

Measurement of neutrino oscillation parameters at T2K using new neutral current pion samples and probing future sensitivities with Hyper-Kamiokande

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Abstract

This thesis explores approaches to improve neutrino oscillation measurements with the T2K experiment and the next-generation Hyper-Kamiokande (Hyper-K). A unique strength of long-baseline accelerator neutrino experiments is to measure δ_{CP} . Observing CP violation at 5σ significance for a large fraction of δ_{CP} values requires significant reductions in systematic uncertainties, particularly in the neutrino-nucleus interaction model, as experiments transition from being statistically to systematically limited. Additionally, these experiments aim to precisely measure the absolute mass difference $|\Delta m_{32}^2|$ and the mixing angle θ_{23} .

Hyper-K's oscillation sensitivity has been evaluated, comparing scaled T2K systematic models with new constraints from the Intermediate Water Cherenkov Detector (IWCD). The IWCD, designed to be vertically movable, measures the neutrino flux at different angles off the beam axis, enabling precise measurements of the relative ν_e and ν_μ interaction cross sections. The impact of new IWCD cross-section models on oscillation sensitivity is presented.

The core of this thesis focuses on implementing new neutral current pion samples into an oscillation analysis. The 1-ring neutral current (NC) $1\pi^0$ events are backgrounds to the oscillated ν_e signal due to their indistinguishable topologies measured at Super-Kamiokande. A 1-ring π^0 sample is reintroduced to recover misidentified ν_e signals, while a high-purity ($\sim 71\%$) and high-efficiency ($\sim 74\%$) 2-ring NC $1\pi^0$ selection improves modelling of NC $1\pi^0$ events. Their kinematic properties are studied, and their impact on oscillation sensitivity is evaluated using a Markov Chain Monte Carlo (MCMC) fitting approach.

A fit with the new samples to T2K neutrino beam data, using 2.14×10^{21} (1.63×10^{21}) proton-on-target in the (anti-)neutrino beam mode, yields a 1σ credible interval for $\delta_{CP} = [-2.513, -0.880]$, excluding CP -conservation at a 90% credible level. These results are consistent with the latest T2K analysis, giving a 74% probability for neutrinos under normal mass ordering and a 68% probability that θ_{23} is in the upper octant.

Declaration of Originality

The research presented in this thesis was conducted between January 2021 and September 2024, with the support of Imperial College London, and through collaboration with T2K and Hyper-K members. Below is a summary of clarification of the work done by collaborators and from my personal contributions:

- Chapters 1-4: A historical and theoretical overview of neutrino physics, the T2K and Hyper-K experiments, and a description of the statistical methods and general strategy for oscillation analysis. Content summarised from other sources have been appropriately referenced.
- Chapter 5: This chapter presents my own analysis work. The work was built upon the VALOR oscillation framework. The IWCD parameterisation and scaled T2K model were provided by the Hyper-K LBL+ND joint analysis group.
- Chapter 6: This chapter presents my own analysis work, using developed $\text{NC}\pi^0$ samples provided by the T2K-SK group without modifications.
- Chapters 7-8: These chapters present my own analysis work. The work was built upon the MaCh3 framework, with raw MC and data input files developed by the T2K-SK group. The Super-K detector systematics were provided by Dr. Dan Martin, the flux systematics by the T2K Beam working group, and the cross-section systematics by the T2K Neutrino Interactions working group, which I updated the cross-section systematics with new parameters.
- Additionally, I contributed to the optical system simulation, calibration analysis, and the onsite DAQ maintenance of the T2K Optical Transition Radiation (OTR) beam monitor during the T2K shut-down period. However, details of this work are not discussed in this thesis.

I hereby declare that all materials presented here are my own work, except where explicitly referenced. This thesis has not been submitted for any other qualification.

Tailin Zhu

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Chapter 1

Neutrino Physics

1.1 Background and Theory

1.1.1 The Standard Model and Weak Interaction

The Standard Model of particle physics (SM) describes the properties and interactions of the fundamental particles discovered thus far, as well as the generation of particle mass via the Higgs mechanism. The Standard Model (SM) particles are divided into fermions (spin- $\frac{1}{2}$) and bosons (integer spin). Fermions include quarks (u, d), (c, s), (t, b) and leptons (e, ν_e), (μ, ν_μ), (τ, ν_τ), as well as their corresponding anti-particle constituents across three generations. Bosons include the four force-carrying gauge bosons – the photon, the $W^\pm$, the Z^0 , and the gluons; as well as the Higgs boson. A summary of the particle configuration and their properties is shown in Fig. 1.1.

The SM interactions are governed by the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)_C$ symmetry describes the conservation of colour charge in Quantum Chromodynamics (QCD) under the strong nuclear force, while the $SU(2)_L \times U(1)_Y$ symmetry is corresponding to the conservation of weak isospin and hypercharge in the electroweak unification of the electromagnetic and weak nuclear forces. The transmission of these forces is mediated by the corresponding gauge bosons.

Gauge symmetry requires all gauge bosons to be massless. However, the observed short range of the weak nuclear force implies that its mediating bosons must be massive. The mass is introduced through the Higgs scalar field, which excites a

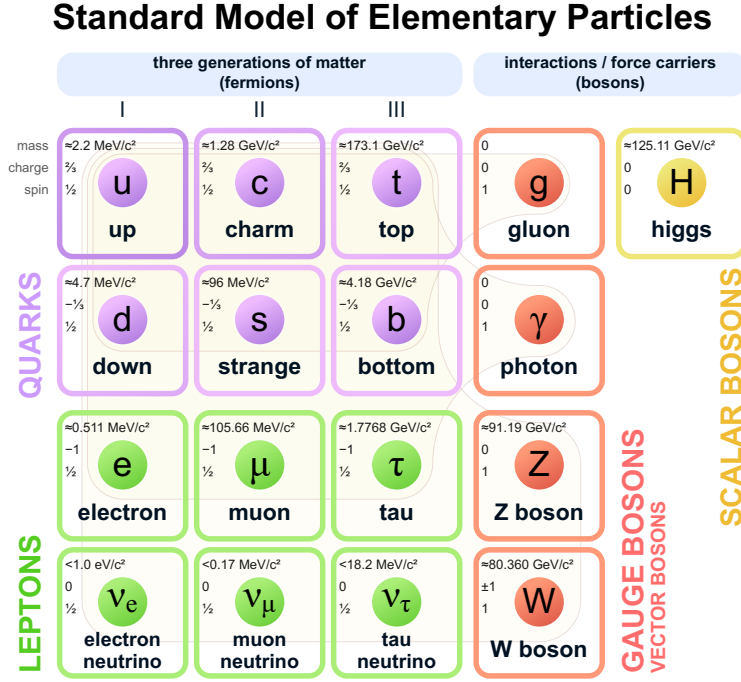


Figure 1.1: Fundamental particles in the Standard Model [1].

field from its ground vacuum state to a non-zero vacuum expectation value (VEV), thereby spontaneously breaking the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry down to $SU(3)_C \times U(1)_{EM}$ [2]. As a result, the weak nuclear force is separated from the electroweak unification, allowing mass terms to be generated for the weak interaction mediator $W^\pm$ and Z^0 bosons. The non-zero VEV field also couples to the fermions, providing a mechanism to predict fermion masses based on their coupling strength to the Higgs boson.

The $SU(2)_L$ and $U(1)_Y$ symmetries in the SM are associated with the three gauge fields (W_1, W_2, W_3) and the B field, respectively. The charged $W^\pm$ bosons are formed by combining the W_1 and W_2 fields, and they mediate the charged current (CC) interactions, which involve transitions between fermions within the same weak isospin doublet. The Z^0 boson and the photon arise from the mixing of the W_3 and B fields. The Z^0 boson mediates neutral current (NC) interactions, which do not alter the weak isospin of the interacting fermions. The mixing of W_3 and B also introduces the weak mixing angle (the Weinberg angle, $\sin^2 \theta_W \approx 0.23$ [3]) into the coupling, causing the different coupling strength of the fermions to the Z^0 boson

and to the $W^\pm$ bosons. This difference will affect the cross sections of NC interactions relative to CC interactions. The photon, which remains massless due to the preserved $U(1)_{EM}$ symmetry, couples to electric charge independently of θ_W .

While the SM interactions primarily rely on continuous symmetries, such as the conservation of gauge and momentum, further development of the SM theory requires incorporating discrete symmetries [4]. The discrete symmetry describes the conservation law when changes of a system are in finite steps, defined as operators when acting on a particle [5]:

1. The charge-conjugate operator, $\hat{C}$, inverses the intrinsic quantum numbers of a particle, thus transforming the particle into its antiparticle.
2. The parity operator, $\hat{P}$, inverses the spatial coordinates, such that, $\tilde{\mathbf{x}} \rightarrow -\tilde{\mathbf{x}}$, which flips the sign of the vector quantities (e.g. momentum) but leaves the axial-vectors (e.g. spin) unchanged. As a consequence, the transformation maps the left-handed and right-handed chiral components of fermion fields onto each other.
3. The time-reversal operator, $\hat{T}$, flips the time such that $t \rightarrow -t$, hence, reverses the direction of motion of the particle.
4. The combined $\hat{C}$ and $\hat{P}$ operates the Charge-Parity (CP) transformation, which transforms an interaction process to the CP -conjugate process, simultaneously maps the particles to their antiparticles and reverts the spatial and chiral states.

The C and P symmetries were found to be conserved in the electromagnetic interaction during the 1940s, followed by the assumption that this conservation is also obeyed in strong and weak interactions. However, in 1956, Yang and Lee proposed for the first time that parity may be violated in weak interactions [6]. This hypothesis was experimentally confirmed in 1957 by Wu's experiment, which demonstrated that parity is maximally violated in weak interactions. The discovery was based on observing a significant up-down asymmetry in the ejected electron directions under a strong magnetic field during the radioactive decays of ^{60}Co [7]. The discov-

ery implies that only the left-handed chiral components of particles and the right-handed chiral components of antiparticles can couple to the weak nuclear force. Feynman and Gell-Mann later developed the formalism of the weak interaction as a Vector–Axial-vector ($V - A$) term to specify the chiral projection, with the projector also being parity-odd to account for the parity violation [8].

A consequence of the chiral properties of particles in weak interactions implies the suppression of certain decay processes, assuming spin and momentum are conserved. Due to relativistic effects, the amount of suppression is determined by the spin and momentum of the mother particle and the mass of the daughter particles from the decay. The suppression phenomenon is confirmed from the measurements of the charged pion decays, assuming tiny mass for the outgoing neutrinos, showing the decay branch ratio of $\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) = 1.23 \times 10^{-4}$ [9].

It was then expected that the combined CP symmetry should hold for weak interactions, as the $V - A$ form is conserved under CP transformation. However, the observed decay of long-life Kaon into $\pi^+\pi^-$ indicated evidence of indirect CP violation in the quark sector [10, 11]. This phenomenon is explained by the fact that the up-type quark couples to a weak eigenstate given by the linear combinations of the down-type quarks. The coupling strengths are linked to a unitary matrix, known as the CKM matrix [12, 13], which describes the mixing between the weak eigenstates and physical quarks (also referred to as mass eigenstates). This matrix contains 3 real parameters and a complex phase under 3 fermion generations. The mixing can lead to neutral meson oscillations, with the CP violation arising from the complex phase. The first direct CP violation in the quark sector was discovered in neutral Kaon decays by the NA31 experiment [14], and later on also measured in the charm and B meson decays [15].

In the lepton sector, particles only couple within the same doublets and do not exhibit the mixing observed in the quark sector. Consequently, CP violation is not expected in the SM interactions between leptons. However, CP violation in the lepton sector can potentially arise from neutrino mixing. This phenomenon goes

beyond the original formulation of the SM, as this requires neutrinos to have mass. Extensions to the SM that address the neutrino mass and its consequences will be discussed in Section 1.1.3.

The combined CPT symmetry is believed to hold in the fundamental level of physics [16], with no experimental evidence being observed for a CPT violation.

1.1.2 Neutrinos in the Standard Model

Neutrinos are particles that do not carry the colour charge or electric charge, and thus can only undergo weak interactions with other particles under the SM frame. Therefore, neutrinos only interact through the CC or NC via exchanges with $W^\pm$ or Z^0 bosons, respectively.

The existence of the neutrino was first postulated in 1930 by Wolfgang Pauli [17] to explain the continuous energy spectrum of electrons produced from the β -decay processes measured by James Chadwick [18]. The radioactive decay kinematics suggest the energy of the radiated particle should be uniquely determined by its mass and the mass of isotopes involved. However, the observed continuous spectrum indicates that part of the electron energy is missed, which can be carried away by a third unobserved particle.

Following Pauli's principle, Enrico Fermi further developed the theory of β -decay, which proposed the neutrinos and electrons as new particles created during the decay process rather than being a part of the nucleus [19]. Fermi concluded from the data that neutrinos should be massless or very light, as the electron spectrum doesn't show a distinct endpoint. As a consequence, the SM was initially built upon the assumption of massless neutrino for simplicity and lacked sensitivity to measure the neutrino mass at the time.

Fermi interpreted the process as a point interaction between 4 fermions that conserves the electric charge and particle flavour, and introduced a coupling constant ($G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$) to characterise the size of interaction cross section and hence the decay rate. At the stage when the SM is developed, the Fermi

coupling constant is used to address the coupling strength ($\propto G_F^2$) between the left-handed particles and right-handed antiparticles, together with the $V - A$ form and the vertex element from the mixing matrix, to determine the total interaction cross section during the weak interaction process.

The detection of neutrinos is experimentally challenging due to the small interaction cross section of neutrinos with particles, as the only possible interaction through the weak nuclear force has a short range of roughly the size of the atomic nuclei. The first direct detection of a neutrino was made by Clyde Cowan and Frederick Reines in 1956, using the neutrino source ($\bar{\nu}_e$) from a nuclear reactor [20]. The $\bar{\nu}_e$ interacts through the inverse β -decay process in a water target volume to produce positrons and neutrons, which were identified from the photon emissions in the positron-electron collision and neutron capture processes. The measured neutrino flux matched predictions based on the interaction cross section described by Fermi's theory, showing good agreement in the existence of neutrinos with the interaction process interpreted by the theory.

The second-generation neutrino (ν_μ) was discovered in 1962 using the decays of charged pions and Kaons produced from a 15 GeV proton beam hitting a fixed target at the Brookhaven Alternating Gradient Synchrotron [21]. Given the hypothesis that only muons and muon neutrinos can be produced from the two-body decays of pions and kaons, Lederman, Schwartz, and Steinberger conducted observations on the leptons produced from the neutrino interactions in an aluminium spark chamber. The observation of 29 muon signal events and only 6 electron-like shower tracks suggests that, to preserve lepton universality, ν_μ and ν_e must be fundamentally different types of particles.

The existence of the third-generation neutrino was postulated after first detecting the τ lepton in 1975 at the SLAC e^+e^- collider [22]. The first direct detection of ν_τ was made in 2000 by the DONUT experiment using an 800 GeV proton beam from the Fermilab Tevatron interacting with a beam dump, where the source of neutrinos came from the decays of charm mesons produced in the beam dump [23]. The τ lep-

ton produced from the neutrino interactions were identified as a track with sharp bend in the track direction due to its short decay range of 2 mm. This distinctive feature also facilitates distinguishing electrons from muons when combined with data from a charged particle spectrometer. It was observed that 4 out of the 203 interaction events showed evidence of no other leptons produced at the primary vertex except the τ lepton, confirming the occurrence of ν_τ interaction.

There is no experimental evidence supporting the existence of fourth-generation fermions. The determination of the number of neutrino generations relies on the relative decay widths of the Z^0 boson into hadrons, leptons, and neutrinos. Experimental measurements have consistently aligned with the three-neutrino-generation hypothesis, as illustrated in Fig. 1.2, providing strong evidence for the existence of only three light neutrino generations under the SM predictions [24].

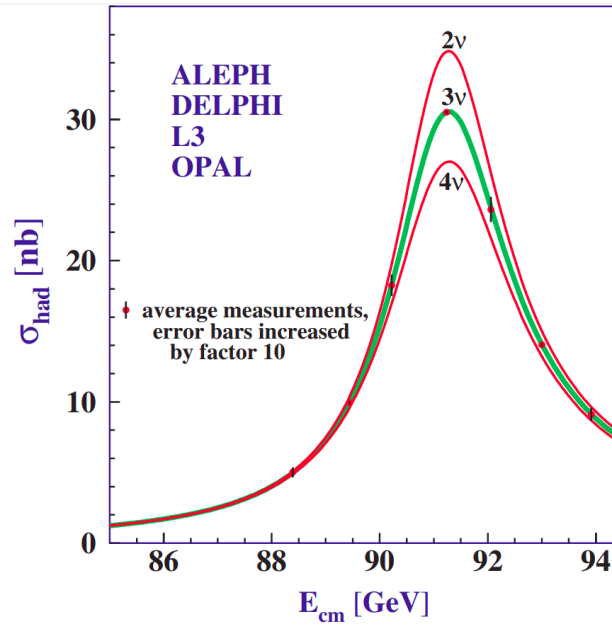


Figure 1.2: The hadron production cross section around the Z^0 resonance measured by the ALEPH, DELPHI, L3, OPAL, SLD dataset [24]. The results show agreement of the data with the hypothesis under SM couplings with three light neutrino generations.

1.1.3 Neutrino Physics Beyond the Standard Model

The Standard Model (SM) has been very successful in predicting the interactions between fundamental particles through three of the four fundamental forces and in assigning mass to particles. However, limitations exist when it comes to explaining certain phenomena observed in the neutrino sector. Initially, the SM posited that neutrinos were massless, as there are no SM interactions involving right-handed neutrinos, effectively implying that right-handed neutrinos do not exist. Without right-handed neutrinos, a Dirac mass term cannot be constructed. However, this assumption is no longer held following the discovery of neutrino oscillation.

Neutrino oscillation is a phenomenon in which neutrinos periodically change their flavours as they travel through space. This occurs because the flavour eigenstates of neutrinos are linear combinations of their mass eigenstates, leading to mixing between these two sets of eigenstates. This mixing can be modelled similarly to quark mixing, using a unitary mixing matrix that includes a complex phase and three mixing angles, directly affecting the oscillation probabilities. As a result, the existence of neutrino oscillation implies that neutrinos must have mass. Furthermore, the complex phase in the mixing matrix can introduce differences in the oscillation probabilities between neutrinos and antineutrinos if CP symmetry is violated here. A formalism of neutrino oscillation, along with current measurements of the oscillation parameters, will be discussed in Section 1.2.

To extend the SM and accommodate neutrino masses, a Dirac mass term is added to the Lagrangian, introducing a Yukawa coupling between the left-handed neutrino and the right-handed neutrino conjugate with the Higgs doublet. A Majorana mass term [25] can also be included, generating a large intrinsic mass for right-handed neutrinos, and explaining the small masses of observed (left-handed) neutrinos through the seesaw mechanism [26]. If such heavy neutrinos exist, CP violation in the neutrino sector could lead to differing decay rates between heavy neutrinos and their conjugate processes, potentially driving leptogenesis in the early

Universe [27]. This leptogenesis could then contribute to baryogenesis, proposed as the mechanism of the matter-antimatter asymmetry observed today [28]. Currently, the precise value of the neutrino mass remains unmeasured, and the exact mechanism responsible for neutrino mass generation is still unclear. The KATRIN experiment has presented the best upper limit of the light electron neutrino mass of 0.8 eV at 90% confidence level (C.L.) [29].

In general, light right-handed neutrino counterparts and heavy neutrinos (either Majorana or higher-generation Dirac type) are considered "sterile", as they do not have SM interactions but may be detectable through rare decays. The detection of NC neutrino-nucleus interactions can also potentially provide indirect evidence of sterile neutrinos. There can be a reduction in the number of expected CC events for a given flavour due to the oscillation of active neutrinos into sterile neutrinos. By comparing the rate of CC events to NC events, a deficit in CC events without a corresponding deficit in NC events would strongly indicate sterile neutrino oscillations. The new NC π^0 samples used in this thesis work is potentially helpful in measuring sterile neutrino oscillations. The energy and baseline at T2K takes the advantage to measure the sterile mixing in the $3 + 1$ oscillation model, with current constraints on the sterile mixing mass difference being $\Delta m_{41}^2 < 3 \times 10^{-3} \text{ eV}^2$ [30]. This provides a foundation for further exploration of potential sterile neutrino signatures in the NC π^0 data.

1.2 Neutrino Oscillation

1.2.1 Three-flavour Oscillation Mechanism

The mixing of neutrino flavour and Dirac mass eigenstates is governed by a 3×3 unitary matrix, known as the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix [31]. Neutrinos are created and interact as flavour eigenstates ν_α ($\alpha \in \{e, \mu, \tau\}$), but propagate as mass eigenstates ν_i ($i \in \{1, 2, 3\}$). Therefore, flavour eigenstates can be

expressed as linear combinations of the mass eigenstates [3],

$$|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle, \quad (1.1)$$

where $U_{\alpha i}^*$ denotes an element in the PMNS matrix describing the mixing strength.

The propagation of the neutrino mass eigenstates can be described by the Schrödinger equation, by defining an initial mass eigenstate $|\nu_i\rangle$, such that

$$\mathcal{H} |\nu_i\rangle = E_i |\nu_i\rangle, \quad (1.2)$$

where $\mathcal{H}$ is the Hamiltonian operator and E_i is the energy of the state $|\nu_i\rangle$. The solution of the state $|\nu_i(t)\rangle$ propagating at time t can thus be solved from

$$i \frac{d}{dt} |\nu_i(t)\rangle = \mathcal{H} |\nu_i(t)\rangle, \quad (1.3)$$

to get a plane-wave solution

$$|\nu_i(t)\rangle = e^{-ip_i \cdot x} |\nu_i\rangle = e^{-i(E_i t - \vec{p}_i \cdot \vec{x})} |\nu_i\rangle, \quad (1.4)$$

where p_i and x are the 4-momentum of the state $|\nu_i\rangle$ and space-time components, respectively. The tiny masses of the neutrinos relative to their energy results in the approximation $|\vec{p}_i| \approx |\vec{p}_j| \approx E$. Hence, the energy E_i of the mass eigenstate $|\nu_i\rangle$ can then be approximated using the binomial expansion:

$$E_i = \sqrt{|\vec{p}_i|^2 + m_i^2} \approx E + \frac{m_i^2}{2E} \quad (1.5)$$

Therefore, the exponential part in Eq.1.4 can be approximated as

$$(E_i t - \vec{p}_i \cdot \vec{x}) \approx (E + \frac{m_i^2}{2E})L - EL = \frac{m_i^2 L}{2E}, \quad (1.6)$$

assuming the distance travelled (a.k.a. the “baseline”) is given by $t \approx L$ for the

ultra-relativistic neutrinos. Eq. 1.4 can thus be written as:

$$|\nu_i(L)\rangle = e^{-i\frac{L}{2E}m_i^2} |\nu_i\rangle. \quad (1.7)$$

Hence the flavour eigenstate after having travelled a distance L can be derived from Eq.1.1

$$|\nu_\alpha(L)\rangle = \sum_{i=1}^3 U_{\alpha i}^* e^{-i\frac{L}{2E}m_i^2} |\nu_i\rangle. \quad (1.8)$$

The oscillation probability is described as the probability of a neutrino starting from flavour eigenstate $|\nu_\alpha\rangle$ but observed as $|\nu_\beta\rangle$ after the mass eigenstate travelled a distance L :

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= |\langle \nu_\beta | \nu_\alpha(L) \rangle|^2 \\ &= \left| \sum_i \sum_\gamma U_{\alpha i}^* U_{\gamma i} e^{-i\frac{L}{2E}m_i^2} \langle \nu_\beta | \nu_\gamma \rangle \right|^2 \\ &= \left| \sum_i U_{\alpha i}^* U_{\beta i} e^{-i\frac{L}{2E}m_i^2} \right|^2 \quad \text{where } \langle \nu_\beta | \nu_\gamma \rangle \equiv \delta_{\beta\gamma} \\ &= \sum_i \sum_j U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-i\frac{L}{2E}\Delta m_{ij}^2}, \end{aligned} \quad (1.9)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ is the mass-squared difference between mass eigenstates $|\nu_i\rangle$ and $|\nu_j\rangle$. The unitarity properties of the PMNS matrix can be applied to rewrite Eq. 1.9 as the most common form for neutrino and anti-neutrino oscillation probabilities:

$$\begin{aligned} P(\nu_\alpha^{(-)} \rightarrow \nu_\beta^{(-)}) &= \delta_{\alpha\beta} - 4 \sum_{i>j}^n \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left(\Delta m_{ij}^2 \frac{L}{4E} \right) \\ &\quad + (-)2 \sum_{i>j}^n \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left(\Delta m_{ij}^2 \frac{L}{2E} \right). \end{aligned} \quad (1.10)$$

From Eq. 1.10, the maximum of the oscillation probability occurs when $(\Delta m_{ij}^2 \frac{L}{4E}) \sim \frac{\pi}{2}$ or $\frac{3\pi}{2}$. Under the three-neutrino-flavour model, only two of the Δm_{ij}^2 parameters are independent, given by Δm_{21}^2 and Δm_{32}^2 .

The PMNS matrix of dimension $N \times N$ can be parameterised by $N(N - 1)/2$ mixing angles and $(N - 1)(N - 2)/2$ physical phase. Under 3-flavour mixing, three mixing angles θ_{12} , θ_{13} , θ_{23} , and one complex phase δ_{CP} are used [3]

$$\begin{aligned} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \quad (1.11)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$. The three mixing angles have physical values $\theta_{ij} \in [0, \frac{\pi}{2}]$, and $\delta_{CP} \in [-\pi, \pi]$ is known as the CP -violating phase.

1.2.2 Matter Effects and Mass Ordering

The oscillation probability derived above assumes that the neutrino mass eigenstates are propagating in the vacuum. In reality, however, neutrino oscillation measurements involve the mass eigenstates propagating through matter. Coherent scatterings between neutrinos and matter can occur, but the interaction channels differ between ν_e and other neutrino flavours due to the composition of matter, which consists of protons, neutrons, and electrons.

All neutrino and antineutrino flavours can interact via NC interactions, but only ν_e can undergo CC interactions because of the presence of electrons in matter. NC interactions do not differentiate between neutrino flavours and, therefore, do not directly affect oscillation probabilities because they do not cause any overall phase shift in the mass eigenstates. However, CC interactions can introduce additional phase shifts in the mass eigenstates, effectively altering the propagation of mass eigenstates.

This asymmetry between ν_e ($\bar{\nu}_e$) and other (anti-)neutrino flavours generates an effective potential $V_{\text{matter}} = +(-)\sqrt{2}G_F n_e$ in addition to the vacuum Hamiltonian [32], where G_F is the Fermi constant and n_e is the electron density. The opposing matter effects between neutrinos and antineutrinos can introduce differences in their oscillation probabilities, apart from effects of δ_{CP} .

The matter effect is the key to resolve the “solar neutrino problem”, where the observed neutrino event rates were significantly lower than the theoretical predictions at the time [33]. As solar neutrinos travel from the core to the outer layers of the Sun, they encounter regions with varying electron densities. This variation alters the composition of the mass eigenstates that make up the ν_e . At certain critical electron densities, the difference between the effective mass eigenstates of ν_e and other neutrino flavours becomes minimal, thereby maximising the oscillation probability. This phenomenon is known as the Mikheyev-Smirnov-Wolfenstein (MSW) resonance [34, 35]. The presence of the matter effect in the Sun and the Earth also provides sensitivity to determining the ordering between the first two neutrino mass eigenstates, $|\nu_1\rangle$ and $|\nu_2\rangle$ [36, 37].

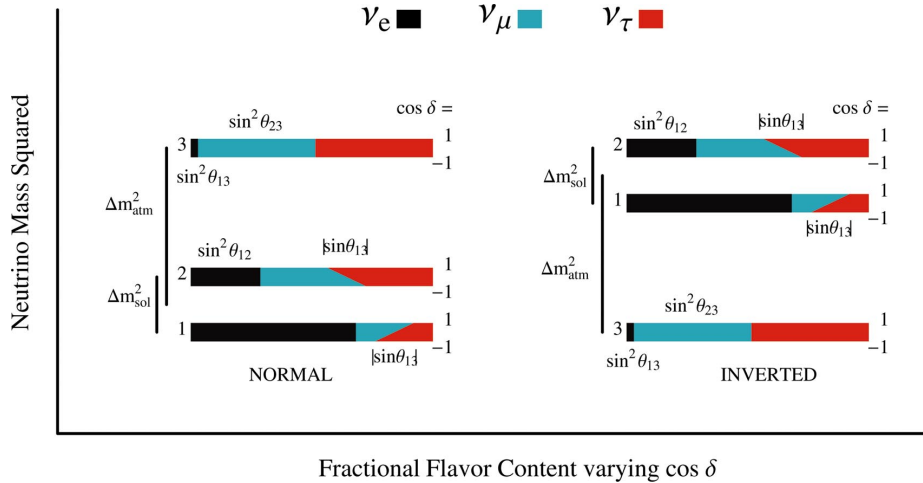


Figure 1.3: The normal and inverted mass orderings, with the colour indicating the probability of finding α flavour in the i^{th} mass eigenstate, with varied CP -violating phase δ [38].

The determination of Δm_{21}^2 has provided two possible mass ordering solutions among the three neutrino states: the normal ordering (NO) $m_1 < m_2 < m_3$ and the inverted

ordering (IO) $m_3 < m_1 < m_2$, making $\Delta m_{32}^2 > 0$ and $\Delta m_{32}^2 < 0$, respectively, as shown in Fig. 1.3. The long baseline experiments would provide sensitivity to the sign of Δm_{32}^2 with a baseline of $\mathcal{O}(1000\text{km})$ under the energy region of $\mathcal{O}(\text{GeV})$ [32] to have significant matter effects from the Earth.

For the oscillation analysis presented in this thesis, the matter effect on the oscillation probability is subdominant, and a constant matter density of 2.6 gcm^{-3} is assumed [39].

1.2.3 Oscillation Phenomenology at T2K

The T2K experiment measures neutrino oscillations using the high energy ν_μ ($\bar{\nu}_\mu$) produced from boosted pion decays. The description of the neutrino beamline can be found in section 2.1. The $(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ disappearance and $(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ appearance events are measured at a distance L (km) away from where the neutrinos are produced. The event rates primarily depend on the disappearance and appearance probabilities in vacuum, derived from Eq. 1.10 [40]:

$$\begin{aligned} P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) &\simeq 1 - 4c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right) \\ &\simeq 1 - \sin^2 2\theta_{23} \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right) \end{aligned} \quad (1.12)$$

$$\begin{aligned} P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) &= 4c_{13}^2 s_{13}^2 s_{23}^2 \cdot \sin^2 \Delta_{31} \\ &\quad + 8c_{13}^2 s_{12} s_{13} s_{23} (c_{12} c_{23} \cos \delta_{CP} - s_{12} s_{13} s_{23}) \cdot \cos \Delta_{32} \cdot \sin \Delta_{31} \cdot \sin \Delta_{21} \\ &\quad - (+) 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta_{CP} \cdot \sin \Delta_{32} \cdot \sin \Delta_{31} \cdot \sin \Delta_{21} \\ &\quad + 4c_{13}^2 s_{12}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2c_{12} c_{23} s_{12} s_{13} s_{23} \cos \delta_{CP}) \cdot \sin^2 \Delta_{21}. \end{aligned} \quad (1.13)$$

where $\Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E}$, and E is the neutrino energy in GeV. The matter effect from the Earth can contribute a 10% perturbation to the oscillation probabilities.

There are a few points to highlight regarding how the oscillation parameters

affect oscillation probabilities and observed event spectra:

- **Disappearance channel:**

- This channel is primarily sensitive to $|\Delta m_{32}^2|$ and $\sin^2 2\theta_{23}$. The amplitude of the oscillation is governed by $\sin^2 2\theta_{23}$. The sensitivity to $|\Delta m_{32}^2|$ comes from the position of the minimum survival probability along energy E , given this modifies the frequency of the oscillation.
- The disappearance requires $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \neq 1$, implies a non-zero Δm_{ij} and a small θ_{13} .
- The measurements of $\sin^2 2\theta_{23}$ do not provide sensitivity to the octant of θ_{23} because $\sin^2 2\theta_{23}$ is symmetric about $\theta_{23} = 45^\circ$.

- **Appearance channel:**

- The oscillation probability in the appearance channel is influenced by $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$, and $\sin \delta_{CP}$. This is sensitive to values of θ_{13} , the octant of θ_{23} , and potential CP -violation effects from the sign flipping of the $\sin \delta_{CP}$ term.
- The amplitude of the oscillation is primarily controlled by $\sin^2 \theta_{13}$. The size of $\sin^2 \theta_{13}$ determines the magnitude of variation in the oscillation probability with changes in δ_{CP} and $\sin^2 \theta_{23}$. Small values of $\sin^2 \theta_{13}$ lead to smaller variations in oscillation probability with respect to δ_{CP} and $\sin^2 \theta_{23}$, and vice versa. Since event rates vary periodically with δ_{CP} , constraints on $\sin^2 \theta_{13}$ from reactor experiments (e.g., Daya Bay [41]) can improve sensitivity to δ_{CP} precision.
- The interplay of these oscillation parameters can lead to degeneracies in the observed event rates. Fig. 1.4 shows the predicted oscillated ν_e and $\bar{\nu}_e$ event rate at T2K for several combinations of the oscillation parameter values, highlighting these degeneracies and the difficulties in disentangling them.

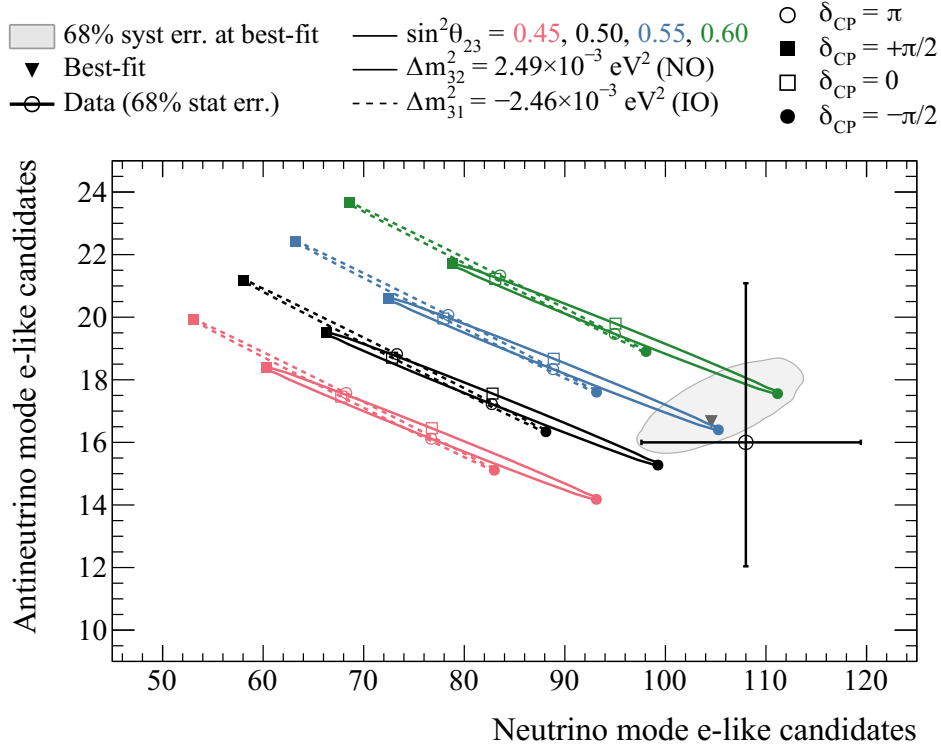


Figure 1.4: The number of ν_e and $\bar{\nu}_e$ events simulated from different oscillation parameter values to show the degeneracy in the parameter values on predicted event rate. The best-fit value over all parameters and the corresponding 1σ contour to the T2K data is indicated [42].

1.2.4 Oscillation Parameter Measurements

Apart from accelerator-based experiments, studies of neutrino oscillations have been conducted using neutrinos produced by the Sun, the Earth's atmosphere, and nuclear reactors, covering oscillation across different channels, energy regions, and baselines. In 1968, the Homestake experiment observed a deficit in the solar neutrino interaction rates, leading to the hypothesis of neutrino oscillation [33]. This hypothesis was later confirmed by atmospheric neutrino measurements at Super-Kamiokande (Super-K) in 1998 [43] and by Solar neutrino measurements at the Sudbury Neutrino Observatory (SNO) in 2001 [44].

Precision measurements of θ_{12} and Δm_{21}^2 were conducted from the ν_e oscillation channel. Solar ν_e were observed via the charged-current elastic neutrino scattering (CCE) processes inside the Super-K detector [45], as well as through interactions

with deuterons observed from SNO [44]. KamLAND further constrained Δm_{21}^2 using long baseline reactor $\bar{\nu}_e$ detected through inverse beta decay processes [37].

High energy atmospheric and accelerator long baseline neutrino experiments have provided sensitivity for measuring θ_{23} and $|\Delta m_{32}^2|$ with the most precise constraints coming from the T2K and NO ν A experiments. However, there remains a discrepancy regarding the preference for normal mass ordering (NO) or inverted mass ordering (IO) [46]. After combining data from accelerator long-baseline, atmospheric, and short-baseline reactor experiments, IO is disfavoured at just below a 3σ significant level [3]. The value of θ_{23} was found to be very close to 45° , but the significance of being in the lower octant is not excluded by more than 3σ [3]. Determining the mass ordering and θ_{23} octant will be one of the primary objectives for future long-baseline accelerator, atmospheric, and reactor experiments, such as Hyper-K, DUNE, and JUNO [47].

The precise measurement of θ_{13} was performed by short-baseline reactor experiments (Daya Bay [48], RENO [49], and Double Chooz [50]) from the $\bar{\nu}_e$ disappearance channel, providing strong constraints on $\sin^2 2\theta_{13}$. Part of the oscillation analysis performed in this thesis will have oscillation sensitivities assessed by applying these constraints.

Precision measurements of δ_{CP} and search for CP -violation in the neutrino sector are the alternative goals of the long baseline experiments which compares event rates between the $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ channels. The current measurement of δ_{CP} from T2K suggests a value close to $-\frac{\pi}{2}$ (or equivalently, $\frac{3\pi}{2}$) [42], excluding CP -conserving values ($\delta = 0$ or π) at more than 90% C.L. However, results from NO ν A favour a δ_{CP} near $\frac{\pi}{2}$ under the normal mass ordering [51].

Detailed reviews of neutrino oscillation measurements using long-baseline neutrinos, atmospheric neutrinos, and reactor neutrinos can be found in Refs. [52, 53, 54]. The most recent results from a global-fit [55] of the oscillation parameters are shown in Table 1.1.

Table 1.1: Current global-fit to the measurements of neutrino oscillation parameters under different mass ordering (if applicable) provided by NuFit [55].

Parameter	NO	IO
Δm_{21}^2 [10^{-5}eV^2]	$7.41^{+0.21}_{-0.20}$	
$\sin^2(\theta_{12})$	$0.307^{+0.012}_{-0.011}$	
$\sin^2(\theta_{13})$ [10^{-2}]	$2.224^{+0.056}_{-0.057}$	$2.222^{+0.069}_{-0.057}$
Δm_{32}^2 [10^{-3}eV^2]	$2.505^{+0.024}_{-0.026}$	$-2.487^{+0.027}_{-0.024}$
$\sin^2(\theta_{23})$	$0.454^{+0.019}_{-0.016}$	$0.568^{+0.016}_{-0.021}$
δ_{CP} [rad]	$-2.23^{+0.68}_{-0.44}$	$-1.51^{+0.41}_{-0.45}$

1.3 Neutrino-Nucleus Interaction

1.3.1 Interaction Types

Neutrinos are measured through observations of the outgoing particles when the neutrinos interact with target nuclei. The CC interactions will produce a charged lepton with the same flavour as the incoming neutrino and can be directly observed by the detector. NC interactions don't produce charged leptons from the primary interaction vertex, therefore, the flavour and kinematic information of the incoming neutrino cannot be reconstructed. The NC channel may act as a background if the outgoing particles produce an indistinguishable visible topology compared to a CC interaction, where details on the possible source and treatment for an oscillation analysis will be described in Chapters 2 and 4. This section will introduce the dominant interaction topologies at T2K.

CC0 π

The CC0 π topology doesn't contain any pions in the outgoing particles from the primary interaction vertex. The interaction types with this topology include the quasi-elastic interactions (CCQE) and the interaction with multiple nucleons (2p2h). Fig. 1.5 shows the CCQE interaction, where the neutrino interacts with a single nucleon to produce a lepton and an outgoing nucleon. The 2p2h interaction instead

has the neutrino interacting with two strongly correlated nucleons or two-nucleon states with meson exchange currents (MEC) and produces two outgoing nucleons, with examples shown in Fig. 1.6. A review of different MEC situations can be found in Ref. [56].

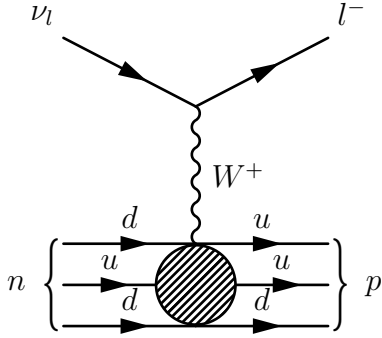


Figure 1.5: The CCQE Feynman diagram.

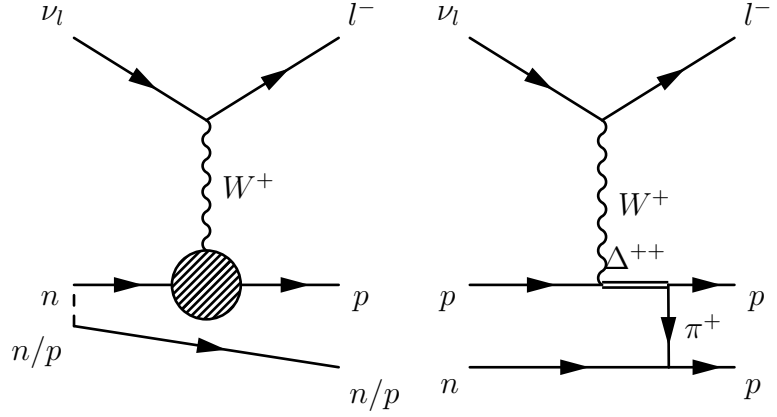


Figure 1.6: Diagrams of the 2p2h interaction, with examples of (left) correlated nucleons and (right) MEC.

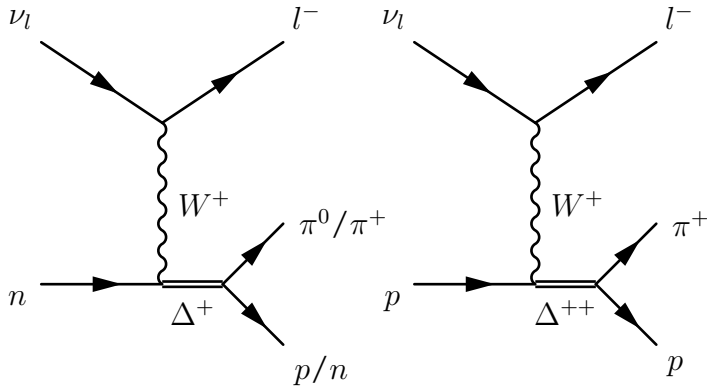


Figure 1.7: The CC resonant pion production.

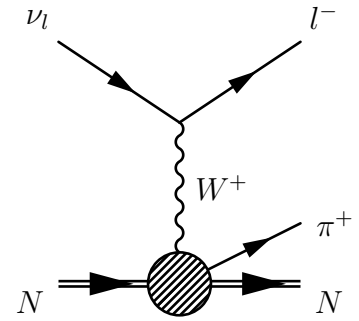


Figure 1.8: The CC coherent pion production.

CC1 π^0 / CC1 $\pi^\pm$

The CC1 π topology includes 1 outgoing pion from the primary interaction vertex. One example of this is when the nucleon is excited into a $\Delta(1232)$ resonance state

when the neutrino interacts with a nucleon, and then decays into a pion and a nucleon. The Feynman diagrams in Fig. 1.7 demonstrate the resonance pion interaction targeted on a neutron and a proton, respectively. Another possible method of pion production comes from the coherent interactions of the neutrino with the whole nucleus (CC_{Coh}), which leaves the nucleus unchanged, as shown in Fig 1.8.

CC DIS

The deep inelastic scattering (DIS) occurs when a neutrino scatters off an individual quark of a nucleon and breaks the nucleon to produce hadronic showers. The interaction becomes dominant at higher-energy neutrinos. In T2K measurements, the hadrons produced through this process can contribute to background events.

$\text{NC}1\pi^0 / \text{NC}1\pi^\pm$

Single pion production from the NC channel can act as the background in the oscillation measurements. Similar to the CC channel, the production of the pion can be from a Δ resonance or a coherent interaction. In particular, the $\text{NC}\pi^0$ events are the main focus for this thesis on studying its kinematic properties and effects on oscillation sensitivities. Figs. 1.9 and 1.10 demonstrate the possible pion production process at the NC channel.

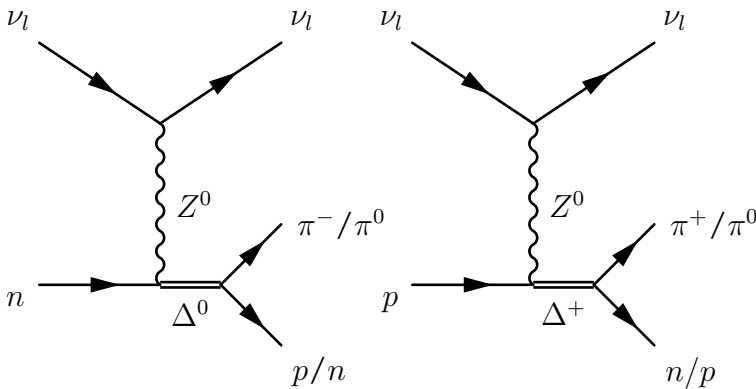


Figure 1.9: The NC resonant pion production.

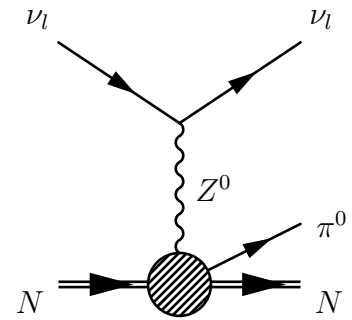


Figure 1.10: The NC coherent pion production.

1.3.2 Nuclear Effects

Nuclear effects are significant for neutrino energies around 1 GeV, where this will involve interactions with the whole nucleon rather than the partons of the nucleon. The modelling of the nuclear effects is a key step to understanding the interaction process inside the nuclear medium and constraining the kinematics of the final state particles from the interaction. The Benhar Spectral Function (SF) [57] method is used for predicting nuclear effects at T2K, which provides a shell-model-based description of the nuclear Fermi motion, the nuclear removal energy and the initial correlations of the nucleons.

There can also be the *final state interactions* (FSI) occurs inside the nucleus, where particles produced from a neutrino-nucleon interaction can re-interact through elastic scattering, charge exchange, additional pion production, or get absorbed by other nucleons. This will change the event kinematic properties or topology when observed by a detector and inaccurately reconstruct the incoming neutrino energy. For the energy region covered at T2K, pion FSI is the dominant uncertainty in oscillation analyses, where the process is simulated using an Intra-Nuclear cascade model, described in Ref. [58].

1.3.3 Interaction Cross Section

The interaction cross section describes the probability for a particular interaction to occur. The cross section of neutrino-nucleon interactions can be derived from theoretical models, usually described by a set of vector and axial vector form factors. The vector form factors describe the electric and magnetic field of the nucleon, constrained by electron scattering experiments; while the axial vector describes the weak interactions. Understanding the values and uncertainties of parameters in the model that affect the cross sections is important to accurately predict the event rates from the different interactions and reconstruct the incoming neutrino information.

The modelling of the interaction cross section for T2K is performed using the

NEUT event generator [59]. Fig. 1.11 and Fig. 1.12 shows the interaction cross sections modelled by NEUT over the true neutrino energy (E_ν), with detailed breakdowns in the CC and NC modes, respectively. The CCQE interactions are dominant at energies below 1 GeV, where the fraction of CC resonance pion productions (CC Res π) gradually increases and becomes dominant above 1 GeV. As the energy approaches 2 GeV, contributions from multi- π and DIS processes become significant.

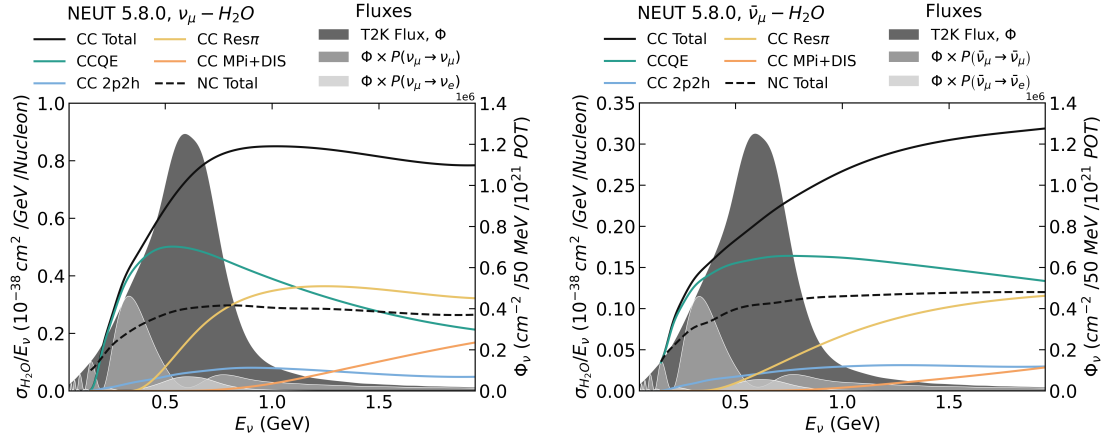


Figure 1.11: The cross sections of neutrino (left) and anti-neutrinos (right) with the nucleus breakdown by CC interaction modes, predicted by the NEUT generator [59]. The distribution in grey peaked around 0.6 GeV is the T2K unoscillated ν_μ flux, together with the predictions of ν_e appearance and ν_μ disappearance shown.

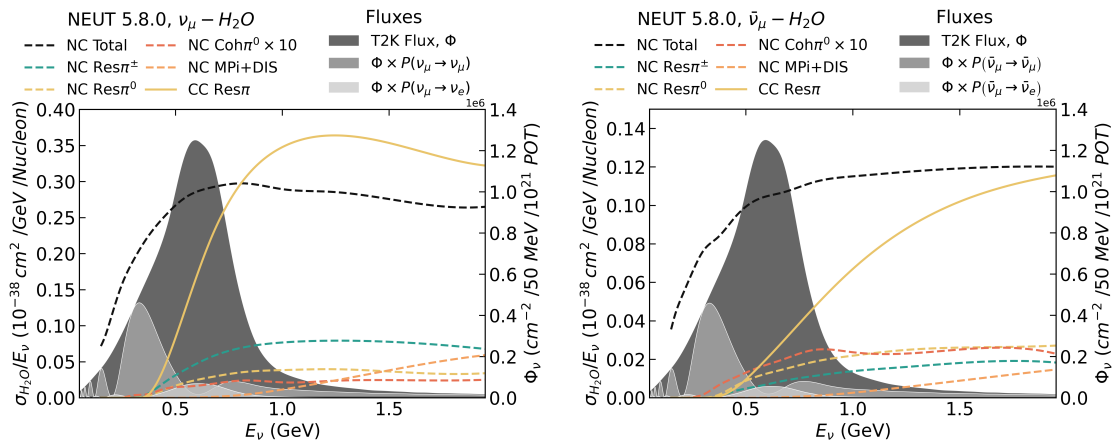


Figure 1.12: The cross sections of neutrino (left) and anti-neutrinos (right) with the nucleus breakdown by NC interaction modes, predicted by the NEUT generator [59]. NC Elastic (NCE) interaction is the dominant mode under NC inclusive, but not particularly showing due to the similar shape as CCQE. The distribution in grey peaked around 0.6 GeV is the T2K unoscillated ν_μ flux, together with the predictions of ν_e appearance and ν_μ disappearance shown.

The NC modes exhibit lower cross sections compared to CC modes. The disparity arises from several factors, including differences in weak interaction coupling constants induced by the Weinberg angle and the Cabbibo angle [12], the Clebsch-Gordan isospin factors associated with the interactions [60], and the overall phase space available for the final states in NC interactions. A more detailed breakdown of the generated cross sections into CC and NC interaction modes is available in Ref. [59].

The primary source of uncertainty in the neutrino-nucleon interaction cross section for the CCQE channel arises from the axial form factor F_A , expanding at $Q^2 = 0$ to give a dipole form [61],

$$F_A(Q^2) = F_A(0) \left(1 + \frac{Q^2}{M_A^{QE^2}} \right)^{-2}, \quad (1.14)$$

where Q^2 is the 4-momentum transfer, $F_A(0)$ is measured from the neutron β decay [62], and M_A^{QE} is a free parameter constrained by the bubble chamber neutrino-deuterium scattering data [63]. The 2p2h interactions cross section is modelled with modified form factors and an energy dependence referred from the Nieves, Martini, and SuSAv2 models [64, 65, 66].

Similarly, the single-pion interaction from the isospin I-3/2 $\Delta(1232)$ resonances can also be described by a dipole form, derived from the Rein-Sehgal model [67],

$$C_A^5(Q^2) = C_A^5(0) \left(1 + \frac{Q^2}{M_A^{Res^2}} \right)^{-2}, \quad (1.15)$$

where $C_A^5(0)$ and M_A^{Res} are the two free parameters. There are also parameters associated with the isospin-1/2 component to model the non-resonance interactions (referred to as the I-1/2 background). For the cross section of the coherent pion production, it turns out the Berger-Sehgal model [68] agrees better with the experimental data.

The DIS and multi- π interaction cross sections are derived from a parton density

distribution determined the energy transferred from the neutrino to the hadronic system and the momentum carried by the scattered quark [58], with the Bodek-Yang correction [69] applied at low Q^2 . This correction refines nucleon structure functions, ensuring a smoother transition from the high- Q^2 perturbative regime to lower Q^2 , where nonperturbative effects like hadron formation and quark confinement dominate. As a result, theoretical predictions align more closely with experimentally observed quark distributions. For the multi- π scenario, the multiplicity AGKY model [70] is employed to tune the number, charge, and momenta of the pions produced.

NC π^0 Cross Section

The modelling of the NC π^0 cross sections heavily depends on the modelling and data of the CC1 π events that are used to constrain the C_A^5 , M_A^{Res} , and I-1/2 background parameters. T2K is good at measuring the CC1 π events using the ND280 detector to have those parameters well-constrained, but it does not fully describe the NC π^0 cross section due to the fundamental differences from the NC and CC interactions. There were previously no specific parameters and event samples used to provide constraints on the NC π^0 cross sections independent of the CC1 π events. The NC π^0 sample selection from the ND280 detector is challenging due to the large background from other charged particles [71].

Development of NC π^0 event samples from the Super-K detector has been ongoing, with their influence gradually integrated into the T2K oscillation analyses [72, 73, 74]. In parallel, an expanded neutrino-nucleus cross-section uncertainty model is under development to more accurately determine NC π^0 interaction cross sections using the new samples. Meanwhile, these NC π^0 samples have been produced with high efficiency and purity, and methods to estimate detector-related systematic uncertainties have been established. Nevertheless, a deeper understanding of the sample characteristics is required to optimise the treatment of these systematic uncertainties. This thesis presents the first steps toward constructing a systematic

uncertainty model for $\text{NC}\pi^0$ events to ensure their proper incorporation in the oscillation analyses.

Experiments like MicroBooNE also carried out measurements of the $\text{NC}\pi^0$ cross sections, a recent result is shown in Fig. 1.13 compared to the predictions from various event generators. The plot for the low proton energy (< 35 MeV) case indicates that the T2K default generator NEUT predicts a lower differential cross section than the measured data at $P_{\pi^0} < 0.3$ GeV and flips over as $P_{\pi^0} > 0.4$ GeV. While for the high proton energy (> 35 GeV) case, NEUT predictions are overall higher than the data and deviated further from the data as P_{π^0} increases.

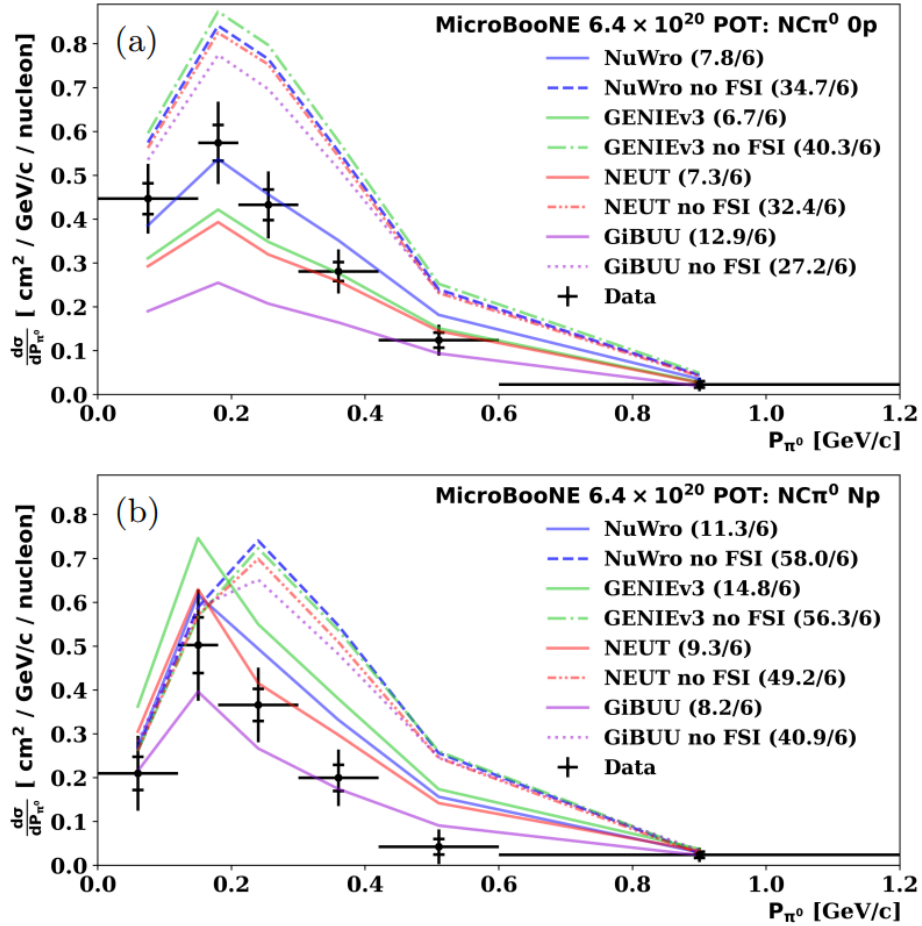


Figure 1.13: The $\text{NC}\pi^0$ differential cross-section measurement from MicroBooNE with (a) no protons with kinetic energy above 35 MeV and (b) at least one proton above 35 MeV in the final states [75]. The statistical (inner bar) and total (extended line) uncertainties are displayed for each data point. The coloured lines show predictions from different generators and the corresponding χ^2/ndf of the fit.

Chapter 2

T2K and Future at Hyper-Kamiokande

T2K (a.k.a. Tokai-to-Kamioka) [76] is a long-baseline neutrino oscillation experiment using an accelerator-produced 0.6 GeV neutrino beam from the Japan Proton Accelerator Research Complex (J-PARC). An overview of the experimental facilities is shown in Fig. 2.1. The beam properties are measured both 280 m from the production point and at the Super-Kamiokande (Super-K / SK) detector after 295 km. The experiment has been measuring neutrino oscillation parameters and CP -violation effects in the neutrino sector.

A next-generation experiment, Hyper-Kamiokande (Hyper-K / HK) [77], is under construction using the (upgraded) J-PARC neutrino beam and the near detector suite from T2K. Hyper-K will be 8.3 times larger than Super-K in fiducial volume and the neutrino beam will be twice as powerful, increasing the neutrino interaction rate by a factor of approximately 17. Hyper-K will also come with an additional intermediate detector 1 km from the production point, IWCD [78], that will measure the neutrino interactions using the same target as the far detector to control the systematic uncertainties in the neutrino flux and interaction modelling.

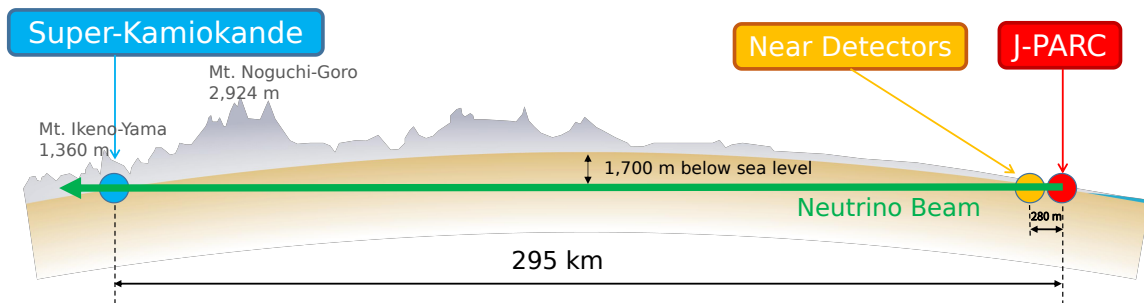


Figure 2.1: A schematic of the T2K neutrino beam from J-PARC to Super-K [76].

2.1 Neutrino Beamline

2.1.1 Beamline Structure

The T2K beamline accelerates protons to 30 GeV using the J-PARC Main Ring (MR) synchrotron, generating a proton beam every 2.48 seconds in 8 bunches (or “spills”), each lasting approximately 80 ns [76]. The proton beam is magnetically focused and bent through the primary beamline to impinge on a 91.4 cm-long monolithic graphite target. This collision produces mesons, which decay into neutrinos throughout the secondary beamline towards the ND280 near detector. A schematic of the T2K beamline is shown in Fig. 2.2.

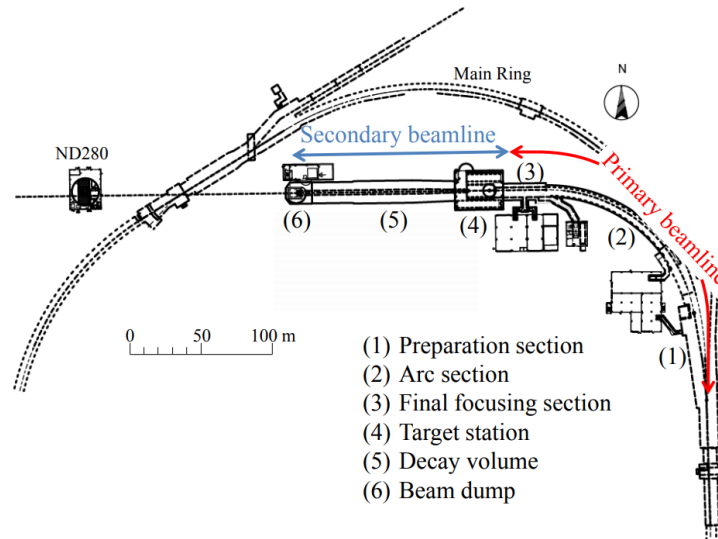


Figure 2.2: The structure of the neutrino beamline at J-PARC [76].

The position, direction, intensity, and size of the proton beam, just upstream of the target, are precisely monitored using electrostatic monitors (ESMs), segmented secondary emission monitors (SSEMs), and an optical transition radiation (OTR) monitor (see Fig. 2.3). These monitors track the beam profile at multiple points to determine its trajectory. The Y position of the beam is constrained by the SSEM19 and the OTR after the vertical bending magnet (FVD2). The OTR monitor, positioned 30 cm upstream of the target, measures the beam width and position with an accuracy

better than $500 \mu\text{m}$ [79]. Accurate Y position measurements are essential for minimising systematic uncertainties in the off-axis neutrino flux directed towards the Super-K detector.

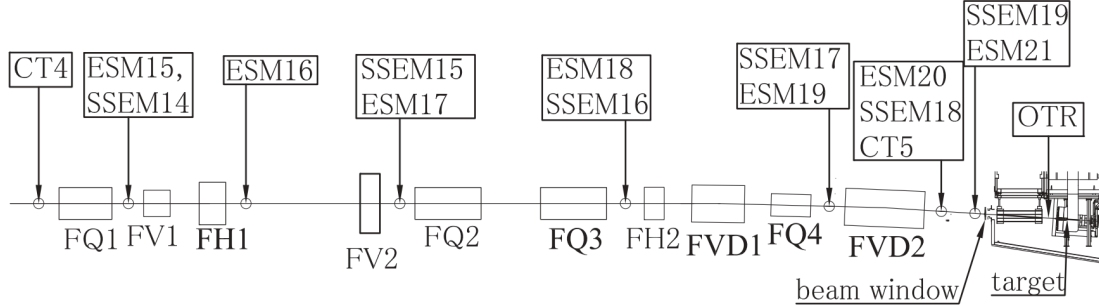


Figure 2.3: The arrangement of proton beam monitors at the final focusing section of the primary beamline upstream the target. [76].

The interaction of the proton beam with the target produces a shower of secondary particles, dominated by charged pions ($\sim 95\%$), kaons ($\sim 5\%$), and a small fraction of muons [80]. These particles are focused or deflected by the first magnetic horn (Horn 1), depending on the chosen polarity of the magnetic field, to produce either a ν -mode or $\bar{\nu}$ -mode beam (see Fig. 2.4). Two additional horns (Horn 2 and Horn 3) further focus or deflect the opposite charged hadron tracks. The hadrons then decay in a 96 m-long decay volume, producing $\bar{\nu}_\mu^{(-)}$ through the processes of $\pi^{+(-)} \rightarrow \mu^{+(-)} \bar{\nu}_\mu^{(-)}$ and $K^{+(-)} \rightarrow \mu^{+(-)} \bar{\nu}_\mu^{(-)}$.

The remaining hadrons and muons with momenta below 5 GeV are absorbed by a beam dump at the end of the decay volume, to maximally purify the $\bar{\nu}_\mu^{(-)}$ beam. Contamination from intrinsic $\bar{\nu}_e^{(-)}$ and wrong-sign components ($\bar{\nu}_\mu$ in ν -mode or ν_μ in $\bar{\nu}$ -mode) can arise from secondary or tertiary particle decays. The contamination aims to be constrained through flux predictions using hadron production models and measurements at the ND280 detector. A GeV-energy $\bar{\nu}_\mu^{(-)}$ can then be produced. The remaining high penetrating muons are monitored using a muon beam monitor (MUMON) [81], located downstream of the beam dump. The MUMON measures the direction of the muon beam with a precision of 0.028 mrad. This measurement can be used to evaluate the stability of the neutrino beam.

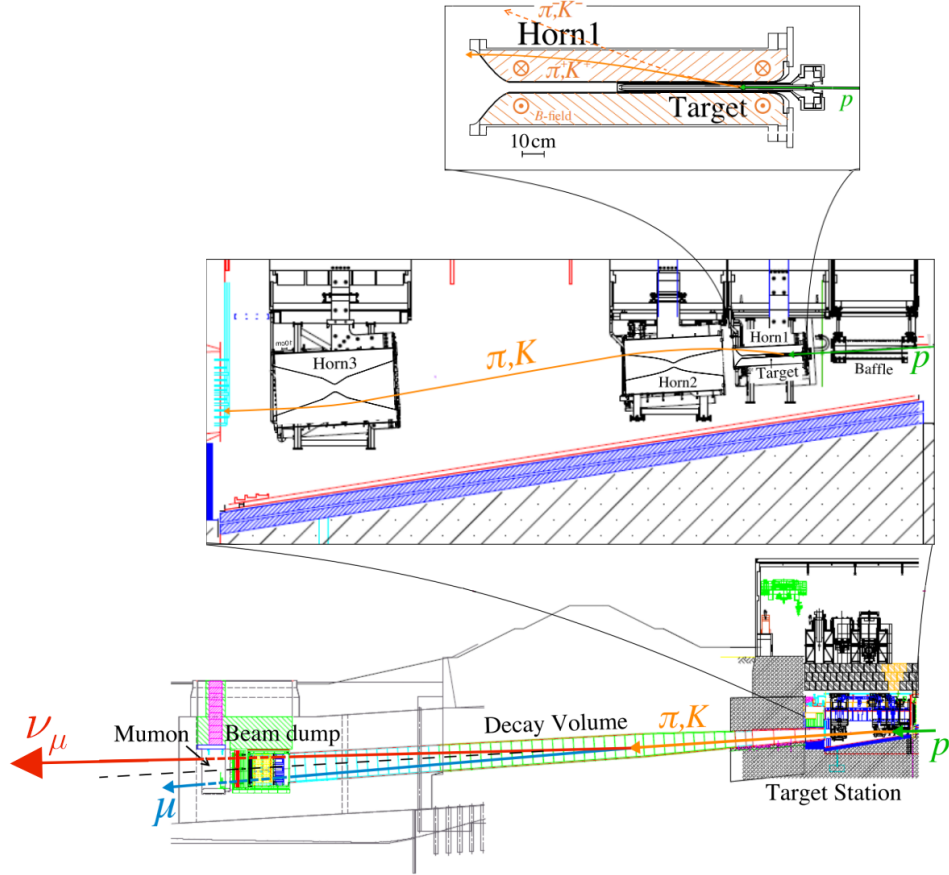


Figure 2.4: A cross-section from the side view of the secondary neutrino beamline, showing the structure from the target station to the muon beam monitor (MUMON) and a sketch indicating the focused and deflected hadron tracks throughout the magnetic horns [76].

2.1.2 Beam Properties

The current beam power has reached a maximum of 800 kW, a significant improvement since Run 11 (515 kW) thanks to the T2K beamline upgrade [82]. The accumulated proton-on-target (POT) from all the Runs is shown in Fig. 2.5. Further upgrades are underway to increase the beam power to 1.3 MW, in preparation for the next-generation Hyper-K experiment – more details see Section 2.4.

The near and far detectors are placed 2.5 degrees off the neutrino beam centre to allow the peak of the neutrino energy spectra narrowing down at 0.6 GeV, where the $(\bar{\nu})_e$ appearance probability (described in Eq. 1.9) can be maximised for a 295 km baseline, as illustrated in Fig. 2.6. Around this energy region, the CCQE interactions

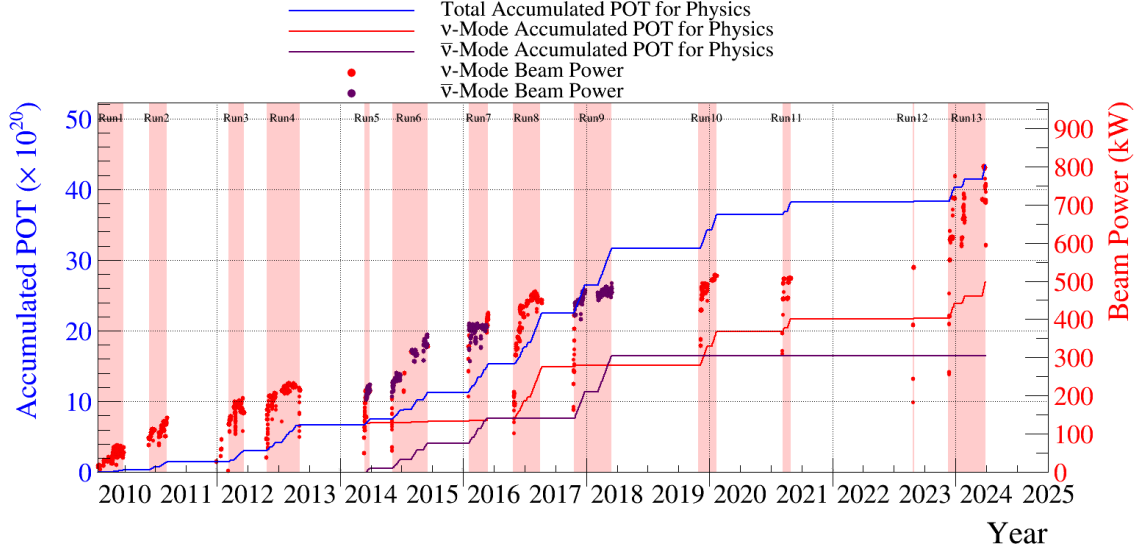


Figure 2.5: The accumulated proton-on-target (POT), beam power and run period for each 'Run' of the J-PARC beam in years [83].

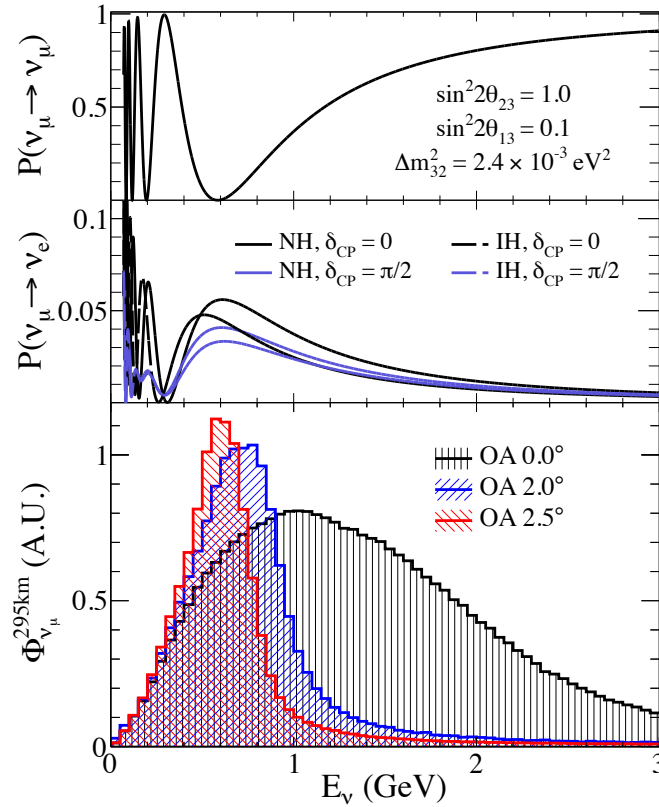


Figure 2.6: (bottom) The unoscillated ν_μ flux at three different off-axis angles (OAA); (top) The ν_e appearance and ν_μ survival probabilities at the 295 km baseline, for $P(\nu_\mu \rightarrow \nu_e)$ under $\delta_{CP} = 0$ and $\frac{\pi}{2}$ with both mass ordering hypotheses (dashed lines for IO). The plots illustrate the alignment between the first oscillation maximum and the peaked flux at 0.6 GeV for the OAA 2.5° case [84].

are dominant (see Fig. 1.11). Since the higher energy neutrinos are more boosted along with the beam axis, interactions from these neutrinos can be filtered out by the off-axis technique, which helps reduce the background from CC and NC interactions that are more difficult to reconstruct.

Near the peak energy, the ν -mode ($\bar{\nu}$ -mode) beam is composed of 97.2% (96.2%) ν_μ ($\bar{\nu}_\mu$), with 0.47% (0.49%) intrinsic ν_e and $\bar{\nu}_e$ contamination, and the remaining dominated by wrong-sign components $\bar{\nu}_\mu$ (ν_μ) [85]. The predicted unoscillated neutrino flux distributions at different baselines (ND280 and Super-K) are shown in Fig. 2.7. The fraction of wrong-sign components is higher in $\bar{\nu}$ -mode compared to ν -mode due to the lower production rate of π^- relative to π^+ [86].

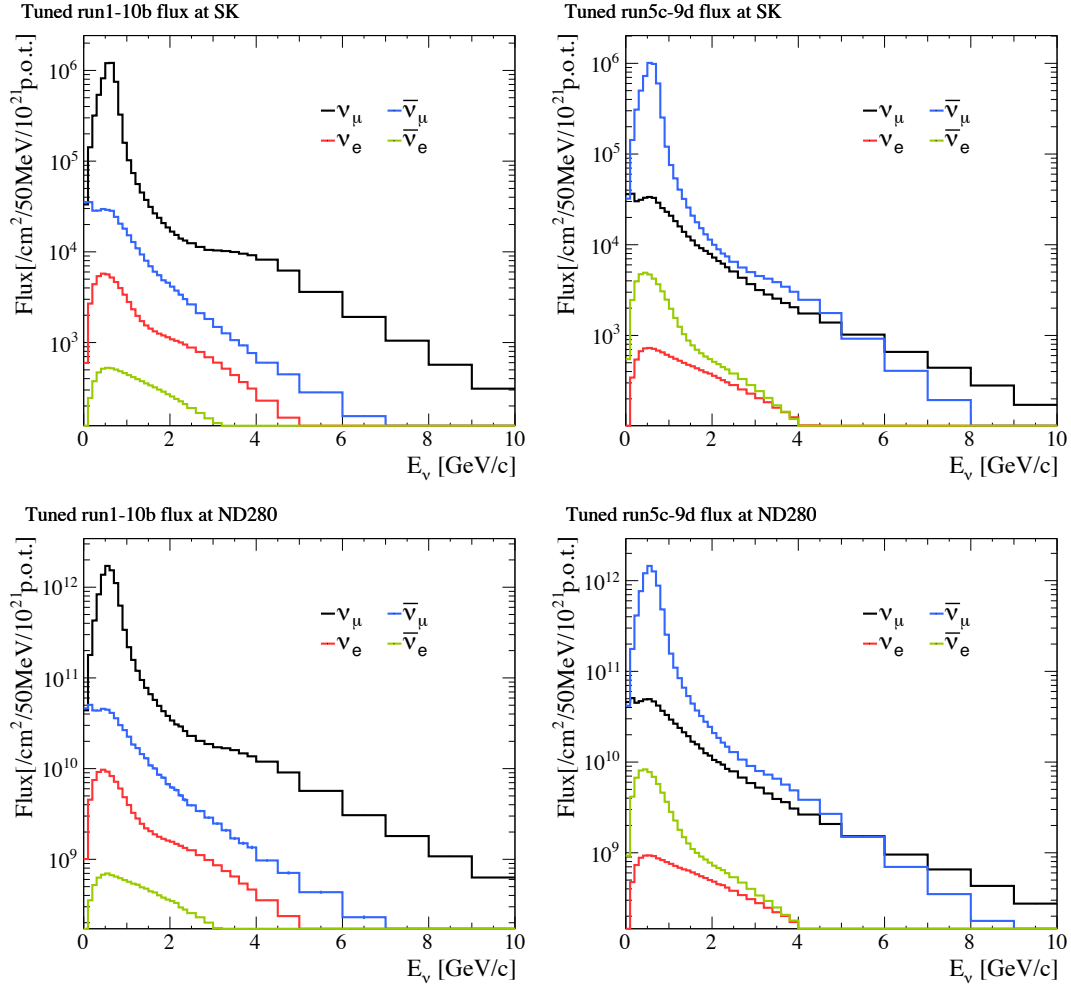


Figure 2.7: The predicted unoscillated neutrino flux at Super-K (top) and ND280 (bottom) tuned with NA61 replica data [86] in ν -mode (left) and $\bar{\nu}$ -mode (right) [87].

2.2 The Near Detector Suite

The near detectors, INGRID and ND280 [42], built 280 m away from the target, aim to measure the neutrino flux before the long baseline oscillations. This helps to predict the neutrino-nucleus interaction event rate at Super-K, and constrain the flux and interaction cross-section uncertainties in the oscillation parameter measurements. A detailed description on constraining these systematic uncertainties using the ND280 data can be found in Ref. [88].

The near detector modules were upgraded during the T2K shut-down period, and neutrino beam data collected after Run 11 utilised the new configuration. However, for the oscillation analysis presented in this work, data from the pre-upgrade configuration is used. This section provides an overview of the detector configuration prior to the upgrade, while details of the upgraded system can be found in Section 2.4.2.

The INGRID detector is positioned along the primary proton beamline axis to monitor neutrino beam profile. It has a cross-shaped configuration with dimensions of 10 m $\times$ 10 m, perpendicular to the beam, covering an angular area of ± 18 mrad (see Fig. 2.8) [89]. INGRID consists of 16 identical modules, each containing 9 iron plates as the neutrino-nucleus interaction target, and 11 scintillator planes for tracking charged particles. INGRID achieves a 10 cm precision for beam centre, corresponding to 0.4 mrad angular precision at 280 m downstream from the target.

The ND280 detector suite is positioned 2.5° off the neutrino beam axis. It is surrounded by a 0.2 T magnet yoke and consists of a π^0 detector (P0D), two fine-grained detectors (FGDs) as the neutrino-nucleus interaction target, consist of scintillator plans with the FGD2 also including layers of water reservoirs. Three time projection chambers (TPCs), filled with argon-based drift gas, are placed between the P0D and FGDs to measure particle tracks, aiding in particle identification and reconstruction. A schematic of ND280 is shown in Fig. 2.8.

The P0D detector consists of 40 scintillator planes interleaved with fillable water

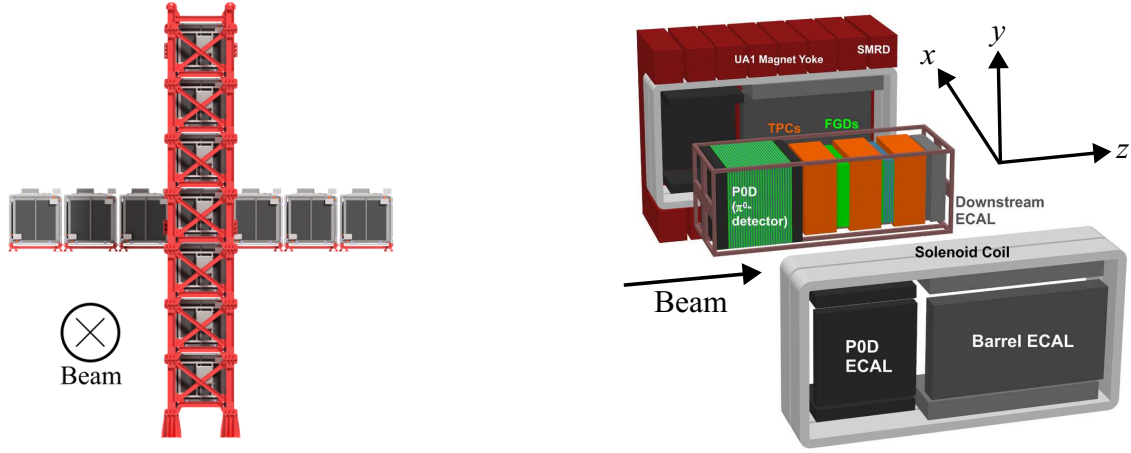


Figure 2.8: (left) The INGRID on-axis near detector measuring the neutrino beam profile; (right) The ND280 off-axis near detector measuring the 2.5° off-axis neutrino flux before the long baseline oscillations [42].

bags, brass, and lead sheets. It is primarily designed to measure $\text{NC}\pi^0$ interactions in water by comparing interaction events with the water bags either filled or empty. Scintillator planes allow the reconstruction of electron and photon tracks from π^0 decays. A previous measurement yielded $106 \pm 41(\text{stat.}) \pm 69(\text{syst.})$ $\text{NC}\pi^0$ events, from data taken with the POD containing water (2.64×10^{20} POT) and without water (3.49×10^{20} POT) [90].

Using the electromagnetic calorimeters (ECals), the charged particles exiting the inner detectors can be further tracked to provide information on particle identification. The energy and direction of the photons can also be measured to help with the event reconstruction of π^0 .

2.3 The Super-Kamiokande Detector (Super-K)

The T2K experiment uses Super-K as the far detector to observe the $\nu_\mu \rightarrow \nu_e$ appearance and ν_μ disappearance events. Super-K is located 295 km from the beam production point. It is a stainless-steel cylindrical tank 1 km underground and about 40 m in height and diameter. It contains a target mass of 50 kton of ultrapure water with 11,146 20-inch photo-multiplier tubes (PMTs) installed on the inner detector (ID)

wall to collect Cherenkov radiation signals with 40% coverage of the ID wall [91]. There is another 2 m thick volume from the outer detector (OD) wall to veto the cosmic ray and other background using 1,885 outward-facing PMTs. A schematic sketch of the Super-K detector is shown in Fig. 2.9.

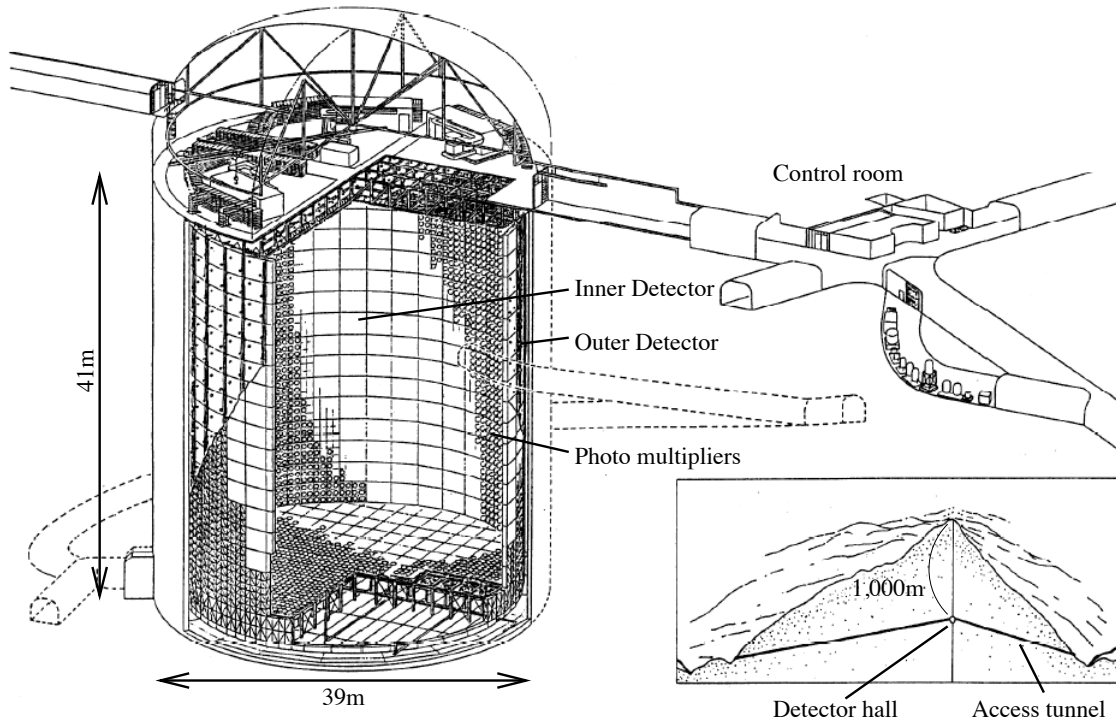


Figure 2.9: The Super-K detector schematic sketch showing the inner and outer detector and the PMT layout [92].

2.3.1 Water Cherenkov Detection Methods

Super-K detects interaction events by observing the Cherenkov radiation emitted from the charged particles produced. A sketch of the process is shown in Fig. 2.10. When the charged particle travels in a medium, light can be produced when the medium is polarised by the charged particle when it returns to the ground state. In water, light travels at a speed of c/n , where n is the refractive index of water and c is the speed of light in the vacuum. For the case when a charged particle travels faster than the speed of light in water, the wavefronts of the light from the polarised medium overlap with each other to form constructive interference, which generates

a light cone until the speed of the particle drops below c/n . The cone of the light can then leave a ring shape when mapped over to the ID wall. The minimum energy required for a charged particle to emit Cherenkov radiation is called the *Cherenkov threshold* of the particle. This will affect the reconstructed event topology of the same interaction type, as low energy large mass charged particles produced may not directly be observed.

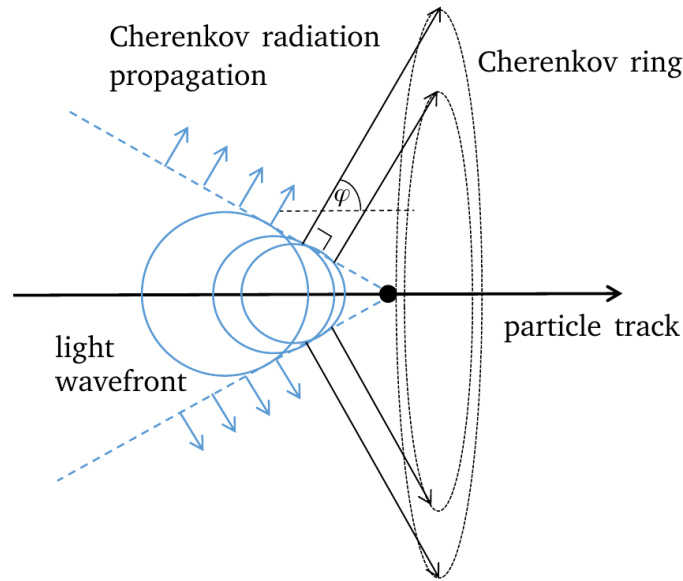


Figure 2.10: The sketch of the Cherenkov radiation emitting process when a charged particle travels faster than the speed of light in the medium.

The Cherenkov ring pattern can provide a range of information to reconstruct an interaction event. The timing distributions of the hits on the PMT can be used to reconstruct the location of the cone vertex, while the ring edges and thickness will provide the direction of the travel of the particle. The momentum of the particle can be reconstructed from the intensity of the hit based on the photo-electric (pe) charge distributions, while this also benefits from the thickness of the ring that provides the distance the particle travelled during the time it is above the Cherenkov threshold.

The pattern of the ring is used for particle identification (PID). The electron-like (e -like) ring has a fuzzy shape due to the frequent scattering with other electrons to create electromagnetic showers, while the muon-like (μ -like) ring is much sharper due to the heavy mass of the muon that penetrates strongly through the medium

without getting scattered. Fig. 2.11 shows the event display of Super-K, emphasising the comparisons between a e -like and a μ -like ring. The other commonly observed particles in Super-K are the photons, which emit 1-ring e -like patterns due to the collinear electron pairs produced from the photons. The hadrons can shower to also produce fuzzy rings at high energies, or two narrow rings from hadronic scatter at lower energies.

Super-K doesn't include a magnetic field to identify the charge of the emitting particle. The identification of the ν or $\bar{\nu}$ is based on the beam mode setup at J-PARC and constraints of the wrong-sign components in the beam from near detectors.

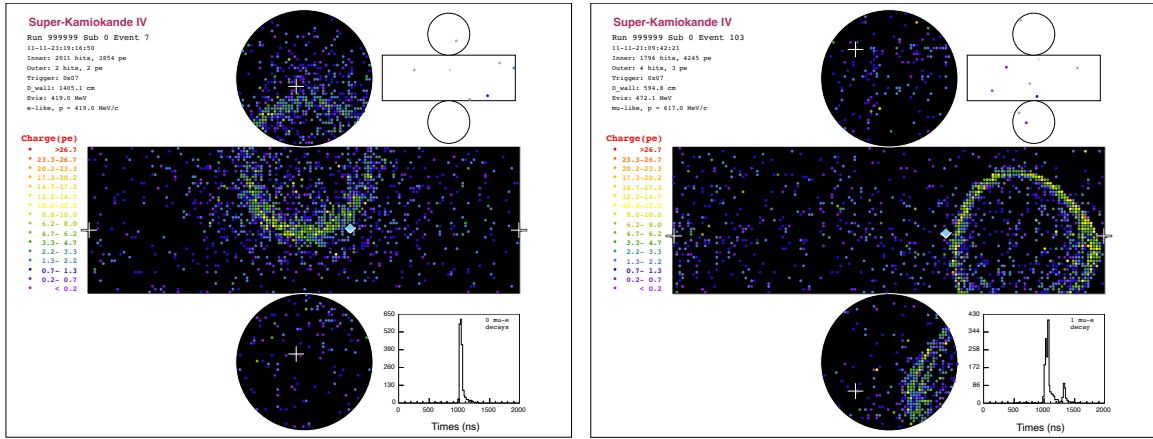


Figure 2.11: Event display of the Super-K detector, showing a typical electron-like ring (left) and muon-like ring (right). Each dot represents the hits on 1 PMT, where the colour represents the photo-electron (pe) charge accumulated. The white crosses indicate the location of the reconstructed vertex. The diamond shows the intersection point from the vertex to the ID wall along the neutrino beam direction. The bottom right shows the distributions of the arrival time of light on the PMT, where in particular it indicates a delayed signal of approximately $2 \mu\text{s}$ for a Michel electron in the muon-like ring case. The top right display shows the hits from the OD [93].

2.3.2 Event Reconstruction

The reconstruction of events from Cherenkov rings is carried out using the fiTQun algorithm [94]. This algorithm is based on a maximum likelihood method, which fits the observed ring pattern to a hypothesised topology. The likelihood is determined by calculating the probability of each PMT either registering a hit or not, as well as

the likelihood of the charge and time associated with the registered hit for a given vertex position, production time, and the particle's momentum/direction. By comparing the likelihood ratios of two different hypotheses, the algorithm determines which topology is more likely to explain the observed topology.

The likelihood calculations start with a single-ring hypothesis and progressively add rings, up to a maximum of six. A new ring is only accepted if the increase in likelihood exceeds a set threshold, ensuring it represents a genuine feature rather than a statistical artifact. Meanwhile, the algorithm is also set to identify whether the ring comes from the primary or secondary interaction vertex based on the timing of the signal. A detailed description of the event reconstruction techniques for Super-K can be found in [95].

The neutrino oscillation analysis selects events that are considered to be clearly reconstructable. There are several criteria needed to qualify an event:

- The event is *fully contained* (FC): the whole process occurs within the ID volume, with less than 16 hits from the OD.
- The event is within the *fiducial volume* (FV): the effective volume within the ID, to ensure some distance between the interaction vertex and the ID wall.
- The event is above certain *visible energy* (E_{vis}): this is calculated from the amount of charge recorded by the detector to select events with showers produced with sufficiently strong Cherenkov radiation.

Once the basic qualification criteria are met, the next step is to identify the event topology. Based on the number of rings, the PID for an event is determined by comparing the likelihood ratios of different topologies. This process helps distinguish between various signal events (e.g., μ -like or e -like), and differentiate signal events from major background events (e.g., μ -like or e -like, μ -like or π^+ -like, e -like or π^0 -like, etc). Below is a set of separations considered for the current and new Super-K samples used in the analysis presented here:

- **The μ/e PID:** $\ln \left(\frac{L_\mu}{L_e} \right) > -0.2 P_{1Re}^{reco}$ makes selections between the two fundamental topologies (fuzzy ring or sharp ring), where it selects the μ -like case and removes the e -like case, as demonstrated in Fig. 2.12. P_{1Re}^{reco} is the reconstructed momentum assuming the 1-ring e -like hypothesis. The cut dependence on P_{1Re}^{reco} is to include high momentum muon events with thicker rings, where these rings can be reconstructed with higher L_e than L_μ and falls into lower momentum regions when assuming 1Re-like.
- **The μ/π^+ PID:** $\ln \left(\frac{L_\mu}{L_{\pi^+}} \right) > -0.15 P_{1R\mu}^{reco}$ selects the μ -like case and removes the π^+ background during 1-ring μ -like event selections, as demonstrated in Fig. 2.13. $P_{1R\mu}^{reco}$ is the reconstructed momentum assuming the 1-ring μ -like hypothesis. The cut dependence on $P_{1R\mu}^{reco}$ is to include high momentum muon events with thicker rings reconstructed to have higher L_{π^+} than L_μ .
- **The e/π^0 PID:** $\ln \left(\frac{L_{\pi^0}}{L_e} \right) < 175 - 0.875 M_{\pi^0}^{reco}$ selects the e -like case and remove the $NC\pi^0$ background during 1-ring e -like event selections, as demonstrated in Fig. 2.14. $M_{\pi^0}^{reco}$ is the reconstructed π^0 mass. An algorithm is used to fit the data with a second ring to obtain the invariant mass from the reconstructed momenta of the two rings, where those close to the π^0 rest mass (135 MeV) are likely to be an $\pi^0 \rightarrow \gamma\gamma$ event with a missing reconstructed Cherenkov ring. Photons can either be boosted in the same direction to produce two overlapped rings, or opposite directions to have one photon absorbed.
- **The ee / 2-other-ring PID:** $\ln \left(\frac{L_{2Rother}}{L_{ee}} \right) < 100$ makes selections of events with visible rings only from the $\pi^0 \rightarrow \gamma\gamma$ for $NC\pi^0$ events, and therefore selects the 2 e -like ring cases and removes 2 other ring cases (see Fig. B.7).

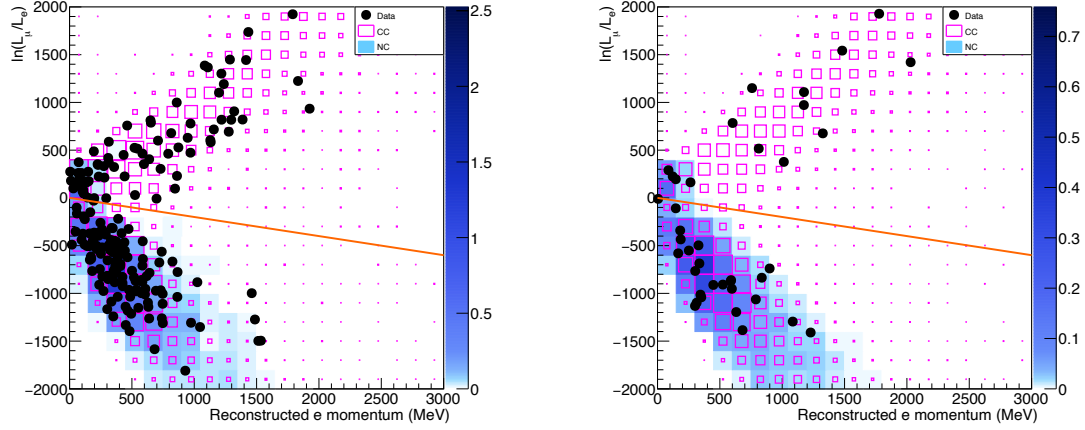


Figure 2.12: Distributions of the e and μ likelihood ratio, as a function of the reconstructed lepton momentum under the 1-ring e -like hypothesis, for the ν -mode (left) and the $\bar{\nu}$ -mode (right). The orange line represents the PID cut for e -like (below) and μ -like (above) events.

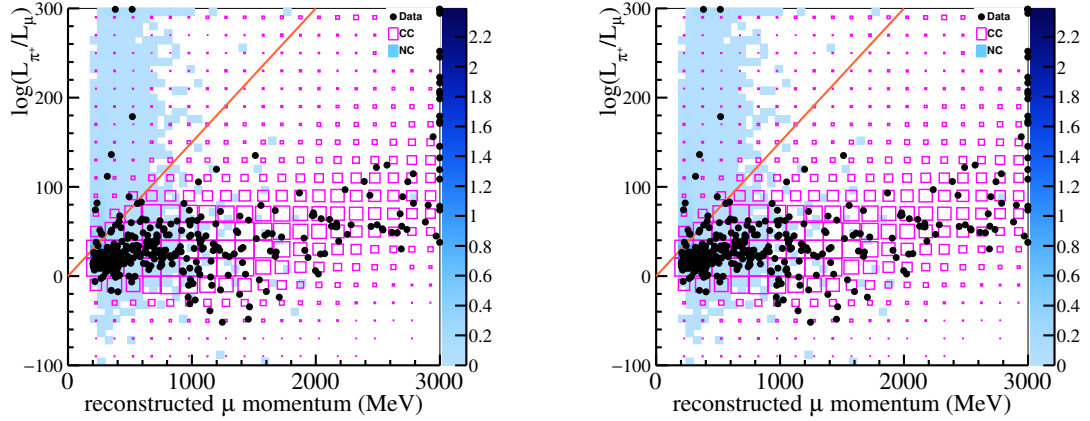


Figure 2.13: Distributions of the μ and π^+ likelihood ratio, as a function of the reconstructed lepton momentum under the 1-ring μ -like hypothesis, for the ν -mode (left) and the $\bar{\nu}$ -mode (right). The orange line represents the PID cut for π^+ -like (below) and μ -like (above) events. Figure reproduced from Ref. [96].

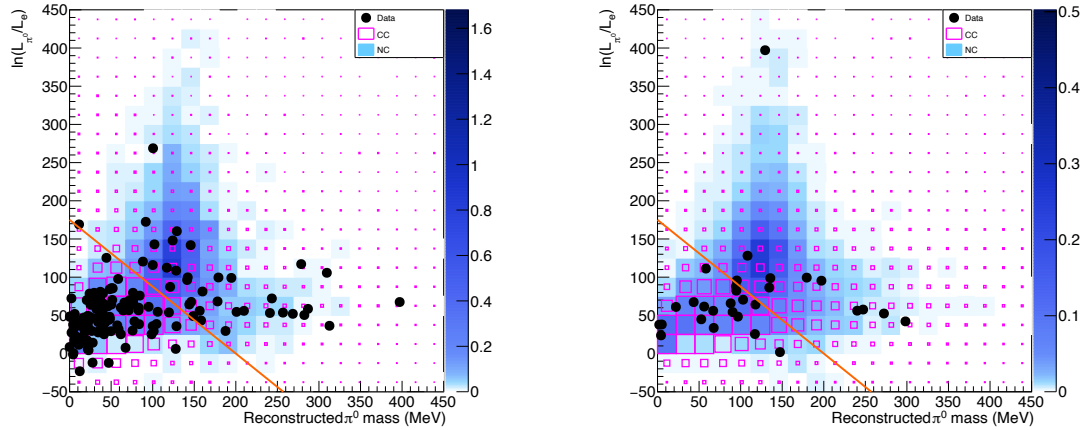


Figure 2.14: Distributions of the e and π^0 likelihood ratio, as a function of the reconstructed π^0 mass, for the ν -mode (left) and the $\bar{\nu}$ -mode (right). The orange line represents the PID cut for e -like (below) and π^0 -like (above) events.

