

RELIABILITY ANALYSIS OF THE FATIGUE RESISTANCE  
OF OFFSHORE STRUCTURES

A Thesis presented for the degree of  
Doctor of Philosophy in the  
Faculty of Engineering of  
the University of London

by

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September 1983

## ABSTRACT

This thesis discusses the application of structural reliability theory to the design and analysis of the fatigue resistance of offshore structures. Structural reliability theory provides methods for analysing civil engineering structures which enable all the uncertainties that affect the safety of such structures to be incorporated into the analysis.

The types of decisions that are made in the offshore industry concerning the fatigue resistance of structures such as oil production platforms are discussed and classified according to the nature of the analysis required. An important class of decision concerned with the detection and repair of fatigue cracks is identified for which a comparatively simple analysis is practicable.

The theoretical basis of structural reliability theory is discussed and some recent criticisms of its use of probability theory are examined and shown to be incorrect. Sources of uncertainty in civil engineering applications are described and the problems inherent in the use of statistical data collected for purposes other than reliability analysis are discussed. The concept of a 'set of nominally identical cases' is shown to be useful.

Models are developed for fatigue resistance and for the behaviour of offshore structures. These models are mainly based on the design methods used in the offshore industry, so that published experimental results can be used to develop the required probabilistic description of the accuracy of these models. Some models are also formulated based on fracture mechanics theories. A wide range of available data is reviewed and analysed, and probabilistic models are formulated.

The methods given in the thesis are illustrated with some calculations which also show the safety levels indicated for some typical circumstances by the data currently available.

### ACKNOWLEDGEMENTS

The author would like to express his thanks to his supervisors, Dr T A Wyatt, and Mr M J Baker for their help and encouragement throughout the time he spent at Imperial College working on reliability theory and its applications.

He is grateful for the financial support from the SERC Marine Technology Programme for the work covered in this thesis, and also to the Department of Transport for their support for his earlier work in reliability analysis.

The author would also like to thank his colleagues, Dr K Ramachandran, Mr A H S Wickham, and Mr J C Nicholls, with whom he enjoyed many fruitful discussions.

Grateful thanks are also due to Mrs S Wright for her efficient typing of the thesis.

## CONTENTS

|                                                                           |        |
|---------------------------------------------------------------------------|--------|
| ABSTRACT                                                                  | 1      |
| ACKNOWLEDGEMENTS                                                          | 2      |
| CONTENTS                                                                  | 3      |
| NOTATION                                                                  | 9      |
| <br>CHAPTER 1 INTRODUCTION                                                | <br>11 |
| 1.1 Objective of Thesis                                                   | 11     |
| 1.2 Decisions concerning the Fatigue Resistance<br>of Offshore Structures | 12     |
| 1.3 Scope of Thesis                                                       | 14     |
| <br>CHAPTER 2 RELIABILITY THEORY AND ITS APPLICATION                      | <br>17 |
| 2.1 Introduction                                                          | 17     |
| 2.2 Basis of Probability Theory                                           | 17     |
| 2.3 Blockley's objections to the use of<br>Probability Theory             | 19     |
| 2.3.1 Introduction                                                        | 19     |
| 2.3.2 Rescher's Paradox                                                   | 21     |
| 2.4 More General Objections to the use of<br>Probability Theory           | 24     |
| 2.5 Basis of Structural Reliability Theory                                | 25     |
| 2.6 Probabilistic Modelling of Basic Variables                            | 29     |
| 2.6.1 Introduction                                                        | 29     |
| 2.6.2 Classification of Uncertainty                                       | 29     |
| 2.6.3 Choice of Distribution Type                                         | 31     |
| 2.6.4 Determination of Parameters for<br>Distributions                    | 35     |
| 2.6.5 The Role of Statistical Uncertainties                               | 37     |
| 2.6.6 Uncertainties in Real Data                                          | 39     |
| 2.7 "Level 2" Algorithm                                                   | 42     |
| 2.7.1 Definition of Problem                                               | 42     |
| 2.7.2 Development of Iterative Routine                                    | 44     |
| 2.7.3 Definition of $\beta$                                               |        |

|           |                                                                                                         |    |
|-----------|---------------------------------------------------------------------------------------------------------|----|
| 2.8       | Calculation of Conditional Probabilities by<br>truncation and re-normalisation of the<br>variable space | 47 |
| 2.8.1     | Conditional Probability of Failure                                                                      | 47 |
| 2.8.2     | Truncation and Re-normalisation                                                                         | 47 |
| 2.8.3     | Choice of Truncation Plane                                                                              | 51 |
| 2.9       | Calculation of Conditional Probabilities<br>for certain special cases                                   | 53 |
| 2.9.1     | Basis of Method                                                                                         | 53 |
| 2.9.2     | Extension of Basic Principle to<br>more than two functions                                              | 56 |
| 2.9.3     | Development of Method                                                                                   | 58 |
| CHAPTER 3 | DETERMINISTIC FATIGUE DESIGN PROCEDURES<br>FOR OFFSHORE STRUCTURES                                      | 63 |
| 3.1       | Introduction                                                                                            | 63 |
| 3.2       | Basis of Fatigue Life Calculations                                                                      | 64 |
| 3.2.1     | General                                                                                                 | 64 |
| 3.2.2     | Fatigue Damage due to Gaussian<br>Processes                                                             | 66 |
| 3.3       | Deep Water Waves                                                                                        | 70 |
| 3.4       | Wave Loading and Structural Response                                                                    | 72 |
| 3.5       | Non-Spectral Methods                                                                                    | 73 |
| 3.5.1     | Introduction                                                                                            | 73 |
| 3.5.2     | Closed Form Solutions                                                                                   | 75 |
| 3.5.3     | Stresses due to Wave Forces on a<br>Vertical Column                                                     | 76 |
| 3.6       | Spectral Methods                                                                                        | 79 |
| CHAPTER 4 | FORMULATION OF FAILURE FUNCTION                                                                         | 81 |
| 4.1       | Introduction                                                                                            | 81 |
| 4.2       | Re-formation of Deterministic Procedures                                                                | 82 |
| 4.3       | Selection of Basic Variables                                                                            | 83 |
| 4.3.1     | Introduction                                                                                            | 83 |
| 4.3.2     | Loading Variables                                                                                       | 84 |
| 4.3.3     | Variables affecting Structural<br>Analysis                                                              | 84 |
| 4.3.4     | Special Features of Spectral Methods                                                                    | 86 |
| 4.3.5     | Use of ratios as basic variables                                                                        | 88 |

|           |                                                                                |     |
|-----------|--------------------------------------------------------------------------------|-----|
| 4.3.6     | Variables modelling the Frequency of Occurrence of Wave Heights and Sea-States | 89  |
| 4.4       | Use of Fracture Mechanics                                                      | 91  |
| 4.4.1     | Introduction                                                                   | 91  |
| 4.4.2     | Application to Offshore Structures                                             | 95  |
| 4.4.3     | Varying Stress Cycles: Threshold Effects                                       | 95  |
| 4.4.4     | Varying Stress Cycles: Cumulative Damage                                       | 96  |
| 4.4.5     | Formulation for Fatigue ending in Fracture                                     | 100 |
| CHAPTER 5 | PROBABILISTIC MODELLING OF BASIC VARIABLES                                     | 103 |
| 5.1       | Wave Climate Descriptions                                                      | 103 |
| 5.1.1     | Introduction                                                                   | 103 |
| 5.1.2     | Distribution Type                                                              | 107 |
| 5.1.3     | Available Data                                                                 | 108 |
| 5.1.4     | Analysis of Data: Distribution of Wave Height                                  | 113 |
| 5.1.5     | Analysis of Data: Persistence of Sea-States                                    | 115 |
| 5.2       | Wave Loading Variables                                                         | 119 |
| 5.2.1     | Introduction                                                                   | 119 |
| 5.2.2     | Distribution Type                                                              | 121 |
| 5.2.3     | Available Data                                                                 | 121 |
| 5.3       | Variables affecting Structural Analysis                                        | 122 |
| 5.4       | Stress Concentration Factors                                                   | 123 |
| 5.4.1     | Introduction                                                                   | 123 |
| 5.4.2     | Distribution Type                                                              | 124 |
| 5.4.3     | Available Data                                                                 | 124 |
| 5.4.4     | Proposed Model                                                                 | 127 |
| 5.5       | S-N Curve Variables                                                            | 127 |
| 5.5.1     | Introduction                                                                   | 127 |
| 5.5.2     | Distribution Type                                                              | 129 |
| 5.5.3     | Available Data                                                                 | 131 |
| 5.5.4     | Analysis of Data                                                               | 133 |
| 5.6       | Miner's Rule Uncertainty                                                       | 138 |
| 5.7       | Damage Reduction Factor                                                        | 139 |
| 5.8       | Corrosion Effects                                                              | 141 |

|            |                                                                                   |     |
|------------|-----------------------------------------------------------------------------------|-----|
| CHAPTER 6  | EXAMPLES AND CALCULATIONS                                                         | 143 |
| 6.1        | Failure of Single Connections                                                     | 143 |
| 6.1.1      | Introduction                                                                      | 143 |
| 6.1.2      | States of Uncertainty                                                             | 144 |
| 6.1.3      | Nominal Design Values                                                             | 145 |
| 6.1.4      | 'Worst' and 'Average' Cases                                                       | 146 |
| 6.1.5      | Results of Calculations                                                           | 147 |
| 6.2        | Failure of more than one connection                                               | 150 |
| 6.2.1      | Introduction                                                                      | 150 |
| 6.2.2      | Degree of Independence                                                            | 151 |
| 6.2.3      | Results                                                                           | 153 |
| 6.3        | Conditional Probabilities of Failure                                              | 154 |
| 6.3.1      | Introduction                                                                      | 154 |
| 6.3.2      | Results                                                                           | 154 |
| CHAPTER 7  | CONCLUSIONS                                                                       | 156 |
| REFERENCES |                                                                                   | 159 |
| TABLES     |                                                                                   |     |
| 5.1        | Data Recovery from m.v. Famita                                                    | 168 |
| 5.2        | Regression Analysis Results for long term<br>wave height distribution             | 169 |
| 5.3        | Regression Analysis Results for long term<br>wave height distribution             | 170 |
| 5.4        | Statistics for n'and t'                                                           | 171 |
| 5.5        | Coefficients of Variation for Wave<br>Description Variables                       | 172 |
| 5.6        | Accuracy of Stress Concentration<br>Factor Formulae - 1                           | 173 |
| 5.7        | Accuracy of Stress Concentration<br>Factor Formulae - 2                           | 174 |
| 5.8        | Summary of Results in table 5.7                                                   | 175 |
| 5.9        | Accuracy of Stress Concentration Factor<br>Formulae - 3                           | 175 |
| 5.10       | Sources of S-N Curve Data                                                         | 176 |
| 5.11       | Summary of Data for S-N Curves                                                    | 177 |
| 5.12       | Regression Analysis results - S-N Curves<br>with different m for each set of data | 181 |
| 5.13       | Regression Analysis results - S-N Curves<br>with constant m throughout            | 185 |

|      |                                                                                                                                       |     |
|------|---------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.14 | Estimated Standard Deviation representing<br>the variation of $\log_{10}K$ within sets of<br>nominally identical cases                | 189 |
| 5.15 | Statistics representing the variations between<br>the values of $\log_{10}K$ found for different sets<br>of nominally identical cases | 189 |
| 5.16 | Accuracy of Miner's Rule                                                                                                              | 190 |
| 6.1  | Data for Reliability Analysis                                                                                                         | 191 |
| 6.2  | Probability of Failure of Single Connections                                                                                          | 192 |
| 6.3  | Failure of Single Connections - $\beta$ values                                                                                        | 193 |
| 6.4  | Sensitivity Factors for Failure of Single<br>Connections                                                                              | 193 |
| 6.5  | Probability of at least one failure in $n$ cases                                                                                      | 194 |
| 6.6  | Probability of all $n$ cases failing                                                                                                  | 194 |
| 6.7  | Probability of at least one of cases $B_1$ to $B_n$<br>failing during required lifetime given that<br>case A fails prematurely        | 195 |
| 6.8  | Probability that all of cases $B_1$ to $B_n$ fail<br>during required lifetime given that case A<br>fails prematurely.                 | 196 |

## FIGURES

|     |                                                                                                         |     |
|-----|---------------------------------------------------------------------------------------------------------|-----|
| 2.1 | Truncated Probability Distribution                                                                      | 197 |
| 2.2 | $u-u'$ Transformation                                                                                   | 197 |
| 2.3 | Example of transformation of Failure Surface<br>by Truncation and Re-normalisation of<br>variable space | 198 |
| 3.1 | Distribution of Peaks and Troughs for<br>Gaussian Process                                               | 199 |
| 5.1 | Combined Scatter Diagram showing data for<br>seven winters at m.v. Famita                               | 200 |
| 5.2 | Combined Scatter Diagram showing "filled"<br>data for seven winters at m.v. Famita                      | 203 |
| 5.3 | Long-term Distribution of Wave Height                                                                   | 206 |
| 5.4 | Scatter Diagram showing mean frequency<br>of occurrence ( $\times 100\ 000$ ) of each sea-state         | 207 |

|      |                                                                                                               |     |
|------|---------------------------------------------------------------------------------------------------------------|-----|
| 5.5  | Scatter Diagram showing Standard Deviation of frequency of occurrence ( $\times 100\ 000$ ) of each sea-state | 208 |
| 5.6  | Scatter Diagram showing Coefficient of Variation of frequency of occurrence of each sea-state                 | 209 |
| 5.7  | Scatter Diagram showing variance/mean for frequency of occurrence ( $\times 10\ 000$ ) of each sea-state      | 210 |
| 5.8  | Comparison of Hazard Functions for typical lognormal and Weibull Distributions                                | 211 |
| 5.9  | Typical S-N Curve data and mean regression lines                                                              | 211 |
| 5.10 | Plot of values of $m$ from data for fatigue life $N_3$ against mean value of $\log \Delta \epsilon$           | 212 |
| 5.11 | Sixteen Fatigue Test Results taken as single group                                                            | 212 |
| 5.12 | Sixteen Fatigue Test Results taken as eight pairs                                                             | 213 |
| 6.1  | Probability of Failure of single connections                                                                  | 214 |
| 6.2  | Probability of at least one failure                                                                           | 215 |
| 6.3  | Probability of all connections failing                                                                        | 216 |
| 6.4  | Conditional Probability of at least one failure                                                               | 217 |
| 6.5  | Conditional Probability of all connections failing.                                                           | 218 |

### NOTATION

Throughout this thesis the functions  $F(\cdot)$  and  $f(\cdot)$  are used exclusively to represent the cumulative probability function and the probability density function respectively. It is general practice in probability literature to use upper and lower case letters to distinguish between the name and the value of a variable; thus the variable  $X$  takes values  $x$ , and hence the value of the probability density function of  $X$  at  $x$  is given by  $F_X(x)$ . This convention is adhered to in Chapter 2, but in subsequent chapters the variables are often those used in offshore engineering, fatigue design, etc., which are conventionally used in either upper or lower case only. This convention is adhered to in this thesis so that the significant wave height, for instance, is always written  $H_S$ , and its probability density function is written  $f_{H_S}(H_S)$ .

Principal items of notation which are used throughout the thesis are given below; other notation is defined in the sections where it is used.

|                        |                                                                              |
|------------------------|------------------------------------------------------------------------------|
| $a$                    | constant used when stress range is proportional to wave height i.e. $S = aH$ |
| $a$                    | crack size                                                                   |
| $a_f$                  | final crack size                                                             |
| $a_i$                  | initial crack size                                                           |
| $a_i$                  | $\partial g_U(\underline{u})/\partial u_i$                                   |
| $d_k$                  | fatigue damage in $k^{\text{th}}$ sea-state                                  |
| $g_U(\underline{u})$   | failure function in standard normal variable space                           |
| $g_X(\underline{x})$   | failure function in basic variable space                                     |
| $m$                    | parameter of S-N curve                                                       |
| $m$                    | mean value (with subscripts as appropriate)                                  |
| $m_0, m_2, \dots, m_n$ | moments of spectrum                                                          |
| $n$                    | number of waves or cycles                                                    |
| $n_{\text{ann}}$       | total number of cycles per year                                              |
| $P_f$                  | probability of failure                                                       |
| $P_f(p_v)$             | probability of failure due to physical variability only                      |
| $r$                    | radius of column (including marine growth)                                   |

|                                 |                                                                           |
|---------------------------------|---------------------------------------------------------------------------|
| $C$                             | constant in Paris crack growth law                                        |
| $C_m$                           | mass (or inertia) coefficient                                             |
| $C_D$                           | drag coefficient                                                          |
| $E[.]$                          | expectation                                                               |
| $H$                             | wave height                                                               |
| $H_g$                           | significant wave height                                                   |
| $K$                             | parameter of S-N curve                                                    |
| $L[.]$                          | likelihood                                                                |
| $M$                             | Miner's rule sum                                                          |
| $N$                             | fatigue life (cycles)                                                     |
| $P[.]$                          | probability                                                               |
| $R$                             | stress ratio (min.stress/max.stress)                                      |
| $S$                             | stress range                                                              |
| $SCF$                           | stress concentration factor                                               |
| $T_p$                           | mean peak to peak period                                                  |
| $T_z$                           | mean zero up-crossing period                                              |
| $Y$                             | fatigue life (years)                                                      |
| $Y_r$                           | required fatigue life (years)                                             |
| $Z$                             | safety margin                                                             |
| $\alpha$                        | constant in formula for stress intensity factor                           |
| $\alpha$                        | parameter of long-term wave height distribution                           |
| $\beta$                         | parameter of long-term wave height distribution                           |
| $\beta$                         | distance from failure surface to origin in standard normal variable space |
| $\gamma$                        | safety factor                                                             |
| $\epsilon$                      | band width of spectrum                                                    |
| $\phi(.)$                       | probability density function of standard normal distribution              |
| $\lambda, \lambda_K, \lambda_w$ | Wirsching's Damage Reduction Factor (see p.69)                            |
| $\lambda_C$                     | factor on Miner's Sum to allow for corrosion effects                      |
| $\sigma$                        | standard deviation (with subscripts as appropriate)                       |
| $\omega$                        | frequency (rad/sec)                                                       |

## CHAPTER 1

### INTRODUCTION

#### 1.1 Objective of Thesis

The aim of this thesis is to discuss the application of structural reliability theory to the design and analysis of the fatigue resistance of offshore structures.

Structural reliability theory aims to make decision-making in civil engineering more rational by quantifying and analysing any uncertainties which ought to influence the decision. They can then be incorporated into calculations for optimum designs, quality control procedures, inspection strategies, etc.

Structural reliability theory differs from the forms of reliability theory used in many other branches of engineering because of the one-off nature of most civil engineering structures, and because the failures which most structural design aims to avoid are very unlikely events (albeit ones with major consequences). It is therefore not practicable to collect data for uncertainty in structural performance by directly testing whole structures. Also in many cases the length of time for which structures are expected to survive and the nature of their loads (especially environmental loads) makes such testing impossible. It is therefore necessary to test smaller structural elements and the validity of structural theories, and then to combine this information to give an assessment of the safety of the structure as a whole. It is this process that is generally referred to as 'structural reliability analysis'.

The degree of confidence that can be placed in such procedures depends not only on their theoretical basis, but also on the availability of suitable test data. Both these aspects of structural reliability analysis are discussed in this thesis in the context of the fatigue resistance of offshore structures.

## 1.2 Decisions concerning the Fatigue Resistance of Offshore Structures

The decisions which concern the fatigue resistance of an offshore structure are decisions on such matters as the dimensions of welded connections, the weld preparation and heat treatment, the inspection methods to be used, the frequency of inspection in service and whether or not to repair cracked welds. Ideally, all the decisions which have to be made at a particular time should be made together as a single optimisation of cost, including the consequences of failure. However the analyses required to support such an optimisation would be so expensive that, if these costs were included, the single optimisation would not be the optimal strategy. For practical reasons therefore most decisions are made by taking into consideration only a limited number of factors. The whole range of possible decision-making procedures can be divided into three groups on the basis of the extent to which each procedure is concerned with the relationship between different fatigue cracks or different locations for possible fatigue cracks.

a) The first type of decision-making procedure is that concerned with only one crack location. Conventional structural design decisions are of this type because each member or connection is designed independently. The designs are based on assumptions about how loads are distributed through the structure which are in turn based on an assumption that the structure remains essentially intact. This assumption is correct until one crack reaches some critical length. It is clearly possible to frame design rules on the basis that reaching this critical length constitutes 'failure', ignoring any reserves of strength that may exist after loads have begun to be redistributed. Hence if reliability analysis were to be used, for example, to adjust design rules of this sort so that different types of members and connections all had the same probability of 'failing' in fatigue before the end of the required life, each reliability analysis would be concerned with the failure of a single connection.

b) The other types of decision-making procedure are those that require the simultaneous consideration of more than one fatigue crack. The most general situation is that of the so-called 'system' failure where one is interested in the behaviour of the whole structure and hence in the sequence of element failures and load redistributions that are necessary to produce total failure in an indeterminate structure. Although such a sequence might start with a fatigue failure, substantial redistributions of load may produce other types of failure as well. Because of the need to consider these other types of failure, and all the different patterns of load distribution that will occur during different possible sequences of failures, the reliability analysis needed for this sort of situation is likely to be extremely complex.

c) For many types of failure, the decision-making situations and appropriate reliability analyses can be classified into one or other of the two groups above. However fatigue failure is a process of slow steady crack growth which is often detectable long before the stage of 'failure' which causes significant redistribution of loads within a structure. There are therefore other decisions concerned, for example, with the frequency with which inspections should be carried out or actions that should be taken following the early detection of a crack, where it is necessary to consider the relationship between the different fatigue cracks (e.g. the extent to which crack growth rates are correlated) but where it can be assumed that cracks will be detected and repaired before they reach the stage of failure that causes significant load redistribution. Thus while the 'failure' of different elements in the structure, defined for example in terms of requiring repair, may be correlated with each other because they all depend on certain features of the design, construction, or load history, the probability of any particular failure occurring is not changed by the actual occurrence of other failures. To assist with decisions of this type calculations might be made for the probability of various members all failing (i.e. requiring repair) during the required life of the structure, or for the probability of such failures given that extensive cracking has been discovered somewhere in the structure.

### 1.3 Scope of Thesis

Because of the complexity of the calculations required in the circumstances described in (b) above, the methods discussed in this thesis will be directed towards the use of reliability analysis in circumstances that come within (a) and (c). These methods will however form part of the necessary basis for the use of reliability methods in more general circumstances.

As remarked above, a review of an application of structural reliability analysis requires discussion of both theoretical aspects and the availability of data relevant to the area of application. The theoretical basis for the use of structural reliability analysis in the context of the fatigue resistance of offshore structures is made up from the concepts and models used in three different areas of engineering science: structural reliability analysis itself, metal fatigue, and the analysis of the behaviour of offshore structures.

Chapter 2 discusses the theoretical basis of structural reliability theory, giving particular attention to a description of the various types of uncertainty that occur in civil engineering. The so-called 'Level 2' method, often used for approximate reliability calculations is derived and some new extensions of this approach are given which are suitable for situations such as those described in paragraph 1.2(c) above.

Chapters 3 and 4 are devoted to the development of the models for fatigue resistance and for the analysis of offshore structures. These models are based on the design methods used in industry. One reason for this is that industrial use suggests that a method is likely to be the best available in terms of balancing the competing claims of accuracy, simplicity, and applicability to the full range of possible cases. A more important reason for using standard design methods is that in formulating a model for use in reliability analysis it is not particularly important that it should be the most accurate model available, but rather that it should be possible to assess its accuracy over the whole range of its application. If experimental studies cannot be made specially then it is necessary to rely on published results to assess the

accuracy of the models used, and this usually makes it necessary to follow existing practice.

Chapter 3 describes the principal characteristics of metal fatigue and the basic principles behind the conventional analysis methods for offshore structures. The common calculation models are developed, initially in fairly general terms, then showing how they can be made increasingly detailed in those circumstances where certain simplifying assumptions are valid. In Chapter 4 these models are reformulated as required for reliability analysis, and the suitability of their parameters for use as the 'basic' variables required by reliability analysis is discussed.

Chapter 5 reviews the data available for the variation of, and uncertainty in, these 'basic' variables. Probabilistic models are formulated for these variables on the basis of the discussion of uncertainty in Chapter 2. Some of this data analysis is more extensive and thorough than that normally undertaken by those responsible for the original experiments.

In Chapter 6 some calculations are performed to illustrate the methods outlined earlier and to indicate the effects in typical calculations of using the data discussed in Chapter 5.

Chapter 7 draws some general conclusions from the previous chapters and indicates possible future developments.

Work published by other authors on the subjects covered by this thesis is reviewed in the relevant chapters. A complete review of all the work published on structural reliability theory, metal fatigue and offshore structures is of course impossible; only Wirsching (with a variety of co-authors)<sup>1,2,3,4</sup> was known to have published specifically on the application of reliability analysis to offshore fatigue when the work in this thesis was being undertaken, and aspects of his work are discussed in several chapters. Recently Karadeniz *et al*<sup>5</sup> have published some results for reliability analyses of the fatigue resistance of two particular designs of offshore structures. Their paper pays considerable attention to the structural analysis of those designs. It does not discuss the derivation of probability

distributions from data in any detail, so it is difficult to compare their work with that presented here, but their general conclusions about the relative importance of the various influences on fatigue life are not greatly different. Other authors' works are discussed where they provide support for the arguments advanced here or provide useful data. In addition some approaches favoured by other authors which have not been followed in this thesis are discussed.

## CHAPTER 2

### RELIABILITY THEORY AND ITS APPLICATION

#### 2.1 INTRODUCTION

The term 'reliability theory' is generally taken to mean the use of probability theory in the analysis of the safety and serviceability, or more generally the reliability, of structures or systems in engineering. The purpose of this chapter is to discuss this probabilistic approach and some of the objections that have been raised to it; and to develop algorithms that are suitable for the type of probabilistic calculations required.

#### 2.2 Basis of Probability Theory

Since the creation of probability theory, many different ways have been devised of deriving or justifying the theory (e.g. Carnap<sup>6</sup>, Keynes<sup>7</sup>, Tribus<sup>8</sup>). Weatherford<sup>9</sup> has recently published a detailed discussion of the principal schools of thought. The approach adopted here is to treat probability theory as an autonomous branch of logic or mathematics with its own definitions and axioms, which can be shown to be useful and practicable when applied to certain problems.

A quantity "probability" is associated with an "event". An "event" is a set (described in terms of normal set theory) which contains one or more of the possible descriptions of the situation under consideration. The three basic axioms are<sup>10</sup>.

- (i) where A is any event, the "probability of A",  $P[A]$ ,  $\geq 0$
- (ii) where S is the certain event, i.e. the set of all possible descriptions,  $P[S] = 1$ .
- (iii) where A and B are two mutually exclusive events,  
 $P[A \cup B] = P[A] + P[B]$

In addition there are two important definitions:

- (i) the "conditional probability" of A given B,  
 $P[A|B]$  is defined by

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

- (ii) two events A and B are said to be "statistically independent" if

$$P[A|B] = P[A]$$

The development of probability follows logically from these axioms, although to be strictly rigorous some additions or modifications are necessary to cover the case where there can be an infinite number of sets within S. As an alternative to this approach it is possible to consider the definition of event A as a proposition, so that probabilities are considered as numerical values assigned to propositions rather than to sets.

The application and interpretation of probability theory is based on intuitive decisions as to the reasonableness or usefulness of the definitions of probability, conditional probability and independence, and the results that flow from the use of the axioms.

One interpretation of probability which is universally accepted is its use as a measure of the relative frequency with which the various possible outcomes of repeated experiments occur. This has the effect of making probability compatible with statistics so that statistical investigations may serve as the basis for the estimation of probabilities. This does not mean however that this "frequentist" interpretation of probability is the only possible one.

Another interpretation which is widely, although not universally, accepted is the "Bayesian" or "subjective" interpretation. Whereas with the frequentist interpretation probability is used as a measure of the innate variability of the situation under consideration, the Bayesian interpretation is based on the use of

probability as a measure of the uncertainty due to lack of knowledge. The probability then depends not directly on the objective reality of the situation under consideration, but on the subjective assessment of the situation by an observer. It can therefore vary between different observers and change with time as more information becomes available. It is against this use of probability that most objections are raised, some of which will now be discussed.

## 2.3 Blockley's Objections to the use of Probability Theory

### 2.3.1 Introduction

The principal source of reasoned objections to the use of probability in structural reliability calculations has been Blockley. In the form in which they are presented in his book<sup>12</sup> the objections apply specifically to the application of probability to "system uncertainty" which he defines as "the uncertainty due to the lack of dependability of a theoretical model when used to predict the behaviour of a proposed structure, assuming a precisely defined set of parameters describing the model". This is contrasted with "parameter uncertainty" which he defines as "that uncertainty which results from uncertain values of the parameters to a perfect system", for which probability is deemed an appropriate measure. This terminology is different from that employed in this thesis, which is given in section 2.6.2

Blockley gives two separate arguments to support his objections. The first is based on Popper's philosophy of scientific discovery<sup>13</sup>, and the second on a paradox due to Rescher<sup>14</sup>. He then discusses the approximate reasoning methods of fuzzy sets and fuzzy logic which he proposes as a substitute for probability. But it is noteworthy that nowhere does he specifically justify these substitutes by demonstrating any superiority in meeting his objections to probability theory.

Popper's view of science is that general scientific theories can never be 'proved' absolutely because of the impossibility of checking all instances to which a theory has applied, does apply, and will in future apply. Experiments can only falsify, or fail to falsify, theories. A good theory then is one that has been repeatedly tested without being falsified. It is natural to want to quantify the extent to which a theory has been successfully tested, but Popper shows that probability is not a useful measure for this and he therefore develops his concept of 'degree of corroboration'.

Blockley takes up these ideas, stating that the probability that Newton's Laws are actually true is very small or indeed zero (evidence favouring Einstein where the two differ) but that nonetheless they are highly dependable because they are highly corroborated. He concludes: "Structural analysis is built on theory which is probably not true, but one which is dependable".

The phrase 'probably not true' is open to considerable objections and many views of the nature of probability and of truth would not consider such phrases meaningful. It is notable that Popper himself does not use this phrase. However the principal problem with Blockley's argument is that he never explains why he thinks the probability of a theory being true is relevant to the assessment of reliability. It is as if he envisages calculations of the form:

$$\begin{aligned}
 &P[\text{Fatigue Life} < 20 \text{ years}] \\
 &= P[\text{Calculated Fatigue Life} < 20 \text{ years} | (\text{Structural} \\
 &\quad \text{model is correct})] \\
 &x P[\text{Structural model is correct}] \qquad (2.1)
 \end{aligned}$$

In fact, no such calculations are made. As will be demonstrated in more detail later (Section 2.6.2), calculations tend to take the form:

$$\begin{aligned}
 &P[\text{Fatigue Life} < 20 \text{ years}] \\
 &= \sum_{\text{all } i} \{ P[(\text{Fatigue Life} < 20 \text{ years}) | (\text{Structural model} \\
 &\quad \text{in error by factor between } f_i \text{ and } f_{i+1})] \\
 &\quad x P[\text{Structural model in error by factor} \\
 &\quad \text{between } f_i \text{ and } f_{i+1}] \} \qquad (2.2)
 \end{aligned}$$

Here instead of  $P[\text{Structural model is correct}]$  we have a series of probabilities,  $P[\text{Structural model in error by factor between } f_i \text{ and } f_{i+1}]$ , which will form a probability distribution on  $f_i$ . According to Blockley, "if we are dealing with a finite number of well-defined events in a sample space then we are dealing with the classical application of the probability of events". In the case given the sample space is clearly defined by all the possible values of  $f_i$ , so by Blockley's own definition probability can be used for "system uncertainty", at least in the approach to it adopted here.

In this formulation there are of course still assumptions present. For example, it will generally be assumed that laboratory studies provide adequate evidence as to the values to be given to the uncertainties:

$P[\text{Structural model in error by factor between } f_i \text{ and } f_{i+1}]$

However when there is doubt as to the validity of any such assumption the process above can be carried out again using whatever evidence is available as to the magnitude of the uncertainties.

In conclusion therefore we are not interested in questions of the form "is this theory true?" or "is this theory dependable?" but we wish to know how dependable the theory is, measured by the accuracy of its predictions, and to incorporate all this information into the calculations; and according to Blockley's own statements, probability theory is suitable for such calculations.

### 2.3.2 Rescher's Paradox

Rescher's paradox approaches the corroboration of a theory or proposition from a different viewpoint. If we have a statement 'A' which is to form the basis of structural calculations (e.g. Newton's laws are correct), then we wish this to have a high probability of being true, i.e.  $P[A]$  should be nearly unity. Now suppose we have a source 'X' and a statement 'B' which is 'X

asserts A' then if X is a reliable source  $P[A|B]$  will also be nearly unity.

$$\text{Since } P[A] = \frac{P[A|B].P[B]}{P[B|A]} \quad (2.3)$$

$$\text{it follows that } P[B|A] \approx P[B] \quad (2.4)$$

Rescher interprets this as meaning that B is nearly independent of A, i.e. X asserts A irrespective of the truth of A. Blockley considers this to be unacceptable and as further evidence that corroboration cannot be measured satisfactorily by probability.

The so-called paradox can however be resolved by considering the complementary events  $\bar{A}$  ('A is untrue') and  $\bar{B}$  ('X does not assert A'). If X is to be a reliable and useful source, as stated in the definition of the problem, it must be taken that X always expresses a definite opinion on A so that  $\bar{B}$  also means in effect 'X asserts  $\bar{A}$ '.

Now if  $P[A]$  is high it automatically follows that  $P[\bar{A}]$  is low. If X is a good source we want a high probability that X predicts correctly and a low probability that X predicts incorrectly, i.e.  $P[A|B]$  and  $P[\bar{A}|\bar{B}]$  should both be high and  $P[\bar{A}|B]$  and  $P[A|\bar{B}]$  should both be low.

Now if  $P[A|B] = P[A]$ , i.e. exact equality, it can be proved that all the following equalities are necessarily true:

$$\begin{array}{ll} P[A|B] = P[A] & P[B|A] = P[B] \\ P[\bar{A}|B] = P[\bar{A}] & P[\bar{B}|A] = P[\bar{B}] \\ P[A|\bar{B}] = P[A] & P[B|\bar{A}] = P[B] \\ P[\bar{A}|\bar{B}] = P[\bar{A}] & P[\bar{B}|\bar{A}] = P[\bar{B}] \end{array} \quad (2.5)$$

so that the independence of  $\bar{A}$  and  $\bar{B}$  is inseparable from the independence of A and B, as one would intuitively expect from the meaning of the word "independent" in this context.

Rescher's concept of "near-independence" is not an accepted part of probability theory so it is necessary to define what the term should be taken as meaning. At least, enlarging on Blockley, it

can be said that it would be unacceptable if the probability of X asserting A, or not, was independent of the truth, or otherwise, of A; i.e. the paradox would still stand if:

$$\begin{aligned}
 &P[B|A] \approx P[B] \\
 \text{and } &P[\bar{B}|A] \approx P[\bar{B}] \\
 \text{and } &P[B|\bar{A}] \approx P[B] \\
 \text{and } &P[\bar{B}|\bar{A}] \approx P[\bar{B}]
 \end{aligned} \tag{2.6}$$

But unlike the case of exact equality we cannot say that all these are true if any one is true. Using the assumptions above we can examine each proposed relationship in turn:

$$\begin{aligned}
 \frac{P[B|A]}{P[B]} &= \frac{P[A|B]}{P[A]} = \frac{\text{high}}{\text{high}} \approx 1 \\
 \frac{P[\bar{B}|A]}{P[B]} &= \frac{P[A|\bar{B}]}{P[A]} = \frac{\text{low}}{\text{high}} \neq 1 \\
 \frac{P[B|\bar{A}]}{P[B]} &= \frac{P[\bar{A}|B]}{P[A]} = \frac{\text{low}}{\text{low}} \approx 1, \text{ perhaps} \\
 \frac{P[\bar{B}|\bar{A}]}{P[B]} &= \frac{P[\bar{A}|\bar{B}]}{P[A]} = \frac{\text{high}}{\text{low}} \neq 1
 \end{aligned} \tag{2.7}$$

which shows that X's assertions are not all 'nearly independent' of the truth of A under the given assumptions. It should be noted that it has been assumed that the probabilities above can all be simultaneously assigned appropriate values. Because we can write:

$$\begin{aligned}
 P[A] &= P[A|B].P[B] + P[A|\bar{B}].P[\bar{B}] \\
 P[\bar{A}] &= P[\bar{A}|B].P[B] + P[\bar{A}|\bar{B}].P[\bar{B}] \\
 P[A] + P[\bar{A}] &= P[B] + P[\bar{B}] = 1
 \end{aligned} \tag{2.8}$$

we do not have complete freedom to assign whatever values we like to these quantities, but any restrictions will make it even more difficult to satisfy all the intuitive requirements of 'near-independence'.

If A has a high probability of being true, then clearly a characteristic of a good source is that it correctly identifies the few occasions on which A is not true. It is the complementary events A and B, whose probabilities do not seem paradoxical, which are most important.

## 2.4 More General Objections to the use of Probability Theory

What seems to lie behind many of the objections commonly raised to the use of probability theory in structural problems is the feeling that unless it is possible to envisage realistically a set of nominally identical cases or a repeated experiment, then terms such as 'probability of failure' seem meaningless. In the case of a large set of identical cases, the probability of failure (defined as the relative frequency with which failure occurs in that set) is a measurable property of that set, at least in retrospect, and it is therefore possible to say (subject to the limits of statistical significance) that a calculated probability was either correct or incorrect. However if it is only really possible to envisage a single structure, which either fails or survives, it is difficult to see how one could ever judge the correctness of the assessed probability; and if it is impossible to say whether a probability is correct or not, then what meaning can be attached to it? Is it not a purely arbitrary number?

This question can be answered by saying that such probabilities are to be interpreted in a "Bayesian" or "subjective" fashion and are only intended as "notional" or "operational" values which will play an intermediate role in a decision-making process, but need not themselves have any other significance. Alternatively<sup>6,8</sup> a calculated probability can be regarded as a logical deduction from the assumptions about structural behaviour and the data used to derive probability distributions for the basic variables. This restricts the role of subjective assessments because the whole process must be logical and rational; any rational person starting from the same basis should come to the same conclusions. In theory this is not a problem, but it is clear that, in practice, much of the available data will have considerable scope for interpretation and that this interpretation cannot always be spelled out as a set of strictly logical deductions. There are strictly systematic methods of interpretation such as the maximum entropy formulism, however for reasons that will be discussed later (Section 2.6.3) this is not considered wholly satisfactory. In any case the question that worries some people still remains, what does the calculated probability actually mean?

Another possible answer to the problem of the meaning of probability in cases which are inherently unrepeatable, which has the advantage of encompassing the frequentist interpretation when that is acceptable, is suggested here. It involves creating a form of frequentist situation by imagining a large number of probability assessments being made for different cases, such as might be made by an insurance underwriter, for instance. A statement of the type "the probability of this structure failing within the next year is  $10^{-4}$ " would be interpreted as "this structure belong to a set of risks whose expected rate of occurrence during the next year is 1 in  $10^4$ ". The point of such an interpretation is that although one can never say that any one calculation was or was not correct, the calculations cannot be wholly arbitrary because they must always aim at satisfying certain criteria. Clearly, for an insurer to stay in business he must assess the risks so that (ignoring the allowance in his premiums for profit) he just breaks even, on a large enough quantity of business. Furthermore assuming the presence of competitors it is necessary that he should aim to break even on any subset of his business, otherwise those parts where he over-estimates the risk and therefore quotes too high a premium will be liable to be taken by his competitors, leaving him to make a loss on those parts where he has under-estimated the risk. The point of this argument is not to provide an accurate discussion of the insurance business, but to demonstrate, first that an interpretation of probability not wholly different from the frequentist interpretation can be used in cases where the idea of repeated tests or large numbers of identical examples is not intellectually appealing, and secondly that even when subjective judgements, or indeed sheer guesswork enter into the calculations there is still a clear objective aim which must be held in mind when making the assessment, namely to produce a calculated probability which will prove to be statistically correct when checked over any (large enough) group of risks with the same calculated probability.

## 2.5 Basis of Structural Reliability Theory

The basis of structural reliability theory is the use of probability as a measure of the uncertainty affecting the

occurrence of those circumstances (e.g. collapse or lack of serviceability) which a structure is designed to avoid. However structural failures are extremely infrequent occurrences, and most structures are built on a one-off basis. There are, therefore, far fewer opportunities to collect meaningful statistics for the actual failures than there are in other branches of engineering.

It is therefore necessary to calculate the probability of these "failure" states from other information; structural reliability analysis is the calculation of probabilities of failure using models of structural behaviour and data on the variability and uncertainty of relevant quantities. Such calculations can be concerned with the failure of the whole or of part of a structure. In the case of fatigue where cracks may grow throughout the life of a structure, calculations concerned with, for example, the inspection of structures and the early detection of cracks may be based on the achievement of states short of what would normally be described as failure; such calculations are not however fundamentally different from those relating to true failure. In the general exposition of the methods, the condition whose probability is being calculated will be referred to as "failure".

The occurrence of failure in a particular case will depend on the values of various quantities such as dimensions, loads, and material properties. Some of these can themselves be expressed as functions of other, more fundamental quantities; for example, material properties can be expressed, in theory at least, as functions of chemical composition and aspects of the process of manufacture. A set of variables,  $\underline{X}$ , termed the "basic variables" are chosen to be the basis of the reliability analysis. They are chosen, on grounds of convenience and availability of data, to cover all the sources of uncertainty that are relevant to the situation being analysed. Conventionally failure is indicated by a quantity  $Z$  where  $Z \leq 0$  means that failure occurs and  $Z > 0$  means that failure does not occur. The dependence of the reliability of the structure on the basic variables is expressed by a function:

$$Z = g(\underline{x}) \quad (2.9)$$

The essence of the method is that this function is deterministic; all the uncertainties associated with the problem are accounted for by assigning probability distributions to the basic variables.

The function  $g(\underline{x})$  does not, of course, need to be a single explicit equation; it can express the outcome of a lengthy series of calculations. If the equations and models which are available for use in  $g(\underline{x})$  are not exact models of reality it is necessary to introduce (an) additional variable(s) into  $g(\underline{x})$  to allow for the discrepancies between theory and practice and to enable  $g(\underline{x})$  to remain wholly deterministic. These "model uncertainties" will be discussed in detail in the next section.

It is normally possible to choose the basic variables so that they are statistically independent of one another. This should be done wherever possible, although calculation methods have been devised<sup>15</sup> to allow mutually dependent variables to be used. The methods outlined below assume all variables are independent.

Given a set of independent basic variables  $X_i, i = 1, n$ , with probability density functions,  $f_{X_i}(x_i)$ , and a description of failure as occurring if and only if

$$Z = g(\underline{x}) \leq 0 \quad (2.10)$$

the joint probability density function of all the variables is given by:

$$\prod_{i=1}^n f_{X_i}(x_i)$$

and hence the probability of failure is given by evaluating

$$\iiint \dots \int \prod_{i=1}^n f_{X_i}(x_i) dx_i \quad (2.11)$$

over the region of the  $n$ -dimensional hyperspace in which  $Z \leq 0$ .

More complex failures can also be handled. For example the probability of the joint occurrence of two failures A and B (i.e. the event  $(\text{Failure A}) \cap (\text{Failure B})$ ) where

$Z_A = g_A(\underline{x}) \leq 0$  indicates failure 'A'

$Z_B = g_B(\underline{x}) \leq 0$  indicates failure 'B'

is found by evaluating the same integral over the region where both  $Z_A \leq 0$  and  $Z_B \leq 0$ , provided that these functions apply irrespective of whether or not the other failure has occurred. This sort of calculation is appropriate for problems of the type discussed in section 1.2(c) above. Some basic variables will occur in both failure functions, some in only one. If at first sight a particular basic variable appears to be included in both functions, it is necessary to check whether both failures really depend on the same value of the variable, or whether they actually depend on two different values. For example, considering plate thickness as such a variable: are the two failures in the same piece of plate, or are they in two different pieces of plate? In the latter case there should be two basic variables, one in each function, albeit both having the same distribution.

The integrals can in theory be evaluated by numerical integration or by Monte-Carlo methods, but in general simpler approximate methods are sufficiently accurate, and may often be the best approach for a given amount of computing. These will be discussed below.

Structural reliability analysis then consists of three basic steps:

- (i) Choice of basic variables and formulation of failure function,  $g(\underline{x})$ .
- (ii) Probabilistic modelling of uncertainties in basic variables to give distributions  $f_{X_i}(x_i)$ .
- (iii) Calculation of probability of failure.

There is little that can be said in general terms about step (i). The final sections of this chapter develop the theoretical bases for steps (ii) and (iii).

## 2.6 Probabilistic Modelling of Basic Variables

### 2.6.1 Introduction

The task of selecting probability distributions to model the basic variables can be divided into three stages: deciding the nature of the uncertainty to be represented, choosing a distribution type, and choosing parameters for the distribution. These three stages will be discussed in turn.

### 2.6.2 Classification of Uncertainty

Three types of uncertainty are normally identified. They are:

#### (i) Physical Variability

Any physical quantity involved in an engineering design, such as a dimension, or a material property, is subject to some unavoidable variation between nominally identical items. Although at the sub-atomic level nature is inherently uncertain, in the engineering context the physical variability is the result of the impossibility of exercising perfect control over manufacturing or fabricating processes and the degree of variability that exists in practice is the result of the quality of workmanship, and of the tolerances allowed by the inspection and control procedures.

#### (ii) Statistical Uncertainty

Models for physical variability are produced by considering data consisting of measurements of the quantity in question. If the amount of data is small, or if it is believed to be imperfect in some way (e.g. the examples measured were not truly selected at random and are therefore not necessarily representative of the whole population) another type of uncertainty, statistical uncertainty, is introduced. It is frequently convenient to express this as uncertainty in the values of the parameters (such as mean and standard deviation) of the distribution describing the physical variability. The statistical uncertainty is a measure of one's confidence in the results derived from measurements or

tests. If probability is only interpreted in a strictly "frequentist" way it cannot be used as a measure of statistical uncertainty. Only with a wider "Bayesian" view of probability can statistical uncertainty be covered.

(iii) Model Uncertainty

In formulating the failure function,  $g(\underline{x})$ , use is made of calculation models for the type of failure being studied. However few, if any, models will always give exactly correct predictions and so a model uncertainty has to be introduced. This represents variations in the quantity:

Value found in experiment

Value given by the calculation model

where the 'value' referred to is the value of empirical constants when the model uses empirical rules, and of additional model uncertainty variables where the model has a theoretical basis. Although its parameters and distribution type will generally be found from experimental results in the same way as the physical variability discussed above, it actually expresses much more complex uncertainties.

If the model uncertainty is assessed separately for every possible combination of every possible value of all the basic variables that are used in the model, then the model uncertainty arises from the physical variability of those quantities which affect the outcome but which are not included explicitly in the calculation model. This procedure would however normally need extremely extensive testing so that it is usual to collect test results together which are not from nominally identical tests. In this case any systematic variations that may exist between the model uncertainties which affect different cases will be incorporated into the assessment. The usual aim of the data analysis is to expose these systematic variations as far as possible, but sometimes the best that can be achieved is a distribution that represents the average performance of the model over a set of circumstances which are believed to reflect actual usage.

A model uncertainty in practice then may compound together several more fundamental sources of uncertainty, and it is necessary when determining its distribution to be clear what is being included and to interpret and use the resulting probability of failure in an appropriate way<sup>16</sup>.

As with the physical variability, shortage of data will lead to statistical uncertainty about the correctness of the distribution functions proposed for a model uncertainty. This statistical uncertainty tends to be greater in the case of model uncertainties because it is often necessary (due to lack of appropriate data) to assume that a distribution derived from one set of data is applicable in circumstances not covered by the original test programme.

As mentioned above the statistical uncertainty is often best expressed as a distribution for the parameters of the distribution that represents the physical variability, although in the case of many model uncertainties it is difficult, if not impossible to separate the two elements.

In the following sections on the choice of distributions it will be assumed that the distribution sought is to represent the physical variability of a basic variable, as exhibited by a set of data which will be assumed to be a random sample from a single population. This is, however, rarely true in practice and section 2.6.5 will discuss the role of statistical uncertainties and practical methods for handling real data.

### 2.6.3 Choice of Distribution Type

There are three different approaches to this problem. They are:

(i) Choice by accuracy of fit to the available data. In this approach a number of different distribution types are chosen and the best values of the required parameters found for each type using the data and one of the methods to be discussed in section 2.6.4. The resulting distributions are then compared with each other and with the data, and the one that gives the best fit is

chosen. The goodness of fit may be judged (a) by graphical methods<sup>17</sup>, (b) by the more formal tests of classical statistics<sup>18</sup>, or (c) by comparing one or more values such as the higher order moments of the distributions and the data. The latter method is only appropriate where there is a very large quantity of data available, which is not often true of structural reliability problems.

