

Planning Capacity Expansion Under Oligopoly: The case of the petrochemical industry

by

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Abstract

In its early stages of development, the petrochemical industry was characterised as being an innovative and R&D intensive industry, enjoying expanding markets and rapid growth, with high profits. By the 1970's the industry was already showing signs of maturity, with an increase in the number of players and slowing growth. Technology was becoming standardised, and the need to exploit economies of scale for new plants was the order of the day. This process continued in the 1980's and 1990's; rapid growth gave way to cyclical demand and the emergence of significant overcapacity. Industry profitability had become an issue. Concern about overcapacity and uncertainty about future demand precipitated a process of restructuring of the industry with changing roles for major players, merger activity, and consolidation.

This thesis explores the relationship between overcapacity and the investment behaviour of firms, and the role of demand uncertainty. A dynamic capacity expansion model of the linear quadratic type is developed and formulated in a game theoretic framework for rival firms serving a single market, and producing a homogenous product. Price is endogenised as an inverse demand function, making the actions of the firms interdependent and capturing the interaction between them. The model allows for adjustment costs; in the empirical simulations economies of scale and learning effects are also accounted for.

Initially formulated for a duopoly, the model is then extended to deal with an oligopoly. It is applied to the US styrene industry which serves as a proxy for the petrochemical industry. The industry is examined over a period of 23 years. Optimal control theory and dynamic programming techniques, are used to obtain optimal solutions, under different investment strategies, for investment and output rates of rival firms. The model performs well in explaining the evolution of industry capacity, production and prices, and most of the overcapacity over the study period.

The investment behaviour of the major styrene producers is best described by a feedback Nash-Cournot strategy, indicating that overcapacity can be attributed to strategic considerations. An examination of market developments also reveals that major US styrene producers invested strategically. Building larger plants to benefit from economies of scale, and increased integration of production operations contributed to raising entry barriers and the low levels of entry into the styrene industry.

While uncertainty causes firms to increase their investment rate in this model, the simulation results show that industry overcapacity could not have been caused solely by concern about demand uncertainty.

To the constellation of stars that shines every night over the dark skies, in its eternal journey south by southeast, over mount Hermon.

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Chapter 1

Introduction

1.1 Overcapacity and the Investment Decision

Capacity expansion modelling, which is the main concern of this thesis, has grown in significance to occupy a small but expanding segment of the industrial organisation literature since the early pioneering contributions of Chenery (1952), Manne (1961), Lucas, Jr. (1967), Arrow (1968), and Nickell (1978). Driven by advances in oligopoly theory, optimal control, and the explosive rise in computing power and memory of new computers, more recent contributions cover dynamic and sophisticated models applied to empirical work in areas such as electric power generation, water and natural gas networks, petrochemicals, and newsprint. It was these same large scale industries which in the first place raised the interest of the pioneering works; due to the importance of economies of scale and the inherent association with overcapacity, or the economic penalties involved in accumulating backlogs of unsatisfied demand in these industries. The high cost of investments involved made it imperative for early modelers and planners to strive to find optimal capacity investment rates especially when firms were facing uncertain demand growth and economies of scale. These issues continue to be the concern and subject of research to the present day. Building on advances in oligopoly theory the models have become more developed and the techniques sophisticated, and a wider coverage of topics and industries has become possible for empirical work as data sourced from industry became available. But some of the old arguments about interpretation of the data continue

to the present day.

Overcapacity in industry has been dealt with extensively in the economic and particularly the industrial organisation literature. While it is accepted in the main stream of the literature that overcapacity is a result of oligopolistic markets, where large scale industries operate, there are different explanations for its causes, which include mainly: concentration of producers, protection of market share against rival firms and entry deterrence by incumbents against potential entrants.

Another explanation which is explored to a lesser extent in the literature is the *lumpiness* of plants resulting from economies of scale, combined with uncertainty about demand growth patterns. Also the decision regarding the timing and size of capacity by different firms contribute to overcapacity when ill timing, or poor planning is involved. Also uncertainty about future demand and the allowance that firms build in their planned capacity is often cited.

In petrochemicals, the topic of this research, investments are huge and sunk. As in all large scale industries exercising an investment decision is a complex process. Decision makers have to contend with different types of uncertainty when evaluating their new investments; which introduces an element of risk into the decision making process. These include, uncertainty about the growth of demand for the output products, or about their prices, or the prices of their raw materials. There is also uncertainty about the cost of investment required for the project, or uncertainty about the volatility of exchange, or interest rates. In this research project, however, we will be concerned with uncertainty about future demand, since it is demand variation which has the largest element of risk for producers of commodity petrochemicals.

Other aspects of the decision process, which petrochemical producers have to contend with, include the timing of new investments or projects to coincide with favourable market conditions. And choosing its capacity so as to capture the benefits of economies of scale, whilst achieving sufficiently high utilisation rates to hold market share, and meet demand expectations which are uncertain.

Finding an optimal path for the investment decision or capacity expansion problem, therefore, requires careful planning, and an understanding of industry dynamics, if the

objectives of maximizing profits, and strategic consideration such as maintaining market share are to be achieved, especially under conditions of uncertainty.

The situation is complicated further if possible actions, or strategies of rival firms are also to be taken into account. To capture these characteristics and find an optimal solution to the investment problem, which explains the presence of overcapacity in the industry, we propose developing a dynamic model of oligopoly for capacity expansion of the game theoretic type. The model, its structure, and how we propose to address the question of optimal investment and overcapacity are outlined in Sections 1.3 and 1.4

In what follows a brief review of the petrochemical industry is given; then a succinct literature coverage of entry deterrence, dynamic game theoretic models, and investment models of the option pricing type relevant to our proposed project is presented. Some of their attributes and short comings are also indicated.

1.1.1 Petrochemicals: A Mature and Oligopolistic Industry

The petrochemical industry is characterised by its oligopolistic structure and practices of its major players; partly due to the history of its development, its integrated nature, and the high degree of know how required in its operations. Also the huge investments involved reduce the number of potential participants in this industry.

Over the last 40 years the petrochemical industry has undergone dramatic changes transforming it from an R&D intensive, highly profitable and rapidly growing, to a technologically mature and sluggish industry, frequently hit by cyclical trends and low profit margins, especially in Western Europe, where overcapacity appears to be an endemic problem.

With the maturity of the petrochemical industry, both in terms of its technology and its markets, overcapacity has become one of its endemic features, which during periods of weak demand, burdens the industry, and causes substantial losses.

Two independent driving forces were shaping the industry: Technological maturity exemplified by the standardisation of process technology available from a large number of engineering companies; and a change in the production cost structure resulting from higher raw materials prices following the oil price rises of the 1970's, and from increased

benefits of economies of scale of much larger plants that were coming on stream.

Freeman (1982a) and Pavitt (1982) provide the evidence for the technological maturity of the industry through a number of studies they conducted. Using patents they evaluate the role of R & D and innovations in changing the structure of the industry, and its interactions with its markets. Studies of US and European industries reveal that high concentration industries such as synthetic fibres, synthetic rubbers, and organic chemicals derivatives was caused by vigorous innovation combined with patent and know-how barriers to imitation. Geroski (1990) found that there was a strong tendency for concentration to fall during the 1970's. He also found that higher concentration had a negative influence on innovation; and greater monopoly power leads to larger time-lagged profit margins. Spitz (1988), Chapman (1991), and Arora, Landau & Rosenberg (1998) survey the historical development and growth of the global petrochemical industry, as the major companies introduced new products and processes.

What followed as a result of these forces, was a process of restructuring within the industry involving the main producers: the chemical majors and the rising chemical arms of major oil companies. This process affected product lines and market share. There was also the emergence of the new comers to the industry, notably in oil producing and the newly industrialised countries. The process of restructuring does not seem to have resolved the problem of overcapacity, which bedevilled the industry during the early eighties. The industry continues to complain about overcapacity and the adverse impact on profitability .

But what are the main causes of this overcapacity? Is it a natural result of the oligopolistic structure of the industry, or more a consequence of the maturity of the markets, increased uncertainty about the economy, and poor planning on the part of the main petrochemical producers? It is important to understand this phenomenon of overcapacity, for the industry has been blaming the high cost of this overcapacity for years for its bad performance and low profitability margins, which could put at risk the future growth of this important industry.

The technological maturity of the industry meant that the process technology was more easily accessible to potential entrants to the market. The industry has arrived

at what Freeman (1982a) describes as *swarming stage* where the industry attracts large numbers of newcomers. Stobaugh (1976) by studying investments in nine products shows that this process has been taking place on a global scale. He also emphasizes the role of chemical majors in globalising the industry through joint ventures.

Fayad & Motamen (1986) point out that in petrochemicals the *swarmers* were mainly the global entrants in the oil producing countries of the Middle East, Canada, Mexico, and the newly industrialised countries such as Brazil and South Korea.

They also observe that the integrated nature of production operations of large volume petrochemical products, and the higher investment requirements for the new vintage of large size plants to capture economies of scale, reduced the number of entrants, and viable competitors which can operate in the industrial markets of Western Europe, the US and Japan to a small number of large and powerful producers. They conclude that despite its technological maturity the structure of the petrochemicals industry remained oligopolistic in nature.

Since the early 1980's, the petrochemical industry appears to have been trapped in a cyclical pattern of rapid investments, overcapacity, followed by low profit margins and a process of capacity rationalisation through mergers and product line shedding. Sutton (1991, 1998) analyses this process by studying the development of market structure for several industries. He provides a general theoretical framework to explain the evolution of industrial markets.

This thesis will investigate the investment behaviour of firms in oligopolistic markets, in the light of recent developments in oligopoly theory; and will address the question of overcapacity in the US petrochemical industry by studying styrene, a commodity petrochemical product over a period which extends from 1974-1996.

1.1.2 Oligopoly Theory and the Problem of Overcapacity

The question of overcapacity in oligopolistic markets goes back a long way. Wallace (1937) considers that oligopolistic rivalry can lead to a rapid rate of investment when one or more firms see an opportunity to gain advantage over rival firms, which leads to overcapacity.

There are two major strands for the causes of overcapacity in the literature: 1) The oligopolistic structure of the industry under demand uncertainty together with the lumpiness of projects; and 2) strategic considerations by the incumbents to deter entry or mobility. Empirical work in this area, however, appears to provide inconclusive evidence as to the causes of overcapacity.

Esposito & Esposito (1974) by studying a large sample of three digit industries found that there was a significant relationship between excess capacity and the concentration ratio of producers. They conclude that loose oligopolies exhibited greater excess capacity than tight oligopolies.

Scherer & Ross (1990) makes two important observations on oligopolistic markets: 1) that they are prone to chronic excess capacity; and 2) that the structure of these markets contribute to cyclical instability in capital investments. He provides examples from the US petrochemical industry (plastics and synthetic fibres) as well as the aluminium and fertilizers industries.

The reasoning for this behaviour, explains Scherer, is as follows: in periods of rapidly growing demand oligopolistic firms will invest heavily in additional capacity collectively for fear of losing market share to their rivals; and this takes place at a rate exceeding demand growth. As a result when excess capacity starts to disrupt pricing disciplines, new investments must be cut back abruptly.

This view, however, that oligopolistic markets naturally result in overcapacity is by no means universal. Richardson (1960) attributes overcapacity even in competitive markets to a failure on the part of firms to respond correctly to market conditions.

The works of Lieberman (1987a, 1987b, 1987c), and Gilbert & Lieberman (1987) on the US chemical industry, and those of Paraskevopoulos (1988) and Paraskevopoulos, Karakitsos & Rustem (1991) on its Western European counterpart support the first view, and find little evidence of entry deterrence behaviour by incumbent firms. While Reynolds (1983, 1986) by studying the US aluminium industry finds contradictory evidence, where excess capacity results from strategic considerations of incumbent firms. Steele (1995) in an empirical study of North American newsprint industry also finds that most firms invested strategically to influence the behaviour of their opponents.

Two observations on the Lieberman (1987a, 1987c, 1989) and Gilbert & Lieberman (1987) works are worth noting. Firstly, the data series analysed covers a period of thirty years, it ends in 1982, which just covers the beginning of the period when the industry was facing excessive overcapacity as a result of the recession, increasing maturity of the markets, and mounting uncertainty about demand growth patterns.

Secondly, a large number of products are studied as one lump. It is more likely that firms would behave differently in product lines where they realise that they have a large share of the market, or possess a technological advantage, than they would in others where the market is more competitive. As such these firms would be more selective about where, and in which products strategic investment would be used effectively.

1.1.2.1 Uncertainty

Evaluating investment decisions under conditions of uncertainty is a complex process. Since the end of the era of macroeconomic stability and continued growth of the 1950's and 1960's, planners in the petrochemical industry had to contend with different forms of uncertainty when making decisions about new investments.

In the mid 1970's uncertainty arose about the security of feedstock supplies and its cost following the oil price rises. In the 1980's and 1990's, it was mainly uncertainty about demand growth patterns and the volatility of interest and exchange rates.

With modern and large scale petrochemical plants where investments are sunk, and costs could run into several hundred million dollars, petrochemical producers have to decide on the timing and capacity of their new investments under conditions of uncertain future demand, while at the same time taking into consideration the strategic objectives of their rivals. To maximise its profits, a petrochemical firm has to find an optimal solution to this investment problem.

But how relevant is uncertainty to the problem of overcapacity and the investment decision? While Reynolds does not deal with the question of uncertainty directly, he attributes to it the presence of excess capacity unexplained by his model.

Paraskevopoulos, Karakitsos & Rustem (1991), on the other hand, introduce the element of uncertainty into their model, but deal with the industry aggregate, ignoring

the interaction between rival firms in the market.

Bell & Campa (1997) study the influence of volatilities on irreversible investments in the chemical processing industry. They find that the effect of input prices, and product demand volatility on investment in capacity to be less significant than exchange rate volatility, especially in Western Europe where a significant share of the output is exported. Exchange rate volatility is found to have a strong negative influence on investment by chemical producers.

It is worth noting that their model instead of considering the scale of investment in capacity, assumes new capacity as any incremental change above 5% of the smallest national capacity for a product; although they differentiate between incremental and greenfield expansions, effectively it is the number of capacity changes that is being modelled, rather than the size of production capacity in place.

1.1.2.2 Entry Deterrence

In another strand of the literature, excess capacity is held for strategic reasons by existing firms to deter entry. A large body of empirical contributions in this area supports the findings of theoretical models. But there are also ample opposing views. The differences on theoretical grounds appeared also early on, as in the contributions of Spence (1977) and Dixit (1980).

Eaton & Lipsey (1981) find that an incumbent firm may have an incentive to hold excess capacity in the pre-entry period to deter entry even if entry is not effectively blocked. In a bargaining model of capacity choice, Osborne & Pitchik (1987) show that collusive firms carry excess capacity to make their threats more damaging, and so that they can obtain more favourable agreement. Maskin (1986*b*) argues that uncertainty forces the incumbent to choose a higher capacity to deter entry than he would under certainty.

Also Spence (1977) models the behaviour of several firms facing the possible entry by a single firm. He argues that existing firms would hold excess capacity for the strategic purpose of deterring entry. In this model Spence assumes that firms would resort to full capacity production if entry actually occurred. But as Dixit (1980) points out this is not

an optimum response. Dixit shows that once entry occurs the incumbent will not carry out its threat of utilising excess capacity. Booth, Kanetkar, Vertinsky & Whistler (1991) counter by arguing that in industries characterised by modest rates of return, and a high degree of uncertainty, the cost of entry deterrence would be prohibitive.

Fudenberg & Tirole (1983) analyse strategic investments by an early entrant in a new market. In this model, by over-investing, the leading firm forces its rival to make its subsequent decisions taking the leader's capital stock as given. And the follower firm is forever deterred from investing to its steady-state reaction curve.

The evidence in the literature on the causes of overcapacity in oligopolistic markets is by no means conclusive. In fact there are conflicting views on this matter.

Lieberman (1987c) in an empirical study of the US chemical industry finds rare evidence of incumbents building excess capacity to deter entry. In this study which covers thirty eight chemical products, he finds that entrants and incumbents exhibited similar investment behaviour.

In an another empirical study of twenty four chemical products Gilbert & Lieberman (1987) show that preemptive investment by large chemical firms has the role of maintaining market share and leads to lower probability of redundant investment by rivals. Rather than leading to an increase in the market share, preemption has the effect of allowing rival firms to coordinate their investments. Paraskevopoulos, Karakitsos & Rustem (1991) find that the signalling mechanism, where companies announce planned investments, contributed to minimising excessive investment with respect to demand forecasts during the early 1980's in the Western European PVC industry.

Where as Reynolds (1983, 1986), in an empirical study of the US aluminium industry, shows evidence of strategic use of overcapacity in this oligopolistic industry. A similar behaviour is uncovered by Steele (1995), in an empirical study of another oligopolistic industry. His findings show that strategic considerations are prevalent in capacity expansions in the North American newsprint industry over a 38 year period. Major firms are unable to deter entry; but instead they utilise mobility deterrence in order to pre-empt rivals' expansions.

Booth, Kanetkar, Vertinsky & Whistler (1991) on the other hand, find that newsprint

producing firms in the North American market were tacitly coordinating their capacity expansions to share the market. Their model estimation supports the price mark-up mechanism, which they find is a function of operating rate in industry. They also discover evidence of price rigidities indicating the existence of a price signalling mechanism to induce firms to adjust their prices to changing market conditions. They uncover regional variations in investment behaviour as well. Higher prices lead to aggressive expansion of capacity in the US where capital stock is homogeneous and new. While in Canada a two-tier age structure of capital cost exists, with old vintage capital stock which is retired when demand is weak, is mobilised into production during periods of peak demand.

However, Bernstein (1992) who studies price margins and capital adjustment in the Canadian pulp and paper industries, finds that firms adjust their capital stock toward long-run equilibrium through successive short-run equilibria. And unlike other studies in this industrial sector, he finds that firms behave competitively in the short run as they adjust towards a long-run equilibrium.

Investment rivalry in the pulp and paper industry is dependent on market structure; investment rules are studied by Christensen & Caves (1997), instead of just looking at investments in projects, they study the factors affecting abandonment of announced projects and devise a set of rules for finding out which are destined for completion. They find that unexpected announcements of new projects by rivals, promote abandonment by announcing firms in less concentrated markets ; while implementation is encouraged in more concentrated ones.

The investment behaviour of manufacturing firms in 15 industrial sectors in the UK is analysed by Hay & Liu (1998). Using panel data, they show that in fragmented sectors behaviour is non-cooperative; while it is co-operativer in dominant group sectors, and competitive in dominant group sectors. In fragmented sectors market shares of firms are found to have declined over time; for dominant firms, on the other hand, market shares are stable, as was the case also for leading firms in a dominant group sector.

The desired outcome from entry deterrence, merger activity and the heightening of investment activity, is an increase in industry concentration, reduced competition and eventually higher profit margins. By studying the UK industry, Cowling & Waterson

(1976) and Clarke, Davies & Waterson (1984) find that profit margins tend to rise in concentrated sectors of industry. While Demsetz (1973) attributes higher concentration and improved profit margins in a sample of US industries to firms achieving a strong degree of efficiency over their competitors, and gaining advantage as result. Smith (1981) also attributes gains of strategic investment, in part, to higher efficiency gains made by incumbent firms.

Besides excess capacity other instruments are available to firms to influence market structure. Davies & Geroski (1997) find that advertising and to a lesser extent R&D and innovation play a major role in affecting the dynamics of market shares and the concentration at the industry level in the UK. The authors employ a model of stochastic firm growth in studying the dynamics of industry structure. They also find that while considerable turbulence in market shares takes place, the concentration on the industry level is fairly stable.

Another observation made by Geroski (1991a) in this respect ought to be pointed out here though; that studies that involve surveys of industry sectors in general find little evidence to support the entry deterrence hypothesis; while case studies of specific industries, are more likely to find evidence of strategic use of overcapacity to influence markets and opponents' investments. There are exceptions here to the general rule as in Sutton (1991). He conducts cross-sectional industry studies on different sectors of industries in different countries, and finds strategic investment behaviour in various forms, including advertising, innovation, and R&D expenditure.

In this regard, the works of Singh, Utton & Waterson (1992, 1998), Smiley (1988), Bunch & Smiley (1992), and Driver (2000) are relevant. These works cover a wide range of mainly small and medium industries where the use of excess capacity may not be the main instrument in influencing the behaviour of opponents, and the market. They find that a significant proportion of firms (nearly two thirds) report the use of different forms of entry deterring strategies including excess capacity, advertising, R&D, patenting and pricing policy to deter new entrant and an even higher proportion to influence their opponents.

Specific industry studies on the other hand such as those of Ghemawat (1984),

Reynolds (1986), Hall (1990), Rosenbaum & Ye (1992), Booth, Kanetkar, Vertinsky & Whistler (1991), Steele (1995), and Mathis & Koscianski (1996) report the use of excess capacity and strategic investment to be prevalent in the industries they study. Others such as Gilbert & Lieberman (1987), Lieberman (1987a, 1987c), and Paraskevopoulos (1988) report more ambiguous findings about the strategic use of investment and excess capacity. Where as a study by Dunne, Roberts & Samuelson (1989) on the US chemical industry found little incidence of entry despite the consistent healthy growth of the industry; and where entry occurred it was by large chemical producers from other sectors expanding into a new product market.

We have more to say about entry deterrence, and these developments in the context of the petrochemical industry in Sections 5.7.1 and 5.7.2, in Chapter 5.

1.1.2.3 Game Theory, Firm Interaction, and Uncertainty

A game theoretic framework allows firms to interact, and to find optimal policies which is a suitable framework for our problem of optimal capacity planning. A dynamic formulation also captures the requirement for subjecting the capital stock to new investment and depreciation over the specified planning horizon of the game. Where the firms or players act as independent economic agents, each trying to maximise its payoff or profits.

There is a rich literature in game theory that deals with investment and capacity planning. The advances made in game theory opened a new horizon for researchers in the field of industrial organisation and in particular those dealing with problems in oligopoly theory. Although the topic has also received its fair share of criticism as well (Sutton 1990). Fudenberg & Tirole (1992) provide a comprehensive review of the game theoretic and dynamic models of oligopoly literature. But most of this literature remains theoretic in nature, with very few empirical contributions particularly in the area of capacity planning.

In their seminal work, Starr & Ho (1969a, 1969b) construct a capacity expansion model for n firms as a state space or differential game. The payoff for each firm is its value function which is a function of its investment rate, its capacity and that of its rivals, which varies over time. Starr and Ho proved that a unique equilibrium investment

strategy that maximizes the payoff for the firms involved can be found for linear quadratic games. In their formulation each player takes into consideration the investment strategies of his rivals.

In one of very few papers that utilize the game theoretic approach in a real case study of an industrial sector, Reynolds (1983, 1986) develops a dynamic model of output and capital investment choice to explore the influence of strategic interaction among existing firms on the incentives for investment. The firms play a Nash-Cournot game, and he tests feedback, open-loop and Stackelberg strategies. Reynolds (1987, 1991), using a model formulation similar to that developed by Starr & Ho (1969a), also examines the impact of variation in adjustment costs on model performance and equilibrium values for rival firms.

Reynolds (1986) utilizes algorithms devised by Kydland (1975, 1977) for computing the Nash equilibrium and to simulate the performance of the US aluminium industry. He uses the model to test firms' investment behaviour, production and output prices. Although demand is taken to be deterministic in this model, Reynolds points out that introducing demand uncertainty, may improve its ability to explain the emergence of excess capacity in the American aluminium industry.

In a related work, Kreps & Scheinkman (1983) develop a two stage model where companies compete on capacity in the long term, and on price in the short term. In their model the firms produce a homogeneous product, and the effects of one firm's output on its rivals profit are analyzed. They show that this type of competition is akin to a Nash-Cournot competition, and equilibrium can be achieved. However, this model assumes that there is no depreciation.

Başar & Olsder (1999) provide a wide ranging treatment and formulate the theoretical framework for solving discrete and differential games of n persons of the noncooperative type. They also deal with situations that involve conditions of uncertainty.

Toivanen & Waterson (2000) provide an illustration of applying a game theoretic model of entry to the fast food industry in the UK. They deal with the problems of identification and implementation in applying such models to an empirical case, which involves the hamburger market as a duopoly between McDonalds and Burger King over

a five year period.

On the other hand, a new branch in the area of evaluating capital investments under conditions of uncertainty has grown out of the option pricing literature. Pindyck (1991) and Dixit & Pindyck (1994) provide an extensive survey of this literature.

Applications to irreversible capital investments by firms are mostly theoretical and include several contributions by Majd & Pindyck (1987), Pindyck (1988), Dixit (1992), Abel, Dixit, Eberly & Pindyck (1996), Lund & Øksendal (1991), Kulatilaka & Perotti (1992, 1998). Problems that involve real life investments include Brennan & Schwartz (1985) and Slade (2001). There is also a wider coverage in the area of real options.

In this literature capital is evaluated as irreversible investments under conditions of uncertainty. Which could result from demand, product prices, capital investment cost, or interest rate fluctuations. These variables are modelled as Weiner processes, there by introducing the element of uncertainty into the investment problem.

Dixit & Pindyck (1994) show several examples of how this type of timing problem can be solved. While Brennan & Schwartz (1985) evaluate the value of a copper mine, when it is open or shut as the price of copper fluctuates. Lund & Øksendal (1991) also includes examples on evaluating investments in natural resources projects such as oil fields.

A common feature of these contributions is that they all deal with a single firm interacting with an exogenous environment. Bertola & Caballero (1991), however, extend their treatment of solving the optimal investment problem for the firm, under the combined effect of uncertain business environment and capital costs, to characterise the aggregate investment behaviour of a large number of firms. But this does not cover the strategic interaction of these firms.

Kulatilaka & Perotti (1992, 1998) undertake this task by examining the optimal timing problem under uncertainty for two firms in a model of Cournot competition, and find that increased volatility favours early investment, an outcome which is in contrast to the results of main stream option theory literature.

1.2 The Research Project Objectives

The purpose of the research project and the model we propose to develop, is to examine persistent overcapacity in the petrochemical industry. In fact we want to find out whether capital investment and overcapacity are being used for strategic purposes by the main industry players. Whether investment or overcapacity is being used strategically is not only measured by the success, or otherwise, of deterring new entrants. Rather, the analysis needs to find out how firms are interacting in the investment and production game. Is investment being used to deter mobility of other firms, dissuade new entrants, or compel them to enter at lower capacity level? And how is the market environment, in terms of concentration and the level of competition, changing? These are the kind of questions that need to be answered.

A model of capacity expansion will be developed in this thesis to study intertemporal developments that take place in an oligopolistic industry. In our model, we intend to test the investment behaviour of firms, and its interaction with industry dynamics, to determine whether capital accumulation, output, and capacity utilisation in the petrochemical industry can be explained or predicted by a dynamic, game theoretic model of the non-cooperative Nash-Cournot type. And to find an appropriate equilibrium concept for the analysis.

Initially, the model will be constructed for two firms producing a homogeneous product, but will be extended to test the performance of the largest petrochemical producers that dominate the US market. Demand uncertainty will also be introduced into the model to determine if it provides a plausible alternative explanation for the excess capacity problem in the petrochemical industry.

It is hoped that this research will further our understanding of firm interaction and investment behaviour and its impact on market structure in oligopolistic industries producing a homogeneous product.

1.3 Methodological Aspects and Outline of Thesis

In this thesis I explore the suitability of dynamic capacity expansion models for studying capacity expansion by firms in a growing industry, and the development of its market structure. I examine how well these developments can be captured by these models; and if well behaved and properly specified, may be they can be developed so that they could be usefully employed by industry in capacity planning implementation.

Empirical testing for strategic investment is difficult to accomplish directly, unless one relies on issuing questionnaires to corporate planning management on the main criteria that determine and drive their strategic planning, as in the studies of Smiley (1988), Bunch & Smiley (1992), and Singh, Utton & Waterson (1991,1998). Since it is usually difficult to ascertain whether an investment is carried out for strategic purposes to influence the actions of opponents, like in situations where firms invest in excess capacity to deter entry or expansion by rival firms, many econometric studies rely on indirect testing, by modelling the correlation of the incidence of entry, or lack of it, and capacity addition over time, together with other relevant economic measures. This is the approach used in Gilbert & Lieberman (1987), Lieberman (1987a, 1987c), and Ghemawat & Caves (1986). Or in the form of announcements for investment in new capacity and the degree of abandonment or success rate in adding new capacity from the initial announcement list, by rivals as employed by Christensen & Caves (1997) in their study of the pulp and newsprint industry, and by Henderson & Cool (2003) in their study of the world petrochemical industry.

Strategic investment behaviour can also be modelled in the form of conjectures that the firms hold about the reaction of their rivals to their actions. The model can then be evaluated econometrically, as in Hall (1990), who investigates Du Pont's capacity expansions in the titanium dioxide industry in a two stage game; or as in Steele (1995) who evaluates a dynamic capacity expansion model for the newsprint industry in North America relying on an Euler equation. Using industry data he then estimates the model coefficients econometrically.

Also Rosenbaum & Ye (1992) study the investment behaviour of Du Pont in the

titanium dioxide industry, by examining the timing and scale of its capacity additions. They test for strategic investment by Du Pont to preempt capacity expansions by rivals, by the deviation of its investments from the cost minimising optimal path. The industry is reduced into a duopoly, where Du Pont is the preemptor and the rivals act as the opponent.

A similar approach is employed by Reynolds (1986) in studying the US aluminium industry, and the investment behaviour of Alcoa (the Aluminium Company of America). Using a quantity game theoretic model, the capacity expansion of each firm is modelled explicitly in a dynamic setting. Strategic investment is envisaged by the use of Stackelberg, open-loop or feedback Nash-Cournot investment strategies; and strategic excess capacity is measured as the divergence between optimal capacity generated by the model, and the actual amount of excess capacity held by industry.

Game theoretic models allow the interaction between firms to be captured directly; and the strategies employed by rival firms to be modelled explicitly. Outputs and investment rates are modelled as control variables that can be selected optimally by competing firms. In a dynamic context, the total output of industry, the price formation, and the evolution of market shares, result explicitly from the intertemporal decisions of the firms that makeup the industry. By examining the different scenarios of possible strategic interaction that can take place between the economic agents, the competitive nature of industrial markets can be studied, and it would be possible to infer from the resulting simulations the economic relationships taking place, and market characteristics of the industry and its products.

Open-loop Nash-Cournot, feedback Nash-Cournot, Stackelberg, and price taking scenarios can be studied independently. Strategic behaviour can be modelled in terms of the type of model adopted; in these models the impact of a rise in the number of firms in the market on total industry output and price may also be examined. These developments cannot be modelled explicitly in this way by cross-section or time series models. Complex interactions between variables of market structure and capital commitment may not be detectable by a general model of a cross-section of industries; nor can such a model be able to filter out the noise of historical accident. Such details may be resolved by detailed

empirical studies of individual markets or industries (Ghemawat & Caves 1986, p. 108).

This approach is followed in this thesis. A game theoretic model of capacity expansion is developed where the firms are modelled explicitly, to capture the interaction between them directly. Each firm (player) maximises a payoff or objective function which is the firm's discounted flow of profits over the duration of the game. Because we are interested in studying industry developments, a dynamic formulation is employed, where demand and capacity evolve over time. Firms' capacities, therefore, are subject to change as a result of additional investment and depreciation.

However, initially static models are studied in Chapter 2, showing the different model formulations and the implication for interacting firms. In each case the corresponding solution concept is discussed, with an illustration of the resulting market shares for the firms concerned. These concepts are developed further in Chapter 3 for a dynamic continuous time setting. A noncooperative Nash-Cournot game is played between the firms; where two types of Nash-Cournot strategies are examined: the open-loop, and the feedback strategies. Adjustment costs are introduced and justification for their use is discussed; and the model formulation is extended from duopoly to an oligopoly of n firms. Using optimal control theory techniques, the main focus of this chapter is two fold: (a) to determine the optimal investment strategy for the two firms, and establish the uniqueness of the solution; and (b) to examine the behaviour of the model and its performance. This is done assuming that firms operate at full capacity. As a result each firm controls one instrument or control variable, which is its investment rate. In this case production is equivalent to capacity in place. The model simulation results presented, are for a duopoly.

In Chapter 4, the assumption that firms operate at full capacity is relaxed, and production costs that were assumed to be negligible in Chapter 3 are added to the model. Each firm's profit maximisation problem is subject to two decision variables the investment and the production rate in every period. Four investment strategy/model concepts are introduced in this chapter: open-loop Nash-Cournot, feedback Nash-Cournot, Stackelberg, and competitive price taking strategies. Using algorithms developed by Kydland (1975, 1977), the model is formulated as a *linear quadratic* problem in each case. The

performance of the model is examined, as before, under different adjustment cost parameters; and the impact of different production costs, adjustment costs, and investment strategies used, on capacity utilisation and market shares is shown. Also, as the number of firms operating in the market increases, it can be illustrated how the model performance shifts to show a more competitive outcome in terms of product prices, profits, and market shares.

The model formulations and solution concepts considered in Chapter 4 are put to test in Chapter 5, in an application to a case study of the US petrochemical industry, namely the investment behaviour of styrene industry. The industry characteristics, and the development of market structure are studied first within a brief historical context, paying particular attention to the investments made by, and the changing roles of the main producers. For evaluating the model and determining the optimal investment and production rates of styrene producers, empirically based investment and production cost functions are estimated, taking into consideration learning effects, and economies of scale achieved in the styrene industry during the period under study. The model simulations, under different investment strategies, are used to explain the incidence of overcapacity; and the presence of certain forms of entry barriers in the styrene industry is also discussed.

In Chapter 6 attention turns to the role of demand uncertainty in affecting capacity accumulation. Using a geometric Brownian motion process to drive the demand, the model is reformulated and solved to see how well demand uncertainty accounts for industry overcapacity. Chapter 7 summarises the main findings and conclusions of the thesis, drawing attention to the limitations and possible extensions of this work. Also presented are some suggestions for future research and for empirical work, in this interesting and promising segment of the industrial organisation literature.

1.4 Model Type, Scope, and Contribution

The model employed in this research project is a dynamic model of capacity expansion of the non-cooperative Nash-Cournot type. It will be built as a linear quadratic game with a finite horizon, in the range of thirty years for the parametric simulations. It

will be formulated as a linear quadratic dynamic game in discrete, or continuous time. The model will be similar in spirit to that used by Reynolds (1986) for the aluminium industry; and where capacity expansion is subject to investment and depreciation.

I propose to extend this approach to the treatment to the US petrochemical industry, by allowing for uncertainty about demand in the model formulation to examine how well uncertainty explains the presence of overcapacity in industry. As far as I am aware no empirical work has been published to date using the present formulation which takes into consideration firm rival interaction in the context of a game theoretic framework.

I will also introduce the following modifications into the model by allowing for economies of scale, and learning effects to be taken into account when evaluating production costs, with different firms having different production costs. Adjustment costs in the production cost function allow for penalties on redundant capacity to be measured directly. This is slightly different from the treatment of Reynolds (1986) where production adjustment cost is treated as a quadratic function evaluated by a Taylor expansion.

The role overcapacity in an entry game of a duopoly is examined in Chapter 2. This is a single shot or a two stage game where the cost of redundant capacity is taken into account to examine using different examples how it impacts on the game outcome under Nash-Cournot and Stackelberg strategies. This is a point which is commonly overlooked in this segment of the literature.

The model structure and how it works is explained in Chapters 2 and 3. A discrete version based on two algorithms by Kydland (1975, 1977) is presented in Chapter 4. Below some of its other features are briefly presented.

In this model firms choose their outputs and investments (capacities) to maximise the value of their profits for that period. And for the dynamic case over the remaining period of the game. Both investment and output are the decision or control variables in this game. Output will be constrained by the capacity in place; and capacity utilisation rate takes the strain of fluctuating demand.

Benefits of economies of scale, or of more efficient processes will be incorporated in the model. So companies operating larger plants have a cost advantage when production is close to design capacity. And depending on plant capacities and the process technologies

employed, costs of production will vary across firms.

Price in the model is endogenised as an inverse demand function, where total demand is considered to be linear in price and a stochastic variable. All firms face the same depreciation rate, and discount rate of profits, which are exogenous. Also the capital stock market is assumed competitive, so firms face the same unit capital acquisition cost for plants of the same capacity and employing same technology.

When uncertainty is introduced into the model the overall demand facing the industry is considered to be known in the current stage but its future is uncertain and is driven by a stochastic process. So when companies plan for their capacity expansions, they are uncertain about future demand but they all hold the same expectations about how it evolves over time.

Different strategies will be considered in the thesis. In the case of feedback strategies each firm chooses its output and investment as a function of the capital stock held by other firms, there by accounting for the subsequent decisions taken by its rivals as a result of its own actions. Other variations such as Stackelberg leadership, or open-loop equilibrium will also be addressed.

Chapter 2

Competition and Entry

Dynamics: Duopoly in a Game

Theoretic Framework

2.1 Introduction: Model Formulation and Roles of Play

This chapter deals with a duopoly where the role of entry, and the impact of the nature of competition between the players on the market is studied. In particular strategic investment in capacity in an entry game between an incumbent firm and an entrant is examined under different assumptions for the firms' costs of production and roles of play. In this game the duopolists maximise their payoffs which are their profits. Consequently, the incumbent has the advantage and the incentive to invest strategically in his capacity to deter the entrant, or to manipulate the market in his favour. The different outcomes that arise in each situation for both firms, and the degree of success in achieving profit maximisation and deterrence are explored.

The market I examine throughout this thesis is that of firms producing a homogeneous commodity product, and serving a single market. These firms engage in noncooperative competition; and in view of the undifferentiated nature of their product, they compete on output. The price clears the market, whatever the firms produce in a period is sold in the market; and there are no inventories accumulated, imports or exports.

The petrochemical industry which is the subject of this research, is served by a large number of engineering and service firms where plant capacities are known, and new investments are widely publicised in advance; investment costs may therefore be considered to be common knowledge; and where as other production costs are firm specific, it may be assumed that industry participants are in a position to estimate to a fair degree of accuracy, the costs involved in the operations of their competitors. Raw materials are widely traded, while utilities, energy, and labour costs are published regularly. The firms can observe the capacity of their rivals, but not the output or production of the product in any single period which remains unknown. But the firms can detect from the price variations what is happening in the market place. Such industry characteristics, give credence to any assumptions made about gaming with complete information, an essential condition in a noncooperative game for arriving at an optimal solution (Shubik 1955). Because the two firms do not observe production in the current period, they can be thought of as acting simultaneously in arriving at the Nash-Cournot equilibrium output, although the actual physical production may be taking place in sequential moves.

However, it should be made clear at this point that in this chapter, as well as in Chapters 3 and 4, in conducting the model simulations, I introduce user induced values for the parameters of the cost and investment functions. But these are chosen such that model costs mimic the cost structures for commodity products in the petrochemical industry. Raw materials and utilities are the dominant elements of the cost of production; and capital investment costs form only a small proportion of overall costs. In Chapters 5 and 6, I employ industry based data to evaluate the parameters of the cost and investment functions, and test the model performance.

The chapter is organised as follows. In Section 2.2 a noncooperative Nash-Cournot model is introduced. Its structure, the equilibrium solution and its basic characteristics are then discussed briefly. In Section 2.3 an optimisation based general approach for evaluating Nash-Cournot equilibria in oligopolistic markets and its main attributes is presented. In Section 2.4 and Section 2.5 the role of strategic investment for the incumbent in a Nash-Cournot game is explored. Asymmetric and symmetric costs are considered for the entrant and incumbent firms. The impact of the role of play on the outcome

of the game, in terms of market share by production or capacity, and profitability are examined for each situation. Initially, under a Nash-Cournot game setting, the two firms are assumed to have identical production costs. Another alternative is tested, where the incumbent has lower investment and production costs than the entrant. For instance the incumbent firm, in view of its experience and well established position in the market, is allowed to assume an advantageous position in terms of its production costs and the size of its initial capacity in place. And the impact of the role of play on the game outcome, and the overcapacity levels that emerge when mobility or entry deterrence strategies are employed are examined. Then the entrant is allowed to enjoy more favourable production costs than the incumbent, as a result of superior technology for instance. Finally the privileged position of the incumbent firm in terms of its ability to choose its capacity first is eliminated; and the two firms are allowed to enter the market simultaneously.

In Section 2.6 another set of games is tested, where the two firms play a Nash-Cournot game and the entrant challenges the position of the incumbent. In this section I also examine the situations where it becomes feasible for the entrant to force the incumbent out of the market.

Alternatively, the game may also be played under a different set of conditions where one of the firms acts as leader, as in Section 2.7. The incumbent firm takes advantage of its superior position of being in the market first to take on the role of leader, and the game is played under a Stackelberg setting. Two cases are examined. In the first, Section 2.7.1, the entrant takes into consideration only the leader's output when deciding how much to produce and chooses capacity independently; and in the second, Section 2.7.2, the entrant allows for both her capacity and output to be subject to the leader's output. Then the entrant takes on the role of leader in Section 2.7.3, and I examine the incumbent's reaction in modifying his output rate and capacity utilisation. In Section 2.7.4 both firms strive for the leadership position. The likely credibility of each of these strategies is discussed in Section 2.8. Finally Section 2.9 concludes the chapter. These simple two player games are discussed further below.

2.2 Noncooperative *Nash-Cournot* Competition and Market Outcome

Strategic investment to gain advantage over business opponents whether in the form of advertising, or R&D expenditure to develop new products and more efficient production processes, or by differentiating products to fill a market niche and attain higher price margins, or in larger capacity for forestall new entrants or existing competitors plans has been studied extensively in the organisation theory literature as the surveys in Geroski (1991a), Nickell (1978), Scherer & Ross (1990), Sutton (1991, 1998), Tirole (1988), and in the PhD theses that are cited in the Introduction and the rest of this thesis attest to.

In particular the area that pertains to the use of strategic investment in excess capacity for entry or mobility deterrence has received considerable attention. Yet the evidence is far from being conclusive in favour of this thesis, or against it, as discussed in Chapter 1, and as we shall see again in Chapter 5. The examples that are considered in this chapter examine strategic interaction in Nash-Cournot, and Stackelberg games between two players using capacity as a instrument. The model parameters used for the two players are chosen such that their cost structures and their differentials, reflect costs in the petrochemical industry. The emphasis is on the roles of play and the extent to which strategic investment in capacity can influence the outcome of the game. And the likelihood of extreme situations such as entry deterrence, or forcing an opponent to exit the market.

Strategic interaction between two firms, and a game in the choice of capacity and output in the context of the chemical industry was first studied by Fathi-Afshar & Rippin (1984). In this chapter I develop this interaction further by allowing for adjustment costs and redundant capacity. I study a richer menu of possible roles between the incumbent and entrant firms; and I emphasize the role of strategic investment in capacity for mobility or entry deterrence, and allow for cost differentials between the two players. Furthermore I examine the credibility of the different strategies that may be followed by the two players.

2.2.1 Model Structure

To begin with it is useful to define some preliminaries and market characteristics some of which have already been mentioned. The market is served by a set N of firms which produce a homogeneous product where $N = \{1, \dots, n\}$; each firm $i \in N$ produces a quantity x_i , where $x_i \geq 0$. The industry production profile is a vector¹ $\mathbf{x} = (x_1, \dots, x_i, \dots, x_n)'$, and the total industry output is $X = \sum_{i=1}^n x_i$, and since we are dealing with a duopoly $n = 2$. Each firm also decides on an investment rate² u_i , and a plant capacity k_i . Obviously in this context, nonnegativity applies to both $u_i \geq 0$, and $k_i \geq 0$. In a single shot game, the firms' main decision involves the choice of production rate, and plant capacity for an entrant; investment rate and depreciation are more relevant to multiperiod or dynamic situations, that are dealt with in following chapters. The structure of the model in its quadratic form is preserved in this chapter to facilitate comparison with later results.

Market price, P , is determined as an inverse demand function, where total industry output determines the product price, $P = h(X)$. Each firm maximises a utility or payoff function, which is its single period profit, w_i ; this function is in turn defined as the firm's revenue less its investment, I_i , and operating costs, C_i .

$$w_i = \max_{\substack{x_i \geq 0 \\ u_i \geq 0}} \{Px_i - C_i - I_i\} \quad \forall i \in N \quad (2.1)$$

Given its capacity k_i , each firm i maximises its profit w_i by selecting its optimal output, x_i , and investment rate, u_i . A new firm that enters the market must also decide upon an optimal plant capacity.

Following Ruffin (1971), Friedman (1986), Dockner (1992), Murphy, Sherali & Soyster (1982), and Nagurney (1988) I also make the following assumptions:

- (a) The inverse demand function that determines the price is assumed to be linear and downward sloping throughout this thesis. In the case of a duopoly, it is given by:

$$P = h(X) = a - b(x_i + x_j) \quad a, b > 0 \quad (2.2)$$

¹A prime on a vector denotes a transpose; $\mathbf{x}$ here is a column vector.

²The role of investment rate will become more prominent in subsequent chapters when the model becomes dynamic, and the evolution of capacity is subject to depreciation and new investment.

prices are limited to be positive, by assuming that $h(0) > 0$, and that there is a limiting output $\bar{X}$ such that $h(\bar{X}) = 0$; and for all $X < \bar{X}$, $h(X) > 0$, and $\frac{dh}{dX} < 0$.

- (b) Both the operating costs, C_i , and the investment costs, I_i , are assumed to be nonnegative, increasing and convex functions that are continuous and twice differentiable. And I assume further, that they are quadratic and take the following form.

$$C_i = f_i x_i + m_i k_i + \frac{1}{2} g_i (x_i - k_i)^2 \quad (2.3)$$

where f_i , m_i , and g_i are positive parameters. The first two terms in equation (2.3) refer to the variable and fixed production costs; while the last quadratic term is an adjustment cost. For the time being we accept this cost as given, but have more to say about the significance of the cost parameters, and about adjustment costs, their use and justification in the literature in Chapter 3; and about their relevance to the petrochemical industry in Sections 5.5.2 and 5.5.5 of Chapter 5. While in general we take $x_i \leq k_i$, small deviations are allowed, where output can exceed stated plant capacity at an additional cost given by the adjustment cost term as indicated in (2.3).

The investment cost function, I_i , also involves an adjustment cost, and takes the following form.

$$I_i = c_i u_i + \frac{1}{2} d_i (u_i - \delta k_i)^2 \quad (2.4)$$

where c_i is the unit cost of capital, d_i is an adjustment cost parameter, and δ is the depreciation rate of capital. All are positive. The first term in equation (2.4) is the cost of investment, and the second is an additional cost incurred for the adjustment of capital.

- (c) The strategy of each firm i is a choice of an output rate, x_i , and an investment rate, u_i , that maximise the firm's profit function. This pair of actions may be represented as a vector $\mathbf{u}_i = (u_i, x_i)'$ (also known as firm i 's control vector). And the strategy set for firm i is the set of all admissible actions, defined as $\mathbf{U}_i$, where $\mathbf{u}_i \in \mathbf{U}_i$. Although $u_i \geq 0$ and $x_i \geq 0$ were assumed, with the implication that

C_i and I_i are defined over the range $[0, \infty)$; it is reasonable however, to assume that rates of production and investment are bounded by upper limits such that $\bar{u}_i < \infty$, and $\bar{x}_i < \infty$, since the market was assumed to be finite, and bounded by $\bar{X}$; also plant capacity is a binding factor on the firm. Therefore the range of admissible actions for each firm, are defined over bounded ranges, with $u_i \in [0, \bar{u}_i]$, and $x_i \in [0, \bar{x}_i]$. As such the strategy set U_i for each firm is also bounded, where $U_i \subset \mathbb{R}_+^2$. Furthermore it is also compact and convex³. Consequently C_i and I_i are bounded.

From the aforementioned we can also deduce that $Xh(X)$ is bounded and concave for all x , and where $x_i \in [0, \bar{x}_i]$, $\forall i \in \mathbf{N}$. In view of (a) above, where P is a linear and decreasing function, $x_i h(X)$ is bounded and concave. This holds also for the more general case where the price function, $h(X)$, is nonincreasing or convex. A formal proof is found in Theorem 3.1, in Friedman (1986, p. 72) and in Murphy, Sherali & Soyster (1982). The profit function w_i , which is defined for all $u_i \in U_i$, is bounded, continuous and concave since as equation (2.1) shows, it is the sum of three concave and bounded functions.

I turn next to the question of the existence of an equilibrium solution for problem (2.1), and its evaluation.

2.2.2 The Noncooperative Nash-Cournot Equilibrium Solution

In a noncooperative game, firms choose their decisions or strategies that maximise their profits, independently. Collusion and binding agreements between firms are ruled out. But two important features affecting firms behaviour are required to hold: first, that each firm acts optimally to maximise its payoff or objective function when reacting to its opponents actions; and second, that firms hold the correct beliefs about their opponents behaviour (Friedman 1986).

A Nash-Cournot equilibrium for our problem is a vector of optimal admissible strategies, $u^* = (u_1^*, \dots, u_i^*, \dots, u_n^*)'$, of output and investment choices of the n firms partici-

³A compact set is a set that is closed, i.e. it contains its boundary, and is bounded in that it can be contained in a ball of finite radius. A convex set has the property that any straight line connecting two points in the set is also contained in the set (Friedman 1986).

pating in the market, that maximises the profit function w_i for each firm, and such that no firm has an incentive to deviate from its choice, by selecting any other strategy from its admissible set of strategies, given that its opponents have made their optimal choices. This can be represented by the following familiar inequality.

$$w_i(u_1^*, \dots, u_i^*, \dots, u_n^*) \geq w_i(u_1^*, \dots, u_i, \dots, u_n^*) \quad \forall u_i \in U_i, \forall i \in N \quad (2.5)$$

The existence of a Nash-Cournot equilibrium rests on meeting conditions (a), (b), and (c) above. It has been established that the strategy set U_i for each firm i is compact and convex; and the payoff or profit function, w_i , is defined, continuous, bounded and concave for all admissible actions $u \in U$, where $U = U_1 \times \dots \times U_i \times \dots \times U_n$, is the strategy space of the game. Therefore by Theorem 3.1 of Friedman (1986) the market admits a Nash-Cournot equilibrium for our problem as stated in (2.1). Since problem (2.1) admits a maximum, then this equilibrium is unique (Murphy, Sherali & Soyster 1982, Theorem 1, p. 96).

Next I turn to evaluate the Nash-Cournot equilibrium for a case where the market is served by a duopoly. The problem at hand is made simple by having a linear inverse demand function. The Two firms act independently, each maximising its profit function with respect to its production and investment rates. However the two firms are interdependent in their actions, as their combined output determines the market price of their product and consequently affects their profitability. Initially, I assume that firm 1 is the incumbent, with a plant of capacity k_1 in place; while firm 2 is the entrant and has to select an appropriate optimal plant capacity, given the state of the market. Expressed in full the two objective functions are shown below:

$$w_1 = \max_{x_1, u_1} (a - b(x_1 + x_2))x_1 - C_1 - I_1 \quad (2.6)$$

$$w_2 = \max_{x_2, u_2, k_2} (a - b(x_1 + x_2))x_2 - C_2 - I_2 \quad (2.7)$$

These are quadratic functions. Optimal values u_1^* , x_1^* , for firm 1 and u_2^* , x_2^* , and k_2^* for

firm 2 can be obtained from the first order conditions, namely:

$$\frac{\partial w_1}{\partial u_1} = -c_1 - d_1(u_1 - \delta k_1) = 0 \quad (2.8)$$

$$\frac{\partial w_1}{\partial x_1} = a - 2bx_1 - bx_2 - f_1 - g_1(x_1 - k_1) = 0 \quad (2.9)$$

$$\frac{\partial w_2}{\partial u_2} = -c_2 - d_1(u_2 - \delta k_2) = 0 \quad (2.10)$$

$$\frac{\partial w_2}{\partial x_2} = a - bx_1 - 2bx_2 - f_2 - g_2(x_2 - k_2) = 0 \quad (2.11)$$

$$\frac{\partial w_2}{\partial k_2} = -m_2 + g_2(x_2 - k_2) + \delta d_2(u_2 - \delta k_2) = 0 \quad (2.12)$$

the first order conditions are linear equations that can be solved simultaneously by variable substitution; or if arranged in matrix form, by simple matrix inversion. Using equations (2.10) and (2.12) in equation (2.11) we eliminate u_2 and k_2 , and obtain a relationship between x_2 and x_1 :

$$x_2 = \frac{1}{2b} (a - f_2 - m_2 - \delta c_2) - \frac{1}{2} x_1 \quad (2.13)$$

we will need this result for the situations when the incumbent firm acts as a leader and can influence the output of the second firm; but for the time being equations (2.13) and (2.9) can be solved to determine x_1 and x_2 . The remaining values of u_1 , u_2 , and k_2 can be easily obtained from the respective equations (2.8), (2.10), and (2.12). The resulting Nash-Cournot equilibrium depends on the model parameters pertaining to the relative costs of the two firms, the demand function, and the size of the incumbent's capacity.

The optimal output for firm 1 is given by equation (2.9). In the process of determining its optimal output, firm 1 assumes that firm 2 will be utilising its optimal output as well given by equation (2.13). Substituting x_2^* from (2.13) into (2.9), and rearranging with some algebraic manipulation we obtain the following relationship:

$$x_1 = \frac{1}{3b + 2g_1} (a + f_2 + m_2 + \delta c_2 - 2f_1) + \frac{2g_1}{3b + 2g_1} k_1 \quad (2.14)$$

The optimal output of firm 1 is dependent on its capacity, its raw materials cost, its rivals costs of production, and the demand function parameters. Where as the optimal

output of firm 2 is dependent on its production costs, demand function parameters, and the output of its rival, firm 1, as illustrated in (2.13).

When firm 1 is operating as a monopolist, it selects a capacity and output that can be determined from first order conditions. These are analogous to equations (2.10)-(2.12) above. I assume initially that the incumbent selects a capacity of $k_1 = 21$; which is equivalent to its monopolistic capacity, when production is constrained by capacity, and the model parameters used are: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 10$, $d_1 = 50$, $f_1 = 10$, $g_1 = 2$, and $m_1 = 5$. These cost parameters apply to a firm when it is an inefficient producer throughout this chapter. I then determine the Nash-Cournot equilibrium that results when the incumbent faces an entrant that has the same costs.

It should be noted that the model parameters used for simulations, are chosen such that raw materials costs dominate the cost of production, as in commodity petrochemicals. And where adjustment costs are restricted to account for only a small proportion of total costs, by appropriate choice of d_i and g_i values.

In general the entrant selects a capacity that is lower than the output rate. This is apparent from the optimality conditions (2.10)-(2.12), which when combined yield a relationship of the following form:

$$k_2 = x_2 - \frac{1}{g_2} (m_2 + \delta c_2) \quad (2.15)$$

The second term on the *rhs* of the above equation is always negative since all costs and model parameters such as adjustment cost are restricted to take positive values. For firm 2 then it is always the case that $k_2 < x_2$, when it selects its optimal output and capacity simultaneously. Without enforcing capacity constraints on the problem, the entrant's capacity will be always overutilised with production exceeding plant capacity.

2.2.3 Capacity Utilisation and the Role of Adjustment Cost

Additional insight may be gained from evaluating the impact of variations in the value of the adjustment cost factor g_i on the capacity utilisation of each firm i . The relationship between the incumbent's choice of capacity and production rate, and that of its opponent

is particularly relevant to dealing with the issue of entry level of the opponent and the instances where the possibility of entry or mobility deterrence can be achieved.

Consider the incumbent with an existing capacity k_1 , since we are assuming that the incumbent chooses his capacity first. By examining equation (2.14), it can be shown that as $g_1 \rightarrow \infty$, the first term on the *rhs* vanishes and the coefficient of k_1 approaches unity. In this situation firm 1's output, x_1 , is equivalent to its capacity in place, k_1 . This can be explained in terms of the role of the adjustment cost. An increase in the adjustment cost factor causes the firm to increase its output so as to minimise the adjustment cost term $\frac{1}{2}g_1(x_1 - k_1)^2$; when output is equal to capacity this term vanishes completely, and production costs, C_i , are minimised.

On the otherhand, when g_1 is very small, adjustment cost becomes insignificant. As $g_1 \rightarrow 0$, the second term on the *rhs* of equation (2.14) which involves k_1 , goes to zero and the first term dominates. In this case firm 1's output approaches a fixed value given by $x_1 = \frac{1}{3b}(a + f_2 + m_2 + \delta c_2 - 2f_1)$. For the case of a duopoly with asymmetric costs⁴, where the entrant is the more efficient firm, firm 1's output becomes, $x_1 = 15.03$. From this outcome it can be deduced that in situations where adjustment costs are negligible, and firm 1 can choose its capacity first, the incumbent can maximise profits when faced with the prospect of entry by firm 2 whose costs it has foreknowledge of, by choosing a capacity $k_1 = x_1 = 15.03$ (for the case of symmetric costs, $k_1 = x_1 = 16.0$) instead of maintaining its monopolistic capacity ($k_1 = 21$), and output⁵, ($x_1 = 21$).

Another observation worthy of note here is that when adjustment costs are low, there is little chance on the part of the incumbent to deter entry by building a large capacity, especially when the entrant firm has lower production costs than the incumbent. Such a strategy would have some degree of success when the incumbent has a very wide cost margin over the entrant; otherwise the incumbent inflicts as much damage on his venture before succeeding in making entry for the would be entrant a loss making venture.

I examine further the choices available to the incumbent firm when it has the advantage of selecting its capacity first. The incumbent operates at full capacity when $\frac{x_1}{k_1} = 1$.

⁴For parameter values of two firms' costs see Figure 2.1.

⁵The interaction between the incumbent and entrant, and choice of optimal capacity and output is discussed in Section 2.4 and Section 2.5. The results of runs *Ncga2* in Table 2.1, p. 52 and *Ncga13* in Table 2.2, p. 58 attest to the above observation.

Dividing equation (2.14) by k_1 and rearranging we obtain a term for the incumbent's capacity when it is fully utilised, which I will denote as *critical* capacity, $\check{k}_1$, given by:

$$\check{k}_1 = \frac{(a + f_2 + m_2 + \delta c_2 - 2f_1)}{3b} \quad (2.16)$$

this term is independent of the adjustment cost factor g_1 ; it is a function of the raw materials cost of the incumbent firm, the cost of production of its opponent, and demand parameters. When the incumbent chooses a capacity $k_1 > \check{k}_1$, he operates with overcapacity since in this region $\frac{x_1}{k_1} < 1$; and similarly when $k_1 < \check{k}_1$, firm 1 overutilises its capacity and production exceeds capacity, since in this region $\frac{x_1}{k_1} > 1$.

Firm 2 produces at an output level of x_2 which derives from the Nash-Cournot competition with firm 1. This rate of production is determined by equation (2.13); it is therefore dependent on firm 2's costs of production and is inversely proportional to the output of firm 1. Firm 1 in turn uses (2.14) to determine its optimal rate of production as a Nash-Cournot player in a production game with firm 2. The optimal output, x_1 , of firm 1 is dependent on its own costs, its opponents raw materials costs and its own capacity, k_1 . When g_1 is very small ($g_1 = 0.02$), the first term in (2.14) dominates. Possessing a very large capacity does not ensure for the incumbent a large market share, since the coefficient of the second term, k_1 , is very small, and production costs determine x_1 . Also when firm 1 employs a capacity well below the critical capacity $\check{k}_1$, production exceeds capacity by a wide margin of more than 2.5 orders of magnitude when the two firms face asymmetric costs. Firm 2 on the otherhand ends up having a *negative* capacity. This is clear from from equation (2.15). At very low values of adjustment cost factor, (e.g. $g_2 = 0.02$), the second term which is always negative, becomes very large and dominates the output term, x_2 . With no capacity constraints in place the results are spurious, producing a negative k_2 . It becomes necessary, when g_i values are very small, to impose capacity constraints on output for any meaningful results to be derived from this exercise.

The choice of the incumbent firm's capacity has an important bearing on its production capability, capacity utilisation, profitability, and also on its opponents capacity

of entry and production potential. Firm 1's choice of capacity $k_1 > \check{k}_1$ ensures that it operates with some overcapacity. In this region it gets a larger market share the larger k_1 is, but this is at the expense of a larger overcapacity which rises as k_1 gets bigger, especially with low values of g_1 . As g_1 increases overcapacity declines, because higher adjustment costs cause the firm to increase its production. Figure 2.1 depicts this variation of overcapacity for the incumbent firm at different levels of capacity k_1 . For values of $k_1 < \check{k}_1$, the incumbent firm, in the absence of capacity constraints, overproduces by operating above its physical capacity. In this region larger g_1 results in firm 1 reducing its output to minimise the adjustment cost. It achieves this by operating the plant at levels closer to its design capacity.

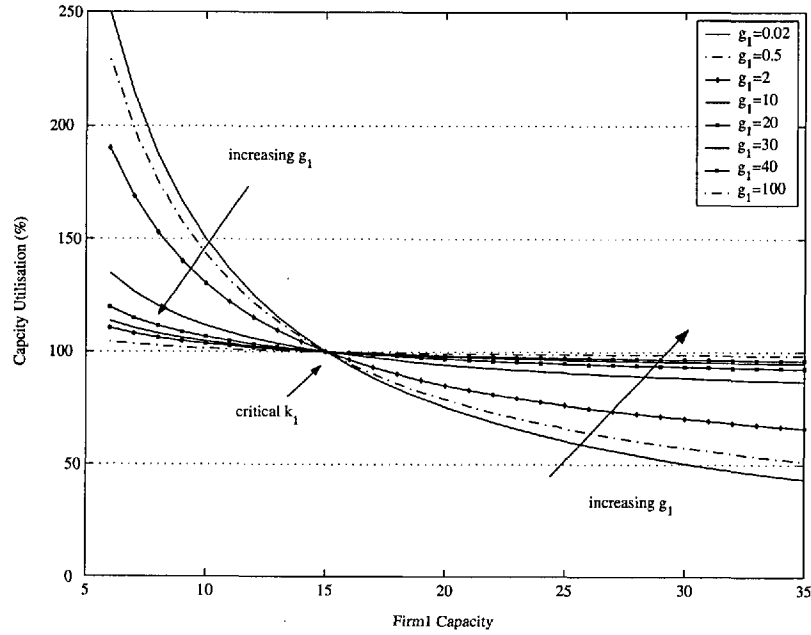


Figure 2.1: Capacity utilisation vs capacity of incumbent firm for a duopoly: parameter values used: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 10$, $c_2 = 2$, $d_1 = 50$, $d_2 = 50$, $f_1 = 10$, $f_2 = 8$, $m_1 = 5$, $m_2 = 2$.

The situation for firm 2 is in the reverse order; for $k_1 > \check{k}_1$ its share of the market declines as the incumbent's capacity level k_1 increases. And because an increase in g_1 causes firm 1 to increase its output in this region, firm 2 reduces its production and choice of capacity level with rising g_2 . While both x_2 and k_2 increase in the region where $k_1 < \check{k}_1$. The impact on capacity utilisation is uniform. As we have seen earlier the entrant always

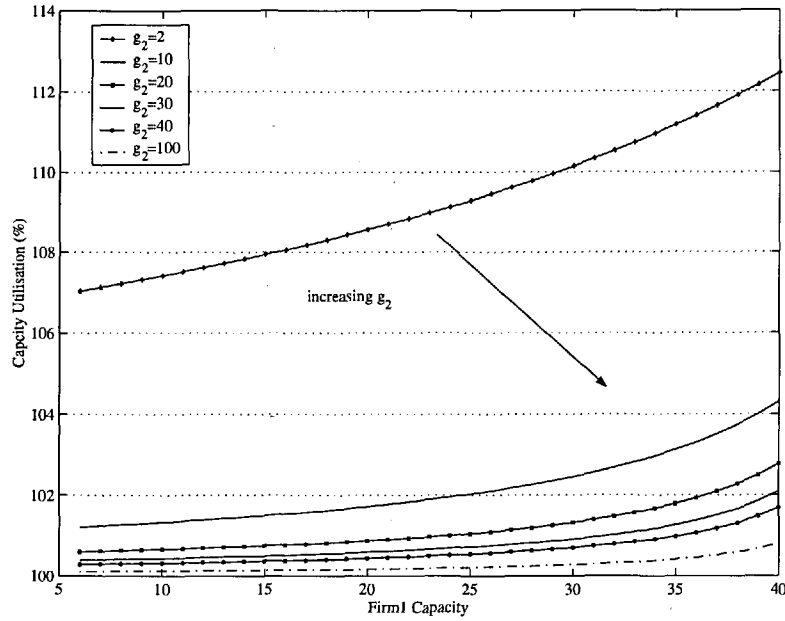


Figure 2.2: Capacity utilisation for entrant vs capacity of incumbent firm for a duopoly: parameter values used: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 2$, $c_2 = 2$, $d_1 = 50$, $d_2 = 50$, $f_1 = 10$, $f_2 = 8$, $m_1 = 5$, $m_2 = 2$.

chooses production higher than capacity; a rise in g_2 , causes overproduction to decline. To minimise adjustment costs firm 2 which is selecting both capacity and output, operates such that capacity utilisation moves closer to the 100% level as shown in Figure 2.2

Firm 2's entry level into the market is dependent on the incumbent's output and capacity. As the incumbent increases its capacity beyond $\check{k}_1$, the entrant's entry level k_2 declines. When the incumbent faces high adjustment costs, its output, x_1 , rises and the entrant therefore enters at an even lower level, and produces at a lower rate.

For intermediate values of g_1 , the impact on the direction of movement of the output can be inferred from the adjustment cost term $\frac{1}{2}g_1(x_1 - k_1)^2$, and the critical capacity $\check{k}_1$. Above this capacity level firm 1 carries overcapacity; and below it overproduces above the capacity in place. In this region its output exceeds capacity, such that $x_1 > k_1$. As g_1 increases, firm 1 increases its output when its capacity is above $\check{k}_1$, to reduce overcapacity and minimise the adjustment cost term. Alternatively output decreases when k_1 is less than $\check{k}_1$, also in order to minimise adjustment costs that are caused by overproduction.

These variations in the capacity utilisation on either side of $\check{k}_1$ are depicted in Fig-

ure 2.1. However, when capacity constraints are enforced, the incumbent firm produces at full capacity, at or below $\check{k}_1$, since enforcing capacity constraints ensures that $x_1 \leq k_1$; therefore when $k_1 \leq \check{k}_1$, we select an output that matches and does not exceed capacity. In this region, the output of firm 1 becomes unaffected by variations in the value of g_1 . All the curve segments to the left of $\check{k}_1$ in Figure 2.1 collapse on the horizontal of full capacity utilisation when capacity constraints are enforced. While the entrant, firm 2, which has to select its optimal capacity and output, would operate at full capacity. When capacity constraints are binding, all the curves in Figure 2.2 would collapse on the base line where capacity utilisation = 100%.

2.3 An Alternative Approach to Compute Nash-Cournot Equilibrium Solution with Binding Capacity Constraints

Finding the equilibrium to our problem was made easy by the quadratic nature of the model, and the linearity imposed on the inverse demand function. Implementing the first order conditions for a Nash-Cournot equilibrium was straight forward as a result of the linearity of equations (2.8)-(2.12). In situations where the demand function is nonlinear, or with higher order cost functions than what was assumed in Section 2.2.1, a more advanced treatment is required to find a Nash-Cournot equilibrium for the oligopoly problem posed in (2.1). A problem of this nature was examined by Murphy, Sherali & Soyster (1982)⁶. The same oligopoly problem was later dealt with by a number of papers including Harker (1984), Dafermos (1983) and Outrata, Kočvara & Zowe (1998). Each employs a variant formulation that relies on variational inequality theory, to construct a computational scheme to obtain an equilibrium solution which is shown to be an equivalent Nash equilibrium for the oligopoly market problem. Different algorithms are used to arrive at the Nash-Cournot equilibrium solution⁷; the main emphasis in these papers is on the efficiency of the algorithms in dealing with the oligopoly problem, and for formula-

⁶The demand function in their paper was of the form $P(X) = 5000^{\frac{1}{\gamma}} X^{-\frac{1}{\gamma}}$, where the price elasticity $\gamma > 1$; the cost function also had a similar form, and the industry was served by 5 firms.

⁷The Nash-Cournot equilibrium solution is not unique in this case, and is dependent on the initial vector of output values used for the players. This problem can be implemented in MATLAB using a nonlinear equation solver and utilising a formulation as in Harker (1984); it takes 5 to 7 iterations to converge to the Nash-Cournot equilibrium in a matter of few CPU seconds.

tions of varying degrees of complexity. Nagurney (1988) discusses different algorithms for solving an oligopoly problem where a large number of firms are involved in the production, and marketing of several commodity products that each sells in different markets. Each market has its own demand function that determines the price. Transportation costs are involved as well for shipments of products between markets. Clearly this is a more complex problem than those dealt with by previous researchers. She develops a linearised Gauss-Seidal decomposition method to compute the Nash-Cournot equilibrium, and her results show that this method is computationally superior to other algorithms.

The literature on the development of generalised Nash-Cournot games and evaluation of their associated equilibria as variational inequality problems is surveyed in Harker (1991). In this paper he also discusses computational methods and provides appropriate algorithms for each problem formulation. The relationship between variational inequality theory and Nash-Cournot game theoretic problems was demonstrated in an earlier paper (Harker 1984), where a diagonalisation/relaxation (also known as nonlinear Jacobi) algorithm based on a contribution by Dafermos (1983) was presented for solving this type of problems. Only the main steps involved in this procedure are outlined below.

2.3.1 Variational Inequality Formulation

The problem of finding a solution for the maximisation problem (2.1), can be dealt with as a finite dimensional variational inequality problem, which involves finding a solution vector $\mathbf{u}^*$, where $\mathbf{u}^* \in U$, and U is a closed and convex set, such that:

$$\mathbf{F}(\mathbf{u}^*)'(\mathbf{u} - \mathbf{u}^*) \geq 0 \quad \forall \mathbf{u} \in U \quad (2.17)$$

where $\mathbf{F}(\mathbf{u})$ is some smooth and monotonic function, $\mathbf{F} : U \rightarrow \mathbb{R}_+^n$, and $\mathbf{F}(\mathbf{u})'$ is the transpose of $\mathbf{F}(\mathbf{u})$. In our formulation of the oligopoly problem, $U \subseteq \mathbb{R}_+^{2n}$, since each player i of the n players controls 2 decision variables, $\mathbf{u}_i = (u_i, x_i)'$. Also define:

$$\mathbf{F}_i(\mathbf{u}^*) = -\frac{\partial w_i(\mathbf{u}^*)}{\partial \mathbf{u}_i}, \quad \text{and,} \quad \mathbf{F}(\mathbf{u}^*) = (\mathbf{F}_1(\mathbf{u}^*)', \dots, \mathbf{F}_i(\mathbf{u}^*)', \dots, \mathbf{F}_n(\mathbf{u}^*)')'$$

where $\mathbf{u}_{N \setminus i}^*$ refers to the vector of optimal decision variables under the control of all other n players excluding player i .

This formulation involves the formation of a smooth function $G(\mathbf{u}, \mathbf{u}) : U \times U \rightarrow \mathbb{R}_+^n$, where $G(\mathbf{u}, \mathbf{u}) = F(\mathbf{u})$, $\forall \mathbf{u} \in U$. The algorithm starts at some chosen initial vector $\mathbf{u}_0 \in U$, and solves sequentially in step s ($s = 1, 2, \dots$) the following version of (2.17) until conversion (see Dafermos (1983) for a discussion and proofs) which is when no player has any incentive to change his control variables:

$$G(\mathbf{u}^{*,s}, \mathbf{u}^{*,s-1})'(\mathbf{u} - \mathbf{u}^{*,s}) \geq 0 \quad \forall \mathbf{u} \in U \quad (2.18)$$

for player i , then we have

$$G_i(\mathbf{u}_i^{*,s}, \mathbf{u}_{N \setminus i}^{*,s-1}) = -\frac{\partial w_i(\mathbf{u}_i^{*,s}, \mathbf{u}_{N \setminus i}^{*,s-1})}{\partial \mathbf{u}_i},$$

equation (2.17) can therefore be represented as follows:

$$\sum_i^n -\frac{\partial w_i}{\partial \mathbf{u}_i}(\mathbf{u}_i^{*,s}, \mathbf{u}_{N \setminus i}^{*,s-1})'(\mathbf{u}_i - \mathbf{u}_i^{*,s}) \geq 0 \quad \forall \mathbf{u} \in U \quad (2.19)$$

Dafermos (1983) and Harker (1984) show that for firm i , the first order conditions for the maximisation of its payoff function w_i as in (2.1), namely:

$$\max_{\mathbf{u}_i} w_i(\mathbf{u}_i, \mathbf{u}_{N \setminus i}) \quad \mathbf{u}_i \in U_i, \forall i \in N, \quad (2.20)$$

can be represented as:

$$-\frac{\partial w_i}{\partial \mathbf{u}_i}(\mathbf{u}_i^*, \mathbf{u}_{N \setminus i})'(\mathbf{u}_i - \mathbf{u}_i^*) \geq 0 \quad \forall \mathbf{u}_i \in U_i, \quad (2.21)$$

where U_i is player i 's set of feasible strategies, and $-\frac{\partial w_i}{\partial \mathbf{u}_i}(\mathbf{u}_i^*, \mathbf{u}_{N \setminus i})$ is the gradient of w_i wrt to player i 's strategy vector $\mathbf{u}_i$, which is the optimal solution to (2.20). In view of the concavity of w_i wrt $\mathbf{u}_i$, this gradient is the direction of ascent towards the solution $\mathbf{u}_i^*$. Once at the solution, no feasible ascent direction exists and conversion is accomplished. Equation (2.19) above is the summation of the individual player's first order conditions

for maximisation of their respective payoff functions. The solution to the variational inequality, is equivalent to finding the equilibrium solution to the Nash-Cournot problem.

2.3.2 Algorithm Procedure and Implementation

There is more than one approach to finding a solution to the problem in (2.20). A direct approach involves implementing the first order optimality conditions directly, and solving a math programme as in Murphy, Sherali & Soyster (1982). Considering that there is an incumbent firm with an existing plant in place, one solves a set of $3n - 1$ equations as in (2.8)-(2.12). For situations where we have complex demand and cost functions, that are of higher order than quadratic functions, this set of equations can be solved using any nonlinear equation solver.

The variational inequality approach adopted in the previous section (Harker 1984), for implementing (2.19) involves solving n optimisation subproblems (2.20) in a serial manner. This approach has advantages in terms of efficiency of implementation, especially when dealing with large scale and highly nonlinear problems, by dividing a large scale problem into n smaller problems. In addition to solving complex oligopoly market problems, this approach can be adopted to solve very large scale traffic and network flow problems; examples of these applications can be found in Nagurney (1988), and Harker (1991) and references therein.

Furthermore an optimisation approach allows the imposition of constraints on the variables of the problem, and to specify feasible regions for the solution. For instance investment and production rates can be limited to be positive and bounded by an upper limit. In particular the outputs of the two firms may be constrained indirectly by specifying that product price remains within a positive region. For our Nash-Cournot problem we solve n optimisation subproblems in the following manner.

$$\max_{\substack{w_i \\ u_i \geq 0 \\ x_i \geq 0}} w_i = Px_i - C_i - I_i \quad u_i \in U_i, \forall i \in N \quad (2.22)$$

$$= \left(a - b(x_i^s + X_{N \setminus i}^{s-1}) \right) x_i^s - C_i - I_i \quad (2.23)$$

subject to:

$$\begin{aligned} x_i^s &\leq k_i^s && \text{(capacity constraint)} \\ x_i^s &\leq \frac{a}{b} - X_{N \setminus i}^{s-1} && \text{(price constraint)} \end{aligned} \quad (2.24)$$

where we are at step or iteration s of the algorithm; and define the production of opponents to firm i as $X_{N \setminus i} = \sum_{\substack{j=1 \\ j \neq i}}^n x_j$, and total industry output becomes $X = x_i + X_{N \setminus i}$.

1. We initiate the algorithm by setting $s = 1$, and choose a starting vector $\mathbf{u}^0$, where $\mathbf{u}^0 = (\mathbf{u}_1^0, \dots, \mathbf{u}_i^0, \dots, \mathbf{u}_n^0)'$.
2. Next solve (2.23) for a new value $\mathbf{u}_i^s = (u_i^s, x_i^s, k_i^s)'$; and repeat procedure for all firms, $i = 1, \dots, n$.
3. Once a new set of $\mathbf{u}_i$ values is obtained, update vector $\mathbf{u}^s$.
4. Evaluate if $|\mathbf{u}^s - \mathbf{u}^{s-1}| \leq \epsilon$, where ϵ is a preset small tolerance value, then stop. The current value of $\mathbf{u}^s$ is the Nash-Cournot equilibrium solution.
5. Else, the latest computed value of $\mathbf{u}^s$, becomes the new starting vector for (2.23); increment s , $s = s + 1$, and go back to 2. This completes one iteration of the algorithm. Continue with this procedure until convergence.

From the above procedure it is clear that the algorithm describes the process of arriving at a Nash-Cournot equilibrium. Each firm i , in turn, takes the output of all other firms $X_{N \setminus i}$ as given, and tries, by solving (2.23), to improve on its payoff function by adjusting its output, investment rate and plant capacity until it finds a new maximising vector $\mathbf{u}_i$. The process is repeated until no firm finds it profitable to change its output profile. The firms perform these actions independently. In step 4 above, vector $\mathbf{u}$ describes all the firms in the industry. The Nash-Cournot equilibrium solution, is arrived at when all the firms attain optimal maximising values that they cannot improve upon in successive steps. The optimal vector we arrive at upon convergence, $\mathbf{u}^* = (\mathbf{u}_1^*, \dots, \mathbf{u}_i^*, \dots, \mathbf{u}_n^*)'$, is the Nash-Cournot equilibrium solution.

Variational inequality methods can be used for dynamic situations. Haurie, Zaccour & Smeers (1990), Pineau (2000), and Pineau & Murto (2000) implement dynamic game

formulations that involve capital accumulation. These works make use of GAMS (2003) software, which seems to be in use in other publications with models dealing with nonlinear functions that appear in the applied game theory literature⁸. Slade (1989, 1994) uses a different formulation for dynamic games, in which firms maximise an objective function that she terms as a *fictitious payoff-function*. The problem is formulated such that the whole oligopolistic industry appears to be maximising a single objective function; and where the firm's objective function embodies the payoffs for all the periods involved (two periods are considered as an illustration in the paper). Slade's approach is similar to that of Murphy, Sherali & Soyster (1982) in that all first order conditions for optimality are solved simultaneously. A specialised proprietary nonlinear optimisation software is used to obtain the Nash-Cournot equilibrium solution. Nagurney (1988) also uses a specialised university software. The last two references however do not involve capital accumulation or depreciation in their models.

In this chapter as well as in the rest of this thesis, I make use of a PC version of MATLAB[®](2000) software to obtain model solutions⁹ for different *Nash equilibria*. MATLAB utilises two approaches for dealing with constrained nonlinear optimisation problems. The one I employ selects a sequential quadratic programming algorithm to solve the maximisation problem. The algorithm moves in the direction of the gradient at each step to arrive at the maximum; and one provides the analytic gradient of the objective function to speed up the convergence process. A brief but clear description of the algorithm used is provided in the MATLAB documentation. But for a more detailed description the reader is referred to Schittkowski (1985/6).

The constraints of (2.24) are formed in matrix form $Au_i \leq b$, and the non-negativity constraints are introduced in the argument of the *maximisation* function as lower and upper bounds on the elements of u_i . In evaluating the cases of Table 2.1 I assume that the incumbent, firm 1, has a capacity of $k_1 = 21$, which is its monopolistic capacity,

⁸GAMS apparently has features of a mathematical programme with equilibrium constraints, that it combines with nonlinear programme models to solve single or multiple-objective function models. Its documentation specifies that the main applications are in economics and game theory.

⁹The optimisation in equation (2.23) is computed using the *fmincon* function; all MATLAB functions are essentially minimisation functions as this software is used mainly as a tool in solving and simulating engineering optimal control problems where the aim is to minimise cost functions. By using the negative of the objective function one converts a minimisation problem into one of maximisation.

and throughout the exercises I use a tolerance value of $\epsilon = 0.001$. In addition the parameter values for the exercises are chosen such that the cost structure corresponds to the prevailing average costs for the basic petrochemical industry. Raw materials and utilities costs are the dominant costs of basic petrochemical products, and depending on the scale of the plant and technology used, can range from 60% to as high as 80% or more, of the production cost. Fixed costs that relate to salaries, and other capital related costs not affected directly by the level of production are usually in the range of 15-40%. And capital investment costs are on average in the range of 5-10% of the total production costs¹⁰ (see Section 5.5.2 of Chapter 5 for an illustration). For dynamic situations one just adds the equation of state describing the evolution of the capacity into a matrix equation describing equality constraints similar to the one above, and the dimensions of the matrices and vectors would be extended to take account of all periods involved. The objective function to be maximised in (2.22) would be reformulated to account for the discounted sum of w_i for all periods.

2.4 Incumbent under threat of entry in a Nash-Cournot game

When the market is served by the incumbent only, firm 1 acts as a monopolist and achieves monopolistic profits. Its optimal choice of plant capacity, k_1 , and output, x_1 , and the corresponding market price and profits, w_1 , are shown in Table 2.1 (for the particular set of model parameters specified in the table footnotes). With its monopolistic capacity fixed at $k_1 = 21$, firm 1 faces an entrant, firm 2, and has to decide whether to maintain, or to change its monopolistic output under the threat of entry. Once entry occurs, and since both the incumbent and the entrant are concerned only about profit maximisation, the two firms engage in a Nash-Cournot game and employ a solution concept based on Section 2.3 (or Section 2.2.2).

I consider two situations; one where the entrant has the same production costs as the

¹⁰These ranges of cost estimates pertain to the commodity sector of the petrochemicals industry; and cover a range of plants of different sizes over a period going back to the 1970's. The wide variation in the range of costs reflects the level of complexity of technology in use, the size of the plant, and the benefits of learning effects and economies of scale over time.

incumbent, and the second where the two firms have asymmetric costs with the entrant being the more efficient firm with lower production costs. This could arise as a result of the entrant employing a more efficient technology and production process for instance. I also allow for two possible scenarios of adjustment costs: low adjustment costs ($g_i = 2$) which indicate that firms can operate plants with significant overcapacity at low penalty cost; and with higher adjustment costs (e.g. $g_i = 20$), firms are penalised more heavily for operating their plants at high rates of overcapacity.

Table 2.1: Duopolistic competition between incumbent and entrant firms in a Nash-Cournot game

	Incumbent				Entrant			Industry		
	x_1	k_1	w_1	$cu_1\%$	x_2	k_2	w_2	X	Price	Profit
<i>Low adjustment cost ($g_1 = 2, g_2 = 2$)</i>										
<i>Mono</i>	21.00	21.00	883.0	100				21.00	58.0	883.0
<i>Ncgs1</i> ⁽¹⁾	18.00	21.00	406.0	86	12.00	12.00	289.0	30.00	40.0	695.0
<i>Ncga2</i> ⁽²⁾	17.42	21.00	344.4	83	13.74	13.74	377.6	31.16	37.7	722.0
<i>High adjustment cost ($g_1 = 20, g_2 = 20$)</i>										
<i>Ncgs3</i> ⁽¹⁾	20.35	21.00	433.4	97	10.83	10.83	235.4	31.18	37.6	668.8
<i>Ncga4</i> ⁽²⁾	20.22	21.00	372.0	96	12.34	12.34	304.5	32.56	34.9	676.6
<i>Ncga5</i> ⁽³⁾	19.64	21.00	92.4	94	11.86	12.34	136.7	31.50	22.0	229.1
<i>Ncga6</i> ⁽⁴⁾	19.41	21.00	-15.7	92	11.62	12.34	71.7	31.03	16.9	55.9
<i>Simultaneous entry</i> ⁽⁵⁾ ($g_1 = 20, g_2 = 20$)										
<i>Ncgs7</i> ⁽¹⁾	14.00	14.00	393.0	100	14.00	14.00	393.0	28.00	44.0	786.0
<i>Ncga8</i> ⁽²⁾	13.03	13.03	340.7	100	15.93	15.93	507.8	28.96	42.1	848.5

Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, $k_1 = 21$. cu_1 refers to capacity utilisation of firm 1. The convention used for the numbering of the different runs, is that in *Ncgs1*, for instance, it refers to a Nash-Cournot run with changing value for g_i , and symmetric costs for both firms. The last digit refers to the run number.

(1) Firms with symmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

(2) Firms with asymmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 2$, $f_2 = 8$, $m_2 = 2$, $d_2 = 50$.

(3) Firms with asymmetric costs as in case *Ncga4* but with lower demand, where $a = 85$.

(4) Firms with asymmetric costs as in case *Ncga4* but with lower demand, where $a = 79$.

(5) When both firms enter the market simultaneously, the assumption is that both start from zero capacity.

Under symmetric production costs and low adjustment cost factors, the incumbent reduces his output to $x_1 = 18$, and operates at a capacity utilisation rate of 86%, as shown in case *Ncgs1*. Firm 2, the entrant, selects an optimal capacity $k_2 = 12$, and operates at full capacity. The industry price and profits fall as a result of entry. The well established firm, firm 1, makes more profits than firm 2. But because firm 2 operates at

full capacity, it appears to be more productive since its capacity is 57% relative to firm 1, while its profits are 71% those of its rival.

When the entrant is more efficient in terms of its lower costs, it enters at a higher capacity than with symmetric costs. In this case the incumbent reduces its output further and operates with a capacity utilisation rate of 83%, but still retains a larger share of the market than the entrant. The entrant operates a smaller plant, and has a smaller market share than the incumbent, but makes higher profits as case *Ncga2* illustrates. Total output is higher than with symmetric costs, and industry profits are higher.

Higher adjustment cost factors, as in cases *Ncgs3* and *Ncga4*, force the incumbent to operate at high utilisation rates. This forces the entrant to enter at even lower capacity levels than before. Because output of the incumbent rises to avoid incurring higher costs of adjustment, industry price drops and so do profits. Prices and profits are lower than the corresponding cases with low adjustment costs. Under this scenario, in case *Ncga4*, the entrant firm does worse than the incumbent even when the former has lower costs. The incumbent firm also makes more profits than it did under case *Ncga2* when it had lower adjustment costs.

This anomalous situation of the high cost producer, firm 1, attaining larger market share and higher profits than its entrant rival, firm 2, arises as a result of the static nature of the game. Firm 1 has a short term advantage that derives from its position as an incumbent with an established production capacity in the market. By selecting a large plant it forces its more efficient rival firm 2 to enter at a smaller scale. Although firm 1 reaps higher profits, firm 2 runs a more efficient operation; it has lower cost per unit output, and generates higher revenue per unit of production capacity.

The advantage that accrues to the entrant from its lower production and investment costs, becomes clear when market conditions deteriorate; the entrant is capable of competing in this environment, and of withstanding adverse market conditions better than the incumbent. Consider the situation of case *Ncga4*, where the two firms have established their production capacities, as indicated in Table 2.1, and the level of demand is at $a = 100$; but now when market demand drops to $a = 85$, both firms suffer due to lower demand as shown in the results of case *Ncga5*. Production for firm 1 declines. It

now operates at a capacity utilisation rate of $cu_1 = 94\%$. Firm 2 also reduces its output. Instead of operating at full capacity it operates at $cu_2 = 96\%$ utilisation rate. Market price declines from $P = 34.9$, to $P = 22.0$; but more importantly the fortunes of the two firms are interchanged. The entrant, firm 2, although it now produces and retains a much smaller market share than its opponent, generates substantially higher profits than the incumbent.

This is a particular feature of oligopolistic markets, where market price in a quantity game, is dictated by the least efficient producer. The more efficient producer benefits from the high market price and despite its smaller market share, can in some instances (as in cases *Ncga2*, *Ncga5*, and *Ncga6*), generate higher profits than the high cost opponent. When demand declines and price drops the least efficient producers that form the tail-end of the supply curve are unable to sustain their inefficient production operation and are the first to exit the market. A further decline in the demand to $a = 79$, (case *Ncga6*) leads to both firms reducing their output, with firm 1 now operating at a capacity utilisation rate of $cu_1 = 92\%$. Price drops to $P = 16.9$, and firm 1 makes losses of $w_1 = -15.74$; while the more efficient firm 2 continues to generate profits, as shown in case *Ncga6*. The advantage for the incumbent firm from holding a large capacity that allows it to obtain a large market share and higher profits, at the expense of a more efficient entrant is sustainable only when demand is high to support high utilisation rates and prices. In the presence of adjustment costs that penalise producers for operating their plants with overcapacity, an inefficient producer that relies on having large scale plant to gain market share can easily find its venture run into difficulty if demand falls and prices decline as the above examples demonstrate.

2.5 Entry or Mobility Deterrence by Incumbent

Instead of the inflexible position of having the incumbent firm stick to its monopolistic capacity in the face of entry, let us consider a situation where the incumbent can pick a plant of capacity k_1 of his choice first, and then the entrant having taken notice of the incumbent's capacity, decides whether to enter the market; and if she does, has to choose

an optimal plant capacity and output.

This is a two-stage game, where in the first stage the incumbent acts as a Stackelberg leader in the choice of capacity; and in the second stage both the incumbent and entrant engage in a Nash-Cournot game in outputs. In this game capacity is fixed; once chosen, it cannot be changed. The incumbent by acting as a Stackelberg leader, can conduct an analysis of the choice of capacity, given the anticipated reaction of the entrant, that maximises his profits; or can even contemplate the situations where it would be possible to deter the entrant by blocking her completely from entering the market.

