

Research Paper

Modulation of droplet trajectory using oscillating electric fields

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ABSTRACT

The use of sinusoidal electric forces to modulate the location of charged fuel droplets in a channel flow configuration is studied as a potential technology to achieve control over fuel–air mixing. Trajectories of a single non-evaporating droplet are first investigated using both an analytical solution based on a linearised drag model and results obtained with a non-linear model for the drag force. Non-dimensional quantities are also introduced to generalise the problem and provide useful information for technology development. Then, the effects of oscillating electric fields on the dispersion and stratification of fuel vapour are investigated using large-eddy simulations. Kerosene is used as fuel, which is relevant for aviation applications. Results show that oscillating electric fields perpendicular to the bulk flow can significantly enhance the spreading of both droplets and fuel vapour, an effect that is more evident with decreasing angular frequency of the oscillation and decreasing flow bulk velocity. The effects of oscillating electric forces on the spreading of droplets become negligible above a given threshold of the angular frequency of the oscillations. Non-dimensional analysis shows that this threshold depends on both the intensity of the electric force and the gas-phase properties, in particular the dynamic viscosity. The distribution of droplets in the channel is characterised by regions of preferential concentration, where fuel vapour mass fraction tends to be higher. Results also demonstrate that the time-averaged local fuel–air mixing is significantly affected by the period of oscillations, evaporation timescale and residence time in the channel. The present investigation further supports the proposal of using electric forces to improve fuel–air mixing in liquid-fuelled combustors, opening up new possibilities for fuel-preparation systems based on electrohydrodynamics.

1. Introduction

To address the urgent demand of transitioning to net zero-carbon aviation and transportation, the possibility of using a variety of different fuels in the same combustion system is becoming crucial. A combustion system able to perform at high efficiency with a variety of fuels is usually referred to as ‘fuel-flexible’. A fuel-flexible system could enable a rapid implementation of newly developed fuels (e.g., sustainable aviation fuels and e-fuels), ensuring a fast transition to the best fuel options to achieve environmental sustainability (Blakey et al., 2011; Kousoulidou and Lonza, 2016; Masri, 2021; Undavalli et al., 2023; Su-ungkavatin et al., 2023). Combustion systems currently used in the aviation sector are quite far from being fuel-flexible. This is mainly because the injection and atomisation of liquid fuel, and subsequent evaporation and mixing, are achieved through complicated injector configurations, which are very sensitive to changes in fuel properties. For example, in injection systems based on pre-filming airblast technology (e.g., Holz et al. (2019), Andreini et al. (2014)), the air flows contribute to both flame stabilisation and atomisation, and are designed to optimise the fuel preparation (i.e., all processes that lead to the

final distribution of fuel vapour) and emissions characteristics. Even small changes in fuel properties could alter the droplet size distribution (Vankeswaram et al., 2023; Undavalli et al., 2023), evaporation timescales and subsequent mixing and flame characteristics, possibly requiring changes in the injection system or combustor geometry to achieve the required emission performance (Sundararaj et al., 2019; Undavalli et al., 2023). To avoid significant changes in the injection and combustor configurations, research on alternative fuels is mainly looking at synthetic fuels with properties similar to kerosene and at the use of blends of kerosene with sustainable fuels, e.g., biofuels (Masri, 2021; Su-ungkavatin et al., 2023).

With the aim of improving atomisation and dispersion of liquid droplets, the injection of fuel droplets with a charge has been proposed (e.g., Gomez (1993), Yule et al. (1995)). Starting from early studies on electrospray ionisation (Fenn et al., 1989; Gomez and Tang, 1994), research has focused on both electrostatic atomisation of typical fuels used in transportation (e.g., Shrimpton and Yule (1999)) and combustion of charged droplets and sprays (e.g., Kyritsis et al. (2004),

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Anderson et al. (2008), Ahmed et al. (2022)). Injecting fuel droplets with a charge is becoming an established technology (e.g., Kourmatzis and Shrimpton (2012), Kourmatzis et al. (2012, 2010), Al-Ahmad et al. (2009)). Charge injection atomisation (Shrimpton and Yule, 1999; Kelly, 1984), which is the most suitable technology for dielectric liquids such as diesel and kerosene, has been demonstrated to be able to deliver mass flow rates suitable for transportation applications, with possibility to control the amount of charge injected in the spray (Shrimpton and Yule, 1999; Rigit and Shrimpton, 2006b,a; Kourmatzis and Shrimpton, 2014). Charged droplet atomisation is typically performed by applying a DC voltage, although the atomisation with time-varying currents has also been investigated (Maheshwari et al., 2009; Kourmatzis and Shrimpton, 2011), highlighting the potential effect of the frequency of oscillating fields or pulses on the atomisation characteristics. More recently, Fredrich et al. (2023) have investigated the use of charged droplets in combination with external electrostatic forces to control the location of fuel droplets and offer an additional degree of freedom to improve mixing. Fredrich et al. (2023) studied the mixing of a spray in cross-flow with external electric forces acting on the droplet both normal and parallel to the bulk velocity of the carrier phase. Results demonstrated that for relatively low flow bulk velocities, the trajectory of the fuel droplets can be significantly changed. Also, in the case of external electric forces opposite to the flow bulk velocity, it is possible to achieve quasi-stabilisation of the droplets, which implies a long residence time to complete evaporation in a confined region of the system (Fredrich et al., 2023). Giusti and Fredrich (2024) further assessed this potential technology by exploring the behaviour of different fuels and also assessing to what extent secondary breakup induced by charge repulsion within the droplet (instabilities triggered at the onset of the so-called Rayleigh limit (Lord Rayleigh, 1882)) could be significant. The configuration with an external electrostatic force parallel to the bulk flow was further investigated (Giusti and Fredrich, 2024), and it was demonstrated that droplet stabilisation and fuel evaporation in a confined space could be achieved with different fuels with no changes (or relatively small changes) in the strength of the external electric field. It was also remarked that the main area of uncertainty for a robust modulation of the droplet trajectories is the accurate evaluation of the net charge in the droplets after atomisation, which directly affects the electric forces, and the relation between charge and droplet size distributions (Rigit and Shrimpton, 2009). It is important to note that a configuration with external electric force parallel to the bulk flow poses technical challenges, since it implies the use of perforated plates to impose the potential difference (Kyritsis, 2020). In addition, it should be noted that stabilising the droplets close to the injection location is beneficial to fully pre-evaporate the fuel but it does not necessarily enhance the mixing rate of the vapour with air.

Previous research on spray mixing enhanced by the use of electric forces to control the trajectory of droplets has mainly focused on the use of external electric fields that are constant in time (Fredrich et al., 2023; Giusti and Fredrich, 2024). However, there is also a possibility of creating electric fields that change in time to further modulate the force acting on the droplets (after atomisation) and enhance the degree of control over the droplet trajectories. With the overall aim of further advancing the understanding of electromagnetic control of charged fuel droplets, the present work investigates the trajectory and evaporation of charged fuel droplets in a channel flow configuration in the presence of time-varying electric fields. Kerosene is considered as fuel, which is relevant for aviation applications. The objectives of the work are to: (i) investigate the effect of sinusoidal electric fields on the trajectory of charged droplets for varying frequency and amplitude of the electric field; (ii) evaluate the effect of sinusoidal electric forces on fuel vapour distribution and quality of mixing. The first objective will be addressed by means of analytical solutions of the dynamics of single droplets and by using a deterministic model to consider a non-uniform gas-phase velocity and to include non-linearities in the formulation of the drag force. As far as the second objective is concerned, large-eddy

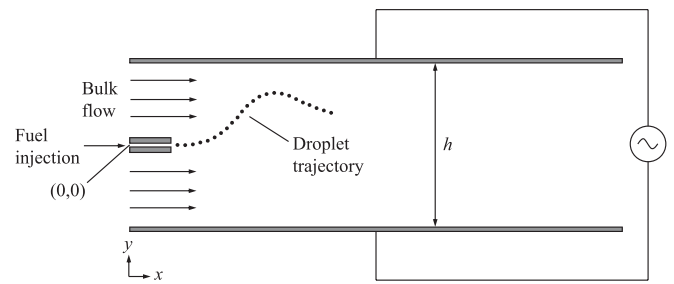


Fig. 1. Schematic of the setup for the modulation of the trajectory of charged droplets by means of time-varying electric fields. In this study, the potential difference applied across the two plates is alternating with a sinusoidal profile. The Cartesian frame of reference, with origin 25 mm upstream of the injection location, used throughout the paper is also shown.

simulations of evaporating droplets are used to compute the vapour distribution. For both objectives, electric forces normal to the flow bulk velocity are considered, which, with reference to a channel flow, can be easily achieved by applying a potential difference between the walls that confine the flow.

The paper is organised as follows. The reference configuration is first presented, followed by the description of the methods used for the investigation. Then, results are presented and discussed. A summary of the main outcomes and conclusions of the present study closes the paper.

2. Configuration

To investigate the effect of a time-varying electric field on the trajectory, evaporation and mixing of charged droplets, a spray injected in the centre of a gas flow between two parallel plates is considered, as schematically shown in Fig. 1. The two plates are separated by a distance h , and a time-varying potential difference is applied to them such that the resulting electric field is in the direction normal to the plates (the electric potential on each plate is uniform). The aim of this configuration is to obtain curvilinear droplet trajectories to increase their spreading in the vertical direction. In addition, the resulting curvilinear trajectory of the droplets could facilitate the evaporation dynamics, without the necessity of an external electric field parallel to the bulk flow, as suggested in previous work (Fredrich et al., 2023; Giusti and Fredrich, 2024), to complete evaporation within a short space. The ultimate goal is to enable a more uniform distribution of the fuel vapour and an increased control over the stratification of the fuel vapour–air mixture at the exit of the channel.

3. Methods

The trajectory of non-evaporating droplets is studied using a deterministic model based on a balance of forces applied to a single droplet. The details of this model are discussed below, followed by an analytical solution of the droplet trajectory for time-varying sinusoidal electric fields. Then, the large-eddy simulation (LES) model is presented, which is used to study spray evaporation and effects of external electric forces on air–fuel vapour mixing.

3.1. Deterministic model

A two-dimensional deterministic model to compute the trajectory of a non-evaporating single droplet is developed from Newton's second law (applied to the droplet), under the assumption that the only two forces acting on the droplet are the aerodynamic drag and the Lorentz force:

$$\frac{du_d}{dt} = \frac{1}{m_d} (F_D + F_L), \quad (1)$$

where $\mathbf{u}_d$ is the droplet velocity, t is the time, m_d is the mass of the droplet, $\mathbf{F}_D$ and $\mathbf{F}_L$ are the drag and Lorentz forces, respectively. The Lorentz (electromagnetic) force on the droplet is given by:

$$\mathbf{F}_L = q_d (\mathbf{E} + \mathbf{u}_d \times \mathbf{B}). \quad (2)$$

where q_d is the net charge of the droplet, $\mathbf{E}$ is the applied electric field and $\mathbf{B}$ is the magnetic flux density. The drag force is modelled as:

$$\mathbf{F}_D = \frac{\pi}{8} \rho_g C_D \mathbf{u}_{\text{rel}} |\mathbf{u}_{\text{rel}}| d^2, \quad (3)$$

where ρ_g is the density of the gas, d is the droplet diameter, $\mathbf{u}_{\text{rel}}$ is the velocity of the gas-phase relative to the droplet, and C_D is the drag coefficient, which is evaluated as follows (Faeth, 1983; Fredrich et al., 2023):

$$C_D = \begin{cases} 0.424, & \text{if } \text{Re}_d > 1000, \\ \frac{24}{\text{Re}_d} \left(1 + \frac{\text{Re}_d^{2/3}}{6} \right), & \text{otherwise.} \end{cases} \quad (4)$$

The drag coefficient is a function of the droplet Reynolds number, $\text{Re}_d = \rho_g |\mathbf{u}_{\text{rel}}| d / \mu_g$, with μ_g being the dynamic viscosity of the gas. Under the assumption of a non-evaporating droplet, Eq. (1) can then be integrated in time to obtain the trajectory of the droplet. For the investigations presented in this paper, the magnetic component of the Lorentz force is neglected (i.e., it is assumed $\mathbf{B} = 0$). The negligible effect of a magnetic field on the droplet trajectory in configurations of practical interest and for realistic values of the magnetic field is further discussed in Appendix C. The electric field is assumed to vary in time with a sinusoidal profile, such that:

$$\mathbf{E} = \bar{E} \sin(\omega t - \theta) \hat{\mathbf{e}}_E, \quad (5)$$

where $\bar{E}$, ω and θ are the amplitude, angular frequency and phase shift of the electric field, respectively; $\hat{\mathbf{e}}_E$ is a unit vector representing the direction of the electric field, which in this work is taken normal to the plates and pointing upward (i.e., a positive electric field is aligned with the y -direction of the Cartesian coordinate system shown in Fig. 1).

