

**Flow field and mixing in stirred vessels
with regular and fractal impellers**

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Abstract

The goal of this thesis is to explore the potential benefits of fractal objects in dynamic mixing applications. To this end, the performance of an impeller with fractal blades was evaluated and compared to the one with regular, rectangular-shaped blades. Direct Numerical Simulations (DNS) were performed to compute the flow and passive scalar fields inside an unbaffled stirred vessel in the transitional flow regime. It was observed that at $Re = 1600$ the mean power consumption decreases by ca. 8% when the regular blades are replaced by fractal ones. The physical explanation for this reduction was provided by comparing several characteristics of the flows generated by the two types of impellers, such as the pressure distribution on blade surfaces, the time-average recirculation pattern and the trailing vortex system in wake of the blades, the radial transport of angular momentum and the distribution of energy dissipation inside the tank. Furthermore, fluctuations were observed in the power consumption with a peak frequency at ca. three times the impeller rotational speed, for both impeller types. It was discovered that these fluctuations are associated with a periodic event in the wake of the blades, which involves alternating growth and decay of the upper and lower cores of the trailing vortex pair as well as up-and-down swinging motion of the radial jet. Moreover, the mixing time required to homogenize an injected passive scalar was evaluated for both impeller types, at $Sc = 1$. It was observed that the fractal impeller can lead to a shorter mixing time by 10 – 12%. This result was explained by the differences in characteristics of flow and scalar fields generated by regular and fractal blades. A simple mathematical model was suggested which is able to approximate the decay rate of the passive scalar fluctuations integrated over the tank volume.

Author's declaration

I hereby declare that this thesis is based on my own research only. No parts of it have been submitted for another degree. All references or other sources of information have been fully acknowledged.

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Salur Başbuğ

Winter is coming!

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Nomenclature

Greek symbols

Γ	molecular diffusivity
Γ_t	eddy diffusivity
ϵ_K	total kinetic energy dissipation rate
ϵ_M	kinetic energy dissipation rate due to the mean velocity gradients
ϵ_T	kinetic energy dissipation rate due to the fluctuating velocity gradients
η	characteristic length scale of scalar diffusion
η^*	nondimensional characteristic length scale of scalar diffusion, $\eta^* = \eta/D$
η_B	Bachelor length scale
η_K	Kolmogorov length scale
μ	dynamic viscosity
ν	kinematic viscosity
ν_t	eddy viscosity
ρ	density
$\boldsymbol{\tau}$	viscous stress tensor
$\vec{\omega}$	angular velocity vector of the impeller
Ω	magnitude of the angular velocity vector of the impeller, $\Omega = 2\pi N$
$\vec{\xi}$	fluid vorticity

Roman symbols

A	area of the iso-scalar surface $c = \bar{c}$
A_W	total area of the outer tank surfaces, $A_W = 1.5 \pi T^2$

c	passive scalar
$\bar{c}$	volume average of the passive scalar over the tank
c'	passive scalar fluctuations around $\bar{c}$
C_D	blade drag coefficient
C_p	local pressure coefficient
CoP	centre of pressure
D	diameter of the regular impeller
E	kinetic energy of the fluid
$\hat{e}$	unit vector
f	frequency
f'	nondimensional frequency, $f' = f/N$
H	height of the tank
k	turbulence kinetic energy
L	angular momentum per unit mass
l_{in}	normalized angular momentum per unit mass evaluated at S_{in}
l_{out}	normalized angular momentum per unit mass evaluated at S_{out}
N	impeller rotational speed
N_p	nondimensional instantaneous power input, $N_p = P/(\rho N^3 D^5)$
$\langle N_p \rangle$	nondimensional time-averaged power input, $\langle N_p \rangle = \langle P \rangle / (\rho N^3 D^5)$
N_q	nondimensional flow rate through the control volume, $N_q = Q/(ND^3)$
p	pressure
P	instantaneous power input
$\langle P \rangle$	time-averaged power input
Pe	Peclet number, $Pe = Re \times Sc$
Q	volumetric flow rate through the control volume
r	radial distance from the axis of the vessel
r_w	radial distance of a wall point from the axis of the vessel
R	radius of the regular impeller
R_{CV}	radius of the control volume
Re	Reynolds number, $Re = ND^2/\nu$

s_{ij}	strain-rate tensor based on instantaneous velocity gradients
s'_{ij}	strain-rate tensor based on fluctuating velocity gradients
S_{ij}	strain-rate tensor based on mean velocity gradients
S_{in}	upper and lower flat surfaces of the control volume
S_{out}	side surface of the control volume
Sc	Schmidt number, $Sc = \nu/\Gamma$
Sc_t	turbulent Schmidt number, $Sc_t = \nu_t/\Gamma_t$
t	time
t^*	nondimensional time, $t^* = tN$ (number of impeller revolutions)
t_{mix}	nondimensional mixing time
T	diameter of the tank
T_{imp}	impeller torque
TI	turbulence intensity
u	instantaneous velocity in the inertial reference frame
u^*	nondimensional instantaneous velocity in the inertial reference frame, $u^* = u/ND$
u'	velocity fluctuations in the inertial reference frame
U	time-averaged velocity in the inertial reference frame
U_{tip}	blade tip velocity, $U_{tip} = \pi ND$
v	instantaneous velocity in the rotating reference frame
v^*	nondimensional instantaneous velocity in the rotating reference frame, $v^* = v/ND$
v'	velocity fluctuations in the rotating reference frame
V	time-averaged velocity in the rotating reference frame
V_{tot}	total volume of the vessel
V_{Δ}	volume confined between two iso-scalar surfaces
$\vec{x}$	position vector with respect to the origin

Subscripts

r	Radial component
-----	------------------

θ	Azimuthal component
x	X-component
y	Y-component
z	Axial or Z-component

Acronyms

CoV	coefficient of variation
CV	control volume
DES	Detached Eddy Simulation
DNS	Direct Numerical Simulation
FVM	Finite Volume Method
LES	Large Eddy Simulation
PDF	Probability Density Function
PIV	Particle Image Velocimetry
PSD	Power Spectral Density
RANS	Reynold's Averaged Navier-Stokes
RMS	Root-Mean-Square

Chapter 1

Introduction

1.1 Motivation

1.1.1 Fractal/multiscale generated flows

In the past decade, fundamental properties of flows generated by fractal/multiscale objects have been studied in some detail.^{7–12} These objects are created by repeating a geometrical pattern (e.g. a square grid-element) by decreasing the size of the pattern at every iteration. An example of the fractal square grids employed by Hurst and Vassilicos¹ as turbulence generators in the wind tunnel experiments is shown in Figure 1.1. Further studies have demonstrated the benefits of fractal/multiscale objects in practical applications, such as the noise reduction,¹³ vortex shedding suppression,^{14–16} heat transfer enhancement¹⁷ and flame control,^{18–20} to name a few.

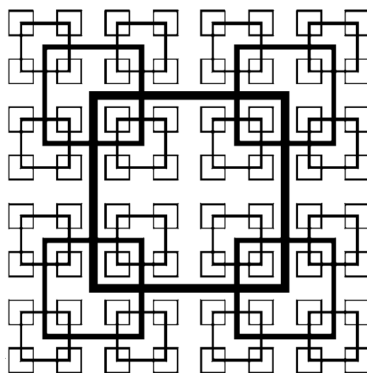


Figure 1.1: One of the fractal square grids used by Hurst and Vassilicos¹ in wind tunnel experiments.

Suzuki et al.²¹ investigated turbulent mixing of dye in a water channel in the wake of the regular and fractal grids having the same blockage ratio (ratio of the area blocked by the grid to the channel cross-section). Experiments were conducted at a high Schmidt number, i.e. $Sc \approx 2100$, where $Sc = \nu/\Gamma$ is based on the kinematic viscosity of water (ν) and the molecular diffusivity (Γ) of dye in water. At the inlet of the channel, they imposed a step profile for the passive scalar (dye concentration) and assessed the vertical profiles of the mean scalar and scalar variance in the lee of the grids. Their results showed that mixing is enhanced when the fractal grid was used instead of the regular grid. The same conclusion was drawn in a computational study²² with direct numerical simulations (DNS) using the same two grids as the aforementioned experimental work, but this time at a low Schmidt number, i.e. $Sc = 0.71$.

The DNS study of Laizet and Vassilicos²³ also compared the turbulent flow in the wake of a regular and a fractal grid of equal blockage ratio. They imposed a constant scalar gradient normal to the mean flow direction at the inlet of the virtual wind tunnel and set $Sc = 0.1$. They observed that the turbulent transfer of scalar fluctuations in the direction of the scalar gradient is larger for the fractal than the regular grid by an order of magnitude downstream of the grids for the majority of the domain examined. The authors linked this improvement to the interaction of wakes created in different sizes, since the fractal grid is composed of square grid-elements of different side lengths and bar thicknesses. Hence, they suggested that fractal objects can be employed to design energy-efficient industrial mixers and heat transfer devices. This conclusion was further elaborated by the same authors in another study²⁴ where they compared this time three different regular and three different fractal grid designs, along with varying Sc and mean scalar gradients. Similar to their previous study, it was observed that the transverse flux of turbulent scalar fluctuations is greater by an order of magnitude in the wake of all types of fractal grids investigated, than the regular ones.

1.1.2 Fractal impellers in stirred vessels

The outcomes summarized above motivated the utilization of fractal/multiscale geometries in the context of stirred vessels. These mixing devices are employed in a wide range of applications in the chemical, pharmaceutical and process industries.²⁵ Reducing power consumption and/or increasing mixing quality improves process efficiency. Modification of blade design has been considered as a means to achieve this objective. Indeed this has been the subject of extensive research in the past decades,^{26–31} but these efforts yielded only modest improvements.^{30,31}

Steiros et al.³ continued work in this direction and proposed a promising new impeller design. They introduced fractal impeller blades, created with a modification of the regular, rectangular-shaped impeller blades. The design of the novel fractal blades was inspired by the fractal plates tested in wind tunnel experiments by Nedić et al.² who demonstrated that fractal plates (shown in Figure 1.2) generate higher turbulence intensity in their wake compared to square plates. More specifically, Steiros et al.³ performed shaft torque measurements to determine the power consumption of four-bladed radial impellers in an unbaffled tank at $Re = 1 - 2 \times 10^5$, where $Re = ND^2/\nu$ is based on the rotational speed (N), impeller diameter (D) and kinematic viscosity of the fluid (ν). Their results demonstrated that the fractal-1 impeller, that uses blades with one fractal iteration (see in Figure 1.3b) has 11 – 12% reduced power consumption compared to the impeller with regular blades (see in Figure 1.3a). When the fractal-2 impeller was employed (with two fractal iterations, see Figure 1.3c), this difference increased to 17 – 20%. The authors also measured the pressure distribution on both sides of the blades using pressure transducers. They found that the center of pressure for a fractal blade is located radially further away from the shaft compared to a regular one. Consequently it was shown that the decreased torque on the fractal blade is due to the lower net pressure force applied onto the blade and *not* due to the reduced moment arm length. Since both types of blades have equal frontal area by construction, it was concluded that the fractal blade has a lower drag coefficient. Furthermore, they tested a two-bladed impeller, where one regular and one fractal blade were

mounted 90° apart. They conducted experiments for both rotational directions, whereby the regular blade was immersed in the wake of the fractal blade and vice versa. They measured exactly the same torque in both experiments. This finding suggests that the wake interaction was *not* the reason for the reduced drag coefficient of a fractal blade.

However, their results did not identify and explain clearly the mechanism which leads to the reduced torque. The answer to this question will be one of the objectives of the current thesis. Moreover, in order to determine whether this decrease in power consumption can potentially lead to an improvement in the process efficiency, the mixing performance of the fractal impeller must be evaluated. This assessment is included in the objectives of the present work as well.

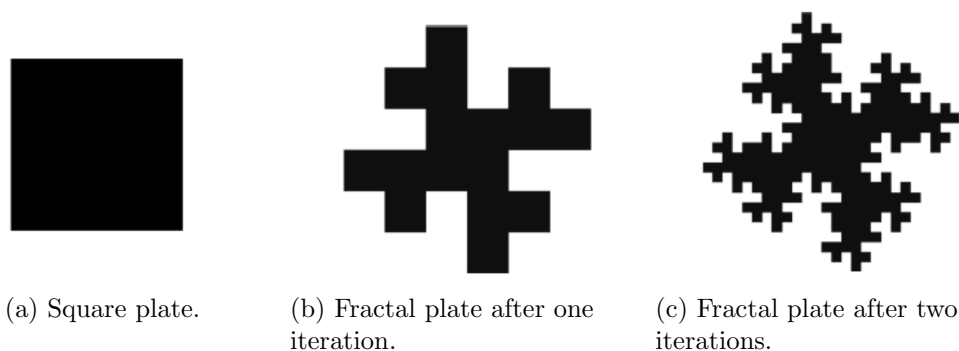


Figure 1.2: The square plate and some of the fractal plates used in experiments of Nedić et al.²

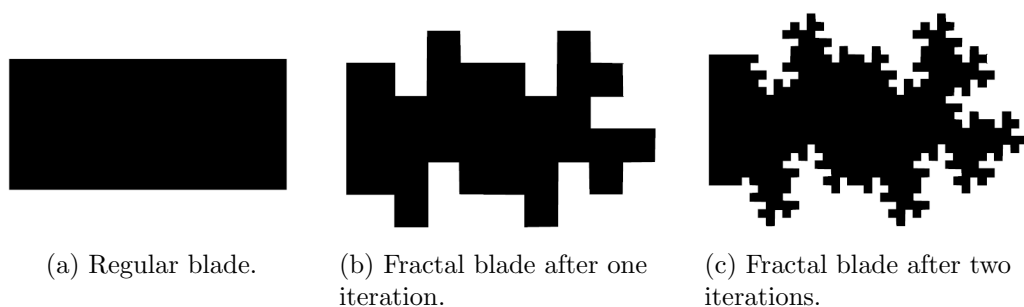


Figure 1.3: The blade shapes with equal frontal area, which were used in experiments of Steiros et al.³ conducted in an unbaffled tank.

1.2 Literature review

Stirred tanks are employed for suspending solids or dispersing gas in liquids, blending of miscible liquids, creating immiscible liquid-liquid dispersions, and also for heating/cooling mixtures.³² Mixing of a passive scalar has been the focus of a large number of studies for fundamental research purposes.^{33–37} A passive scalar is transported by the flow, but does not react back, i.e. it does not affect the flow field in any dynamical sense. In order to evaluate the mixing performance of a stirred vessel, it is common practice to inject a passive scalar and measure the time required for its homogenization. This is especially suitable for predicting the blending time of miscible liquids, or for the study of heat transfer applications, since these cases do not involve a phase interface. Mixing operations are performed in the turbulent, transitional or laminar regime, mainly depending on the viscosity of the fluid, since this can be as low as e.g. $10^{-3} \text{ Pa} \cdot \text{s}$ for water or as high as e.g. $150 \text{ Pa} \cdot \text{s}$ for corn syrup.³² According to *Handbook of Industrial Mixing*, among these regimes the most limited understanding is for transitional regime.³²

1.2.1 Characteristics of laminar flows in stirred vessels

The only route for effective mixing in the laminar regime is the generation of chaotic flow.³² It is characterized by the exponential rate of stretching of fluid elements and the growth of material lines^{32,38} (instead of the linear rate seen in a non-chaotic flow, e.g. the laminar pipe flow). This can be achieved by means of periodic forcing,³³ such as the impeller motion in a stirred tank^{39,40} or repetitive static elements in a mixing pipe.⁴¹ These settings lead to stretching and folding of initially segregated materials, hence increase the intermaterial area density and decrease the thickness of material striations.^{39–41} The significance of the intermaterial area density (and its distribution) stems from its direct influence on the mixing rate.⁴² Indeed, the local reaction rate depends on the intermaterial area density for diffusion-controlled reactions.⁴³

In chaotic flows, the intermaterial area density can grow exponentially and the exponent can be used to characterize the mixing rate of the process. The

spatial distributions of the intermaterial area density and the striation thickness have identical statistical properties. They are highly non-uniform, yet remain self-similar, since they change everywhere at the same rate.^{44–47} This is because chaotic flows possess an intrinsic invariant field of orientations which are time-independent and determined by the stretching field.⁴⁸ These properties of chaotic flows are observed initially in simple 2D-systems,^{38,49} but then also in complex 3D-systems, such as stirred vessels,^{39,40} both experimentally and numerically. It must be noted that when Peclet number (Pe) is very high, diffusion is slow compared to advection and very fine structures remain intact for a relatively long time before diffusion acts, where $Pe = Re \times Sc$. On the other hand, for $Pe \approx 1$ diffusion can erase patterns, such as striations, as fast as advection generates them.³²

1.2.2 Characteristics of turbulent flows in stirred vessels

Turbulent flow is preferred for industrial processes due to its ability to transport and mix more effectively.⁵⁰ The scales of segregation are reduced by the turbulent eddies.⁵¹ In a fully turbulent flow at high Re , the nondimensional parameters converge to a constant value. For instance, it is established that the nondimensional mixing time (normalized with the period of the impeller rotation, henceforth called t_{mix}) and the power number (defined as $\langle N_p \rangle = \langle P \rangle / (\rho N^3 D^5)$, where P is the power input and angular brackets $\langle \rangle$ represent the time-averaging operation) are constant for a given geometry.^{25,52,53} The mean⁵⁴ and fluctuating⁵⁵ velocity profiles collapse in fully turbulent regime regardless of Re , when normalized with the blade tip speed. Engineers benefit from these aspects of fully turbulent flows when a stirred vessel design needs to be scaled up or down.³²

1.2.3 Studies of mixing in unbaffled stirred vessels

There are several applications conducted in the laminar regime, e.g. the ones involving high viscosity fluids. In general, unbaffled tanks are preferred in the laminar regime since baffles cause dead zones in the laminar flow field and decrease the mixing performance,⁵² although baffles are found beneficial in the turbulent

regime. Applications of unbaffled tanks are also preferred in food and pharmaceutical industries since vessel cleaning in place is necessary.⁵⁶ Other examples are the bioreactor operations using shear-sensitive cell cultures where mixing must be at low speeds to prevent cell death,^{57,58} or the crystallization processes where the baffles can lead to particle attrition.^{59,60} When a suspended solid phase is present, a higher mass transfer coefficient between the solid and liquid phases can be obtained in unbaffled vessels with the same power consumption.⁶¹

Some experimental studies used an unbaffled tank with an open surface, which leads to a surface depression due to the highly swirling motion of the fluid.^{56,62,63} Rousseaux et al.⁶² suggested that the injection into the forced vortex zone was favourable for the fast mixing of the reagents and the consequent chemical reaction. Cabaret et al.⁶³ measured the power and the mixing time of eight different types of impellers in an unbaffled vessel. They observed that $\langle N_p \rangle$ scales with Re^{-a} , with $a = 1$ for $Re < 10$ and equal for all impellers types, whereas $a < 1$ for $10 < Re < 400$ and different for all impeller types. Moreover, all impeller types exhibited a decreasing trend of t_{mix} against Re for $10 < Re < 40$, but Sc was not stated. Assirelli et al.⁵⁶ compared the performance of a vessel under baffled and unbaffled conditions. Having the same power consumption, which was realised using different impeller speeds, the unbaffled tank delivered either a higher mixing efficiency than the baffled tank, or the equal efficiency, depending on the injection location. However, the maximum achievable power of an unbaffled tank was limited by the surface vortex reaching the impeller and the consequent mechanical damage.

Yang et al.⁶⁴ assessed a possible improvement of the mixing efficiency of an unbaffled stirred tank using detached eddy simulations (DES). DES method is based on the large eddy simulation (LES), but uses a Reynold's averaged Navier-Stokes (RANS) model near the walls to avoid the high resolution required by LES in the near-wall region. They compared the performance of pitched-blade turbines with centrally and eccentrically (radially shifted) located shafts. They used 15 monitoring points to determine t_{mix} , which matched with the experimental results within 20%, which were already obtained at the same locations. Moreover, they

reported a significant decrease in t_{mix} thanks to the eccentricity of the shaft, but did not report on the change in the power consumption due to this modification.

1.2.4 Studies of mixing in baffled stirred vessels

Baffled tanks are predominantly preferred in mixing applications (especially for turbulent mixing), except for the special cases mentioned in Section 1.2.3. Radial impellers discharge the fluid towards the vessel walls with a strong swirling motion. Wall baffles suppress the fluid swirl (also called the solid body rotation), transform the tangential motion into axial motion, therefore improve the axial mixing between the top and bottom of the tank.⁵² The decrease in the tangential velocity results in a higher relative velocity between the recirculating fluid and the impeller. This increases the power drawn by the impeller, the shear rate near the impeller and the turbulence inside the tank.³² The central vortex and free surface depression observed in the uncovered vessels, are also prevented by the wall baffles.

Because of the favourable characteristics mentioned in the above paragraph, the literature regarding the mixing of passive scalars in stirred vessels is dominated by studies conducted with baffled tanks operated at the turbulent regime. There are both experimental^{65–70} and numerical studies available. Almost all of the numerical investigations used turbulence modelling, such as RANS models^{71–74} or LES.^{75–78}

Pivotal experimental studies from 50's onwards^{65–68} endeavoured to develop empirical correlations or scaling laws for the mixing time and required energy. The most extensive investigation is perhaps that of Grenville.⁶⁹ The aforementioned author performed a series of experiments in a baffled tank stirred with four-bladed flat and pitched blade turbines, using an electrolyte tracer for conductivity measurements at three probe points, for Re values in the range $200 < Re < 5 \times 10^4$. The obtained mixing times based on different probe points agreed well in the turbulent regime, whereas they varied significantly in the transitional regime and the longest time was measured at the probe behind a baffle. For the turbulent regime and a tank with equal height and diameter, the following correlation was developed to estimate the mixing time: $t_{mix} = 5.2 \langle N_p \rangle^{-1/3} (T/D)^2$, where T and D are the dia-

meters of the tank and impeller, respectively. Since then, this correlation was cited repeatedly by other sources.^{32, 53, 76, 79, 80} For the transitional regime, the suggested correlation was: $t_{mix} = 183^2 \langle N_p \rangle^{-2/3} Re^{-1} (T/D)^2$. The surprising implication of these correlations was that the impellers with equal T/D and $\langle N_p \rangle$ would result in the same t_{mix} irrespective of the impeller type.³² The interpretation was that the mean rate of the energy dissipation inside the tank determines t_{mix} , hence only the power input suffices.^{32, 53} The value of Sc was not mentioned in this report. Using the available data^{69, 81} it can be calculated that Sc was $O(10^5)$ for the cases at the lowest Re considered, and it declined to $O(10^3)$ as Re was increased, due to the decreased fluid viscosity. Therefore, Pe was always $O(10^7)$, meaning that the diffusion was very slow compared to the advection.

The introduction of RANS models provided a new tool, faster and cheaper than experimental methods, whereby all scales of turbulence are modelled. For the study of mixing in stirred vessels, the $k-\epsilon$ model was most widely used.⁷¹⁻⁷⁴ When the eddy viscosity (ν_t) was computed with a turbulence model, the turbulent Schmidt number ($Sc_t = \nu_t/\Gamma_t$) was used to predict the eddy diffusivity Γ_t . Sc_t can be based on experience or determined with trial and error.⁷² Montante et al.⁷² showed that the choice of Sc_t was critically important in order to get correct results of the mixing processes when the $k-\epsilon$ model was used. The value giving the best agreement with experiments, could also depend on the grid resolution of the simulation.⁷⁴ Although the $k-\epsilon$ model could predict the mean velocity profiles with reasonable success, the results on the mixing processes were deemed unreliable.^{75, 76}

In the LES approach, only the influence of smallest eddies (smaller than a pre-defined filter size) are modelled and large eddies are directly resolved in space and time.⁵⁰ LES studies of the flow field in stirred vessels were already published in mid 90's, first by Eggels,⁸² then by Derksen and van den Akker.^{83, 84} It took almost ten more years until Yeoh et al.⁷⁵ employed this method for the assessment of t_{mix} in a stirred tank and compared this against experimental results and empirical correlations. In order to estimate t_{mix} , they took the entire scalar field inside the tank into account, instead of using probe points. They reported that the discrepancies

with respect to the references they compared their results with, varied between 9% and 53%, with an average of 18%. They observed that any estimate of t_{mix} using a single probe point would vary significantly across the vessel, leading to less realistic results. The discrepancies were attributed to this observation, since experiments evaluated only a number of locations. Hartmann et al.⁷⁶ also used the LES method to estimate t_{mix} in a baffled tank stirred with a Rushton turbine. They positioned the monitoring points in accordance to the experiments of Distelhoff et al.⁷⁰ and the numerical results agreed within 30% with the measured values.

All aforementioned LES studies used an injection of a passive scalar to investigate the mixing process, but with a different criteria to determine the time for the complete homogenization. The selected values of Sc_t varied between 0.7 and 0.8. Although LES is the most reliable numerical approach for high Re considering the available computational resources, it still requires the selection of a suitable value for Sc_t and it is not yet proven to provide an excellent agreement with experiments regarding the estimation of t_{mix} .

Derksen^{80,85} performed DNS of the flow in a baffled tank stirred with a pitched-blade turbine, and investigated the blending process of two miscible fluids, that is governed by the same set of equations as the mixing of a passive scalar. The author assessed the influences on t_{mix} stemming from the density differences⁸⁰ and viscosity differences⁸⁵ of the two fluids, at $Re = 3000, 6000$ and 12000 , and $Sc = 1000$. The simulations were initiated with the tank having the upper and lower halves filled with the two different fluids. Although the flow field could be resolved almost down to its smallest scales (i.e. Kolmogorov length scale, η_K), the same was not possible for the scalar field. The smallest scale of the scalar field is referred to as the Batchelor length scale (η_B) and is a factor of $\sqrt{Sc} \approx 30$ smaller than η_K in the aforementioned studies. Therefore, the simulated diffusion process was affected by the numerical diffusion determined by the grid resolution.⁸⁵ For instance at $Re = 12000$, t_{mix} computed with 168×10^6 grid cells was ca. 13% higher than t_{mix} computed with 6×10^6 grid cells. Taking this into account, the latter

one was regarded as a good compromise between computational resources and accuracy.⁸⁵ For the fluids with no density and viscosity difference, t_{mix} was found equal to ca. 48, 40 and 35 for the three Re considered, decreasing with increasing Re .⁸⁰ This trend neither corresponds to the expected one in the turbulent regime, where $t_{mix} = const.$, nor the one in the transitional regime, where t_{mix} should scale with Re^{-1} according to Grenville,⁶⁹ but it is somewhat between the two.

1.2.5 The importance of trailing vortices in stirred vessels

Trailing vortices are observed in the wake of impeller blades used in stirred vessels. Pioneering investigations on this subject date back to early 70's. Takashima and Mochizuki⁸⁶ placed a camera rotating with the impeller below the tank, in order to visualize the flow field in the rotating reference frame around several impeller types where tracer particles were injected. They provided a schematic diagram illustrating the radial jet on the suction side of a blade as well as the horizontal helical vortices at the upper and lower edges of the blade which give rise to the trailing vortices in the discharge flow. They stated that these trailing vortices were "a kind of Kármán's vortex series". This work was confirmed and improved by the more extensive studies⁸⁷ where a similar measurement technique was applied to examine the flow around a Rushton turbine in a baffled tank. This configuration has been commonly employed for gas-liquid dispersion applications. According to Van't Riet and Smith,⁸⁸ the power consumption and the dispersion performance are mainly determined by the flow in the impeller region. The prominent feature of this flow field is the trailing vortex pair, where gas bubbles are drawn due to low pressure and pass through a region of high shear rate. Their vortex pair idealization slightly differed from that of Takashima and Mochizuki⁸⁶ mainly because of the differences in the impeller geometry, e.g. the inner vertical edge of the Rushton turbine blades. It was observed that the trailing vortices were initially formed at Reynolds number values in the range $150 < Re < 250$ and it was claimed that the flow around the impeller was turbulent even at $Re = 300$.

In addition to the determination of the vortex axis location at different Reyn-

olds numbers, Van't Riet and Smith⁸⁸ also derived analytical expressions for the velocity and pressure profiles within the vortex core and found them to be in qualitatively good agreement with their experimental results. On the other hand, the unsteady behaviour of the trailing vortices was only referred to as an “erratic movement”. In a later study, Van't Riet et al.⁸⁹ detected that the blade-passage frequency dominates the energy spectra of the velocity fluctuations in the impeller region and these fluctuations were termed as pseudo-turbulence. Therefore, it was suggested to obtain the turbulence parameters like the eddy size, turbulence intensity and energy spectra from the measurements acquired in the rotating frame e.g. with probes rotating with the stirrer.⁸⁹

Numerous researchers^{90–94} continued the study of the turbulent flow inside stirred vessels with a focus on trailing vortices. Apart from predicting the mean-velocity profiles and turbulence characteristics for the flow around the impeller, they endeavoured to locate the mean path of vortex cores, because many important flow properties are associated with the trailing vortex region. The various predictions of vortex trajectories were compared by Escudié et al.⁹⁴ and collapsed well near the impeller blade, however discrepancies increased with growing distance from the blade. In order to visualise the vortex core, three methods were listed by Escudié and Liné.⁹⁵ The first is the determination of the location where the axial velocity is equal to zero. The second utilizes the isosurfaces of vorticity with an arbitrary fixed value to visualise the coherent structures and their centreline can be assumed as the vortex trajectory. A third option is to employ the vortex identification method suggested by Jeong and Hussain,⁹⁶ also known as λ_2 -criterion. Here λ_2 is the second eigenvalue of the symmetric tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ (assuming $\lambda_3 \geq \lambda_2 \geq \lambda_1$), where $\mathbf{S}$ and $\mathbf{\Omega}$ are the symmetric and antisymmetric parts of the velocity gradient tensor, respectively. The region where λ_2 is strictly negative corresponds to the vortex core and an arbitrarily selected threshold can be avoided.

Trailing vortices are the source of turbulence energy inside stirred vessels.⁹⁴ Delafosse et al.⁹⁷ and Derksen and Van den Akker⁸⁴ stated that these structures

are associated with high levels of turbulence, and emphasized their importance for mixing applications. These aspects motivated extensive studies, mainly concerned with averaged properties such as location and size of vortices, as cited above, or with turbulence properties like anisotropy, time and length scales or dissipation rates.^{98–102} On the other hand, the dynamical behaviour of the trailing vortices has received far less attention. This aspect will be addressed in the present thesis.

Besides the aforementioned cases with baffled tanks, trailing vortices were also studied in the context of unbaffled tanks.^{103–106} Sharp et al.¹⁰⁵ conducted particle image velocimetry (PIV) measurements in an unbaffled tank stirred with a Rushton turbine, and Alcamo et al.¹⁰⁶ performed LES using a similar setup. Both studies (independent from each other) made a comparison between their predictions of trailing vortex trajectories and the published results obtained in baffled tanks. They reported very good agreement between baffled and unbaffled cases.

1.3 Objectives

As mentioned earlier, fractal objects exhibited characteristics which were deemed beneficial or promising for industrial applications. The present work builds on the existing literature on the fractal generated flows and stirred vessels, and combines those in order to explore the potential advantages. The main objective is to assess the performance of fractal objects in a dynamic mixing application, more specifically the impeller equipped with the fractal blades as introduced by Steiros et al.,³ and compare its performance to the one with regular (rectangular-shaped) blades. To this end, DNS of the flow and passive scalar fields inside an unbaffled vessel stirred with a four-bladed radial impeller have been performed at $Re = 320$ and $Re = 1600$, and $Sc = 1$.

It should be noted that stirred tanks are predominantly employed in industrial applications for liquid systems at $Sc \geq O(10^3)$.³² Therefore, the study of mixing at $Sc = 1$ is not directly representative of industrial applications. The selection of $Sc = 1$ is mainly due to the computational limitations, since a higher Sc would decrease the size of the characteristic scales of the passive scalar fluctuations,

require a finer resolution and an excessive computational power for the present DNS study. Nevertheless, the results obtained at $Sc = 1$ will serve as the first step of the performance comparison between the regular and fractal impellers, before the more practical and challenging cases are investigated.

Since DNS can provide accurate and detailed information of the flow and scalar fields, this can be harnessed to conduct a rigorous analysis. Consequently, any performance difference between regular and fractal impellers observed can be scrutinized and provide a better insight on how to design efficient impellers. The main goals of this thesis can be summarized as follows:

- To study the flow fields generated by the regular and fractal impellers using DNS results, hence improve the scarce knowledge regarding the transitional regime in stirred tanks.
- To assess whether the observed reduction in power consumption reported for the turbulent regime is also valid in transitional regime, and explain the physical mechanism responsible for the difference.
- To analyse the fluctuations observed in the power consumption of the impellers and demonstrate their links with flow instabilities.
- To estimate the mixing time of an injected passive scalar in the tank stirred with regular and fractal impellers, in order to assess the mixing performance of these impellers.
- To investigate the differences in the scalar fields generated by both impeller types.

1.4 Outline

The present thesis is divided into six chapters, the first of which is this introduction. Chapter 2 describes the numerical methodology and the flow configuration employed. The code validation and grid convergence studies are also presented in the same chapter. Chapter 3 considers the angular momentum and energy

balances inside the tank. Afterwards, the differences between the regular and fractal impellers are analysed with respect to (i) the shaft torque, (ii) the angular momentum transport from the impeller to the flow field, (iii) energy dissipation characteristics. Chapter 4 is dedicated to the investigation of instabilities of the impeller power and the flow field. The study of passive scalar field and mixing performance is presented in Chapter 5. Finally, Chapter 6 summarizes the main findings of the thesis and proposes directions for future research.

Chapter 2

Numerical methodology and validation

2.1 Introduction

The DNS approach is capable to produce high fidelity, well-resolved data of velocity and pressure in space and time. If the discretization is fine enough to capture smallest scales of the flow field and also $Sc = 1$, then the smallest scales of the scalar field are also resolved. Such a time-accurate and comprehensive database allows a detailed analysis of the wake flow patterns, and their effect on the pressure field around the impeller, the distribution of energy dissipation in the whole tank and the mixing process. There are few other DNS studies of the flow in baffled^{80,85,107} and unbaffled⁴ stirred tanks, but with different goals to those of the present work.

This chapter starts with the description of the numerical methodology, followed by the validation of the code. To this end, a DNS study previously published was used, and this case was simulated using our code in order to compare the results with the reference results. Afterwards the flow configuration of the current study is illustrated and the grid convergence study is presented. Finally, the results obtained with this configuration are compared with experimental results.

2.2 Numerical methodology

The incompressible in-house code “Pantarhei” was used to obtain the DNS results presented in this thesis. It is based on the finite-volume method in unstructured grids, and was previously employed for LES of the flow in a stirred vessel,^{75,108} for DNS of the flow around an airfoil¹⁰⁹ and the flow past a single square grid-element.¹¹⁰ The code was parallelized using the PETSc,¹¹¹ MPI and Hypr libraries. The PISO algorithm was adopted for the pressure-velocity coupling. Second-order central differencing scheme was selected for the spatial discretization, and implicit second-order Euler method for time-marching. A very detailed description of the code was provided by Paul.¹¹²

The Navier-Stokes equations were solved in a reference frame rotating with the impeller. Consequently, the momentum equations take the following form:

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} - \rho[\vec{\omega} \times (\vec{\omega} \times \vec{x}) + 2\vec{\omega} \times \vec{v}], \quad (2.1)$$

where $\vec{v}$ is the instantaneous velocity vector in the rotating frame, $\boldsymbol{\tau}$ is the viscous stress tensor and $\vec{\omega}$ the angular velocity of the frame. The origin of the coordinate system is located on the impeller axis at the mid-height of the vessel. The position vector with respect to the origin is denoted by $\vec{x}$. Vector $\vec{\omega}$ points in the axial direction and has magnitude $\Omega = 2\pi N$. The last two terms of Equation 2.1 represent the centrifugal and Coriolis accelerations, respectively, and were treated as source terms during the solution procedure. Since the surrounding walls of the tank have a relative motion with respect to the impeller, a moving wall boundary condition was imposed on the tank walls (i.e. top, bottom and side walls). A prescribed velocity (equal to Ωr_w , where r_w is the radial distance of a wall point to the axis of the vessel) was imposed on these external walls to represent the relative motion with respect to the impeller. All solid surfaces have a no-slip condition. There is an exception for the top wall for the validation case only (see Section 2.3). In all cases considered, the tank is closed at the top and it is assumed that a wall is present. Therefore, free-surface depression was not considered. Also the vessels

contain a single phase fluid. When the simulations started from scratch, the fluid was assumed to be at rest in the absolute reference frame, which corresponds to a solid-body rotation in the rotating reference frame. Hence the velocity field over the flow domain was initiated with solid-body rotation. The equations were solved in the Cartesian coordinate system, where the z-axis is the axis of rotation, x- and y-axes are parallel to the impeller blades (considering four-bladed impellers, as shown in Figure 2.3b). The velocity components were transformed to a cylindrical coordinate system during the post-processing stage when needed.