(ii) Choice based on qualitative arguments about the nature of the basic variable. Many distribution types were originally created to represent the variations and uncertainties that occur in certain well-defined instances. For instance, the normal and log-normal distributions arise from the central limit theorem and the extreme value distributions from consideration of the asymptotic behaviour of the distribution of extreme values. Hence it should often be possible to select suitable distribution types from consideration of the nature of the basic variable being studied. This approach will be used extensively in this thesis.

(iii) Use of distribution types capable of generating a wide variety of forms. A natural development of the first approach given above is to seek a general method in which a large number of different distribution forms are brought together into a single system, and the most appropriate distribution found simply by consideration of the data. One classical version of this approach is due to Pearson<sup>19</sup> who uses the first four moments of the data to select from a total of twelve different distributions (three main types and nine transitional types). This approach cannot be used with confidence when there is only a limited quantity of data, because of the unreliability of the calculated values of the higher moments. The Pearson distributions are all solutions of the differential equation

$$\frac{d}{dx} (\log(f_X(x))) = \frac{a + x}{b_0 + b_1x + b_2x^2} \quad (2.12)$$

which was arrived at by Pearson by consideration of the general forms that distributions take. It includes the normal distribution as a special case. It is clearly however a somewhat arbitrary

form, and other families of distributions could be created by using other assumptions.

Jaynes<sup>20</sup> has attempted to avoid this problem by using the concept of maximising entropy to generate distributions. Following Shannon's work on Information Theory, entropy is used as a measure of the lack of knowledge or information about a situation. Jaynes' principle is to find the distribution that maximises the entropy subject only to the constraints imposed by the known facts of the situation (including the data). This distribution is then held to be the only one that carries no information other than that contained in the data, and hence it incorporates no arbitrary assumptions.

The most widely quoted solutions obtained by this method are for distributions of discrete-valued variables where the constraints on the maximisation that represent the available information are in the form of expectations of various functions. Hence the task is to find the probability,  $p_i$ , for each value  $x_i$ , by maximising the entropy  $H$ , which is given by:

$$H = -\sum_i p_i \cdot \ln p_i \quad (2.13)$$

$$\text{subject to } \sum_i p_i = 1 \quad (2.14)$$

$$\text{and } \sum_i p_i \cdot g_k(x_i) = G_k \text{ for } k = 1 \text{ to } m \quad (2.15)$$

where  $g_k(\cdot)$  are the functions whose expectations,  $G_k$ , are the constraints on the distributions.

The solution to this is:

$$p_i = \exp(\lambda_0 + \sum_k \lambda_k \cdot g_k(x_i)) \quad (2.16)$$

where the parameters  $\lambda_k$  are found by substituting for  $p_i$  in equation 2.15 and solving.

For continuous variables the problem is formulated thus:

$$\text{Maximise } H = -\int f(x) \cdot \ln \frac{f(x)}{m(x)} \cdot dx \quad (2.17)$$

$$\text{subject to } \int f(x) \cdot dx = 1 \quad (2.18)$$

$$\text{and } \int f(x) \cdot g_k(x) \cdot dx = G_k$$

where  $f(x)$  is the distribution to be found,  $m(x)$  is termed the invariant measure function and the integrals are made over the range of  $x$  values over which  $x$  is defined. The solution is given by

$$f(x) = m(x) \cdot \exp(\lambda_0 + \sum_k \lambda_k \cdot g_k(x)) \quad (2.19)$$

The invariant measure function can be thought of as the distribution of complete ignorance, since in the absence of any other constraints

$$f(x) = \frac{m(x)}{\int m(x) \cdot dx} \quad (2.20)$$

It might be thought that the concept of 'complete ignorance' is too vague to be useful, but in Jaynes' hands it proves to be extremely powerful<sup>21</sup>. Some of the arguments he employs are extremely subtle and thus leave considerable doubt as to whether they are suitable for widespread use. In addition, other authors writing on the same subject do not always come to the same conclusions (e.g. Bernardo<sup>22</sup>). Thus while one may accept that the method of maximising entropy removes the arbitrariness implicit in other methods if the function  $m(x)$  is given and if the constraints are of the form given above, it is much more difficult to accept that no arbitrariness is present in the choice of  $m(x)$  and the constraints.

The real constraint on the distribution ought to be that the available data are random samples from it. There seems to be no way, however, of using this idea directly. It would seem from the published work of Jaynes, and others, that the only way in which data can be used is to give values for the expectations of some functions, and the choice of functions must be essentially arbitrary.

If  $m(x)$  is taken as constant and the expectations used are the mean and variance, then a normal distribution results. The arguments of Jaynes<sup>23</sup> would seem to suggest that  $m(x) = 1/x$  is appropriate for a physical quantity which cannot be negative and which has physical dimensions. If the mean and variance of  $\log(x)$  are used as constraints then a lognormal distribution results with  $m(x)$  constant or equal to  $1/x$ . However if  $m(x) = 1/x$  is used with the

mean and variance of  $x$  as constraints then the resulting calculations involve integrals with infinite integrands, and no solution is possible. The range of solutions could be increased further by considering further moments and different functions.

#### 2.6.4 Determination of Parameters for Distributions

The commonest, and simplest, method used to determine parameters for a probability distribution of a specified type is to fix them so that the moments of the distributions equal the moments of the data. The number of moments that can be matched this way is of course equal to the number of parameters that the distribution uses. The widespread use of this method is probably the result of the general use of quantities such as mean and standard deviation, skewness and kurtosis, which are derived from moments, to characterise sets of statistics, and of the fact that the Normal Distribution uses the mean and standard deviation as parameters in its usual algebraic presentation. There is however no reason other than convenience why this method should be used.

In statistics the generally preferred method is that of maximum likelihood<sup>24</sup>. If a variable  $X$  has a distribution  $f_{X|\underline{\theta}}(x, \underline{\theta})$  with parameters  $\underline{\theta}$  and if  $n$  random samples have been taken with values  $\underline{z} = (x_1, x_2, \dots, x_n)$ , then the likelihood of  $\underline{\theta}$ , given the known values of  $x$  is given by

$$L(\underline{\theta}|\underline{z}) = \prod_{i=1}^n f_{X|\underline{\theta}}(x_i, \underline{\theta}) \quad (2.21)$$

i.e. it is proportional to the probability that the values  $x_i$  will occur, if the parameters are  $\underline{\theta}$ . Hence the values of  $\underline{\theta}$  to be used are those that maximise the likelihood, i.e. the solutions of the equations

$$\frac{\partial}{\partial \theta_j} \left[ \prod_{i=1}^n f_{X|\underline{\theta}}(x_i, \underline{\theta}) \right] = 0 \quad \text{for all } j. \quad (2.22)$$

Although the maximum likelihood method is widely accepted and has much to commend it, it is not beyond criticism. For instance, Benjamin and Cornell<sup>25</sup> show that for a normal distribution, the maximum likelihood estimator for the standard deviation is not the unbiased estimator that is commonly used.

The underlying purpose of all the methods of choosing parameters is to use the available data as a means of estimating the properties of the population of which they are assumed to be a random sample. Clearly however any method for doing this is subject to some uncertainty, and in the Bayesian approach this uncertainty is explicitly recognised by giving a probability distribution to the parameters  $\underline{\theta}$  of the distribution. Initially, a "prior" distribution  $f_{\underline{\theta}}'(\underline{\theta})$  is created to represent whatever information about  $\underline{\theta}$  is available before the data itself is considered. Then a "posterior" distribution for  $\underline{\theta}$  given  $\underline{z}$  is calculated from the prior distribution and the data,  $\underline{z} = (x_1, x_2, \dots, x_n)$ , by the use of Bayes' Theorem:

$$f_{\underline{\theta}|\underline{z}}''(\underline{\theta}, \underline{z}) = \frac{1}{N} \cdot P[\underline{z}|\underline{\theta}] \cdot f_{\underline{\theta}}'(\underline{\theta})$$

$$\text{where } N = \int_{-\infty}^{+\infty} \dots P[\underline{z}|\underline{\theta}] \cdot f_{\underline{\theta}}'(\underline{\theta}) \cdot d\theta_1 \cdot d\theta_2 \dots \quad (2.23)$$

$P[\underline{z}|\underline{\theta}]$  is the probability of getting the sample  $\underline{z}$ , given  $\underline{\theta}$ , and is the same as the likelihood function above,

$$\begin{aligned} P[\underline{z}|\underline{\theta}] &= L(\underline{\theta}|\underline{z}) \\ &= \prod_{i=1}^n f_{X_i|\underline{\theta}}(x_i, \underline{\theta}) \end{aligned} \quad (2.24)$$

There are two questions to be answered in using this approach: what prior distribution should be used, and what use should be made of the posterior distribution?

In general the Bayesian approach of prior information and subsequent data is rather artificial because the information does not come sequentially like this. One approach is to try to find prior distributions which can be said to represent "complete ignorance". This is the same idea as the invariant measure function discussed in the previous section, and is therefore subject to the same difficulties and disagreements. An alternative approach is to assume that the prior and posterior distributions have the same form, the use of Bayes Equation only changing the parameters. This leads to the theory of conjugate priors<sup>26</sup>.

There are a number of ways in which the posterior distribution can be used to indicate suitable values of  $\underline{\theta}$  to use as parameters for the distribution of  $X$ . The most obvious are:

- (i) Using the values of  $\underline{\theta}$  which give the posterior density function its maximum value.
- (ii) Calculating the marginal distribution of each  $\theta_j$ , and using its mode.
- (iii) Using the expectation of each  $\theta_j$ :

$$E[\theta_j] = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \dots f''_{\underline{\theta}|\underline{z}}(\underline{\theta}, \underline{z}) \cdot \theta_j \cdot d\theta_1 \cdot d\theta_2 \dots \quad (2.25)$$

It is clear that method (i) used with  $f'_{\underline{\theta}}(\underline{\theta})$  uniform on all  $\theta_j$  gives the maximum likelihood method, which is therefore seen merely as one possible realisation of a wider set of methods.

However, if one of these methods is adopted, it is clear that information about statistical uncertainty is being ignored in the subsequent calculations. The next section discusses the role that statistical uncertainty should play in reliability analysis.

#### 2.6.5 The Role of Statistical Uncertainties

In the previous section it was shown how uncertainty in deductions made from sample data could be represented by probability distributions. These took the form of distributions for the parameters,  $\underline{\theta}$ , of the distributions of the random physical variability of a basic variable,  $X$ ; i.e. the physical uncertainty in  $X$  is described by the conditional distribution,  $f_{X|\underline{\theta}}(x, \underline{\theta})$  and the statistical uncertainty by  $f_{\underline{\theta}}(\underline{\theta})$ . It is clearly possible to combine these distributions to give an unconditional distribution for  $X$ , thus:

$$f_X(x) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \dots f_{X|\underline{\theta}}(x, \underline{\theta}) \cdot f_{\underline{\theta}}(\underline{\theta}) \cdot d\theta_1 \cdot d\theta_2 \dots \quad (2.26)$$

This gives a probability for any value  $x$  which is, in effect, a weighted average of the values given by the various possible distributions of the physical variability; the weighting being given by the probability assigned to each distribution by the distribution of  $\underline{\theta}$ . This unconditional distribution is therefore the best to use to predict the probability of some value of  $x$  occurring, hence its common name of the 'predictive' distribution. Aitchison and Dunsmore<sup>27</sup> give extensive examples of its use.

In a reliability analysis some statistical uncertainties may exist for all or most of the basic variables,. Taken together these uncertainties are the source of statistical uncertainty in the results of reliability calculations which used only the physical variabilities (including physical variability elements of model uncertainties). The aim of a reliability calculation may be to find one or more probabilities of failure, or, it may go beyond that to the calibration of safety factors, for example. It is possible to carry out such calculations for physical variability only, repeating the calculations with a series of different values for the parameters  $\underline{\theta}$ . Then the distributions of  $\underline{\theta}$  could be used to assess the overall statistical uncertainty in these results, leading to statements such as "we are 90% confident that the probability of failure due to physical variability lies between this value and that value". In algebraic terms, the probability of failure due to physical variability,  $P_F(PV)$  is a function of  $\underline{\theta}$ , where  $\underline{\theta}$  is now taken as the vector of all the parameters of all the physical variability distributions.

$$P_F(PV)(\underline{\theta}) = \int_{z < 0} \dots \int_{i=1}^n f_{X_i|\underline{\theta}}(x_i, \underline{\theta}) \cdot dx_i \quad (2.27)$$

One interesting quantity to calculate is the expectation of  $P_F(PV)$ . If the parameters  $\underline{\theta}$  are all mutually independent:

$$E[P_F(PV)] = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} P_F(PV)(\underline{\theta}) \cdot \prod_{j=1}^m f_{\theta_j}(\theta_j) \cdot d\theta_j \quad (2.28)$$

$$= \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \int_{z < 0} \dots \int_{i=1}^n f_{X_i|\underline{\theta}}(x_i, \underline{\theta}) \cdot dx_i \cdot \prod_{j=1}^m f_{\theta_j}(\theta_j) \cdot d\theta_j \quad (2.29)$$

If the parameters,  $\Theta$ , are not independent, the appropriate joint density functions must be incorporated into the above equations.

If the relationship between the X variables and  $\Theta$  variables is such that while more than one  $\Theta$  may condition an X, each  $\Theta$  conditions only one X, i.e. the same statistical uncertainty does not apply to more than one basic variable, then,

$$E[P_f(PV)] = \int_{z>0} \dots \int f_{X_1}(x_1) \cdot f_{X_2}(x_2) \dots dx_1 \cdot dx_2 \dots (2.30)$$

i.e. the expectation of  $P_f(PV)$  is the value of  $P_f$  that is calculated by integration over  $z < 0$  using the unconditional predictive distributions of the X variables.

#### 2.6.6 Uncertainties in Real Data

For many of the variables that occur in the analysis of the fatigue resistance of tubular joints, data is likely to come in the form of measurements on a very wide range of different geometries with often only one or two results for each. A similar situation exists in many areas of structural engineering.

If one is interested in one particular geometry, and if there are a large number of results available for exactly that case, it will in general be possible to find a suitable probability distribution to fit the data, and it will be reasonable to believe that this distribution represents accurately the random variability in the quantity in question, due to those factors which are not fully controlled by the processes of manufacture and fabrication within the tolerances used.

If the number of test results available is small, but they are a genuinely random sample of cases which are "nominally identical" to the specified case which the basic variable is to represent, then the situation is as in section 2.6.4 where the random variability is subject to statistical uncertainty due to the sampling effect.

Frequently, however, there may be little or no data for the particular case of interest. In such circumstances it is necessary to use data for other broadly similar, but not nominally identical cases. But clearly there is some uncertainty about the correctness of this action. It is unlikely that the type of distribution used in one case will be seriously incorrect in another, but the distribution parameters are likely to differ from case to case. Therefore this uncertainty is best expressed as a distribution on the parameters of the random variability distribution, and it can therefore be considered as an extra element in the statistical uncertainty discussed above. In these circumstances the methods used to determine distributions will depend on exactly what data are available. If for example there is a reasonably large quantity of data for one set of nominally identical cases (but not the case of interest) then this may be used to indicate the scatter of results expected within any set of nominally identical cases, while the mean value for one particular geometry would be determined by the limited data available for that particular geometry. In such a case the statistical uncertainty might not be considered to be very large. By contrast, if no data at all were available for the specified case for which a distribution is required, then it must be assumed that the parameters  $\theta$  could take any of the values found in those cases for which data is available. On the assumption that this data covered a wide enough range of cases to reveal the complete extent of the variation in  $\theta$ , the distribution of  $\theta$  would be such that the unconditional, predictive, distribution of the basic variable would cover all the available results.

In the preceding paragraphs, the case discussed was where the basic variable whose distribution was being sought represented some aspect of the behaviour of one particular specified geometry. In many problems however one will want to discuss the reliability of a tubular joint without specifying the geometry or details of the joint. The distribution for such a case is obtained by combining the distributions for natural variability within nominally identical batches in proportion to the usage of those

batches in practice. This may be virtually the same as the 'predictive' distribution of the 'no data for the specified case' situation discussed in the preceding paragraph, but with the 'usage' distribution in place of the statistical uncertainty.

It is clear from this discussion that the concept of a set of 'nominally identical cases' plays an important role in the methods used in this thesis. When considering, for example, the reliability of an engineering design, this concept is used to define a set of cases (e.g. structures or structural elements) which, as far as the designer is concerned, are identical. They have the same nominal dimensions and are subject to the same specification for materials and workmanship. The differences that exist between the members of such a set are those that are allowed (in practice rather than in theory) by the tolerances and quality control procedures. These differences, being essentially uncontrolled, are characterised by a genuinely random variability. The differences between sets are, at a fundamental physical level, deterministic, but may be the cause of uncertainties in structural reliability resulting from a lack of knowledge about these physical differences, and from the combining of different sets into the mixed populations which occur in real structures or test programmes.

In practice the amount of data available is often limited, having been obtained for purposes other than reliability analysis. Its interpretation is therefore difficult and the formal procedures outlined above for choosing distributions and parameters do not have the theoretical justification that they have when the data is a genuine random sample from a single distribution. It is therefore more rational to proceed on an *ad-hoc* basis taking full account of what is known about the origin of the data. However, the basic system given above still applies. In this system there are three distributions to be found:

- (1) the distribution that describes the random variability of the basic variable that occurs within a set of nominally identical cases.
- (2) the distributions of the parameters of (1) which, depending on the context represent the statistical uncertainty in the parameters of distribution (1) or the usage of different cases.
- (3) the 'predictive' distribution of the basic variable which is a combination of the distributions (1) and (2) (as given by equation 2.26).

## 2.7 "Level 2" Algorithm

### 2.7.1 Definition of Problem

For the case of a set of  $n$  independent random variables, the probability of failure,  $p_f$ , is to be calculated from

$$P_f = \iiint_{Z < 0} \dots \int_{i=1}^n f_{X_i}(x_i) dx_i \quad (2.27)$$

as discussed in 2.5 above. If the probability density functions are all normal and  $Z$  is a linear function of all the  $X_i$ , then the probability of failure may be evaluated exactly. The possibility of using this fact as the basis for devising approximate methods of calculation for other circumstances has been widely discussed and accepted since the late 1960's. These methods have been called "Level 2 methods of safety checking". The reason for this terminology and a detailed account of the early methods is given in reference 28. The early methods suffered from the drawback that the result of the calculation depended on the form chosen for the failure function  $Z = g(\underline{X})$  at points away from the failure boundary  $Z = 0$ .

Subsequent developments by Hasofer & Lind<sup>29</sup> and Rackwitz & Fiessler<sup>30</sup> removed this problem, and Horne<sup>31</sup> produced a useful algorithm. An alternative formulation was shown by Baker & Flint<sup>32</sup> to lead to the same result; and the following analysis gives a much simplified derivation of the same algorithm.

A set of  $n$  standard normal variables  $\underline{U}$  is related to the set of basic variables,  $\underline{X}$ , by:

$$\Phi(u_i) = F_{X_i}(X_i) \quad (2.28)$$

where  $\Phi(\cdot)$  is the standard normal cumulative probability function.

$$\begin{aligned} \text{Hence } \phi(u_i)du_i &= f_{X_i}(x_i)dx_i \\ \text{and } P_f &= \iiint_{\text{Failure Region}} \dots \int_{u=1}^n \phi(u_i)du_i \end{aligned} \quad (2.29)$$

The failure region is defined by  $Z < 0$  where

$$\begin{aligned} Z &= g(x_1, x_2, \dots, x_i, \dots) \\ &= g(F_{X_1}^{-1}(\Phi(u_1)), \dots, F_{X_i}^{-1}(\Phi(u_i)), \dots) \\ &= g_U(u_1, u_2, \dots, u_i), \text{ say} \\ &= g_U(\underline{u}) \end{aligned} \quad (2.30)$$

If  $g_U(\underline{u})$  is linear in  $\underline{u}$ , then the failure boundary  $Z=0$  is a hyper-plane in the  $n$ -dimensional  $U$ -space. And if the perpendicular distance from this plane to the origin is  $\beta$  then

$$P_f = \Phi(-\beta) \quad (2.31)$$

For non-linear functions, the assumption is made that a good approximation to the true probability of failure by using this same equation with  $\beta$  as the shortest distance from the failure boundary to the origin.

$$\text{ie } P_f = \Phi(-\beta) \text{ where } \beta = \left( \sum_{i=1}^n u_i^{*2} \right)^{\frac{1}{2}} \quad (2.32)$$

and where  $\underline{u}^*$  is the point (sometimes called the "design point") on  $Z=0$  closest to the origin of the  $U$ -space.

Using a Lagrange multiplier,  $\lambda$ ,  $\underline{u}^*$  is given by the solution of:

$$g_U(\underline{u}^*) = 0 \quad (2.33)$$

$$\text{and} \quad \frac{\partial}{\partial u_i} \left[ \left( \sum_1 u_i^2 \right)^{\frac{1}{2}} + \lambda g_U(\underline{u}) \right] = 0 \text{ at } \underline{u} = \underline{u}^* \text{ for all } i \quad (2.34)$$

$$\text{This gives } u_i^* \left( \sum_1 u_i^{*2} \right)^{-\frac{1}{2}} + \lambda \frac{\partial g_U(\underline{u})}{\partial u_i} \Big|_{\underline{u} = \underline{u}^*} = 0$$

$$\text{whence} \quad \frac{u_i^*}{\frac{\partial g_U(\underline{u})}{\partial u_i} \Big|_{\underline{u} = \underline{u}^*}} = \text{constant for all } i \quad (2.35)$$

The solution of 2.33 and 2.35 gives  $\underline{u}^*$ , and hence  $\beta$  and  $P_f$ .

#### 2.7.2 Development of Iterative Routine

An iterative routine to find the point  $\underline{u}^*$  is created by starting at some arbitrary point  $\underline{u}$ , and calculating there:

$$Z = g_U(\underline{u}) \quad (2.36)$$

$$\text{and} \quad a_i = \frac{\partial g_U(\underline{u})}{\partial u_i} \quad \text{for all } i. \quad (2.37)$$

$a_i$  is normally calculated numerically rather than algebraically using small increments of each  $U_i$  in turn. A new point  $\underline{u} + \Delta \underline{u}$  is found, which is defined as being the point that would be the design point if  $g_U(\underline{u})$  were a linear function with the calculated values of  $a_i$ . This is assumed to be a closer approximation to the true design point than the initial  $\underline{u}$ , and the process is then repeated until convergence is achieved. While it has not been proved mathematically that this procedure will always converge, it has proved satisfactory in practice in a wide range of cases.

Expressing this argument in mathematical terms, the linearised failure function will take values

$$Z = a_0 + \sum_1 a_i u_i \quad \text{at } \underline{u}$$

and  $0 = a_0 + \sum_i a_i(u_i + \Delta u_i)$  at  $\underline{u} + \Delta \underline{u}$

$$\text{Hence } Z = -\sum_i a_i \Delta u_i \quad (2.38)$$

Now from 2.35 at  $\underline{u} + \Delta \underline{u}$ :

$$\frac{u_i + \Delta u_i}{a_i} = \text{constant, say } K$$

$$\text{i.e. } u_i + \Delta u_i = a_i K$$

Multiplying by  $a_i$  and summing over all  $i$ :

$$\sum_i a_i u_i + \sum_i a_i \Delta u_i = \sum_i a_i^2 K$$

Substituting from 2.38:

$$\sum_i a_i u_i - Z = \sum_i a_i^2 K$$

$$K = \frac{\sum_i a_i u_i - Z}{\sum_i a_i^2}$$

$$\text{Hence } u_i + \Delta u_i = \frac{(\sum_i a_i u_i - Z)}{\sum_i a_i^2} \quad (2.39)$$

It should be noted that the calculation is carried out entirely in terms of the transformed  $U$  variables, not the original  $X$  variables, except that in practice it is convenient to write the failure function in terms of the original  $X$  variables. Hence the procedure is to carry out the iteration in the  $U$ -space, but when it is required to find a value of  $Z$ , the appropriate values of  $u_i$  are transformed by equation 2.28 into  $x_i$  and then substituted into 2.10 to obtain  $Z$ .

### 2.7.3 The definition of $\beta$

The definition of  $\beta$  and  $P_f$  given in equation 2.32 means that if  $\beta$  is taken as the positive root,  $P_f$  will always be less than 0.5. This is of course normally the case, but it is advisable to review the definition of  $\beta$  so that other cases will be covered adequately if they should arise. If the equation

$$P_f = \Phi(-\beta) \quad (2.40)$$

is to apply in all cases then we require

$$\beta = \left[ \sum_{i=1}^n u_i^{*2} \right]^{1/2} \text{ when the origin of the U-space is in the "non-failure" or "safe" region,}$$

and  $\beta = - \left[ \sum_{i=1}^n u_i^{*2} \right]^{1/2} \text{ when the origin of the U-space is in the "failure" region}$  (2.41)

Alternatively, at the end of each stage of the iterative calculation the "current estimate" of  $\beta$  is given by:

$$\begin{aligned} \beta &= \left[ \sum_1 (u_i + \Delta u_i)^2 \right]^{1/2} \\ &= \left[ \sum_1 a_i^2 \frac{(\sum_1 a_i u_i - Z)^2}{(\sum_1 a_i^2)^2} \right]^{1/2} \\ &= \pm \frac{(\sum_1 a_i u_i - Z)}{(\sum_1 a_i^2)^{1/2}} \end{aligned} \quad (2.42)$$

Now at or near the correct solution,  $Z$  is small, so

$$\beta \approx \pm \frac{\sum_1 a_i u_i}{(\sum_1 a_i^2)^{1/2}}$$

and it is found that in the usual case of small probabilities of failure,  $a_i > 0$  and  $u_i < 0$  for resisting variables, and  $a_i < 0$  and  $u_i > 0$  for loading variables, so that for a positive value of  $\beta$  we should use

$$\beta \approx - \frac{\sum_1 a_i u_i}{(\sum_1 a_i^2)^{1/2}} \quad (2.43)$$

However, if the origin lies in the "failure" region,  $a_i$  and  $u_i$  will always have the same sign. Therefore if we define  $\beta$  by the equation:

$$\beta = - \frac{(\sum_1 a_i u_i - Z)}{(\sum_1 a_i^2)^{1/2}} \quad (2.44)$$

then it will automatically have the correct sign.

## 2.8 Calculation of Conditional Probabilities by truncation and re-normalisation of the variable space

### 2.8.1 Conditional Probability of Failure

The method given in section 2.7 for calculating an approximate probability of failure has been tested in a wide variety of situations<sup>16</sup> and has proved to be simple and effective, and generally sufficiently accurate. The accuracy is due to the rapid decline in the value of the joint probability density function away from the design point in the failure region, so that the difference between the true failure boundary and the linearised one is of limited significance. The accuracy will not be so good if the design point is close to the origin ( $P_f = 0.5$ ) as the joint probability density function is much flatter in this region. This may be important when considering some conditional failure events.

The method of section 2.7 cannot be used to calculate conditional probabilities of failure (e.g.  $P[\text{Failure A}|\text{Failure B}]$ ) of the sort discussed in section 2.5 above where Failure A and Failure B are partly correlated because some of the variables that determine failure are the same for both. In this section and the next, two new methods will be presented, to cover such calculations in the particular circumstances (see section 1.2(c)) where the form of the failure function for each failure is unaffected by the occurrence of the other failure; for example where the occurrence of one failure does not lead to a significant redistribution of loads.

These methods are essentially developments of the standard "Level 2" method of section 2.7. The first is generally applicable, the second can only be used in certain cases, but is expected to be more accurate. Neither is likely to be quite as accurate on average as the standard "Level 2" method of section 2.7 and the extent to which they have been used and tested is limited to the examples given in this thesis.

### 2.8.2 Truncation and Renormalisation

The first method is derived from the approach used for basic

variables whose range of possible values is truncated by some action such as testing or inspection. If a basic variable  $X$ , say a material strength, has an initial cumulative distribution function,  $F_X(x)$ , but before incorporation into the structure undergoes an ideal test which ensures that material whose strength is below  $x_T$  is not incorporated into the structure, then the distribution of the material actually incorporated will be given by:

$$F_X'(x) = 0 \quad x \leq x_T$$

$$F_X'(x) = \frac{F_X(x) - F_X(x_T)}{1 - F_X(x_T)} \quad x > x_T \quad (2.45)$$

as shown in fig. 2.1. This can be transformed into a standard normal variable,  $U'$ , by the usual process:

$$\Phi(u') = F_X'(x)$$

However it clearly makes no difference if the transformation is done first:

$$\Phi(u) = F_X(x)$$

followed by the truncation:

$$\Phi(u') = 0 \quad \text{ie } u' = -\infty \quad u \leq u_T$$

$$\Phi(u') = \frac{\Phi(u) - \Phi(u_T)}{1 - \Phi(u_T)} \quad u > u_T \quad (2.46)$$

where  $\Phi(u_T) = F_X(x_T)$

This process can therefore be regarded as the truncation and renormalisation of a standard normal variable; the variable  $U$  is truncated at  $u_T$  and then "renormalised" into the variable  $U'$ .

The standard normal variable  $U'$ , representing the truncated variable  $X$ , can then be incorporated into a full probability of failure calculation with a set of other  $U$ -variables representing

all the other basic variables in the problem. Once again it clearly makes no difference whether the variable is regarded as being truncated and renormalised before the whole problem is set up, or after, provided the correct transformations are included in the calculations of values of  $Z$ . In the latter case, the failure surface is envisaged in the multi-dimensional standard normal space (or  $U$ -space) including the untruncated variable as one of the orthogonal axes of the  $U$ -space, and then the whole space is truncated at a hyper-plane normal to this axis and the remaining space is then renormalised by transforming the values of this one co-ordinate in accordance with the formulae above. This distorts the failure surface and the use of the method of 2.7 in the renormalised space gives an estimate for the probability of failure with the truncated variable.

However it is clear that because of the rotational symmetry of the  $U$ -space it is possible to do this transformation in any direction, not just along a direction representing a given basic variable. If, therefore, we wish to calculate the probability of Failure A given Failure B and if Failure B can be represented by a hyper-plane in the  $U$ -space of all the variables used in A or B, then truncation of the  $U$ -space on the Failure B surface, and renormalisation will give a modified Failure A surface, on which the use of the standard Level 2 calculation will give an estimate of the required conditional probability of failure.

It is clear that if both A and B have non-linear failure surfaces then the use of two linearisations will in general make the calculations less accurate than the simple Level 2 method. In addition the choice of a point on surface B at which to linearise it needs some care. This problem will be discussed after the truncation and renormalisation procedure has been described mathematically.

The plane at which the truncation takes place is described by saying that it is normal to a line through the origin which has direction cosines,  $b_i$ , and which runs from  $-\infty$  in the non-failure region to  $+\infty$  in the failure region. The plane of truncation cuts this line at a distance  $\beta_T$  from the origin and it will be seen

that because the line has been taken as directed from the non-failure region to the failure region the probability of failure given by the truncation plane for the conditioning event is given by:

$$P_f = \Phi(-\beta_T) = 1 - \Phi(\beta_T) \quad (2.47)$$

Any point in the  $U$ -space can be given an effective co-ordinate in the  $\underline{b}$  direction by

$$u_{eff} = \sum_i b_i u_i \quad (2.48)$$

It is this value that is transformed to  $u'_{eff}$  by the renormalisation. By inspection of fig 2.2, we have either

$$\Phi(u'_{eff}) = \frac{\Phi(u_{eff}) - \Phi(\beta_T)}{1 - \Phi(\beta_T)} \quad \text{for cases conditional on failure} \\ \text{ie } u_{eff} > \beta_T$$

$$\text{or } \Phi(u'_{eff}) = \frac{\Phi(u_{eff})}{\Phi(\beta_T)} \quad \text{for cases conditional on non-failure} \\ \text{ie } u_{eff} < \beta_T \quad (2.49)$$

Now  $u'_{eff}$  is related to a complete set of renormalised variables by

$$u'_{eff} = \sum_i b_i u'_i$$

The change in  $u_{eff}$  is given by  $(u'_{eff} - u_{eff})$  in the  $\underline{b}$  direction so the change in any  $u_i$  is given by

$$u'_i - u_i = b_i (u'_{eff} - u_{eff}) \quad (2.50)$$

Now in practice the calculation of the conditional probability of failure will be performed in the standard normal  $u'$ -space, and when it is required to evaluate the safety margin,  $Z$ , at some point  $\underline{u}'$  it is necessary to transform  $\underline{u}'$  to  $\underline{u}$  and thence to  $\underline{x}$  for substitution in  $Z = g(\underline{x})$ . Hence the transformations required are from  $\underline{u}'$  to  $\underline{u}$ , thus:

$$u_i = u'_i + b_i [\Phi^{-1}\{1 - \Phi(\beta_T)\} \cdot \Phi(\sum_j b_j u'_j) + \Phi(\beta_T)] - \sum_j b_j u'_j$$

when conditioned on failure

$$u_i = u_i' + b_i[\Phi^{-1}\{\Phi(\sum_1 b_i u_i') \cdot \Phi(\beta_T)\} - \sum_1 b_i u_i']$$

when conditioned on non-failure.

(2.51)

Fig. 2.3 illustrates the method described in the section with a simple lexample, in which the failure indicated by

$$Z_A = u_1 - u_2 + 3/2$$

is conditioned by the failure indicated by

$$Z_B = u_1 + 2.0$$

The figure shows that the unconditional failure probabilities for A and B are 0.00135 ( $\beta=-3$ ) and 0.023 ( $\beta=-2$ ) respectively. The conditional failure probability of A given B is 0.033 ( $\beta=-1.84$ ).

### 2.8.3 Choice of truncation plane

The remaining problem is the choice of a point on the conditioning failure surface at which to linearise the surface. If the conditioning failure surface is linear in U-space, then of course there is no problem. If its curvature is small, then it is not particularly important which point is chosen. If the failure surface is significantly non-linear, i.e. if the choice of linearisation point has a significant effect on the calculated conditional probability, then it is possible that the non-linearity is such that the calculated probability would be in error to a significant degree whichever linearisation point was chosen.

Consider a problem in the form of a requirement to calculate the probability of failure 'A' occurring (described by  $Z_A = g_A(\underline{u}) \leq 0$ ) conditional on failure 'B' ( $Z_B = g_B(\underline{u}) \leq 0$ ) also occurring. The following points on  $g_B(\underline{u}) = 0$  can be suggested as linearisation points:

- (i) the closest point on  $g_B(\underline{u})=0$  to the origin, ie the "design point",  $\underline{u}^*_B$ .

- (ii) the closest point on  $g_B(\underline{u})=0$  to the design point,  $\underline{u}_A^*$ , on  $g_A(\underline{u})=0$ .
- (iii) the closest point on the intersection  $g_A(\underline{u}) = g_B(\underline{u})=0$  to the origin.
- (iv) the closest point on the intersection  $g_A(\underline{u}) = g_B(\underline{u})=0$  to  $\underline{u}_A^*$ .

If using each of these points in turn gives similar results then it is not necessary to make a choice between them. If, however, the results differ from each other by what is considered, in the context of the calculation being undertaken, to be an excessive amount then it is not clear, from the work done to date, which, if any, should be preferred. Studies of particular examples may shed some light on this in the future, and it may be possible to frame theoretical arguments that indicate that one point will be, on average, better than the others.

For any chosen linearisation point (say  $\underline{u}_L$ ) on  $g_B(\underline{u})=0$ , the required values of  $\beta_T$  and  $b_i$  can be determined as follows:

$$\text{If } a_{Li} = \left. \frac{\partial g_B(\underline{u})}{\partial u_i} \right|_{\underline{u}=\underline{u}_L}$$

$$\text{and } z_L = g_B(\underline{u}_L) = 0$$

$$\text{then from equation 2.44 } \beta_T = - \frac{\sum_1 a_{Li} u_{Li}}{(\sum_1 a_{Li}^2)^{1/2}} \quad (2.52)$$

$$\text{and } b_i = \frac{u_{Li} + \Delta u_{Li}}{\beta_T} = - \frac{a_{Li}}{(\sum_1 a_{Li}^2)^{1/2}} \quad (2.53)$$

Algorithms suitable for use in calculating the suggested linearisation points can be found by similar methods to that given in section 2.7.2. For example, to find point (iv) the aim is to minimise

$$(\sum_1 (u_i - u_{Ai}^*)^2)^{1/2}$$

subject to  $Z_A - Z_B = 0$  instead of minimising  $(\sum_1 (u_i^2))^{1/2}$  subject to  $Z = 0$ .

## 2.9 Calculation of Conditional Probabilities for certain special cases

### 2.9.1 Basis of Method

The method outlined in section 2.8 is a general method for calculating conditional probabilities, but it is likely to be of limited accuracy if the functions are non-linear. In practice many of the situations that occur will be rather simpler; in particular a common situation will be where failures A and B have the same (or very similar) failure functions, but where some of the variables will take the same value in both cases (perfect correlation between corresponding variables in A and B) while others take two independent values, although the probability distribution may be the same for both. This suggests that it may be possible to find more accurate methods of calculation for such cases by exploiting their particular features.

The method is discussed below is suitable for cases where it is possible to put all the independent variables together in a function which is part of the failure function and which does not include any of the fully correlated variables.

The method is approached by considering a case with only one variable (say  $x_i$ ) taking separate values for case A and B ( $x_{iA}$  and  $x_{iB}$ ), which otherwise have identical failure functions, i.e.

$$Z_A = g(x_1, x_2, \dots x_i \dots x_{iA})$$

$$Z_B = g(x_1, x_2, \dots x_i \dots x_{iB}) \quad (2.54)$$

The variables  $x_{iA}$  and  $x_{iB}$  are effectively independent random variables which may or may not be identically distributed. The probability of both failures occurring is given by

$$P[\text{Failure A} \cap \text{Failure B}]$$

$$= \int \int \dots \int f_{X_1}(x_1) \cdot f_{X_2}(x_2) \dots f_{X_{IA}}(x_{IA}) \cdot f_{X_{IB}}(x_{IB}) dx_1 dx_2 \dots dx_{IA} dx_{IB} \quad (2.55)$$

both  $Z_A < 0$

and  $Z_B < 0$

Now  $Z$  is generally a monotonically increasing or decreasing function of each of the basic variables. If this is true for  $Z_A$  as a function of  $x_{IA}$ , and if  $x_{IA}$  is in fact a resisting variable, ie

$$\frac{\partial g(\underline{x})}{\partial x_{IA}} > 0$$

then for any set of basic variables ( $x_1, x_2 \dots x_i \dots$  excluding  $x_{IA}$ ) there will be one value of  $x_{IA}$ , say  $\xi$ , for which  $Z_A = 0$ , and if  $x_{IA} < \xi$ , then  $Z_A < 0$ .

Similar statements can be made about  $Z_B$  and  $x_{IB}$  because the two failure functions are identical.  $\xi$  is of course a function of all the other basic variables. Equation 2.55 can be rewritten:

$$P[\text{Failure A} \cap \text{Failure B}]$$

$$= \int_{-\infty}^{+\infty} \dots \int \left\{ \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} f_{X_{IA}}(x_{IA}) \cdot f_{X_{IB}}(x_{IB}) dx_{IA} dx_{IB} \right\} \cdot f_{X_1}(x_1) f_{X_2}(x_2) \dots dx_1 dx_2 \dots$$

$$\text{Now } \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} f_{X_{IA}}(x_{IA}) \cdot f_{X_{IB}}(x_{IB}) dx_{IA} dx_{IB}$$

$$= \int_{-\infty}^{\xi} f_{X_{IA}}(x_{IA}) dx_{IA} \cdot \int_{-\infty}^{\xi} f_{X_{IB}}(x_{IB}) dx_{IB}$$

$$= F_{X_{IA}}(\xi) \cdot F_{X_{IB}}(\xi) \quad (2.56)$$

Hence if we define  $F_{X_I}(\xi) = F_{X_{IA}}(\xi) \cdot F_{X_{IB}}(\xi)$  (2.57)

$$= \int_{-\infty}^{\xi} f_{X_I}(\xi) d\xi$$

then  $P[\text{Failure A} \cap \text{Failure B}]$

$$= \iint_{Z \leq 0} f_{X_1}(x_1) f_{X_2}(x_2) \dots f_{X_I}(x_I) \cdot dx_1 \cdot dx_2 \dots dx_I \quad (2.58)$$

where  $Z = g(x_1, x_2, \dots, x_I)$  (2.59)

Thus a single 'level 2' type calculation using the variable  $X_I$  whose distribution is generated from those of  $X_{IA}$  and  $X_{IB}$  by equation 2.57 will find the required probability.

It should be noted that this result is conditional on the assumption that  $X_I$  is a resisting variable. For a loading variable similar calculations lead to the same result but with the distribution of  $x_I$  given by

$$(1 - F_{X_I}(\xi)) = (1 - F_{X_{IA}}(\xi)) \cdot (1 - F_{X_{IB}}(\xi)) \quad (2.60)$$

In practice using the algorithm of section 2.7, working in the standard normal  $u$ -space, it is necessary, when wishing to calculate the value of  $Z$  at some  $\underline{u}$  to first calculate  $x_I$  from the equivalent standard normal variable  $u_I$  by solving

$$\Phi(u_I) = F_{X_{IA}}(x_I) \cdot F_{X_{IB}}(x_I) \quad (2.61)$$

$$\text{or } (1 - \Phi(u_I)) = (1 - F_{X_{IA}}(x_I)) \cdot (1 - F_{X_{IB}}(x_I)) \quad (2.62)$$

as appropriate and then substitute into equation 2.59 to find  $Z$ .

### 2.9.2. Extension of Basic Principle to more than two functions.

An examination of the arguments above shows that this principle can be extended so that

$$P[\text{Failure A} \cap \text{Failure B} \cap \text{Failure C} \cap \dots \cap \text{Failure N}]$$

where the safety margins for each failure are defined by the functions:

$$\begin{aligned} Z_A &= g(x_1, x_2 \dots x_i \dots x_{IA}) \\ Z_B &= g(x_1, x_2 \dots x_i \dots x_{IB}) \\ &\vdots \\ Z_N &= g(x_1, x_2 \dots x_i \dots x_{IN}), \end{aligned} \quad (2.63)$$

can be found by using the Level 2 method on

$$Z = g(x_1, x_2 \dots x_i \dots x_I) \quad (2.64)$$

with either

$$F_{X_I}(\xi) = F_{X_{IA}}(\xi) \cdot F_{X_{IB}}(\xi) \dots F_{X_{IN}}(\xi) \quad (2.65)$$

$$\text{or } (1 - F_{X_I}(\xi)) = (1 - F_{X_{IA}}(\xi)) \cdot (1 - F_{X_{IB}}(\xi)) \dots (1 - F_{X_{IN}}(\xi)) \quad (2.66)$$

depending on whether  $x_I$  is a resisting variable or loading variable respectively.

There is a clear similarity between these calculations and the use of minimum and maximum distributions for basic variables:

if  $F_{X_{IA}}(\xi) = F_{X_{IB}}(\xi) = \dots = F_{X_{IN}}(\xi) = F_{X_{IP}}(\xi)$ , say

$$\text{so that } F_{X_I}(\xi) = F_{X_{IP}}^n(\xi) \quad (2.67)$$

then equations 2.64 and 2.65 (where  $X_I$  is a resisting variable) can be interpreted as describing a failure in which the variable  $X_I$  represents the maximum of  $n$  independent samples from the "parent distribution",  $F_{X_{IP}}(\cdot)$ .

Another calculation that can be made in a similar way is that for the probability that at least one of a series of cases fails. This is the complementary event to all the cases not failing, so for two cases A and B (with same definitions as above):

$$\begin{aligned} & P[\text{at least one of A or B fails}] \\ & = 1 - P[A \text{ does not fail} \cap B \text{ does not fail}] \end{aligned} \quad (2.68)$$

$$= 1 - \int \int \dots \int_{\substack{\text{both } Z_A > 0 \\ \text{and } Z_B > 0}} f_{X_1}(x_1) \dots f_{X_{IA}}(x_{IA}) \cdot f_{X_{IB}}(x_{IB}) \cdot dx_1 \cdot dx_2 \dots dx_{IA} \cdot dx_{IB}$$

$$= 1 - \int \int \dots \left\{ \int \int \dots f_{X_{IA}}(d_{IA}) \cdot f_{X_{IB}}(x_{IB}) \cdot d_{X_{IA}} \cdot d_{X_{IB}} \right\} \cdot f_{X_1}(x_1) \dots dx_1 \quad (2.69)$$

for  $X_I$  as a resisting variable

$$= 1 - \int \int \dots \left\{ \int f_{X_I}(x_I) d_{X_I} \right\} \cdot f_{X_1}(x_1) \dots dx_1 \dots \quad (2.70)$$

if  $1 - F_{X_I}(\xi) = (1 - F_{X_{IA}}(\xi)) \cdot (1 - F_{X_{IB}}(\xi))$

$$\begin{aligned} & = 1 - \int \int \dots f_{X_1}(x_1) \dots f_{X_I}(x_I) \cdot dx_1 \dots dx_I \\ & = \int \int \dots f_{X_1}(x_1) \dots f_{X_I}(z_I) \cdot dx_1 \dots dx_I \end{aligned} \quad (2.71)$$

$Z > 0$

Hence this probability may also be calculated by the Level 2 method as before (c.f. equation 2.58) but with the two alternative forms for  $F_{X_I}(\cdot)$  interchanged as between resisting and loading variables.

Further developments are made possible by combining failures already discussed. For example, to calculate

$P[\text{at least one out of cases } B_1, B_2 \dots B_n \text{ fails} \mid \text{case A fails}]$

$$= \frac{P[\text{at least one out of cases } B_1, B_2 \dots B_n \text{ fails} \cap \text{case A fails}]}{P[\text{case A fails}]} \quad (2.72)$$

the numerator can be found using

$$F_{X_I}(\xi) = F_{X_{IA}}(\xi) \cdot \left[ 1 - \prod_{i=1}^n (1 - F_{X_{IB_i}}(\xi)) \right] \quad (2.73)$$

if  $X_I$  is a resisting variable and assuming, as in all the above calculations, that the failure functions for the different failures differ in one variable only.

Some apparently similar probabilities cannot be evaluated by this method. For instance, attempting to calculate  $P[A \text{ fails} \cap B \text{ does not fail}]$  where failures A and B are as in equation 2.54 will lead to the following in place of equation 2.56:

$$\begin{aligned} & \int_{-\infty}^{\xi} \int_{\xi}^{\infty} f_{X_{IA}}(x_{IA}) \cdot f_{X_{IB}}(x_{IB}) \cdot dx_{IA} \cdot dx_{IB} \\ &= F_{X_{IA}}(\xi) \cdot (1 - F_{X_{IB}}(\xi)) \end{aligned} \quad (2.74)$$

which is zero for both  $\xi = -\infty$  and  $\xi = +\infty$  and hence cannot be expressed as a cumulative distribution on  $\xi$ . The integral cannot then be re-expressed in the form required for the Level 2 type of calculation.

This difficulty can be overcome by writing:

$$\begin{aligned} & P[A \text{ fails} \cap B \text{ does not fail}] \\ &= P[A \text{ fails}] - P[A \text{ fails} \cap B \text{ fails}] \end{aligned} \quad (2.75)$$

in which case both parts can be evaluated using the method above. Similarly many other probabilities, such as that for exactly  $m$  out of  $n$  cases failing can be evaluated by a series of calculations which all fit the pattern required for use of the Level 2 method. However the number of separate calculations may be large and the results may be small differences between very similar numbers and hence seriously in error as a result of the approximations in the Level 2 method. This development has not therefore been used for any of the calculations in this thesis.

### 2.9.3 Development of method

In the foregoing sections the failures have differed from one another only to the extent of depending on different values of one particular variable.

The same basic principle can be applied to the calculation of

$$P[\text{failure A} \cap \text{failure B}]$$

where

$$Z_A = g(x_1, x_2, \dots, x_{IA}) \text{ and } x_{IA} = g_A(x_{A1}, x_{A2}, \dots, x_{Ai} \dots)$$

$$Z_B = g(x_1, x_2, \dots, x_{IB}) \text{ and } x_{IB} = g_B(z_{B1}, x_{B2}, \dots, x_{Bi} \dots) \quad (2.76)$$

i.e.  $x_{IA}$  and  $x_{IB}$  are no longer basic variables with given distributions, but are functions of other basic variables. The variables in  $g_A(\cdot)$  are independent of those in  $g_B(\cdot)$ ; in fact  $g_A(\cdot)$  and  $g_B(\cdot)$  could be totally different functions.