3.2. Dimensionless parameters

The trajectory of the droplet generally depends on the bulk velocity (U_b) and thermophysical properties of the carrier phase, on the density (ρ_d), diameter and injection velocity of the droplet, as well as on the characteristics of the external force. For the configuration shown in Fig. 1 and for a given droplet diameter, the external electric force is determined by the droplet charge density, $\rho_{q,d} = 6q_d/(\pi d^3)$, and the characteristics of the external electric field, i.e. amplitude, angular frequency and phase shift (see Eq. (5)). By non-dimensional analysis, it is possible to reduce the number of variables to a smaller set of non-dimensional quantities. The analysis presented in this paper for non-evaporating droplets will make use of the non-dimensional quantities reported in Table 1. For evaporating droplets, a non-dimensional evaporation time should also be considered as further discussed in Section 3.5. Also, note that if a spray is considered in place of a single droplet, additional non-dimensional quantities may be required to include physical phenomena present in a spray (e.g., droplet repulsion).

3.3. Analytical model

If the drag force depends linearly on the velocity and the carrier-phase velocity is assumed uniform, the three components of Newton's second law applied to the droplet are independent, and Eq. (1) can be solved analytically to find the droplet trajectory. To linearise the problem, the drag force is estimated using Stokes' law:

$$\mathbf{F}_D = 3\pi\mu_g d \mathbf{u}_{\text{rel}}. \quad (6)$$

It should be noted that Stokes' law is only valid for small droplet Reynolds number, Re_d . Therefore, the accuracy of the analytical solution is expected to increase with decreasing Re_d . By considering an

Table 1

Dimensionless groups used for the analysis of the trajectory of non-evaporating droplets with a sinusoidal external electric field in the configuration shown in Fig. 1. U_b , ρ_g and μ_g indicate the flow bulk velocity, density and dynamic viscosity, respectively, of the carrier phase; d , ρ_d and $\rho_{q,d}$ represent diameter, density and charge density of the droplet; h is the height of the channel; U_0 and V_0 are the horizontal and vertical components of the droplet injection velocity; x_0 is the coordinate of the injection location along the x -axis; $\bar{E}$ and ω are the amplitude and angular frequency of the sinusoidal electric field; t represents the time and x_d is the coordinate of the droplet location along the x -axis. In the analysis performed with both the deterministic and analytical models, injection is assumed to be at $t = 0$. Note that the phase shift, θ , could also be added to the list of parameters.

Symbol	Relation	Comments
Re_b	$\frac{\rho_g h U_b}{2\mu_g}$	Bulk flow Reynolds number
Π_ω	$\frac{\rho_d \omega d^2}{\mu_g}$	Non-dimensional angular frequency
Π_E	$\frac{\bar{E} \rho_{q,d} \rho_d d^3}{\rho_g^2}$	Non-dimensional electric force
Π_{U_0}	$\frac{U_0}{U_b}$	Non-dim. droplet injection velocity in x -direction
Π_{V_0}	$\frac{V_0}{\omega d}$	Non-dim. droplet injection velocity in y -direction
Π_ρ	$\frac{\rho_d}{\rho_g}$	Density ratio
Π_d	$\frac{d}{h}$	Non-dimensional droplet diameter
$\tilde{t}$	ωt	Non-dimensional time
$\tilde{x}_d$	$\frac{x_d \omega}{U_b}$	Non-dim. droplet location along the x -axis
$\tilde{x}_0$	$\frac{x_0 \omega}{U_b}$	Non-dim. injection location along the x -axis

electric field in the form given in Eq. (5) and assuming a negligible magnetic field, the resulting linear equation can be solved to find the position of the droplet with respect to time. With reference to Fig. 1, an electric field aligned with the vertical direction, y , normal to the bulk flow, is considered. In addition, a uniform horizontal flow velocity (equal to U_b , i.e. boundary layer effects are neglected) is assumed. Droplets are injected at $t = 0$ in the centre of the channel, i.e. at $y = 0$, at a location x_0 along the x -axis, with a horizontal component of velocity equal to U_0 and a vertical component of velocity V_0 . The solution of the equation for the location of the droplet in the horizontal, x_d , and vertical, y_d , directions gives:

$$\tilde{x}_d - \tilde{x}_0 = \tilde{t} - \frac{1}{a} \Pi_\omega \left(1 - \Pi_{U_0} \right) \left[1 - \exp \left(-\frac{a \tilde{t}}{\Pi_\omega} \right) \right], \quad (7)$$

$$\begin{aligned} \frac{y_d}{d} = & \frac{\Pi_E}{a \Pi_\omega^3 + a^3 \Pi_\omega} \left[(a^2 + \Pi_\omega^2) \cos \theta \right. \\ & - (\Pi_\omega \cos \theta + a \sin \theta) \Pi_\omega \exp \left(-\frac{a \tilde{t}}{\Pi_\omega} \right) \\ & \left. - a \Pi_\omega \sin (\tilde{t} - \theta) - a^2 \cos (\tilde{t} - \theta) \right] \\ & + \frac{1}{a} \Pi_\omega \Pi_{V_0} \left[1 - \exp \left(-\frac{a \tilde{t}}{\Pi_\omega} \right) \right], \end{aligned} \quad (8)$$

with $a = 18$ and all the other dimensionless quantities defined as in Table 1. The dimensional solution of the droplet trajectory is further detailed in Appendix A. Note that in the limit of linearised drag force, the vertical electric force does not affect the droplet location in the x -direction, which is dependent only on the horizontal component of the injection velocity and flow bulk velocity. In addition, since the initial velocity of the droplet has no components in the z -direction and both the external force and the gas flow velocity also have no components in this direction, the droplet trajectory is in a plane normal to z (i.e., the droplet does not move along z).

It is also of interest to study the trajectory of the droplet a long time after injection (i.e., the 'steady-state' behaviour of the system), which is obtained from the analytical solution by taking the limit of large t . The droplet movement after the initial transient can be characterised by the amplitude of its oscillations, A , and the offset of these in the y -direction from the injection position. The symbol Δy_0 will be used in the

following to indicate this offset. Starting from the analytical solution, Eq. (8), and assuming $\Pi_{V_0} = 0$, these quantities can be calculated as:

$$\frac{A}{d} = \frac{|\Pi_E|}{a\Pi_\omega^3 + a^3\Pi_\omega} \left(a\sqrt{a^2 + \Pi_\omega^2} \right), \quad (9)$$

$$\frac{\Delta y_0}{d} = \frac{\Pi_E}{a\Pi_\omega^3 + a^3\Pi_\omega} (a^2 + \Pi_\omega^2) \cos \theta. \quad (10)$$

Note that the amplitude of the oscillations does not depend on the phase shift, whereas the absolute value of the offset is maximum for $\theta = 0$ and π and equal to zero in case of $\theta = \pi/2$ and $3\pi/2$.

3.4. Large-eddy simulations

Simulations of droplet evaporation in a channel flow are performed using the large-eddy simulation approach, with the same methodology discussed in Fredrich et al. (2023). A modified version of OpenFOAM-v7 is used for the simulations (Fredrich et al., 2023). The sub-grid stress tensor is modelled using the constant Smagorinsky model, with van Driest damping function (van Driest, 1956) for near-wall treatment. As far as droplets are concerned, a two-way coupled Eulerian–Lagrangian approach is used. The droplets are assumed to be spherical with only drag and external electric forces acting on them. All the other forces as well as secondary breakup, including breakup of droplets induced by the effect of charge repulsion within the droplet, are neglected in the present analysis. Drag is computed with the same formulation used for the deterministic model for non-evaporating droplets discussed in Section 3.1, with the droplet Reynolds number evaluated with the film properties (Yuen and Chen, 1976) to take into account the effect of evaporation on the aerodynamic interactions. Evaporation is modelled using a rapid mixing model (Miller et al., 1998), with Ranz–Marshall correlation for droplet heat transfer (Ranz and Marshall, 1952; Miller et al., 1998). The presence of charges in the droplet is assumed not to affect the mass transfer at the droplet–gas interface (Shrimpton and Laonual, 2006). In addition, since secondary breakup phenomena are neglected, the increase in droplet evaporation rate due to charge-induced fragmentation as the droplet reaches the Rayleigh limit is not included in the model. Charge-induced secondary breakup has been shown to have a relatively small effect on the total evaporation time since the mass lost by the droplet during the breakup tends to be small (Shrimpton and Laonual, 2006; Kourmatzis, 2017). However, further work is needed to assess the breakup mechanism in the case of charged droplets with a relatively high relative velocity with respect to the bulk flow. Note also that we assume that the charge of the droplet stays constant during the entire evaporation process. Charge-induced breakup generally leads to a loss of charge of the parent droplet which could affect the trajectory under external electric fields, a phenomenon that should be investigated in future work. As far as the electrostatic force is concerned, the electric potential field, ϕ , is first solved with an Eulerian approach and then used to compute the electric field necessary to determine the Lorentz force (see Fredrich et al. (2023) for details). In all the simulations reported in this study, electric repulsion forces between droplets have been neglected. These forces are small for the fuel flow rate and droplet charge investigated here. However, they may not be negligible for higher fuel flow rates or higher values of net charges of the droplets.

Equations of motion are integrated using second-order discretisation schemes in space and the Crank–Nicolson scheme for time discretisation. A second-order accurate scheme is also used to solve the electric potential field, whereas first-order explicit schemes are used for the droplet motion. The simulation domain reproduces the configuration in Fig. 1, and is illustrated in Fig. 2. A channel of length, L , equal to 0.3 m is considered, with parallel plates separated by a distance, h , of 0.1 m. The width of the channel is 0.1 m. The domain is discretised with a hexahedral mesh of $150 \times 50 \times 50$ cells. Uniform cells size is imposed along the x and z directions, while the height of the cells along the y -direction decreases from the centre of the channel towards the walls.