2.3 Validation of the code

It was essential to validate that our approach is capable of successfully predicting the flow field inside a stirred vessel. To this end, the case studied by Verzicco et al.⁴ was employed as a reference and recomputed using our in-house code for comparison of the results. These authors performed DNS of the flow field agitated by an eight-bladed radial impeller inside an unbaffled stirred tank. The flow Reynolds number was $Re = 1042$. The geometrical details of the tank are displayed in Figure 2.1. In the top wall for the validation case, a free-slip boundary condition was implemented (instead of no-slip) in order to match the boundary conditions of the reference work.⁴ The grid utilized here was generated to be as similar as possible to that described in the aforementioned paper. The reference work used the immersed boundary method for the modelling of the solid surfaces whereas the present study utilized a body-fitted mesh; this led to differences in the shape of the cells around the impeller. However, the resolution was kept the same with ca. 2 million cells discretising the domain.

The same paper provided time-averaged and root-mean-square velocity profiles along radial lines at different axial positions which were compared with the experimental results of Dong et al.⁵ who acquired data using the same configuration. Figure 2.2 illustrates the comparison between these two reference works and our DNS results. The experiments were conducted with a free-surface flow that leads to a certain surface deformation due to the rotation of the flow. This

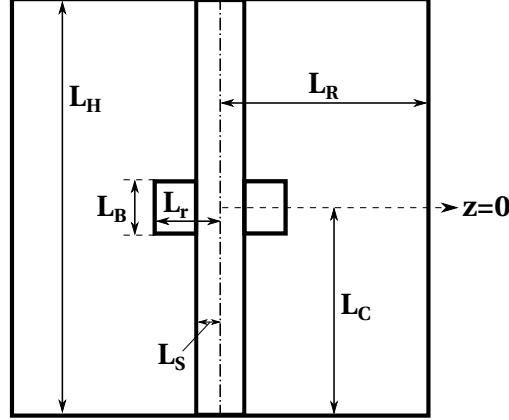


Figure 2.1: The configuration investigated by Verzicco et al.⁴ All dimensions are normalized with the impeller radius L_r : $L_S = 0.32$, $L_B = 0.8$, $L_C = L_R = L_H/2 = 4$.

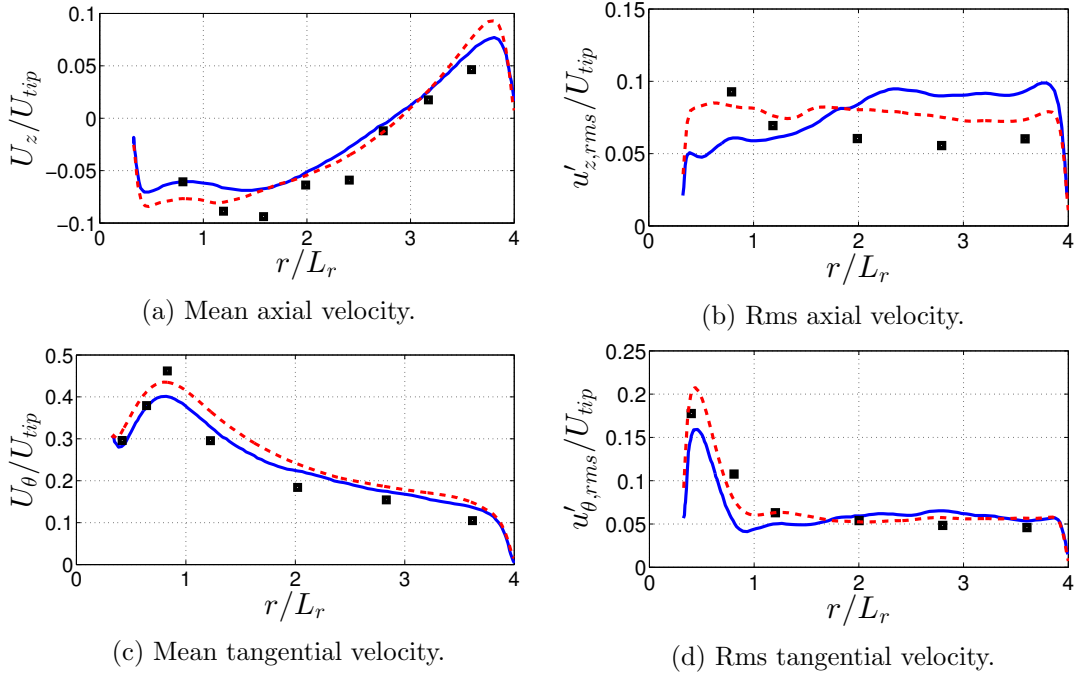


Figure 2.2: Profiles of radial and tangential velocity components along a radial line in a vertical plane in the middle of two blades, at $z/L_H = 0.2$. The dashed red lines denote the present DNS results, straight blue lines the DNS of Verzicco et al.⁴ and the square symbols the experimental data of Dong et al.⁵ The velocities are normalized with the blade tip speed $U_{tip} = \Omega L_r$ and radial distance with the impeller radius, L_r .

was neglected by both simulations, which may explain the deviations between the DNS results and the experimental data. Some discrepancies between the two DNS data sets probably stem from the different methodologies used for the modelling of solid surfaces (immersed boundary method vs. body-fitted finite volume method).

The conventional immersed boundary method as employed by the reference work cannot enforce the no-slip condition exactly on solid surfaces,¹¹³ hence the viscous forces applied by the impeller and tank walls may not be precisely computed leading to some differences in the profiles. As a general conclusion, our DNS results are in satisfactory agreement with the reference numerical predictions and the experimental data. Nevertheless, a rigorous analysis of the grid convergence is presented in Section 2.5 using the four-bladed impeller. The balances of angular momentum and kinetic energy are assessed in Chapter 3.

2.4 Flow configurations examined

The stirred tank investigated in the present study has a cylindrical geometry of equal height and diameter, with a four-bladed impeller located at mid-height. Figures 2.3a and 2.3b show vertical and horizontal views of the vessel and the impeller, where the impeller clearance and diameter with respect to the tank size are displayed. The blade height and blade thickness are $1/10$ and $1/100$ of the tank height, respectively. Flows agitated by two types of impeller blades with equal frontal area are examined in this work. They have exactly the same shapes as the regular blade and the fractal blade with one iteration used in the experimental study of Steiros et al.³ and shown earlier in Figures 1.3a and 1.3b, respectively. As seen in Figure 1.3b, the radius of the fractal impeller is not uniform. The furthest tip is located at $r/R = 1.1$, where R is the (constant) radius of the regular impeller. For normalization purposes, the dimensions of the regular impeller are used throughout this thesis.

Although the main focus of this work is on four-bladed impellers, also simulations with regular one-bladed and two-bladed impellers are performed to support and complement the results of the four-bladed case (presented in Chapter 4). This allows to assess the effect of the wake interaction between the blades of the four-bladed impeller. In the case of the two-bladed impeller, the angle between the blades is equal to 180° . All the other dimensions remain unchanged.

Flows inside stirred vessels with both regular and fractal impellers were simu-

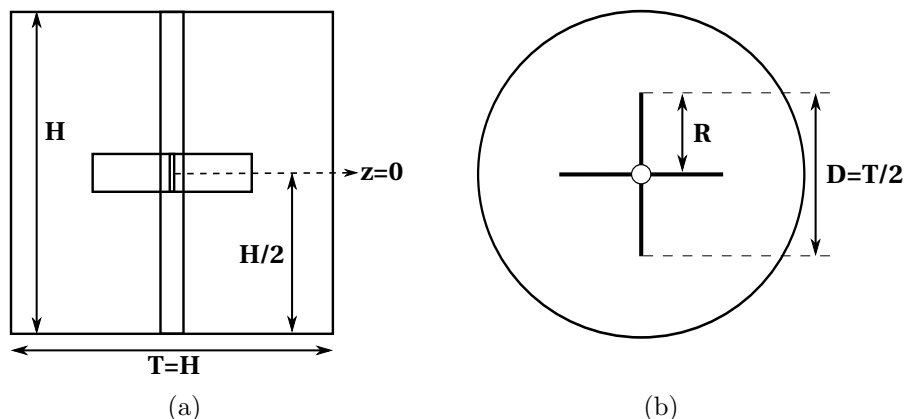


Figure 2.3: Tank geometry and dimensions. a) Vertical view along the axis, b) horizontal view at the mid-height.

lated at two Reynolds numbers, $Re = 320$ and $Re = 1600$. Inspection of the power number variation with Re shows the flow regime in which our simulations are, see Figure 2.4. The empirical correlation provided by Furukawa et al.⁶ which takes all the geometrical details of the tank into account except the blade thickness, was employed to plot $\langle N_p \rangle$ vs. Re . On the left end of the curve, where Re is very low, there is inverse proportionality between $\langle N_p \rangle$ and Re , which is a characteristic of the laminar regime. Fully turbulent regime is observed on the right end of the curve and is distinguished by the Re -independence of $\langle N_p \rangle$, i.e. a plateau of the curve. Although a fully turbulent flow throughout the tank requires very high Reynolds number,¹¹⁴ the flow closer to the impeller can become turbulent even at moderate Re .¹⁰⁷ The two Reynolds numbers considered in the present work are indicated in Figure 2.4 with vertical dashed lines and correspond to the transitional regime. However, some regions of the flow field can still be locally turbulent, especially in the impeller discharge region.

Although the present study investigates flows in the transitional regime, some concepts are still used, which were originally introduced in the context of fully turbulent flows, such as the rms values of velocity fluctuations, turbulence intensity, or turbulent dissipation. These are defined and employed later in the text, in order to make a comparison between different cases or to evaluate the role and contribution of velocity fluctuations in several phenomena. The conclusions reached in

the present study using these parameters do not require the assumption of a fully turbulent flow field, unless otherwise stated.

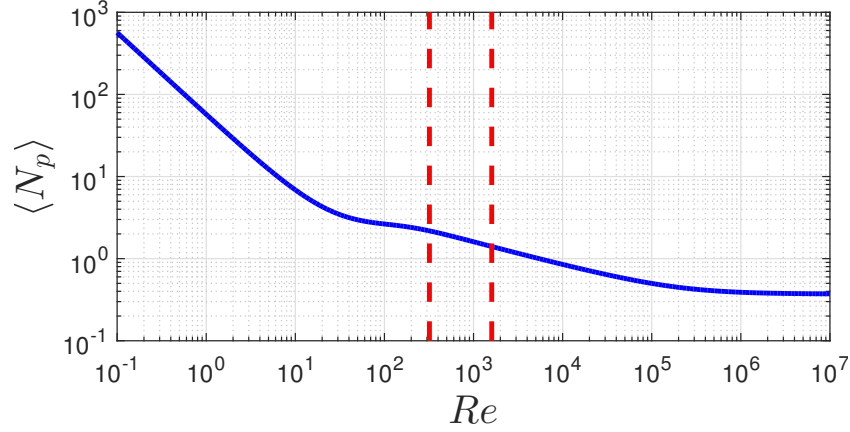


Figure 2.4: Power number as a function of Reynolds number based on the empirical correlation given by Furukawa et al.⁶ The vertical dashed lines mark the Reynolds numbers considered in the present work, i.e. $Re = 320$ and 1600 .

2.5 Grid convergence study

A grid convergence study was performed for both types of impeller, for $Re = 320$. For the regular impeller, the computational domain was discretized using two meshes consisting of 4×10^6 and 13×10^6 hexahedral cells. The simulation was initially run with the coarser grid. Afterwards, the grid was refined, especially near the surfaces and in the impeller discharge region. The developed flow field obtained with the coarse grid was interpolated to the fine grid and served as initial condition. The same procedure was followed for the case of fractal impeller, using meshes with 9×10^6 and 21×10^6 cells. The increased cell numbers for the case of fractal blades is a natural consequence of the more complex blade shape compared to the regular blades. The fine grids used for both cases (13×10^6 or 21×10^6) resolve the blade thickness with 7 cells. The number of cells per blade height are 45 for regular and 66 for fractal blades. All four cases were run for a time equivalent to 80 impeller revolutions to collect statistics used in this thesis, which started after 20 revolutions of flow development. Each impeller revolution was resolved using 1500 time steps if one of the coarse grids was used (4×10^6 or 9×10^6) or 3000

time steps if one of the fine grids was used (13×10^6 or 21×10^6). This ensured that the maximum Courant number did not exceed 0.5 in any cell throughout the domain. Figure 2.5 shows the coarser one of the two grids used for the regular impeller case and the finer one of the two grids used for the fractal impeller case at $Re = 320$, in order to exhibit the body-fitted meshing approach employed (only one quarter of the horizontal plane is shown).

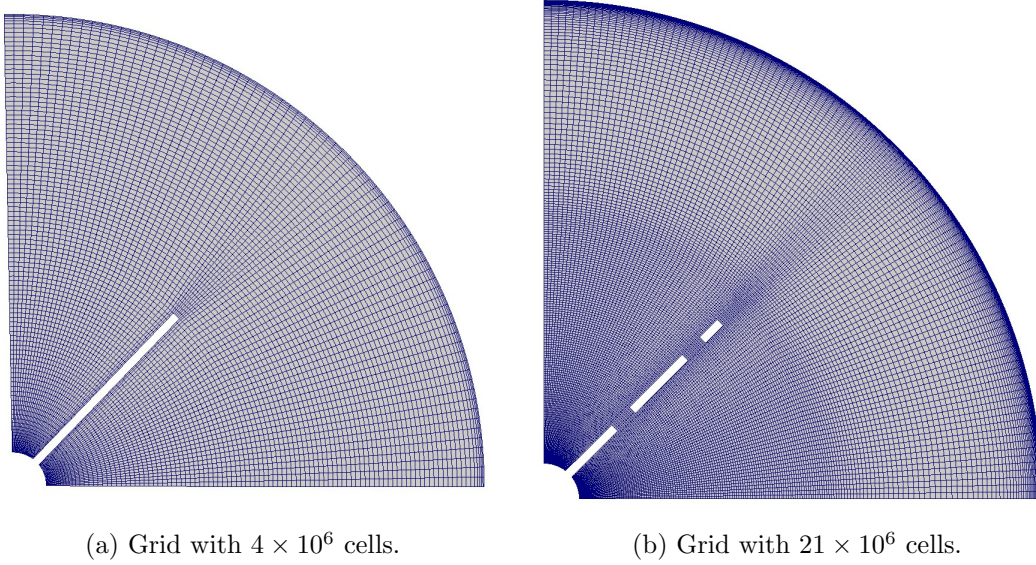


Figure 2.5: Examples of the meshes for regular (a) and fractal (b) impellers.

Figure 2.6 exhibits profiles of the mean and rms velocities, turbulence kinetic energy and the dissipation of turbulence kinetic energy, taken from the tank with the fractal impeller. Here, results computed using the grids with 9×10^6 and 21×10^6 cells are compared. The data were extracted along a radial line from the shaft to the tank wall, at the mid-height and at 45° behind a blade. Since there are four blades, the time-averaged profile was obtained with the average of all four planes (at 45° behind the blades) to increase the number of samples and improve the statistical convergence. The mean and rms velocities were normalized with the blade tip speed ($U_{tip} = \pi ND$), the turbulence kinetic energy with U_{tip}^2 , the dissipation of turbulence kinetic energy with $N^3 D^2$ and the radial distance with the blade radius (R). The same factors of normalization were also adopted wherever needed in the remainder of the thesis. The tangential velocity component was transformed from the relative to the inertial reference frame by adding Ωr ,

where r is the distance from the axis of rotation. Since Ωr is constant in time, the rms velocities or turbulence properties did not require a similar transformation.

It can be clearly seen that the profiles of mean velocities collapse very well. The other profiles are also in good agreement, including the profile of the dissipation of turbulence kinetic energy which requires very high resolution to capture correctly. It can be concluded that the refinement from 9×10^6 to 21×10^6 cells did not lead to a significant change in the results and therefore no further refinement was necessary.

Another criterion to assess grid resolution is the ratio of the cell size to the Kolmogorov length scale (η_K). This length scale is defined as $\eta_K = (\frac{\nu^3}{\varepsilon_T})^{1/4}$ where $\varepsilon_T = 2\nu \langle s'_{ij}s'_{ij} \rangle$ ($s'_{ij} = \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)$ is the strain rate tensor based on fluctuating velocity gradients). It should be noted that η_K represents the smallest length scales in a turbulent flow,⁵⁰ whereas the present study deals with transitional flows. Still, η_K is evaluated here assuming that it characterizes the length scales of fluctuating velocity gradients, in order to assess the size of the grid cells with respect to the aforementioned length scales. For both of the fine grids used at $Re = 320$ (with 13×10^6 and 21×10^6 cells), the ratio of the cell size (cubic root of the cell volume) to η_K is below 2 throughout the entire tank volume. Also, the wall resolution on the impeller surfaces for these grids is such that the distance between the near-wall cell center and the surface is less than unity in terms of wall units ($y^+ < 1$). On the tank walls, the grids are even finer ($y^+ < 0.1$). Considering the good agreement of the results obtained with the course and fine grids of the fractal impeller case (shown in Figure 2.6), it can be deduced that the mesh with 13×10^6 cells provides sufficient resolution for the regular impeller case. The results of the cases at $Re = 320$ presented for the remainder of this thesis belong to the simulations performed using the finer grids, unless otherwise stated.

For $Re = 1600$, grids with 60×10^6 and 70×10^6 cells were used for the regular and fractal impellers, respectively. Both grids resolve the blade thickness with 13 cells. The number of cells per blade height are 64 for regular and 96 for fractal blades. One impeller revolution was resolved using 5000 time steps for both cases.

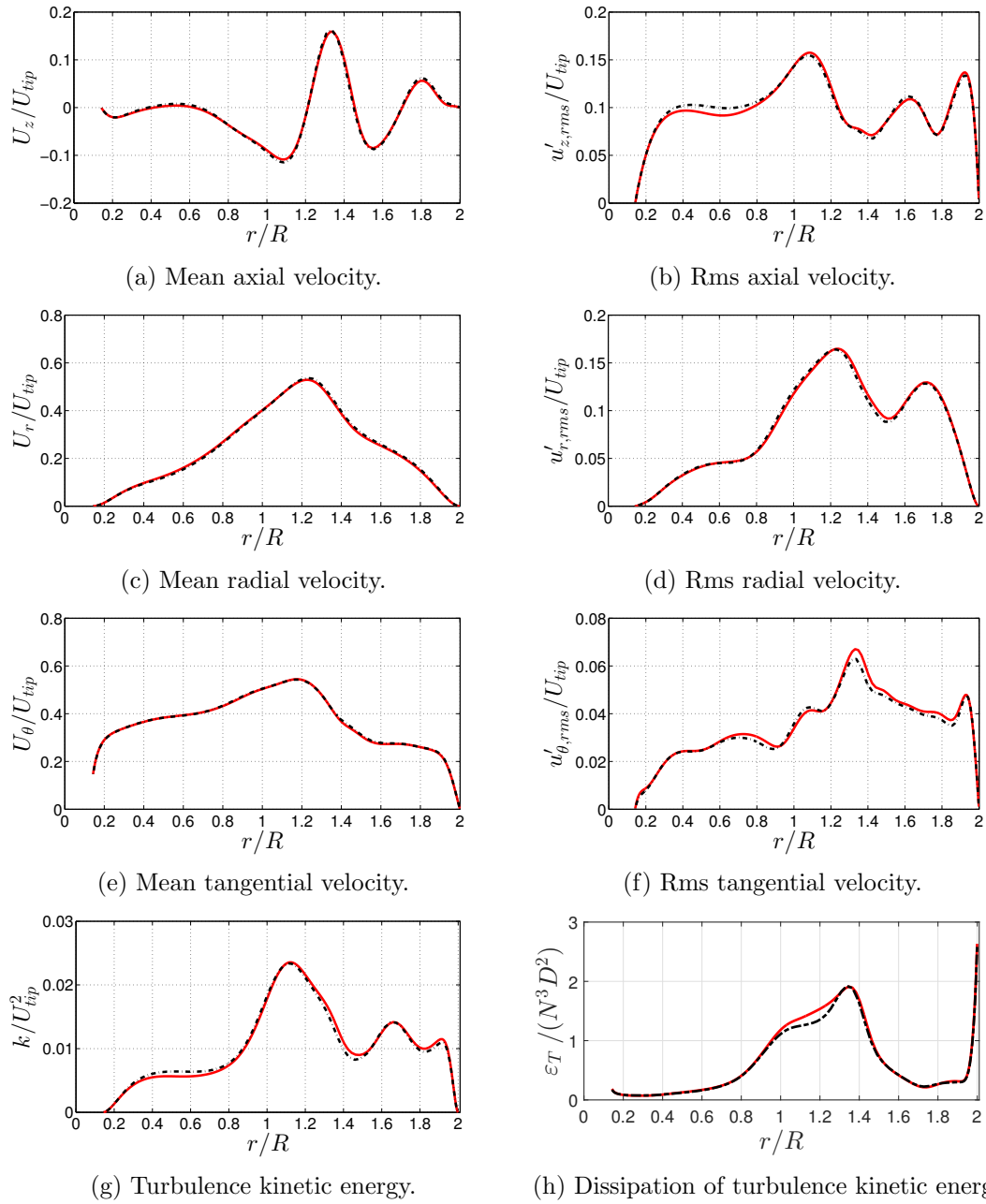


Figure 2.6: Profiles along a radial line at the mid-height of the tank with fractal impeller at 45° behind the blade, at $Re = 320$. Black dashed lines denote results obtained with 9×10^6 cells and red lines with 21×10^6 cells.

To obtain a developed flow field, the simulations at $Re = 1600$ were initially run with the grids used at low Re cases for 20 impeller revolutions. Afterwards, the flow field was transferred to the grids designed for the high Reynolds number and the computations were performed for 20 revolutions for the flow to adapt to the new grid, and a further 100 impeller revolutions to collect statistics used in this

thesis.

It was not affordable to make a grid convergence study at this Reynolds number, due to the larger grid sizes and smaller time steps. (The list of computational resources used for the simulations can be found in Appendix A.) Nevertheless, the cell size to the Kolmogorov length scale ratio was again kept below 2 for both types of impeller, also at $Re = 1600$. Moreover, the same wall resolution as the one described above was achieved at the high Reynolds number cases (i.e. $y^+ < 1$ on the impeller surfaces and $y^+ < 0.1$ on the tank walls).

2.6 Comparison with experimental results

The numerical results at $Re = 1600$ are compared with data acquired by means of phase-locked planar PIV at $Re = 1.5 \times 10^5$. The experiments were performed using blades of the same shape in an unbaffled tank of the same dimensions. The only difference was that the tank was hexagonal instead of cylindrical, but this difference is not expected to affect the results in the near impeller region. More details on the stirred tank setup and experimental technique can be found in Steiros et al.¹¹⁵

The instantaneous velocity vector in the absolute reference frame is denoted by $\vec{u}$ ($\vec{u} = \vec{v} + \vec{\omega} \times \vec{x}$) and can be decomposed in mean and fluctuating components as $\vec{u} = \vec{U} + \vec{u}'$; the same decomposition is applied to $\vec{v}$ as well later in the text. The azimuthal velocity component was not accessible in the aforementioned experiment. Using the radial and axial velocity fluctuations only, u'_r and u'_z respectively, the turbulence intensity (TI) is defined as follows:

$$TI = \frac{\sqrt{\langle u'^2_r \rangle + \langle u'^2_z \rangle}}{U_{tip}} . \quad (2.2)$$

Profiles of TI obtained numerically and experimentally are compared in Figure 2.7 along a radial line 30° behind the regular impeller, at an axial position where the center of the upper trailing vortex core is located, where the mean azimuthal vorticity, i.e. $(\nabla \times \vec{U}) \cdot \hat{e}_\theta$, is at its highest. This point is preferred to a fixed axial

coordinate since the location of vortex cores depends on Re .

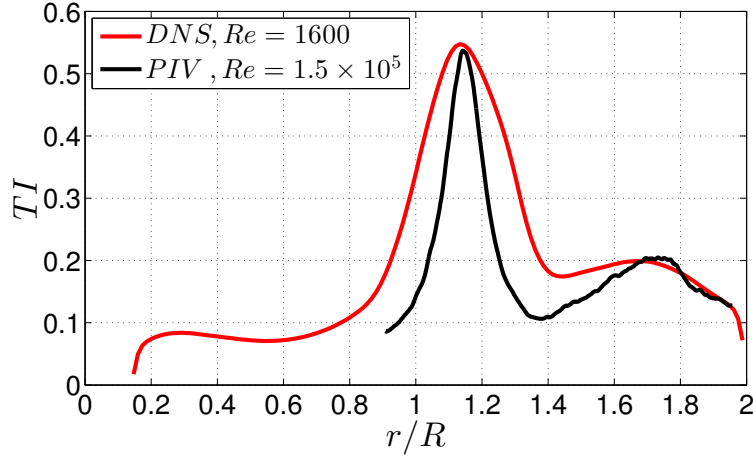


Figure 2.7: Profiles of turbulence intensity (TI) along a radial line 30° behind the regular impeller, at the axial position where the mean azimuthal vorticity is highest in the upper trailing vortex core.

As seen in Figure 2.7, the two curves are in qualitative agreement despite the large difference in Re (two orders of magnitude). The peaks of TI indicate the locations of the vortex cores and their radial positions (between $r/R = 1.1 - 1.2$) are in agreement. A second, but less prominent peak, appears around $r/R = 1.7$ due to the wake of the preceding blade, and in this region the curves collapse. Yoon et al.⁵⁴ investigated the Reynolds number scaling of the flow in an unbaffled tank stirred with a Rushton turbine using phase-locked stereoscopic PIV measurements for a range of Re values between 4×10^3 and 78×10^3 . They reported that the vortex core diameter decreases as Re rises. The results shown in Figure 2.7 are in accordance with their observation, the peaks of TI become sharper and narrower with increasing Re .

Raju et al.⁵⁵ conducted a similar study (using the same configuration as Yoon et al.⁵⁴) and measured the rms of the components of $\vec{u}'$ in a plane normal to the radial direction close to a blade tip. They reported that the rms values (averaged over the measurement plane) decrease slowly with increasing Re , from $Re = 4 \times 10^3$ to ca. $Re = 4 \times 10^4$. For larger Re , the rms values remained constant. The experimental results presented in Figure 2.7 were obtained for a Re higher than the aforementioned threshold for Re -independence. The moderately higher

level of TI at $Re = 1600$ is also in accordance with the findings of Raju et al.⁵⁵ In conclusion, the present numerical results are in qualitative agreement with experimental findings. The observed discrepancies are attributed to the large difference in Reynolds number.

2.7 Conclusions

The code is validated using the experimental and numerical reference studies investigating the flow in a vessel agitated with an eight-bladed radial turbine. The reliability of the DNS results dealing with four-bladed impellers were verified using the following criteria: (i) A grid convergence study showed that refinement from 9×10^6 to 21×10^6 cells does not affect the quality of the results, for the case of fractal impeller at $Re = 320$. (ii) The ratio of cell size to Kolmogorov length scale was kept less than 2 throughout the domain, the resolution of near-wall regions fulfil $y^+ < 1$ on the impeller surfaces and $y^+ < 0.1$ on the tank walls. (iii) The comparison with the experimental results are in good agreement despite the large difference in Re .

Chapter 3

Reduced energy consumption by means of fractal impellers

3.1 Introduction

In this chapter, the flows around both regular and fractal impellers are analysed in detail. The aim is to answer the following open questions: Does the fractal impeller draw less power than the regular impeller when operated in the transitional regime, as it was already observed³ in the turbulent regime? If this is the case, how can one explain the physical mechanism behind this observation? How is this related to the flow patterns in the wake? The physical understanding from such an investigation can lead to further improvements in impeller design in the future.

The chapter starts with the examination of the integral balances of angular momentum and kinetic energy. Following this, the pressure distribution on regular and fractal blades and its relation to the flow is presented and analysed. The investigation also considers the transport of angular momentum and how it is affected by vortical wake structures, the distribution of dissipation in the wake and the whole vessel, as well as the frequency content of dissipation. The chapter is closed with a summary and main conclusions. (Part of the results of the present chapter is accepted for publication in AIChE Journal.)

3.2 Angular momentum balance

The power drawn by an impeller is equal to the impeller torque (T_{imp}) times its angular velocity (Ω). Therefore, the accuracy of the computation of T_{imp} must be assessed, before proceeding with the investigation of the power input. To this end, the angular momentum balance inside the tank is examined. Our incompressible Navier-Stokes finite-volume solver satisfies the conservation of momentum and mass by construction. However, the angular momentum and the kinetic energy balance must be examined separately.

The global angular momentum balance is evaluated with respect to the center of mass of the entire fluid volume which is located on the axis of rotation. The corresponding conservation equation can be derived by taking the cross product of the location vector ($\vec{x}$) with the Navier-Stokes equations, already expressed in rotating reference frame (see Equation 2.1). The integration of the resulting equation over the entire volume takes the following form:

$$\int_V \vec{x} \times \left[\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v} \otimes \vec{v}) \right] dV = \int_V \vec{x} \times [-\nabla p + \nabla \cdot \boldsymbol{\tau} - \rho(\vec{\omega} \times (\vec{\omega} \times \vec{x}) + 2\vec{\omega} \times \vec{v})] dV. \quad (3.1)$$

The focus is on the balance of the axial component of the angular momentum, that is parallel to the axis of impeller rotation, hence directly linked to the power draw. The cross product of $\vec{x}$ with the Coriolis term is projected on the axial direction as follows:

$$-2[\vec{x} \times (\vec{\omega} \times \vec{v})] \cdot \hat{e}_z = -2[\vec{\omega}(\vec{x} \cdot \vec{v}) - \vec{v}(\vec{x} \cdot \vec{\omega})] \cdot \hat{e}_z = -2\Omega v_r(\vec{x} \cdot \hat{e}_r). \quad (3.2)$$

The final form on the right-hand side of Equation 3.2 when integrated over the tank volume gives zero. The cross product of $\vec{x}$ with the centrifugal term becomes $(\vec{x} \times \vec{\omega})(\vec{x} \cdot \vec{\omega})$ and has no axial component. The advective term in Equation 3.1 integrates to zero, as well. The terms related to the pressure gradient and the stress tensor can be reduced to surface integrals. Moreover, the time derivative

can be taken out of the integral since the control volume is fixed. The final form (Equation 3.3) demonstrates that the time derivative of the integrated angular momentum is equal to the net torque applied to the fluid by the solid surfaces (i.e. the impeller, shaft and tank walls).

$$\frac{\partial}{\partial t} \int_V (\vec{x} \times \rho \vec{v}) dV \cdot \hat{e}_z = \left[\int_S -p (\vec{x} \times d\vec{S}) + \int_S \vec{x} \times (\boldsymbol{\tau} \cdot d\vec{S}) \right] \cdot \hat{e}_z. \quad (3.3)$$

Furthermore, it can be shown that the calculation of the torque applied to the fluid by any surface gives the same result in inertial and rotating reference frames. This is because the pressure field and the viscous stress tensor ($\boldsymbol{\tau}$) are invariant with respect to the transformation between these two frames. To prove this, the relations between the velocities in both frames are written, where $\vec{u}$ and $\vec{v}$ are the velocities in the inertial and the rotating frames, respectively:

$$u_r = v_r, \quad u_\theta = v_\theta + \Omega r, \quad u_z = v_z. \quad (3.4)$$

Consequently, the only velocity gradient that can be affected by the transformation is the gradient of the tangential velocity component with respect to the radial direction. There are only two components of the stress tensor associated with this velocity gradient, namely $\tau_{r\theta}$ and $\tau_{\theta r}$. In fact, these are equal due to the symmetry of the tensor:

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right]. \quad (3.5)$$

When u_θ is replaced with $v_\theta + \Omega r$ in Equation 3.5, it can be shown that $\tau_{r\theta}$ computed using the velocities in inertial or relative reference frames are equal, due to the following equality:

$$\frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) = \frac{\partial}{\partial r} \left(\frac{v_\theta + \Omega r}{r} \right) = \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} + \Omega \right) = \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right). \quad (3.6)$$

If averaged for long enough time, the unsteady term in Equation 3.3 goes to zero since the system is closed and the angular momentum cannot increase indefinitely. Therefore, the net torque applied to the fluid must also be equal to zero in the

time-average sense. The blades and the shaft apply torque on the fluid in the positive sense of rotation (for $\Omega > 0$) and the tank walls in the negative sense due to the wall friction. The time-derivative term being equal to zero, the torque applied by the impeller and the shaft must balance the oppositely directed torque applied by the tank walls.

This balance was also mentioned by Gimbun et al.¹¹⁶ in their paper on the numerical study of turbulent flow in a stirred vessel agitated by a Rushton turbine. Their results indicated an imbalance between the impeller torque and the wall torque of 11% and 2% obtained by means of DES and LES methods, respectively.

Here, the time-derivative term is also included in the assessment of the instantaneous balance (Equation 3.3). The fluid angular momentum is integrated over the tank volume and its time derivative is calculated using a second-order central differencing scheme. T_{imp} is computed from the surface integrals of the pressure and the shear stress applied on the fluid by the blades and the shaft. The torque applied by the tank walls is likewise calculated with the surface integration of the shear stress. These quantities are normalized with $(\rho N^3 D^5)/\Omega$, giving rise to the nondimensional torque, N_T . (When T_{imp} is normalized this way, $\langle N_T \rangle$ is equivalent to $\langle N_p \rangle$.)

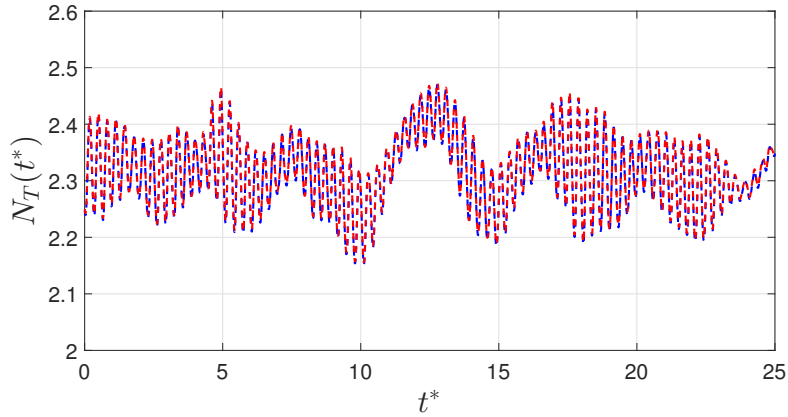


Figure 3.1: Torque balance for the regular impeller at $Re = 320$. Blue dashed line indicates the impeller torque and red dashed line the sum of the time-derivative term and torque applied by the tank walls.

In Figure 3.1 the normalized torque is plotted against the nondimensional time $t^* = tN$, i.e. the number of revolutions, for the case of the regular impeller at

$Re = 320$. To assess the balance, the torque applied by the impeller is compared to the sum of the other two terms, namely the time-derivative term and the torque applied by the tank walls. The two curves collapse very well indicating that the balance is satisfied instantaneously. The small discrepancy is equal to 0.15% and is constant in time. All balance results are summarized in Table 3.1.

	$Re = 320$	$Re = 1600$
Regular impeller	0.15%	0.32%
Fractal impeller	0.29%	1.5%

Table 3.1: Torque imbalance for different cases.

As stated earlier, the spatial resolution at the tank walls is ten times finer in wall units than on the impeller surfaces. This fact results in an accurate computation of the torque applied by the tank walls. Moreover, the resolution on the tank walls is identical for both regular and fractal impeller cases, whereas the torque imbalance in the fractal impeller case is higher than the regular impeller case, especially at $Re = 1600$. This suggests that the small errors most probably stem from the computation of the impeller torque. It is likely that the pressure computed at the edges of the blades include errors due to the high velocity gradients emerging as the fluid particles turn around sharp corners. These gradients increase as the Reynolds number gets higher; hence this conclusion is in accordance with the increased errors at $Re = 1600$. It must be noted that the torque applied by the impeller is higher than at the wall, for the fractal impeller at $Re = 1600$, and this leads to the difference of 1.5%. Nevertheless, the imbalance is small for all cases. This result gives us confidence about the quality of our predictions of the power drawn by the impeller.

