Design and

Operations of Enterprise-wide Process Networks under Uncertainty

by

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Abstract

In recent years there has been great industrial and academic interest in enterprise-wide supply chains and their design and operation issues because of their economic impact under increasingly competitive and narrow profit margin environments. Enterprise-wide networks involve various types of decision-making problems, which range from traditional operation problems such as short-term scheduling of an individual process, to newly focused problems such as long-term planning of entire process networks. Many issues relating to these problems pose new challenges for the process systems engineering community. The presence of uncertainty is one important issue which makes decision-making in supply chains a very difficult problem. This thesis investigates a number of such decision-making problems of enterprise-wide process networks in the presence of uncertainty.

First, a multi-level programming framework is presented for planning of enterprise-wide supply chains. Supply chain problems inherently exhibit multi-level decision network structures, which can be mathematically represented using multi-level mathematical programming principles. Particularly, a bilevel decision-making problem considering uncertainty is formulated in the context of enterprise-wide supply chain optimization with one level corresponding to a plant-specific planning problem and the other to a distribution network problem. In order to solve the resulting bilevel programming problem, efficient solution methodologies are proposed based on parametric programming techniques for the deterministic as well as stochastic case.

Second, a novel methodology is proposed for the short-term operation planning under uncertainty using parametric programming techniques. Uncertainty, such as in processing times and equipment availabilities, is incorporated into scheduling models, which are then transformed into multi-parametric mixed integer programming (mp-MILP) problems. A solution framework based on parametric optimization algorithms is then established to address these problems. A key advantage of the

proposed methodology is that the complete map of optimal schedules can be obtained as a function of varying parameters: re-scheduling can thus be performed via simple function evaluations without any further optimization.

Throughout the thesis, a number of examples are used to demonstrate the key concepts and highlight their potential for novel solutions.

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In private life, my family is the first. After being a father, I miss my father more than ever. But I feel he is always with me. My mother knows that too and her love makes me carry out this work. Because my parents-in-law also think of me as their own son, I could survive in a foreign country. More than anyone, I owe this work to my wife. Younghee, I love you. My daughter, Chelsea ! Thank you very much for believing me.

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Chapter 1

Introduction and Project Objectives

This chapter looks at problems associated with enterprise-wide process networks against the background of current manufacturing environments. Two particular issues, supply chain management under uncertainty and scheduling under uncertainty are identified and illustrated through examples. These examples are then elaborated in order to generate ideas to overcome them. The objectives of the project are established and the outline of the thesis is presented.

1.1 Introduction

In recent years there has been great industrial and academic interest in enterprise-wide supply chains and their design and operation. This interest is driven by their potential in the current competitive manufacturing environments, which have the following features:

- Uncertainty exists at all levels.

  Disturbances can change the entire environment unexpectedly. For example, the petrochemical industry has experienced an enormous variation in the price of raw materials as well as end products. This variation is the typical source of uncertainty. In addition, fast appearance of new products and their corre-

sponding new markets also drive overall environments to become more uncertain. Industries are therefore forced to operate over a wide range of conditions in response to uncertainty.

- Competition is becoming global.

  Barriers controlled by legislation have been lowered to allow foreign and international companies to compete with domestic companies relatively freely. Many enterprises also have plants located in different geographical regions. As a result, products are selected on a more competitive basis.

- Customers drive the economy.

  Saturated markets are reversing the driving force of the economy from the supplier to the customer. Customers are becoming more demanding and sophisticated in their expectation regarding quality, delivery and service. Manufacturers need to pay considerably more attention on customers.

- Process operations are becoming more difficult.

  Environmental regulations are being strengthened globally. More complex and expensive methodologies are therefore required, which in turn is making operating conditions to become more restricted. The presence of uncertainty exacerbates this difficulty.

In order to survive in such competitive environments, traditional process engineering communities have generally tried to classify the above difficult features into a relatively large number of narrow categories. Each category is then focused and solved separately, assuming the others known and fixed. Some such categories are:

- How resources for a particular process are prepared and utilized to meet a fixed demand
- How resources for a process are distributed, over a certain time horizon, to minimize the consumption of resources
- How products are delivered from place to place after production

However, this traditional approach is not desirable for two reasons; in the first place, real industrial problems rarely fall entirely within one single category and secondly, the other categories which are assumed to be fixed are actually dynamic and uncertain. It is therefore necessary to take into account multiple aspects of entire process networks simultaneously. The concept of managing enterprise-wide process networks or, as it is often called, *Supply Chain Management*, has mainly developed as a consequence of this competitive pressure. Some general definitions of supply chain in the open literature are as follows:

- The supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers (Ganeshan and Harrison, 1995).

- Supply chain management is about getting a smooth and efficient flow from raw material to finished goods in your customer's hands. It is a concept which is increasingly replacing traditional fragmented management approaches to buying, storing and moving goods (Department of Trade and Industry, U.K.[1]).

- Supply chain management is the process of effectively managing the flow of materials and finished goods from vendors to customers using manufacturing facilities and warehouses as potential intermediate stops (Senguta and Turnbull, 1996).

As implied in the above definitions, supply chain management is different from the traditional approach in that various activities in a supply chain are considered not as separate issues but as hierarchically connected parts which should be incorporated.

A number of challenges arise in introducing the concept of supply chain management in the operations of actual enterprise-wide process networks:

Because supply chain elements are hierarchically connected, their activities affect and are affected by others. It is quite natural to stress that these activities should be incorporated in their overall management. However the incorporation of various elements should follow the practice that elements generally have their

[1] http://www.dti.gov.uk

own objectives which are mutually indifferent and can conflict each other. For instance, raw material suppliers want to maximize the consumption of raw materials in plants, while the plants themselves want to minimize this within the limits of meeting orders; plants want to maximize the amounts of products delivered to warehouses, while warehouses want to control the amounts in order to prepare for future variations. Therefore, this indifference in decision-making principles of individual elements should be taken into account in the supply chain management.

It should be also noted that uncertainty is an important issue in the design and operation of supply chains. Because each element affects and is affected by other elements, a variation in one element may be a critical factor in other elements and, ultimately, in entire process networks. Therefore uncertainty should be considered not only in the operation of individual elements but also in the design and planning of enterprise-wide process networks.

Because of the presence of uncertainty, successful supply chain management is thus dependent on robust operations of individual elements as well as planning of enterprise-wide supply chains with the consideration of independent interests of individual elements. This thesis will hereafter focus these two particular issues of enterprise-wide supply chain management, supply chain planning under uncertainty and short-term operation of individual processes under uncertainty.

1.2 Motivating Examples

Two examples will be presented to highlight some of the challenges associated with previously raised issues and to generate ideas to overcome them. The first example addresses how presences of multiple elements in a supply chain should be incorporated in a decision-making problem and the second discusses how to operate each individual plant to meet its assigned tasks in the face of uncertainty.

1.2.1 Motivating Example 1. Enterprise-wide Design Problem

Consider a manufacturing enterprise involving two plants $PL1$, $PL2$ and one distribution centre DC, with two products, A and B (see Figure 1.1).

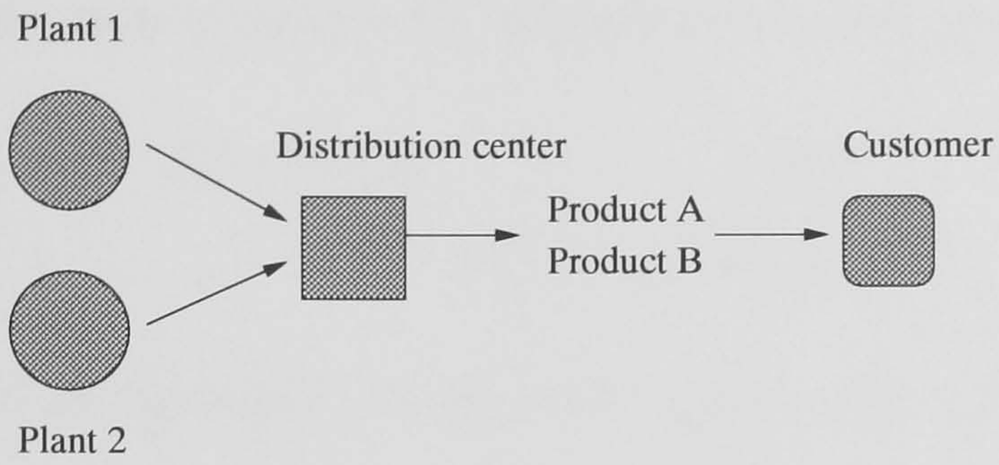


Figure 1.1: Process Configuration of the Enterprise in Motivating Example 1

The enterprise aims to minimize the overall operating cost which consists of a production cost and a distribution cost. Based on the notation in Table 1.1, the operation model may be mathematically formulated as follows:

Table 1.1: Notation for Motivating Example 1

variable	description
Y_{1A}	production amount A in plant P1 (ton)
Y_{1B}	production amount B in plant P1 (ton)
Y_{2A}	production amount A in plant P2 (ton)
Y_{2B}	production amount B in plant P2(ton)
X_A	inventory holding of A in DC (ton)
X_B	inventory holding of B in DC (ton)
parameter	description
θ_A	demand amount A (ton)
θ_B	demand amount B (ton)

The objective of the production part is to minimize its manufacturing and delivery cost to distribution centres which can be typically formulated as follows:

$$Z_{PC} = 1.5X_A + 2X_B + 7Y_{1A} + 3Y_{1B} + 10Y_{2A} + 6Y_{2B} \tag{1.1}$$

The production part is subject to the following constraints:

(a) Common resources are shared by both plants:

$$Y_{1A} + Y_{1B} + Y_{2A} + Y_{2B} \leq 500 \tag{1.2}$$

(b) Some resources may be controlled by individual plant conditions:

$$2Y_{1A} + Y_{1B} \leq 200 \tag{1.3}$$

$$Y_{2A} + Y_{2B} \leq 250 \tag{1.4}$$

(c) Production levels achieved by both plants should not be lower than the inventory capacity:

$$Y_{1A} + Y_{2A} \geq X_A \tag{1.5}$$

$$Y_{1B} + Y_{2B} \geq X_B$$

On the other hand, the objective of the distribution centre is to minimize its inventory holding including material handling cost, and distribution cost from warehouses to markets. Such a distribution cost may be formulated as follows:

$$\min Z_{DC} = 15X_A + 13X_B + 3Y_{1A} + 2Y_{1B} + 3.5Y_{2A} + 2.5Y_{2B} \tag{1.6}$$

The distribution part is subject to the following constraints:

(a) Inventory levels are limited by their overall capacity:

$$3X_A + 2X_B \leq 500 \tag{1.7}$$

(b) Inventory levels should meet demands:

$$X_A \geq \theta_A \tag{1.8}$$

$$X_B \geq \theta_B$$

According to past research on supply chain modeling such as Tsiakis *et al.*(2001), which sums both costs at the same time, this problem may be transformed into the following optimization model:

$$\min Z_{PC} + Z_{DC} =$$

$$\{1.5X_A + 2X_B + 7Y_{1A} + 3Y_{1B} + 10Y_{2A} + 6Y_{2B}\}$$

$$+\{15X_A + 13X_B + 3Y_{1A} + 2Y_{1B} + 3.5Y_{2A} + 2.5Y_{2B}\}$$

$$s.t. \quad 3X_A + 2X_B \leq 500, \tag{1.9}$$

$$X_A \geq \theta_A,$$

$$X_B \geq \theta_B,$$

$$Y_{1A} + Y_{1B} + Y_{2A} + Y_{2B} \leq 500$$

$$2Y_{1A} + Y_{1B} \leq 200,$$

$$Y_{2A} + Y_{2B} \leq 250,$$

$$Y_{1A} + Y_{2A} \geq X_A,$$

$$Y_{1B} + Y_{2B} \geq X_B,$$

$$X_A, X_B, Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B} \geq 0$$

However, this model does not fully represent actual operations. In practice, the production part makes its decision without the complete information on the distribution part and that can be only known after the corresponding decisions on plants have been made. Therefore it is more realistic to transform this type of problem as follows:

$$
\begin{aligned}
\min_{X_A, X_B} Z_{DC} \quad &= 15X_A + 13X_B + 3Y_{1A} + 2Y_{1B} + 3.5Y_{2A} + 2.5Y_{2B} \\
s.t. \quad & 3X_A + 2X_B \leq 500 \\
& X_A \geq \theta_A \\
& X_B \geq \theta_B \\
& Y_{1A} + Y_{2A} \geq X_A \\
& Y_{1B} + Y_{2B} \geq X_B \\
\min_{Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B}} Z_{PC} \quad &= 1.5X_A + 2X_B + 7Y_{1A} + 3Y_{1B} + 10Y_{2A} + 6Y_{2B} \\
s.t. \quad & Y_{1A} + Y_{1B} + Y_{2A} + Y_{2B} \leq 500 \\
& 2Y_{1A} + Y_{1B} \leq 200 \\
& Y_{2A} + Y_{2B} \leq 250 \\
& Y_{1A} + Y_{2A} \geq X_A \\
& Y_{1B} + Y_{2B} \geq X_B \\
& X_A, X_B, Y_{1A}, Y_{1B}, Y_{2A}, Y_{2B} \geq 0
\end{aligned}
\tag{1.10}
$$

where the distribution part sets parameters influencing the decisions on the production part, while in turn the distribution decision is affected by the outcome of the production part.

Classes of problem **(1.10)** are denoted as a bi-level (or multi-level in general) programming problem, where an optimization problem is constrained by another problem. A large number of industrial problems may be represented by such bilevel principles, *e.g.* (i) hierarchical decision making policy problems where policy makers at the top level influence the decisions of private individuals and companies, or (ii) energy distribution problems where consumption of private companies is affected by imported resources controlled by government policy. Little research has been done on bilevel programming in the design and operation of enterprise-wide process networks though there are significant incentives to construct a decision-making framework based on bilevel programming principles.

In order to solve bilevel programming problems of such importance, some researchers presented a number of methodologies in the chemical engineering sphere. For instance, Clark and Westerberg (1983) proposed two algorithms for computing

local optima of non-linear bilevel programming problems and updated their algorithms in their succeeding works (Clark and Westerberg, 1990). Visweswaran *et al.* (1996) proposed a global optimization algorithm for bilevel linear and quadratic programming problems. However these studies are limited in that (i) only local optimal solutions are obtained (Clark and Westerberg, 1983, 1990) and (ii) complex computations with additional variables and constraints should be undergone (Visweswaran *et al.*, 1996). Therefore it is fully motivating to propose an efficient global optimization methodology.

1.2.2 Motivating Example 2. Scheduling under Uncertainty

Consider a manufacturing process involving three products in three serial stages. It is known that the process is operated in a zero-wait policy where a task in a stage should be finished for the next to start (see Figure 1.2). Average processing times of products in various stages are known as in Table 1.2. The objective of the process is to minimize a makespan, which is the completion time of the last produced product at the last stage.

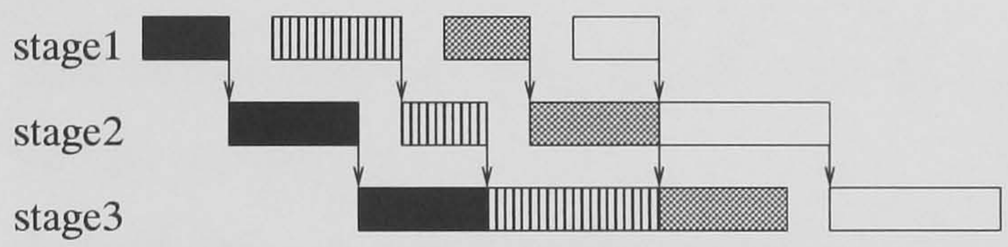


Figure 1.2: Illustrative Gantt Chart of a Zero-Wait Batch Process

Table 1.2: Average Processing Time Data (I) for Motivating Example 2

	Processing time (hrs)		
Product	Stage 1	Stage 2	Stage 3
P1	10	20	5
P2	15	8	12
P3	20	7	9

This type of short-term operation planning problem has been formulated into a number of mixed integer linear programming (MILP) problems (Ku and Karimi, 1988; Rajagopalan and Karimi, 1989; Jung *et al.*, 1994; Moon *et al.*, 1996). For instance, one recently proposed formulation is as follows (Jung *et al.*, 1994) based on notation in Table 1.3:

Table 1.3: Notation for Motivating Example 2

Indices	
i	time slot $(1, \cdots, \mathrm{N})$
l	product $(1, \cdots, \mathrm{N})$
k, j	stage $(1, \cdots, \mathrm{M})$
Parameters	
P_{lk}	processing time of product l in stage k
Variables	
C_{ij}	completion time of i time slot product in stage j
y_{li}	binary variable; 1 if product l is made at ith time slot, otherwise 0

$$Min \quad C_{N,M} \tag{1.11a}$$

$$\sum_{l=1}^{N} y_{li} = 1, \ \forall\, i \tag{1.11b}$$

$$\sum_{i=1}^{N} y_{l,i} = 1, \ \forall\, l \tag{1.11c}$$

$$C_{i,j} \geq C_{i-1,M} - \sum_{l=1}^{N} \sum_{k=(j+1)}^{M} P_{l,k} y_{l,(i-1)} + \sum_{l=1}^{N} \sum_{k=j}^{M} P_{l,k} y_{l,i}, \quad j = 1, \cdots M-1, \ , y_{l,0} = 0 \forall i \tag{1.11d}$$

$$C_{i,M} \geq C_{i-1,j} + \sum_{l=1}^{N} \sum_{k=j}^{M} P_{l,k} y_{l,i}, \quad i = 2, \cdots N, \ j = M \tag{1.11e}$$

where $P_{l,k}$ is the processing time of product l on stage k. It is assumed that transfer and set-up times are negligible in the above problem but the proposed methodology can be also used for the one including them.

(1.11b) and **(1.11c)** ensure that specific products are produced at specific time instances (slots) and in a specific sequence by ensuring that product l is made at the ith sequence (or the product in slot i). **(1.11d)** and **(1.11e)** determine completion times of individual products at different stages, based on the selected

sequence and the zero-wait process condition. The optimal solution of this problem with processing time data in Table 1.2 corresponds to 61 hrs with the corresponding optimal sequence of [P2 → P1 → P3].

In reality, there exist variations such as fluctuations in the quality of raw materials, uncertainty in product demands, equipment break-downs, etc. For instance, let us assume that some of processing times at stages 1 and 2 change due to uncertainty as can be seen in Table 1.4. Then, application of the above mathematical model with the data in Table 1.4 yields the optimal makespan of 91 hrs with the corresponding optimal sequence of [P1 → P3 → P2]. [P2 → P1 → P3] which was previously computed as optimal becomes sub-optimal with the makespan of 92 hrs in the changed condition.

Table 1.4: Varied Processing Time Data (II) for Motivating Example 2

	Processing time (hrs)		
Product	Stage 1	Stage 2	Stage 3
P1	11	36	5
P2	16	16	12
P3	22	15	9

In the case of the data in Table 1.5, the optimal sequence is again changed into [P1 → P2 → P3] with a corresponding makespan of 99 hrs.

Table 1.5: Varied Processing Time Data (III) for Motivating Example 2

	Processing time (hrs)		
Product	Stage 1	Stage 2	Stage 3
P1	11	40	5
P2	16	18	12
P3	22	17	9

Table 1.6: Variation of the Optimal Schedule depending on Uncertainty for Motivating Example 2

Data	makespan (hrs)	optimal schedule
Table 1.2	61	[P2 → P1 → P3]
Table 1.4	91	[P1 → P3 → P2]
Table 1.5	99	[P1 → P2 → P3]

As can be seen in Table 1.6 which summarizes the above three cases, the optimal schedule may vary on the basis of the varying processing time. According to the conventional methods presented in the literature (Cott and Macchietto, 1989; Kanakamedala *et al.*, 1994; Schilling and Pantelides, 1997; Vin and Ierapetritou, 2000), the entire scheduling problem has to be solved again and again in response to occurrences of these variations to obtain a new optimal schedule. This is computationally expensive. There is a need for more computationally efficient and pro-active methodologies in order to solve this short-term scheduling problems under uncertainty.