Below, I examine a set of examples, under different scenarios of cost structures for the incumbent and entrant firms. The first assumes that both firms have identical costs; and the second corresponds to a situation where the incumbent has the advantage of lower investment and production costs. While in the third, the entrant takes on the role of the more efficient producer which possesses lower costs. Also four values of adjustment costs ranging from low ($g_i = 2$), to medium ($g_i = 10$, $g_i = 20$), to high ($g_i = 100$) are considered for each scenario. These adjustment costs correspond to the respective situations where when it takes low values ($g_i = 2$) it would be least costly to operate plants at low capacity utilisation rates; and when it is high it indicates the high cost of overcapacity. In each situation, the examples help to illustrate the nature of strategic interaction between the two firms and the likely market equilibrium that could emerge.

Since the incumbent has the opportunity to select his capacity first, he can therefore carry out an analysis to find the best starting capacity to confront the entrant with, that maximises his profits, given his knowledge of the costs of production of both players. In these exercises the differentials in costs between the two firms for raw materials, fixed costs and investment costs when they run asymmetric operations, are chosen within reasonable limits. Although for commodity petrochemical products it is unlikely that producers would have differentials in raw materials costs that are as high as those considered here. Nonetheless the exercises serve to illustrate the extent and possible influence of incumbent's capacity on the outcome of the Nash-Cournot production game that ensues.

In carrying out an analysis of the optimal choice of capacity, the incumbent starts from

an initial capacity, say $k_1 = 6$, and finds the Nash-Cournot solution to his production game with the entrant. Both find the optimal output and investment rates that sustain the profitability of their operations using the variational inequality formulation discussed in the previous section. The incumbent then selects a new capacity; so he increments k_1 to $k_1 = 7$, and conducts the same exercise for finding the new Nash-Cournot solution. And so on until the incumbent finds the best choice of k_1 that maximises his profits. Here the incumbent is acting like a Stackelberg leader in its choice of capacity. The results are discussed below and shown in Figures 2.3-2.5.

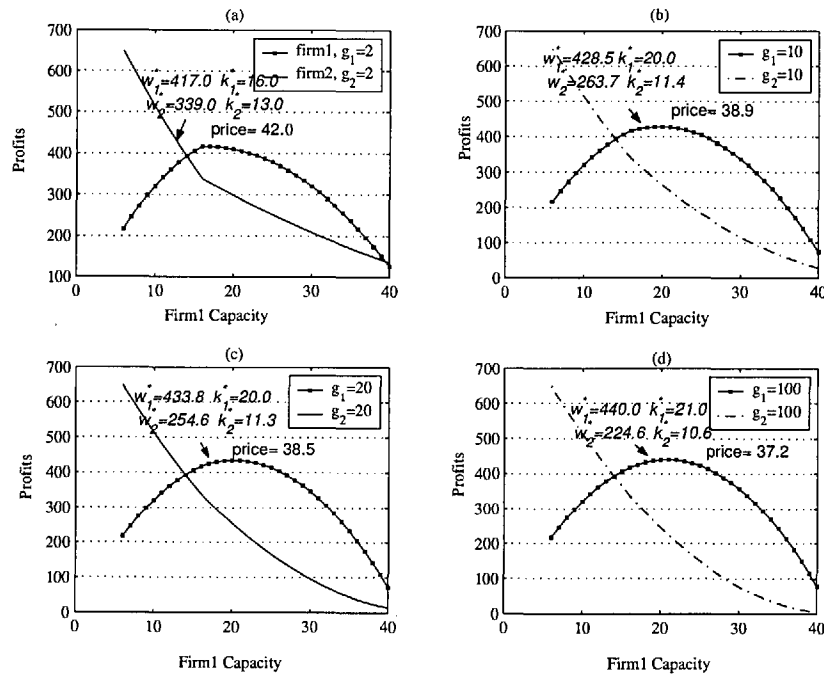


Figure 2.3: Incumbent's capacity as a mobility deterrence in a Nash-Cournot game against an entrant under symmetric costs: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 10$, $c_2 = 10$, $d_1 = 50$, $d_2 = 50$, $f_1 = 10$, $f_2 = 10$, $m_1 = 5$, $m_2 = 5$.

2.5.1 Symmetric Costs: incumbent searching for k_1^*

The two firms have symmetric costs when the incumbent and entrant have access to the same technology, raw materials, and utilities. They also have the same investment cost per unit of capacity, c_i , which is assumed here to be constant and independent of the scale of plant. This assumption will be relaxed in later chapters to allow for economies

of scale and learning to be taken into consideration.

With low adjustment costs, $g_i = 2$, the incumbent is unable to block the entrant from the market as the results of the Nash-Cournot analysis show in pane (a) of Figure 2.3. It is worth noting that each pair of values represent the profits of the two firms when they are in a Nash-Cournot equilibrium. In the figure they are plotted against the capacity chosen by the incumbent. In this exercise, the *Nash equilibrium* for the incumbent that maximises his profits corresponds to a choice of capacity of $k_1 = 16$. Firm 1 operates at full capacity and generates profits of $w_1 = 417.0$. Firm 2 enters the market with a capacity of $k_2 = 13$, and generates profits of $w_2 = 339.0$. At this point market price is $P = 42$. These results indicate that it is worthwhile for the incumbent to use a lower capacity than his monopolistic capacity; both firms now achieve higher profits than cases *Ncgs1* and *Ncgs3* of Table 2.1, p. 52. Also there are two instances at which the two firms generate equal profits. When they have equal capacities of $k_i = 14$, each; and when $k_1 = 39.5$, and $k_2 = 8.3$. In the latter case firm 1 operates at a capacity utilisation of $cu_1 = 65\%$ and the two firms generate profits of $w_i = 139.0$, each.

Firm 1 is unable to deter firm 2 from entering the market by installing a very large capacity. Beyond $k_1 = 40$, the profits of firm 1 start to decline at a faster rate than firm 2; eventually firm 1 would be making losses before it can make the entry of firm 2 a loss making venture. Firm 1 can, however, follow a mobility deterrence policy. When it chooses a capacity in the range of $k_1 = 25 - 31$, its profits remain above 300; while firm 2 enters with a lower capacity, and its profits are pushed down into the range of 200. However, the two firms can do better if they stick to their profit maximising policy where they achieve the highest profits by sticking to their *Nash equilibrium* at $k_1 = 16$, and $k_2 = 13$. With higher adjustment costs ($g_i = 10$, $g_i = 20$, and $g_i = 100$) the incumbent firm operates at higher rate, and installs larger capacity. The Nash-Cournot equilibrium capacity for firm 1 rises to $k_1 = 20$, and $k_1 = 21$, respectively. Higher adjustment costs force a rise in utilisation rate for the incumbent. The entrant as a result enters at a lower capacity and takes a slightly smaller share of the market than before. However, total industry output rises, and as a result market price drops, and industry profitability declines, as shown in panes (b), (c), and (d) of Figure 2.3. These

Table 2.2: Capacity preemption by incumbent in a Nash-Cournot game under different cost structures

	g_i	Incumbent				Entrant			Industry		
		x_1	k_1	w_1	$cu_1\%$	x_2	k_2	w_2	X	Price	Profit
<i>Symmetric costs⁽¹⁾: incumbent and entrant have same costs of production</i>											
<i>Ncgs9</i>	2	16.00	16.00	417.0	100	13.00	13.00	339.0	29.00	42.0	756.0
<i>Ncgs10</i>	10	19.08	20.00	428.5	95	11.46	11.46	263.7	30.54	38.9	692.2
<i>Ncgs11</i>	20	19.48	20.00	433.8	97	11.26	11.26	254.6	30.74	38.5	688.4
<i>Ncgs12</i>	100	20.85	21.00	440.0	99	10.57	10.57	224.6	31.57	37.1	664.6
<i>Asymmetric costs⁽²⁾: entrant is more efficient producer</i>											
<i>Ncga13</i>	2	15.00	15.00	362.5	100	14.95	14.95	447.0	29.95	40.1	809.5
<i>Ncga14</i>	10	17.32	18.00	371.8	96	13.57	13.57	380.5	30.89	37.8	752.3
<i>Ncga15</i>	20	18.48	19.00	376.3	97	13.21	13.21	349.0	31.69	36.6	725.3
<i>Ncga16</i>	100	18.88	19.00	381.4	99	13.01	13.01	338.4	31.89	36.2	719.8
<i>Asymmetric costs⁽³⁾: incumbent is more efficient producer</i>											
<i>Ncga17</i>	2	18.40	21.00	528.5	87	11.80	11.80	279.5	30.20	39.6	808.0
<i>Ncga18</i>	10	21.54	23.00	551.8	93	10.23	10.23	229.0	31.77	36.5	762.1
<i>Ncga19</i>	20	22.17	23.00	559.6	96	9.91	9.91	197.5	32.08	35.8	757.2
<i>Ncga20</i>	100	23.79	24.00	568.5	99	9.11	9.11	166.9	32.90	34.2	735.4

Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, k_1 is free.

(1) Firms with symmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

(2) Firms with asymmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 2$, $f_2 = 8$, $m_2 = 2$, $d_2 = 50$.

(3) Firms with asymmetric costs: $c_1 = 2$, $f_1 = 8$, $m_1 = 2$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

results are also summarised in Table 2.2.

Overcapacity may be used strategically by the incumbent. A combination of large capacity and low utilisation rate by the incumbent can act to reduce the profitability and entry level of its rival when adjustment costs are low, or moderate ($g_i = 2$, $g_i = 10$) as in panes (a) and (b) of Figure 2.3. When firm 1 employs a capacity of $k_1 = 40$, firm 2 is faced with the following options. When firm 1 operates with an overcapacity of 36%, firm 2 enters at a capacity of about $k_2 = 8$ ($g_1 = 2$, in pane (a) and the rest of Figure 2.3, the capacity for firm 2 is not shown), and generates profits of $w_2 = 135.5$; and with overcapacity of 14%, firm 2 enters at a capacity of $k_2 = 3.8$ ($g_1 = 10$, pane(b)), and receives profits of $w_2 = 29.4$.

Lower levels of overcapacity achieve the same effect under regimes of higher adjustment costs. An overcapacity of only 8% is needed by firm 1 when $g_i = 20$, which leaves firm 2 with a capacity of about $k_2 = 2.6$, and profits of $w_2 = 14.2$; and with $k_1 = 40$ and $g_i = 100$, firm 2 is virtually deterred from entry into the market. But this is achieved at the cost of lower profitability; since with large capacity and high adjustment costs,

the incumbent operates at higher utilisation rates, which forces down market price and profits.

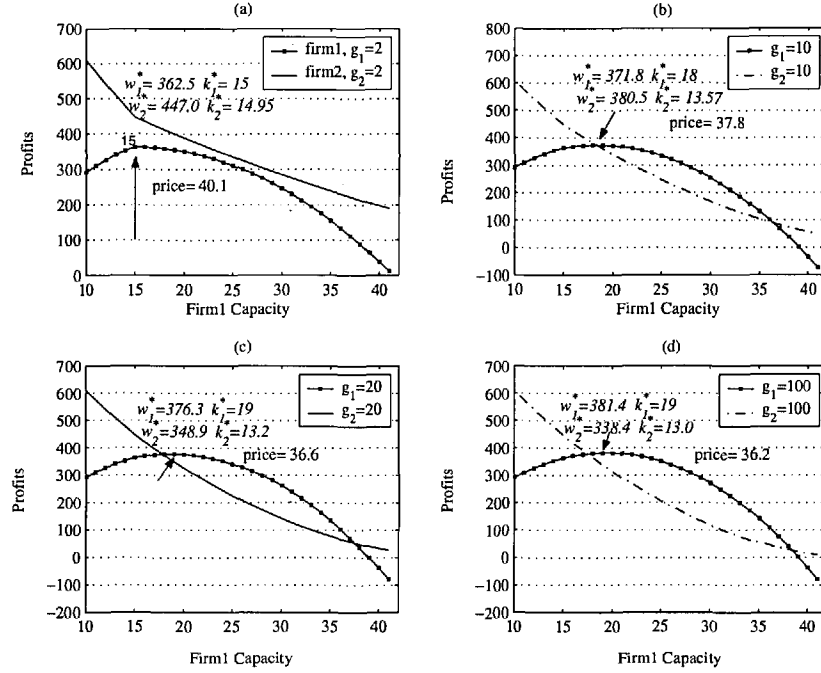


Figure 2.4: Incumbent's capacity as a mobility deterrence in a Nash-Cournot game against an efficient entrant under asymmetric costs: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 10$, $c_2 = 2$, $d_1 = 50$, $d_2 = 50$, $f_1 = 10$, $f_2 = 8$, $m_1 = 5$, $m_2 = 2$.

2.5.2 Asymmetric Costs: entrant as the more efficient producer

An interesting feature of the power of incumbency is illustrated in this example, where the entrant is a producer that enjoys lower costs of production than the incumbent firm. The entrant may be an innovator with a new more efficient technology, or the owner of a process that utilises cheaper raw materials, and may also have lower investment costs. Yet when the entrant is faced with a market dominated by an established incumbent, her entry into, and share of the market is largely influenced by the prevailing market conditions. These conditions are dictated by the less efficient incumbent, when firm 2 accepts the leadership position of capacity choice by firm 1; which in turn influences the distribution of market shares.

The incumbent carries out an analysis of his choices of capacity to select an optimal

k_1^* , that will maximise his profits in the face of the more efficient potential entrant. The results that are shown in Figure 2.4, and summarised in Table 2.2 indicate that the profit maximising Nash-Cournot equilibria in this case are lower, under the different adjustment costs, than what they were when both firm's had symmetric costs; which is expected given the entrant's lower costs. In each case, firm 2 now can enter at a larger capacity than before, and as a result has larger production and market share. There is a rise in total industry output, and market price is lower, but industry profitability as a whole is improved.

In this situation, the incumbent cannot deter the entry of firm 2 by employing a large capacity. In fact as the incumbent's capacity increases to $k_1 > 38$, it starts making losses; while firm 2 can still enter the market, albeit at a low scale, and make some profits as panes (c-d) in Figure 2.4 illustrate. Despite the entrant's lower production costs, the incumbent, in view of his advantage of being in the market first, is capable of capturing a market share of about 60%, when adjustment costs are on the high side.

It should be noted, however, that the incumbent can pursue a policy of mobility deterrence against firm 2 as illustrated especially in panes (c) and (d). Instead of opting for the Nash-Cournot equilibrium that maximises its profits at $k_1 = 19$, firm 1 can choose a larger capacity in the range of $k_1 = 25 - 33$ and force a substantial reduction in the expected profits of its opponent thereby reducing its growth potential. Consider the situation when $g_i = 20$ (pane (c)) and the incumbent opts for $k_1 = 28$, instead of $k_1 = 19$. In this case firm 1 operates with an overcapacity of 6% and its profits drop to $w_1 = 299$, a decline of about 20%. While for the entrant the expected profits drop to $w_2 = 173$, a decline of more than 50%. A similar proportional decline in the fortunes of the entrant arises also when adjustment costs are high ($g_i = 100$) as can be envisaged from pane (d).

2.5.3 Asymmetric Costs: incumbent as the more efficient producer

An incumbent is more likely to benefit from his established position in the market than a potential entrant. There are several ways in which he do so, to reduce his costs, expand his market share and improve profits at the expense of opponents. Being in the

market first, an incumbent can take advantage by forming a large network of suppliers to market his product and create a strong relationship with customers. Building strategically located storage centres and distribution channels reduce the average cost of delivered product to users. Also experience gained over the years by the incumbent in operating production facilities more efficiently contribute to reducing waste, and to improving operating conditions for optimal output. There are cost savings that can be made from reducing investment and capital related costs through learning and technological advances in constructing and developing more efficient plants.

These issues are discussed further in Chapter 5, where empirically determined cost savings from the petrochemical industry are also introduced in finding Nash-Cournot equilibria for an oligopoly. For the time being I consider an investment game between two firms. I assume that the incumbent firm has lower production and investment costs than its entrant opponent.

The cost differential between the two firms is the same as that for the cases considered above in Section 2.5.2, and as shown in Table 2.2, where in this case the incumbent is the more efficient producer.

The incumbent derives additional advantage from being the low cost producer with an established presence in the market. This is apparent from the results of the incumbent's search for optimal capacity to forestall the potential entrant. Under different adjustment cost scenarios the incumbent obtains larger capacity and market share than the entrant. And his profit and market share are consistently higher than what was possible under the cost structures of Section 2.5.1 and Section 2.5.2 as illustrated by the results shown in Table 2.2. Because he is the more efficient producer, the incumbent dominates the production capacity in the market; and retains on average a market share of 64% to 72%. The total industry output is also higher, and consequently prices are lower. But the industry as a whole benefits and profits increase; this indicates that the industry is operating in a region of the market where demand is elastic.

The results of the incumbent's searches for optimal capacity are also shown in the plots of Figure 2.5. Firm 1 obtains consistently higher profits than firm 2; and these profits are higher than what firm 1 obtained when the entrant was the more efficient producer,

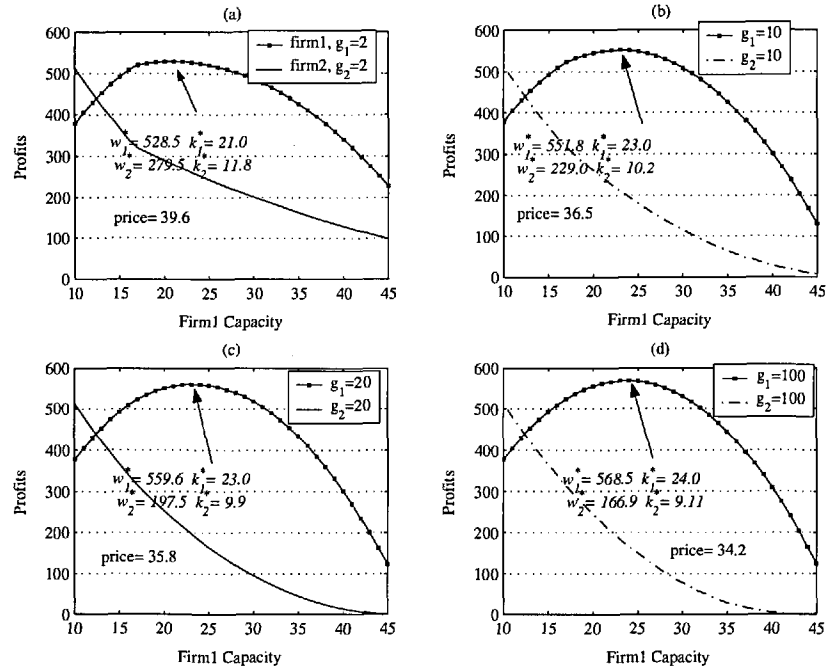


Figure 2.5: Incumbent's capacity as a mobility deterrence in a Nash-Cournot game against a less efficient entrant under asymmetric costs: $a = 100$, $b = 2$, $\delta = 0.10$, $c_1 = 2$, $c_2 = 10$, $d_1 = 50$, $d_2 = 50$, $f_1 = 8$, $f_2 = 10$, $m_1 = 2$, $m_2 = 5$.

by about 45% to nearly 50%. More significant is that the incumbent now dominates the market; where his profits range from being 1.9 times those of the entrant ($g_i = 2$) as in pane (a), to 3.4 times ($g_i = 100$) as in pane (d). This outcome is also indicative of the advantages that the incumbent enjoys, from having an established operation in the market, and from lower production costs.

The pattern of competition which is prevalent in large scale commodity industries such as petrochemicals, is similar in many respects to what is presented in this section. Incumbent firms in such industries benefit from lower production and investment costs that derive from economies of scale, advances in process technology, learning and experience gained over the years in operation and production. Economies of scale and overcapacity allow the incumbent firm to gain market share at the expense of its opponents of smaller producers, and potential entrants. Larger profits and experience gained in industrial production enable the incumbent firm to finance future expansions more readily, and to stay ahead in the investment game.

The incumbent is unable to deter firm 2 from entering the market when adjustment costs are low ($g_i = 2$) as illustrated in pane (a) of Figure 2.5; the incumbent nevertheless, is capable of mounting a mobility deterrence strategy. Using a combination of large plant and some overcapacity the incumbent forces firm 2 to enter at a low capacity and accept modest profits. For instance when firm 1 chooses a capacity of $k_1 = 36$, and operates with an overcapacity of 32% it receives profits of $w_1 = 410.9$, on the other hand firm 2 enters with a capacity of $k_2 = 8.8$, and receives profits of $w_2 = 155.9$.

Mobility deterrence becomes more feasible when adjustment costs rise. Firm 1 can even deter its opponent, firm 2, from the market with moderately higher adjustment costs ($g_i = 20$). As demonstrated by pane (c), when firm 1 increases its capacity to $k_1 = 45$, firm 2 is unable to enter the market and sustain a profitable operation. And when the adjustment cost rises to $g_i = 100$, the incumbent can deter the entrant when he installs a capacity of $k_1 = 40$, as demonstrated by pane (d). However in all cases the incumbent receives much lower profits when a mobility or entry deterrence strategy is followed instead of following a profit maximising Nash-Cournot strategy.

2.6 Strategic Investment in Capacity by Entrant: mobility deterrence and exit

In a similar exercise to that conducted by the incumbent in Section 2.5, the entrant can search over the choice of capacity that maximises her profits given that the incumbent's capacity has already been chosen. The entrant attempts to improve on the outcome of her profits obtained in Section 2.5.1-2.5.3. I also examine situations where mobility deterrence or exit of one of the players may arise under the three different production cost structures already discussed; but I consider only two of the four possible costs of adjustment, that I designate for this purpose as low ($g_i = 2$), and high ($g_i = 20$). In each case, the incumbent's capacity is considered fixed at the value obtained for the Nash-Cournot equilibrium that maximised his profits. These are taken in turn for each cost structure reflecting the relative efficiency of the two firms at the corresponding adjustment cost as shown in Table 2.2, and the entrant tries to find a new Nash-Cournot equilibrium

that maximises her profits.

2.6.1 Symmetric Costs: firm 2 searching for k_2^*

Under symmetric costs and low adjustment costs it was found in Section 2.5.1 that the best that the incumbent can do is to select a plant capacity of $k_1 = 16$, and expect to generate profits of $w_1 = 417$ as in case *Ncgs9* of Table 2.2. The entrant, instead of selecting a capacity of $k_2 = 13$, can carry out an analysis of the best possible outcome by searching over her optimal capacity k_2^* that maximises her profits given that the incumbent's capacity is now fixed as just indicated. The results of this Nash-Cournot game are shown in pane (a) of Figure 2.6. The entrant can improve her profitability by selecting a plant capacity of $k_2 = 15$, and producing an output of $x_2 = 14.9$, giving her a larger market share than in case *Ncgs9*. Her profitability rises by 3.2% to $w_2 = 349.8$. The incumbent's output declines to $x_1 = 15.4$, and his profitability by 14% to $w_1 = 358.2$, mainly as a result of smaller market share and a lower market price which now stands at $P = 39.5$. These results are also summarised in run *Ncgsj9* in Table 2.3.

The figure also shows that the two firms share the market and attain equal profits when the entrant firm selects a capacity of $k_2 = 16$. Both firms, however, obtain slightly lower profits of $w_i = 346$, each. Also the entrant can obtain higher profits than the incumbent when she selects a capacity in the range of $k_2 = 17$, to $k_2 = 28$. But in all of these cases the two firms would be better-off if they retain their Nash-Cournot equilibrium profit maximising capacities.

It was shown in Section 2.5.1 and in pane (a) of Figure 2.3 that the incumbent can resort to a mobility deterrence strategy successfully. By selecting a capacity of $k_1 = 25$, and operating with an overcapacity of 14%, the incumbent reduces the entry level of firm 2 to $k_2 = 11.2$ (capacity of firm 2 is not shown in Figure 2.3), and its profits to $w_2 = 251.9$. As a result firm 1's profits deteriorate also, falling from $w_1 = 417$, to $w_1 = 378.5$; and price falls to $P = 38.4$. However, the entrant can still counter and obtain slightly better results by independently selecting a capacity of $k_2 = 13$. She gets profits of $w_2 = 261$, which are about 3.6% higher as illustrated in pane (b) of Figure 2.6. Neither firm is able to dislodge the other from the market by installing a large enough capacity. The two

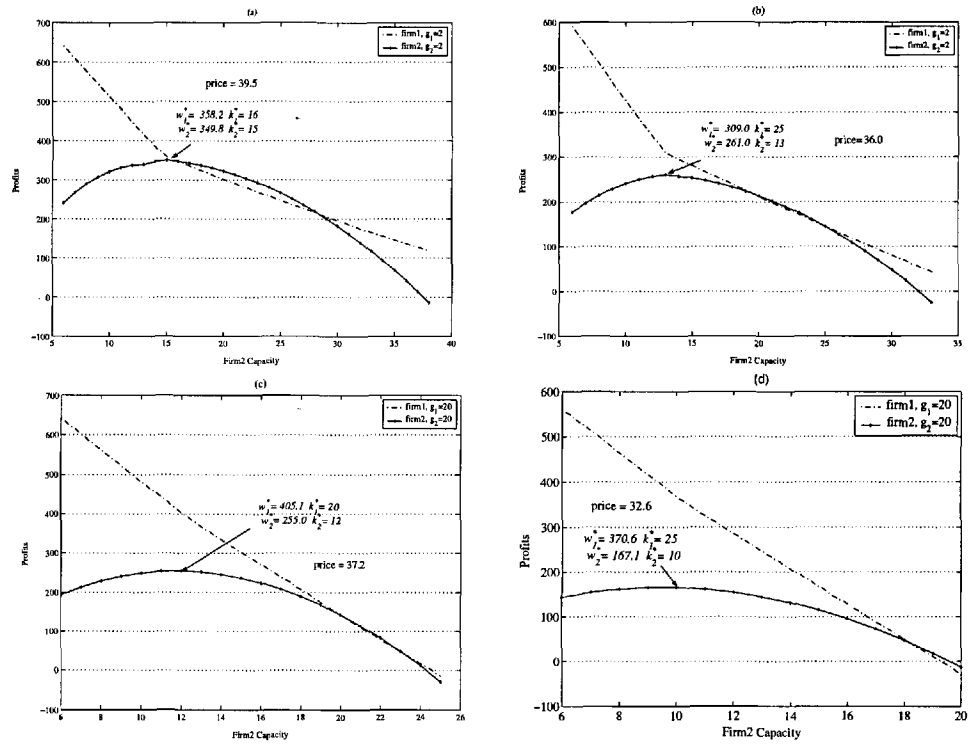


Figure 2.6: Nash-Cournot game in a duopoly under symmetric costs: k_1 is fixed and firm 2 is searching for optimal capacity.

firms are better-off following a profit maximising Nash-Cournot strategy; since a larger capacity installed by the incumbent to deter its opponent, results in both firms having their profits deteriorate by similar orders of magnitude if the entrant chooses to fight for larger market share. This would also indicate that once entry occurs, and if the entrant wants to expand market share, the incumbent would be more accommodating.

When adjustment costs are high ($g_i = 20$) the incumbent performs even better. We saw in run *Ncgs11* of Table 2.2 that firm 1 selected an optimal capacity of $k_1 = 20$, and made profits of $w_1 = 433.8$. The search over firm 2's optimal capacity and a new Nash-Cournot equilibrium is illustrated in pane (c) of Figure 2.6, and the results are summarised in run *Ncgsj11*, Table 2.3. The entrant makes negligible improvement in her profitability (compared to case *Ncgs11*) by selecting a capacity of $k_2 = 12$, and obtaining $w_2 = 255$. The incumbent's profits drop about 6.6% to $w_1 = 405.1$ as a result of the challenge by the entrant, which is much smaller than the decline with low adjustment

costs. It can be seen also that Firm 2 can also enter with a capacity of $k_2 = 20$, and share the market with the incumbent. The two firms obtain equal profits of $w_i = 145.6$. Both firms, however, are better-off retaining the Nash-Cournot equilibrium capacities where they both generate much higher profits.

Table 2.3: Entrant searching for k_2^* in a Nash-Cournot game against incumbent under different cost structures⁽¹⁾

	g_i	Incumbent				Entrant			Industry		
		x_1	k_1	w_1	$cu_1\%$	x_2	k_2	w_2	X	Price	Profit
<i>Symmetric costs⁽²⁾: incumbent and entrant have same costs of production</i>											
<i>Ncgsj9</i>	2	15.37	16.00	358.2	96	14.87	15.00	349.8	30.24	39.5	708.0
<i>Ncgsj11</i>	20	19.42	20.00	405.1	97	12.00	12.00	255.0	31.42	37.2	660.1
<i>Asymmetric costs⁽³⁾: entrant is more efficient producer</i>											
<i>Ncgaj13</i>	2	14.62	15.00	327.7	97	16.12	17.00	453.7	30.74	38.5	781.4
<i>Ncgaj15</i>	20	18.42	19.00	348.3	97	13.96	14.00	349.5	32.38	35.2	697.8
<i>Asymmetric costs⁽⁴⁾: incumbent is more efficient producer</i>											
<i>Ncgaj17</i>	2	17.75	21.00	458.0	85	13.75	14.00	288.2	31.50	37.0	746.2
<i>Ncgaj19</i>	20	22.17	23.00	555.8	96	10.00	10.00	197.7	32.17	35.7	753.4

Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, k_1 is fixed.

⁽¹⁾ Compare results in this table with corresponding runs in Table 2.2, p. 58.

⁽²⁾ Firms with symmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

⁽³⁾ Firms with asymmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 2$, $f_2 = 8$, $m_2 = 2$, $d_2 = 50$.

⁽⁴⁾ Firms with asymmetric costs: $c_1 = 2$, $f_1 = 8$, $m_1 = 2$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

A search by the entrant for an optimal capacity does not affect the outcome of the game when the incumbent follows a mobility deterrence strategy. Because of high adjustment costs the incumbent operates at high utilisation rate making the mobility deterrence more effective. As illustrated in pane (d) of Figure 2.6, the incumbent selects a capacity of $k_1 = 25$, and forces the entrant to accept a capacity of $k_2 = 10$, and reduced profits of $w_2 = 167.1$ as a result. This amounts to a loss of 34% for the entrant compared to what she could get when the incumbent follows a profit maximising strategy as in case (c) of Figure 2.6. The incumbent's profits, however, drop by only 8.5%, from $w_1 = 405.1$, to $w_1 = 370.6$.

Under conditions of high adjustment costs, the incumbent can curtail the market share, profits, and growth potential of the entrant. In addition, by installing a sufficiently large enough capacity, the incumbent firm can practically exclude the entrant from the market, as illustrated by panes (c) and (d) of Figure 2.3, p. 56. When firm 1 selects a plant capacity $k_1 = 40$, the entrant is unable to make any significant entry. This however

is achieved at a price. The incumbent will have to accept much lower profits of about $w_1 = 73.4$. If the incumbent reduces output to allow prices to rise, he risks making the market attractive to a potential entrant.

2.6.2 Efficient Entrant: firm 2 searching for k_2^*

An efficient entrant, with lower investment and production costs than the incumbent, obtains a larger market share when she follows a strategy of selecting her capacity independently, and searches over for an optimal capacity that maximises her profits. When adjustment costs are low and overcapacity is inexpensive, the incumbent reduces his operating rate and allows for a larger share for firm 2 (compared with the Nash-Cournot equilibrium results of pane (a) of Figure 2.4).

As shown in pane (a) of Figure 2.7 firm 2 is able to increase her capacity to $k_2 = 17$, and obtains profits of $w_2 = 453.7$. The entrant is also capable of increasing her market share at the expense of her opponent, and of diminishing his profits. But this is also at the expense of reducing her own profits.

Despite her superior production and investment costs, the entrant is unable to force the incumbent out of the market, since as firm 2's capacity rises ($k_2 > 42$), the entrant's profits decline at a faster rate than that of the incumbent.

This situation changes when the incumbent follows a mobility deterrence strategy by employing a larger capacity ($k_1 = 25$) to reduce the entry level of firm 2. Both firms obtain lower profits. More importantly firm 2 can now enter with a large capacity ($k_2 = 36$), and force the incumbent out of the market as illustrated in pane (b) of Figure 2.7. In order to achieve this outcome the entrant has to operate with a large overcapacity (39%), and ends up retaining much lower profits of $w_2 = 76.9$.

The incumbent firm benefits from high adjustment costs. Because of the high cost of overcapacity the incumbent firm operates at high utilisation rate and retains a larger share of the market ($k_1^* = 19$), than the more efficient entrant ($k_2^* = 14$). The two firms, however, realise almost equal profits at the new *Nash equilibrium* as shown in pane (c) of Figure 2.7.

Furthermore, the incumbent can now mount a strategy of mobility deterrence suc-

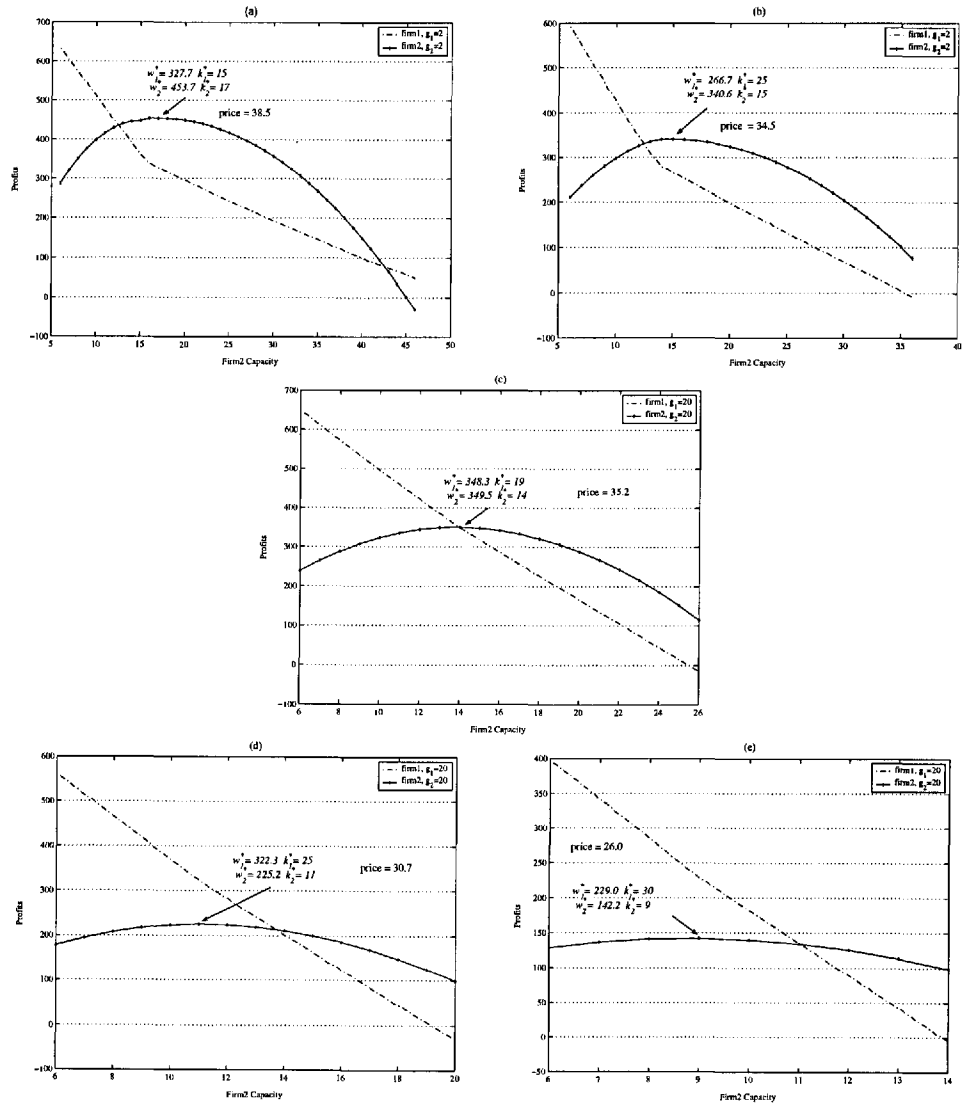


Figure 2.7: Nash-Cournot game in a duopoly under asymmetric costs where entrant is more efficient: k_1 is fixed and firm 2 is searching for optimal capacity.

cessfully as illustrated in pane (d) of Figure 2.7 (also see pane (c) of Figure 2.4). When firm 1 selects a capacity of $k_1 = 25$, in the first stage of the game, it receives much higher profits than firm 2, and forces it to enter at a lower capacity level of $k_2 = 11$. As a result the incumbent's profits decline by about 7.5% (compared to his profit maximising *Nash equilibrium* position of pane (c) of Figure 2.7), while the profits of firm 2 drop by more than 35% to $w_2 = 225.2$. However the incumbent's position now becomes vulnerable and may be challenged by the more efficient entrant. The latter can enter the market at a

capacity level of $k_2 = 20$ and employ a capacity utilisation rate of 93%, thereby forcing the incumbent into a loss making position and eventual exit. The entrant in this case expects to receive lower profits of about $w_2 = 100$.

Incumbency endows firm 1 with an advantage even when it operates a more costly plant than its opponent. Despite its inefficiency the incumbent firm obtains a larger market share and higher profits than the more efficient entrant. The incumbent is unable to deter the entry of firm 2, but his mobility deterrence strategy takes him a long way. When he selects a capacity of $k_1 = 30$, the best that the entrant can do in a single shot game is to enter with a capacity $k_2 = 9$, and receive profits of $w_2 = 144.2$, which are lower by some 60% of what was possible under the *Nash equilibrium* strategy of pane (c) of Figure 2.7. But there are other alternatives available to the entrant as depicted in pane (e) of Figure 2.7. The entrant can select a capacity $k_2 = 14$, and force firm 1 out of the market completely. In this case she receives profits of $w_2 = 98.8$; but the entrant can now act as a monopolist in the market. It is immediately clear that a dynamic setting provides a richer menu of options for the entrant than is available in a static game. The entrant can choose a large capacity to force a less efficient incumbent out of the market, or to increase market share at his expense gradually, and eventually arrive at a new market equilibrium that is more advantageous to the entrant. This type of games is examined further in the following chapters.

2.6.3 Efficient Incumbent: firm 2 searching for k_2^*

By searching over a profit maximising capacity the entrant can improve on her position when the incumbent is the more efficient producer. When adjustment costs are low and overcapacity is inexpensive ($g_i = 2$), the incumbent selects a capacity of $k_1 = 21$, to maximise his profits (see pane (a) of Figure 2.5). The entrant takes the capacity of firm 1 as given and tries to find optimal values for her capacity and output.

The results of firm 2's search over its profit maximising capacity against the more efficient incumbent are shown in pane (a) of Figure 2.8. By following this strategy the entrant gains a larger share of the market. Her capacity rises by about 18%, from $k_2 = 11.8$, to $k_2 = 14$; and now produces an output of $x_2 = 13.75$. The incumbent

accepts a reduction in output from $x_1 = 18.40$, to $x_1 = 17.75$, and his profits decline to $w_1 = 458.0$; while firm 2 gets a profit of $w_2 = 288.2$, which is higher by about 5% than what she gets under case *Ncga17*. The reason for the small rise in the profit margin of firm 2 is due to the fact that the two firms output turns out to be higher under this scenario and as a result industry price drops to $P = 37.0$ (compared with $P = 39.6$).

It is apparent from pane (a) of Figure 2.5 also, that the incumbent can follow a mobility deterrence strategy. By installing a capacity of $k_1 = 30$, he forces the entrant to select a capacity of $k_2 = 12$, when she searches over a profit maximising capacity. The results are shown in pane (b) of Figure 2.8; the entrant now obtains lower profits of $w_2 = 208$. However the incumbent should be careful about installing a larger capacity in the hope of curtailing the share of the entrant further. As pane (c) of Figure 2.8 illustrates when the incumbent increases his capacity to $k_1 = 45$, his profits decline to $w_1 = 172.4$, instead of $w_1 = 458$, that he obtains if he persists with his Nash-Cournot profit maximising strategy; and the entrant's optimal capacity declines to $k_2 = 8$, and receives profits of $w_2 = 102.3$. More importantly, the entrant can also encounter by installing a capacity of $k_2 = 15$, where upon the incumbent becomes liable to run a loss making operation, and would eventually be forced out of the market.

The position of the entrant becomes worse when adjustment costs are high. Because overcapacity is costly in this case, the incumbent's plant operates at high loading rates, leaving the entrant with little room for manoeuvre. When $g_i = 20$, the incumbent selects an optimal capacity of $k_1 = 23$, and operates at a utilisation rate of over 96% (see case *Ncga19* of Table 2.2). The entrant by searching for an optimal capacity can scarcely improve on her position. Firm 2's optimal capacity of $k_2 = 10$, and optimal profits of $w_2 = 197.7$, illustrated in pane (d) of Figure 2.8 are almost identical to those of case *Ncga19* which refer to a standard Nash-Cournot game played with firm 1. The latter's profits and market share are also practically unchanged.

The incumbent can mount a successful mobility deterrence strategy as demonstrated by the results of pane (e) of Figure 2.8. When the incumbent chooses a plant capacity $k_1 = 30$, and operates with an overcapacity of 6%, the best that the entrant can do now is to select a capacity of $k_2 = 7$, which is 30% lower than before. The incumbent's

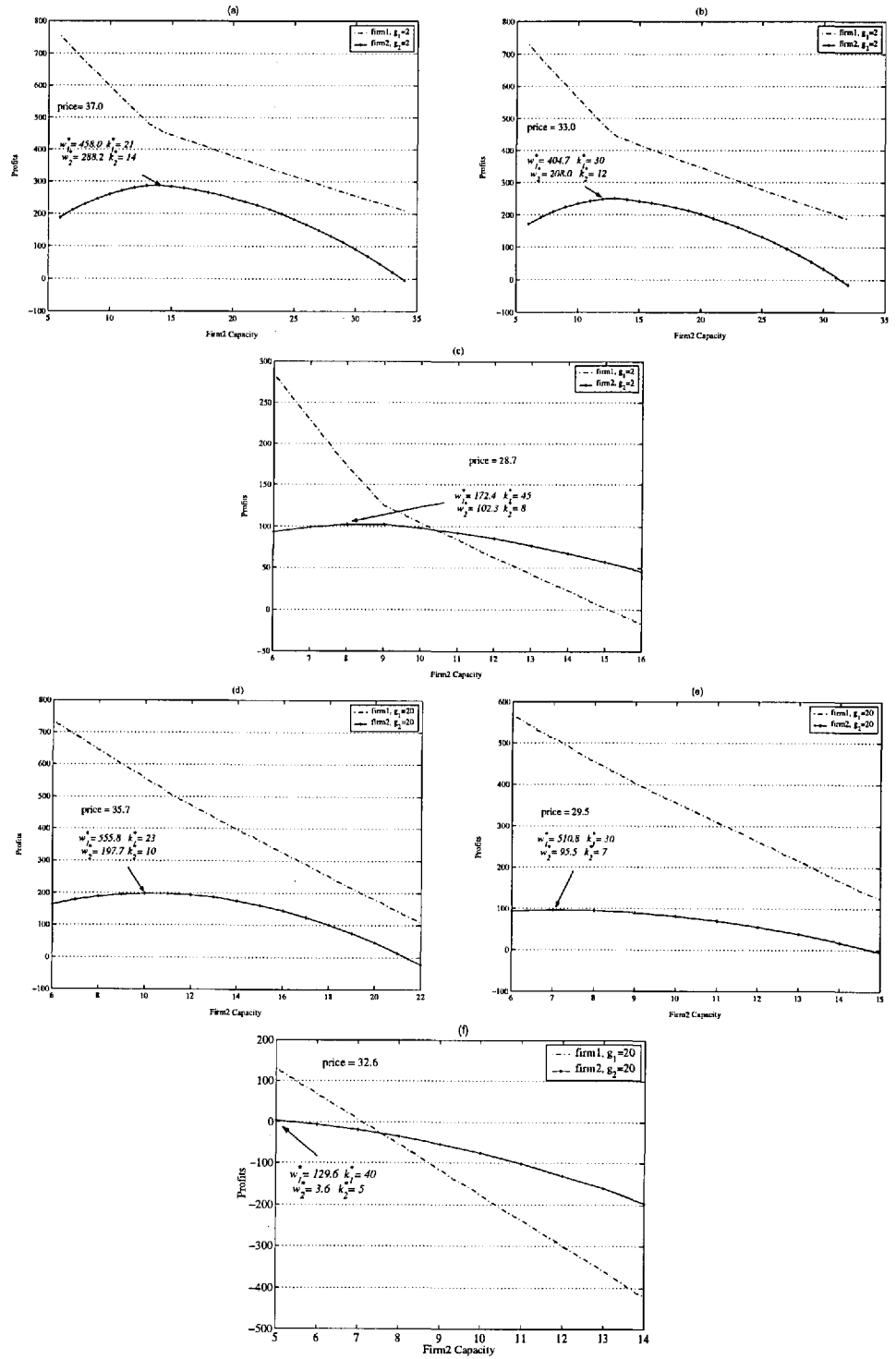


Figure 2.8: Nash-Cournot game in a duopoly under asymmetric costs where incumbent is more efficient: k_1 is fixed and firm 2 is searching for optimal capacity.

profits drop by 8% to $w_1 = 510.8$, while the entrant's profits are reduced by about 52% to $w_2 = 95.5$. Total industry output rises and the market price drops as a result from $P = 35.7$, to $P = 29.5$.

2.7 Follow the Leader: Stackelberg game between Incumbent & Entrant

A Stackelberg solution is considered when one of the firms acts as a leader. In this section two cases are evaluated. The first when the incumbent firm acts as the leader; here the entrant react in one of two ways. By choosing her capacity independently and acting as a follower in output; or by making her choice of capacity and output to be subject to the leader's influence. And the second when the entrant, in turn, acts as leader. The equilibrium solutions are evaluated under conditions of binding capacity, that is when $x_i \leq k_i$ is enforced, where output is not allowed to exceed capacity.

A Stackelberg equilibrium is a Nash equilibrium where the leader acts first by choosing the strategy u_i^* that maximises his profits. The follower reacts rationally to the leader's action by selecting optimal strategy $u_j^* = R_j(u_i^*)$ that maximises her profits, taking as given the leader's choice of optimal strategy u_i^* . Where the notation $R_j(u_i^*)$ refers to the rational reaction of the follower firm j when firm i selects strategy u_i^* . The pair of strategies $(u_i^*, R_j(u_i^*))$ is said to constitute a Stackelberg equilibrium if it satisfies the following inequality (Simaan & Cruz 1973b, Sherali, Soyster & Murphy 1983). A definition for a Stackelberg equilibrium solution for a more general problem can be found in Başar & Olsder (1999, p. 133):

$$w_i(u_i^*, u_j^*) \geq w_i(u_i, u_j^*), \quad \forall u_i \in U_i, \quad i \neq j, \quad \forall i = 1, 2. \quad (2.25)$$

I will concentrate on the optimal outputs of the two firms. In this context $R_2(x_1)$ is the reaction function of firm 2, which determines the optimal response, or reaction of this firm to every output x_1 produced by the leader, firm 1. A pair of output strategies by the two firms (x_1^*, x_2^*) , or equivalently $(x_1^*, R_2(x_1^*))$, constitutes a Stackelberg equilibrium

if the two outputs satisfy the solution of the two maximisation equations stated below.

$$w_1 = \max_{\mathbf{u}_1} P(x_1, R_2(x_1))x_1 - C_1(x_1, k_1) - I_1(u_1, k_1) \quad \forall \mathbf{u}_1 \in U_1 \quad (2.26)$$

$$w_2 = \max_{\mathbf{u}_2} P(x_1, R_2(x_1))R_2(x_1) - C_2(x_2, k_2) - I_2(u_2, k_2) \quad \forall \mathbf{u}_2 \in U_2 \quad (2.27)$$

Considering the case for the outputs, the two equations become:

$$\frac{\partial w_1}{\partial x_1} = P + \left(\frac{\partial P}{\partial x_1} + \frac{\partial P}{\partial R_2(x_1)} \cdot \frac{\partial R_2(x_1)}{\partial x_1} \right) x_1 - \frac{\partial C_1}{\partial x_1} - \frac{\partial I_1}{\partial x_1} = 0 \quad (2.28)$$

$$\frac{\partial w_2}{\partial R_2(x_1)} = P + \frac{\partial P}{\partial R_2(x_1)} R_2(x_1) - \frac{\partial C_2}{\partial R_2(x_1)} - \frac{\partial I_2}{\partial R_2(x_1)} = 0 \quad (2.29)$$

These two equations are equivalent to equations (2.6) and (2.7) of the Nash-Cournot problem of Section 2.2.2. The inverse demand function was defined to be strictly decreasing in Section 2.2.1; moreover equations (2.28) and (2.29) involve the maximisation of strictly concave functions over two bounded, compact and convex sets, since we have already defined $x_i \in [0, \bar{x}_i]$, (see item (c) in Section 2.2.1). A unique optimum exists, and the solution $(x_1^*, R_2(x_1^*))$ of the above system is a Stackelberg equilibrium (for a formal proof refer to Theorem 1 and Lemma 1 in Sherali, Soyster & Murphy (1983), or to Başar & Olsder (1999, p. 131)).

2.7.1 Entrant Acting as Stackelberg Follower in Output

Initially the leader is assumed to influence only the output of the follower, firm 2. The latter selects her investment rate and plant capacity independently. When the leader selects an output rate x_1 , the follower firm 2 takes this output as given and reacts in a Cournot fashion to leader's output. The leader realising this, knows that firm 2 will take his output as given and tries to find an optimal output x_1^* , that maximises his profits given firm 2's optimal response $R_2(x_1^*)$. Which he obtains from equation (2.26). Firm 2 on the other hand, finds her optimal output $x_2^* = R_2(x_1^*)$ in the usual way by solving (2.27).

The term $\frac{\partial R_2(x_1)}{\partial x_1}$ refers to the optimal variational response in the output of firm 2

to marginal changes in the output of the leader. Since firm 2 is acting as a Cournot follower, its optimal reaction to the output of firm 1 is obtained from the reaction curve of firm 2 as given in equation (2.11) p. 39. When plant capacity is chosen independently by firm 2, k_2 is not influenced by x_1 and in this case from (2.11) we can derive:

$$\frac{\partial R_2(x_1)}{\partial x_1} = \frac{\partial x_2}{\partial x_1} = -\frac{b}{(2b + g_2)} \quad (2.30)$$

inserting this value in equation (2.28) and rearranging we obtain:

$$(a - f_1 + g_1 k_1) + (-2b + \frac{b^2}{(2b + g_2)} - g_1)x_1 - bR_2(x_1) = 0 \quad (2.31)$$

for the follower firm the output of its opponent is taken as given, and firm 2 in finding its optimal output $x_2^* = R_2(x_1^*)$, utilises equation (2.29) which is the same as (2.11). Optimal investment rate, u_2^* , and capacity, k_2^* , are also obtained from same equations as (2.10) and (2.12). Using these results in equation (2.29) and rearranging we obtain an equation which is the same as (2.13), p. 39.

$$R_2(x_1) = \frac{1}{2b}(a - f_2 - m_2 - \delta c_2) - \frac{1}{2}x_1 \quad (2.32)$$

The vector of optimal output strategies $\mathbf{x}^* = (x_1^*, R_2(x_1^*))$, which constitutes the Stackelberg equilibrium for the two firms, can be obtained by solving (2.31) and (2.32). The results are shown in case *Skgsi1* in Table 2.4. The leader, firm 1, performs better than the follower. Its profits are better than when the two firms played Nash-Cournot strategies under the same conditions in Table 2.1, case *Ncgs1*. These results are restated in Table 2.4. Firm 2 performs worse because its role as a follower allows it a smaller market share.

2.7.2 Entrant's Capacity and Output Subject to Leader's Influence

A situation may arise where a new firm which is considering entry into a market served by a well established incumbent, becomes cautious. It could be because the entrant is venturing into a new market with which she is not familiar; or because what is being

considered is a new product or process, that the entrant may wish to give due consideration to the scale of entry into the new market. The entrant in these circumstances would wish to enter at a small scale, so as not to ‘upset’ the market and precipitate an adverse reaction from the incumbent. The entrant in this case behaves as a follower, and would take into consideration the output of the leader when choosing her output and plant capacity.

The problem at hand is still one of finding solutions for the maximisation problems of equations (2.26) and (2.27). In addition to having the output of firm 2 dependent on the leader’s output, $x_2 = R_2(x_1)$, we also have firm 2’s capacity dependent on its output, $k_2(x_2)$, and as such is indirectly dependent of the leader’s output.

Evaluating the optimal output for firm 2 proceeds as in Section 2.2.2; if we allow for the optimal investment rate and capacity to be taken into consideration first, we obtain the following expression:

$$g_2(x_2 - k_2) = m_2 + \delta c_2 \quad (2.33)$$

using this result in equation (2.29) we obtain equation (2.13). The relationship between $R_2(x_1)$ and x_1 in this equation takes into consideration the influence on the optimal choice of capacity k_2 as well. The term $\frac{\partial R_2(x_1)}{\partial x_1}$ can then be obtained by differentiation of (2.13):

$$\frac{\partial R_2(x_1)}{\partial x_1} = \frac{\partial x_2}{\partial x_1} = -\frac{1}{2} \quad (2.34)$$

using this result in (2.28) we obtain an equation for the optimal output of the leader firm:

$$a - \frac{3}{2}bx_1 - bR_2(x_1) - f_1 - g_1(x_1 - k_1) \quad (2.35)$$

the Stackelberg equilibrium of output strategies, $x^* = (x_1^*, R_2(x_1^*))$, for the two firms can be obtained by solving equations (2.35) and (2.32). In this case the entrant will do worse than before, since it gets an even smaller market share and plant capacity as illustrated by case *Skgsi2*, in Table 2.4.

2.7.3 The Entrant Acting as Leader

An entrant in many circumstances may not wish to act as follower upon entering a new market. Instead, an entrant may assume the role of leader and expects that the incumbent would act as a follower. This may arise when the entrant is in possession of a more efficient production process, or has access to cheaper raw materials that give her an advantage over the incumbent. Taking on the role of Stackleberg leader enables the entrant to benefit from her advantageous costs of production. This is more likely in situations when the entrant is a large and well established firm in the industry entering the market for a new product segment; or is the subsidiary of a large producer in a similar industry, and is therefore adept in dealing with manufacturing and marketing of new products.

The roles taken by the two firms in the previous section are now reversed. Firm 2 now acts as leader while firm 1 accepts the role of the Cournot follower. The entrant selects her optimal output rate, x_2^* , first, and the incumbent reacts by selecting an optimal response, $R_1(x_2^*)$. The two firms try to find a solution that satisfies the maximisation to the following two equations:

$$w_1 = \max_{u_1} P(R_1(x_2), x_2)R_1(x_2) - C_1(x_1, k_1) - I_1(u_1, k_1) \quad \forall u_1 \in U_1 \quad (2.36)$$

$$w_2 = \max_{u_2} P(R_1(x_2), x_2)x_2 - C_2(x_2, k_2) - I_2(u_2, k_2) \quad \forall u_2 \in U_2 \quad (2.37)$$

which in turn when profits are being maximised with respect to outputs, become:

$$\frac{\partial w_1}{\partial R_1(x_2)} = a - 2bR_1(x_2) - bx_2 - f_1 - g_1(R_1(x_2) - k_1) = 0, \quad (2.38)$$

$$\frac{\partial w_2}{\partial x_2} = P + \left(\frac{\partial P}{\partial x_2} + \frac{\partial P}{\partial R_1(x_2)} \cdot \frac{\partial R_1(x_2)}{\partial x_2} - \frac{\partial C_2}{\partial x_2} - \frac{\partial I_2}{\partial x_2} \right) = 0. \quad (2.39)$$

Note that the term $\frac{\partial R_1(x_2)}{\partial x_2}$ is obtained from the Cournot reaction of the follower incumbent, namely equation (2.38), which yields:

$$\frac{\partial R_1(x_2)}{\partial x_2} = -\frac{b}{(2b + g_1)}, \quad (2.40)$$

and remembering that when the entrant is optimising her choice of capacity, k_2 , we also have:

$$g_2(x_2 - k_2) = m_2 + \delta c_2 \quad (2.41)$$

Using those two terms in (2.39) we obtain:

$$\frac{\partial w_2}{\partial x_2} = (a - f_2 - m_2 - \delta c_2) - bR_1(x_2) + (-2b + \frac{b^2}{2b + g_1})x_2 = 0. \quad (2.42)$$

The Stackelberg equilibrium $x^* = (R_1(x_2^*), x_2^*)$, where $R_1(x_2^*) = x_1^*$, is obtained by solving equations (2.38) and (2.42), and finding the optimal values. These values are depicted in Table 2.4, case *Skgsj3*. The entrant now installs a capacity of $k_2 = 15$, and produces at full capacity. The incumbent, firm 1, reduces its output to $x_1=17$, to allow for the leader's increased output. It so happens that the two firms obtain equal profits. The entrant generates higher profits than when she acted as a follower in cases *Skgsi1* and *Skgsi2*.

2.7.4 Entrant and Incumbent Acting as Leaders: Stackelberg Disequilibrium

It was shown in Section 2.7.2 and Section 2.7.3 above, that each firm generates more profits when it takes on the role of leader with its opponent accepting the role of follower, than the case of both firms engaging in a Nash-Cournot game. This is clear from the outcome of profits generated by the leader in Table 2.4, in runs *Skgsi1* and *Skgsi2* where the incumbent takes on the leadership role; and in run *Skgsj3* where the entrant plays leader. In each of those cases the leader obtains higher profits than the cases when he takes on the role of follower, or when both firms engage in a Nash-Cournot game as illustrated in case *Ncgs1*.

These results are corroborated by the findings of Vives (1999), Dastidar (2003), and Anderson & Engers (1992). The latter find that in an oligopoly of m firms, the Stackelberg equilibrium is more competitive than the equivalent Nash-Cournot equilibrium. The reason is that in a hierarchical Stackelberg game each firm selects a higher output in an attempt to induce remaining firms to cut back on output. As a result prices are

higher under Nash-Cournot competition and firms make more profits.

In general this is an expected outcome for quantity games with linear and downward sloping demand curves. Dockner, Feichtinger & Mehlmann (1989) illustrate a similar outcome for a fisheries game. Gal-Or (1985) and Dowrick (1986) generalise this result for situations where firms are engaged in quantity games, and have downward sloping reaction functions. Coverage of the more recent literature concerning these issues can be found in Vives (1999) and Dastidar (2003).

But the question of which of the firms takes on the role of leader, when the leader-follower roles are determined endogenously in the game remains largely unresolved. The leader-follower relationship is instable, as originally pointed out by von Stackelberg, since each firm struggles to become leader (Matsumura 2002). The dichotomy of roles and its implications for the payoffs of players is illustrated in Dowrick (1986), who argues that a Cournot solution is a Nash equilibrium in outputs, but not in the choice of roles. However, Dowrick is inclined to the view that a Stackelberg equilibrium is a more likely outcome in situations of asymmetric costs as a compromise solution to avoid a “leadership warfare” or a price war; and further that firms might be expected to collude in seeking a more profitable solution. While a Stackelberg solution is more likely in situations where one firm enjoys sufficiently large cost advantages over its rival, that the follower accepts the weaker role as it expects its rival to take a strong position and win a price a war.

Matsumura (1999) shows under restrictive conditions¹¹ in a quantity game where the roles are determined endogenously, that a Stackelberg equilibrium is plausible for a duopoly. While for a price game Anderson & Engers (1994) demonstrate that a Stackelberg equilibrium with sequential moves exists.

The roles of play for the firms, in all the games that are considered in this thesis are imposed exogenously as we have seen, and there is no ambiguity about what role each player takes.

However, consider a situation which we have not dealt with so far, where both firms want to act as leaders, and each expects its opponent to act as a Nash-Cournot follower.

¹¹In his model firms produce in m periods but sell their output simultaneously in the m th period. There is no discounting and the firms are unable to observe deviations from the optimal path. The author shows that a Cournot or Stackelberg equilibrium is feasible for a duopoly, but a hierarchical Stackelberg equilibrium for an oligopoly is not.

Let us see how the two would perform.

When both firms act as Stackelberg leaders, with each assuming that its opponent will take on the role of Nash-Cournot follower, the two firms overproduce and find themselves in a worse situation than any of the other strategies that have been considered. The equilibrium solution to this strategy is known as Stackelberg disequilibrium, or what Dowrick (1986) refers to as “leadership warfare”. We have already seen that when firm 1 acts as leader, while holding the expectation that firm 2 will behave as a Nash-Cournot follower in output, the following relationship describing the influence of the incumbent’s output on that of the entrant was obtained from (2.30):

$$\frac{\partial R_2(x_1)}{\partial x_1} = \frac{\partial x_2}{\partial x_1} = -\frac{b}{(2b + g_2)}$$

furthermore, the optimal output for firm 1 has to satisfy equation (2.31), which is restated below:

$$(a - f_1 + g_1 k_1) + (-2b + \frac{b^2}{(2b + g_2)} - g_1)x_1 - bR_2(x_1) = 0$$

and similarly for firm 2, when it acts as a Stackelberg leader assuming that the incumbent will act as a Nash-Cournot follower. The relationship describing the influence of firm 2 on that of the incumbent was shown above in (2.40) to be:

$$\frac{\partial R_1(x_2)}{\partial x_2} = -\frac{b}{(2b + g_1)},$$

the optimal output for the entrant firm when it acts as a Stackelberg leader has to satisfy equation (2.42):

$$\frac{\partial w_2}{\partial x_2} = (a - f_2 - m_2 - \delta c_2) - bR_1(x_2) + (-2b + \frac{b^2}{2b + g_1})x_2 = 0.$$

remembering that at equilibrium $x_1^* = R_1(x_2^*)$, and also $x_2^* = R_2(x_1^*)$. The optimal output values for this Stackelberg disequilibrium are obtained by solving equations (2.31) and (2.42). Also for firm 2 the optimal capacity can be obtained from equation (2.41), and optimal investment rate from (2.8), p. 39.

The optimal results when the incumbent’s capacity is fixed at $k_1 = 21$, are shown in

case *skgsd4* in Table 2.4. It has to be noted that the capacity constraints on output are imposed for all the runs that appear in the table.

Table 2.4: Entrant searching for k_2^* against incumbent under symmetric costs and different investment strategies

	Incumbent				Entrant			Industry		
	x_1	k_1	w_1	$cu_1\%$	x_2	k_2	w_2	X	Price	Profit
<i>Stackelberg games with incumbent acting as leader</i> ⁽¹⁾										
<i>Skgsi1</i>	20.80	21.00	440.5	99	10.62	10.62	226.4	31.42	37.2	666.9
<i>Skgsi2</i>	21.00	21.00	442.0	100	10.50	10.50	221.5	31.50	37.0	663.5
<i>Stackelberg game with entrant acting as leader</i> ⁽²⁾										
<i>Skgsj3</i>	17.00	21.00	301.0	81	15.00	15.00	301.0	32.00	36.0	602.0
<i>Stackelberg game both incumbent and entrant acting as leaders</i> ⁽³⁾										
<i>Skgsd4</i>	19.59	21.00	329.2	93	13.77	13.77	239.0	33.36	33.3	568.2
<i>Skgsd5</i>	18.98	21.00	274.7	90	15.00	15.00	230.1	33.98	31.3	504.8
<i>Incumbent and entrant engaged in Nash-Cournot games</i>										
<i>Ncgs1</i> ⁽⁴⁾	18.00	21.00	406.0	86	12.00	12.00	289.0	30.00	40.0	695.0
<i>Ncgs2</i> ⁽⁵⁾	17.38	21.00	339.7	83	13.88	14.00	298.5	31.26	37.5	638.2
<i>Ncgs3</i> ⁽⁶⁾	17.25	21.00	326.7	82	14.25	15.00	295.2	31.50	37.0	621.9

Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, $g_i = 2$, $k_1 = 21$. Capacity constraints are enforced in all cases, and both firms have symmetric costs: $c_1 = 10$, $f_1 = 10$, $m_1 = 5$, $d_1 = 50$; $c_2 = 10$, $f_2 = 10$, $m_2 = 5$, $d_2 = 50$.

⁽¹⁾In case *Skgsi1* the leader, firm 1, influences only the choice of follower's output; and in case *Skgsi2* the leader influences the follower's choice of capacity and output.

⁽²⁾The entrant, firm 2, acts as leader and assumes that firm 1 will act as a Nash-Cournot follower in output.

⁽³⁾Each of the two firms act as leader assuming its opponent will act as a Nash-Cournot follower. Firm 2 chooses its capacity and output accordingly in case *Skgsd4*. While in case *Skgsd5* firm 2 selects its capacity independently at $k_2 = 15$.

⁽⁴⁾The two firms play a Nash-Cournot game with firm 2 selecting its optimal output and capacity. This is the same run as in Table 2.1.

⁽⁵⁾Unlike in *Ncgs1*, the entrant in this case searches for optimal capacity independently.

⁽⁶⁾In this run the entrant acts as a leader in the choice of capacity as in case *Skgsj3*; and then the two firms play a Nash-Cournot game in the choice of optimal outputs.

2.8 Which Optimal Strategy?

The problem at hand that we are dealing with is one where an incumbent firm has already installed a plant, with a capacity fixed at $k_1 = 21$, and is facing the entry of a new firm into the market. The question that needs to be addressed is what output strategy should the incumbent follow? And for the entrant, the strategy she follows will have a bearing on both her capacity and output.

In evaluating *Nash equilibria* we assume that the players hold the correct beliefs about

their opponents (Friedman 1986). The players stick to the roles they are assigned to, for the equilibrium solution we evaluate to be meaningful. So in case *Skgsi1*, of Table 2.4 when the incumbent takes on the role of leader, and makes his choice of optimal output on the basis that the entrant will behave as a Nash-Cournot follower, the entrant, firm 2, is aware of this and makes her choice of optimal output and capacity on the basis of her role as a follower with the expectation that firm 1 will play the role of leader who will stick to the declared optimal output x_1^* .

When making their choices of which strategy to follow, both firms would evaluate the alternatives that are available. Whether to play a Stackelberg or a Nash-Cournot game, and what role each would take. I also examine two cases where there is a breakdown in the agreement over the roles of leadership between the two firms. The results of the different alternatives are shown in Table 2.4.

By accepting the role of follower, firm 2 obtains the smallest market share and profits when it allows the choice of both its capacity and output to be subject to the leader's influence as cases *Skgsi1* and *Skgsi2* illustrate. Firm 1 operates at full capacity and generates its highest profits possible. Firm 2, as case *Skgsj3* shows, performs much better when it acts as a Stackelberg leader, and selects a capacity of $k_2 = 15$, assuming that the incumbent will behave as a Nash-Cournot follower. The entrant's market share rises by more than 40% and generates as much profits as the incumbent. The incumbent is worse off, his operating rate declines to $cu_1 = 81$, and profits drop by more than 31%. The price is more competitive than before, and profits for the industry decline.

However, there is no reason under these conditions for the incumbent to accept the role of follower. By sticking to a Stackelberg strategy, firm 1 performs much better even if the entrant chooses to act as leader as well. In case *Skgsd4* firm 1 retains a larger market share and obtains more profits than case *Skgsj3* where it was acting as follower. Firm 2 is worse off now; but is still better off than when it acted as a follower. It generates more profits than in cases *Skgsi1* and *Skgsi2*. It is unlikely therefore that the entrant should elect to act as a follower, because she is better off choosing a Stackelberg or a Nash-Cournot strategy. Both firms perform better when the entrant chooses a capacity of $k_2 = 12$, and the two firms engage in a Nash-Cournot game, as in case *Ncgs1*. Firm 2

in this case is still better off than being a follower. The two firms obtain higher profits than when they engage in Stackelberg warfare. In the latter case, market price is more competitive, and industry profits are lower. Under a Nash-Cournot game, the industry is more disciplined. Prices and industry profitability are high.

Case *Skgsd5* is the worst case scenario. The entrant chooses her capacity according to a Stackelberg strategy assuming the incumbent will behave as a follower as in *Skgsj3*. The incumbent instead produces following a Stackelberg strategy. Both firms obtain less profits than the previous case *Skgsd4*. They can do still worse if they both produce at full capacity. The entrant however, is still making more profits than when she choose the role of follower. The entrant would do better in a Nash-Cournot game if she chose her capacity independently by searching over a profit maximising capacity as in case *Ncgks2*. Firm 2 now has a larger market share and its profits rise to $w_2 = 298.5$; while the profits for firm 1 drop to $w_1 = 339.7$. These profits are still better than what the incumbent would get under the Stackelberg warfare cases.

The entrant would perform better by following a Stackelberg like strategy in the choice of her capacity. And if she is unsure about whether the incumbent would resort to a Stackelberg production strategy, by sticking to her choice of capacity of $k_2 = 15$, the entrant would ensure a better outcome. A Stackelberg warfare under this choice is damaging to both firms as the results of case *Skgsd5* illustrate. The incumbent is unlikely to carry out the threat of resorting to Stackelberg output if firm 2 selects a capacity of $k_2 = 15$. This strategy is not credible. Because both firms would do better by resorting to Nash-Cournot production strategy. The two firms will learn that once their capacities are fixed, they can earn more by reducing their outputs in accordance with the outcome of the Nash-Cournot equilibrium values of case *Ncgks3*.

In a quantity game, firms engage in struggle for the choice of capacity (or capital (Anderson & Engers 1992, p.129); (Friedman 1986)). Capacity is the strategic asset that enables the firms to compete for market share. And overcapacity is like the strategic reserve that allows the firms to make their threats of reverting to higher output credible. For if firm 2 were to choose a smaller capacity there is no guarantee that firm 1 will reduce its output to accommodate the increased share of its rival .

2.9 Analysis of Results and Concluding Remarks

In this chapter I examined the role of investment in capacity for strategic gain in a noncooperative game of a duopoly producing a homogenous product. Using a model of simple structure, that involves quadratic adjustment costs that penalise the firm for operating with idle capacity, I also briefly explored under what conditions it becomes feasible to deter mobility or entry into a market, and the levels of sustainable overcapacity required.

The examples examined for different competitive positions between the incumbent and entrant reveal that because of adjustment costs firms care about the size of plant capacity they build. Two factors are taken into consideration when deciding on production schedule and the overcapacity they carry. First, production is limited by the capacity, and how much product is produced affects market price and the firm's profitability. Second, the cost of overcapacity is influenced by adjustment costs and its level is dictated by the desire to accommodate a new entrant, or the willingness to have enough production capability to force an opponent out of the market. A profit maximising firm tries to balance these two opposing factors.

Depending on the cost structures of the two firms, their relative efficiency, and the cost of investment, the firms determine whether mobility or entry deterrence is a desirable or feasible option. Either of the two firms can pursue such a policy to its advantage.

Adjustment costs are often not taken into consideration in studies concerned with the strategic role of overcapacity. The models in Dixit (1979, 1980), and Spence (1977, 1979) overlook the role of the cost of idle capacity. Dockner (1992) includes quadratic adjustment costs that are a function output but do not account for variations in capacity utilisation and overcapacity. The cost of idle capacity has an important bearing on how far the firms can go in investing in additional capacity for entry or mobility deterrence; and determines whether employing overcapacity for strategic effect is feasible.

It was shown in Section 2.5 that under a regime of low adjustment costs, it is not possible for an incumbent to deter entry into the market even when the incumbent is the more efficient producer. Nevertheless, the exposé of different examples throughout the

chapter illustrate that the incumbent can take advantage of his ability to move first, by investing strategically. By choosing a large plant capacity in the first stage of the game, an incumbent invests strategically to influence the second stage of the game, and forces the entrant to accept a smaller market share. Slade (1989), Fudenberg & Tirole (1992), Reynolds (1991), and Dockner (1988, 2000) provide different representations of a similar investment strategy, in the form of feedback investment strategies that will be dealt with in the following chapter.

The results of Table 2.1, p. 52, and Table 2.2, p. 58, show this effect. Despite the entrant's more favourable production and investment costs, the incumbent still dominates the market and generates more profits, because of his higher capacity. The results of the simultaneous entry cases of *Ncgs7* and *Ncgsa8* in Table 2.1, depict the outcome that would emerge in the market if the incumbent were to forgo his larger capacity. The two firms would retain identical profits and market shares when their costs are symmetric. When the entrant is more efficient, firm 2 would dominate the market and obtain higher profits.

An implementation of an algorithm developed by Harker (1984), to compute Nash-Cournot equilibria using a variational inequality formulation was illustrated. This formulation can deal with more complicated market situations and highly nonlinear problems, such as those discussed in Harker (1984, 1991) and Nagurney (1988). The use of optimisation techniques in this formulation, facilitates the explicit imposition of capacity and other linear or nonlinear constraints on the problem. Outrata, Kočvara & Zowe (1998) show how constraints like upper limits on the output of the leader or on total industry can be imposed, whilst finding equilibrium solutions for oligopoly problems under Nash-Cournot or Stackelberg strategies.

When adjustment costs are high the use of capacity investment by the incumbent for mobility or entry deterrence is rendered more effective. It becomes harder for the entrant to enter the market. High adjustment costs make idle capacity very expensive, and the incumbent operates at high utilisation rate; as a result the entrant firm has to accept a smaller market share, and is forced to enter at lower capacity.

In situations where costs are symmetric or the incumbent is the more efficient pro-

ducer, entry deterrence becomes possible with the incumbent holding very high capacity. Such a strategy is unlikely to be attractive with prevailing differentials in costs of production. A drop in demand, or arrival of a determined entrant puts the incumbent at risk (see cases *Ncga5* and *Ncga6*, Table 2.1). Mobility deterrence is more likely. Strategic use of capacity by the incumbent has to be used with caution especially if the entrant has more advantageous costs of production. If the incumbent selects too high a capacity, the entrant could become indifferent between choosing a profit maximising strategy at the Nash-Cournot equilibrium capacity, and opting for a large enough capacity to force her opponent out of the market (see Figure 2.7).

It is evident from pane (f), in Figure 2.8, that when the incumbent installs a capacity of $k_1 = 40$, the entrant enters with a capacity of $k_2 = 5$ and is barely making any profits. If the incumbent firm raises its capacity level slightly, the entrant will effectively be deterred from entry into the market. But the question is what would happen if the entrant installs a capacity of $k_2 = 10$? It is clear that in this situation, both firms will be making losses. And although the incumbent is the more efficient producer, he will incur substantially higher losses than the entrant. This then raises the question of whether the incumbent would still be willing to maintain such a high capacity and risk incurring as a result heavy losses in the face of a determined entrant; or would he rather revise his capacity level plans and accommodate his less efficient rival? This analogy becomes more compelling when the entrant has more efficient or similar costs of production.

In the examples examined in this chapter, engaging in entry or mobility deterrence was found to be costly to both firms even when the incumbent enjoyed fairly large cost advantages in terms of investment and production costs. In all the cases considered, both firms obtain higher payoffs if they stick to their Nash-Cournot equilibrium capacities. A strategy of entry deterrence is justified only when the cost differentials between the two firms are substantial to allow one firm to dominate the market while running a profitable operation. Firms may follow such a strategy when markets are still growing. Or if there are economies of scale and learning effects that can be exploited, in the expectation of reducing costs and capturing larger market share of an expanding market. The rationale behind this behaviour is that firms forgo short term profits for longer term rewards. A

dynamic setting is more suited to the study of such market situations. The petrochemical and commodity products industry has several documented cases that attest to similar behaviour; Du Pont with titanium dioxide (Ghemawat 1984, Hall 1990, Rosenbaum & Ye 1992), Dow Chemicals with magnesium (Lieberman 1987c), Alcoa and aluminium ingot (Reynolds 1986), Kellogg and breakfast cereals (Schmalensee 1978, Sutton 1991), and many other examples given in Scherer & Ross (1990) and Sutton (1991, 1998).

Under a low adjustment costs regime, where the cost of idle capacity is low, the incumbent can afford to invest strategically to influence his opponent. By installing a large capacity the incumbent aims to gain a large market share and earn higher profits. Overcapacity becomes prevalent when one of the firms follows a mobility deterrence strategy to force its opponent to reduce its capacity or output level. Consequently, social welfare improves when the firms engage in mobility deterrence, or one of them assumes the role of Stackelberg leader. The market becomes more competitive and prices fall. Where as when they follow profit maximising Nash-Cournot strategies, the market becomes more disciplined. Under Nash-Cournot strategies, industry output is lower, prices rise and both firms' profitability improves.

It was also found that the strategy adopted by a firm in choosing its role as a leader or follower, has an important bearing on its market share and profitability. Since a Stackelberg leader performs better than its follower, the entrant by playing the role of leader gains in market share and profits. It is natural therefore that each firm struggles for the leadership position (Dowrick 1986), but when there is a breakdown in the agreement over the roles, both firms will suffer as shown by the results of the Stackelberg disequilibrium (see Table 2.4, runs *Skgsd4* and *Skgsd5*). At least by opting for a Stackelberg-like strategy in the choice of capacity, the entrant improves her standing in the market even when the duopolistic firms resort to Nash-Cournot competition in outputs. This conclusion appears to point in the direction of the suggestion by Anderson & Engers (1992) and Matsumura (1999) that more attention be paid to Stackelberg models.

Chapter 3

Capacity Expansion and Competition in a Dynamic Context: a linear quadratic model

Having discussed an entry and competition game in a static setting in the previous chapter, I turn to analyse the capacity expansion problem in a dynamic setting as a differential game, with the firms engaging in a Nash-Cournot competition. Before discussing the necessary conditions for convergence and uniqueness of the solution of the game, I address the model formulation as a linear quadratic problem for n firms (for the simulations, the model is implemented for a duopoly; this can be easily extended to an oligopoly of three or more firms).

Linear quadratic (LQ) games became widely used in economic modelling for their tractability and the possibility of obtaining analytical solutions for the optimal stabilising strategies. More importantly LQ games may be used to represent a more general type of models by means of linear quadratic approximations along the optimal solution of these general type models.

LQ games are characterised by having linear quadratic objective functions, and linear state equations; that is equation systems that are linear in the state of the game and the control variables. With quadratic investment and production costs, it is imperative that

the objective function is quadratic.

Another feature of these models is the presence of adjustment costs. The adjustment costs can be interpreted as deviations of some economic indicators, that may be represented by the state variable or control variable, from optimal target values or some long run equilibrium level. These deviations are accounted for as quadratic cost functions that penalise the objective function which the player is trying to maximise. The player attempts to minimise the sum of these deviations from the desired target values.

LQ games were first studied by Starr & Ho (1969a, 1969b) and showed that these games are *normal*; that is they admit unique equilibrium solutions in open-loop and feedback Nash strategies if the differential equations of the game admit feasible trajectories.

Capacity expansion models, which face a linear downward sloping demand curve for their output, fall necessarily into this category (Reynolds 1986, 1987, 1991, Hanig 1986). Other areas where LQ games find application include macroeconomic policy coordination (e.g. Kydland 1974, and Dockner, Jorgensen, Long & Sorger 2000, who model games between the Federal Reserve Board, or central banks, and government fiscal policy), optimal policy stabilisation (e.g. Fershtman & Muller 1984, Fershtman & Kamien 1987, 1990, Pindyck 1973), R&D competition for patents of new products (Kamien & Schwartz 1991), gasoline price wars (Slade 1992b), advertising campaigns (Slade 1990, 1992a, 1996), and international trade (Karp & Perloff 1989). Within the class of cooperative games, LQ models have been used to model power generation planning and electricity exchange between different states or across countries (Pineau & Murto 2000, Haurie & Zaccour 1986, Haurie, Zaccour & Smeers 1990), pollution control (Jorgensen & Zaccour 2001, Feenstra, Kort & de Zeeuw 2001). And in the energy field but in a noncooperative framework they have been used to model natural gas and oil projects development and supply contracts (Haurie, Zaccour & Smeers 1990, Salant 1982); and finally LQ games have been used to model optimal policies for fishing problems, or in the extraction of renewable or nonrenewable natural resources (Levhari & Mirman 1980).

The capacity expansion games we analyse are of the LQ type, with an equation of motion that is linear in the state and the control variable. Another significant feature of these games is that they admit unique solutions when certain conditions on the strategies

of the players are met which are dealt with in Sections 3.4.1 and 3.4.2.

The role of adjustment costs, which are introduced into the model formulation, and justification for their use in the literature, is discussed first in Section 3.1 below. Then the model formulation, its main components, and the optimality conditions for a single company and their economic interpretation are presented in Sections 3.2 and 3.3. Two types of models and their solutions are considered for the Nash-Cournot game, the open-loop, and the feedback model. The significance of the different information structures imposed on the formulations of the three models is discussed, and the implications for the equilibrium solutions of the models is highlighted in Section 3.4. In the last section the feedback Nash-Cournot model is solved in discrete time to show the stability of the solution attained, which is not always clear in continuous time due to limitations in the convergence of the algorithms used, in some cases when certain parameter values of the model are applied; this reveals itself in the resulting oscillations around the equilibrium values. And finally the chapter concludes with a discussion and conclusion sections.

3.1 Adjustment Costs

The role of adjustment costs in influencing the investment policy of firms has long been recognised, and was first studied by Gould (1968), Treadway (1970), and Nickell (1978). Since then many more studies that incorporate adjustment costs have been carried out; other than capacity accumulation models, adjustment costs are now widely used in models of price adjustment (Pindyck 1982, 1988, Slade 1992*a*, 1996, Fershtman & Kamien 1987, 1990), pollution and environmental control (Feenstra, Kort & de Zeeuw 2001), and international trade (Karp & Perloff 1989).

Adjustment costs result from changes in the capital held by the firm. In this context capital is treated as a quasi-fixed input where it is possible to increase (or reduce) the capital factor, subject to a positive adjustment cost whenever the firm adjusts its capital structure, and undertakes new investment (or if it disinvests some of its capital).

Theocharis (1960) found that a noncooperative Nash-Cournot oligopoly problem with linear inverse demand and adjustment cost functions has a stable solution for a duopoly,

but becomes unstable when the number of firms involved is more than two; McManus & Quandt (1961), and Fisher (1961) showed that with quadratic adjustment cost functions, this oligopoly problem has a stable Nash equilibrium solution for n firms.

Adjustment costs arise as a result of internal reorganisation within the firm when new investments are undertaken by devoting resources and personnel from existing operations to plan new ventures and develop new products, to oversee the installation of new machinery and equipment, expansion of production lines, training of staff and operators (Kort 1988). An important role played by adjustment costs is in controlling the size of the firm, when these costs are convex the implication is that it is more costly for the firm to adjust its capital more quickly than slowly (Pindyck 1982, 1988). Adjustment costs eliminate instantaneous jumps in capital to equilibrium in the form of the bang-bang solution, allowing inter-temporal developments to be undertaken by the model. Because these costs rise with increasing investment, capacity expansion is not instantaneous in one jump, but recurs over time, and takes place slowly. This type of capacity adjustment is more consistent with firms capacity decisions.

The literature is abound with studies of investment models with quadratic adjustment costs, very few of these, however, include empirical investigation of industry and oligopolistic markets. Karp & Perloff (1989) study the international rice market in a game theoretic framework. Along the same line but adding a dynamic dimension to the problem Reynolds (1986) studies the development of the US aluminium market. Also Steele (1995) studies the US-Canadian newsprint industry, and Booth, Kanetkar, Vertinsky & Whistler (1991) the paper and pulp industry in north America; both paper industry studies make allowance for adjustment costs in their models and report that these costs are real and substantial, and give some empirical estimates for adjustment costs in the respective industries. Although adjustment costs can be substantial, they are usually a small proportion of the total investment costs involved when firms undertake new capacity expansions. Several companies attribute falls in earnings, or lower operating rates, to down time and diversion of resources associated with capital projects from normal operations.¹

¹Steele (1995, p.66) reports that at least two of the companies he studied attribute in their annual reports for 1987 and 1988 a drop in their earnings and operating rates to the expansions and new capital

Large scale industries where projects have long lead times (for development and construction), are more likely to be prone to these costs when undertaken by firms. Paper and newsprint industries, petrochemicals, power generation, aluminium and many others are typical industries where the scale of investment and size of operations are important. Empirical as well as theoretical studies that cover these industries, some of which have already been mentioned, employ investment models that are characterized by the inclusion of convex adjustment costs. The vast majority of these utilise quadratic adjustment costs in their formulation (Reynolds 1987, 1991, Karp & Perloff 1989, Steele 1995, and Hanig 1986); a few which involve non-quadratic adjustment costs resort to quadratic approximations along the optimal solution path when estimating the model (Reynolds 1986, Kort 1988, 2001). In this study I opt for using quadratic adjustment costs in the model formulation. But before closing this topic it is worth noting the following points regarding the role of adjustment costs, and the influence they bear on the performance of investment models in oligopolistic industries.

The relationship between the responsiveness of the adjustment of prices and changing costs in concentrated industries is somewhat ambiguous. But there is a large body of literature and empirical studies (Eichner 1973, Philips 1980, 1983, Dixon 1983, and Fershtman & Kamien 1987) that support the view that the more concentrated the industry, the *less* rapidly will cost variations be transmitted into prices, and price movements tend to be sluggish. Ginsburgh & Michel (1988) develop a model for an oligopolistic industry using a linear industry demand curve, and a general form of adjustment costs. Starting with adjustment costs on output changes of the firms (which may be interpreted in terms of capacity adjustment), they develop from Cournot equilibrium quantities of firms, a formulation for explaining price variations in response to costs. They find that quadratic adjustment costs in concentrated industries lead to a decrease in the speed of adjustment of prices, since concentration has a positive effect on price inertia. On the other hand, as the numbers of firms, n , increases, prices adjust more *quickly*; and in a competitive setting when $n \rightarrow \infty$ the adjustment cost term vanishes and prices adjust *instantaneously* despite, the presence of adjustment costs. It is this property, of

projects undertaken by the two companies. One reported a drop of 5% in earnings, due to down time associated with capital projects, while the other saw a fall in its operating rate to 96% for similar reasons.

quadratic adjustment costs, that imparts on oligopolistic models predictions that are coherent with economic theory, which makes them appealing to economic modelers in formulating dynamic models.

Ginsburgh and Michel also analyse adjustment costs that are not quadratic but are more general in form, and show how the number of firms in the industry and the economies of scale of the adjustment cost function influence the degree of price responsiveness. They show that different outcomes that appear in the literature can be explained by their model of adjustment costs. What is significant is that adjustment costs play an important role in both dynamic and static models of industry and cannot be ignored.

3.2 Model Formulation and Specification

In this model, firms (or players) produce a homogeneous product, and attempt to maximise the value of their discounted net profits over the duration of the game, from initial time, t_0 , until final time, t_f . The game in this case is dynamic, and involves the interaction between the players over the entire lifespan of the game, as their capacities, costs and outputs evolve over time.

As an illustration, it will be considered that the market is served by two firms only, i , and l . Each firm faces a dynamic optimisation problem of choosing its investment rate and output in each period to maximise the stream of profits it generates over the planning horizon given the state of demand, and at the same time taking into consideration the output of the other firm; for firm i this may be represented by the following notation:

$$J_i = \max_{u_i} \int_{t_0=0}^{t_f} e^{-\beta t} w_i(t) dt + e^{-\beta t} \varphi(k_i(t_f)) \quad (3.1)$$

$$\text{subject to: } \dot{k}_i = f_i(u_i(t), k_i(t)) = u_i(t) - \delta k_i(t) \quad (3.2)$$

$$k_i(t_0) = k_{i,0} \text{ a given known value, and } k_i(t_f) \text{ is free} \quad (3.3)$$

where:

$$w_i(t) = \pi_i(t) - I_i(t) \quad (3.4)$$

$$\pi_i(t) = P(x_i(t), x_l(t)) x_i(t) - C_i(x_i(t), k_i(t)) \quad (3.5)$$

$$I_i(t) = c_i u_i(t) + \frac{1}{2} d_i (u_i(t) - \delta k_i(t))^2 \quad (3.6)$$

$$P(t) = a - b(x_i(t) + x_l(t)) \quad (3.7)$$

a, b	Demand function parameters.
β	Risk free rate of interest.
$c_i(t)$	Unit cost of capital for firm i at time t .
$C_i(t)$	Operating costs of firm i at time t .
d_i	Adjustment cost parameter of firm i .
δ	Depreciation rate of capital.
$I_i(t)$	Investment cost of firm i at time t .
$k_i(t)$	State variable or capacity of firm i at time t .
$P(t)$	Output price, or inverse demand function.
$\pi_i(t)$	Operating profit of firm i at time t .
$\varphi(k_i(t_f))$	Salvage value of capital at final time t_f .
t_0, t_f	Initial time and final time of the planning period.
$u_i(t)$	Investment rate of firm i at time t .
$w_i(t)$	Short-run profit of firm i at time t .
$x_i(t), x_l(t)$	Outputs of firms i and l respectively at time t .

The integral term on the right hand side of equation (3.1), is simply the discounted sum of short run profits for firm i over the entire planning horizon; the second term is the salvage value of the capital assets held by the firm in the final period t_f . Capital stock or capacity $k_i(t)$ evolves and accumulates according to the linear *state equation* (3.2); where u_i is the investment rate at time t , and δ is the depreciation rate of capital. Both, the control, u_i , and state variable, $k_i(t)$, are assumed to be piecewise continuous and differentiable functions of time; and the control influences the profit function directly and indirectly through the state.

Short Run Profit

The short run, or single period net profit $w_i(t)$ for firm i , in period t is defined by (3.4), where $\pi_i(t)$ and $I_i(t)$ are the respective single period operating profit and cost of investment for firm i at time t . $w_i(t)$, is a concave function, that is continuous and twice differentiable.

Operating Profit

Equation (3.5) states that the operating profit, $\pi_i(t)$, is the revenue (the price $P(x_i(t), x_l(t))$ multiplied by the output x_i of firm i) less the operating costs, $C_i(x_i(t), k_i(t))$. Both $\pi_i(t)$ and $C_i(t)$ are continuous and twice differentiable. $\pi_i(t)$ is concave, while $C_i(t)$ is convex.