3.3 Kinetic energy balance

In order to examine the kinetic energy balance, the terms of the conservation equation of the total kinetic energy must be evaluated. This equation is derived by taking the inner product of the velocity vector with the incompressible momentum equations. Hence, the aforementioned equation takes the following form

(formulated in the inertial frame):

$$\frac{\partial E}{\partial t} + u_j \frac{\partial E}{\partial x_j} = -\frac{u_i}{\rho} \frac{\partial p}{\partial x_i} + \nu u_i \frac{\partial^2 u_i}{\partial x_j^2}, \quad (3.7)$$

where $E = \frac{1}{2} u_i u_i$. The last term of the equation, which represents the viscous effects, can be decomposed into diffusive and dissipative parts as:

$$\nu u_i \frac{\partial^2 u_i}{\partial x_j^2} = \nu \frac{\partial^2 E}{\partial x_j^2} - \underbrace{\nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j}}_{\tilde{\varepsilon}}. \quad (3.8)$$

The last term in Equation 3.8, i.e. the dissipative term, is called pseudo-dissipation⁵⁰ and is symbolized by $\tilde{\varepsilon}$. However, it is common practice to add $\frac{\partial u_i}{\partial x_j} \cdot \frac{\partial u_j}{\partial x_i}$ to the Laplace term and subtract the same expression from the pseudo-dissipation term, in order to obtain the following form:

$$\nu u_i \frac{\partial^2 u_i}{\partial x_j^2} = 2\nu \frac{\partial}{\partial x_j} (u_i s_{ij}) - \underbrace{2\nu (s_{ij} s_{ij})}_{\varepsilon}. \quad (3.9)$$

s_{ij} is the strain-rate tensor defined as:

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (3.10)$$

The formulation of the dissipation (ε) in Equation 3.9 has the advantage of being the same in both frames considered here, unlike the form referred to as pseudo-dissipation ($\tilde{\varepsilon}$). This is because the strain-rate tensor takes the same values in the two frames considered here, as explained in the previous section. It must be noted that the computed dissipation of total kinetic energy includes not only the turbulence dissipation but also the contribution of the mean velocity gradients. This contribution is insignificant at fully turbulent regime. However, given that the present study deals with transitional Reynolds numbers, the contribution of the mean velocity gradients may not be negligible.

The roles of the other terms can be summarized as follows: If the pressure term in Equation 3.7 is evaluated in the inertial frame and integrated over the

entire tank volume, it is the major source of power input. The distribution of the work done by the pressure term is normalized by the time and volume averaged power input, $\langle P \rangle / V$. This is shown in Figure 3.2 displaying that most of the energy is injected in the impeller discharge region. The red regions in this plot illustrate where the flow is accelerated by a favourable pressure gradient, hence the flow kinetic energy is increased. It can be noted that the rate of energy injection locally is as high as 300 times the volume average of the power input, especially on the suction side of the blades. The blue regions indicate that the flow is also decelerated strongly due to adverse pressure gradient.

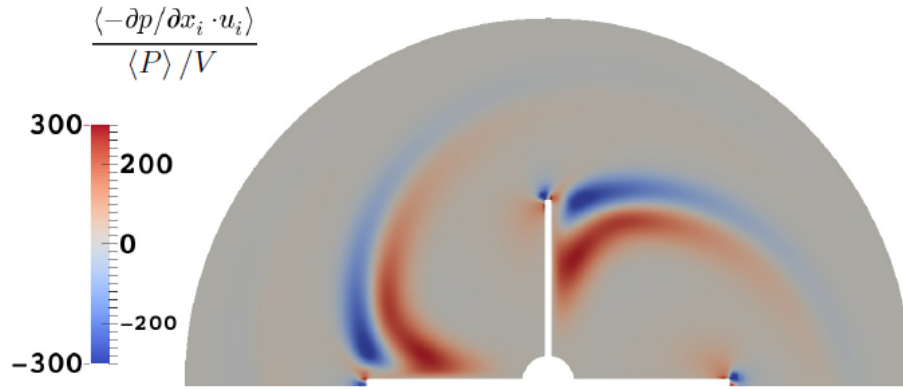


Figure 3.2: Contour plot of the work done by the pressure term in the inertial frame at the mid-height of the tank normalized by the time and volume averaged power input, for the regular impeller at $Re = 320$. (Since this quantity is computed with time averaging in the relative frame and transformation to the inertial frame, the contour plot is equivalent to a phase-averaged field obtained in the inertial frame.)

The volume integral of the pressure term can be reduced to a surface integral which is non-zero only on the pressure and the suction sides of the impeller blades. The volume integral of the diffusion term can also take the form of a surface integral that quantifies the work done by the viscous forces applied on the surfaces of the impeller and the shaft. The sum of these two terms is equal to the product of T_{imp} with Ω , i.e. to the power input. The integral of the advective term is equal to zero since the vessel is a closed volume and energy cannot be advected into or out of the tank. The volume integral of the time-derivative term is non-zero at any instant when the power input exceeds the dissipation or vice versa, but its time average converges to zero.

Recall that the Navier-Stokes equations are solved in the rotating reference frame where the extra terms due to Coriolis and centrifugal accelerations are added. However, the Coriolis term is by definition perpendicular to the velocity vector, hence cannot affect the energy balance. The work done by the centrifugal force can locally become the largest term of the equation, but its integral over the volume is equal to zero. Therefore, these two terms do not play a role in the global energy balance. However, there are further differences with respect to the energy balance in the inertial frame. Not only the advection, but also the pressure term integrates to zero, since the blade surfaces are not moving in the rotating frame. The only source of power is the shear stress applied by the moving walls of the tank. As already mentioned, the moving walls apply equal torque to the impeller in time-average sense, and after multiplication with Ω , this leads to the same power input as the impeller. In fact, this is in accordance with the earlier observation about the invariance of the dissipation term ($\varepsilon = 2\nu s_{ij}s_{ij}$). In conclusion, the energy balance is of course also satisfied in the rotating reference frame. The remainder of this section considers the energy balance computed in the inertial frame.

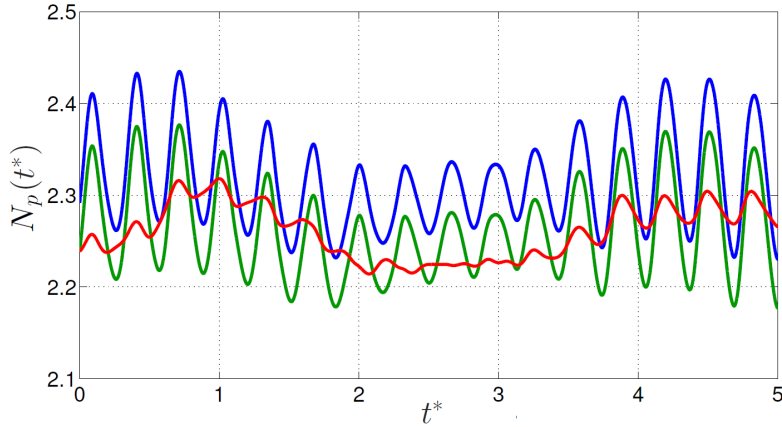


Figure 3.3: Instantaneous power draw, $P(t^*) = \Omega T_{imp}$ (blue line), $\int \varepsilon dV$ i.e. integral of dissipation in the whole domain (red line) and $\int (\varepsilon + dE/dt^*)dV$ (green line), for the regular impeller at $Re = 320$. All curves are plotted after normalization, such as: $N_p(t^*) = P(t^*)/(\rho N^3 D^5)$.

Figure 3.3 shows the time evolution of the integrated and normalized power input and dissipation over a relatively short time interval. Since any instantaneous difference between the source and sink would lead to an increase or decrease of

the total kinetic energy, the time derivative of energy is also integrated over the volume and added to the dissipation, as shown with the green curve. When this is compared with the power injection of the impeller, the two curves (shown with the blue and green lines) agree well, except for a small discrepancy of ca. 2% that remains constant in time. The same discrepancy also appears when the time-averaged power input and the time-averaged dissipation are compared. It is possibly due to the numerical dissipation and discussed later in this section. Furthermore, both blue and green curves illustrate synchronized fluctuations with multiple peaks per revolution, similar to that observed in the time evolution of the torque shown in Figure 3.1. This high frequency fluctuation is less evident in the integral dissipation (shown with the red line) but the addition of the time derivative of the energy brings the fluctuations to the level of the impeller power. Moreover, the plot demonstrates that there is an indication of a superimposed signal with much lower frequency. These aspects are analysed and discussed in Chapter 4.

The statistical convergence of the running averaged power number is shown in Figure 3.4, for both regular and fractal impellers at $Re = 320$. It is observed that any change in the running averaged values of $\langle N_p \rangle$ in the period after 40 revolutions is less than $\pm 0.16\%$ for the regular and $\pm 0.30\%$ for the fractal impeller, with respect to the final values of averaging. Hence the averaging is finalized after 80 revolutions and assumed sufficiently converged.

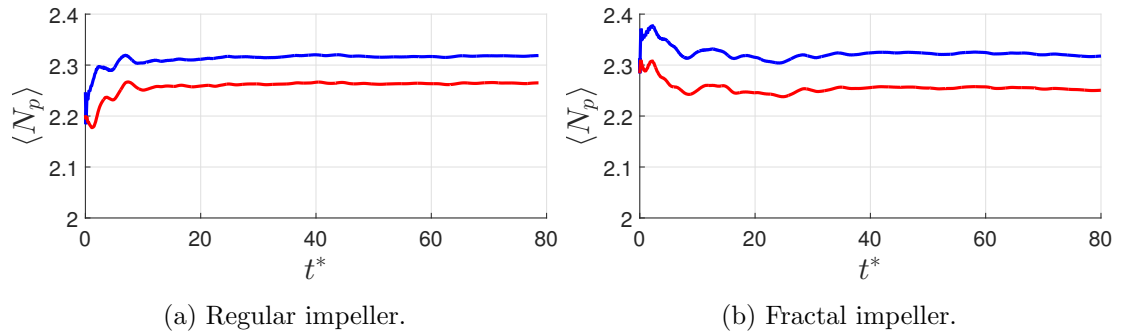


Figure 3.4: Running average of the power number over number of revolutions at $Re = 320$, based on impeller torque (blue line) and on dissipation (red line).

Table 3.2 provides the values of $\langle N_p \rangle$ for the cases at $Re = 320$ averaged for 80

revolutions, and compares the values based on the power input of the impeller and the volume integral of dissipation. The results based on the impeller power and obtained with the fine grids suggest that the power consumption is not affected by the difference in the blade shape for this Reynolds number. Also the relatively coarse grid of 9×10^6 cells provides a good prediction of $\langle N_p \rangle$, whereas the coarser grid used for the regular impeller slightly overshoots the value of $\langle N_p \rangle$ with respect to the finer one. On the other hand, the computations based on the dissipation provide an estimate for $\langle N_p \rangle$ that is slightly lower than the impeller power in all cases. This probably stems from the numerical dissipation of the computational method. As expected, the finer grids provide a more accurate energy balance. For the regular impeller, the energy balance is better than the fractal impeller, when the results of the fine grids are compared, despite the higher number of cells used for the fractal impeller. The reason may lie in the flow field differences around the impeller. In contrast to the two large trailing edge vortices observed behind the blades of the regular impeller, a number of smaller vortices emerge in the wake of the ragged edges of the fractal blades. Since the energy containing structures are smaller in this case, this may require a finer grid to correctly capture the dissipative scales.

Blade type, mesh size	Reg., 4×10^6	Reg., 13×10^6	Fra., 9×10^6	Fra., 21×10^6
Impeller power	2.36	2.32	2.31	2.32
Dissipation	2.24	2.27	2.20	2.25
Difference	5.2%	2.2%	4.9%	3.1%

Table 3.2: Averaged power numbers for the cases computed at $Re = 320$ and the balance between energy injection and dissipation. Reg. stands for regular and Fra. for fractal.

It is also noted that the contribution of the shear stress to the impeller torque is only 4% for the regular impeller, whereas it is 8% for the fractal impeller at $Re = 320$. This difference is expected because a fractal blade has ca. twice the perimeter length of a regular blade, as seen in Figure 1.3. If only pressure forces were taken into account, one would obtain a lower $\langle N_p \rangle$ with the fractal impeller by ca. 4%, which is compensated by the increased skin friction drag. On the other

hand at $Re = 1600$, the contribution of the shear stress is an order of magnitude lower than at $Re = 320$, and it therefore has no significant influence on the power consumption.

For the simulations at $Re = 1600$, the statistical convergence of the power number is shown in Figure 3.5 and the final values are listed in Table 3.3. Any change in the running averaged values of $\langle N_p \rangle$ in the period after 60 revolutions is less than $\pm 0.37\%$ for the regular and $\pm 0.48\%$ for the fractal impeller, with respect to the final values of averaging. Hence, the computations are ended after 100 revolutions and assumed statistically converged.

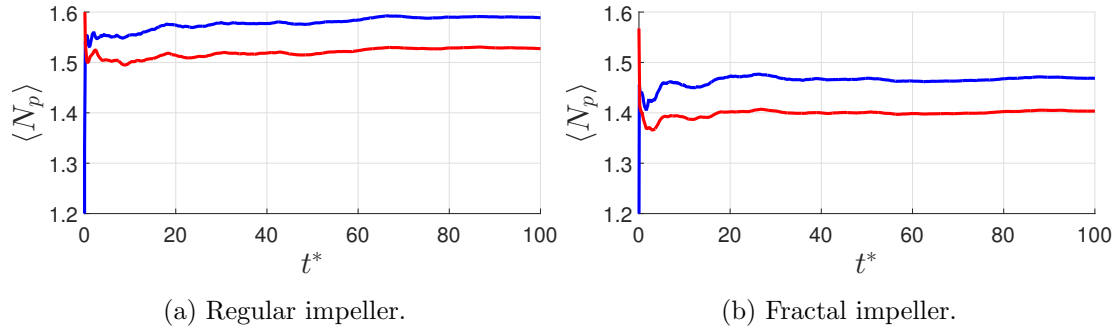


Figure 3.5: Running average of the power number over number of revolutions at $Re = 1600$, based on impeller torque (blue line) and on dissipation (red line).

Unlike the simulations at the lower Re , the results at $Re = 1600$ demonstrate a notable difference when the power numbers of the regular and fractal impellers are compared. This difference is comparable to the one observed in the laboratory experiments of Steiros et al.³ They measured the power draw using exactly the same two impeller shapes as considered in the present study, in a hexagonal unbaffled tank. They observed 11 – 12% lower $\langle N_p \rangle$ for the fractal impeller compared to the regular impeller at $Re = 1 - 2 \times 10^5$. The numerical estimates at $Re = 1600$ suggest a reduction of power consumption of ca. 8% based on the impeller torque, when the fractal blades are utilized. This finding is promising for the efficiency improvement of industrial applications in case the mixing performance of the impeller is also enhanced or remains unchanged. An effort to understand this reduction in power is made in the following sections of this chapter.

Note that the difference between the power draw and the integral dissipation

is around 4% for the cases at $Re = 1600$ (see Table 3.3). This corresponds to a slightly larger energy imbalance compared to that achieved at $Re = 320$, yet still acceptably small.

Blade type, mesh size	Regular, 60×10^6	Fractal, 70×10^6
Impeller power	1.59	1.47
Dissipation	1.53	1.41
Difference	3.8%	4.2%

Table 3.3: Averaged power numbers for the cases computed at $Re = 1600$ and the balance between energy injection and dissipation.

Finally, numerically predicted power numbers are compared with the empirical correlation employed to display the variation of $\langle N_p \rangle$ vs. Re in Figure 2.4. This curve provides the following estimates: $\langle N_p \rangle = 2.18$ at $Re = 320$ and $\langle N_p \rangle = 1.41$ at $Re = 1600$, for the geometry of the regular impeller used in this work. These are in qualitative agreement with our results but are lower than our estimates based on the impeller power, by 6% at $Re = 320$ and 12% at $Re = 1600$. The details of the experiments or the uncertainty of the correlation are not provided by Furukawa et al.⁶ However, considering the high number of independent parameters such as the tank and impeller diameters, heights, clearance etc., a degree of discrepancy is inevitable. Moreover, the blade thickness is not considered by this correlation, and is known to have a strong influence on the power number. For example, Rutherford et al.¹¹⁷ suggested a correlation between the power number and the blade thickness of a Rushton turbine under fully turbulent regime: $\langle N_p \rangle = 6.405 - 55.673(t/D)$, where t/D is the blade thickness to impeller diameter ratio. For instance, if this ratio is reduced from 2% to 1%, $\langle N_p \rangle$ would have an increase of ca. 10% according to this correlation. Although this equation is not directly applicable to our case, it emphasizes the influence of the blade thickness. Taking this into account, the empirical correlation given by Furukawa et al.⁶ can only be considered for approximate comparison.

The DNS study conducted by Verzicco et al.⁴ which was already used for code validation purposes, reported ca. 18% difference between the power input and the integral energy dissipation. The DNS study by Gillissen and Van den

Akker¹⁰⁷ investigated the flow in a baffled tank stirred with a Rushton turbine at $Re = 7300$, using the lattice-Boltzmann method and 2.9×10^9 grid cells. They reported an energy imbalance of 3%. Considering the high resolution used in the above-mentioned work, having a similar quality in terms of energy balance is regarded as satisfactory. Note that their work employed a uniform Cartesian grid, whereas our simulations utilized a body-fitted unstructured grid which allows a much better cell distribution. This partly explains the smaller mesh sizes in our cases, besides other important factors like the lower Re and lack of baffles in the tank.

3.4 Pressure distribution on blade surfaces

Considering that both regular and fractal impellers rotate with the same angular velocity, any difference in the power is a consequence of a different torque on the impeller. Therefore, the analysis starts with an investigation of pressure distribution on the blade surfaces. The time-averaged pressure difference between the suction and pressure sides is normalized to define a local pressure coefficient, C_p , as follows:

$$C_p = \frac{\langle \Delta p \rangle}{0.5 \rho U_{tip}^2}. \quad (3.11)$$

The distribution of C_p over the blades surface is illustrated in Figure 3.6, and qualitatively it agrees well with the experimental results.³ The maximum value for the regular blade is close to 1, whereas for the fractal blade it reaches up to 0.9. In the experiments³ with $Re = 1 - 2 \times 10^5$, the maximum value was less than 0.75. It is expected that this quantity will be lower at higher Re , because $\langle N_p \rangle$ is lower as well. The radial positions of C_p maxima are denoted with vertical dashed lines in Figure 3.6. These are located at $r/R = 0.84$ and 1.01 for the regular and fractal blades, respectively. From the C_p distributions reported in the aforementioned experimental study, it appears that the maxima are located in similar positions, but the exact locations were not stated.

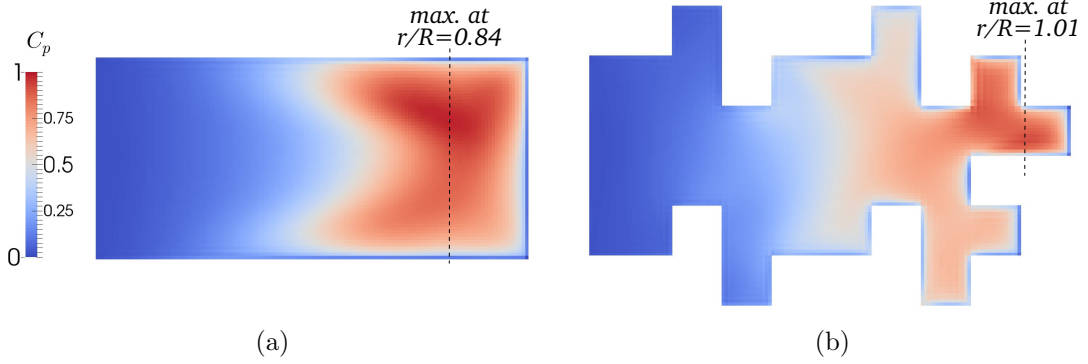


Figure 3.6: C_p distribution on the surface of (a) regular and (b) fractal blade, at $Re = 1600$. The radial positions of maximum C_p are indicated with vertical dashed lines.

From the surface integral of C_p , the drag coefficient can be computed as $C_D = (\int C_p dA) / \int dA$. The values of C_D from the present DNS study and the experiments³ at $Re = 10^5$ are listed in Table 3.4. Note that C_D is based on pressure forces only, i.e. viscous forces are excluded. The center of pressure, CoP , is also included in Table 3.4. It is defined as the net torque divided by the net force applied on the blade, and it is normalized with the impeller radius, R . As expected, C_D decreases with the increasing Re , a behaviour similar to that of $\langle N_p \rangle$. Moreover, CoP moves radially further from the axis as Re rises. At every Re , the fractal blade has a lower C_D than the regular blade. At $Re = 320$, the larger value of CoP and the more significant viscous forces on the fractal blade compensate for the lower C_D , so that both blades apply the same torque on the fluid. At the higher Re numbers, the reduced pressure drag force on the fractal impeller is the dominant factor that explains the lower torque with respect to the regular impeller.

Blade type	Reg.	Fr.	Reg.	Fr.	Reg.	Fr.
Re	320	320	1600	1600	10^5 (exp.)	10^5 (exp.)
C_D	0.597	0.538	0.406	0.363	0.172	0.153
CoP/R	0.698	0.728	0.735	0.736	0.820	0.844

Table 3.4: Drag coefficients of both types of blades at $Re = 320$, 1600 and 10^5 . Reg. stands for regular and Fr. for fractal impeller. CoP denotes the radial position of the center of pressure. The values at $Re = 10^5$ are taken from Steiros et al.³

In order to gain more insight on the observed pressure distributions, now the attention is turned to the velocity fields around the blades. Figure 3.7a shows the

mean azimuthal velocity profiles acquired along two radial lines, at 10° upstream of the pressure side and 10° downstream of the suction side (as depicted in Figure 3.7b). These profiles were obtained at the mid-height of the tank, in the relative reference frame, at $Re = 1600$. Since the impeller rotation is in the positive azimuthal direction, as indicated with an arrow in Figure 3.7b, the relative azimuthal velocities (V_θ) will have negative values on the pressure side along the entire profile. For the radial positions up to half of the blade radius ($r/R < 0.5$), the velocity magnitudes are less than 10% of U_{tip} for both types of blades on both sides. This explains why C_p values are very low on the inner half of the blades, as seen in Figure 3.6.

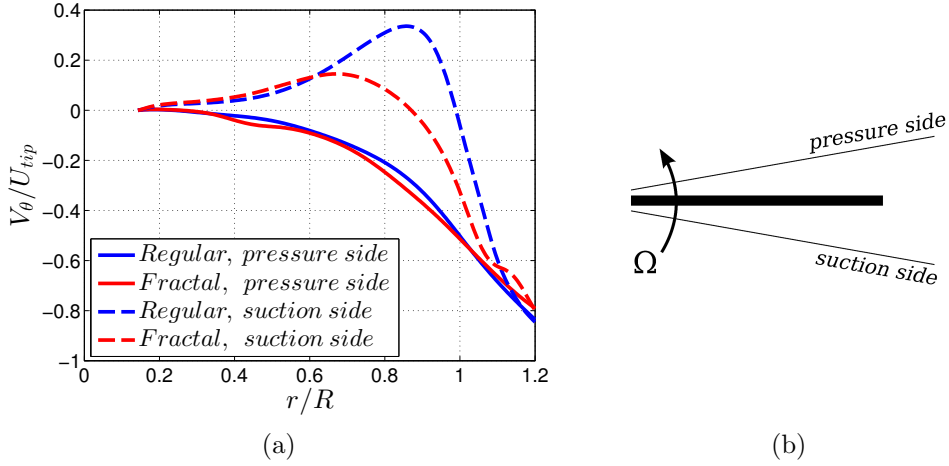


Figure 3.7: (a) Time-averaged azimuthal velocity profiles along a radial line 10° upstream of the pressure side and 10° downstream of the suction side of regular and fractal blades, at the tank mid-height in relative frame at $Re = 1600$. (b) Illustration of the radial lines along which the profiles were acquired and the direction of rotation.

The profiles of V_θ on the pressure sides of both blades collapse (solid lines in Figure 3.7a). This is consistent with the results of experiments³ using a two bladed impeller, where one regular and one fractal blade are mounted 90° apart. The measured torque was the same when the impeller rotated in either direction, immersing the regular or fractal blade in the wake of the other. This led to the conclusion that the wake interaction is *not* the reason of the reduced drag coefficient of a fractal blade. Here it is elaborated further and shown that this is because the upstream velocity profiles are almost independent of the type of the

preceding blade. On the other hand, the velocities downstream of the blades are significantly different (dashed lines in figure 3.7a). This may seem contradictory if one expects that the downstream profile of a blade must interact with the following blade. In our observation, there is a strong radial jet along the suction side of the blades which carries the fluid in the wake radially away. The part of the flow field immediately upstream of the blade is mainly advected axially from the top and bottom of the tank towards the mid-height along the shaft, before interacting with the pressure side. Therefore, the profiles on the pressure side are strongly affected by the angular momentum of the fluid that is in the bulk of the flow far from the impeller. If the bulk flow had higher angular momentum (i.e. higher positive azimuthal velocity in absolute frame, as it is observed at increased Re^{52}), then the profiles of V_θ on the pressure side would have a smaller magnitude (i.e. lower velocity relative to the blade). This would lead to a weaker stagnation on the pressure side and a lower drag coefficient, as it is seen in Table 3.4 for growing Re .

Here the flow separation zone in the wake of a blade is defined as the region where V_θ is positive, i.e. towards the blade suction side (see Figure 3.7b). This region extends until about $r/R = 1$ in the wake of the regular impeller (blue dashed line in Figure 3.7a). Moreover, the magnitude of V_θ reaches a maximum value of $0.34 U_{tip}$. On the other hand, in the wake of the fractal impeller V_θ becomes negative again around $r/R = 0.88$ and the maximum magnitude is only $0.16 U_{tip}$, indicating a smaller and weaker separation zone.

The separation zone as defined above is illustrated in Figures 3.8a and 3.8c in a three dimensional view in the wakes of regular and fractal blades, respectively. Taking all four blades into account, the size of the separated region is 14.1% and 13.2%, respectively, of the impeller swept volume ($\pi R^2 H/10$). The larger separating zone in the wake of a regular blade (by about 7%) causes a lower pressure on the suction side, resulting in a higher form drag. At first sight, this finding seems to contradict the results of Nedić et al.² who performed wind tunnel experiments with square and fractal plates placed normal to an incoming flow.

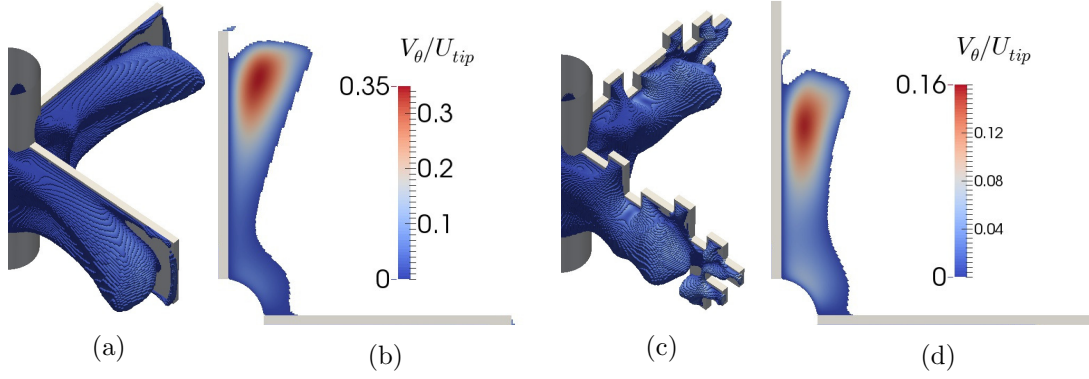


Figure 3.8: Flow separation regions (where $V_\theta > 0$) behind the blades at $Re = 1600$, (a) 3-D view for regular impeller, (b) contours of V_θ in a cross section of the volume shown in (a), (c) 3-D view for fractal impeller, (d) contours of V_θ in a cross section of the volume shown in (c).

The shapes of the plates were similar to the blades used in the present study, as shown in Figure 1.2. In this flow setting, they measured larger C_D values for the fractal plates compared to the square plates. However, this flow setting is different to the one examined in the present thesis. While the front stagnation flow patterns may be qualitatively similar, the wakes have markedly different behaviour. The rotation of the blade creates a strong radial jet, that is absent when the object is placed normal to the flow. The presence of the radial jet changes entirely the wake properties, resulting in different trends in the C_D values.

Figures 3.8b and d illustrate contours of V_θ in a cross section of the separation zones at the tank mid-height. The dark red colour indicates the location with the highest V_θ . This location is radially further away from the axis for the regular blade compared to the fractal blade, and matches with the radial location where the highest C_p was observed in Figure 3.6a, at $r/R = 0.84$. On the other hand, the fractal blade has the highest C_p near the tip of the blade, as seen in Figure 3.6b, which does not coincide with the location of the strongest separation. This can be explained as follows: Since the tip of the fractal blade extends radially further (up to $r/R = 1.1$) than the tip of the regular blade (up to $r/R = 1$), the approaching relative velocity is higher, leading to higher stagnation pressure on the pressure side. This is the dominant factor that determines the location of highest C_p for the fractal blade.

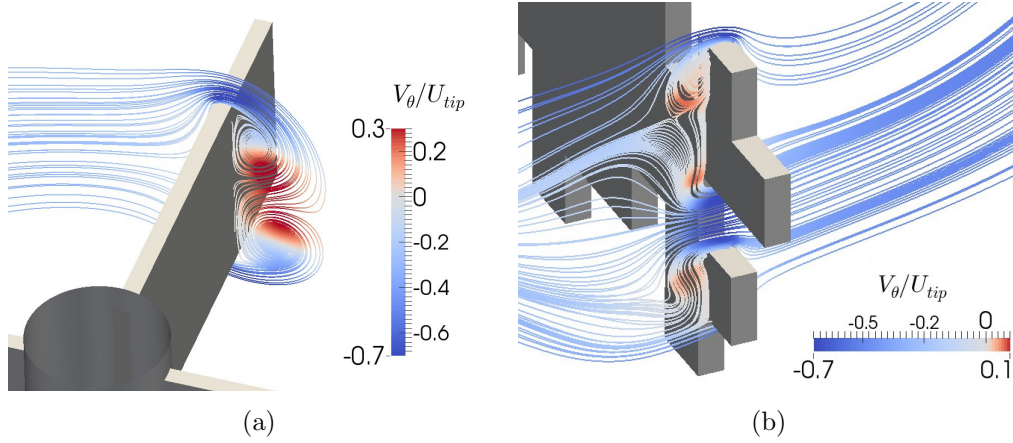


Figure 3.9: Recirculation zone behind the blades at $Re = 1600$, (a) regular (b) fractal blade.

In order to analyse further the mechanism that leads to a smaller separation region in the suction side of the fractal blades, the streamlines around both types of blades are illustrated in Figure 3.9. To generate the streamlines, the radial velocity component is subtracted from the mean flow field and only the mean axial and azimuthal components are used. In other words, the effect of radial jet is not illustrated in these figures. The streamlines are coloured according to V_θ . As seen in Figure 3.9 a), the flow passing along the upper and lower edges of the regular blade recirculates in the wake and turns towards the suction side. With the influence of the radial jet along the suction side of the blade, the recirculating fluid is carried towards the blade tip, resulting in a spiraling motion that gives rise to the trailing vortex pair.

In the case of the fractal impeller, the concave parts of the blade perimeter allow the upcoming fluid with high azimuthal velocity, hence high momentum, to enter into the recirculation region formed in the wake. The jets penetrate and brake the two large recirculation zones, leading to multiple smaller recirculation zones.

This has a profound influence on the coherent trailing vortex structures present in the wake of the blades. Vortex cores can be visualised with the help of the λ_2 -criterion introduced by Jeong and Hussein.⁹⁶ In this criterion, λ_2 is the second (intermediate) eigenvalue of the symmetric tensor $\mathbf{S}^2 + \mathbf{\Omega}^2$ (with $\lambda_3 \geq \lambda_2 \geq \lambda_1$),

where $\mathbf{S}$ and $\mathbf{\Omega}$ are the symmetric and antisymmetric parts of the velocity gradient tensor, respectively. This is one of the widely used methods to illustrate trailing vortices, employed also by Escudié and Liné⁹⁵ and Sharp et al.¹⁰⁵ Figure 3.10 shows the contours of normalized λ_2 on a plane 15° behind the blades, at $Re = 1600$. The borders of the impeller swept volume are marked with black lines and the blue regions indicate the vortex cores. In the wake of the regular blade, there are two large trailing vortex cores, as expected. On the other hand, five separate cores appear in the wake of the fractal blade (excluding the smaller ones closer to the shaft), which are about half the size of the cores in the wake of the regular blade. Since the recirculation region is penetrated by jets through the concave edges of the fractal blade as shown in Figure 3.9b, the coherent structures are broken into multiple trailing vortices.

It can be concluded that the wake properties are strongly affected by the blade design, as illustrated above. The pressure distribution on the suction side of blades is closely connected to the trailing vortex structures that emerge in the flow separated wake region. This opens the possibility to design blades with favourable characteristics that reduce the drag coefficient and impeller torque, without changing the total area.

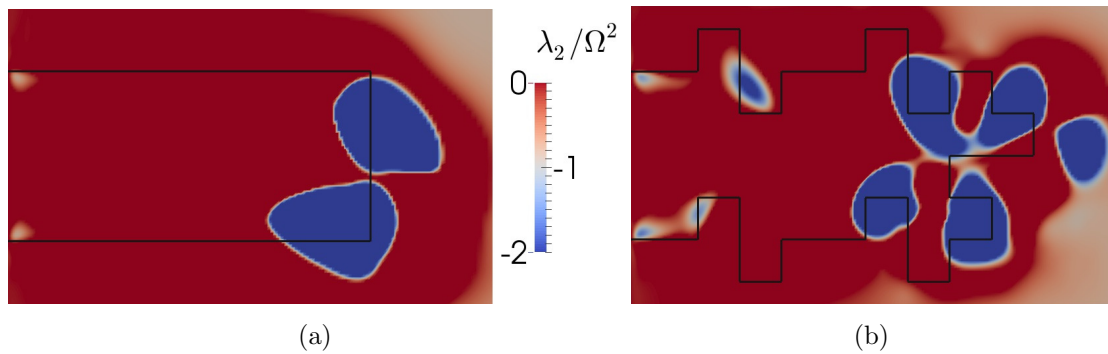


Figure 3.10: Vortex cores are illustrated using the contours of λ_2/Ω^2 on a plane 15° behind the blades at $Re = 1600$, (a) regular blade, (b) fractal blade. The blue regions in the figures display the vortex cores. Black lines indicate the borders of the impeller swept volume.

3.5 Transport of angular momentum

As a result of the above differences in the pressure distribution on the blade surfaces, the regular impeller applies a higher torque to the volume of fluid in the vessel compared to the fractal impeller at $Re = 1600$. Therefore, the flux of angular momentum away from the impeller must be higher. Some questions naturally arise: Which differences in the flow field around the impeller result in the higher angular momentum flux? How is this related to the modification in the blade shape or in the trailing vortex system? In this section I try to answer these questions.