If the approach of 2.9.1 were used, the problem would arise of what distributions to use for  $x_{IA}$  and  $x_{IB}$  in equation 2.61 or 2.62. These have to be derived somehow from the  $f_A(\cdot)$  and  $f_B(\cdot)$  functions and the distributions of the  $x_{Ai}$  and  $x_{Bi}$  variables.

$$\text{Now } F_{x_{IA}}(x_I) = P[x_{IA} < x_I]$$

$$\begin{aligned} &= \int \int \dots \int_{x_{IA} < x_I} f_{x_{A1}}(x_{A1}) \cdot f_{x_{A2}}(x_{A2}) \dots dx_{A1} dx_{A2} \dots \\ &= \int \int \dots \int_{Z'_A, 0} f_{x_{A1}}(x_{A1}) f_{x_{A2}}(x_{A2}) \dots dx_{A1} dx_{A2} \dots \quad (2.77) \end{aligned}$$

$$\text{where } Z'_A = g_A(x_{A1}, x_{A2}, \dots) - x_I \quad (2.78)$$

Hence the Level 2 approach could be used to evaluate  $F_{x_{IA}}(x_I)$  given a value for  $x_I$ . At any point  $\underline{u}_A = (u_{A1}, u_{A2}, \dots)$  it is possible to evaluate  $Z'$  and  $a_{Ai}$  given by

$$a_{Ai} = \frac{\partial Z'_A}{\partial u_{Ai}}$$

and hence estimate the required probability by

$$F_{x_{IA}}(x_I) = \Phi(-\beta_A) \quad (2.79)$$

$$\text{where } \beta_A = - \frac{\sum a_{Ai} u_{Ai} - Z'_A}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \quad (2.80)$$

$$= - \frac{\sum a_{Ai} u_{Ai} - g_A(\underline{x}_A) + x_I}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \quad (2.81)$$

The summations are over all the  $X_{Ai}$  variables;  $X_I$  is not included.

With a similar calculation for  $X_{IB}$  leading to  $\beta_B$  equations 2.61 and 2.62 become

$$\Phi(u_I) = \Phi(-\beta_A) \cdot \Phi(-\beta_B) \text{ for } X_I \text{ as a resisting variable} \quad (2.82)$$

$$\text{and } \Phi(-u_I) = \Phi(\beta_A) \cdot \Phi(\beta_B) \text{ for } X_I \text{ as a loading variable} \quad (2.83)$$

These equations give the relationship between  $U_I$  and  $X_I$ . Thus if  $x_I$  is changed to  $x_I + \Delta x_I$ ,  $u_I$  changes to  $u_I + \Delta u_I$  as given by (for resisting variable case):

$$\Phi(u_I + \Delta u_I) = \Phi \left[ -\beta_A + \frac{\Delta x_I}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \right] \cdot \Phi \left[ -\beta_B + \frac{\Delta x_I}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \right] \quad (2.84)$$

and for small changes:

$$\begin{aligned} \Phi(u_I) \delta u_I &= \Phi(-\beta_A) \cdot \Phi(-\beta_B) \cdot \frac{\delta x_I}{(\sum a_{Bi}^2)^{\frac{1}{2}}} \\ &+ \Phi(-\beta_A) \cdot \Phi(-\beta_B) \cdot \frac{\delta x_I}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \\ &= \left[ \frac{\Phi(-\beta_A) \cdot \Phi(-\beta_B)}{(\sum b_{Bi}^2)^{\frac{1}{2}}} + \frac{\Phi(-\beta_A) \cdot \Phi(-\beta_B)}{(\sum a_{Ai}^2)^{\frac{1}{2}}} \right] \delta x_I \end{aligned} \quad (2.85)$$

The algorithm may be summarised as follows.

(1) Given the functions

$$Z = g(x_1, x_2, \dots, x_i, \dots, x_I) \quad (2.86)$$

$$Z'_A = g_A(x_{A1}, x_{A2}, \dots, x_{Ai}, \dots) - x_I$$

$$Z'_B = g_B(x_{B1}, x_{B2}, \dots, x_{Bi}, \dots) - x_I$$

and probability distributions  $F_{X_i}(x_i)$  for all  $x_i$ ,  $x_{Ai}$  and  $x_{Bi}$ , but not for  $x_I$ ; and given also starting values for all  $u_i$ ,  $u_{Ai}$ ,  $u_{Bi}$  and for  $x_I$ ;

Calculate:  $a_i = \frac{\partial Z}{\partial u_i}$ ,  $a_{Ai} = \frac{\partial Z'_A}{\partial u_{Ai}}$ ,  $a_{Bi} = \frac{\partial Z'_B}{\partial u_{Bi}}$

and  $Z$ ,  $Z'_A$ ,  $Z'_B$ .

(2) Calculate  $\beta_A$  from equation 2.80 and  $\beta_B$  from equivalent equation.

(3) Calculate  $u_I$  from equation 2.82 or 2.83 as appropriate to nature of  $x_I$  in equation 2.86.

(4) Using a convenient small step size  $\delta u_I$ , calculate  $\delta x_I$  from equation 2.85 (or equivalent form for loading variables), and evaluate

$$Z + \delta Z = g(x_1, x_2, \dots, x_i, \dots, (x_I + \delta x_I)) \quad (2.87)$$

and hence  $a_I = \frac{\delta Z}{\delta u_I}$  (2.88)

(5) Calculate new values for  $u_i$  to start next iteration

$$u_i + \Delta u_i = a_i \frac{\sum a_{Ai} u_{Ai} + a_i u_i - Z}{\sum a_{Ai}^2 + a_i^2} \quad (2.89)$$

and  $u_I + \Delta u_I = a_I \frac{\sum a_{Ai} u_{Ai} + a_I u_I - Z}{\sum a_{Ai}^2 + a_I^2}$  (2.90)

(6) Calculate change in  $x_I$  corresponding to  $\Delta u_I$  from

$$\Delta x_I = \Delta u_I \cdot \frac{\delta x_I}{\delta u_I} \quad (2.91)$$

or more precisely by solving equation 2.84 (or equivalent for loading variable

(7) Calculate new values for  $u_{Ai}$  by incorporating change in  $x_I$  into usual Level 2 algorithm, thus:

$$u_{Ai} + \Delta u_{Ai} = a_{Ai} \frac{(\sum a_{Ai} u_{Ai} - Z'_A + \Delta x_I)}{\sum a_{Ai}^2} \quad (2.92)$$

and calculate new values for  $u_{Bi}$  similarly.

(8) Repeat from (1) using new values of  $u_i$ ,  $u_{Ai}$ ,  $u_{Bi}$  and  $x_I$ .

When the algorithm has converged the joint probability of Failure A and Failure B is given by

$$P[\text{Failure A} \cap \text{Failure B}] = \Phi(-\beta)$$

where  $\beta = (\sum u_i^2 + u_I^2)^{\frac{1}{2}}$

and hence the required conditional probability can be calculated.

Since the basic principle here is the same as in section 2.9.1, it follows that the extensions of 2.9.2 are all possible, using the appropriate modifications such as equations 2.65 and 2.66, to equations 2.82, 2.83 and 2.84.

## CHAPTER 3

### DETERMINISTIC FATIGUE DESIGN PROCEDURES FOR OFFSHORE STRUCTURES

#### 3.1 Introduction

The purpose of this chapter is to outline the deterministic calculation methods that are used in the offshore industry for the analysis of fatigue resistance.

These methods are all 'deterministic' in the sense that they aim to calculate a single value for the estimated life of the structure which can then be compared with the specified or required life, or to achieve the same aim indirectly. This is in contrast to the 'probabilistic' approach of this thesis where the life is uncertain and the aim is to assess this uncertainty. It should be noted, however, that some of the procedures that are here called deterministic are sometimes described by others as probabilistic. This is because probability functions are used in these procedures to describe the relative numbers of waves of different heights and periods that are expected to occur during the structure's life. In such cases the probability functions are not being used to model uncertainty because it is assumed that waves occur in exactly the proportions given by the functions, and that the fatigue damage accumulates over all the waves.

The principal cause of fatigue damage in offshore structures is wave action. In certain parts of some structures other effects such as wind action and vibrations caused by machinery or wave slamming may be important, but these causes are not considered further in this thesis.

A number of accounts have been published (e.g. refs 33, 34, 35) of the methods used in the offshore industry to design and check offshore structures against fatigue due to wave action. Certifying authorities<sup>36</sup> and other responsible bodies<sup>37</sup> also publish requirements and guidance on fatigue calculations.

These methods of analysis are discussed at greater length by Brebbia and Walker<sup>38</sup>. The basis of the calculation methods for fatigue resistance of welded joints is given by Gurney<sup>39</sup>, and a detailed account of all fatigue phenomena is given by Frost *et al*<sup>40</sup>. These references discuss the justification for the use of various models in terms of their practicality, theoretical correctness, and degree of experimental verification. Where no specific reference is given, definitions, assumptions and equations are taken from these sources.

### 3.2 Basis of Fatigue Life Calculations

#### 3.2.1 General

This section summarises the basic assumptions in the fatigue resistance calculation models. The normal basis of such calculations is the use of Miner's Rule and experimental fatigue life data from constant amplitude tests; although recently more data related to offshore loading conditions has been produced.

One important pre-requisite for the Miner's Rule approach is the definition of a single reference stress whose variation is assumed to determine the fatigue life. For connections between tubular members the stress used is the so-called 'hot-spot' stress. This was originally defined, rather loosely, as the maximum stress at the intersection of the tubular members, but a precise definition is now used (in the U.K. at least), which is discussed in section 5.4.3. The reason for using the hot-spot stress is that it is hoped that this quantity will adequately characterise the stress concentration produced by the geometry of the joint so that different joints with similar hot-spot stress histories will have similar fatigue lives, so that in design the same relationship between hot-spot stress statistics and fatigue life may be used for all joints. Normal design procedures are based on this assumption, although Gurney<sup>42</sup> has suggested that an additional allowance be made for the thickness of the metal.

The other important pre-requisite is a definition of a 'fatigue cycle'. This presents no problem for constant amplitude load cycles, but in other, more general, cases it is not clear how

cycles should be defined. This is discussed again below.

Constant amplitude load cycles are specified by the stress range and mean stress. Test data therefore give the fatigue life for a particular type of joint as a function of these two parameters. For welded connections the effect of the mean applied stress is considerably reduced by the presence of residual welding stresses, and it is normal to assume that fatigue life is independent of mean stress. With these assumptions the fatigue life,  $N$ , is a function of stress range,  $S$ , only and the commonest model used for this relationship is  $NS^m = K$ . For design purposes the constants  $m$  and  $K$  are chosen to give a lower bound on all or most of the constant cycle test results.

For offshore work it is convenient to define the number of cycles on an annual basis and take Miner's Rule as:

$$\sum_1 \frac{Yn_1}{N_1} = 1 \quad (3.1)$$

where  $Y$  = fatigue life in years

$n_1$  = number of cycles of range  $S_1$  per year

$N_1$  = fatigue life for cycles of range  $S_1$ ,

and where the summation is taken over a finite number of discrete or discretised stress ranges  $S_1$ .

This can also be expressed by invoking the concept of fatigue 'damage' and saying that each cycle contributes an amount of 'damage' equal to  $1/N_1$  to a total of 'cumulative damage' which is unity at failure.

Other formulations which are useful for developing particular calculation models are:

$$(\text{Life in years}) \times (\text{annual damage rate}) = 1$$

$$\text{where annual damage rate} = \sum_1 \frac{n_1}{N_1} \quad (3.2)$$

and  $(\text{Life in years}) \times (\text{number of cycles/year})$

$$\times (\text{average damage/cycle}) = 1 \quad (3.3)$$

where average damage/cycle =  $\frac{1}{n_{ann}} \sum_i \frac{n_i}{N_i}$  if there are  $n_{ann}$  cycles per year

$$\begin{aligned}
 &= \sum_i \frac{1}{N_i} \cdot \frac{n_i}{n_{ann}} \\
 &= \sum_i (\text{damage due to one cycle of range } S_i) \\
 &\quad \times (\text{probability that one cycle, chosen completely at random, has range } S_i) \quad (3.4)
 \end{aligned}$$

For cycles of stress range  $S_i$ , assuming  $N_i S_i^m = K$ :

$$\text{damage/cycle} = \frac{1}{N_i} = \frac{S_i^m}{K} \quad (3.5)$$

Now  $m$  is usually in the range from 2 to 4 and is always greater than one, so that larger cycles are proportionately more damaging than small ones. It is important, therefore, when determining stress ranges to use a procedure that correctly, or conservatively, identifies the larger stress ranges. Suitable procedures are the Rainflow and Reservoir<sup>43</sup> methods. These are used on stress histories, but are too complex to be used to give a distribution of stress ranges in algebraic form when the stress history is specified in some general algebraic way. The most important example of this is where the stress history is a Gaussian Process, and this case will be examined in the next section.

### 3.2.2 Fatigue Damage due to Gaussian Processes

If the variation of stress with time at a stress concentration,  $\sigma(t)$ , (where the mean value of  $\sigma(t)$  is zero) can be described mathematically as a stationary Gaussian process with frequency spectrum  $S_{\sigma\sigma}(\omega)$ , the question arises of whether it is possible to determine mathematically, from this spectrum, a distribution of stress range for use with S-N curves and Miner's Rule.

A number of results can be obtained<sup>44</sup> directly from the spectrum, or rather from its moments:

$$m_n = \int_0^\infty \omega^n S_{\sigma\sigma}(\omega) d\omega \quad (3.6)$$

(1) The mean period between positive up-crossings of  $\sigma=0$  by the function  $\sigma(t)$  is given by:

$$T_Z = 2\pi \left[ \frac{m_0}{m_z} \right]^{1/2} \quad (3.7)$$

(2) The mean period between peak values of  $\sigma(t)$  is given by:

$$T_P = 2\pi \left[ \frac{m_z}{m_4} \right]^{1/2} \quad (3.8)$$

(3) A measure of the "band width" of  $S_{\sigma\sigma}(\omega)$  is given by a parameter  $\epsilon$  where

$$\epsilon = \left[ \frac{m_z^2}{1 - m_0 m_4} \right]^{1/2} \quad (3.9)$$

$$\text{where } 0 \leq \epsilon \leq 1 \text{ and } T_P = T_Z(1-\epsilon^2)^{1/2} \quad (3.10)$$

(4) The distribution of peak values,  $\sigma_P$ , of  $\sigma(t)$  is given by:

$$\begin{aligned} f_{\sigma_P}(\sigma_P) = & \frac{\epsilon}{(2\pi m_0)^{1/2}} \cdot \exp \left[ -\frac{1}{2} \frac{\sigma_P^2}{m_0 \epsilon^2} \right] \\ & + (1-\epsilon^2)^{1/2} \cdot \frac{\sigma_P}{m_0} \cdot \phi \left[ \frac{\sigma_P(1-\epsilon^2)^{1/2}}{m_0^{1/2} \epsilon} \right] \cdot \exp \left[ -\frac{\sigma_P^2}{2m_0} \right] \end{aligned} \quad (3.11)$$

$$F_{\sigma_P}(\sigma_P) = \Phi \left[ \frac{\sigma_P}{m_0^{1/2} \epsilon} \right] - (1-\epsilon^2)^{1/2} \cdot \phi \left[ \frac{\sigma_P(1-\epsilon^2)^{1/2}}{m_0^{1/2} \epsilon} \right] \cdot \exp \left[ -\frac{\sigma_P^2}{2m_0} \right] \quad (3.12)$$

where  $\Phi(\cdot)$  is the standard normal cumulative probability function.

Similarly the distribution of the trough values,  $\sigma_t$ , of  $\sigma(t)$  is given by the mirror image of this function about  $\sigma=0$ :

$$\begin{aligned} \text{e.g. } f_{\sigma_t}(\sigma_t) = & \frac{\epsilon}{(2\pi m_0)^{1/2}} \cdot \exp \left[ -\frac{1}{2} \frac{\sigma_t^2}{m_0 \epsilon^2} \right] \\ & - (1-\epsilon^2)^{1/2} \cdot \frac{\sigma_t}{m_0} \cdot \phi \left[ \frac{\sigma_t(1-\epsilon^2)^{1/2}}{m_0^{1/2} \epsilon} \right] \cdot \exp \left[ -\frac{\sigma_t^2}{2m_0} \right] \end{aligned} \quad (3.13)$$

None of the available results gives any information about the sequence of peaks and troughs; no method is known, for instance, for finding the joint distribution of a peak value and the following trough. It is, of course, possible to simulate stress

histories with spectrum  $S_{\sigma\sigma}(\omega)$  and use a procedure such as the rainflow method to find stress ranges. In this method the number of cycles is equal to the number of peaks and troughs, and each peak and each trough is used once, to define the upper or lower stress for one cycle. A simple, conservative, approach is to take the peaks and troughs given by equations 3.11 and 3.13 and pair them in such a way as to give the maximum calculated damage.

Because of the symmetry between the distributions of peaks and troughs, maximum damage will be obtained by pairing peaks with equal and opposite troughs, except where this is impossible because a cycle cannot be formed with its trough higher than its peak. The peaks and troughs that correspond to the overlap of the distributions (see fig. 3.1) are therefore combined to produce cycles of zero stress range.

If a particular spectrum applies for (long) time  $T$ , then the number of peak stresses between  $\sigma$  and  $\sigma+\delta\sigma$  will be

$$\frac{T}{T_p} f_{\sigma_p}(\sigma) \cdot \delta\sigma$$

and the number of troughs will be

$$\frac{T}{T_p} f_{\sigma_t}(\sigma) \cdot \delta\sigma \quad (3.14)$$

Hence the number of cycles with stress range between  $S=2\sigma$  and  $S+\delta S=2(\sigma+\delta\sigma)$  will be

$$\begin{aligned} & \frac{T}{T_p} (f_{\sigma_p}(\sigma) - f_{\sigma_t}(\sigma)) \delta\sigma \quad \sigma > 0 \\ &= \frac{T}{T_p} \sqrt{(1-\epsilon^2)} \frac{\sigma}{m_0} \exp\left[\frac{-\sigma^2}{2m_0}\right] \delta\sigma \\ &= \frac{T}{T_p} \sqrt{(1-\epsilon^2)} \frac{S}{2m_0} \exp\left[\frac{S^2}{8m_0}\right] \frac{\delta S}{2} \end{aligned} \quad (3.15)$$

and the number of cycles of zero stress range will be

$$2 \frac{T}{T_p} F_{\sigma_p}(0) = \frac{T}{T_p} \left[ 1 - \sqrt{(1-\epsilon^2)} \right] \quad (3.16)$$

The number of non-zero cycles is given by

$$\frac{T}{T_p} \sqrt{(1-\epsilon^2)} = \frac{T}{T_z} \quad (2.17)$$

and their distribution by

$$f_S(S) = \frac{S}{4m_0} \exp\left[-\frac{S^2}{8m_0}\right]$$

$$F_S(S) = 1 - \exp\left[-\frac{1}{2} \left[\frac{S/2}{m_0}\right]^2\right] \quad (3.18)$$

Hence the number of damaging cycles can be taken from the upcrossing rate and assumed to be Rayleigh distributed.

In narrow-band spectra the energy is concentrated in a small range of frequencies and hence  $\epsilon$  is close to zero.  $\sigma(t)$  takes the form of a sine wave whose amplitude and period vary slowly. Thus a peak stress is always followed by a trough of similar value and the numbers of peaks and troughs is almost equal to the number of zero crossings. In these circumstances it is clearly reasonable to take the number of cycles in time  $T$  as  $T/T_2$  and the distribution of  $S/2$  as being the same as that of  $\sigma_p$ . These considerations lead directly to equation 3.18, showing that the pairing for maximum damage procedure is equivalent to an assumption that the spectrum is narrow-banded, with the same values of  $m_0$  and  $m_2$  as the original spectrum.

The narrow-banded assumption is often used in design, often apparently without any awareness of its conservative nature. More conservative methods have in fact been suggested<sup>34</sup>.

Wirsching and Light<sup>4</sup> have investigated the degree of conservatism in a narrow band assumption for a wide variety of spectra. They calculate values for a damage reduction factor  $\lambda$ , assuming that the rainflow method gives a correct assessment of fatigue damage.  $\lambda$  generally has a value in the range from 0.7 to 1.0; this will be discussed further in section 5.7.

One particular advantage of the narrow-band assumption is that it only uses two spectral moments  $m_0$  and  $m_2$ , thus avoiding the difficulties which can be encountered in the use of certain wide-band spectra. For example, if the stress spectrum takes a form similar to the Pierson-Moskowitz wave spectrum (see section 3.3 below), the fourth moment,  $m_4$ , of the spectrum is infinite, and hence  $\epsilon$  equals 1.0 and  $T_p$  is zero.

### 3.3 Deep Water Waves

Ocean waves exert forces on submerged structures, and as will be outlined below (section 3.5.3) these forces are currently calculated using the velocities and accelerations which the wave would have if undisturbed by the structure.

It can be shown<sup>45</sup> that in deep water small waves can take a sinusoidal form with the amplitude of particle motions decreasing exponentially with increasing depth. For small wave heights the differential equations and boundary conditions that are used to model deep water waves can be taken as linear and so different sinusoidal waves can be added together to give further admissible solutions. For large waves the boundary condition at the surface is non-linear and more complex solutions are necessary; however for the wave heights of importance in fatigue calculations a linear solution is generally used.

This suggests that a random sea might be modelled as a stationary Gaussian process. This has been shown<sup>44</sup> to be a good model, and although conditions are not strictly stationary, they change sufficiently slowly for it to be possible to model them as a succession of stationary processes, each persisting for some hours.

Conventional wave recorders take readings at regular intervals (e.g. 3 hours) so it is assumed that each set of measurements defines a "sea-state" which persists for the same length of time.

Sea-states are normally described by two parameters:

- (1) The significant wave height,  $H_s$ . This was originally defined as the average height of the highest third of the waves. Some authors now define  $H_s$  as  $4\sigma_\eta$  where  $\sigma_\eta$  is the standard deviation of the water surface elevation. This is discussed in more detail in section 5.1.3.
- (2) The zero up-crossing wave period,  $T_z$ , i.e. the mean time between up-crossings of the mean water level.

Data on the occurrence of sea-states is normally displayed in the form of a "scatter diagram", i.e. a bivariate histogram giving the number of occurrences of pairs of values of  $H_s$  and  $T_z$ . If the number of occurrences of each pair is divided by the total number of records available then the diagram gives the proportion of time,  $t_k$ , for which the  $k^{\text{th}}$  sea-state persisted.

A fuller description of the sea-state is given by the spectrum of sea-surface elevation,  $S_{\eta\eta}(\omega)$ . The two forms most commonly used are the Pierson-Moskowitz<sup>46</sup> and JONSWAP<sup>47</sup> Spectra. In both cases the original form did not use the parameters  $H_s$  and  $T_z$ . For example the Pierson-Moskowitz Spectrum was originally given as

$$S_{\eta\eta}(\omega) = \frac{\alpha g^2}{\omega^5} e^{-\beta \left[ \frac{g}{\omega U} \right]^4}$$

where  $\alpha$  and  $\beta$  are constants ( $\alpha = 0.0081$ ,  $\beta = 0.74$ ) and  $U$  is the mean wind velocity 64 feet above the sea surface.

In this form it was designed to give the spectrum for a fully developed sea caused by the wind-speed  $U$ . It can however be generalised as

$$S_{\eta\eta}(\omega) = A\omega^{-5} e^{-B\omega^{-4}}$$

which will represent a sea-state described by the parameters  $H_s$  and  $T_z$  if<sup>33</sup>

$$A = \frac{H_s^2 T_z}{8\pi^2} \left[ \frac{T_z}{2\pi} \right]^{-5}, \quad B = \frac{1}{\pi} \left[ \frac{T_z}{2\pi} \right]^{-4}$$

Conventional wave recording devices simply record the variation of sea surface elevation at a fixed point. They cannot therefore give any information on the direction in which waves are travelling. In the absence of such information it is necessary to make the conservative assumption, in wave loading calculations for instance, that the waves are concentrated in the most unfavourable direction. If, however, data are available, directional spectra can be used to give more accurate calculations,

### 3.4 Wave Loading and Structural Response

There are three principal steps in the calculations that relate waves to fatigue crack growth: the calculation of the wave forces, the structural analysis, and the calculation of the hot-spot stresses.

Morison's Equation is normally used to model the wave forces on the members of jacket structures. This expresses the force on an element as the sum of two forces: the inertia force and the drag force. Although these forces each have some theoretical basis, the equation is essentially empirical and it has been developed so that it can be used in a much wider range of situations than originally envisaged<sup>48</sup>. An example of its use is given in section 3.5.3.

The response of the structure to wave loading is calculated elastically, and if the sea-states contain significant amounts of energy at any of the natural frequencies of the structure, a dynamic analysis is required. The type of analysis undertaken has a significant effect on the whole of the fatigue life calculation; this will be discussed in sections 3.5 and 3.6.

The structural analysis gives values for forces and moments at connections. The hot-spot stresses are calculated by multiplying stresses based on nominal section properties by stress concentration factors. These factors are given in the form of parametric equations, such as those of Kuang *et al*<sup>49</sup>, which are derived from finite element analyses or model tests. This is discussed in more detail in section 5.4.

There are various effects which make the analysis of offshore structures difficult. The principal effects are:

- (1) the non-linearity of the drag force in Morison's Equation.
- (2) the variation in water level which introduces a further non-linearity into the calculation of the total force on a structure.

- (3) the fact that the variations in the wave forces on different parts of a structure are not in phase so that the total force on a structure is dependent on wavelength.
- (4) the dependence of the response of the structure on the relationship between its natural frequencies and the frequencies present in the wave; if there is a substantial dynamic response, this will depend on the damping which includes the effect of the non-linear drag force again.

Only a full time-domain simulation of structural behaviour based on representative wave histories could take full account of all these effects. Simpler methods are however possible in most cases.

Because the sea-state can be represented by an energy spectrum, and because a spectrum of hot-spot stress variation can be used in a fatigue life calculation, it is natural to look to spectral methods of analysis. These have been extensively studied and used, and are discussed in section 3.6. The main drawbacks are the need to linearise the relationship between wave height and stress, and the complexity of the calculations. Simpler (non-spectral) methods are possible if the structure is not dynamically sensitive to wave action; that is if the stresses present at any time depend only on the forces acting on the structure at that time.

### 3.5 Non-Spectral Methods

#### 3.5.1 Introduction

If a dynamically insensitive structure is subjected to a sinusoidal wave, height  $H$  and period  $T$ , the variation of a stress within that structure will have a wave form of the same period as the incident wave, although possibly deformed in shape by non-linearities and shifted in phase. In general the stress range  $S$  will be a function of both  $H$  and  $T$ . Section 3.5.2 describes a situation in which a simple linear form  $S=aH$  would apply.

In a narrow-band sea-state (band width parameter,  $\epsilon$ ,  $\ll 1$ ) the wave energy is concentrated into a narrow range of frequencies and

the wave takes the form of a sine wave whose amplitude changes slowly with time. In these circumstances it is clearly possible to use the response to sinusoidal waves to give a relationship between  $S$ ,  $H$  and  $T$  which is equally applicable to each separate cycle in the wave history of the narrow-band sea-state. It is therefore only necessary to know the number of waves of each height and period that occur, on average, in a year, to be able to calculate the average damage rate; it is not necessary to discuss the wave history in terms of sea-states. If waves of height  $H_i$  and period  $T_i$  cause stress cycles of range  $S_i$ , and hence 'damage' of  $1/N_i$  per wave, and if there are  $n_i$  such waves per year, then equations 3.2 may be expressed as:

$$\begin{aligned} \text{annual damage rate} &= \sum_i \frac{1}{N_i} \cdot n_i \\ &= \sum_i \frac{S_i^m}{K} \cdot n_i \quad \text{if } NS^m = K \end{aligned} \quad (3.14)$$

If it is possible, without undue conservatism, to regard the stress range produced by a given wave height as independent of the wave period, then the number of cycles,  $n_i$ , need only be expressed as a function of  $H_i$ . This is done most conveniently by using the long term probability density function of  $H$ ,  $f_H(H)$ , and the number of waves per year  $n_{ann}$ .

$$\text{Hence, annual damage rate} = n_{ann} \int_0^{\infty} \frac{S^m}{K} f_H(H) dH \quad (3.15)$$

This form of calculation is sufficiently simple that closed form solution of the integral exists if  $f_H(H)$  and  $S$  as a function of  $H$  take certain forms<sup>50</sup>. These solutions are developed in the next section.

If the wave energy spectrum is wide-band, the arguments above do not all hold. However it is clear from section that conservative calculations may be made by assuming that the spectra is narrow-band, using a damage reduction factor such as that proposed by Wirsching and Light<sup>4</sup> if desired.

### 3.5.2 Closed Form Solutions

If the relationship between stress-range and wave height is given by:

$$S = aH \quad (3.16)$$

and the S-N curve is often the form:

$$NS^m = K$$

and if the long-term distribution of wave heights has a two-parameter Weibull distribution:

$$F_H(H) = 1 - e^{-(H/\beta)^k} \quad H \geq 0 \quad (3.17)$$

$$f_H(H) = \frac{k}{\beta} \left[ \frac{H}{\beta} \right]^{k-1} e^{-(H/\beta)^k}$$

$$\begin{aligned} \text{then expected damage/cycle} &= \int_0^{\infty} \frac{(aH)^m}{K} f_H(H) dH \\ &= \int_0^{\infty} \frac{(aH)^m}{K} \frac{kH^{k-1}}{\beta^k} e^{-(H/\beta)^k} dH \\ &= \frac{a^m \beta^m}{K} \Gamma\left(1 + \frac{m}{k}\right) \end{aligned} \quad (3.18)$$

$$\text{where } \Gamma(p) = \int_0^{\infty} t^{p-1} e^{-t} dt \quad (p > 0)$$

If the long-term distribution of wave height has an exponential distribution, then  $k=1$  so

$$\begin{aligned} \text{expected damage/cycle} &= \frac{a^m \beta^m}{K} \Gamma(1+m) \\ &= \frac{a^m \beta^m m!}{K} \text{ if } m \text{ is an integer.} \end{aligned} \quad (3.19)$$

Data on wave heights analyzed in section 5.1 is best fitted by a modified form of exponential distribution:

$$\left. \begin{aligned} F_H(H) &= 1 - \alpha e^{-H/\beta} \\ f_H(H) &= \frac{\alpha}{\beta} e^{-H/\beta} \end{aligned} \right\} \begin{aligned} &\text{for } H_0 \leq H \leq \infty \\ &\text{where } H_0 = \beta \ln \alpha \text{ and} \\ &\alpha \text{ is a constant slightly} \\ &\text{greater than 1.} \end{aligned} \quad (3.20)$$

This gives expected damage/cycle

$$= \int_{H_0}^{\infty} \frac{(aH)^m}{K} \frac{\alpha}{\beta} e^{-H/\beta} dH \quad (3.21)$$

If  $m$  is an integer equation 3.21 has an analytical solution.

For instance for  $m=3$ ;

$$\begin{aligned} \text{expected damage/cycle} &= \frac{a^3 \beta^3 3!}{K} \left\{ 1 + \ln \alpha + \frac{1}{2} \ln^2 \alpha + \frac{1}{6} \ln^3 \alpha \right\} \\ &\approx \frac{a^3 \beta^3 3! \alpha}{K} \end{aligned} \quad (3.22)$$

More generally, writing  $\omega = H/\beta$ ,  $\omega_0 = H_0/\beta = \ln \alpha$  one obtains

$$\begin{aligned} \text{expected damage/cycle} &= \frac{a^m \beta^m \alpha}{K} \int_{\omega_0}^{\infty} \omega^m e^{-\omega} d\omega \\ &= \frac{a^m \beta^m \alpha}{K} \left\{ \Gamma(1+m) - \int_0^{\omega_0} \omega^m e^{-\omega} d\omega \right\} \\ &= \frac{a^m \beta^m \alpha}{K} \left\{ \Gamma(1+m) - \frac{\omega_0^{m+1}}{m+1} + \frac{\omega_0^{m+2}}{m+2} - \frac{\omega_0^{m+3}}{2(m+3)} + \dots \right\} \\ &\approx \frac{a^m \beta^m \alpha \Gamma(1+m)}{K} \end{aligned} \quad (3.23)$$

whence the error caused by ignoring the terms in  $\omega_0$  may be calculated and found to be generally very small. For instance, with  $m = 3.5$  and  $\alpha = 1.35$  (these are typical values from chapter 5), the bracketed term is:

$$\begin{aligned} \Gamma(4.5) - \frac{0.3^{4.5}}{4.5} + \frac{0.3^{5.5}}{5.5} - \frac{0.3^{6.5}}{6.5} + \dots \\ \approx 11.63 - 9.86 \times 10^{-4} + 2.42 \times 10^{-4} - 6.14 \times 10^{-5} + \dots \end{aligned}$$

### 3.5.3 Stresses due to Wave Forces on a Vertical Column

The use of the simple relationship  $S=aH$  arises from circumstances such as the following:

For modest sized waves of height  $H$  and period  $T$  in deep water, linear wave theory<sup>45</sup> gives the horizontal components of water velocity and acceleration at depth  $h$  as

$$\text{velocity} = \frac{1}{2} \omega H e^{-kh} \sin \omega t \quad (3.24)$$

$$\text{acceleration} = \frac{1}{2} \omega^2 H e^{-kh} \cos \omega t \quad (3.25)$$

where  $k = \omega^2/g$  and  $\omega = 2\pi/T$

Hence the horizontal inertia and drag forces per unit length on a cylinder of radius  $r$  are given by Morison's Equation as

$$\frac{1}{2} C_m \rho \pi r^2 \omega^2 H e^{-kh} \cos \omega t \quad (3.26)$$

$$\text{and } \frac{1}{4} C_D \rho r \omega^2 H^2 e^{-2kh} |\sin \omega t| \sin \omega t \text{ respectively} \quad (3.27)$$

If the cylinder extends vertically from the sea-bed to the surface the amplitude of the total horizontal inertia force is given by

$$\int_0^d \frac{1}{2} C_m \rho \pi r^2 \omega^2 H e^{-kh} dh \text{ where } d \text{ is the depth}$$

$$= \frac{1}{2} C_m \pi \rho g r^2 H \text{ for very deep water} \quad (3.28)$$

Similarly the amplitude of the total drag force is given by

$$\int_0^d \frac{1}{4} C_D \rho r \omega^2 H^2 e^{-2kh} dh$$

$$= \frac{1}{8} C_D \rho g r H^2 \quad (3.29)$$

Hence taking typical values for  $C_D$  and  $C_m$  of 0.6 and 2.0 respectively:

$$\frac{\text{Drag force amplitude}}{\text{Inertia force amplitude}} = \frac{1}{4\pi} \frac{C_D}{C_m} \frac{H}{r}$$

$$\approx \frac{1}{4\pi} \frac{0.6}{2.0} \frac{H}{r}$$

$$\approx 0.024 \frac{H}{r} \quad (3.30)$$

With H around 5 to 10 metres for waves of greatest interest and r of the order of 1 metre it is clear that in terms of total loading on a vertical cylinder the drag force is much less important than the inertia force, and that bearing in mind the difference in phase the amplitude of the total horizontal loading will be given reasonably accurately by the inertia loading alone. Thus the stress at any joint where loading is determined principally by the total horizontal load on one column will have the characteristics of the inertia force in equation 3.28 - namely that it is proportional to the wave height, to  $C_m$ , and to the square of the member radius, but independent of wave frequency.

For example, at a hot-spot in a bracing member near the bottom of a jacket structure and neglecting secondary moments due to joint rigidity, one could use the form  $S=aH$  where a is given by

$$a = (SCF) \times 2 \times \frac{1}{A} \times F_1 \times F_2 \times \left(\frac{1}{2}C_m\pi\rho g r^2\right) \quad (3.31)$$

where (SCF) = stress concentration factor

$$= \frac{\text{hot-spot stress}}{\text{axial stress in bracing member}}$$

and  $A$  = Steel cross section area of bracing member

$r$  = Effective radius of a column (including marine growth for load calculation)

$$F_1 = \frac{\text{Axial load in bracing member}}{\text{Total Base Shear on Platform}}$$

$$F_2 = \frac{\text{Total Base Shear on Platform}}{\text{Base Shear Force for single column of radius } r}$$

The factor of 2 allows for the reversal of stress between equal tensile and compressive stresses as the wave passes, and the factor  $F_1$  is the result of the structural analysis of the structure. The factor  $F_2$  should be a function of the wave period  $T$  because the forces on the columns of a platform are not all in phase with one another, so the total force at one instant of time depends on the wavelength and hence on  $T$ . It is assumed here that a single representative, or conservative, value is used.

Bringing equations 3.3, 3.23 and 3.31 together gives:

$$\text{Fatigue Life} = \left[ \frac{n_{\text{ann}} \propto \Gamma(1+m)}{K} \cdot \left[ \frac{2\beta(\text{SCF})F_1F_2}{A} \cdot \frac{1}{2} C_m \rho g r^2 \right]^m \right]^{-1} \quad (3.32)$$

### 3.6 Spectral Methods

If the structure is dynamically sensitive then it is necessary to calculate the response sea-state by sea-state because it is impossible to separate the responses to the individual waves from one another. To avoid extensive time-domain simulations, this requires the use of spectral methods. The result of such an analysis is an energy spectrum for each sea-state of the stress at the hot-spot under investigation. These spectra will almost inevitably have a high bandwidth parameter, so arguments similar to those in section 3.2.2 will be applicable here to the derivation of stress-cycle statistics from the spectra. These statistics are summed over all the sea-states to calculate the fatigue life.

If the annual damage rate in the  $k^{\text{th}}$  sea-state is  $d_k$ , then the actual annual damage rate is given by

$$\sum_k t_k d_k$$

where  $t_k$  is the proportion of a typical year for which sea-state  $k$  persists and hence the life is

$$1 / \sum_k t_k d_k$$

Statistics for  $t_k$  are supplied in the form of a scatter diagram (a bivariate histogram of  $t_k$  in terms of  $H_s$  and  $T_z$ ). Because of the complexity of the calculations of  $d_k$ , and because there is no accepted algebraic description of scatter diagrams, the calculation of

$$\sum_k t_k d_k$$

is performed over a finite set of discrete sea-states. There is no equivalent closed-form solution, such as that described in section 3.5.2 above.

The annual damage rate in the  $k^{\text{th}}$  sea-state,  $d_k$ , (if that sea-state persisted all year) can be expressed as:

$$d_k = \sum_{\text{all } i} \left( \begin{array}{l} \text{Number of cycles/year if } k^{\text{th}} \text{ sea-state persisted all} \\ \text{year} \end{array} \right) \times \begin{array}{l} \text{(damage due to one cycle of range } S_i) \\ \text{(probability in } k^{\text{th}} \text{ sea-state that one cycle, chosen} \\ \text{at random, has range } S_i) \end{array} \quad (3.33)$$

If, for example, it is assumed that the S-N curve has the form  $NS^m = K$  and if the upper-bound approach given in section 3.2.2 is used, with a damage reduction factor  $\lambda_k$ ,

$$d_k = \int_0^{\infty} \left[ 365.25 \times 24 \times 60 \times 60 \times \frac{1}{2\pi} \frac{\gamma_{m_2}}{\gamma_{m_0}} \right] \lambda_k \left[ \frac{S^m}{K} \right] \left[ \frac{S}{4m_{0k}} e^{-1/2 \cdot \frac{S^2}{4m_{0k}}} \right] dS$$

$$= \frac{365.25 \times 24 \times 60 \times 60}{2\pi K} .8^{m/2} \lambda_k \Gamma(m/2+1) m_{2k}^{1/2} m_{0k}^{1/2(m-1)}$$

where  $m_{0k}$ ,  $m_{2k}$  are the zero<sup>th</sup> and second moments of the energy spectrum of hot-spot stress under the wave loading of the  $k^{\text{th}}$  sea-state ( $S_{\sigma\sigma k}(\omega)$ ), i.e.

$$m_{n_k} = \int_0^{\infty} \omega^n S_{\sigma\sigma k}(\omega) d\omega$$

Now,

$$S_{\sigma\sigma k}(\omega) = |\alpha(\omega)|^2 \cdot S_{\eta\eta k}(\omega) \quad (3.35)$$

where  $S_{\eta\eta k}(\omega)$  is the energy spectrum of sea surface elevation in the  $k^{\text{th}}$  sea-state, and  $\alpha(\omega)$  is the transfer function which relates wave height and hot-spot stress. The essence of this method is that this transfer function is a function of wave frequency ( $\omega$ ) but not of wave height, so that wave height and hot-spot stress are assumed to be directly proportional to one another at a given  $\omega$ . The need to linearise what is essentially a non-linear relationship (because of the drag force and sea surface effects, for instance) and the difficulty of carrying out a full model analysis are the major drawbacks of this approach. The approximations and simplifications used to overcome these difficulties inevitably lead to some inaccuracies in the calculations. Various methods have been suggested<sup>35, 51</sup> which attempt to minimise these inaccuracies.

## CHAPTER 4

### FORMULATION OF FAILURE FUNCTION

#### 4.1 Introduction

The purpose of this chapter is to discuss, for the case of failure by fatigue, the formulation of the failure function,  $Z = g(\underline{x})$ , that is required for the calculation of probabilities of failure as discussed in Chapter 2. The first formulations to be discussed are derived from the deterministic methods of design and analysis outlined in Chapter 3. Then some further developments are given (in section 4.5) incorporating the concepts of fracture mechanics.

The basic requirement that the failure function,  $g(\underline{x})$ , must meet is that it should be formulated in such a way that  $g(\underline{x}) \leq 0$  indicates failure and  $g(\underline{x}) > 0$  indicates non-failure.

The simplest way to satisfy this requirement in the case of fatigue failure is to take the safety margin  $Z$ , as a measure of the reserve of life, i.e.

$$Z = (\text{Actual Life}) - (\text{Required Life}) \quad (4.1)$$

so that  $Z \leq 0$  if the actual life is too short. An alternative formulation which leads to a more quickly converging solution in many of the problems to be discussed later is

$$Z = \log(\text{Actual Life}) - \log(\text{Required Life}) \quad (4.2)$$

The required life may be a specified number of years, or it could be expressed as "for as long as the structure is required", in which case, for an oil production platform, it could be developed as a function of basic variables such as the size of the oil-field, operating costs, world oil prices, and taxation levels, all of which are subject to considerable uncertainty. This approach will not be explored further in this thesis and the required life will be taken simply as  $Y_r$  years.

#### 4.2 Re-formulation of Deterministic Procedures

The deterministic procedures given in Chapter 3 are based on Miner's Rule: the idea that failure occurs when the cumulative damage reaches unity. Experiments have revealed a wide range of variation in the performance of this rule. This may conveniently be allowed for in the formulation of the problem by the introduction of an additional basic variable,  $M$ , describing the uncertainty in the operation of the Miner's Rule under variable amplitude loading. Thus equation 3.2 for instance becomes

$$(\text{Life in years}) \times (\text{annual damage rate}) = M \quad (4.3)$$

and hence the failure function can be written

$$Z = \frac{M}{(\text{annual damage rate})} - Y_r \quad (4.4)$$

The various equations given in Chapter 3 can now be easily reformulated. For instance, using the most general formulation, given in equation 3.2,

$$Z = \frac{M}{\sum_i (1/N_i) n_i} - Y_r \quad (4.5)$$

More specifically, if the stress spectrum is narrow-banded, the structure dynamically insensitive, with a linear relationship between  $S$  and  $H$ , and assuming  $NS^m = K$ , then equation 3.18 gives

$$Z = \frac{MK}{n_{ann} a^m \rho^m \Gamma(1+m/k)} - Y_r \quad (4.6)$$

and equation 3.23 given

$$Z = \frac{MK}{n_{ann} a^m \rho^m \alpha \Gamma(1+m)} - Y_r \quad (4.7)$$

Equations 4.6 and 4.7 differ only in the form used for the long-term wave height distribution.

For the particular case where the stress range is calculated from the wave force on a single vertical column, equation 3.32 gives (with the addition of the reduction factor  $\lambda$ )

$$Z = \frac{MKA^m}{\lambda n_{ann} \alpha \Gamma(1+m) [\beta (SCF) F_1 F_2 C_m \rho g r^2]^m} - Y_r \quad (4.8)$$

Similarly for the spectral methods outlined in section 3.6, the general form is

$$Z = \frac{M}{\sum_k t_k d_k} - Y_r \quad (4.9)$$

and, more specifically, equation 3.34 gives

$$Z = \frac{2\pi MK}{(365.25 \times 24 \times 60 \times 60) \cdot 8^{\frac{1}{2}m} \cdot \Gamma(1+\frac{1}{2}m) \sum_k (t_k \cdot m_{z_k}^{\frac{1}{2}} \cdot m_{o_k}^{\frac{1}{2}(m-1)} \cdot \lambda_k)} - Y_r \quad (4.10)$$

for the case where  $NS^m = K$  and  $m_{o_k}$  and  $m_{z_k}$  are moments of the hot-spot stress spectrum for the  $k^{th}$  sea-state.

#### 4.3 Selection of Basic Variables

##### 4.3.1 Introduction

For the purposes of reliability analysis the failure function has to be formulated in terms of the chosen basic variables; that is those quantities for which probability distributions are to be provided. The choice of which quantities to use as basic variables is to some extent a compromise between those for which data is most readily available, and those that lead to a failure function which will be easy to use.

The functions given above (equations 4.5 to 4.10), being derived from the deterministic design methods, essentially represent the final stage of a much longer calculation procedure. Some of the quantities they contain, such as the Miner's Rule model uncertainty,  $M$ , and the S-N curve parameters,  $m$  and  $K$ , are

suitable for use as basic variables in that experimental data is widely available for these quantities. In other cases further consideration is necessary.

#### 4.3.2 Loading Variables

Loading variables, such as  $C_m$  in equation 4.8 may appear to be suitable for use as basic variables, but the situation is not straightforward. Quantities such as  $M$ ,  $m$  and  $K$ , representing the calculation of strengths, need apply only to the particular detail which is being checked. If the failure function is formulated (as in equation 4.8) using a single quantity  $C_m$ , then this represents the cumulative effect of loading all over the structure, which includes horizontal members, vertical members and inclined members, in various different parts of the wave, and different distributions for the uncertainty in  $C_m$  may be indicated for different members by the experimental data. In addition if  $C_m$  is found to depend on parameters of the wave such as Reynold's Number or the Keulegan-Carpenter Number then it will vary with time as various sea-states succeed one another. It is extremely unlikely however that sufficient data would be available to enable all these effects to be modelled accurately, and the resulting calculations would certainly be extremely complex. A realistic reliability analysis will therefore treat a quantity like  $C_m$  as a single basic variable, but the problems raised above have to be considered in creating the distribution to be used for such a variable. This will be discussed further in section 5.2.

Similar arguments apply to other loading variables such as the drag coefficient  $C_D$  and the effective radius (including marine growth),  $r$ .

#### 4.3.3 Variables which affect the Structural Analysis

The structural analysis of an offshore structure is generally a long and complex process because of the complexity of the structure itself. The results of an analysis depend on the

dimensions of the structure and its members and the properties of the materials. They also depend on quantities such as the effective stiffness of the tubular joints, and although in theory these quantities are determined by the dimensions and material properties of the structure, in practice they are regarded as additional variables because of the lack of accurate models with which to calculate them from the dimensions of the joints.

The stiffness of foundations will affect the outcome of some structural analyses and for dynamically sensitive structures the effective mass of the structure (including the added mass indicated by the coefficient  $C_m$ , and the marine growth) and the drag force effects which determine the damping will also be important.

These quantities do not appear explicitly in any of the formulations given by equations 4.5 to 4.10. The results of the structural analysis are included in the evaluation of quantities such as  $a$  in 4.6 and 4.7,  $F_1$  in 4.8, and  $m_{0k}$  and  $m_{2k}$  in 4.10. Of these, only  $F_1$  is concerned only with the structural analysis, the others also cover aspects such as the calculation of wave force and the use of stress concentration factors to obtain the hot-spot stress.

The many quantities that determine the outcome of a structural analysis will all be subject to some uncertainty, and could be taken as basic variables. This would make the reliability calculation extremely lengthy, because whether it is done by numerical integration, the Monte-Carlo method, or the Level 2 method, the calculation involves many evaluations of the failure function using different values of the basic variables, and each of these evaluations requires a new structural analysis.

An important question, therefore, is whether there are any significant uncertainties in the structural analysis. If there are not, then it is possible to arrange the calculations so that the structural analysis is only carried out once and the results, in the form, for example, of influence line coefficients, are used throughout the reliability analysis.

If there are significant uncertainties affecting the structural analysis then it may still be possible to avoid carrying out full structural analyses throughout the reliability calculation. This is done by undertaking a preliminary study of the effects on the results of the structural analyses of the significant uncertainties, which are then allowed for in the reliability analysis in a simpler way. One method is to introduce a model uncertainty variable expressing all the uncertainties in the structural analysis. Alternatively, if there are only a few variables which are subject to a significant degree of uncertainty, it may be possible to find simple empirical functions relating the results of the structural analysis to these variables so that they can be used as basic variables.

#### 4.3.4 Special Features of Spectral Methods

In the case of the spectral methods that lead to equation 4.10 the results of the wave loading and the structural analysis are contained within the two moments  $m_{0k}$  and  $m_{2k}$ . These are defined by

$$m_{\eta k} = \int_0^{\infty} \omega^n S_{\sigma\sigma k}(\omega) d\omega \quad (4.11)$$

$$\text{where } S_{\sigma\sigma k}(\omega) = |\alpha_k(\omega)|^2 S_{\eta\eta k}(\omega) \quad (4.12)$$

where  $S_{\eta\eta k}(\omega)$  is the wave energy spectrum for the  $k^{\text{th}}$  sea-state

and  $\alpha_k(\omega)$  is the transfer function for the  $k^{\text{th}}$  sea-state

This formulation ignores the problem of the directionality of the waves. Vughts and Kinra<sup>34</sup> give a more elaborate formulation which includes this.