The selected events are binned in specific reconstructed kinematic variables when proceeding to the oscillation analysis. The samples can be binned in the reconstructed neutrino energy to describe the changes to the neutrino flux from the oscillation effects. The reconstructed neutrino energy (E_{reco}) can be calculated assuming an interaction type [96]:

- CCQE with the nucleon at rest:

$$E_{reco} = \frac{(M_n - E_b)E_l - m_l^2/2 + M_n E_b - E_b^2/2 + (M_p^2 - M_n^2)/2}{M_n - E_b - E_l + P_l \cos(\theta_{beam})}, \quad (2.1)$$

where M_n (M_p) is the mass of the neutron (proton) which are exchanged for the ν ($\bar{\nu}$) interaction processes; m_l is the mass of the charged lepton; P_l , E_l , and θ_{beam} are the reconstructed momentum, energy and the scattering angle from the beam of the charged lepton, and E_b is the binding energy of the nucleon in oxygen (27 MeV). The charged lepton is assumed to be an electron for the e -like sample and a muon for the μ -like sample.

- CCRES with the nucleon at rest, specifically used for the events with a π^+ produced and assumed to be from a $\Delta(1232)$ resonance:

$$E_{reco} = \frac{2M_n E_l + M_\Delta^2 - M_n^2 - m_l^2}{2(M_n - E_l + p_l \cos(\theta_{beam}))}, \quad (2.2)$$

where the variables are defined the same way as in Eq. 2.1, apart from the M_Δ being the mass of the Δ particles. The nuclear binding energy is not included in this case due to its elimination by the Δ recoil.

NC π^0 Reconstruction

The reconstruction of the neutral current π^0 events and the available observables are different from the charged current case. From Fig. 1.9 and 1.10, the only possible outgoing charged particles are protons, which are mostly below the Cherenkov threshold (~ 1 GeV in momentum) due to their heavy mass. However, the π^0 has a

mean lifetime of $(8.43 \pm 0.13) \times 10^{-17}$ s and decays into two photons with a branch fraction of $(98.823 \pm 0.034)\%$ [3], and therefore, $\text{NC}\pi^0$ events are mainly observed from the Cherenkov rings induced by the photons. Since π^0 decays can produce at least one photon with momentum roughly 2 MeV, this photon is always capable of generating electron-positron pairs above the Cherenkov threshold through pair production and Compton scattering. As a result, these events are always considered visible.

The primary observables are the momenta and direction of the photons, and then the 4-momentum of the π^0 can be calculated. The reconstructed momentum of the photon (P_γ^{reco}) is calculated from the reconstructed momentum of the ring assuming an e -like ring (P_{1Re}^{reco}), which represents the sum of the momenta of all the collinear electrons produced from the photons, and therefore,

$$\mathbf{p}_\gamma = |\mathbf{p}_\gamma| \mathbf{r}_\gamma, \quad (2.3)$$

$$P_\gamma^{\text{reco}} = |\mathbf{p}_\gamma| = E_\gamma = E_{1Re}^{\text{reco}} = \sqrt{m_e^2 + P_{1Re}^{\text{reco}2}}, \quad (2.4)$$

where m_e is the mass of the electron and $\mathbf{r}_\gamma$ is the reconstructed direction of the ring. The reconstructed π^0 momentum ($P_{\pi^0}^{\text{reco}}$) and mass ($M_{\pi^0}^{\text{reco}}$) can then be

$$P_{\pi^0}^{\text{reco}} = |\mathbf{p}_{\pi^0}| = |\mathbf{p}_{\gamma_1} + \mathbf{p}_{\gamma_2}|, \quad (2.5)$$

$$M_{\pi^0}^{\text{reco}} = \sqrt{p_{\gamma_1} \cdot p_{\gamma_2}} = \sqrt{2|\mathbf{p}_{\gamma_1}||\mathbf{p}_{\gamma_2}| - 2\mathbf{p}_{\gamma_1} \cdot \mathbf{p}_{\gamma_2}}, \quad (2.6)$$

where $p_{\gamma_{1(2)}} = (E_{\gamma_{1(2)}}, \mathbf{p}_{\gamma_{1(2)}})$ is the 4-momentum of the photons. For the new $\text{NC}\pi^0$ samples in this analysis, $P_{\pi^0}^{\text{reco}}$ is used as the sample binning as this can be more correlated with the interaction kinematics and ring dynamics, compared to $M_{\pi^0}^{\text{reco}}$ that is only centred around the π^0 rest mass. For the events reconstructed as 1-ring, the method to obtain $P_{\pi^0}^{\text{reco}}$ is similar to the $M_{\pi^0}^{\text{reco}}$ case, where a fit to a second ring is performed assuming a 2-ring hypothesis.

2.4 Next Generation: Hyper-Kamiokande (Hyper-K)

2.4.1 Beam Upgrade

The beamline upgrade aims to increase the beam power to 1.3 MW by the start of the Hyper-K experiment [77]. This will be achieved by enhancing the beam repetition rate and minimising beam losses and instabilities. Upgrades to the power supplies and Radio Frequency (RF) cavity systems will reduce the time between beam spills from 2.48 s to 1.16 s and increase the number of protons per spill from 2.5×10^{14} up to 3.2×10^{14} , as shown in Fig. 2.15.

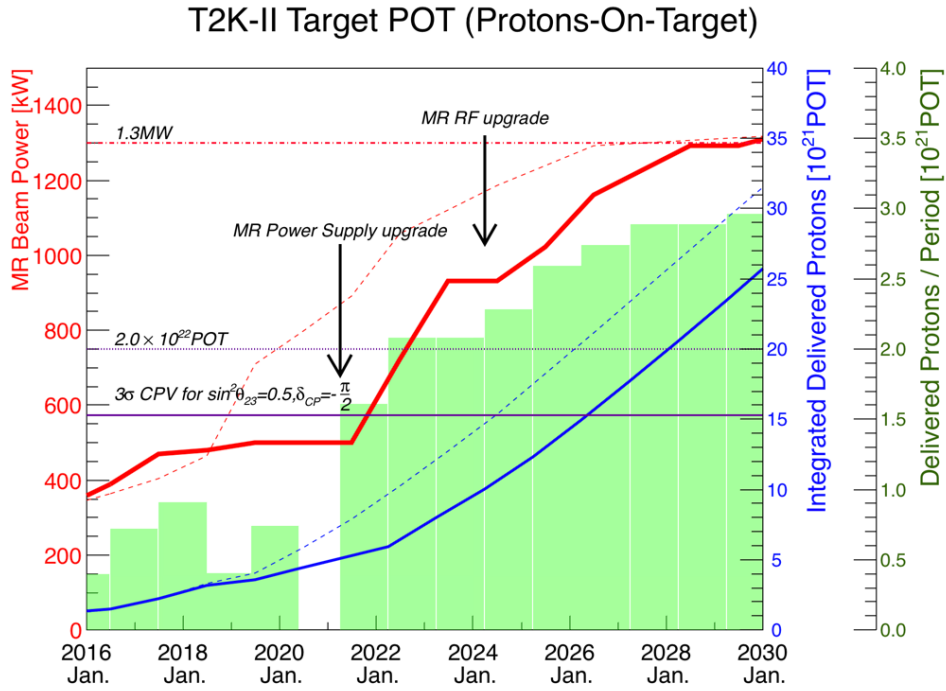


Figure 2.15: The J-PARC beam upgrade schedule, indicating the expected beam power increase and accumulated proton-on-target (POT) through to year 2030 [82].

To accommodate the increased beam power, upgrading the magnetic horns is essential for better focusing of hadrons with the desired charges. The current in the three magnetic horns has been increased from 250 kA (used for all Runs 1-11 data) to regularly operate at 320 kA during the 2024 beam upgrade, which is expected to reduce the wrong-sign neutrino flux by 5–10% [82].

2.4.2 ND280 Upgrade

The major upgrade to the near detector suite involves the ND280 modules, specifically replacing the P0D detector with a super-FGD and two high-angle TPCs (HA-TPCs) modules, and six Time of Flight planes (TOF) surrounding both Super-FGD and HA-TPCs. [97].

The super-FGD consists of 2 million 1 cm^3 scintillator cubes, each embedded with three wavelength-shifting (WLS) optical fibres, which connect the cubes in a grid-like structure. The fibre ends are connected to multi-pixel photon counters (MPPCs) for readout. This design allows the super-FGD to track charged particles in 3D. Two high-angle TPCs are positioned above and below the super-FGD to track the momentum of high-angle particles as they exit the super-FGD. The TOF system improves particle identification, especially at lower energies, by measuring travel time between detector parts. It also helps suppress background and provides precise timing for better event reconstruction when combined with super-FGD 3D tracking.

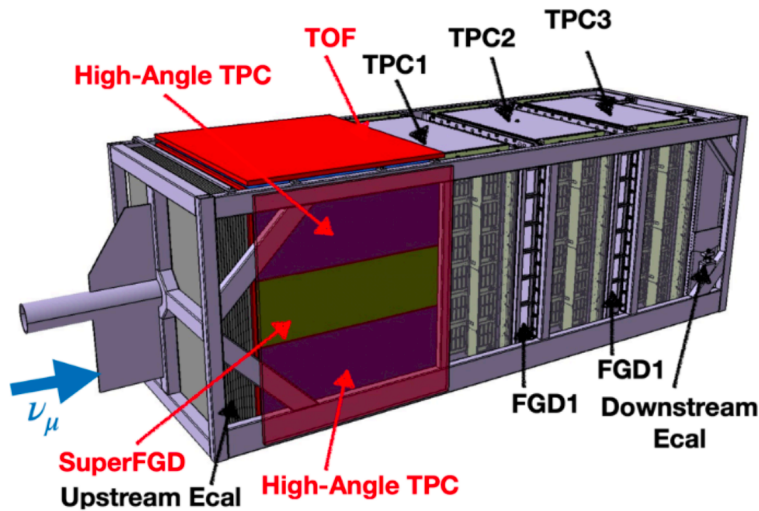


Figure 2.16: The ND280 upgrade modules, operating as 2.5° off-axis near detector for T2K-II and Hyper-K [97].

2.4.3 The Intermediate Water Cherenkov Detector

The intermediate water Cherenkov detector (IWCD) is a new detector for Hyper-K [77]. It is designed to be vertically movable along 52.2 m spanning over $1^\circ - 4^\circ$ from the neutrino beam axis, as shown in Fig. 2.17. It is proposed to be 1 km away from the beam production point with a 300 t fiducial volume. It has a similar detecting and event reconstruction method as Super-K due to the similar detector geometry. For this analysis, the IWCD geometry was defined with a 6 m height and 8 m diameter, installed with about 500 “multi-PMT” (mPMT) modules to detect the Cherenkov radiation. Each mPMT module contains 19 8-cm diameter PMTs, which is designed to improve the vertex resolution of the neutrino-nucleus interactions.

With the excellent electron identification powers supported by the improved photosensors and the vertically movable ability, IWCD can provide high statistics electron (anti-)neutrino and muon (anti-)neutrino samples. The vertically movable feature allows for the measurement of unoscillated muon neutrino flux at different positions, providing neutrino samples with energy spectra peaking between 0.4 GeV and 1 GeV.

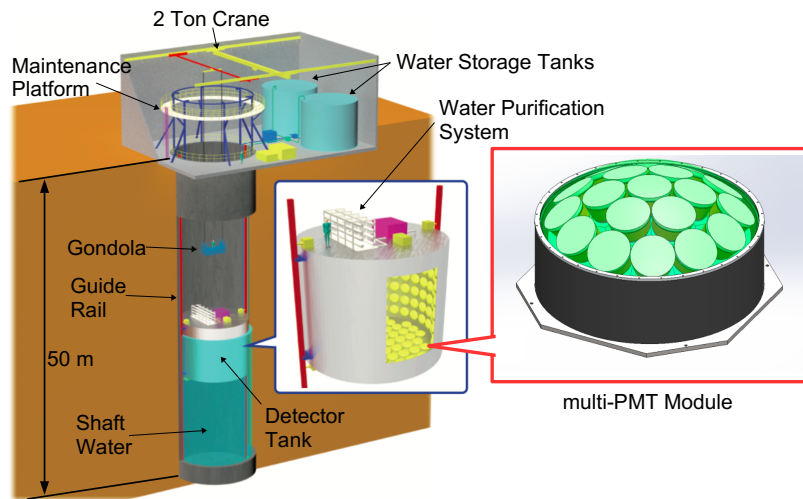


Figure 2.17: The IWCD detector, showing the vertically movable feature and the “multi-PMT” module [98].

Fig. 2.18 demonstrates the ν_μ and ν_e energy spectra dependences over the 1° , 2.5° ,

and 4° off-axis angles (OAA). The ν_μ spectra measured from different OAA positions can be combined linearly to produce arbitrary spectrum shapes that match the interested reconstructed spectra and extract the constitutions of the interaction modes for a particular spectrum shape.

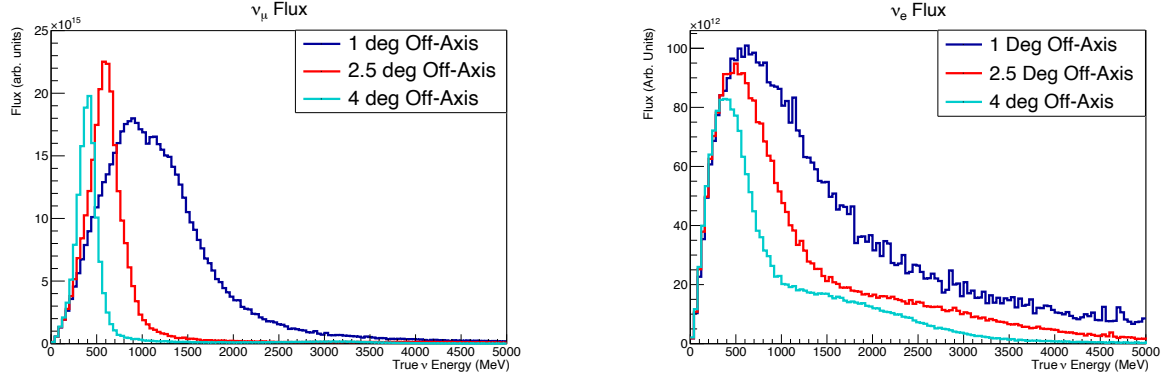


Figure 2.18: (left) The true ν_μ (left) and ν_e (right) energy spectra dependence on the three off-axis angles [99].

High purity large ν_e and $\bar{\nu}_e$ samples can be provided, especially at further OAA, where 18,000 and 24,000 events are expected with 10-year data. These samples are contributed by the 1% intrinsic ν_e contamination produced by the muon and kaon decays and help constrain the dominated relative ν_e and ν_μ interaction cross-section uncertainties on the neutrino event predictions,

$$\frac{\sigma(\nu_e)/\sigma(\nu_\mu)}{\sigma(\bar{\nu}_e)/\sigma(\bar{\nu}_\mu)} \equiv \frac{\nu_e}{\bar{\nu}_e}, \quad (2.7)$$

where the current T2K theory-driven error is 3% [100]. The impact of IWCD on constraining such uncertainty is studied and described in Chapter 5. The measurement of the intrinsic ν_e events also helps constrain the intrinsic ν_e flux, which is a background that enters the far detector for the oscillated ν_e events.

Another motivation of IWCD is to constrain the “feed-down” events, where CC non-QE events with higher true neutrino energy can be reconstructed to lower energy events with water Cherenkov detectors due to the indistinguishable visible topologies with the CCQE interactions (e.g. 2p2h). This can change the shape of the reconstructed ν_e sample spectra for the sub-GeV energy regions and affect neutrino

cross-section measurements around the oscillation maximum. Therefore, a large fraction of neutrino events with reconstructed energies greater than 1 GeV (CC non-QE dominated) is required to constrain the feed-down components. Events with higher incoming neutrino energy can be measured more effectively using a large OAA span, compared to the fixed 2.5° case.

2.4.4 The Hyper-K Detector

The Hyper-K detector is a large-scale water Cherenkov detector and the primary detector for the Hyper-K project, designed to measure oscillated long-baseline neutrino-nucleus interactions. Similar in design to Super-K (described in Section 2.3), Hyper-K features an increased water target volume from 50 kton to 258 kton, with a fiducial volume of 187 kton. The detector will have dimensions of 68 m in diameter and 71 m in height, with approximately 20,000 new 20-inch PMTs planned for installation [101]. Additionally, mPMT modules, similar to those designed for the IWCD (see Section 2.4.3), will be placed between the large PMTs to provide additional photo-coverage. Currently, the response of the mPMT is being studied using a prototype Water Cherenkov Test Experiment and a test beam from CERN [102]. The PMT configuration in Hyper-K is expected to double the photo-detection efficiency while maintaining a similar dark rate and offering better charge and timing resolutions. The OD will also be equipped with more PMTs compared to Super-K, improving the background veto capability.

The increased statistics from Hyper-K will significantly improve constraints on the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance measurements, as well as improve measurements of the NC π^0 events that constrain the neutrino flux uncertainties and the NC π^0 interaction cross sections. The NC π^0 samples selected at Super-K and their application to oscillation analysis, as presented in this thesis, are expected to serve as a basis for future analyses at Hyper-K. An assessment of oscillation sensitivities using the Hyper-K detector scale, along with constraints from the IWCD detector, is presented in Chapter 5 of this thesis.

Chapter 3

Statistical Methods

Measuring oscillation parameters involves employing various statistical inferences to interpret the results. The significance level of the results is associated with some probabilities for given constraints from the data and the model. There are two common interpretations of probability: the *frequentist* and *Bayesian* probability. The frequentist probability takes the frequency of the occurrence of an event over an infinite number of experiments, assigning the probability of seeing the event as the ratio of occurrence times to the number of experiments. In contrast, the Bayesian probability is linked to the degree of belief for the event to occur under certain hypotheses from the outcomes of some random processes.

The two statistical inferences are associated with different treatments when evaluating the parameters in the model based on the likelihood information between the model and the data. The frequentist approach aims to identify the parameter values when the resulting likelihood is maximised to evaluate a model that is most probable to contain the true values of the parameters, such a process of finding the maximal likelihood is called *profiling*. The Bayesian approach applies the Bayes theorem to calculate the posterior probability, given a set of prior probabilities and a likelihood. The posterior information of each parameter is *marginalised* from all parameters other than this one. The choice of the methods will primarily depend on the size of the parameter space, the prior information and the available inputs to the analysis.

3.1 The Likelihood Function

The analysis at T2K and Hyper-K uses MC (Monte-Carlo simulated data) to predict the oscillated neutrino flux at the far detector. The MC takes the simulated neutrino-nucleus interaction kinematics, and applies weights to account for the variations affected by the systematic uncertainties and oscillation probability. The final MC predictions will depend on a few hundreds of parameters, including the oscillation parameters to be measured and the nuisance parameters to model the systematic uncertainties.

The MC and the data are binned in histograms using the reconstructed kinematic information of the events, such that the number of occurrences of events in bin i can be described using a Poisson probability density function (p.d.f),

$$f(n_i^{obs}|\lambda_i) = \frac{(\lambda_i)^{n_i^{obs}} e^{-\lambda_i}}{n_i^{obs}!}, \quad (3.1)$$

where n_i^{obs} is the observed occurrence and λ_i is the expected number. The expected number can be set to equal the MC predicted event rate $\lambda_i = n_i^{pred}$, to compare the observations with the MC. The measure of how well the MC matches the observations can be obtained from the ratio of the likelihood of getting n_i^{pred} for a given n_i^{obs} to the saturated hypothesis [103, 104], i.e. when $n_i^{pred} = n_i^{obs}$:

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\theta}|D) = \frac{\prod_i^{\text{bins}} f(n_i^{obs}|n_i^{pred}(\mathbf{x}, \boldsymbol{\theta}))}{\prod_i^{\text{bins}} f(n_i^{obs}|n_i^{pred}(\mathbf{x}, \boldsymbol{\theta}) = n_i^{obs})} = \frac{\prod_i^{\text{bins}} (n_i^{pred}(\mathbf{x}, \boldsymbol{\theta}))^{n_i^{obs}} e^{-n_i^{pred}(\mathbf{x}, \boldsymbol{\theta})}}{\prod_i^{\text{bins}} (n_i^{obs})^{n_i^{obs}} e^{-n_i^{obs}}}, \quad (3.2)$$

in which $\mathcal{L}(\mathbf{x}, \boldsymbol{\theta}|D)$ is a function of nuisance parameters $\mathbf{x}$ and parameters of interest $\boldsymbol{\theta}$ for given data D , with an interval of $[0, 1]$. In practice, the likelihood ratio is represented in the negative log form,

$$-\ln \mathcal{L}(\mathbf{x}, \boldsymbol{\theta}|D) = \sum_i^{\text{bins}} \left(n_i^{pred}(\mathbf{x}, \boldsymbol{\theta}) - n_i^{obs} + n_i^{obs} \ln \left(\frac{n_i^{obs}}{n_i^{pred}(\mathbf{x}, \boldsymbol{\theta})} \right) \right). \quad (3.3)$$

Due to the limited statistics on the available data, prior constraints on predicting the

true values and the uncertainties for the nuisance parameters can be provided from external studies, such as developing from the interaction model theory, calibration data and external flux and interaction cross-section measurements. A multivariate Gaussian penalty will be added to the likelihood due to the changes in the nuisance parameter values from their prior predicted values, weighted by the prior uncertainties:

$$-\ln \mathcal{L}_{penalty} = \frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \cdot \mathbf{C}^{-1} \cdot (\mathbf{x} - \mathbf{x}_0), \quad (3.4)$$

where $\mathbf{x}_0$ is the prior-predicted values of the nuisance parameters and $\mathbf{C}$ is the covariance matrix from the prior uncertainties of the parameters. The combined log-likelihood function

$$-\ln \mathcal{L}_{tot}(\mathbf{x}, \boldsymbol{\theta}) \equiv -\ln \mathcal{L}(\mathbf{x}, \boldsymbol{\theta}|D) - \ln \mathcal{L}_{penalty}, \quad (3.5)$$

will be used to proceed with the analysis of the methods described in the following sections. $\ln \mathcal{L}(\mathbf{x}, \boldsymbol{\theta}|D)$ is sometimes also referred as the sample log-likelihood, while $\ln \mathcal{L}_{penalty}$ is the parameter log-likelihood.

3.2 Frequentist Approach

3.2.1 Profiling

Frequentists use profiling to estimate the best fit and error of the parameter value, and test the hypotheses of different models over the null hypothesis. The best fit of parameters is obtained when the likelihood ratio between the MC and the data is maximised, in which case this is the same as to minimise the negative log-likelihood ratio described in Section 3.1 over the parameter space. For a given parameter of interest, θ_0 , the parameter is set to be at a range of points to explore the likelihood of different predicted hypotheses. For each point of θ_0 , the negative log-likelihood ratio is minimised with keeping θ_0 fixed at the point to obtain the best-fit values of

other parameters. Here the treatment of other parameters becomes, for example, systematic parameters treated as nuisance parameters with prior constraints and other unknown oscillation parameters are set with some random initial values and varied completely free.

The minimisation of the negative log-likelihood ratio $-\ln \mathcal{L}_{tot}$ is performed using MINUIT [105] with the MIGRAD algorithm. The post-fit uncertainties and covariances are evaluated from the HESSE algorithm, which produces a covariance matrix by inverting the matrix of the second derivatives of $-\ln \mathcal{L}_{tot}$ at the best-fit point. For systematic parameters that are not Gaussian-behaved, the MINOS algorithm can provide a better estimation of the asymmetric errors, by scanning over the parameter values to find the differences to its best-fit values from the calculation of $\Delta\chi^2$ while keeping other parameters fixed at the best fit. The points at where $\Delta\chi^2 = 1$ on each side of the best fit correspond to the lower and upper error bound of the 1σ variations.

3.2.2 1-Dimensional Confidence Intervals

The frequentist way of representing the uncertainties from the fit results requires defining confidence regions of the parameters from a critical $\Delta\chi^2$ value. In this thesis, the study chose to use the *constant- $\Delta\chi^2$* approach [3] and evaluate the confidence region from a 1-dimensional confidence interval. The constant- $\Delta\chi^2$ approach only works well for Gaussian-behaved parameters, which is restricted in many cases, as in reality the parameters can not always be Gaussian. However, it can be computationally efficient when performing analysis in simple cases such as sensitivity studies with fitting the MC data to itself (later referred to as an “Asimov” dataset). For other cases such as likelihood functions with complex behaviours or fitting to multiple parameters, which is out of the scope of the sensitivity study discussed in the thesis, Feldman-Cousin methods can be a more stable and reliable way than the constant- $\Delta\chi^2$ method.

For 1-dimensional confidence interval, the $\Delta\chi^2$ at each fixed point of the pa-

parameter θ_0 is constructed from its minimised negative log-likelihood distributions:

$$\begin{aligned}\Delta\chi^2(\theta_0, fix) &= \chi^2(\theta_0, fix) - \chi^2(\theta_0, bf) \\ &= -2 \ln \mathcal{L}(\theta_0, fix; (\mathbf{x}, \boldsymbol{\theta})_{bf}) - (-2 \ln \mathcal{L}((\theta_0; \mathbf{x}, \boldsymbol{\theta})_{bf})),\end{aligned}\quad (3.6)$$

where $-\ln \mathcal{L}(\theta_0, fix; (\mathbf{x}, \boldsymbol{\theta})_{bf})$ is the minimum at each fixed θ_0 with other parameters at the best-fit and $-\ln \mathcal{L}((\theta_0; \mathbf{x}, \boldsymbol{\theta})_{bf})$ is the global minimum by also setting θ_0 free with all the parameters at their best-fit values.

From a frequentist view, the aim is to find how frequently the measured parameter has its true value contained in some interval ranges after the experiment has been performed an infinite number of times. The $\Delta\chi^2(\theta_0, fix)$ described in Eq. 3.6 quantifies how much worse the fit is when the measured parameter is constrained to certain values compared to when it's allowed to vary freely and find the best-fit value. Hence, a more significant likelihood deviation of the constrained value from the best fit means a higher probability for the true value to be closer to the best fit than the constrained value. Thus, the probability of containing the true value is related to a critical value $\Delta\chi^2(\theta_0, fix)_{critical} = N^2$, such that the true value is within the $N\sigma$ interval over the parameter region for $\Delta\chi^2(\theta_0, fix) < \Delta\chi^2(\theta_0, fix)_{critical}$. Alternatively, the probability is equivalent to the confidence level (C.L.) on the true values to be within the $N\sigma$ interval, where such an interval is called the confidence interval. For Gaussian-behaved parameters with one parameter fit, $N = 1, 2, 3$ corresponds to the 68%, 95.5%, and 99.7% C.L. contours, respectively. The $\Delta\chi^2$ values corresponding to the $N\sigma$ contours for constructing higher dimensional confidence intervals can be found in Ref. [106].

3.3 Bayesian Approach with Markov Chain Monte Carlo

3.3.1 Posterior Probability and Marginalisation

In Bayesian statistics, all model parameters are treated as random variables (no distinctions between parameters of interest and the nuisance parameters) with prior information on their nominal values and uncertainties. The prior information is used to construct prior probability distributions $P(\mathbf{x})$ for each model parameter $\mathbf{x}$. For the T2K analysis, most systematic parameters are given a Gaussian prior, with most oscillation parameters having flat priors across their allowed range.

Unlike the frequentist way to update the parameters by minimising the log-likelihood, the goal is to update the prior probability density to get posterior probability distributions $P(\mathbf{x}|D)$ according to the observed data, where $P(\mathbf{x}|D)$ reflects the updated degree of beliefs about the parameters $\mathbf{x}$ after observing the data D . The $P(\mathbf{x}|D)$ can be evaluated using the Bayes theorem [107],

$$P(\mathbf{x}|D) = \frac{P(D|\mathbf{x})P(\mathbf{x})}{P(D)} = \frac{P(D|\mathbf{x})P(\mathbf{x})}{\int P(D|\mathbf{x})P(\mathbf{x})d\mathbf{x}}, \quad (3.7)$$

where $P(D|\mathbf{x})$ represents the probability of observing the data D given the value of the parameter $\mathbf{x}$, and is equivalent to the likelihood of $\mathbf{x}$ to explain the data $\mathcal{L}(\mathbf{x}|D)$, which is equivalent to the definition of the likelihood ratio in Eq. 3.2. $P(D)$ is a constant term describing the marginalised likelihood of observing the data under all possible parameter values, weighted by their respective prior probabilities. Hence, the posterior probability distribution can be represented using the definition of $\mathcal{L}_{tot}$ in Eq.3.5, such that

$$-\ln P(\mathbf{x}|D) \propto -\ln(P(D|\mathbf{x})P(\mathbf{x})) \propto -\ln \mathcal{L}_{tot}(\mathbf{x}), \quad (3.8)$$

with the prior probability distribution $P(\mathbf{x})$ being the penalty term described in Eq. 3.4, and $\mathcal{L}_{tot}(\mathbf{x})$ can be interpreted as how well a set of parameters $\mathbf{x}$ is supported by

the data. In this case, the proportionality can directly reflect the underlying posterior probability distribution from calculations of the likelihood distribution over the parameter space, with the likelihood distribution eventually converging to a stationary distribution after a certain number of random throws of the parameter values. The methods to approximate the likelihood function and propagate the parameters leading to the convergence are described in Section 3.3.2.

The posterior probability distribution of individual parameters can be evaluated from the marginalisation of the joint posterior distribution, using the integral of $\mathcal{L}_{tot}(\mathbf{x}, \boldsymbol{\theta})$ over all parameters except the ones of interest

$$\mathcal{L}_{tot}(\boldsymbol{\theta}) = \int \mathcal{L}_{tot}(\mathbf{x}, \boldsymbol{\theta}) d\mathbf{x}, \quad (3.9)$$

where $\boldsymbol{\theta}$ is the set of parameters of interest and $\mathbf{x}$ is the nuisance parameter set. This will produce a marginalised posterior distribution with a dimension equal to the size of $\boldsymbol{\theta}$.

3.3.2 Markov Chain Monte Carlo (MCMC)

The likelihood function for the neutrino oscillation analysis covers a few hundred nuisance parameters, making it analytically difficult to understand the behaviour of the function over the whole parameter space. The region of most interest is around where the likelihood is maximised to evaluate the best fit of the parameters. For a high-dimensional space, the different regions in the parameter space can be widely explored by random sampling of the parameters, and efficiently identifying the region with the highest likelihood to accumulate an approximated distribution around this region.

The Monte Carlo technique is commonly used to approximate the distribution of a function with many parameters by performing repeated random samplings. The probability of obtaining a specific value can often be determined by integrating its likelihood function over a small interval region that contains the value, which

works well for simple functions with small parameter space, that can be integrated analytically and calculated efficiently. However, for the complex likelihood functions encountered in this thesis, which also involves a very high-dimensional parameter space, this analytical approach becomes difficult to evaluate. In such cases, a numerical method like the *Metropolis-Hastings* algorithm [108] is used to determine when and how parameter values should be updated and added to the distribution.

The Metropolis-Hastings algorithm is one of the Monte Carlo techniques that propagates the parameter values through Markov chains (MCMC), rather than sampling randomly across the entire parameter space. A Markov chain is a stochastic process that undergoes an order of transitions from one point to another in the parameter space [109]. At each step of the transition, the proposed position for the next step only depends on the position of the current point, rather than any previous points. The starting point of the Markov chain is given by a set of randomly generated parameter values, where for any starting point the probability of transit to any other points is non-zero. Throughout the iterations, the acceptance of a step will be justified through some criteria, so the chains are likely to propagate towards a higher posterior probability area over the parameter space. The chains will eventually progress towards a convergence state, with the steps proposed being independent of where the initial point is. When this is achieved, it is said to reach a stationary distribution such that the sampling probability to different points does not depend on the number of steps iterated. Therefore, all the subsequent steps will be sampled from the stationary distribution, which is equivalent of saying that the probability of obtaining any particular value over the parameter space remains fixed according to the current probability distribution. The Markov chain can then accumulate to a distribution after enough sampling and closely approximate the underlying posterior distributions to conclude the fit outcomes of the parameters.

The Metropolis-Hastings algorithm is well-established in constructing Markov chains to find the stationary distribution around the highest probability density re-

gion of the parameter space, which relies on some influences applied to the random walk in the parameter space by accepting or rejecting a step proposed. The chain will propagate to the next step when the proposal is accepted, or stay at the same step for the next proposal if the current proposal is rejected.

For a Markov chain, parameter values at a new step are proposed from the parameters $\mathbf{x}_i$ at the current step i :

$$\mathbf{x}_p = \mathbf{x}_i + \text{random}(f(\mathbf{x}_p|\mathbf{x}_i)), \quad (3.10)$$

where the second term is a random number determined from the probability of proposing a state $\mathbf{x}_p$ given $\mathbf{x}_i$. For the T2K neutrino oscillation analysis in this thesis, $f(\mathbf{x}_p|\mathbf{x}_i)$ used is a multivariant Gaussian distribution centred at $\mathbf{x}_i$, with the width given by the prior uncertainties and correlations corresponding to these parameters, using a covariance matrix $\mathbf{C}$ as described in Eq. 3.4. The second term can also be referred to as the step size, to describe the variations of the parameters between two steps. In some cases, the step size for individual parameters can be tuned to ensure efficient exploration of the parameter space. If the step size is too small, the algorithm may take a long time to explore the parameter space, leading to slow convergence. On the other hand, if the step size is too large, the algorithm may skip the regions of high probability density, resulting in low sampling efficiency and falls to convergence.

The acceptance or rejection of a proposed step is determined by an *acceptance probability* [109], which is always within the bounds $[0, 1]$ and does not require a bounding function that constrains sampling within the target distribution. The acceptance probability depends on the relative posterior probability distribution of the parameter at the current step $\mathbf{x}_i$ and the proposed step $\mathbf{x}_p$,

$$\alpha \equiv \min \left[1, \frac{P(\mathbf{x}_p|D)f(\mathbf{x}_i|\mathbf{x}_p)}{P(\mathbf{x}_i|D)f(\mathbf{x}_p|\mathbf{x}_i)} \right], \quad (3.11)$$

where the function $f(\mathbf{x}_i|\mathbf{x}_p)$ describes the probability of proposing a state $\mathbf{x}_i$ given

$\mathbf{x}_p$, and vice versa for $f(\mathbf{x}_p|\mathbf{x}_i)$. For simplicity, using the multivariate Gaussian distribution makes the solution symmetric $f(\mathbf{x}_i|\mathbf{x}_p) = f(\mathbf{x}_p|\mathbf{x}_i)$. Hence, the acceptance rate can be simplified to give

$$\alpha = \min \left[1, \frac{P(\mathbf{x}_p|D)}{P(\mathbf{x}_i|D)} \right] = \min \left[1, \frac{\mathcal{L}_{tot}(\mathbf{x}_p)}{\mathcal{L}_{tot}(\mathbf{x}_i)} \right] \quad (3.12)$$

by adapting the proportionality relationship in Eq. 3.8. Finally, a random number u is uniformly generated from an interval $[0, 1]$ that determines whether $\mathbf{x}_p$ is accepted for the next step

$$\mathbf{x}_{i+1} = \begin{cases} \mathbf{x}_p & \text{for } \alpha \geq u, \\ \mathbf{x}_i & \text{for } \alpha < u. \end{cases} \quad (3.13)$$

So the proposal with a higher likelihood than the current step will always be accepted, but for a lower likelihood, it will only be accepted with some probability α . In general, the step to the region with a high likelihood is more likely to be accepted than to the region with a low likelihood, so it can accumulate more points at the high likelihood region and vice versa to be proportional to the posterior probability distribution.

Once a step is accepted, all the parameter values will be updated to the new accepted position and appended to the chain. If the proposed step is rejected, the current position will be appended to the chain. Then a new circle will be started to repeat the process of proposing a new step from the current step and determining whether to append the new step or the current step to the chain.

3.3.3 Consequences of the MCMC

Several properties of the Markov Chain can be used to ensure the chain follows an expected behaviour in an analysis. This particularly involves checking the proposed values of different quantities along the step numbers (a.k.a. traces) and correla-

tions in the steps to see if a dedicated convergence region and shape is probed and whether the overall acceptance rate of the steps becomes stable.

The period before the Markov chain reaches a stationary state is called *burn-in*, which encompasses an unstable probability to propose the points distributed in the parameter space when sampling the likelihood function. Therefore, the burn-in period may cause bias in the distributed points. The points from the burn-in period usually get removed by looking at the traces of the log-likelihood and parameters as a function of the step numbers, from before where a stable distribution starts. After the burn-in period, the chain is expected to move backwards and forth around the highest probability area to show a stabilised fluctuation of the likelihood, with the probability of transiting to each state converging to a fixed value.

Although the proposals for the next step only depend on the current state, the current state can still influence the sampling towards subsequent steps. A measurement of the influence from the current state to the states after a lag of the steps is the *auto-correlation* between the two states. The auto-correlation of a parameter with a chain length N and a step lag l from the current step i can be represented as [109]

$$C_x = \frac{1}{\sigma_x^2} \frac{1}{N-l} \sum_{i=1}^{N-l} (x_i - \bar{x})(x_{i+l} - \bar{x}), \quad (3.14)$$

where $\bar{x}$ and σ_x^2 are the mean and the variance of the sampled values of parameter x over the chain, respectively. In order to sample with states under enough degree of independence, it is crucial to ensure the auto-correlation between the states drops within a reasonable small lag of steps. If high auto-correlations last for too many steps, this can still lead to inefficient exploration of the parameter space, under-sampling or over-sampling of certain regions, even if the post-burn-in status of the chain has been reached.

In practice, the burn-in and auto-correlation length can be adjusted by tuning the step size between the two states, as described in Eq. 3.10. In this case, high auto-correlation over long step lag indicates the step size is too small, leading to the

region explored being too narrow relative to the entire space. Increasing the step size can have the auto-correlation dropped within a short lag, and reduce the number of steps required to predict the posterior distribution under successive independent samplings. The step size can be adjusted accordingly based on the traces of the parameters to also ensure a reasonable burn-in length, while not being too large to be trapped at the low likelihood region with a large fraction of proposed states being rejected.

3.3.4 Posterior Distribution and Credible Interval

Given the posterior probability distribution accumulated using the MCMC method, the next step is to decide how to extract the information on the best-fit point and evaluate the degree of belief in the results for it to accurately represent the true value. As mentioned in section 3.3.1, all nuisance parameters will be marginalised to produce the posterior probability distribution for the parameter of interest. In practice with the T2K analysis, the distribution of $\mathcal{L}_{tot}(\theta)$ described in Eq. 3.9 is marginalised in binned regions, with the magnitude of a single bin at $[\theta_1 = \Theta^1, \theta_2 = \Theta^2, \dots, \theta_m = \Theta^m]$ over the space of parameter of interest calculated as

$$L[\Theta^1, \Theta^2, \dots, \Theta^m] = \sum_{X^1=1}^{\dim(x_1)} \sum_{X^2=1}^{\dim(x_2)} \dots \sum_{X^n=1}^{\dim(x_n)} L[X^1, X^2, \dots, X^n, \Theta^1, \Theta^2, \dots, \Theta^m], \quad (3.15)$$

where m and n are the number of parameters of interest and nuisance parameters, respectively. The integers $\Theta^i \in [1, \dim(\theta_i)]$ and $X^i \in [1, \dim(x_i)]$ are the indices of the binned regions over parameters θ_i and x_i . In the case of a high dimensional phase space (e.g. a few hundreds for T2K analysis), a chain with steps $\mathcal{O}(10^8)$ will be required to ensure enough points are distributed in each of the bin regions and provide a smooth distribution, especially when the bin width is set to be small for precise estimation of the probability density over the parameter space.

Given the posterior probability distribution, the method to get the best fit for one parameter or the joint best fit of multiple parameters is to look at which bin con-

tains the most number of points. The best-fit values for a parameter can be different when viewed from its single posterior distribution and joint posterior distributions with other parameters of interest, due to the marginalisation effects. Bias can be introduced from marginalisation due to correlations between the parameter of interest and nuisance parameters when the parameters are not all Gaussian-behaved. Hence, marginalising to higher dimensions allows for a broader representation of the parameters of interest, by capturing the underlying correlation between parameters and enabling resolutions on the complex behaviour of the probability distribution. Examples of the bias due to marginalisation effects will be apparent when discussing the results from the T2K oscillation analysis (in Chapters 7 - 8).

From the Bayesian view, the uncertainty of the best-fit value can be extracted from a surrounding credible interval. A method of defining the credible interval is from the interval that contains a certain fraction (β) of the posterior density from the marginalised posterior distribution,

$$\int_I \mathcal{L}_{tot}(\boldsymbol{\theta}) d\boldsymbol{\theta} = \beta, \quad (3.16)$$

for a given set of parameters of interest $\boldsymbol{\theta}$ and interval I over the parameter space. Hence, the probability of the true value of the parameter falling into a $\beta \times 100\%$ credible interval is just equal to $\beta \times 100\%$.

For T2K oscillation analysis, the credible interval around the highest posterior density (HPD) regions is used, where every point within the interval has a higher posterior density than any point outside the interval. The calculation involves summing the likelihood bins, starting from the maximum bin and sequentially adding bins with decreasing density until the cumulative value reaches the critical threshold β . The regions containing these summed bins define the quoted $\beta \times 100\%$ credible interval. In cases where the posterior distribution includes distinct HPD regions, the credible interval may encompass multiple disconnected regions, which are then combined to account for the $\beta \times 100\%$ posterior density.

Chapter 4

Oscillation Analysis Techniques

The thesis will include two separate neutrino oscillation analyses from different perspectives. One is to look at the impact of the new systematic uncertainty constraints for the future Hyper-K experiment, and the other one, as the main analysis of the thesis, is to look at the impact of new event samples using the T2K data.

In Chapter 3, the statistical inference for fitting the MC with data in an oscillation analysis was introduced, where the two analyses respectively choose to use the frequentist and Bayesian approaches. This chapter will describe the strategy to construct different inputs required to complete the oscillation analysis, including methods of generating the MC and constructing the event samples for the MC and the data, followed by treatments for the systematic uncertainties in the modelling. Finally, methods to check and validate the results will be described. The focus will be on the methods shared in common for the two oscillation analyses. There will also be some brief highlights of the new implementations that are generally relevant for both analyses. However, exact versions and specifics of the MC prediction generation, sampling, and systematic uncertainty models for the two analyses will be addressed in their individual analysis chapters.

4.1 Monte-Carlo (MC) Event Predictions

As discussed in Chapter 3, neutrino oscillation parameters are measured by identifying combinations of the values over the space that produce the MC most likely to predict the observed data. These parameters are used to tune the simulations of the neutrino beam production, interactions of the neutrinos with the nuclei in the detector, and the responses from the detector.

The neutrino beam simulation starts by modelling the proton beam hitting the target. The proton beam monitors upstream of the target capture the beam profile and related information, which is used to define the initial kinematic properties of the proton beam. The intensity, energy, and direction of the proton beam are then used to generate the hadrons and secondary interactions inside the target, where the kinematics of the interactions are simulated using FLUKA version 2011.2c.6 [110]. Next, the propagation of the particles outside the target, their interactions, and decays to neutrinos are tracked through the horn fields using the GEANT3-based framework JNUBEAM [111]. Eventually, each neutrino can be assigned properties on its flavour, energy, and direction of travel to model the flux distribution.

The generated neutrino flux is then passed to the interaction event generator NEUT 5.6.4 [59], to simulate the different types of interaction processes at ND280 and Super-K and the kinematic information of the outgoing particles. The fraction of different event types generated depends on the relative cross sections of the energy region that the flux corresponds to, adapting the prior knowledge from the cross-section model. Section 1.3.3 has provided an overview of the cross-section model.

The generated interaction events in both detectors are then propagated with the detector geometry simulations using GEANT3 or GEANT4-based frameworks to obtain the detector responses to the particles produced in the interaction. The events can be reconstructed based on the simulated detector response, applied in the same way to both simulation and data. The details of the reconstruction techniques for different detectors can be found in Chapter 2.

At this stage, a nominal MC prediction can be generated with all the oscillation and nuisance parameters assigned a central value, where variations of the parameters will result in changes to the event kinematics and rate. The process of applying the variations to the event kinematics and rate according to the variations of the parameters is known as *reweighting* of the MC. Details on the reweighting methods of the systematic parameters will be described in Section 4.3. Before proceeding with the fit in the oscillation analysis, the flux, and cross-section used to generate the MC need to be tuned. This tuning is based on the T2K beamline data for the flux and on non-T2K cross-section measurements for the cross-section model [87].

For the T2K and Hyper-K oscillation analysis, both the MC and the data are sampled for specific event topologies, with the events binned in the reconstructed kinematic variables. These samples are dominated by events from particular oscillation channels or interaction modes using the reconstructed information. Depending on which detector interaction events are observed, different samples can be used to constrain certain systematic uncertainties or are directly sensitive to the oscillation parameters.

Near detector samples like ND280 or IWCD use the high statistics unoscillated neutrino flux to constrain neutrino interaction cross sections and flux for the neutrino beam. The far detector samples are mainly sensitive to the changes in the flux due to neutrino oscillations, but can also provide extra constraints on the interaction cross sections when having better sensitivities to detect certain interaction types compared to near detectors. In the next section, choices and selection criteria of the ND280 and Super-K samples used for a recent T2K oscillation analysis will be described, with also a brief overview of the role of the new samples presented in this thesis.

4.2 Event Samples

4.2.1 ND280 Samples

The ND280 detector measures the unoscillated neutrino flux 280 m from the neutrino production point, so no oscillation weights are applied to the ND280 MC simulation. This will provide ND280 a large number of ν_μ ($\bar{\nu}_\mu$) events to enhance the constraints on the cross sections and flux. ND280 samples used for the oscillation analysis are based on the topology of the interaction events from ν_μ ($\bar{\nu}_\mu$) only. The ability of ND280 to measure the charge of the emitted particles can also help identify the wrong-sign components from the beam.

The current selection for T2K oscillation analysis uses 22 samples in total, 11 each for the FGD1 and FGD2 [112]. All samples must have the reconstructed interaction vertex included in the fiducial volume of the FGDs. The ν beam mode samples only contain ν_μ interaction events, while the $\bar{\nu}$ beam mode contains both events from ν_μ (wrong-sign components) and $\bar{\nu}_\mu$ selected separately for the three topologies. The samples for larger wrong-sign components in the $\bar{\nu}$ beam mode are necessary to distinguish the wrong-sign flux from the right-sign flux predicted for Super-K, providing constraints for the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation rate. The samples are split based on the event topologies: **CC0 π -0p**, **CC0 π -Np**, **CC1 π** , **CC-Other-0 γ** , and **CC-Photon** for the ν -beam mode (5 samples); **CC0 π** , **CC1 π** , and **CC-Other** for the $\bar{\nu}$ -beam mode (3 + 3 samples).

The CC0 π samples select events with one reconstructed track as a $\mu^\pm$ and without any $e^\pm$, $\pi^\pm$, π^0 , or photon tracks reconstructed. Specifically, the CC0 π -0p sample has no reconstructed protons in the event while the CC0 π -Np sample has protons tagged in the FGD or TPCs. The CC1 π case includes a π^+ or π^- track depending on the charge of the muon, so CC1 π^+ for the ν_μ events and CC1 π^- for the $\bar{\nu}_\mu$ events. The CC-Other includes all other events that are not part of the rest categories such as having tracks of multi-pions, π^0 , or opposite charged $\pi^\pm$, while the CC-Other-

0γ will exclude events with photons. The CC-Photon tags all the reconstructed ν_μ events with photons, where those events are dominated by DIS interactions and resonant interactions that produce π^0 particles. A detailed description of the selection criteria can be found in Ref. [113].

The ND280 samples are binned in reconstructed μ momentum and the angle of the outgoing μ to the incoming neutrino beam. Fig. 4.1 shows an example of binned kinematic distributions for the ν_μ CC 0π - $0p$ sample before fitting the MC to data. Details of the binning information can be found in Appendix C.1.1. The ND280 samples will be used for the T2K oscillation analysis presented in Chapters 7-8.

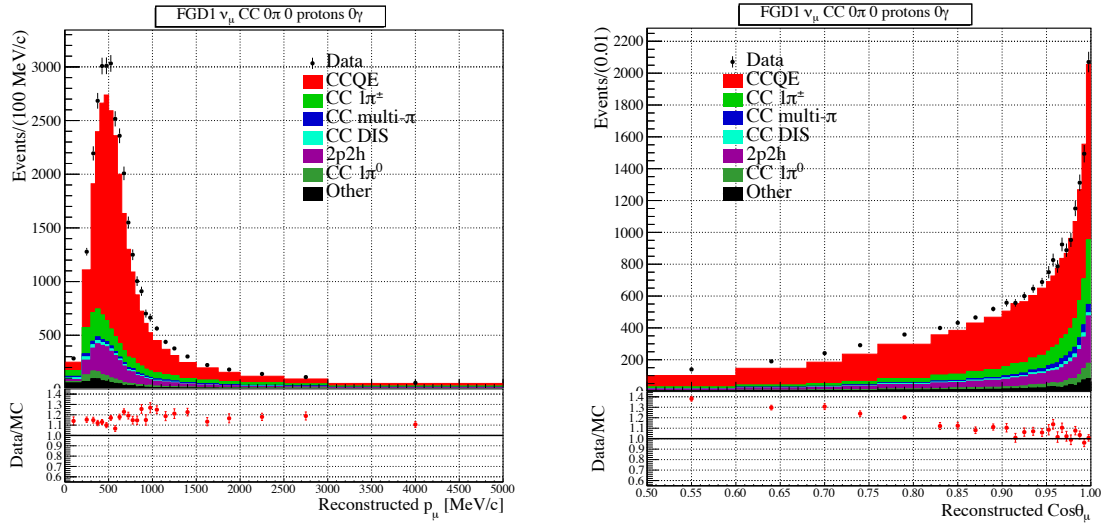


Figure 4.1: An example of the ND280 sample (the CC 0π - $0p$ sample) selected for the T2K oscillation analysis presented in Chapters 7 and 8, showing the events binned in multiple kinematic variables, reco. P_μ and reco. $\cos \theta_\mu$ [114].

4.2.2 Super-K Samples

Super-K aims to measure the oscillated ν_μ and ν_e flux. As described in Section 2.3.2, Super-K identifies the flavour of the interacting neutrinos and interaction modes by looking at the Cherenkov rings emitted from the outgoing particles. Therefore, events are selected based on the rings being μ -like or e -like respectively. The selection is performed using reconstructed kinematic information obtained by fiTQun

[94], with the event required to be fully contained (FC) in the detector fiducial volume (FV) – FCFV means that number of hits in the OD < 16 and the event vertex is reconstructed within the detector fiducial volume.

Five Super-K Samples

The T2K oscillation analysis described in Ref. [42] uses 5 reconstructed event samples at Super-K: 1-ring μ -like (**1R μ**), 1-ring e -like (**1Re**) for both ν and $\bar{\nu}$ beam modes; the 1-ring $e + 1$ decay e (**1Re1de**) for the ν beam mode only. The fully contained fiducial volume (FCFV) criteria for the 5 samples are summarised in table 4.1.

Table 4.1: The FCFV cuts for the 5 Super-K samples, where the ‘OD hits’ is the number of hits to the OD PMTs, ‘ToWall’ is the vertex distance from the ID wall in the direction of travel, and ‘DWall’ is the vertex distance from ID wall.

	1R μ	1Re	1Re1de
OD hits	< 16	< 16	< 16
ToWall	> 250 cm	> 170 cm	> 270 cm
DWall	> 50 cm	> 80 cm	> 50 cm

The 1R μ samples are selected to measure the ν_μ disappearance and the 1Re samples are for the $\nu_\mu \rightarrow \nu_e$ appearance, which both aim to have the CCQE events as the signal events. The 1R μ sample selects the topology of a single sharp ring, and allows an above-threshold decayed electron produced from the muons to show a delayed fuzzy ring signal. The 1Re sample selects the topology of a single fuzzy ring.

The 1Re1de sample makes use of the CC1 π^+ interactions to provide additional signals for ν_e appearance events, where the π^+ is assumed to be produced from the Δ resonance decays and usually below the Cherenkov threshold when induced by ν_e . Thus, visible rings can only be observed when the π^+ decays to electrons via muons as a delayed fuzzy ring signal. The 1Re1de sample is only selected for the ν beam mode, whereas for the $\bar{\nu}$ beam mode, not enough events are available to construct such a sample due to the higher nuclear capture rate of the muons produced from

the π^- decay.

The selection criteria are performed the same way for the events in ν and $\bar{\nu}$ beam modes. Table 4.2 shows a summary of the cut criteria applied. Detailed descriptions of the selection method and visualisations of the selected samples can be found in Refs. [42] and [96].

Table 4.2: The topological cuts for the 5 Super-K samples. The PID criteria for Super-K events are part of the reconstruction strategy described in Section 2.3.2. P_μ is the reconstructed momentum of the outgoing μ , E_{vis} is the total charge collected from the visible rings, E_{reco} is the reconstructed neutrino energy assuming the dominated interaction mode in the sample.

	1R μ	1Re	1Re1de
No. of decayed electron	0 or 1	0	1
No. of reconstructed ring	1	1	1
Kinematics	$P_\mu > 170 \text{ MeV}$	$E_{\text{vis}} > 75 \text{ MeV}, E_{\text{reco}} < 1250 \text{ MeV}$	
PIDs of reconstructed ring (accept/reject)	I. μ/e II. $\mu/\pi^\pm$	I. e/μ II. e/π^0	

The 1R μ sample is binned as a function of the reconstructed neutrino energy. The 1Re and 1Re1de samples are binned in a 2-dimensional kinematic space of the reconstructed neutrino energy (E_{reco}), and the angle between the primary electron from the interaction and the incoming neutrino (θ_{reco}). The additional dimension is to help distinguish the e -like rings produced from the $\nu_\mu \rightarrow \nu_e$ signal events and background events with the same topology. Details of the binning information can be found in Appendix C.1.2. These samples will be adapted for the Hyper-K oscillation analysis in Chapter 5 and for the T2K oscillation analysis in Chapters 7-8, where details of the sample kinematic distributions will be discussed in due chapters.

New Neutral Current π^0 Samples at Super-K

The neutral current (NC) π^0 samples are the new samples being studied and implemented for the T2K oscillation analysis in this thesis. The NC π^0 interactions are only visible from the rings produced due to the two photons from the decay of the π^0 , as

shown in Fig. 4.2. In most cases, the propagation of each photon produces a fuzzy Cherenkov ring, showing two separate rings at the same time. But in the scenarios of highly overlapped rings or missing rings (see Fig. 4.2), the $\text{NC}\pi^0$ events can be reconstructed as the 1-ring e -like topology, where as mentioned before, a e/π^0 PID cut is then applied during the $1\text{R}e$ selections. However, while a majority of the π^0 background has been separated, this e/π^0 PID cut also removes a fraction of the signal ν_e appearance events that are misidentified as π^0 -like. The oscillation sensitivity is expected to be improved from more signal ν_e events. Therefore, the portion that is rejected from e/π^0 PID separation in the $1\text{R}e$ selection is re-selected as the 1-ring π^0 ($1\text{R}\pi^0$) sample to accumulate the additional ν_e events.

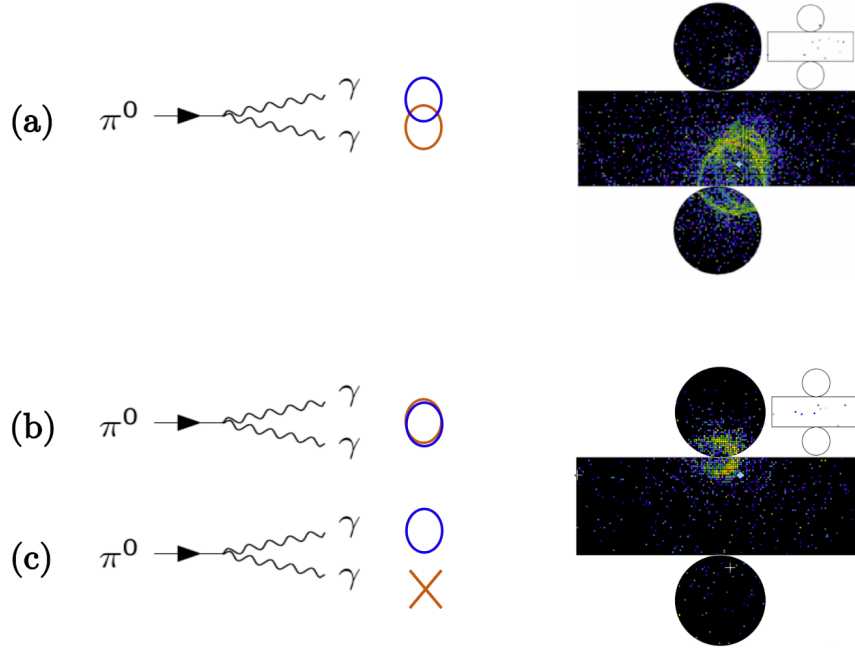


Figure 4.2: Possible topologies of photons decaying from the π^0 produced in $\text{NC}\pi^0$ interaction, include: **(a)** Clearly separated rings; **(b)** Overlapped rings from boosted photons; **(c)** Only 1 visible ring due to one photon being absorbed. Examples of Super-K event displays for $\text{NC}\pi^0$ events are provided, with the top image demonstrating case **(a)**, while the bottom image likely represents the response from either case **(b)** or **(c)** showing a single fuzzy ring – indistinguishable from a e -like ring.

Since the $1\text{R}\pi^0$ sample contains a large fraction of the $\text{NC}\pi^0$ background events, a separate sample is required to control the $\text{NC}\pi^0$ events. This control sample selects $\text{NC}\pi^0$ events with 2 clear photon rings as the 2-ring π^0 ($2\text{R}\pi^0$) sample. Super-K is

sensitive to these events due to its large detector mass and the low energy threshold for photon reconstruction, and can provide a larger event sample even though Super-K is 295 km from the neutrino source.

There are in total 4 $\text{NC}\pi^0$ samples selected: $1\text{R}\pi^0$ and $2\text{R}\pi^0$ samples in both the ν and $\bar{\nu}$ beam modes. The selection criteria for $1\text{R}\pi^0$ samples match the same as for 1Re samples, except inverting the e/π^0 PID cut. The $2\text{R}\pi^0$ samples instead select 2-ring e -like events and use slightly different e/π^0 PID cut criteria to have the optimal selection efficiency. Table 4.3 summarises the topological cuts applied to obtain these samples, where the event distributions at each selection level can be found in Appendix B.1. More details on the properties of selected $\text{NC}\pi^0$ samples and how they impact the oscillation analysis will be described in Chapters 6-8.

Table 4.3: The topological cuts for the new $\text{NC}\pi^0$ samples. The PID criteria for these samples are separately described in Chapter 6. E_{vis} is the total charge collected from the visible rings.

	$1\text{R } \pi^0$	$2\text{R } \pi^0$
No. of decayed electron	0	0
No. of reconstructed ring	1	2
Kinematics	$E_{\text{vis}} > 75 \text{ MeV}$	$E_{\text{vis}} > 75 \text{ MeV}$
PIDs of reconstructed ring (accept/reject)	I. e/μ II. π^0/e	I. ee/Other II. π^0/e

4.3 Uncertainties in the MC Predictions

4.3.1 Neutrino Flux Model

The simulated neutrino flux has the following uncertainties: the hadronic interactions and modelling with the target, beam profiling and the direction of the off-axis beam, the magnetic horn current and field, the horn alignment with the target, and the number of incoming protons. The beam profiling and off-axis angle direction uncertainties are calculated from the uncertainties in measurements from beam moni-

tors at J-PARC. The hadron production and consecutive interactions are constrained using the external hadron production data from NA61/SHINE data [115, 87], adapting a full-size replica of the T2K target. The neutrino flux MC generated from JNUBEAM is tuned according to the NA61/SHINE data before proceeding to the neutrino interaction event generation at NEUT. Details of the flux uncertainty calculations for a recent T2K oscillation analysis can be found in Refs. [42, 116], with a summary for the Super-K ν_μ flux prediction shown in Fig. 4.3.

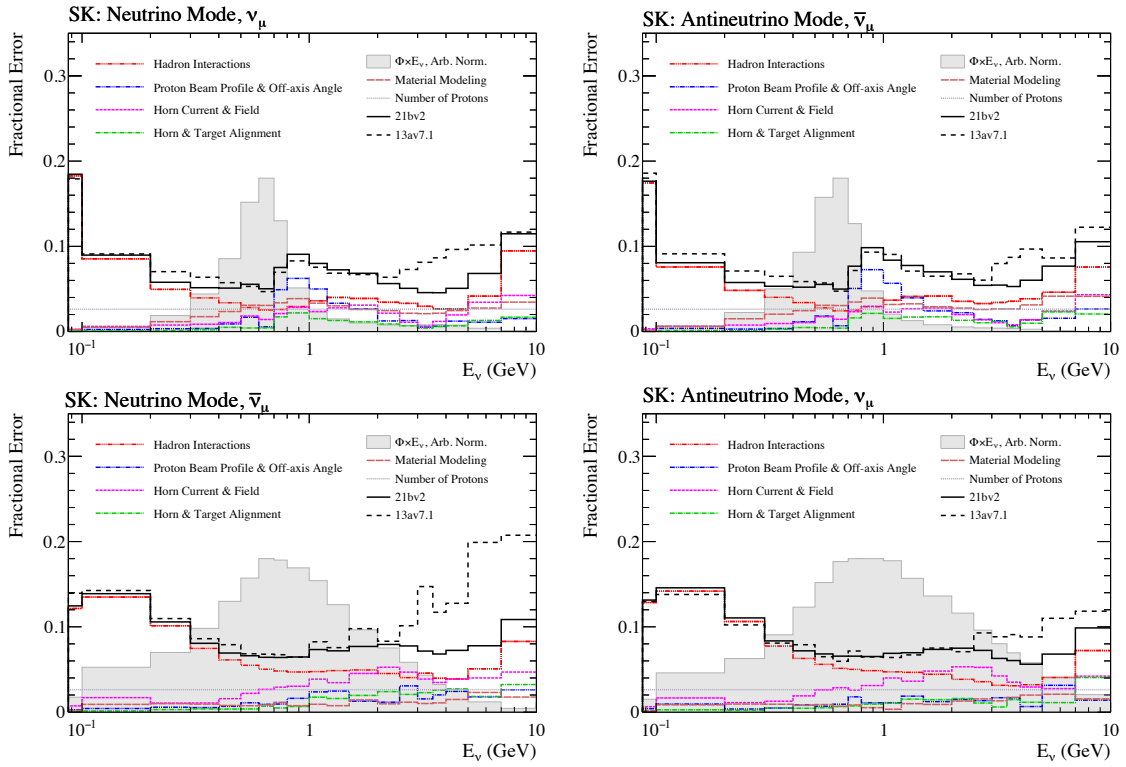


Figure 4.3: The uncertainties on the right-sign (top) and wrong-sign (bottom) Super-K flux prediction in the ν -mode (left) and $\bar{\nu}$ -mode (right) [116]. The solid black line represents the total flux uncertainty in the T2K oscillation analysis presented in this thesis, while the dashed black line corresponds to the total uncertainty from the previous T2K analysis [85]. The shape of the flux is represented by the grey shaded distribution.

For the oscillation analysis, flux uncertainties are parameterised across different E_ν regions, separated by horn currents and neutrino flavours for both ND280 and SK. The uncertainties and correlations between parameters are derived from the total uncertainties contributed by various components described earlier. The fractional uncertainties for each parameter are then calculated and used as prior constraints

during the fitting of parameters in the oscillation analysis. Variations in these parameters will then directly normalise the events in the MC.

4.3.2 Neutrino Cross Section Model

The interaction cross-section uncertainties are handled by three different types of parameters:

- **Shape parameters:** These include parameters that directly model the interaction cross sections for event generation in NEUT, as well as other parameters that affect either the shape or normalisation of the events. Those parameters are theoretically motivated, with the uncertainties from the modelling. The parameters modelling the nuclear effects and interaction cross sections introduced in section 1.3 will be of this type. Some of these parameter values and uncertainties can be constrained from external experimental measurements.
- **Normalisation parameters:** parameters to simply normalise the events. Those parameters are usually constrained by the experimental data, either from external measurements or measured by T2K detectors.
- **Shift parameters:** parameters to shift the events to different reconstructed kinematic bins. This can arise from uncertainties in the outgoing particle kinematics during the interaction process. The only relevant cross-section parameter of this type for the current oscillation analysis is to account for the uncertainties in the nucleon binding energy.

The variations in shape and normalisation parameters are applied as weights to reweight the event, similar to how flux normalisation parameters are handled. Shift parameters, on the other hand, directly modify the event kinematics, causing events to migrate between different reconstructed kinematic bins.

For the shape parameters, due to the non-linear dependence of the interaction cross-section on these parameters, the resulting event weights from the parame-

ter variations are not linear as well. Therefore, specific response functions are constructed to describe the event rate variations corresponding to different parameter values for each interaction mode involved. The function is built upon the response of the event weight at points of the $\pm 1\sigma$, $\pm 2\sigma$, and $\pm 3\sigma$ variations, with a smooth curve interpolated between those points using the TSpline3 class in ROOT [117].

For the ND280 MC reweighting, the event-by-event weight responses are calculated and stored for each event. In the case of SK MC, the response is calculated and stored based on both the true and reconstructed variables. The MC reweighting is applied on an event-by-event basis, with weights assigned according to the kinematic bin to which the event belongs. An example of a response function illustrating the parameter variations is shown in Fig. 4.4.

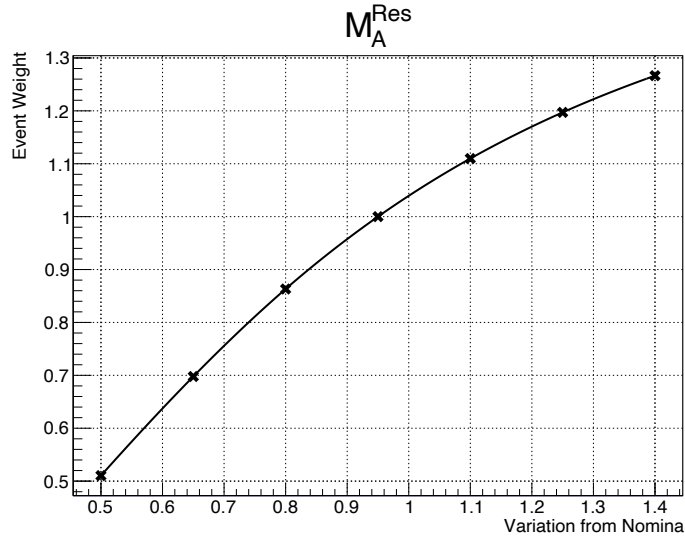


Figure 4.4: The non-linear response function on weighting the MC event from given variations of a parameter. The 7 points show the pre-calculated responses of the event rate at the nominal and the $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$ variations for interpolating the curve.

There is a long list of cross-section uncertainty parameters that correspond to the different interaction types, neutrino modes, and functionalities. Some of these parameters are not impacted by the improvements to the oscillation analyses discussed in this thesis, and therefore will not be covered in detail. An in-depth explanation of the cross-section uncertainties used in the T2K oscillation analysis can be found in Ref. [42]. A brief summary of the parameter central values, uncertainties, and

their properties is provided in Appendix A.1. The cross-section parameters that are directly relevant to the work presented in this thesis will be discussed in detail in the corresponding analysis chapters.

4.3.3 Detector Systematics

Detector systematics involve uncertainties in particle identification (PID) and the reconstruction of neutrino-nucleus interaction events. There are also uncertainties from hadron secondary interactions and from photonuclear effects (primarily in Super-K). Additionally, the ND280 detector has the uncertainties related to magnetic field distortions, sand muon backgrounds, and the FGD mass, that are intrinsic to the detector setup [42]. For Super-K, apart from these mentioned above, there is also an energy scale uncertainty of 2.1% [118].

In the T2K experiment, detector systematics are constrained using well-understood control samples (e.g., cosmic ray muon data for ND280, and cosmic ray / atmospheric neutrino data for Super-K) and the calibration data, which provide information on the detector response of known particles. Comparisons of data with MC simulations help identify biases and uncertainties in the detector response.

Detector systematics can cause events to migrate in or out of specific selections and shift the reconstructed kinematics of the particles. Such uncertainties can be classified as either efficiency-like that depends on the particle's kinematics or normalisation-like that are related to the sample properties (e.g. topologies and PID) [42]. A detailed description of the method used to generate the uncertainties and covariances of the detector systematic parameters for the samples used in this analysis can be found in Ref. [114] for ND280-related (used in the T2K oscillation analysis here) and Ref. [118] for Super-K-related (which serves as the starting point for Hyper-K and T2K oscillation analyses). In the Hyper-K and T2K oscillation analysis presented here, updated Super-K detector systematic models are used, with further details provided in Sections 5.2 and 7.3, respectively.

4.4 Oscillation Parameters for MC

The central values of the oscillation parameters used to generate the MC simulations are chosen based on the best-fit values from previous measurements. This approach ensures that the sensitivity studies are performed close to the true values of the oscillation parameters. During a data fit, parameters with priors set to these best-fit values help reduce the complexity of finding the optimal fit for the unmeasured parameters, improving the overall efficiency of the fitting process. The Asimov A dataset has been used for the analyses presented, as summarised in Table 4.4.

Table 4.4: The oscillation parameter central value defined in “Asimov A”-type datasets.

Parameter	AsimovA 2022	AsimovA 2020
$\sin^2(\theta_{12})$	0.307	0.307
$\sin^2(\theta_{23})$	0.561	0.528
$\sin^2(\theta_{13})$	0.022	0.0218
Δm_{21}^2 (eV ²)	7.53×10^{-5}	7.53×10^{-5}
Δm_{32}^2 (eV ²)	2.494×10^{-3}	2.509×10^{-3}
δ_{CP}	-1.601	-1.601

4.5 Validation Methods

Overall, inputs required for a complete oscillation analysis will include the sampled MC, the prior central values, uncertainties, correlations for systematic and oscillation parameters, and the response functions for the MC. A set of methods can be used to check whether the inputs applied and parameter values evaluated are as expected and validate the results from new implementations. The following methods contain a standard checking procedure for the validation studies:

- The event rate predictions at SK over different samples, interaction modes, and oscillation modes. The event rates should be consistent if the calculations are

made using the same input values, e.g. the POT, systematic and oscillation parameter values, and the MC generation method from SK. This is a basic approach to validate any changes made to the code.

- The changes in the event rate prediction and the event spectra due to the variations of the parameter. The event spectra are expected to be tweaked further from the nominal by an amount according to the event weight response from the parameter variation. The tweaked spectra with the $\pm 1\sigma$ and $\pm 3\sigma$ variations of the parameters are usually checked.
- The negative log-likelihood (nLLH) distribution for individual parameters, where definitions of the nLLH are introduced in Section 3.1. Both the sample nLLH and parameter nLLH are calculated by varying a specific parameter while keeping all other parameters fixed at their central values. This method generates a distribution of nLLH values across a range of parameter variations. When scanning over a dataset generated with the same central values, the minimum of this distribution is expected to align with the prior value of this parameter. The width of the nLLH distribution reflects how closely the uncertainty prior to the fit aligns with the post-fit result, and indicates the extent to which the data can constrain the parameter compared to the prior.

4.5.1 The Simulated Data Study

The simulated data study aims to fit the MC to a well-understood simulated dataset. Before fitting the MC events to the real data, the whole fitting process and expected sensitivities can be examined with simulated datasets from known parameter values, i.e. a different set of MC-predicted event spectra. Usually the MC dataset is generated with tweaked central values for certain parameters to test the robustness of converging to the true values of the parameters when fitting to real data. For a valid fitting process, the postfit parameter values are expected to be close to the input central values to produce the MC dataset. There are two types of simulated

datasets that can be used to test the fitting process: the *Asimov* dataset and the fake dataset.

The Asimov dataset is obtained by setting all parameters to their prior value, and thus this is to fit the MC predicted events to itself (the so-called *Asimov fit* [119]). The Asimov fit method is the most straightforward method for reflecting the convergence of the fitting and the behaviour of the posterior results. Parameters are expected to remain centred around their priors, with their postfit uncertainties close to the 1σ width of the total nLLH distributions. The Asimov fits can also be used for assessing sensitivities in measuring oscillation parameters from new event samples or new systematic constraints.

The ‘fake’ data method uses generated datasets assuming a different value from the prior for some particular systematic parameters. This is to model the fitting procedure with a dataset different from the initial MC dataset. This can test the robustness of the fitting progress in matching the MC predictions to the different event rates and whether it is intended to approach the parameter values used for generating the fake data. This can directly reflect what behaviour will potentially be expected in a real data fit.

Chapter 5

Oscillation Sensitivities with New IWCD Constraints at Hyper-Kamiokande

As a successor of T2K, the Hyper-Kamiokande (Hyper-K / HK) experiment aims to accumulate large ν_μ disappearance and $\nu_\mu \rightarrow \nu_e$ appearance datasets to measure δ_{CP} precisely, and constraining better the octant and value of θ_{23} , and the absolute value of Δm_{32}^2 . The CPV will be expected to be discovered at 5σ level for 25% of true δ_{CP} values within 3 years. Detailed descriptions of the Hyper-K long-baseline oscillation sensitivity studies can be found in Ref. [77] and [40].

This chapter describes the methods of performing a sensitivity study on the oscillation parameter with potential new constraints on the interaction cross sections from the Intermediate Water Cherenkov Detector (IWCD) for Hyper-K. The study aims to assess how much the uncertainties on the oscillation parameters can be improved from the impact of systematic uncertainty improvement, especially when measurements are no longer statistically limited.

The analysis was performed using the VALOR [120] oscillation fitting framework. The strategy of constructing new IWCD systematic parameters and how they proceeded to the fitting framework will be described. Finally, the impact of the IWCD constraints relative to the T2K cross-section model constrained by ND280 will be described.

5.1 Analysis Strategy

One of the dominant systematic uncertainties to affect the oscillation sensitivity is the interaction cross sections of neutrinos with the oxygen nuclei, especially the difference in the cross sections between ν_μ ($\bar{\nu}_\mu$) and ν_e ($\bar{\nu}_e$). As discussed in Section 2.3.2, IWCD can measure the relative ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$ cross section very well, and reduce the $\nu_e/\bar{\nu}_e$ uncertainty (defined in Eq. 2.7) using a model-independent approach.

There are two types of systematic parameters constructed to describe IWCD cross-section uncertainties. One is for ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$ cross sections, and parameterised in true neutrino energy (E_ν). The other type is to constrain the “feed-down” events described in Section 2.4.3, where they are parameterised to model the difference between the E_ν and the reconstructed energy of those events assuming CCQE interactions (reco. E_ν^{CCQE}), defined in Eq. 2.1. This difference is modelled using the difference between E_ν and the energy calculated from the MC truth charged lepton momentum and scattering angle of the interaction assuming CCQE interaction, denoting this as $E_\nu - \text{true } E_\nu^{CCQE}$. The true E_ν^{CCQE} has the same form as in Eq. 2.1, except that the P_l , E_l , and θ_{beam} becomes the true kinematic variables instead. This parameter type uses a 2-dimensional space to describe the dependence of $E_\nu - \text{true } E_\nu^{CCQE}$ on E_ν . The additional dimension will bring an extra degree of freedom to the cross-section model, which causes an increase in the cross-section uncertainties.

The parameterisations of other cross-section and flux, and the Hyper-K detector systematic uncertainties are used based on the T2K 2018 model [121] with slight modifications. The 2p2h ^{12}C to ^{16}O normalisation uncertainty is not included due to the water-only target at IWCD and thus has the parameter fixed. These parameters have nominal values set to the ND280 postfit dataset [85].

Apart from the cross-section and flux systematics, there are two types of detector systematics for IWCD. One is to describe the uncertainties in the pile-up efficiency loss ($\sim 10\%$)[98], and the other is the energy scale uncertainty similar to the one used for Super-K (2-5%). Due to the large neutrino flux at IWCD, multiple inter-

action events can happen from the same beam bunch, which becomes more difficult to reconstruct compared to the single-interaction case. Thus the MC is weighted down to account for the statistics of the single-interaction events only, according to the estimated pile-up factor at different off-axis angle (OAA) positions of the detector. In the IWCD fit, the detector systematic parameters are the nuisance parameters, while the cross-section and flux parameters are the parameters to be measured.

Due to the similarity in the detection methods of IWCD and Super-K, IWCD uses similar selection criteria to get the $1R\mu$ and $1Re$ signal samples using the reconstructed information from fiTQun. It also makes use of the 2-ring $NC\pi^0$ samples to control the π^0 background due to its good performance in measuring overlapped rings from the high-resolution mPMTs. Each of the 3 topologies is selected for the ν and $\bar{\nu}$ beam modes. The selections are performed to cover events at different OAA spans to understand the effects of the size of the span range on the oscillation sensitivities. The signal samples are binned in the reconstructed neutrino energy, while the $NC\pi^0$ samples are binned in the reconstructed π^0 mass. A detailed description of the selection methods and description of the samples can be found in Ref. [73].

During the fitting process at VALOR, constraints on systematic parameters are progressively applied using data from the near and far detectors. This analysis aims to assess the impact of IWCD data on constraining cross-section and flux uncertainties. The postfit uncertainties from the IWCD fit are subsequently passed to a fit using Hyper-K far detector data for oscillation measurements. Both the IWCD and Hyper-K fits are performed by profiling a binned Poisson likelihood, as described in Sections 3.1 and 3.2.

The oscillation sensitivity is evaluated from fits to an Asimov dataset according to the Asimov A (2020) dataset (in Table. 4.4), generated at different values for the parameter of interest, while other oscillation parameters and nuisance parameters remain at their central values. During the fit, $\sin^2(\theta_{12})$ and Δm_{21}^2 are fixed at their nominal values, and $\sin^2(\theta_{13})$ is treated as a nuisance parameter based on the reactor constraints. The remaining oscillation parameters are left free, with $\sin^2 \theta_{23}$ and δ_{CP}

assigned starting values unless they are the target parameters of the fit, while the Δm_{32}^2 is fixed to the normal mass ordering. Multiple starting values are used to reduce the risk of the fit getting stuck in a local minimum in the likelihood space. The starting point that maximises the likelihood will be selected for evaluating the postfit distributions.

The likelihood is calculated for each sample by comparing MC and data across reconstructed kinematic bins, according to Eq. 3.3. The MC predictions in the r^{th} reconstructed bin can be obtained from the sum over the template bins, where

$$N_r^\beta = T_{r,g_{ES}} \sum_m \sum_t \sum_{(t,r')} P_t^{\alpha \rightarrow \beta} T_{m,r',g}(S_{m,t,r',f} \Phi_{m,t,r'}^\alpha). \quad (5.1)$$

$\Phi_{m,t,r'}^\alpha$ is the tuned flux of an event before oscillation (indicated by α), with interaction mode m in the t^{th} true kinematic bin, and is reconstructed to be in the $(r')^{th}$ bin. $S_{m,t,r',f}$ is the total multiplicative weight from the systematic parameters $\mathbf{f}$ that applies to the events with modes m and bins (t, r') . $T_{m,r',g}$ represents the migration of events with mode m between different reconstructed kinematic bins r' , when parameter $\mathbf{g}$ is applied. $P_t^{\alpha \rightarrow \beta}$ is the $\nu_\alpha \rightarrow \nu_\beta$ oscillation probability calculated based on the true neutrino energy and kinematics in bin t . $T_{r,g_{ES}}$ applies a shift in the energy scale, often due to energy calibration uncertainties or detector mis-calibration, to adjust the final prediction in the r^{th} reconstructed bin.

The method for applying the systematic parameter variations to the MC involves binning the MC in a multi-dimensional template that span both the systematic parameter binning and the sample binning of the MC. When a systematic parameter is varied, it reweights the events associated with particular interaction modes, beam modes, and oscillation channels that the parameter controls, leading to changes in the event rate for specific bins within the template. As discussed in Section 4.3.2, both the shape and normalisation parameters apply direct weight variations to events, while shift parameters cause events to migrate between bins due to changes in reconstructed kinematics.

For this analysis, the template binning is thus 4-dimensional over E_ν , E_ν -true E_ν^{CCQE} , E_{reco} , and θ_{reco} . The E_ν -true E_ν^{CCQE} template is aligned with the systematic parameter binning to optimise the efficiency of the fit.

5.2 Inputs for Oscillation Analysis

5.2.1 Postfit Constraints

The analysis is performed based on a POT exposure of 0.675×10^{21} (2.025×10^{21}) per year for the (anti-)neutrino beam, with statistics from a 10-year data taking assumed. There are also several strategies to model the systematics for Hyper-K, where the cases presented in this thesis are summarised below.

- **The T2K-2018 systematics:**

This is to use the same systematics constraints used for the 2018 T2K oscillation analysis [122], but with the Hyper-K 10-year data assumed. The aim is to show the impact on the oscillation sensitivity of any new model constructed by comparing it with this one.

- **The “improved” systematics:**

Scaling down the uncertainty on flux and cross-section systematics according to the relative increase in neutrino beam exposure from T2K to Hyper-K, with $\sqrt{1/N}$, where $N = 8.63$ considering the relative increase in the beam exposure for Hyper-K and T2K [40]. The same increase in the atmospheric neutrino data is assumed to estimate the SK detector systematics. No parameter was allowed to have an uncertainty of less than 1%. A reduction of the $\nu_e/\bar{\nu}_e$ error from 3.6% to 1% is applied to assess its impact. Expected additional constraints from ND280 Upgrade and IWCD on the cross-section uncertainties are included as [40]:

- A factor of 3 reduction on all non-QE uncertainties.
- A factor of 2.5 reduction on all QE uncertainties.

- A factor of 2 reduction on all $\bar{\nu}$ uncertainties.
- A reduction in neutral current uncertainties to 10%.

- **The IWCD systematics:**

This uses the model for the new IWCD cross-section parameters described in Section 5.1. No prior constraints are applied to the 2-D cross-section parameters, allowing the neutrino cross-section to be unconstrained by external inputs and purely informed by the IWCD data. The Hyper-K detector systematic uncertainties take the same scaling as the “improved” systematics, with the prior covariance matrix applied to the fit shown in Fig. 5.2.

Sensitivities from different OAA spans are considered, including the 1° - 4° , 1.5° - 4° , and the fixed- 2.5° configurations. The impact is studied for both a 10-year exposure and a reduced 70% exposure (accounting for a potential delay in IWCD operation). Fig. 5.1 presents the post-IWCD-fit uncertainties and covariance of the cross-section and flux parameters for the 1° - 4° case.

A summary of the systematic uncertainty parameterisation is provided in Table 5.1. Detailed definitions of parameters from the T2K 2018 model and the binning scheme for all the parameters can be found in Appendix A.

Table 5.1: The parameterisation of systematic uncertainties for Hyper-K oscillation analysis. The parameter corresponds to the $2p2h$ ^{12}C to ^{16}O cross-section normalisation uncertainty from the T2K 2018 model is fixed for this analysis.

Parameter	Interaction Types	Binning	Size
Flux from T2K 2018	ν or $\bar{\nu}$ all types	E_ν	50
Xsec. modified from T2K 2018	CC or NC	E_ν	18
IWCD 1-D $\nu_e + \bar{\nu}_e$ norm. xsec.	$\nu_e(\bar{\nu}_e)$ CC	E_ν	10
IWCD 2-D $\nu + \bar{\nu}$ norm. xsec.	$\nu_\mu(\bar{\nu}_\mu)$ CC + $\nu_e(\bar{\nu}_e)$ CC	E_ν , $E_\nu - \text{true } E_\nu^{CCQE}$	58
SK detector syst. (T2K 2018)	CC or NC	E_{reco}	44
SK energy scale	all	E_{reco}	1

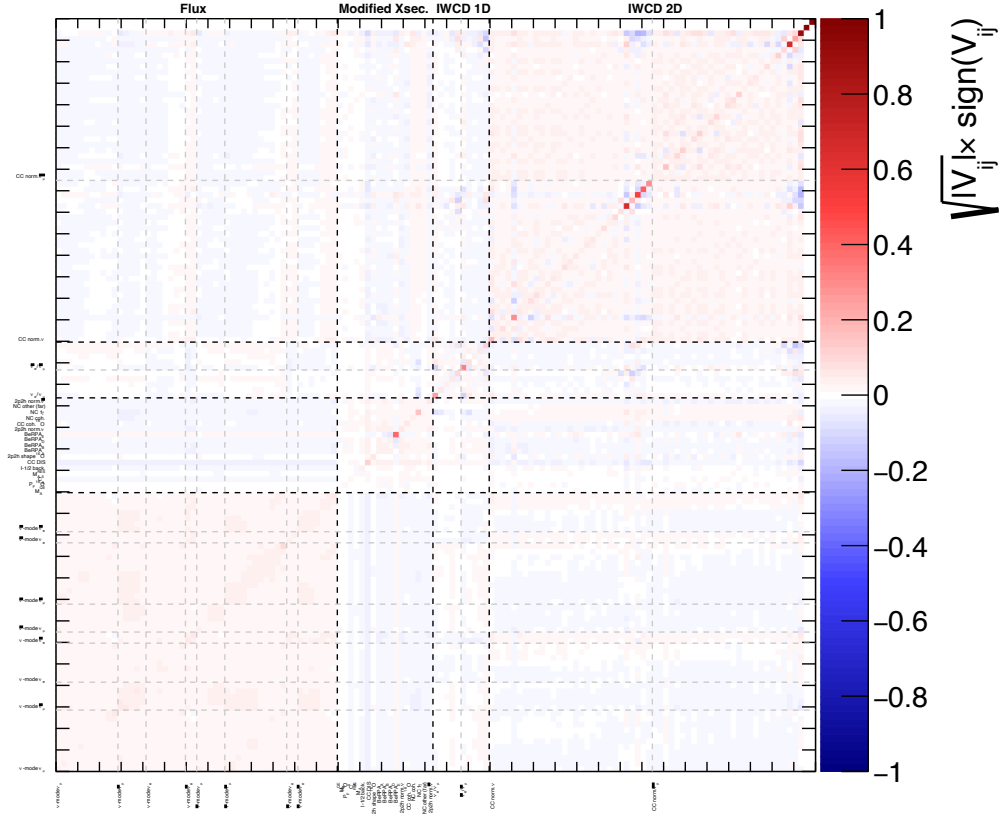


Figure 5.1: The post-IWCD-fit cross-section and flux covariance matrix containing the 135 parameters described in Table 5.1.

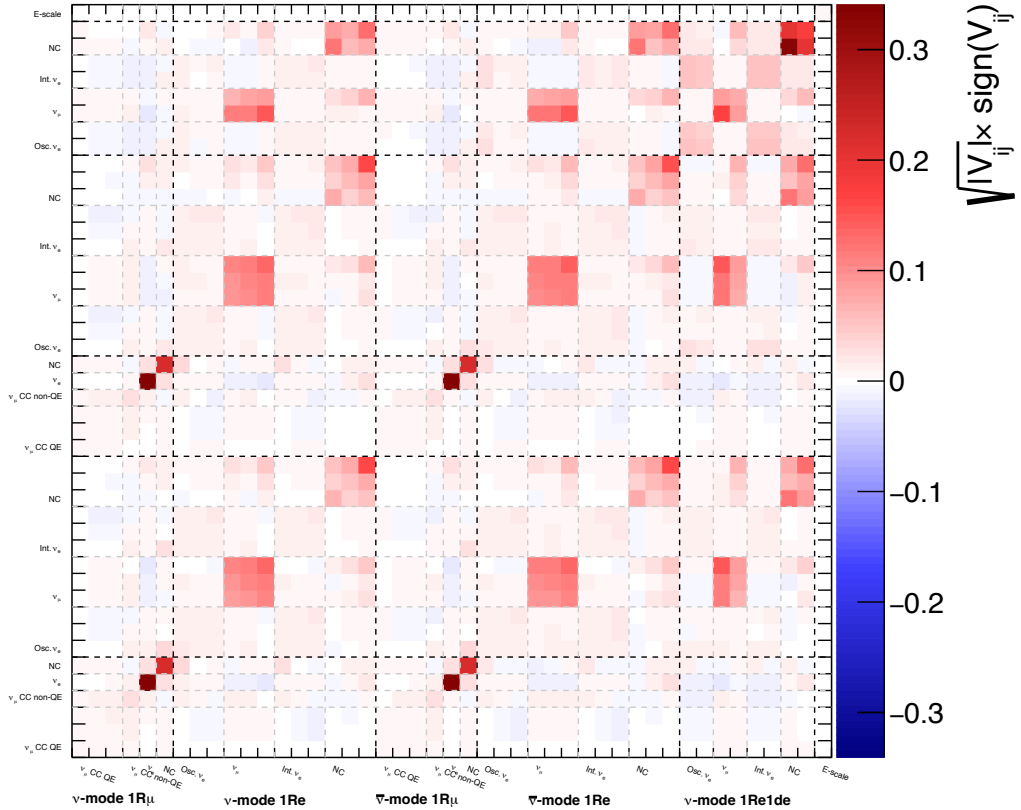


Figure 5.2: The prior Hyper-K detector covariance matrix scaled from the T2K2018 Super-K detector systematics, including the 45 parameters described in Table 5.1.

5.2.2 The Hyper-K MC

The Hyper-K far detector uses the same event selection as the five existing Super-K samples described in Section 4.2.2, with data scaled for a 10-year exposure and an 8.3 times larger detector size. The MC spectra are illustrated in Fig. 5.3. With 10 years of Hyper-K data, a total of approximately 9,300 (12,300) events are expected for the $1R\mu$ samples in the ν ($\bar{\nu}$) beam mode, while for the $1Re$ and $1Re1de$ samples, around 2,700 (1,600) events are expected.

The event spectra over E_ν -true E_ν^{CCQE} are displayed in Fig. 5.4. The distributions of the CCQE events are centred around $E_\nu = \text{true } E_\nu^{CCQE}$, showing the majority of the CCQE events occur near the true E_ν^{CCQE} . For events located to the right of the peak, the assumed true E_ν^{CCQE} is less than E_ν , which may be attributed to energy loss to non-leptonic particles produced during the interaction. For the events to the left of the peak, the produced lepton could be boosted to higher energies when interacting with an energetic nucleus, resulting in true E_ν^{CCQE} being greater than E_ν . The $1Re1de$ sample shows a shifted distribution above zero, indicating increased energy loss to non-leptonic particles. This shift suggests that processes like resonance production, expected from the dominant $CC1\pi$ mode, contribute to the observed energy loss.

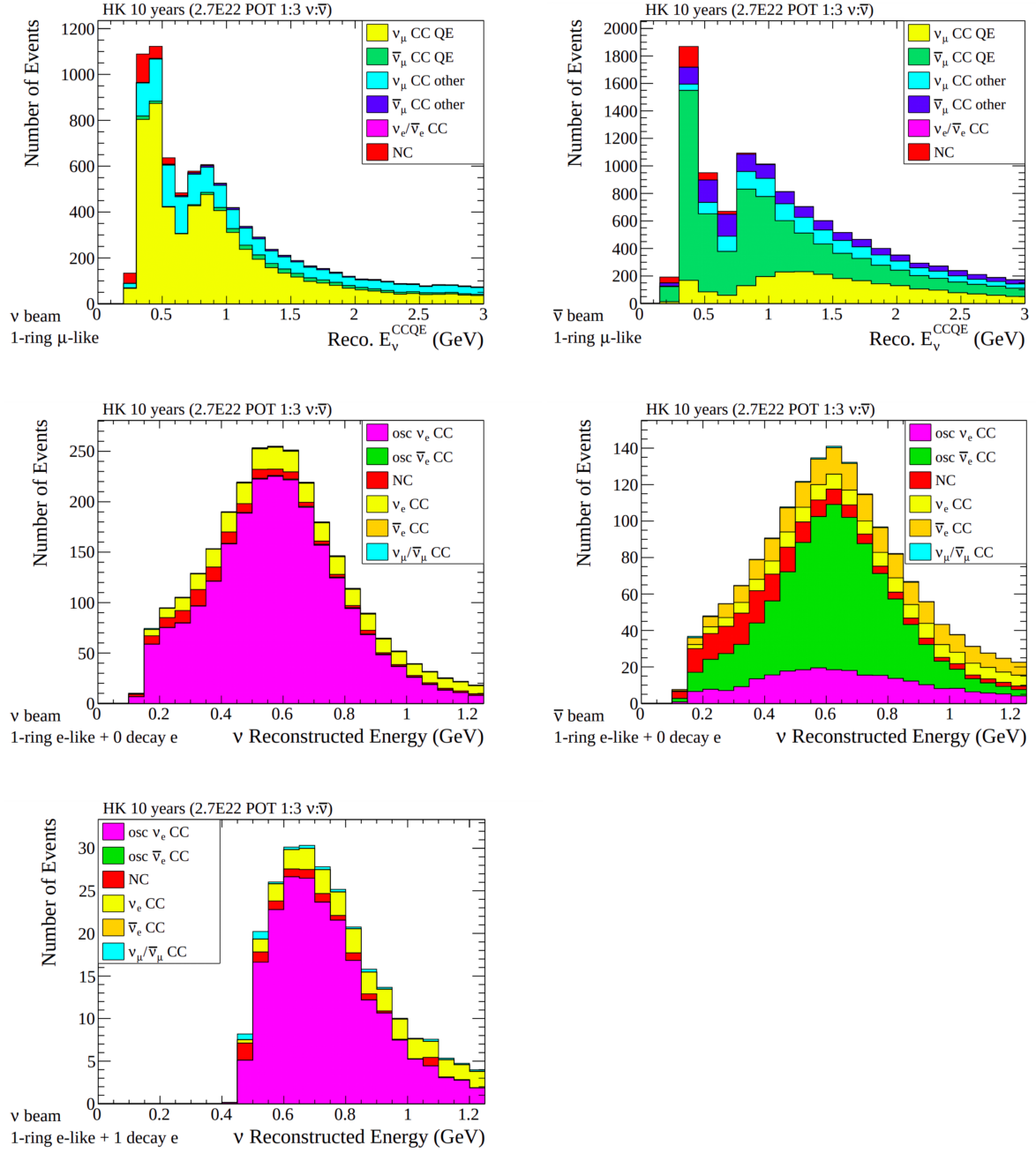


Figure 5.3: The breakdown of spectra for the 5 Hyper-K samples in reconstructed neutrino energy, assuming normal ordering $\delta_{CP} = 0$, $\sin^2 \theta_{23} = 0.528$, $\sin^2 \theta_{13} = 0.0218$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ [123].

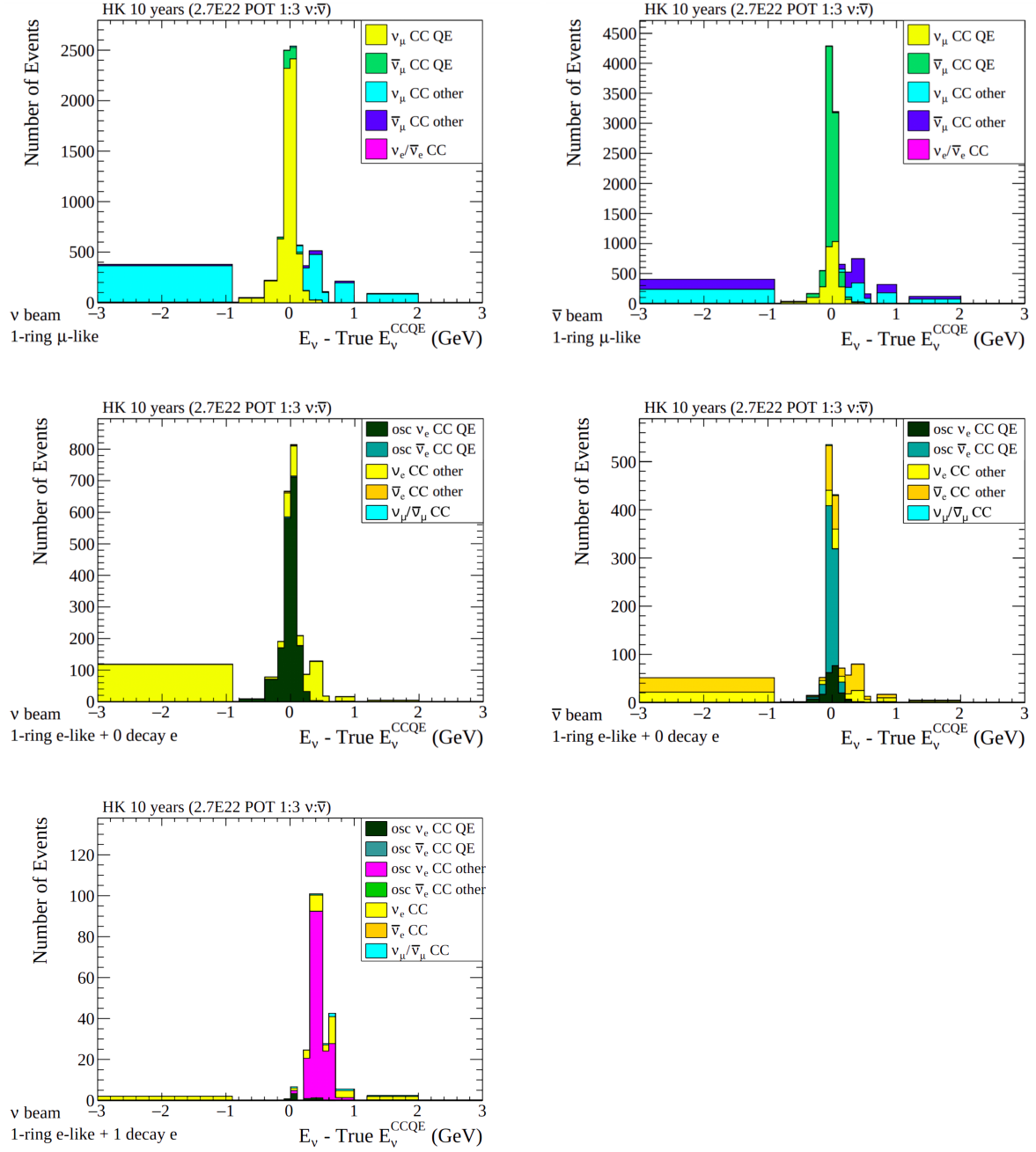


Figure 5.4: The breakdown of spectra in ν interaction modes for the 5 Hyper-K samples in $E_\nu - \text{true } E_\nu^{\text{CCQE}}$, assuming normal ordering $\delta_{CP} = 0$, $\sin^2 \theta_{23} = 0.528$, $\sin^2 \theta_{13} = 0.0218$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$.

5.2.3 Impact from IWCD Constraints

The impact on the $\nu_e/\bar{\nu}_e$ cross-section uncertainties from different IWCD detector positions is shown in Table 5.2, comparing to the T2K 2018 and the “Improved” systematics model. The values reflect the relative uncertainties for the ν -mode and the $\bar{\nu}$ -mode 1Re events when only varying the ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$ parameters in the cross-section model. The results show a reduction in uncertainties compared to the T2K 2018 model, even with only 70% of the IWCD data, and further reductions by up to 1.4% when achieving the same statistics. However, wider OAA spans lead to slightly larger errors, likely due to lower purity of the ν_e events in the more on-axis regions where the fraction of the Kaon decay products is lower at the more on-axis region, ended up with fewer ν_e events than ν_μ .

Table 5.2: Relative ν -mode and the $\bar{\nu}$ -mode 1Re event rate uncertainties due to ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$, with post-ND/IWCD-constraints applied, and assuming normal ordering, $\delta_{CP} = -1.885$, $\sin^2 \theta_{23} = 0.532$, $\sin^2 \theta_{13} = 0.0212$, $\Delta m_{32}^2 = 2.452 \times 10^{-3} \text{ eV}^2$.

Model	Uncertainty
T2K 2018	4.9%
HK “Improved”	2.7%
IWCD 2.5°	3.5%
IWCD 1°-4°	3.8%
IWCD 1.5°-4°	3.6%
IWCD 1°-4°, 70%	4.5%
IWCD 1.5°-4°, 70%	4.3%

The overall event rate uncertainties for each sample are summarised in Table 5.3, comparing the T2K 2018, “improved”, and IWCD systematics. Flux and cross-section uncertainties remain the dominant contributors to the $\nu/\bar{\nu}$ 1Re event rate error. IWCD constraints reduce this error by half compared to T2K 2018, and by about 40% with 70% exposure, while the “improved” model reduces it by 60%. The 1°-4° configuration performs better than the fixed 2.5° OAA.

Errors in 1R μ samples are reduced by over 25%, reaching 40% for the 1.5°-4° case. However, the 70% exposure scenario leads to larger ν_μ errors, especially in the $\bar{\nu}$ -mode, for the 1.5°-4° span compared to the 1°-4° span.

Table 5.3: Event rate uncertainties from variations of the systematic parameters, assuming normal ordering, $\delta_{CP} = -1.885$, $\sin^2 \theta_{23} = 0.532$, $\sin^2 \theta_{13} = 0.0212$, $\Delta m_{32}^2 = 2.452 \times 10^{-3} \text{ eV}^2$.

	ν_μ samples			ν_e samples			
	ν -mode	$\bar{\nu}$ -mode		ν -mode 1Re-like	$\bar{\nu}$ -mode 1Re-like	ν -mode 1Re1de-like	ν -mode/ $\bar{\nu}$ -mode 1Re-like
Error Source (HK Detector Syst.)							
T2K 2018	3.22%	2.76%		4.14%	4.39%	17.77%	2.06%
HK "Improved"	1.23%	1.13%		1.40%	1.61%	4.87%	0.54%
Error Source (Flux+Xsec.)	ν -mode	$\bar{\nu}$ -mode		ν -mode CCQE-like	$\bar{\nu}$ -mode CCQE-like	ν -mode CC1 π -like	ν -mode/ $\bar{\nu}$ -mode CCQE-like
T2K 2018	3.27%	2.95%		4.33%	4.37%	4.99%	4.52%
HK "Improved"	0.53%	0.52%		1.70%	1.54%	1.78%	1.96%
IWCD 2.5°	2.17%	1.85%		2.56%	2.05%	2.86%	2.51%
IWCD 1°-4°	2.23%	2.15%		2.45%	2.34%	3.18%	2.50%
IWCD 1.5°-4°	1.90%	1.89%		2.27%	2.18%	2.85%	2.30%
IWCD 1°-4°, 70%	2.24%	2.12%		2.65%	2.54%	3.35%	2.83%
IWCD 1.5°-4°, 70%	2.29%	5.26%		2.59%	2.42%	3.26%	2.69%
Error Source (All Systematics)	ν -mode	$\bar{\nu}$ -mode		ν -mode CCQE-like	$\bar{\nu}$ -mode CCQE-like	ν -mode CC1 π -like	ν -mode/ $\bar{\nu}$ -mode CCQE-like
T2K 2018	4.63%	4.10%		5.97%	6.25%	18.49%	4.95%
HK "Improved"	1.34%	1.24%		2.19%	2.24%	5.16%	2.03%
IWCD 2.5°	2.49%	2.18%		2.90%	2.60%	5.61%	2.57%
IWCD 1°-4°	2.58%	2.44%		2.80%	2.85%	5.76%	2.55%
IWCD 1.5°-4°	2.26%	2.20%		2.67%	2.71%	5.63%	2.39%
IWCD 1°-4°, 70%	2.56%	2.42%		3.02%	3.02%	5.90%	2.92%
IWCD 1.5°-4°, 70%	2.59%	5.31%		2.94%	2.89%	5.86%	2.76%

5.3 Impact on Oscillation Sensitivity

A set of Asimov fits was performed against different oscillation parameters and true-value assumptions to evaluate the impact of the IWCD constraints, compared to the T2K 2018 and the “improved” systematic models. The resulting $\Delta\chi^2$ functions for the oscillation parameters are shown in Fig. 5.5 with the models.