Investment Cost

The investment cost, $I_i(t)$, in (3.6) is a quadratic function due to the presence of adjustment costs. The first term is the cost of investment, where u_i is the rate of investment, and c_i is the unit cost of capital. The second term represents the cost of adjustment, where d_i is the speed of adjustment cost parameter. In this formulation it is assumed that rapid investment is penalised as discussed in Section 3.1 above. c_i , and d_i may be estimated from available historical industry data; the values of d_i used will be chosen such that the adjustment costs represent a reasonably small proportion of the total investment costs. δ and β the respective depreciation and interest rate, are assumed to be constant and have the same value for both firms; their value will be based on estimates acceptable in industry practice. $I_i(t)$, is a convex, continuous and twice differentiable function. It is increasing in the two variables in its argument, u_i and k_i .

Price as an Inverse Demand Function

Demand is assumed to be linear, and downward sloping. This is a typical assumption to make and is widely used in this type of models (Reynolds (1986, 1987), Kulatilaka & Perotti (1992), Karp & Perloff (1989), Bensoussan & Lesourne (1980)). There are no inventories in this model so total output is equivalent to market demand. Price of output is endogenised as an inverse demand function, and this price clears the market.

Price is based on a linear inverse demand function, and is given by $P(x_i, x_l)$ in equation (3.7), where a and b are two parameters with $a > 0$, and $b > 0$. x_i and x_l are the outputs of firms i and l respectively.

In the empirical sections of this thesis, the inverse demand function will be estimated econometrically from historical market data for the product under consideration; but for the time being both parameters, a and b , will be estimated such that demand is downward sloping. Different values are used to test the linear quadratic model.

A Simplified Model

To make the notation easier to follow, and for the sake of the problem exposition, I make the following simplifying assumptions:

1. Operating costs are assumed to be negligible and can be ignored for the time being. This assumption will be relaxed in later chapters.
2. The firms operate their plants at full capacity, as such each firm's output is equivalent to capacity in place; total demand is then equivalent to total capacity of the two firms. This is a legitimate assumption, especially when complete reversibility is assumed, with the firms adjusting their capacity according to demand variability.

In the absence of operating costs, operating profits π_i are equivalent to revenue, which is price times output. Here we are considering output to be equivalent to capacity, and therefore k_i replaces x_i in the above functions and similarly k_l replaces x_l .

The problem at hand is one where firms i and l , each is trying independently to maximize its respective value J_i and J_l , which are their respective discounted sum of net profits over the time horizon under consideration, from initial time t_0 to final time t_f , subject to the capital accumulation equations and boundary conditions as shown below:

$$J_i(t_0) = \max_{u_i} \int_{t_0}^{t_f} e^{-\beta t} ((a - b(k_i + k_l))k_i - c_i u_i - \frac{1}{2} d_i (u_i - \delta k_i)^2) dt + \varphi(k_i(t_f)) \quad (3.8)$$

$$\text{subject to: } \dot{k}_i = f_i(u_i(t), k_i(t)) = u_i(t) - \delta k_i(t) \quad (3.9)$$

$$k_i(t_0) = k_{i0}, \text{ a given known value, and } k_i(t_f) \text{ is free} \quad (3.10)$$

and similarly for firm l

$$J_l(t_0) = \max_{u_l} \int_{t_0}^{t_f} e^{-\beta t} (a - b(k_i + k_l))k_l - c_l u_l - \frac{1}{2} d_l (u_l - \delta k_l)^2 dt + \varphi(k_l(t_f)) \quad (3.11)$$

$$\text{subject to: } \dot{k}_l = f_l(u_l(t), k_l(t)) = u_l(t) - \delta k_l(t) \quad (3.12)$$

$$k_l(t_0) = k_{l0} \text{ a given known value, and } k_l(t_f) \text{ is free} \quad (3.13)$$

Each of the two firms is facing an optimal control problem of capacity growth; when serving a single market the interdependence of the two firms derives from the fact that the price of their product is determined by the combined output that they both put on the market.

3.3 The Single Firm Optimal Capacity Growth Problem

The problem posed by equations (3.1), (3.2), and (3.3) for a single firm has been dealt with extensively in the literature as an optimal control problem, a calculus of variations problem or as a class of optimal control problems of the dynamic programming type. In this chapter we deal with the capacity expansion as a general form of a linear quadratic optimal control problem, and in Chapter 4 it is solved using dynamic programming techniques.

There is a rich literature dealing with dynamic problems of growth dating back to the 1950's when the theory of the *maximum principle* was developed by Pontryagin, Boltyanskii, Gamkrelidze & Mishchenko (1962), and *dynamic programming* by Bellman (1957). Chenery (1952), Jorgenson (1963), Leland (1972), Manne (1961), Nickell (1978), Arrow (1968), and Abel (1981, 1983) were among the first contributors to the implementation of these techniques in the economic literature of the firm. Formulation of the solution to this economic problem based on a derivation for an optimal control problem, by Bryson &

Ho (1969) is shown in appendix A. Also Kamien & Schwartz (1991), and Conrad & Clark (1987) provide a lucid exposition of different classes of these economic problems. Both Kamien & Schwartz (1991) and Dreyfus & Law (1977) show the relationship between the various dynamic formulations of optimal control problems: the maximum principle, dynamic programming and the calculus of variations.

The solution of the capacity expansion problem of the firm, as an optimal control problem, essentially involves the formation of the Hamiltonian, and then maximising it. This is carried out by meeting the optimality and the boundary conditions, while at the same time the state equation for capacity growth is satisfied. The Hamiltonian is formed by appending the state equation (3.2), to the short run profit of the integrand of (3.1), as shown below:

$$H_i = w_i(k_i(t), k_j(t), u_i(t)) + \lambda(t)f(k_i(t), u_i(t)) \quad (3.14)$$

where $\lambda(t)$ is a costate variable introduced as a dynamic multiplier; it plays a role similar to that of the Lagrangian multipliers of static optimisation. It is assumed to be a continuous and differentiable function of time. As shown in appendix A, $\lambda(t)$ represents the *shadow price* of a marginal unit of capital ($\lambda(t) = \frac{\partial J_i(t)}{\partial k_i}$). The resulting conditions of optimality, when the firm follows an optimal policy of investment and output, are summarised by the following three equations (see appendix A):

$$\frac{\partial H_i}{\partial u_i} = 0 \quad (3.15)$$

$$\frac{\partial H_i}{\partial \lambda} = \dot{k}_i \quad (3.16)$$

$$\beta\lambda(t) - \dot{\lambda}(t) = \frac{\partial H_i}{\partial k_i} \quad (3.17)$$

Which upon expansion yield the following three equations:

$$\frac{\partial w_i(t)}{\partial u_i(t)} + \lambda(t) \frac{\partial f(t)}{\partial u_i} = 0 \quad (3.18)$$

$$\frac{\partial H_i}{\partial \lambda} = \dot{k}_i = u_i - \delta k_i \quad (3.19)$$

$$\beta \lambda(t) - \dot{\lambda}(t) = \frac{\partial w_i(t)}{\partial k_i(t)} + \lambda(t) \frac{\partial f(t)}{\partial k_i} \quad (3.20)$$

3.3.1 Optimality Conditions: An Economic Interpretation

Solving these three equations simultaneously, determines the optimal trajectories of the decision variable, $u_i(t)$ (which is the investment rate), the costate variable $\lambda(t)$, and in turn the optimal capacity growth (which is given by equation (3.19)), over the entire planning horizon.

These solution equations of the firm's optimal problem, have a consistent economic interpretation; equation (3.18) is a statement of the economic principle that the marginal value of capital equals its marginal benefit when the firm follows, at every instant of time, an optimal policy of output and investment. The first term in the equation, is the marginal effect of a change in the decision variable, $u_i(t)$, on the short-run profit $w_i(t)$; the second term is the marginal effect of the current decision variable on the rate of growth of capital ($\dot{k}_i(t) = f(u_i, k_i)$) multiplied by the price of capital which is valued at its marginal worth $\lambda(t)$. So when following an optimal policy, the firm selects its decision variable (investment rate) such that the marginal immediate gain, just counter balances the marginal long-run cost; it should be pointed out that $\lambda(t)$ is, by definition, the marginal value of capital when an optimal policy of capital accumulation is being followed (i.e. $\lambda(t) = \frac{\partial J^*}{\partial k}$).

Equation (3.19) just defines the rate of accumulation of capital as the gross investment in each period less the depreciation of capital.

Where as the economic interpretation of equation (3.20), can be viewed as a relationship between the rate of depreciation of capital, and the influence of a marginal unit of capital on short-run profits, and the accumulation of the stock of capital. When $\dot{\lambda}(t)$ is defined as the rate of appreciation of the marginal value of capital, then $-\dot{\lambda}(t)$ is the rate

of depreciation of the marginal value of capital. And the equation simply requires that when the firm is following an optimal path of capital accumulation, the rate at which a unit of capital depreciates, is equal to the sum of its contribution to the short-run profits, $\frac{\partial w_i(t)}{\partial k_i(t)}$, and its contribution to the increase in the value of the capital stock, (since the second term on the right hand side is the marginal rate of change of capital accumulation multiplied by the marginal value of capital $\lambda(t)$), less the interest on the marginal unit of capital, that is $\beta\lambda(t)$ (Dorfman 1969).

The problem of finding the optimal investment rate, and consequently the optimal capital growth path of the firm has been reduced to one of finding the solution to equations (3.18)-(3.20). With initial capacity known at the start of the planning horizon, the investment rate $u_i(t_0)$, is obtained from equation (3.18) as a function of $\lambda(t_0)$. The value of $\lambda(t_0)$ is obtained by solving equation (3.20) backward in time, starting from the boundary condition for the marginal value of capital in the final period. Having obtained $\lambda(t_0)$, which is the marginal value of capital in the initial period, it becomes easy to evaluate the optimal investment rate required in the first period. Since $k_i(t_0)$ and $u_i(t_0)$ are both known now, the optimal value of the capital $k_i(t)$ in the next period may be obtained from equation (3.19). This equation describes the evolution of the capital growth forward in time from the initial period until the end of the planning horizon. The cycle of finding the values of $u_i(t)$, $\lambda(t)$, and $k_i(t)$, is then repeated. This becomes more evident when the problem is solved in discrete time.

In pursuing such an investment policy, the firm is in fact finding the optimal investment and capacity growth path at every instant of time over its entire planning period. Also from an economic perspective, the requirement for an equilibrium, of equating long-run marginal cost of capital to its marginal benefit, is being met at every instant of time, through the solution of the equations (3.18)-(3.20) as discussed above. This solution is essentially the dynamic path of the economic optimisation performed by the firm at each point over the period from t_0 to t_f , as it adjusts its capital from its initial position to the long run equilibrium level. Due to the presence of adjustment costs, capital growth and the rate of adjustment take place slowly over time, thereby reflecting the inter-temporal nature of the investment process.

3.4 Dynamic Duopolistic Competition: A Linear Quadratic Representation

In the previous section we considered a situation where firm l , was assumed to have fixed output choices or output path, over the entire duration of the planning horizon; while the other firm, i , was trying to maximise its profits through the choice of an optimal investment and production strategy over the same period. When firm i faces a residual demand, it operates as if it is the only player in the market. To achieve its goal, we found that firm i solves an optimal control problem; and that the solution to this problem is feasible if the optimality conditions, (3.18)-(3.20) are satisfied, and the equation of motion, (3.2), admits a solution.

When firm's l output and investment rate are not fixed, and the two firms are trying simultaneously to maximise their discounted flow of profits, and at the same time find an optimal investment path to meet an inverse demand function defined by equation (3.7), the two firms are engaged in a differential game. The problem is no longer a straightforward optimisation problem that each is trying to solve. In a game theoretic framework, each optimising firm has to take into account the role and actions of its opponents; the type of interaction between different firms in oligopolistic industries has an important bearing on the market they serve. The price of their products is sensitive to the total output that they put on the market. When deciding how much to produce, the firm has to take into consideration how its output will affect the price, and also how its opponents will respond. An added dimension in a dynamic setting is the role of investment in affecting capacity, since each firm's output is bounded by the capacity it has in place. Current investments affect the capability for future output; for each firm therefore the actions of its opponents become an integral part of its problem formulation. Outputs affect the price level and the firm's profitability; this is accounted for in each firm's objective function. Where as investments in capacity affect the state variable and the evolution of the game through the state equation, the actions affecting the state equation of the game, therefore, constitute an indirect influence of the opponents on the profitability of the firm in future periods.

The interaction of the firms in the market place, and the interdependence of their objective functions or profitability define the game. Interdependence of the profitability outcome results from two aspects: (a) through the outputs, since the combined output of the firms determines the market price, and (b) through the rate of investment that each commits in every period with its bearing on changing the market structure. The optimising firm faces an optimal control problem that takes into account its own actions that affect the control variables that are under its own influence, and at the same time it allows for the actions of its opponents which it anticipates in response to its own actions. Other firms will be doing the same.

Solution to the differential game entails each firm solving an optimal control problem within the framework of a particular information structure that governs the game. In order to characterise the solution of the game there is a need to define the information structure that is imposed on the game. That is to identify the decision rules that govern the pattern of play or actions of the firms when choosing their outputs or investments. In addition the nature of interaction between the firms needs to be identified.

The models that we study in this chapter and in the thesis as a whole are of the noncooperative type. While we first deal with a duopoly, we consider noncooperative play between the firms in an oligopolistic industry; this choice was made since later on we consider a situation for a homogeneous commodity petrochemical product that is served by several companies; the assumption being that it would be unrealistic to discipline cooperation between these companies over long periods, extending over the life of the game. It has to be borne in mind also that we are dealing with games of pure strategy only, and hence the firms cannot switch from one type of strategy to another. It is also assumed that all players follow the same rules; so when the game is open-loop, then all players follow this strategy. No allowance is made for some to follow open-loop while others follow a different strategy.

Two types of information structures are studied for the game utilising different strategies for the players, namely the open loop Nash-Cournot, and the feedback Nash-Cournot, leading to different model formulations. For each model the information structure that is adopted is chosen such that it reflects the information that is available to the players

about the state of the game and the strategies that govern the rules of play. Rules like how the players interact as the game evolves, and how they influence and are influenced by the state of the game.

Under the open-loop strategy the players' actions are a function of time only, and the firms choose their actions for the entire game from the outset. Where as with feedback strategy type model, the firms utilise strategies that are a function of the state and time. Under Stackelberg rules one of the firms acts as a leader, and the rest act as followers. The leader gets to move first, while the followers take the action of the leader as given when it is their turn to act and move to optimise their actions.

3.4.1 Open-loop Nash Equilibrium

In the single shot Nash-Cournot games that were considered in Chapter 2, the firm chooses its best strategy in response to that of its opponent, assuming that this opponent has selected its best strategy. So given its opponents capacity and the level of demand, the firm's strategy would be to choose a capacity and an output that will maximise its profits, given the output of the opponent which has been chosen, by the opponent, to maximise his profits.

A dynamic game is a multistage game where the time duration of each stage is infinitely small (Başar & Olsder 1999). The choice of output and investment in a dynamic game is made many times over. The general notion of an open-loop dynamic game, is that firms' strategies are dependent on time and the initial state at the start of the game. This argument presupposes that each firm knows from the outset the optimal profit maximising choices that are made by its opponents, and that it makes its own optimal choice for investment and output accordingly for the entire game.

Each of the two firms faces an optimal control problem which is described by equations (3.8)-(3.13). For the two firms the problem may be formulated in a more general representation as a *linear quadratic control problem* as shown below. Using matrix and vector notation, the system dynamics defined by the differential equations (3.9) and (3.12) for the capital accumulation of the two firms is combined in equation (3.22); and similarly for the boundary conditions of equations (3.10) and (3.13). Also by assuming that all

costs and the inverse demand function parameters a and b are constant over the entire duration of the game, the matrices and vectors of equation(3.21), namely $\mathbf{Q}_i$, $\mathbf{R}_i$, $\mathbf{r}_i$, and $\mathbf{S}_i$ become time invariant. These assumptions are made to see how the dynamics of the system for the two firms perform, even when all costs and variables are kept constant. Also it may be considered that the capital at the end of the game has no value, and therefore the salvage value of capital is zero. This assumption will not affect the quality of the results in any major way, especially when the costs are assumed to be constant over time, and the duration of the game is long.

$$J_i = \max_{\mathbf{u}_i} \int_{t_0}^{t_f} e^{-\beta t} \left(\frac{1}{2} \mathbf{k}' \mathbf{Q}_i \mathbf{k} + \frac{1}{2} \mathbf{u}' \mathbf{R}_i \mathbf{u} + \mathbf{r}'_i \mathbf{u} + \mathbf{u}' \mathbf{S}_i \mathbf{k} + \mathbf{q}'_i \mathbf{k} \right) dt \quad (3.21)$$

for $i = 1 \dots n$

subject to the following equation which describes the rate of change of capacity:

$$\dot{\mathbf{k}} = f(\mathbf{k}, \mathbf{u}) = \mathbf{A}\mathbf{k} + \mathbf{B}\mathbf{u} \quad (3.22)$$

and the boundary conditions: $\mathbf{k}(t_0) = \mathbf{k}_0$ and $\mathbf{k}(t_f)$ is free. $\lambda_i(t_f) = \mathbf{0}$ since at this time we are assuming that the salvage value $\varphi(t_f)$ is zero.

$$\text{Where } \mathbf{k} = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}, \quad \lambda_i = \begin{pmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix}$$

The prime denotes a transpose, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}_i$, $\mathbf{R}_i$, and $\mathbf{S}_i$ are symmetric square matrices, $\mathbf{r}_i$, and $\mathbf{q}_i$ are vectors of appropriate dimensions.

The game of prescribed fixed duration $[t_0, t_f]$, under consideration admits an open-loop Nash equilibrium provided some conditions are met that are considered below. Also existence of the solution and its uniqueness rest on the linear quadratic nature of the game, and the affine strategies that are used by the players. In what follows we make use of Theorems 6.11 and 6.12 of Başar & Olsder (1999), and then proceed in the following

section to show that the necessary conditions for the equilibrium can be found if the solution to a set of coupled matrix Riccati equations exists.

Proposition 1 *For an n -person differential game of prescribed fixed duration $[t_0, t_f]$, let*

(i) $f(t, \cdot, u_i, \dots, u_n)$ *be continuously differentiable on $\mathbf{R}^n, \forall t \in [t_0, t_f]$,*

(ii) $w_i(t, \cdot, u_i, \dots, u_n)$ *and $\varphi_i(\cdot)$ be continuously differentiable on $\mathbf{R}^n, \forall t \in [t_0, t_f], i \in \mathbf{N}$.*

Then if $\{v_i^*(t, \mathbf{k}_0) = u_i^*(t); i \in \mathbf{N}\}$ provides an open loop Nash equilibrium solution, and $\{\mathbf{k}^*, t_0 \leq t \leq t_f\}$ is the corresponding state trajectory, there exist n costate functions $\lambda_i(\cdot) : [t_0, t_f] \rightarrow \mathbf{R}^n, i \in \mathbf{N}$, such that the following relations are satisfied:

$$\dot{\mathbf{k}}^*(t) = f(t, \mathbf{k}^*(t), u_1^*, \dots, u_n^*), \quad \mathbf{k}^*(t_0) = \mathbf{k}_0, \quad (3.23)$$

$$v_i^*(t, \mathbf{k}_0) \equiv u_i^*(t) = \max_{u_i} H_i(t, \lambda_i, \mathbf{k}^*, u_1^*, \dots, u_{i-1}^*, u_i, u_{i+1}^*, \dots, u_n^*) \quad (3.24)$$

$$\beta \lambda_i(t) - \dot{\lambda}_i(t) = \frac{\partial H_i(t, \lambda_i, \mathbf{k}^*, u_1^*, \dots, u_n^*)}{\partial \mathbf{k}} \quad (3.25)$$

$$\lambda_i(t_f) = \frac{\partial \varphi(\mathbf{k}^*(t_f))}{\partial \mathbf{k}} \quad (3.26)$$

where

$$H_i(\cdot) = w_i(\cdot) + \lambda_i' f(\cdot), \quad t \in [t_0, t_f], \quad i \in \mathbf{N}.$$

Proof. The proof is found in that for Theorem 6.11, in Başar & Olsder (1999, p.310-311), and related theorems that make use of the maximum principle for continuous time control systems. ■

Proposition 2 *For an n -person affine-quadratic differential game with $\mathbf{Q}_i \leq \mathbf{0}, \mathbf{R}_i \leq \mathbf{0}, (i \in \mathbf{N})$, let there exist a solution set $\{\Lambda_i; i \in \mathbf{N}\}$ to the coupled matrix Riccati equations*

$$\begin{aligned} \dot{\Lambda}_i + \Lambda_i \mathbf{A} - (\beta - \mathbf{A}') \Lambda_i + \mathbf{Q}_i - (\Lambda_i \mathbf{B} + \mathbf{S}) \tilde{\mathbf{R}}^{-1} (\tilde{\mathbf{S}} + \mathbf{B}' \tilde{\Lambda}) &= \mathbf{0} \\ \Lambda_i(t_f) &= \frac{\partial \varphi(t_f)}{\partial \mathbf{k}}, \quad (i \in \mathbf{N}). \end{aligned} \quad (3.27)$$

Then the differential game admits an open-loop Nash equilibrium solution given by

$$v^*(t, \mathbf{k}_0) \equiv u^*(t) = -\tilde{\mathbf{R}}^{-1} \left(\tilde{\mathbf{r}} + \tilde{\mathbf{S}} \mathbf{k} + \mathbf{B}' (\tilde{\Lambda} \mathbf{k} + \tilde{\gamma}) \right) \quad (3.28)$$

where $\{\gamma_i; i \in \mathbf{N}\}$ solve uniquely the set of linear differential equations:

$$\begin{aligned} \dot{\gamma}_i - (\beta - A') \gamma_i + q_i - (\Lambda_i B + S) \tilde{R}^{-1} (\tilde{r} + B' \tilde{\gamma}) &= 0 \\ \gamma_i(t_f) &= 0, \quad (i \in \mathbf{N}). \end{aligned} \quad (3.29)$$

and $k^*(\cdot)$ denotes the Nash equilibrium state trajectory, generated by:

$$k^* = e^{(A+BZ_2)t} k(t_0) + \int_{t_0}^{t_f} e^{(A+BZ_2)(t-\tau)} B Z_1 d\tau \quad (3.30)$$

where $\tilde{R}^{-1}$, $\tilde{r}$, and $\tilde{S}$ are defined in (3.36)-(3.40), $\tilde{\lambda}$, $\tilde{\Lambda}$, $\tilde{\gamma}$ are defined in (3.42)-(3.45), and Z_1 and Z_2 are defined in (3.55) and (3.56) respectively.

Proof. The proof follows the same lines as Theorem 6.12 in Başar & Olsder (1999). In Section 3.4.1.1 below the optimality conditions satisfying Proposition 1 are sketched out. The Riccati equation solution satisfying (3.27) and (3.29) is given in Section 3.4.1.2; and for the optimal state (vector of capacities) trajectory satisfying (3.30) is provided in Section 3.4.1.3 ■

3.4.1.1 Model Solution: Open-loop Nash Strategy

First we define the Hamiltonian for each firm i :

$$H_i = \frac{1}{2} k' Q_i k + \frac{1}{2} u' R_i u + r'_i u + u' S_i k + q'_i k + \lambda'_i (A k + B u) \quad (3.31)$$

for $i = 1, \dots, n$

Then each firms tries to maximise its Hamiltonian by choosing an investment and output strategy that maximises its profits for the duration of the game. The conditions of optimality require that equations (3.32)-(3.34) are satisfied:

$$\frac{\partial H_i}{\partial u_i} = 0 \quad (3.32)$$

$$\frac{\partial H_i}{\partial \lambda_i} = \dot{k} = A k + B u \quad (3.33)$$

$$\beta \lambda_i - \dot{\lambda}_i = \frac{\partial H_i}{\partial k} \quad (3.34)$$

Now taking the derivative of the Hamiltonian in (3.31) wrt $\mathbf{u}$ we obtain:

$$\frac{\partial H_i}{\partial \mathbf{u}} = \mathbf{R}_i \mathbf{u} + \mathbf{r}_i + \mathbf{S}_i \mathbf{k} + \mathbf{B}'_i \boldsymbol{\lambda}_i \quad (3.35)$$

and by applying optimality condition (3.32), which indicates that player i , controls only his own investment rate u_i , we get:

$$0 = \frac{\partial H_i}{\partial u_i} = \mathbf{R}_{ii} u_i + r_{ii} + \mathbf{S}_{ii} \mathbf{k} + \mathbf{B}'_{ii} \boldsymbol{\lambda}_i$$

from the above we conclude, provided $\mathbf{R}_i$, $\mathbf{R}_l$ and $\mathbf{R}_k$ are invertible so that:

$$u_i^* = -\mathbf{R}_{ii}^{-1}(r_{ii} + \mathbf{S}_{ii} \mathbf{k} + \mathbf{B}'_{ii} \boldsymbol{\lambda}_i) \text{ for } i = 1, \dots, n \quad (3.36)$$

and similarly for the other players we get:

$$u_l^* = -\mathbf{R}_{ll}^{-1}(r_{ll} + \mathbf{S}_{ll} \mathbf{k} + \mathbf{B}'_{ll} \boldsymbol{\lambda}_l) \text{ for } i \neq l, i = 1, \dots, n \quad (3.37)$$

$$u_n^* = -\mathbf{R}_{nn}^{-1}(r_{nn} + \mathbf{S}_{nn} \mathbf{k} + \mathbf{B}'_{nn} \boldsymbol{\lambda}_n) \text{ for } i \neq n, i = 1, \dots, n \quad (3.38)$$

where u_i^* , u_l^* , and u_n^* are the optimal investment strategies for firms i , l and n respectively.

Similarly, the optimal investment strategy for the remaining firms may be obtained.

Therefore we can define an optimal strategy vector $\mathbf{u}^*$, for all the firms involved:

$$\mathbf{u}^* = \begin{pmatrix} u_i^* \\ u_l^* \\ \vdots \\ u_n^* \end{pmatrix} = - \begin{pmatrix} \mathbf{R}_{ii}^{-1} \\ \mathbf{R}_{ll}^{-1} \\ \vdots \\ \mathbf{R}_{nn}^{-1} \end{pmatrix} \cdot \begin{pmatrix} r_{ii} + \mathbf{S}_{ii} \mathbf{k} + \mathbf{B}'_{ii} \boldsymbol{\lambda}_i \\ r_{ll} + \mathbf{S}_{ll} \mathbf{k} + \mathbf{B}'_{ll} \boldsymbol{\lambda}_l \\ \vdots \quad \vdots \quad \ddots \\ r_{nn} + \mathbf{S}_{nn} \mathbf{k} + \mathbf{B}'_{nn} \boldsymbol{\lambda}_n \end{pmatrix} \quad (3.39)$$

which may be represented in a more compact form:

$$\mathbf{u}^* = -\tilde{\mathbf{R}}^{-1} (\tilde{\mathbf{r}} + \tilde{\mathbf{S}} \mathbf{k} + \mathbf{B}' \tilde{\boldsymbol{\lambda}}) \quad (3.40)$$

from (3.40) it is clear that in order to find the optimal investment strategy, $\mathbf{u}^*$, all we have to do, is determine the vector value of $\tilde{\boldsymbol{\lambda}}$. Now let's assume that $\boldsymbol{\lambda}_i$ is linear in $\mathbf{k}$

and has the following form:

$$\lambda_i = \begin{pmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix} = \Lambda_i \mathbf{k} + \gamma_i \quad (3.41)$$

Where Λ_i and γ_i are a $n \times n$ matrix, and a $n \times 1$ vector respectively, that we need to evaluate. The equation is shown in expanded form below:

$$\lambda_i = \begin{pmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \vdots & \phi_{nn} \end{pmatrix} \mathbf{k} + \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

similarly we have for firm l , a costate variable equation which is linear in the state, namely:

$$\lambda_l = \begin{pmatrix} \lambda_{l1} \\ \lambda_{l2} \\ \vdots \\ \lambda_{ln} \end{pmatrix} = \Lambda_l \mathbf{k} + \gamma_l \quad (3.42)$$

and for firm k :

$$\lambda_k = \begin{pmatrix} \lambda_{k1} \\ \lambda_{k2} \\ \vdots \\ \lambda_{kn} \end{pmatrix} = \Lambda_k \mathbf{k} + \gamma_k \quad (3.43)$$

Therefore $\tilde{\lambda}$ has the following form:

$$\tilde{\lambda} = \begin{pmatrix} \lambda_{ii} \\ \lambda_{ll} \\ \vdots \\ \lambda_{nn} \end{pmatrix} = \begin{pmatrix} \Lambda_{ii} \\ \Lambda_{ll} \\ \vdots \\ \Lambda_{nn} \end{pmatrix} \mathbf{k} + \begin{pmatrix} \gamma_{ii} \\ \gamma_{ll} \\ \vdots \\ \gamma_{nn} \end{pmatrix} \quad (3.44)$$

where Λ_{ii} refers to the i th row of matrix Λ_i ; Λ_{ll} refers to the l th row of matrix Λ_l , and so Λ_{nn} refers to the n th row of matrix Λ_n . Similarly γ_{ii} refers to the i th element of the vector γ_i , and so on for the l th, and k th elements. As such we define a new vector $\tilde{\lambda}$ that may be represented in the following form:

$$\tilde{\lambda} = \tilde{\Lambda}k + \tilde{\gamma}$$

and u^* of equation (3.40) can now be written as:

$$u^* = -\tilde{R}^{-1} \left(\tilde{r} + \tilde{S}k + B' \left(\tilde{\Lambda}k + \tilde{\gamma} \right) \right) \quad (3.45)$$

where the tilde sign, ' $\sim$ ', refers to formulated matrices and vectors, whose rows or elements are defined from across the corresponding rows and elements of the matrices and vectors of the different players, and in the order ' ii ', ' ll ', ' nn ', as was shown above.

3.4.1.2 Riccati Equations Formulation

Having found the optimal investment strategy, we turn to the second condition of optimality that needs to be satisfied, namely equation (3.34). The first term on the *rhs* of this equation, $\frac{\partial H_i}{\partial k}$, can be obtained by differentiating (3.31):

$$\frac{\partial H_i}{\partial k} = Q_i k + S_i u + q_i + A' \lambda_i \quad (3.46)$$

We now turn to evaluate equation (3.34) using the above result.

$$\beta \lambda_i - \dot{\lambda}_i = Q_i k + S_i u + q_i + A' \lambda_i \quad (3.47)$$

It has already been stated that the costate variable λ_i is assumed to be linear in the state, and has the form as shown in (3.41). Taking the time derivative of this equation, we obtain:

$$\dot{\lambda}_i = \dot{\Lambda}_i k + \Lambda_i \dot{k} + \dot{\gamma}_i$$

Also from (3.22) we have $\dot{\mathbf{k}} = \mathbf{A}\mathbf{k} + \mathbf{B}\mathbf{u}$, so we can write:

$$\dot{\lambda}_i = \dot{\Lambda}_i \mathbf{k} + \Lambda_i \mathbf{A} \mathbf{k} + \Lambda_i \mathbf{B} \mathbf{u} + \dot{\gamma}_i \quad (3.48)$$

where $\mathbf{u}$ is evaluated at its optimal value, or more accurately $\mathbf{u} = \mathbf{u}^*$. Then substituting the value of $\dot{\lambda}_i$ from the above equation, and the values of λ_i and $\mathbf{u}^*$ from (3.41) and (3.45) respectively in (3.47) and rearranging we obtain:

$$\begin{aligned} &(\beta - \mathbf{A}')\Lambda_i \mathbf{k} + (\beta - \mathbf{A}')\gamma_i - \dot{\Lambda}_i \mathbf{k} - \Lambda_i \mathbf{A} \mathbf{k} - \mathbf{Q}_i \mathbf{k} - \mathbf{q}_i - \dot{\gamma}_i \\ &- (\Lambda_i \mathbf{B} + \mathbf{S}_i)(-\mathbf{R}^{-1})(\tilde{\mathbf{r}} + \tilde{\mathbf{S}}\mathbf{k} + \mathbf{B}'(\tilde{\Lambda}\mathbf{k} + \tilde{\gamma})) = 0 \end{aligned} \quad (3.49)$$

The necessary condition for the above equation to have a solution is when the coefficients of the capacity vector $\mathbf{k}$ and the independent matrices, and vectors are zero. So next we collect the coefficients of the $\mathbf{k}$ terms, and the independent terms to get two equations as shown below:

$$\{(\beta - \mathbf{A}')\Lambda_i - \Lambda_i \mathbf{A} - \dot{\Lambda}_i - \mathbf{Q}_i - (\Lambda_i \mathbf{B} + \mathbf{S}_i)(-\tilde{\mathbf{R}}^{-1})(\tilde{\mathbf{S}} + \mathbf{B}'\tilde{\Lambda})\}\mathbf{k} = 0 \quad (3.50)$$

$$(\beta - \mathbf{A}')\gamma_i - \mathbf{q}_i - \dot{\gamma}_i - (\Lambda_i \mathbf{B} + \mathbf{S}_i)(-\tilde{\mathbf{R}}^{-1})(\tilde{\mathbf{r}} + \mathbf{B}'\tilde{\gamma}) = 0 \quad (3.51)$$

Now by rearranging the above two equations we obtain two coupled Riccati like equations in Λ_i and γ_i that satisfy the optimality conditions of equations (3.32)-(3.34).

$$\dot{\Lambda}_i = (\beta - \mathbf{A}')\Lambda_i - \Lambda_i \mathbf{A} - \mathbf{Q}_i + (\Lambda_i \mathbf{B} + \mathbf{S})\tilde{\mathbf{R}}^{-1}(\tilde{\mathbf{S}} + \mathbf{B}'\tilde{\Lambda}) \quad (3.52)$$

$$\dot{\gamma}_i = (\beta - \mathbf{A}')\gamma_i - \mathbf{q}_i + (\Lambda_i \mathbf{B} + \mathbf{S})\tilde{\mathbf{R}}^{-1}(\tilde{\mathbf{r}} + \mathbf{B}'\tilde{\gamma}) \quad (3.53)$$

Similarly for firm l we obtain two Riccati like equations of the same form as (3.52) and (3.53). These are the matrix Riccati equations (3.27) and (3.29) of Proposition 2 whose solutions determine the unique solution of our game.

These are nonlinear differential equations that are solved numerically. In the case of a duopoly we obtain four equations that we solve simultaneously. It is found that they admit a stable solution. Having found the values of Λ_i and γ_i , and Λ_l and γ_l , we

can determine the corresponding values of $\tilde{\Lambda}$ and $\tilde{\gamma}$, using equation (3.44). The optimal investment strategy u^* , which is a vector of the investment strategies for all firms can now be evaluated for the entire game using equation (3.45).

3.4.1.3 Optimal Investment Strategy and Capacity Growth for the Open-loop Case

It is clear now that the optimal investment strategy is easily obtained for the entire period from t_0 to t_f , by applying (3.45), since all the elements of the matrix $\tilde{\Lambda}$, and vector $\tilde{\gamma}$, are now known from the solution of the Riccati equations. With the optimal path of investment, u^* at hand, what remains to be done is to find the optimal path of the capacity, k^* . This can be done by solving the differential equation (3.22); which is discussed below.

Bearing in mind that $B' = I$, then (3.45) may be written in the following form:

$$u^* = Z_1 + Z_2 k \quad (3.54)$$

where,

$$Z_1 = -\tilde{R}^{-1}(\tilde{r} + \tilde{\gamma}) \quad (3.55)$$

$$Z_2 = -\tilde{R}^{-1}(\tilde{S} + \tilde{\Lambda}) \quad (3.56)$$

Now we can rewrite (3.22) using the optimal strategy for the game u^* . This equation describes the optimal path of the state which is the capacities for all the firms involved, as a result of following an optimal investment strategy.

$$\begin{aligned} \dot{k} &= Ak + Bu^* \\ &= Ak + BZ_1 + BZ_2 k \\ &= (A + BZ_2)k + BZ_1 \end{aligned} \quad (3.57)$$

This last equation is an in-homogeneous differential equation, since the second term on

the *rhs*, i.e. $\mathbf{BZ}_1$, is non-zero and is independent of $\mathbf{k}$; this equation has a solution of the following form which is attributed to Athans & Falb (1966):

$$\mathbf{k}^* = e^{(\mathbf{A} + \mathbf{BZ}_2)t} \mathbf{k}(t_0) + \int_{t_0}^{t_f} e^{(\mathbf{A} + \mathbf{BZ}_2)(t-\tau)} \mathbf{BZ}_1 d\tau \quad (3.58)$$

Equations of this form have no analytical solution, but they can be solved numerically; this was accomplished using numerical integration.

As an illustration, a duopoly model is solved. The following parameter values for the base case of the model simulation were used: for the inverse demand function, $a = 100$, $b = 2$; while for the unit capital investment cost for the two firms, $c_1 = 10$, $c_2 = 1$; and the speed of adjustment costs were assumed to be equal for the two firms, $d_1 = 50$, $d_2 = 50$; a depreciation rate of $\delta = 0.10$, and an interest rate on capital borrowing of $\beta = 0.05$. The parameters were chosen such that for normal situations, the operating profit for the firms is positive and the results that are obtained are meaningful. A wide range of values is tested in several runs to check the performance of the model.

Starting with the symmetric costs case, where the two firms have the same investment costs, $c_1 = 10$, and $c_2 = 10$, it is found that the two firms converge to the same equilibrium, where their capacities equalise, despite having different capacities at the start of the game, as shown in Figures 3.1 and 3.2.

For the asymmetric case, where the capital investment costs are, $c_1 = 10$, $c_2 = 1$, the results are shown in Figures 3.3-3.9. Each figure contains 4 panes. Panes (a), show the development of capacity and equilibrium values for the two firms; panes (b) the current value profits; panes (c) the investment rates, and panes (d) the investment cost functions. Before discussing the significance of these results, I turn to consider the case of feedback Nash-Cournot strategies and finding an equilibrium solution in the next section; and then I discuss the results for both models in Section 3.4.3.

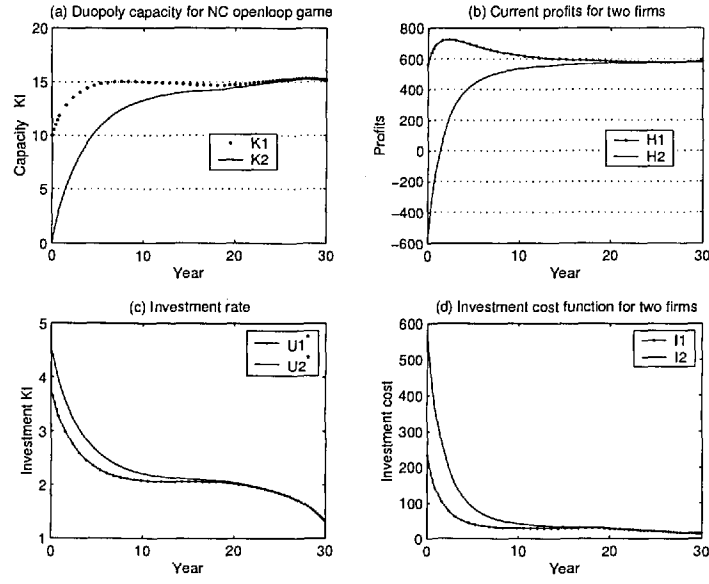


Figure 3.1: Investment and output profile for symmetric costs open-loop equilibrium: parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 10$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $\beta = 0.05$ $k_i(t_0) = 10$, $k_l(t_0) = 0$.

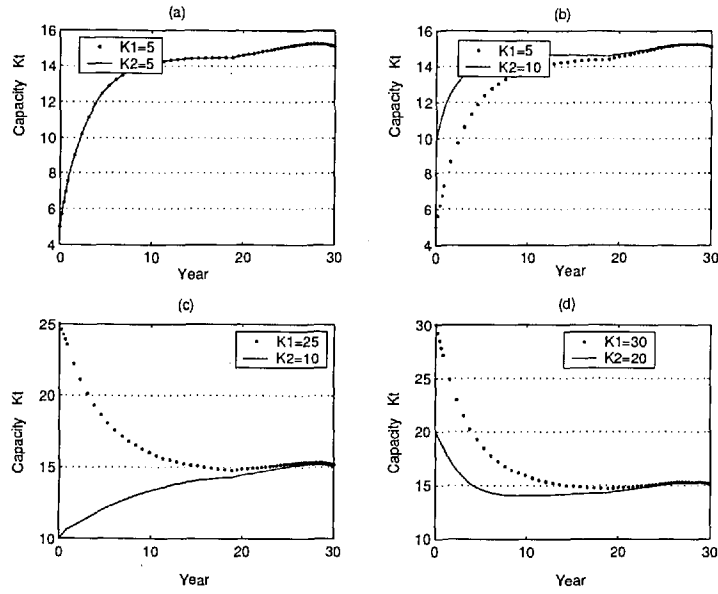


Figure 3.2: Capacity profile for symmetric costs open-loop equilibrium. Initial capacities for the two firms vary in each case: parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 10$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $\beta = 0.05$

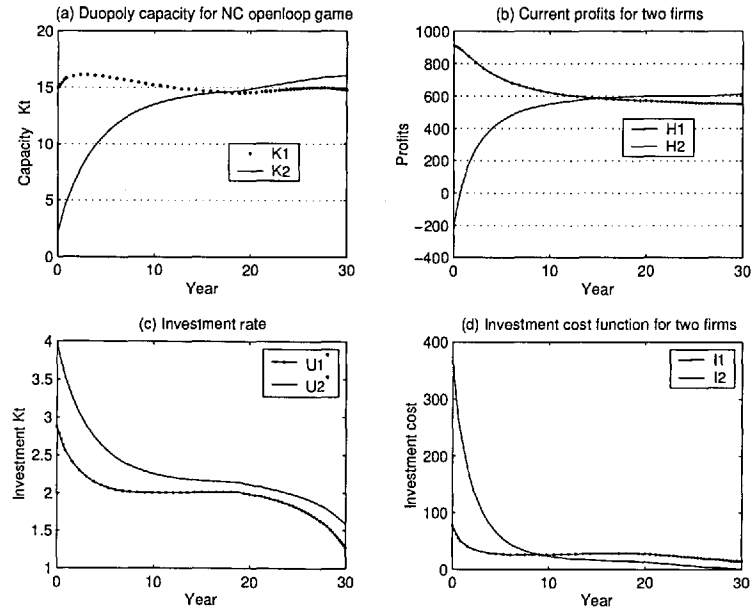


Figure 3.3: Investment and output profile for open-loop equilibrium: parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 1$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $\beta = 0.05$ $k_i(t_0) = 15$, $k_l(t_0) = 2$.

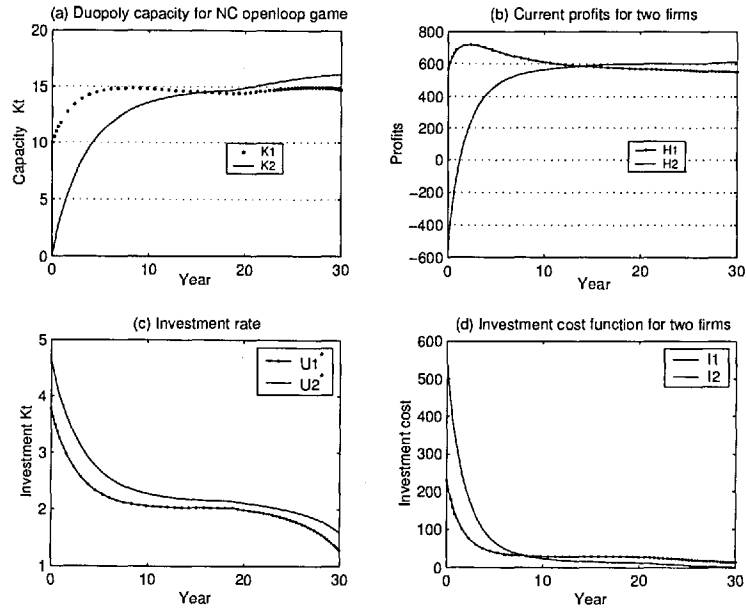


Figure 3.4: Investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $k_i(t_0) = 10$, $k_l(t_0) = 0$.

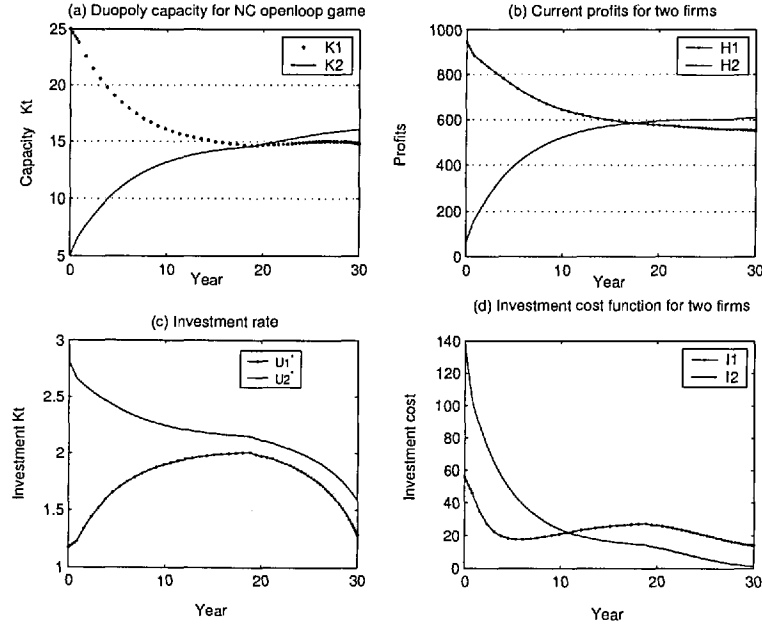


Figure 3.5: Investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $k_i(t_0) = 25$, $k_l(t_0) = 5$.

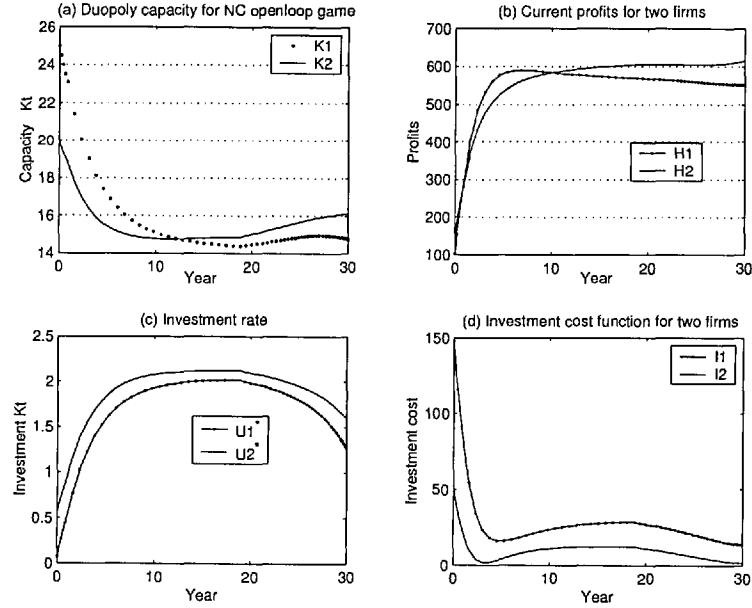


Figure 3.6: investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $k_i(t_0) = 25$, $k_l(t_0) = 20$.

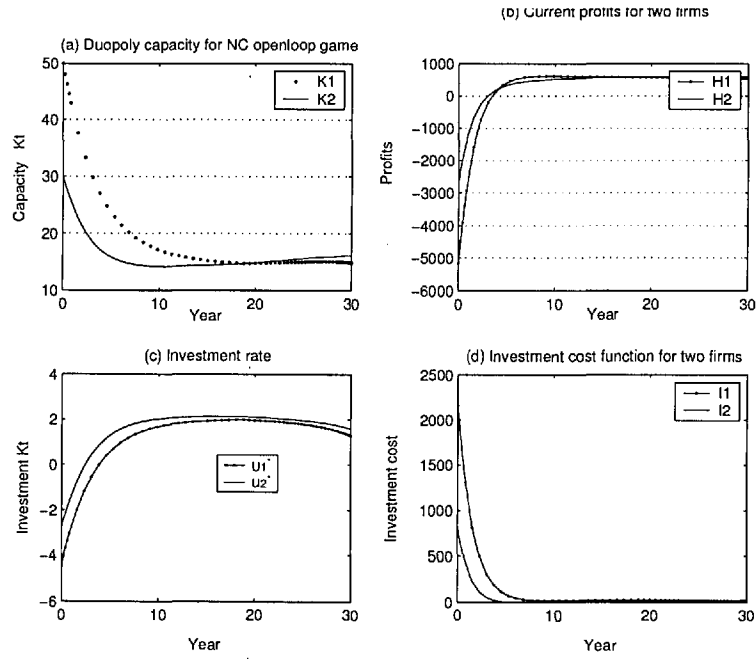


Figure 3.7: Investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $k_i(t_0) = 50$, $k_l(t_0) = 30$.

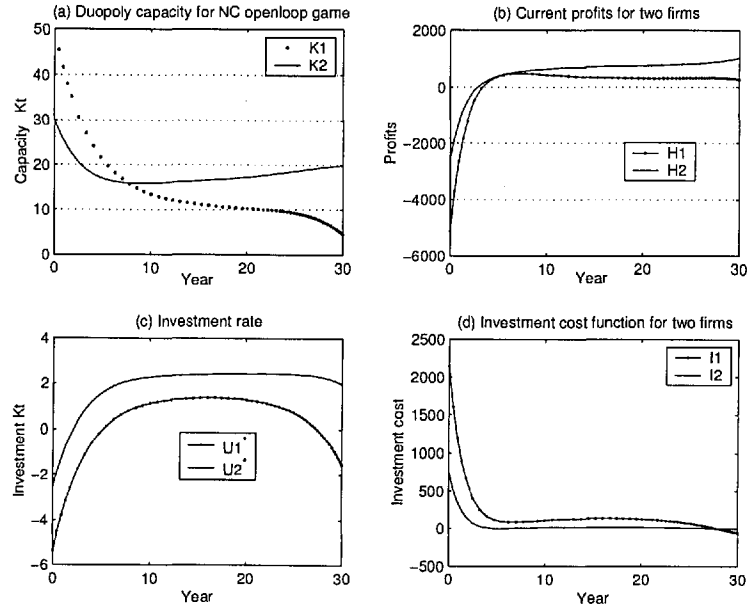


Figure 3.8: Investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $c_1 = 100$, $c_2 = 1$, and $k_i(t_0) = 50$, $k_l(t_0) = 30$.

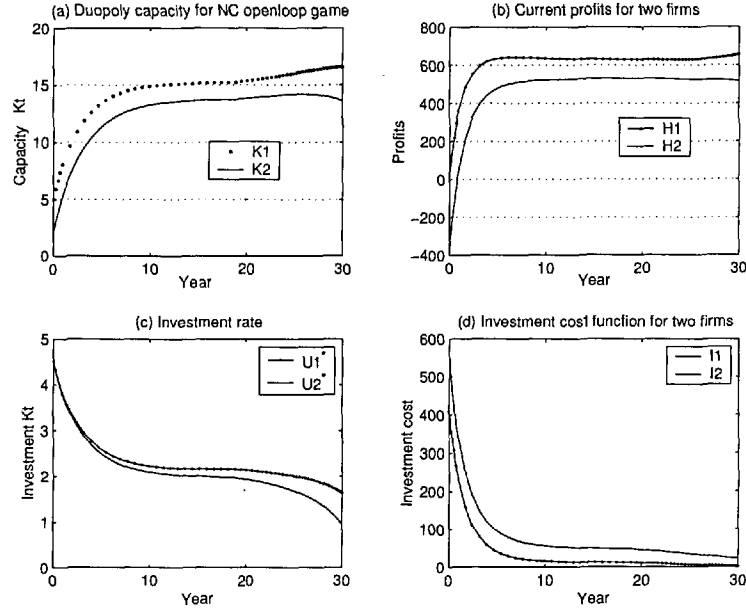


Figure 3.9: Investment and output profile for open-loop equilibrium: parameter values used: same as in Fig.3.3 except for $c_1 = 1$, $c_2 = 20$, and $k_i(t_0) = 5$, $k_l(t_0) = 2$.

3.4.2 Feedback Nash Equilibrium

With feedback Nash strategy, also referred to as Markov perfect Nash strategy in the literature, the firms follow strategies that are dependent on the state of the game and time. In Markov, or memoryless feedback games, the history of previous play or the previous actions of the players is not considered on its own merit. The evolution of the history of the game is embodied in the current state of the game itself, since each firm's strategy is a function of the state. The current state of the game which is the vector of the capacities held by the firms is the result of the previous actions in terms of investments made in previous periods up to the present.

A memoryless feedback, or Markov perfect Nash equilibrium is a subgame perfect equilibrium, in the sense that the firms follow strategies that are decision rules which constitute Nash strategies over any part of the subgame; that is when starting from any period over the interval of the original game, the investment strategy leads to the Nash equilibrium.

Furthermore, what the firms care about is only the profits that each generates over

the horizon of the game. As before, each of the firms faces an optimal control problem similar to that of Section 3.4.1, when choosing its output and investment strategy that maximise its profits over the planning horizon of the game $[t_0, t_f]$. The profit function for each firm is described by an equation exactly as in (3.21), and the system dynamics which refers to the capacity change of all firms is given by an equation the same as (3.22) for the open-loop model. All other assumptions about boundary conditions, time invariancy of costs, demand level and zero salvage value for capital are the same.

$$J_i = \max_{u_i} \int_{t_0}^{t_f} e^{-\beta t} \left(\frac{1}{2} \mathbf{k}' \mathbf{Q}_i \mathbf{k} + \frac{1}{2} \mathbf{u}' \mathbf{R}_i \mathbf{u} + \mathbf{r}'_i \mathbf{u} + \mathbf{u}' \mathbf{S}_i \mathbf{k} + \mathbf{q}'_i \mathbf{k} \right) dt \quad (3.59)$$

for $i = 1 \dots n$

subject to the following equation which describes the rate of change of capacity:

$$\dot{\mathbf{k}} = f(\mathbf{k}, \mathbf{u}) = \mathbf{A}\mathbf{k} + \mathbf{B}\mathbf{u} \quad (3.60)$$

and the boundary conditions: $\mathbf{k}_{(t_0)} = \mathbf{k}_0$ and $\mathbf{k}_{(t_f)}$ is free. $\lambda_i(t_f) = \mathbf{0}$ since at this time we are assuming that the salvage value $\varphi_{(t_f)}$ is zero.

$$\text{Where } \mathbf{k} = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}, \quad \lambda_i = \begin{pmatrix} \lambda_{i1} \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix}$$

The prime denotes a transpose, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}_i$, $\mathbf{R}_i$, and $\mathbf{S}_i$ are symmetric square matrices, $\mathbf{r}_i$, and $\mathbf{q}_i$ are vectors of appropriate dimensions.

The main difference with the open-loop model is in the pattern of interaction between the firms reflected in the decision rules of investment that govern the game now. In addition to taking its opponents investment decisions in response to its own investment into account when trying to find the optimal values of investment by each profit maximising firm, the investment rate at every instant of time is now a function of state of the game which is the capacities held by all the firms.

In feedback strategy, the decision variable of every player is explicitly dependent on its own capacity and that of its opponents. All firms realise that the capacities they hold will influence future decisions about how much each invests in the future. Investment decisions have a direct bearing on future profits of each firm and its competitors. This realisation influences the pattern of investment, and gives rise to a strategic aspect to the investment decision and about how much capacity to hold. If the investment decision of each firm is inversely proportional to the capacity held by its competitors, then by holding more capacity the firm would reduce the investment rate of its competitors in the next instant of time, or the next period, thereby reducing its capacity holding.

The optimal control problem facing each firm, and the necessary conditions for a solution to the game is described by another two propositions adapted from Theorems 6.15 and 6.16 and related definitions and remarks due to Başar & Olsder (1999).

Proposition 3 *For an n -person differential game of prescribed fixed duration $[t_0, t_f]$, let*

(i) *$f(t, \cdot, u_1, \dots, u_n)$ be continuously differentiable on $\mathbf{R}^n \times \mathbf{U}^1 \times \dots \times \mathbf{U}^n, \forall t \in [t_0, t_f]$,*

(ii) *$w_i(t, \cdot, u_1, \dots, u_n)$ and $\varphi_i(\cdot)$ be continuously differentiable on $\mathbf{R}^n \times \mathbf{U}^1 \times \dots \times \mathbf{U}^n$, and $\mathbf{R}^n$ respectively, $\forall t \in [t_0, t_f], i \in \mathbf{N}$.*

Then if $\{v_i^*(t, \mathbf{k}_0) = u_i^*(t); i \in \mathbf{N}\}$ provides a feedback no-memory Nash equilibrium solution, such that $v_i^*(t, \mathbf{k}_0)$ is continuously differentiable on $\mathbf{N} \forall t \in [t_0, t_f], i \in \mathbf{N}$, and $\{\mathbf{k}^*, t_0 \leq t \leq t_f\}$ is the corresponding state trajectory, there exist n costate functions $\lambda_i(\cdot) : [t_0, t_f] \rightarrow \mathbf{R}^n, i \in \mathbf{N}$, such that the following relations are satisfied:

$$\dot{\mathbf{k}}^*(t) = f(t, \mathbf{k}^*(t), u_1^*, \dots, u_n^*), \quad \mathbf{k}^*(t_0) = \mathbf{k}_0, \quad (3.61)$$

$$v_i^*(t, \mathbf{k}^*, \mathbf{k}_0) \equiv u_i^*(t) = \max_{u_i} H_i(t, \lambda_i, \mathbf{k}^*, u_1^*, \dots, u_{i-1}^*, u_i, u_{i+1}^*, \dots, u_n^*), \quad (3.62)$$

$$\beta \lambda_i(t) - \dot{\lambda}_i(t) = \frac{\partial}{\partial \mathbf{k}} H_i(t, \lambda_i, \mathbf{k}^*, v_1^*(t, \mathbf{k}^*, \mathbf{k}_0), \dots, v_{i-1}^*(t, \mathbf{k}^*, \mathbf{k}_0), u_i^*, v_{i+1}^*(t, \mathbf{k}^*, \mathbf{k}_0), \dots, v_n^*(t, \mathbf{k}^*, \mathbf{k}_0)) \quad (3.63)$$

$$\lambda_i(t_f) = \frac{\partial \varphi(\mathbf{k}^*(t_f))}{\partial \mathbf{k}}, i \in \mathbf{N} \quad (3.64)$$

where

$$H_i(\cdot) = w_i(\cdot) + \lambda_i' f(\cdot), \quad t \in [t_0, t_f], \quad i \in \mathbf{N}.$$

Proof. The proof follows from that for Theorem 6.15, in Başar & Olsder (1999, pp.320-321), and related theorems that make use of the maximum principle for continuous time control systems, as in Theorem 6.11. ■

Corollary 4 For an n -person affine-quadratic differential game with $Q_i \leq 0, R_i \leq 0, (i \in \mathbf{N})$, let there exist a solution set $\{\Lambda_i; i \in \mathbf{N}\}$ to the coupled matrix Riccati equations:

$$\dot{\Lambda}_i + \Lambda_i A - (\beta - A') \Lambda_i + Q_i - (\Lambda_i B + S) \tilde{R}^{-1} (\tilde{S} + B' \tilde{\Lambda}) - (\tilde{\Lambda}' B + \tilde{S}') (\tilde{R}^{-1})' \underline{\Lambda}_{il} = 0 \quad (3.65)$$

$$\Lambda_i(t_f) = \frac{\partial \varphi(t_f)}{\partial k}, \quad (i \in \mathbf{N}). \quad (3.66)$$

Then under the Markov perfect solution, the differential game admits a feedback Nash equilibrium solution affine in the state, given by:

$$v^*(t, k) \equiv u^*(t, k) = -\tilde{R}^{-1} (\tilde{r} + \tilde{S}k + B' (\tilde{\Lambda}k + \tilde{\gamma})) \quad (3.67)$$

where $\{\gamma_i; i \in \mathbf{N}\}$ solve uniquely the set of differential equations:

$$\dot{\gamma}_i - (\beta - A') \gamma_i + q_i - (\Lambda_i B + S) \tilde{R}^{-1} (\tilde{r} + B' \tilde{\gamma}) - (\tilde{\Lambda}' B + \tilde{S}') \tilde{R}^{-1} \underline{\gamma}_{il} = 0 \quad (3.68)$$

$$\gamma_i(t_f) = 0, \quad (i \in \mathbf{N}). \quad (3.69)$$

and $k^*(\cdot)$ denotes the Nash equilibrium state trajectory, generated by:

$$k^* = e^{(A+BZ_2)t} k(t_0) + \int_{t_0}^{t_f} e^{(A+BZ_2)(t-\tau)} B Z_1 d\tau \quad (3.70)$$

where $\tilde{R}^{-1}, \tilde{r}, \tilde{S}$ are defined in (3.36)-(3.40), $\tilde{\lambda}, \tilde{\Lambda}, \tilde{\gamma}$ are defined in (3.42)-(3.45), Z_1 and Z_2 are defined in (3.55)-(3.56).

Proof. See Corollary 6.5 in Başar & Olsder (1999, p.323); and by the solvability of the matrix Riccati equations in Section 3.4.2.2, also by the restrictions on the negative definiteness of matrices Q_i , R_i and the restriction to affine strategies imposed on the game (refer to Remark 6.16 p. 324 in Başar & Olsder (1999)), the game admits a unique solution in feedback strategies given by (3.77) p. 121 and state trajectory given by (3.93) as shown in Section 3.4.2.3 ■

3.4.2.1 Model Solution: Feedback Nash Strategy

Finding a solution for the present game involves the familiar steps of defining the Hamiltonian for each player, and satisfying the optimality conditions set out in Proposition 3:

$$H_i = \frac{1}{2} \mathbf{k}' Q_i \mathbf{k} + \frac{1}{2} \mathbf{u}' R_i \mathbf{u} + \mathbf{r}'_i \mathbf{u} + \mathbf{u}' S_i \mathbf{k} + \mathbf{q}'_i \mathbf{k} + \lambda'_i (\mathbf{A} \mathbf{k} + \mathbf{B} \mathbf{u}) \quad (3.71)$$

$$\text{for } i = 1, \dots, n$$

The conditions of optimality require that equations (3.72)-(3.74) are satisfied:

$$\frac{\partial H_i}{\partial u_i} = 0 \quad (3.72)$$

$$\frac{\partial H_i}{\partial \lambda_i} = \dot{\mathbf{k}} = \mathbf{A} \mathbf{k} + \mathbf{B} \mathbf{u} \quad (3.73)$$

$$\beta \lambda_i - \dot{\lambda}_i = \frac{\partial H_i}{\partial \mathbf{k}} + \frac{\partial \mathbf{u}}{\partial \mathbf{k}} \frac{\partial H_i}{\partial \mathbf{u}} \quad (3.74)$$

These are the same equations as (3.32)-(3.34) for the open-loop Nash game of Section 3.4.1.1. The main difference is in the extra last term in the costate equation (3.74). Recall that each player's strategy under feedback or Markov perfect Nash rules, is a function of the state of the game. The Hamiltonian of player i in equation (3.63) includes strategies of other players that are functions of time and the current state $\mathbf{k}^*$; the partial derivative of the Hamiltonian wrt $\mathbf{k}$ in this equation receives contributions from the dependence of remaining players strategies on the current state of the game. The costate variable λ_i accounts for these contributions, which in turn influences the optimal investment strategy of firm i as depicted in equation (3.63) of Proposition 3.

The term $\frac{\partial \mathbf{u}}{\partial \mathbf{k}} \cdot \frac{\partial H_i}{\partial \mathbf{u}}$ in (3.74) is exactly this contribution expressed in matrix form.

The first term of this expression is the influence of the state on the investment strategy of the competitors, and the second is the influence of the competitors investment strategy on the firm i 's profitability through its Hamiltonian.

Deriving the optimal investment strategy u_i^* using (3.72), is the same as for the open-loop model. By going through exactly the same steps of differentiating the Hamiltonian in (3.71), wrt u_i as before where each firm i has control only over its investment rate, u_i , we get:

$$u_i^* = -R_{ii}^{-1}(r_{ii} + S_{ii}k + B'_{ii}\lambda_i) \text{ for } i = 1, \dots, n \quad (3.75)$$

$$u_l^* = -R_{ll}^{-1}(r_{ll} + S_{ll}k + B'_{ll}\lambda_l) \text{ for } i \neq l, l = 1, \dots, n \quad (3.76)$$

where u_i^* , and u_l^* are the optimal investment strategies for firms i and l respectively. Similarly, the optimal investment strategy for the remaining firms may be obtained. Therefore we can define an optimal strategy vector $\mathbf{u}^*$, for all the firms involved:

$$\mathbf{u}^* = -\tilde{R}^{-1} \left(\tilde{r} + \tilde{S}k + B' \left(\tilde{\Lambda}k + \tilde{\gamma} \right) \right) \quad (3.77)$$

which has the same form as equation (3.45) for the open-loop equilibrium.

3.4.2.2 Riccati Equations Formulation

Deriving the Riccati equations for the feedback Nash equilibrium involves the evaluation of the differential equation of the costate variable (3.74); and this will be done starting from the *rhs* of the equation, with the term $\frac{\partial \mathbf{u}}{\partial \mathbf{k}} \cdot \frac{\partial H_i}{\partial \mathbf{u}}$ where the vector $\mathbf{u}$ is evaluated at its optimal value, and $\frac{\partial \mathbf{u}}{\partial \mathbf{k}}$ can therefore be obtained by differentiating equation (3.77).

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial \mathbf{k}} &= -\tilde{S}'\tilde{R}^{-1} + (B'\tilde{\Lambda})'(-\tilde{R}^{-1})' \\ &= -\tilde{S}'(\tilde{R}^{-1})' - \tilde{\Lambda}'B(\tilde{R}^{-1})' \end{aligned}$$

Therefore:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{k}} = (\tilde{S}' + \tilde{\Lambda}'B)(-\tilde{R}^{-1})' \quad (3.78)$$

Where,

$$\frac{\partial \mathbf{u}}{\partial \mathbf{k}} = \frac{\partial \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}}{\partial \mathbf{k}} = \begin{pmatrix} \frac{\partial u_1}{\partial k_1} & \frac{\partial u_2}{\partial k_1} & \cdots & \frac{\partial u_n}{\partial k_1} \\ \frac{\partial u_1}{\partial k_2} & \frac{\partial u_2}{\partial k_2} & \cdots & \frac{\partial u_n}{\partial k_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial u_1}{\partial k_n} & \frac{\partial u_2}{\partial k_n} & \cdots & \frac{\partial u_n}{\partial k_n} \end{pmatrix} \quad (3.79)$$

It has also been established that:

$$\frac{\partial H_i}{\partial \mathbf{u}} = \begin{pmatrix} \frac{\partial H_i}{\partial u_1} \\ \frac{\partial H_i}{\partial u_2} \\ \vdots \\ \frac{\partial H_i}{\partial u_n} \end{pmatrix} = \mathbf{R}_i \mathbf{u} + \mathbf{r}_i + \mathbf{S}_i \mathbf{k} + \mathbf{B}_i' \boldsymbol{\lambda}_i \quad (3.80)$$

If the matrices on the *rhs* are expanded then it can be verified by inspection that when i utilises an optimal investment strategy u_i^* :

$$\frac{\partial H_i}{\partial \mathbf{u}} = \begin{pmatrix} \frac{\partial H_i}{\partial u_1} \\ \frac{\partial H_i}{\partial u_2} \\ \vdots \\ \frac{\partial H_i}{\partial u_n} \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix} \quad (3.81)$$

since by definition $\frac{\partial H_i}{\partial u_i} = 0$.

We can now turn to evaluate the last term on the *rhs* of (3.74), namely: $\frac{\partial \mathbf{u}}{\partial \mathbf{k}} \cdot \frac{\partial H_i}{\partial \mathbf{u}}$, using (3.78) and (3.81) to obtain:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{k}} \cdot \frac{\partial H_i}{\partial \mathbf{u}} = \begin{pmatrix} \frac{\partial u_1}{\partial k_1} & \frac{\partial u_2}{\partial k_1} & \cdots & \frac{\partial u_n}{\partial k_1} \\ \frac{\partial u_1}{\partial k_2} & \frac{\partial u_2}{\partial k_2} & \cdots & \frac{\partial u_n}{\partial k_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial u_1}{\partial k_n} & \frac{\partial u_2}{\partial k_n} & \cdots & \frac{\partial u_n}{\partial k_n} \end{pmatrix} \begin{pmatrix} \frac{\partial H_i}{\partial u_1} \\ \frac{\partial H_i}{\partial u_2} \\ \vdots \\ \frac{\partial H_i}{\partial u_n} \end{pmatrix}$$

$$\begin{aligned}
&= (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})' \begin{pmatrix} 0 \\ \lambda_{i2} \\ \vdots \\ \lambda_{in} \end{pmatrix} \\
&= (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})' \underline{\lambda}_{il} \quad l = 1, \dots, n, l \neq i
\end{aligned} \tag{3.82}$$

And since it was assumed that λ_i is linear in $\mathbf{k}$, we can assume the same for $\underline{\lambda}_{il}$, and so we may write:

$$\underline{\lambda}_{il} = \underline{\Lambda}_{il} \mathbf{k} + \underline{\gamma}_{il}$$

or in expanded form:

$$\begin{pmatrix} 0 \\ \lambda_{il} \\ \lambda_{ik} \\ \vdots \\ \lambda_{in} \end{pmatrix} = \begin{pmatrix} 0 \\ \underline{\Lambda}_{il} \mathbf{k} + \underline{\gamma}_{il} \\ \underline{\Lambda}_{ik} \mathbf{k} + \underline{\gamma}_{ik} \\ \ddots \\ \underline{\Lambda}_{in} \mathbf{k} + \underline{\gamma}_{in} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \phi_{31} & \phi_{32} & \dots & \phi_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{pmatrix} \mathbf{k} + \begin{pmatrix} 0 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_n \end{pmatrix}$$

Therefore we have:

$$\frac{\partial u}{\partial \mathbf{k}} \cdot \frac{\partial H_i}{\partial u} = (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})' (\underline{\Lambda}_{il} \mathbf{k} + \underline{\gamma}_{il}) \tag{3.83}$$

Then we turn to $\frac{\partial H_i}{\partial \mathbf{k}}$ the first term on the *rhs* of (3.74); this can be obtained by differentiating (3.71):

$$\frac{\partial H_i}{\partial \mathbf{k}} = \mathbf{Q}_i \mathbf{k} + \mathbf{S}_i u + \mathbf{q}_i + \mathbf{A}' \lambda_i \tag{3.84}$$

Now from (3.83) and (3.84) it is possible to evaluate the *rhs* of equation (3.74). Next we turn to evaluate the *lhs* of (3.74). It has already been stated that the costate variable λ_i is assumed to be linear in the state, and has the form as shown in (3.85) below.

$$\lambda_i = \Lambda_i \mathbf{k} + \gamma_i \tag{3.85}$$

Taking the time derivative of this equation, we obtain:

$$\dot{\lambda}_i = \dot{\Lambda}_i k + \Lambda_i \dot{k} + \dot{\gamma}_i$$

Also from (3.60) we have $\dot{k} = Ak + Bu$, so we can write:

$$\dot{\lambda}_i = \dot{\Lambda}_i k + \Lambda_i Ak + \Lambda_i Bu + \dot{\gamma}_i \quad (3.86)$$

where u is evaluated at its optimal value, or more accurately $u = u^*$. From the above equations we are now able to evaluate the *lhs* of (3.74), and therefore we write (3.74), using (3.83) and (3.84) for the *rhs* as follows:

$$\beta \lambda_i - \dot{\lambda}_i = Q_i k + S_i u + q_i + A' \lambda_i + (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})'(\underline{\Lambda}_{il} k + \underline{\gamma}_{il})$$

substituting the value of $\dot{\lambda}_i$ from (3.86) in the above equation, we get:

$$\beta \lambda_i - \dot{\Lambda}_i k - \Lambda_i Ak - \Lambda_i Bu - \dot{\gamma}_i = Q_i k + S_i u + q_i + A' \lambda_i + (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})'(\underline{\Lambda}_{il} k + \underline{\gamma}_{il})$$

which we rearrange in the following two steps:

$$\begin{aligned} \beta \lambda_i - A' \lambda_i - \dot{\Lambda}_i k - \Lambda_i Ak - \Lambda_i Bu - S_i u - Q_i k - q_i - \dot{\gamma}_i \\ - (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})'(\underline{\Lambda}_{il} k + \underline{\gamma}_{il}) = 0 \end{aligned}$$

$$\begin{aligned} (\beta - A') \lambda_i - \dot{\Lambda}_i k - \Lambda_i Ak - (\Lambda_i B + S_i) u - Q_i k - q_i - \dot{\gamma}_i \\ - (\tilde{S}' + \tilde{\Lambda}' B)(-\tilde{R}^{-1})'(\underline{\Lambda}_{il} k + \underline{\gamma}_{il}) = 0 \quad (3.87) \end{aligned}$$

Substituting the values of λ_i and u^* from (3.85), and (3.77) respectively, we obtain:

$$\begin{aligned}
 & (\beta - A')\Lambda_i k + (\beta - A')\gamma_i - \dot{\Lambda}_i k - \Lambda_i A k - Q_i k - q_i - \dot{\gamma}_i \\
 & - (\Lambda_i B + S_i)(-\tilde{R}^{-1})(\tilde{r} + \tilde{S}k + B'(\tilde{\Lambda}k + \tilde{\gamma})) \\
 & - (\tilde{S}' + \tilde{\Lambda}'B)(-\tilde{R}^{-1})'(\underline{\Lambda}_{il}k + \underline{\gamma}_{il}) = 0
 \end{aligned} \tag{3.88}$$

For the above equation to admit a solution, the coefficients of the capacity vector k , and the independent matrices and vectors, must be zero. So next we collect the coefficients of the k terms, and the independent terms to get two equations as shown below:

$$\begin{aligned}
 & \{(\beta - A')\Lambda_i - \Lambda_i A - \dot{\Lambda}_i - Q_i - (\Lambda_i B + S_i)(-\tilde{R}^{-1})(\tilde{S} + B'\tilde{\Lambda}) \\
 & - (\tilde{S}' + \tilde{\Lambda}'B)(-\tilde{R}^{-1})'\underline{\Lambda}_{il}\}k = 0
 \end{aligned} \tag{3.89}$$

$$\begin{aligned}
 & (\beta - A')\gamma_i - q_i - \dot{\gamma}_i - (\Lambda_i B + S_i)(-\tilde{R}^{-1})(\tilde{r} + B'\tilde{\gamma}) \\
 & - (\tilde{S}' + \tilde{\Lambda}'B)(-\tilde{R}^{-1})'\underline{\gamma}_{il} = 0
 \end{aligned} \tag{3.90}$$

Now by rearranging the above two equations we obtain two Riccati like equations in Λ_i and γ_i that satisfy (3.72)-(3.74) (*c.f. equations (3.52) and (3.53)*):

$$\begin{aligned}
 \dot{\Lambda}_i = & (\beta - A')\Lambda_i - \Lambda_i A - Q_i + (\Lambda_i B + S)\tilde{R}^{-1}(\tilde{S} + B'\tilde{\Lambda}) + \\
 & (\tilde{\Lambda}'B + \tilde{S}')(\tilde{R}^{-1})'\underline{\Lambda}_{il}
 \end{aligned} \tag{3.91}$$

$$\dot{\gamma}_i = (\beta - A')\gamma_i - q_i + (\Lambda_i B + S)\tilde{R}^{-1}(\tilde{r} + B'\tilde{\gamma}) + (\tilde{\Lambda}'B + \tilde{S}')(\tilde{R}^{-1})'\underline{\gamma}_{il} \tag{3.92}$$

In the n player game we obtain $2n$ similar Riccati equations. For the duopoly game there are four of these equations. So for firm l we obtain two Riccati equations of the same form as (3.91) and (3.92). The four equations can then be solved simultaneously.

It was found that they admit a stable unique solution. The conditions set by equations (3.65) and (3.68) of Corollary 4 are satisfied.

From the solution of the Riccati equations, we obtain the values of Λ_i and γ_i , and Λ_l and γ_l , and the corresponding values of $\tilde{\Lambda}$ and $\tilde{\gamma}$. With these values it is now possible to find the optimal investment strategy $\mathbf{u}^*$, using (3.77), which is a vector of the investment strategies for all firms for the entire game.

3.4.2.3 Optimal Investment Strategy and Capacity Growth for Feedback Case

While the optimal controls for the open-loop and feedback Nash games are the same as in equations (3.45) p. 108 and (3.77) p. 121 respectively, the capacity equilibria they admit are different. Different strategies lead to different Riccati equations and different solutions.

The unique solutions of the Riccati equations give rise to unique optimal equilibrium paths for the rate of investment and capacities in the two games. The accumulation of capital develops forward in time from the initial known capacity level k_0 at t_0 the starting time of the game. This path is dependent on the values of $\tilde{\Lambda}$ and $\tilde{\gamma}$, which for the feedback game now have extra terms involving $\underline{\Lambda}_{il}$ and $\underline{\gamma}_{il}$, in the respective Riccati equations (3.91) and (3.92).

The derivation for the optimal capacity growth for the feedback case is similar to that of the open-loop case of Section 3.4.1.3. The optimal trajectory of the vector of capacities is obtained by solving the system dynamics equation (3.60) which leads to the same equation as in the open-loop case:

$$\mathbf{k}^* = e^{(\mathbf{A} + \mathbf{B}\mathbf{Z}_2)t} \mathbf{k}(t_0) + \int_{t_0}^{t_f} e^{(\mathbf{A} + \mathbf{B}\mathbf{Z}_2)(t-\tau)} \mathbf{B}\mathbf{Z}_1 d\tau \quad (3.93)$$

where $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are defined as before. Since the initial capacities at time t_0 for all firms are known, the optimal trajectory of capacity growth is obtained by solving the above equation forward in time until t_f by numerical integration. The results for several runs are shown, for a duopoly model, below in Figures 3.10-3.14.

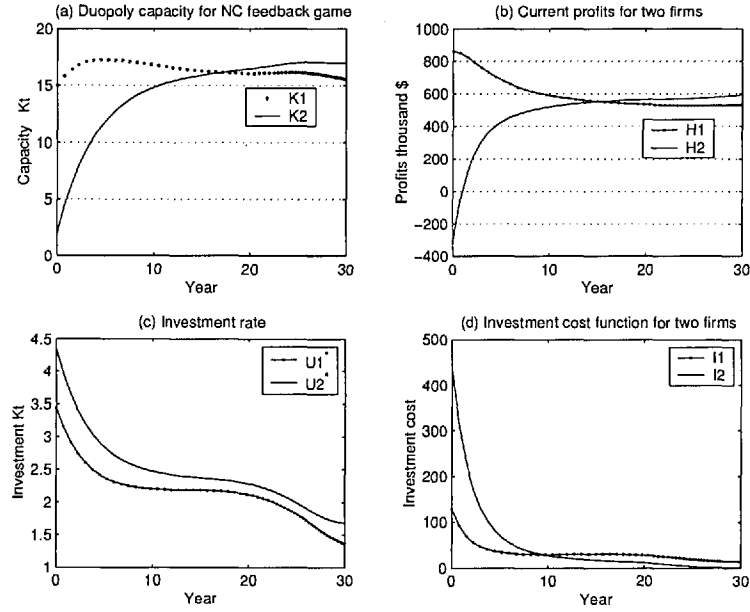


Figure 3.10: Investment and output profile for feedback equilibrium: parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 1$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $\beta = 0.05$ $k_i(t_0) = 15$, $k_l(t_0) = 2$.

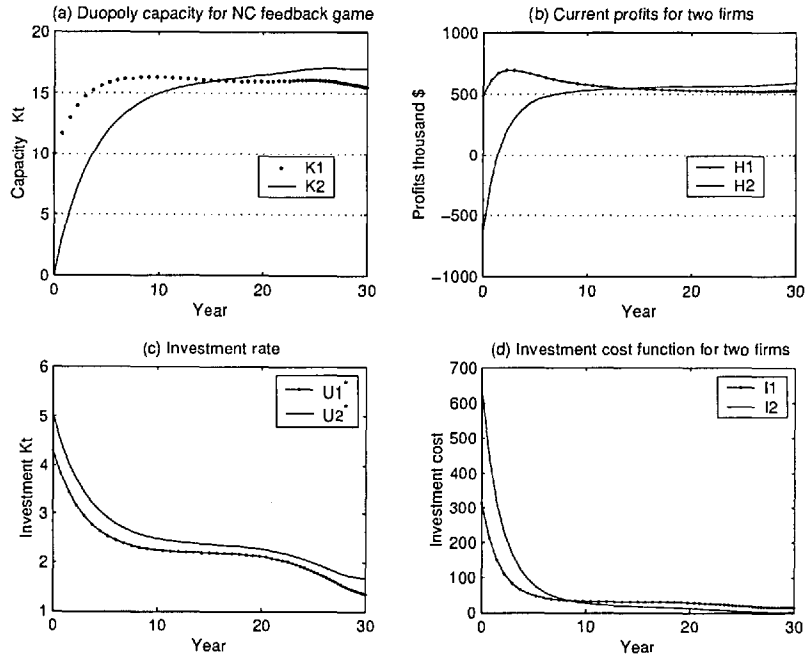


Figure 3.11: Investment and output profile for feedback equilibrium: parameter values used: same as in Fig.3.10 except for $k_i(t_0) = 10$, $k_l(t_0) = 0$.

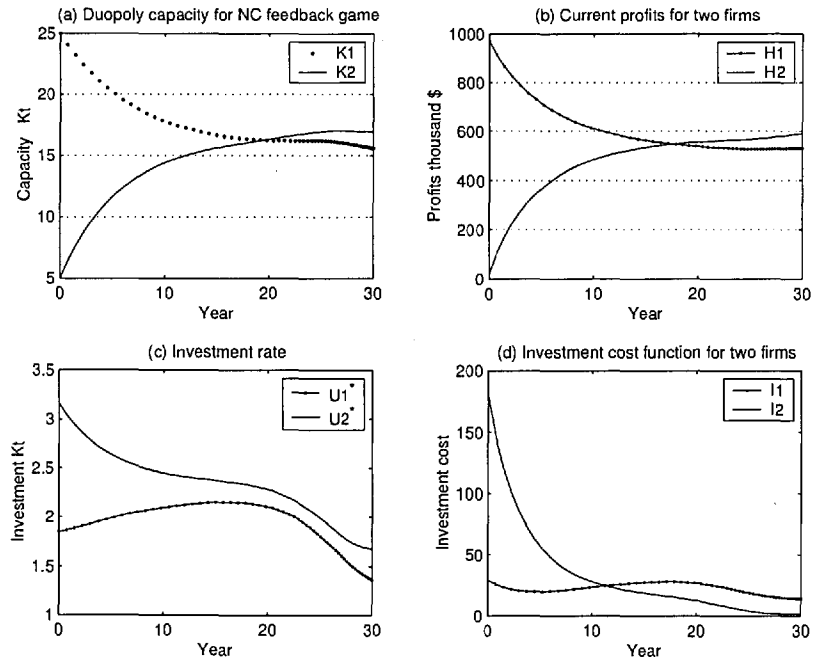


Figure 3.12: Investment and output profile for feedback equilibrium: parameter values used: same as in Fig.3.10 except for $k_i(t_0) = 25, k_l(t_0) = 5$.

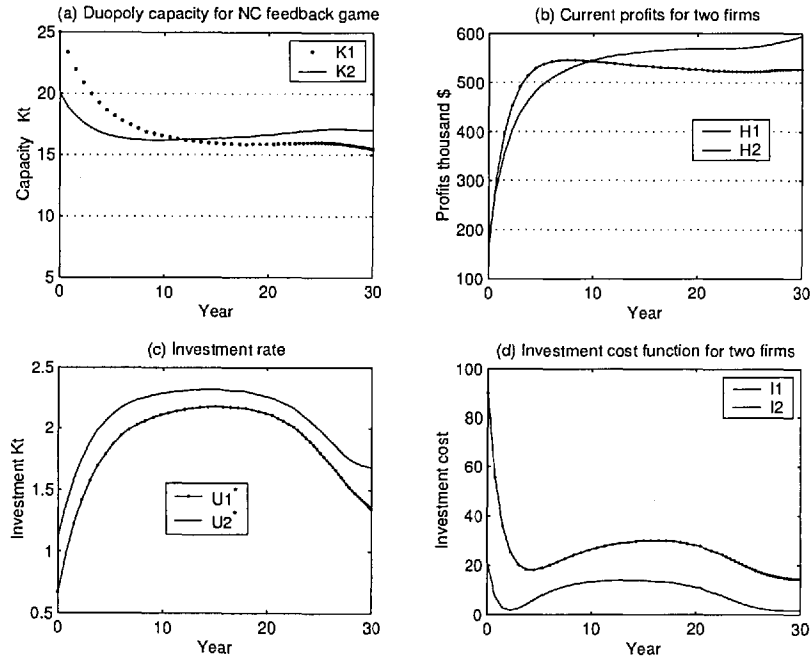


Figure 3.13: Investment and output profile for feedback equilibrium: parameter values used: same as in Fig.3.10 except for $k_i(t_0) = 25, k_l(t_0) = 20$.

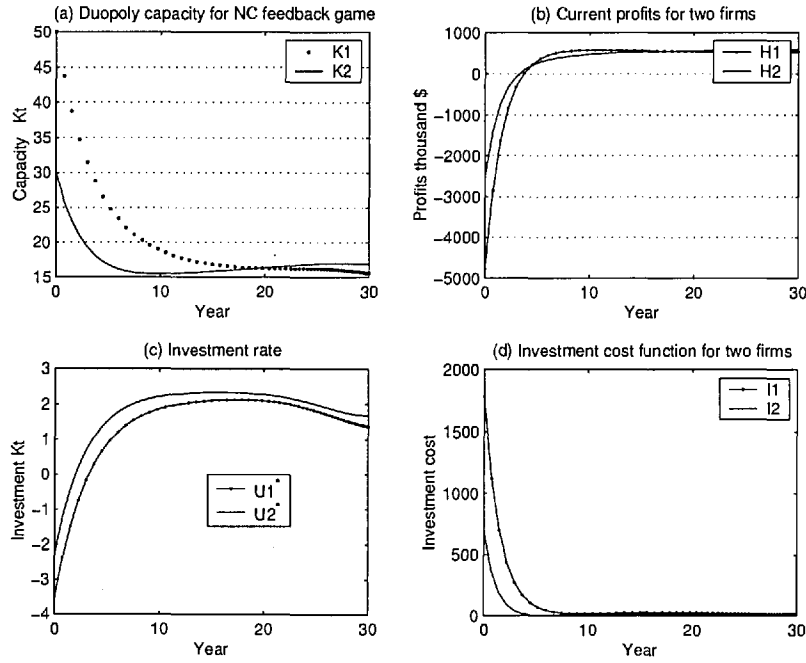


Figure 3.14: Investment and output profile for feedback equilibrium: parameter values used: same as in Fig.3.10 except for $k_i(t_0) = 50, k_l(t_0) = 30$.

3.4.3 Discussion of Results

Both open-loop and feedback Nash strategies for the *linear quadratic* model of fixed duration admit unique equilibria. The state trajectories converge to stable stationary capacity levels for the two firms in both cases. To examine the convergence of the model, several runs were conducted, in which variations in the following model parameters were considered:

- The starting values at time t_0 , for the vector of capacities of the two firms.
- The unit costs of capital investment.
- The speed of adjustment costs.
- The inverse demand function parameters.

The runs are divided into two classes. In the first, it was assumed that the two firms face identical costs of investment; while in the second, the firms face asymmetric costs; the smaller firm is assumed to have more advantageous costs of investment than the

second larger firm. Another case is considered where the larger firm is more efficient in terms of investment costs.

It was shown in Section 3.4.1 and Section 3.4.2 that there exists a unique pair of investment strategies that lead to stable *Nash equilibria* for the capacity expansion problem formulated as a *linear quadratic* Nash-Cournot game of fixed duration. The uniqueness of the solution is contingent on finding a solution to a set of matrix Riccati equations. Indeed, these equations admit unique solutions for both open-loop and memoryless feedback, or Markov perfect models, provided the players are confined to strategies that are affine in the state. The solution is an adaptation of a general formulation for this type of models by Başar & Olsder (1999). Engwerda (1998b,1998a) on the other hand showed that for a *linear quadratic* game an open-loop *Nash equilibrium* may exist for situations where the Riccati equations do not admit a solution.

Existence of unique solutions for *linear quadratic* differential games is due to Starr & Ho (1969a). Papavassilopoulos & Olsder (1984) also demonstrated the existence of linear feedback solution for finite horizon *linear quadratic* games; Kydland (1975) showed the existence and uniqueness of linear feedback strategies for *linear quadratic* discrete time dynamic games of finite horizon. Reynolds (1987) showed the existence of a solution for an infinite horizon *linear quadratic* game for a duopoly; using a quadratic value function he solves the Bellman equations and shows the stability and uniqueness of the solution. Capacity adjustment in the Reynolds model is subject to adjustment costs that are quadratic in the gross investment ($I_i = cu_i + \frac{1}{2}d u_i^2$). The model developed in this chapter is defined for a fixed duration, and the adjustment costs are quadratic in the net investment undertaken by the firm, ($I_i = cu_i + \frac{1}{2}d(u_i - \delta k_i)^2$).