The transfer functions  $\alpha_k(\omega)$  covers the effects of three 'transfers':

- (i) from the wave to the wave loads
- (ii) from the loads to the member stresses
- (iii) from the member stresses to the hot-spot stress.

Part (i) of this corresponds to  $F_z(1/2C_m\rho g r^2)$  in the simpler formulation, but in general also includes some effect of the drag force. It is a function of the frequency  $\omega$  and the best linearisation is different for each sea-state. Part (ii) is the result of the structural analysis, including the effects of dynamic response. This depends on the natural frequencies and damping coefficients of the structure, and hence not only on the stiffness but also on the effective mass of the structure (which includes effects of  $C_m$  and marine growth) and on drag effects.  $\alpha(\omega)$  is not now proportional to  $C_m$ . To avoid a complete structural analysis at every stage of the iterative process it would be necessary to find approximate equations representing the dependence of the results of the structural analysis on the required basic variables. Such calculations are clearly specific to each structure and there is no generally agreed form that these equations should take. Part (iii) of the transfer function is the stress concentration effect. In equation 4.8 only one stress concentration factor was used. In general the hot-spot stress is given by

$$\text{Hot-spot stress} = \sum_1 (SCF)_i \cdot \sigma_i \quad (4.13)$$

where the summation is over stresses due to axial loading, bending about two axes, etc. This summation is complicated by the fact that the hot-spot may not be in the same position for each stress. As with the wave coefficients in section 4.3.2 a composite effective stress concentration factor can be devised and  $\alpha(\omega)$  would be proportional to such a quantity. It follows therefore that the moments  $m_{0k}$  and  $m_{2k}$  will both be proportional to the square of this effective stress concentration factor, and hence the summation

$$\sum (t_k m_{2k}^{\frac{1}{2}} m_{0k}^{\frac{1}{2}(m-1)} \lambda_k)$$

in equation 4.10 will be proportional to  $(SCF)^m$ ; the same relationship as held in equation 4.8. This is the only element in  $\alpha(\omega)$  whose effect can be described with such generality.

Since sea-states are specified by their significant wave height and zero-up-crossing wave period, the wave energy spectrum  $S_{\eta\eta k}(\omega)$  for each sea-state is a function of the appropriate values of these two parameters. The function, which might for instance take the Pierson-Moskowitz<sup>46</sup> or JONSWAP<sup>47</sup> form, will be subject to some uncertainty. This can be treated by taking the various 'constants' in these spectrum formulae as basic variables.

#### 4.3.5 Use of Ratios as Basic Variables

It is convenient to express basic variables as non-dimensional quantities which are factors on the values used in deterministic design calculations. This can give the resulting calculations a considerable degree of generality.

For example, taking the formulation of equation 4.7, but in the log form of equation 4.2, and using superscript V to indicate a basic random variable and superscript D to indicate a deterministic constant:

$$\begin{aligned} Z &= \log \left[ \frac{M^{V_K V}}{\lambda^V (a^V \beta^V)^{mV} \alpha^V \Gamma(1+m^V)} \right] - \log (Y_r^D) \\ &= \log \left[ \frac{M^{V_K V}}{\lambda^V (a^V \beta^V)^{mV} \alpha^V \Gamma(1+m^V) Y_r^D} \right] \end{aligned} \quad (4.14)$$

If in a deterministic design calculation the calculated life is  $Y_d$ , then from the corresponding deterministic form, equation 3.23:

$$Y_d^D = \frac{M^{D_K D}}{\lambda^D (a^D \beta^D)^{mD} \alpha^D \Gamma(1+m)} \quad (4.15)$$

Introducing a superscript R is to indicate a basic variable which is a non-dimensional factor on the deterministic design value, i.e. for a quantity X:

$$X^R = \frac{X^V}{X^D} \quad (4.16)$$

then substituting equation 4.15 into the bracket of equation 4.14 therefore gives:

$$Z = \log \left[ \frac{M^{R_K R} \Gamma(1+m^D)}{\lambda^{R_\alpha R} \Gamma(1+m^V)} \frac{(a^D \beta^D)^{m^D}}{(a^V \beta^V)^{m^V}} \cdot \frac{Y_d^D}{Y_r^D} \right] \quad (4.17)$$

If it is now assumed that m may be taken as a constant ( $m^V = m^D = m$ ), then

$$Z = \log \left[ \frac{M^{R_K R}}{\lambda^{R_\alpha R} (a^R \beta^R)^m} \cdot \frac{Y_d^D}{Y_r^D} \right] \quad (4.18)$$

giving all the basic variables in ratio form. This can be extended to the form of equation 4.8:

$$Z = \log \left[ \frac{M^{R_K R} (A^R)^m}{\lambda^{R_\alpha R} [\beta^R (SCF)^{R_{F_1}} R_{F_2} R_{C_m} (r^R)^2]^m} \cdot \frac{Y_d^D}{Y_r^D} \right] \quad (4.19)$$

assuming that the quantities,  $n_{ann}$ ,  $\rho$ ,  $g$  are taken as constants.

Finally

$$\frac{Y_d^D}{Y_r^D} = \frac{\text{Design Life}}{\text{Required Life}} = \text{Safety Factor on Life}, \quad (4.20)$$

so that equation 4.19 is the failure function for a very wide set of actual structures and circumstances.

#### 4.3.6 Variables modelling the Frequency of Occurrence of Wave Heights and Sea-States

In the simplest of the non-spectral formulations (equation 4.5) the quantities  $n_i$  give the number of waves of height  $H_i$  per year. In the subsequent developments (equations 4.6 to 4.8), the numbers

of waves of various heights have been described by an algebraic formula so that it is parameters of the formula that appear in the equation:  $\beta$  and  $k$  in equation 4.6,  $\alpha$  and  $\beta$  in equations 4.7 and 4.8. These algebraic formulae take the form of probability distributions and could be thought of as such in that they can be used to give the probability that a wave, chosen at random, will have a particular height. Here, however, they are used to give the relative frequency with which the various wave heights will occur during the fatigue life. The incorporation of such a distribution into a failure function implies an assumption that the general form of the distribution is correct. The parameters of the distribution, however, may be uncertain quantities and can be treated as basic variables. However, where a frequency distribution has two or more parameters, those that are conventionally used would not necessarily be mutually independent when considered as basic variables. If this were the case, it would be necessary to find a joint probability density function for them, or to seek an alternative set of parameters that would be independent.

In the case of equations 4.9 and 4.10 the quantities  $t_k$  give the time spent in each sea-state. Values for  $t_k$  can be obtained from scatter diagrams which give  $t_k$  in terms of the significant wave height ( $H_s$ ) and mean zero-upcrossing period ( $T_z$ ). However, there is no accepted formula for representing the relationship between  $t_k$ ,  $H_s$  and  $T_z$  algebraically, so that the approach used above for  $n_i$  cannot be readily adopted for  $t_k$ .

An alternative approach to the representation of wave description uncertainty, which is applicable to both  $n_i$  and  $t_k$  is to take these as basic variables. The problem is that they cannot all be independent of one another. For instance,  $t_k$  represents the fraction of time spent in sea-state  $k$ , so by definition,

$$\sum_k t_k = 1.0 \quad (4.21)$$

A similar constraint applies to  $n_i$  because the total number of waves per year is determined by the average wave period and is certainly subject to much less uncertainty than are the individual  $n_i$  values.

If the values of the variables  $t_k$  are assumed to be stochastically independent, subject only to the constraint of equation 4.22, then it is possible to modify the Level 2 method to give results which will satisfy this constraint by introducing a second failure function that has to be zero at the 'design point':

$$Z_B = \sum_k t_k - 1.0 \quad (4.22)$$

An alternative approach is to take a deterministic value for each  $t_k$  and introduce a single basic variable outside the summation, i.e. replacing

$$\sum_k t_k d_k$$

in equation 4.9 by

$$t' \sum_k t_k d_k$$

where only  $t'$  is a basic variable. Similarly

$$\sum_i (1/N_i) n_i$$

could become

$$n' \sum_i (1/N_i) n_i$$

in equation 4.5 with deterministic  $n_i$  and  $n'$  as a basic variable. These methods will be discussed further in section 5.1

#### 4.4 Use of Fracture Mechanics

##### 4.4.1 Introduction

The science of fracture mechanics was developed initially to quantify the phenomenon of brittle fracture where failure occurs by the rapid propagation of a crack starting from an existing defect in the material. Early work was by Griffiths<sup>52</sup> and modern fracture mechanics dates from the introduction of the concept of

the stress intensity factor by Irwin<sup>53,54</sup>. Subsequent developments have also included concepts such as the crack opening displacement<sup>55</sup> and J contour integral<sup>56</sup>, and aim to explain all the modes of crack extension under load, including the effects of ductility.

Since fracture mechanics is concerned with the extension of cracks it is not surprising that many investigators have sought to use this approach as the basis for a more penetrating insight into fatigue than is obtained from the consideration of S-N curves and the test data that support them. The major advance has been the measurement by a variety of techniques<sup>57</sup> of the rate of crack propagation per cycle ( $da/dn$ ) during constant cycle tests, and the establishment of relationships between this rate and various other parameters. The most widely accepted and used relationship is due to Paris<sup>58</sup> and takes the form:

$$\frac{da}{dn} = C\Delta K^m \quad (4.23)$$

where  $\Delta K$  is the range of stress intensity factor and  $C$  and  $m$  are empirical constants.  $\Delta K$  is given by:

$$\Delta K = \alpha S\sqrt{\pi a} \quad (4.24)$$

where  $\alpha$  is a constant (generally close to unity) which takes account of the geometry of the test piece,  $S$  is the stress range (based on a reference stress defined to suit the specimen being tested) and  $a$  is a measurement of crack size, normally the crack depth for an edge crack or surface crack and half the crack width for an internal crack.

Various authors (see ref. 59 for summary) have suggested theoretical bases for such a relationship, which lead to particular values (usually integers) for the index  $m$ . Nevertheless, Equation 4.23 remains an essentially empirical rule with  $m$  usually taking values in the range 2 to 4, with no clear indications of an underlying integer value.

In the case of constant stress cycle tests this relationship is consistent with a common form of S-N curve.

$$\text{Given } \frac{da}{dn} = C \alpha^m S^m \pi^{m/2} a^{m/2} \quad (4.25)$$

$$\text{i.e. } a^{-m/2} da = C \alpha^m S^m \pi^{m/2} dn$$

If the crack extends from length  $a_i$  to  $a_f$  during  $N$  cycles,

$$\frac{a_f^{1-m/2} - a_i^{1-m/2}}{1 - \frac{m}{2}} = C \alpha^m S^m \pi^{m/2} N$$

$$\text{i.e. } NS^m = \frac{a_i^{1-m/2} - a_f^{1-m/2}}{\left[\frac{m}{2} - 1\right] C \alpha^m \pi^{m/2}} = K \quad (4.26)$$

which shows that the S-N curve parameter  $K$  is related to more fundamental properties of the material, and of the geometry of the particular case under examination.

It is often found in constant stress cycle tests that stresses below a certain threshold level will not cause cracks to propagate. In the fracture mechanics view it is considered more likely that a threshold value of range of stress intensity factor ( $\Delta K_{th}$ ) would exist as a fundamental property of the material and the Paris crack growth law equation 4.23) can be modified<sup>60, 61</sup> to include this:

$$\text{either } \frac{da}{dn} = C(\Delta K - \Delta K_{th})^m \quad (4.27)$$

$$\text{or } \frac{da}{dn} = C(\Delta K^m - \Delta K_{th}^m) \text{ for } \Delta K \geq \Delta K_{th} \quad (4.28)$$

Another modification is based on the suggestion that the end of a fatigue test occurs when the actual value of stress intensity factor first reaches the critical value,  $K_C$ , at which fracture occurs in tests conducted under conditions of steadily increasing load, and that the crack growth accelerates as this event comes

closer, due to local fractures on parts of the crack front. If the fatigue cycles are from  $K_{\min}$  to  $K_{\max}$  then,

$$\Delta K = K_{\max} - K_{\min} \quad (4.29)$$

and the stress ratio  $R$  is defined by

$$R = \frac{K_{\min}}{K_{\max}} \quad (4.30)$$

$$\text{hence } K_{\max} = \frac{\Delta K}{1-R} \quad (4.31)$$

The Forman<sup>62</sup> equation gives

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1-R)K_C - \Delta K} \quad (4.32)$$

which fulfills these requirements.

A combined form can be created incorporating both the threshold effect at the beginning of the life and fracture effect at the end,

$$\text{e.g. } \frac{da}{dn} = \frac{C(\Delta K^m - \Delta K_{th}^m)}{(1-R)K_C - \Delta K} \quad (4.33)$$

These more complex forms have some experimental support, but none has yet become established as the best available and the simpler Paris form is still widely used. The more complex formulae do not integrate as neatly as the Paris equation but S-N curves for constant cycle tests can be derived from them if required.

#### 4.4.2 Application to Offshore Structures

There are a number of problems involved in using the equations above in the study of offshore fatigue problems. For instance the effect of the geometry of tubular joints is usually to give crack propagation behaviour that is markedly different from that observed in simple fatigue test specimens. As the crack extends away from the hot-spot it moves rapidly into areas of lower stress and it is therefore found that crack propagation occurs at a much more uniform rate during the life than is given by a formula such as equation 4.25.

There is also a problem in defining a suitable measure of crack size. In the early part of a test, it is the propagation of the crack through the plate thickness that is of greatest interest, but subsequently it is the propagation of the crack along the weld that is significant.

These problems have been extensively studied by Dover<sup>63</sup> who is developing methods of overcoming them. However, at the present time fracture mechanics methods are not normally used in design calculations. Nevertheless they may have something more to offer in reliability studies because of the insight they give into the nature of the uncertainties in fatigue behaviour. For instance equation 4.26 shows how the S-N curve parameter K can be explained in terms of more fundamental properties and hence where the variations found in fatigue test results may originate.

#### 4.4.3 Varying Stress Cycles: Threshold Effects

The discussion so far has been in terms of constant stress cycle tests and equations that seek to model the results of such tests. A further problem in the use of the fracture mechanics approach is how to handle varying stress cycles. This will be considered fully in the next section. Before that, however, an example will be given of how the fracture mechanics point of view affects the way in which varying stress cycles are handled, even when the calculation method used is that of S-N curves and Miner's Rule. The S-N curves used in conventional design calculations are

normally obtained from the results of constant stress cycle tests and often show a threshold level of stress range below which no crack growth occurs. Now if this threshold value of stress range is a basic property of the material then only stress ranges above this level need to be considered in the cumulative damage calculations. However, if it is the threshold value of stress intensity factor range that is a fundamental property, then whenever a cycle occurs that causes the crack to extend, the apparent threshold level in stress terms decreases, so that a greater proportion of the cycles become damaging. Thus the threshold is gradually eroded. It is for this reason that threshold stress ranges are not used in offshore fatigue calculations. They use either a straight line S-N curve (in log-log space) or one with a shallower (but still non-zero) slope for low values of stress range.

#### 4.4.4 Varying Stress Cycles: Cumulative Damage

Measurements of crack growth rate during tests where specimens are subjected to sequences of cycles of varying stress range (and sometimes varying mean stress as well) have shown that fatigue lives can be seriously affected by the order in which cycles occur and it is clear that the cumulative damage hypothesis of Miner's Rule can be very incorrect. These sequence effects are likely, however, to be less marked where the loading is due to natural environmental loads whose behaviour is stationary, at least in the short term, than where the loading sequences are more precisely controlled as, for example, in the case of the effects of take-off and landing in aircraft.

Ideally fatigue tests should be carried out with sequences of loads following the pattern to be experienced by the structure under consideration. This is particularly relevant in the case of broad-band spectrum loads, where the definition of cycles is not simple and where methods such as the rainflow method give cycles superimposed on other cycles making an accurate calculation method based on cycle-by-cycle crack extension seem impossible. However, while tests have been made with wave spectrum loadings, the variety of forms that broad-band spectra can take is so large that results of general applicability are not available.

In the absence of such tests it is of course possible to proceed by ignoring the sequence effects that undoubtedly occur, hoping that they will, on average, cancel out over the large number of cycles involved. The calculations below are made on this basis but it is important to realise that much of the advantage of the fracture mechanics approach may have been lost in these approximations, and the results will be of limited applicability.

The first necessary assumption is that crack growth can be represented by an equation such as equations 4.23, 4.27, 4.28, 4.32 or 4.33, or more generally

$$\frac{da}{dn} = g(S,a) \quad (4.34)$$

implying that a cycle of stress range  $S$  will extend a crack of length  $a$  by amount  $g(S,a)$  irrespective of the magnitudes of previous cycles.

Given a sequence of stress cycles, this formula can be used to find the total crack extension by applying it to each cycle in turn. However, since the crack extension caused by the  $n^{\text{th}}$  cycle is calculated from the crack length at the beginning of that cycle it is thus affected by all the previous cycles so that the total crack extension over the sequence of cycles is unlikely to be independent of the order in which they occur. This is indeed intuitively sensible, since if the formula used includes a threshold effect in terms of  $\Delta K_{th}$  then one would expect it to make a difference whether the smallest cycles occur at the beginning or the end of the sequence.

The use of equation 4.34 does not therefore guarantee that the actual sequence of loads will not affect total crack extension. This does not matter if a known sequence of cycles is to be used or if it is possible to simulate sequences of cycles from given distributions. However in Level 2 type calculations it is necessary to be able to write down a deterministic failure function with all the probabilistic information contained in the probability distributions given for the variables in the failure function. This is clearly impracticable using the sum of millions

of crack extensions due to the individual cycles, each of which depends on all those that have gone before.

Hence for the application of Level 2 methods it is necessary to be able to write an explicit practicable formula for the whole fatigue life. This can be done easily only if either

(i) the order of cycles is immaterial, so that the life depends only on the long-term distribution of cycle sizes,

or

(ii) the loading process is sufficiently stationary that the crack growth history is always close to that given by the average or expected growth rate, in which case the effect of the randomness of the cycles, within the limits imposed by the stationarity condition, will have little effect on the life.

Condition (i), that the order of the cycles is immaterial, is essentially the cumulative damage hypothesis of Miner's Rule. If this condition is to hold, then it must be possible at any stage in the life to define a measure of "damage",  $Q(a)$ , which is a function of the crack length at that stage, but which is independent of the previous history of the crack. But if the  $n^{\text{th}}$  cycle extends the crack from  $a_{n-1}$  to  $a_n$ , then the increment of "damage" must depend only the stress range of the  $n^{\text{th}}$  cycle,  $S_n$ , and not in any way on the crack length, because otherwise this cycle would have a different effect if it occurred at another stage in the process.

$$\text{i.e. } Q(a_n) - Q(a_{n-1}) = \text{function of } S_n \quad (4.35)$$

$$\text{Now } Q(a_n) - Q(a_{n-1}) = \frac{dQ(a)}{dn} \text{ at } a = a_n$$

$$= \frac{dQ(a)}{da} \cdot \frac{da}{dn} \quad (4.36)$$

$$\text{Hence } \frac{da}{dn} = (\text{function of } S_n) \times \left[ \frac{dQ(a)}{da} \right]^{-1}$$

$$\text{or generally } \frac{da}{dn} = (\text{function of } S) \times (\text{function of } a) \quad (4.37)$$

Now if  $S$  and  $a$  are linked by  $\Delta K = \alpha S/\pi a$  it is clear that only the Paris crack growth law (equation 4.23) fits this criterion.

Hence this is the only one of the crack growth formulae given above which is strictly consistent with the cumulative damage hypothesis, given the assumption that these equations can be used with varying stress cycles.

It is therefore possible to combine equations 4.7 and 4.26 to give a failure function

$$Z = \frac{M (a_i^{1-m/2} - a_f^{1-m/2})}{\lambda n_{ann} a_s^m \beta^m \alpha_\omega \Gamma(1+m) (\frac{m}{2}-1) C \alpha_C^m \pi^{m/2}} - Y_r \quad (4.38)$$

where  $a_s = a$  from equation 3.16

$\alpha_\omega = \alpha$  from equation 3.20

$\alpha_C = \alpha$  from equation 4.24

and other symbols as previously defined.

It is of course also possible to devise S-N curves from the other suggested crack growth formulae and to use these in cumulative damage calculations, but this does imply making contradictory assumptions in different parts of the calculation.

However if condition (ii) applies, then such calculations may be reasonable. Ocean waves can be regarded as stationary processes both in the short term and in terms of annual statistics, so that over a reasonably long life this condition may be reasonably applicable, although this could only be fully tested by extensive numerical simulations.

Condition (ii) implies that it is possible, in effect, to replace the actual sequence of crack extensions with an equal number of "expected" extensions given by

$$E \left[ \frac{da}{dn} \right] = \int_0^\infty g(S, a) \cdot f_S(S) dS \quad (4.39)$$

where  $f_S(S)$  is the long-term distribution of stress range. Hence for the "replacement" cycles:

$$\frac{da}{dn} = \int_0^\infty g(S, a) f_S(S) dS \quad (4.40)$$

$$dn = \frac{da}{\int_0^{\infty} g(S,a) f_S(S) dS}$$

$$\text{Writing } G(a) = \int_0^{\infty} g(s,a) f_S(S) dS \quad (4.41)$$

$$\text{gives } n = G(a) - G(a_1) \quad (4.42)$$

and the life is given by

$$N = G(a_f) - G(a_1) \quad (4.43)$$

Hence if there are  $n_{ann}$  cycles per year the failure function is given by

$$Z = \frac{G(a_f) - G(a_1)}{n_{ann}} - Y_r \quad (4.44)$$

With the appropriate functions for  $g(S,a)$  and  $f_S(S)$  this will also lead to equation 4.38. However other forms of  $g(S,a)$  are equally applicable here, for instance equations 4.27 or 4.28 incorporating the threshold effect. This approach is however of limited validity when failure is defined in terms of brittle fracture at critical stress intensity factor,  $K_C$ , (equations 4.32 and 4.33) rather than in terms of attaining some final crack length. What is of most interest in this case is the possibility that a large stress cycle will cause premature failure. This problem can only be formulated in the way required for Level 2 analysis if a number of special assumptions are made, as demonstrated in the next section.

#### 4.4.5 Formulation for Fatigue ending in Fracture

The basic assumption is that the process of stress cycling is sufficiently regular for equation 4.42 to be used to give the crack growth as a steady deterministic process, not significantly affected by random variations. The peak stress,  $\sigma$ , for each cycle is taken as a random variable with long-term distribution  $f_\sigma(\sigma)$  and the peak stresses for each cycle are assumed to be independent. Failure occurs when the stress intensity factor  $\alpha\sigma\sqrt{\pi a}$  calculated from the current value of  $a$  exceeds  $K_C$ ,

$$\text{i.e. } \sigma > \frac{K_C}{\alpha\sqrt{\pi a}} \quad (4.45)$$

Now from equation 4.42  $a = G^{-1}(G(a_1)+n)$

$$= H(n), \text{ say.} \quad (4.46)$$

Hence after  $n$  cycles, given particular values of  $K_C$ ,  $\alpha$  and all the variables that govern the function  $H(n)$ , the probability of surviving the next load cycle is the probability that

$$\sigma < \frac{K_C}{\alpha\sqrt{\pi H(n)}}$$

i.e.  $F_\sigma \left[ \frac{K_C}{\alpha\sqrt{\pi H(n)}} \right]$  (4.47)

The probability of surviving all cycles from  $n=1$  to  $n=N_r$  is the product of the individual probabilities for each  $n$ . Hence if the probability of failure is defined as the probability of failing to reach  $N_r$  cycles, it is given by

$$1 - \prod_{n=1}^{N_r} F_\sigma \left[ \frac{K_C}{\alpha\sqrt{\pi H(n)}} \right] = P_{fc}, \text{ say} \quad (4.48)$$

For ease of computation this may be written

$$P_{fc} = 1 - \exp \left\{ \sum_{n=1}^{N_r} \ln \left[ F_\sigma \left[ \frac{K_C}{\alpha\sqrt{\pi H(n)}} \right] \right] \right\} \quad (4.49)$$

$$\text{or } P_{fc} = 1 - \exp \left\{ \int_0^{N_r} \ln \left[ F_\sigma \left[ \frac{K_C}{\alpha\sqrt{\pi H(n)}} \right] \right] dx \right\} \quad (4.50)$$

which can be evaluated numerically using a small number of values of  $x$ .

This probability of failure is of course conditional on the values chosen for  $K_C$ ,  $\alpha$ , and all the variables that affect  $H(n)$ . Before it can be de-conditioned it is necessary to rearrange the calculation into a form similar to a simple Level 2 calculation, i.e. using a standard normal variable,  $u'$  say, and a failure function.

$$\text{If } P_{fc} = 1 - \Phi(u_{fc}) = \int_{u_{fc}}^{\infty} \phi(u') du' \quad (4.51)$$

$$\text{ie } u_{fc} = \Phi^{-1} \{ 1 - P_{fc} \} \quad (4.52)$$

then the probability of failure is given by

$$\int_{Z < 0} \phi(u') du' \quad (4.53)$$

where  $Z < 0$  when  $u' > \Phi^{-1}\{1 - P_{fc}\}$ , so that  $Z$  could be given by:

$$Z = \Phi^{-1} \left\{ \exp \left[ \int_0^{N_T} \ln \left[ F_{\sigma} \left[ \frac{K_C}{\alpha \sqrt{\pi H(x)}} \right] \right] dx \right] \right\} - u' \quad (4.54)$$

Hence if the vector  $\underline{X}$  represents all the other basic variables and  $\underline{U}$  their standard normal equivalents (see section 2.7) then

$$P[\text{Failure}] = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} P[\text{Failure} | \underline{X}] \cdot f_{X_1}(x_1) \cdot f_{X_2}(x_2) \dots dx_1 \cdot dx_2 \dots \quad (4.52)$$

$$= \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \left\{ \int_{Z < 0} \phi(u') du' \right\} \cdot f_{X_1}(x_1) \cdot f_{X_2}(x_2) \dots dx_1 \cdot dx_2 \dots$$

$$= \int \int \dots \int_{Z < 0} \phi(u') \cdot \phi(u_1) \phi(u_2) \dots du' \cdot du_1 \cdot du_2 \dots \quad (4.56)$$

so that Level 2 calculations may be made in the  $u' + \underline{u}$  space using equation 4.54 as the failure function.

The failure function of equation 4.54 can be developed further given more detailed information on the structure and its behaviour. For instance making similar assumptions to those used before, i.e.

$$S = a_S H$$

$$F_H(H) = 1 - \alpha_{\omega} \exp(H/\beta)$$

$$g(S, a) = C(\alpha_C S \gamma(\pi a))^m$$

$$\text{and if in addition } \sigma = \frac{1}{2} a_S H \quad (4.57)$$

$$\text{then } Z = \Phi^{-1} \left\{ \exp \left[ \int_0^{N_T} \ln \left[ 1 - \alpha_{\omega} \exp \left[ \frac{2K_C}{\beta a_S \alpha_C \gamma \pi H(x)} \right] \right] dx \right] \right\} - u' \quad (4.58)$$

$$\text{where } H(x) = \left\{ a_1^{1-m/2} + C \alpha_C^m a_S^m \pi^{m/2} \alpha_{\omega} \beta \left( 1 - \frac{m}{2} \right) \Gamma(1+m) x \right\}^{\frac{1}{1-m/2}} \quad (4.59)$$

## CHAPTER 5

### PROBABILISTIC MODELLING OF BASIC VARIABLES

#### 5.1 Wave Climate Descriptions

##### 5.1.1 Introduction

As discussed in the previous chapter, the failure functions include a summation over a set of different wave heights or sea-states; uncertainties in the description of the wave climate are therefore uncertainties in the number of times each wave height occurs during the life of the structure, or in the length of time for which each sea-state persists.

Probabilistic models are needed to express two different types of uncertainty: the natural variability of the wave climate and the statistical uncertainty due to inadequate data. These can be combined into a 'predictive' distribution as described in section 2.6.6.

The principal variation of wave climate is of course seasonal but this is not relevant here because the assumption is made that calculations will generally be concerned with whole years, and that any fractions of a year that may appear in the calculations can be adjusted to allow for appropriate seasonal effects. It is a matter of common observation that the weather varies from one year to the next. However most platform calculations involve periods of twenty years or more, and it seems reasonable that the variations observed from year to year will have little effect when aggregate twenty year figures are calculated. Longer term variations in the weather certainly occur but these are small effects which occur very slowly and are the cause of much debate among climatologists. Wave records in deep water do not exist for long enough for any conclusions to be drawn about long-term variations. Where long sequences of weather records do exist (e.g. for temperature and wind-speed at inland or coastal sites where some records go back for centuries), changes of instrumentation and local topography make their interpretation difficult<sup>64</sup>. While

it may be true that the wave climate will change measurably during the lifetime of platforms being installed today there seems to be no reason to expect such change to be anything other than negligibly small in its effects on the fatigue life of the platform when compared with many of the other uncertainties discussed in this thesis. An estimate will be made below of the variation from year to year and of the extent to which this will affect longer term aggregate statistics.

In the early days of the exploitation of North Sea oil reserves, data about waves were extremely scarce. The situation is rapidly improving with the installation of wave recording devices on platforms, but in the early days of any new offshore oil field it is likely that wave data will be limited, and calculations for a proposed platform may have to be made on the basis of a year or two's records at a site possibly some distance from that of the proposed platform. This leads to some uncertainty about the appropriateness of the available statistics and, in addition, records obtained from wave recording devices are often seriously incomplete, leaving further doubt as to the accuracy with which the available statistics represent the true situation. An attempt has been made to estimate these uncertainties from some typical data.

The failure functions outlined in chapter 4 are based on either the number of waves ( $n_i$ ) per year of various heights ( $H_i$ ) or the proportion of time ( $t_k$ ) spent in each sea-state (specified by  $H_s$  and  $T_z$ ). Three alternative types of basic variable have been suggested (see section 4.3.6):

- (1) using all the  $n_i$  or  $t_k$  quantities as basic variables
- (2) using a single variable  $n'$  or  $t'$  as a factor on the summation
- (3) expressing  $n_i$  or  $t_k$  as functions of some other basic variables.

These three methods are all suitable for describing both the random variability of the wave climate and the statistical uncertainty due to inadequate or incomplete data. The following features govern their usefulness in practice:

Method (1): This has a large number of basic variables which are all theoretically correlated with one another. Deriving a suitable model is likely to be difficult, and it would be cumbersome to use in reliability calculations.

Method (2): This is much simpler to use, but the probabilistic model for  $n'$  or  $t'$  will depend to some extent on the particular problem being analysed. It may be necessary to reformulate the model for each reliability calculation.

Method (3): This is probably the best method if a good model exists relating  $n_i$  to  $H_i$  or  $t_k$  to  $H_{sk}$  and  $T_{zk}$ . This requirement is generally held to be satisfied for  $n_i$  but not for  $t_k$ .

These points will now be elaborated in more detail. The difficulties involved in method (1) are related to the partial correlation of the variables and will be discussed in terms of the  $t_k$  variables, although similar arguments apply to  $n_i$ . First the sum of all the  $t_k$ 's must be unity. This difficulty can be overcome in the Level 2 approach, as discussed in section 4.3.6. The full problem can be seen by considering a series of annual scatter diagrams for a particular location. A second series of diagrams can be produced in which the number of separate sea-states is halved, each sea-state in the second series of diagrams being produced by combining two adjacent sea-states on the first set of diagrams. The number of occurrences of each sea-state on each diagram in the second series is therefore given by adding the number of occurrences of its two component sea-states on the corresponding diagrams in the first series. The variation from year to year shown by the second series of diagrams will be less than that shown by the first. Further reductions in the number of separate sea-states on the diagrams will reduce the scatter until, of course, for the whole diagram there is no scatter at all. Since the reliability is more sensitive to some sea-states than others, what is required is a model that results in the correct degree of uncertainty in the summation; an idea which leads to the formulation of method (2). It is possible to formulate a model for the joint variation of all the  $t_k$ 's, perhaps by finding distributions for each one individually and then taking their standard normal transformations as the marginal distributions of a multivariate normal distribution. This model would however be

awkward to use, and has not been pursued since model (2) is considered preferable.

To use method (2) the failure function is reformulated; for example an expression such as  $\Sigma(1/N_i).n_i$  in equation 4.5, in which all the  $n_i$  are taken as basic variables in method (1), is replaced by  $n'\Sigma(1/n_i).n_i$  where the  $n_i$  are now taken as deterministic and  $n'$  is the only basic variable. Data, however, still comes in the form of observations of the variability of  $n_i$ , the frequency of occurrence of wave height  $H_i$ , so it is necessary to derive data for the effective variation in  $n'$  which results from the actual variation in  $n_i$ . If there are records for years  $j=1$  to  $J$ , and if there are  $n_{ij}$  waves of height  $H_i$  in year  $j$ , and if the deterministic values used for  $n_i$  is  $n^*_i$ , then the value of  $n'$  for year  $j$ ,  $n'_j$ , is given by

$$n'_j \cdot \sum_i \left( \frac{1}{N_i} \right) n^*_i = \sum_i \left( \frac{1}{N_i} \right) n_{ij} \quad (5.1)$$

Now if  $N_i S_i^m = K$  and  $S_i = a H_i$

$$\text{then } n'_j = \frac{\sum_i H_i^m n_{ij}}{\sum_i H_i^m n^*_i} \quad (5.2)$$

Hence a value  $n'_j$  is calculated from each year's data, and a model for the variation of  $n'$  can then be derived. Although such a model applies strictly only to the particular circumstances assumed it is not unreasonable to assume that the variation and uncertainty found in this way will be similar to that of other cases as well.

A similar approach is possible for finding a model for  $t'$  from data for  $t_k$ , the proportion of time spent in sea-state  $k$ , but in this case

$$t'_j \sum_k t^*_k d_k = \sum_k t_{kj} d_k \quad (5.3)$$

Now  $d_k$  is the annual damage rate for the  $k^{\text{th}}$  sea-state and is in general a function of many basic variables so a different calculation of  $t'_j$  could be made for each possible set of values of  $d_k$ . A general model for  $t'$  can only be found if a number of assumptions are made. But if for instance it is assumed that the stress-cycles that occur during the  $k^{\text{th}}$  sea-state are proportional

to  $H_{sk}$  and comparatively insensitive to  $T_{zk}$  then  $d_k$  will be proportional to  $H_{sk}^m$ , so that a model for  $t'$  can be derived from

$$t'_{ij} = \frac{\sum_k H_{sk}^{m_{t'_{kj}}}}{\sum_k H_{sk}^{m_{t'_{k}}}} \quad (5.4)$$

where  $t'_{k}$  are the deterministic values used in the reliability analysis. This approach is not correct if dynamic response is important.

Method (3) is only currently practicable for the method based on wave heights rather than sea-states. The discrete  $n_i$  quantities are replaced by a continuous distribution giving the relative frequencies of occurrence of wave height,  $H$ , given, for example, by

$$F_H(H) = 1 - \alpha e^{-H/\beta} \quad \beta \ln \alpha \leq H \leq \infty \quad (5.5)$$

$$f_H(h) = \frac{\alpha}{\beta} e^{-H/\beta}$$

The parameters  $\alpha$ ,  $\beta$  are then the basic variables, but they are not necessarily independent. This does not matter if the formulation of equation 4.7 is used because  $\alpha\beta^m$  can be treated as a single variable in this case.

The probabilistic models to be found below are therefore for the parameters  $\alpha$  and  $\beta$  of the long-term distribution of wave height and  $n'$ . These can then be compared in their effects on reliability calculations. For the reasons given above no general model for the scatter diagram/sea-state approach can be formulated but section 5.1.4 gives some potentially useful results.

#### 5.1.2 Distribution Type

The parameters  $\alpha$  and  $\beta$  of the long-term wave height distribution are subject to the restrictions  $\alpha > 0$  and  $\beta > 0$ , and in practice it appears that  $\alpha$  is always greater than 1. The choice of parameters  $\alpha$  and  $\beta$  was purely arbitrary and alternatives such as  $\alpha/\beta$ ,  $1/\beta$  etc could have been used instead. These two points suggest that the most suitable distribution type would be

log-normal for  $(\alpha-1)$  and  $\beta$ ; these distributions are defined over the admissible range of values and some of the other possible parameters will also then have log-normal distributions. The available data is not sufficient to indicate the correct form of the distributions so in the absence of other indications log-normal distributions will be assumed.

The variables  $n'$  and  $t'$  are in effect weighted averages which suggests they should be normally distributed, however they must be greater than zero so a log-normal distribution would be better, introducing a degree of skewness if the coefficient of variation is high.

#### 5.1.3 Available Data

In order to estimate the year to year variability and the inaccuracy that could result from using a very limited quantity of data, it was necessary to obtain data for one location for as many years as possible. It was also considered that the data should come from a location in the central or northern North Sea so that any conclusions drawn would be as applicable as possible to the British offshore work. The only source of data which satisfied these aims was the shipborne wave recorder on the m.v. *Famita*, data from which was supplied by the Institute of Oceanographical Sciences (IOS).

The m.v. *Famita* is a Norwegian vessel which serves as both a meteorological station and as a rescue vessel when the need arises. When on station its location is 57°30'B 3°00'E (i.e. within 100 miles or so of fields such as Ekofisk and Forties), where the depth of water is approximately 66 metres<sup>65</sup>. The vessel normally operates only during the winter, generally covering the period October to March at least.

The vessel is fitted with a shipborne wave recorder<sup>66</sup> maintained by the Institute of Oceanographical Sciences. Twelve-minute samples of the variation of surface elevation are taken every three hours and recorded on paper chart rolls. Wave records are considered usable only when the vessel was on station during the

recording period and not when it was under way for rescue purposes. For this reason a large number of wave records has been missed, and unfortunately it is probable that these lost records would have contained many high waves associated with the storm conditions which necessitated the rescue operations. IOS have not attempted to quantify or correct this effect.

Data has been supplied by IOS derived from the records of seven winters from 1969/70 to 1976/77; no data has been supplied for 1970/71 because very few records were successfully made during that winter. The data have been trimmed to cover only the six months from October to March inclusive in those cases where the recorder functioned for a longer period. This means that, ignoring leap years, 1456 records were potentially available. The actual periods of operation and numbers of records is given in table 5.1. As can be seen about 70% of the possible records are normally available, but in the second winter, 1971/72, recording started over a month late and less than 40% of the 1456 potential records for a full six months were obtained.

From each twelve minute record the significant wave height  $H_g$ , and the zero-up-crossing period,  $T_z$ , were calculated, as described by Tucker<sup>67</sup>. These results were then displayed in the form of scatter diagrams (bivariate histograms) for each of the seven winters in which the calculated values of  $H_g$  and  $T_z$  were grouped into intervals of 0.5m and 0.5 seconds respectively. The data were supplied in this form by IOS and are given in fig 5.1.

In order to obtain some information about the variation of wave climate with distance, a scatter diagram was obtained from IOS for the six months October 1973 to March 1974 for the Stevenson weather ship in the northern North Sea (61°20'N 0°00'E), with the aim of comparing this with the same period at Famita. Unfortunately the Stevenson recorder only recorded 566 valid records during this period and subsequent data was still subject to commercial confidentiality. It was felt that these data were insufficient to make any analysis worthwhile. This research will be possible in the future when more data is available.

The wave recorders used on m.v. Famita and Stevenson give no information on the direction of travel of the waves, and where necessary it is assumed in the calculations that the waves all travel in a single direction.

The principal problem in the use of scatter diagrams is the definition, estimation and use of the term "significant wave height",  $H_g$ . The original definition<sup>68</sup> was that  $H_g$  was the mean height of the highest third of the waves, and the notation  $H_{1/3}$  was also used. However it was not usually made clear how the wave heights were defined, and except for narrow band cases this is not obvious. Tucker<sup>69</sup> defines the wave heights as the difference in level between each crest and the preceding trough. He admits however that some of his colleagues prefer to define wave height as the difference in level between the highest crest and the lowest trough that occur between successive zero up-crossings. These two definitions will give different numbers and sizes of waves, the disparity being most marked in the number of small waves indicated for wide band spectra.

If the wave spectrum is narrow-banded then it can be shown from the properties of the Rayleigh distribution that  $H_g = 4/\sqrt{m_0}$ , where  $m_0$  is the zero<sup>th</sup> moment of the energy spectrum and that therefore  $H_g = 4\sigma$  where  $\sigma$  is the r.m.s. value of the sea surface elevation above still water level.

Although it is possible to obtain  $H_g$  statistically by measuring all the wave heights on a wave record (given a definition of wave height), this calculation is rather time consuming. Tucker's method<sup>67</sup>, which is much quicker, uses only the two highest crests and the two lowest troughs, and gives two estimates of the r.m.s. value  $\sigma$ . The equation  $H_g = 4\sigma$  is then used to give  $H_g$ . This relationship is of course only true for narrow-band processes where there is, in addition, no ambiguity in the definition of wave height, but such calculations are held generally to be reasonably accurate for most actual spectra.

Nowadays<sup>65</sup>  $H_g$  is actually defined as  $4\sigma$  and it is then described as approximating to the mean height of the one third highest waves,

the approximation improving with narrowing of the spectrum. It is still calculated from chart records in the same way, although other recording devices that enable  $\sigma$  to be measured directly are sometimes used.

Therefore, although scatter diagrams are plotted in terms of  $H_S$ , it is more accurate to think of them as plots of  $\sigma$  or  $m_0$ .

Data for actual wave heights are not readily available and so have to be inferred from the scatter diagrams by making suitable assumptions about the distribution of wave heights within sea-states.

For narrow-band spectra the distribution of wave height,  $H$ , is given by the Rayleigh distribution:

$$F_H(H) = 1 - \exp\left[-\frac{1}{2} \frac{(H/2)^2}{m_0}\right] \quad (5.6)$$

$$= 1 - \exp\left[-2 \left[\frac{H}{H_S}\right]^2\right]$$

$$\text{hence } f_H(H) = \frac{4H}{H_S^2} \exp\left[-2 \left[\frac{H}{H_S}\right]^2\right] \quad (5.7)$$

If a scatter diagram is defined by a discrete set of significant wave heights,  $H_{Si}$ , and a discrete set of zero up-crossing periods,  $T_{zj}$ , and if  $t_{ij}$  is the ratio of the number of times sea-state  $ij$  is recorded to the total number of records on the diagram, then during a (long) period of time  $T$ :

$$\text{time spent in sea-state } ij = t_{ij}.T$$

$$\text{number of waves in sea-state } ij = \frac{t_{ij}.T}{T_{zj}}$$

assuming that the number of waves is defined by the number of zero up-crossings not by the number of peaks and troughs.

number of waves in all sea-states with significant wave height  $H_{Si}$

$$= \sum_j \frac{t_{ij}.T}{T_{zj}} \quad (5.8)$$

Now if the significant wave height is  $H_{S1}$ , the proportion of waves whose height exceeds  $H$  is given by:

$$1 - F_{H|H_{S1}}(H) = \exp(-2H^2/H_{S1}^2)$$

Hence during time  $T$  the total number of waves whose height exceeds  $H$  is given by:

$$\sum_j \left\{ \exp(-2H^2/H_{S1}^2) \cdot T \cdot \frac{t_{1j}}{T_{Zj}} \right\}$$

Hence the long-term distribution of  $H$  is given by:

$$F_H(H) = 1 - \frac{\sum_j \left\{ \exp(-2H^2/H_{S1}^2) \cdot \frac{t_{1j}}{T_{Zj}} \right\}}{\sum_j \left( \frac{t_{1j}}{T_{Zj}} \right)} \quad (5.9)$$

Strictly, this calculation applies only to narrow-band distributions, but it is shown in section 3.2.2 that, for the purpose of fatigue calculations, broader band spectra can be conservatively taken as narrow-banded, and the circumstances in which data for wave height distributions are required are those in which the spectrum either is, or is assumed to be, narrow-banded.

With regard to the data from m.v. Pamita, it is likely that the large number of missing records will be a major cause of the differences that are observed between the data for the various available years. To give some idea of the extent of this effect, an alternative set of data was prepared in which the number of records in each diagram was exactly 1456; the new diagrams being prepared from the old by 'filling in' the missing records in such a way as to minimise the differences between the years in each position on the diagram.

If  $n_{jk}$  is the number of occurrences on the  $j^{\text{th}}$  scatter diagram of sea-states with significant wave height  $H_{Sk}$  and zero-up-crossing period  $T_{Zk}$ , then for each diagram (i.e. for each value of  $j$ ) the number of missing records is given by

$$1456 - \sum_k n_{jk} \quad (5.10)$$

The extra records theoretically available in a leap year are ignored. The procedure adopted was as follows:

- (1) Calculate the 'deficit' for each record below the average for that place on the diagram:

$$d_{jk} = \frac{1}{7} \sum n_{jk} - n_{jk} \quad (5.11)$$

- (2) For each scatter diagram (i.e. for each  $j$ ) in turn:
  - (a) Find the place ( $k=k_{\max}$ ) where  $d_{jk}$  has its maximum value ( $d_{\max}$ ), and find also the second largest value of  $d_{jk}$  ( $d'_{\max}$ ).
  - (b) Increase  $n_{jk_{\max}}$  by the smallest integer which is greater than  $d_{\max} - d'_{\max}$ , and reduce the number of missing records to be distributed by the same amount.
  - (c) Using the new value of  $n_{jk_{\max}}$  adjust the 'deficits' all the values of  $j$ , thus:

$$d_{jk_{\max}} = \frac{1}{7} \sum n_{jk_{\max}} - n_{jk_{\max}} \quad (5.12)$$

Stage (2) is repeated until all the available 'missing' records have been distributed. There are then seven diagrams in each of which the total number of occurrences is 1456. The results of this process are given in fig. 5.2.

One clear characteristic of these results is that, at most positions on diagrams, all but one of the diagrams have very similar values, the other one having a larger value. In all but the most exceptional circumstances the original diagrams and the 'filled-in' diagrams will give upper and lower bounds on the differences between the years.

#### 5.1.4 Analysis of Data: Distribution of Wave Height.

Fig. 5.3 shows the complementary cumulative distribution of the wave height  $H$  obtained from the seven sets of data by use of equation 5.9. They are plotted in the log-linear form, that is fig. 5.3 is a graph of  $H$  against  $\ln(1-F_H(H))$  so that if the assumed form is correct a straight line should result, thus:

$$\text{If } F_H(H) = 1 - \alpha e^{-H/\beta}$$

$$\text{then } H = \beta[\ln \alpha - \ln(1-F_H(H))] \quad (5.13)$$

Tables 5.2 and 5.3 give values of  $\alpha$  and  $\beta$  which are the results of linear regression analyses on these curves and values for the mean and standard deviation of  $\alpha$  and  $\beta$ , and  $\alpha\beta^m$  where  $m$  is taken as 3.5 (see section 5.5). These statistics are given for all seven sets of data taken together and also with the second set omitted. This year had very poor data recovery (see table 5.1) and can be seen in fig. 5.3 to have a markedly different distribution for  $H$ .

The regression analysis has to be carried out over a limited range of values of wave height; fig. 5.3 shows that for small values of  $H$  the curves are clearly non-linear and a lower limit was taken at 1.0m. The upper limit is more difficult to choose. For high values of  $H$  the value of  $F_H(H)$  is governed by the upper tail of the Rayleigh Distribution used for wave heights within sea-states and by whether or not the occasional extreme sea states were recorded, and it is therefore less directly based on the statistical evidence. Two alternative upper limits were used, 4.0m in table 5.2 and 6.0m in table 5.3 to test the sensitivity of the results to the choice of limit; these heights are exceeded by approximately 5% and 1% of the waves respectively. The results in table 5.5 show that the 4m upper limit gives a smaller range of results, which means that using this more limited range values as the basis of the regression analysis gives a more reliable procedure for estimating  $\alpha\beta^m$ .

It would be possible to weight the values of  $H$  to express their relative importance to the reliability analysis, but the calculations would then lose their generality. Such weighting is included in the method 2 ( $n'$ ) approach. Linear regression analysis was used because the maximum likelihood method does not work with this form of distribution.

Values for the coefficient of variation of  $n'$  and  $\alpha\beta^m$  are given in table 5.5  $n'$  is calculated from equation 5.2. In practice the value of the coefficient of variation that represents the random variability from year to year will be between that derived from the original data and that derived from the filled data. Assuming

no correlation between consecutive years the coefficient of variation for the random variability between 20 year periods will be a factor of  $\sqrt{20}$  less than that for individual years, i.e. between 0.043 (original data) and 0.014 ('filled' data).

If wave height data for both of two years follows equation 5.5 then the data for the sum of the two years does not have exactly this same form. It is not therefore possible to say exactly how the variability of  $\alpha$  and  $\beta$  between twenty year periods will relate to the variability between years. However since the coefficients of variation for  $\alpha\beta^m$  are similar to those for  $n'$  it does not seem likely that the choice between the different approaches will be of great importance in assessing the contribution of various forms of uncertainty to the overall reliability. It is thus concluded that the random variability of the wave climate contributes an uncertainty with a coefficient of variation of some two or three per cent for a typical required life of 20 years.