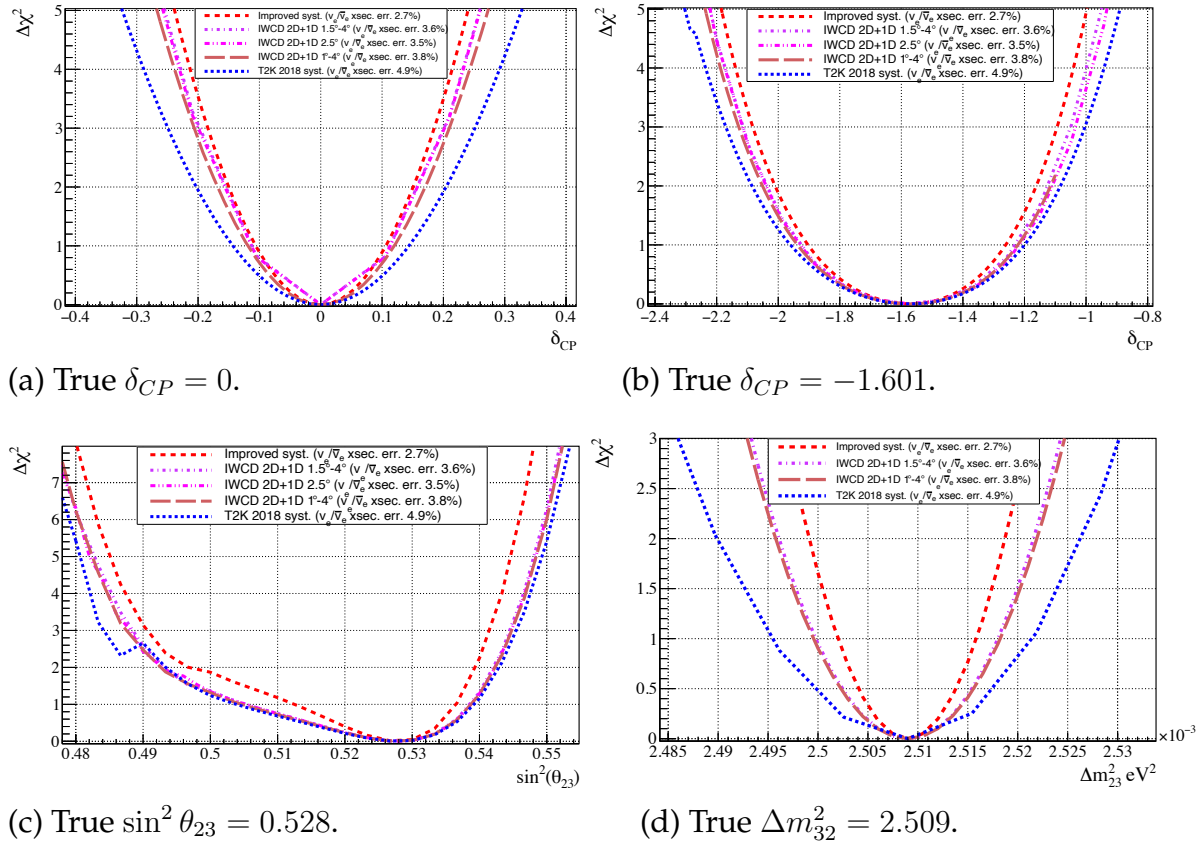


Figure 5.5: Hyper-K sensitivities of (a) and (b) δ_{CP} , (c) $\sin^2 \theta_{23}$, and (d) Δm_{32}^2 , after 10-year run time using the T2K 2018, “improved”, and IWCD constrained models, assuming normal ordering, $\sin^2 \theta_{23} = 0.528$, $\delta_{CP} = -1.601$, $\sin^2 \theta_{13} = 0.0218$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$.

Fig. 5.5 (a) and (b) show the expected precision for δ_{CP} with the true δ_{CP} set at 0° and -90° , respectively. The 1σ error decreases from 8.5° (22°) to $<7.3^\circ$ ($<21^\circ$) for true $\delta_{CP}=0^\circ$ (true $\delta_{CP}=-90^\circ$), reflecting an improvement of at least 14% (5%) with the IWCD constraints. A more significant improvement is observed for true δ_{CP} at the CP -conserved value. The achievable precision on δ_{CP} can be evaluated from the

partial derivative error $\partial P(\nu_\mu \rightarrow \nu_e)/\partial \delta_{CP}$, derived from the oscillation probability in Eq. 1.13 [40].

At $\delta_{CP} = 0$, the CP -odd term ($\sin \delta_{CP}$) in $P(\nu_\mu \rightarrow \nu_e)$ vanishes, while its derivative ($\cos \delta_{CP}$) is maximal. As a result, the difference in the oscillation probabilities for $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance channels depends primarily on the event rate differences between ν_e and $\bar{\nu}_e$, and the precision of δ_{CP} is largely influenced by the $\nu_e/\bar{\nu}_e$ cross-section uncertainty.

At $\delta_{CP} = -90^\circ$, the CP -odd term dominates, making the precision of δ_{CP} more dependent on shape differences in the ν_e and $\bar{\nu}_e$ spectra. These shape uncertainties arise from various factors beyond the $\nu_e/\bar{\nu}_e$ cross-section uncertainty, including uncertainties in event reconstruction and the statistical fluctuations from the data.

Improvements in $\sin^2 \theta_{23}$ are relatively less significant than for δ_{CP} , as shown in Fig. 5.5 (c). This is likely due to fitting towards the local minima, which will be discussed in more detail in Section 5.4.

Finally, Δm_{32}^2 shows a more obvious improvement, with its uncertainty reduced from 0.51% to 0.39% under IWCD constraints, as shown in Fig. 5.5 (d).

Fig. 5.6 shows the significance levels to exclude CP -conservation, defined as the minimum $\sqrt{\Delta\chi^2}$ among $\delta_{CP} = 0$ and $\pm\pi$ in the $\Delta\chi^2$ distributions against different true δ_{CP} assumptions. The reduction of the $\nu_e/\bar{\nu}_e$ cross-section error shows significant improvements, where $>58\%$ (74%) of true δ_{CP} have CPV at 5σ (3σ) with the IWCD constraints, about 10% (3%) higher than the T2K 2018 systematics.

The sensitivities of wrong octant exclusion for θ_{23} is shown in Fig. 5.7. For each assumed true $\sin^2 \theta_{23}$, fits are performed by limiting the $\sin^2 \theta_{23}$ to the opposite octant of the selected true value and obtain the minimum $\sqrt{\Delta\chi^2}$. The wrong octant exclusion is more significant to the left of the $\sin^2 \theta_{23} = 0.5$, indicating it is more likely to be identified as the correct octant when $\theta_{23} < 45^\circ$. The reason for this is due to the maximal disappearance in the ν_μ spectra happens when $\sin^2 \theta_{23} = 0.511$ rather than 0.5, so that the changes become more steady when the disappearance is getting closer to the case with 0.5 right after 0.511. The IWCD constraints have

reduced the region of where the octant cannot be identified ($< 3\sigma$ exclusion) to (0.47, 0.56), compared to the T2K 2018 model of (0.46, 0.57). It is aimed to approach the “improved” systematics case by reducing the freedom in 2D IWCD parameters and adding the ND280 upgrade constraints together with the IWCD ones.

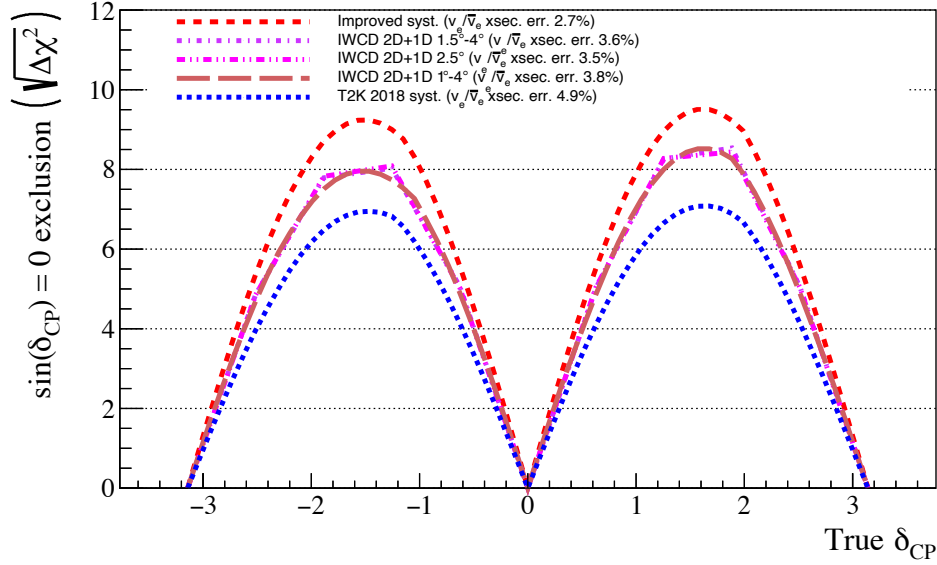


Figure 5.6: Significance level to exclude CP conservation at Hyper-K after 10-year run time using the T2K 2018, “improved”, and IWCD constrained models, assuming normal ordering, $\sin^2 \theta_{23} = 0.528$, $\sin^2 \theta_{13} = 0.0218$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$.

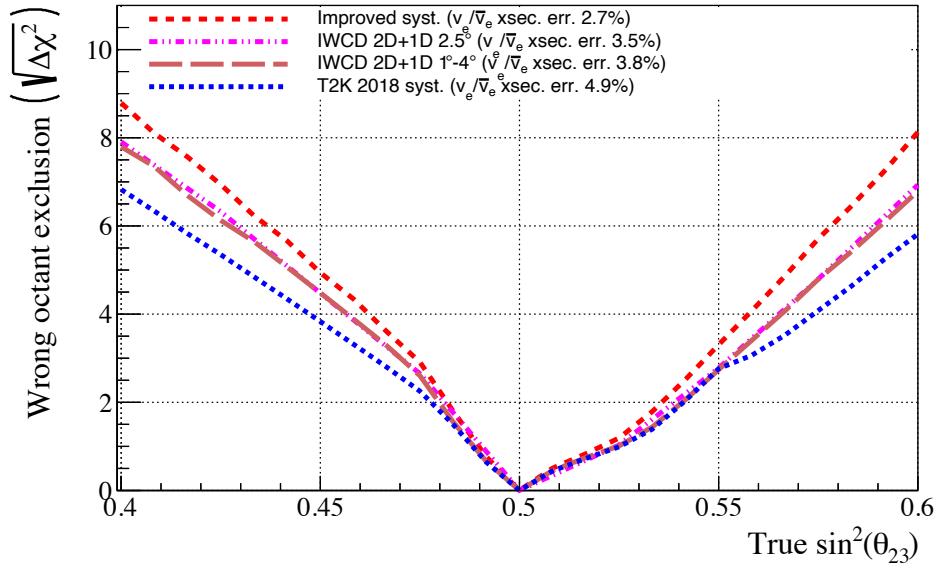


Figure 5.7: Significance level to exclude the wrong $\sin^2 \theta_{23}$ octant at Hyper-K after 10-year run time using T2K 2018, “improved”, and IWCD constrained models, assuming normal ordering, $\delta_{CP} = -1.601$, $\sin^2 \theta_{13} = 0.0218$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$.

5.4 Discussions

According to the sensitivity plots presented in the previous section, most fits converge smoothly, as reflected by the continuous $\Delta\chi^2$ distributions. However, there is a notable exception for $\sin^2 \theta_{23}$, as shown in Fig. 5.5(c). The observed discontinuity in the $\Delta\chi^2$ suggests that the fit may have fallen into and become stuck at a local minimum in the parameter space.

Fig. 5.8 illustrates the best-fit value of δ_{CP} as a function of fixed $\sin^2 \theta_{23}$, comparing two true values of $\sin^2 \theta_{23}$: 0.528 and 0.5. The analysis is performed both with and without systematic uncertainties included.

In the absence of systematic uncertainties, the fits produce smooth curves that align well with the expected δ_{CP} value corresponding to the true minimum, as predicted by the Asimov fit. However, for the true $\sin^2 \theta_{23} = 0.528$ case, a significant discrepancy appears to the left of the true value especially around $\sin^2 \theta_{23} = 0.49$, where the discontinuities occurred.

In the case of $\sin^2 \theta_{23} = 0.5$, the best-fit δ_{CP} exhibits oscillatory behaviour as a function of $\sin^2 \theta_{23}$, with multiple closely distributed local minima. This behaviour further indicates that systematic effects or statistical noise create complex correlations between $\sin^2 \theta_{23}$ and δ_{CP} . These correlations make it more difficult to reliably identify the true global minimum for δ_{CP} , as the proximity of local minima can obscure the fit. This correlation complicates the fitting process and emphasises the need for careful optimisation and precise modelling of systematic uncertainties to ensure accurate parameter extraction.

A possible way of solving this is to investigate the starting value of δ_{CP} to have it find the true minima quicker (but this will be difficult if the local minima are distributed very densely). Another way which has been demonstrated to work [40] is to perform the fit in Cholesky space rather than in the normal parameter space. The Cholesky decomposition is more efficient in describing the degrees of freedom for a set of correlated parameters.

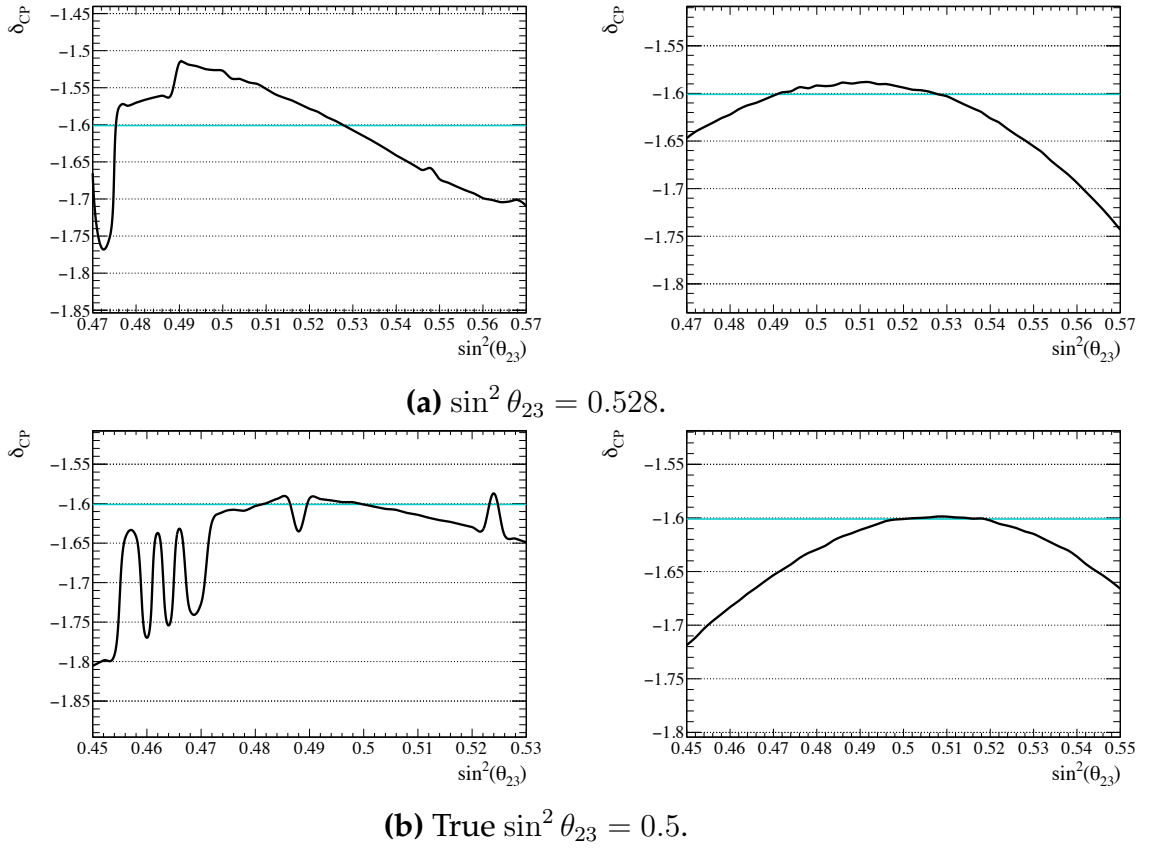


Figure 5.8: The best-fit values of δ_{CP} during the $\sin^2 \theta_{23}$ fit against two different true $\sin^2 \theta_{23}$ value. The left and right columns are fitted with and without systematic parameters, respectively. The cyan line is the true δ_{CP} value used (-1.601).

5.5 Outlook

In general, IWCD has been demonstrated with great potential to increase the sensitivity in neutrino oscillation measurements. Comparisons on the oscillation sensitivities between T2K 2018 and Hyper-K with IWCD were made, showing 3.8% $\nu_e/\bar{\nu}_e$ cross-section experimental error and the improvements on the 4.9% T2K theory errors, which is half a milestone away from the aimed sensitivity for Hyper-K.

Fake data studies can help identify the potential improvements of the systematic model and detector design in increasing sensitivities. Improvements could also be revised based on the uncertainties from the MC events, the sample selection criteria, or the background of the samples.

Chapter 6

New Neutral Current Pion Samples at Super-K

Section 4.2.2 introduced the new $\text{NC}\pi^0$ samples from the Super-K detector. The inclusion of the $1\text{R}\pi^0$ sample is expected to increase 7% (11%) of the oscillated ν_e events from the data available for the ν ($\bar{\nu}$) beam modes. These additional statistics are expected to enhance of the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) appearance and improve sensitivities to δ_{CP} , $\sin^2 \theta_{13}$, and octant of θ_{23} .

The $1\text{R}\pi^0$ events are the major background for the ν_e CCQE signal events. Currently, the T2K systematic model lacks specific parameters to account for uncertainties in the NC component of the single pion production cross-section, and these uncertainties are primarily constrained by CC interactions.

The $\text{NC}\pi^0$ samples are gradually being developed for use in oscillation analyses at T2K. At this stage, control samples are selected to include π^0 events with high purity and efficiency. Corresponding Super-K detector systematic uncertainties have been developed based on Super-K atmospheric neutrino data. The next step before incorporating these samples into oscillation analysis is to investigate the kinematic properties of the samples and to develop uncertainty parameters for the $\text{NC}\pi^0$ cross sections that are currently missing from the T2K analysis.

6.1 $\text{NC}\pi^0$ Sample Properties

6.1.1 $\text{NC}\pi^0$ Interaction Kinematics

The sample properties can be checked through the true kinematics of the events. Fig. 6.1 illustrates a kinematic diagram of an $\text{NC}\pi^0$ interaction, which is the primary interaction mode contributing to $\text{NC}1\pi^0$ events. For $\text{NC}\pi^0$ interactions, the only observable kinematic variable is the 4-momentum of the outgoing π^0 , if the π^0 survives from the FSI process. Another topologically indistinguishable process is the NC Coherent (NCCoh) interaction, which also appears as a single π^0 . Additionally, other interaction types may produce a single π^0 through FSI processes, potentially influencing the $\text{NC}\pi^0$ event rates in the selected samples. A detailed breakdown of the samples and the potential effects of different components will be discussed in the next section.

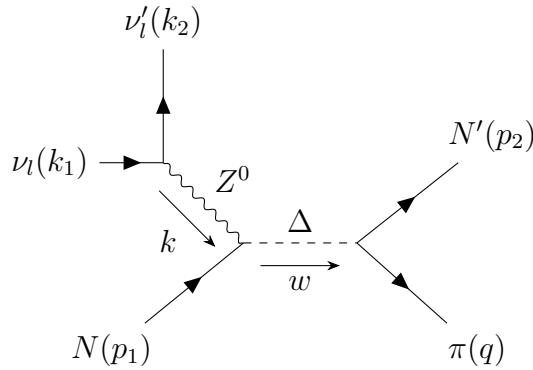


Figure 6.1: The kinematic diagram of the $\text{NC}\pi^0$ interaction, including labels for the 4-momenta of the incoming, intermediate, and outgoing particles.

The kinematics of the interaction process can be described using the variables Q^2 and W . Here, Q^2 represents the squared 4-momentum transfer of the weak boson, and W represents the 4-momentum of the outgoing nucleon or resonance state. For

the $NC\pi^0$ interaction, Q^2 and W can be calculated as follows:

$$Q^2 = -k \cdot k = -(k_1 - k_2)^2 \approx 2k_1 \cdot k_2 = 2E_1 E_2 (1 - \cos(\theta_{beam})), \quad (6.1)$$

$$W = \sqrt{\omega \cdot \omega} = \sqrt{(k + p_1)^2} \approx \sqrt{-Q^2 + M_N^2 + 2M_N(E_1 - E_2)}, \quad (6.2)$$

where the neutrino mass term is relatively small and ignored for the calculation, M_N is the mass of the nucleon, E_1 and E_2 are the energies of the incoming and outgoing neutrinos, and θ_{beam} is the angle between the two neutrinos.

6.1.2 Kinematic Distributions Breakdown

For events with a given topology, it is crucial to determine whether the event originates from the expected interaction modes or from background modes that result in the same final state topology. Additionally, events with different true topologies may be reconstructed as the same event topology at Super-K. To understand the composition of the events, breakdowns and comparisons of the samples into different final state topologies and interaction modes are studied.

The fractions of possible true final state topologies that could produce the observed patterns of one or two reconstructed e -like rings are listed in Table 6.1. The $2R\pi^0$ sample identifies $1\pi^0$ events as the signal, while the $1R\pi^0$ sample considers $1\pi^0$ events as background for the signal ν_e events with the 1 visible e ($1e$) topology. The breakdown is provided for both primary and secondary particles produced from the neutrino-nucleus interaction.

Among the true 2-secondary-visible-photon events in the $2R\pi^0$ samples, 74.6% are produced from primary π^0 decay, while only 2.2% originate from other particles, such as $NC\eta$ events. When comparing the $1\pi^0$ event fraction of 76.9% with the fraction of π^0 -decayed photons, a 2.3% discrepancy is observed. This discrepancy arises from contributions by other π^0 decay channels and a fraction of π^0 events that produce very low-energy photons (less than 2 MeV) in the MC simulation. These photon events are either challenging to reconstruct or may result from peculiarities

Table 6.1: The event fractions of possible true topologies to produce the 1-ring or 2-ring e-like pattern for the $NC\pi^0$ samples. The kinematic distributions corresponding to these modes can be found in Appendix B.2.2-B.2.3.

Vertex	Topology	ν -mode $2R\pi^0$	$\bar{\nu}$ -mode $2R\pi^0$	ν -mode $1R\pi^0$	$\bar{\nu}$ -mode $1R\pi^0$
Primary	1 π^0	76.9%	76.2%	47.8%	41.9%
	1 visible e	2.5%	2.1%	30.7%	37.9%
	1 visible γ	-	-	0.1%	0.1%
	Other	20.6%	21.7%	21.4%	20.1%
Secondary (visible only)	2 primary- π^0 -decayed γ	74.6%	73.7%	45.0%	39.8%
	1 primary- π^0 -decayed γ	-	-	0.3%	0.2%
	2 other γ	2.2%	2.6%	0.2%	0.1%
	1 e	0.1%	-	0.1%	0.1%

in the GEANT4 simulation.

The background contributes over 20% in the 2R samples, primarily from NC multi- π and DIS processes, which produce multiple visible rings and are challenging to reconstruct accurately. There is also a small fraction of $NC\pi^0$ events that do not result in π^0 final states, primarily due to the pion FSI effects. Alternatively, FSI can also produce charged pion events with a $1\pi^0$ topology.

For the $1R\pi^0$ samples, the true background fraction and corresponding interaction modes are generally similar to those in the $2R\pi^0$ case, but with the rings more boosted, leading to reconstruction as 1R. In the case of the primary $1e$ topology, the interaction modes breakdown for the 2R and 1R samples are similar, with approximately 79% being CCQE, 10% being 2p2h, and 7% being the $CC1\pi^+$ background. Contributions from the primary 1γ and secondary $1e$ events are negligible in both the 1R and 2R samples.

The interaction mode breakdown for the primary $1\pi^0$, $1e$, and background events as a function of W are shown in Figs. 6.2 and 6.3 for the ν -mode $2R\pi^0$ and $1R\pi^0$ samples, respectively. The events are peaked around the proton/neutron mass and the $\Delta(1232)$ and $N(1535)$ resonances, as expected from the given interaction modes.

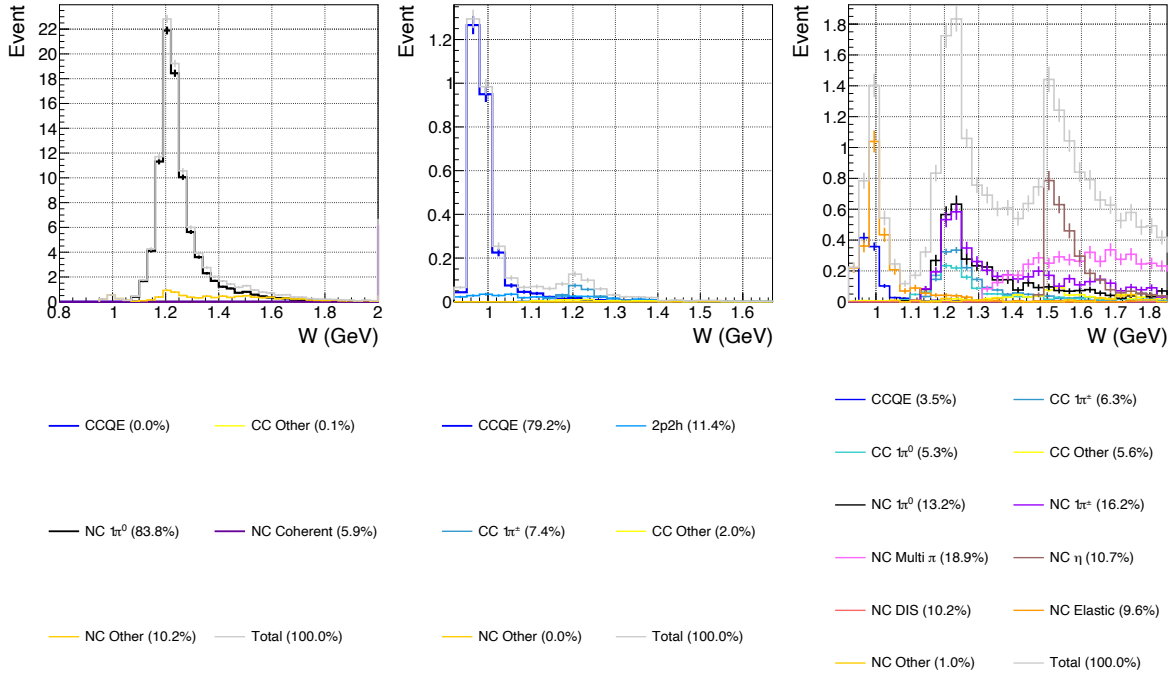


Figure 6.2: The $2R \pi^0$ sample distributions by interaction modes against W with primary final state topologies of (left) 1 visible π^0 , (middle) 1 visible e , and (right) other topologies from the ν beam mode. The numbers in the brackets indicate the fraction of the events under the current topology.

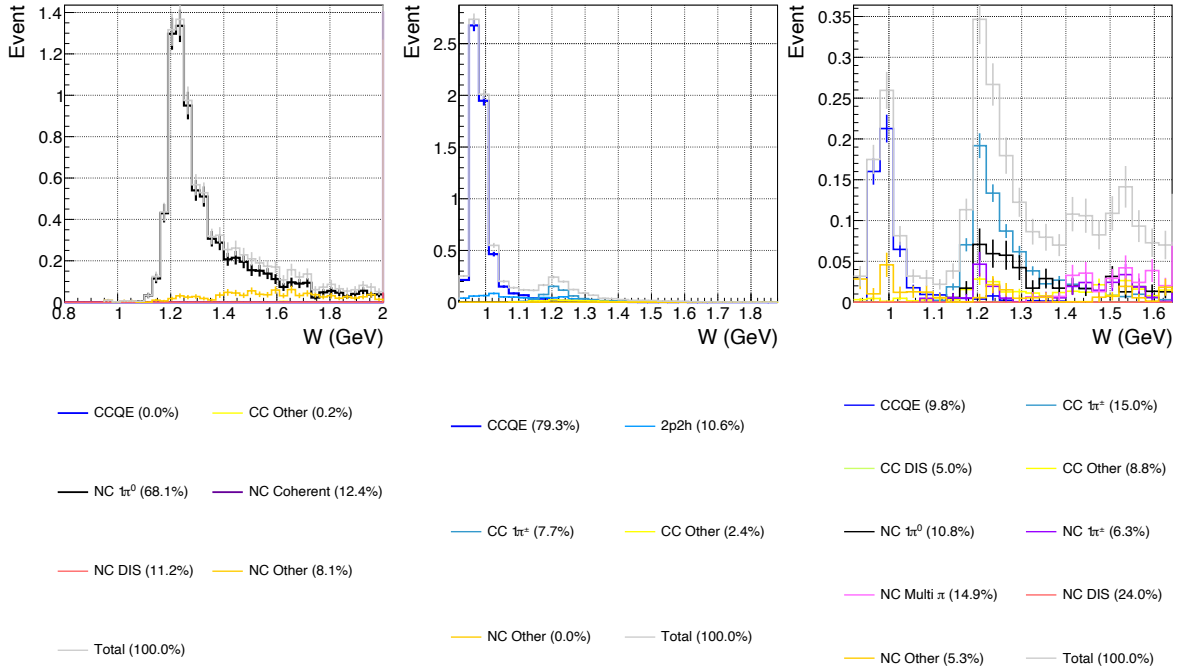


Figure 6.3: The $1R \pi^0$ sample distributions by interaction modes against W with the primary final state topologies of (left) 1 visible π^0 , (middle) 1 visible e , and (right) other topologies from the ν beam mode. The numbers in the brackets indicate the fraction of the events under the current topology.

For the $1\pi^0$ topology, contributions mainly come from pion production via the $\Delta(1232)$ resonance. There is a tiny fraction around the nucleon mass in the $2R\pi^0$ case due to NC Elastic background, possibly influenced by FSI effects. In contrast, the 1R sample shows a broader resonance peak, which could be due to higher momentum $\Delta(1232)$ or $\Delta(1440)$ production, leading to more boosted π^0 decays. The NCCoh mode is located at much higher W values due to interactions with the entire nucleus, peaked around the mass of the oxygen nucleus. Overall, 69% of the $2R\ NC\pi^0$ events have been correctly reconstructed, while about 2% of events resulted in other final states but were still reconstructed as $2R\pi^0$.

The total number of events and the fractions for each interaction mode are presented in Table 6.2. The $\nu(\bar{\nu})$ -mode $2R$ sample has 71.1% (70.8%) being the signal π^0 events. The high purity of the π^0 events makes this sample with potentially highly sensitivity in constraining $NC\pi^0$ interaction cross sections when fitting to neutrino oscillation data.

Most background interaction modes contribute only 3% or less. However, the NC $\pi^\pm$ and multi- π backgrounds contribute slightly more, ranging from 6.5% to 8%. These events exhibit very similar reconstructed kinematic properties to the $NC\pi^0$ events, potentially affecting measurements of $NC\pi^0$ cross sections if the uncertainties associated with these events are not properly estimated. The relevant uncertainties will be discussed in Section 6.2.

The $1R\pi^0$ sample has selected 26.4% (29.2%) of CCQE events in the ν ($\bar{\nu}$) beam modes, with over 90% (95%) of these events exhibiting a true $1e$ topology. $NC\pi^0$ is the primary background and is expected to be constrained by the $2R\pi^0$ samples. Other significant NC backgrounds come from NC multi- π (5%) and DIS events (10%(8%) in $\nu(\bar{\nu})$ -mode), with multi- π events reconstructed to have π^0 momentum similar to that of CCQE events. NCDIS predominantly contributes at high π^0 momentum and does not overlap with $NC\pi^0$ events. Contributions from $CC\pi^\pm$ and $2p2h$ are less significant in comparison to their fractions in the $1Re$ samples, so no additional concerns are anticipated based on the current modelling of these events.

Table 6.2: The event fractions of the possible true interaction modes breakdown for the $NC\pi^0$ samples. The kinematic distributions corresponds to these modes can be found in Appendix B.2.1.

Interaction	ν -mode $2R\pi^0$	$\bar{\nu}$ -mode $2R\pi^0$	ν -mode $1R\pi^0$	$\bar{\nu}$ -mode $1R\pi^0$
CCQE	2.7%	1.9%	26.4%	29.2%
CC2p2h	0.4%	0.3%	3.5%	4.6%
CC $1\pi^\pm$	1.5%	1.6%	5.6%	8.9%
CC $1\pi^0$	1.2%	1.0%	1.2%	1.4%
CCmulti π	0.5%	0.7%	0.9%	1.3%
CCDIS	0.4%	0.4%	1.1%	0.8%
CCcoh	-	0.1%	0.2%	0.4%
CC 1γ	-	-	-	-
CCOther	0.1%	0.1%	0.1%	0.1%
NC $1\pi^0$	67.2%	62.1%	34.8%	27.9%
NCCoh	4.5%	8.6%	5.9%	8.4%
NC $1\pi^\pm$	6.5%	6.4%	2.6%	2.5%
NCmulti π	7.0%	8.1%	5.6%	5.0%
NCDIS	2.5%	2.5%	10.5%	8.1%
NC 1γ	-	-	0.2%	0.2%
NC η	2.4%	3.0%	0.4%	0.4%
NCES	2.9%	3.0%	0.8%	0.8%
NCOther	0.1%	0.1%	0.1%	0.1%
Total Events	135.99	41.36	23.68	8.86

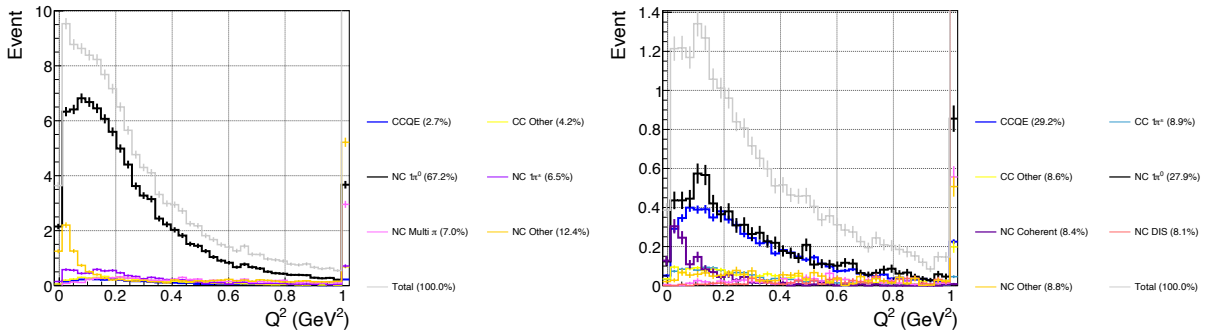


Figure 6.4: The distributions of the 2R (left) and 1R $NC\pi^0$ samples breakdown into interaction modes against Q^2 in the ν beam mode.

The Q^2 distributions for the 2R and 1R samples are shown in Fig. 6.4. The event rate decreases with increasing Q^2 for all cases due to the relationship between Q^2 and the neutrino-nucleus interaction form factors as described in Section 1.3.3. Coherent interactions only occur in events at low Q^2 .

The overall selection efficiency from the FCFV level for the control and signal samples is summarised in Table 6.3. The efficiency for π^0 events remains nearly constant across E_ν and Q^2 , but it varies with respect to W and π^0 momentum. This variation is due to the different ring patterns produced by photons, which are influenced by the kinematics of the final state particles. In this case, 2R $NC\pi^0$ events start with high efficiency but decrease as W or π^0 momentum increases, while the opposite trend is observed for 1R $NC\pi^0$ events. Fig. 6.5 illustrates the differences in shape from the normalised efficiency distributions. Overall, the combined efficiency of the 2R and 1R samples remains flat against W or π^0 momentum.

Table 6.3: The signal event selection efficiency from the FCFV level in the π^0 control samples ($2R\pi^0$) and oscillated ν_e samples ($1R\pi^0 + 1Re$). The efficiency distributions can be found in Appendix B.2.4.

Signal	NC π^0 + NCCoh		Osc. ν_e CCQE	
	ν -mode 2R	$\bar{\nu}$ -mode 2R	ν -mode 1R	$\bar{\nu}$ -mode 1R
Efficiency	74.8%	73.5%	73.3%	62.9%

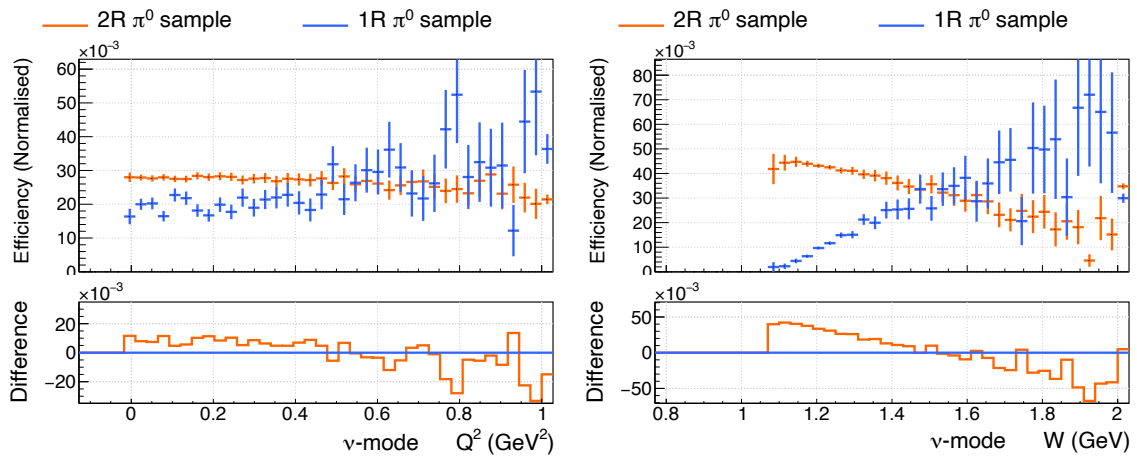


Figure 6.5: The comparison of the shape in the efficiency (from FCFV) between the 2R $NC\pi^0$ and 1R $NC\pi^0$ events against Q^2 and W in the ν beam mode.

6.1.3 Sample Binning for Oscillation Analysis

For the oscillation analysis, samples are binned based on reconstructed π^0 momentum ($P_{\pi^0}^{reco}$). Fig. 6.6 shows the breakdown of 2R and 1R samples into true interaction modes as a function of $P_{\pi^0}^{reco}$. In the 2R samples, the dominant mode is $NC\pi^0$, with all background modes contributing less than 20%, until the $P_{\pi^0}^{reco}$ reaches 1.5 GeV. Beyond this point, the fraction of $NC\pi^0$ events drops below 20% to have similar amount as the individual background modes, while the fraction of NCDIS events starts to increase. Similarly, for the 1R sample, the π^0 background becomes less significant compared to other interaction modes above this momentum threshold.

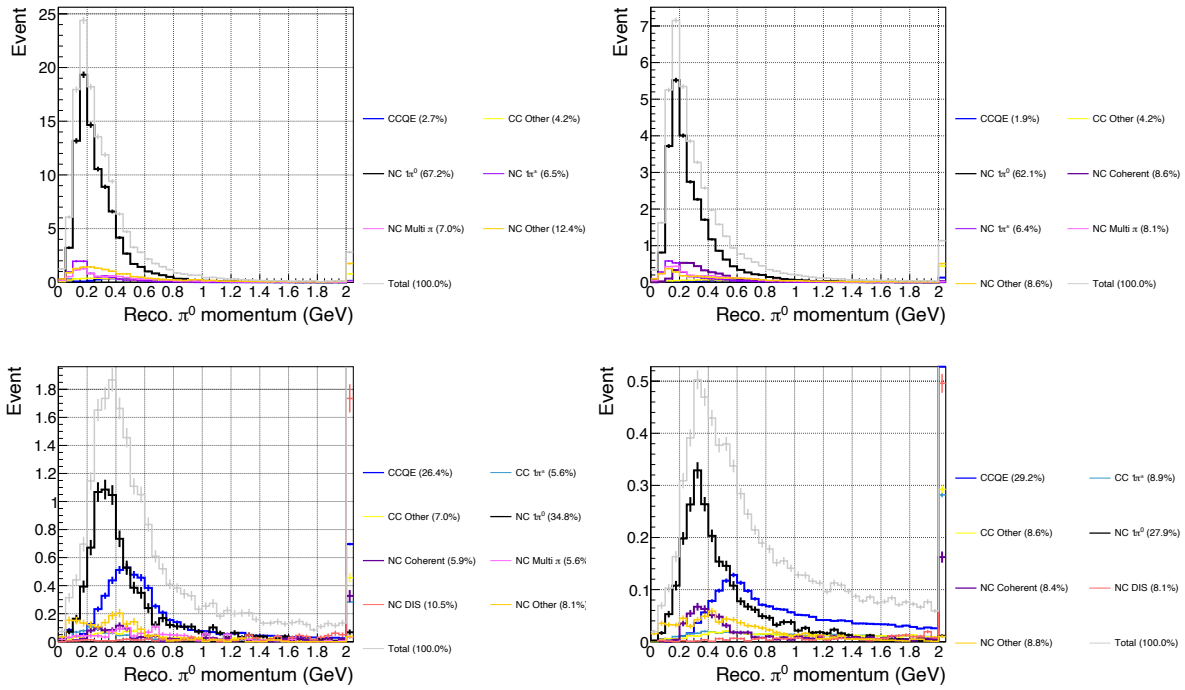


Figure 6.6: The distributions of the $NC\pi^0$ samples breakdown into interaction modes against reconstructed π^0 momentum, where the top row is the $2R\pi^0$ and the bottom row is the $1R\pi^0$ samples. The ν and $\bar{\nu}$ beam mode is shown in the left and right column, respectively.

$NC\pi^0$ events dominate around the same $P_{\pi^0}^{reco}$ regions in both the 2R and 1R samples. However, the shape of the 1R π^0 events is broader and peaked at slightly higher $P_{\pi^0}^{reco}$, as expected due to the more boosted observed photons. In the 1R sample, the fraction of CCQE events becomes significant after $P_{\pi^0}^{reco} = 0.2$ GeV, with 95% of these

events contributed by those with a primary $1e$ final state. The peak of CCQE events occurs between 0.4-0.6 GeV, corresponding to the region around 0.6 GeV in E_ν .

The fact that CCQE events peak at higher $P_{\pi^0}^{reco}$ compared to $\text{NC}\pi^0$ events is also expected, as the outgoing electron should have momentum close to the incoming neutrino energy. The outgoing lepton from a neutrino-nucleus interaction, being much lighter than the recoiling nucleon or resonant state, can take most of the incoming neutrino's energy. In contrast, the pion must share the energy with both the lepton and the decayed nucleon from the resonance, leading to lower momentum than the electron.

A selection of 2D kinematic distributions for the primary $1\pi^0$ and $1e$ events is displayed in Fig. 6.7 and Fig. 6.8, illustrating the relationship between $P_{\pi^0}^{reco}$ and the true event kinematics. The first two columns highlight the differences in energy transfer from the incoming neutrino to the π^0 and electrons.

For $1\pi^0$ events, $P_{\pi^0}^{reco}$ primarily depends on E_ν , rather than Q^2 in both the 2R and 1R cases, with events to the right of the first Δ resonance peak mainly associated with high E_ν . In contrast, the $1e$ events exhibit a stronger correlation (anticorrelation) between $P_{\pi^0}^{reco}$ and E_ν (Q^2), with the oscillation maximum centred around 0.5 GeV in $P_{\pi^0}^{reco}$. There is almost no dependence on W for the $1e$ events, as most of the energy is transferred to the recoil nucleon.

There are several reasons to support the choice of $P_{\pi^0}^{reco}$ as the sample binning variable, in addition to the primary motivations discussed in Section 2.3.2. Based on the MicroBooNE measurements shown in Fig. 1.13, the $\text{NC}\pi^0$ interaction cross section exhibits a strong dependence on π^0 momentum. Consequently, new uncertainties on the cross sections could potentially be constructed using the true π^0 momentum of the events. The right columns of Figs. 6.7 and 6.8 demonstrate that the majority of events have closely matched true and reconstructed π^0 momentum. However, a small fraction of the $2R\pi^0$ sample is reconstructed with lower π^0 momentum, possibly due to photon absorption or contributions from $\text{NC}\pi^\pm$ events.

Overall, this suggests that $P_{\pi^0}^{reco}$ is as sensitive to $\text{NC}\pi^0$ interaction cross sections

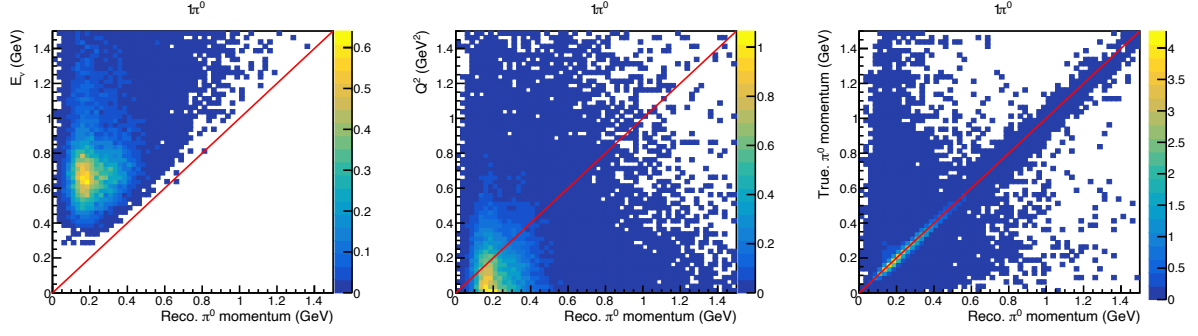


Figure 6.7: The relationships between the reconstructed π^0 momentum and true kinematics for the $1\pi^0$ events in the ν -mode $2R\pi^0$ sample. E_ν is the incoming neutrino energy and Q^2 is the 4-momentum transfer to Z^0 or W . Similar effects applied for the $\bar{\nu}$ -mode.

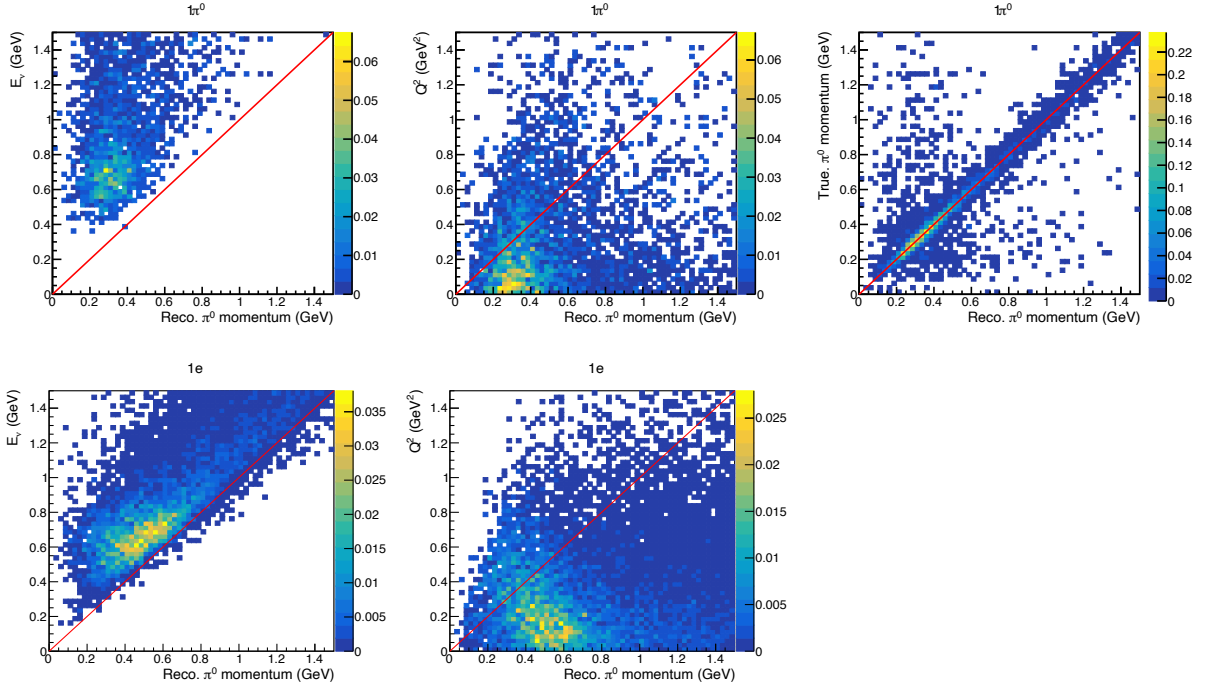


Figure 6.8: The relationships between the reconstructed π^0 momentum and true kinematics for the $1\pi^0$ and $1e$ events in the ν -mode $1R\pi^0$ sample. E_ν is the incoming neutrino energy and Q^2 is the 4-momentum transfer to Z^0 or W . Similar effects applied for the $\bar{\nu}$ -mode.

as the true π^0 momentum. Therefore, it is expected to effectively capture variations in events binned in $P_{\pi^0}^{reco}$, influenced by cross-section uncertainties from parameters based on the true π^0 momentum.

Given the observed differences in the shapes of the kinematic distributions between the $2R$ and $1R$ $NC\pi^0$ events, it is necessary to understand whether these dis-

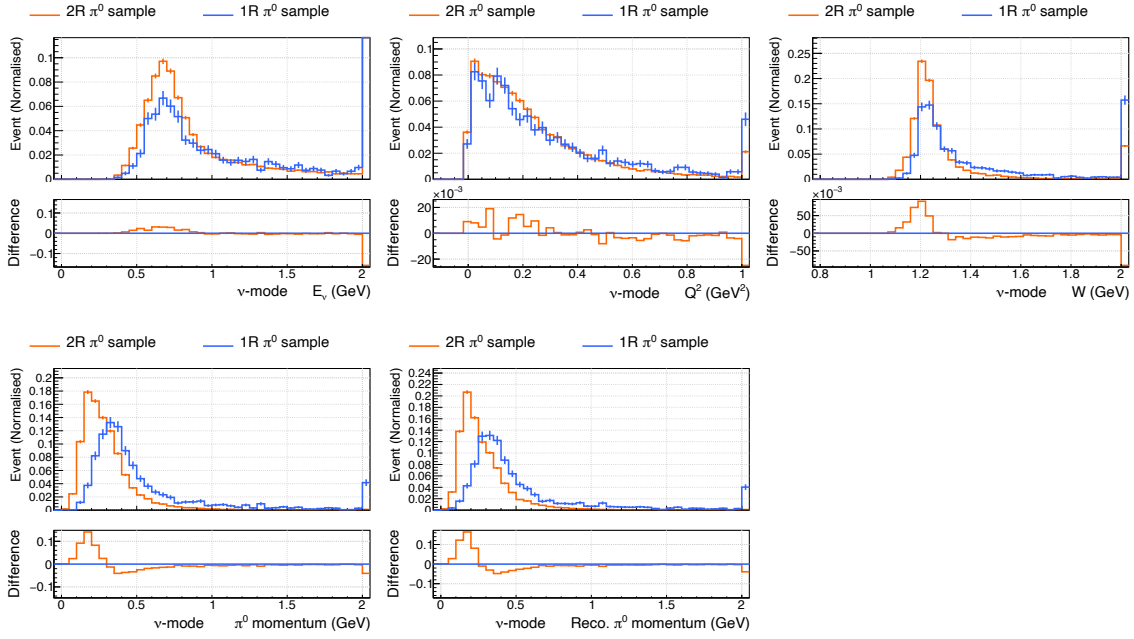


Figure 6.9: Normalised distributions overlapping the 2R and 1R $NC\pi^0$ events (ν -mode) to indicate the shape differences against the truth and reconstructed kinematic information from the primary interaction vertex.

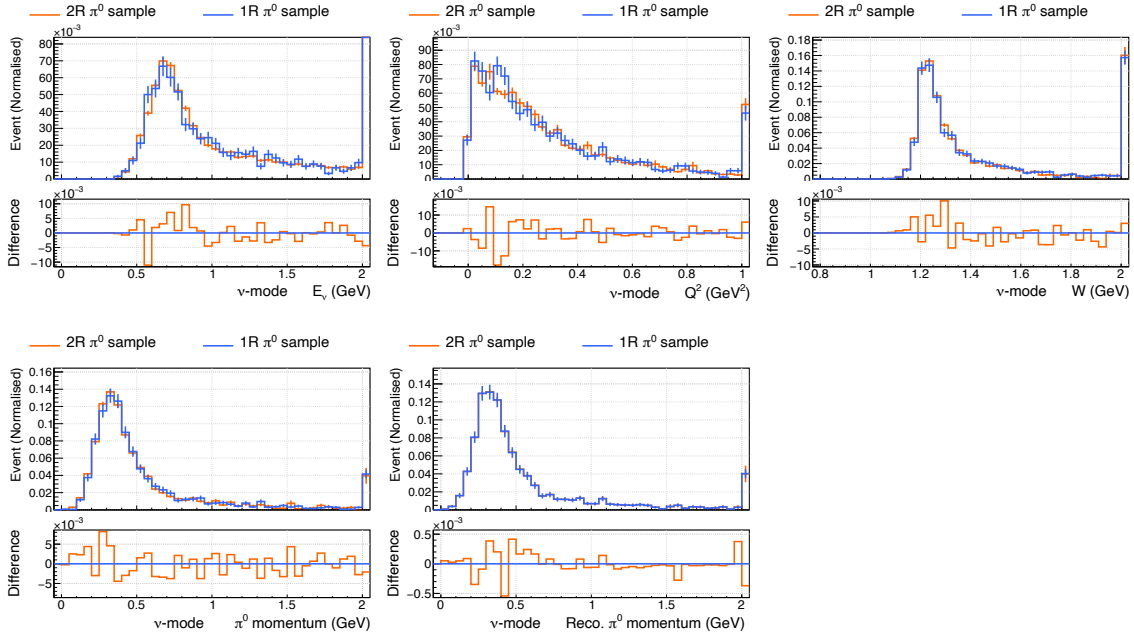


Figure 6.10: Normalised distributions overlapping the 2R and 1R $NC\pi^0$ events (ν -mode) after weighting by the 1R/2R efficiency ratio of the reconstructed π^0 momentum, showing matching shapes for all the truth kinematics.

crepancies are fully correlated with the differences under P_{π^0} and $P_{\pi^0}^{reco}$. If there are discrepancies beyond those accounted, it may be necessary to provide the events an-

other degree of freedom to vary, depending on where this occurs within the phase space. Fig. 6.9 shows a set of comparisons between the shapes of the 2R and 1R $\text{NC}1\pi^0$ ($\text{NC}\pi^0 + \text{NC Coh modes}$) events across several kinematic variables.

A weight is applied to the 2R $\text{NC}\pi^0$ events to match the shape of the 1R $\text{NC}\pi^0$ events in $P_{\pi^0}^{\text{reco}}$. The weight for events in a particular bin i is calculated from the efficiency ratio of that bin:

$$\mathcal{R}_i = \frac{\text{Eff}_{1\text{R}}^i}{\text{Eff}_{2\text{R}}^i} = \frac{N_{1\text{R}}^i}{N_{1\text{R}}^{\text{FCFV}, i}} \frac{N_{2\text{R}}^{\text{FCFV}, i}}{N_{2\text{R}}^i}, \quad (6.3)$$

$$E_{2\text{R}}^{ij'} = \mathcal{R}_i E_{2\text{R}}^{ij} \quad (6.4)$$

where N^{FCFV} and N represent the distributions of $P_{\pi^0}^{\text{reco}}$ before and after the event topology and likelihood cuts from the FCFV level, respectively. $E_{2\text{R}}^{ij}$ is the rate of the j^{th} event in bin i of $N_{2\text{R}}$. In this case, the weighted events $E_{2\text{R}}^{ij'}$ binned in $P_{\pi^0}^{\text{reco}}$ will then have a shape that matches exactly the shape of $N_{1\text{R}}$.

If the shape differences in other kinematic variables are fully correlated with those in $P_{\pi^0}^{\text{reco}}$, the weighted events binned by these variables should also exhibit a matched shape. Fig. 6.10 shows the weighted events binned by $P_{\pi^0}^{\text{reco}}$ and the true variables. The well-matched shape in $P_{\pi^0}^{\text{reco}}$ validates the correct implementation of the weights. The shapes of the true variables also show a good level of matching compared to the case before the weights were applied, indicating that using the 1D $P_{\pi^0}^{\text{reco}}$ binning for the samples will be sufficient.

6.2 $\text{NC}\pi^0$ Cross-section Uncertainties

6.2.1 Current Parameterisation

The cross-section parameterisation for the T2K oscillation analysis presented here is developed from a previous model described in Ref. [124]. The prior covariance matrix of $\text{NC}\pi^0$ related parameters in the model is shown in Fig. 6.11. The $\text{NC}\pi^0$

events are affected by uncertainties from the following parameters:

- C_A^5 and M_A^{Res} : These are the axial form factors for resonant single pion production, as described in Eq. 1.15. The central values and prior uncertainties are tuned according to fits to the ANL and BNL neutrino-nucleon scattering data [124].
- $I_{1/2}$ and $I_{1/2}$ low P_π : These parameters represent the isospin-1/2 non-resonant component for single pion production. The $I_{1/2}$ low P_π case specifically affects antineutrino events with a low-momentum negative pion. The central values and prior uncertainties are also based on fits to the ANL and BNL data [124].
- Resonant E_b : This parameter accounts for uncertainties due to non-zero binding energy during single pion production. The central values and prior uncertainties are estimated to be slightly higher than the binding energy in CCQE interactions, considering a stronger optical potential for pion production [124].
- Rein-Saghal (RS) Δ decay: This parameter accounts for uncertainties in the shape of the Δ resonance decay kinematics. No specific prior value or uncertainty has been estimated for this parameter.
- π -FSI parameters: These parameters represent uncertainties in the pion final state interaction (FSI) processes within the nucleus. Prior information is provided from NEUT cascade model fits to the $\pi^\pm$ -C interaction data [58]. The uncertainties are parameterised separately for effects on quasi-elastic pion scattering off nucleons, additional pion production, pion absorption probabilities, and pion charge exchange.
- NC coh.: This parameter describes normalisation uncertainties for NC coherent π^0 events, with prior information estimated based on MINER ν A data [125].
- NC other SK: This parameter describes normalisation uncertainties for background events dominated by NC multi-pion and NC DIS interactions. The treatment is based on MINOS measurements of the CC interactions [126].

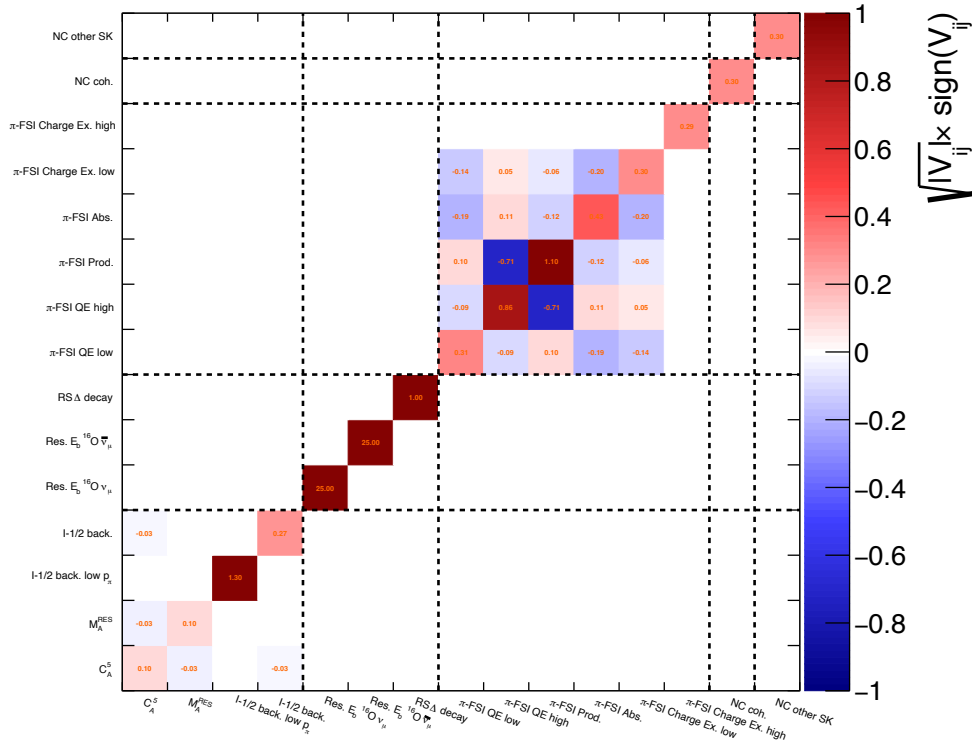


Figure 6.11: The partial covariance matrix of the cross-section model prior to the T2K data constraints, with parameters related to uncertainties in the $NC1\pi^0$ events.

6.2.2 The New Normalisation Parameters

As mentioned earlier, the parameters described above are constrained primarily from the CC interaction measurements and have cases of CC and NC events treated equally. It is motivated to include additional normalisation parameters specifically to describe the uncertainties for NC interactions to account for potential differences from the CC case, particularly when the NC background is significant.

Since the $2R\pi^0$ samples have been selected to constrain the cross-section uncertainties for the $NC1\pi^0$ background events in the 1R samples, additional normalisation uncertainties can be introduced to the $NC1\pi^0$ events and controlled with the large $2R\pi^0$ dataset. This approach allows for more accurate modelling of the ν_e signal events in the 1R sample.

A small portion of $1\pi^0$ $NC\pi^\pm$ events, resulting from FSI processes, contributes approximately 4% of the 2R sample and 2.5% of the 1R sample near the oscillation

maximum. The cross-section uncertainties for $1\pi^0 NC\pi^\pm$ events are typically in the range of 30%–40%, while FSI uncertainties are approximately 10%–15%. When combined, the resulting fractional uncertainties for the 2R and 1R samples are estimated to be around 1.5% and 0.7%, respectively. Therefore, given their small contribution relative to the dominant $NC\pi^0$ and $NCCoh$ processes, the overall impact of $1\pi^0 NC\pi^\pm$ events on the modelling is considered negligible, with existing constraints on single pion production uncertainties deemed sufficient.

Due to the shape differences between the $2R\pi^0$ and $1R\pi^0$ distributions (Fig. 6.9) and efficiencies (Fig. 6.5), multiple parameters covering different true π^0 momentum regions are required to avoid underestimating or overestimating variations in the $1R\pi^0$ events.

Fig. 6.12 illustrates a possible strategy for splitting the kinematic region to apply reasonable variations to 1R events based on parameters constrained by the 2R events, considering their relative event rates and efficiencies. Four normalisation parameters are constructed for over the P_{π^0} momentum:

- **I. [0, 0.2] GeV:** In this region, the number of 2R events increases faster than 1R events, and ended up with large (small) 2R (1R) events. The efficiencies are nearly flat for both the 2R and 1R case, so a single normalisation parameter is sufficient to describe the uncertainty in this region.
- **II. [0.2, 0.4] GeV:** In this region, both the number and efficiency of 2R events decrease, while they increase for the 1R events. The rate of change in efficiency is nearly constant for both cases. As a result, 1R events may be underestimated on the lower side of the region and overestimated on the higher side. This effect can be mitigated by splitting the region into more bins; however, doing so introduces more uncertainties due to the more parameters and fewer events per bin.
- **III. [0.4, 0.9] GeV:** In this region, both 2R and 1R events decrease at a similar rate. The efficiency for the 1R sample remains nearly flat, so variations will primarily be driven by the 2R efficiency.

- **IV. [0.9, 1.5] GeV:** This region has few events for both 2R and 1R cases.
- **Above 1.5 GeV:** The purity of $NC\pi^0$ events in this region drops significantly, falling below that of other background modes, which becomes difficult to constrain the π^0 uncertainty. Given the low purity and the fact that ν_e events in this region are predominantly intrinsic rather than oscillated signals, events from this region are not included in the oscillation analysis presented here, and no normalisation parameters are constructed.

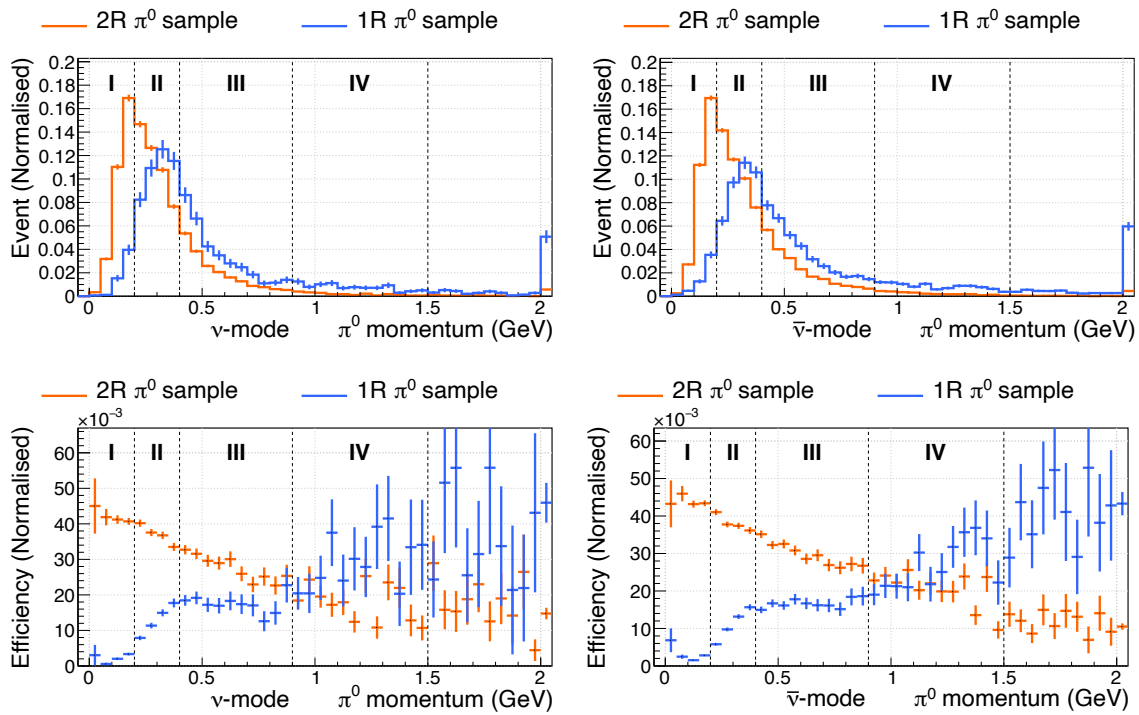


Figure 6.12: The comparison of the shape in the distribution efficiency (from FCFV) between the 2R $NC\pi^0$ and 1R $NC\pi^0$ events against the true π^0 momentum in the ν and $\bar{\nu}$ beam modes. The new cross-section uncertainties are parameterised based on the regions labelled.

The prior uncertainties on these parameters are estimated based on the differences between the $NC\pi^0$ cross-section measurements and predictions from event generators. Fig. 6.13 shows the fractional difference for $NC\pi^0$ events between the K2K and MicroBooNE data and the GENIE predictions. The events in the 0.2-0.4 GeV range are more tightly constrained with plenty of data, while the 0-0.2 GeV and 0.2-0.9 GeV regions have fewer data points and larger uncertainties. The region above 0.9

GeV contains very few data points makes the error hard to estimate.

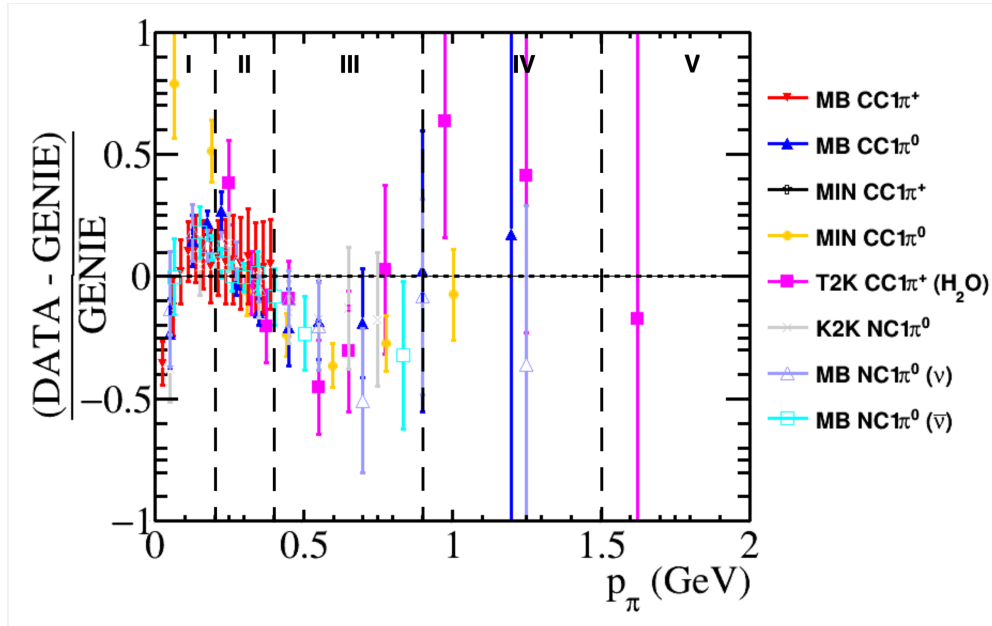


Figure 6.13: The fractional difference between the predictions from GENIE and measurements from the MicroBooNE (MB), MINER ν A (MIN), T2K, and K2K experiments for the neutrino-nucleus interaction cross sections with single pion productions, as a function pion momentum [127].

The prior uncertainties are estimated based on the values, uncertainties, and number of data points for the different regions shown in Fig. 6.13. The estimated uncertainties are summarised in Table 6.4. The events from the $\bar{\nu}$ -mode are treated similarly to the ν -mode events due to their comparable shapes, efficiency variations, and fractional differences between the predictions and data.

Table 6.4: The uncertainties for the new $NC\pi^0$ normalisation parameters prior to the T2K data fit, estimated according to previous measurements and generator predictions.

Parameter	I	II	III	IV
Uncertainty	50%	20%	50%	100%

Chapter 7

Oscillation Sensitivities with the New Neutral Current Pion Samples

Chapter 6 introduced strategies of using new $\text{NC}\pi^0$ samples to constrain NC single pion production cross-section uncertainties. This chapter will focus on discussing implementations of these samples in an oscillation analysis, as well as their impact on systematic uncertainties and sensitivities to oscillation measurements.

Unlike the Hyper-K oscillation analysis described in Chapter 5, this analysis employs Bayesian statistical inference using the Markov Chain Monte Carlo (MCMC) techniques introduced in Chapter 3. This method is more efficient for fitting within a large parameter space and complex likelihood compared to the gradient descent method, especially in dealing with the multiple minima scenarios. In this case, the proposed values of the systematic parameters from each MCMC step will be applied on an event-by-event basis, rather than the bin-by-bin approach used in the Hyper-K analysis.

The analysis is conducted using the MaCh3 framework [128], with sensitivity studies performed with and without the new $\text{NC}\pi^0$ samples. Additionally, the impact of the new π^0 normalisation parameters is investigated using simulated datasets. In MaCh3, fitting to both near detector and far detector data is done simultaneously.

A brief description of the fitting strategy and the introduction to the samples can be found in Sections 7.1 and 7.2, followed by the systematic parameter model specifics in Section 7.3. The impact of the new $\text{NC}\pi^0$ samples and new systematic parameters on oscillation sensitivities, along with related validation work, are described in Sections 7.4-7.7.

7.1 Analysis Strategy

For the T2K oscillation analysis, fits are performed using the joint ND280 and Super-K data to constrain the entire systematic parameter space simultaneously. The MC events are varied individually based on the proposed parameter values from each MCMC step, depending on which interaction modes, kinematic regions, beam modes, or target nucleus types the uncertainties apply to. As introduced in Section 3.3.2, the probability of accepting or rejecting the proposed values depends on the relative likelihood between the previous and current step. In this analysis, the negative log-likelihood (nLLH) is derived according to Eq. 3.5:

$$\begin{aligned}
 -\ln \mathcal{L}_{tot}(\mathbf{x}, \boldsymbol{\theta}) = & \sum_i^{\text{bins}} n_i^{p,ND}(\mathbf{x}, \boldsymbol{\theta}) - n_i^{o,ND} + n_i^{o,ND} \ln \left(\frac{n_i^{o,ND}}{n_i^{p,ND}(\mathbf{x}, \boldsymbol{\theta})} \right) + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
 & + \sum_i^{\text{bins}} n_i^{p,SK}(\mathbf{x}, \boldsymbol{\theta}) - n_i^{o,SK} + n_i^{o,SK} \ln \left(\frac{n_i^{o,SK}}{n_i^{p,SK}(\mathbf{x}, \boldsymbol{\theta})} \right) \\
 & + \sum_j^{\text{syst.}} \frac{1}{2} (\mathbf{x} - \mathbf{x}_0)_j^T \cdot \mathbf{C}_j^{-1} \cdot (\mathbf{x} - \mathbf{x}_0)_j \\
 & + \frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^T \cdot \mathbf{C}_\theta^{-1} \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0),
 \end{aligned} \tag{7.1}$$

where the first term represents the nLLH from the ND280 samples and includes a scaling factor, β , with associated uncertainties, σ_{β_i} , to account for statistical uncertainties in the ND280 MC simulation, following the Barlow-Beeston model [129]. The second term is the nLLH from the Super-K data. The third term is the sum of penalty terms for different systematic uncertainties j , including flux, cross-section, near detector, and far detector uncertainties, with $\mathbf{C}_j$ being the prior oscillation fit covariance matrix for each type. The fourth term is a penalty term for the oscillation parameters, as their values are proposed in the same way as for the systematic parameters. The covariance matrix $\mathbf{C}_\theta$ is obtained from the prior uncertainties of the oscillation parameters, without correlations included here. For oscillation parameters, prior uncertainties are only set on $\sin^2 \theta_{12}$ and Δm_{21}^2 according to experimental

measurements, while flat priors, bounded by physically allowed values, are used for other oscillation parameters.

The predicted event rate in the i^{th} bin of the ND280 and Super-K samples is calculated after directly reweighting the individual events:

$$n_i^{p,ND} = n_r^\alpha = \int_r T_{m,r',g}(S_{m,t,r',f}\Phi_{m,t,r'}^\alpha)dV_r, \quad (7.2)$$

$$n_i^{p,SK} = n_r^\beta = \int_r T_{m,r',g}(P_t^{\alpha\rightarrow\beta}S_{m,t,r',f}\Phi_{m,t,r'}^\alpha)dV_r. \quad (7.3)$$

$\Phi_{m,t,r'}^\alpha$ is the tuned flux of an event with interaction mode m , truth kinematics t and reconstructed kinematics r' , before oscillation (indicated by α). $S_{m,t,r',f}$ is the multiplicative weight applied to the events based on the variations of parameters f . $P_t^{\alpha\rightarrow\beta}$ is the $\nu_{\alpha\rightarrow\beta}$ oscillation probability for events with true kinematics t . Shifts $T_{m,r',g}$ are then applied to the reconstructed kinematics of the weighted flux according to the shift parameters g . The event rate of the bin r is the total weight integrated over the sample binning variable V_r within the range covered by the bin r .

During the fit, the values of parameters from each MCMC step are proposed based on the estimated prior central values and uncertainties. The prior central values for normalisation parameters are set to one, while specifically motivated central values are used for cross-section parameters originating from the theoretical models (see Appendix A.1). The prior central values and uncertainties for the oscillation parameters are provided in Table 7.1.

Table 7.1: The prior central value and uncertainties of the oscillation parameters used in the T2K oscillation analysis. The “flat” refers to a uniform prior distribution.

Parameter	Prior	Uncertainty
$\sin^2(\theta_{12})$	0.307	0.013
$\sin^2(\theta_{13})$ woRC	0.022	Flat
$\sin^2(\theta_{13})$ wRC	0.022	0.0007
$\sin^2(\theta_{23})$	0.561	Flat
Δm_{21}^2 (eV ²)	7.53×10^{-5}	0.18×10^{-5}
Δm_{32}^2 (eV ²)	2.494×10^{-3}	Flat
δ_{CP}	-1.601	Flat

The $\sin^2 \theta_{13}$ is primarily treated as a free parameter during the fit. However, when constraints from reactor neutrino data are applied, the posterior distribution is weighted by a likelihood function based on the $\sin^2 \theta_{13}$ values proposed at each step:

$$\text{RCweight} = \frac{\text{new prior}}{\text{old prior}} = \exp \left(-\frac{1}{2} \left(\frac{\text{proposed} - \text{new central}}{\sigma_{\text{new}}} \right)^2 \right), \quad (7.4)$$

where the Gaussian term represents how likely the proposed value is, given the central value of the reactor constraint and its associated uncertainty σ_{new} . The fit cases with and without the reactor constraints will be referred to as “wRC” and “woRC”, respectively.

7.2 MC and Data at Super-K

The analysis uses Super-K data from Runs 1-11, with a proton-on-target (POT) exposure of 2.14×10^{21} (1.63×10^{21}) in the $\nu(\bar{\nu})$ -beam mode. The ν -mode has an 8.3% increase compared to the latest public T2K oscillation measurements [42]. The ND280 data and samples used in the fit remain unchanged, with 1.15×10^{21} (0.83×10^{21}) POT accumulated from Runs 2-9. A description of the ND280 samples for the analysis presented here and in Chapter 8 can be found in Ref. [114]. Table 7.2 shows the integrated MC event rates and data for the Super-K samples, where further breakdowns of event rates into interaction modes and oscillation channels are described in Appendix C.2.

The MC before the fit and the data are shown in Fig. 7.1 for the 5 Super-K samples, and in Fig. 7.2 for the new $\text{NC}\pi^0$ samples. The sample binning scheme of the 5 Super-K remains unchanged from the previous analysis. The $\text{NC}\pi^0$ samples are binned versus reconstructed π^0 momentum, with the bin boundaries chosen:

- To have the sample bin boundaries not cross over the boundaries of the π^0 normalisation parameters to maximise sensitivity to individual parameter variations.
- To have finer binning around the oscillation maximum, which corresponds to the

area around 500 MeV in the reconstructed π^0 momentum.

- To avoid having 0 events in the MC predictions.

In Fig. 7.2, both the $2R\pi^0$ and $1R\pi^0$ samples in the ν -mode show a trend where the MC predicts a higher number of events at lower π^0 momentum and fewer events at higher momentum compared to the observed data. This discrepancy may be due to an overestimation of the NC pion production cross-sections, due to estimation based on CC interactions, and an underestimation of NC multi- π or NC DIS backgrounds at higher energies. In contrast, the discrepancy is smaller in the $\bar{\nu}$ -mode, possibly due to the lower cross-section in this mode, making it less sensitive to the differences, or other factors.

For all samples, the MC predictions and the data are statistically compatible, as indicated by a χ^2 test and the corresponding p-value. The p-value represents the probability of obtaining a χ^2 value greater than the observed χ^2 due to statistical fluctuations in the data prior to performing any fit. The results are summarised in Table 7.3.

Table 7.2: The MC event rates and data at Super-K with 3.77×10^{21} POT. The MC assumes normal ordering with $\delta_{CP} = -1.601$, $\sin^2 \theta_{23} = 0.561$, $\sin^2 \theta_{13} = 0.022$, and $\Delta m_{32}^2 = 2.494 \times 10^{-3} \text{ eV}^2$.

Sample	MC (Unosc.)	MC (osc.)	T2K Runs 1-11 data
ν -mode $1R\mu$	1368.4	332.31	357
$\bar{\nu}$ -mode $1R\mu$	398.76	127.15	137
ν -mode $1Re$	21.22	96.09	102
$\bar{\nu}$ -mode $1Re$	6.67	16.94	16
ν -mode $1Re1de$	3.30	11.07	19
ν -mode $2R \text{ NC}\pi^0$	137.72	134.76	127
$\bar{\nu}$ -mode $2R \text{ NC}\pi^0$	41.27	41.06	41
ν -mode $1R \text{ NC}\pi^0$	17.09	20.49	22
$\bar{\nu}$ -mode $1R \text{ NC}\pi^0$	5.71	6.61	5

Table 7.3: Summary of the χ^2 test results before fitting the MC to the T2K data. The χ^2 values are calculated using the definition $\chi^2 = -2 \ln \mathcal{L}$ based on definition in Eq. 3.3. The number of degrees of freedom (ndf) corresponds to the number of non-zero bins in each sample's spectrum. The p-values represent the probability of observing a χ^2 value smaller than the calculated χ^2 under statistical fluctuations, given the ndf.

Sample	χ^2/ndf	p-value
ν -mode $1R\mu$	47.5/58	0.99
$\bar{\nu}$ -mode $1R\mu$	81.1/50	0.83
ν -mode $1Re$	9.9/10	0.89
$\bar{\nu}$ -mode $1Re$	3.64/5	0.87
ν -mode $1Re1de$	7.18/3	0.31
ν -mode $2R \text{ NC}\pi^0$	9.42/5	0.45
$\bar{\nu}$ -mode $2R \text{ NC}\pi^0$	1.57/4	0.94
ν -mode $1R \text{ NC}\pi^0$	5.92/5	0.71
$\bar{\nu}$ -mode $1R \text{ NC}\pi^0$	1.29/4	0.96

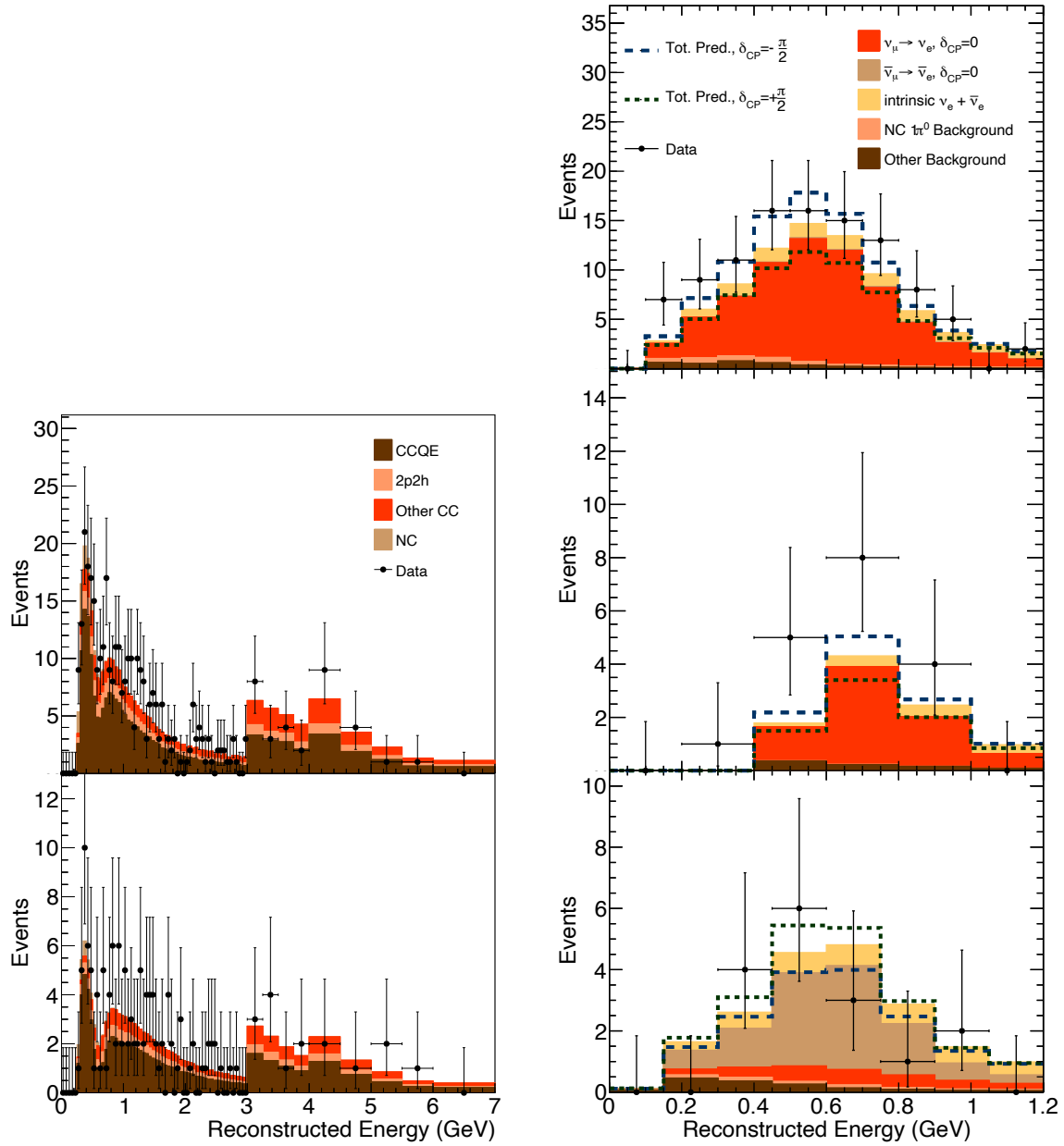


Figure 7.1: The MC and data distributions before the oscillation fit for the 5 Super-K samples. The left column shows the $1R\mu$ samples in ν -mode (top) and $\bar{\nu}$ -mode (bottom), with breakdowns into CCQE signals and various backgrounds. The MC production used in these plots assumes normal ordering with $\delta_{CP} = -1.601$, $\sin^2 \theta_{23} = 0.561$, $\sin^2 \theta_{13} = 0.022$, and $\Delta m_{32}^2 = 2.494 \times 10^{-3} \text{ eV}^2$. The right column presents the ν -mode $1Re$ (top), $1Re1de$ (middle), and $\bar{\nu}$ -mode $1Re$ (bottom) samples, showing breakdowns by oscillation channels and background contributions. The shape of the e -like spectra for true $\delta_{CP} = 0, \pm\pi/2$ are also indicated.

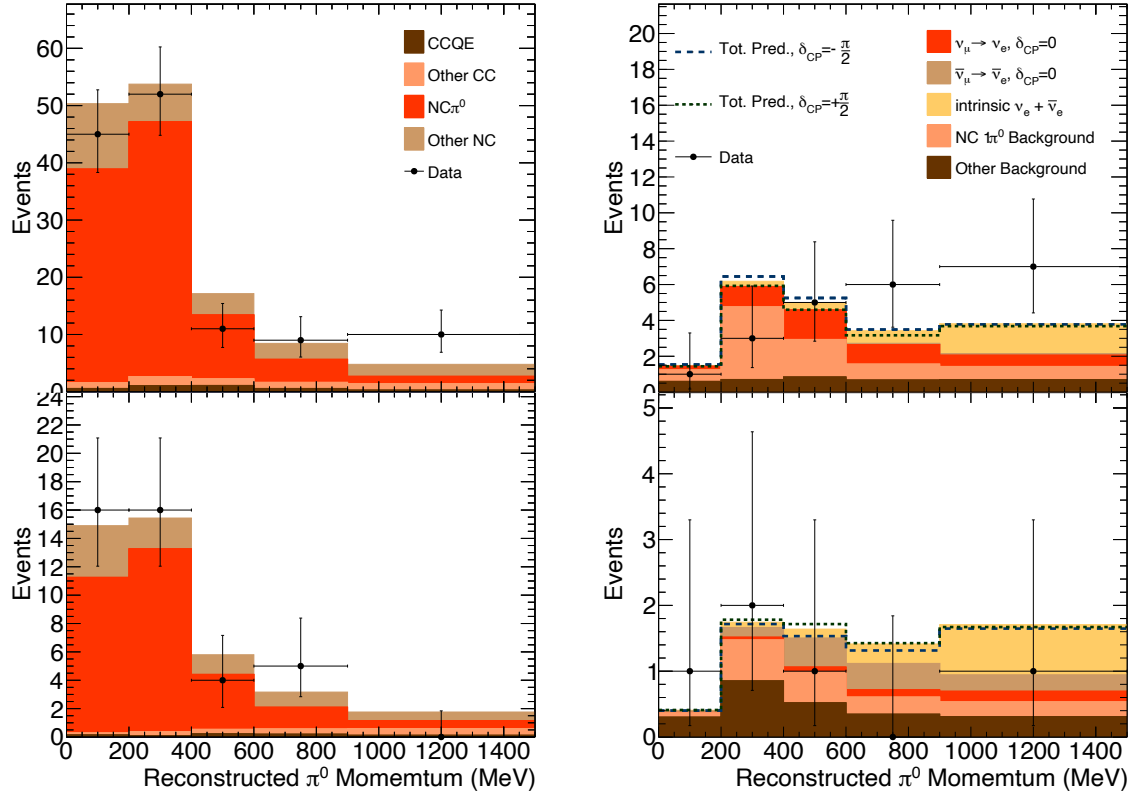


Figure 7.2: Overlap of MC and data before the oscillation fit for the new Super-K $\text{NC}\pi^0$ samples. The left column shows the $2\text{R}\pi^0$ samples in ν -mode (top) and $\bar{\nu}$ -mode (bottom), with breakdowns into $\text{NC}\pi^0$ events and backgrounds. The MC assumes normal ordering with $\delta_{CP} = -1.601$, $\sin^2 \theta_{23} = 0.561$, $\sin^2 \theta_{13} = 0.022$, and $\Delta m_{32}^2 = 2.494 \times 10^{-3} \text{ eV}^2$. The right column shows the $1\text{R}\pi^0$ samples in ν -mode (top) and $\bar{\nu}$ -mode (bottom), with breakdowns by oscillation channels and background contributions for spectra shapes of true $\delta_{CP} = 0, \pm\pi/2$.

7.3 Systematic Uncertainties

For this analysis, the parameterisation and prior constraints of the flux and existing cross-section parameters remain unchanged from the previous T2K oscillation analysis described in Ref. [42]. The four new π^0 normalisation parameters have been added to this version to form an updated cross-section covariance matrix. A description of the full cross-section parameterisation and the prior constraints for the parameters are available in Appendix A.1. Additionally, the near detector systematic uncertainties were not specifically studied in this analysis, as the model and selections of near detector samples also remain unchanged, and no additional effects on those parameters are expected as a result. A description of treatments to the

near detector uncertainties used for this analysis can be found in Ref. [130].

The Super-K detector systematic uncertainties have been updated with additional parameters for the new $\text{NC}\pi^0$ samples. The parameters are set to have the same bin boundaries as the sample under the reconstructed π^0 momentum, categorised into $\text{NC}1\pi^0$ events ($\text{NC}\pi^0 + \text{NCCoh}$), CC events, and NC other events. A comparison of the uncertainties from the previous T2K oscillation analysis with the ones for the current analysis is shown in Fig. 7.3. The new model exhibits larger uncertainties compared to the previous one, primarily due to a different method of calculating errors from the atmospheric neutrino fit. The uncertainties and covariance of the updated Super-K detector systematics are derived from the standard deviation of the best-fit values across multiple MCMC chains. However, these chains tend to converge on multiple minima, leading to significant deviations in the predicted MC event rates and, consequently, larger uncertainties. More details on this study can be found in Ref. [74].

A few Super-K detector parameters have uncertainties exceeding 100%, resulting in numerous negative proposed values during oscillation fits, which are meaningless since event rates are normalised by these parameters. These large uncertainties are likely to be inaccurately estimated by the current method and could exaggerate the uncertainties of other Super-K detector parameters due to the strong correlations – the prior covariance of the parameters is shown in Fig. 7.4. Given that only 1% of the total events can be affected by each of these parameters, a cap of 100% uncertainty is applied to the parameters to reduce the impact of the overestimation while still allowing for reasonable event rate variations to be applied.

Nevertheless, a 100% error can still result in small fractions of negatively proposed parameter values. If such steps are rejected solely due to the values being negative, it could limit the exploration of the parameter space and introduce bias into the posterior distributions of the correlated parameters. To address this, such steps are accepted or rejected based on the acceptance probability as other cases, but instead of applying a negative weight to events, a weight of zero is used.

7.4 Log-likelihood (LLH) Scans

7.4.1 Impact on Systematic Parameters

As mentioned in Section 6.2.1, several existing cross-section parameters are relevant to the cross-section of $\text{NC}1\pi^0$ production. The negative log-likelihood (nLLH) scans on these parameters were performed to assess the impact from the new $\text{NC}\pi^0$ samples. Fig. 7.5 highlights distributions of the nLLH for the resonance pion production form factors C_A^5 and M_A^{Res} , and normalisation parameters for NC coherent π^0 events and the multi- π and DIS-dominated NC background at Super-K.

It shows that $2\text{R}\pi^0$ events are able to improve the sensitivity for the form factors as expected, but effects are negligible compared to the constraining power of ND280. The impact on constraining the NC Coh normalisation is more significant with the $2\text{R}\pi^0$ samples compared to ND280, but the small number of events still limits sensitivity to NCCoh cross sections. What is more relevant for this analysis, however, is the cross section of the $\text{NC}1\pi^0$ events rather than individual NCCoh modes, which is set to be included in the π^0 normalisation uncertainties with the new parameters. Additionally, the impact of background NC events on the $2\text{R}\pi^0$ samples is shown to be the most significant among all the samples, though these background events are not well-constrained at this stage. Given that this background constitutes about 15% of the total events, the overall impact is expected to be small.

The impact of the samples on the new π^0 normalisation parameters is shown in Fig. 7.6. The $2\text{R}\pi^0$ samples effectively constrain these parameters in regions where the π^0 background and ν_e signal events are most dominant. However, the samples are not sensitive to the parameter corresponding to the 0.9-1.5 GeV P_{π^0} region due to the small number of events in that range. As expected, the gap between the sensitivities of the 2R and 1R samples increases at higher P_{π^0} , given the inverse relationship in their efficiency shapes. The sensitivity is highest in the 0.2-0.4 GeV region, where both the 2R and 1R samples peak. Overall, the parameters can be constrained up to

a 1σ variation of 22%, 15%, and 30% for the first three π^0 normalisation parameters.

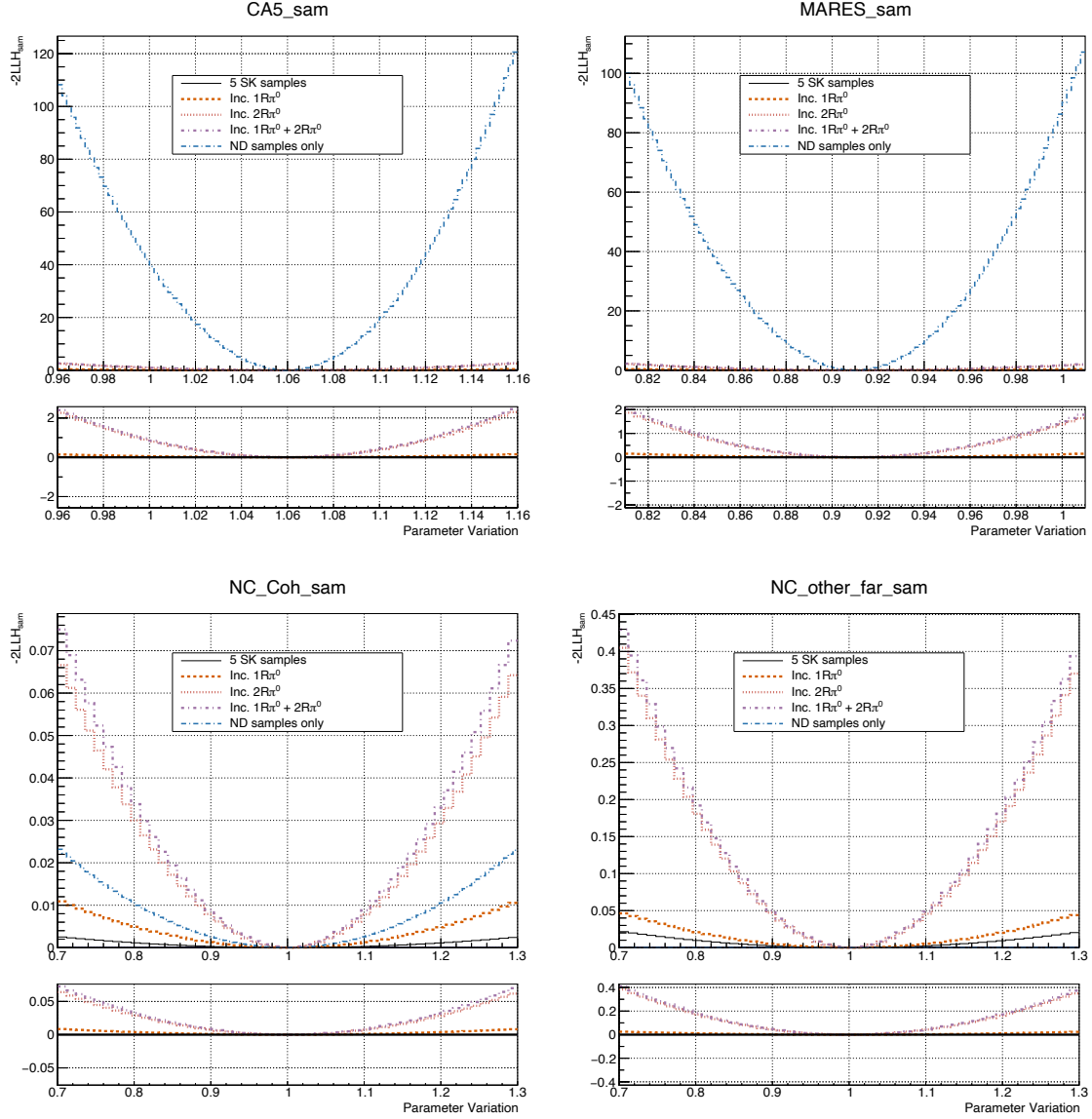


Figure 7.5: Comparisons of the sample nLLH for the single pion production form factors C_A^5 (top-left) and M_A^{Res} (top-right), as well as normalisation uncertainties for NC coherent π^0 production (bottom-left) and NC background events (bottom-right), using 5 Super-K samples (black), inclusions of the new NC π^0 samples (red), and the constraints from ND280 alone (blue). The upper part of each plot shows the distribution of the LLH scans from each sample combination, while the lower part highlights the impact of the new samples compared to the 5 Super-K samples.

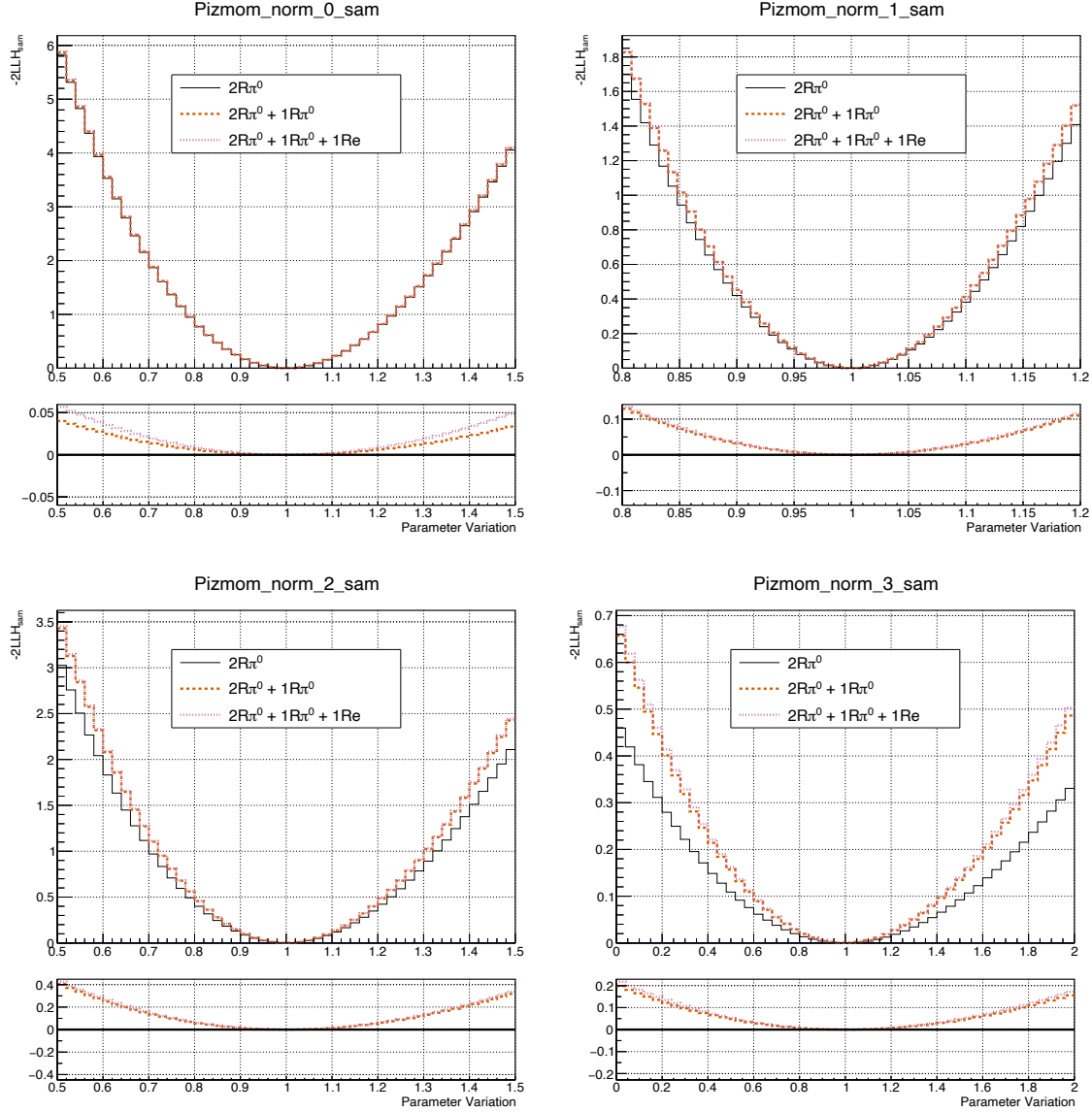


Figure 7.6: Comparisons of the nLLH scan for the 4 new $\text{NC}1\pi^0$ normalisation parameters when the normalisation is applied to events from combinations of the $2R\pi^0$, $1R\pi^0$ or $1Re$ samples. The upper part of each plot shows the distribution of the LLH scans from each sample combination, while the lower part highlights the further impact of the 1R samples apart from the 2R case.