1.3 Project Aims and Thesis Outline

As derived by these examples, the main objectives of this thesis are:

- Construction of a modeling framework to address hierarchical decision-making problems of the enterprise-wide process networks under uncertainty
- Development of an efficient methodology to compute the solution of such decision-making problems
- Development of a pro-active short-term scheduling framework to provide a robust decision support system for individual processes under uncertainty

The rest of this thesis is organized as follows:

- In chapter 2, a review of recent work on enterprise-wide process networks is made with particular emphasis on modeling and solution methodologies.

- In chapter 3, a novel solution methodology is developed in order to compute the global optimization solution of deterministic bi-level programming problems for linear and quadratic cases using parametric programming techniques.

- In chapter 4, enterprise-wide process network planning problems are transformed into bilevel programming problems under uncertainty, which are then solved by proposing a methodology based on the one proposed in chapter 3.

- In chapter 5, a pro-active scheduling system is constructed for short-term scheduling problems involving uncertainty and a solution methodology is presented to solve such corresponding problems using parametric programming techniques.

- Some concluding remarks and directions for future work are given in chapter 6.

Chapter 2

Literature review

This chapter reviews the previous research on design and operation of enterprise-wide process networks under uncertainty. The review will be made with two main emphases. First, the way in which industrial situations are transformed into mathematical programming problems is outlined with a special focus on scheduling under uncertainty and supply chain management under uncertainty. Then methodologies to solve the resulting problems under uncertainty are briefly reviewed with a focus on parametric programming. Throughout the survey of the previous research, key features are discussed and potential limitations are identified to highlight research opportunities, which will be addressed in the remainder of this thesis.

2.1 Modeling of Enterprise-wide Process Network under Uncertainty

Enterprise-wide process networks involve numerous complex situations, which may be transformed into various types of engineering problems requiring advanced methodologies. Those problems have generally been formulated as mathematical optimization models with a set of mathematical relationships (e.g. equalities, inequalities, logical conditions). Considerable effort has been put into how to transform those problems into mathematical models and how to solve the resulting problems.

The literature here can be mainly categorized into short-term scheduling and long-term planning depending on the time scale of the problems. The former is

mainly interested in calculating how resources are distributed over a short-time period for a specific process while the latter is concerned with computing process design and capacity planning issues over a relatively long time period for a wide range of enterprise-wide process networks.

2.1.1 Short-term Operation Scheduling under Uncertainty

Short-term operations should be planned in the face of varying process-, model- or market- related parameters with a view to maximizing profits. There are three major ways of dealing with uncertainty for the short-term operation of a process.

One simple and direct strategy is to install additional equipment which can be used in the case of uncertainty (Karimi and Reklaitis, 1985a, 1985b). However, this is often not practical because it requires additional high investment costs which result in low availability during normal conditions. Besides, the uncertainty involved in the operation of the additional units cannot be dealt with using this method.

Another way is to manipulate the process condition such that it uses a fixed schedule in the condition of uncertainty (Onogi et al., 1986; Hvala et al., 1993). An example of this type of manipulation is adjusting the operation parameters in order to use the original schedule. However, it cannot be applied generally because there is not much room for modification in actual complex processes.

The third is reactive scheduling (see representative references in Table 2.1) which is a way to address uncertainty issues in (typically on-line) scheduling applications in response to uncertainty. The main idea of reactive scheduling is to repeatedly solve deterministic scheduling problems whenever a variation occurs: new schedules are computed and implemented based upon newly realized parameters (for example, from on-line measurements).

Studies of reactive scheduling in the literature can be summarized in the following three ways:

First, the computational issue is the major obstacle to improving the performance of reactive scheduling. Because of the need for constant re-computations, reactive scheduling approaches may become computationally expensive. Thus, some of the studies in the literature have attempted to accelerate the computational performance of the underlying deterministic scheduling problem by shifting the starting time of

Table 2.1: Reactive Scheduling at a glance

Uncertainty	Research
processing time	Cott and Macchietto (1989); Kanakamedala *et al.* (1994); Huercio *et al.*(1995); Ishii and Muraki (1996)
equipment availability	Kanakamedala *et al.* (1994); Schilling and Pantelides (1997); Vin and Ierapetritou (2000)
batch size variability	Cott and Macchietto (1989)
rush order	Vin and Ierapetritou (2000)

jobs in a schedule (Cott and Macchietto, 1988), limiting the search area for a new solution (Kanakamedala *et al.*,1994), relaxing the constraints (Vin and Ierapetritou, 2000) or resolving the scheduling problem in a hierarchical way (Schilling and Pantelides, 1997, Sand et al.,2000). Considering the scale of conventional scheduling problems, the heavy computational load still poses a new challenge for reactive scheduling.

Second, most studies focus on how to minimize the effect of disturbances only *after* their occurrence. Little attention has been given to the more positive approach such as predicting a potential variation and the corresponding optimal schedule in response to the variation. Ishii and Muraki (1997) noticed the importance of predicting the process state in reactive scheduling but their work leaves unanswered the question of how the prediction can be made and realized in the framework of the reactive schedule.

Third, most studies approach the reactive scheduling problem as a subproblem of plant operation management. The management of plant operations generally consists of (i) production planning (at the highest level), (ii) scheduling of various products (at the intermediate level), and (iii) process control (at the lowest level)

(Cott and Macchietto, 1989; Kanakamedala, Reklaitis and Venkatasubramanian, 1994). Research on reactive scheduling has focused on the intermediate level by re-computing new completion times of tasks (Cott and Macchietto, 1989; Ishii and Muraki, 1997). However, in order to respond to uncertainty systematically, it would be necessary to incorporate all three levels, and this has not been fully investigated.

2.1.2 Long-term Operation Planning under Uncertainty

Long-term operation planning problems are concerned with determining how processes should be operated over a relatively long time period. These problems aim to compute initial process capacities as well as their potential expansion plans subject to varying circumstances typically demands of products.

Recently there has been great industrial and academic research on enterprise-wide supply chain planning which incorporates various operations of the entire process network. A review on the long-term planning is thereafter surveyed with a specific focus on supply chain planning problems after the brief discussion of studies on conventional single process planning issues.

Single Process Planning under Uncertainty

Planning models are generally constructed aiming to compute the optimal production levels or production configurations which maximize profits subject to various constraints such as (i) production capacities with or without capacity expansions, (ii) demands, (iii) availabilities, (iv) inventory requirements, and (v) material balance.

The production configuration may take the form of selecting particular units or processing types in response to varying external conditions, which are, in most cases, uncertain demands (Sahinidis *et al.*, 1989; Ierapetritou and Pistikopoulos, 1994). Consequently, most planning models are mathematically formulated into a mixed integer linear programming (MILP) problem. This can be explained by the fact that the major focus of the planning model involves the discrete decision such as which unit to select or when to expand the current process capacity by what amount etc. The corresponding MILP problem is generally of a large problem size. Therefore, various algorithms and computational methods have been introduced to

improve solution performances such as cutting planes, Benders decomposition and heuristics (Sahinidis and Grossmann, 1991a; 1991b; 1992).

On the other hand, novel methodologies such as parametric optimization (Pistikopoulos and Dua, 1998), and stochastic programming (Liu and Sahinidis, 1996; Clay and Grossmann, 1997; Petkov and Maranas, 1997; Ahmed and Sahinidis, 1998) have been proposed in order to explicitly address uncertainty involved in planning problems.

Methodologies developed for general chemical process planning have been implemented to a number of specific applications such as utility system (Kim and Han, 2001; Iyer and Grossmann, 1997, 1998b), refinery planning (Zhang *et al.*,2001) and oil field infrastructure planning (van den Heever, 2000a, 200b; Iyer *et al.*, 1998).

See Table 2.2 for major previous studies on the single process planning in chemical engineering community.

Table 2.2: Past Research on Single Process Planning at a glance

Authors	Year	formulation	uncertainty	major contributions
Sahinidis *et al.*	1989	MILP	multiperiod	cutting plane, benders decomposition
Sahinidis and Grossmann	1991	MILP	multiperiod	variable disaggregation technique
Shah and Pantelides	1991	MILP	multiperiod	scenario-based, campaign planning
Ierapetritou *et al.*	1994	mp^a-LP, MILP	stoch. prog.	flexibility, maximum regret
Ierapetritou *et al.*	1996	MINLP	stoch. prog.	feasibility, decomposition, Gauss quadrature
Liu and Sahinidis	1996	MILP	multiperiod	cutting plane, lot sizing problem, polyhedral
Liu and Sahinidis	1996	MILP	stoch. prog.	two stage, decomposition
Ierapetriou and Pistikopoulos	1996	MINLP	stoch. prog.	value of perfect information, flexibility Gaussian quadrature
Iyer and Grossmann	1997	MILP	multiperiod	application(utility system), decomposition shortest path algorithm
Petkov and Maranas	1997	MINLP	stoch. prog.	chance constraint
Clay and Grossmann	1997	LP	stoch. prog.	successive disaggregation algorithm
Iyer and Grossmann	1998a	MILP	multiperiod	application(utility system), bilevel (design, planning)
Acevedo and Pistikopoulos	1998	MINLP	stoch. prog.	Gaussian quadrature, parallel computing
Ahmed and Sahinidis	1998	MILP	stoch. prog.	robustness
Bok *et al.*	1998	MINLP	stoch. prog.	application(petrochemical network), robustness
Pistikopoulos and Dua	1998	*mp*-MILP	para. prog.	computing all optimal solutions
Iyer and Grossmann	1998b	MILP	multiperiod	application(utility system)
Iyer *et al.*	1998	MILP	*deter*[b]	application(offshore oil field), sequential decomposition
van den Heever *et al.*	1999	MINLP	*deter*	application(oil field infrastructure) disjunctive prog., outer approximation
Asprey *et al.*	1999	*n.s.*[c]	simulation	artificial intelligence, flammable chemical control operation procedure synthesis
Bernado *et al.*	1999	NLP	stoch. prog.	Gauss cubature formula, robustness
Georgiadis and Pistikopoulos	1999	NLP	stoch. prog.	Gaussian quadrature, robustness, signal to noise ratio
Linninger *et al.*	2000	*n.s.*	simulation	Monte Carlo simulation, chance constraints, waste management, trade off btn cost, flexibility
Lee and Malone	2000a	NLP	*deter*	simulated annealing, waste minimization, solvent recovery system
Lee and Malone	2000b	NLP	*deter*	simulated annealing, zero wait, network flow shop
Pinto *et al.*	2000	MILP	*deter*	STN, scheduling
Ahmed and Sahinidis	2000	MILP	*deter*	approximation, heuristic, probabilistic analysis
van den Heever *et al.*	2000	MINLP	multiperiod	dynamic programming, (dis)aggregation application(oil field infrastructure)
Ahmed *et al.*	2000	NLP	stoch. prog.	algorithm by Ierapetritou and Pistikopoulos(1994) is improved
Pistikopoulos *et al.*	2001	MILP	multiperiod	preventive maintenance, Scheduling equipment failure rate
Kim and Han	2001	NLP	multiperiod	application(utility system), dynamic programming, heuristic
Lee and Malone	2001	NLP	simulation	Monte Carlo sampling, flexibility, scheduling
Orçuns *et al.*	2001	MINLP	*deter*	paint production

mp^a:multi-parametric, $n.s.^b$: not specified, $deter^c$: deterministic

Supply Chain Planning under Uncertainty

In recent years supply chain planning problems have been actively studied due to a need to incorporate multiple supply chain activities with a view to increasing efficiencies of the entire supply chain in the current competitive environment (see Table 2.3 for its representative references).

Table 2.3: Supply Chain Literature at a glance

Research	Year	model	solution method	Uncertainty	major contributions
Voudouris	1996	MILP	optimization	*deter*[a]	application(fine chemical) efficiency - manpower and floor-space
Wilkinson	1996	MILP	optimization	*deter*	scheduling, aggregated algorithm
Dimitriadis	1997	MILP	optimization	*deter*	scheduling, RTN framework, rolling horizon algorithm
McDonald and Karimi	1997	MILP	optimization	multiperiod	mid-term planning, varying time scale
McDonald	1998	MILP	optimization	*deter*	lot sizing problem, Planning and scheduling
Gupta and Maranas	1999	MILP	optimization	multiperiod	largrangian relaxation
Gupta and Maranas	2000	MINLP	optimization	stoch. prog.	compared with Monte Carlo sampling rolling horizon simulation
Gupta *et al.*	2000	MINLP	optimization	stoch. prog.	chance constraints, customer satisfaction
Zhou *et al.*	2000	LP	optimization	*deter*	sustainability, multi-objective (goal prog. + relaxing constraints)
Applequist *et.al*	2000	NLP	optimization	stoch. prog.	balance of risk and profit
Flores *et.al*	2000	*n.s.*[b]	simulation		control(MPC)
Bok *et.al*	2000	MILP	optimization	multiperiod	bilevel decomposition Norton and Grossmann (1994)
Bose and Pekny	2000	n.s.	simulation		control(MPC)
Gjerdrum *et al.*	2001	MILP	optimization	*deter*	fair profit distribution, Nash optimum
Tsiakis *et al.*	2001	MILP	optimization	multiperiod	scenarios, flexible transfer cost
Papageorgiou *et.al*	2001	MILP	optimization	*deter*	application(pharmaceuticals), tax, scale-up cost
Perea-Lopez *et.al*	2001	n.s.	simulation		control, decentralized model

deter[a] :deterministic

n.s.[b] :not specified

The active research trends may be categorized into the following three research directions.

First, some researchers addressed the basic principles which should be involved in the supply chain management. Backx *et al.* (1998) described the current manufacturing environment as transient and pointed out that manufacturing plants have to adapt themselves to the transient environment to fully exploit their economic

potential. They claimed that the operating strategy of enterprises should thereafter be also intentionally dynamic. As an alternative strategy, they presented an idea of dissecting the operation of a supply chain into problems of several levels. But the resulting problem becomes very complex. Decomposition and coordination techniques are suggested as a possible way to solve this complex problem. McDonald (1995) presented the idea of designing enterprises as a whole and not as a localized problem of a specific unit. This work discusses advantages and disadvantages of using global information technologies for operating enterprises. Rosen (1998) reviewed currently used supply chain analysis methodologies in industries such as optimization, simulation and heuristics. The author proposed that an ideal direction for the practical supply chain decision-making would be to combine mathematical programming optimization and simulation.

Along with above studies in academia, there have been lots of industrial activities in establishing industrial standard of supply chain management. For instance an institute called Supply Chain Council[1] which was organized by leading companies such as Dow Chemical, Merck, Texas instruments, Compaq and Federal express in 1996 introduced an industrial model termed supply chain operations reference model (SCOR). The institute developed the model from the practical perspective; they defined common supply chain management processes and matched these processes against "best practice" examples and benchmarked performance data as well as optimal software applications. The objectives of the council is to find a tool for measuring both supply chain performance and the effectiveness of supply chain re-engineering, as well as testing and planning for future process improvement.

Secondly, there is an increasing number of activities which formulate supply chain issues into mathematical optimization models. This is the most widely used approach which mainly starts from a simple model of a single plant and expand to the large scale model of multiple plants, warehouses etc. Tsiakis *et al.* (2001) presented a supply chain design model for steady state continuous processes. Their supply chain model was based on determining the connections between multiple plants, warehouses, and markets. Uncertainty is involved as a form of multiple scenarios

[1] http://www.supply-chain.org

in their model. However the number of scenarios in their application is limited, so the actual performance of their model in representing actual supply chain is not examined. McDonald and Karimi (1997a, 1997b) presented models for planning and scheduling of parallel semi-continuous processes. Their model is expanded for supply chains in succeeding works of Gupta and Maranas (2000), Gupta, Marana and McDonald (2000). Zhou *et al.*(2000) used a multi-objective programming method to incorporate multiple conflicting objectives such as economic, material, social and environmental sustainability within a supply chain but the issue of how to incorporate conflicting interests of elements is not covered in their work. Papageorgiou *et al.* (2001) explored strategies for launching new pharmaceutical products considering capacity expansions of manufacturing processes. They presented an optimization model for pharmaceutical companies covering a range of aspects from the experimental level to the future process network planning level. Gjerdrum *et al.* (2001) studied the compromise of profits between the supplier and the demand in a supply chain using the game-theoretic equilibrium, which is based on constraints guaranteeing minimum profit levels. Wilkinson (1996) and Dimitriadis (1997) were concerned with how to solve scheduling problems of a large scale process network. They tried to aggregate a large number of small time periods into relatively assembled time periods and transformed the original large scale problem into problems with the reasonably tractable size. The results of the aggregated problems are then disaggregated to fit into the detail specification.

Although many studies have been undertaken, there is still room for improvement in transforming supply chain issues into optimization models. First, it should be noted that most models are not entirely based on actual practice. Since individual supply chain activities are governed by separate decision-makers which have their own objectives, their operation and control should be also based on such multi-perspectives. However, previously proposed models generally assume that all activities are governed globally without considering such multiple principles.

Furthermore the assumption forces decisions of individual elements to be based on the same level of information and this is against practices. Considering the fact that multiple elements with mutually independent objectives are generally connected in a hierarchical way, it is natural to assume that some elements in a supply chain

may possess less information, while other possess more complete information. However, little research has been done actually addressing such different information availabilities in the context of supply chain planning problems.

Finally the uncertainty is an important issue which makes the decision-making involved in supply chains very difficult. Parameters in supply chains, for instance, processing times, utility coefficients, delivery and inventory costs, etc., are generally subject to uncertainty. Values of these parameters cannot be perfectly known because they will be only decided in the future.

Therefore uncertainty should be explicitly considered in supply chain planning problems. Otherwise, the resulting decisions may be ineffective and even infeasible.

In the literature, typical methodologies for computing problems under uncertainty such as scenario-based multi-period formulation (Tsiakis et al., 2001), stochastic programming formulation (Gupta et al., 2000) have been proposed. Some researchers conceived the actual supply chain as a dynamic system subject to uncertainty and tried to emulate control concepts in the supply chain dynamics such as simple control laws (Perea et al., 2000) and basic model predictive control concepts (Bose and Pekny, 2000; Flores et al., 2000). Financial investment techniques are even introduced in order to handle the financial risks associated with the design and planning of supply chains under uncertainty (Applequist *et al.*, 2000). Although they tried to evaluate the expected profits under variances, a simplified criteria is used in their model to avoid computational complexities. Therefore effects of the simplified against the rigorous that can indicate the full potential of their approach were not investigated.

As can be seen in the above discussion, there is increased interest on the supply chain and their design and operation issues. However most work is only concerned with constructing mathematical models as a simple expansion of single models or simulating the dynamics of the supply chain in a simple way. There is a need to construct a modeling framework that incorporates multiple conflicting decision-makers under uncertainty. In the next section, past research on such a modeling framework is reviewed with a focus on bilevel programming.

2.1.3 Bilevel Programming

Many industrial situations involve more than one group which are inter-connected in a hierarchical structure. Each group may be an individual or an agency which has an independent, perhaps conflicting, objective. These situations of multiple decision-makers can be mathematically modeled into a multi-level programming problem (Clark and Westerberg 1983, 1990) or multiobjective programming. Because the solutions by multi-objective programming are compromising without fully representing the decision-making of real practices (Clark and Westerberg, 1990), this review will focus on multi-level programming, particularly bilevel programming problem which is for the case of two decision makers. The bilevel programming problem refers to an optimization problem (which is called a higher level decision problem, *a leader*'s problem or an outer problem) that is constrained by another optimization problem (which is called a lower level decision problem, *a follower*'s problem or an inner problem).

The two problems are connected in a way that the leader's problem sets parameters influencing the follower's problem and the leader's problem, in turn, is affected by the outcome of the follower's problem.

To compare the two problems in terms of scope of information, a follower makes decisions using only its local information while a leader does using the complete information including the follower's possible reaction to the leader's decision. This is an important feature which can contribute to addressing the complicated industrial situations which may be difficult to be modeled by other modeling methodologies. Therefore it has been applied in extensive and diverse areas (See some of representative references in the open literature in Table 2.4). Good reviews of bibliography on BLPPs can be found in Vicente and Calamai (1994).