The uniqueness of the solution is clear from Figures 3.3(a)-3.7(a) for open-loop strategy, where the capacities of the two firms i and l (or respectively 1 and 2) converge to the *Nash equilibrium* which is maintained until the end of the game. This equilibrium is unique, in the sense that no matter at what initial capacity levels the two firms start the game, by following the optimal investment strategies u_i^* , u_l^* they will eventually converge to the same equilibrium. And similarly for the feedback case as shown in Figures 3.10(a)-3.14(a).

When the two firms incur symmetric costs, both end up having identical capacities and market shares, as they converge to the Nash equilibrium. Reynolds (1987) findings are also consistent with this outcome.

The intertemporal nature of investment and changing market shares of the two firms is also discernable from these figures. Due to the presence of adjustment costs, the firms adjust their capacities slowly over time towards the equilibrium level.

Another feature which is apparent from these figures is that towards the end of the game, both firms realise that the useful life of additional investment is very short, and marginal earnings are too low to cover rising marginal adjustment costs (Kort 1988). They reduce the investment rate from the level required to compensate for depreciating capital, and their capacities decline, or dip slightly, from the equilibrium level. This pattern of investment is shown in *panes (c)* of the figures for all the cases considered.

When firm i has a larger market share than firm l at time t_0 , initially the smaller firm invests at a higher rate than the larger firm i , until the two firms become comparable in size, and the investment rates stabilize to maintain a capacity level that is capable of meeting market demand. As can be seen from Figures 3.3, and 3.4 for the open-loop model, and also Figures 3.10 and 3.11 for the feedback, the capacities of the two firms converge, to the *Nash equilibrium* level, which is maintained until the end of the game.

The economic interpretation of this result is that in a duopoly the firms realise that by controlling the output they can influence the price. Since both firms are only concerned about maximising their profits, when the smaller more efficient firm, l , expands its capacity by utilising a high investment rate, the larger firm, i , maintains a low investment level, and consequently a low output such that the price is not adversely affected.

In Figure 3.5 firm i , which starts from a high level of capacity, allows its capital to depreciate, while firm l invests at a high rate until they both stabilise when the *Nash equilibrium* is reached. As firm i 's capacity and market share decline, its profits decline initially and then stabilise; while firm l invests heavily raising its market share and profits. When both firms have capacities far in excess of market demand as in Figures 3.6 and 3.7 both firms reduce their capacities to arrive at the *Nash equilibrium*. As the two firms allow their capital to depreciate, they reduce their costs and the profitability rises. In

Figure 3.7 both firms disinvest initially in order to reduce their overcapacities, and losses. Investment then stabilises as they approach the *Nash equilibrium*. Initial investment, which is negative, may be interpreted as the two firms selling their capital. In the present model there are no limits on the investment which may take positive as well as negative values.

The firms act strategically by following feedback Nash strategies of investment and output. Unlike with the open-loop, there is no pre-commitment over any period under feedback strategy solution, instead the investment strategy is dependent on the current state of the game represented by the pair of current capacities held by the two firms. Since each firm's investment strategy is decreasing in the capacity held by its rival, each firm has an incentive to overinvest in an effort to preempt its rival in future periods, or more accurately for the remaining period of the game.

By overinvesting the firm induces its rival to invest less, as the firm realises that its investment strategy and that of its rival, is a function of the current state of the game. The reaction of the other firm, by following the same strategy of overinvesting in an effort to influence its rival's investment decision, gives rise to a pair of capacity equilibria for the two firms that is higher than that attained under open-loop strategy. This is true for all the cases tested as shown in Figures 3.10-3.14, pp. 127-129, and their counterparts for the open-loop in Figures 3.3-3.7, pp. 113-115.

It is noticeable also from these figures that feedback Nash strategies result in more competitive outcomes than open-loop strategies. When the firms follow feedback strategies investment is higher, leading to higher capacity and output equilibrium levels for both firms than for the open-loop counterpart. Higher outputs result in lower prices, and both firms achieve lower profits as depicted in *panes (b)* of the figures.

Also Table 3.1 shows the capacity *Nash equilibria* attained for several runs where the two firms have asymmetric capacity investment costs ranging from 0.01 to 100. Some of the results of the asymmetric costs cases are somewhat intriguing. Assuming that firm i is the smaller of the two firms, and that it incurs higher investment costs, it is found that by utilising the same optimal investment strategy as in (3.45), or (3.77), i will continue to invest rapidly initially, to catch up with firm l , despite its higher costs. Eventually

i , ends up having a capacity, and consequently a market share, close to that of l . The discrepancy in the market shares between the two firms is dependent on the magnitude of the difference in the cost structure of their investment costs. When i has an investment cost 10 to 100 times that of l , its equilibrium capacity is nearly equal to that of l , or in the worst case about half that of firm l .

But on the face of it this result runs counter to standard findings of examples given in economics textbooks; which is that market shares of firms should reflect directly their cost efficiency. However, it should be remembered that these examples refer to static situations. In a dynamic setting the outcome is different. Again the duopoly analysis is relevant here. As long as there are profits to be made and firm i can cover its costs, it will continue to invest at a high rate despite its higher costs.

On the other hand if firm i were to start from a very high level of capacity; then as l invests, i will reduce its capacity to accommodate the increase in its rivals capital additions. This continues until the *Nash equilibrium* is reached. But why should i make these sacrifices? Again the analogy is derived from oligopoly theory. In a market served by a duopoly both firms realise their interdependence in affecting the market price; the leading firm realises that unless it accommodates its rival, the price will crash. Firm i has an incentive to reduce its capacity to support prices and maintain profitability.

Table 3.1: Nash equilibria under different investment costs^(b)

Investment cost		Feedback		Open-loop	
c_1	c_2	k_1	k_2	k_1	k_2
0.02	0.01	15.19	15.18	13.78	13.78
2	1	15.08	15.18	13.70	13.77
10	1	16.23	16.51	15.01	15.44
30	3	14.62	16.87	13.47	15.11
40	10	13.80	16.70	11.80	13.80
50	0.5	13.36	17.73	12.49	15.71
100	1	10.34	19.51	10.27	16.81
100	10	10.15	18.65	9.89	15.30
100	20	11.05	18.32	10.80	16.32

^(b) parameter values used: $a = 100$, $b = 2$, $d_1 = 50$, $d_2 = 50$, $r = 0.05$, $\delta = 0.10$.

The results of the table show that on average the equilibrium capacity levels attained under feedback strategy are 5% - 22% higher than the open-loop capacity levels; reflecting the strategic behaviour of firms when they follow feedback (or Markov perfect) Nash strategies.

A decrease in demand elasticity ($b = 4$) results in *Nash equilibria* at lower capacity levels ($k_1 = 7.57, k_2 = 8.55$) than those of the base case. Where as when $b = 0.5$, indicating a demand curve with higher demand elasticity, the pair of equilibrium capacities are much higher, in the range of $k_1 = 53.9$, and $k_2 = 61.2$. As shown in Table 3.2, despite the variation in the demand curve slope, the *Nash equilibria* obtained for the two firms are consistent with previous runs where the two firms i and l attain capacities that are comparable in size, and where feedback capacities are higher than their open-loop counterparts.

Table 3.2: Nash equilibria under different inverse demand parameters^(†)

b	Feedback		Open-loop	
	k_1	k_2	k_1	k_2
0.5	64.97	64.67	62.19	62.29
2.0	15.60	16.89	14.84	16.04
4.0	7.57	8.55	7.21	8.07

[†] parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 1$, $d_1 = 50$, $d_2 = 50$, $r = 0.05$, $\delta = 0.10$.

Variations in the adjustment cost parameter d appear to have a small impact on the equilibrium values for both the feedback and open-loop strategies, as shown by the small variation in the values of k_1 and k_2 in Table 3.3.

Table 3.3: Nash equilibria under different speed of adjustment costs^(*)

Adjustment Cost		Feedback		Open-loop	
d_1	d_2	k_1	k_2	k_1	k_2
0.05	0.05	13.68	15.99	12.14	13.96
0.50	0.50	13.90	16.03	12.41	14.04
50	50	16.23	16.51	15.01	15.44
100	100	16.11	16.29	14.76	14.87

^(*) parameter values used: $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 1$, $r = 0.05$, $\delta = 0.10$.

3.5 Conclusions

The dynamic capacity expansion problem faced by a duopoly is formulated in this chapter as a Nash-Cournot differential game with convex adjustment costs. Using optimal control techniques, the optimal investment strategy that maximises the profits for each firm is found by solving a system of coupled matrix Riccati equations. These investment strategies are referred to as the *golden* investment rule by Conrad & Clark (1987); they are given by equations (3.45), p. 108, and (3.77), p. 121. The optimal investment strategies induce unique equilibrium solutions for the state trajectories under both the open-loop and feedback, or Markov perfect, information structures. It was demonstrated that when firms follow feedback strategies, they hold larger capacities at the steady state equilibrium than when they follow open-loop strategies, indicating strategic intent, by attempting to influence the investment behaviour of their opponents. This result was shown to be robust under different conditions of investment costs, adjustment costs, and demand curve variations.

Unlike the one shot games of Chapter 2, the advantage of preemptive investment by the incumbent to deter mobility in the long run is ineffective due to the dynamic nature of the investment game, depreciation and the reversibility of capital. In the Spence (1979) model, the first mover firm has an advantage over its rival, and early investment has a preemptive effect on the outcome of the game. Fudenberg & Tirole (1983) by developing the same model show that in a Markov perfect equilibrium, the firm that has a head start can deter entry or mobility, by overinvesting causing its rival to reduce its investment.

In this chapter, the capacity equilibria that are generated by following open-loop or feedback Nash-Cournot investment strategies, are independent of the pair of capacities the two firms possess at the start of the game. From the foregoing, it may be concluded that in this model small firms increase their market share rapidly, provided that adjustment costs are reasonable, while larger firms make allowance for the growth of their smaller competitors. This indicates that profit maximising firms accommodate their rivals when following Nash-Cournot investment strategies.

Despite large differentials in initial capacities at the start of the game between the

two firms, at equilibrium they equalise in size and end up having equal shares of the market when they face symmetric costs. In this chapter, it was found that even in cases when there are substantially large differences in investment costs between the two firms, their capacities converge in the long run to comparable sizes. The discrepancy in the market shares between the two firms, nevertheless, is dependent on the magnitude of the difference in their cost structures.

Investment costs play a small role in affecting the equilibrium values, when these are only a small proportion of total costs, or are small in comparison to the revenue. In the petrochemical industry the production of most bulk products involves processes where the investment cost is a small fraction of the cost of production; which may explain the existence of inefficient producers in such oligopolistic markets and why they are able to survive for long periods; and suggests that firms follow Nash-Cournot strategies.

Chapter 4

Optimal Output and Investment with Binding Capacity

4.1 Introduction

The assumptions made in Chapter 3, that the firms face production at full capacity, and negligible production costs are relaxed in this chapter. Also capital is allowed to depreciate in this chapter; thereby imparting some capital reversibility on the problem when the investment rate falls behind the annual depreciation of capital. Relaxing the assumption of operating at full capacity, increases the dimension of the problem; instead of controlling just one control variable, each firm now has to decide on two control variables in every period: an optimal investment rate, and an optimal production rate. Each firm solves an optimal control problem with a two dimensional control variable. We resort to a discrete time formulation for the problem, using an algorithm attributed to Kydland (1975). A discrete time formulation of the problem facilitates applying the model to empirical testing using industry data, which will be the subject of the next two chapters.

In addition to the the Nash-Cournot competition that takes place between the two firms discussed in Chapter 3, we look at a Stackelberg game in this chapter, where one of the firms takes on the role of the leader in the market. The role of the leader may be alternated between the incumbent and the entrant. The leadership position may

be assumed by one of the firms for a variety of reasons. It could be due to historical reasons like the role of a long established producer in the market, with its marketing network and sales outlets, or perhaps with the added benefit of accumulated technical and production know how. Another would be technological advantage like the ownership of an efficient proprietary production process, or access to cheap raw materials. Yet another reason could be due to a more efficient management structure rapidly responsive to market developments and industry dynamics; also more innovative, skillful and creative workforce enjoying good industrial relation with its management gives companies a lead over their competitors. The Stackelberg model is dealt with in Section 4.4

The rest of this chapter is organised as follows. Both types of strategies are considered for the Nash-Cournot model, the open-loop, and feedback strategies, with an analysis of the likely credible outcome. The feedback strategy for the model is considered first in Section 4.2, and the open-loop in Section 4.3. The performance of the dynamic model under different market structures, and the role of variations in adjustment cost parameters, are studied in Section 4.5. Section 4.6 concludes the chapter.

4.2 Nash-Cournot Duopoly with Two Decision Variables and Feedback Strategies

This section deals with the case where each of the two firms have to contend with two decision variables, namely, the output, x , and the investment rate, u , in each period, and use Nash-Cournot feedback strategies. Furthermore, the equilibrium solution that we are seeking for the investment problem will be a feedback equilibrium; since in each period, the optimisation is carried out on the basis of the current state of the world. Each firm solves an optimal control problem, assuming a feedback information pattern, where the permissible strategies are dependent on the current value of the state and time (Başar & Olsder 1999). This will become clear from the exposition of the problem which follows in the next section.

The strategies are decision rules about actions that are undertaken in each period. In the present context, these actions concern the production rates (or outputs), and the

investments in capacity that the firms pursue over the planning horizon under consideration. Past actions of the players and the history of the evolution of game to the present state are not relevant, since in every period the players base their decisions on the current state; this gives rise to the subgame perfectness property of the feedback game. When the firms follow these type of strategies, the game starting from any designated time within the time horizon of the game, constitutes from that time until the remaining part of the game, a perfect game. A Nash-Cournot feedback equilibrium is reached when each firm solves its control problem, and neither firm has an incentive to deviate from its course of action. For instance the actions of the firms regarding their outputs are interdependent, since in a duopolistic market, their combined production determines the price. So when each of the two firms solves its optimal control problem, it makes an assumption about action of its competitor, that that firm is following an optimal policy of outputs. And when each arrives at its optimal solution, neither has an incentive to deviate from this outcome since neither can unilaterally benefit from any deviation. This amounts to solving two optimal control problems for the two firms simultaneously. The question of whether firms decide on the timing of their moves alternately or simultaneously, when timing choice is made endogenously (i.e. the choice of timing of production or investment in every period is made by the firms and not imposed on the structure of the game) in a Nash-Cournot setting game similar in nature to the one analysed here was discussed in Maskin & Tirole (1987). They demonstrate that when firms move alternately, they have a tendency to produce higher quantities leading to lower profits than if they were to move simultaneously. The firms have a joint incentive to be in a simultaneous mode. In the presence of relatively low interest rates, the gain to firms from moving simultaneously is more significant, and simultaneity of moves therefore prevails.

The problem is formulated in a *dynamic programming* framework. By construction, this formulation ensures that decisions are optimised from the present time until the final period under consideration irrespective of any previous actions or decisions made (Bellman 1957)¹. The planning horizon for the problem we consider is a finite one; but a sufficiently long horizon will be assumed, to warrant usefulness for the study of empirical

¹Dynamic programming is used for the case where a feedback solution is required. Alternatively it would have been possible to use differential game formulation for the problem as we did in Chapter 3.

work or long range planning. Bearing this in mind, and the fact we are considering interest rates that are small in value, the salvage value of capital at the end of the game would be insignificant, and may be safely assumed to be zero. Finite horizon problems of the type studied here, in general, have no closed form solution. They can be formulated using either Riccati equations, or dynamic programming formulation, and then numerical methods are used to solve for the optimal control solution.

Most of the work in this area, however, assume infinite planning horizon. Cyert & de Groot (1984) paper is one of the earliest to consider a finite horizon for this problem. Maskin & Tirole (1987) show that the finite horizon equilibrium strategies converge to their infinite horizon counterparts as the planning horizon is extended; giving an indication of the robustness of the nature of equilibrium solution. Kydland (1975) derived a game theoretic solution for a similar problem for n players, where each controls a single decision variable. Reynolds (1986), extended this formulation, by considering developments in the US aluminium industry, to a problem where each firm i controls two decision variables: investment rate, u_i , and output, x_i . Here we follow the same methodology, which in reality is a dynamic programming problem with two decision variables. The algorithm is described in detail in Kydland (1975) for the Nash-Cournot game, and for the Stackelberg game in Kydland (1977). The theorems and their proofs are found therein, and the reader is referred to the two papers. Here, only reference is made to the theorems, and the basic structure of the algorithms is sketched out. It is worth noting also, that the formulation in Kydland is for problems where the players are concerned about minimising costs. So instead of maximising, the problems that players contend with in Kydland are those of minimisation over the horizon of the game.

4.2.1 Model Description and Structure

As in chapter 3, the market is served by a set N of firms or players, where $N = \{1, \dots, n\}$, and in a duopoly we have $n = 2$; each player $i \in N$ has a vector control variable $\mathbf{u}_{i,t}$, such that $\mathbf{u}_{i,t} = \{u_{i,t}, x_{i,t}\}'$, comprising its investment and output rates respectively. Where $\mathbf{u}_i \in U_i$, and U_i is a compact and convex set of admissible controls, $U_i \in \mathbb{R}^2$. The planning horizon of the game is finite, and it is defined to cover $[1, T]$.

In each period t , the net profit for each firm, $w_{i,t}$ is defined to be a quadratic function; this is so because $\pi_{i,t}$, $C_{i,t}$, and $I_{i,t}$ are all quadratic, and time dependent; but initially for the duopoly simulation runs we assume that they are time invariant:

$$w_i(t) = \pi_i(t) - I_i(t) \quad \forall i \in \mathbf{N}, \forall t \in [1, T]$$

where the operating profit is the revenue less the production costs:

$$\pi_i(t) = (a - b(x_{i,t} + x_{j,t}))x_{i,t} - C_{i,t}(x_{i,t}, k_{i,t})$$

and the production cost function, $C_{i,t}$ is quadratic in output, $x_{i,t}$, and capital, $k_{i,t}$.

$$C_{i,t}(x_{i,t}, k_{i,t}) = f_i x_{i,t} + m_i k_{i,t} + \frac{1}{2} g_i (x_{i,t} - k_{i,t})^2$$

f_i and m_i are the unit variable, and fixed costs respectively; these are the costs of materials, supplies and capital related costs per period. They cover all costs of activities such as those involved in the day to day running of the plant, production and sales operations of the product. The firm is penalised by the last quadratic term in the equation, when output deviates appreciably from the optimal design capacity of the plant; and g_i is an output adjustment cost parameter. The significance of the adjustment cost element of the production cost function, and the relevance for its inclusion in the context of the petrochemical production costs is expounded upon in the next chapter. In most studies of dynamic models, the adjustment cost element of the production cost function, is assumed to be quadratic in gross change of output (Reynolds 1987, 1991, Karp & Perloff 1993a, 1993b); this effectively means that adjustment costs are increasing with output; and at higher rates of output the firms are penalised by paying higher costs of adjustment. In the present formulation of the model, when the production rate is at the optimal level, the adjustment cost is zero. Only when production deviates from optimal capacity do firms get penalised; and the cost of adjustment is proportional to the deviation irrespective of the level of production at which firms are operating. Also as before, the investment cost, $I_i(u_i, k_i)$ is quadratic in the investment rate, u_i , and capacity,

k_i , and includes an adjustment cost element as defined in Chapter 3

$$I_i(u_{i,t}, k_{i,t}) = c_{i,t}u_{i,t} + \frac{1}{2}d_i(u_{i,t} - \delta k_{i,t})^2$$

Following Kydland (1975), Reynolds (1986), and Slade (1995), the state vector $\mathbf{y}_t$, in each period, is defined to include the output, the rate of investment and the capital stock of both firms. $\mathbf{y}_t$, therefore describes the state of the industry in each period t . The net profit $w_{i,t}(\mathbf{y}_t)$, now takes the following compact form, as a result of redefining the state vector²; which is a linear quadratic function of the state:

$$w_{i,t}(\mathbf{y}_t) = \mathbf{p}'_{i,t}\mathbf{y}_t + \frac{1}{2}\mathbf{y}'_t\mathbf{Q}_{i,t}\mathbf{y}_t \quad \text{for } i, j=\{1,2\}, i \neq j \quad (4.1)$$

the vectors $\mathbf{p}_{i,t}$, and matrices $\mathbf{Q}_{i,t}$, have the appropriate sizes of 6×1 and 6×6 respectively; for a duopoly the state vector, $\mathbf{y}_t \in \mathbb{R}^{3n}$, since for each player the state has three elements: its capacity and the two control variables of production and investment; $\mathbf{p}_{i,t}$, and $\mathbf{Q}_{i,t}$ contain the equation parameters of the model. Capital is subject to depreciation, and the evolution of capacity is described by: $k_{i,t+1} = u_{i,t} + (1-\delta)k_{i,t}$. which is analogous to equation (3.12) of Chapter 3. It simply states that the capacity in the next period is equivalent to the current gross investment, $u_{i,t}$, undertaken by the firm plus the net capacity (after depreciation) in place in the current period. Both firms are undergoing this process. The development of capacity over the planning horizon, as represented by the state, is shown in equation (4.2). It describes the evolution of the state, for both firms, as a function of the state in the previous period, and the current decision vector of investment and output rates.

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t \quad (4.2)$$

where the state vector $\mathbf{y}'_{t-1} = (u_1, x_1, k_1, u_2, x_2, k_2)_{t-1}$, and the control vector, $\mathbf{u}'_t = (u_1, x_1, u_2, x_2)_t$. Also $\mathbf{A}$ and $\mathbf{B}$, are two matrices with the respective dimensions of 6×6 and 6×4 . Chow (1981) provides a detailed exposition of the construction of these

²I use interchangeably the notation i, j and respectively 1, 2 to refer to the two firms, which is an acceptable and common practice in the game theoretic literature.

equations in the context of the optimal control of dynamical economic systems. The objective of each firm is to maximise the flow of its profits over the planning period, by selecting optimal values for the investment and output, that it controls, while at the same time taking into consideration the actions and strategy of the rival firm. J_i is the net present value of current and future profits generated by the firm as a result of following an optimal policy over the planning period. β is the discount factor, where $\beta = \frac{1}{1+r}$, and r is the risk free per period discount rate³, namely:

$$J_i = \max_{x_{i,t}, u_{i,t}} \sum_{t=1}^T \beta^{t-1} w_i(\mathbf{y}_t) \quad \text{for } i \text{ and } j, i \neq j \quad (4.3)$$

We can introduce a dynamic programming formulation for the problem; where the stream of profits generated by firm i at any time t as a result of holding a capital stock described by $\mathbf{y}_t$, is expressed by the value function, $V_{i,t}(\mathbf{y}_{t-1})$; which comprises two elements: the profits generated in the current period $w_{i,t}(\mathbf{y}_t)$, and the future flow of profits that its capital stock will continue to generate, when the firm follows an optimal policy of investment and output until the final period of the game, T . This future flow of profits is expressed as a value function for the next period, and is discounted to the present by the discount factor β . But the capital stock is dynamic, and is changing in every period as a result of depreciation and capital investment; and the maximisation of (4.4) is subject to the state equation (4.2).

$$V_{i,t}(\mathbf{y}_{t-1}) = \max_{x_{i,t}, u_{i,t}} (w_i(\mathbf{y}_t) + \beta V_{i,t+1}(\mathbf{y}_t)) \quad \forall i \in \mathbf{N}, \forall t \in [1, T] \quad (4.4)$$

By construction, a dynamic programming implementation of the planning problem for capacity expansion is akin to a feedback strategy, since at every stage each firm reacts to the state variable at time t when deciding on its future policy and the selection of its optimal decision variables; perhaps to illustrate this point, it's worth noting here Bellman's principle of optimality which states that:

³The interest rate is assumed to be constant over all periods, and all firms discount their profits at the same rate which appears to be the norm in the literature; although it may be argued that different companies may have different discounting rates, depending on the value of their capital, and the ability to raise funds internally. In the present formulation of the model this could be easily incorporated.

“An optimal policy has the property that, whatever the initial state and decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.” (Intriligator 1971)

To solve the capacity expansion problem, we have to specify the value function. Equation (4.4) suggests that the value function is a function of the state in place in the previous period, which reflects mainly the existing capital in place and production capability. Since we are dealing with a quadratic problem where $w_i(\mathbf{y}_t)$, is quadratic, we assume that the value function is quadratic in the state, and has the following form:

$$V_{i,t}(\mathbf{y}_{t-1}) = \nu_{i,t} + \mathbf{r}'_{i,t}\mathbf{y}_{t-1} + \frac{1}{2}\mathbf{y}'_{t-1}\mathbf{S}_{i,t}\mathbf{y}_{t-1} \quad (4.5)$$

where $\nu_{i,t}$, $\mathbf{r}_{i,t}$ and $\mathbf{S}_{i,t}$ are unknown and have yet to be determined. If it can be shown that such a function exists, and that it satisfies the optimality conditions for both firms, then the problem is solved. *Dynamic programming* suggests a recursive procedure, so we can also write the value function for the next period, $t + 1$:

$$V_{i,t+1}(\mathbf{y}_t) = \nu_{i,t+1} + \mathbf{r}'_{i,t+1}\mathbf{y}_t + \frac{1}{2}\mathbf{y}'_t\mathbf{S}_{i,t+1}\mathbf{y}_t \quad (4.6)$$

Now using equations (4.1) and (4.6) in (4.4), we obtain:

$$V_{i,t}(\mathbf{y}_{t-1}) = \max_{x_{i,t}, u_{i,t}} [\beta\nu_{i,t+1} + (\mathbf{p}'_t + \beta\mathbf{r}'_{i,t+1})\mathbf{y}_t + \frac{1}{2}\mathbf{y}'_t(\mathbf{Q}_t + \beta\mathbf{S}_{i,t+1})\mathbf{y}_t] \quad (4.7)$$

then substituting the value of $\mathbf{y}_t$ from (4.2), we get the value function for period t in the desired form, as a function of $\mathbf{y}_{t-1}$, the state variable in the previous period:

$$V_{i,t}(\mathbf{y}_{t-1}) = \max_{x_{i,t}, u_{i,t}} [\beta\nu_{i,t+1} + (\mathbf{p}'_{i,t} + \beta\mathbf{r}'_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t) + \frac{1}{2}(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t)'(\mathbf{Q}_{i,t} + \beta\mathbf{S}_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t)] \quad (4.8)$$

and similarly for firm j we have:

$$V_{j,t}(\mathbf{y}_{t-1}) = \max_{x_{j,t}, u_{j,t}} [\beta \nu_{j,t+1} + (\mathbf{p}'_{j,t} + \beta \mathbf{r}'_{j,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t) + \frac{1}{2}(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t)'(\mathbf{Q}_{j,t} + \beta \mathbf{S}_{j,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t)] \quad (4.9)$$

The two equations (4.8) and (4.9) are now compatible with their counterparts for the value functions $V_{i,t}(\mathbf{y}_{t-1})$, and $V_{j,t}(\mathbf{y}_{t-1})$. They are of the same form as in (4.5), and are expressed in terms of the state in the previous period; this procedure, as we shall see in Section 4.2.3 below, facilitates solving the recursive procedure for the unknown coefficients, $\nu_{i,t}$, $\mathbf{r}_{i,t}$ and $\mathbf{S}_{i,t}$, that are essential for the solution.

4.2.2 Optimal Production and Investment Rates

Next we set about to find the optimal investment and production strategy for the two firms, for the entire period from t to T . For clarity, in the following few steps, the maximisation is carried out individually for each control variable for the two firms. To perform the maximisation *wrt* $u_{i,t}$, the investment rate for firm i , equation (4.8) is expanded to yield:

$$V_{i,t}(\mathbf{y}_{t-1}) = \max_{x_{i,t}, u_{i,t}} [\beta \nu_{i,t+1} + (\mathbf{p}'_{i,t} + \beta \mathbf{r}'_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{b}_1 u_{i,t} + \mathbf{B}^1 \mathbf{u}_{i,t}^1) + \frac{1}{2}(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{b}_1 u_{i,t} + \mathbf{B}^1 \mathbf{u}_{i,t}^1)'(\mathbf{Q}_{i,t} + \beta \mathbf{S}_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{b}_1 u_{i,t} + \mathbf{B}^1 \mathbf{u}_{i,t}^1)] \quad (4.10)$$

where the following notation has been introduced: $\mathbf{b}_1$ stands for the first column of matrix $\mathbf{B}$, and where $\mathbf{B}^1 = [\mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4]$, and $\mathbf{u}_{i,t}^1 = [x_{i,t}, u_{j,t}, x_{j,t}]$; taking the differential *wrt* $u_{i,t}$ we obtain for player i :

$$\mathbf{b}'_1(\mathbf{p}_{i,t} + \beta \mathbf{r}_{i,t+1}) + \mathbf{b}'_1(\mathbf{Q}_{i,t} + \beta \mathbf{S}_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{b}_1 u_{i,t} + \mathbf{B}^1 \mathbf{u}_{i,t}^1) = 0 \quad (4.11)$$

which is the same as:

$$\mathbf{b}'_1(\mathbf{p}_{i,t} + \beta \mathbf{r}_{i,t+1}) + \mathbf{b}'_1(\mathbf{Q}_{i,t} + \beta \mathbf{S}_{i,t+1})(\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_{i,t}) = 0 \quad (4.12)$$

and for the maximisation *wrt* the second control variable, $x_{i,t}$, the output for firm i we obtain:

$$b'_2(p_{i,t} + \beta r_{i,t+1}) + b'_2(Q_{i,t} + \beta S_{i,t+1})(Ay_{t-1} + Bu_{i,t}) = 0 \quad (4.13)$$

And similarly for the second firm, j , when the maximisation *wrt* the choice of investment rate, $u_{j,t}$, and output rate, $x_{j,t}$, is performed, we obtain the two respective equations:

$$b'_3(p_{j,t} + \beta r_{j,t+1}) + b'_3(Q_{j,t} + \beta S_{j,t+1})(Ay_{t-1} + Bu_{j,t}) = 0 \quad (4.14)$$

$$b'_4(p_{j,t} + \beta r_{j,t+1}) + b'_4(Q_{j,t} + \beta S_{j,t+1})(Ay_{t-1} + Bu_{j,t}) = 0 \quad (4.15)$$

combining the four equations, they may be represented in matrix form as:

$$\begin{bmatrix} b'_1(p_{i,t} + \beta r_{i,t+1}) \\ b'_2(p_{i,t} + \beta r_{i,t+1}) \\ b'_3(p_{j,t} + \beta r_{j,t+1}) \\ b'_4(p_{j,t} + \beta r_{j,t+1}) \end{bmatrix} + \begin{bmatrix} b'_1(Q_{i,t} + \beta S_{i,t+1}) \\ b'_2(Q_{i,t} + \beta S_{i,t+1}) \\ b'_3(Q_{j,t} + \beta S_{j,t+1}) \\ b'_4(Q_{j,t} + \beta S_{j,t+1}) \end{bmatrix} (Ay_{t-1} + Bu_t) = 0 \quad (4.16)$$

which may be written in more compact form by introducing the vector h_t , and matrix H_t :

$$h_t + H_t(Ay_{t-1} + Bu_t) = 0 \quad (4.17)$$

and solving for u_t , the optimal control vector of outputs and investment rates for the two firms in each period t , we get:

$$u_t = -(H_t B)^{-1}[h_t + H_t A y_{t-1}] \quad (4.18)$$

$$= -(H_t B)^{-1}h_t - (H_t B)^{-1}H_t A y_{t-1} \quad (4.19)$$

$$= d_t + E_t y_{t-1} \quad (4.20)$$

where the following notation has been used:

$$\mathbf{d}_t = -(\mathbf{H}_t \mathbf{B})^{-1} \mathbf{h}_t \quad (4.21)$$

$$\mathbf{E}_t = -(\mathbf{H}_t \mathbf{B})^{-1} \mathbf{H}_t \mathbf{A} \quad (4.22)$$

$$\mathbf{H}_t = \begin{bmatrix} b'_1(Q_{i,t} + \beta S_{i,t+1}) \\ b'_2(Q_{i,t} + \beta S_{i,t+1}) \\ b'_3(Q_{j,t} + \beta S_{j,t+1}) \\ b'_4(Q_{j,t} + \beta S_{j,t+1}) \end{bmatrix} \text{ and similarly } \mathbf{h}_t = \begin{bmatrix} b'_1(p_{i,t} + \beta r_{i,t+1}) \\ b'_2(p_{i,t} + \beta r_{i,t+1}) \\ b'_3(p_{j,t} + \beta r_{j,t+1}) \\ b'_4(p_{j,t} + \beta r_{j,t+1}) \end{bmatrix} \quad (4.23)$$

It is clear from the above how $\mathbf{H}_t$ and $\mathbf{h}_t$ can be constructed for the two firms at each time interval t for the entire planning horizon. By *Theorem 1* of Kydland (1975, p. 325), the necessary conditions for the existence of a maximising optimal $\mathbf{u}_t$ in equation (4.20), are:

- i. $\mathbf{H}_t \mathbf{B} < \mathbf{0} \quad \forall t \in [1, T]$
- ii. $|\mathbf{H}_t \mathbf{B}| \neq \mathbf{0} \quad \forall t \in [1, T]$

Due to the *linear quadratic* nature of the the problem, and since $\mathbf{Q}_{i,t}$, and $\mathbf{Q}_{j,t}$ are both semi-definite negative, (i) is met. The revenues $\pi_{i,t}$, and $\pi_{j,t}$, are concave in the output arguments as a result of a downward sloping inverse demand function, and the convexity of the cost functions, making the functions $w_{i,t}(\mathbf{y}_t)$, and $w_{j,t}(\mathbf{y}_t)$, quadratic and concave in both output and investment rates. All the elements along the leading diagonals of $\mathbf{Q}_{i,t}$, and $\mathbf{Q}_{j,t}$, are negative. This ensures that the optimal control vector of output and investment maximises the value function of the respective firms. For a formal proof along these lines, see Outrata, Kočvara & Zowe (1998, Chapter 12). They provide proofs for both the Nash-Cournot and the Stackelberg models.

And (ii) is necessary to ensure that matrix $\mathbf{H}_t \mathbf{B}$ is invertible. Furthermore, this $\mathbf{u}_t$ is the equilibrium solution to the problem, which is also unique for the class of strategies that are linear in the state.

As can be seen from equation (4.20), the optimal controls of output and investment for the two firms in $\mathbf{u}_t$, are dependent on the state variable, $\mathbf{y}_{t-1}$, in the previous period. In a duopoly this gives rise to each firm trying to influence the actions of its rival firm; because the state, by definition, includes the capacities of the two firms and their controls, and in a feedback Nash-Cournot set-up, the firms make allowance for the decisions of their rivals in every period when deciding on their actions; if a firm increases its investment a little bit more above the equilibrium level in this period, it can force its rival to reduce its investment and output in the next period (Fudenberg & Tirole 1992); both firms engage in this investment rivalry. This preemptive behaviour on the part of the players is reinforced by the feedback structure of the game, and the interaction between the firms. Investment, output and the capacity, as expected, will be found to be higher under feedback rules than under the open-loop.

While the planning problem appears to be solved, since the optimal control vector for the industry, $\mathbf{u}_t$ in (4.20) has been found as a function of the state in the previous period, $\mathbf{y}_{t-1}$ which is known; $\mathbf{d}_t$ and $\mathbf{E}_t$ are functions of $\mathbf{S}_{i,t+1}$, $\mathbf{r}_{i,t+1}$, $\mathbf{S}_{j,t+1}$, and $\mathbf{r}_{j,t+1}$ (as can be seen from (4.23)), that are valued in the next period, and have yet to be determined. Typically in a *dynamic programming* formulation the problem is solved backward in time, and the values for the coefficients of the value functions $V_{i,t+1}(\mathbf{y}_t)$ and $V_{j,t+1}(\mathbf{y}_t)$ are obtained starting from the boundary conditions in the final period. This is shown below.

4.2.3 The Recursion Procedure and Optimal Capacity Expansion

We can now proceed to show how the recursion equations for $\nu_{i,t}$, $\mathbf{r}_{i,t}$, and $\mathbf{S}_{i,t}$ are obtained so that the value function V_t for each firm is evaluated. In the following few steps, to avoid clutter, the subscript i will be dropped. Now if we substitute (4.20) into the right hand side of (4.8), and (4.5) for the left hand side, we obtain an equation of the value function when the firm implements an optimal policy of output and investment, depicted by $\mathbf{u}_t$ of equation (4.20), and whose two sides are in terms of the state variable

in the previous period:

$$\begin{aligned}
\nu_t + \mathbf{r}'_t \mathbf{y}_{t-1} + \frac{1}{2} \mathbf{y}'_{t-1} \mathbf{S}_t \mathbf{y}_{t-1} = & \\
& \beta \nu_{t+1} + (\mathbf{p}'_t + \beta \mathbf{r}'_{t+1})(\mathbf{A} \mathbf{y}_{t-1} + \mathbf{B}(\mathbf{d}_t + \mathbf{E}_t \mathbf{y}_{t-1})) \\
& + \frac{1}{2} (\mathbf{A} \mathbf{y}_{t-1} + \mathbf{B}(\mathbf{d}_t + \mathbf{E}_t \mathbf{y}_{t-1}))' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) \\
& (\mathbf{A} \mathbf{y}_{t-1} + \mathbf{B}(\mathbf{d}_t + \mathbf{E}_t \mathbf{y}_{t-1})) \quad (4.24)
\end{aligned}$$

and upon rearranging the right hand side of this equation we obtain:

$$\begin{aligned}
\nu_t + \mathbf{r}'_t \mathbf{y}_{t-1} + \frac{1}{2} \mathbf{y}'_{t-1} \mathbf{S}_t \mathbf{y}_{t-1} = & \beta \nu_{t+1} + (\mathbf{p}'_t + \beta \mathbf{r}'_{t+1})(\mathbf{B} \mathbf{d}_t + (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1}) \\
& + \frac{1}{2} (\mathbf{B} \mathbf{d}_t + (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1})' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{B} \mathbf{d}_t + (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1}) \quad (4.25)
\end{aligned}$$

Now the two sides of the above equation describe the value function; the equality implies that the coefficients of the independent, first order, and second order terms in $\mathbf{y}_{t-1}$ on both sides of the above equation are equal, and we can write the following equalities, that are recursive difference equations, namely: for the independent terms

$$\nu_t = \beta \nu_{t+1} + (\mathbf{p}'_t + \beta \mathbf{r}'_{t+1}) \mathbf{B} \mathbf{d}_t + \frac{1}{2} (\mathbf{B} \mathbf{d}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) \mathbf{B} \mathbf{d}_t \quad (4.26)$$

$$= \beta \nu_{t+1} + (\mathbf{p}_t + \beta \mathbf{r}_{t+1} + \frac{1}{2} (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) \mathbf{B} \mathbf{d}_t)' \mathbf{B} \mathbf{d}_t \quad (4.27)$$

and for the first order terms in $\mathbf{y}_{t-1}$, we get

$$\begin{aligned}
\mathbf{r}'_t \mathbf{y}_{t-1} = & (\mathbf{p}_t + \beta \mathbf{r}_{t+1})' (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1} + \frac{1}{2} (\mathbf{B} \mathbf{d}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1} \\
& + \frac{1}{2} ((\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1})' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) \mathbf{B} \mathbf{d}_t \\
= & (\mathbf{p}_t + \beta \mathbf{r}_{t+1})' (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1} + (\mathbf{B} \mathbf{d}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1}
\end{aligned}$$

which upon factorisation may be written as:

$$\mathbf{r}'_t \mathbf{y}_{t-1} = [(\mathbf{p}_t + \beta \mathbf{r}_{t+1})' + (\mathbf{B} \mathbf{d}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1})] (\mathbf{A} + \mathbf{B} \mathbf{E}_t) \mathbf{y}_{t-1}$$

taking the transpose of both sides of the above equation, we get:

$$\begin{aligned} \mathbf{y}'_{t-1} \mathbf{r}_t &= \mathbf{y}'_{t-1} (\mathbf{A} + \mathbf{B}\mathbf{E}_t)' [(\mathbf{p}_t + \beta \mathbf{r}_{t+1})' + (\mathbf{B}\mathbf{d}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1})]' \\ &= \mathbf{y}'_{t-1} (\mathbf{A} + \mathbf{B}\mathbf{E}_t)' [(\mathbf{p}_t + \beta \mathbf{r}_{t+1}) + (\mathbf{Q}_t + \beta \mathbf{S}_{t+1})' \mathbf{B}\mathbf{d}_t] \end{aligned}$$

therefore we can write the equation for $\mathbf{r}_t$:

$$\mathbf{r}_t = (\mathbf{A} + \mathbf{B}\mathbf{E}_t)' [\mathbf{p}_t + \beta \mathbf{r}_{t+1} + (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) \mathbf{B}\mathbf{d}_t] \quad (4.28)$$

where $(\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) = (\mathbf{Q}_t + \beta \mathbf{S}_{t+1})'$ since $\mathbf{Q}_t$ and $\mathbf{S}_{t+1}$ are both symmetric.

Also, for the second order terms in $\mathbf{y}_{t-1}$, we have:

$$\begin{aligned} \frac{1}{2} \mathbf{y}'_{t-1} \mathbf{S}_t \mathbf{y}_{t-1} &= \frac{1}{2} ((\mathbf{A} + \mathbf{B}\mathbf{E}_t) \mathbf{y}_{t-1})' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{A} + \mathbf{B}\mathbf{E}_t) \mathbf{y}_{t-1} \\ &= \frac{1}{2} \mathbf{y}'_{t-1} (\mathbf{A} + \mathbf{B}\mathbf{E}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{A} + \mathbf{B}\mathbf{E}_t) \mathbf{y}_{t-1}, \end{aligned}$$

and we may conclude that the $\mathbf{S}_t$ recursion is as follows:

$$\mathbf{S}_t = (\mathbf{A} + \mathbf{B}\mathbf{E}_t)' (\mathbf{Q}_t + \beta \mathbf{S}_{t+1}) (\mathbf{A} + \mathbf{B}\mathbf{E}_t). \quad (4.29)$$

Having obtained the recursion equations for $\mathbf{S}_t$, $\mathbf{r}_t$, and ν_t from (4.29), (4.28), and (4.27) respectively, the entire sequence can be solved backward in time from the final period T to any initial period t . As such the value function $V_t(\mathbf{y}_{t-1})$ can be evaluated from (4.5); and it has been established that it is quadratic in the state variable, $\mathbf{y}_{t-1}$. Also the optimal strategy for investment, u_i , and production, x_i , for each firm i has been obtained in (4.20). It is clear that this strategy is linear in the state; $\mathbf{d}_t$ and $\mathbf{E}_t$ may be evaluated for each period from the variables given by the recursion equations thus obtained.

The boundary conditions for the problem remain to be defined, and will now be shown below. Since $V_{i,T+1} = 0$, we can immediately deduce that:

$$\mathbf{S}_{i,T+1} = \mathbf{0}, \quad \mathbf{r}_{i,T+1} = \mathbf{0}, \quad \nu_{i,T+1} = 0$$

and consequently we have,

$$\mathbf{S}_{i,T} = (\mathbf{A} + \mathbf{B}\mathbf{E}_T)' \mathbf{Q}_{i,T} (\mathbf{A} + \mathbf{B}\mathbf{E}_T) \quad (4.30)$$

$$\mathbf{r}_{i,T} = (\mathbf{A} + \mathbf{B}\mathbf{E}_T)' [\mathbf{p}_{i,T} + (\mathbf{Q}_{i,T}) \mathbf{B} \mathbf{d}_T] \quad (4.31)$$

$$\nu_{i,T} = (\mathbf{p}_{i,T} + \frac{1}{2} \mathbf{Q}_{i,T} \mathbf{B} \mathbf{d}_T)' \mathbf{B} \mathbf{d}_T. \quad (4.32)$$

It is clear that $\mathbf{d}_T$ and $\mathbf{E}_T$, are now evaluated for the final period, T . Also $\mathbf{H}_T$ and $\mathbf{h}_T$ are defined as in (4.23), from the corresponding matrices and vectors, $\mathbf{Q}_{i,T}$, $\mathbf{p}_{i,T}$, and $\mathbf{Q}_{j,T}$, and $\mathbf{p}_{j,T}$, which include the variables for prices and costs, for both firms in the final period.

$$\begin{aligned} \mathbf{d}_T &= -(\mathbf{H}_T \mathbf{B})^{-1} \mathbf{h}_T, & \mathbf{E}_T &= -(\mathbf{H}_T \mathbf{B})^{-1} \mathbf{H}_T \mathbf{A} \\ \mathbf{H}_T &= \mathbf{B}' \mathbf{Q}_T, & \mathbf{h}_T &= \mathbf{B}' \mathbf{p}_T \end{aligned}$$

And for the previous period we can immediately write:

$$\begin{aligned} \mathbf{d}_{T-1} &= -(\mathbf{H}_{T-1} \mathbf{B})^{-1} \mathbf{h}_{T-1} & \mathbf{E}_{T-1} &= -(\mathbf{H}_{T-1} \mathbf{B})^{-1} \mathbf{H}_{T-1} \mathbf{A} \\ \mathbf{H}_{T-1} &= \mathbf{B}' (\mathbf{Q}_{T-1} + \beta \mathbf{S}_T) & \mathbf{h}_{T-1} &= \mathbf{B}' (\mathbf{p}_{T-1} + \beta \mathbf{r}_T) \end{aligned}$$

and the recursion equations can be developed as:

$$\mathbf{S}_{i,T-1} = (\mathbf{A} + \mathbf{B}\mathbf{E}_{T-1})' (\mathbf{Q}_{i,T-1} + \beta \mathbf{S}_{i,T}) (\mathbf{A} + \mathbf{B}\mathbf{E}_{T-1}) \quad (4.33)$$

$$\mathbf{r}_{i,T-1} = (\mathbf{A} + \mathbf{B}\mathbf{E}_{T-1})' [\mathbf{p}_{i,T-1} + \beta \mathbf{r}_{i,T} + (\mathbf{Q}_{i,T-1} + \beta \mathbf{S}_{i,T}) \mathbf{B} \mathbf{d}_{T-1}] \quad (4.34)$$

$$\nu_{i,T-1} = [\mathbf{p}_{i,T-1} + \beta \mathbf{r}_{i,T} + \frac{1}{2} (\mathbf{Q}_{i,T-1} + \beta \mathbf{S}_{i,T}) \mathbf{B} \mathbf{d}_{T-1}]' \mathbf{B} \mathbf{d}_{T-1} \quad (4.35)$$

This process is developed for all periods t , until the initial time, utilising equations (4.33), (4.34), and (4.35). In this problem we are assuming that the firms have full foresight of how demand develops, and all the costs incurred are known for the entire planning period. Therefore all the $\mathbf{p}_{i,t}$ vectors, and $\mathbf{Q}_{i,t}$ matrices, are known. We obtain the values $\mathbf{S}_{i,t}$, $\mathbf{r}_{i,t}$, and $\nu_{i,t}$ for all periods. This is done for both firms; and the equations are solved simultaneously, as a *two point boundary value problem*. We are now ready to obtain the

solution. The evolution of the state of the game is obtained by applying the optimal control, $\mathbf{u}_t$, of (4.20) to the state difference equation (4.2); with the initial state, $\mathbf{y}_0$, of the game is known, the first period optimal control vector comprising the investment and output rates is determined from (4.20), which in turn is employed in (4.2) to determine the state of the second period. Since we have already determined the values of $\mathbf{h}_t$, $\mathbf{H}_t$, $\mathbf{d}_t$, and $\mathbf{E}_t$, for all periods, this procedure alternates forward in time until the last period T .

4.2.4 The Role of Strategic Investment in Feedback Nash-Cournot Games

The preemptive effect of each firms investment strategy on its competitor, when the firms are engaged in a feedback Nash-Cournot rivalry, can be seen by considering the marginal effect of each firms capacity on the strategy of the other. When feedback strategies are employed, each firm's control is decreasing in its rival's capacity in the previous period. By differentiating equation (4.20) wrt to $\mathbf{y}_{t-1}$ we get $\frac{\partial \mathbf{u}_t}{\partial \mathbf{y}_{t-1}} = \mathbf{E}_t$, and the marginal effect of a change in the state on the optimal control, is embodied in matrix $\mathbf{E}_t$. Upon closer examination of the partial differentiation given below, of the two vectors, where:

$$\frac{\partial \mathbf{u}_t}{\partial \mathbf{y}_{t-1}} = \begin{pmatrix} \frac{\partial u_1}{\partial u_1} & \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial k_1} & \frac{\partial u_1}{\partial u_2} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial k_2} \\ \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial k_1} & \frac{\partial x_1}{\partial u_2} & \frac{\partial x_1}{\partial x_2} & \frac{\partial x_1}{\partial k_2} \\ \frac{\partial u_2}{\partial u_1} & \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial k_1} & \frac{\partial u_2}{\partial u_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial k_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial k_1} & \frac{\partial x_2}{\partial u_2} & \frac{\partial x_2}{\partial x_2} & \frac{\partial x_2}{\partial k_2} \end{pmatrix} \quad (4.36)$$

and from the duopoly simulation we have:

$$\mathbf{E}_{t=4} = \begin{pmatrix} 0 & 0 & -0.0373 & 0 & 0 & -0.0176 \\ 0 & 0 & +0.3257 & 0 & 0 & -0.1144 \\ 0 & 0 & -0.0176 & 0 & 0 & -0.0373 \\ 0 & 0 & -0.1144 & 0 & 0 & +0.3257 \end{pmatrix}, \quad (4.37)$$

it can be noted, that the third column of $\mathbf{E}_t$ gives the marginal effect of a change in the first firm's capacity, k_1 , on the optimal control vector,; and the last column is the marginal

effect of the second firm's capacity, k_2 . Each firm's capacity has a larger influence on its own controls than on its rival's, and because the costs and capacities are identical for both firms, the influences are symmetrical. Investment rates are decreasing with own capacity for both firms ($\partial u_1/\partial k_1 = \partial u_2/\partial k_2 = -0.0373$); this is because the firms invest heavily initially, and then their investment rates decline as they approach their steady state feedback Nash-Cournot equilibrium⁴. The influence of firm 1's capacity on its rival's investment rate is smaller, $\partial u_2/\partial k_1 = -0.0176$. As expected the output is increasing with each firm's capacity ($\partial x_1/\partial k_1 = 0.3257$), but is decreasing with its rival's ($\partial x_2/\partial k_1 = -0.1144$). By increasing its capacity through overinvesting in this period, a firm can reduce its rival's investment and consequently its rival's output in the next period, giving it a bigger share of the market, and higher profits. The second firm behaves in exactly the same way, and both firms end up with higher capacity and output than if they were not behaving strategically. It is paradoxical that a strategy designed to preempt rivals and increase output and profits at their expense, results in a more competitive environment, with lower prices and profits. But this is also a rational behaviour. By overinvesting the firms ensure that they have some reserve capacity in case there is an unexpected surge in demand; a capacity that they can use to punish their rivals if they misbehave, by producing more and lowering prices if the need arises. Furthermore, the presence of overcapacity in the industry, gives the impression of a loose oligopoly which is more competitive; and the impression of a tough industry, which in turn may act as a potential barrier to entrants.

⁴See Kort (1988) for the case of a similar analysis of a single firm, when there are no constraints on investment rate.

4.3 Open-loop Model Solution

4.3.1 Model Formulation in Open-loop Information Structure

In the open-loop formulation each firm chooses its investment and output rates that maximise its flow of profits for the entire game. The firms commit from the outset, to the optimal path of control variables that each will use for the entire game, considering that its opponents will be following the same strategy of committing to an optimal path. Each firm i , selects its vector of controls $\mathbf{u}_{i,t} = (x_{i,t}, u_{i,t})'$, $\forall t \in [1, T]$

$$J_i = \max_{\mathbf{u}_{i,1}, \dots, \mathbf{u}_{i,T}} \sum_{t=1}^T \beta^{t-1} w_i(\mathbf{y}_t), \quad \forall i \in \mathbf{N} \quad (4.38)$$

and remembering that $w_i(\mathbf{y}_t)$ is quadratic in the state variable $\mathbf{y}_t$ and has the form as in (4.1), the above equation may be written out in expanded form for all periods:

$$J_i = \max_{\mathbf{u}_{i,1}, \dots, \mathbf{u}_{i,T}} \begin{pmatrix} p_{i,1} \\ \beta p_{i,2} \\ \beta^2 p_{i,3} \\ \vdots \\ \beta^{T-1} p_{i,T} \end{pmatrix}' \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \vdots \\ \mathbf{y}_T \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \vdots \\ \mathbf{y}_T \end{pmatrix}' \begin{bmatrix} Q_{i,1} & 0 & 0 & \dots & 0 \\ 0 & \beta Q_{i,2} & 0 & \dots & 0 \\ 0 & 0 & \beta^2 Q_{i,3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \beta^{T-1} Q_{i,T} \end{bmatrix} \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \vdots \\ \mathbf{y}_T \end{pmatrix} \quad (4.39)$$

Subject to a difference equation which describes the evolution of the state of the game; the state equation (or equation of motion) of the game, which written in terms of the initial state at the start of the game, is represented by:

$$\mathbf{y}_t = A^t \mathbf{y}_0 + \sum_{i=1}^t A^{t-1} B \mathbf{u}_i, \quad t = 1, \dots, T \quad (4.40)$$

This equation of motion also may be expanded for all periods, as shown below, where $\mathbf{u}'_1 = (u_1, x_1, u_2, x_2, \dots, u_n, x_n)_{t=1}$, is the vector of controls of all players in period 1:

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \vdots \\ \mathbf{y}_T \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \mathbf{A}^3 \\ \vdots \\ \mathbf{A}^T \end{pmatrix} \mathbf{y}_0 + \begin{bmatrix} \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}^2\mathbf{B} & \mathbf{AB} & \mathbf{B} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{A}^{T-1}\mathbf{B} & \mathbf{A}^{T-2}\mathbf{B} & \dots & \mathbf{A}^2\mathbf{B} & \mathbf{AB} & \mathbf{B} \end{bmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \\ \vdots \\ \mathbf{u}_T \end{pmatrix} \quad (4.41)$$

performing the maximisation by each firm *wrt* to its controls of investment and output rate over all periods, we obtain the following matrix equation:

$$\begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \\ \vdots \\ \mathbf{h}_T \end{pmatrix} + \begin{pmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \mathbf{H}_3 \\ \vdots \\ \mathbf{H}_T \end{pmatrix} \left[\begin{pmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \mathbf{A}^3 \\ \vdots \\ \mathbf{A}^T \end{pmatrix} \mathbf{y}_0 + \begin{bmatrix} \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{A}^2\mathbf{B} & \mathbf{AB} & \mathbf{B} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{A}^{T-1}\mathbf{B} & \mathbf{A}^{T-2}\mathbf{B} & \dots & \mathbf{A}^2\mathbf{B} & \mathbf{AB} & \mathbf{B} \end{bmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \\ \vdots \\ \mathbf{u}_T \end{pmatrix} \right] = \mathbf{0} \quad (4.42)$$

in the above equation each $\mathbf{h}_t$ and $\mathbf{H}_t$ are a vector and a matrix containing elements from each player in period t as illustrated below:

$$\mathbf{h}_t = \begin{pmatrix} b'_1 \beta^{t-1} p_{i,t} \\ b'_2 \beta^{t-1} p_{i,t} \\ b'_3 \beta^{t-1} p_{j,t} \\ b'_4 \beta^{t-1} p_{j,t} \\ \vdots \\ b'_{n-1} \beta^{t-1} p_{n,t} \\ b'_n \beta^{t-1} p_{n,t} \end{pmatrix}, \quad \mathbf{H}_t = \begin{pmatrix} b'_1 \beta^{t-1} Q_{i,t} \\ b'_2 \beta^{t-1} Q_{i,t} \\ b'_3 \beta^{t-1} Q_{j,t} \\ b'_4 \beta^{t-1} Q_{j,t} \\ \vdots \\ b'_{n-1} \beta^{t-1} Q_{n,t} \\ b'_n \beta^{t-1} Q_{n,t} \end{pmatrix} \quad (4.43)$$

writing equation (4.42) in a more compact form we get:

$$\mathfrak{h} + \mathcal{H}\mathcal{A} \mathbf{y}_0 + \mathcal{H}\Phi \mathbf{u} = \mathbf{0} \quad (4.44)$$

by rearranging this equation, we obtain the optimal control vector of investment and output rates, $\mathcal{U}$, for all firms for the entire duration of the game, $\forall t \in [1, T]$.

$$\mathcal{U} = -(\mathcal{H}\Phi)^{-1}\mathfrak{h} - (\mathcal{H}\Phi)^{-1}\mathcal{H}\mathcal{A}y_0 \quad (4.45)$$

This indicates that the optimal strategy of the open-loop game is linear in the initial state, and (4.45) is analogous to (4.20) for the feed back strategy model. In fact we can write (4.45) in the same form as shown below although the values of d_t and E_t in the two equations are different due to the different information structure of the two games:

$$\mathcal{U} = d_t + E_t y_0 \quad (4.46)$$

where:

$$d_t = (\mathcal{H}\Phi)^{-1}\mathfrak{h} \quad (4.47)$$

$$E_t = (\mathcal{H}\Phi)^{-1}\mathcal{H}\mathcal{A} \quad (4.48)$$

The evolution of optimal path of the state of the game can now be obtained by utilising the optimal investment and output rate of (4.46), which we could have written as $\mathcal{U}^*$, in (4.41):

$$\mathcal{Y} = \mathcal{A}y_0 + \Phi\mathcal{U}^* \quad (4.49)$$

It is clear from (4.46), and the above equation that the evolution of the optimal control, and the optimal state are dependent on the initial state and the time. Each firm solves its optimal control problem, by selecting from the outset the vector of its optimal controls for all periods as specified in equation (4.38).

4.3.2 Duopoly Under Competitive Market Conditions

In a competitive market the firms operate as price takers. These markets are usually served by a large number of small firms, none of which is big enough to influence demand or market price. Under such market conditions, each firm produces as much product as

it can sell, and takes it that the price is dictated by the market. The firm does not care what other firms are doing; it plans its production and investment under the assumption that its decisions do not affect other firms. To illustrate how the firms interact under these conditions, we consider below the market for a duopoly first. The analysis is then extended to n firms in Section 4.5.3, p. 172, with the results shown in Table 4.4.

Retaining the quadratic structure of the game, the objective function for each firm is described by the same equations as before. We make a slight modification to a perfectly competitive market, served by two firms only, as just described above. In that price is not constant for all periods, but rather it is linear, downward sloping, and is dependent on total output. Price is described by the same equation as before (equation (3.7) of Chapter 3):

$$P(x_i, x_j, t) = a - b(x_i(t) + x_j(t)), \quad \forall i = 1, 2 \ i \neq j, \quad (4.50)$$

Using the same notation as in previous sections, the single period profit is described by:

$$w_i(P, x_i, u_i, k_i, t) = \pi_i(P, x_i, k_i, t) - I_i(u_i, k_i, t) \quad \forall i \in \mathbf{N} = \{1, 2\}, \forall t \in [1, T]$$

The operating profit $\pi_i(t)$ is the revenue (which is simply price times firm i 's output) less the production costs:

$$\pi_i(P, x_i, k_i, t) = P(t)x_{i,t} - C_{i,t}(x_{i,t}, k_{i,t})$$

and the production cost function, $C_{i,t}$ is quadratic in output, $x_{i,t}$, and capital, $k_{i,t}$. When each firm is trying to determine its optimal output, it maximises its profit function wrt to its output, assuming that the price in that period is constant; and since $P(x_i, x_j, t)$ is constant, we have the following:

$$\frac{\partial w_i(t)}{\partial x_{i,t}} = P(t) - \frac{\partial C_{i,t}}{\partial x_{i,t}} - \frac{\partial I_{i,t}}{\partial x_{i,t}} = 0, \quad (4.51)$$

in equilibrium $\frac{\partial I_{i,t}}{\partial x_{i,t}} = 0$, and we have the familiar result for each firm where price equals marginal cost. And similarly for firm 2 we have: $P(t) = \frac{\partial C_{j,t}}{\partial x_{j,t}}$.

Because the two firms are price takers, each firm operates under the assumption that

it does not have any bearing on the decisions of the other firm. The problem is therefore structured in exactly the same way as an open-loop game. The only difference is that the firms now take the price as constant and do not therefore interact as before. Each firm's decision is made independent of the other; and the optimality condition for output now follows equation (4.51). The equations and problem solution follows exactly the same steps as in Section 4.3.1.

4.4 Stackelberg Game for n Firms with Two Decision Variables

4.4.1 Introduction

Another noncooperative Nash equilibrium concept, describing the interaction between rival players, or competing firms in an oligopolistic market, is the Stackelberg equilibrium that was first introduced by von Stackelberg, which later on gained wide acceptance in oligopoly theory. The main difference between a Stackelberg and a Nash game, is that in a Stackelberg game, the market is characterised by the presence of a leader, who has the advantage of making the first move by announcing his or her strategy first. And the leader has the advantage of taking into consideration the response of the rivals when making her decision. The leader gets to decide on her action first, and then the followers response follows. A Stackelberg equilibrium is arrived at when each player has no incentive to change his or her decision.

In the context of this research, among the set of players, the leader knows that when each follower is evaluating what investment, and production, policy to follow that will maximise his objective function (which is the the value function of profits), he will take as given the leader's declared investment and production policy over the duration of the game. This derives from the fact that the leader gets to act first. Knowing this, the leader would therefore select the policy, (from her set of permissible strategies), to which she knows that her opponents would react optimally, that maximises her value function of profits.

This problem is tackled in the following section. In every period, the leader announces

his production and investment strategy; the followers, having observed this, will each try to find his optimal response. So the followers' optimal control problem of profit maximisation is solved first. Then taking the followers' optimal response into account, the leader evaluates and selects her optimal policy that maximises her value function over the duration of the game.

A question which arises in Stackelberg games is that of the credibility, and commitment of the leader's declared decision. For this reason a feedback strategy is more sensible for all players. It is possible for instance for the leader to announce a strategy of investments and outputs; the followers then take this into account and evaluate their best response to the leaders declared optimal strategy. But once this has been done, and the game started, the leader may find it to her advantage to change her policy from what has been announced, and the followers actions are now, not a best response to the leader's present production and investment strategy and therefore not optimal. This makes a feedback strategy more credible, and feasible as an equilibrium concept. In this instance, both the leader and her rivals can follow strategies based on decision rules that are contingent on the current state and time; and can therefore modify their decisions as the game evolves, as opposed to an open-loop trajectory of outputs and investments. For this reason, only the feedback information structure is considered in the following section.

4.4.2 Model Formulation for Stackelberg Game of n Firms

The formulation of the problem follows the same structure as in Section 4.2.1 for the feedback Nash-Cournot duopoly, but now we have a market served by a set $\mathbf{N}$ of players, and the information structure, and sequence of play is different, reflecting the roles taken by the respective firms; once again, the i th player controls two decision variables u_i , and x_i , the investment rate and the output respectively, and has a capacity of k_i , where $i \in \mathbf{N} = \{1, \dots, n\}$ since the market is now served by n firms. Let's restate some essential notation; for the industry we define a control vector $\mathbf{u}_t$, where $\mathbf{u}_t = (u_1, x_1, u_2, x_2, \dots, u_n, x_n)'_t$, and where the prime denotes a transpose; the corresponding state vector is $\mathbf{y}_t = (u_1, x_1, k_1, u_2, x_2, k_2, \dots, u_n, x_n, k_n)'_t$. In this section it is

assumed that the 1st firm is the leading firm which acts as a Stackelberg leader, and the $n - 1$ remaining firms act as noncooperative Cournot followers.

The problem in terms of the value function for each firm i is the same as equation (4.8) of the feedback model of Section 4.2.1:

$$V_{i,t}(y_{t-1}) = \max_{u_{i,t}, x_{i,t}} \{ \beta \nu_{i,t+1} + (\dot{p}'_{i,t} + \beta r'_{i,t+1})(Ay_{t-1} + Bu_t) + \frac{1}{2}(Ay_{t-1} + Bu_t)'(Q_{i,t} + \beta S_{i,t+1})(Ay_{t-1} + Bu_t) \} \quad (4.52)$$

Since in the present formulation of the problem each firm has two control variables, it is obvious that $u_t \in \mathbb{R}^{2n}$, and $y_t \in \mathbb{R}^{3n}$; and where A , and B are two matrices of dimensions $3n \times 3n$ and $3n \times 2n$ respectively. Also if we define b_j as the j th column of matrix B , then it may be represented by $B = [b_1, b_2, b_3, b_4, \dots, b_{2n}]$. Following Kydland (1977), I also introduce the following additional notation to simplify the exposition of the equations developed in this section. Let $\rho_{i,t} = p_{i,t} + \beta r_{i,t+1}$ and $\Theta_{i,t} = Q_{i,t} + \beta S_{i,t+1}$. Also let $B^l = [b_1, b_2]$, and $B^f = [b_3, b_4, \dots, b_{2n}]$. And similarly let the leader's control variable be represented by $u^l = (u_1, x_1)'$, and for the follower firms, by $u^f = (u_2, x_2, u_3, x_3, \dots, u_n, x_n)'$. Then (4.52) may be represented in the following form:

$$V_{i,t}(y_{t-1}) = \max_{u_{i,t}, x_{i,t}} \{ \beta \nu_{i,t+1} + \rho'_{i,t}(Ay_{t-1} + Bu_t) + \frac{1}{2}(Ay_{t-1} + Bu_t)'\Theta_{i,t}(Ay_{t-1} + Bu_t) \} \quad (4.53)$$

The information structure of a Stackelberg game dictates that in each period, the leader makes her decision first and then the followers having observed the decision of the leader, make their decisions, while taking the decision of the leader into consideration. The followers act non-cooperatively, as if they are in a Nash-Cournot game among themselves. They make their decisions independently of each other and act simultaneously⁵.

⁵Other variations could be envisaged for the mode of play of the followers, one would be that there is a hierarchical mode of play between the followers in which case some of the players could be made to act before others, in a sequence of a Stackelberg hierarchy similar to a model studied by Matsumura (1999). Binmore (1992, p. 293), in the tradition of game theorists, considers that the distinction between Stackelberg and Nash-Cournot play is in the sequence and timing of play.

4.4.3 Follower and Leader Optimal Controls

When the $n-1$ follower firms consider the maximisation problem of their value functions, they take the decision of firm 1, the leader, as given. The necessary condition for the selection of the optimal investment rate, $u_{j,t}$ for each follower firm $j \in \mathbf{N} \setminus \{1\}$ ⁶ that maximises its value function, is obtained by differentiating equation (4.53) wrt $u_{j,t}$ to obtain:

$$b'_{2j-1}\rho_{j,t} + b'_{2j-1}\Theta_{j,t}(Ay_{t-1} + Bu_t) = 0 \quad (4.54)$$

and for the selection of optimal output, $x_{j,t}$ we get:

$$b'_{2j}\rho_{j,t} + b'_{2j}\Theta_{j,t}(Ay_{t-1} + Bu_t) = 0 \quad (4.55)$$

where for both equations $j \in \mathbf{N} \setminus \{1\} = \{2, \dots, n\}$; the $2n-2$ equations for the $n-1$ follower firms may be written in matrix form as follows:

$$h_t + H_t(Ay_{t-1} + Bu_t) = 0 \quad (4.56)$$

where the following notation has been introduced:

$$h_t = \begin{bmatrix} b'_3\rho_{2,t} \\ b'_4\rho_{2,t} \\ \vdots \\ b'_{2n-1}\rho_{n,t} \\ b'_{2n}\rho_{n,t} \end{bmatrix}, \quad \text{and} \quad H_t = \begin{bmatrix} b'_3\Theta_{2,t} \\ b'_4\Theta_{2,t} \\ \vdots \\ b'_{2n-1}\Theta_{n,t} \\ b'_{2n}\Theta_{n,t} \end{bmatrix} \quad (4.57)$$

recall that $B = [B^l, B^f]$, and $u_t = (u_t^l, u_t^f)'$, so the above matrix equation (4.56) for the follower firms, may be expanded to get:

$$h_t + H_t(Ay_{t-1} + B^l u_t^l + B^f u_t^f) = 0 \quad (4.58)$$

⁶This means the set $\mathbf{N}$ excluding firm 1, and is just a notation that refers to the set of rivals of firm 1, which is the set of followers.

which may be rearranged in the following steps to obtain the optimal control variables vector $\mathbf{u}_t^f$, for the follower firms. It can be noted that each is a function of the leader firm's control vector of output and investment rate, $\mathbf{u}_t^l$:

$$\mathbf{H}_t \mathbf{B}^f \mathbf{u}_t^f = -\mathbf{H}_t \mathbf{A} \mathbf{y}_{t-1} - \mathbf{H}_t \mathbf{B}^l \mathbf{u}_t^l - \mathbf{h}_t \quad (4.59)$$

$$\mathbf{u}_t^f = -(\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{H}_t \mathbf{A} \mathbf{y}_{t-1} - (\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{H}_t \mathbf{B}^l \mathbf{u}_t^l - (\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{h}_t \quad (4.60)$$

$$= \mathbf{G}_t \mathbf{y}_{t-1} + \boldsymbol{\eta}_t \mathbf{u}_t^l + \boldsymbol{\gamma}_t \quad (4.61)$$

where the following notation has been used:

$$\mathbf{G}_t = -(\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{H}_t \mathbf{A}$$

$$\boldsymbol{\eta}_t = -(\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{H}_t \mathbf{B}^l$$

$$\boldsymbol{\gamma}_t = -(\mathbf{H}_t \mathbf{B}^f)^{-1} \mathbf{h}_t$$

for the dominant 1st firm, the value function is defined by an equation in the same form, and structure as the other firms, as follows:

$$V_{1,t}(\mathbf{y}_{t-1}) = \max_{\mathbf{u}_{1,t}, \mathbf{x}_{1,t}} (w_1(\mathbf{y}_t) + \beta V_{1,t+1}(\mathbf{y}_t)) \quad (4.62)$$

subject to the state equation (4.2), and equation (4.61), since the leading firm knows how the follower firms would react to its own actions, and would therefore take their response into account when choosing its optimal investment and output strategy. Once again we assume for the leading firm, a function which is quadratic in the state; its value function may be written as follows (*c.f.* equation (4.7)):

$$V_{1,t}(\mathbf{y}_{t-1}) = \max_{\mathbf{u}_{1,t}, \mathbf{x}_{1,t}} \{ \beta \nu_{1,t+1} + \boldsymbol{\rho}'_{1,t} \mathbf{y}_t + \frac{1}{2} \mathbf{y}'_t \boldsymbol{\Theta}_{1,t} \mathbf{y}_t \} \quad (4.63)$$

where $\rho_{1,t} = p_{1,t} + \beta r_{1,t+1}$ and $\Theta_{1,t} = Q_{1,t} + \beta S_{1,t+1}$. Now substituting the value for y_t using the state equation, (4.2) we get:

$$V_{1,t}(y_{t-1}) = \max_{u_t^l} \{ \beta \nu_{1,t+1} + \rho'_{1,t} (A y_{t-1} + B u_t) + \frac{1}{2} (A y_{t-1} + B u_t)' \Theta_{1,t} (A y_{t-1} + B u_t) \} \quad (4.64)$$

and by expanding the vector of the control variables we obtain:

$$V_{1,t}(y_{t-1}) = \max_{u_t^l} \{ \beta \nu_{1,t+1} + \rho'_{1,t} (A y_{t-1} + B^l u_t^l + B^f u_t^f) + \frac{1}{2} (A y_{t-1} + B^l u_t^l + B^f u_t^f)' \Theta_{n,t} (A y_{t-1} + B^l u_t^l + B^f u_t^f) \} \quad (4.65)$$

Since the leader knows that the $n - 1$ follower firms will choose their optimal investment rates and outputs subject to 4.61, we substitute the optimal value for u_t^f to obtain the leaders value function when the followers are making their optimal decisions:

$$V_{1,t}(y_{t-1}) = \max_{u_t^l} \{ \beta \nu_{1,t+1} + \rho'_{1,t} (A y_{t-1} + B^l u_t^l + B^f (G_t y_{t-1} + \eta_t u_t^l + \gamma_t)) + \frac{1}{2} (A y_{t-1} + B^l u_t^l + B^f (G_t y_{t-1} + \eta_t u_t^l + \gamma_t))' \Theta_{1,t} (A y_{t-1} + B^l u_t^l + B^f (G_t y_{t-1} + \eta_t u_t^l + \gamma_t)) \} \quad (4.66)$$

which upon rearrangement yields:

$$V_{1,t}(y_{t-1}) = \max_{u_t^l} \{ \beta \nu_{1,t+1} + \rho'_{1,t} [(A + B^f G_t) y_{t-1} + (B^f \eta_t + B^l) u_t^l + B^f \gamma_t] + \frac{1}{2} [(A + B^f G_t) y_{t-1} + (B^f \eta_t + B^l) u_t^l + B^f \gamma_t]' \Theta_{1,t} [(A + B^f G_t) y_{t-1} + (B^f \eta_t + B^l) u_t^l + B^f \gamma_t] \} \quad (4.67)$$

Next we carry out the maximisation *wrt* the leader's control variables vector of investment rate and output, by taking the differential of the above equation *wrt* u_t^l to get:

$$(B^f \eta_t + B^l)' \rho_{n,t} + (B^f \eta_t + B^l)' \Theta_{n,t} [(A + B^f G_t) y_{t-1} + (B^f \eta_t + B^l) u_t^l + B^f \gamma_t] = 0 \quad (4.68)$$

by rearranging the above equation the optimal controls vector for the leader firm is obtained:

$$\begin{aligned} \mathbf{u}_t^l = & -[(\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{n1,t} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)]^{-1} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' (\boldsymbol{\rho}_{1,t} + \boldsymbol{\Theta}_{1,t} \mathbf{B}^f \boldsymbol{\gamma}_t) \\ & - [(\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{1,t} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)]^{-1} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{1,t} (\mathbf{A} + \mathbf{B}^f \mathbf{G}_t) \mathbf{y}_{t-1} \end{aligned} \quad (4.69)$$

to simplify the notation, the following two terms are introduced:

$$\mathbf{d}_{1,t} = -[(\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{1,t} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)]^{-1} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{1,t} (\mathbf{A} + \mathbf{B}^f \mathbf{G}_t) \quad (4.70)$$

$$\phi_{1,t} = -[(\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' \boldsymbol{\Theta}_{1,t} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)]^{-1} (\mathbf{B}^f \boldsymbol{\eta}_t + \mathbf{B}^l)' (\boldsymbol{\rho}_{1,t} + \boldsymbol{\Theta}_{1,t} \mathbf{B}^f \boldsymbol{\gamma}_t) \quad (4.71)$$

and equation (4.69) for the optimal control vector, $\mathbf{u}_t^l$, of the leading firm reduces to a simpler form:

$$\mathbf{u}_t^l = \mathbf{d}_{1,t} \mathbf{y}_{t-1} + \phi_{1,t} \quad (4.72)$$

which shows that the optimal control vector of the leader firm is a linear function of the state variable of the previous period. Having found the optimal investment and output rates for the leader firm, $\mathbf{u}_t^{*l}$, we can reevaluate the equation for the optimal control vector of the follower firms by inserting (4.72) in (4.61):

$$\mathbf{u}_t^f = \mathbf{G}_t \mathbf{y}_{t-1} + \boldsymbol{\eta}_t (\mathbf{d}_{1,t} \mathbf{y}_{t-1} + \phi_{1,t}) + \boldsymbol{\gamma}_t \quad (4.73)$$

$$= (\mathbf{G}_t + \boldsymbol{\eta}_t \mathbf{d}_{1,t}) \mathbf{y}_{t-1} + (\boldsymbol{\eta}_t \phi_{1,t} + \boldsymbol{\gamma}_t) \quad (4.74)$$

Therefore the optimal control vector of investment and output rates for all the firms for the entire game, $\mathbf{u}_t^*$, may now be constructed by combining the two equations (4.74) and (4.72):

$$\mathbf{u}_t = \begin{pmatrix} \mathbf{u}_t^l \\ \mathbf{u}_t^f \end{pmatrix} = \begin{bmatrix} \mathbf{d}_{1,t} \\ \mathbf{G}_t + \boldsymbol{\eta}_t \mathbf{d}_{1,t} \end{bmatrix} \mathbf{y}_{t-1} + \begin{bmatrix} \phi_{1,t} \\ \boldsymbol{\eta}_t \phi_{1,t} + \boldsymbol{\gamma}_t \end{bmatrix} \quad (4.75)$$

which may be written, as shown in (4.76) below, in a more compact form by introducing further notation, namely $\mathbf{D}_t$, and $\boldsymbol{\phi}_t$ whose values are obvious:

$$\mathbf{u}_t = \mathbf{D}_t \mathbf{y}_{t-1} + \boldsymbol{\phi}_t \quad (4.76)$$

As expected, the optimal control is linear in the state variable, and dependent on the state of the previous period. The coefficients D_t and ϕ_t are dependent on the values of G_t , $d_{1,t}$, η_t , $\phi_{1,t}$, and γ_t when the controls of the leader and followers are in equilibrium, as is evident in (4.75). Which in turn are obtained from H_t , and h_t . As can be inspected from the equations above, the values for $d_{1,t}$, $\phi_{1,t}$, h_t , and H_t , are dependent on the respective values of $\rho_{i,t}$, and $\Theta_{i,t}$ for all players. But all of these coefficients are transient coefficients and they depend on the values of $\nu_{i,t+1}$, $r_{i,t+1}$ and $S_{i,t+1}$. These are valued in the next period, as is evident from the their subscripts. So the coefficients for the optimal controls in the present t period are obtained backward in time from the boundary conditions in the final period. The optimal control vector u_t , cannot be evaluated as the state y_{t-1} is not yet known. Instead, the coefficients D_t and ϕ_t are evaluated at every stage and recorded backward in time until the first period where the initial state y_0 , is known. The whole procedure rests on evaluating the sequence of the values of ν_i , r_i and S_i for all periods.

4.4.4 Recursion Procedure and Optimal Capacity Expansion

The analysis which follows for the evaluation of the recursion equations $\nu_{i,t}$, $r_{i,t}$, and $S_{i,t}$ is similar to that for the Nash-Cournot game in Section 4.2.3. As before, the value function $V_{i,t}(y_{t-1})$ has been assumed to be quadratic in the state variable, and of having the same structure as in equation (4.5). Similarly, the maximisation problem is formulated as in equation (4.8) p. 144. The problem is then developed as in equation (4.24), where instead we will use the optimal control vector for the Stackelberg problem which we have just obtained in equation (4.76). Therefore we first get:

$$V_{i,t}(y_{t-1}) = \nu_{i,t} + r'_{i,t}y_{t-1} + \frac{1}{2}y'_{t-1}S_{i,t}y_{t-1}. \quad (4.77)$$

Also the optimal value for $V_{i,t}$ may be obtained by inserting the optimal control vector from (4.76) into (4.53):

$$\begin{aligned} V_{i,t}(\mathbf{y}_{t-1}) = & \beta \nu_{i,t+1} + \boldsymbol{\rho}'_{i,t} (\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}(\mathbf{D}_t\mathbf{y}_{t-1} + \boldsymbol{\phi}_t)) \\ & + \frac{1}{2} (\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}(\mathbf{D}_t\mathbf{y}_{t-1} + \boldsymbol{\phi}_t))' \boldsymbol{\Theta}_{i,t} (\mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}(\mathbf{D}_t\mathbf{y}_{t-1} + \boldsymbol{\phi}_t)) \end{aligned} \quad (4.78)$$

By following the same procedure as in Section 4.2.1, of equating the independent, the first order, and the second order terms in $\mathbf{y}_{t-1}$ on both sides of equations (4.77) and (4.78) we obtain three recursive equations for $\nu_{i,t}$, $\mathbf{r}_{i,t}$, and $\mathbf{S}_{i,t}$, as was done in equations (4.33)-(4.35):

$$\mathbf{S}_{i,t-1} = (\mathbf{A} + \mathbf{B}\mathbf{D}_{t-1})' \boldsymbol{\Theta}_{i,t-1} (\mathbf{A} + \mathbf{B}\mathbf{D}_{t-1}) \quad (4.79)$$

$$\mathbf{r}_{i,t-1} = (\mathbf{A} + \mathbf{B}\mathbf{D}_{t-1})' [\boldsymbol{\rho}_{i,t-1} + \boldsymbol{\Theta}_{i,t-1} \mathbf{B}\boldsymbol{\phi}_{t-1}] \quad (4.80)$$

$$\nu_{i,t-1} = \beta \nu_{i,t} + [\boldsymbol{\rho}_{i,t-1} + \frac{1}{2} \boldsymbol{\Theta}_{i,t-1} \mathbf{B}\boldsymbol{\phi}_{t-1}]' \mathbf{B}\boldsymbol{\phi}_{t-1} \quad (4.81)$$

where $i \in \{1, \dots, n\}$ and $t \in [1, T]$; since the game ends at T , and the salvage value of capital at the end of the game is assumed zero, we have $V_{i,T+1}(\mathbf{y}_T) = 0$. Therefore we can deduce that:

$$\mathbf{S}_{i,T+1} = \mathbf{0}, \quad \mathbf{r}_{i,T+1} = \mathbf{0}, \quad \text{and} \quad \nu_{i,T+1} = 0 \quad (4.82)$$

and the recursion equations for the final period are obtained by inserting these values in $\boldsymbol{\rho}_{i,T}$ and $\boldsymbol{\Theta}_{i,T}$, as shown below :

$$\mathbf{S}_{i,T} = (\mathbf{A} + \mathbf{B}\mathbf{D}_T)' \mathbf{Q}_{i,T} (\mathbf{A} + \mathbf{B}\mathbf{D}_T) \quad (4.83)$$

$$\mathbf{r}_{i,T} = (\mathbf{A} + \mathbf{B}\mathbf{D}_T)' [\mathbf{p}_{i,T} + \mathbf{Q}_{i,T} \mathbf{B}\boldsymbol{\phi}_T] \quad (4.84)$$

$$\nu_{i,T} = [\mathbf{p}_{i,T} + \frac{1}{2} \mathbf{Q}_{i,T} \mathbf{B}\boldsymbol{\phi}_T]' \mathbf{B}\boldsymbol{\phi}_T \quad (4.85)$$

Once the sequence of values for $\nu_{i,t}$, $\mathbf{r}_{i,t}$, and $\mathbf{S}_{i,t}$, has been obtained for the entire planning horizon $t \in [1, T]$, the problem would be solved. This is done sequentially backward in time, starting from the final period T , for each cycle of values in the following sequence until $t = 1$:

1. Note that since this is a deterministic problem, all the costs and the inverse demand function parameters are known for all players. We have all vectors and matrices $\mathbf{p}_{i,t}$, and $\mathbf{Q}_{i,t}$, $\forall i \in \mathbf{N}$, $\forall t \in [1, T]$ stored. Also store the time invariant matrices $\mathbf{A}$ and $\mathbf{B}$.
2. At $T + 1$ the coefficients $\nu_{i,T+1}$, $\mathbf{r}_{i,T+1}$, and $\mathbf{S}_{i,T+1}$ are valued at zero.
3. Next evaluate $\rho_{i,T}$ and $\Theta_{i,T}$ for the leader and follower firms. $\mathbf{h}_T$, $\mathbf{H}_T$, and the values of $\mathbf{G}_T$, η_T , γ_T are evaluated from (4.57) and (4.61).
4. From the above step we can evaluate $\mathbf{d}_{1,T}$, and $\phi_{1,T}$ from equations (4.70)-(4.71).
5. Once this is done, the values of $\mathbf{D}_T$ and ϕ_T can be evaluated, from equation (4.75) and stored.
6. The values of $\mathbf{S}_{i,T}$, $\mathbf{r}_{i,T}$, and $\nu_{i,T}$ can now be obtained using equations (4.83)-(4.85). This completes one cycle of values.
7. Now we can evaluate the next set of coefficients $\mathbf{S}_{i,T-1}$, $\mathbf{r}_{i,T-1}$, and $\nu_{i,T-1}$, for the previous period $T - 1$, by going back to step (3), and repeating the same procedure for values of $T - 1$ instead, and using equations (4.79)-(4.81). Continue with steps 4-6 for all periods, t until start of the game at period 1, storing the values of $\mathbf{D}_t$, and ϕ_t as we go along.

Having obtained the sequence of values for $\mathbf{D}_t$, and ϕ_t for the entire game, the evolution of the state variable and the optimal control vectors may be evaluated. Since the state variable, $\mathbf{y}_0$, at the start of the game is known, the optimal control vector, $\mathbf{u}_{t=1}^*$, of investment and output rates for the next period can be obtained for both firms from (4.76). The evolution of capacity may be obtained from the state vector using the state equation of motion (4.2). With the state vector for the next period known, the optimal investment and output rates for the second period are again obtained using (4.76), and this process is repeated forward in time until the final period in the game. The optimal evolution of investments and outputs for the firms is obtained from (4.76), and that of the capacity from the state equation in (4.2).

4.5 Model Performance and Simulation Results

Several runs were conducted to test the effect of changes in the investment adjustment cost parameter, d , and the production adjustment cost parameter, g , on the model performance in terms of the production, capacity, capacity utilisation levels of the two players at steady state, and its impact on price. The results are shown in Tables 4.1, and 4.2. Except for asymmetries in the raw materials costs, the two firms are assumed to have identical costs in all the runs. The first player is more efficient with lower raw materials cost of $f_1 = 8$, and for the second, $f_2 = 10$. The rest of the parameters used, are those of the model base case, and are shown in the footnotes to the tables. Several observations can be made about the behaviour of the model. The type of competition and its bearing on the market structure at equilibrium is examined below.

4.5.1 Adjustment Cost of Investment - d Effect

The following observations can be made about the model behaviour when d takes different values.

- Firm 1 which has a lower cost of raw materials holds a higher capacity at equilibrium, and makes higher profits as expected in all the cases (see values of w_1 and w_2). At low values of d as in cases 1-3, both firms overproduce at a slightly higher rate than the capacity held.
- As the d values rise, both firms now operate holding overcapacity, as in cases 4-7, where the overcapacity which ranges between 4% – 8%. The equilibrium capacities of both firms, and their respective production rates increase, and the price drops. In case 5 where $d = 50$ the firms register the highest capacity at $k_1 = 17.63$ and $k_2 = 16.46$.
- When the adjustment cost rises beyond $d = 50$, increases in d value start to have a negative effect on the equilibrium values of capacity and production for the two firms. In case 8, where $d = 5000$, it becomes very costly for the players to invest, equilibrium is at a lower level, and now production exceeds capacity.

This behaviour is due to dynamics of the adjustment cost factor. Reynolds (1987) and Karp & Perloff (1993a) have discussed the role of adjustment costs for infinite games, with particular attention given to the influence of limit values on investment behaviour; when the d parameter tends to zero, and on the other hand as it rises to approach infinity. When d is very large, firm i 's rival finds it too costly to respond to changes in firm's i output; and the preemptive incentive for firm i to use overcapacity is small. As $d \rightarrow \infty$, the steady state of the feedback model tends to the open-loop steady state. The reason is that the preemptive incentive of holding excess capacity disappears with the rising cost of adjustment costs. The equilibrium capacities for the two firms therefore decline. On the other hand, for small d values, firm i 's rival can easily respond to changes in output, and the steady states of open-loop and feedback models are different. When $d = 0$, there are no adjustment costs, and the static model is obtained. In this case the open-loop and feedback model results coincide. But when d is very small the two equilibria are not the same; even when $d \rightarrow 0$ the steady state equilibrium of the feedback model declines but remains higher than the open-loop equilibrium (Karp & Perloff 1993a). Reynolds (1991) also performs a similar analysis, and arrives at similar conclusions for a continuous time game.

Table 4.1: the effect of investment adjustment cost (d) on feedback NC equilibria at steady-state⁽¹⁾

	d	x_1	k_1	x_2	k_2	$cu_1\%$	$cu_2\%$	price	w_1	w_2
1	0.001	15.45	14.81	14.45	13.81	104	105	40.2	479	419
2	0.05	15.49	14.93	14.48	13.92	104	104	40.1	478	418
3	0.50	15.66	15.62	14.62	14.54	100	101	39.4	473	413
4	5.0	16.02	16.98	14.91	15.77	94	95	38.1	461	400
5	50	16.18	17.63	15.10	16.46	92	92	37.6	454	396
6	100	16.03	17.38	15.11	16.43	92	92	37.8	454	398
7	400	15.78	16.44	15.12	16.09	96	94	38.3	458	408
8	5000 ⁽²⁾	15.61	15.48	14.66	14.59	101	100	39.5	472	415

⁽¹⁾Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, $c_1 = 2$, $c_2 = 2$, $d_1 = d_2$, $m_1 = 1$, $m_2 = 1$, $f_1 = 8$, $f_2 = 10$, $g_1 = 2$, $g_2 = 2$, $T = 30$. cu_1 and cu_2 refer to capacity utilisation rates of firms 1 and 2 respectively.

⁽²⁾The time horizon for this case has been extended to $T = 300$, to allow the firms to arrive at their steady state.

Table 4.2: the effect of production adjustment cost (g) on feedback NC equilibria at steady-state^(†)

	d	x_1	k_1	x_2	k_2	$cu_1\%$	$cu_2\%$	price	w_1	w_2
1	0.5	15.71	15.72	14.66	14.66	100	100	39.3	472	411
2	5.0	16.33	16.51	15.15	15.32	99	99	37.0	454	392
3	50	16.74	17.04	15.52	15.80	98	98	35.7	438	380
4	100	16.66	16.94	15.52	15.79	98	98	35.9	439	378

^(†)Parameter values used: same as in table(4.1) except for $g_1 = 20$, and $g_2 = 20$.