7.4.2 Impact on Oscillation Parameters

The nLLH scans of oscillation parameters can be used to estimate sensitivities achievable with the current statistics and to assess the impact of additional signal events from the new samples. The 1σ and 2σ credible intervals over two parameters are shown in Fig. 7.7. Correlations between parameters are also indicated, where these correlations are induced by degeneracies in the ν_e appearance and ν_μ disappearance oscillation probabilities.

The contours from including the $\text{NC}\pi^0$ samples are generally tighter compared to those with only the 5 Super-K samples. The $\text{NC}\pi^0$ samples primarily contribute to improving signal and background modelling in the ν_e appearance channels. Since these channels have less direct impact on Δm_{32}^2 and $\sin^2 \theta_{23}$, the inclusion of $\text{NC}\pi^0$ samples shows negligible effects on the constraint of these parameters. As expected from including more ν_e signal events with the $1\text{R}\pi^0$ samples, a slight improvement can be observed in the joint precision of δ_{CP} and $\sin^2 \theta_{13}$.

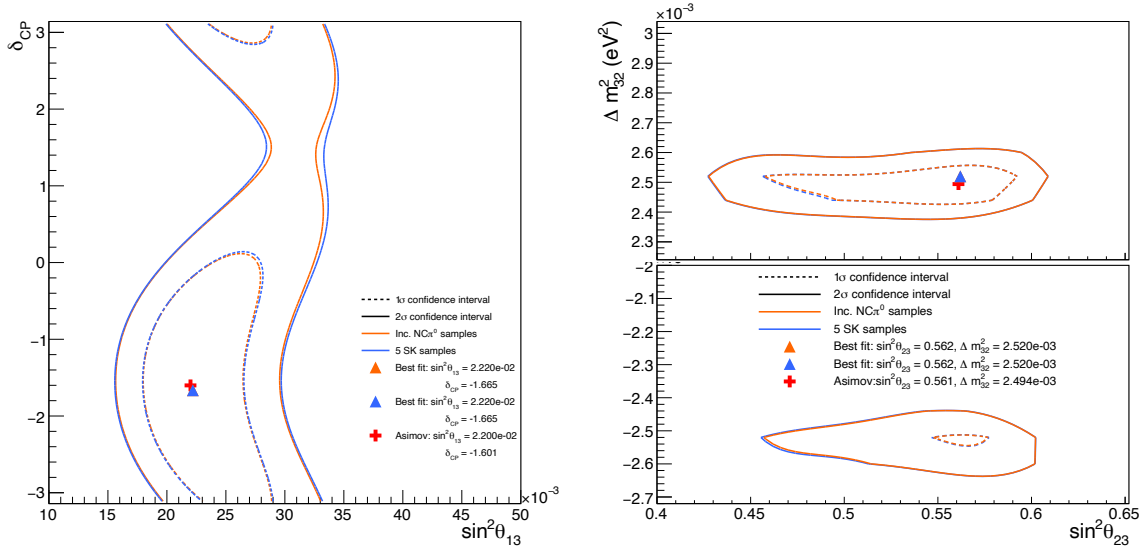


Figure 7.7: Comparisons of the 1σ ($-2\text{LLH} = 2.30$) and 2σ ($-2\text{LLH} = 6.18$) variations of 2D nLLH scans for the oscillation parameters against Asimov datasets, with and without including new $\text{NC}\pi^0$ samples. The left figure shows relative sensitivities to ν_e appearance, assuming normal ordering, $\Delta m_{32}^2 = 2.494 \times 10^{-3} \text{ eV}^2$, and $\sin^2 \theta_{23} = 0.561$. The right figure shows relative sensitivities to ν_μ disappearance, assuming $\delta_{CP} = -1.601$ and $\sin^2 \theta_{13} = 0.022$. The “best fit” corresponds to the minimum of the 2D nLLH distribution for the grid of points scanned.

Asimov Fits to Oscillation Parameters Only

A cross-check was performed by fitting the MC to the Asimov dataset without varying the systematic parameters. Fig. 7.8 shows the 1σ and 2σ posterior credible intervals obtained from the fits after 40M proposed steps. The best-fit point is obtained from the highest posterior distribution (HPD) point.

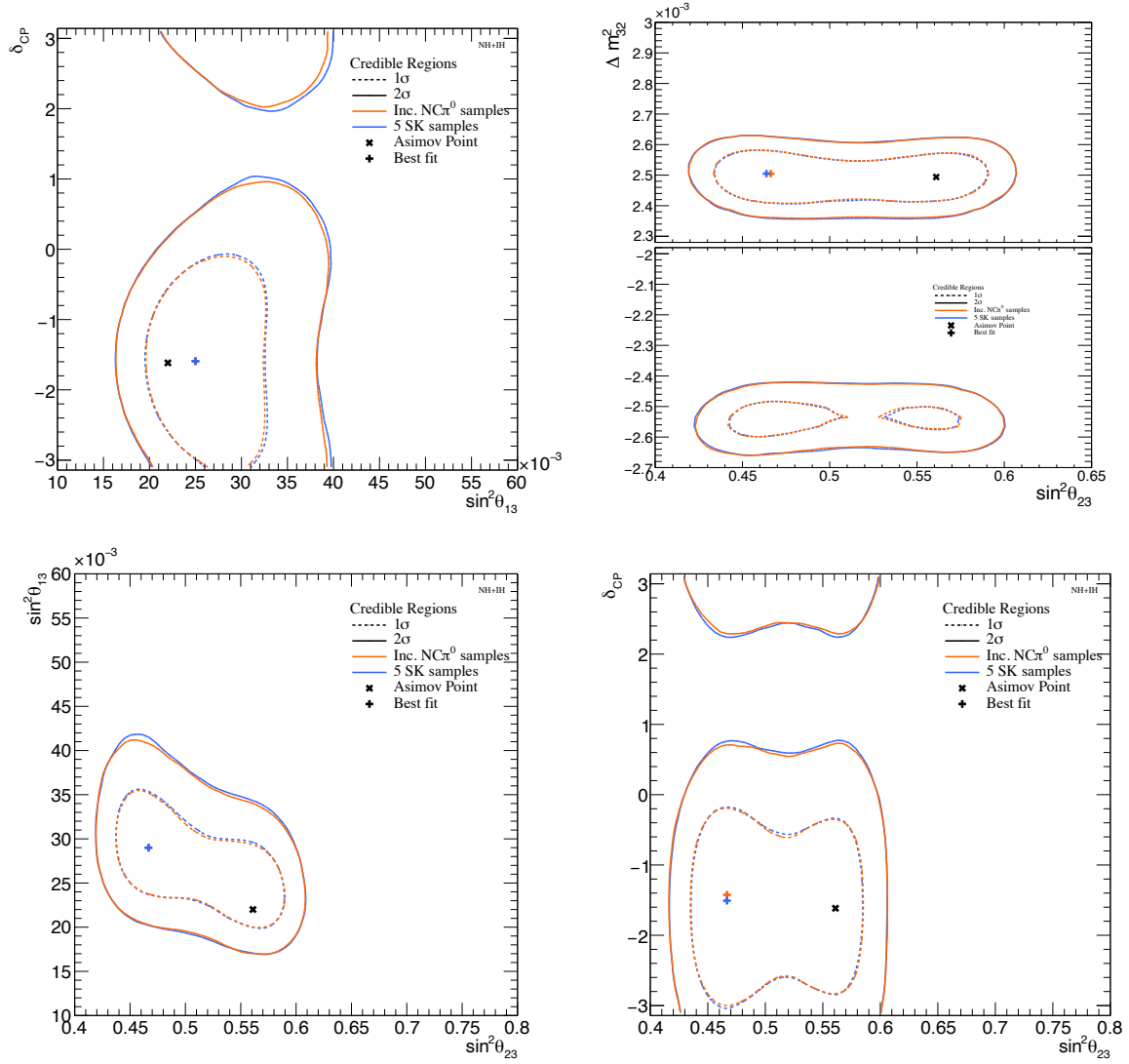


Figure 7.8: Comparisons of the 2D credible intervals of oscillation parameters from an Asimov fit without varying systematic parameters, constrained by 5 SK samples and the newly included $\text{NC}\pi^0$ samples.

The plot shows that the best-fit of $\sin^2 \theta_{23}$ lies in the lower octant region, symmetric to the Asimov value around the maximum ν_μ disappearance point at $\sin^2 \theta_{23} = 0.511$. This outcome is due to the nature of degeneracies in the ν_μ disappearance probability, which makes the spectra indistinguishable between the two θ_{23} octants. The

preference for the lower octant rather than the upper octant could be due to the fact that the region below 0.511 including a larger portion of the physically allowed values for θ_{23} . This larger region provides more parameter space to explore, increasing the likelihood of finding a better fit in the lower octant during the MCMC sampling process. As a result, there is a higher probability of steps accumulating in the lower octant, leading to the best fit to be found in that region.

The preference for $\sin^2 \theta_{23}$ in the lower octant results in a shift towards larger values of $\sin^2 \theta_{13}$ to balance variations in the MC predictions. Consequently, the best fit of $\sin^2 \theta_{13}$ also deviates from the input value, with the credible intervals stretched horizontally. In contrast, the changes in $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are less correlated to δ_{CP} and Δm_{32}^2 which have them remain close to their input values.

Fig. 7.9 illustrates the sampling with the normal mass ordering under $0.54 < \sin^2 \theta_{23} < 0.58$ (around the input value of 0.561), where only a range of the upper octant $\sin^2 \theta_{23}$ values was explored. In this case, the shape of the ν_e appearance contours and amount of variations from including the new samples match the case seen in the 2D nLLH scans of the two parameters in Fig. 7.7. The best fit of $\sin^2 \theta_{13}$ remains close to the input value of 0.022 as expected.

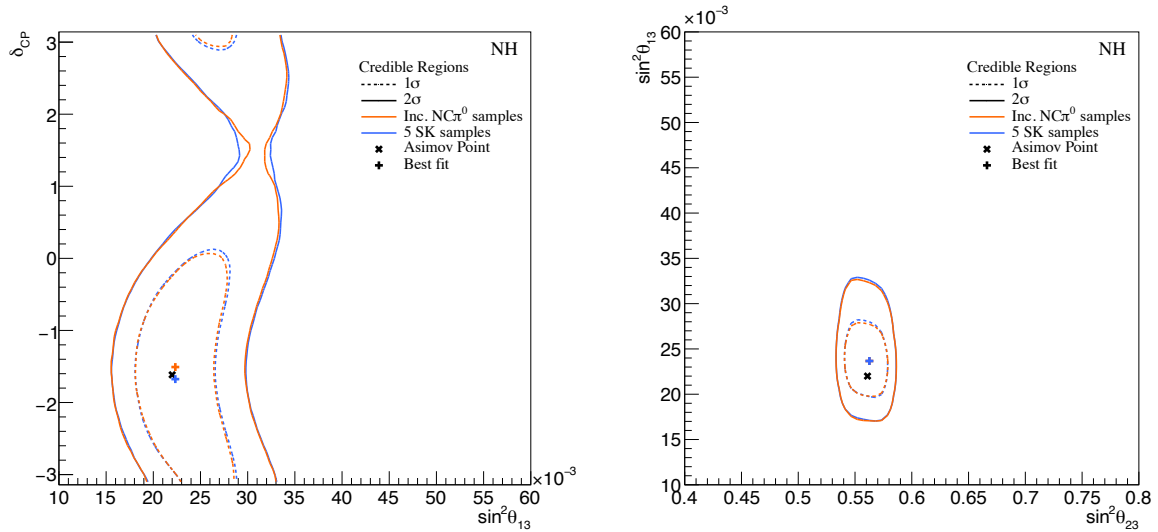


Figure 7.9: Comparisons of the 2D credible intervals from an Asimov fit without varying systematic parameters for the steps with proposed values around the true $\sin^2(\theta_{23})$ space (restricting the space to be within $0.54 < \sin^2 \theta_{23} < 0.58$), constrained by 5 SK samples and the newly included $NC\pi^0$ samples.

7.5 Simulated Data Fits

7.5.1 Overview

Several fits were performed using simulated datasets to assess the impact of the new samples and effects from the new systematic parameterisation. About 160 M post burn-in steps are accumulated for the each simulated data fit. The posterior credible intervals from the fits are compared to evaluate the impact on the oscillation sensitivities. Furthermore, the postfit uncertainties and correlations for the parameters are also checked to improve the understanding of the variations.

MCMC Chain Properties

The log-likelihood distributions and auto-correlations of the MCMC chains were checked for the fits against all the systematic parameters, in order to understand the sampling progress. If the chain progresses too slowly to reach equilibrium (i.e., the probabilities of proposing parameter values become stable) or if the proposed step remains highly correlated after many iterations from the initial step, it may indicate that the step size for the parameters is not properly set. Section 3.3.3 has provided an overview related to step size tuning.

As shown in Fig. 7.10, the chains have reached equilibrium after approximately 50,000 steps. The auto-correlations quickly drop below 0.2 and remain very low for the majority of the parameters. However, few parameters exhibit slower convergence. This is likely due to the data's lack of sensitivity to these parameters.

The step sizes for the existing parameters remain the same as those used in the previous T2K oscillation analysis. Therefore, parameters with slower convergence should not raise new concerns for the fits in this analysis. The step sizes for the new systematic parameters including π^0 normalisation and Super-K detector normalisation, and the oscillation parameters, have been tuned accordingly.

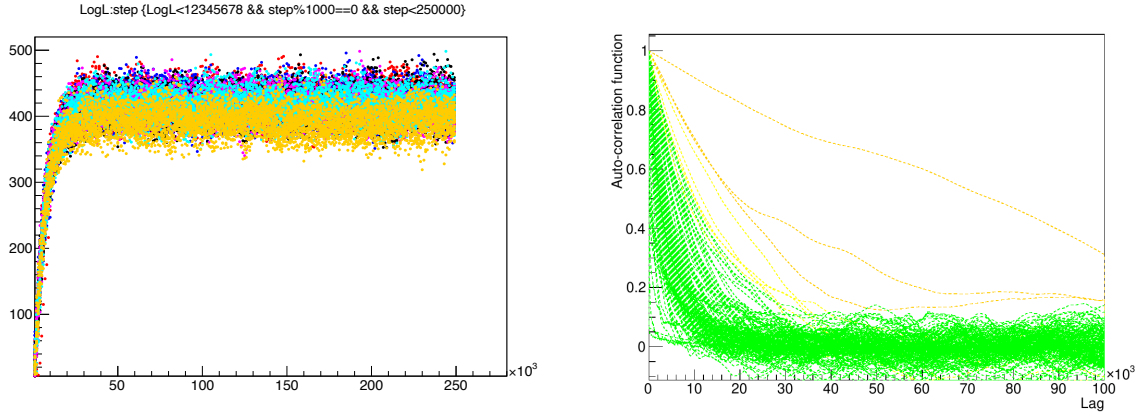


Figure 7.10: MCMC chain diagnostic plots: (left) The total log-likelihood against MCMC steps overlapped for several fits, showing a steady distribution after 50×10^3 steps; (right) An example of the auto-correlation distributions overlapped for all the parameters in a fit, where the color of the lines are determined from the minimum value obtained between 0 and 25,000 lags, with green for minimum less than 0.2, yellow for 0.4, and orange for 0.8.

7.5.2 Impact from New Super-K Detector Parameterisation

Two Asimov fits were performed with the 5 Super-K samples to assess the impact of the larger uncertainties introduced by the new Super-K detector systematics, as described in Section 7.3. Fig. 7.11 and Fig. 7.12 compare the 1σ and 2σ credible intervals between the old and new detector systematics under normal and inverted mass ordering, respectively. The plots show results both with and without constraints from reactor neutrino data on $\sin^2 \theta_{13}$, with the left and right columns corresponding to these cases.

For the normal mass ordering without reactor constraints, $\sin^2 \theta_{13}$ credible intervals expand significantly with the large Super-K detector uncertainties, while smaller effects are observed for δ_{CP} and $\sin^2 \theta_{23}$. The larger systematic variations also absorb some correlations between $\sin^2 \theta_{13}$ and δ_{CP} to eliminate the bending shape.

After applying the reactor constraint, the 1σ and 2σ variations of $\sin^2 \theta_{13}$ align more closely. However, compared to the case with the old detector systematics, the δ_{CP} variations do not shrink alongside $\sin^2 \theta_{13}$, maybe due to the weaker correlation between these parameters. Additionally, more values of $\sin^2 \theta_{23}$ are observed in the upper octant with the best fit approaching the input value, though the impact from

the new detector systematics case on this shift is smaller, likely due to the weaker correlation with $\sin^2 \theta_{13}$.

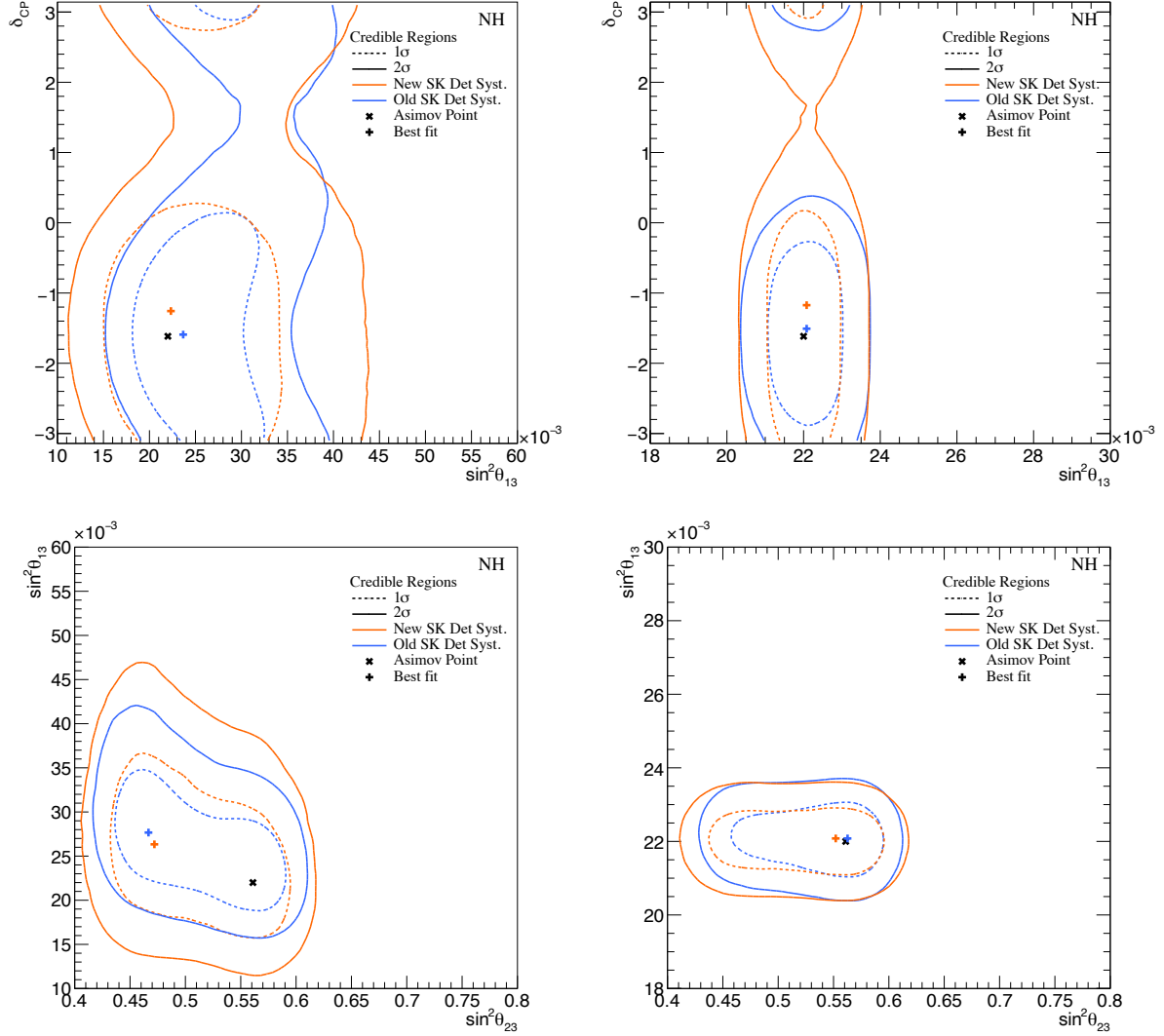


Figure 7.11: Comparisons of the 2D credible intervals of δ_{CP} , $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ from a full Asimov fit under the normal mass ordering and constrained by the 5 SK samples. The Super-K detector systematic parameterisation was changed between the two fits. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

Fig. 7.12 shows that the inverted mass ordering exhibits a similar pattern to the normal ordering case discussed above. However, shifts in $\sin^2 \theta_{13}$ towards larger values are smaller for the new detector systematics from this mass ordering switching.

Fig. 7.13 illustrates the impact of the new Super-K detector systematics on the ν_μ disappearance channel. Similar to the previous cases, the octant of θ_{23} is less sensitive

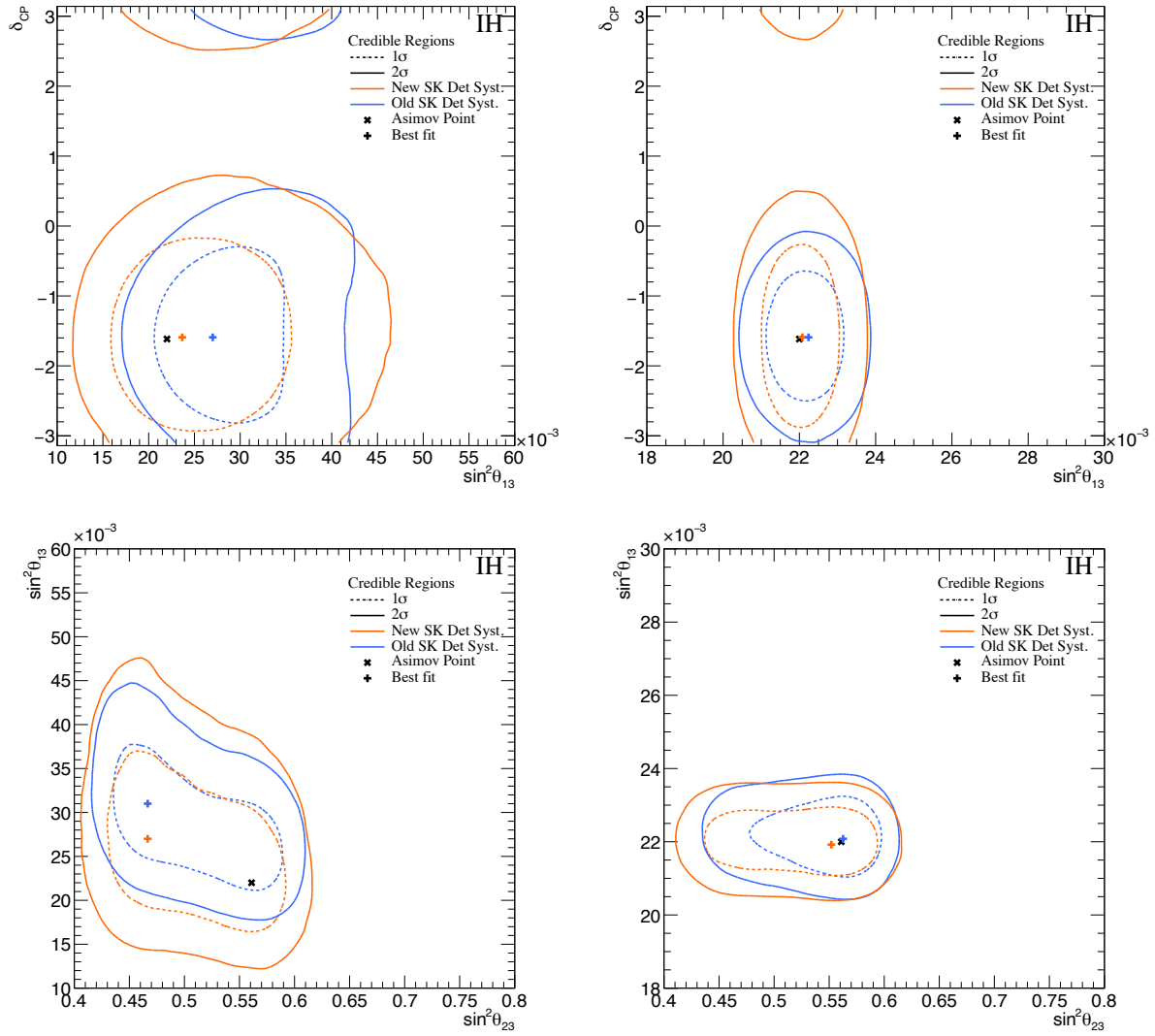


Figure 7.12: Comparisons of the 2D credible intervals of δ_{CP} , $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ from a full Asimov fit under the inverted mass ordering and constrained by the 5 SK samples. The Super-K detector systematic parameterisation was changed between the two fits. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

to the reactor constraints. While the larger systematics have expanded the contours, the variations are smaller compared to the ν_e appearance case.

The correlations between the systematic parameters with the Δm_{32}^2 and $\sin^2 \theta_{23}$ are weak, potentially due to the stronger constraints provided by the ν_μ disappearance data, thus the variations induced by each parameter will be small compared to the data. Here, most of the uncertainty in $\sin^2 \theta_{23}$ arises due to the correlations with $\sin^2 \theta_{13}$.

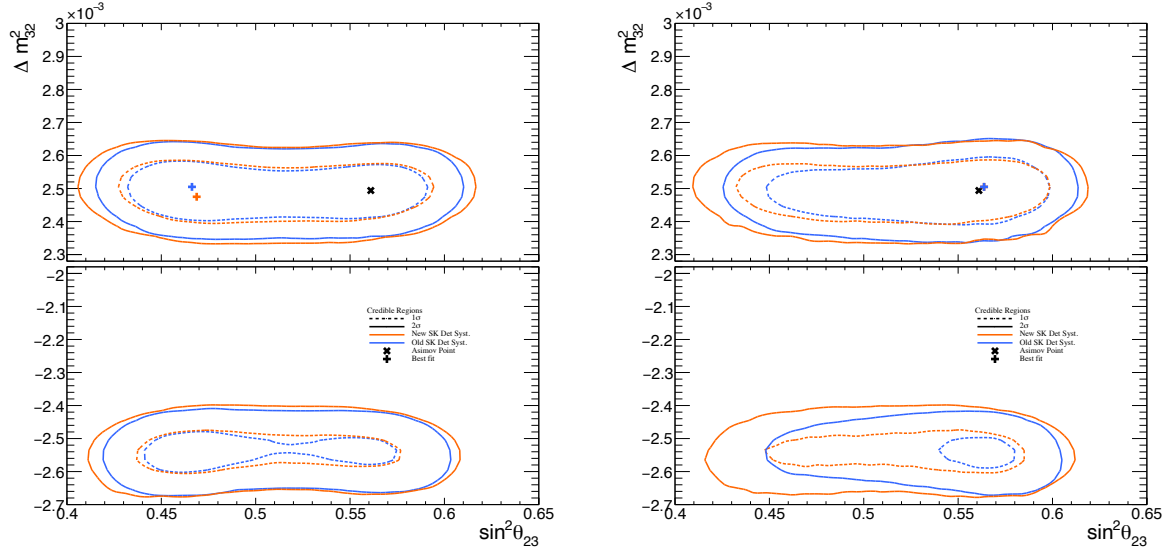


Figure 7.13: Comparisons of the ν_μ disappearance 2D credible intervals from a full Asimov fit constrained by the 5 SK samples. The Super-K detector systematic model was changed between the two fits. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

The main oscillation parameter that has strong correlations with the detector systematic parameters is $\sin^2 \theta_{13}$. Table 7.4 shows that the strong correlation ($|\text{corr}| > 0.2$) occurs when ν_e events dominate, especially in regions around the oscillation maximum. These correlations primarily explain the large variations in the credible interval of $\sin^2 \theta_{13}$ from detector systematic parameters with the large uncertainties.

Table 7.4: The postfit correlation of $\sin^2 \theta_{13}$ with systematic parameters from an Asimov fit for $|\text{correlation}| \geq 0.2$. The majority of these parameters are the Super-K detector systematics.

Type	Event	Region	Correlation
SK Det.	ν -mode 1Re Osc. ν_e	$0.00 \leq E_\nu^{reco.} < 0.35 \text{ GeV}$	-0.27
	ν -mode 1Re Osc. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.57
	ν -mode 1Re Osc. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.55
	ν -mode 1Re Int. ν_e	$0.00 \leq E_\nu^{reco.} < 0.35 \text{ GeV}$	-0.27
	ν -mode 1Re Int. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.57
	ν -mode 1Re Int. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.44
	$\bar{\nu}$ -mode 1Re Osc. ν_e	$0.00 \leq E_\nu^{reco.} < 0.35 \text{ GeV}$	-0.21
	$\bar{\nu}$ -mode 1Re Osc. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.57
	$\bar{\nu}$ -mode 1Re Osc. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.47
	$\bar{\nu}$ -mode 1Re Int. ν_e	$0.00 \leq E_\nu^{reco.} < 0.35 \text{ GeV}$	-0.23
	$\bar{\nu}$ -mode 1Re Int. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.57
	$\bar{\nu}$ -mode 1Re Int. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.36
	ν -mode 1Re1de Osc. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.35
	ν -mode 1Re1de Osc. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.44
	ν -mode 1Re1de Int. ν_e	$0.35 \leq E_\nu^{reco.} < 0.80 \text{ GeV}$	-0.35
	ν -mode 1Re1de Int. ν_e	$0.80 \leq E_\nu^{reco.} < 1.25 \text{ GeV}$	-0.33
	ν -mode 2R π^0 CC	$0.40 \leq P_{\pi^0}^{reco.} < 0.60 \text{ GeV}$	0.25
	ν -mode 1R π^0 CC	$0.00 \leq P_{\pi^0}^{reco.} < 0.20 \text{ GeV}$	0.22
	ν -mode 1R π^0 CC	$0.20 \leq P_{\pi^0}^{reco.} < 0.40 \text{ GeV}$	0.51
	ν -mode 1R π^0 CC	$0.40 \leq P_{\pi^0}^{reco.} < 0.60 \text{ GeV}$	0.51
	ν -mode 1R π^0 NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40 \text{ GeV}$	0.40
	$\bar{\nu}$ -mode 1R π^0 CC	$0.00 \leq P_{\pi^0}^{reco.} < 0.20 \text{ GeV}$	0.22
	$\bar{\nu}$ -mode 1R π^0 CC	$0.20 \leq P_{\pi^0}^{reco.} < 0.40 \text{ GeV}$	0.51
	$\bar{\nu}$ -mode 1R π^0 CC	$0.40 \leq P_{\pi^0}^{reco.} < 0.60 \text{ GeV}$	0.50
	$\bar{\nu}$ -mode 1R π^0 NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40 \text{ GeV}$	0.32
Osc.	$\sin^2 \theta_{23}$	-	-0.33

7.5.3 Including the $2R\pi^0$ Samples

The inclusion of the $2R\pi^0$ samples in the fit results in a slight improvement in constraining $\sin^2 \theta_{13}$, with the effect being more pronounced in the 2σ credible interval. This improvement is likely due to better constraints on the Super-K detector systematic parameters associated with the $2R\pi^0$ events. These parameters may be directly correlated with $\sin^2 \theta_{13}$ or with other Super-K detector parameters, resulting in the overall uncertainties being better constrained. Due to the much smaller correlations with the Super-K detector systematics from the ν_μ disappearance channel, the variations in the credible interval is negligible.

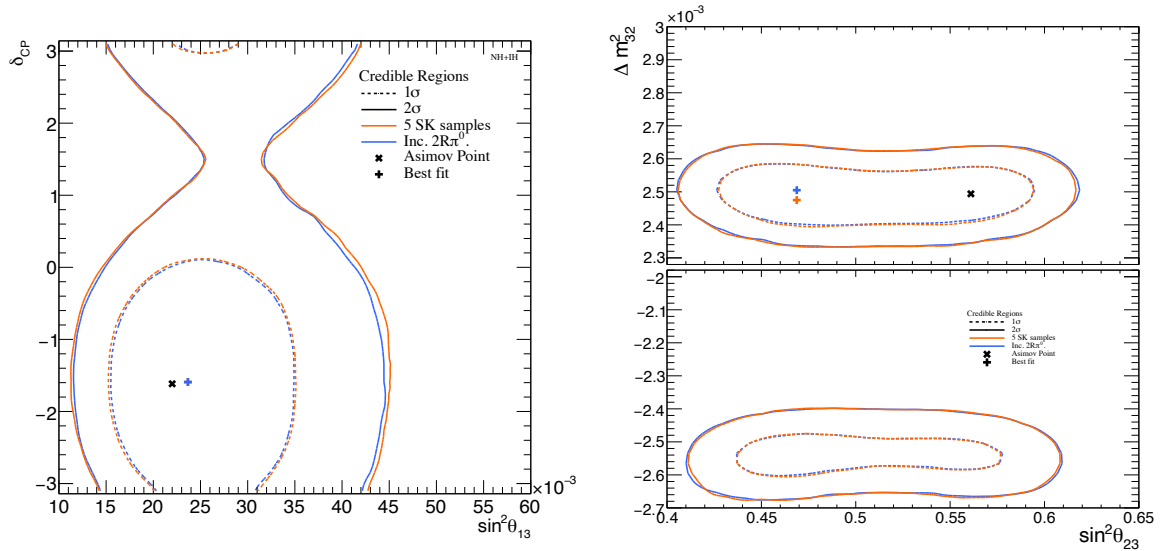


Figure 7.14: Comparisons of the ν_e appearance and ν_μ disappearance 2D credible intervals from a full Asimov fit combining both mass ordering constrained by the 5 SK samples and the case including the $2R\pi^0$ samples.

7.5.4 Including Both $2R\pi^0$ and $1R\pi^0$ Samples

Comparing the fit results with only the $2R\pi^0$ samples to the case where the $1R\pi^0$ samples are also included, a more significant improvement in $\sin^2 \theta_{13}$ is observed, as shown in Fig. 7.15. The inclusion of the $1R\pi^0$ samples better constrains the Super-K detector uncertainties. Additionally, the $1R\pi^0$ samples include more oscillated ν_e events. Together, these factors enhance the correlations between oscillation pa-

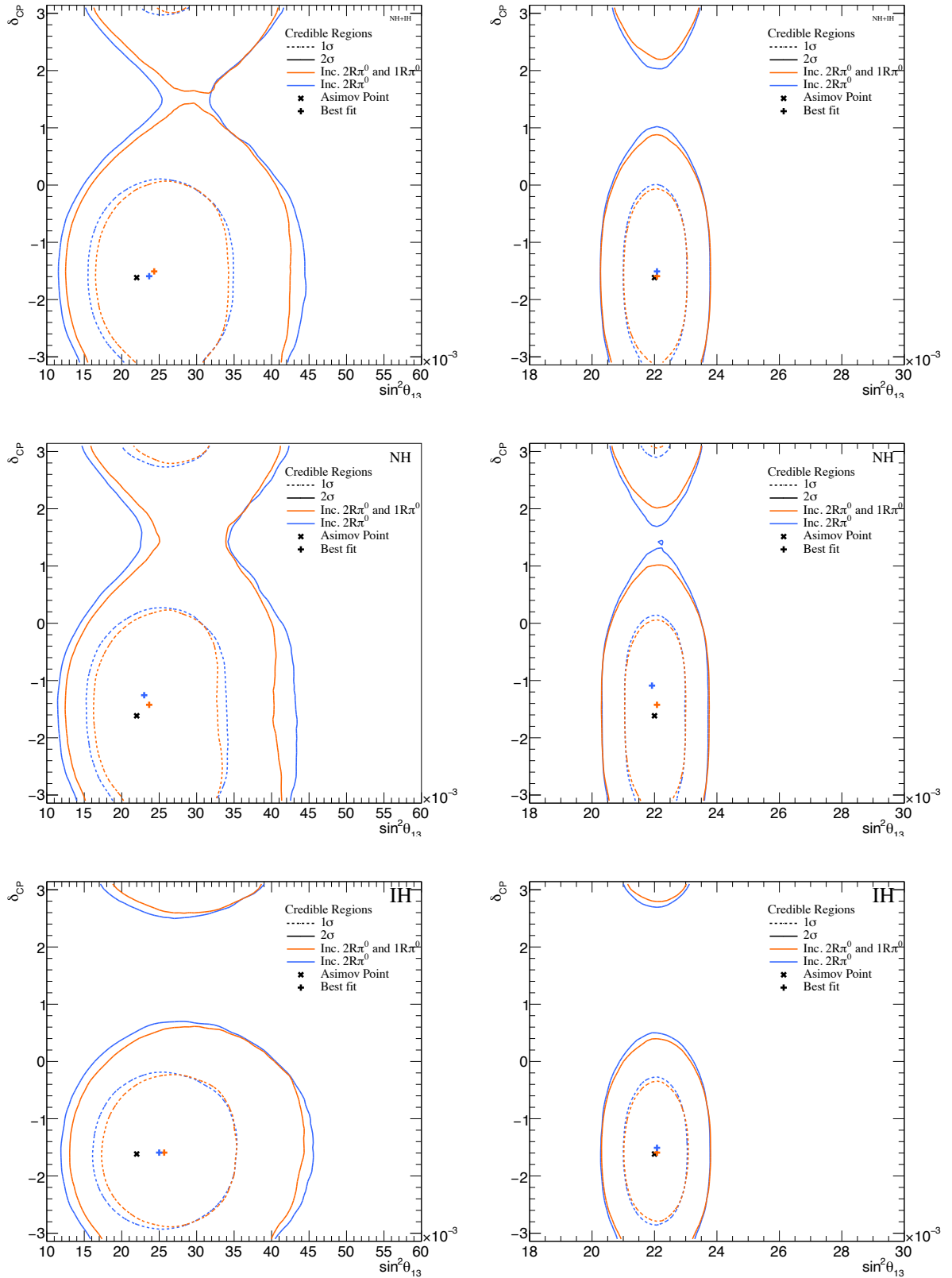


Figure 7.15: Comparisons of the ν_e appearance 2D credible intervals from a full Asimov fit constrained by the inclusion of $2R\pi^0$ samples only and the case also including the $1R\pi^0$ samples apart from the 5 SK samples. The middle and bottom plots separate distributions under normal and inverted mass ordering. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

rameters. This leads to δ_{CP} being more tightly constrained when reactor constraints are applied to $\sin^2 \theta_{13}$, particularly in the normal mass ordering case. Additionally, for the inverted mass ordering case, the ν_e appearance contours shift toward higher $\sin^2 \theta_{13}$ values.

A slight tightening of the ν_μ disappearance contours is also observed, as shown in Fig. 7.16. The sensitivity to the θ_{23} octant is enhanced compared to the previous cases when reactor constraints are re applied to $\sin^2 \theta_{13}$.

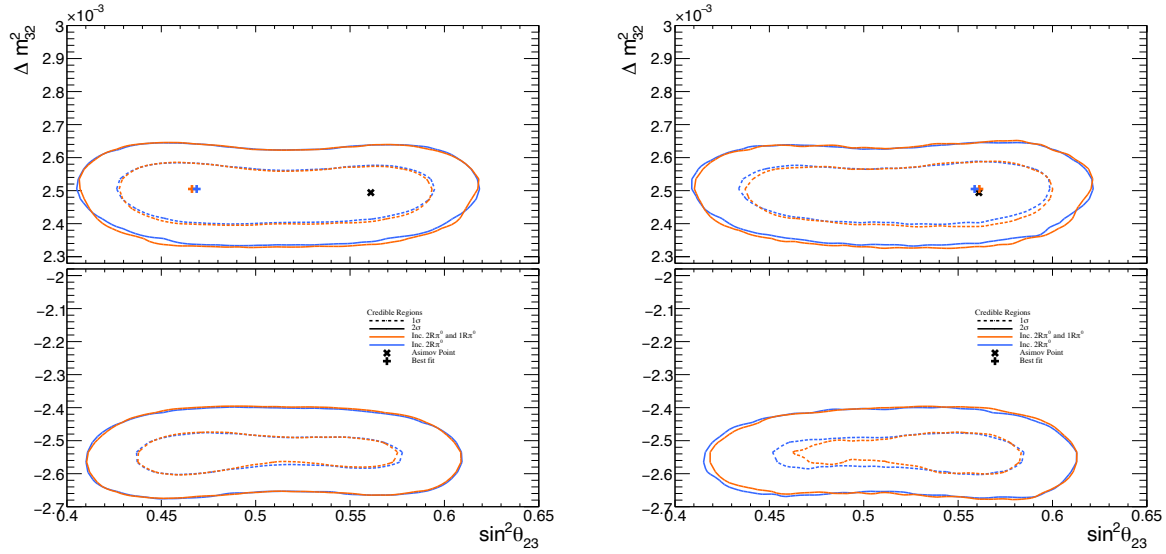


Figure 7.16: Comparisons of the ν_μ disappearance 2D credible intervals from a full Asimov fit constrained by the inclusion of $2R\pi^0$ samples only and the case also including the $1R\pi^0$ samples apart from the 5 SK samples. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

Fig. 7.17 shows the postfit uncertainties on the Super-K detector parameters, comparing the results with and without the inclusion of the $NC\pi^0$ samples in the fit. The majority of the parameters are centred close to the input values, with some small deviations for parameters with large prior uncertainties and those correlated with them. For instance, parameters 79-81 (corresponding to CC events in the lower momentum bin of $1R\pi^0$ samples) exhibit such deviations, as a result of the MCMC propagation and the corresponding highest posterior distribution point (HPD). Meanwhile, parameters 74-76, which correspond to the $NC\pi^0$ events in the same kinematic region, are affected to balance this variation. This effect is particularly noticeable when the $NC\pi^0$ samples are included, as this better constrains parameters 74-76

and become more sensitive to the variations induced by parameters 79-81.

Moreover, the inclusion of the new $\text{NC}\pi^0$ samples reduces the uncertainties for the corresponding Super-K detector parameters, leading to improvements in the parameters related to NC and ν_e CC events in other samples. Many of these parameters exhibit a strong correlation with $\sin^2 \theta_{13}$, as shown in Table 7.4, which aligns with the expected variations in the ν_e appearance contours.

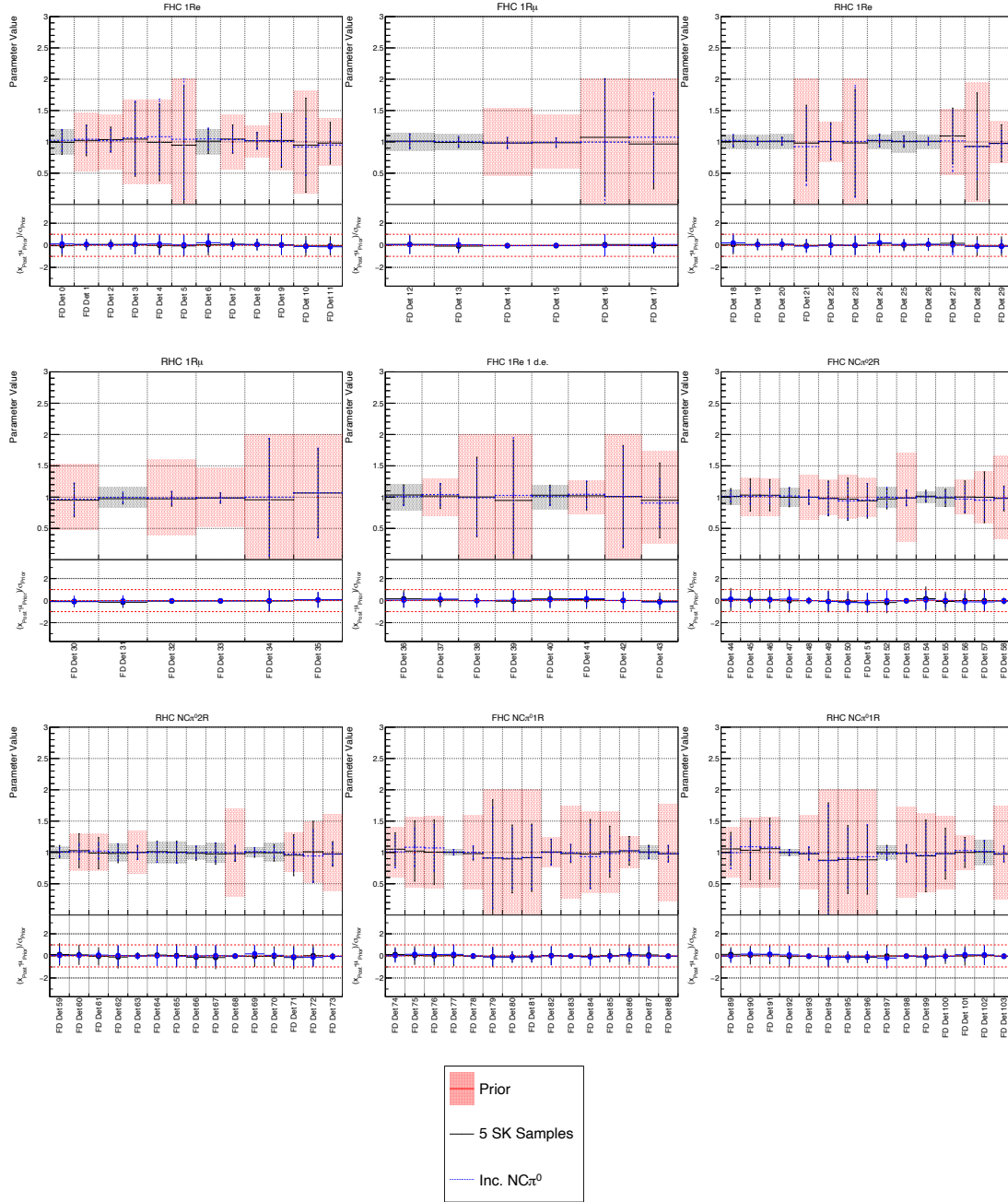


Figure 7.17: The Super-K detector systematic uncertainties prior to and post the Asimov fit, with and without the new $\text{NC}\pi^0$ samples.

7.5.5 Including the New Cross-Section Parameters

As shown in Fig. 7.18, the credible intervals resulting from the inclusion of the new π^0 normalisation parameters remain largely unchanged, with only a slight and negligible widening. This indicates that the inclusion of these parameters has minimal impact on the behaviour of other parameters during the fit, as expected when fitting against the Asimov dataset. It also indicates the new parameters are not greatly reducing the sensitivities of the new NC π^0 samples added to the fit.

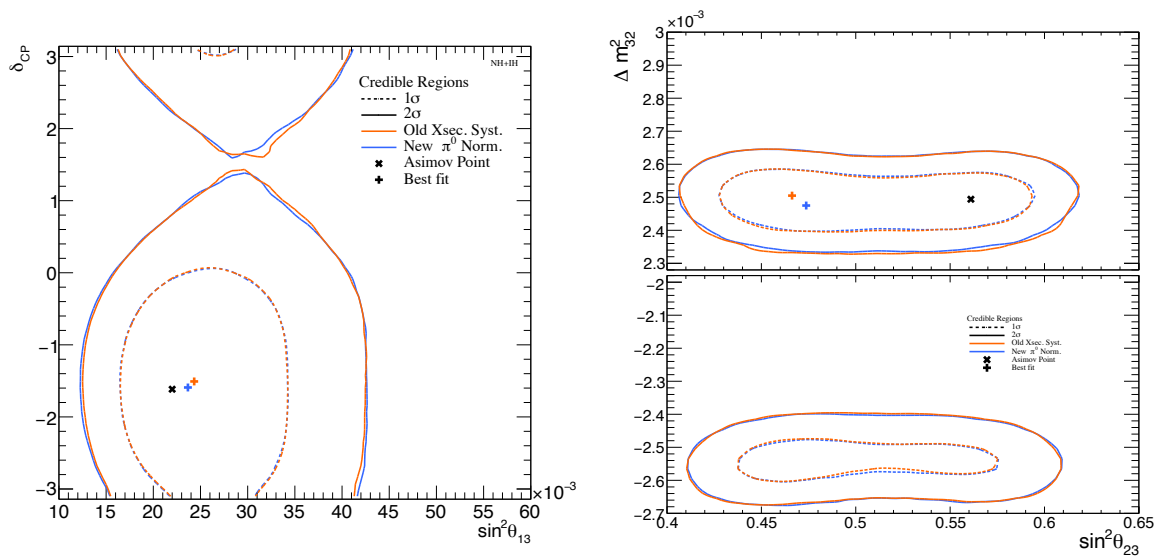


Figure 7.18: Comparisons of the ν_e appearance and ν_μ disappearance 2D credible intervals from a full Asimov fit combining both mass ordering constrained by all the SK samples before and after including the new π^0 normalisation parameters.

7.6 Postfit Impact on the New π^0 Parameters

7.6.1 Asimov Fits

The results for the four π^0 normalisation parameters from an Asimov fit are presented in Fig. 7.19. The first three parameters remain close to the input value of one, while the fourth parameter shifts slightly further due to its large prior uncertainty and the limited constraining power from the available events. The parameters

corresponding to P_{π^0} ranges of 0-0.2 GeV and 0.4-0.9 GeV have their uncertainties constrained to a similar extent as estimated from the parameters' nLLH distributions (as shown in Fig. 7.6). However, the parameter corresponding to the 0.2-0.4 GeV region did not see much improvement from its prior uncertainty and did not reach the estimated improved 15% uncertainty with the nLLH scan.

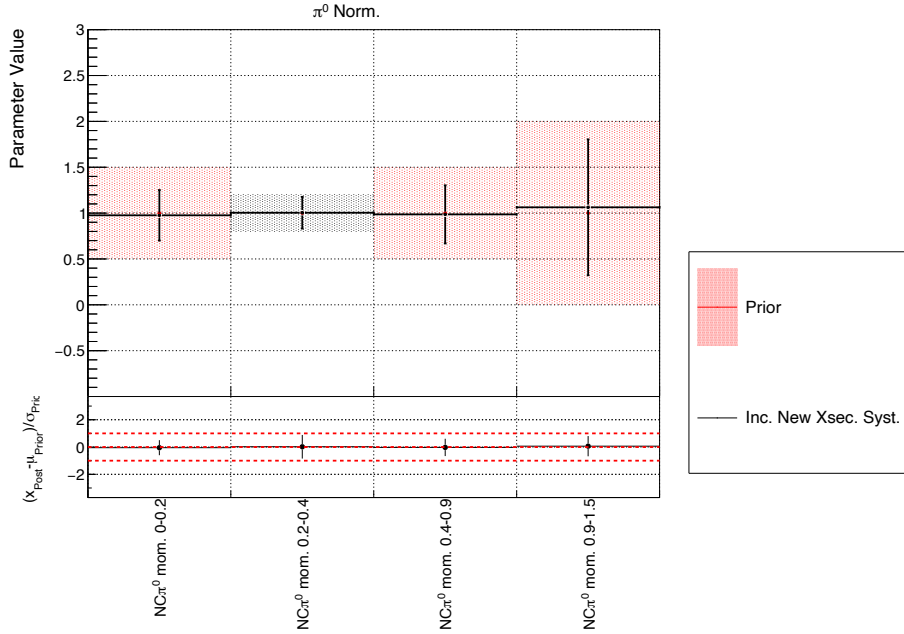


Figure 7.19: Results of the new π^0 normalisation parameters from an Asimov fit.

As previously discussed, the posterior distribution of a parameter may be influenced by correlations with other parameters. The postfit correlations for the four π^0 normalisation parameters are shown in Fig. 7.20, with details of the correlations greater than 0.2 listed in Tables 7.5 – 7.7.

The strongest correlation effects are observed for the π^0 normalisation parameter in the 0.2-0.4 GeV region. As shown in Table 7.6, significant correlations arise from parameters associated with NC events in the overlapped kinematic regions with P_{π^0} of 0.2-0.4 GeV, as well as parameters that are correlated with these through the detector covariance. Due to these correlations, an increase in detector uncertainties can weaken the constraints on the π^0 normalisation parameter.

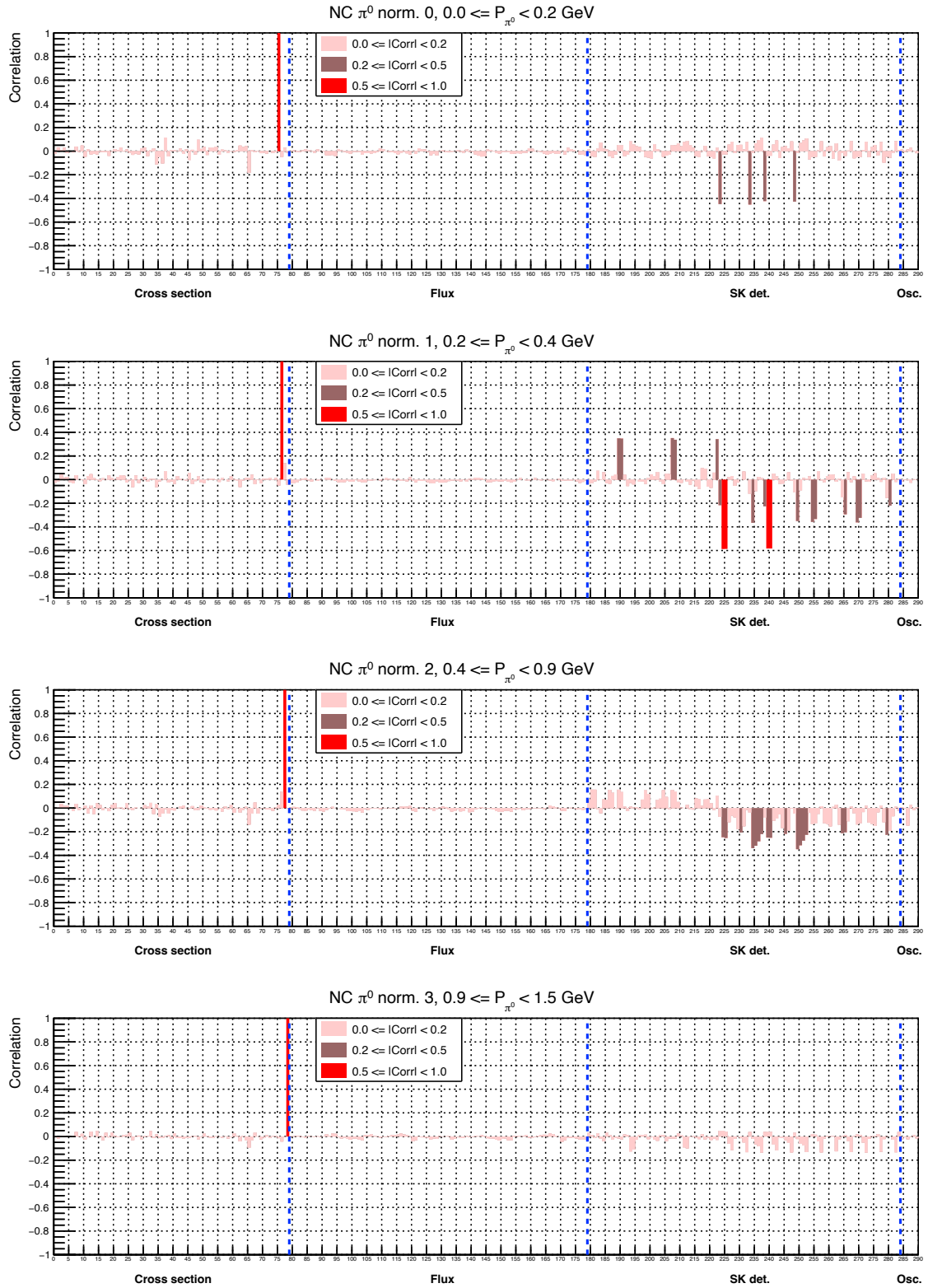


Figure 7.20: The postfit correlations of the new π^0 normalisation parameters with all other parameters from an Asimov fit. The majority of the correlations are contributed by the Super-K detector systematics.

Table 7.5: Parameters with postfit correlation absolute value ≥ 0.2 for π^0 normalisation parameter corresponding to $0.0 \leq P_{\pi^0} < 0.2$ GeV.

Type	Event	Region	Correlation
SK Det.	ν -mode $2R\pi^0$ NC π^0	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.45
	ν -mode $2R\pi^0$ NC other	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.45
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.42
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.43

Table 7.6: Parameters with postfit correlation absolute value ≥ 0.2 for π^0 normalisation parameter corresponding to $0.2 \leq P_{\pi^0} < 0.4$ GeV.

Type	Event	Region	Correlation
SK Det.	ν -mode 1Re NC	$0.35 \leq E_{\nu}^{reco.} < 0.80$ GeV	0.35
	ν -mode 1Re NC	$0.80 \leq E_{\nu}^{reco.} < 1.25$ GeV	0.35
	$\bar{\nu}$ -mode 1Re NC	$0.35 \leq E_{\nu}^{reco.} < 0.80$ GeV	0.35
	$\bar{\nu}$ -mode 1Re NC	$0.80 \leq E_{\nu}^{reco.} < 1.25$ GeV	0.34
	ν -mode 1Re1de NC	$0.80 \leq E_{\nu}^{reco.} < 1.25$ GeV	0.34
	ν -mode $2R\pi^0$ NC π^0	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.22
	ν -mode $2R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.59
	ν -mode $2R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.59
	ν -mode $2R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.37
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.00 \leq P_{\pi^0}^{reco.} < 0.20$ GeV	-0.23
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.58
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.58
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.35
	ν -mode $1R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.36
	ν -mode $1R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.33
	ν -mode $1R\pi^0$ NC other	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.29
	$\bar{\nu}$ -mode $1R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.36
	$\bar{\nu}$ -mode $1R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.32
	$\bar{\nu}$ -mode $1R\pi^0$ NC other	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.22

Table 7.7: Parameters with postfit correlation absolute value ≥ 0.2 for π^0 normalisation parameter corresponding to $0.4 \leq P_{\pi^0} < 0.9$ GeV.

Type	Event	Region	Correlation
SK Det.	ν -mode $2R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.25
	ν -mode $2R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.25
	ν -mode $2R\pi^0$ CC	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.21
	ν -mode $2R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.34
	ν -mode $2R\pi^0$ NC other	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.32
	ν -mode $2R\pi^0$ NC other	$0.60 \leq P_{\pi^0}^{reco.} < 0.90$ GeV	-0.28
	ν -mode $2R\pi^0$ NC other	$P_{\pi^0}^{reco.} \geq 0.9$ GeV	-0.22
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.25
	$\bar{\nu}$ -mode $2R\pi^0$ NC π^0	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.25
	$\bar{\nu}$ -mode $2R\pi^0$ CC	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.22
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.35
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.31
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$0.60 \leq P_{\pi^0}^{reco.} < 0.90$ GeV	-0.28
	$\bar{\nu}$ -mode $2R\pi^0$ NC other	$P_{\pi^0}^{reco.} \geq 0.9$ GeV	-0.23
	ν -mode $1R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.21
	ν -mode $1R\pi^0$ NC other	$0.40 \leq P_{\pi^0}^{reco.} < 0.60$ GeV	-0.21
	$\bar{\nu}$ -mode $1R\pi^0$ NC other	$0.20 \leq P_{\pi^0}^{reco.} < 0.40$ GeV	-0.23

7.6.2 Fake Data Tests

The response of the π^0 normalisation parameters was tested using the datasets simulated with alternative central values for these parameters. The goal was to determine whether the parameters could be fit to their true values if they differ from the prior assumptions. This helps in understanding the potential behaviour of these parameters during a fit with real data.

Two separate simulated datasets were used for the testing. The first dataset applies the same adjustments to all parameters, while the second introduces more complexity in the central value distribution. The central values used for these tests are listed in Table 7.8. The best-fit values and postfit uncertainties for the π^0 normalisation parameter and correlated parameters are shown in Fig. 7.21.

In the case of "Fakedata A", parameters were pulled to values above one but

Table 7.8: The tweaked central values for the new $\text{NC}\pi^0$ normalisation parameters prior to the fit for generating fake datasets.

Parameter	I	II	III	IV
Fakedata A	1.2	1.2	1.2	1.2
Fakedata B	1.1	1.3	0.9	1.2

did not successfully reproduce the true value of 1.2. Potentially, variations in these parameters are absorbed by other parameters that affect the same events. This behaviour was particularly observed in FD Det 45, 60, 75, and 90 and their strongly correlated parameters in Fig. 7.21. These parameters correspond to Super-K detector uncertainties for events with $P_{\pi^0}^{reco}$ of 0.2-0.4 GeV, and thus reduce the changes to the π^0 normalisation parameter for events in the same region.

In contrast, the more complex setup in "Fakedata B" has the $2\text{R}\pi^0$ FD Det parameters remaining close to the prior central values, while the ones from $1\text{R}\pi^0$ events were pulled less significantly. However, the parameter of 0.2-0.4 GeV region is still not sensitive to the data. The parameters of 0-0.2 GeV and 0.4-0.9 GeV were fit more accurately to their true values, which could be due to the smaller correlation effects, and weaker constraints on the priors relative to their true values.

A fit was also performed using "Fakedata B" with the Super-K detector systematic parameters fixed to assess the impact of cross-section, flux, and oscillation parameters. This fit shows that the parameter of 0-0.2 GeV region was tweaked further from its true value, while the 0.2-0.4 GeV parameter remained unchanged, and the larger $P_{\pi^0}^{reco}$ parameters were tweaked accurately. This could be due to the true value for the 0.2-0.4 GeV region being too far from the prior, causing the fit to adjust other parameters instead to minimise the likelihood penalty.

Overall, the study suggests that the π^0 parameters are less likely to be accurately predicted if the prior error is underestimated or if the amount of normalisation needed across different regions is similar. The exact impact of real data on these parameters and possible improvements to the parameterisation require further investigation, but are beyond the scope of this analysis.

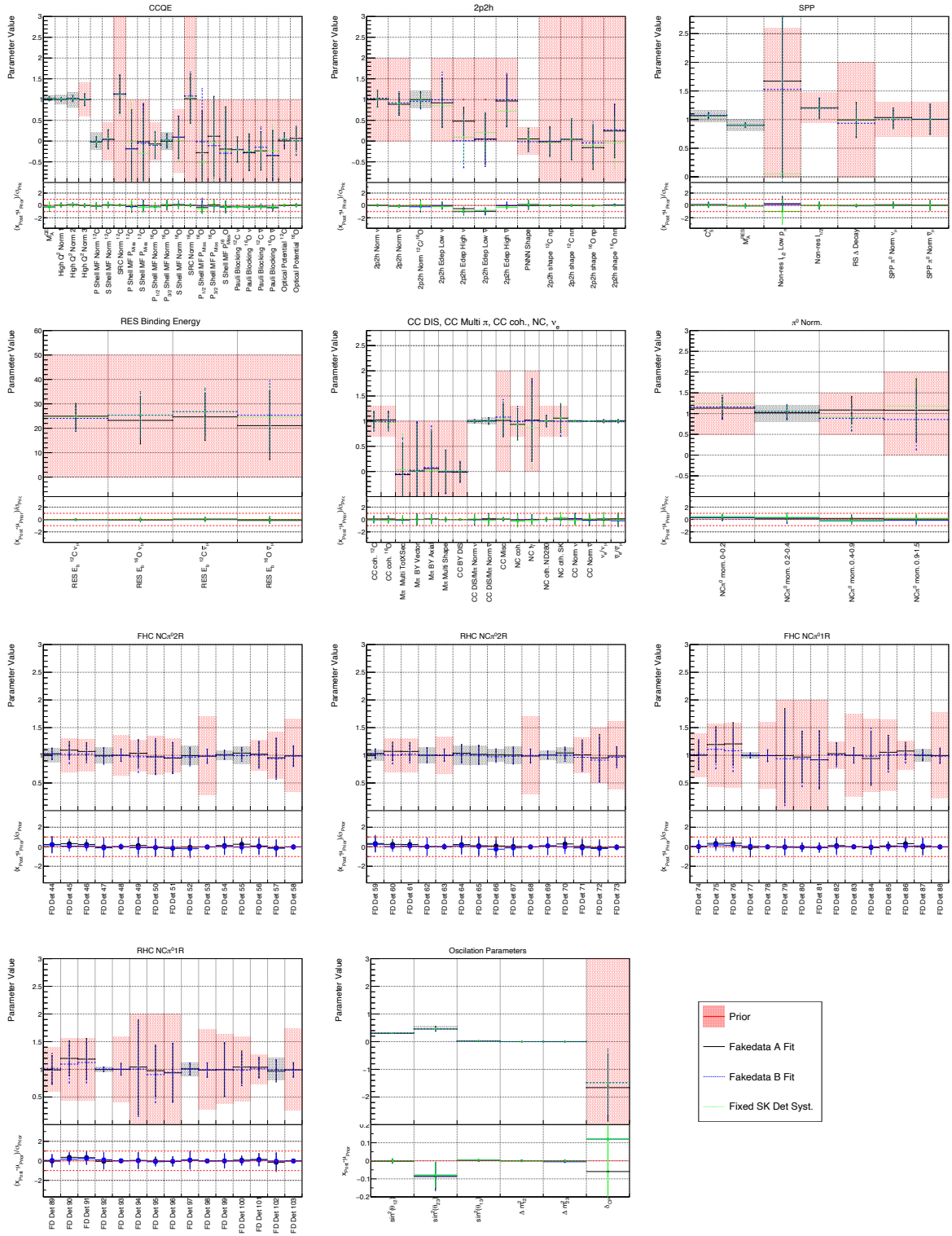


Figure 7.21: Results on the selected cross-section and Super-K detector systematic parameters from fake datasets.

7.7 Sensitivity to the Oscillation Model

The marginalised 1D posterior distributions of the oscillation parameters are shown in Fig. 7.22 to highlight the changes on specific oscillation parameters. Tables 7.9 and 7.10 summarise the changes in the 1σ credible intervals resulting from the inclusion of $\text{NC}\pi^0$ samples, for cases without and with reactor constraints, respectively. The most significant impacts are observed on the upper limit of δ_{CP} and the lower limit of $\sin^2 \theta_{13}$. The application of reactor constraints also slightly improves the ν_μ disappearance parameters under the normal ordering assumption.

Table 7.9: Oscillation parameters' highest posterior point (HPD) and 1σ credible intervals from 1D Asimov posterior distributions, without reactor constraint on $\sin^2 \theta_{13}$. Variations on the 1σ C.I. between the cases of 5 Super-K samples and inclusion of $\text{NC}\pi^0$ samples are indicated.

	5 SK Samples		Inc. $\text{NC}\pi^0$		
NO	HPD	1σ C.I.	HPD	1σ C.I.	Δ 1σ C.I.
δ_{cp}	-1.162	[-2.890, -0.314]	-1.602	[-2.890, -0.377]	-2.4%
$\sin^2 \theta_{23}$	0.472	[0.448, 0.568]	0.472	[0.448, 0.568]	-
$\sin^2 \theta_{13}$	0.0236	[0.0181, 0.0318]	0.0244	[0.0192, 0.0314]	-10.9%
$\Delta m_{32}^2 \times 10^3$	2.493	[2.430, 2.550]	2.493	[2.430, 2.550]	-
IO	HPD	1σ C.I.	HPD	1σ C.I.	Δ 1σ C.I.
δ_{cp}	-1.602	[-2.450, -0.628]	-1.477	[-2.450, -0.691]	-3.5%
$\sin^2 \theta_{23}$	0.464	[0.444, 0.564]	0.468	[0.444, 0.560]	-3.3%
$\sin^2 \theta_{13}$	0.0244	[0.0188, 0.0325]	0.0255	[0.0200, 0.0325]	-8.8%
$\Delta m_{32}^2 \times 10^3$	-2.541	[-2.598, -2.478]	-2.541	[-2.598, -2.484]	-5.0%
Both	HPD	1σ C.I.	HPD	1σ C.I.	Δ 1σ C.I.
δ_{cp}	-1.602	[-2.765, -0.440]	-1.602	[-2.702, -0.440]	-2.7%
$\sin^2 \theta_{23}$	0.464	[0.444, 0.568]	0.472	[0.448, 0.564]	-6.5%
$\sin^2 \theta_{13}$	0.0240	[0.0185, 0.0321]	0.0247	[0.0192, 0.0318]	-7.4%
$\Delta m_{32}^2 \times 10^3$	2.493	[-2.580, -2.502] $\cup$ [2.418, 2.568]	2.493	[-2.580, -2.502] $\cup$ [2.418, 2.562]	-2.6%

The fractions of the posterior distributions under the two mass orderings and θ_{23} octants are shown in Table 7.11. Overall, a preference for the normal ordering and upper octant is observed, with the inverted ordering and lower octant being the least

likely. The inclusion of $\text{NC}\pi^0$ samples and the application of reactor constraints can both stretch the preferences for these two cases further, while the other two cases are much less sensitive to these changes.

Table 7.10: Oscillation parameters' highest posterior distribution point (HPD) and 1σ credible intervals from 1D Asimov posterior distributions, with reactor constraint on $\sin^2 \theta_{13}$. Variations on the 1σ C.I. between the cases of 5 Super-K samples and inclusion of $\text{NC}\pi^0$ samples are indicated, where $\Delta 1\sigma \text{ C.I.} = 1\sigma \text{ C.I.}(\text{Inc. NC}\pi^0) - 1\sigma \text{ C.I.}(5 \text{ SK Samples})$.

	5 SK Samples		Inc. $\text{NC}\pi^0$		
NO	HPD	1σ C.I.	HPD	1σ C.I.	$\Delta 1\sigma \text{ C.I.}$
δ_{cp}	-1.162	[-2.827, -0.377]	-1.728	[-2.702, -0.440]	-7.7%
$\sin^2 \theta_{23}$	0.551	[0.466, 0.579]	0.554	[0.470, 0.583]	-
$\sin^2 \theta_{13}$	0.0220	[0.0213, 0.0227]	0.0220	[0.0213, 0.0228]	+7.1%
$\Delta m_{32}^2 \times 10^3$	2.493	[2.436, 2.556]	2.493	[2.436, 2.550]	-5.0%
IO	HPD	1σ C.I.	HPD	1σ C.I.	$\Delta 1\sigma \text{ C.I.}$
δ_{cp}	-1.539	[-2.450, -0.691]	-1.539	[-2.388, -0.754]	-7.1%
$\sin^2 \theta_{23}$	0.551	[0.467, 0.579]	0.551	[0.478, 0.583]	-6.2%
$\sin^2 \theta_{13}$	0.0221	[0.0213, 0.0228]	0.0220	[0.0213, 0.0228]	-
$\Delta m_{32}^2 \times 10^3$	-2.541	[-2.598, -2.478]	-2.547	[-2.598, -2.484]	-5.0%
Both	HPD	1σ C.I.	HPD	1σ C.I.	$\Delta 1\sigma \text{ C.I.}$
δ_{cp}	-1.728	[-2.702, -0.503]	-1.477	[-2.576, -0.503]	-5.7%
$\sin^2 \theta_{23}$	0.551	[0.467, 0.579]	0.554	[0.478, 0.583]	-6.2%
$\sin^2 \theta_{13}$	0.0220	[0.0213, 0.0227]	0.0220	[0.0213, 0.0228]	+7.1%
$\Delta m_{32}^2 \times 10^3$	2.493	[-2.574, -2.502] $\cup$ [2.418, 2.568]	2.493	[-2.574, -2.508] $\cup$ [2.418, 2.568]	-2.7%

Table 7.11: Comparison of the probabilities for the NO ($\Delta m_{32}^2 > 0$) and IO ($\Delta m_{32}^2 < 0$), as well as the lower (LO) and upper octant (UO) of θ_{23} from an Asimov fit.

	woRC						wRC					
	5 SK Samples			Inc. $\text{NC}\pi^0$			5 SK Samples			Inc. $\text{NC}\pi^0$		
	LO	UO	Sum	LO	UO	Sum	LO	UO	Sum	LO	UO	Sum
NO	0.296	0.341	0.637	0.230	0.434	0.664	0.241	0.402	0.643	0.230	0.434	0.664
IO	0.176	0.188	0.364	0.114	0.222	0.336	0.134	0.223	0.357	0.114	0.222	0.336
Sum	0.472	0.529		0.344	0.656		0.469	0.531		0.375	0.625	

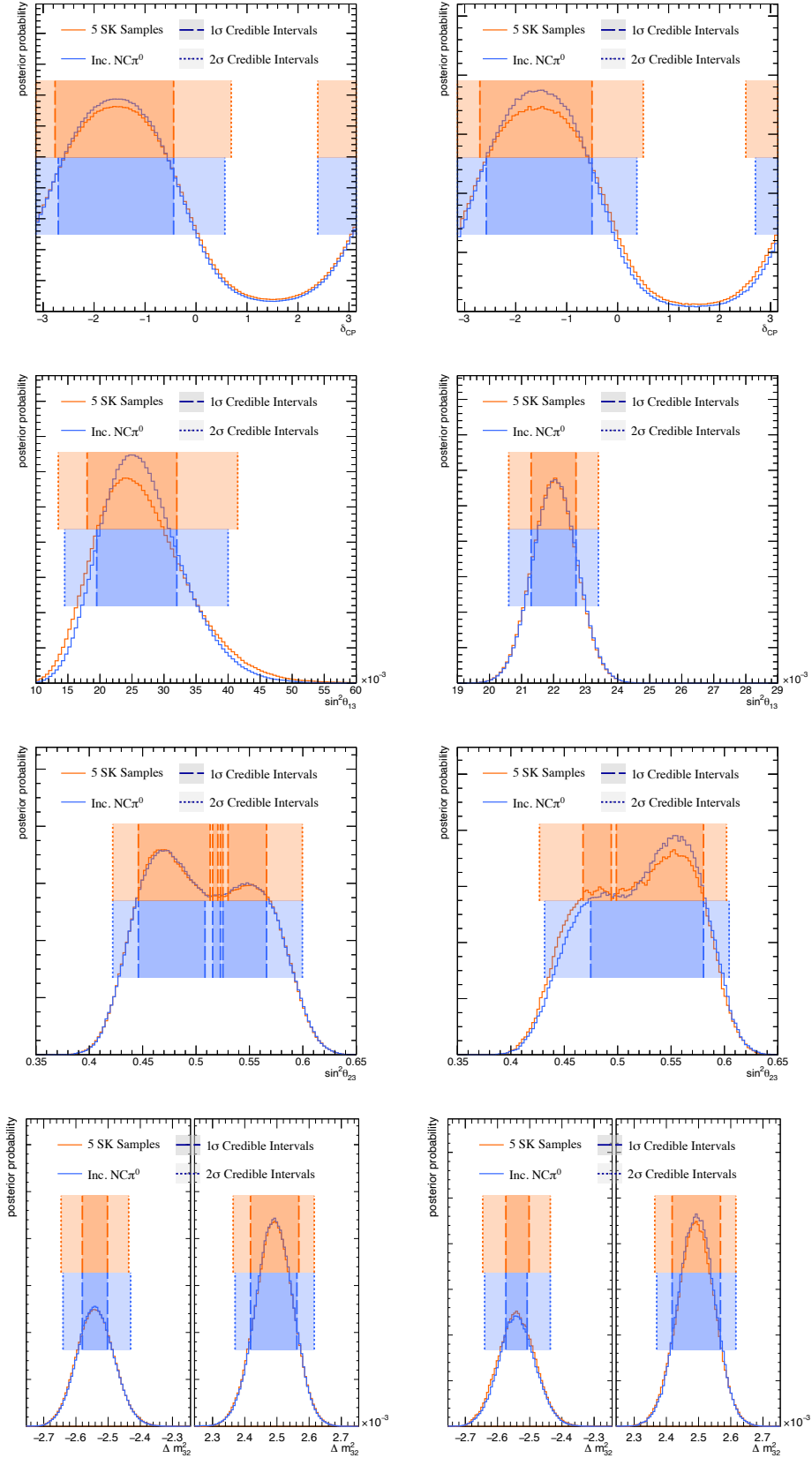


Figure 7.22: Comparisons of the 1D posterior distributions and credible intervals of the oscillation parameters from a full Asimov fit constrained by the 5 SK samples and the inclusion of $\text{NC}\pi^0$ samples. The left and right columns are the cases without and with the reactor constraints applied to θ_{13} .

Chapter 8

Results on Oscillation Parameters using T2K Runs 1-11 Data

This chapter transitions from the simulated data study and validation efforts discussed in Chapter 7 to the actual measurements of oscillation parameters using the full T2K Runs 1-11 dataset. The chapter builds on the methodologies previously established with the MaCh3 framework.

The results presented will include both analyses with and without the reactor constraints applied to $\sin^2 \theta_{13}$. By comparing these results, the chapter aims to illustrate the impact of the new $\text{NC}\pi^0$ samples and the updated systematic uncertainty model on the precision of the oscillation parameter measurements.

8.1 Analysis Strategy

The oscillation analysis procedure using the beam neutrino data collected by T2K follows the same methods used for simulated data fits. The details of the data and breakdowns of distributions into samples were introduced earlier in Section 7.2, alongside the corresponding MC predictions prior to the fit. A comparison of the postfit MC predictions with the observed data and an evaluation of the goodness of fit are provided before discussing the results on the oscillation parameters.

The postfit MC predictions are generated by randomly selecting a step in the MCMC chain, and calculating the event distributions for each sample using the values of the oscillation and nuisance parameters at that step. This process is repeated 5,000 times picking up parameter values from various MCMC steps. The event spectra are binned and have a mean and RMS calculated for each bin from the 5,000 spectra, to obtain the final predicted spectrum for each Super-K sample.

The goodness-of-fit is evaluated using a p-value [131], defined as the fraction of random datasets generated from the prediction that yield a $\chi^2_{\text{rnd data}}$ larger than the observed χ^2_{data} . The χ^2 values are computed from the negative log-likelihood ($-2 \ln \mathcal{L}$), comparing the prediction to the data and to random datasets statistically fluctuated using a Poisson distribution based on the predicted event counts [132]. This approach quantifies how much of the discrepancy between data and prediction is attributable to statistical fluctuations.

The MaCh3 posterior predictive p-value assesses how likely the observed data are under the post-fit model if the experiment were repeated [114]. Parameter values are sampled from the MCMC posterior distribution (after burn-in), predictions are generated for each draw, and statistical fluctuations are applied. A test statistic is calculated between the prediction and the data, as well as between the prediction and its own fluctuation. A draw is considered likely if the test statistic for the data is more favorable than for the fluctuation. The p-value is the fraction of such favorable draws across the 5,000 trials.

Fits were performed using all the Super-K samples, with comparisons made to the cases using only the 5 Super-K samples to assess the impact of the extra data from the $\text{NC}\pi^0$ samples. About 250 M post-burn-in steps were accumulated for each data fit. No additional step size tuning of the parameters was necessary, as the step sizes used in the simulated data fits were found to be sufficient.

8.2 Fits to T2K Data Only

8.2.1 Postfit Predictions

The postfit MC predictions for the 1Re , $2\text{R}\pi^0$, and $1\text{R}\pi^0$ samples are displayed in Fig. 8.1. The predictions for other Super-K samples, which are not significantly impacted by this analysis, can be found in Appendix C.6.1. It shows that the MC predictions and data are in agreement for the majority of bins.

For the 1Re events, potential discrepancies are observed only in bins with low data statistics, which are likely due to statistical fluctuations as the χ^2 test shows good agreement. In the $2\text{R}\pi^0$ samples, discrepancies between the MC predictions and data are observed in the final bins, which contain a small fraction of $\text{NC}\pi^0$ events. The opposing variations required between the ν -mode and $\bar{\nu}$ -mode constrain the flexibility of the MC to match both datasets, limiting its ability to fully account for the discrepancies. This limitation also affects the final bins of the $1\text{R}\pi^0$ samples, where an under-prediction persists in the ν -mode.

Additionally, the variations allowed by the π^0 normalisation parameters do not fully resolve to have shape matched between data and MC for the ν -mode $1\text{R}\pi^0$ sample, possibly because the $\bar{\nu}$ -mode $1\text{R}\pi^0$ sample shows good agreement with the data in this region. This suggests that the observed effects are likely statistical in nature rather than systematic, as supported by the goodness-of-fit χ^2 test.

The p-value calculation was conducted to evaluate the compatibility between the best-fit MC prediction and the observed data. Figure 8.2 shows the χ^2 distribu-

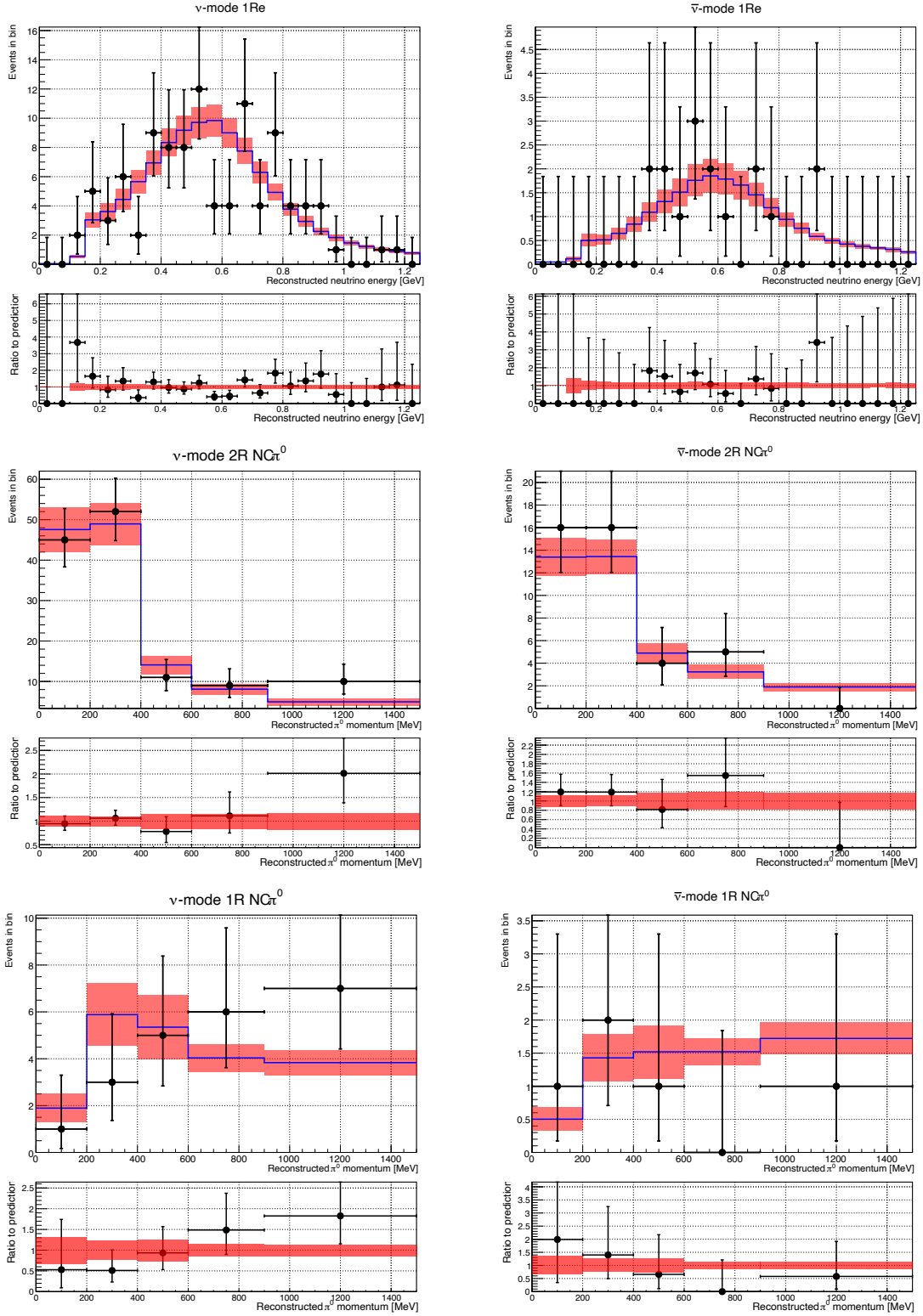


Figure 8.1: The postfit MC predictions for the 1Re (top), 2R π^0 (middle), and 1R π^0 (bottom) samples are shown for both the ν -mode (left) and $\bar{\nu}$ -mode (right), fit to the T2K Runs 1-11 data without reactor constraints applied to $\sin^2 \theta_{13}$. The mean predictions (blue line) and the 1 σ variations (red band) are derived from 5,000 MC distributions throughout the MCMC chain. The data is represented by black points, with statistical errors estimated using the Gamma quantile method [133].

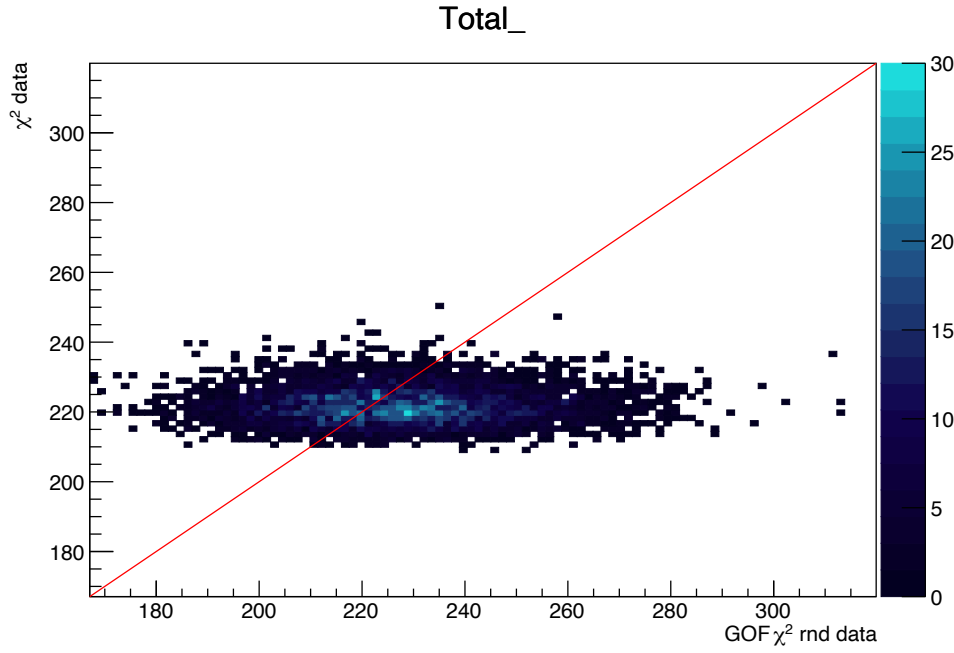


Figure 8.2: The χ^2 distribution of 5,000 MC predictions compared to data and random fake data, combining all the samples, without reactor constraints applied to $\sin^2 \theta_{13}$. The z-axis represents number of throws per bin. The p-value is calculated from the fraction of events under the $\chi^2_{\text{rnd data}} = \chi^2_{\text{data}}$ line.

Table 8.1: Breakdown of goodness-of-fit p-values from shape-based and total rate-based χ^2 calculations when fitting to T2K data without reactor constraints applied to $\sin^2 \theta_{13}$, averaged over 10 runs to account for statistical fluctuations in fake data throws. The results are shown for cases with and without the π^0 normalisation uncertainties applied to the 1Re samples.

Sample	π^0 Norm. to 1Re		π^0 Norm. Excl. 1Re	
	Shape	Total Rate	Shape	Total Rate
ν -mode 1R μ	0.528 (7)	0.451 (6)	0.527 (5)	0.442 (5)
$\bar{\nu}$ -mode 1R μ	0.824 (6)	0.542 (5)	0.822 (4)	0.540 (7)
ν -mode 1Re	0.161 (6)	0.497 (4)	0.157 (6)	0.501 (6)
$\bar{\nu}$ -mode 1Re	0.645 (8)	0.334 (6)	0.653 (6)	0.349 (5)
ν -mode 1Re1de	0.578 (5)	0.122 (5)	0.574 (5)	0.115 (5)
ν -mode 2R NC π^0	0.278 (6)	0.530 (8)	0.276 (6)	0.528 (6)
$\bar{\nu}$ -mode 2R NC π^0	0.318 (3)	0.481 (7)	0.302 (5)	0.480 (6)
ν -mode 1R NC π^0	0.377 (6)	0.574 (8)	0.400 (4)	0.579 (8)
$\bar{\nu}$ -mode 1R NC π^0	0.608 (6)	0.460 (4)	0.589 (8)	0.451 (4)
Total	0.606 (8)	0.417 (6)	0.599 (7)	0.414 (6)

tions, where the likelihood is calculated as the sum of differences across each bin in the sample spectra. This approach yields a shape-based total p-value of 60.6%, indicating a very good fit between the predicted and observed distributions in terms of their shapes. Additionally, a rate-based p-value was calculated by comparing the total number of events in each sample, resulting in a total p-value of 41.7%, which assesses the agreement based on the overall event rates.

The p-values were calculated individually for each sample, with statistical fluctuations included for the final decimal, as shown in Table 8.1. The numbers are generally showing agreement between the MC prediction and the data. The ν -mode 1Re sample has the lowest shape-based p-value among all the samples. To assess whether this outcome is influenced by the application of π^0 normalisation uncertainties to the 1Re samples, an additional fit was performed without applying these uncertainties. The p-values from both fits yielded the same values under the statistical fluctuations. Given the very small number of $\text{NC}\pi^0$ events in the 1Re samples (see Tables C.12 and C.13 in the Appendix), the variations in χ^2 values due to the π^0 normalisation uncertainties are expected to be less than 0.1, implying negligible changes in the p-values. Therefore, applying the π^0 normalisation uncertainties to the 1Re events does not introduce any incompatibilities in the fit.

8.2.2 Posterior Credible Intervals

The best-fit and the 1-3 σ posterior credible intervals of oscillation parameters from the marginalised 2D posterior distributions, fit over all the Super-K samples including the $\text{NC}\pi^0$ samples, are shown in Fig. 8.3. The results indicate only slight correlations between δ_{CP} and $\sin^2 \theta_{13}$ for both mass ordering cases. This is likely due to the large Super-K detector uncertainties and correlations with $\sin^2 \theta_{13}$, which tend to absorb some event variations – a situation also observed in the Asimov fit case.

The best-fit value of $\sin^2 \theta_{13}$ is found to be slightly higher than the reactor central value of 0.022, while $\sin^2 \theta_{23}$ is observed to lie in the lower octant, consistent with the Asimov fit results. Moreover, the δ_{CP} values exhibit a slight preference for different

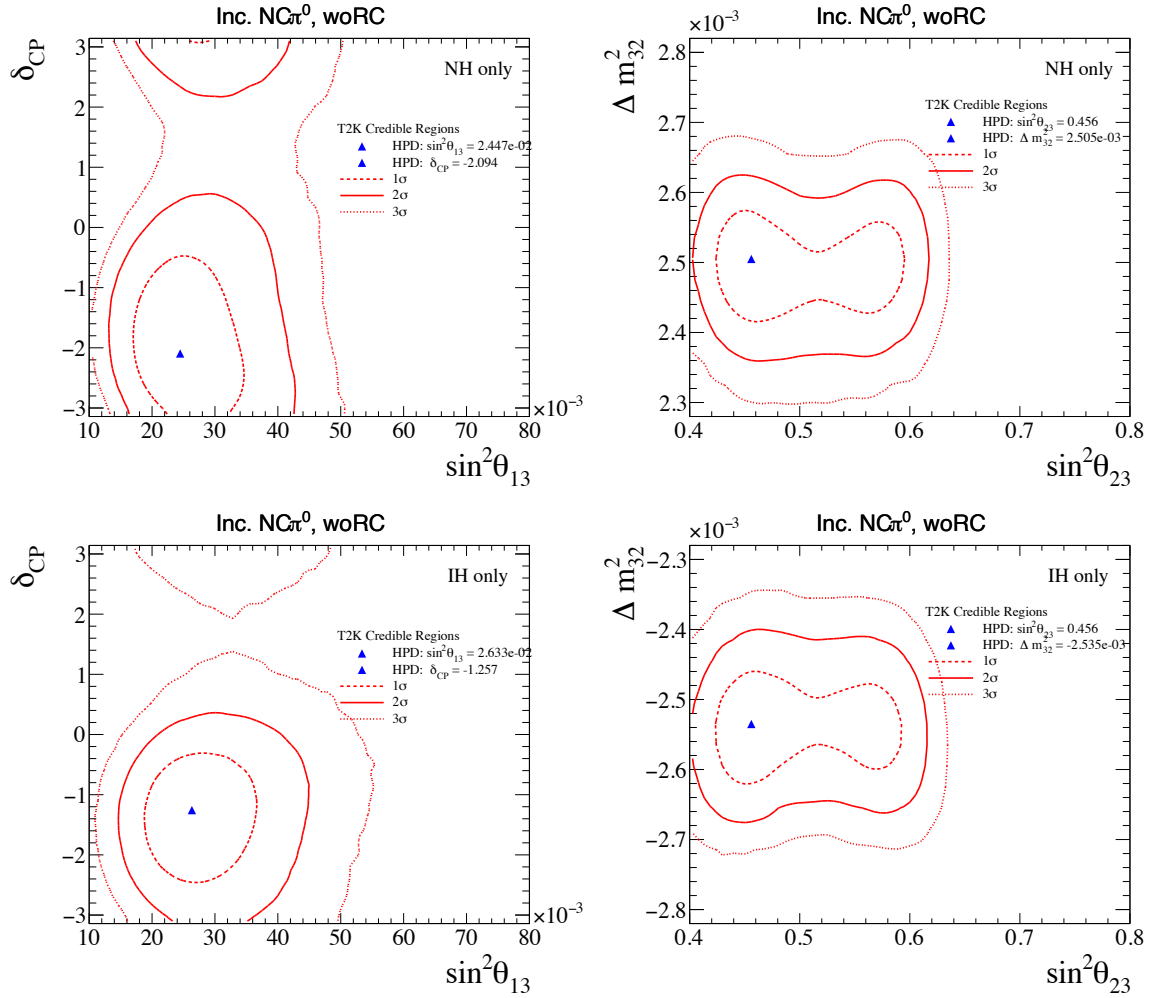


Figure 8.3: The 1-3 σ 2D credible intervals of the ν_e appearance and ν_μ disappearance parameters fit using the T2K Runs 1-11 data under the normal (top) and inverted (bottom) mass ordering over all the Super-K samples, without reactor constraints applied to $\sin^2 \theta_{13}$. The HPD value is indicated for each parameter.

regions depending on the mass ordering assumptions, with a tendency to favour regions symmetrically below and above the maximal CP -violation point ($-\frac{\pi}{2}$) for normal and inverted mass orderings, respectively.

8.2.3 Impact from the $NC\pi^0$ Samples

Fig. 8.4 highlights the impact of adding the $NC\pi^0$ samples on the uncertainties of the oscillation parameters. The inclusion of these samples better constrains the Super-K detector uncertainties, leading to improved measurements of $\sin^2 \theta_{13}$. The correlation between $\sin^2 \theta_{13}$ and δ_{CP} is particularly enhanced in the normal ordering

case, results in a slightly stretched contour along δ_{CP} . Due to the weak correlation of δ_{CP} with other parameters (see Appendix C.5.2), the impact of the $\text{NC}\pi^0$ samples to the δ_{CP} is small, consistent with observations from the Asimov fits.

The shape of the ν_μ disappearance contours shown in Fig. 8.4, which exhibit smaller correlations with other parameters, is similar to the Asimov fit results without systematic uncertainties (see Fig. C.11 with separated mass orderings). The posterior probability is driven further away from the $\sin^2 \theta_{13} = 0.511$ point after including the $\text{NC}\pi^0$ samples, with the best-fit point shifting slightly to the left in both mass orderings.

8.3 Fits to T2K Data with Reactor Constraint

8.3.1 Postfit Predictions

When applying the reactor constraints on $\sin^2 \theta_{13}$, the MC predictions show reduced error bands for the $1Re$ and $1R\pi^0$ samples. A slightly smaller number of predicted events is observed around the oscillation maximum, which is expected since the reactor central value of $\sin^2 \theta_{13}$ is about 10% lower than the best-fit value observed from the T2K data. Despite these adjustments, the MC predictions remain in agreement with the data, at a level compatible to the case without the reactor constraints applied.

The changes in p-values are not significant and covered by the statistical fluctuations. This outcome is expected, given that the oscillated ν_e events can vary by approximately 10% with the 10% change in $\sin^2 \theta_{13}$. The p-values and χ^2 distributions calculated with the reactor constraints can be found in the Appendix C.6.1.

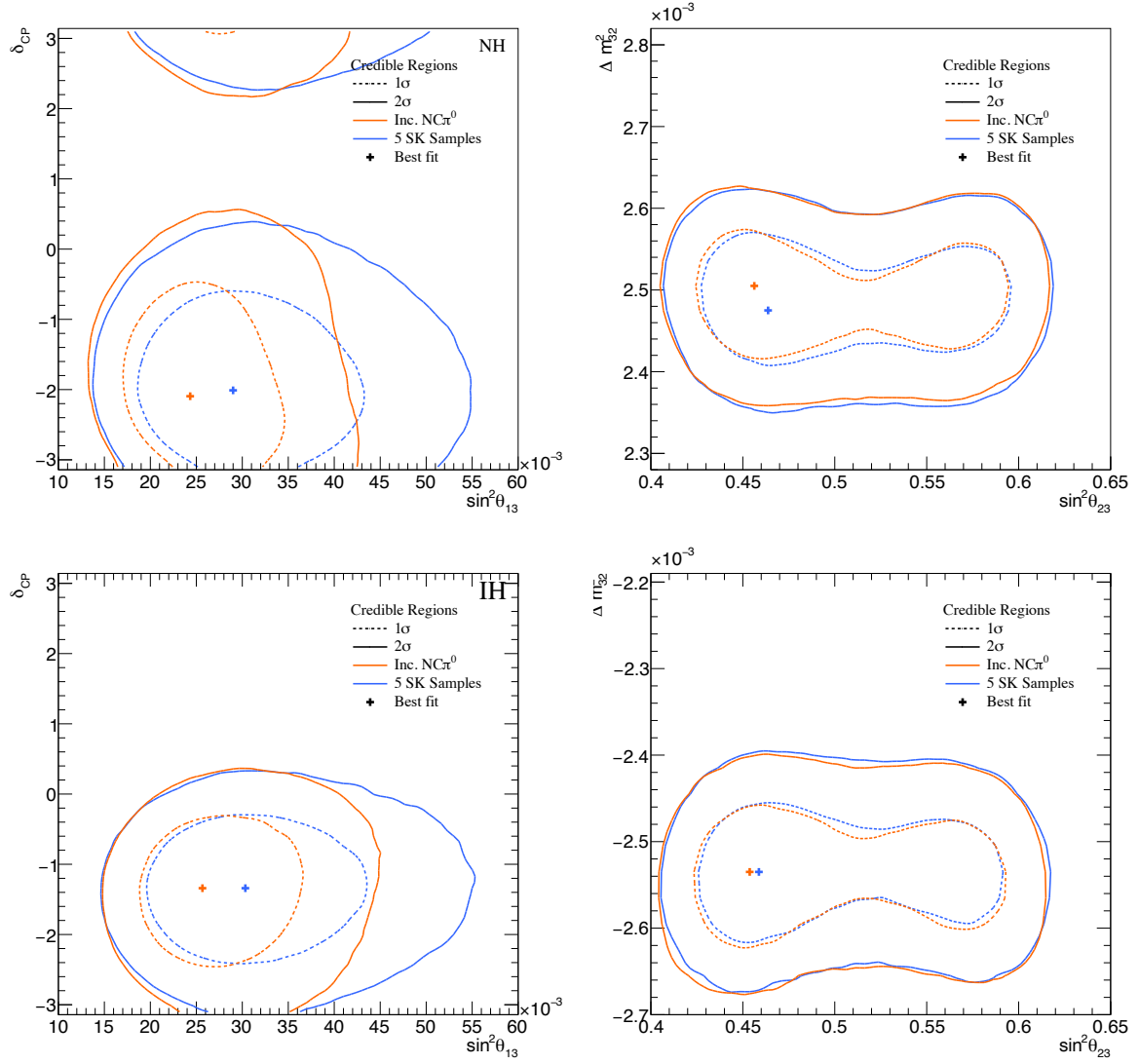


Figure 8.4: Comparisons of the 2D credible intervals of the ν_e appearance and ν_μ disappearance parameters fit using the T2K Runs 1-11 data under the normal (top) and inverted (bottom) mass ordering, over the 5 Super-K samples and the case including the $NC\pi^0$ samples, without reactor constraints applied to $\sin^2 \theta_{13}$.