Table 2.4: Representative Applications of Bilevel Programming at a glance

Area	**Subject**	**Reference**
Economy	central planning with resource distribution	Cassidy et al.(1971)
	utility pricing and planning	Hobbs and Nelson (1992)
Civil eng.	transportation network design	Clegg et al. (2000) Boyce and Mattsson (1999) Chiou (1999) Migdalas (1995)
Environ. eng.	waste treatment planning with facility locations	Amouzegar and Moshirvaziri (1999)
Finance	tax credit for industry	Bard et al. (2000)
Chem. eng.	process design with phase equilibrium	Clark and Westerberg (1983, 1990)
	flexibility analysis	Grossmann and Floudas (1987)
	process design with control	Brengel and Seiderm (1992)
	plant design under uncertainty	Ierapetritou and Pistikopoulos (1996)
	flexibility and feasibility	Floudas et al. (2001)

In the literature, research on solving BLPPs has mainly focused on a linear case (See representative solution methodologies in Table 2.5) and can be categorized into two major groups. In the first group, the original problem is transformed into a single optimization problem by employing the Karush-Kuhn-Tucker optimality condition of the lower level problem (Visweswaran et al., 1996). Research in the other group utilizes enumeration techniques based on the fact that an optimal solution to the bilevel problem is a basic feasible solution of the linear constraints involved in the lower and upper level, and consequently must occur at an extreme point of the feasible set (Bard and Moor, 1990).

Both methodologies require many additional variables and complex computations which involve many iterations. Many algorithms and applications presented so far can be found in Bard(1998) and test problems can be found in the recent book by Floudas et al.(1999). Many references on complexity issues of BLPPs can be found in Gümüs and Floudas (2001).

Table 2.5: Major Solution Methodologies of Bilevel Programming at a glance

Subject	**Key feature**	**Reference**
LP	enumeration (vertex enumeration)	Candler and Townsley (1982)
	enumeration (K-th best algorithm)	Bialas (1984)
	enumeration (branch and bound)	Bard and Moore (1990)
	reformulation (branch and bound)	Bard and Falk (1982)
	reformulation (complementary pivoting)	Judice and Faustino (1992)
	reformulation (global optimization)	Visweswaran et al. (1996)
	reformulation (parametric programming, global optimization)	Ryu et al. (2002)
QP	branch and Bound (global optimization)	Visweswaran et al. (1996)
	descent method	Vicente et al. (1994)
	reformulation (parametric programming, global optimization)	Ryu et al. (2002)
NLP	grid search	Bard (1983, 1984)
	simulated annealing	Sahin and Ciric (1998)
	trust region	Marcotte et al. (2001)
	penalty function	Aiyoshi and Shimizu (1981)
MILP	branch and Bound	Moore and Bard (1990)
	simulated annealing	Sahin and Ciric (1998)

In order to fully utilize the potential of BLPPs that is such an important modeling framework, an efficient solution methodology is of critical importance. Many approaches have been proposed but most of them do not allow problems to be solved to global optimality because of the non-convex nature which is caused by the interaction between solution spaces of the two problems (Visweswaran et al., 1996).

2.2 Solution Approaches to Problems under Uncertainty

Many process engineering problems involve varying parameters. These varying parameters can be, for example, attributed to fluctuations in resources, market requirements and prices which can affect the feasibility and economics of a project (Dua *et al.*, 2002). If the problems are solved without considering the presence of uncertainty, the resulting solutions may be not robust or be even infeasible. Major approaches proposed so far to handle such problems can be classified into three types: multiperiod optimization, stochastic programming and parametric programming based on the description of the varying parameters.

2.2.1 Multiperiod Optimization

Multiperiod optimization approaches problems under uncertainty by discretizing a uncertain parameter, such as expected demands, into multiple periods of operation with a specific realization in each period. The original problem under uncertainty is then transformed into a deterministic one, whose objective is to find the best strategy that maximizes the net present value over the whole period. As can be seen in Table 2.2 and Table 2.3 of the previous section, this approach is one of the most widely used in process systems engineering, specifically, in the area of process planning and synthesis (Sahinidis *et al.*, 1989; Shah and Pantelides, 1991; Liu and Sahinidis, 1996; Iyer and Grossmann, 1997, 1998a, 1998b; van den Heever *et al.*, 2000; Pistikopoulos et al., 2001). However, there are some limitations. Apart from the computational difficulties due to the explosion of the problem size, another drawback of this approach is that if the forecasted values are given by ranges, rather

than a number of discrete points (which usually is the case), the planning problem has to be resolved for all the expected values that lie within the forecasted range (Pistikopoulos and Dua, 1998).

2.2.2 Stochastic Programming

Problems where uncertain parameters are given with their probability distribution functions can be solved by stochastic programming. The stochastic programming is concerned with computing the best solution that can hedge against multiple future outcomes (Birge and Louveaux, 1997). The aim is to identify the single "most preferred" policy so as to maximize an "average" profit criterion (or minimize economic risk) over possible future outcomes or scenarios. As can be seen in Table 2.2, it has been applied to solve many process systems engineering problems, particularly in the area of process planning and design (Ierapetritou *et al.*, 1994, 1996; Ierapetritou and Pistikopoulos, 1996; Petkov and Maranas, 1997; Clay and Grossmann, 1997; Acevedo and Pistikopoulos, 1998; Ahmed and Sahinidis, 1998; Bok *et al.*, 1998; Bernado *et al.*, Georgiadis and Pistikopoulos, 1999; Ahmed *et al.*, 2000).

Although stochastic programming solutions provide significant practical advantages over deterministic models, one drawback of this approach is that the identified single planning strategy may be sub-optimal and infeasible for other scenarios. It may be more useful to have information on the values of uncertain parameters for a number of planning strategies rather than obtain a single strategy which tries to capture all the scenarios based on an "average" performance criterion (Pistikopoulos and Dua, 1998). In order to tackle the infeasibility issue, a concept of robustness has been introduced by some researchers (Bok et al., 2000).

2.2.3 Parametric Programming

The aim of parametric programming is to obtain the optimal solution as a function of the parameters. Parametric programming provides a complete map of the optimal solution as a function of the varying parameters. Compared to the previous two approaches, application of parametric programming in solving design and operation problems has been relatively limited. This may be attributed to the lack of research

on theory and algorithm on parametric programming for a wide range of problems. For instance, it was only relatively recent that algorithms were proposed that can solve a problem involving multiple varying parameters or mixed integers (Acevedo and Pistikopoulos, 1997; Dua and Pistikopoulos, 2000). However various theoretic and algorithmic works for a wide range of parametric programs have been recently reported. Particularly, Pistikopoulos and co-workers in Imperial College London have proposed a series of parametric programming techniques and softwares (some of which are listed in Table 2.6 and 2.7). Therefore it may be said that the principles of parametric programming have been fairly established. This thesis will hereafter focus on parametric programming techniques in solving process systems problems under uncertainty.

Table 2.6: Parametric Programming at a glance

Subject	**Reference**
mp-MILP[1]	Acevedo and Pistikopoulos (1997, 1999); Pistikopoulos and Dua (1998); Dua and Pistikopoulos (1998b, 2000); Pistikopoulos *et al.*(2000a);
mp-QP[2]	Dua *et al.* (2002); Bemporad *et al.*(2000a, 2000b, 2002); Pistikopoulos *et al.* (1999, 2000a, 2000b, 2002);
mp-MIQP	Dua *et al.*, (2002); Sakizlis *et al.* (2001a)
mp-MIGOP[3]	Dua *et al.* (1999)
p-MINLP	Acevedo and Pistikopoulos (1996); Papalexandri and Dimkou (1998); Pertsinidis *et al.* (1998)
mp-MINLP	Acevedo and Pistikopoulos (1996); Dua and Pistikopoulos (1998a); Pistikopoulos and Dua (1998); Hené *et al.*(2002)

[1]multi-parametric Mixed Integer Linear Programming

[2]multi-parametric Quadratic Programming

[3]multi-parametric Global Optimization Programming

Table 2.7: Representative Applications of Parametric Programming

Application area	**Reference**
multi-objective optimization	Pistikopoulos and Grossmann (1988); Pertsinidis(1992); Papalexandri and Dimkou (1998)
process synthesis	Acevedo and Pistikopoulos (1996); Dua and Pistikopoulos (1998b)
process planning	Pistikopoulos and Dua (1998)
process scheduling	Ryu and Pistikopoulos (2001)
flexibility analysis	Bansal *et al.*(2000a, 2001)
dynamic optimization	Sakizlis *et al.*(2001a, 2001b, 2001c)
on-line control & optimization	Pistikopoulos *et al.* (2000, 2001) Bemporad *et al.*(2002) Sakizlis *et al.*(2001e, 2002b, 2002c) Kakalis *et al.* (2002)
hybrid systems	Acevedo and Pistikopoulos (1997) Sakizlis *et al.* (2001d, 2001f, 2002a)

2.3 Summary

This chapter reviewed the previous literature on the design and operation of enterprise-wide process networks under uncertainty. A review of mathematical modeling approaches to capture enterprise-wide process network issues has been presented with a brief survey of the different methodologies used to solve the problems. Key features have been discussed and some potential limitations identified in order to highlight research opportunities, which will be further investigated in the following chapters.

Chapter 3

A Parametric Optimization Based Global Optimization Approach for Bilevel Programming Problems

This chapter presents a global optimization approach to bilevel programming problems using parametric programming techniques. The proposed approach transforms the bilevel problem into a family of single optimization problems, which can be solved to global optimality for linear-linear, linear-quadratic, quadratic-linear, and quadratic-quadratic bilevel models.

3.1 Introduction

Many industrial situations involve several groups which are inter-connected in a hierarchical structure. Each group may be an individual or an agency which has independent, perhaps conflicting, objectives. These situations can be mathematically modeled into a multi-level programming problem (Clark and Westerberg 1983, 1990). When there are two decision makers, this can be formulated as a BiLevel Programming Problem (BLPP) which is generally of the following form:

$$\begin{aligned} \min_{x} \; & F(x,y) \\ s.t. \; & G(x,y) \leq 0 \\ \min_{y} \; & f(x,y) \\ s.t. \; & g(x,y) \leq 0 \end{aligned} \tag{3.1}$$

where $x \in X$ and $y \in Y$ and X and Y are compact and polyhedral regions. $F(x,y)$ is called a higher level decision problem, namely *a leader*'s problem or an outer problem and $f(x,y)$ is called a lower level decision problem, *a follower*'s problem or an inner problem.

As can be seen in **(3.1)**, a BLPP refers to an optimization problem that is constrained by another optimization problem. The two problems are connected in such a way that the leader's problem sets parameters influencing the follower's problem and the leader's problem, in turn, is affected by the outcome of the follower's problem.

To compare the two problems in terms of scope of information, a follower makes decisions using only its local information while a leader does so using the complete information including the follower's possible reaction to the leader's decision. This is an important feature which contributes to addressing complicated industrial situations which may be difficult to model using other modeling methodologies. Therefore the BLPP has been applied in extensive and diverse areas. A bibliography and good reviews of works on BLPPs can be found in Vicente and Calamai (1994).

In the literature, research on solving BLPPs has mainly focused on the linear case in the following two major ways. One aims to transform the original problem into a single optimization problem by employing the Karush-Kuhn-Tucker optimality condition of the lower level problem (Visweswaran et al., 1996). The other utilizes enumeration techniques based on the fact that an optimal solution to a bilevel problem is a basic feasible solution of the linear constraints involved at the lower and upper level and consequently must occur at an extreme point of the feasible set (Bard and Moor, 1990). Both methodologies require many additional variables and complex computations which involve many iterations.

In order to fully utilize the potential of BLPPs which is such an important mod-

eling framework, an efficient solution methodology is of critical importance. Because most approaches proposed so far do not allow problems to be solved to global optimality due to the fact that the solution space of the BLPP is non-convex (Visweswaran et al., 1996), this chapter proposes a novel global optimization methodology to solve the bilevel programming problem.

The rest of this chapter is organized as follows: By re-visiting some key features of BLPPs, a novel idea to solve BLPPs will be introduced. Based on the idea, a new global optimization methodology is presented and illustrated for the case of bilevel linear and quadratic programming problems. A selection of numerical examples in the literature are then solved to demonstrate the major advantage of the proposed methodology.

3.2 Theory and Algorithm

The following terminologies relating to BLPPs in the literature are worth noting again. First, the relaxed feasible set of BLPPs, Ω is defined as follows:

$$\Omega = \{(x, y) : G(x, y) \leq 0, g(x, y) \leq 0\}$$

If there are no $(x, y) \in \Omega$, then the BLPP is infeasible. This chapter hereafter assumes that there exist solutions satisfying the above set.

The feasible set of the inner problem, $S(x)$ is defined for every $x \in X$ as :

$$S(x) = \{y | g(x, y) \leq 0\}.$$

The rational reaction set of the inner problem, $RR(x)$ is defined for every $x \in X$ as:

$$RR(x) = \{y \in arg\, min f(x, y) | y \in S(x)\}.$$

The BLPP feasible set (or namely the induced, or inducible region), IR is defined as follows :

$$IR = \{(x, y) : (x, y) \in \Omega, y \in RR(x)\}.$$

The challenge associated with solving BLPPs is mainly caused by the fact that the BLPP feasible set becomes non-convex (Visweswaran et al., 1996). However, this difficulty may be avoided if x and y are an affine function. The inducible region can be obtained as a set of explicit functions of variable, x. The issue is then how to compute the functions and the corresponding boundary conditions which validate the functions.

As an alternative method, this chapter introduces parametric programming techniques which have been mainly used for computing the solutions of problems involving uncertain parameters. This is a novel idea because there is little research to our knowledge employing parametric programming techniques in solving BLPPs although it is widely known that the feasible set of the inner problem is *parametric* in terms of the decision variables of the outer problem.

In order to achieve that, it is assumed that $f(x,y)$ and $g(x,y)$ in **(3.1)** are twice continuously differentiable in x and y and variables of the leader's problem, x meet the basic sensitivity theorem in the follower's problem (Dua et al., 2002). For the detail of the parametric programming techniques, refer to Dua et al.(2002).

At first, the proposed methodology will be illustrated for the case of a bilevel linear programming problem which is of the following form:

$$\begin{aligned} \min_x F(x,y) &= c_1^T x + d_1^T y \\ s.t.\ \ & A_1 x + B_1 y \leq b_1 \\ \min_y f(x,y) &= \ c_2^T x + d_2^T y \\ s.t.\ \ & A_2 x + B_2 y \leq b_2 \end{aligned} \tag{3.2}$$

where $x \in X \subseteq R^{n1}, y \in Y \subseteq R^{n2}$, $c_1, c_2 \in R^{n1}, d_1, d_2 \in R^{n2}, b_1 \in R^j, b_2 \in R^{j'}, A_1 \in R^{j \times n1}, A_2 \in R^{j' \times n1}, B_1 \in R^{j \times n2}$, and $B_2 \in R^{j \times n2}$.

In order to obtain the explicit expression of the inducible region, consider the inner problem of the above BLPP:

$$\begin{aligned} \min_y \ \ & c_2^T x + d_2^T y \\ s.t. \ \ & A_2 x + B_2 y \leq b_2 \end{aligned} \tag{3.3}$$

(3.3) can be transformed into the following parametric programming problem by assuming variables x as parameters:

$$\begin{aligned} \min_{y} \quad & d_2^T y + c_2^T x \\ s.t. \quad & B_2 y \leq b_2 - A_2 x \end{aligned} \tag{3.4}$$

Problem **(3.4)** corresponds to the following general class of a multi-parametric Linear Programming (mp-LP) problem :

$$\begin{aligned} z(x) = \min_{y} \; & c'^T y + ct'^T x \\ s.t. \; & A'y \leq b' + F'x \end{aligned} \tag{3.5}$$

where A', F' are constant matrices and b', c', ct' are constant vectors.

By solving **(3.5)** using parametric programming techniques, a complete profile of optimal solutions of **(3.3)** can be obtained as an explicit function of x, variables of the outer problem with the corresponding valid boundary conditions, which is as follows:

$$y = \begin{cases} \xi_1(x) = m_1 + n_1 x & \text{if } H_1 x \leq h_1, \\ \xi_2(x) = m_2 + n_2 x & \text{if } H_2 x \leq h_2, \\ \quad \vdots & \quad \vdots \\ \xi_k(x) = m_k + n_k x & \text{if } H_k x \leq h_k, \end{cases} \tag{3.6}$$

where k denotes the number of computed parametric solutions, H_k is a constant matrix, h_k is a constant vector, and the corresponding valid boundary condition of the kth parametric solution, $H_k x \leq h_k$ is called a critical region, CR^k, which is defined as the feasible space of parameter where a solution remains optimal (Dua, 2000).

Using the explicit expression of the inducible region in the form of **(3.6)**, a bilevel programming problem **(3.2)** is transformed into a family of single optimization problems which are as follows:

$$
\begin{aligned}
& & z_1 &= \min_x c_1^T x + d_1^T \xi_1(x), \\
& s.t. & A_1 x + B_1 \xi_1(x) &\leq b_1, \\
& & H_1 x &\leq h_1, \\
& & z_2 &= \min_x c_2^T x + d_2^T \xi_2(x), \\
& s.t. & A_2 x + B_2 \xi_2(x) &\leq b_2, \\
& & H_2 x &\leq h_2, \\
& & \vdots & \\
& & z_k &= \min_x c_k^T x + d_k^T \xi_k(x), \\
& s.t. & A_k x + B_k \xi_k(x) &\leq b_k, \\
& & H_k x &\leq h_k.
\end{aligned} \tag{3.7}
$$

Solutions of the above single LP problem correspond to the local optima of the original BLPP because parametric programming techniques search all solution spaces of the parametric problem (Dua et al, 2002). Therefore the original bilevel programming problems are solved to global optimality by solving the above single linear programming problems.

The proposed methodology can be applied for the case of bilevel quadratic programming problems. If the inner problem has a quadratic objective function, we can formulate the problem as a multi-parametric Quadratic Programming (mp-QP) problem which corresponds to the following general class of mp-QP problem :

$$z(x) = \min_{y} c^T y + ct^T x + \frac{1}{2} y^T Q y \tag{3.8}$$
$$s.t.\ \ A'y \leq b' + F'x$$
$$x_{min} \leq x \leq x_{max}$$

where Q is an $(n \times n)$ symmetric positive definite constant matrix, A', F' are constant matrices and b' is a constant vector. Using mp-QP algorithm such as one by Dua et al.(2002), the remaining procedure is the same as the one for the linear case.

In the next section, numerical examples will be solved to illustrate the proposed methodology.

3.3 Numerical Examples

Five numerical examples will be solved. First, two bilevel linear programming problems will be solved and then examples of quadratic problems will be illustrated.

3.3.1 Example 1 : Linear-Linear case(i)

This problem is taken from Bard (1983) and Visweswaran et al.(1996):

$$\begin{aligned} & \min_x F(x,y) = x + y \\ s.t. \quad & -x \leq 0 \\ & \min_y f(x,y) = -5x - y \\ s.t. \quad & -x - 0.5y \leq -2 \\ & -0.25x + y \leq 2 \\ & x + 0.5y \leq 8 \\ & x - 2y \leq 2 \\ & -y \leq 0 \end{aligned}$$

Based on the proposed methodology, the inner problem is transformed into the following mp-LP problem:

$$\begin{aligned} & \min_y -y - 5x \\ s.t. \quad & \begin{bmatrix} -0.5 \\ 1 \\ 0.5 \\ -2 \\ -1 \end{bmatrix} y \leq \begin{bmatrix} -2 \\ 2 \\ 8 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0.25 \\ -1 \\ -1 \\ 0 \end{bmatrix} x \end{aligned} \tag{3.9}$$

The application of parametric programming techniques yields the following optimal parametric solution of **(3.9)**:

$$y = \begin{cases} 0.25x + 2 & \text{if } 0.8889 \leq x \leq 6.22 \\ -2x + 16 & \text{if } 6.22 \leq x \leq 6.8 \end{cases}$$

As a result, the leader's problem corresponds to the following two LP problems :

$$\begin{aligned} & \min_x 1.25x + 2 \\ & s.t \ 0.8889 \leq x \leq 6.22 \end{aligned} \tag{3.10}$$

$$\min_{x} -x + 16 \tag{3.11}$$
$$s.t\ 6.22 \leq x \leq 6.8$$

The optimal solution of problem **(3.10)** is 3.111 when (x,y) is (0.8889, 2.222) and the optimal solution of problem **(3.11)** is 9.2 when (x,y) is (6.8, 2.4). Therefore the global optimum is 3.111 which is the same as that in the previous studies. The computation time in solving **(3.9)** is 0.93 sec in the SUN Ultrasparc using the software implementation by Pistikopoulos et al.(1999).