Table 4.3: the effect of roles and adjustment cost on Stackelberg equilibria at steady-state⁽¹⁾

	f_1	f_2	g	x_1	k_1	x_2	k_2	$cu_1^{(2)}$	cu_2	price	$w_1^{(3)}$	w_2
1	8	10	2	20.88	20.66	12.51	13.44	101	93.1	33.2	501	273
2	10	10	2	19.92	19.69	13.03	14.02	101	93.0	34.1	456	296
3	10	10	20	18.98	19.03	13.90	14.07	99.7	98.7	34.2	437	320
4	8	10	20	19.94	20.00	13.55	13.69	99.7	98.8	33.4	482	296
5	10	8	20	18.46	18.51	14.73	14.92	99.7	98.7	33.6	414	359
6 ⁽⁴⁾	10	10	5	21.18	20.96	11.94	11.96	101	99.8	33.8	478	269

⁽¹⁾Firm 1 acts as Stackelberg leader, and firm 2 as follower. Parameter values used: $a = 100$, $b = 2$, $r = 0.05$, $\delta = 0.10$, $c_1 = 2$, $c_2 = 2$, $m_1 = 1$, $m_2 = 1$, $d_1 = 5$, $d_2 = 5$, $g_1 = g_2$, $T = 30$.

⁽²⁾ cu_1 and cu_2 refer to percentage capacity utilisation of firms 1 and 2 respectively.

⁽³⁾ w_1 and w_2 refer to single period profitability of firms 1 and 2 respectively.

⁽⁴⁾ $d_1 = d_2 = 0.5$.

4.5.2 Adjustment Cost of Production - g Effect

A substantial increase in the value of the production adjustment cost parameter, g , makes the holding of excess capacity by the firms more expensive; the adjustment cost for production becomes more significant, and accounts for a higher share of the cost of production. The firms adjust their output and investment to minimise this effect. As the value of g rises from 2 to 20, both firms increase their utilisation of capacity as shown in Table 4.2. At low values of d ($d = 0.5$ and $d = 5.0$), output and capacity rise for both firms (*c.f.* results of equivalent d value cases 3 and 4 of Table 4.1); and the price drops from 39.4 and 38.1 to 39.3 and 37.0.

At higher values of d ($d = 50$, and $d = 100$), adjustment of capacity becomes costly. When g rises to 20, the two firms hold less capacity in steady state equilibrium than before (*c.f.* cases 5 and 6 of Table 4.1), to minimise the higher cost of overcapacity. Output

risks, but capacity drops slightly, resulting in the combined effect of a higher capacity utilisation rate. The drop in price is now more pronounced, falling to 35.7 and 35.9 for the two cases respectively. It should be noted that in all of these cases, while variations in the adjustment costs had a direct bearing on capacity utilisation, the market shares of the firms in terms of their production, or capacities, were more or less unchanged. Market shares remained around 52% and 48%, for firms 1, and 2 respectively.

A profoundly different outcome is obtained when one of the firms acts as a leader, and the other takes the role of a follower. The results for the Stackelberg game, shown in Table 4.3, indicate that which firm takes the role of the leader, has a direct bearing on its market share and the outcome of the game. Consider for instance the results of case 1 in Table 4.3, and case 4 in Table 4.1; where the costs of the firms in the two cases are identical. When the two firms engage in a Nash-Cournot competition (case 4), firm 1 which has a slightly more advantageous raw materials cost, has a capacity of $k_1 = 16.98$ while firm 2 has a capacity of $k_2 = 15.77$. When firm 1 now acts as a leader (case 1), its capacity rises to $k_1 = 20.88$, while firm 2 the follower, has a capacity of only $k_2 = 13.44$. The leader, firm 1, has now increased its market share from 52% to 61% and can generate 80% more profits than its competitor ($w_1 = 501$, $w_2 = 273$); while the follower's share has dropped to a mere 39%. The market is more competitive as the two firms produce more under the Stackelberg mode of play, and the price drops from 38.1 to 33.2.

When the two firms have the same raw materials costs (case 2 in Table 4.3, $f_1 = 10$, $f_2 = 10$) and with firm 1 still acting as a leader, the equilibrium capacities ($k_1 = 19.69$, $k_2 = 14.02$) show that firm 1 still retains its dominant position and controls more than 58 percent of the market. It also generates 54 percent higher profits than firm 2. Firm 2 now has a slightly higher capacity than in case 1. But the two firms together produce slightly less than before, and the price rises to 34.1.

Case 3 differs from case 2, in that both firms now have a higher production adjustment cost (g rises from 2, to 20). The influence of higher g value here, is similar in its performance to that under the Nash-Cournot model, where the firms try to minimise the deviation from optimal capacity. Firm 2 now increases its production and operates at higher capacity utilisation ($cu_2 = 98.7$); the profit gap between the two firms, is smaller

than before.

Another example of the influence of a higher adjustment cost g , is case 4 where the leader, firm 1, has a raw materials cost advantage over its rival (*c.f.* case 1), the two firms modify their production and capacities to minimise the cost of overcapacity, and the results are comparable to those of case 1.

Case 4 in Table 4.3 is comparable to case 2 in Table 4.2 for the Nash-Cournot game; both cases have identical costs, and have a high production adjustment cost ($g = 20$), but now firm 1 which has a lower raw materials cost is acting as leader. The leadership position of firm 1 gives it an advantage over firm 2, its Stackelberg production and capacity ($x_1 = 19.94$, $k_1 = 20$) is much higher than what it can achieve as a Nash-Cournot rival ($x_1 = 16.33$, $k_1 = 16.51$). Firm 2 does worse as a follower, and its production and capacity drop (from $x_2 = 15.15$, $k_2 = 15.32$ to $x_2 = 13.55$, $k_2 = 13.69$); and does worse in terms of profits as well, which drop from $w_2 = 391$, to $w_2 = 296$.

The advantage that the leadership position imparts on the firm that gets to make the first move is illustrated further in case 5 in Table 4.3. The leader, firm 1, is now disadvantaged in terms of its raw materials costs ($f_1 = 10$), which are higher than the follower ($f_2 = 8$). Despite of this, the leader, still dominates the market ($x_1 = 18.46$, $k_1 = 18.51$) and achieves much higher profits than its rival ($w_1 = 414$, $w_2 = 359$). Note also that the leader, firm 1, does only marginally worse, as a result of its higher raw materials costs, than when it had lower or the same raw materials costs as its rival follower (compare these results for firm 1, with those it had in cases 4 and 3).

4.5.3 The Number of Firms and Roles of Play

The impact of the type of rivalry that takes place between two firms serving a duopolistic market, on the industry equilibrium capacity and its competitiveness is shown in Figure 4.1. When the firms operate under the assumption that they ignore each others production and act as price takers, they generate the highest capacity level and operate as a competitive industry. For the remaining models, the industry equilibrium capacity, and competitiveness decline in the following descending order, for Stackelberg, feedback Nash-Cournot and open-loop Nash-Cournot markets. The Stackelberg leader does bet-

ter than the follower, in terms of market share and profitability, but the industry is more competitive, than the feedback Nash-Cournot industry as we saw in Tables 4.2 and 4.3. Because the follower does worse under the Stackelberg model, and since industry profitability (the sum of profits of the two players) is also worse off, the Stackelberg dominance in this model is only a weak dominance. Each player would prefer to be in the dominant position (Dockner, Feichtinger & Mehlmann 1989). In Chapter 2, Section 2.8, and Table 2.4, it was shown that the entrant firm obtains better results when it acts in a Stackelberg fashion in choosing its capacity. The entrant firm obtains higher profits upon selecting a Stackelberg strategy. Cases *Skgsd4* and *Skgsd5* illustrated that the entrant is better off acting as leader, even under a Stackelberg disequilibrium scenario, than choosing the role of follower.

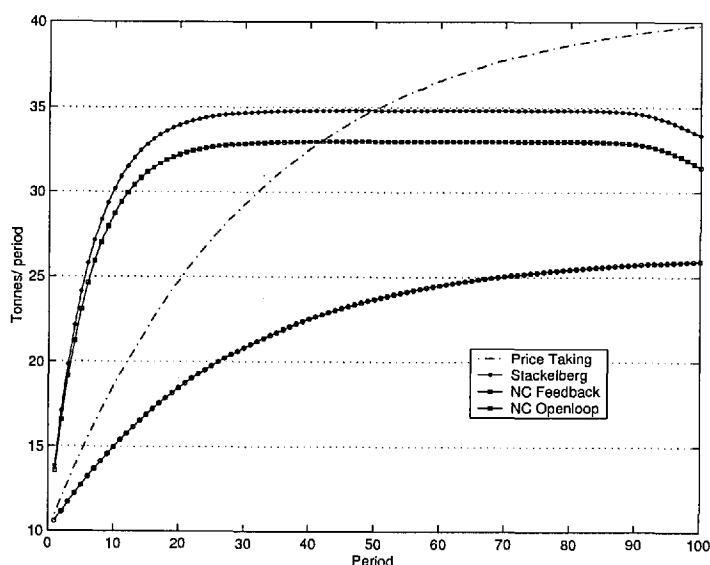


Figure 4.1: The impact of game information structure on capital accumulation for a duopoly: parameter values used: $a = 100$, $b = 2$, $c_1 = 2$, $c_2 = 2$, $d_1 = 50$, $d_2 = 50$, $f_1 = 8$, $f_2 = 10$, $g_1 = 2$, $g_2 = 2$, $m_1 = 1$, $m_2 = 1$, $\delta = 0.10$, $r = 0.05$, $k_1(t_0) = k_2(t_0) = 5$.

Another aspect of the model behaviour to be examined is the impact of an increase in the number of firms in the market on the equilibrium and total industry capacity and output. It can be seen in Figure 4.2 that as more firms enter the market the equilibrium capacity of the firms playing a feedback Nash-Cournot game, rises as well. Output also increases and prices drop. The industry becomes more competitive. When the market is

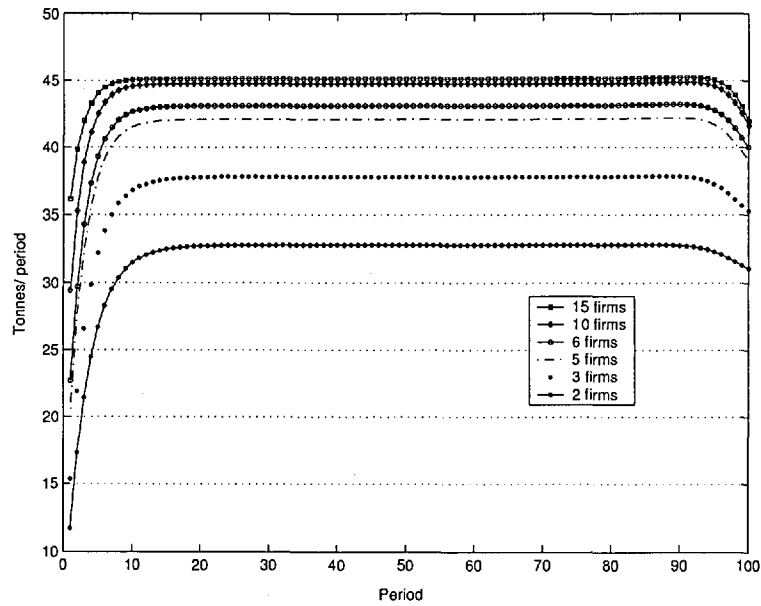


Figure 4.2: Equilibrium capacity for an oligopolistic industry in a Nash-Cournot game: parameter values used: $a = 100$, $b = 2$, $c_1 = 2$, $d_1 = 50$, $f_1 = 8$, $g_1 = 20$, $m_1 = 1$, $\delta = 0.10$, $r = 0.05$, $k_1(t_0) = 5$. All firms have the same costs.

served by a larger number of firms, (the cases of 30, or 35 firms) the industry output of the open-loop Nash-Cournot, feedback Nash-Cournot, and price taking models, at steady state, start to converge; indicating that the market is approaching competitive market conditions. The results are shown in Table 4.4.

4.6 Conclusions

A dynamic model of capacity expansion, where firms make decisions about optimal investment and production in each period, was presented in this chapter. The model solution is based on two algorithms developed by Kydland (1975, 1977), that employ dynamic programming techniques. The four main modes of competition between firms, that are most prevalent in the industrial organisation literature are developed: Feedback Nash-Cournot, open-loop Nash-Cournot, and Stackelberg type strategies for an oligopolistic market setting; and the more competitive model where firms behave as price takers.

I obtain *unique* solutions for all the models, when the firms employ a class of strategies

Table 4.4: increasing number of firms and competitiveness of industry output at steady-state^(†)

n	Open-loop	Feedback	Price-taking
2	29.26	32.35	43.42
3	32.98	37.01	43.89
5	36.66	40.88	43.98
6	37.71	41.78	44.00
10	40.38	43.35	44.00
15	41.25	43.93	44.00
20	41.90	44.28	44.00 ^(‡)
25	42.31	44.43	44.00
30	42.58	44.51	44.00
35	42.78	44.58	44.00

^(†)Parameter values used: $a = 100$, $b = 2$, $c = 2$, $d = 100$, $f = 8$, $g_1 = 20$, $m_1 = 2$, $\delta = 0.10$, $r = 0.05$, $T = 100$. All firms have the same costs.

^(‡) As the number of firms increases, the dimension of the problem becomes too large; for the price taking and open-loop models when $n \geq 20$ in this case the planning horizon is reduced to $T = 60$ to circumvent PC memory problems. It is expected that as n and d increase, the output of the three models would converge.

that are linear in the state; I also show the optimal capital accumulation path for the competing firms. In all the models, the optimal control strategies for investment and output, have the same form but they embody different interactions that characterise the roles of play for the players in each game. The results are consistent with the findings of main stream literature on quantity games (Dockner, Feichtinger & Mehlmann 1989, Başar & Olsder 1999, Karp & Perloff 1993a, Reynolds 1987, 1991).

Under price taking conditions, investment and production rivalry between the firms is most competitive. The firms accumulate the highest levels of capacity, production levels are high, while prices drop to their lowest levels. The open-loop strategies generate the lowest capacity profiles, highest prices and highest profits for the firms. Because the firms have to commit to trajectories of optimal outputs and investments for the entire planning horizon, the open-loop Nash-Cournot, as a solution concept is difficult to implement. Despite their appeal to firms from a profitability point of view, open-loop strategies suffer from time inconsistency (Başar & Olsder 1999). It would be difficult to enforce an open-loop strategy, as a planning policy, in case one of the firms deviates from the originally computed optimal plans at a later stage in the game. Such policies are more likely to be practicable when long term contracts are involved, such as those present

in natural gas agreements, or in bilateral or international agreements on pollution and environmental control for instance.

The feedback strategies of the Nash-Cournot and the Stackelberg models, on the other hand, are time consistent. The firms are able to react to changes in the state in every period; for the strategies are now decision rules and not predetermined trajectories (Başar & Olsder 1999). A Stackelberg game is more competitive than feedback Nash-Cournot game; but a firm taking the role of Stackelberg leader is shown to have an advantage in terms of market share and profits. The leader's position therefore should reflect a significant production cost advantage, or some other strategic factor. Otherwise why should the followers concede to the leader market advantages at their expense?

The type of strategy chosen, affects the market structure more profoundly than a reasonable production cost differential. The models analysed in this chapter may be adopted for empirical testing. Their construction is such that each model can be adapted to accommodate several states and control variables for each player. It is also flexible in allowing for variations in different costs, across time and firms. It is possible to allow for firms to enter, or exit at any stage during the game. The major weakness of the model, however, is the inability to place constraints or bounds on the control variables.

By virtue of its dynamic nature, the model may be used to study the degree of competitiveness of oligopolistic industries, and the evolution of market structure. By estimating the prevailing price/demand relationship, and production costs for a specific product market, the different oligopolistic models examined in this chapter can be used to simulate the market data. Market shares, output and capacity evolution, prices and company profits can be generated. Using these results, the oligopolistic power of the market can be examined, and compared to the more competitive case of price taking behaviour. In the next chapter a single market for a chemical product is studied using the model concepts examined thus far.

Chapter 5

The US Styrene Market: Structure, Evolution, and Investment Behaviour 1974-1996

Keep silent, because the world of silence is a vast fullness.

Do not beat the drum of words. The word is only an empty drum.

—Andrew Harvey (1994), *The Way of Passion*, p. 122

5.1 Introduction

The different formulations of the models developed in Chapter 4 are now tested using industry data from the US petrochemical industry, to see how well they perform in explaining the capacity expansion of the styrene industry and its development over the period 1974-1996 for which data is available. Initially I look at the main characteristics of styrene as a product, and its main uses in Section 5.2. In this section I also give a brief description of the chemistry of the product and a synopsis of the processes for its production; followed by an outline of the main global developments of the industry and the role of the major styrene producers in these developments. In Section 5.3 more

attention is paid to industry developments in the US market and the evolution of market structure. Then I present and estimate a formulation for the inverse demand function in Section 5.4, and for the production costs in Section 5.5.2. In addition, for the cost function, this section includes estimates based on industry data for cost savings resulting from learning and the experience curve, and also the those generated by the gains from economies of scale.

In Section 5.6 the model simulations for the styrene industry are presented showing the performance of the model that best describes the investment behaviour of the major US styrene producers, and deduce whether strategic intent is present. Then in Section 5.7 I discuss other aspects of strategic investment and entry or mobility barriers that are prevalent in oligopolistic industries and the industrial organisation literature, and evaluate in a heuristic fashion the extent of their use, or the presence of some of these strategic attributes and forms of investment behaviour in the styrene industry. Previous empirical work covering this area is discussed in Section 5.7.2 with particular reference made to studies that cover the petrochemical industry. In the last section I round-up by presenting some concluding remarks.

5.2 Styrene: Product, Chemistry, and Market Characteristics

5.2.1 Market and Product Characteristics

Styrene is an intermediate chemical product, produced on a large scale around the world, with a total output that exceeded 22 million tonnes in 2002. Styrene is produced in liquid form, and is transported in large tankers for industrial use. For producers involved in small scale operations such as the production of unsaturated polyester, it is delivered in regular size steel drums. Styrene is a highly flammable product; its production involves a series of chemical processes that operate at temperatures and pressures near the explosive limit of these products; this perhaps explains why accidents, though an unlikely event in the industry, take place at times disrupting the production process and forcing plants to

shut down¹.

The reader is unlikely to encounter or recognise styrene in its basic form; but like with many chemicals and plastics that are produced by modern chemical industry, we encounter styrene products and derivatives in products and items that we use in our everyday life. One of the main end uses for styrene is in packaging products. Perhaps the most discernible are the white foamed cups used in coffee shops at campuses and elsewhere, which are made from polystyrene, the plastic polymer derived from styrene. The disposable glasslike transparent plastic cups used for drinking water and refreshments, used initially on flights but now widely available in eating outlets, are also made from polystyrene; as well as the white plastic pots for yoghurts and similar products. The packaging of burgers and the like in fast food restaurants is mostly made from foamed-sheet polystyrene.

Most electrical and electronic items we buy such as TV sets, HiFi's, computers and laptops come packaged in a white styrofoam. Several of the body or internal plastic parts of these products are made from plastics material containing styrene such as acrylonitrile butadiene styrene, (ABS). Numerous household items contain ABS and polystyrene plastics; the interior plastic parts of refrigerators are made predominantly from this type of plastics. Rubber tyres and many of the rubber products we use also contain styrene butadiene rubber, (SBR). Styrene derivatives have a wide range of application, and enter into different industries. Figure 5.1 depicts the major downstream uses of styrene by type of product, which shows polystyrene in the leading position with a share of 62% of styrene consumption. With high-content styrene plastics such as ABS and styrene acrylonitrile (SAN), and styrene butadiene latex trailing at 8% each. SBR accounts now for a mere 4%, and unsaturated polyester for 6%. Polystyrene is used in the construction industry where extruded styrofoam sheets and boards are used for insulation. And unsaturated polyester is an essential constituent of oil based paints that are used in industrial and marine applications.

The pattern of consumption of styrene by end use and economic activity in the US is

¹The latest in the styrene industry was Nova's explosion at its plant at Bayport, Texas in 2003, Chemical Marketing Reporter 23 June, 2003. Arco had an accident in 1990 at one of its plants, and Dow also had one in the mid 1980's.

shown in Table 5.1. The packaging segment dominates the consumption of styrene and accounts for 30% of all styrene consumed in the US market; while construction, household and consumer goods, electronic and electrical products account for about 10% each. The transportation sector accounts for 6% and recreation for 5%. Other miscellaneous uses of styrene account for 28% indicating the wide variety of applications styrene products have in different sectors of the economy.

Table 5.1: US consumption of styrene by end use – 1998^(a)
(Percent)

Packaging	Construction	Household Consumer Products	Electrical Electronic Appliances	Transportation	Recreation	Other	Total
30	10	10	11	6	5	28	100

^(a)Source: Chemical Economics Handbook, SRI, Menlo Park, California, USA.

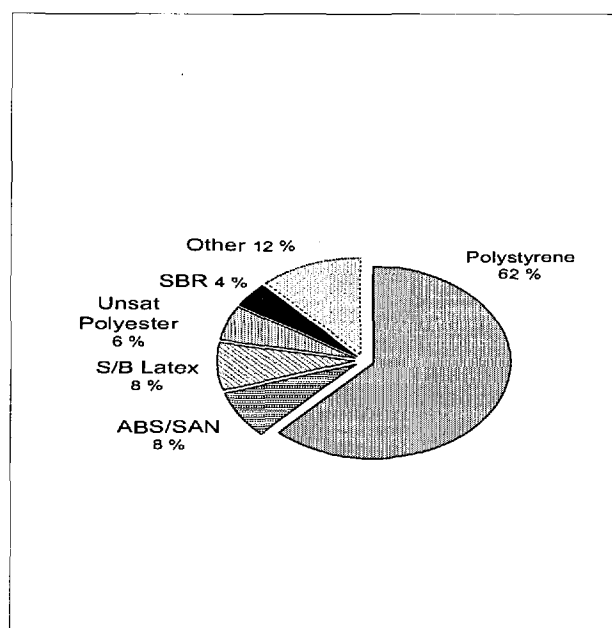


Figure 5.1: Pattern of styrene consumption in the US -1998

5.2.2 Chemistry and Process Technology

Styrene is produced from ethylene and benzene in two stages. Initially ethylene and benzene are reacted in the first stage to produce ethylbenzene, and the second stage involves the production of styrene by dehydrogenation of ethylbenzene to crude styrene, followed by purification and stabilisation of the purified styrene final product. Ethylbenzene and styrene units are almost always installed together in matching capacities because nearly all of the ethylbenzene produced is converted to styrene. Ethylbenzene's other uses such as for solvent applications, or as an intermediate for other products are insignificant. As a result there are no commercial sales of ethylbenzene on the market to speak of. It is advantageous also to integrate the two stages of styrene production from an energy saving point of view, since the first stage is exothermic generating a surplus of energy; while the second is endothermic requiring a significant input of energy.

The alkylation reaction between benzene and ethylene takes place over an acidic catalyst. In the older processes such as those developed by Monsanto, Dow, BASF and one by Union Carbide in cooperation with the engineering company Badger, the reaction takes place in the liquid phase over an aluminium chloride catalyst. The Union Carbide-Badger process was widely used in the 1960's and 1970's; 20 plants employing this process were built around the world. Other improvements were made by Monsanto in the 1970's on a similar process. UOP the engineering company also developed another process which became available for licensing. The improvements that were being made affected the process design and catalyst efficiency. Another process was developed in 1980 by Mobil, and Badger a process engineering company. Their process which became known as the Mobil-Badger vapour-phase ethylbenzene process, uses a zeolite catalyst which has the advantage of being noncorrosive and environmentally more benign than the older aluminium chloride process. The first plant to be commercialised based on this process was built for Hoechst at Bayport, Texas. This was a single train plant with an annual capacity of 450 thousand tonnes/year.

Most plants built in the 1990's were based on this process. About 35 plants based on this process were built around the world by 1995, with a combined capacity of 10 million tonnes/year. ABB Lummus Engineering developed a similar liquid phase process

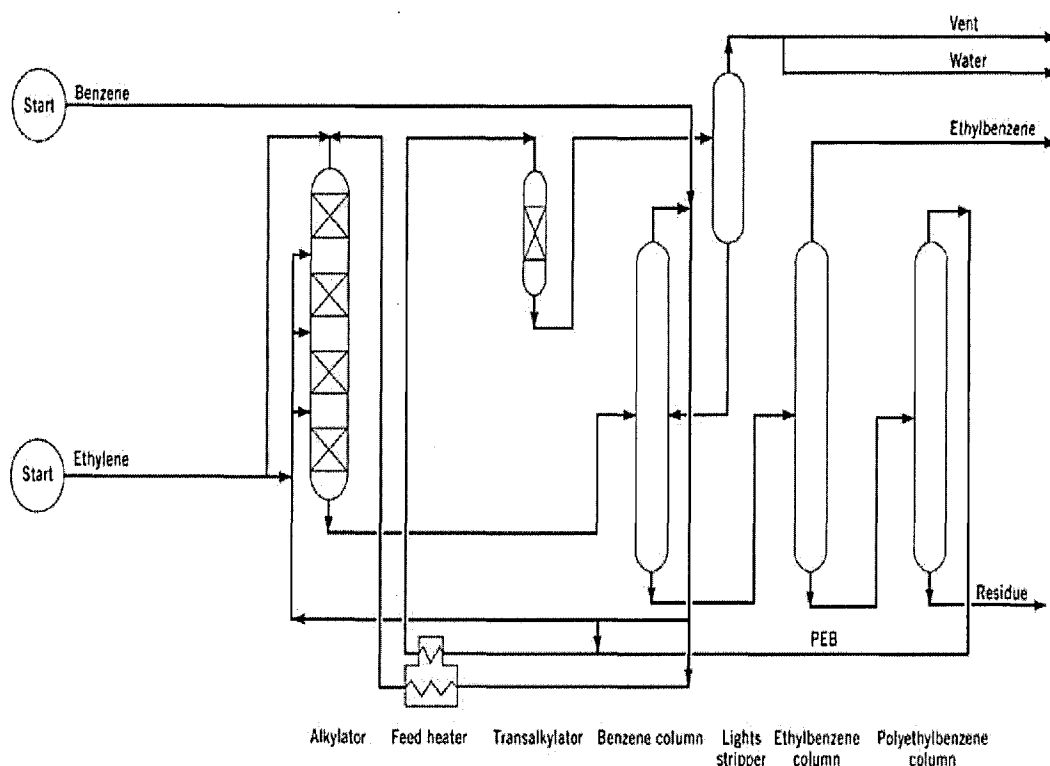
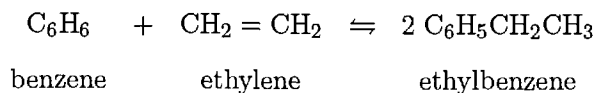


Figure 5.2: Ethylbenzene production flow chart

that uses zeolite catalyst in 1990, and had commercialised three plants based on this process by mid-1995 in Japan. The overall yield to ethylbenzene from the zeolite based processes is estimated to be 98-98.5%, for the ABB Lummus process, and 99.5% for the Mobil-Badger process; with the residue accounting for most of the losses.

The alkylation reaction that takes place in all of these processes is described below:

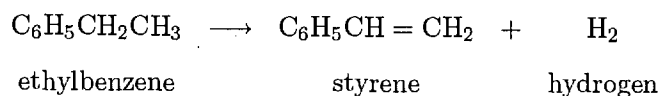


In most processes the benzene and ethylene used are of very high purity to minimise the formation of byproducts and the formation of impurities that may poison the catalyst. Consumption figures of raw materials consumption vary depending on the type of process in use and its efficiency, the catalyst used and operating conditions. On average about 0.305 tonnes of ethylene, and 0.838 tonnes of benzene are consumed for each tonne of

styrene produced. Excess benzene to ethylene is used, which favours the forward reaction to form ethylbenzene and to ensure that all the ethylene is reacted, which minimises the formation of polyethylbenzenes (PEB) and other alkylated products. Unreacted benzene, and Polyethylbenzene are separated and recycled to complete the reaction. Excess benzene is recycled back to the reactor with fresh benzene and ethylene; while ethylbenzene product is purified and sent to storage as shown in Figure 5.2 which is based on the Mobil-Badger² process.

There are two routes for the production of styrene, both of which start from ethylbenzene. The first is the dominant process in the US and around the world, and accounts for about 86% of world wide styrene capacity. This a straight forward process based on ethylbenzene dehydrogenation, and produces a single product. Its main attributes are presented below. The second route involves a more complicated process, that involves the combined production of propylene oxide and styrene, and is known as the PO-SM process.

The dehydrogenation of ethylbenzene to styrene takes place on a metal oxide catalyst at high temperature in the presence of steam, and relatively low pressure since low pressure favours the forward reaction to styrene, and produces hydrogen which is usually used as a fuel in the process. The reaction is shown below:



The rate of reaction is highly temperature dependent; high temperature favours dehydrogenation, but also increases byproducts in side reactions, and decreases styrene selectivity. The reaction selectivity to styrene is high at 97%. The main byproducts in the dehydrogenation reactor are toluene and benzene; they account for about 2% and 1% respectively of the styrene yield loss. However both are recovered and purified, in the process; they contribute to recover some of the cost of lost ethylbenzene. The section of a typical process for the production of crude styrene is shown in Figure 5.3. There

²The source of information for the chemical technology in this section is the Kirk-Othmer Encyclopedia of Chemical Technology, and SRI.

are several processes for the production of styrene in operation, the design concepts are essentially the same but they differ in some of the details, and the catalyst in use. Dow and BASF use their own proprietary catalysts and their own processes, whereas the majority of licensed processes rely on supplies from specialised catalyst manufacturers such as Criterion, United Catalysts, and the Sud-Chemie Group. There are two main processes available for licensing for styrene production. The Fina-Badger process licensed by Badger, and the Monsanto process licensed by ABB Lummus; and most of the licensed plants under construction around the world since the 1990's rely on one of these two processes. Several factors are taken into consideration by the licensee when selecting a process, namely, the capital cost involved, the yield on raw materials consumed, plant reliability, energy efficiency, product purity, consumption of catalysts and other chemicals, and process flexibility in addition to other factors.

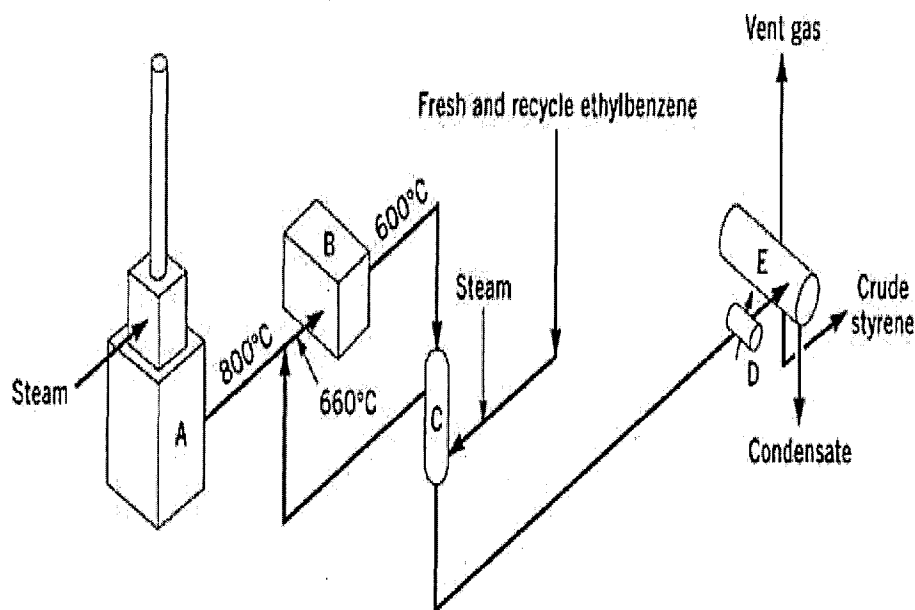


Figure 5.3: Ethylbenzene dehydrogenation production flow chart. A, steam superheater; B, reactor section; C, feed-effluent exchanger; D, condenser; and E, settling drum.

Modern plants have an average ethylbenzene to styrene yield of 89-92% which corresponds to 1.12-1.14 tonnes of ethylbenzene per tonne of styrene produced. Because polymer grade styrene of high purity is required, an elaborate distillation system is in-

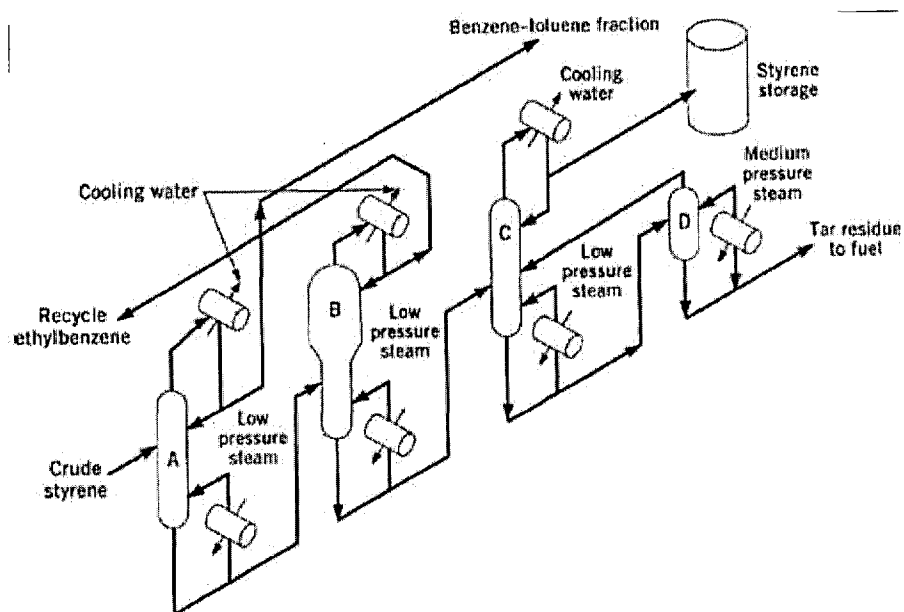


Figure 5.4: Styrene production and purification flow chart. A, benzene-toluene column; B, ethylbenzene recycle column; C, styrene finishing column; and D, residue finishing.

involved in the production process. Styrene polymerises readily, so an inhibitor is added to suppress its formation within the process. Styrene is distilled in several stages to remove all impurities and because it has a boiling point close to that of ethylbenzene making the distillation more difficult. Benzene and toluene are recovered in the overhead of the benzene-toluene column as shown in Figure 5.4 which is based on the Fina-Badger process. Ethylbenzene is recovered, distilled and recycled back to the reactor. The styrene is sent to a further distillation step in the styrene finishing column where it is collected at the top as a purified product and is sent to storage. The bottoms from this column are further processed to recover any styrene and inhibitor chemicals. The heavy residue is used as a fuel in the process. The design of the process as can be seen, optimises the recovery of energy used in the distillation section.

The second route to styrene production involves the peroxidation of propylene and ethylbenzene, with the coproduction of styrene and propylene oxide, in what also became to be known as the PO-SM (propylene oxide - styrene monomer) process. This process was developed by Halcon International and was commercialised in Spain in 1973 in a joint

venture between Enpetrol of Spain, and Arco. Starting from ethylbenzene and propylene, in an oxidative reaction that involves three reaction steps, where ethylbenzene is oxidised with air and then reacted with propylene to form propylene oxide and styrene. The reactions in this process are complex and involve several reactors and processing units. The recovery and purification of the products is also complex due to the presence of oxygenates. Extensive distillation is required to purify the final products in this process involving the use of some specialised equipment. And the whole operation is capital intensive. Capital costs for this process are appreciably higher than the conventional dehydrogenation route. But the sales revenues from propylene oxide product more than compensates for this. On average the overall costs for the two processes are comparable, although somewhat more favourable to the PO-SM process. Styrene is a byproduct in this process, but more styrene is produced than propylene oxide; on average 2.4 tonnes of styrene are produced for each tonne of propylene oxide. In the US this process was operated by Arco, which is now owned by Lyondell, at two plants at Channelview, Texas. This process now accounts for about 21% of the styrene capacity in the US. It has achieved continued growth at the expense of the dehydrogenation process since the mid 1980's, not only in the US but also in other parts of the world. Its expansion however is limited by the demand prospects for propylene oxide. In the US it is expected that its share could rise to 30% in the coming few years.

Shell also developed a similar process in 1979 based on the same concept, and has plants in the Netherlands, and Singapore that utilise this process. In Spain, Repsol, which now owns the joint venture that started in 1973 with Arco and Halcon, embarked on a plan in 1997 to expand its production by building a new larger plant (the planned project had a capacity of 150 thousand tonnes/year of propylene oxide and 340 thousand tonnes/year of styrene) at a new site at Taragona in Spain, using the original process. Arco disputed Repsol's rights to use the technology, and the case was referred by the latter to the European Competition Commission³ which provisionally decided that Arco's attempts to stop Repsol building its plant breached European Union competition regulations. Before a final ruling was made, the two companies had settled their dispute through

³*Repsol, Arco Settle Propylene Oxide Dispute*, Chemical Market Reporter, 22 December, 1997. See also other related articles in 1st December, 20 October, 28 July and 4 April issues of the same journal.

an arbitration panel set up by the International Chamber of Commerce. After some delay, Repsol's new plant came on stream in 2000. It should be noted that none of these companies licenses this technology to third parties; but some joint ventures have been formed by Shell in Singapore, Korea, and in Japan. And more recently by Lyondell with Bayer in the Netherlands.

In 1980 a Russian company, known as Nizhnekamskneftekhim, developed a process similar to the Arco process; and has been operating a plant of about 140 thousand tonnes/year. Dow bought the licensing rights from the Russian company in 1998, but as yet does not operate any plant using this process. Announcements to build a new world scale plant on the gulf coast in the US, using this technology have been made on several occasions, but construction dates continued to be postponed⁴. Dow is in a position now to utilise either technology. It is also a leading world producer of propylene oxide, where it utilises a conventional technology using a chlorine based process. It has plants in the US, Europe and other parts of the world where it produces both styrene and propylene oxide using its own processes.

There are other processes as well for the production of styrene. Dow developed a new technology that converts butadiene from C₄ streams of refinery fluidised catalytic crackers, or naphtha based ethylene crackers into ethylbenzene and then to styrene. The process has been demonstrated at pilot scale, and is thought to be competitive with conventional processes if butadiene is available in sufficiently large quantities. No commercial plants based on this process have been built yet.

5.2.3 Major Producing Companies and Global Developments

5.2.3.1 The Role of Major Producing Companies

Dow, BASF, Monsanto, Shell and Union Carbide were among the first companies that became involved in the production of styrene and its derivatives in the world. Union

⁴Dow has been planning a world scale plant for styrene in the US since the mid 1990's. Upon the acquisition of the PO-SM technology from Russia, it announced in 2000 that it was planning a world scale plant to come on stream in mid 2004 with a capacity of 250 kt/y propylene oxide, and 575 kt/y styrene, at one of its sites on the US gulf coast. A year later however, Dow announced it was delaying the project.

Dow to Build World Scale POSM Plant, Chemical Market Reporter, 17 July, 2000, p. 3. *Dow Delays Construction of Gulf Coast POSM Site*, Chemical Market Reporter, 2 April, 2001, p. 3.

Carbide and Monsanto were responsible for developing process designs that are still being licensed by process engineering companies like Badger and Lummus. However, both companies exited styrene production in the US. Union Carbide was the first to exit in 1979; Monsanto followed by the mid 1980's, and within a few years it sold all its styrene derivatives production facilities in the US and around the world, as the company was moving away from commodity chemicals and into specialities and biotechnology.

Dow, Shell and BASF on the other hand, continue to expand their production capabilities in North America, Western Europe and around the world. They are the three world leading firms in styrene production, in that order, as shown in Table 5.2.

Dow is the most diversified of styrene producers in terms of its geographical coverage and range of styrene derivatives it produces; and is also well integrated upstream into the production of ethylene and benzene. It has a large production complex of 1 million tonnes/year of styrene at Terneuzen, the Netherlands, and a large production plant of about 650 thousand tonnes/year at Freeport, Texas in the US. It also produces styrene at several locations in different parts of the world, in joint ventures or through local subsidiaries in South America, the Far East and China. And has polystyrene plants in even more countries. As part of a rationalisation programme Dow closed down its small styrene plants in Australia in 1994, and in Canada in 1999. But over the same period it expanded its styrene plant at Terneuzen in the Netherlands⁵, and took over a petrochemical site at Schkopau in former Eastern Germany⁶ where it built a 200 thousand tonnes/ year plant.

In the Middle East also Dow was a forerunner to be picked for a planned 500 thousand tonnes/year styrene plant which was later built in 2005, jointly with Petrochemical Industries Company of Kuwait⁷; Dow had already become an effective joint venture partner in existing petrochemical projects in the country upon acquiring Union Carbide and its global assets in 2001. Union Carbide had a joint venture in Kuwait for the production of ethylene, polyethylene and ethylene glycol since 1997. At one stage Dow and BASF

⁵ *Dow Hikes Capacity Around the Globe*, Chemical Week, 29 March, 1995. Also same journal issue of 9 October, 1996, p. 20; and 17 September, 1997, p. 36.

⁶ Chemical Week, 12 February, 1997, p. 16.

⁷ Chemical Week, 26 June, 2002, p. 14.

were considering a joint venture for a 500 thousand tonnes/year styrene plant in Brazil⁸, where it already has a small styrene plant at Camaçari, with a capacity of 150 thousand tonnes/year, and has styrene derivative projects in other parts of the country and in neighbouring countries. In the US Dow continues to lay claim to a world scale plant of nearly 600 thousand tonnes/year that has been under consideration since 1995⁹. Dow has allowed its share of the US styrene market to decline over the years, but it compensated by expanding its operations in other parts of the world. Although it is the largest world producer, Dow continues to be overall short on styrene, and resorts to buying on the merchant market to supplement its requirements of the product for its downstream operations.

Table 5.2: Production capacity of major world styrene producers.

(Thousand tonnes/year)					
Company	1995	%	Company	2003	%
Dow (US)	1844	10.4	Dow (US)	2224	9.4
Arco (US)	1145	6.4	Shell (UK)	2166	9.1
Shell (UK)	1110	6.2	BASF (Germany)	1618	6.8
BASF (Germany)	870	4.9	Lyondell (US)	1480	6.2
Cos-Mar (US)	862	4.8	TotalFinaELf (France)	1173	4.9
Chevron (US)	771	4.3	Nova (Canada)	1030	4.3
Sterling (US)	726	4.1	BP Amoco	803	3.4
Elf Atochem (France)	680	3.8	Sterling (US)	772	3.3
Huntsman (US)	672	3.8	Samsung (S. Korea)	670	2.8
EniChem (Italy)	550	3.1	Enichem	660	2.8
Others	8551	48.1	Others	11125	46.9
Total	17781	100	Total	23722	100

Shell also is a global styrene producer and has a wide geographical distribution of its production facilities. For a long time Shell was involved in the production of a wide range of styrene based products. In the mid 1970's Shell exited the US styrene market, but has continued to operate ethylene and benzene production units, which are closely integrated with its refining operations, in the country. A decade later it restarted styrene production at Scotford in Canada. In 1997 the facility was expanded to a capacity of 450

⁸Dow, BASF Plan World Scale Styrene Unit in Brazil, Chemical Week, 23 August, 2000, p. 7; also issue of 24 October, 2001, p. 12.

⁹Dow Hikes Capacity Around the Globe, Chemical Week, 29 March, 1995, p. 13, also Dow to build World Scale PO-SM Plant, Chemical Market Reporter, 17 July, 2000. p. 3

thousand tonnes/year. In addition Shell has styrene production plants in Germany, the Netherlands, Saudi Arabia and Singapore. In Germany, Shell operated Rheinische Olefin Werke a 50/50 joint venture with BASF, at Wesseling in the Nordrhein-Westfalen region, since the early 1970's. Its UK styrene operation at Carrington was phased out by 1978. A year later it started using its newly developed propylene oxide and styrene monomer (PO-SM) process at a 315 thousand tonnes/year plant at Moerdijk, in the Netherlands. In 1985 it established SADAF, a 50/50 joint venture with the Saudi Arabian Basic Industries Company (SABIC) for a 400 thousand tonnes/year plant at Jubail in Saudi Arabia. The project is set next to a large refinery which is also a joint venture between Shell and Petromin, a local oil refining company. The project was expanded in 2000 to more than 1 million tonnes/year. A third expansion¹⁰ for a plant with a capacity of 600 thousand tonnes/year is planned at the same site; it is expected to come on stream in 2006.

Shell employs the conventional ethylbenzene dehydrogenation technology at its projects in Canada, Germany and Saudi Arabia. In Singapore Shell uses its peroxidation technology, where in 1997 it established Seraya Chemicals, at Pulau Seraya where it also has a large refining complex. The project which has a capacity of 140 thousand tonnes/year of propylene oxide and 315 thousand tonnes/year of styrene, started as a joint venture between Shell (70%) and Yuka Seraya (30%) a subsidiary of Mitsubishi Petrochemical Company. Two years later the project came under Shell's total ownership when its partner pulled out¹¹. In 2002 the project capacity was upgraded to 400 thousand tonnes/year. In China Shell also operates a similar but larger PO-SM project which serves the growing requirements of the huge Chinese market.

Singapore serves as a refining and petrochemical regional centre for Shell in the Far East. It supplies styrene to its polystyrene and other styrenic plastics projects, as well as to its customers in the region. However, by 1999 Shell was preparing to sell off its polystyrene operations in Europe and around the world. It had already exited styrene and polystyrene production in the US, and by 2000 it completed the sale of its polystyrene

¹⁰ *SADAF to Award Technology License for Styrene Plant*, Chemical Week, 21 January, 2004.

¹¹ Chemical Week, 25 October, 1999, p. 3.

business to Nova¹² Chemicals of Canada. The deal included Shell's plants in the Netherlands, France, and the UK in Europe where it was the largest producer of expandable polystyrene, as well as its smaller plants in Chile, China and Japan. It kept smaller scale styrene copolymer operations for styrene based plastics and elastomers which command high margins. By the late 1990's Shell was concentrating on producing basic chemicals like ethylene, propylene and their derivatives, as well as benzene and styrene. Products that complement its refining operations, where it has an advantage in raw materials, or a technological lead in processes and specialised products.

Since the mid 1990's Shell has undertaken a massive capacity expansion drive for its styrene and propylene oxide operations around the world. In addition to its large scale plants in the Netherlands, Saudi Arabia, and Singapore, and building on its previous experience with BASF, Shell formed Basell a joint venture owned equally by the two companies. Basell brought on stream two world scale plants in succession; both plants use Shell's PO-SM technology, one at Moerdijk in the Netherlands with a styrene capacity of 565 thousand tonnes/year in 1999; and another in Singapore with a capacity of 550 thousand tonnes/year that started production in 2002. In the same year the two firms closed down their old plant at Wesseling, in Germany¹³.

BASF undertook another expansion in 2002 when it brought on stream a 550 thousand tonnes/year styrene plant that uses the conventional dehydrogenation technology at its petrochemical complex at Ludwigshafen, Germany. The new plant was simply a replacement of an older one, of about the same size, at the same site that was closed down a year later. Like Dow, BASF is a well integrated styrene producer with a product portfolio that covers a wide range of styrene derivative products, and is also integrated upstream into styrene raw materials. It has a presence in styrene production in China, where it has a small plant of 120 thousand tonnes/year; and in 2001 it acquired a 320 thousand tonnes/year styrene plant at Ulsan from a South Korean company¹⁴. BASF has a large styrenics complex nearby, that produces 500 thousand tonnes/year of polystyrene

¹²Chemical Week, 15 September, 1999, p. 22; also issue of 9 February, 2000, p 12. Shell was also selling off other chemical product lines as well; see Chemical Week issues of 2 June, 1999, and 23 and 30 December, 1998, p. 7.

¹³*BASF and Basell Trim Styrene Output*, Chemical Market Reporter, 4 February, 2002.

¹⁴*BASF Acquires Styrene Plant from SK Evertec*, 4 July, 2001, p. 26.

and styrene plastic products. It remains short on styrene in South Korea, and the Asian region due to its large scale production of derivatives in several countries. The company also has large scale styrene derivatives operations in the US (its styrene requirements in 1998 amounted to 500 thousand tonnes/year); and in South America it produces styrene in Brazil where it has a plant of 120 thousand tonnes/year, at Cubatao; but BASF is also active in styrene products in more than one country in the region.

Arco through its association with Halcon International, the engineering company that first developed the peroxidation process that produces propylene oxide and styrene, became within a few years the fourth largest producer of styrene in the world; and was able to acquire full ownership of the process in 1982. The first plant using this process was built in 1973 in Spain with a capacity of 80 thousand tonnes/year; and a second plant was built in Japan and operated by Nihon Oxirane in 1976 with a larger capacity of 225 thousand tonnes/year. In 1977 as the process technology for the process had become well established, a larger plant was constructed at Channelview, Texas, with a total capacity of 454 thousand tonnes/year, and operated by Oxirane, the joint venture between Arco and Halcon. When Oxirane became insolvent in 1982, the company was eventually taken over by Arco¹⁵. Arco also operated smaller styrene plants that use the ethylbenzene dehydrogenation technology in the US. The project was expanded in 1991 to 635 thousand tonnes/year; and in 1992 a second plant was built on the same site with a capacity of over 500 thousand tonnes/year in association with Nova and BASF who have a share in the styrene output. Arco's core business is in propylene oxide and butyl alcohols such as MTBE¹⁶ (a fuel additive), and remains mainly a merchant styrene

¹⁵Ralph Landau, the founder of Halcon Scientific Design, the company that was responsible for developing nine major chemical processes including the peroxidation process for propylene oxide and styrene, tells an interesting story of how this came about and how economics had triumphed over technology (Arora, Landau & Rosenberg 1998). Oxirane had been a successful joint venture operation with Arco; it had plants in the Spain, Japan, the US, and the Netherlands generating good profits. Halcon's contribution to their 50% share of the joint venture had been mainly their technology. To finance its share of the last large PO-SM plant built in Texas, Halcon took a loan of US\$ 230 million at 11% interest. By 1979 a new chief, Paul Volker, was appointed to the Federal Reserve Board who moved to limit the money supply and curb inflation, and interest rates rose suddenly to 21%. Oxirane's cash flow almost disappeared; and Halcon could not cope with the situation. They were forced to sell their share of the venture, and its technology back to their partner Arco (its main business being in oil) who had no difficulty in raising the cash. Dr. R. Landau who had been a pioneer process engineer learned his lesson, and later became a professor of Economics at Stanford University.

¹⁶It should be noted that the peroxidation process is flexible and can produce with propylene oxide, either styrene when ethylbenzene is used, or butyl alcohol when propylene with isobutene is used instead.

producer. Its polystyrene plants in Pennsylvania were sold to Nova in 1996. And in 1998 the company was taken over by Lyondell and capacity at its Channelview plants were expanded to 1.27 million tonnes/year. Since 1992 Arco had enjoyed the position of the largest styrene producer in the US.

However, Arco's attempts to become the world's largest styrene producer¹⁷ were thwarted by Shell's quick move to build a large plant at Moerdijk with BASF, as part of Basell which came on stream in 1999, a year before Arco's planned project.¹⁸; and a second plant for the Basell joint venture in Singapore in 2002. Dow also increased its capacity for propylene oxide at its plant at Stadt in Germany by 205 thousand tonnes/year, to 625 thousand tonnes/year in 1998. Following the settlement of its dispute with Repsol regarding the use of its technology, Repsol also brought its new styrene plant of 340 thousand tonnes/year on stream in 2000. Arco and its successor Lyondell, kept on delaying¹⁹ the construction of its PO-SM plant in Europe, as a result of the newly established capacity by its rivals, and its inability to secure raw materials and a partner to lift styrene from the project. The company also feared that the new capacity if badly timed may cause overcapacity in the European market. Eventually Lyondell formed a joint venture with Bayer and the project was commissioned in 2004, at Maasvlakte, the Netherlands, with a combined capacity of 285 thousand tonnes/year of propylene oxide and 635 thousand tonnes/year of styrene.

Chevron ranks among the top world styrene producers. It acquired Gulf Oil in 1985, together with its styrene plant at St. James, Louisiana. The plant has been expanded incrementally since 1991. Its capacity in 2003 was increased to 953 thousand tonnes/year. Chevron uses about half of its styrene output for polystyrene production, and sells the balance on the merchant market. It has a small styrene plant of 100 thousand tonnes/year in China. The company merged with Phillips Oil in 2000, in the wave of merger activity that was taking place in the oil sector, and its chemical arm now is known as Chevron Phillips Chemicals. The company has a large petrochemical plant in Saudi Arabia, where historically it had oil production activity. It owns this petrochemical project

¹⁷Chemical Week, 2 August, 1995, p. 14, also in issues of 27 March, 1996, p. 18, and 17 February, 1997, p. 4.

¹⁸Shell BASF Joint Venture Could Crimp Arco's Plans, Chemical Week, 28 February, 1996, p. 14.

¹⁹Lyondell May Move Proposed Arco PO-SM Plant to the US, Chemical Week, 17 August, 1998, p. 7.

that produces benzene among its products, jointly with local investors. Its output goes mainly for export. A second joint venture was formed with private investors in 2000 for producing benzene, ethylbenzene and a plant for 600 thousand tonnes/year of styrene²⁰. The new project is expected to come on stream at Jubail in 2006, with the bulk of its output destined for the export markets. Chevron has another large scale project for 500 thousand tonnes/year of styrene, planned for Venezuela in 2007. The project is part of a joint venture with Pequiven, a local subsidiary of the state oil company. The project will be located at Paraguaná where Chevron operates a large oil refinery, and its output will be mostly exported. When both of these projects become operational Chevron will increase its world styrene capacity to more than 1.5 million tonnes/year, thereby ensuring its position among the top styrene producers.

Nova Chemicals (a Canadian company involved in the production of styrene, and other basic chemicals and plastic products) emerged from being a small petrochemical producer in the 1970's to become one of the top producers of styrene and polystyrene in the world by the end of the 1990's. The company owns a styrene plant at Sarnia, in Canada, which the company expanded to 430 thousand tonnes/year in 1999. A year earlier, Nova merged with Huntsman giving it control over its styrene plant at Bayport, Texas, and its polystyrene operations around the world. It also took over polystyrene production facilities from Shell in 2000 making it one of the world's largest polystyrene producers as noted above. It also had an equity in Lyondell's second plant at Channelview, and a tolling arrangement, securing for itself long term supply of styrene product. Although Nova has large polystyrene production operations in Europe and Asia, it still lacks a styrene production base in Europe; but it has arrangements for swapping the product with other companies, by supplying them with styrene in the North American market.

BP took-over Amoco in 1994. It raised the capacity of its styrene plant at Texas City, Texas, by about 90 thousand tonnes to 453 thousand tonnes/year, but has not invested in any major styrene expansion in the US. Its position in that country has declined since the early 1980's. However, in 1999 BP Amoco acquired from Hüls a 350 thousand tonnes

²⁰ *CPChem to Build Styrene Plant in Saudi Arabia*, Chemical Market Reporter, 5 July, 2004, p. 2; and Chemical Week, 17 April, 2002, p. 9.

styrene plant at Marl in Germany. A year later it closed an old 140 thousand tonnes/year plant at Port Talbot in the UK, and signed a long term contract for the supply of styrene with Basell in the Netherlands, after some discussions about a similar arrangement with Arco's planned project at Rotterdam²¹.

The picture that emerges from the global styrene industry and its major players is one of growing markets, joint ventures and mergers, and continued technological development. It is a dynamic industry that has been changing, restructuring, and growing.

Also, there appears to be a reversal in the trend for diversification, that was followed by most companies in the 1970's and the 1980's, when chemical subsidiaries of major oil companies diversified downstream into petrochemical production; while major chemical producers such as Du Pont, Dow, Monsanto, and BASF among others, sought to acquire a foothold in oil exploration and refining operations when oil prices were rising in the mid and late 1970's, and together with the associated concerns about the reliability of their supply (Fayad & Motamen 1986). With the rising cost of developing new oil ventures in more recent times, several oil companies such as Shell and BP Amoco were shedding some of the less efficient chemical ventures to raise cash for oil operations²². By the late 1990's there was a return to the more specialised companies as opposed to the diversified ventures of the previous two decades²³. Nova also divested its energy related assets (such as its share in the Canadian natural gas liquids supplier Dynegy²⁴) at about the same time when it was completing its deals with Huntsman, and was in the process of acquiring Shell's assets for polystyrene. This was a necessary move for Nova, so that it can raise the required capital to enable it to finance the acquisition of styrene related assets from the two companies.

Chemical subsidiaries of major oil companies expanded their role as major suppliers of styrene product on the world market since the 1970's. Five of the eight largest styrene producers in the world are owned by oil companies. Shell made the largest strides in

²¹ *Shell-BP Styrene Deal Blurs Future of Baglan Bay*, Chemical Week, 13 December, 1996, p. 22; and Chemical Market Reporter, 11 November, 1996.

²² *Changing Perspectives: Restructuring and Global Strategies are Creating Opportunities for Petrochemical Companies in 21st Century*, Chemical Market Reporter, EPCA Supplement, September, 1999, p. 24.

²³ For an insight into the changing corporate strategies within chemical majors see the account of Chandler, Hikino & Mowery (1998) and Bower (1986).

²⁴ *Nova Completes Shell PS Deal, Says Goodbye to Dynegy*, Chemical Week, 9 February, 2000.

expanding globally during the 1990's. By 2003, it grew to rival Dow's position as the world's largest producer. It achieved this by exiting polystyrene production, and investing instead in the production of styrene and its raw materials; and by forming joint ventures in different countries. Shell located its modern and large scale plants in countries with growing markets, or where it had integrated refining operations like in the Netherlands, Saudi Arabia and Singapore. Lyondell, the successor to Arco which is an oil related company, also made a large leap becoming the fourth largest styrene producer in as little as about two decades. I discuss further the development of the US styrene market, and the role of the main producers in shaping the structure of the industry in that market, in Section 5.3 below.

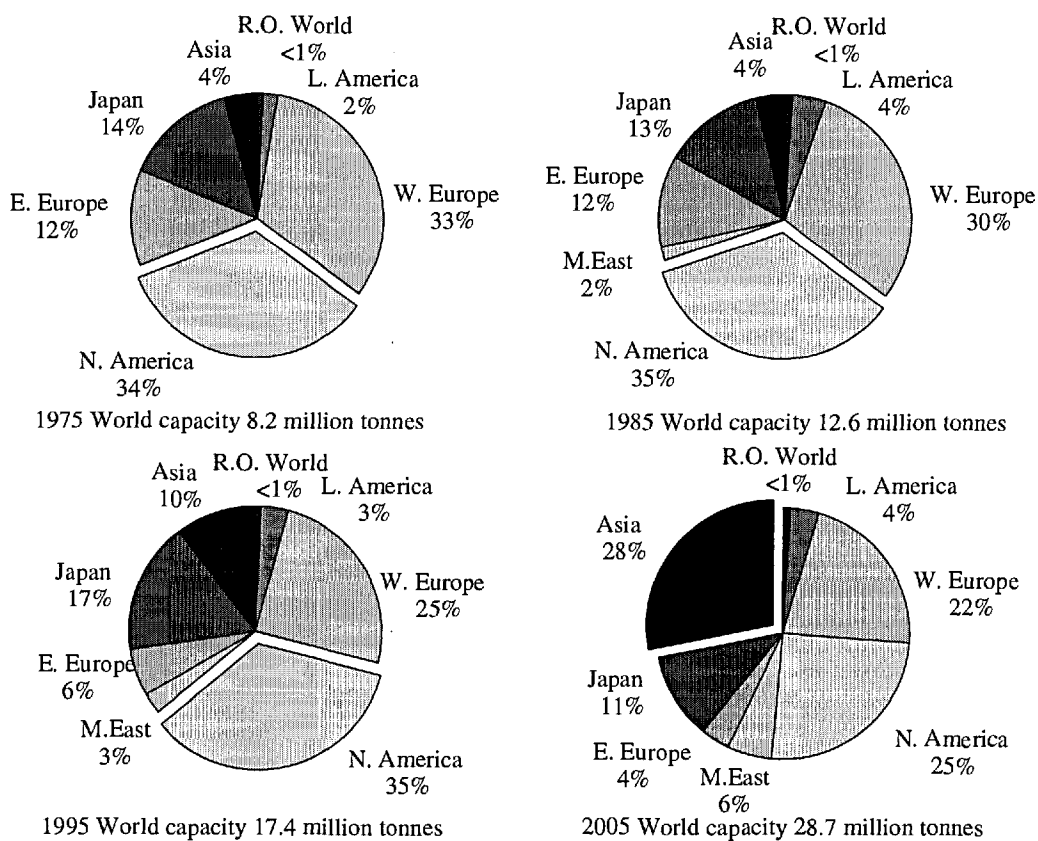


Figure 5.5: World styrene capacity by region.

5.2.3.2 World Regional Developments

The world styrene market grew from 8.2 million tonnes in 1975 to 17.4 million tonnes in 1995, and is expected to reach more than 28.7 million tonnes in 2005. An increase of 20 million tonnes within the pace of three decades. Figure 5.5 shows how different regions of the world fared over this period. The production and consumption of styrene and its products, has been concentrated since its development in the 1940's, in North America and Western Europe, where most of industrial activity and economic growth in the world had been taking place. The two regions accounted for 67% to 50% of world capacity until 1995. Japan, on the other hand, had a share which was roughly half the size of that of North America. Throughout this period North America, especially the US, was a net exporter of styrene to both Western Europe and Japan, with annual exports averaging between 0.5 million to 1 million tonnes/year. By the 1990's, N. American plants had on average larger capacities than their European counterparts, and were becoming smaller in number. Japan on the other hand, had a larger number of producers with smaller plants on average, than both their European and American counterparts.

In Western Europe, the Netherlands overtook Germany as the largest styrene producer by 1995. A few years later, in the small region between Rotterdam, and Antwerp (Belgium) the world's four leading styrene producers Dow, Shell, BASF and Lyondell were operating just over 50% of West European capacity. Germany's share in percentage terms, has steadily declined from its dominant position in the early 1970's, although its capacity has increased slightly in volume terms. The styrene producers are attracted to the Netherlands due to the extensive refining capacity, and developed petrochemical infrastructure, with good storage and transportation facilities for petrochemical and oil products in the region. Over the last decade, Spain has emerged as a large producer of styrene. Repsol, its major petrochemical producer now has a capacity of 0.5 million tonnes/year. Enichem in Italy has a capacity of 0.6 million tonnes/year, and Total-Fina-Elf has a capacity of 0.7 million tonnes/year in France. The French company also has a 50% share in a large styrene operation Cos-Mar, in the US, making it one of the large styrene producers in the world. Notwithstanding that BP and Shell are UK based companies, that rank among the top styrene producers in the world, the two companies

closed down their operations in the UK over the last few years. As indicated above, they moved their operations to other regions of the world with access to larger growing markets, and where production costs are more competitive. Styrene production has declined in the UK, from 420 thousand tonnes/year in 1975, to a mere 60 thousand tonnes/year at present.

While styrene markets in North America and Western Europe continued to grow, their share of world styrene production and consumption, was declining steadily notably since 1995. This trend was caused by enormous growth in Asia, excluding Japan, which had by this time become much faster, and larger in absolute terms as well, than any other region in the world. By 1997 styrene capacity in the countries of the Far East had exceeded that of Japan; and by 2003 Asia (excluding Japan), had become the largest market for styrene in the world, overtaking North America, and Western Europe. By 2005, this region had a total styrene capacity of about 8 million tonnes/year. Figure 5.5 illustrates, the changing structure of market shares in the different regions. The largest contributors to this growth were South Korea, Taiwan, China, Singapore and in later years Thailand. The South Korean styrene market approached in size that of Japan. Growth in these countries was driven by expanding industries, namely electronics and automotive production, as well as the rise in personal income and expenditure, which drive personal consumption of electrical and electronic goods, cars, packaging for food, and consumer goods in general. It has already been mentioned how the major styrene producers established production facilities in different countries in Asia, and contributed to the growth in supply of styrene. Local companies had also invested heavily in styrene production especially in South Korea, Taiwan, China and Thailand. Despite the massive scale of investment in styrene production in China, the country at present, continues to be a major importer of styrene, with imports of about 1-2 million tonnes/year.

5.3 The Styrene Industry Stages of Development & Restructuring

As an intermediate chemical product, styrene's fortunes are tied to the performance of its derivatives, mainly polystyrene, styrene copolymers (products such as ABS and other styrenic plastics), rubbers and other resins. Styrene has no other uses except for insignificant volumes as an additive in specialised oils, and antioxidants for rubber.

Polystyrene²⁵ was invented in 1925 by I.G. Farben²⁶ and production started on small commercial scale in 1930. Synthetic rubbers based on styrene followed, and production expanded during the war driven by the demand for tyre production and cable covers and related materials. Production at this stage was small; and the real demand take-off for these products did not arrive until after the war. Styrene products have been around for more than seventy years now. Although these products and their properties have changed and improved over the years they are essentially the same materials using styrene as a raw material. Like other innovations and products, the development of styrene can be seen to follow the innovation/product life cycle originated by Kuznets (1972, p. 74) and developed by van Duijn (1984). According to this theory, the product life cycle maps four phases of the known S-shaped growth curve, namely:

1. *Innovation and Introduction,*
2. *Growth,*
3. *Maturity,*
4. *Decline or Extended Maturity.*

²⁵Polystyrene was one of the first plastic products to be commercialised. It was developed at least two years ahead of PVC which was introduced in 1932; and before the other major plastics materials such as polyethylenes (LDPE in 1939; LLDPE and HDPE in 1958).

²⁶I. G. Farben the German chemical giant of the time, was responsible for the invention of many other chemical products. The period of the 1930's and 1940's was rich in innovation activity, which saw the invention of polyvinyl chloride and styrenic rubbers (I.G. Farben), polyethylene (ICI, Britain), nylon (Du Pont, USA), and silicones (Dow-Corning, USA). Innovation activity was not confined to chemicals only, but covered also other sectors as well, such as: synthetic fibres, oil refining, medicine, electronics and telecommunication, and aircraft engines (the interested reader could refer to Freeman (1982a, 1984) and Arora, Landau & Rosenberg (1998) for more on this topic). I. G. Farben was broken up into three companies following WW II, to form Bayer, BASF and Hoechst, which were later to become among the largest chemical companies in the world.

Life cycles of innovations and products vary widely due to the diverse nature of the products, and their markets. The length of the different phases can also be quite variable. van Duijn (1984) estimates the average product life cycle at about half a century or more even before it starts to decline; and Kuznets (1972) gives an average of about 20 years for the introductory phase of the product, and much more for its diffusion phase which includes its growth and maturity. Due to its versatile properties styrene markets continue to grow in the US, albeit at modest rates now, and at much more faster growth rates worldwide, especially in the developing economies of east Asia. It appears that styrene is going through an extended growth phase, with no market decline in sight, after nearly 80 years of its invention, and well over more than half a century of its commercialisation on a large scale.

There is an intricate relationship between the innovation and duration of product life cycle phases, technological activity and economic growth. Whole industries had been developed as a result of the early discoveries of synthetic resins and plastics, not only those that are involved in the production of these materials, but also the downstream producers of their derivatives for packaging, recreation products, construction materials, automotive and electronic parts and many more related industries that evolved around these products. R&D centres were also established to develop new processes, and products with better properties. Furthermore a substantial engineering industry developed around all of these industries from the large process engineering firms that build the chemical plants, to the makers of plastics machinery of injection moulding, blow moulding, and film making equipment that produce the plastic end products needed by everyday life uses, and the economy. With economic growth and infrastructural development during the 1950's and 1960's, there was a rise in the demand for capital products and rapid expansion of consumer goods; this generated a multiplier effect that lead to an expanding plastics industry and the chemicals needed to make them. The development of the chemical industry was driven by strong innovation and R&D *push* on the supply side, and equally strong *pull* on the demand side²⁷. The relevance of the product life cycle stages

²⁷This is a type of Schumpeterian model of innovation and product development. See Freeman (1982a) for a detailed account of the role of innovations and patents in the development of the chemical industry and its interaction with macroeconomic activity.

to the developments in the industry, is that the *growth* period attracts more producers to the industry and this impacts on the structure of the industry and its competitiveness, and on the innovation activity and regeneration of further demand for its products.

In his study of entry and market dynamics Geroski (1990) finds that entry takes place mostly during the growth stage of a product cycle, when swarms of firms enter a new market especially when entry barriers are low. But also that it is not confined to this stage, as considerable entry also takes place during the mature stage of a product when technology becomes more accessible and available for licensing. In this stage, the production process becomes standardised and the market large enough to absorb new producers with sufficiently large plants that utilise economies of scale. Innovative activity when combined with patents and know-how barriers can lead to concentration of markets as was the case in organic chemicals, synthetic fibres and pharmaceuticals. But also vigorous innovative activity can cause concentration to fall when several successful companies break into the market, by developing new products or alternative processes, a process that appears to have taken place in the synthetic fibres and plastics industry in the 1970's (Geroski 1990).

The development of the US styrene industry is reviewed below in three stages, that relate to the historical milestones that heralded important structural changes within the industry, than to any other major development. The progress of the styrene industry should be viewed in the context of the overall changes that were taking place in the petrochemical industry, and the interactive forces at work. For instance, techno-economic developments would continuously reshape the industry, where the roles of the main contenders change in lieu with market forces, and reflect the dynamics of the industry. Also the competitive positions of firms are influenced by their capital assets, know-how and technology in possession, and access to raw materials. Integration downstream insures marketing outlets, and generates value added for the firm's products. On the other hand increased competition may result from a rise in the number of market players, sluggish growth or a slack in demand; or may be from a strategy by one or more competitors to gain market share at the expense of their rivals. As a result, the new environment creates ripe conditions for a new wave of restructuring in the industry. One of the ways

in which the industry can restore profitability is when some firms under the pressure of market exit the industry, or when a wave of takeovers, and mergers reduces the number of players. The pressures of competition is eased upon the reorganisation of the market. Also at times the firms may find a variety of ways to collude either directly or indirectly, to overcome adverse market conditions. One way is when firms enter into illegal price fixing arrangements, or the formation of cartels that allocate market shares, and prop-up prices. The continuous interaction of these factors in a dynamic environment, help to shape the structure of the industry and affect the behaviour of its firms.

5.3.1 The Growth Phase: Pre-1974 Period

Following the Second World War there was rapid economic growth during the 1950's and 1960's driven by massive infrastructure projects at home as well as for reconstruction in Europe and Japan. The production of capital and consumer goods was at an all time high, and demand for synthetic materials and plastics products, which was growing at 2-3 times the rate of growth of GDP, was in double figures throughout this period. The production of cars, trucks, automotive equipment and tyres, witnessed a rapid increase. Also production of electrical and electronic products, household appliances, and numerous office equipment, all of which contain parts made from plastic and synthetic resins was rising rapidly. Demand for plastics and synthetic products and their chemical raw materials, such as styrene, was growing fast. Styrene showed a corresponding increase in production over this period. In the US its output rose from about 800 thousand tonnes in 1960, to 1.3 million tonnes in 1965, and 1.8 million tonnes in 1970. An increase of 2.5 fold over a span of one decade.

The number of styrene producers and their capacities over this period mirrored the rapid rise in demand. Eight styrene producers were already in operation in the US market in the 1950's. Of those only two, Dow Chemicals and Union Carbide, had large plants with capacities of 170-180 thousand tonnes/year. The rest had much smaller capacities in the range of 10-70 thousand tonnes/year. Over the next few years five new producers entered the styrene market; four of those were companies involved in oil refining, and three entered with large capacities: Amoco brought a plant on stream in 1965 with an

annual capacity of 363 thousand tonnes/year, while Cos-Mar entered the styrene market in 1968 with an annual capacity of 227 thousand tonnes/year, and Gulf Oil in 1971, with an annual capacity of 238 thousand tonnes/year. Suntime Refining brought a small plant (36 thousand tonnes/year) on stream in 1962, and Borg-Warner (61 thousand tonnes/year) in 1964. By 1973 the number of styrene producers had risen from eight to thirteen.

The involvement of some oil companies in styrene production stems in part from their chemical operations in benzene production which is closely tied to oil refining. Benzene is the main raw material for the production of ethylbenzene, the precursor for styrene. During this period, nearly half of the styrene capacity was controlled by companies involved in oil refining. These include: Amoco, Cosden, Fina (the joint venture partner in the Cos-Mar operation), Gulf, Shell, El-Paso, and Suntime Refining. This trend of oil companies involvement in the production of basic chemicals, and plastics materials will be reinforced in the 1970's following the oil price rises that changed the cost structure of production costs in favour of the oil companies.

5.3.2 The Turbulent Years: 1974-1990

At the same time as the oil price rises of the 1970's were taking effect, two other driving trends were at work that will have a profound impact on the petrochemical industry: (a) the slowdown in the rate of growth of demand for petrochemicals and plastics materials, and (b) the developments at the technological front which lead engineering companies to successfully build plants with much larger capacities than before. Following the rapid growth of synthetic materials in the 1960's, and their wide acceptance as substitutes of traditional materials, as well as for many new applications in different sectors of the economy, petrochemical markets were becoming mature. By the early 1970's economic growth was showing signs of slowing down. Production of petrochemicals and manufacturing output even declined in the recessionary year of 1975. But this was only a temporary blip, growth recovered quickly the following year. It was realised by now that petrochemical markets would grow at more modest rates than in previous decades, and prices and profit margin were on a downward trend. However, new producers were still

being attracted into the highly profitable US petrochemical market.

Several factors contributed to reduce production costs, and would eventually play a part in dampening prices. Firms benefited from scale economies of larger capacity plants, in that the investment and fixed costs per tonne of product were drastically reduced. Also, learning effects together with advances in process design and catalysts meant that styrene producers could get higher yields from their plants, and lower their costs. Overcapacity was becoming more evident in the industry as new capacity additions were becoming larger in size and demand growth was slowing with the increased maturity of end-use markets. Additional entry intensified the effects of overcapacity especially during periods of economic downturn; as the demand for styrene and other basic petrochemicals had now become closely tied to the business cycle. In such an environment competition between the market players is heightened, and price levels and profit margins tend to suffer as a result.

Two major petrochemical companies entered the styrene industry on a large scale. Hoechst started styrene production in 1980 with a plant of 408 thousand tonnes/year, at Bayport, Texas. And Arco brought on stream a large plant in 1981, with an annual capacity of 454 thousand tonnes at Channelview, Texas. A new technology was used at this plant, originally designed for the production of propylene oxide, which also co-produces styrene²⁸. More styrene is produced than propylene oxide, the ratio being about 2.4 tonnes of styrene for each tonne of propylene oxide produced; although Arco's main interest is in propylene oxide, as part of its large urethane chemicals, one of its core business units.

The two companies followed different paths of development. Arco a producer of basic petrochemical products, purchased Sinclair-Kopper's 100 thousand tonnes/year plant at Monaca, Pennsylvania, and would continue to expand its styrene operations in later years. On the other hand, Hoechst was already established as a diversified chemical company and one of the largest in the world; its operations cover a wide range of products, in performance and speciality chemicals, pharmaceuticals as well as plastics, fibres, and basic

²⁸The technology was developed by Halcon a US chemical engineering company, and was first commercialised in Spain in 1974, in a joint venture between Atlantic Richfield, Arco's mother company, and Empresa Nacional del Petroleo. This venture would operate under Repsol Quimica the Spanish petrochemical producer.

chemicals. In 1988 it merged with Foster Grant, acquiring its 400 thousand tonnes/year styrene plant at Baton Rouge, Louisiana. But a year later Hoechst exited the styrene business by selling its Bayport plant to Huntsman Chemical, and the Baton Rouge plant to Deltech Corporation. Both Huntsman and Deltech were essentially polystyrene producers that found an opportunity to move upstream into styrene production. Deltech as mentioned earlier did not manage to get into styrene production, and the styrene plant it acquired was kept idle for a long time.

Three more major chemical companies exited the styrene industry during this period. Shell shut down its plant in Torrance, California, in 1974. It exited all styrene and its major derivatives operations in the US; although it continued its involvement in the large scale production of the main raw materials ethylene and benzene, and high value added styrenic products such as latexes and specialised styrene based plastics. In 1984 Shell reestablished a large styrene production base at Scotford, Canada, with a 306 thousand tonnes/year plant making it the largest styrene producer in the country. Union Carbide also, closed down its 136 thousand tonnes/year plant in Texas, in 1979. Monsanto, on the other hand sold its large styrene operation (680 thousand tonnes/year) also in Texas, to Sterling Chemicals in 1986.

Further rationalisation of styrene capacity took place involving the following companies. Gulf Chemicals' 272 thousand tonnes/year plant at St. James, Louisiana was sold to Chevron's chemical subsidiary in 1984. And El Paso sold its 145 thousand tonnes/year plant at Odessa, Texas to Rexene Corporation in 1989. Several other smaller ventures also were closed down or exchanged hands. Sinclair-Koppers' other styrene plant at Houston, Texas was sold to United States Steel company in 1980; but the latter exited the styrene industry completely by 1983, closing down its small operation. Borg-Warner, Cosden and Suntime-Refining also exited styrene production by closing down their small operations, all of which were located in Texas, the centre of oil refining and petrochemical production in the US.

But what lies behind this considerable activity in the styrene industry? The process of entry and exit into markets, and the survival rate of firms is a rather complex process; there is a large body of literature on this topic which has been studied by Geroski

(1991a, 1991b, 1995), Gort & Klepper (1982, 1996, 2000), Scherer & Ross (1990), and Sutton (1998). However in the context of the developments of the petrochemical industry during the period under study, the causes of increased activity can be traced to three main factors. First, the rise into prominence of the role of financial markets in investment funding in the 1980's, gave rise to increasing pressure on corporations for more efficiency, and the requirement for higher returns on capital. Stock markets were experiencing phenomenal growth and returns; chemical companies had to satisfy their shareholders with adequate returns on their stocks. This led to particular emphasis on the virtues of comparative advantage and specialisation of firms. A wave of takeovers, mergers and reorganisation of product lines and corporate structures swept the petrochemical industry, and affected all industrial sectors as well²⁹. Second, there was a high degree of uncertainty about security of raw materials supply and concern about higher energy costs at the time. The larger firms established integrated production operations by expanding upstream into raw materials; and moved into downstream markets as well, to secure higher added value for their products. Some of the main styrene producers for instance got involved in styrene derivatives like polystyrene and other products, as well as investing in processing and fabrication operations such as packaging products, or autoparts and the like. Only Hoechst among the major players was not integrated into the production of styrene precursors, and purchased all its ethylene and benzene requirements on the merchant market or through long-term contracts. Third, the pressures of increased competition combined with emergence of large scale operations squeezed smaller producers by forcing them out of the market, or by acquiring their operations all together by rivals.

This period was marked by two major entries into the styrene industry. Arco, and Hoechst which sold its operations to Huntsman by the end of this period; and by the exit of four major petrochemical producers (Shell, Union Carbide, Monsanto and Hoechst). The number of producers rose at first, which heightened the competition in the market, but then declined to 8 producers. Total capacity closures amounted to more than 700 thousand tonnes/year, and a further 260 thousand tonnes/year (Dow's Midland, MI,

²⁹For an account of these developments in the petrochemical industry, and corporate priorities of management at the time see Bower (1986), Stobaugh (1988), and Arora, Landau & Rosenberg (1998).

plant and Arco's plant at Pennsylvania) were placed on standby bringing total retired capacity to about 1 million tonnes. But all major existing companies, notably Arco, increased their capacity over the period. Capacity rose by 1.9 million tonnes, from 2.4 million tonnes/year in 1974 to 4.3 million tonnes/year in 1990.

Strategic decisions that affect the direction of future development of the four major petrochemical producers lay behind their exit from the styrene industry. Shell and Union Carbide are both strong in basic petrochemical products in terms of production capability and technological base in these areas. The two firms are among the world leaders in these products and license their technology for many products, to third parties on a global scale. They exited styrene and concentrated on polyolefins such as polyethylenes and polypropylene plastics, ethylene oxide and ethylene glycols. Union Carbide continued also to operate vinyl acetate production which is an ethylene based product; while Shell exited styrene production in the US market, but reestablished a base across the border in Canada a few years later, and retained its position as a major global producer of styrene.

On the other hand, Hoechst and Monsanto moved in the opposite direction; like many companies that abandoned basic petrochemicals and moved downstream into higher value added products. Both companies concentrated on life science products which were seen as having high growth potential, and promising new markets in the nineties and beyond. Hoechst with its strong base in pharmaceuticals and speciality chemicals concentrated its operations in these areas, and a few years later would spin Aventis (a joint venture formed in 1999 between Hoechst and Rhone Poulanc of France) a life science company. Monsanto sold its petrochemical operations around the world and concentrated on establishing a strong R&D base since the mid 1980's, to become a leader in genetically modified foods and agrochemicals.

5.3.3 Consolidation Period: Post 1990

Merger and takeover activity continued, albeit with a lower number of transactions, but on a larger scale and level than before. The companies and assets involved at this stage were much larger than during the 1980's and 1990's. Also new capacity additions were significant in terms of volume, especially during the opening years of the decade, and

following the exceptionally high performance of 1988, when a wide range of petrochemical products achieved record profits. Over a million tonnes of new capacity was built by three producers. Arco's new plant of 510 thousand tonnes/year was built in 1992 on the same site at Channelview, Texas. In the same year, Chevron also brought on stream a large plant with a capacity of more than 410 thousand tonnes/year at the site of its existing operations in Louisiana, thereby more than doubling its capacity, which rose to 771 thousand tonnes/year. Westlake Styrene Corporation entered the market as a new player, with a small capacity of 160 thousand tonnes/year. Westlake was formed as a joint venture between Taita Chemical Company (Taiwan), BTR Nylex (Australia), Sumitomo (Japan) and the Westlake Group (USA). Most of the production at Westlake is exported to the the shareholders of the company, who are involved in the production of styrene derivatives in the Far East, with very small sales in the US styrene merchant market. Also smaller capacity expansions were made by Cos-Mar, Huntsman and Sterling in 1991 which totalled about 350 thousand tonnes/year.

Three major takeovers took place during 1998 and 1999, involving Lyondell Chemical Company³⁰ which acquired Arco Chemicals entire assets including styrene. Arco had already shut its old facility at Monaca, Pennsylvania in 1992. The other takeover was by Nova Chemicals which is a Canadian based company. It entered the US styrene market in a big way when it purchased from Huntsman its styrene plant located at Bayport, Texas in December 1998³¹. Nova was already established as a large polystyrene producer in the US market. A year earlier Huntsman had acquired Rexene's small styrene plant at Odessa, Texas that was not involved in this takeover; and it continued to operate independently until it was closed down in 2002. The third move involved BP's merger with Amoco which took place in 1999; and this involved oil operations as well as chemicals including styrene. Following the takeover, Nova was quoted as reviving plans to debottleneck the facility at Bayport, Texas, to increase its capacity to 681 thousand tonnes/year. The original plan to expand capacity dates back to 1996 when it was announced by Huntsman at the time that it was planning this capacity expansion, but was shelved a

³⁰Lyondell is also owned by Atlantic Richfield Corporation, the main shareholder of Arco.

³¹Chemical Week, 24 February, 1999, p. 8.

Table 5.3: Production capacity growth of major US styrene producers^(a).

(Thousand tonnes/year)								
Company	1960	%	Company	1975	%	Company	1985	%
Dow	254	35.4	Monsanto	590	21.9	Monsanto	680	18.9
Monsanto	172	24.0	Foster Grant	399	14.8	Dow	590	16.4
Sinclair	79	11.0	Amoco	381	14.1	Cos-Mar	590	16.4
Shell	73	10.2	Dow	295	10.9	Arco	554	15.4
Union Carbide	54	7.5	Cos-Mar	270	10.0	Hoechst	408	11.3
Foster Grant	54	7.5	Sinclair	254	9.4	Amoco	363	10.1
El Paso	23	3.2	Gulf	238	8.8	Chevron	272	7.6
Cosden	9	1.3	Union Carbide	136	5.0	El Paso	145	4.0
—	—	—	Others	132	4.9	—	—	—
Total	718	100	Total	2695	100	Total	3602	100
$C_4^{(b)}$	—	80.5	C_4	—	61.8	C_4	—	67.0
C_6	—	95.5	C_6	—	81.2	C_6	—	88.4

Company	1995	%	Company	2000	%	Company	2005	%
Arco	1145	21.4	Lyondell	1270	22.1	Lyondell	1270	18.7
Cos-Mar	906	17.0	Cos-Mar	906	15.8	Dow	1219	18.0
Sterling	726	13.6	Sterling	772	13.4	Cos-Mar	1136	16.8
Chevron	680	12.7	Chevron	726	13.4	Chevron	953	14.1
Dow	644	12.1	Dow	644	11.2	Nova	794	11.7
Huntsman	567	10.6	Nova	600	10.5	Sterling	772	11.4
Amoco	366	6.9	BP-Amoco	453	7.9	BP-Amoco	453	6.7
Westlake	160	3.0	Westlake	180	3.1	Westlake	180	2.7
Rexene	145	2.7	Huntsman	145	2.5	–	–	–
Total	5339	100	Total	5741	100	Total	6777	100
$C_4^{(b)}$	–	64.7	C_4	–	64.8	C_4	–	67.6
C_6	–	87.4	C_6	–	86.4	C_6	–	90.7

^(a) Capacities for 2005 include expansions that are planned, or under construction.

^(b) C_4 and C_6 refer to the concentration ratios of the largest 4 and 6 firms of industry capacity.

year later when world prices of styrene began to falter in 1997. Nova³² moved quickly a few years later following an explosion at its Bayport plant to announce further styrene capacity expansions. It started building a major addition to its plant on the same site that was expected to increase its existing capacity by about one third, making Nova one of the largest styrene producers in the US. When this plant came on stream in 2005, Nova³³ had amassed a total styrene capacity of 0.80 million tonnes/year. Cos-Mar also

³² *Styrene Market Quiet Following Nova Explosion*, Chemical Market Reporter, 23 June, 2003, p. 1.

³³ Nova also has a styrene plant at Sarnia, Ontario, in Canada with a capacity of 430 thousand tonnes/year. This gives the company a total capacity of 1.2 million tonnes/year to service its operations in the North American market, placing it on a par with Dow and Lyondell.

announced a large expansion equivalent to 25% of its existing capacity, which was to come on stream in the third quarter of 2004. Upon completion, its capacity rose to more than 1.1 million tonnes/year, making it the third largest styrene producer in the US after Lyondell and Dow which retain the lead position. With these developments taking place, Dow Chemicals started building a new plant at the Gulf Coast, Texas, with a rated capacity of 575 thousand tonnes/year. The plant as expected became fully operational in 2005. Dow had been planning for this project since 1995, but kept on postponing its implementation citing unsuitable market conditions. Upon completion of all these expansions, the US styrene capacity rose to an estimated 6.7 million tonnes/year by 2005.

Once again, since Huntsman exited in 2002, the US styrene industry is being served by eight producers. The number of producers in the market has since the 1970's, oscillated between eight to fourteen firms. By 2005 the largest producers, Lyondell, Dow, and Cos-Mar had comparable market shares, as measured by their capacities, of about 17%-18% of the US market. Those top three are followed closely by Chevron, Nova and Sterling, and with BP-Amoco and Westlake as distant followers. The capacity shares of the major US styrene producers over different time frames are depicted in Table 5.3. What is noticeable over this period, is that nearly 1.5 million tonnes of styrene capacity had been added since 1990, and a further one million tonnes was scheduled for completion by 2005. The net effect of the restructuring process following takeovers and mergers, has been a decline in the number of firms serving the styrene market; and the wave of expansions that followed each stage of restructuring resulted in the top four, or top six producers enlarging their market share each time. In 2005 the largest 6 styrene producers accounted for about 91% of the market, whereas in 1986 their share was a little less than 87%, and only 81% back in 1975; indicating the emergence of a tight oligopoly in the US styrene industry following its consolidation over the last decade.

5.4 Estimation of Inverse Demand for Styrene

Demand for styrene is derived demand. As an intermediate chemical product, styrene is used as a raw material for the production of a variety of other products that are needed by the economy. Polystyrene, styrenic plastics such as ABS, styrene based synthetic rubbers and latexes, and polyester resins are the main uses of styrene. The demand for these products is driven by the general wellbeing and performance of the economy. Polystyrene's are consumed mainly in packaging, the construction industry, and electrical and electronic equipment manufacture. The automotive sector, together with electrical equipment and cables industry, on the other hand, drive the demand for ABS plastics and synthetic rubbers. Unsaturated polyesters, which end up in paints and fibreglass, also are strongly influenced by the performance of automotive production, the construction industry, and the output of machinery and capital goods.

Like many primary and intermediate products such as metals and paper, demand for petrochemical products especially those at the commodity end of the market, is strongly influenced by the business cycle. fluctuating raw materials costs, the level of industry capacity utilisation, new entry into the market, or eminent large scale expansion of production capacity by existing producers impact on price variability. Also shifts in the competitive behaviour of firms brought about by a changing market structure, or the introduction of new more efficient technology would inevitably disturb prices in the market, and bring about a new equilibrium. As a result many of these products suffer from cyclical instability where the degree of price fluctuations is usually much higher than that for demand variability.³⁴

Two additional factors contribute to the market cyclicity, namely the short-run inelasticity of styrene demand to changes in price. The cost of styrene and its derivatives make only a small part of the cost of the final product which incorporates styrene. This in particular applies to electrical and electronic products, automotive parts and to many packaging applications that use polystyrene. In other applications there is little competitive alternatives that producers can switch to quickly. Styrene butadiene rubber

³⁴See the special issue of *Resources Policy*, Volume 17, 1991, and references therein for a review of this topic. Also Davutyan & Roberts (1994) provide some evidence of the interrelationship between the duration of metal price cycles and economic performance.

