1976; Yura et al., 1978) or (ii) the determination of the elastic buckling moment of the beam with a reduced stiffness (Trahair & Kitipornchai, 1972; Trahair & Hancock, 2004; Trahair, 2010; Kucukler et al., 2015a; Kucukler et al., 2015b; Kucukler & Gardner, 2019). The former approach has traditionally been adopted in structural steel design standards (AS 4100, 1998; EN 1993-1-1, 2005; AISC 360-16, 2016; prEN 1993-1-1, 2019) owing to its suitability for application through hand calculations (Kucukler et al., 2015b). In this section, the Eurocode provisions (EN 1993-1-1, 2005; EN 1993-1-4, 2015; prEN 1993-1-1, 2019; prEN 1993-1-4, 2020) for the LTB assessment of steel and stainless steel beams are described.

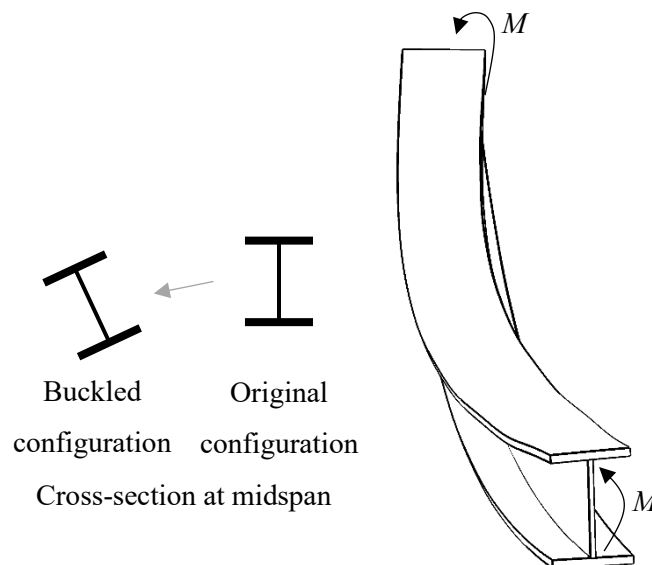


Figure 2.2 I-section beam undergoing lateral-torsional buckling under uniform bending

2.2.3.1 EN 1993-1-1 design rules for laterally-unrestrained beams

The current version of EN 1993-1-1 (2005) considers two cases for the LTB assessment of steel beams – (i) a general case and (ii) a specific case, for rolled sections or equivalent welded sections. In both cases, the member buckling resistance $M_{b,Rd}$ is given by Eq. (2.1):

$$M_{b,Rd} = \chi_{LT} \frac{M_{c,Rk}}{\gamma_{M1}} \quad (2.1)$$

where χ_{LT} is the buckling reduction factor, which considers the adverse effects of LTB on the ultimate cross-section resistance of a steel beam, γ_{M1} is the partial safety factor for member buckling, and $M_{c,Rk}$ is the characteristic (unfactored) cross-section bending moment resistance equal to the product of the yield stress f_y and the major axis section modulus W_y (i.e. $M_{c,Rk} = W_y f_y$); W_y is taken as the plastic section modulus $W_{pl,y}$ for Class 1 and 2 cross-sections, the elastic section modulus $W_{el,y}$ for Class 3 cross-sections and an effective section modulus $W_{eff,y}$ for Class 4 cross-sections.

a. General case

The general case is applicable to beams with any cross-section type. In this approach, the LTB reduction factor χ_{LT} is calculated as:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1 \quad (2.2)$$

where

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right], \quad (2.3)$$

in which α_{LT} is the imperfection factor determined on the basis of the cross-section depth-to-width (h/b) ratio and $\bar{\lambda}_{LT}$ is the non-dimensional slenderness, given by:

$$\bar{\lambda}_{LT} = \sqrt{M_{c,Rk} / M_{cr}}, \quad (2.4)$$

in which M_{cr} is the elastic critical buckling moment determined either through numerical methods using a linear buckling analysis (LBA) or using approximate equations, for example those given in technical report CEN TR 1993-1-103 (2017); the latter approach has been

adopted in this study. The current version of EN 1993-1-4 (2015) also adopts the buckling curves described above for the design of stainless steel beams, but with $\alpha_{LT} = 0.76$ (for welded I-sections) and a threshold (plateau) slenderness value of 0.4 instead of 0.2.

b. Specific case

The specific case is applicable to steel beams with rolled sections or equivalent welded sections. The LTB reduction factor χ_{LT} can be determined as follows:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}} \text{ but } \chi_{LT} \leq \begin{cases} 1 \\ 1/\bar{\lambda}_{LT}^2 \end{cases} \quad (2.5)$$

where

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} \left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0} \right) + \beta \bar{\lambda}_{LT}^2 \right] \quad (2.6)$$

in which β is a modification factor and $\bar{\lambda}_{LT,0}$ is the threshold (plateau) slenderness value, for which the value of 0.4 is recommended (EN 1993-1-1, 2005). For non-uniform bending cases, in addition to the consideration through M_{cr} , the applied LTB reduction factor may also be modified to take account of the full influence of the bending moment distribution (EN 1993-1-1, 2005). The formulation of the specific case is based on extensive experimental results (Byfield & Nethercot, 1998), statistical work (King, 2001) and numerical studies (Greiner et al., 2000).

Both the general case and specific case are modified Ayrton-Perry type formulae, calibrated for LTB through modification of the imperfection factor α_{LT} (Taras, 2010). For both hot-rolled and welded I-sections, the values of α_{LT} prescribed for use with Eqs. (2.5) and (2.6) are higher than those for Eqs. (2.2) and (2.3).

2.2.3.2 prEN 1993-1-1 and prEN 1993-1-4: Doubly symmetric I- and H-sections

Unlike the flexural buckling curves, the LTB curves outlined in Section 2.2.3.1 are not fully mechanically coherent (Taras & Greiner, 2010). To address this issue, Taras and Greiner (Taras, 2010; Taras & Greiner, 2010) proposed a new set of formulae for the LTB design of steel members, as given by Eqs. (2.7)-(2.8), that are due to be included in the upcoming prEN 1993-1-1 (2019),

$$\chi_{LT} = \frac{f_M}{\phi_{LT} + \sqrt{\phi_{LT}^2 - f_M \bar{\lambda}_{LT}^2}} \leq 1 \quad (2.7)$$

with

$$\phi_{LT} = 0.5 \left[1 + f_M \left(\left(\frac{\bar{\lambda}_{LT}}{\bar{\lambda}_z} \right)^2 \alpha_{LT} (\bar{\lambda}_z - 0.2) + \bar{\lambda}_{LT}^2 \right) \right] \quad (2.8)$$

where f_M is a factor that accounts for the influence of the bending moment diagram and $\bar{\lambda}_z$ is the normalised member slenderness for minor axis flexural buckling. The definition of the imperfection factor α_{LT} was also amended, becoming a function of the major $W_{el,y}$ and minor axis $W_{el,z}$ elastic section moduli of the beam cross-section, as listed in Table 2.1.

Table 2.1 Imperfection factor α_{LT} for lateral-torsional buckling of doubly symmetric I- and H-sections (prEN 1993-1-1, 2019; prEN 1993-1-4, 2020)

Material	Grade	Limits	α_{LT}
Hot-rolled steel	-	$t_f \leq 40 \text{ mm}$	$0.12 \sqrt{W_{el,y} / W_{el,z}} \leq 0.34$
		$h/b > 1.2$	
		$t_f > 40 \text{ mm}$	$0.16 \sqrt{W_{el,y} / W_{el,z}} \leq 0.49$
		$h/b \leq 1.2$	$0.16 \sqrt{W_{el,y} / W_{el,z}} \leq 0.49$
Stainless steel	Austenitic	-	$0.37 \sqrt{W_{el,y} / W_{el,z}} \leq 1.10$
	Duplex	-	$0.23 \sqrt{W_{el,y} / W_{el,z}} \leq 0.76$
	Ferritic	-	$0.27 \sqrt{W_{el,y} / W_{el,z}} \leq 0.76$

The new provisions shift the transition point between buckling curves (i.e. α_{LT} value) from $h/b = 2.0$ to 1.2 to reflect the h/b ratio at which a change in residual stress magnitude is assumed (ECCS, 1984), while the geometric factor $\sqrt{W_{el,y} / W_{el,z}}$ considers the influence of varying cross-section geometry and reduces the scatter of the capacity predictions (Taras, 2010; Taras & Greiner, 2010). An upper limit is also defined, taken as the imperfection factor for minor axis buckling α_z , on the basis that the compression flange of a very deep section with (comparatively) low torsional rigidity should behave similarly to a column undergoing minor axis buckling (Taras, 2010).

Taking the same expressions (i.e. Eqs. (2.7) and (2.8)), Fortan and Rossi (2020) determined suitable α_{LT} values for application in the LTB design of stainless steel members; the proposals,

given by Table 2.1, are due to be included, together with the indicated upper bounds, in the upcoming Eurocode prEN 1993-1-4 (2020).

2.3 The Continuous Strength Method (CSM)

2.3.1 CSM base curve and cross-section slenderness

The Continuous Strength Method (CSM) is a deformation-based approach to structural design that allows a continuous, rational and accurate treatment of material nonlinearity i.e. the partial spread of plasticity, strain hardening and inelastic force/moment redistribution, depending on the cross-section slenderness. The CSM was proposed by Gardner (2008), and has been applied to the design of stainless steel (Ashraf et al., 2008; Afshan & Gardner, 2013), carbon steel (Yun et al., 2018a; Yun et al., 2018b) and aluminium alloy (Ashraf & Young, 2011; Su et al., 2014; Su et al., 2016) structural members, as well as planar steel frames (Fieber et al., 2020). The CSM has two key features: (i) a base curve, described in the present subsection, that defines the maximum strain ε_{csm} that a cross-section can experience prior to its failure; this is presented as a multiple of the yield strain ε_y and determined based on the cross-section slenderness $\bar{\lambda}_p$, defined below, and (ii) an appropriate constitutive model describing the stress-strain response of the structural material, presented below. The CSM can be used as an alternative to the cross-section classification concept and provides a more consistent and continuous treatment of the influence of local instabilities on the ultimate resistances of cross-sections ranging from Class 1 to Class 4.

For use in manual design calculations, in conjunction with a simplified elastic, linear hardening material model for cold-formed steel, stainless steel or aluminium (Afshan & Gardner, 2013), or a quad-linear stress-strain model for hot-rolled steels (Yun & Gardner, 2017), the CSM base curve is given by Eqs. (2.9) and (2.10), as shown in Figure 2.3.

$$\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}} \quad \text{but } \leq \Omega \quad \text{for } \bar{\lambda}_p \leq 0.68 \quad (2.9)$$

$$\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} = \left(1 - \frac{0.222}{\bar{\lambda}_p^{1.05}} \right) \frac{1}{\bar{\lambda}_p^{1.05}} \quad \text{for } 0.68 < \bar{\lambda}_p \leq 1.0 \quad (2.10)$$

The CSM base curve is split into two parts, and the transition point between non-slender and

slender cross-sections is set at $\bar{\lambda}_p = 0.68$ (Afshan & Gardner, 2013). For non-slender cross-sections ($\bar{\lambda}_p \leq 0.68$), the CSM strain limit is greater than or equal to the yield strain (i.e. $\varepsilon_{\text{csm}}/\varepsilon_y \geq 1$), given by Eq. (2.9), allowing the rational exploitation of the spread of plasticity and strain hardening. For slender cross-sections ($0.68 < \bar{\lambda}_p \leq 1.0$), the CSM strain limit is less than the yield strain (i.e. $\varepsilon_{\text{csm}}/\varepsilon_y < 1$), as given by Eq. (2.10).

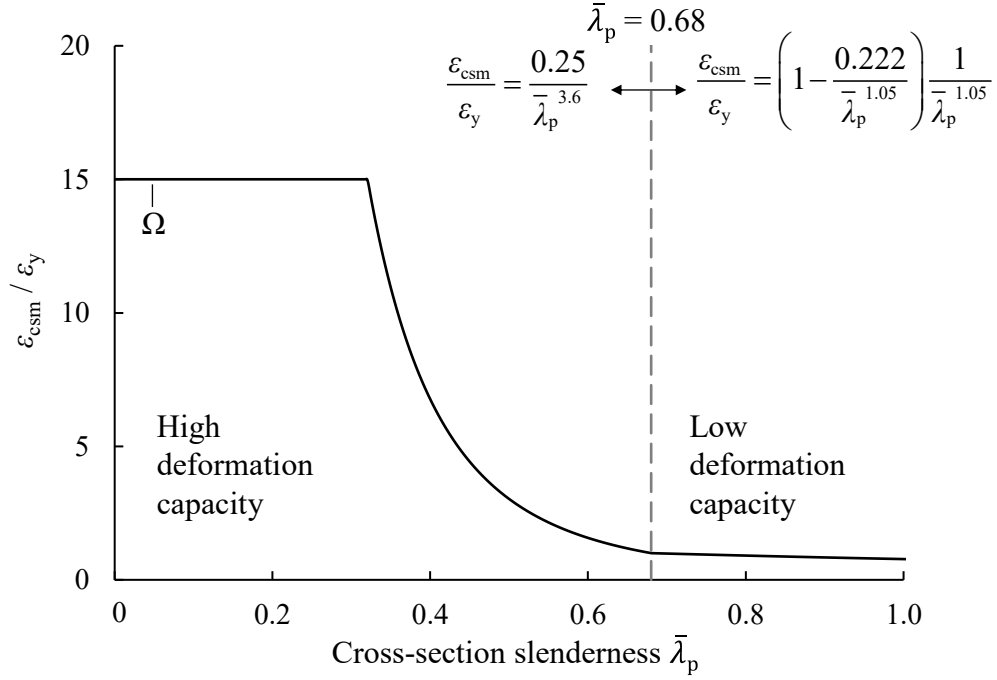


Figure 2.3 CSM base curve: a continuous relationship between the cross-section slenderness $\bar{\lambda}_p$ and its deformation capacity expressed by the normalised limiting strain $\varepsilon_{\text{csm}}/\varepsilon_y$.

For use in design by second order inelastic analysis, where the rounded stress-strain response of materials such as cold-formed steel (Gardner & Yun, 2018), stainless steel (Arrayago et al., 2015) and aluminium alloys (Yun et al., 2020) is explicitly modelled, an adjustment to the CSM base curve is required. Eqs. (2.11) and (2.12) describe the adjusted CSM base curve for materials exhibiting a rounded stress-strain response.

$$\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}} + \frac{0.002}{\varepsilon_y} \quad \text{but } \leq \Omega \quad \text{for } \bar{\lambda}_p \leq 0.68 \quad (2.11)$$

$$\frac{\varepsilon_{\text{csm}}}{\varepsilon_y} = \left(1 - \frac{0.222}{\bar{\lambda}_p^{1.05}}\right) \frac{1}{\bar{\lambda}_p^{1.05}} + \frac{0.002(\sigma_{\text{Ed,max}} / f_y)^n}{\varepsilon_y} \quad \text{for } 0.68 < \bar{\lambda}_p \leq 1.0 \quad (2.12)$$

In Eqs. (2.9)-(2.12), the upper limit Ω for $\varepsilon_{\text{csm}}/\varepsilon_y$ is a project specific design parameter that

defines the maximum permitted level of plastic deformation, for which the value of 15 is recommended, remaining within the EN 1993-1-1 (2005) ductility requirements; $\sigma_{Ed,max}$ is the maximum stress in the cross-section; the cross-section slenderness $\bar{\lambda}_p$ is determined using Eq. (2.13), where $\sigma_{cr,cs}$ is the elastic local buckling stress of the full cross-section, which can be calculated numerically (e.g. through the finite strip software CUFSM (Li & Schafer, 2010b)), or using the formulae developed by Gardner et al. (2019a).

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr,cs}}} \quad (2.13)$$

2.3.2 Material modelling

The second key feature of the CSM is the definition of an accurate and appropriate material model. Hot-rolled steel and stainless steel have distinctively different material characteristics. While hot-rolled steel has an elastic stage with a clear yield point, followed by a plateau and then a strain hardening region, stainless steel is characterised by a rounded stress-strain response with no clear yield point and significant strain hardening. In this study, for hot-rolled steel, the quad-linear stress-strain model, developed by Yun and Gardner (2017) and included in prEN 1993-1-14 (2019), is used, as illustrated in Figure 2.4 (a). The three input parameters for the material model are the Young's modulus E , yield stress f_y and ultimate stress f_u . The stress-strain (σ - ε) relationship over the full range is defined by:

$$\sigma = \begin{cases} E\varepsilon & \text{for } \varepsilon \leq \varepsilon_y \\ f_y & \text{for } \varepsilon_y < \varepsilon \leq \varepsilon_{sh} \\ f_y + E_{sh}(\varepsilon - \varepsilon_{sh}) & \text{for } \varepsilon_{sh} < \varepsilon \leq C_1\varepsilon_u \\ f_{C_1\varepsilon_u} + \frac{f_u - f_{C_1\varepsilon_u}}{\varepsilon_u - C_1\varepsilon_u}(\varepsilon - C_1\varepsilon_u) & \text{for } C_1\varepsilon_u < \varepsilon \leq \varepsilon_u \end{cases} \quad (2.14)$$

where the strain hardening strain ε_{sh} is given by:

$$\varepsilon_{sh} = 0.1 \frac{f_y}{f_u} - 0.055 \quad \text{but } 0.015 \leq \varepsilon_{sh} \leq 0.03 \quad (2.15)$$

the ultimate strain ε_u is determined from:

$$\varepsilon_u = 0.6 \left(1 - \frac{f_y}{f_u} \right) \quad \text{but } \varepsilon_u \geq 0.06 \quad (2.16)$$

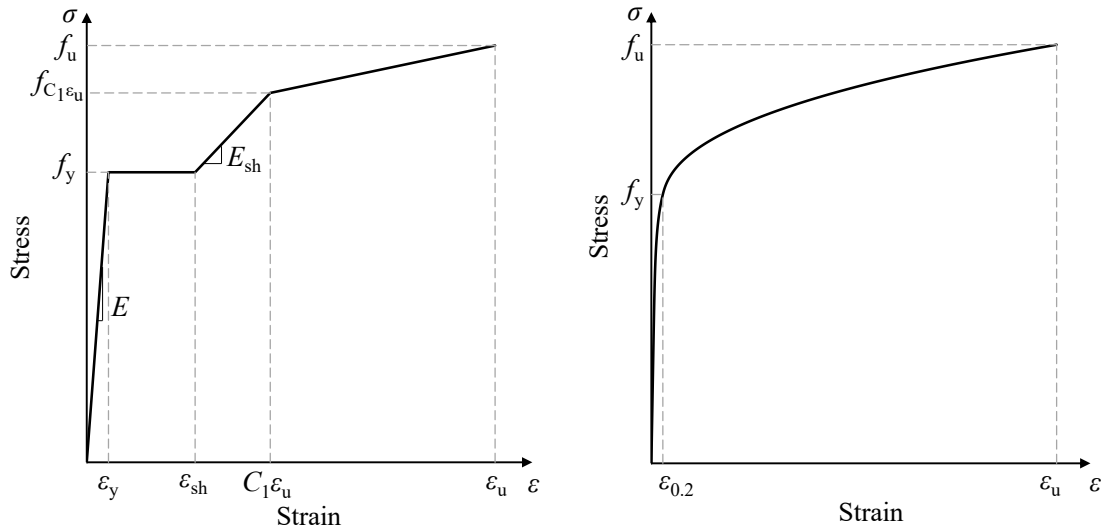
and the strain hardening modulus E_{sh} is determined from:

$$E_{sh} = \frac{f_u - f_y}{C_2 \varepsilon_u - \varepsilon_{sh}} \quad (2.17)$$

The constants C_1 and C_2 are given by Eqs. (2.18) and (2.19), respectively.

$$C_1 = \frac{\varepsilon_{sh} + 0.25(\varepsilon_u - \varepsilon_{sh})}{\varepsilon_u} \quad (2.18)$$

$$C_2 = \frac{\varepsilon_{sh} + 0.4(\varepsilon_u - \varepsilon_{sh})}{\varepsilon_u} \quad (2.19)$$



(a) Quad-linear material model for hot-rolled steel (Yun & Gardner, 2017) (b) Two-stage Ramberg-Osgood material model for stainless steel (Arrayago et al., 2015)

Figure 2.4 Material stress-strain models adopted for (a) hot-rolled steel and (b) stainless steel

For stainless steel, the two-stage Ramberg-Osgood material model (Mirambell & Real, 2000; Rasmussen, 2003; Arrayago et al., 2015) was adopted in this study, as given by Eqs. (2.20)-(2.21) and illustrated in Figure 2.4 (b), where f_y is the yield stress, taken as 0.2% proof stress, $\varepsilon_{0.2}$ is the total strain at the yield stress f_y , equal to $0.002 + f_y/E$, E_y is the tangent modulus at the 0.2% proof stress, as given by Eq. (2.22) and n and m are the strain hardening exponents.

$$\varepsilon = \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{f_y} \right)^n \quad \text{for } \sigma \leq f_y \quad (2.20)$$

$$\varepsilon = \varepsilon_{0.2} + \frac{\sigma f_y}{E_y} + \left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u f_y}{E_y} \right) \left(\frac{\sigma f_y}{f_u f_y} \right)^m \quad \text{for } f_y < \sigma \leq f_u \quad (2.21)$$

$$E_y = \frac{E}{1 + 0.002n \frac{E}{f_y}} \quad (2.22)$$

2.4 Design by second order inelastic analysis

Second order inelastic analysis, also referred to as geometrically and materially nonlinear analysis with imperfections (GMNIA) or advanced inelastic analysis, is an efficient structural analysis method to verify the strength capacity of structures, which accounts for both plasticity and buckling failure modes and is able to accurately capture the load-deformation response of structures (Marques et al., 2013). Since the influence of geometrical and material nonlinearity, and geometric and material (i.e. residual stresses) initial imperfections, on structural behaviour is taken account of through GMNIA, global buckling behaviour can be directly and accurately captured, and thus individual member design checks are not required, which streamlines the design process. Recently, on account of its widely recognised benefits and with improvements in computational power and efficiency, design by GMNIA is becoming widespread in practical design and research studies (Liew et al., 2000; Chan & Chen, 2005; Buonopane & Schafer, 2006; Gardner et al., 2019b).

Owing to the high computational efficiency, GMNIA is commonly carried out using beam FE models. However, beam finite elements are not able to capture cross-section instabilities, i.e. local buckling is not accounted for and rotation capacities are unlimited. Hence, in current practice (e.g. EN 1993-1-1, 2005), cross-section instabilities are considered through the concept of cross-section classification, where cross-sections are placed into discrete classes that define the resistance and permitted analysis type, based on specified plate width-to-thickness ratios, as detailed in Section 2.2.1. An alternative, more accurate, approach is to apply strain limits determined from the Continuous Strength Method (CSM) base curve (see Section 2.3.1) to the maximum compressive normal strain within cross-sections, to capture the influence of local buckling on the resistance of the cross-sections, as well as control the level of spread of plasticity, moment redistribution and strain hardening. The method of design by advanced inelastic analysis together with the adoption of the CSM strain limits has been developed and verified against carbon steel (Fieber et al., 2019a; Fieber et al., 2020) and stainless steel (Walport et al., 2020a) structures and will be further developed in this study.

2.5 Equivalent imperfections for design by second order analysis

The load-carrying capacity of steel members is affected by member imperfections, particularly initial geometric imperfections (e.g. out-of-straightness) and material imperfections (e.g. residual stresses). These two imperfections arise during the manufacturing and fabrication processes, and the influence of both must be accounted for in design. The required imperfection amplitude for use in design by advanced analysis depends on (i) the analysis type, (ii) the cross-section failure criterion and (iii) the shape of the imperfection modelled. Geometric imperfections can either be included in an analysis through direct modelling of the deformed geometry or through the scaling of a suitable eigenmode. In the former case, the relative simplicity of defining the deformed geometry is retained without the encumbrance of an additional linear buckling analysis, while in the latter case, more accurate results are generally achieved. For beam FE models, explicit modelling of residual stresses is possible though less practical. To overcome this issue, EN 1993-1-1 (2005) allows the use of ‘equivalent’ imperfections that implicitly account for the combined effects of geometric imperfections and residual stresses in a second order analysis. EN 1993-1-1 (2005) and the upcoming prEN 1993-1-1 (2019) include recommendations for equivalent imperfections for use in design by GNIA, for both flexural buckling design and out-of-plane stability design. With regards to the design by GMNIA, Walport et al. (2020b) provided recommendations for equivalent imperfections for use in the flexural buckling design, and the present thesis follows a similar methodology and derives equivalent imperfections for use in out-of-plane stability design.

2.6 Concluding remarks

This chapter had summarised key aspects relevant to this thesis, including the traditional design methods of steel structures, at cross-section and member levels. In the following chapters, based on extensive numerical studies, these traditional design methods will be reassessed by comparison against benchmark FE results and by comparison with the design method developed in this study, i.e. design by second order inelastic analysis with strain limits. The essential background information of the proposed design method, such as CSM base curve, material modelling and equivalent imperfections has been described. Further literature will be introduced and examined in more detail within the appropriate chapters.

Chapter 3

Elastic local buckling of three-flanged cross-sections

3.1 Introduction

There are a number of scenarios in which structural members with cross-sections having three flanges are utilised in practice, including in longitudinally-stiffened plate girders (see Figure 3.1 (a)) and in the haunch and apex regions of portal frames (see Figures 3.1 (b) and (c)). In the former case, the members are prismatic and the formulae developed in this chapter are directly applicable. In the latter case, the members are tapered and account should be taken of the variation in cross-section geometry along the member length; the approach of initially neglecting tapering effects at the cross-section level in the determination of the elastic local buckling stress, then allowing for the variation in cross-section properties at the global member level through GMNIA using beam finite elements with stepped properties along the member length, will be explored in Chapter 4. This is analogous to the current practice of classifying and determining effective section properties for individual cross-sections without considering either the member length or the gradient of forces or cross-section geometry along the member length (EN 1993-1-1, 2005). Following this approach, the resulting resistance predictions have been shown to be extremely accurate in comparison to shell FE results, confirming the validity of the approach up to tapering angles of 15°.

As detailed in Section 2.2.1, in current steel design specifications, such as EN 1993-1-1 (2005) and AISC 360-16 (2016), the resistance and deformation capacity of structural cross-sections are determined with reference to the concept of cross-section classification, whereby limiting width-to-thickness ratios for the individual cross-section plates (assuming simply-supported boundary conditions along their adjoined edges) are defined to assess the extent to which local

buckling influences the behaviour of the full cross-section. In reality, the plate elements within a cross-section interact with each other and enhanced cross-section capacities can be exploited when element interaction is explicitly considered. Recently established design methods, such as the Direct Strength Method (DSM) (AISI, 2006; Schafer, 2008) and the CSM (Gardner, 2008; Gardner et al., 2011; Afshan & Gardner, 2013), are able to account for the beneficial effects of element interaction by using the elastic local buckling stress of the full cross-section to determine the cross-section slenderness and hence the cross-section resistance. While there has been a number of studies into the effects of element interaction on the elastic local buckling response of single I-sections (Johnson, 1985; Möller, 1995; Bedair, 2009; Seif & Schafer, 2010; Han & Lee, 2016; Gardner et al., 2019a), no studies have, to the knowledge of the author, considered the effect of element interaction in three-flanged sections.

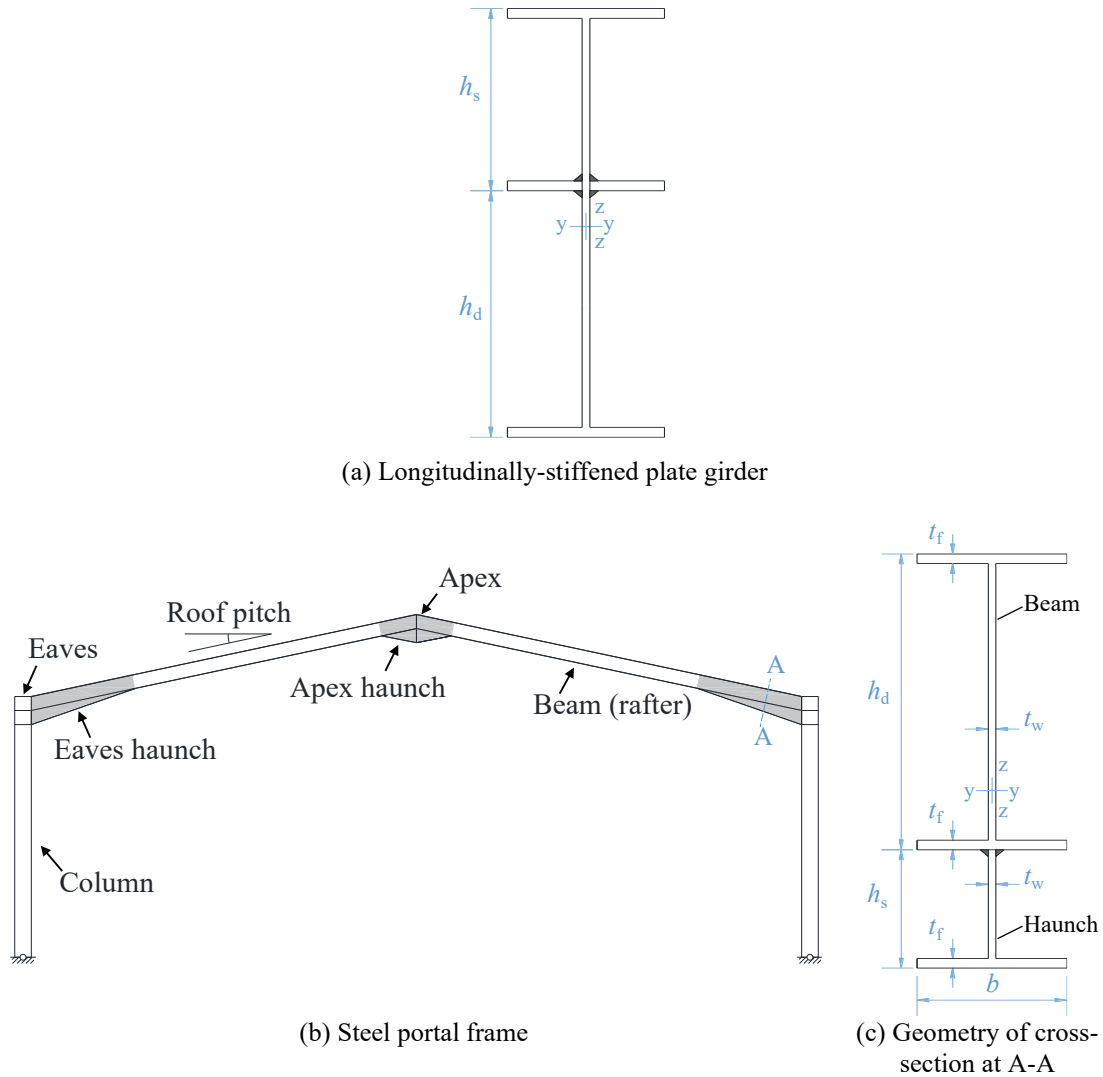


Figure 3.1 Examples of steel members with three-flanged cross-sections

In this chapter, the formulae for calculating the elastic local buckling stress of single I-sections subjected to combined compression and major axis bending developed in Gardner et al. (2019a) are extended to three-flanged sections. Note that although three-flanged section members are, when used in the eaves and apex regions of portal frames, typically tapered and subjected to moment gradients, the cross-sectional behaviour is the focus of the present chapter, i.e. the elastic local buckling response of uniformly-loaded prismatic members with three-flanged cross-sections is examined. The influence of tapering (Marques, 2012; Marques et al., 2013; Kucukler & Gardner, 2018; Tankova et al., 2018; Kucukler & Gardner, 2019) and moment gradients (Lay, 1964; Lay & Galambos, 1964; Fieber et al., 2019a) have been investigated elsewhere at the member level. The predicted elastic local buckling stresses are compared with results obtained from finite strip analysis performed using CUFSM (Li & Schafer, 2010b) and with elastic buckling stresses calculated on an element-by-element basis (i.e. ignoring the influence of element interaction). Finally, implementation of the developed expressions is demonstrated through a worked example.

3.2 Elastic local buckling of plates and I-sections

3.2.1 Elastic buckling of isolated plates

The elastic local buckling stress $\sigma_{cr,p}$ of a rectangular flat plate of width b and thickness t , made of an elastic material with Young's modulus E and Poisson's ratio ν , is given by Eq. (2.23), where the plate buckling coefficient k depends on three principal factors: (1) the plate aspect ratio, (2) the restraint conditions along the plate edges and (3) the distribution of axial stress applied to the plate.

$$\sigma_{cr,p} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (2.23)$$

Considering the typical dimensions of structural members, plate elements within cross-sections are generally sufficiently long that the influence of the plate aspect ratio is negligible. The restraint conditions along the longitudinal plate edges are generally idealized as simply-supported or fixed. In most practical applications, the boundary conditions along the longitudinal edges of individual cross-section plates lie between these idealised limits and can be represented by an elastic rotational spring (Lu et al., 2017). In general, as the level of restraint

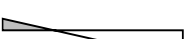
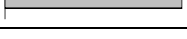
along the longitudinal plate edges increases (i.e. approaching fixed boundary conditions) the buckling coefficient k , and thus the local buckling stress, increases (Lundquist & Stowell, 1941; Lundquist & Stowell, 1942), while the local buckling half-wavelength reduces (Fieber et al., 2019b).

The applied stress distribution across the loaded edges of the plates within a cross-section typically varies from pure compression to pure bending. Uniform compression is the most severe loading condition and thus results in the lowest elastic local buckling stress. As the applied stress distribution becomes increasingly non-uniform, the local buckling stress of the plate increases. The parameter ψ , defined in Eq. (2.24), is typically used to represent linearly varying applied stress distributions across the loaded edge of a plate, where σ_1 and σ_2 are the maximum and minimum stress applied to the plate respectively, with compressive stresses taken as positive. A value of $\psi = 1$ corresponds to pure compression, while $\psi = -1$ corresponds to pure bending.

$$\psi = \frac{\sigma_2}{\sigma_1} \quad (2.24)$$

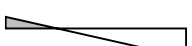
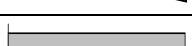
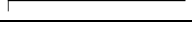
Explicit expressions for the plate buckling coefficient k as a function of the applied stress distribution ψ are provided in EN 1993-1-5 (2005) for internal and outstand plates with simply-supported boundary conditions along the adjoining edges, as summarized in Table 3.1. These expressions are applicable to plates subjected to stress distributions where $-3 \leq \psi \leq 1$; this range was found to be sufficient to cover the web plates of all the three-flanged cross-sections subjected to major axis bending (or major axis bending plus compression) considered in this study. Note that the stress distribution across the outstand flanges is always uniform (i.e. $\psi_f = 1$) for any combination of major axis bending and compression.

Table 3.1 Expressions for the buckling coefficient k for internal and outstand plates with simply-supported (SS) boundary conditions along the adjoining edges – adopted from EN 1993-1-5 (2005).

Stress distribution (compression positive, shaded grey)		$\psi = \sigma_2 / \sigma_1$	Buckling coefficient k
Internal plates (SS – SS)		1	$k = 4$
		$1 > \psi > 0$	$k = 8.2 / (1.05 + \psi)$
		0	$k = 7.81$
		$0 > \psi > -1$	$k = 7.81 - 6.29\psi + 9.78\psi^2$
Outstand plates (SS – free)		-1	$k = 23.9$
		$-1 > \psi \geq -3$	$k = 5.98 (1 - \psi)^2$
Outstand plates (SS – free)		1	$k = 0.43$
		1	$k = 0.43$

For plates with fixed boundary conditions along the adjoining edges, Gardner et al. (2019a) developed equivalent expressions to those presented in EN 1993-1-5, as set out in Table 3.2. The expressions were calibrated against the results of finite strip analysis CUFSM (Li & Schafer, 2010b), but were limited to stress distributions in which $-1 \leq \psi \leq 1$. While this is adequate for single I-sections, in three-flanged cross-sections, lower ψ values often arise in the web. Hence, using the same method as employed in Gardner et al. (2019a), an additional expression for the plate buckling coefficient k for internal elements with fixed boundary conditions has been derived herein for $-3 \leq \psi \leq -1$, as shown in Figure 3.2 and included in Table 3.2.

Table 3.2 Expressions for the buckling coefficient k for internal and outstand plates with fixed boundary conditions along the adjoining edges – adopted from Gardner et al. (2019a).

Stress distribution (compression positive, shaded grey)		$\psi = \sigma_2 / \sigma_1$	Buckling coefficient k
Internal plates (Fixed – Fixed)		1	$k = 6.97$
		$1 > \psi > 0$	$k = 14.29 / (1.05 + \psi)$
		0	$k = 13.6$
		$0 > \psi > -1$	$k = 13.6 - 11.5\psi + 14.5\psi^2$
Outstand plates (Fixed – free)		-1	$k = 39.6$
		$-1 > \psi \geq -3$	$k^a = 9.9 (1 - \psi)^2$

^aProposed extension

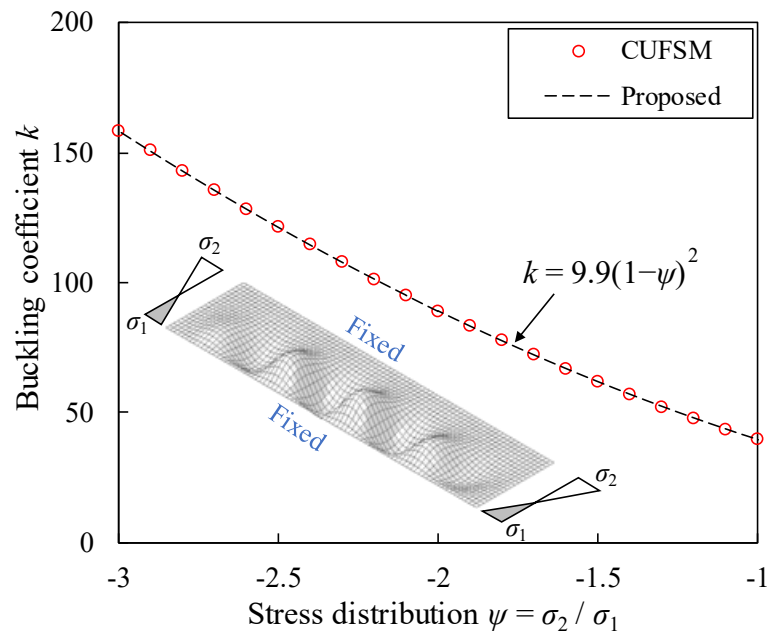


Figure 3.2 Buckling coefficient k for internal elements with fixed boundary conditions along longitudinal edges with $-3 \leq \psi < -1$

3.2.2 Elastic local buckling of I-sections

3.2.2.1 Introduction

When isolated plates are combined to form structural cross-sections, the interaction between the plate elements generally results in the elastic local buckling stress of the full cross-section exceeding that of the most slender constituent plate element with simply-supported boundary conditions along the adjoined edges. There are two primary causes of element interaction in structural cross-sections: (1) differences in elastic buckling stresses between isolated plates and (2) incompatibility in local buckling half-wavelengths between isolated plates. In the former case, the non-critical plates (i.e. the isolated plates with the higher elastic local buckling stresses) are able to provide rotational restraint to the edges of the critical plate (i.e. the isolated plate with the lowest elastic local buckling stress). In the latter case, the interconnectivity between the plates prevents them from buckling into their natural isolated buckling mode shapes, hence resulting in an enhanced elastic buckling stress; this effect is particularly pronounced in cross-sections composed of both internal and outstand elements (e.g. I-sections), which can have very different local buckling half-wavelengths when considered in isolation.

3.2.2.2 Existing formulae

Gardner et al. (2019a) developed expressions to calculate the elastic local buckling stress of full cross-sections, allowing for the effects of element interaction. The underlying concept is that the elastic local buckling stress of a full cross-section $\sigma_{cr,cs}$ is bound by the elastic local buckling stresses of the isolated critical plates with simply-supported $\sigma_{cr,p}^{SS}$ and fixed $\sigma_{cr,p}^F$ boundary conditions along the adjoined edges, as given by Eq. (2.25).

$$\sigma_{cr,cs} = \sigma_{cr,p}^{SS} + \zeta \left(\sigma_{cr,p}^F - \sigma_{cr,p}^{SS} \right) \quad (2.25)$$

The lower bound elastic local buckling stress $\sigma_{cr,p}^{SS}$ is taken as the minimum elastic local buckling stress of the constituent flange or web plate elements forming the cross-section, assuming simply-supported boundary conditions along the adjoined edges, denoted $\sigma_{cr,f}^{SS}$ and $\sigma_{cr,w}^{SS}$ respectively, as given in Eq. (2.26). Similarly, the upper bound stress $\sigma_{cr,p}^F$ is equal to the minimum of the elastic local buckling stresses of the flange $\sigma_{cr,f}^F$ and the web $\sigma_{cr,w}^F$ with fixed boundary conditions along the adjoined longitudinal edges, as given in Eq. (2.27). Note that the

calculation of the local buckling stresses of the individual plates is based on the centreline geometry of the cross-section, as indicated in Figure 3.3 for a single I-section, where the adopted notation is also illustrated.

$$\sigma_{cr,p}^{SS} = \min(\sigma_{cr,f}^{SS}, \sigma_{cr,w}^{SS}) \quad (2.26)$$

$$\sigma_{cr,p}^F = \min(\sigma_{cr,f}^F, \sigma_{cr,w}^F) \quad (2.27)$$

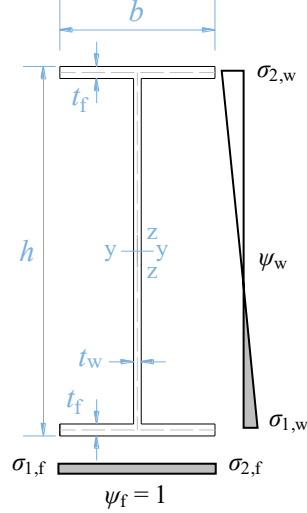


Figure 3.3 Geometry, notation and stress distribution in an I-section under compression and major axis bending

The degree of element interaction is defined through the interaction coefficient ζ , which ranges from 0 (i.e. no element interaction, corresponding to the simply-supported lower bound) and 1 (i.e. full element interaction, corresponding to the upper bound of fixed boundary conditions along the adjoined edges of the critical plate). Gardner et al. (2019a) back-calculated the level of element interaction within a cross-section by rearranging Eq. (2.25) into Eq. (2.28), and hence developed predictive expressions for ζ for different cross-section profiles and loading conditions.

$$\zeta = \frac{\sigma_{cr,cs} - \sigma_{cr,p}^{SS}}{\sigma_{cr,p}^F - \sigma_{cr,p}^{SS}}, \text{ but } 0 \leq \zeta \leq 1 \quad (2.28)$$

For the case of single I-sections subjected to combined compression and major axis bending, the interaction coefficient ζ is given by Eq. (2.29) and (2.30) for flange and web critical cases (defined through the parameter ϕ below) respectively.

$$\text{For } < 1 \text{ (i.e. flange critical), } \zeta = 0.15 \frac{t_f}{t_w} \geq \frac{t_w}{t_f} (0.4 - 0.25), \text{ but } 0 \leq \zeta \leq 1 \quad (2.29)$$

$$\text{For } \phi \geq 1 \text{ (i.e. web critical), } \zeta = \frac{t_f}{t_w} \left(0.45 - \frac{0.3}{\phi^2} \right), \text{ but } 0 \leq \zeta \leq 1 \quad (2.30)$$

The parameter ϕ , defined by Eq. (2.31), is the ratio of the elastic local buckling stress of the isolated and simply-supported flange to that of the web, and is used to determine whether the flange or the web is critical. Values of ϕ less than unity correspond to flange critical cases, suggesting that the web is able to provide rotational restraint to the flange. Conversely, values of ϕ greater than unity suggest that the web plate is critical and that the flange can provide rotational restraint to the web. When ϕ has a value close to unity, the isolated and simply-supported flange and web plates have similar elastic local buckling stresses and the degree of element interaction is typically low. Conversely, as ϕ approaches zero (flange critical) or very large values (web critical), the level of element interaction increases.

$$\phi = \frac{\sigma_{cr,f}^{SS}}{\sigma_{cr,w}^{SS}} \quad (2.31)$$

This means of calculating the elastic local buckling stress of I-sections is straightforward and explicit, and has already been successfully applied within a recently developed method of design by advanced inelastic analysis with strain limits (Fieber et al., 2019a; Gardner et al., 2019b; Fieber et al., 2020).

3.2.2.3 Extension of existing formulae to a broader range of stress distribution ratios

The existing formulae given in Section 3.2.2.2 for calculating the elastic local buckling stress of I-sections (Gardner et al., 2019a) were developed for single I-sections under combined compression and major axis bending, in which the stress distribution ratio in the web ψ_w always lies between -1 and 1. However, in three-flanged cross-sections, the stress distribution in the web can be such that $\psi_w < -1$. Hence, it is necessary to evaluate the applicability of the existing formulae in this range of stress distributions. The elastic local buckling stresses calculated using Eq. (2.25) for 100 different I-sections subjected to web stress distribution ratios in the range of $-2 \leq \psi_w < -1$ were compared against results obtained from finite strip analysis (FSA) performed using the software CUFSM v4.05 (Li & Schafer, 2010b). In the analyses, the cross-sections were modelled based on centreline dimensions; to accurately capture the deformation shape and provide the benchmark results, based on the preliminary sensitivity study, each flange

and web was discretized into 8 strips across the plate width. The following material properties were used: Young's modulus $E = 210000$ MPa and Poisson's ratio $\nu = 0.3$. The mean value of the ratios of the elastic local buckling stresses obtained from Eq. (2.25) to those obtained from FSA, i.e. $\sigma_{cr,cs}^{Eq} / \sigma_{cr,cs}^{FSA}$, was found to be equal to 1.013, with a coefficient of variation (CoV) 0.035; the maximum and minimum values of $\sigma_{cr,cs}^{Eq} / \sigma_{cr,cs}^{FSA}$ were 1.080 and 0.927 respectively. Thus, it is concluded that the range of applicability of the formulae presented in Gardner et al. (2019a) for calculating the elastic local buckling stress of I-sections can be extended to stress distribution ratios where $-2 \leq \psi_w \leq 1$.

3.2.2.4 Extension of existing formulae to shallow I-sections

The formulae presented in Gardner et al. (2019a) were established based on standard I-sections listed in European and American catalogues, leading to a normalized dimensional range of applicability of $0.89 \leq (h/t_w)(t_f/b) \leq 6.36$. However, I-sections of shallower proportions can exist in longitudinally-stiffened and haunched beams, and the accuracy and suitability of the expressions developed in Gardner et al. (2019a) need to be assessed for shallower I-sections in which $(h/t_w)(t_f/b) < 0.89$. Comparisons of the predictions of elastic local buckling stresses obtained from Eq. (2.25) are therefore made against those obtained from finite strip analysis performed using CUFSM. During the finite strip analysis of some of the shallower I-sections, global buckling had to be inhibited by the application of suitable boundary conditions at the web-to-flange junction (see Figure 3.4) to enable the development of local buckling. In some cases, the signature curve obtained using CUFSM displayed no distinct local buckling minimum, as shown in Figure 3.4. This issue can be solved by using the approach “FSM@cFSM- L_{cr} ”, proposed by Li and Schafer (Li & Schafer, 2010a), in which the constrained finite strip method (i.e. cFSM) is first employed to determine the half-wavelength L_{cr} corresponding to the pure local buckling mode; the critical load factor is then taken as that obtained from an unconstrained finite strip analysis at the same half-wavelength L_{cr} . Note that this approach provides critical load factor predictions close to those predicted by the point of inflection on the signature curve (Gardner et al., 2019a). Comparisons between the predicted values of elastic buckling stress $\sigma_{cr,cs}^{Eq}$ determined from Eq. (2.25) and the numerical values $\sigma_{cr,cs}^{FSA}$ determined using CUFSM for a series of shallow I-sections under different loading conditions ranging from pure compression to pure major axis bending are shown in Figure 3.5 (a). It can be seen that elastic buckling stresses calculated using Eq. (2.25) $\sigma_{cr,cs}^{Eq}$ become

increasingly conservative relative to the numerical results as the value of $(h/t_w)(t_f/b)$ reduces. This conservatism is overcome by adjusting the interaction coefficient ζ in the range $(h/t_w)(t_f/b) < 0.89$. The difference between the back-calculated interaction coefficients obtained from CUFSM ζ^{FSA} and the values calculated from Eqs. (2.29) and (2.30) ζ^{Eq} is shown in Figure 3.5 (b). This difference can be predicted using the fitted parabola $\zeta^{\text{FSA}} - \zeta^{\text{Eq}} = 0.4 \left[0.89 - (h/t_w)(t_f/b) \right]^2$, which can be used to modify the existing expression. This equation provides an approximate lower bound to the numerical data. The flange typically becomes the critical plate element in shallow I-sections; hence, to calculate ζ in the range $\phi < 1$, Eq. (2.29), which remains applicable in the range $0.89 \leq (h/t_w)(t_f/b) \leq 6.36$, can be supplemented with Eq. (2.32). As shown in Figure 3.5 (a), the accuracy of predictions is markedly improved using the modified equations.

For $\phi < 1$ and $0.89 \leq \frac{h}{t_w} \frac{t_f}{b} \leq 6.36$,

$$\zeta = 0.15 \frac{t_f}{t_w} \phi \geq \frac{t_w}{t_f} (0.4 - 0.25\phi), \text{ but } 0 \leq \zeta \leq 1 \quad (2.29)$$

For $\phi < 1$ and $\frac{h}{t_w} \frac{t_f}{b} < 0.89$,

$$\zeta = \max\left(0.15 \frac{t_f}{t_w} \phi, \frac{t_w}{t_f} (0.4 - 0.25\phi)\right) + 0.4 \left(0.89 - \frac{h}{t_w} \frac{t_f}{b} \right)^2, \text{ but } 0 \leq \zeta \leq 1 \quad (2.32)$$

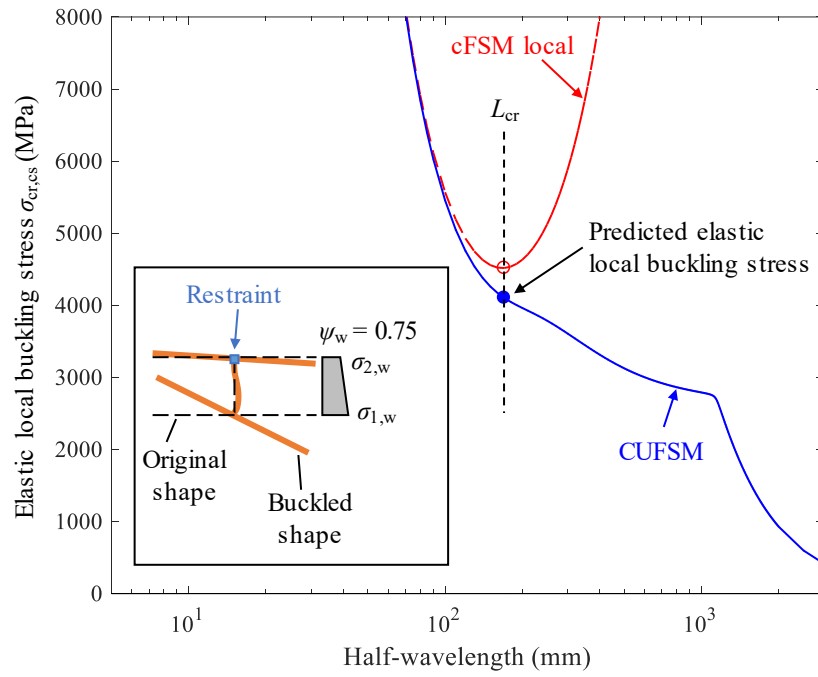
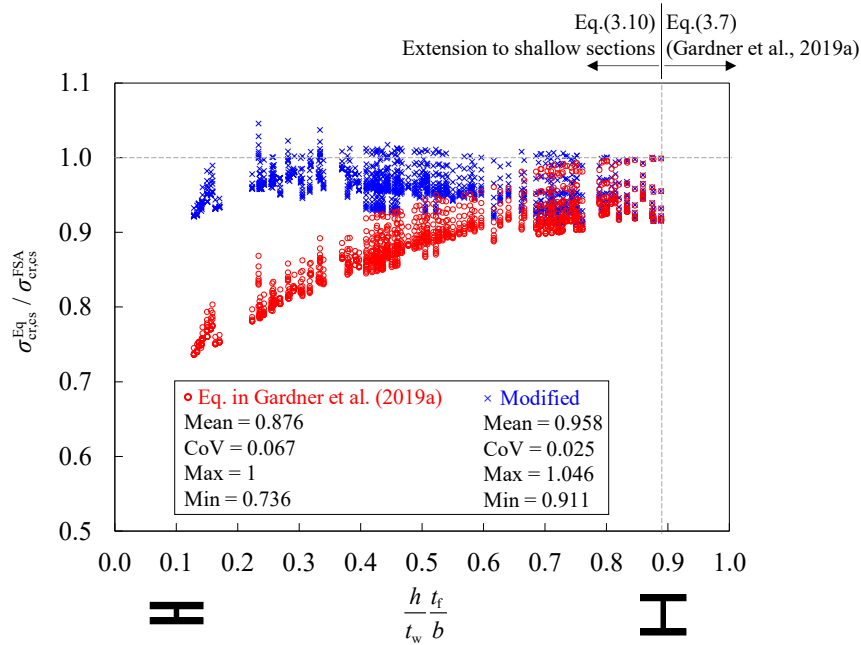
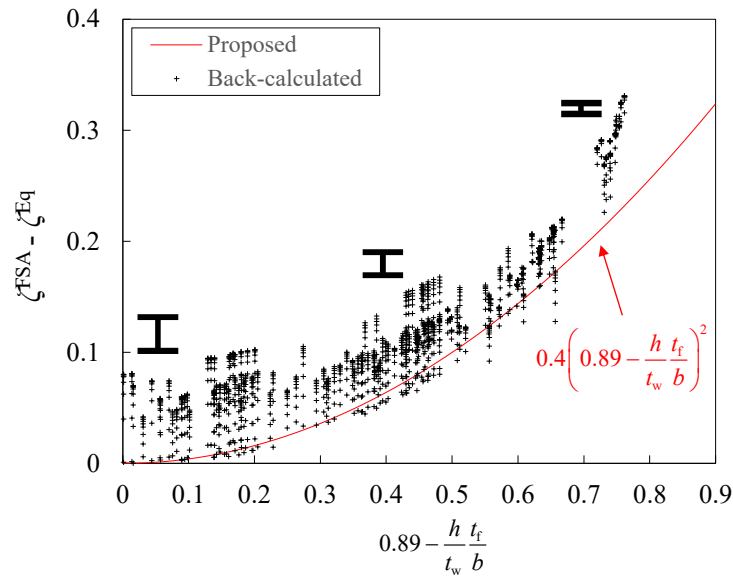


Figure 3.4 Signature curves obtained using cFSM and CUFSM and method to determine elastic local buckling stress in the absence of a distinct minimum in CUFSM results



(a) Calculated elastic local buckling stresses using equations in Gardner et al. (2019a) or modified equations, compared against results obtained from FSA



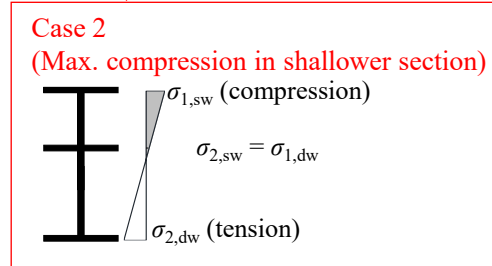
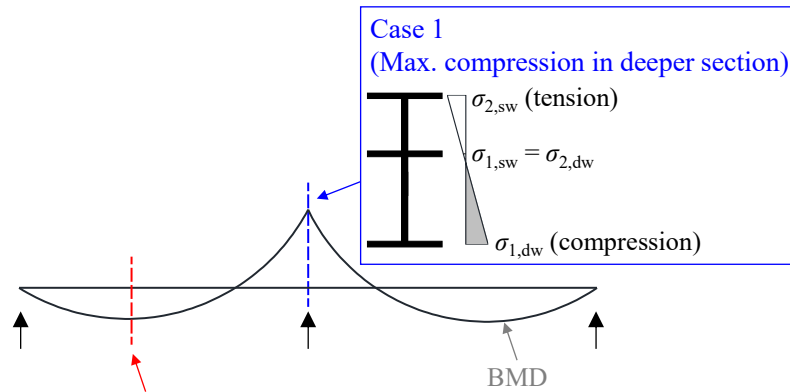
(b) Calibration of proposed modification factor for shallow I-sections

Figure 3.5 Elastic local buckling stresses and interaction coefficient ζ for shallow I-sections with $\frac{h}{t_w} \frac{t_f}{b} < 0.89$

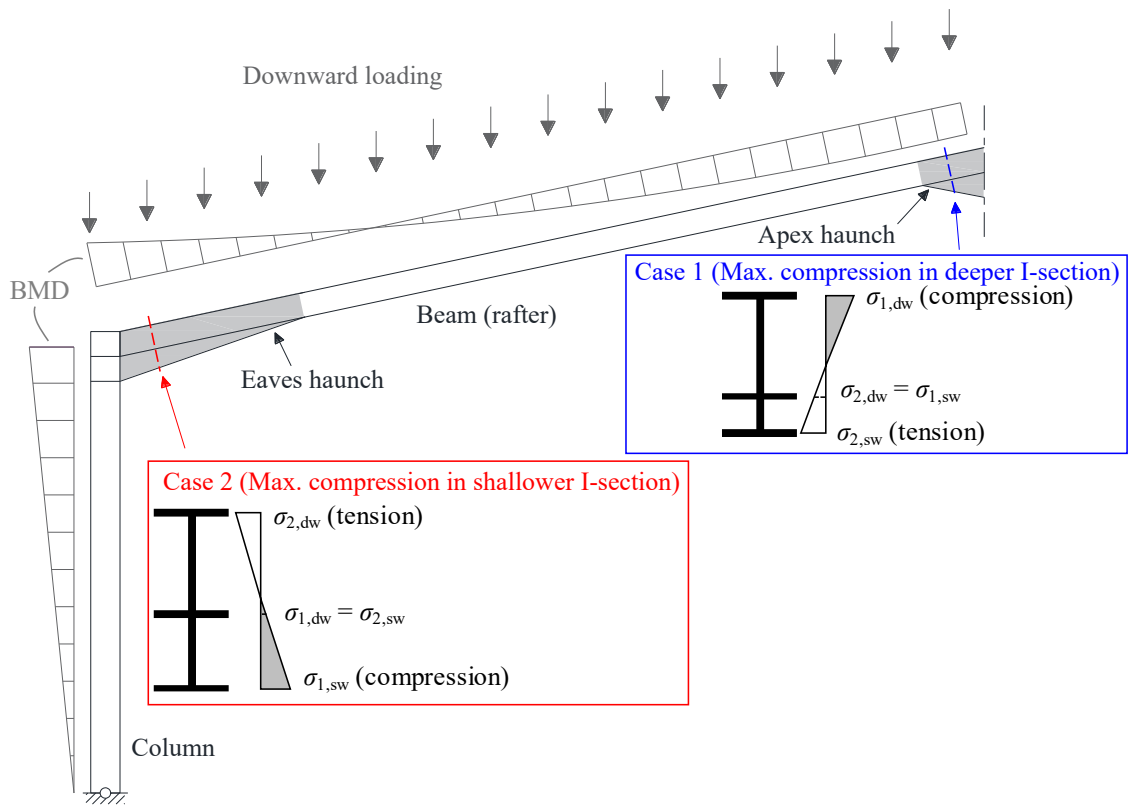
3.3 Elastic local buckling behaviour of three-flanged cross-sections

3.3.1 Loading cases and concept

Three-flanged cross-sections can experience either sagging or hogging bending moments e.g. in the span or support regions for continuous, longitudinally-stiffened plate girders (see Figure 3.6 (a)) and in the haunch or apex regions of portal frames subjected to gravity loading (see Figure 3.6 (b)) or wind uplift (see Figure 3.6 (c)). Accordingly, the stress distribution in three-flanged cross-sections can be categorised into one of two states: (i) Case 1, corresponding to the maximum axial compression occurring within the deeper I-section and (ii) Case 2, corresponding to the maximum axial compression occurring within the shallower I-section. These two states are illustrated in Figure 3.6.



(a) Continuous and longitudinally-stiffened plate girder



(b) Portal frames under downward loading

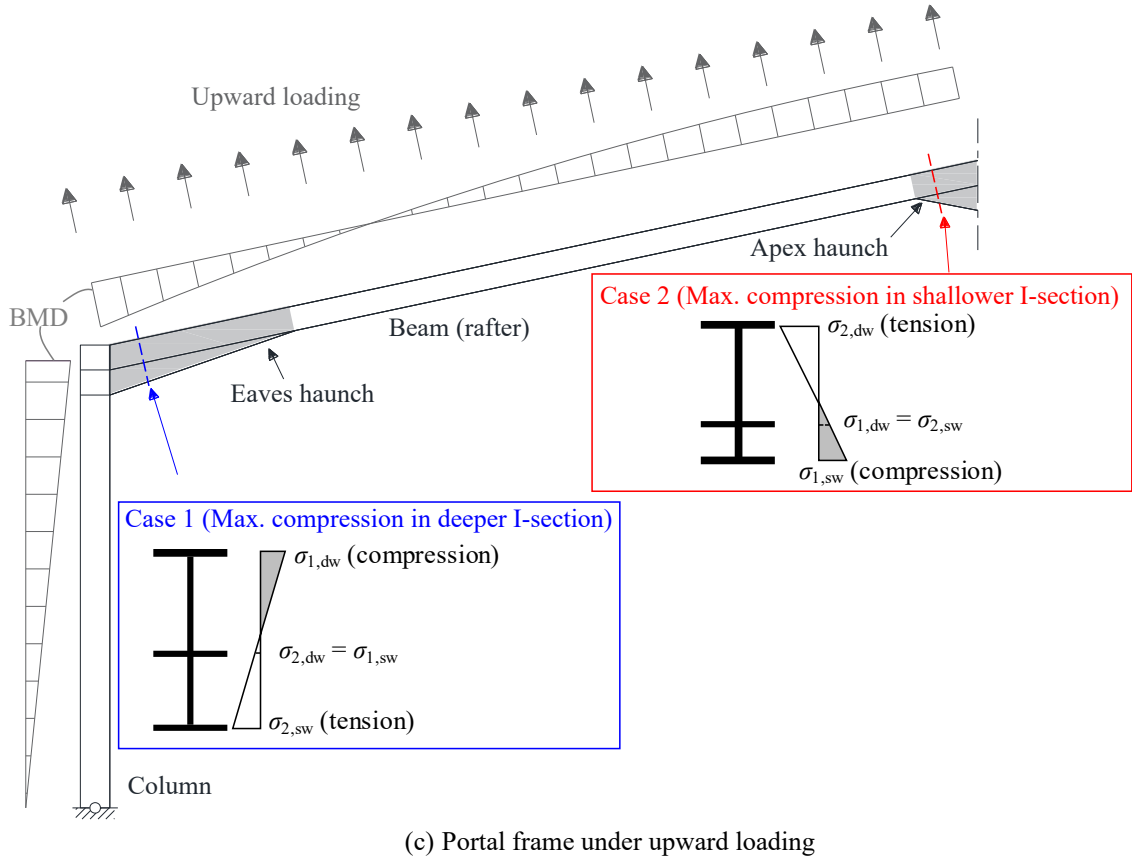


Figure 3.6 Typical loading conditions with corresponding bending moment diagrams (BMD) of (a) the continuous and longitudinally-stiffened plate girder and (b) the beam (rafter) of portal frames under downward loading and (c) upward loading

Considering these two cases, a similar approach to that developed by Gardner et al. (2019a) for calculating the elastic local buckling stress of single I-sections is employed herein for three-flanged cross-sections. First, the three-flanged cross-section is considered as two single I-sections, i.e. the deeper I-section and the shallower I-section, that share a common middle flange; the compound section is then classified as either deeper section critical or shallower section critical, depending on which has the lower critical local buckling load factor. The above scenarios are summarised in Table 3.3.