To this end, a control volume (CV) around the impeller is considered as shown in Figure 3.11 and the angular momentum balance in this CV is computed. The impeller can be thought of as a source of angular momentum which is then transported through the borders of the CV and is eventually lost at the tank walls due to viscous stresses. The general form of the angular momentum balance in this CV is:

$$T_{imp} = \left\langle \int_{S=S_{in}+S_{out}} (\vec{x} \times \vec{u}) \rho \vec{u} \cdot d\vec{S} - \int_{S=S_{in}+S_{out}} \vec{x} \times (\boldsymbol{\tau} \cdot d\vec{S}) \right\rangle \cdot \hat{e}_z, \quad (3.12)$$

where T_{imp} is obtained from integration of moment distribution due pressure and viscous forces over the impeller blade (the major contributor is the pressure force). The first term in the right hand side is the net angular momentum flux through the boundary S of the CV, and the second term is the moment due to viscous stresses on S . Vector $\hat{e}_z$ is the unit vector along the impeller axis and S_{in} is the part of the CV's surface normal to $\hat{e}_z$ (both planar surfaces) whereas S_{out} is the remaining cylindrical part. Using the DNS data, both sides of Equation 3.12 are computed separately and balance up to three significant digits. Although the viscous term is also taken into account during the evaluation of this balance, it constitutes a small portion of the right-hand side of the equation. This term is about 2 – 4% of the total transport at $Re = 320$ and this percentage is an order of magnitude smaller at $Re = 1600$.

Equation 3.12 is very general and it contains the effects of spatial variation of

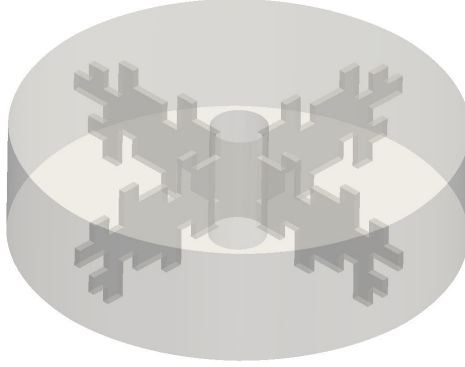


Figure 3.11: Control volume around the fractal impeller.

velocity as well as turbulence (due to time averaging operation, $\langle \rangle$). Ignoring the viscous term and assuming that the velocity is uniform and stationary, a simplified form can be obtained, known as the Euler's turbine equation¹¹⁸

$$T_{imp} = \rho Q (L_{S_{out}} - L_{S_{in}}), \quad (3.13)$$

where Q is the volumetric flow rate through the CV (due to continuity $Q_{S_{in}} = Q_{S_{out}} = Q$), while $L_{S_{in}}$ and $L_{S_{out}}$ stand for the angular momentum per unit mass at the inlet and outlet of the CV, respectively. The product $\rho Q (L_{S_{out}} - L_{S_{in}})$ denotes the net angular momentum flux through the boundary of the CV and it is the simplified form of the first term on the right hand side of Equation 3.12.

Equation 3.13 indicates that, in order to examine in more detail the differences between the standard and fractal impellers, it is instructive to investigate the net flow rate through the CV. The dimensions of the CV are selected such that it has the smallest possible volume that can contain the fractal impeller, allowing a small gap of $0.01 R$ between edges of the blade and the CV borders, hence $R_{CV} = 1.11 R$ (see Figure 3.11). For a fair comparison, the CV used for the regular impeller has exactly the same dimensions as the one used for the fractal impeller. The flow rate Q can be computed by the integration of $U_r \hat{e}_r \cdot \vec{n}$ over the side surface of the CV (hereafter S_{out}), where $\vec{n}$ is the unity vector normal to the surface pointing outwards from the CV (equal to $\hat{e}_r$ because the volume is cylindrical). This gives the outflow from the CV, since strong radial jets generated by the blades carry the

fluid from the impeller towards the walls. Q can be also computed from integration of $U_z \hat{e}_z \cdot \vec{n}$ over the upper and lower flat surfaces of the CV (hereafter S_{in}); that would give the inflow which is equal to the outflow (with opposite sign).

	Regular	Fractal	Difference
$Re = 320$	0.577	0.621	7.3%
$Re = 1600$	0.434	0.481	10.3%

Table 3.5: Normalized flow rate, $N_q = Q/(ND^3)$, through the CV in Figure 3.11. The difference between the flow rates of regular and fractal impellers is presented in the rightmost column.

The values of the normalized flow rate, $N_q = Q/(ND^3)$, are reported in Table 3.5 for the four cases examined. It is noted that N_q decreases with Re , which is in agreement with the trend presented by Nagata⁵² for an unbaffled tank with an eight-bladed paddle impeller. He showed that N_q increases rapidly in the laminar regime and reaches a peak at $Re \approx 90$, where $N_q \approx 1$. This is followed by a slow decrease over transitional and turbulent regimes down to $N_q \approx 0.45$ at $Re = 10^6$. Since a four-bladed impeller is used in the present study, the values are expected to be somewhat lower compared to those of Nagata, hence the quantities presented in Table 3.5 are reasonable.

On the other hand, it may be surprising that the fractal impeller has at $Re = 320$ ca. 7% higher N_q compared to the regular impeller with equal $\langle N_p \rangle$, and at $Re = 1600$ ca. 10% higher N_q despite having 8% lower $\langle N_p \rangle$. This difference in N_q in favour of the fractal impeller is even larger at $Re = 1.5 \times 10^5$, according to the PIV study of Steiros et al.¹¹⁵ This property of the fractal impeller has the potential to accelerate the stirring of a scalar injected into the fluid, which is especially desirable for low to moderate Re , where the macro-mixing plays an important role. The study of the mixing properties of fractal impellers will be the subject of Chapter 5.

To evaluate the dependence of this result on the size of the CV, the mean radial velocity averaged over S_{out} for varying CV-radius is plotted in Figure 3.12. Since this calculation corresponds to averaging U_r in the azimuthal direction, it is the same in both absolute and relative reference frames. The radial flow rate

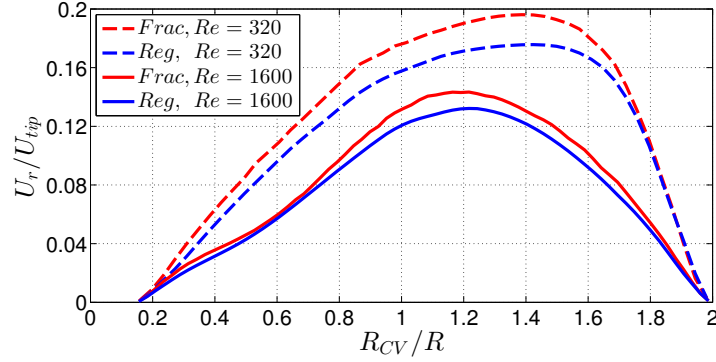


Figure 3.12: Radial velocity averaged over S_{out} and in time, for varying R_{CV} .

is consistently higher for the fractal impeller over a broad range of CV radii ($1 < R_{CV}/R < 1.6$). It is deduced therefore that the higher power number of the regular impeller at $Re = 1600$ is not related to an increased flow rate, but to a larger difference of the fluid angular momentum per unit mass between S_{in} and S_{out} . Therefore, the crucial quantity to evaluate is the transport of angular momentum into and out of the CV per unit flow rate.

The aforementioned transport occurs via two mechanisms: advective transport and viscous transport due to the shear stresses (refer to Equation 3.12), but the latter has a negligible contribution. The advective transport of angular momentum over S_{in} and S_{out} is normalized with the mass flow rate (ρQ) and made nondimensional using the angular momentum (per unit mass) at the blade tip ($R U_{tip}$), as follows:

$$l_{in} = \frac{|\int_{S_{in}} \rho r \langle u_{\theta} u_z \rangle \hat{e}_z \cdot d\vec{S}|}{\rho Q R U_{tip}}, \quad l_{out} = \frac{\int_{S_{out}} \rho r \langle u_{\theta} u_r \rangle \hat{e}_r \cdot d\vec{S}}{\rho Q R U_{tip}}. \quad (3.14)$$

l_{in} is defined in terms of the absolute value of the integral in Equation 3.14, since this integral results in a negative value indicating momentum entering into the CV. Therefore the net normalized advective transport is $\Delta l = l_{out} - l_{in}$. The values of l_{in} , l_{out} and Δl are listed in Table 3.6 for both Re and impeller types.

When the two values of Re are compared, it is seen that l_{in} is significantly higher at $Re = 1600$, meaning that the fluid enters the CV with a higher angular momentum. This can be explained as follows: While the radially discharged flow

		l_{in}	l_{out}	$\Delta l = l_{out} - l_{in}$
$Re = 320$	Reg.	0.159	0.557	0.398
	Frac.	0.177	0.536	0.359
				=> 10% difference
$Re = 1600$	Reg.	0.288	0.656	0.368
	Frac.	0.283	0.589	0.306
				=> 18% difference

Table 3.6: The values of l_{in} and l_{out} as defined by Equation 3.14. The values of Δl obtained with regular and fractal impellers are compared and the percentage difference is also shown.

circulates back to the impeller, it loses part of its angular momentum due to shear stresses applied by tank walls. This effect of the wall shear stress weakens with growing Re , leading to a higher l_{in} . If there were baffles on the tank walls, l_{in} would be much smaller, since viscous forces on the walls are strongly augmented by pressure forces on the baffle surfaces normal to the azimuthal fluid motion. This would lead to a much larger Δl , hence larger impeller torque and $\langle N_p \rangle$. Moreover, Table 3.6 shows that l_{out} also increases with Re , but not as much as l_{in} , resulting in a lower Δl at $Re = 1600$ for both types of impellers. Consequently, the computed $\langle N_p \rangle$ decreases with Re .

At both Reynolds numbers, the regular impeller yields a higher Δl with respect to the fractal impeller. However, at $Re = 320$ this difference is compensated mainly by the higher flow rate of the fractal impeller, and also with the inclusion of the small contribution of the viscous transport over S_{out} (since CV-borders are very close to the tip of the fractal blade, the mean velocity gradients are high at S_{out} near the blade tip). Taking everything into account, at the low Re cases both types of impeller have the same torque and $\langle N_p \rangle$. On the other hand at $Re = 1600$, the 18% higher Δl of the regular impeller is only partially balanced by the 10% lower flow rate. Consequently, the regular impeller requires 8% higher power consumption compared to the fractal impeller.

In order to provide more insight as to why Δl is larger for the regular impeller and why this difference grows with Re , the advective transport term is decomposed in two parts, representing the contributions of the mean and fluctuating velocities.

So the term $r \langle u_\theta u_r \rangle$ is decomposed as follows:

$$r \langle u_\theta u_r \rangle = r U_\theta U_r + r \langle u'_\theta u'_r \rangle. \quad (3.15)$$

Using this expression, one can evaluate separately $l_{out,mean}$, which is the contribution due to mean velocities (i.e. $r U_\theta U_r$), and $l_{out,turb}$, the contribution due to turbulent fluctuations (i.e. $r \langle u'_\theta u'_r \rangle$).

$$l_{out,mean} = \frac{\int_{S_{out}} r U_\theta U_r \hat{e}_r \cdot d\vec{S}}{Q R U_{tip}}, \quad l_{out,turb} = \frac{\int_{S_{out}} r \langle u'_\theta u'_r \rangle \hat{e}_r \cdot d\vec{S}}{Q R U_{tip}}. \quad (3.16)$$

Both terms are separately integrated over the surface S_{out} of the CV and normalized as in Equation 3.16. The radial profiles of $l_{out,mean}$ and $l_{out,turb}$ are illustrated in Figure 3.13 where they are evaluated over S_{out} with a varying radius (R_{CV}). A similar decomposition is also performed for l_{in} to evaluate $l_{in,mean}$ and $l_{in,turb}$ separately, but it is not discussed here because the contribution of $l_{in,turb}$ is found to be ca. 1% or less.

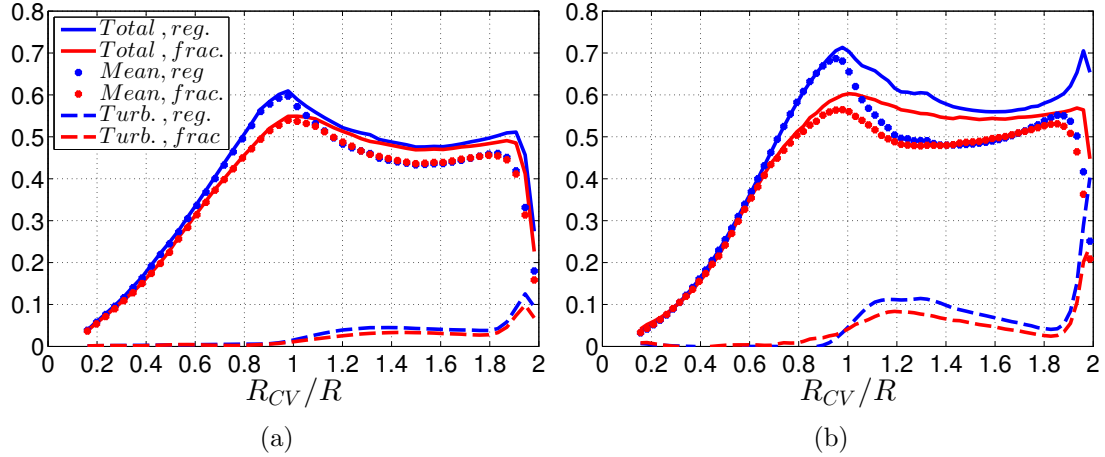


Figure 3.13: l_{out} (Total), $l_{out,mean}$ (Mean) and $l_{out,turb}$ (Turb.) computed at S_{out} with varying radial positions (R_{CV}) for regular and fractal impellers (a) at $Re = 320$, (b) at $Re = 1600$.

Figure 3.13 shows that for both impeller types and at both Re , $l_{out,mean}$ is larger than $l_{out,turb}$, therefore it determines the form of the l_{out} profile. Since $l_{out,mean}$ has in the denominator the flow rate for normalization, which is basically the integral of U_r over S_{out} , it might have been assumed that the profile of $l_{out,mean}$ corresponds to

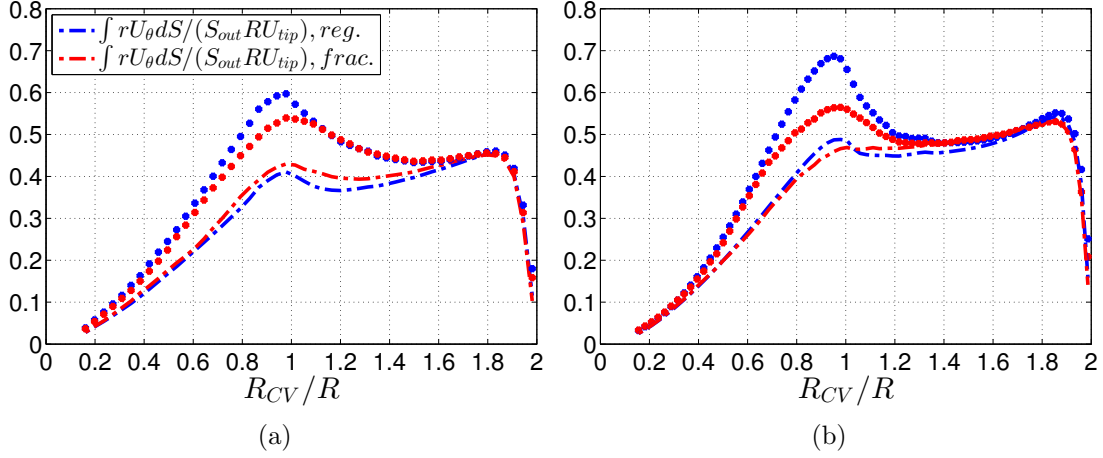


Figure 3.14: $l_{out,mean}$ shown with dots as in Figure 3.13 and compared with mean angular momentum (rU_θ) averaged over S_{out} with varying radial positions (R_{CV}) for regular and fractal impellers that is shown with dash-dotted lines (a) at $Re = 320$, (b) at $Re = 1600$.

the profile of angular momentum (rU_θ) averaged over S_{out} . This would be correct if U_r and U_θ had a homogeneous spatial distribution over S_{out} . To see if this is the case, angular momentum (rU_θ) is averaged over S_{out} with varying radius (R_{CV}) and is normalized with RU_{tip} . The so-obtained radial profile of angular momentum is illustrated in Figure 3.14 with dash-dotted lines and is compared with the profile of $l_{out,mean}$. It is seen in Figures 3.14 a) and b), that the angular momentum increases monotonically from the shaft until the blade tip (for $R_{CV}/R \leq 1$) as in the core of a forced vortex, then remains approximately constant until the near-wall region (for $1 < R_{CV}/R < 1.9$) which is expected in a free vortex. On the other hand, the curves of $l_{out,mean}$ have a higher slope until $R_{CV}/R \approx 1$, where they reach their maxima and have a significant surplus compared to the average angular momentum, for both impeller types and Re numbers. This surplus is due to the strong spatial correlation between U_r and U_θ in the impeller discharge region. Especially on the suction side of the blades near the blade tip ($0.8 < R_{CV}/R < 1$), the highest radial and azimuthal velocities are observed approximately in the same region. This spatial correlation is stronger for the regular impeller, hence it has a higher $l_{out,mean}$, despite the similar angular momentum profiles of both impeller types. But the difference between the regular and fractal impellers is observed only until a certain radial position, $R_{CV}/R \approx 1.1 - 1.2$, where the profiles of $l_{out,mean}$ of

both impellers merge. At $Re = 1600$, the profiles of $l_{out,mean}$ also collapse with the profiles of angular momentum at $R_{CV}/R \approx 1.2$. This indicates that the spatial distributions of U_r and U_θ over S_{out} are homogenized. This happens approximately at the radial position where $l_{out,turb}$ completes its growth and reaches a plateau (see Figure 3.13b). It can be deduced that the strong fluctuations (i.e. $\langle u'_\theta u'_r \rangle$) enhance the homogenization of mean velocities. However, at $Re = 320$ this homogenization occurs later, at $R_{CV}/R \approx 1.6$, possibly due to the lack of turbulent mixing of momentum.

Focusing on Figure 3.13 and comparing the results at both Re numbers, several aspects can be clarified. At $Re = 320$, the difference between the l_{out} -profiles of regular and fractal impellers is very small (except for the radial range of $0.7 < R_{CV}/R < 1.1$, that is mainly due to $l_{out,mean}$). At $Re = 1600$, on the other hand, the difference between the l_{out} -profiles is larger for $R_{CV}/R > 0.6$. Since $l_{out,mean}$ -profiles of regular and fractal impellers collapse at $R_{CV}/R \approx 1.2$, any difference in the transport mechanism is due to $l_{out,turb}$ for $R_{CV}/R > 1.2$. Most importantly, $l_{out,turb}$ grows almost threefold when Re is increased from 320 to 1600. Actually at both Re , regular impeller yields ca. 30% higher $l_{out,turb}$ than the fractal impeller for the range of radial positions $1.1 < R_{CV}/R < 1.6$, but at $Re = 320$ the influence of $l_{out,turb}$ is not crucial. In conclusion, at $Re = 1600$, $l_{out,turb}$ accounts for the difference in the transport of the angular momentum away from the impeller when the regular and fractal impellers are compared. The larger shaft torque of regular impeller, hence the larger source of angular momentum inside the CV, is balanced in this way with the larger transport over S_{out} .

The experiments of Steiros et. al³ demonstrated 11 – 12% higher torque for the regular impeller compared to the fractal impeller at $Re = 1 - 2 \times 10^5$. Therefore, it can be expected that $\langle u'_\theta u'_r \rangle$ grows further with Re , hence the contribution of $l_{out,turb}$ grows as well, until a fully turbulent regime is reached.

In addition to the radial range discussed above, where $l_{out,turb}$ plays an important role, the exact location can be determined in the flow field, where the value of $r \langle u'_\theta u'_r \rangle$ reaches its maximum level. This is illustrated in Figure 3.15 with the

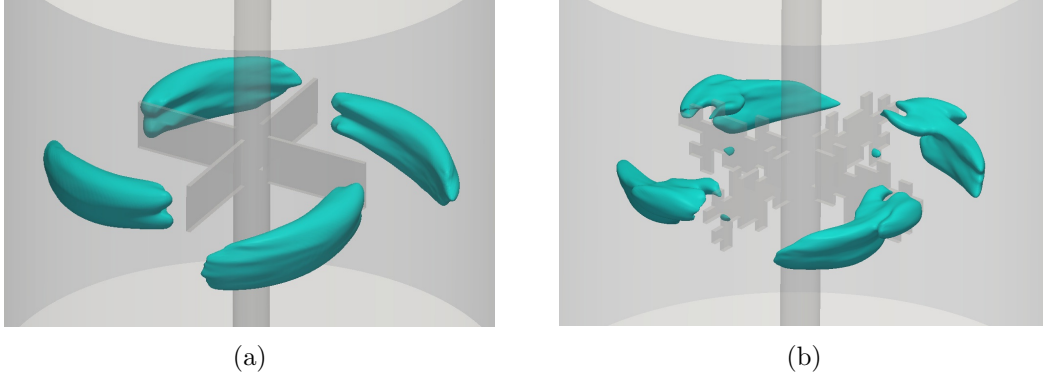


Figure 3.15: The region in the flow field where $r \langle u'_\theta u'_r \rangle / (RU_{tip}^2) > 2\%$ at $Re = 1600$ for (a) regular, (b) fractal impeller.

isosurfaces of $r \langle u'_\theta u'_r \rangle / (RU_{tip}^2) = 2\%$, at $Re = 1600$ for both impeller types. In the volume contained in these isosurfaces, the value of $r \langle u'_\theta u'_r \rangle / (RU_{tip}^2)$ reaches a maximum of 14% for the regular impeller and 6% for the fractal impeller. It is remarkable that the regions indicated by these isosurfaces coincide exactly with the location of trailing vortices in the wake of the blades. Therefore it is deduced that these coherent structures account for the turbulent transport of the angular momentum radially away from the impeller, quantified by $l_{out,turb}$.

The two large trailing vortices observed in the wake of the regular blades are replaced with multiple and weaker vortices in the wake of the fractal blade, as shown earlier in Figure 3.10. Still, the integral of turbulent kinetic energy, i.e. $0.5 \langle u'_i u'_i \rangle$, over the entire volume is equal for both impellers at $Re = 1600$, but it is rather distributed in the fractal impeller case. On the other hand, the correlation between the radial and azimuthal velocity fluctuations, hence $l_{out,turb}$, is lower around the fractal impeller, due to the break-down of trailing vortices.

It was previously shown that the larger recirculation region in the wake of the regular blade accounts for the lower pressure on the suction side, hence the higher form drag. With the help of Figure 3.9 it was also illustrated that this recirculation leads to the formation of the trailing vortices. Finally, it is shown that these coherent structures have a strong influence on the transport of angular momentum. Taking these into account, it can be concluded that the differences in the form drag, impeller torque and transport of the angular momentum between

the regular and fractal impellers are directly connected via the modification of the trailing vortices.

3.6 Energy dissipation characteristics

The time-average of the power draw of the impeller must be balanced by the time-average of the total energy dissipation over the entire tank. Indeed, the total dissipation in the tank is 8% lower when the fractal impeller is employed instead of the regular impeller (see Table 3.3). In the previous section, the mechanism leading to the differences in impeller torque and angular momentum transport has been analysed. In this section, it is untangled how, and in which part of the flow field, the dissipation of kinetic energy differs between the two impellers. The study focuses mainly on $Re = 1600$ and considers time-average quantities. The time-average aspect of this section's study is stressed because, as shown in the Appendix B, both the power and the total energy dissipation fluctuate in time and there is a cascade time-lag between them.

The dissipation of the total kinetic energy (ε_K) can be decomposed in two parts: $\varepsilon_K = \varepsilon_T + \varepsilon_M$, where $\varepsilon_T = 2\nu \langle s'_{ij} s'_{ij} \rangle$ and $\varepsilon_M = 2\nu S_{ij} S_{ij}$ are the turbulent and mean velocity dissipation, respectively ($s'_{ij} = \frac{1}{2} \left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right)$ and $S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)$ are strain rate tensors based on fluctuating velocity gradients and mean velocity gradients, respectively). At $Re = 320$, the share of ε_T in ε_K integrated over the tank is only around 32%, whereas this value rises to 61% at $Re = 1600$. Figure 3.16 illustrates radial profiles of ε_K and ε_T calculated by averaging over concentric cylindrical surfaces with varying radii, for both impeller types at $Re = 1600$. The values are normalized with $N^3 D^2$. The calculation is performed for the entire tank height (see Figure 3.16a) and for the middle one third of the tank height (see Figure 3.16b). The latter part of the tank ($-1/6 < z/H < 1/6$) is significant, because this is where ca. 66% of the total dissipation and 75% of the turbulent dissipation occurs (for $Re = 1600$). In Figure 3.16 the radial profiles are weighted with r/R to emphasize the increasing share of a cylindrical surface in the total volume as its radius increases.

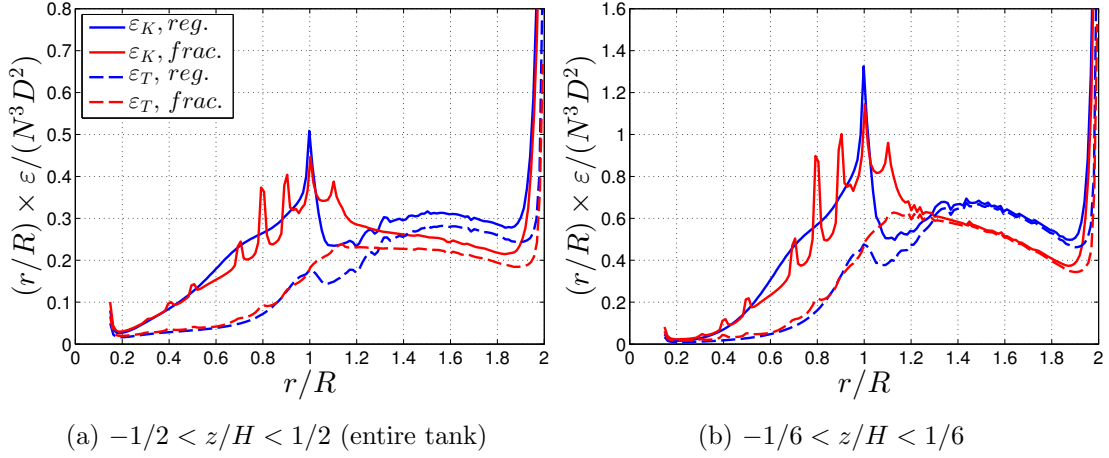


Figure 3.16: Radial profiles of dissipation obtained by averaging over concentric cylindrical surfaces with varying radii at $Re = 1600$ for both regular (reg.) and fractal (frac.) impellers, (a) for the entire height of tank, (b) only over the middle one third of the tank height. The averaged dissipation is normalized with $N^3 D^2$ and weighted with r/R .

It can be seen in Figure 3.16 that ε_K is dominated by ε_M between the shaft and the blade tip ($r/R \leq 1$). For the regular impeller, it increases monotonically until a sharp peak at $r/R = 1$ while it presents multiple spikes for the fractal impeller, one spike for every axial edge of the blade (refer to Figure 3.6b for the shape of the fractal blade). The turbulent part of the dissipation grows significantly in the radial range between $0.8 < r/R < 1.3$ where trailing vortices emerge. In the bulk of the flow, $1.3 < r/R < 1.9$, ε_T accounts for almost the entire dissipation. Especially in Figure 3.16b representing the middle one third of the tank height, the difference between ε_K and ε_T is imperceptible. This is also the part of the flow field where the blue curve (indicating the regular impeller) is consistently higher than the red curve (denoting the fractal impeller). The maximum values of the profiles of both ε_K and ε_T are observed at the tank wall at $r/R = 2$ (not visible in Figure 3.16), that is ca. twice as high as the peak value seen at $r/R = 1$. However, the high level of dissipation near the wall is confined within a volume of very small radial extent.

Figure 3.16 suggests that the 8% difference in the total dissipation between the two impeller types is mainly due to the turbulent dissipation observed in the radial range $1.3 < r/R < 1.9$. In order to further substantiate this conclusion, the profiles

of ε_K and ε_T are volume-integrated. Figure 3.17 shows at every radial position (r/R) the volume-integrated dissipation from 0 to this point for the entire height of the tank. The resulting value is normalized with $N^3 D^5$ so that it corresponds to the power numbers listed in Table 3.3, when the curves representing the integral of ε_K reach $r/R = 2$.

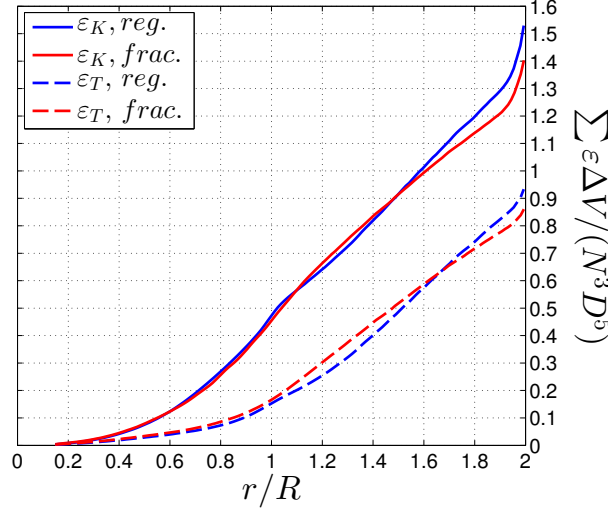


Figure 3.17: Cumulative integrals of ε_K and ε_T at $Re = 1600$ for both impeller types.

Figure 3.17 shows that the cumulative integrals of ε_K are equal for both impeller types up to ca. $r/R = 1.1$, i.e. the tip of the fractal blade. From this point until ca. $r/R = 1.9$ the straight and dashed lines continue almost parallel to each other, indicating that any increase in the integral of ε_K is mainly due to ε_T . The integral of turbulent dissipation until ca. $r/R = 1.3$ is higher for the fractal impeller case, which implies that the smaller trailing vortices in the wake of the fractal blade dissipate over a shorter distance from the impeller. From $r/R = 1.3$ to ca. $r/R = 1.9$, the blue dashed line has a higher slope than the red dashed line and this is where the surplus of dissipation in the regular impeller case is observed with respect to the fractal impeller case. Finally, the curves present a sharp increase in the near-wall region, but the difference between the blue and red curves does not change over this last part. It can be concluded that the two large coherent structures in the wake of the regular blade dissipate further from the impeller, in other words survive longer before decaying, and remove a higher amount of energy

during this process.

The above argument is made more clear when contour plots of the turbulent dissipation ε_T are examined in the vicinity of the trailing vortices. These are displayed in Figure 3.18 over a vertical cross-section of the entire domain at 30° downstream of the blades for $Re = 1600$. The highest values are observed in the region where trailing vortices emerge. The higher level of ε_T in the wake of the regular impeller is notable with the darker red colour when compared to the fractal impeller case; the latter has a rather dispersed distribution of ε_T . The intermediate values of ε_T between the wall and the vortex cores mark the wake of the succeeding blades. This quantity drops quickly at least two orders of magnitude as the fluid moves in the (positive or negative) axial direction further from the mid-height of the tank.

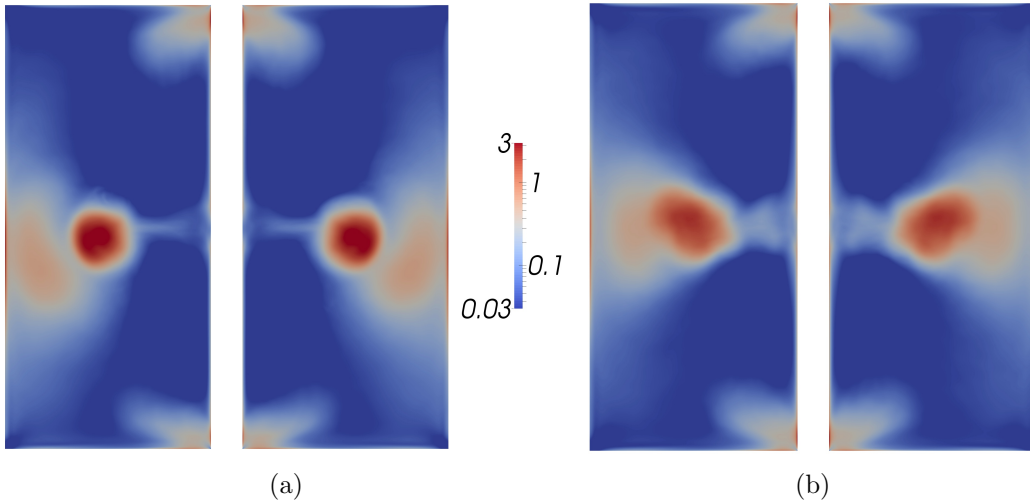


Figure 3.18: Contours of $\varepsilon_T/(N^3 D^2)$ at $Re = 1600$ at 30° downstream of the blade wakes for (a) regular, (b) fractal impeller.

When $\varepsilon_T(t) = 2\nu s'_{ij} s'_{ij}$ is analysed in the frequency domain, further differences are observed in the wakes of regular and fractal blades. By dropping the averaging operation, the $\varepsilon_T(t)$ can be considered as a function of time. In order to evaluate this quantity, all nine velocity gradients $(\frac{\partial u_i}{\partial x_j})$ are recorded at three probe points on-the-fly at every time step and used to compute the components of the s'_{ij} tensor. The locations of the aforementioned probes are illustrated in Figure 3.19 with cross symbols, along with the paths of the vortex cores at $Re = 1600$ for both impeller

types. In order to show the vortex path, vortex cores are determined based on the λ_2 -criterion⁹⁶ and the radial coordinate is averaged over their cross-section at various angles behind the blade. Also the schematic image of impeller blades is illustrated in the same figure. The probe points will be referred to as probe-1,2,3; the numbering follows the increasing x-location displayed in Figure 3.19, i.e. with the increasing downstream distance away from the blade. The locations of probe-1,2,3 in cylindrical coordinates are ca. $r/R = 1.1, 1.3, 1.8$ at $30^\circ, 45^\circ$ and 90° behind the blade, respectively. In order to obtain the power spectral density (PSD) of $\varepsilon_T(t)$, the nine components of the s'_{ij} tensor are normalized with N , their PSDs are computed separately and added in the frequency domain. These PSDs are illustrated in Figure 3.20 over a nondimensional frequency $f' = f/N$, for both impeller types and evaluated at the aforementioned probe points.

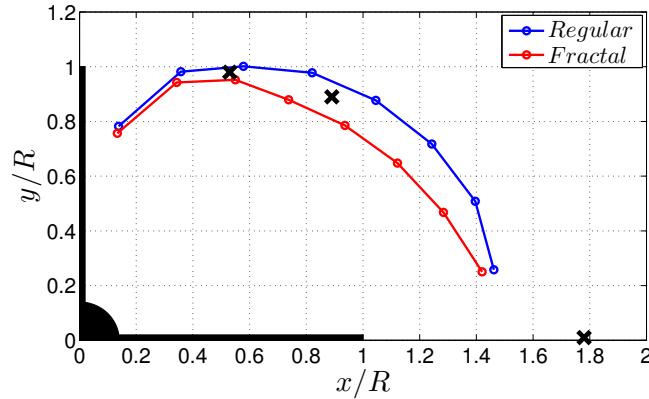


Figure 3.19: Trailing vortex paths for both impeller types at $Re = 1600$ are shown with the lines and the locations of probe points are marked with cross symbols: probe-1,2,3 where the numbering follows the increasing x-location. A sketch of the impeller blades improves the clarity of the figure.

When any one of the spectra in Figure 3.20 is integrated over the frequency, the result is proportional to the value of $\langle s'_{ij}s'_{ij} \rangle$ at this point. For instance, at probe-2 shown in Figure 3.20b, the values of ε_T of both cases are equal, hence the integrals of the blue and red curves. On the other hand, it is notable that the red curve is higher than the blue curve over frequencies in the range of ca. $3 < f' < 100$, whereas it is lower over the range of lower frequencies. This means that the flow field in the wake of a fractal blade has at this point (probe-2) a surplus of dissipation for the fluctuations at higher frequencies with respect to

the wake of a regular blade, which is balanced in the latter case with a surplus of dissipation for the fluctuations at lower frequencies. Moreover, at probe-3 the turbulent dissipation is ca. 40% higher for the regular impeller, mainly due to the difference at low frequencies as seen in Figure 3.20c. This point is in the part of the flow, where the radial profile of the dissipation illustrates a significant difference between the regular and fractal impeller cases, as shown earlier in Figure 3.16. In addition to the region in the flow field, which accounts for the surplus of dissipation in the regular impeller case, it is also demonstrated now that this surplus is mainly due to the difference of the fluctuations at low frequencies.