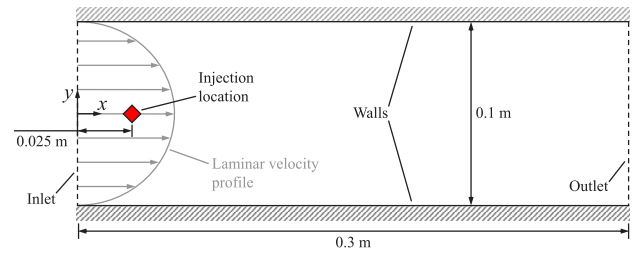


Fig. 2. Schematic of the computational domain used for large-eddy simulations. Relevant dimensions are directly indicated in the diagram, together with boundaries and the frame of reference used for post-processing. The red marker indicates the location where droplets are injected. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

A fully developed velocity profile is applied at the inlet boundary with no turbulent fluctuations. In each simulation, a time step that ensures a maximum Courant number lower than 0.8 is adopted. The droplet injection location is placed 0.025 m downstream of the inlet boundary, at mid-distance between the two plates and at mid-distance between the two lateral boundaries. The injector is assumed to be infinitely small compared to the height of the channel and therefore not able to alter the velocity profile. This assumption is considered sufficient for the scope of this paper, which aims to provide an assessment of the capability of time-varying electric fields to improve the dispersion of droplets. The modelling of the velocity profile and injection configuration should be improved in the future, once the geometry of the injector is better defined. Walls are modelled as adiabatic with no-slip condition. Pressure is imposed at the outlet of the domain with a wave-transmissive boundary condition. Periodic conditions are used for the lateral sides of the domain. Droplets are assumed to bounce if they impact the walls. The external electric field is modelled by imposing a uniform time-varying potential on the upper plate of the channel, while keeping the bottom plate at ground potential, i.e. $\phi = 0$ on the bottom plate. Therefore, the potential imposed on the top plate has the expression: $\phi = \Delta\bar{\phi} \sin(\omega t)$, with $\Delta\bar{\phi}$ such that $\Delta\bar{\phi}/h = \bar{E}$. All simulations are initialised with a fully developed velocity field. Simulations are advanced for 5 flow-through times (evaluated as the length of the channel divided by the flow bulk velocity) after droplet injection before initialising the computation of flow field statistics (time averages and root-mean-square of the fluctuations). Statistics are collected over 10 flow-through times.

3.5. Gas-phase and fuel conditions

In all computations performed in this work, the gas phase consists of pre-heated air at atmospheric pressure, $p_g = 101325$ Pa, with a temperature of 600 K. The fuel chosen for this study is kerosene, a fuel relevant for aviation applications. At these gas-phase conditions, the fuel evaporation rate constant, K , in quiescent conditions, evaluated from d²-law approximation using the same methodology and code used in Fredrich and Giusti (2022), is $0.176 \text{ mm}^2/\text{s}$ (see Fig. 3). This value of the evaporation rate constant is used in the following to estimate the evaporation timescale.

For large-eddy simulations, a fuel flow rate of 0.05 g/s is imposed. Droplets have an initial diameter, d_0 , of $50 \text{ }\mu\text{m}$ (a monodisperse distribution is assumed) and a charge density of 1 C/m^3 . Note that the charge density given by the Rayleigh limit could be even higher for this droplet size (Giusti and Fredrich, 2024). The evaporation timescale corresponding to the selected initial droplet diameter, evaluated as $\tau_{\text{ev}} = d_0^2/K$, is 14.2 ms. Two different flow bulk velocities, U_b , are investigated. The values of U_b are selected such that the evaporation timescale of the droplets is a fraction of the residence time of the droplets in the investigated domain, i.e. $\tau_{\text{ev}} U_b / L < 1.0$. Values of $\tau_{\text{ev}} U_b$ equal to $L/30$ and $4L/30$ are selected, which correspond to a flow bulk

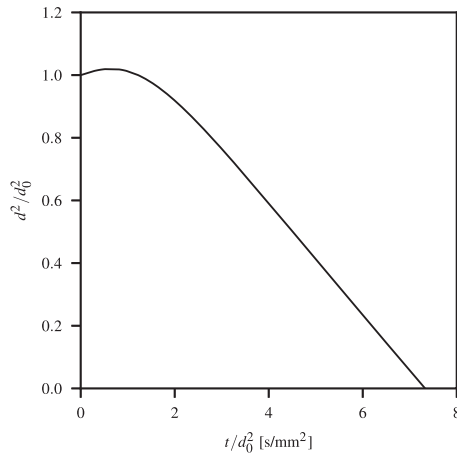


Fig. 3. Time evolution of the square of the droplet diameter computed with the approach discussed in Fredrich and Giusti (2022) for a kerosene droplet in air at $p_g = 1$ atm and $T_g = 600$ K, and initial droplet temperature of 300 K.

velocity of 0.71 m/s and 2.82 m/s, respectively. These velocities correspond to a bulk flow Reynolds number, Re_b (based on $h/2 = 0.05$ m, see also Table 1), of 687 and 2750, respectively. These values are below the critical Reynolds number for Poiseuille flow (Orszag, 1971), so a laminar velocity profile is modelled, further justifying the choice of no velocity fluctuations imposed at the inlet boundary. Note that, since a laminar flow is simulated, in principle there is no necessity of turbulence modelling. However, interactions between droplets and the gas phase could lead to fluctuations of the velocity field of the carrier phase. Given the relatively low frequency of the electric forces (see below), the large-eddy simulation approach was considered adequate to capture the large-scale effects of the droplet motion on the carrier phase. Note also that the relatively coarse mesh used in this study was found sufficient to reproduce the main characteristics of the flow field. Simulations performed with a more refined mesh (not shown here) showed the same trends, with some differences on the distribution of the fuel vapour, which are mainly related to the effect of the grid size on the evaporation dynamics (Sontheimer et al., 2021). The fuel injection temperature is 300 K. The fuel injection velocity is set as horizontal with magnitude equal to the unladen fully developed flow at the centre of the channel (i.e., 1.06 m/s and 4.23 m/s, for the two flow bulk velocities selected for this study). Note that the fuel mass flow rate is the same for both flow bulk velocities; therefore, a more diluted spray and fuel vapour are expected in the case of higher air flow bulk velocity. For the sake of completeness, large-eddy simulations results for $U_b = 2.82$ m/s and same air-to-fuel ratio as in the case with $U_b = 0.71$ m/s are reported in Appendix B. The electric field is defined with $\bar{E} = 1 \times 10^6$ V/m. Cases with $\omega = 300, 500$ and 1000 rad/s are investigated. These values of angular frequency allow for a maximum vertical movement of the droplets smaller than $h/2$, which is important to avoid impingement of the droplets on the channel walls. This is further discussed in Section 4.1. Note that the selected angular frequencies of the electric field oscillations allow us to investigate cases with the period of the oscillations (equal to $2\pi/\omega$) either smaller or larger than the evaporation timescale. The non-dimensional numbers corresponding to these simulation conditions are summarised in Table 2 (the initial droplet diameter is used to characterise the size of the droplet).

4. Results

4.1. Non-evaporating droplets

The trajectory of a non-evaporating droplet computed using both the deterministic model (see Section 3.1) and the analytical solution

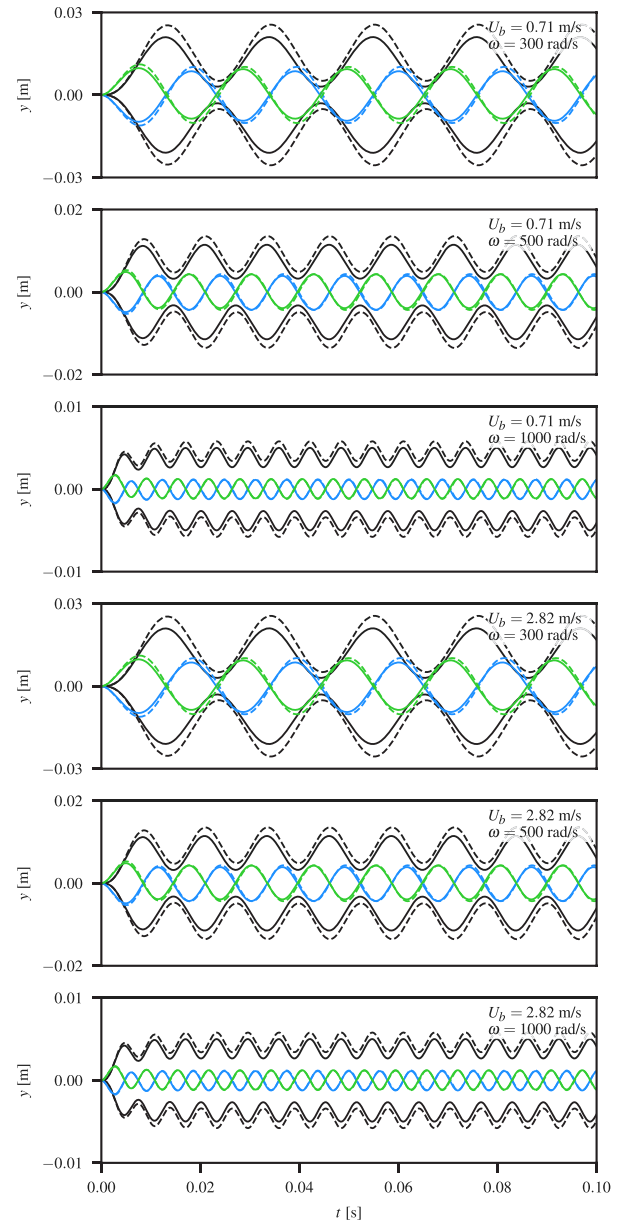


Fig. 4. Variation of droplet y position in time as calculated using the deterministic model (solid lines) and the analytical solution (dashed lines). The six cases shown correspond to the conditions given in Table 2. The black lines correspond to phase shifts of 0 and π , which give the maximum offset magnitude from the injection position, while $\theta = \pi/2$ and $\theta = 3\pi/2$ are illustrated in blue and green, respectively, for clarity. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(see Section 3.3) is first investigated. Figs. 4 and 5 show the trajectory of the droplet over time and space, respectively, for different values of flow bulk velocity and angular frequency of the oscillations, and a selected droplet charge density and amplitude of the electric field (same conditions as used in the large-eddy simulations). For the deterministic model, a laminar velocity profile has been used, whereas for the analytical model the flow bulk velocity is used for the carrier phase velocity (assumed uniform in the derivation). In all cases, droplets are injected horizontally with an initial velocity equal to the gas velocity at the injection point. Trajectories for different values of the phase shift angle, θ , are reported. Trajectories are observed to be sinusoidal, with the centre of oscillation being offset from the injection position for most values of phase shift. The offset varies depending on the phase

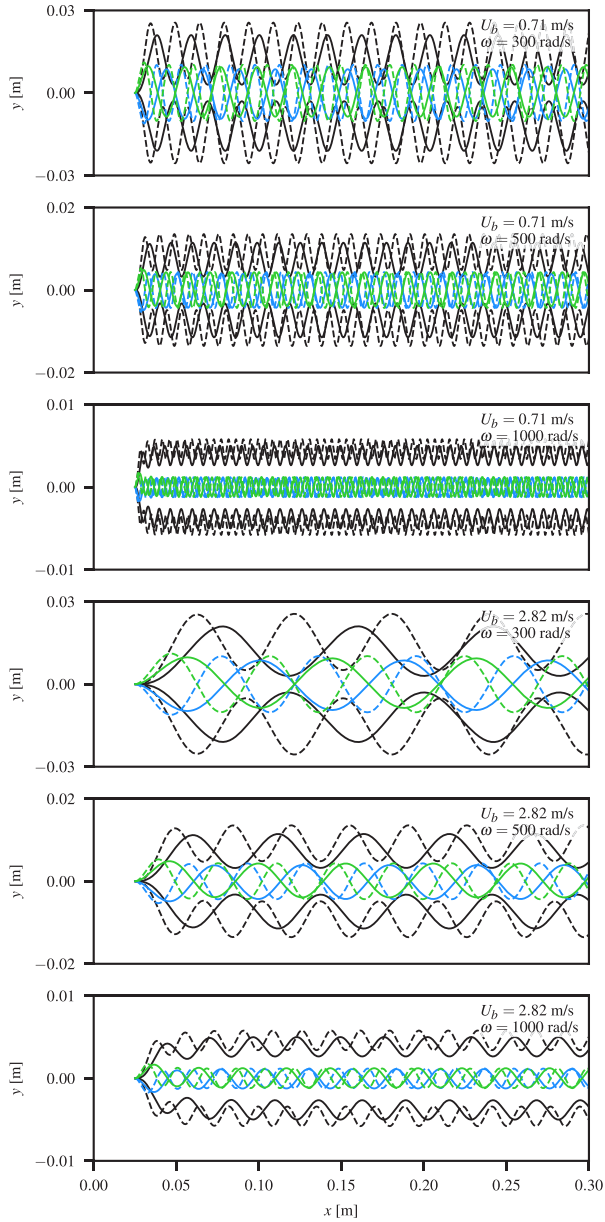


Fig. 5. Single droplet trajectories corresponding to the cases illustrated in Fig. 4. For the analytical model, the x position is determined using Eq. (7), which assumes a uniform velocity profile equal to the flow bulk velocity. The black lines correspond to phase shifts of 0 and π , which give the maximum offset magnitude from the injection position, while $\theta = \pi/2$ and $\theta = 3\pi/2$ are illustrated in blue and green, respectively, for clarity. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

shift of the external field when the droplet is injected, which gives the droplets a natural tendency to spread out across the height of the channel, as their steady-state oscillations are centred around different heights (both above and below the injection point). In all cases, the maximum displacement (absolute value) in the y -direction is obtained for droplets injected with $\theta = 0$ and π . This displacement, as well as the amplitude of the oscillations, tends to be higher when evaluated with the analytical solution compared to the prediction of the deterministic model. This suggests that in general the non-linear effects of the drag cannot be neglected. As far as the trajectory of the droplet in the x - y plane is concerned (see Fig. 5), the number of complete oscillations within the domain increases with decreasing flow bulk velocity. The solutions obtained with analytical and deterministic models show a