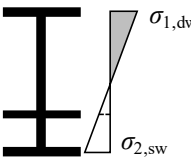
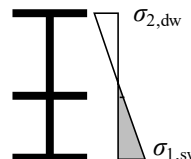
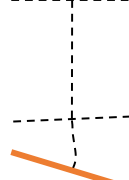
Similar to the interaction between the flange and web plates within single I-sections, in the case of three-flanged cross-sections, the deeper I-section and the shallower I-section also interact with each other. This is because the non-critical part of the three-flanged cross-section (i.e. either the deeper or the shallower section) is able to provide rotational restraint to the critical part. In the proposed approach, the interaction between the plates within the deeper and shallower I-sections are captured through the interaction coefficients, ζ_d and ζ_s respectively,

determined from Eq. (2.29) or (2.30). The combined interaction, both within and between the two cross-section parts (i.e. the deeper I-section and the shallower I-section), are then determined from the compound interaction coefficient ζ_{d+s} . For deeper section critical scenarios, ζ_{d+s} is calculated from Eq. (2.33), where ζ_s^* represents the influence of the shallower section on the deeper section; for shallower section critical scenarios, ζ_{d+s} is calculated from Eq. (2.34), where ζ_d^* represents the influence of the deeper section on the shallower section. Note that in both deeper section and shallower section critical scenarios, the compound interaction coefficient for the three-flanged cross-section ζ_{d+s} maintains a value between 0 and 1.

$$\text{For deeper section critical scenarios, } \zeta_{d+s} = \zeta_d + \zeta_s^*, \text{ but } 0 \leq \zeta_{d+s} \leq 1 \quad (2.33)$$

$$\text{For shallower section critical scenarios, } \zeta_{d+s} = \zeta_s + \zeta_d^*, \text{ but } 0 \leq \zeta_{d+s} \leq 1 \quad (2.34)$$

Table 3.3 Local buckling shapes of three-flanged cross-sections in Case 1 and Case 2. The orange coloured plate represents the critical element.

	Deeper section critical		Shallower section critical	
	Web critical	Flange critical	Web critical	Flange critical
Case 1 (Max. compression in deeper section) 			N/A	
Case 2 (Max. compression in shallower section) 				

* Although the middle flange cannot be the critical plate element, this ‘artificial’ case is nonetheless considered as part of the calculation process – see Section 3.3.3 for further details.

3.3.2 Finite strip analysis

Values of the aforementioned compound interaction coefficients ζ_{d+s} were back-calculated from the results of finite strip analyses performed on three-flanged cross-sections; the analyses were carried out using the software CUFSM v4.05 (Li & Schafer, 2010b). Corresponding single deeper and shallower sections were also analysed under the same stress conditions as in the three-flanged cross-section to aid the assessment of the results. For straightforward interpretation of the results, the maximum compressive stress applied to the three-flanged cross-

sections was taken equal to unity, such that the obtained values of the critical load factor were equal to the elastic local buckling stress.

For each of Cases 1 and 2, 4500 three-flanged cross-sections consisting of 100 different deeper sections and 5 different shallower sections under 9 different applied stress distributions were analysed. The considered deeper cross-sections included standard hot-finished European (IPE, HEA, HEB, UKC, UKB) and American (W, S, HP) I-sections. In all cases, the same section size was used for the deeper and shallower I-sections, and a series of different ratios of shallower section height to deeper section height h_s/h_d were considered: doubly symmetric three-flanged cross-sections ($h_s + t_f = h_d$), and $h_s/h_d = 0.9, 0.7, 0.4, 0.1$. Nine stress distributions ranging from pure compression to pure major axis bending were considered. The superscript and subscript notation adopted herein to distinguish between the various critical stresses is shown in Figure 3.7, where compressive stresses are taken as positive.

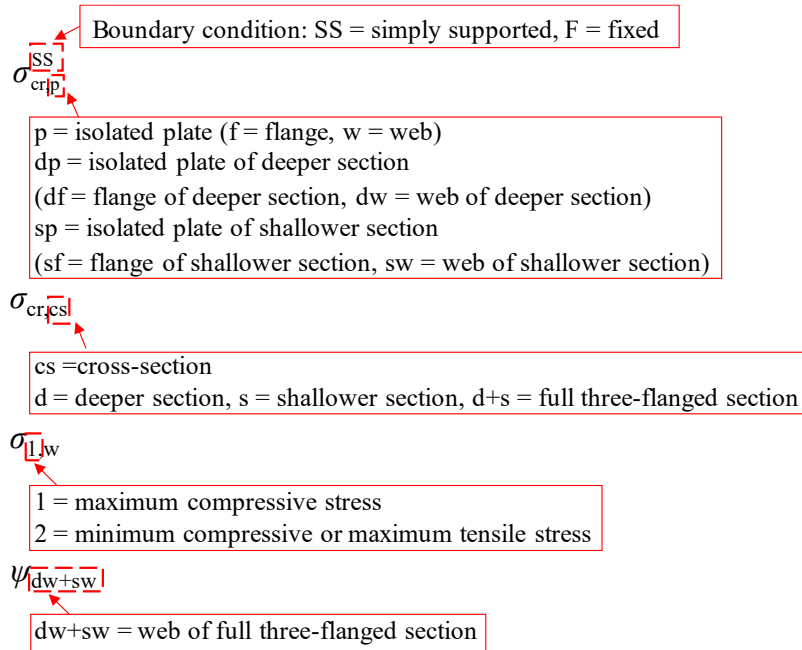


Figure 3.7 Definitions of superscript and subscript notation

3.3.2.1 Case 1 - maximum compression in deeper I-section

For Case 1, the overall linearly varying stress distribution applied over the depth of the three-flanged cross-section is described through the parameter ψ_{dw+sw} , which is equal to the ratio of the stress at the outer flange of the shallower section to that at the outer flange of the deeper section, i.e. $\sigma_{2,sw}/\sigma_{1,dw}$. The stress distributions, local buckling shapes and critical local buckling load factors of (a) a three-flanged cross-section and (b) the corresponding single deeper I-

section and single shallower I-section are shown in Figure 3.8. In Case 1, the critical load factor of the deeper section is always the same or lower than that of the shallower section, i.e. $\alpha_{cr,d} \leq \alpha_{cr,s}$. The shallower section can therefore provide restraint to the deeper section. The influence of the shallower section on the deeper section can be seen through the difference between the critical load factor of the three-flanged cross-section $\alpha_{cr,d+s}$ and that of the corresponding single deeper I-section $\alpha_{cr,d}$. This is shown in Figure 3.9, for a series of three-flanged cross-sections with different h_s/h_d ratios under a range of loading conditions. It can be seen from the figure that, for this case, the influence of the shallower section on the deeper section is small, with a the maximum improvement in critical load factor of 10%; this is because the maximum compressive stress and thus the critical region of the three-flanged cross-section is located towards the outer flange of the deeper section, while the rotational restraint from the shallower section is provided to the middle flange; for the same reason, the highest $\alpha_{cr,d+s}/\alpha_{cr,d}$ ratios (i.e. the scenarios in which the shallower section has the greatest influence) arise in the more compression dominated load cases where ψ_{dw+sw} approaches unity. Note that for some doubly symmetric three-flanged cross-sections under pure compression, the deeper and shallower sections buckle simultaneously, and with the common middle flange, the compound three-flanged cross-section buckles at a lower stress than each single I-section.

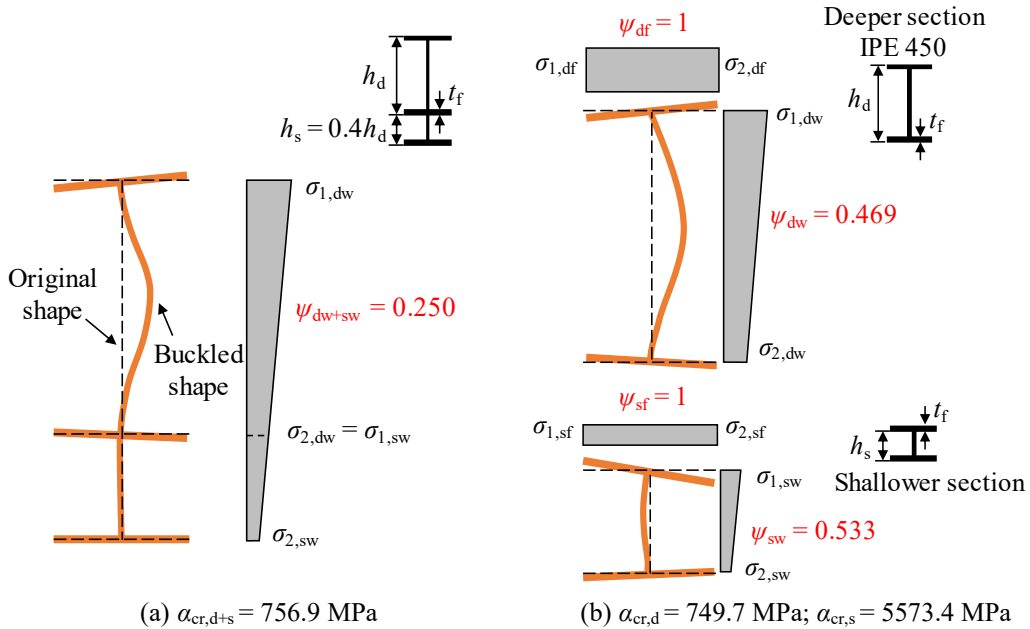


Figure 3.8 A Case 1 (deeper section critical) example of stress distributions, local buckling shapes and critical local buckling load factors of (a) a three-flanged cross-section and (b) the corresponding single deeper and shallower I-sections

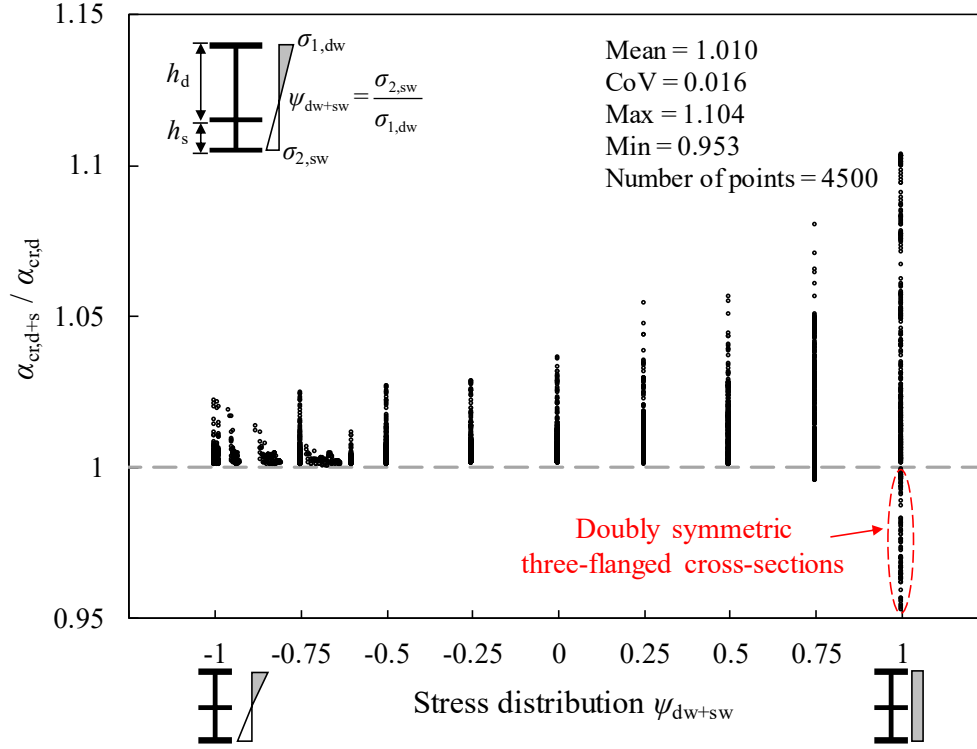


Figure 3.9 CUFSM results showing influence of shallower I-section on local buckling stress of three-flanged cross-section in Case 1

3.3.2.2 Case 2 - maximum compression in shallower I-section

For Case 2, where the maximum compressive stress arises in the haunch, the parameter $\psi_{dw+sw} = \sigma_{2,dw}/\sigma_{1,sw}$ is used to describe the linearly varying stress distribution applied over the depth of the three-flanged cross-section, as shown in Figures 3.10 and 3.11. Figure 3.10 shows an example of a shallower section critical situation, where $\alpha_{cr,s} < \alpha_{cr,d}$, while in Figure 3.11, the deeper section is critical, i.e. $\alpha_{cr,d} \leq \alpha_{cr,s}$.

For shallower section critical scenarios in Case 2, the influence of the deeper section on the critical shallower section is small, with a maximum improvement in critical load factor of less than 6%, as shown in Figure 3.12, where the difference between the critical load factor of a series of the three-flanged cross-sections $\alpha_{cr,d+s}$ and their corresponding critical single shallower I-sections $\alpha_{cr,s}$ are presented. This is because the maximum compressive stress and thus the critical region of the three-flanged cross-section is located towards the outer flange of the shallower section, while the rotational restraint from the deeper section is provided to the middle flange.

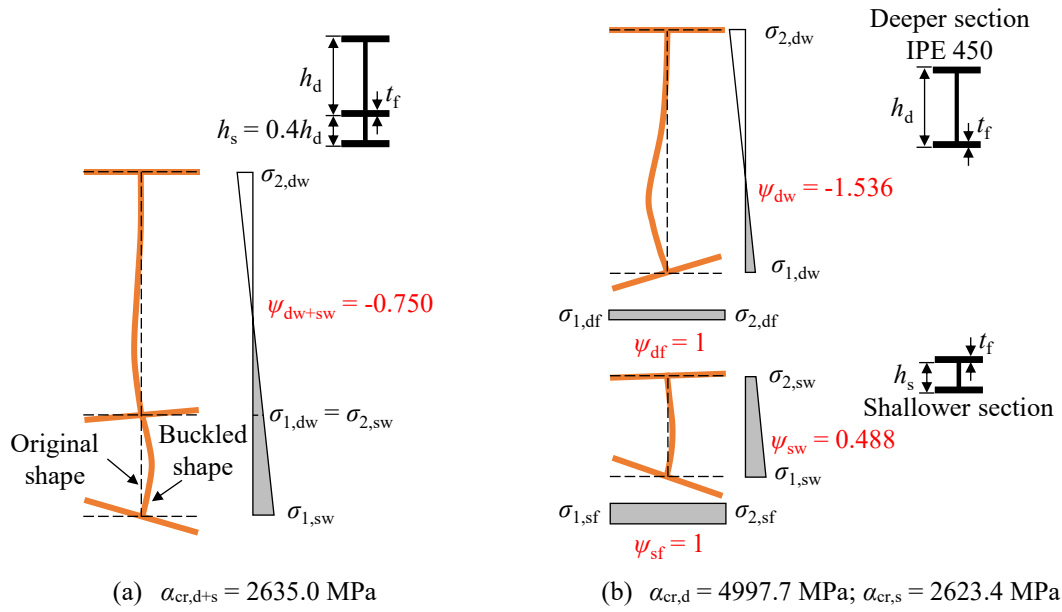


Figure 3.10 A Case 2 (shallower section critical) example of stress distributions, local buckling shapes and critical local buckling load factors of (a) a three-flanged cross-section and (b) the corresponding single deeper and shallower I-sections

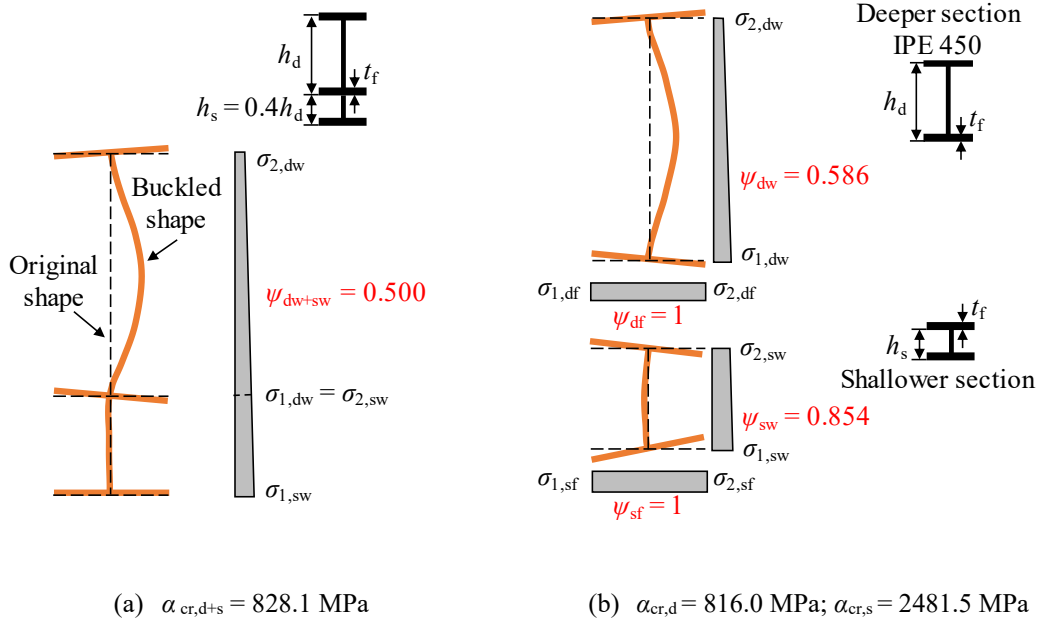


Figure 3.11 A Case 2 (deeper section critical) example of stress distributions, local buckling shapes and critical local buckling load factors of (a) a three-flanged cross-section and (b) the corresponding single deeper and shallower I-sections

In contrast, for deeper section critical scenarios in Case 2, the shallower section can offer significant restraint to the deeper section, resulting in improvements in critical load factors of up to about 40%, as shown in Figure 3.13, where the difference between the critical load factor of a series of the three-flanged cross-sections $\alpha_{cr,d+s}$ and their corresponding critical single

deeper I-sections $\alpha_{cr,d}$ are presented. This is because the maximum compressive stress and thus the critical region of the three-flanged cross-section is located towards the middle flange, and the rotational restraint from the adjoined shallower section is also provided to this region.

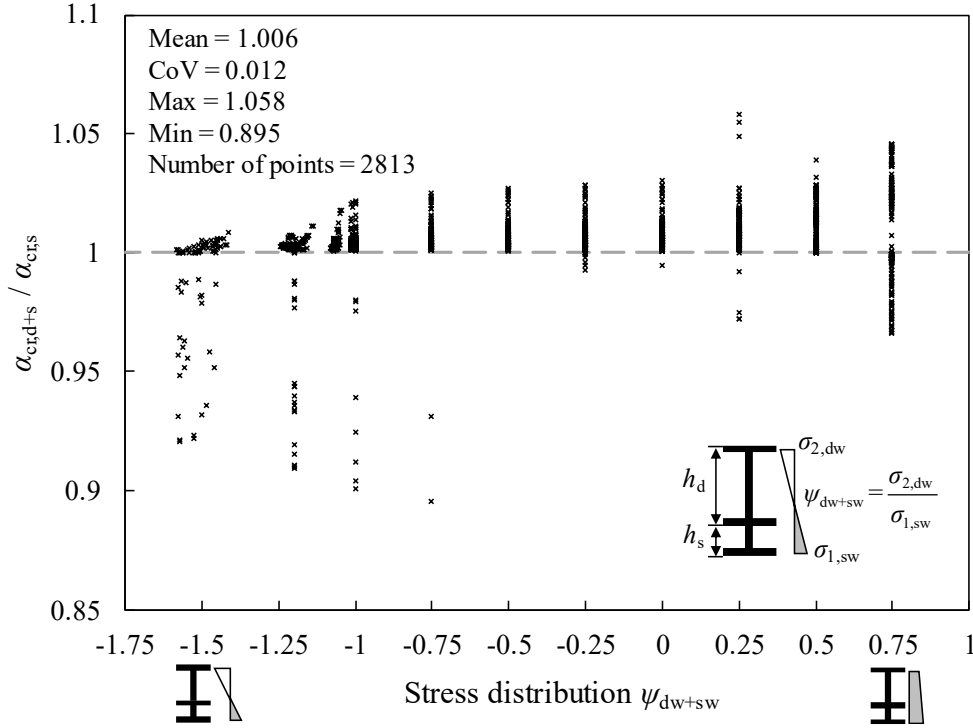


Figure 3.12 CUFSM results showing influence of deeper section on critical shallower section, for shallower section critical scenarios in Case 2

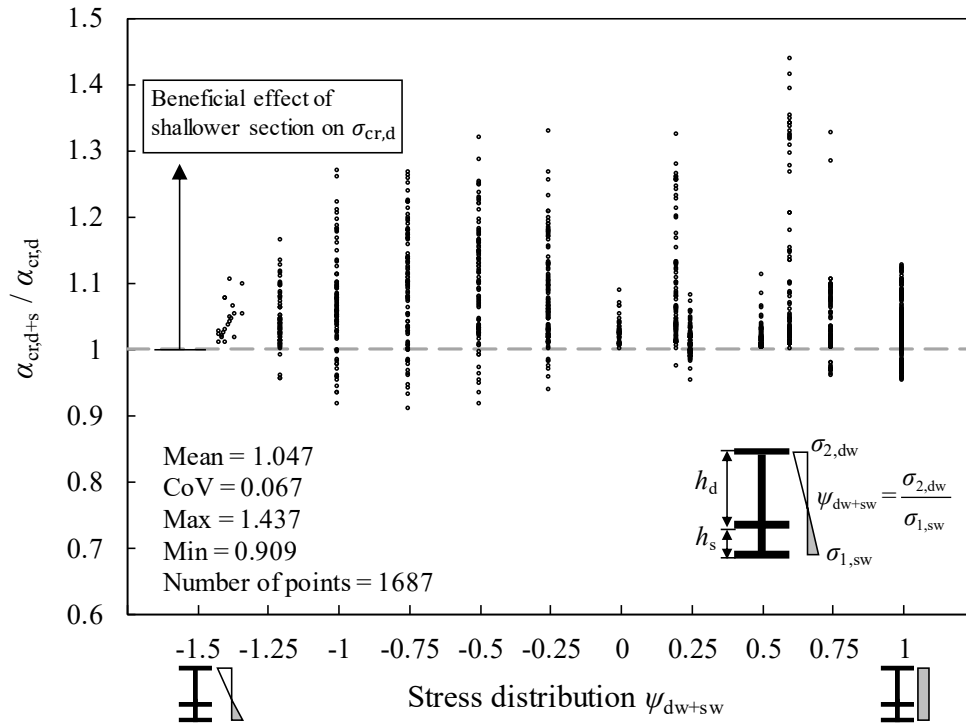


Figure 3.13 CUFSM results showing influence of shallower section on critical deeper section, for deeper section critical scenarios in Case 2 with varying stress distributions

It can be seen from Figures 3.12 and 3.13 that, as in Case 1, there are also some scenarios in Case 2 where the compound three-flanged cross-section buckles at a lower stress than the critical single I-section; all these scenarios arise where $\alpha_{cr,d} \approx \alpha_{cr,s}$, as explained in Section 3.3.2.1.

3.3.3 Influence of shallower section in deeper section critical scenarios in Case 2

The results presented in Figure 3.13 show that the shallower section can offer significant restraint to the deeper section in deeper section critical scenarios in Case 2. This is explored further in this subsection. First, the deeper section critical scenarios in Case 2 are further categorised into either deeper section web critical or deeper section flange critical, depending on which isolated plate within the deeper section has the lower elastic local buckling stress (assuming simply-supported boundary conditions along the adjoined edges): $\alpha_{cr,d} \leq \alpha_{cr,s}$ and $\phi_d = \sigma_{cr,df}^{SS} / \sigma_{cr,dw}^{SS} \geq 1$ implies that the deeper section web is critical, while $\alpha_{cr,d} \leq \alpha_{cr,s}$ and $\phi_d < 1$ indicates that the deeper section flange is critical. Note that the latter classification does not mean the deeper section flange is the critical plate element within the compound three-flanged cross-section; this ‘artificial’ case is only considered as part of the analysis and calculation process.

The restraining influence of the shallower section on the deeper section can be assessed by inspecting the buckling coefficient k of its critical plate element. This is illustrated in Figure 3.14, where several pairs of back-calculated buckling coefficients for the web within the single deeper I-section k_{dw} (using Eq. (2.35)) and the web within the three-flanged cross-section k_{dw+sw} (using Eq. (2.36)) are plotted for deeper section web critical cases. Both the k_{dw} and k_{dw+sw} values lie between the lower bound value of the buckling coefficient for isolated plates with simply-supported boundary conditions along the adjoined edges and the upper bound value for isolated plates with fixed boundary conditions along the adjoined edges, as obtained from Tables 3.1 and 3.2. However, the k_{dw+sw} values are consistently higher than the k_{dw} values, and closer to the upper bound; this illustrates that the rotational restraint afforded to the deeper section by the shallower section, enhances the buckling coefficient of the critical plate element towards the upper bound value for fixed boundary conditions.

$$k_{dw} = \sigma_{cr,d} / \left[\frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{h_d - t_f} \right)^2 \right] \quad (2.35)$$

$$k_{dw+sw} = \sigma_{cr,d+s} \psi_{sw} / \left[\frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{h_d - t_f} \right)^2 \right] \quad (2.36)$$

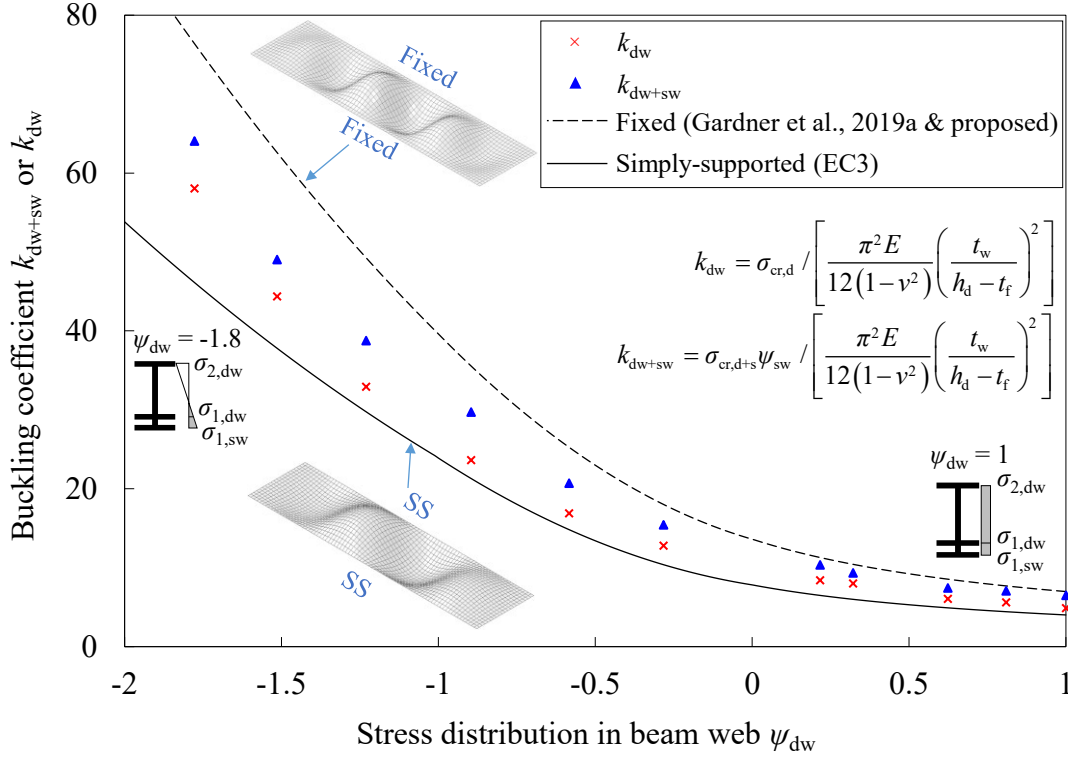


Figure 3.14 Back-calculated buckling coefficients of the web in single deeper I-sections k_{dw} and in corresponding three-flanged cross-sections k_{dw+sw} (when the web in deeper sections is the critical plate); the lower and upper bound buckling coefficient values of the isolated web with simply-supported and fixed boundary conditions along the longitudinal edges are also shown.

The enhancement in the elastic buckling stresses of the deeper section due to the rotational restraint from the shallower section in a series of three-flanged cross-sections is assessed in Figure 3.15, where the ratio $\alpha_{cr,d+s}/\alpha_{cr,d}$ is presented in relation to the parameter $\phi_d = \sigma_{cr,df}^{SS} / \sigma_{cr,dw}^{SS}$. It can be seen that in three-flanged cross-sections with very large ϕ_d values (deeper section web critical) or where ϕ_d approaches zero (deeper section flange critical), the beneficial effect of the shallower section is small (i.e. $\alpha_{cr,d+s} \approx \alpha_{cr,d}$); this is because in the corresponding single deeper I-sections, the web (for high ϕ_d values) or flange (for low ϕ_d values) are already close to fixed boundary conditions along their adjoined edges (i.e. at the web-to-flange junction) and thus there is little or no scope for further increases to their elastic local buckling stresses. Conversely, in three-flanged cross-sections with ϕ_d values close to unity, the beneficial effect of the shallower section is significant, with increases in elastic buckling stresses of up to about 40%; this is because in the corresponding single deeper I-sections, the web and flange have

approximately equal elastic local buckling stresses, hence the boundary conditions at the web-flange junction are close to simply-supported and the rotational restraint provided by the adjoining shallower section is highly beneficial.

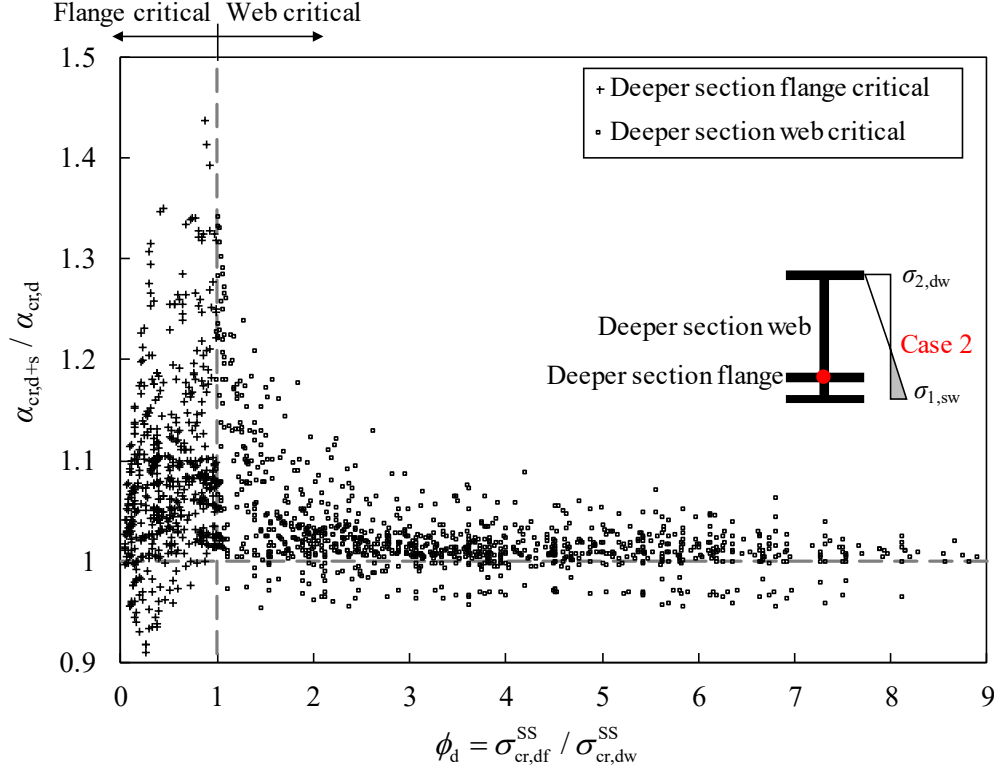


Figure 3.15 CUFSM results showing influence of shallower section on critical deeper section, for deeper section critical scenarios in Case 2 with varying ϕ_d values

3.3.4 Determination of ζ_s^* for deeper section critical scenarios in Case 2

In order to capture quantitatively the influence of the shallower section on the deeper section in deeper section critical scenarios in Case 2, the interaction coefficient ζ_s^* is introduced; this is used to represent the difference between the interaction coefficient to evaluate flange-web interaction within the single deeper I-section ζ_d and that for the compound three-flanged cross-section ζ_{d+s} , as expressed by Eq. (2.37).

$$\zeta_s^* = \zeta_{d+s} - \zeta_d \quad (2.37)$$

The interaction coefficients ζ_d and ζ_{d+s} can be back-calculated using Eqs. (2.38) and (2.39), according to the elastic buckling stresses of the single deeper I-section $\sigma_{cr,d}$ and three-flanged cross-section $\sigma_{cr,d+s}$, as obtained from finite strip analysis.

$$\zeta_d = \frac{\sigma_{cr,d} - \sigma_{cr,dp}^{SS}}{\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS}}, \text{ but } 0 \leq \zeta_d \leq 1 \quad (2.38)$$

$$\zeta_{d+s} = \frac{\sigma_{cr,d+s} \psi_{sw} - \sigma_{cr,dp}^{SS}}{\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS}}, \text{ but } 0 \leq \zeta_{d+s} \leq 1 \quad (2.39)$$

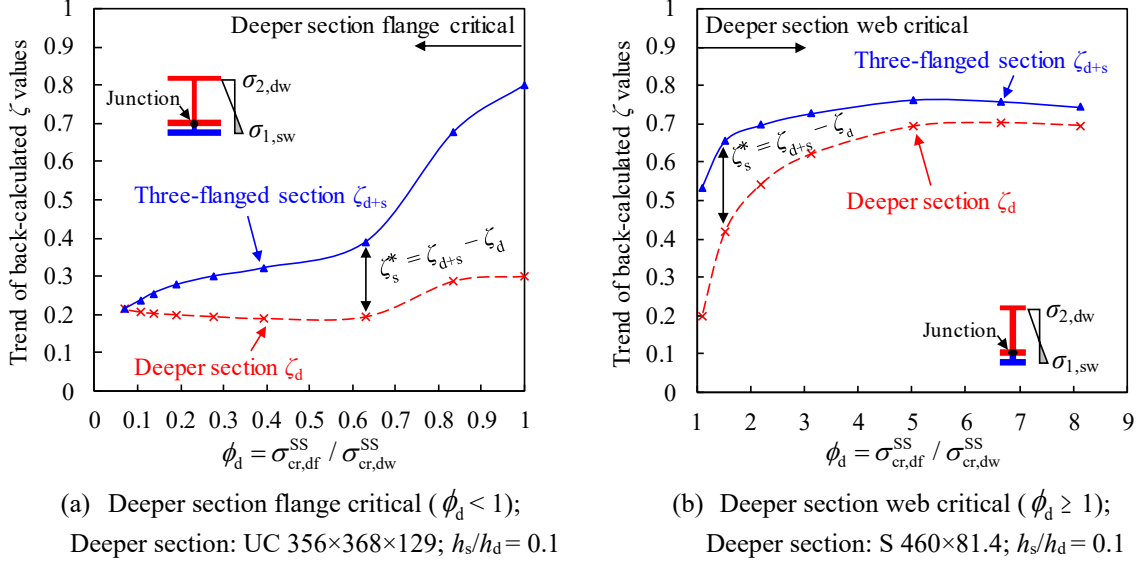


Figure 3.16 Example trends of back-calculated interaction coefficients for single deeper I-sections ζ_d and corresponding three-flanged cross-sections ζ_{d+s}

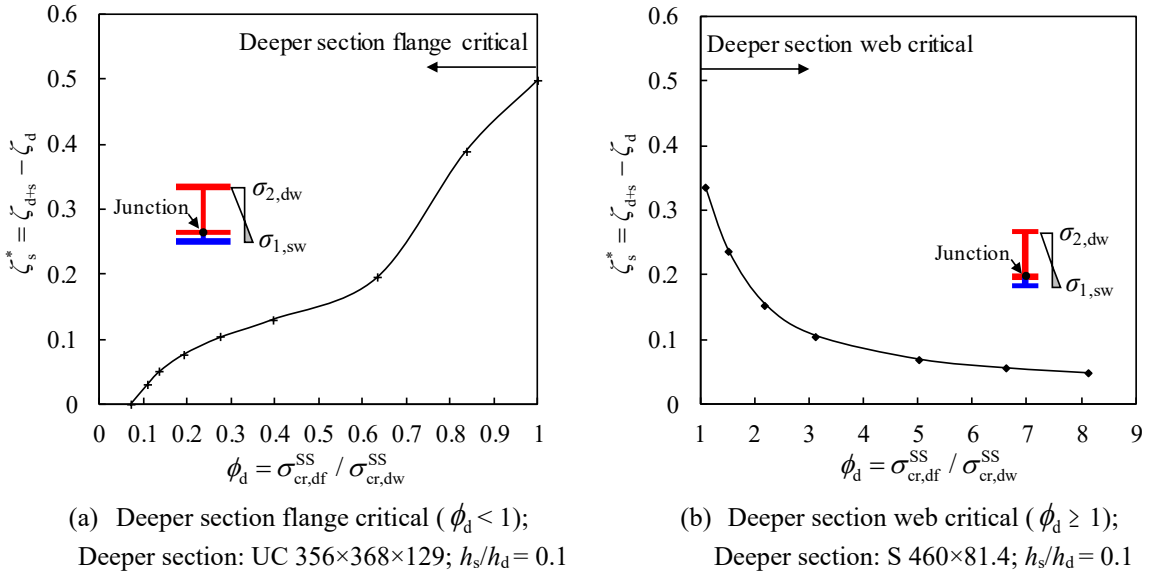


Figure 3.17 CUFSM results showing influence of shallower section on critical deeper section through interaction coefficient ζ_s^* , for deeper section critical scenarios in Case 2

As shown in Figure 3.16 (a) and (b) for deeper section flange critical and deeper section web critical scenarios respectively, both ζ_d and ζ_{d+s} lie between 0 and 1, i.e. as discussed above, although the shallower section clearly provides benefit through rotational restraint to the deeper

section (indicated by the ζ_{d+s} values being greater than the corresponding ζ_d values), the maximum value of the elastic buckling stress of the three-flanged cross-section is limited to that of the critical individual plate element with fixed boundary conditions along the adjoined edges. As shown in Figure 3.17, the difference between these two coefficients, i.e. ζ_s^* , exhibits a consistent trend, i.e. lower ζ_s^* values arise in the three-flanged cross-sections with very high or low (approaching 0) ϕ_d values, while higher ζ_s^* values appear in the three-flanged cross-sections with ϕ_d values close to unity.

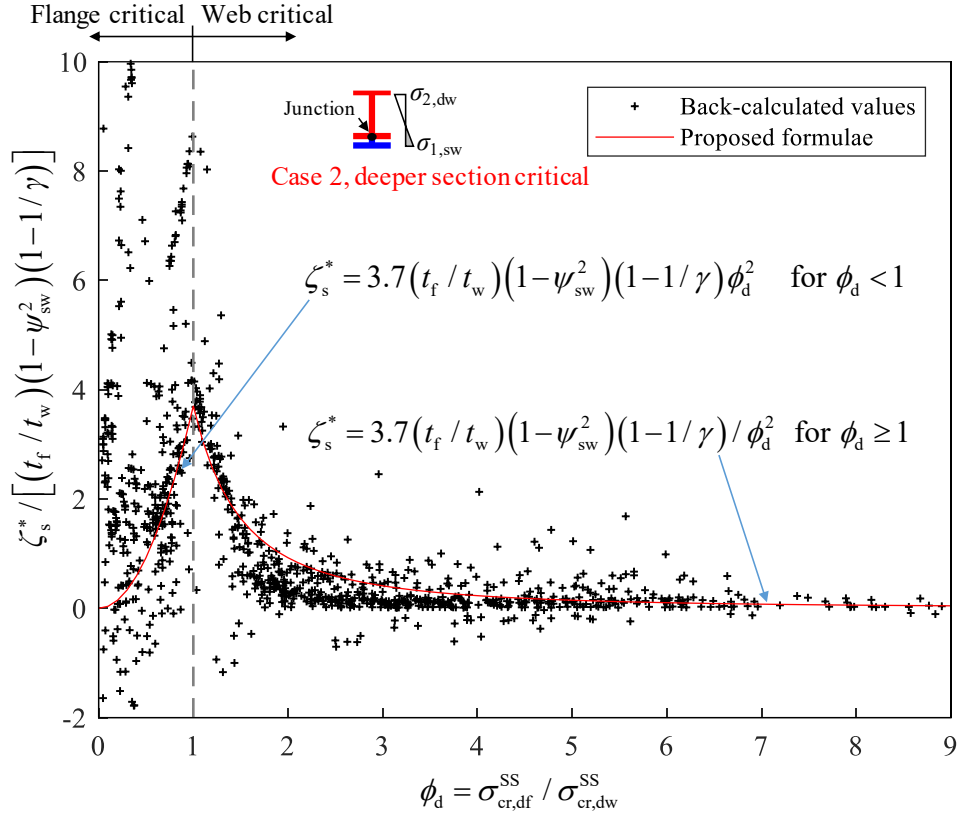


Figure 3.18 Back-calculated normalised values of ζ_s^* from CUFSM and proposed predictive equations for deeper section critical scenarios in Case 2

Figure 3.18 shows the back-calculated normalised ζ_s^* values for all deeper section critical results in Case 2. According to the trend of the results, predictive equations to determine ζ_s^* are proposed for both the deeper section flange critical ($\phi_d < 1$) and deeper section web critical ($\phi_d \geq 1$) scenarios, as expressed by Eq. (2.40). The formulae, which are plotted in Figure 3.18, are dependent on the ratio of the elastic buckling stress of the shallower section to that of the deeper section $\gamma = \alpha_{cr,s}/\alpha_{cr,d}$, the stress distribution in the shallower section web ψ_{sw} and the ratio of flange to web thickness t_f/t_w , and ensure continuity of predictions between Case 1 and Case 2

and across the full range of loading scenarios.

$$\zeta_s^* = \begin{cases} 3.7 \frac{t_f}{t_w} (1 - \psi_{sw}^2) \left(1 - \frac{1}{\gamma}\right) \phi_d^2 & \text{for } \phi_d < 1 \\ 3.7 \frac{t_f}{t_w} (1 - \psi_{sw}^2) \left(1 - \frac{1}{\gamma}\right) / \phi_d^2 & \text{for } \phi_d \geq 1 \end{cases} \quad (2.40)$$

3.4 Development of formulae for predicting elastic local buckling stress of three-flanged cross-sections

3.4.1 Development of predictive expressions

Based on the behaviour described above, formulae for predicting the elastic local buckling stress of three-flanged cross-sections are developed herein. The overall framework of the predictive expressions is illustrated in Figure 3.19. First, the case in question (i.e. Case 1 or Case 2) is identified.

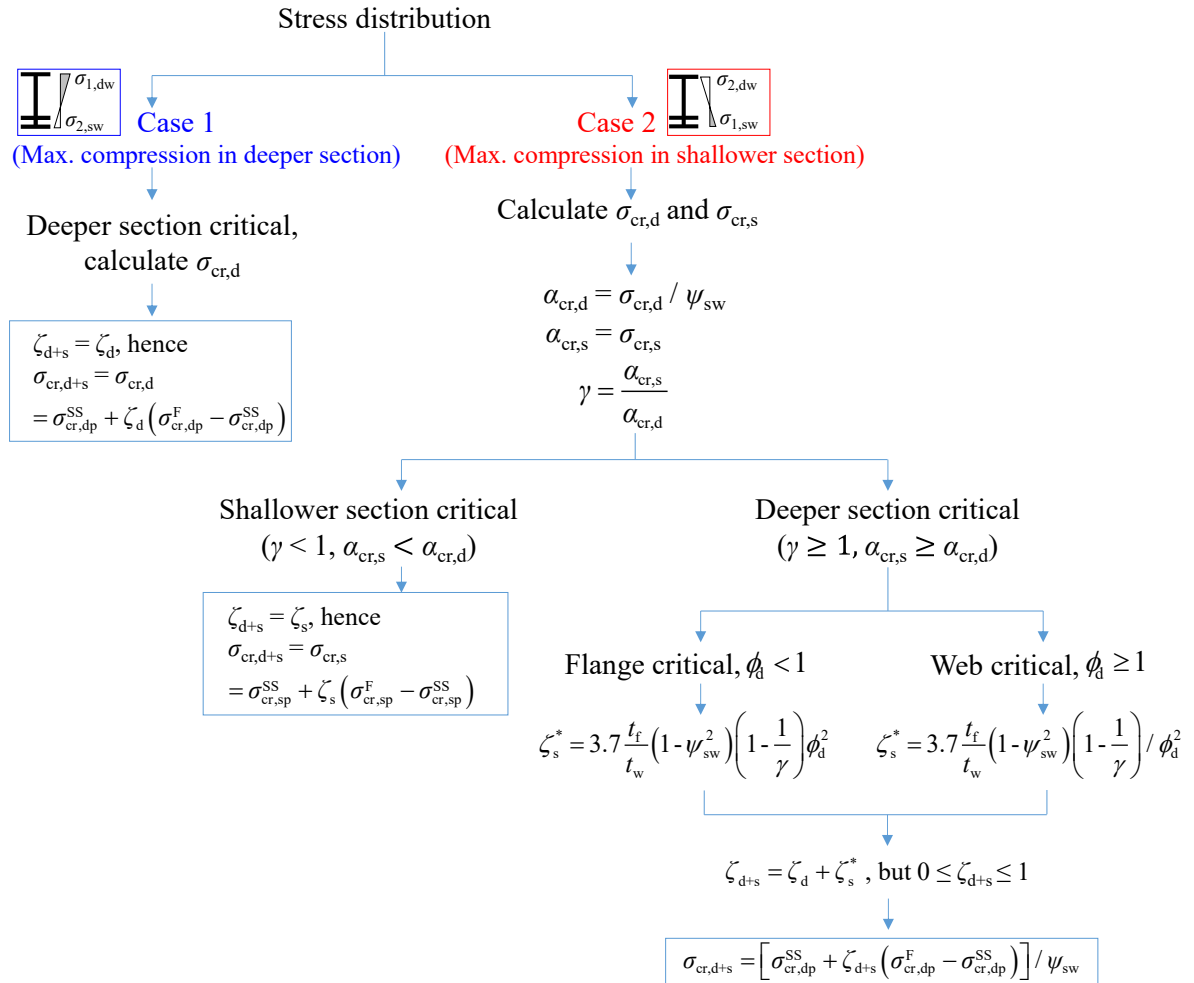


Figure 3.19 Proposed framework for calculating the elastic local buckling stress of three-flanged cross-sections

For Case 1 (maximum compression in deeper I-section), the outer portion of the deeper section is always the critical part of the three-flanged cross-section, and the influence of the shallower section is small. Thus, it is assumed that $\zeta_{d+s} = \zeta_d$ (i.e. $\zeta_s^* = 0$), and the elastic local buckling stress of the three-flanged cross-section is given by:

$$\sigma_{cr,d+s} = \sigma_{cr,d} = \sigma_{cr,dp}^{SS} + \zeta_d \left(\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS} \right). \quad (2.41)$$

For Case 2 (maximum compression in shallower I-section), first, Eq. (2.25) is used to calculate elastic local buckling critical stresses of the single deeper I-section $\sigma_{cr,d}$ and the single shallower I-section $\sigma_{cr,s}$; then the critical load factors of these two sections are calculated: $\alpha_{cr,d} = \sigma_{cr,d}/\psi_{sw}$ and $\alpha_{cr,s} = \sigma_{cr,s}$, identifying whether the shallower ($\gamma = \alpha_{cr,s}/\alpha_{cr,d} < 1$) or the deeper ($\gamma = \alpha_{cr,s}/\alpha_{cr,d} \geq 1$) section is critical.

For the shallower section critical scenarios, the influence of the deeper section is small; hence it is assumed that $\zeta_d^* = 0$, thus $\zeta_{d+s} = \zeta_s$, and the elastic local buckling stress of the three-flanged cross-section is given by:

$$\sigma_{cr,d+s} = \sigma_{cr,s} = \sigma_{cr,sp}^{SS} + \zeta_s \left(\sigma_{cr,sp}^F - \sigma_{cr,sp}^{SS} \right). \quad (2.42)$$

For the deeper section critical scenarios, first the critical plate within the deeper cross-section (i.e. either the deeper section flange or web) is identified according to the value of ϕ_d calculated from Eq. (2.31); the influence of the shallower section on the deeper section is then determined through ζ_s^* using Eq. (2.40). Finally, the compound interaction coefficient ζ_{d+s} and the elastic local buckling stress $\sigma_{cr,d+s}$ of the three-flanged cross-section are given by:

$$\zeta_{d+s} = \zeta_d + \zeta_s^*, \text{ but } 0 \leq \zeta_{d+s} \leq 1 \quad (2.43)$$

$$\sigma_{cr,d+s} = \left[\sigma_{cr,dp}^{SS} + \zeta_{d+s} \left(\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS} \right) \right] / \psi_{sw} \quad (2.44)$$

3.4.2 Evaluation of developed proposal

In this section, the accuracy of the predictions of the elastic local buckling stresses of three-flanged cross-sections using the formulae developed herein is evaluated against results obtained using CUFSM; the predictions are also compared with elastic buckling stresses calculated on an element-by-element basis (i.e. ignoring the influence of element interaction). These comparisons are presented in Table 3.4.

Table 3.4 Comparisons of the predicted elastic local buckling stresses of three-flanged cross-sections calculated using the proposed formulae and on an element-by-element basis (i.e. isolated critical plates having simply-supported boundary conditions along the adjoined edges) relative to numerical (CUFSM) results. For each of Cases 1 and 2, the number of analysed cross-sections was as follows: 100 European and American I-sections $\times$ 5 height ratios (h_s/h_d) $\times$ 9 stress distributions (ψ_{dw+sw} values).

Prediction/CUFSM		Proposal	Isolated critical plate
Case 1	Mean	0.945	0.663
	CoV	0.036	0.099
	Max	1.080	0.920
	Min	0.749	0.540
Case 2	Mean	0.943	0.641
	CoV	0.046	0.109
	Max	1.120	0.820
	Min	0.711	0.418

3.4.2.1 Case 1 - maximum compression in deeper I-section

For three-flanged cross-sections where the maximum compression is within the deeper I-section (i.e. Case 1), it can be seen in Table 3.4 that the developed formulae provide accurate and safe-sided predictions of the numerical results: the mean value of the ratios of the predicted to numerical (CUFSM) elastic local buckling stresses of the three-flanged cross-sections is 0.945 with a CoV of 0.036, compared to an equivalent mean prediction of 0.663 with a CoV of 0.099 when the elastic local buckling stress is calculated on an element-by-element basis, i.e. the developed formulae offer an average improvement in accuracy of about 30%. The variation in prediction accuracy with ϕ_d is shown in Figure 3.20, and the frequency distributions for the predictions, which can be seen to be approximately symmetrical, are shown in Figure 3.21 (a).

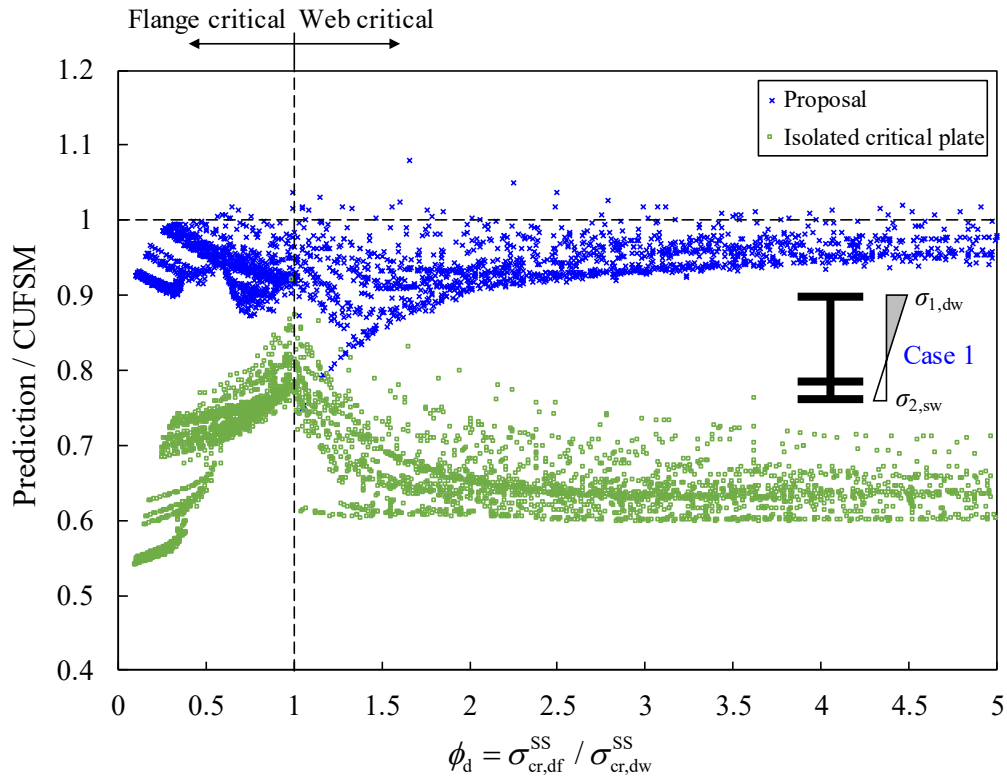


Figure 3.20 Assessment of accuracy of predictions of elastic local buckling stresses of three-flanged cross-sections in Case 1 using proposed formulae and element-by-element approach (i.e. isolated critical plates having simply-supported boundary conditions along the adjoined edges) relative to numerical (CUFSM) results

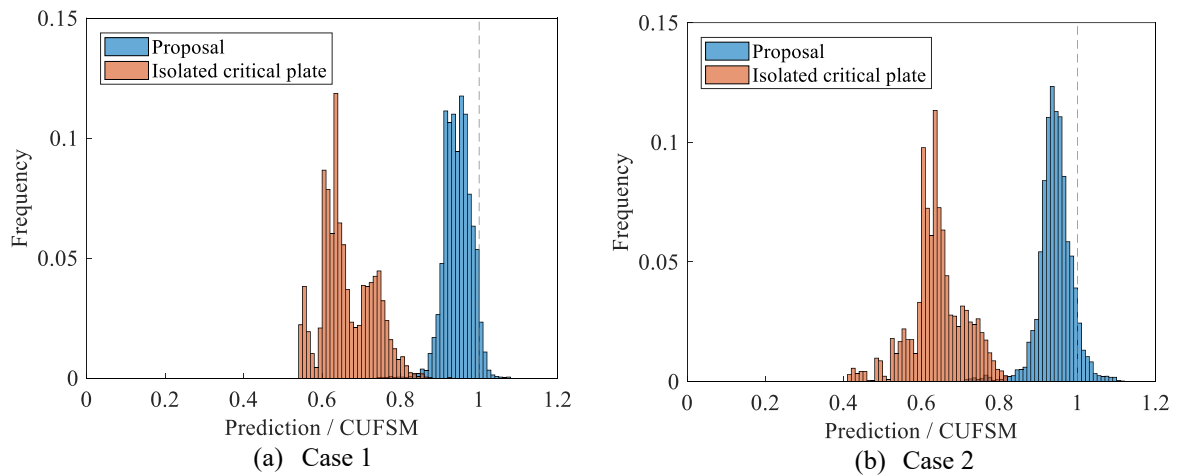


Figure 3.21 Frequency distributions for predicted elastic local buckling stresses of three-flanged cross-sections in (a) Case 1 and (b) Case 2 using proposed formulae and element-by-element approach (i.e. isolated critical plates having simply-supported boundary conditions along the adjoined edges) relative to numerical (CUFSM) results

3.4.2.2 Case 2 - maximum compression in shallower I-section

For three-flanged cross-sections where the maximum compression is within the shallower I-section (i.e. Case 2), similar improvements in prediction accuracy of around 30% are achieved using the developed formulae in comparison to calculations performed on an element-by-element basis; the scatter of the predictions is also dramatically improved, as indicated by the CoV values given in Table 3.4, and the comparisons shown in Figure 3.22 for the shallower section critical scenarios and Figure 3.23 for the deeper section critical scenarios. The improvement in accuracy achieved through the use of the modified equation (Eq. (2.32)) for shallow shallower I-sections is small (around 2%) on average, but can be significant (over 10%) in some cases. The frequency distributions for the predictions, which also can be seen to be approximately symmetrical, are shown in Figure 3.21 (b). For both Case 1 and Case 2, the frequency distributions of the predictions achieved using the developed formulae are a bit more symmetrical than those of the calculations performed on an element-by-element basis.

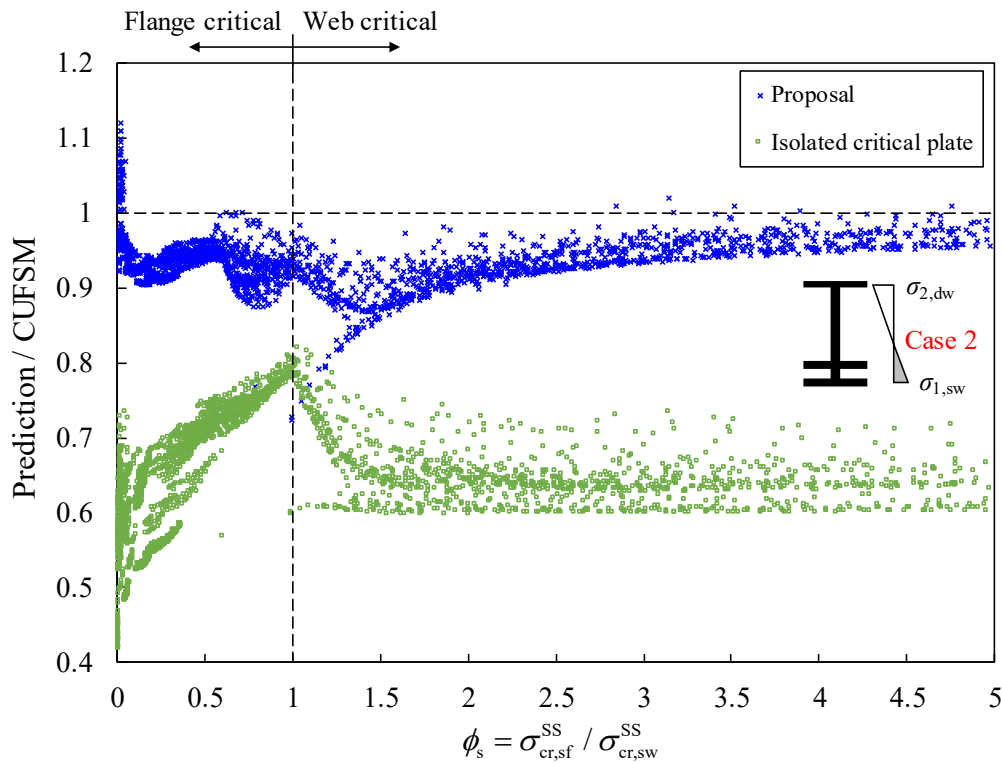


Figure 3.22 Assessment of accuracy of predictions of elastic local buckling stresses of three-flanged cross-sections for shallower section critical scenarios in Case 2 using proposed formulae and element-by-element approach (i.e. isolated critical plates having simply-supported boundary conditions along the adjoining edges) relative to numerical (CUFSM) results

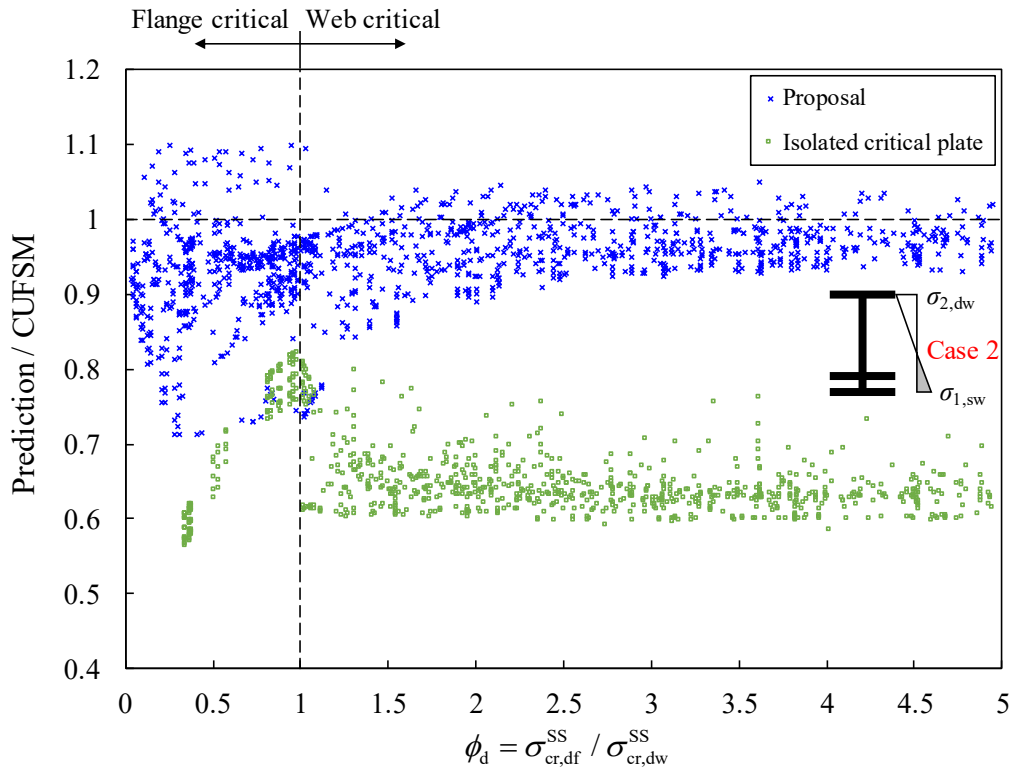


Figure 3.23 Assessment of accuracy of predictions of elastic local buckling stresses of three-flanged cross-sections for deeper section critical scenarios in Case 2 using proposed formulae and element-by-element approach (i.e. isolated critical plates having simply-supported boundary conditions along the adjoining edges) relative to numerical (CUFSM) results

3.4.3 Worked example

A worked example is now presented to demonstrate application of the developed method for the determination of the elastic local buckling stress of a three-flanged cross-section in Case 2. Note that the centreline geometry of the considered cross-section is used and fillets and corner radii are ignored.

A three-flanged cross-section made up of a UB 686×254×125 beam profile in S355 steel with $E = 210000$ MPa and $\nu = 0.3$ subjected to a combination of compression $N_{Ed} = 1292$ kN and major axis bending $M_{y,Ed} = 1000$ kNm is considered, as shown in Figure 3.24. The maximum compression arises in the shallower section and we are thus considering Case 2. The distance between the neutral axis and the centreline of the top, bottom and middle flanges are denoted z_d , z_s , z_m , respectively, while A_{d+s} and I_{d+s} are the area and major axis second moment of area of the compound (deeper + shallower) section. The stress distribution within the cross-section is first determined based on the applied loading and the section geometry, with compressive stresses taken as positive.