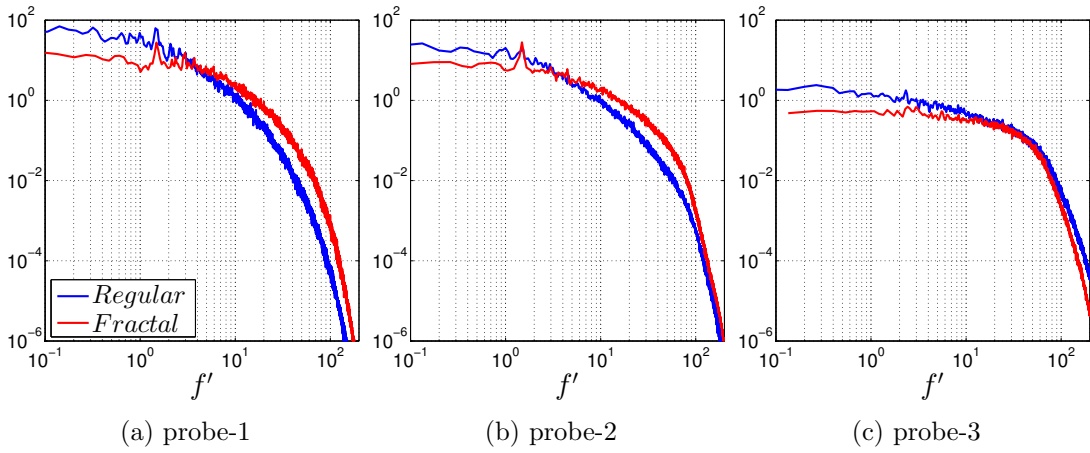


Figure 3.20: PSDs of $\varepsilon_T(t)$ at the three probe points marked in Figure 3.19 with cross symbols, for $Re = 1600$.

In order to illustrate the change in spectra along the vortex paths, Figure 3.21 compares the PSDs of $\varepsilon_T(t)$ at the three probe points, separately for both impeller types. As the fluid moves further from the blade, there is a substantial decay in the spectral density at low frequencies (for ca. $f' < 20$) in both cases. Meanwhile for the regular impeller, the spectral density increases for the part at high frequencies, i.e. $f' > 20$. This can be a shift of the dissipation from lower frequencies partially to higher frequencies. If it is assumed that the dissipation at higher frequencies is mainly due to vortical structures at smaller scales, it can be interpreted that this shift is due to the cascade of the energy from large to small scales. It might be the case that the energy reaches down to smallest scales only after a certain time and distance from the regular blade, due to the large initial size of vortices. Hence, at

probe-3 there is an increased dissipation for $f' > 20$, compared to probe-1 and 2, see Figure 3.21a. On the other hand, for the fractal impeller this part of spectra ($f' > 20$) does not change much along the vortex path, see Figure 3.21b, and is very similar to what is seen at probe-3 of the regular impeller. This part might have developed much earlier in terms of time and distance for the fractal impeller, due to the smaller length scale of vortices. Nevertheless, this interpretation requires the support of spectra in the scale-space before drawing firm conclusions.

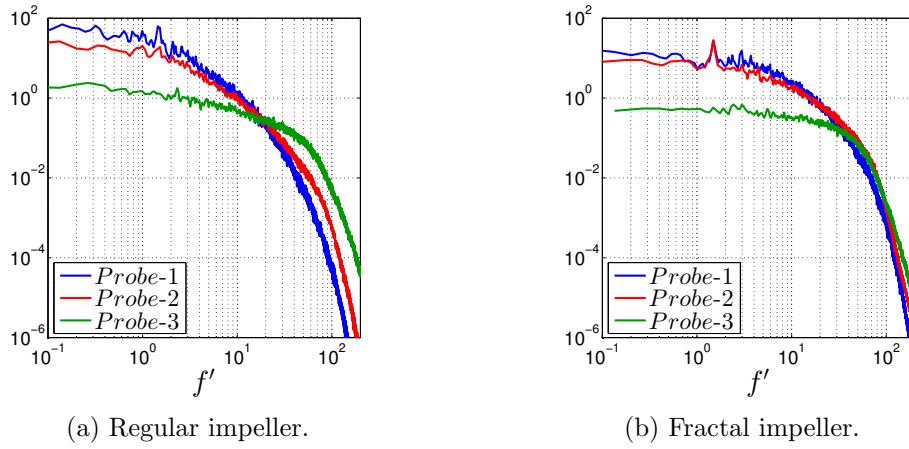


Figure 3.21: PSDs of $\varepsilon_T(t)$ at the three probe points compared separately for regular and fractal impeller.

Using the assumptions of the last paragraph, the difference at low frequencies seen in Figure 3.20c, which leads to 40% higher dissipation for the regular impeller at probe-3, can be linked to the larger size of its trailing vortices. This would be in accordance with the earlier observations, that the larger trailing vortices decay further from the impeller and dissipate higher energy.

In conclusion, the same mechanism leading to a higher drag coefficient of the blades and the higher transport of angular momentum, is also responsible for the dissipation of the higher power drawn by the regular impeller. The modification in the blade shape is able to significantly change the trailing vortex structures, hence alter the impeller power as well the dissipation in the flow field. This result is a promising example of how such intrinsic properties might be tuned by means of modifications on the blade shape, for instance to match specific process requirements.

3.7 Conclusions

First of all, the torque applied to the fluid by the impeller was found to be in balance with the torque applied by the tank walls, with an error less than 0.3% at $Re = 320$ and 1.5% at $Re = 1600$. The imbalance between the energy injection and dissipation was less than 3.1% at $Re = 320$ and 4.2% at $Re = 1600$. Values of $\langle N_p \rangle$ obtained with DNS are somewhat lower than the estimates given by the empirical correlation of Furukawa et al.⁶ (by 6% at $Re = 320$ and 12% at $Re = 1600$).

Previous experiments³ have shown that the fractal impeller has 11 – 12% reduced torque compared to the regular impeller at $Re = 1 - 2 \times 10^5$. A similar difference of 8% was also found in the DNS results for a lower Reynolds number ($Re = 1600$). In order to explain the origin of this difference, a detailed analysis is conducted of the flow inside the tank stirred with regular and fractal impellers using the DNS data.

It is detected that the drag coefficient of the fractal blade is distinctly lower than that of the regular blade at $Re = 1600$ (in agreement with experiments at higher Re ³), leading to the torque reduction. In order to analyse this outcome, the relation between the pressure distribution on the blades and the velocity field around the blades is investigated. It is demonstrated that profiles of V_θ upstream of both blade types are close to each other; the main difference is in the wake of the blades. The volume of the flow separation region is 7% smaller, and the maximum magnitude of V_θ towards the suction side is ca. 50% lower in the wake of the fractal blade compared to the wake of the regular blade. These differences emerge since the concave edges of the fractal blade allow the upcoming fluid to penetrate into the separation region. It is also shown that the recirculation pattern on the suction side is directly connected with the generation of trailing vortices. While there are two coherent structures behind a regular blade, the fractal blade has multiple and smaller trailing vortices.

Since any difference in the impeller torque is directly reflected in the transport of angular momentum from the impeller to the fluid, this quantity was also compared between the two blade types at $Re = 1600$, using a control volume (CV)

around the impeller. First, it is noted that the fractal impeller yields a 10% higher mass flow rate through the borders of the CV, despite having 8% lower $\langle N_p \rangle$. The net transport of angular momentum per unit flow rate is 18% lower for the fractal impeller. This aspect outweighs its 10% higher flow rate and explains the 8% lower transport of angular momentum with respect to the regular impeller. Elaborating further, the radial advective transport per unit flow rate out of the CV is decomposed into mean and turbulent transport, i.e. $l_{out,mean}$ based on $rU_\theta U_r$ and $l_{out,turb}$ based on $r \langle u'_\theta u'_r \rangle$, respectively. Although $l_{out,mean}$ has a larger contribution in the total l_{out} , the difference between the regular and fractal impellers is mainly due to the role of $l_{out,turb}$ which is ca. 30% higher for the regular impeller. Furthermore, the three dimensional isosurfaces of $r \langle u'_\theta u'_r \rangle$ coincide exactly with trailing vortices, showing that these structures are responsible for the turbulent transport. It can be inferred that the alteration of trailing vortices is the link between the reduction in the torque and the reduction in the transport of angular momentum.

As the power draw is 8% lower for the fractal impeller, so must be the integral dissipation as well. This quantity includes the contribution of mean velocity gradients (ε_M) and the turbulent dissipation (ε_T). It is noted that ε_T is concentrated in the vicinity of trailing vortices and it makes up 61% of the total dissipation of the entire tank, at $Re = 1600$. It is observed that the reduction of 8% in the total dissipation, when the fractal impeller is compared to the regular impeller, is mainly due to the difference in ε_T in the radial range of $1.3 < r/R < 1.9$. The frequency distribution of the dissipation was assessed at 3 probe points along the vortex paths. It is illustrated that the wake of the regular impeller contains higher dissipation compared to the wake of the fractal impeller due to fluctuations at low frequencies ($f' < 20$), which accounts for the difference in the bulk of the flow ($1.3 < r/R < 1.9$).

It is concluded that by means of modifications in the blade shape it is possible to influence crucially the trailing vortex structures. Consequently, the drag coefficient of the blades, the impeller torque and the energy dissipation characteristics can be altered, in order to tune these according to process requirements and optimise the

impeller design. In the case of fractal impeller, the reduction of $\langle N_p \rangle$, the increase in N_q and the equal level of integrated turbulent kinetic energy with respect to the regular impeller are promising findings for an improved process efficiency.

Chapter 4

Power fluctuations due to flow instabilities

4.1 Introduction

In the previous chapter, the mechanism was explained that is causing the torque differences between regular and fractal impellers in time average sense, and the link between the trailing vortices and the mean power consumption was established. The purpose of this chapter is to characterize the dynamical behaviour, i.e. the periodic features of the trailing vortex structures, which have an impact on the time dependence of the power consumption of a four-bladed radial impeller. The results related to the power draw fluctuations are presented and the link with the trailing vortex structures is discussed.

The chapter starts with the analysis of instabilities observed in the tank stirred with the regular four-bladed impeller at $Re = 320$. The discussion continues with the study of the cases with regular one-bladed and two-bladed impellers, in order to understand the role of the blade number and wake interaction during the creation of instabilities. Following this, it is assessed how these instabilities are affected when the regular impeller is replaced with the fractal impeller. Afterwards, the influence of the increase in Re from 320 to 1600 is investigated. Finally, the chapter is closed with a summary and main conclusions. (The content of the present chapter has been published in Bašbuğ et al.¹¹⁹)

4.2 Fluctuations in impeller power

To the best of author's knowledge, the fluctuations in the power consumption and their possible fluid mechanical origin have not been discussed in the literature for the present configuration. Labbé et al.¹²⁰ and Bramwell et al.¹²¹ studied experimentally the fluctuations of power injected into the turbulent flow confined in the cylindrical space between two coaxial counter-rotating disks with eight blades mounted vertically on the surfaces of the disks. Both papers mainly focused on the probability density function (PDF) of the power fluctuations and reported a non-Gaussian distribution skewed towards low values of power. They emphasized the good collapse of PDF of the reduced variable $(P(t) - \langle P \rangle)/P_{rms}$ for different Re . Labbé et al.¹²⁰ evaluated the PSD of power fluctuations, where they observed only the peaks related to the impeller rotation frequency and its harmonics, beside a flat region at lower frequencies. They also mentioned that power fluctuations are related to the dynamics of the large scale flow, but did not provide a clear explanation of this relation.

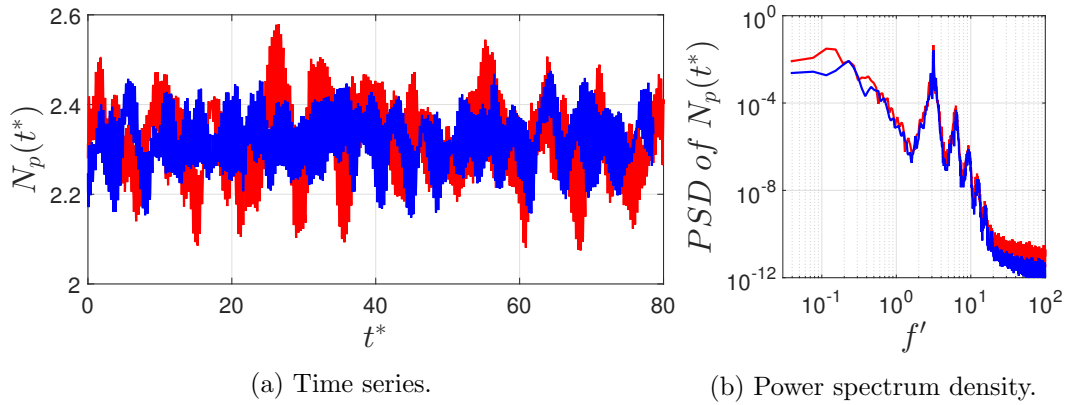


Figure 4.1: $N_p(t^*)$ of regular (blue line) and fractal impellers (red line) compared at $Re = 320$, over (a) time and (b) frequency domains.

The power fluctuations for the regular impeller are already shown in previous chapter in Figure 3.3 over a short period of time providing a close look at the signal. In Figure 4.1a, the variation of $N_p(t^*)$ for 80 revolutions is displayed, obtained at $Re = 320$ for both regular and fractal impellers. It is observed that the fractal impeller results in higher extrema of $N_p(t^*)$, despite $\langle N_p \rangle$ being equal

for both impellers at this Re (see Table 3.2). Consequently, the standard deviation of $N_p(t^*)$ normalized by its mean is equal to 3.7% in the fractal and 2.7% in the regular impeller case. On the other hand, the power spectra of both cases collapse very well for the frequencies in the range $2 < f' < 20$. Considering that the signal at the very high frequencies ($f' > 20$) contains very little energy, the main PSD difference appears to be at the very low frequencies. This suggests that the extrema observed in the $N_p(t^*)$ values of the fractal impeller are due to the low frequency fluctuations ($f' < 2$) where the largest difference appears at $f' < 0.2$. All observations regarding the low frequency fluctuations are summarized and discussed later in Section 4.7.

Furthermore, it is also evident in Figure 4.1b that the strong fluctuations appear at the frequency $f' \approx 3$ in both cases regardless of the different blade shapes. Below it is attempted to explain the mechanism leading to this peak. To this end, the regular four-bladed impeller at $Re = 320$ is considered as the baseline case. After a detailed analysis of this case, the other ones are discussed in order to highlight similarities and differences in the results.

4.3 Analysis of the regular four-bladed impeller

The fluctuations in power are mainly a reflection of the pressure fluctuations applied on the blade surfaces, since it was previously determined in Section 3.3 that the viscous forces on the blades have a very minor contribution to the impeller torque. These fluctuations are directly linked to the unsteady motion of the trailing vortices. But before their dynamics is analysed, first the shape and position of the time-averaged coherent structures are examined using the mean velocity field, at $Re = 320$.

As mentioned before, one way to illustrate the trailing vortices is to use the λ_2 -criterion introduced by Jeong and Hussain.⁹⁶ Escudié and Liné⁹⁵ simplified this method so that the procedure can be applied on the planar PIV measurements obtained on the radial-axial plane. They ignored the derivatives in the tangential direction as well as derivatives of the tangential velocity component (except $\partial v_\theta / \partial \theta$

which was estimated using conservation of mass). This simplification reduces the order of the polynomial equation from third to second, which must be solved in order to calculate the eigenvalues of $\mathbf{S}^2 + \Omega^2$ tensor and decreases the computational cost of this method significantly. Escudié and Liné⁹⁵ obtained the vortex size and location by means of this simplified approach and found the results to be in very good agreement with the original method introduced by Jeong and Hussain.⁹⁶ The simplified method is utilized to visualise the trailing vortices, as seen in Figure 4.2. Choosing lower values of λ_2 provides smaller surfaces closer to the vortex cores. Here the isosurfaces of $\lambda_2/\Omega^2 = -0.5$ are selected for a clearer illustration of trailing vortices rather than $\lambda_2 < 0$, where Ω is the impeller angular velocity.

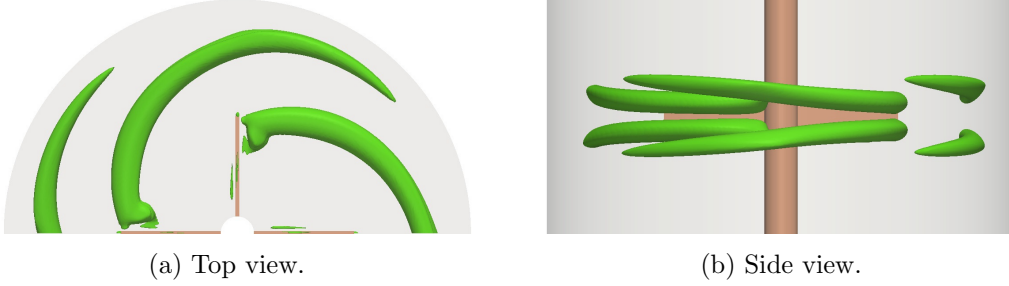


Figure 4.2: Trailing vortices illustrated by means of simplified λ_2 -criterion, where $\lambda_2/\Omega^2 = -0.5$, at $Re = 320$.

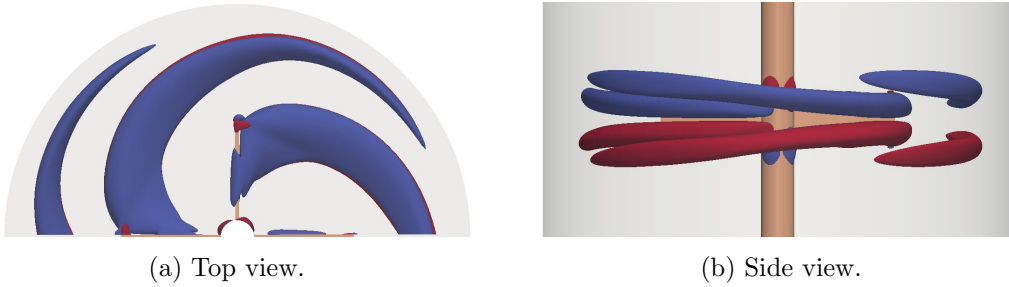


Figure 4.3: Trailing vortices illustrated with the isosurfaces of normalized tangential vorticity, $\xi_\theta/\Omega = 1.5$ (red) and $\xi_\theta/\Omega = -1.5$ (blue), at $Re = 320$.

In the literature, the mean path of the trailing vortices emerging from a six-bladed Rushton turbine has been illustrated in many studies and a comparison between the results of some of them was carried out by Escudié et al.⁹⁴ The structures observed around the four-bladed impeller are qualitatively in agreement with what was observed by these authors (see Figure 4.2). Trailing vortices are

formed right behind the edges of the blades, initially propagate normal to the blade and then become curved as they are carried with the flow that circulates around the impeller. The side view of the vortices in Figure 4.2b shows that the vortices are also displaced in the axial direction away from the mid-plane of the tank while being carried in the tangential direction. Since the radial jets are directed upwards or downwards after hitting the tank walls, the mean flow carries the upper and lower branches of trailing vortices along with their motion in the positive or negative axial direction, respectively. Moreover, the time-resolved data shows simultaneous up-and-down motion of the vortex pair as well as broken-up parts of isosurfaces impinging on the tank walls.

As an alternative, the trailing vortices are also illustrated by means of the isosurfaces of the tangential component of the vorticity displayed in Figure 4.3. This has delivered similar structures to those obtained with the simplified λ_2 -criterion. In addition to that, the positive or negative values of tangential vorticity indicate the counter-rotation of the vortex pair (positive tangential direction points towards counter-clockwise sense of rotation, for an observer from the top as in Figures 4.2a and 4.3a).

In order to understand the relationship between the fluctuations in the power number and the trailing vortices, the velocity signal was recorded at several probe points in the impeller discharge region. It was noticed that the observed peak frequency in the velocity fluctuations can vary with the probe location. Figure 4.4 shows the PSD of the axial velocity fluctuations and the PSD of the radial velocity fluctuations recorded at probe-2 that lies between a vortex pair at the mid-height plane, hence equidistant from both cores ($r = 1.3R$, $\theta = 45^\circ$ and $z = 0$, as shown in previous chapter in Figure 3.19). The strongest peak found in the PSD of the radial velocity fluctuations at this point is at $f' = 3.16$ that is exactly the same peak frequency seen in the PSD of the power number fluctuations. On the other hand, the PSD of the axial velocity fluctuations has a peak frequency at $f' = 1.58$ that is exactly one half of the aforementioned value. Spectra obtained at probe-4 are shown in Figure 4.5. This point is slightly above probe-2 so that it is placed in the

core of the upper trailing vortex ($r = 1.3R$, $\theta = 45^\circ$ and $z = 0.02H$). At this point, both axial and radial velocity components oscillate with a dominant frequency of $f' = 1.58$. Moreover, the magnitude of the PSD of the radial component is almost an order of magnitude higher in the vortex core, i.e. in probe-4 with respect to probe-2, as seen in Figure 4.5b.

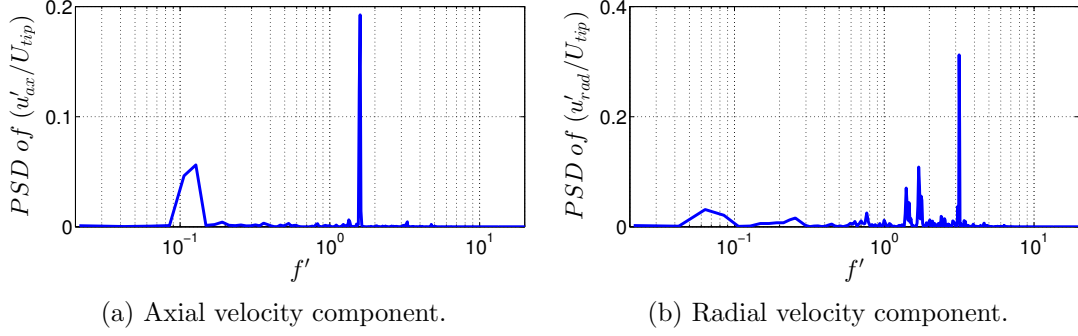


Figure 4.4: PSD of velocity fluctuations at the probe-2 located at $r = 1.3R$, $\theta = 45^\circ$ and $z = 0$, as shown in Figure 3.19.

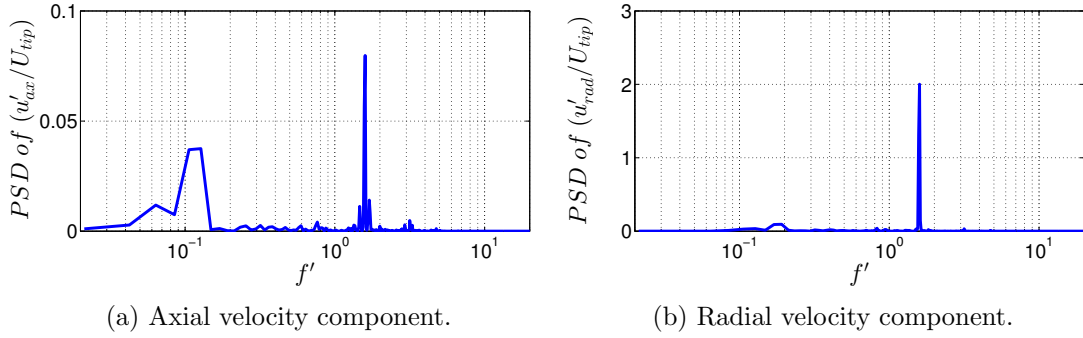


Figure 4.5: PSD of velocity fluctuations at the probe-4 located at $r = 1.3R$, $\theta = 45^\circ$ and $z = 0.02H$ (slightly above the probe-2, inside the upper vortex core).

In fact, what is captured at probe-4 is the frequency at which the two cores of the vortex pair grow and shrink in an alternating manner. The axial velocity component fluctuates with the same frequency as the unsteadiness in both probes i.e. at $f' = 1.58$. This is similar to the fluctuating lift force on an object due to the vortex shedding in its wake. The radial velocity component at the mid-height is effected by both vortices equally. Therefore this component oscillates with double the instability frequency, like the drag force on an object with vortex shedding. Note that the flow field does not contain the periodic fluctuations due

to the blade passage that was termed pseudo-turbulence,⁸⁹ since the probes have a fixed position with respect to the impeller.

The snapshots shown in Figure 4.6 cover half of the cycle of the unsteadiness at frequency $f' = 1.58$. In other words, Figures 17a,c,e,g correspond to time instants $t = 0, T^*/6, T^*/3, T^*/2$, respectively, with steps of $\Delta t = T^*/6$, with T^* being the period of the vortex unsteadiness. Figures 4.6a,c,e,g provide information about the generation and evolution of the trailing vortices. The streamlines passing by the upper and lower edges of the blade (shown with blue and red lines, respectively) initially turn towards the suction side of the blade and then bend radially towards the blade tip, due to the radial jet along the suction side of the blade (shown with green lines). Near the blade corners, the blue and red streamlines start rolling around the trailing vortex cores, which the radial jet joins as well, after passing between the vortex pair. It is observed that the upper and the lower vortex cores grow and attenuate in an alternating manner (shown with yellow surfaces) while the radial jet swings up and down.

The data analysis also demonstrates that the pressure on the upper or lower half of the suction side decreases while the radial jet is twisted upwards or downwards, respectively. In other words, the radial jet is directed towards the region on the suction side where the pressure is the lowest as illustrated in Figures 4.6b,d,f,h. This dynamic behaviour of the low pressure region explains the connection between the cycle of vortex unsteadiness and fluctuations in the torque applied on the blade.

So far the discussion has provided some understanding of the phenomena occurring in the wake of one blade. However, the total power consumption is determined by all blades and can also be affected by the phase relation of the instabilities in the wakes of different blades. An instantaneous picture of the coherent structures of two consecutive blades is shown in Figure 4.7 suggesting that the cycle of unsteadiness with the frequency $f' = 1.58$ occurs in every blade with a phase shift of 180° with respect to the preceding (or succeeding) blade. However, the drag force on blades appears with double the frequency ($f' = 3.16$), which results in torque fluctuations at $f' = 3.16$. These are in the same phase for all blades, which

is analysed in the following paragraphs.

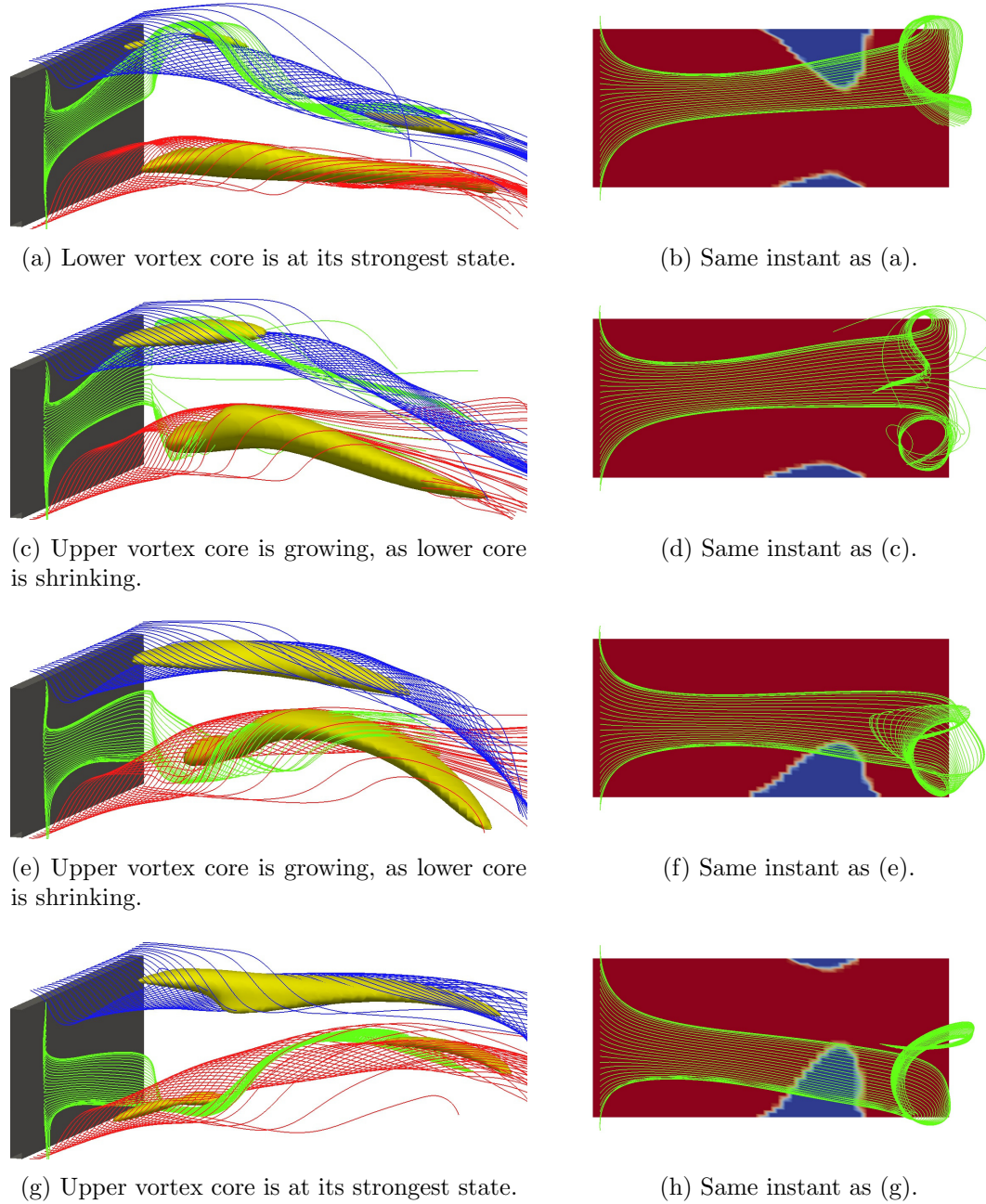


Figure 4.6: (a),(c),(e),(g) Isosurfaces of normalized tangential vorticity of $|\xi_\theta/\Omega| = 10$ (yellow surfaces) used to illustrate vortex cores. The blue, red and green lines in the images display the streamlines obtained by putting seed points along the upper, lower and inner edges of the blade, respectively. The blade is shown with grey colour. (b),(d),(f),(h) Suction side of the blade is coloured according to the pressure distribution. Red and blue regions illustrate where the pressure is above and below a certain threshold, respectively.

To evaluate the phase shift between the torque fluctuations on different blades,

the torque caused by each one of the four blades were recorded. The blades are referred to with increasing numbers in clockwise direction such that blade 2 encounters the wake of blade 1 etc. Figure 4.8a displays the PSD of the torque signal of all blades evaluated separately. As expected, all blades yield identical PSD spectra with a peak at $f' = 3.16$ due to the above discussed phenomenon and another peak at a much lower frequency around $f' = 0.22$. Nevertheless, the strongest peak of the PSD appears at $f' = 1.70$ which was not observed in the PSD of the power draw. As seen in Figure 4.8b, when the torques applied on each one of the four blades are added together, this peak disappears. This suggests the existence of a different instability frequency that is not reflected in the power fluctuations, since it cancels out as the signals from different blades are superimposed.

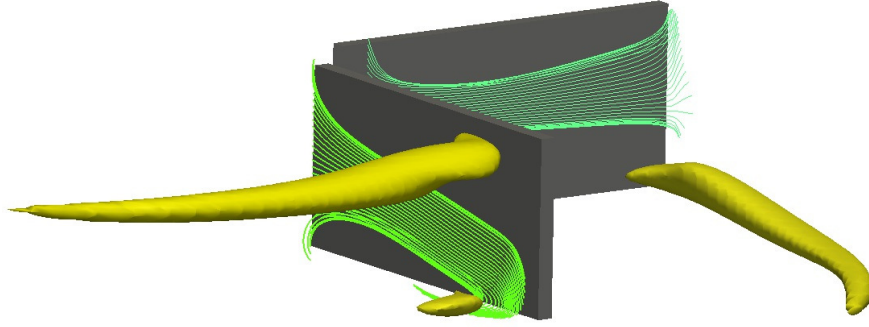


Figure 4.7: Streamlines and isosurfaces of normalized tangential vorticity are shown as in Figure 4.6 for two blades with 90° angle. Note that the impeller has four blades, but the shaft and the two other blades are removed from the figure for clarity of visualisation.

To further investigate this aspect, the phase relation between the blades 1 and 2 is analysed with the help of Figure 4.8c. The blue line in Figure 4.8c shows the PSD of the sum of the torque signals applied on blades 1 and 2. It exhibits the same characteristics of the sum of all blades. On the other hand, the red line represents the PSD of the difference of the torques applied on blades 1 and 2 and demonstrates only the peak at the frequency $f' = 1.70$. This points at a phase difference of 180° between the torque fluctuations on blades 1 and 2 at the frequency $f' = 1.70$ and explains why they cancel each other out.

Furthermore, the phase relation between the blades 1 and 3 is examined in the

same way. Figure 4.8d is the same as Figure 4.8c but with torque data from blade 3 instead of blade 2. When the torque data from blades 1 and 3 are added, this amplifies the signals at all frequencies. Moreover, they cancel almost completely when subtracted (if the weaker peaks are ignored). This means the signal component at $f' = 1.70$ on blade 3 is shifted a further 180° with respect to blade 2. If the impeller had only the blades 1 and 3, the unsteadiness at $f' = 1.70$ would dominate the fluctuations of the power consumption since it has a much higher amplitude than frequency $f' = 3.16$.

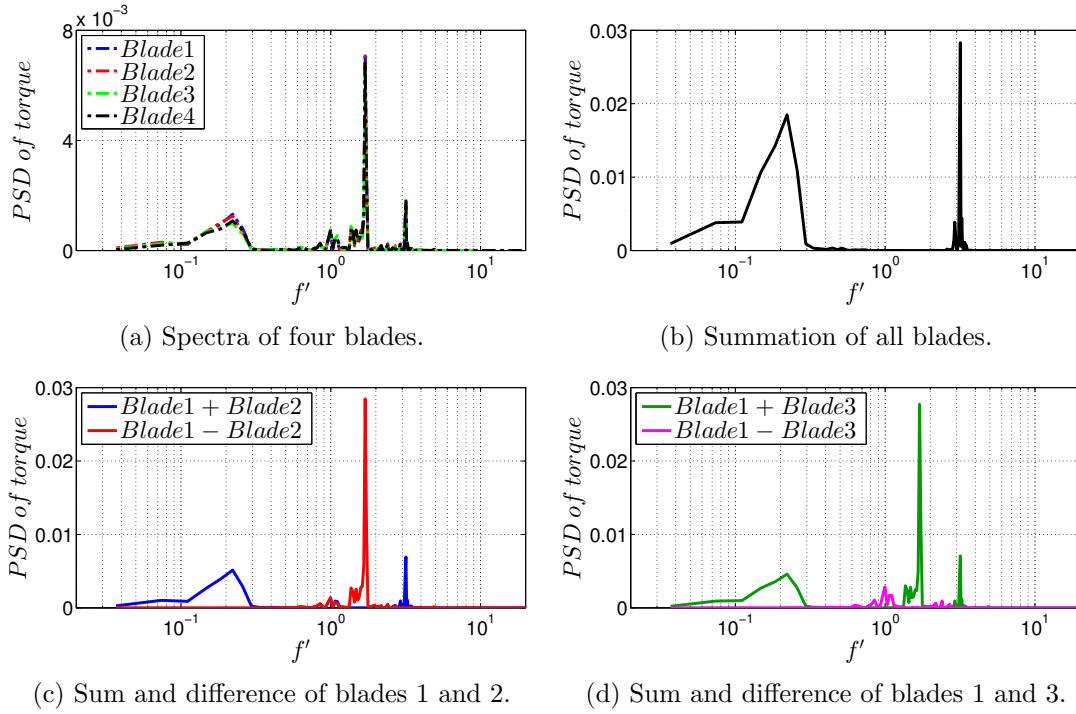


Figure 4.8: PSDs of the torque fluctuations applied on the regular blades.