(SBR) which enters into the production of tyres and other rubber products, competes with natural rubber; and for unsaturated polyester producers have little choice but to use styrene which enters the manufacturing process in small proportions anyway. A rise in the price of styrene would therefore have a proportionally lower impact on the cost of the final products containing styrene. Unless styrene price rises are large and perceived to be permanent, consumers of styrene and its direct derivatives have little incentive to switch to alternatives, which in general would involve modifications to equipment and process of production at a not insignificant cost to manufacturers.

The other factor, is the inelasticity of supply of styrene product in the short-run. The supply of styrene by producers is constrained by their production capacity in place; as the level of production approaches capacity, it becomes more and more difficult, and costly for producers to expand their output. Modification of the plant in the form of debottlenecking³⁵, becomes necessary to expand output further; or it may even require a major capital investment in a new plant, which in the chemical industry on average, takes about 1-2 years to complete, and several years of planning prior to that. Expanding output in a big way is not always a feasible alternative in the short-run in petrochemical production.

This behaviour of cycling prices associated with shifts in demand and the extent of capacity utilisation, is characteristic of many petrochemical products, and commodity industries. It is seen in many primary metals such as copper, aluminium, lead and even gold. Figure 5.6 which is taken from Vogely (1985) depicts this process; an analysis of the price cycles of major primary metals can be found in Eggert (1991). The figure shows how price rises rapidly as output reaches full capacity utilisation. A shift in demand upwards from $D_m D_m$ to $D_p D_p$ due to a small output increase from X_m to X_p brings about a dramatic rise in price from P_m , to P_p which lies on the intersection of demand curve $D_p D_p$, and the supply curve as it approaches the capacity limit.

In general, the price of most petrochemical products rises significantly when production approaches full capacity utilisation. Sedriks (1994a, 1994b) observed a similar

³⁵The term debottlenecking is used in the petrochemical industry to describe a capacity expansion which involves a modification in the production process to circumvent bottlenecks that allow production increases by installing some additional equipment and process modification. It usually involves major but not very significant investment to achieve.

phenomenon for the petrochemical industry, where prices for many petrochemical products were observed to follow a 'hokey-stick' pattern. With price rising significantly when capacity utilisation rises above the 90% mark. In the paper industry, *anchor* price levied by producers, is achieved when strong demand prevails, which is when operating rates are sustained at more than 93% (Rich 1978). Christensen & Caves (1997) also confirm this threshold level for capacity utilisation in the paper and pulp industry, beyond which price rises significantly.

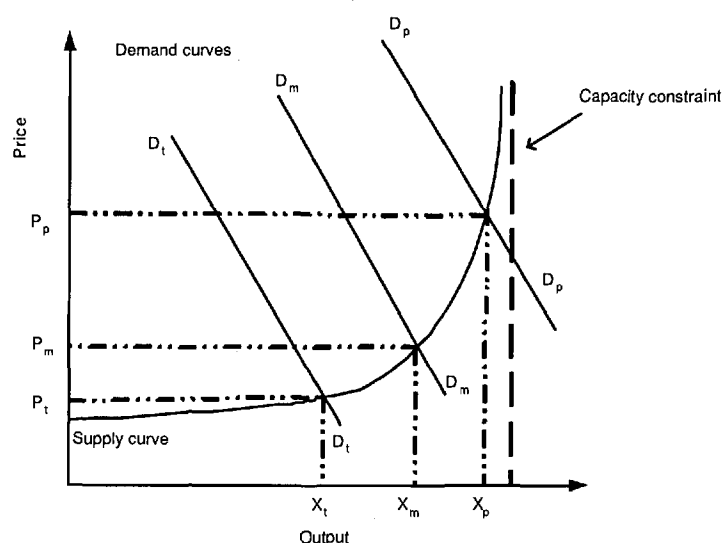


Figure 5.6: Cyclical demand and price instability in the petrochemical industry.

Different approaches have been pursued for estimating demand for oligopolistic industries. The most widely used is the econometric approach of evaluating demand as a function of GNP or GDP, industrial production, price of product and its competing alternatives, and some other factors that influence its consumption. Another approach involves input-output analysis originally pioneered by Leontief (1951) for economic planning and resource allocation. In petrochemicals, demand for the west European polyvinyl chloride industry has been econometrically modelled in logarithmic form (Paraskevopoulos 1988). And demand for several plastic products has been modelled using input-output techniques for the UK (Fayad & Motamen 1984).

Booth, Kanetkar, Vertinsky & Whistler (1991) estimate an inverse demand function

for newsprint in Canada using a logarithmic function relating consumption of newsprint to its wholesale price, the real price of substitute inputs and GDP. Steen & Salvanes (1999) on the other hand, estimate a demand function for an oligopoly fishery market in salmon for Norway. The model they use is a modified version of an earlier model that was first developed by Bresnahan (1982) and Lau (1982). While the aforementioned models may perform well for long run estimates of demand and prices, and may prove useful for forecasting purposes. They are all complex in structure, and are not suitable for our *linear quadratic* model. Most of these models involve non-linearities in the form of logarithms and lagged differences of the price, or other demand factors in their formulation, and cannot be incorporated in our dynamic model.

It is quite likely that the demand for styrene is nonlinear in the independent or explanatory variables that drive its consumption. But in keeping with the requirements of *linear quadratic* modelling, and the rich literature in this area, a linear downward sloping inverse demand function, of the form we have seen in previous chapters, is used. Our purpose here is not to estimate a demand function for forecasting purposes, but rather to facilitate, within reasonable and accurate bounds, the modelling of firm behaviour within the framework of a dynamic model. A linear downward sloping demand function is consistent with economic theory; where price declines as production increases, and more product is placed on the market.

The inverse demand model that is adopted in this thesis, has been studied in a game theoretic framework by many authors, including Boyer & Moreaux (1986), Reynolds (1986, 1987, 1991), Steele (1995), Slade (1994, 1995, 1996) and Vives (1988). It is given in equation (5.1), and below I explain its characteristics, and how it is estimated from industry data.

$$P(t) = a(t) - b(t)X(t) \quad (5.1)$$

where³⁶: $a(t) = a_0 e^{\alpha t}$, $b(t) = b_0 e^{\beta t}$, and $X(t) = \sum_{i=1}^n x_i(t)$. By expressing $a(t)$ and $b(t)$ in exponential form, we allow for nonlinearity and time variation. The inverse demand function can now shift in time, while retaining its linearity in the total output, $X(t)$, of

³⁶Gaskins, Jr. (1971, p. 309) uses a formulation for a dominant firm where the demand function involves a similar exponential term.

the major oligopoly players that dominate the styrene industry and determine its price. $x_i(t)$ is output of an individual major producer, and n is the number of those producers. Another property of this formulation of a and b is worth noting. At the initial time, $a = a_0$, and $b = b_0$; and because α and β are both negative (as we shall find later), as t increases, both a and b decline in value, and the price shows a falling trend. This is a property consistent with learning effects, where products prices fall with increasing cumulative output.

A variant of this formulation for the demand has been used by Lloyd (1979, p. 223), to demonstrate market penetration. He also shows how market saturation is obtained as t approaches infinity, and cumulative production stabilises at a constant value asymptotically. Rosenbaum & Ye (1992) utilise the same formulation to that used here for modelling titanium dioxide prices, in their effort to study Du Pont's investment behaviour in that industry³⁷. Below we follow the same steps to evaluate the inverse demand function as specified in equation (5.1) from industry data on output and prices for styrene; and then, using a simple ordinary least square (OLS) regression, determine the values for $a(t)$, and $b(t)$ for all periods. $X(t)$ is the total output of the six major US styrene producers, in any year t . The fringe firms are treated as price takers and therefore would not affect the market price³⁸. They just supply whatever product they can accommodate on the market at the prevailing market price dictated by the oligopolistic producers. Both $a(t)$ and $b(t)$ are nonlinear and time varying, they are estimated below from industry data on prices and production. Prices are measured in constant 1990 US dollars, and output in thousands of metric tonnes. By allowing $a(t)$ and $b(t)$ to be time varying, the demand function takes care of fluctuating demand which is witnessed in many petrochemical products markets. Figure 5.6 shows the shifts in demand and the associated price and output variations. When output rises to approach capacity in place, as point X_p , on the intersection of the supply curve and demand curve, $D_p D_p$, price rises dramatically to P_p . Starting from an estimate for the price elasticity of demand for styrene, ε , for the

³⁷Rosenbaum & Ye (1992) do not give any justification for using such a demand function, nor do they make any comment about its properties.

³⁸The underlying assumption here is that fringe producers and importers are price takers; and exports are assumed to be sold at the same price prevailing in the US market. The total output of the 6 major producers determines the price of styrene in the US market as an inverse demand function.

period under study, we can determine the values for $a(t)$ and $b(t)$, using equations (5.3), and (5.2). Since by definition, $\varepsilon \equiv -\frac{dX(t)}{dP(t)} \cdot \frac{P(t)}{X(t)}$, from (5.1) we estimate $\varepsilon = \frac{P(t)}{b(t)X(t)}$. This equation can be rearranged to give $b(t)$ in terms of the price of styrene, its price elasticity and output as shown in (5.2) below. Substituting this in equation (5.1) again and rearranging we obtain a relationship for $a(t)$ as in (5.3).

$$b(t) = \frac{P(t)}{\varepsilon X(t)} \quad (5.2)$$

$$a(t) = P(t)\left(1 + \frac{1}{\varepsilon}\right) \quad (5.3)$$

And since we assumed exponential relationships for both $a(t)$ and $b(t)$, we can run an OLS regression using the calculated yearly values we have just obtained, and the time index for the following two equations:

$$\ln(a(t)) = \alpha_0 + \alpha_1 t \quad (5.4)$$

$$\ln(b(t)) = \beta_0 + \beta_1 t \quad (5.5)$$

from the regression results we obtain the values for $\alpha_0 = \ln(\hat{a}_0)$, and $\alpha_1 = \hat{\alpha}$. And $\beta_0 = \ln(\hat{b}_0)$, and $\beta_1 = \hat{\beta}$. This defines the inverse demand function of (5.1) in terms of its parameters $a(t)$ and $b(t)$.

There are no published values for the price elasticity of demand for styrene that are readily available. But we know that styrene is now a *mature* product that has been in use in the US market since it was introduced on a large scale in the 1950's. And as indicated above, like other intermediate products, demand for styrene and its derivatives is inelastic, suggesting a price elasticity of demand $\varepsilon < 1$. It is likely that ε for styrene falls in the range exhibited by other similar intermediate products such as titanium dioxide, PVC or newsprint paper. Table (5.4) shows the price elasticity of demand values range from a low of 0.25 to 1.37, and for PVC at 0.76. For the base case runs of the model, I use a value of $\varepsilon = 0.70$. Other values are also tested, in which allowance is also made for $\varepsilon > 1$, to check how well the model performs in explaining industry developments if styrene demand were price elastic.

Table 5.4: Price elasticity of demand for various commodity industries^(a).

Author	Year	Industry	Elasticity, ϵ
Booth et. al.	1991	Newsprint	$-0.25 \sim -0.50$
Steele	1995	Newsprint	-0.42
Townsend & Uhler ^(b)	1986	Newsprint	$-0.66 \sim -1.37$
Rosenbaum and Ye	1992	TiO_2	-0.50
Hall	1990	TiO_2	$-0.25 \sim -0.44$
Paraskevopoulos	1988	PVC	-0.76

^(a)The price elasticities are negative, indicating a downward sloping demand curve. In equation (5.1) p. 214, demand for styrene is defined as downward sloping, and the price elasticity for styrene used is an absolute value.

^(b)Quoted in Steele (1995).

From the data series for the period 1974-1996 of annual styrene prices, the output of the six major styrene producers, $X(t)$, and the price elasticity of demand, estimated at $\epsilon = 0.70$, for the base case of the model, we calculate annual values for $b(t)$ and $a(t)$, from equations (5.2) and (5.3). These are shown in Table B.1 in Appendix B. It should be borne in mind that styrene importers, and the fringe firms are treated as price takers and therefore would not affect the market price. They just supply whatever product they can accommodate on the market at the prevailing market price dictated by the oligopolistic producers.

The OLS regression results for equations (5.3) and (5.2) using the yearly values we have just obtained for $a(t)$ and $b(t)$, are shown in Table 5.5. The statistics show that, the results are acceptable; although the R^2 value for the $\ln(a(t))$ equation are somewhat low in view of the short time series we have, and the large swings in prices. The Durbin-Watson values for the two equations are acceptable. And the t -statistic values, show that a two tailed t -test at the 1% significant level, rejects the null hypothesis of zero coefficients for both equations; the validity of the equation coefficients can therefore be ascertained with a good degree of confidence. The standard errors for the independent variables are shown in parentheses.

From these results the estimated values are obtained; for $\hat{a}(t)$, we have $\hat{a}_0 = \exp(\alpha_0) = e^{56.18}$, and $\hat{\alpha} = \alpha_1 = -0.0245$, and similarly for $\hat{b}(t)$, we get $\hat{b}_0 = \exp(\beta_0) = e^{136.48}$, and $\hat{\beta} = \beta_1 = -0.0692$. The parameters of the inverse demand equation given by (5.1), are

now defined for all periods from 1974 to 1996, and are given in Table B.2 in Appendix B. It can be seen from Figure 5.7 showing the values of estimated and actual prices for styrene, that the derived demand equation just estimated performs well. This equation can now be readily used in the model for the price of styrene.

Table 5.5: Regression results for styrene derived demand equations^(†).

$\ln(a(t))$	=	$56.1790 - 0.0245 \times t$
		(13.2318) (0.0066)
t -statistic		4.246 - 3.678
R^2	=	0.39, $F = 13.53$, $DW = 1.47$
$\ln(b(t))$	=	$136.4851 - 0.0692 \times t$
		(14.2217) (0.0072)
t -statistic		9.597 - 9.665
R^2	=	0.81, $F = 93.42$, $DW = 1.57$

(†) Figures in parenthesis are for standard errors

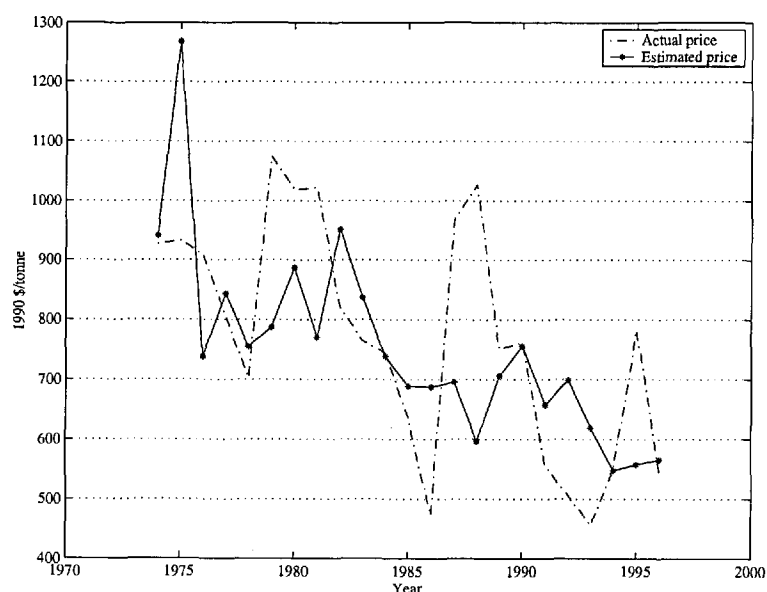


Figure 5.7: Styrene prices: Actual and estimated price 1974-1996.

5.5 Production Cost Estimation and Sources of Data

Production cost data and pricing are confidential. Several empirical studies use production costs data, that would have otherwise been proprietary, but had become available as a result of court cases. In a study of the aluminium industry, Reynolds (1986) relied on data for US aluminium industry that became available as a result of the infamous Alcoa³⁹ antitrust court case. He also made use of long-run average cost estimates per tonne of aluminium, from a world wide United Nations study covering a large number of aluminium smelters in various countries. Similarly, Rosenbaum & Ye (1992) study the investment behaviour of Du Pont in the titanium dioxide industry, utilising data which emanate from the court case against it. Paraskevopoulos (1988) used capacity and cost of production data from Chem Systems, also Henderson & Cool (2003) made use of data sets from Chem Systems and Tecnon for their empirical analysis of capacity expansions of the worldwide petrochemical industry; while Lieberman (1987a, 1987b, 1987c) relied on the database of SRI. On the other hand, Steele (1995) relied on capacity and investment cost data from a combination of annual reports of North American newsprint companies, government statistical reports, and from the chemical processing industry. These are some of the major industry consulting firms that publish multiclient and sometimes elaborate studies that provide a benchmark for industry planners to evaluate their performance and plan investments. These studies include supply curves, production costs, price projections and profitability profiles for different product groups. Some firms specialise in the study of metals, and even in a single product like aluminium (*Anthony Bird and Associates*). Producing companies and clients from industry who participate by supplying data and then by subscribing to these studies at substantial cost, have an incentive to do so. Their incentive is in having a benchmark against which they can rank themselves; and in having an independent and reliable third party that provides an overview of the industry and its developments. Firms can evaluate their cost and business competitive position against their rivals in the industry.

This thesis is not any different. I make use of the SRI databases. Styrene is one of the products that has been studied extensively and for which a good data set is avail-

³⁹The Aluminium Company of America. See also Scherer & Ross (1990) for a brief account.

able. Capacity on the firm level, and aggregate output data is available from the *World Petrochemical Program*; investment and production cost estimates based on engineering design, and process specific parameters, form the core of the *Process Engineering Program* which covers most of the major chemical products. Furthermore, perhaps justification for relying on consultants' data can be put in the words of Scherer (1995, p. 348) when commenting on a paper on industry productivity based on a study by McKinsey & Company:

“..I am led to conclude that the study must be taken quite seriously and that the marriage of economists' methodological skills with a consulting firm's vast knowledge of industry detail can be a fruitful one.”

The basis of arriving at these cost estimates is discussed further below.

5.5.1 Capacity & Output Estimation at the Firm Level

While capacity for individual plants held by petrochemical producers is published and is usually widely publicised, production data is available only at the country level. Government agencies, and specialised industry consultants collect this data, but it is published at the aggregate level.⁴⁰ Capacity and production data for styrene is based on the SRI World Petrochemical Program estimates, that are collected on annual basis and are widely used by the industry planners. Capacity data is available at the firm and plant level; each firm's capacity refers to end year capacities for individual plants held by the firm that are updated and aggregated as the effective rated capacity for each product.

These are then aggregated for each country and region of the world.⁴¹

⁴⁰In Europe also, associations such as the European Council for the Federations of the Chemical Industry (*CEFIC*) based in Brussels, collect capacity and output data from its chemical and petrochemical producer members. Production data on the country level only is published, and it is the stated policy of *CEFIC* to destroy production data on the company level, once the aggregates are produced.

⁴¹It must be emphasized however, that in many instances some plants are placed on standby, and are not in effective use by the firm. Plants may also be mothballed when they are put out of service for prolonged periods of time; before they are replaced or shut down completely. This information filters through via trade journal articles, and industry forums, and is usually known within that particular industry. Capacities for such plants is usually not included in the aggregated capacity for the year. Dow Chemicals placed its small styrene plant at Midland, Michigan on standby from 1981 until it was finally shut down in 1986. Until 2003, Deltech Corporation was still being reported to have a plant of 270 kt/year capacity at Baton Rouge, Louisiana, which had been dormant since mid 1980's. Only when there is an unusually high upturn in demand in a particular year, or if a problem arises with existing facilities such as an accident that puts an operating facility out of action, which is not uncommon in

The output of the six major styrene producers that will be needed to solve the dynamic model, is estimated based on the capacity held by each firm, multiplied by the industry average capacity utilisation rate for each year. The same data for output is also used in deriving the inverse demand function equations.

5.5.2 The Cost Function Form and Evaluation

In the study of the economics of the firm, and in the industrial organisation literature in general, the emphasis in studying production processes is on the use of production functions that describe the relationship between the output of the production process, or the firm, and its factor inputs, such as capital, labour and other inputs, that are determined by the type and nature of the production process involved. Data about the production process and the cost elements involved, are obtained from the technical information specific to the process. In economic theory, Cobb-Douglas and CES type production functions are predominantly used; other aspects pertaining to the production process such as whether increasing, decreasing, or constant returns to scale are involved, are also specified. For industry studies, the production function parameters are then determined by statistical analysis of time series data for the instrumental variables that are specified for the production function.

Alternatively, cost functions may be estimated and used directly. In practice, measuring cost curves is often more convenient than estimating production functions, since the accounting data are normally cast in money terms (Walters 1963). The use of engineering studies to estimate cost functions is not new. Chenery (1949) and Johnston (1952) were among the first to investigate the use of engineering data in industrial process functions. Chenery studied transmission of natural gas by pipeline by employing principles of gas engineering. He found a relationship for estimating the cost of transporting gas as a function of pipe diameter, its thickness, compression ratio and horsepower. Johnston (1952) estimated the cost of electricity generation for power plants, as a function of the output and size of plants, and generated different types of average and marginal cost

the petrochemical industry, that mothballed or standby plants are serviced and brought on stream. For instance Arco restarted its 100 kt/y styrene plant in Louisiana, and increased its capacity to nearly 500 kt/y at Channelview, Texas in 1988 to meet high demand (*Polysar Mulls styrene Restart*, European Chemical News, 25 April, 1988, p. 10).

curves. Moore (1959) applied similar techniques using chemical engineering principles, and found that there are increasing returns to scale for chemical plants. He also estimated that for each doubling in capacity, the plant investment costs rise by only 60%; which later became to be known as the 0.6 rule, that is widely used in the processing industries. Furthermore Smith (1961) provides a detailed study of the interrelationships between investment theory, cost and production functions and the engineering approaches. He shows how learning can lead to substantial reductions in labour requirement as cumulative output doubles.

Wej, Russell & Swatzlander (1979) show that in many chemical processes cost functions can be approximated, or estimated in a quadratic form. And Walters (1963, p. 45) discusses the use of a cost function which involves a penalty when output deviates from the optimal design output at which the short run average cost is minimised. Which is analogous to the concept of adjustment cost, and the formulation of the cost function used in this thesis.

More recently, Morrison (1988) applied a generalised Leontief restricted cost function in a study of US and Japanese manufacturing industries. She assumed quasi-fixed inputs cost function as a basis for estimation. Watkins & Berndt (1992) on the other hand, incorporate a quadratic cost function with adjustment costs in their investment model. Both are applied in a dynamic context. Morrison studies investment and infers capital-labour ratio variations between the US and Japanese economies. While Watkins & Berndt (1992) are concerned about capital investment in manufacturing and the implication for energy demand in the US. Also Karp & Perloff (1989, 1993b) use quadratic cost functions to study international trade in the rice and coffee markets in the context of game theoretic models for an oligopoly of exporters. They use the model to study the oligopolistic power of major exporters.

In developing the model for this chapter, I make use of cost functions, which are more adept for the study of an operational industry. This allows for the direct estimation of factor inputs involved in the production process. Hall (1990) follows a similar methodology in estimating production costs for the titanium dioxide industry. These factor inputs may vary somewhat, from firm to firm even if the same process is used, depending for

instance on operating conditions at the plant, the experience and technical know how of the operators, and the purity and quality of raw materials. They may also vary for the same firm over time as a result of how the process is run, or due to improvements or learning effects as more experience is gained by operating labour. For raw materials consumption average figures are used in this study, that are derived from engineering estimates of reaction kinetics of the production process in use, and corrected in accordance with available industry data and practice. The figures are based on the estimates of the *Process Economics Program*, (PEP), of SRI. For the purpose of economic evaluation it is assumed that firms utilising the same process have the same consumption figures of raw materials and face similar costs for factor inputs such as wages, number of operators and overhead costs.

For chemical plants it is reasonable to consider fixed proportions technology where consumption figures for materials, utilities, the number of operating labour required, and other production input requirements are known and may be based on engineering contractors data. The variations between companies do not warrant a significant impact on the economic cost estimate averages that are used in this evaluation.

In general, capacity estimates for chemical plants are rated on size of the reactor, and main processing equipment. The plant is operated at an optimal profile that yields the highest output, with the most efficient use of inputs, raw materials and least waste. It is possible, however, to increase the output beyond this optimal capacity, but this would be at a cost in the efficiency of the conversion of the raw materials, or higher consumption of other inputs such as utilities for instance. Similarly when the plant is operated over long periods, at utilisation rates well below the rated design capacity, unit production cost of output would be higher than when the plant is operated optimally (Wei, Russell & Swatzlander 1979). An allowance in the cost function is made to account for these variations which is similar in form to adjustment costs.

The cost function which has already been alluded to in Section 4.2.1, Chapter 4, is restated below:

$$C_i(x_i, k_i) = f_i x_i + m_i k_i + \frac{1}{2} g_i (x_i - k_i)^2 \quad (5.6)$$

The first term, $f_i x_i$, in the production cost function comprises the variable cost element; this accounts for costs such as raw materials, utilities and other inputs whose consumption, or requirements, vary in direct proportion to production. Where f_i is the aggregate cost of those inputs measured in constant 1990 US dollars per tonne of styrene produced; and x_i is firm i 's annual output of styrene in thousands of tonnes. Similarly, $m_i k_i$, is the associated fixed cost element, which covers costs that are incurred by the firm irrespective of variations in the level of production, these are usually measured in proportion to the scale of operations of the firm. Where, k_i is the capacity of firm i in year t , measured in thousands of tonnes, and m_i is the average fixed cost per tonne of styrene product, also measured in constant 1990 dollars. The estimated unit fixed, and variable costs for each firm, reflect the scale of operations and technology employed by the firm.

A breakdown of the investment costs, labour, raw materials requirements and other factor inputs, and the derived production costs per tonne of styrene are shown in Tables B.3 and B.4 in Appendix B. They refer to typical (hypothetical) plants that could have been built in the US in 1993. Table B.3 shows the production economics for a typical technologically advanced plant, employing latest developments in process technology, and benefiting from the largest economies of scale achievable in industry. While Table B.4 refers to the economics of older and smaller scale plants that are less efficient.

This simple form specification for the cost function allows for the inclusion in the analysis, firms that employ different processes and technologies. Since $f_i x_i$ is equivalent to the product of two vectors; that of the unit costs of each variable input, and the requirements for each input per tonne of styrene produced. And similarly the fixed cost, $m_i k_i$ can be represented by its constituent elements.

The last term in the above equation, is the adjustment cost, or penalty that the firm incurs. This term is zero when the firm is operating optimally, with its output equivalent to its design capacity; it increases dramatically when its output deviates substantially from its optimal capacity.

The short-run cost functions in the form indicated above, are U -shaped, a property consistent with economic theory assumptions. Figure 5.8 depicts a typical quadratic cost function with specified parameters. It shows the behaviour of the cost function

with different combinations of its variables. As can be seen production costs follow a *U*-shaped trajectory in capacity, and output along every vertical plane that cuts through the 3D diagram. The arrow in the figure indicates the optimal combinations of output and capacity that yield minimum cost.

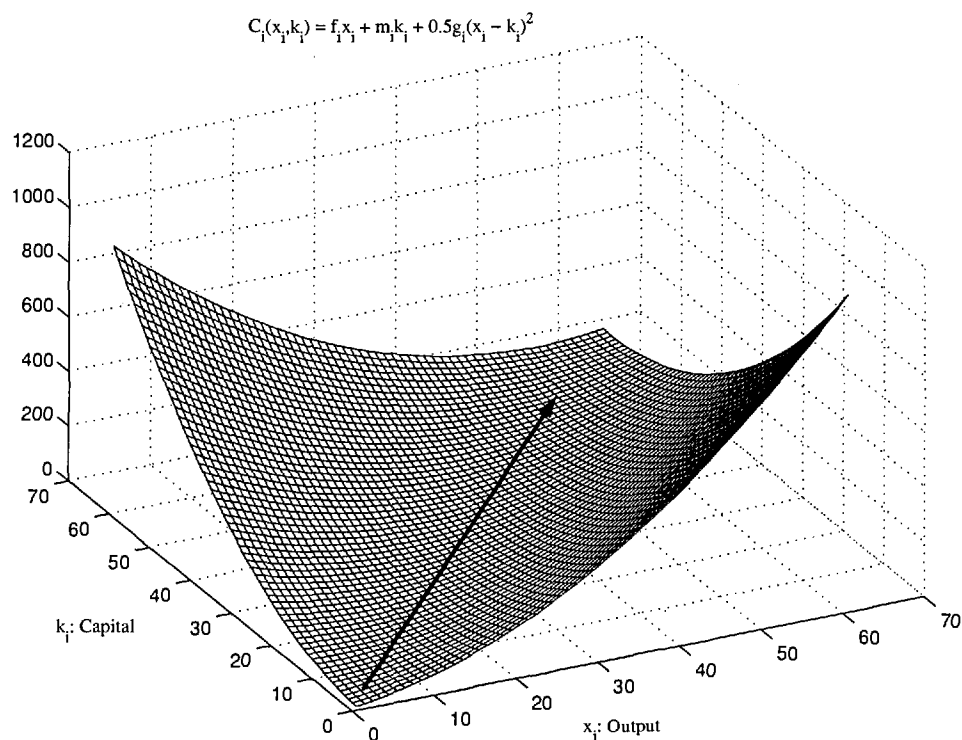


Figure 5.8: Variable cost function, with the following parameter values: $f = 8$, $m = 2$, $g = 2$.

5.5.3 Learning and the Experience Curve

The concept of *experience curve*, where unit production costs decline by a certain proportion with every doubling of output, resulting from the accumulation of experience, was first formalised by Hirsch (1952), and then vigorously seized upon and marketed as a management tool by the Boston Consulting Group (BCG), in the 1960's and 70's. BCG promoted this concept on the basis of industrial evidence to support the relationship between declining production costs and cumulative output. In addition, in many products it is found that prices decline with time and increasing cumulative output. By this time

the electronics and chemical industries, were subjected to intensive scrutiny for learning effects (Baden-Fuller 1983).

This phenomenon was observed earlier on, around the 1930's, in the aircraft manufacturing industries, where it was found that cumulative learning had a profound impact on workers efficiency in the production of aircraft frames. Several aircraft producing companies were already using the learning curve as a production planning instrument, to plan production schedules for new contracts on different models, and to negotiate their cost with the military (Andress 1954). Applications to other industries such as automobiles and computers followed suit (Abernathy & Wayne 1974).

In the machine tools industry Hirsch (1952) estimated that each doubling of cumulative output, was associated with a saving in direct labour of 18.5%, as a result of learning and progress in technical knowledge. The chemical industry, where economies of scale are present, due to the involvement of large scale operations in production processes of most bulk products, and where innovation, R&D, and technical progress are important features, is a prime candidate for the implementation of learning curve concepts to cost saving improvements at different stages of product development and production; whether these involve operational aspects in the form of more efficient use of raw materials, energy consumption, process design, or efficient use of labour and other essential resources. The study by Hirschmann (1964) is one of the earliest applications of learning curves in oil processing and chemical production industries. The main concern of these studies was the projection of production costs of products, and their prices. For managers this became a means for evaluating new investments, and a tool in the arsenal of planners contemplating entry into new markets.

In a study of 82 chemical products, Stobaugh & Townsend (1975) found evidence of learning effects on the long run trend of declining prices of chemical products. In addition to cumulative production, the scale of production operations, and the rise in the number of producers, were found to be the main contributing factors in affecting the decline in prices of chemical products. On average prices were found to decline by about 8% annually over the period of study which covered five years.

Ghemawat (1985) identifies three sources of cost reductions related to the experience

curve: exogenous factors, resulting from improvements in general and technical knowledge, the exploitation of economies of scale (this could be exogenous or endogenous), and basic improvements learned from the firms cumulated output (where the firm makes improvements in the way the production process operates; improves product design, and increased labour and factory efficiencies). Experience gained in-house through improvements resulting from the firms accumulated production, allows the firm to take advantage of the cost reductions in a strategic fashion. Du Pont's demonstrated this in its titanium dioxide chloride process it developed in the 1970's. This was a difficult process, and none of its competitors could come up with a similar efficient process. Du Pont took advantage of this situation by keeping on expanding capacity ahead of demand. By doing so, it benefited from the cost advantages it acquired, while its competitors had to climb a steep learning curve, and Du Pont effectively kept on expanding its market share at their expense (Ghemawat 1985).

Building investment strategy on the basis of anticipated cost reductions, however, could be disastrous. Several factors may contribute to this: mis-specifying the slope of the experience curve, overestimating demand growth prospects for the product, or underestimating the competitive position and investment behaviour of rivals. Ghemawat points out several examples where this happened in industry, including Monsanto's troubled acrylonitrile venture. The production process for this chemical product was characterised as having an 87% experience curve; to take advantage of this, Monsanto pursued a policy of rapid expansions in capacity, but the expected large cost reductions did not materialise, leaving the company with large losses.

But there were spectacular successes as well, especially in the electronics, and fibre optics industries. Experience curve based strategies, are most effective when a product is still in its rapid growth phase, and where product demand is price sensitive. In such a situation doubling of output takes place every few years, allowing the firm to reduce prices and propel further demand growth and output expansion.

5.5.3.1 Learning with Cumulative Output

The effect of learning gained through increasing cumulative production on cost reductions is captured by an exponential relationship, depicted in the equation below:

$$C = \Gamma X^{-\xi} \quad (5.7)$$

where C is the cost, and X is the cumulative output of the firm or the industry. These equations are usually evaluated from industry data. When plotted on a logarithmic graph equation (5.7) produces a downward sloping straight line with y-intercept Γ , and a slope of ξ . Most of the cost saving resulting from learning with cumulative production affects the direct fixed costs of production operations (which relate to cost item m_i in the production cost function of equation (5.6)). These costs pertain to items such as operating labour, maintenance labour and materials requirements, overhead costs and sales operations, as shown in Table 5.6. On the basis of styrene process data, covering the period since 1967 and measured in constant US dollars, the estimated values for $\Gamma = 1.12$, and $\xi = 0.35$, are obtained as shown in Figure 5.9. This indicates that the US styrene industry has an experience curve of $S = e^{-\xi \ln(2)} = 78\%$. Every doubling of cumulative output of styrene, results in a 22% reduction in fixed costs. In the decade from 1967 to 1977 when world styrene output increased by five fold, fixed costs were nearly halved in real terms from US\$ 109 to about US\$ 48 per tonne of styrene product respectively; by 1993 a considerable saving in costs was achieved, when it had dropped to just over US\$ 37. But as the industry becomes more mature and growth slows down, it takes longer and longer for output to double, and the extent of cost saving achievable diminishes, as production and process improvement possibilities are exhausted. On the basis of the experience curve for styrene fixed costs, a time series data set is generated for two types of plants. Producers with small plants employing old, average technology; and the second is typical of styrene producers with large scale plants utilising latest frontier technology. These costs were checked against data obtained through industry wide surveys conducted for companies involved in styrene production⁴². Many styrene

⁴²This was a detailed large study, conducted by SRI for several chemical products including styrene. Its aim was to construct industry supply curves, and estimate costs of production, and profitability profiles

Table 5.6: Styrene production costs and learning effects.^(a)

(1990 US \$/tonne styrene)

	1967	1973	1977	1993
<i>Plant Capacity (kt/y)</i>	<i>90</i>	<i>270</i>	<i>450</i>	<i>900</i>
Oper. Labour	31.05	10.93	6.99	2.72
Material Supplies	10.86	4.72	4.02	3.05
Overheads	38.71	13.04	10.32	6.36
Sales & Marketing	28.97	26.54	26.46	25.14
Total Fixed Costs	109.60	55.23	47.80	37.26
Utilities	12.68	6.77	6.54	17.43
Raw materials	486.16	495.25	501.33	473.16
Investment Costs ^(b)	75.83	32.12	23.86	21.00
Production Cost	684.26	589.37	579.52	548.85

^(a)The data refer to processes for the production of styrene starting from ethyl benzene, using similar technology of production but allowing for process improvements.

^(b) Investment costs are apportioned according to the plant capacity, and are equivalent to depreciation charge. They reflect both increases in efficiency due to increasing economies of scale and learning in process technology.

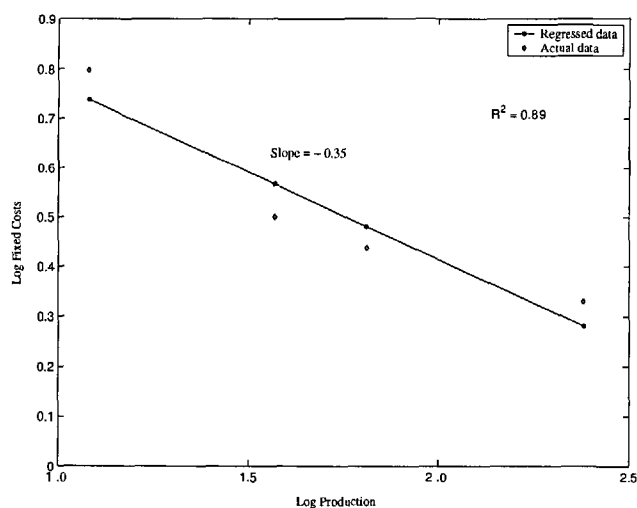


Figure 5.9: Fixed costs and learning effects for styrene production: 1967-1993.

producers operate more than one plant and these have different vintages. The fixed costs are illustrated for the two types of processes in columns (6) and (9) of Table 5.7. The production cost data are representative of plants in the top and lower ends of the for petrochemical producers.

distribution of plants in operation in industry. The fixed costs used in the model are based on plants operating at full capacity, rather than the average industry capacity utilisation rate. Deviations from full capacity are accounted for by the adjustment cost element, the last term of equation (5.6) p. 223, as discussed in Section 5.5.5 below. Like with many chemicals, process improvements and technological advances were made during this period which affected the performance of styrene plants. Raw materials and utilities consumption were reduced as a result of improvements in process and engineering design; and the introduction of new, more efficient catalysts. This has a positive influence on the variable cost element represented by f_i in equation (5.6). However due to fluctuating prices of raw materials and energy, the variable costs in columns (5) and (8) of Table 5.7 show more variability than fixed costs.

5.5.3.2 The Impact of Economies of Scale

The significant increases in operating labour productivity, and declining fixed costs was not only a result of learning and experience gained through years of operating styrene plants; but also due to economies of scale and rising styrene plant capacities. Improvements in design and technical advances together with continuous developments in process engineering and know how, allowed the exploitation of economies of scale, by pushing the frontier of styrene plant capacities further and further. Over the period from 1967 to 1993 styrene plant capacities have increased by ten fold, from 90 thousand tonnes/year to 900 thousand tonnes/year respectively. Larger plants require only marginal increases in the number of operators, maintenance labour and overhead support, to perform the same tasks as when operating smaller plants. This translates into considerable savings when projected into average cost per tonne styrene produced in the larger plant as shown in Table 5.6 (see also Tables B.3 and B.4 in Appendix B).

Economies of scale for capital investment obey similar laws of declining costs as the experience curve, exemplified by an equation analogous to equation (5.7):

$$\frac{C_i}{C_j} = \left(\frac{k_i}{k_j} \right)^\zeta \quad \text{where, } 0 < \zeta < 1. \quad (5.8)$$

Table 5.7: Styrene production costs based on experience curve estimates for straggler and pacesetter producers.⁽¹⁾

Straggler											Pacesetter									
Conv.		US	Variable					Fixed Cost		Fixed Cost		Variable		Fixed Cost		Fixed Cost				
Factor		CU	Styrene Price		Cost	100% CU	Avg. ICU		Cost		100% CU		Avg. ICU		Cost		100% CU		Avg. ICU	
Year	to 1990\$	(%)	US\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t	1990\$/t
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)										
1974	2.445	82	379	927	675	188	229	631	87	106										
1975	2.227	82	419	933	673	180	220	626	83	101										
1976	2.093	78	434	909	645	171	219	598	78	100										
1977	1.962	78	410	805	599	163	209	555	74	95										
1978	1.829	87	384	702	560	156	179	513	70	80										
1979	1.680	90	639	1074	732	149	166	689	67	74										
1980	1.541	78	661	1019	914	144	185	859	64	82										
1981	1.405	76	728	1022	896	139	183	844	62	82										
1982	1.321	72	622	821	747	134	186	702	59	82										
1983	1.271	83	602	765	707	130	157	666	57	69										
1984	1.226	93	606	743	616	126	135	581	55	59										
1985	1.191	94	534	635	541	123	131	514	54	57										
1986	1.160	100	408	473	389	119	119	368	52	52										
1987	1.125	98	860	967	489	116	118	465	50	51										
1988	1.089	104	944	1027	504	112	108	478	48	46										
1989	1.044	99	719	750	562	109	110	534	47	47										
1990	1.000	85	761	761	542	106	125	519	45	53										
1991	0.963	80	580	559	440	104	130	420	44	55										
1992	0.937	81	540	506	384	101	125	368	43	53										
1993	0.917	88	498	457	353	99	113	338	42	48										
1994	0.898	96	616	553	376	96	100	360	41	43										
1995	0.876	96	894	783	368	94	98	351	39	41										
1996	0.856	99	637	545	346	92	93	331	38	38										
1997	0.838	91	565	473	358	89	98	342	37	41										
1998	0.822	90	466	383	278	87	97	265	36	40										

⁽¹⁾ Production costs are based on processes starting from ethylene and benzene raw materials. Straggler producer refers to less efficient small scale producer; and pacesetter refers to producers operating larger and more modern plants.

⁽²⁾ CU refers to capacity utilisation, and ICU refers to industry capacity utilisation.

For styrene plants, the scale exponent, ζ , is in the range of 0.69 - 0.81. This indicates that building a plant with double the capacity, k_i , results in an increase of only 61-75% in investment cost. In processing industries, equipment capable of handling volumetric flows are involved, such as vessels, tanks, pumps and piping. Producing this type of machinery with double the size, requires a much smaller increase in the use of materials

and equipment. Infrastructure requirements, and design and construction are essentially the same, which explains the significant savings in capital investment costs for the larger plants. Silberston (1972) provides an account of the range of economies of scale factors from different industries and the significance of minimum efficient scale plants. However, the benefits from economies of scale are not limitless. In the same way that experience curves flatten out with the attainment of very large production outputs, the realisation of economies of scale are also subject, beyond a certain size, to diminishing returns (see Christensen & Greene (1976) for an analysis along these lines applied to the US electric power generation industry). Advances in process design and engineering of equipment, is subject to physical and technical constraints. Very large size reactors, vessels and equipment are limited by their structural stability. In the design of larger plants, an array of technical factors has to be taken into consideration, such as reaction kinetics, process control, and thermodynamic stability of the chemical processes which determine the maximum attainable size of reactors and main equipment in the plant.

Economic factors come into play and may form a limiting upper barrier on the economically efficient scale of plants. For instance beyond a certain point due to operating conditions, larger size equipment may require special design to reinforce structural stability, with increasing costs creeping in. Larger plants carry higher risks to the firm, in case of accidental, or natural hazards leading to disruption of production and loss of most of its production capability. Larger plants, and more concentrated industrial operations on a single site, are more complicated to manage and run. Labour productivity may also be lower in much larger institutions (Scherer & Ross 1990). The combination of these factors plays a leading role in determining the size of efficient scale of production. More often this is referred to as the minimum efficient scale (MES) plant. This is defined as the smallest scale plant at which minimum unit costs are attained. With technological advances in process engineering, chemical technology and process control, the size of MES plants continued to rise since the 1950's, but made larger leaps in the 1970's and beyond. In steel, cement, fertilisers and most process industries the size of plants increased several fold. In ethylene the main building block for the chemical industry, the size of plants rose from 200 thousand tonnes/year in the late 1960's, to 400-600 thousand tonnes/year

in the early 1980's, and 800 thousand tonnes/year by the 1990's. And presently it is possible for modern plants to be built with capacities approaching one million tonnes/year. Styrene also followed a similar trend. Plant capacities increased from about 100 thousand tonnes/year in the late 1960's, to about 300-450 thousand tonnes/year in the 1970's; and rising again in the 1990's to 900 thousand tonnes/year. Investment costs per tonne of styrene produced, declined from about US\$ 76 in the 1960's to US\$ 21 in the 1990's as depicted in Table 5.6.

However, the dominant cost element was raw materials costs. These costs were on average about 85% of the total cost of styrene production over the same period. Whereas investment costs represent on average between less than 4% to about 5.5% of styrene overall cost. Despite the fluctuations in the real cost of raw materials and utilities prices, which are in part related to the energy and basic petrochemicals prices;⁴³ the overall cost of styrene production continued to decline in real terms.⁴⁴

Firms realise the benefits of economies of scale that lead to falling unit costs with rising production, and larger plant sizes. When deciding on the size of their new investments, they take into consideration a range of techno-economic factors, that dictate the choice of plant capacity, which vary with product and process, and are dependent on the economic environment and the market characteristics in which they operate. The range of plant capacities brought on stream by the main styrene producers, over the years is indicative of this process and shows how plant capacities edged higher.

5.5.4 Investment Cost Estimation

In their large majority, petrochemical plants are designed, engineered and constructed by specialised engineering contracting firms that operate independently in the chemical industry. Some producers get involved in the basic design of the process and production

⁴³This is only partially the case since the supply/demand situation also affects the cost of ethylene and benzene, the two main raw materials for the production of styrene. On several occasions the prices of ethylene and benzene on the one hand, and energy prices would move in opposite directions as was the case in 1986-1988, and again in 1995, when chemical markets were tight.

⁴⁴This cost saving was not always passed on in the form of lower prices for styrene product, due to the oligopolistic nature of the industry and the integrated market structure where many styrene producers are vertically integrated into the production of ethylene and benzene raw materials; and also downstream into the production of styrene derivatives such as polystyrene and other products. Several of the larger styrene players are also involved in the fabrication and conversion business, and the consumer end of the market.

of proprietary catalyst, especially when a significant element of in-house process development has taken place. Several processes are available for licensing for the production of styrene; and equally a number of engineering contracting companies are active in the business of their construction on a global scale. Petrochemical plants are capital intensive and costly. Typically a greenfield plant costs several hundred millions of dollars, requires a few years in planning, and its construction takes 2-3 years to complete. Producers announce their new investments in advance, and publicise their planned capacities making clear their intentions to their rivals in a form of a signalling game. The capacities, and in most cases the costs, of plants under construction are published in trade journals such as Chemical Week, Hydrocarbon Processing, and European Chemical News. These characteristics suggest that a reasonably competitive environment exists for this segment of the industry. All firms face equal capital investment costs, *ceteris paribus*, when deciding on plans for new petrochemical projects such as styrene.

Petrochemical plant costs are tracked by a number of institutions that publish construction cost indices for the industry. These cost indices are based on detailed and disaggregated engineering estimates of the main equipment of petrochemical plants. They also account for escalating cost influences to which petrochemical plants are subject to, such as: inflationary pressure, market conditions for machinery suppliers and equipment vendors, and the extent of construction activity by main contractors. In this study, yearly unit investment costs for styrene plants are estimated on the basis of the PEP, (*Process Economics Programme*), Chemical Industry investment cost index published by SRI and is shown in column (2) of Table B.5, for the period under consideration. The index uses 1958 as the base year with a value of 100; by 1996 it stood at a value of 591. Starting from basic design, and process engineering of the major processes for styrene production, investment cost estimates are made using patents, published licensors documentation, and engineering contractors data. This detailed calculation is carried out for pivotal points in time (as shown in Table 5.6 for instance), where typical plant capacities, reflecting MES plant capacity for the period under consideration, are assumed. Investment costs for larger, and smaller plant capacities can then be made, scaling up or down by applying the appropriate scale factor exponent, as in equation (5.8) discussed in the previous

section. For the intervening years, the PEP construction cost index is used to estimate and update the investment costs to the current year. Unit capital investment costs, c_i for firm i which appear in the quadratic investment cost function that we discussed in Chapter 4, are generated by dividing total capital investment cost, C_i by plant annual capacity, k_i and are measured in US\$ per tonne styrene per year. The economic life of the plant is considered to be 10 years for economic evaluation purposes. The investment cost equation is restated below:

$$I_i = c_i u_i + \frac{1}{2} d_i (u_i - \delta k_i)^2 \quad (5.9)$$

Economies of scale are exogenously determined by the engineering companies serving the petrochemical industry. Two types of producers are considered in the model. They reflect the status of plant capacities that existed in the industry as it developed over the years. The largest four styrene producers operate modern *frontier* technology plants, that benefit from economies of scale with smaller unit investment costs. They are designated as *pacesetters*. While the remaining two producers operate smaller plants utilising older type technology referred to as *straggler* producers. The unit capital investment costs are converted into constant 1990 US dollars, by applying a GDP based deflator index. Table B.5 lists these values.

5.5.5 Adjustment Costs Revisited

The arguments justifying the inclusion of adjustment costs in cost functions, when quasi-fixed capacity is subjected to change as a result of capital investment, are not just steeped in theoretical formulations, but have recently been the subject of empirical evaluation and measurement in different economies and industrial sectors (for a survey of the recent literature on empirically determined adjustment costs see Chakrabarti (1999, 2004).

Further to the coverage of this topic in Section 3.1 of Chapter 3, there is a growing body of evidence suggesting that capital accumulation is subject to adjustment costs⁴⁵. Watkins (1993) estimates adjustment costs based on an empirical study of four sectors of

⁴⁵Rothschild (1971) stands out in arguing for concave adjustment costs, but provides no empirical evidence to this effect.

Canadian industry, covering: manufacturing, iron and steel, textiles, and transportation equipment. The study finds variations in adjustment costs among different sectors of industry; but on average, total manufacturing as a sector shows that adjustment costs constituted 23-33% of total costs. Also Carmichael & Ng (1992) find disparities in adjustment costs between regions for Canadian manufacturing industries. Firms incur a positive adjustment cost in the process of attaining long-run equilibrium. In a study of the Greek manufacturing industry over the period 1958-80, Zanas (1991) finds that as much as 8.5% of output is foregone by firms due to adjustment costs as new investments are undertaken. Adjustment costs levels, evidently, vary in different sectors of the economy. Also a study of 521 US firms (Chakrabarti 1999), shows significant differences in adjustment costs associated with the introduction of new plants and equipment, between capital-intensive and labour-intensive industries. The estimated cost of adjustment is significantly higher in the former than the latter.

Summers (1981) studies trends in the business cycle and capital formation using empirical data from the US stock market. He introduces adjustment costs into an investment cost function which he employs in the analysis of taxation policy, and the rate of cumulative corporate investment. His model examines the impact of changes in tax regimes and inflation, on savings and the rate of accumulation of capital. But his approach of introducing estimated adjustment costs has its critics as well; especially the notion of the ad hoc nature of these costs, and the proportionally high level of adjustment costs relative to total costs⁴⁶.

Fudenberg & Tirole (1992) argue that different firms have different abilities in processing and evaluating information, giving them a leadership role, as they are in a better position to exploit opportunities for investment, or in running their operations, than other firms. Considering this point from a different angle, it can be reasoned that firms in the same industry, do not necessarily face adjustment processes with the same adjustment costs; rather it would be reasonable to argue that more efficient firms have lower adjustment costs, imparted on them by their advanced information handling and man-

⁴⁶Barry Bosworth (Summers 1981, p. 131) criticises the ad hoc inclusion by Summers of an adjustment cost that rises with the rate of investment which is similar in principle to the models of Gould (1968), Lucas, Jr. (1967), and Treadway (1970). James Tobin (Summers 1981, p. 134) argues against the high proportion accounted for by the adjustment costs as these raise the total cost by a factor of 2-3 times.

agerial skills. For how else can one explain the changing fortunes of firms, their profit cycles, and of their lost opportunities, investment failures, and successful achievements, that are reported frequently in financial media? These are firms that operate in the same markets and have the same line of business, but differ in their performance.

In view of this, it would not be unreasonable to use in the investment model a range of adjustment cost parameters for different firms, instead of just applying a uniform value for all. This modification will introduce an element of differentiation into the adjustment process, and costs of firms, which does not impart a strong advantage in favour of the beneficiary firm, that the Stackelberg leadership role would.

The adjustment toward the long-run equilibrium takes place through successive short-run moves, and the adjustment cost covers the cost of the deviation between short-run and long-run equilibrium (Bernstein 1992, and Chakrabarti 1999, 2004). Quantity games are identified by Friedman (1986) as games of competition in capacity. In this study of capital accumulation, I am considering two types of adjustment as taking place. In the short-run the firms compete on production, and in the long-run they compete on capacity. In our model, the adjustment costs that the firms incur, for production and investment, are limited so that the cost of adjustment in any one year does not exceed 40% of the total cost for that year. So in the case of capital investment, as can be seen from equation (5.9), the value of adjustment cost term $\frac{1}{2}d_i(u_i - \delta k_i)^2$ does not exceed 40% of the value of I_i . It should be noted that this is the maximum value in the fixed cost differential between operating at full capacity, and at the average industry capacity utilisation for styrene producers (compare columns 6 and 7, and 9 and 10 of Table 5.7). In fact, the results of the model reveal that for most years, the values for the adjustment cost share are much lower. This is achieved by taking the adjustment cost coefficients, g_i , for the production cost function, of equation (5.6), and d_i the adjustment cost coefficient for investment of equation (5.9), as free parameters. The adjustment cost coefficients are used to calibrate the model. It was shown in Section 4.5.1 and Section 4.5.2 of Chapter 4, that the model results are robust for a wide range of values of g_i and d_i , and that these do not affect the equilibrium outcome or market shares⁴⁷. The market shares are more

⁴⁷The results for market shares have been verified, as can be figured out from the output and capacity values, but are not shown in Tables 4.1 and 4.2.

dependent on the decision rules governing the type of interaction between the firms.

5.6 Investment Behaviour in the US Styrene Industry

5.6.1 Setting the Case for Optimal Planning in the Styrene Industry

So far in this chapter I have developed and estimated the derived demand function for styrene, and the investment and production costs for typical styrene producers operating in the US market.

Demand is exogenously driven by economic growth, the demand for styrene derivatives, and exports. And all firms have full knowledge of how demand is evolving over time. So the price elasticity of demand for styrene, which is assumed to be constant, is known together with the parameters of the inverse demand function (that is $\epsilon = 0.70$, and the estimated parameters $\hat{a}$ and $\hat{b}$ of the inverse demand equation (5.1) p. 214, presented in Table B.2). Each firm is trying to maximise its net present value of profits over the time horizon from 1974 to 1996 covering a total of 23 years. The US is a net exporter of styrene to the rest of the world; and the largest six producers have on average accounted for 86% of the total US output. These firms operate in an oligopolistic market and must realise their interdependence. In a Nash-Cournot setting their joint output determines the price of styrene product. The firms do not need to observe the output of their rivals, each can observe the market price, and what is required is just knowledge of the mechanism dictating the price formation. That is knowledge of the demand function and its parameters. Because it was assumed that the price at which the firms sell is market clearing in every period, there are no inventories carried over from year to year.

How much should each firm produce to maximise its profits? This problem gets resolved by each firm solving an optimal control problem of finding the output that maximises the net present value of its profits over the period under consideration. But since the price is dependent on the total output of all producers, how much the firm produces also depends on the type of interaction with other firms, and how they behave. This behaviour is dictated by the rules defining the interplay between the firms and the conjectures they hold about each other. Which brings us back to the notion of

the information structure of the game, and the strategies that the firms follow (Başar & Olsder 1999, Fudenberg & Tirole 1983) that was discussed in Chapter 3. The firms act strategically in an attempt to influence the actions of their rivals when they utilise feedback strategies of output and investment; and non-strategically when they choose open-loop strategies. When one of the firms acts as a leader in the market, a Stackelberg model arises as in Section 4.4 of Chapter 4. The leader firm gets to act first, and decides on how much to produce, while the followers face the residual demand. If the market were price competitive, each firm would act as a price taker, and the problem facing the firm is as described in Section 4.3.2 of Chapter 4. These four scenarios describe the possible market environment in which the styrene producers interact in a quantity game. There may be other more complex relationships at work, such as that some firms switching between different strategies over time; like selecting between equilibria of competing on prices, and then on capacity additions (Kirman & Masson 1986, Davidson & Deneckere 1990, Benoît & Krishna 1987). Our attention however, is restricted to pure strategies in accordance with the rich theoretical and empirical literature of dynamic Nash-Cournot games (Başar & Olsder 1999, Fudenberg & Tirole 1992, Dockner 1988).

In the following section I solve for an optimal solution of the capacity expansion problem of these four models, utilising the production costs of the US styrene industry to see which model describes best the industry developments. By examining the results of these models, I could infer from the performance of that model which best describes the industry developments, the competitive environment and investment behaviour of the main participants in the styrene industry over the period of study. The performance of the models is examined in terms of how well they predict the following criteria: the development of capacity and firms' market shares, in meeting the production schedule, as well as the evolution of price and overcapacity over the 1974-1996 period.

In a dynamic environment, the firm faces changing costs and variable demand. Its capacity is subject to change as a result of depreciation, and additional new investment. The costs faced by the firms are changing because of the intertemporal nature of the industry, which is captured by the model, and the experience gained by the firms through involvement in production. The firms undergo a learning process which affects two cost

aspects:

1. Plant construction and engineering design, embodying latest technological improvements, and economies of scale that affect mostly investment costs (Section 5.5.3.2).
2. And experience gained in operating and running of the plants which affects both fixed costs, and materials consumption which in turn impacts on variable costs in a small way (Section 5.5.3.1).

The styrene industry cost savings of 22% in fixed costs, are not far from those reported by Hirsch (1952) for machine tools, and Hirschmann (1964) for oil and chemical processing industries. Both types of cost savings are allowed for in the model of this thesis. Fixed costs savings based on the experience curve for styrene, and gains in investment costs associated with economies of scale. Dynamic capacity expansion models that allow for learning include Paraskevopoulos (1988) for the West European PVC industry. Also Steele (1995) allows for economies of scale for the North American newsprint industry. Pineau & Murto (2000) evaluate investment costs for different types of power generation plants. Reynolds (1986) on the other hand, relies on long-run average cost curve function to evaluate production costs faced by individual firms, where all firms face the same costs that are time dependent.

Further to selecting their outputs, the firms have to decide on how much to invest to maintain an efficient, reliable capacity level, and to be able to meet the requirements of rising demand. Each firm, therefore, chooses two decision variables in each period, the output and the investment rate, which is represented by a control vector $\mathbf{u}'_{i,t} = (u_{i,t}, x_{i,t})$. Because of the presence of depreciation, the rate of change of capital from one period to the next is a function of the gross investment less the depreciation of capital, which is assumed to be at a rate of $\delta = 10\%$ per year. This is represented in matrix form as a state equation, shown below. The equation forms a constraint on the dynamic capital expansion problem for each firm:

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{u}_t$$

The firms maximise the flow of their profits over the life time of the project, which is

described by an objective function, J_i , where:

$$J_i = \max_{x_{i,t}, u_{i,t}} \sum_{t=1}^T \beta^{t-1} w_i(\mathbf{y}_t) \quad \forall i \in \mathbf{N} \quad (5.10)$$

in this context, by taking the entire planning horizon into consideration, and allowing the state to be subject to the state equation, the firms strive to find an optimal solution for their investment, and output rates, that maximises their profits in the long-run.

For the cases of open-loop and price taking strategies, the firms select the entire sequence of their optimal controls of output and investment strategies for the entire game. That is the maximisation in J_i , for each firm i , is subject to $\mathbf{u}_{i,1} \dots \mathbf{u}_{i,T}$, as was shown in Section 4.3.1 and Section 4.3.2 in Chapter 4. When firms, on the other hand, employ feedback Nash-Cournot (as in Section 4.2.1) and Stackelberg strategies, the objective function for each firm i is expressed as a value function $V_{i,t}(\mathbf{y}_{t-1})$, and is given by an equation equivalent to equation (4.4), p. 143:

$$V_{i,t}(\mathbf{y}_{t-1}) = \max_{x_{i,t}, u_{i,t}} (w_i(\mathbf{y}_t) + \beta V_{i,t+1}(\mathbf{y}_t)), \quad \forall i \in \mathbf{N}, \forall t \in [1, T] \quad (5.11)$$

It was assumed that the value function is quadratic in the state variable, $\mathbf{y}_{t-1}$ (given by equation (4.5)). The value function in the above equation describes in each period, t , the sum of the short-run profits, and long-run profits from the next period until the final period of the planning horizon. It was shown in Chapter 4, how the optimal path of the control variables describing the evolution of outputs and investments, for each of the four models is formulated and obtained. In all the cases the control variable, $\mathbf{u}_t$, for the game was found to be linear in the state variable, $\mathbf{y}_{t-1}$. While the optimal controls are similar in form, its determination in each case is dependent on the type of interaction that takes place between the firms in the market place, and the price formation. For the case when open-loop and price taking strategies are employed, the optimal strategy of output, and investment rates for all firms, over all periods of the game, $\mathbf{U}$, is obtained as a function of the state variable at the start of the game, $\mathbf{y}_0$. It was given by equation

(4.46), p. 156, shown again below:

$$\mathbf{u} = \mathbf{E}_t \mathbf{y}_0 + \mathbf{d}_t \quad (5.12)$$

As discussed in Section 4.3.1, the firms decide on their optimal strategies for the entire planning horizon from the outset. The optimal controls for the feedback Nash-Cournot and Stackelberg cases, on the other hand, as shown in equations (4.20), p. 146, and (4.76), p. 164, have also a similar form. However, the optimal strategy, $\mathbf{u}_t$, is now a vector of the optimal controls for all the firms in period t ; and is a linear function of the state variable in the previous period, $\mathbf{y}_{t-1}$. For the feedback Nash-Cournot game we have:

$$\mathbf{u}_t = \mathbf{E}_t \mathbf{y}_{t-1} + \mathbf{d}_t \quad (5.13)$$

and for the Stackelberg game we have:

$$\mathbf{u}_t = \mathbf{D}_t \mathbf{y}_{t-1} + \boldsymbol{\phi}_t \quad (5.14)$$

It is also obvious in the above equations, that the formation and dimensions of $\mathbf{E}_t$ and $\mathbf{d}_t$ for the open-loop and feedback Nash-Cournot strategies are not the same, a point that was emphasised in Section 4.2.2 and Section 4.3.1. And similarly $\mathbf{D}_t$ and $\boldsymbol{\phi}_t$ for the optimal controls of the Stackelberg strategy. Note also, that with feedback Nash-Cournot and Stackelberg optimal strategies, the optimal control is not taken over the entire planning horizon; but is robust and can respond to changes in the state from period to period, as it is a function of the state in previous period, $\mathbf{y}_{t-1}$, which is not the case in the open-loop Nash-Cournot strategies, where it is dependent on the initial state, and time. The firms can respond to the actions of their opponents in every period, as the game evolves. In a Stackelberg game, the optimal control has two components, $\mathbf{u}'_t = (\mathbf{u}^l_t, \mathbf{u}^f_t)$, respectively, the leader's vector of optimal controls, and the followers'. The followers take the leaders actions as given, and are accounted for in the formation of their optimal strategy. This was depicted in equation (4.75), on page 164 (see Section 4.4.3, Chapter 4).

Once the optimal strategies are obtained for each game, all that remains to be done is to implement the optimal strategy in the state equation. The optimal state, and therefore, the optimal capacity required in each period, may be obtained from the initial state, y_0 , at the start of the game, until the final period, T , in the game. With the optimal output, investment and capacity known in every period, for all four games, the main criteria for assessment of model performance, namely the evolution of styrene price, industry capacity and output, overcapacity, and market shares of industry major producers and fringe firms, may be evaluated. The pattern of investment behaviour by the main firms in the industry, is laid bare for assessment.

The importance of economies of scale, and increasing size of modern petrochemical plants has an important bearing on the presence of overcapacity in the industry. If the firms follow a competitive investment policy without any consideration for the actions of their rivals, the overall industry capacity would be expected to be quite high especially in later years when plant sizes were rising as exemplified by the MES plants. Overcapacity on the industry level in such a case should be particularly high, considering the given demand levels. Prices also would show discernable levels of decline as a result of firms competing on production in an effort to gain market share. Profit margins in the presence of such a strategy would be low for prolonged periods as a result.

Alternatively, when the firms allow for their rivals' investments in their planning, and are more accommodating towards them, the emerging overcapacity is expected to be on the low side despite the large size of modern plants. The firms do not have to cooperate directly; such a pattern may occur non-cooperatively through a signalling game. Firms announce their planned investments in advance, and the coordination of capital investment gets resolved in the longer term (Christensen & Caves 1997).

In the petrochemical industry coordination of investment, particularly when large scale greenfield plants are involved, has been noted to reduce the incidence of excessive overcapacity (Gilbert & Lieberman, 1987; Lieberman 1987b, 1987c). This coordination appears to work well in tight oligopolies; when a large number of producers is present in the market, and demand growth is sluggish, the system may breakdown and large overcapacity becomes a problem. The persistent overcapacity of the West European

PVC industry during the 1970's and early 1980's has been attributed to such a failure (Paraskevopoulos 1988; Paraskevopoulos & Pitelis 1995).

5.6.2 Model Implementation

The US styrene industry has been dominated since the 1970's by 6 firms. Despite entry by some firms and the exit of others over the years, these involved mostly a change of ownership of existing operations; and the market continued to operate as an oligopoly. Since we are trying to capture the investment behaviour of the leading companies, I formulate the model for six firms (by setting $N = 6$). Taking into consideration the distribution of the firms by size in the industry, I allow for four large firms to operate as *pacesetter* producers, and the other two as *straggler* producers, where the latter operate smaller plants. The roles of which firms are in the lead position may change over time, as we have seen in Table 5.3; and this formulation allows for size distribution of the major producers in the industry to be preserved; we assess, in this model, the impact on industry performance through the interaction of individual firms.

The model was implemented, and solved for the four different models by applying the corresponding strategies. In all the cases it was assumed, and for all firms, that the salvage value of the capital investment, recoverable at the end of the planning horizon to be zero. In view of the long planning horizon of 23 years, a fraction of the cost of equipment that may be recoverable would not affect the net present value of cash flows, especially when a discount rate of $\delta = 10\%$, was applied throughout the models. We have to bear in mind also that the cost of equipment accounts for only a small proportion of the total production costs. In this situation the revenues from production operations dominate.

For each firm i , the costs and inverse demand parameters of the quadratic net profit function, $w_{i,t}(y_t)$, were saved in vectors $p_{i,t}$, and matrices $Q_{i,t}$, as the firms were assumed to have full knowledge of all costs and the evolution of demand in advance, for all periods (refer to equation (4.1), in Section 4.2.1, of Chapter 4). The optimal feedback Nash-Cournot and Stackelberg strategies were evaluated by implementing the recursion procedure for $S_{i,t}$, $r_{i,t}$, and $\nu_{i,t}$, that solve the two point boundary problem

for each firm⁴⁸. The procedure for the feedback Nash-Cournot game was discussed in Section 4.2.3. And a similar procedure for the Stackelberg game, in Section 4.4.4, where also the steps for the algorithm were presented. The values for $S_{i,t}$, $r_{i,t}$, and $\nu_{i,t}$, were obtained from equations (4.29), (4.28), and (4.27) respectively, for the feedback Nash-Cournot game (see page 150.). And similarly for the Stackelberg game from the corresponding equations (4.79), (4.80), and (4.81) respectively (see page 166). Starting from the final period, T , and using the boundary values $S_{i,T}$, $r_{i,T}$, and $\nu_{i,T}$, where zero salvage value was assumed, the recursion was solved backward in time, to the starting period, where $t_1 = 1974$. Along the way the values of E_t and d_t , would have been saved for the feedback Nash-Cournot model, and D_t and ϕ_t for the Stackelberg model, as these would be needed to evaluate the optimal strategies of the two models respectively. This was done by implementing these values in the state equation using the vector of initial conditions, y_0 . These initial conditions that refer to the major firms' capacities and outputs in 1974, together with the model's other parameters for the base case, are shown in Table 5.8.

The Stackelberg model involved one firm taking on the role of leader, irrespective of how firm ranking was changing since the purpose of the exercise is to study the changing structure of the market, the investment behaviour of the firms, and type of interaction between them. The leader's production and investment costs were the same as the other three pacesetters; but the leader had the advantage of deciding first on production and investment, before the remaining five followers take their actions.

Implementing the open-loop and price taking models, is simpler. It involves generating large matrices using the stored values of $p_{i,t}$ and $Q_{i,t}$, of the profit function, and A , and B of the state equation (see equation (4.42) on page 155). Deriving the optimal strategy U , for the entire 1974-1996 period was achieved by multiplying and inverting a number of large size matrices, which represent E_t and d_t , as shown in equation (4.45). Implementing the optimal strategy, is then straightforward, and may be achieved in one step. This also involves the multiplication of very large size matrices. To obtain the op-

⁴⁸The algorithm can be made to run more efficiently by imposing a matrix structure; instead of solving the equations for $S_{i,t}$, $r_{i,t}$, and $\nu_{i,t}$, individually for each firm, they may be stacked in matrix form, MATLAB runs more efficiently this way. In this format the algorithm can also deal with different number of firms with minimum modification.

Table 5.8: Model parameters and initial values for base case simulations – 1974.^(†)

Firm i	Capacity	Production	N_{cfb7}		N_{col7}		S_{kfb7}	
	k_i (kt/y)	x_i (kt/y)	d_i	g_i	d_i	g_i	d_i	g_i
1	254	254	0.5	20	0.03	20	0.5	2.0
2	400	400	0.5	20	0.03	20	5.0	0.5
3	590	590	0.5	20	0.03	20	5.0	0.5
4	270	270	0.5	20	0.03	20	30	2.0
5	380	380	0.5	20	0.03	20	5.0	0.5
6	181	181	0.5	20	0.03	20	30	2.0
Interest rate,		r	10 %					
Depreciation rate,		δ	10 %					
Price elasticity,		ε	0.70					
Number of firms,		n	6					
Planning horizon,		T	23 years					
Industry capacity,		(kt/y)	2580					
Industry production,		(kt/y)	2701					
Capacity utilisation,		CU	105 %					

timal evolution of investment, production and capacity expansion I solve equation (4.49) using the initial values of Table 5.8. Steady state may not be reached as the industry equilibrium is being disturbed regularly but the firms are in Nash-Cournot equilibrium in every period.

5.6.3 Model Performance and Simulation Results

In this section I present the results of the dynamic capacity expansion model, as it is applied to the US styrene industry, over a planning horizon from 1974 to 1996. The period covers two oil price shocks, and a number of entries, exits and mergers. Few studies employing dynamic game theoretic models, have been developed in the literature that deal with actual industries empirically. These include Reynolds (1986), Salant (1982), Haurie & Zaccour (1986), and Pineau & Murto (2000). Most of the work in this area remains of theoretic nature. However, there is increasing interest in the application of dynamic models to industry studies.

Calibration of the model was carried out by adjusting the values of d_i and g_i the adjustment cost parameters for investment and output respectively for each firm i , such that the cost of adjustment was on average within acceptable limits of 40%; and keeping in

mind the need to obtain output, and prices consistent with developments in industry. The model is driven endogenously in terms of selecting the optimal outputs and investment rates for each period, given the exogenous demand as dictated by the inverse demand function. The model is deemed to meet the total industry output, if the cumulative output generated by the model over the planning period, is equivalent to the cumulative output of actual industry.

The simulation results for the different models are shown in Table 5.9. Spurious results were obtained for the price taking model and the results for this model have been eliminated from inclusion in the table. The first set of results refers to the base case with a price elasticity of demand, $\varepsilon = 0.70$, and includes the Nash-Cournot open-loop model (*Ncol7*), the Nash-Cournot feedback model (*Ncfb7*), and the Stackelberg model (*Skfb7*). The other two sets refer to the case of $\varepsilon = 0.50$ where demand for styrene is less elastic, and when it is more elastic with $\varepsilon = 1.20$. The cases where firms use feedback Nash-Cournot strategies, namely *Ncfb5*, *Ncfb7*, and *Ncfb12*, appear to give better results in terms of overall output, styrene price, capacity utilisation and market shares. The table gives only average values, where run *Ncfb7*, for the feedback Nash-Cournot strategies, gives the best performance. An examination of the full results confirms this, in terms of all the measures mentioned above; and the results for the base case *Ncfb7* are shown Figures 5.10- 5.15.

The performance of the model in terms of the output, and capacity of the major six producers is shown in Figure 5.10. The model performs very well, by generating optimal output and investment values for major six producers for all periods except for a short period for the years 1979-1981. In periods of low demand the firms can reduce their capital investments, drastically by disinvestment, or by slowing down their investment rate; and because of depreciation, capacity declines slowly in subsequent periods. This can be seen in Figure 5.10, where the model generated capacity for the major six firms, tracks the actual capacity of these six firms in industry (model M6, and actual M6 in the figure), as it declines and rises during the period from 1974 to 1988; and then increases more steadily thereafter.

Capacity is utilised in full when the six firms employ optimal strategies as shown by

Table 5.9: Model simulation results for US six major styrene producers.

	Model/Actual Output	Industry Average	Model Average	Model Average	Industry Excess	Strategic Excess	$C_1^{(d)}$	C_4
	Ratio (%)	Price (1990 \$/t)	Price (1990 \$/t)	CU ^(a) (%)	Capacity ^(b) (%)	Capacity ^(c) (%)	(%)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<i>Ncol7</i>	95	771	839	101.6	10.9	14.4	16.5	66.0
<i>Ncfb7</i>	100	771	768	100.5	10.9	9.1	16.6	66.7
<i>Skfb7</i>	96	771	806	90.5	10.9	4.9	37.1	69.2
<i>Ncol5</i>	93	771	900	101.6	10.9	16.1	15.4	61.8
<i>Ncfb5</i>	99	771	783	99.8	10.9	9.7	15.8	64.7
<i>Ncol12</i>	99	771	774	101.6	10.9	10.4	19.0	76.9
<i>Ncfb12</i>	100	771	736	100.7	10.9	9.2	14.0	66.9

^(a) Model average capacity utilisation is defined as the percentage ratio of model output against model capacity, averaged for all years.

^(b) Industry excess capacity is defined as the difference between actual industry capacity and industry output divided by actual industry capacity, averaged over all years.

^(c) Strategic excess capacity is defined as the percentage ratio of actual industry capacity less model capacity divided by actual industry capacity, averaged over all years.

^(d) C_1 and C_4 are the respective market shares of industry capacity of the largest firm, and largest four firms, generated by the model. Also note the case numbering, *Ncol7* refers to Nash-Cournot Open-loop model, where demand elasticity = 0.7.

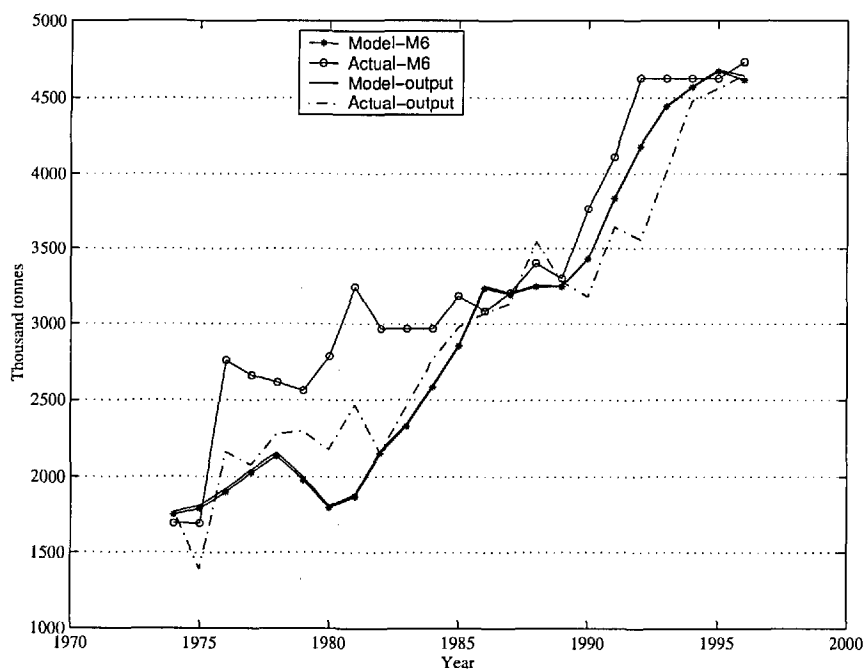


Figure 5.10: Model vs industry capacity and output for the major-6 styrene producers for feedback NC game (*Ncfb7*).

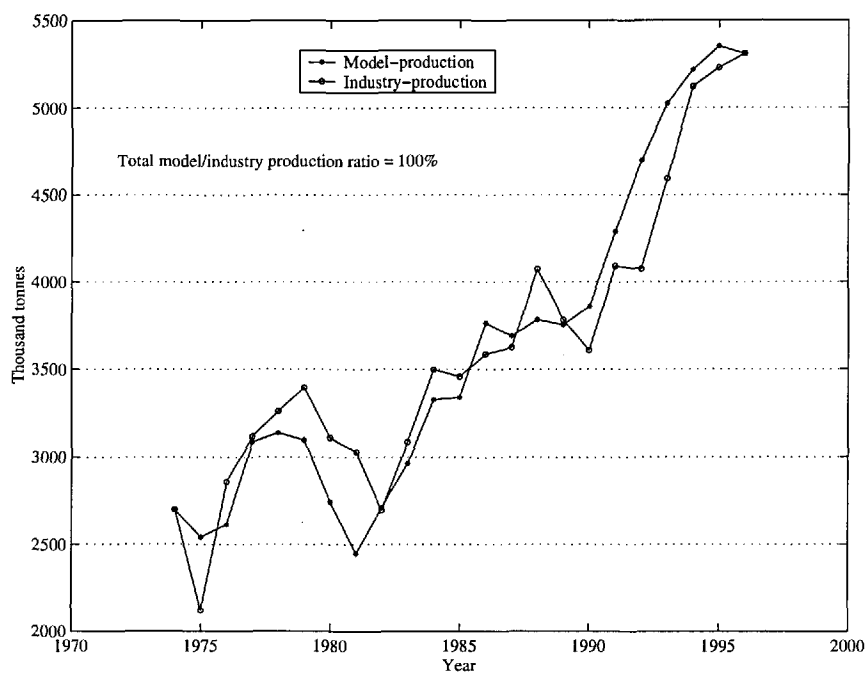


Figure 5.11: Model vs industry output for feedback NC game (*Ncfb7*).

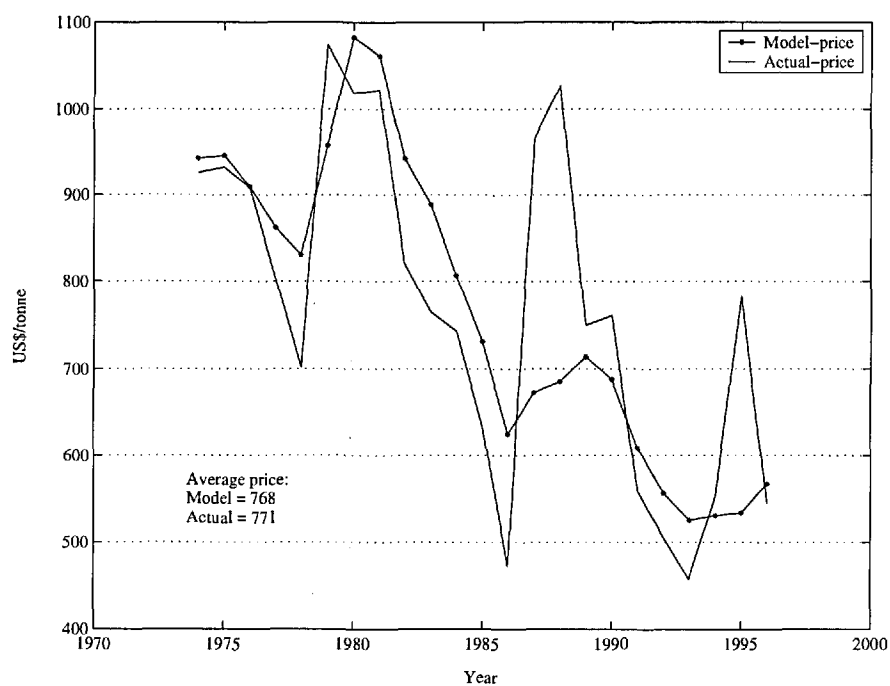


Figure 5.12: Model price vs actual price for feedback NC game (*Ncfb7*).

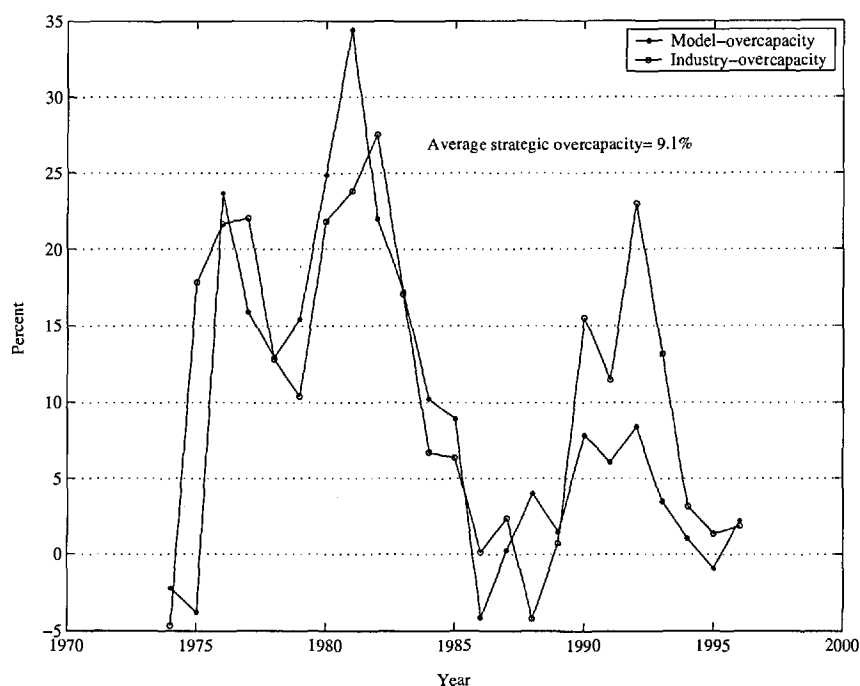


Figure 5.13: Model vs industry overcapacity for feedback NC game (*Nc**f**b7*).

the model output and capacity in the figure. In terms of total output, which is obtained by adding the production of the major six firms and the fringe producers, the model production over the entire period under study, meets the actual industry production as shown in Figure 5.11.

On the other hand, Figure 5.12 shows that the model generated prices were in good agreement with actual prices. The average prices for the model and industry over the whole period are within a very close range (model average price of US\$ 768/tonne as compared with an actual average price of US\$ 771/tonne). Model prices track actual prices that prevailed in the US market over the period under study, as the figure illustrates. One observation worthy of note is that the model predicted prices follow the cycle of industry prices, but show slightly less variability than actual prices during the periods where the styrene market was tight, like in the years of 1988 and 1995. During periods of high capacity utilisation, and rising demand, markets are usually tight and the expectation of possible product shortages drive the prices up, leading to divergence between actual and model prices. Expectations play an important role in affecting prices during such

periods, and are only calmed down when the prospects of additional capacity coming on stream, are substantiated by credible announcements, or irrevocable investments towards this aim, like performing detailed engineering studies, or commencing groundwork construction preparations by industry participants, for new projects or plant expansions.

The cyclical movement of overcapacity observed in industry, is also explained well by the model, when the firms follow feedback Nash-Cournot strategies as in case *Ncfb7*. This is shown in Figure 5.13. The significance of, and how overcapacity is generated by the model, should be made clear once again at this point. By following optimal investment and output strategies, the six firms minimize the incidence of overcapacity; their output and capacity are almost identical as depicted by Figure 5.10. Model overcapacity is the percentage difference between the actual capacity held by the styrene industry, and the optimal capacity generated by the model for the industry (the sum of the capacities of the major-6 firms and of the fringe producers). This is strategic overcapacity, since it exceeds the optimal capacity that the firms need to be able to meet industry demand when they follow optimal strategies that minimise their costs, and maximise their profits. It was discussed earlier, that incumbent firms hold strategic excess capacity to influence the investment behaviour of their rivals, so that their opponents invest less in the next period; or as an instrument to punish them if they deviate from the industry equilibrium by overproducing, or if they attempt to expand their market shares in a large way. Another possibility also, is that overcapacity is held to meet unexpected rises in demand, attributed to market uncertainty.

On average strategic excess capacity held by the styrene industry that the model generates,⁴⁹ was about 9.1% over the entire period under study, as compared to an average industry overcapacity of 10.9%. Our model explains most of the overcapacity that was seen in the styrene industry. It performs well in explaining strategic overcapacity as compared to that of Reynolds (1986), where the model could explain less than 2% of strategic overcapacity⁵⁰, when the industry average was about 9%. In Paraskevopoulos

⁴⁹In fact the strategic overcapacity held by the major-6 producers is higher; when fringe firms are excluded and overcapacity is measured against the capacity of the M6 producers their strategic overcapacity amounts to about 11.8%.

⁵⁰This may be attributed to two factors: the first is that of the production and investment cost estimates for styrene used in this thesis are closer to industry costs than those used by Reynolds for aluminium. He relied on two old studies for estimating costs which are derived indirectly. A UN study which gave

(1988) substantial excess capacity was generated by the model, which was attributed to nonstrategic factors, such as the industry's inability to reduce capacity because of exit costs and exit barriers, and significant economies of scale.

Before closing the discussion on overcapacity the following observations on Figure 5.13 ought to be made. The negative overcapacities that appear around the years of 1974 and 1988 indicate that the industry was overproducing, and exceeding its capacity. In some years this can be achieved when production exceeds the tolerance of optimal limits of equipment set by designers; or by postponing annual maintenance turnarounds, a practice which takes place in industry during periods of exceptionally high demand, and when capacity utilisation for the industry is at its limit. During the recessionary period of 1980-1982 when demand fell, the industry managed to maintain profitability, and support prices, by reducing the capacity utilisation rate, which fell to about 72%-78%. The negative overcapacities for the model around the years of 1975 and 1986, indicate that the model capacities for the six firms, exceeded their actual industry capacities.

The model and actual industry market shares for the major-6, and fringe producers, are shown in Figures 5.14 and 5.15. The first is based on shares in terms of firms' capacities; and shows that the models predictions were much better for the period from 1984 onwards, than the previous decade, which experienced a turbulent development path. What is significant also is the persistent decline in the share of the fringe producers from about 34% of industry capacity in 1974, to just over 12% by 1996. Figure 5.15 illustrates the same trend based on model generated outputs. The major-6 producers, expand their market share at the expense of the fringe. The figure shows the evolution of market shares for each firm. One final point to note, is that on average larger firms have a market share of about 18%, while the two smaller producers have a market share of 7%. The concentration ratios $C_4 = 66.7\%$, and $C_6 = 86\%$, which measure the capacity market shares of the largest four producers, and six producers respectively, as predicted by the model, were in agreement with industry values (see also Table 5.3 p.209). Because it was assumed that the pacesetter producers have the same costs, these firms eventually

long-run average costs for aluminium, from which he derived his production costs. And another study for estimating investment costs. The second relates to estimating the derived demand equation. The diversion between his model price predictions and industry prices illustrate that the demand equation may have been misspecified.

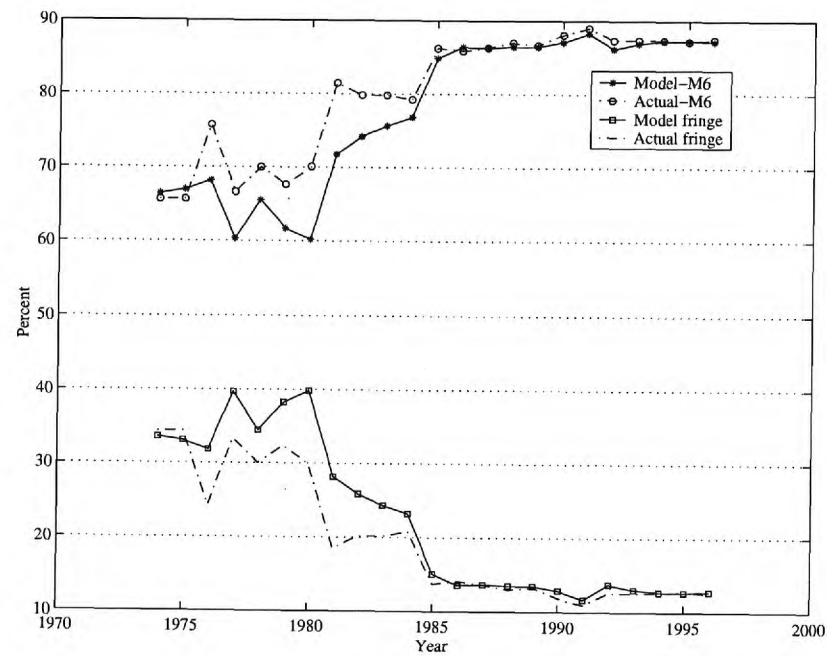


Figure 5.14: Model vs industry market shares of major-6 and fringe producers for feed-back NC game (*Ncfb7*).