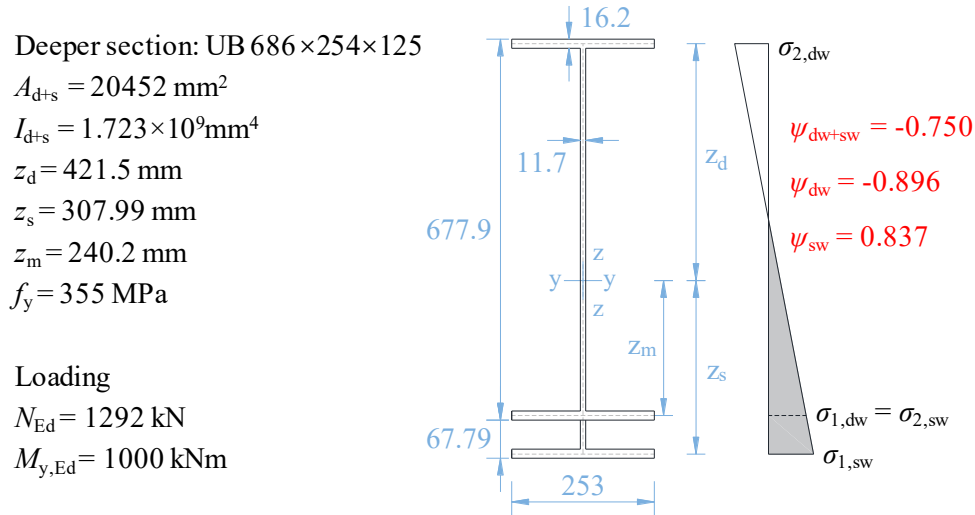


Figure 3.24 Worked example: geometry and applied stress distribution in a three-flanged cross-section (all dimension in mm)

The applied stress at the top flange $\sigma_{2,dw}$ is:

$$\begin{aligned}
 \sigma_{2,dw} &= \frac{N_{Ed}}{A_{d+s}} - \frac{M_{Ed}z_d}{I_{d+s}} \\
 &= \frac{1292 \times 10^3}{20452} - \frac{1000 \times 10^6 \times 421.5}{1.723 \times 10^9} \\
 &= -181.4 \text{ MPa}.
 \end{aligned} \tag{2.45}$$

The applied stress at the bottom flange $\sigma_{1,sw}$ is:

$$\begin{aligned}
 \sigma_{1,sw} &= \frac{N_{Ed}}{A_{d+s}} + \frac{M_{Ed}z_s}{I_{d+s}} \\
 &= \frac{1292 \times 10^3}{20452} + \frac{1000 \times 10^6 \times 307.99}{1.723 \times 10^9} \\
 &= 241.9 \text{ MPa}.
 \end{aligned} \tag{2.46}$$

The applied stress at the middle flange $\sigma_{1,dw}$ is:

$$\begin{aligned}
 \sigma_{1,dw} &= \sigma_{2,sw} = \frac{N_{Ed}}{A_{d+s}} + \frac{M_{Ed}z_m}{I_{d+s}} \\
 &= \frac{1292 \times 10^3}{20452} + \frac{1000 \times 10^6 \times 240.2}{1.723 \times 10^9} \\
 &= 202.5 \text{ MPa}.
 \end{aligned} \tag{2.47}$$

Hence, the stress distributions in the web of the three-flanged cross-section ψ_{dw+sw} , the single

deeper I-section ψ_{dw} and the single shallower I-section ψ_{sw} are respectively equal to:

$$\psi_{dw+sw} = \frac{\sigma_{2,dw}}{\sigma_{1,sw}} = \frac{-181.4}{241.9} = -0.750, \quad (2.48)$$

$$\psi_{dw} = \frac{\sigma_{2,dw}}{\sigma_{1,dw}} = \frac{-181.4}{202.5} = -0.896, \quad (2.49)$$

$$\psi_{sw} = \frac{\sigma_{2,sw}}{\sigma_{1,sw}} = \frac{202.5}{241.9} = 0.837. \quad (2.50)$$

The flange plates are all in pure compression; hence:

$$\psi_{df} = \psi_{sf} = 1.0. \quad (2.51)$$

The elastic local buckling stress of the single deeper I-section is now calculated. First, the elastic local buckling stresses of the isolated deeper section flange and web with simply-supported and fixed boundary conditions are calculated. The buckling coefficients k are obtained from Tables 3.1 and 3.2.

For the deeper section flange ($\psi_{df} = 1$), $k_{df}^{SS} = 0.43$ and $k_{df}^F = 1.25$. Hence:

$$\sigma_{cr,df}^{SS} = k_{df}^{SS} \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_f}{0.5b} \right)^2 = 0.43 \times \frac{210000\pi^2}{12(1-0.3^2)} \left(\frac{16.2}{0.5 \times 253} \right)^2 = 1338.5 \text{ MPa} \quad (2.52)$$

$$\sigma_{cr,df}^F = k_{df}^F \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_f}{0.5b} \right)^2 = 1.25 \times \frac{210000\pi^2}{12(1-0.3^2)} \left(\frac{16.2}{0.5 \times 253} \right)^2 = 3890.9 \text{ MPa} \quad (2.53)$$

For the deeper section web ($\psi_{dw} = -0.896$), $k_{dw}^{SS} = 21.30$ and $k_{dw}^F = 35.54$. Hence:

$$\sigma_{cr,dw}^{SS} = k_{dw}^{SS} \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{h_d - t_f} \right)^2 = 21.30 \times \frac{210000\pi^2}{12(1-0.3^2)} \left(\frac{11.7}{677.9 - 16.2} \right)^2 = 1263.9 \text{ MPa} \quad (2.54)$$

$$\sigma_{cr,dw}^F = k_{dw}^F \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t_w}{h_d - t_f} \right)^2 = 35.54 \times \frac{210000\pi^2}{12(1-0.3^2)} \left(\frac{11.7}{677.9 - 16.2} \right)^2 = 2108.9 \text{ MPa} \quad (2.55)$$

Since $\phi_d = \sigma_{cr,df}^{SS} / \sigma_{cr,dw}^{SS} = 1.059 \geq 1$, the web is the critical element in the deeper section, and the

interaction coefficient for the deeper section ζ_d is given by:

$$\zeta_d = \frac{t_f}{t_w} \left(0.45 - \frac{0.3}{\phi_d^2} \right) = \frac{16.2}{11.7} \times \left(0.45 - \frac{0.3}{1.059^2} \right) = 0.253. \quad (2.56)$$

The elastic local buckling stresses of the critical plate in the deeper section with simply-supported and fixed boundary conditions are respectively equal to:

$$\sigma_{cr,dp}^{SS} = \min(\sigma_{cr,df}^{SS}, \sigma_{cr,dw}^{SS}) = 1263.9 \text{ MPa} \quad (2.57)$$

$$\sigma_{cr,dp}^F = \min(\sigma_{cr,df}^F, \sigma_{cr,dw}^F) = 2108.9 \text{ MPa} \quad (2.58)$$

Thus, the elastic local buckling stress of the deeper section is:

$$\sigma_{cr,d} = \sigma_{cr,dp}^{SS} + \zeta_d (\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS}) = 1263.9 + 0.253 \times (2108.9 - 1263.9) = 1477.7 \text{ MPa}. \quad (2.59)$$

Applying the same process, the elastic local buckling stress of the single shallower I-section can be obtained. The buckling coefficients of the isolated shallower section flange and web plates with simply-supported and fixed boundary conditions are $k_{sf}^{SS} = 0.43$, $k_{sf}^F = 1.25$, $k_{sw}^{SS} = 4.35$, $k_{sw}^F = 7.57$. The corresponding elastic local buckling stresses of the isolated shallower section flange and web plates are equal to $\sigma_{cr,sf}^{SS} = 1338.5 \text{ MPa}$, $\sigma_{cr,sf}^F = 3890.9 \text{ MPa}$, $\sigma_{cr,sw}^{SS} = 24593.8 \text{ MPa}$, $\sigma_{cr,sw}^F = 42798.9 \text{ MPa}$. Since $\phi_s = \sigma_{cr,sf}^{SS} / \sigma_{cr,sw}^{SS} = 0.054 < 1$, the flange is the critical element in the shallower section. Since the dimension ratio of this shallower section $((h_s + t_f)/t_w)(t_f/b) = 0.460 < 0.89$, the interaction coefficient is calculated using Eq. (2.32) as follows.

$$\begin{aligned} \zeta_s &= \max\left(0.15 \frac{t_f}{t_w} \phi_s, \frac{t_w}{t_f} (0.4 - 0.25 \phi_s)\right) + 0.4 \left(0.89 - \frac{h_s + t_f}{t_w} \frac{t_f}{b}\right)^2 \\ &= \max\left(0.15 \times \frac{16.2}{11.7} \times 0.054, \frac{11.7}{16.2} (0.4 - 0.25 \times 0.054)\right) + 0.4 \left(0.89 - \frac{83.99}{11.7} \frac{16.2}{253}\right)^2 \\ &= 0.353 \end{aligned} \quad (2.60)$$

Thus, the elastic local buckling stress of the shallower I-section is given by:

$$\sigma_{cr,s} = \sigma_{cr,sp}^{SS} + \zeta_s (\sigma_{cr,sp}^F - \sigma_{cr,sp}^{SS}) = 1338.5 + 0.353 \times (3890.9 - 1338.5) = 2239.5 \text{ MPa}. \quad (2.61)$$

Hence, the critical load factors of the deeper and shallower I-sections are respectively equal to $\alpha_{cr,d} = \sigma_{cr,d}/\psi_{sw} = 1765.5$ MPa and $\alpha_{cr,s} = \sigma_{cr,s} = 2239.5$ MPa. Since $\gamma = \alpha_{cr,s}/\alpha_{cr,d} = 1.268 \geq 1$, the scenario is classified as deeper section critical. Considering the beneficial effect from the shallower section on the critical deeper section as follows,

$$\zeta_s^* = 3.7 \frac{t_f}{t_w} (1 - \psi_{sw}^2) \left(1 - \frac{1}{\gamma}\right) / \phi_d^2 = 3.7 \frac{16.2}{11.7} (1 - 0.837^2) \left(1 - \frac{1}{1.268}\right) / 1.059^2 = 0.289 \quad (2.62)$$

the compound interaction coefficient of the three-flanged cross-section is equal to:

$$\zeta_{d+s} = \zeta_d + \zeta_s^* = 0.253 + 0.289 = 0.542. \quad (2.63)$$

Hence, the elastic local buckling stress of the three-flanged cross-section is given by:

$$\begin{aligned} \sigma_{cr,d+s} &= \left[\sigma_{cr,dp}^{SS} + \zeta_{d+s} (\sigma_{cr,dp}^F - \sigma_{cr,dp}^{SS}) \right] / \psi_{sw} \\ &= \left[1263.9 + 0.542 \times (2108.9 - 1263.9) \right] / 0.837 \\ &= 2057.2 \text{ MPa}. \end{aligned} \quad (2.64)$$

The elastic local buckling stress of this three-flanged cross-section obtained from CUFSM is 2104.4 MPa, only 2.2% higher than the predicted value. On an element-by-element basis, the critical elastic local buckling stress of the three-flanged cross-section, (as controlled by the shallower section flange) is 1338.5 MPa, 34% below the prediction attained using the proposed formulae. Note that although the deeper section web has a lower elastic buckling stress than the shallower section flange, the deeper section web is subjected to a lower compressive stress and is not the critical element (as indicated by the higher critical load factor). The presented calculation involves a number of steps, but the formulae are both transparent in their purpose and straightforward, making them readily suitable for programming.

3.5 Concluding remarks

Formulae for calculating the elastic local buckling stress of three-flanged cross-sections have been developed and presented herein. The geometry and loading of the studied cross-sections have been assumed to be constant along the member length, i.e. the influence of tapering and moment gradients has not been considered, but the current work provides the basis for exploration of these effects in future research. The formulae were benchmarked against numerical results generated by means of finite strip analysis on a total of 9000 three-flanged

cross-sections subjected to combined loading conditions ranging from pure compression to pure major axis bending; corresponding single deeper and shallower sections were also analysed under same stress conditions as in the three-flanged cross-section to aid the assessment of the results. It was found that, similar to the element interaction observed between the flange and web within single I-sections, in the case of three-flanged cross-sections, the deeper and shallower sections also interact with each other, i.e. the non-critical part is able to provide rotational restraint to the critical part. Two cases were considered – Case 1, where the maximum compression arises in the deeper section and Case 2, where the maximum compression arises in the shallower section; predictive formulae were developed for both cases, allowing for the interaction both within and between the deeper and shallower I-sections. Using the developed formulae, predictions of the elastic local buckling stress of three-flanged cross-sections were shown to be typically within 5% of the numerically obtained values, around 30% higher than calculations performed on an element-by-element basis. The formulae can be used to facilitate the more efficient design of structures that feature three-flanged cross-sections, such as continuous and longitudinally-stiffened plate girders and portal frames, and their application was demonstrated through a worked example.

Chapter 4

Design of web-tapered steel I-section members

4.1 Introduction

Web-tapered I-section members are commonly used in steel construction to enhance structural efficiency. Typically, cross-sections of greater depth are arranged to coincide with the regions of larger internal forces within the structure, such as at the eaves and apexes of steel portal frames, as shown in Figure 4.1. In keeping with the design of structures comprising uniform members, the design of structures featuring tapered members also generally involves two steps: (i) the determination of the internal forces and moments within the structure through a structural analysis and (ii) the performance of a series of strength and stability checks on the individual members. The structural analysis is typically carried out using beam finite elements, which are not able to capture cross-section instabilities, i.e. local buckling is not accounted for and rotation capacities are unlimited. Hence, in current practice (e.g. EN 1993-1-1, 2005), cross-section instabilities are considered through the concept of cross-section classification, as detailed in Section 2.2.1. This does however lead to artificial steps between different classes in the resistance predictions of steel members and systems. Tapered steel members are particularly prone to the shortcomings of the cross-section classification concept since their cross-sections continuously vary along the member length and often fall into different classes, leading to abrupt changes in load carrying capacity depending upon the location of the most heavily utilised cross-section, which may change for different loading combinations and analysis types.

With the aim of overcoming the above drawbacks, an alternative design approach for tapered steel members performed using advanced analysis is presented in this chapter. In the proposed approach, a second order inelastic analysis i.e. GMNIA of the tapered member is carried out using beam finite elements; the ultimate strength of the member is then determined based on either (i) the load factor corresponding to the attainment of the CSM strain limit within the

member or (ii) the peak load factor, after which the load versus deformation response descends, whichever occurs first. The proposed approach exploits the benefits of advanced analysis such as its ability to (i) furnish very accurate ultimate strength predictions for individual members and systems, (ii) provide realistic structural failure modes and (iii) avoid the need for conducting member design checks, which can be particularly complex for tapered steel members (Butler & Anderson, 1963; Prawel et al., 1974; Shiomi & Kurata, 1984). The beneficial influence of strain gradients along the member lengths on the local cross-section strength is also allowed for by limiting averaged compressive strains over the local buckling half-wavelength rather than peak compressive strains at a specific cross-section using the CSM; this results in improved capacity predictions. The accuracy of the proposed design approach is verified against results from nonlinear shell FE modelling as well as a number of experiments on tapered members with various taper ratios and member slenderness values. Furthermore, it is shown that the proposed design approach leads to a considerably more direct and practical way of designing tapered steel members relative to traditional design methods provided in design standards such as EN 1993-1-1 (2005).

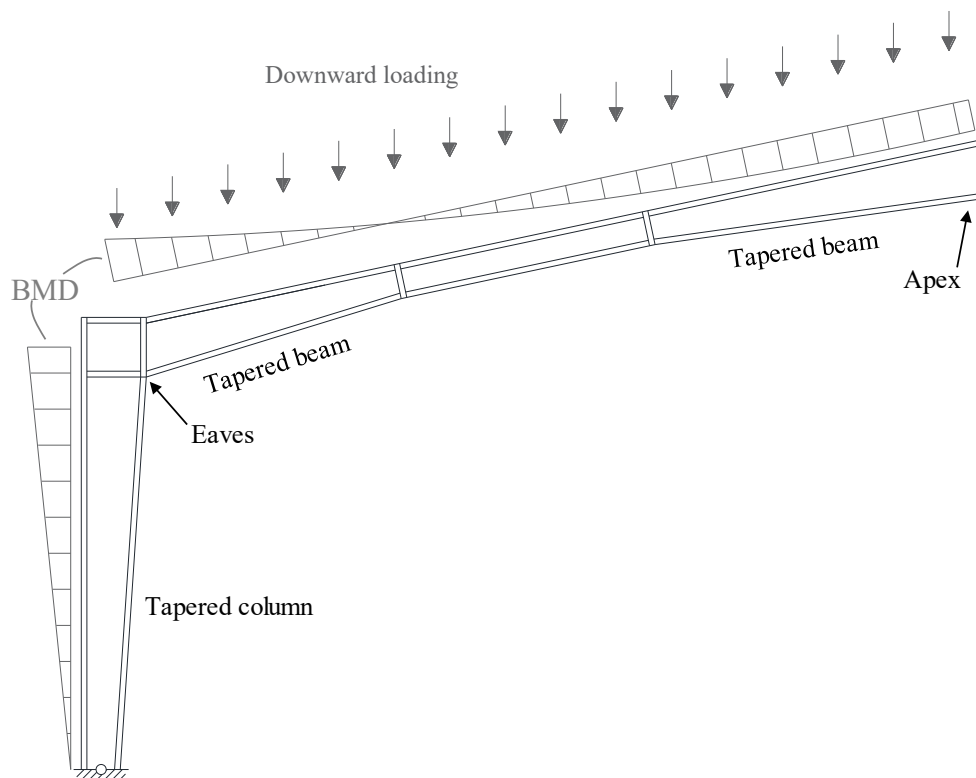


Figure 4.1 Downward loading with corresponding bending moment diagram (BMD) on a typical portal frame with tapered beams and tapered columns

4.2 Design by advanced analysis using beam elements and CSM strain limits

4.2.1 Advanced analysis with CSM strain limits for uniform members

4.2.1.1 Concept

To overcome the shortcomings of traditional steel design methods as described in Section 2.2, as well as responding to the changing landscape with regards to the sophistication of available structural analysis software and computational power, Fieber et. al (2019a) proposed a new approach to structural steel design for uniform members. The new design method involves: (i) using beam FE models to perform GMNIA and (ii) using either the load factor that corresponds to the attainment of the CSM strain limit or the peak load factor, whichever occurs first, to define the ultimate resistance of individual members or structures. Through GMNIA with beam elements, member instabilities can be captured directly, but local instabilities cannot. Thus, the CSM strain limits are adopted to account for local buckling effects; the maximum compressive strains are checked against the corresponding CSM strain limits at all cross-sections in the structure at each load increment. The key aspects of CSM are illustrated in Section 2.3.

4.2.1.2 Strain averaging approach

In previous research (Driscoll & Beedle, 1957; Lay & Galambos, 1964; Gioncu & Petcu, 1997; Gardner et al., 2010; Wang et al., 2016; Meng & Gardner, 2020b), it has been observed that laterally-restrained steel beams subjected to moment gradients exhibit greater cross-section resistances than the same beams under uniform bending. This has been ascribed to the beneficial effects of strain gradients along member lengths on the local stability of cross-sections (Fieber et al., 2019a), i.e. the critical cross-section receives support from the adjacent less heavily loaded cross-sections. It was shown in Fieber et al. (2019a) that this effect could be accurately captured by limiting averaged rather than peak compressive strains to the CSM strain limit. The length of member of which to average the strains was taken as the elastic local buckling half-wave length $L_{b,cs}$ that can be obtained numerically e.g. using the finite strip method software CUFSM (Li & Schafer, 2010b) or through the expressions presented in Fieber et al. (2019b).

4.2.2 Extension to tapered members

4.2.2.1 Procedure for application of CSM strain limits in GMNIA of tapered members

The extension of the aforementioned new design approach to tapered members is described in this section. In the proposed design method, the first step is to use beam element models to perform GMNIA of the tapered member. In this study, the finite element analysis package Abaqus (2017) was used to carry out the GMNIA simulations. A common method for analysing tapered members is to divide them into a sufficient number of prismatic segments (Kaehler et al., 2011); at least twenty is recommended (Chan et al., 2018). Although the influence of the inclined flanges of the tapered members on their internal stress distributions is neglected in this approach, it has been established that, for tapering angles typically encountered in practice ($<15^\circ$), this influence is negligible and, hence the in-plane behaviour of tapered steel members can be accurately modelled using a series of prismatic beam elements (Marques, 2012). Thus, in the implementation of the proposed design method in this chapter, the analysed tapered members, all of which had a taper angle of less than 15° , were modelled as stepped members with prismatic cross-sections for each element, as shown in Figure 4.2. The cross-section properties of each element were taken equal to the corresponding cross-section properties at the midspan of the element. The shear deformable prismatic Timoshenko beam element for open cross-sections, referred to as B31OS in Abaqus (2017), was adopted; this element is capable of accounting for transverse shear deformations and modelling the effects of torsion and warping in open-sections, as studied in this thesis. To accurately capture the spread of plasticity, each web and flange plate was discretised into 33 section points along the width. To enable the correct application of the strain averaging approach, used to consider the beneficial influence of strain gradients along the member lengths, the element sizes were taken as less than or equal to the shortest local buckling half-wavelength $L_{b,cs}$ determined for all cross-sections within the considered tapered member, in accordance with the recommendations of Fieber et al. (2019a).

The quad-linear stress-strain model for hot-rolled steels (Yun & Gardner, 2017) – see Section 2.3.2) – was used for the studied tapered steel members, which were assumed to be formed by the welding of hot-rolled steel plates. Unless otherwise indicated, grade S355 steel was used in all cases considered, thus the three required parameters for the material model were taken as the Young's modulus $E = 210000$ MPa, the yield stress $f_y = 355$ MPa and the ultimate stress $f_u = 510$ MPa. The engineering stress-strain relationship was transformed into the true stress-strain

relationship. Since the explicit modelling of residual stress in the design of steel structures with advanced analysis can be somewhat impractical, the equivalent imperfections provided in prEN 1993-1-1 (2019), as developed by Lindner et al. (2018), to consider the combined influence of both geometric imperfections and residual stresses, were adopted in the implementation of the proposed design method. Further discussion on this issue and improved equivalent bow imperfection magnitudes for use specifically in GMNIA are presented in Walport et al. (2020b). The geometric imperfections were assigned to the beam FE models in the shape of the lowest global buckling modes from prior LBA. The investigated tapered members were pin-ended and laterally restrained in all the considered cases to suppress lateral-torsional buckling effects.

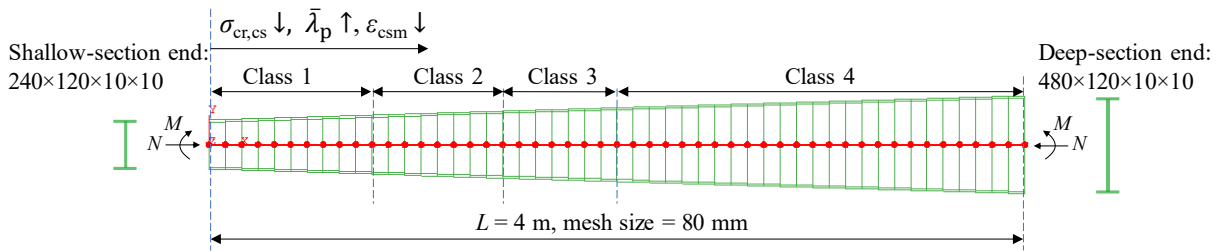


Figure 4.2 Variation in cross-section classification along the length of a tapered member under combined axial compression and uniform major axis bending

Following the beam element analysis of the tapered member, the ultimate capacity can be determined according to the proposed procedure illustrated in Figure 4.3.

- First, based on the first-order internal force distribution, the local buckling stress of the full cross-sections of all prismatic beam elements along the member length $\sigma_{cr,cs,m}$ (where m denotes the number of the beam element) is obtained (Li & Schafer, 2010b; Gardner et al., 2019a).
- Then, using Eq. (2.13), the corresponding cross-section slendernesses $\bar{\lambda}_{p,m}$ along the member length are determined.
- According to the value of $\bar{\lambda}_{p,m}$, the strain limit for each element $\epsilon_{csm,m}$ is obtained from the base curve.
- If the strain averaging approach is applied, the local buckling half-wavelength of each cross-section $L_{b,cs,m}$ needs to be calculated, using CUFSM (Li & Schafer, 2010b) or the expressions provided in Fieber et al. (2019b); the average strain for each element $\epsilon_{Ed,av,m}$ is taken as the average value of strains $\epsilon_{Ed,m}$ over the corresponding local buckling half-wavelength $L_{b,cs,m}$, as shown in Figure 4.4. Note that only beam elements fully located within $L_{b,cs,m}$ are considered in the strain averaging approach. It should be emphasised that,

unlike in prismatic members, strain gradients occur along the lengths of tapered members even under uniform loading.

- Finally, it is necessary to determine (i) the load increment p at which the average strain at any cross-section m_0 attains the corresponding CSM strain limit, i.e. $\varepsilon_{Ed,av,m_0,p} \geq \varepsilon_{csm,m_0}$ and (ii) the load increment j (if any) at which the peak load factor is reached. If $j < p$, the member is assumed to have failed primarily due to global instability, and the ultimate load carrying capacity α_u is taken as the peak load factor from the GMNIA α_{peak} . On the other hand, if $j > p$, the tapered member is assumed to have failed due to reaching the cross-section capacity, and the load factor at which the strain limit is attained is adopted as the ultimate member resistance α_u .

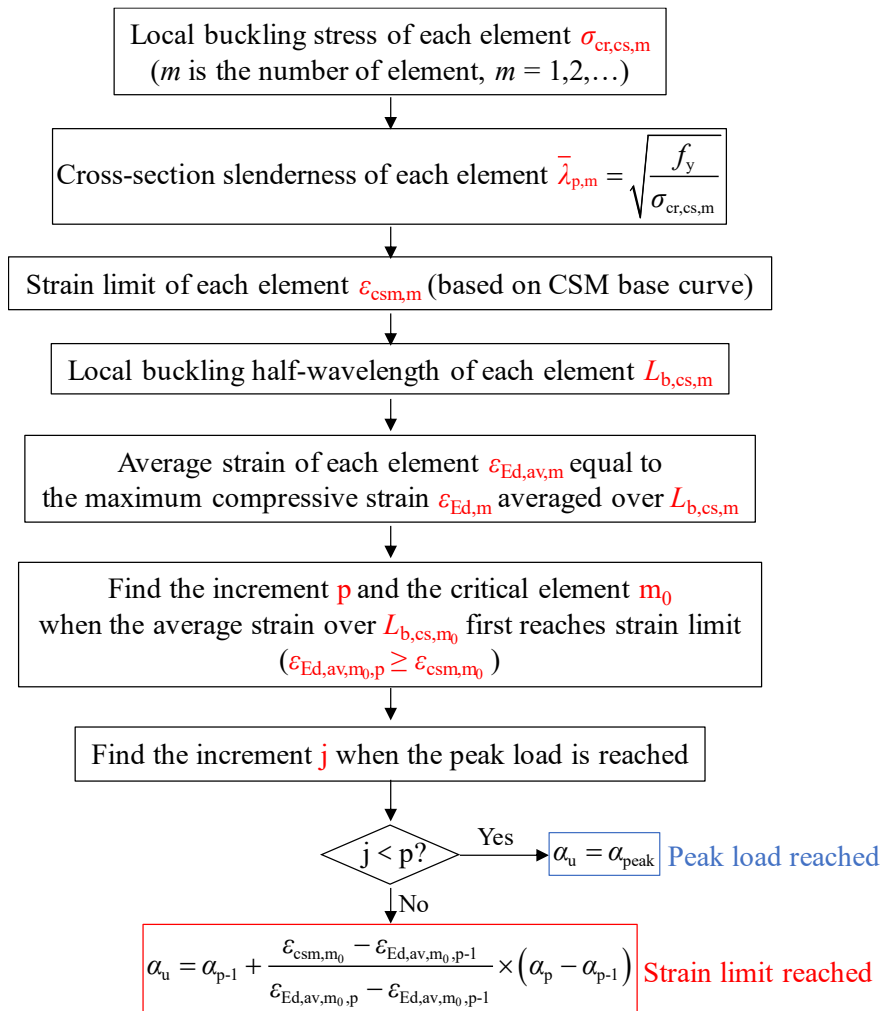
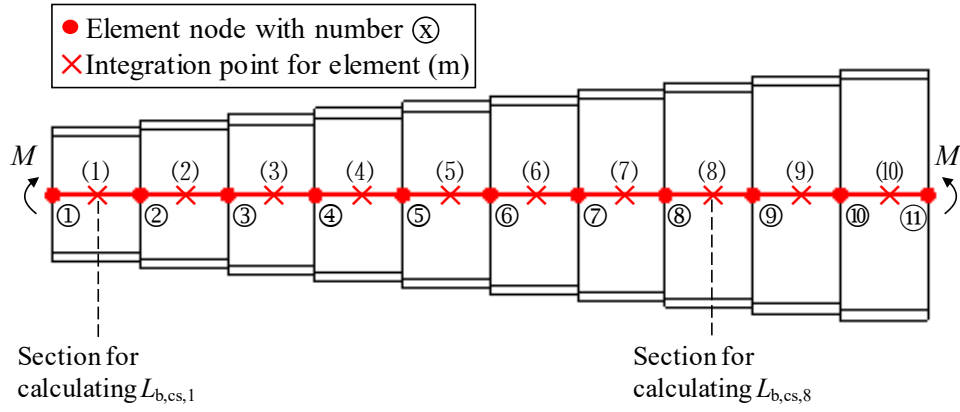
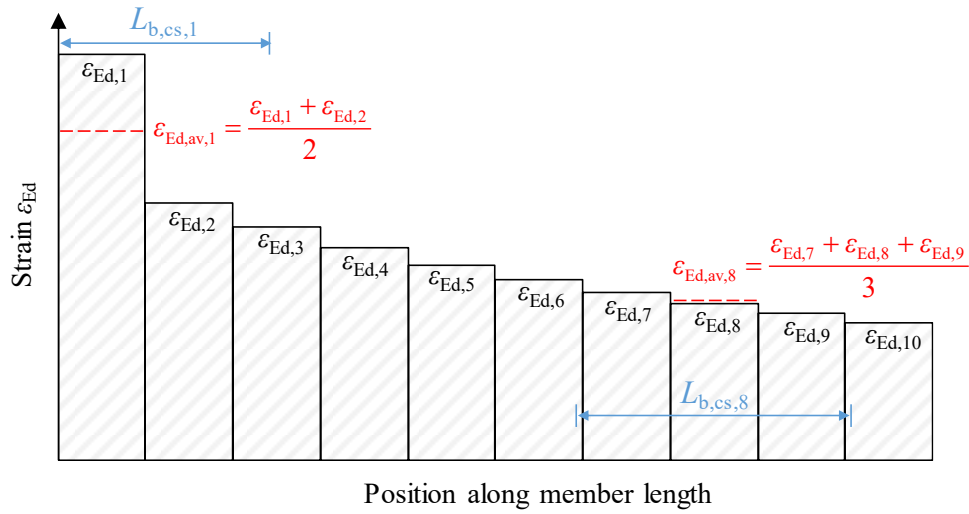


Figure 4.3 Process to apply the method of beam element advanced analysis with CSM strain limit to tapered members (where α_u , α_{peak} , and α_p are the predicted ultimate load factor, the peak load factor, and the load factor at increment p respectively)



(a) Beam FE model of a tapered member



(b) Strain distribution along the member length

Figure 4.4 Schematic representation of the strain averaging approach along a member with 10 beam elements; the average strains for elements 1 and 8, $\varepsilon_{Ed,av,1}$ and $\varepsilon_{Ed,av,8}$ respectively, are shown.

4.2.2.2 Illustrative examples

The benefits of the proposed design method can be best demonstrated through an illustrative example. Figure 4.5 presents the normalised moment-strain response from a beam FE model of a tapered member under uniform bending. In the figure, the end bending moment M , is normalised by the full plastic moment capacity at the shallow end of the tapered member $M_{pl,s}$; the strain plotted is the average maximum compressive strain over the local buckling half-wavelength at the critical beam element (m_0) ε_{Ed,av,m_0} , normalised by the yield strain ε_y . In the beam element GMNIA of this tapered member, no peak load factor was reached, and the end bending moment continuously increased. Thus, this member was deemed to have failed when the CSM strain limit was reached at the critical cross-section. The CSM strain limit at the critical cross-section ε_{cs,m_0} was equal to $7.89\varepsilon_y$; when ε_{Ed,av,m_0} reached $\varepsilon_{cs,m_0} = 7.89\varepsilon_y$, the analysis was terminated and the bending moment capacity of the beam was calculated as $M_{u,prop}$.

$= 1.053M_{pl,s}$. The corresponding benchmark shell FE model of the same member (whose development and validation is described in detail in Section 4.3) reached a peak bending moment of $M_{u,shell} = 1.112M_{pl,s}$, following the occurrence of local buckling in the compression flange at the critical cross-section; of course local buckling is explicitly captured in the shell FE model and hence application of the CSM strain limit is neither appropriate nor required. Compared to the capacity obtained from the benchmark shell FE model, the prediction obtained using the proposed design method of beam element GMNIA with strain limits is only 5% lower. On the other hand, the ultimate load carrying capacity of the member according to EN 1993-1-1 (using GNIA plus cross-section checks) is signified by reaching the plastic moment capacity $M_{pl,s}$ at the shallow end of the member, which is approximately 10% below the shell FE result. Note that in this example, and all the considered cases in this study, the strain outputs, elastic local buckling stresses and corresponding defined CSM strain limits, are all compatibly determined at, or calculated based on, the centreline of the wall thickness, as recommended in Fieber et al. (2019a).

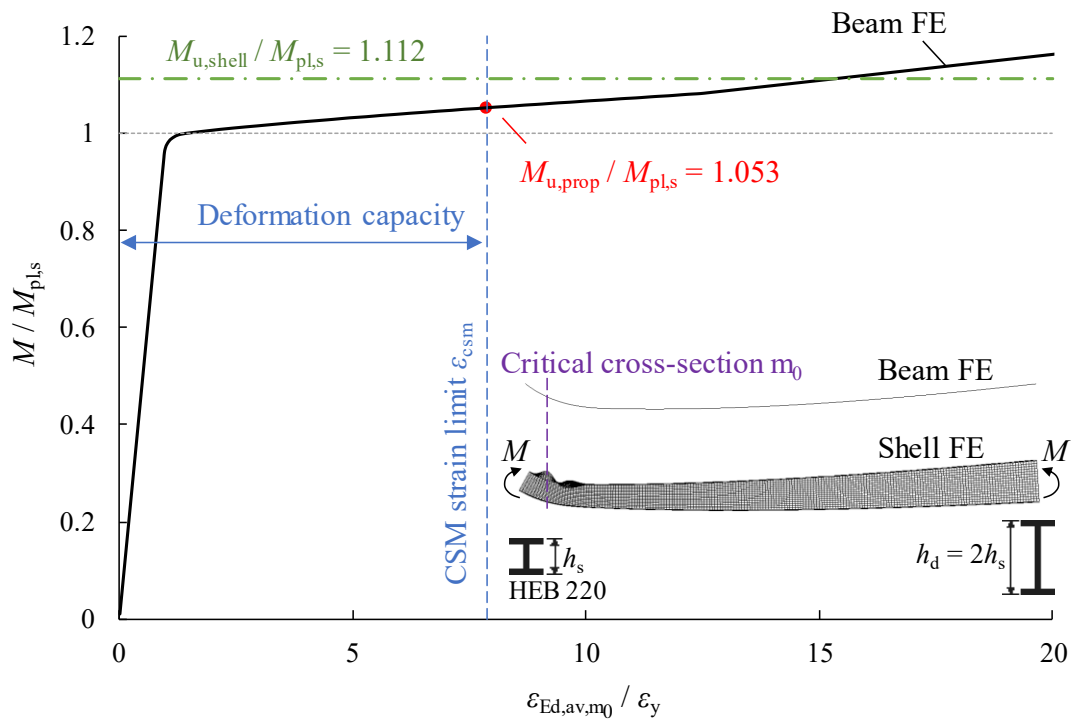


Figure 4.5 An illustrative example of the proposed design method using advanced analysis with strain limits on a tapered member under uniform bending (member length $L = 5300$ mm)

A further illustrative example of the proposed method applied to a tapered column is presented in Figure 4.6, which shows (i) the variation of the CSM strain limit ϵ_{csm} (because of the changing geometry), the total compressive strain $\epsilon_{Ed,I+II}$, the first order strain $\epsilon_{Ed,I}$ and the second order strain $\epsilon_{Ed,II}$ along the member length at the load increment when the CSM strain limit is first

reached, and (ii) the corresponding variation in the strain utilisation ratio along the member length. The taper ratio γ (i.e. the ratio of the deep-section height h_d to the shallow-section height h_s) of this tapered column is equal to 5, and the major axis normalised member slenderness $\bar{\lambda}_y$, which is defined as the square root of the axial yield load of the shallow end cross-section $N_{pl,s}$ divided by the elastic buckling load N_{cr} is equal to 0.8 (i.e. $\bar{\lambda}_y = \sqrt{N_{pl,s} / N_{cr}} = 0.8$). As can be seen from Figure 4.6, in the proposed design method, (i) the second order effects can be accurately and directly captured through the GMNIA, (ii) the critical cross-section can be determined in a straightforward manner and (iii) the contribution of the first order and second order effects on the strain utilisation can be clearly observed. Considering the strain distribution at the ultimate state, it is even possible to optimise the geometry of a tapered member in terms of material use by aiming to achieve a uniform strain utilisation ratio along the member length.

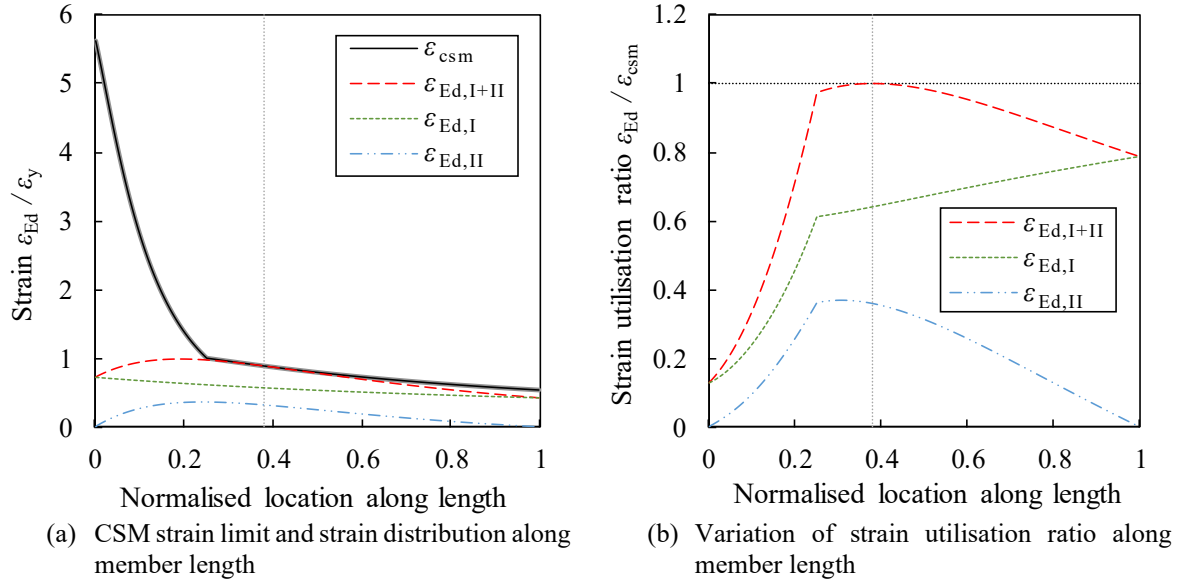


Figure 4.6 (a) CSM strain limit and strain distribution and (b) variation of strain utilisation ratio at the ultimate state (i.e. when $\epsilon_{Ed,I+II} = \epsilon_{csm}$) along the length of a tapered column, where the shallow-section end is an HEB 220, the taper ratio $\gamma = 5$ and $\bar{\lambda}_y = 0.8$

4.3 Benchmark shell finite element modelling

4.3.1 Modelling approach

The accuracy of the proposed design approach is verified for a large number of tapered members against results obtained from benchmark shell FE models. The finite element analysis software Abaqus (2017) was used to create the shell FE models in this study. The element type S4R, a four-noded shell element taking into account transverse shear deformation and finite

membrane strains with reduced integration and a large-strain formulation, which has been successfully employed in previous study for similar applications (Kucukler & Gardner, 2018; Fieber et al., 2019a; Kucukler et al., 2019; Xing et al., 2020), was used to create the models. All cross-sections were subdivided into 16 elements along the flange width and the web depth. The element number along the longitudinal axis of a tapered member was defined such that the element aspect ratio at the midspan cross-section was close to unity. The Simpson integration method was used, and five integration points were employed through the thickness of the shell elements (Abaqus, 2017).

The quad-linear stress-strain model developed by Yun and Gardner (2017) for hot-rolled steel was used in the models. The Poisson's ratio was taken as $\nu = 0.3$ in the elastic range and $\nu = 0.5$ in the plastic range. As required by Abaqus (2017), for shell FE models, the engineering stress-strain relationships were transformed into the true stress-strain relationships.

Beam multi-point constraints were used to couple the web and two flange plates making up the investigated members. To avoid overlapping of the web and flange plates, the web nodes were offset by half the flange thickness at the top and bottom, in line with the approach adopted in (Kucukler et al., 2015a; Kucukler et al., 2015b; Kucukler & Gardner, 2018). Since this chapter focuses only on the in-plane behaviour of tapered steel members, lateral-torsional buckling effects were suppressed by laterally restraining the models at the web-to-flange junctions along their lengths. Boundary conditions and concentrated forces and moments were applied at the member ends by defining coupling constraint relationships. The ECCS (1984) residual stress pattern illustrated in Figure 4.7, which is recommended for steel members fabricated by the welding of individual steel plates (Kucukler & Gardner, 2018; Kucukler & Gardner, 2019), was adopted in the FE models. Global geometric imperfections were assigned to the models in the shape of the lowest global buckling modes. Unless otherwise indicated, the magnitudes of the global geometric imperfections were taken as $L/1000$, where L is the member length. As shown in Figure 4.8, local imperfections were applied to the shell FE models by adopting a series of sinusoidal subpanel imperfections complying with the recommendations provided in Annex C of EN 1993-1-5 (2006). For the cases where the web plate was more susceptible to local buckling than the flange plates, i.e. when $\sigma_{cr,w} < \sigma_{cr,f}$ (where $\sigma_{cr,w}$ and $\sigma_{cr,f}$ are the elastic local buckling stresses of the isolated web and flange plates respectively with simply-supported boundary conditions along the adjoined edges), the magnitudes of the local web imperfection was taken as $1/200$ of the web height h_w (i.e. $h_w/200$). Similarly, for the cases where the flange

plates were more susceptible to local buckling than the web plate, the magnitude of the local flange imperfection was taken as $1/100$ of the flange widths b (i.e. $b/100$). The local imperfection magnitudes of the non-critical plate elements were defined such that the web-to-flange junctions remained at 90° . The half-wavelengths $L_{b,cs}$ of the local imperfections were obtained from the expressions in Fieber et al. (2019b). Since tapered members have variable cross-section depths along their lengths, there are different local buckling half-wavelengths for each cross-section; to reflect this, the local buckling half-wavelengths $L_{b,cs}$ used to represent the local imperfections were also varied along the member length, as explained in Figure 4.9. Starting at the deep end of the member, the local buckling half-wavelengths $L_{b,cs,n}$ were calculated at sequential cross-sections (separated by the distances $L_{b,cs,n}$) until the shallow end of the member was reached. In the event that the sum of the local buckling half-wavelengths (i.e. $\sum L_{b,cs,n}$) was not exactly equal to the member length L , which was generally the case, an integer number of half-waves was ensured by uniformly stretching or contracting the $L_{b,cs,n}$ values by a factor x , such that the ratio $L/\sum xL_{b,cs,n}$ was equal to unity – see Figure 4.9.

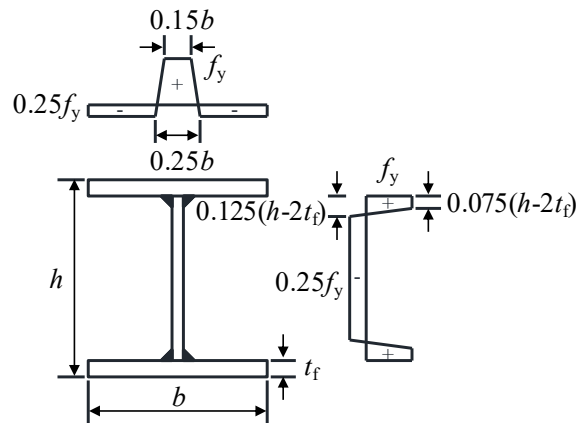


Figure 4.7 Residual stress pattern applied to shell FE models (+ve = tension; -ve = compression)

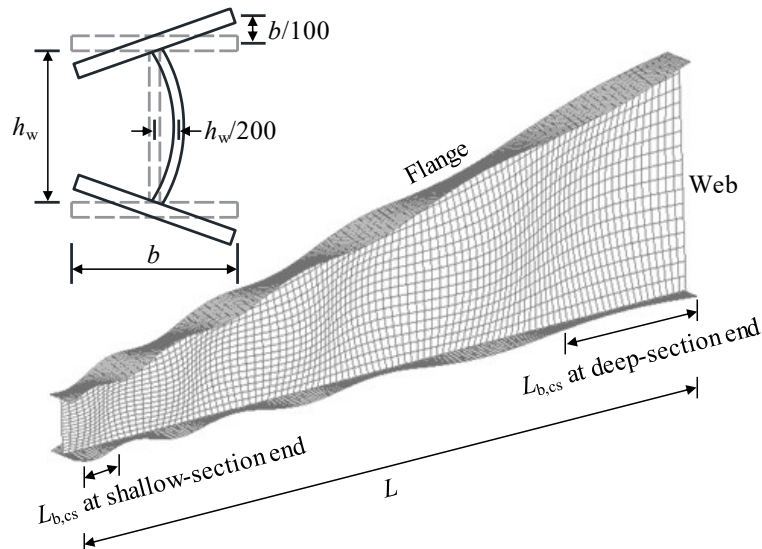


Figure 4.8 Definition of local geometric imperfections along the member length

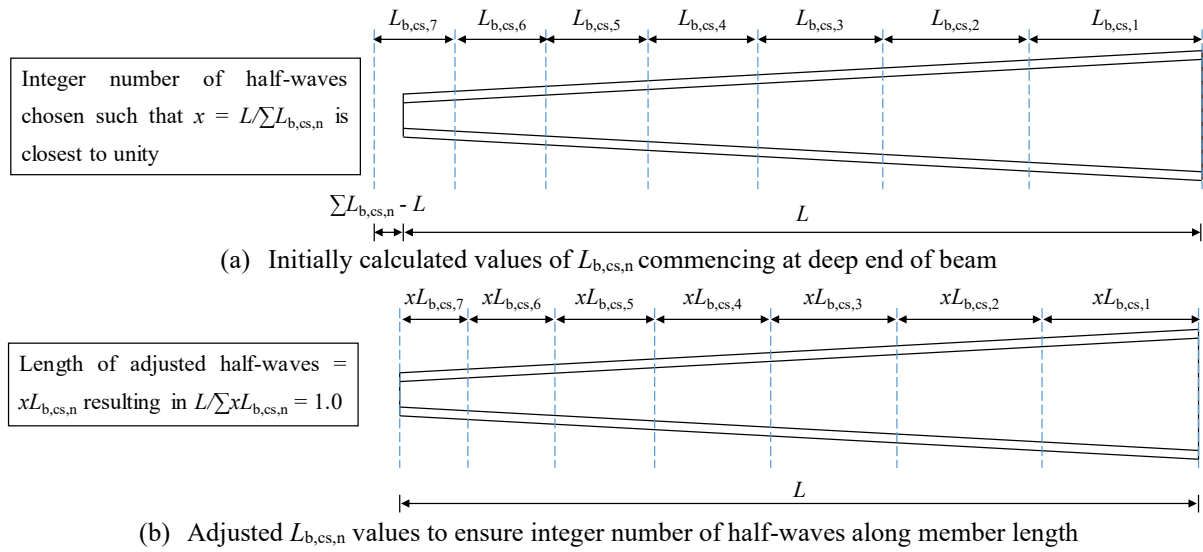


Figure 4.9 Illustration of procedure adopted in the application of local imperfections

4.3.2 Validation of shell FE models

The shell FE models were validated against the results from eighteen tapered member experiments collected from the literature. In the experimental studies (Prawel et al., 1974; Shiomi & Kurata, 1984; Cristutiu et al., 2012; Tankova et al., 2018b) considered for the validation of the shell FE models developed herein, tapered members with different taper ratios were tested, including columns (Tankova et al., 2018b), beams under different bending moment gradients (Prawel et al., 1974) and beam-columns (Shiomi & Kurata, 1984; Cristutiu et al., 2012) which had either (i) no intermediate lateral restraint (NR), (ii) lateral restraints only (LR) or (iii) both lateral and torsional restraints (TR) between the end supports. Although the focus of the present study is the in-plane behaviour and design of tapered steel members, a few experiments on tapered members without out-of-plane restraints and susceptible to lateral-torsional buckling were also included in the validation study; in these cases, the boundary conditions applied in the FE models mirrored those adopted in the tests. In the validation study, the measured magnitudes of the global geometric imperfections from the considered experiments were applied to the FE models in the shape of their lowest global elastic buckling modes. For the cases where the geometric imperfection magnitudes were not reported, the lowest global elastic buckling modes were scaled to 1/1000 of the member lengths. A summary of the results of the validation study, including the mean, coefficient of variation (CoV), minimum and maximum values of the ratios of the ultimate load carrying capacities determined using the shell FE models to those obtained from the experiments ($\alpha_{u,shell}/\alpha_{u,test}$), are given in Table 4.1. As can be seen from the table, the shell FE models are able to provide ultimate

strength predictions very close to those observed in the physical experiments for tapered columns, beams and beam-columns.

Table 4.1 Summary of validation of shell FE models, and comparison of results obtained from the proposed design method and Eurocode 3 relative to experimental results. LR = lateral restraints; NR = no restraints; TR = lateral and torsional restraints. The ultimate load factors obtained from the tests, shell FE modelling, proposed design method and Eurocode 3 are denoted $\alpha_{u,test}$, $\alpha_{u,shell}$, $\alpha_{u,prop}$ and $\alpha_{u,EC3}$, respectively.

Reference	Test	Member shape	Member type	Taper ratio γ	Material	Lateral restraints	$\alpha_{u,shell}/\alpha_{u,test}$	$\alpha_{u,prop}/\alpha_{u,test}$	$\alpha_{u,EC3}/\alpha_{u,test}$
Tankova et al. (2018b)	C1	V	Column	4	S355	LR	0.993	0.862	0.779
	C2	V	Column	2			0.945	0.830	0.829
	C3	L	Column	3			0.974	0.859	0.781
	BC	L	Beam-column	3			0.990	*	*
Prawel et al. (1974)	LB-3	L	Beam	2.67	ASTM A242	LR	0.982	*	*
	LB-5						0.937	0.816	0.804
	LB-6						1.012	*	*
Shiomi & Kurata (1984)	IT-1.4-1	V	Beam-column	1.36	SS-41	LR	1.024	0.760	0.698
	IT-1.6-2			1.50			1.013	0.756	0.688
	IT-1.8-3			1.76			0.998	0.734	0.634
	IT-2.2-5			2.15			1.103	0.811	0.702
	IT-2.4-6			2.30			1.040	0.775	0.678
Cristutiu et al. (2012)	C1-8-NR	L	Beam-column	2.4	S355	NR	0.981	*	*
	C1-8-LR					LR	1.015	*	*
	C1-8-TR					TR	0.958	0.904	0.803
	C2-6-NR					NR	1.017	*	*
	C2-6-LR					LR	1.038	*	*
	C2-6-TR					TR	1.029	0.934	0.910
Mean							1.003	0.822	0.755
CoV							0.039	0.077	0.108
Max							1.103	0.934	0.910
Min							0.937	0.734	0.634
Total number of tests							18	11	11

*Test specimens exhibited large out-of-plane deformations and are therefore outside the scope of the in-plane design methods examined in the present study

Figure 4.10 shows experimental and numerical load-displacement curves for the tapered column specimen C1 tested in Tankova et al. (2018b), while Figure 4.11 shows experimental and numerical moment-rotation curves for the tapered beam-column specimens with lateral and torsional restraints (C1-8-TR) and without lateral restraints (C2-6-NR) tested in Cristutiu et al.

(2012). As can be seen from the figures, the load-deformation paths obtained from the shell FE models developed in this study closely follow the corresponding experimentally determined paths; this indicates that the developed models are able to replicate the observed structural response of tapered members and can be used to generate benchmark data to evaluate the accuracy and safety of the proposed design approach in this study.

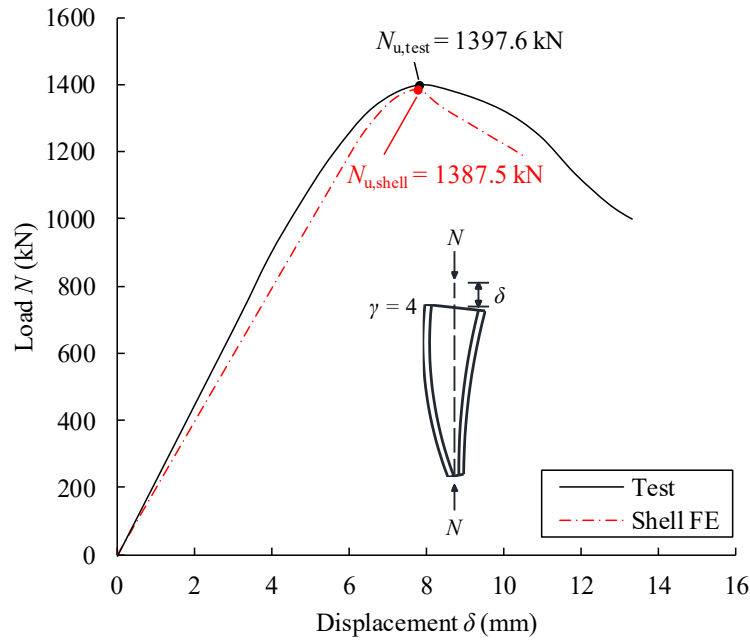


Figure 4.10 Comparison of experimental and numerical load-displacement curves of the tapered column specimen C1 tested in Tankova et al. (2018b)

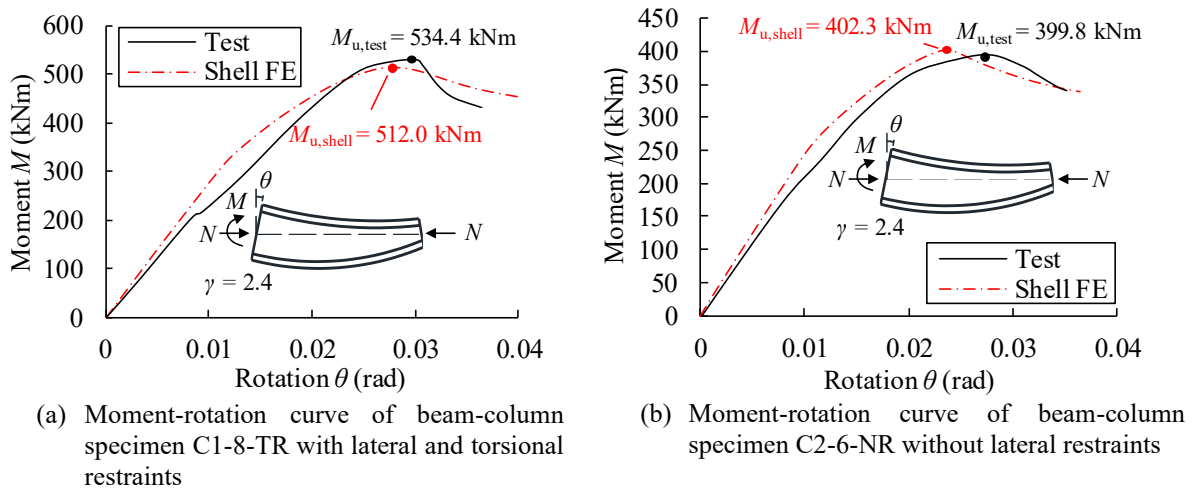


Figure 4.11 Comparison of experimental and numerical moment-rotation curves of beam-columns tested in Cristutiu et al. (2012) (a) specimen C1-8-TR with lateral and torsional restraints; (b) specimen C2-6-NR without lateral restraints.

4.4 Application of proposed design method to members under pure compression, uniform bending and combined compression and uniform bending

In this section, the accuracy of the proposed design method of advanced analysis with strain limits, implemented in beam FE models, is assessed against the results from the benchmark shell FE models for tapered members subjected to compression, uniform bending and combined compression and uniform bending. The accuracy of the traditional design method presented in EN 1993-1-1 (2005) for tapered members, i.e. second order elastic analysis with equivalent imperfections (GNIA) plus cross-section checks, is also presented in order to highlight the benefits brought about by the proposed design approach. It should be noted though that even greater benefits arise when performing system-level, rather than member-level, design.

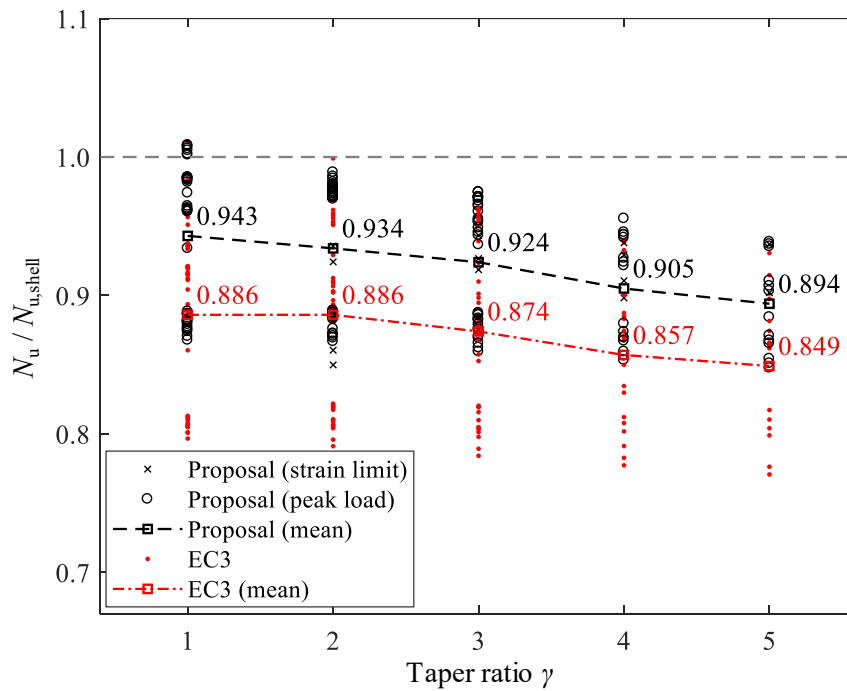
As listed in Table 4.2, the investigations carried out in this section cover a range of normalised slenderness values $\bar{\lambda}_y = \sqrt{N_{pl,s} / N_{cr}} = 0.2, 0.5, 0.8, 1.2, 1.5$ and a range of taper ratios $\gamma = h_d/h_s = 1, 2, 3, 4, 5$. A series of different shallow end cross-section geometries were also considered – the employed cross-section geometries were those of European HEM 100, HEM 120, HEB 140, IPE 160, HEM 180, HEB 200, HEB 220, HEB 240 profiles, but without the presence of the fillet radii. Note that the maximum cross-section slenderness $\bar{\lambda}_p$ along the length of the tapered members did not exceed 1.5.

Table 4.2 Summary of varied parameters considered and accuracy of the proposed design method and Eurocode 3 in comparison to the results from benchmark shell FE models for tapered members under axial compression, uniform bending, and combined of compression and uniform bending

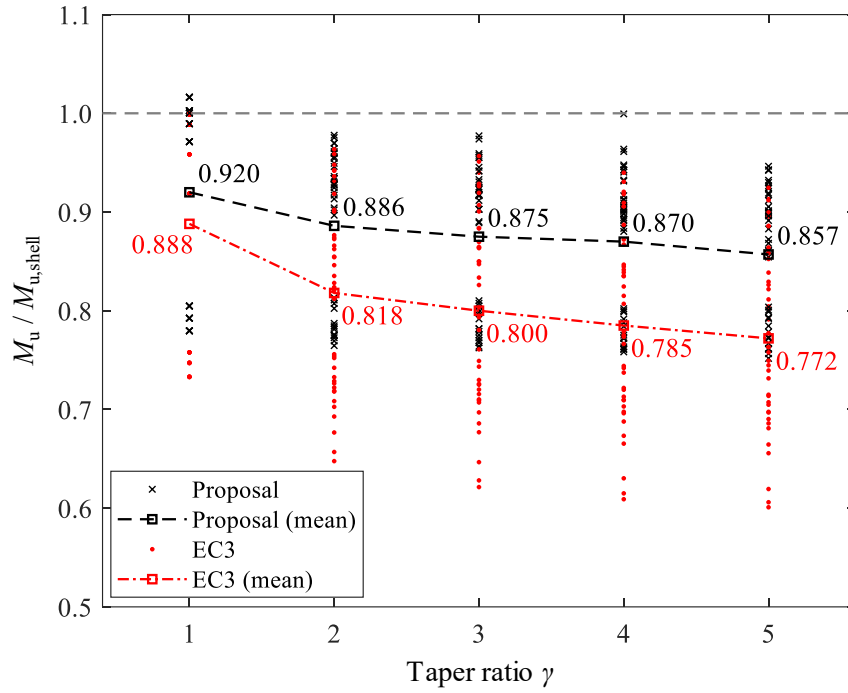
Loading condition	Member slenderness $\bar{\lambda}_y$	Taper ratio γ	Shallow-end cross-section	No. of cases N	Design method	$\alpha_w/\alpha_{u,shell}$			
						Mean	CoV	Max	Min
Compression	0.2, 0.5, 0.8, 1.2, 1.5	1, 2, 3, 4, 5	HEM 100, HEM 120, HEB 140, IPE 160, HEM 180, HEB 200, HEB 220, HEB 240	200	Proposed	0.926	0.052	1.009	0.848
					EC3	0.875	0.068	1.011	0.771
Bending			Proposed	0.881	0.092	1.016	0.752		
				EC3	0.813	0.130	1.000	0.601	
Compression + bending			2000	Proposed	0.950	0.074	1.066	0.641	
				EC3	0.888	0.112	1.051	0.469	

4.4.1 Tapered steel members subjected to pure compression

Figure 4.12 (a) and Table 4.2 show that the ultimate capacities of 200 the tapered columns (5 values of $\bar{\lambda}_y \times 5$ taper ratios $\times 8$ cross-sections) predicted using the proposed design method generally agree well with those obtained from the benchmark shell element models. With increasing taper ratios, the accuracy of the ultimate resistances obtained using the proposed design method decreases slightly. However, even for the tapered columns with a taper ratio of $\gamma = 5$ (which are generally rarely used in practice), the average value of the ratios of the resistance predictions determined using the proposed design method to those obtained from the benchmark shell FE models, i.e. $N_{u,prop}/N_{u,shell}$, is equal to 0.894, indicating that even for very large taper ratios, the proposed method provides accurate and safe-sided ultimate strength predictions. In contrast, the predictions obtained from Eurocode 3 (GNIA + cross-section checks) are more conservative and scattered, highlighting the significantly more accurate and consistent ultimate strength predictions achieved when the proposed method is used for the design of tapered columns.