The peak at the frequency $f' = 1.70$ is also present in the PSD of the radial velocity fluctuations acquired at probe-2 on the mid-height plane, previously shown in Figure 4.4b. This is the second dominant peak in the PSD after $f' = 3.16$. However, the other probe point located in the vortex core does not capture any fluctuation at the frequency $f' = 1.70$. On the other hand, none of the power spectra of the axial velocity fluctuations acquired at these probe points have this peak frequency. These findings imply that there might be a different phenomenon (besides the unsteadiness of the trailing vortex pair), which mainly occurs on

the radial-tangential plane around the mid-height leading to an instability at the frequency $f' = 1.70$. This can be linked to the axial vortices in the wake of the blades. Previously, isosurfaces of tangential vorticity were used to illustrate the trailing vortex pair. On the other hand, the isosurfaces of axial vorticity have a quite different shape, as shown in Figure 4.9. Using the mean velocity field, their time-averaged structures are displayed in Figure 4.9a indicating an elongated part of the isosurfaces behind the blade tip on the mid-height plane. When the instantaneous structures are observed for several revolutions, it has been detected that this extended part of the isosurfaces grow and detach with the frequency $f' = 1.70$. Moreover, this occurs in the wake of blades 1 and 3 (or 2 and 4) simultaneously, which agrees with the phase relations of the blades explained above. This can be seen in Figures 4.9b and 4.9c. These results suggest that the torque fluctuations at $f' = 1.70$ can stem from the instabilities of the axial vortices generated on the blade tips. However, the frequency of this mechanism is not reflected in the power draw fluctuations of the four-bladed impeller, which is the main focus this study. Therefore, a more elaborate analysis of this mechanism is not pursued further at this stage.

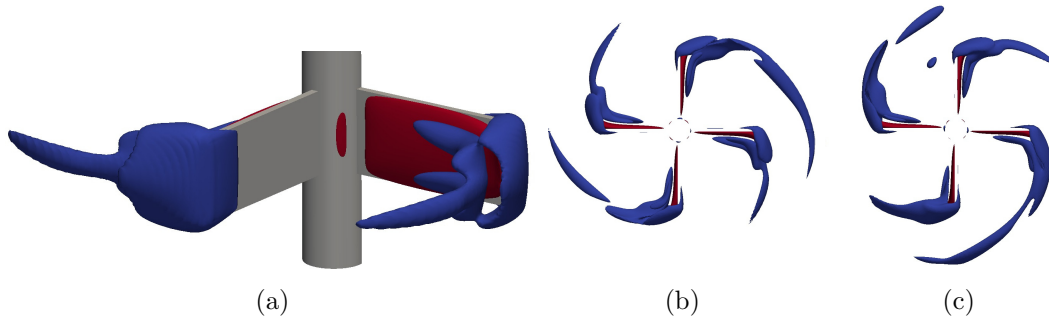


Figure 4.9: Isosurfaces of normalized axial vorticity; $\xi_z/\Omega = 3.5$ shown with red surfaces and $\xi_z/\Omega = -3.5$ with blue surfaces. a) Time-averaged structures, b) and c) instantaneous structures. The impeller is displayed only in the figure on the left side with the grey colour.

4.4 Analysis of the regular one-bladed impeller

In the case of a regular four-bladed impeller, two types of instability mechanisms were observed, which are associated with the periodic deformation of the coherent vortical structures. Moreover, a specific phase relation was detected between the wakes of consecutive blades. This may stem from an interaction between the structures emanating from different blades. In order to examine whether the generation of instabilities depends on such an interaction, additional simulations were conducted using a regular one-bladed impeller. This investigation is conducted using only the regular blade, but not the fractal one. This is because the fluctuations in power input appear at the same frequency for both impeller types and it is assumed that the same instability mechanism is responsible for this phenomenon in both cases.

The blue curve in Figure 4.10a displays the power number as a function of the number of revolutions for a one-bladed impeller at $Re = 320$. The initial period with large values of $N_p(t^*)$ is a common phenomenon observed with every type of impeller and is due to the starting flow conditions.¹²² After this jump, $N_p(t^*)$ decreases and exhibits a plateau. It might have been expected that $N_p(t^*)$ fluctuates in time as observed in the four-bladed case. However, this one-bladed case at $Re = 320$ returns a steady laminar flow field around the impeller. Trailing vortex structures emerge again in the wake of the blade but remain static. After approximately 70 revolutions, randomly distributed spatial fluctuations were artificially superimposed to the velocity field. Even though the magnitude of these artificial fluctuations were up to 15% of the local velocity magnitude, they did not trigger a self-sustaining instability. The absence of any unsteadiness in the one-bladed $Re = 320$ case strikingly contrasts the strong fluctuations present in four-bladed $Re = 320$ case seen in Figure 4.1 and implies that the wake interactions have a decisive role in the development of the unsteady behaviour of the coherent structures.

The same simulation was also performed at an increased Reynolds number, i.e. at $Re = 1600$. As shown by the red curve in Figure 4.10a, the signal of the power

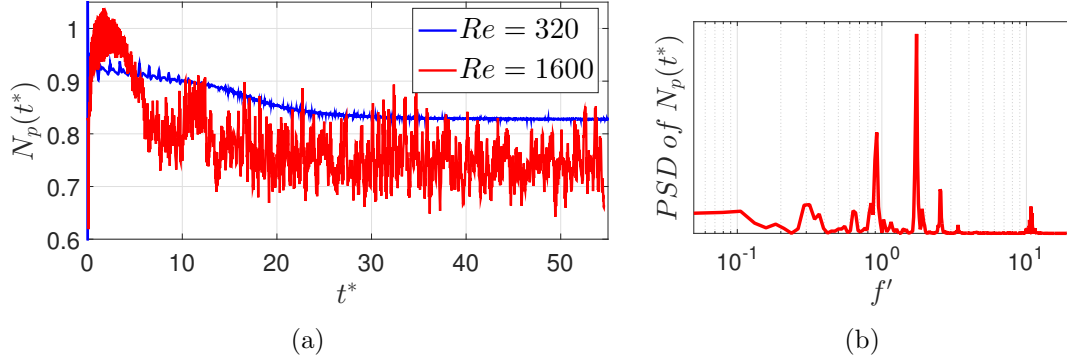


Figure 4.10: $N_p(t^*)$ of the one-bladed impeller plotted against the number of impeller revolutions (a), and its PSD for $Re = 1600$ (b).

number exhibits strong fluctuations at this Reynolds number. The PSD of power number shows that the dominant peak appears at $f' = 1.74$ (see Figure 4.10b). This might be a consequence of the same phenomenon at $f' = 1.70$ linked to the axial vorticity, which was discussed in the context of the four-bladed impeller in Section 4.3 and shown with Figure 4.9. The slight increase in the frequency (from $f' = 1.70$ to $f' = 1.74$) can be attributed to the slightly increased relative velocity that the blade tip is exposed to because of the absence of the other blades.

On the other hand, up-and-down swinging motions of the radial jet do *not* arise in this case, neither the frequency associated with this event (i.e. $f' = 1.58$). Figure 4.11 illustrates an instantaneous snapshot of the streamlines in the wake of the blade. Note that the radial jet (shown with green lines) is divided into two streams, and these are rolled around the upper and lower vortices. This is unlike the behaviour observed in the four-bladed case where the radial jet swings between upper and lower vortices and rolls around them in an alternating manner, as demonstrated in Figure 4.6. The lack of this type of unsteadiness cannot be attributed to the increased Reynolds number since this is observed in the case of four-bladed impeller also at $Re = 1600$, and is discussed later in Section 4.8. This result supports the conclusion that the interaction between the disturbances generated by multiple blades is essential for the onset of the unsteadiness observed in the four-bladed case at $f' = 1.58$.

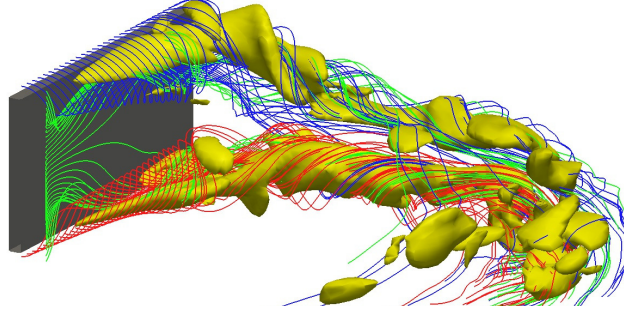


Figure 4.11: Yellow surfaces indicate the instantaneous isosurfaces of normalized tangential vorticity of $|\xi_\theta/\Omega| = 15$ in the wake of the one-bladed impeller at $Re = 1600$. The blade and instantaneous streamlines are as in Figure 4.6.

4.5 Analysis of the regular two-bladed impeller

At $Re = 320$, the flow around the regular one-bladed impeller is steady without fluctuations. In the tank stirred with a regular two-bladed impeller (with 180° between the blades), the instabilities emerge but grow gradually with a much longer transition time when compared to the four-bladed case. Indeed the axial velocity component recorded at probe-2 illustrates this slow growth. As seen in Figure 4.12, the axial velocity fluctuations grow after about 60 revolutions and carry on increasing for another 30 revolutions or so.

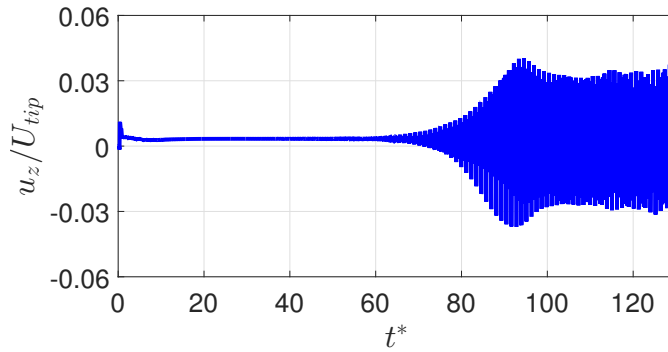


Figure 4.12: Instantaneous axial velocity normalized with blade tip speed and plotted against the number of impeller revolutions, acquired at probe-2 ($r = 1.3R$, $\theta = 45^\circ$ and $z = 0$).

Consequently, the power number undergoes a similar transition. In the period between 20 and 80 revolutions, $N_p(t^*)$ fluctuates only with the frequency $f' = 1.74$ with a weak amplitude. However, this amplitude increases gradually during the mentioned period, as shown in Figure 4.13a. After the axial velocity fluctuations

have been amplified after about 90 revolutions, the power number undergoes a transition, as shown in Figure 4.13b. Shortly after 90 revolutions, additional frequencies are superimposed on the fluctuations of $N_p(t^*)$.

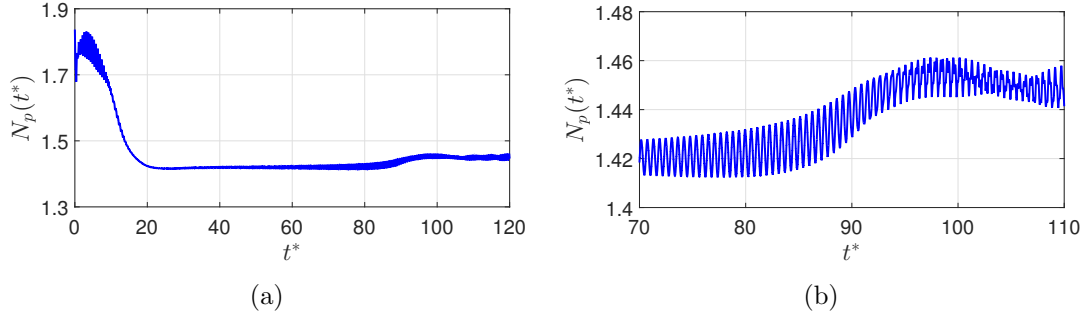


Figure 4.13: $N_p(t^*)$ plotted against the number of impeller revolutions (a), closer look at the transition (b), for the regular two-bladed impeller case.

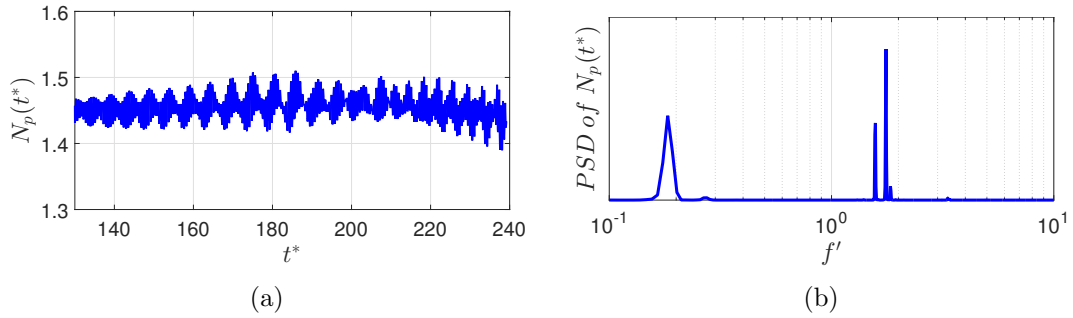


Figure 4.14: $N_p(t^*)$ plotted against the number of impeller revolutions after the transition (a), PSD of $N_p(t^*)$ using the signal after 130 revolutions (b), for the regular two-bladed impeller case.

The trend of the power number is illustrated separately in Figure 4.14a for the time after the advent of additional frequencies. It is observed that the low frequency modulation of $N_p(t^*)$ starts in an organized periodic manner and slowly becomes rather chaotic as the simulation proceeds, as it is also seen in $N_p(t^*)$ of four-bladed cases. The PSD of the signal, which is obtained using only the part after 130 revolutions, is shown in Figure 4.14b. The highest peak seen here is at $f' = 1.74$, the same frequency already detected before the transition. The low frequency fluctuations are at $f' = 0.18$ and presumably emerge due to the macro-instabilities. The third peak seen in the PSD is at $f' = 1.58$. It is observed that this peak is caused by the same phenomenon which lead to the fluctuations of $N_p(t^*)$ at

$f' = 3.16$ in the four-bladed impeller case. This was associated with the alternating growth of trailing vortex pair and up-and-down swinging motion of the radial jet. This event occurs in the two-bladed impeller case only with half the frequency (i.e. $f' = 0.79$) when compared to the four-bladed case, which was detected with the help of the PSDs of the velocity components acquired in the trailing vortices. Consequently, its impact on $N_p(t^*)$ of the two-bladed impeller is at $f' = 1.58$. This suggests that the number of blades has a direct influence on the frequency of this event. Moreover, it initiates after ca. 100 revolutions simultaneously with the low frequency instabilities, suggesting that this phenomenon might be triggered by the onset of macro-instabilities. On the other hand, the unsteadiness linked to the axial vorticity appears at around $f' = 1.70$ regardless of the number of impeller blades.

4.6 Analysis of the fractal impeller

At $Re = 320$, the cases with the regular and fractal impellers display several common features such as equal power numbers (in time-average sense) and the common peak frequency of power fluctuations, despite different blade geometries. The mechanism leading to the fluctuations in power is already explained in the context of the regular impeller, which is associated with the dynamical behaviour of the trailing vortex pair in the wake of the blades. Also an unsteadiness linked to the axial vorticity was observed, which has a footprint on the torque signal applied to the individual blades (with $f' = 1.70$) but not on the total power draw, since this signal is cancelled out due to the phase difference between the blades.

When the same analysis was conducted in the context of the fractal impeller, it revealed several differences in the torque applied on the blades with respect to the regular impeller case. The sum of the torque applied on the fractal blades displays fluctuations with a sharp peak frequency at $f' = 3.14$ and a second peak at $f' = 0.12$, as indicated in Figure 4.15b.

The high frequency peak in the power draw fluctuations is $f' = 3.16$ in the regular impeller case, and $f' = 3.14$ in the fractal impeller case. The difference of

$\Delta f' = 0.02$ is of the order of magnitude of the frequency resolution of the PSD, hence it is negligible. The mechanism leading to this peak frequency is discussed above in the context of regular impeller. This phenomenon was also observed with the same nondimensional frequency in the fractal impeller case regardless of the fact that the multiple edges of the fractal blades generate a higher number of smaller vortices when compared to the regular impeller. This can be seen in Figure 4.16a where the isosurfaces of tangential vorticity are illustrated using a relatively high value of vorticity which makes it possible to detect the vortex cores. Isosurfaces of lower vorticity levels are shown in Figures 4.16b and 4.16c. These are suggesting that the multiple trailing vortices tend to interact and merge downstream of the blade.

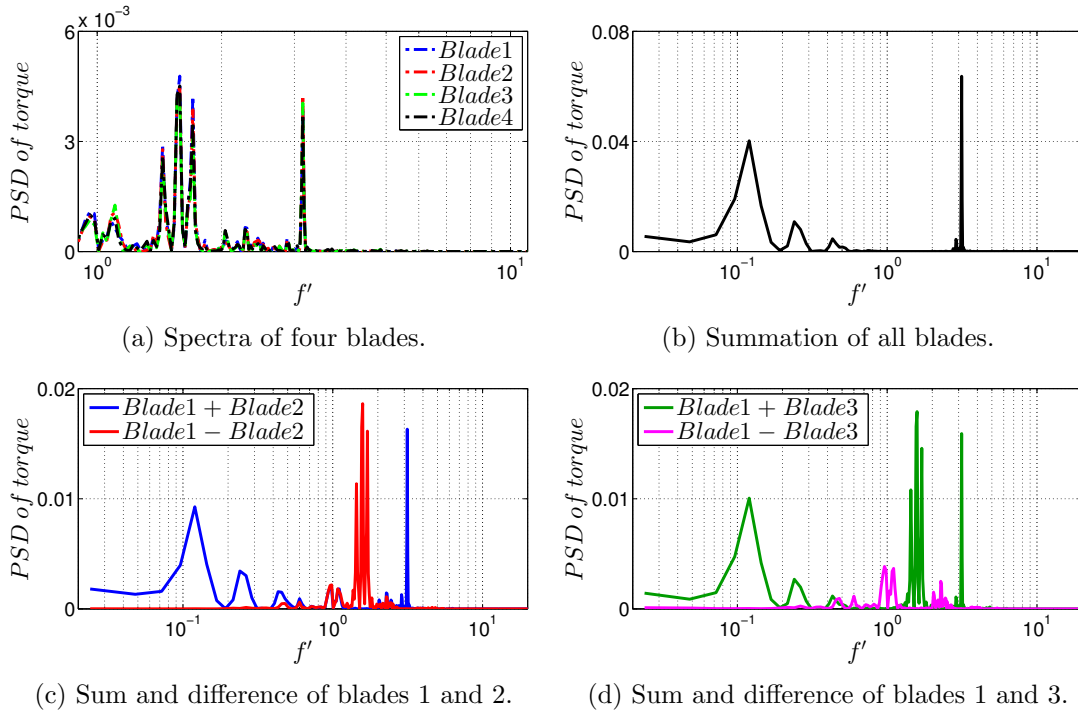


Figure 4.15: PSDs of the torque fluctuations applied on the fractal blades.

Figure 4.15a exhibits the PSD of the torque signal of four fractal blades evaluated separately, focusing on the frequency range $1 < f' < 10$ for clarity of illustration. Besides the peak at $f' = 3.14$, there are three other peaks with comparable energy levels at $f' = 1.44$, $f' = 1.57$ and $f' = 1.70$. The peak at $f' = 1.57$ stems from the same event which gives rise to the peak at $f' = 3.14$. Unlike regular

blades, fractal blades are not symmetrical with respect to the mid-height plane. Consequently, the upper trailing vortex grows larger than the lower one, as seen in Figure 4.16. Moreover, the upper half of the blade has a larger radial extent than the lower half. Possibly due to these factors the unsteadiness with the frequency $f' = 1.57$ has a stronger impact on the torque generated on the upper half of the blade compared to the lower half. Therefore the PSD of the torque applied to a blade has peaks at both frequencies $f' = 1.57$ and $f' = 3.14$.

Furthermore, it is seen that adding the torque applied on two consecutive blades cancels out the signal with $f' = 1.57$, as shown in Figure 4.15c with the blue line (considering that blade 1 and blade 2 are any two blades with 90° angle between them). This stems from the phase difference between the cycles of both blades. When the low pressure region on the suction side of blade 1 is shifted towards the upper half of the blade, on blade 2 it is shifted towards the lower half. After a half cycle, the described roles of blade 1 and blade 2 are exchanged. When these two instants are compared, the addition of the torque applied on these two blades is equal. In other words, both half cycles apply equally strong torque on the impeller. Therefore, the PSD of the sum of blade 1 and blade 2 presents only the peak at $f' = 3.14$ and no peak at $f' = 1.57$.

The other peak at $f' = 1.70$ was already noted and discussed during the analysis of the regular impeller case. The peak at $f' = 1.44$ is also associated with the same phenomenon as the one at $f' = 1.70$, namely linked to the instability of the axial vortices emanating from the blade tip. As shown in Figure 4.17, upper and lower corners of the blade have two separate structures of axial vorticity. Possibly due to different length scales, these vortices lead to fluctuations in torque with two different frequencies. These are in anti-phase when blade 1 and blade 2 are compared and cancel out after the torque applied to these two blades are added, as seen in Figure 4.15c. However, when any two blades with 180° are considered, as in Figure 4.15d, it is detected that the signals with frequencies $f' = 1.44$ and $f' = 1.70$ are in phase. This phase relation is also observed in the regular impeller case for the unsteadiness caused by the axial vorticity.

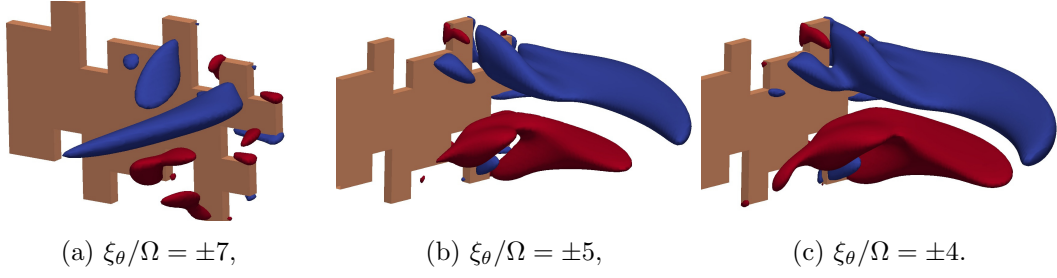


Figure 4.16: Isosurfaces of tangential vorticity based on the time-averaged flow field (positive and negative values of vorticity are displayed with red and blue colours, respectively).

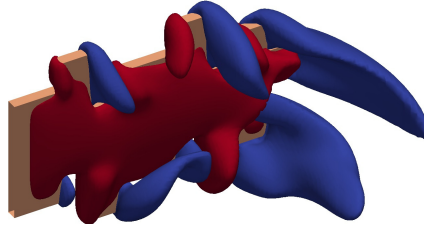


Figure 4.17: Isosurfaces of axial vorticity equal to $\xi_z/\Omega = \pm 3.5$ based on the time-averaged flow field (positive and negative values of vorticity are displayed with red and blue colours, respectively).

4.7 Notes about low frequency fluctuations

The PSD of $N_p(t^*)$ of the fractal impeller contains a peak at $f' \approx 0.12$ and in the regular impeller case a peak is detected at $f' \approx 0.24$ (see Figure 4.1b). The low frequency modulation of $N_p(t^*)$ of the regular impeller seen in Figure 3.3 has a period of ca. four revolutions, possibly related to the aforementioned peak at $f' \approx 0.24$. The same peaks are also observed in the PSDs of the torque applied on regular and fractal blades, shown in Figures 4.8 and 4.15, respectively. Moreover, the PSDs of axial velocity component acquired in probe points in the discharge region of both regular and fractal impellers show a peak at $f' \approx 0.12$ (shown in Figures 4.4a and 4.5a for the regular impeller case). These low frequency fluctuations may be related to macro-instabilities (MI), discussed at length by many authors.^{123–126} Galetti et al.¹²⁷ reported that MIs are observed at $f' = 0.106$ for Re values in the range $400 < Re < 6300$, based on experiments conducted in a baffled tank stirred with a Rushton turbine with $D/T = 1/3$. This frequency agrees with the results of further studies concerning pitched-blade turbines^{128,129} and Rushton

turbines^{130, 131} conducted in the aforementioned range of Re . Although the tank and impeller geometries studied in the present study are different than the cited papers, the observed low frequency peak in the PSD of axial velocity at $f' \approx 0.12$ is close to the MI frequencies reported in the literature. Note that in the regular impeller case the power input has a peak at $f' \approx 0.24$, whereas the axial velocity at $f' \approx 0.12$, which might be explained as follows: MIs in the upper and lower halves of the tank might be effecting the impeller with exactly the same magnitude because of the symmetry with respect to the mid-height plane, making an impact on the impeller torque with double the frequency of MIs, i.e. $f' \approx 0.24$. On the other hand, the fractal impeller does not have the same symmetry property, hence $f' \approx 0.12$ is directly reflected in the torque and power fluctuations. The detailed investigation of low frequency fluctuations are left out of the scope of this work. This would require very long time series in terms of number of revolutions to be able to draw a decisive conclusion, and that would be very expensive to obtain using DNS.

4.8 Effect of higher Reynolds number on four-bladed impellers

It was already determined that the upper and lower trailing vortices grow in an alternating manner while the radial jet swings up and down at a frequency of $f' = 1.57 - 1.58$, for the four-bladed cases at $Re = 320$. The results obtained at $Re = 1600$ using both regular and fractal four-bladed impellers illustrated that the same type of periodic motion of the trailing vortex pair is maintained for the higher Reynolds number. Moreover, this phenomenon occurs at a very similar frequency to what was observed at $Re = 320$. To be precise, the frequency is shifted slightly downwards to $f' = 1.50$ for the cases at $Re = 1600$. Steiros et al.³ utilized exactly the same two impeller shapes as considered in the present study and conducted time-resolved pressure measurements using pressure transducers fixed on the blade surfaces at $Re = 1 - 2 \times 10^5$. They reported that the spectral analysis of the pressure signal has a peak around $f' = 1.4$ with a high magnitude at

blade corners and argued that trailing vortices account for this peak. Considering the decreasing trend of the f' -value with Re , the simulation and experimental results are consistent.

Figures 4.18a and 4.18b illustrate the PSDs of the velocity fluctuations in radial and tangential directions, respectively, acquired at probe-4 located inside the upper trailing vortex in the wake of a fractal blade ($r = 1.3R$, $\theta = 45^\circ$ and $z = 0.02H$). Especially the radial component has a pronounced peak at both Reynolds numbers at the frequencies mentioned above. In both figures the green curve, which represents $Re = 320$, has a more prominent peak compared to the blue curve standing for $Re = 1600$, with an energy density more than twice as high as the case at $Re = 1600$. Although this difference is not apparent due to the log-scaled ordinate of the figure, the log-log plot is preferred in order to illustrate the distribution of energy content over the higher frequencies.

It is clear that at $Re = 320$ the magnitude of PSD decreases steeply at higher frequencies and only the large scale structures, namely the trailing vortices are energetic. When the Reynolds number is raised, the energy density at higher frequencies is increased, i.e. the energy of smaller scales. For instance, at $f' = 15$, which is one order of magnitude greater than the peak frequency, the energy density is ca. four orders of magnitude higher at $Re = 1600$ when compared with the case at $Re = 320$.

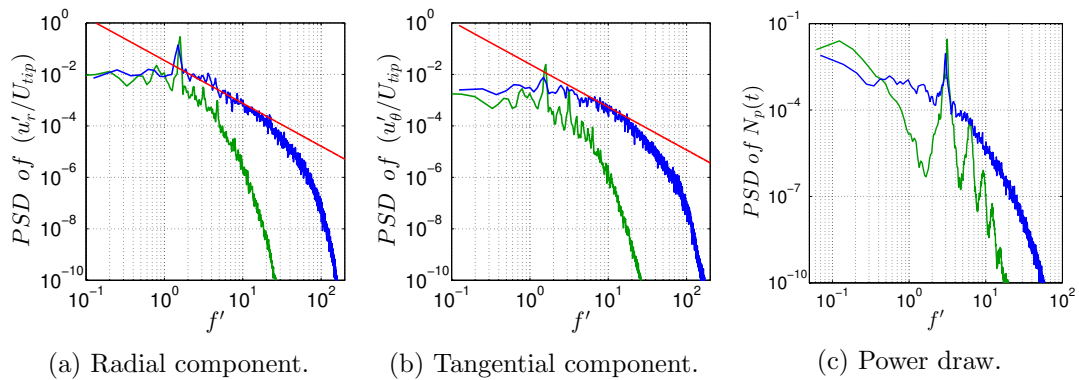


Figure 4.18: a) and b) PSDs of velocity fluctuations at probe-4 inside the upper trailing vortex ($r = 1.3R$, $\theta = 45^\circ$ and $z = 0.02H$), c) PSD of fluctuations of power draw, for the fractal impeller case at $Re = 320$ (green line) and $Re = 1600$ (blue line). The red line shows the slope of $-5/3$ for comparison.

Moreover, the energy spectra of velocity fluctuations follow a slope of $-5/3$ in a small extent of frequencies, which was predicted by Kolmogorov¹³² for homogeneous and isotropic turbulence. This was also observed in the context of stirred vessels by other authors: Güntel and Weber¹³³ studied the flow in a baffled vessel agitated by a Rushton turbine and captured the turbulent fluctuations in the impeller discharge region for the range of $Re = 1 - 3 \times 10^4$. Their work demonstrated energy spectra with a slope of $-5/3$ over about 1.5 decades of the frequency space. Lee and Yianneskis⁹² also reported the same observation for a similar configuration at $Re = 4 \times 10^4$, despite the lack of isotropy and homogeneity in the studied cases. Taking these into account, the presented results imply that the flow in the wake of the blade may become locally turbulent already at $Re = 1600$.

If the part of the energy spectrum with a slope of $-5/3$ is considered as the cascade of turbulence energy towards smaller scales, this cascade starts around the frequency which is associated with the unsteadiness of trailing vortices. It was already emphasized that the trailing vortices account for the turbulence kinetic energy inside stirred vessels.^{84,94,97} However, the process of turbulence production is not completely explained. Our findings provide insights about how the velocity fluctuations are introduced by the trailing vortices and can help to explain the mechanism of turbulence production inside stirred vessels.

Figure 4.18c compares the PSDs of the power draw fluctuations of the fractal impeller at both Reynolds numbers. It can be noted that the same peak frequency is kept at the increased Reynolds number beside the slight shift from $f' = 3.14$ to $f' = 3.0$ which is exactly twice the frequency of the trailing vortex pair unsteadiness for the reasons explained earlier for the lower Re case. This demonstrates that the direct connection between the power consumption and the motion of trailing vortices is not broken at the increased Reynolds number while the impeller discharge flow shows indications of local turbulence. All the aspects discussed in this section are common for the regular and fractal impeller cases. This implies that the results are not dependent on the specific blade shape and may be observed even when different types of impellers are employed.

4.9 Conclusions

Strong fluctuations were detected in the power draw of four-bladed impellers, at the frequencies $f' = 3.16$ and $f' = 3.14$, for regular and fractal impellers, respectively, for the cases at $Re = 320$. It was determined that these fluctuations are linked to an unsteadiness in the wake of the blades which involves an up-and-down swinging motion of the radial jet while the two cores of the trailing vortex pair grow and decay in an alternating manner. It was also noted that the frequency of the power fluctuations is equal to the double of the frequency of the described periodic event. This is because both of the semi-cycles of the unsteadiness have a similar impact on the torque applied to the blade. Moreover, the fluctuations in torque applied to the separate blades are in the same phase, hence they are superimposed linearly.

Even though the regular and fractal blades have quite different shapes, the above described unsteadiness occurs with almost the same frequency in both cases, implying that it may appear in cases of other impeller geometries, as well. The frequency of power fluctuations was shifted to $f' = 3.0$ for both cases at $Re = 1600$. Considering that $\langle N_p \rangle$ of the regular impeller decreases from 2.3 to 1.6 when Re increases from $Re = 320$ to $Re = 1600$, it can be concluded that the frequency of power fluctuations is not affected from the change in Re as much as $\langle N_p \rangle$.

It was also observed that the blade number has a strong influence on the above described unsteadiness. The flow field around the one-bladed impeller remained steady at $Re = 320$ with a constant power draw over time. Even though it become unsteady at $Re = 1600$, the aforementioned phenomenon did not arise. In the two-bladed impeller case at $Re = 320$, this motion initiated ca. after 100 revolutions, but this time the unsteadiness frequency was exactly the half of the frequency seen in the four-bladed case. Moreover, its advent occurred simultaneously with the advent of low frequency fluctuations in power draw, possibly due to the so-called macro-instabilities. This implies that macro-instabilities have the potential to trigger an unsteadiness affecting the radial jet and trailing vortices, at a higher frequency than their own.

Chapter 5

Reduced mixing time by means of fractal impellers

5.1 Introduction

The reduction in the energy consumption achieved with the fractal impeller does not assure the improvement of the process efficiency. In order to fully assess the mixing efficiency of a fractal with respect to a regular impeller, the mixing times must be compared as well. To this end, a passive scalar is injected in the flow field and its evolution is computed. The mixing performance is assessed using the time required to homogenize the distribution of the passive scalar. Moreover, the factors are analysed which affect the mixing time and lead to differences when regular and fractal impellers are compared. In addition to that, the impact of the increasing Re is assessed while staying in the transitional regime.

This chapter is organized as follows: First, the scalar transport equation is introduced, the details of the scalar injection and the methods used for the assessment of the scalar field are presented. DNS results of the mixing time with regular and fractal blades are illustrated. Afterwards, the impacts of the impeller type and Re on the mixing time are discussed. A simple model is proposed that can estimate the decay rate of scalar fluctuations. Finally, the chapter is closed with a summary and main conclusions. (The content of the present chapter is under review for publication in Physical Review Fluids.)

5.2 Scalar transport equation and injection details

The convection-diffusion equation for the evolution of a passive scalar in incompressible form reads:

$$\frac{\partial c}{\partial t} + v_i \frac{\partial c}{\partial x_i} = \Gamma \frac{\partial}{\partial x_i} \left(\frac{\partial c}{\partial x_i} \right), \quad (5.1)$$

where the passive scalar is denoted by c and the diffusion coefficient by Γ . The equation has the same form for both inertial and rotational reference frames. Using the length scale D (the impeller diameter), the velocity scale ND and the time scale $1/N$ (i.e. the period of one impeller revolution), Equation 5.1 can be nondimensionalized and takes the following form:

$$\frac{\partial c}{\partial t^*} + v_i^* \frac{\partial c}{\partial x_i^*} = \frac{1}{Pe} \frac{\partial}{\partial x_i^*} \left(\frac{\partial c}{\partial x_i^*} \right), \quad (5.2)$$

where $v_i^* = v_i/ND$, $t^* = tN$, $x_i^* = x_i/D$, $Pe = Re \times Sc$. For all simulations, Sc was taken equal to 1, hence the smallest scales of the flow and scalar fields are equal. Equation 5.1 was discretized using the finite volume method (FVM) and solved after the flow field was computed at every time step. A second-order central differencing scheme was used for spatial discretization and an implicit second-order Euler method for time-marching. A zero normal-gradient boundary condition was imposed at the walls for the scalar field.

After the flow field has had time to fully develop and the impeller power has converged to a constant value, the passive scalar was injected on the suction side of only one of the impeller blades using a constant flux boundary condition. The injection period was equal to a quarter of an impeller revolution for all simulations (see Figure 5.1). Since the regular and fractal blades have the same frontal area, the total amount of scalar injected was the same. This method of injection does not introduce any momentum to the flow field and allows for the total amount of scalar to be precisely controlled.

The suction side of the blade was selected for the injection because the strong radial jet along this side of the blade advects the scalar away from the source and the up-and-down swinging motion of the radial jet (identified in Chapter 4) can

redistribute the passive scalar in the axial direction. Moreover, the wake of the blade is the region with the highest velocity fluctuations. This region includes the trailing vortices which may be utilized to accelerate mixing. It was already emphasized in previous studies^{130,134,135} that mixing is enhanced when the injection is located in the region with strongest instabilities.

In order to estimate the simulation time and memory requirements for computing the mixing time and to evaluate the computational quality of the solution, the homogenization of an injected passive scalar was simulated once for each case (for both impeller types at $Re = 320$ and 1600). These initial computations are referred to as "the pilot runs", used only for the aforementioned tests and not for the conclusions regarding the mixing time.

The homogenization in a stirred vessel is a random process and is affected by the instantaneous flow field present at the time of injection. In order to take this influence into account for the mixing time evaluation, five different scalars are injected with a time lag equal to two impeller revolutions between the starting points of two consecutive injections. Although all five scalars were present in the same flow field instantaneously, they were solved separately and did not affect each other.