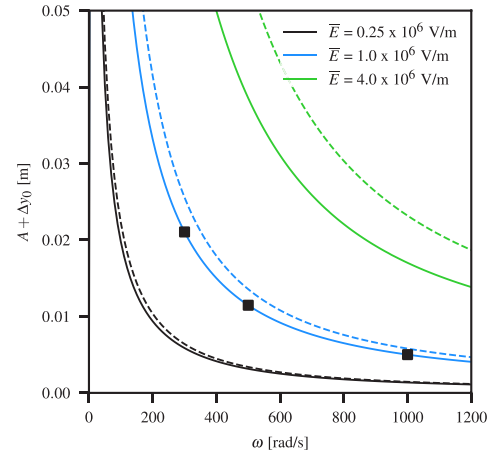


Fig. 6. Maximum droplet displacement in y from the injection location plotted against ω for various $\bar{E}$ in the case of $V_0 = 0$ and $U_b = 0.71$ m/s. Values calculated using the deterministic model and the analytical solution are shown with solid and dashed lines, respectively. The electric field conditions given in Table 2 are indicated with black squares.

Table 2

Conditions investigated in the large-eddy simulations: both dimensional and the corresponding non-dimensional quantities (as defined in Table 1) are given. The initial droplet diameter, d_0 , is taken as reference length scale for the droplet. The ratio between the evaporation timescale and residence time in the channel, $\tau_{ev} U_b / L$, as well as the ratio between the evaporation timescale and the period of oscillations of the electric field, $\tau_{ev} \omega / (2\pi)$, are also given. In total, six different cases have been simulated, corresponding to the combinations of flow bulk velocities and angular frequencies of the electric field oscillations.

Quantity	Values	Non-dim. quan.	Values
U_b , m/s	0.71; 2.82	Re_b	687; 2750
ω , rad/s	300; 500; 1000	Π_ω	20.2; 33.6; 67.1
$\bar{E}$, V/m	1×10^6	$\frac{\tau_{ev} U_b}{L}$	0.033; 0.133
$ \rho_{q,d} $, C/m ³	1	$\frac{\tau_{ev} \omega}{2\pi}$	0.68; 1.13; 2.26
d_0 , μ m	50	Π_E	111×10^3
$T_{d,0}$, K	300	Π_ρ	1376.6
U_0 , m/s	$\frac{3}{2} U_b$	Π_{U_0}	1.5
V_0 , m/s	0	Π_{V_0}	0
p_g , atm	1	Π_d	5×10^{-4}
T_g , K	600		

horizontal shift that tends to increase with increasing distance from the injection location. The trajectory predicted by the deterministic model is in general more elongated along the x -direction compared to the analytical solution. This is caused by the different flow velocity profiles assumed by each model: the uniform velocity profile equal to the flow bulk velocity used in the analytical solution has lower values of velocity in the central region of the channel, where the droplet trajectories are located, compared to the laminar velocity profile used in the deterministic model. These effects are more evident for higher values of the angular frequency, since the maximum displacement along y is lower for these cases, therefore making the droplets much more confined along the central region of the channel where the difference between the bulk velocity and the laminar profile is larger. This behaviour is similar for both flow bulk velocities. Overall, results suggest that the analytical solution provides a reasonable estimate of the vertical spreading of the droplets; however, for an accurate evaluation of the droplet location along the channel, a model that considers the actual velocity profile of the carrier phase is needed.

An increase in the strength of the electric force, obtained either by increasing the droplet charge density or by increasing the amplitude of the electric field, leads to an increase in the vertical movement of the droplets, with also some effects on the trajectory along x if a non-linear

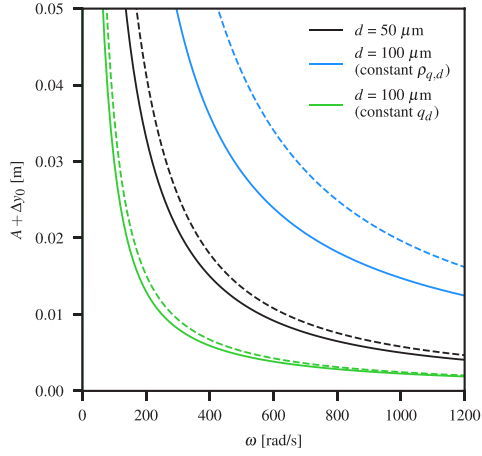


Fig. 7. Maximum droplet displacement in y from the injection location plotted against ω for $\bar{E} = 10^6$ V/m in the case of $V_0 = 0$ and $U_b = 0.71$ m/s. The droplet diameter is increased from a reference value of $50 \mu\text{m}$ to $100 \mu\text{m}$, assuming the droplet to have either a constant charge density or a constant net charge. Values calculated using the deterministic model and the analytical solution are shown with solid and dashed lines, respectively.

model for drag is considered. The effect on the amplitude and offset along y is demonstrated in Fig. 6, which shows the maximum vertical movement of the droplets as a function of the angular frequency. The conditions simulated in the large-eddy simulations are indicated by markers. Note that these conditions have been selected such that the maximum vertical movement of non-evaporating droplets is lower than half the height of the channel. The effect of an increase in droplet diameter is shown in Fig. 7 for the cases of constant net charge and constant droplet charge density. If the net charge is kept constant, an increase in the diameter leads to a decrease in both the amplitude of the oscillations and the vertical offset of the trajectory due to an increased drag force while the electric force stays the same. Conversely, if the droplet charge density is kept constant, both the amplitude of the oscillations and the offset of the trajectory increase, since the increase in electric force (due to the increase in net charge) is not balanced by an equal increase in drag force (Fredrich et al., 2023). These results are further summarised in Fig. 8 where the non-dimensional amplitude and vertical offset of the droplet trajectory are reported. Changes of thermophysical properties of the gas phase affect the oscillation of the droplet trajectory. In particular, an increase in the dynamic viscosity of the gas generally leads to lower amplitude of the oscillations and lower offset with respect to the channel centreline (note that both Π_E and Π_ω depend on μ_g , c.f. Table 1). It is important to remark that all results discussed in this section are based on one-way coupled models, i.e. there is no effect of the droplet on the gas phase. Also, note that if the droplet injection velocity has a component perpendicular to the bulk flow, i.e. V_0 is different from zero, much higher deviations between the analytical solution and the deterministic model are observed (not shown here) due to the high Reynolds number at injection which renders Stokes' law inapplicable. In addition, a component of the injection velocity perpendicular to the bulk flow moves the trajectories of the droplets in the direction of V_0 , an effect that increases with increasing value of the velocity component.

By assuming a continuous injection (e.g., this is the case in a gas turbine engine), i.e. a uniform distribution of the phase shift θ , it is possible to compute the probability to find a droplet at a given location in space. Fig. 9 shows such probability for the various cases investigated in this work. The results were obtained with the deterministic model and an imposed laminar profile across the height of the channel. The probability is characterised by peaks along the centre of the channel, which tend to correspond to the minima of the droplet trajectories for $\theta = 0$ and $\theta = \pi$ (see Fig. 5). These peaks are separated by regions of low

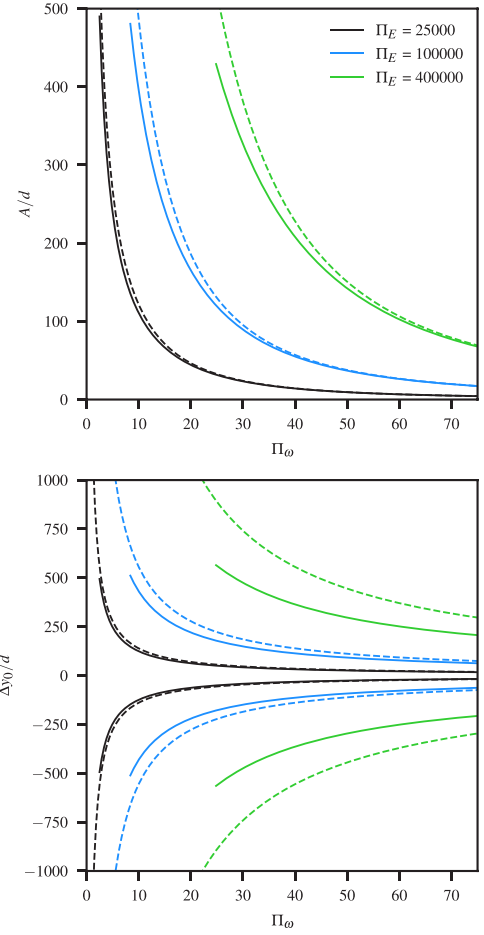


Fig. 8. Amplitude and maximum offset in y observed in the steady-state single droplet trajectory, both non-dimensionalised with the droplet diameter, plotted against Π_ω for $\Pi_{V_0} = 0$. Solid and dashed lines represent the deterministic model and analytical solution, respectively. For the deterministic model, values are computed from the droplet trajectory between $t = 0.1$ s and $t = 0.2$ s (where transient effects are negligible); results for the case $Re_b = 687$ and values of Π_{U_b} , Π_ρ and Π_d as reported in Table 2 are used. Note that for the deterministic model the solution exists only for $(A + |\Delta y_0|)/d < 10^3$, which implies a droplet confined within the channel's walls.

probability along the centre, which are confined by symmetric branches of higher probability with a sine-like shape. The distance between peaks decreases with decreasing flow bulk velocity and increasing angular frequency of the electric field.