(a) Compression



(b) Uniform bending

Figure 4.12 Comparison of the ratios of the ultimate load carrying capacities obtained using the proposed design approach and Eurocode 3 to those obtained from the benchmark shell element models versus taper ratio γ for tapered members under (a) axial compression and (b) uniform major axis bending

4.4.2 Tapered steel members subjected to pure uniform bending

Figure 4.12 (b) and Table 4.2 present comparisons of the ultimate bending capacities of the considered 200 tapered beams obtained using the proposed design method and EN 1993-1-1 (2005) against the results from their benchmark shell element model counterparts. According to the proposed design method, all the tapered beams failed by reaching their CSM strain limits. As can be seen from Figure 4.12 (b), the predicted ultimate bending moment capacities are generally very accurate, though in some cases, the predictions are somewhat lower than those obtained from the shell FE models, e.g. $M_{u,prop}/M_{u,shell} \approx 0.8$. These conservative predictions occurred for tapered members with very stocky ($\bar{\lambda}_{p,cr} \leq 0.32$) critical cross-sections (i.e. the cross-sections where the strain limit was attained), as shown in Figure 4.13. However, if the upper strain limit $\varepsilon_{csm}/\varepsilon_y$ (see Section 2.3.1) adopted for these members is relaxed from 15 to 30 (i.e. if additional strain hardening is allowed for these members), higher and thus more accurate bending moment resistances can be attained. Hence, although to prevent excessive deformations, the upper strain limit of $15\varepsilon_y$ is usually recommended, if higher strains can be tolerated at the ultimate limit state, the upper limit of $30\varepsilon_y$ can be adopted to improve accuracy.

In contrast with the proposed design approach, the EN 1993-1-1 (2005) approach (in which moments cannot exceed the plastic moment capacity) is generally overly-conservative, thus indicating that the proposed design approach leads to considerably more accurate ultimate strength predictions for tapered members under bending.

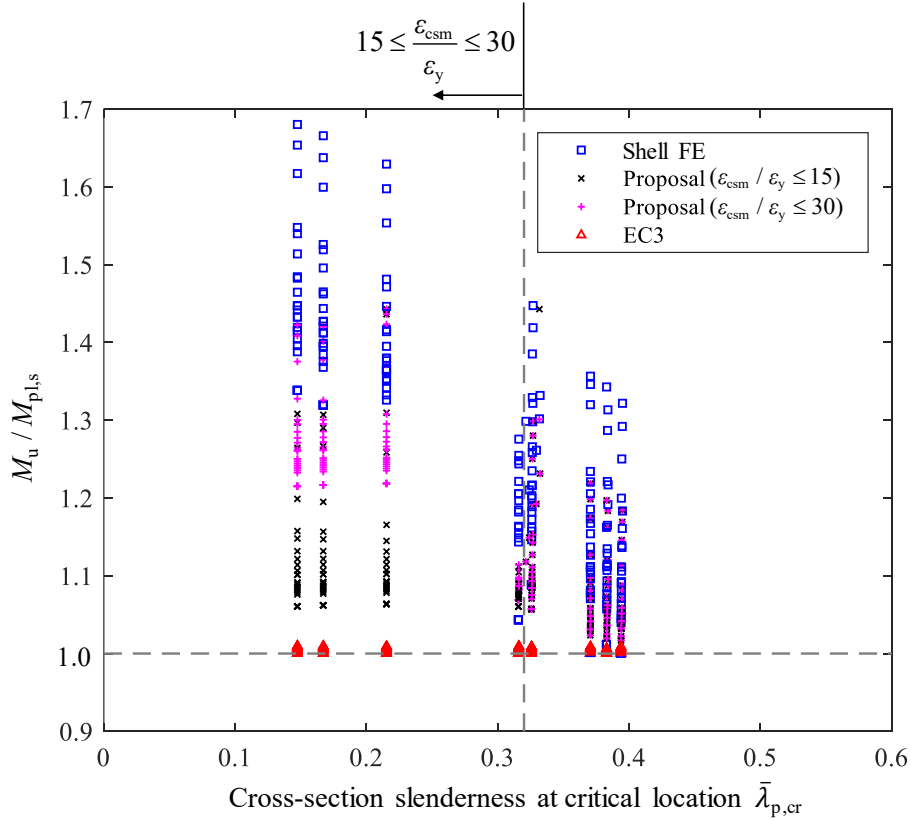


Figure 4.13 Comparison of the ratios of the ultimate capacities obtained from shell FE, the proposed design approach and Eurocode 3 to that corresponding to the attainment of the plastic moment at the shallow end cross-section $M_u/M_{pl,s}$ versus cross-section slenderness at the critical location $\bar{\lambda}_{p,cr}$ for tapered members under uniform bending; all tapered beams are governed by the CSM strain limit, rather than peak load

The high accuracy of the proposed design method is achieved partly through the adoption of the strain averaging approach, which allows for the beneficial effect of local strain gradients along the member length on the resistance of cross-sections, as explained in Section 4.2.1.2. This is illustrated in Figure 4.14, which shows the variation of the CSM strain limit ϵ_{csm} , the maximum longitudinal compressive strain ϵ_{Ed} (at the mid-thickness of the compression flange) and the average maximum longitudinal compressive strain over the local buckling half-wavelength $\epsilon_{Ed,av}$ at the load increment when the CSM strain limit is first reached. According to EN 1993-1-1 (2005), the critical cross-section of this tapered beam is at the shallow end, which falls into the Class 1 category. Thus, the ultimate resistance of this beam arises when the

bending moment at the shallow end of the member reaches the plastic moment resistance of the shallow end cross-section i.e. $M_{u,EC3}/M_{pl,s} = 1.000$; this is however a significant underestimation of the ultimate strength of the beam, determined as $M_{u,shell}/M_{pl,s} = 1.322$ from the benchmark shell FE model. Using the proposed design method, but without employing strain averaging, the ultimate resistance of the beam is calculated as $M_{u,prop}/M_{pl,s} = 1.010$; this corresponds to the point at which the CSM strain limit is reached at the shallow end cross-section, which has a cross-section slenderness equal to $\bar{\lambda}_p = 0.39$. On the other hand, if the strain averaging approach is applied in the implementation of the proposed design method by limiting $\varepsilon_{Ed,av}$ over $L_{b,cs}$ (rather than ε_{Ed}) to ε_{csm} , the beneficial influence of the strain gradient along the beam length is considered; this results in an ultimate design resistance of the beam of $M_{u,prop}/M_{pl,s} = 1.184$, which represents an improvement of 15% relative to the ultimate resistance determined according to EN 1993-1-1 (2005).

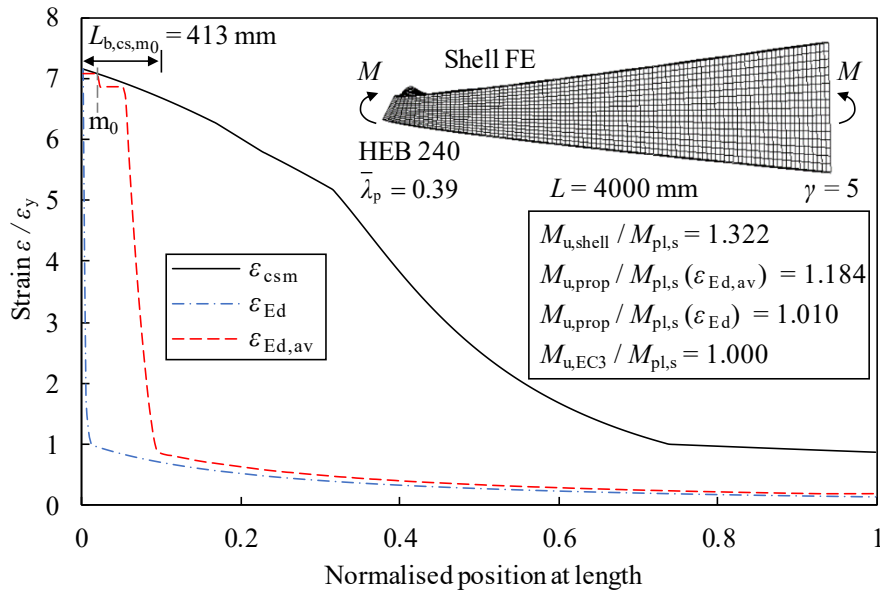


Figure 4.14 Variation of CSM strain limit ε_{csm} , maximum longitudinal compressive strain ε_{Ed} (for the load level at which $\varepsilon_{Ed} = \varepsilon_{csm}$) and average maximum longitudinal compressive strain $\varepsilon_{Ed,av}$ (for the load level at which $\varepsilon_{Ed,av} = \varepsilon_{csm}$), along the length of a tapered member under uniform bending. The critical element m_0 when strain averaging is considered (i.e. that in which $\varepsilon_{Ed,av}$ first reaches ε_{csm} , signifying failure) is close to the shallow end of the beam, mirroring the failure location indicated by the benchmark shell FE model. The local buckling half-wavelength for the cross-section at this location $L_{b,cs,m_0} = 413$ mm. The average strain for the critical element ε_{Ed,av,m_0} is obtained by averaging the maximum longitudinal compressive strains for all elements (of which there are 28 in the present example) that lie entirely within L_{b,cs,m_0} , as illustrated in Figure 4.4.

4.4.3 Tapered steel members subjected to combined compression and uniform bending

In this subsection, the accuracy of the proposed design method of advanced analysis with strain limits implemented by beam elements is assessed for 2000 tapered members ($5 \bar{\lambda}_y \times 5 \gamma \times 8$ cross-sections $\times 10$ N to M ratios) under uniform bending plus axial compression. As shown in Figure 4.15 (a) and Table 4.2, the proposed method is able to provide very accurate capacity predictions with a mean value of 0.950 and a CoV of 7.4% for the ratios of the ultimate load carrying capacities determined by the proposed method to those obtained from the benchmark shell FE models. On the other hand, EN 1993-1-1 (2005) yields less accurate and more scattered predictions with a mean value of 0.888 and a CoV of 11.2%. Note that EN 1993-1-1 (2005) is particularly conservative for tapered members with large taper ratios γ , as shown in Figure 4.15 (b). Comparing Figure 4.15 (a) and Figure 4.15 (b) shows that the proposed method provides more reliable ultimate strength predictions relative to EN 1993-1-1 (2005), where the majority of the predictions obtained through the proposed design approach are very close to those obtained from the benchmark shell FE models.

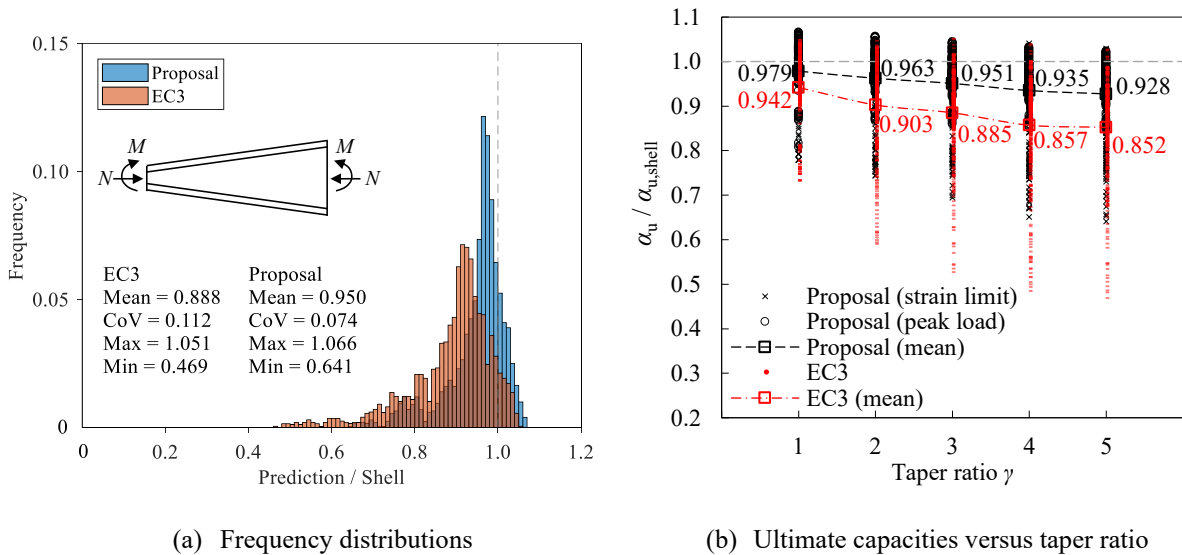


Figure 4.15 Comparison between the ratios of the ultimate capacities predicted by the proposed design method and Eurocode 3 to those obtained from benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ for tapered members under uniform bending plus axial compression

Normalised moment-axial force interaction diagrams for a series of tapered members with HEB 140 and HEB 220 sections at the shallow ends and subjected to combined compression and major axis bending are shown in Figure 4.16. According to EN 1993-1-1 (2005), the critical

cross-sections (i.e. the cross-sections where the cross-section resistance is attained) of these tapered members all fall into the Class 1 category, and thus the ultimate resistances of these members were determined using the linear plastic bending moment-axial force (M-N) interaction equation provided in the standard and the corresponding imperfection amplitudes provided in prEN 1993-1-1 (2019) for a linear plastic cross-section strength check. The EN 1993-1-1 (2005) strength predictions for these members are rather conservative. By contrast, the proposed design method is able to accurately capture the behaviour observed in the benchmark shell FE models. According to the proposed design approach, mirroring the shell FE models, for the short members and for the members where bending is dominant, the load carrying capacities are governed by reaching the CSM strain limit at the critical cross-sections, while for the longer members, where global instability effects are dominant, the ultimate capacities correspond to the peak load factor obtained from the beam element analyses.

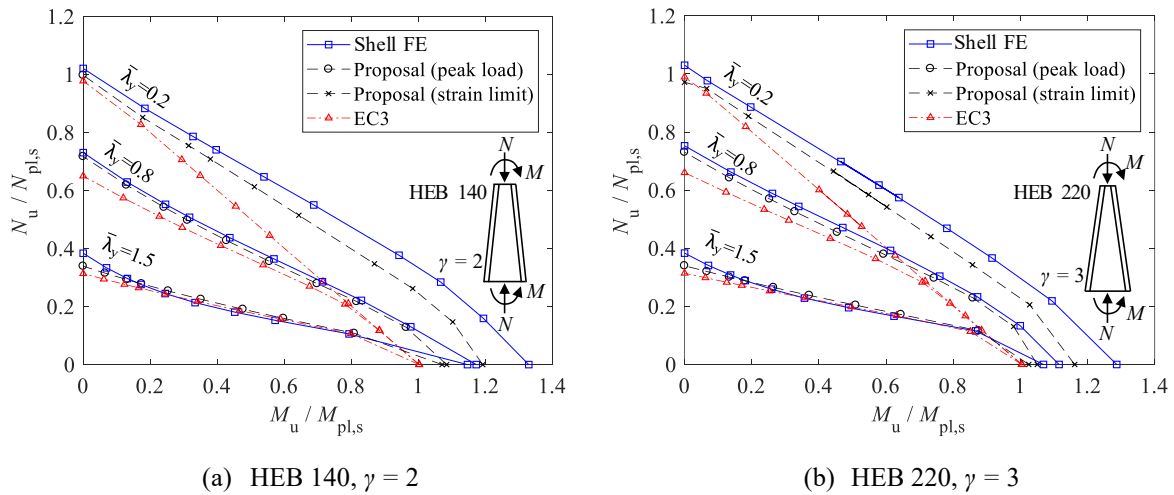


Figure 4.16 Comparison of the ultimate strengths determined using the proposed design approach against benchmark shell element models and Eurocode 3 for tapered members with different member slendernesses under axial compression and uniform major axis bending

Similar observations can also be made in Figure 4.17, which presents comparisons between the ratios of the ultimate capacities determined using the proposed design method and EN 1993-1-1 (2005) to those determined from the benchmark shell FE models ($\alpha_u / \alpha_{u,shell}$) versus the cross-section slenderness at the critical location $\bar{\lambda}_{p,cr}$ for all the analysed tapered members. The critical location refers to the cross-section where the strain limit is attained or the shallow end cross-section when the ultimate capacity is predicted by the peak load factor. For the tapered members with critical cross-sections of intermediate slenderness ($0.32 \leq \bar{\lambda}_{p,cr} \leq 0.5$), the improved accuracy and enhanced resistances achieved using the proposed design method

relative to EN 1993-1-1 (2005) arise due to the rational exploitation of the spread of plasticity and the allowance for local moment gradients through strain averaging. For the tapered members with critical cross-sections of stocky proportions ($\bar{\lambda}_{p,cr} \leq 0.32$), the proposed design method also allows for material strain hardening, which leads to further improvements in the accuracy of the resistance predictions. For the members with very stocky critical cross-sections, if the upper strain limit is increased from $15\varepsilon_y$ to $30\varepsilon_y$, the accuracy of the proposed design method improves even further, as shown in Figure 4.18. For the tapered members with slender critical cross-sections ($\bar{\lambda}_{p,cr} \geq 0.68$), the ultimate resistances predicted by the proposed design method are close to those obtained from EN 1993-1-1 (2005), as shown in Figure 4.17. However, note that the cumbersome process of the determination of the effective section properties for Class 4 cross-sections is not required in the application of the proposed design method.

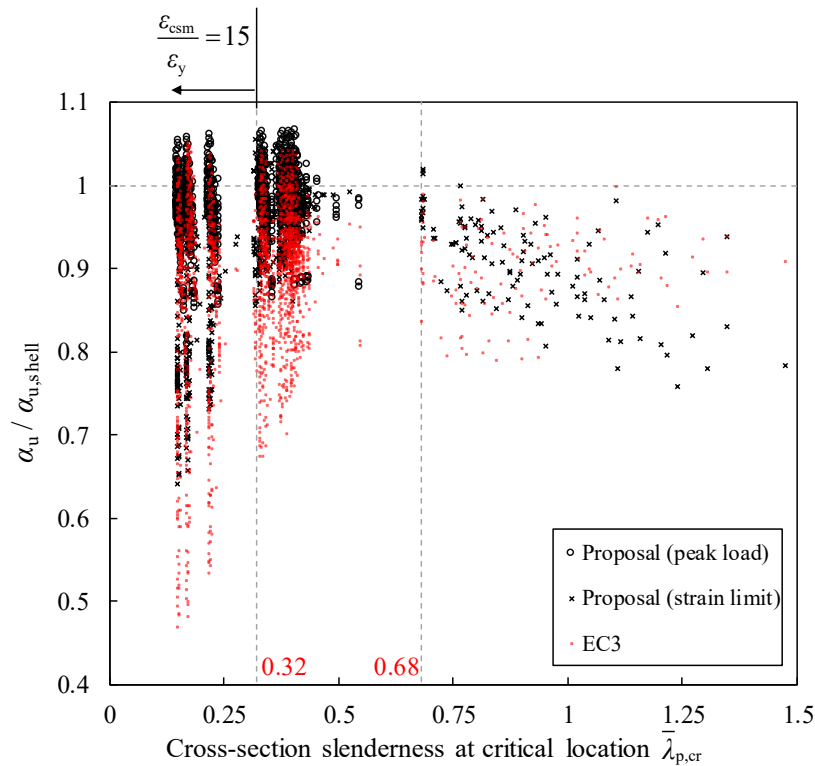


Figure 4.17 Comparison of the ratios of the ultimate capacities obtained from proposed design approach and Eurocode 3 to those obtained from benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ versus cross-section slenderness at the critical location $\bar{\lambda}_{p,cr}$ for tapered members under combined compression and uniform major axis bending

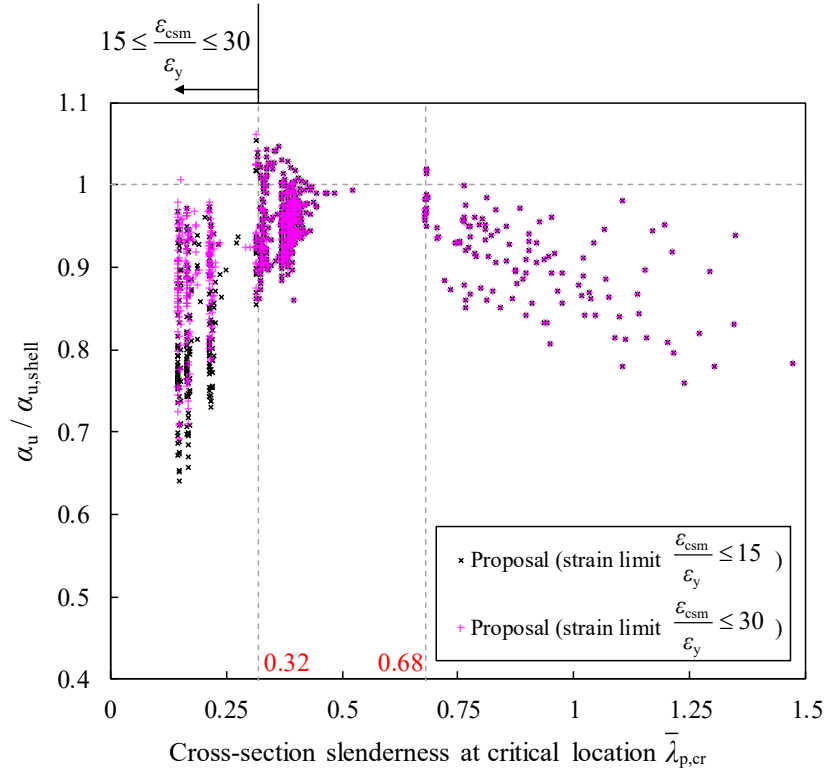


Figure 4.18 Ratios of the ultimate load carrying capacities predicted by the proposed design method using $15\epsilon_y$ or $30\epsilon_y$ as the maximum strain limit to those determined from the benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ for tapered members under combined compression and uniform major axis bending, considering only cases when the strain limit, rather than the peak load, governed.

Similar to EN 1993-1-1 (2005), in which cross-section classification is typically based on cross-section stress distributions determined from a first order elastic analysis of a steel member, in this study, cross-section slendernesses $\bar{\lambda}_p = \sqrt{f_y / \sigma_{cr,cs}}$ and hence the corresponding CSM strain limits ϵ_{csm} are also calculated using local buckling stresses $\sigma_{cr,cs}$ corresponding to the stress distribution from a first order elastic analysis. Typically, CSM strain limits ϵ_{csm} based on cross-section stress distributions determined from the GMNIA of members will result in an increase in deformation capacity, because the ratio of bending moment to axial force generally increase during the analysis as second order effects become increasingly dominant. This results in a more favourable stress distribution for local buckling, with increasing values of $\sigma_{cr,cs}$ leading to lower values of slenderness $\bar{\lambda}_p$ and hence increased deformation capacities. This is illustrated in Figure 4.19, which shows that higher strain limits and improved accuracy can be achieved when the stress distribution associated with the deformed geometry (i.e. allowing for second order effects) is considered in the determination in $\sigma_{cr,cs}$ (and hence $\bar{\lambda}_p$ and $\epsilon_{csm}/\epsilon_y$), but at the expense of greater calculation effort. Note that the slightly unconservative results in Figure 4.18

are not affected by this issue, since these members are either relatively short or their critical cross-sections are located near to one of the ends of the members; in both cases, second order effects are not significant.

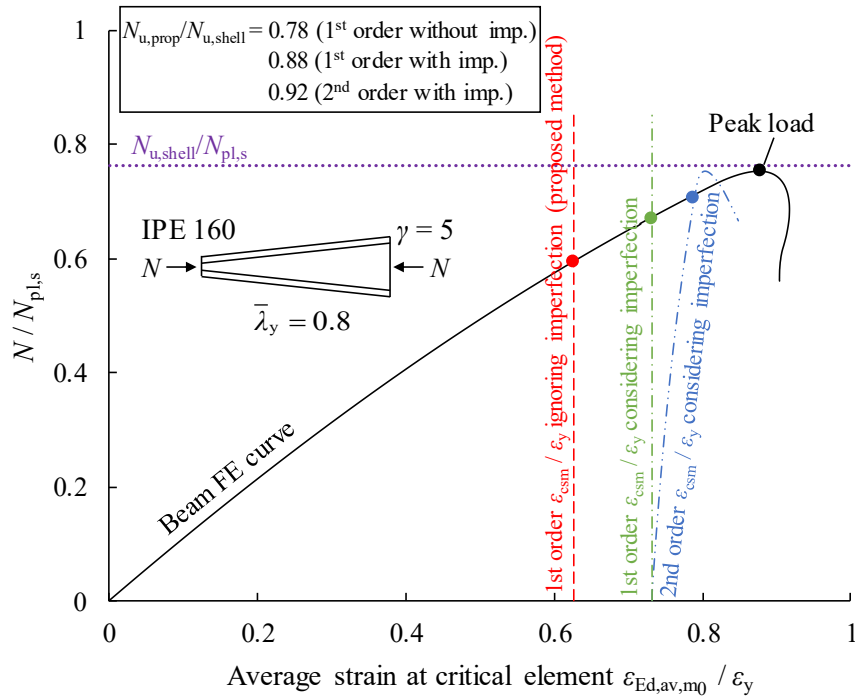


Figure 4.19 An illustrative example to show the difference between the ultimate member resistances predicted by the proposed design method using strain limits determined based on first order or second order internal force/moment distributions

4.5 Application of proposed design method to members under non-uniform bending and combined compression and non-uniform bending

In this section, the proposed design method of advanced analysis with strain limits is applied to 1680 tapered members subjected to different shapes of bending moment diagrams along their lengths, with or without axial compression. As summarised in Table 4.3, the following parameters were varied in the study: (i) the ratio of applied compression to bending moment, (ii) the ratio ψ of the bending moment applied at the shallow end of the member M_s to that applied at the deep end M_d , with values of $\psi = M_s/M_d = -1, -0.5, 0$ and 0.5 , as shown in Figure 4.20, (iii) the normalised member slenderness with values of $\bar{\lambda}_y = 0.5, 1$ and 1.5 , (iv) the taper ratio with values of $\gamma = 1, 2, 3, 4$ and 5 and (v) the cross-section profile at the shallow end of the tapered members, whose geometric properties were taken as the same as those of European

HEM 100, HEM 120, HEB 140, IPE 160, HEM 180, HEB 220 and HEB 240 cross-sections. For all the considered tapered members, the maximum cross-section slenderness $\bar{\lambda}_p$ along the length did not exceed 1.5.

Table 4.3 Summary of varied parameters considered and accuracy of the proposed design method and Eurocode 3 in comparison to the results from benchmark shell FE models for tapered members under non-uniform bending with or without axial compression

Moment ratio ψ	Member slenderness $\bar{\lambda}_y$	Taper ratio γ	Shallow-end cross-section	No. of cases N	Design method	$\alpha_u/\alpha_{u,shell}$			
						Mean	CoV	Max	Min
-1	0.5, 1.0, 1.5	1, 2, 3, 4, 5	HEM 100, HEM 120, HEB 140, IPE 160, HEM 180, HEB 220, HEB 240	420	Proposed	0.922	0.073	1.035	0.706
					EC3	0.848	0.099	0.985	0.597
-0.5				420	Proposed	0.918	0.083	1.095	0.703
					EC3	0.832	0.106	1.028	0.586
0				420	Proposed	0.947	0.063	1.057	0.730
					EC3	0.907	0.066	1.027	0.681
0.5				420	Proposed	0.961	0.078	1.064	0.709
					EC3	0.913	0.094	1.042	0.630
Total				1680	Proposed	0.937	0.077	1.095	0.703
					EC3	0.875	0.101	1.042	0.586

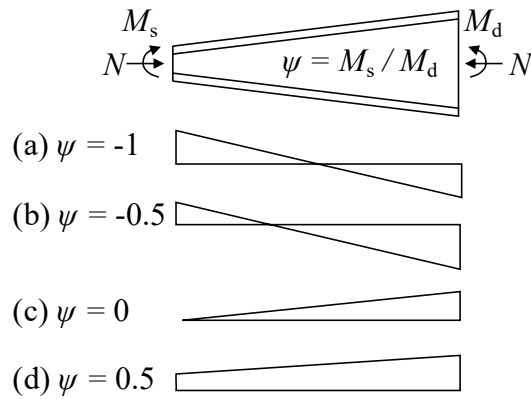


Figure 4.20 Load cases considered for tapered members under combined compression and non-uniform bending along the member length

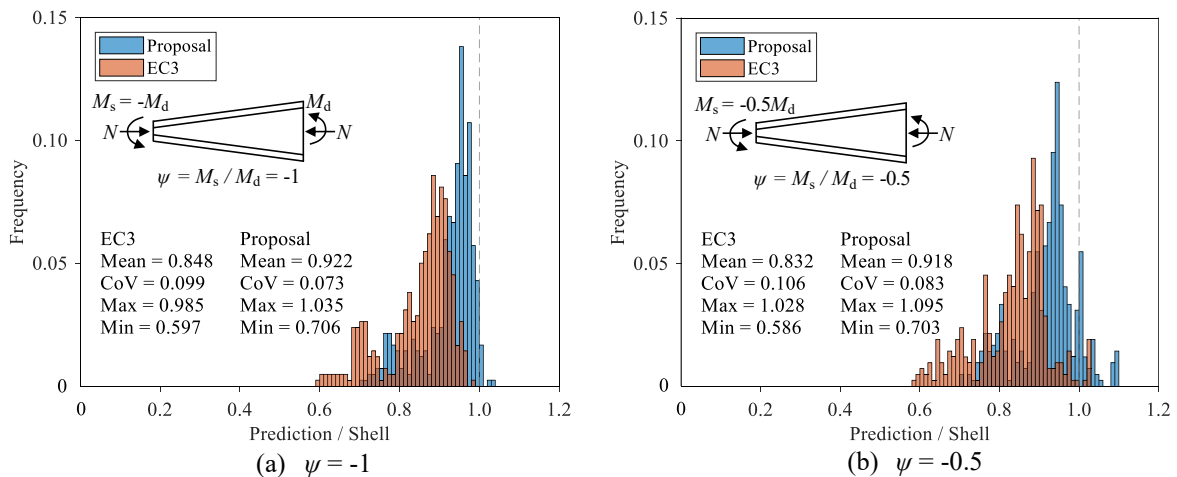
The presence of a moment gradient implicitly means that a tapered member is also subjected to shear forces, and high shear forces may negatively influence the ultimate load carrying capacities of tapered steel members. To allow for this effect, in this study, the approach proposed in Fieber et al. (2019a) was adopted; thus, when the design shear force $V_{y,Ed}$ exceeded half of the plastic shear force capacity of the cross-section $V_{pl,y}$, the interaction between bending

and shear was accounted for through a reduction factor ρ_{csm} applied to the CSM strain limit, as described in Fieber et al. (2019a). The expression for the determination of the reduction factor ρ_{csm} is given by Eq. (2.65). The CSM shear reduction factor ρ_{csm} utilises the shear reduction factor defined in EN 1993-1-1 (2005) ρ , which is given by Eq. (2.66). Note that separate cross-section shear capacity and shear buckling checks are still required in the application of the proposed design method which can be carried out using the provisions of EN 1993-1-1 (2005) and EN 1993-1-5 (2006).

$$\rho_{\text{csm}} = \begin{cases} 1 & \text{for } V_{y,\text{Ed}} \leq 0.5V_{\text{pl},y} \\ \frac{0.5}{0.5 + \rho} & \text{for } V_{y,\text{Ed}} > 0.5V_{\text{pl},y} \end{cases} \quad (2.65)$$

$$\rho = \left(\frac{2V_{y,\text{Ed}}}{V_{\text{pl},y}} - 1 \right)^2 \quad (2.66)$$

Frequency distributions of the ratios of the ultimate resistances predicted by the proposed design method and EN 1993-1-1 (2005) to those determined from the benchmark shell FE models are shown in Figure 4.21. As can be seen from the figure, the proposed design method provides accurate and generally safe-sided resistance predictions for tapered members subjected to compression plus non-uniform bending. As presented in Figure 4.21 and Table 4.3, relative to EN 1993-1-1 (2005) (i.e. GNIA plus cross-section checks), the proposed design method offers an average of 4% to 9% improvement in accuracy, depending on the loading case considered, as well as a consistent reduction in the scatter of the predictions. As shown in Figure 4.22, the traditional method presented in EN 1993-1-1 (2005) becomes increasingly conservative for larger taper ratios γ .



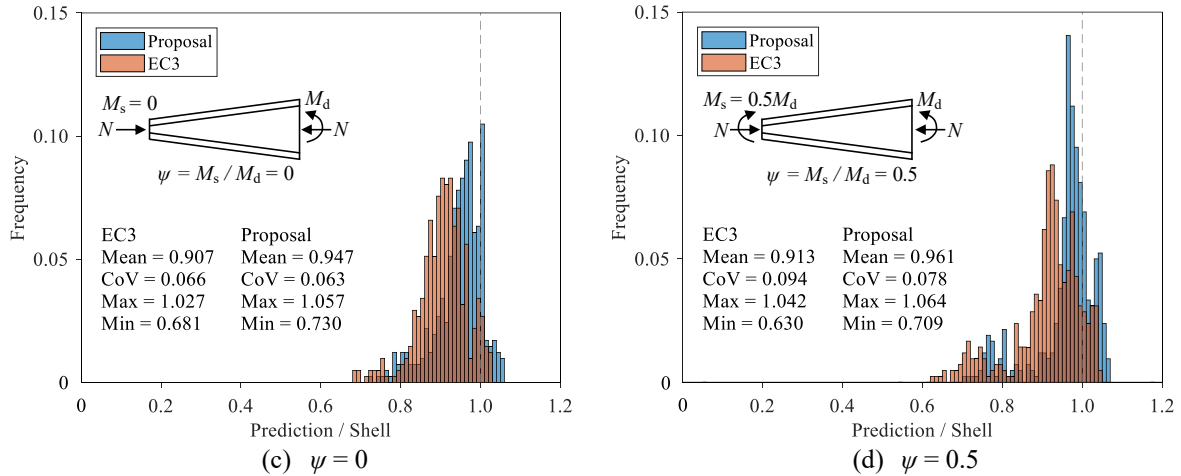


Figure 4.21 Frequency distributions of the ratios of the ultimate capacities predicted using the proposed design method and Eurocode 3 to those obtained from the benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ for tapered members under combined compression and non-uniform bending

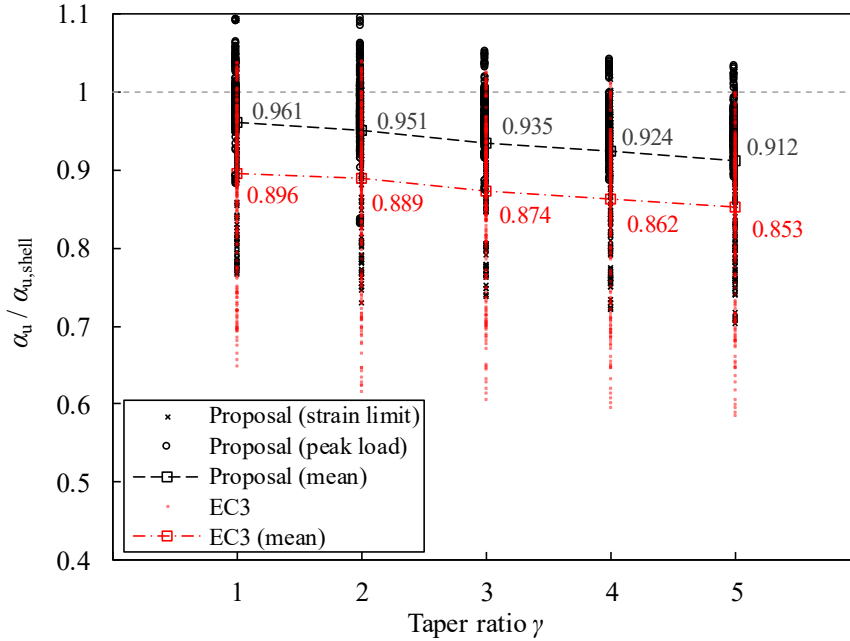


Figure 4.22 Comparison between the ratios of the ultimate load carrying capacities predicted using the proposed design method and Eurocode 3 to those determined from the benchmark shell FE models versus taper ratio γ for tapered members under combined compression and non-uniform bending

Normalised bending moment-axial force interaction diagrams for a series of tapered members subjected to compression plus non-uniform bending are shown in Figure 4.23. In the figure, $M_{u,s}$ and $M_{u,d}$ are the ultimate bending moments at the shallow and deep ends of the tapered members respectively, and $M_{pl,s}$ and $M_{pl,d}$ are the plastic bending moment resistances of the cross-sections at the shallow and deep ends. According to EN 1993-1-1 (2005), the critical cross-sections (i.e. the cross-sections where the internal bending moment and axial compression

reached the ultimate cross-section resistance) of these members fall into the Class 1 category; hence, the ultimate load carrying capacities of these tapered members are determined using the linear plastic bending moment-axial force (M-N) interaction equations provided in EN 1993-1-1 (2005). Similar to the observations made in Section 5 for tapered members under compression plus uniform bending, the design method presented in EN 1993-1-1 (2005), i.e. GNIA plus cross-section checks, is rather conservative for tapered members under compression plus non-uniform bending. On the other hand, the proposed design method offers more accurate but still generally safe-sided strength predictions.

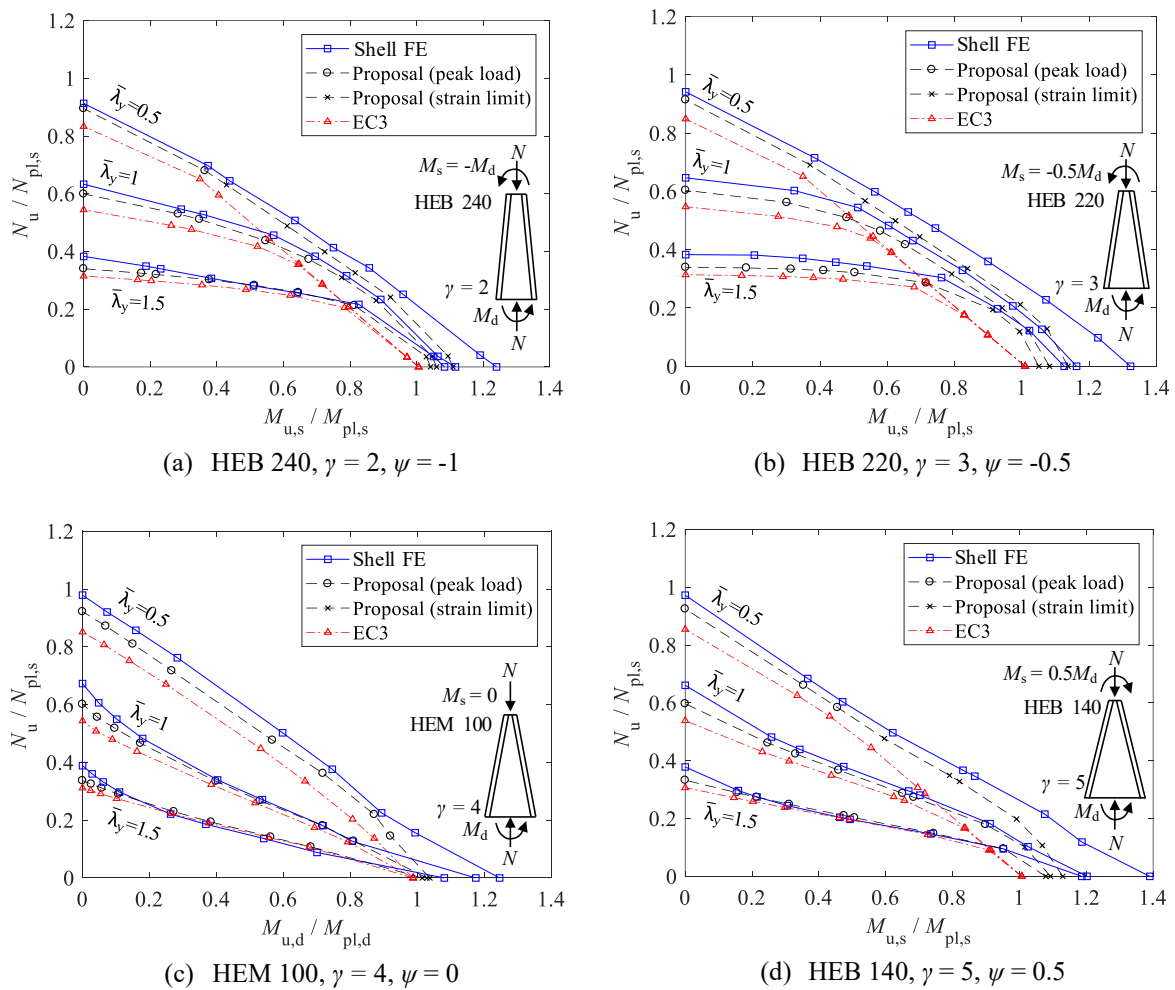


Figure 4.23 Comparison of the ultimate strengths determined using the proposed design approach with the benchmark shell FE models and Eurocode 3 for tapered members with different member slendernesses under axial compression and non-uniform major axis bending

Similar observations can also be made based on Figure 4.24, which presents comparisons between the ratios of the ultimate capacities predicted using the proposed design method and EN 1993-1-1 (2005) to those determined from the benchmark shell FE models ($\alpha_u / \alpha_{u,shell}$) versus

the cross-section slenderness at the critical location $\bar{\lambda}_{p,cr}$ for all the analysed tapered members. The critical location refers to the cross-section where the strain limit is attained or the shallow end cross-section when the ultimate capacity is predicted by the peak load factor. It can be seen from Figure 4.24 that, similar to the observations made in Section 4.4, for the tapered members with critical cross-sections of intermediate slenderness ($0.32 \lesssim \bar{\lambda}_{p,cr} \lesssim 0.5$), the improved accuracy and enhanced resistances achieved using the proposed design method relative to EN 1993-1-1 (2005) arise due to the rational exploitation of the spread of plasticity and the allowance for local moment gradients through strain averaging. For the tapered members with critical cross-sections of stocky proportions ($\bar{\lambda}_{p,cr} \lesssim 0.32$), the proposed design method also allows for strain hardening, which results in further improvements in the accuracy of the resistance predictions. For the members with very stocky critical cross-sections, if the upper strain limit is increased from $15\varepsilon_y$ to $30\varepsilon_y$, the accuracy of the proposed design method improves even further, as shown in Figure 4.25. For the tapered members with slender critical cross-sections ($\bar{\lambda}_{p,cr} \geq 0.68$), the ultimate resistances predicted by the proposed design method are close to those obtained from EN 1993-1-1 (2005).

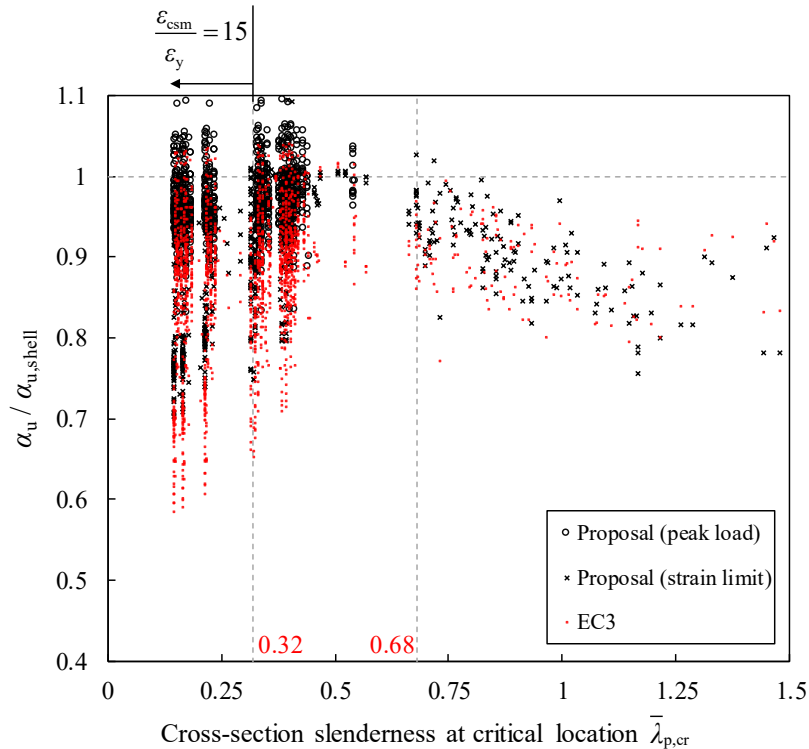


Figure 4.24 Comparison of the ratios of the ultimate load carrying capacities obtained using the proposed design approach and Eurocode 3 to those obtained from the benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ versus cross-section slenderness at the critical location $\bar{\lambda}_{p,cr}$ for tapered members under combined compression and non-uniform bending

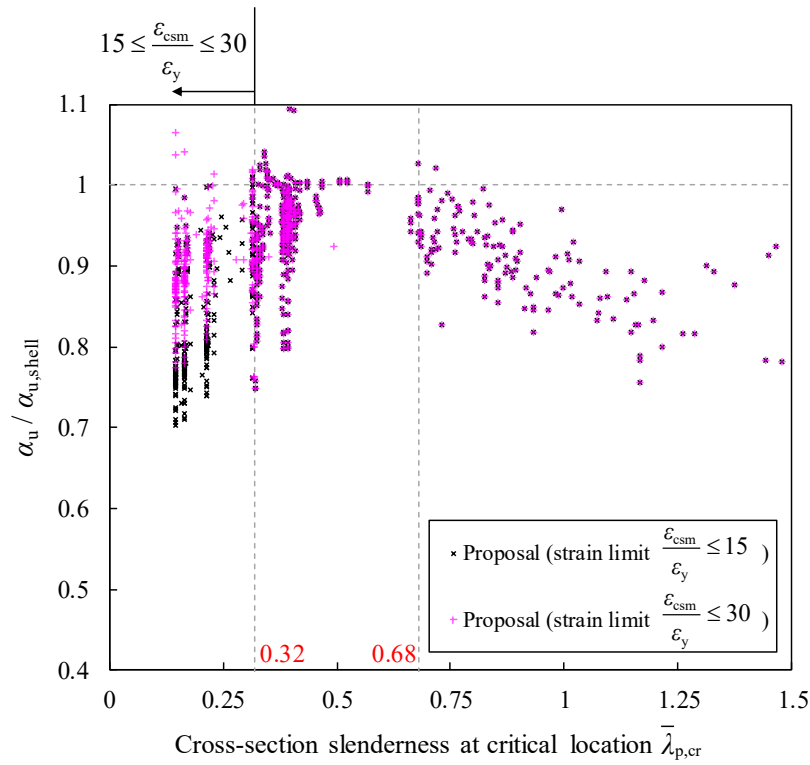


Figure 4.25 Ratios of ultimate load carrying capacities predicted using the proposed design method using $15\epsilon_y$ or $30\epsilon_y$ as the maximum strain limit to those obtained from the benchmark shell FE models $\alpha_u/\alpha_{u,shell}$ for tapered members under non-uniform bending, considering only cases when the strain limit, rather than the peak load, governed.

In addition to the verification of the accuracy of the proposed design approach against the results from the benchmark shell FE models, its accuracy is also assessed against the results obtained from available experiments on tapered members under uniform and non-uniform bending reported in the literature Prawel et al. (1974), Shiomi & Kurata (1984), Cristutiu et al. (2012) and Tankova et al. (2018b), as shown in Table 4.1. As can be seen from the table, the ultimate strengths determined using the proposed design method are somewhat conservative, but substantially more accurate and consistent than EN 1993-1-1 (2005). The conservatism is attributed to the use of the lowest eigenmode affine geometric imperfections with equivalent imperfection magnitudes in the proposed design method, both of which are more severe than the actual measured imperfections in the test specimens.

4.6 Concluding remarks

A consistent design method for tapered steel members performed by advanced inelastic analysis using beam finite elements with CSM strain limits has been put forward in this chapter. The

proposed design method was applied to a total of 2000 tapered members subjected to compression, uniform bending and combined compression and uniform bending, and also to a total of 1680 tapered members under non-uniform bending and combined axial compression and non-uniform bending. Various taper ratios, geometrical properties, loading conditions and member slenderness values were considered. The accuracy of the proposed design method was extensively verified against results from nonlinear shell FE modelling as well as a number of experiments on tapered members. It was shown that the proposed method provides consistently more accurate ultimate strength predictions relative to the traditional design method provided in EN 1993-1-1 (2005) for the design of tapered steel members owing to (i) taking account of the interaction between cross-section elements in the determination of local slenderness, (ii) allowing for strain hardening in stocky cross-sections, (iii) exploiting partial plastification in cross-sections and (iv) considering the beneficial effects of strain gradients along the member lengths. The proposed method also precludes the need for cross-section classification, the cumbersome process of determining effective section properties for Class 4 cross-sections, individual member buckling design checks and the calculation of effective lengths in the determination of the ultimate strengths of tapered steel members.

Chapter 5

Equivalent imperfections for out-of-plane stability design by GMNIA

5.1 Introduction

Steel beams are often susceptible to out-of-plane instability effects, such as lateral-torsional buckling (LTB), as shown in Figure 2.2. The influence of LTB on the resistances of steel beams is traditionally accounted for in design standards (EN 1993-1-1, 2005; prEN 1993-1-1, 2019) through the use of buckling curves. These curves implicitly consider the influence of imperfections in the determination of the buckling reduction factor, as detailed in Section 2.2.3. Alternatively, the LTB design of members can be undertaken more directly by performing a second order analysis with imperfections, through which the member out-of-straightness is explicitly modelled and individual member buckling checks can be avoided.

As illustrated in Section 2.5, the influence of both geometric and material (i.e. residual stresses) initial imperfections must be accounted for in design. For modelling convenience, so-called ‘equivalent’ imperfections are typically utilised, which consider the combined influence of both geometric imperfections and residual stresses. While EN 1993-1-1 (2005) and the upcoming prEN 1993-1-1 (2019) include recommendations for equivalent imperfections for use in second order elastic analysis (i.e. GNIA), there are currently no provisions for second order inelastic analysis (i.e. GMNIA).

Equivalent imperfections have been developed by Walport et al. (2020b) for use in design by second order inelastic analysis of structural members for flexural buckling. The present study follows a similar methodology and derives equivalent imperfections for use in design by second order inelastic analysis of structural members for LTB.

5.2 Equivalent imperfections for use in design by second order elastic analysis

Equivalent imperfections implicitly account for the combined effects of geometric and material imperfections. They may be applied in an analysis in two ways: (1) through the scaling of the lowest eigenmode or (2) through the direct modelling of a bow imperfection, and with an amplitude either determined by back-calculation from the relevant buckling curve or taken from a set of prescribed tabulated values. The equivalent imperfections developed for use in second order elastic analysis (i.e. GNIA) are described in this section.

5.2.1 Back-calculated equivalent imperfections

5.2.1.1 EN 1993-1-1 back-calculated equivalent imperfections

For the design by GNIA of a steel beam susceptible to LTB, EN 1993-1-1 (2005) recommends an equivalent imperfection with a lateral bow shape and an amplitude ke_0 , where e_0 is the equivalent imperfection for the weak axis flexural buckling design of the corresponding member in compression and $k = 0.5$. The amplitude e_0 can be back-calculated from the flexural buckling curves given in EN 1993-1-1 (2005), as given by:

$$e_0 = \alpha_z \left(\bar{\lambda}_z - 0.2 \right) \frac{W_z}{A} \quad \text{for } \bar{\lambda}_z \geq 0.2 \quad (5.1)$$

In Eq. (5.1), α_z is the imperfection factor for minor axis flexural buckling, W_z is the minor axis section modulus and A is the cross-sectional area. This equation was derived on the basis of a linear M-N cross-section failure criterion, under axial load N and second order minor axis bending moment M_z , as described in Lindner et al. (2016). The linear elastic and plastic cross-section M-N interaction curves for axial force and minor axis bending moment are given by Eqs. (5.2) and (5.3), respectively:

$$\frac{N}{N_{pl}} + \frac{M_z}{M_{el,z}} = 1 \quad (5.2)$$

$$\frac{N}{N_{pl}} + \frac{M_z}{M_{pl,z}} = 1 \quad (5.3)$$

where N_{pl} is the axial cross-section resistance, and $M_{el,z}$ and $M_{pl,z}$ are the elastic and plastic minor axis bending moment resistances. For LTB design by GNIA, the linear elastic and plastic cross-section interaction curves are given by Eqs. (5.4) and (5.5), respectively (prEN 1993-1-1, 2019; Stroetmann & Fominow, 2019):

$$\frac{M_y}{M_{el,y}} + \frac{M_z}{M_{el,z}} + \frac{B}{B_{el}} = 1 \quad (5.4)$$

$$\frac{M_y}{M_{pl,y}} + \frac{M_z}{M_{pl,z}} + \frac{B}{B_{pl}} = 1 \quad (5.5)$$

where M_y and M_z are the major and minor axis bending moments, B is the bimoment, $M_{el,y}$ and $M_{pl,y}$ are the elastic and plastic major axis bending moment resistances and B_{el} and B_{pl} are the elastic and plastic warping bimoment resistances (Vayas et al., 2019), as given for I-sections by:

$$B_{el} = t_f b^2 f_y (h - t_f) / 6 \quad (5.6)$$

$$B_{pl} = t_f b^2 f_y (h - t_f) / 4 \quad (5.7)$$

where h , b and t_f are the overall depth, flange width and flange thickness of the I-section.

For flexural buckling, use of the back-calculated equivalent imperfection amplitudes – $e_{0,el,b-c}$ (with a linear elastic cross-section check – Eq. (5.2)) and $e_{0,pl,b-c}$ (with a linear plastic cross-section check – Eq. (5.3)) – in design by GNIA yields the same flexural buckling resistances as those obtained from the flexural buckling curves provided in EN 1993-1-1 (2005). However, for LTB design by second order elastic analysis, the equivalent imperfections – $ke_{0,el,b-c}$ and $ke_{0,pl,b-c}$ – do not directly correspond to the results obtained from the buckling curves, and have been shown to be inaccurate for use in LTB design by GNIA, with the value of $k = 0.5$ deemed to be too small (Snijder et al., 2018).

5.2.1.2 Equivalent imperfections back-calculated from LTB curves in prEN 1993-1-1

According to the analytical formulation (Taras, 2010; Taras & Greiner, 2010) behind the new LTB curves given in prEN 1993-1-1 (2019), the amplitude of the equivalent imperfection for use in GNIA $e_{0,LT}$ (Snijder et al., 2016) can be back-calculated from:

$$e_{0,LT} = \alpha_{LT} \left(\bar{\lambda}_z - 0.2 \right) \frac{W_z}{A} \quad \text{for } \bar{\lambda}_z \geq 0.2 \quad (5.8)$$

where α_{LT} is the imperfection factor for LTB, determined from Table 2.1. Eq. (5.8) was derived on the basis of linear cross-section failure criterion under uniform major axis bending moment M_y , second order minor axis bending moment M_z and warping bimoment B (Taras, 2010), as given by Eqs. (5.4) and (5.5). Employment of the elastic $W_{el,z}$ and plastic $W_{pl,z}$ minor axis section moduli in Eq. (5.8) provides the equivalent imperfections $e_{0,LT,el,b-c}$ and $e_{0,LT,pl,b-c}$, for use in GNIA with linear elastic (Eq. (5.4)) and plastic (Eq. (5.5)) cross-section verifications, respectively. Since the derivation of the LTB curves in prEN 1993-1-1 (2019) was performed on the basis of eigenmode-affine imperfections, these back-calculated equivalent imperfections should also be defined through the scaling of the eigenmode in GNIA.

Figure 5.1 (a) and (b) present the normalised second order internal moments at the critical cross-section and linear cross-section interaction curves for members subjected to uniform major axis bending, obtained from GNIA and GMNIA employing equivalent imperfections $e_{0,LT,el,b-c}$ and $e_{0,LT,pl,b-c}$, as well as the prEN 1993-1-1 member buckling resistances M_{EC3} (see Section 2.2.3.2). Figure 5.1 (a) presents the results for a Class 3 cross-section, which utilises a linear elastic cross-section interaction curve, while Figure 5.1 (b) presents the results for a Class 2 cross-section, which utilises a linear plastic cross-section interaction curve. In both cases, the second order elastic moment path with back-calculated imperfections intersects the interaction curve exactly at the buckling resistance M_{EC3} , though this is not always the case due to the post-critical response of slender beams described below. However, as in the case of flexural buckling (Walport et al., 2020b), the back-calculated equivalent imperfections determined for use in LTB design by GNIA are not suitable for use in GMNIA. Depending on the design case, the GNIA-derived equivalent imperfections can result in both over-predictions and under-predictions of resistance. In Figure 5.1 (a), due to the spread of plasticity through the cross-section after first yield and before the attainment of the peak load, the member resistance obtained from GMNIA with $e_{0,LT,el,b-c}$ is higher than the result obtained from GNIA with $e_{0,LT,el,b-c}$. In Figure 5.1 (b), due to the effective double-counting of the influence of material nonlinearity, utilising the plastic equivalent imperfection $e_{0,LT,pl,b-c}$ in GMNIA results in a lower member resistance than that obtained from GNIA.

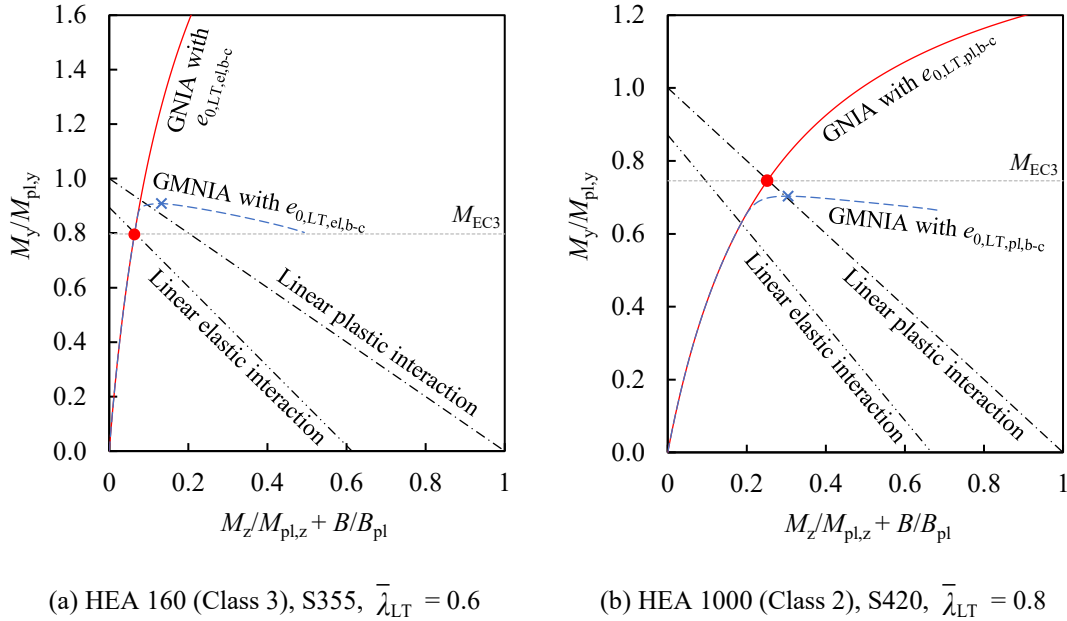


Figure 5.1 Normalised second order internal moments at the critical cross-section and linear cross-section interaction curves for members subjected to uniform major axis bending, obtained from GNIA and GMNIA using the back-calculated (a) elastic $e_{0,LT,el,b-c}$ and (b) plastic $e_{0,LT,pl,b-c}$ equivalent imperfections compared against the prEN 1993-1-1 member buckling resistance M_{EC3}

Note that, for some slender members, the ultimate resistance can exceed the elastic buckling moment M_{cr} . This is because after large torsional deformations, the cross-sectional minor axis bending capacity effectively becomes a lower bound to the LTB resistance (Taras, 2010). This behaviour is captured in a second order analysis. However, the derivation of the LTB curves presented in prEN 1993-1-1 (2019) and thus the equivalent imperfections back-calculated from the LTB curves, as determined by Eq. (5.8), do not account for this “post-critical” load-carrying mechanism (Taras & Greiner, 2010). Therefore, member resistances obtained from GNIA or GMNIA using back-calculated equivalent imperfections can be substantially exceed the prEN 1993-1-1 buckling resistance M_{EC3} (van der Aa, 2015), but only after large torsional deformations. This is shown in Figure 5.2, for a hot-rolled S420 steel HEA 100 member with $\bar{\lambda}_{LT} = 1.0$ subjected to uniform bending. It can be seen that the member resistances obtained from GNIA and GMNIA with a back-calculated equivalent imperfection $e_{0,LT,pl,b-c}$ for the linear plastic cross-section interaction curve are higher than the Eurocode resistance M_{EC3} by 14% and 12%, respectively.

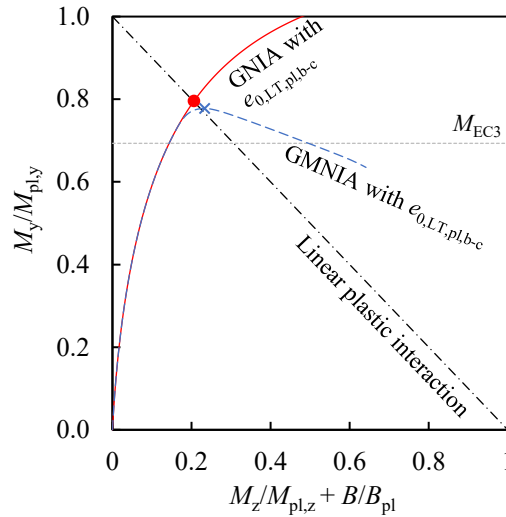


Figure 5.2 Normalised second order internal moments at the critical cross-section and linear plastic cross-section interaction curve for a hot-rolled S420 steel HEA 100 (Class 1) member with $\bar{\lambda}_{LT} = 1.0$ subjected to uniform major axis bending, obtained from GNIA and GMNIA using the back-calculated plastic equivalent imperfection $e_{0,LT,pl,b-c}$ compared against the prEN 1993-1-1 member buckling resistance M_{EC3}

5.2.2 Tabulated equivalent imperfections

5.2.2.1 EN 1993-1-1 tabulated equivalent imperfections

According to EN 1993-1-1 (2005), when using lateral bow equivalent imperfections of amplitude ke_0 in design by second order elastic analysis, e_0 may, as a simpler alternative to performing the back-calculation described above, be taken from the tabulated values listed in Table 5.1 of EN 1993-1-1 (2005) for the weak axis flexural buckling design of the corresponding member in compression; in prEN 1993-1-1 (2019), revised tabulated equivalent imperfections for LTB design by second order elastic analysis are provided (Stroetmann & Lindner, 2010), as described below.