If one year's data of the quality of that obtained from m.v. Famita were used as the basis of a design, then the error in the value of  $n'$  or  $\alpha\beta^m$  used would have a coefficient of variation of about 20%. With better quality data this is reduced. In particular it is worth noting that if year 2 is excluded on the grounds that data with such a low recovery rate would not be used on its own as a basis for design, then the coefficient of variation is reduced to 16%. It can be seen from table 5.5 that with the 'filled' data the error could have a coefficient of variation of up to 6%, but this would of course be reduced by using more than one year's data. It is proposed that this statistical uncertainty in  $n'$  or  $\alpha\beta^m$  should be taken to have a coefficient of variation of either 5% or 20% to test the effect of different qualities of data.

#### 5.1.5 Analysis of Data: Persistence of Sea-states

For the reasons given earlier (see section 5.1.1), no general model exists to describe the uncertainty in the wave behaviour when the failure function is formulated on a sea-state basis. However a number of observations have been made during the

analysis of the available data and are noted below as contributions that may prove useful in creating models for particular circumstances.

If the sensitivity of the failure function to the sea-state is of a very simple form, then equation 5.3 shows how values of  $t'$  can be obtained from the data. The values found for each of the seven winters and for the original data and the 'filled' data are given in table 5.4. The coefficients of variation are 0.213 for the original data and 0.075 for the 'filled' data. These figures are slightly greater than the comparable figures for  $n'$  which are calculated in a similar though not identical, way. It appears that the Rayleigh distribution assumption incorporated in the  $n'$  calculation, and the further assumptions in the  $\alpha\beta^m$  calculation have the effects of smoothing out some of the irregularities in the scatter diagram. Interestingly this effect is more marked in the 'filled' data than in the original.

On any scatter diagram the apparent proportion of time spent in the  $k^{\text{th}}$  sea-state ( $t_k$ ) is found by dividing the number of records in the  $k^{\text{th}}$  sea-state by the total number of records on the diagram. Given a series of diagrams it is then possible to find the mean ( $\bar{t}$ ) and standard deviation ( $S_t$ ) for each sea-state. These values are tabulated in figs 5.4 and 5.5. It can be seen that generally  $S_t$  is larger where  $\bar{t}$  is large. However if the coefficients of variation  $S_t/\bar{t}$  are calculated (fig 5.6) these are largest where  $\bar{t}$  is small. Fig 5.7 tabulates values of  $S_t^2/\bar{t}$  and it can be seen that the variation of this quantity does not exhibit the same degree of regularity seen in figs 5.4 to 5.6. This suggests that a distribution to represent the variation of  $t_k$  from year to year should be one in which  $\sigma^2/m$  is a parameter. This is true for the binomial distribution and the gamma distribution. Neither of these however can represent a continuous variable limited to the range 0 to 1. The Beta distribution satisfies this requirement and if the mean is small  $\sigma^2/m$  is dependent on only one of the parameters. The distribution is given by

$$f_X(x) = \frac{\Gamma(R)}{\Gamma(r) \cdot \Gamma(R-r)} \cdot x^{r-1} \cdot (1-x)^{R-r-1} \quad (5.14)$$

$$\begin{aligned}
 m_X &= \frac{r}{R} \\
 \sigma_X^2 &= \frac{r(R-r)}{R^2(R+1)} \\
 \text{Therefore } \frac{\sigma_X^2}{m_X} &= \frac{R-r}{R(R+1)} \quad (5.15)
 \end{aligned}$$

But if  $m_X \ll 1$ , then  $r \ll R$

$$\text{so } \frac{\sigma_X^2}{m_X} \approx \frac{1}{R+1} \quad (5.16)$$

which suggests that the distributions of  $t_k$  might be represented by beta distributions which had the same  $R$  for all  $k$ , but different  $r$  for each  $k$ . Some theoretical justification can be given for this but it depends on a different interpretation of sea-states from that implied by scatter diagrams. Since values of  $H_s$  and  $T_z$  are determined from twelve minute records they can be regarded as properties of the sea at an instant of time, which will vary with time. A particular sea-state comes into existence when the instantaneous values of  $H_s$  and  $T_z$  cross the boundary in  $H_s - T_z$  space that defines that sea-state, and persists for as long as they remain within that space. In reality therefore both the number of occurrences of each sea-state per year, and the duration of each occurrence will be variables. The scatter diagram simplifies this by assuming that the duration of each recorded sea-state is the same (3 hours usually). The variation is therefore in the number of recorded occurrences only and any model based on this will deal in discrete numbers and will not lead readily to a model for a continuously variable  $t_k$ . If however the contrary simplification is made, that is the number of occurrences of each sea-state is assumed constant from year to year, but their durations are variable, and if the arrival of each new sea-state is assumed to be modelled by a Poisson process with mean inter-arrival time  $1/\lambda$ , then if there are  $n$  occurrences of sea-state  $k$  out of a total of  $N$ :

duration of  $n$  sea-states= $T$  which is gamma-distributed:  $G(n, \lambda)$

duration of  $N-n$  sea-states= $T_0 - T$  which is gamma-distributed:  $G(N-n, \lambda)$

now  $t = T/T_0$  = fraction of time spent in specified sea-state

$$\text{therefore } f_t(t) \cdot \delta t = P[ t < T/T_0 < t + \delta t ] \quad (5.17)$$

$$= \int_T P[t < T/T_0 < t + \delta t \mid T] \cdot P[T]$$

Now if  $t < T/T_0 < t + \delta t$

$$\text{then } \frac{1}{t} > \frac{T_0}{T} > \frac{1}{t + \delta t}$$

$$\text{For small } \delta t \quad \frac{1}{t + \delta t} \approx \frac{1}{t} \left[ 1 - \frac{\delta t}{t} \right],$$

$$\text{hence } \frac{T}{t} - T > T_0 - T > \frac{T}{t} - T - \frac{T}{t^2} \delta t \quad (5.18)$$

$$\text{therefore } f_t(t) dt = \int_0^{\infty} f_{T_0 - T} \left[ \frac{T}{t} - T \right] \cdot \frac{T}{t^2} \cdot dt \cdot f_T(T) \cdot dT \quad (5.19)$$

$$\text{Now } f_{T_0 - T}(x) = \frac{\lambda^{N-n} \cdot x^{N-n-1} \cdot e^{-\lambda x}}{\Gamma(N-n)} \quad (5.20)$$

$$\text{and } f_T(x) = \frac{\lambda^n \cdot x^{n-1} \cdot e^{-\lambda x}}{\Gamma(n)} \quad (5.21)$$

Substituting equations 5.20 and 5.21 into 5.19 gives

$$\begin{aligned} f_t(t) &= \frac{1}{t^2} \cdot \frac{\lambda^{N-n}}{\Gamma(N-n)} \cdot \left[ \frac{1-t}{t} \right]^{N-n-1} \cdot \frac{\lambda^n}{\Gamma(n)} \cdot \int_0^{\infty} T^{N-1} \cdot e^{-\lambda T/t} \cdot dT \\ &= \frac{1}{t^2} \cdot \frac{\lambda^{N-n}}{\Gamma(N-n)} \cdot \left[ \frac{1-t}{t} \right]^{N-n-1} \cdot \frac{\lambda^n}{\Gamma(n)} \cdot \Gamma(N) \cdot \left[ \frac{t}{\lambda} \right]^N \\ &= \frac{\Gamma(N)}{\Gamma(N-n) \cdot \Gamma(n)} \cdot (1-t)^{N-n-1} t^{n-1} \end{aligned} \quad (5.22)$$

i.e.  $t$  is beta distributed with  $R=N$ ,  $r=n$ ,  $m_t = \frac{n}{N}$

$$\text{and } \sigma^2_t = \frac{n(N-n)}{N^2(N+1)}$$

Because of the different interpretation of sea-states the quantities  $n$  and  $N$  cannot be derived directly from the data but should be chosen to give the best fit. If this is done with one value of  $N$  but a different  $n$  ( $=n_k$ ) for each variable,  $t_k$ , then the means and variances satisfy the following equations:

$$\sum_k m_{t_k} = \sum_k \frac{n_k}{N} = 1$$

$$\frac{\sigma^2_{t_k}}{m_{t_k}} = \frac{1}{N+1} \quad \text{which is the same for all } k$$

While this model is satisfactory as far as it goes, it does not of course include any assessment of the correlation of the  $t_k$  variables for adjacent sea-states. What is required is a joint probability distribution for all the  $t_k$  which has the beta distributions as the marginal distributions for each  $t_k$ ; no suitable form is known, and its use in practice would involve many difficulties which would only be justified if these variables were a major source of uncertainty.

## 5.2 Wave Loading Variables

### 5.2.1 Introduction

In earlier chapters two methods were outlined for specifying the waves that strike an offshore structure. This can be done directly by specifying the heights of individual waves, in which case the waves are assumed to approximate to regular sinusoidal waves, or indirectly by specifying the significant wave height and zero-crossing period of the prevailing sea-state, in which case the properties of the individual waves themselves are found by assuming that they can be modelled as a Gaussian process.

In the former case the principal variables that affect the calculation of the wave load, given the wave height, are the mass coefficient,  $C_m$ , the drag coefficient,  $C_D$ , and the relevant structural dimensions. The dimensions do not normally need to be modelled as probabilistic variables because the uncertainty about their values is small. But marine growth can produce a significant (but uncertain) increase in the effective diameter of tubular members and therefore needs to be modelled.

There will also be some uncertainty in the wave theory by which velocities and accelerations are derived from wave height, and in the use of Morison's Equation to calculate the total wave force. These uncertainties may however be included in the uncertainties found in experimental determinations of  $C_m$  and  $C_D$ ; this will depend on the exact nature of the tests undertaken and the measurements taken. In addition, the waves that are most significant in causing fatigue damage are comparatively modest in

size so that linear wave theory will not show the clear inaccuracies that are observed in calculations for lifetime maximum waves; and the inertia force is generally the major component of the total force so that errors in the Morison Equation model and errors caused by the non-linearity of the drag force are of limited importance. One or more of these considerations will apply in most of the simple cases for which calculations are to be made, so models have not been developed for these uncertainties.

When the sea-state is specified rather than the waves, another possible source of error and uncertainty is in the modelling of the wave spectrum for given significant wave height and zero-crossing period. This may be important in broad-banded spectra, however if the spectrum is narrow-banded the significant wave height and zero-crossing period are sufficient to define those aspects of the wave which are most important, namely the number and distribution of the heights of the wave crests and troughs. The effect of broad-banded spectra can be covered by the damage reduction factor proposed by Wirsching which will be discussed later (see section 5.7).

For present purposes therefore, models are required for the uncertainties in  $C_m$ ,  $C_D$  and marine growth.

For the hydrodynamic parameters  $C_m$  and  $C_D$  the major source of uncertainty is in the choice of values to use in design from the range of values found in tests. The true value of  $C_m$  (or  $C_D$ ) for a part of a structure in a particular wave or sea-state is a function of a large number of variables. Many of these such as the size and orientation of the members and various aspects of the wave (velocities, accelerations etc) are determined by the structural design and the specification of the waves or sea-states. Others, such as the roughness of the members are not determined by the design and are therefore true random variables. It is also likely that many of the differences observed between test results are caused by difficulties of measurement and systematic differences between different test methods. It is therefore the inadequacy of the data that causes most of the uncertainty in  $C_m$  and  $C_D$ , but

there is some random variability. It is also clear that the complexity of the situation, particularly with regard to the loading of members situated in water whose flow is disturbed by other members means that the lack of knowledge in this case is almost unavoidable.

Marine growth, however, is probably determined by factors which are largely independent of the actual design, except perhaps its location which governs sea temperatures and currents which may be important. An investigation of the origin of marine growth is beyond the scope of this thesis, but it seems reasonable to say that both physical variability and lack of knowledge will contribute to the overall uncertainty.

#### 5.2.2 Distribution Type

From hydrodynamic considerations it is clear that  $C_m > 1.0$  and  $C_D > 0.0$ . If the marine growth is expressed by a factor

$$\mu = \frac{\text{Effective diameter including marine growth}}{\text{Original diameter of steel member}} \quad (5.23)$$

then clearly  $\mu > 1.0$ . This suggests that a distribution type such as log-normal (with false origin for  $C_m$  and  $\mu$ ) should be used. It can also be argued that the effective values of these quantities as far as the overall structural behaviour is concerned are weighted averages of the values that apply to particular parts so that the central limit theorem would imply that normal distributions should be used. On balance log-normal distributions seem most appropriate but the arguments are far from conclusive. The available data itself is too limited in quantity to form an adequate basis for choosing the distribution type.

#### 5.2.3. Available Data

Since the aspects of offshore structures under consideration in this section are not unique to the analysis of fatigue resistance, the results of earlier work can be used. Baker and Ramachandran<sup>71</sup> have reviewed the available data and concluded that reasonable values for the mean and standard deviation respectively are 1.8

and 0.27 for  $C_m$  and 0.75 and 0.225 for  $C_D$ . These figures can be used to represent the predictive distribution including all the sources of uncertainty. As remarked above the random physical variability within the limits of a specified design is thought to be quite a small part of this, but no data has been found to indicate its magnitude. It is suggested that a standard deviation of 0.05 be used to test the sensitivity of the computed reliability to a much reduced uncertainty.

Baker and Ramachandran give mean of 0.16m and standard deviation of 0.06m for the thickness of marine growth at mean water level (where the largest wave forces occur), averaged throughout the structure's life, allowing for periodic cleaning. For the purposes of example calculations the structure legs may be taken as 3m diameter giving the ratio of effective diameter to nominal a mean of 1.11 and standard deviation of 0.04.

### 5.3 Variables affecting the Structural Analysis

As discussed in section 4.3.3, there are two stiffness variables that may need to be modelled in an offshore fatigue reliability analysis. These are the stiffnesses of the tubular joints and the foundations.

The importance of the tubular joint stiffness has been generally realised only recently in the offshore industry<sup>73</sup> and little work has been done to quantify this effect, although some has now been reported by Fessler and Spooner<sup>74</sup>. Some parametric formulae are available in the DnV rules<sup>75</sup> and the uncertainty arises from doubts about the accuracy of these formulae, or any other proposed formulae, to all the cases in which they are used. The large number of possible geometries for tubular intersections means that it is unlikely that test results, or accurate calculations, will be available for every case required, so practical formulae have to be based on generalisations from particular cases. The circumstances where the resulting uncertainty is most important is where a calculated hot-spot stress is particularly sensitive to the stiffness of one joint, and it will be less important where the stresses at a connection are largely due to the axial loads rather than the moments in a member.

In view of the comparative recentness of the appreciation of the significance of tubular joint stiffness, and the lack of work carried out on this subject, it is not possible at present to formulate a model for these stiffnesses with any degree of confidence. However, real tubular joints are less stiff than the joints usually assumed in computer analyses and this will generally mean that hot-spot stresses are less than calculated and hence that fatigue resistance is greater than calculated. Analyses which ignore the stiffness uncertainty will therefore tend to be conservative, in that real structures may have an additional (uncertain) margin of resistance beyond that modelled. For these reasons it has been thought better not to include the connection stiffness uncertainty in the present analyses.

At the design stage the uncertainty about the stiffness of the foundations of the completed structure will be considerable. Many of the most important sites for fatigue damage are concerned, however, with forces in bracing members and other members where the sensitivity of the fatigue resistance to foundation stiffness will often be small, particularly in dynamically insensitive structures. It is proposed therefore that soil stiffness should not be included in the examples discussed in this thesis, although it would clearly have to be included in some parts of a full reliability analysis of an offshore structure. In such cases the soil stiffness model would be derived from the available geotechnical data and such studies are beyond the scope of this thesis.

#### 5.4 Stress Concentration Factors

##### 5.4.1 General Introduction

The stress concentration factors (SCFs) used in design calculations are calculated using parametric equations such as those of Kuang<sup>49</sup> et al, Gibstein<sup>76</sup>, and Wordsworth and Smedley<sup>77,78</sup>. These are derived by generalising from the results of model tests or finite element calculations, and are periodically revised, and extended. Although any one such formula should be a good fit to the tests or calculations on which it is

based, the complexity of the situations under consideration makes it inevitable that the agreement with measurements on other models, or on real structures, will be less good. There is thus some uncertainty in the representation of the true stress concentration by the calculated SCF and it is this model uncertainty that requires a probabilistic model to represent it.

To be more precise about the origin of the uncertainty, it can be said that the true stress concentration should not vary much between measurements on a set of nominally identical cases. This is because the stress concentration is an elastic phenomenon and it should only vary in such circumstances as a result of the small differences in shapes, wall thickness and weld sizes which occur between nominally identical joints. The principal cause of uncertainty is the variation within the total population of possible geometries which the calculation models do not accurately represent. Differences in experimental technique may also contribute. This means that, in terms of the discussion in 2.6.6, distribution (i) has negligible scatter so that the distributions (ii) and (iii) are identical; it is this distribution that needs to be determined.

#### 5.4.2 Distribution Type

Since the uncertainty under consideration is not the result of a well-defined random process, there is no distribution type which is clearly more appropriate than any other. All that can be said is that SCFs are always positive numbers so that the ratio of Experimental SCF to Calculated SCF will also always be positive. In these circumstances a lognormal distribution can sensibly be used.

#### 5.4.3 Available Data

Most researchers working in this field have published figures and given opinions on the accuracy of the various parametric formulae. There are however considerable difficulties in using these comparisons as a basis for a probabilistic model uncertainty. The principal problems are:

(1) In comparisons made some years ago the stress concentration that was to be calculated was not well defined, and in an area of large stress gradients the position of strain gauges and the form of finite elements could make considerable differences to the results obtained. More recently, this difficulty has been reduced (at least within the UKOSRP work) by the use of the linear extrapolation technique<sup>79,81</sup>.

(2) Although the factors are intended to be stress concentration factors it is of course strains that are actually measured on models and some authors also give strain concentration factors (abbreviated SNCF). Generally however a clear distinction is not made between the two. Their relationship does of course depend on the other stresses present but it appears<sup>82</sup> that this is not always taken into consideration and systematic differences between different authors' comparisons may be due to different treatments of this problem.

(3) Some of the parametric formulae in use have been revised and extended from time to time, and so different comparisons are not therefore necessarily based on exactly the same formulae.

(4) The cases considered are generally determined by the test specimens available and there is no reason to believe that the relative proportions of different geometries in the comparisons are in any way typical of actual practice. In fact, virtually all the comparisons are on much simpler joints than are generally necessary in real structures.

In the light of these problems, all that can be done is to examine the data available to try to obtain some ideas (however vague) about the nature and order of magnitude of this model uncertainty; but the assessment of the data will inevitably have a large subjective element.

The results discussed below have not therefore been examined in great detail to see to what extent they are comparable with each other. In examining them most effort has been spent in looking at the scatter of the results. The variations in mean values are

likely to be the result of differences of approach and are not necessarily so significant. In any case errors in the mean values given by a formula are easily corrected, the scatter is intrinsic in the form of equation used.

Comparisons by Gibstein<sup>76</sup> show values for the ratio Experimental SCF/Calculated SCF falling within  $\pm 25\%$  of unity and he states: 'The middle average root square error in the axial load and OPB (out-of-plane bending) is barely 10% indicating very good agreement indeed ..... The less satisfactory agreement in IPB (in-plane bending) must be contributed (sic) to the sparse data base and its sensitivity to scatter.'

In contrast, later work by Gurney<sup>83</sup> on some of the results of the UKOSRP tests show a range of experimental SCF's from half to twice the value calculated by the Kuang formulae. Numerical values estimated from Gurney's figure 2 are given in table 5.6, with some calculated means and standard deviations. Some of this large range of results is due to the use of the Kuang formulae outside their proper range of applicability. Such results should not however be totally ignored since in the absence of other information designers may be forced to use the formulae in this way. The calculated standard deviations show how the scatter can be reduced by taking smaller, more homogeneous, sets of results.

This is also illustrated by the figures shown in tables 5.7, 5.8 and 5.9, where there are clear systematic differences between the ratios for the chord and brace, and it is possible to suspect the existence of other systematic trends.

Irvine<sup>79</sup> states that 'for the use of appropriate methods (i.e. parametric equations with parametric validity range or a particular stress analysis exercise (sic)) one would expect accuracy generally to within  $\pm 10\%$ , with a certainty that inaccuracies will not exceed 20%.' In view of the results quoted above this appears to be rather optimistic, even bearing in mind the 'appropriate methods' qualification. It would be more reasonable to say that if the errors arising from a proposed rule are generally less than 10% and always less than 20%, then the rule is considered acceptable. If they fall outside this then a

revision of the rule would be considered necessary. This seems a sensible view to adopt considering the scatters that still occur even in the more homogeneous sets of results.

#### 5.4.4 Proposed Model

It is appropriate, therefore, in reliability calculations to use two alternative models: one representing the 'acceptable' range which covers many cases already, and is the ultimate aim of the development of these formulae, and one representing the full range of results that may occur in practice, including the use of rules in strictly inadmissible cases. It will then be possible to compare the effects of these two models on the overall reliability calculations. It is suggested that standard deviations of 0.1 and 0.3 are appropriate to represent these two cases.

The logic behind the first case suggests that it should have a distribution limited to the range  $1.0 \pm 0.2$ . If a lognormal distribution is used the results of the calculation should be checked to ensure that the part of the distribution lying outside this range has not played a significant part in the answer. This can be done by checking the final  $x^*$  value for the SCF model uncertainty variable.

### 5.5 S-N Curve Variables

#### 5.5.1 Introduction

In earlier chapters, where an algebraic form for the S-N curve has been used, it has been of the form  $NS^m = K$ , where  $N$  is the number of cycles to failure caused by loading cycles with stress range  $S$ .  $m$  and  $K$  are empirical constants. This model is used (with Miner's Rule) as a basis for the assessment of fatigue life under variable amplitude loading. For the reasons discussed in section 5.6, this division does cause difficulty in the derivation of probabilistic models, but at present it is made necessary by the nature of the available data. The present section is concerned with the accuracy with which constant stress cycle fatigue behaviour can be modelled.

The main inaccuracy known to exist in the  $NS^m=K$  equation is that fatigue cracks do not grow at all, and hence no failure occurs, if the stress range is less than some threshold value. Most investigators have found however that the transition between stress ranges where cracks grow in accordance with this model (i.e.  $NS^m=K$ ) and those where no growth occurs at all is extremely abrupt, and hence the model is usually held to apply for all  $S$  above the threshold value. In any particular case, of course, the actual value of this threshold will be subject to considerable uncertainty. An attempt will be made in the data analysis to show that for tests which do give failure, the  $NS^m=K$  equation is an acceptable representation of the available test results for the fatigue lives of tubular joints.

When the constant amplitude S-N curve is to be used as part of a variable amplitude calculation, the actual threshold value of stress range is not of much interest as it is eroded by the crack-growth process as explained in section 4.4.1. It has been suggested<sup>53</sup> that an equation  $NS^{m+2}=K'$  should be used for  $S$  less than the constant amplitude threshold to cover this effect. In order to keep the calculations in this thesis comparatively simple and comprehensive this approach has not been used here and the form  $NS^m=K$  is assumed to apply for all  $S$ . Hence the value of the threshold stress range is not relevant and the parameters  $m$  and  $K$  are the only variables for which probabilistic models are required.

Since, by their origin,  $m$  and  $K$  are simply empirical constants, the possibility exists that they may both be dependent on the same fundamental properties of the connection being tested and hence the distributions which model the uncertainties in their values should be correlated. Establishing such correlations with confidence is difficult with test results obtained for purposes other than reliability analysis. However it is commonly found that  $m$  does not differ much over quite a wide variety of situations, so that one possible course of action is to use a single constant value of  $m$  in all cases and take  $K$  as the only uncertain quantity.

### 5.5.2 Distribution Type

The variables  $m$  and  $K$  are both positive quantities and there is some reason to believe that  $m \geq 2$ ; most, if not all, test results satisfy this, any exceptions being of doubtful validity or relating to unusual circumstances. In addition there are theoretical fracture mechanics arguments that support  $m=2$  and which may therefore model part of the crack growth mechanism<sup>59</sup>. This suggests that both  $m$  and  $K$  should be represented by a distribution such as the log-normal or Weibull with a false origin for  $m$ .

The suggestion of a log-normal distribution for  $K$  is given support by the argument that the S-N curve is normally displayed and analysed in terms of  $\log S$  and  $\log N$ , so that  $K$  will often feature in calculations as  $\log K$ , which will therefore be normally distributed. In addition, fracture mechanics calculations (see equation 4.25) suggest that  $K$  can be expressed as the product of a number of more fundamental variables, a process which would tend to make  $K$  lognormally distributed by the operation of the Central Limit Theorem.

If  $m$  is taken as a deterministic constant and  $K$  is lognormally distributed, then, for given stress range  $S$  the fatigue life is lognormally distributed. It has been stated (for example in ref 84), however, that lognormal distributions should not be used to model variations in fatigue life. The argument is based on the use of the hazard function  $h_X(x)$  defined by:

$$h_X(x) = \frac{f_X(x)}{1-F_X(x)} \quad (5.24)$$

For a lognormal distribution this function rises to a peak and then decreases as  $x$  increases as shown in fig. 5.8. The hazard function is interpreted when calculated from the distribution of fatigue life as

$$\begin{aligned} & \frac{P[\text{specimen will fail in } n^{\text{th}} \text{ cycle}]}{P[\text{specimen does not fail before } n^{\text{th}} \text{ cycle}]} \\ &= P[\text{specimen will fail in } n^{\text{th}} \text{ cycle} \text{ given that it does not fail before}] \end{aligned} \quad (5.25)$$

It is argued that the increasing damage due to fatigue means that this probability should always increase with increasing  $n$  and that the lognormal distribution, where this does not happen, 'violates our physical understanding of progressive deterioration resulting from the fatigue process'<sup>84</sup>. The Weibull distribution is preferred as its hazard function always increases with increasing  $n$ . It is also sometimes preferred because it arises from considerations of 'weakest link' failures, but this is not applicable where the location of the fatigue crack is as restricted as it is in the tubular connections used in offshore structures.

The hazard function is often used in the discussion of 'wearing-out' processes, and if the initial state is always the same and the uncertainty arises from random variations in the way in which the 'wearing-out' occurs, then it may be a useful tool since it is reasonable to say that the longer failure is delayed, the more one would expect it to occur within a given time or number of cycles. The evidence of fracture mechanics crack growth testing however is that fatigue crack growth under constant cycle testing is a fairly steady, predictable process and that the uncertainties are in the variables such as the parameters that govern this process and the initial crack size. If failure does not occur within a certain number of cycles from the start of the test some inferences can be made about the values of these variables, but this does not necessarily imply that the probability of failure in the next cycle increases with every cycle that does not cause failure. For example, given the possible existence of a threshold value of stress intensity factor range (see section 4.4.1), as a specimen survives for an increasing number of cycles, the suspicion will grow that this connection has been made extremely well with a good weld profile and no flaws, and that the fatigue life may be virtually unlimited. In such a case the log-normal distribution's hazard function actually seems better than that of the Weibull distribution.

The core of the argument is whether fatigue failure is seen as a random occurrence or as the culmination of an essentially deterministic process, albeit one whose governing parameters are

subject to great uncertainty. It should be noted that the arguments above in favour of dismissing the hazard function objections to the log-normal distribution are strongly influenced by the circumstances encountered in offshore structures. In other situations such as the initiation of fatigue cracks in smooth specimens or the propagation of cracks in specimens containing many flaws, such as ceramics, the hazard function argument may be more applicable and the Weibull distribution may be preferred to the log-normal.

### 5.5.3 Available Data

Recent testing programmes, in the United Kingdom and elsewhere, have produced a considerable amount of data on the fatigue resistance of offshore structures. Three aspects of the offshore situation make it different from those for which large quantities of test data already existed. These are the size and geometry of the tubular connections, the marine environment and the stress history produced by wave loading and dynamic response. Because of the difficulty involved in realistic testing of tubular joint specimens, these have generally only been tested with constant amplitude loading, in air, and only a limited range of the simpler joint geometries has been used. The effects of variable amplitudes and corrosion have been investigated using smaller simpler test-pieces. This section will be limited to a discussion of the determination of S-N curves for tubular geometries under constant amplitude loading in air. The effects of variable amplitude loading and corrosion will be discussed in later sections.

The aim behind the use of stress concentration factors in the design and analysis of tubular connections is that all the different geometries and loading arrangements should be served by a single S-N curve. It is clear from an examination of the data that while many of the differences may have been taken care of in this way there are still significant differences between different sets of results, where the members of each set are all nominally identical, except for stress range, but the sets differ in some significant way from each other. This is illustrated in fig 5.9

where there is a clear difference between the axial loading cases and in-plane bending cases for otherwise identical T-joint tests. It is therefore necessary to collect sets of results with identical geometry and loading arrangements if the true nature of the uncertainties present is to be understood.

The available data is given in table 5.11 with each set of nominally identical tests placed together. Ideally this would be taken to the extent of dividing the results into sets according to stress range as well as all the other aspects, and analysing each set independently so that the accuracy of the conventional S-N curve would be easily seen. However, few sets have more than two or three test results, and it is therefore necessary to assume the conventional form of S-N curve in the calculations. The sources of the data are given in table 5.10 and with the exception of tests where the specimen or loading was in some way unique, all the results of tubular connection tests published in refs 72 and 80 have been included. These references contain the most up-to-date results of the United Kingdom Offshore Steels Research Project (UKOSRP) and similar programmes in Italy, Holland and Norway, that are available at the time of writing.

These programmes have been coordinated and the results as presented are on a common basis. Instead of the stress range, the hot-spot strain range  $\Delta\epsilon$  is recorded, calculated by extrapolation from two strain gauge readings near the hot-spot as specified by refs 79 and 81. Four stages of 'failure' are recorded, although not all have been given for each test. Missing data are recorded as zero in the tables. The four stages of failure are given by<sup>85</sup>

- 1) 15% reduction in strain-gauge measurements
- 2) First crack observation
- 3) Crack through full thickness of tube wall
- 4) End of test (often governed by limitations of the testing machine).

The recorded lives  $N_1$ ,  $N_2$ ,  $N_3$ ,  $N_4$  are the number of cycles to each stage from the start of the test. All tests were continued to at least  $10^7$  cycles and specimens that remained unbroken are recorded

as such. Generally this implies that no cracking at all had occurred. Some of these unbroken specimens were retested under higher loads as indicated in the table, and it is suspected<sup>86</sup> that in at least one case (E2A) the results then obtained are unreliable because some cracking had occurred in the previous test.

#### 5.5.4 Analysis of Data

If the data is to be represented by an equation of the form  $N \Delta \epsilon^m = K$  where  $m$  is a constant and  $K$  is a log-normally distributed variable, and if

$$y = \log(N) \quad x = \log(\Delta \epsilon) \quad A = \log(K) \quad (5.26)$$

where all logarithms are to base 10, then

$$y + mx = A \quad (5.27)$$

and  $A$  is a normally distributed variable. The conventional method<sup>87</sup> for determining estimates for  $m$  and the mean and standard deviation of  $A$  is to minimise the sum of the squared differences between values of  $y$  given by  $y = A - mx$  for given  $x$ , and the experimentally determined value of  $y$  for the same  $x$ . If the data is represented by pairs of results ( $x_i = \log \Delta \epsilon_i$ ,  $y_i = \log N_i$ , for  $i = 1$  to  $n$ ), the total square deviation is given by

$$E = \sum_1 (y_i - (A - mx_i))^2 \quad (5.28)$$

The values of  $m$  and  $A$  are given by

$$\frac{\partial E}{\partial m} = 0 \quad \text{and} \quad \frac{\partial E}{\partial A} = 0 \quad (5.29)$$

$$\text{which gives } m = - \frac{\sum_1 x_i y_i - \frac{1}{n} \sum_1 x_i \cdot \sum_1 y_i}{\sum_1 x_i^2 - \frac{1}{n} (\sum_1 x_i)^2} \quad (5.30)$$

$$\text{and } A = \frac{1}{n} (\sum_1 y_i + m \sum_1 x_i) \quad (5.31)$$

For deterministic  $m$ ,  $A$  is taken as normally distributed with mean  $\bar{A}$  and standard deviation given by

$$\left[ \frac{E}{n-2} \right]^{\frac{1}{2}} \quad (5.32)$$

where  $(n-2)$  is used to give an unbiased estimate<sup>88</sup>. This means that the standard deviation is undefined unless there are at least three data pairs.

Table 5.12 gives the results of this analysis. For each set of results and for each stage of failure (where results are available) the table gives the calculated value of  $m$  and  $\log(K)$  (i.e.  $A$ ) that describe the mean regression line, and the estimated standard deviation. The latter measures the scatter of results about the mean regression line and because  $m$  is taken as deterministic is also the standard deviation for  $A$ . The values for  $\log(K)$  are compatible with  $\Delta\epsilon$  measured as microstrain not actual strain.

Some sets of data give values of  $m$  of the expected order of magnitude (between 2 and 4), but as should be expected with such limited data, some of the results are strongly influenced by the variability of fatigue life.

As a check on the validity of the  $N\Delta\epsilon^m=K$  model it is noted that if the true relationship is different it would appear as a curve in  $\log(N) - \log(\Delta\epsilon)$  space. One would therefore expect the experimentally determined values of  $m$  in table 5.12 to show some dependence on the values of  $\Delta\epsilon$  used in the tests from which each value of  $m$  was determined. Fig. 5.10 shows (for fatigue life  $N_3$ ) a plot of corresponding values of  $m$  and the average value of  $\log(\Delta\epsilon)$  for the tests in each set. As can be seen there is no noticeable correlation suggesting that any inaccuracy in the form  $N\Delta\epsilon^m=K$  is too small to be detected in data of the nature of that analysed here.

It is clear from table 5.12 that most of the available data cannot be usefully used in the way in which it is analysed in this table. The conventional way of obtaining more reliable results is to use more data in each regression analysis, by grouping sets of data for different cases together. In the analyses by Gurney<sup>83</sup> of the UKOSRP data, all the test results for each chord wall thickness are analysed together. There are therefore a large number of different cases taken together. The results of such an analysis can be misleading because they can be affected by the stress

ranges that happened to have been used for testing. An example of this is given in figs. 5.11 and 5.12, which show some results of an earlier analysis<sup>89</sup> of some UKOSRP data, consisting of tests on eight pairs of nominally identical specimens. Fig. 5.11 shows these results taken as a single set. Fig. 5.12 shows each pair of nominally identical specimens joined by a line; these lines are remarkably parallel but their slope is clearly different from that obtained by taking the results as a single set.

Data such as this clearly indicates that a preferable procedure is to assume that the same value of  $m$  will apply to every set of nominally identical cases so that only  $A$  is determined for each set individually. If this approach is invalid it should show by giving larger values for the standard deviation. The results of this approach are given in table 5.13; separate calculations were done for each stage of failure, on the following basis.

If there are data for  $J$  sets of results, indicated by  $j = 1$  to  $J$ , then  $J$  different regression lines are required of the form

$$y + mx = \bar{A}_j \quad (5.33)$$

and if for each  $j$  there are  $K_j$  data pairs  $(x_{jk}, y_{jk})$  then the deviation in  $y$  for the test  $(j,k)$  is given by

$$y_{jk} - (\bar{A}_j - m.x_{jk})$$

and the total squared deviation is given by

$$E = \sum_j \sum_k (y_{jk} - \bar{A}_j + m.x_{jk})^2 \quad (5.34)$$

The required values of  $m$  and  $\bar{A}_j$  is given by

$$\frac{\partial E}{\partial m} = 0 \text{ and } \frac{\partial E}{\partial \bar{A}_j} = 0 \text{ for all } j \quad (5.35)$$

$$\text{which lead to } m = - \frac{\sum_j (S_{XY_j} - S_{X_j} S_{Y_j} / K_j)}{\sum_j (S_{XX_j} - S_{X_j}^2 / K_j)} \quad (5.36)$$

$$\text{and } \bar{A}_j = \frac{1}{K_j} (m.S_{X_j} + S_{Y_j}) \quad (5.37)$$

$$\begin{aligned}
\text{where } S_{X_j} &= \sum_k x_{jk} & S_{Y_j} &= \sum_k y_{jk} \\
S_{XX_j} &= \sum_k x_{jk}^2 & S_{XY_j} &= \sum_k x_{jk} \cdot y_{jk}
\end{aligned} \tag{5.38}$$

and the standard deviation for  $A_j$  is then given by

$$\left[ \frac{1}{(K_j-1)} \sum_k (y_{jk} - \bar{A}_j + mx_{jk})^2 \right]^{\frac{1}{2}} \tag{5.39}$$

As can be seen from table 5.13 this procedure gives very similar values of  $m$  for each definition of failure. The standard deviations are generally of the same order as before, this similarity being assisted by the use of  $(K_j-1)$  in place of  $(K_j-2)$  used before, which tends to offset the increase in error caused by the use of a less than optimum value of  $m$ . The justification for using  $(K_j-1)$  is that  $m$  is determined by a very large set of results so that in the choice of  $A_j$  and the standard deviation there is effectively only one degree of freedom where there were two before. Only two cases show standard deviations larger than the largest for the same definition of failure in table 5.12. This confirms that the assumption of a single value of  $m$  throughout is not inconsistent with the available data. The four values of  $m$  range from 3.30 to 3.73 and  $N_3$ , which is the least ambiguous definition of failure, comes in the middle of the range at 3.51. It is therefore proposed that 3.5 be taken when a single deterministic value of  $m$  is required for use in calculations.

If it is further assumed that the standard deviations of the populations from which the data are sampled are all identical, the best estimate of these is given by

$$\left[ \frac{1}{\sum_j K_j - 1} \sum_{jk} (y_{jk} - \bar{A}_j + mx_{jk})^2 \right]^{\frac{1}{2}} \tag{5.40}$$

The results of such calculations are given in table 5.14.  $N_1$  was only recorded in a few cases so this value of standard deviation should not be considered particularly reliable. The value for  $N_2$  being larger than those for  $N_3$  and  $N_4$  is probably influenced by the definition of  $N_2$  (first crack observation) which clearly is not very precise. It is suggested therefore that the standard deviation of  $\log K$  representing the random physical variability

within a specified geometry and loading should be taken as 0.22, on the basis of the values for  $N_3$  and  $N_4$ .

The calculated values of  $\bar{A}$  vary considerably and table 5.15 gives statistics for these variations. Each value is equally weighted initially, but results are also given with certain outlying values omitted. With this modification all the calculated standard deviations are of the order of 0.45 which suggests that this is a reasonable value to use for the uncertainty about which mean line to use when no data is available for the required case.

If  $A$  is normally distributed about its mean with standard deviation 0.22 and if the mean is normally distributed with standard deviation 0.45, then the overall predictive distribution of  $A$  is also normal with standard deviation given by

$$\sqrt{(0.45^2 + 0.22^2)} = 0.5$$

Standard deviations of 0.22, and 0.5 for  $\log_{10}K$  correspond to coefficients of variation of 0.54 and 1.66 for  $K$ .

It is important to note that this two stage procedure for obtaining the standard deviation for a mixed population of cases will give a larger standard deviation than is obtained by treating the results as a single set, as in fig. 5.11 for example. This is particularly important if the standard deviation is to be used in the derivation of a design curve, because taking a mixed population as a single set will give a design curve which is actually less conservative than was intended.

An example of this is the proposed 'T' curve<sup>90</sup> which is intended to be mean minus two standard deviations of  $\log N$  for fatigue life  $N_3$ . It uses an index  $m=3$  and gives a life of  $1.8 \times 10^5$  cycles at 200 N/mm<sup>2</sup> and  $7.5 \times 10^6$  cycles at 60 N/mm<sup>2</sup>. Comparing these lives with the model proposed in this section and table 5.15 for the predictive distribution of  $N_3$ , shows that the 'T' curve lies at mean minus 1.5 and 2.0 standard deviations at 200 N/mm<sup>2</sup> and 60 N/mm<sup>2</sup> respectively. The implications of this will be considered further in chapter 6.

## 5.6 Miner's Rule Uncertainty

Miner's Rule is the method by which test results for constant amplitude fatigue tests are used to predict the life of structures subjected to variable amplitude loading. The main justification for this rule is that it is intuitively reasonable and generally seems to work well in practice. The main exception to this is where precisely prescribed sequences of loads are used including a very small number of heavy overloads, and considerable errors in life calculations have been found. The essentially random nature of wave loading does not have such an effect but there is a serious problem in determining what the actual magnitude of errors is. This arises from the variability found above in the S-N curve for a particular connection. It has been shown (see 5.5.4) that in constant amplitude tests  $\log_{10}K$  has a standard deviation of around 0.22 due to the random variability of those physical features of a tubular connection that are not controlled exactly in the normal fabrication process.

The ideal experimental procedure would be to test a series of connections under various load sequences and calculate the quantity

$$\frac{\text{Fatigue Life in test}}{\text{Calculated fatigue life using Miner's Rule}}$$

for each test to obtain data on uncertainty in Miner's Rule. However the calculation also requires an S-N curve and of course it is impossible to have constant amplitude test data for an exactly identical connection. The best that can be done is to use the mean regression line obtained from tests on nominally identical connections, but this means that the variability in the true constant amplitude behaviour will be included in the assessment of Miner's Rule. This means of course that the variations observed in variable amplitude tests should be greater than those observed in constant amplitude tests. Wirsching<sup>3</sup> has extensively investigated the uncertainty in Miner's Rule on this basis, and with offshore applications in mind, and has found cases in which the variable amplitude tests show less variation than constant amplitude tests on similar specimens, implying that the

S-N curve uncertainty should be reduced when being used with Miner's Rule, and that there is no additional uncertainty associated with Miner's Rule. However he does not consider the evidence for this to be conclusive, and on the basis of the available evidence he suggests that the Miner's Rule model uncertainty should be log-normally distributed with median 1.15 and coefficient of variation 0.3; these figures are however to be considered preliminary. In the examples in chapter 6 the effect of this assumption and also of taking Miner's Rule as correct will be investigated.

Within the UKOSRP program and similar European programmes, tests using variable and constant amplitude loadings have been compared, and assessments of the accuracy of Miner's Rule have been made. The conclusions are summarised in table 5.16. As can be seen the implication is that the mean Miner's Rule total at failure should be less than unity, in contrast to Wirsching's figure, but the range of results suggests that his coefficient of variation of 0.3 may be satisfactory. It is difficult to draw more precise conclusions because the tests themselves and the analysis of their results were not carried out with reliability analysis in mind. In particular, the results are generally presented as an attempt to correlate fatigue life with the r.m.s. stress range, and to establish that the use of Miner's Rule with an appropriate design S-N curve (not necessarily established from constant amplitude tests on similar specimens) gives a safe estimate of fatigue life. The appraisals of Miner's Rule in table 5.16 are not described in detail in the cited reference, no attempt is made to compare the variability observed in constant amplitude and variable amplitude tests. These results do not therefore seem to add any reliable useful information to that given by Wirsching.

### 5.7 Damage Reduction Factor

The damage reduction factor ( $\lambda$ ) proposed by Wirsching and Light<sup>4</sup> is defined as follows

$$\lambda = \frac{\text{Fatigue damage due to specified loading process}}{\text{Fatigue damage due to equivalent narrow-band process}} \quad (5.41)$$

The equivalent narrow-band process has the same r.m.s. value and zero-crossing rate as the actual loading process. In terms of the moments of their spectra, this means that  $m_0$  and  $m_2$  (as defined by equation 4.11) are the same for both processes. Cartwright and Longuet-Higgins<sup>44</sup> have shown that the distribution of peaks and troughs depends on  $m_0$ ,  $m_2$  and  $m_4$  only so there is only one effective degree of freedom in the broad-band process (as far as peak and trough heights are concerned) once  $m_0$  and  $m_2$  have been defined. There is no general method for calculating the stress ranges required for fatigue analysis (e.g. as specified by the rain flow algorithms) but there is no reason to believe that the same three moments alone are sufficient to describe the distribution of stress-ranges. Nevertheless it is reasonable to expect that for given  $m_0$  and  $m_2$ ,  $m_4$  will be the most useful other quantity to know when trying to predict  $\lambda$ . It will be more convenient to discuss this as the dependence of  $\lambda$  on the band-width parameter  $\epsilon$ , or the irregularity factor  $\alpha$ , given by

$$\epsilon = (1 - \alpha^2)^{\frac{1}{2}} = \left[ \frac{m_0 m_4 - m_2^2}{m_0 m_4} \right]^{1/2} \quad (5.42)$$

Given the distribution of stress-ranges the damage depends also on the S-N curve. If threshold effects and mean stress effects are of limited significance the most important feature of the S-N curve will be the index  $m$ . Hence the best strategy is to investigate the dependence of  $\lambda$  on  $m$  and  $\epsilon$  for a wide variety of cases in the hope that knowing these two will enable  $\lambda$  to be predicted to a reasonable degree of accuracy. Wirsching and Light<sup>4</sup> report the results of such a study which assumed the rainflow algorithm to give a correct measure of damage for a sequence of cycles generated from a specified spectrum. The results show that  $\lambda$  falls rapidly from its maximum of 1.0 as  $\epsilon$  increases, but that for  $\epsilon$  greater than 0.4 or so,  $\lambda$  is largely independent of  $\epsilon$ . For  $\epsilon > 0.5$  and  $m$  around 3.5 values of  $\lambda$  fall mainly within the range 0.75 to 0.85. The mean value is about 0.8 and the standard deviations about 0.025 (see ref 4 fig 4). Since  $\lambda$  is a continuous variable in the range  $0 \leq \lambda \leq 1$  a Beta distribution might be most suitable; it would only be possible to speak of a distribution being 'correct' if there was a precisely defined population of broad-band spectra to consider.

Although some of this observed variation may be the result of differences between actual realisations of the same random process, most of it is the result of 'losing' the full description of the spectrum and using a few moments only in the calculations. The distribution of the variable  $\lambda$  therefore represents lack of knowledge; a lack which in this case (unlike most others) could be avoided if more elaborate calculation procedures were adopted.

Although it would be possible for designers to use a deterministic value of  $\lambda$  less than unity, the general impression given by published accounts is that the narrow-band assumption is commonly made so the use of  $\lambda$  in reliability analysis will show an additional margin of safety beyond that envisaged by the designer.

#### 5.8 Corrosion Effects

The various testing programmes on the effect on fatigue behaviour of the corrosive marine environment have shown that the effects of this environment can be very complicated and that they depend on a large number of factors. It has not proved simple to create an environment in the laboratory which represents the real conditions of offshore structures as far as all the possibly relevant aspects of the physics, chemistry and biology of sea-water are concerned. Some of the conclusions reached by investigators are as follows:

- (1) For constant amplitude tests at stress ratios  $R=0$  and  $R=-1$ , testing in sea-water with total or intermittent immersion, and with or without cathodic protection does not give results significantly different from tests in air, except that there is less evidence of a threshold stress range for crack growth<sup>91</sup>.
- (2) At higher values of  $R$  (0.7 and 0.85) crack growth rates can be up to 6 times higher than at lower  $R$  values in freely corroding conditions. In air the  $R$  value has little effect. Cathodic protection modifies this behaviour to a limited extent<sup>92</sup>.

- (3) Crack growth rates are very sensitive to temperature; a small increase in temperature can give a large increase in crack growth rate. This may mean that cracks near the water line or near hot risers are more vulnerable to fatigue than otherwise expected<sup>92</sup>.
- (4) Under narrow-band loading, fatigue life is reduced by up to 40% by freely corroding conditions. In some, but not all, cases cathodic protection improves the fatigue life<sup>93</sup>.
- (5) Pauses in testing allow cracks to be blunted by corrosion, thus prolonging the fatigue life. Calm spells may therefore be beneficial to real structures<sup>94</sup>.

It is difficult to draw general conclusions from such studies at this stage. It seems likely that corrosive conditions will affect the fatigue resistance of real structures, but really applicable research results do not seem to be available yet. The effect will probably depend on the corrosion protection measures employed, on the location of the connection within the structure, and perhaps on the geographical location of the platform. But it seems possible that much of the variation in fatigue behaviour observed in the laboratory is the result of the particular conditions employed in each test and that there is much less variation between real structures or parts of structures. To test the sensitivity of reliability calculations to effects of the order of magnitude found for narrow-band loading (see (4) above) a factor could be applied to the calculated fatigue life with, say, a mean of 0.8 and a standard deviation of 0.15.

## CHAPTER 6

### EXAMPLES AND CALCULATIONS

#### 6.1 Failure of Single Connections

##### 6.1.1 Introduction

Chapter 1 discussed the situations in which the making of decisions on matters concerning fatigue resistance would be assisted by the use of reliability analysis. In this chapter some calculations are discussed which illustrate these uses and show the results that arise in typical circumstances from the data presented in chapter 5.