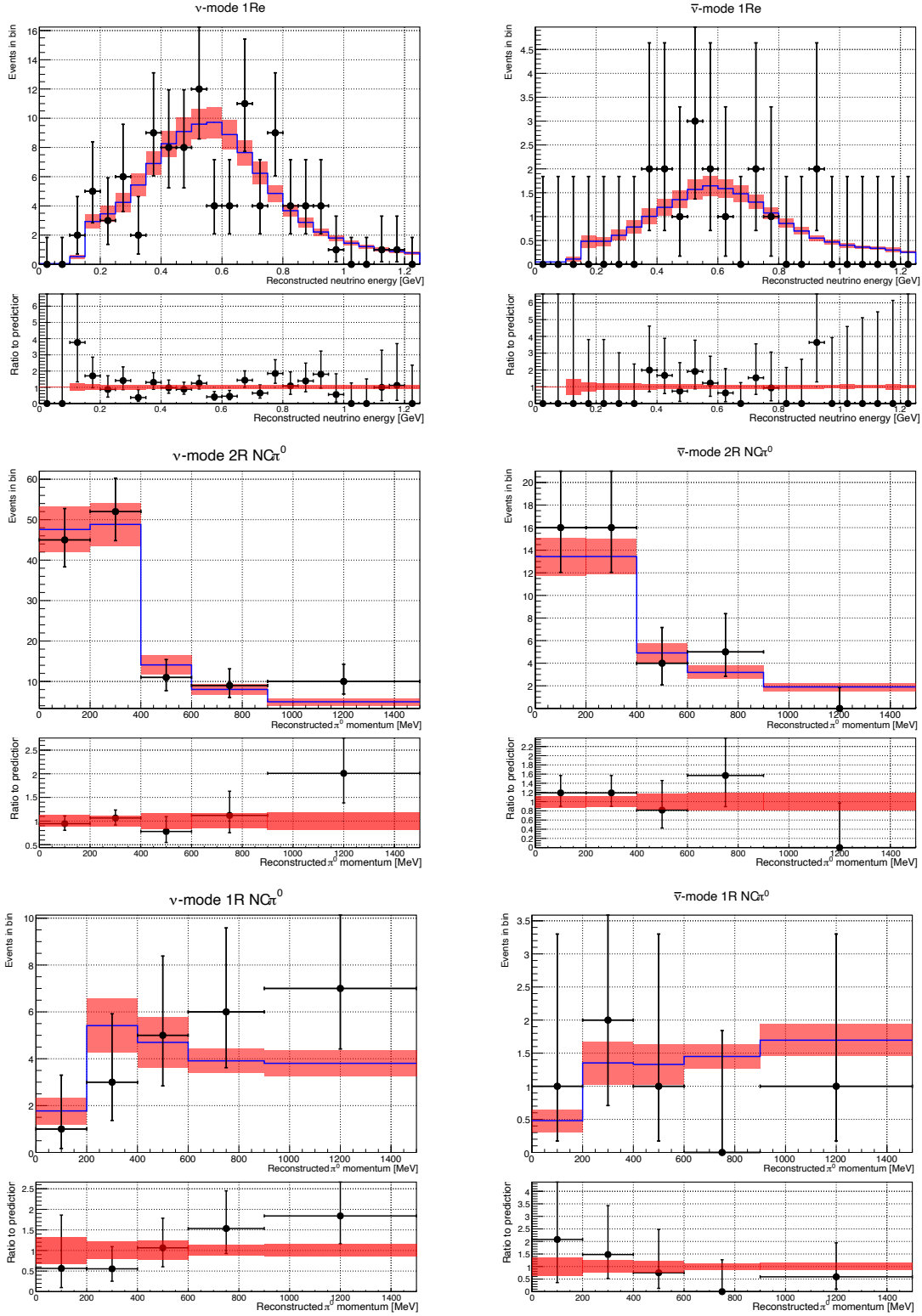


Figure 8.5: The postfit MC predictions for the 1Re (top), 2R π^0 (middle), and 1R π^0 (bottom) samples are shown for both the ν -mode (left) and $\bar{\nu}$ -mode (right), fit to the T2K Runs 1-11 data with reactor constraints applied to $\sin^2 \theta_{13}$. The mean predictions (blue line) and the 1σ variations (red band) are derived from 5,000 MC distributions throughout the MCMC chain. The data is represented by black points, with statistical errors estimated using the Gamma quantile method [133].

8.3.2 Posterior Credible Intervals

Impact from the Reactor Constraints

Fig. 8.6 shows effects on the oscillation parameters from applying the reactor constraints. $\sin^2 \theta_{13}$ is tightly constrained to align with the reactor central value, with a best-fit value of $0.0221^{+0.0007(0.0006)}_{-0.0008}$ for the normal (inverted) mass ordering from the marginalised 2D posterior distribution.

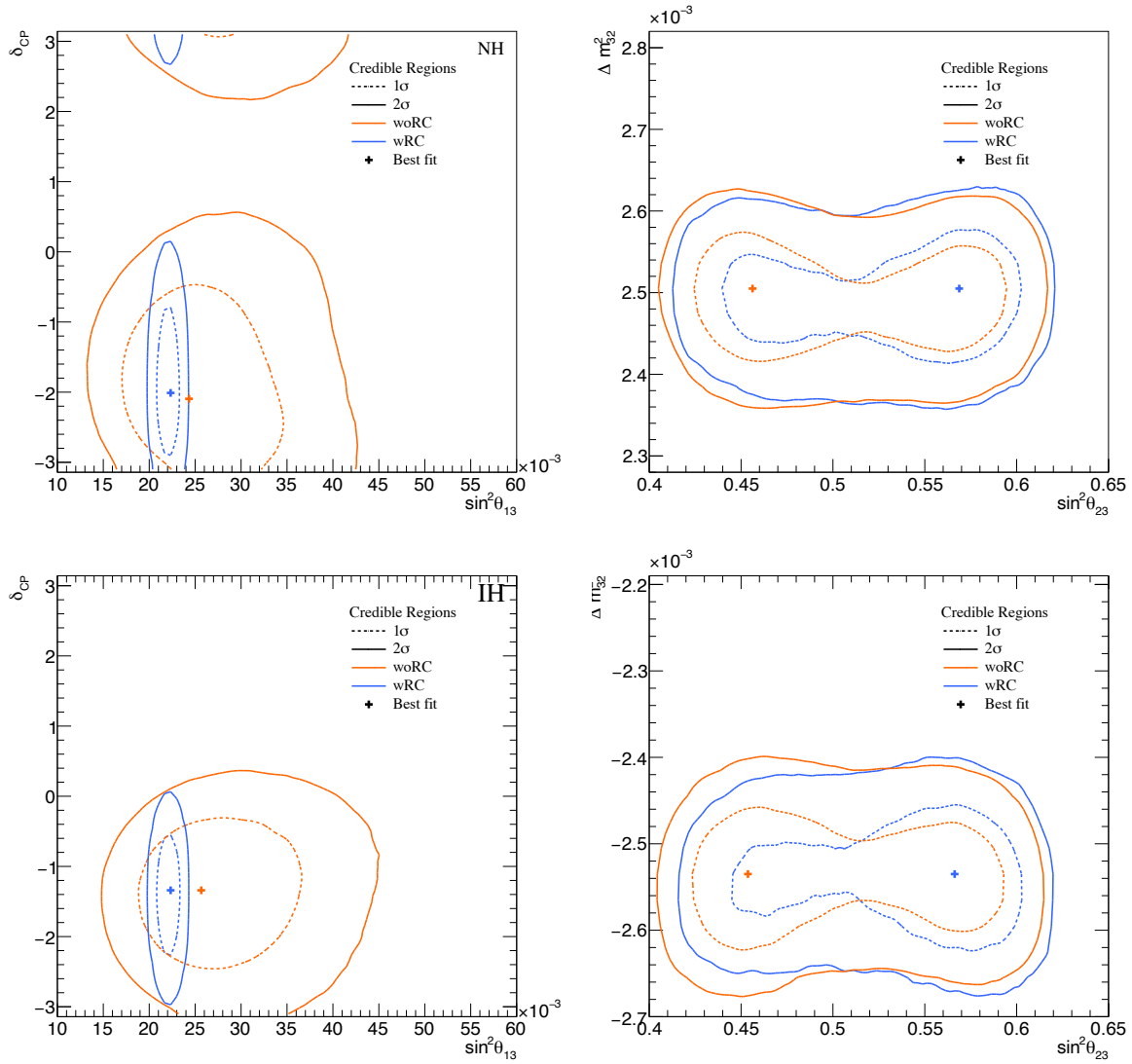


Figure 8.6: Comparisons of the 2D credible intervals of the ν_e appearance and ν_μ disappearance parameters fit using T2K Runs 1-11 data under the normal (top) and inverted (bottom) mass ordering, over all Super-K samples, with and without reactor constraints applied to $\sin^2 \theta_{13}$.

There is a small reduction in the uncertainties of δ_{CP} , with the extent of this reduction depending on the level of correlation with $\sin^2 \theta_{13}$, as indicated by the different variations observed from the normal and inverted mass ordering. No significant changes are observed in the best-fit value of δ_{CP} .

The best fit of $\sin^2 \theta_{23}$ is flipped about 0.511 without observable changes across Δm_{32}^2 . The upper octant preference is enhanced with the upper octant credible interval of size similar to the lower octant before applying the reactor constraints.

Impact from the Data and New Samples

The impact on oscillation parameters from the $\text{NC}\pi^0$ samples is shown in Fig. 8.7, which is comparable to the observations from the Asimov fit (see Fig. 7.15). In the data fit, a slight expansion of the δ_{CP} uncertainties is observed at the upper edge of the 2D credible intervals for the normal mass ordering from including $\text{NC}\pi^0$ samples, while no significant changes are seen in the inverted mass ordering case. This behaviour aligns with the stretched δ_{CP} contours observed when reactor constraints are not applied (see Fig. 8.4).

The application of reactor constraints also appears to have eliminated the correlations between δ_{CP} and $\sin^2 \theta_{13}$. Given these weak correlations over the entire parameter space, the small variations in the contours with and without the $\text{NC}\pi^0$ samples are indeed expected, particularly in light of the small effects observed in the Asimov fits without systematic uncertainties. Additionally, statistical fluctuations from the limited data may further obscure any noticeable new impact. Despite these minor variations, the tightening of the contour results in the δ_{CP} best fit being varied oppositely when the $\text{NC}\pi^0$ samples are included, under the two mass orderings.

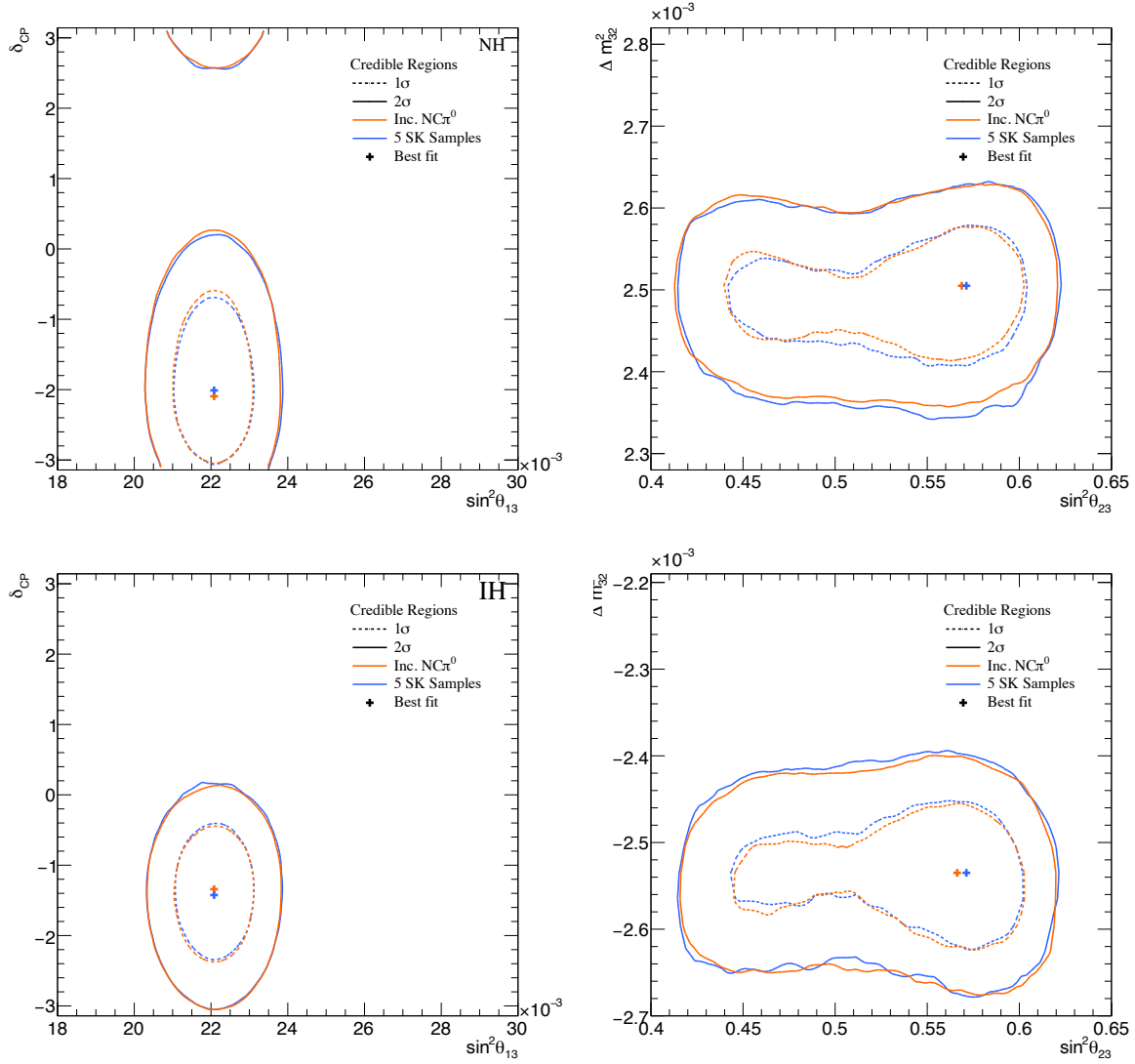


Figure 8.7: Comparisons of the 2D credible intervals of the ν_e appearance and ν_μ disappearance parameters fit using the T2K Runs 1-11 data under the normal (top) and inverted (bottom) mass ordering, over the 5 Super-K samples and the case including the $\text{NC}\pi^0$ samples, with reactor constraints applied to $\sin^2 \theta_{13}$.

8.4 Systematic Parameters Postfit

The impact of the data and $\text{NC}\pi^0$ samples on constraining the systematic parameters is illustrated in Figs. 8.8-8.10 for the selected parameters. The complete list is available in Fig. C.17 in the Appendix. The cases with and without the $\text{NC}\pi^0$ samples are compared to assess the impact from these samples and ensure they do not introduce unexpected biases in the parameter estimations. Results from the Asimov

fit with the $\text{NC}\pi^0$ samples are also shown as a reference for the expected size of the postfit uncertainties.

Fig. 8.8 highlights the cross-section parameters for neutral pion production (as introduced in Section 6.2). Most of these parameters are pulled from their prior central values. The deviations are overall consistent with and without the $\text{NC}\pi^0$ samples. This consistency suggests that parameters governing pion production are primarily constrained by charged current pion events measured at ND280. The impact from $\text{NC}\pi^0$ events generally falls within expectations, given the variations seen in the likelihood (LLH) scans of these parameters (see Fig. C.9 and Fig. 7.5).

Fig. 8.9 shows that parameter variations on flux uncertainties are systematically higher at low energies and lower at high energies. This likely reflects systematic uncertainties exist in flux and interaction modelling. These biases may stem from uncertainties in hadron interactions or insufficient constraints from beamline measurements (referred from Fig. 4.3). Nuclear effects, such as Fermi motion or final-state interactions, could further amplify discrepancies by altering the initial and final-state kinematics. At high energies, mismodeling of resonance production, DIS, or the transition between these regimes may contribute. A detailed investigation is needed to quantify these contributions.

The neutral current pion production was tuned using the new π^0 normalisation parameters according to the $\text{NC}\pi^0$ data. These parameters exhibit sensitivities to the $\text{NC}\pi^0$ samples, with results indicating an underestimation at lower P_{π^0} and overestimation for P_{π^0} above 0.4 GeV. This pattern is consistent with observations from MicroBooNE, where data were compared with NEUT predictions, as shown in Fig. 1.13. However, the size the predictions deviated from data is different from the MicroBooNE case, which may be attributed to strong correlations with the Super-K detector systematics, as indicated in Fig. 7.20.

Fits from data normalise the flux predictions by a similar amount, as shown in Fig. 8.9. The $2\text{R}\pi^0$ samples at Super-K, which do not distinguish between oscillated neutrino flavours, are expected to provide constraints on the unoscillated flux pre-

dictions. Additional effects on the flux parameters have been observed when the $\text{NC}\pi^0$ samples are included, despite the much smaller number of events compared to the ND280 samples.

The Super-K detector systematics are overall better constrained by the inclusion of the $\text{NC}\pi^0$ samples, as shown in Fig. 8.10. There is less impact observed for the $1\text{R}\mu$ samples, which is expected since these events are less correlated with those from the $\text{NC}\pi^0$ samples. The energy scale is driven consistently by the data with and without the $\text{NC}\pi^0$ samples.

The parameters that control lower momentum $\text{NC}\pi^0$ events from the $2\text{R}\pi^0$ samples do not show significant deviations from their nominal values (FD Det 45, 46, 60, 61), indicating that neutral pion production is more effectively constrained by the new π^0 normalisation parameters rather than being absorbed into these parameters.

Notably, despite the constraints provided by the $2\text{R}\pi^0$ samples, $\text{NC}\pi^0$ events in the $1\text{R}\pi^0$ samples are pulled lower than the nominal values (FD Det 75, 76, 90, 91). In contrast, the CC events (FD Det 79-81, 94-96) are pulled higher. Alternatively, in 1Re samples, while ν_e events are pulled lower (FD Det 0-2, 6-8, 18-20, 24-26), the NC events are higher (FD Det 9-11, 27-29). This discrepancy could potentially be attributed to lower e/π^0 PID resolutions in the data compared to the MC when separating the 1Re and $1\text{R}\pi^0$ samples. Events near the cut may not be consistently selected between the MC and data, leading to this deficit (see Fig. B.11 in the Appendix for the data and MC distributions close to the cut). Additionally, mis-modeling in the detector response simulation and reconstruction algorithms could potentially cause an excess of $\text{NC}\pi^0$ events and a simultaneous deficit of ν_e signal events.

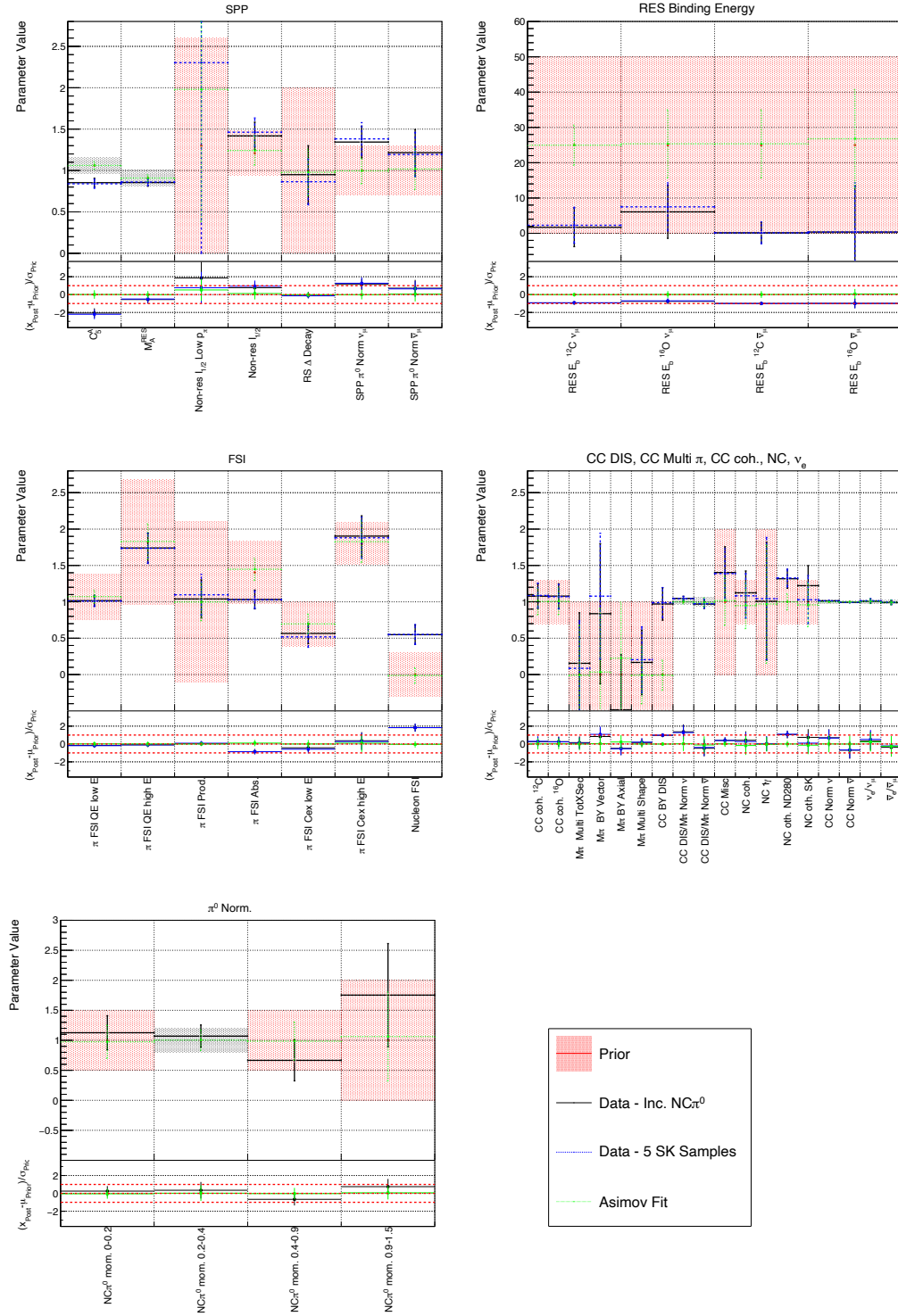


Figure 8.8: Results on the pion production related cross-section systematic parameters, comparing with and without $\text{NC}\pi^0$ samples using T2K Runs 1-11 data, and the Asimov fit case over all the Super-K samples.

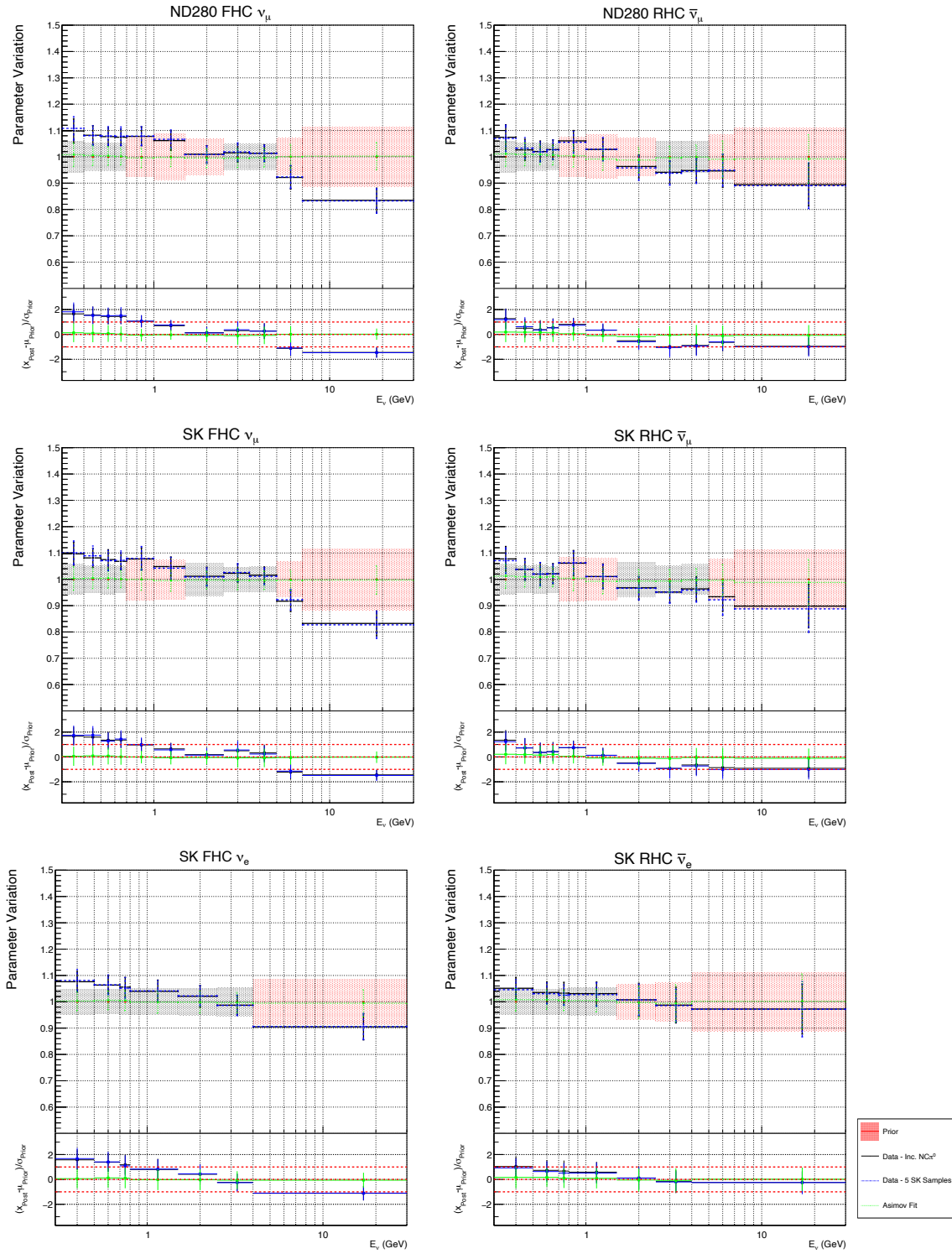


Figure 8.9: Results on the selected flux systematic parameters, comparing with and without $\text{NC}\pi^0$ samples using T2K Runs 1-11 data, and the Asimov fit case over all the Super-K samples.

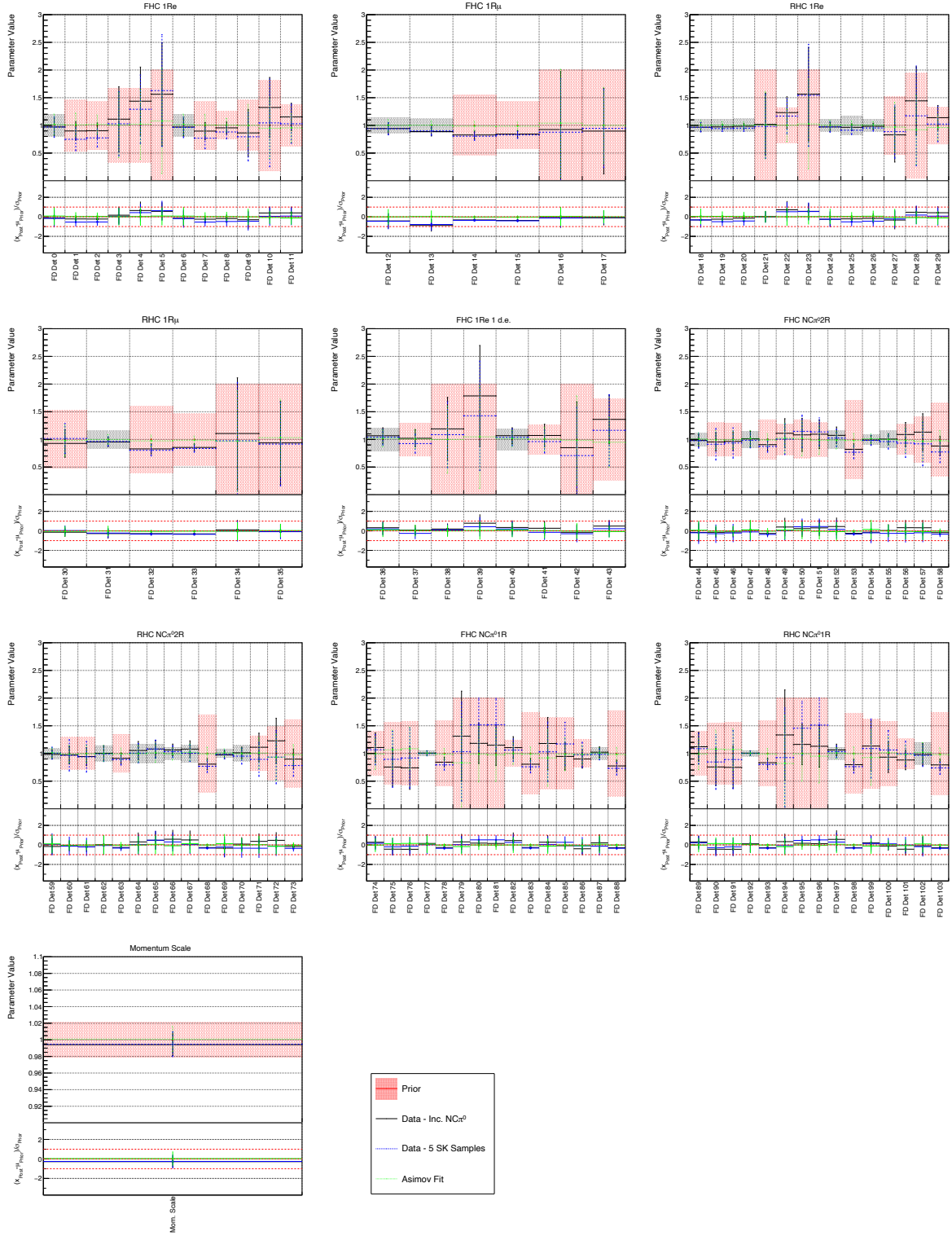


Figure 8.10: Results on the Super-K detector systematic parameters, comparing with and without $\text{NC}\pi^0$ samples using T2K Runs 1-11 data, and the Asimov fit case over all the Super-K samples.

8.5 Summary of the Results

Overall, the analysis demonstrates a good match between the MC predictions and the data with the inclusion of new $\text{NC}\pi^0$ selections using the updated systematic uncertainty model. The inclusion of the $\text{NC}\pi^0$ samples has significantly constrained the Super-K detector uncertainties and reduced the uncertainties on the strongly correlated oscillation parameter $\sin^2 \theta_{13}$. The large Super-K detector uncertainties also absorb variations in the events and diminish the correlations between oscillation parameters, particularly between $\sin^2 \theta_{13}$ and δ_{CP} . As a result, δ_{CP} becomes less influenced by variations in $\sin^2 \theta_{13}$. In contrast, $\sin^2 \theta_{23}$ remains sensitive to the variations in $\sin^2 \theta_{13}$, shifting the preference from lower octant to upper octant when reactor constraints are applied to $\sin^2 \theta_{23}$.

The results on oscillation parameters extracted from 1D posterior distributions of individual parameters are summarised in Table 8.2. The comparisons of 1D credible intervals with and without the reactor constraints are illustrated in Figs. 8.12 (5 SK samples) and 8.13 (Inc. $\text{NC}\pi^0$ samples). The results show a preference of δ_{CP} values that violates the CP symmetry, with CP -conserving values excluded at nearly the 2σ level when the reactor constraints are applied. The combined mass ordering scenario for δ_{CP} indicates a value approaching the maximal CP -violation point of $-\pi/2$. Additionally, Fig. 8.11 shows that values close to $\sin \delta_{CP} = 1$ are excluded at the 99% credible level, with CP -conservation ($\sin \delta_{CP} = 0$) excluded from the 90% credible interval.

Although $\sin^2 \theta_{23}$ appears to be more sensitive to $\sin^2 \theta_{13}$ than δ_{CP} , as observed from the 2D credible intervals, this is not reflected in the 1D credible interval variations due to the complexities of octant degeneracies. In this analysis, the lower octant is not completely ruled out of the 1σ and 2σ regions under the reactor constraints, despite whether the $\text{NC}\pi^0$ samples are included, as shown in Figs. 8.12 and 8.13.

Table 8.2: Oscillation parameters' highest posterior point (HPD) and 1σ credible intervals from 1D data fit posterior distributions over all Super-K samples, with and without reactor constraint on $\sin^2 \theta_{13}$. Variations on the 1σ C.I. from applying the reactor constraint are indicated, where $\Delta 1\sigma \text{ C.I.} = 1\sigma \text{ C.I. (Inc. NC}\pi^0) - 1\sigma \text{ C.I. (5 SK Samples)}$.

	woRC		wRC		$\Delta 1\sigma \text{ C.I.}$
NO	HPD	$1\sigma \text{ C.I.}$	HPD	$1\sigma \text{ C.I.}$	
δ_{cp}	-2.105	[-2.890, -1.005]	-2.042	[-2.702, -1.068]	-13.3%
$\sin^2 \theta_{23}$	0.453	[0.429, 0.586]	0.566	[0.448, 0.598]	-4.8%
$\sin^2 \theta_{13}$	0.0251	[0.0196, 0.0314]	0.0221	[0.0213, 0.0228]	-87.3%
$\Delta m_{32}^2 \times 10^3$	2.487	[2.442, 2.544]	2.487	[2.442, 2.544]	-
IO	HPD	$1\sigma \text{ C.I.}$	HPD	$1\sigma \text{ C.I.}$	
δ_{cp}	-1.288	[-2.073, -0.628]	-1.414	[-2.073, -0.754]	-8.7%
$\sin^2 \theta_{23}$	0.453	[0.429, 0.583]	0.566	[0.459, 0.601]	-7.3%
$\sin^2 \theta_{13}$	0.0266	[0.0215, 0.0337]	0.0222	[0.0214, 0.0228]	-88.5%
$\Delta m_{32}^2 \times 10^3$	-2.535	[-2.592, -2.490]	-2.553	[-2.592, -2.490]	-
Both	HPD	$1\sigma \text{ C.I.}$	HPD	$1\sigma \text{ C.I.}$	
δ_{cp}	-1.791	[-2.639, -0.817]	-1.791	[-2.513, -0.880]	-10.3%
$\sin^2 \theta_{23}$	0.453	[0.429, 0.586]	0.566	[0.448, 0.598]	-4.8%
$\sin^2 \theta_{13}$	0.0251	[0.0204, 0.0321]	0.0221	[0.0214, 0.0228]	-88.0%
$\Delta m_{32}^2 \times 10^3$	2.487	[2.424, 2.562] $\cup$ [-2.562, -2.514]	2.487	[2.418, 2.568] $\cup$ [-2.556, -2.532]	-50% (NO), +8.7% (IO)

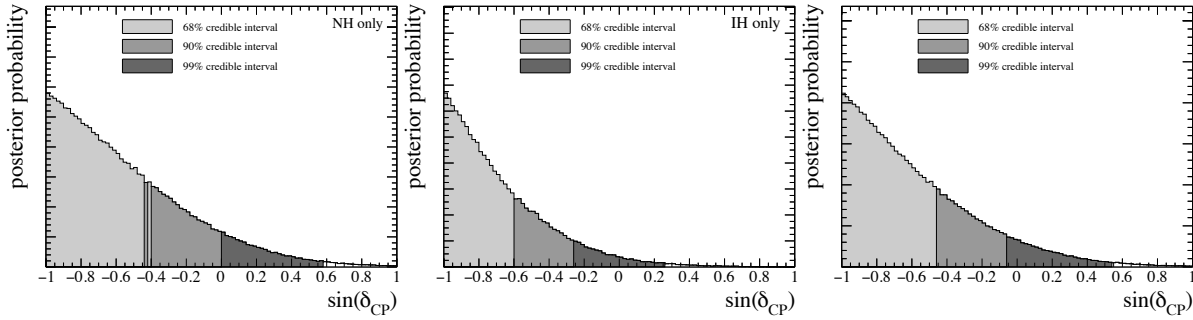


Figure 8.11: The 1D posterior distributions and credible intervals over $\sin \delta_{CP}$, reweighted with a flat prior in δ_{CP} from T2K Runs 1-11 data fit with the 5 SK samples and the inclusion of $\text{NC}\pi^0$ samples, and the reactor constraints on $\sin^2 \theta_{13}$ applied. The left, middle, and right columns are the cases with NO, IO, and both mass ordering.

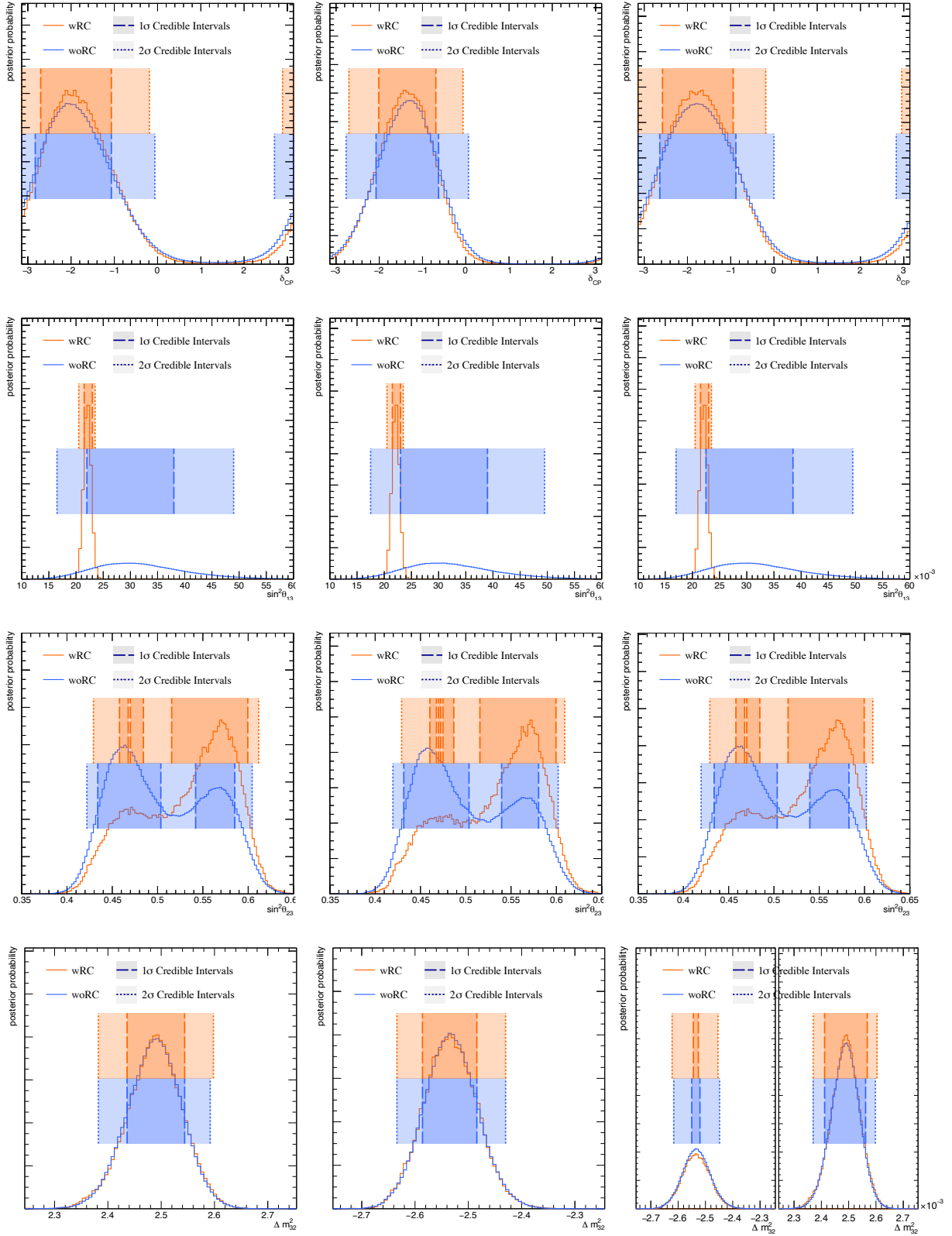


Figure 8.12: Comparisons of the 1D posterior distributions and credible intervals of the oscillation parameters using T2K Runs 1-11 data, overlapped with and without reactor constraints on $\sin^2 \theta_{13}$ applied, using the 5 SK samples only. The left, middle, and right columns are the cases with NO, IO, and both mass ordering.

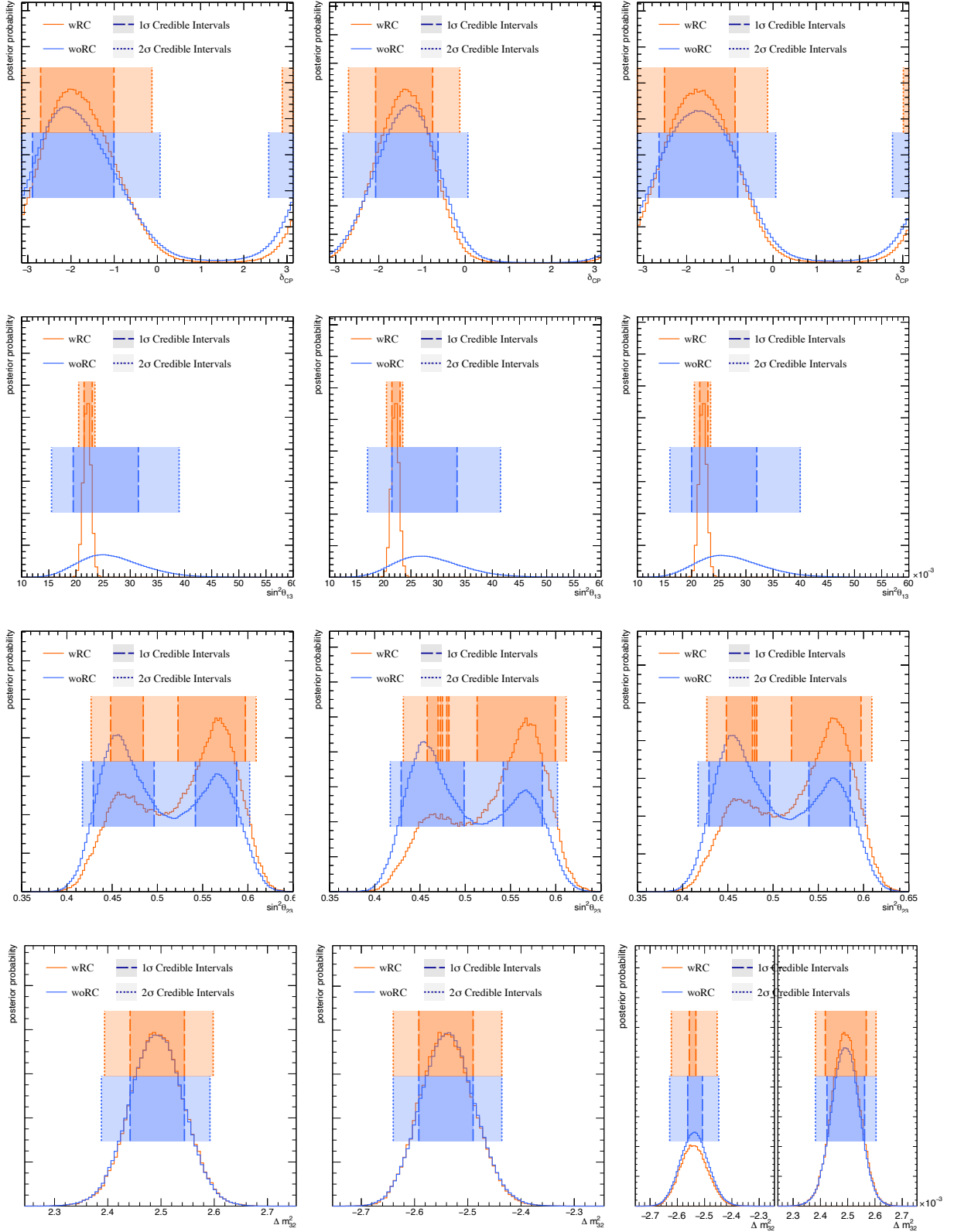


Figure 8.13: Comparisons of the 1D posterior distributions and credible intervals of the oscillation parameters using T2K Runs 1-11 data, overlapped with and without reactor constraints on $\sin^2 \theta_{13}$ applied, using the 5 SK samples and the inclusion of $\text{NC}\pi^0$ samples. The left, middle, and right columns are the cases with NO, IO, and both mass ordering.

The probability for a particular combination of mass ordering and θ_{23} octant being true, derived from the data fit, is shown in Tables 8.3 and 8.4, for fits without and with $\text{NC}\pi^0$ samples. A slight preference is observed for normal mass ordering and the lower octant for fits with $\text{NC}\pi^0$ samples, with a probability of 34.9% when constrained by T2K data only. The application of reactor constraints shifts the preference to normal ordering and the upper octant with a probability of 49.5%, slightly lower than the case without $\text{NC}\pi^0$ samples. This preference is observed to be stronger (53.2%) for the case without $\text{NC}\pi^0$. The Bayes factors are also calculated assuming equal priors on both mass orderings and θ_{23} octants, showing ‘substantial’ evidence against IO and LO, though this evidence is not strong enough to conclusively exclude these cases according to Jeffrey’s scale [134].

Table 8.3: Comparison of the probabilities for the NO ($\Delta m_{32}^2 > 0$) and IO ($\Delta m_{32}^2 < 0$), as well as the lower (LO) and upper octant (UO) of θ_{23} from the T2K Runs 1-11 data fit, using the 5 Super-K samples only.

	woRC				wRC			
	LO	UO	Sum		LO	UO	Sum	
NO	0.368	0.370	0.738	B(NO/IO)	0.226	0.532	0.758	B(NO/IO)
IO	0.137	0.125	0.262	= 2.8	0.071	0.171	0.242	=3.1
Sum	0.505	0.495			0.297	0.703		
	B(UO/LO) = 1.0				B(UO/LO) = 2.4			

Table 8.4: Comparison of the probabilities for the NO ($\Delta m_{32}^2 > 0$) and IO ($\Delta m_{32}^2 < 0$), as well as the lower (LO) and upper octant (UO) of θ_{23} from the T2K Runs 1-11 data fit, using the 5 Super-K samples and inclusion of the $\text{NC}\pi^0$ samples.

	woRC				wRC			
	LO	UO	Sum		LO	UO	Sum	
NO	0.349	0.336	0.685	B(NO/IO)	0.246	0.495	0.741	B(NO/IO)
IO	0.167	0.148	0.315	= 2.2	0.074	0.185	0.259	= 2.9
Sum	0.516	0.484			0.320	0.680		
	B(UO/LO) = 0.94				B(UO/LO) = 2.1			

Tables 8.5 compare the results of this analysis with recent official T2K results, both with and without reactor constraints. While this analysis uses Run 11 data, it was

developed based on samples and a systematic uncertainty model from the official Runs 1-10 data [42]. The official Run 11 results include additional Super-K samples and systematic parameters not present in this analysis [112]. Presenting both the official results with and without Run 11 data helps isolate the impact of this analysis from other variations.

In the case without reactor constraints, an improvement in the 1σ uncertainty of δ_{CP} is observed. However, this improvement may be due to the absorption of event variations by larger Super-K detector uncertainties, as mentioned earlier. Parameters such as $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$, which are strongly correlated, exhibit increased uncertainties due to the different models used between this analysis and the official one. With reactor constraints, the uncertainties and best fit for δ_{CP} become more comparable to the official Run 11 analysis results. However, the reactor constraints have a weaker effect on $\sin^2 \theta_{23}$ in this analysis, as shown by an increased percentage difference from 100% to 114%, likely due to reduced correlations between the oscillation parameters.

Table 8.5: Results on oscillation parameters extracted from 1D posterior distributions, compared to official T2K results with and without the Run 11 beam data. Percentage differences of the 1σ intervals from the official results to the results of the analysis are indicated.

woRC	This analysis	Official T2K w/o Run 11		Official T2K w/ Run 11	
δ_{cp}	$-1.79^{+0.97}_{-0.85}$	$-2.09^{+1.37}_{-0.71}$	-13%	$-2.01^{+1.26}_{-0.82}$	-13%
$\sin^2 \theta_{23}$	$0.45^{+0.14}_{-0.02}$	$0.49^{+0.06}_{-0.02}$	+100%	$0.48^{+0.07}_{-0.01}$	+100%
$\sin^2 \theta_{13}$	$0.025^{+0.007}_{-0.005}$	$0.024^{+0.007}_{-0.002}$	+55%	$0.024^{+0.006}_{-0.003}$	+55%
$\Delta m_{32}^2 \times 10^3$	$2.49 [2.42, 2.56]$ $\cup [-2.56, -2.51]$	$2.46 [2.42, 2.54]$ $\cup [-2.57, -2.50]$	+17% (NO), -29% (IO)	$2.51 [2.44, 2.57]$ $\cup [-2.59, -2.51]$	+7.6% (NO), -38% (IO)
wRC					
δ_{cp}	$-1.79^{+0.91}_{-0.72}$	$-1.97^{+0.96}_{-0.54}$	+8.7%	$-1.85^{+0.84}_{-0.75}$	+2.5%
$\sin^2 \theta_{23}$	$0.57^{+0.03}_{-0.12}$	$0.55^{+0.02}_{-0.05}$	+114%	$0.54^{+0.02}_{-0.05}$	+114%
$\sin^2 \theta_{13}$	$0.022^{+0.001}_{-0.001}$	$0.022^{+0.001}_{-0.001}$	-	$0.022^{+0.001}_{-0.001}$	-
$\Delta m_{32}^2 \times 10^3$	$2.49 [2.42, 2.57]$ $\cup [-2.56, -2.53]$	$2.49 [2.41, 2.55]$	+7.1% (NO)	$2.50 [2.42, 2.59]$	-11% (NO)

Chapter 9

Conclusion

The T2K, and the next-generation Hyper-K experiment, aim to search for CP -violation through the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) appearance and to better constrain oscillation parameters $|\Delta m_{32}^2|$ and $\sin^2 \theta_{23}$, and the θ_{23} octant. As data accumulates, Hyper-K is also expected to provide better intrinsic constraints on $\sin^2 \theta_{13}$ from this channel.

This thesis has focused on evaluating the impact of systematics on the oscillation parameters by introducing new constraints on the $\nu_e/\bar{\nu}_e$ uncertainties through IWCD and the new $\text{NC}\pi^0$ samples at Super-K. A fit to the T2K Runs 1-11 data was carried out, incorporating these new $\text{NC}\pi^0$ samples and the corresponding updated systematic uncertainty model for the $\text{NC}\pi^0$ events.

In Chapter 5, the impact of the $\nu_e/\bar{\nu}_e$ uncertainties on the oscillation parameters was assessed using an expanded cross-section model according to the capability of IWCD. This model involved splitting the parameterisation of the relative ν_e/ν_μ ($\bar{\nu}_e/\bar{\nu}_\mu$) uncertainties into different E_ν regions. Special consideration was also provided to the normalisation accounting for the differences between E_ν and reco. E_ν^{CCQE} , by using a set of parameters corresponding to distinct regions in the E_ν vs $E_\nu - \text{true } E_\nu^{CCQE}$ space. Asimov fits performed under the new IWCD constraints showed improved sensitivity to oscillation measurements when comparing with the current T2K theory-driven model. However, challenges arose during the fit process, as finding the true minimum of the likelihood became more difficult with increased statistics. The expanded parameter space also made the fit more computationally expensive when using the profiling method. Further studies are needed to explore the treatment of systematic uncertainties in the context of large datasets. Specifically, the parameterisation of systematics can be improved to reduce correlations between

parameters, potentially improving the efficiency and accuracy of the fit process.

Another aspect that needs improvement in the oscillation analysis is the accurate prediction of background events, which becomes increasingly important as statistics grow. The $1R(NC)\pi^0$ events represent a major background for the ν_e ($\bar{\nu}_e$) in the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) appearance channel. Previously, these $NC\pi^0$ events were controlled by parameters constrained using only the CC pion production measurements at ND280. However, this approach can introduce biases in predicting $NC\pi^0$ events, as the form factors and interaction parameters may differ between CC and NC interactions.

In Chapter 6, a possible approach to normalise $NC\pi^0$ events has been explored using the high-efficiency and purity $2R(NC)\pi^0$ samples selected at Super-K. The normalisation was controlled by relating the kinematic properties of $2R\pi^0$ events to those of $1R\pi^0$ events. Leveraging this approach, the previously rejected events during e/π^0 cut of the $1Re$ samples selection have also been added to the oscillation analysis as the $1R\pi^0$ samples to provide additional ν_e signal events.

The impact on normalising the $NC\pi^0$ events using the $2R\pi^0$ samples followed a close pattern from the data prediction discrepancy observed by MicroBooNE. However, the postfit prediction suggests potential mis-modelling in the ν -mode $2R\pi^0$ samples, where further investigation on cross-section model or the event normalisation parameterisation are needed to improve accuracy in these regions.

In Chapter 7, the impact of the $NC\pi^0$ samples on oscillation sensitivity was evaluated using the Markov Chain Monte Carlo (MCMC) approach to fit over a large multi-variable parameter space. Constraints on the credible intervals were observed alongside the implementations of both $2R\pi^0$ and $1R\pi^0$ samples, with more pronounced effects in the $\nu_\mu \rightarrow \nu_e$ appearance channel following the inclusion of the $1R\pi^0$ samples. These improvements are likely due to better constraints on the Super-K detector systematic uncertainties associated with these samples, as well as the additional ν_e signal events. Future improvements could focus on optimising the method for calculating Super-K detector uncertainties, refining the binning method

among the sample binning, normalisation parameter binning, and the Super-K detector binning for the new samples, to reduce the correlations between them and more effectively control event variations.

Finally, in Chapter 8, the $\text{NC}\pi^0$ samples were included in a fit to the T2K data to extract oscillation parameters, incorporating all the available data before the ND280 upgrade. The $\text{NC}\pi^0$ samples demonstrated an impact on the flux uncertainties. The interpretation of the impact on oscillation parameters remains challenging due to large statistical fluctuations in the data. Nevertheless, the results showed compatibility with the newest official T2K findings regarding δ_{CP} both with and without reactor constraints applied.

With the current systematic uncertainty model, the Hyper-K sensitivity study with 10-year data predicts a 3σ (5σ) exclusion of CP conservation for 70% (48%) of true δ_{CP} values and a 3σ wrong octant exclusion for $\sin^2 \theta_{23}$ values outside the (0.46, 0.57) range. In the future, alongside improvements from IWCD constraints, there is bright potential for applying the $\text{NC}\pi^0$ selection at Hyper-K. This can benefit from making it a control sample for the far detector flux, and the $\text{NC}\pi^0$ background events in the Hyper-K and IWCD samples, improving the intrinsic ν_e measurements at IWCD and ν_e signal data for oscillation measurements.

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Appendix A

Systematic Parameterisation

A.1 Flux and Cross Section Models

The flux model comes with 50 parameters for the ND280 flux and another 50 for the Super-K systematics, both have the same binning scheme. The covariance matrix used for the T2K oscillation analysis is shown in Fig. A.1. The parameters are binned in true neutrino energy E_ν (GeV) :

- ν -mode ν_μ : [0, 0.4, 0.5, 0.6, 0.7, 1.0, 1.5, 2.5, 3.5, 5.0, 7.0, 30.0];
- ν -mode $\bar{\nu}_\mu$: [0, 0.7, 1.0, 1.5, 2.5, 30.0];
- ν -mode ν_e : [0, 0.5, 0.7, 0.8, 1.5, 2.5, 4.0, 30.0];
- ν -mode $\bar{\nu}_e$: [0, 2.5, 30.0];
- $\bar{\nu}$ -mode ν_μ : [0, 0.7, 1.0, 1.5, 2.5, 30.0];
- $\bar{\nu}$ -mode $\bar{\nu}_\mu$: [0, 0.4, 0.5, 0.6, 0.7, 1.0, 1.5, 2.5, 3.5, 5.0, 7.0, 30.0];
- $\bar{\nu}$ -mode ν_e : [0, 2.5, 30.0];
- $\bar{\nu}$ -mode $\bar{\nu}_e$: [0, 0.5, 0.7, 0.8, 1.5, 2.5, 4.0, 30.0];

The T2K oscillation analysis performed in Chapters 7 and 8 uses a cross-section model of 79 parameters, with the 75 parameters from the model used in oscillation analysis described in Ref. [135], and 4 being the new π^0 normalisation parameters. The parameters are listed as in Table A.1. A full description of the cross-section uncertainties from this model can be found in Ref. [42]. The cross-section parameters used for Hyper-K oscillation analysis presented in Chapter 5 have 18 parameters, which is an earlier version of the T2K cross-section model, as described in Ref. [58].

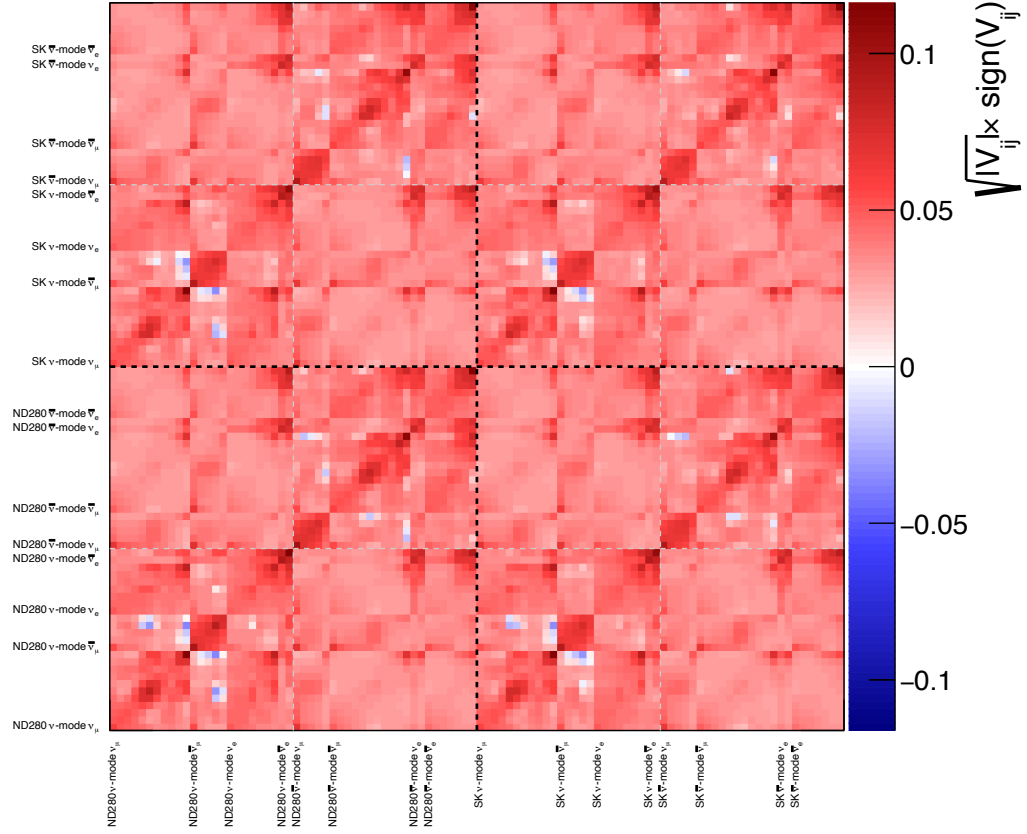


Figure A.1: The prior flux covariance for the T2K oscillation analysis presented in Chapters 7 and 8.

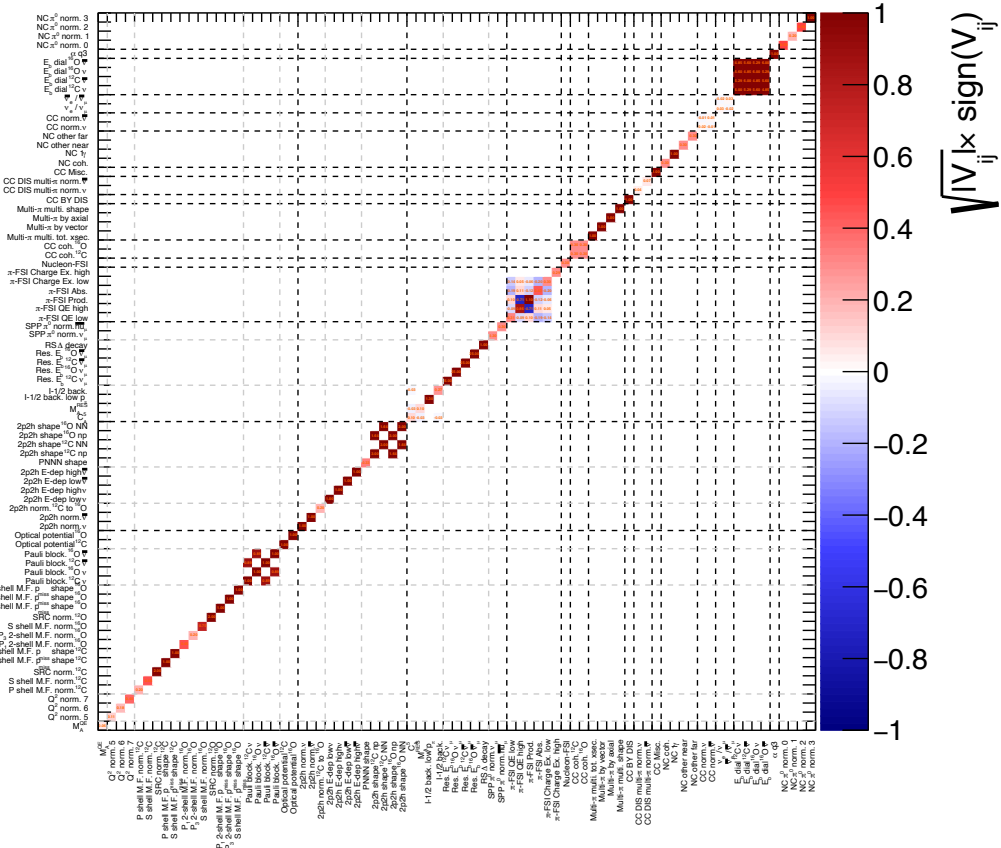


Figure A.2: The prior cross-section covariance for the T2K oscillation analysis presented in Chapters 7 and 8.

Table A.1: A summary of the cross-section parameters for the T2K oscillation analysis presented in Chapters 7 and 8 [135].

Parameter	Validity Range	Generated Value	Prior Mean	Prior Error	Type	Unit
M^{QE}	all	1.21	1.03	0.06	shape	GeV/c ²
Q^2 norm 0.25-0.50 GeV ²	all	1	1	0.11	norm	—
Q^2 norm 0.50-1.00 GeV ²	all	1	1	0.18	norm	—
Q^2 norm > 1.00 GeV ²	all	1	1	0.4	norm	—
PShell MF Norm C	all	0	0	0.2	shape	—
SShell MF Norm C	all	0	0	0.35	shape	—
SRC Norm C	all	1	1	2	shape	—
PShell MF PMiss C	all	0	0	1	shape	—
SShell MF PMiss C	all	0	0	1	shape	—
P1 2Shell MF Norm O	all	0	0	0.2	shape	—
P3 2Shell MF Norm O	all	0	0	0.45	shape	—
SShell MF Norm O	all	0	0	0.75	shape	—
SRC Norm O	all	1	1	2	shape	—
P1 2Shell MF PMiss O	all	0	0	1	shape	—
P3 2Shell MF PMiss O	all	0	0	1	shape	—
SShell MF PMiss O	all	0	0	1	shape	—
Pauli Blocking C ν	all	0	0	1	shape	—
Pauli Blocking O ν	all	0	0	1	shape	—
Pauli Blocking C $\bar{\nu}$	all	0	0	1	shape	—
Pauli Blocking O $\bar{\nu}$	all	0	0	1	shape	—
Optical Potential C	0-1	0	0	1	shape	—
Optical Potential O	0-1	0	0	1	shape	—
2p2h norm ν	> 0	1	1	1	norm	—
2p2h norm $\bar{\nu}$	> 0	1	1	1	norm	—
2p2h norm CtoO	> 0	1	1	0.2	norm	—
2p2h Edep low E_ν	0-1	1	1	1	shape	—
2p2h Edep high E_ν	0-1	1	1	1	shape	—
2p2h Edep low $E_{\bar{\nu}}$	0-1	1	1	1	shape	—
2p2h Edep high $E_{\bar{\nu}}$	0-1	1	1	1	shape	—
PN-NN Shape	-1-1	0	0	0.33	shape	—
2p2h shape C np	-1-1	0	0	1	shape	—
2p2h shape C NN	-1-1	0	0	1	shape	—
2p2h shape O np	-1-1	0	0	1	shape	—
2p2h shape O NN	-1-1	0	0	1	shape	—
C^5	all	1.01	0.96	0.15	shape	—
M_{A}^{RES}	all	0.95	1.07	0.15	shape	—
1- π bkg. low p_π	all	1.3	0.96	1.3	shape	—
1- π bkg.	all	1.3	0.96	0.27	shape	—
RES E_b C ν_μ	0-25	0	25	25	shape	—
RES E_b O ν_μ	0-25	0	25	25	shape	—
RES E_b C $\bar{\nu}_\mu$	0-25	0	25	25	shape	—
RES E_b O $\bar{\nu}_\mu$	0-25	0	25	25	shape	—
RS Delta Decay	0-1	1	1	1	shape	—
SPP Pi0 ν_μ	all	1	1	0.3	norm	—
SPP Pi0 $\bar{\nu}_\mu$	all	1	1	0.3	norm	—
π -FSI QE	all	1.069	1.069	0.313	shape	—
π -FSI QE High	all	1.824	1.824	0.859	shape	—
π -FSI Hadron production	all	1.002	1.002	1.101	shape	—
π -FSI Absorption	all	1.404	1.404	0.432	shape	—
π -FSI Charge Exchange	all	0.697	0.697	0.305	shape	—
π -FSI Pion Charge Exchange	all	1.8	1.8	0.288	shape	—
Nucleon FSI	-1-1	0	0	0.3	shape	—
CC Coh. C	> 0	1	1	0.3	norm	—
CC Coh. O	> 0	1	1	0.3	norm	—
MPi Multi TotXsec	all	0	0	1	shape	—
MPi BY Vector	all	0	0	1	shape	—
MPi BY Axial	all	0	0	1	shape	—
MPi Multiplicity shape	all	0	0	1	shape	—
CC BY DIS	all	0	0	1	shape	—
CC DIS MultiPi ν	> 0	1	1	0.035	norm	—
CC DIS MultiPi $\bar{\nu}$	> 0	1	1	0.065	norm	—
CC Misc.	> 0	1	1	1	norm	—
NC Coh.	> 0	1	1	0.3	norm	—
NC 1 γ	> 0	1	1	1	norm	—
NC other near	> 0	1	1	0.3	norm	—
NC other far	> 0	1	1	0.3	norm	—
CC norm ν	> 0	1	1	0.02	norm	—
CC norm $\bar{\nu}$	> 0	1	1	0.01	norm	—
ν_e / ν_μ	> 0	1	1	0.0282843	norm	—
$\bar{\nu}_e / \bar{\nu}_\mu$	> 0	1	1	0.0282843	norm	—
E_b C ν	-10-15	0	2	6	function	MeV
E_b C $\bar{\nu}$	-10-15	0	0	6	function	MeV
E_b O ν	-10-15	0	4	6	function	MeV
E_b O $\bar{\nu}$	-10-15	0	0	6	function	MeV
αq^3	0-1	0	0	1	function	—
π^0 norm. 0	0-0.2 (P_{π^0})	1	1	0.5	norm	—
π^0 norm. 1	0.2-0.4 (P_{π^0})	1	1	0.2	norm	—
π^0 norm. 2	0.4-0.9 (P_{π^0})	1	1	0.5	norm	—
π^0 norm. 3	0.9-1.5 (P_{π^0})	1	1	1	norm	—

A.2 The Expanded IWCD Normalisation Parameters

The 10 1-D and 58 2-D normalisation parameterisation from the IWCD cross-section model is implemented at VALOR for the Hyper-K oscillation sensitivity studies presented in Chapter 5.

The IWCD 1-D parameterisation is given below (GeV):

E_ν : [0, 0.3, 0.6, 1.0, 1.5, 5.0], for both the ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$ normalisation.

The IWCD 2-D parameterisation is defined as (also shown in Fig. A.3):

Normalisation on ν events:

- E_ν : [0.2, 0.4]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.4, -0.1, 0.3];
- E_ν : [0.4, 0.5]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.7, -0.1, 0.2, 0.5];
- E_ν : [0.5, 0.6]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.8, -0.1, 0.1, 0.5];
- E_ν : [0.6, 0.7]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.9, -0.1, 0.1, 0.6];
- E_ν : [0.7, 0.8]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-1.1, -0.1, 0.1, 0.3, 0.7];
- E_ν : [0.8, 1.0]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-1.3, -0.2, 0.1, 0.3, 1.0];
- E_ν : [1.0, 1.2]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-1.5, 0.2, 1.2];
- E_ν : [1.2, 1.5]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-1.5, 0.2, 1.6];
- E_ν : [1.5, 2.0]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-1.0, 1.0];

Normalisation on $\bar{\nu}$ events:

- E_ν : [0.2, 0.4]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.3, 0, 0.3];
- E_ν : [0.4, 0.5]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.4, -0.1, 0.1, 0.4];
- E_ν : [0.5, 0.6]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.5, -0.1, 0.1, 0.5];

- E_ν : [0.6, 0.7]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.6, -0.1, 0.1, 0.6];
- E_ν : [0.7, 0.8]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.8, -0.1, 0.1, 0.3, 0.7];
- E_ν : [0.8, 1.0]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.9, -0.2, 0.1, 0.3, 0.9];
- E_ν : [1.0, 1.2]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.9, 0.2, 1.1];
- E_ν : [1.2, 1.5]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.8, 0.2, 1.2];
- E_ν : [1.5, 2.0]; $E_\nu - \text{true } E_\nu^{CCQE}$: [-0.6, 0.75, 1.4];

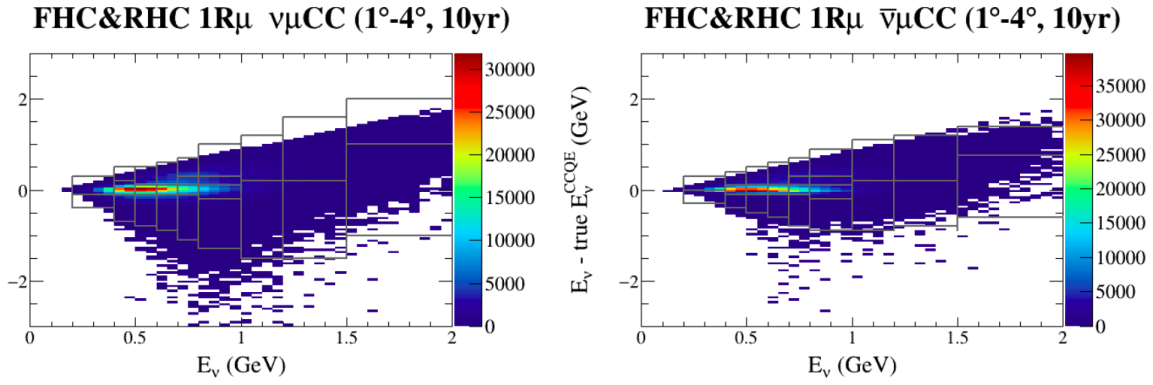


Figure A.3: The IWCD 2-D ν (left) and $\bar{\nu}$ (right) normalisation parameterisation for the Hyper-K oscillation analysis presented in Chapter 5. The uncertainty is calculated with an 1° - 4° off-axis angle span assumed.

A.2.1 The IWCD Samples

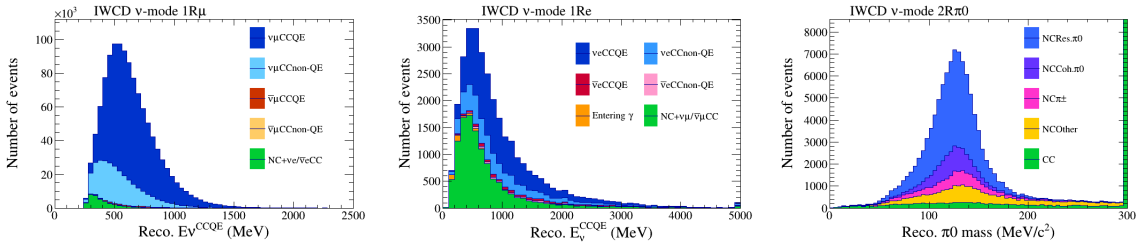


Figure A.4: The IWCD samples (in the ν -mode) used to constrain the new cross-section parameters proceeding to the Hyper-K oscillation analysis described in Chapter 5. Figures are provided by R. Akutsu (KEK).

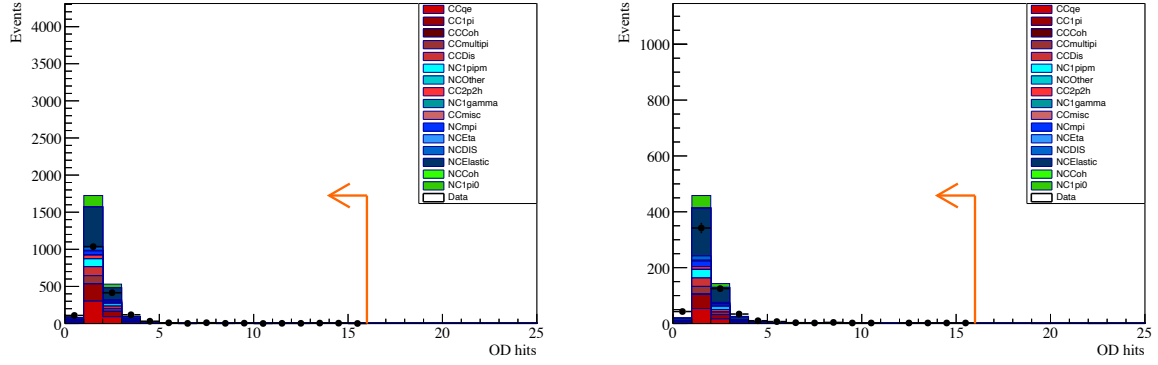
Appendix B

The NC π^0 Sample Selection

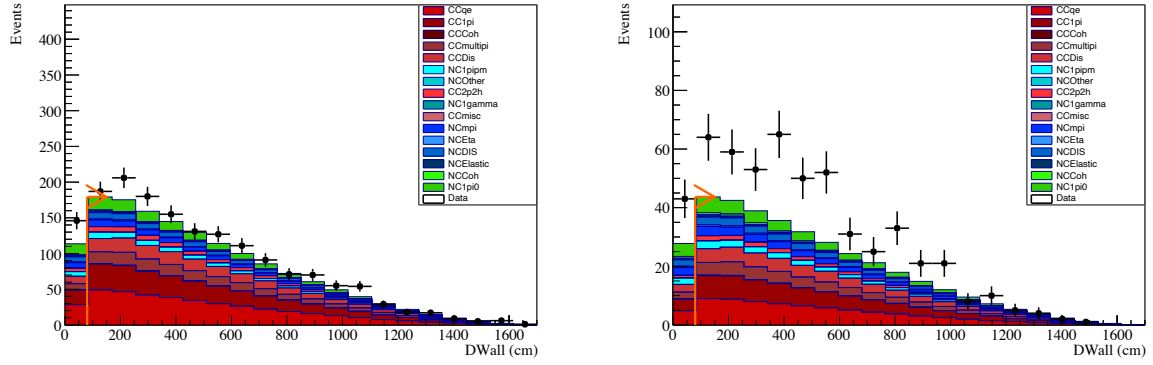
B.1 Sample Selection Distributions

The section demonstrates the MC and T2K Runs 1-11 data at each selection level for the NC π^0 samples described in Chapter 6. The plots on the left show the distributions of the ν -mode samples, while the $\bar{\nu}$ -mode samples are on the right. The MC distributions are broken down into different interaction modes.

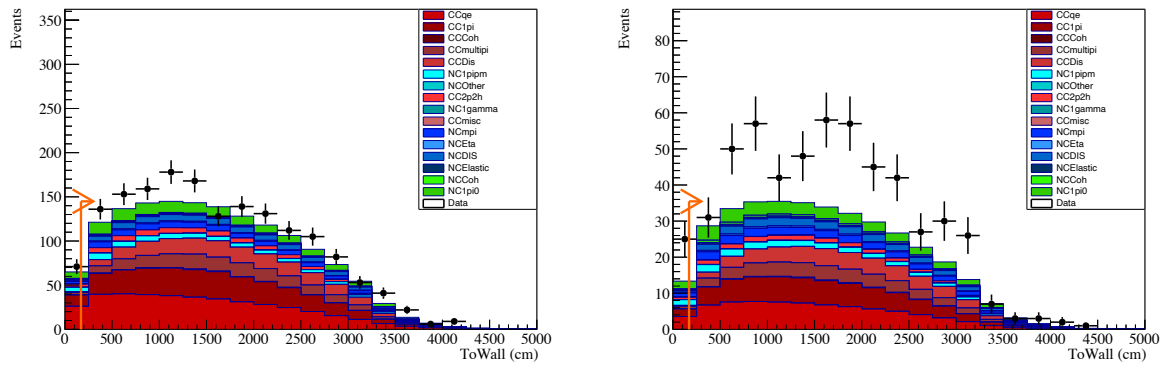
1. Outer detector hits < 16.

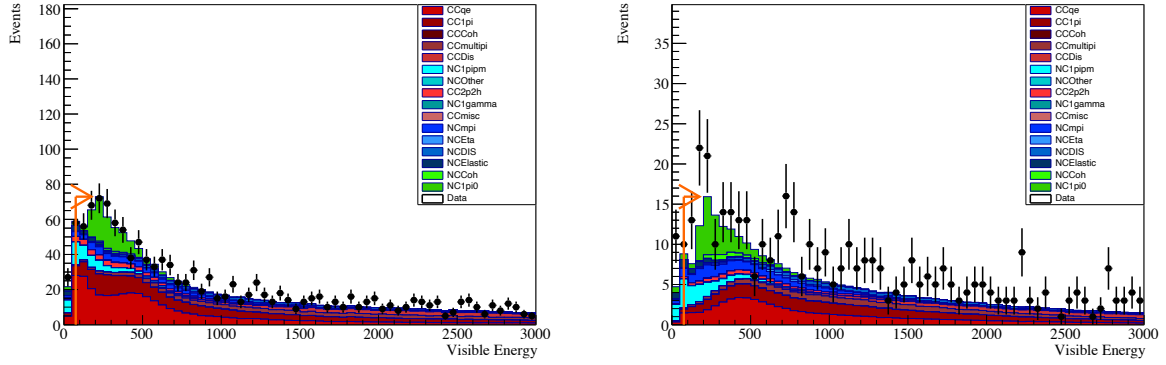
Figure B.1: NC π^0 FCFV selection - OD hits.

2. Vertex distance from the ID wall > 80 cm.

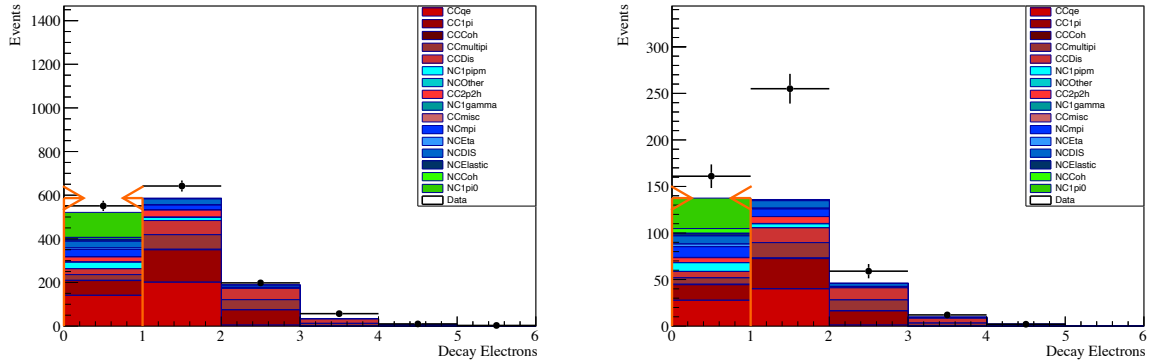
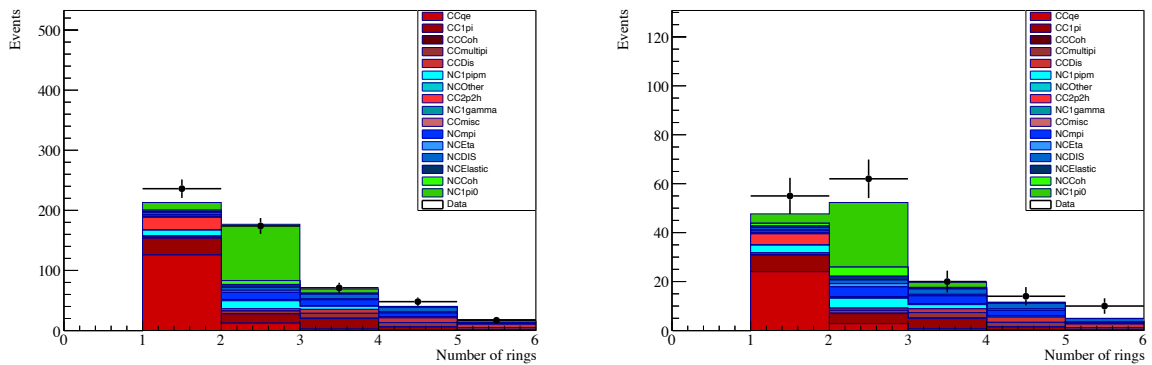
Figure B.2: NC π^0 FCFV selection - vertex distance from the ID wall.

3. Vertex distance from ID wall in direction of travel > 170 cm.

Figure B.3: NC π^0 FCFV selection - Vertex distance from ID wall in direction of travel.

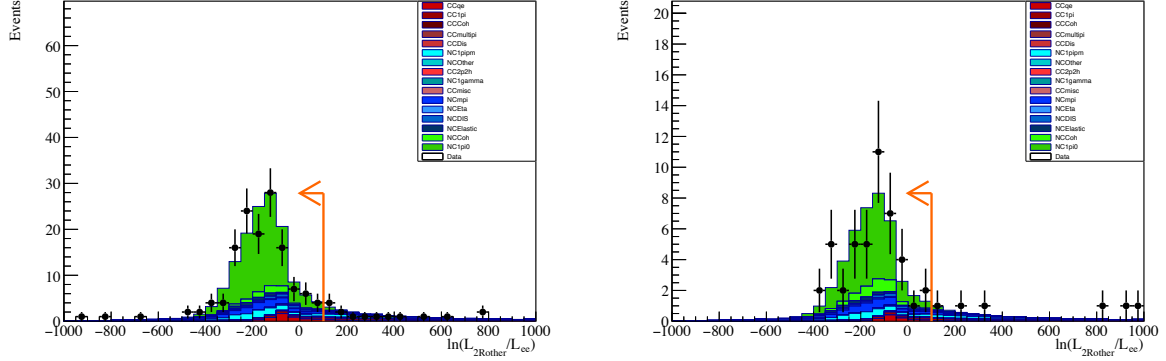
4. Visible energy in ID > 75 MeV.Figure B.4: NC π^0 FCFV selection - Visible energy at ID.

5. No decay electrons.

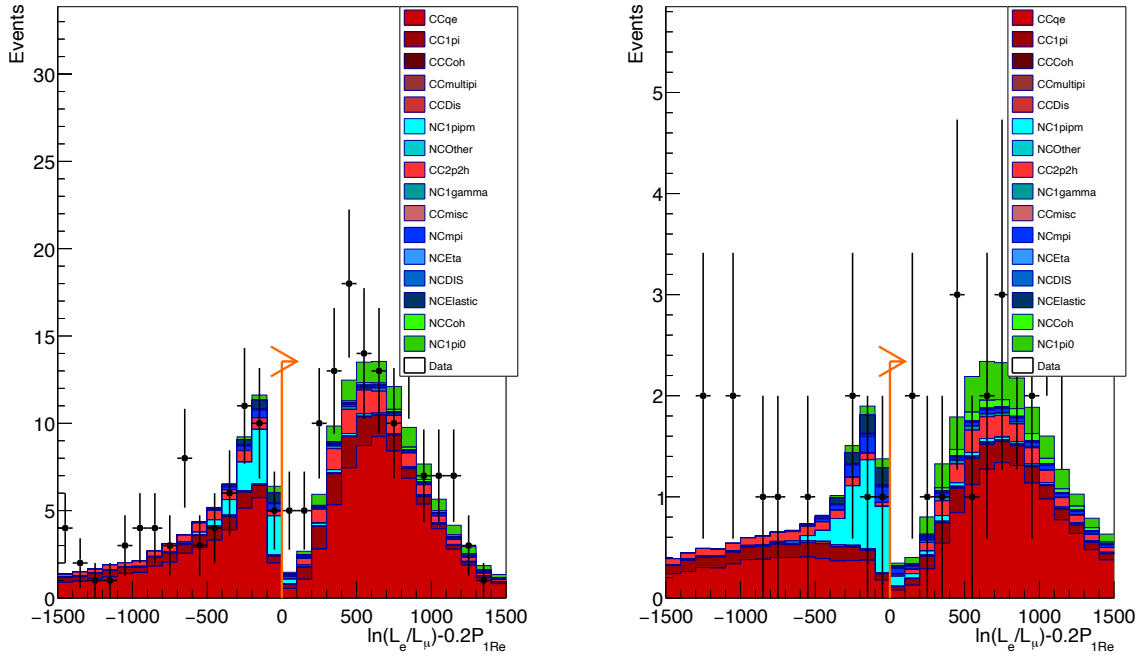
Figure B.5: NC π^0 topology selection - number of decay electrons.6. 1 or 2 reconstructed rings (for $1R\pi^0$ or $2R\pi^0$)Figure B.6: NC π^0 topology selection - number of reconstructed rings.

7. Ring likelihood selection I

$$2R\pi^0: \ln\left(\frac{L_{2R\pi^0}}{L_{ee}}\right) < 100$$

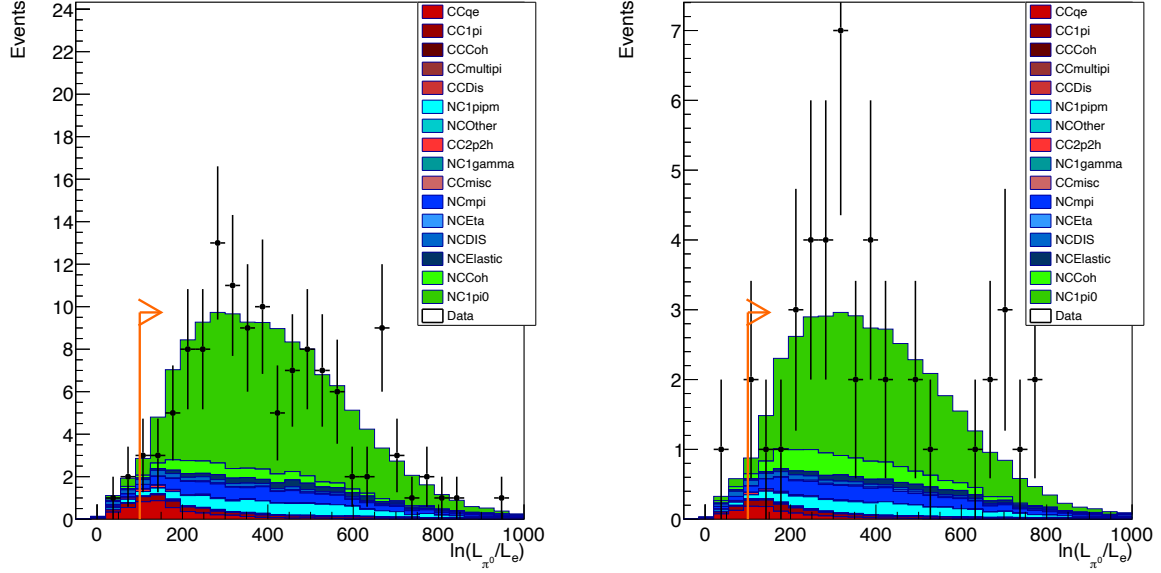
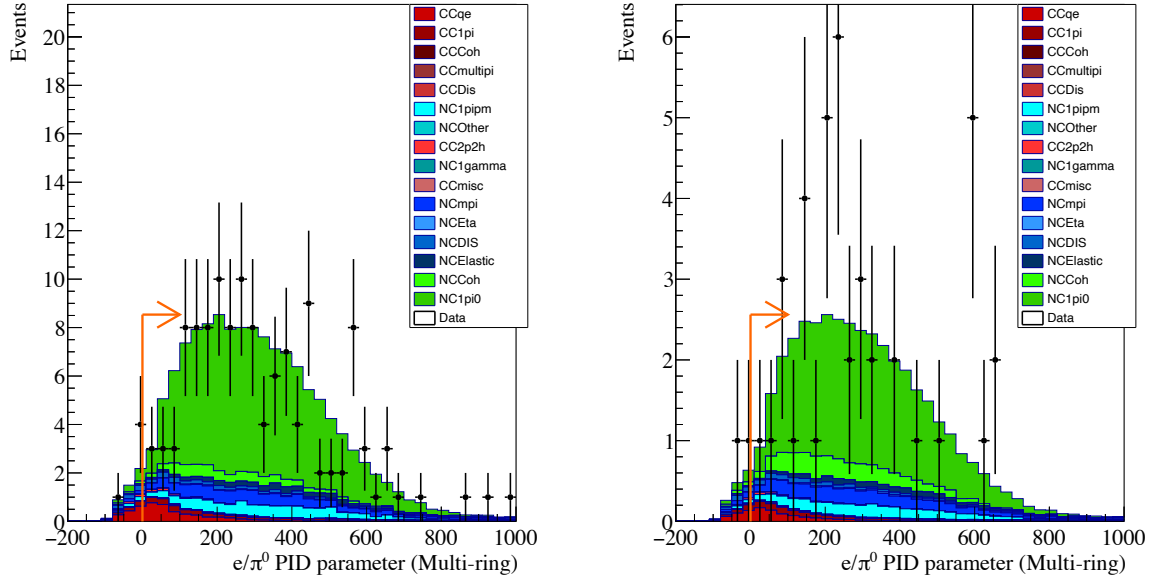
Figure B.7: $2R\pi^0$ topology selection - ee /other separation.

$$1R\pi^0: \ln\left(\frac{L_e}{L_\mu}\right) - 0.2P_{1Re}^{reco} > 0$$

Figure B.8: $1R\pi^0$ topology selection - e/μ separation.

8. Ring likelihood selection II

$$2R\pi^0: \ln\left(\frac{L_{\pi^0}}{L_e}\right) > 100$$

Figure B.9: $2R\pi^0$ topology selection - π^0/e separation.Figure B.10: $2R\pi^0$ topology selection - π^0/e PID ($\ln\left(\frac{L_{\pi^0}}{L_e}\right) - 100$).

$$1R\pi^0: \ln\left(\frac{L_{\pi^0}}{L_e}\right) > 175 - 0.875M_{\pi^0}^{reco}$$

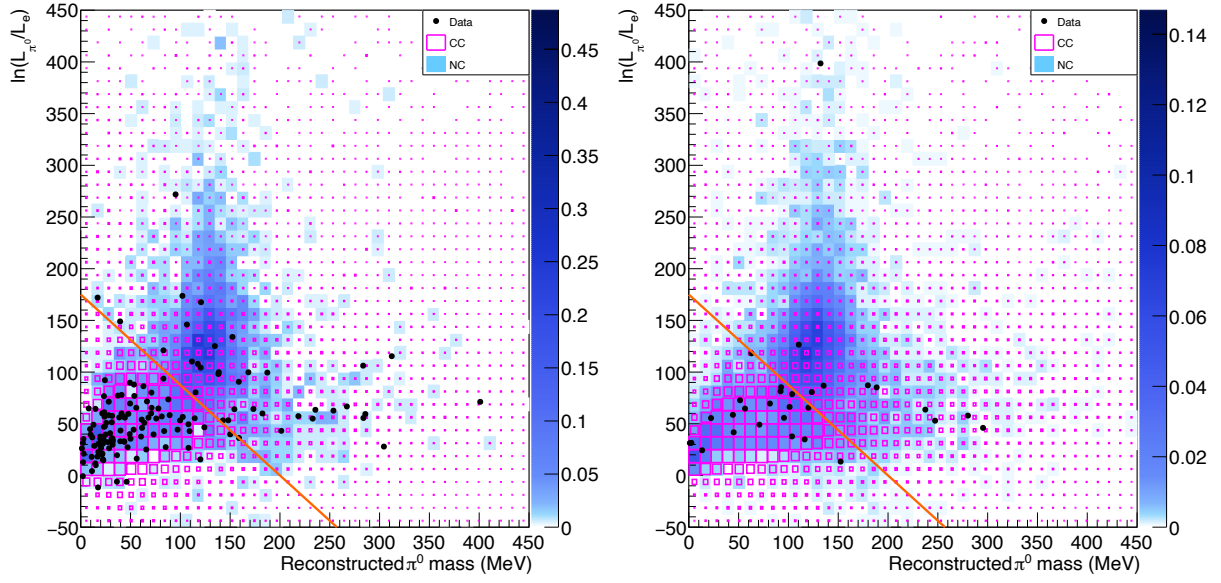


Figure B.11: $1R\pi^0$ topology selection - π^0/e separation.

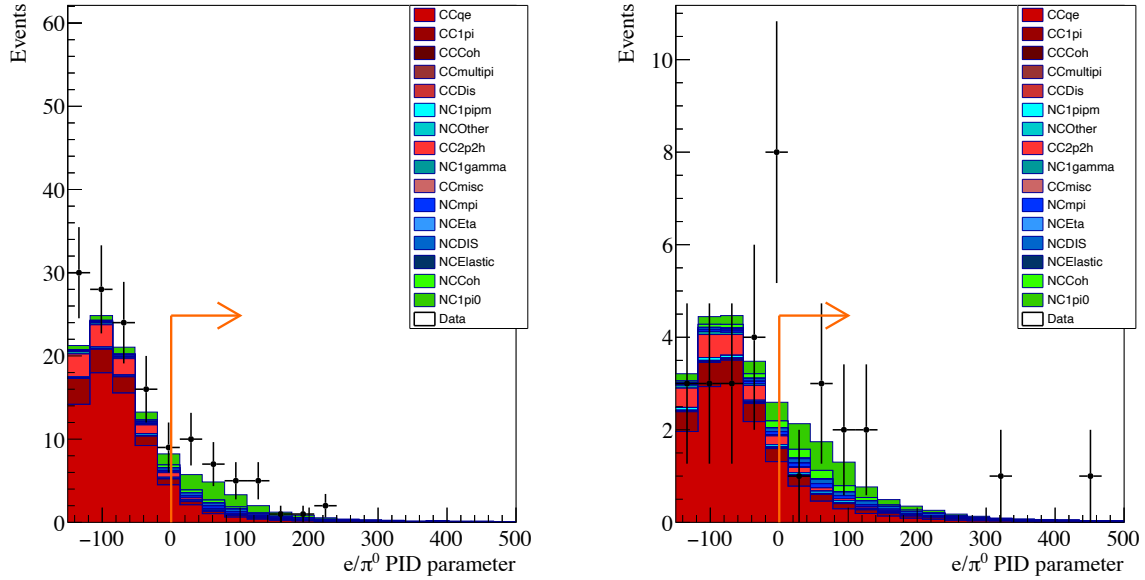
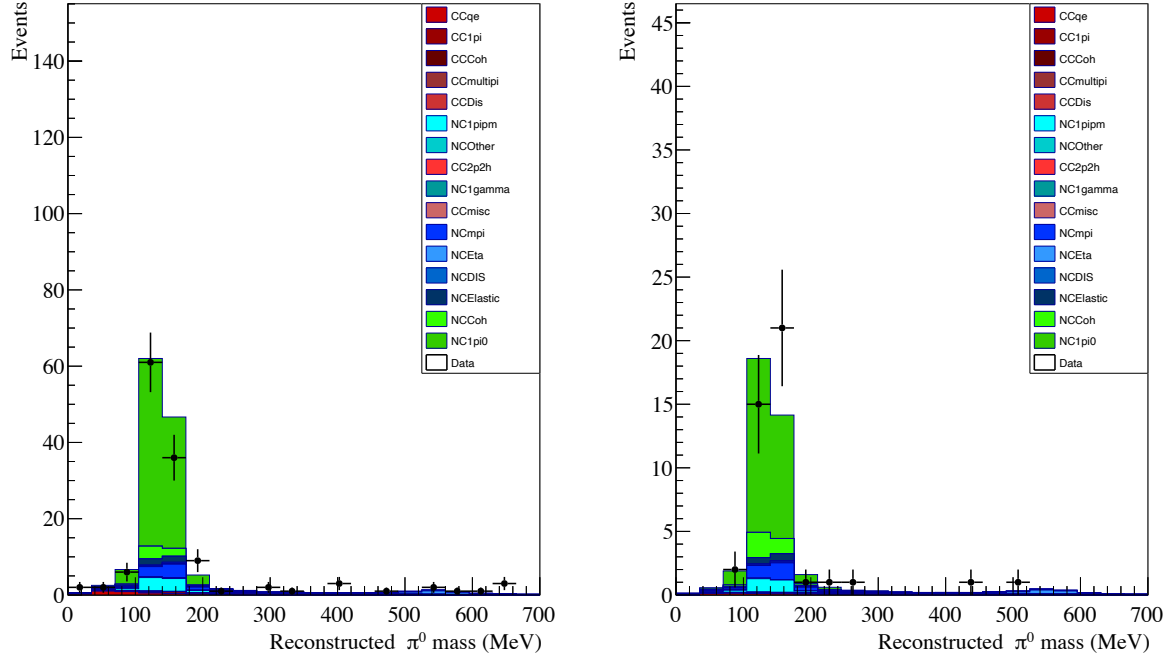
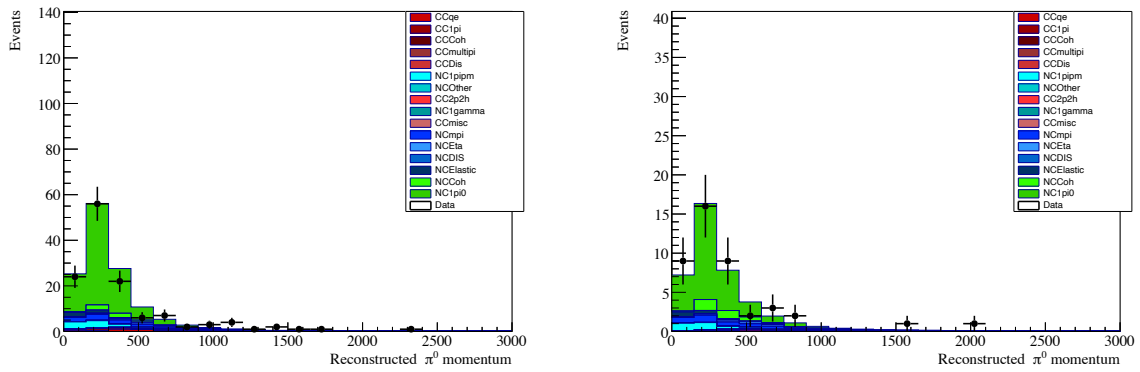


Figure B.12: $2R\pi^0$ topology selection - π^0/e PID ($\ln\left(\frac{L_{\pi^0}}{L_e}\right) - 175 + 0.875M_{\pi^0}^{reco}$).

9. Selected sample distributions

 $2R\pi^0$:Figure B.13: $2R\pi^0$ distribution in reconstructed π^0 mass.Figure B.14: $2R\pi^0$ distribution in reconstructed π^0 momentum.

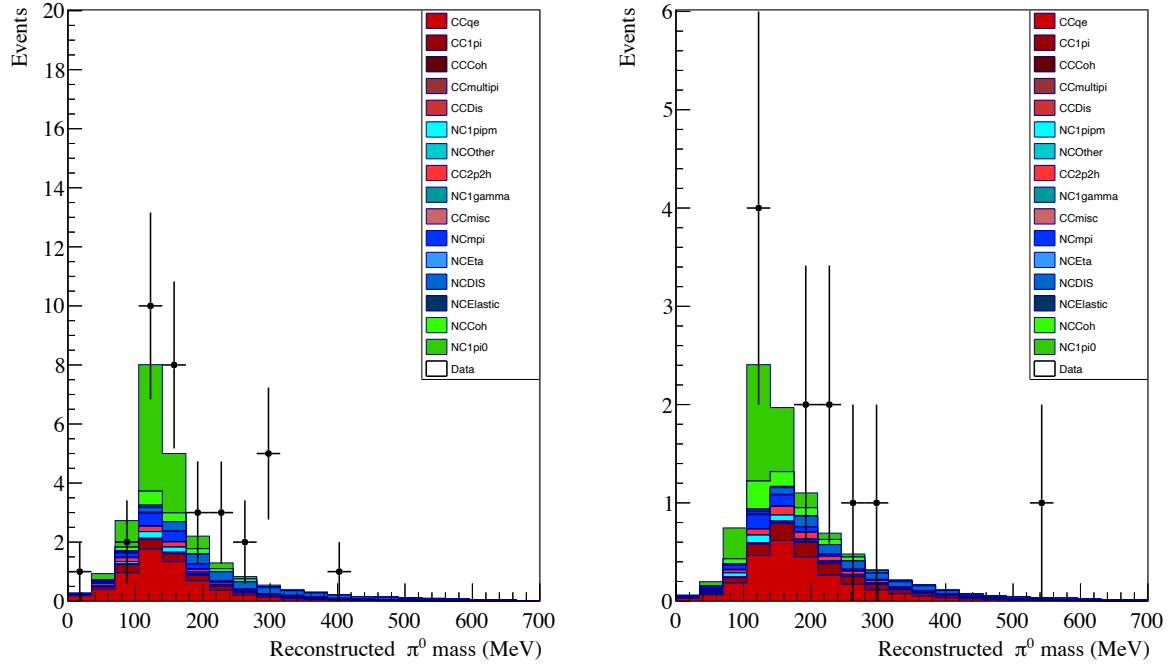
$1R\pi^0$:


Figure B.15: $1R\pi^0$ distribution in reconstructed π^0 mass.