5.2.2.2 prEN 1993-1-1 tabulated equivalent imperfections

According to prEN 1993-1-1 (2019), equivalent imperfection amplitudes $e_{0,LT}$ for use in LTB design by GNIA can be determined from Eq. (5.9):

$$e_{0,LT} = \frac{\beta_{LT} L}{\varepsilon} \quad (5.9)$$

where β_{LT} is the reference relative bow imperfection for LTB, values of which are provided in tabular form (Stroetmann & Lindner, 2010; prEN 1993-1-1, 2019), L is the member length and

ε is the material parameter given by:

$$\varepsilon = \sqrt{\frac{235}{f_y}} \quad (5.10)$$

The imperfection amplitude may be applied in the form of a lateral bow. Note that, unlike the corresponding equivalent imperfections for flexural buckling, given by Eq. (5.11), that are dependent on not only the reference relative bow imperfection for flexural buckling β but also the imperfection factor α (Lindner et al., 2018), Eq. (5.9) is not dependent on the LTB imperfection factor α_{LT} . The factor β_{LT} in Eq. (5.9) does however depend on the adopted cross-section failure criterion (linear elastic or linear plastic) and the geometrical depth-to-width ratio h/b of the cross-section.

$$e_0 = \frac{\alpha \beta L}{\varepsilon} \quad (5.11)$$

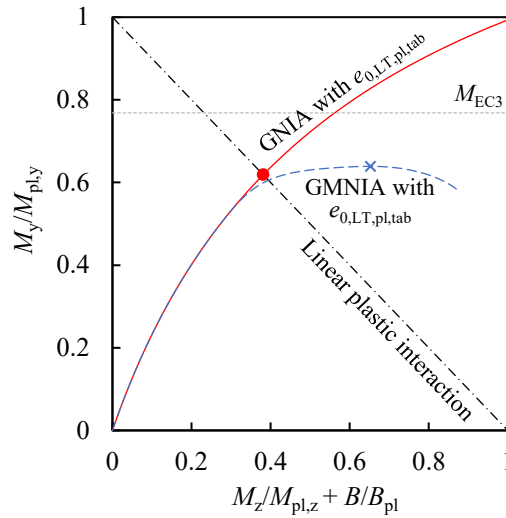


Figure 5.3 Normalised second order internal moments at the critical cross-section and linear plastic cross-section interaction curve for a hot-rolled S420 steel IPE 140 (Class 1) member with $\bar{\lambda}_{LT} = 0.8$ subjected to uniform major axis bending, obtained from GNIA and GMNIA using the tabulated plastic equivalent imperfection $e_{0,LT,pl,tab}$ compared against the prEN 1993-1-1 member buckling resistance M_{EC3}

The use of Eq. (5.9) for LTB design by second order elastic analysis has been shown in previous research (Kindmann & Beier-Tertel, 2010; van der Aa, 2015) to provide somewhat conservative resistance predictions. This is highlighted in Figure 5.3, which shows the normalised load paths and capacity predictions of a hot-rolled S420 steel IPE 140 member with $\bar{\lambda}_{LT} = 0.8$ subjected

to uniform bending. Employing the tabulated imperfection for use with the plastic interaction curve determined from Eq. (5.9) $e_{0,LT,pl,tab}$ in both GNIA and GMNIA results in conservative resistance predictions, up to 17% lower than the prEN 1993-1-1 member buckling resistance M_{EC3} . These tabulated imperfection amplitudes have been derived on the basis of second order elastic analysis and, as for the back-calculated imperfection amplitudes and as in the case of flexural buckling (Walport et al., 2020b), are generally unsuitable for use in design by second order inelastic analysis (GMNIA).

5.3 Generation of benchmark finite element data

In this section, benchmark finite element (FE) data, against which the required equivalent imperfections for use in the out-of-plane stability design of beams by second order inelastic analysis (i.e. GMNIA) can be calibrated, are generated. The benchmark data were obtained following the recommendations set out in (Fieber et al., 2019a; Gardner et al., 2019b; Walport et al., 2020a), featuring the GMNIA of beam FE models with geometric imperfections, residual stresses and strain limits to simulate cross-section failure. The key components of the procedure followed to obtain the benchmark data, including validation against physical experiments, are described in this section.

5.3.1 Modelling approach

The finite element analysis software Abaqus (2017) was used to carry out the GMNIA simulations. The beam element B31OS, which allows for the effects of torsion and warping in open-sections, was used (Mohri et al., 2003; Sabat, 2009; Sahraei, 2017). Fork-end support conditions, allowing warping deformations but fully restraining twisting at the supports, were employed for all members.

The investigations carried out in this section cover: (i) different grades of hot-rolled steel (S235, S355 and S420) and stainless steel (austenitic, duplex and ferritic) members, (ii) different ratios of the major axis bending moments applied at the beam ends $\psi = 1, 0.5, 0, -0.5$ and -1 , (iii) a range of normalised LTB slenderness values $\bar{\lambda}_{LT} = 0.4, 0.6, 0.8, 1.0, 1.2$ and 1.4 , and (iv) a series of different cross-section geometries – European HEB 100, HEB 400, HEB 700, HEB 1000, IPE 100, IPE 140, HEM 200, HEA 100, HEA 800 profiles – covering a range of cross-section slenderness values $\bar{\lambda}_p$ between 0.14 and 0.48. Note that the influence of the fillet radii

was not included in either the benchmark models described in this section or the design models described in later sections. Beams with closed cross-sections, such as square or circular hollow sections, are not generally vulnerable to LTB (Ellobody, 2014), and thus were not assessed in this study.

5.3.2 Material modelling

The material parameters adopted in this study are summarised in Table 5.1. For hot-rolled steel, the three steel grades, S235, S355 and S420, were considered. The quad-linear stress-strain model (see Section 2.3.2), developed by Yun & Gardner (2017) and included in prEN 1993-1-14 (2019), was used. For stainless steel, three families – austenitic (A), duplex (D) and ferritic (F) – were considered; for each family, one typical grade was chosen. The standardised material properties for numerical studies defined by Afshan et al. (2019) were employed, as listed in Table 5.1. The two-stage Ramberg-Osgood material model (Mirambell & Real, 2000; Rasmussen, 2003; Arrayago et al., 2015) was used, as detailed in Section 2.3.2.

Table 5.1 Overview of adopted material parameters for the FE models (Yun & Gardner, 2017; Afshan et al., 2019)

Material	Grade	Young's modulus E (N/mm ²)	Yield (0.2% proof) stress f_y (N/mm ²)	Ultimate stress f_u (N/mm ²)	Strain hardening strain ϵ_{sh}	Ultimate strain ϵ_u	Strain hardening exponent n	Strain hardening exponent m
Hot-rolled Steel	S235		235	360	0.015	0.21		
	S355	210000	355	510	0.015	0.18	-	-
	S420		420	520	0.026	0.12		
Stainless steel	A		280	580		0.50	9.1	2.3
	D	200000	530	770	-	0.30	9.3	3.6
	F		320	480		0.16	17.2	2.8

5.3.3 Geometric imperfections and residual stresses

Equivalent imperfections applied in a shape of single lateral bow have been shown to be unsuitable for use in loading cases with a low proportion of bending moment at midspan (e.g. antisymmetric bending) (Stroetmann & Fominow, 2019). Therefore, initial geometric imperfections in the form of the first LTB eigenmode, scaled to 1/1000 of the unbraced member lengths L were defined in the benchmark FE models. In this study, initial twist imperfections were not included in the beam FE models; this is deemed suitable since initial twist imperfections have been shown to have negligible influence on the ultimate lateral-torsional

buckling resistances of beams (Greiner et al., 2000; Couto & Vila Real, 2019) and are stated to not generally be needed in EN 1993-1-1 (2005). Residual stresses were also introduced into the FE models. Residual stress patterns in carbon steel and stainless steel sections differ due to the different mechanical and thermal properties resulting from the specific chemical compositions and microstructure (Gardner & Ng, 2006). For hot-rolled carbon steel, the residual stress pattern given by ECCS (1984) was utilised, as illustrated in Figure 5.4 (a). For stainless steel, the residual stress pattern for welded I-sections developed by Yuan et al. (2014) was adopted, as illustrated in Figure 5.4 (b). The residual stresses were introduced into the beam FE models using the SIGINI user subroutine (Abaqus, 2017), by defining the initial stresses at section points prior to the loading step. To accurately represent the residual stress distributions, each web and flange plate was assigned 33 section points for the hot-rolled steel profiles (Kucukler et al., 2016) and 41 sections points for the stainless steel profiles (Walport et al., 2019) along the width, respectively. Corresponding plastic strains were also assigned in the case of the stainless steel models (Kucukler et al., 2019).

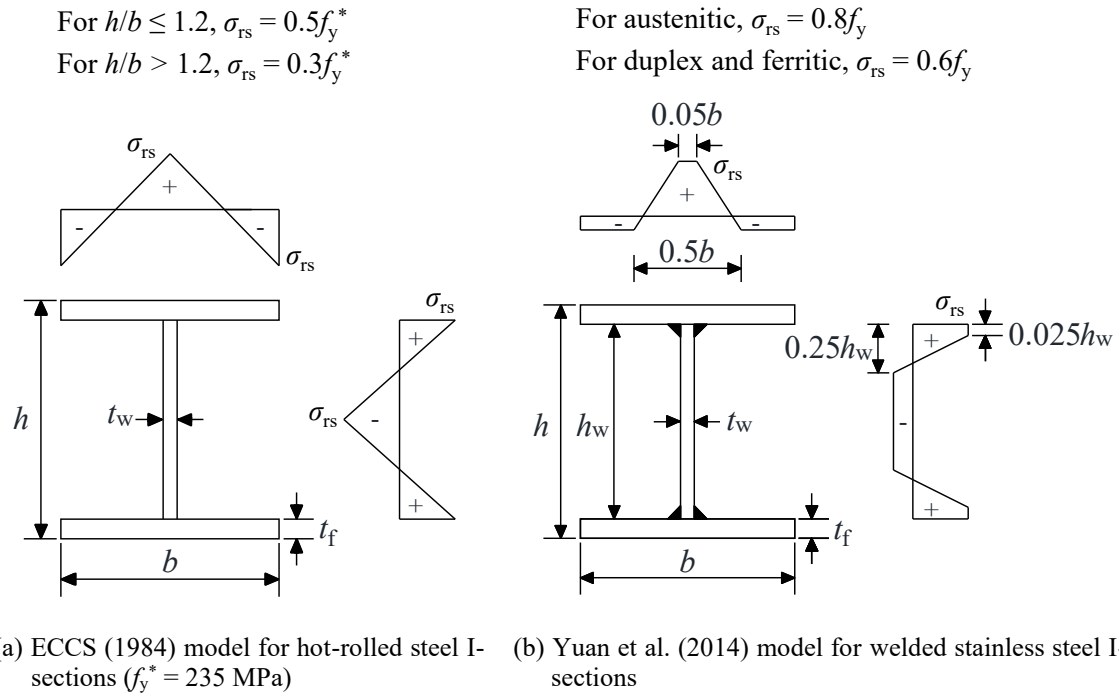


Figure 5.4 Residual stress patterns adopted for (a) hot-rolled steel and (b) welded stainless steel I-sections (+ve = tension; -ve = compression)

5.3.4 Generation of benchmark data

In this study, the method of design by advanced inelastic analysis together with the adoption of the CSM strain limits, as detailed in Section 4.2.1, is adopted to generate the benchmark data.

In the design method, a GMNIA of the structure using beam finite elements is first performed, to directly capture global member instabilities (e.g. LTB); the ultimate capacity is then defined based on either (i) the peak load factor obtained from the analysis or (ii) the load factor at which the CSM strain limit is attained at any cross-section in the considered member or structure, whichever occurs first. To take account of the beneficial influence of local strain gradients along the member lengths on cross-section resistance, the strain averaging approach (see Section 4.2.1.2) was applied; the lengths of the beam elements were taken as less than or equal to the corresponding elastic local buckling half-wavelengths $L_{b,cs}$ of the cross-sections, which can be obtained numerically, e.g. through the finite strip software CUFSM (Li & Schafer, 2010b), or through the expressions presented in Fieber et al. (2019b), which were adopted in the present study. Note that the strain limit is applied only to the strains resulting from the applied load (i.e. not the strains arising from the residual stresses). Relative to the traditional design calculations, this design method provides more accurate and consistent member capacity predictions (Fieber et al., 2019a).

5.3.5 Validation of beam FE models

Although the aforementioned beam FE modelling approach has been successfully employed in numerous previous investigations (Fieber et al., 2019a; Walport et al., 2020a; Walport et al., 2020b), in the present study, prior to being used to generate benchmark data, the approach is further validated against the results from 44 experiments reported in the literature (Dux & Kitipornchai, 1983; Lindner & Glitsch, 2004; Tusnin & Prokic, 2015a; Tusnin & Prokic, 2015b; Schaper et al., 2019; Demirhan et al., 2020), focusing on beams experiencing LTB. The loading configurations included (i) 3-point bending and 4-point bending (Dux & Kitipornchai, 1983), (ii) 3-point bending with eccentrically applied vertical loading, leading to additional torsion (Lindner & Glitsch, 2004; Tusnin & Prokic, 2015a; Tusnin & Prokic, 2015b; Schaper et al., 2019) and (iii) concentrated loading applied at the free-end of cantilever beams (Demirhan et al., 2020). The boundary and loading conditions of the beam FE models were consistent with those employed in the tests. The measured global geometric imperfection amplitudes and residual stresses were incorporated into the FE models where reported; the shape of the geometric imperfections were defined as the first LTB eigenmode. If no measured imperfection amplitudes or residual stresses were provided, the values from Section 5.3.3 were employed. The results of the validation study, including the mean and coefficient of variation (CoV) values of the ratios of the ultimate resistances obtained from beam FE models $\alpha_{u,FE}$ to those obtained

in the experiments $\alpha_{u,\text{test}}$ (i.e. $\alpha_{u,\text{FE}}/\alpha_{u,\text{test}}$), are summarised in Table 5.2. It can be seen that the beam FE models developed in this study are able to provide ultimate strength predictions that are consistently very close to experimental results.

Table 5.2 Summary of validation study for beam finite element models against experimental results from literature

Reference	Load configuration	No. of tests	$\alpha_{u,\text{FE}}/\alpha_{u,\text{test}}$	
			Mean	CoV
Dux & Kitipornchai (1983)	3-point bending; 4-point bending	9	0.969	0.015
Lindner & Glitsch (2004)	3-point bending with eccentricity	13	0.951	0.028
Tusnin & Prokic (2015a, 2015b)	3-point bending with eccentricity	6	0.898	0.055
Schaper et al. (2019)	3-point bending with eccentricity	7	1.006	0.036
Demirhan et al. (2020)	Cantilever beams	9	1.009	0.140
Total		44	0.968	0.081

In Figure 5.5, the experimental and numerical load-twist and load-vertical displacement curves for a sample of beams tested under eccentric 3-point bending in Tusnin & Prokic (2015a), Tusnin & Prokic (2015b) and Schaper et al. (2019) are shown, where P is the applied load, ϕ and w are the twist and vertical displacement at midspan, respectively, and $P_{u,\text{test}}$ is the ultimate load obtained from the experiments. It can be seen from the figures that the beam FE models accurately capture the load-deformation response observed in the experiments, and are therefore considered to be suitable for the generation of benchmark data, against which the required equivalent imperfections can be determined.

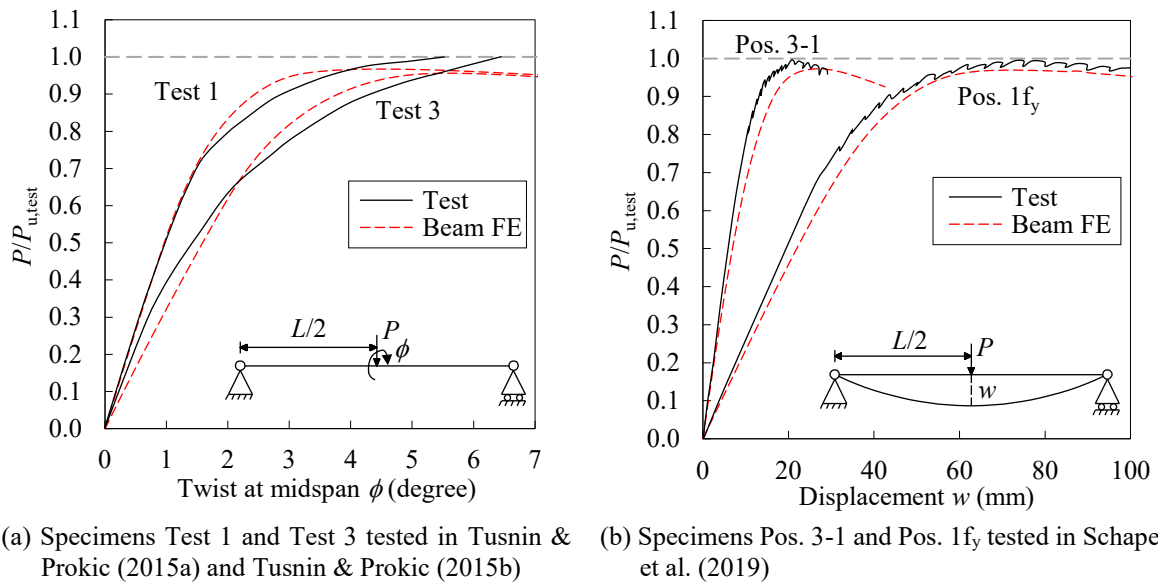


Figure 5.5 Comparison of experimental and numerical load-deformation curves of members subjected to 3-point bending with eccentrically applied vertical loading at midspan, tested in Tusnin & Prokic (2015a), Tusnin & Prokic (2015b) and Schaper et al. (2019)

5.4 Required equivalent imperfections for design by second order inelastic analysis

In this section, the required equivalent imperfections for use in the out-of-plane stability design of steel members by second order inelastic analysis are calculated by iteratively running beam FE models with varying initial imperfection amplitude e_0 until the obtained ultimate member resistance coincides with the benchmark data to within 0.5%. This is illustrated in Figure 5.6 for two example beams under uniform bending. The beam FE models used to back-calculate the required equivalent imperfections were developed following the same approach as the benchmark FE models except with a varying amplitude of geometric imperfection and no residual stresses.

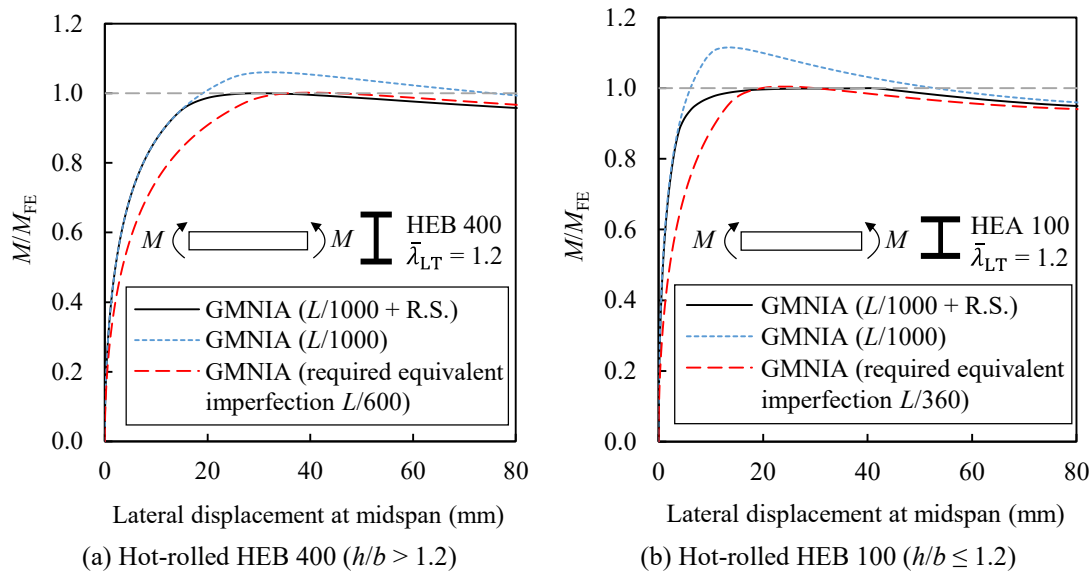


Figure 5.6 Required equivalent imperfections $e_{0,req}$ calculated iteratively to obtain ultimate bending moment M_u within 0.5% of the benchmark ultimate bending moments M_{FE} obtained from GMNIA of members with geometric imperfection magnitudes of $L/1000$ and residual stresses (R.S.)

It should be noted that the required equivalent imperfections calculated herein were introduced into the FE models through the scaling of the first LTB eigenmode, as employed in the benchmark FE models. The equivalent imperfection applied in a shape of single lateral bow has been shown to be unsuitable for use in loading cases with a low proportion of bending moment at midspan (e.g. antisymmetric bending) (Stroetmann & Fominow, 2019). A similar conclusion was reached in the study of Ebel (2014) in which the shape of the first LTB eigenmode was recommended for equivalent imperfections. The new design curves for LTB in prEN 1993-1-1

(2019) (see Section 2.2.3.2) were also developed based on imperfections in the shape of the first LTB eigenmode (Taras, 2010).

The amplitude of the geometric imperfections introduced into benchmark FE models was equal to $L/1000$. Since the equivalent imperfections take account of the combined effects of geometric imperfections and residual stresses, the required values $e_{0,\text{req}}$ must be larger than $L/1000$. Figure 5.6 (a) and (b) show the required equivalent geometric imperfection amplitudes $e_{0,\text{req}}$ for two S355 hot-rolled steel members subjected to uniform bending, with HEB 400 and HEA 100 cross-sections, respectively, and $\bar{\lambda}_{\text{LT}} = 1.2$. It can be seen that the HEA 100 cross-section, which has a depth-to-width ratio h/b less than 1.2, needs a higher required equivalent imperfection than that of the member with the HEB 400 cross-section (i.e. $L/360 > L/600$), where h/b is greater than 1.2. This is because, as shown in Figure 5.4 (a), for hot-rolled steel I-section members, larger residual stresses exist in cross-sections with $h/b \leq 1.2$ than with $h/b > 1.2$, resulting in a more detrimental effect on member resistances, and consequently requiring larger equivalent imperfection amplitudes. This is also reflected in the upcoming prEN 1993-1-1 (2019), which includes a higher LTB imperfection factor α_{LT} for the LTB curves of cross-sections with $h/b \leq 1.2$.

Figures 5.7 and 5.8 show the required values of the non-dimensional equivalent imperfections $j_{\text{req}} = L/e_{0,\text{req}}$ for hot-rolled and stainless steel members, respectively, subjected to a range of bending moment distributions along the member length ($\psi = 1, 0.5, 0, -0.5$ and -1), achieved through varying the ratio of the applied end moments (i.e. $\psi = M_2/M_1$, where M_1 and M_2 are the bending moments applied at each end of the beam) versus member slenderness $\bar{\lambda}_{\text{LT}}$. It should be noted that members with different member slenderness have varying sensitivity to geometric imperfections. This is illustrated in Figure 5.9, which shows the ratio of ultimate load carrying capacities obtained from GMNIA with CSM strain limits M_u , with varying non-dimensional geometric imperfection values $j = L/e_0$ ranging from 100 to 1000, to the benchmark results M_{FE} , for two hot-rolled steel HEB 400 members with member slenderness $\bar{\lambda}_{\text{LT}} = 0.4$ and 1.0 , respectively, subjected to uniform bending. It can be observed that the more slender member (with $\bar{\lambda}_{\text{LT}} = 1.0$), for which LTB is more pronounced, is more sensitive to the geometric imperfections. It can be seen that with decreasing geometric imperfection amplitude, there is a clear increase in ultimate resistances. By contrast, the stockier member (with $\bar{\lambda}_{\text{LT}} = 0.4$) is much less sensitive to geometric imperfections; when the imperfection e_0 varies from $L/100$ to $L/1000$,

the difference between the ultimate bending moment resistances is less than 10%. In the range of $L/1000 \leq e_0 \leq L/500$, the ultimate resistances are almost constant and all are within 0.5% of the benchmark results. This confirms that for stocky members, variation in the initial geometric imperfection has little influence on the ultimate resistance.

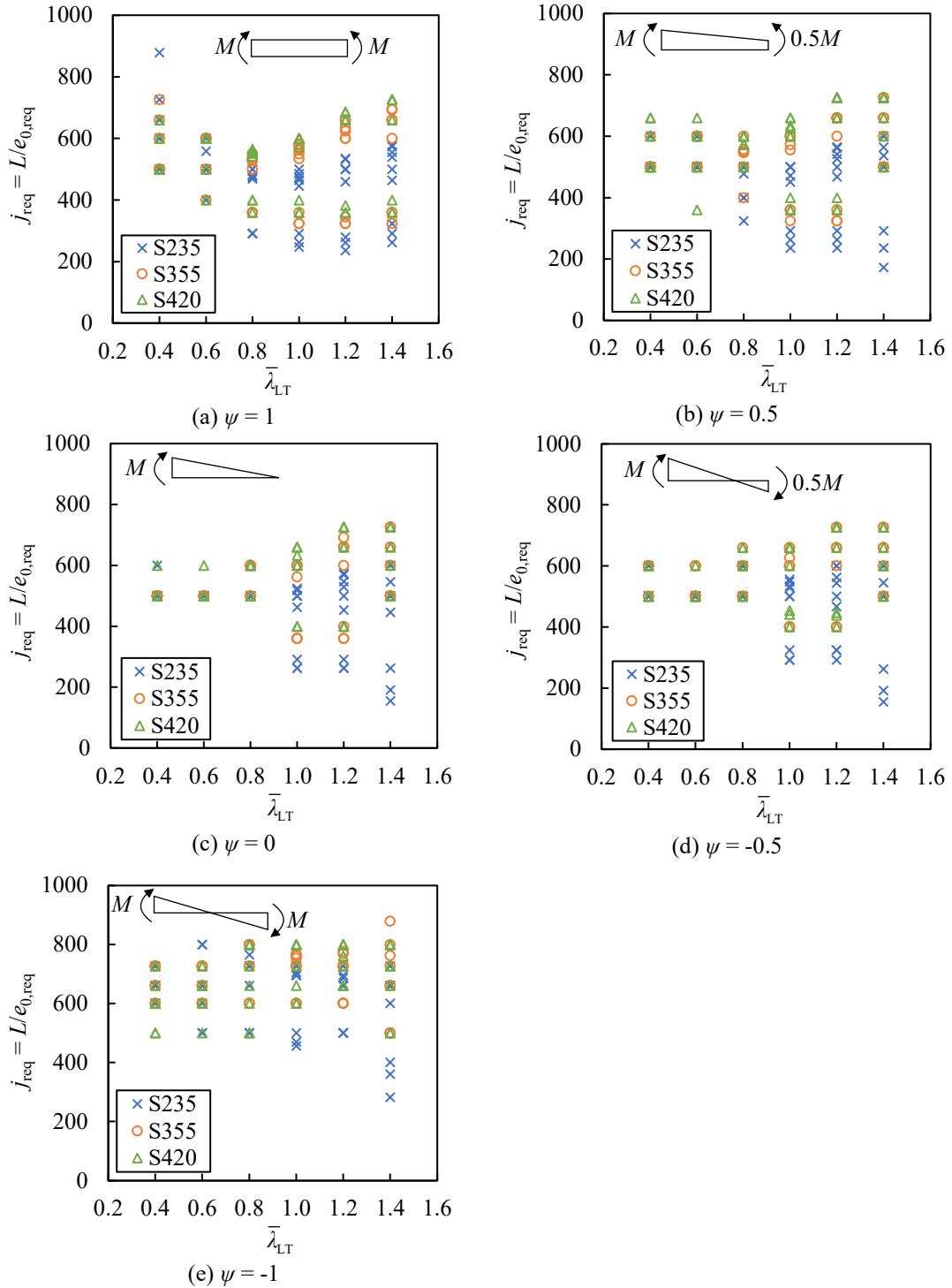


Figure 5.7 Required values of non-dimensional equivalent imperfections $j_{\text{req}} = L/e_{0,\text{req}}$ for hot-rolled steel members subjected to different bending moment distributions along the member length ($\psi = 1, 0.5, 0, -0.5$ and -1), achieved by changing the ratio of the applied end moments $\psi = M_2/M_1$

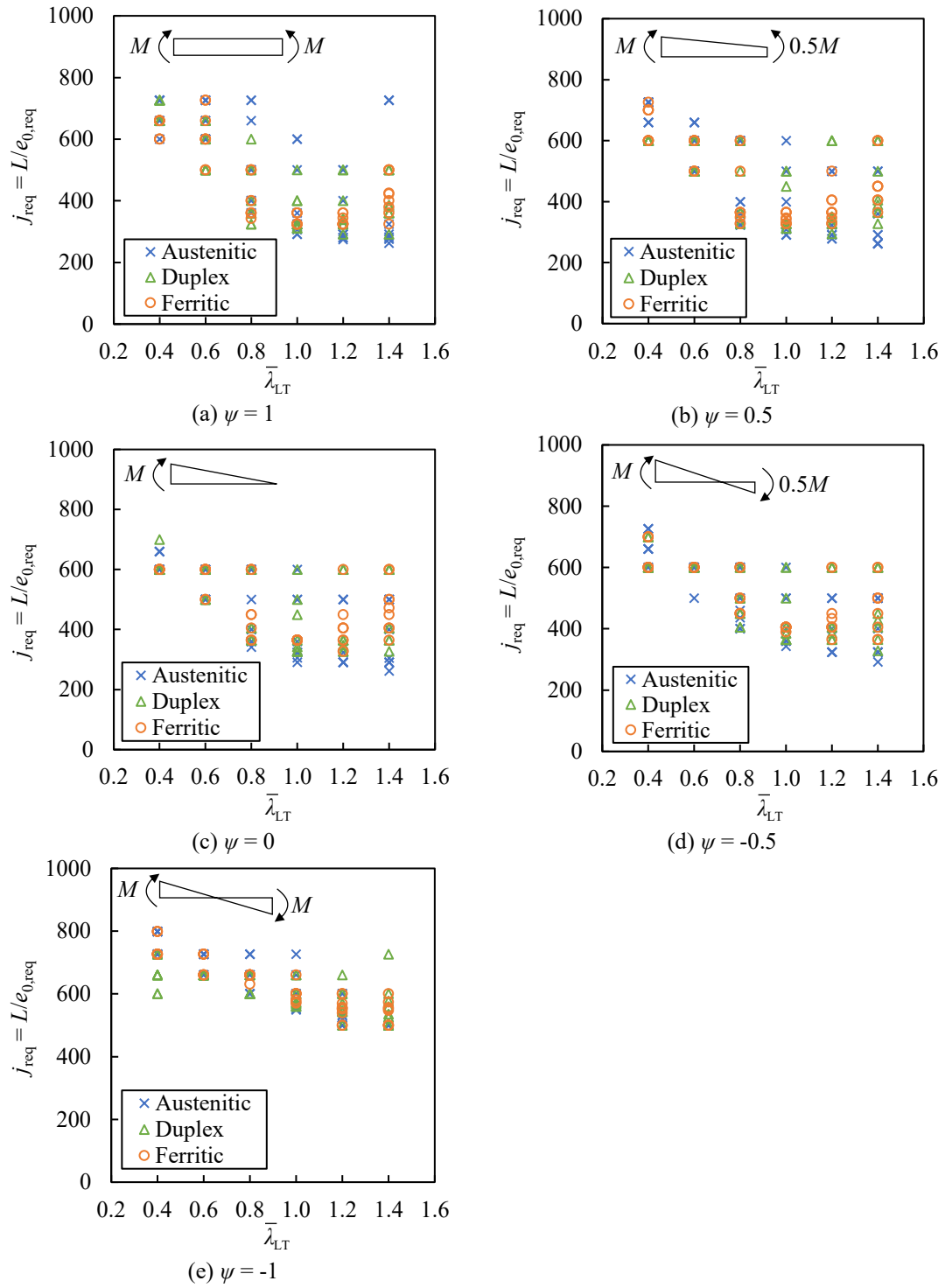


Figure 5.8 Required values of $j_{\text{req}} = L/e_{0,\text{req}}$ for stainless steel members subjected to different bending moment distributions along the member length ($\psi = 1, 0.5, 0, -0.5$ and -1), achieved by changing the ratio of the applied end moments $\psi = M_2/M_1$

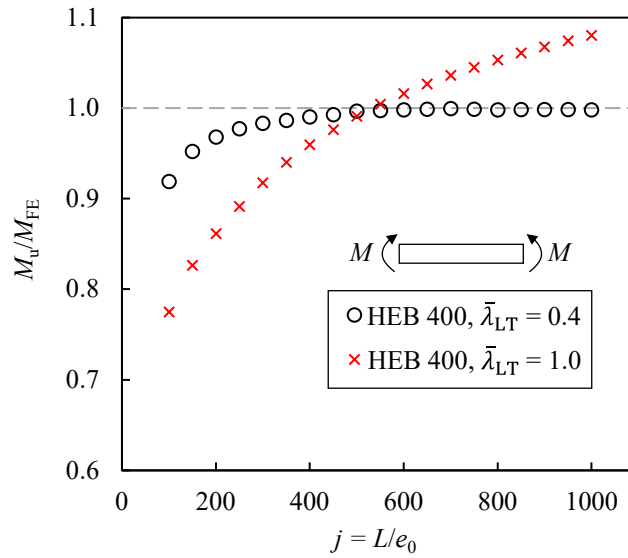


Figure 5.9 Sensitivity to geometric imperfections of hot-rolled S355 steel HEB 400 members with LTB slenderness $\bar{\lambda}_{LT} = 0.4$ and 1.0 subjected to uniform bending

In this study, as in Walport et al. (2020b), it is proposed that the imperfection factor is incorporated into the definition of the equivalent imperfection amplitude $e_{0,LT}$, to capture the varying influence of residual stresses for different cross-section types. Since the influence of geometric properties is directly captured by GMNIA using three-dimensional beam finite elements, the employed imperfection factor does not require inclusion of the geometric parameter $\sqrt{W_{el,y} / W_{el,z}}$ discussed in Section 2.2.3.2, and is proposed to be simply taken as α_z (i.e. the imperfection factor for minor axis flexural buckling), as given in Table 5.3, which generally corresponds to the upper limit of α_{LT} provided in prEN 1993-1-1 (2019) and prEN 1993-1-4 (2020). This ensures that the severity of the influence of residual stresses is captured, but without inheriting the additional compensatory effects included in α_{LT} for calibration of the LTB curves.

Table 5.3 Imperfection factor α_z for flexural buckling of doubly symmetric I- and H-sections (prEN 1993-1-1, 2019; prEN 1993-1-4, 2020)

Material	Limits		α_z
Hot-rolled steel	$h/b > 1.2$	$t_f \leq 40$ mm	0.34
		$t_f > 40$ mm	0.49
	$h/b \leq 1.2$		0.49
Stainless steel	-		0.76

Both Eqs. (5.9) and (5.11) include the material parameter ε , to allow for the influence of

material yield strength on the required equivalent imperfections in GNIA. However, since the influence of the material nonlinearity is directly captured by GMNIA, the equivalent imperfections for LTB design are no longer required to be a function of ε . This mirrors the provisions in Walport et al. (2020b) for flexural buckling design by GMNIA.

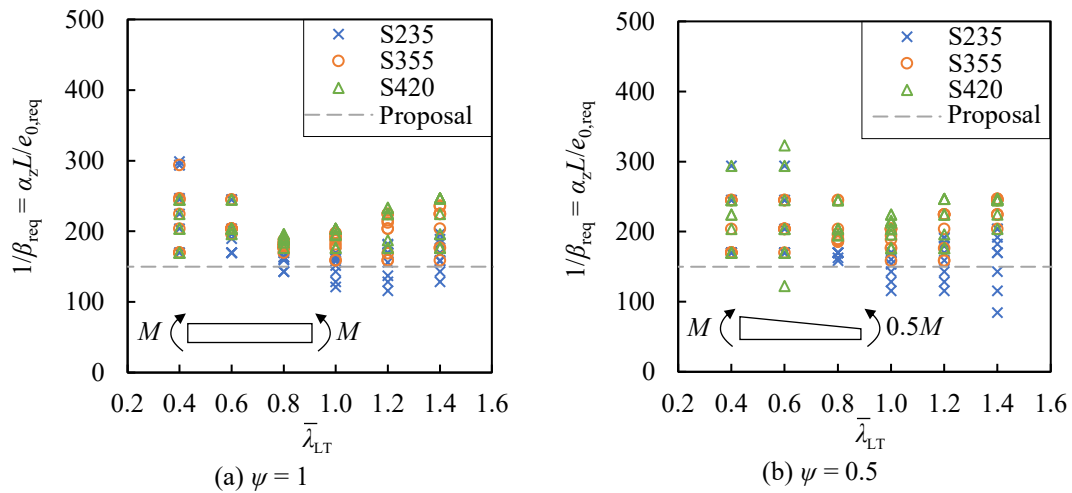
Hence, adopting the same format of the expressions for flexural buckling (Walport et al., 2020b), the equivalent imperfections proposed in this study are a function of the imperfection factor α_z and a reference relative imperfection β_{LT} , as given by Eq. (5.12), where α_z is determined from Table 5.3.

$$e_{0,LT} = \alpha_z \beta_{LT} L \quad (5.12)$$

Based on the back-calculated required equivalent imperfections $e_{0,req}$, the required values of the reference relative imperfection β_{req} can be determined as:

$$\beta_{req} = e_{0,req} / \alpha_z L \quad (5.13)$$

Figures 5.10 and 5.11 show the β_{req} values for hot-rolled steel and stainless steel members, respectively, for a range of cross-sections, member slenderness values and bending moment gradients. With the introduction of the imperfection factor α_z , the required values of $1/\beta_{req}$ are less scattered than the required values of $j_{req} = L/e_{0,req}$, as shown in Figures 5.7 and 5.8. This confirms the validity of including the imperfection factor α_z in the definition of $e_{0,LT}$.



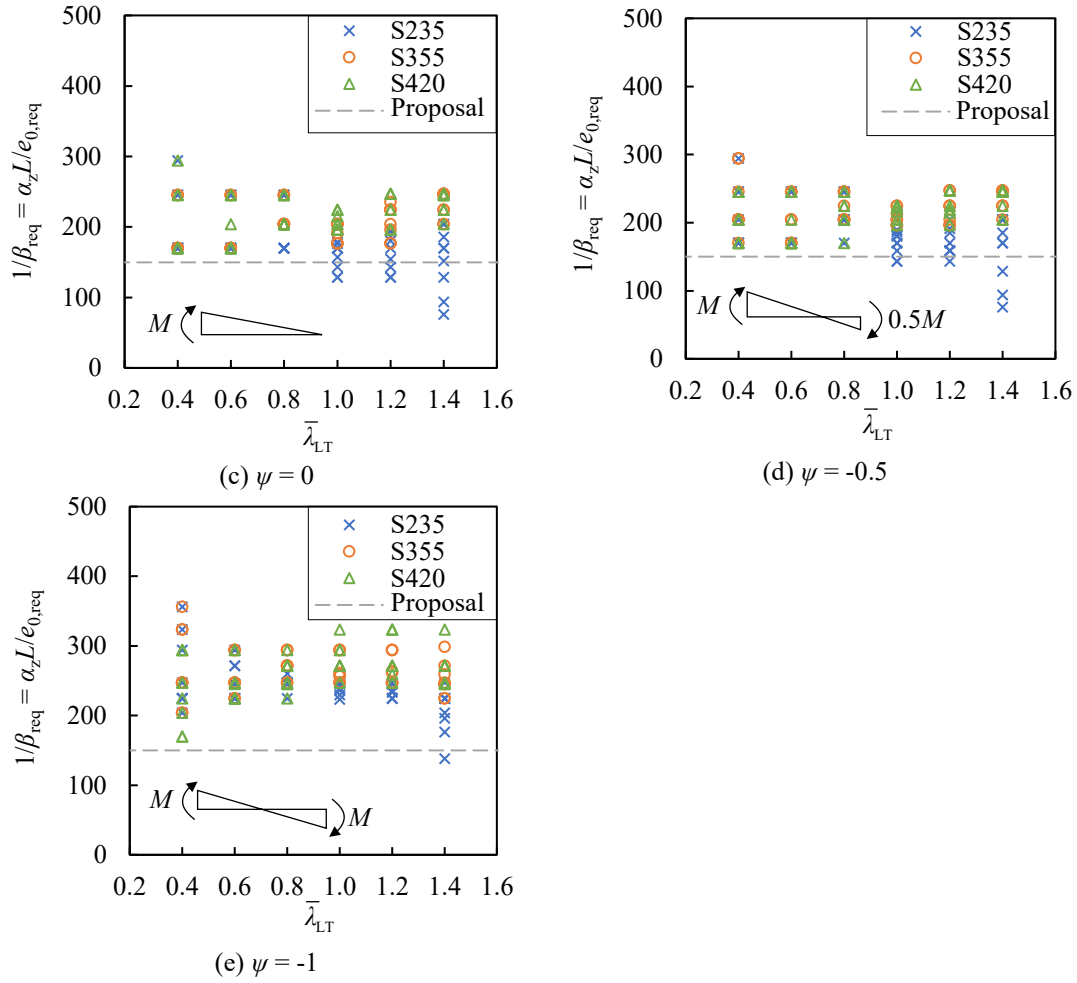
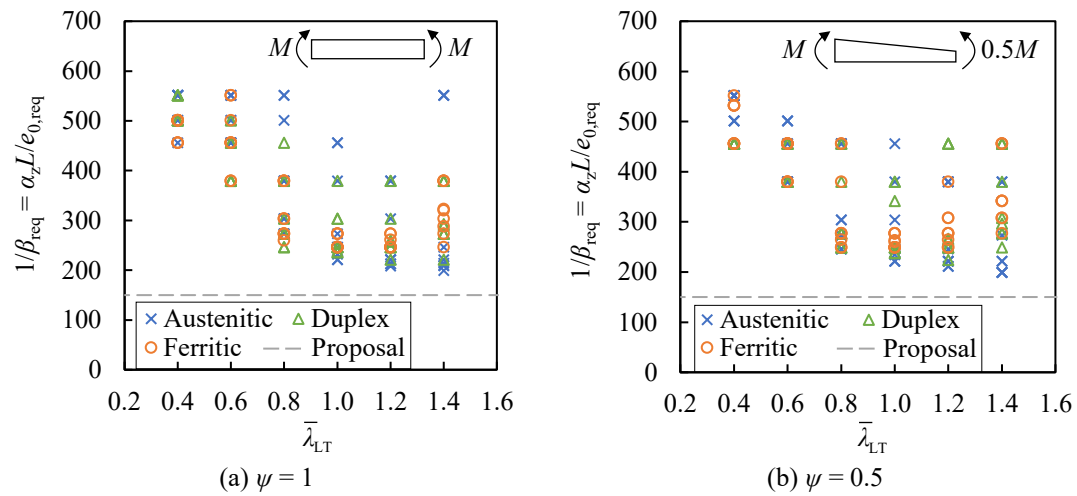


Figure 5.10 Required values of $1/\beta_{\text{req}}$ for hot-rolled steel members subjected to different bending moment distributions along the member length ($\psi = 1, 0.5, 0, -0.5$ and -1), achieved by changing the ratio of the applied end moments $\psi = M_2/M_1$



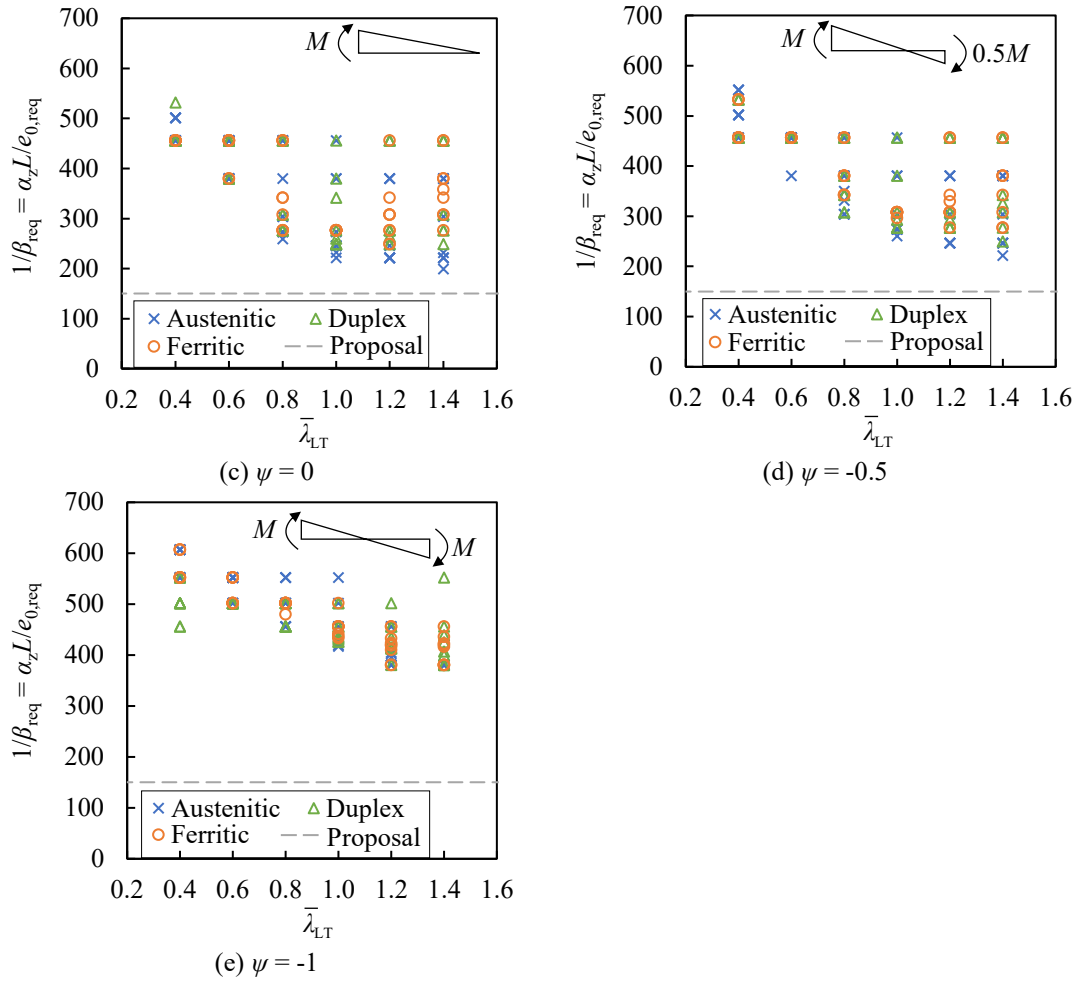


Figure 5.11 Required values of $1/\beta_{\text{req}}$ for stainless steel members subjected to different bending moment distributions along the member length ($\psi = 1, 0.5, 0, -0.5$ and -1), achieved by changing the ratio of the applied end moments $\psi = M_2/M_1$

5.5 Design recommendations and evaluation

In this section, recommendations are made for equivalent imperfections for use in the out-of-plane stability design of steel and stainless steel members by GMNIA. Two proposals for equivalent imperfection amplitudes are made for: (1) the scaling of the first LTB eigenmode, denoted $e_{0,\text{mod}}$ and (2) for direct modelling of lateral bow imperfection shapes, denoted $e_{0,\text{bow}}$. The ultimate strength predictions obtained through the use of the recommended equivalent imperfections are compared against both the benchmark FE results and Eurocode calculated resistances. The reliability of the recommendations is then assessed, and their application is demonstrated through worked examples. Note that in the present study, CSM strain limits are applied to capture cross-section failure, but cross-section checks can alternatively and conservatively be used.

5.5.1 Design recommendations

5.5.1.1 Mode imperfection $e_{0,\text{mod}}$

Based on the required imperfections determined in Section 5.4 and shown in Figures 5.10 and 5.11, the reference relative imperfection value of $\beta_{\text{LT}} = 1/150$ is proposed for use in LTB design by GMNIA of members subjected to different bending moment gradients, for both hot-rolled steel and stainless steel, and for all the member slenderness values and cross-sections examined herein, as given by Eq. (5.14).

$$e_{0,\text{mod}} = \alpha_z \beta_{\text{LT}} L = \alpha_z L / 150 \quad (5.14)$$

A single value of β_{LT} is chosen for all cases, independent of the member slenderness $\bar{\lambda}_{\text{LT}}$, in order to retain simplicity. A certain level of conservatism is therefore accepted for some geometries (Lindner et al., 2018; Walport et al., 2020b), as indicated in Section 5.4.

It can be seen in Figures 5.10 and 5.11 that the proposed recommendation of $\beta_{\text{LT}} = 1/150$ provides a safe-sided approximation to the majority of the required values of β_{req} . This value is also consistent with that provided in Walport et al. (2020b) for use in flexural buckling design by GMNIA; in both studies, the structural response and influence of varying geometric characteristics and material properties is directly captured in the GMNIA and the influence of different residual stresses is captured through the imperfection factor (α or α_z).

5.5.1.2 Lateral bow imperfection $e_{0,\text{bow}}$

The proposed equivalent imperfection detailed above is recommended for use with the first LTB eigenmode. However, in practical design situations, it is somewhat cumbersome to conduct a linear buckling analysis (LBA) prior to GMNIA to obtain the LTB eigenmode; furthermore, the appropriate isolation of members within a structure and the choice of eigenmode is not always straightforward. Therefore, an alternative design recommendation, whereby equivalent imperfections are applied through direct modelling as lateral bow imperfections of amplitude $e_{0,\text{bow}}$, is also made herein. As mentioned in Section 5.4, an imperfection in the shape of a half-sine wave is unsuitable for bending moment diagrams with a low proportion of bending moment at midspan (Stroetmann & Fominow, 2019). Hence, to cover the range of possible bending moment diagrams, a combination of two shapes of lateral imperfections is proposed in this study: (1) a half-sine wave, which is similar to the deformed

shape of members under uniform bending ($\psi = 1$), and (2) a full-sine wave, corresponding to the deformation profile of members under antisymmetric bending ($\psi = -1$). The amplitude of the full-sine wave imperfection is taken as 70% of that of the half-sine wave imperfection. Hence, a lateral equivalent bow imperfection $e_{0,\text{bow}}$ is defined as the summation of a half-sine wave with imperfection amplitude $e_{0,\text{bow},1}$, as determined from Eq. (5.15), and a full-sine wave with imperfection amplitude $e_{0,\text{bow},2}$, as determined from Eq. (5.16). This proposal is illustrated in Figure 5.12, and given in Table 5.4.

$$e_{0,\text{bow},1} = \alpha_z \beta_{\text{LT},1} L = \alpha_z L / 150 \quad (5.15)$$

$$e_{0,\text{bow},2} = \alpha_z \beta_{\text{LT},2} L = \alpha_z L / 215 \quad (5.16)$$

Note that the amplitude of the half-sine wave bow imperfection is consistent with that for flexural buckling (Walport et al., 2020b). Note also that the summation of these two imperfections is not symmetric, and thus should be applied in practice in the most unfavourable direction and form.

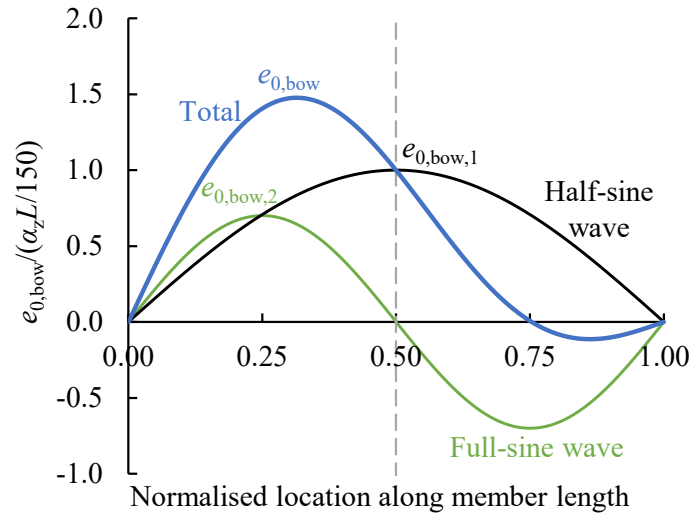


Figure 5.12 Direct modelling of lateral bow equivalent imperfection $e_{0,\text{bow}}$ through summation of a half-sine wave with amplitude $e_{0,\text{bow},1} = \alpha_z L / 150$ and a full-sine wave with amplitude $e_{0,\text{bow},2} = \alpha_z L / 215$

Table 5.4 Proposed equivalent geometric imperfections for out-of-plane stability design of steel and stainless steel members by GMNIA

Imperfection	Shape	β_{LT}	Illustration
$e_{0,mod}$	LTB eigenmode*	1/150	
$e_{0,bow}$	Lateral bow	1/150 (half-sine wave) + 1/215 (full-sine wave)	

*Eigenmode can be with or without twist with minimal effect on results

5.5.2 Evaluation of design recommendations

The proposed mode $e_{0,mod}$ and lateral bow $e_{0,bow}$ equivalent imperfections are assessed in this section for application with GMNIA. The assessment was carried out on 1134 steel members and 1134 stainless steel members, each considering three material grades, nine cross-sections, six member slenderness values, the seven load cases indicated in Section 5.3.1, as well as the load cases of uniformly distributed loading (UDL) and a concentrated central point load.

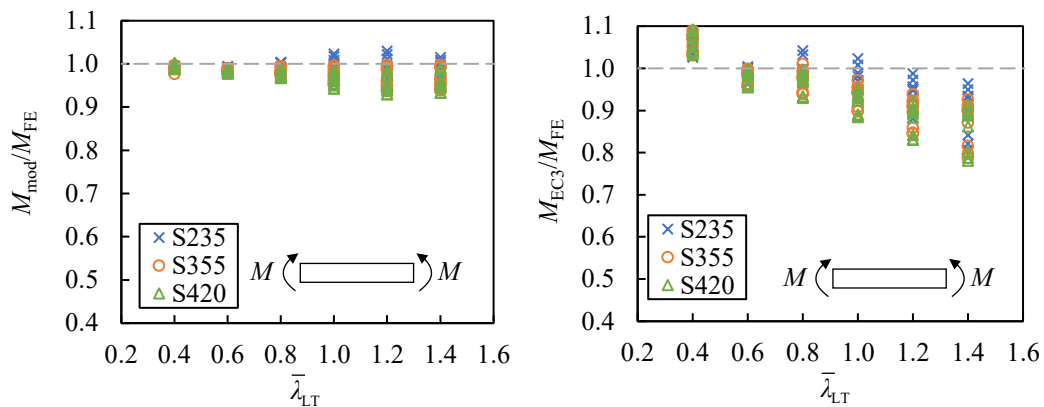
5.5.2.1 Mode imperfection $e_{0,mod}$

Figure 5.13 shows comparisons of the ultimate resistance predictions obtained from (1) the proposed design approach M_{mod} using GMNIA with the eigenmode shape of equivalent imperfection and $e_{0,mod}$, (2) the prEN 1993-1-1 (2019) LTB curves M_{EC3} and (3) the benchmark FE models (GMNIA with eigenmode imperfections of amplitude $L/1000$ plus residual stresses) M_{FE} , for the examined hot-rolled steel members subjected to different loading conditions. The mean values of the member resistance predictions obtained using the different design approaches relative to the benchmark FE results are summarised in Table 5.5. It can be seen that the Eurocode predictions are overly conservative for slender members (e.g. $\bar{\lambda}_{LT} \geq 1.2$); this is because the “post-critical” load-carrying mechanism in slender members (see Section 5.2.1.2) was not allowed for in the derivation of the buckling curves provided in prEN 1993-1-1 (2019). For some stocky members ($\bar{\lambda}_{LT} = 0.4$), the Eurocode predictions lie slightly on the unsafe side. The resistance predictions obtained from design by GMNIA using the proposed equivalent imperfections are more accurate and less scattered than those obtained using the Eurocode 3

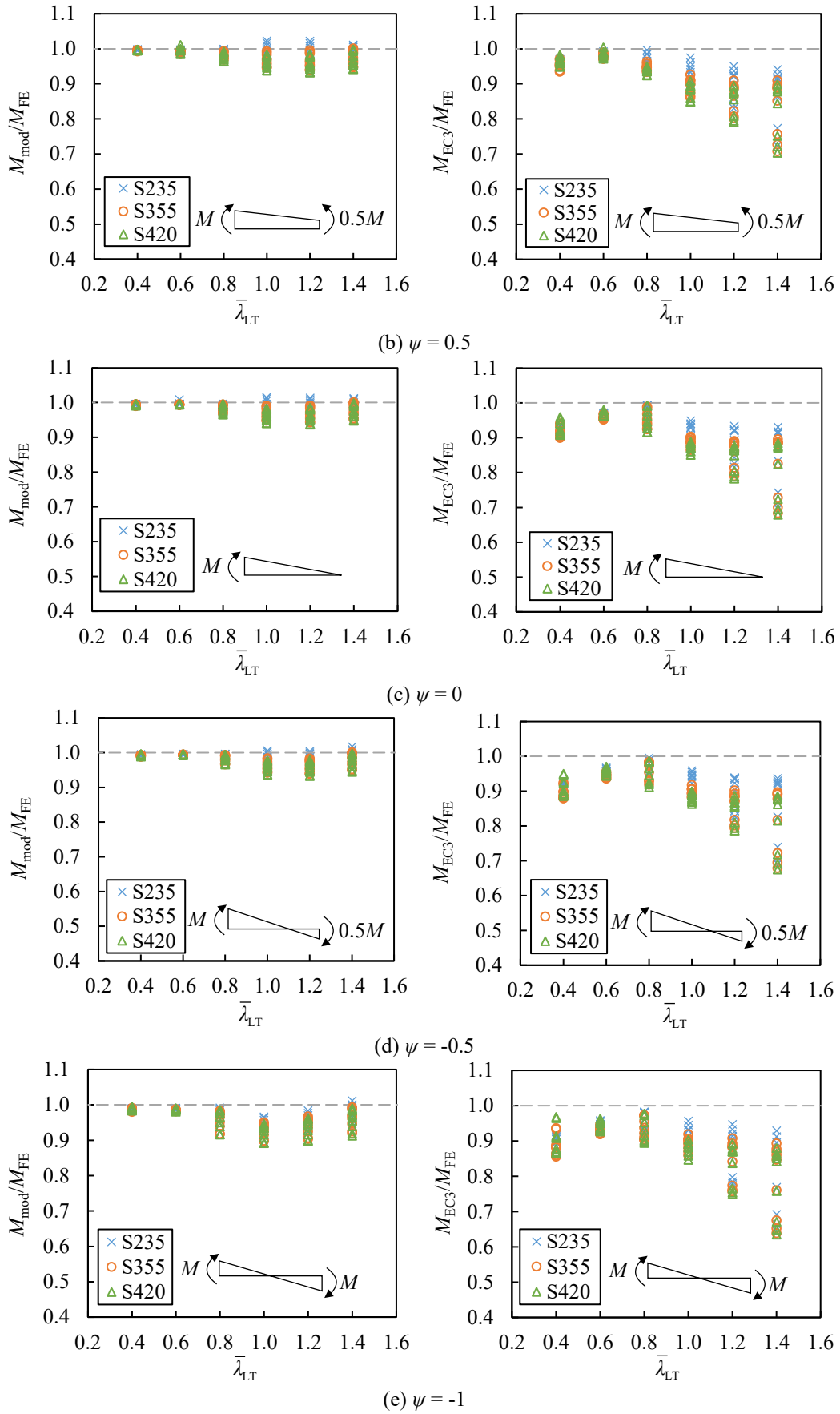
buckling curves. The large majority of the predictions obtained using the proposed equivalent imperfections are safe-sided (i.e. $M_{\text{mod}}/M_{\text{FE}} \leq 1$). For stainless steel members, similar conclusions can also be made, as shown in Figure 5.14. Compared with the design method in prEN 1993-1-4 (2020), design by GMNIA employing the proposed equivalent mode imperfections $e_{0,\text{mod}}$ provides more accurate member resistance predictions while remaining safe-sided. Note that initial twist was not introduced into the beam FE models in this study, as explained in Section 5.3.3.