3.3.2 Example 2 : Linear-Linear case (ii)

This problem is taken from Liu and Hart (1994).

$$\begin{aligned} \min_x F(x,y) &= -x - 3y \qquad (3.12)\\ s.t. \quad -x &\leq 0\\ -y &\leq 0\\ \min_y f(x,y) &= y\\ s.t. -y &\leq 0\\ -x + y &\leq 3\\ x + 2y &\leq 12\\ 4x - y &\leq 12 \end{aligned}$$

The inner problem is transformed into the following mp-LP problem:

$$\begin{aligned} &\min_y y \qquad (3.13)\\ s.t. &\begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix} y \leq \begin{bmatrix} 3 \\ 12 \\ 12 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ -4 \\ 0 \end{bmatrix} x \end{aligned}$$

The parametric solutions of problem **(3.13)** is as follows:

$$y = \begin{cases} 0 & \text{if } 0 \leq x \leq 3 \\ 4x - 12 & \text{if } 3 \leq x \leq 4 \end{cases}$$

The solution of the resulting leader's problems are summarized in Table 3.1. The global solution corresponds to -16, which is the same as that in the previous study. The computation time is 0.68 sec in the SUN Ultrasparc using the software implementation by Pistikopoulos et al.(1999).



Table 3.1: Result of Example 2

	Critical region	
Variable	CR1	CR2
x	3	4
y	0	4
Function	CR1	CR2
F	-3	-16
f	0	4

3.3.3 Example 3 : Linear-Quadratic case

Consider the following bilevel linear-quadratic programming problem :

$$\min_x F(x,y) = x + y_1 + y_2$$

$$s.t. \quad 0 \le x \le 5$$

$$x + 2y_1 - y_2 \le 10$$

$$\min_y f(x,y) = (y_1 - 1)^2 + (y_2 - 1)^2 + x$$

$$s.t. \quad -2x + y_1 + 4y_2 \le 16$$

$$2x + 3y_1 - 2y_2 \le 48$$

$$-2x + y_1 - 3y_2 \le -12$$

$$0 \le y_1 \le 4$$

$$0 \le y_2 \le 4$$

The parametric solutions of the inner problem are summarized in Table 3.2.

As a result, the leader's problem corresponds to three LP problems and their optimal solutions are summarized in Table 3.3.

The global optimum solution corresponds to 2.5 when (x, y_1, y_2) is (0, 0, 2.5). The computation time is 0.83 sec in the SUN Ultrasparc using the software implementation by Pistikopoulos et al.(1999).

Table 3.2: Parametric Solutions of the follower's problem in Example 3

No.	Critical region	Parametric solution
CR1	$0 \leq x \leq \frac{2}{3}$	$y_1 = 0$
		$y_2 = 2.5$
CR2	$\frac{2}{3} \leq x \leq 4$	$y_1 = 0.2x + 1.2$
		$y_2 = -0.6x + 4.4$
CR3	$4 \leq x \leq 5$	$y_1 = 2$
		$y_2 = 2$

Table 3.3: Result of Example 3

	Critical region		
Variable	CR1	CR2	CR3
x	0	0.666	4
y_1	0	1.333	2
y_2	2.5	4	2

Function	CR1	CR2	CR3
F	2.5	6	8
f	3.25	9.778	6

3.3.4 Example 4 : Quadratic-Linear case

$$\begin{aligned}
&\min_x F(x,y) = (x_1-1)^2 + (x_2+1)^2 && (3.14)\\
s.t. \quad & 0 \le x_1 \le 5\\
& 0 \le x2 \le 5\\
&\min_y f(x,y) = 3y_1 - y_2 - 2x_1 + x_2\\
s.t. \quad & 2y_1 + 3y_2 \le 14 + 3x_1 + x_2\\
& -y_1 + y_2 \ge 3 + x_1\\
& 2y_1 - y_2 \ge 5 + x_2\\
& 0 \le y_1 \le 30\\
& 0 \le y_2 \le 30
\end{aligned}$$

The parametric solutions of the inner problem of **(3.14)** are summarized in the following Table:

Table 3.4: Parametric Solutions of the follower's problem in Example 4

No.	Critical region	Parametric solution
CR1	$1.25x_1 + x_2 \le 3.25$	$y_1 = x_1 + x_2 + 2$
	$-\frac{5}{3} \le x_1 - x_2 \le -1$	$y_2 = 2x_1 + x_2 - 1$
CR2	$\frac{5}{3}x_1 + x_2 \le 1$	$y_1 = 0.4x_2 + 2.6$
		$y_2 = -0.2x_2 + 0.2$

The corresponding outer problems are solved and their results are summarized in Table 3.5. The global optimum is 1.16 when (x_1, x_2, y_1, y_2) is (0.6, 0, 2.6, 0.2). The computation time is 0.91 sec in the SUN Ultrasparc using the software implementation by Pistikopoulos et al. (1999).

Table 3.5: Result of Example 4

	Critical region	
Variable	CR1	CR2
x_1	0	0.6
x_2	1	0
y_1	3	2.6
y_2	0	0.2
Function	CR1	CR2
F	5	1.16
f	10	6.4

3.3.5 Example 5 : Quadratic-Quadratic case

Consider the following bilevel programming problem which has a quadratic function at the inner and outer problem :

$$\begin{aligned} \min_{x} F(x,y) &= (x-5)^2 + (2y+1)^2 \\ s.t. \quad & -x \leq 0 \\ \min_{y} f(x,y) &= (y-1)^2 + (x-1)^2 \\ s.t. \quad & -3x + y \leq -3 \\ & x - 0.5y \leq 4 \\ & x + y \leq 7 \\ & -y \leq 0 \end{aligned} \tag{3.15}$$

The parametric solutions of the inner problem by the proposed methodology are as follows:

$$y = \begin{cases} 3x - 3 & \text{if } 1 \leq x \leq \frac{5}{3} \\ 2 & \text{if } \frac{5}{3} \leq x \leq 5 \end{cases}$$

As a result, the corresponding leader's problem is formulated as the following two single optimization problems:

$$\min_x (x-5)^2 + (6x-5)^2 \qquad (3.16)$$
$$s.t. \quad 1 \leq x \leq \frac{5}{3}$$

$$\min_x (x-5)^2 + 25 \qquad (3.17)$$
$$s.t. \quad \frac{5}{3} \leq x \leq 5$$

The optimal solutions of **(3.16)** and **(3.17)** are 17 when (x,y) is (1,0) and 25 when (x,y) is (5,2) respectively. Therefore the global optimum corresponds to 17 when (x,y) is (1,0). The computational time is 0.85 sec in the SUN Ultrasparc using the software implementation by Pistikopoulos et al. (1999).

3.3.6 Remarks

From the results of the above examples, a number of important observations and remarks can be made.

First, it would be necessary to evaluate the performance of the proposed method to others. Visweswaran et al. (1996) provided numbers of LPs in solving bilevel example problem 1 using their methodology. The global optimal solution is obtained by solving seven LP problems over two iterations based on their research. The proposed methodology computes the same solution using four LP problems over two steps (two for solving the inner mp-LP problem and two for corresponding outer problems). It might be said that the proposed methodology is more efficient than the previous methods because it requires the smaller number of LPs. More research seems to be continued to evaluate the efficiency of the proposed methodology.

Computational statistics for all examples are summarized in Table 3.6.

Table 3.6: Computational Statistics of Examples

	number of LPs	number of QPs
Example 1(LP-LP)	4	
Example 2(LP-LP)	4	
Example 3(LP-QP)	3	3
Example 4(QP-LP)	2	2
Example 5(QP-QP)		3

Second, the proposed methodology does not require additional variables such as Lagrange multipliers and additional constraints (Visweswaran et al.,1996).

Third, the proposed methodology can be applied to computing the solution of other classes of bilevel programming problems such as bilevel programming problems under uncertainty which will be addressed in the next chapter, bilevel programming problem with discrete variables, or multi-level programming problems.

3.4 Conclusion

This chapter has presented a novel global optimization methodology for bilevel linear and quadratic programming problems. By transforming the original bilevel problem into a family of single optimization problems using parametric programming techniques, the problem can be solved to global optimality for linear-linear, linear-quadratic, quadratic-linear, and quadratic-quadratic cases. A number of numerical examples have been solved to demonstrate the efficiency of the proposed methodology.

Chapter 4

A Bilevel Programming Framework for Enterprise-wide Process Network Planning under Uncertainty

A bilevel programming framework is presented to address enterprise-wide supply chain optimization problems considering uncertainty. We first describe how such problems can be modeled as bilevel programming problems and then present an effective solution strategy based on parametric programming techniques.

4.1 Introduction

Supply chains typically involve multiple enterprise-wide activities, from the procurement of the raw materials, through a series of process operations, to the distribution of end-products to customers. It is not surprising that their design and operation issues pose a number of important theoretical, technical and practical challenges, which have started to receive increasing attention in academia and industry. However little attention has been given to actual supply chain principles, particularly (i) hierarchical decision structures from local, independent to global, centralized objectives, which are often conflicting each other, and (ii) incomplete data and in-

formation subject to uncertainty involved in characteristics at the various levels of the hierarchy i.g. demand forecasts, raw material availabilities, etc.

In order to bridge the gap between the industrial practices and the lack of corresponding research, we propose an approach that directly captures their multilevel and uncertainty aspects based on bilevel optimization principles. The solutions of the resulting stochastic bilevel programming problems are obtained by proposing an effective solution strategy based on parametric programming techniques.

4.2 Supply Chain Planning - A Bilevel Optimization Model

Because supply chain elements are hierarchically connected, their activities affect and are affected by others. It is quite natural to stress that these activities should be incorporated in the management of enterprise-wide process networks. The incorporation of various elements should follow the practice that elements in a supply chain generally have their own objectives which mutually conflict each other. For instance, raw material suppliers want to maximize the consumption of raw materials in plants, while the plants themselves want to minimize this within the limits of meeting orders; plants want to maximize the amounts of products delivered to warehouses, while warehouses want to control the amounts in order to prepare for future variations. Therefore, this indifference in decision-making principles of individual elements should be taken into account in the supply chain management. In view of multiple enterprise activities in actual supply chains, their planning problems can be naturally posed as bilevel optimization models.

As described in the previous chapter, bilevel programming problems refer to hierarchical optimization problems that are constrained by another optimization problem. It is often used to describe situations involving several indifferent groups which are inter-connected in a hierarchical structure. Each group may correspond to an individual or an agency, often with a corresponding independent objective. The two problems are inter-connected: the outer problem sets parameters influencing the inner problem; the outer problem, in turn, is affected by the outcome of the inner problem.

The proposed bilevel programming framework will be illustrated for the following supply chain planning based on the notation in Table 4.1.

Consider a manufacturing enterprise-wide process network consists of a production part involving L plants $(1, \cdots, L)$, and a distribution part, involving W inventory warehouse$(1, \cdots, W)$ for products I $(1, \cdots, I)$.

Table 4.1: Notation for Supply Chain Planning Model

Indices	
i	product $(1, \cdots, I)$
l	plant $(1, \cdots, L)$
w	warehouse $(1, \cdots, W)$
r	market $(1, \cdots, R)$
Parameters	
M_{ri}	demand of product i at market r
α_{li}	capacity coefficient of product i at plant l
β_{li}	resource coefficient of product i at plant l
γ_{wi}	resource coefficient of product i at warehouse w
tr_{wri}	transportation cost of product i from warehouse w to r
R_w	Available capacity at inventory w
a_{li}	production cost coefficient for product i at plant l
b_{lwi}	transportational cost coefficient for product i from plant l to warehouse w
Variables	
$PCOST$	cost involved in a production part
$DCOST$	cost involved in a distribution part
Y_{lwi}	production amount of i at plant l for warehouse w
X_{wri}	inventory of product i at warehouse w for market r

Individual production and distribution model of the enterprise can be mathematically modelled respectively as follows:

4.2.1 A Production Model

A production part of supply chains is typically subject to the following constraints:

- Production amount of all plants should meet the level required by their demand which is various warehouses:

$$\sum_{l=1}^{L} Y_{lwi} \geq \sum_{r=1}^{R} X_{wri} \quad \forall w, i \tag{4.1}$$

- Operations of individual plants can be limited by own plant capacities:

$$\sum_{i=1}^{I} \sum_{w=1}^{W} \alpha_{li} Y_{lwi} \leq P_l \quad \forall l \tag{4.2}$$

- Commonly used resources may be shared by all plants:

$$\sum_{l=1}^{L} \sum_{w=1}^{W} \beta_{li} Y_{lwi} \leq Q_i \quad \forall i \tag{4.3}$$

An operating objective of production parts is to minimize their costs, which typically consists of its manufacturing cost and distribution cost between plants and warehouses:

$$\min PCost = \sum_{l=1}^{L} \sum_{w=1}^{W} \sum_{i=1}^{I} a_{li} Y_{lwi} + \sum_{l=1}^{L} \sum_{w=1}^{W} \sum_{i=1}^{I} b_{lwi} Y_{lwi} \tag{4.4}$$

Operations of production parts can thus be formulated as the following mathematical programming problem:

$$\begin{aligned} \min PCost = & \sum_{l=1}^{L} \sum_{w=1}^{W} \sum_{i=1}^{I} a_{li} Y_{lwi} + \sum_{l=1}^{L} \sum_{w=1}^{W} \sum_{i=1}^{I} b_{lwi} Y_{lwi} \\ s.t. & \sum_{l=1}^{L} Y_{lwi} \geq \sum_{r=1}^{R} X_{wri} \quad \forall w, i \\ & \sum_{i=1}^{I} \sum_{w=1}^{W} \alpha_{li} Y_{lwi} \leq P_l \quad \forall l \\ & \sum_{l=1}^{L} \sum_{w=1}^{W} \beta_{li} Y_{lwi} \leq Q_i \quad \forall i \end{aligned} \tag{4.5}$$

4.2.2 A Distribution Model

A distribution part is typically subject to the following constraints:

- Sums of individual warehouses' holding should meet demands in markets:

$$\sum_{w=1}^{W} X_{wri} \geq M_{ri} \quad \forall r, i \tag{4.6}$$

- Each inventory warehouse has its own limited capacity:

$$\sum_{r=1}^{R} \sum_{i=1}^{I} \gamma_{wi} X_{wri} \leq R_w \quad \forall w \tag{4.7}$$

The following indicates an objective function of warehouses:

$$\min \ DCost = \sum_{w=1}^{W} \sum_{i=1}^{I} h_{wi} X_{wri} + \sum_{w=1}^{W} \sum_{r=1}^{R} \sum_{i=1}^{I} tr_{wri} X_{wri}, \tag{4.8}$$

where the first term denotes inventory holding cost including material handling cost at warehouses and the second indicates transportation cost from warehouses to markets.

Operations of inventory parts can thus be formulated as the following mathematical programming problem:

$$\begin{aligned} \min \ DCost = & \sum_{w=1}^{W} \sum_{i=1}^{I} h_{wi} X_{wri} + \sum_{w=1}^{W} \sum_{r=1}^{R} \sum_{i=1}^{I} tr_{wri} X_{wri} \\ s.t. & \sum_{w=1}^{W} X_{wri} \geq M_{ri} \quad \forall r, i \\ & \sum_{r=1}^{R} \sum_{i=1}^{I} \gamma_{wi} X_{wri} \leq R_w \quad \forall w \end{aligned} \tag{4.9}$$

Note that the decisions of the distribution part are based on those of the production part: for example, inventory policies are made using the outcome of production decisions. Similarly, decisions on the production part are affected by parameters which are decided by the distribution part: for example, production levels are decided from given information regarding the inventory conditions. Therefore the

overall supply chain planning model can be posed as the following bilevel optimization problem:

$$
\begin{aligned}
\min \ DCost = & \sum_{w=1}^{W}\sum_{i=1}^{I} h_{wi}X_{wri} + \sum_{w=1}^{W}\sum_{r=1}^{R}\sum_{i=1}^{I} tr_{wri}X_{wri} \\
s.t. & \sum_{w=1}^{W} X_{wri} \geq M_{ri} \quad \forall r,i \\
& \sum_{r=1}^{R}\sum_{i=1}^{I} \gamma_{wi}X_{wri} \leq R_w \quad \forall w \\
\min PCost = & \sum_{l=1}^{L}\sum_{w=1}^{W}\sum_{i=1}^{I} a_{li}Y_{lwi} + \sum_{l=1}^{L}\sum_{w=1}^{W}\sum_{i=1}^{I} b_{lwi}Y_{lwi} \\
s.t. & \sum_{l=1}^{L} Y_{lwi} \geq \sum_{r=1}^{R} X_{wri} \quad \forall w,i \\
& \sum_{i=1}^{I}\sum_{w=1}^{W} \alpha_{li}Y_{lwi} \leq P_l \quad \forall l \\
& \sum_{l=1}^{L}\sum_{w=1}^{W} \beta_{li}Y_{lwi} \leq Q_i \quad \forall i
\end{aligned}
$$

where the inner problem corresponds to the production optimization problem and the outer problem to the distribution optimization problem.

4.3 A Solution Methodology for Bilevel Programming Problems under Uncertainty

By denoting X_{wri} as x, Y_{lwi} as y, **(4.2.2)** may be recast as the following bilevel programming problem:

$$
\begin{aligned}
\min_{x} F(x,y) &= c_1^T x + d_1^T y \\
s.t. \quad & A_1 x + B_1 y \leq b_1 \\
\min_{y} f(x,y) &= c_2^T x + d_2^T y \\
s.t. \quad & A_2 x + B_2 y \leq b_2
\end{aligned}
\tag{4.10}
$$

where $x \in X \subseteq R^{n1}, y \in Y \subseteq R^{n2}$, $c_1, c_2 \in R^{n1}, d_1, d_2 \in R^{n2}, b_1 \in R^{j}, b_2 \in R^{j'}, A_1 \in R^{j \times n1}, A_2 \in R^{j' \times n1}, B_1 \in R^{j \times n2}$,and $B_2 \in R^{j' \times n2}$.

The solution of **(4.10)** can be computed using the methodology described in 3.2.

When uncertainty widely existing in actual supply chains, such as in demand forecast, equipment availability etc., is incorporated and denoted as θ, **(4.2.2)** may be recast as the following bilevel programming problem under uncertainty:

$$\begin{aligned} \min_{x} F(x,y) &= c_1^T x + d_1^T y + ct_1^T \theta \\ s.t. \quad & A_1 x + B_1 y \leq b_1 + K_1 \theta \\ \min_{y} f(x,y) = \quad & c_2^T x + d_2^T y + ct_2^T \theta \\ s.t. \quad & A_2 x + B_2 y \leq b_2 + K_2 \theta \end{aligned} \tag{4.11}$$

where $\theta \in \Theta \subset R^m, ct_1, ct_2 \in R^m, K_1 \in R^{i \times m}$ and $K_2 \in R^{i' \times m}$.

For the solution of **(4.11)**, a novel solution methodology consisting of the following three steps are proposed based on the metholodgy described for deterministic bilevel programming problems in 3.2.

Step 1 Formulate the inner optimization problem as a multi-parametric linear programming (mp-LP) problem by regarding variables of the outer problem as parameters:

$$\begin{aligned} \min_{y} \quad & d_2^T y + (c_2^T \quad ct_2^T) \begin{pmatrix} x \\ \theta \end{pmatrix} \\ s.t. \quad & B_2 y \leq b_2 + (-A_2 \quad K_2) \begin{pmatrix} x \\ \theta \end{pmatrix} \\ & x^L \leq x \leq x^U \\ & \theta^L \leq \theta \leq \theta^U \end{aligned} \tag{4.12}$$

which can be transformed into the following general class of multi-parametric linear programming problem:

$$\begin{aligned} &\min_X c^T X + ct^T \theta \\ &s.t.\ \ A'X \leq b' + F'\theta \\ &\theta_{min} \leq \theta \leq \theta_{max} \end{aligned} \tag{4.13}$$

where A', F' are constant matrices and c, ct, b' are constant vectors.