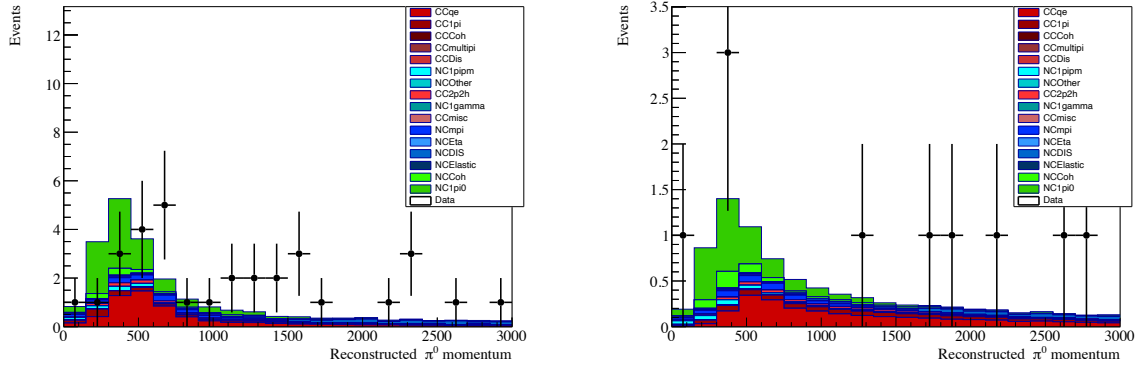


Figure B.16: $1R\pi^0$ distribution in reconstructed π^0 momentum.

B.2 Kinematic Distributions

B.2.1 Breakdown by Interaction Modes

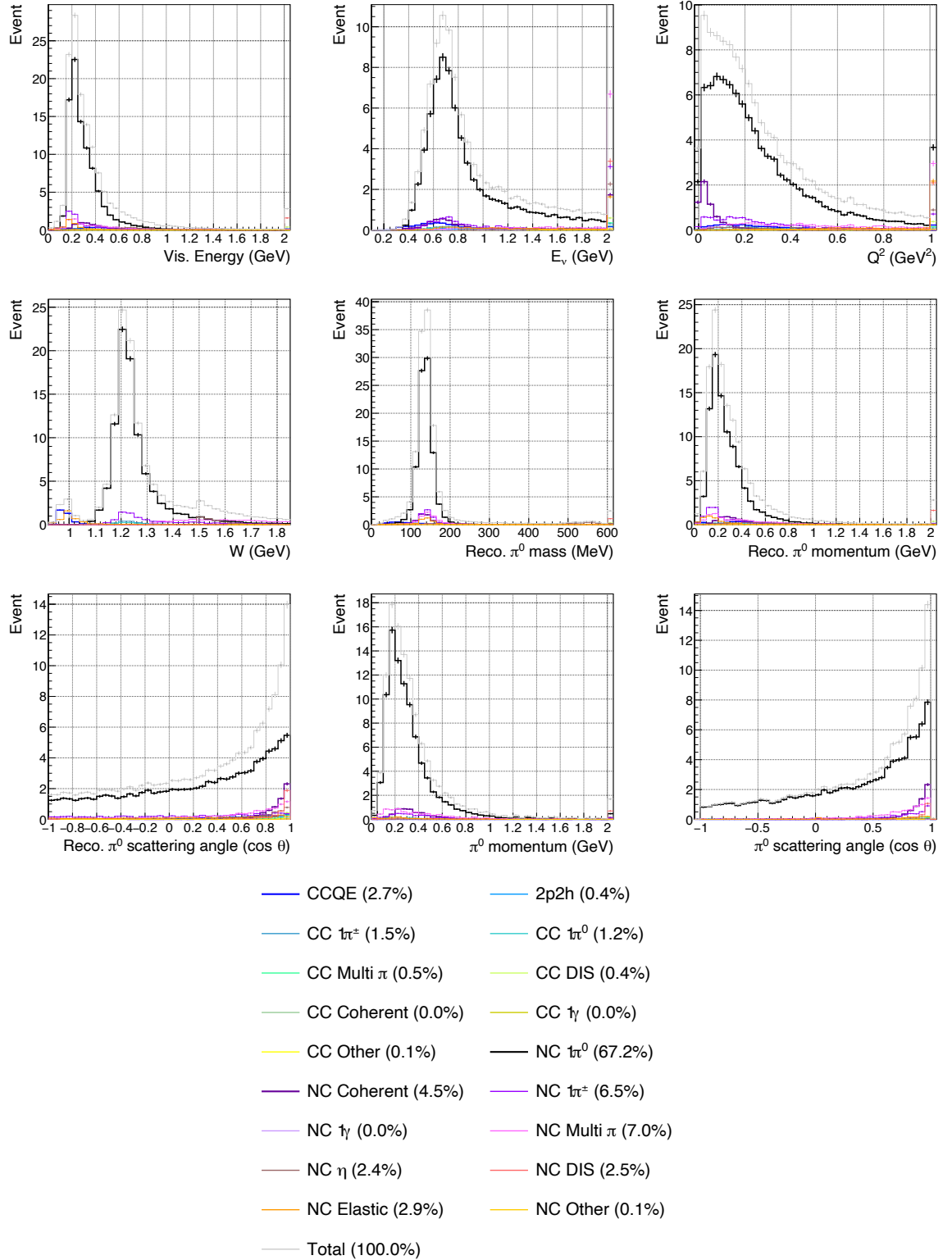
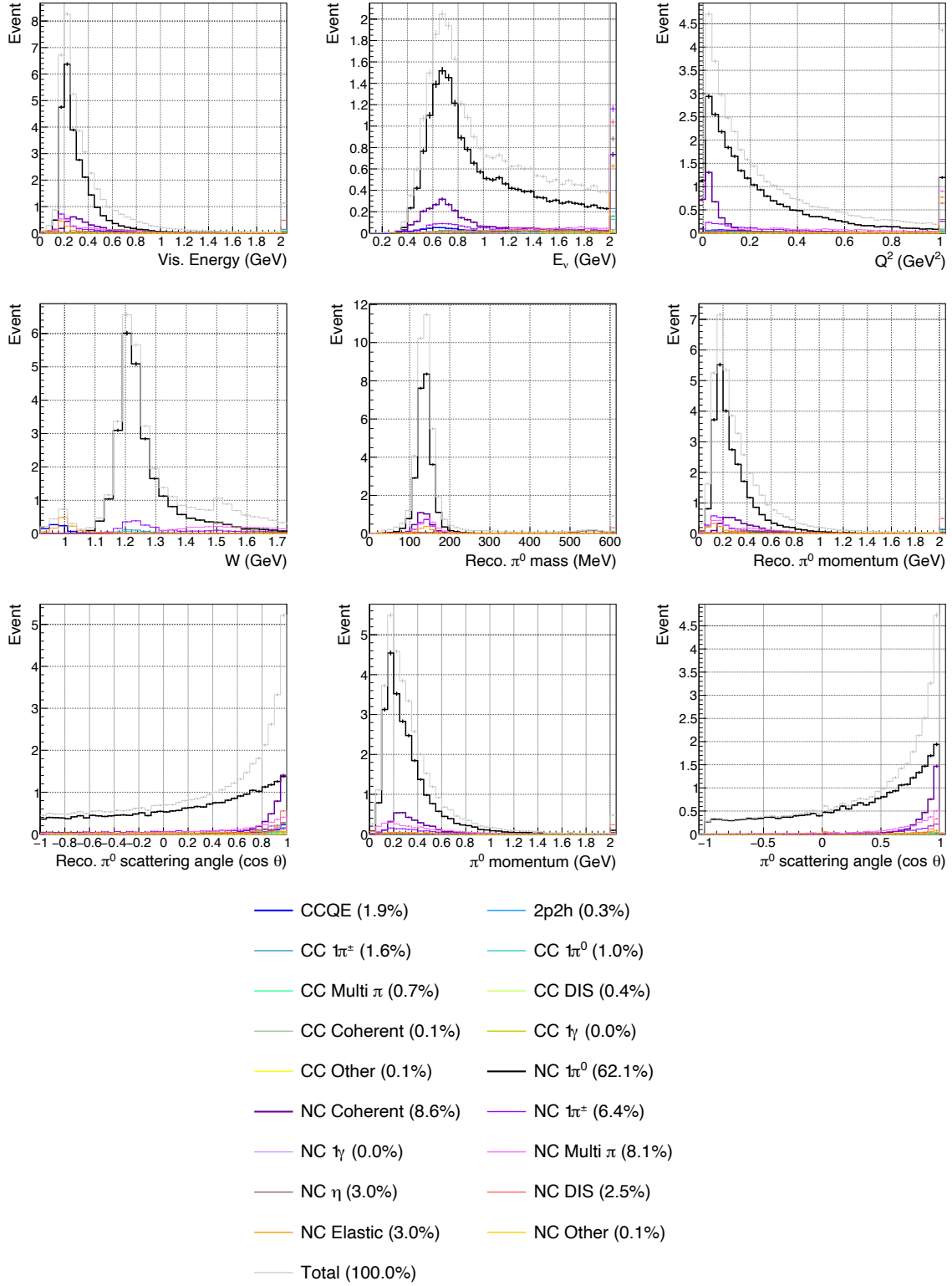
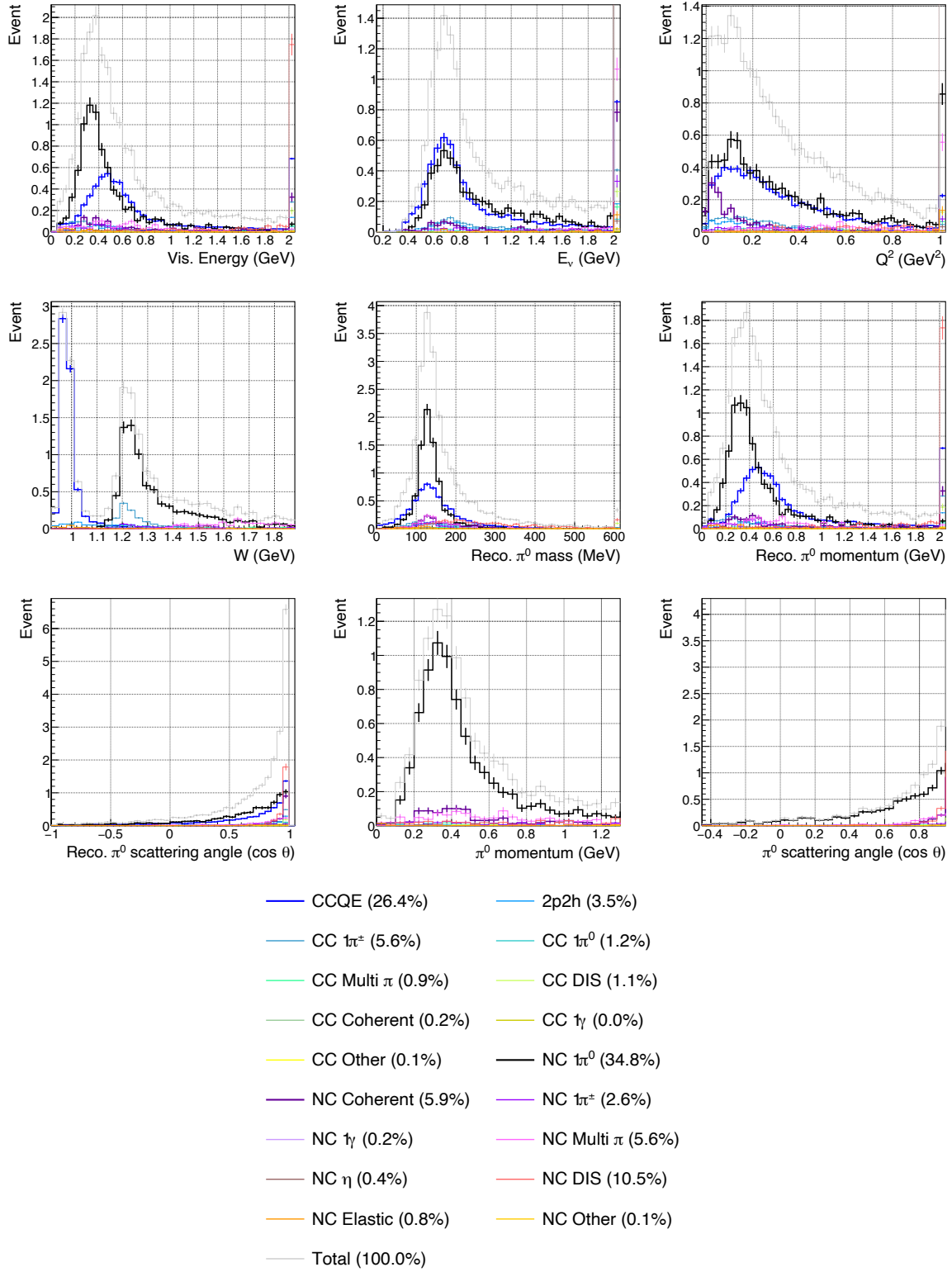
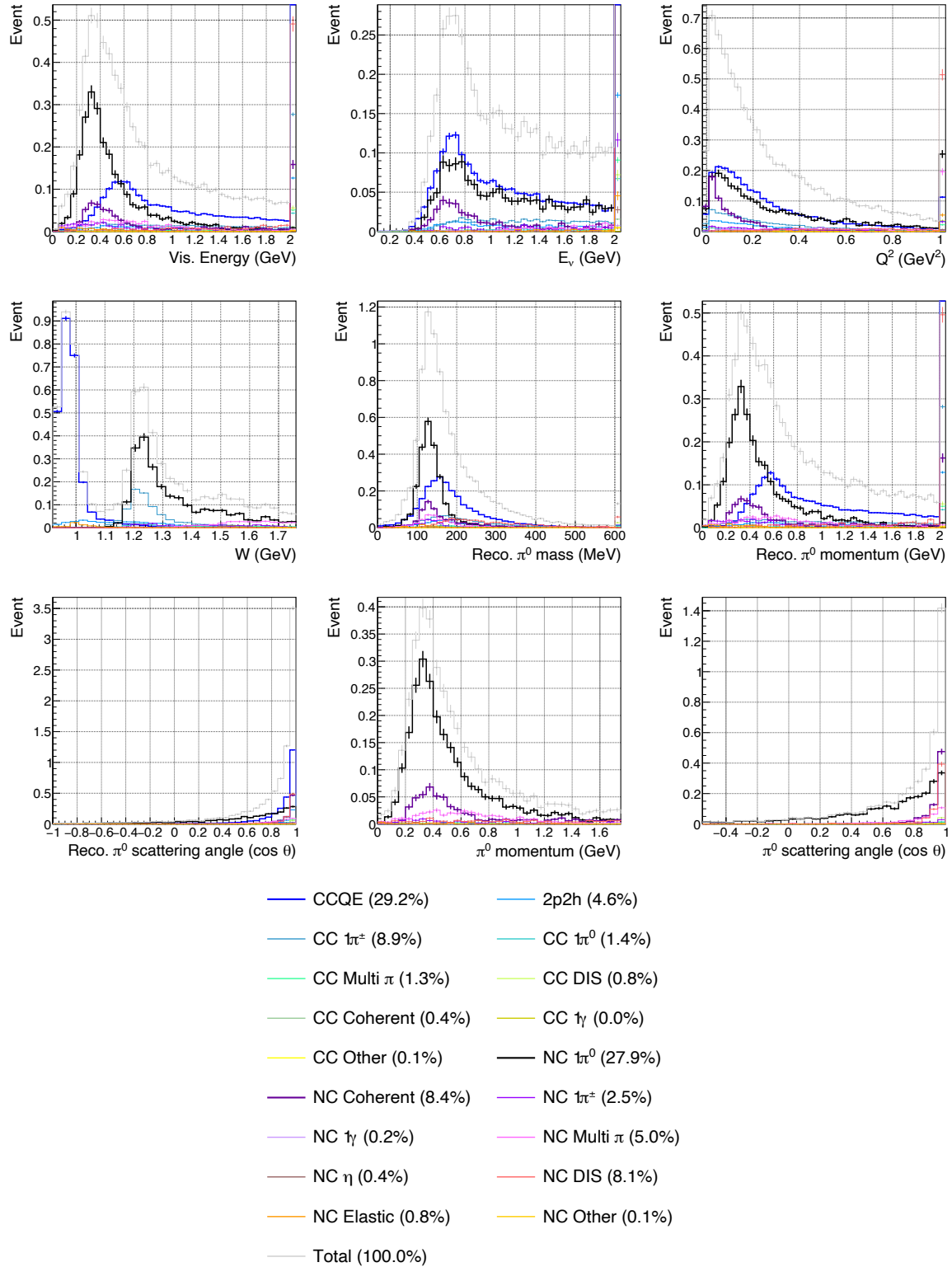


Figure B.17: The 2R π^0 sample by interaction modes in the ν beam mode.

Figure B.18: The 2R π^0 sample by interaction modes in the $\bar{\nu}$ beam mode.

Figure B.19: The 1R π^0 sample by interaction modes in the ν beam mode.

Figure B.20: The $1R \pi^0$ sample by interaction modes in the $\bar{\nu}$ beam mode.

Breakdown by Interaction Modes for Single Primary π^0 Events

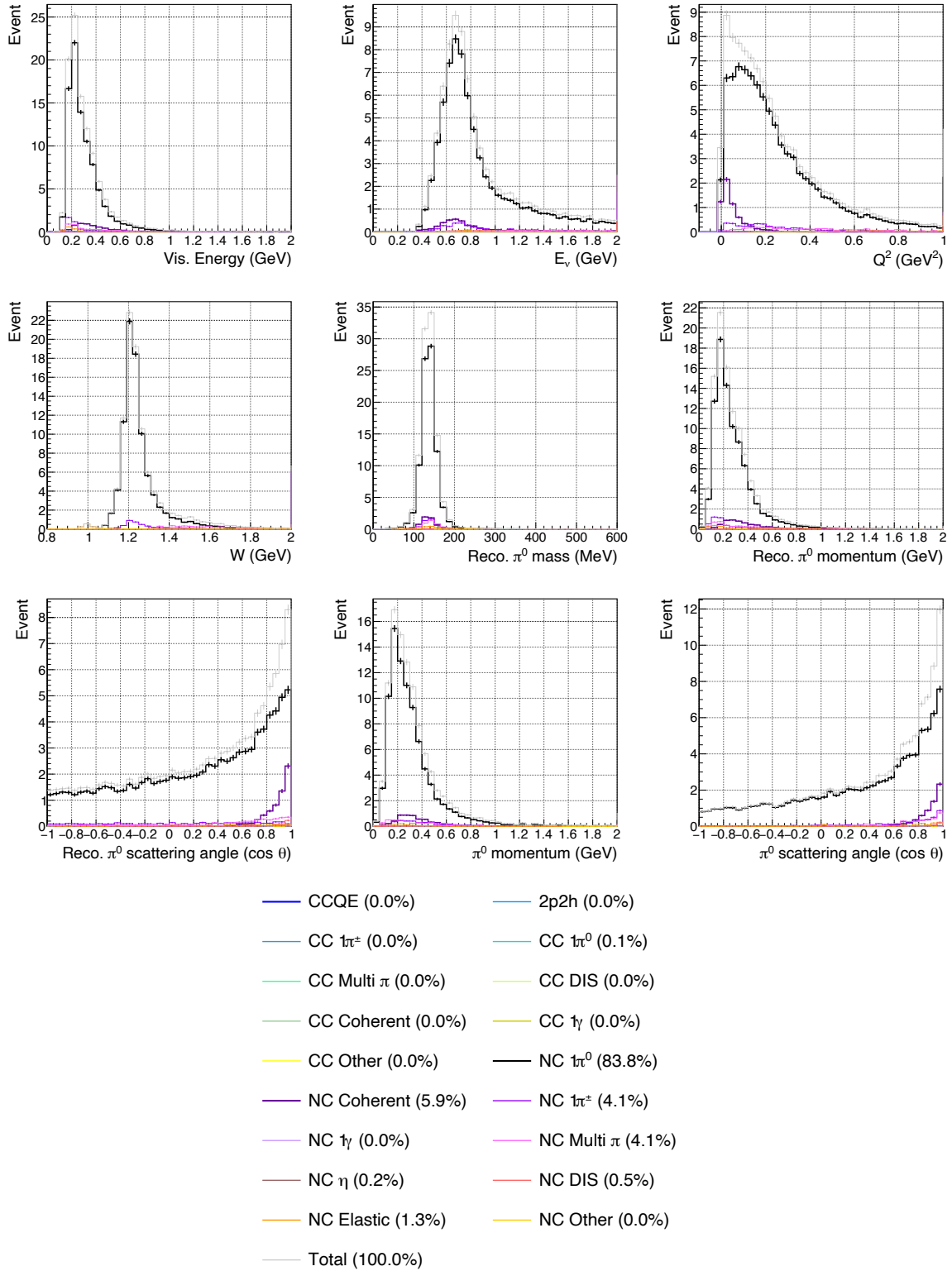


Figure B.21: The 2R π^0 sample by interaction modes with a single visible π^0 in the final state from the primary interaction vertex in the ν beam mode.

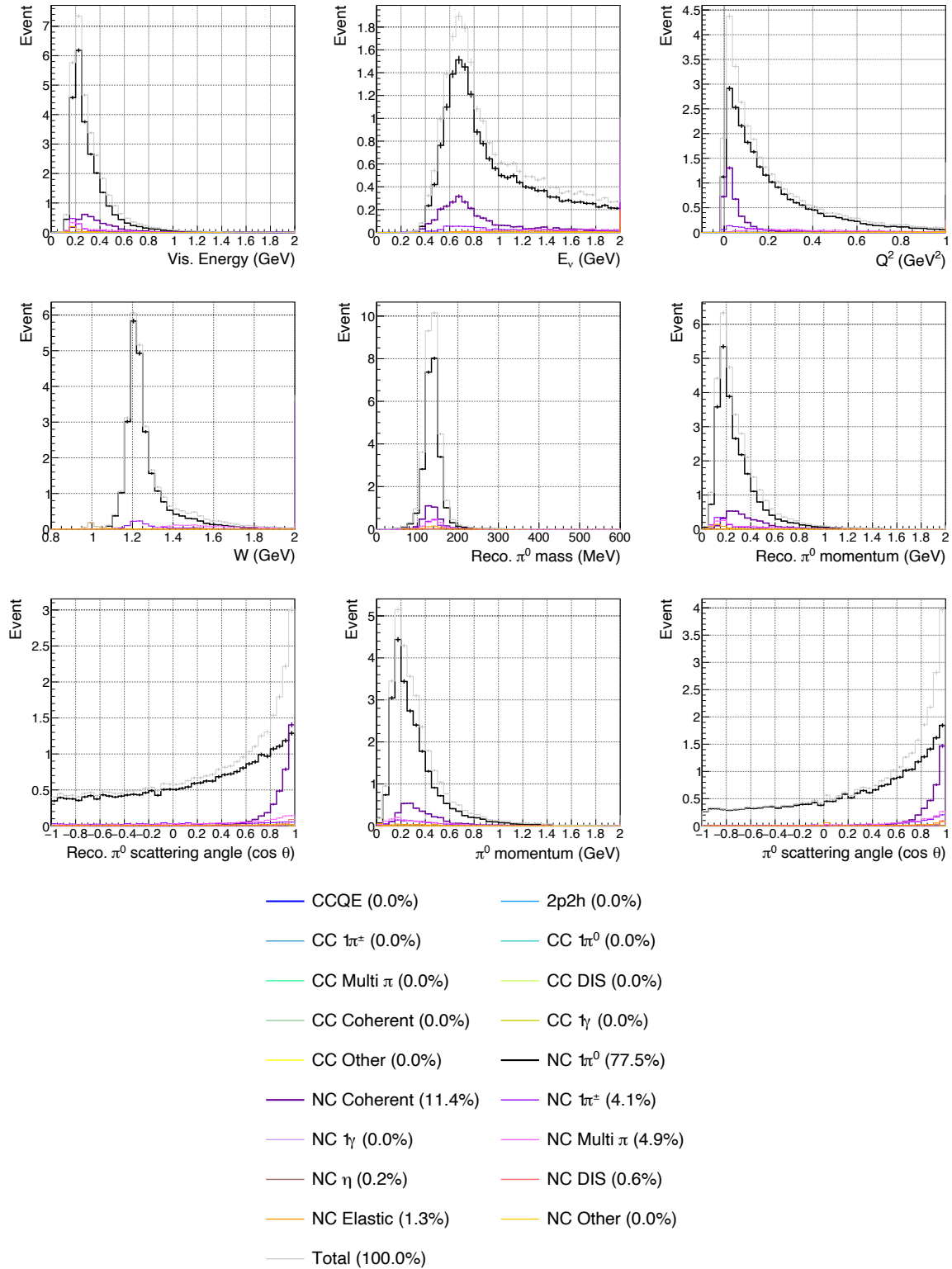


Figure B.22: The 2R π^0 sample by interaction modes with a single visible π^0 in the final state from the primary interaction vertex in the $\bar{\nu}$ beam mode.

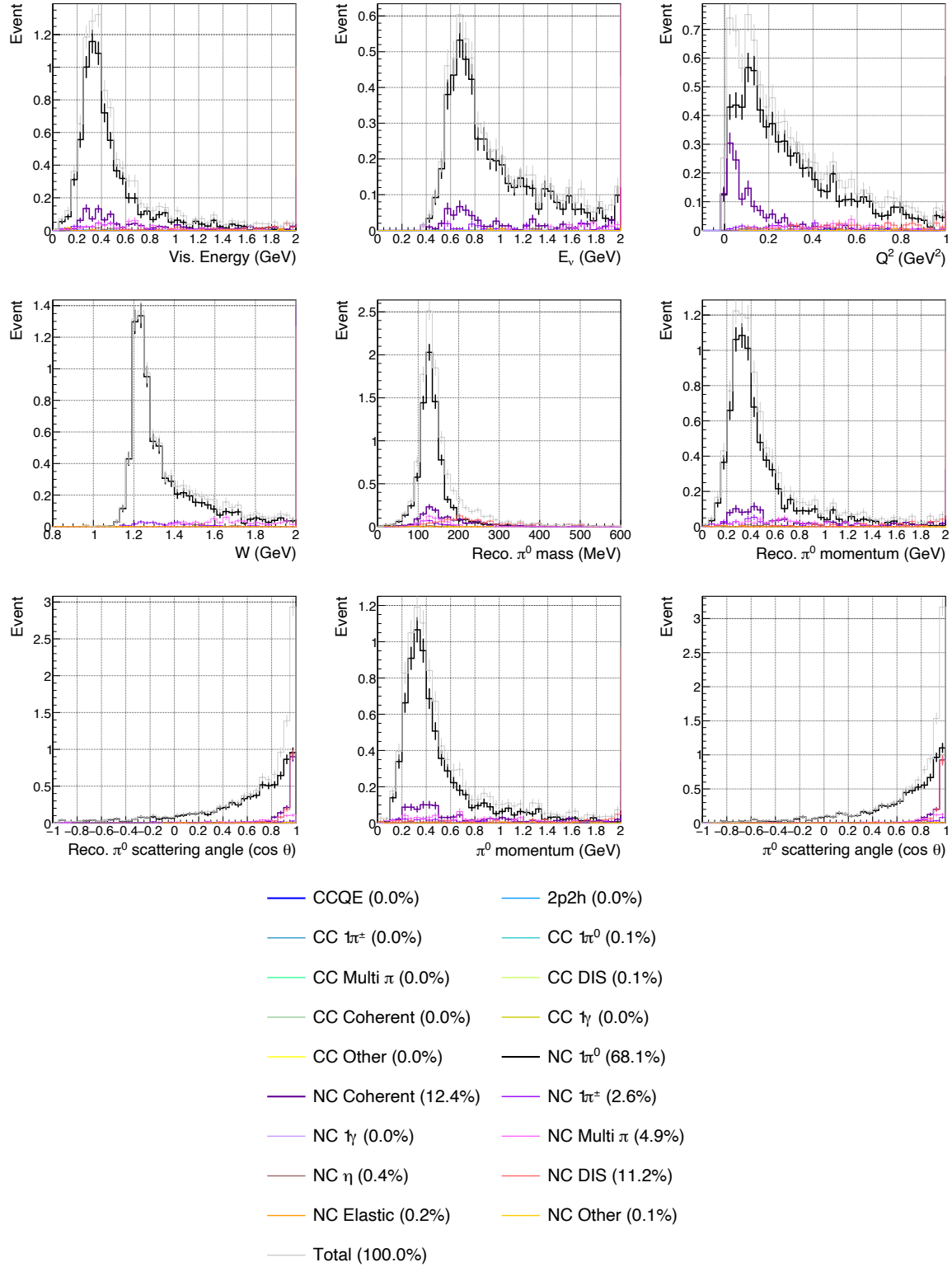


Figure B.23: The 1R π^0 sample by interaction modes with a single visible π^0 in the final state from the primary interaction vertex in the ν beam mode.

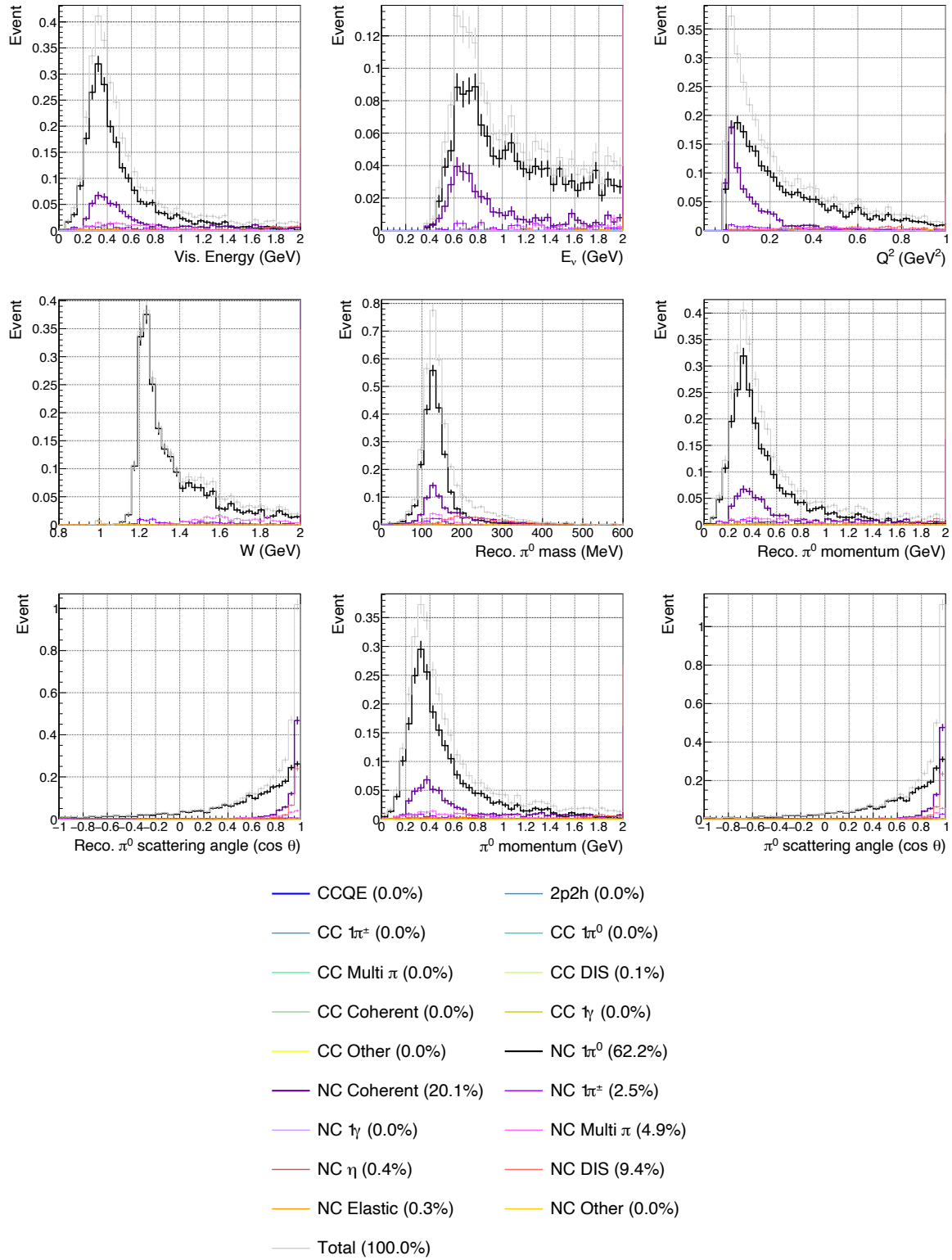


Figure B.24: The 1R π^0 sample by interaction modes with a single visible π^0 in the final state from the primary interaction vertex in the $\bar{\nu}$ beam mode.

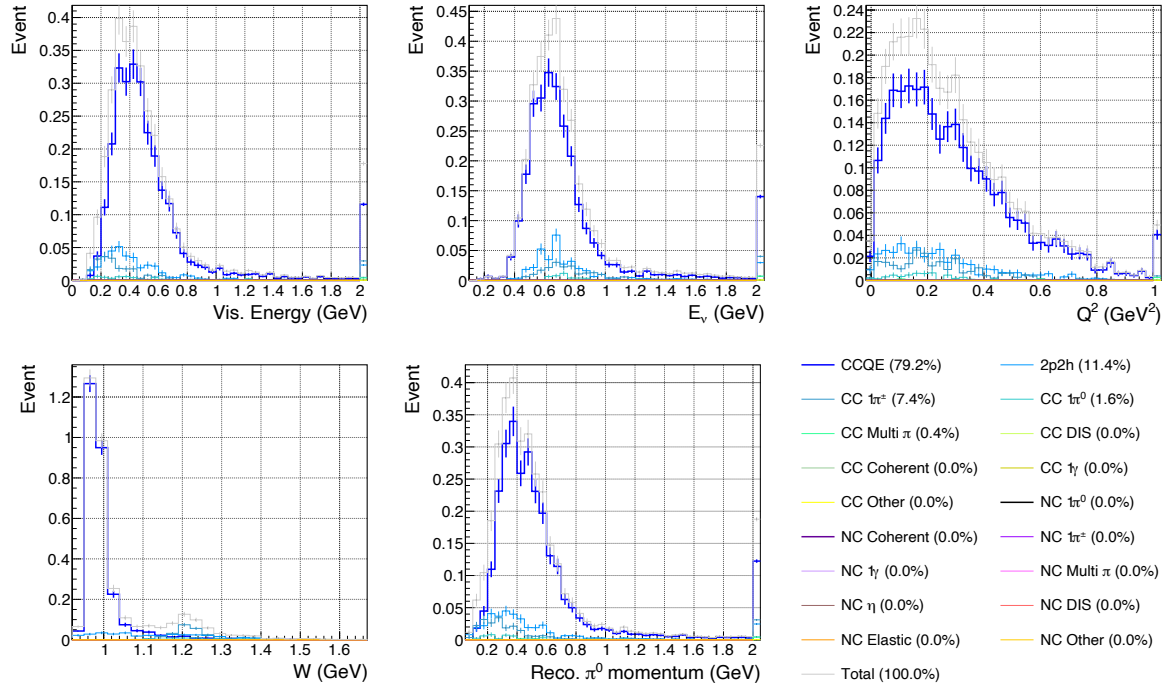
Breakdown by Interaction Modes for Single Primary e Events

Figure B.25: The $2R \pi^0$ sample by interaction modes with a single visible electron in the final state from the primary interaction vertex in the ν beam mode.

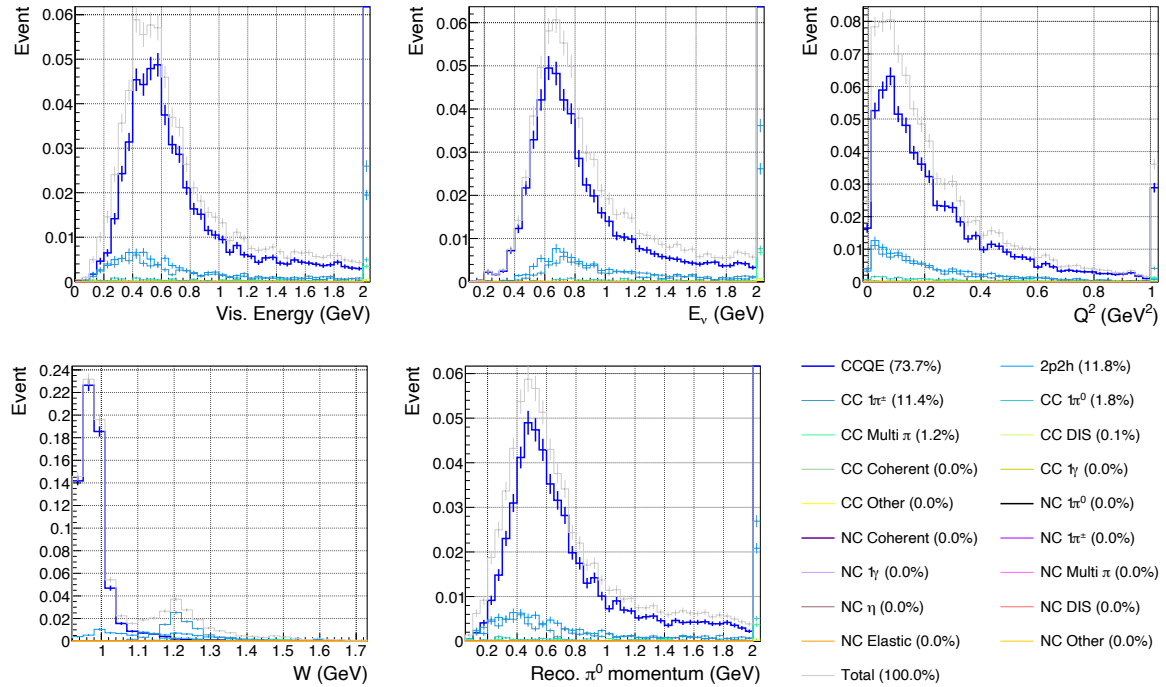


Figure B.26: The $2R \pi^0$ sample by interaction modes with a single visible electron in the final state from the primary interaction vertex in the $\bar{\nu}$ beam mode.

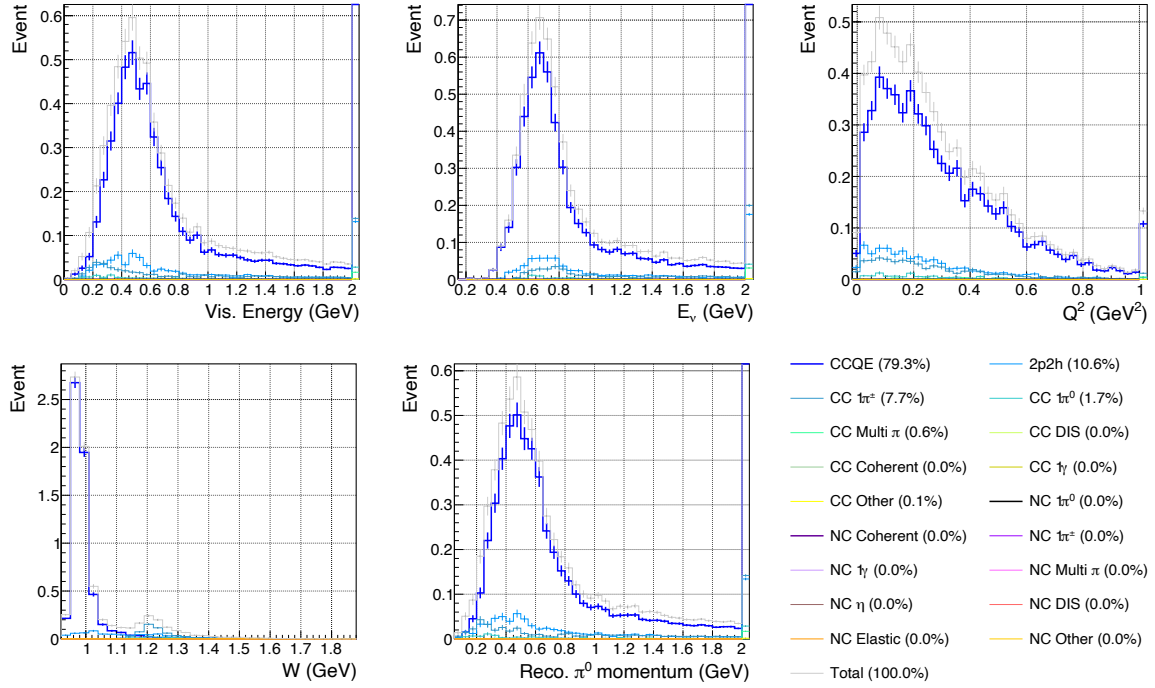


Figure B.27: The $1R \pi^0$ sample by interaction modes with a single visible electron in the final state from the primary interaction vertex in the ν beam mode.

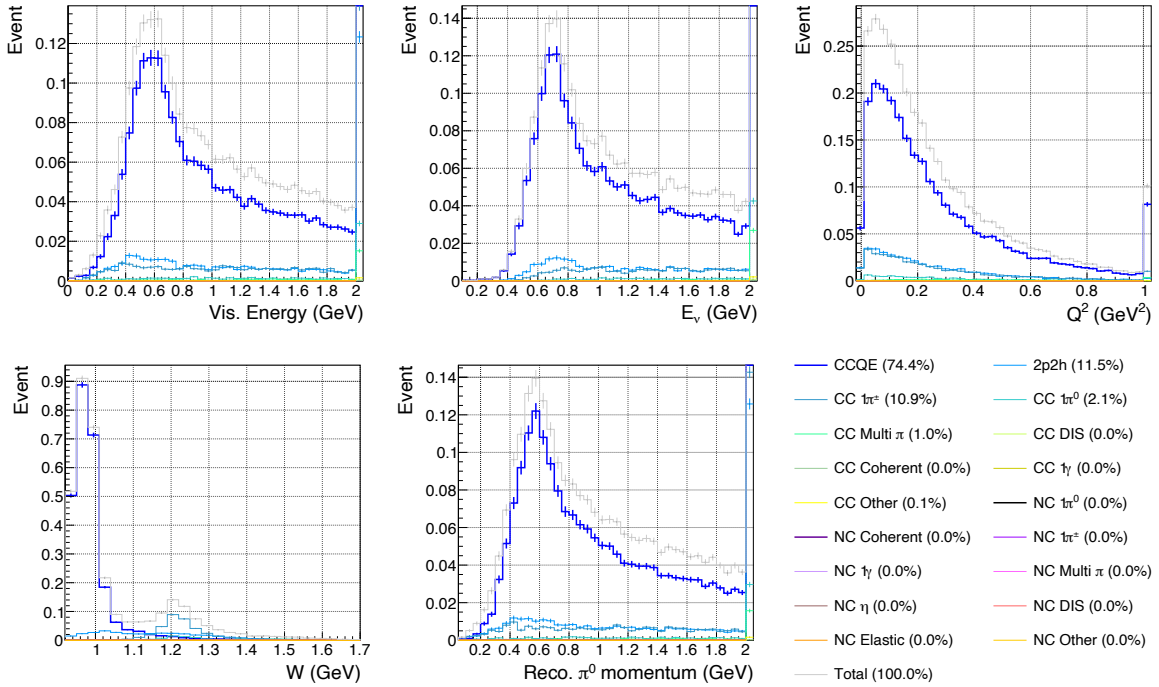


Figure B.28: The $1R \pi^0$ sample by interaction modes with a single visible electron in the final state from the primary interaction vertex in the $\bar{\nu}$ beam mode.

Breakdown by Interaction Modes All Background Topologies (no $1e$, $1\pi^0$)

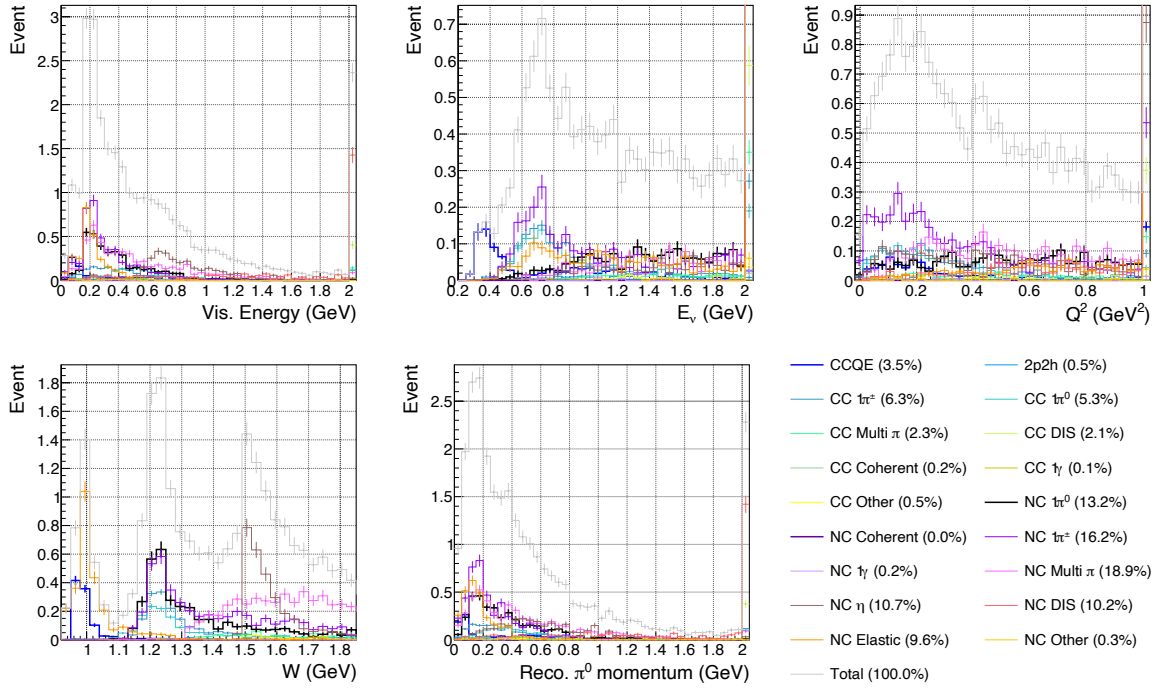


Figure B.29: The $2R \pi^0$ sample by interaction modes from background topologies in the ν beam mode.

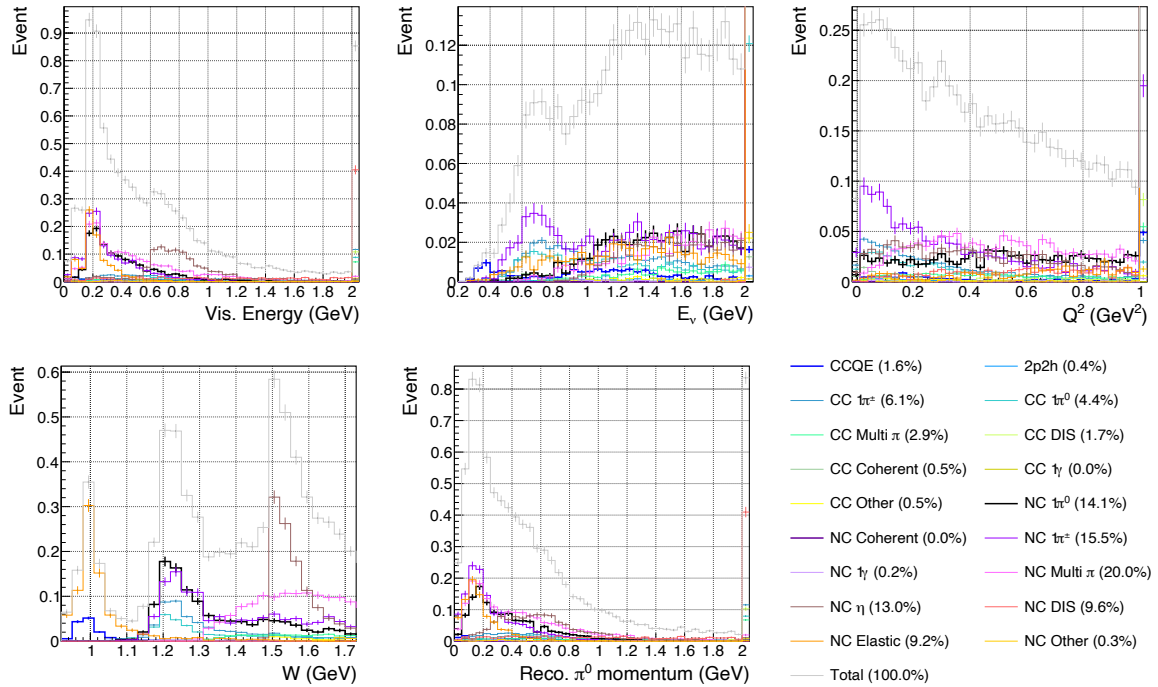


Figure B.30: The $2R \pi^0$ sample by interaction modes from background topologies in the $\bar{\nu}$ beam mode.

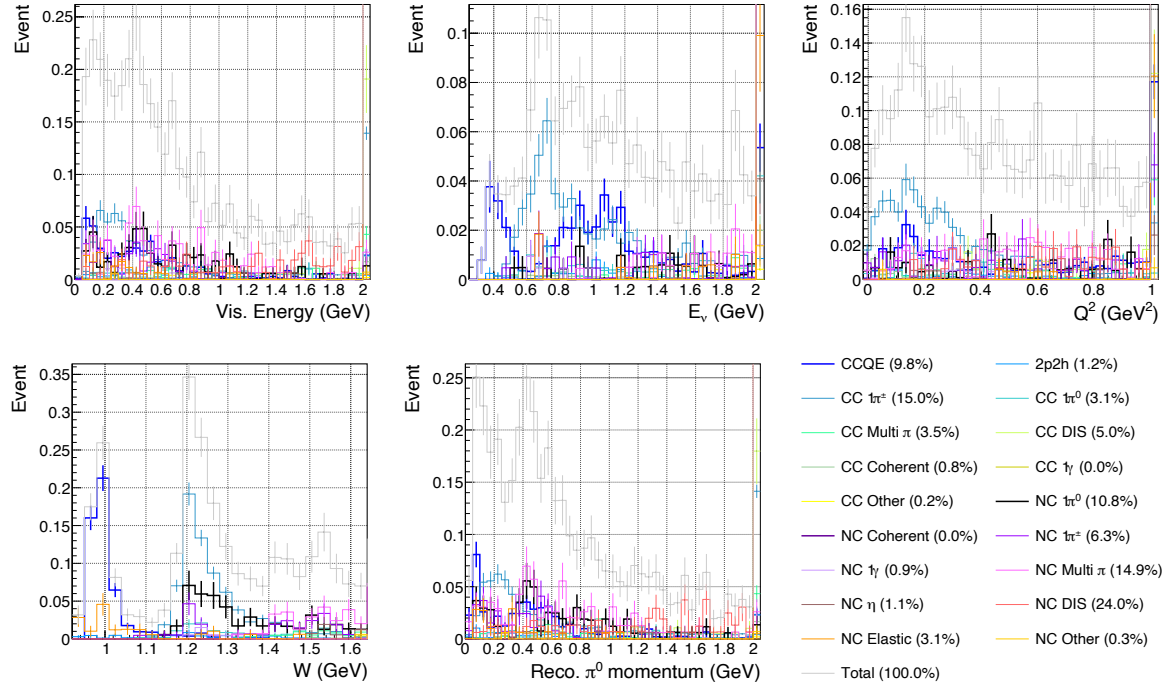


Figure B.31: The 1R π^0 sample by interaction modes from background topologies in the ν beam mode.

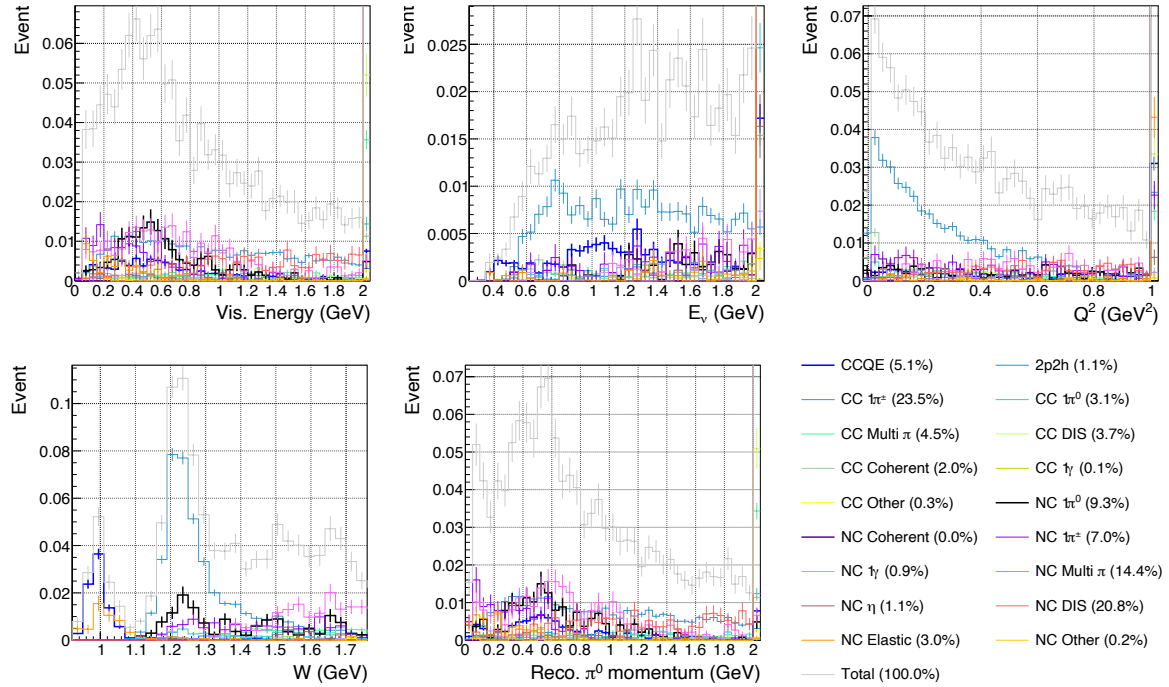


Figure B.32: The 1R π^0 sample by interaction modes from background topologies in the $\bar{\nu}$ beam mode.

B.2.2 Breakdown by Primary Topology

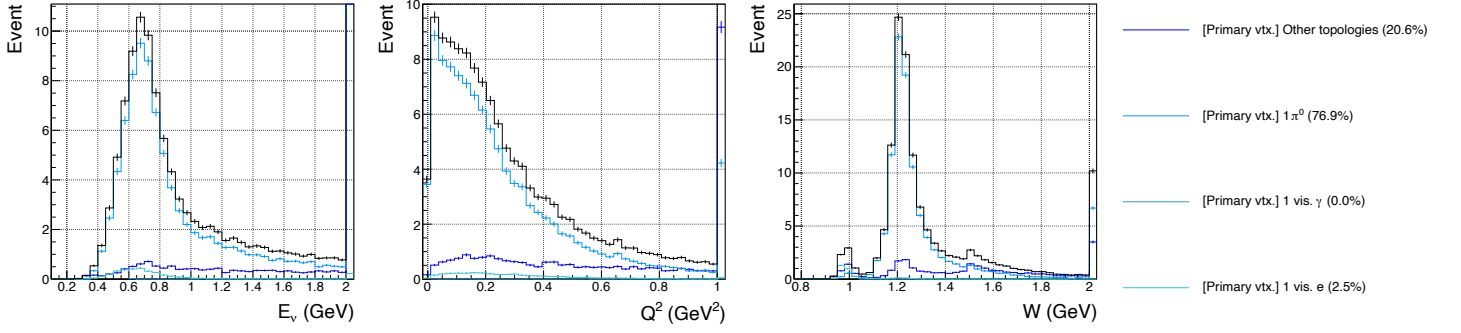


Figure B.33: The 2R π^0 sample by primary final state topologies in the ν beam mode.

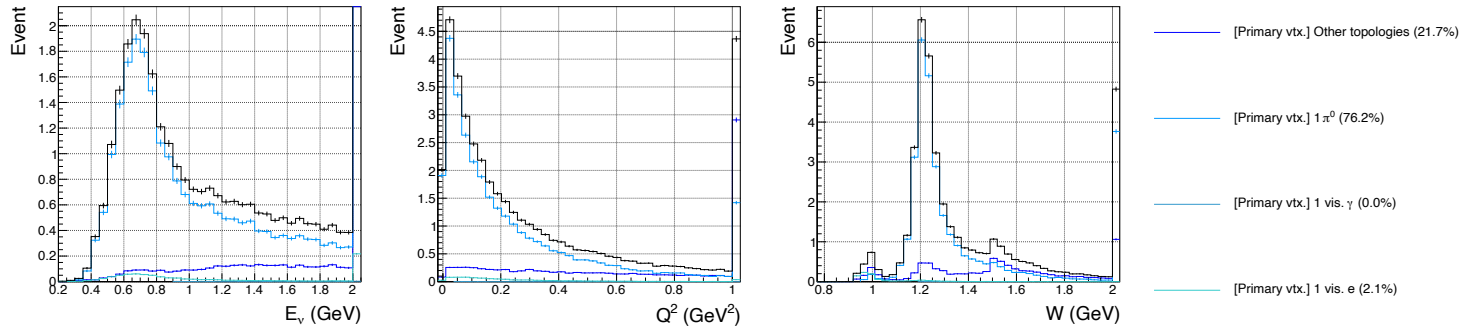


Figure B.34: The 2R π^0 sample by primary final state topologies in the $\bar{\nu}$ beam mode.

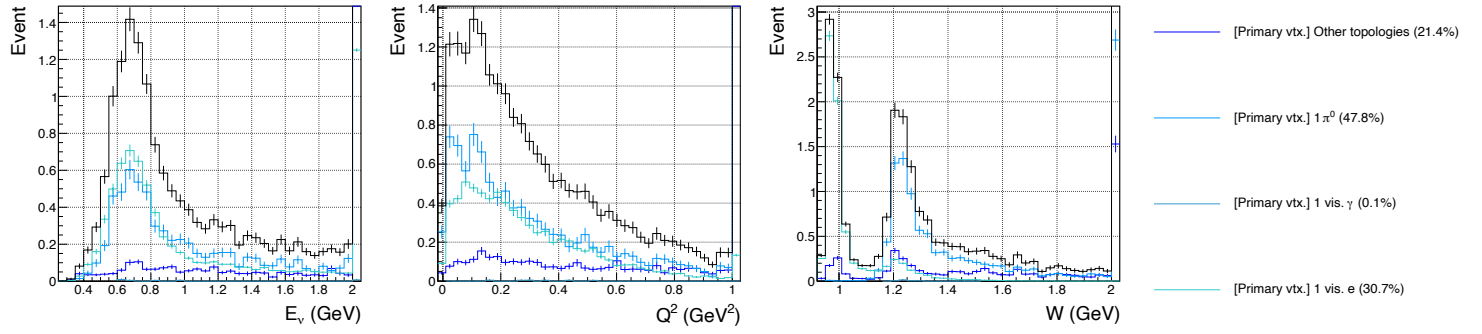


Figure B.35: The 1R π^0 sample by primary final state topologies in the ν beam mode.

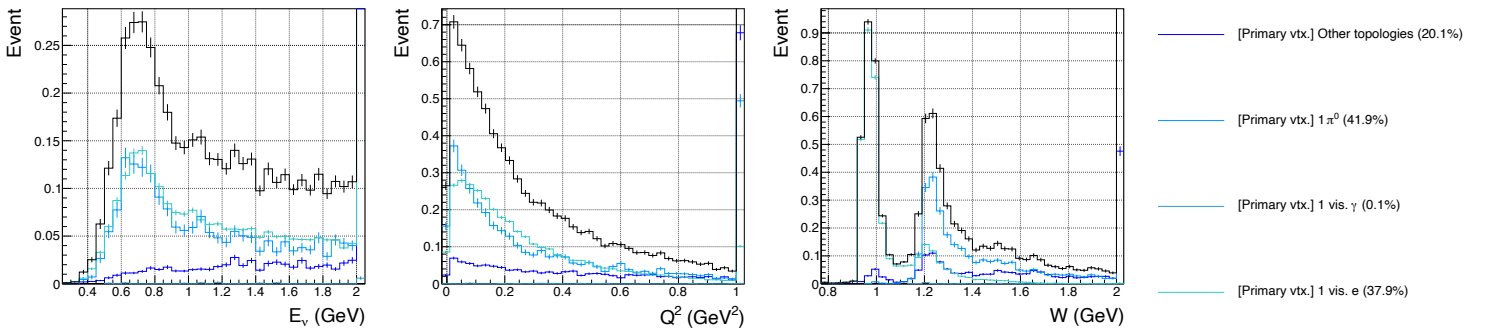


Figure B.36: The 1R π^0 sample by primary final state topologies in the $\bar{\nu}$ beam mode.

Breakdown by Primary- π^0 with 2-Secondary- γ Topology

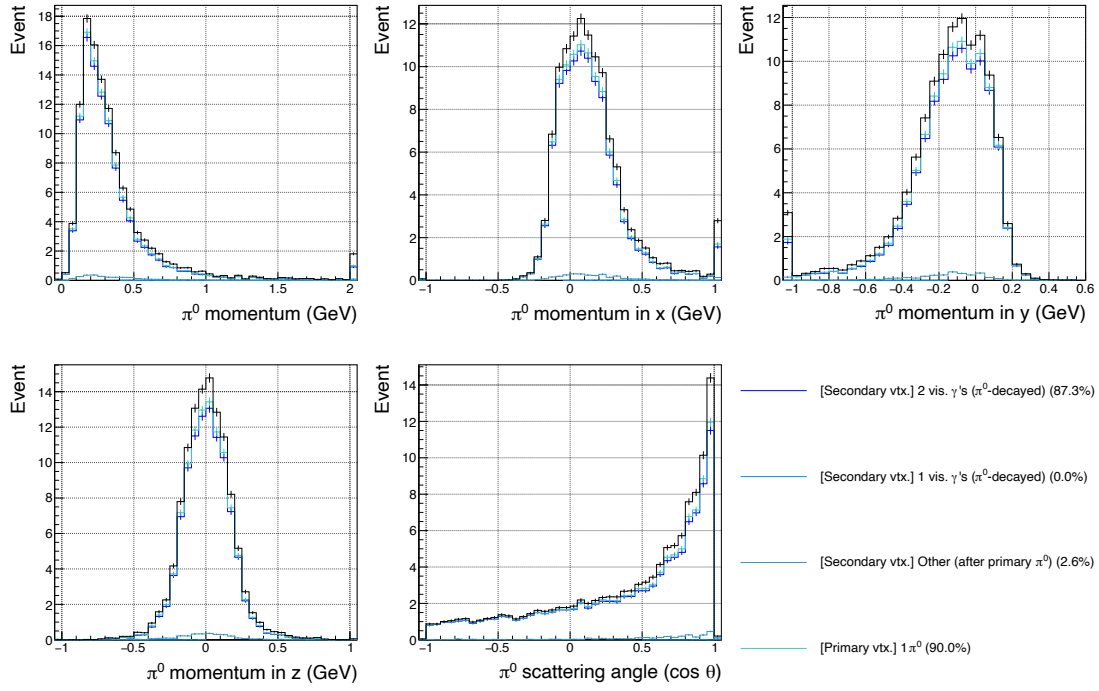


Figure B.37: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with π^0 kinematics).

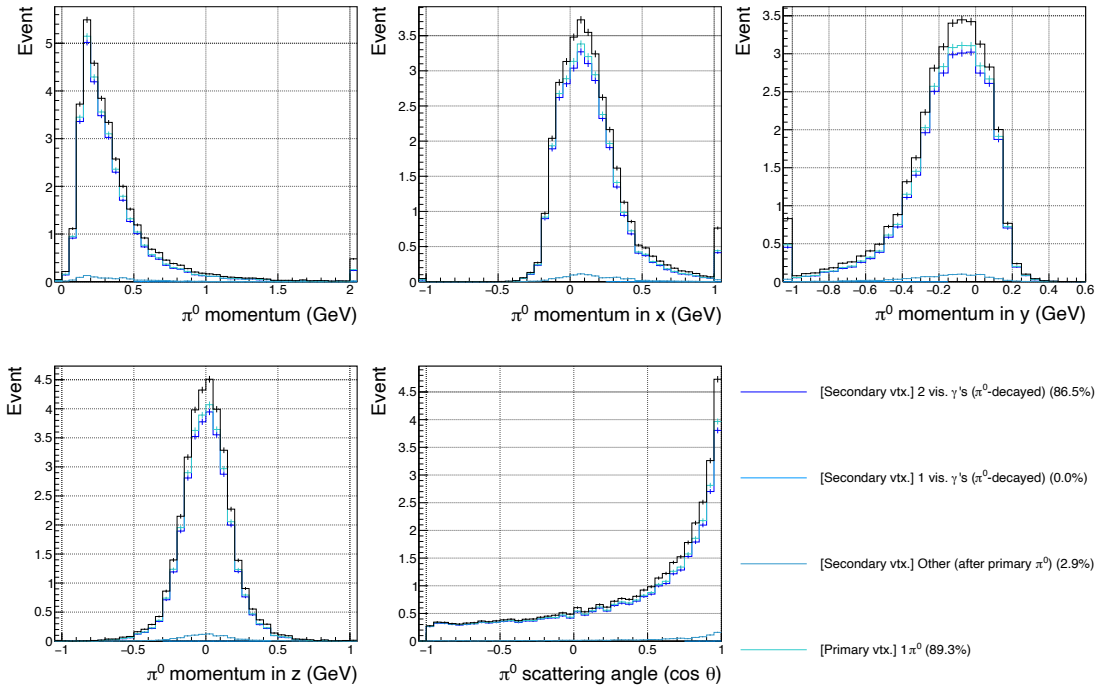


Figure B.38: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with π^0 kinematics).

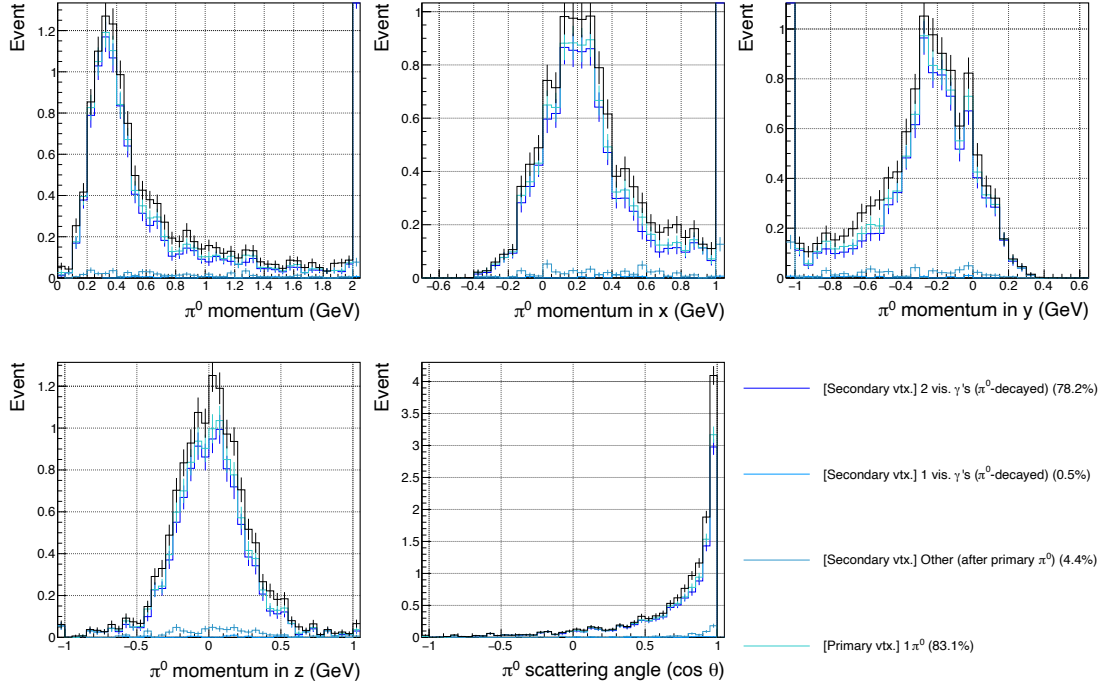


Figure B.39: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with π^0 kinematics).

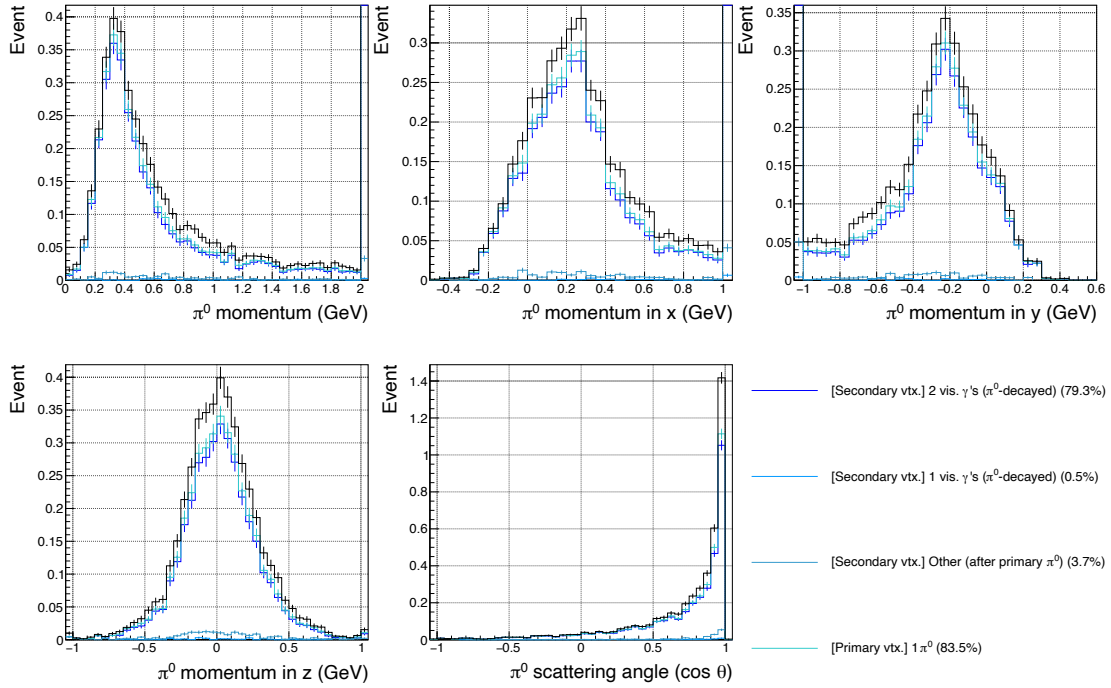


Figure B.40: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with π^0 kinematics).

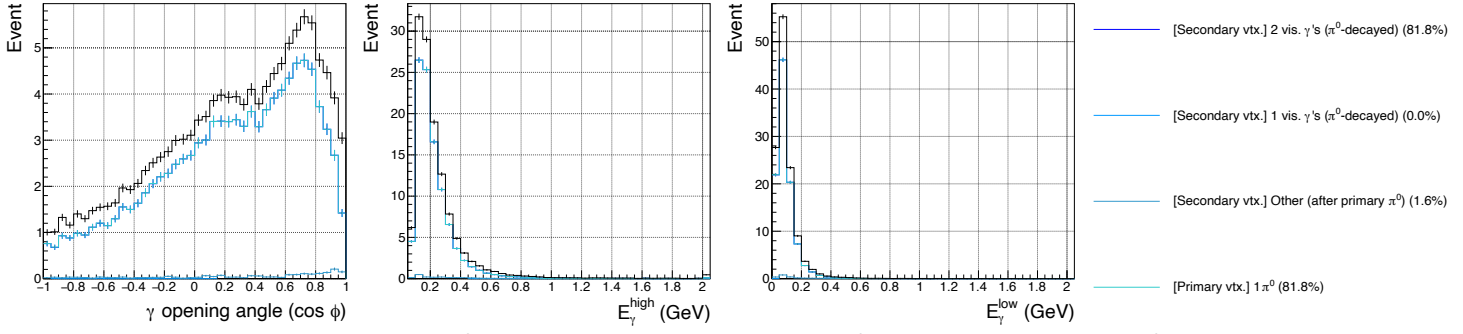


Figure B.41: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with 2- γ kinematics).

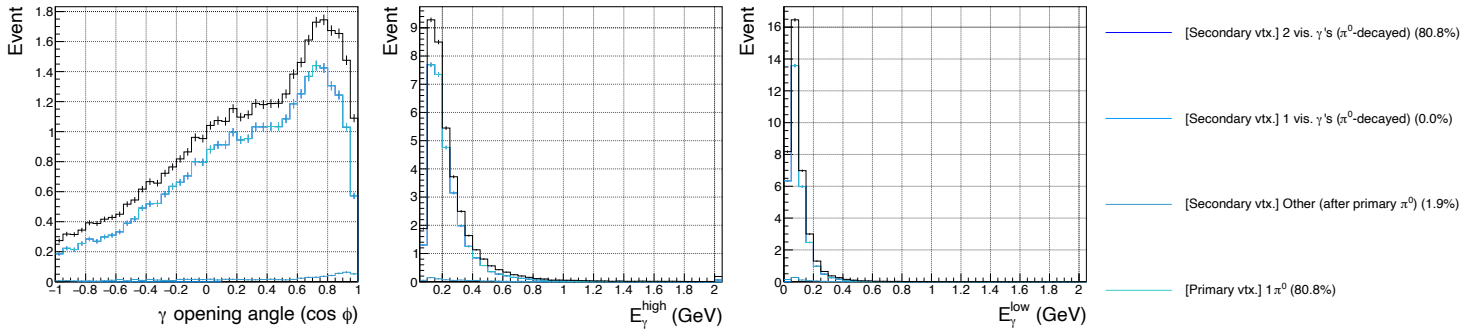


Figure B.42: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with 2- γ kinematics).

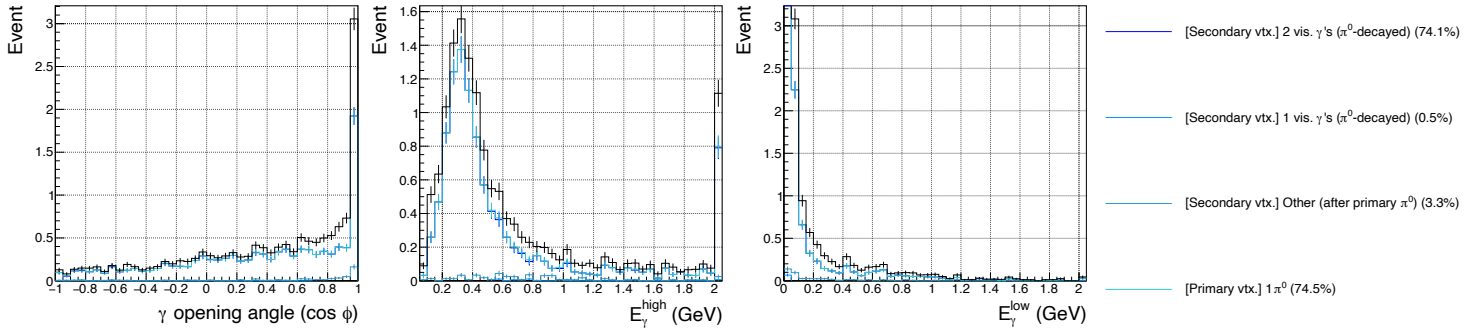


Figure B.43: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with 2- γ kinematics).

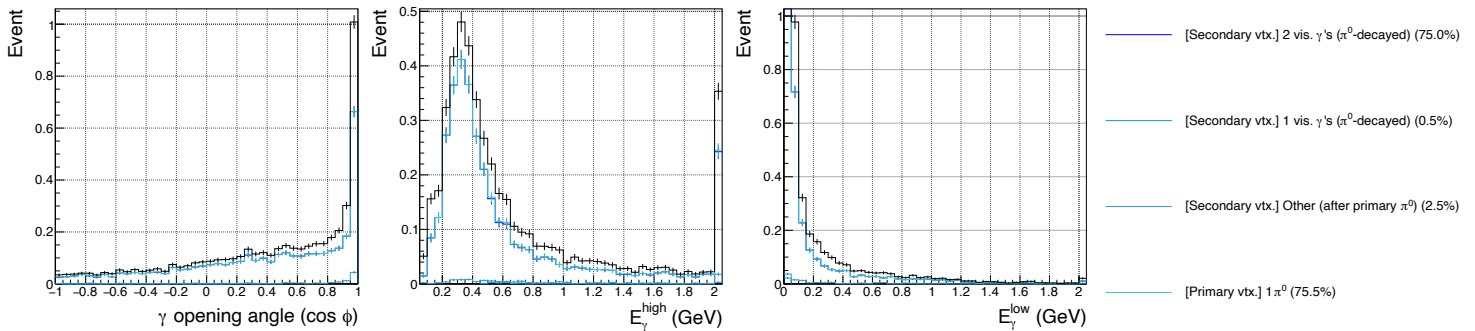


Figure B.44: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with 2- γ kinematics).

Breakdown by Topology in Reconstructed Kinematics

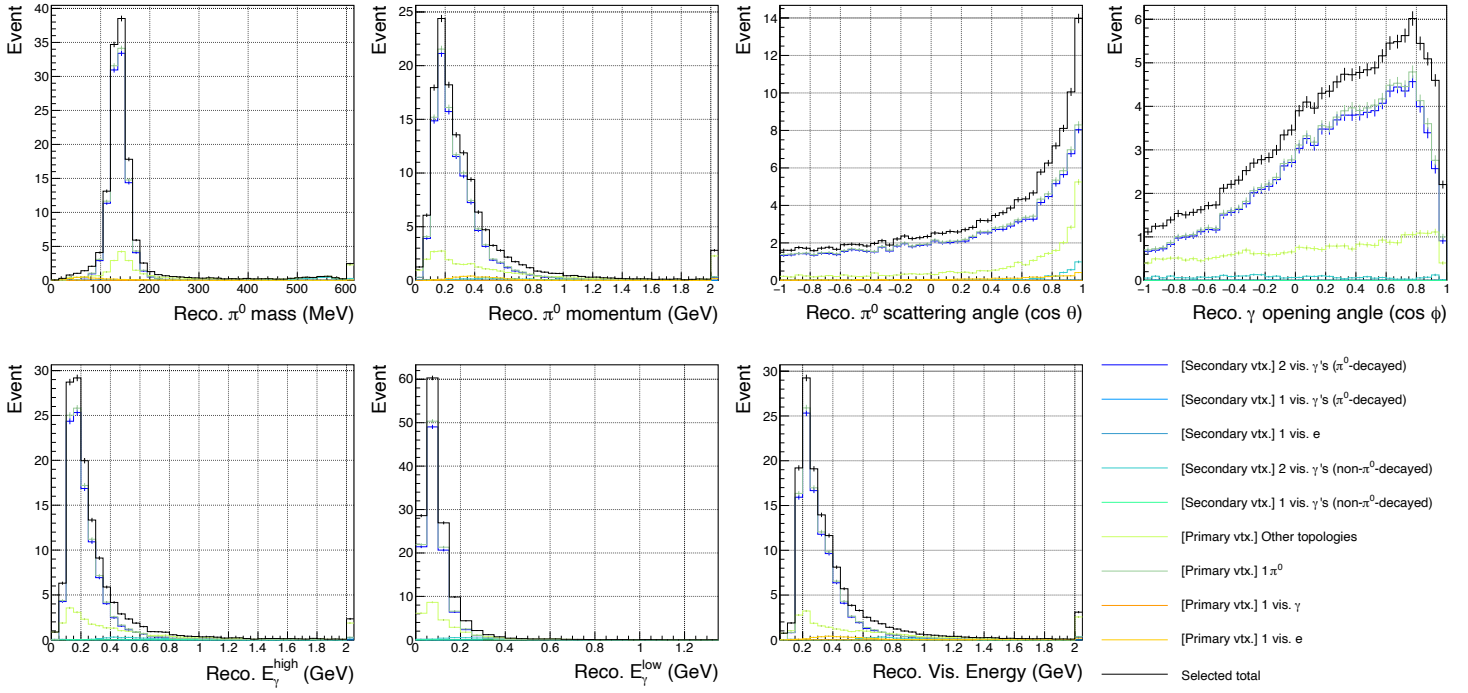


Figure B.45: The $2R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

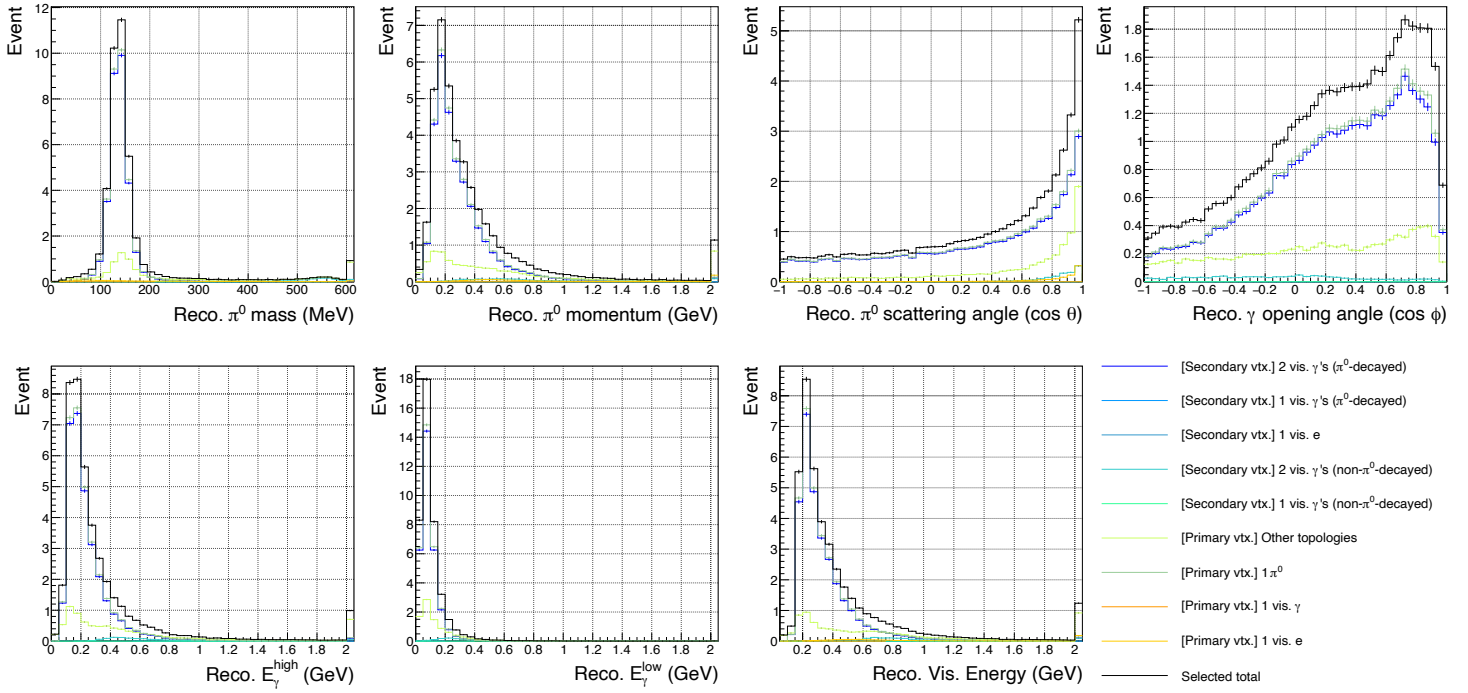


Figure B.46: The $2R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

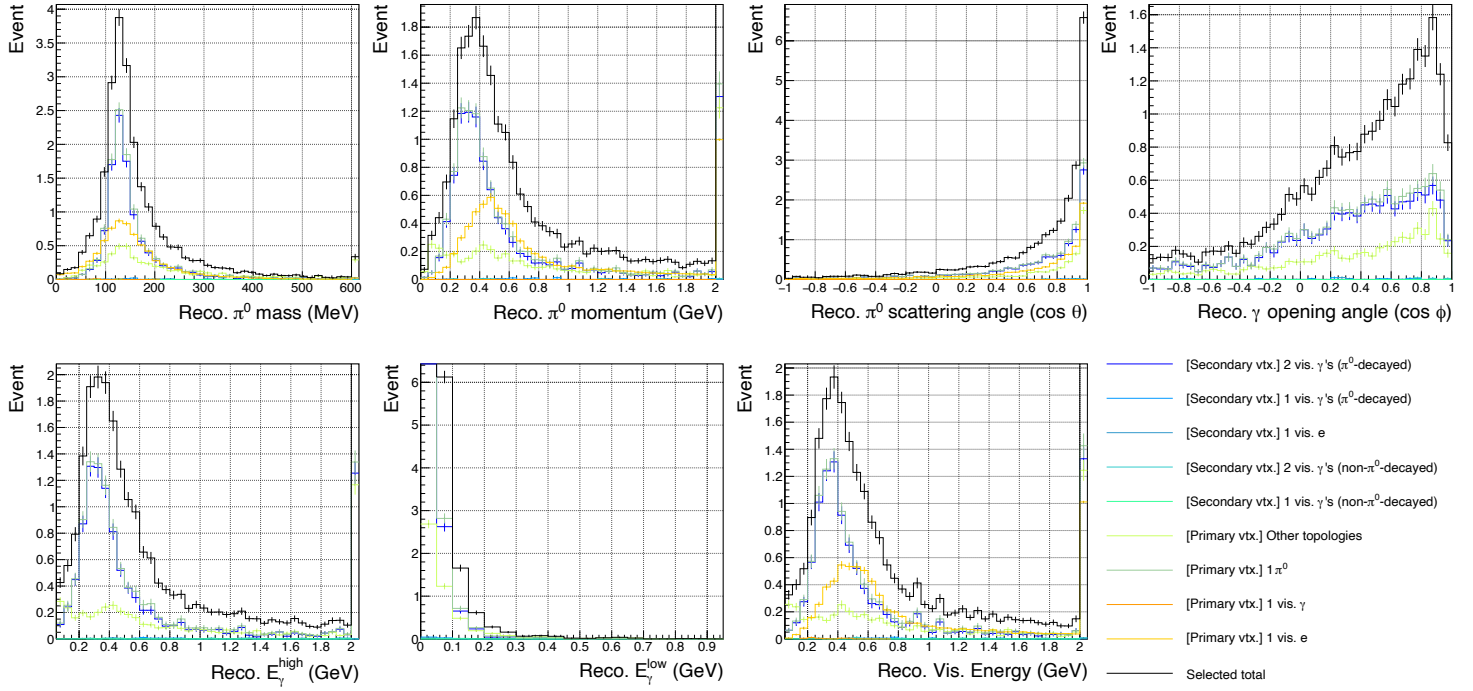


Figure B.47: The 1R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

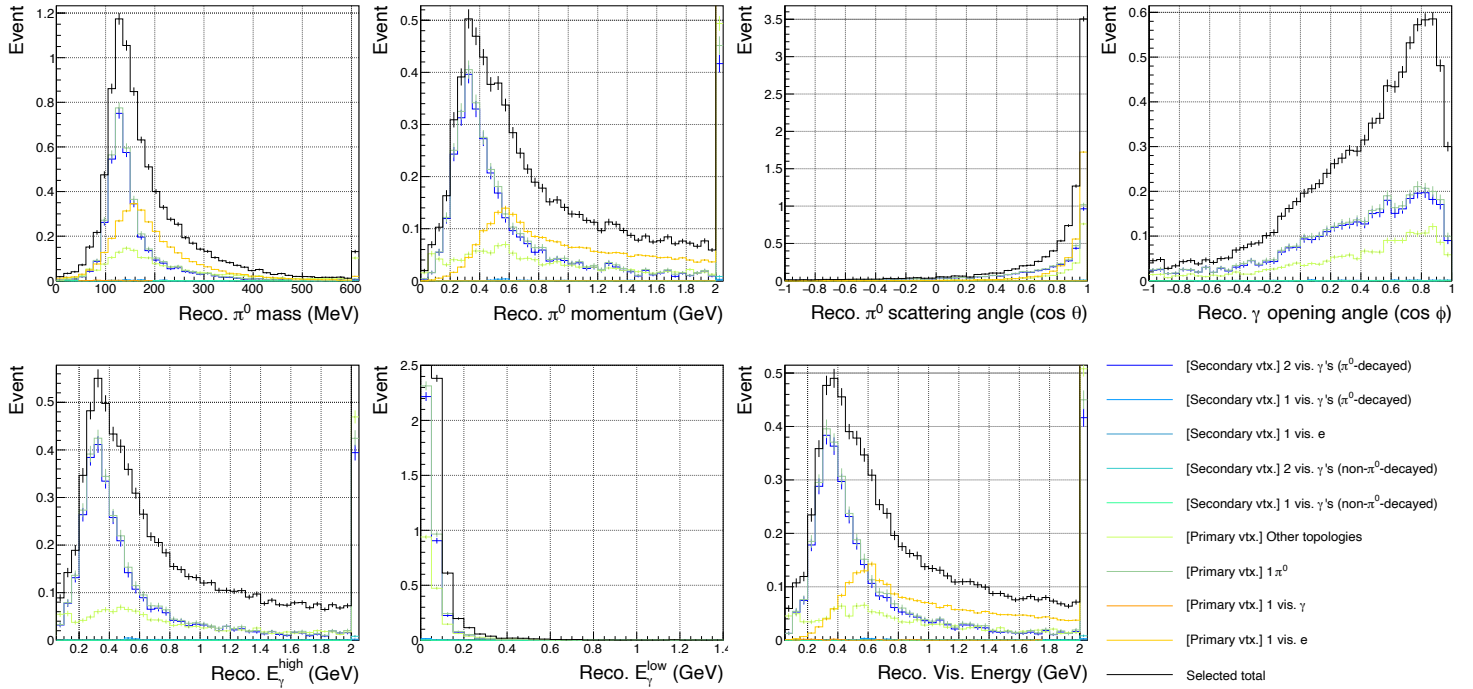


Figure B.48: The 1R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

Breakdown by Topology with True $\text{NC}\pi^0$ Modes

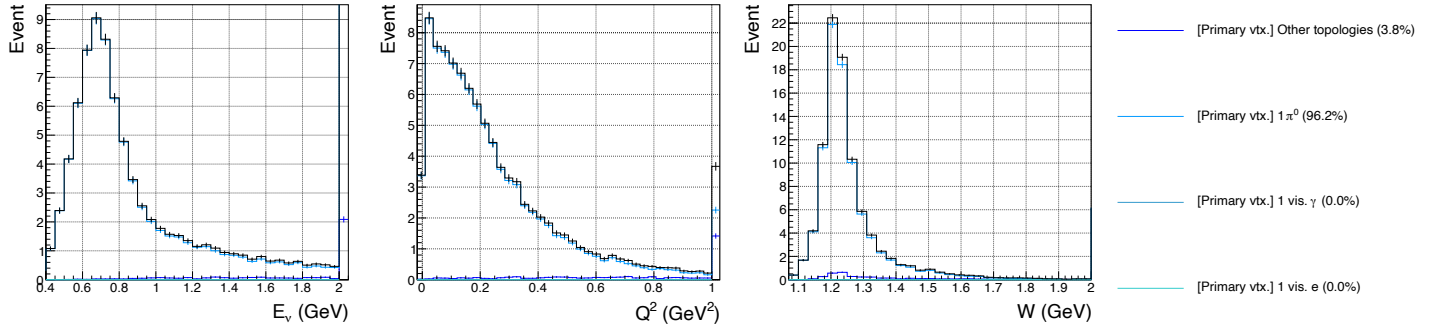


Figure B.49: The 2R true $\text{NC}\pi^0$ (+ NC Coh) events by primary final state topologies in the ν beam mode.

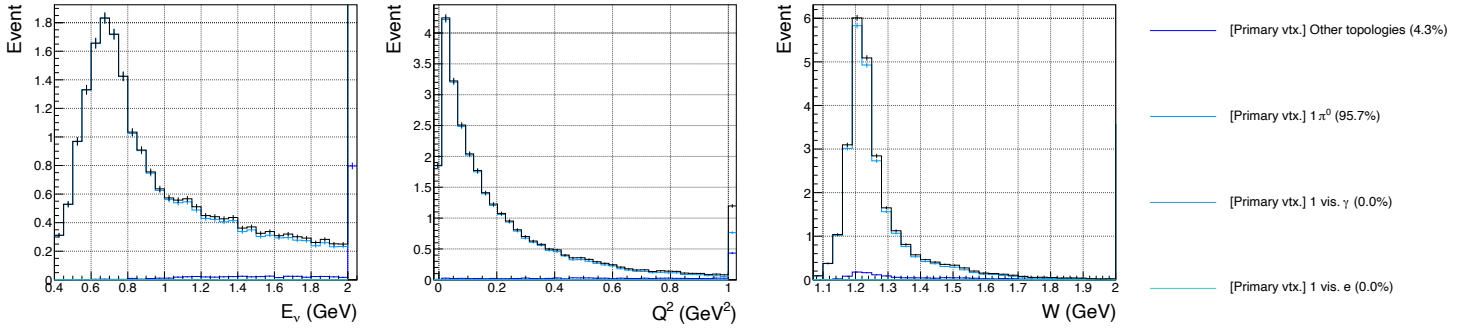


Figure B.50: The true $\text{NC}\pi^0$ (+ NC Coh) events by primary final state topologies in the $\bar{\nu}$ beam mode.

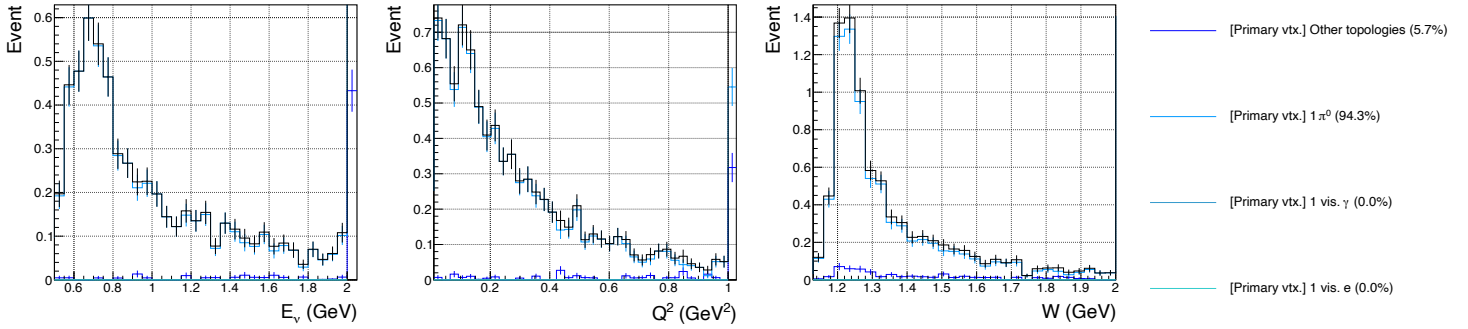


Figure B.51: The 1R true $\text{NC}\pi^0$ (+ NC Coh) events by primary final state topologies in the ν beam mode.

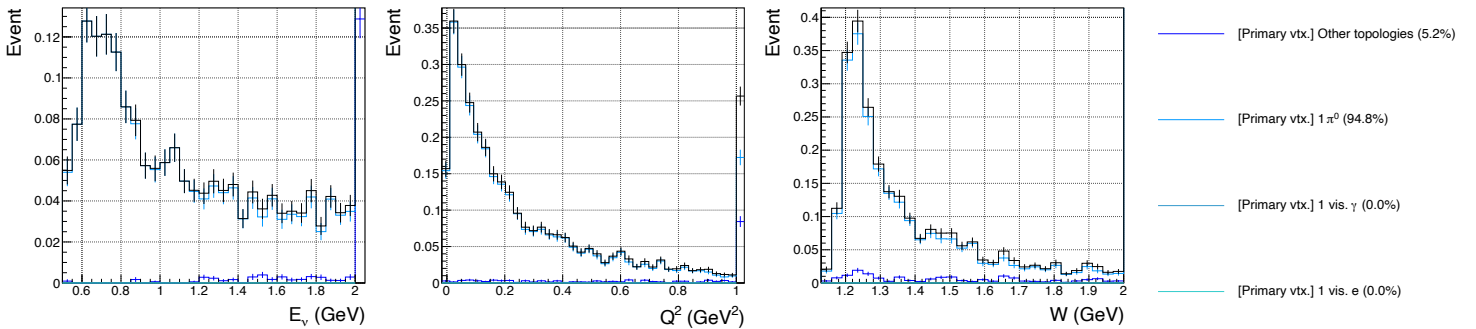


Figure B.52: The 1R true $\text{NC}\pi^0$ (+ NC Coh) events by primary final state topologies in the $\bar{\nu}$ beam mode.