4.2. Fuel vapour distribution

The vapour distribution in the domain, which was investigated using two-way coupled large-eddy simulations, is discussed next. Fig. 10 shows the instantaneous droplet locations for the cases investigated in this work. For reference, cases with no electric fields are also reported. Close to the injection location, the droplets oscillate with an amplitude similar to the values found using the analytical solution and the deterministic model (e.g., cf. Figs. 5 and 6). As the droplets move further downstream and become smaller due to evaporation, the trajectory deviates from the one computed for non-evaporating droplets. A smaller diameter leads to lower drag forces for the same gas–droplet relative velocity. Since the electric force acting on the droplets does not change between evaporating and non-evaporating cases (a constant droplet charge is assumed), a decrease in the diameter leads to an increase in the vertical movement of the droplets, i.e. to increasing amplitude of the oscillations as the droplets move downstream in the channel. The droplet trajectory tends to become less regular far from

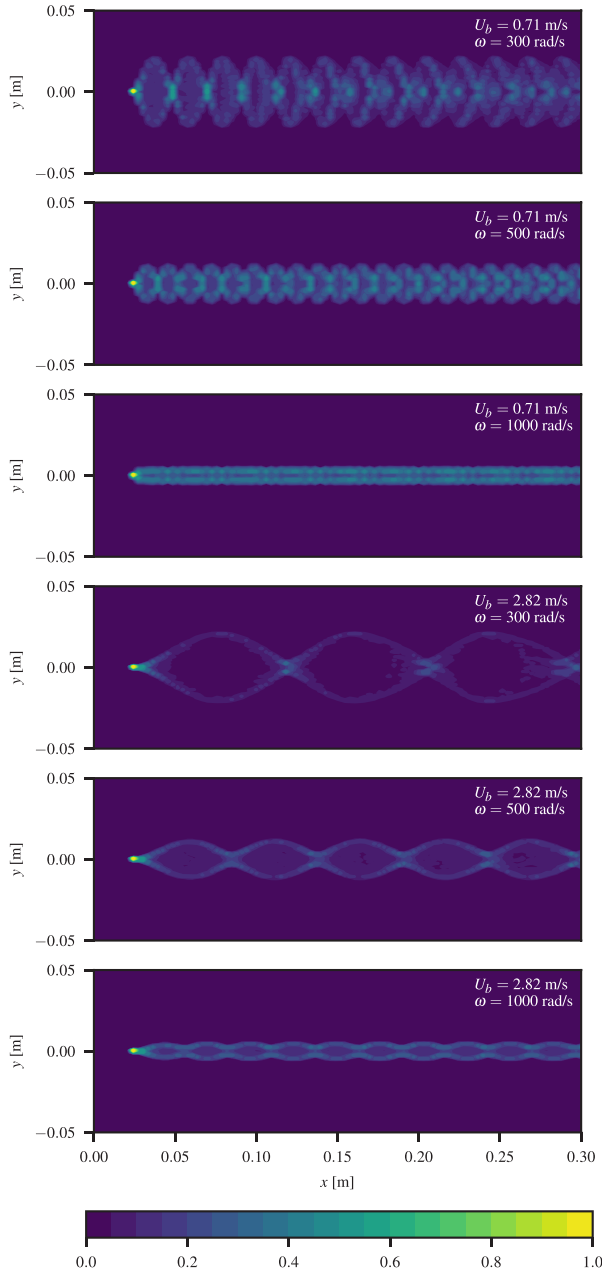


Fig. 9. Probability of a droplet trajectory passing through each point in space, calculated using the deterministic model for each of the cases given in Table 2. The probability was computed from a sample of 100 droplets with uniform distribution of the phase shift angle. For each location in space the number of droplets visiting such location is counted. The probability is given by the ratio between droplet count and total number of droplets in the sample.

the injection location. Note that in the LES simulations, the droplet trajectory is also affected by the local change in gas-phase density and velocity due to evaporation. Note also that the trajectory of droplets without an external electric field is simply a horizontal line in the centre of the domain.

It is also interesting to evaluate the droplet residence time for the various conditions investigated here. Fig. 11 shows the cross correlation between diameter and residence time (age of the droplet after injection) for the various cases investigated in this work. The residence time of the droplet in the domain significantly increases compared to the nominal evaporation timescale, τ_{ev} . This is due to two-way coupling effects between droplets and the gas phase. With the given fuel mass

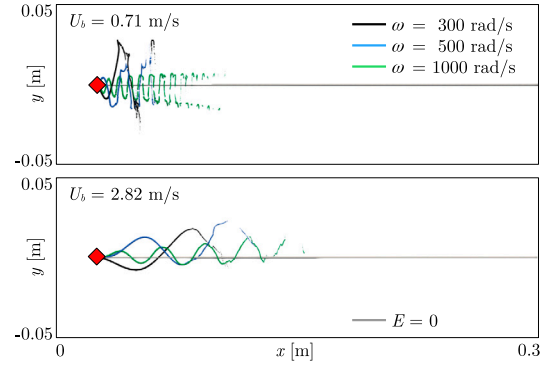


Fig. 10. Instantaneous spray location for the various conditions investigated in this study. For $U_b = 0.71$ m/s, droplets are sampled at $\omega t/(2\pi) \approx 300$; for $U_b = 2.82$ m/s, droplets are sampled at $\omega t/(2\pi) \approx 80$. For the case with no electric field, the droplet location after ten flow-through times is shown. The red marker indicates the injection location. The size of the droplets is scaled based on the droplet diameter. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

flow rate, the evaporation significantly changes the characteristics of the carrier phase, leading to an increase in the mass fraction of fuel vapour and a decrease in gas-phase temperature (see also Figs. 13 and 16) along the droplet trajectory. Both of these effects lead to a decrease in the evaporation rate, which is partly compensated for by the increase in evaporation due to a relative velocity between the droplet and the gas phase. As reference, Fig. 11 also shows the residence time of droplets from simulations without an external electric field, with and without two-way coupling. In case of two-way coupled simulations, the accumulation of fuel vapour and decrease in temperature along the centre of the domain lead to a relatively high evaporation time. The droplets exit the domain with a diameter that is very close to the injection value (note that the maximum residence time is equal to the flow-through time along the centre of the domain). If two-way coupling is deactivated, the correlation between residence time and droplet diameter simply follows the evaporation curve of a droplet in a carrier phase with constant composition and temperature, with values of total evaporation time similar to those evaluated for a droplet in a quiescent medium (i.e., Fig. 3). This further corroborates the reliability of the evaporation model used in the LES framework. Note also that the residence time, i.e. the total evaporation time, of the droplets increases with increasing frequency of the oscillation of the external electric field. This is consistent with the decrease in spread of the droplets with increasing ω , which leads to higher concentration of fuel vapour and lower temperature of the gas phase along the spray trajectory. Similarly, the droplet residence time (total evaporation time) decreases with increasing flow bulk velocity. This is consistent with higher dilution of the spray in the case with higher flow bulk velocity (it is reminded that simulations are performed with constant fuel mass flow rate), which leads to relatively smaller accumulation of fuel vapour and decrease in temperature of the gas phase along the spray trajectory.

Fig. 12 shows the time-averaged fuel vapour mass fraction and x -component of the gas-phase velocity in a streamwise cross section without the presence of an external electric field for the two flow bulk velocities investigated in the present work. The fuel vapour is concentrated along the centre of the domain with a time-averaged value that increases towards the outlet of the domain and negligible fluctuations (not shown here). If an oscillating electric field is applied, the spreading of the fuel vapour increases as a result of the modulation of the droplet trajectory. This is demonstrated in Figs. 13 to 16, which show the time-averaged values and root-mean-square (rms) of the fluctuations of the fuel vapour mass fraction, selected gas-phase velocity components and gas-phase temperature in the mid-plane normal to z for the various cases studied in this work. In general, the spreading of the fuel vapour

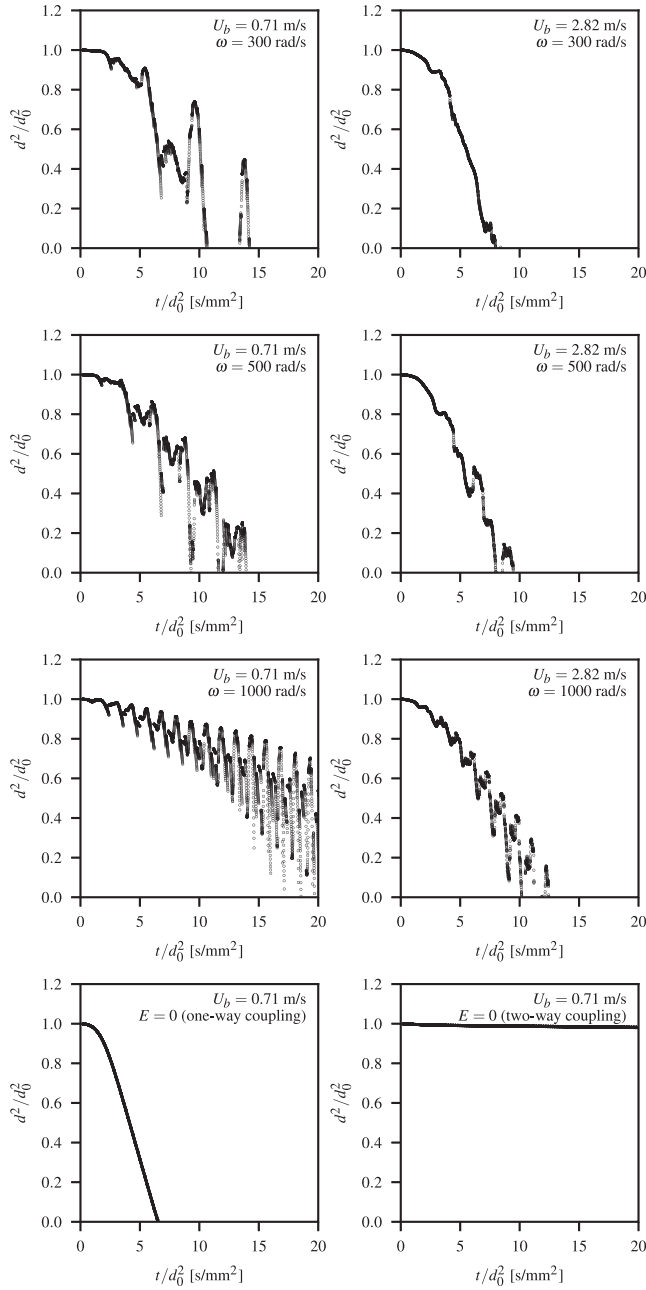


Fig. 11. Scatter plot of d^2 over droplet residence time (droplet age since injection) evaluated at the same time instants used for Fig. 10 for each of the cases given in Table 2. Cases with no electric field and one-way or two-way coupling between droplet and gas phase are also reported.

increases with decreasing angular frequency of the oscillations and decreasing flow bulk velocity as also illustrated in Fig. 17, which shows the fuel vapour distribution at the outlet of the domain, and Fig. 18, which shows the fuel mass fraction profile along y at the channel's exit. It should also be noted that low flow bulk velocities tend to give higher local peaks of time-averaged fuel mass fraction in the region close to the injection location. Since simulations were run at constant fuel mass flow rate, this is consistent with the decrease in fuel-to-air ratio with increasing flow bulk velocity. Results with constant fuel-to-air ratio are demonstrated in Appendix B. The peak value of fuel vapour mass fraction increases with increasing angular frequency of the electric field due to the lower amplitude of the oscillation of the droplet trajectory. The location of these peaks seems to be correlated with the areas

of high probability of finding droplets (cf. Fig. 9). In those regions, the fluctuations of the fuel mass fraction also tend to be maximum. Peaks either at the centre of the channel or in the areas of maximum deviation of the trajectory along the y -direction are observed. The exact location of those peaks is the result of a balance between period of the oscillations and evaporation time, which, as already observed, is strongly affected by the two-way coupling with the flow. Note also that in the case of low flow bulk velocity and low angular frequency of the oscillation, accumulation of fuel vapour close to the wall is observed. After evaporation is completed, the fuel vapour is simply advected by the flow and further diluted with air by the action of molecular and turbulent diffusion induced by velocity fluctuations (see Figs. 14 and 15). Analysis of the fuel vapour distribution at the exit of the domain, shown in Figs. 17 and 18, suggests that a more uniform distribution of the fuel over the entire cross section requires low-frequency oscillations of the electric field. Higher flow bulk velocities with constant fuel mass flow rate lead to a more dilute fuel vapour, which is also more confined in the region close to the centre of the channel.