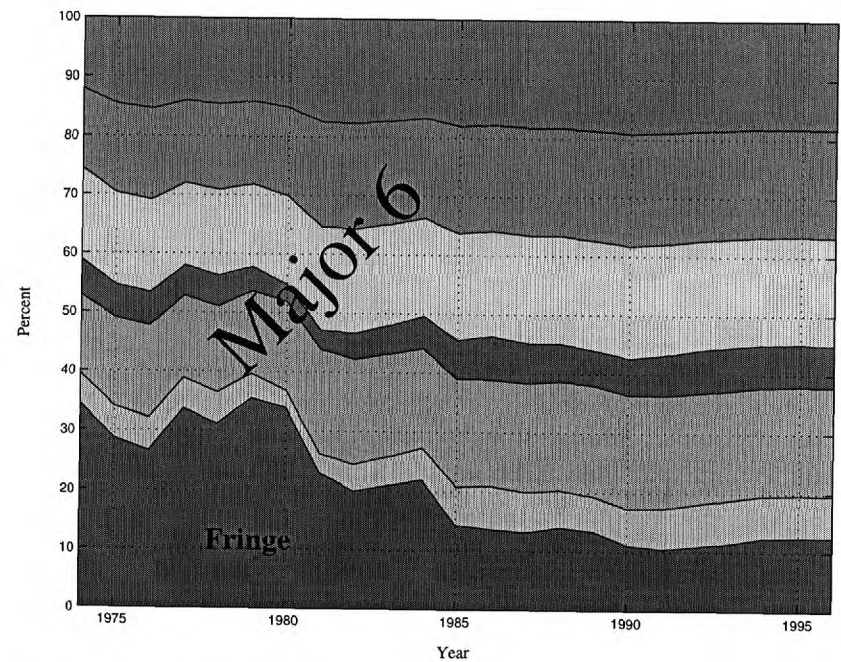


Figure 5.15: Model market shares of major-6 and fringe producers for feedback NC game (*Ncfb7*).

converge, and end up having equal sizes.

The styrene industry profitability is illustrated in Figure 5.16. It shows the model predicting actual industry price margins fairly accurately. Price margin is defined as product price less cash costs (variable plus fixed costs; or the equivalent of production costs without the cost of investment. See Table B.3.); and is a measure of operational profitability. While chemical products prices have, in general, been reported to decline over time due to economies of scale and learning (Stobaugh & Townsend, 1975; Lieberman, 1984), styrene product prices have been undergoing cyclical movement, with what appears to be a downward trend. While price margins have been showing large swings as well, they have consistently, been fairly high for a commodity chemical product. On average, the styrene price margin measured in real values, was US\$ 168/tonne, but could be as high as US\$ 500/tonne, and US\$ 400/tonne, as witnessed in 1988, and 1995 respectively. Integrated styrene producers can be expected to make even more profits, as they can even out the swings in raw material prices of benzene and ethylene. These firms do not have to buy these products at market prices. They can be supplied directly from within the same organisation. What the figure illustrates, is that over the 1974-2002 period⁵¹, styrene prices and margins can be seen to move in cycles. Except for a short period⁵² around the mid 1980's, price margins moved within a band of about US\$ 100/tonne to US\$ 300/tonne, with some swings overshooting the barriers at times. There does not appear to be any discernable trend for falling margins.

The open-loop model results for the base case price elasticity of demand $\varepsilon = 0.70$, (case *Ncol7*) are shown in Figures 5.17-5.21. The pattern of the model results is similar to that of the feedback model, but somewhat slightly less consistent with industry developments than case *Ncfb7*. The capacity and output evolution of the major six firms, as shown in Figure 5.17, is lower for most of the years up to 1990, in particular during the periods of 1978-1982, and 1987-1989. The firms make full use of their capacity, and produce slightly above their held capacities for most of the time. But in general the industry

⁵¹More up-to-date data became available on styrene prices and its precursors recently. The remaining variable and fixed costs were estimated on the basis of existing data used in the model.

⁵²This period included a recession around 1982, and Arco and Hoechst establishing their operations in the US styrene market.

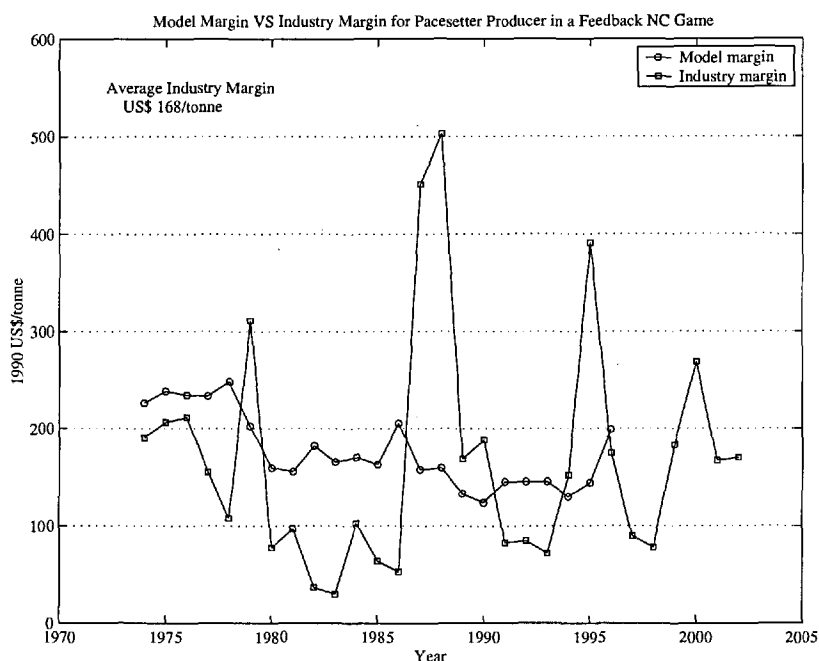


Figure 5.16: Model vs industry price margin for pacesetter producer in a feedback NC game (*Ncfb7*).

meets only 95% of the required demand as shown in Figure 5.18. The model generated prices follow the same trend as the actual industry prices (Figure 5.19). They are on average higher than industry prices with an average price of US\$ 839/tonne, compared to an industry price of US\$ 771/tonne. This is expected as the model predicts lower outputs, and consequently the model generated prices, are higher than when the firms follow feedback strategies (compare with Figure 5.12).

The overcapacity for the open-loop model in this case, is also more pronounced than before as illustrated by Figure 5.20, p. 258. The average model overcapacity for the styrene industry is higher as well. The average strategic overcapacity for the industry generated by the model is 14.4%, compared with an actual industry overcapacity of 10.9%. The market shares of the major-6 and fringe producers, are depicted in Figure 5.21. The fringe firms have a slightly higher share than before; but the market shares of the main firms are in general consistent with industry developments.

The model results when the major-6 firms use Stackelberg strategies for our base case where elasticity of demand $\varepsilon = 0.70$, are shown in case *Skfb7* of Table 5.9. One of the

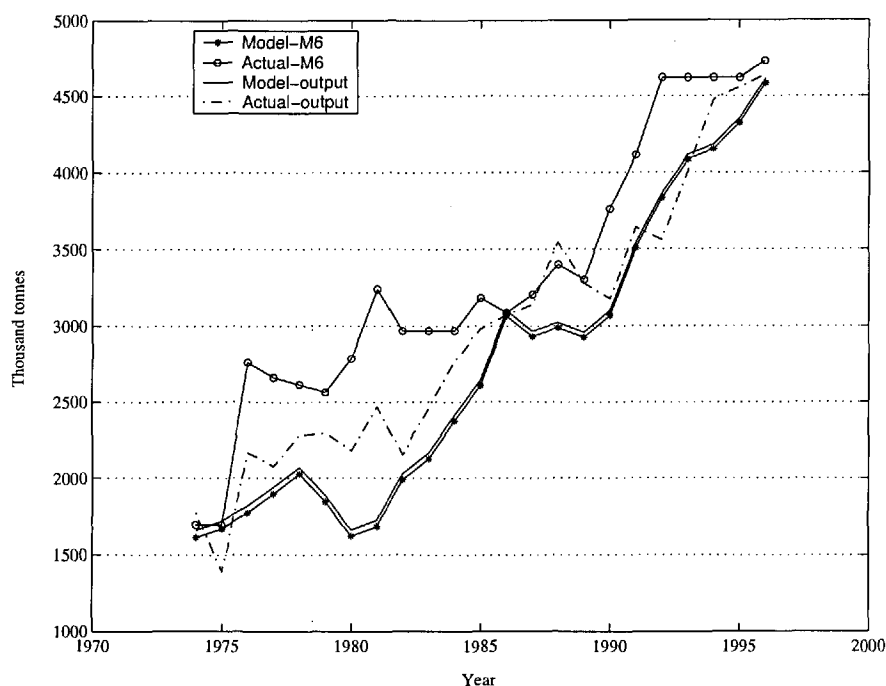


Figure 5.17: Model vs industry capacity and output for the major-6 styrene producers for open-loop NC game (*Ncol7*).

firms now assumes the role of a Stackelberg leader. All four pacesetter firms, including the leader, are assumed to have the same investment and production costs; and the starggler firms their usual higher costs. The firms accumulation of capacity in this case is lower than that of the industry. The firms in this model are able to generate production that satisfies 96% of demand. Prices as a result, are in general higher; and an average styrene price of US\$ 806/tonne is achieved. The figures for this case are similar in pattern to those seen in previous cases and are not reproduced here. What is significant, however, is that the leader firm accumulates far higher capacity than was the case by any firm in industry. The leader's share of the product market ranges from about 28% to 52% as shown in Figure 5.22. In the US styrene industry, the highest market share in terms of capacity that was held by Arco and then Lyondell was about 22% during the 1995-2000 period (see Table 5.3, page 248). Furthermore, the leader has almost full utilisation of capacity, whereas the followers, have considerable redundant capacity. The Stackelberg strategies in this instance do not give a realistic representation of market developments

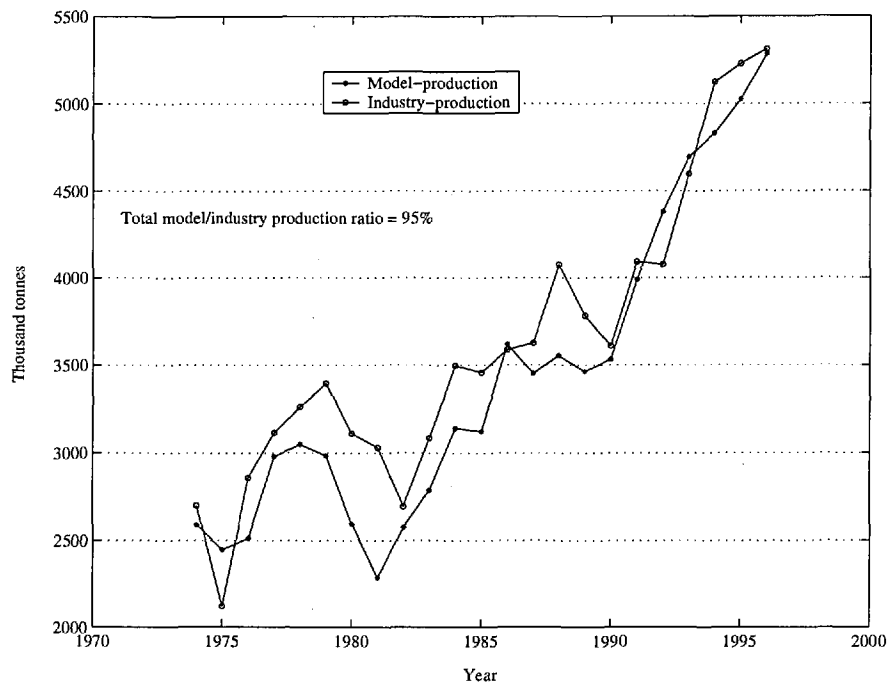


Figure 5.18: Model vs industry output for open-loop NC game ($Ncol7$).

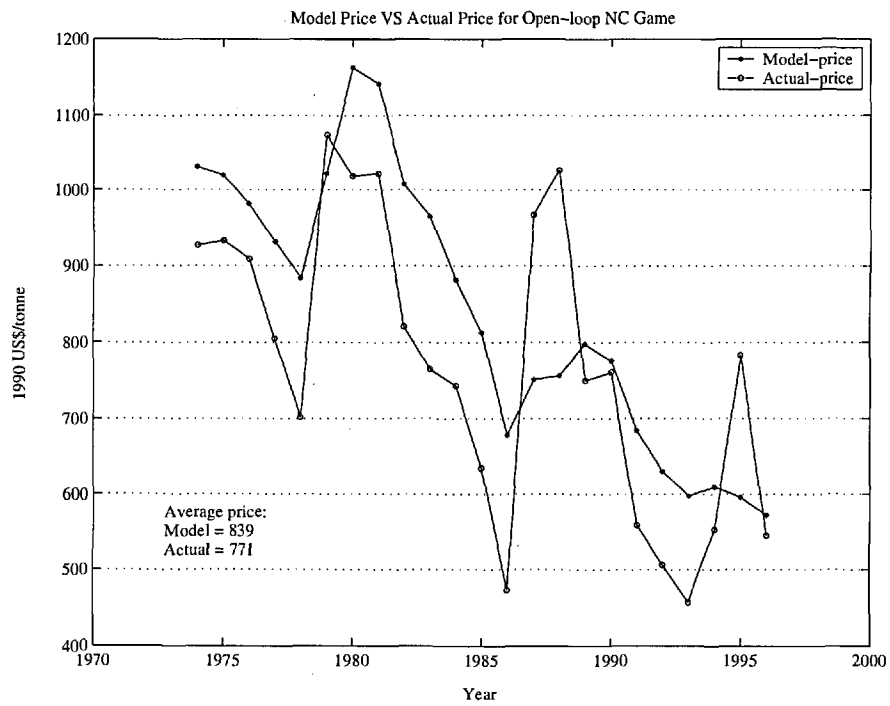


Figure 5.19: Model price vs actual price for open-loop NC game ($Ncol7$).

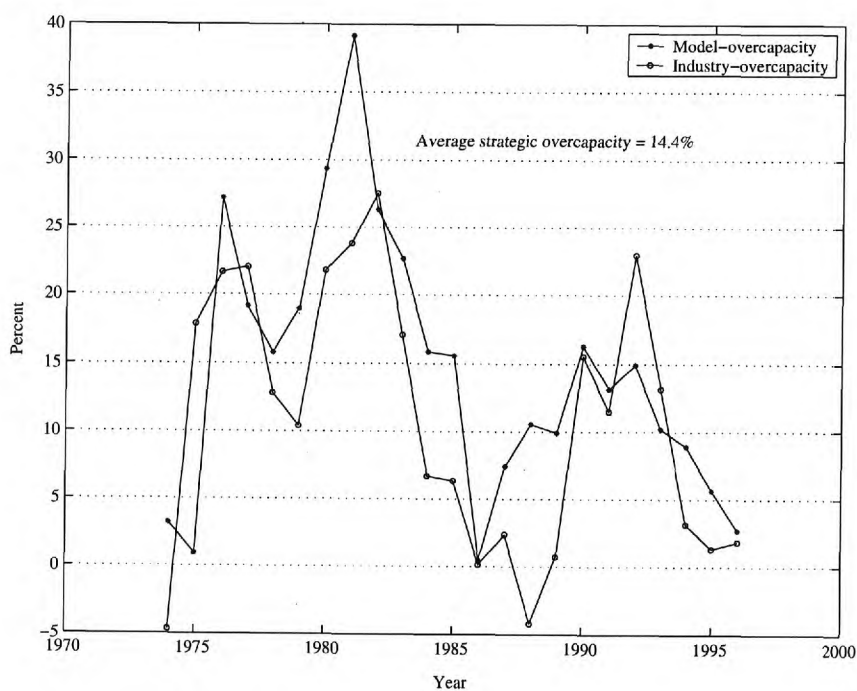


Figure 5.20: Model vs industry overcapacity for open-loop NC game ($Ncol7$).

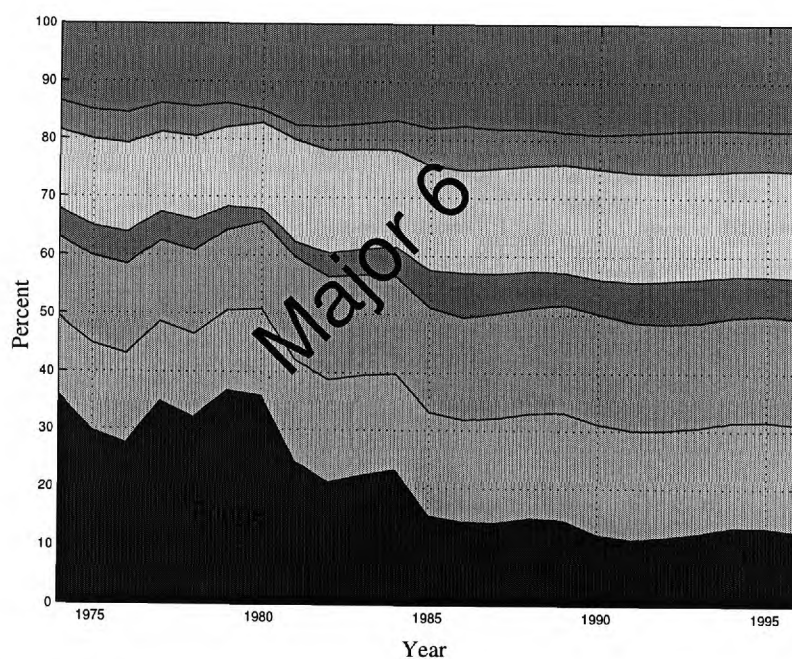


Figure 5.21: Model market shares of major-6 and fringe producers for open-loop NC game ($Ncol7$).

in the US styrene industry.

The other cases when a lower price elasticity $\varepsilon = 0.50$, is assumed show that the feedback strategy model performed better than the open-loop strategy model. As shown in Table 5.9, case *Ncfb5* meets 99% of demand requirements, compared to 93% for case *Ncol5*. Prices are also on average higher than that of the styrene industry average, but perform better than the open-loop case, *Ncol5*, where the average price was US\$ 900/tonne styrene. For the situation when styrene is assumed to be price elastic with a price elasticity of $\varepsilon = 1.20$, the open-loop strategy case *Ncol12*, appears to perform well, with average styrene price of US\$ 774/tonne styrene. Strategic excess capacity of 10.4%, and average market shares seem to be within acceptable ranges. The performance in terms of price and meeting output demand, give the impression that case *Ncol12* is as good as *Ncfb7*. But a closer examination of the data reveals that this is not the case. Figure 5.23, shows that the capacity of the six producers is far lower over the period from 1979-1984. And while the performance in terms of model generated prices is rather good as Figure 5.24 illustrates (model average price of US\$ 774/ tonne, compared to an actual industry price of US\$ 771/tonne); market shares and capacities generated for smaller producers reveal a serious flaw. Figure 5.25 shows that the smaller producers have smaller shares than was predicted by *Ncfb7*, and for the years from 1979-1984, and from 1989-1990, the smaller producers (namely firm 2 and firm 4 in the figure) have their capacities, and market shares completely wiped out. This apparently compensates by making the average values, for the major-6 in the model for this case, to appear better than what the actual model prediction are.

A final point to note about the results, is that in all the cases examined, there was strategic excess capacity present. This is an indication that the firms in industry held capacity, in excess of what is required to meet demand, while the industry participants were pursuing optimal investment strategies.

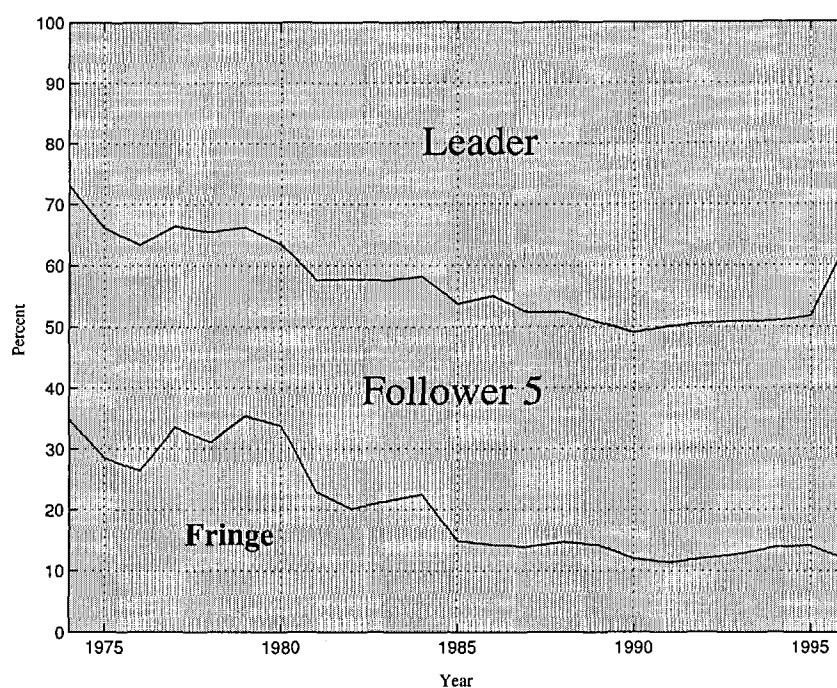


Figure 5.22: Model market shares of leader, five followers and fringe producers for Stackelberg game (*Skfb7*).

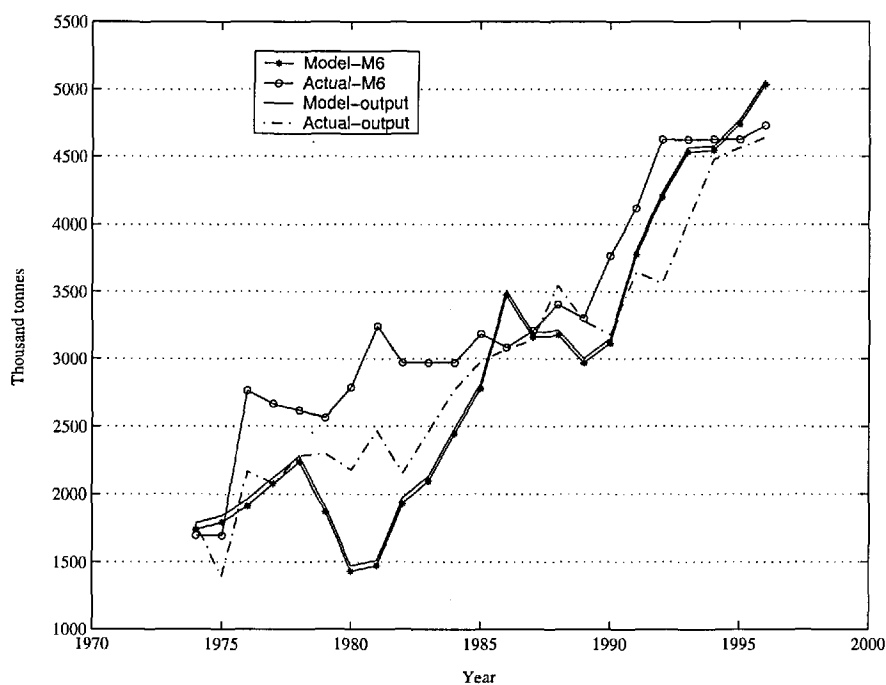


Figure 5.23: Model vs industry capacity and output for the major-6 styrene producers for open-loop NC game (*Ncol12*). Price elasticity of demand, $\varepsilon = 1.2$.

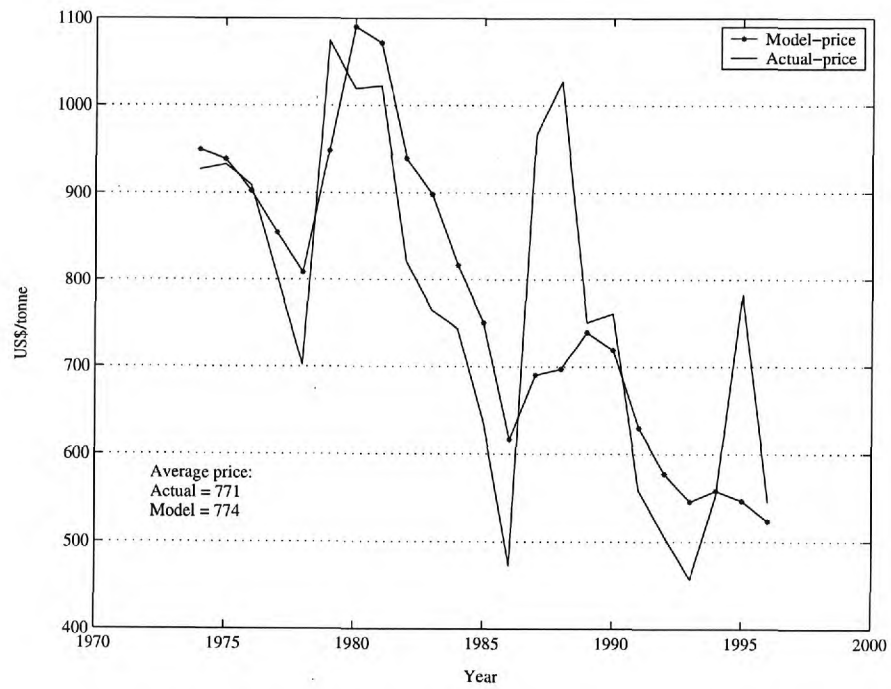


Figure 5.24: Model price vs actual price for open-loop NC game (*Ncol12*). Price elasticity of demand, $\varepsilon = 1.2$.

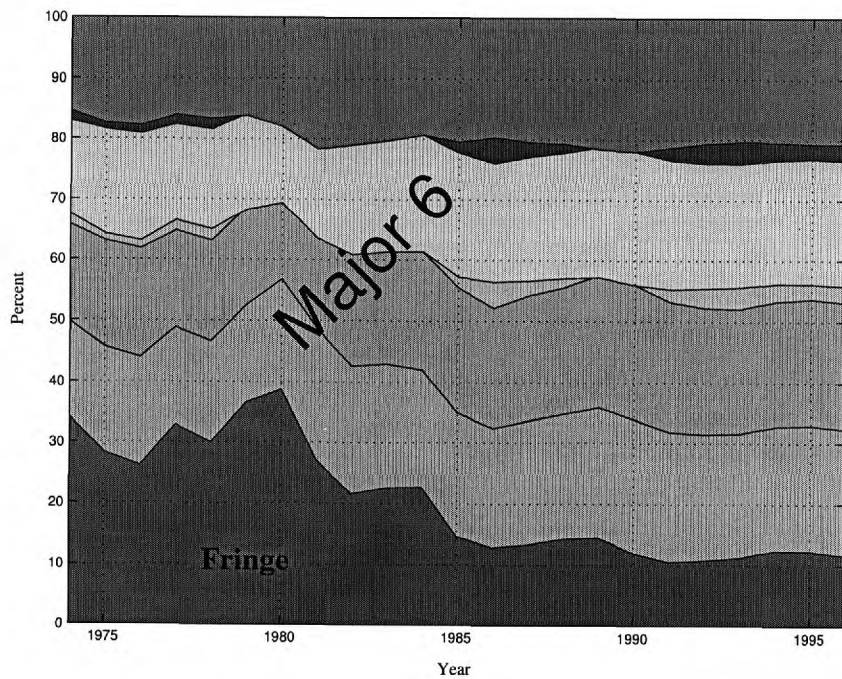


Figure 5.25: Model market shares of major-6 and fringe producers for open-loop NC game (*Ncol12*). Price elasticity of demand, $\varepsilon = 1.2$.

5.7 Entry and Determinants of Market Structure

Oligopoly theory suggests that growing markets have the propensity to attract new entrants. Firms expand into new markets when they find an opportunity to generate profits, or a niche which existing firms may not be able to fulfil quickly when the market is growing fast. Entry reduces the incumbent firms' profit margins. According to this line of argument, incumbent firms invest in a manner to discourage entry; since an increase in the number of producers raises the level of competition. Eventually, with a sufficiently large number of producers, price cost margins may be driven down to the level of competitive markets, and may even go down to zero (see simulation results in Section 4.5.3, Chapter 4; and Bresnahan & Reiss (1991) for a diagrammatic analysis, and evaluation of entry thresholds with constant marginal costs).

The response of firms to entry, or the threat of entry, is by no means uniform. It is dependent on several factors that pertain to characteristics of the market and the technology in use, the height of entry barriers, and the historical experience of the producing firms. Being a dynamic process, entry is not only affected by the market and the incumbent firms behaviour, but it also affects the market structure in which it operates, changing and modifying it, and in the process altering the dynamics of entry itself. Consider for instance the development of a new efficient process for a product, which becomes available for licensing by the engineering company that developed it, may attract a large number of new entrants into a growing market. This leads to a heightened level of competition, and may force some of the leading established firms out of their positions, making the market more fragmented and competitive. On the other hand, a rise in prominence of economies of scale may favour leading firms. As the product market becomes mature, larger firms can expand at the expense of smaller producers who fail to modernise, by investing in more efficient larger plants. The industry becomes more concentrated when smaller producers are squeezed out of the market; while entry is discouraged by low growth, and the rise in the size of investment required for large scale plants. As a result the market becomes, perhaps, less competitive.

Entry is important in the development of market structure and the evolution of firms

investment behaviour, with significant implications for competitiveness and profitability. It is not surprising therefore, that a significant proportion of the industrial organisation literature is devoted to the study of the process of entry and entry barriers, in the form of both theoretical and empirical studies. The recent literature on entry and market structure is reviewed in Geroski (1991a, 1995), Davies & Geroski (1997), Caves (1998), and Sutton (1998). The traditional view is that the incumbent firms counter the threat of entry into their market by employing a variety of measures, with excess capacity as the leading instrument, to discourage entry, or at least impel it to take place at a lower scale. Implicit in the use of excess capacity in this context, is the threat that the incumbents can resort to overproduction forcing prices down if entry were to occur. Excess capacity is the tool that makes this threat credible, as was modelled earlier on by Spence (1977, 1979). Sutton (1998) emphasizes the role of other measures such as advertising, or R&D to heighten the level of investment competition, that eventually lead to market restructuring. He shows how this was achieved in the aircraft industry by Airbus and Boeing. By investing heavily in R&D, and by introducing new aircraft models that covered all market classes and niches, competition was heightened and the market restructuring that followed, forced other competitors to be merged, or hastened their exit from the industry.

Two types of entry barriers are referred to in the industrial organisation literature: *exogenous* barriers, which are those that are not necessarily under the direct control of the firm, but affect the market as whole, such as new innovations, technological developments, economies of scale, and market shocks. The other type are the *endogenous* barriers, which are subject to the direct control of the firm, such as the investment in excess capacity, level of expenditure on R&D activity or advertising, integration of the production process, and others (Smith 1981, Mathis & Koscianski 1996, Geroski 1991a).

exogenous barriers are by implication non-strategic in nature, since all firms are affected equally by them. A new technology for instance, unless proprietary like when it is developed by the firm itself, is available to all market participants. Lumpiness of plants in petrochemicals resulted mostly from technological advances in the processing industries. So when firms invest in a new large scale plant causing overcapacity in the market

place, it may be argued that this is not a strategic investment in overcapacity; but the timing, and scale of new investments could be employed for strategic intent. Unexpected demand shocks or demand uncertainty also affect the level of investment, or production rate by the firm; the excess capacity that may result, is not considered as capacity held for strategic considerations.

Whereas *endogenous* barriers entail strategic intent by the firm to gain advantage and influence the behaviour of its opponents. Excess capacity to deter entry requires the firm to devote additional investment in larger plants. To gain market share the firm has to invest directly in advertising, or in a new differentiated product. Or it has to intensify its R&D expenditure and effort, to come up with new products or more efficient processes. All of these instruments require the firm to take direct action to bring about the desired results. The choice of which of these strategic instruments to utilise, depends on the characteristics of industry and role of its member firms.

The next two subsections (Section 5.7.1 and Section 5.7.2) look into some of these barriers and their pertinent use by petrochemical producers and the styrene industry, drawing attention where appropriate to empirical work in other industries.

5.7.1 Strategic Investment and Entry Barriers

Ever since Bain's (1956) seminal work on firms' investment in barriers to deter entry, there has been considerable interest on the theoretical and empirical levels, in the strategic use of entry barriers in industrial economics. There is ample evidence to suggest that firms in general, attach a great deal of importance to strategic considerations regarding entry by other firms into their markets (Singh, Utton & Waterson 1998). In a survey of 293 US manufacturing firms, Smiley (1988) finds that 58% of firms make frequent use of strategic instruments to deter entry into mature markets. And in a similar survey of about the same number of UK firms Singh, Utton & Waterson (1998) also report that just under two thirds of firms attempt to use entry deterring strategies towards entry of new products, and the percentage rises to three quarters for competition against existing firms. The common instruments between the two studies, which are also the most widely studied in the literature include: excess capacity, advertising, R&D and patenting, and

pricing policy. Both studies found that advertising and R&D were the most widely used by firms to dissuade entry and protect their market shares; while that of excess capacity was least used among their respondents (the percentage who rated excess capacity as an important instrument to preempt rivals was 22% in Smiley (1988); and in Singh, Utton & Waterson (1998) about 25% considered capacity creation a strategic instrument, and 8% would use excess capacity deliberately to make entry for other firms difficult).

Other cross-section industry studies in this area, also find similar results, Davies & Geroski (1997) in a study of 200 UK firms over a period of 7 years, also found that advertising and to a lesser extent R&D play a major role in affecting the dynamics of market shares and industry concentration levels. The relative importance of each of these instruments to the respondents has more to do with the nature of the products, the characteristics of the production process and markets of each industry, and the scale of operations involved than with the significance and suitability of capacity or advertising as an entry deterrence instrument in general.

Such cross-section studies, are perhaps ambitious in trying to generalise the findings of theoretical models like those of Spence (1977, 1979), and Dixit (1979, 1980) on strategic use of excess capacity in entry deterrence, to industry in general. Their samples usually cover a diversified group of mainly small or medium scale industries, and do not explore the characteristics of the industries being investigated. This becomes apparent when the industries are grouped by categories, and the question centres on the factors affecting the use of specific strategies, as performed by Bunch & Smiley (1992). The analysis is performed on the same set of industries studied in Smiley (1988); and where new and existing product markets are examined separately. They find that firms utilise preemption strategies using capital expansion in concentrated and growing markets that are populated by large firms. Another feature, is that capacity expansion to preempt rivals is the strategy of choice for large firms with deep pockets when demand is still growing. The strategy is employed more frequently when larger firms enjoy a greater cost differential over smaller firms. Which suggests that markets where economies of scale are important, are likely to carry more excess capacity than other markets. On the other hand, R&D and patenting are used more in research intensive industries where firms are large, with

large budgets for this type of activity. Here the markets are characterised by small minimum efficient scale operations relative to the market size, and advertising budgets are relatively large. Advertising and reputation are also more widely used to deter entry in research intensive industries, and/or industries where MES operations are small.

It is more likely that preemptive capacity expansion would be used as a strategic instrument for entry or mobility deterrence in markets of homogeneous commodity products such as petrochemicals, fertilisers, cement, sugar, salt, or aluminium. Other types of structural barriers that may be employed for this purpose, originally proposed by Bain (1956), were studied by Caves & Porter (1977). Besides excess capacity, these include limit pricing, patented technology and know-how, setup or sunk costs, product differentiation, and vertical integration. Investment coordination also, was identified to be closely related to excess capacity (Smith 1981). Five of these barriers are discussed below, indicating where appropriate their relevance to the styrene industry.

- **Excess capacity** in the form of unused production capacity makes a threat of price war against entrant firms credible, and represents an entry discouraging investment by the incumbent firms. It may be less costly for the incumbent firm to acquire an entrant, than to engage in a price war; but this may increase the likelihood of future entry whereas the prospect of a price war reduces it. Investment in entry deterring capacity protects not just the investing firm, but also its oligopolistic rivals as well; excess capacity fosters the interdependence of oligopolistic firms, in their effort to protect their market from entrants. Excess capacity is useful in cartel like situations, since market shares are quite often assigned according to capacity (Scherer & Ross 1990). Investment in excess capacity to retaliate may pay whether it is used or not, because of its deterring effect (Wenders 1971). Even when excess capacity does not lead to the creation of a credible threat, it may still act as a barrier to entry by distorting the perceptions of potential entrants about the expected returns and risks, to the extent of diverting their potential investments into other industries (Hilke 1984).

The strategic use of excess capacity for entry deterrence in the empirical literature

has already been alluded to in the works of Reynolds (1986) on the US aluminium industry, by Ghemawat (1984) and Rosenbaum & Ye (1992) on titanium dioxide, and Steele (1995) on the North American newsprint industry. In a study of the US titanium industry, Mathis & Koscianski (1996) conduct an empirical test of the impact of excess capacity on the probability of entry. While the authors were unable to establish proof of intent to preempt entry (in the sense that incumbents deviate from profit maximising behaviour in order to create a barrier to entry) due to the lack of necessary detailed cost information necessary for this kind of analysis, their results, however, remain significant. There was little sign of entry into the industry; building of capacity by incumbents appears to have preempted entry and thus served as a barrier to entry. Whether by intent or not, the incumbents investment in excess capacity managed to achieve the desired result of a form of entry deterrence.

Similarly in the empirical model of oligopoly capacity and pricing decisions and of entrant responses by Masson & Shaanan (1986), the results do not provide evidence that oligopolists deliberately install excess capacity to deter entry, but they support the hypothesis that both lower prices and excess capacity inhibit entry. Furthermore their results support limit pricing, and suggest that limit pricing firms may exploit the entry inhibiting effects of unintended excess capacity by raising their limit prices.

Strategic investment can take different forms other than simply the use of excess capacity as modelled earlier on by Spence (1979); advertising and R&D are important elements of strategic investment responses by firms in the signalling game in the face of entrants as shown by the studies of Singh, Utton & Waterson (1998) and Davies & Geroski (1997) of the UK industry, and that of Smiley (1988) in the US.

It is more likely that excess capacity will be used as an instrument of mobility, or entry deterrence where it would be guaranteed to give the desired effect. This is particularly true of industries with highly concentrated markets, and where large

firms dominate. Gilbert & Lieberman (1987), Lieberman (1987a) and Bunch & Smiley (1992) find that use of excess capacity occurred in products where concentration was high. But this is exactly the point; capacity in capital intensive industries is generally expensive and costly to hold. Oligopolists act rationally by utilising investments in industries, and products where use of excess capacity would pay-off. There is nothing to be gained from overinvesting and holding excess capacity when the market structure does not support tight oligopolistic interaction between the firms. When other barriers are present (such as restricted access to technology, or large sunk costs) excess capacity would be less likely to be used on a large scale.

Also, the strategy of build and scrap, observed in petrochemicals (Lieberman 1989, Paraskevopoulos 1988) allows larger producers to stay ahead in the game of capacity expansion, by retiring older plants and replacing them with larger plants, that also employ latest technology. Styrene producers were following a similar strategy, especially Dow, Shell and BASF. By following this strategy the incumbent firms attempt to preempt their rivals; but effectively succeed more in preempting new entrants. A study of the US manufacturing industry covering a 20 year period (1963-1982), by Dunne, Roberts & Samuelson (1989) find that this is generally the case in the chemical industry. Despite the strong growth of chemicals and large entry, smaller firms fail to make as strong an impact on chemical industry structure and its competitiveness, as similar firms did in other manufacturing sectors, where entrant firms after 15-19 years, are on average about 20% larger than the average size firm in the industry. In chemicals after 20 years, entrants account for 28% of industry output, as compared to 42% for entrants in the total manufacturing industries.

- **Investment coordination** in capital intensive and large scale industries, allows the firms to avoid excessive buildup of capacity; capacity is added gradually instead of the otherwise ruinous race to install capacity to meet expected growth opportunities. This is done by regulating the addition of new capacity in an informal way,

through the signalling game in the form of announcements about prospective new investments, allowing other firms to reconsider the timing of their own investment plans. According to Smith (1981) this form of strategic behaviour can result in improved resource allocation, because it improves the firms ability to forecast the state of aggregate supply, and minimises the effect of supply coordination failure. Strategic investment can take different forms, besides capacity, it includes product differentiation, innovation, futures markets long term contracts, and government regulations which facilitate for firms the coordination of the supply side of the market. It is recognised also that this type of strategic investment, and its coordination can create barriers to entry (Caves & Porter 1977).

In petrochemicals Gilbert & Lieberman (1987) and Paraskevopoulos (1988) found that the signalling and announcement game had been used extensively in coordinating the addition of new capacity especially in the concentrated segments of the market. In the paper and pulp industry Christensen & Caves (1997) also show how the announcements made by firms play an important role in determining who abandons, and who goes ahead with investments. Henderson & Cool (2003) find that firms in the petrochemical industry learn from their past mistakes to time their greenfield investments avoiding redundant capacity, and improving their profitability.

Also in the styrene industry firms make use of announcements at different industry forums, and in the trade journals to signal to rivals their investment intentions about new planned capacity. An examination of this literature reveals that firms make allowance in their plans for their rivals' investments. Sometimes a firm would revise its original plans in terms of capacity, or the timing of new investment to reduce adverse effects on expected market supply demand balance. Some recent examples were discussed briefly in Section 5.2.3, like Lyondell's project in the Netherlands, and Dow's claim to a project on the Gulf Coast in the US.

Sometimes the firms would go further in their coordination plans and form joint ventures, especially when these involve very large scale plants like in the case of

Cos-Mar, the joint venture between GE Plastics and Fina Oil in the US, and the equity arrangement that Nova and BASF had in one of Lyondell's plants in Texas. More recently, Shell and BASF formed 'Basell' a company that runs their two joint ventures in Singapore and the Netherlands. Lyondell and Bayer formed a similar joint venture in the Netherlands as well.

Besides coordination of capacity investments, firms can coordinate the supply of their raw materials. Other than normal market transactions, firms can secure raw materials supply, either by direct investment in vertically integrated upstream operations, or through long term contracts with raw materials suppliers. The firms in this case negotiate directly the conditions of future supply, regarding volume and the pricing mechanism. Long term contracting is a form of strategic investment, since the parties involved enter into an agreement that facilitates the coordination of supply (Smith 1981), by allowing the seller in this case to plan capacity expansions without the risk of running an idle operation. These contracts are usually publicised in advance, so that all market participants become aware of its implications and would evaluate any expansion plans accordingly. In the styrene industry several arrangements of this nature are known to have taken place.

Lyondell and Sterling are basically merchant producers, but the two firms are also involved in long term contracts, or toll production, where a percentage of the output would be earmarked for supply to another company. Nova for instance had an equity share in one of Lyondell's Channelview plants for 180 thousand tonnes/year, and another equivalent volume on a long term contract basis. BASF also has an equity share in the same plant.

In more recent times another arrangement that avoids open market transaction mechanisms, was devised by some of the major players in the global styrene industry. BASF which is the third largest styrene producer in the world, has significant styrene derivative operations in the North American market (its requirements in the US market alone were about half a million tonnes in 1999), arrived at an arrangement with Nova Chemicals where the two firms would swap styrene product

in two different markets. BASF⁵³ would supply styrene to Nova in Europe. Nova had acquired several polystyrene plants in recent years including those that were part of the deal with Huntsman in late 1998, and from Shell in 2000; as a result Nova was short in styrene in Europe. In exchange Nova agreed to supply BASF with styrene from its expanding styrene capability in the US and Canada, to meet BASF's requirements in the North American market where it lacks a styrene production base.

- *Sunkness of costs and investment irreversibility*

Setup costs are defined as the minimal required level of outlays, in the form of sunk cost, incurred by an entrant into industry. Because these costs are sunk and irreversible, they can constitute an impediment to new entrants. In the petrochemical industry for instance, these costs are determined exogenously, by the engineering and processing industries. In its simplest form this setup cost can be identified as the cost of a minimum efficient scale (MES) plant for that particular industry. Hilke (1984) argues that investment in excess capacity has the effect of converting fixed costs into sunk costs for new entrants, because additional investment in excess capacity reduces the market for second hand equipment.

Advertising and R&D activity play the same role as capacity in Cournot games; since the expenditure on developing new differentiated products, and on advertising is a form of a sunk cost that raises the level of competition between firms. Schmalensee (1978) and Sutton (1991) demonstrate the mechanism of how this type of activity operates as a barrier to entry in different industries. For Schmalensee (1978) the lack of noticeable entry by new firms into the market of ready to eat cereals over a long period given the growth and profitability of the industry, is an indication of the existence of some form of impediment or barrier to entry. During the period from 1940 to mid 1960's the industry was highly concentrated; the largest four firms controlled at least 85 percent of sales, and the top six firms 95 percent of the market.

⁵³ *Nova Chemicals and BASF to Swap Styrene*, Chemical Week, 7 August, 2002. p. 7.

Ollinger & Fernandez-Cornejo (1998) find that regulation (environmental rules and regulations relating to clean water and air) and research costs for the pesticide industry in the US, constitute a form of setup or sunk costs for the firms. They derive relationships similar to those of Sutton (1991), between the number of firms in industry and the level of market competition based on sunk costs. They show that non-price competition resembles price competition in the limit, where a decline in industry demand or an increase in the toughness of competition in the form of a rise in R&D expenditure, regulatory or capital costs, negatively affects the number of firms operating in industry.

Setup costs have a bearing on market structure. Using the terminology of Sutton (1991), an increase in the setup cost, σ , leads to higher concentration in the market. He measures this in terms of the dimensionless ratio, S/σ , where, S , is the market size measured in money value for that particular product for instance, relative to the setup cost, σ . In general, Sutton⁵⁴ finds that for a wide range of commodity industries there is an inverse relationship between S/σ , and industry concentration. An increase in the setup cost leads the market to support a smaller number of larger firms. In the US styrene market, I find, using investment cost data of Table 5.6, and the market size based on styrene prices measured in constant US dollars, that over the period between 1973 and 1993, which is comparable to the planning horizon of this study, the value of S/σ ⁵⁵ declined from 14.2 in 1973 to 8.8 in 1993. This pattern for styrene industry is consistent with Sutton's theory, which predicts a rise in the concentration ratio as the ratio of market size/setup cost declines.

- **Product differentiation** reduces the interdependence among firms, and between the products of the incumbents and those of the potential entrant. When sellers brand their products, and emphasize their specific characteristics, they force the entrant to make extra outlays in the form of advertising and promotional activity to gain a foothold in a crowded market. As a consequence, the entrant faces

⁵⁴See Sutton (1991, chp. 4), where a wide range of industries including salt and sugar, are analysed across different countries.

⁵⁵In this exercise, σ for each year, was measured as the investment cost for an MES plant for that year in constant 1990 US\$; and S was obtained by multiplying the styrene price also in 1990 US\$ by the production for that year.

higher costs, and has to fight existing brands to gain market share⁵⁶. Also when incumbents differentiate their products, they insulate their market from that of their competitors, and all their promotional efforts go towards the niche market they created for each particular product. This has two desirable effects. The first, is that differentiation reduces the firm's concern about capacity expansion by rivals, as the firm can expand capacity to meet the demand for its own product which operates as a segregated submarket. And the second, is that differentiated products facilitate pricing differentials, as the products are not directly comparable to their competing substitutes produced by other firms. Because of its differentiated attributes, the firm can raise the expected value of the product, and the consumers become willing to pay a higher premium for the new product.

In the petrochemical industry the vast majority of products are sold as commodity products, with the exception of a few specialised resins and fibres. But even here petrochemical producers have attempted to introduce some differentiation. Each of the major chemical companies for instance sells its plastic resin products such as polystyrene, PVC, polypropylene, or polyethylene under its own brand name. Nevertheless, these are effectively commodity products sold according to their technical specification that are known to the plastics converters. There was an attempt in the late 1980's to introduce differentiated plastics resins in the polyethylene segment (LLDPE and HDPE) using copolymers such as alpha-olefins⁵⁷ that sell at a slightly higher premium. This end of the market, however, remained insignificant and constituted only a small share of the total sales volume of these resins. Most companies introduced their range of these products; but the plastics resins market remained essentially a commodity market. Moreover, intermediate chemical products such as styrene, are homogenous products that cannot be differentiated and

⁵⁶For an elaborate account of the use of product differentiation and advertising as a barrier to entry see the paper by Schmalensee (1978) on breakfast cereals, and also Sutton (1991) for an account of a range of industries. And Sutton (1998) discusses the use of R&D activity.

⁵⁷These are polyethylene products that are produced using the same process of production and in the same equipment as for conventional polyethylenes, but in this case a copolymer such as butene or hexene is added to ethylene and the reaction takes place over metallocene catalysts. These copolymers are members of a family of products known as alpha-olefins, which in turn are also produced from ethylene. Their use results in producing highly linear polymers with enhanced properties.

are sold as such.

- **Vertical integration** acts as an entry barrier by raising investment levels. A potential entrant faces uncertainty about supplies of raw materials and highly variable spot market prices if entry is considered only at the downstream stage of the operation; and the prospect of much higher investment requirement if he considers entering as an integrated producer (Caves & Porter 1977). When firms integrate their operations, they are effectively ensuring security of supply of raw materials for their industrial venture, and/or the market outlet for their products. Also an element of market forces distortion is introduced, as integration seals a firm's transactions from open market operations, hiding real costs and margins of profits. It can also be used for transfer pricing in the chain of various sections of the production process, or between divisions of the firm with plants in different countries. Vertical integration facilitates the use of price discrimination among its buyers in oligopolistic industries (Tirole 1988, p. 141); an integrated firm for instance can sell its intermediate product at a higher price to other firms on the market, but to its downstream subsidiaries competing with other firms at a lower price, allowing it to reap high profits for its product, and giving its downstream operations an edge over their competitors.

A high degree of integration can be found in the US styrene industry, with the top styrene producers integrated upstream into raw materials production, and downstream into styrene derivatives as depicted in Table 5.10. Chevron, Dow, and Nova are fully integrated in their upstream and downstream operations. Both Dow and Nova are among the top world styrene producers, but in the US they were in deficit in 1999 due to their large styrene derivative production. They continued to supplement their requirements through long term arrangements with merchant styrene suppliers such as Lyondell and Sterling. Cos-Mar also is fairly integrated, if one considers the operations of the joint venture partners, Fina Oil⁵⁸ and GE Plastics, that make up the company. Fina produces benzene as part of its refining operations, and is also a large producer of polystyrene. GE Plastics on the other hand,

⁵⁸Fina Oil is now part of the TotalFinaElf Oil Group.

is involved in the production of polystyrene and other styrene derivatives such as ABS resin and SAN styrenic resins. Overall Cos-Mar's output is totally consumed by its joint venture partners.

Lyondell is integrated upstream into ethylene and benzene production, but has no downstream operations mainly because its interest lies in the propylene oxide segment of the business. Prior to being overtaken by Lyondell, Arco divested its polystyrene assets that were taken over by Huntsman. Sterling, on the other hand, is the only major styrene producer that is not integrated either into raw materials or derivatives of styrene. All its output is sold on the merchant market mainly for export, and in long term contracts where about 35% is sold on contract basis, mainly to BP-Amoco and Bayer.

Table 5.10: US styrene producers and degree of integration of operations - 1999
(Thousand tonnes/year)

	Styrene Capacity	Styrene Precursors ^(†)			Styrene Requirements ^(‡)			Net Styrene
		Ethylene	Benzene	Ethyl-benzene	Poly-styrene	Other	Total	
Lyondell	1225	X	X	X				1225
Cos-Mar	907			X				907
Chevron	771	X	X	X	349		349	422
Sterling	771			X				771
Dow	644	X	X	X	590	268	857	-213
Nova	567	X	X	X	1034		1034	-467
BP Amoco	454	X	X	X				454
Westlake	160	X		X				160
Huntsman	145	X		X	84		84	61

^(†) Upstream integration into styrene precursors is indicated by X.

^(‡) Styrene requirements are based on derivative capacities.

This situation perhaps explains Sterling Chemicals' difficult position as a midsize styrene producer, which is not integrated in terms of its feedstock or downstream derivatives. Sterling was more susceptible to adverse effects of styrene's cyclical markets. By decreasing its reliance on export markets, and looking for domestic long term contract customers Sterling was trying to minimise the effect of spot market fluctuations on its profit margins. The firm was also considering takeover

bids⁵⁹ from its competitors during mid 1990's. It remains possible that at a future date, Sterling will be merged or taken over by one of its competitors, or another firm seeking further integration.

5.7.2 Petrochemicals and the Case for Strategic Investment

Firm strategies should be evaluated on the basis of the outcomes that the firm actions and strategies produce. Since in general, one should assume that the management, in their investment behaviour, are rational decision makers, and their investments are designed to achieve the goals that they set for their company (Scherer & Ross 1990, p. 128).

Empirical studies on investment behaviour in the petrochemical industry include the important studies of Gilbert & Lieberman (1987), Lieberman (1987a, 1987c), and Paraskevopoulos (1988). In these works, the authors find that generally there is little evidence of incumbents success in investing preemptively in capacity to deter entry into their markets. Their findings are consistent with Nash-Cournot type pattern of investment where smaller firms are more likely to add capacity, while larger firms are more careful in adding new capacity to avoid deteriorating market conditions where they have substantially more at stake. They suggest that firms in the petrochemical industry did not invest strategically and attribute excess capacity to exogenous factors such as market variability, and lumpiness of investments.

However, the following points are worthy of mention here. Gilbert & Lieberman (1987), and Lieberman (1987c) study a cross-section of petrochemical products over a period extending from the late 1950's to 1982. The products belong to different sub-sectors of the petrochemical industry. The technology, market structure, and market characteristics for these products can vary widely. For many products the process technology and production knowhow is not easily obtainable (such as toluene diisocyanate (TDI), methyl-diphenyl-diisocyanate (MDI), acrylonitrile, terephthalic acid, vinyl acetate, propylene oxide, and pentaerythritol). In addition a high degree of market integration, where the major firms are vertically integrated from the production of raw materials, to the downstream production of derivatives, would make entry rather diffi-

⁵⁹ *Is Sterling Chemicals Open for Bids?*, Chemical Week, 5 April, 1995, p. 9. Also *Sterling Situation Remains Unclear*, Chemical Week, 3 April, 1996, p. 9.

cult. The investment behaviour and response of incumbents to the threat of entry in market segments of these products would differ markedly from those that do not share these attributes, like ethylene and its thermoplastics derivatives for instance, or fertilisers. It would not be surprising even if the behaviour of the same firm, that is active in both types of these products, differs in each market in accordance with the product attributes and its market characteristics. Without distinguishing between these product groups and controlling for their attributes with respect to the firms' investment behaviour, and the likely long term influence on concentration, the results are likely to be liable to bias⁶⁰.

Furthermore, the data coverage until 1982, extends over the *growth* phase for most of these products. Also entry on a large scale was taking place during this period. But it just covers the period when the market growth was beginning to slow down and overcapacity becoming more apparent as a result of the increased importance of economies of scale, and arrival of the era of large scale plants. It misses the rest of the 1980's and the period when considerable merger activity and market restructuring was taking place. It should be noted also that products, where the firms operated in concentrated markets and overcapacity was noted to be chronically high⁶¹, were studied separately as well. But here also, citing production decline resulting from obsolete plants and the effects of higher oil prices as the main causes of overcapacity, the period of study was confined to cover only up to 1973, thereby omitting one third of the sample, where chronic overcapacity was noted but had occurred in the post 1973 period.

Upon examining the West European PVC industry, Paraskevopoulos (1988) found that the investment behaviour of larger firms was accommodating towards the smaller ones. He shows how excessive investment and rivalry between the large number of producers lead to widespread overcapacity and severe price competition. As the markets stagnated in the early 1980's combined with significant capacity additions, price cut-

⁶⁰See Scherer & Ross (1990, pp. 645-650) review and discussion of empirical models on the relationship between market structure, innovation and investment in R&D. In particular the influence of proper specification of low-opportunity and high-opportunity industries on the direction of relationships with respect to industry concentration. Also see Geroski (1990).

⁶¹Chronic excess capacity is defined in Lieberman (1987c), by the strict requirement that capacity utilisation for the products concerned falls below 80% for five successive years or more. Capacity utilisation fell below 80% for 35% of the observations prior to 1973 but only 15% were classified as chronic. Of the ten products that were found to have exhibited chronic excess capacity, for three products only, was excess capacity used strategically. And of those, for one product only was entry successfully deterred.

ting between the major producers was rife; but planners realised that there is a huge financial price to pay resulting from price competition. It was a question of who could bear this situation and absorb financial losses before the weaker competitors, exited the industry and taking with them the excess capacity⁶². This situation was exacerbated by the reluctance of companies to exit the industry first, and were instead waiting for other competitors to make the hard decisions (Paraskevopoulos 1988, p. 198).

The situation was remedied and profitability somehow restored as a result of the restructuring process which ensued. During the period between 1980 and 1983 the number of West European producers declined from 23 to 14 in vinyl chloride (the raw material for PVC production), and from 27 to 16 in PVC⁶³, and almost one million tonnes of capacity was closed down. The largest 4 firms capacity rose from 39% to 50%. Like Gilbert & Lieberman (1987), he discounts the possibility that investment was used strategically in the West European PVC industry, and concludes by attributing overcapacity to the lumpiness of modern plants, demand variability, and poor planning of new investments manifested in many firms jumping on the investment bandwagon at some stages during the development of the industry.

Gilbert & Lieberman (1987) also find a bandwagon effect whereby smaller firms' investments undermine larger firms' investments that would have had a preemptive effect. Their results reject the round-robin pattern of investment where firms take it in turn to maintain market shares. They, however, find both elements of preemptive behaviour and a tendency to maintain market shares. Where the role of short term preemptive investment is to coordinate new investments which minimises redundant capacity and has the long term effect of maintaining market shares. This is achieved through the role played by the firms with large market share, which have a more pronounced potential for preemption and in moderating fluctuations in industry capacity than their smaller opponents. This accommodating behaviour is found by Lieberman (1987a) to be prevalent in

⁶²European Chemical News June 22, 1982.

⁶³The number of PVC producers in Western Europe declined further to only 10 producers by 2002. PVC prices in Europe soared by 40% in the last few months, but producers were still unhappy with margins. Some were seeking opportunities for mergers in order to benefit from greater economies of scale. A large proportion of Western Europe producers were considered to be relatively small in comparison with PVC players in the US, and suffer from poor access to raw materials supply. Chemical Market Reporter, 13 May, 2002, p. 2.

concentrated chemical industries, where incumbent firms coordinated their investments avoiding excess capacity especially when investments in greenfield projects were undertaken. But incumbents reacted with hostility towards investments by entrants, by adding capacity following entry. He finds that excess capacity developed in concentrated industries following entry, indicating the strategic use of excess capacity to deter mobility rather entry.

Another interesting feature uncovered by Lieberman (1984) for the same set of products is that prices in concentrated markets showed a lower propensity for decline. Whereas in less concentrated markets prices declined sharply and were influenced equally by new investments by incumbent and entrants alike. The incidence of coordinated investments by incumbents, combined with low rates of unutilised capacity and maintenance of high prices in concentrated industries raises concern about the possibility of tacit collusion taking place for some of these products⁶⁴. Neither Gilbert & Lieberman (1987), Lieberman (1987c), nor Paraskevopoulos (1988) consider or test for collusion in their models.

Collusion did occur on several occasions in the West European chemical industry during the 1970's and 1980's. There were reports in the trade journals about some companies discussing the need for crisis cartel to deal with the special difficulties facing the industry⁶⁵. In the fibre industry a cartel operated by the eleven major Western European producers between 1978 and 1980, with the intention of reducing overall capacity in such a way as to preserve the existing market shares (Shaw & Shaw 1983). Also, in polyethylene, polypropylene and polyvinyl chloride (PVC), price fixing arrangements were uncovered to have been taking place, over the period between 1982 and 1986. Several of the largest West European petrochemical producers were party to these arrangements. The EU Commission brought court charges against these companies, and were subsequently fined⁶⁶.

⁶⁴Christensen & Caves (1997) make these observations as well, and give a fuller and more critical account of their implications.

⁶⁵Solvay and Ato-Chem were attempting the formation of a plastics cartel at a time when overcapacity was affecting the major petrochemical producers in Europe and the threat of imports was looming during a recessionary period. *Financial Times*, *Plastics Cartel*, 23 July 1982, p. 2; also *The Economist*, 24 July, 1982, p. 58.

⁶⁶*Financial Times*, 2 May, 1986. Also *European Chemical News* issues of 4 November, 1991; 9 March, and 16 March, 1992.

Suggestions about the formation of a crisis cartel for the petrochemical industry continued to appear in the press; there were discussions at the Association of Petrochemical Producers in Europe in this regard

The majority of these large petrochemical companies operate in markets on both sides of the Atlantic, and globally. Both Paraskevopoulos (1988) and Lieberman (1987a) fail to take these developments into consideration and seem to suggest that the industries they study were competitively priced. Gilbert & Lieberman (1987, p. 118) show that in concentrated sections of the petrochemical industry capacity could be used to preempt rivals' expansions; but were unsure about whether such behaviour was intentional or profitable. Declining prices and low capacity utilisation rates need not always be signs of a competitive industry (Caves & Porter 1977). The concerned industries continued to invest even during periods of severe overcapacity. Companies with strong competitive advantage took this opportunity to retire old plants, and replace them with new more efficient and larger plants employing the latest technology. They also engaged in a series of mergers and product line swaps. As a result, small and the less efficient producers were forced out of the market in the process⁶⁷. All these moves can, if anything, be classified as strategic moves in the long-run scheme of events. They aim to reduce the number of industry players, thereby easing the pressure of competition and improving profitability. Although Stigler (1964, p. 45) takes a more acute view, and considers outright merger of firms as the most comprehensive form of collusion.

Industry consolidation through the acquisition of opponents capacity, or preemptive investment that eventually leads to the exit of competitors, is one way of avoiding engagement in illegal practices such as price fixing arrangements when competition is strong. In any case, price coordination is rather difficult to achieve in fragmented industries and

which prompted Karel Van Miert the European Commissioner for Competition, to address the issue in a long article, restating the Commission's industrial policy. It was made clear that the Commission was ready to assess the situation provided it was satisfied that the situation did not arise from a temporary dip in the business cycle, and that a major restructuring of capacity was necessary. He alluded to the recent anticompetitive arrangements and subsequent fines, and that the Commission will not hesitate to take similar measures to uphold the principles of competition law. *Competition Policy in the EC*, European Chemical News, European Review, December issue, 1993, pp. 7-9.

⁶⁷In addition to the account of Paraskevopoulos (1988) of the restructuring process in the VCM and PVC industries, other petrochemical products were subject to a similar phenomenon, and in most cases the same group of large petrochemical companies were involved. Also these arrangements included the geographical redistribution of markets. For instance in 1992, ICI swapped its worldwide nylon assets with Du Pont's acrylics as part of a deal to rationalise their assets (European Chemical News, 1 September, 1992, p. 5). Five years later in a deal estimated at US\$ 3 billion, Du Pont acquired ICI's polyester fibres and PET (polyethylene terephthalate the plastic widely used for carbonated drinks bottles) worldwide operations, as ICI continued to concentrate its operations in speciality chemicals and away from commodities. An account of those developments for the world petrochemical industry can be consulted in Arora, Landau & Rosenberg (1998, chp. 13), Chapman (1991), Stobaugh (1988), and Fayad & Motamen (1986, chp. 2,3).

is usually short lived. In the US where antitrust laws are pursued more stringently than in other parts of the world⁶⁸, this was the industry preferred strategy to avoid the intervention of the Federal Trade Commission (Scherer & Ross 1990, chp. 10). Consolidation following severe competition and market restructuring, produces the desired effect of improving price margins, and restoring profitability. The developments in the US styrene industry (as in Section 5.3 above) appear to follow a similar pattern.

Another view, is that in some industries, firms find it preferable to engage in price wars than to buyout competitors, as this gives the impression of a tough industry with a competitive environment (Fudenberg & Tirole 1983; Wenders 1971). However, when capital investment outlays are very large, and raw materials costs high, which is particularly the case with basic petrochemicals, this kind of activity causes the engaging firms to suffer substantial losses with damaging effects on their profitability, and their ability to finance future investments. Coordinating investments, and capacity expansions becomes an incentive when only a small number of players is involved, and the industry has good growth prospects. The main players are encouraged to cooperate, however implicitly, by the prospect of maintaining, or expanding their market shares, together with minimising price erosion in a growing market.

It is interesting to note that of the world's top three styrene producers, Dow, BASF and Shell, only Dow operates directly in the US market, which until recently was the largest world market for styrene. Dow has been following an investment strategy which is accommodating towards its rivals in the US for the last three decades by allowing its market share to fall over long periods, as its rivals expand their capacities (see Table 5.3). At the same time Dow has been expanding its global investments in styrene making it the world's largest producer. Similarly BASF and Shell operate on a global scale when deciding on where investments are made.

In the industrial organisation literature there is no shortage of evidence of price co-ordination, and of stories of cartel formation in different industries over the years. The histories of the salt and sugar industries give a perfect example of homogenous product industries; the long histories of the development of these industries extending over

⁶⁸The exception being the case of Microsoft Corporation, where it was acquitted in the US, but was heavily fined in Europe for its monopolistic practices.

more than a century, provided a good number of studies which analyse the economic evolution of the two industries⁶⁹. In the US the Federal Trade Commission would investigate the industry activities, issuing new regulations and fines to discipline the salt industry throughout the 1920 and 1930's. It continued to intervene in the 1950's and 1960's also, against cases of price fixing arrangements by some of the leading companies. In many instances, US salt regional markets were controlled to the tune of more than 90%, by the top three producers.

Also in the UK, following a wave of investments and mergers in the 1950's and 1960's, the salt industry became effectively a duopoly with two firms controlling 95% of the market. The Mergers Commission intervened in 1986 to regulate prices after an investigation found that there was a lack of effective competition. More recently, a case brought to the high court by the NHS (National Health Service) against five generic drug producers, for allegations of fixing prices and allocating market shares, was making headlines. Involved in the case, although indirectly, was the pharmaceuticals global producer Glaxo-SmithKline. The case⁷⁰ is still in its early stages, but its details are worthy of note as a typical example of coordination in a concentrated market where the stakes are high.

Several other examples abound from the US regarding the monopolistic practices of some companies. The Justice Department challenged the dominant position of two major companies, IBM and AT&T in their respective markets⁷¹. Court cases were brought in 1975 against IBM's control of the computer market, and AT&T's of the communications

⁶⁹See Sutton (1991, chp. 6) for a discussion of these industries, and cited references for further account.

⁷⁰The health secretary lodged a lawsuit on behalf of the NHS, seeking damages for £120 million from 5 generic drugs companies in the UK. The NHS accuses these companies of fixing the price of the generic equivalent of the anti-ulcer drug Zantac, marketed by Glaxo-SmithKline. The case arose when the patent on Zantac expired in 1997, and the 5 new producers entered the UK market but prices remained practically unchanged. Glaxo-SmithKline had supplied the largest of the five producers with a generic version of the drug, and was not directly involved in price fixing arrangements. But according to the filed case, documents seized by the Office of Fair Trading appear to show that the 5 companies mistakenly believed that Glaxo-SmithKline was in the cartel with them, and they set aside 34% of the UK generic market for Glaxo-SmithKline, and attempted to fix prices. Top selling drugs protected by patents are very important for pharmaceutical companies. Zantac's sales in 1996 of £2.2 billion accounted for almost a quarter of Glaxo-SmithKline's total sales (*Glaxo ensnared in £120m lawsuit over drugs 'cartel'*, Sunday Times, 22 August, 2004, p. 7).

In 1988 Zantac accounted for as much as 49% of Glaxo's pharmaceuticals sales, and Tagamat a rival ulcer treatment drug produced by SmithKline accounted for 51% of its sales. The two companies merged in 2000 during the wave of pharmaceutical companies mergers which began in the second half of the 1990's. An interesting and detailed account analysing pharmaceutical companies marketing strategies, and oligopolistic practices, covering Zantac and similar drugs appears in Sutton (1998, chp. 8).

⁷¹Refer to Scherer & Ross (1990, pp. 459-464) and cited references for a detailed account of these developments.

and related equipment market. By 1982 the case against IBM was dropped due to fast technological advances that were being made and that brought about the mini and personal computers and changed the market for computers; while AT&T was broken up into three constituent companies. Earlier on, Alcoa was also forced to divest some of its aluminium assets, following its aggressive investment strategy of preempting its much smaller rivals in the 1950's, that left it in a dominant position in the US aluminium market (Reynolds 1983, 1986). Du Pont's conduct in titanium dioxide, a chemical used as a whitener in the paints industry, was similar to that of Alcoa; its successive investments were shown to preempt its rivals giving it a leading edge and a large controlling market share (Ghemawat 1984, Hall 1990, Scherer & Ross 1990, Rosenbaum & Ye 1992). Du Pont was found to be following a limit pricing policy that would starve its competitors of the necessary funds to compete with it effectively, and was subsequently forced by the Federal Trade Commission in the late 1970's, to divest two of its plants and to license its technology to its competitors. On a previous occasion also, Du Pont was ordered to license its nylon technology, in which it also had a dominant market share in the 1950's, to Monsanto without receiving any licensing fees from the latter (Arora, Landau & Rosenberg 1998, p. 92). More recently an antitrust case was brought against Du Pont Dow Elastomers, a 50-50 joint venture between the two companies, in the US District Court of Columbia for price fixing of rubber chemicals⁷². The company reached a settlement, subject to court approval, with buyers of polychloroprene a synthetic rubber used widely in tyres, to pay them US\$ 36 million plus attorney's fees for alleged overcharges from 1999 to 2003.

Collusive behaviour can be found in other industries and a wide range of countries as well; besides OPEC in oil, cartels were operating in one way or another and with different degrees of success, in many countries and in different industries⁷³: In steel (Yamawaki 1984,1985, Baker 1989), aluminium, and cement (Steen & Sørsgard 1999), to name only a few besides those noted above in petrochemicals. The point to note here is not necessarily that the US styrene industry operated in collusion, but that firms,

⁷²*Price-Fixing Investigations Continue*, Chemical Market Reporter, 28 June, 2004, p. 2.

⁷³See also cartels that were operating in the US and other countries surveyed in Scherer & Ross (1990, chp. 12)

which are after all industrial establishments whose main objective is to generate profits for their shareholders, have a tendency to take advantage of opportunities that arise to collude directly or indirectly; and an oligopolistic market is more conducive to this behaviour, where it is more likely to lead to success, especially in situations where a small number of players are involved. In view of the experience of recent history, and firms' behaviour in oligopolistic industries, the findings of studies of these industries that overlook strategic intentions, or possible forms of collusion of its participants that operate in highly concentrated markets, should be treated with caution. The onus is on the regulatory authorities to detect and deal with these incidents when they arise, as has been the case on numerous occasions.

5.8 Concluding Remarks

Studies of investment behaviour of firms, that are concerned with processes of exit and entry, tell only half the story when they neglect to account for the impact of investment on changes in market structure. By and large, these studies confine their attention to measuring the determinants of the process of entry, in relation to the presence of over-capacity. Capital investment in new capacity secures for the firm its claim for long term share of the market. Output is bound by capacity. And like with all large scale processing industries, in the petrochemicals industry expansion of capacity and its adjustment is slow, and requires planning and substantial investment of capital.

In industries that are characterised by economies of scale and learning effects, such as petrochemicals, raising capacity allows the firm to move further down the long-run average cost curve. A firm with a larger capital stock (plant) has a competitive advantage over its smaller rivals. With a larger plant not only can it produce at a lower cost, but it can also accumulate larger production volumes to its credit, and gain learning experience more quickly; which in turn enables the firm to reduce its costs further, at a higher rate than its competitors. Also additional capacity gives the firm the capability to service its customers when demand rises unexpectedly. A firm may be willing to incur higher costs of carrying excess capacity, so that it can meet unexpected rises in demand. Furthermore,

excess capacity in an industry gives outsiders the impression that the industry is a loose oligopoly prone to price wars and cyclical bouts of low margins and possible losses, which discourages entry into the industry (Kirman & Masson 1986).

With these characteristics in mind, I proceeded in this chapter to examine the investment behaviour of the six major styrene producers that dominated the US styrene industry since 1974, using the dynamic capacity expansion model, and four different scenarios of investment strategies, ranging from competitive price taking environment, to open-loop Nash-Cournot, feedback Nash-Cournot, and Stackelberg leadership, that were discussed in Chapter 4. The model captures the interaction between the different firms, and the competitive nature of the market directly.

Technological advances in chemical engineering and process technology, that had a profound impact on increasing the size of petrochemical plants, were found to be present in the styrene industry. The sizes of plants increased by a factor of ten fold over a period of three decades, and resulted in a considerable reduction of the unit investment cost required per tonne of styrene produced. Furthermore, learning effects where firms through their experience in production operations, introduce cost saving measures were at work in the styrene industry.

In formulating the model for the simulations of the styrene industry developments, both of these effects were taken into consideration explicitly. When evaluating the investment cost for each firm, the economies of scale that were implemented by the styrene industry, were accounted for in each firm's investment cost function. Cost savings on the basis of a styrene industry learning curve, were also accounted for in each firm's production costs as they changed over time.

Using empirical data from the styrene industry, the simulation results show that the investment behaviour of the major styrene producers, is best described by the feedback Nash-Cournot model. This suggests that the firms were pursuing strategic investment policies. In all the cases considered, the firms in industry held more capacity than would be necessary to meet demand, had the firms been following optimal investment strategies that maximise their profits (these results are consistent with those found by Reynolds (1986) in the aluminium industry; but in the case of styrene, the model has better

predictive power in explaining most of the industry overcapacity). This outcome was found to be consistent when different values of price elasticity of demand were examined, ranging from strongly inelastic demand ($\epsilon = 0.50$), to market conditions of moderately elastic demand ($\epsilon = 1.20$).

The feedback Nash-Cournot model provided a better representation of capacity accumulation and expansion than the open-loop and Stackelberg models. The feedback Nash-Cournot strategy for our base case (*Ncfb7*), explains most of the excess capacity experienced by industry. The evolution of capacity and styrene price it generates, are consistent with industry developments. If we are to believe the game theory story on strategic behaviour in Nash-Cournot games and the [explanation for the] incidence of overcapacity, then by following investment feedback strategies, the firms exercise a strategic intent to influence the investment behaviour of their competitors. Excess capacity in this respect may be used to preempt rivals expansions, as well as to deter entry, or mobility of *nouvo* producers. When all the firms implement such a strategy the outcome is industry wide overcapacity.

Notwithstanding the strategic behaviour, excess capacity may also be plausibly used by firms to counter the effect of uncertainty of demand, an element which has been assumed away in this chapter. We have allowed, like all deterministic models, the firms to have full foresight of future demand. In the next chapter I examine the effect of demand uncertainty on the capacity expansion of profit maximising firms, and on the extent of industry overcapacity.

We have also examined developments in the styrene industry since 1974. The industry continued to grow at high rates over this period, with capacity rising from just under 2.7 million tonnes in 1975, to more than 5.7 million tonnes in 2000; and will increase by another one million tonnes, if all the expansions now underway or planned, come on stream by 2005. However, despite the considerable activity apparently taking place in the industry, only two entries may be classified as *nouvo* entries into the US styrene market, namely, Arco and Westlake. The rest were mergers or takeovers involving new ownership of existing producers and their facilities. All firms took advantage of an expanding market to build new capacity over this period; but the market growth opportunities, did not

lead to an increase in the number of industry players. Two trends appear to have taken place in parallel: (a) Small firms expanded their capacity in a Nash-Cournot fashion. Cos-Mar and Chevron moved from being small producers in the early 1970's, to join the league of top producers in a series of expansions (see Table 5.3). Nova joined the race a decade later; and more recently moved to become one of the global styrene players. Each of these companies now holds a market position comparable to that of the traditional leading companies in the styrene market, such as Lyondell and Dow. Their new positions were reinforced by the new capacities that came on stream in 2004 and 2005. And (b) the increased level of activity in the industry heightened the level of competition, which eventually lead to the exit of a number of smaller producers from the market. A restructuring process was precipitated, with some of the large players such as Monsanto, Shell, Union Carbide and Hoechst exiting the US styrene market and repositioning themselves in other more profitable segments of the chemical industry (Monsanto and Hoechst), or by reorganising their global operations and moving into other geographical markets (Shell). Following the second wave of mergers in the 1990's, the industry became more concentrated; and profit margins remained high.

In view of the large market opportunities that the growing styrene industry offers, and very low level of entry over long periods, can we deduce that the industry incumbents have succeeded in reducing the level of entry, and in deterring mobility, by employing excess capacity? Our model results suggest that the firms invested strategically, and excess capacity was persistently present in the industry. Entry deterrence was not total in the styrene industry as entry deterrence models would suggest; but entry was effectively minimal. On theoretical grounds also, as suggested by Dixit (1980), the incumbents would not carry out their threats once entry occurred. In a dynamic setting Dixit's argument becomes even more convincing. In a growing market, growth opportunities make it easier and less painful for incumbents to accommodate a new entrant. By following Nash-Cournot strategies, the firms can generate more profits, and there is less likelihood of resorting to a price war.

Other factors, that hindered entry, were at work as well. Merger activity lead to industry consolidation, leaving a smaller number of firms active in the market. The in-

dustry is now served by eight producers. With a smaller number of players, coordination of investment activity, and production discipline is by far easier than in loose oligopolies, where agreements are more difficult to enforce, and are prone to breaking down. Coordination in the styrene industry was taking place at more than one level. As well as engaging in the signalling and announcement game for new capacity investments; some of the firms would cooperate more directly on the supply side of the industry by arranging long term contracts, swapping agreements for styrene product, or through direct joint investment in large scale projects.

Another two characteristic features of the petrochemical industry linked to entry barriers, that were strongly evident in the styrene industry, are the vertical integration of production operations, and economies of scale. Although with the advent of large scale plants, significant cost savings in production costs could be achieved, and unit investment cost per tonne of styrene product were much lower; the sheer size of new large plants meant that the absolute capital investment required for the new projects were rising all the time. This trend places smaller producers under additional financial pressure, should they want to stay competitive and keep pace with industry developments. Integration of production operations, upstream to secure supply of raw materials (mainly ethylene, but also benzene for larger producers) by direct investment in their own facilities, or through long term contracts, was compounded by the effect of economies of scale, and the requirement for large investments in the form of setup costs. Capital investment costs for new large styrene plants, run into hundreds of millions of dollars. Financial burdens of this magnitude can be afforded only by large industrial enterprises, and is beyond the reach of most small industry players. Also, when major styrene producers integrate downstream into styrene derivatives not only do they secure higher value added for their styrene product, but they also crowd out potential suppliers, such as prospective entrants, from the market.

Chapter 6

Capacity Expansion Under Demand Uncertainty

6.1 Introduction

Whereas the firms in previous chapters have been treated as being able to foresee future demand developments, giving them the chance to plan for future capacity expansions accurately; in this chapter the demand is treated as stochastic. The firms hold rational expectations about the stochastic process that drives the demand for the homogenous product they produce. That is the level of demand in the current period is known to all firms but its future value is uncertain; however, the mean and variance of the stochastic process are known to all firms. I adopt a geometric Brownian motion process (GBM) with time invariant mean and variance. This type of process is particularly suited for the problem tackled in this thesis, and has been widely used to model securities prices in financial markets as well as in the investment literature to model commodities prices, or the value of an investment in a project producing a product that is subject to a demand that follows a GBM process (Pindyck 1982, 1988, 1991, McDonald & Siegel 1985, Brennan & Schwartz 1985, Kulatilaka & Perotti 1992, Dixit & Pindyck 1994, Slade 2001). In addition to its appeal from an analytical point of view, a GBM process describes very well the evolution of economic developments. Firms may have an idea about the general level of demand, especially in a growing industry where the trend may be known, but it

would fluctuate along its path; and this is what a GBM process models.

Uncertainty about other variables could also be envisaged as well. Uncertainty about raw materials costs, or capital investment costs has important implications for the profitability of firms. And would no doubt have a bearing on the investment, or production decision of the firm. But in the context of planning capacity expansions, it is the expected level of demand for its products that is of crucial importance for the firm. Most petrochemical firms are integrated in their operations, and could pass, with relative ease, rises in raw materials costs onto down stream products; or they can absorb these rises in the chain of products involved in the production process. Also capital investment costs per unit of product produced, form only a small proportion of the final product total cost. For the investing firm, the way demand evolves in the future, and any uncertainty about the level of demand remain the main causes of concern.

The rate of growth of demand (and consequently price through the inverse demand function) is being modelled with a certain degree of confidence in this chapter. The firms are concerned about the rate of growth of demand, from its current level to the not too distant future as a means for planning their investments or production schedules. And this has the obvious bearing on the rate of capacity utilisation and the rate of capital accumulation, with direct implications for profitability.

6.2 Demand Uncertainty and Capacity Planning in the Literature

The literature on the optimal capacity held by a firm facing uncertainty about demand, output price, or costs has been dealt with using different approaches. The investment behaviour of the firm, is contingent on whether capital is reversible or irreversible; also the presence of adjustment costs and the form of the production function have an impact on the direction of the investment-uncertainty relationship.

One strand of the literature involves adjustment costs, and capital reversibility such as in Abel (1983), Caballero (1991), and Abel & Eberly (1994). In these works, increased uncertainty facing industry leads firms to increase their investment. Pindyck (1982, 1988)

also finds that a firm facing convex adjustment costs, and depreciating capital when subjected to stochastic demand or a price process of the GBM type, has an incentive to hold more capital. Under these conditions it is optimal for the firm to increase its stock of capital with increasing uncertainty.

Whereas in the option pricing literature uncertainty leads to the opposite effect. Firms in this case are faced with uncertainty about the price of their output which is determined exogenously, and is subject to a GBM process. Like in the financial literature increasing volatility of price increases the option value of the project, and as a result firms hold on investment awaiting more information. This leads to a decline in investment. Capital is assumed to be irreversible in this case. And the main concern for firms in this strand of the literature is the relationship between price volatility and the timing of investment at the optimal price that maximises the value of the firm. A good survey of this literature is found in Pindyck (1991); and Dixit & Pindyck (1994) provide an extensive coverage with a wide range of examples of different applications.

Abel & Eberly (1997) extend their previous work to the case of capital irreversibility; they find that irreversibility does not change the investment behaviour of firms, but it reduces its value. Similarly, Kabilia (1991) find a generalised solution to the optimal investment problem of the firm under uncertainty and irreversibility. Uncertainty is introduced in general form into the profit function, and can take the form of demand uncertainty, stochastic product price, input price uncertainty, or a random disturbance into the production function or cost function. The problem is formulated into a generalised Hamilton-Jacobi-Bellman equation. They¹ find the decision rule for investment, when the inverse demand function is subject to a Brownian motion process; and show that there is a risk premium due to uncertainty and irreversibility which leads to smaller optimal capacity under those conditions, than under reversible capital. They go on further to show that Pindyck's (1988) results of the firm's investment under uncertainty and irreversibility are a special case of their general result (Theorem 3, p. 183).

Baldursson (1998) studies the behaviour of an oligopoly facing uncertain demand and irreversible capital. He derives optimal investment rules in the forms of a trigger price

¹Kabilia is an acronym for a group of researchers at the Central Bureau of Statistics, Oslo, Norway, and the Institute of Mathematics, University of Oslo.

at which firms can enter the market, as the industry develops from a monopoly to a competitive industry.

Despite considerable advances made in theoretical work, empirical work in the area of capacity expansion under uncertain demand remains fairly small. Letteri (2000) introduces uncertain demand in the context of a water resource planning problem, and utilise an option pricing approach as in Dixit & Pindyck (1994). In her case, the dynamic capacity expansion problem is carried out for a single firm. Pineau & Murto (2000) evaluate capacity expansion for different types of power generation plants in the Finnish market. They apply the open-loop stochastic S-adapted solution concept, introduced by Haurie, Zaccour & Smeers (1990), to solve the investment problem and find optimal capacity expansion scenarios. In both of these studies uncertainty cause firms to increase investment and accumulate more capacity.

Paraskevopoulos, Karakitsos & Rustem (1991) evaluate capacity planning in the West European PVC industry under demand uncertainty. Bell & Campa (1997) study the US chemical industry in an uncertain environment. They find that firms' investment is more sensitive to fluctuations in foreign exchange rates; and increased uncertainty reduces investment and leads firms to expand into foreign markets.

6.3 Model Formulation for a Duopoly Under Demand Uncertainty

Once again for clarity of exposition, I now consider the case where the market is served by a duopoly, and make the same assumptions about the two firms as in Chapter 3, and follow the same steps in formulating the problem. To remind ourselves, the firms produce at full capacity, such that production is equivalent to capacity in place; and they face negligible production costs. Both firms are risk neutral and have existing capacities installed as before; but they are now faced with uncertain demand. An inverse demand function is again considered to represent the price that clears the market. The derived demand equation (3.7) of Chapter 3 now has the scalar a replaced by a random variable

θ , and is represented by the following equation:

$$P = \theta - b(k_1 + k_2) \quad (6.1)$$

All terms and assumptions are defined as before, and the two firms' capacities are k_1 and k_2 . We assume that the random variable θ follows a GBM process:

$$d\theta = \mu\theta dt + \sigma\theta dz \quad (6.2)$$

where μ is the expected growth rate of θ per unit of time, and σ is the standard deviation of the proportional change in θ per unit of time; and where θ is non-negative, that is $\theta \in [0, \infty)$. In the above equation σdz represents the variability in the path of the expected growth rate of θ ; where z is a Weiner process with a zero drift rate, and a variance of 1.0. The rate of change of z over a short period of time dt is given by $dz = \epsilon\sqrt{dt}$, where ϵ is a random draw from a standard normal distribution.

Before proceeding, I make the following assumptions about the two firms:

1. The market is served by two firms only, and derived demand is given exogenously and is driven by a GBM process as in equation (6.2) above. Price is market clearing, and there are no inventories held from one period to another.
2. The firms are risk neutral, and discount their future profits at the risk free interest rate r .
3. Both μ and σ are time invariant, and are constant over the entire planning horizon from t_0 to T . Changes in θ are exogenously generated; and its initial value θ_0 , at time t_0 , is known to both firms, however, its future values are unknown, but the firms know its distribution which follows equation (6.2) above.
4. Costs are known with certainty for all firms for the entire planning horizon.

Since the price of the output follows a random process, the profit of each firm also will now become random, and follows the same process. The value of the firm $J_i(t_0)$, and how capacity evolves have the same formulation as equations (3.1) and (3.2) defined in

Section 3.2 of Chapter 3. With the value function now involving a random process, the equation should include the expectation operator, as we now take the expected value of the random process as it evolves over time.

$$J_i = \max_{u_i} E\left\{\int_{t_0}^T e^{-rt} w_i dt + F_i(k, T)\right\} \quad i = 1, 2 \quad (6.3)$$

subject to

$$\dot{k}_i = u_i - \delta k_i \quad (6.4)$$

where:

- J_i = value of firm i at time t .
- w_i = single period net profit for firm i in period t .
- $F_i(k, T)$ = salvage value of capital for firm i
at the end of the planning horizon T .
- r = interest or discount rate.
- k_i = capacity of firm i .
- u_i = investment rate of firm i .
- δ_i = discount or capital depreciation rate.

The value of the project, J_i consists of two parts, a cash flow which is now uncertain represented by the integral term, and a reward or salvage value that the firm can recover at the end of the planning horizon T ; this is represented by the second term $F_i(k, T)$. This second term is not a function of any random variable, and is therefore deterministic, although its actual value is unknown. Its value can be obtained from the boundary conditions. In view of the long planning horizon contemplated, the salvage value term is assumed to be zero. Making this assumption does not affect the project value in any significant way.

Equivalently the above equation can therefore be written as:

$$J_i = \max_{u_i} \left\{ \int_{t_0}^T E[e^{-rt} w_i] dt \right\} \quad (6.5)$$

Let us concentrate on the cash flow term. Recall that net profit is defined as the operating

profits less investment cost:

$$w_i(t) = \pi_i(t) - I_i(t) \quad i = 1, 2 \quad (6.6)$$

The project is treated now as a series of European call options; in each period the firm pays $I_i(t)$ and receives a claim of $\pi_i(t)$ on future revenues, and generates a flow of $w_i(t)$ of profits. It can be assumed that the firm will produce as long as it covers a certain fixed cost $C_i(t)$, which is known with certainty. To avoid making losses the firm's decision to produce is conditional on covering its fixed cost, and has the option to shut down. Otherwise the firm will not produce and it makes zero profits. As such the value of the per period profit of this uncertain cash flow is given by:

$$E[w_i] = E \max[\pi_i - I_i, 0] \quad i = 1, 2 \quad (6.7)$$

Now since the price follows a GBM due to the influence of θ , the per period profit w_i , which is linear in θ , also follows a GBM. The value of firm i profits w_i , can be estimated conditional on the information available at the initial time t_0 , if the probability density function, $g(\theta)$, for the GBM process can be obtained. But it is known that the probability density function for a log-normally distributed random variable can be easily derived from that of a normal distribution², and is shown in equation (6.11) below (McDonald & Siegel 1985).

Starting from the GBM for the inverse demand, where we know the initial value of θ_0 and its distribution i.e the two parameters μ and σ , we have:

$$\frac{d\theta}{\theta} = \mu dt + \sigma dz \quad (6.8)$$

First, we make a change of variable $\Omega = \ln \theta$, and then apply Ito's Lemma³ (see Dixit &

²by the transformation $f_n(x) \rightarrow f_{log}(x) = \frac{1}{x} f_n(\ln x)$ (Harris & Stocker 1998, pp. 797-798).