5.3 Evaluation of scalar field

The volume averaged passive scalar is calculated as:

$$\bar{c} = \frac{\int c dV}{V_{tot}}, \quad (5.3)$$

where V_{tot} denotes the total volume of the vessel. Note that c is a function of space and time, $c = c(x_i, t)$, whereas $\bar{c}$ is a function of time only during the injection (after the end of injection it becomes constant). In all simulations, the final value of $\bar{c}$ was equal to $\bar{c}_\infty = 0.04$ after injection stopped. Figure 5.1 shows that $\bar{c}$ increases linearly during the injection period and remains precisely constant afterwards. Figure 5.1 is independent of the impeller type and Re . The conservation of the

total amount of scalar was not ensured when an immersed boundary method was used to mimic the solid surfaces in the LES study of Hartmann et al.⁷⁶ In the aforementioned study, even after the injection stopped, the total scalar content continued to rise with a constant rate, which became steeper with growing impeller size. The authors related this issue to a problem with the scalar conservation on the impeller surface. This problem is avoided in the present study because the FVM conserves scalar by construction.

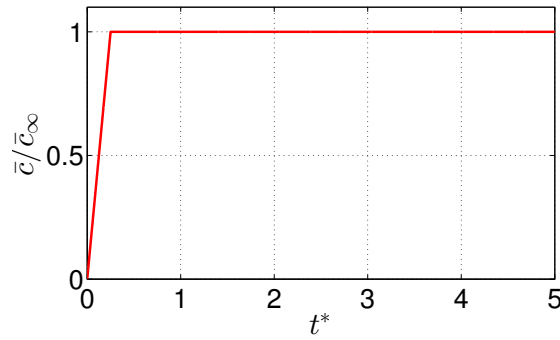


Figure 5.1: Injection and conservation of passive scalar ($\bar{c}_\infty$ denotes the final value of $\bar{c}$ in the tank).

The scalar can be decomposed into its volume average, $\bar{c}$ (that is also the long time limit value at every point after complete mixing), and the fluctuations around $\bar{c}$ as follows: $c = \bar{c} + c'$. The degree of mixing can be evaluated using the coefficient of variation (CoV) that is defined as the standard deviation of concentration measurements divided by the mean concentration.³² This parameter was also referred to as coefficient of mixing,⁷⁶ or intensity of segregation.⁵¹ It was commonly evaluated using a number of monitoring points or measurement locations.^{32,51,76} In the present study, the main objective is to compare the mixing performance of the regular and fractal impellers as objectively and comprehensively as possible without reference to particular locations in the flow. Therefore, the CoV is computed using the information in the entire tank as follows:

$$CoV = \frac{1}{\bar{c}} \sqrt{\frac{\int (c')^2 dV}{V_{tot}}}, \quad (5.4)$$

When the value of CoV drops down to 5%, the scalar can be considered well

mixed in a typical industrial application.³² This degree of mixing means that the standard deviation of the spatial distribution is equal to 5% of the mean value. In the present study, the nondimensional time required to reach this degree of mixing will be referred to as t_{mix} . This value will be employed to make a comparison between different cases. Another approach to evaluate the mixing time utilizes the ratio of the mixed volume to the tank volume, as done by Yeoh et al.⁷⁵ The so-called mixed volume is defined as the region where the scalar is around $\bar{c}$ within a selected discrepancy, e.g. $\pm 5\%$. Consequently, the time required to reach the state where $0.95 < c/\bar{c} < 1.05$ over 95% of the tank volume, was referred to as t_{95} .

There is a relation between the two measures of mixing. Assuming a normal distribution around the mean value, 68% of the data is expected to lie within ± 1 standard deviation. For the scalar field this means that $c/\bar{c} = 1 \pm CoV$ for 68% of the tank. Similarly, 95% of the data lies within ± 2 standard deviations, i.e. the equation $c/\bar{c} = 1 \pm 2 \times CoV$ applies to 95% of the tank volume, at any instant for any value of CoV .

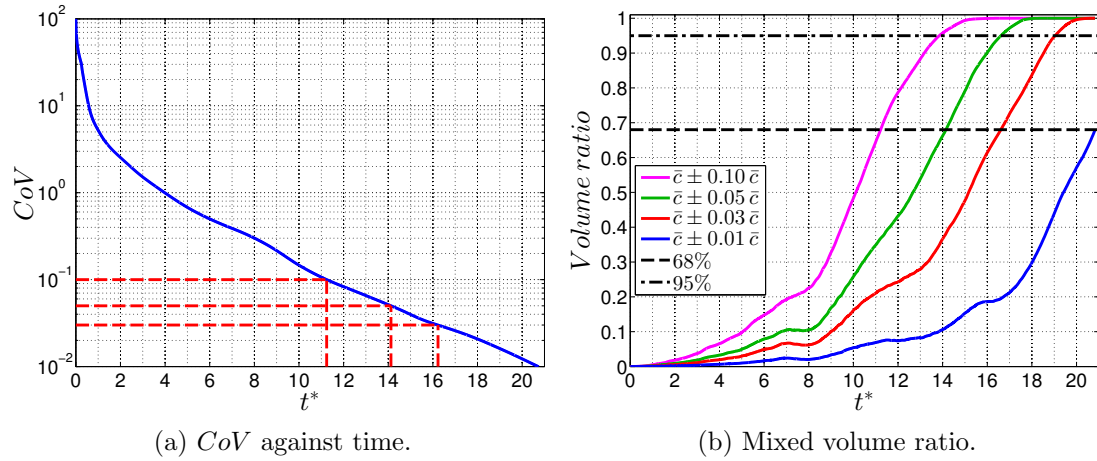


Figure 5.2: Mixing times based on two different criteria.

Figure 5.2a shows the trend of CoV against time for the pilot run with the fractal impeller at $Re = 320$. The value of CoV drops to 0.1, 0.05, 0.03, 0.01 at times 11.2, 14.1, 16.2, 20.7, respectively, as shown with red dashed lines in Figure 5.2a (except for the last value). The four curves in Figure 5.2b illustrate the ratio of the volume where c is around $\bar{c}$ to within $\pm 10, 5, 3$ and 1% (calculated

with the same scalar field used for Figure 5.2a). These curves reach 68% at the instant when CoV drops to the value of the selected discrepancy. For instance, when $CoV = 0.05$ (i.e. $t^* = 14.1$), the green curve in Figure 5.2b ($\bar{c} \pm 0.05\bar{c}$) reaches 68%, and the magenta curve ($\bar{c} \pm 0.10\bar{c}$) arrives at 95%. Moreover, when $CoV = 0.01$ (i.e. $t^* = 20.7$), the blue curve ($\bar{c} \pm 0.01\bar{c}$) reaches 68%, and the red curve ($\bar{c} \pm 0.03\bar{c}$) arrives at almost 100%, since almost the entire volume is within ± 3 standard deviations. In brief, it is observed that the data of CoV provides sufficient information to predict the trend of all four curves displayed in Figure 5.2b. Therefore, the mixing time will be evaluated using the trend of CoV in the remainder of this study.

In order to assess the quality of the numerical resolution, the conservation equation of scalar variance, $(c')^2$, is utilized. When Equation 5.2 is multiplied with c' , and averaged over the volume, and assuming that the passive scalar injection has already stopped, the following equation is obtained:

$$-\frac{1}{2} \frac{d}{dt^*}(\overline{c'^2}) = \frac{1}{Pe} \overline{\frac{\partial c'}{\partial x_i^*} \frac{\partial c'}{\partial x_i^*}}, \quad (5.5)$$

where the overbar denotes the volume averaging operation. The right-hand side of this equation denotes the dissipation rate of the scalar variance averaged over the entire tank, and quantifies the rate of homogenization dictated by scalar gradients. This term is recorded at every time step on-the-fly, along with the value of $\overline{c'^2}$. The left-hand side of Equation 5.5 is computed in the post-processing stage, using a second-order central differencing scheme for the time derivative. If the resolution is not fine enough to resolve the smallest scales of the scalar field, there will be an imbalance in Equation 5.5 with the left-hand side taking a larger value than the right-hand side due to the numerical dissipation. This can affect the prediction of t_{mix} as determined by Derksen⁸⁵ and mentioned earlier.

The discrepancy between the two sides of Equation 5.5 is computed for both impeller types and Re . This discrepancy is ca. 1% at $Re = 320$ and ca. 4% at $Re = 1600$ (in average over time). Both sides of Equation 5.5 are plotted against t^* in Figure 5.3 for the pilot run with the fractal impeller at $Re = 1600$. Since

these terms decay exponentially, a discrepancy of 4% does not lead to a discernible difference in terms of their time evolution. Similarly, an error of 4% in CoV will not affect the prediction of t_{mix} . These results and the conservation of the total amount of passive scalar (shown earlier in Figure 5.1) ensure the computational quality of the simulations.

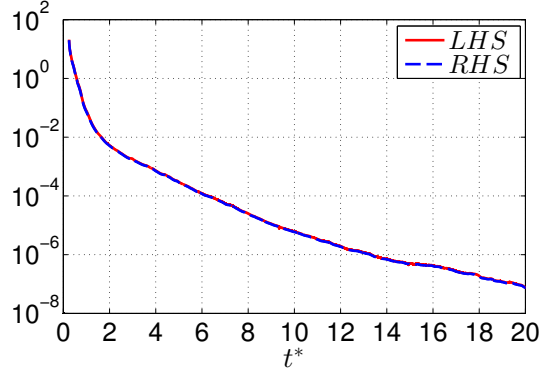


Figure 5.3: The two sides of Equation 5.5 for the fractal impeller at $Re = 1600$.

5.4 Mixing times

In the present study, the mixing time (t_{mix}) is defined as the time that the homogenization process takes until CoV drops to 5%. Figures 5.4a-d demonstrate the CoV -curves of the five injections, for both impellers at $Re = 320$ and 1600. Although the five passive scalars are injected with a time lag of two impeller revolutions, for clarity of comparison, the time is reset to $t^* = 0$ at the beginning of each injection.

When CoV is determined using a limited number of probe or measurement points, the homogenization curves exhibit strong oscillations^{69,76} due to the fluctuations at the probe points. In the present cases, all curves in Figure 5.4 are monotonically decreasing with a smooth trend. This is a consequence of the volume integration over the entire tank whereby locally extreme values are averaged out.

Figures 5.4a-d demonstrate that the mixing time of the five injections are spread between ca. 14.5 and 20.5 revolutions for the regular impeller, and between ca. 13.5 and 18 revolutions for the fractal impeller, taking results of both Re into

account. The mixing time cannot be precisely predicted and varies between different realisations depending on the instant of the injection. This variation may stem from the interactions between the injected passive scalar and the large scale instabilities observed in the flow field discussed in Sections 4.3, 4.6, 4.7 and 4.8. Such an interaction may have an impact on each injection at a different magnitude due to the transient nature of the injection and the homogenization, hence leading to a variation of mixing times of the separate injections. Moreover, the findings suggest that the spread of mixing times is relatively narrower for the fractal compared to the regular impeller. The difference between the maximum and minimum t_{mix} is ca. 6 revolutions for the regular and 4.5 for the fractal impeller. This result indicates an improvement in the predictability of t_{mix} when the fractal impeller is used.

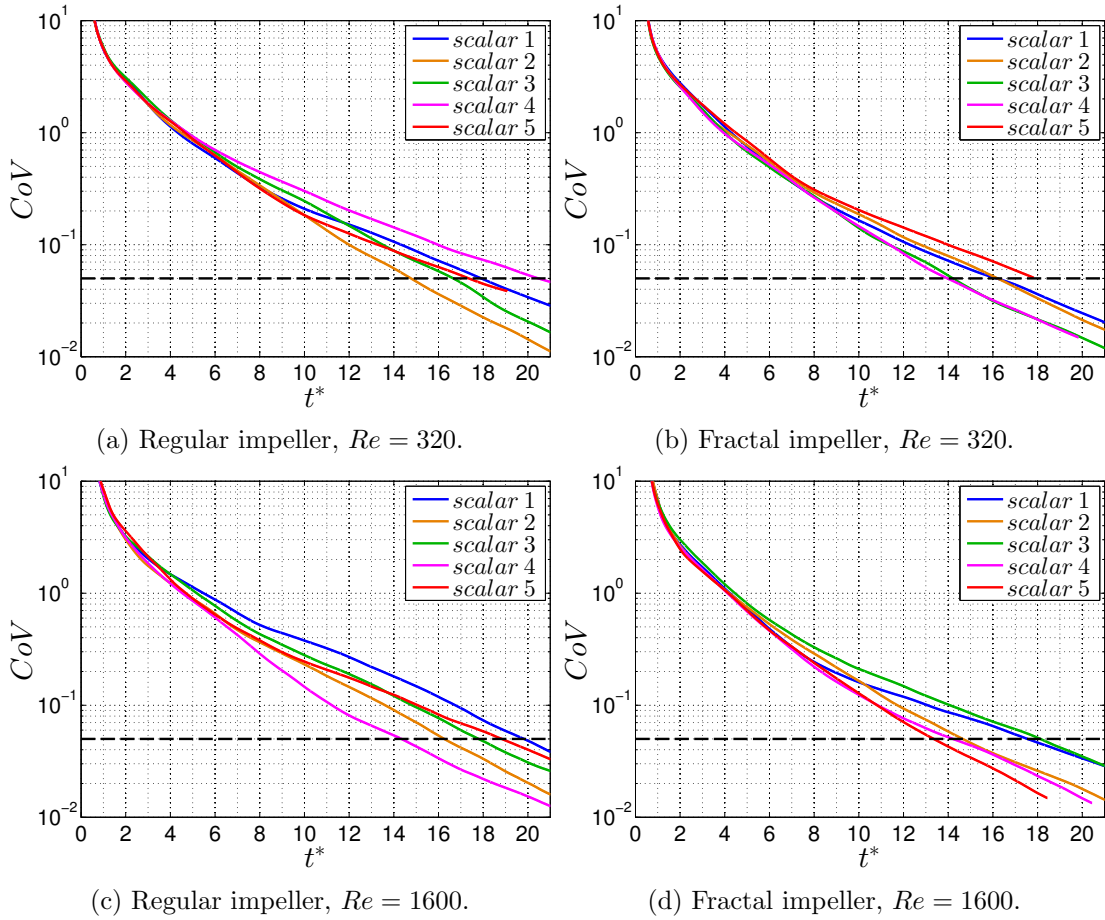


Figure 5.4: Variation of CoV for five scalars shown for regular and fractal impellers at both Re . The dashed horizontal line indicates $CoV = 5\%$.

In order to compare the mixing performance of the two impellers, the CoV -curves of the five realizations are averaged and the resulting curves are displayed in Figure 5.5. The average values of t_{mix} are presented in Table 5.1 along with the percentage difference between the two impeller types. The effect of impeller design is evident. The fractal impeller results in a shorter mixing time, by 10.3% at $Re = 320$ and by 12.0% at $Re = 1600$. To evaluate the mixing performance objectively, these findings must be combined with the power consumption. At $Re = 320$, both impeller types draw equal power (with $\langle N_p \rangle = 2.32$). Therefore, any decrease in the estimated t_{mix} for the fractal impeller is equivalent to an improvement in the mixing performance. At $Re = 1600$, $\langle N_p \rangle$ decreases from 1.59 to 1.47 when the regular blades are replaced by fractal ones, leading to a power reduction of ca. 8%, as presented in Chapter 3. Since the total energy consumption required to mix is proportional to $t_{mix} \times \langle N_p \rangle$ (for a fixed N), this value is estimated to be ca. 20% lower for the fractal impeller with respect to the regular one. This promising improvement provides motivation for further investigation of the fractal impellers' performance in other mixing applications in future research.

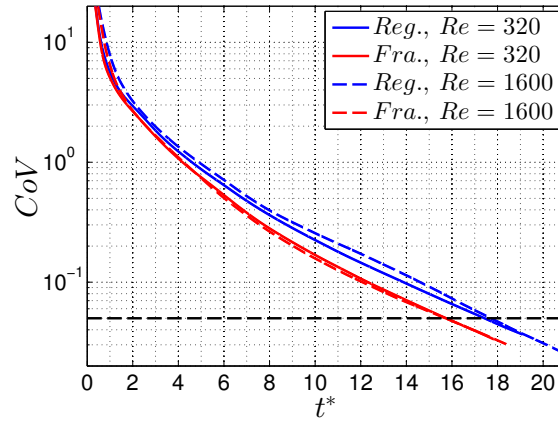


Figure 5.5: Average curves of CoV of the five scalars and comparison between the four cases. Black dashed line indicates $CoV = 5\%$. Reg. stands for the regular and Fra. for the fractal impeller.

It can be argued that five realisations of passive scalar homogenization may not be sufficient to obtain a firm conclusion about t_{mix} considering the spread of data, as seen in Figure 5.4. However, the influence of the impeller design on the trend of the averaged CoV -curves is very similar at both Re , as shown in 5.5. This

	Regular	Fractal	Difference
$Re = 320$	17.4	15.7	10.3%
$Re = 1600$	17.7	15.7	12.0%

Table 5.1: Average mixing time t_{mix} . The difference between the regular and fractal impellers is presented in the rightmost column.

decrease of t_{mix} at both Re , when the regular blades are replaced by fractal ones, improves the reliability of the findings.

The results show that the increase in the Re from 320 to 1600 does not significantly affect the trend of CoV -curves or t_{mix} . This seems counter-intuitive considering that the range of Re examined is in the transitional regime. Indeed, the results showed that the power number decreases significantly when Re is increased from 320 to 1600, e.g. from 2.32 to 1.59 for the regular impeller. As mentioned earlier, Grenville⁶⁹ suggested for the transitional regime the following relation: $t_{mix} = 183^2 \langle N_p \rangle^{-2/3} Re^{-1} (T/D)^2$. With $Re = 320$ and $\langle N_p \rangle = 2.32$, this relation gives $t_{mix} = 240$. With $Re = 1600$ and $\langle N_p \rangle = 1.59$, it results in $t_{mix} = 61$, considering the parameters of the regular impeller. Clearly, the values of t_{mix} given by Grenville's relation are much higher than our numerical predictions. Moreover, this relation predicts a decrease in t_{mix} with increasing Re . These two major differences between the estimates obtained with Grenville's relation and the simulations can stem from the large difference in Sc which determines the influence of the diffusion term in the scalar transport equation (see Equation 5.2). Sc was between $O(10^3)$ and $O(10^5)$ in experiments of Grenville⁶⁹ whereas it was equal to 1 in our simulations. This influence will be discussed later in detail.

In the next section, the characteristics of the flow and scalar fields are identified that determine the mixing process, and physical explanations are provided for the results presented in this section.

5.5 Analysis of mixing times

5.5.1 Evolution of the iso-scalar surfaces

In order to shed more light on the mixing times for regular and fractal impellers, the evolution of iso-scalar surfaces are investigated. These surfaces are by definition normal to the scalar gradient driving the diffusive mixing. The fluid advection can stretch or fold the isosurfaces, hence increase the area for the diffusive flux to operate.¹³⁶ The iso-scalar surfaces $c = \bar{c}$ are used, since such iso-surfaces exist throughout the entire process until full homogenization. It can be imagined that the flow domain is divided into two subdomains: one is where c is higher than the equilibrium value (i.e. $c > \bar{c}$) and another is where c is lower (i.e. $c < \bar{c}$). The diffusion process can transport c from one to the other subdomain only through the iso-scalar surfaces $c = \bar{c}$. Therefore, the investigation of this surface can be used for the interpretation of the mixing process, and in the explanation of the different mixing times observed between different injections and impeller types. This analysis starts with an illustration of the scalar field using contours and iso-scalar surfaces.

Figure 5.6a illustrates iso-contours of $c/\bar{c}$ at the mid-height plane of the tank with the regular impeller at $Re = 1600$, half a revolution after the start of injection. The passive scalar, injected on the suction side of the blade, is clearly carried along the path of the trailing vortices. The iso-scalar surface $c = \bar{c}$ at the same instant is shown in Figure 5.6b. Although the scalar is still profoundly segregated at this stage, the isosurfaces indicate the patterns of stretching and folding by the flow field before the scalar is dispersed in the whole tank.

The scalar field two revolutions after the start of injection is illustrated in Figure 5.7. The contour plot of $c/\bar{c}$ on the vertical plane 30° behind a blade is shown in Figure 5.7a. It indicates that the strong radial jets have already transported the passive scalar towards the side walls of the tank. Smaller scales of the flow field are also emerging in the scalar field, as demonstrated in Figure 5.7b that displays the iso-scalar surface. This has led to a significant increase in the area of isosurfaces, which can be noted when Figures 5.6b and 5.7b are compared. (Figures similar to

5.6 and 5.7 were also created for the case with the fractal impeller. However, they do not illustrate qualitatively different patterns, hence they were not included in the discussion.)

The growth of the aforementioned surface area over the entire computational time is illustrated in Figure 5.8 for all cases. The curves corresponding to different injections start from their actual injection times, i.e. $t^* = 0, 2, 4, 6, 8$, so that their behaviour can be compared because the same instantaneous flow field is advecting all five scalars simultaneously. For normalization purposes, the total area of the outer wall surfaces is used (top, bottom and side walls), which is $A_W = 1.5 \pi T^2$ (T is the tank diameter equal here to tank height, see Figure 2.3). The five curves, corresponding to the five injections, are averaged and illustrated in Figure 5.9, in order to highlight the effect of the impeller type and Re .

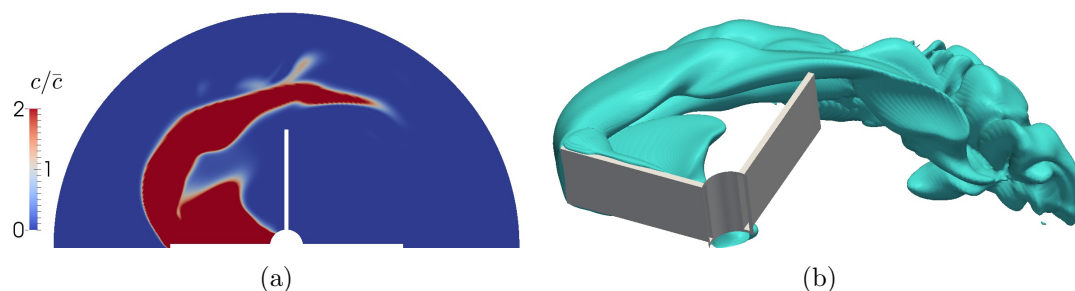


Figure 5.6: (a) Contour plot of $c/\bar{c}$ at the mid-height plane. (b) The iso-scalar surface $c = \bar{c}$. Regular impeller at $Re = 1600$, half a revolution after the start of injection.

As seen in Figure 5.8, the iso-scalar surfaces grow continuously during the initial two revolutions after the injection. Figure 5.9 shows that on average A/A_W is equal to 0.6 for $Re = 320$, and to 1.3 for $Re = 1600$, at the end of the first two revolutions regardless of the impeller type. Moreover, the curves of every realization collapse very well during the aforementioned period for $Re = 320$, and for $Re = 1600$ they start to deviate after ca. 1.5 revolutions. (This can be seen more clearly when all curves are plotted together starting from $t^* = 0$ that is not shown here.) In other words, the initial growth rate of the iso-scalar surfaces (i.e. the slope of curves in Figure 5.8) is equal for every realizations, and only different between the cases at different Re . This can be explained as follows: The segregated

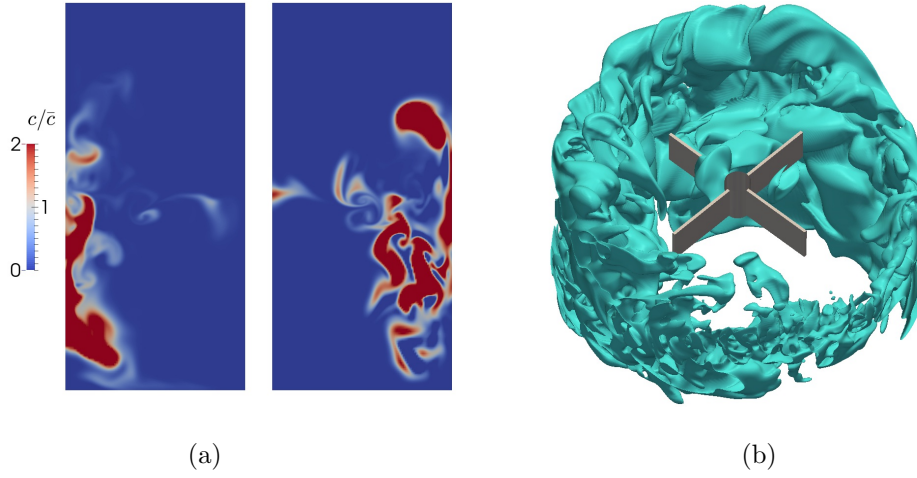


Figure 5.7: (a) Contour plot of $c/\bar{c}$ on the vertical plane 30° behind a blade (the cross-section of the shaft is left vertically blank at the centre of the plot). (b) The iso-scalar surface $c = \bar{c}$. Regular impeller at $Re = 1600$, two revolutions after the start of injection.

state of the passive scalar seen in Figure 5.6 becomes dispersed as shown in Figure 5.7 and the length scales of the flow field determine the level of dispersion (i.e. the scale of segregation), hence the area of iso-scalar surfaces. Since the length scales of the flow field become smaller at higher Re , the initial growth rate of A/A_W gets steeper, which is also evident in Figure 5.9.

This dispersion, and the consequent rapid stretching, is also the reason of the steep decrease of the CoV values during the initial period (see Figure 5.4). Similarly, the curve of $-\frac{1}{2} \frac{d}{dt}(\overline{c'^2})$ (see Figure 5.3) exhibits a sharp drop for the first two revolutions, that is followed by a milder slope for the remainder of the process.

After the initial dispersion, the passive scalar starts to circulate inside the tank along with the flow. At the same time, the areas of iso-scalar surfaces exhibit strong fluctuations as seen in Figure 5.8. One can distinguish some fast fluctuations with a small amplitude and several peaks per revolution, which are easily recognized over the periods when A/A_W does not exhibit a steep change. This can be noted for instance in Figure 5.8a on the curve of the scalar-1 at $t^* = 6 - 8$, $t^* = 10 - 12$, or $t^* = 18 - 20$. These oscillations in A/A_W possibly stem from the instabilities of the trailing vortices and the radial jet observed at the frequency $f' \approx 3$ and discussed earlier in Chapter 4. In addition to the fast fluctuations in A/A_W , the curves in Figure 5.8 exhibit larger peaks which appear irregularly and over large

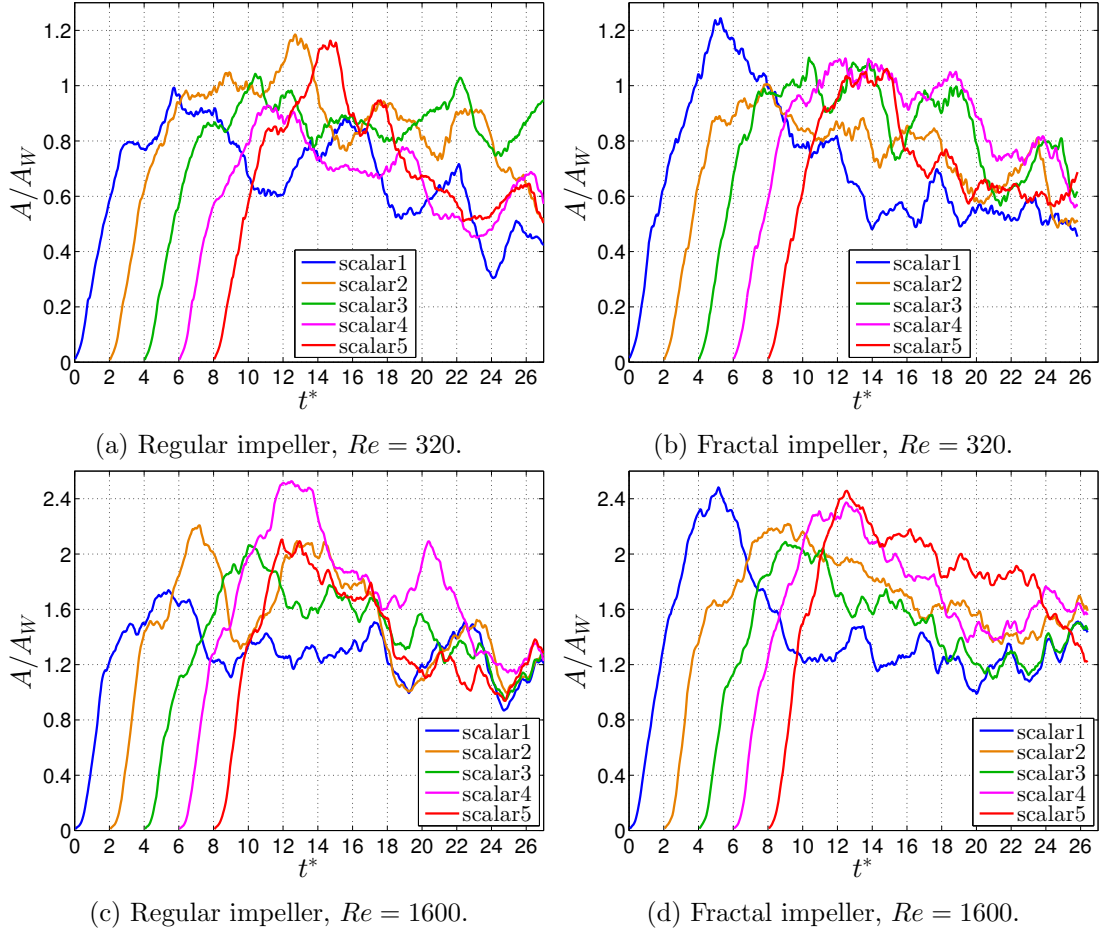


Figure 5.8: Normalized area of iso-scalar surfaces for five realizations in all cases.

time scales such as several revolutions. It is noted that these can appear in curves of several different scalars simultaneously. For instance in Figure 5.8a, the curves of scalar-1 and scalar-3 exhibit the same trend from $t^* = 20$ to 24, although they are injected with a time difference equal to four revolutions; this can also be noted in Figure 5.8c between the curves of scalar-1 and scalar-2 from $t^* = 18$ to 26 where both curves simultaneously exhibit a long wave. This phenomenon can be related to the slow instabilities already reported in Chapter 4 that appear at $f' \approx 0.12$ in the PSD of the axial velocity component.

Focusing on Figure 5.9 and comparing the results of averaged curves, several interesting features can be detected. It is seen that the fractal impeller leads to a larger area of iso-scalar surfaces on average than the regular impeller, at both Re . This can explain the lower t_{mix} predicted for the fractal impeller. The correlation

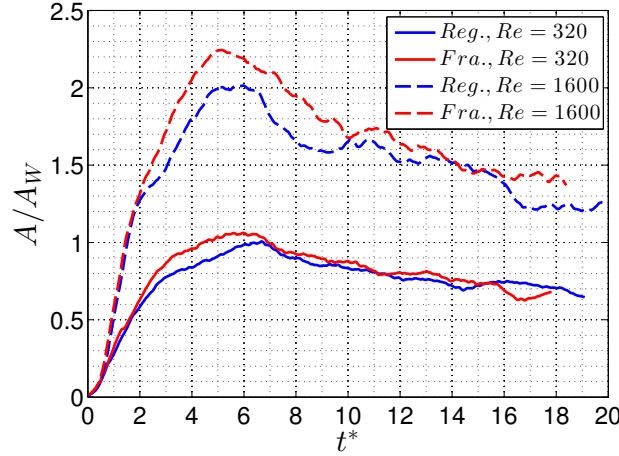


Figure 5.9: Area of isosurfaces averaged for the five injection in all cases.

between A/A_W and t_{mix} is assessed at the end of this and in the next subsections.

Moreover, it is observed that the area of iso-scalar surfaces are ca. twice as large at $Re = 1600$ than at $Re = 320$. This is a consequence of the changes in the velocity field as Re is increased, since the flow field is strongly Re -dependent in the transitional regime. Also the scalar gradients are expected to be steeper at $Re = 1600$, since the iso-scalar surfaces are more stretched. One can expect that these characteristics enhance mixing and result in a lower t_{mix} . However, the increase in Re has another impact that must be taken into account. There is a factor $1/(Re Sc)$ in front of the diffusion term of Equation 5.2 (the increase in Re is realized in simulations by setting a lower ν , and is accompanied by a proportional decrease in Γ , since Sc is kept constant). Therefore, the increase in Re weakens the diffusion term, which counteracts to the expected mixing enhancement due to larger isosurfaces. Although the increase in Re from 320 to 1600 creates larger isosurfaces and steeper gradients, this does not decrease t_{mix} , because the molecular diffusion becomes weaker at $Re = 1600$.

In cases like Grenville's experiments,⁶⁹ where Sc is very high (e.g. $O(10^3)$ – $O(10^5)$ in the aforementioned study), the diffusion term is very small compared to the advection term. It is already known that in the fully turbulent regime, where Sc and Re are very high and flow field is fully developed, any increase in Re does not affect t_{mix} .^{32,52} Taking these into account, when Re is increased in

the transitional regime, the changes in the diffusive term can be insignificant (due to high Sc) compared to the drastic development of the velocity field and the increase in the area of isosurfaces. Therefore, the aforementioned author might have observed $t_{mix} \sim Re^{-1}$ in the transitional regime due to the increasing area of isosurfaces. Comparing Grenville's results with the present results, it can be inferred that the Re -scaling of t_{mix} in the transitional regime can be strongly affected by Sc .

In what follows, it is tried to relate the area of iso-scalar surfaces $c = \bar{c}$ with the rate of homogenization. First, a qualitative evaluation is conducted and the area of iso-scalar surfaces of the five injections in one case are compared, e.g. for the regular impeller at $Re = 1600$. To this end, Figure 5.10 illustrates the running averages of the aforementioned isosurfaces. When this is considered together with Figure 5.4c showing the CoV -curves of the same case, it is observed that the scalar injections with the higher value of averaged A/A_W also present faster mixing. Moreover, the trends of the CoV -curves in Figure 5.4c seem to be related with the trends of A/A_W of the corresponding scalars. For instance, scalar-5 (red curve) which has a lower CoV than scalar-3 (green curve) until $t^* = 12$, has also a larger averaged A/A_W . After $t^* = 12$, the averaged A/A_W of scalar-5 drops below the curve representing scalar-3. Eventually, scalar-3 presents a larger average A/A_W and exhibits a faster mixing than scalar-5. Next, the decrease of the CoV is correlated with the area of iso-scalar surfaces using a simple mathematical model.

5.5.2 Decay rate scaling

As defined by Equation 5.4, CoV is proportional to the square root of $(\overline{c'^2})$. Moreover, Equation 5.5 shows that the negative time derivative of $(\overline{c'^2})$, i.e. the decay rate, depends crucially on the scalar gradients. Now it is assessed if the volume integral of the decay rate can be characterized only by the scalar gradients in the vicinity of the iso-scalar surface $c = \bar{c}$. The gradient of c normal to the aforementioned surface can be approximated using a length scale characteristic of scalar diffusion, hereafter denoted by η , and the corresponding increment of c . η is

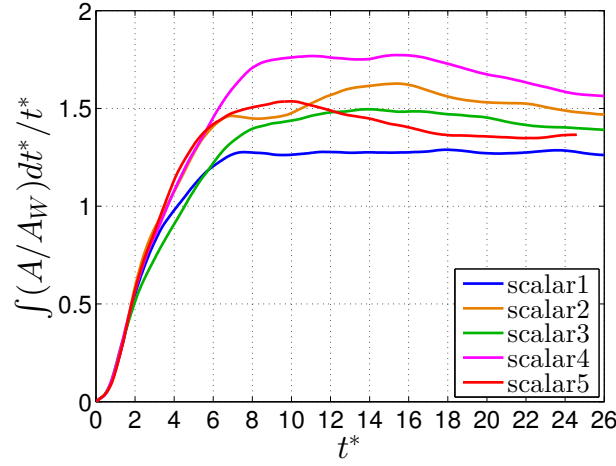


Figure 5.10: Running average of the normalized area of isosurfaces for regular impeller at $Re = 1600$.

defined as the average distance between two iso-scalar surfaces $c = c_1$ and $c = c_2$, where c_1 and c_2 deviate only a little from $\bar{c}$, i.e. $c_1 = \bar{c} + \Delta c$ and $c_2 = \bar{c} - \Delta c$ with a small enough Δc . Since $c_1 - c_2 = 2\Delta c$, the aforementioned gradient is approximated with $2\Delta c/\eta$. After nondimensionalization ($\eta^* = \eta/D$), this is substituted into Equation 5.5 which takes the following form:

$$-\frac{1}{2} \frac{d}{dt^*} (\overline{c'^2}) = \frac{1}{Pe} \overline{\frac{\partial c'}{\partial x_i^*} \frac{\partial c'}{\partial x_i^*}} \sim \frac{1}{Pe} \left(\frac{2\Delta c}{\eta^*} \right)^2. \quad (5.6)$$

The average diffusion length η is calculated as follows: $\eta = V_\Delta/A$, where A is the area of the iso-scalar surface $c = \bar{c}$, and V_Δ is the volume confined between the two iso-scalar surfaces, $c = c_1$ and $c = c_2$. The value selected for Δc needs to be very small compared to $\bar{c}$, since the focus is on the gradients on the iso-scalar surface $c = \bar{c}$. On the other hand, the model is set up with an effort to make it independent of this choice. When the selected value (Δc) is increased, the volume V_Δ grows as well, hence the length scale $\eta = V_\Delta/A$ increases. Therefore, the ratio $2\Delta c/\eta$ is not expected to exhibit a strong dependence on Δc , provided Δc is small enough. In order to confirm this, the influence of Δc is tested below.