It is important to note that the fluctuations of the droplets also lead to changes in the mean velocity profile and to fluctuations of all velocity components, which further contribute to the mixing of fuel vapour. These fluctuations are high close to the injection location (before full evaporation of the droplets), where the volume fraction of the droplets is higher, and their rms value tends to increase with decreasing angular frequency of the electric field oscillations. Changes in the mean flow are also more evident at low angular frequency of the electric field, which highlights how in general the system tends to damp high frequency oscillations. All these effects are strongly coupled with droplet trajectory, evaporation and dispersion of the fuel vapour. This is also evident from the temperature of the gas phase (see Fig. 16), which shows a significant decrease in the areas of fuel vapour release. This decrease in temperature affects the evaporation rate of droplets as well as the transport of momentum. Fluctuations of temperature and fuel vapour mass fraction generally correlate well with fluctuations of the velocity components. Note that this study was mainly focused on the effects of frequency of oscillation and flow bulk velocity on the droplet trajectory and fuel vapour distribution. As also indicated by the non-dimensional numbers introduced in the present work, there are multiple quantities that may further affect the droplet trajectory. To achieve control of the stratification of the fuel vapour, future work should focus on the development of a comprehensive dataset that allows all thermophysical properties of the carrier phase and characteristics of the electric field to be considered. Studies with different fuel mass flow rates, as well as investigations of the effects of droplet characteristics, fuel type, repulsion forces between droplets and spray cone angle (i.e., increased dispersion of the droplets at injection), should also be conducted. Interactions between the external electrostatic field used for trajectory control and the electric field used for atomisation, as well as the effect of charge-induced breakup on the trajectory of the droplets and fuel preparation, are other areas of investigation that require further attention. It is also worth noting that the flow bulk velocities studied in the present work are relatively low and far from velocities typically found in current engine technologies (e.g., gas-turbine engines). High flow bulk velocities decrease the residence time of the droplets in the channel and also extend in space the region affected by the initial transient of the droplets (affected by injection conditions) before achieving an oscillatory behaviour. Therefore, practical use of the proposed technology may require the development of new concepts for fuel preparation based on relatively low flow bulk velocities, which consider the space available in a given engine configuration and the evaporation timescales.

5. Concluding remarks

The effect of sinusoidal electric fields on the dispersion of charged droplets and fuel vapour in a channel flow configuration has been

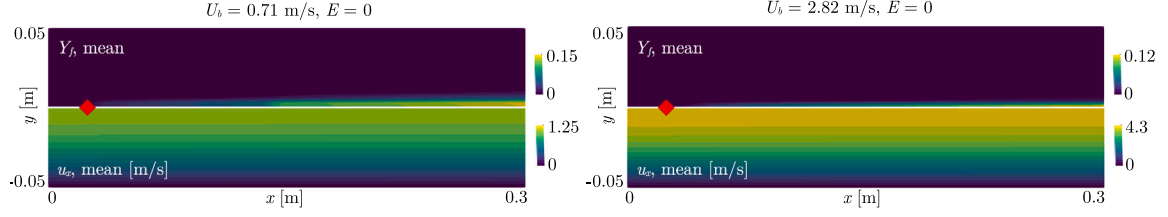


Fig. 12. Time-averaged fuel vapour mass fraction (Y_f) and x -component of the gas-phase velocity (u_x) in the mid-plane normal to z for the two flow bulk velocities investigated in the present work without external electric fields. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

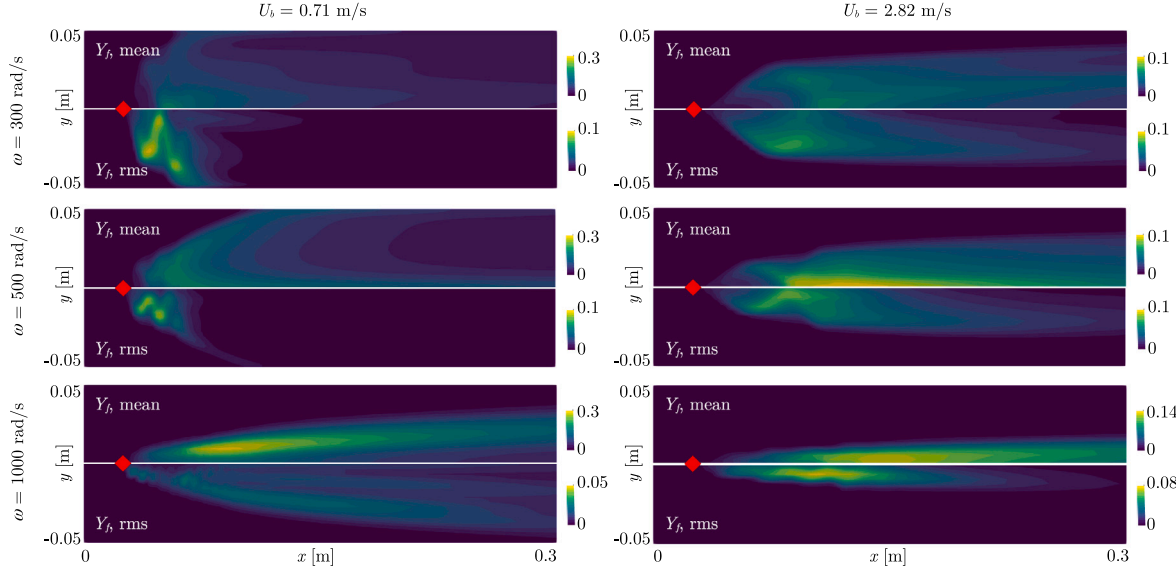


Fig. 13. Time-averaged values and rms of the fluctuations of fuel vapour mass fraction in the mid-plane normal to z for each of the cases given in Table 2. Results for $U_b = 0.71$ m/s are reported on the left; results for $U_b = 2.82$ m/s are reported on the right; results from top to bottom have an increasing angular frequency of the electric field oscillations. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

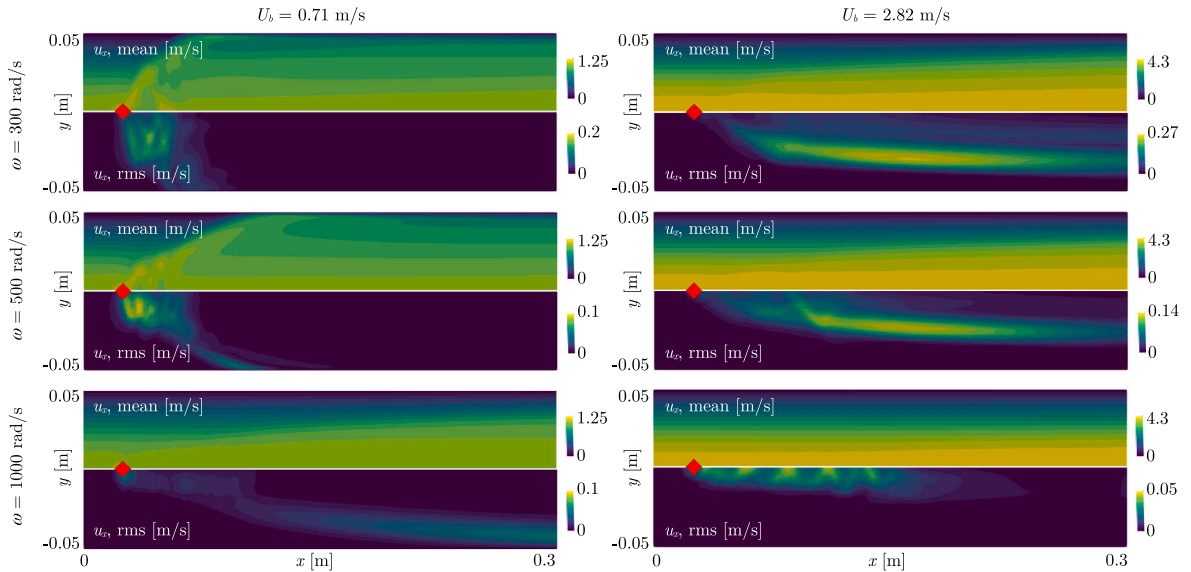


Fig. 14. Time-averaged values and rms of the fluctuations of the x -component of the gas-phase velocity (u_x) in the mid-plane normal to z for each of the cases given in Table 2. Results for $U_b = 0.71$ m/s are reported on the left; results for $U_b = 2.82$ m/s are reported on the right; results from top to bottom have an increasing angular frequency of the electric field oscillations. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

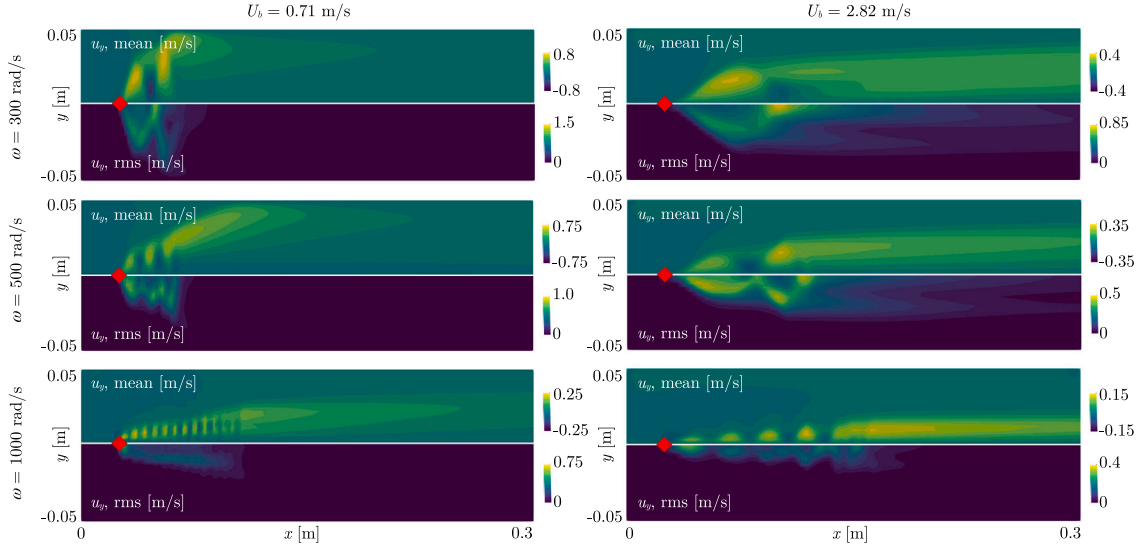


Fig. 15. Time-averaged values and rms of the fluctuations of the y -component of the gas-phase velocity (u_y) in the mid-plane normal to z for each of the cases given in Table 2. Results for $U_b = 0.71$ m/s are reported on the left; results for $U_b = 2.82$ m/s are reported on the right; results from top to bottom have an increasing angular frequency of the electric field oscillations. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