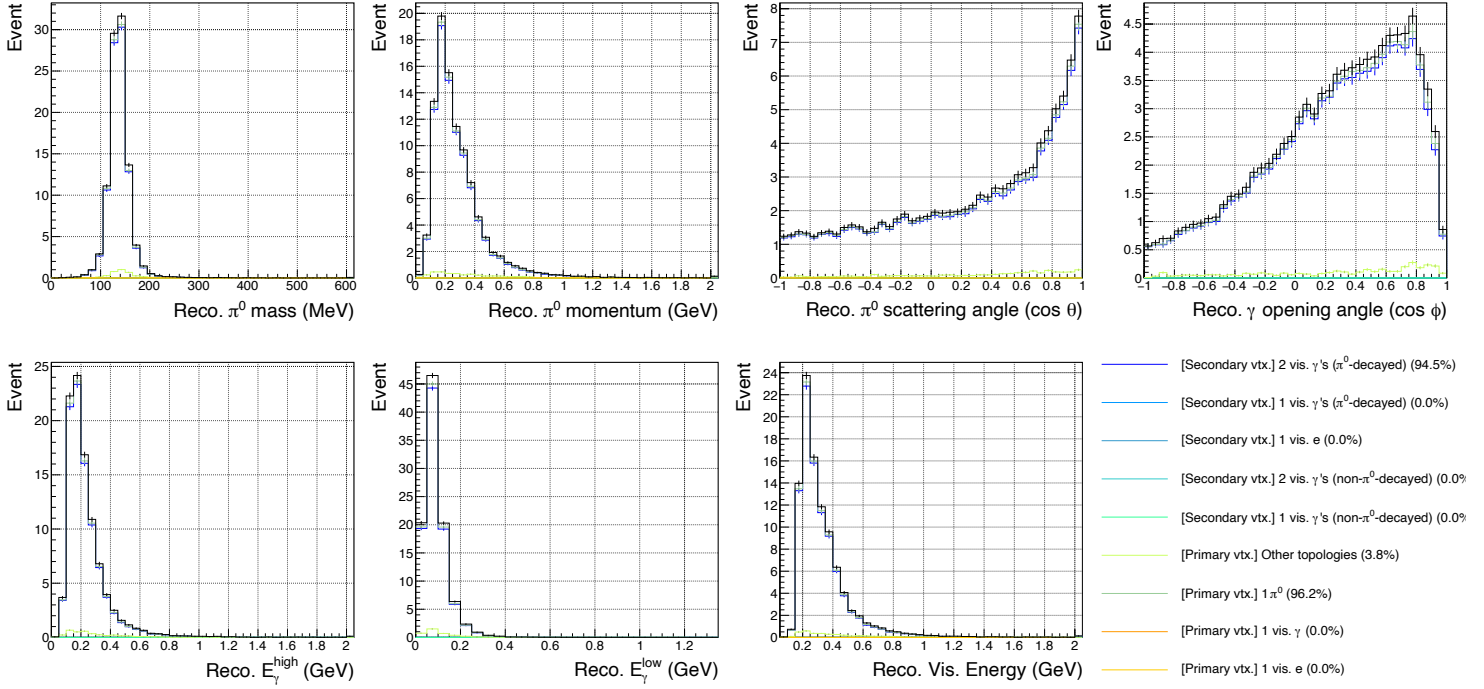


Figure B.53: The 2R true $\text{NC}\pi^0$ (+ NC Coh) events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

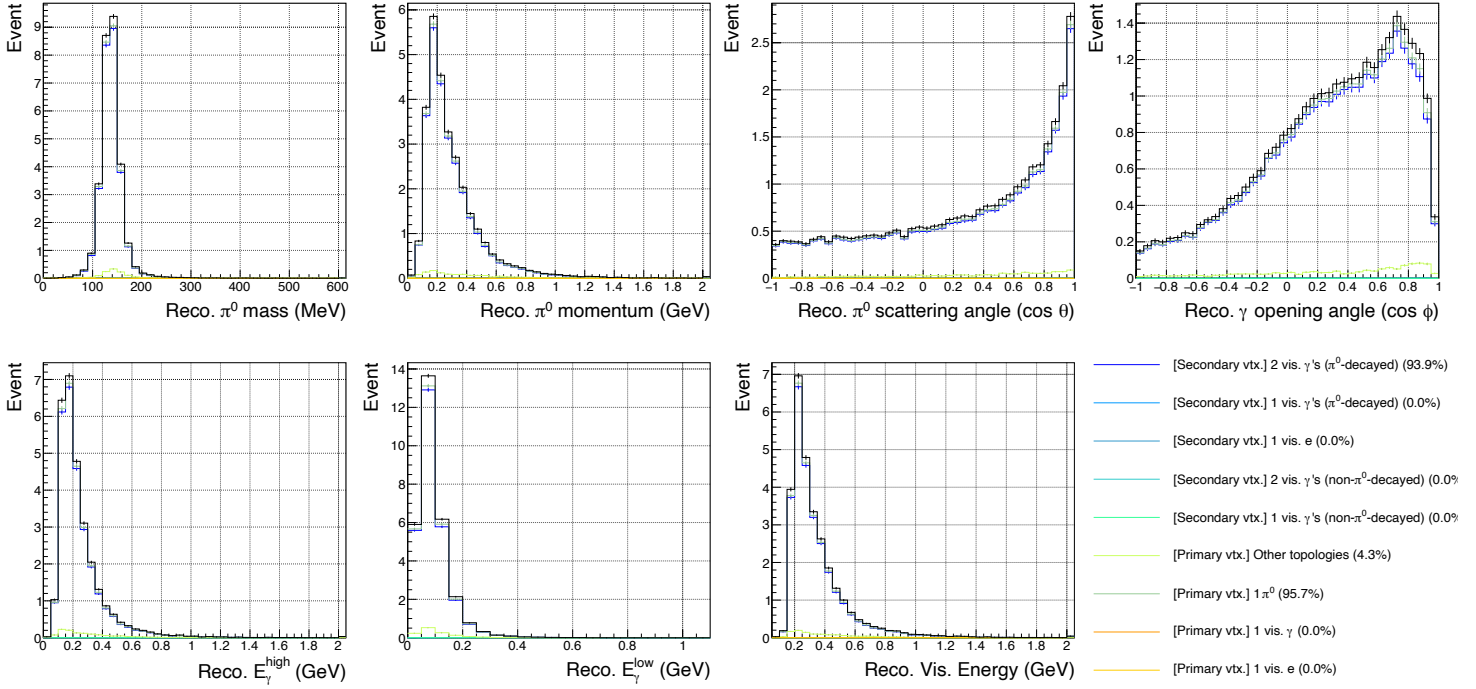


Figure B.54: The 2R true $\text{NC}\pi^0$ (+ NC Coh) events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

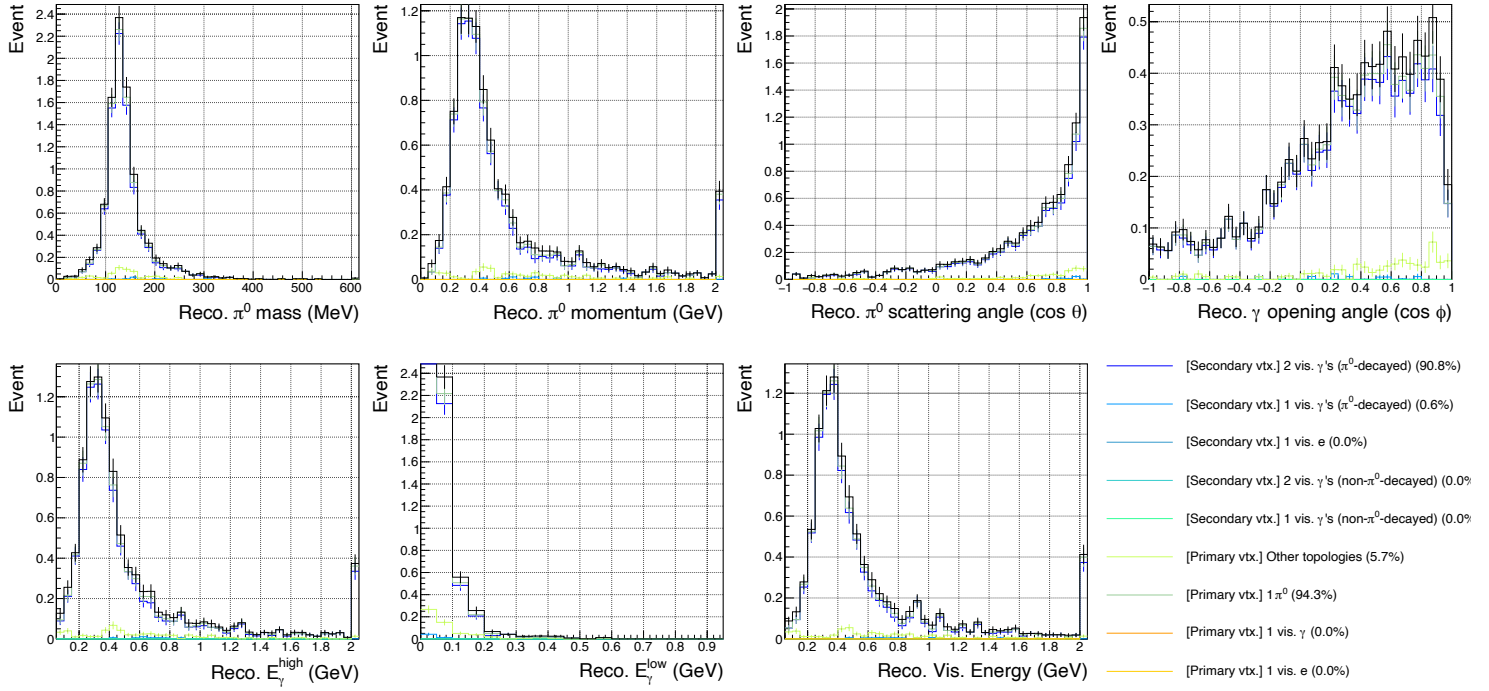


Figure B.55: The 1R true $\text{NC}\pi^0$ (+ NC Coh) events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

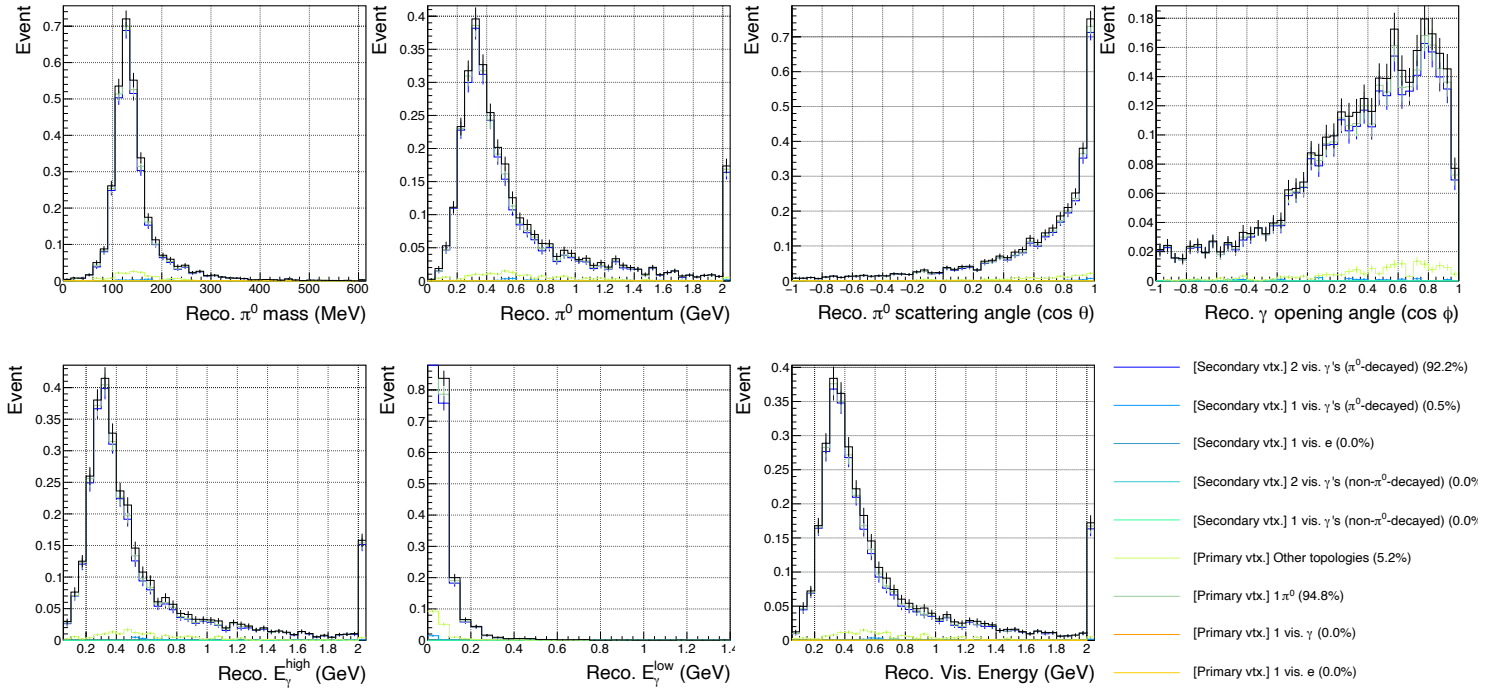
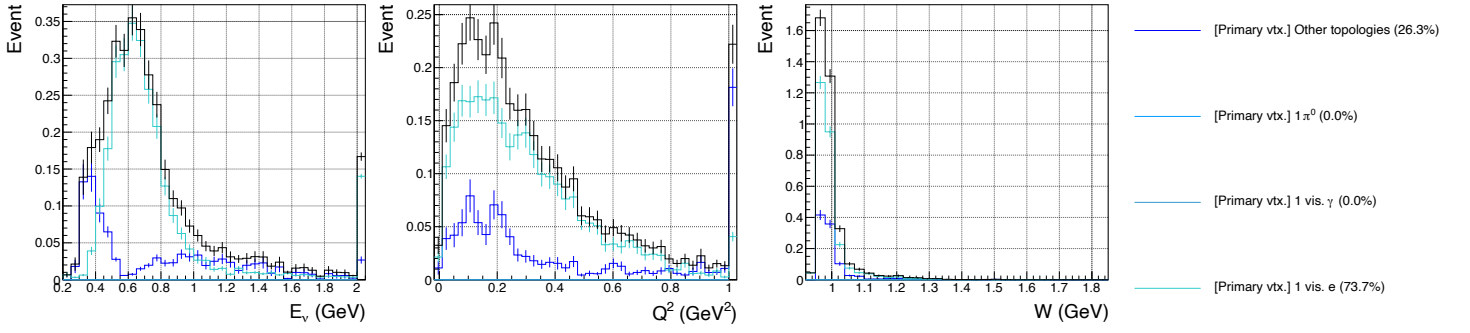
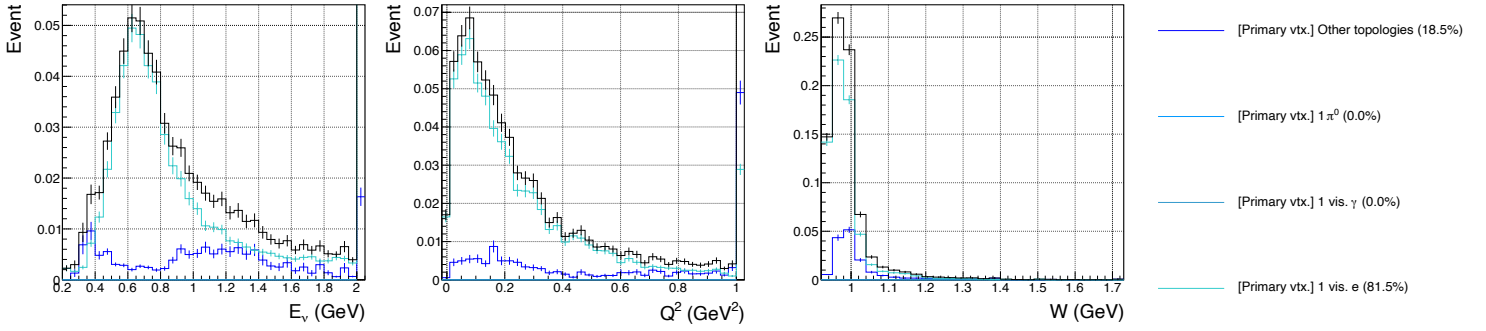
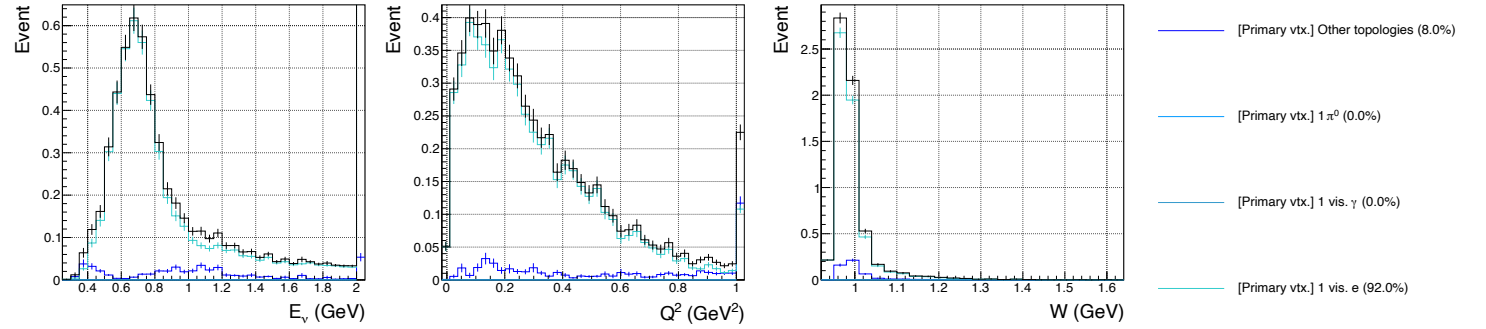
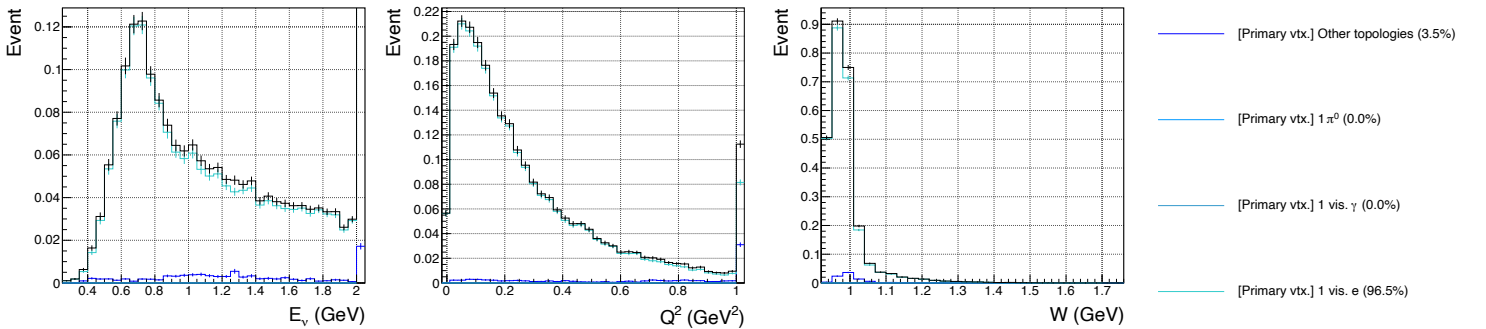


Figure B.56: The 1R true $\text{NC}\pi^0$ (+ NC Coh) events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

Breakdown by Topology with true CCQE modes

Figure B.57: The 2R true CCQE events by primary final state topologies in the ν beam mode.Figure B.58: The true CCQE events by primary final state topologies in the $\bar{\nu}$ beam mode.Figure B.59: The 1R true CCQE events by primary final state topologies in the ν beam mode.Figure B.60: The 1R true CCQE events by primary final state topologies in the $\bar{\nu}$ beam mode.

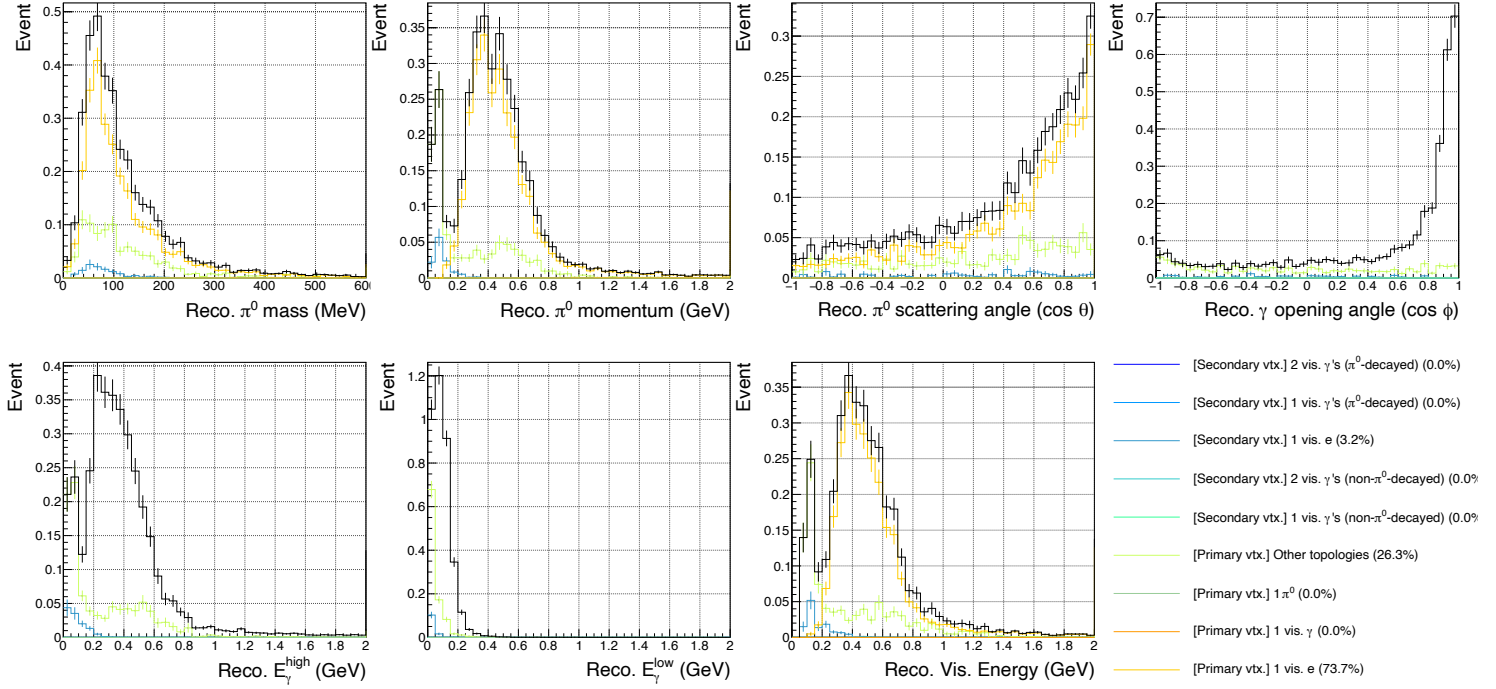


Figure B.61: The 2R true CCQE events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

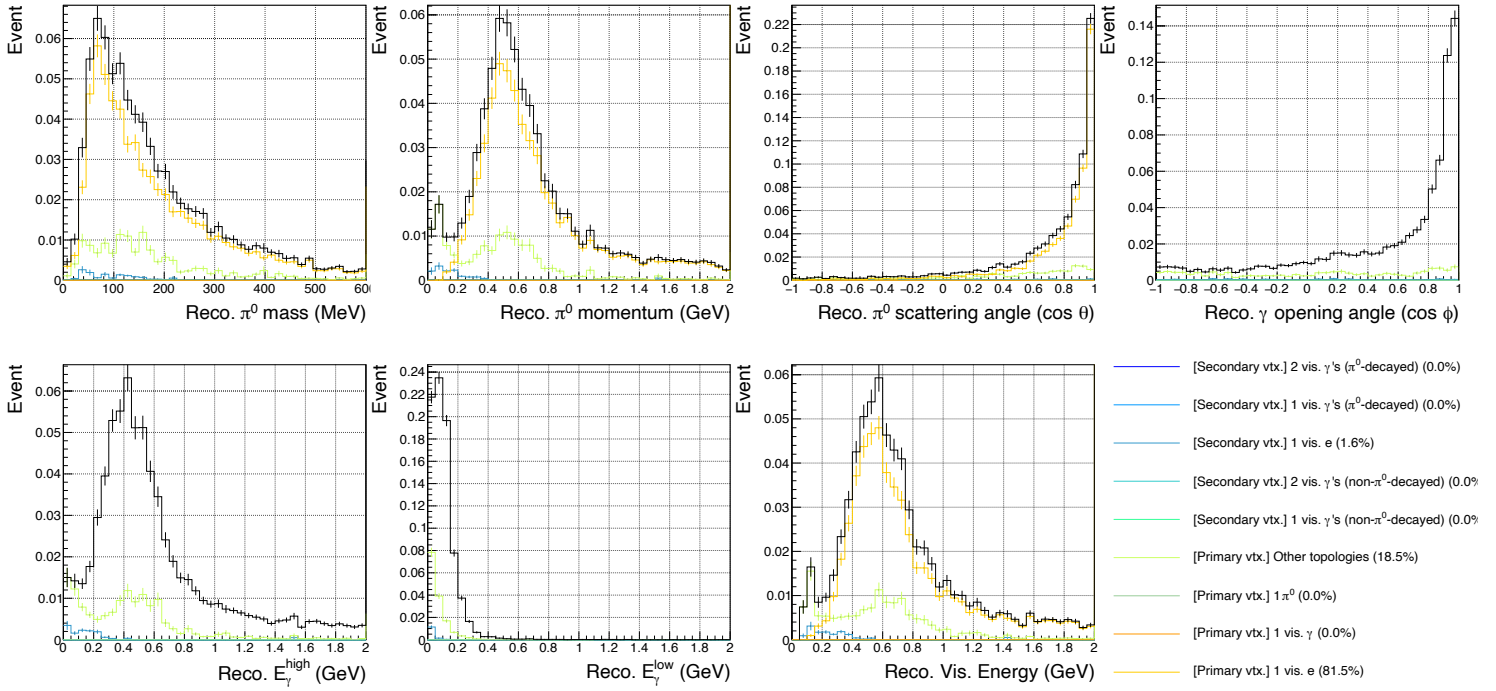


Figure B.62: The 2R true CCQE events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

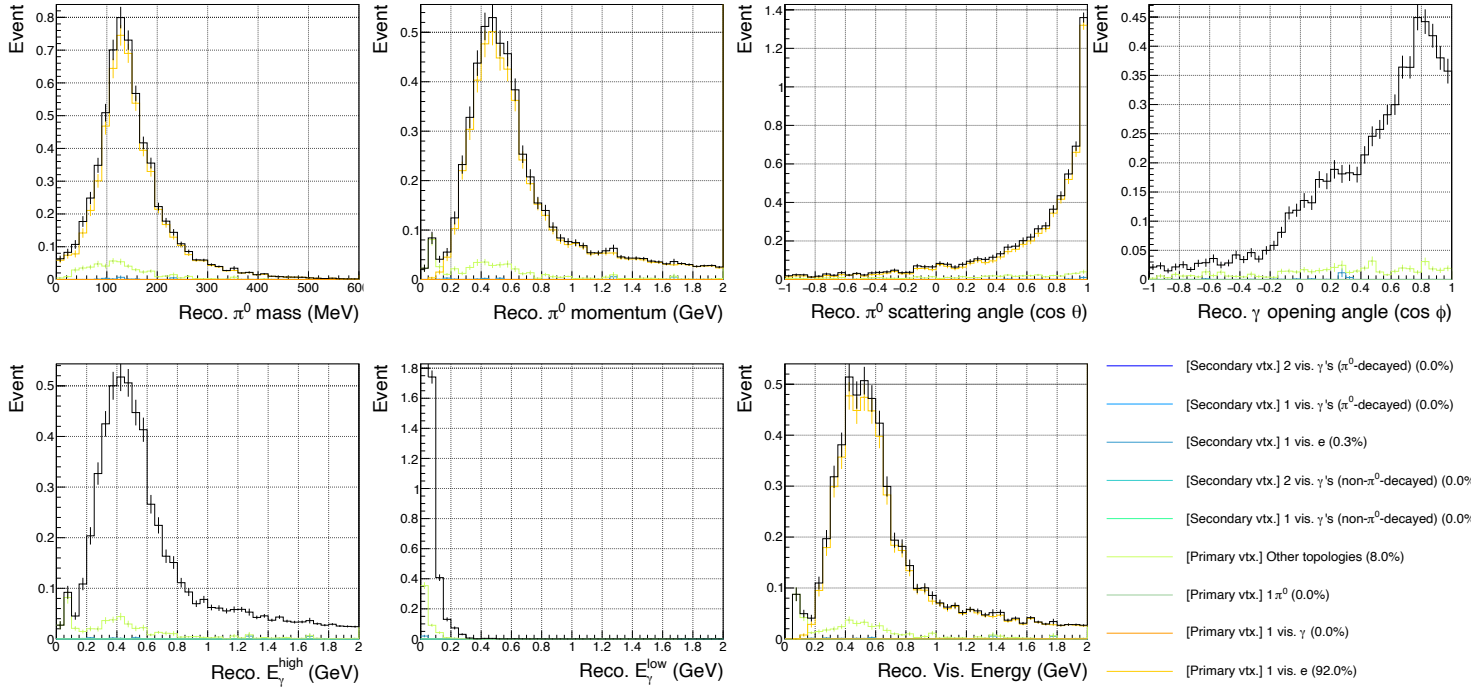


Figure B.63: The 1R true CCQE events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

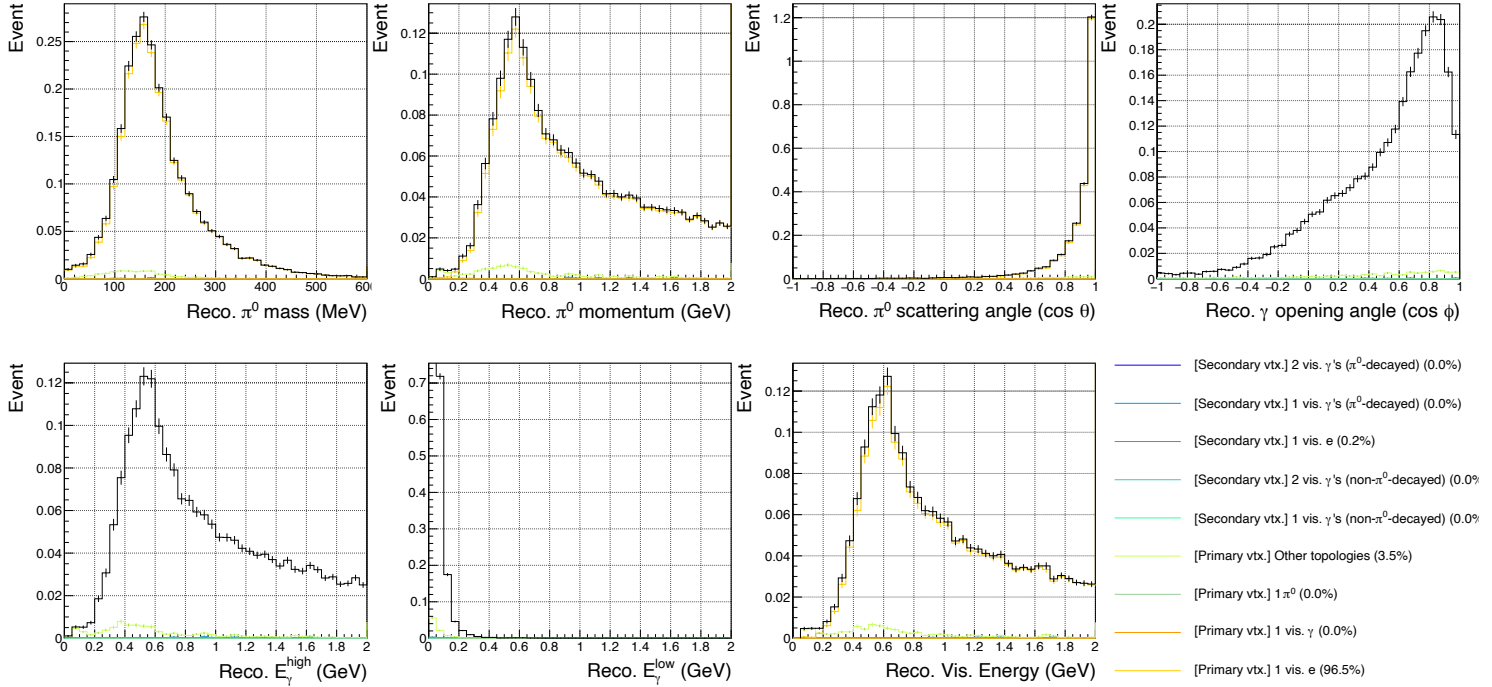


Figure B.64: The 1R true CCQE events by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

B.2.3 Breakdown by Topology (Purity Distributions)

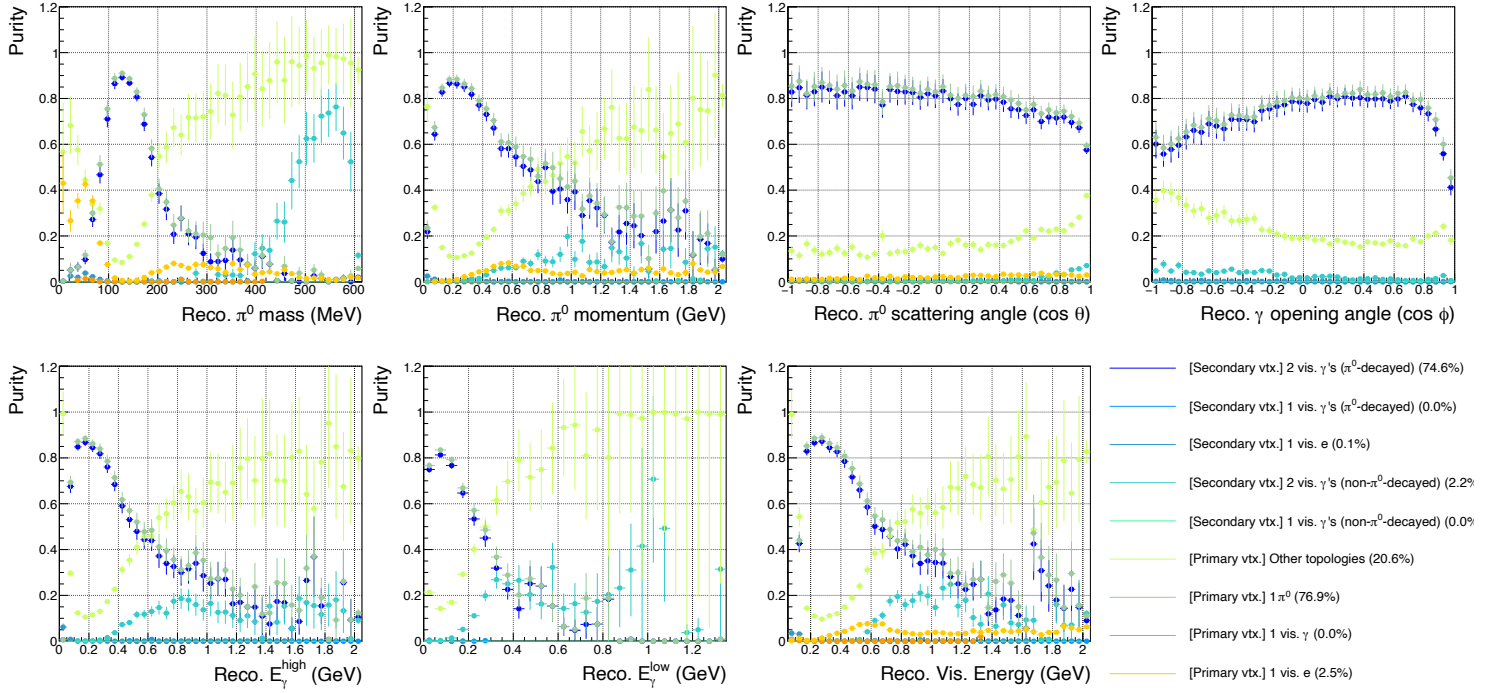


Figure B.65: The 2R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

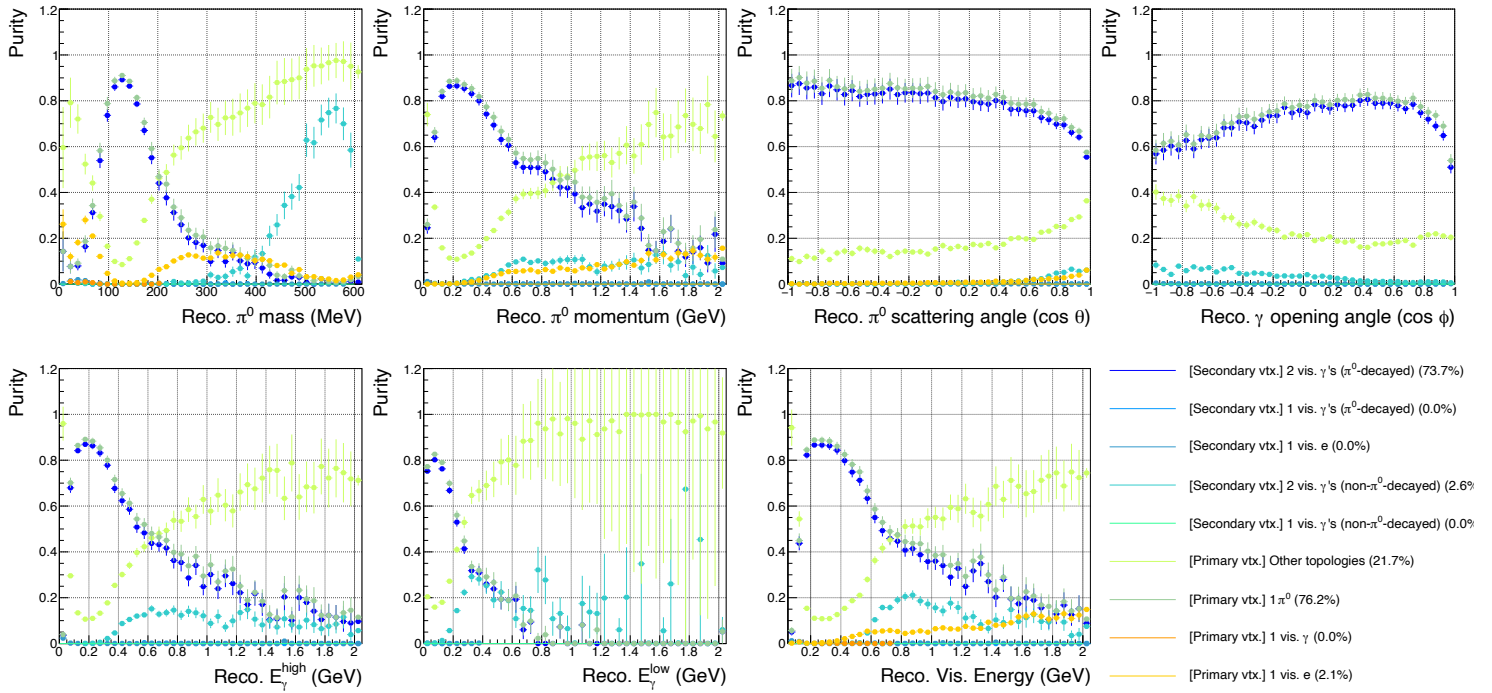


Figure B.66: The 2R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

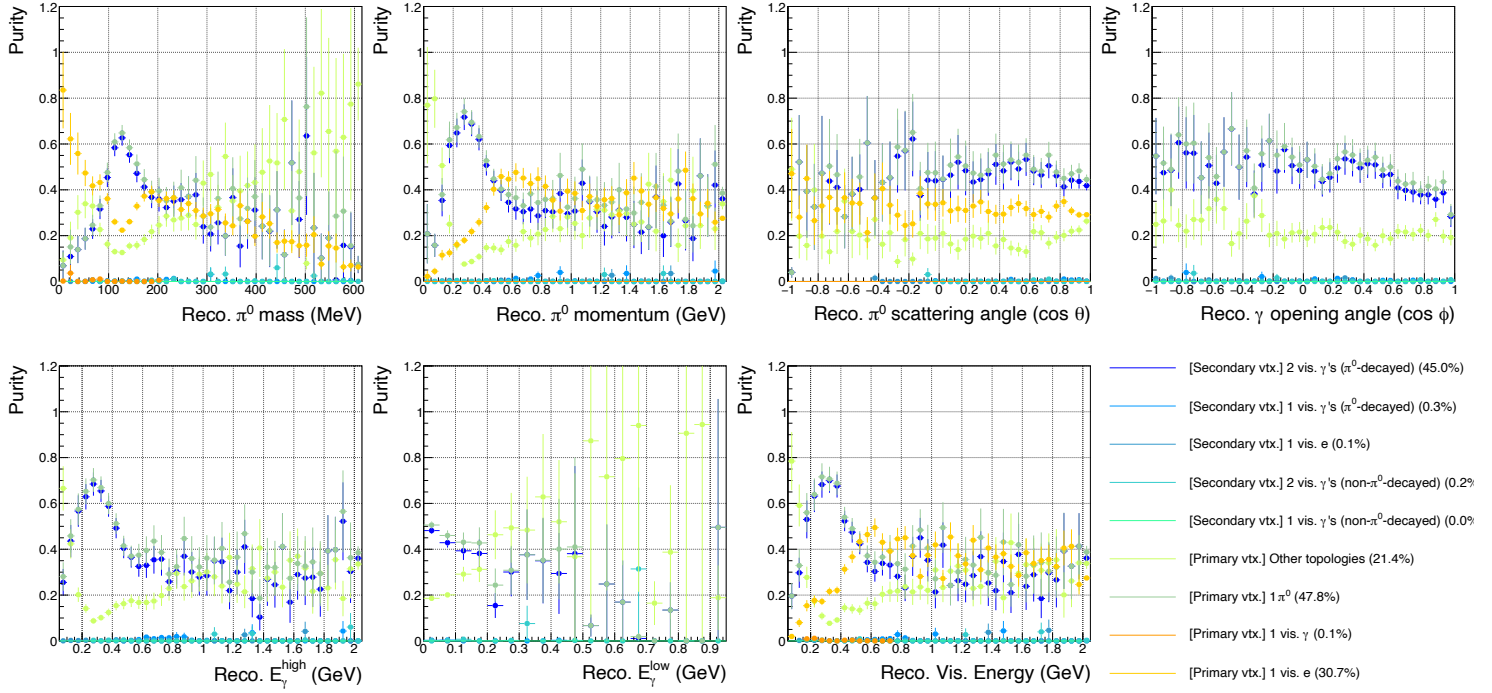


Figure B.67: The 1R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

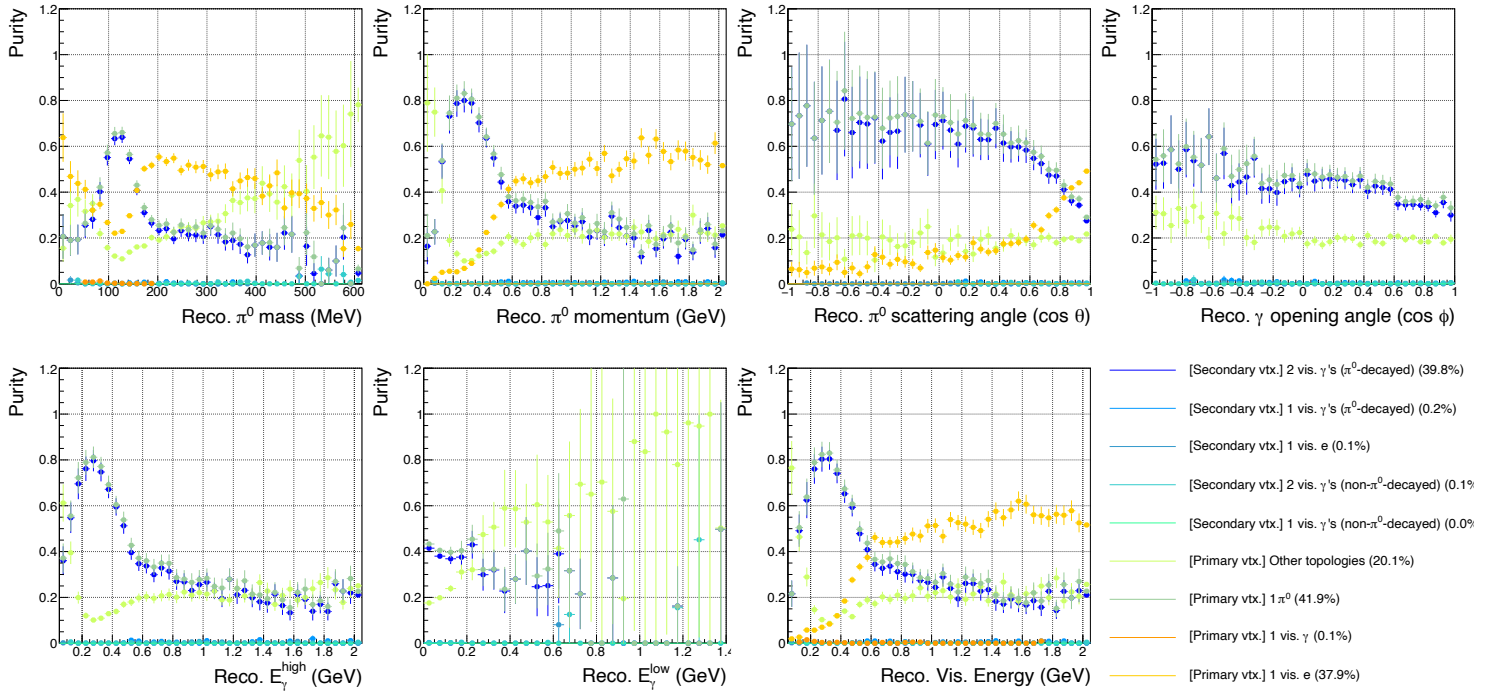


Figure B.68: The 1R π^0 sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

B.2.4 Breakdown by Topology (Efficiency Distributions)

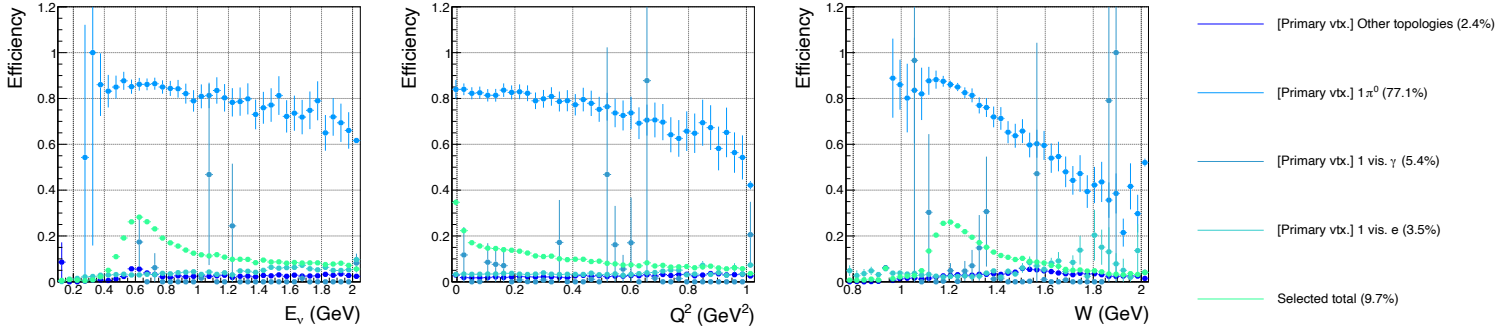


Figure B.69: The 2R π^0 sample by primary final state topologies in the ν beam mode.

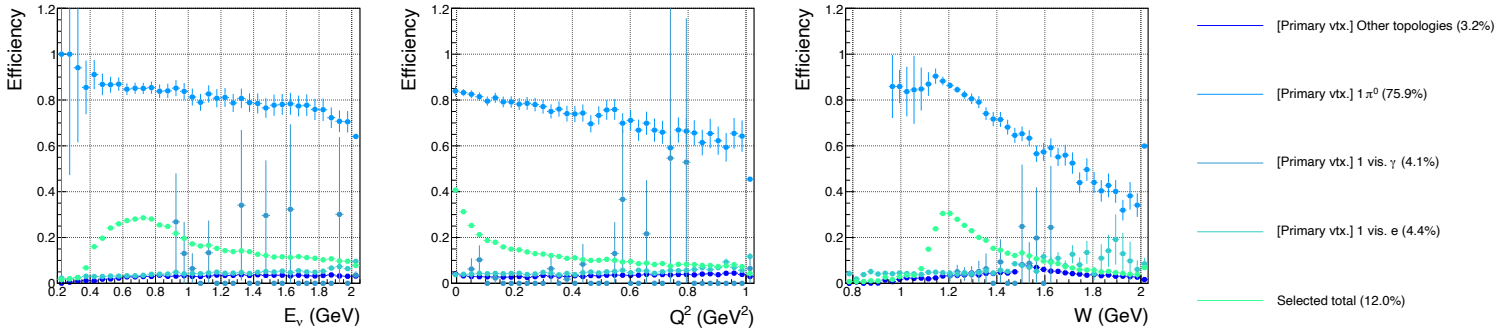


Figure B.70: The 2R π^0 sample by primary final state topologies in the $\bar{\nu}$ beam mode.

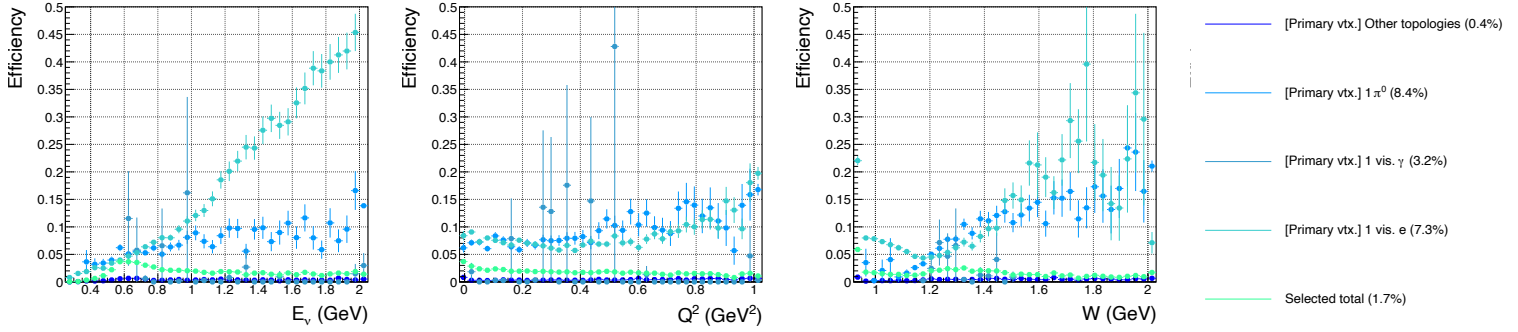


Figure B.71: The 1R π^0 sample by primary final state topologies in the ν beam mode.

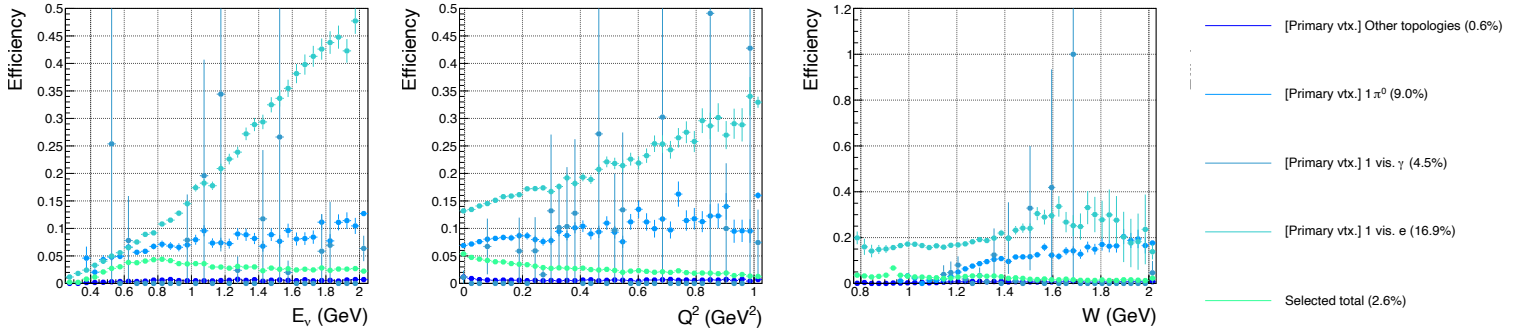


Figure B.72: The 2R π^0 sample by primary final state topologies in the $\bar{\nu}$ beam mode.

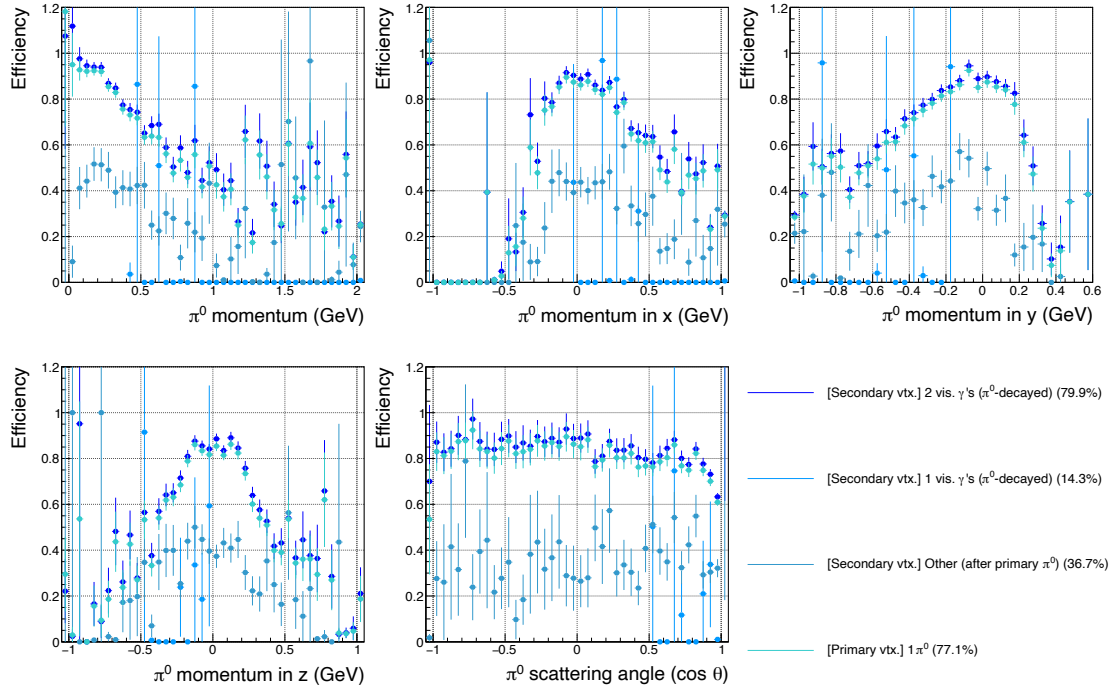
Efficiency with the Primary- π^0 Kinematics

Figure B.73: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with π^0 kinematics).

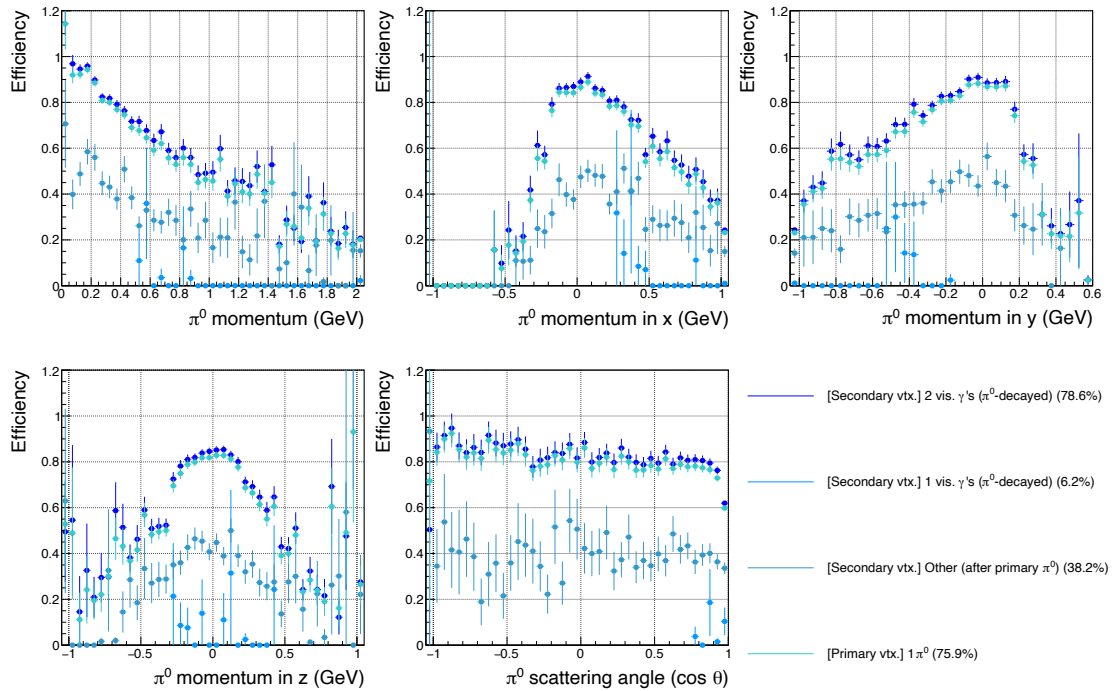


Figure B.74: The 2R π^0 sample with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with π^0 kinematics).

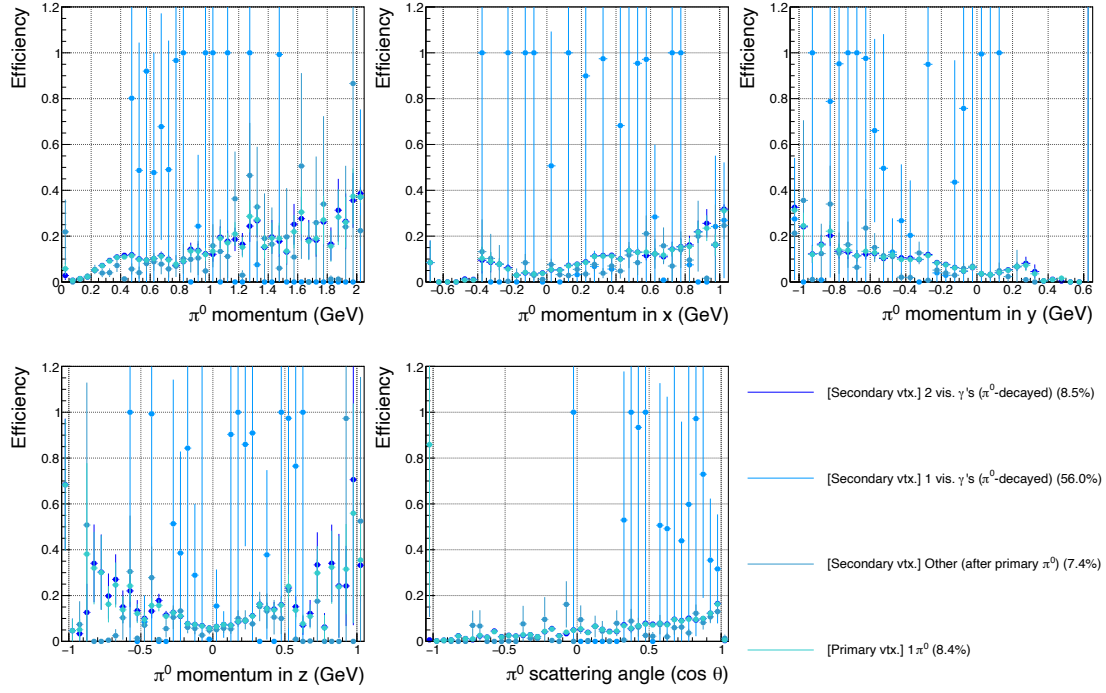


Figure B.75: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the ν beam mode (with π^0 kinematics).

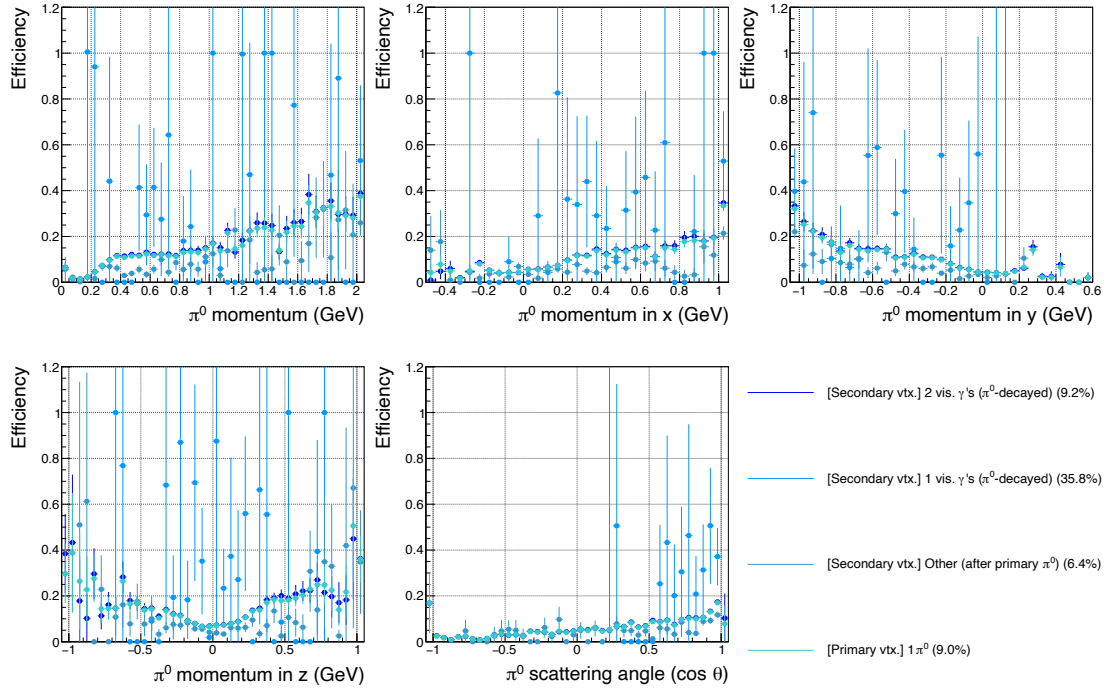


Figure B.76: The 1R π^0 samples with 1-primary- π^0 and 2-secondary- π^0 -decayed-photon topologies in the $\bar{\nu}$ beam mode (with π^0 kinematics).

Efficiency by Topology in Reconstructed Kinematics

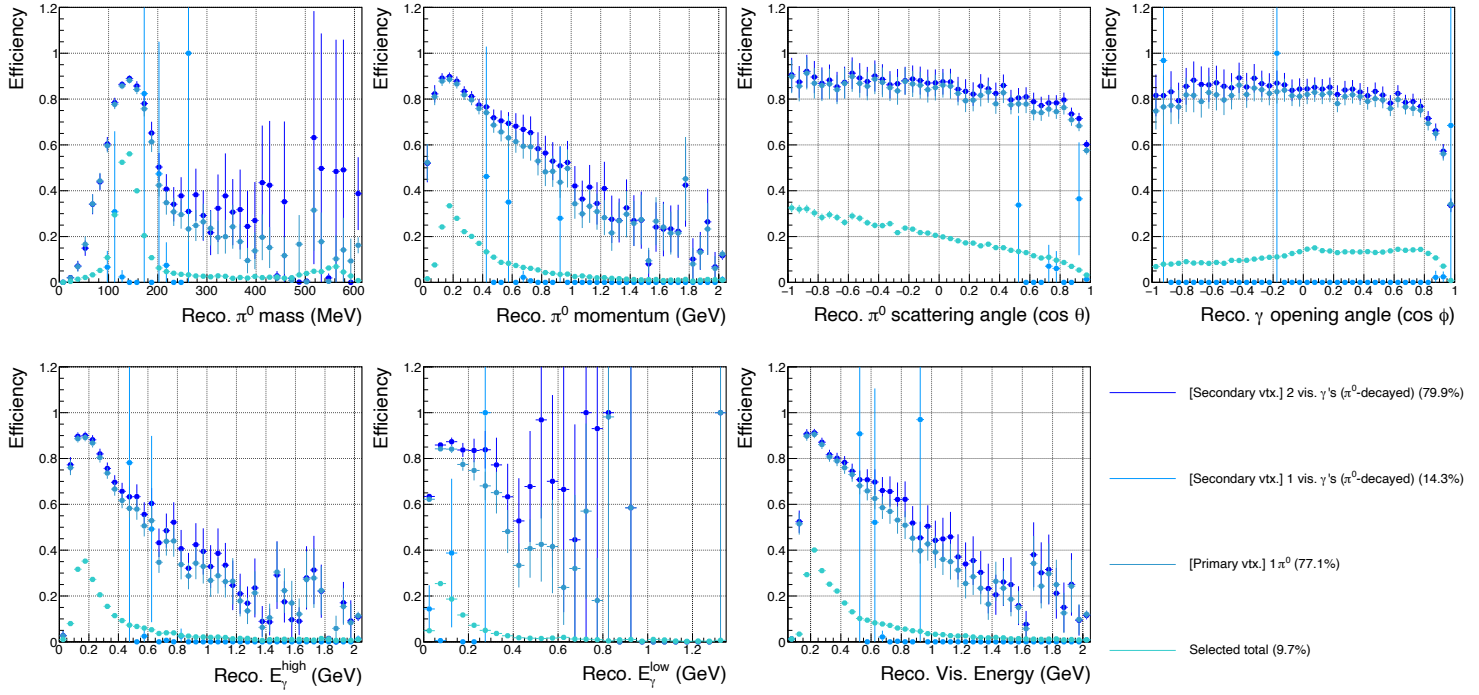


Figure B.77: The $2R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

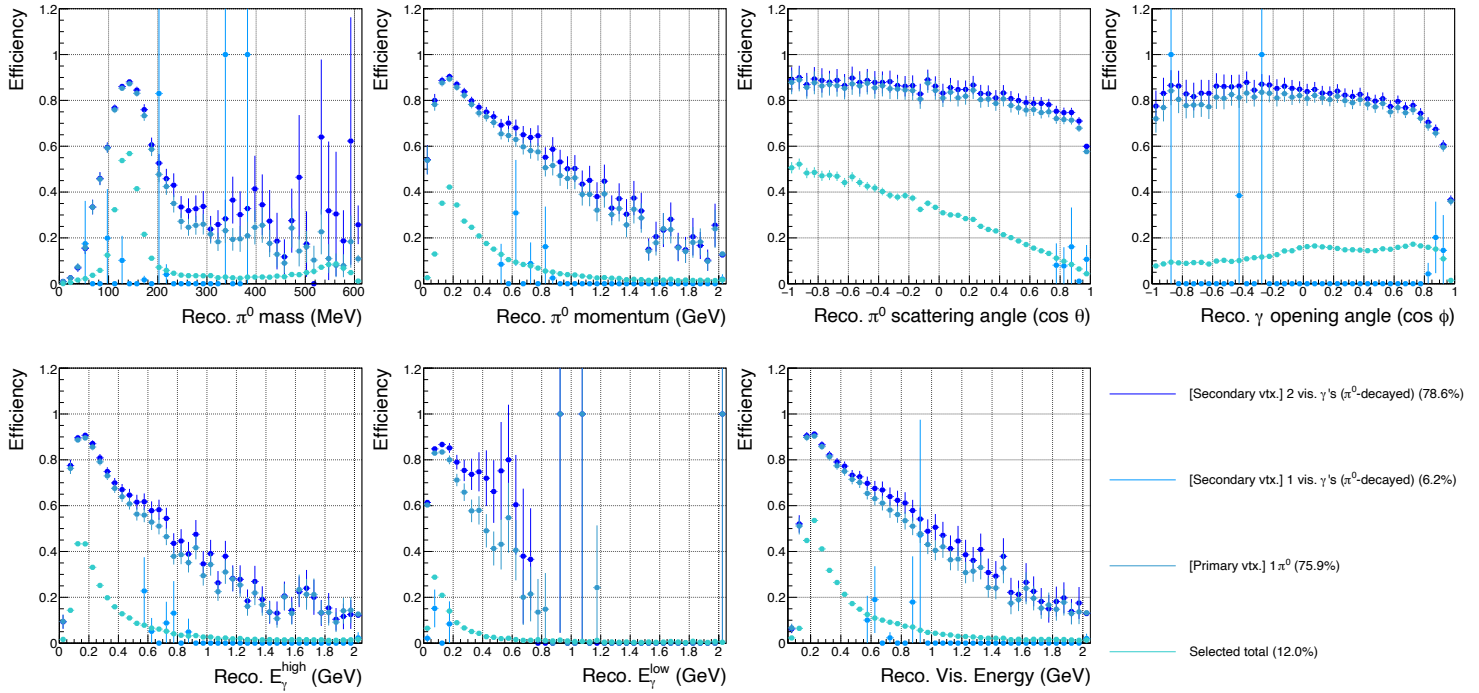


Figure B.78: The $2R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

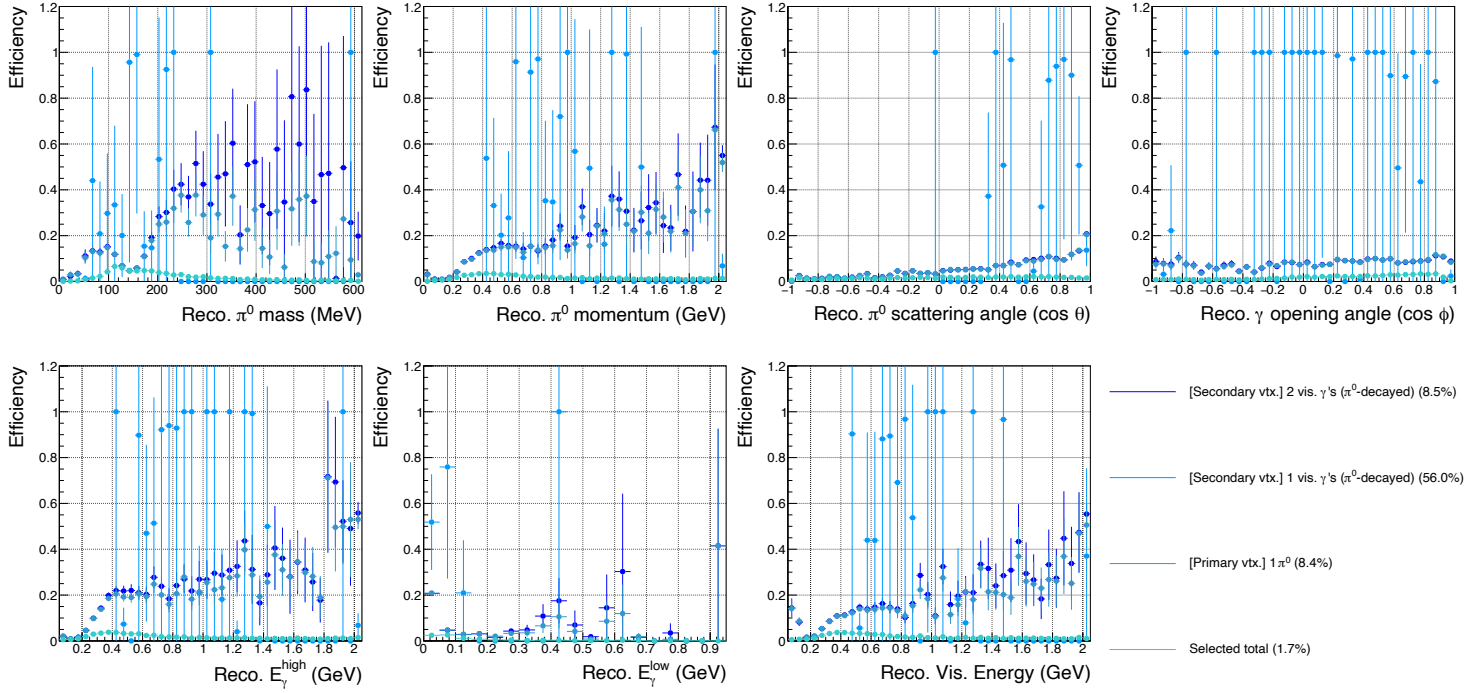


Figure B.79: The $1R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the ν beam mode.

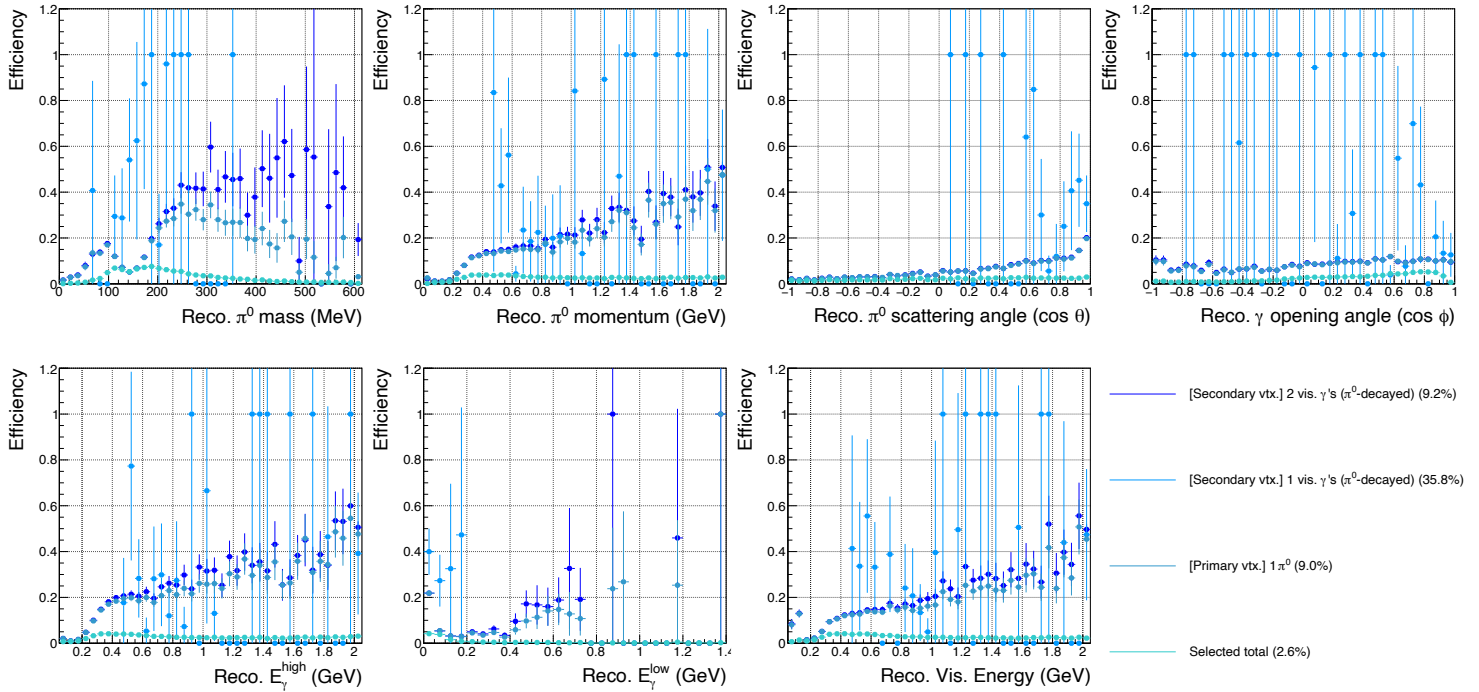
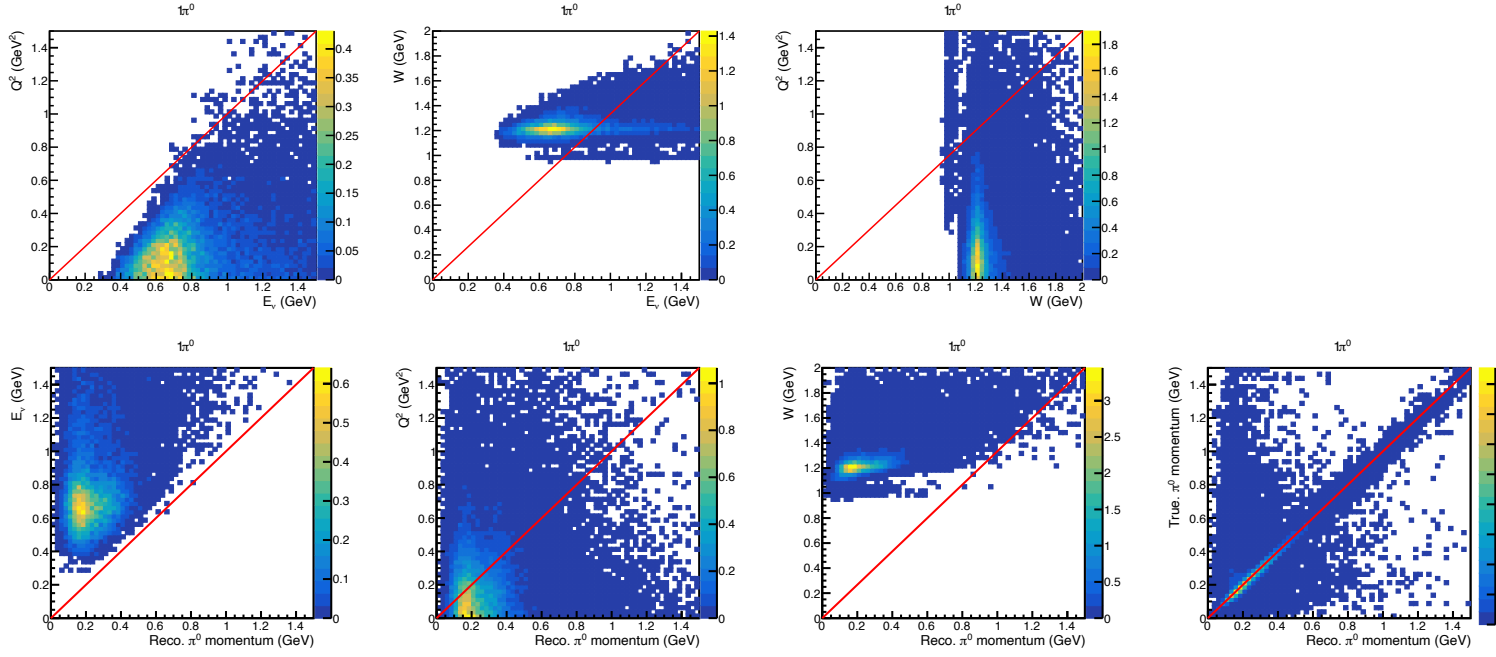
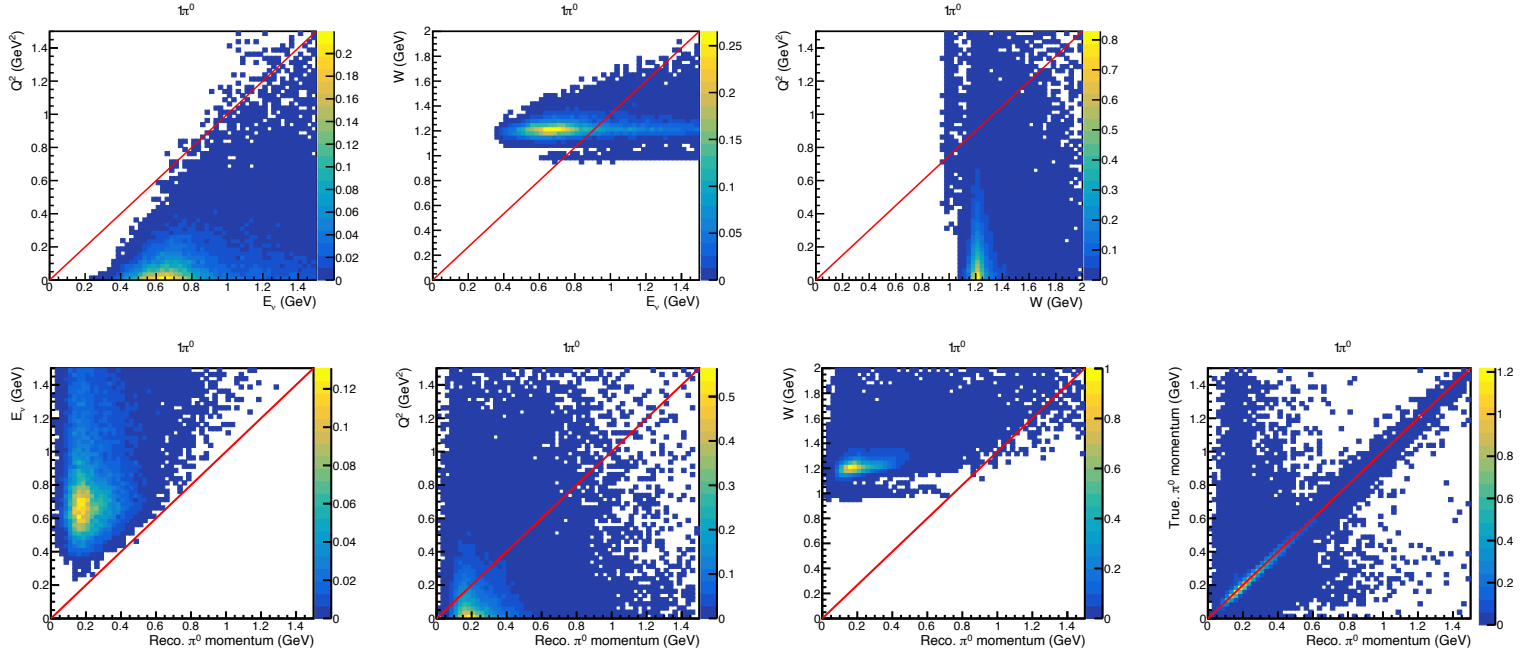
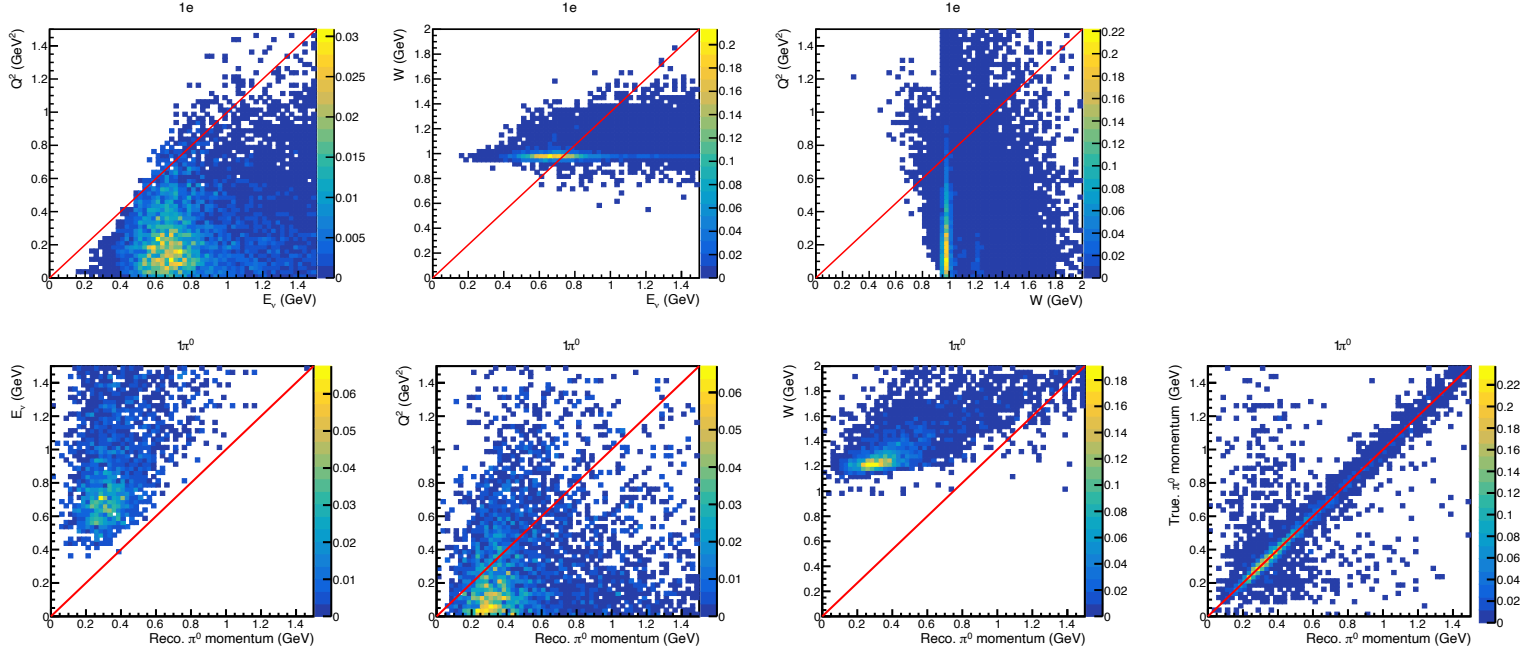
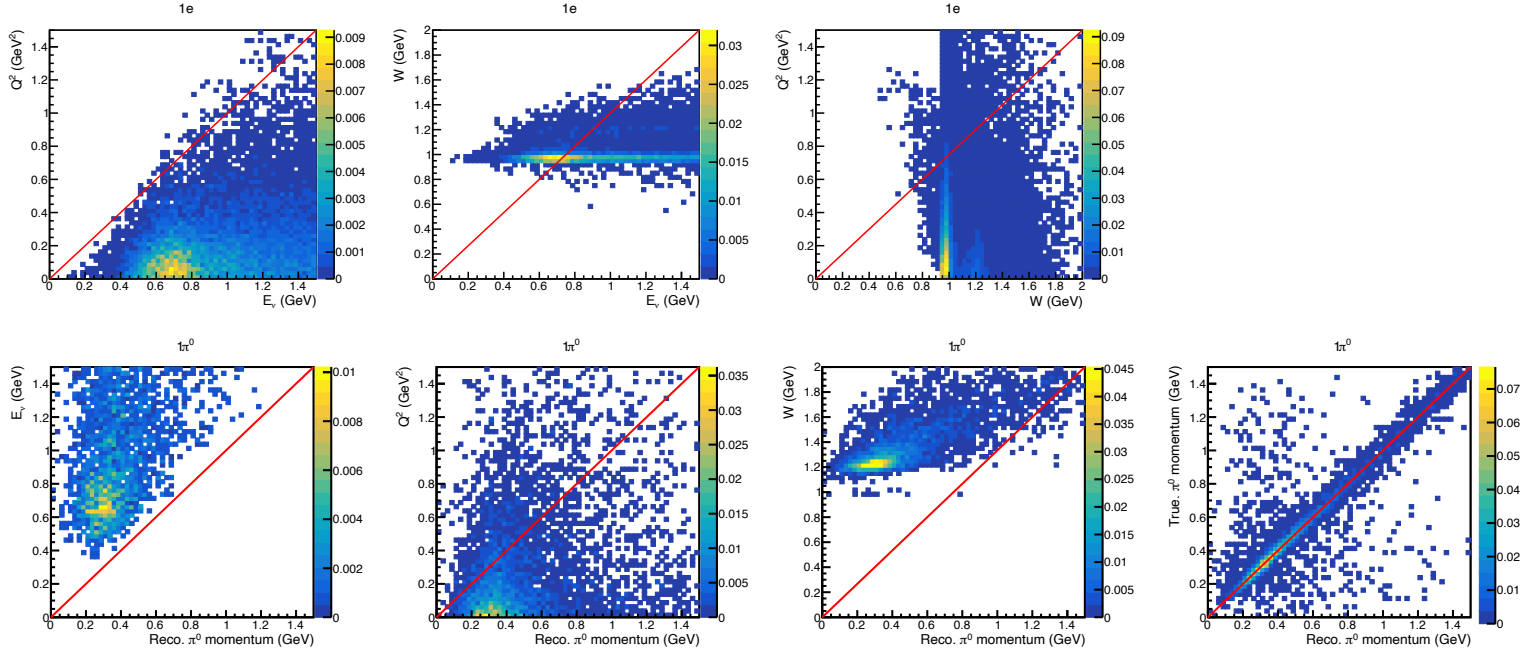
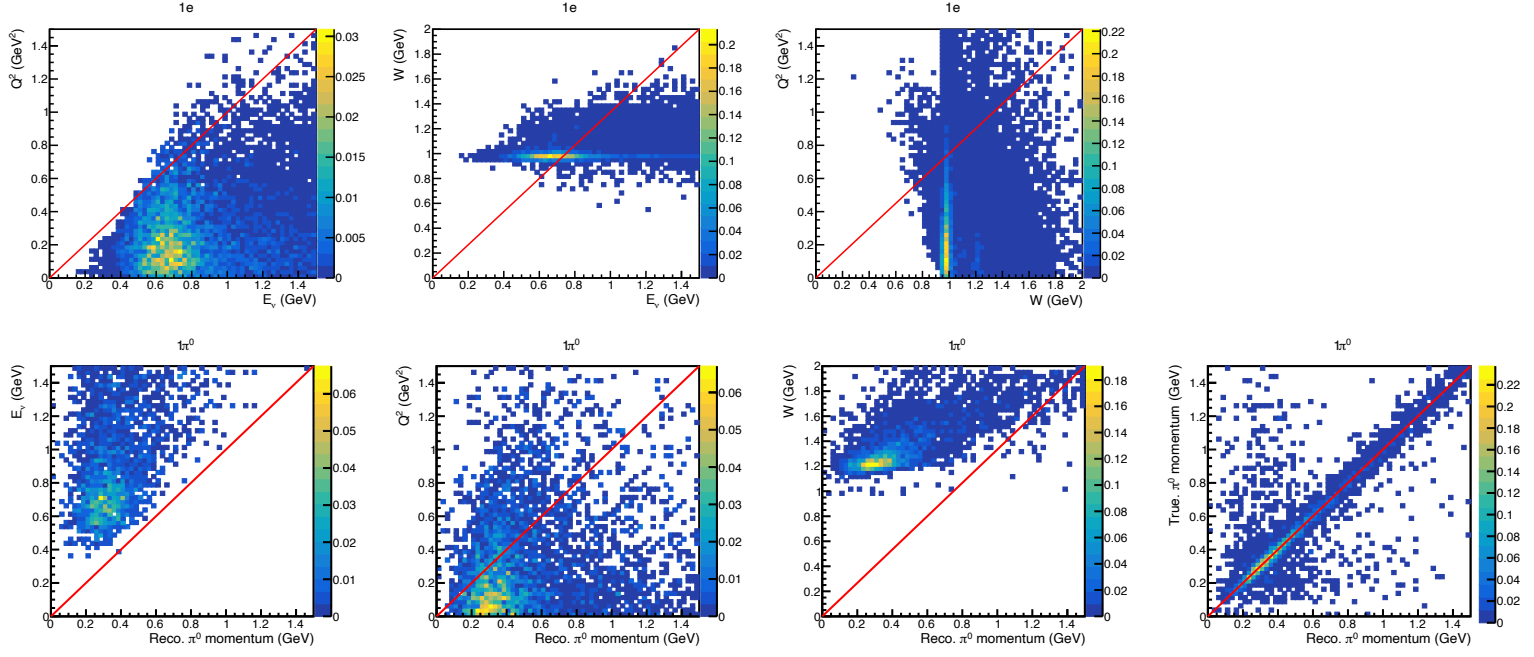
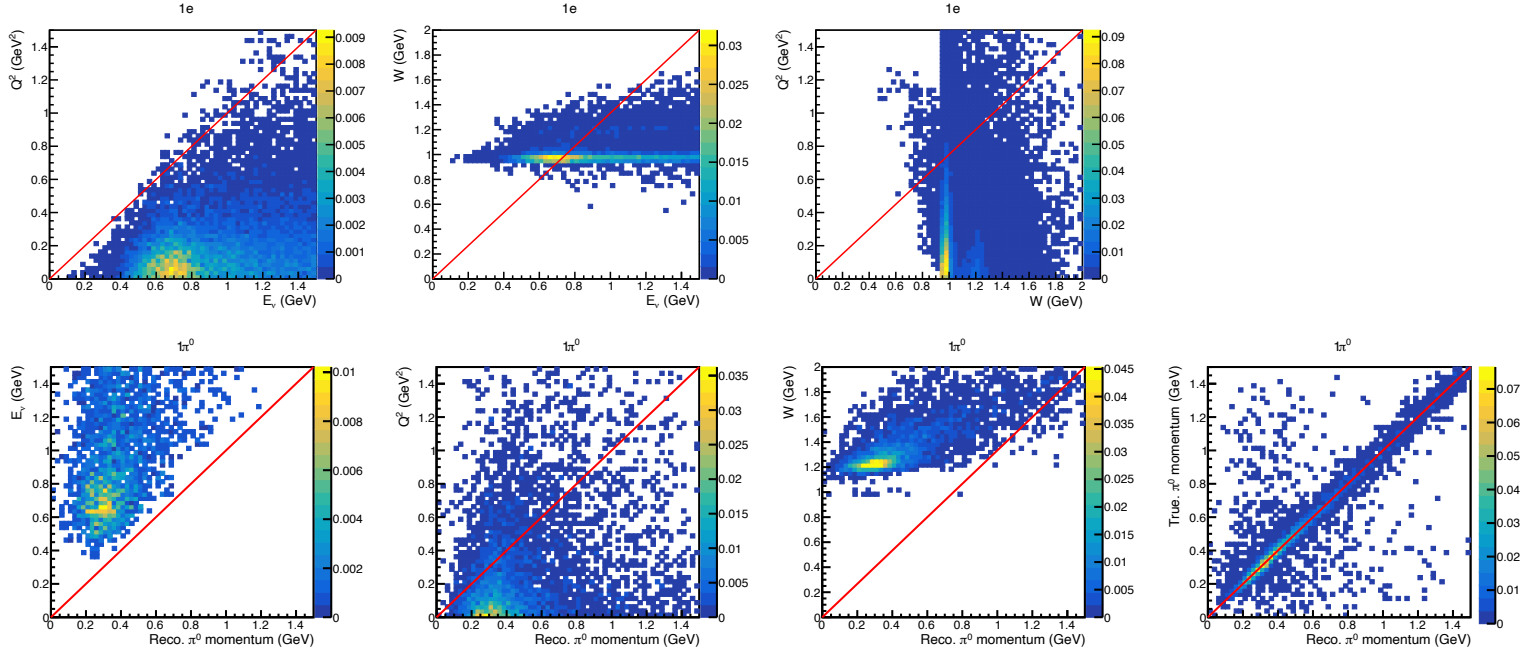


Figure B.80: The $1R \pi^0$ sample by true topologies that can produce 1 or 2 e -like rings from the primary or secondary vertices in the $\bar{\nu}$ beam mode.

B.2.5 2D Kinematic Distributions for $1\pi^0$ and $1e$ EventsFigure B.81: The 2D distributions of the 2R $1\pi^0$ final state in the ν beam mode.Figure B.82: The 2D distributions of the 2R $1\pi^0$ final state in the $\bar{\nu}$ beam mode.

Figure B.83: The 2D distributions of the 1R $1\pi^0$ final state in the ν beam mode.Figure B.84: The 2D distributions of the 1R $1\pi^0$ final state in the $\bar{\nu}$ beam mode.

Figure B.85: The 2D distributions of the 1R $1e$ final state in the ν beam mode.Figure B.86: The 2D distributions of the 1R $1e$ final state in the $\bar{\nu}$ beam mode.

B.2.6 Shape Comparisons for $\text{NC}\pi^0$ Events

Reconstructed and Truth Information

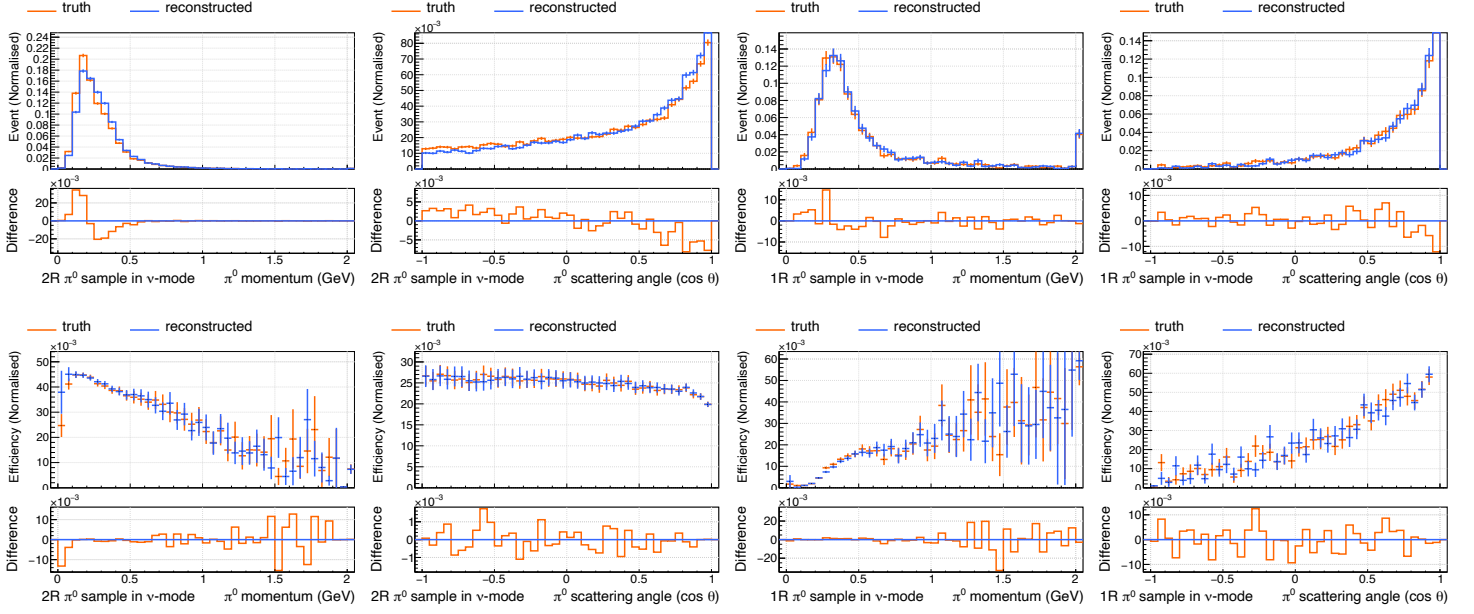


Figure B.87: Normalised distributions overlapping the reconstructed and truth π^0 kinematics for the 2R and 1R $\text{NC}\pi^0$ events in the ν beam mode.

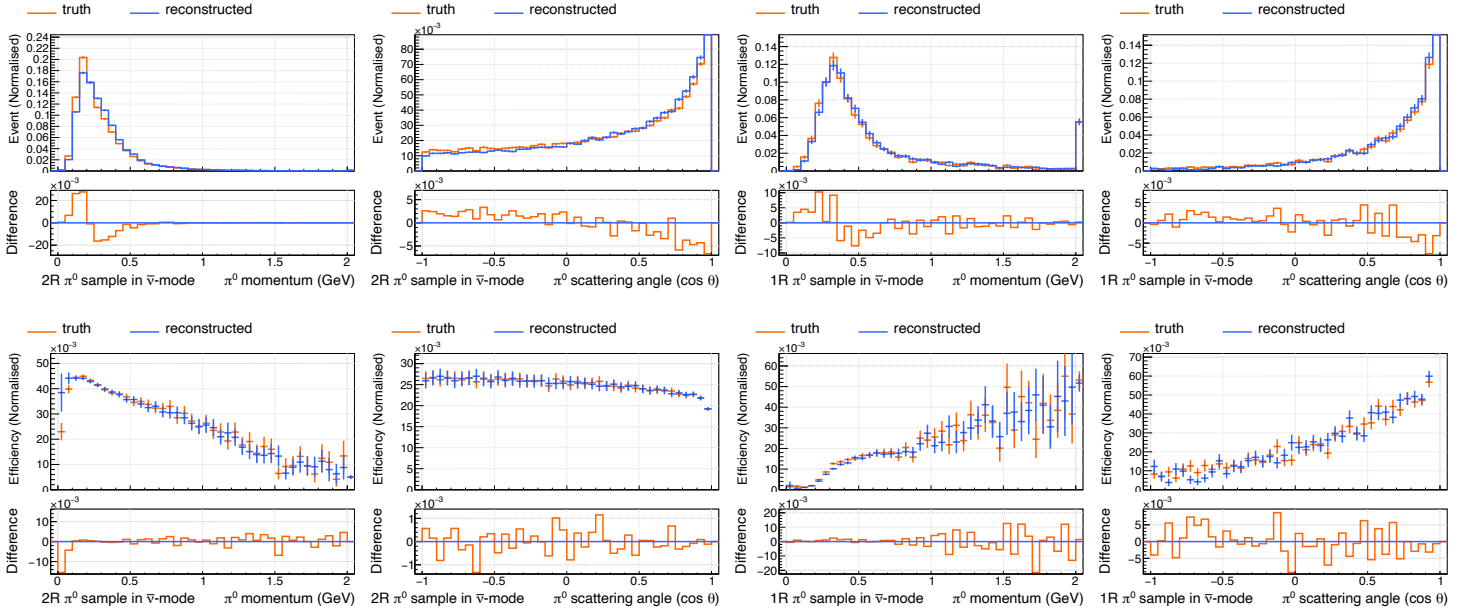


Figure B.88: Normalised distributions overlapping the reconstructed and truth π^0 kinematics for the 2R and 1R $\text{NC}\pi^0$ events in the $\bar{\nu}$ beam mode.

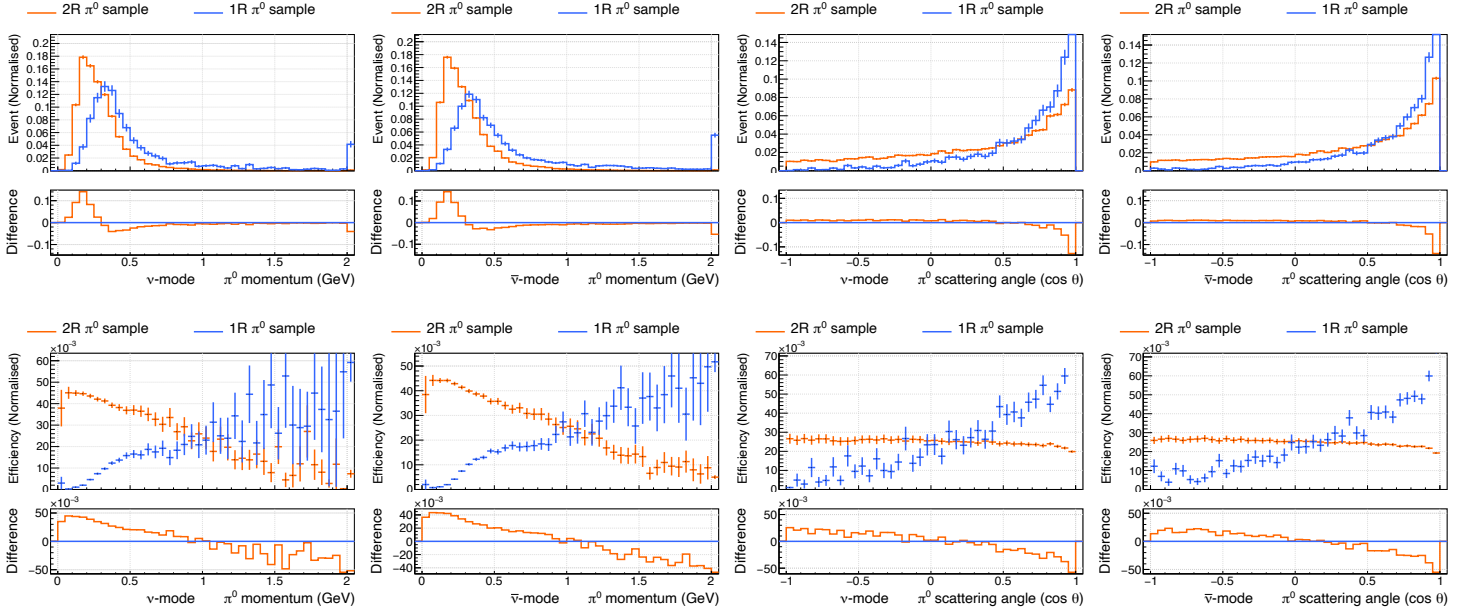
The 2R π^0 and 1R π^0 Samples

Figure B.89: Normalised distributions overlapping the 2R and 1R NC π^0 events for the true π^0 kinematics in the ν and $\bar{\nu}$ beam modes.

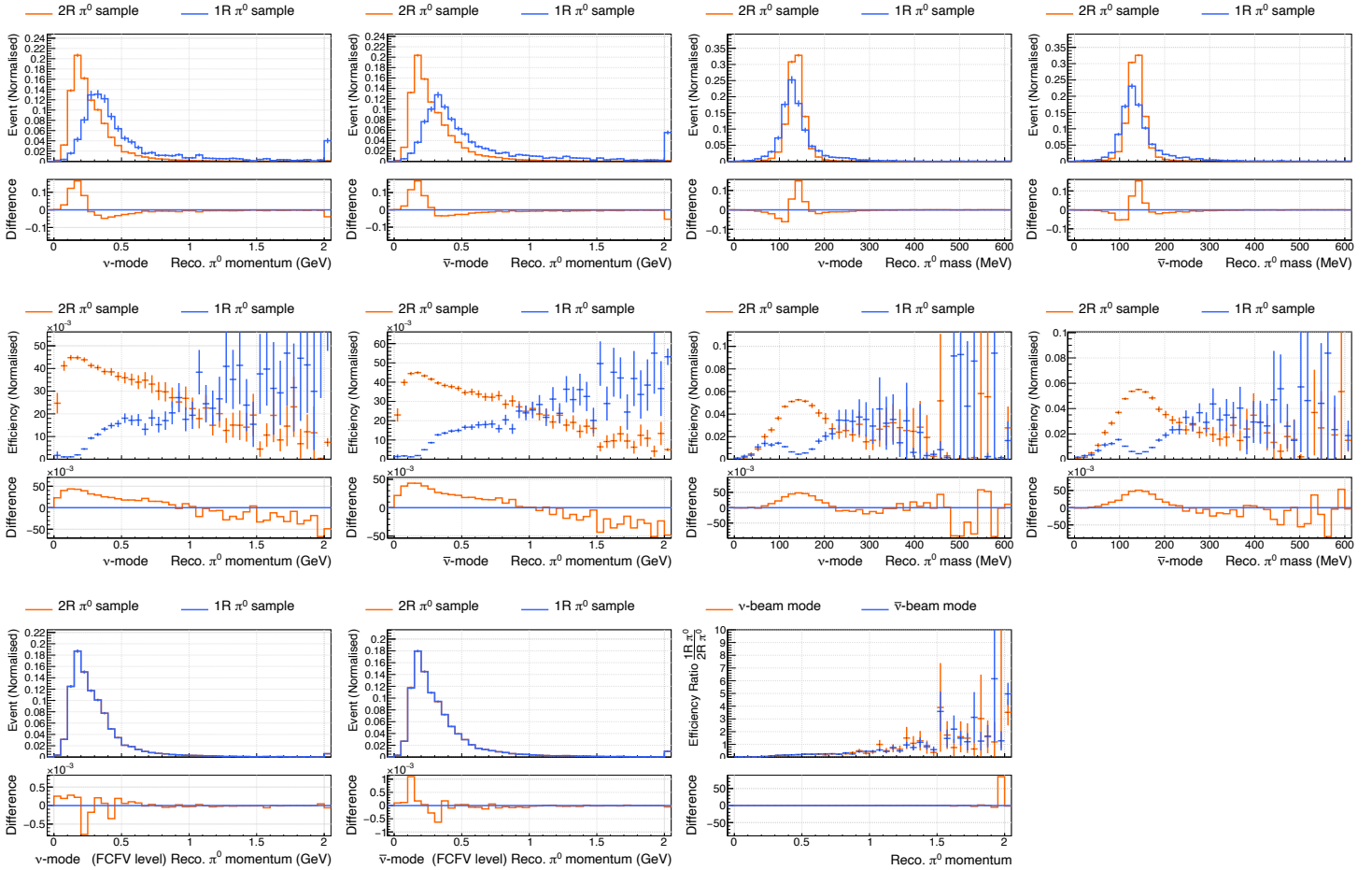


Figure B.90: Normalised distributions overlapping the 2R and 1R NC π^0 events for the reconstructed π^0 kinematics in the ν and $\bar{\nu}$ beam modes.