Step 2. Solve problem **(4.12)** using multi-parametric LP algorithm such as one suggested by Dua (2000), the corresponding parametric solution may take the following form:

$$y = \begin{cases} \xi_1(x,\theta) = l_1 + m_1 x + n_1 \theta & \text{if } H_1 x \leq h_1 + I_1 \theta, \\ \xi_2(x,\theta) = l_2 + m_2 x + n_2 \theta & \text{if } H_2 x \leq h_2 + I_2 \theta, \\ \quad \vdots & \quad \vdots \\ \xi_k(x,\theta) = l_k + m_k x + n_k \theta & \text{if } H_k x \leq h_k + I_k \theta, \end{cases} \tag{4.14}$$

where k denotes the number of the computed parametric solutions, l_k is a constant parameter, m_k and n_k are constant vectors. H_k, I_k are constant matrices and h_k is constant vector. $H_k x \leq h_k + I_k \theta$ is called a critical region, CR^k which is the corresponding valid boundary condition of the kth parametric solution (Dua, 2000).

Step 3. Using **(4.14)**, the outer problem may be transformed into a number of single parametric optimization problems as follows:

$$
\begin{aligned}
z_1 &= \min_x c_1^T x + d_1^T \xi_1(x,\theta) \\
s.t.\ \ A_1 x + B_1 \xi_1(x,\theta) &\le b_1 + K_1 \theta \\
H_1 x \quad &\le h_1 + I_1 \theta \\
\\
z_2 &= \min_x c_2^T x + d_2^T \xi_2(x,\theta) \\
s.t.\ \ A_2 x + B_2 \xi_2(x,\theta) &\le b_2 + K_2 \theta \\
H_2 x \quad &\le h_2 + I_2 \theta \\
&\vdots \\
z_k &= \min_x c_k^T x + d_k^T \xi_k(x,\theta) \\
s.t.\ \ A_k x + B_k \xi_k(x,\theta) &\le b_k + K_k \theta \\
H_k x \quad &\le h_k + I_k \theta
\end{aligned}
\tag{4.15}
$$

Each of above problems corresponds to a multi-parametric linear programming problem. By solving those single problems, all local optimal solutions of the original BLPP are obtained. Consequently the global optimum is determined because parametric programming techniques search the whole solution space.

We can also solve other types of bilevel programming problems, such as one whose inner problem takes the quadratic form as **(4.16)** by formulating it as the multi-parametric Quadratic Programming (mp-QP) problem (Dua et al., 2002) :

$$
\begin{aligned}
&\min_X c^T X + \frac{1}{2} X^T Q X + ct^T \theta \\
&s.t.\ \ A'X \le b' + F'\theta \\
&\theta_{min} \le \theta \le \theta_{max}
\end{aligned}
\tag{4.16}
$$

where Q is an $(n \times n)$ symmetric positive definite constant matrix.

In the next section, the proposed methodology will be demonstrated in a numerical example.

4.4 Numerical Example

Consider a manufacturing enterprise involving three plants with two warehouses, two markets and two products, A and B. Refer to Figure 4.1 which shows the configuration of this enterprise.

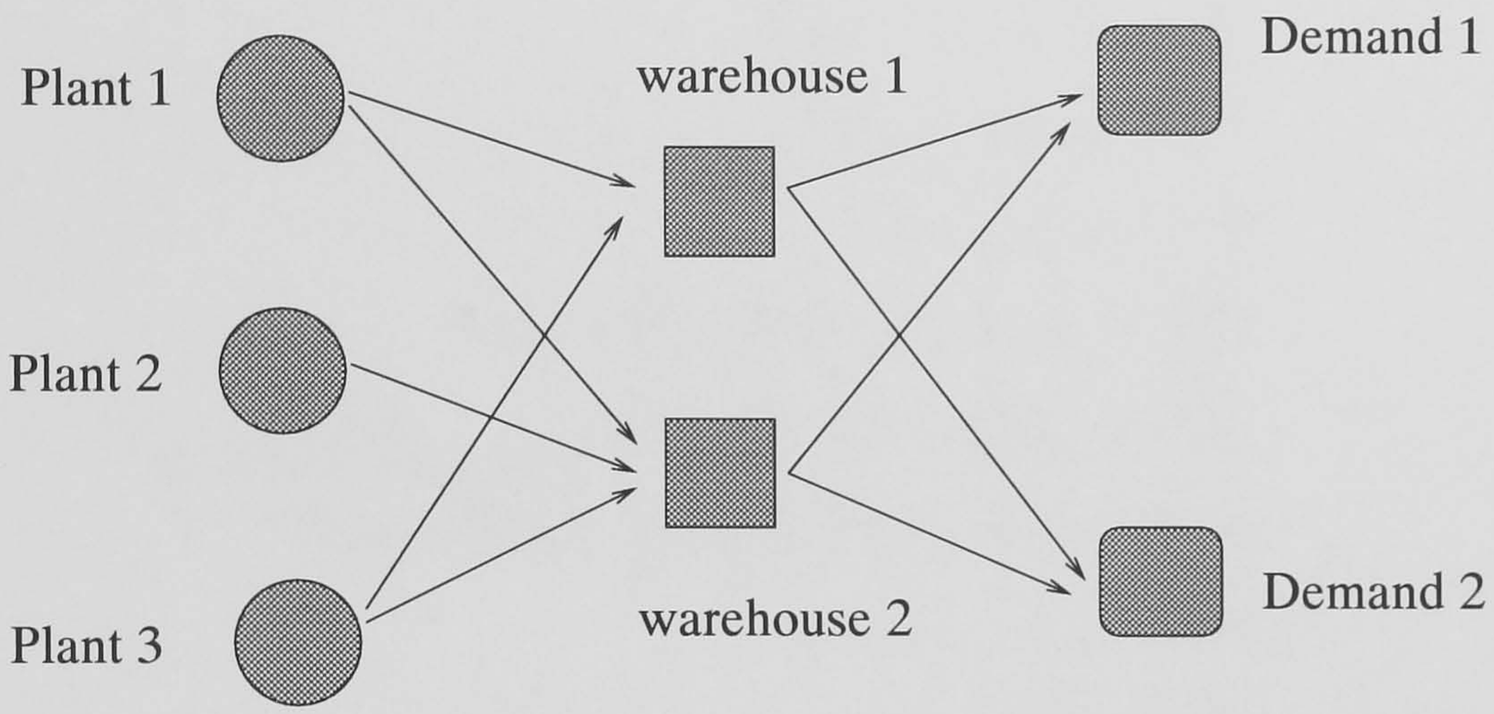


Figure 4.1: Process Configuration of the Enterprise in Example 2

While amount of product B at demand 1 and product A in demand 2 are assumed to be known as 170 and 100 tons respectively, amount of product A at demand 1 (θ_1) and amount of product B at demand 2 (θ_2) are assumed to be uncertain as follows:

$$50\ tons \leq \theta_1 \leq 80\ tons,\ \ 50\ tons \leq \theta_2 \leq 80\ tons$$

The following numerical problem can be formulated based on the proposed model.

$$\min_{X_{1A},X_{1B},X_{2A},X_{2B}} 10X_{1A} + 7X_{1B} + 8X_{2A} + 9X_{2B}$$
$$+Y_{1A} + 3Y_{1B} + 2Y_{2A} + Y_{2B} + 2Y_{3A} + 4Y_{3B}$$
$$s.t.\ X_{1A} + X_{2A} \geq \theta_1 + 100$$
$$X_{1B} + X_{2B} \geq \theta_2 + 170$$
$$3X_{1A} + 2X_{1B} \leq 800$$
$$4X_{2A} + X_{2B} \leq 700$$
$$4X_{1A} + X_{1B} + 2X_{2A} + 5X_{2B} \leq 1500$$
$$\min_{Y_{1A},Y_{1B},Y_{2A},Y_{2B},Y_{3A},Y_{4A}} 3X_{1A} + 2X_{1B} + 3X_{2A} + 5X_{2B}$$
$$+7Y_{1A} + 3Y_{1B} + 10Y_{2A} + 6Y_{2B} + 5Y_{3A} + 6Y_{3B}$$
$$s.t.\ 2Y_{1A} + Y_{1B} \leq 300$$
$$Y_{2A} + 5Y_{2B} \leq 250$$
$$3Y_{3A} + 2Y_{3B} \leq 400$$
$$Y_{1A} + Y_{2A} + Y_{3A} \geq X_{1A} + X_{2A}$$
$$Y_{1B} + Y_{2B} + Y_{3B} \geq X_{1B} + X_{2B}$$
$$Y_{1A} + 3Y_{1B} + 4Y_{2A} + 2Y_{2B} + Y_{3A} + 3Y_{3B} \leq 1000$$
$$50 \leq \theta_1, \theta_2 \leq 80$$
$$0 \leq X_{1A}, X_{1B}, X_{2A}, X_{2B} \leq 300$$

The solutions of the above supply chain problem is described in the following Table in terms of warehouse decision variables and uncertain parameters.

Table 4.2: Parametric Solutions of Example using warehouse variables and uncertain parameters

CR	Critical region	Operation plan	
		Plant	Warehouse
CR1	$2X_{1A} + X_{1B} + 2X_{2A} + X_{2B} < 566.7$ $2\theta_1 + \theta_2 \leq 196.7$	$Y_{1A} = X_{1A} + X_{2A} - 133.3$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = Y_{2B} = Y_{3B} = 0$ $Y_{3A} = 133.3$	$X_{1A} = X_{2B} = 0$ $X_{1B} = \theta_2 + 170$ $X_{2A} = \theta_1 + 100$
CR2	$X_{1A} + 1.1X_{1B} + X_{2A} + 1.1X_{2B} \leq 462.5$ $-2X_{1A} - X_{1B} - 2X_{2A} - X_{2B} \leq -566.7$ $\theta_1 \geq 75$	$Y_{1A} = -0.5X_{1B} - 0.5X_{2B} + 150$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = X_{1A} + 0.5X_{1B} + X_{2A} + 0.5X_{2B} - 283.3$ $Y_{2B} = Y_{3B} = 0, Y_{3A} = 133.3$	$X_{1A} = \theta_1 - 75$ $X_{1B} = \theta_2 + 170$ $X_{2A} = 175$ $X_{2B} = 0$
CR3	$X_{1A} + 1.1X_{1B} + X_{2A} + 1.1X_{2B} \leq 462.5$ $-2X_{1A} - X_{1B} - 2X_{2A} - X_{2B} \leq -566.7$ $2\theta_1 + \theta_2 \leq 196.7$	$Y_{1A} = -0.5X_{1B} - 0.5X_{2B} + 150$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = X_{1A} + 0.5X_{1B} + X_{2A} + 0.5X_{2B} - 283.3$ $Y_{2B} = Y_{3B} = 0, Y_{3A} = 133.3$	$X_{1A} = 0$ $X_{1B} = \theta_2 + 170$ $X_{2A} = -0.5\theta_2 + 198.3$ $X_{2B} = 0$
CR4	$X_{1A} + 1.1X_{1B} + X_{2A} + 1.1X_{2B} \leq 462.5$ $-2X_{1A} - X_{1B} - 2X_{2A} - X_{2B} \leq -566.7$ $-2\theta_1 - \theta_2 \leq -196.7$ $\theta_1 \leq 75$	$Y_{1A} = -0.5X_{1B} - 0.5X_{2B} + 150$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = X_{1A} + 0.5X_{1B} + X_{2A} + 0.5X_{2B} - 283.3$ $Y_{2B} = Y_{3B} = 0, Y_{3A} = 133.3$	$X_{1A} = 0$ $X_{1B} = \theta_2 + 170$ $X_{2A} = \theta_1 + 100$ $X_{2B} = 0$
CR5	$-X_{1A} - 1.1X_{1B} - X_{2A} - 1.1X_{2B} \leq -462.5$ $X_{1A} + X_{1B} + X_{2A} + X_{2B} \leq 471.1$ $\theta_1 \leq 75$ $-X_{1A} + 2X_{1B} - X_{2A} + 2X_{2B} \leq 433.3$	$Y_{1A} = 0.8X_{1A} + 0.4X_{1B} + 0.8X_{2A} + 0.4X_{2B} - 220$ $Y_{1B} = -1.6X_{1A} - 0.8X_{1B} - 1.6X_{2A} - 0.8X_{2B} + 740$ $Y_{2A} = 0.2X_{1A} - 0.4X_{1B} + 0.2X_{2A} - 0.4X_{2B} + 86.7$ $Y_{2B} = 1.6X_{1A} + 1.8X_{1B} + 1.6X_{2A} + 1.8X_{2B} - 740$ $Y_{3A} = 133.3, Y_{3B} = 0$	$X_{1A} = 0$ $X_{1B} = -0.9\theta_1 + 322.2$ $X_{2A} = \theta_1 + 100$ $X_{2B} = 0$
CR6	$-X_{1A} - 1.1X_{1B} - X_{2A} - 1.1X_{2B} \leq -462.5$ $X_{1A} + X_{1B} + X_{2A} + X_{2B} \leq 471.1$ $-X_{1A} + 2X_{1B} - X_{2A} + 2X_{2B} \leq 433.3$ $-\theta_1 \leq -75$	$Y_{1A} = 0.8X_{1A} + 0.4X_{1B} + 0.8X_{2A} + 0.4X_{2B} - 220$ $Y_{1B} = -1.6X_{1A} - 0.8X_{1B} - 1.6X_{2A} - 0.8X_{2B} + 740$ $Y_{2A} = 0.2X_{1A} - 0.4X_{1B} + 0.2X_{2A} - 0.4X_{2B} + 86.7$ $Y_{2B} = 1.6X_{1A} + 1.8X_{1B} + 1.6X_{2A} + 1.8X_{2B} - 740$ $Y_{3A} = 133.3, Y_{3B} = 0$	$X_{1A} = \theta_1 - 75$ $X_{1B} = -0.9\theta_1 + 322.2$ $X_{2A} = 175$ $X_{2B} = 0$
CR7	$X_{1A} - 2X_{1B} + X_{2A} - 2X_{2B} \leq -433.3$ $X_{1A} + 3X_{1B} + X_{2A} + 3X_{2B} \leq 1050$ $\theta_1 \leq 60$	$Y_{1A} = X_{1A} + X_{2A} - 133.3$ $Y_{1B} = -X_{1A} - 2X_{1B} - X_{2A} - 2X_{2B} + 1000$ $Y_{2A} = Y_{3B} = 0$ $Y_{2B} = X_{1A} + 3X_{1B} + X_{2A} + 3X_{2B} - 1000$ $X_{3A} = 133.3$	$X_{1A} = X_{2B} = 0$ $X_{1B} = 0.5\theta_1 + 266.7$ $X_{2A} = \theta_1 + 100$

Table 4.2 can be summarized into the following Table 4.3 by describing the critical regions in terms of uncertain parameters.

Table 4.3: Parametric Solutions of Example using uncertain parameters

CR	Critical region	objective function
CR1	$2\theta_1 + \theta_2 \leq 196.7$	$9\theta_1 + 10\theta_2 + 2450$
CR2	$75 \leq \theta_1 \leq 80$	$12\theta_1 + 10.5\theta_2 + 2485$
CR3	$2\theta_1 + \theta_2 \leq 196.7$	$5.5\theta_2 + 3615$
CR4	$-2\theta_1 - \theta_2 \leq -196.7$ $\theta_1 \leq 75$	$10\theta_1 + 10.5\theta_2 + 2635$
CR5	$50 \leq \theta_1 \leq 75$	$0.6\theta_1 + 4233.2$
CR6	$75 \leq \theta_1 \leq 80$	$2.6\theta_1 + 4083.2$
CR7	$50 \leq \theta_1 \leq 60$	$9\theta_1 + 3900.1$

According to Table 4.3, multiple operation decisions are obtained for the same uncertain parameter range. For example, two different operation decisions with outer objective function values of $12\theta_1+10.5\theta_2+2485$ and $2.6\theta_1+4083.2$ are obtained for the range of $75 \leq \theta_1 \leq 80$. In order to compute minimum of the two, the following mp-LP problem can be formulated:

$$\min_{\theta_1,\theta_2} F$$

$$s.t. \quad F \leq 12\theta_1 + 10.5\theta_2 + 2485$$

$$F \leq 2.6\theta_1 + 4083.2$$

$$75 \leq \theta_1 \leq 80$$

$$50 \leq \theta_2 \leq 80$$

Similar problems are formulated for whole parametric ranges and their optimal solutions are summarized in Table 4.4 and graphically shown in Figure 4.2.

Table 4.4: Final Parametric Solutions of Example

FCR	Critical region	Plant	Warehouse	Objective function
FCR1	$2\theta_1 + \theta_2 \leq 196.7$ $50 \leq \theta_1 \leq 80$ $50 \leq \theta_2 \leq 80$	$Y_{1A} = X_{1A} + X_{2A} - 133.3$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = Y_{2B} = Y_{3B} = 0$ $Y_{3A} = 133.3$	$X_{1A} = X_{2B} = 0$ $X_{1B} = \theta_2 + 170$ $X_{2A} = \theta_1 + 100$	$9\theta_1 + 10\theta_2 + 2450$
FCR2	$75 \leq \theta_1 \leq 80$ $50 \leq \theta_2 \leq 80$	$Y_{1A} = -0.5X_{1B} - 0.5X_{2B} + 150$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = X_{1A} + 0.5X_{1B}$ $+X_{2A} + 0.5X_{2B} - 283.3$ $Y_{2B} = Y_{3B} = 0, Y_{3A} = 133.3$	$X_{1A} = \theta_1 - 75$ $X_{1B} = \theta_2 + 170$ $X_{2A} = 175$ $X_{2B} = 0$	$12\theta_1 + 10.5\theta_2 + 2485$
FCR3	$-2\theta_1 - \theta_2 \leq -196.7$ $50 \leq \theta_1 \leq 75$	$Y_{1A} = -0.5X_{1B} - 0.5X_{2B} + 150$ $Y_{1B} = X_{1B} + X_{2B}$ $Y_{2A} = X_{1A} + 0.5X_{1B}$ $+X_{2A} + 0.5X_{2B} - 283.3$ $Y_{2B} = Y_{3B} = 0, Y_{3A} = 133.3$	$X_{1A} = 0$ $X_{1B} = \theta_2 + 170$ $X_{2A} = \theta_1 + 100$ $X_{2B} = 0$	$10\theta_1 + 10.5\theta_2 + 2635$

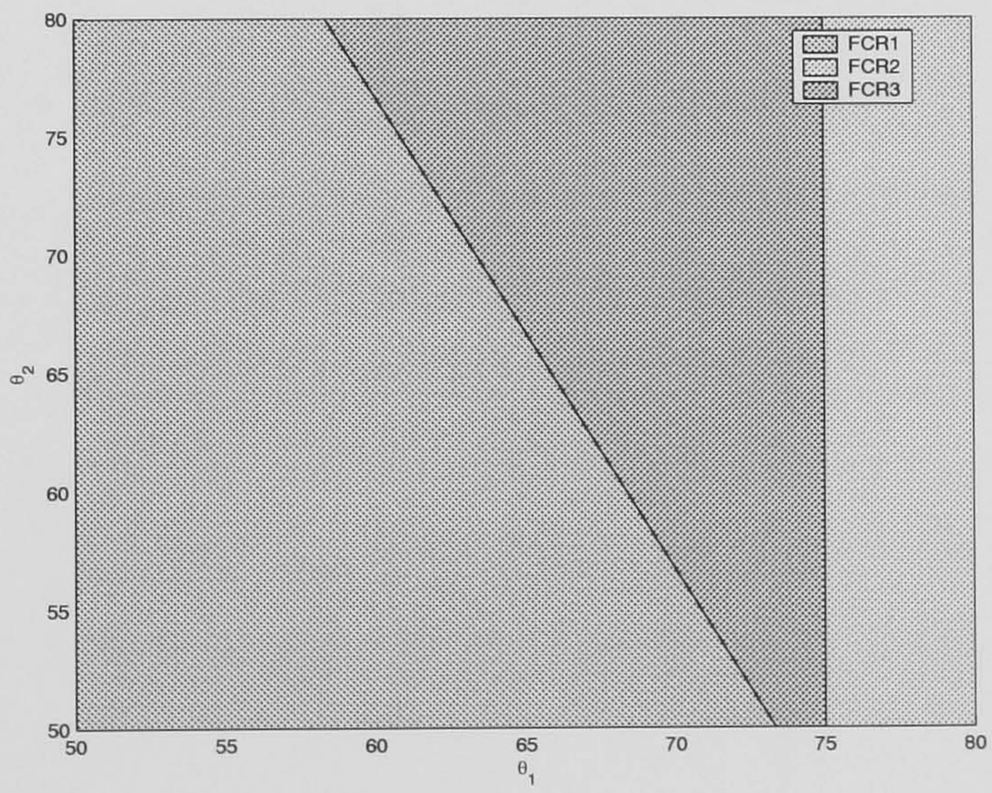


Figure 4.2: Graphical Representation of Final Parametric Solutions

From the result described in the Table and Figure, optimal operation strategies under varying demand are obtained. This can provide an important insight on the supply chain operation under uncertainty.