The first type of calculation is for the probability of failure of a single connection. To give these calculations the greatest generality they are presented in the ratio format discussed in section 4.3.5. The failure function used is based on that given in equation 4.8, modified to include all the uncertainties discussed in chapter 5. This function is used to calculate the probability that the actual life will be less than some required life, and the ratio format can be used to allow the calculations to represent any well-defined stage of the crack growth process as 'failure'. In this approach it is necessary to use the same deterministic value of  $m$ , the S-N curve index, for both the design S-N curve and the probabilistic model of fatigue resistance under constant cycle loading. The failure function takes the form:

$$Z = \log \left[ \frac{\lambda_C^R M^R K^R \gamma}{\lambda_w^R n^R ((SCF)^R C_m^R (r^R)^2)^m} \right] \quad (6.1)$$

where the superscript R indicates that the variable is the ratio of real value to the value used in design calculations, and where:

$m, K$  are the S-N curve parameters ( $m$  taken as deterministic)

$\gamma$  is the safety factor, i.e. the ratio of fatigue life (in years) as calculated in deterministic design calculations to the minimum required life.

$\lambda_w$  Is Wirsching's damage reduction factor, allowing for the effect of broad-band spectra.

$M$  is the Miner's Rule Sum at failure and is used here only to cover uncertainty in the relationship between variable amplitude and constant amplitude tests in air.

$\lambda_c$  is a factor to allow for the reduction in Miner's Rule Sum at failure due to corrosion effects.

$n'$  is the effect of uncertainty in the wave history

(SCF) is the stress concentration factor

$C_m$  is the inertia (or mass) coefficient

$r$  is the effective radius, allowing for effects of marine growth.

#### 6.1.2 States of Uncertainty

It is clear from the discussion in chapter 5 that the uncertainties present in fatigue calculations originate from many different sources, and that, depending on the problem under consideration, different values should be given to the parameters of the distributions. The number of possible combinations of uncertainties that might be made is therefore impracticably large. To allow some useful results to be presented, three 'states of uncertainty' have been defined representing three different situations in which a reliability analysis might be carried out. In addition, in considering the description of S-N curve data two cases, termed 'worst' and 'average' have been defined. The effects of Miner's Rule uncertainty and corrosion are excluded from the uncertainty classification and are tested separately, because of the doubts discussed in chapter 5 about the nature and role of these uncertainties.

**Uncertainty State 1:** This is the narrowest definition and includes only the random variability that is completely beyond the

designer's control. It is assumed that testing on models (or even on the real structure) has enabled the forces generated by given waves and the stress concentration factors to be determined exactly, and that simulation and rainflow calculations have enabled accurate cycle counts to be made for representative stress histories. This leaves only the variation of S-N curve parameter  $K$  between nominally identical connections, the uncertainty caused by marine growth, and the uncertainty in  $n'$  which expresses the difficulty in predicting future weather exactly, however much data on past weather is available.

Uncertainty State 2: This is a wider definition of uncertainty and includes, in addition to the items in state 1, those aspects of lack of knowledge which seem unavoidable in a realistic design office situation. Thus there is some uncertainty about  $C_m$  and the stress concentration factor and more uncertainty than before in  $n'$ . Nevertheless it is assumed that some data for  $C_m$ , SCF and  $K$  are available that relates closely to the situation under consideration.

Uncertainty State 3: This is the broadest of all and includes all the uncertainties suggested in chapter 5. It therefore covers the case of a connection designed on purely nominal values where there is no appropriate data available.

### 6.1.3 Nominal Design Values

In order to use the ratio format outlined above, deterministic design values are required for each variable. In uncertainty states 1 and 2, values for certain of the variables are assumed to be known exactly so that there is no uncertainty and the true value is used for design. Where there is uncertainty, represented by the probabilistic models found in chapter 5, a deterministic design value has to be specified. Those used for  $K$  will be discussed in section 6.1.4.

For the wave history factor  $n'$ , the effective radius (including marine growth) and the stress concentration factor, the mean values were taken as design values. This is based on the

Addendum to page 146

The data analysis in section 5.5.4 leads to a value for the standard deviation of  $\log_{10}K$  of 0.22 for the variation within a set of nominally identical cases, so this value is used for uncertainty states 1 and 2, and 0.50 for the overall uncertainty used in uncertainty state 3.

For uncertainty state 3 and for the 'average' case in states 1 and 2, the difference between the mean and design values of  $\log_{10}K$  is two of the larger (0.50) standard deviations. For the 'worst' case in states 1 and 2 this difference is two of the smaller (0.22) standard deviations. Hence in state 3 and in the 'worst' case of states 1 and 2, the design value of  $\log_{10}K$  is at mean minus two standard deviations of the distribution used for  $\log_{10}K$  in the analysis. In the 'average' case of states 1 and 2, the design value is at mean minus  $2 \times 0.5/0.22 = 4.545$  standard deviations.

assumption that deliberately conservative estimates are not used for these quantities. By contrast  $C_m$  was taken as 2.0 which is a high value when compared with the available data. It is the theoretical value found in potential flow calculations; it is widely used as a result, and is specified in the DnV rules<sup>36</sup>. It was also assumed that Miner's Rule was used without any modification in the calculations, so that design values for  $M$  and  $\lambda_c$  are 1.0. The DnV rules specify a number of different values to be used for Miner's Sum at failure in particular circumstances, but this is assumed to be covered here by the safety factor  $\gamma$ . Finally it was assumed that design calculations would be made using the narrow-band assumption, so that the deterministic design value for  $\lambda_w$  is 1.0.

#### 6.1.4 'Worst' and 'Average' Cases

The principal restriction imposed by the ratio format is that the design S-N curve and the probabilistic model describing the results of constant cycle tests must have the same deterministic value of  $m$ . This means that in the traditional log S - log N plot the design curve is parallel to the line indicating the mean value of log N for each S and since the standard deviation of log N (i.e. of log K) has been assumed constant, then the design curve falls at the same fractile of log N for all values of S. A common aim of those responsible for producing design curves is that they should fall at mean minus two standard deviations.

In uncertainty states 1 and 2 the standard deviation used represents only the variation found within a set of nominally identical cases, whereas in state 3, it represents the overall uncertainty in the fatigue resistance of a tubular joint for which no specific data is available.

In state 3 the design curve is taken at mean minus two standard deviations (of log N). For states 1 and 2 two cases are defined: an 'average' case where the mean is the same as in state 3, and a 'worst' case where the mean is just two of the (smaller) standard deviations from the design curve.

One advantage of this approach is that it is not necessary to define which stage of 'failure' is to be used as the basis of design, provided the same stage is used in all parts of the calculations.

In mathematical terms, the design value of  $K$ ,  $K_{des}$ , is given by

$$\ln K_{des} = m_{\ln K} - 2\sigma_{\ln K} \quad (6.2)$$

where  $m_{\ln K}$ ,  $\sigma_{\ln K}$  are the mean and standard deviation respectively of  $\ln K$ .

The mean value of  $K^R = K/K_{des}$  is given by

$$m_{K^R} = \frac{m_K}{K_{des}} = \frac{\exp(\frac{1}{2}\sigma_{\ln K}^2 + m_{\ln K})}{K_{des}} \quad (6.3)$$

$$= \exp\left[\frac{1}{2}\sigma_{\ln K}^2 + m_{\ln K} - (m_{\ln K} - 2\sigma_{\ln K})\right]$$

$$= \exp\left[\frac{1}{2}\sigma_{\ln K}^2 + 2\sigma_{\ln K}\right]$$

$$= 3.13 \quad \text{if} \quad \sigma_{\log_{10} K} = 0.22$$

$$= 19.40 \quad \text{if} \quad \sigma_{\log_{10} K} = 0.50$$

$$V_K = (\exp(\sigma_{\ln K}^2) - 1)^{\frac{1}{2}} \quad (6.4)$$

$$= 0.54 \quad \text{if} \quad \sigma_{\log_{10} K} = 0.22$$

$$= 1.66 \quad \text{if} \quad \sigma_{\log_{10} K} = 0.50$$

The coefficient of variation  $V_K$  is unaffected by the use of ratio forms for  $K$ , so the standard deviations are:

for Uncertainty State 3,  $\sigma = 1.66 \times 19.40 = 32.21$

for Uncertainty States 1 and 2, 'worst' case,  $\sigma = 0.54 \times 3.13 = 1.69$

" " " " " 'average' case,  $\sigma = 0.54 \times 19.40 = 10.48$

#### 6.1.5 Results of Calculations

Using the data of table 6.1, the method of section 2.7 and the failure function of equation 6.1, gives values for the probability

of failure of a single connection as shown in table 6.2. The corresponding values of  $\beta$  (see equation 2.44) are in table 6.3. Values of  $\alpha$ , the sensitivity factor (normalised values of  $a_i$ ) are given in table 6.4.

It is clear from table 6.2 that the Miner's Rule uncertainty has little impact on the reliability, confirming Wirsching's conclusions<sup>2</sup>. The corrosion effect is more important, increasing the probability of failure by a factor of the order of 3 or 4 in most cases. This is not however a major effect by comparison with some of the other effects noted here.

The most important variable is  $K$ , which has the largest coefficient of variation, but in some cases some of the loading variables are also important. This is because, although their coefficients of variation are small compared to that of  $K$ , their effect is increased considerably when the variable is raised to the  $m^{\text{th}}$  power.

Many of the probabilities of failure are quite high by normal structural standards. This is because the design values of some of the variables were not taken at low fractiles of the distribution and because the large scatter of  $K$  and the nature of the failure function mean that a safety factor of 2 on life is not large in its effects. It is equivalent to a factor of only  $2^{1/3.5}$  ( $\approx 1.2$ ) on stress. In practice many of the joints on a real structure will have very long design lives because the design is determined by other limit states. It is also clear that using the lower uncertainty levels, only cases close to the 'worst' case will have these high probabilities of failure.

The effect of changes in uncertainty state and safety factor are illustrated in fig. 6.1 The lack of a simple consistent pattern of change between the three uncertainty states is because the changes from state 1 to 2 and from 2 to 3 include not only the addition of further uncertainty into the calculations, but also the introduction of variables with conservative design values. For instance, in state 1  $C_m$  is taken as being known exactly, whereas in state 2 the design is 4 standard deviations above the mean value.

Such biased quantities offset the increasing standard deviations and cause the probabilities of failure to be much more consistent (for the 'worst' cases at least) than might have been expected.

There is clearly an arbitrary element in the choice of the design values, although it is believed that they represent the sort of approximations and assumptions used in design. A wider range of results would have been obtained using other design values.

The general conclusion to be drawn from these results is that reliability levels in these types of connection can be quite low because of the large uncertainties in many aspects of the designs, although there are good reasons why many connections will in practice have much higher reliabilities. It must be remembered that 'failure' here is defined as exceeding the stage of crack growth used to define the design S-N curve, and may be only a fraction of the life to total failure.

As remarked above the principal limitation imposed on the calculations by using the ratio formulation is that the same deterministic value of  $m$  has to be used for the design S-N curve and the probabilistic model. There are two important situations in which a different formulation would be necessary. The first is where the design curve has a different value of  $m$  from the probabilistic model, and the second is where both the design curve and the probabilistic model use a higher value of  $m$  for lower stress ranges to allow for the threshold effect. It is possible to obtain some insight into these cases by considering the results obtained in the ratio formulation calculations.

If different values of  $m$  are used in the S-N curve and probabilistic model then this means that the design curve does not fail at a consistent fractile in the distribution of  $N$ . The result obtained in any particular calculation will depend on how close the design curve comes to the mean life over the range of values of  $S$  which play a significant role in the calculations. In section 5.5.4 it was shown that the proposed 'T' curve lies within two standard deviations of the mean value using the probabilistic model for uncertainty state 3. It follows that this curve will give lower reliabilities than those given here.

The argument that a higher value of  $m$  should be used below the stress range threshold for crack growth in constant cycle tests applies to the derivation of both the probabilistic model and the design curve. If the relationship between these two is the same for stress ranges above and below the threshold, the change in  $m$  will only have a limited effect on the results of the calculations; specifically it will increase the sensitivity to variables such as stress concentration factor which are raised to the  $m^{\text{th}}$  power in the calculations, at the expense of the other variables.

## 6.2 Failure of more than one connection

### 6.2.1 Introduction

So far the calculations in this chapter have been concerned with the failure of a single welded connection, but the overall safety of a structure depends on the integrity of a large number of connections whose fatigue resistances are unlikely to be wholly independent. As discussed in chapter 1, because total fatigue failure is preceded by extensive and often visible crack growth it is possible to discuss the correlation of the early stages of 'failure' in different connections without having to consider changes in overall structural behaviour which follow total failure of a connection. In these circumstances the failure function for a particular connection does not depend on whether or not other failures have occurred and it is only the degree of correlation between the basic variables that is relevant to these early stages of failure.

Some of the calculations which might be considered useful would be extremely complicated to carry out, but to establish the orders of magnitude of the effects of correlation between the variables that govern the lives of various connections, some simple calculations can be made. It is clear from the discussion in chapter 2 that the two probabilities which are easiest to find, given a set of  $n$  similar connection, are the probability of all  $n$  failing, and the probability of at least one failing.

### 6.2.2 Degree of Independence

The connections are all assumed to be described by the same failure function, representing some early stage of failure, so the difference between the connections is only in the values of the basic variables. The extreme, purely theoretical, case is where the behaviour and resistances of the connections are totally correlated, so that the values of the basic variables are identical for all the connections. In these cases the connections would all fail simultaneously, and hence the probabilities that all fail and that at least one fails will equal the probability that one single connection fails.

In practice there must be a degree of independence between different connections, even if they are nominally identical. For the sake of example three degrees of independence have been defined as follows.

'Slight' Independence: In this case the  $n$  cases under consideration are assumed to be nominally identical connections on the same structure and situated so that the loads on the connections all have the same origins (e.g. exactly the same value of  $C_m$  applies to all the connections). They are therefore independent only to the extent of the variation in  $K$  between nominally identical cases. The uncertainties are all taken at the values given in table 6.1 for Uncertainty Level 3 with a safety factor,  $\gamma$ , of 2.0, except that  $K$  is expressed as  $K_1.K_2$  where  $K_1$  represents the variation within a set of nominally identical cases and  $K_2$  represents the variation between different cases.

$$\text{If } K = K_1.K_2 \quad (6.5)$$

$$\text{then } \ln K = \ln K_1 + \ln K_2 \quad (6.6)$$

$$m \ln K = m \ln K_1 + m \ln K_2 \quad (6.7)$$

$$\text{and } \sigma_{\ln K}^2 = \sigma_{\ln K_1}^2 + \sigma_{\ln K_2}^2 \quad (6.8)$$

If  $K_1$  is the variation within a set of nominally identical cases, superimposed on the variation between cases, then

$$m_{\ln K_1} = 0$$

$$\text{and } m_{\ln K} = m_{\ln K_2}$$

From the data in chapter 5,

$$\sigma_{\log_{10} K_1} = 0.22, \sigma_{\log_{10} K_2} = 0.45$$

hence  $V_{K_1} = 0.54$  and  $V_{K_2} = 1.38$

$$m_{K_1} = \exp \left[ \frac{1}{2} \sigma_{\ln K_1}^2 + m_{\ln K_1} \right] = 1.137$$

$$\sigma_{K_1} = 1.137 \times 0.54 = 0.614$$

$K_2$  is to be expressed in the ratio form, so that mean value of  $K_2$  as a ratio is given by:

$$\frac{m_{K_2}}{K_{des}} = \frac{\exp(\frac{1}{2} \sigma_{\ln K_2}^2 + m_{\ln K_2})}{K_{des}} \quad (6.9)$$

but  $\ln K_{des} = m_{\ln K} - 2\sigma_{\ln K}$  as before (equation 6.2)

$$\begin{aligned} \text{hence } \frac{m_{K_2}}{K_{des}} &= \exp \left[ \frac{1}{2} \sigma_{\ln K_2}^2 + 2(\sigma_{\ln K_1}^2 + \sigma_{\ln K_2}^2)^{\frac{1}{2}} \right] \quad (6.10) \\ &= 17.066 \end{aligned}$$

$$\frac{\sigma_{K_2}}{K_{des}} = 17.066 \times 1.38 = 23.60$$

Since only  $K_1$  takes independent values in each case the same Level 2 method can be used as in section 6.1 but with the distributions for  $K_1$  modified in accordance with section 2.9.2.

'Considerable' Independence: At this level it is assumed that the variables  $K$  and the stress concentration factor, which are the two variables related specifically to the design of the individual connections, take independent values in each case, but that the other variables, which are more concerned with the whole structure and its history, are taken with the same values for each of the  $n$  cases. Because two variables are now assumed to take independent values, one of the new algorithms given in chapter 2 is required. Because it is possible to separate the failure function into two parts, thus (omitting the  $R$  superscripts in equation 6.1):

$$Z = \log \left[ \frac{\lambda_c M x_I \gamma}{\lambda_w n' (C_m r^2)^m} \right] \quad (6.11)$$

$$Z_A=Z_B= \dots = \frac{K}{(SCF)^m} - x_I \quad (6.12)$$

which reduce to the correct failure functions when  $Z_A = Z_B = \dots = 0$ , the algorithm of section 2.9.3 may be used. The following form was found to be computationally easier:

$$Z = \log \left[ \frac{\lambda_C M e^{x_I} \gamma}{\lambda_W n' (C_m r^2)^m} \right] \quad (6.13)$$

$$Z_A=Z_B= \dots = \log \left[ \frac{K}{(SCF)^m} \right] - x_I \quad (6.14)$$

**Total Independence:** This is the extreme case (included for interest rather than because of its applicability) in which there is no correlation at all between connections. If  $p_1$  is the probability of failure of a single connection then the probability of failure of  $n$  such joints is  $p_1^n$  and the probability of at least one failure in a set of  $n$  is  $1-(1-p_1)^n$ .

### 6.2.3 Results

The results of these calculations are given in tables 6.5 and 6.6, for various values of  $n$ , and they are plotted in figs 6.2 and 6.3. In these graphs the points have been joined by continuous lines, but of course these lines are really a set of discrete points at integer values of  $n$ .

The general form of the results is as one would expect. One notable feature is that while the 'considerable' and 'total' independence cases are very similar when calculating the probability of at least one failure, there is a considerable divergence between them for large  $n$  when calculating the probability of all failing. The limited degree of correlation present clearly becomes significant in this situation.

It is also clear that there is a wide range of possible results between the 'slight' and 'considerable' cases, so that the exact degree of correlation will be of great importance in practice.

### 6.3 Conditional Probabilities of Failure

#### 6.3.1. Introduction

One particular effect of the correlation between cases that is likely to be of interest is the effect of the discovery of a failure. The calculations below apply to the situation where one joint is inspected after a certain percentage of the required life has elapsed (20%, 50%, 100% are the figures used); it is found to have 'failed' by the definition used in the initial design, but without affecting the load carrying capacity of the structure or the distribution of loads within the structure. If there are  $n$  similar joints in the structure, what is the probability that at least one of these (or, alternatively, all of these) have already failed or will fail during the remainder of the required life? The failure function and data are the same as used in section 6.2.

Clearly if the connections are all totally independent of one another these probabilities are unchanged from those found in section 6.2 because the failure of one joint gives no information about the others. For the 'slight' and 'considerable' independence cases the conditional probabilities are calculated using various forms of the algorithms of section 2.9 to find the required joint probabilities, and using the following relationships:

$$\begin{aligned}
 &P[\text{at least one of } n \text{ cases fails} | \text{one particular case fails}] \\
 &= \frac{P[\text{at least one of } n \text{ cases fails} \cap \text{one particular case fails}]}{P[\text{One particular case fails}]} \\
 &\text{and } P[\text{all } n \text{ cases fail} | \text{one particular case fails}] \\
 &= \frac{P[\text{all } n \text{ cases fail} \cap \text{one particular case fails}]}{P[\text{one particular case fails}]} \\
 &= \frac{P[\text{all } n + 1 \text{ cases fail}]}{P[\text{one case fails}]}
 \end{aligned}$$

#### 6.3.2 Results

The results of these calculations are given in tables 6.7 and 6.8 and in figs 6.4 and 6.5. Here again the range between the results for 'slight' and 'considerable' independence is very wide,

covering a substantial part of the range between zero and total independence. The definitions of the 'slight' and 'considerable' cases is such that this range should be approximately the same as the range of correlations found in a real structure. It therefore seems that in practice the failure of one connection should cause action to be taken to check the connections that bear the closest similarity to the failed one, but that there is much less reason to be concerned about fundamentally different types of connection, although the probability of finding one or more other failures becomes large when there are a large number of such connections.

If the correlation between different connections in a real structure could be adequately quantified and modelled, calculations such as these could be developed and used to optimise the frequency of inspections, including determining the most rational response to the discovery of cracks.

## CHAPTER 7

### CONCLUSIONS

The preceding chapters have reviewed the basis of structural reliability analysis and its application to the quantification of the uncertainties that affect the fatigue resistance of offshore structures. Some recent criticisms of the use of probability theory in such calculations were discussed and found to be invalid, and it was shown that a number of different interpretations can be placed on the concept of probability which enable it to be applied in areas such as structural engineering. The classification of types of uncertainty was discussed and methods were given for the probabilistic modelling of random variables. The difficulties that result from having to use the sort of data commonly available were analysed, and it was shown that the concept of a set of nominally identical cases was particularly useful in identifying the nature of the uncertainties present.

A simple derivation was given for the so-called 'Level 2' method of reliability analysis and this was extended to provide two methods for the calculation of conditional failure probabilities.

Various methods used in the fatigue analysis of offshore structures were outlined and formulated as required for reliability analysis, and in the course of this work a proof was given of the upper bound nature of the narrow-band spectrum assumption. The formulations included the application of fracture mechanics methods, although these have not yet won wide acceptance in the offshore industry.

Calculations were made using the simpler formulation which were shown to be applicable to certain realistic situations. In other situations more complex calculations would be unavoidable. One of these is where it is necessary to consider more complicated interactions between connections than those considered in chapter 6, including, in the extreme case, the full sequence of element failures leading to total failure of a highly redundant structure. The other situation is where the structural analysis process is

subject to uncertainty because effects such as uncertainty in soil stiffness have to be considered. This will apply particularly to structures which show significant dynamic sensitivity so that the various parameters that influence the damping are important. Further investigation of these two areas should form a major part of future work. The problem of analysing 'systems' failures is receiving attention from many workers in reliability theory: the investigation of the effects of complex structural behaviour on reliability is best done by studying real structures (or designs) in detail; this is discussed again below.

The general conclusion of the calculations is that the most important single variable is the S-N curve parameter K, but that any other variable which has a major effect on the calculations of loads or stresses will also be important because the  $NS^m=K$  relationship (or whatever similar form is used) effectively magnifies uncertainties in S, the stress range, so that such variables will have a much larger effect on the overall uncertainty than one would expect by comparing their coefficients of variation with that of K. By comparison, uncertainties in the Miner's Rule sum and wave loading effects were shown to be comparatively unimportant.

This conclusion of course implies that the accurate assessment of coefficients of variation and other probability distribution parameters is important for K and several of the other variables. It was also shown that in assessing the effects of correlation of crack growth, the degree of independence between the different connections is very important and a wide variety of results is possible within a realistic range of situations. The results also show that it can be very important to define clearly the nature of the uncertainty which is relevant to the situation being analysed.

This dependence on the accuracy and relevance of the data means that the usefulness of reliability analysis in decision-making situations will depend on the availability of data. Although it is always possible to justify the use of reliability analysis because it makes the most rational use of whatever data is

available, in practice if conclusions are reached and decisions made which go beyond what would have been done anyway on the basis of common sense, it is necessary for them to have a sound basis in relevant empirical data. The problem is that data collection for the behaviour of offshore structures (including fatigue resistance) rarely takes any account of the needs of reliability analysis, since the aim is usually to cover the widest possible range of cases with perhaps only one test per case. Statistical analysis of test results is often extremely perfunctory and can be very misleading.

One important part of future developments of this work therefore is the procurement and analysis of data on all aspects of the uncertainties that affect fatigue in offshore structures, concentrating, of course, on those found to be most important. In particular it will be important to obtain data on the actual usage of the different structural elements so that this can be compared with the usage in laboratory tests and the results of those tests weighted appropriately. These tasks will be easier if future work can be tied to specific structures so that questions of what is and what is not relevant can be answered more easily. The use of specific examples, although it will mean that the conclusions lack some generality, will enable aspects of fatigue resistance which depend on details of structural configuration and loading to be investigated in greater depth. It should be possible to clarify differences in fatigue resistance between different parts of the structure, for instance differences between members, such as main columns and vertical plane bracing which carry the whole structural loading, and members such as horizontal bracing frames which are subject to more local effects, or differences between members which are always totally submerged and those near the surface subject to intermittent immersion and wave slam.

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Table 5.1 Data Recovery from m.v. Famita

| Reference Number | Winter  | Exact Period of recovery | Number of records | Percentage data recovery |
|------------------|---------|--------------------------|-------------------|--------------------------|
| 1                | 1969/70 | 3/10/69-29/3/70          | 1055              | 72.5%                    |
| 2                | 1971/72 | 4/11/71-31/3/72          | 567               | 38.9%                    |
| 3                | 1972/73 | 1/10/72-31/3/73          | 1020              | 70.1%                    |
| 4                | 1973/74 | 2/10/73-30/3/74          | 941               | 64.6%                    |
| 5                | 1974/75 | 1/10/74-29/3/75          | 1052              | 72.3%                    |
| 6                | 1975/76 | 12/10/75-31/3/76         | 1092              | 75.0%                    |
| 7                | 1976/77 | 1/10/76-31/3/77          | 951               | 65.3%                    |

Table 5.2 Regression Analysis Results for long-term wave height distribution. (Regression on values of H from 1.0m to 4.0m inclusive).

| Data Set               | Original Data    |                  |                     | "Filled" Data    |                  |                     |
|------------------------|------------------|------------------|---------------------|------------------|------------------|---------------------|
|                        | $\alpha$         | $\beta$          | $\alpha\beta^{3.5}$ | $\alpha$         | $\beta$          | $\alpha\beta^{3.5}$ |
| 1                      | 1.40             | 1.28             | 3.36                | 1.34             | 1.26             | 2.98                |
| 2                      | 1.96             | 1.08             | 2.56                | 1.33             | 1.24             | 2.85                |
| 3                      | 1.33             | 1.29             | 3.24                | 1.33             | 1.26             | 3.02                |
| 4                      | 1.31             | 1.30             | 3.30                | 1.31             | 1.27             | 3.00                |
| 5                      | 1.37             | 1.24             | 2.91                | 1.30             | 1.28             | 3.09                |
| 6                      | 1.36             | 1.20             | 2.58                | 1.32             | 1.25             | 2.90                |
| 7                      | 1.32             | 1.14             | 2.09                | 1.31             | 1.25             | 2.84                |
| Mean*                  | 1.44<br>(1.35)   | 1.22<br>(1.24)   | 2.86<br>(2.91)      | 1.32<br>(1.32)   | 1.26<br>(1.26)   | 2.95<br>(2.97)      |
| Standard*<br>Deviation | 0.233<br>(0.036) | 0.084<br>(0.063) | 0.47<br>(0.50)      | 0.015<br>(0.015) | 0.014<br>(0.013) | 0.093<br>(0.090)    |

\*Bracketed figures are calculated by excluding data set 2.

Table 5.3 Regression Analysis Results for long-term wave height distribution. (Regression on values of H from 1.0m to 6.0m inclusive).

| Data Set               | Original Data   |                 |                     | "Filled" Data    |                  |                     |
|------------------------|-----------------|-----------------|---------------------|------------------|------------------|---------------------|
|                        | $\alpha$        | $\beta$         | $\alpha\beta^{3.5}$ | $\alpha$         | $\beta$          | $\alpha\beta^{3.5}$ |
| 1                      | 1.47            | 1.25            | 3.21                | 1.35             | 1.25             | 2.96                |
| 2                      | 2.38            | 0.99            | 2.26                | 1.31             | 1.25             | 2.89                |
| 3                      | 1.28            | 1.38            | 3.98                | 1.32             | 1.27             | 3.03                |
| 4                      | 1.30            | 1.31            | 3.32                | 1.27             | 1.29             | 3.10                |
| 5                      | 1.39            | 1.23            | 2.86                | 1.23             | 1.32             | 3.25                |
| 6                      | 1.31            | 1.22            | 2.66                | 1.27             | 1.28             | 3.01                |
| 7                      | 1.30            | 1.15            | 2.12                | 1.28             | 1.27             | 2.92                |
| Mean*                  | 1.49<br>(1.34)  | 1.22<br>(1.26)  | 2.91<br>(3.02)      | 1.29<br>(1.29)   | 1.28<br>(1.28)   | 3.02<br>(3.04)      |
| Standard*<br>Deviation | 0.40<br>(0.073) | 0.13<br>(0.080) | 0.65<br>(0.64)      | 0.039<br>(0.042) | 0.023<br>(0.024) | 0.12<br>(0.12)      |

\* Bracketed figures are calculated by excluding data set 2.

Table 5.4 Statistics for n' and t' (normalised to mean value of 1.0)

| Data Set | n'            |               | t'            |               |
|----------|---------------|---------------|---------------|---------------|
|          | Original Data | "Filled" Data | Original Data | "Filled" Data |
| 1        | 1.13          | 0.95          | 1.16          | 0.94          |
| 2        | 0.79          | 0.96          | 0.72          | 0.96          |
| 3        | 1.16          | 0.95          | 1.14          | 0.92          |
| 4        | 1.24          | 1.05          | 1.28          | 1.06          |
| 5        | 1.01          | 1.11          | 1.01          | 1.13          |
| 6        | 0.95          | 1.01          | 0.97          | 1.03          |
| 7        | 0.73          | 0.97          | 0.73          | 0.97          |
| Mean     | 1.00          | 1.00          | 1.00          | 1.00          |
| S.D.     | 0.194         | 0.060         | 0.213         | 0.075         |

Table 5.5 Coefficients of Variation for Wave Description Variables

| Variable                               | Original Data | "Filled" Data |
|----------------------------------------|---------------|---------------|
| $\alpha\beta^m$ based on H=1.0 to 4.0m | 0.17          | 0.03          |
| $\alpha\beta^m$ based on H=1.0 to 6.0m | 0.22          | 0.04          |
| n'                                     | 0.19          | 0.06          |
| t'                                     | 0.21          | 0.08          |



Table 5.7 Accuracy of Stress Concentration Factor Formulae - 2

The figures below are taken from Dover, Choudhury and Dharmavasan<sup>95</sup>, Tables II and III.

| Load Case        | Position     | SCF<br>(measured)* | SCF from<br>Kuang Formula | <u>Measured SCF</u><br>Calculated SCF |
|------------------|--------------|--------------------|---------------------------|---------------------------------------|
| <b>T joints:</b> |              |                    |                           |                                       |
| Axial            | Chord Saddle | 8.76               | 9.26                      | .946                                  |
| "                | Brace Saddle | 6.8                | 10.22                     | .665                                  |
| OPB              | Chord Saddle | 8.6                | 7.02                      | 1.225                                 |
| "                | Brace Saddle | 5.63               | 7.54                      | .747                                  |
| IPB              | Chord Crown  | 2.74               | 2.51                      | 1.092                                 |
| "                | Brace Crown  | 1.83               | 2.66                      | .688                                  |
| <b>Y joints:</b> |              |                    |                           |                                       |
| Axial            | Chord Saddle | 6.9                | 5.81                      | 1.188                                 |
| "                | Brace Saddle | 6.13               | 5.95                      | 1.030                                 |
| OPB              | Chord Saddle | 5.34               | 3.36                      | 1.589                                 |
| "                | Brace Saddle | 4.13               | 3.93                      | 1.051                                 |
| IPB              | Chord Crown  | 1.92               | 2.45                      | .784                                  |
| "                | Brace Crown  | 1.70               | 2.42                      | .702                                  |
|                  |              |                    | Mean                      | .98                                   |
|                  |              |                    | Standard<br>Deviation     | .28                                   |

\*Average of acrylic and steel model results for T-joints.

Only acrylic models were used for Y-joints.

Table 5.8 Summary of results in table 5.7

|         |              | Mean | Range |
|---------|--------------|------|-------|
| T-joint | Chord Saddle | 1.09 | .28   |
| "       | Brace Saddle | 0.70 | .08   |
| Y-joint | Chord Saddle | 1.19 | .81   |
| "       | Brace Saddle | 0.93 | .35   |

Table 5.9 Accuracy of Stress Concentration Factor Formulae - 3

| Joint | SNCF measured |       | SCF calculated<br>(Kuang formulae) |       | <u>Measured SNCF</u><br>Calculated SCF |       |
|-------|---------------|-------|------------------------------------|-------|----------------------------------------|-------|
|       | chord         | brace | chord                              | brace | chord                                  | brace |
| A     | 11.0          | 7.5   | 8.7                                | 10.0  | 1.264                                  | .750  |
| B     | 4.7           | 4.4   | 4.9                                | 7.1   | .959                                   | .620  |
| C     | 1.6           | 1.7   | 2.2                                | 3.9   | .727                                   | .436  |
| D     | 10.0          | 7.0   | 9.8                                | 10.3  | 1.020                                  | .680  |
| E     | 3.8           | 3.2   | 4.3                                | 6.1   | .884                                   | .525  |
| A'    | 3.04          | 1.84  | 2.9                                | 2.5   | 1.048                                  | .736  |
| B'    | 1.45          | 1.45  | 1.6                                | 2.2   | .906                                   | .659  |
| C'    | 0.62          | 1.33  | 0.9                                | 2.2   | .689                                   | .605  |
| D'    | 2.72          | -     | 2.9                                | 2.5   | .938                                   | -     |
| E'    | 1.56          | 1.71  | 1.6                                | 2.2   | .975                                   | .777  |
|       |               |       | Mean                               |       | .941                                   | .643  |
|       |               |       | Standard Deviation                 |       | .162                                   | .111  |

Table 5.10 Sources of S-N Curve Data

| I*                | Country of Origin | EEC Conf <sup>80</sup><br>Papers No. | ICE Conf <sup>72</sup><br>Papers No |
|-------------------|-------------------|--------------------------------------|-------------------------------------|
| 1-78 <sup>+</sup> | UK                | PS2, ST 10.2                         | 10                                  |
| 79-98             | Netherlands       | PS2, ST 8.4                          |                                     |
| 99-108            | Italy             | PS2, ST 10.1                         |                                     |
| 109-116           | Norway            | PS2                                  |                                     |
| 117-122           | UK                | ST 8.2                               | 10                                  |
| 123-147           | UK                |                                      | 9, 10                               |

\* Cross-reference number to tables 5.11, 5.12 and 5.13.

<sup>+</sup> This data has been revised. Paper ST 10.2 contains the later figures and PS2 the earlier ones. ICE Conf Paper 10 contains the later figures in its final published edition, but the pre-conference pre-print contained the same figures as Paper PS2. The later values are used in the analysis. The tests referenced I = 57 to 78 are described in some references as series E1, E2, F1 and F2 (as given in tables 5.11-5.13) but in some references they are given as series 41/, 42/, 43/, and 44/.

Table 5.11 Summary of Data for S-N Curves

| I  | TEST NO. | NOTES | MICRO-STRAIN RANGE | N1 | FATIGUE N2 | LIVES N3 | N4       |
|----|----------|-------|--------------------|----|------------|----------|----------|
| 1  | 17/1     |       | 1049.              | 0  | .799E+06   | .153E+07 | .203E+07 |
| 2  | 17/3     |       | 798.               | 0  | .270E+07   | .520E+07 | .590E+07 |
| 3  | 17/4     |       | 1624.              | 0  | .459E+05   | .165E+06 | .200E+06 |
| 4  | 17/2     | U     | 780.               | 0  | 0          | 0        | 0        |
| 5  | 17/2     | R     | 1340.              | 0  | .540E+05   | .324E+06 | .373E+06 |
| 6  | 17/12    |       | 1100.              | 0  | .383E+06   | .839E+06 | .118E+07 |
| 7  | 17/6     |       | 1529.              | 0  | .112E+06   | .490E+06 | .997E+06 |
| 8  | 17/7     |       | 1037.              | 0  | .108E+07   | .210E+07 | .516E+07 |
| 9  | 17/8     |       | 883.               | 0  | .370E+07   | 0        | .639E+07 |
| 10 | 17/9     |       | 1566.              | 0  | .980E+06   | .143E+07 | .155E+07 |
| 11 | 17/10    | U     | 1028.              | 0  | 0          | 0        | 0        |
| 12 | 17/10    | R     | 1727.              | 0  | .100E+07   | .146E+07 | .175E+07 |
| 13 | 17/11    |       | 1713.              | 0  | .120E+07   | .148E+07 | .187E+07 |
| 14 | 18/1     |       | 1421.              | 0  | .126E+06   | .490E+06 | .542E+06 |
| 15 | 18/5     | U     | 836.               | 0  | 0          | 0        | 0        |
| 16 | 18/5     | R     | 1418.              | 0  | .100E+07   | .140E+07 | .164E+07 |
| 17 | 18/7     |       | 1000.              | 0  | 0          | 0        | .540E+07 |
| 18 | 18/8     | U     | 707.               | 0  | 0          | 0        | 0        |
| 19 | 18/8     | R     | 1649.              | 0  | .208E+06   | .387E+06 | .422E+06 |
| 20 | 18/9     |       | 1560.              | 0  | 0          | .220E+06 | .270E+06 |
| 21 | 18/12    |       | 1826.              | 0  | .460E+05   | .217E+06 | .236E+06 |
| 22 | 18/15    |       | 1132.              | 0  | .210E+06   | .415E+06 | .543E+06 |
| 23 | 18/4     |       | 1564.              | 0  | .600E+06   | .890E+06 | .197E+07 |
| 24 | 18/6     | U     | 893.               | 0  | 0          | 0        | 0        |
| 25 | 18/6     | R     | 1474.              | 0  | .700E+06   | .100E+07 | .186E+07 |
| 26 | 18/10    |       | 1674.              | 0  | .130E+07   | .204E+07 | .251E+07 |
| 27 | 18/13    |       | 2574.              | 0  | .699E+05   | .222E+06 | .265E+06 |
| 28 | 18/14    |       | 1935.              | 0  | .299E+06   | .660E+06 | .110E+07 |
| 29 | 18/16    |       | 1154.              | 0  | .580E+07   | 0        | .860E+07 |
| 30 | 19/11    |       | 1000.              | 0  | .752E+06   | .820E+06 | .107E+07 |
| 31 | 19/12    |       | 750.               | 0  | .157E+07   | .189E+07 | .253E+07 |
| 32 | 19/13    |       | 1500.              | 0  | 0          | .362E+06 | .419E+06 |
| 33 | 20/4     | U     | 600.               | 0  | 0          | 0        | 0        |
| 34 | 20/4     | R     | 1500.              | 0  | .420E+05   | .235E+06 | .356E+06 |
| 35 | 20/11    |       | 1500.              | 0  | .225E+06   | 0        | .400E+06 |
| 36 | 20/2     | U     | 800.               | 0  | 0          | 0        | 0        |
| 37 | 20/2     | R     | 1600.              | 0  | .290E+06   | .337E+06 | .393E+06 |
| 38 | 20/5     |       | 1000.              | 0  | .185E+06   | .518E+06 | .850E+06 |
| 39 | 20/6     |       | 600.               | 0  | .823E+07   | .100E+08 | .104E+08 |
| 40 | 37/3     |       | 1370.              | 0  | .620E+05   | .144E+06 | .153E+06 |
| 41 | 37/5     |       | 814.               | 0  | .590E+06   | .100E+07 | .150E+07 |
| 42 | 37/9     |       | 511.               | 0  | .239E+07   | .695E+07 | .873E+07 |
| 43 | 37/1     |       | 1090.              | 0  | .790E+05   | .290E+06 | .500E+06 |
| 44 | 37/7     |       | 519.               | 0  | .227E+07   | .100E+08 | .144E+08 |
| 45 | 37/13    |       | 677.               | 0  | .337E+06   | .729E+06 | .144E+07 |

Continued on next sheet

Table 5.11 continued

| I  | TEST NO. | NOTES | MICRO-STRAIN RANGE | FATIGUE LIVES |          |          |          |
|----|----------|-------|--------------------|---------------|----------|----------|----------|
|    |          |       |                    | N1            | N2       | N3       | N4       |
| 46 | 37/8     |       | 1090.              | 0             | .510E+06 | .120E+07 | .153E+07 |
| 47 | 37/10    |       | 736.               | 0             | .606E+06 | .379E+07 | .618E+07 |
| 48 | 37/12    |       | 1610.              | 0             | 0        | 0        | .334E+06 |
| 49 | 38/4     |       | 882.               | 0             | 0        | .535E+06 | .678E+06 |
| 50 | 38/5     |       | 633.               | 0             | 0        | 0        | .288E+07 |
| 51 | 38/8     |       | 1160.              | 0             | 0        | 0        | .391E+06 |
| 52 | 38/1     |       | 900.               | 0             | .914E+06 | .211E+07 | .386E+07 |
| 53 | 38/7     |       | 1170.              | 0             | 0        | 0        | .156E+07 |
| 54 | 38/2     |       | 650.               | 0             | 0        | 0        | .300E+07 |
| 55 | 38/3     |       | 858.               | 0             | .409E+06 | .610E+06 | .170E+07 |
| 56 | 38/6     |       | 550.               | 0             | .100E+07 | .180E+07 | .532E+07 |
| 57 | E1B      | U     | 459.               | 0             | 0        | 0        | 0        |
| 58 | E1B      | R     | 1060.              | 0             | .412E+06 | .760E+06 | .879E+06 |
| 59 | E1C      |       | 663.               | 0             | .332E+06 | .140E+07 | .232E+07 |
| 60 | E1A      |       | 326.               | 0             | .219E+07 | .210E+08 | .262E+08 |
| 61 | E1D      |       | 461.               | 0             | .111E+07 | .346E+07 | .490E+07 |
| 62 | E2B      |       | 373.               | 0             | .170E+07 | .479E+07 | .635E+07 |
| 63 | E2C      |       | 304.               | 0             | .250E+07 | .146E+08 | .153E+08 |
| 64 | E2A      | U     | 192.               | 0             | 0        | 0        | 0        |
| 65 | E2A      | R     | 447.               | 0             | .960E+05 | .530E+06 | .900E+06 |
| 66 | E2D      |       | 341.               | 0             | .120E+07 | .223E+07 | .332E+07 |
| 67 | F1A      | U     | 308.               | 0             | 0        | 0        | 0        |
| 68 | F1A      | R     | 1170.              | 0             | .106E+05 | .210E+06 | .240E+06 |
| 69 | F1B      | U     | 340.               | 0             | 0        | 0        | 0        |
| 70 | F1B      | R     | 580.               | 0             | .762E+05 | .240E+07 | .333E+07 |
| 71 | F1C      | U     | 267.               | 0             | 0        | 0        | 0        |
| 72 | F1C      | R     | 1080.              | 0             | .105E+06 | .172E+06 | .245E+06 |
| 73 | F1D      |       | 349.               | 0             | .250E+07 | .750E+07 | .889E+07 |
| 74 | F2C      |       | 720.               | 0             | .300E+06 | .120E+07 | .275E+07 |
| 75 | F2D      | U     | 486.               | 0             | 0        | 0        | 0        |
| 76 | F2D      | R     | 1455.              | 0             | .800E+06 | .147E+06 | .230E+06 |
| 77 | F2A      |       | 875.               | 0             | .836E+06 | .116E+07 | .213E+07 |
| 78 | F2B      |       | 694.               | 0             | 0        | .212E+07 | .732E+07 |

Continued on next sheet

Table 5.11 continued

| I   | TEST NO. | NOTES | MICRO-STRAIN RANGE | N1       | FATIGUE N2 | LIVES N3 | N4       |
|-----|----------|-------|--------------------|----------|------------|----------|----------|
| 79  | NL1      |       | 2230.              | .125E+05 | 0          | .600E+05 | .630E+05 |
| 80  | NL2      |       | 745.               | .100E+08 | .120E+08   | .120E+08 | .130E+08 |
| 81  | NL3      |       | 1325.              | .290E+07 | .260E+07   | .300E+07 | .330E+07 |
| 82  | NL5      |       | 818.               | .350E+06 | .330E+06   | .680E+06 | .780E+06 |
| 83  | NL6      |       | 737.               | .420E+06 | .400E+06   | .130E+07 | .130E+07 |
| 84  | NL7      |       | 737.               | .440E+06 | .350E+06   | .840E+06 | .110E+07 |
| 85  | NL9      |       | 818.               | .320E+06 | .280E+06   | .760E+06 | .100E+07 |
| 86  | NL13     |       | 370.               | .190E+07 | .210E+07   | .410E+07 | .500E+07 |
| 87  | NL15     |       | 615.               | .370E+06 | .500E+06   | .950E+06 | .130E+07 |
| 88  | NL18     |       | 870.               | .120E+07 | .260E+07   | .300E+07 | .360E+07 |
| 89  | NL19     |       | 1990.              | .200E+05 | .300E+05   | .600E+05 | .740E+05 |
| 90  | NL22     |       | 1070.              | .110E+06 | .150E+06   | .880E+06 | .950E+06 |
| 91  | NL23     |       | 760.               | .170E+07 | .150E+07   | .200E+07 | .240E+07 |
| 92  | NL24     |       | 1285.              | .310E+06 | .250E+06   | .330E+06 | .370E+06 |
| 93  | NL25     |       | 1445.              | .430E+06 | .430E+06   | .470E+06 | .480E+06 |
| 94  | NL26     |       | 1010.              | .153E+07 | .630E+06   | .150E+07 | .170E+07 |
| 95  | NL30     |       | 913.               | 0        | .570E+06   | .100E+07 | .120E+07 |
| 96  | NL33     |       | 783.               | .140E+07 | .140E+07   | .240E+07 | .290E+07 |
| 97  | NL37     |       | 621.               | .450E+07 | .390E+07   | .670E+07 | .810E+07 |
| 98  | NL38     |       | 594.               | .520E+07 | .480E+07   | .780E+07 | .850E+07 |
| 99  | IIE      |       | 3500.              | .550E+04 | .780E+04   | .170E+05 | .210E+05 |
| 100 | IVC      |       | 2850.              | .550E+04 | .780E+04   | .190E+05 | .210E+05 |
| 101 | ID       |       | 2450.              | .170E+05 | .230E+05   | .500E+05 | .590E+05 |
| 102 | IIID     |       | 2500.              | .130E+05 | .230E+05   | 0        | .590E+05 |
| 103 | IIC      |       | 1100.              | .850E+05 | .950E+05   | .500E+06 | .700E+06 |
| 104 | IVD      |       | 1060.              | .600E+05 | .900E+05   | .380E+06 | .700E+06 |
| 105 | IIIE     |       | 2080.              | 0        | .300E+05   | .600E+05 | .790E+05 |
| 106 | IVE      |       | 1960.              | .190E+05 | .220E+05   | .480E+05 | .790E+05 |
| 107 | IID      |       | 1530.              | .330E+05 | .220E+05   | .620E+05 | .290E+06 |
| 108 | IIIC     |       | 1475.              | .920E+05 | .750E+05   | 0        | .290E+06 |
| 109 | NJR1     | U     | 980.               | 0        | 0          | 0        | 0        |
| 110 | NRP3     |       | 1390.              | .560E+05 | .750E+05   | 0        | .580E+06 |
| 111 | NOR5     |       | 1100.              | .450E+06 | .650E+06   | 0        | .160E+07 |
| 112 | NJR6     |       | 960.               | .110E+06 | .200E+06   | 0        | .110E+07 |
| 113 | NOR2     |       | 1740.              | 0        | .300E+05   | 0        | .210E+06 |
| 114 | NJR4     |       | 1085.              | .120E+06 | .160E+07   | 0        | .870E+06 |
| 115 | NORA     |       | 1307.              | 0        | 0          | 0        | .590E+06 |
| 116 | NOR3     |       | 715.               | .190E+07 | .230E+07   | 0        | .400E+07 |