Table 5.5 Summary of mean values of member resistance predictions obtained using design methods with LTB eigenmode equivalent imperfections M_{mod} , lateral bow equivalent imperfections M_{bow} and prEN 1993-1-1 LTB curves M_{EC3} , normalised by benchmark FE results M_{FE} , for all considered cases

Load case	No.	Hot-rolled steel			Stainless steel		
		$M_{\text{mod}}/M_{\text{FE}}$	$M_{\text{bow}}/M_{\text{FE}}$	$M_{\text{EC3}}/M_{\text{FE}}$	$M_{\text{mod}}/M_{\text{FE}}$	$M_{\text{bow}}/M_{\text{FE}}$	$M_{\text{EC3}}/M_{\text{FE}}$
$\psi = 1$	162	0.980	0.968	0.957	0.952	0.943	0.890
$\psi = 0.5$	162	0.982	0.938	0.918	0.954	0.905	0.841
$\psi = 0$	162	0.984	0.932	0.906	0.957	0.888	0.828
$\psi = -0.5$	162	0.982	0.938	0.904	0.951	0.891	0.830
$\psi = -1$	162	0.962	0.989	0.886	0.902	0.940	0.798
UDL	162	0.973	0.971	0.958	0.947	0.944	0.877
Point load	162	0.970	0.975	0.947	0.943	0.946	0.862



(a) $\psi = 1$



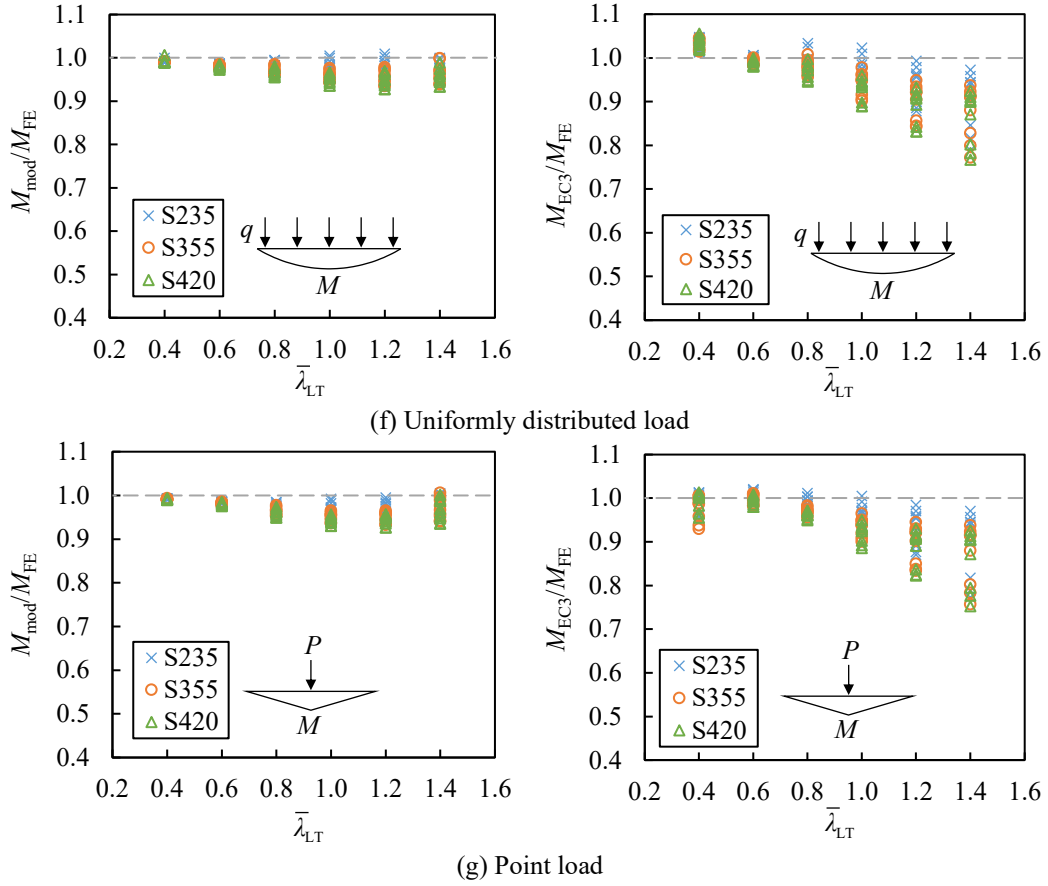
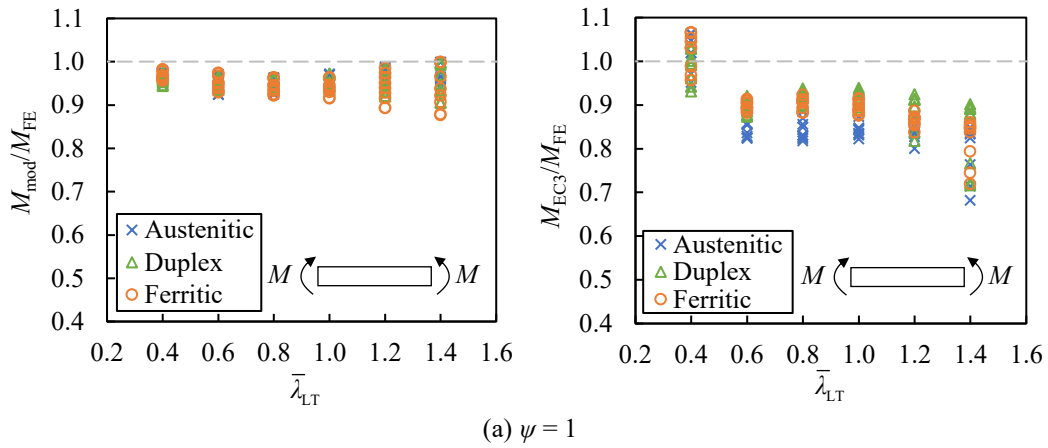
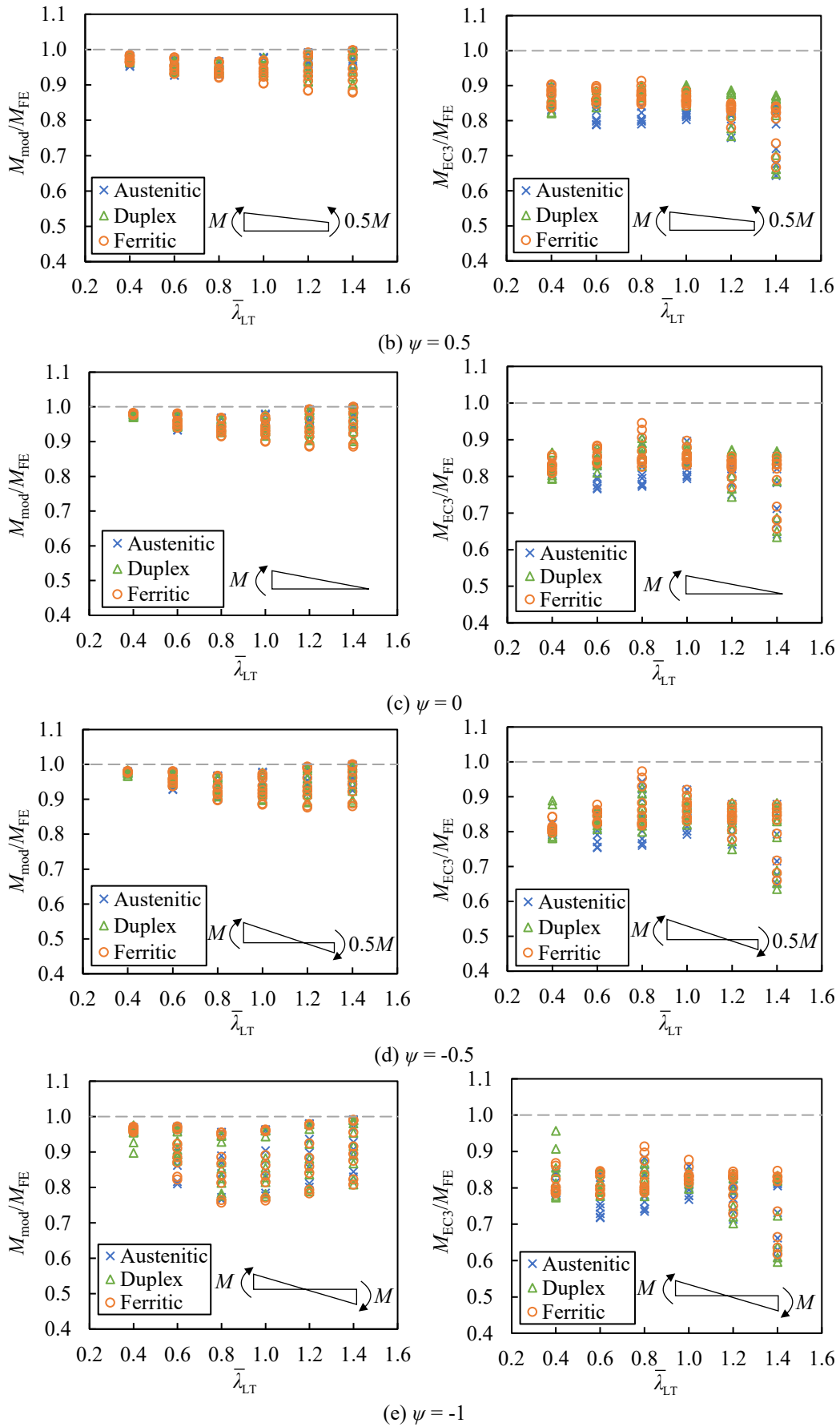


Figure 5.13 Comparisons of the ultimate resistance predictions obtained using the proposed equivalent imperfections based on the first LTB eigenmode M_{mod} and using the prEN 1993-1-1 LTB curves M_{EC3} , with the benchmark results M_{FE} , for hot-rolled steel members subjected to different loading conditions





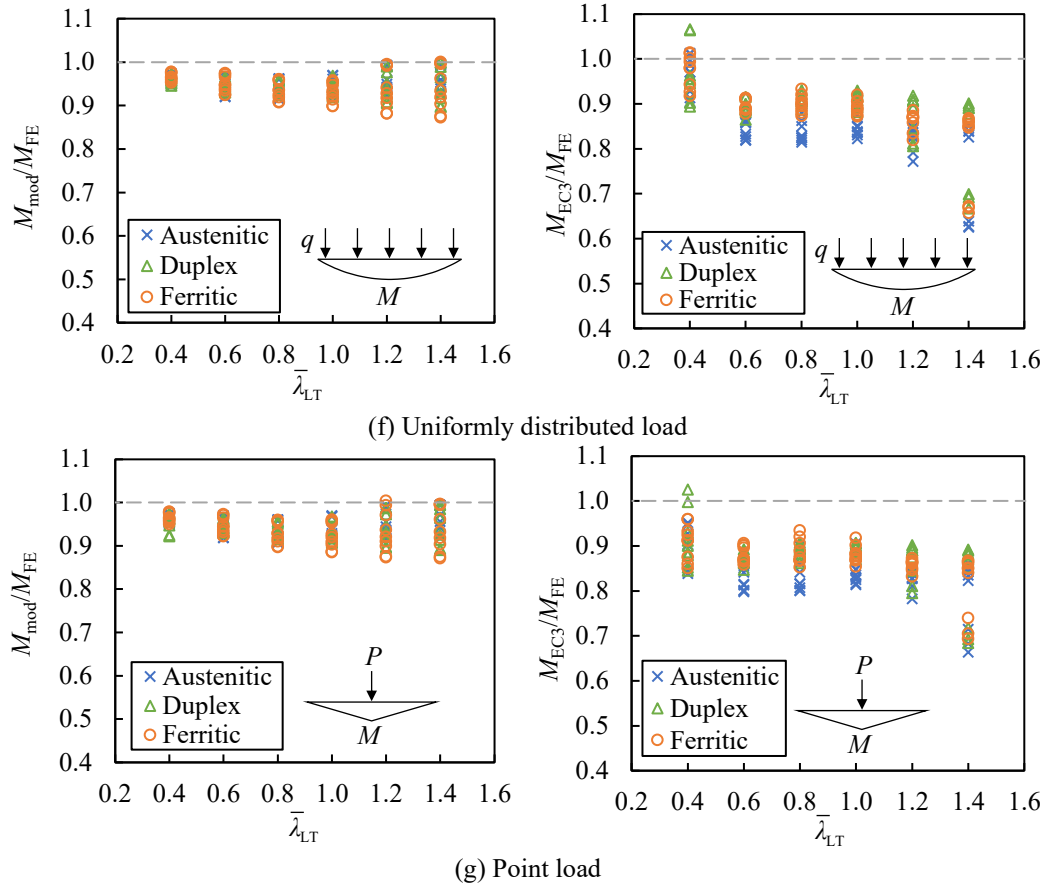


Figure 5.14 Comparisons of the ultimate resistance predictions obtained using the proposed equivalent imperfections based on the first LTB eigenmode M_{mod} and using the prEN 1993-1-4 LTB curves M_{EC3} , with the benchmark results M_{FE} , for stainless steel members subjected to different loading conditions

5.5.2.2 Lateral bow imperfection $e_{0,bow}$

The proposal for the lateral bow equivalent imperfection $e_{0,bow}$ was assessed in the same manner as above. Figures 5.15 and 5.16 show comparisons of the ultimate resistance predictions obtained from (1) GMNIA with the proposed lateral bow equivalent imperfection M_{bow} , (2) the prEN 1993-1-1 LTB curves M_{EC3} and the benchmark FE models M_{FE} , for hot-rolled steel and stainless steel members, respectively, subjected to different loading conditions. Comparing the results presented in Figures 5.13-5.16 and Table 5.5, it can be seen that, in general, for both steel and stainless steel members, the mode equivalent imperfection provides the most accurate resistance predictions. The lateral bow imperfection provides more conservative resistance predictions in the cases where $\psi = 0$ and -0.5 ; in these cases, the shape of the applied combined lateral bow imperfections is similar to the LTB eigenmodes but the bow equivalent imperfection amplitude $e_{0,bow}$ is more severe than $e_{0,mod}$. Nonetheless, employing the lateral bow imperfection in GMNIA provides very good results, with more accurate predictions than the Eurocode design

calculations, and with the benefit of not requiring an LBA for obtaining the LTB eigenmode.

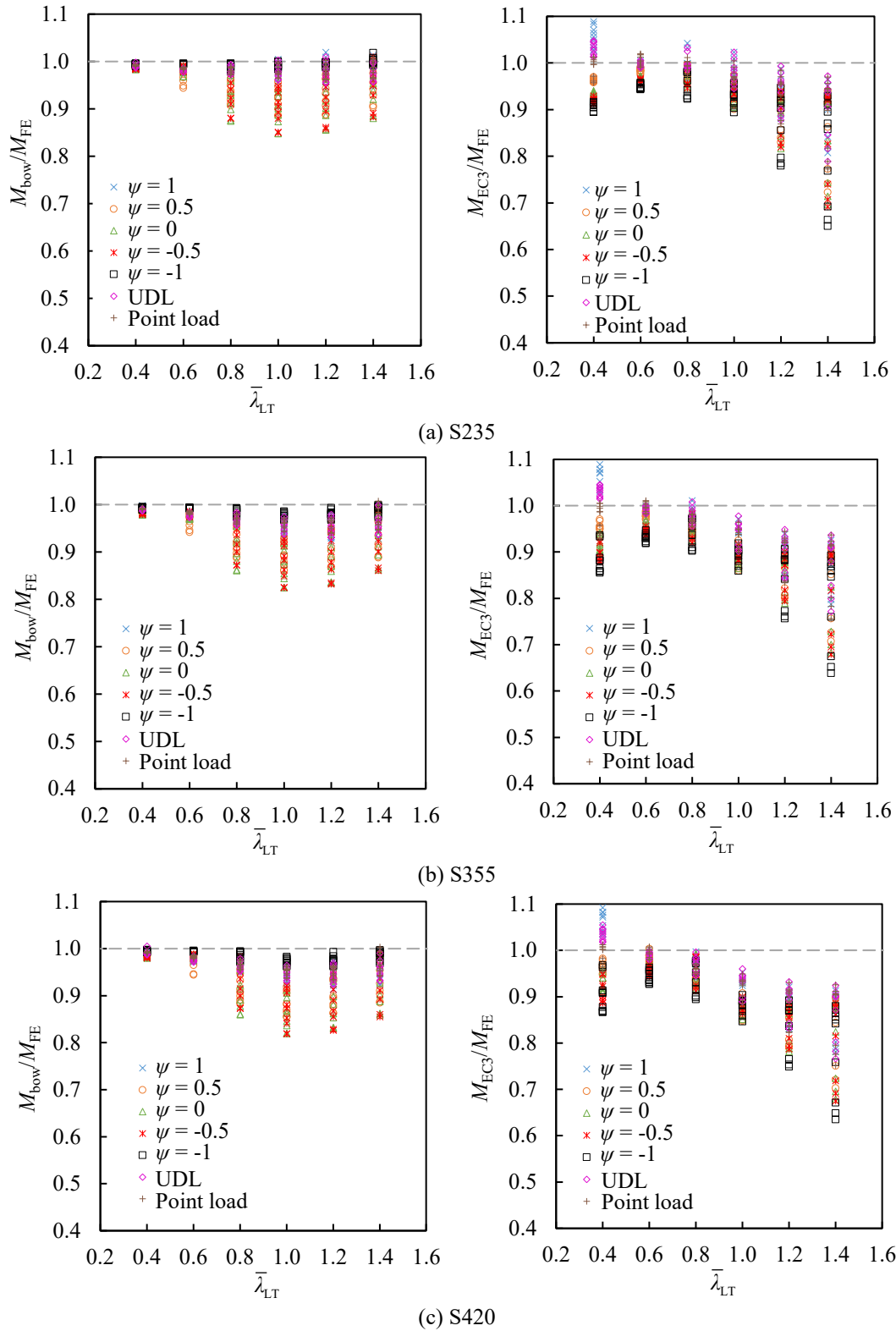


Figure 5.15 Comparisons of the ultimate resistance predictions obtained using the proposed equivalent imperfections based on the combined half and full sine wave lateral bow shapes M_{bow} and using the prEN 1993-1-1 LTB curves M_{EC3} , with the benchmark results M_{FE} , for hot-rolled steel members subjected to different loading conditions

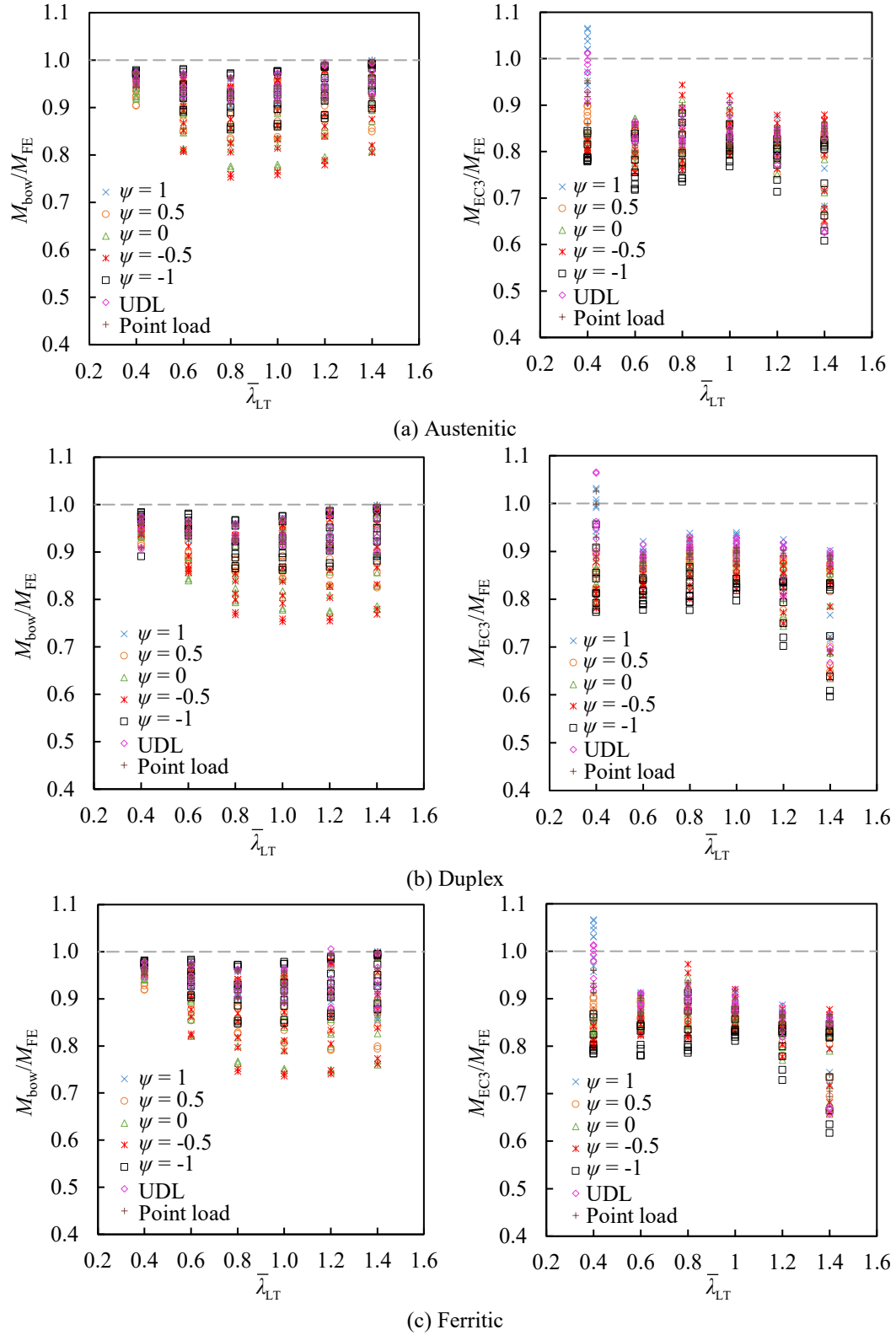


Figure 5.16 Comparisons of the ultimate resistance predictions obtained using the proposed equivalent imperfections based on the combined half and full sine wave lateral bow shapes M_{bow} and using the prEN 1993-1-4 LTB curves M_{EC3} , with the benchmark results M_{FE} , for stainless steel members subjected to different loading conditions

5.5.3 Reliability analysis

The reliability and required partial safety factor γ_{M1}^* for the proposed equivalent imperfections for use in design by GMNIA were evaluated through the first order reliability method (FORM) set out in EN 1990 (2002) and further explained by Afshan et al. (2015). According to EN 1990 (2002), typical buildings fall into reliability class RC2 with a 50-year design life and a target reliability index of 3.8 for ultimate limit state design. The derivation of partial safety factors are based on this value. The recommended values of the partial safety factors γ_{M1} are 1.0 for steel in prEN 1993-1-1 (2019) and 1.1 for stainless steel in prEN 1993-1-4 (2020).

The ultimate bending moment capacities of members experiencing LTB are dependent on the basic variables – yield stress f_y , major axis plastic section modulus $W_{pl,y}$ and Young's modulus E . The influence of these variables change with the member proportions for each numerical simulation. The dependency of the member resistance on the basic variables f_y , $W_{pl,y}$ and E can be derived for each numerical simulation following the method set out in Afshan et al. (2015), presented as the exponents c , d and e to be applied to the basic variables (i.e. f_y^c , $W_{pl,y}^d$, E^e), as given by Eqs. (5.17), (5.18) and (5.19), respectively, where $M_{1.05f_y}$, $M_{1.05W_{pl}}$ and $M_{1.05E}$ are the ultimate bending moment capacities obtained from the numerical analysis with the yield stress multiplied by 1.05, with the major axis plastic section modulus multiplied by 1.05 and with the Young's modulus multiplied by 1.05, respectively, and M_{f_y} , $M_{W_{pl}}$ and M_E are the original ultimate bending moment capacities.

$$c = \frac{\ln(M_{1.05f_y} / M_{f_y})}{\ln(1.05f_y / f_y)} \quad (5.17)$$

$$d = \frac{\ln(M_{1.05W_{pl}} / M_{W_{pl}})}{\ln(1.05W_{pl,y} / W_{pl,y})} \quad (5.18)$$

$$e = \frac{\ln(M_{1.05E} / M_E)}{\ln(1.05E / E)} \quad (5.19)$$

Figure 5.17 shows values of the exponents c , d and e obtained for members subjected to uniform bending. Figure 5.17 (a) shows the values obtained using the proposed mode equivalent imperfections $e_{0,mod}$ for the considered hot-rolled steel members, while Figure 5.17 (b) shows the values obtained using the proposed lateral bow equivalent imperfections $e_{0,bow}$ for the considered stainless steel members. In general, with increasing member slenderness $\bar{\lambda}_{LT}$, the

dependency on the yield stress f_y (represented by exponent c) decreases, while the dependency on the major axis plastic section modulus $W_{pl,y}$ (represented by exponent d) and the Young's modulus (represented by exponent e) increases. This reflects the transition from in-plane bending, dominated by plasticity to elastic buckling, controlled by Young's modulus, with increasing slenderness, together with a general dependency on the section geometry. However, it can be seen from Figure 5.17 that there are a few outliers (highlighted in red dashed circles) not consistent with the above statements. These are due to the “post-critical” load-carrying mechanism mentioned in Section 5.2.1.2: the ultimate resistance of slender members can exceed the elastic critical buckling moment M_{cr} and approach the minor axis bending resistance. Hence, for these slender members, the dependencies on the yield stress and geometric properties (presented by exponents c and d) approach unity, while the dependency on the Young's modulus (presented by exponent e) reduces.

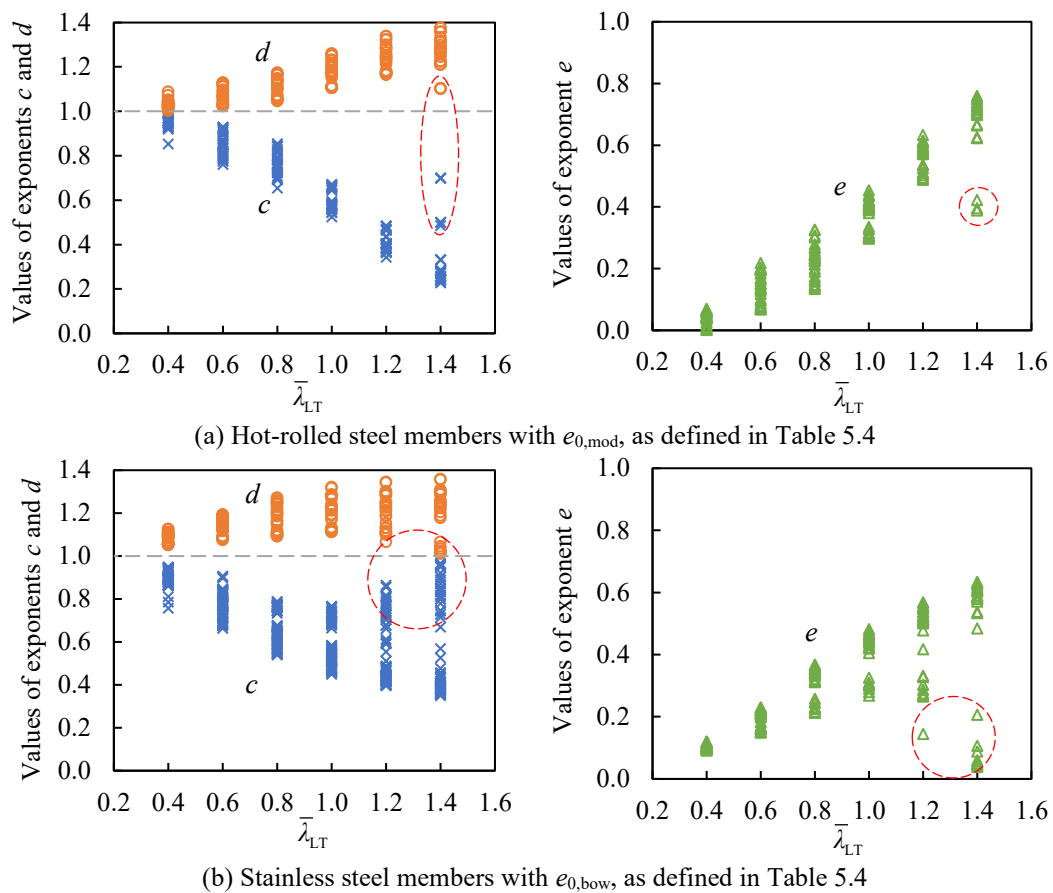


Figure 5.17 Values of exponents c , d and e reflecting the dependency of the ultimate member resistances on the yield stress f_y , plastic major axis section modulus $W_{pl,y}$ and Young's modulus E , respectively, obtained through the design method (a) using the proposed imperfections $e_{0,mod}$ defined using the LTB eigenmode for the considered hot-rolled steel members and (b) using the proposed lateral imperfections $e_{0,bow}$ for the considered stainless steel members, subjected to uniform bending.

The adopted values of material overstrength $f_{y,\text{mean}}/f_{y,\text{nom}}$ (i.e. the ratio of mean to nominal yield strength) and the CoV of yield strength V_{fy} were specified according to prEN 1993-1-1 (2019) for steel and reference Baddoo et al. (2020) for stainless steel; the CoV of geometric properties V_{Wpl} was determined according to Afshan et al. (2015); the CoV of Young's modulus V_E was taken as 0.03 for both materials (prEN 1993-1-1, 2019). The values assumed herein are listed in Tables 5.6 and 5.7. Details of the assumed statistical distributions of the input parameters in the EN 1990 (2002) reliability analysis may be found in SAFEBRICKTILE (2016). The combined coefficient of variation V_{rt} of the material and geometric basic variables is given by Eq. (5.20) (Meng & Gardner, 2020a).

$$V_{rt} = \sqrt{(cV_{fy})^2 + (dV_{Wpl})^2 + (eV_E)^2} \quad (5.20)$$

Table 5.6 Summary of the reliability analysis results for the proposed approach applied to hot-rolled steel members subjected to different loading conditions assessed against benchmark FE results

Load case	Grade	No.	$f_{y,\text{mean}}/f_{y,\text{nom}}$	V_{fy}	V_{Wpl}	V_E	Mode imperfection			Lateral imperfection		
							$e_{0,\text{mod}}$			$e_{0,\text{bow}}$		
							b	V_δ	γ_{MI}^*	b	V_δ	γ_{MI}^*
$\psi = 1$	S235	54	1.25	0.055	0.029	0.03	1.008	0.014	1.013	1.020	0.017	1.005
	S355	54	1.20	0.050	0.029	0.03	1.025	0.018	1.024	1.037	0.024	1.018
	S420	54	1.20	0.050	0.029	0.03	1.030	0.021	1.021	1.043	0.026	1.016
$\psi = 0.5$	S235	54	1.25	0.055	0.029	0.03	1.007	0.013	1.016	1.055	0.036	1.004
	S355	54	1.20	0.050	0.029	0.03	1.022	0.019	1.028	1.073	0.045	1.028
	S420	54	1.20	0.050	0.029	0.03	1.026	0.022	1.029	1.076	0.049	1.034
$\psi = 0$	S235	54	1.25	0.055	0.029	0.03	1.007	0.012	1.013	1.064	0.050	1.025
	S355	54	1.20	0.050	0.029	0.03	1.020	0.017	1.028	1.081	0.059	1.056
	S420	54	1.20	0.050	0.029	0.03	1.023	0.020	1.029	1.084	0.063	1.058
$\psi = -0.5$	S235	54	1.25	0.055	0.029	0.03	1.011	0.012	1.009	1.057	0.049	1.029
	S355	54	1.20	0.050	0.029	0.03	1.022	0.018	1.025	1.074	0.058	1.057
	S420	54	1.20	0.050	0.029	0.03	1.025	0.021	1.026	1.077	0.062	1.059
$\psi = -1$	S235	54	1.25	0.055	0.029	0.03	1.033	0.025	1.000	1.006	0.006	1.007
	S355	54	1.20	0.050	0.029	0.03	1.043	0.030	1.003	1.014	0.009	1.022
	S420	54	1.20	0.050	0.029	0.03	1.045	0.034	1.008	1.015	0.011	1.023
UDL	S235	54	1.25	0.055	0.029	0.03	1.016	0.012	1.008	1.018	0.013	1.007
	S355	54	1.20	0.050	0.029	0.03	1.031	0.018	1.020	1.033	0.019	1.019
	S420	54	1.20	0.050	0.029	0.03	1.037	0.020	1.017	1.039	0.021	1.017
Point load	S235	54	1.25	0.055	0.029	0.03	1.021	0.013	1.001	1.016	0.011	1.004
	S355	54	1.20	0.050	0.029	0.03	1.034	0.020	1.017	1.029	0.017	1.019
	S420	54	1.20	0.050	0.029	0.03	1.038	0.022	1.016	1.033	0.020	1.017

Table 5.7 Summary of the reliability analysis results for the proposed approach applied to stainless steel members subjected to different loading conditions assessed against benchmark FE results

Load case	Grade	No.	$\frac{f_{y,mean}}{f_{y,nom}}$	V_{fy}	V_{Wpl}	V_E	Mode imperfection			Lateral imperfection		
							$e_{0,mod}$			$e_{0,bow}$		
							b	V_δ	γ_{MI}^*	b	V_δ	γ_{MI}^*
$\psi = 1$	A	54	1.20	0.050	0.029	0.03	1.046	0.020	1.004	1.057	0.029	1.008
	D	54	1.10	0.030	0.029	0.03	1.051	0.022	1.040	1.060	0.027	1.042
	F	54	1.15	0.055	0.029	0.03	1.056	0.030	1.047	1.068	0.039	1.055
$\psi = 0.5$	A	54	1.20	0.050	0.029	0.03	1.042	0.020	1.011	1.101	0.053	1.024
	D	54	1.10	0.030	0.029	0.03	1.049	0.026	1.051	1.106	0.052	1.061
	F	54	1.15	0.055	0.029	0.03	1.055	0.033	1.056	1.118	0.067	1.080
$\psi = 0$	A	54	1.20	0.050	0.029	0.03	1.040	0.021	1.014	1.125	0.077	1.069
	D	54	1.10	0.030	0.029	0.03	1.047	0.028	1.056	1.134	0.079	1.118
	F	54	1.15	0.055	0.029	0.03	1.051	0.034	1.061	1.141	0.090	1.133
$\psi = -0.5$	A	54	1.20	0.050	0.029	0.03	1.047	0.028	1.016	1.123	0.083	1.090
	D	54	1.10	0.030	0.029	0.03	1.053	0.033	1.061	1.134	0.088	1.150
	F	54	1.15	0.055	0.029	0.03	1.056	0.038	1.064	1.137	0.095	1.151
$\psi = -1$	A	54	1.20	0.050	0.029	0.03	1.111	0.077	1.081	1.063	0.044	1.033
	D	54	1.10	0.030	0.029	0.03	1.117	0.076	1.124	1.066	0.044	1.074
	F	54	1.15	0.055	0.029	0.03	1.117	0.082	1.128	1.067	0.048	1.077
UDL	A	54	1.20	0.050	0.029	0.03	1.051	0.023	1.004	1.055	0.027	1.007
	D	54	1.10	0.030	0.029	0.03	1.057	0.026	1.041	1.061	0.028	1.043
	F	54	1.15	0.055	0.029	0.03	1.062	0.035	1.052	1.066	0.038	1.056
Point load	A	54	1.20	0.050	0.029	0.03	1.055	0.026	1.005	1.051	0.024	1.006
	D	54	1.10	0.030	0.029	0.03	1.063	0.029	1.042	1.060	0.027	1.043
	F	54	1.15	0.055	0.029	0.03	1.067	0.038	1.053	1.063	0.036	1.053

It should be noted that, as recommended in Walport et al. (2020a) and Walport et al. (2020b), in the implementation of the design method through second order inelastic analysis (i.e. GMNIA), the Young's modulus E should be taken as the characteristic value (i.e. the fifth percentile), rather than the mean value. Hence, in this study, the characteristic values of $E = 200000 \text{ N/mm}^2$ for steel and $E = 191000 \text{ N/mm}^2$ for stainless steel were adopted (prEN 1993-1-1, 2019; prEN 1993-1-14, 2019; prEN 1993-1-4, 2020; Walport et al., 2020b) in the performed GMNIA design calculations.

The mean correction factor b was calculated using a modified definition based on the average ratio of the benchmark resistance r_e to the predicted resistance r_t , as given by Eq. (5.21), instead of the least-squares method recommended in EN 1990 (2002). This prevents the derived value

of b from being biased toward the data points with higher resistance values (Meng et al., 2020; Meng & Gardner, 2020a).

$$b = \frac{1}{n} \sum_{i=1}^n \frac{r_{e,i}}{r_{t,i}} \quad (5.21)$$

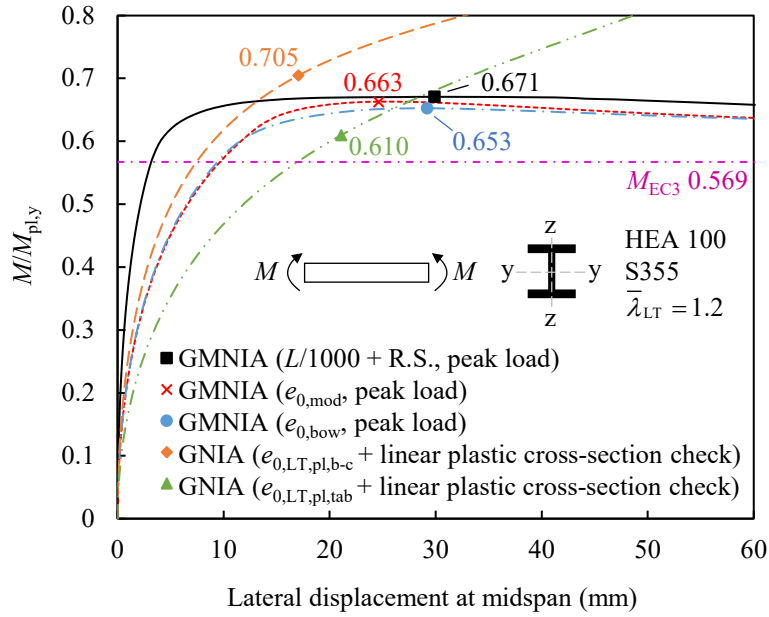
The key reliability analysis results for all considered cases are reported in Tables 5.6 and 5.7, including the mean correction factor b , the coefficient of variation of the results obtained using the proposals relative to the benchmark FE results V_δ and the required value of partial safety factor γ_{M1}^* . All values of b are greater than 1, indicating the mean predictions obtained using two proposals are safe-sided; this is shown in Table 5.5. The required values of the partial safety factor γ_{M1}^* are generally lower than the target values of 1.0 for steel members (prEN 1993-1-1, 2019) and 1.1 for stainless steel members (prEN 1993-1-4, 2020), and are all in line with the recommendations given by SAFEBRICKTILE (2016; HOLLOSSTAB, 2019), which allows a small exceedance of the target values to allow for the influence of the combined coefficient of variation V_r incorporating the variability of the model and the basic variables. Note that if the values of $f_{y,mean}/f_{y,nom}$ and V_{fy} are taken as 1.30 and 0.060 instead of 1.20 and 0.050 for austenitic stainless steel and taken as 1.20 and 0.045 instead of 1.15 and 0.055 for ferritic stainless steel (as assumed in the reliability analyses performed in Afshan et al. (2015)), the calculated values of γ_{M1}^* still satisfy the SAFEBRICKTILE (2016; HOLLOSSTAB, 2019) requirements. Therefore, the two proposals for equivalent imperfections are deemed to be suitable for use in the out-of-plane stability design of steel and stainless steel members by GMNIA, in conjunction with the recommended values of the partial safety factor γ_{M1} i.e. 1.0 for steel and 1.1 for stainless steel.

5.5.4 Worked examples

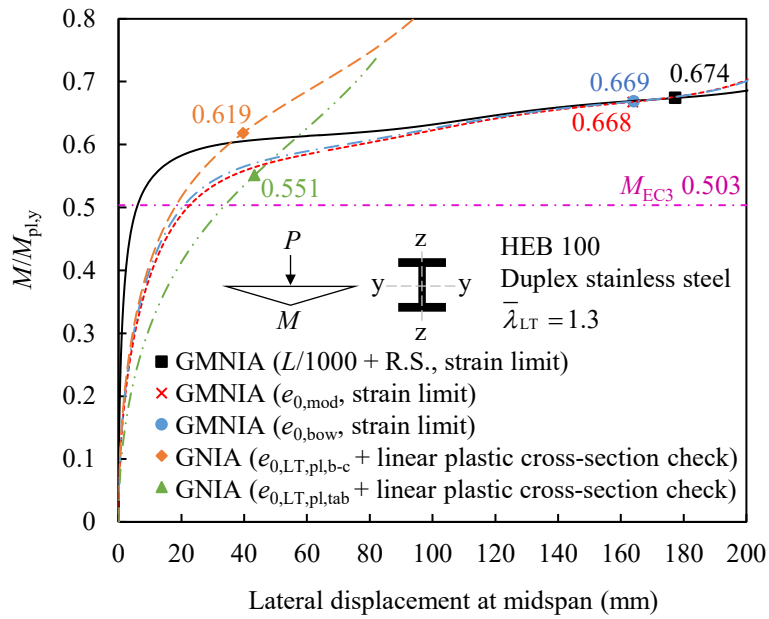
Two worked examples are presented in this section to illustrate the implementation of the proposed equivalent imperfections in design by second order inelastic analysis.

The first worked example considers a beam made of grade S355 steel ($f_y = 355$ MPa, $f_u = 510$ MPa, characteristic value of $E = 200000$ N/mm² and $\nu = 0.3$) with a hot-rolled HEA 100 cross-section and a length of $L = 4685$ mm, which corresponds to a relative LTB member slenderness $\bar{\lambda}_{LT} = 1.2$; the considered beam is subjected to uniform major axis bending moment M , as shown in Figure 5.18 (a).

The second worked example considers a beam made of duplex stainless steel ($f_y = 530$ MPa, $f_u = 770$ MPa, $n = 9.3$, $m = 3.6$, reduced value of $E = 191,000$ N/mm² and $\nu = 0.3$) with a cross-section equivalent to an HEB 100 and a length of $L = 5690$ mm, which corresponds to a relative LTB member slenderness $\bar{\lambda}_{LT} = 1.3$; the considered beam is subjected to a concentrated central point load P , generating an internal major axis bending moment M at midspan, as shown in Figure 5.18 (b).



(a) Worked example 1: HEA 100, S355, $\bar{\lambda}_{LT} = 1.2$



(b) Worked example 2: HEA 100, duplex stainless steel, $\bar{\lambda}_{LT} = 1.3$

Figure 5.18 Load-deformation paths and ultimate member resistance predictions for worked examples obtained from different design approaches

Figure 5.18 (a) shows the normalised applied bending moment for worked example 1 versus lateral displacement paths of the considered beams, as well as the resulting ultimate load predictions obtained from: (1) GMNIA with $e_0 = L/1000 + \text{residual stress (R.S.)}$, i.e. the benchmark FE result, (2) GMNIA with $e_0 = e_{0,\text{mod}}$, (3) GMNIA with $e_0 = e_{0,\text{bow}}$, (4) GNIA with $e_0 = e_{0,\text{LT,pl,b-c}}$, (5) GNIA with $e_0 = e_{0,\text{LT,pl,tab}}$ and (6) the member buckling check given in prEN 1993-1-1 (2019) (see Section 2.2.3.2) M_{EC3} . The corresponding graph and ultimate load predictions for worked example 2, with the normalised internal major axis bending moment at midspan (i.e. $M/M_{\text{pl,y}}$) plotted on the vertical axis, is shown in Figure 5.18 (b). A summary of the results is given in Table 5.8.

Table 5.8 Summary of comparisons between ultimate member resistances obtained from different design methods for worked examples

Imperfection e_0	Analysis	Cross-section check	Example 1		Example 2	
			$M_u/M_{\text{pl,y}}$	M_u/M_{FE}	$M_u/M_{\text{pl,y}}$	M_u/M_{FE}
Benchmark $L/1000 + \text{R.S.}$	GMNIA	CSM strain limit	0.671	-	0.674	-
$e_{0,\text{mod}}$	GMNIA	CSM strain limit	0.663	0.988	0.668	0.991
$e_{0,\text{bow}}$	GMNIA	CSM strain limit	0.653	0.973	0.669	0.993
$e_{0,\text{LT,pl,b-c}}$	GNIA	Linear plastic check	0.705	1.051	0.619	0.918
$e_{0,\text{LT,pl,tab}}$	GNIA	Linear plastic check	0.610	0.909	0.551	0.818
Member buckling check M_{EC3}	-	-	0.569	0.848	0.503	0.746

Considering the first worked example, the prediction obtained from GNIA using the back-calculated equivalent imperfection $e_{0,\text{LT,pl,b-c}}$ with a linear plastic cross-section check is located on the unsafe side of the benchmark result and overpredicts the resistance by 5%. The prediction obtained from GNIA using the tabulated equivalent imperfection $e_{0,\text{LT,pl,tab}}$ with a linear plastic cross-section check is safe-sided but overly conservative compared with the benchmark FE result M_{FE} , with $M_{e_{0,\text{LT,pl,tab}}} = 0.909M_{\text{FE}}$. Use of the proposed equivalent imperfections in GMNIA provides close and safe-sided predictions compared with the benchmark FE result M_{FE} with $M_{\text{mod}}/M_{\text{FE}} = 0.988$ and $M_{\text{bow}}/M_{\text{FE}} = 0.973$. Use of the proposed equivalent imperfections in GMNIA provides an increase in capacity of about 14%, compared to the Eurocode design prediction M_{EC3} for the case considered in this worked example. The failure of this member in GMNIA was stability governed (i.e. due to the attainment of the peak load prior to the CSM strain limit being reached).

Similar observations also can be made for the second worked example. Again, use of the

proposed equivalent imperfections in GMNIA provides close and safe-sided predictions compared with the benchmark FE result M_{FE} , with $M_{mod}/M_{FE} = 0.991$ and $M_{bow}/M_{FE} = 0.993$, corresponding to an increase in capacity of about 25%, 7% and 17% compared to the Eurocode design prediction M_{EC3} , the predictions obtained from GNIA using the back-calculated equivalent imperfections $e_{0,LT,pl,b-c}$ and the predictions obtained from GNIA using the tabulated equivalent imperfections $e_{0,LT,pl,tab}$, respectively. In this worked example, GMNIA failure was governed by the CSM strain limit. Note that in both worked examples, the member resistances obtained from GNIA using the back-calculated equivalent imperfection $e_{0,LT,pl,b-c}$ are higher than the buckling resistances M_{EC3} , since the “post-critical” load-carrying mechanism in slender members (see Section 5.2.1.2) was not allowed for in the development of the LTB curves provided in prEN 1993-1-1 (2019).

5.6 Concluding remarks

Equivalent imperfections consider the combined influence of initial geometric imperfections and residual stresses. Current design specifications include recommendations for equivalent imperfection amplitudes for use in design by GNIA. However, no appropriate provisions are given for equivalent imperfections for use in design by GMNIA. In this study, calibrated against the benchmark results generated from GMNIA with both geometric imperfections and residual stresses, two shapes and corresponding amplitudes of equivalent imperfections for use in the out-of-plane stability design of steel and stainless steel members by GMNIA have been developed: (1) the first LTB eigenmode with an amplitude $e_{0,mod} = \alpha_z L/150$ and (2) a lateral bow imperfection $e_{0,bow}$ combining of a half-sine wave with an amplitude $e_{0,bow,1} = \alpha_z L/150$ and a full-sine wave with an amplitude $e_{0,bow,2} = \alpha_z L/215$. The minor axis flexural buckling imperfection factor α_z , which is generally equal to the upper bound value on α_{LT} in prEN 1993-1-1 (2019) and prEN 1993-1-4 (2020), is utilised in the proposed imperfection amplitudes to capture the adverse influence of residual stresses, but without inheriting the additional compensatory effects included in α_{LT} for calibration of the LTB curves. In the design by GMNIA using the developed equivalent imperfections, cross-section checks or CSM strain limits can be used to capture cross-section failure; the latter more accurate approach is adopted in this study.

The accuracy and suitability of the proposed equivalent imperfections were assessed against

benchmark FE results for 1134 steel members and 1134 stainless steel members, each considering three material grades, nine cross-sections, six member slenderness values and seven load cases. The resistance predictions obtained using GMNIA with the proposed equivalent imperfections (both $e_{0,\text{mod}}$ and $e_{0,\text{bow}}$) were shown to be accurate, safe-sided and less scattered than those obtained from the Eurocode lateral-torsional buckling curves.

The suitability of using the existing recommended values of partial safety factors (i.e. $\gamma_{M1} = 1.0$ for steel members and 1.1 for stainless steel members) in conjunction with the proposed imperfections was demonstrated by reliability analysis. The proposals are due to be included in the new upcoming Eurocode for design by finite element analysis – prEN 1993-1-14 (2019).

Chapter 6

Out-of-plane stability design of steel beams

6.1 Introduction

Thus far, the method of design by second order inelastic analysis with CSM strain limits has been established and developed for the in-plane design of steel structures and structural components (Fieber et al., 2019a; Fieber et al., 2020). In this chapter, its scope is extended to the scenarios in which out-of-plane stability effects govern, with a focus on steel I-section beams susceptible to lateral-torsional buckling (LTB). A wide range of cases is considered, including variation in cross-section properties, member slendernesses and bending moment diagram shapes. Application of the proposed design approach to structural steel members subjected to pure torsion, combined bending and torsion, and combined shear, bending and torsion is also illustrated. The design capacity predictions are assessed relative to the results of benchmark shell FE models, which are themselves validated against existing experiments in the literature.

6.2 Design of steel beams by GMNIA using beam elements with CSM strain limits

The proposed method of design by GMNIA with strain limits involves: (i) carrying out a GMNIA of the steel member using beam finite elements and (ii) using the load factor that corresponds to the attainment of the peak load α_{peak} or the CSM strain limit α_{csm} , whichever occurs first, to define the ultimate resistance. Development of the primary components of the proposed design method for the out-of-plane stability assessment of steel beams is explained in this section.

6.2.1 FE modelling of steel structures using beam finite elements

The proposed design method is implemented using beam finite elements. In this study, the finite element analysis software Abaqus (2017) was used to carry out the GMNIA simulations. The beam element referred to as B31OS in Abaqus (2017), which is capable of modelling the effects of torsion and warping in open-sections, was used. Thirty-three section integration points were employed along the widths of each web and flange plate to accurately capture the spread of plasticity through the cross-section. To enable the application of the strain averaging approach, which is used to take account of the beneficial influence of local strain gradients along the member lengths on cross-section resistance, as described in Section 4.2.1.2, the lengths of the beam elements were defined to be less than or equal to the corresponding local buckling half-wavelengths $L_{b,cs}$ of the cross-sections of the modelled beams. The quad-linear material model introduced in Section 2.3.2 was employed, with the engineering stress-strain curve converted in true stress and strain. Unless otherwise indicated, grade S355 steel has been used in all the cases considered in this study. Nominal values of material strengths and geometries, together with a characteristic value of the Young's modulus ($E = 200000$ MPa (prEN 1993-1-1, 2019)) were employed in the implementation of the proposed design approach, as recommended in Walport et al. (2020a) and Walport et al. (2020b). The combined influence of the geometric imperfections and residual stresses was accounted for through the use of equivalent geometric imperfections (prEN 1993-1-1, 2019), as is recommended for the practical implementation of the proposed design method. The equivalent imperfections were modelled through the scaling of the first LTB eigenmode with the amplitude proposed in Section 5.5.1.1 on the out-of-plane design of steel members by GMNIA. In this study, all the investigated steel beams had fork-end support conditions, allowing warping deformations but fully restraining twisting about the longitudinal axis of the members at the supports.

6.2.2 Procedure for application of CSM strain limits in GMNIA

In the proposed design method, after performing GMNIA with beam FE models, CSM strain limits are applied to account for the influence of local buckling on the response. To account for the beneficial effect of local strain gradients on the local stability of cross-sections, the strain averaging approach is also applied (see Section 4.2.1.2). The key aspects of CSM are illustrated in Section 2.3.

For steel I-section beams susceptible to lateral-torsional buckling, internal second order torsion arises upon the application of the major axis bending moments. The total internal torsion moment T_{Ed} at any given cross-section has two components: (i) the St. Venant torsion T_{St} with associated St. Venant torsion shear stresses τ_{St} and (ii) the warping torsion T_w with associated warping torsion normal stresses σ_w and warping torsion shear stresses τ_w , as shown in Figure 6.1. In the implementation of the proposed design method by the GMNIA of steel beams with beam finite elements, the normal strains ε_w resulting from the warping torsion normal stresses σ_w can be directly extracted and the total maximum compressive normal strains within cross-sections due to the combination of bending and warping can be checked against the CSM strain limits. However, the St. Venant torsion τ_{St} and the warping torsion shear stresses τ_w still need to be considered; the treatment of shear stresses arising from either primary or secondary torsion in the proposed design method is provided in Section 6.5.

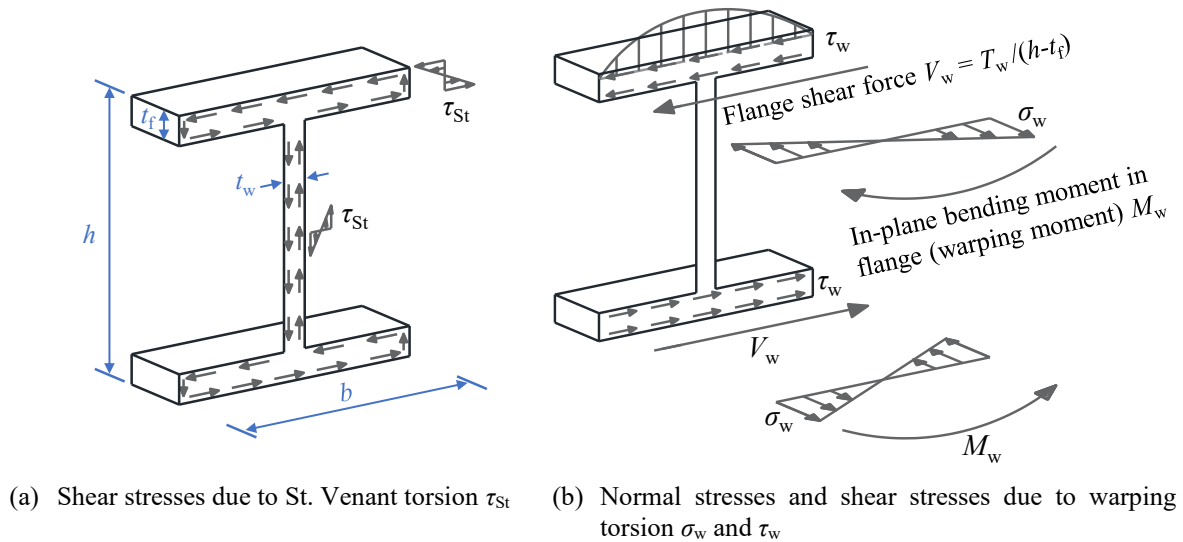


Figure 6.1 Shear stress and normal stress distributions due to (a) St. Venant torsion and (b) warping torsion

As shown in Figure 6.2, the procedure for the application of the proposed design approach to steel beams susceptible to LTB is as follows:

- I. Based on the first order internal force distribution, determine the elastic local buckling stresses of the cross-sections $\sigma_{cr,cs}$ at the middle of each element along the beam length (Li & Schafer, 2010b; Gardner et al., 2019a).
- II. Using Eq. (2.13), calculate the corresponding cross-section slendernesses $\bar{\lambda}_p$.

- III. Using the calculated $\bar{\lambda}_p$ values, determine the strain limits ε_{csm} for the cross-sections at the middle of each element using the CSM base curve given by Eqs. (2.9) and (2.10). For elements under high shear force and/or torsion, reduce the strain limits considering the shear effect from shear force and/or torsion (see Sections 6.4.2 and 6.5.1.3).
- IV. If the strain averaging approach is used, calculate the local buckling half-wavelengths of the cross-sections $L_{b,cs}$ on the basis of the first order internal force distribution, either using numerical methods (e.g. CUFSM (Li & Schafer, 2010b)) or the expressions provided in Fieber et al. (2019b) and determine the average maximum compressive normal strain for each element $\varepsilon_{\text{Ed,av,m}}$ as the average of the peak compressive strains $\varepsilon_{\text{Ed,m}}$ over the local buckling half-wavelengths $L_{b,cs}$, as shown in Figure 6.3. Note that in the calculation of $\varepsilon_{\text{Ed,av,m}}$, only the elements that lie fully within $L_{b,cs}$ should be considered.
- V. Determine (i) the load increment j (if any) at which the peak load factor is reached and (ii) the load increment p at which the average strain at any cross-section m_0 attains the CSM strain limit, i.e. $\varepsilon_{\text{Ed,av,m}_0,p} \geq \varepsilon_{\text{csm}}$ or $\varepsilon_{\text{Ed,av,m}_0,p} \geq \rho_{\text{csm},m_0} \varepsilon_{\text{csm}}$ or $\varepsilon_{\text{csm},V,m_0}$ for members under high shear and/or torsion (reduction is applied to the CSM strain limit ε_{csm} for the latter – see Sections 6.4.2 and 6.5.1.3). If j is less than p , the member is deemed to fail primarily due to global instability i.e. lateral-torsional buckling, and the characteristic resistance factor α_{Rk} is taken as the peak load factor from the GMNIA α_{peak} (i.e. $\alpha_{\text{Rk}} = \alpha_{\text{peak}}$); otherwise, the member is deemed to fail by attaining the cross-section capacity, and the load factor at which the CSM strain limit is attained α_{csm} is adopted as the characteristic resistance factor α_{Rk} (i.e. $\alpha_{\text{Rk}} = \alpha_{\text{csm}}$).
- VI. As recommended in prEN 1993-1-1 (2019), Walport et al. (2020a) and Walport et al. (2020b), in the implementation of the proposed design method, the Young's modulus should be taken as the characteristic value of $E = 200000$ MPa, and the resulting resistance should be assumed to be the characteristic resistance, to which a partial safety factor ($\gamma_{\text{M1}} = 1.0$) should be applied. Hence, finally, the design resistance factor of the member α_{Rd} is equal to the characteristic resistance factor α_{Rk} divided by the partial safety factor γ_{M1} , as given by Eq. (6.1).

$$\alpha_{\text{Rd}} = \alpha_{\text{Rk}} / \gamma_{\text{M1}} \quad (6.1)$$

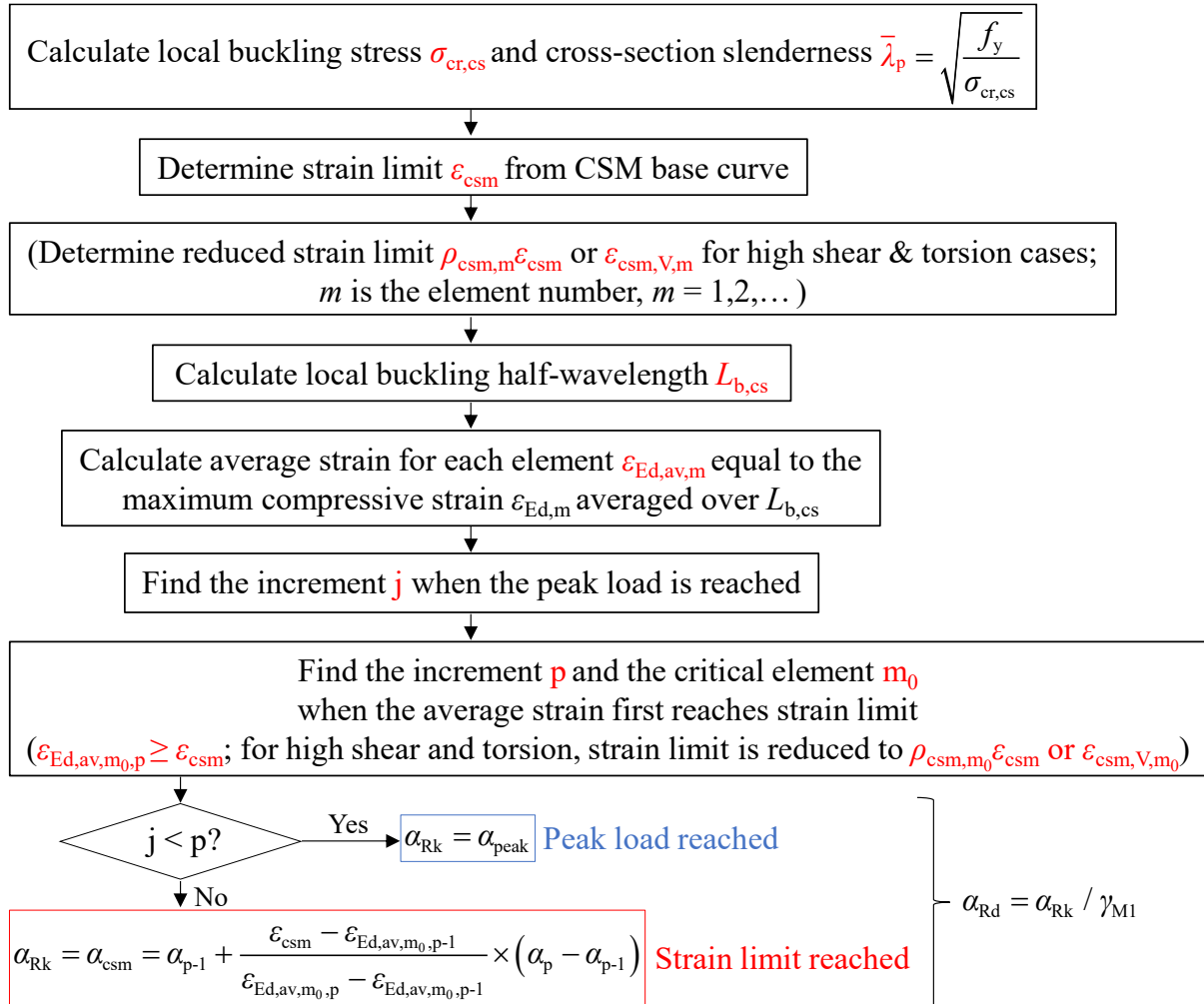


Figure 6.2 Procedure to apply the proposed method of design by advanced analysis with CSM strain limits using beam elements to steel beams susceptible to LTB (where α_p is the load factor at increment p)

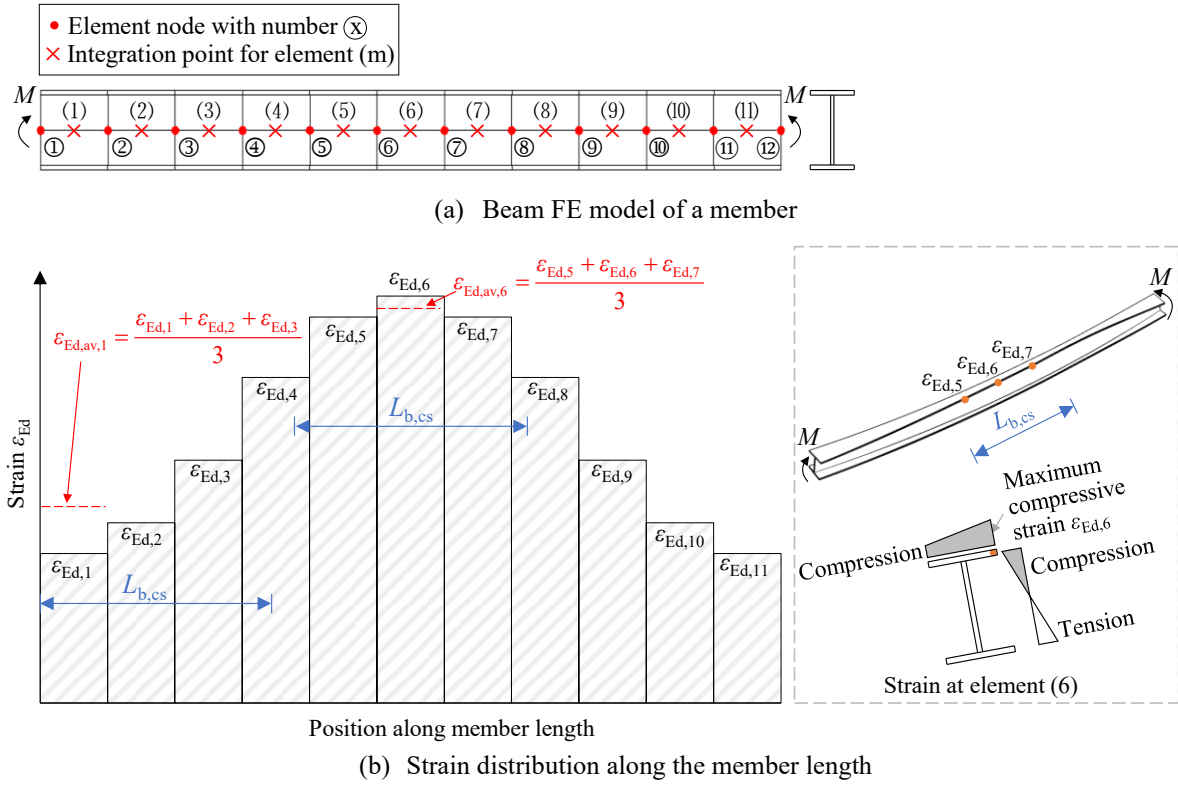


Figure 6.3 Schematic representation of the strain averaging approach along a member with 11 beam elements; the average strains for elements 1 and 6, $\epsilon_{Ed,av,1}$ and $\epsilon_{Ed,av,6}$ respectively, are shown.

6.3 Benchmark shell finite element modelling

6.3.1 Modelling approach

In this study, the accuracy of the proposed design approach for the LTB assessment of steel beams is verified against the results from shell FE modelling considering various cross-section proportions, member slendernesses and loading conditions. The shell FE models were created using the finite element analysis software Abaqus (2017). The four-noded general purpose S4R shell element was used. Each web and flange plate of the cross-sections was subdivided into 16 elements. The number of elements along the member lengths was defined such that the element aspect ratios were close to unity. Beam multi-point constraints were employed to connect the web and two flange plates. To avoid overlapping of the web and flange plates, the web plates were offset by half the flange thicknesses. Fork-end support conditions enabling warping deformations but preventing twist were applied at the member ends by the application of coupling constraints.

The quad-linear stress-strain model developed by Yun & Gardner (2017) for hot-rolled steel

(see Section 2.3.2) was employed to define the material stress-strain response. The Poisson's ratio was taken as $\nu = 0.3$ in the elastic range and $\nu = 0.5$ in the plastic range. As required by Abaqus (2017), the engineering stress-strain relationships were transformed into true stress-strain curves.

The ECCS (1984) residual stress pattern for hot-rolled steel sections illustrated in Figure 5.4 (a) was adopted in the shell FE models. Unless otherwise indicated, initial geometric imperfections in the shape of the first LTB eigenmode with an amplitude of 1/1000 of the unbraced lengths were incorporated into the FE models. As shown in Figure 6.4, local geometric imperfections were also applied to the shell FE models by adopting a series of sinusoidal subpanel imperfections. For the cases where the web plate was more susceptible to local buckling (see Section 4.3.1), the magnitude of the local web imperfection was taken as 1/200 of the web heights h_w (i.e. $h_w/200$). Similarly, for the cases where the flange plates were more susceptible to local buckling, the magnitudes of the local flange imperfections were taken as 1/100 of the flange widths b (i.e. $b/100$). The local imperfection magnitudes of the non-critical plate elements were defined such that the web-to-flange junctions remained at 90° . The local geometric imperfections were applied to the shell FE models using the local buckling half-wavelengths $L_{b,cs}$, which were obtained from the expressions provided in Fieber et al. (2019b). It should be noted that in the case of some members with very short span lengths, the shell FE models did not exhibit a peak load; in these cases, the failure was defined when the tangent stiffness of the model degraded to 1% of its initial stiffness, similar to the approach proposed by dos Santos et al. (2018) and adopted in Fieber et al. (2019a).