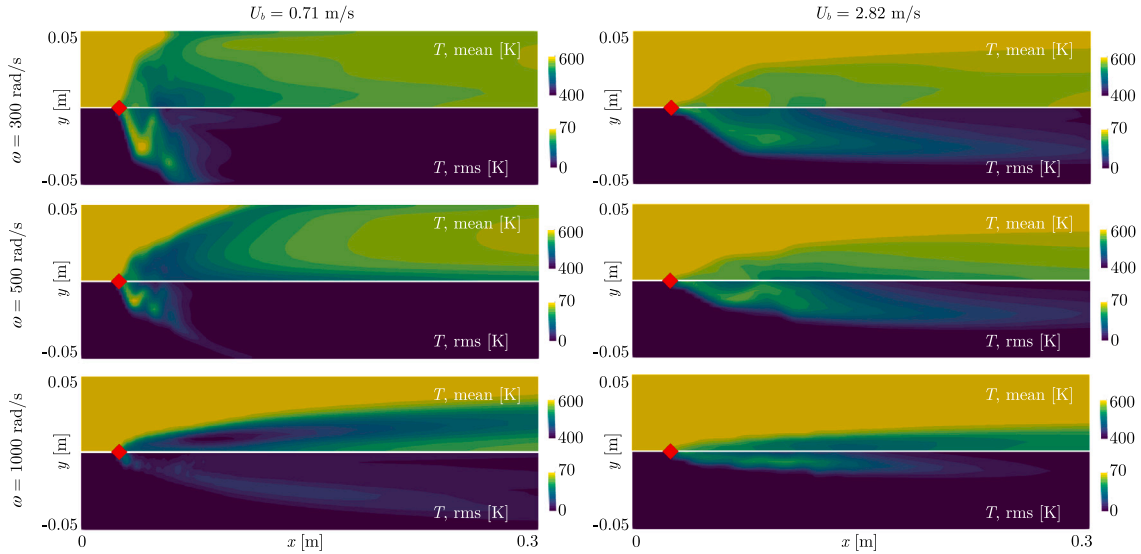


Fig. 16. Time-averaged values and rms of the fluctuations of gas-phase temperature in the mid-plane normal to z for each of the cases given in Table 2. Results for $U_b = 0.71$ m/s are reported on the left; results for $U_b = 2.82$ m/s are reported on the right; results from top to bottom have an increasing angular frequency of the electric field oscillations. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

investigated using both analytical and numerical solutions. It is demonstrated that for sufficiently low angular frequencies of the electric field, droplet trajectories show significantly large-amplitude oscillations that allow them to spread over the entire height of the channel. The amplitude of these oscillations increases with decreasing angular frequency of the electric field. It is also shown that depending on the phase shift at the injection time instant, droplet trajectories may present an offset in the direction of the electric field which allows them to further spread over the height of the channel. The result is a non-uniform distribution of droplets in the channel with a probability in space characterised by peaks along the centre of the channel separated by lobes. A non-dimensional solution for the trajectory is also proposed, which highlights the dependence of the oscillations on both droplet and gas-phase properties, with the dynamic viscosity of the gas being one of the main properties affecting the amplitude and offset of the oscillations. To enable a spread of droplets over the channel height,

a relatively low value of the flow bulk velocity is needed such that the residence time of the droplets in the channel is higher than the evaporation timescale. Simulations that include evaporation, performed with constant fuel mass flow rate, show that the spreading of the fuel vapour across the height of the channel increases with decreasing flow bulk velocity and decreasing angular frequency of the electric field oscillations. Oscillations of the droplet trajectories also induce local fluctuations of the flow field quantities, which tend to be higher for lower angular frequency of the electric field. The evaporation time of droplets increases with increasing frequency of the oscillations and decreasing flow bulk velocity as a result of accumulation of fuel vapour along the spray trajectory and related evaporative cooling. These results suggest that the stratification of the fuel vapour at the exit of the channel is the result of interconnected physical phenomena, which should be accurately evaluated to achieve control of the fuel vapour distribution. Overall, this study demonstrates the possibility of using

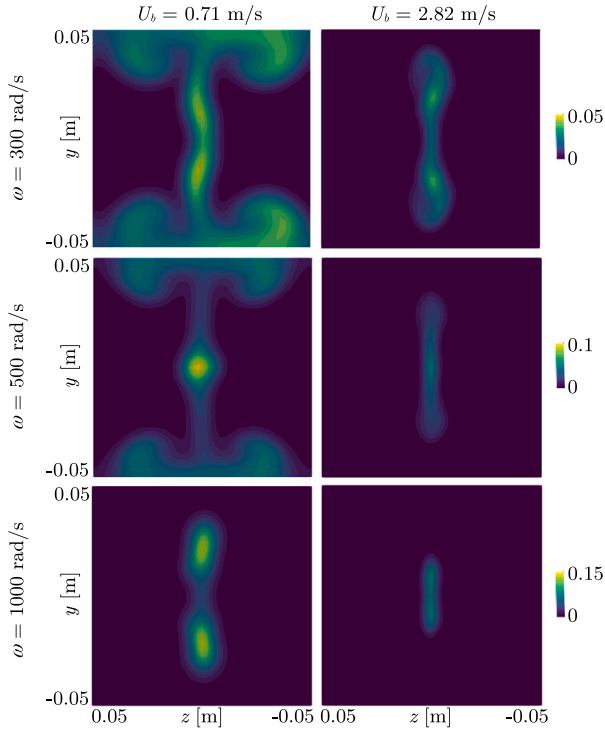


Fig. 17. Time-averaged fuel vapour mass fraction in a cross section with normal x and located at $x = 0.3$ m. Results for each of the cases given in Table 2 are reported. Results for $U_b = 0.71$ m/s are reported on the left; results for $U_b = 2.82$ m/s are reported on the right; results from top to bottom have an increasing angular frequency of the electric field oscillations.

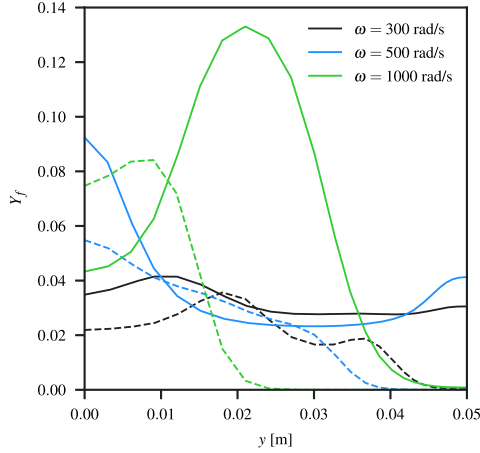


Fig. 18. Profile of time-averaged fuel vapour mass fraction along y at the outlet of the channel for the cases in Fig. 17. Bulk velocities of 0.71 m/s and 2.82 m/s are indicated by solid and dashed lines, respectively.

time-varying electric fields for the control of fuel vapour stratification, which could open up possibilities for novel technologies for liquid fuel preparation.

CRediT authorship contribution statement

Thomas H. Lawson: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation. **Andrea Giusti:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Andrea Giusti reports financial support was provided by UK Research and Innovation. Thomas H. Lawson reports financial support was provided by UK Research and Innovation. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Linearised equations for droplet trajectory

In this section, the ordinary differential equations (and related initial conditions) solved to obtain the final expressions reported in Eqs. (7) to (10) are given. The solution is reported here in dimensional form. Eq. (1) is solved analytically by inserting the drag force given by Stokes' law (Eq. (6)). In the x -direction, with a constant and uniform flow bulk velocity, and $m_d = \pi \rho_d d^3 / 6$, the equation to be solved is given as:

$$\frac{d^2 x_d}{dt^2} = \frac{1}{m_d} \left[3\pi \mu_g d \left(U_b - \frac{dx_d}{dt} \right) \right]. \quad (\text{A.1})$$

With initial conditions such that the droplet is injected at a position x_0 with velocity U_0 (i.e., $x_d(t=0) = x_0$; $dx_d/dt(t=0) = U_0$), the solution for the droplet position in x is:

$$x_d - x_0 = U_b t - \frac{m_d}{C} (U_b - U_0) \left[1 - \exp\left(-\frac{Ct}{m_d}\right) \right], \quad (\text{A.2})$$

where $C = 3\pi \mu_g d$.

The Lorentz force is calculated using Eq. (2), with zero magnetic component and electric component varying with the profile in Eq. (5). The equation to be solved for the droplet position in the y -direction is therefore:

$$\frac{d^2 y_d}{dt^2} = \frac{1}{m_d} \left[q_d \bar{E} \sin(\omega t - \theta) - 3\pi \mu_g d \frac{dy_d}{dt} \right], \quad (\text{A.3})$$

where $q_d = \pi \rho_{q,d} d^3 / 6$. Initial conditions are imposed such that the droplet is injected at the centre of the channel with a y velocity of V_0 (i.e., $y_d(t=0) = 0$; $dy_d/dt(t=0) = V_0$). The solution in dimensional form is:

$$\begin{aligned} y_d = & \frac{q_d \bar{E}}{m_d^2 \omega^3 + C^2 \omega} \left[\left(C + \frac{m_d^2 \omega^2}{C} \right) \cos \theta \right. \\ & - (m_d \omega \cos \theta + C \sin \theta) \frac{m_d \omega}{C} \exp\left(-\frac{Ct}{m_d}\right) \\ & \left. - m_d \omega \sin(\omega t - \theta) - C \cos(\omega t - \theta) \right] \\ & + \frac{m_d}{C} V_0 \left[1 - \exp\left(-\frac{Ct}{m_d}\right) \right]. \end{aligned} \quad (\text{A.4})$$

The steady-state amplitude and offset of the oscillations are obtained by assuming large t (exponential terms tend to zero) and $V_0 = 0$. The amplitude is taken as the maximum value of the oscillating terms and offset is taken as the value of the constant terms.

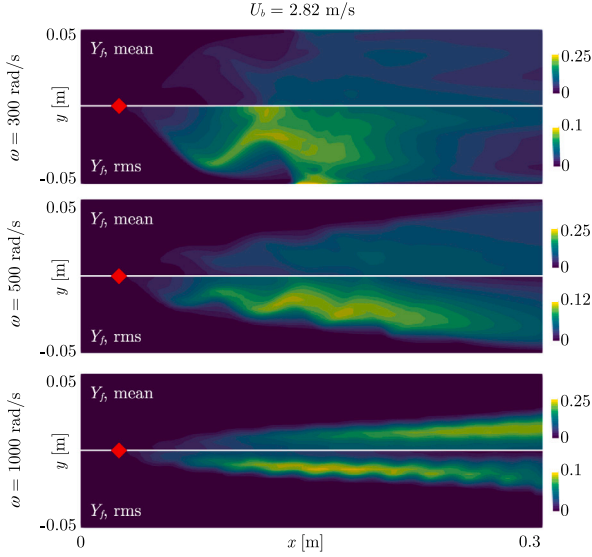


Fig. B.19. Time-averaged values and rms of the fluctuations of fuel mass fraction in the mid-plane normal to z for $U_b = 2.82$ m/s and same air-to-fuel ratio as the cases with $U_b = 0.71$ m/s given in Table 2. Results from top to bottom have an increasing angular frequency of the electric field oscillations. The red marker indicates the injection location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Appendix B. Results for constant air-to-fuel ratio

Figs. B.19 and B.20 show the results obtained with $U_b = 2.82$ m/s and the same air-to-fuel ratio as the case at $U_b = 0.71$ m/s discussed in Section 4.2. Apart from the increase in fuel mass flow rate, all the other conditions are the same as reported in Table 2. Comparison with results reported in Section 4.2 suggests that the distribution of fuel mass fraction at the exit of the channel obtained with constant air-to-fuel ratio tends to be less dependent on the flow bulk velocity. However, it is still observed that the spreading of the fuel vapour increases with decreasing flow bulk velocity. This is attributed to a balance between advection and diffusion of the fuel vapour, which changes with varying flow bulk velocity, as well as to the different evolution of the spray trajectories and velocity fluctuations (not shown here) along the channel. As also observed by comparing Fig. B.19 with Fig. 13, evaporation is completed much closer to the injection location in the case of low flow bulk velocity, which, together with the larger flow-through time, gives a much larger space and residence time to further enhance the mixing between air and fuel vapour.

Appendix C. Evaluation of magnetic field effects

The possibility of using a uniform and constant external magnetic field to control the trajectory of charged droplets (through the Lorentz force, see also Section 3.1) was also investigated. The reference configuration is schematically shown in Fig. C.21. A solenoid around a circular duct creates a uniform magnetic field inside the flow. Droplets are injected with a velocity component normal to the magnetic field, such that the combined effect of drag force and magnetic action could possibly result in a helical trajectory. The aim is to enhance the spreading of droplets and fuel vapour over the duct cross section.