Continued on next sheet

Table 5.11 continued

| I   | TEST NOTES<br>NO. | MICRO-<br>STRAIN<br>RANGE | FATIGUE LIVES |          |          |          |
|-----|-------------------|---------------------------|---------------|----------|----------|----------|
|     |                   |                           | N1            | N2       | N3       | N4       |
| 117 | 707/1             | 1190.                     | 0             | .180E+06 | 0        | .160E+07 |
| 118 | 707/2             | 1160.                     | 0             | .620E+06 | 0        | .400E+07 |
| 119 | 709/1             | 1300.                     | 0             | .230E+06 | 0        | .130E+07 |
| 120 | 709/2             | 1110.                     | 0             | .290E+06 | 0        | .230E+07 |
| 121 | 711/1             | 930.                      | 0             | .350E+06 | 0        | .120E+08 |
| 122 | 711/2             | 1310.                     | 0             | .720E+05 | 0        | .170E+07 |
| 123 | 701/2             | 1730.                     | 0             | 0        | 0        | .850E+06 |
| 124 | 701/3             | 1210.                     | 0             | 0        | 0        | .230E+07 |
| 125 | 702/1             | 1430.                     | 0             | .380E+06 | .120E+07 | .230E+07 |
| 126 | 702/2             | 1290.                     | 0             | .640E+05 | .610E+06 | .170E+07 |
| 127 | 702/3             | 1580.                     | 0             | .190E+06 | .470E+06 | .880E+06 |
| 128 | 703/1             | 885.                      | 0             | .400E+06 | .450E+07 | .910E+06 |
| 129 | 703/2             | 1465.                     | 0             | .660E+05 | .870E+06 | .140E+07 |
| 130 | 703/3             | 1180.                     | 0             | 0        | 0        | .840E+07 |
| 131 | 704/1             | 1470.                     | 0             | .980E+05 | .580E+06 | .190E+07 |
| 132 | 704/2             | 1220.                     | 0             | .350E+06 | .210E+07 | .450E+07 |
| 133 | 704/3             | 1390.                     | 0             | .360E+06 | .400E+07 | .650E+07 |
| 134 | 705/1             | 1370.                     | 0             | .110E+06 | .720E+06 | .200E+07 |
| 135 | 705/2             | 1520.                     | 0             | .460E+06 | .110E+07 | .200E+07 |
| 136 | 705/3             | 1710.                     | 0             | .800E+05 | .340E+06 | .380E+06 |
| 137 | 621/1             | 785.                      | 0             | 0        | 0        | .120E+07 |
| 138 | 621/2             | 1190.                     | 0             | 0        | 0        | .810E+06 |
| 139 | 621/3             | 592.                      | 0             | 0        | 0        | .970E+07 |
| 140 | 622/1             | 1360.                     | 0             | 0        | 0        | .370E+06 |
| 141 | 622/2             | 795.                      | 0             | 0        | 0        | .150E+07 |
| 142 | 622/3             | 583.                      | 0             | 0        | 0        | .590E+07 |
| 143 | 623/1             | 677.                      | 0             | .100E+07 | .480E+07 | .150E+08 |
| 144 | 623/2             | 1490.                     | 0             | .270E+05 | .200E+06 | .630E+06 |
| 145 | 624/1             | 832.                      | 0             | 0        | 0        | .450E+07 |
| 146 | 624/2             | 1610.                     | 0             | 0        | 0        | .750E+05 |
| 147 | 624/3             | 1215.                     | 0             | 0        | 0        | .290E+06 |

NOTES:

U=UNBROKEN

R=RETEST OF UNBROKEN TESTPIECE  
AT HIGHER STRESS RANGE

Table 5.12 Regression Analysis results - S-N Curves with  
different m for each set of data

| I  | TEST<br>NO. | M  |       |       |       | LOG(K) |       |       |       | S.D. FOR LOG(K) |     |     |     |
|----|-------------|----|-------|-------|-------|--------|-------|-------|-------|-----------------|-----|-----|-----|
|    |             | N1 | N2    | N3    | N4    | N1     | N2    | N3    | N4    | N1              | N2  | N3  | N4  |
| 1  | 17/1        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 2  | 17/3        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 3  | 17/4        | 0  | 5.81  | 4.88  | 4.81  | 0      | 23.35 | 20.89 | 20.78 | 0               | .12 | .04 | .08 |
| 5  | 17/2        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 6  | 17/12       | 0  | 9.93  | 4.82  | 5.04  | 0      | 35.77 | 20.59 | 23.82 | 0               | 0   | 0   | 0   |
| 7  | 17/6        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 8  | 17/7        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 9  | 17/8        | 0  | 6.27  | 3.75  | 3.54  | 0      | 25.00 | 17.63 | 17.30 | 0               | .07 | 0   | .11 |
| 10 | 17/9        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 12 | 17/10       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 13 | 17/11       | 0  | -1.05 | -2.28 | -1.59 | 0      | 2.62  | 5.25  | 1.10  | 0               | .06 | .00 | .02 |
| 14 | 18/1        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 16 | 18/5        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 17 | 18/7        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 19 | 18/8        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 20 | 18/9        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 21 | 18/12       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 22 | 18/15       | 0  | 2.82  | 1.76  | 4.11  | 0      | 14.21 | 11.21 | 18.77 | 0               | .50 | .30 | .33 |
| 23 | 18/4        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 25 | 18/6        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 26 | 18/10       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 27 | 18/13       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 28 | 18/14       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 29 | 18/16       | 0  | 5.20  | 3.07  | 4.08  | 0      | 22.57 | 15.89 | 19.40 | 0               | .22 | .22 | .14 |
| 30 | 19/11       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 31 | 19/12       |    |       |       |       |        |       |       |       |                 |     |     |     |
| 32 | 19/13       | 0  | 2.56  | 2.36  | 2.58  | 0      | 13.55 | 13.04 | 13.79 | 0               | 0   | .05 | .04 |
| 34 | 20/4        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 35 | 20/11       | 0  | 0     | 0     | 0     | 0      | 0     | 0     | 0     | 0               | 0   | 0   | 0   |
| 37 | 20/2        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 38 | 20/5        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 39 | 20/6        | 0  | 3.47  | 3.49  | 3.36  | 0      | 16.27 | 16.53 | 16.25 | 0               | .73 | .42 | .28 |
| 40 | 37/3        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 41 | 37/5        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 42 | 37/9        | 0  | 3.72  | 3.93  | 4.11  | 0      | 16.49 | 17.46 | 18.09 | 0               | .12 | .04 | .05 |
| 43 | 37/1        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 44 | 37/7        |    |       |       |       |        |       |       |       |                 |     |     |     |
| 45 | 37/13       | 0  | 4.35  | 4.44  | 4.26  | 0      | 18.04 | 18.80 | 18.51 | 0               | .25 | .47 | .38 |

| I  | TEST NO. | M  |       |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |    |    |     |
|----|----------|----|-------|------|------|--------|-------|-------|-------|-----------------|----|----|-----|
|    |          | N1 | N2    | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2 | N3 | N4  |
| 46 | 37/8     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 47 | 37/10    |    |       |      |      |        |       |       |       |                 |    |    |     |
| 48 | 37/12    | 0  | .44   | 2.95 | 3.74 | 0      | 7.05  | 15.04 | 17.53 | 0               | 0  | 0  | .02 |
| 49 | 38/4     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 50 | 38/5     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 51 | 38/8     | 0  | 0     | 0    | 3.33 | 0      | 0     | 0     | 15.75 | 0               | 0  | 0  | .12 |
| 52 | 38/1     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 53 | 38/7     | 0  | 0     | 0    | 3.45 | 0      | 0     | 0     | 16.79 | 0               | 0  | 0  | 0   |
| 54 | 38/2     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 55 | 38/3     |    |       |      |      |        |       |       |       |                 |    |    |     |
| 56 | 38/6     | 0  | 2.01  | 2.43 | 2.51 | 0      | 11.51 | 12.92 | 13.59 | 0               | 0  | 0  | .05 |
| 58 | E1B      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 59 | E1C      | 0  | -0.46 | 1.30 | 2.07 | 0      | 4.22  | 9.82  | 12.20 | 0               | 0  | 0  | 0   |
| 60 | E1A      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 61 | E1D      | 0  | 1.96  | 5.20 | 4.84 | 0      | 11.27 | 20.40 | 19.58 | 0               | 0  | 0  | 0   |
| 62 | E2B      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 63 | E2C      | 0  | 1.89  | 5.45 | 4.30 | 0      | 11.08 | 20.69 | 17.86 | 0               | 0  | 0  | 0   |
| 65 | E2A      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 66 | E2D      | 0  | 9.33  | 5.31 | 4.82 | 0      | 29.71 | 19.79 | 18.74 | 0               | 0  | 0  | 0   |
| 68 | F1A      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 70 | F1D      | 0  | 2.81  | 3.47 | 3.75 | 0      | 12.65 | 15.97 | 16.88 | 0               | 0  | 0  | 0   |
| 72 | F1C      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 73 | F1D      | 0  | 2.81  | 3.34 | 3.18 | 0      | 13.53 | 15.37 | 15.03 | 0               | 0  | 0  | 0   |
| 74 | F2C      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 76 | F2D      | 0  | -1.39 | 2.98 | 3.53 | 0      | 1.49  | 14.61 | 16.52 | 0               | 0  | 0  | 0   |
| 77 | F2A      |    |       |      |      |        |       |       |       |                 |    |    |     |
| 78 | F2B      | 0  | 0     | 2.60 | 5.33 | 0      | 0     | 13.72 | 22.00 | 0               | 0  | 0  | 0   |

Table 5.12 continued

| I   | TEST NO. | M    |      |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |     |     |     |
|-----|----------|------|------|------|------|--------|-------|-------|-------|-----------------|-----|-----|-----|
|     |          | N1   | N2   | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2  | N3  | N4  |
| 79  | NL1      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 80  | NL2      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 81  | NL3      | 6.03 | 2.66 | 4.79 | 4.82 | 24.62  | 14.71 | 21.03 | 21.15 | .81             | 0   | .49 | .51 |
| 82  | NL5      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 83  | NL6      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 84  | NL7      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 85  | NL9      | 2.40 | 1.99 | 2.33 | 2.91 | 12.52  | 11.29 | 12.64 | 14.41 | .02             | .05 | .04 | .07 |
| 86  | NL13     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 87  | NL15     | 3.22 | 2.82 | 2.88 | 2.65 | 14.55  | 13.58 | 14.00 | 13.51 | 0               | 0   | 0   | 0   |
| 88  | NL18     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 89  | NL19     | 4.95 | 5.39 | 4.73 | 4.69 | 20.63  | 22.27 | 20.38 | 20.36 | 0               | 0   | 0   | 0   |
| 90  | NL22     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 91  | NL23     | 8.00 | 6.73 | 2.40 | 2.71 | 29.29  | 25.57 | 13.21 | 14.18 | 0               | 0   | 0   | 0   |
| 92  | NL24     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 93  | NL25     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 94  | NL26     | 4.00 | 1.48 | 3.69 | 3.95 | 18.14  | 10.18 | 17.20 | 18.03 | .26             | .23 | .26 | .23 |
| 95  | NL30     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 96  | NL33     | 0    | 5.85 | 5.70 | 5.74 | 0      | 23.07 | 22.87 | 23.09 | 0               | 0   | 0   | 0   |
| 97  | NL37     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 98  | NL38     | 3.25 | 4.67 | 3.42 | 1.08 | 15.74  | 19.64 | 16.38 | 9.94  | 0               | 0   | 0   | 0   |
| 99  | IIE      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 100 | IVC      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 101 | ID       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 102 | IIID     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 103 | IIC      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 104 | IVD      | 2.22 | 2.11 | 2.87 | 3.14 | 11.60  | 11.38 | 14.36 | 15.37 | .14             | .14 | .12 | .12 |
| 105 | IIIE     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 106 | IVE      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 107 | IID      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 108 | IIIC     | 4.34 | 1.65 | .38  | 4.28 | 18.54  | 9.87  | 5.99  | 19.05 | .27             | .27 | .08 | .07 |
| 110 | NOR3     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 111 | NOR5     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 112 | NOR6     | 2.60 | 3.37 | 0    | 2.01 | 13.08  | 15.62 | 0     | 12.15 | .58             | .54 | 0   | .21 |
| 113 | NOR2     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 114 | NOR4     | 0    | 8.42 | 0    | 3.01 | 0      | 31.76 | 0     | 15.07 | 0               | 0   | 0   | 0   |
| 115 | NORA     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 116 | NORB     | 0    | 0    | 0    | 3.17 | 0      | 0     | 0     | 15.66 | 0               | 0   | 0   | 0   |

Table 5.12 continued

| I   | TEST NO. | M  |       |      |       | LOG(K) |        |       |        | S.D. FOR LOG(K) |     |     |     |
|-----|----------|----|-------|------|-------|--------|--------|-------|--------|-----------------|-----|-----|-----|
|     |          | N1 | N2    | N3   | N4    | N1     | N2     | N3    | N4     | N1              | N2  | N3  | N4  |
| 117 | 707/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 118 | 707/2    | 0  | 48.44 | 0    | 35.89 | 0      | 154.23 | 0     | 116.57 | 0               | 0   | 0   | 0   |
| 119 | 709/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 120 | 709/2    | 0  | 1.47  | 0    | 3.61  | 0      | 9.93   | 0     | 17.36  | 0               | 0   | 0   | 0   |
| 121 | 711/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 122 | 711/2    | 0  | 4.62  | 0    | 5.70  | 0      | 19.25  | 0     | 24.01  | 0               | 0   | 0   | 0   |
| 123 | 701/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 124 | 701/3    | 0  | 0     | 0    | 2.78  | 0      | 0      | 0     | 14.95  | 0               | 0   | 0   | 0   |
| 125 | 702/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 126 | 702/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 127 | 702/3    | 0  | -5.43 | 1.24 | 3.21  | 0      | -11.91 | 9.77  | 16.32  | 0               | .44 | .29 | .23 |
| 128 | 703/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 129 | 703/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 130 | 703/3    | 0  | 3.57  | 3.26 | -1.22 | 0      | 16.14  | 16.26 | 2.60   | 0               | 0   | 0   | .70 |
| 131 | 704/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 132 | 704/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 133 | 704/3    | 0  | 5.58  | 4.81 | 3.31  | 0      | 22.85  | 21.28 | 16.94  | 0               | .32 | .53 | .34 |
| 134 | 705/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 135 | 705/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 136 | 705/3    | 0  | 1.73  | 3.53 | 3.78  | 0      | 10.72  | 17.05 | 18.20  | 0               | .56 | .27 | .14 |
| 137 | 621/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 138 | 621/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 139 | 621/3    | 0  | 0     | 0    | 3.36  | 0      | 0      | 0     | 16.12  | 0               | 0   | 0   | .38 |
| 140 | 622/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 141 | 622/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 142 | 622/3    | 0  | 0     | 0    | 3.20  | 0      | 0      | 0     | 15.55  | 0               | 0   | 0   | .12 |
| 143 | 623/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 144 | 623/2    | 0  | 4.58  | 4.03 | 4.02  | 0      | 18.96  | 18.08 | 18.55  | 0               | 0   | 0   | 0   |
| 145 | 624/1    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 146 | 624/2    |    |       |      |       |        |        |       |        |                 |     |     |     |
| 147 | 624/3    | 0  | 0     | 0    | 6.26  | 0      | 0      | 0     | 24.89  | 0               | 0   | 0   | .14 |

Table 5.12 continued

Table 5.13 Regression Analysis results - S-N Curves with  
constant m throughout

| I  | TEST<br>NO. | M    |      |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |     |     |     |
|----|-------------|------|------|------|------|--------|-------|-------|-------|-----------------|-----|-----|-----|
|    |             | N1   | N2   | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2  | N3  | N4  |
| 1  | 17/1        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 2  | 17/3        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 3  | 17/4        | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.72 | 16.72 | 17.25 | 0               | .40 | .21 | .19 |
| 5  | 17/2        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 6  | 17/12       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.34 | 16.54 | 17.09 | 0               | .40 | .08 | .13 |
| 7  | 17/6        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 8  | 17/7        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 9  | 17/11       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.95 | 16.89 | 17.64 | 0               | .37 | .03 | .08 |
| 10 | 17/9        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 12 | 17/10       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 13 | 17/11       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 16.66 | 17.47 | 18.01 | 0               | .11 | .09 | .12 |
| 14 | 18/1        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 16 | 18/5        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 17 | 18/7        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 19 | 18/8        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 20 | 18/9        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 21 | 18/12       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 22 | 18/15       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.74 | 16.75 | 17.33 | 0               | .43 | .30 | .30 |
| 23 | 18/4        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 25 | 18/6        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 26 | 18/10       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 27 | 18/13       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 28 | 18/14       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 29 | 18/16       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 16.45 | 17.32 | 18.01 | 0               | .30 | .20 | .13 |
| 30 | 19/11       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 31 | 19/12       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 32 | 19/13       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.74 | 16.50 | 17.04 | 0               | .07 | .18 | .17 |
| 34 | 20/4        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 35 | 20/11       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.48 | 0     | 17.18 | 0               | .52 | 0   | .04 |
| 37 | 20/2        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 38 | 20/5        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 39 | 20/6        | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.77 | 16.59 | 17.12 | 0               | .52 | .30 | .21 |
| 40 | 37/3        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 41 | 37/5        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 42 | 37/9        | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.28 | 16.24 | 16.76 | 0               | .12 | .09 | .10 |
| 43 | 37/1        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 44 | 37/7        |      |      |      |      |        |       |       |       |                 |     |     |     |
| 45 | 37/13       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.04 | 16.15 | 16.79 | 0               | .24 | .37 | .29 |

| I  | TEST NO. | M    |      |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |      |     |     |
|----|----------|------|------|------|------|--------|-------|-------|-------|-----------------|------|-----|-----|
|    |          | N1   | N2   | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2   | N3  | N4  |
| 46 | 37/8     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 47 | 37/10    |      |      |      |      |        |       |       |       |                 |      |     |     |
| 48 | 37/12    | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.50 | 16.69 | 17.26 | 0               | .34  | .07 | .02 |
| 49 | 38/4     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 50 | 38/5     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 51 | 38/8     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 16.69 | 0               | 0    | 0   | .10 |
| 52 | 38/1     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 53 | 38/7     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 17.39 | 0               | 0    | 0   | .02 |
| 54 | 38/2     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 55 | 38/3     |      |      |      |      |        |       |       |       |                 |      |     |     |
| 56 | 38/6     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.17 | 15.98 | 16.81 | 0               | .18  | .15 | .12 |
| 58 | E1B      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 59 | E1C      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.22 | 16.27 | 16.84 | 0               | .54  | .32 | .23 |
| 60 | E1A      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 61 | E1D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 14.74 | 16.01 | 16.51 | 0               | .14  | .18 | .13 |
| 62 | E2B      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 63 | E2C      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 14.66 | 15.79 | 16.23 | 0               | .09  | .12 | .04 |
| 65 | E2A      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 66 | E2D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 14.09 | 15.13 | 15.71 | 0               | .50  | .15 | .10 |
| 68 | F1A      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 70 | F1D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 14.08 | 16.08 | 16.61 | 0               | .11  | .01 | .02 |
| 72 | F1C      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 73 | F1D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 14.92 | 15.84 | 16.36 | 0               | .17  | .06 | .16 |
| 74 | F2C      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 76 | F2D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.63 | 16.19 | 16.90 | 0               | 1.01 | .11 | .03 |
| 77 | F2A      |      |      |      |      |        |       |       |       |                 |      |     |     |
| 78 | F2D      | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 16.34 | 17.16 | 0               | 0    | .06 | .12 |

Table 5.13 continued

| I   | TEST NO. | M    |      |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |     |     |     |
|-----|----------|------|------|------|------|--------|-------|-------|-------|-----------------|-----|-----|-----|
|     |          | N1   | N2   | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2  | N3  | N4  |
| 79  | NL1      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 80  | NL2      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 81  | NL3      | 3.73 | 3.30 | 3.51 | 3.65 | 17.46  | 16.64 | 17.04 | 17.52 | .79             | .11 | .46 | .45 |
| 82  | NL5      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 83  | NL6      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 84  | NL7      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 85  | NL9      | 3.73 | 3.30 | 3.51 | 3.65 | 16.35  | 15.07 | 16.05 | 16.57 | .04             | .05 | .05 | .06 |
| 86  | NL13     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 87  | NL15     | 3.73 | 3.30 | 3.51 | 3.65 | 15.91  | 14.86 | 15.69 | 16.19 | .08             | .07 | .10 | .16 |
| 88  | NL18     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 89  | NL19     | 3.73 | 3.30 | 3.51 | 3.65 | 16.01  | 15.75 | 16.57 | 17.11 | .31             | .53 | .31 | .26 |
| 90  | NL22     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 91  | NL23     | 3.73 | 3.30 | 3.51 | 3.65 | 16.65  | 15.43 | 16.49 | 16.98 | .45             | .36 | .12 | .10 |
| 92  | NL24     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 93  | NL25     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 94  | NL26     | 3.73 | 3.30 | 3.51 | 3.65 | 17.29  | 15.82 | 16.64 | 17.12 | .18             | .22 | .18 | .17 |
| 95  | NL30     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 96  | NL33     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.62 | 16.46 | 16.97 | 0               | .12 | .10 | .10 |
| 97  | NL37     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 98  | NL38     | 3.73 | 3.30 | 3.51 | 3.65 | 17.06  | 15.83 | 16.63 | 17.09 | .01             | .02 | .00 | .04 |
| 99  | IIE      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 100 | IVC      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 101 | ID       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 102 | IIID     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 103 | IIC      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 104 | IVD      | 3.73 | 3.30 | 3.51 | 3.65 | 16.59  | 15.33 | 16.45 | 17.07 | .36             | .29 | .19 | .15 |
| 105 | IIIE     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 106 | IVE      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 107 | IID      |      |      |      |      |        |       |       |       |                 |     |     |     |
| 108 | IIIC     | 3.73 | 3.30 | 3.51 | 3.65 | 16.57  | 15.21 | 16.21 | 17.02 | .19             | .25 | .23 | .07 |
| 110 | NJR3     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 111 | NJR5     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 112 | NJR6     | 3.73 | 3.30 | 3.51 | 3.65 | 16.53  | 15.42 | 0     | 17.17 | .42             | .38 | 0   | .20 |
| 113 | NJR2     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 114 | NJR4     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.70 | 0     | 17.10 | 0               | .74 | 0   | .09 |
| 115 | NURA     |      |      |      |      |        |       |       |       |                 |     |     |     |
| 116 | NJRB     | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 17.09 | 0               | 0   | 0   | .09 |

Table 5.13 continued

| I   | TEST<br>No. | M    |      |      |      | LOG(K) |       |       |       | S.D. FOR LOG(K) |     |     |     |
|-----|-------------|------|------|------|------|--------|-------|-------|-------|-----------------|-----|-----|-----|
|     |             | N1   | N2   | N3   | N4   | N1     | N2    | N3    | N4    | N1              | N2  | N3  | N4  |
| 117 | 707/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 118 | 707/2       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.66 | 0     | 17.62 | 0               | .35 | 0   | .25 |
| 119 | 709/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 120 | 709/2       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.58 | 0     | 17.49 | 0               | .09 | 0   | .00 |
| 121 | 711/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 122 | 711/2       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.25 | 0     | 17.77 | 0               | .14 | 0   | .22 |
| 123 | 701/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 124 | 701/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 17.69 | 0               | 0   | 0   | .10 |
| 125 | 702/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 126 | 702/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 127 | 702/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.64 | 16.92 | 17.71 | 0               | .49 | .23 | .16 |
| 128 | 703/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 129 | 703/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 130 | 703/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.30 | 17.02 | 17.53 | 0               | .04 | .04 | .73 |
| 131 | 704/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 132 | 704/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 133 | 704/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.71 | 17.22 | 18.03 | 0               | .24 | .38 | .24 |
| 134 | 705/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 135 | 705/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 136 | 705/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.72 | 16.98 | 17.82 | 0               | .40 | .19 | .10 |
| 137 | 621/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 138 | 621/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 139 | 621/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 16.97 | 0               | 0   | 0   | .27 |
| 140 | 622/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 141 | 622/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 142 | 622/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 16.89 | 0               | 0   | 0   | .12 |
| 143 | 623/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 144 | 623/2       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 15.13 | 16.53 | 17.46 | 0               | .31 | .13 | .09 |
| 145 | 624/1       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 146 | 624/2       |      |      |      |      |        |       |       |       |                 |     |     |     |
| 147 | 624/3       | 3.73 | 3.30 | 3.51 | 3.65 | 0      | 0     | 0     | 16.88 | 0               | 0   | 0   | .39 |

Table 5.13 continued

**Table 5.14** Estimated Standard Deviation representing the variation of  $\log_{10}K$  within sets of nominally identical cases.

|                              | Stage of Fatigue Failure |       |       |       |
|------------------------------|--------------------------|-------|-------|-------|
|                              | N1                       | N2    | N3    | N4    |
| Number of test results       | 30                       | 106   | 94    | 133   |
| Estimated Standard Deviation | 0.371                    | 0.353 | 0.213 | 0.215 |

**Table 5.15** Statistics representing the variations between the values of  $\log_{10}K$  found for different sets of nominally identical cases.

Note: each value of  $\log_{10}K$  in table 5.13 has been given equal weighting in calculating these statistics.

|                                  | Stage of Fatigue Failure |         |        |        |
|----------------------------------|--------------------------|---------|--------|--------|
|                                  | N1                       | N2      | N3     | N4     |
| Number of values of $\log_{10}K$ | 10                       | 40      | 35     | 48     |
| Mean                             | 16.72 <sup>+</sup>       | 15.45   | 16.47  | 16.86  |
| Modified Mean                    | -                        | 15.52** | 16.51* | 16.89* |
| Standard Deviation               | 0.456                    | 0.541   | 0.492  | 0.484  |
| Modified Standard Deviation      | -                        | 0.451** | 0.440* | 0.444* |

\* These results have been modified by excluding one outlier: the value of  $\log_{10}K$  for tests E2A and E2D.

\*\* These results have been modified by excluding two outliers: the values of  $\log_{10}K$  for tests E2A and E2D, and F1A and F1B.

+ This value is higher than the means for N2 and N3 because it is derived from a limited number of test results which gave above average values of  $\log_{10}K$  for all stages of failure.

Table 5.16 Accuracy of Miner's Rule

| Country of Origin | Paper number<br>in ref. 80 | Range of Miner's Rule Sum<br>at failure |
|-------------------|----------------------------|-----------------------------------------|
| Netherlands*      | { PS 5<br>ST 7.3           | 0.75 to 1.0<br>1.0 to 1.3               |
| United Kingdom    | ST 7.2                     | 0.5 to 1.1                              |
| Germany           | PS 5                       | 0.4 to 0.8                              |

- \* In the author's interpretation these two papers give contradictory conclusions from the same tests. There is insufficient discussion of the origin of these figures to enable the problem to be resolved.

Table 6.1 Data for Reliability Analysis

| Variable                       | Distribution Type                  | Uncertainty State |          |       |          |       |          |
|--------------------------------|------------------------------------|-------------------|----------|-------|----------|-------|----------|
|                                |                                    | 1                 |          | 2     |          | 3     |          |
|                                |                                    | m                 | $\sigma$ | m     | $\sigma$ | m     | $\sigma$ |
| n'                             | lognormal                          | 1.0               | 0.025    | 1.0   | 0.05     | 1.0   | 0.2      |
| C <sub>m</sub>                 | lognormal<br>(origin=1.0/2.0=0.5)  | 1.0               | 0        | 0.9   | 0.025    | 0.9   | 0.135    |
| r                              | lognormal<br>(origin=1.0/1.11=0.9) | 1.0               | 0.04     | 1.0   | 0.04     | 1.0   | 0.04     |
| SCF                            | lognormal                          | 1.0               | 0        | 1.0   | 0.1      | 1.0   | 0.3      |
| m                              | constant                           | 3.5               | —        | 3.5   | —        | 3.5   | —        |
| K { worst case<br>average case | lognormal                          | 3.13              | 1.69     | 3.13  | 1.69     | 19.40 | 32.21    |
|                                | lognormal                          | 19.40             | 10.48    | 19.40 | 10.48    |       |          |
| $\lambda_w$                    | beta                               | 1.0               | 0        | 1.0   | 0        | 0.8   | 0.025    |
|                                |                                    | m $\sigma$        |          |       |          |       |          |
| M                              | lognormal                          | 1.15    0.3       |          |       |          |       |          |
| $\lambda_c$                    | lognormal                          | 0.8     0.15      |          |       |          |       |          |

Note: data is for variables (excluding m) expressed as ratios to design value. M and  $\lambda_c$  are not classified under uncertainty levels.

Table 6.2 Probability of Failure of Single Connections

| Uncertainty State<br>Fatigue Detail Case                  | 1                    |                      | 2                    |                      | 3                    |
|-----------------------------------------------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|                                                           | 'worst'              | 'average'            | 'worst'              | 'average'            |                      |
| Excluding Miner's<br>rule and corrosion<br>uncertainties: |                      |                      |                      |                      |                      |
| $\gamma=1.0$                                              | $3.5 \times 10^{-2}$ | $1.0 \times 10^{-5}$ | $1.8 \times 10^{-2}$ | $6.7 \times 10^{-6}$ | $2.7 \times 10^{-2}$ |
| 2.0                                                       | $2.1 \times 10^{-3}$ | $3.8 \times 10^{-7}$ | $1.2 \times 10^{-3}$ | $2.5 \times 10^{-7}$ | $9.8 \times 10^{-3}$ |
| 4.0                                                       | $8.3 \times 10^{-5}$ | $1.5 \times 10^{-8}$ | $5.4 \times 10^{-5}$ | $9.8 \times 10^{-9}$ | $3.0 \times 10^{-3}$ |
| For $\gamma=2.0$ :                                        |                      |                      |                      |                      |                      |
| Data as above                                             | $2.1 \times 10^{-3}$ | $3.8 \times 10^{-7}$ | $1.2 \times 10^{-3}$ | $2.5 \times 10^{-7}$ | $9.8 \times 10^{-3}$ |
| " " " plus<br>Miner's Rule<br>uncertainty                 | $2.4 \times 10^{-3}$ | $4.6 \times 10^{-7}$ | $1.3 \times 10^{-3}$ | $3.1 \times 10^{-7}$ | $8.8 \times 10^{-3}$ |
| Data as above plus<br>Corrosion<br>uncertainty            | $7.8 \times 10^{-3}$ | $1.7 \times 10^{-6}$ | $4.2 \times 10^{-3}$ | $1.1 \times 10^{-6}$ | $1.5 \times 10^{-2}$ |

Table 6.3 Failure of Single Connections -  $\beta$  values

| Uncertainty State<br>Fatigue Detail Case           | 1       |           | 2       |           | 3    |
|----------------------------------------------------|---------|-----------|---------|-----------|------|
|                                                    | 'worst' | 'average' | 'worst' | 'average' |      |
| Excluding Miner's Rule and corrosion uncertainties |         |           |         |           |      |
| $\gamma=1.0$                                       | 1.81    | 4.26      | 2.10    | 4.35      | 1.92 |
| 2.0                                                | 2.86    | 4.95      | 3.03    | 5.03      | 2.34 |
| 4.0                                                | 3.77    | 5.54      | 3.87    | 5.62      | 2.75 |
| For $\gamma=2.0$ :                                 |         |           |         |           |      |
| As above                                           | 2.86    | 4.95      | 3.03    | 5.03      | 2.34 |
| " " plus Miner's Rule uncertainty                  | 2.83    | 4.91      | 3.01    | 4.99      | 2.37 |
| As above plus Corrosion uncertainty                | 2.42    | 4.65      | 2.63    | 4.73      | 2.18 |

Table 6.4 Sensitivity Factors for Failure of Single Connections

| Variable       | Uncertainty State |           |         |           |       |
|----------------|-------------------|-----------|---------|-----------|-------|
|                | 1                 |           | 2       |           | 3     |
|                | 'worst'           | 'average' | 'worst' | 'average' |       |
| n'             | -.035             | -.023     | -.064   | -.045     | -.118 |
| C <sub>m</sub> | -                 | -         | -.127   | -.089     | -.338 |
| r              | -.695             | -.885     | -.593   | -.824     | -.172 |
| scf            | -                 | -         | -.450   | -.317     | -.611 |
| K              | .719              | .464      | .652    | .459      | .684  |
| $\lambda_w$    | -                 | -         | -       | -         | -.018 |

Table 6.5 Probability of at least one failure in n cases

| n   | Degree of independence between cases |              |       |
|-----|--------------------------------------|--------------|-------|
|     | Slight                               | Considerable | Total |
| 2   | .014                                 | .019         | .019  |
| 5   | .020                                 | .045         | .048  |
| 10  | .026                                 | .085         | .093  |
| 30  | .036                                 | .209         | .255  |
| 100 | .049                                 | .457         | .625  |

Table 6.6 Probability of all n cases failing

| n   | Degree of independence between cases |                       |                        |
|-----|--------------------------------------|-----------------------|------------------------|
|     | Slight                               | Considerable          | Total                  |
| 2   | $5.6 \times 10^{-3}$                 | $2.9 \times 10^{-4}$  | $9.5 \times 10^{-5}$   |
| 5   | $3.0 \times 10^{-3}$                 | $5.6 \times 10^{-7}$  | $8.8 \times 10^{-11}$  |
| 10  | $2.0 \times 10^{-3}$                 | $5.6 \times 10^{-9}$  | $7.8 \times 10^{-21}$  |
| 30  | $1.2 \times 10^{-3}$                 | $2.2 \times 10^{-11}$ | $4.7 \times 10^{-61}$  |
| 100 | $7.0 \times 10^{-4}$                 | $3.1 \times 10^{-13}$ | $8.0 \times 10^{-202}$ |

Table 6.7 Probability of at least one of cases  $B_1$  to  $B_n$  failing during required lifetime given that case A fails prematurely.

| n  | Degree of independence between cases B <sub>1</sub> to B <sub>n</sub> |     |      |                |      |      |
|----|-----------------------------------------------------------------------|-----|------|----------------|------|------|
|    | 'Slight'                                                              |     |      | 'Considerable' |      |      |
|    | Proportion of required lifetime during which case A fails             |     |      |                |      |      |
|    | 20%                                                                   | 50% | 100% | 20%            | 50%  | 100% |
| 1  | .99                                                                   | .84 | .57  | .045           | .035 | .029 |
| 2  | 1.0                                                                   | .95 | .72  | .086           | .067 | .056 |
| 4  | 1.0                                                                   | .99 | .85  | .16            | .13  | .11  |
| 9  | 1.0                                                                   | 1.0 | .96  | .30            | .25  | .21  |
| 29 | 1.0                                                                   | 1.0 | 1.0  | .64            | .55  | .49  |
| 99 | 1.0                                                                   | 1.0 | 1.0  | .94            | .89  | .85  |

Table 6.8 Probability that all of cases  $B_1$  to  $B_n$  fail during required lifetime given that case A fails prematurely.

| n  | Degree of independence between cases B <sub>1</sub> to B <sub>n</sub> |     |      |                       |                       |                       |
|----|-----------------------------------------------------------------------|-----|------|-----------------------|-----------------------|-----------------------|
|    | 'Slight'                                                              |     |      | 'Considerable'        |                       |                       |
|    | Proportion of required lifetime during which Case A fails             |     |      |                       |                       |                       |
|    | 20%                                                                   | 50% | 100% | 20%                   | 50%                   | 100%                  |
| 1  | .99                                                                   | .84 | .57  | 4.5x10 <sup>-2</sup>  | 3.5x10 <sup>-2</sup>  | 2.9x10 <sup>-2</sup>  |
| 2  | .98                                                                   | .74 |      | 4.6x10 <sup>-3</sup>  | 2.9x10 <sup>-3</sup>  | 2.1x10 <sup>-3</sup>  |
| 4  | .97                                                                   | .63 | .31  | 2.1x10 <sup>-4</sup>  | 9.9x10 <sup>-5</sup>  | 5.8x10 <sup>-5</sup>  |
| 9  | .93                                                                   | .50 | .21  | 3.9x10 <sup>-6</sup>  | 1.3x10 <sup>-6</sup>  | 5.8x10 <sup>-7</sup>  |
| 29 | .85                                                                   | .34 | .12  | 2.8x10 <sup>-8</sup>  | 6.3x10 <sup>-9</sup>  | 2.3x10 <sup>-9</sup>  |
| 99 | .72                                                                   | .22 | .072 | 5.1x10 <sup>-10</sup> | 9.8x10 <sup>-11</sup> | 3.2x10 <sup>-11</sup> |

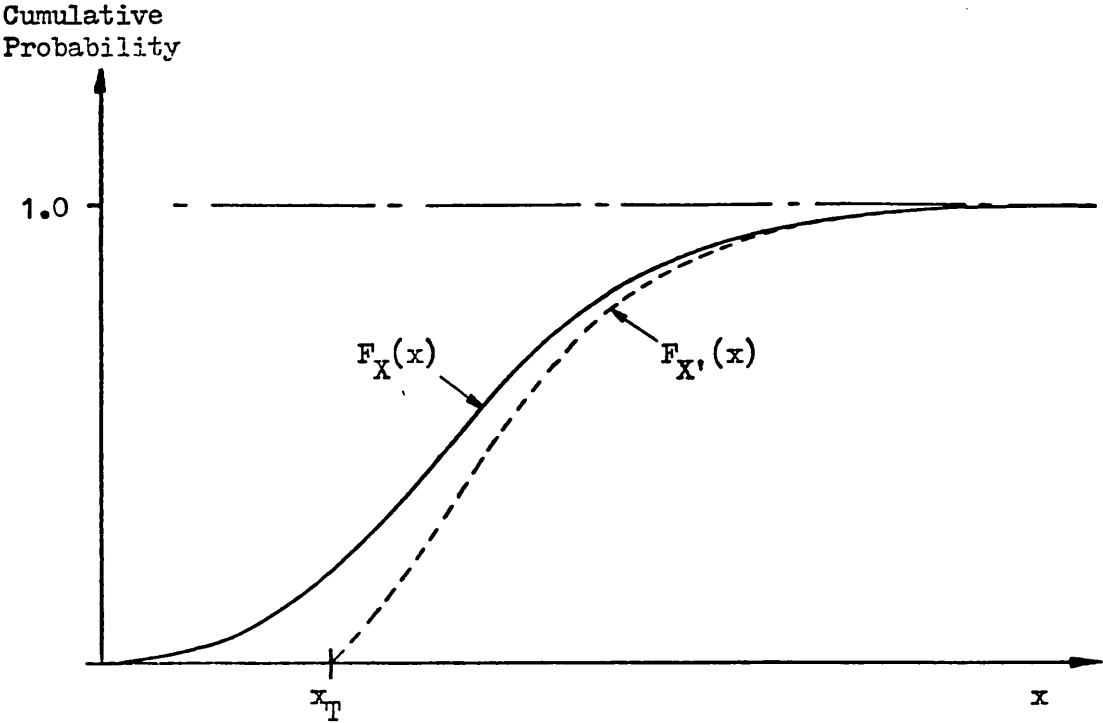


Fig. 2.1 Truncated Probability Distribution

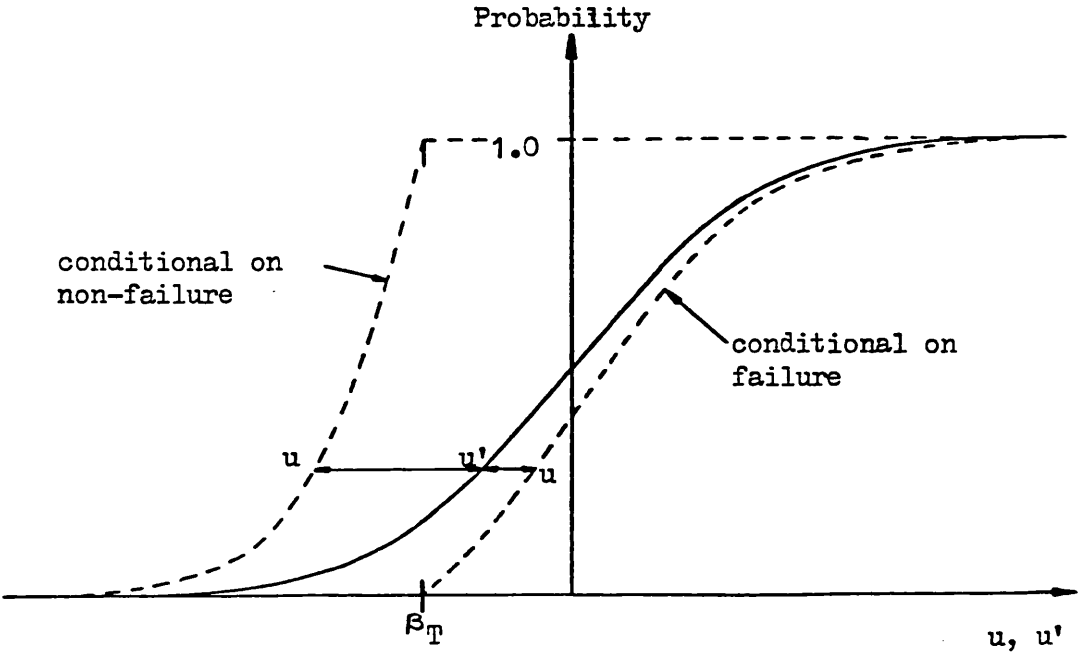


Fig. 2.2  $u-u'$  Transformation

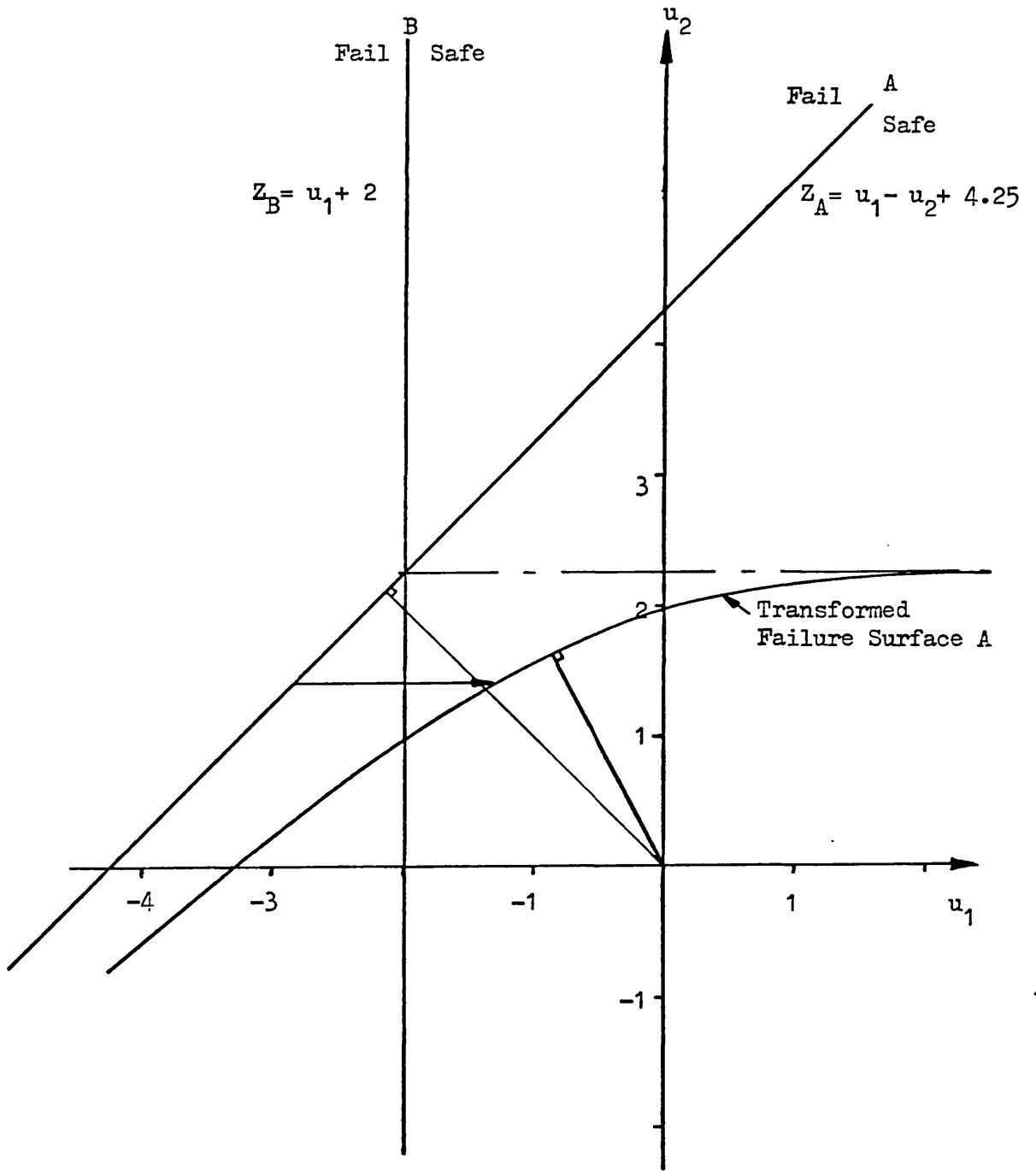


Fig. 2.3 Example of transformation of Failure Surface  
by Truncation and Re-normalisation of variable space