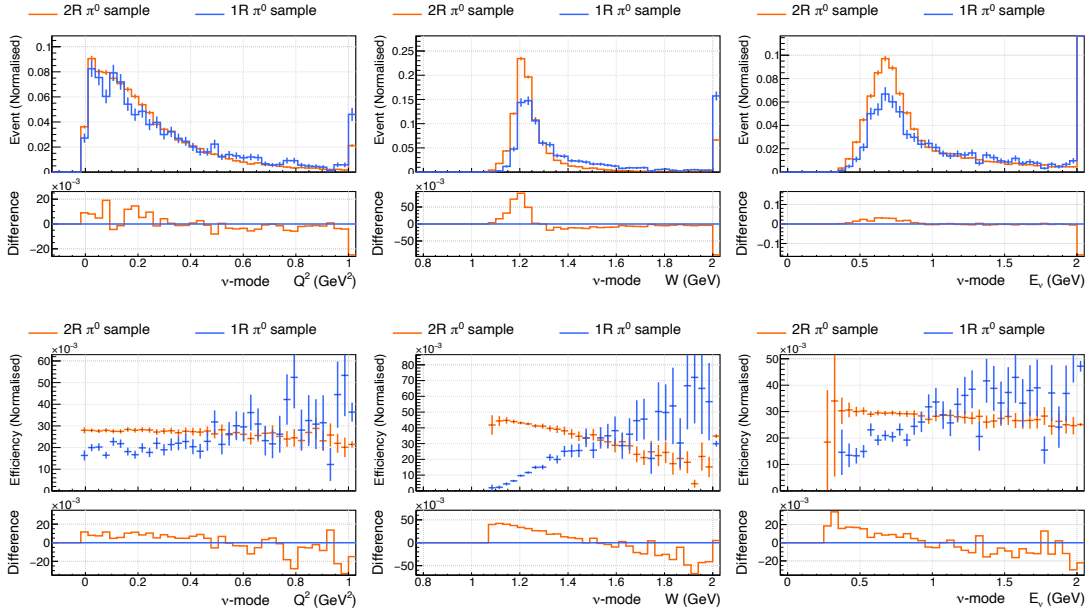


Figure B.91: Normalised distributions overlapping the 2R and 1R NC π^0 events for Q^2 , W , and E_ν in the ν beam mode.

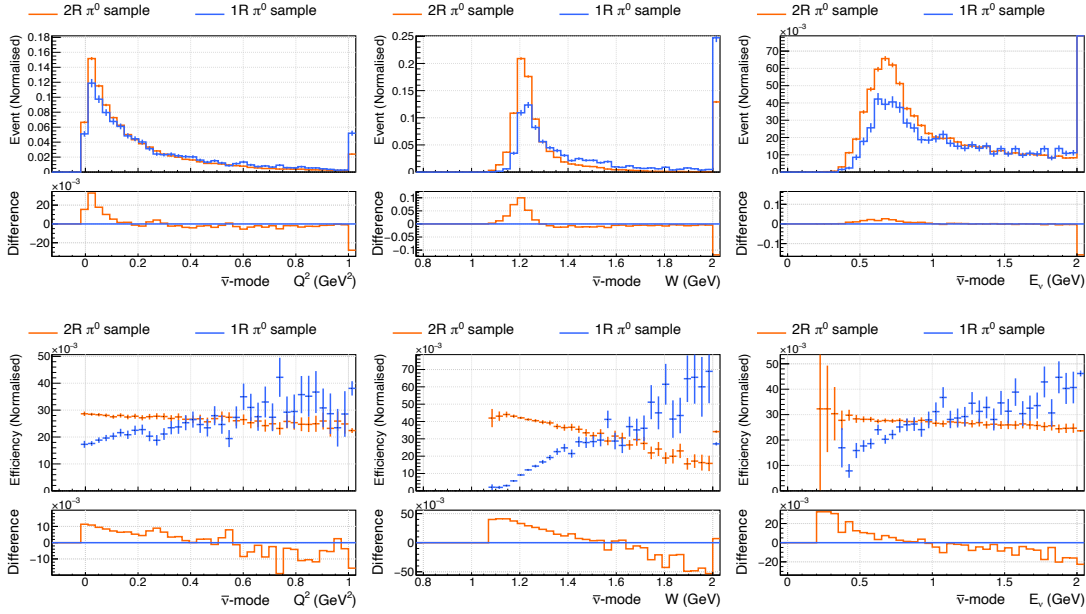


Figure B.92: Normalised distributions overlapping the 2R and 1R NC π^0 events for Q^2 , W , and E_ν in the $\bar{\nu}$ beam mode.

The 2R π^0 (weighted) and 1R π^0 Samples

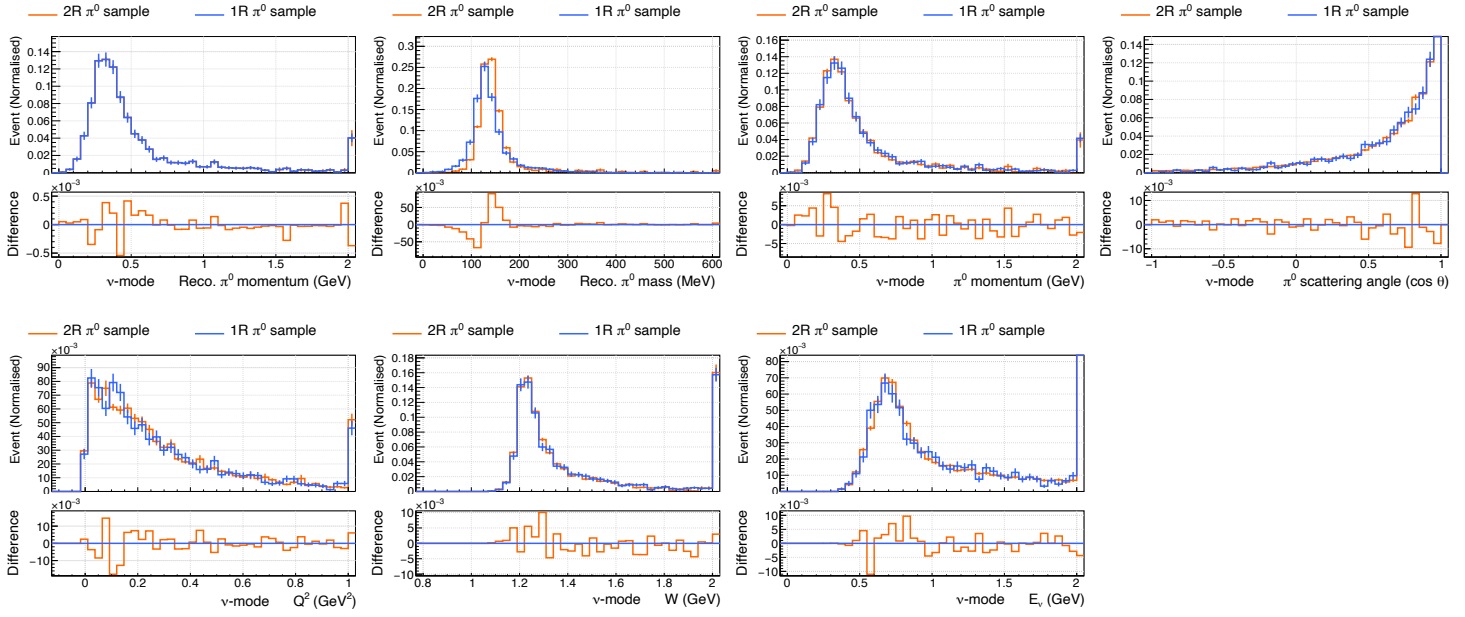


Figure B.93: Normalised distributions overlapping the weighted-2R and 1R $\text{NC}\pi^0$ events in the ν beam mode. The weight is given by the 1R/2R $\text{NC}\pi^0$ efficiency ratio over the reconstructed π^0 momentum.

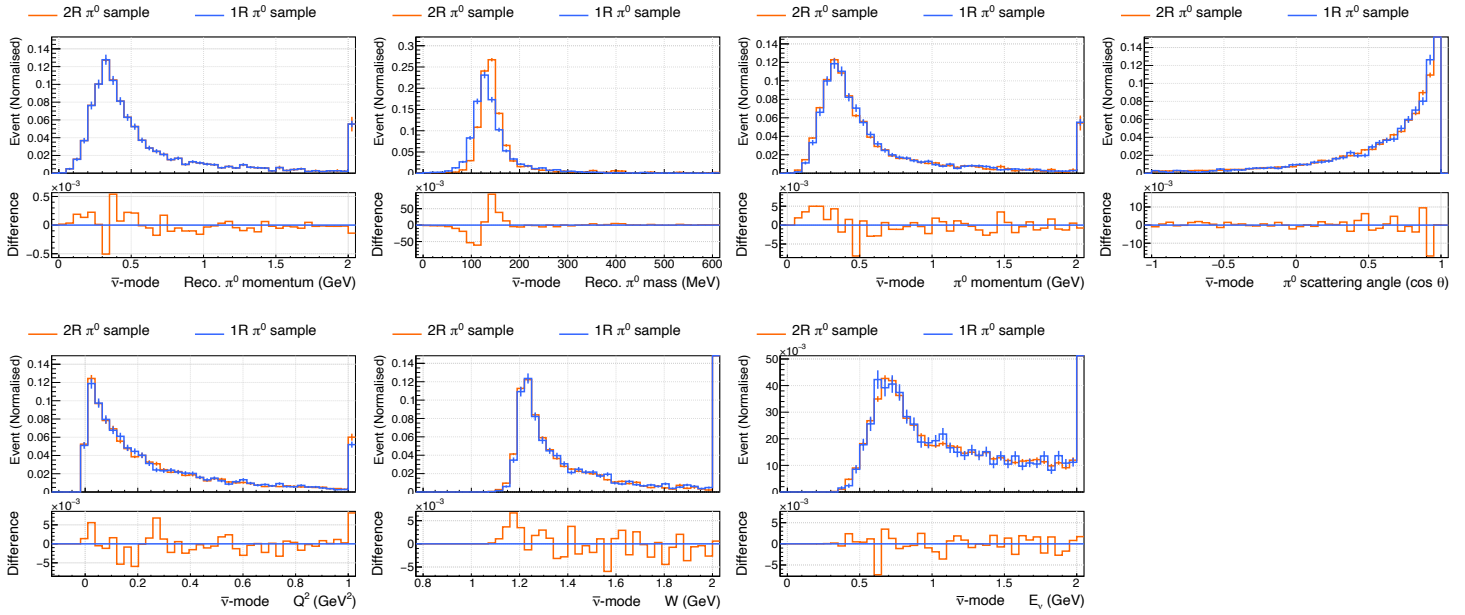


Figure B.94: Normalised distributions overlapping the weighted-2R and 1R $\text{NC}\pi^0$ events in the $\bar{\nu}$ beam mode. The weight is given by the 1R/2R $\text{NC}\pi^0$ efficiency ratio over the reconstructed π^0 momentum.

Appendix C

Additional Information for T2K Oscillation Analysis

C.1 Sample Binning

C.1.1 ND280 Sample Binning

The FGD1 and FGD2 samples have the same binning scheme:

- **ν -mode ν_μ CC0 π -0p:**

p_μ (MeV): [0, 200, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 950, 1000, 1100, 1200, 1300, 1500, 1750, 2000, 2500, 3000, 5000, 30000]

$\cos \theta_\mu$: [-1.0, 0.5, 0.6, 0.68, 0.72, 0.76, 0.82, 0.84, 0.86, 0.88, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.955, 0.96, 0.965, 0.97, 0.975, 0.98, 0.985, 0.99, 0.995, 1.0]

- **ν -mode ν_μ CC0 π -Np:**

p_μ (MeV): [0, 250, 350, 400, 450, 500, 550, 600, 650, 700, 800, 900, 1000, 1100, 1200, 1300, 1500, 1600, 1750, 2000, 2750, 5000, 30000]

$\cos \theta_\mu$: [-1.0, 0.55, 0.65, 0.75, 0.8, 0.85, 0.88, 0.9, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.988, 1.0]

p_{proton} (MeV): [0, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 1000, 1200, 1500, 2000, 3000, 5000, 30000]

$\cos \theta_{\text{proton}}$: [-1, 0.5, 0.6, 0.7, 0.75, 0.8, 0.85, 0.90, 0.92, 0.94, 0.96, 0.98, 0.99, 1]

- **ν -mode ν_μ CC1 π :**

p_μ (MeV): [0, 300, 350, 400, 500, 600, 650, 700, 750, 800, 900, 1000, 1100, 1200, 1500, 2000, 5000, 30000]

$\cos \theta_\mu$: $[-1.0, 0.6, 0.68, 0.74, 0.79, 0.85, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.0]$

p_π (MeV): $[0, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000]$

$\cos \theta_\pi$: $[-1, 0.7, 0.8, 0.85, 0.90, 0.92, 0.94, 0.96, 0.97, 0.98, 0.99, 1]$

- **ν -mode ν_μ CC-Other-0 γ :**

p_μ (MeV): $[0, 350, 500, 600, 650, 700, 800, 900, 1000, 1150, 1250, 1500, 2000, 5000, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.6, 0.7, 0.8, 0.85, 0.88, 0.9, 0.92, 0.94, 0.96, 0.98, 1.0]$

- **ν -mode ν_μ CC-Photon:**

p_μ (MeV): $[0, 300, 400, 500, 600, 650, 700, 750, 800, 900, 1000, 1100, 1250, 1500, 1600, 1750, 2000, 2500, 3000, 5000, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.6, 0.7, 0.76, 0.8, 0.84, 0.86, 0.88, 0.89, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 0.995, 1.0]$

- **$\bar{\nu}$ -mode $\bar{\nu}_\mu$ CC0 π :**

p_μ (MeV): $[0, 300, 400, 500, 550, 600, 650, 700, 750, 800, 900, 1000, 1100, 1200, 1500, 2000, 4000, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.6, 0.7, 0.8, 0.85, 0.9, 0.92, 0.93, 0.94, 0.95, 0.96, 0.965, 0.97, 0.975, 0.98, 0.985, 0.99, 0.995, 1.0]$

- **$\bar{\nu}$ -mode $\bar{\nu}_\mu$ CC1 π :**

p_μ (MeV): $[0, 500, 700, 900, 1300, 2500, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.7, 0.8, 0.9, 0.94, 0.96, 0.98, 0.99, 1.0]$

- **$\bar{\nu}$ -mode $\bar{\nu}_\mu$ CC-Other:**

p_μ (MeV): $[0, 600, 800, 1000, 1250, 1500, 2000, 4000, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.7, 0.8, 0.85, 0.9, 0.93, 0.95, 0.97, 0.98, 0.99, 1.0]$

- **$\bar{\nu}$ -mode ν_μ CC0 π :**

p_μ (MeV): $[0, 300, 500, 700, 800, 900, 1250, 1500, 2000, 4000, 30000]$

$\cos \theta_\mu$: $[-1.0, 0.7, 0.8, 0.85, 0.88, 0.9, 0.92, 0.94, 0.96, 0.97, 0.98, 0.99, 1.0]$

- **$\bar{\nu}$ -mode ν_μ CC1 π :**

p_μ (MeV): [0, 600, 800, 1500, 30000]

$\cos \theta_\mu$: [-1.0, 0.7, 0.8, 0.86, 0.9, 0.94, 0.96, 0.97, 0.98, 0.99, 1.0]

- **$\bar{\nu}$ -mode ν_μ CC-Other:**

p_μ (MeV): [0, 600, 1000, 1250, 2000, 4000, 30000]

$\cos \theta_\mu$: [-1.0, 0.7, 0.8, 0.86, 0.9, 0.93, 0.95, 0.97, 0.99, 1.0]

C.1.2 Super-K Sample Binning

- **1R μ :**

Reco. E_ν^{CCQE} (GeV): [0.0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.7, 0.75, 0.8, 0.85, 0.90, 0.95, 1.0, 1.05, 1.10, 1.15, 1.20, 1.25, 1.30, 1.35, 1.40, 1.45, 1.50, 1.55, 1.60, 1.65, 1.70, 1.75, 1.80, 1.85, 1.90, 1.95, 2.0, 2.05, 2.10, 2.15, 2.20, 2.25, 2.30, 2.35, 2.40, 2.45, 2.50, 2.55, 2.60, 2.65, 2.70, 2.75, 2.80, 2.85, 2.90, 2.95, 3.00, 3.25, 3.5, 3.75, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0, 9.0, 10.0, 30.0];

- **1Re and 1Re1de:**

Reco. E_ν^{CCQE} (GeV): [0., 0.05, 0.1, 0.15, 0.2, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.0, 1.05, 1.1, 1.15, 1.20, 1.25];

Reco. θ_l : [0., 10.0, 20.0, 30.0, 40.0, 50.0, 60.0, 70.0, 80.0, 90.0, 100.0, 110.0, 120.0, 130.0, 140.0, 180.0];

- **2R π^0 and 1R π^0 :**

Reco. P_{π^0} (MeV): [0., 200., 400., 600., 900., 1500.];

C.2 Super-K MC Event Rates

This section shows the breakdown of the MC events presented in Table 7.2 for the T2K oscillation analysis. The MC event rates and data at Super-K with 3.77×10^{21} POT. The MC assumes normal ordering with $\delta_{CP} = -1.601$, $\sin^2 \theta_{23} = 0.561$, $\sin^2 \theta_{13} = 0.022$, and $\Delta m_{32}^2 = 2.494 \times 10^{-3} \text{ eV}^2$.

C.2.1 Unoscillated Event Rates

Table C.1: Breakdown for sample: ν -mode $1R\mu$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	1005.92802	0.00504	28.00794	0.00037	1033.94138
CC $1\pi^\pm$	123.11504	0.00471	5.42241	0.00010	128.54225
CC Coh	1.43582	-	0.23715	0.00005	1.67301
CCmulti π	10.14576	0.00110	0.62812	0.00004	10.77502
CC DIS	0.45831	-	0.01713	-	0.47545
NC $1\pi^0$	0.98239	0.02935	0.03669	0.00265	1.05106
NC $1\pi^\pm$	8.21626	0.19346	0.33600	0.01786	8.76358
NC Coh	-	0.00055	-	-	0.00055
NC other	1.09799	0.03644	0.05386	0.00407	1.19236
2p-2h	150.77072	0.00179	5.59880	0.00005	156.37136
NC 1γ	0.00919	-	-	-	0.00919
CC Misc	0.77585	-	0.04502	-	0.82087
NC multi π	1.42885	0.06967	0.09207	0.00582	1.59641
NC DIS	0.14906	0.00575	0.00471	0.00038	0.15990
CC $1\pi^0$	22.32724	-	0.70767	0.00005	23.03496
Total	1326.84050	0.34786	41.18756	0.03143	1368.40734

Table C.2: Breakdown for sample: $\bar{\nu}$ -mode $1R\mu$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	60.72758	0.00146	237.22147	0.00192	297.95243
CC $1\pi^\pm$	16.34597	0.00049	19.98663	0.00043	36.33353
CC Coh	0.17046	-	1.47417	-	1.64463
CCmulti π	2.36912	0.00013	1.43021	0.00009	3.79955
CC DIS	0.09441	-	0.04158	-	0.13598
NC $1\pi^0$	0.12626	0.00553	0.13538	0.00281	0.26998
NC $1\pi^\pm$	1.05458	0.04084	1.43070	0.03419	2.56031
NC Coh	0.00060	-	-	-	0.00060
NC other	0.27363	0.01080	0.12282	0.00475	0.41201
2p-2h	13.31351	0.00039	35.94799	0.00034	49.26222
NC 1γ	-	-	-	-	-
CC Misc	0.20530	-	0.09988	0.00008	0.30526
NC multi π	0.34553	0.01505	0.22682	0.00870	0.59611
NC DIS	0.03310	0.00146	0.01173	0.00045	0.04674
CC $1\pi^0$	2.86087	0.00013	2.58236	-	5.44336
Total	97.92093	0.07628	300.71174	0.05376	398.76271

Table C.3: Breakdown for sample: ν -mode 1Re.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	1.50414	8.25446	0.03506	0.35897	10.15263
CC $1\pi^\pm$	0.20645	1.70068	0.00896	0.10752	2.02360
CC Coh	-	0.01751	0.00047	0.00529	0.02327
CCmulti π	0.03509	0.18623	0.00128	0.00970	0.23230
CC DIS	-	0.00711	-	0.00036	0.00747
NC $1\pi^0$	2.88809	0.06221	0.12497	0.00705	3.08232
NC $1\pi^\pm$	0.52647	0.01665	0.02386	0.00147	0.56846
NC Coh	0.32956	0.00664	0.03275	0.00190	0.37085
NC other	0.22907	0.00622	0.00775	0.00089	0.24393
2p-2h	0.20887	1.53229	0.00646	0.08306	1.83068
NC 1γ	1.56372	0.03681	0.07005	0.00341	1.67398
CC Misc	0.00916	0.01293	0.00016	0.00089	0.02315
NC multi π	0.31149	0.01205	0.01776	0.00149	0.34279
NC DIS	0.10227	0.00214	0.00163	0.00009	0.10612
CC $1\pi^0$	0.15730	0.37343	0.00194	0.01366	0.54634
Total	8.07169	12.22736	0.33311	0.59573	21.22789

Table C.4: Breakdown for sample: $\bar{\nu}$ -mode 1Re.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	0.09154	1.03024	0.34961	1.74505	3.21644
CC $1\pi^\pm$	0.02607	0.26981	0.04801	0.34104	0.68493
CC Coh	-	0.00207	0.00439	0.02183	0.02829
CCmulti π	0.00460	0.04146	0.00447	0.01947	0.07000
CC DIS	0.00061	0.00074	-	0.00008	0.00144
NC $1\pi^0$	0.36911	0.01535	0.47089	0.01207	0.86742
NC $1\pi^\pm$	0.09319	0.00327	0.07529	0.00194	0.17368
NC Coh	0.04447	0.00154	0.14060	0.00277	0.18937
NC other	0.03238	0.00175	0.02051	0.00055	0.05519
2p-2h	0.01999	0.23593	0.04980	0.32557	0.63129
NC 1γ	0.17449	0.00683	0.27659	0.00537	0.46328
CC Misc	0.00113	0.00387	-	0.00203	0.00704
NC multi π	0.09567	0.00448	0.04642	0.00228	0.14885
NC DIS	0.01114	0.00098	0.00223	0.00013	0.01448
CC $1\pi^0$	0.01040	0.05732	0.00965	0.04221	0.11958
Total	0.97480	1.67564	1.49846	2.52239	6.67129

Table C.5: Breakdown for sample: ν -mode 1Re1de.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	0.39324	0.00826	0.00905	-	0.41055
CC $1\pi^\pm$	0.28171	1.30599	0.00843	0.00153	1.59766
CC Coh	0.00907	0.02957	0.00049	0.00024	0.03936
CCmulti π	0.08759	0.14359	0.00161	0.00330	0.23608
CC DIS	-	0.00271	0.00016	0.00005	0.00291
NC $1\pi^0$	0.02908	0.00053	0.00081	-	0.03041
NC $1\pi^\pm$	0.13379	0.00617	0.00549	0.00069	0.14614
NC Coh	-	-	-	-	-
NC other	0.06075	0.00406	0.00539	0.00059	0.07078
2p-2h	0.07709	0.00895	0.00244	0.00040	0.08888
NC 1γ	-	-	-	-	-
CC Misc	0.06924	0.00450	0.00017	0.00010	0.07400
NC multi π	0.15338	0.00685	0.01485	0.00151	0.17658
NC DIS	0.03053	0.00107	0.00229	0.00004	0.03393
CC $1\pi^0$	0.34976	0.04145	0.00260	0.00100	0.39481
Total	1.67522	1.56369	0.05376	0.00944	3.30211

Table C.6: Breakdown for sample: ν -mode 2R π^0 .

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	3.10635	0.53940	0.05640	0.03265	3.73481
CC $1\pi^\pm$	2.50091	0.63760	0.03357	0.05745	3.22952
CC Coh	0.02855	0.01020	0.00336	0.00287	0.04499
CCmulti π	0.59991	0.28894	0.01419	0.01976	0.92281
CC DIS	0.14649	0.04331	0.00162	0.00116	0.19258
NC $1\pi^0$	87.11064	1.75251	3.30912	0.18144	92.35372
NC $1\pi^\pm$	8.08056	0.20955	0.36509	0.02275	8.67795
NC Coh	5.38869	0.11157	0.46993	0.02300	5.99319
NC other	6.56395	0.25028	0.33365	0.02136	7.16924
2p-2h	0.66369	0.10730	0.01324	0.00953	0.79375
NC 1γ	0.11141	0.00535	0.00554	0.00071	0.12301
CC Misc	0.18420	0.03362	0.00337	0.00185	0.22304
NC multi π	8.53961	0.32923	0.46249	0.03361	9.36493
NC DIS	1.20615	0.05129	0.03457	0.00303	1.29504
CC $1\pi^0$	3.21476	0.33696	0.02363	0.02749	3.60283
Total	127.44587	4.70712	5.12977	0.43866	137.72142

Table C.7: Breakdown for sample: $\bar{\nu}$ -mode $2R\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	0.17365	0.09986	0.52117	0.09698	0.89166
CC $1\pi^\pm$	0.20466	0.13051	0.10441	0.12485	0.56443
CC Coh	0.00411	0.00128	0.01827	0.00919	0.03286
CCmulti π	0.15020	0.07014	0.04043	0.03367	0.29445
CC DIS	0.02934	0.01135	0.00548	0.00134	0.04752
NC $1\pi^0$	10.02308	0.40564	15.57664	0.31735	26.32272
NC $1\pi^\pm$	1.20376	0.05371	1.38091	0.03438	2.67275
NC Coh	0.61071	0.02381	2.83527	0.04581	3.51560
NC other	1.47720	0.06781	0.89738	0.03188	2.47428
2p-2h	0.04413	0.02128	0.12006	0.02119	0.20665
NC 1γ	0.01863	0.00026	0.01215	-	0.03104
CC Misc	0.04627	0.00716	0.01056	0.00303	0.06703
NC multi π	2.10412	0.09723	1.03358	0.04223	3.27715
NC DIS	0.33582	0.01558	0.06855	0.00269	0.42264
CC $1\pi^0$	0.22661	0.06852	0.10577	0.05757	0.45847
Total	16.65229	1.07417	22.73064	0.82216	41.27926

Table C.8: Breakdown for sample: ν -mode $1R\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	1.48318	1.51972	0.03086	0.15640	3.19016
CC $1\pi^\pm$	0.37941	0.60434	0.01203	0.08606	1.08184
CC Coh	-	0.00701	0.00080	0.00312	0.01093
CCmulti π	0.12164	0.12880	0.00382	0.01190	0.26616
CC DIS	0.04800	0.01358	0.00033	0.00031	0.06222
NC $1\pi^0$	7.45441	0.20493	0.32048	0.02069	8.00052
NC $1\pi^\pm$	0.53366	0.01901	0.03260	0.00230	0.58756
NC Coh	0.82551	0.02315	0.09111	0.00499	0.94475
NC other	0.24950	0.00740	0.01612	0.00075	0.27376
2p-2h	0.19325	0.29263	0.00581	0.03599	0.52768
NC 1γ	0.07965	0.00200	0.00522	0.00028	0.08715
CC Misc	0.00528	0.01209	0.00047	0.00066	0.01850
NC multi π	1.14112	0.04552	0.05444	0.00387	1.24495
NC DIS	0.38089	0.01049	0.01053	0.00096	0.40286
CC $1\pi^0$	0.27689	0.10427	0.00505	0.01104	0.39725
Total	13.17238	2.99493	0.58965	0.33933	17.09629

Table C.9: Breakdown for sample: $\bar{\nu}$ -mode $1\text{R}\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	Total
CCQE	0.07811	0.29770	0.28588	0.40508	1.06678
CC $1\pi^\pm$	0.03889	0.13660	0.03298	0.17223	0.38070
CC Coh	-	0.00193	0.00088	0.00829	0.01110
CCmulti π	0.02315	0.03608	0.00453	0.01888	0.08264
CC DIS	0.00710	0.00323	0.00267	0.00063	0.01363
NC $1\pi^0$	1.11350	0.04713	1.23754	0.03108	2.42925
NC $1\pi^\pm$	0.09717	0.00566	0.10691	0.00352	0.21326
NC Coh	0.12356	0.00556	0.39079	0.00791	0.52782
NC other	0.06245	0.00264	0.03386	0.00134	0.10028
2p-2h	0.01871	0.06792	0.03999	0.07389	0.20050
NC 1γ	0.01534	0.00052	0.01267	0.00066	0.02918
CC Misc	0.00342	0.00244	0.00365	0.00127	0.01078
NC multi π	0.27644	0.01494	0.11385	0.00479	0.41001
NC DIS	0.09574	0.00489	0.02576	0.00052	0.12691
CC $1\pi^0$	0.05001	0.02328	0.02247	0.01691	0.11267
Total	2.00360	0.65050	2.31443	0.74698	5.71551

C.2.2 Oscillated Event Rates

Table C.10: Breakdown for sample: ν -mode $1R_\mu$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	196.20013	0.00480	13.42115	0.00036	0.04159	0.00025	209.66828
CC $1\pi^\pm$	45.36872	0.00439	3.49287	0.00009	0.04747	0.00007	48.91362
CC Coh	0.58861	-	0.12681	0.00005	-	0.00002	0.71549
CCmulti π	8.08915	0.00109	0.51342	0.00004	0.00204	-	8.60574
CC DIS	0.43756	-	0.01631	-	-	-	0.45387
NC $1\pi^0$	0.98239	0.02935	0.03669	0.00265	-	-	1.05106
NC $1\pi^\pm$	8.21626	0.19346	0.33600	0.01786	-	-	8.76358
NC Coh	-	0.00055	-	-	-	-	0.00055
NC other	1.09799	0.03644	0.05386	0.00407	-	-	1.19236
2p-2h	38.86982	0.00164	3.02380	0.00005	0.01243	0.00007	41.90781
NC 1γ	0.00919	-	-	-	-	-	0.00919
CC Misc	0.62015	-	0.03643	-	-	-	0.65658
NC multi π	1.42885	0.06967	0.09207	0.00582	-	-	1.59641
NC DIS	0.14906	0.00575	0.00471	0.00038	-	-	0.15990
CC $1\pi^0$	8.14161	-	0.46905	0.00005	0.00805	0.00001	8.61876
Total	310.19948	0.34714	21.62317	0.03140	0.11158	0.00042	332.31318

Table C.11: Breakdown for sample: $\bar{\nu}$ -mode $1R_\mu$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	26.54536	0.00142	54.83521	0.00182	0.00080	0.00288	81.38748
CC $1\pi^\pm$	9.86180	0.00047	8.67957	0.00042	0.00083	0.00030	18.54338
CC Coh	0.10924	-	0.36184	-	-	0.00030	0.47139
CCmulti π	1.90536	0.00012	1.13599	0.00008	0.00007	0.00015	3.04176
CC DIS	0.08911	-	0.03971	-	-	-	0.12882
NC $1\pi^0$	0.12626	0.00553	0.13538	0.00281	-	-	0.26998
NC $1\pi^\pm$	1.05458	0.04084	1.43070	0.03419	-	-	2.56031
NC Coh	0.00060	-	-	-	-	-	0.00060
NC other	0.27363	0.01080	0.12282	0.00475	-	-	0.41201
2p-2h	7.00325	0.00037	9.53302	0.00033	0.00060	0.00038	16.53795
NC 1γ	-	-	-	-	-	-	-
CC Misc	0.16586	-	0.07902	0.00008	-	-	0.24496
NC multi π	0.34553	0.01505	0.22682	0.00870	-	-	0.59611
NC DIS	0.03310	0.00146	0.01173	0.00045	-	-	0.04674
CC $1\pi^0$	1.70602	0.00012	1.21050	-	0.00014	-	2.91678
Total	49.21970	0.07619	77.80231	0.05363	0.00243	0.00400	127.15827

Table C.12: Breakdown for sample: ν -mode 1Re.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.29444	7.66102	0.01125	0.33769	58.64090	0.41312	67.35841
CC $1\pi^\pm$	0.06372	1.59479	0.00483	0.10240	7.60787	0.08620	9.45981
CC Coh	-	0.01632	0.00019	0.00500	0.07371	0.00521	0.10043
CCmulti π	0.01904	0.17999	0.00078	0.00941	0.12100	0.00355	0.33378
CC DIS	-	0.00703	-	0.00036	0.00051	0.00001	0.00791
NC $1\pi^0$	2.88809	0.06221	0.12497	0.00705	-	-	3.08232
NC $1\pi^\pm$	0.52647	0.01665	0.02386	0.00147	-	-	0.56846
NC Coh	0.32956	0.00664	0.03275	0.00190	-	-	0.37085
NC other	0.22907	0.00622	0.00775	0.00089	-	-	0.24393
2p-2h	0.05613	1.43014	0.00290	0.07884	8.77576	0.07863	10.42241
NC 1γ	1.56372	0.03681	0.07005	0.00341	-	-	1.67398
CC Misc	0.00031	0.01248	0.00010	0.00086	0.01355	0.00021	0.02752
NC multi π	0.31149	0.01205	0.01776	0.00149	-	-	0.34279
NC DIS	0.10227	0.00214	0.00163	0.00009	-	-	0.10612
CC $1\pi^0$	0.02714	0.35004	0.00103	0.01306	1.59019	0.01024	1.99170
Total	6.41147	11.39451	0.29987	0.56392	76.82348	0.59718	96.09043

Table C.13: Breakdown for sample: $\bar{\nu}$ -mode 1Re.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.03282	0.95878	0.05650	1.63845	2.03397	6.13978	10.86031
CC $1\pi^\pm$	0.01372	0.25453	0.01878	0.32231	0.38279	0.81998	1.81211
CC Coh	-	0.00196	0.00080	0.02051	0.00290	0.07106	0.09723
CCmulti π	0.00301	0.04021	0.00286	0.01882	0.01998	0.01377	0.09864
CC DIS	0.00056	0.00073	-	0.00008	0.00015	-	0.00152
NC $1\pi^0$	0.36911	0.01535	0.47089	0.01207	-	-	0.86742
NC $1\pi^\pm$	0.09319	0.00327	0.07529	0.00194	-	-	0.17368
NC Coh	0.04447	0.00154	0.14060	0.00277	-	-	0.18937
NC other	0.03238	0.00175	0.02051	0.00055	-	-	0.05519
2p-2h	0.00943	0.22153	0.01015	0.30681	0.36569	0.95816	1.87177
NC 1γ	0.17449	0.00683	0.27659	0.00537	-	-	0.46328
CC Misc	0.00038	0.00375	-	0.00197	0.00164	0.00062	0.00836
NC multi π	0.09567	0.00448	0.04642	0.00228	-	-	0.14885
NC DIS	0.01114	0.00098	0.00223	0.00013	-	-	0.01448
CC $1\pi^0$	0.00417	0.05412	0.00359	0.04000	0.08374	0.09246	0.27808
Total	0.88455	1.56979	1.12522	2.37405	2.89087	8.09582	16.94030

Table C.14: Breakdown for sample: ν -mode 1Re1de.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.04998	0.00768	0.00138	-	0.05616	0.00005	0.11524
CC $1\pi^\pm$	0.05576	1.21074	0.00218	0.00148	8.40304	0.00075	9.67396
CC Coh	0.00072	0.02722	0.00004	0.00024	0.16107	0.00009	0.18938
CCmulti π	0.04599	0.13760	0.00093	0.00317	0.13962	0.00144	0.32876
CC DIS	-	0.00267	0.00013	0.00004	0.00034	-	0.00319
NC $1\pi^0$	0.02908	0.00053	0.00081	-	-	-	0.03041
NC $1\pi^\pm$	0.13379	0.00617	0.00549	0.00069	-	-	0.14614
NC Coh	-	-	-	-	-	-	-
NC other	0.06075	0.00406	0.00539	0.00059	-	-	0.07078
2p-2h	0.00976	0.00846	0.00060	0.00039	0.02069	0.00020	0.04009
NC 1γ	-	-	-	-	-	-	-
CC Misc	0.00548	0.00434	0.00004	0.00010	0.00235	0.00006	0.01236
NC multi π	0.15338	0.00685	0.01485	0.00151	-	-	0.17658
NC DIS	0.03053	0.00107	0.00229	0.00004	-	-	0.03393
CC $1\pi^0$	0.04151	0.03874	0.00093	0.00096	0.17348	0.00100	0.25661
Total	0.61674	1.45612	0.03505	0.00920	8.95673	0.00360	11.07745

Table C.15: Breakdown for sample: ν -mode 2R π^0 .

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.69196	0.50977	0.02318	0.03123	2.24864	0.02355	3.52834
CC $1\pi^\pm$	0.48529	0.61156	0.01990	0.05570	1.20895	0.02533	2.40673
CC Coh	0.01106	0.00975	0.00198	0.00276	0.01988	0.00237	0.04779
CCmulti π	0.36519	0.28279	0.01075	0.01938	0.09387	0.00369	0.77566
CC DIS	0.13504	0.04292	0.00148	0.00115	0.00387	0.00007	0.18453
NC $1\pi^0$	87.11064	1.75251	3.30912	0.18144	-	-	92.35372
NC $1\pi^\pm$	8.08056	0.20955	0.36509	0.02275	-	-	8.67795
NC Coh	5.38869	0.11157	0.46993	0.02300	-	-	5.99319
NC other	6.56395	0.25028	0.33365	0.02136	-	-	7.16924
2p-2h	0.11987	0.10145	0.00655	0.00917	0.35045	0.00441	0.59190
NC 1γ	0.11141	0.00535	0.00554	0.00071	-	-	0.12301
CC Misc	0.10574	0.03242	0.00232	0.00180	0.02812	0.00047	0.17088
NC multi π	8.53961	0.32923	0.46249	0.03361	-	-	9.36493
NC DIS	1.20615	0.05129	0.03457	0.00303	-	-	1.29504
CC $1\pi^0$	0.56674	0.32151	0.01434	0.02654	0.84150	0.01146	1.78209
Total	119.48188	4.62196	5.06089	0.43364	4.79529	0.07135	134.46500

Table C.16: Breakdown for sample: $\bar{\nu}$ -mode $2R\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.06547	0.09545	0.16463	0.09191	0.10773	0.25118	0.77638
CC $1\pi^\pm$	0.09779	0.12575	0.03351	0.12007	0.08804	0.15573	0.62089
CC Coh	0.00283	0.00125	0.00554	0.00876	0.00123	0.02263	0.04224
CCmulti π	0.10416	0.06872	0.02759	0.03283	0.02336	0.01118	0.26784
CC DIS	0.02635	0.01124	0.00505	0.00133	0.00096	0.00007	0.04499
NC $1\pi^0$	10.02308	0.40564	15.57664	0.31735	-	-	26.32272
NC $1\pi^\pm$	1.20376	0.05371	1.38091	0.03438	-	-	2.67275
NC Coh	0.61071	0.02381	2.83527	0.04581	-	-	3.51560
NC other	1.47720	0.06781	0.89738	0.03188	-	-	2.47428
2p-2h	0.01842	0.02033	0.02815	0.02018	0.02050	0.03985	0.14744
NC 1γ	0.01863	0.00026	0.01215	-	-	-	0.03104
CC Misc	0.02918	0.00699	0.00595	0.00295	0.00308	0.00163	0.04978
NC multi π	2.10412	0.09723	1.03358	0.04223	-	-	3.27715
NC DIS	0.33582	0.01558	0.06855	0.00269	-	-	0.42264
CC $1\pi^0$	0.09244	0.06586	0.04084	0.05512	0.05739	0.08582	0.39747
Total	16.20998	1.05964	22.11574	0.80749	0.30228	0.56809	41.06322

Table C.17: Breakdown for sample: ν -mode $1R\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.24238	1.44612	0.01134	0.15031	3.94194	0.09146	5.88356
CC $1\pi^\pm$	0.09485	0.58340	0.00670	0.08363	0.69878	0.03117	1.49853
CC Coh	-	0.00677	0.00036	0.00301	0.01224	0.00148	0.02386
CCmulti π	0.09359	0.12630	0.00286	0.01169	0.03238	0.00163	0.26845
CC DIS	0.04479	0.01346	0.00030	0.00031	0.00112	0.00003	0.06000
NC $1\pi^0$	7.45441	0.20493	0.32048	0.02069	-	-	8.00052
NC $1\pi^\pm$	0.53366	0.01901	0.03260	0.00230	-	-	0.58756
NC Coh	0.82551	0.02315	0.09111	0.00499	-	-	0.94475
NC other	0.24950	0.00740	0.01612	0.00075	-	-	0.27376
2p-2h	0.02985	0.28093	0.00233	0.03479	0.52882	0.01546	0.89219
NC 1γ	0.07965	0.00200	0.00522	0.00028	-	-	0.08715
CC Misc	0.00396	0.01183	0.00036	0.00065	0.00320	0.00017	0.02017
NC multi π	1.14112	0.04552	0.05444	0.00387	-	-	1.24495
NC DIS	0.38089	0.01049	0.01053	0.00096	-	-	0.40286
CC $1\pi^0$	0.07086	0.10066	0.00270	0.01075	0.12340	0.00280	0.31117
Total	11.24501	2.88195	0.55745	0.32898	5.34189	0.14421	20.49949

Table C.18: Breakdown for sample: $\bar{\nu}$ -mode $1R\pi^0$.

Mode	ν_μ	ν_e	$\bar{\nu}_\mu$	$\bar{\nu}_e$	ν_μ sig	$\bar{\nu}_\mu$ sig	Total
CCQE	0.02467	0.28519	0.06904	0.38621	0.26746	0.68509	1.71767
CC $1\pi^\pm$	0.02125	0.13246	0.01205	0.16639	0.07260	0.13842	0.54317
CC Coh	-	0.00187	0.00001	0.00796	0.00157	0.01145	0.02286
CCmulti π	0.01744	0.03545	0.00312	0.01851	0.00726	0.00333	0.08512
CC DIS	0.00648	0.00320	0.00227	0.00062	0.00028	0.00006	0.01292
NC $1\pi^0$	1.11350	0.04713	1.23754	0.03108	-	-	2.42925
NC $1\pi^\pm$	0.09717	0.00566	0.10691	0.00352	-	-	0.21326
NC Coh	0.12356	0.00556	0.39079	0.00791	-	-	0.52782
NC other	0.06245	0.00264	0.03386	0.00134	-	-	0.10028
2p-2h	0.00719	0.06556	0.00794	0.07105	0.04489	0.08801	0.28465
NC 1γ	0.01534	0.00052	0.01267	0.00066	-	-	0.02918
CC Misc	0.00220	0.00239	0.00301	0.00124	0.00082	0.00039	0.01004
NC multi π	0.27644	0.01494	0.11385	0.00479	-	-	0.41001
NC DIS	0.09574	0.00489	0.02576	0.00052	-	-	0.12691
CC $1\pi^0$	0.02484	0.02264	0.00806	0.01636	0.01238	0.01394	0.09823
Total	1.88828	0.63011	2.02688	0.71816	0.40726	0.94069	6.61138

C.2.3 Oscillated and Unoscillated Event Spectra

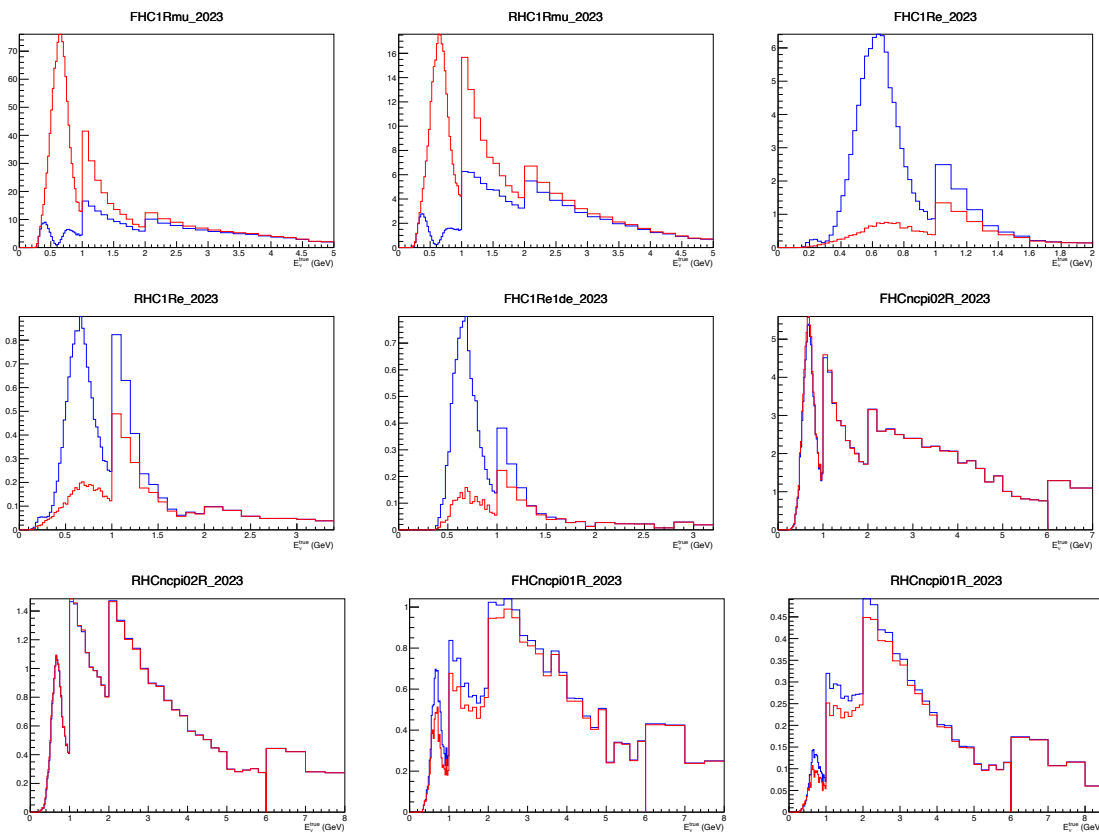


Figure C.1: Comparisons of the oscillated (blue) and unoscillated (red) spectra for the Super-K samples used in the T2K oscillation analysis presented in Chapters 7 and 8.

C.3 Event Rate Variations by New Parameters

C.3.1 Cross-section Parameters

$\text{NC}\pi^0$ Normalisation

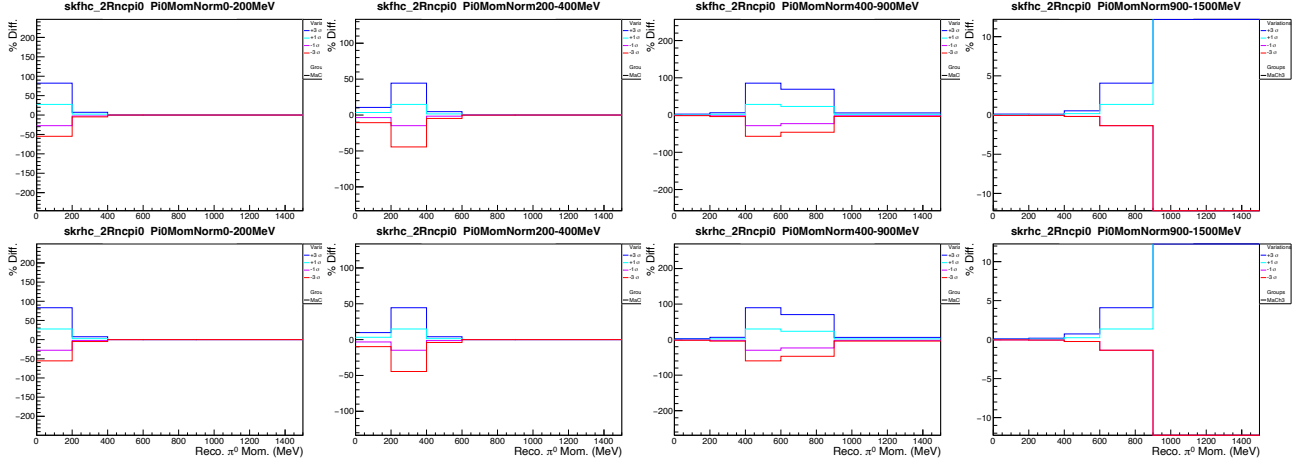


Figure C.2: The tweaked spectra of the $2R\pi^0$ samples from varying $\text{NC}\pi^0$ norm 1-4.

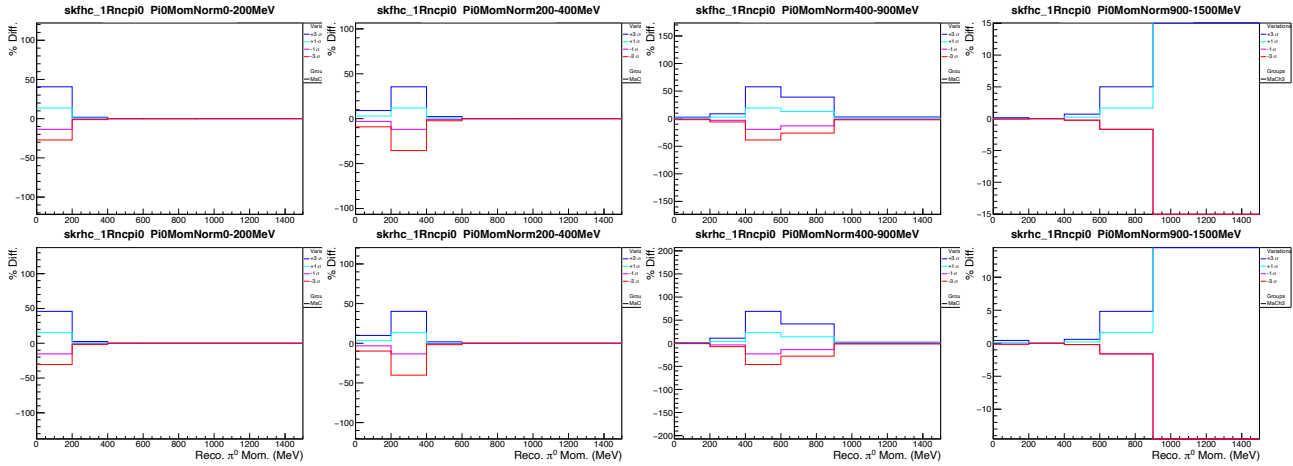


Figure C.3: The tweaked spectra of the $1R\pi^0$ samples from varying $\text{NC}\pi^0$ norm 1-4.

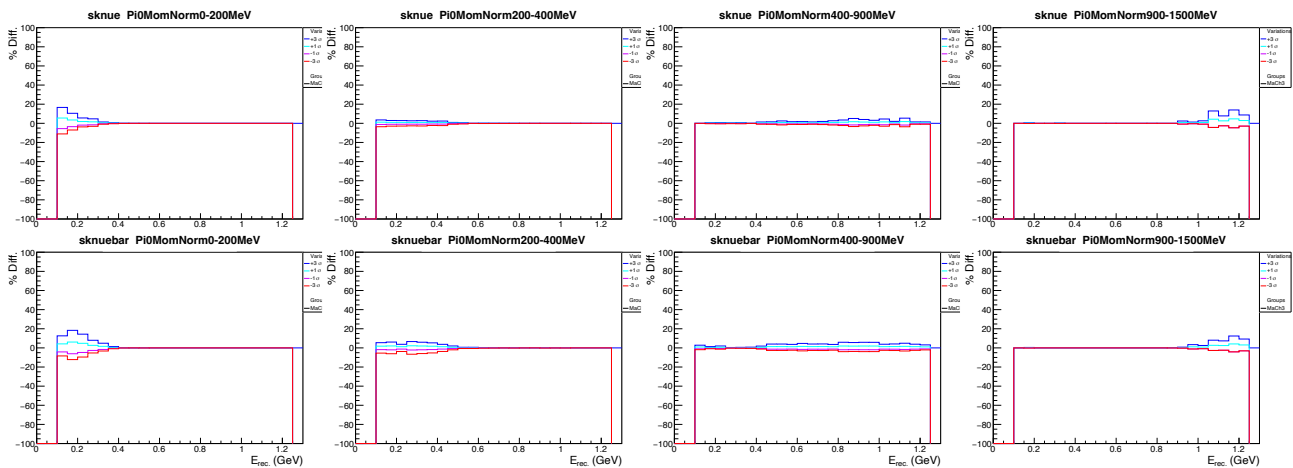


Figure C.4: The tweaked spectra of the $1R_e$ samples from varying $\text{NC}\pi^0$ norm 1-4.

C.3.2 Super-K Detector Parameters

$2R\pi^0$ Reconstruction Uncertainties

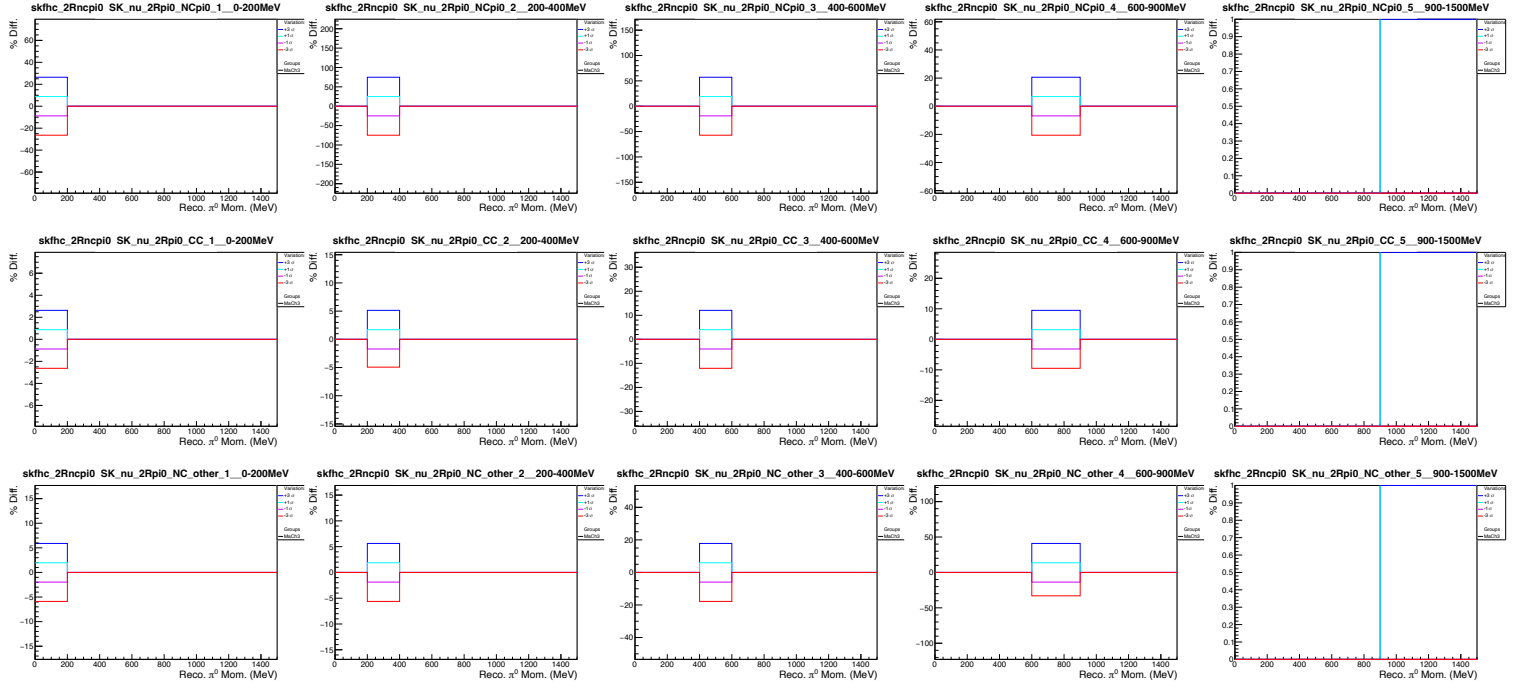


Figure C.5: The tweaked spectra of the $2R\pi^0$ samples from the $2R\pi^0$ reconstruction uncertainties (ν -mode).

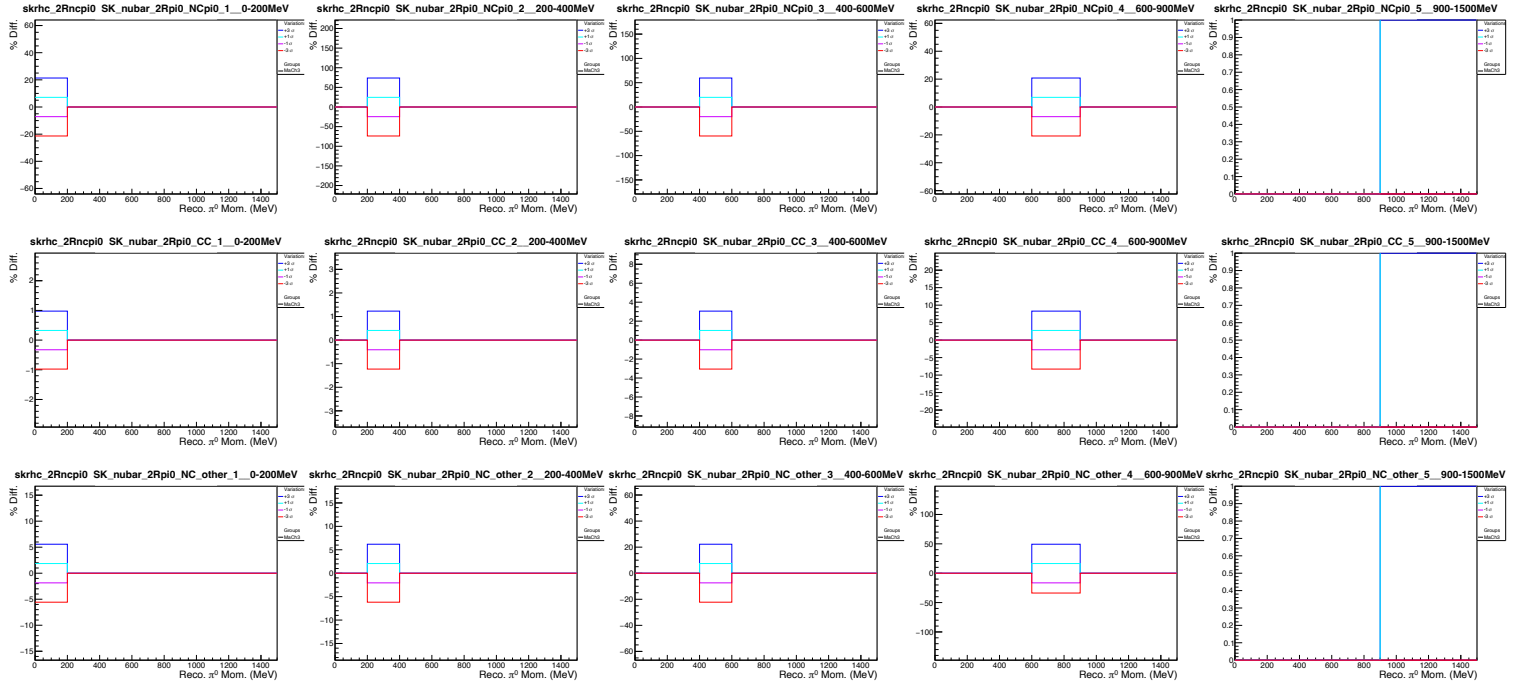


Figure C.6: The tweaked spectra of the $2R\pi^0$ samples from the $2R\pi^0$ reconstruction uncertainties ($\bar{\nu}$ -mode).

1R π^0 Reconstruction Uncertainties

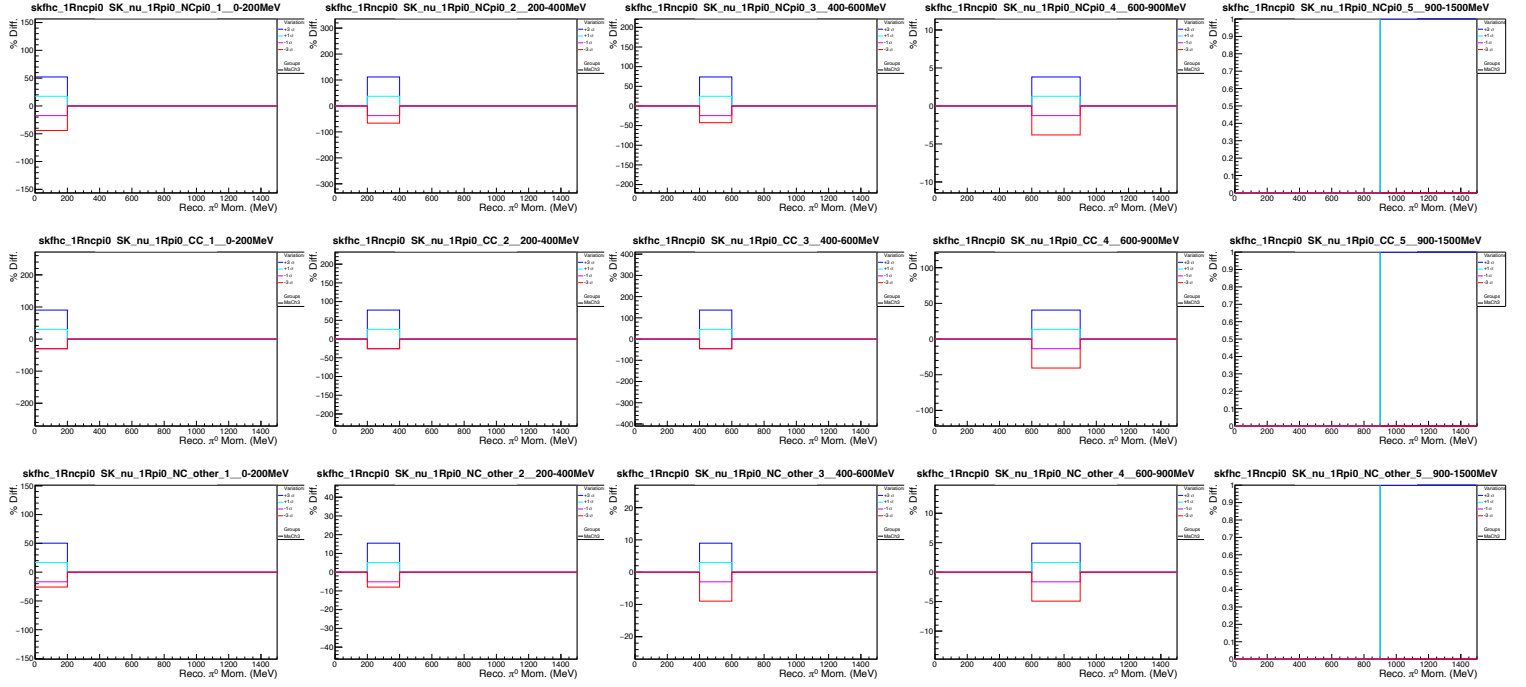


Figure C.7: The tweaked spectra of the 1R π^0 samples from the 1R π^0 reconstruction uncertainties (ν -mode).

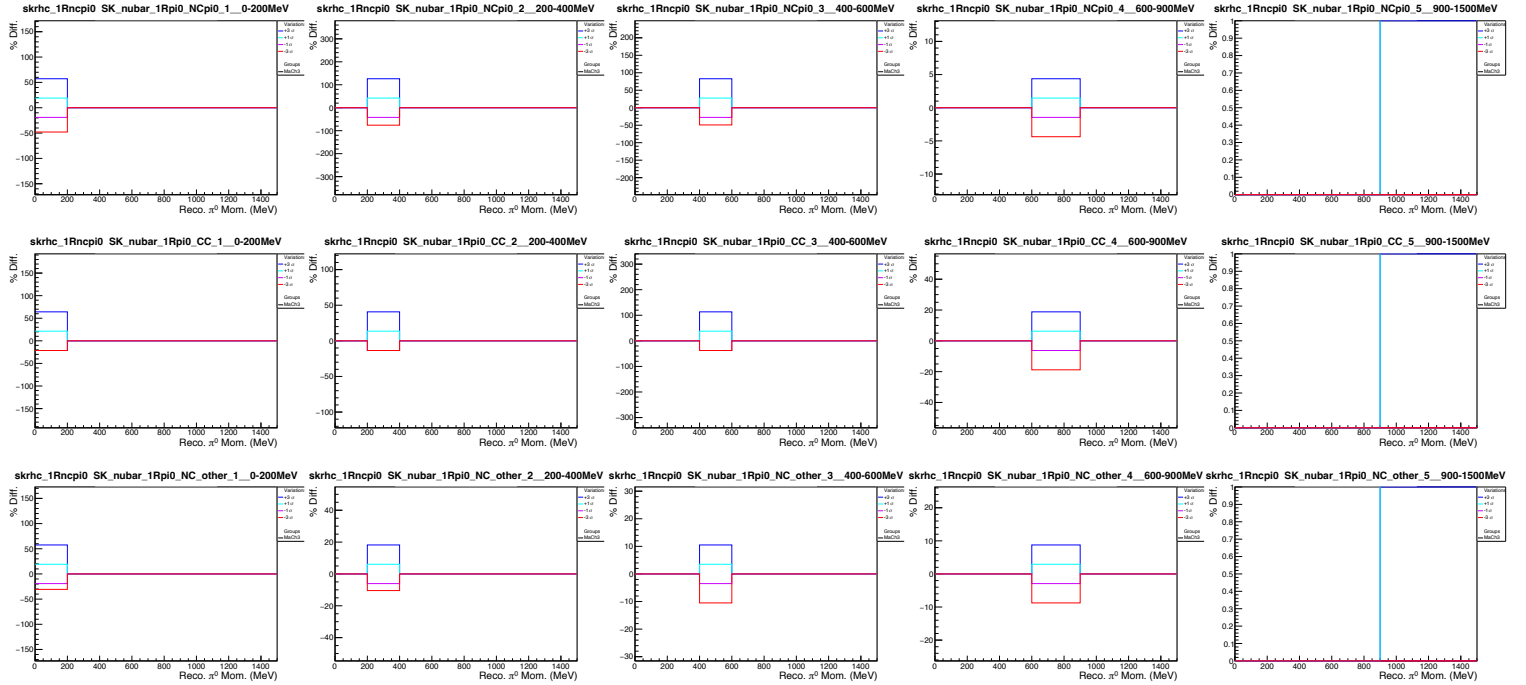


Figure C.8: The tweaked spectra of the 1R π^0 samples from the 1R π^0 reconstruction uncertainties ($\bar{\nu}$ -mode).

C.4 Additional Log-likelihood Scans

C.4.1 Largely Impacted Systematic Parameters

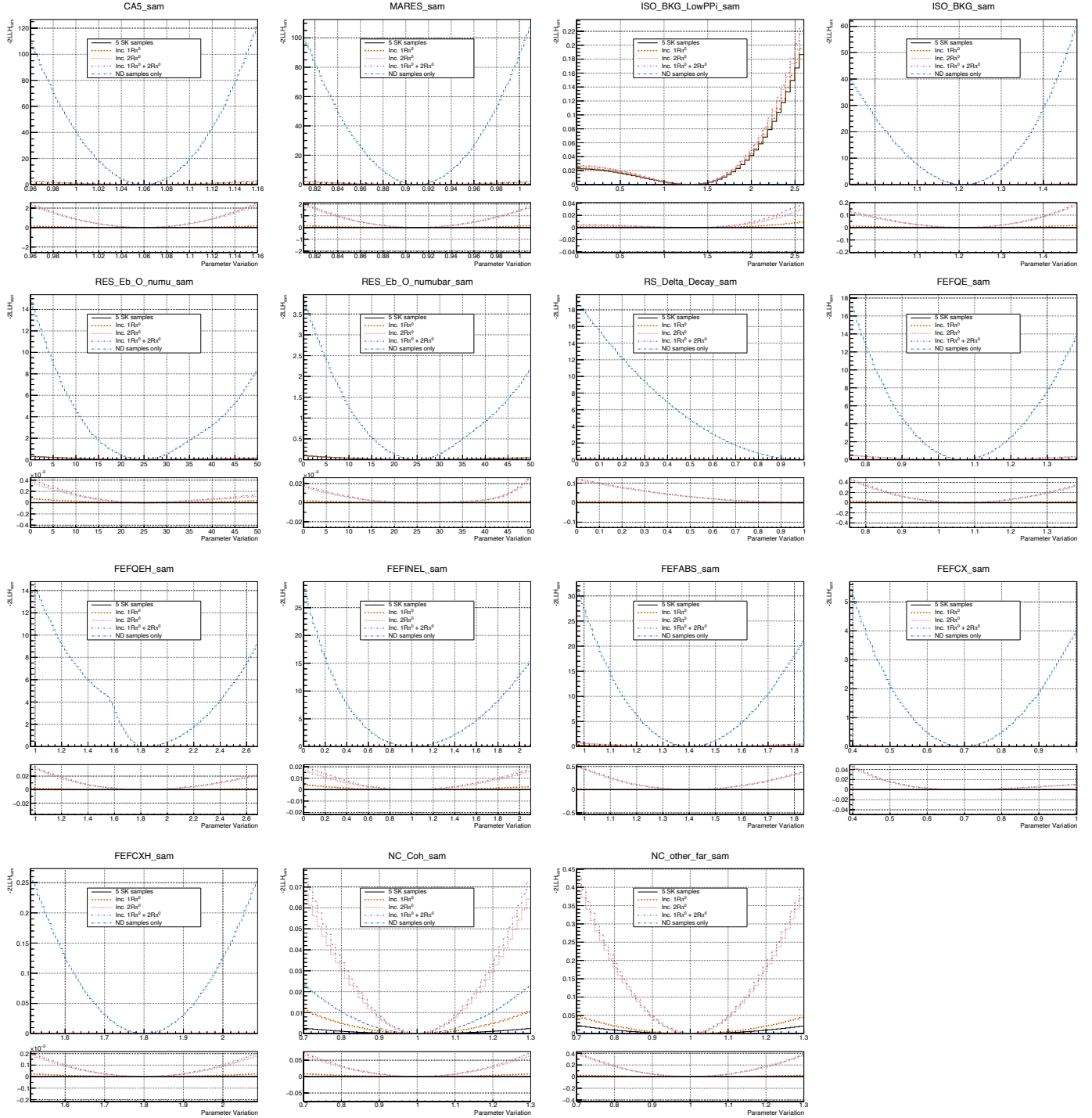


Figure C.9: The LLH scans of the cross-section parameters that are largely impacted from adding the new NC π^0 samples.

C.4.2 1-D Scans for Oscillation Parameters

The $\text{NC}\pi^0$ samples have shown an observable impact on δ_{CP} but not significant on other oscillation parameters from the 1-D LLH scans.

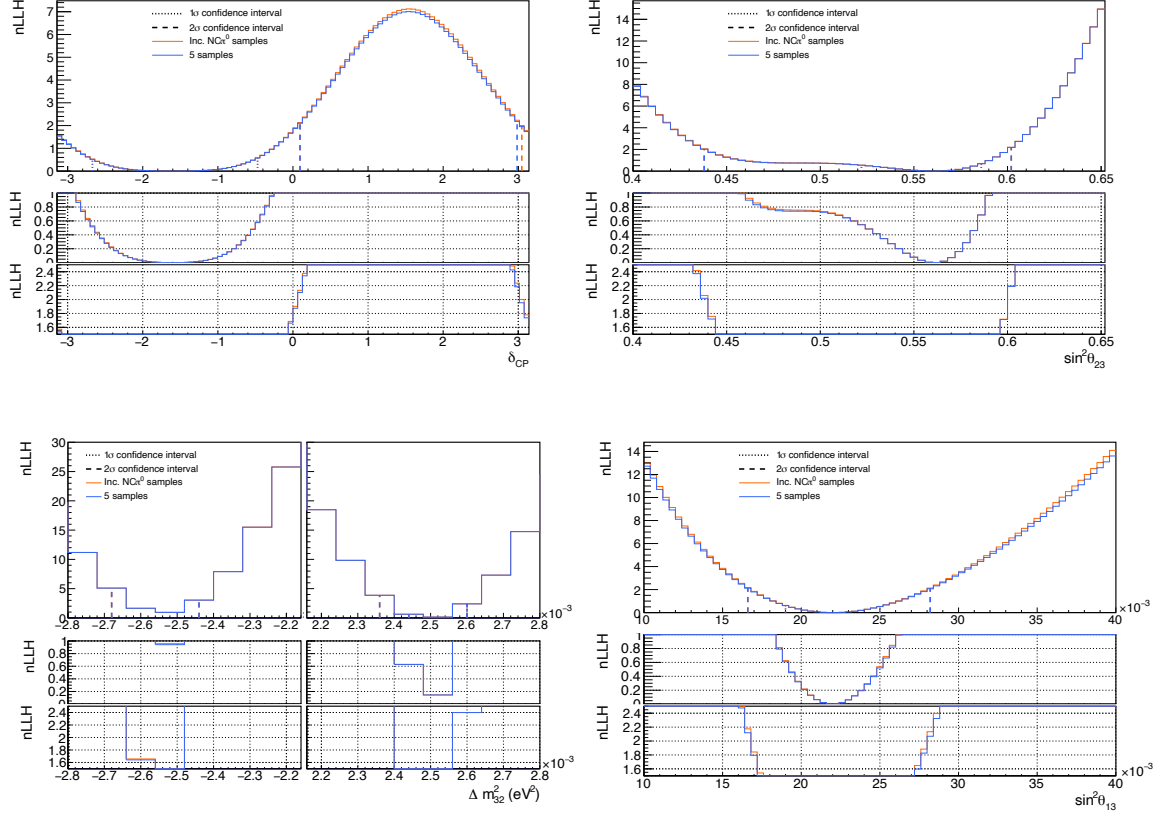


Figure C.10: The 1D LLH scans of oscillation parameters comparing 5 Super-K samples and with the new $\text{NC}\pi^0$ samples included. The bottom plots show the region around the 1σ and 2σ confidence levels.

C.5 Asimov Fits

C.5.1 Additional Posterior Distributions

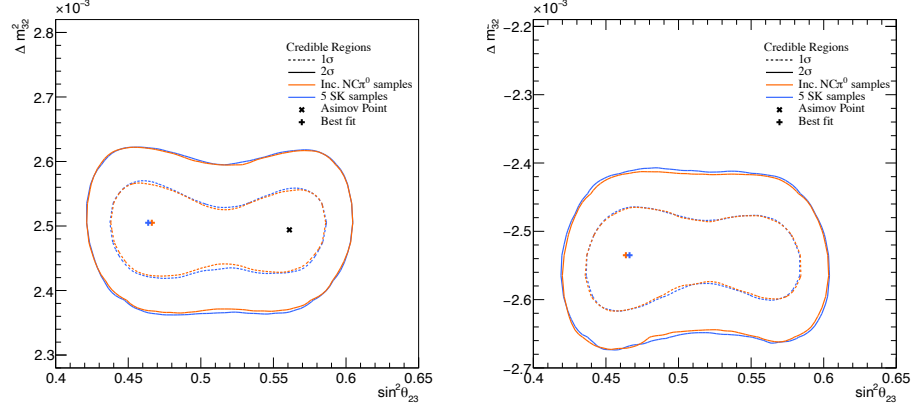


Figure C.11: The comparison of 2D credible intervals obtained from an Asimov fit with normal (left) and inverted (right) mass orderings. The fit is performed without varying the systematic uncertainties, using the 5 SK samples and the newly included $\text{NC}\pi^0$ samples. The reactor constraints are not applied to $\sin^2(\theta_{13})$.

C.5.2 Additional Postfit Correlations

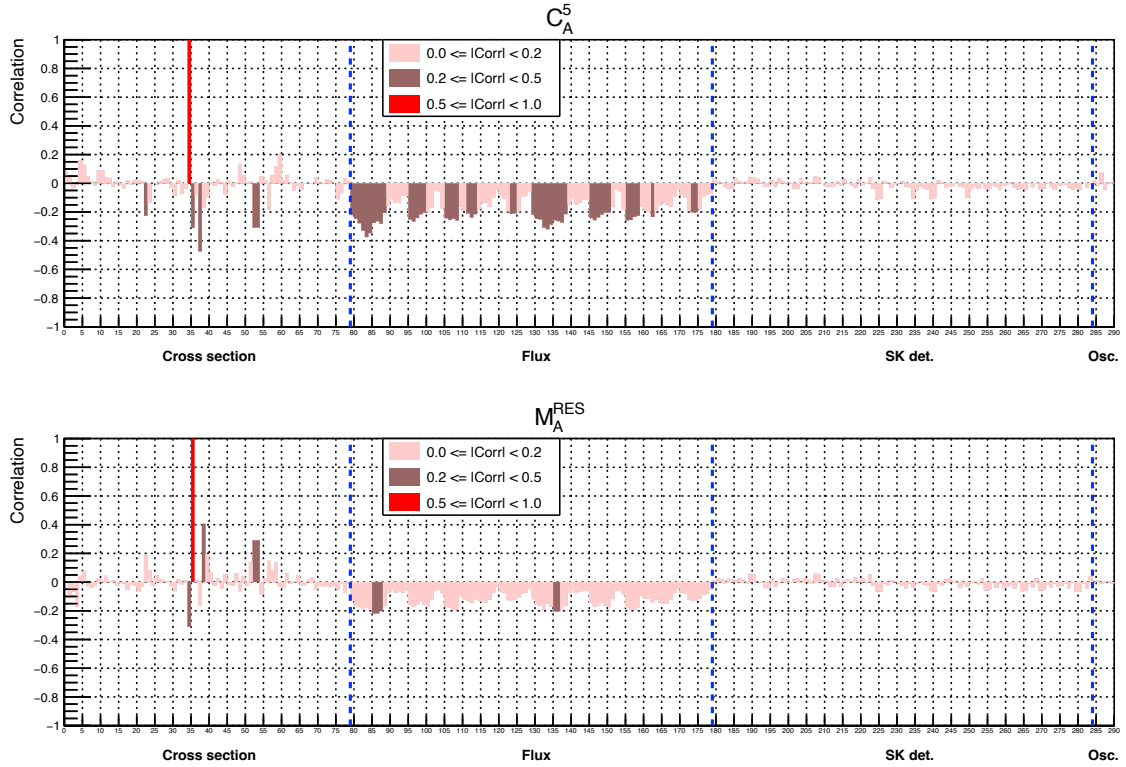


Figure C.12: The postfit correlation of the resonance single pion production form factors from an Asimov fit.

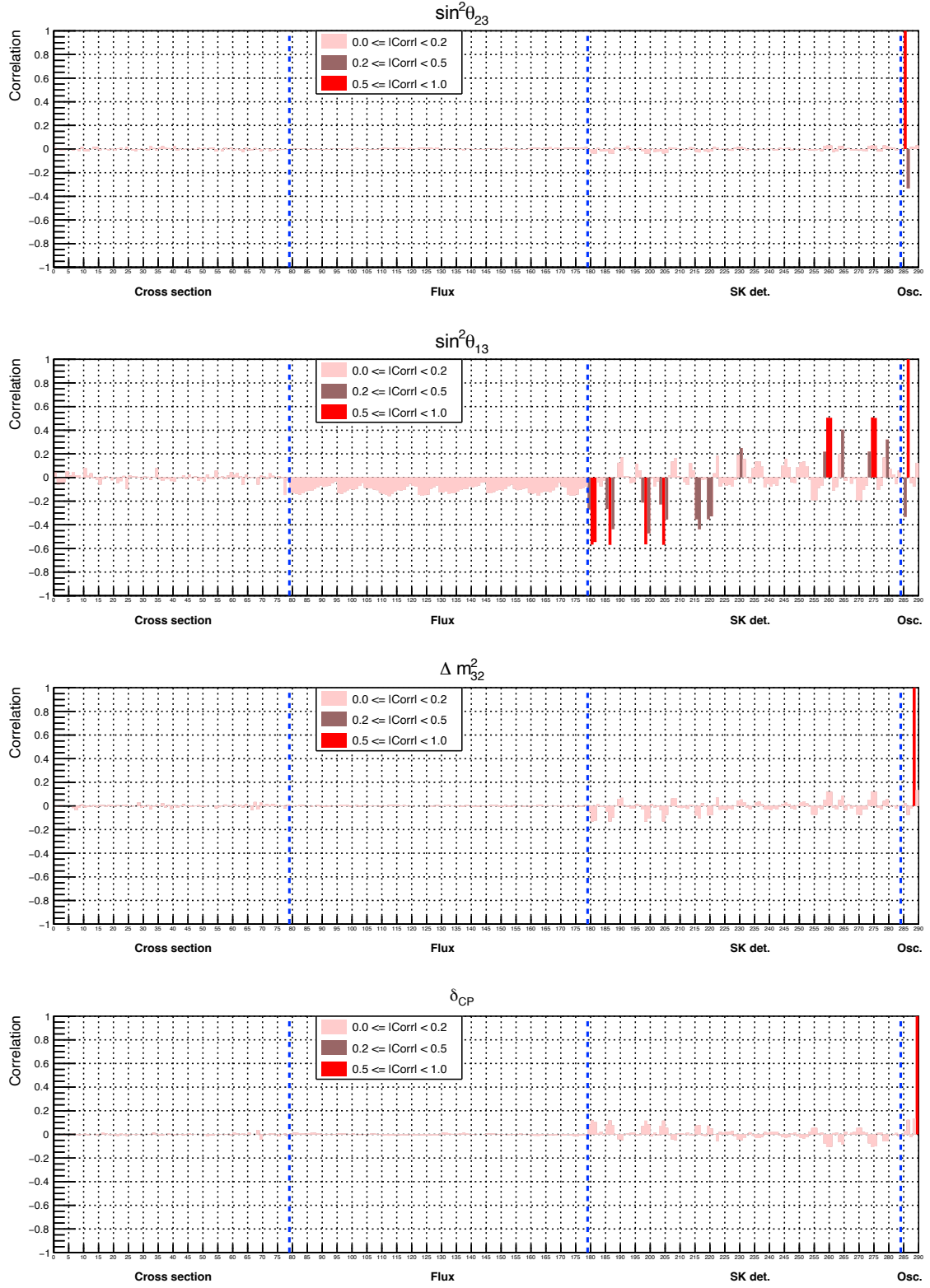


Figure C.13: The postfit correlation of oscillation parameters from an Asimov fit.

C.5.3 Additional Postfit Uncertainties

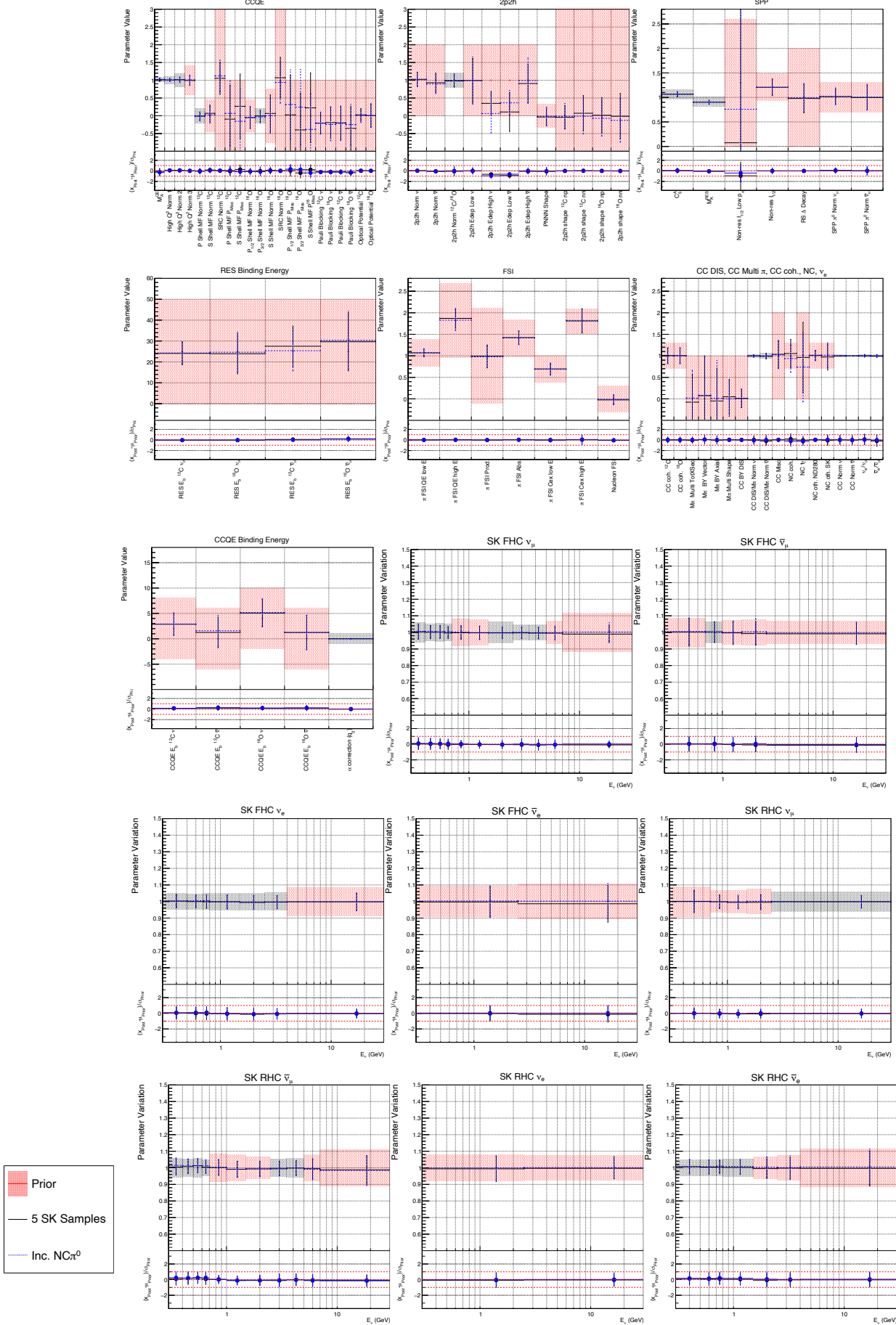


Figure C.14: The results on the Super-K flux and old cross-section parameters from an Asimov fit.

C.6 Data Fits

C.6.1 Postfit Prediction for $1R_\mu$ and $1Re1de$ Samples

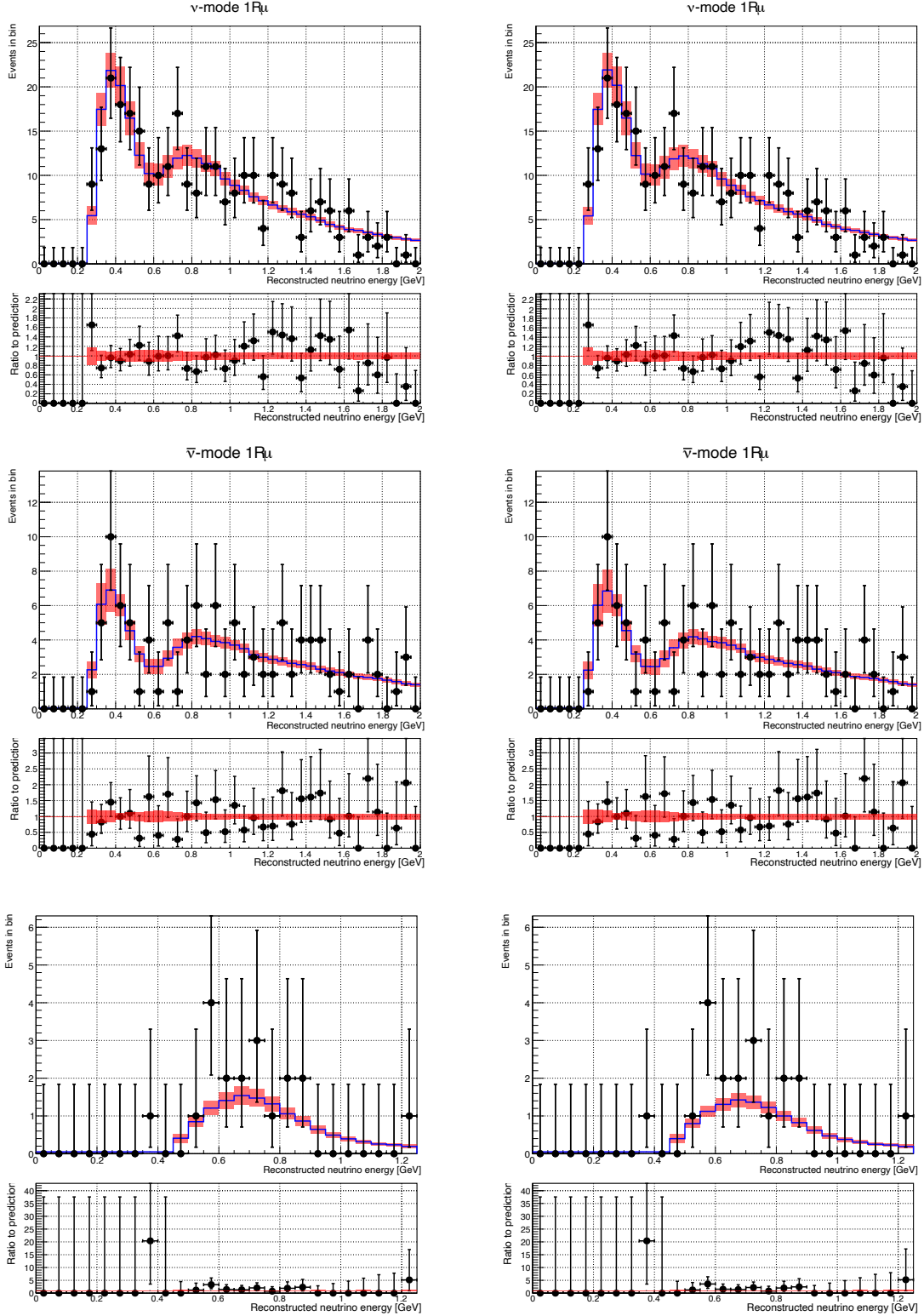


Figure C.15: The postfit MC predictions for the $1R_\mu$ (top and middle), and $1Re1de$ samples (bottom) fit to the T2K Runs 1-11 data without (left) and with (right) reactor constraints applied to $\sin^2 \theta_{13}$.

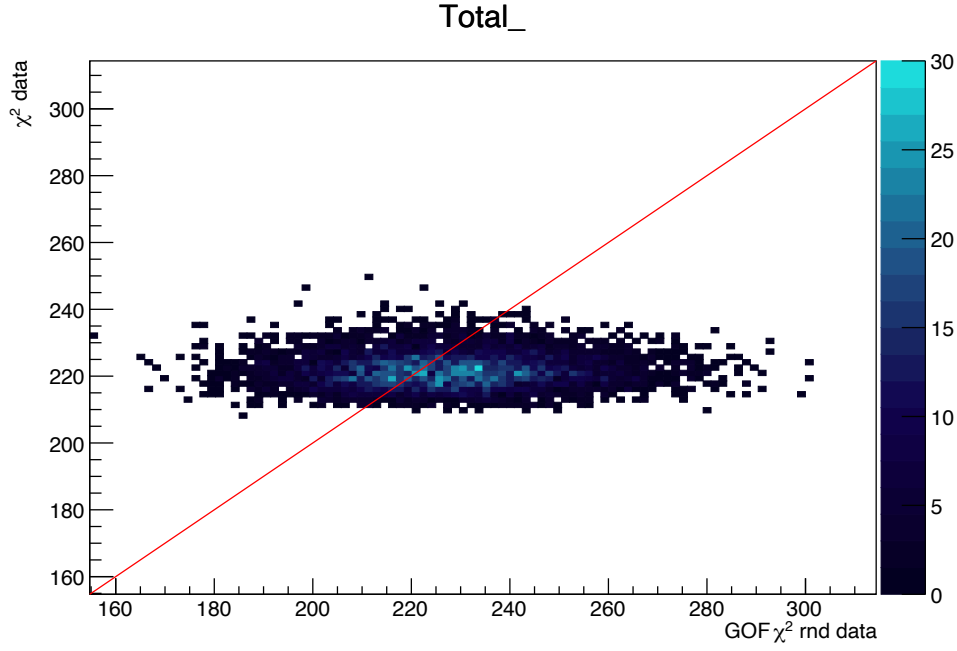


Figure C.16: The χ^2 distribution of 5,000 MC predictions compared to data and random fake data, with reactor constraints on $\sin^2 \theta_{13}$.

Table C.19: Breakdown of goodness-of-fit p-values from shape-based and total rate-based χ^2 calculations when fitting to T2K data with reactor constraints applied to $\sin^2 \theta_{13}$, averaged over 10 runs to account for statistical fluctuations in fake data throws. The results are shown for cases with and without the π^0 normalisation uncertainties applied to the 1Re samples.

Sample	π^0 Norm. to 1Re		π^0 Norm. Excl. 1Re	
	Shape	Total Rate	Shape	Total Rate
ν -mode 1R μ	0.529 (6)	0.451 (5)	0.530 (9)	0.444 (4)
$\bar{\nu}$ -mode 1R μ	0.825 (6)	0.546 (4)	0.822 (7)	0.538 (5)
ν -mode 1Re	0.158 (5)	0.494 (7)	0.158 (5)	0.504 (7)
$\bar{\nu}$ -mode 1Re	0.645 (4)	0.334 (7)	0.652 (4)	0.351 (4)
ν -mode 1Re1de	0.583 (6)	0.124 (5)	0.573 (6)	0.119 (6)
ν -mode 2R NC π^0	0.278 (6)	0.533 (6)	0.270 (6)	0.531 (5)
$\bar{\nu}$ -mode 2R NC π^0	0.318 (6)	0.482 (6)	0.306 (6)	0.478 (6)
ν -mode 1R NC π^0	0.378 (6)	0.569 (6)	0.400 (7)	0.579 (4)
$\bar{\nu}$ -mode 1R NC π^0	0.608 (6)	0.456 (7)	0.595 (8)	0.458 (6)
Total	0.599 (5)	0.416 (6)	0.597 (5)	0.414 (4)

C.6.2 Additional Postfit Uncertainties

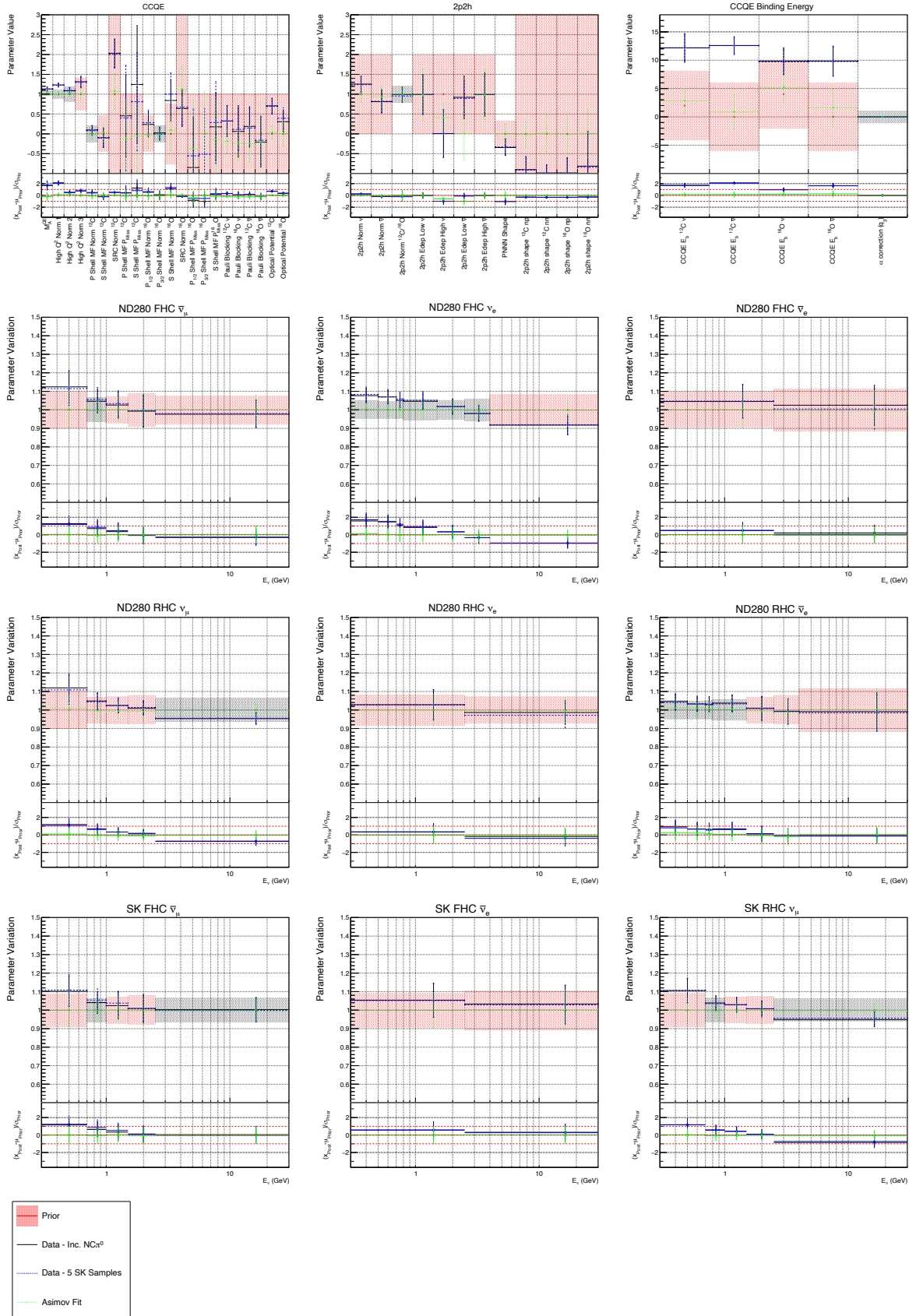


Figure C.17: Results on the systematic and oscillation parameters from T2K Runs 1-11 data fit.

C.6.3 Oscillation Parameter Postfit Correlations

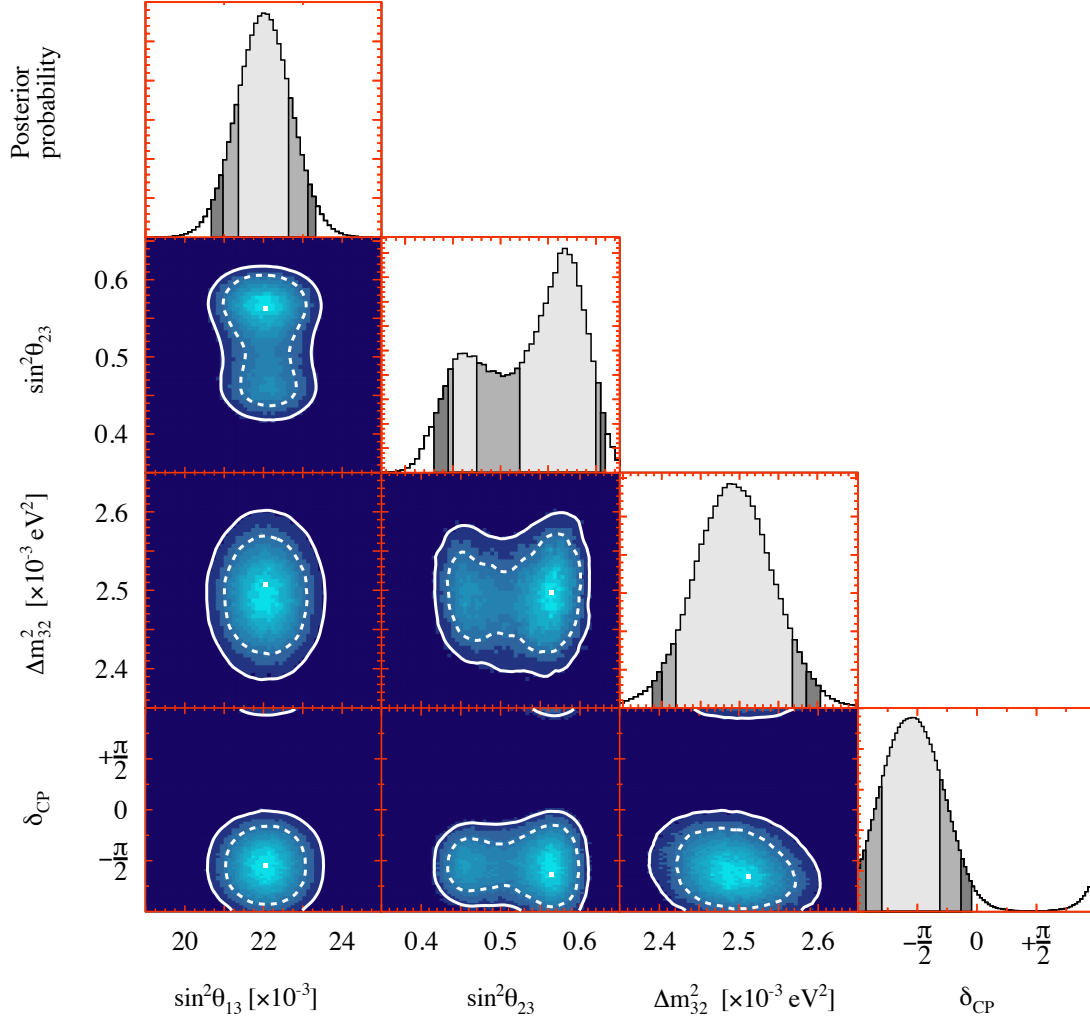


Figure C.18: Plots showing the correlations between any two of the free oscillation parameters results from this analysis constrained by the T2K Runs 1-11 data, indicating the 1σ and 2σ credible intervals, with the reactor constraints applied to $\sin^2\theta_{13}$.