³Ito's Lemma when applied to a function of θ , $\Omega = f(\theta) = \ln \theta$, where θ follows equation (6.8) states that:

$$df(\theta) = d\Omega = \frac{\partial f(\theta)}{\partial \theta} d\theta + \frac{1}{2} \frac{\partial^2 f(\theta)}{\partial \theta^2} (d\theta)^2$$

remembering that $\frac{\partial \ln \theta}{\partial \theta} = \frac{1}{\theta}$, $\frac{\partial^2 \ln \theta}{\partial \theta^2} = -\frac{1}{\theta^2}$, and using equation (6.8), while eliminating higher order terms we obtain (6.9).

Pindyck 1994; for a more detailed treatment of the topic with examples see Dixit 1993, and Harrison 1985). We obtain the stochastic equation for $\ln\theta$, namely:

$$\ln\theta_t - \ln\theta_0 = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz \quad (6.9)$$

This can be written as:

$$d\Omega = \nu dt + \sigma dz \quad (6.10)$$

which is normally distributed. So $\Omega = \ln\theta_t$ has a mean $\nu = \ln\theta_0 + (\mu - \frac{1}{2}\sigma^2)t$ and a variance $s^2 = \sigma^2 t$. Since $\ln\theta_t$ is normally distributed then the density function $g(\theta)$, for θ , which is log-normally distributed, is written formally as $g(\theta_t; \nu, s, \theta_0)$ and is given by:

$$g(\theta) = \frac{1}{\sqrt{2\pi}\theta s} \exp\left(-\frac{1}{2}\left(\frac{\ln\theta - \nu}{s}\right)^2\right) \quad (6.11)$$

With the density function of the log-normally distributed process θ_t known, we can now proceed to estimate the expected value of profits:

$$\begin{aligned} w_i(t) &= \int_{C_i}^{\infty} (\pi_t - I_t) \frac{1}{\sqrt{2\pi}\theta s} \exp\left(\frac{1}{2}\left(\frac{\ln\theta - \nu}{s}\right)^2\right) d\theta \\ &= \int_{C_i}^{\infty} [(\theta - b(k_i + k_j))k_i - I_i] \frac{1}{\sqrt{2\pi}\theta s} \exp\left(\frac{1}{2}\left(\frac{\ln\theta - \nu}{s}\right)^2\right) d\theta \quad i \neq j, i = 1, 2 \end{aligned} \quad (6.12)$$

Now let $\frac{\ln\theta - \nu}{s} = y$, so $\theta = e^{sy + \nu}$ and $d\theta = se^{sy + \nu} dy$. Substituting in the above equation of the expected value of profits we get:

$$w_i(t) = \int_{\frac{\ln C_i - \nu}{s}}^{\infty} [(e^{sy + \nu} - b(k_i + k_j))k_i - I_i] \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \quad (6.13)$$

$$w_i(t) = \int_{\frac{\ln C_i - \nu}{s}}^{\infty} k_i \frac{e^{sy + \nu} e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy - \int_{\frac{\ln C_i - \nu}{s}}^{\infty} [bk_i(k_i + k_j) + I_i] \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \quad (6.14)$$

Once again we let $\gamma = y - s$, which implies that $y = \gamma + s$ and $dy = d\gamma$, and reevaluate

the above equation to get:

$$w_i(t) = \int_{\frac{\ln C_i - \nu}{s} - s}^{\infty} \frac{k_i}{\sqrt{2\pi}} e^{s(\gamma+s)+\nu} e^{\frac{-(\gamma+s)^2}{2}} d\gamma - \int_{\frac{\ln C_i - \nu}{s}}^{\infty} [bk_i(k_i + k_j) + I_i] \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy \quad (6.15)$$

$$= \int_{\frac{\ln C_i - \nu}{s} - s}^{\infty} \frac{k_i}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} e^{\frac{s^2+2\nu}{2}} d\gamma - \int_{\frac{\ln C_i - \nu}{s}}^{\infty} [bk_i(k_i + k_j) + I_i] \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy \quad (6.16)$$

$$= k_i e^{\nu + \frac{s^2}{2}} \int_{\frac{\ln C_i - \nu}{s} - s}^{\infty} \frac{e^{\frac{-\gamma^2}{2}}}{\sqrt{2\pi}} d\gamma - \int_{\frac{\ln C_i - \nu}{s}}^{\infty} [bk_i(k_i + k_j) + I_i] \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy \quad (6.17)$$

$$= k_i e^{\nu + \frac{s^2}{2}} N\left(s - \frac{\ln C_i - \nu}{s}\right) - [bk_i(k_i + k_j) + I_i] N\left(\frac{\nu - \ln C_i}{s}\right) \quad (6.18)$$

$$= k_i e^{\nu + \frac{s^2}{2}} N(d_1) - [bk_i(k_i + k_j) + I_i] N(d_2) \quad (6.19)$$

where we defined, $N(d_1) = \int_{-\infty}^{d_1} \frac{e^{\frac{-\gamma^2}{2}}}{\sqrt{2\pi}} d\gamma$, and $N(-d_1) = 1 - N(d_1)$. Also $d_1 = s - \frac{\ln C_i - \nu}{s}$, and $d_2 = \frac{\nu - \ln C_i}{s}$; or equivalently $d_2 = d_1 - s$. Now since $\nu = \ln \theta_0 + (\mu - \frac{1}{2}\sigma^2)t$, and $s^2 = \sigma^2 t$, by substituting for s and ν in the above equations for d_1 and d_2 , we obtain the value of the firm profits in terms of the known parameters of the original GBM equation.

$$d_1 = s - \frac{\ln C_i - \nu}{s} = \frac{\sigma^2 t - \ln C_i + \ln \theta_0 + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$\text{so } d_1 = \frac{\ln \frac{\theta_0}{C_i} + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{t} = \frac{\ln \frac{\theta_0}{C_i} + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

therefore using these values in the above equation for the expected value of the firm profits we obtain:

$$w_i(t) = \theta_0 k_i e^{\mu t} N(d_1) - [bk_i(k_i + k_j) + I_i] N(d_2) \quad (6.20)$$

where all the values θ_0 , C_i , σ and μ are known. Also d_1 and d_2 can now be estimated. It can be seen that this equation has the same form as the value of the project in McDonald & Siegel (1985); which is also analogous to the Black-Scholes equation (Black & Scholes 1973).

It is possible therefore to evaluate the expected values of future profits for each firm conditional on the known initial conditions. This entails summing the discounted values of the expected future profits, $w_i(t)$, up to time T , each firm will then be able to determine the net present value of its investment $J_i(t_0)$, by implementing equation (6.3) subject to the state equation (6.4), that is:

$$J_i = \max_{u_i} \left\{ \int_{t_0}^T e^{-rt} (\theta_0 k_i e^{\mu t} N(d_1) - [bk_i(k_i + k_j) + I_i]N(d_2)) dt \right\}, \quad i = 1, 2 \quad (6.21)$$

subject to

$$\dot{k}_i = u_i - \delta k_i \quad (6.22)$$

The problem can then be formulated as a two point boundary value problem, as in Section 3.4.2 of Chapter 3, and a solution is then obtained by solving a set of Riccati equations. The optimal investment strategy vector, $\mathbf{u}^* = (u_1, u_2)'$ for both firms is given by an equation similar to (3.77) (see Section 3.4.2.1, p. 121), but now involves stochastic terms.

$$\mathbf{u}^* = -\tilde{\mathbf{R}}^{-1} \left(\tilde{\mathbf{r}} + \tilde{\mathbf{S}}\mathbf{k} + \mathbf{B}' \left(\tilde{\mathbf{\Lambda}}\mathbf{k} + \tilde{\gamma} \right) \right) \quad (6.23)$$

Bearing in mind now that the value of $w_i(t)$ is changing continuously with time, as given by equation (6.20). From the solution of the Riccati equations we can determine the values of $\tilde{\mathbf{\Lambda}}$ and $\tilde{\gamma}$, and therefore the optimal value $\mathbf{u}^*$. Finding the optimal path of the capacities for both firms is then obtained by a straightforward implementation of $\mathbf{u}^*$ in equation (6.4). The results are shown in Figure 6.1 below. Firm 2 which starts from a lower capital stock (in this case it is assumed to enter the market at t_0 , with $k_2(t_0) = 0$; while firm 1 starts from a capacity of $k_1(t_0) = 10$) invests initially at a higher rate than firm 1 as pane (b) of the figure illustrates. It can be seen also that the investment rate curve follows a concave-convex shape. This is because when the two firms follow optimal investment strategies, they start from a high rate of investment initially, which then declines slowly and stabilises as the two firms accumulate more capacity. Towards the end of the life of the project, the investment rate starts to decline again. This results from the fact that the marginal value of any additional unit of capital declines due to its

short life before the project's economic life comes to a close.

Eventually firm 2 will catch up with firm 1; the capacities of the two firms converge and they share the market, since both have the same costs. Because of uncertainty the two firms invest more, and they achieve equilibrium at a capacity level of about 25 kt/year each, which is significantly higher than under deterministic demand. A comparison of the impact of demand uncertainty on capacity accumulated by the duopoly, with the results when demand conditions are deterministic is shown in Figure 6.2. These results are consistent with the findings of Kulatilaka & Perotti (1998), where increasing uncertainty increases the value of the project. This is because of the convexity of the value of the project *wrt* θ , the stochastic variable. Note that the optimal control vector $\mathbf{u}^*$ involves the demand stochastic variable θ ; and the value function is quadratic in $\mathbf{u}^*$ (see equation (3.21), or (3.71)). As uncertainty about demand increases, the value of the project, which involves θ^2 , increases at a faster rate because of its convexity. For the capacity expansion problem of the duopoly in our case, uncertainty about demand therefore causes the firms to invest in more capacity than under deterministic demand. The cost of investment is deterministic and bounded, whereas the demand expectations are increasing with time, and the firm has the option to shut down to avoid costs exceeding revenues when demand fails to materialise.

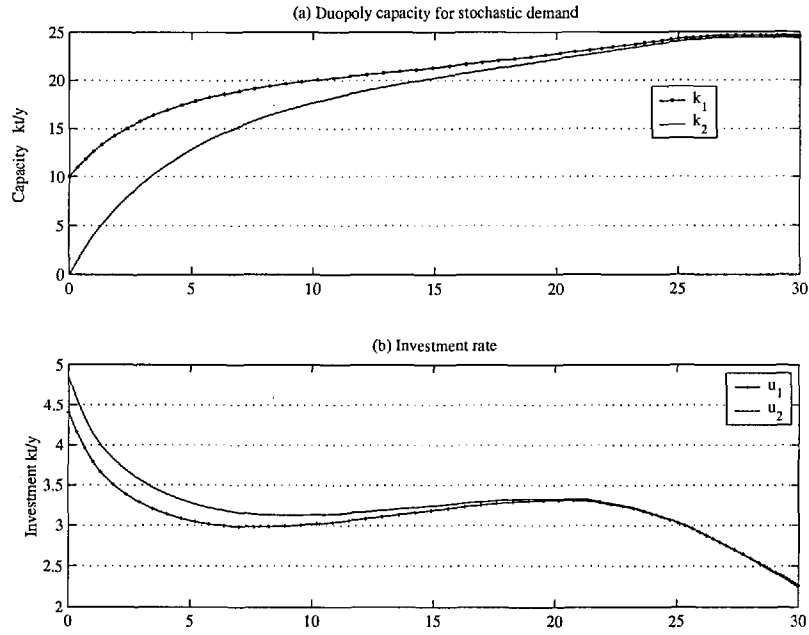


Figure 6.1: Optimal capacity expansion and investment for a duopoly facing stochastic demand: $\mu = 0\%$, $\sigma = 15\%$, $\theta_0 = 100$, $b = 2$, $c_1 = 10$, $c_2 = 10$, $C_i(t) = 70$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $r = 0.05$, $k_1(t_0) = 10$, $k_2(t_0) = 0$.

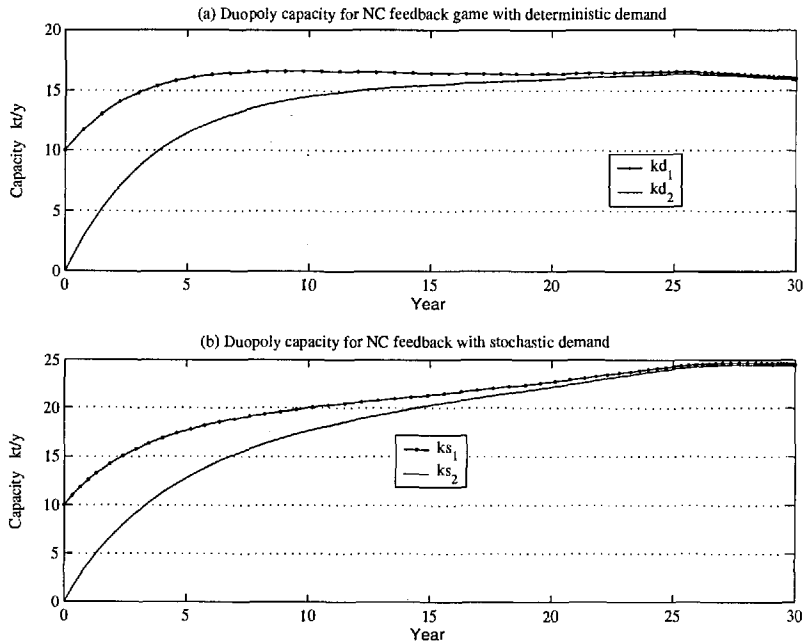


Figure 6.2: Optimal capacity expansion for a duopoly facing (a) deterministic and (b) stochastic demand conditions: $\mu = 0\%$, $\sigma = 15\%$, $\theta_0 = 100$, $a = 100$, $b = 2$, $c_1 = 10$, $c_2 = 10$, $C_i(t) = 70$, $d_1 = 50$, $d_2 = 50$, $\delta = 0.10$, $r = 0.05$, $k_1(t_0) = 10$, $k_2(t_0) = 0$.

6.4 Styrene Industry Optimal Capacity with Uncertain Demand

Uncertainty about the demand for styrene affects the overall demand facing the six major styrene producers (referred to as major-6 or M6 in the figures of this chapter). As in the case for duopoly, all the firms hold the same expectations about how the demand is evolving. Because the firms do not hold any inventory, their total output is equivalent to the demand in each period. I will look at two approaches in which demand is affected by the uncertainty process. In the first I will follow the approach of the option pricing literature where the inverse demand intercept a follows a GBM process as in Section 6.3; while in the second I will consider that the quantity of styrene product demanded follows a GBM process.

6.4.1 Uncertainty Affecting Inverse Demand Parameters

Consider the approach followed in the option pricing literatures as in Pindyck (1991), Dixit & Pindyck (1994) and Kulatilaka & Perotti (1992). This is a situation where the intercept of the inverse demand function, a , follows a GBM process as in Section 6.3, equation (6.20). All firms are therefore aware of how the demand is evolving and have full knowledge of its parameters:

$$da = \mu a dt + \sigma a dz \quad (6.24)$$

Using 1974 as the base year the major styrene producer firms know the starting values $a_{1974} = 2399$, and $b_{1974} = 1.04$. The styrene produced by these firms over the period 1974-1996 grew at a compounded average annual rate of 4.3%. I will assume that the major-6 firms had full knowledge of this growth potential for styrene so, $\mu = 4.3\%$, and that they had expected a modest variation of demand $\sigma = 15\%$.

Following the analysis of Section 6.3, suppose that each firm will keep its plant running and stay in production as long as its revenues can recover a portion of its fixed costs, which is the cost of its operating labour, $C_{i,l} = m_{i,l}(t)k_i$. Where $m_{i,l}$ is the operating labour cost per tonne of styrene for a medium size plant; and $m_{i,-l}$, represent the remaining

unit fixed costs. The revenues are now stochastic and follow (6.24). As before each firm has full knowledge of the remaining costs, which are now assumed to be known with certainty. To simplify the calculations I will assume that all firms, irrespective of their size, face the same fixed cost that each needs to recover to stay in operation.

To estimate the expected value of each firm's per period profits we follow the same steps as in Section 6.3, to obtain:

$$w_i(t) = a_{1974} x_i e^{\mu t} N(d_1) - [bx_i X + m_{i,t} k_i] N(d_2) - f_i x_i - m_{i,-l} k_i - \frac{1}{2} g_i (x_i - k_i)^2 - c_i u_i - \frac{1}{2} d_i (u_i - \delta k_i)^2 \quad (6.25)$$

$$\text{where } d_1 = \frac{\ln \frac{a_{1974}}{c_{i,t}} + (\mu + \frac{1}{2} \sigma^2) t}{\sigma \sqrt{t}}, \quad \text{and} \quad d_2 = d_1 - \sigma \sqrt{t} = \frac{\ln \frac{a_{1974}}{c_{i,t}} + (\mu - \frac{1}{2} \sigma^2) t}{\sigma \sqrt{t}},$$

The value of the per period profit for each firm therefore evolves with time in accordance with the above equation. $w_i(t)$, can be easily estimated since the moments of the stochastic process, and all the firm costs are known. We can follow the same steps as in Chapter 5, Section 5.6 and derive an equation equivalent to equation (5.11). The optimal output and investment rates for the industry can then be evaluated under this state of demand uncertainty as before. The results are illustrated in Figure 6.3.

It is clear that over the period 1986-1996 the firms would invest and produce at higher rates than what was seen in industry if they were allowing for uncertainty in their plans. The model also under performs over the period from 1974-1985. A comparison is shown in Figure 6.4, between the model results with that of actual industry, and the results of the deterministic case *Ncfb7* of Chapter 5. The figure shows that capacity accumulated by the major-6 styrene producers would have been higher under uncertainty especially for the latter period under study.

6.4.2 Uncertainty Affecting Demand

In this section I will consider the case where the demand for styrene is driven by a GBM process described below, where $X = \sum_{i=1}^6 x_i$, and x_i is firm i 's output:

$$\frac{dX}{X} = \mu dt + \sigma dz \quad (6.26)$$

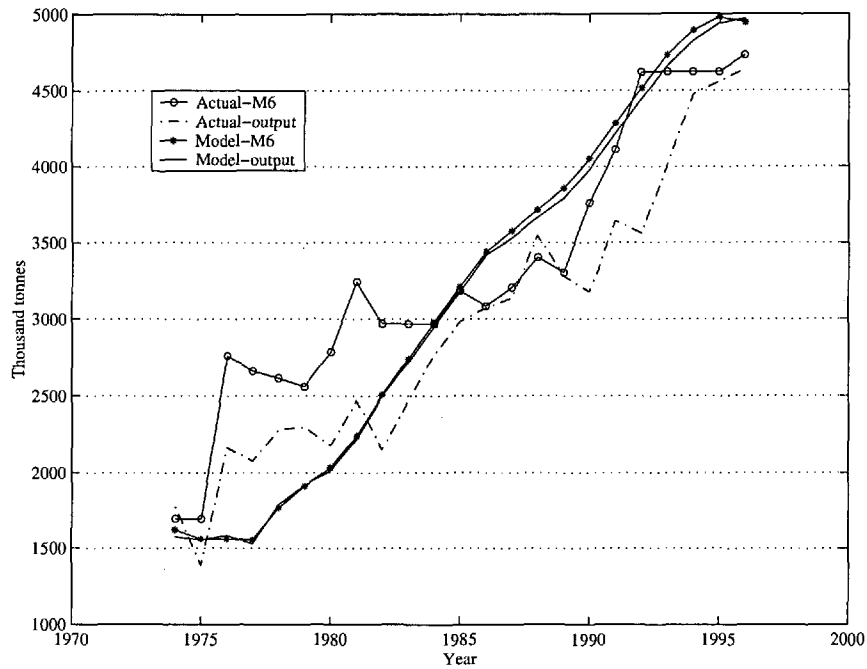


Figure 6.3: Model vs industry capacity and output for major-6 styrene producers facing stochastic demand: $a_{1974} = 2399$, $b_{1974} = 1.04$, $\mu = 4.3\%$, $\sigma = 15\%$.

the values for μ and σ can be estimated from the production of the six firms over the period 1974-1996 (data is given in Table B.1 in Appendix B). The underlying assumption is that over this period demand was driven by the GBM process just described. Equation (6.26) is normally distributed, finding the mean, μ , and standard deviation, σ , is therefore straightforward. It was found that $\mu = 5.37\%$, and $\sigma = 14.76\%$. Over a short period of time, Δt , equation (6.26), can be written in discrete form:

$$\Delta X_t = X_0 \mu \Delta t + X_0 \sigma \epsilon \sqrt{\Delta t} \quad (6.27)$$

where X_0 is the initial level of demand, and ϵ is a random draw from a standardised normal distribution. With all firms holding the same expectations about how future demand evolves, the entire sequence of values for X_t may be derived from the above equation, since $X_0 = X_{1974}$, μ , and σ are all known, and ϵ can be sampled from a standardised normal distribution. Recall also that the inverse demand equation (5.1)

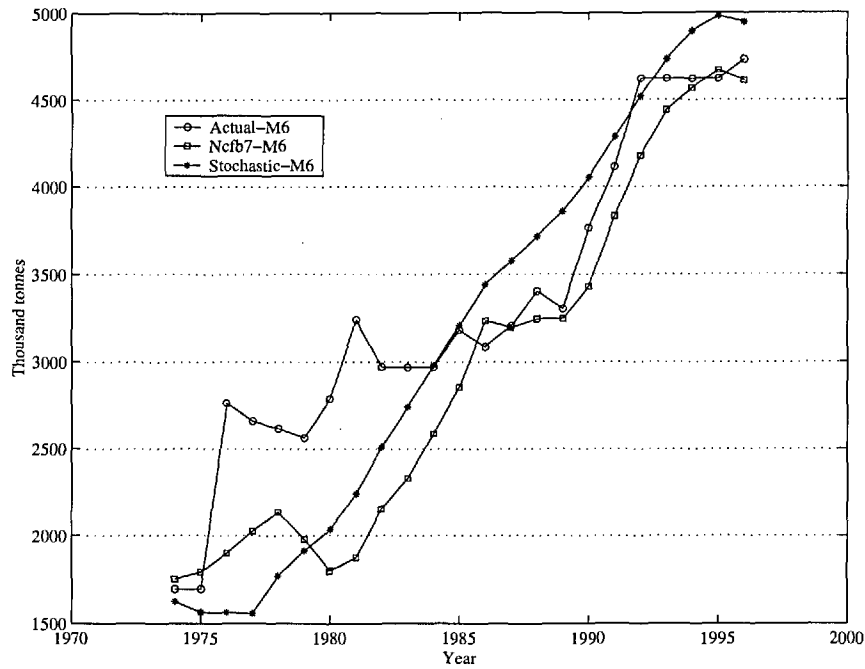


Figure 6.4: Capacity evolution for deterministic and uncertain demand for major-6 styrene producers: $a_{1974} = 2399$, $b_{1974} = 1.04$, $\mu = 5.37\%$, $\sigma = 14.76\%$.

which determines the price in every period still holds:

$$P(t) = a(t) - b(t)X(t)$$

considering that the price elasticity of demand $\varepsilon = -0.70$, the same procedure is followed as in Section 5.4 to estimate the values of $a(t)$ and $b(t)$. An OLS regression was performed to obtain the values for $\hat{a}_0$, $\hat{b}_0$, $\hat{\alpha}$, and $\hat{\beta}$. However, a 2-stage OLS regression, which accounts for serial correlation in the errors of regression variables, was found to improve the results slightly in terms of higher R^2 value, and slightly better Durbin-Watson statistics. The latter test was adopted. The values for $\hat{a}(t)$, and $\hat{b}(t)$ were evaluated as before; and the results are shown in Table 6.1. With the realisation of demand resolved, finding the optimal investment and output rates can proceed following the same way as in Chapter 5, Section 5.6. The results and main findings of the model when the firms follow feedback Nash-Cournot strategies are discussed next.

Table 6.1: Estimated derived demand function parameters with uncertain demand^(†)

	$X_s(t)$	ΔX	ϵ	$a(t)$	$b(t)$	$\hat{a}(t)$	$\hat{b}(t)$
1974	1773.0	-172.02	-1.02	2251.29	0.747	2399.71	1.038
1975	1601.0	-8.94	-0.40	2265.86	0.833	2339.96	0.950
1976	1592.0	126.30	0.17	2207.57	0.816	2281.70	0.869
1977	1718.3	62.82	-0.12	1955.00	0.669	2224.89	0.795
1978	1781.2	375.41	1.06	1704.86	0.563	2169.49	0.728
1979	2156.6	37.70	-0.25	2608.29	0.711	2115.47	0.666
1980	2194.3	-373.66	-1.52	2474.71	0.663	2062.80	0.609
1981	1820.6	100.38	0.01	2482.00	0.802	2011.44	0.558
1982	1921.0	123.39	0.07	1993.86	0.611	1961.36	0.510
1983	2044.4	205.30	0.32	1857.86	0.535	1912.52	0.467
1984	2249.7	286.78	0.50	1804.43	0.472	1864.90	0.427
1985	2536.5	614.70	1.28	1542.14	0.358	1818.47	0.391
1986	3151.2	-85.58	-0.55	1148.71	0.214	1773.19	0.358
1987	3065.6	282.63	0.26	2348.43	0.451	1729.04	0.328
1988	3348.2	173.29	-0.01	2494.14	0.438	1685.99	0.300
1989	3521.5	-112.50	-0.58	1821.43	0.304	1644.01	0.274
1990	3409.0	1257.99	2.14	1848.14	0.319	1603.08	0.251
1991	4667.0	73.16	-0.26	1357.57	0.171	1563.16	0.230
1992	4740.2	-731.62	-1.41	1228.86	0.152	1524.24	0.210
1993	4008.5	1262.55	1.77	1109.86	0.163	1486.29	0.192
1994	5271.1	536.33	0.33	1343.00	0.150	1449.28	0.176
1995	5807.4	-647.35	-1.12	1901.57	0.193	1413.20	0.161
1996	5160.1	0	0.62	1323.57	0.151	1378.01	0.147

^(†)Columns $\hat{a}(t)$ and $\hat{b}(t)$ are obtained from their exponential equations, by applying the regression results of equations equivalent to (5.4) and (5.5) p. 216.

6.5 Demand Uncertainty and Overcapacity

Demand uncertainty was found in Section 6.3 to lead the two duopolistic firms to invest in more capacity. Optimal capacity for the major-6 styrene producers under stochastic demand exceeds the optimal capacity obtainable under deterministic conditions. A comparison with the best case scenario *Ncfb7*, where the firms follow feedback Nash-Cournot strategies, that was discussed in Chapter 5 illustrates this, as shown in Figure 6.5 (compare with Figure 5.10, p. 248). As in the deterministic case the model under performs over the period between 1975-1985. But over the remaining period, the model shows that optimal feedback Nash-Cournot strategies allow the styrene firms to invest and produce more than what was found in industry; and higher than the capacity and

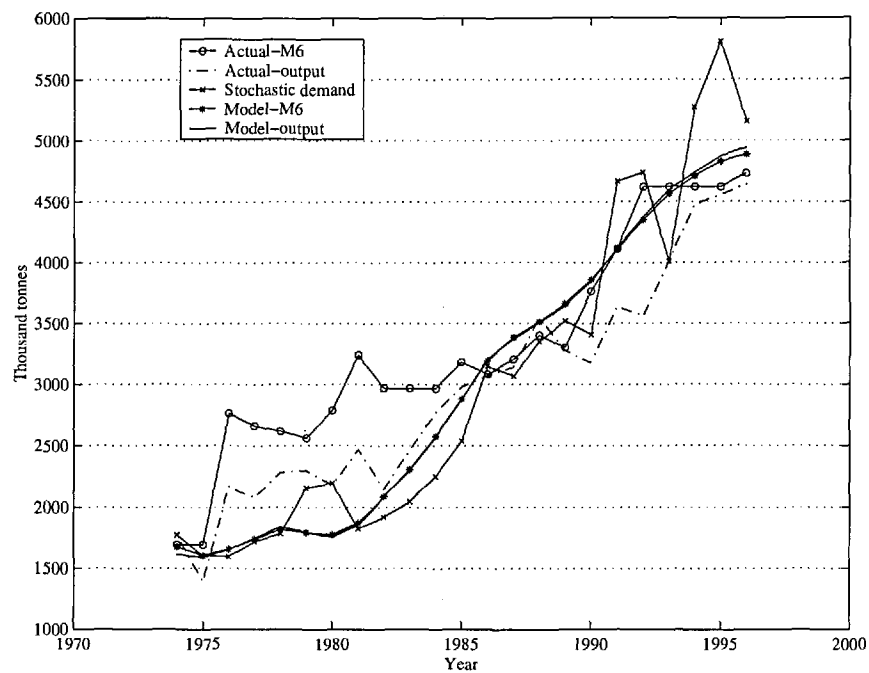


Figure 6.5: Model vs industry capacity and output for major-6 styrene producers facing stochastic demand: $\mu = 5.37\%$, $\sigma = 14.76\%$.

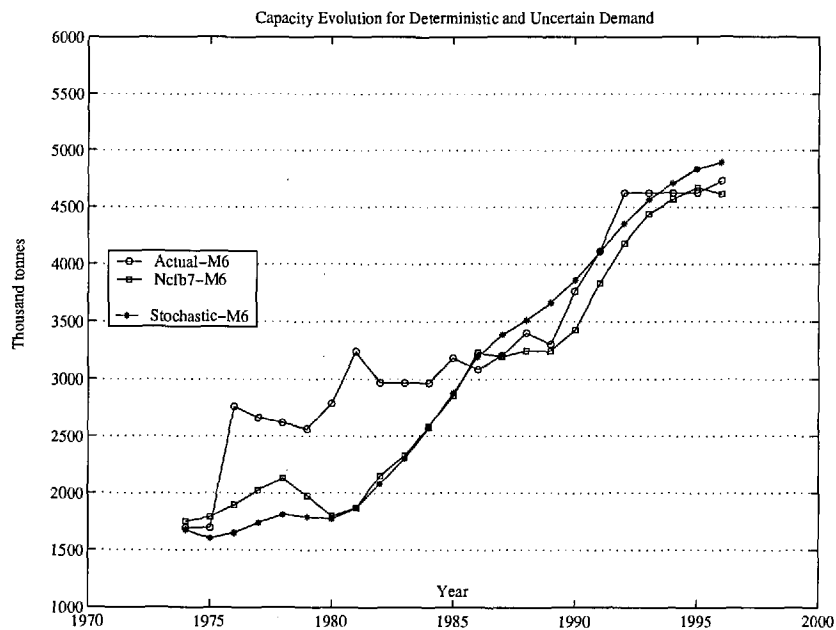


Figure 6.6: Capacity evolution for deterministic and uncertain demand for major-6 styrene producers: $\mu = 5.37\%$, $\sigma = 14.76\%$.

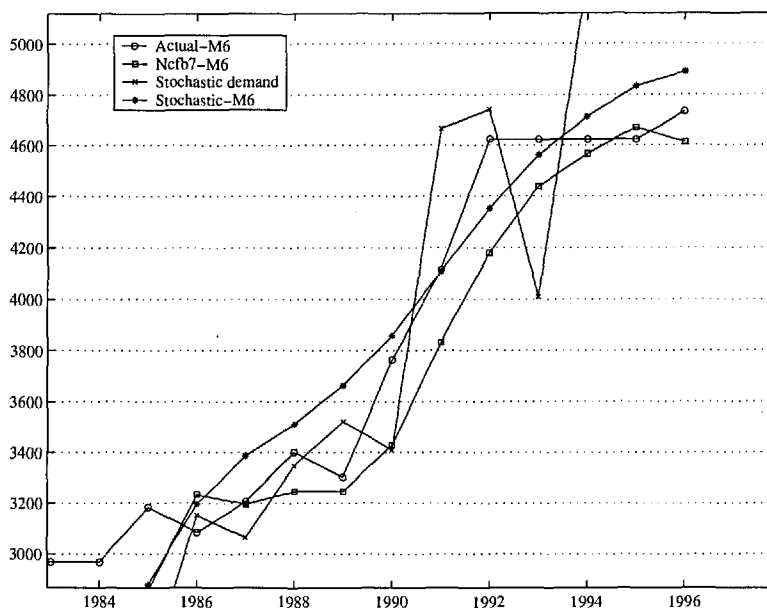


Figure 6.7: Capacity evolution for deterministic and uncertain demand for major-6 styrene producers 1985-1996: $\mu = 5.37\%$, $\sigma = 14.76\%$.

output levels that were obtained in Chapter 5 for the optimal solution obtained under feedback Nash-Cournot strategies, case *Ncfb7*, and deterministic demand. For clarity the capacity levels for the actual industry, and the optimal solutions for the feedback Nash-Cournot strategy under deterministic and stochastic demand are shown again in Figure 6.6.

Figure 6.7 just shows the same data with stochastic demand on a slightly larger scale, for the period from 1985 onwards. The capacity held by the major styrene producers under stochastic demand (stochastic-M6 in the figure) is larger than that with deterministic demand (Ncfb7-M6). The discussion in Section 5.3 showed that the industry by this time was consolidating, and rationalising capacity through acquisitions, and merger activity; and overcapacity was becoming more manageable than in the previous decade. We therefore concentrate on this period to explore the relationship between uncertainty, overcapacity and strategic investment.

The probability distribution of stochastic demand realisation is put in perspective in

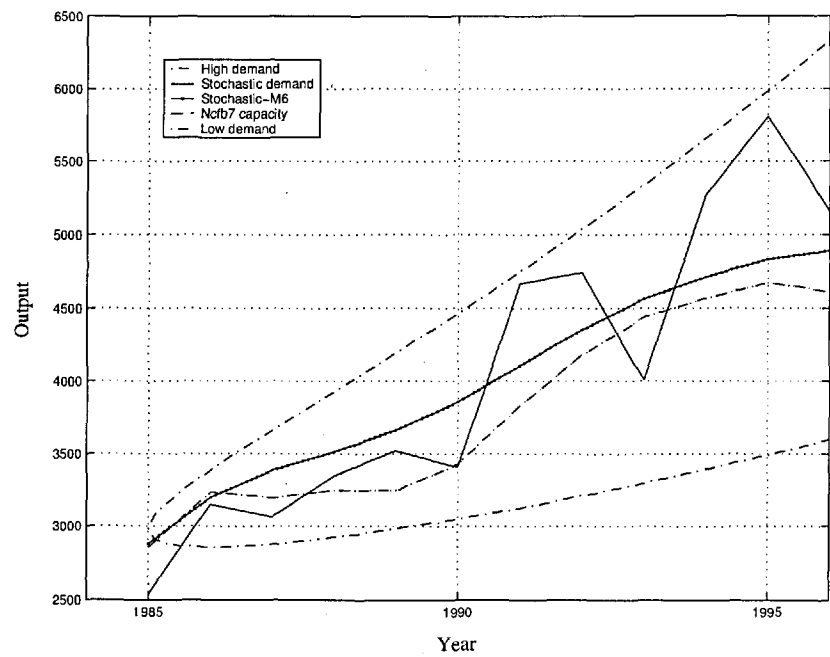


Figure 6.8: Optimal capacity for major-6 styrene producers with deterministic and uncertain demand 1985-1996.

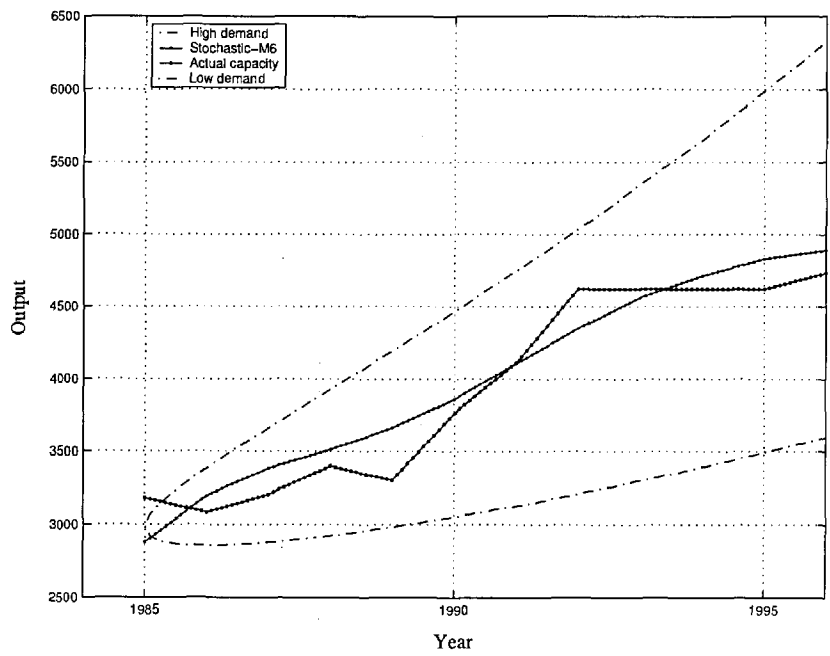


Figure 6.9: Model vs industry capacity for major-6 styrene producers with demand uncertainty 1985-1996.

Figure 6.8, which shows the same results for optimal capacity under stochastic demand, and for case *Ncfb7* from Chapter 5. Stochastic demand is shown within the 95% probability range, and depicted as high demand and low demand from the year 1985. This is the range within $\pm 2\sigma$ from the expected mean of stochastic demand. A similar illustration comparing the model performance for feedback Nash-Cournot strategies when demand is stochastic, with actual industry capacity is shown in Figure 6.9. The optimal capacity accumulated by the styrene producers, if they were allowing for stochastic demand, on average exceeds the actual capacity held by these firms. Even under mild conditions of market variability, $\sigma = 14.7\%$, the firms would hold more capacity than what they actually held in industry. If the firms were particularly concerned about demand uncertainty, and were to allow for market demand volatility of 30% - 40%, industry overcapacity under these conditions would be much higher.

The findings here are similar to those of Kulatilaka & Perotti (1992, 1998). Unlike in the real option literature where an increase in uncertainty increases the option value of the project, leading firms to hold on to their options thereby delaying their investments. In this case the firms decide on whether to invest or not in each period; and because the value of the project is rising firms decide to invest.

The main reason for increased investment with demand uncertainty, is that unit capacity costs are small relative to other costs; and the benefits in case demand turns out to be high, by far exceed the cost of redundant capacity. The losses side in this situation is bound by the investment cost, but the benefits from investment, are unbounded and more than compensate for its cost with expected realisations of stochastic demand. The firms invest more due to the convexity of the profit function with respect to the random variable which increases with increasing uncertainty.

Paraskevopoulos (1988) found that uncertainty about demand variability did not explain the large levels of uncertainty in the West European PVC industry during the late 1970s and early 1980's. The results of Paraskevopoulos, Karakitsos & Rustem (1991) are also interesting. They find that increased demand uncertainty with cautious behaviour explains the incidence of overcapacity. Whereas a more risky behaviour which involves a scrap and build policy by PVC producers results in higher rates of investment and higher

capacity utilisation. This also points out in the direction that when firms follow risky investment behaviour, their investment decisions involve strategic considerations.

6.6 Conclusions

In this chapter I have examined the impact of demand uncertainty on the investment decision of firms. Demand was assumed to follow a GBM process where firms have full knowledge of the current state of demand, but its future evolution is uncertain. Firms were assumed to be risk neutral. It was also assumed that all firms hold the same expectations about demand and the stochastic process whose properties are common knowledge. The mean and variance of the process were considered to be time invariant as well. Variations in demand realisations in this respect affect firms' production and investment rates proportionately.

In the present formulation, while demand is driven exogenously by the stochastic process, price is still being determined endogenously as a linear inverse demand function. The price level is affected by how much product the firms aggregately put on the market.

Following McDonald & Siegel (1985), the project was evaluated as a series of European call options, where the firm has the option to shut down if demand falls and its revenues fall short of meeting a minimum level or a certain fixed cost. The simulation results for the duopoly in Section 6.3 showed that uncertainty has a positive influence on investment. It increases the value of the firm which encourages investment. Firms accumulate higher levels of capacity under uncertainty. The main reason for this outcome is that firm value is convex in the stochastic variable.

Two approaches were used to evaluate the performance of the styrene industry under conditions of demand uncertainty. In both cases it was found that even with modest values of demand variability, the capacity and output of the major-6 styrene producers exceed the levels of the industry by a large margin after 1985. The rises in capacity investment rates indicate that if firms were acting in their investment choices to account for uncertainty about demand, they would have accumulated higher levels of capacity. On average industry capacity utilisation would have been lower than what industry expe-

rienced especially during 1986-1996 which was a period restructuring and rationalisation for the industry. Investment coordination allowed the styrene industry to avoid excessive addition of capacity which explains why industry capacity utilisation rates were above 90% for most of this period (see Table B.1, in Appendix B).

Chapter 7

Conclusions

*Just so every one in matters of doctrine
gives a different description of the hidden subject.
A philosopher expounds it in one way,
and a critic at once refutes his propositions.
A third censures both of them;
a fourth spends his life in traducing the others.*

*This truth and that truth cannot be all true,
yet all of them are not entirely astray in error.*

*If there were no truth, how could falsehood exist?
Falsehood derives its plausibility from truth.
It's the desire of right that makes men buy wrong.*

*He who accepts everything as true is a fool,
but he who says all is false is a knave.*

—Jalaludin Rūmi, AD 1207-1273. The Mathnawi,
p. 98

7.1 Modelling Optimal Capacity Expansion, Firm Interaction, & Overcapacity

I stated from the outset that the main objective of this research is the study of capacity expansion, optimal policies, and the investment behaviour of firms in the petrochemical industry. More specifically the relation between the pattern of investment in petrochemicals, and the presence of overcapacity. In particular, the purpose has been to examine the role of investment as an instrument of mobility or entry deterrence; and to what extent is overcapacity the result of strategic investment. Or is it merely the result of

sluggish growth in demand and lumpy capacity expansion; poor planning by firms, or uncertainty about future demand?

Another objective has been to examine the drivers of growth of the industry and the factors that shaped its development; and whether a mathematical model can be formulated to capture interfirm competition and explain the evolution of the industry in terms of output growth, capacity shares, and market prices.

Understanding the nature of interaction between the firms which make up the industry is a significant first step that can shed some light on the underlying forces that are in operation, and how they shape the investment process and the market in which they operate. Firm interaction and the pattern of investment play an important role in how industry develops, and can explain important aspects of market structure. For this purpose different patterns of interaction between firms were explored throughout the thesis. To capture more accurately the decision making process of firms and their interaction, it was thought that this can be done by employing a game theoretic approach where the firms are modelled as independent agents, and allowing them to interact directly in the market place. Game theoretic models are more suited for the study of oligopolistic industries. A model of capacity expansion was developed in this thesis in which firms maximise their profits subject to new investment and depreciation. The model is a dynamic model with adjustment costs. It is the presence of capacity adjustments, that derive from capital investment and depreciation of capital, which give the model its dynamic property. Allowing the firms to have different cost structures and investment rates, gives a better representation of firm investment behaviour, and is more consistent with the doctrines of microeconomic and oligopoly theory. This formulation also facilitates for the model to capture the intertemporal developments that take place in an oligopolistic industry; where firms act independently but their interaction, in terms of output, and investment behaviour are interdependent and impact on market structure.

Unlike static or '*bang-bang*' type dynamic models where capacities adjust instantaneously, firms in industry add capacity periodically and in increments. This time dependence of capacity adjustment is captured by a dynamic model, and the presence of adjustment costs facilitates the regulation of capacity expansion and limits the growth

in the size of firms, giving the model a relatively more accurate representation of firm behaviour and industrial development. Further more, adjustment costs were shown to be present in sizable measure in various industries; and have been estimated empirically.

The interdependence of firms' actions derives from the homogeneity of their product. Most petrochemicals are indistinguishable homogenous products. It was natural to assume that firms play Cournot production games and compete on output, rather than on price. In an oligopolistic setting, especially when a small number of firms dominate the industry, with insignificant imports, the firms realise their interdependence via output which determines the market price for their product. Price therefore obtains as an inverse demand function, whose parameters can be estimated.

Firms face two types of costs: production and investment related costs. These costs are measured directly as cost functions instead of using an econometrically determined production function formulation, and are time dependent for the dynamic case. Production costs comprise variable costs and fixed costs. Variable costs relate to production operations. For petrochemicals they are dominated by the cost of raw materials and utilities, which vary widely from year to year, rendering empirical estimation of function parameters impractical. Fixed costs pertain to items such as labour, capital and plant related assets, and general non-production operations. On the basis of a unit of output, these costs are prone to decline due to benefits of economies of scale and learning that characterise large scale processing industries such as petrochemicals. The present formulation makes it possible to account for these cost reductions in the model, depending on firm size and the relevant point in time.

In common with many studies in the dynamic and game theoretic literature the model I use is of the linear quadratic type. Both the production and investment cost functions contain a quadratic adjustment cost term. Justification for, and an appraisal of these costs was discussed at length in the thesis, namely in Sections 2.2.3, 3.1, 4.5.1, 4.5.2 and 5.5.5. The combination of the quadratic adjustment costs and the linear nature of transition equation which describes the evolution of capacity (as a function subject to investment and depreciation), impart the linear quadratic property on the capacity expansion model.

It is worth noting that the quadratic adjustment costs adopted in our model are formed such that the firm is penalised for deviations from an optimal path at two instances: in the production cost function when production deviates away from optimal capacity, and in the cost function for deviations of investment rate from its optimal path. With the exception of Reynolds (1983, 1986) and Steele (1995), most models ignore the role of redundant overcapacity which is a real cost that needs to be accounted for in empirical work (see Chapter 5, Table 5.7, p. 231). Other studies (as in Dockner (1992), Reynolds (1987, 1991), and others) consider adjustment costs that penalise the quadratic of output rates. This formulation presumes full capacity utilisation; and does not account for redundant capacity. In a way it corresponds to scenarios of very high costs of adjustment which appear in Chapter 2. However, it was illustrated in that chapter how the cost of redundant capacity could impact on firm investment behaviour.

Adjustment costs account for a small proportion of total production, or investment costs. This was accounted for by controlling the adjustment cost parameters g_i , and d_i in the different model formulations. In the empirical part they were treated as free parameters. Nevertheless, the dominance of the variable cost over the fixed cost element of the production cost function for petrochemical products; and the small contribution of investment cost to the total cost of production per unit of output, were respected in all parameterised model simulations conducted, by careful selection of appropriate values for model parameters.

Because different strategies and situations of static and dynamic settings are considered, corresponding formulations of the model and solution concepts are constructed and then evaluated in the thesis. But the basic structure of the model, where each firm strives to maximise an objective function of profits, remains the same as presented in Chapter 2, Section 2.2.1. The performance and robustness of the theoretical model is examined in Chapters 2 - 4. Nash-Cournot and Stackelberg strategies for an entry game under a duopoly in a single or a two stage game are discussed in Sections 2.2.2 and 2.7, Chapter 2. Unique optimal equilibrium solutions are obtained in all cases. In Chapter 3 the duopoly problem is tackled under a dynamic continuous time setting. The model formulation is described in Section 3.2, while the equilibrium solutions for the open-loop

strategy problem is presented in Section 3.3, and that of the feedback strategy problem in Section 3.4.2. A discrete version of the dynamic model is examined in Chapter 4. The model formulation and solution, based on two algorithms due to Kydland (1975, 1977), is presented in Section 4.2.1 for the Nash-Cournot problem; and in Section 4.4.2 for the Stackelberg problem. Open-loop and feedback strategies are also examined; as well as a competitive market situation where all firms act as price takers. The model formulations examined in Chapter 4 form the basis for testing the empirical data for the US styrene market in Chapter 5.

The study of strategic behaviour of firms in oligopolistic industries centers on the following question: Is investment in capacity being used strategically? The purpose for strategic investment in capacity is to deter new entrants from the market; or perhaps it can provide the concerned firm with the capability that allows it to influence the investment behaviour of opponents.

Using the model developed in this thesis, the arguments of Dixit (1980) and Spence (1977) on entry deterrence for a duopoly whose production and investment cost structures replicate those of petrochemicals were examined in Chapter 2 for a static setting; and in Chapter 3 for a dynamic situation. It was demonstrated in the exercises of the two chapters that in spite of large cost differentials between an incumbent and entrant firm, entry deterrence under low or moderate adjustment costs is not feasible. Further more, even mobility deterrence strategies have to be exercised with caution. The profits of both firms deteriorate significantly as a result of such a strategy; and they would soon realise that they are better-off playing profit maximising Nash-Cournot strategies as Figures 2.3-2.5, and Figures 2.6- 2.8 illustrate. For under certain situations like when the entrant is the more efficient producer, a mobility deterrence strategy followed aggressively by the incumbent could become a liability, that may lead to its exit from the market. Such an eventuality could take place even when the incumbent firm is the more efficient producer, and the latter opts for too large a capacity to influence its opponent. It would be precipitated by: (1) the entrant firm choosing to enter the market with a large capacity, thereby forcing the incumbent to reduce output in a big way (see for example Section 2.6, namely, Figure 2.7 and Figure 2.8, pane (c)); (2) or if demand falls, thereby leaving the

incumbent exposed to operating under large overcapacity and low prices (see case *Ncga6* of Table 2.1, Section 2.4).

However, although entry deterrence under those conditions is unlikely, the advantage gained by the first mover has been demonstrated in Chapter 2. By investing preemptively in capacity the incumbent firm obtains larger market share and better profits than the entrant, as the results summarised in Tables 2.1, p. 52, and 2.2, p. 58, show. Even if the incumbent runs a less efficient production operation, he can still obtain higher profits than a more efficient entrant when adjustment costs are high. The incumbent invests strategically to influence the entrant's choice of capacity and output. And by acting strategically he operates with overcapacity as shown in Tables 2.1- 2.3. The incumbent's investment/overcapacity pays-off especially when the entrant follows profit maximising Nash-Cournot strategies.

Nonetheless, it was demonstrated that the entrant could improve on her outcome of the game by adopting a Stackelberg-like investment strategy. Because the cost of overcapacity is inexpensive under low adjustment costs, when the entrant selects a larger capacity than her Nash-Cournot equilibrium capacity, the incumbent reduces his output, and operates under a larger overcapacity, allowing the entrant to obtain higher market share and profits than before. In the petrochemical industry, notably in products where economies of scale have been exploited extensively, the investment cost per tonne of output produced is very small; which suggests that petrochemical firms operating in an oligopolistic market, are more likely to invest in overcapacity for strategic purposes; and suggests also that large producers would be willing to accommodate expansions by entrants, or by their smaller rivals.

However, since holding excess capacity for the purpose of deterring entry may be questionable in view of Dixit's (1980) model; and with the outcome of the Nash-Cournot results of Chapters 2 and 3, where the firm which holds excess capacity for the purpose of retaliating by increasing its production and depressing prices should entry take place, eventually accommodates the higher output of its, usually, smaller competitor. Why should firms hold strategic excess capacity? Yet it makes more sense that the firms hold some excess capacity lest the need for its use may arise; to discipline opponents if they

overproduce or deviate from existing market equilibrium. The presence of excess capacity indicates to competitors the ability of the industry for meeting future demand and alerts the competitors to modify their expansion plans by going for a lower scale expansion or entry. Overcapacity also gives the impression of a loose oligopoly which is prone to price wars and tough competition.

Also if overcapacity is to have any credible form of deterrent effect, it should be readily available at hand for use; rather than wait for the entrants to come along, and then retaliate by building a large plant to flood the market with overproduction. The larger players would be hurt to a larger degree by depressed prices and should be more sensitive to market stability. If they decide to engage in a price war it should be short lived. In petrochemicals, raw materials and variable costs dominate production costs; and it is not in the interest of producers to depress prices for long. Larger producers stand to lose more in this case. Another possible reason for the incumbent to build and hold excess capacity, is that it acts as a reserve for use in case of any unforeseen upsurge in demand; which also doubles as a deterrent, lest it influences the novice opponent not to enter, and if he does, that would be done at a lower scale. The excess capacity is, therefore, like the option that the incumbent holds for retaliation, but that may never be exercised.

In Chapter 3 it was shown that the incumbent, or the larger firm, reduces its capacity to allow for its smaller rival to enter and expand its market share. Here also the incumbent's overcapacity pays-off since, because of time-dependent adjustment of capacity, the incumbent benefits from its advantageous position by having a larger market share, and higher profits, for a longer period before the two firms arrive at their Nash-Cournot steady state equilibrium (see Chapter 3, Figures 3.3- 3.5, and Figures 3.10- 3.12). Nevertheless, the difference between a static and a dynamic situation was also demonstrated. The lead enjoyed by the incumbent as a result of preemptive investment in the first stage of the game, is ineffective in the long term in the face of a determined entrant. In a dynamic setting the two firms' capital stock is subject to investment and depreciation; a more efficient entrant can invest heavily and overtake the incumbent firm in market share at steady state in a Nash-Cournot game, despite the latter's initial lead. Or the two firms

can share the market equally when they face symmetric costs.

On the otherhand the type of strategy adopted by the players has an important bearing on the outcome of the game, in terms of their market shares and profits. A Stackelberg strategy gives the leader a profound advantage over its rival that could easily exceed the benefits of a sizable cost differential under a Nash-Cournot strategy. Which type of optimal strategy the firms adopt would depend on the market situation, like the relative size of production cost differentials between the firms, elasticity of demand for their product and other related issues. For instance an entrant firm facing an incumbent that had invested strategically in a large capacity, would perform better by adopting a Stackelberg-like investment strategy when choosing plant capacity. By doing so the entrant firm lays claim to a larger market share, and the incumbent accommodates the expansion by its rival by reducing its own output. Once their plant capacities are fixed, both firms would then learn that by reducing their outputs to Nash-Cournot levels they would earn higher profits (see Sections 2.6 and 2.8).

Optimal investment and dynamic interaction between firms under different strategies that was examined in Chapters 3 and 4, illustrated how firms accumulate larger capacities and produce more under feedback than open-loop strategies. Feedback strategies involve Stackelberg-like investment and production behaviour on the part of firms in order to influence their opponents. Feedback strategies are therefore symbiotic of strategic intent. And this is what we test for in the empirical part of the thesis, to determine which type of investment strategy best describes the investment behaviour of firms in the styrene industry.

7.2 Investment Behaviour in US Styrene Industry

The issue of identifying the nature of the investment game in a typical oligopolistic industry has been dealt with in this thesis, using the US styrene market as a proxy for the US petrochemical industry. This was done at two levels: (1) A mathematical model of capacity expansion was formulated in a game theoretic framework, and solved by applying optimal control theory and dynamic programming techniques to obtain opti-

mal investment and production policies. Using industry data, four different investment strategies were employed for model simulations to explore how well each explains industry developments and the presence of overcapacity. The type of strategy used indicates whether investment in capacity is being used strategically to influence the behaviour of opponents; and determines the pattern that best explains the interaction between firms in the investment and production game.

(2) Due to limitations in how far one can go in implementing mathematical models to describe complex industry relationships and techno-economic phenomena, however, some attention has also been devoted to a qualitative study of market developments in the styrene industry to augment the results of the model simulations, with an analysis of the underlying forces that are taking place and shaping the evolution of the industry. For some aspects of industry characteristics are market specific and may pertain to regional peculiarities. The history of how the market and its major firms have developed, and uncertain events such as the introduction of a new technology or process, or the sudden rise of energy costs, can influence the environment in which firms operate, and the course of how industry and market evolve. This is what is described in the literature as accidents of history, or of uncertainty (Caves 1998), that shakeout markets as a result of entry or exit of firms. Or the statistical regularities of firm growth and determinants of firm size and survival rates, that impact on market structure and competitiveness of industry (Mansfield 1962, Sutton 1997).

The global development of the petrochemical industry had been dealt with elsewhere in the literature (Freeman 1982a, Stobaugh 1988, Fayad & Motamen 1986, Chapman 1991, Arora, Landau & Rosenberg 1998). That for the styrene was briefly discussed in Section 5.3. The different stages of development of the styrene industry conform to the product life cycle of a typical petrochemical product; where a small number of producers are involved in the development and innovation stage. In this stage the product commands high prices but modest volumes are produced. The growth phase attracts more firms into the industry lured by expanding markets and the prospects of high profits. Continuous improvements in process technology and larger volumes demanded by expanding markets drive the rapid rise in the size of plants to benefit from economies of

scale; together with cost savings that are achieved due to the process of learning enable firms to reduce costs and prices, thereby fuelling further growth in demand. As the markets mature, growth slows down and the addition of new capacity faces the industry with a challenging task due to the increased number of firms in the market, and the enormous size of modern plants. Overcapacity becomes more prevalent in this phase putting pressure on industry profitability, especially during periods of faltering demand.

The US styrene production rose from 0.8 million tonnes in 1960 to about 5.40 million tonnes in 2000. Similarly capacity increased from 2.70 million tonnes/year in 1974 to 5.74 million tonnes/year in 2000, and further to 6.78 million tonnes/year in 2005. On a world scale the increase was even more dramatic, rising from 8.2 million tonnes/year in 1975, to 28.7 million tonnes/year in 2005. Engineering firms played a pivotal role in developing the process technology for petrochemicals including styrene and its precursors such as ethylene and benzene, all of which achieved enormous economies of scale. Styrene plant capacities for instance increased by ten fold, from about 90 thousand tonnes/year in the late 1960's to 900 thousand tonnes/year by the early 1990's. The rise in production and capacity was also accompanied by a learning process from which producing firms benefited and were able to reduce fixed production costs by about 22%, for every doubling in output. The combination of maturing markets with lower rates of growth in demand for styrene products, and the enormous size of modern plants meant that overcapacity occurs more frequently.

The US styrene industry witnessed several instances of entry and exit of firms, but over the years there was a core of six firms that dominated the market. In the 1970's smaller styrene producers were placed under pressure as a result of the new competitive environment brought about by the advent of larger scale plants and the need to modernise production operations. Some firms exited the market while others were taken over by their larger rivals. In the 1980's some of the major chemical producers such as Monsanto and Hoechst exited the styrene market. Both firms were following a concerted strategy of abandoning the commodity chemicals market, and moving into higher value added products in the life sciences and speciality chemicals markets. Shell also exited the US styrene market in a move aimed at reorganising its global chemicals operations.

The market was undergoing a restructuring process. On the otherhand, oil chemical companies during the 1980's and 1990's consolidated their position in the US styrene market; with Chevron, BP chemicals, and Arco expanding their shares of the market in a big way. They were aided in this by integrating their oil processing and petrochemical operations. By the mid 1990's some of the smaller players, such as Huntsman and Nova Chemicals, that were essentially polystyrene fabricators moved upstream into styrene production. However by the turn of the century the major oil companies, notably Shell and BP, were shedding their petrochemical operations on a global scale, and concentrating instead on oil exploration, and production operations such as refining and processing with its huge financial requirements.

The considerable entry and exit activity which characterised the styrene industry gives the impression of a dynamic and competitive industry. But unless this is translated into an increase in the supply base, lower prices, and lower profit margins it cannot be said that a structural change to make the industry more competitive has taken effect. Despite the considerable activity that was taking place in the US styrene industry, several of these involved only a change of ownership of existing facilities where closing down older plants, modernising and building larger ones took place. In two instances there were nouveau entry into the market. The industry went through a process of restructuring and became more concentrated as a result. Essentially the core of six producers continued to account for most of the styrene produced.

With this in mind, the US styrene industry was modelled as an oligopoly of six major firms, and a fringe of producers. Using industry data over a 23 year period starting from 1974, optimal solutions for the capacity expansion model that was developed in the thesis were obtained for the six firms for four different investment strategies: open-loop Nash-Cournot, feedback Nash-Cournot, Stackelberg, and competitive price taking.

Styrene price that had been assumed to be determined endogenously as a linear function of the output of the six firms, was then estimated econometrically as an inverse demand function over the period of study. Allowing the coefficients of the linear inverse demand function to take exponential form gave an improved estimate of the cyclical styrene prices (see Appendix B, Table B.2). Investment and production costs were ap-

portioned in accordance with the size of the firm. As such larger firms were assigned lower unit costs considering that they operated larger and more modern efficient plants than their smaller counterparts. Economies of scale and learning effects were also taken into consideration. These costs declined in real value for all firms over the planning horizon.

Assuming that styrene producers in the real world follow optimal investment strategies, and that the capacity expansion model captures with a fair degree of accuracy the investment behaviour of the six major styrene producers, the simulation results of Section 5.6.3 show that investment behaviour of the major players in the US styrene industry can best be represented by a strategic investment strategy. The feedback Nash-Cournot strategy gave superior results to other strategies and predicted most of the overcapacity that was observed in industry. It also provided a good representation of the evolution of industry output and capacity growth, and of market shares over the planning horizon. The styrene price predicted by the model when firms employ a feedback Nash-Cournot strategy follows closely the trend of actual market prices for styrene. Furthermore, feedback Nash-Cournot strategy is more consistent with industry developments, since it allows the participants to react to changes in their markets and to their opponents' actions as the market structure evolves over time.

It is possible for overcapacity to emerge naturally in an industry characterised by economies of scale, and whose markets are dependent on the variations of the business cycle. It is possible also that concern about uncertainty of future demand leads firms to hold additional capacity. So not only by the simulation results of the model can the use of strategic investment and overcapacity by styrene firms be established. An examination of market developments reveals more about the firms' investment behaviour and the role of overcapacity. These developments illustrate that investment in the petrochemical industry, and the styrene industry in particular, has been used for strategic ends. It is more likely also that overcapacity has served a similar purpose. The results of Chapter 6 show that the introduction of a modest measure of uncertainty into the demand function, induces firms to invest at higher rates, leaving them with higher levels of overcapacity than the optimal results of the capacity expansion model, and what had been seen in industry. If the firms were accounting for uncertainty of demand, capacity levels in the

styrene industry would have been much higher. Overcapacity in the US styrene industry over the study period was not very large, on average it was about 10%. For strategic overcapacity to be effective, it does not need to be large either. In order to punish their opponents firms require only a small margin of overcapacity; since in an oligopoly price wars to discipline the errant producers are expected to be short lived. And due to the low elasticity of demand it takes only a small increase in output to oversupply the market and bring about a large drop in prices.

The petrochemical industry went through episodes where demand faltered; following the oil price rises of the 1970's, and during the early 1980's there were periods when economic slowdown caused the emergence of significant overcapacity levels. And also in the early 1990's when a recessionary period coincided with rising plant capacities, which compounded the effects of overcapacity. Nevertheless, the petrochemical industry's 'disease' of *chronic* overcapacity cannot be blamed on poor planning and the high exit costs. Several studies (Bower 1986, Paraskevopoulos 1988, Stobaugh 1988, Fayad & Motamen 1986, Henderson & Cool 2003) document instances which show that during periods when overcapacity was prevalent, and industry participants were experiencing low profitability and falling prices, investment in new capacity continued. Companies were swapping product lines, building larger plants, and engaging in merger activity and joint ventures. Smaller firms were forced to exit, while some of the large players moved altogether into higher value added products, and the industry was becoming concentrated.

The major players in this industry are sophisticated in their decisions. These firms have been at this game for a long time. They play the signalling and waiting game. The timing of when to add modern large scale plants becomes a tool to heighten the capacity competition and force weaker opponents out of the market¹. Or to accommodate

¹Considerable anecdotal evidence exists about large firms in the energy and chemical sectors having a full understanding of the significance of oligopolistic markets and how they operate effectively. To quote just two examples, back in 1992 an executive of one of Europe's largest gas producing firms, in a presentation he made to an energy conference held at the Petroleum Institute in London, was advocating an efficient EU utility market dominated by four or five producers. Another executive from one of Europe's largest chemical companies in a presentation made to the GCC's Fifth Petrochemical Products Conference held in Kuwait in 1997, drew attention to oligopolistic pricing, and demonstrated how petrochemical prices were improving as the number of producers declined following the wave of merger activity that was taking place in Europe.

opponents' expansion plans, and to allow capacity utilisation to tighten and prices to rise. And there is a process of learning in this respect which has become an important aspect of the strategic management literature (Henderson & Cool 2003).

There was coordination in the styrene industry as was the case in other petrochemicals. In the absence of investment coordination, overcapacity would have been more profound than what had been experienced by the industry. Firms in oligopolistic markets avoid excessive overcapacity since it leads to a breakdown in market discipline and depresses prices. Investment coordination was taking place in a variety of ways. Indirectly by the announcement of planned projects in advance in the form of a signalling game, which allowed firms to delay the implementation, and sometimes downsizing the scale of projects from original plans, to accommodate the investments of their rivals and avoid excessive incidence of redundant overcapacity. And sometimes cooperation between firms took place directly in the form of production sharing arrangements, long term contracts, or by forging alliances in joint ventures for new large scale projects at home and abroad.

Also a number of merger activities that took place in styrene involved taking over rivals' production plants, to be modernised, or closed down so that it can be replaced by larger and more modern facilities shortly afterwards. Restructuring and merger activity contributed to reducing the number of styrene producers; which translated into a more disciplined behaviour in the US styrene market. Styrene prices declined somewhat over the study period, but also showed a cyclical trend with sharp price surges. More importantly, however, is that profit margins remained high for the most part.

Larger producers benefited from the massive rise in the size of new plants which exploited economies of scale, and allowed them to reduce unit costs. This contributed to lowering of petrochemical product prices. But it also meant that setup costs were rising due to the very large cost in dollar terms for new projects. Larger producers integrated their production operations, upstream into the raw materials for styrene as well as downstream into styrene derivatives. This market trend escalated the financial requirements for firms if they wanted to stay ahead in the investment game, and favoured larger producers with deep pockets. It created a natural entry barrier and restricted the

number of possible entrants into the US market. Most new entrants are to be found in global markets with expanding markets for styrene like the Far East, China and Brazil. Many of these involved joint venture partnerships between some of the leading US styrene producers with local partners; others were in countries where a comparative advantage in production exists, due to extensive oil processing operations, such as in the Netherlands, Saudi Arabia, Singapore, and Malaysia.

7.3 Directions for Future Work

In this study I have dealt with only one petrochemical product. There is a need to conduct more research into a wider range of basic petrochemical products similar to styrene, to uncover the nature of investment behaviour that was taking place in the industry. Several products have been through a similar process of restructuring and consolidation which lead their markets to become more concentrated. These include among others: ethylene glycol and other basic raw materials for polyester fibres such as terephthalic acid. Acetic acid and vinyl acetate, toluene diisocyanate (TDI), vinyl chloride and PVC, pentaerythritol, propylene oxide, and polypropylene. Some of the major producers that are involved in styrene, are also among the leading producers of these petrochemical products.

One aspect alluded to in Chapter 5 was the possibility of a form of collusion taking place in the industry; but this was not tested for. In the game theoretic model formulation followed in this thesis, a type of models known as trigger punishment models may be utilised. In these models when one of the players cheats, the rest revert from collusive behaviour to Nash-Cournot behaviour for the rest of the game to punish the cheater. While solutions may be obtained for this situations they are more difficult to implement for empirical work due to the fact that collusion in real world situations does not operate along these behavioural lines. If punishment were to occur in the form of a price war, or reversion to Nash-Cournot production levels, this would be for a limited period until another equilibrium is restored into the market. Testing for collusion with these models also requires detailed data on production and prices on a monthly rather than on annual basis.

The model developed in this thesis may be a useful tool for planning purposes. It provides the basic steps required for evaluating optimal investment rates, and possible scenarios of market shares and price projections that could arise when firms follow optimal investment strategies. It could form the basis for a model that allows a firm to assess its investment plans; or regulatory authorities monitoring market developments, and the investment and production behaviour of firms. There are a number of important limitations that still need to be tackled first, however. The model in this thesis does not allow for financial constraints which may be an important determinant in limiting a firm's ability to grow and expand its investment commitments. Introducing endogenous financing and allowing for financial constraints on the firm, would improve the model performance and make it more consistent with a real world situation. The works of Bensoussan & Lesourne (1980) and Kort (1994, 1988) are relevant in this respect. Following these model formulations, it would be possible to allow firms to accumulate losses, and induce endogenous exit from the market in a dynamic setting; thereby facilitating for market structure to develop with time as a result of firm behaviour.

Inventories play an important role in smoothing out periodic variations and unexpected surges in demand. And in allowing firms to plan for plant turnarounds for maintenance and similar operations that require a halt in production. They can be useful also in restraining or easing pressure on prices when the supply/demand balance becomes restrained. Adding inventories into the model formulation, would make it more realistic, and is consistent with industry practice. Endogenising economies of scale and cost savings for first mover as in Kulatilaka & Perotti (1998) would be a more interesting way of modelling strategic investment in capacity.

Finally a better way to evaluate the attitude of firms to uncertainty would be to use past industry forecasts for expected growth in demand, as a benchmark around which a margin of uncertainty can be added. If evolution of industry capacity falls within this expected range firms would be reacting to uncertainty of demand. Otherwise if actual industry capacity exceeds these expected projections by a sizable margin, then it may be concluded that firms are building a strategic element into their capacity plans as well as allowing for demand uncertainty.

Appendix A

Optimality Conditions for Capacity Expansion of a Single Firm

The problem facing a profit maximising firm with quasi-fixed capacity subject to the following two equations is one of optimal control. The derivation of optimality conditions shown briefly below, is based on Bryson & Ho (1969).

$$J_i = \max_{u_i} \int_{t_0=0}^{t_f} e^{-\beta t} w_i(k_i(t), u_i(t)) dt + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.1})$$

$$\dot{k}_i(t) = f(k_i(t), u_i(t)), \quad k_i(t_0) \text{ is given} \quad (\text{A.2})$$

The objective of firm i is to maximise $J_i(t_0)$, the discounted sum of its flow of profits over the planning horizon $[t_0, t_f]$, and a salvage value term recovered at the end of the final period, t_f . J_i is the sum of values that are discounted back to the initial period, t_0 , at a constant discount rate β . The per period profit $w_i(k_i(t), u_i(t))$ ¹ is a linear quadratic function of market output, $(k_i(t))$, its capacity $(k_i(t))$, and investment rate, $u_i(t)$. The salvage value term $\varphi_i(k_i(t_f))$, is a function of the capacity attained in the final period.

The capacity of firm i , evolves according to differential equation (A.2), which is linear in its capacity $k_i(t)$, and investment rate $u_i(t)$. The problem is to find the function $u_i(t)$ that maximises J_i over the planning horizon. This is done by the following steps. Adjoin equation (A.2) to J_i with a multiplier function $\mu(t)$:

$$J_i = \int_{t_0=0}^{t_f} [e^{-\beta t} w_i(k_i(t), u_i(t)) + \mu(t) \{f(k_i(t), u_i(t)) - \dot{k}_i(t)\}] dt + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.3})$$

note that the second term of the integrand which multiplies μ is zero; and dropping the arguments where the notation is clear, we get:

¹It is assumed here that firm l , has chosen its capacity and output optimally, and therefore we can write k_l^* , so for this purpose k_l^* , can be treated by firm i as constant; also the firms produce at full capacity so output is equivalent to capacity

$$J_i = \int_{t_0}^{t_f} [e^{-\beta t} w_i(k_i, u_i) + \mu f(k_i, u_i) - \mu \dot{k}_i] dt + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.4})$$

$$= \int_{t_0}^{t_f} [\mathcal{H}_i(k_i, u_i, \mu) - \mu \dot{k}_i] dt + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.5})$$

where we introduced the Hamiltonian, $\mathcal{H}_i$, in (A.5), which is defined as:

$$\mathcal{H}_i(k_i, u_i, \mu) = e^{-\beta t} w_i(k_i, u_i) + \mu(t) f(k_i, u_i) \quad (\text{A.6})$$

Integrating the second term ($\mu \dot{k}_i$) of the integrand in equation (A.5) by parts, and rearranging we obtain:

$$J_i = \int_{t_0}^{t_f} \mathcal{H}_i dt - [\mu k_i]_{t_0}^{t_f} + \int_{t_0}^{t_f} \dot{\mu} k_i dt + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.7})$$

$$= \int_{t_0}^{t_f} [\mathcal{H}_i + \dot{\mu} k_i] dt + \mu(t_0) k_i(t_0) - \mu(t_f) k_i(t_f) + e^{-\beta t} \varphi_i(k_i(t_f)) \quad (\text{A.8})$$

Now consider the variation in J_i due to a small variation in the control variable u_i for the fixed times, t_0 and t_f , and rearranging the terms to get:

$$\delta J_i = \int_{t_0}^{t_f} \left[\left(\frac{\partial \mathcal{H}_i}{\partial k_i} + \dot{\mu} \right) \delta k_i + \frac{\partial \mathcal{H}_i}{\partial u_i} \delta u_i \right] dt + [\mu \delta k_i]_{t_0} + \left[\left(e^{-\beta t} \frac{\partial \varphi_i}{\partial k_i} - \mu \right) \delta k_i \right]_{t_f} \quad (\text{A.9})$$

for a maximum δJ_i should vanish, so we choose the multiplier function $\mu(t)$ to cause the coefficients of δk_i to vanish in the above equation, namely:

$$\dot{\mu}(t) = -\frac{\partial \mathcal{H}_i}{\partial k_i} \quad (\text{A.10})$$

and its boundary condition of:

$$\mu(t_f) = e^{-\beta t} \frac{\partial \varphi_i}{\partial k_i} \quad (\text{A.11})$$

and δJ_i becomes:

$$\delta J_i = \mu(t_0) \delta k_i(t_0) + \int_{t_0}^{t_f} \frac{\partial \mathcal{H}_i}{\partial u_i} \delta u_i dt \quad (\text{A.12})$$

Since the initial capacity $k_i(t_0)$ is a given fixed value at time t_0 , then $\delta k_i(t_0)$ is zero, and there is an optimal value $\mu(t_0)$. For a maximum, δJ_i should be zero for an arbitrary $\delta u_i(t)$, and therefore the integral should vanish, so we have:

$$\frac{\partial \mathcal{H}_i}{\partial u_i} = 0 \quad (\text{A.13})$$

We have, so far, established the necessary conditions for a stationary point of δJ_i . A sufficient condition for the stationary point to be a maximum, would be to consider

the second order expansion of $\delta^2 J_i$, and to show that this value is semi-definite negative for any value δu_i . The derivation is somewhat long (the reader is referred to Bryson and Ho Chapters 3 and 6). The sufficiency condition for the chosen control u_i to yield a maximum, is that in addition to $\frac{\partial \mathcal{H}_i}{\partial u_i} = 0$, we must also have $\frac{\partial^2 \mathcal{H}_i}{\partial u_i^2} \leq 0$. In our formulation $\mathcal{H}_i$ is *linear quadratic* which is concave with respect to the control and the state variables, and this requirement is fulfilled. Furthermore, the problem in this case has a global maximum.

In conclusion, the necessary conditions for solving the maximisation problem posed by equations (A.1) and (A.2), are summarised in the following differential equations:

$$\dot{k}_i(t) = f(k_i(t), u_i(t)) \quad (\text{A.14})$$

$$\dot{\mu}(t) = -\frac{\partial \mathcal{H}_i}{\partial k_i} = -e^{-\beta t} \frac{\partial w_i}{\partial k_i} - \mu \frac{\partial f}{\partial k_i} \quad (\text{A.15})$$

where the maximising $u_i(t)$ is obtained by solving the following equation:

$$\frac{\partial \mathcal{H}_i}{\partial u_i} = e^{-\beta t} \frac{\partial w_i}{\partial u_i} + \mu \frac{\partial f}{\partial u_i} = 0 \quad (\text{A.16})$$

and subject to the boundary conditions for equations (A.14) and (A.15) that are given respectively by:

$$k_i(t_0) \quad \text{given} \quad (\text{A.17})$$

$$\mu(t_f) = e^{-\beta t} \frac{\partial \varphi_i}{\partial k_i} \quad (\text{A.18})$$

So far the values we have been working with are discounted back to the initial time t_0 , the Hamiltonian $\mathcal{H}_i(t)$, and the multiplier, $\mu(t)$, which is the *shadow price* or marginal value of capital, are discounted values. Very often it would be more convenient to work with current values instead. The benefit of using current values becomes apparent when dealing with problems where previous decisions were not optimal, and we are trying to find optimal values from current period onwards. Below we set about to make this transformation.

Recall that the Hamiltonian given by (A.6), may be rewritten in the following form:

$$\mathcal{H}_i(k_i, u_i, \mu) = e^{-\beta t} [w_i(k_i, u_i) + e^{\beta t} \mu(t) f(k_i, u_i)] \quad (\text{A.19})$$

also introduce a current value multiplier, $\lambda(t)$, which is defined as:

$$\lambda(t) = e^{\beta t} \mu(t) \quad (\text{A.20})$$

and therefore equation (A.19) becomes:

$$\mathcal{H}_i(k_i, u_i, \mu) = e^{-\beta t} [w_i(k_i, u_i) + \lambda(t) f(k_i, u_i)] \quad (\text{A.21})$$

$$H_i(k_i, u_i, \lambda) = e^{\beta t} \mathcal{H}_i(k_i, u_i, \mu) = w_i + \lambda(t) f(k_i, u_i) \quad (\text{A.22})$$

where we have introduced a current value Hamiltonian, H_i . If we differentiate (A.20) wrt time and substitute the values of $\lambda(t)$, $\dot{\mu}(t)$ from equations (A.20) and (A.15) where

necessary we get:

$$\dot{\lambda}(t) = \beta e^{\beta t} \mu(t) + e^{\beta t} \dot{\mu}(t) \quad (\text{A.23})$$

$$= \beta \lambda(t) - \frac{\partial w_i}{\partial k_i} - \lambda \frac{\partial f}{\partial k_i} \quad (\text{A.24})$$

substituting from (A.22) into (A.24) we obtain:

$$\dot{\lambda}(t) - \beta \lambda(t) = -\frac{\partial H_i}{\partial k_i} \quad (\text{A.25})$$

Also if we rearrange the current value Hamiltonian, H_i , given by (A.22) and differentiate wrt the control $u_i(t)$, and using the result from (A.16), we get:

$$\frac{\partial \mathcal{H}_i}{\partial u_i} = \frac{\partial(e^{-\beta t} H_i)}{\partial u_i} = 0 \quad (\text{A.26})$$

$$= e^{-\beta t} \frac{\partial H_i}{\partial u_i} = 0 \quad (\text{A.27})$$

and since $e^{-\beta t} > 0$, $\Rightarrow \frac{\partial H_i}{\partial u_i} = 0$, and by using (A.22), we have:

$$\frac{\partial H_i}{\partial u_i} = \frac{\partial w_i}{\partial u_i} + \lambda \frac{\partial f}{\partial u_i} = 0 \quad (\text{A.28})$$

Also taking derivative of (A.22) wrt the current value *shadow price* of capital:

$$\dot{k}_i = \frac{\partial H_i}{\partial \lambda} = f(k_i, u_i) \quad (\text{A.29})$$

Therefore, we have established the necessary conditions of optimality resulting from the choice of optimal investment rate, $u_i(t)$, in terms of the current value Hamiltonian H_i , and current value costate variable λ . These equations are (A.25), (A.28), and (A.29)

The boundary condition referring to the initial capacity (state) given by (A.17), remains unchanged. While the second boundary condition of the state at the final time described by (A.18) is modified using (A.20). The resulting two boundary conditions are shown in (A.33) in the table below:

Conditions of Optimality

$$\frac{\partial H_i}{\partial u_i} = \frac{\partial w_i}{\partial u_i} + \lambda \frac{\partial f}{\partial u_i} = 0 \quad (\text{A.30})$$

$$\dot{k}_i = \frac{\partial H_i}{\partial \lambda} = f(k_i, u_i) \quad (\text{A.31})$$

$$\dot{\lambda}(t) - \beta \lambda(t) = -\frac{\partial H_i}{\partial k_i} \quad (\text{A.32})$$

$$\text{where } k_i(t_0), \text{ is a given known value, and } \lambda(t_f) = \frac{\partial \varphi_i}{\partial k_i} \quad (\text{A.33})$$

Appendix B

Market Data for US Styrene Producers

Table B.1: Capacity and market data for the US styrene industry and major styrene producers^(†)

	Styrene Price (1990 \$/t)	Industry Capacity (kt/y)	Industry Production (kt/y)	Capacity Major 6 (kt/y)	Industry CU (%)	Production Major 6 (kt/y)	$a(t)$	$b(t)$
1974	927	2580	2701	1694	105	1773	2251	0.75
1975	933	2580	2120	1694	82	1392	2266	0.96
1976	909	3648	2858	2762	78	2164	2208	0.60
1977	805	3995	3115	2662	78	2076	1955	0.55
1978	702	3739	3260	2617	87	2282	1705	0.44
1979	1074	3789	3395	2563	90	2296	2608	0.67
1980	1019	3977	3110	2787	78	2179	2475	0.67
1981	1022	3977	3030	3241	76	2469	2482	0.59
1982	821	3719	2695	2969	72	2152	1994	0.55
1983	765	3719	3085	2969	83	2463	1858	0.44
1984	743	3749	3497	2969	93	2769	1804	0.38
1985	635	3692	3457	3184	94	2981	1542	0.30
1986	473	3592	3578	3084	100	3072	1148	0.22
1987	967	3714	3635	3206	98	3138	2348	0.44
1988	1027	3910	4075	3402	104	3546	2494	0.41
1989	750	3810	3782	3302	99	3278	1821	0.33
1990	761	4272	3608	3764	85	3179	1848	0.34
1991	559	4621	4091	4113	80	3641	1358	0.22
1992	506	5291	4076	4623	81	3561	1229	0.20
1993	457	5291	4595	4623	88	4015	1110	0.16
1994	553	5291	5123	4623	96	4476	1343	0.18
1995	783	5302	5230	4623	96	4560	1902	0.25
1996	545	5411	5310	4732	99	4644	1324	0.17

^(†) Values for columns $a(t)$ and $b(t)$ are obtained from equations (5.3) and (5.2) p. 216, where the price elasticity of demand, $\varepsilon = 0.7$.

Table B.2: Estimated demand function parameters for the six major styrene producers^(†)

	$\hat{a}(t)$	$\hat{b}(t)$	Production for Major 6 (kt/y)	Estimated Price (1990 \$/t)	Actual Price (1990 \$/t)
1974	2375	0.81	1773	941	927
1975	2317	0.75	1392	1267	933
1976	2261	0.70	2164	738	909
1977	2206	0.66	2076	843	805
1978	2153	0.61	2282	754	702
1979	2101	0.57	2296	787	1074
1980	2050	0.53	2179	887	1019
1981	2000	0.50	2469	770	1022
1982	1952	0.46	2152	952	821
1983	1904	0.43	2463	837	765
1984	1858	0.40	2769	738	743
1985	1813	0.38	2981	688	635
1986	1769	0.35	3072	687	473
1987	1726	0.33	3138	695	967
1988	1685	0.31	3546	597	1027
1989	1644	0.29	3278	706	750
1990	1604	0.27	3179	755	761
1991	1565	0.25	3641	658	559
1992	1527	0.23	3561	699	506
1993	1490	0.22	4015	619	457
1994	1454	0.20	4476	548	553
1995	1419	0.19	4560	558	783
1996	1385	0.18	4644	566	545

^(†) Columns $\hat{a}(t)$ and $\hat{b}(t)$ are obtained from their exponential equations, by applying the regression results of equations (5.4) and (5.5) p. 216; and the estimated styrene price is obtained from equation (5.1).

Table B.3: Production economics for styrene by a pacesetter producer⁽¹⁾ - USA 1993

PEP Cost Index	535			
Process Type:	Third generation vapour phase alkylation			
Capacity (tonne/year)	644,000			
Capital cost (million US\$)				
Battery limits investment (BLI)			121.10	
Off-sites investment			76.20	
Total fixed capital (TFC)			197.20	
Production Costs	Use/Tonne styrene	Unit cost	\$/t	Thousand \$/year
Raw materials				
Ethylene	0.2843 tonne	\$ 406/t	115.4	74,300
Benzene	0.7882 tonne	\$ 291/t	229.4	147,700
Catalyst & chemicals		\$ 5.00/t	5.00	3,200
Total raw materials			349.8	225,300
By-product credits				
Toluene	(0.0194) tonne	\$ 231/t	(4.5)	(2,900)
Total credits			(12)	(2,900)
Utilities				
Cooling water	85.1 m ³	2.12 ¢/m ³	1.8	1,200
Process water	0.67 m ³	20.4 ¢/m ³	0.1	100
Electricity	97.0 kWh	4.03 ¢/kWh	3.9	2,500
Inert gas	2.95 m ³	2.24 ¢/m ³	0.1	50
Fuel	6.98 GJ	\$ 2.36/GJ	16.5	10,600
Steam	0.06 tonne	\$ 10.10/t	0.6	400
Total Utilities			23.0	14,800
Variable Cost			372.8	240,100
Operating labour	6 operators/shift	\$ 27.18/h	2.2	1,400
Control laboratory	20% of oper. labour		0.4	300
Maintenance labour	1.6%/year of BLI		2.4	1,600
Maintenance materials	2.4%/year of BLI		3.6	2,300
Operating supplies	15% of oper. labour		0.3	200
Subtotal			9.0	5,800
Plant overhead	80% of total labour		4.1	2,600
Taxes and insurance	2%/year of TFC		6.1	3,900
G&A ⁽¹⁾ costs	5% of styrene value		26.3	16,900
Subtotal			36.5	23,500
Fixed Cost			45.5	29,300
Cash Cost			418.3	269,400
Depreciation	10%/year of TFC		30.6	19,700
Cash Cost + Depr.			448.9	289,100
ROI ⁽²⁾ before taxes	25%/year of TFC		76.6	49,300
Product Value			525.5	338,400

⁽¹⁾Pacesetter refers to producer employing modern and efficient plant. All costs are measured in current US \$.

⁽²⁾Refers to general, sales and administrative costs.

⁽³⁾Refers to return on investment.

Table B.4: Production economics for styrene by a straggler producer⁽¹⁾ - USA 1993

PEP Cost Index	535			
Process Type:	Liquid phase alkylation ($AlCl_3$)			
Capacity (tonne/year)	145,000			
Capital Cost (million US\$)				
Battery limits investment (BLI)	65.10			
Off-sites investment	41.00			
Total fixed capital (TFC)	106.10			
Production costs	Use/tonne styrene	Unit cost	\$/t	Thousand \$/year
Raw materials				
Ethylene	0.3100 tonne	\$ 406/t	125.8	18,300
Benzene	0.8210 tonne	\$ 291/t	238.9	34,700
Catalysts and chemicals		\$10.50/t	10.5	1,500
Total raw materials			375.2	54,500
By-product credits				
Toluene	(0.0520) tonne	\$ 231/t	(12)	(1,700)
Total credits			(12)	(1,700)
Utilities				
Cooling water	73.9 m ³	2.12 ¢/m ³	1.6	200
Process water		20.4 ¢/m ³	0.00	0.00
Electricity	91.7 kWh	4.03 ¢/kWh	3.7	500
Inert gas	4.1 m ³	2.24 ¢/m ³	0.1	0
Fuel	4.3 GJ	\$ 2.36/GJ	10.1	1,500
Steam	0.6 tonne	\$ 10.10/t	6.4	900
Total Utilities			21.9	3100
Variable Cost			385.1	55,900
Operating labour	8 operators/shift	\$ 27.18/h	13.1	1,900
Control laboratory	20% of oper. labour		2.6	400
Maintenance labour	2.5%/year of BLI		9.0	1,300
Maintenance materials	2.5%/year of BLI		9.0	1,300
Operating supplies	15% of oper. labour		2.0	300
Subtotal			35.7	5,200
Plant overhead	80% of total labour		19.8	2,900
Taxes and insurance	2%/year of TFC		14.6	2,100
G&A ⁽²⁾ costs	5% of styrene value		37.4	5,400
Subtotal			71.8	10,400
Fixed Cost			107.5	15,600
Cash Cost			492.6	71,500
Depreciation	10%/year of TFC		73.1	10,600
Cash Cost + Depr.			565.7	82,100
ROI ⁽³⁾ before taxes	25%/year of TFC		182.8	26,500
Product Value			748.5	108,600

⁽¹⁾Straggler refers to producer using small scale plant with old technology. All costs are measured in current US \$.

⁽²⁾Refers to general, sales and administrative costs.

⁽³⁾Refers to return on investment.

Table B.5: Unit capital investment costs for styrene plants^(a)

Year	Conv.	PEP	Capital Investment Cost			
	Factor	CI	Straggler		Pacesetter	
	to 1990\$	Index	(US\$/t)	(1990\$/t)	(US\$/t)	(1990\$/t)
	(1)	(2)	(3)	(4)	(5)	(6)
1974	2.445	208	12.70	31.04	28.42	69.49
1975	2.227	242	14.77	32.90	33.07	73.64
1976	2.093	251	15.32	32.07	34.30	71.78
1977	1.962	267	16.30	31.98	36.48	71.58
1978	1.829	282	17.24	31.53	38.59	70.57
1979	1.680	315	19.22	32.28	43.01	72.26
1980	1.541	357	21.81	33.61	48.82	75.23
1981	1.405	404	24.65	34.63	55.17	77.52
1982	1.321	420	25.62	33.84	57.35	75.75
1983	1.271	427	26.08	33.14	58.37	74.19
1984	1.226	437	26.65	32.67	59.66	73.14
1985	1.191	440	26.88	32.01	60.16	71.65
1986	1.160	444	27.09	31.42	60.64	70.34
1987	1.125	458	27.96	31.46	62.59	70.42
1988	1.089	484	29.57	32.20	66.19	72.08
1989	1.044	511	31.16	32.53	69.75	72.82
1990	1.000	523	31.92	31.92	71.46	71.46
1991	0.963	538	32.82	31.60	73.46	70.74
1992	0.937	545	33.26	31.16	74.44	69.75
1993	0.917	553	33.74	30.94	75.52	69.25
1994	0.898	564	34.41	30.90	77.02	69.17
1995	0.876	585	35.72	31.29	79.96	70.04
1996	0.856	591	36.04	30.85	80.68	69.06
1997	0.838	605	36.94	30.96	82.69	69.30
1998	0.822	613	37.40	30.74	83.72	68.82

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