4.5 Concluding remarks

This chapter has presented a bilevel programming framework to capture conflicting interests of involving supply chain elements in the context of supply chain planning problems. Solutions of the corresponding bilevel programming problems under uncertainty have been computed by proposing a methodology based on parametric optimization techniques.

Chapter 5

Short-term Operation Scheduling : A Proactive Scheduling Approach

Previous chapters were concerned with how to incorporate multiple supply chain activities under uncertainty. In order to design and operate overall supply chains, the importance of individual processes should not be neglected. This chapter therefore presents a novel methodology to solve short-term operation planning problems under uncertainty. The uncertainty present in process parameters such as processing times and equipment availabilities is incorporated into scheduling models, which are then transformed into multi-parametric Mixed Integer Linear Programming (mp-MILP) problems. A solution procedure based upon recently proposed state-of-the-art mp-MILP algorithms is then discussed. The proposed methodology contributes to the construction of a *proactive* operation framework under uncertainty.

5.1 Introduction

Considerable effort has been made during the last two decades in the area of scheduling of chemical processes. The effort is motivated by the significant impact of scheduling upon on-time delivery of products as well as efficient utilization of resources. It is therefore a subject that has been extensively studied in the process systems engineering community (see, for example, reviews by Reklaitis, 1991; Pantelides, 1994; Shah, 1998).

Most previous research has focused on the "deterministic" scheduling problem, i.e. the case where process-, model- or market- related parameters are assumed to be known and fixed. The resulting schedule is an off-line deterministic solution because it is based on fixed parameters which are disconnected from the actual dynamic condition. In reality, variations typically exist such as fluctuations in the quality of raw materials, uncertainty in product demands, equipment break-downs, etc. In the presence of such real uncertainty, any "optimal" deterministic schedule may not be robust or even feasible.

One way to address uncertainty in (typically on-line) scheduling applications is reactive scheduling. The main idea of the reactive scheduling is to repeatedly solve deterministic scheduling problems whenever a variation occurs: new schedules are computed and implemented based upon newly realized parameters (for example, from on-line measurements). Studies on the reactive scheduling in the literature can be mainly summarized in the following two ways:

First, most studies focus on how to minimize the effect of disturbances after their occurrences. Little attention has been given to the more positive, *'proactive'* approach such as predicting new optimal schedules in response to potential variations. Ishii and Muraki (1997) noticed the importance of predicting the process state in the reactive scheduling but their work leaves unanswered the question of how the prediction can be made and realized in the framework of the reactive scheduling system.

Second, the computational issue has been the major obstacle to improving the performance of reactive scheduling. Because of needs for constant re-computations, reactive scheduling approaches may become computationally expensive. Thus, some of the previous studies have attempted to accelerate the computational performance of solving the underlying deterministic scheduling problem, for instance, by shifting the starting time of the new schedule (Cott and Macchietto, 1988), limiting the search area of scheduling optimization (Kanakamedala *et al.*,1994), relaxing the constraints (Vin and Ierapetritou, 2000) or resolving the scheduling problem in a hierarchical way (Schilling and Pantelides, 1997). Considering the scale of conventional scheduling problems, the heavy computation issue still poses a new challenge for reactive scheduling.

From the above discussion of the previous studies, it can be concluded that there is a need for an alternative, proactive approach which would provide optimal reactive schedules in a cheap and fast way.

Recently, Pistikopoulos and co-workers in Imperial College London presented a series of parametric programming techniques and softwares. For example, Dua and Pistikopoulos (2000) describe a state-of-the-art solution procedure for a Mixed Integer Linear Programming (MILP) Problem that includes binary variables and multiple right-hand side parameters that are bounded within pre-specified intervals. The techniques have been applied to a variety of applications for the process systems engineering problems under uncertainty. In this chapter, the scheduling problem under uncertainty will be revisited using the techniques.

The rest of this chapter is organized as follows :

Key issues of the scheduling problem are briefly addressed. Then the scheduling problem under uncertainty is formulated as a parametric programming problem; a solution procedure based on the recently proposed state-of-the-art parametric programming techniques is then discussed. Two types of scheduling problems under uncertainty: (i) a process in Unlimited Intermediate Storage (UIS) policy, (ii) a process in Zero-Wait (ZW) policy are presented to illustrate the potential of the proposed methodology.

5.2 Scheduling of Processes with UIS policy considering Uncertainty

In this section, we will present the key idea of using parametric programming techniques to address scheduling problems under uncertainty. A multiproduct batch scheduling model with unlimited intermediate storage (UIS) policy (Ryu and Lee, 1997) will be employed and uncertainty in processing time and equipment availability, which are the most frequently and widely recognized types, will be considered.

5.2.1 A UIS Scheduling Model

A scheduling model generally involves two types of constraints, sequencing and assignment. Sequencing constraints typically denote which products are produced in different time instances (slots) and in what sequence, while assignment constraints normally determine completion times of various products at different stages based on the selected sequence.

Based on the notation in Table 5.1, the deterministic scheduling model employed in this study corresponds to the following MILP problem:

Table 5.1: Notation for UIS Scheduling Model

Indices	
i	time slot $(1, \cdots, N)$
k	product $(1, \cdots, N)$
j	stage $(1, \cdots, M)$
Parameters	
n_j	number of units in stage j
P_{kj}	processing time of product k in stage j
$\theta 1_{kj}$	uncertain processing time of product k in stage j
$\theta 2_{ij}$	uncertain delay in availability of ith time slot in stage j
$\theta 1^L_{kj}$, $\theta 2^L_{ij}$	lower bound
$\theta 1^U_{kj}$, $\theta 2^U_{ij}$	upper bound
Variables	
C_{ij}	completion time of product in ith time slot at stage j
y_{ik}	1 if product k is made at ith time slot, otherwise 0
w_{ikj}	auxiliary variable for bilinear term $y_{i,k}\theta 1_{k,j}$

$$z = Min\, C_{N,M} \tag{5.1a}$$

$$s.t. \sum_{i=1}^{N} y_{ik} = 1,\ \forall k \tag{5.1b}$$

$$\sum_{k=1}^{N} y_{ik} = 1, \quad \forall i \tag{5.1c}$$

$$C_{ij} \geq C_{i,j-1} + \sum_{k=1}^{N} y_{ik} P_{kj},\ \ \forall i,\ j > 1, \tag{5.1d}$$

$$C_{ij} \geq C_{i-n_j,j} + \sum_{k=1}^{N} y_{ik} P_{kj},\ \ \forall j, i > n_j. \tag{5.1e}$$

Constraint **(5.1b)** ensures that each product is assigned to only one time slot in a sequence, while **(5.1c)** ensures that only one product is assigned to every time slot. **(5.1d)** and **(5.1e)** indicate that a product in a stage can only be processed if the product and the corresponding unit are available at the same time. The objective function **(5.1a)** is to minimize a makespan, C_{NM}, which is the completion time of the last product in the last stage.

5.2.2 Uncertainty in Processing Times

By considering processing times in problem **(5.1)**, P_{kj}, as uncertain parameters, the above problem can be reformulated as the following multi-parametric mixed-integer linear programming (mp-MILP) problem (after linearizations):

$$
\begin{aligned}
z = Min \;\; & C_{NM} \\
s.t. \;\; & \sum_{i=1}^{N} y_{ik} = 1, \; \forall k \\
& \sum_{k=1}^{N} y_{ik} = 1, \quad \forall i \\
C_{ij} \geq C_{i,j-1} + & \sum_{k=1}^{N} w_{ikj}, \; j > 1, \; \forall i \\
C_{ij} \geq C_{i-n_j,j} + & \sum_{k=1}^{N} w_{ikj}, \; i > n_j, \; \forall j, \\
\theta 1_{kj} - \theta 1_{kj}^{U}(1 - y_{ik}) & \leq w_{ikj}, \; \forall i,k,j \\
\theta 1_{kj} - \theta 1_{kj}^{L}(1 - y_{ik}) & \geq w_{ikj}, \; \forall i,k,j \\
y_{ik}\theta 1_{kj}^{L} \leq w_{ikj} & \leq y_{ik}\theta 1_{kj}^{U}, \; \forall i,k,j \\
\theta 1_{kj}^{L} \leq \theta 1_{kj} & \leq \theta 1_{kj}^{U}, \; \forall i,k,j
\end{aligned}
\tag{5.2}
$$

where $\theta 1_{kj}$ are the varying processing time parameters of product k at stage j; $\theta 1_{kj}^{L}$ and $\theta 1_{kj}^{U}$ are fixed (known) lower and upper bounds.

5.2.3 Uncertainty in Equipment Availabilities

By considering equipment availabilities in problem **(5.1)** as uncertain, problem **(5.1)** can be reformulated as the following mp-MILP problem, **(5.3)** :

$$
\begin{aligned}
&Min\ C_{NM} \\
&s.t. \sum_{i=1}^{N} y_{ik} = 1, \quad \forall k \\
&\sum_{k=1}^{N} y_{ik} = 1, \quad \forall i \\
&C_{ij} \geq C_{i,j-1} + (\sum_{k=1}^{N} y_{ik} P_{kj} + \theta 2_{ij}), \quad \forall i, j > 1 \\
&C_{ij} \geq C_{i-n_j,j} + (\sum_{k=1}^{N} y_{ik} P_{kj} + \theta 2_{ij}), \quad \forall k, i > n_j \\
&\theta 2_{ij}^{L} \leq \theta 2_{ij} \leq \theta 2_{ij}^{U}, \quad \forall i, j
\end{aligned}
\tag{5.3}
$$

where $\theta 2_{ij}$ denote the varying time parameters for the unavailability of time slot i in stage j; $\theta 2_{ij}^{L}$ and $\theta 2_{ij}^{U}$ are lower and upper bounds.

A similar formulation can be derived and used in the following mp-MILP problem **(5.4)** for the case when both processing times and equipment availabilities are considered uncertain:

$$
\begin{aligned}
& z = Min \;\; C_{N,M} \\
s.t. \;\; & \sum_{i=1}^{N} y_{i,k} = 1, \; \forall k \\
& \sum_{k=1}^{N} y_{ik} = 1, \quad \forall i \\
C_{ij} \geq C_{i,j-1} + & \sum_{k=1}^{N} w_{ikj} + \theta 2_{ij}, \; j > 1, \; \forall i \\
C_{ij} \geq C_{i-n_j,j} + & \sum_{k=1}^{N} w_{ikj} + \theta 2_{ij}, \; i > n, \forall j, \\
\theta 1_{kj} - \theta 1^{U}_{kj}(1 - y_{ik}) & \leq w_{ikj}, \quad \forall i,j,k \\
\theta 1_{kj} - \theta 1^{L}_{kj}(1 - y_{ik}) & \geq w_{ikj}, \quad \forall i,j,k \\
y_{ik}\theta 1^{L}_{kj} \leq w_{ikj} & \leq y_{ik}\theta 1^{U}_{kj}, \quad \forall i,j,k \\
\theta 1^{L}_{kj} \leq \theta 1_{kj} & \leq \theta 1^{U}_{kj}, \quad \forall k,j \\
\theta 2^{L}_{ij} \leq \theta 2_{ij} & \leq \theta 2^{U}_{ij}. \quad \forall k,j
\end{aligned}
\tag{5.4}
$$

where $\theta 1_{k,j}$ denote the varying processing time parameters of product k at stage j and $\theta 2_{i,j}$ denote the varying time parameters for the unavailability of time slot i in stage j; $\theta 1^{L}_{k,j}$ and $\theta 2^{L}_{i,j}$ are their fixed (known) lower bounds and $\theta 1^{U}_{k,j}$ and $\theta 2^{U}_{i,j}$ are upper bounds respectively.

5.2.4 A Solution Procedure

Problems **(5.2)**, **(5.3)**, and **(5.4)** correspond to the following general class of mp-MILP problems, **(5.5)**:

$$\begin{aligned} z(\theta) = \min_{x,y} \; & c^T x + d^T y \\ s.t. \; & Ax + Ey \leq b + F\theta \\ & \theta_{min} \leq \theta \leq \theta_{max} \\ & x \in X; y \in \{0,1\}^m \end{aligned} \tag{5.5}$$

For the solution of **(5.5)**, algorithms and software have been recently proposed by Pistikopoulos and co-workers. An algorithm involving the following steps have been proposed by Dua and Pistikopoulos (1998b, 2000):

Step 1 (Initialization). Define an initial region of θ with the best upper bound $\hat{z}^*(\theta) = \infty$, and obtain an initial integer structure $\bar{y}$ by solving the initial MILP whose θ are treated as variables.

$$\begin{aligned} z = \min_{x,y,\theta} \; & c^T x + d^T y \\ s.t. \; & Ax + Ey \leq b + F\theta \\ & \theta_{min} \leq \theta \leq \theta_{max} \\ & x \in X; y \in \{0,1\}^m \end{aligned} \tag{5.6}$$

where θ is a free variable. The solution of **(5.6)** is given by $y = \bar{y}$

Step 2 (Multi-parametric LP Problem) For each region with a new integer structure, $\bar{y}$:

(a) Solve the multi-parametric LP subproblem **(5.7)** to obtain a set of parametric upper bounds $\hat{z}(\theta)$ and corresponding valid regions which will be called critical regions (CRs).

$$\begin{aligned} \hat{z}(\theta) = \min_{x} \; & c^T x + d^T \bar{y} \\ s.t. \; & Ax + E\bar{y} \leq b + F\theta \\ & \theta_{min} \leq \theta \leq \theta_{max} \\ & x \in X \end{aligned} \tag{5.7}$$

(b) If $\hat{z}(\bar{y}, \theta) \leq \hat{z}^*(\theta)$ for some region of θ, update the best upper bound function, $\hat{z}^*(\theta)$, and the corresponding integer solutions, y^*.

(c) If infeasibility is found in some region CR, go to Step 3.

Step 3 (Master Problem) For each region CR^i, formulate and solve the deterministic MILP master problem by (i) treating θ as a variable bounded in the region CR, (ii) introducing an integer cut, and (iii) introducing a parametric cut $z \leq \hat{z}(\theta)^i$. Return to Step 2 with new integer solutions and corresponding CRs.

$$\begin{aligned} z = \min_{x,y,\theta} \; & c^T x + d^T y \\ s.t. \; & Ax + Ey \leq b + F\theta \\ & d^T y + c^T x \leq \hat{z}(\theta)^i \\ & \sum_{j \in J^{ik}} y_j^{ik} - \sum_{j \in L^{ik}} y_j^{ik} \leq |J^{ik}| - 1, k = 1, \ldots, K^i \\ & \theta \in CR^i \; ; \; x \in X \; ; \; y \in \{0,1\}^m \end{aligned} \tag{5.8}$$

where $J^{ik} = (j|y_j^{ik} = 1)$ and $L^{ik} = (j|y_j^{ik} = 0)$, and $|J^{ik}|$ is the cardinality of J^{ik} and $|J^{ik}|$ is the number of integer solutions that have already analyzed in CR^i. Note that the inequality, $d^T y + c^T x \leq \hat{z}(\theta)^i$, represents a parametric cut for the identification of a new integer solution which has not been explored before; the inequality, $\sum_{j \in J^{ik}} y_j^{ik} - \sum_{j \in L^{ik}} y_j^{ik} \leq |J^{ik}| - 1$, corresponds to integer cuts prohibiting previous integer solutions from appearing again.

Step 4 (Convergence) The algorithm terminates when the solution of the deterministic MILP problem is infeasible for each region CR. The final solution is given by the current upper bounds $\hat{z}^*(\theta)$ in corresponding CRs.

The details of the algorithm can be found in works by Dua and Pistikopoulos (1998b, 2000).

5.2.5 Numerical Examples

Based on the proposed approach with the above algorithm, two numerical examples are solved to illustrate the potential of the proposed approach. The first example considers processing time uncertainty and the second considers both processing time and equipment availability uncertainty at the same time.

Example 1

Consider a manufacturing process consisting of two stages, *stage 1* and *stage 2* for producing three products A, B and C. The process has one unit for each stage. Table 5.2 shows the processing times of products at each stage.

Table 5.2: Processing Time Data for Example 1

	Processing time, P_{kj} (hr/batch)	
	Stage (j)	
Product(k)	stage 1(j=1)	stage 2 (j=2)
A	3	5
B	θ_1	θ_2
C	6	3

Processing times of B at both stages, P_{B1} and P_{B2} are assumed to be uncertain parameters, θ_1 and θ_2 which are bounded as follows:

$$4\,hrs \leq \theta_1 \leq 7\,hrs, \quad 3\,hrs \leq \theta_2 \leq 8\,hrs.$$

The application of our proposed algorithm in section 5.2.4 yields the results summarized in Table 5.3 and graphically depicted in Figure 5.1. Note that in this

example, schedule [A-B-C] is a robust optimal because it is an optimal solution for the entire range of variations. From the computational viewpoint, the results were achieved by solving 8 MILPs (via GAMS/CPLEX (Brooke *et al.*, 1996) in 0.25 s) and 9 mp-LP problems (in 0.25 s) on SUN Ultrasparc workstation.