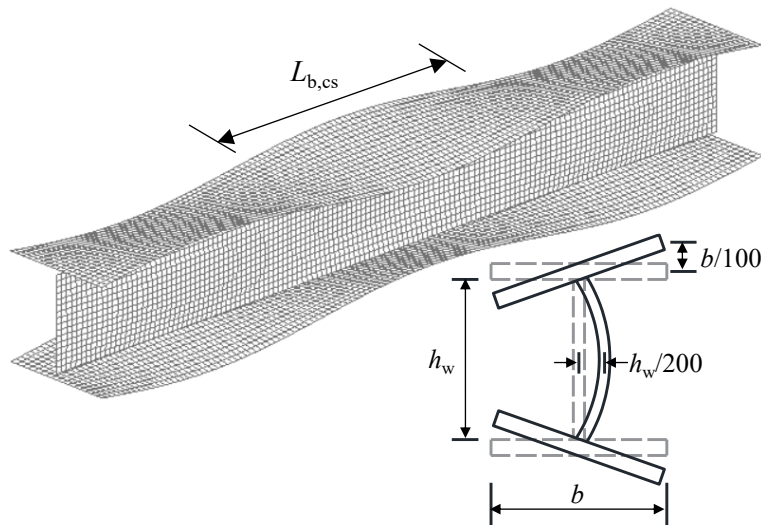


Figure 6.4 Local geometric imperfections applied to shell FE models

6.3.2 Validation of shell FE models

The shell FE models developed in this study were validated against the results from 59 experiments on beams experiencing LTB collected from the literature (Dux & Kitipornchai, 1983; Lindner & Glitsch, 2004; Estabrooks & Grondin, 2008; Ozbasaran et al., 2015; Tusnin & Prokic, 2015a; Tusnin & Prokic, 2015b; Schaper et al., 2019; Demirhan et al., 2020). The loading configurations included (i) 3-point bending and 4-point bending (Dux & Kitipornchai, 1983), (ii) concentrated loading applied at the free-end of cantilever beams (Ozbasaran et al., 2015; Demirhan et al., 2020) and (iii) 3-point bending with eccentrically applied vertical loading, leading to additional torsion (Lindner & Glitsch, 2004; Estabrooks & Grondin, 2008; Tusnin & Prokic, 2015a; Tusnin & Prokic, 2015b; Schaper et al., 2019). The boundary and loading conditions employed in the tests were replicated in the FE models. The geometric imperfection was modelled through the scaling of the first LTB eigenmode. Where reported, the measured global geometric imperfection amplitudes were incorporated into the FE models. For the cases where the geometric imperfection magnitudes were not reported, a magnitude of 1/1000 of the laterally unbraced lengths were used. A summary of the validation study, including the mean and coefficient of variation (CoV) values of the ratios of the ultimate load carrying capacities determined using the shell FE models $\alpha_{u,shell}$ to those obtained from the experiments $\alpha_{u,test}$ (i.e. $\alpha_{u,shell}/\alpha_{u,test}$), is given in Table 6.1. As can be seen from the table, the shell FE models created in this study are able to provide ultimate strength predictions that are very close to those observed in the physical experiments.

Table 6.1 Summary of validation study for shell FE models against experimental results from literature

Reference	Load configuration	No. of tests	$\alpha_{u,shell}/\alpha_{u,test}$	
			Mean	CoV
Dux & Kitipornchai (1983)	3-point bending; 4-point bending	9	0.959	0.020
Ozbasaran et al. (2015)	cantilever beams	9	0.921	0.068
Demirhan et al. (2020)	cantilever beams	9	1.075	0.127
Lindner & Glitsch (2004)	3-point bending with eccentricity	13	0.969	0.027
Estabrooks & Grondin (2008)	3-point bending with eccentricity	6	0.982	0.015
Tusnin & Prokic (2015a, 2015b)	3-point bending with eccentricity	6	0.901	0.061
Schaper et al. (2019)	3-point bending with eccentricity	7	0.970	0.023
Total		59	0.971	0.084

In Figure 6.5, the experimental and numerical load-vertical displacement and load-twist curves for a sample of the beams tested under eccentric 3-point bending in Tusnin & Prokic (2015a),

Tusnin & Prokic (2015b) and Schaper et al. (2019) are shown, where P is the applied load, f and w are the twist and vertical displacement at midspan respectively and $P_{u,\text{test}}$ is the ultimate load obtained from the experiments. As can be seen from the figures, the numerical load-deformation paths obtained from the shell FE models closely follow the load-deformation paths observed in the experiments, indicating that the shell FE models created in this study are able to accurately replicate the structural response of steel beams undergoing LTB and can be used to generate benchmark data to evaluate the accuracy and safety of the proposed design approach.

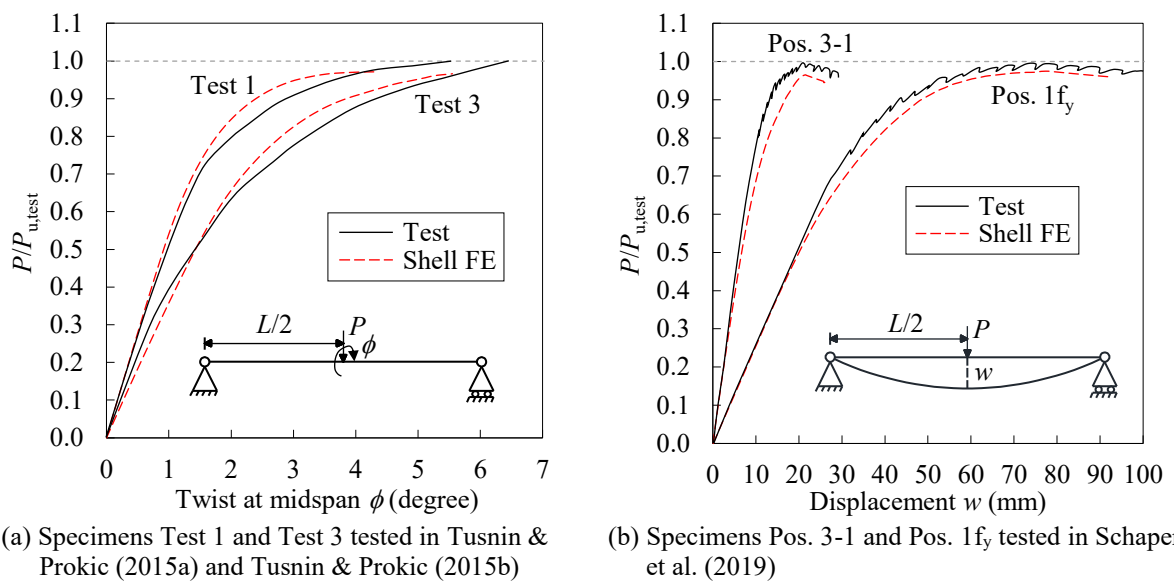


Figure 6.5 Comparison of experimental and numerical load-deformation curves of steel beams subjected to 3-point bending with eccentrically applied vertical loading at midspan, tested in Tusnin & Prokic (2015a), Tusnin & Prokic (2015b) and Schaper et al. (2019)

6.4 Accuracy of proposed design method for steel beams susceptible to LTB

In this section, the accuracy of the proposed design method is assessed against the benchmark results obtained from shell FE modelling for steel beams subjected to uniform and non-uniform bending. For comparison, the accuracy of the traditional design method provided in prEN 1993-1-1 (2019) for the LTB assessment of steel beams is also investigated.

6.4.1 Beams under uniform bending

The accuracy of the proposed design approach is assessed for steel I-section beams under

uniform bending in this subsection, covering a broad range of normalised member LTB slenderness values $0.2 \leq \bar{\lambda}_{LT} = \sqrt{M_{c,Rk} / M_{cr}} \leq 1.8$ and a large number of different cross-section geometries including 10 IPE, 10 HEB and 10 HEA European profiles, where the cross-section slendernesses $\bar{\lambda}_p$ ranged between 0.26 and 0.60 (i.e. $0.26 \leq \bar{\lambda}_p = \sqrt{f_y / \sigma_{cr,cs}} \leq 0.60$).

Figure 6.6 shows an illustrative example of the application of the proposed method to a steel beam with an HEA 260 cross-section under uniform bending. The normalised moment-outer fibre compressive strain response from the shell and beam FE models are displayed in Figure 6.6. In the figure, the applied end bending moment M is normalised by the major axis plastic moment capacity of the cross-section $M_{pl,y}$ (i.e. $M/M_{pl,y}$), while the strain is the average maximum compressive strain over the local buckling half-wavelength for the critical beam element m_0 (which is the element at the midspan in this case) ε_{Ed,av,m_0} normalised by the yield strain ε_y . In the beam element model, the average strain at the midspan reaches the CSM strain limit $\varepsilon_{csm} = 1.80\varepsilon_y$ prior to the attainment of the peak load factor. Thus, this member is deemed to fail when the allowable CSM strain limit is reached, with a corresponding bending moment capacity $M_{u,prop} = 0.480M_{pl,y}$. The corresponding benchmark shell FE model reached a maximum bending moment of $M_{u,shell} = 0.482M_{pl,y}$ following the occurrence of local buckling of the compression flange at midspan, which is only 0.4% higher than the ultimate bending moment resistance determined using the proposed method $M_{u,prop}$, illustrating the very high accuracy of the proposed design approach. Note that if cross-section failure, as defined by the CSM strain limit, was to be ignored, an unconservative prediction of the bending resistance, as given by the peak moment from the beam element model of $M_{peak} = 0.515M_{pl,y}$ would be obtained. The ultimate bending capacity predicted by the traditional design method in prEN 1993-1-1 (2019) $M_{u,EC3} = 0.392M_{pl,y}$ is conservative, 19% lower than the benchmark shell FE result. Note also that in this example and all the considered cases in this study, the strain outputs and CSM strain limits are determined at the centreline of the wall thicknesses of the modelled I-sections, as described in detail in Fieber et al. (2019a).

Figure 6.7 shows the normalised ultimate bending capacities $M_u/M_{pl,y}$ of all considered 300 steel beams undergoing LTB determined through the benchmark shell FE models, the proposed design method and prEN 1993-1-1 (2019) for different LTB slendernesses $\bar{\lambda}_{LT}$. With increasing LTB slendernesses $\bar{\lambda}_{LT}$, steel beams become more susceptible to LTB and exhibit lower ultimate bending moment resistances.

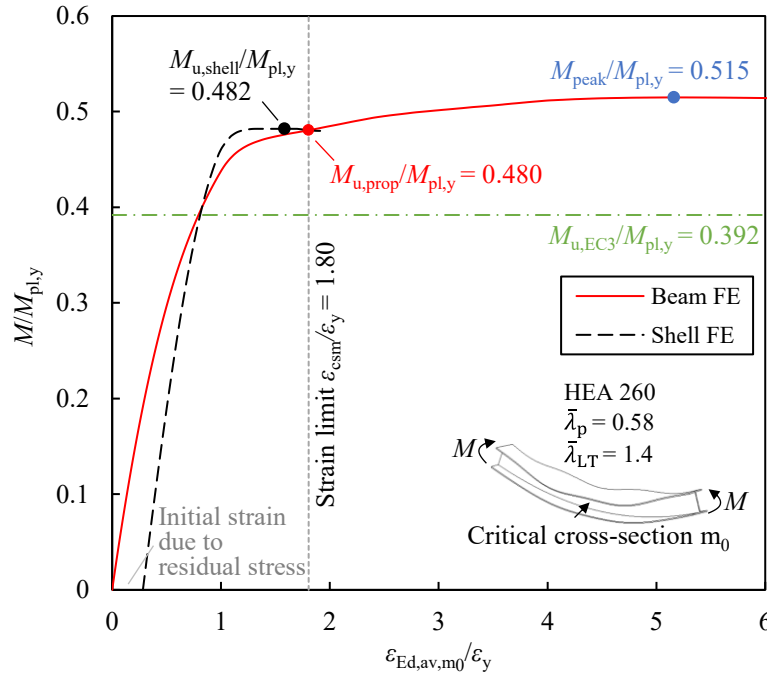


Figure 6.6 Illustrative example of the application of the proposed method of design by advanced analysis with strain limits to a steel beam with an HEA 260 cross-section and non-dimensional LTB slenderness $\bar{\lambda}_{LT}$ of 1.4 under uniform bending

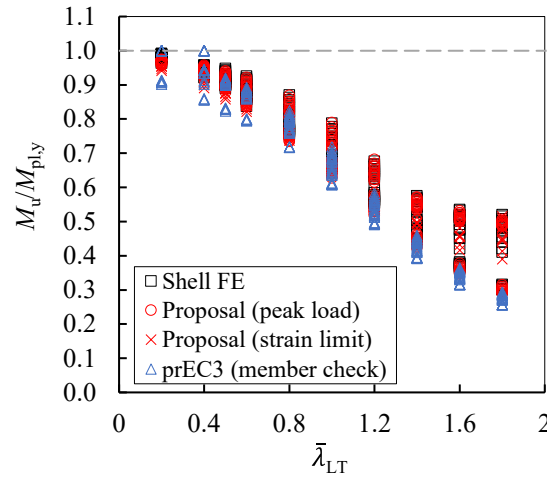


Figure 6.7 Normalised ultimate bending moment capacities of steel beams under uniform bending obtained using the shell FE models, proposed design method and prEN 1993-1-1

In Figure 6.8, the ultimate bending moment resistances M_u of the considered range of steel beams predicted using the proposed design method and prEN 1993-1-1 (2019) are compared against those obtained from the GMNIA of the benchmark shell FE models $M_{u,shell}$. As shown in Figure 6.8 (a) and Table 6.2, the proposed method is able to provide very accurate capacity predictions – the mean value of the ratios of the ultimate resistances determined through the proposed method $M_{u,prop}$ to those determined from the shell FE models $M_{u,shell}$ (i.e. $M_{u,prop}/M_{u,shell}$) is equal to 0.984 with a CoV value of 0.016. On the other hand, prEN 1993-1-

1 (2019) yields less accurate and more scattered predictions, with a mean value of 0.901 and a CoV value of 0.137 for the ratios of ultimate resistances determined using prEN 1993-1-1 (2019) to those obtained from the shell FE models. Note that prEN 1993-1-1 (2019) leads to some rather unsafe results for beams with $\bar{\lambda}_{LT} = 0.4$ and some overly conservative results for slender members (e.g. $\bar{\lambda}_{LT} \geq 1.4$), as shown in Figure 6.8 (b).

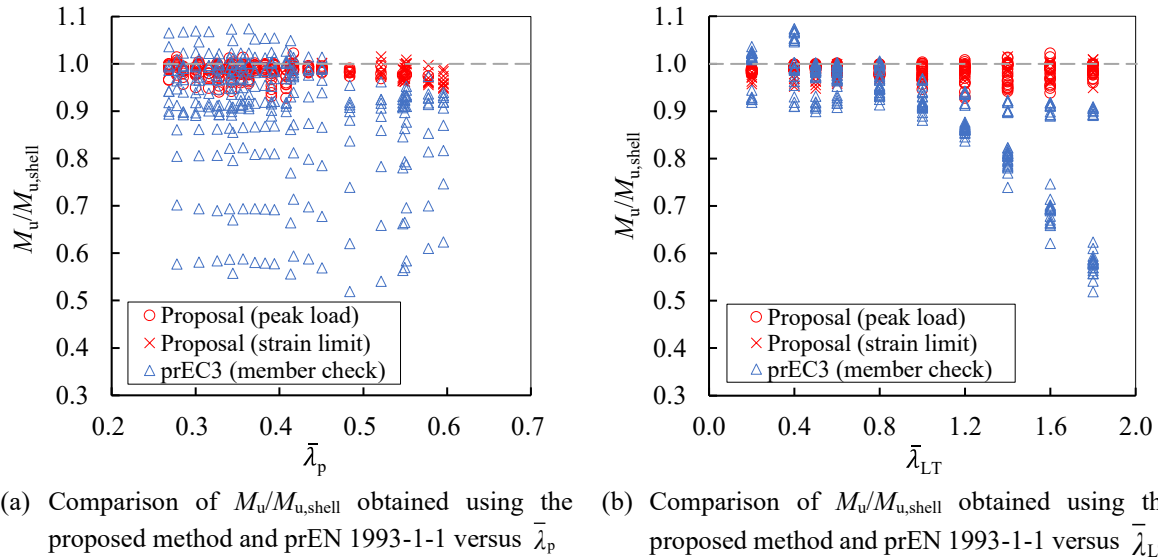


Figure 6.8 Comparison of the ratios of the ultimate bending moment capacities obtained using the proposed design approach and prEN 1993-1-1 to those obtained from the benchmark shell FE models versus (a) cross-section slenderness $\bar{\lambda}_p$ and (b) LTB slenderness $\bar{\lambda}_{LT}$ for beams under uniform bending

Table 6.2 Summary of parameters considered in parametric studies and accuracy of proposed design method and prEN 1993-1-1 relative to results of benchmark shell FE models for steel beams under uniform and non-uniform bending

Moment ratio	Member slenderness	Cross-section	Cross-section slenderness	No. of cases	Design method	$M_u/M_{u,shell}$			
ψ	$\bar{\lambda}_{LT}$		$\bar{\lambda}_p$	N		Mean	CoV	Max	Min
1	0.20 – 1.80	10 IPE 10 HEB 10 HEA	0.26 – 0.60	300	Proposed	0.984	0.016	1.022	0.928
				prEC3	0.901	0.137	1.074	0.519	
0.5	0.35 – 1.40			300	Proposed	0.982	0.031	1.010	0.889
				prEC3	0.904	0.085	0.990	0.674	
0	0.30 – 1.21			300	Proposed	0.988	0.022	1.010	0.888
				prEC3	0.910	0.063	1.000	0.721	
-0.5	0.26 – 1.17			300	Proposed	0.989	0.021	1.011	0.891
				prEC3	0.908	0.054	1.005	0.748	
-1	0.25 – 1.13			300	Proposed	0.971	0.036	1.011	0.877
				prEC3	0.914	0.051	1.010	0.786	
Total				1500	Proposed	0.983	0.027	1.022	0.877
					prEC3	0.907	0.084	1.074	0.519

6.4.2 Beams under moment gradients

In addition to steel beams under uniform bending, the accuracy of the proposed design method is also assessed for 1200 steel beams undergoing LTB and subjected to different bending moment gradients along their lengths. As summarised in Table 6.2, the following parameters were considered: (i) the ratio of the bending moments applied at the beam ends ψ equal to 0.5, 0, -0.5 and -1 (i.e. $\psi = 0.5, 0, -0.5$ and -1), (ii) the normalised LTB slenderness $\bar{\lambda}_{LT}$, ranging between 0.25 and 1.40 (i.e. $0.25 \leq \bar{\lambda}_{LT} \leq 1.40$), and (iii) the cross-section geometry, by considering 10 IPE, 10 HEB and 10 HEA European profiles, thereby covering a broad range of cross-section slenderness values $\bar{\lambda}_p$ between 0.26 and 0.60 (i.e. $0.26 \leq \bar{\lambda}_p \leq 0.60$).

A bending moment gradient within a steel beam leads to the development of vertical shear forces along its length; the presence of high shear forces may adversely influence the bending moment capacity of the beam. To allow for this effect, when the design shear force $V_{y,Ed}$ exceeds half the plastic major axis shear resistance of the cross-section $V_{pl,y}$, a reduction factor ρ_{csm} , as determined by Eq. (4.1), is applied to the CSM strain limit ε_{csm} (see Section 4.5). Note that separate cross-section shear capacity and shear buckling checks are still required, which can be carried out using the relevant provisions of EN 1993-1-1 (2005) and EN 1993-1-5 (2006). Even though there exist lateral (minor axis) shear forces within the cross-sections of steel beams experiencing LTB, it was observed that these shear forces are consistently very small in the large number of steel beams considered in this study. Thus, no reduction to the CSM strain limits due to the presence of high minor axis shear forces is considered necessary in practical LTB design situations.

Comparisons of the ratios of the ultimate bending moment capacities predicted by the proposed design method and prEN 1993-1-1 (2019) to those determined from the benchmark shell FE models for members under different shapes of bending moment diagrams are shown in Figure 6.9. Both limiting failure conditions i.e. attainment of the peak load factor and the CSM strain limits were exhibited among the considered design cases. Figure 6.9 shows that the proposed design method generally provides very accurate and safe-sided resistance predictions for different cross-section slendernesses $\bar{\lambda}_p$. As presented in Figure 6.9 and Table 6.2, relative to prEN 1993-1-1 (2019), the proposed design method offers an average improvement of approximately 8% in accuracy, as well as a consistent reduction in the scatter of the predictions.

As shown in Figure 6.10, for beams with relatively high member slendernesses (e.g. $\bar{\lambda}_{LT} \geq 1.0$), prEN 1993-1-1 (2019) provides overly-conservative resistance predictions, with the predictions becoming increasingly conservative with increasing beam slendernesses $\bar{\lambda}_{LT}$; prEN 1993-1-1 (2019) is also overly-conservative for stocky members (e.g. $\bar{\lambda}_{LT} \leq 0.6$) due to the neglect of the influence of material strain hardening on the ultimate resistances. By contrast, as shown in Figure 6.11, the proposed design method offers considerably improved accuracy by allowing for the beneficial influence of both material strain hardening and local bending moment gradients on cross-section capacity.

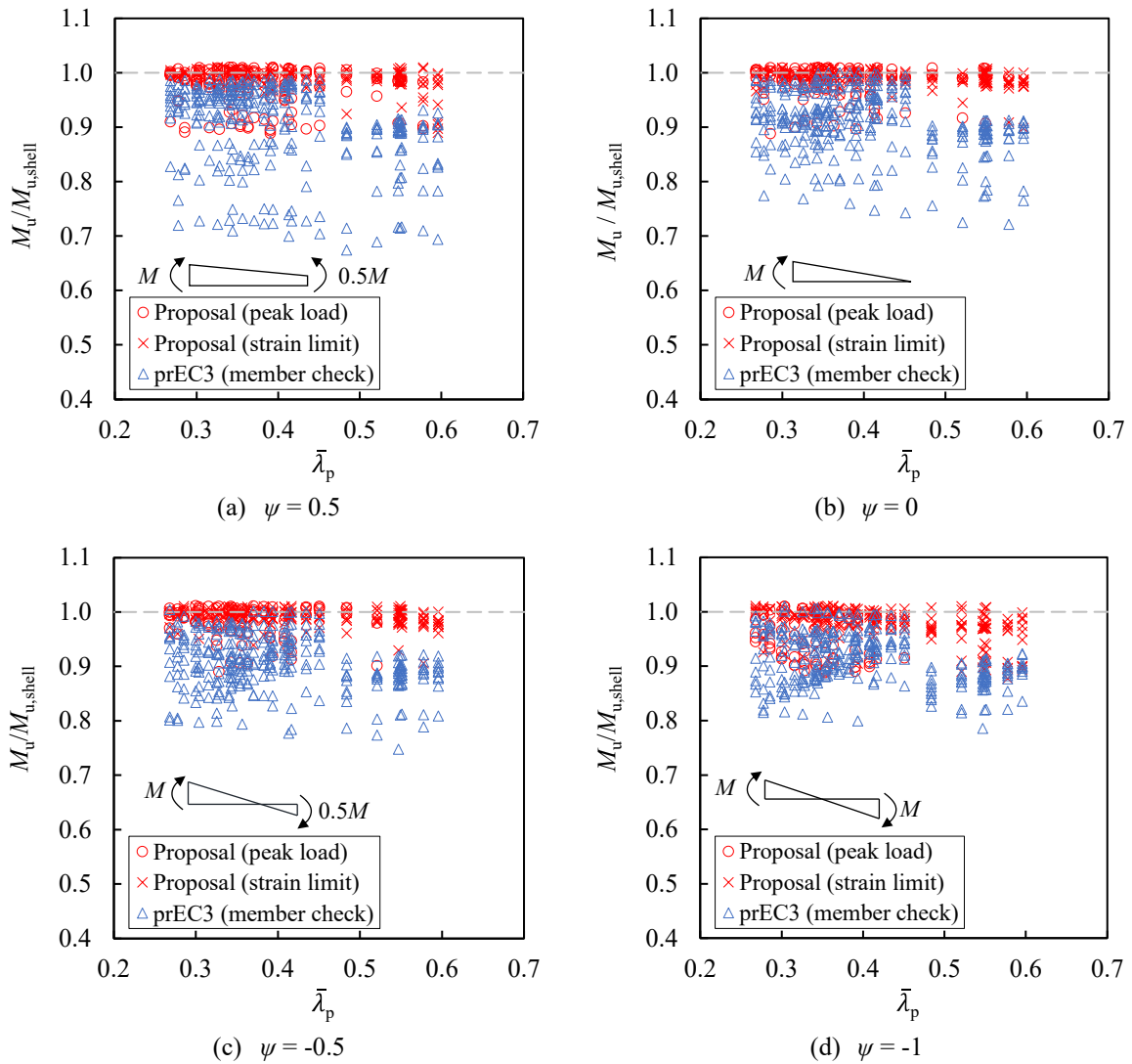


Figure 6.9 Comparison of the ratios of the ultimate bending moment capacities obtained using the proposed design approach and prEN 1993-1-1 to those obtained from the benchmark shell FE models versus cross-section slenderness $\bar{\lambda}_p$ for steel beams under non-uniform bending

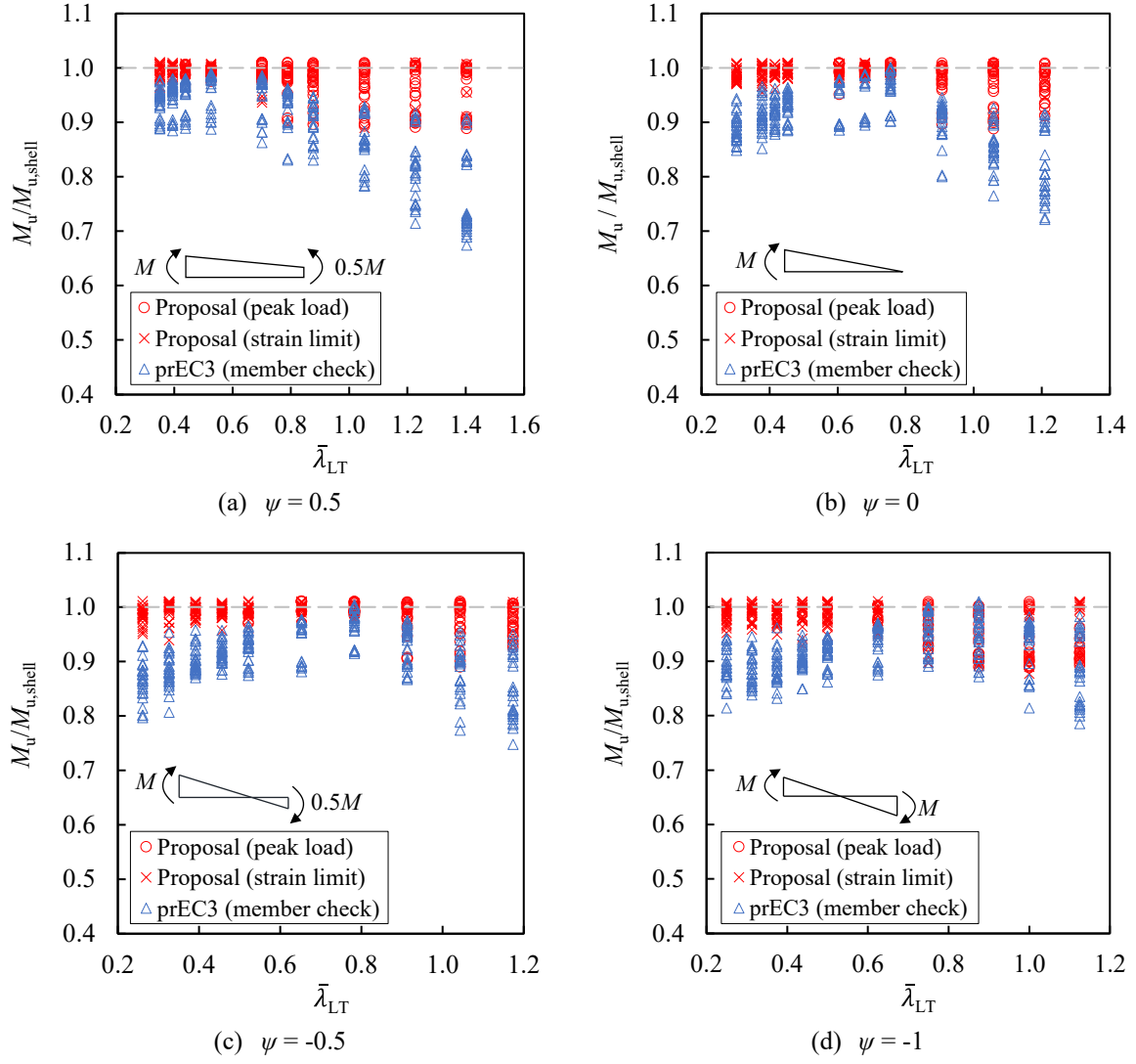
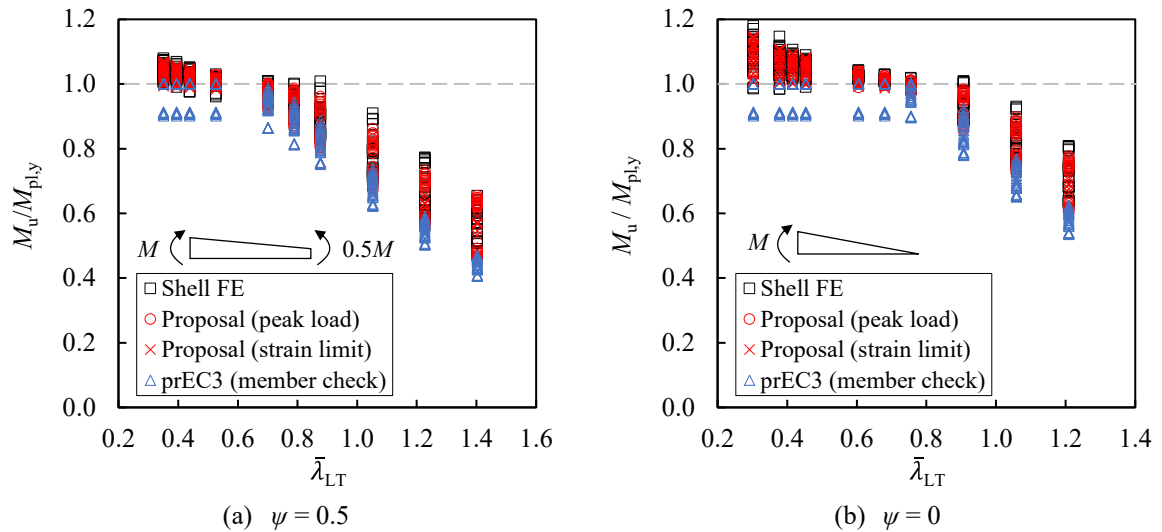


Figure 6.10 Comparison of the ratios of the ultimate bending moment capacities obtained using the proposed design approach and prEN 1993-1-1 to those obtained from the benchmark shell FE models versus member slenderness for LTB $\bar{\lambda}_{LT}$ for steel beams under non-uniform bending



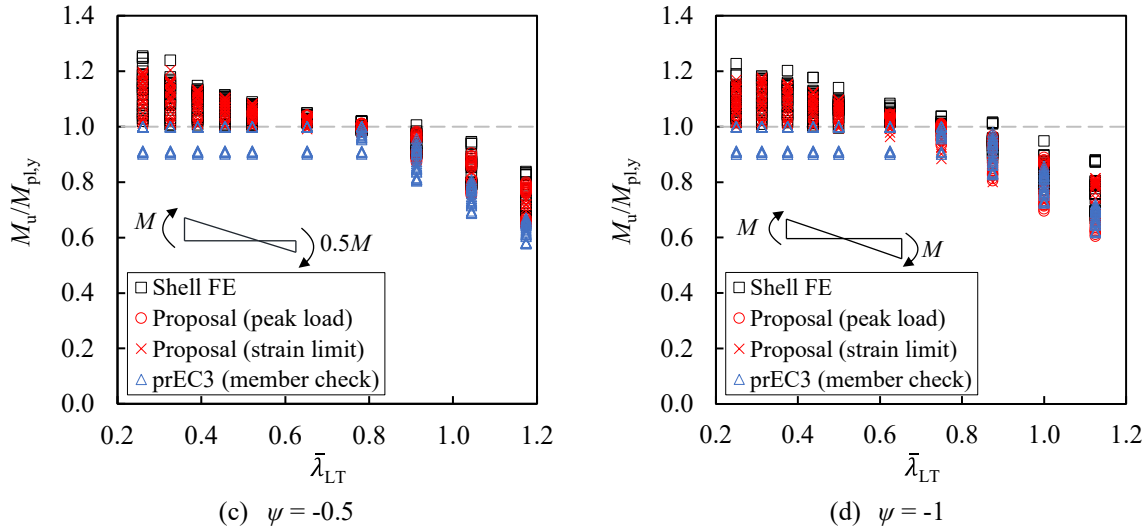


Figure 6.11 Normalised ultimate bending moment capacities of steel members under non-uniform bending obtained from the proposed design method, shell FE models and prEN 1993-1-1 versus member slenderness for LTB $\bar{\lambda}_{LT}$

The influence of the adoption of the strain averaging approach on the accuracy of the resistance predictions obtained using the proposed design method is explored in Figure 6.12; this figure shows the ratios of the predicted ultimate bending capacities obtained from the proposed design method adopting the strain averaging approach $M_{u,prop}(\epsilon_{Ed,av})$ and without employing strain averaging $M_{u,prop}(\epsilon_{Ed})$ (i.e. $M_{u,prop}(\epsilon_{Ed,av})/M_{u,prop}(\epsilon_{Ed})$) for steel beams subjected to different bending moment gradients and failing due to the attainment of the CSM strain limits. In typical I-section steel beams, relative to the normal strains resulting from the applied major axis bending moments, the normal strains caused by second order warping torsion and second order minor axis bending moments, which vary along the member lengths, are small. Thus, as can be seen in Figure 6.12, for members under uniform bending (i.e. $\psi = 1$), the strain averaging approach does not have a significant influence on the ultimate resistance predictions, since there is no variation in the major axis bending moments along the member lengths; only the second order warping torsion and the second order minor axis bending moments vary along the member lengths, and their influence on the development of the normal strains is considerably lower than that of the major axis bending. On the other hand, for steel beams subjected to non-uniform major axis bending, applying the strain averaging approach leads to enhanced ultimate bending moment resistance predictions, which can be up to 14% higher than those determined without adopting the strain averaging approach.

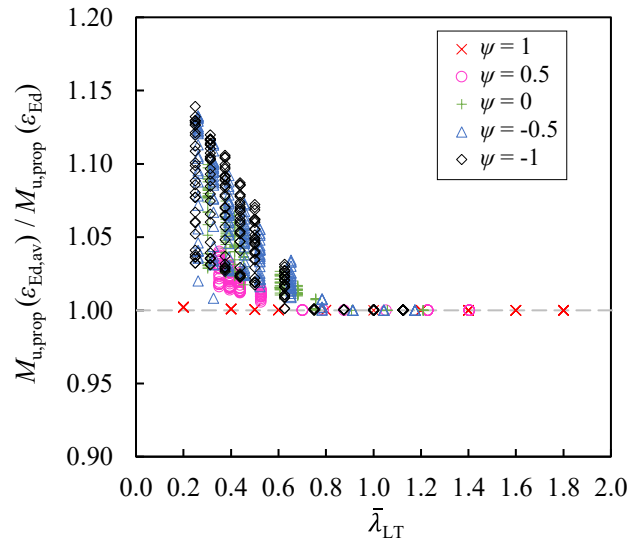


Figure 6.12 Ratios between the ultimate bending capacities determined using the proposed design method with strain averaging (using $\epsilon_{Ed,av}$) and without strain averaging (using ϵ_{Ed})

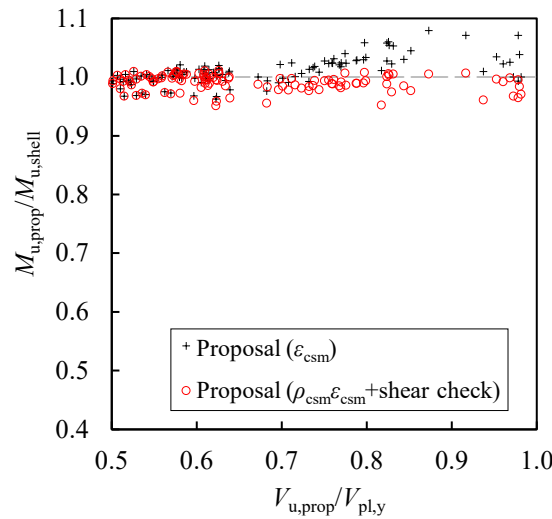


Figure 6.13 Comparison between the ultimate bending moment capacities of steel beams under high shear predicted using the proposed design method with unreduced strain limit ϵ_{csm} and with reduced strain limit $\rho_{csm}\epsilon_{csm}$ plus shear check against the shell FE results

As previously indicated, high shear forces can adversely affect the bending resistances of steel beams. In Figure 6.13, comparisons between the ultimate bending moment resistances determined through the proposed design method with or without the application of the shear force reduction factor ρ_{csm} to the CSM strain limits are presented, considering the cases where the shear forces within the beams exceeded half of their cross-section shear resistances $V_{u,prop}/V_{pl,y} > 0.5$; note that in all the cases, failure was due to the attainment of the CSM strain limits in the beams. As can be seen from Figure 6.13, if the adverse influence of the high shear

forces on the bending capacities is ignored, the predictions obtained from the proposed method can become unconservative. On the other hand, if the reduction factor ρ_{CSM} is applied to the CSM strain limits for the cases where $V_{y,\text{Ed}}/V_{\text{pl},y} > 0.5$, the unconservative predictions shift to the safe side, indicating the importance of applying the shear force reduction factor ρ_{CSM} to the CSM limits for steel beams where the shear forces exceed half of the cross-section shear resistances $V_{y,\text{Ed}}/V_{\text{pl},y} > 0.5$.

6.5 Consideration of torsion in the proposed design method

In steel structures, pure torsion is relatively unusual; typically, torsion arises in steel members in combination with bending. As indicated in Section 6.2.2, in the implementation of the proposed design method, normal strains ε_w resulting from warping torsion normal stresses can be included in the total normal strains that can be directly extracted from the analysis and checked against the CSM strain limits. However, in this approach, the influence of St. Venant torsion τ_{St} and warping torsion shear stresses τ_w is not directly considered. The consideration of these shear effects due to (first or second order) torsion in the application of the proposed design method is described in this section.

6.5.1 Proposed approach for the consideration of torsion effects

6.5.1.1 Internal torsion effects and torsional resistances of steel cross-sections

In the proposed design approach, St. Venant torsion and warping torsion shear effects are taken into account through considering the ratios of the internal St. Venant torsion $T_{\text{St,Ed}}$ to the cross-section St. Venant torsion resistance $T_{\text{St,Rk}}$ (i.e. $T_{\text{St,Ed}}/T_{\text{St,Rk}}$) and the internal warping torsion $T_{w,\text{Ed}}$ to the cross-section warping torsion resistance $T_{w,\text{Rk}}$ (i.e. $T_{w,\text{Ed}}/T_{w,\text{Rk}}$). Since the total internal torsion T_{Ed} within steel members can be directly extracted from GMNIA using beam finite elements, but some FE software packages (e.g. Abaqus (2017)) do not enable the extraction of the individual St. Venant $T_{\text{St,Ed}}$ and warping $T_{w,\text{Ed}}$ torsion components, the use of Eq. (6.2) and Eq. (6.3) is recommended for the determination of the internal St. Venant $T_{\text{St,Ed}}$ and warping torsion $T_{w,\text{Ed}}$ within steel elements, respectively (prEN 1993-1-1, 2019).

$$T_{\text{St,Ed}} = GI_t \phi' \quad (6.2)$$

$$T_{w,\text{Ed}} = T_{\text{Ed}} - T_{\text{St,Ed}} \quad (6.3)$$

In Eq. (6.2), G is the shear modulus, I_t is the St. Venant torsion constant, and ϕ' is the first derivative of twist with respect to distance along the member length, which can be approximately taken as the ratio of the difference of the twists ϕ at the two end nodes of a beam element $\Delta\phi$ to the element length l (i.e. $\phi' = \Delta\phi/l$). The accuracy of this approximate approach is presented in Figure 6.14. As can be seen from the figure, for all pure torsion, pure bending and combined bending and torsion cases, the warping moments determined from beam FE models using the proposed approximate approach agree well with the values obtained from shell FE models.

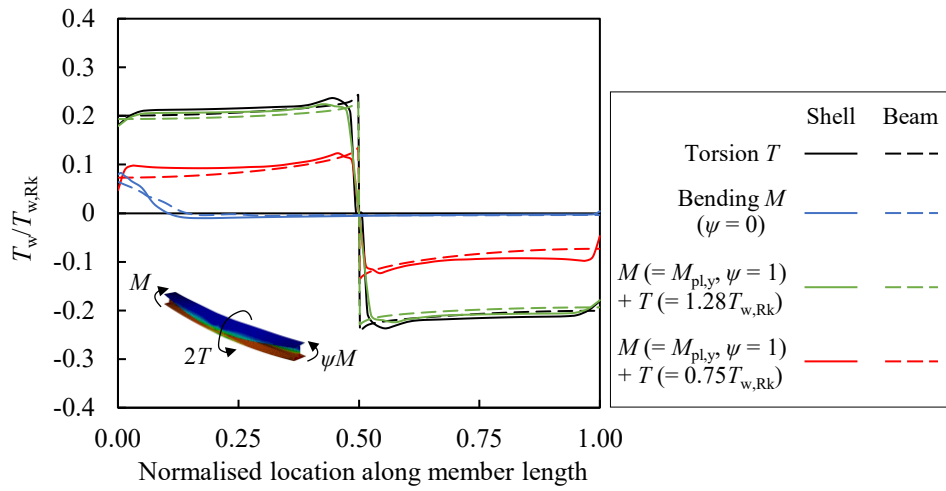


Figure 6.14 Comparison of the normalised warping moment determined from beam FE models against those from shell FE models for pure torsion, pure bending and combined bending and torsion cases

For St. Venant torsion, the associated shear stresses change sign and vary linearly across the wall thickness for open sections and thus can be considered to have no effect on local buckling (Trahair et al., 2008). Therefore, the cross-section St. Venant torsion resistance $T_{St,Rk}$ of all open-sections can be approximated by the plastic St. Venant section resistance (Trahair et al., 2008), which can be expressed by Eq. (6.4) for I-sections, as given in Pi & Trahair (1995), or can be approximated by Eq. (6.5), as recommended in Trahair et al. (2008), where τ_y is the shear yield stress which is equal to $f_y/\sqrt{3}$ and h , b , t_f and t_w are the overall depth, breadth, flange thickness and web thickness of the I-section, as shown in Figure 6.1.

$$T_{St,Rk} = \tau_y \left[bt_f^2 \left(1 - \frac{t_f}{3b} \right) + \frac{(h - 2t_f)t_w^2}{2} + \frac{t_w^3}{6} \right] \quad (6.4)$$

$$T_{St,Rk} \approx \tau_y \left[bt_f^2 + \frac{(h - 2t_f)t_w^2}{2} \right] \quad (6.5)$$

For the case of warping torsion, the cross-section warping torsion resistance $T_{w,Rk}$ is taken as elastic warping torsion resistance $T_{w,el}$ in accordance with Seaburg & Carter (2003) in this study. As shown in Figure 6.1, the shear force due to warping V_w can be approximated by resolving the warping torsion T_w into an equivalent force couple acting within the flanges $V_w = T_w/(h-t_f)$ (Hughes et al., 2011). By elastic analysis, the shear stress at the centre of the flange can be approximated by Eq. (6.6) (Seaburg & Carter, 2003).

$$\tau_w = \frac{1.5V_w}{bt_f} \quad (6.6)$$

Hence, the warping torsion resistance $T_{w,Rk}$ of an I-section, taken as $T_{w,el}$, can be determined as given by Eq. (6.7).

$$T_{w,Rk} = \tau_y bt_f (h - t_f) / 1.5 \quad (6.7)$$

Note that the plastic warping resistance for I-sections is simply given by $T_{w,pl} = \tau_y bt_f (h - t_f)$.

6.5.1.2 Interaction of shear and torsion

According to prEN 1993-1-1 (2019), for a cross-section subjected to combined torsion and shear, the plastic cross-section shear resistance should be reduced considering the adverse influence of shear stresses due to torsion. An expression for the reduced plastic vertical shear resistance $V_{pl,y,T}$ of I-sections in the presence of St. Venant torsion $T_{St,Ed}$ was developed in Mirambell et al. (2016), as given by Eq. (6.8). This expression considers the interaction between the shear stresses due to vertical shear force $\tau_{y,Ed}$, which are uniformly distributed through the web thickness, and the shear stresses due to St. Venant torsion $\tau_{St,Ed}$, which vary linearly through the web thickness, as shown in Table 6.3, and forms the basis of the shear-torsion interaction formula given in prEN 1993-1-1 (2019).

$$\text{For I-section, } V_{pl,y,T} = \sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} V_{pl,y} \quad (6.8)$$

Considering the different shear stress distributions through web thicknesses, as shown in Table 6.3, the equivalent expressions (Mirambell et al., 2016) for the determination of the reduced shear resistances of channels and square or rectangular hollow sections (SHS/RHS) subjected to combined torsion and shear were also set out in Mirambell et al. (2016), as given by Eqs.

(6.9) and (6.10).

$$\text{For channel section, } V_{pl,y,T} = \left(\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} - \frac{T_{w,Ed}}{T_{w,Rk}} \right) V_{pl,y} \quad (6.9)$$

$$\text{For SHS/RHS, } V_{pl,y,T} = \left(1 - \frac{T_{St,Ed}}{T_{St,Rk}} \right) V_{pl,y} \quad (6.10)$$

Table 6.3 Shear stress distribution across plate thickness of different types of cross-sections subjected to vertical shear force $V_{y,Ed}$, lateral shear force $V_{z,Ed}$ and torsion moment T_{Ed}

I-section	Channel section	SHS/RHS

Clearly the ratio of the applied shear force to cross-section shear resistance (i.e. $V_{y,Ed}/V_{pl,y,T}$) has to be less than or equal to 1.0. In this study, the coefficients η_y and η_z are used to represent the utilisation of a cross-section under combined shear and torsion for shear forces acting through the cross-section depth (major axis bending cases) and the cross-section width (minor axis bending cases), respectively. The proposed expressions for the determination of η_y and η_z are provided in Table 6.4 for different cross-sections. The expressions for the determination of η_y in the cases where shear forces act through the cross-section depths were developed considering the shear stress distribution through the web thickness and using Eqs. (6.8)-(6.10). Similarly, to determine η_z for the cases where shear forces act through the cross-section widths, the interaction of the shear stresses due to lateral shear force $\tau_{z,Ed}$, St. Venant torsion $\tau_{St,Ed}$ and warping torsion $\tau_{w,Ed}$ arising in the flanges, as presented in Table 6.3, was considered. Note that in Table 6.4, $V_{y,Ed}$ and $V_{z,Ed}$ are the vertical and lateral applied shear force respectively; $V_{y,Rk}$ and $V_{z,Rk}$ are the vertical and lateral cross-section shear resistances, taken as the plastic cross-

section shear resistances $V_{pl,y}$ and $V_{pl,z}$ equal to $A_v\tau_y$, where A_v is the shear area determined according to prEN 1993-1-1 (2019), and $T_{St,Rk}$ and $T_{w,Rk}$ are the cross-section St. Venant torsion and warping torsion resistances, respectively. $T_{St,Rk}$ can be determined using Eq. (6.5) for I-sections and channel sections and determined as $T_{St,Rk} = 2tA_0\tau_y$ for SHS/RHS, where t and A_0 are the section thickness and area enclosed by the mean perimeter (Hughes et al., 2011), respectively; $T_{w,Rk}$ can be determined using Eq. (6.7) for I-sections and determined as $T_{w,Rk} = T_{w,el} = t_f I_w \tau_y / S_w$ for channel sections, where t_f , I_w and S_w are the flange thickness, cross-section warping constant and warping statical moment (Galambos, 1968), respectively (Hughes et al., 2011). In the application of the proposed design method, shear checks for cross-sections should be performed to assess that Eq. (6.11) is satisfied.

$$T_{St,Ed}/T_{St,Rk} \leq 1.0, \eta_y \leq 1.0 \text{ and } \eta_z \leq 1.0 \quad (6.11)$$

Note that for cross-sections subjected to shear and/or torsion in which the web slenderness $\bar{\lambda}_w$ exceeds $0.83/\eta$, where $\eta = 1.2$ is the shear area factor (EN 1993-1-5, 2006), shear buckling should be accounted for in the determination of the cross-section shear resistances $V_{y,Rk}$ and $V_{z,Rk}$ and the warping torsion resistance $T_{w,Rk}$ according to EN 1993-1-5 (2006).

Table 6.4 Expressions for calculating the factors η_y and η_z for considering the combined effects of shear stresses due to shear force and torsion for different types of cross-sections

	Major axis bending	Minor axis bending
I-section	$\eta_y = \frac{V_{y,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} V_{y,Rk}} \leq 1$	$\eta_z = \frac{V_{z,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} V_{z,Rk}} + \frac{T_{w,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} T_{w,Rk}} \leq 1$
Channel section	$\eta_y = \frac{V_{y,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} V_{y,Rk}} + \frac{T_{w,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} T_{w,Rk}} \leq 1$	$\eta_z = \frac{V_{z,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} V_{z,Rk}} + \frac{T_{w,Ed}}{\sqrt{1 - \frac{T_{St,Ed}}{T_{St,Rk}}} T_{w,Rk}} \leq 1$
SHS/RHS	$\eta_y = \frac{V_{y,Ed}}{\left(1 - \frac{T_{St,Ed}}{T_{St,Rk}}\right) V_{y,Rk}} \leq 1$	$\eta_z = \frac{V_{z,Ed}}{\left(1 - \frac{T_{St,Ed}}{T_{St,Rk}}\right) V_{z,Rk}} \leq 1$

6.5.1.3 Reduced CSM strain limits for the consideration of shear effects

Similar to the approach adopted in Section 6.4.2, the use of reduced CSM strain limits $\varepsilon_{csm,V}$ to take account of the interaction between bending, shear and torsion is recommended in the application of the proposed method, as given by:

$$\varepsilon_{\text{csm},V} = \varepsilon_{\text{csm}} - \rho_y(\varepsilon_{\text{csm}} - \mu_{y,\text{fl}}\varepsilon_y) - \rho_z(\varepsilon_{\text{csm}} - \mu_{z,w}\varepsilon_y) \quad (6.12)$$

where ρ_y and ρ_z are the reduction factors ranging from 0 to 1.0 to allow for the influence of high vertical shear (including that resulting from torsion) and high lateral shear (including that resulting from torsion) on the ultimate cross-section resistance, and $\mu_{y,\text{fl}}$ and $\mu_{z,w}$ are given by:

$$\mu_{y,\text{fl}} = \frac{M_{\text{fl}}}{M_{\text{el},y}} \quad (6.13)$$

$$\mu_{z,w} = \frac{M_w}{M_{\text{el},z}}. \quad (6.14)$$

In Eqs. (6.13) and (6.14), $\mu_{y,\text{fl}}$ is the ratio of the cross-section major axis bending moment resistance M_{fl} considering the flanges alone (i.e. neglecting the presence of the web) to the elastic major axis bending resistance of the full cross-section $M_{\text{el},y}$ (i.e. $M_{\text{fl}}/M_{\text{el},y}$) and $\mu_{z,w}$ is the ratio of the cross-section minor axis bending moment resistance determined neglecting the presence of the flanges M_w to the elastic minor axis bending resistance of the full cross-section $M_{\text{el},z}$ (i.e. $M_w/M_{\text{el},z}$); these ratios are used to approximate the effectiveness of a cross-section in bending when the shear area (i.e. the web for major axis bending and the flanges for minor axis bending) is fully utilised in shear. M_{fl} can be calculated using Eq. (6.15); M_w (and hence $\mu_{z,w}$) can be conservatively taken equal to 0 for I-sections, due to the insignificant contribution of the web to the cross-section minor axis bending resistance.

$$M_{\text{fl}} = bt_f(h - t_f)f_y \quad (6.15)$$

Hence, for I-sections, the expression for calculating the reduced CSM strain limits $\varepsilon_{\text{csm},V}$ can be simplified to Eq. (6.16).

$$\text{For I-sections, } \varepsilon_{\text{csm},V} = \varepsilon_{\text{csm}} - \rho_y(\varepsilon_{\text{csm}} - \mu_{y,\text{fl}}\varepsilon_y) - \rho_z\varepsilon_{\text{csm}} \quad (6.16)$$

prEN 1993-1-1 (2019) accounts for the interaction between vertical shear effects (arising from shear force and torsion) and major axis bending by reducing the cross-section bending capacities for the cases where the ratio of the applied shear force to the plastic cross-section shear resistance exceeds 0.5. The reduced bending moment capacity is determined using a reduced yield strength $(1 - \rho_{y,\text{EC3}})f_y$ for the shear area, where the reduction factor $\rho_{y,\text{EC3}}$ is given by Eq. (6.17).

$$\rho_{y,EC3} = \begin{cases} 0 & \text{for } \frac{V_{y,Ed}}{V_{pl,y,T}} \leq 0.5 \\ \left(\frac{2V_{y,Ed}}{V_{pl,y,T}} - 1 \right)^2 & \text{for } 0.5 < \frac{V_{y,Ed}}{V_{pl,y,T}} \leq 1.0 \end{cases} \quad (6.17)$$

In this study, the reduction factor ρ_y used in Eqs. (6.12) and (6.16) adopts a similar format, as given by Eq. (6.18), where the major axis shear-torsion interaction factor η_y is presented in Table 6.4 and illustrated in Section 6.5.1.2.

$$\rho_y = \begin{cases} 0 & \text{for } \eta_y \leq 0.5 \\ (2\eta_y - 1)^2 & \text{for } 0.5 < \eta_y \leq 1.0 \end{cases} \quad (6.18)$$

In prEN 1993-1-1 (2019), for members subjected to significant shear forces through their cross-section widths, the yield strength of the shear area is reduced by $\rho_{z,EC3}$, which is determined using Eq. (6.19) for I-sections and channels and Eq. (6.20) for SHS/RHS. Note that the von Mises yield criterion was considered in the development of the expression for I-sections and channel sections (Knobloch et al., 2019) and that the reduction to the yield strength is applied when the ratio of the applied shear force $V_{z,Ed}$ to the plastic shear resistance ($V_{z,Ed}/V_{pl,z}$) is higher than 0.25.

$$\text{For I-sections and channel sections, } \rho_{z,EC3} = \begin{cases} 0 & \text{for } \frac{V_{z,Ed}}{V_{pl,z}} \leq 0.25 \\ 1 - \sqrt{1 - \left(\frac{V_{z,Ed}}{V_{pl,z}} \right)^2} & \text{for } 0.25 < \frac{V_{z,Ed}}{V_{pl,z}} < 1.0 \\ 1 & \text{for } \frac{V_{z,Ed}}{V_{pl,z}} = 1.0 \end{cases} \quad (6.19)$$

$$\text{For SHS/RHS, } \rho_{z,EC3} = \begin{cases} 0 & \text{for } \frac{V_{z,Ed}}{V_{pl,z}} \leq 0.5 \\ \left(\frac{2V_{z,Ed}}{V_{pl,z}} - 1 \right)^2 & \text{for } 0.5 < \frac{V_{z,Ed}}{V_{pl,z}} \leq 1.0 \end{cases} \quad (6.20)$$

Following the format adopted in prEN1993-1-1 (2019), in this study, for I-sections and channel sections, Eq. (6.21) is recommended for the determination of the reduction factor ρ_z used in Eqs. (6.12) or (6.16), as given by

$$\text{For I-sections and channel sections, } \rho_z = \begin{cases} 0 & \text{for } \eta_z \leq 0.25 \\ 1 - \sqrt{1 - \eta_z^2} & \text{for } 0.25 < \eta_z < 1.0 \\ 1 & \text{for } \eta_z = 1.0 \end{cases} \quad (6.21)$$

For SHS/RHS, the reduction factor ρ_z can be determined using Eq. (6.22).

$$\text{For SHS/RHS, } \rho_z = \begin{cases} 0 & \text{for } \eta_z \leq 0.5 \\ (2\eta_z - 1)^2 & \text{for } 0.5 < \eta_z \leq 1.0 \end{cases} \quad (6.22)$$

In both Eqs. (6.21) and (6.22), the minor axis shear-torsion interaction factor η_z should be determined from Table 6.4.

6.5.2 Accuracy of proposed design approach for laterally-restrained beams

In this subsection, the accuracy of the proposed approach for the design of laterally-restrained steel beams under in-plane bending and shear without the presence of torsion is investigated. Figure 6.15 (a) shows the reducing CSM strain limit $\varepsilon_{\text{csm},V}/\varepsilon_y$ determined using Eq. (6.16), with increasing $V_{y,Ed}/V_{pl,y}$ ratios for steel beams made of grade S355 steel with an HEB 100 cross-section and subjected to in-plane 3-point major axis bending, while Figure 6.15 (b) illustrates the ultimate capacities of the beams with different normalised member lengths $L/L_{b,cs}$ obtained from the shell FE models $M_{u,\text{shell}}$ and the proposed design method $M_{u,\text{prop}}$ using the unreduced ε_{csm} and reduced $\varepsilon_{\text{csm},V}$ CSM strain limits plus the shear check. It can be seen from the figure that, for beams with high shear forces, the application of the proposed design method without the consideration of the shear effects may lead to unconservative member capacity predictions; on the other hand, employing the reduced strain limits and shear checks shift these unconservative predictions to the safe side.

Similar conclusions can also be drawn from the results of a number of I-section members (including 10 IPE, 10 HEB and 10 HEA profiles with 10 varied member lengths) subjected to 3-point major axis bending shown in Figure 6.16. Figure 6.16 (a) shows the normalised moment-shear interaction obtained from shell FE models and the proposed design method using the unreduced ε_{csm} and reduced $\varepsilon_{\text{csm},V}$ CSM strain limits plus the shear check, where V_u is the ultimate shear capacity obtained from different design methods. It can be seen from the figure that in bending-dominated cases, the beneficial effects from the local moment gradients outweigh the negative influence from shear effects; thus, with increasing shear forces, increases

in the bending moment capacities can be observed (Fieber et al., 2019a). On the other hand, in the shear-dominated cases, reducing the CSM strain limits considering high shear effects and performing shear checks moves unsafe predictions that can be observed without any shear checks to the safe side; this also can be seen from Figure 6.16 (b), which shows a comparison between the member capacities obtained from the proposed method using ϵ_{csm} and $\epsilon_{\text{csm},V}$ plus the shear check, normalised by the corresponding shell FE results, versus the value of η_y at the attainment of the peak load factor or ϵ_{csm} .

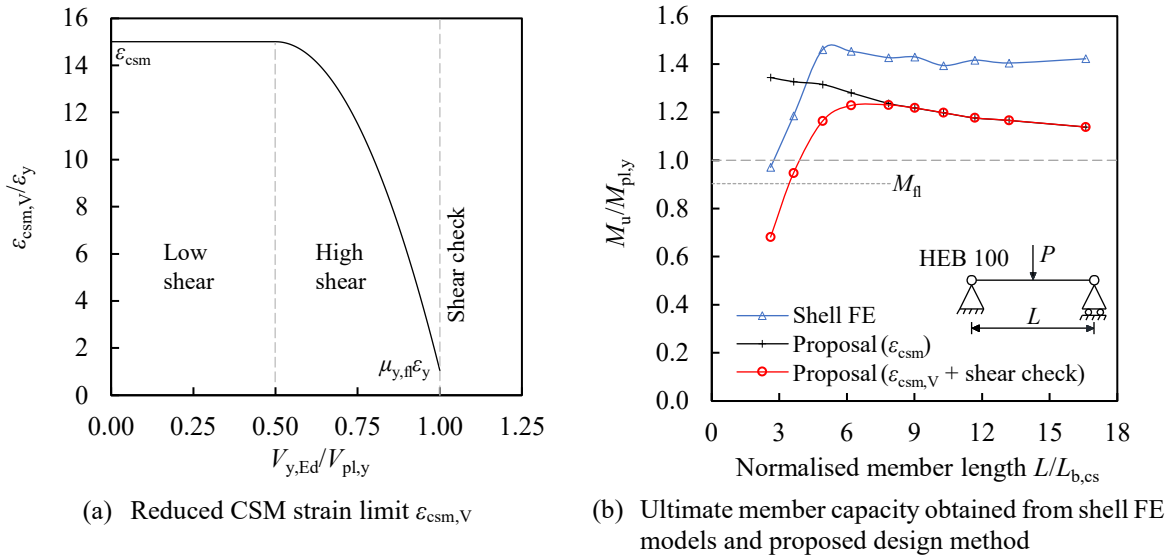


Figure 6.15 (a) Reduced CSM strain limit $\epsilon_{\text{csm},V}$ and (b) ultimate member capacity of members made of grade S355 steel and with HEB 100 cross-section subjected to in-plane 3-point major axis bending obtained from shell FE models and proposed design method with unreduced ϵ_{csm} and reduced $\epsilon_{\text{csm},V}$ CSM strain limits plus shear check

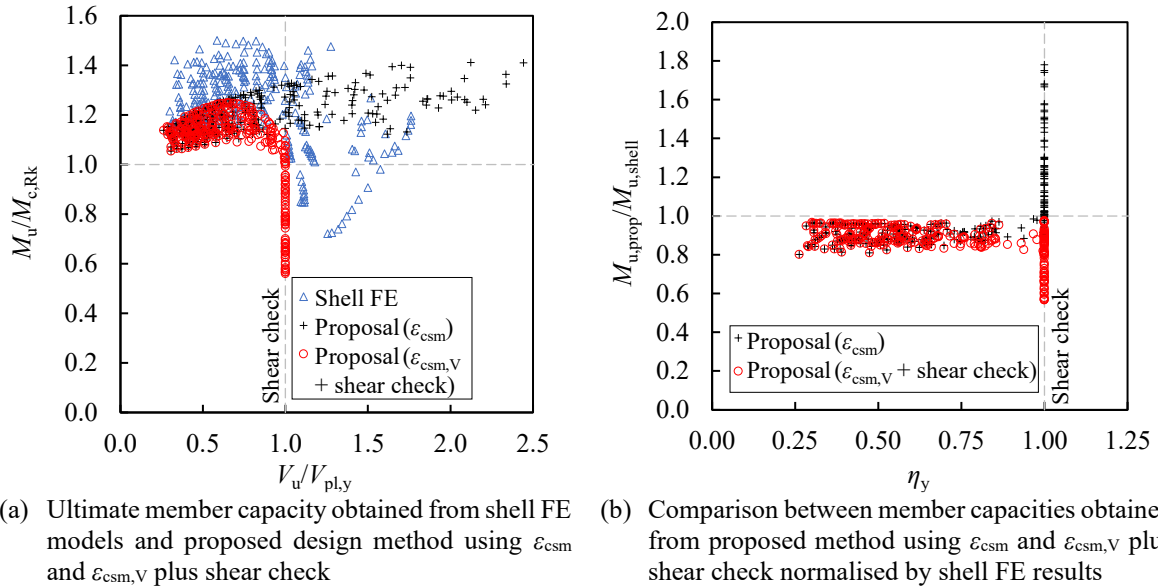


Figure 6.16 Comparison between the ultimate member capacities of I-section members subjected to in-plane 3-point major axis bending obtained from shell FE models and proposed design method using unreduced ϵ_{csm} and reduced $\epsilon_{\text{csm},V}$ CSM strain limits plus shear check

Figure 6.17 shows comparisons between member capacity predictions obtained using the proposed design method $M_{u,prop}$ with unreduced CSM limits ϵ_{csm} and reduced CSM limits $\epsilon_{csm,V}$ plus the shear check, which are normalised by shell FE results for I-section members (including 10 IPE, 10 HEB and 10 HEA profiles with 4 different member lengths) subjected to 3-point minor axis bending and 38 SHS/RHS members subjected to 3-point major and minor axis bending, versus the value of η_y or η_z at the attainment of the peak load factor or ϵ_{csm} . As can be seen from the figure, the application of the proposed design method with $\epsilon_{csm,V}$ provides safe and accurate member capacity predictions.