The investigation of the droplet trajectory is performed using the deterministic model discussed in Section 3.1. For this configuration, a magnetic field parallel to the bulk flow is imposed and the electric field is assumed equal to zero, i.e. $E = 0$. From the expression of the Lorentz force, see Eq. (2), it is clear that to impose a magnetic force, the velocity of the droplet must not be parallel to the direction of the magnetic

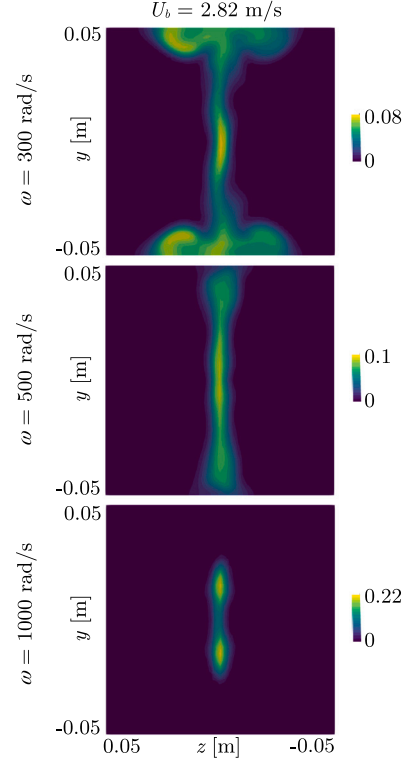


Fig. B.20. Time-averaged fuel vapour mass fraction in a cross section with normal x and located at $x = 0.3$ m for $U_b = 2.82$ m/s and same air-to-fuel ratio as the cases with $U_b = 0.71$ m/s given in Table 2. Results from top to bottom have an increasing angular frequency of the electric field oscillations.

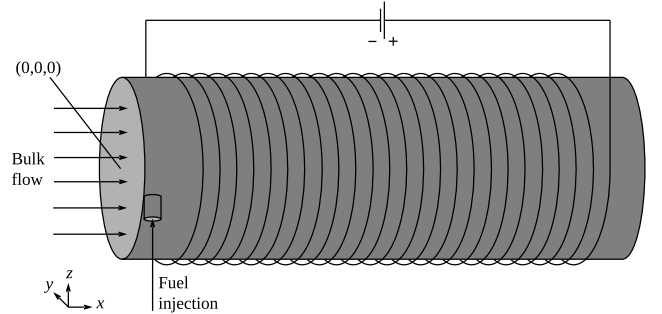


Fig. C.21. Schematic of the configuration to study the effects of a uniform and constant magnetic field on the droplet trajectory. A solenoid is coiled around a cylindrical duct and fuel is injected in the positive z direction. The origin of the frame of reference used for post-processing is located at the centre of the duct inlet.

field. Therefore, in the configuration investigated in the present work the droplet is injected perpendicular to the flow bulk velocity such that the initial magnetic force for a given injection velocity is maximum. The equation of motion of the droplet (i.e., Eq. (1)) is solved in three dimensions. The non-dimensional quantities used for the analysis of this case are given in Table C.3. Since a solenoid around the duct is considered, the magnetic flux density, B , in practical applications could be evaluated using:

$$B = \mu_0 \frac{NI}{l} \hat{e}_B, \quad (C.1)$$

where μ_0 is the permeability of free space; N , I and l are the number of turns on, current through, and length of the solenoid, respectively; $\hat{e}_B$ is the direction of the resulting magnetic flux density (parallel to the axis of the duct).

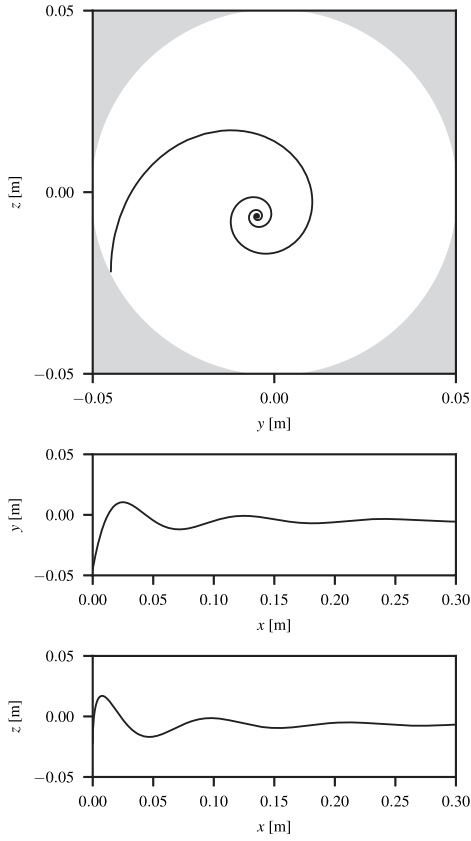


Fig. C.22. Theoretical single droplet trajectory in a uniform and constant magnetic field for $U_b = 10$ m/s, $V_0 = 25$ m/s, $\rho_{q,d} = 2$ C/m³, $d = 100$ μ m, and $|\mathbf{B}| = 2.09 \times 10^5$ T. The fuel is kerosene and the carrier phase is air at atmospheric pressure and a temperature of 600 K. Note that the magnetic field required to produce this trajectory is orders of magnitude greater than can be achieved in practice.

Table C.3

Dimensionless groups used for the analysis of the configuration of Fig. C.21. U_b , ρ_g and μ_g indicate the flow bulk velocity, density and dynamic viscosity, respectively, of the carrier phase; d , ρ_d and $\rho_{q,d}$ represent diameter, density and charge density of the droplet; D is the diameter of the duct; V_0 is the droplet injection velocity assumed to be aligned with the z -direction; $|\mathbf{B}|$ is the magnitude of the magnetic flux density.

Symbol	Relation	Comments
Re_b	$\frac{\rho_g U_b D}{\mu_g}$	Bulk flow Reynolds number
Π_B	$\frac{ \mathbf{B} \rho_{q,d} d}{\rho_g V_0}$	Non-dimensional magnetic field
Re_{d,V_0}	$\frac{\rho_g V_0 d}{\mu_g}$	Non-dimensional injection velocity
Π_ρ	$\frac{\rho_d}{\rho_g}$	Density ratio
Π_U	$\frac{U_b}{V_0}$	Velocity ratio

As mentioned above, since the magnetic force is exerted perpendicular to both the velocity of the droplet and the magnetic field, which is parallel to the flow bulk velocity, a helical droplet trajectory is expected. This is demonstrated in Fig. C.22, which shows results obtained for a cylindrical duct of length $l = 0.3$ m and diameter 0.1 m, initial droplet velocity of 25 m/s in the positive z direction, and injection location at the wall of the duct at $y = -0.045$ m. The velocity of the carrier phase, assumed at atmospheric pressure and a temperature of 600 K, was set to be uniform with a magnitude of 10 m/s. Kerosene droplets of diameter 100 μ m and charge density 2 C/m³ are considered. Note that the resulting net charge of the droplet is close to the Rayleigh limit (Giusti and Fredrich, 2024), therefore the effect of the magnetic field is assessed close to the limit of the maximum charge that a droplet could hold. This means that any increase in the

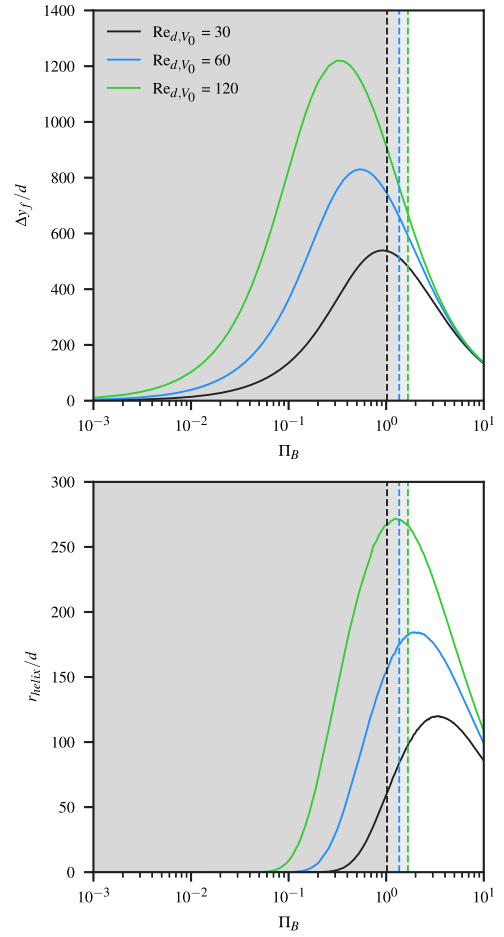


Fig. C.23. Final droplet y position (relative to the injection location) and helix radius, both non-dimensionalised with d and plotted against Π_B . The helix radius is taken as r when $\theta - \theta_{initial} = \pi$ in cylindrical polar coordinates centred on the final position. Results for different values of Re_{d,V_0} are reported as indicated by the legend. The shaded region denotes cases where the droplet trajectory impinges on the wall of the selected duct. The vertical dashed lines mark the value of Π_B above which impingement is avoided and a helical trajectory is formed for each value of Re_{d,V_0} .

intensity of the magnetic force could only result from an increase in either droplet velocity or strength of the magnetic field.

To obtain the trajectory shown in Fig. C.22, a magnetic flux density $|\mathbf{B}| = 2.09 \times 10^5$ T is required. This value is orders of magnitude greater than any value that can be feasibly produced with current technology (such a strong magnetic flux density would require $NI = 5 \times 10^{10}$ A). For lower magnetic field strengths, magnetic forces become negligible when compared to drag.

The trajectory of droplets is further examined in Fig. C.23, where the radius of the helical motion and final y position of the droplet (movement perpendicular to the injection velocity) are shown as a function of the non-dimensional magnetic field and the injection velocity Reynolds number (see Table C.3). For low values of the magnetic field, the magnetic forces acting on the droplet are too low to change its trajectory and the droplet will impinge on the wall of the selected duct without the formation of helical motion (this is indicated by the grey region in Fig. C.23). Only for very large values of Π_B it is possible to establish helical motion. It is clear that for realistic values of magnetic field (e.g., $\Pi_B \sim 10^{-3}$), the deviation of the droplet from its natural trajectory is negligible. This illustrates that, with current technology and with realistic configurations, magnetic fields cannot be used for the control of the trajectory of charged droplets. In addition, any magnetic field generated by the movement of charged droplets

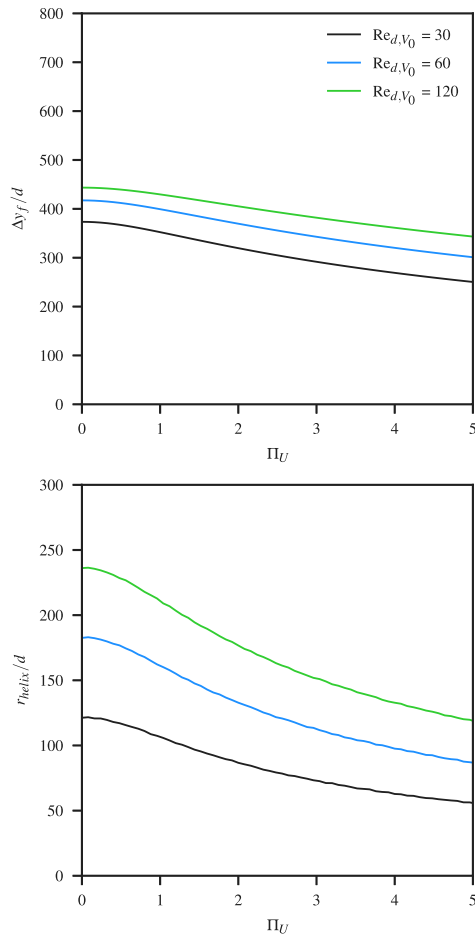


Fig. C.24. Final droplet y position (relative to the injection location) and helix radius, both non-dimensionalised with d and plotted against Π_U . Π_B is fixed at 2.85 such that impingement on the walls of the duct is avoided for all cases.

(very weak current densities) is negligible, therefore its effect on the droplet trajectory can be neglected.

For the sake of completeness, Fig. C.24 shows the values of the radius of the helical motion and the final offset along the y -direction for a sufficiently large Π_B and different values of the velocity ratio, Π_U (see Table C.3). Results suggest that in conditions for which helical motion develops and for a given value of the injection velocity, the radius of the helix and the offset from the injection location reduce with increasing flow bulk velocity.

Data availability

Data will be made available on request.

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