This test is conducted using the data of scalar-1 of the regular impeller at $Re = 320$. The rightmost term in Equation 5.6 is computed using several different values for Δc and compared in Figure 5.11a. The four dashed lines with different

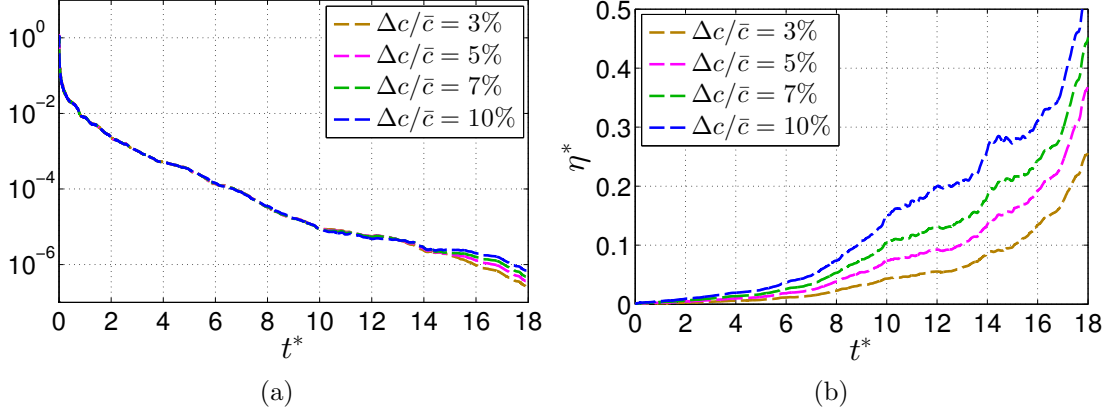


Figure 5.11: (a) The rightmost term in Equation 5.6 is computed using four different values of Δc . (b) The trend of η^* against t^* . For scalar-1 of the regular impeller at $Re = 320$.

colors are obtained with $\Delta c/\bar{c} = 3, 5, 7$ and 10%. Figure 5.11a illustrates that these curves do collapse until ca. $t^* = 15$. After $t^* = 15$, the four curves start to deviate which can be explained as follows. Until ca. $t^* = 15$, the length scales η^* based on different values of Δc exhibit a very slow growth as shown in Figure 5.11b. After $t^* = 15$, these diffusive length scales grow quickly and at $t^* = 18$ the value of CoV has dropped to 0.05 which corresponds to the well-mixed state (see Figure 5.4a). These results suggest that in the last stage shortly before complete homogenization, the average distance between the isosurfaces $c_1 = \bar{c} + \Delta c$ and $c_2 = \bar{c} - \Delta c$ grows quickly and the scalar gradients normal to the iso-scalar surface $c = \bar{c}$ cannot be linearised accurately using $2\Delta c/\eta$.

The test case discussed above has shown that the model does not exhibit a significant dependence on the selected value of Δc (for $\Delta c/\bar{c} < 10\%$, and for the regular impeller at $Re = 320$), until the last 3 revolutions before the well-mixed state. Therefore, it is set to $\Delta c/\bar{c} = 5\%$ for the rest of the discussion. When this value of Δc and $\eta = V_\Delta/A$ are plugged into the model, it takes the following form:

$$\frac{1}{Pe} \left(\frac{2\Delta c}{\eta^*} \right)^2 = \frac{(0.1 \bar{c} D)^2}{Pe} \left(\frac{A}{V_\Delta} \right)^2. \quad (5.7)$$

This is computed for all five realizations and the curves corresponding to the five realizations are averaged. Moreover, the five curves obtained with the exact

calculation of the decay rate are averaged, as well. The model and the exact calculation are compared in Figure 5.12 for every case studied. The results show that the model is able to predict the trend of the decay rate correctly except for the first two revolutions. During the first two revolutions, the model underestimates the decay rate severely. It was already shown in Figures 5.6 and 5.7 that the passive scalar is significantly segregated during this period. There are very steep gradients inside the segregated regions, away from iso-scalar surfaces $c = \bar{c}$ which encircle the segregated bulks. It is likely that the diffusion due to the internal gradients outweigh the gradients near the iso-scalar surface $c = \bar{c}$ in this initial phase.

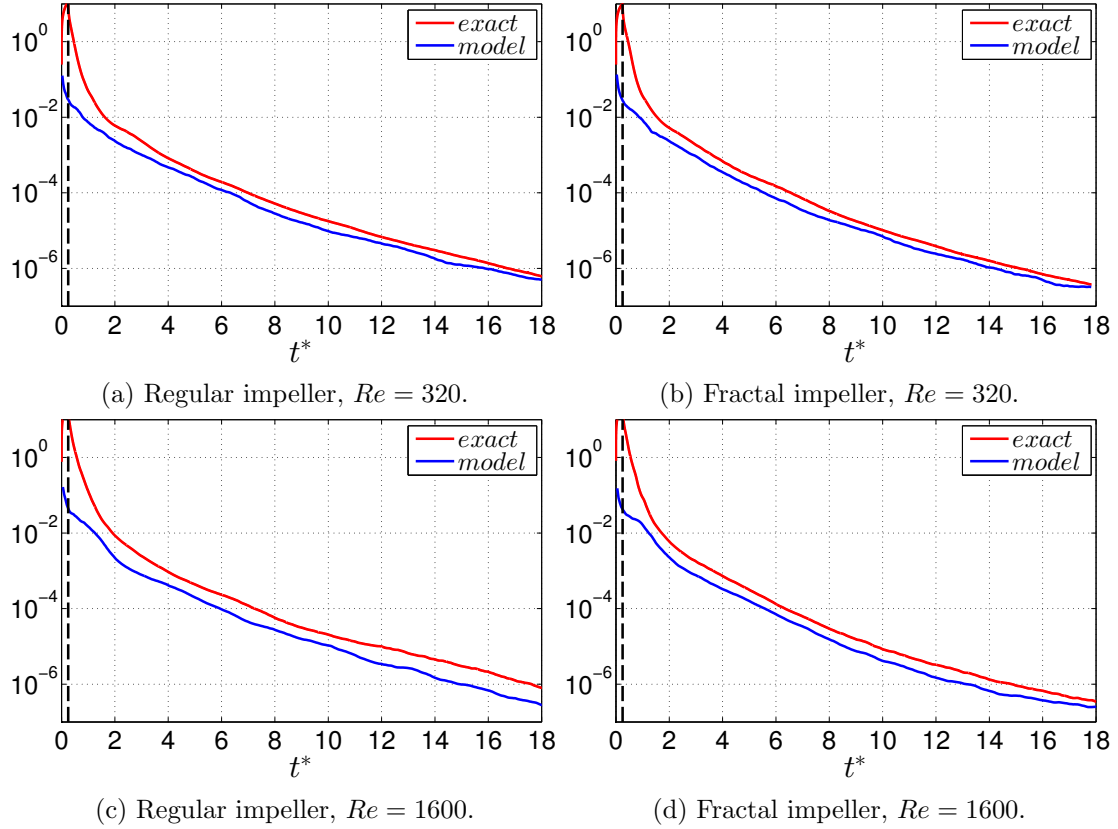


Figure 5.12: Exact calculation of the decay rate (exact) and the model based on $\Delta c/\bar{c} = 5\%$ (model) as expressed in Equation 5.7. The curves obtained with five realizations are averaged in all cases. The vertical dashed lines show the time when the injection stopped.

If the curves generated with the model are scaled up with a constant factor and collapse with the curve of the decay rate, this would show that the decay

rate scales with $(A/V_\Delta)^2$. To evaluate this scaling factor, the ratio of the exact calculation of the decay rate to the model is computed as follows:

$$K = \frac{\overline{\frac{\partial c'}{\partial x_i^*} \frac{\partial c'}{\partial x_i^*}}}{\left(\frac{2\Delta c}{\eta^*}\right)^{-2}} \quad (5.8)$$

This ratio, K , is plotted in Figure 5.13 against time. After ca. $t^* = 3$, the curves converge to the band between 1.5 and 2, except for the curve representing the regular impeller at $Re = 1600$, which fluctuates between 2 and 3. The fluctuations of K are probably the result of the low number of samples, since only five realizations are available. Although the nondimensional value of the decay rate changes over six orders of magnitude (see Figure 5.12), the ratio K remains relatively constant after ca. $t^* = 3$. The model based on the area of the isosurfaces is therefore able to predict at least the order of magnitude of the decay rate over a very broad range.

These findings suggest that for the scalar fields with larger iso-scalar surfaces, the mixing is enhanced. It was already shown in Figure 5.9 that the fractal impeller leads to a larger iso-scalar surface area on average, at both Re . It was inferred that this can be the reason of the shorter t_{mix} provided by the fractal impellers. The simple model developed in this section supports this conclusion.

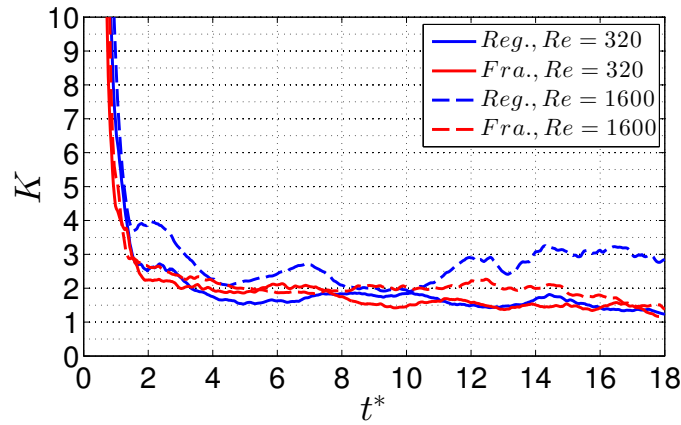


Figure 5.13: The ratio of the exact decay rate to the model (as expressed in Equation 5.8), for both impellers and Re .

5.5.3 Spatial distribution of isosurfaces

In this subsection, the regions inside the tank are determined where the iso-scalar surfaces are frequently created, and the patterns are analysed which the iso-scalar surfaces illustrate while they are stretched. From this it can be inferred which aspects of the flow field have the most dominant influence on the growth of isosurfaces.

In order to detect the location of the iso-scalar surfaces $c = \bar{c}$, a new parameter is defined as a function of space and time as follows: In any grid cell and time step, $g(x_i, t)$ is defined to be equal to 1 if the iso-scalar surface $c = \bar{c}$ is present in that instant in that cell, else it is equal to 0. In practice, the presence of the isosurface in a cell is detected, if the value of $c(x_i, t)$ fulfils the following inequality in that cell: $0.99 < c/\bar{c} < 1.01$. $g(x_i, t)$ can assume only one of the two values: 0 or 1. This is computed and averaged over a certain period of time. The time-averaged value is denoted by $\langle g \rangle$, and it is a function of space. Physically, it is the fraction of time when the isosurface $c = \bar{c}$ is present at a given location. The higher the value, the higher the probability of appearance of the isosurface $c = \bar{c}$.

The calculation and averaging started at $t^* = 2$, since the continuous growth of the isosurfaces during the first two revolutions was attributed to the dispersion of the initially segregated bulk of the passive scalar. Hence, this period exhibits different characteristics compared to the rest of the homogenization process and is excluded from the analysis presented in this subsection. The averaging continued for the duration of 9 impeller revolutions, i.e. over the period $t^* = 2 - 11$. Therefore, the computation covers only the time until the length scale η^* starts growing rapidly (see Figure 5.11b). The calculation of $\langle g \rangle$ is performed for both impeller types, but only for scalar-1 at $Re = 320$. The result of this calculation gives the spatial distribution of $\langle g \rangle$ illustrated in Figure 5.14 on a vertical cut-plane 15° behind the blades.

As seen in Figure 5.14, the highest values of $\langle g \rangle$ appear along the radial jet, along the tank walls and the impeller shaft. It is noted that these patterns of $\langle g \rangle$ show similarity to the mean flow circulation inside the tank. For comparison, the mean flow field is illustrated in Figure 5.15 on the same plane 15° behind the

blades. The contours in that figure display the magnitude of the projected and normalized mean velocity, i.e. $\sqrt{U_r^2 + U_z^2}/U_{tip}$, where U_r and U_z are the mean radial and axial velocity components, respectively, and U_{tip} denotes the blade tip velocity. The arrows show the direction of the mean velocity vector projected on the plane. The comparison between contours in Figure 5.14 and the flow patterns in Figure 5.15 suggests that the iso-scalar surfaces are stretched along the mean flow circulation. Especially, the contours of $\langle g \rangle$ near the top and bottom walls of the tank show that the iso-scalar surfaces are curved along the trajectory of the mean flow. These findings indicate that the mean circulation has a significant impact on the stretching of the iso-scalar surfaces.

Comparing Figures 5.14a and 5.14b in order to reach a conclusion about specific differences between the regular and fractal impellers is avoided, but these are only used to evaluate the overall patterns. This is because these figures are obtained using only one injection (scalar-1).

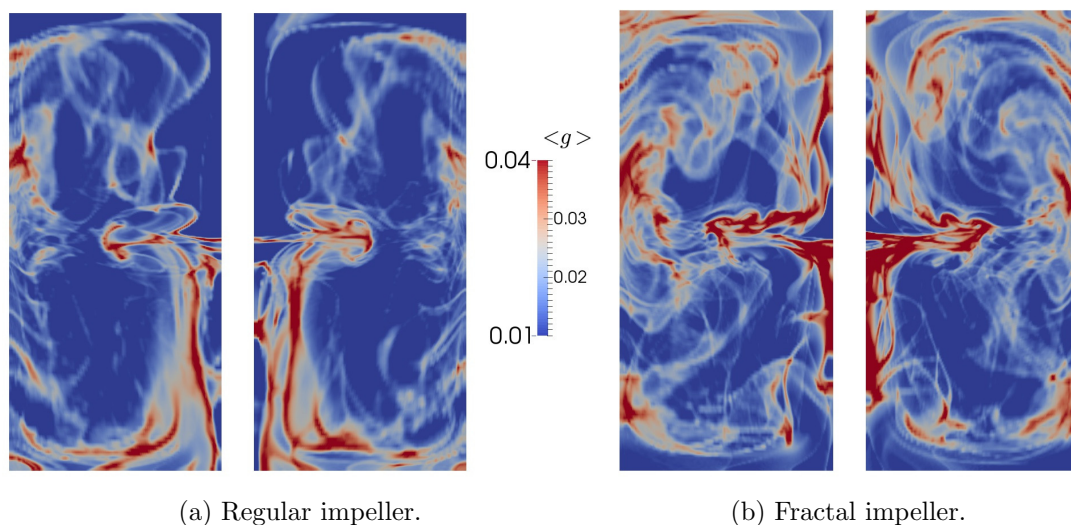


Figure 5.14: Contours of $\langle g \rangle$ on a cut-plane 15° behind the blades, at $Re = 320$ (the cross-section of the shaft is left blank).

Taking into account that the main flow circulation plays an important role in the stretching of the iso-scalar surfaces, this effect can be augmented if the flow rate is increased. It was already shown in Chapter 3 using a control volume around the impeller, that the fractal impeller results in a higher flow rate with respect to the regular impeller, by 7.3% at $Re = 320$ and by 10.3% at $Re = 1600$. This

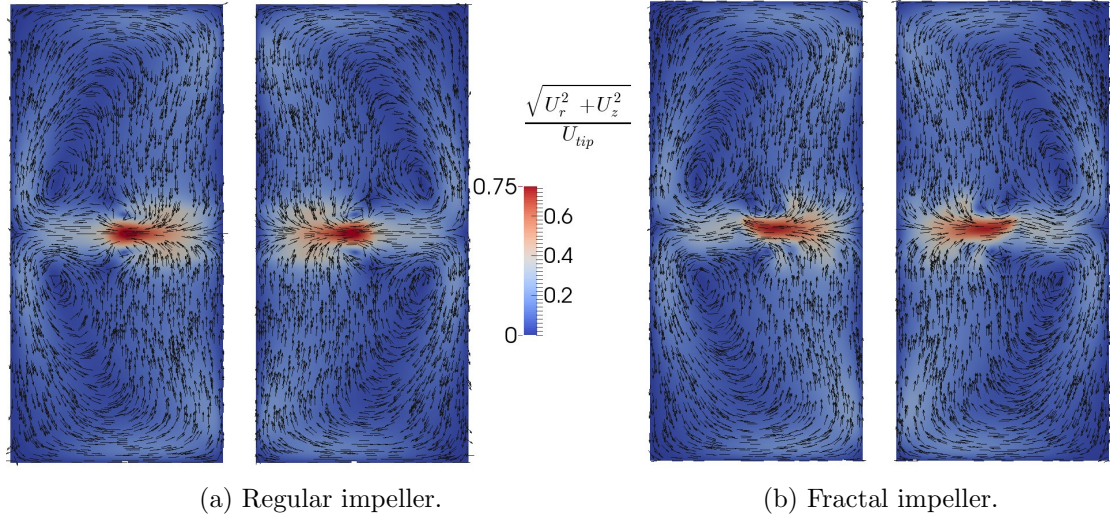


Figure 5.15: The arrows show the direction of the mean velocity projected on the cut-plane (15° behind the blades), and the contours display its magnitude normalized with the blade tip speed, at $Re = 320$ for both impeller types (the cross-section of the shaft is left blank).

increase in flow rate can be the major factor leading to larger area of the iso-scalar surfaces for the fractal impeller at both Re , as shown in Figure 5.9. As a results of these factors, the homogenization of the injected passive scalar is accelerated when the fractal impeller is used instead of the regular impeller.

5.6 Conclusions

It was already demonstrated that there is a reduction in power consumption of an unbaffled stirred vessel when regular blades were replaced with fractal blades, by $11 - 12\%$ at $Re = 1 - 2 \times 10^5$ (as shown by Steiros et al.³) and by 8% at $Re = 1600$, whereas there was no difference at $Re = 320$ (as shown in Section 3.3). In order to make a complete performance comparison, the present study has evaluated the mixing time required to homogenize an injected passive scalar with the regular and fractal impellers, by means of DNS at $Re = 320$ and 1600 , where $Sc = 1$.

The nondimensional mixing time, t_{mix} is determined based on the time when the value of CoV drops to 5% , where CoV is computed using the entire scalar field. In each case, i.e. for both impeller types and Re , five separate scalars are

injected with a time lag equal to two impeller revolutions. In order to compare different cases, the CoV -curves of the five realizations are averaged and the average t_{mix} is determined. Based on these results, the fractal impeller leads to a shorter mixing time by 10.3% at $Re = 320$ and by 12.0% at $Re = 1600$. Since the total energy consumption required for the mixing process is proportional to $t_{mix} \times \langle N_p \rangle$ (for a fixed N), this is estimated to be 10.3% lower at $Re = 320$ and 20% lower at $Re = 1600$, for the fractal impeller with respect to the regular one. These findings give motivation for future studies which may assess the fractal impeller's performance in other mixing applications.

When the performance of a certain impeller type at two different Re is compared, it is observed that Re does not affect t_{mix} significantly. This behaviour is expected at fully turbulent regime³² whereas the considered Re values are in the transitional regime. In order to explain this result as well as the different mixing times obtained with the regular and fractal impellers, the evolution of the iso-scalar surfaces $c = \bar{c}$ is analysed. These surfaces characterize the amount of contact area over which the molecular diffusion can act. The aforementioned surfaces are computed for the five realisations in each case. It is observed that these surfaces grow almost linearly during ca. the first two revolutions in every case, and the slope of this growth only depends on the Re . Afterwards, they stabilize around a certain level and fluctuate around that. These fluctuations can be linked to the large scale instabilities observed in the flow field discussed in Sections 4.3, 4.6, 4.7 and 4.8. When the time-averaged area of isosurfaces are compared among the five realizations obtained in the same case, it is determined that the realizations which result in a larger area, do also lead to a shorter t_{mix} . Moreover, when the isosurface areas of the five realizations are averaged for each case and compared, it is noted that the fractal impeller generates larger iso-scalar surfaces on average with respect to the regular impeller at both Re . This can be the reason of the faster mixing presented by the fractal impeller.

Furthermore, the area of the isosurfaces grows ca. twice as large at $Re = 1600$ than at $Re = 320$. This stems from the changes in the velocity field as Re rises.

On the other hand, the diffusive term of the scalar transport equation has a factor $1/(ReSc)$ in front of it, therefore the increase in Re weakens the diffusion. Our findings suggest that the decreasing strength of the molecular diffusion counteracts to the effect of the increasing area of isosurfaces. Therefore, t_{mix} is not significantly affected by the increase in Re . However, this behaviour can be different for the cases with very high Sc and Re , where the passive scalar diffusion is much weaker than its advection.

In order to provide a quantitative correlation between the decay of CoV and the area of iso-scalar surfaces (A), a simple mathematical model is devised. This model approximates the integral decay rate using the scalar gradients in the vicinity of the iso-scalar surfaces $c = \bar{c}$. The results suggest that the decay rate of $\overline{c'^2}$ scales with A^2 , and support the aforementioned conclusion that t_{mix} can be linked to the area of the isosurfaces.

Finally, the regions in the flow field are determined where the aforementioned surfaces are created predominantly, and the patterns of their growth. It is noted that these surfaces are stretched along the mean flow circulation inside the vessel, i.e. along the radial jets, the tank walls and the impeller shaft. It has already been shown in Chapter 3 that the fractal impellers generate a higher flow rate with respect to the regular impeller, by 7.3% at $Re = 320$ and by 10.3% at $Re = 1600$. This difference in the flow rate can be the dominant factor leading to the larger average area of the iso-scalar surfaces in the tank with the fractal impeller.

Chapter 6

Concluding remarks

6.1 Reduced energy consumption

Flow and scalar fields in an unbaffled stirred vessel agitated by a four-bladed radial impeller is investigated by using Direct Numerical Simulations (DNS) at $Re = 320$ and 1600 . Previous research³ showed that the mean power drawn by a fractal impeller (with one fractal iteration) is ca. $11 - 12\%$ lower compared to the regular impeller at $Re = 1 - 2 \times 10^5$, yet without a clear explanation of the mechanism causing this reduction. Similarly, a difference of 8% was also detected in present DNS results at $Re = 1600$, employing exactly the same design of the aforementioned regular and fractal blades. Therefore, the first goal of this study was to provide the physical explanation for the reduction in power.

It was observed that vortical wakes behind the blades have qualitative differences. While two large trailing vortices form behind the regular blade, multiple small vortices appear behind the fractal blade. This is due to the geometric shape of the latter blade type; high speed jets form in the gaps around the blade perimeter that penetrate inside the recirculation zone in the wake and break up the trailing vortices into smaller ones. This results in changes in the time-average recirculation pattern on the suction side. The volume of the separation region is 7% smaller in the wake of the fractal blades which can explain the lower drag coefficient of the fractal blade, consequently the lower torque. The latter information is equivalent to a decreased transport of angular momentum from the impeller to

the fluid; this difference stems from the reduced turbulent transport induced by the smaller trailing vortices. The major difference in the turbulent dissipation is seen in the impeller discharge region in the radial range of $1.3 < r/R < 1.9$. It is also noted that the fluctuations at relatively low frequencies ($f' < 20$) account for the difference in the turbulent dissipation.

It is concluded that the modification of the blade perimeter can have a strong influence on the trailing vortex structures, consequently on the impeller torque and energy dissipation characteristics. In the example of the fractal impeller, this change results in a lower $\langle N_p \rangle$ and higher N_q .

6.2 Analysis of power fluctuations

It was noted that the power consumptions of (four-bladed) regular and fractal impellers fluctuate with peak frequencies $f' = 3.16$ and $f' = 3.14$, respectively, at $Re = 320$. This frequency drops slightly to $f' = 3.0$ at $Re = 1600$, for both impeller types. It was discovered that the fluctuations in power stem from a periodic event in the wake of the blades, that affects the pressure distribution on blade surfaces. This event involves up-and-down swinging motion of the radial jet as well as alternating growth and decay of the upper and lower trailing vortex cores. Both of the semi-cycles of this unsteadiness have a similar impact on the impeller torque. Therefore, the frequency of power fluctuations is equal to the double of the frequency of the described periodic event.

Since this phenomenon was observed in the wake of both regular and fractal impellers with almost the same frequency regardless of their geometric differences, it is inferred that the exact blade shape does not have a strong influence on it and a similar, periodic unsteadiness might appear in the wake of different types of impeller blades as well. Moreover, it was discovered by means of the simulations using one-bladed and two-bladed regular impellers, that the number of the blades has a direct influence on the onset and the frequency of the unsteadiness of trailing vortices.

6.3 Reduced mixing time

In order to complete the performance assessment of the fractal impeller with respect to the regular impeller, the mixing times of injected passive scalars were evaluated, for both impeller types, at $Re = 320$ and 1600 . To this end, five separate passive scalars were injected into the tank with time lags of two impeller revolutions between consecutive injections. The homogenization curves of five scalars were averaged to obtain the mixing time (t_{mix}) for every case. The results showed that t_{mix} can be reduced by ca. 10% at $Re = 320$, and by ca. 12% at $Re = 1600$, when regular blades were replaced by fractal blades. Taking also the power drawn by both impellers into account, $\langle N_p \rangle \times t_{mix}$ is ca. 10% lower at $Re = 320$, and ca. 20% lower at $Re = 1600$, when fractal impeller was employed.

In order to explain several aspects of the mixing results, the evolution of the iso-scalar surface $c = \bar{c}$ was investigated for every injected scalar (the area of this surface is denoted by A). Three sorts of comparisons can be considered as follows: (i) When the five injections of a particular case were compared, it was noted that the larger A , the faster is the homogenization of the passive scalar, qualitatively speaking. (ii) When the aforementioned surface areas generated by the five separate injections were averaged and compared among different cases, it was observed that the flow induced by the fractal impeller creates a larger A in average with respect to the regular impeller at both Re , which can account for the reduced t_{mix} . (iii) Increasing Re from 320 to 1600 results in an expansion of A more than twice, yet it is also equivalent to an increase in Pe , since $Sc = 1$ for cases at both Re . While the expansion of A (as a consequence of the increase in Re) is expected to accelerate the mixing, the rise of Re also weakens the diffusion term. These two aspects of increasing Re have counteracting influences on t_{mix} , which consequently lead to almost equal t_{mix} at $Re = 320$ and 1600 .

A simple mathematical model is developed, that can be used to quantify the relation between A and t_{mix} . The goal of this model is to predict the decay rate of integral scalar fluctuations (i.e. $\overline{c'^2}$) using the gradients near the iso-scalar surface $c = \bar{c}$. The findings suggest that the decay rate of $\overline{c'^2}$ scales with A^2 .

This is in accordance with the previous assumption that the differences in t_{mix} can be explained by evaluating A . Further investigation showed that the stretching patterns of A matches with the course of the mean flow circulation. Taking this into account, the larger A (in average of five injections) created by the fractal impeller with respect to the regular one is attributed to its larger flow rate (N_q).

6.4 Suggestions for future studies

Low values for Re and Sc were selected for the sake of the simulation accuracy, yet most of the industrial applications are performed at much higher Re and Sc . Still, the findings are promising for the application of fractal impellers and provide plenty of material for future studies which can be built on this work.

It would be especially interesting to evaluate the mixing performance of the fractal impeller in fully turbulent regime and in baffled tanks. The commonly used Rushton turbine can also be equipped with blades of fractal design, in order to contrast their performance with the standard rectangular blades. Such investigations of the fractal impeller's mixing performance may involve reacting flows or can be conducted with non-Newtonian fluids.

The simple mathematical model introduced in Section 5.5.2 and used to approximate the decay rate of the passive scalar fluctuations, can be assessed in cases of higher Re and Sc . It would be interesting to illustrate if this model or a similar one is valid under fully turbulent flow conditions.

The post-processing approaches used in Chapter 3 to investigate the difference in power drawn by regular and fractal impellers, can also be employed by future studies aiming to analyse the performance of various impeller types.

Periodic fluctuations of the radial jet and trailing vortices are detected with a frequency at ca. three times the impeller rotational speed. It is still an open question whether this unsteadiness will be observed in the wakes of different blade types, or whether its frequency can be correlated to the geometrical details of the blade and Re .

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Appendix A

Table of important parameters

Tank height (H)	0.45
Tank diameter (T)	H
Impeller diameter (D)	$0.5 \times H$
Blade height	$0.1 \times H$
Blade thickness	$0.01 \times H$
Impeller shaft diameter	$0.04 \times H$
Impeller angular velocity (Ω)	4
Impeller rotational speed (N)	$2/\pi$
Schmidt number (Sc)	1

Table A.1: Table of parameters and dimensions common for all cases.

Blade type	Reg.	Fra.	Reg.	Fra.	Reg.	Fra.
Re	320	320	320	320	1600	1600
Number of grid cells $\times 10^{-6}$	4	9	13	21	60	70
Time steps per impeller revolution	1500	1500	3000	3000	5000	5000
Number of CPU's used for simulations	160	160	240	240	1728	1728
CPU hours used per impeller revolution	280	670	1730	2660	11850	13100
Number of impeller revolutions simulated to collect statistics	80	80	80	80	100	100

Table A.2: Table of case specific parameters for the six cases simulated. Reg. stands for regular and Fra. for fractal.

Appendix B

Time-lag between energy injection and energy dissipation

The time-average values of the impeller power and integral dissipation have been shown to be in balance, except for a numerical dissipation of ca. 4% at $Re = 1600$ (see Table 3.3). The fluctuations in time of the power number, obtained from pressure integration on the impeller surface (denoted by $N'_p(t^*)$), and of the volume integral of the instantaneous dissipation (denoted by $N'_\varepsilon(t^*)$) are plotted in Figure B.1 against the number of revolutions. (Their time-averaged values are subtracted from the instantaneous quantities, $N_p(t^*)$ and $N_\varepsilon(t^*)$, in order to present only their fluctuations.)

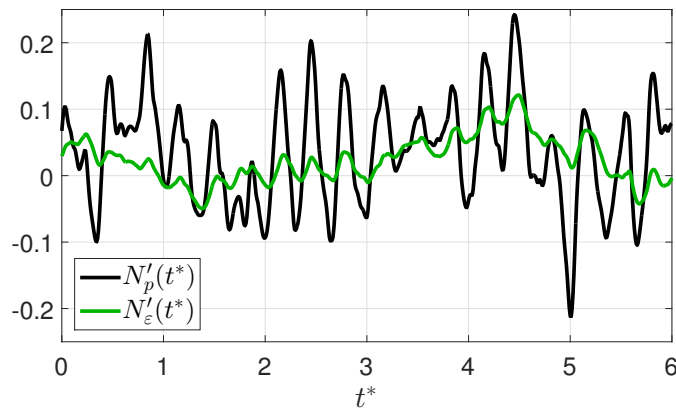


Figure B.1: Fluctuations of power consumption are plotted for fractal impeller at $Re = 1600$. $N'_p(t^*)$ is based on the power drawn by the impeller and $N'_\varepsilon(t^*)$ is based on the volume integral of the total dissipation over the tank volume.

It is seen in Figure B.1 that $N'_p(t^*)$ presents strong fluctuations with multiple peaks per revolution. As discussed in Chapter 4, these fluctuations are directly linked to the unsteady motion of trailing vortices and up-and-down swinging motion of the radial jet, at a frequency $f' = 3$ at $Re = 1600$, for both regular and fractal impellers. The amplitude of these fluctuations are weaker for $N'_\varepsilon(t^*)$ (green curve). The instantaneous difference between impeller power and integral dissipation is equal to the time derivative of the kinetic energy integrated over the tank volume, as the energy balance dictates. A detail, admittedly difficult to notice in Figure B.1, is that there is a time-lag between the peaks of the two quantities. In other words, the peaks in the dissipation follow the peaks in the impeller power with a certain time delay. This delay can be evaluated with the help of the two-time cross-correlation function between the two signals:

$$Cor(\tau) = \frac{\langle N'_p(t^*) N'_\varepsilon(t^* + \tau) \rangle}{\sqrt{\langle N'_p(t^*)^2 \rangle \langle N'_\varepsilon(t^*)^2 \rangle}}, \quad (\text{B.1})$$

where angular brackets $\langle \rangle$ represent the time-averaging operation. The correlation $Cor(\tau)$ is plotted in Figure B.2a for both impeller types. The value of τ , at which $Cor(\tau)$ attains the global maximum value characterizes the time-lag between $N'_p(t^*)$ and $N'_\varepsilon(t^*)$. A closer look at the peaks is provided in Figure B.2b. These peaks appear at $\tau_r = 0.051$ for the regular impeller and $\tau_f = 0.026$ for the fractal impeller. In Figure B.2b, these are indicated with vertical dashed lines in the same color as the corresponding curves. It is noteworthy that τ_r is twice as large as τ_f .

These time-lags characterise the time required for the kinetic energy to cascade from the injection length-scales to the smallest scales where it dissipates. In our cases, highest turbulence production is observed in trailing vortex cores and these represent the most energetic scales in the flow. Taking this into account, the time-lag of the dissipation may be longer for the regular impeller due to the larger scales at which the energy is injected. It is also remarkable, though, that $Cor(\tau)$ presents a much more distinct peak for the fractal impeller than for the regular one. The fractal impeller seems to sharpen the cascade time around a particular

value whereas the range of cascade times appears to be quite wide for the regular impeller. This observation will require a dedicated future study of its own.

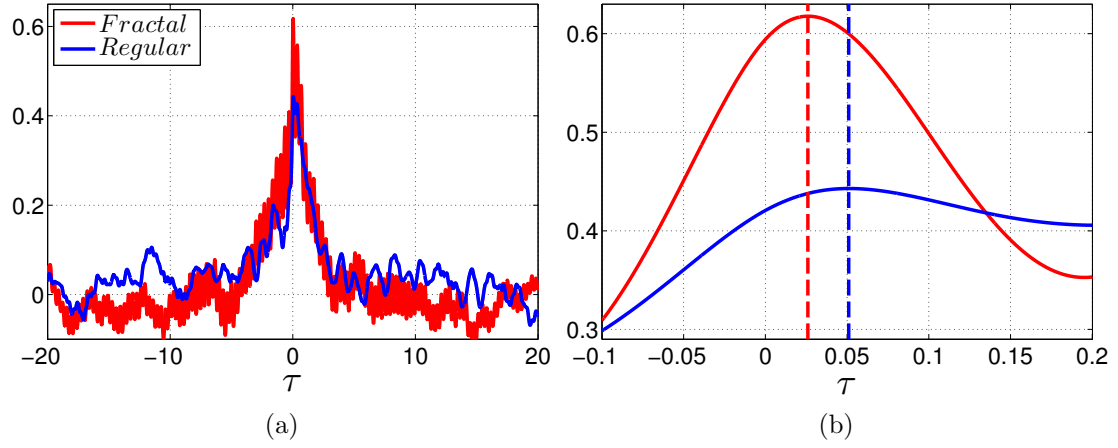


Figure B.2: Time-correlation $Cor(\tau)$, defined in Equation B.1, plotted against τ . a) Comparison between regular and fractal impellers, b) zoom on the peaks.