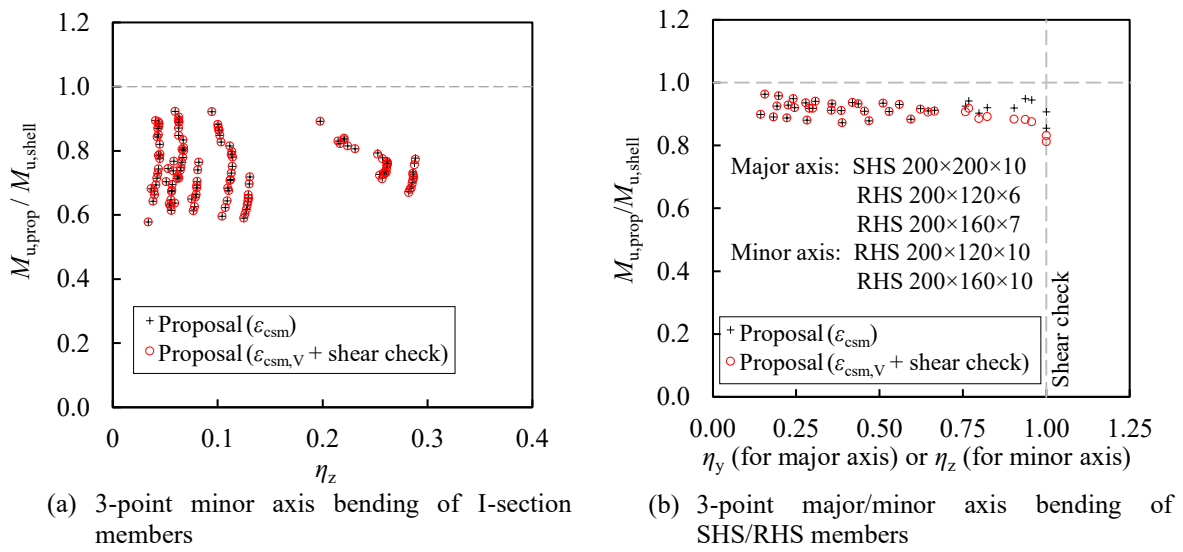


Figure 6.17 Comparison between member capacity predictions obtained using ϵ_{csm} and $\epsilon_{csm,V}$ plus shear check normalised by shell FE results for (a) I-section members subjected to 3-point minor axis bending and (b) SHS/RHS members subjected to 3-point major or minor axis bending

6.5.3 Accuracy of proposed design approach for laterally-unrestrained beams

In this subsection, the accuracy of the proposed design approach for beams subjected to bending alone and susceptible to LTB, where torsion is induced as a second order effect, is investigated. Figure 6.18 shows a comparison between the ultimate resistance predictions obtained from the proposed design method $M_{u,prop}$ using the reduced strain limit $\epsilon_{csm,V}$ determined from Eq. (6.16), which considers the influence of shear stresses resulting from applied shear forces and torsion (second order torsion in this case), and using the reduced strain limit proposed in Fieber et al. (2019a) $\rho_{csm}\epsilon_{csm}$ (see Section 6.4.2), which only considers the influence of shear stresses from applied shear for all studied beams subjected to non-uniform bending. It can be seen from the figure that the two approaches lead to very close predictions (differences are within 5%) of

member capacities. Since the consideration of shear effects due to second order torsion generally has only a slight influence on the strength predictions of steel beams subjected to bending alone, the formula (Eq. (4.1)) employed in Section 6.4.2, which only considers the influence of shear stresses from applied shear forces, can also be used in these cases as it provides accurate and safe member capacity predictions.

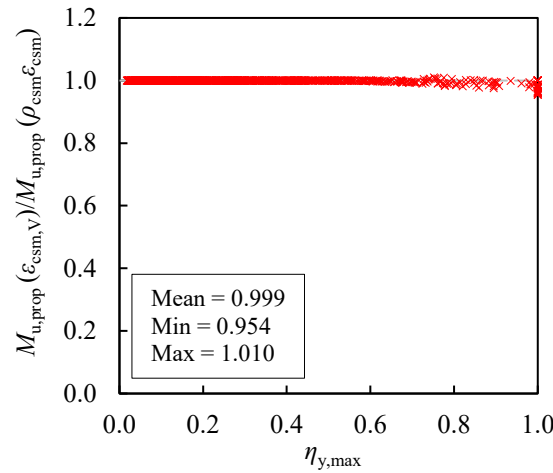


Figure 6.18 Comparison between the ultimate member capacities obtained from proposed design method using the reduced strain limit considering high shear-torsion effects proposed in this study $\epsilon_{csm,v}$ and the reduced strain limit proposed in Fieber et al. (2019a) $\rho_{csm}\epsilon_{csm}$ only considering shear effects due to shear force for all studied beams subjected to non-uniform bending

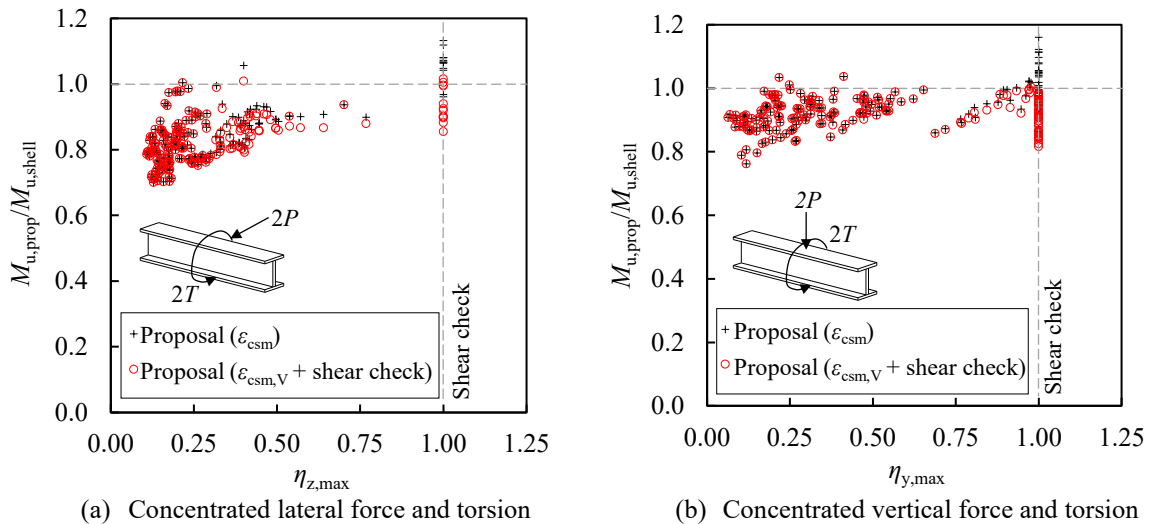
6.5.4 Accuracy of proposed design approach for members subjected to torsion

In this subsection, the accuracy of the proposed design method is assessed for steel beams subjected to torsion (directly applied rather than as a consequence of instability), considering different cross-sections, member lengths and loading conditions.

In the assessment of the proposed design approach for steel beams with I-sections and subjected to torsion, (i) 200 steel beams subjected to concentrated lateral load $2P$ (generating an internal shear force V_z) and concentrated torsion $2T$ at the midspan with the ratio of $0.5V_zL/T = 2, 5, 10$ and 20 , (ii) 200 steel beams subjected to concentrated vertical load $2P$ (generating an internal shear force V_y) and concentrated torsion $2T$ at the midspan with the ratio of $(T/T_{St,Rk})/(0.5V_yL/M_{pl,y}) = 0.4, 1, 4, 7$ and 10 , (iii) 200 steel beams subjected to uniform bending M and concentrated torsion $2T$ at the midspan with the ratio of $(T/T_{St,Rk})/(M/M_{pl,y}) = 0.4, 1, 4, 7$ and 10 and (iv) 180 extreme cases where steel beams were subjected to only concentrated torsion moments $2T$ at the midspan were considered.

In Figure 6.19, the ultimate capacities obtained from the proposed design method using unreduced ε_{csm} and reduced $\varepsilon_{\text{csm,V}}$ strain limits as well as shear checks versus the maximum shear-torsion interaction factors $\eta_{y,\text{max}}$ or $\eta_{z,\text{max}}$ along the member lengths at failure due to attainment of the peak loads or strain limits ε_{csm} without considering shear effects are illustrated. Note that in the figure, the ultimate strengths determined through the proposed design method are normalised by the benchmark shell FE results. As can be seen from Figure 6.19, the consideration of shear effects due to torsion now has a significant influence on the strength predictions of the considered members subjected to torsion. Particularly, for the case of members under pure torsion, the neglect of the shear effects from torsion in the application of the proposed method can lead to unsafe torsion resistance predictions $T_{\text{u,prop}}$, where the predictions can be up to 85% higher than the torsion resistances obtained from the shell FE models $T_{\text{u,shell}}$. On the other hand, as can be seen from Figure 6.19, if the shear check and the reduced strain limits $\varepsilon_{\text{csm,V}}$, which take account of the shear effect from torsion are employed, the great majority of the unsafe predictions shift to the safe side with only few predictions remaining slightly on the unsafe, but with acceptable margins (within 7%).

In addition to I-section steel members, 70 SHS/RHS members subjected to combined concentrated (vertical or lateral) load $2P$, generating a shear force V , and concentrated torsion $2T$ at the midspan and 15 SHS/RHS members subjected to pure torsion were also studied; the results are presented in Figure 6.20. Similar to the conclusions drawn from the results for I-section members, Figure 6.20 shows that it is necessary to consider the influence of high shear arising from the application of torsion and employ the reduced strain limits $\varepsilon_{\text{csm,V}}$ plus the shear check in the application of the proposed design method, to achieve accurate and safe ultimate capacity predictions.



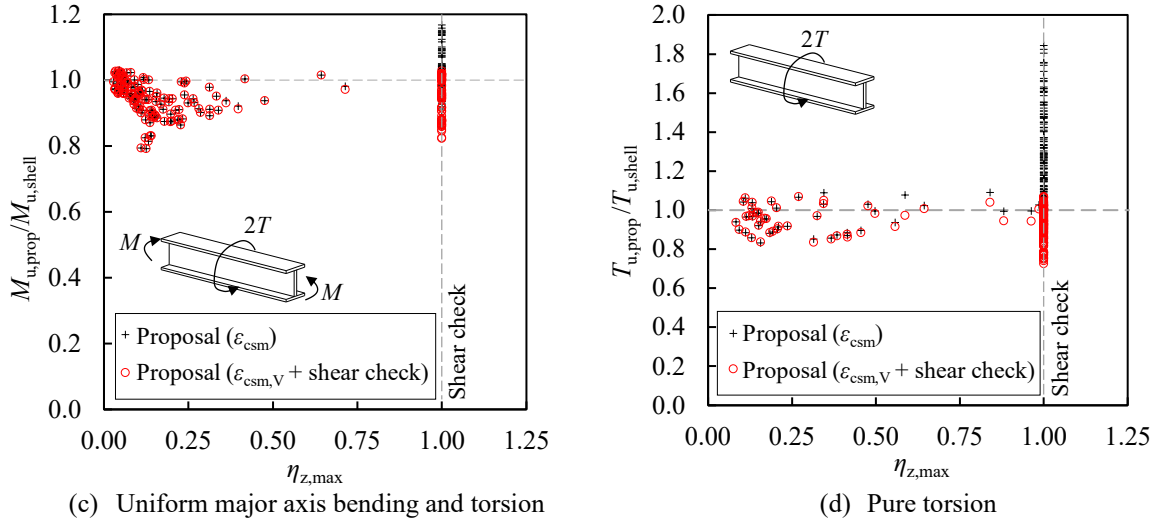


Figure 6.19 Comparisons between the ultimate capacity predictions determined through the proposed method using unreduced ϵ_{csm} and reduced $\epsilon_{\text{csm,V}}$ CSM strain limits plus shear check normalised by benchmark shell FE results for I-section members subjected to (a) concentrated lateral force and torsion, (b) concentrated vertical force and torsion, (c) uniform bending and torsion and (d) pure torsion

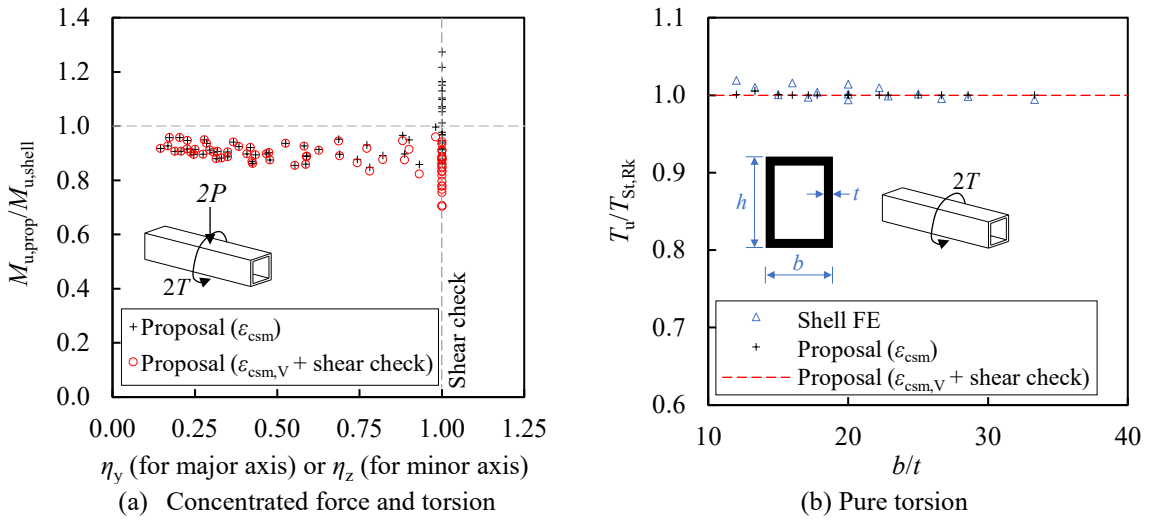


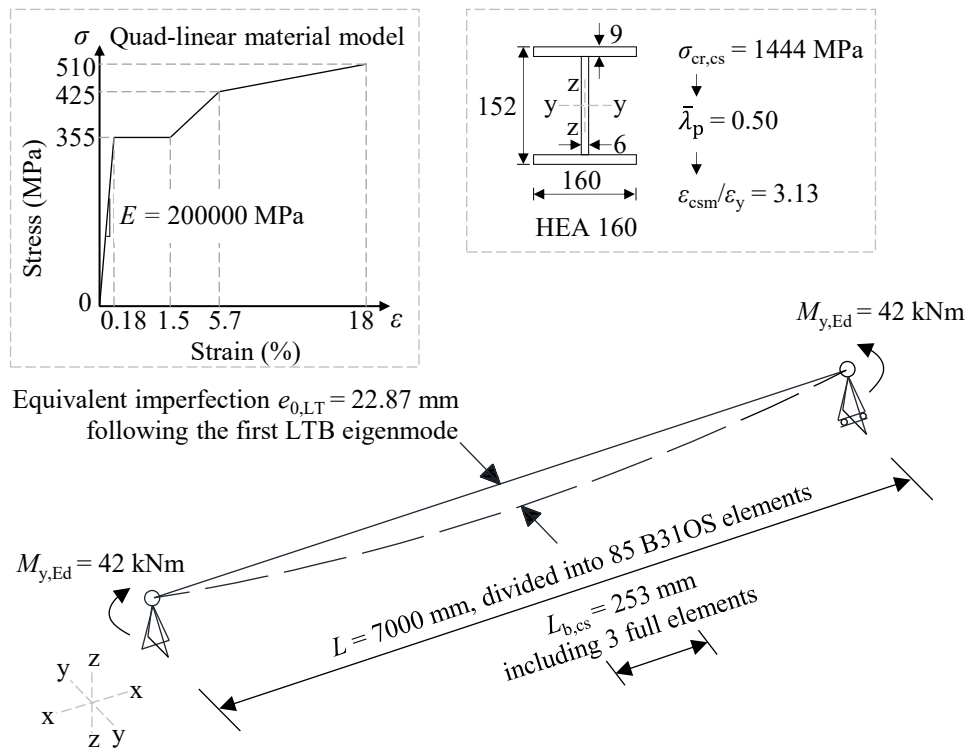
Figure 6.20 Comparisons between the ultimate capacity predictions determined through the proposed method using unreduced ϵ_{csm} and reduced $\epsilon_{\text{csm,V}}$ CSM strain limits plus shear check against benchmark shell FE results of SHS/RHS members subjected to (a) concentrated vertical or lateral force and torsion and (b) pure torsion

It should be noted that, in the application of the proposed design method, for members subjected to combined major or minor axis bending and torsion, the elastic local buckling stress $\sigma_{\text{cr,cs}}$ used to determine the cross-section slenderness $\bar{\lambda}_{\text{p}}$ and hence the CSM strain limit were calculated considering the normal stress distributions arising from major or minor axis bending moments. For the pure torsion cases, since the only normal stresses induced in the member are warping torsion normal stresses, which are distributed within each flange in a similar fashion to that

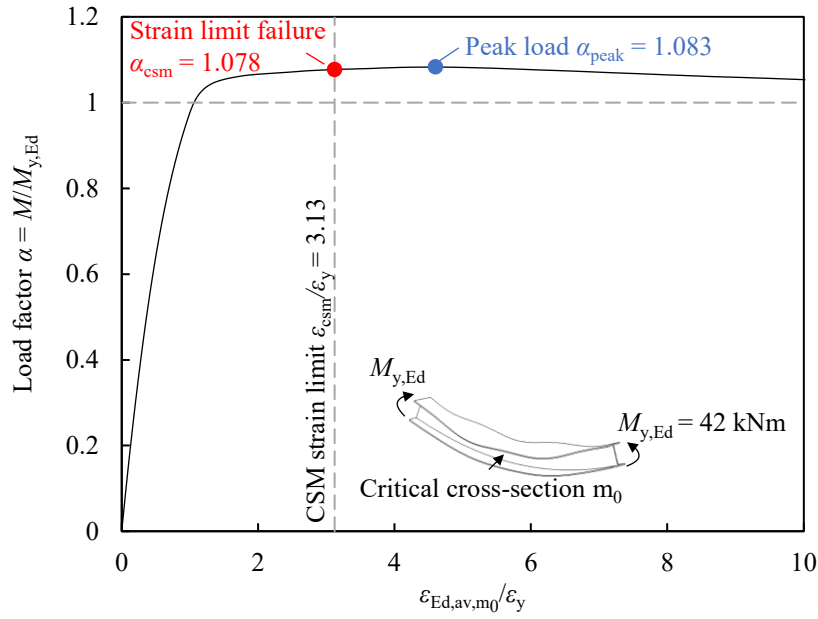
when the cross-section is subjected to minor axis bending, as shown in Figure 6.1, the elastic local buckling stress $\sigma_{cr,cs}$ was calculated assuming that the cross-section is under minor axis bending. For members under torsion, local buckling half-wavelengths $L_{b,cs}$ used to average strains were conservatively taken as the lower value of the local buckling half-wavelengths of the single web and single flange plates with fixed boundary conditions, which is the lower bound envelope of the local buckling half-wavelength of full cross-sections (Fieber et al., 2019b).

6.6 Worked example

A worked example is presented in this section to illustrate the implementation of the proposed design method. The example considers a beam made of grade S355 steel ($f_y = 355$ MPa, $f_u = 510$ MPa, characteristic value of $E = 200000$ MPa and $\nu = 0.3$) with a hot-rolled HEA 160 cross-section and a length of 7000 mm (i.e. $L = 7000$ mm); the considered beam is subjected to major axis bending moments $M_{y,Ed} = 42$ kNm applied at the two ends, as shown in Figure 6.21.



(a) Application of the proposed design method



(b) Load path and load factors at the attainment of peak load factor and CSM strain limit ϵ_{csm}

Figure 6.21 Worked example: design of a hot-rolled steel beam made of grade S355 steel with an HEA 160 cross-section (dimensions shown in mm) under uniform bending

The proposed design approach requires a GMNIA of the member using beam finite elements to be performed, as described in Section 6.2.1. As shown in Figure 6.21 (a), which summarises the application of the proposed method to the considered beam, the quad-linear material model described in Section 2.3.2, converted into a true stress-strain relationship was adopted; 85 B310S beam elements were used to model the member; the equivalent imperfection was modelled through the scaling of the first LTB eigenmode with an amplitude of $e_{0,LT} = \alpha_z L / 150 = 22.87$ mm, where α_z is the imperfection factor taken as 0.49 for the considered cross-section in accordance with the recommendations made in Section 5.5.1.1. After performing GMNIA, the design resistance factor of the beam can be obtained following the steps set out below.

I. Calculation of full cross-section elastic local buckling stress $\sigma_{cr,cs}$

The expressions developed by Gardner et al. (2019a) are used to determine the elastic local buckling stress of the full cross-section $\sigma_{cr,cs}$. The plate buckling coefficients for the isolated flange under uniform compression with simply-supported and fixed boundary conditions are equal to $k_f^{SS} = 0.43$ and $k_f^F = 1.25$, respectively. Likewise, the plate buckling coefficients for the isolated web under pure bending with simply-supported and fixed boundary conditions are equal to $k_w^{SS} = 23.9$ and $k_w^F = 39.6$, respectively. Using these buckling coefficients, the corresponding elastic buckling stresses of the isolated flange and web are determined as $\sigma_{cr,f}^{SS} =$

983.7 MPa, $\sigma_{cr,f}^F = 2859.7$ MPa, $\sigma_{cr,w}^{SS} = 7605.6$ MPa, $\sigma_{cr,w}^F = 12601.8$ MPa. The lower and upper bounds to the full cross-section local buckling stress are thus equal to $\sigma_{cr,p}^{SS} = 983.7$ MPa and $\sigma_{cr,p}^F = 2859.7$ MPa. The interaction coefficient ζ , to account for the effect of element interaction (Gardner et al., 2019a), is given by:

$$\zeta = \max\left(0.15 \frac{t_f}{t_w} \phi, \frac{t_w}{t_f} (0.4 - 0.25 \phi)\right) = 0.245 \quad (6.23)$$

where $\phi = \sigma_{cr,f}^{SS} / \sigma_{cr,w}^{SS} = 0.129$. The full cross-section elastic local buckling stress $\sigma_{cr,cs}$ is determined as

$$\sigma_{cr,cs} = \sigma_{cr,p}^{SS} + \zeta (\sigma_{cr,p}^F - \sigma_{cr,p}^{SS}) = 1444 \text{ MPa}. \quad (6.24)$$

Note that the finite strip analysis software CUFSM (Li & Schafer, 2010b) gives a full cross-section elastic local buckling stress of 1506 MPa.

II. Calculation of cross-section slenderness $\bar{\lambda}_p$

Using Eq. (2.13), the cross-section slenderness is determined as $\bar{\lambda}_p = \sqrt{f_y / \sigma_{cr,cs}} = 0.50$.

III. Calculation of CSM strain limit ε_{csm}

Based on the cross-section slenderness, the CSM strain limit can be calculated using Eqs. (2.9) and (2.10). For this example, with $\bar{\lambda}_p = 0.50$, the CSM strain limit $\varepsilon_{csm}/\varepsilon_y$ is equal to 3.13.

IV. Calculation of elastic local buckling half-wavelength $L_{b,cs}$

The expressions developed by Fieber et al. (2019b) are used to calculate the elastic local buckling half-wavelength $L_{b,cs}$ of the beam. The lower and upper bounds to the local buckling half-wavelength are given by $L_{b,p}^F = 132$ mm and $L_{b,p}^{SS} = 292.4$ mm. Using these values, the elastic local buckling half-wavelength of the full cross-section is determined as:

$$L_{b,cs} = L_{b,p}^{SS} - \zeta (L_{b,p}^{SS} - L_{b,p}^F) = 253 \text{ mm}. \quad (6.25)$$

Note that the finite strip analysis software CUFSM (Li & Schafer, 2010b) provides an elastic local buckling half-wavelength of 258 mm. Based on the calculated local buckling half-

wavelength $L_{b,cs} = 253$ mm, within which 3 beam elements fully lie, the average maximum compressive normal strain for each finite element $\varepsilon_{Ed,av,m}$ along the length of the considered beam can be determined.

V. Determination of failure mode and characteristic resistance factor

The load increments at the attainment of the peak load factor and CSM strain limit in the GMNIA can now be determined. Figure 6.21 (b) shows the load factor α (i.e. $M/M_{y,Ed}$) versus average maximum compressive strain at the critical location (i.e. the midspan in this case) path of the considered beam. It can be seen that in this case, the average strain at the midspan reaches the CSM strain limit prior to the attainment of the peak load factor. Thus, the member is deemed to fail at the load factor $\alpha_{csm} = 1.078$, and the characteristic resistance factor α_{Rk} is equal to 1.078 (i.e. $\alpha_{Rk} = 1.078$). Note that at the failure moment, the shear-torsion factors $\eta_y = 0$ and $\eta_z = 0.013$ are very small; thus, the shear due to torsion is negligible, and there is no need to reduce the strain limits.

VI. Determination of design resistance factor

As mentioned in Section 6.2.2, the application of the partial safety factor ($\gamma_{M1} = 1.0$ for member stability calculations) to the characteristic resistance factor is necessary to obtain the design resistance factor, as given by Eq. (6.1). Hence, in this worked example, the design resistance factor of the member is given by:

$$\alpha_{Rd} = \alpha_{Rk} / \gamma_{M1} = 1.078 / 1.0 = 1.078 > 1.0. \quad \therefore \text{Pass} \quad (6.26)$$

The beam is thus able to withstand the applied forces and its design bending moment resistance is equal to $M_{Rd} = \alpha_{Rd} M_{y,Ed} = 45.26$ kNm, according to the proposed method. On the other hand, using the lateral-torsional buckling design expressions provided in prEN 1993-1-1 (2019), the design bending moment resistance of the beam is calculated as 36.83 kNm, which is equal to 88% of the applied bending moment, indicating that the beam is not able to withstand the applied bending moment. It is worth noting that the benchmark shell FE model of the considered beam provides a resistance factor of $\alpha_{u,shell} = 1.079$ (i.e. $M_{u,shell} = 45.31$ kNm), which is very close to the ultimate resistance factor obtained through the proposed design method.

6.7 Concluding remarks

A new method for the design of steel beams against LTB performed by advanced inelastic

analysis using beam finite elements with CSM strain limits has been put forward in this chapter. The proposed method is performed by carrying out a GMNIA of a steel member using beam finite elements and defining the ultimate resistance as (i) the peak load factor α_{peak} after which the load-deformation curve descends or (ii) the load factor that corresponds to the attainment of the CSM strain limit α_{csm} , whichever occurs first. Shell FE models able to replicate the LTB response of steel beams were created and validated against experimental results from the literature. Using the validated shell FE models, the accuracy of the proposed design method was extensively verified for 300 steel I-section beams subjected to uniform bending and 1200 steel beams subjected to non-uniform bending, considering a range of European cross-section profiles, member slendernesses and bending moment gradients. It was shown that the proposed method consistently provides more accurate ultimate strength predictions relative to the traditional beam design method provided in prEN 1993-1-1 (2019). The high accuracy of the proposed design method derives from (i) recognising the interaction between cross-section elements during local buckling through the use of the cross-section slendernesses in the determination of the CSM strain limits, (ii) allowing for strain hardening, (iii) exploiting of partial plastification in Class 3 cross-sections and (iv) considering the beneficial effects of local strain gradients along the member lengths. The proposed method also removes the need for cross-section classification, individual member buckling checks and the calculation of effective lengths in the determination of the ultimate strengths of members, thereby significantly streamlining the structural steel design process and providing the ultimate resistances and failure modes of steel members and systems directly. In addition to steel beams under bending alone, the proposed method was also developed for and applied to steel I-section members subjected to (i) bending and shear, (ii) bending and torsion, (iii) bending, shear and torsion and (iv) pure torsion, where the safety and accuracy of method were illustrated. For pure bending cases, the consideration of shear effects due to second order torsion had only a slight influence on strength predictions, and thus can be eliminated in the proposed design method, while for members subjected to primary torsion actions, shear effects due to torsion have to be taken into account. Provisions for considering shear-torsion effects for SHS/RHS and channel sections were also presented in this chapter.

Chapter 7

Conclusions and further research

7.1 Conclusions

A comprehensive study into the behaviour and stability design of steel structures has been presented in this thesis. In this final chapter, the work and findings of the present study are briefly summarised, and suggestions for further research are also provided.

An introduction and overview of current structural steel stability design approaches developed in recent years is first presented in Chapters 1 and 2, identifying potential areas of improvement of the traditional methods at both cross-section and member levels. The essential knowledge of the proposed method for the stability design of steel structures is also provided, such as the CSM base curve, material modelling and equivalent imperfections. To overcome the shortcomings presented in traditional steel design methods, more safe and efficient methods for stability design of steel structures by second order inelastic analysis are developed in Chapters 3-6.

The first topic addressed in this thesis relates to the stability design at cross-section level, i.e. elastic local buckling of cross-sections. In current structural steel design specifications, the local buckling of cross-sections is typically treated on an element-by-element basis, with the boundary conditions along the adjoined longitudinal edges of the individual plates assumed to be simply-supported. In reality, cross-sections buckle locally as a whole and the individual plate elements interact. Accounting for the beneficial effects of this element interaction within cross-sections, explicit formulae to predict the elastic local buckling stress of full cross-sections of common profiles, including I-sections, have recently been developed (Gardner et al., 2019a). In Chapter 3, the existing formulae for single I-sections (Gardner et al., 2019a) are extended to cover the case of three-flanged cross-sections that arise in longitudinally-stiffened plate girders

and in the haunch and apex regions of portal frames. The formulae were benchmarked against numerical results generated by means of finite strip analysis on a large number of three-flanged cross-sections subjected to combined loading conditions ranging from pure compression to pure major axis bending. Similar to the element interaction observed between the flange and web within single I-sections, in the case of three-flanged cross-sections, the deeper and shallower sections were also found to interact with each other. Predictive formulae were developed allowing for the interaction both within and between the deeper and shallower I-sections. Using the developed formulae, predictions of the elastic local buckling stress of three-flanged cross-sections were shown to be typically within 5% of the numerically obtained values, around 30% higher than calculations performed on an element-by-element basis. The formulae can be used to facilitate the more efficient design of structures that feature three-flanged cross-sections, such as continuous and longitudinally-stiffened plate girders and portal frames.

At the member level, to overcome the shortcomings of traditional design methods, as well as responding to the changing landscape with regards to the sophistication of available structural analysis software and computational power, recently a new approach to structural steel design has been developed (Fieber et al., 2019a). In the design approach, GMNIA of the steel member is carried out and the ultimate strength of the member is signified by reaching either the strain limit defined according to the Continuous Strength Method (CSM) or the peak load factor, whichever occurs first. To consider the beneficial effect of strain gradients along the lengths of members on local cross-section resistances, the strains are averaged over the local buckling half-wavelength. In Chapter 4, the scope of this design approach is extended to laterally-restrained web-tapered steel members. The proposed design method is applied to tapered members subjected to compression, uniform bending and combined compression and uniform bending, and also to tapered members under non-uniform bending and combined axial compression and non-uniform bending. Various taper ratios, geometrical properties, loading conditions and member slenderness values are considered. The accuracy of the proposed design approach is verified against results from nonlinear shell FE modelling as well as a number of experiments on tapered members. The proposed method provides more accurate and consistent ultimate strength predictions than the traditional design method provided in EN 1993-1-1 (2005).

Attention then turns to the development of the proposed design approach for scenarios in which out-of-plane stability effects, with a focus on lateral-torsional buckling (LTB), govern. For

modelling convenience, so-called ‘equivalent’ imperfections are typically utilised, which consider the combined influence of both geometric imperfections and residual stresses. Equivalent imperfections for use in out-of-plane stability design by GNIA are provided in the current design specifications. However, no appropriate provisions are given for equivalent imperfections for use in design by GMNIA. Hence, to make the proposed method of design by second order inelastic analysis with strain limits applicable to the scenarios where out-of-plane stability effects (e.g. LTB) govern, the corresponding equivalent imperfections for use in GMNIA need to be established first. In Chapter 5, two shapes and corresponding amplitudes of equivalent imperfections for use in the out-of-plane stability design of steel and stainless steel members by GMNIA are developed: (1) the first LTB eigenmode with an amplitude $e_{0,\text{mod}} = \alpha_z L/150$ and (2) a lateral bow imperfection $e_{0,\text{bow}}$ combining of a half-sine wave with an amplitude $e_{0,\text{bow},1} = \alpha_z L/150$ and a full-sine wave with an amplitude $e_{0,\text{bow},2} = \alpha_z L/215$. The minor axis flexural buckling imperfection factor α_z , which is generally equal to the upper bound value on α_{LT} in prEN 1993-1-1 (2019) and prEN 1993-1-4 (2020), is utilised in the proposed imperfection amplitudes to capture the adverse influence of residual stresses, but without inheriting the additional compensatory effects included in α_{LT} for calibration of the LTB curves. In the design by GMNIA using the developed equivalent imperfections, cross-section checks or CSM strain limits can be used to capture cross-section failure; the latter more accurate approach is adopted in this thesis. Relative to the traditional Eurocode design calculations, employing the proposed LTB imperfections in GMNIA provides generally more accurate member resistance predictions, while remaining safe-sided relative to the benchmark FE results. The reliability of the design provisions is demonstrated through statistical analysis, where it is shown that partial safety factors of 1.0 and 1.1 for steel and stainless steel, respectively, can be safely adopted. The proposals are due to be included in the new upcoming Eurocode for design by finite element analysis – prEN 1993-1-14 (2019).

In Chapter 6, employing the equivalent imperfections proposed in Chapter 5, an accurate and consistent approach to the out-of-plane stability design of steel beams and structures utilising second order inelastic analysis with strain limits is proposed. The proposed method involves performing GMNIA using beam finite elements and employing CSM strain limits to determine cross-section failure. Using the validated shell FE models, the accuracy of the proposed design method is extensively verified for a large number of steel I-section beams subjected to uniform bending and non-uniform bending, considering a range of European cross-section profiles,

member slendernesses and bending moment gradients. It is shown that the proposed method consistently provides more accurate ultimate strength predictions relative to the traditional LTB design method provided in prEN 1993-1-1 (2019). In addition to steel beams under bending alone, the proposed method is also developed for and applied to steel I-section members subjected to (i) bending and shear, (ii) bending and torsion, (iii) bending, shear and torsion and (iv) pure torsion, where the safety and accuracy of method were illustrated. For pure bending cases, the consideration of shear effects due to second order torsion had only a slight influence on strength predictions, and thus can be eliminated in the proposed design method, while for members subjected to primary torsion actions, shear effects due to torsion have to be taken into account. Provisions for considering shear-torsion effects for SHS/RHS and channel sections are also presented in Chapter 6.

Overall, the stability design method developed in this thesis, performed by second order inelastic analysis with strain limits, consistently provides more accurate ultimate strength predictions relative to the traditional design method. The high accuracy of the proposed design method derives from (i) recognising the interaction between cross-section elements during local buckling through the use of the cross-section slendernesses in the determination of the CSM strain limits, (ii) allowing for strain hardening, (iii) exploiting of partial plastification in cross-sections and (iv) considering the beneficial effects of local strain gradients along the member lengths. The proposed method also removes the need for cross-section classification, individual member buckling checks and the calculation of effective lengths in the determination of the ultimate strengths of members, thereby significantly streamlining the structural steel design process.

7.2 Suggestions for further research

On the basis of the present thesis, further research could be conducted. The proposed method of design by second order inelastic analysis with CSM strain limits is particularly beneficial for bending dominated framed structures, such as typical steel portal frames. It would be worthwhile to extend the scope of the proposed design method to steel portal frames. Typically, in these types of structures, cross-sections of greater depth are arranged to coincide with the regions of large internal forces. Hence, these structures are generally made of tapered members or incorporate three-flanged cross-sections at the haunch and apex regions. This thesis has

provided the basic components for the extension to steel portal frames; Chapter 3 has provided the formulae for calculating the elastic local buckling stress of three-flanged cross-sections, which is required to determine the CSM strain limit in the application of the design method; Chapter 4 has confirmed the applicability of the design method of laterally-restrained web-tapered members. It is worthwhile to extend the applicability of the proposed design method to full portal frames. However, to enable the application of the strain averaging approach in the design method for portal frames, the formulae for calculating the local buckling half-wavelength of three-flanged cross-sections need to be developed first, which take account of the element interaction both within and between the deeper and shallower I-sections, similar to the approach employed in Chapter 3.

For the proposed design method to have wider applicability for scenarios in which out-of-plane stability effects govern, further work could be conducted. While Chapter 5 provides the equivalent imperfections for out-of-plane stability design by GMNIA, only bending cases are investigated, albeit with different bending moment diagrams arising from end moment loading, uniformly distributed loading and concentrated central point loading. It is worthwhile to verify the applicability of the equivalent imperfections developed in Chapter 5 or develop new equivalent imperfections for the stability design by GMNIA of steel beam-columns, which are subjected to combined compression and bending and thus may be susceptible to combined flexural buckling and lateral-torsional buckling.

Correspondingly, the out-of-plane stability design method developed in Chapter 6 is also only verified on members subjected to combined bending, shear and torsion, but without compression. Additional compression may increase the second order effects and induce more severe buckling behaviour of members. Extension to beam-columns, which are also subjected to compression simultaneously and may experience a combination of flexural buckling and lateral-torsional buckling, would be valuable.

Finally, to make the proposed design method a complete design tool, it would be worthwhile to investigate its applicability to three-dimensional steel frames, which are more complex than individual members and planar frames and require further research. Additionally, whether various beam-to-column connections, such as pinned, rigid and semi-rigid connections, influence the applicability of the proposed design method would also be of interest.

Furthermore, the present thesis mainly focused on I-section members, which are more

susceptible to out-of-plane failure. While Chapter 6 also presents the provisions for considering shear-torsion effects for SHS/RHS and channel sections, providing the initial feasibility of the proposed design method for steel members made of these sections, a more comprehensive study on the out-of-plane stability design of steel members and structures made of SHS/RHS and channel sections could be conducted.

References

- Abaqus. (2017). *ABAQUS/standard user's manual, version 6.17*. Dassault Systèmes Simulia Corp, USA.
- AISC. (2016). ANSI/AISC 360-16 – *Specification for structural steel buildings*. Illinois: American Institute of Steel Construction.
- AISI. (2006). *Direct strength method (DSM) design guide*. Committee on Specifications for the Design of Cold-Formed Steel Structural Members, American Iron and Steel Institute (AISI).
- AS 4100. (1998). *Standards Australia, Steel Structures*. Australian Building Codes Board, Sydney.
- Afshan, S., Francis, P., Baddoo, N. R., & Gardner, L. (2015). Reliability analysis of structural stainless steel design provisions. *Journal of Constructional Steel Research*, 114, 293–304.
- Afshan, S., & Gardner, L. (2013). The continuous strength method for structural stainless steel design. *Thin-Walled Structures*, 68, 42–49.
- Afshan, S., Zhao, O., & Gardner, L. (2019). Standardised material properties for numerical parametric studies of stainless steel structures and buckling curves for tubular columns. *Journal of Constructional Steel Research*, 152, 2–11.
- Arrayago, I., Real, E., & Gardner, L. (2015). Description of stress-strain curves for stainless steel alloys. *Materials and Design*, 87, 540–552.
- Ashraf, M., Gardner, L., & Nethercot, D. A. (2008). Structural stainless steel design: Resistance based on deformation capacity. *Journal of Structural Engineering*, 134(3), 402–411.
- Ashraf, M., & Young, B. (2011). Design formulations for non-welded and welded aluminium columns using Continuous Strength Method. *Engineering Structures*, 33(12), 3197–3207.
- Baddoo, N. R., Meza, F. J., & Gardner, L. (2020). Proposed overstrength for AISC 370 and ASCE 8, SCI Report.
- Bedair, O. (2009). Stability of web plates in W-shape columns accounting for flange/web interaction. *Thin-Walled Structures*, 47(6–7), 768–775.
- Buonopane, S. G., & Schafer, B. W. (2006). Reliability of steel frames designed with advanced analysis. *Journal of Structural Engineering*, 132(2), 267–276.
- Butler, D. J., & Anderson, G. B. (1963). The elastic buckling of tapered beam-columns.

Welding Research Supplement, 29–36.

Byfield, M. P., & Nethercot, D. A. (1998). An analysis of the true bending strength of steel beams. *Proceedings of the ICE-Structures and Buildings*, 128(2), 188–197.

CEN/TR 1993-1-103. (2017). *Eurocode 3: Design of Steel Structures – Part 1-103: Elastic Critical Buckling of Members*. Brussels, European Committee for Standardization (CEN).

Chan, S. L., & Chen, W. F. (2005). Advanced analysis as a new dimension for structural steel design. *Advanced Steel Construction*, 1, 87–102.

Chan, S. L., Liu, S. W., & Liu, Y. P. (2018). Second-order direct analysis of steel structures made of tapered members. *Proceedings of the 12th International Conference on Advances in Steel-Concrete Composite Structures (ASCCS 2018)*, University of Politecnica Valencia, Valencia, Spain, 75–82.

Couto, C., & Vila Real, P. (2019). Numerical investigation on the influence of imperfections in the lateral-torsional buckling of beams with slender I-shaped welded sections. *Thin-Walled Structures*, 145, 106429.

Cristutiu, I. M., Nunes, D. L., & Dogariu, A. I. (2012). Experimental study on laterally restrained steel columns with variable I cross sections. *Steel and Composite Structures*, 13(3), 225–238.

Demirhan, A. L., Eroğlu, H. E., Mutlu, E. O., Yılmaz, T., & Anil, Ö. (2020). Experimental and numerical evaluation of inelastic lateral-torsional buckling of I-section cantilevers. *Journal of Constructional Steel Research*, 168, 105991.

dos Santos, G. B., Gardner, L., & Kucukler, M. (2018). A method for the numerical derivation of plastic collapse loads. *Thin-Walled Structures*, 124, 258–277.

Driscoll Jr., G. C., & Beedle, L. S. (1957). The plastic behavior of structural members and frames. *Welding Journal*, 36(6), 275-s.

Dux, P. F., & Kitipornchai, S. (1983). Inelastic beam buckling experiments. *Journal of Constructional Steel Research*, 3(1), 3–9.

Ebel, R. (2014). Systemabhängiges Tragverhalten und Tragfähigkeiten stabilitätsgefährdeter Stahlträger unter einachsiger Biegebeanspruchung (System-dependent bearing characteristics and load bearing capacities of steel beams under uniaxial bending), Dissertation, Ruhr-Universität Bochum.

ECCS. (1976). *Manual on stability of steel structures*. European Convention for Constructional Steelwork (ECCS). Report number: 22.

ECCS. (1984). *Ultimate limit state calculation of sway framed with rigid joints*. European Convention for Constructional Steelwork (ECCS). Report number: 33.

Ellobody, E. (2014). *Finite element analysis and design of steel and steel-concrete composite bridges*, Butterworth-Heinemann, USA.

- EN 1090-2. (2008). *Execution of Steel Structures and Aluminium Structures – Part 2: Technical Requirements for Steel Structures*. Brussels, European Committee for Standardization (CEN).
- EN 1990. (2002). *Eurocode – Basis of structural design*. Brussels, European Committee for Standardization (CEN).
- EN 1993-1-1. (2005). *Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings*. Brussels, European Committee for Standardization (CEN).
- EN 1993-1-4: 2006 + A1: 2015. (2015). *Eurocode 3: Design of steel structures – Part 1-4: General rules – Supplementary rules for stainless steels*. Brussels, European Committee for Standardization (CEN).
- EN 1993-1-5. (2006). *Eurocode 3: Design of steel structures – Part 1-5: Plated structural elements*. Brussels, European Committee for Standardization (CEN).
- Ermopoulos, J. C. (1997). Equivalent buckling length of non-uniform members. *Journal of Constructional Steel Research*, 42 (2), 141–158.
- Estabrooks, B. G., & Grondin, G. Y. (2008). Combined bending and torsion of steel I-shaped beams, Structural Engineering Report 276, University of Alberta.
- Fieber, A., Gardner, L., & Macorini, L. (2019a). Design of structural steel members by advanced inelastic analysis with strain limits. *Engineering Structures*, 199, 109624.
- Fieber, A., Gardner, L., & Macorini, L. (2019b). Formulae for determining elastic local buckling half-wavelengths of structural steel cross-sections. *Journal of Constructional Steel Research*, 159, 493–506.
- Fieber, A., Gardner, L., & Macorini, L. (2020). Structural steel design using second-order inelastic analysis with strain limits. *Journal of Constructional Steel Research*, 168, 105980.
- Fortan, M., & Rossi, B. (2020). Lateral Torsional Buckling of Welded Stainless-Steel I-Profile Beams: Design and Reliability. *Journal of Structural Engineering*, 146(12), 04020280.
- Galambos, T. V. (1968). *Structural Members and Frames*, Prentice-Hall, Englewood Cliffs, NJ.
- Gardner, L. (2008) The continuous strength method. *Proceedings of the Institution of Civil Engineers – Structures and Buildings*. 161 (3), 127–133.
- Gardner, L., Fieber, A., & Macorini, L. (2019a). Formulae for calculating elastic local buckling stresses of full structural cross-sections. *Structures*, 17, 2–20.
- Gardner, L., & Ng, K. T. (2006). Temperature development in structural stainless steel sections exposed to fire. *Fire Safety Journal*, 41(3), 185–203.
- Gardner, L., Saari, N., & Wang, F. (2010). Comparative experimental study of hot-rolled and cold-formed rectangular hollow sections. *Thin-Walled Structures*, 48(7), 495–507.
- Gardner, L., Wang, F. C., & Liew, A. (2011). Influence of strain hardening on the behaviour

- and design of steel structures. *International Journal of Structural Stability and Dynamics*, 11(5), 855–875.
- Gardner, L., & Yun, X. (2018). Description of stress-strain curves for cold-formed steels. *Construction and Building Materials*, 189, 527–538.
- Gardner, L., Yun, X., Fieber, A., & Macorini, L. (2019b). Steel Design by Advanced Analysis: Material Modeling and Strain Limits. *Engineering*, 5(2), 243–249.
- Gioncu, V., & Petcu, D. (1997). Available rotation capacity of wide-flange beams and beam-columns Part 2. Experimental and numerical tests. *Journal of Constructional Steel Research*, 43(1-3), 219-244.
- Greiner, R., Salzgeber, G., & Ofner, R. (2000). New lateral-torsional buckling curves χ_{LT} – Numerical simulations and design formulae, ECCS TC8 report no. 2000-14, European Commission for Constructional Steelwork, Brussels.
- Han, K. H., & Lee, C. H. (2016). Elastic flange local buckling of I-shaped beams considering effect of web restraint. *Thin-Walled Structures*, 105, 101–111.
- HOLLOSSTAB. (2019). Overall-slenderness based direct design for strength and stability of innovative hollow sections. *Grant Agreement Number: RFCS-2015-709892, Deliverable D7.2 - Reliability Analysis of the Design Proposals and Determination of Partial Factors*.
- Hughes, A. F., Iles, D. C., & Malik, A. S. (2011). SCI P385 Design of Steel Beams in Torsion.
- Johnson, D. (1985). An investigation into the interaction of flanges and webs in wide flange shapes. *Proceedings SSRC Annual Technical Session*, 395–405, Cleveland (OH).
- Kaehler, R. C., White, D. W., & Kim, Y. D. (2011). *Frame design using web-tapered members, Design Guide 25*, American Institute of Steel Construction (AISC) and Metal Building Manufacturers Association (MBMA), Chicago.
- Kindmann, R., & Beier-Tertel, J. (2010). Geometrische ersatzimperfektionen für das biegedrillknicken von trägern aus walzprofilen - Grundsätzliches. *Stahlbau*, 79(9), 689–697.
- King, C. M. (2001). Design moment resistance at $\bar{\lambda}_{LT} = 0.4$. ECCS TC8 paper no. 2001-18, European Commission for Constructional Steelwork, Brussels.
- Knobloch, M., Kuhlmann, U., Lindner, L., Jörg, F., & Winkler, R. (2019). Effect of shear forces on the bending moment resistance – Proposal of Amendments on prEN 1993-1-1:2018 Cl.8.2.8, CEN/TC 250/SC 3/WG 1 N 272.
- Kucukler, M., & Gardner, L. (2018). Design of laterally restrained web-tapered steel structures through a stiffness reduction method. *Journal of Constructional Steel Research*, 141, 63–76.
- Kucukler, M., & Gardner, L. (2019). Design of web-tapered steel beams against lateral-torsional buckling through a stiffness reduction method. *Engineering Structures*, 190, 246–261.
- Kucukler, M., Gardner, L., & Macorini, L. (2014). A stiffness reduction method for the in-plane design of structural steel elements. *Engineering Structures*, 73, 72–84.

- Kucukler, M., Gardner, L., & Macorini, L. (2015a). Flexural-torsional buckling assessment of steel beam-columns through a stiffness reduction method. *Engineering Structures*, 101, 662–676.
- Kucukler, M., Gardner, L., & Macorini, L. (2015b). Lateral-torsional buckling assessment of steel beams through a stiffness reduction method. *Journal of Constructional Steel Research*, 109, 87–100.
- Kucukler, M., Gardner, L., & Macorini, L. (2016). Development and assessment of a practical stiffness reduction method for the in-plane design of steel frames. *Journal of Constructional Steel Research*, 126, 187–200.
- Kucukler, M., Xing, Z., & Gardner, L. (2019). Behaviour and design of stainless steel I-section columns in fire. *Journal of Constructional Steel Research*, 165, 105890.
- Lay, M. G. (1964). *Some studies of flange local buckling in wide-flange shapes*. Fritz Engineering Laboratory Report No. 297.10. Lehigh University.
- Lay, M. G., & Galambos, T. V. (1964). *The inelastic behavior of beams under moment gradient*. Fritz Engineering Laboratory Report No. 297.12, Lehigh University.
- Li, Z., & Schafer, B. W. (2010a). Application of the finite strip method in cold-formed steel member design. *Journal of Constructional Steel Research*, 66(8–9), 971–980.
- Li, Z., & Schafer, B. W. (2010b). Buckling analysis of cold-formed steel members with general boundary conditions using CUFSM: Conventional and constrained finite strip methods. *Proceedings of the Twentieth International Specialty Conference on Cold-Formed Steel Structures*, 17–31.
- Liew, J. Y. R., Chen, W. F., & Chen, H. (2000). Advanced inelastic analysis of frame structures. *Journal of Constructional Steel Research*, 55(1–3), 245–265.
- Lindner, J., & Glitsch, T. (2004). Vereinfachter Nachweis für I- und U-Träger beansprucht durch doppelte Biegung und Torsion. *Stahlbau*, 73(9), 704–715.
- Lindner, J., Kuhlmann, U., & Jörg, F. (2018). Initial bow imperfections ϵ_0 for the verification of Flexural Buckling According to Eurocode 3 Part 1-1 – additional considerations. *Steel Construction*, 11(1), 30–41.
- Lindner, J., Kuhlmann, U., & Just, A. (2016). Verification of flexural buckling according to Eurocode 3 part 1-1 using bow imperfections. *Steel Construction*, 9(4), 349–362.
- Lu, Y., Li, W., Zhou, T., & Wu, H. (2017). Novel local buckling formulae for cold-formed C-section columns considering end condition effect. *Thin-Walled Structures*, 116, 265–276.
- Lundquist, E., & Stowell, E. (1941). *Critical compressive stress for at rectangular plates supported along all edges and elastically restrained against rotation along the unloaded edges*. Special Report 189, National Advisory Committee for Aeronautics.
- Lundquist, E., & Stowell, E. (1942). *Critical compressive stress for outstanding flanges*. Report No. 734, National Advisory Committee for Aeronautics.

- Marques, L. (2012). *Tapered steel members: flexural and lateral-torsional buckling*, PhD thesis, University of Coimbra.
- Marques, L., Simões Da Silva, L., Greiner, R., Rebelo, C., & Taras, A. (2013). Development of a consistent design procedure for lateral-torsional buckling of tapered beams. *Journal of Constructional Steel Research*, 89, 213–235.
- Marques, L., Taras, A., Simões Da Silva, L., Greiner, R., & Rebelo, C. (2012). Development of a consistent buckling design procedure for tapered columns. *Journal of Constructional Steel Research*, 72, 61–74.
- Meng, X., & Gardner, L. (2020a). Behavior and design of normal- and high-strength steel SHS and RHS columns. *Journal of Structural Engineering*, 146(11), 04020227.
- Meng, X., & Gardner, L. (2020b). Cross-sectional behaviour of cold-formed high strength steel circular hollow sections. *Thin-Walled Structures*, 156, 106822.
- Meng, X., & Gardner, L. (2020c). Simulation and design of semi-compact elliptical hollow sections. *Engineering Structures*, 202, 109807.
- Meng, X., Gardner, L., Sadowski, A. J., & Rotter, J. M. (2020). Elasto-plastic behaviour and design of semi-compact circular hollow sections. *Thin-Walled Structures*, 148, 106486.
- Mirambell, E., Bordallo, J., & Real, E. (2016). Torsion and its interaction with other internal forces in EN 1993-1-1 - a new approach. *Steel Construction*, 9(3), 240–248.
- Mirambell, E., & Real, E. (2000). On the calculation of deflections in structural stainless steel beams: An experimental and numerical investigation. *Journal of Constructional Steel Research*, 54(1), 109–133.
- Mohri, F., Brouki, A., & Roth, J. C. (2003). Theoretical and numerical stability analyses of unrestrained, mono-symmetric thin-walled beams. *Journal of Constructional Steel Research*, 59(1), 63–90.
- Möller, M. (1995). *Local flange buckling*, PhD thesis, Lulea University of Technology.
- Nethercot, D. A., & Trahair, N. S. (1976). Inelastic lateral buckling of determinate beams. *Journal of the Structural Division-ASCE*, 102(4), 701–717.
- Ozbasaran, H., Aydin, R., & Dogan, M. (2015). An alternative design procedure for lateral-torsional buckling of cantilever I-beams. *Thin-Walled Structures*, 90, 235–242.
- Pi, Y. L., & Trahair, N. S. (1995). Inelastic torsion of steel I-beams. *Journal of Structural Engineering*, 121(4), 609–620.
- Prawel, S., Morell, M., & Lee, G. (1974). Bending and buckling strength of tapered structural members. *Welding Research Supplement*, 75–84.
- prEN 1993-1-1. (2019). *Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings*. Brussels, European Committee for Standardization (CEN).

- prEN 1993-1-14. (2019). *Eurocode 3: Design of steel structures – Part 1-14: Design by FE analysis*. Brussels, European Committee for Standardization (CEN).
- prEN 1993-1-4. (2020). *Eurocode 3: Design of steel structures – Part 1-4: General rules – Supplementary rules for stainless steels*. Brussels, European Committee for Standardization (CEN).
- Raftoyiannis, I. G., & Ermopoulos, J. C. (2005). Stability of tapered and stepped steel columns with initial imperfections. *Engineering Structures*, 27(8), 1248–1257.
- Rasmussen, K. J. R. (2003). Full-range stress-strain curves for stainless steel alloys. *Journal of Constructional Steel Research*, 59(1), 47–61.
- Sabat, A. K. (2009). *Lateral-torsional buckling analysis of steel frames with corrugated webs*, Master thesis, Chalmers University of Technology.
- SAFEBRICKTILE. (2016). Standardization of safety assessment procedures across brittle to ductile failure modes. *Grant Agreement Number: RFSR-CT-2013-00023, Deliverable D1.1 - Guideline for the Safety Assessment of Design Rules for Steel Structures in Line with EN 1990*.
- Sahraei, A. (2017). *Advances in lateral torsional buckling analysis of beam-columns and plane frames*, PhD thesis, University of Ottawa.
- Schafer, B. W. (2008). Review: The Direct Strength Method of cold-formed steel member design. *Journal of Constructional Steel Research*, 64(7–8), 766–778.
- Schaper, L., Jörg, F., Winkler, R., Kuhlmann, U., & Knobloch, M. (2019). The simplified method of the equivalent compression flange: Development based on LTB tests and residual stress measurements. *Steel Construction*, 12(4), 264–277.
- Seaburg, P. A., & Carter, C. J. (2003). *Torsional Analysis of Structural Steel Members*, AISC Steel Design Guide Series No.9.
- Seif, M., & Schafer, B. W. (2010). Local buckling of structural steel shapes. *Journal of Constructional Steel Research*, 66(10), 1232–1247.
- SEMI-COMP. (2009). *Plastic member capacity of semi-compact steel sections — a more economic design*. European Commission. Report number: EUR 23735.
- Shiomi, H., & Kurata, M. (1984). Strength formula for tapered beam-columns. *Journal of Structural Engineering*, 110(7), 1630–1643.
- Simões da Silva, L., Tankova, T., & Martins, J. P. (2019). Stability design of steel structures: From members to plates and shells. *Proceedings of the International Colloquia on Stability and Ductility of Steel Structures (SDSS 2019)*, Prague, Czech Republic. CRC Press, 2019.
- Snijder, H. H., van der Aa, R. P., Hofmeyer, H., & van Hove, B. W. E. M. (2016). Design imperfections for steel beam lateral torsional buckling. *Proceedings of the International Colloquium on Stability and Ductility of Steel Structures, SDSS 2016*, 549–556.
- Snijder, H. H., van der Aa, R. P., Hofmeyer, H., & van Hove, B. W. E. M. (2018). Lateral

- torsional buckling design imperfections for use in non-linear FEA. *Steel Construction*, 11(1), 49–56.
- Stroetmann, R., & Fominow, S. (2019). Parameter studies on imperfections for the LTB-design of members based on EN 1993-1-1. *Steel Construction*, 12(4), 299–308.
- Stroetmann, R., & Lindner, J. (2010). Knicknachweise nach DIN en 1993-1-1. *Stahlbau*, 79(11), 793–808.
- Su, M. N., Young, B., & Gardner, L. (2014). Deformation-based design of aluminium alloy beams. *Engineering Structures*, 80, 339–349.
- Su, M. N., Young, B., & Gardner, L. (2016). The continuous strength method for the design of aluminium alloy structural elements. *Engineering Structures*, 122, 338–348.
- Tankova, T. (2018). *Stability design of steel columns, beams and beam-columns: behaviour, general formulation and reliability*, PhD thesis, University of Coimbra.
- Tankova, T., Martins, J. P., Simões da Silva, L., Marques, L., Craveiro, H. D., & Santiago, A. (2018a). Experimental lateral-torsional buckling behaviour of web tapered I-section steel beams. *Engineering Structures*, 168, 355–370.
- Tankova, T., Martins, J. P., Simões da Silva, L., Simões, R., & Craveiro, H. D. (2018b). Experimental buckling behaviour of web tapered I-section steel columns. *Journal of Constructional Steel Research*, 147, 293–312.
- Tankova, T., Simões da Silva, L., & Marques, L. (2018). Buckling resistance of non-uniform steel members based on stress utilization: General formulation. *Journal of Constructional Steel Research*, 149, 239–256.
- Taras, A. (2010). *Contribution to the development of consistent stability design rules for steel members*, PhD thesis, Technical University of Graz.
- Taras, A., & Greiner, R. (2010). New design curves for lateral-torsional buckling-Proposal based on a consistent derivation. *Journal of Constructional Steel Research*, 66(5), 648–663.
- Trahair, N. S. (2010). Steel cantilever strength by inelastic lateral buckling. *Journal of Constructional Steel Research*, 66(8–9), 993–999.
- Trahair, N. S., Bradford, M. A., Nethercot, D. A., & Gardner, L. (2008). *The behaviour and design of steel structures to EC3 (4th edition)*. Taylor & Francis.
- Trahair, N. S., & Hancock, G. J. (2004). Steel member strength by inelastic lateral buckling. *Journal of Structural Engineering*, 130(1), 64–69.
- Trahair, N. S., & Kitipornchai, S. (1972). Buckling of inelastic I-beams under uniform moment. *Journal of the Structural Division-ASCE*, 98(11), 2551–2566.
- Tusnin, A. R., & Prokic, M. (2015a). Experimental research of I-beams under bending and torsion actions. *Magazine of Civil Engineering*, 53(1), 24–31.

- Tusnin, A. R., & Prokic, M. (2015b). Selection of parameters for I-beam experimental model subjected to bending and torsion. *Procedia Engineering*, 111, 789–796.
- van der Aa, R. P. (2015). *Numerical assessment of the design imperfections for steel beam lateral torsional buckling*. Master thesis, Eindhoven University of Technology.
- Vayas, I., Ermopoulos, J., & Ioannidis, G. (2019). *Design of Steel Structures to Eurocodes*.
- Walport, F., Gardner, L., & Nethercot, D. A. (2020a). Design of structural stainless steel members by second order inelastic analysis with CSM strain limits. *Thin-Walled Structures*, 159, 107267.
- Walport, F., Gardner, L., & Nethercot, D. A. (2020b). Equivalent bow imperfections for use in design by second order inelastic analysis. *Structures*, 26, 670–685.
- Walport, F., Gardner, L., Real, E., Arrayago, I., & Nethercot, D. A. (2019). Effects of material nonlinearity on the global analysis and stability of stainless steel frames. *Journal of Constructional Steel Research*, 152, 173–182.
- Wang, J., Afshan, S., Gkantou, M., Theofanous, M., Baniotopoulos, C., & Gardner, L. (2016). Flexural behaviour of hot-finished high strength steel square and rectangular hollow sections. *Journal of Constructional Steel Research*, 121, 97–109.
- Yuan, H. X., Wang, Y. Q., Shi, Y. J., & Gardner, L. (2014). Residual stress distributions in welded stainless steel sections. *Thin-Walled Structures*, 79, 38–51.
- Yun, X., & Gardner, L. (2017). Stress-strain curves for hot-rolled steels. *Journal of Constructional Steel Research*, 133, 36–46.
- Yun, X., Gardner, L., & Boissonnade, N. (2018a). The continuous strength method for the design of hot-rolled steel cross-sections. *Engineering Structures*, 157, 179–191.
- Yun, X., Gardner, L., & Boissonnade, N. (2018b). Ultimate capacity of I-sections under combined loading – Part 2: Parametric studies and CSM design. *Journal of Constructional Steel Research*, 148, 265–274.
- Yun, X., Wang, Z., & Gardner, L. (2020). Full-range stress-strain curves for aluminum alloys. *Journal of Structural Engineering*, 147(6), 04021060.
- Yura, J. A., Ravindra, M. K., & Galambos, T. V. (1978). The bending resistance of steel beams. *Journal of the Structural Division-ASCE*, 104(9), 1355–1370.
- Xing, Z., Kucukler, M., & Gardner, L. (2020). Local buckling of stainless steel plates in fire. *Thin-Walled Structures*, 148, 106570.