Table 5.3: Parametric Solutions of Example 1

Notation	Critical region (CR)	Optimal sequence	Makespan (hrs)
CR1	$4 \leq \theta_1 \leq 5$ $5 \leq \theta_2 \leq 8$ $\theta_1 + 1 \leq \theta_2$	$A - B - C$	$\theta_2 + 11$
CR2	$5 \leq \theta_1 \leq 7$ $6 \leq \theta_2 \leq 8$	$A - B - C$	$\theta_1 + \theta_2 + 6$
CR3	$4 \leq \theta_1 \leq 7$ $5 \leq \theta_2 \leq 8$ $\theta_1 + 1 \geq \theta_2$	$A - B - C$	$\theta_1 + 12$
CR4	$\{4 \leq \theta_1 \leq 5$ $3 \leq \theta_2 \leq 4\}$, $\{5 < \theta_1 < 7$, $3 < \theta_2 \leq 4\}$,	$A - B - C$, $B - A - C$	$\theta_1 + 12$
CR5	$5 \leq \theta_1 \leq 7$ $\theta_2 = 3$	$A - B - C$, $B - A - C$, $A - C - B$	$\theta_1 + 12$

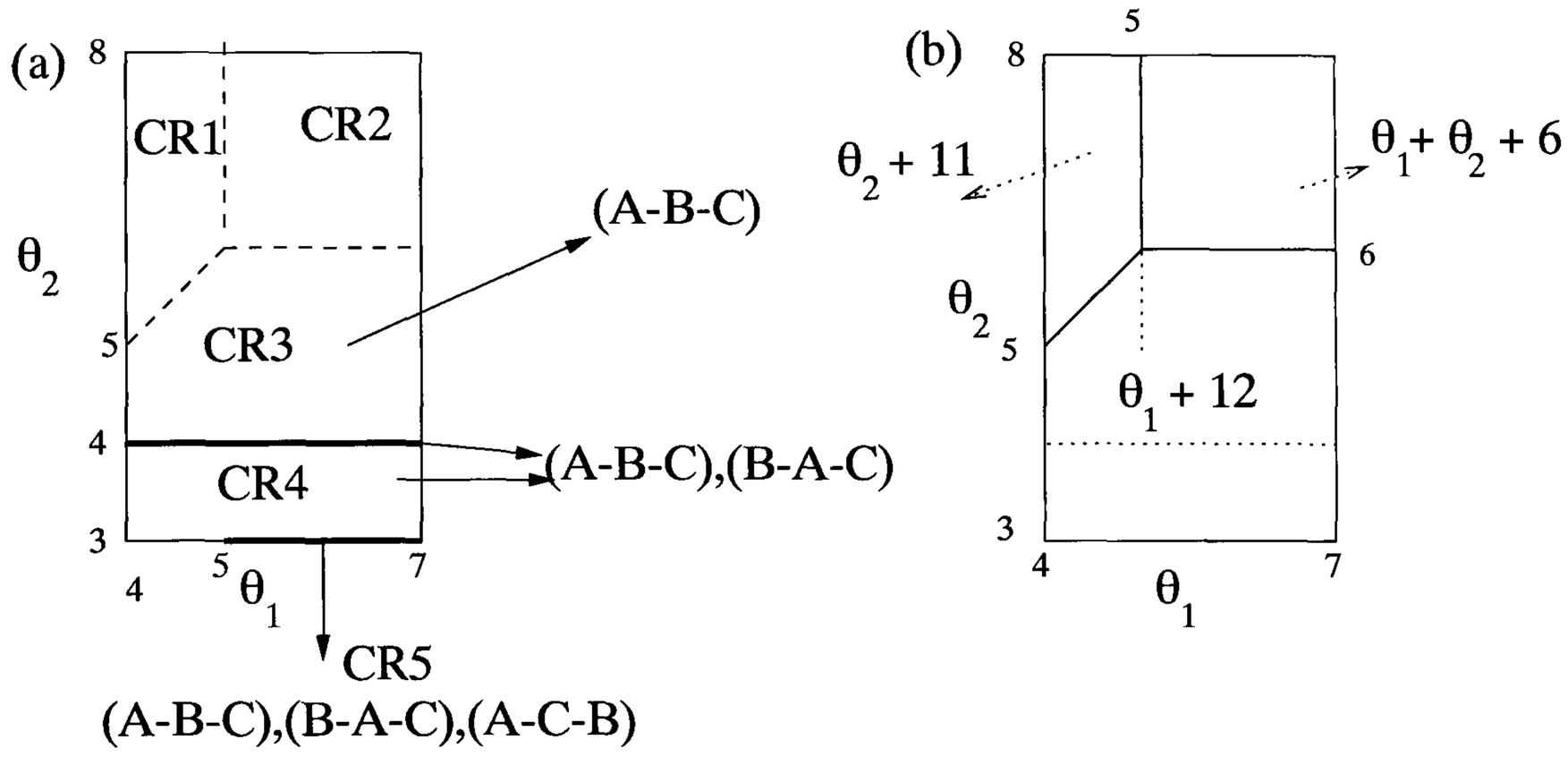


Figure 5.1: Result of Example 1: (a) Optimal Sequences and (b) Final Parametric Solutions

Example 2

Consider a manufacturing process involving five products, A, B, C, D and E in the three stages, *mixing*, *reaction* and *separation*. The *reaction* stage consists of two units, *reactor*1 and *reactor*2 while other stages have one unit (see Figure 5.2). Processing times of products in three stages are shown in Table 5.4.

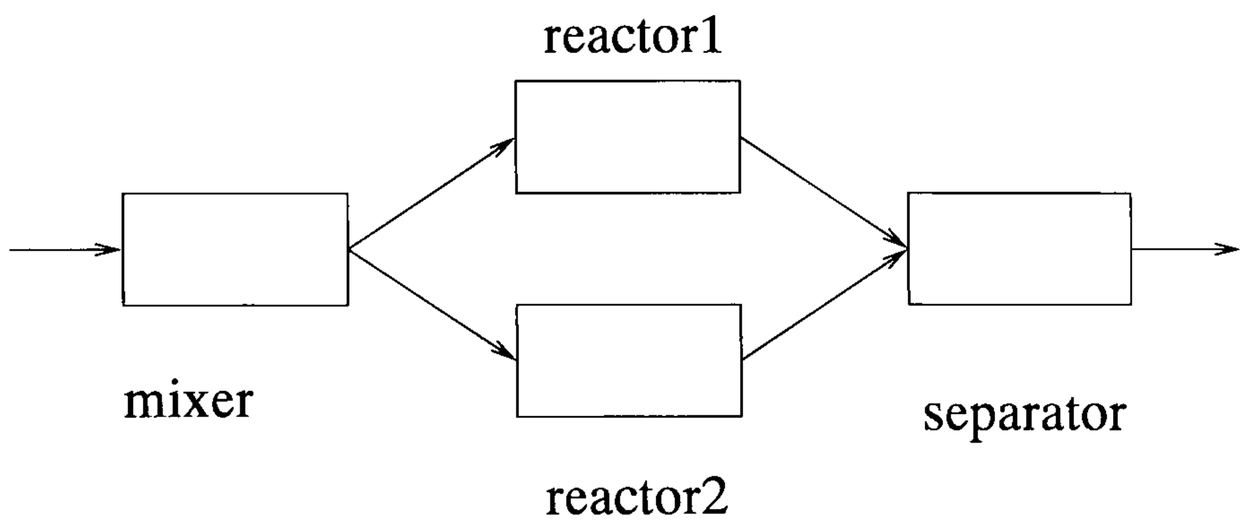


Figure 5.2: Process Configuration for Example 2

Table 5.4: Processing Time Data for Example 2

	Processing time (hrs/batch)		
Product	mixing	reaction	separation
A	6	25	11
B	9	14	17
C	12	θ_1	5
D	14	11	16
E	15	20	8

Uncertainty is involved in the processing time of product C at the reaction stage, $8\ hrs \leq \theta_1 \leq 20\ hrs$, and the availability of *reactor 1* which may be unavailable after finishing the first task and before starting the second task for as long as 30 hrs, $0 \leq \theta_2 \leq 30\ hrs$.

The application of our proposed algorithm in section 5.2.4 yields the results summarized in Table 5.5 and graphically depicted in Figure 5.3. The results were obtained by solving 33 MILPs (via GAMS/CPLEX (Brooke *et al.*, 1996) in 1.32 s)

and 12 mp-LP problems (in 5.25 s) on a SUN Ultrasparc workstation, based on the software implementation developed by Pistikopoulos *et al.*(2000).

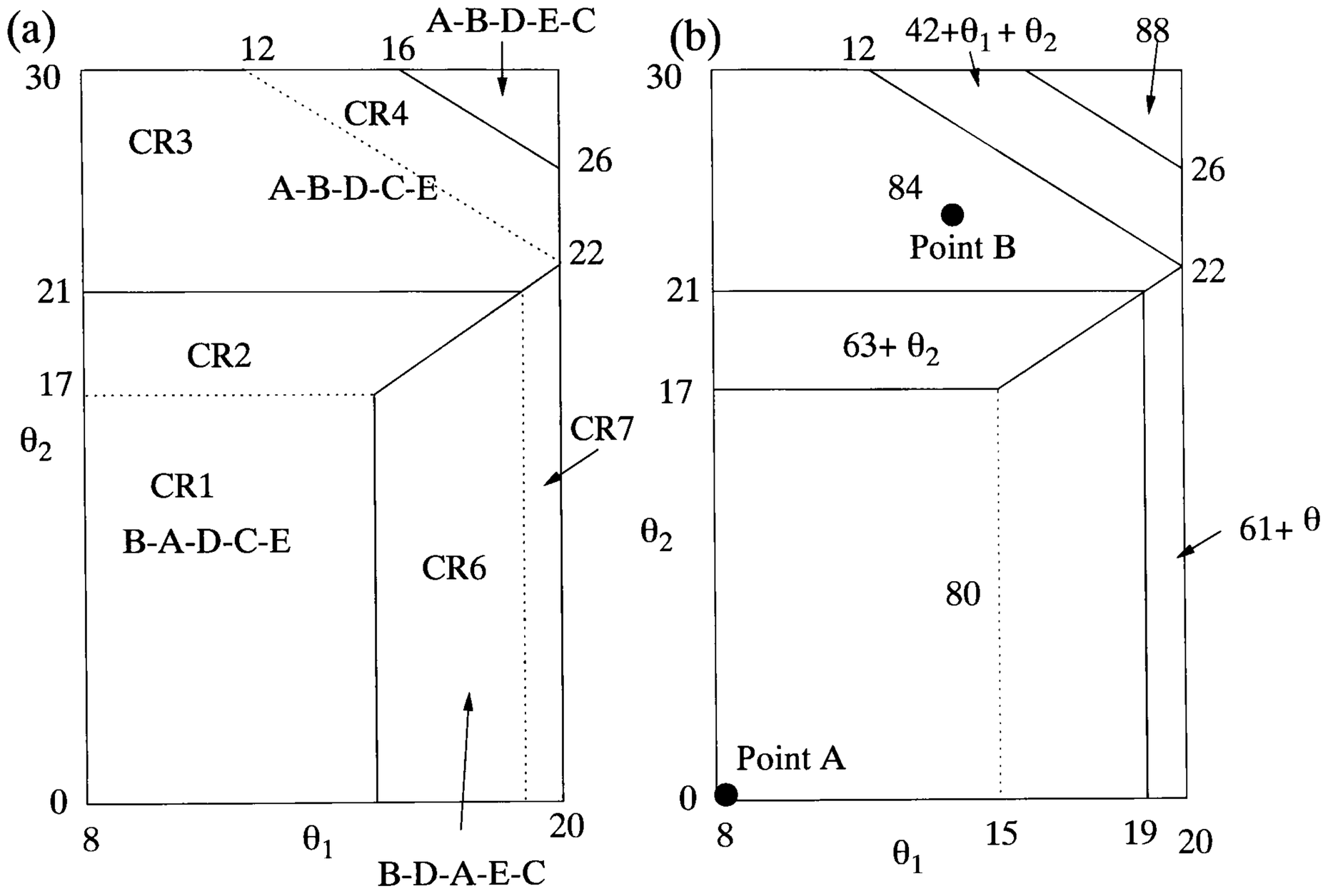


Figure 5.3: Results of Example 2: (a) Optimal Sequences in different Critical Regions and (b) corresponding Optimal Makespans

Table 5.5: Parametric Solutions of Example 2

Notation	Critical region (CR)	Optimal sequence	Makespan (hrs)
CR1	$8 \leq \theta_1 \leq 15$ $0 \leq \theta_2 \leq 17$	$B-A-D-C-E$	80
CR2	$8 \leq \theta_1 \leq 19$ $17 \leq \theta_2 \leq 21$ $\theta_1 + 2 \leq \theta_2$	$B-A-D-C-E$	$63 + \theta_2$
CR3	$8 \leq \theta_1 \leq 20$ $21 \leq \theta_2 \leq 30$ $\theta_1 + 2 \leq \theta_2$ $\theta_1 + \theta_2 \leq 42$	$A-B-D-C-E$	84
CR4	$12 \leq \theta_1 \leq 20$ $22 \leq \theta_2 \leq 30$ $42 \leq \theta_1 + \theta_2$ $\theta_1 + \theta_2 \leq 46$	$A-B-D-C-E$	$42 + \theta_1 + \theta_2$
CR5	$16 \leq \theta_1 \leq 20$ $26 \leq \theta_2 \leq 30$ $46 \leq \theta_1 + \theta_2$	$A-B-D-E-C$	88
CR6	$15 \leq \theta_1 \leq 19$ $0 \leq \theta_{21} \leq 21$ $\theta_2 \leq \theta_1 + 2$	$B-D-A-E-C$	80
CR7	$19 \leq \theta_1 \leq 20$ $0 \leq \theta_2 \leq 22$ $\theta_2 \leq \theta_1 + 2$	$B-D-A-E-C$	$61 + \theta_1$

5.2.6 Remarks

A number of important observations and remarks can be made from the results of above two numerical examples.

(i) The proposed methodology can derive a complete map of optimal schedules explicitly as a function of parametric variations in corresponding critical regions. For the case of example 2, if no uncertainty is involved, i.e. processing time of C at *reaction* stage is 8 hrs and all equipment operated normally, the optimal schedule corresponds to [B-A-D-C-E] with a corresponding makespan of 80 hrs (see point A, in Figure 5.3, CR1). Then, assume that the processing time of C at the reaction stage changes to 14 hrs and *reactor 1* is unavailable for 24 hrs. This new condition is substituted into the constraints defining critical regions in Table 5.5 and it only satisfies CR3 (see point B in Figure 5.3, CR3). In this case the optimal schedule corresponds to [A-B-D-C-E] with a corresponding makespan of 84 hrs, which is the same as that obtained by resolving the whole scheduling problem.

(ii) The change in the scheduling policy can be obtained without any need for further computations (i.e. re-solving the scheduling problem); it can be achieved by a simple function evaluation (see Table 5.3 for example 1 and Table 5.5 for example 2).

(iii) The proposed methodology presents a complete map of all optimal schedules against potential occurrences of uncertainty *before* the start of the process, thereby constructing a *proactive* scheduling system.

(iv) Though sizes of the presented examples are relatively small, the insight conveyed by them can be significant for the scheduling problem under uncertainty.

5.3 Scheduling of Zero-wait Batch Processes under Processing Time Uncertainty

5.3.1 Introduction

A number of low volume, high value-added products are manufactured using a batch process. In operations of the batch process for some special products e.g. biological or pharmaceutical products, it often occurs that either the intermediate product is unstable for storage and must immediately be processed by the next stage, or no storage vessels are available. This type of batch process is called Zero-Wait (ZW) and is illustrated using a Gantt chart in Figure 5.4.

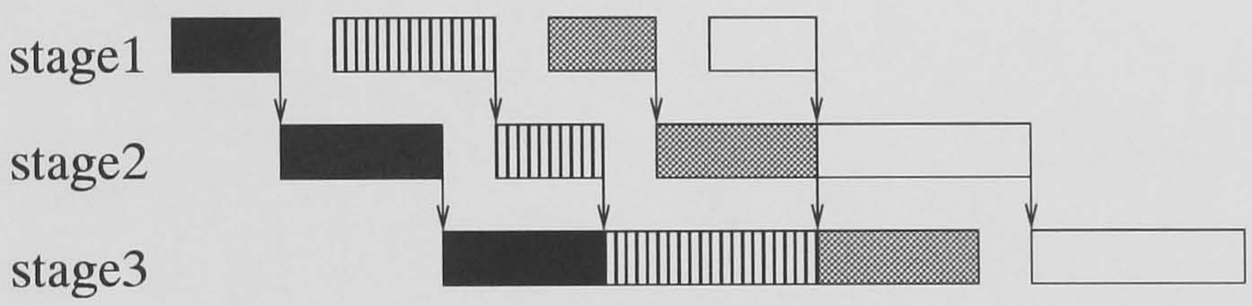


Figure 5.4: Illustrative Gantt Chart of a Zero-Wait Batch Process

Scheduling of the ZW batch process is an important research issue which has attracted attention of the process systems engineering community due to economic motivation of meeting demands for high value-added products (Ku and Karimi, 1988; Rajagopalan and Karimi, 1989; Birewar and Grossmann, 1989; Pekny and Miller, 1991; Jung *et al.*, 1994; Moon *et al.*, 1996).

Most of the past work on ZW scheduling has only been concerned with constructing mathematical models in order to effectively compute the optimal schedule on the assumption that all parameters are known for certain. In fact, zero wait processes are generally subject to uncertainty due to various reasons such as fluctuations in the quality of raw materials, variations in product demands, equipment breakdowns, etc. In the presence of uncertainty, an 'optimal' deterministic schedule for fixed parameters may not, in fact, be optimal or even feasible.

5.3.2 A Zero-wait Scheduling Model

A number of deterministic mathematical programming models have been proposed to address ZW scheduling problems (Ku and Karimi, 1988; Rajagopalan and Karimi, 1989; Jung *et al.*, 1994; Moon *et al.*, 1996). In this chapter, the following mixed integer linear programming (MILP) model proposed by Jung *et al.*(1994) is employed based on notation in Table 5.6:

Table 5.6: Notation for ZW Scheduling Model

Indices	
i	time slot $(1, \cdots, \mathrm{N})$
l	product $(1, \cdots, \mathrm{N})$
k, j	stage $(1, \cdots, \mathrm{M})$
Parameters	
P_{lk}	processing time of product l in stage k
θ_{lk}	uncertain processing time of product l in stage k
θ_{lk}^{L}	lower bound
θ_{lk}^{U}	upper bound
Variables	
C_{ij}	completion time of i th product in stage j
y_{li}	binary variable;
	1 if product l is made at ith time slot, otherwise 0
w_{lki}	auxiliary variable for bilinear term $y_{li}\theta_{lk}$

$$Min \quad C_{NM} \tag{5.9a}$$

$$\sum_{l=1}^{N} y_{l,i} = 1, \ \forall\, i \tag{5.9b}$$

$$\sum_{i=1}^{N} y_{l,i} = 1, \ \forall\, l \tag{5.9c}$$

$$C_{ij} \geq C_{i-1,M} - \sum_{l=1}^{N}\sum_{k=j+1}^{M} P_{lk} y_{l,(i-1)} + \sum_{l=1}^{N}\sum_{k=j}^{M} P_{lk} y_{li}, \quad j = 1, \cdots M-1, y_{l,0} = 0, \ \forall i \tag{5.9d}$$

$$C_{iM} \geq C_{i-1,j} + \sum_{l=1}^{N}\sum_{k=j}^{M} P_{lk} y_{li}, \quad i = 2, \cdots N, \ j = M \tag{5.9e}$$

where $P_{l,k}$ is the processing time of product l on stage k. It is assumed that transfer and set-up times are negligible but the proposed methodology can also include them.

(5.9b) and **(5.9c)** ensure that specific products are produced at specific time instances (slots) and in a specific sequence by ensuring that product i is made at the kth sequence (or the product in slot k). **(5.9d)** and **(5.9e)** determine completion times of individual products at different stages, based on the selected sequence and the zero-wait process condition.

By incorporating processing times, $P_{l,k}$ as uncertain parameters, the above problem can be reformulated as the following multi-parametric mixed integer linear programming (mp-MILP) problem after linearization:

$$
\begin{aligned}
& Min \quad C_{N,M} \\
\sum_{l=1}^{N} y_{l,i} \quad &= \quad 1, \ \ \forall\, i \\
\sum_{i=1}^{N} y_{l,i} \quad &= \quad 1, \ \ \forall\, l \qquad (5.10) \\
C_{i,j} \quad &\geq \quad C_{i-1,M} - \sum_{l=1}^{N} \sum_{k=(j+1)}^{M} w_{l,k,i-1} + \sum_{l=1}^{N} \sum_{k=j}^{M} w_{l,k,i}, \\
& \qquad j = 1, \cdots, M-1, \ \ \forall\, i \\
C_{i,M} \quad &\geq \quad C_{i-1,j} + \sum_{l=1}^{N} \sum_{k=j}^{M} w_{l,k,i} \quad i = 2, \cdots, N, \ \ j = M \\
\theta_{l,k} - \theta^{U}_{l,k}(1 - y_{l,i}) \quad &\leq \quad w_{l,k,i} \leq \theta_{l,k} - \theta^{L}_{l,k}(1 - y_{l,i}), \\
y_{l,i}\theta^{L}_{l,k} \quad &\leq \quad w_{l,k,i} \leq y_{l,i}\theta^{U}_{l,k},
\end{aligned}
$$

where $\theta_{l,k}$ are the varying processing time parameters and $\theta^{L}_{l,k}$ and $\theta^{U}_{l,k}$ are fixed (known) lower and upper bounds.

Problem **(5.10)** corresponds to the following general class of mp-MILP problem :

$$
\begin{aligned}
& z(\theta) = \min_{x,y} c^T x + d^T y \\
s.t. \ \ & Ax + Ey \leq b + F\theta \\
& \theta_{min} \leq \theta \leq \theta_{max} \\
& x \in X; y \in \{0,1\}^m
\end{aligned} \qquad (5.11)
$$

The solution procedure described in 5.2.4 is also applied to solve the problem.

5.3.3 Numerical Examples

Example 1

Consider a zero wait process involving three products in three serial stages. The objective is to minimize the makespan.

Uncertain processing times are identified as a linear function of uncertain parameters as can be seen in Table 5.7.

Table 5.7: Processing Times for Example 1

	Processing time (hrs)		
Product	Stage 1	Stage 2	Stage 3
P1	$10 + \theta_1$	$20 + 2\theta_2$	5
P2	$15 + \theta_1$	$8 + \theta_2$	12
P3	$20 + 2\theta_1$	$7 + \theta_2$	9

The boundaries of the uncertain parameters are as follows:

$$0 \leq \theta_1 \leq 5\, hrs, \quad 0 \leq \theta_2 \leq 12\, hrs.$$

The application of the presented model and algorithm yields the result summarized in Table 5.8 and graphically depicted in Figure 5.5. The solution procedure consists of 7 MILP problems via GAMS/CPLEX (Brooke *et al.*, 1996) (0.41s) and 3 mp-LP problems (3.84s) on a Sun Ultrasparc workstation based on the software implementation developed by Pistikopoulos *et al.* (2000).

Table 5.8: Parametric Solutions of Example 1

Notation	Critical region	Optimal makespan (hrs)	Optimal sequence
CR1	$\theta_1 \geq \theta_2$	$61 + 4\theta_1 + \theta_2$	$P2 \rightarrow P1 \rightarrow P3$
CR2	$\theta_1 \leq \theta_2$ $\theta_2 - \theta_1 \leq -2$	$61 + 2\theta_1 + 3\theta_2$	$P2 \rightarrow P1 \rightarrow P3$
CR3	$-2 \leq \theta_2 - \theta_1 \leq 6$	$59 + \theta_1 + 4\theta_2$	$P2 \rightarrow P1 \rightarrow P3$
CR4	$6 \leq \theta_2 - \theta_1 \leq 8$	$65 + 2\theta_1 + 3\theta_2$	$P1 \rightarrow P3 \rightarrow P2$
CR5	$\theta_2 - 2\theta_1 - 9 \leq 0$ $\theta_1 + 8 \leq \theta_2$	$57 + \theta_1 + 4\theta_2$	$P1 \rightarrow P3 \rightarrow P2$
CR6	$\theta_2 - 2\theta_1 - 9 \geq 0$	$66 + 3\theta_1 + 3\theta_2$	$P1 \rightarrow P2 \rightarrow P3$

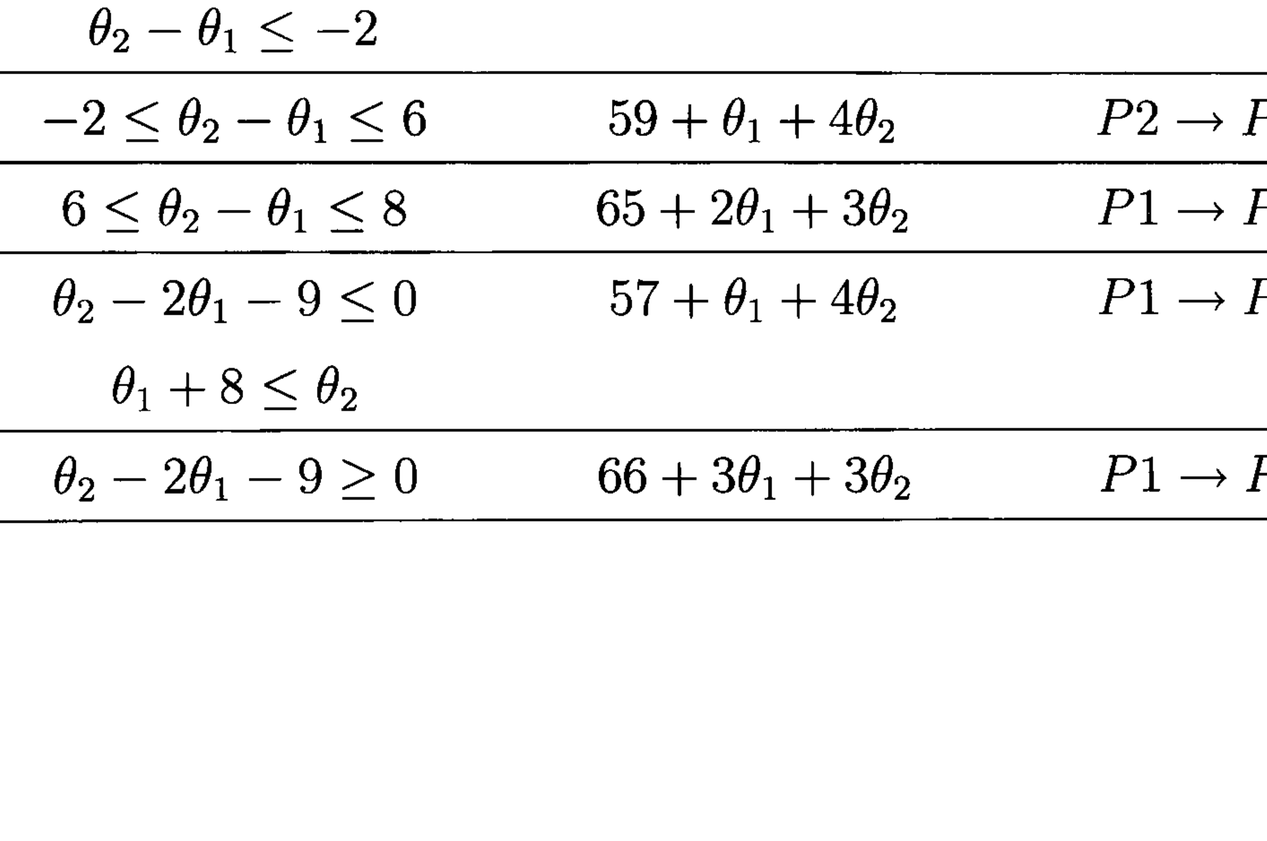


Figure 5.5: Result of Example 1. (a) Divisions of Critical Region and (b) Optimal Sequence

Example 2

Consider a zero wait process involving four stages and four products. Processing times of products in stages are listed in Table 5.9. The objective is to minimize the makespan.

Table 5.9: Processing Times for Example 2

	Processing time (hrs)			
Product	Stage 1	Stage 2	Stage 3	Stage 4
P1	$10 + \theta_1$	$20 + 2\theta_2$	5	4
P2	$15 + \theta_1$	$8 + \theta_2$	12	10
P3	$20 + 2\theta_1$	$7 + \theta_2$	9	5
P4	$8 + 2\theta_1$	$15 + 3\theta_2$	20	9

Boundaries of the uncertain parameters are as follows:

$$0 \leq \theta_1 \leq 5\, hrs, \quad 0 \leq \theta_2 \leq 12\, hrs.$$

The application of the proposed model and algorithm yields the results summarized in Table 5.10 and graphically depicted in Figure 5.6. The solution procedure consists of 16 MILP problems via GAMS/CPLEX (Brooke *et al.*, 1996) (3.32s) and 9 mp-LP problems (12.31s) on SUN Ultrasparc workstation based on the software implementation developed by Pistikopoulos *et al.* (2000).

5.3.4 Discussion

Some remarks can be made from observing the results of the above examples.

First of all, as already confirmed in the previous section, the proposed methodology provides a complete map of potential optimal schedules as a simple function of varying parameters (see Table 5.8, Example 1 and Table 5.10, Example 2). Most previous research on zero-wait scheduling was only concerned with reducing the time needed to solve the entire scheduling problem by speeding up the calculation time *after* all parameters are revealed. However, by establishing a complete map

Table 5.10: Parametric Solution of Example 2

CR	Critical region	Optimal makespan (hrs)	Optimal sequence
CR1	$\theta_2 \geq 2\theta_1 + 11$	$91 + 4\theta_1 + 5\theta_2$	$P4 \rightarrow P2 \rightarrow P3 \rightarrow P1$
CR2	$\theta_2 \leq 2\theta_1 + 11$ $\theta_2 \geq \theta_1 + 8$	$80 + 2\theta_1 + 6\theta_2$	$P4 \rightarrow P3 \rightarrow P2 \rightarrow P1$
CR3	$\theta_2 \leq \theta_1 + 8$ $\theta_2 \geq \theta_1 + 4.5$ $\theta_2 \geq 2\theta_1 + 1$	$88 + 3\theta_1 + 5\theta_2$	$P4 \rightarrow P3 \rightarrow P2 \rightarrow P1$
CR4	$\theta_2 \geq 8$ $2\theta_1 \leq \theta_2 \leq 2\theta_1 + 1$	$87 + \theta_1 + 6\theta_2$	$P2 \rightarrow P4 \rightarrow P3 \rightarrow P1$ $P1 \rightarrow P2 \rightarrow P4 \rightarrow P3$
CR5	$\theta_1 + 2 \leq \theta_2 \leq \theta_1 + 4.5$ $\theta_2 \geq 2\theta_1$ $\theta_2 \leq 8$	$79 + \theta_1 + 7\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR6	$\theta_1 \leq \theta_2 \leq 2\theta_1$ $\theta_2 \geq 2$	$79 + 3\theta_1 + 6\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR7	$\theta_2 \leq 2$ $(2\theta_1 - 4)/3 \leq \theta_2 \leq \theta_1$	$83 + 5\theta_1 + 2\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR8	$\theta_2 \leq \theta_1$ $\theta_2 \geq 2$	$79 + 5\theta_1 + 4\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR9	$\theta_2 \leq 2$ $\theta_2 \geq \theta_1$	$83 + \theta_1 + 5\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR10	$\theta_1 \leq \theta_2 \leq 2\theta_1, \theta_1 \leq 2$	$83 + 3\theta_1 + 4\theta_2$	$P2 \rightarrow P4 \rightarrow P1 \rightarrow P3$
CR11	$\theta_2 \leq 2$ $\theta_1 - 4 \leq \theta_2$ $\theta_2 \leq (2\theta_1 - 4)/3$	$87 + 3\theta_1 + 5\theta_2$	$P1 \rightarrow P2 \rightarrow P4 \rightarrow P3$
CR12	$\theta_2 \leq \theta_1 - 4$	$53 + 7\theta_1 + 9\theta_2$	$P1 \rightarrow P2 \rightarrow P4 \rightarrow P3$

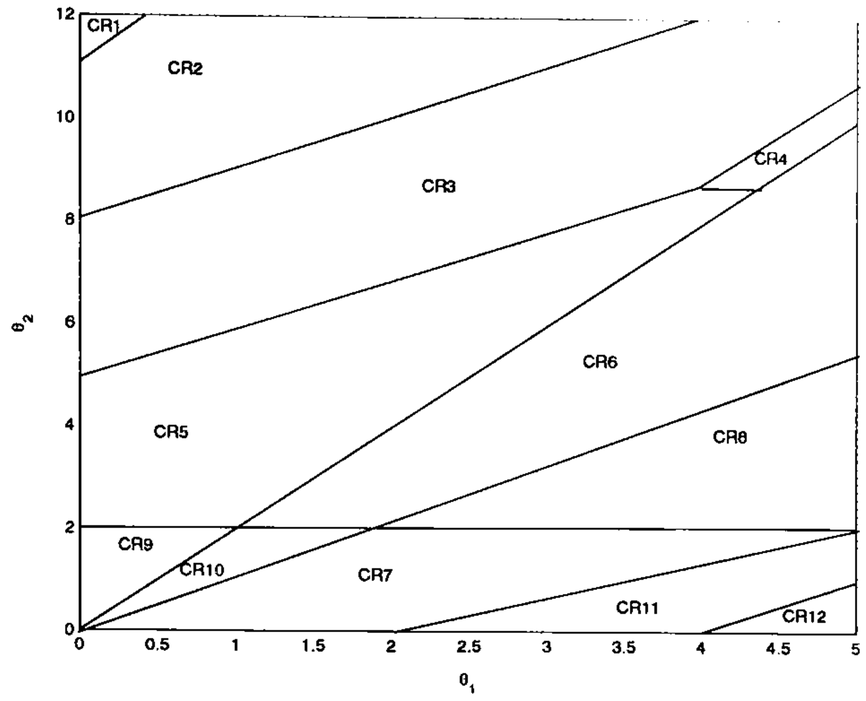


Figure 5.6: Division of the Critical Region of Example 2

of optimal schedules as a function of those varying parameters *prior to* operations of processes, a more significant reduction in terms of computational time has been achieved.

Secondly, the proposed methodology can provide not only optimal schedules under uncertainty but also valuable maintenance management strategies in response to uncertainty. For instance, see the solutions of Example 1 in Table 5.8 and Figure 5.5. If the uncertain parameter, (θ_1, θ_2) is expected to be (0,5.5), the imminent priority of the operation should be minimizing θ_2. If θ_2 becomes longer than 6 hrs with an increase of θ_1, its corresponding optimal sequence changes from the current optimal schedule $[P2 \rightarrow P1 \rightarrow P3]$ to a new schedule $[P1 \rightarrow P3 \rightarrow P2]$. Considering the fact that it is generally recommended to reduce the adjustment of the production schedule to a minimum in actual practice (Kanakamedala *et al.*, 1994), this work may provide valuable insight on operation strategies in the face of uncertainty.

Finally, the proposed methodology is illustrated using two types of specific scheduling problems. Other types of formulation such as State-Task-Network (STN) or Resource-Task-Network (RTN) (Pantelides, 1994; Shah, 1998) may be used for solving scheduling of other cases such as multipurpose processes. The issue would be then how to address the computational load caused by a large number of binary variables in solving corresponding mp-MILP problems.

5.4 Conclusion

This chapter has proposed a novel methodology for the construction of the proactive scheduling systems using parametric programming techniques. The uncertainty present in processing times and equipment availabilities is incorporated into scheduling models, which are then transformed into multi-parametric mixed integer linear programming (mp-MILP) problems. A solution procedure based upon recently proposed state-of-the-art mp-MILP algorithms has been then discussed. A key advantage of the proposed methodology is that the complete map of optimal schedules can be obtained as a function of varying parameters; re-scheduling can thus be performed via simple function evaluations without any further optimization. Therefore a *proactive* scheduling system can be constructed using the proposed methodology. Two scheduling examples which are scheduling problem in UIS policy and Zero-Wait policy have been presented to illustrate the potential of the proposed methodology.

Chapter 6

Conclusions and Future Directions

6.1 Conclusions

Various types of decision-making problems arise in the design and operation of enterprise-wide process networks, ranging from short-term scheduling of an individual process to design and planning of enterprise-wide process networks. Many issues relating to these problems pose new challenges for the process systems engineering community. This thesis has investigated two particular issues involved in enterprise-wide process networks under uncertainty in order to construct efficient decision-support systems.

Because enterprise-wide process networks generally involve multiple elements, this thesis introduced bilevel programming principles in order to address their complex hierarchical decision-making issues. In Chapter 3, a methodology is proposed for computing the solution of deterministic bilevel programming problems using parametric programming techniques. The bilevel programming problem generally means an optimization problem that is constrained by another optimization problem. It is an important modeling framework that can address various real world industrial problems and its potential on chemical engineering applications has been also reported in the literature. Despite of its importance, there is still room for improvement for computing the solution of the problems in terms of requiring many additional variables and constraints, and computational iterations. As an alternative way, we have proposed an approach which transforms the bilevel problem into

a family of single optimization problems using parametric optimization techniques. This means the original problem can be solved to global optimality for linear-linear, linear-quadratic, quadratic-linear, and quadratic-quadratic cases in an efficient way.

This thesis applied bilevel programming principles into the planning of enterprise-wide process networks. In Chapter 4, a bilevel programming framework is proposed for the enterprise-wide supply chain planning under uncertainty. A supply chain generally involves many participating elements which operate according to their own objectives. Because those objectives often conflict with each other, a special type of modeling methodology is required but as yet little has been done in this field. As an alternative approach, a bilevel programming framework is proposed in which one level corresponds to a plant planning problem and the other to a distribution network problem. The resulting bilevel programming problem under uncertainty is computed by proposing an effective solution strategy based on the methods for deterministic bilevel problems, which are described in the previous chapter.

The presence of uncertainty is also an important issue in the short-term operation planning, which should not be neglected in the overall enterprise-wide process networks. In Chapter 5, a novel methodology is proposed to solve short-term operation planning problems under uncertainty using parametric programming techniques. The uncertainty present in process parameters such as processing times and equipment availabilities is then incorporated into scheduling models, which are transformed into multi-parametric mixed integer linear programming (mp-MILP) problems. A solution procedure based upon recently proposed mp-MILP algorithms is then discussed. A key advantage of the proposed methodology is that the complete map of optimal schedules can be obtained as a function of varying parameters; re-scheduling can thus be performed via a simple function evaluation without any further optimization. Therefore, the proposed methodology contributes to the construction of a *proactive* operation framework under uncertainty, and ultimately, to robust design and operation of entire process networks.

The key contributions of this thesis can be summarized as follows:

- The development of a new global optimization methodology for the solution of deterministic bilevel programming problems. A major advantage is that it solves bilevel problems to global optimality for linear-linear, linear-quadratic, quadratic-linear and quadratic-quadratic cases without using additional variables and constraints.

- The formulation of bilevel programming problems under uncertainty. A novel methodology for the solutions of bilevel programming problems under uncertainty is developed using parametric programming techniques.

- An application of bilevel programming principles to supply chain optimization problems. A key advantage is that conflicting interests of multiple supply chain elements are encapsulated realistically in the proposed framework.

- The development of a proactive modeling framework to scheduling problems under uncertainty. A key advantage is that a complete map of optimal schedules can be obtained as a function of varying parameters; re-scheduling is then obtained via simple function evaluation without further optimization.

6.2 Future Directions

From the knowledge and insight gained throughout this thesis, some suggestions for future work are as follows.

- On-line scheduling and control

  The proposed parametric optimization methodology which were used for scheduling problems under uncertainty can be introduced for the construction of an on-line reactive scheduling framework, for instance, such as MPC-like forms. Recently on-line control and optimization has been actively studied using parametric optimization *e.g.* Pistikopoulos et al. (2000, 2001). Because control variables are computed as a function of state variables in their research, expensive computations to compute the new control variable are reduced into a simple function evaluation.

Because on-line scheduling optimization also requires computing a new schedule in response to dynamic process conditions, the proposed approach may be implemented in order to compute a newly updated schedule and its corresponding control actions as a function of state variables without needs of re-computation.

- Bilevel programming problems with discrete variables

  The proposed parametric optimization-based methodology can be introduced to solve other types of bilevel programming problems such as bilevel programming problems with discrete variables, where a follower's problem with discrete variables can be computed as a function of outer problem's variables using multi-parametric mixed integer linear programming algorithm such as one by Dua and Pistikopoulos (2000). Then the outer problem is transformed into a family of single Mixed Integer Linear Programming problems. The solutions of the original bilevel problem with discrete variables could be thus obtained by solving these problems.

- Multi-level programming problem

  The proposed methodology can be also expanded to solve a multi-level programming problem where more than two optimization problems are connected in a hierarchical way. The solutions of multi-level programming problems (for instance m-level programming problem, $m > 2$) could be also computed using the proposed methodology in the following way. The variables of the lowest problem (*1st* level) can be computed as a function of variables of other upper problems, then the original m level problem is transformed into a family of (*m-1*) level programming problems. By repeating the procedure, the original problem can be formulated as a family of single optimization problems.

- Demand supply chain management

  This thesis models enterprise-wide process networks using a bilevel framework with a focus on production and distribution planning. Most research in the area of process systems engineering including this work assumes demands as external sources of disturbances. In fact, demands and their prices in the market may be varied according to strategies of suppliers, *i.g.* promotional offer

or sale price can boost demands. Therefore a recently focused issue in management science is how to manipulate prices and production levels, as is called *demand supply chain management* (Hoover Jr. *et al.*, 2001). As parametric programming computes control variables as a function of state variables in the context of optimization problems under uncertainty, we may construct a modeling framework using the methodology in chapter 5, where prices of products are computed (expressed) as a function of process parameters such as production amounts and market states. The introduction of the proposed framework can thus provide valuable operational strategies for proactive demand supply chain management.

- Aggregation in solving large-scale Problems using Parametric Programming
  Parametric programming techniques have been mainly applied to solve relatively small scale problems consisting of less than five uncertain parameters and less than hundreds of constraints. In order to introduce the techniques in computing large-scale industrial problems, the major challenge may be the corresponding expensive computation. As a potential strategy to overcome the challenge, we may (i) take advantage of the special structure of the problem or (ii) introduce aggregation / disaggregation strategies.

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2. J. Ryu, V. Dua and Pistikopoulos, E.N. (2002) Proactive Scheduling under Uncertainty: A Parametric Optimization Approach, *in preparation for Ind. Eng. Chem. Res.*

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