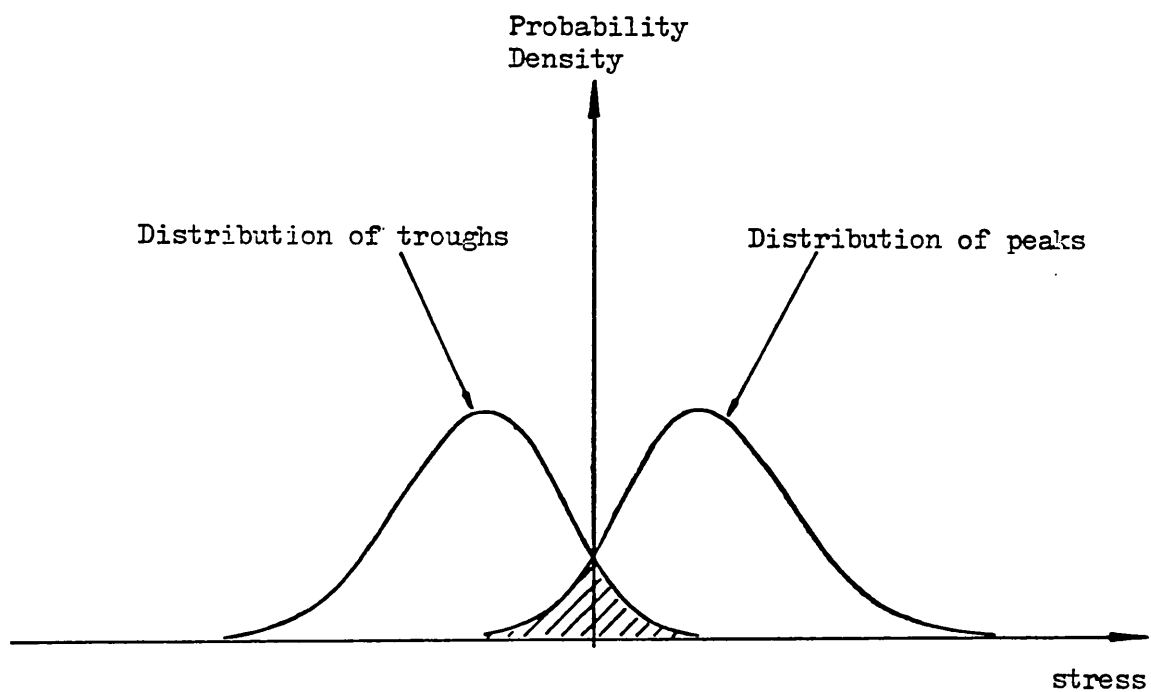


Fig. 3.1 Distribution of Peaks and Troughs for Gaussian Process

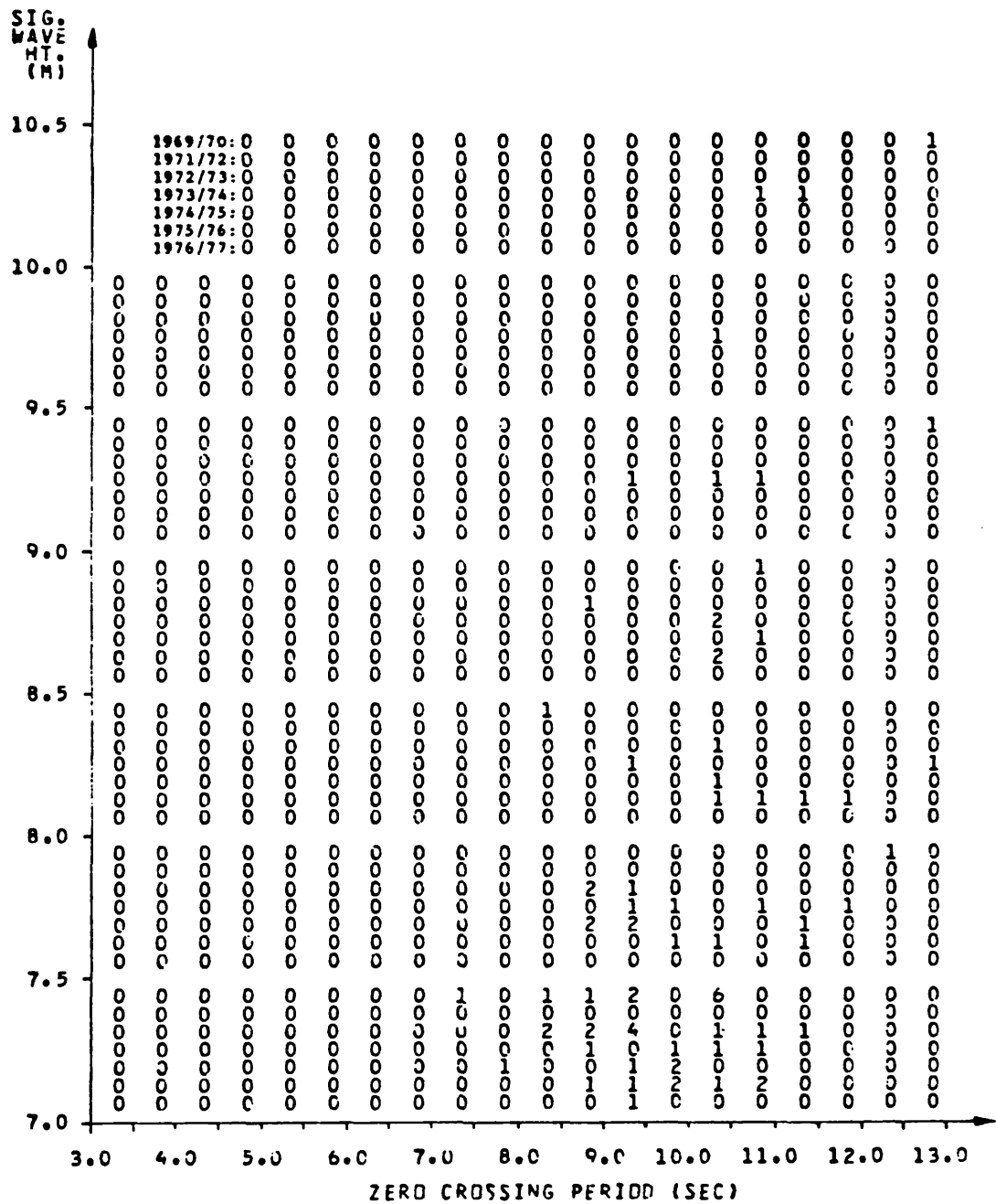


DIAGRAM CONTINUED ON NEXT PAGE

Fig. 5.1 Combined Scatter Diagram showing data for seven winters at m.v. Famita

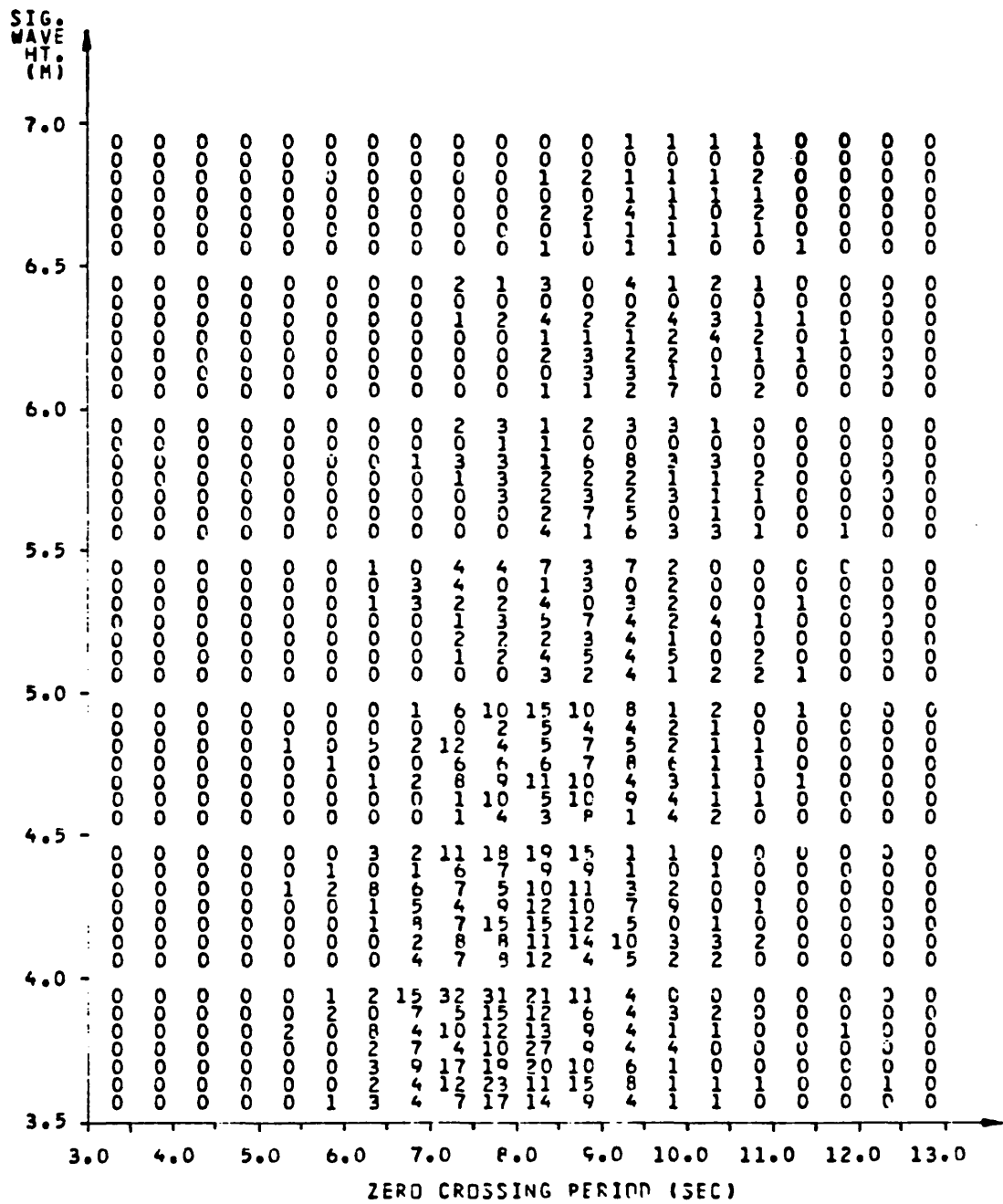


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Fig. 5.1 continued

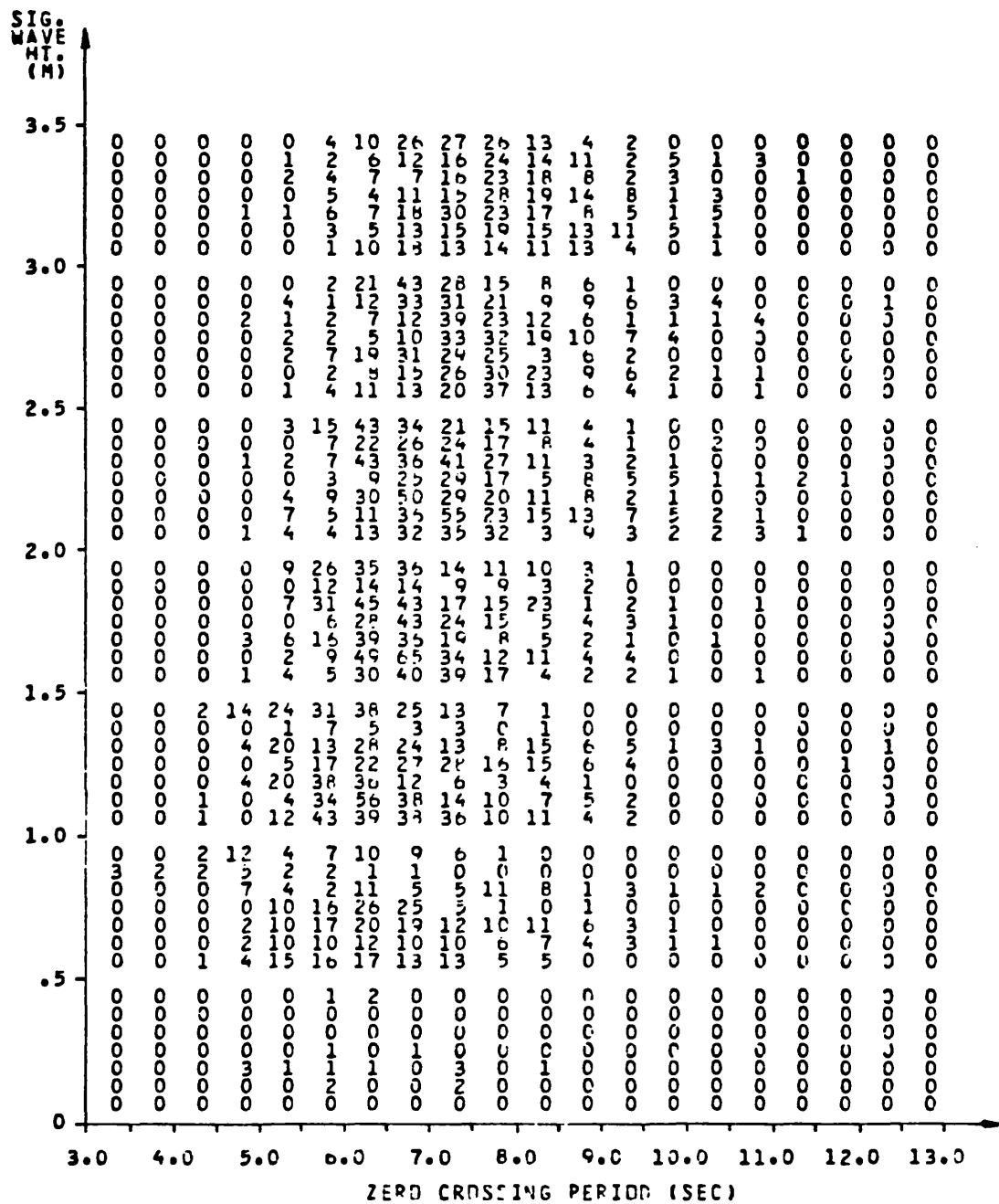


Fig. 5.1 continued

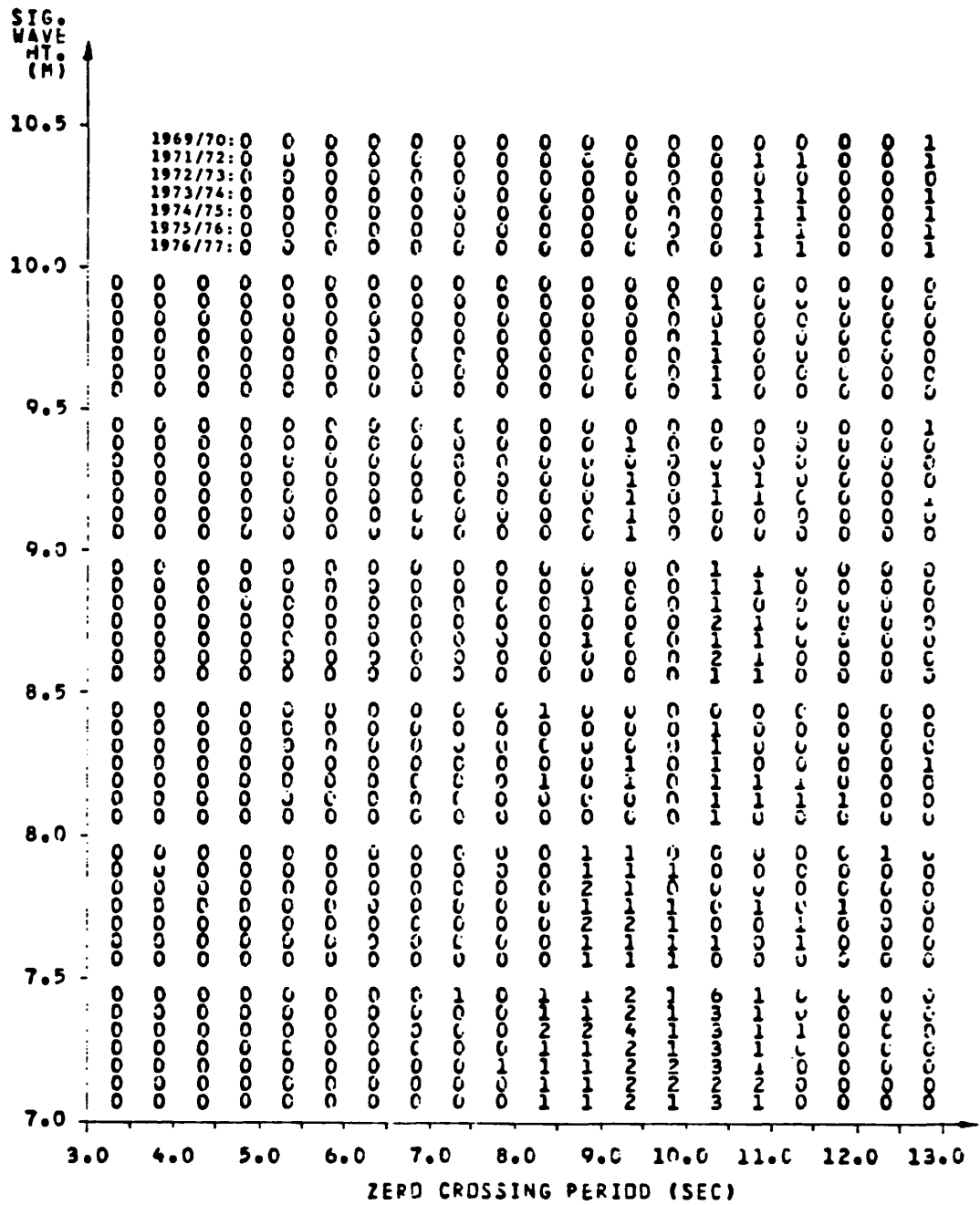


DIAGRAM CONTINUED ON NEXT PAGE

Fig. 5.2 Combined Scatter Diagram showing "filled" data for seven winters at m.v. Famita.

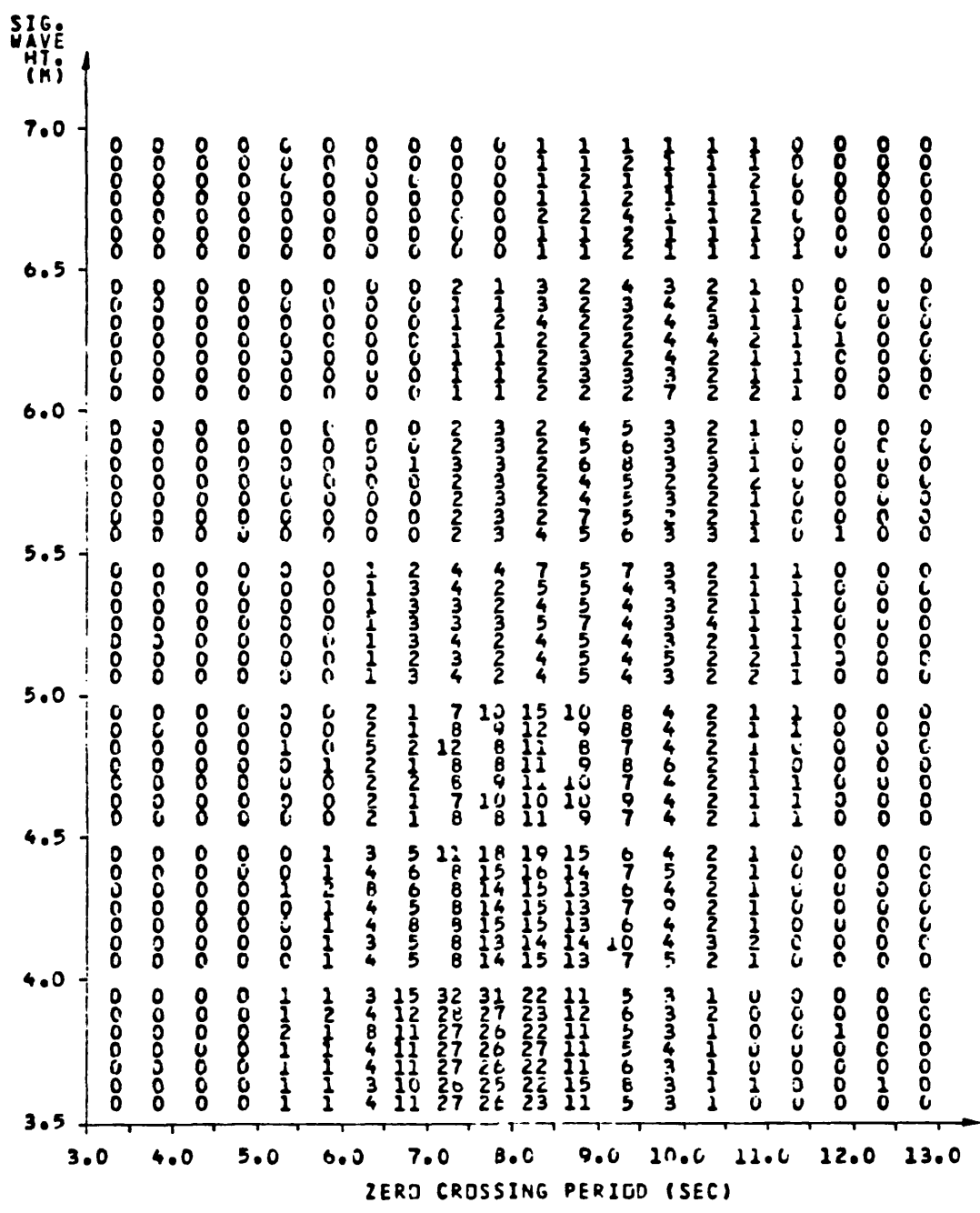


DIAGRAM CONTINUED ON NEXT PAGE

Fig. 5.2 continued

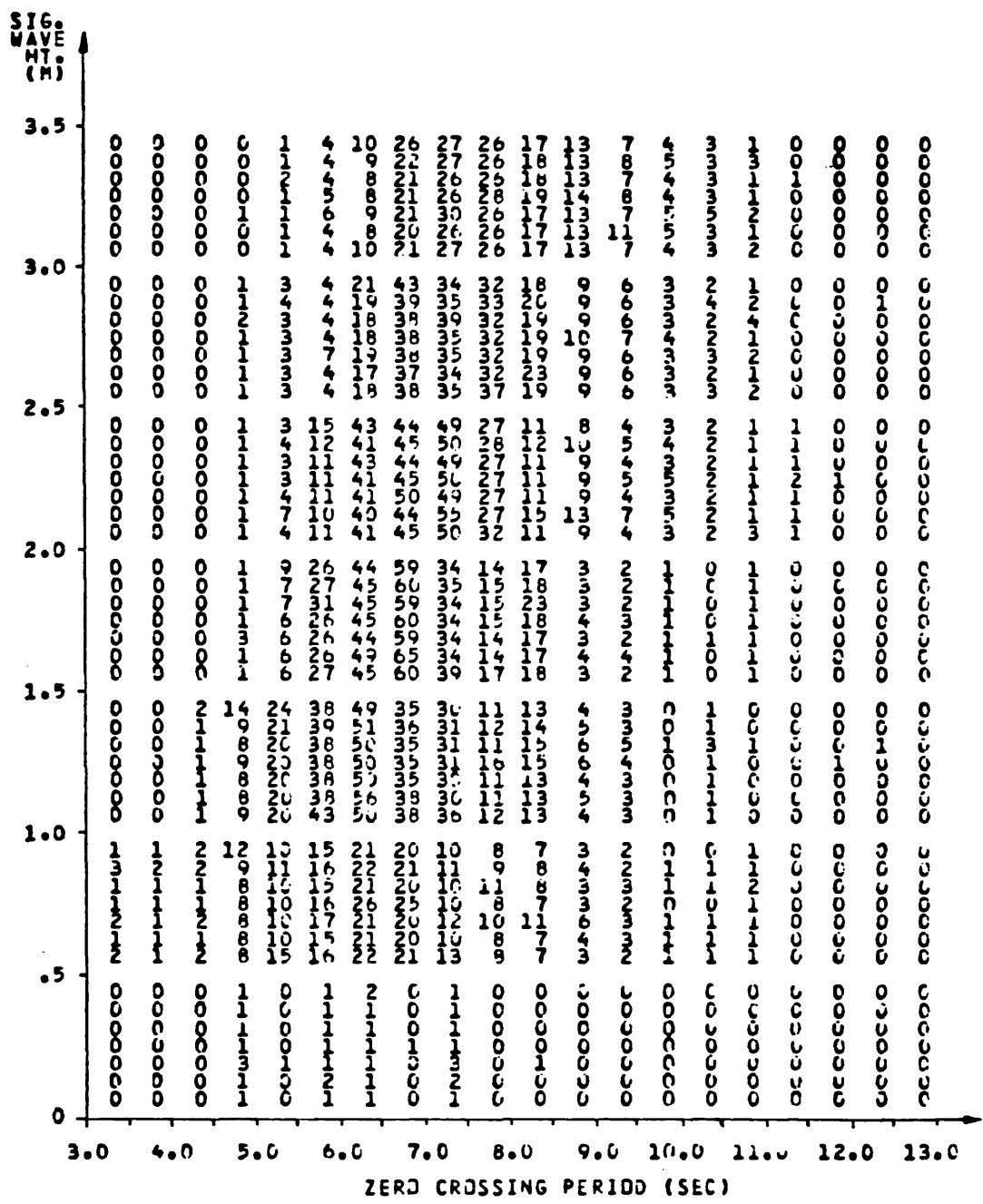


Fig. 5.2 continued

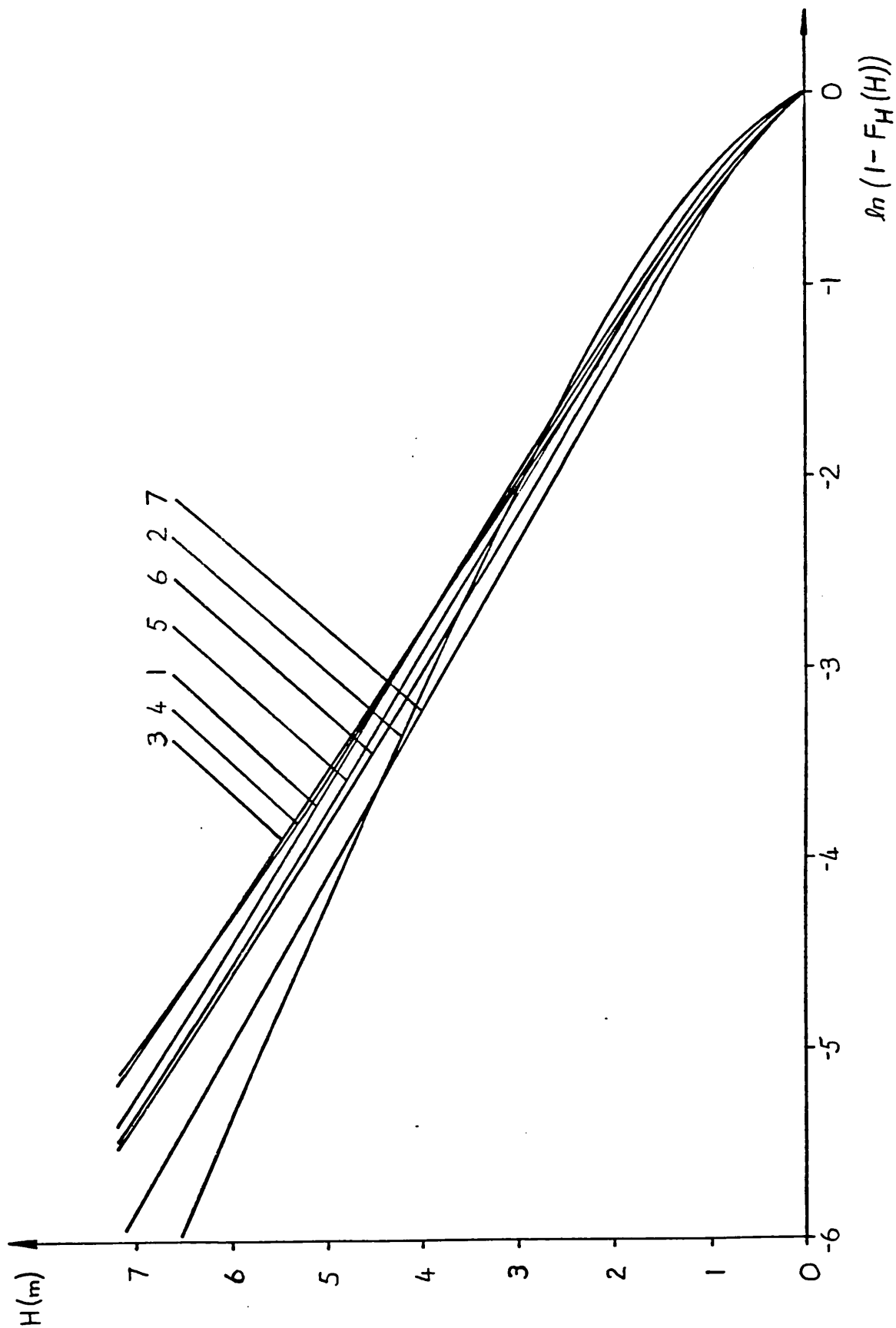


Fig. 5.3 Long-term distribution of Wave Height.

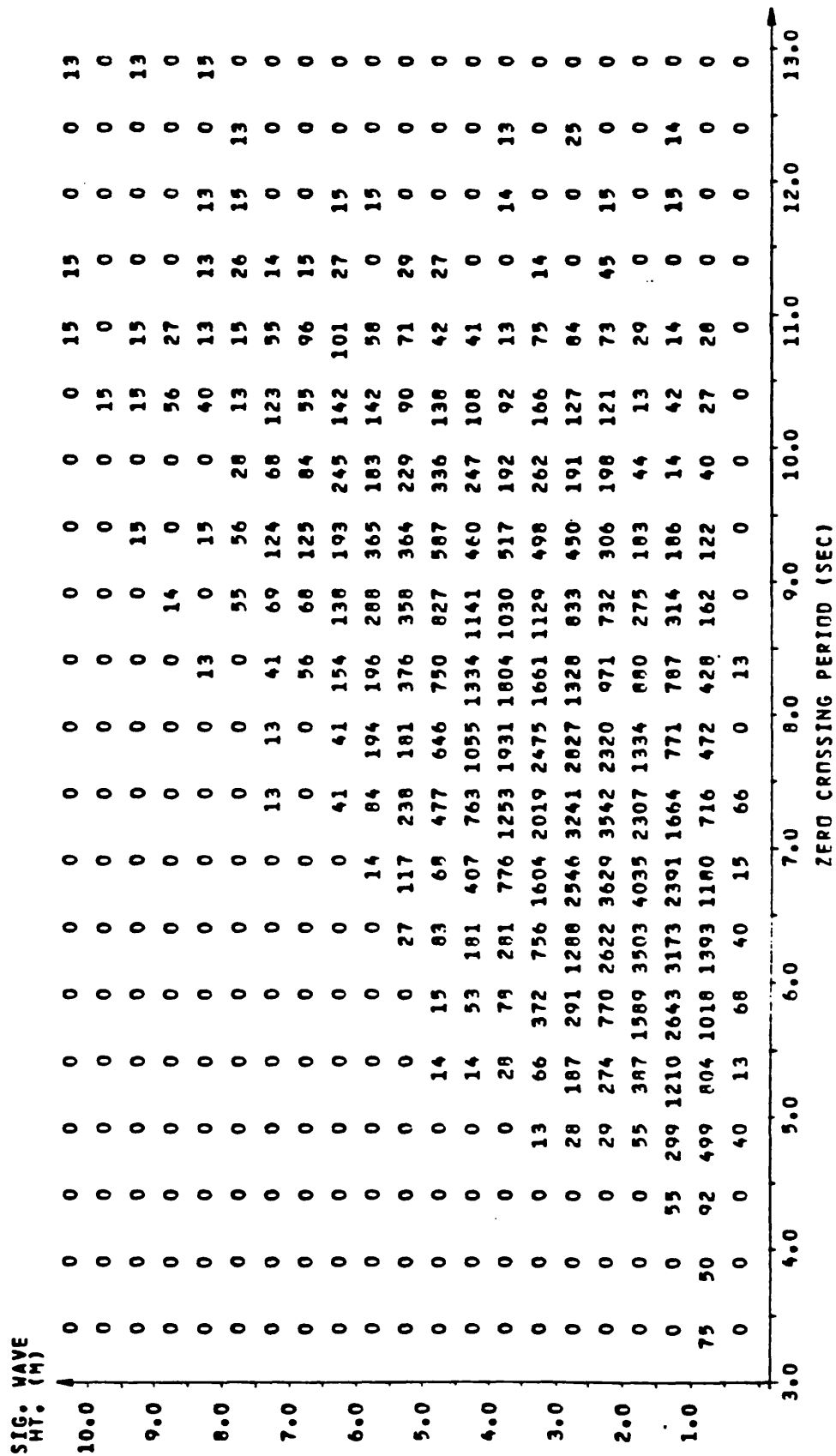


Fig. 5.4 Scatter Diagram showing mean frequency of occurrence (x 100000) of each sea-state.

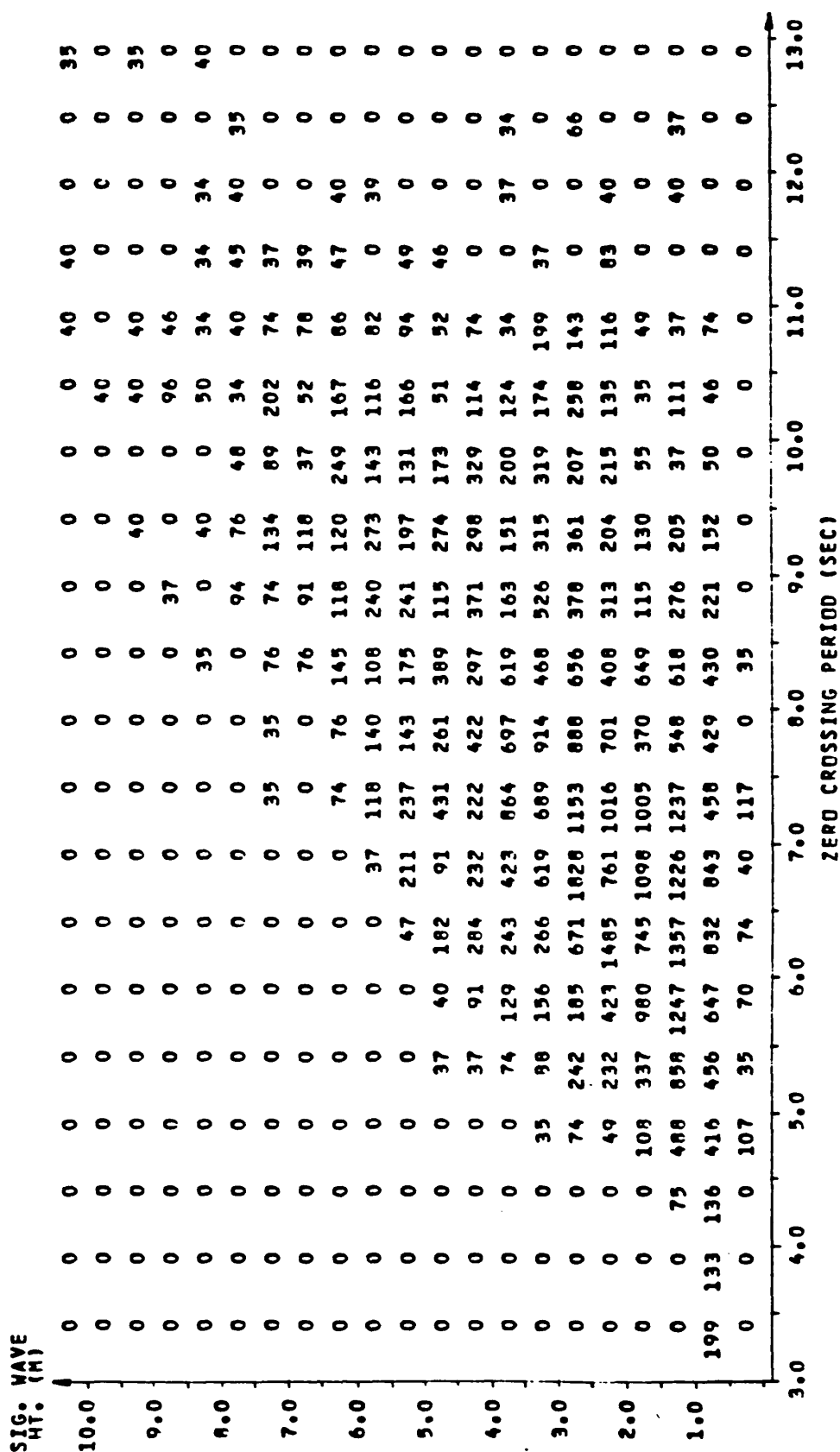


Fig. 5.5 Scatter Diagram showing standard deviation of frequency of occurrence (x100000) of each sea-state.

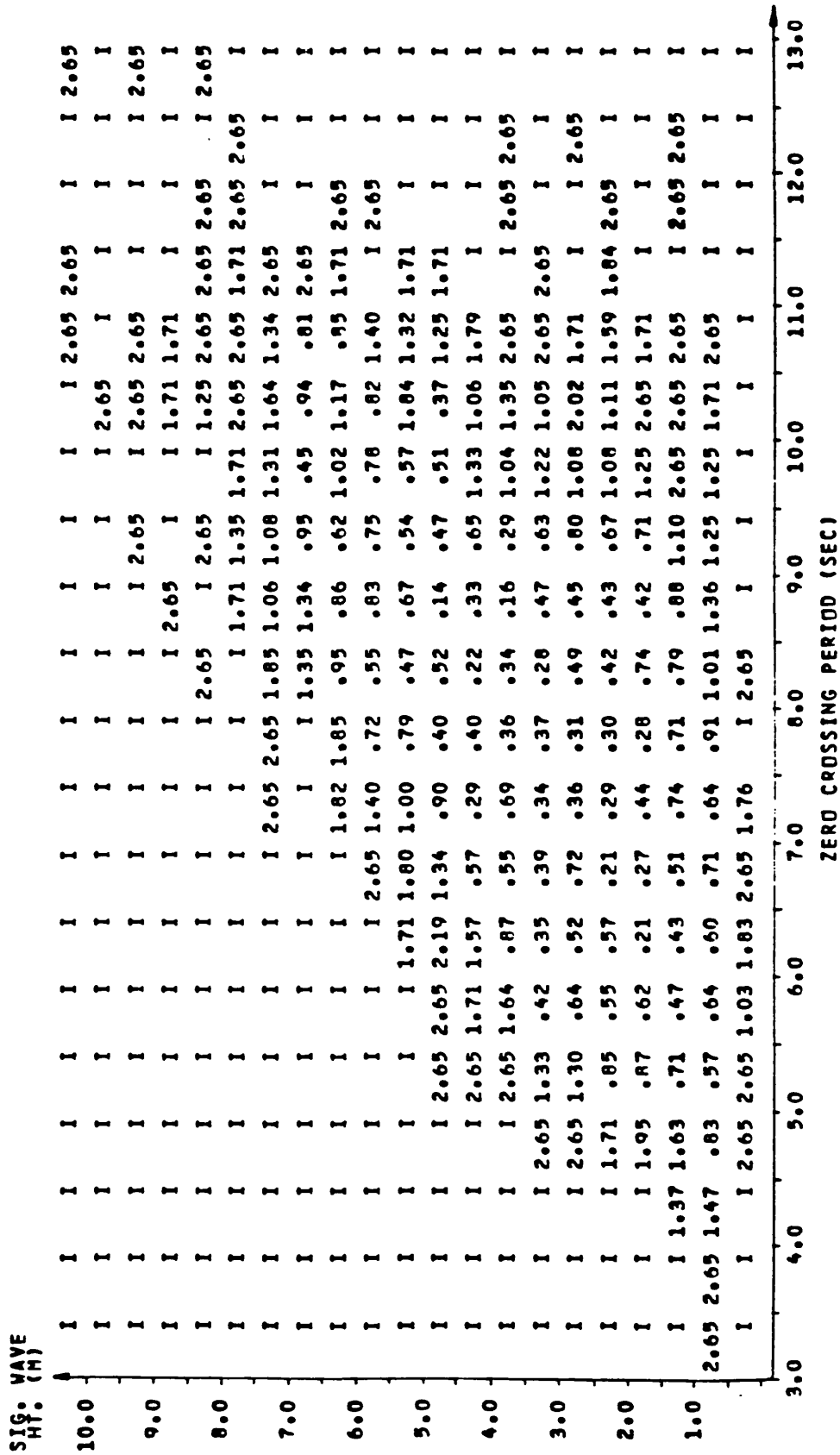


Fig. 5.6 Scatter Diagram showing coefficient of variation of frequency of occurrence of each sea-state.

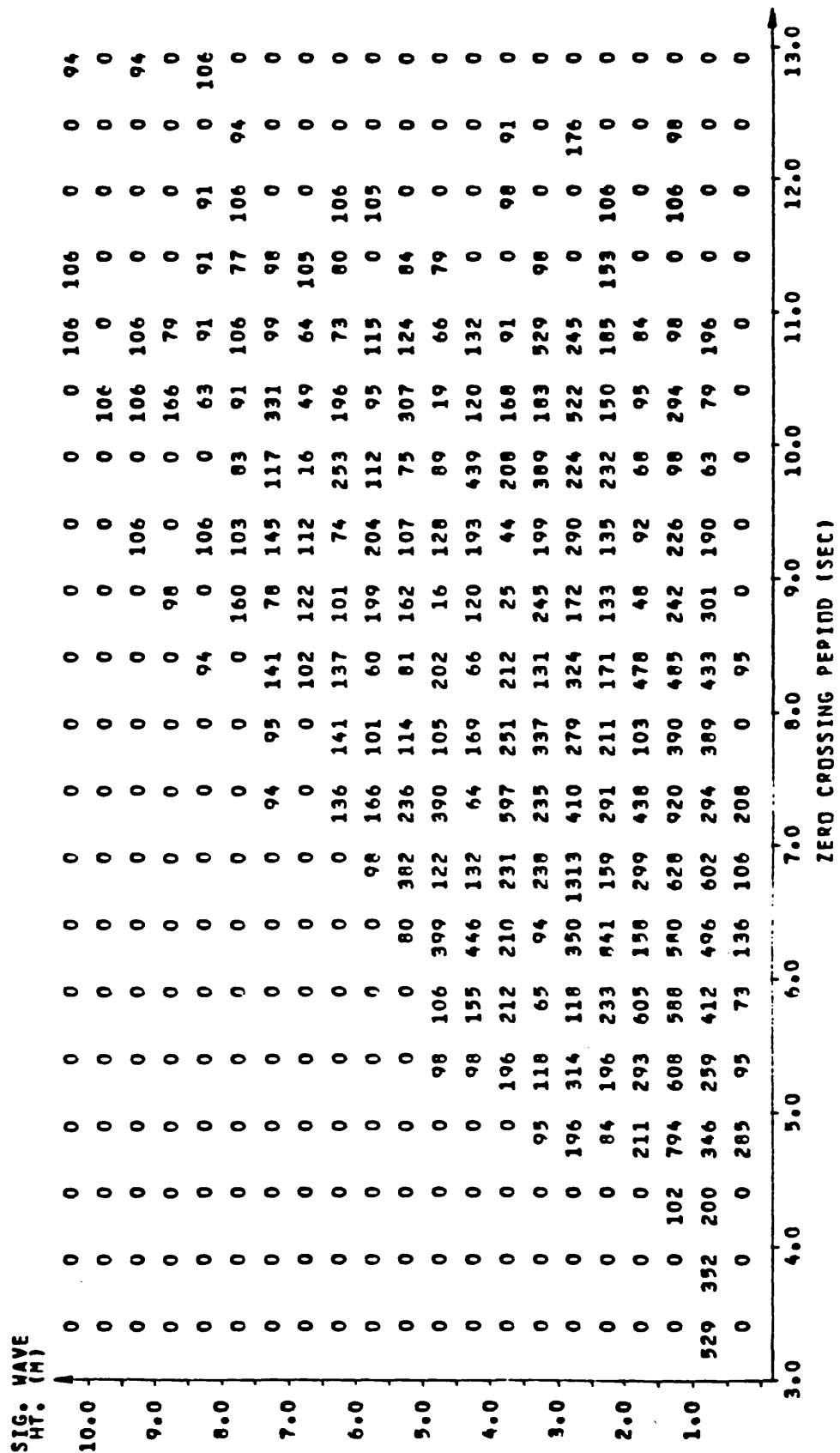


Fig. 5.7 Scatter Diagram showing variance/mean for frequency of occurrence (x 10000) of each sea-state.

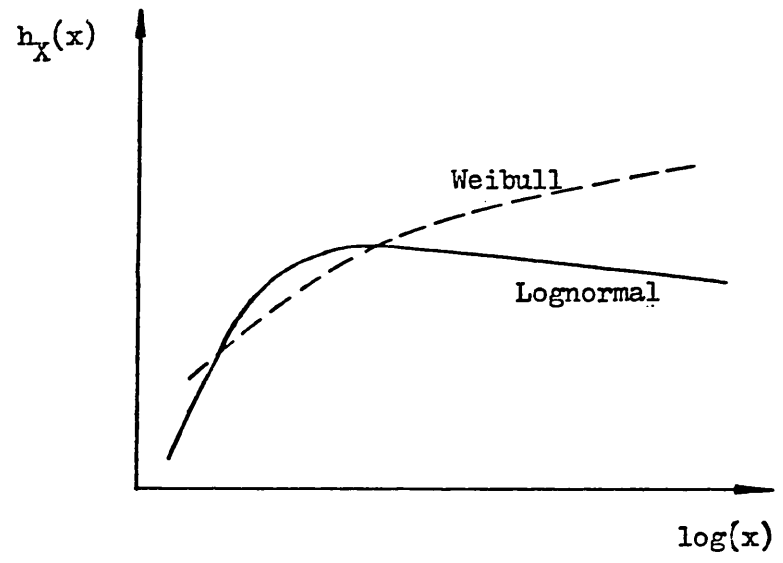


Fig. 5.8 Comparison of Hazard Functions for typical Lognormal and Weibull Distributions

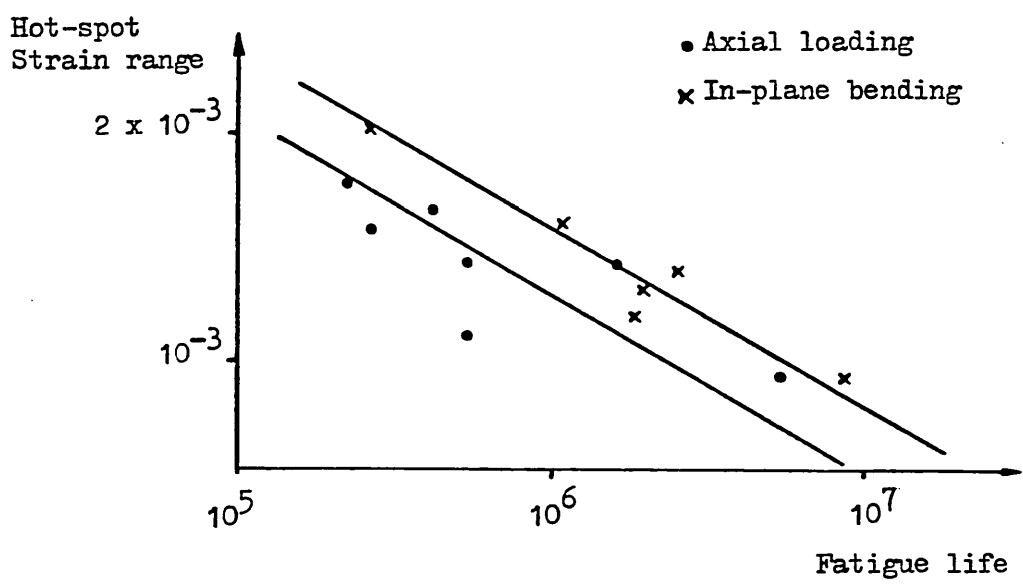


Fig. 5.9 Typical S-N Curve data and mean regression lines

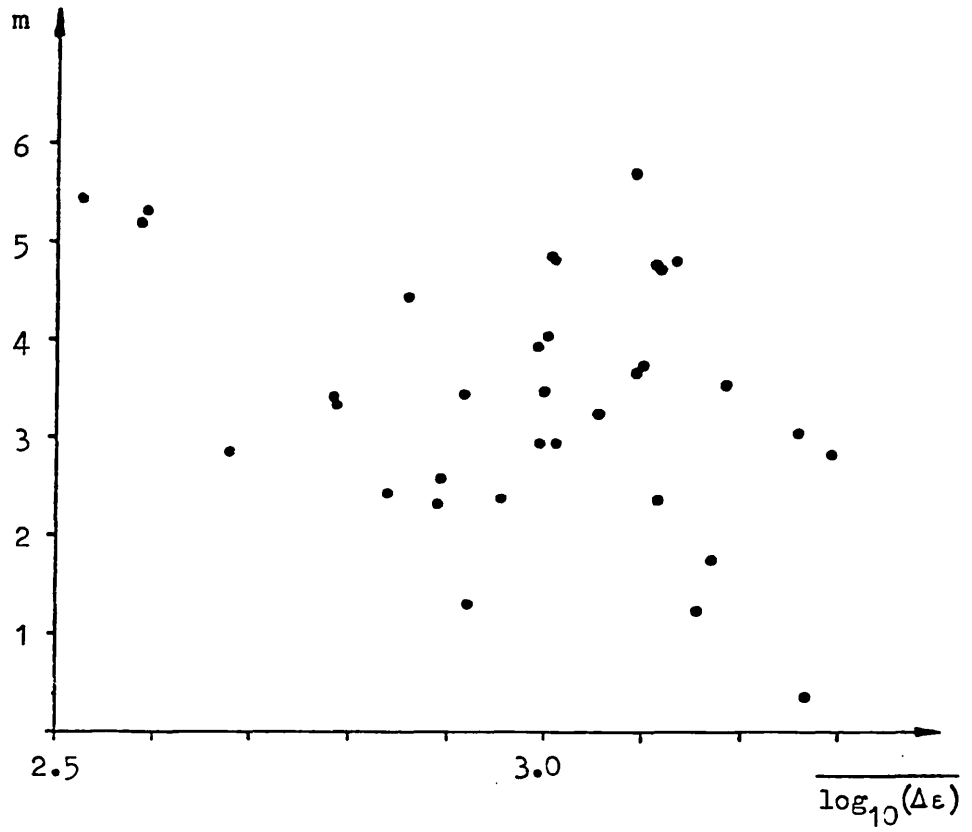


Fig. 5.10 Plot of values of  $m$  from data for fatigue life  $N_3$  against mean value of  $\log \Delta \epsilon$

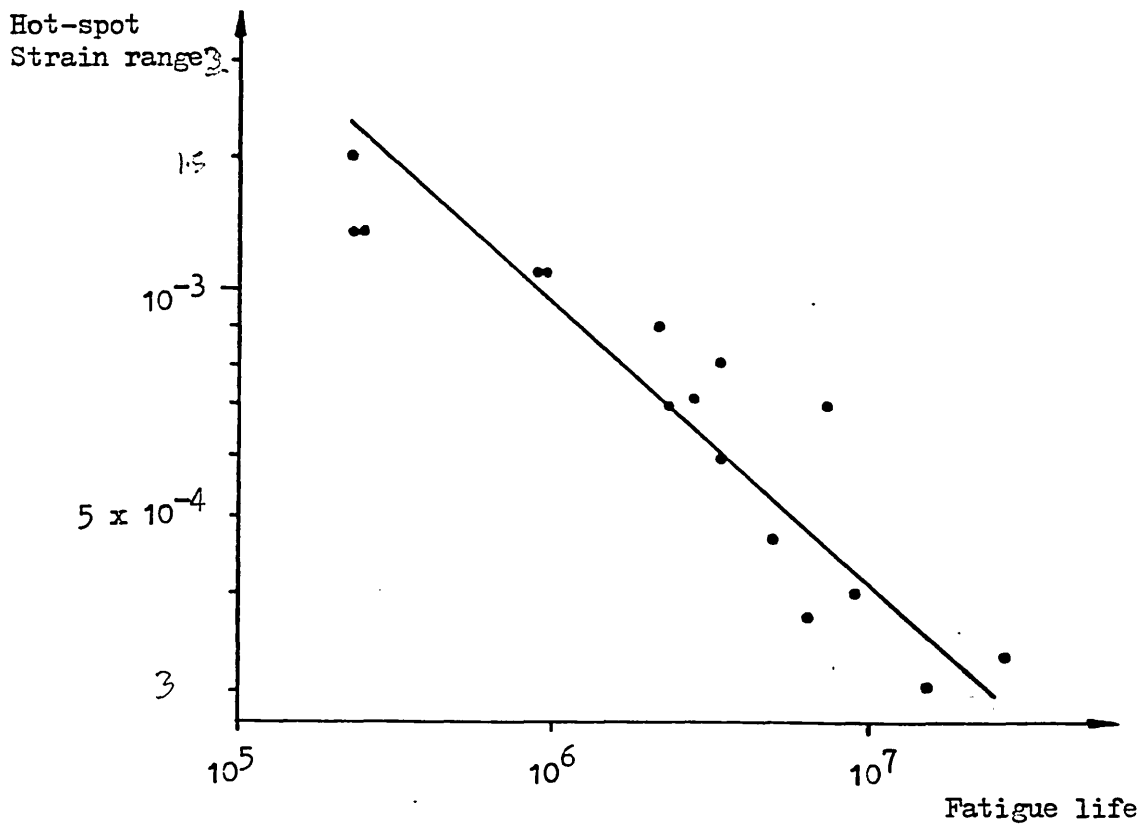


Fig. 5.11 Sixteen Fatigue Test Results taken as a single group.

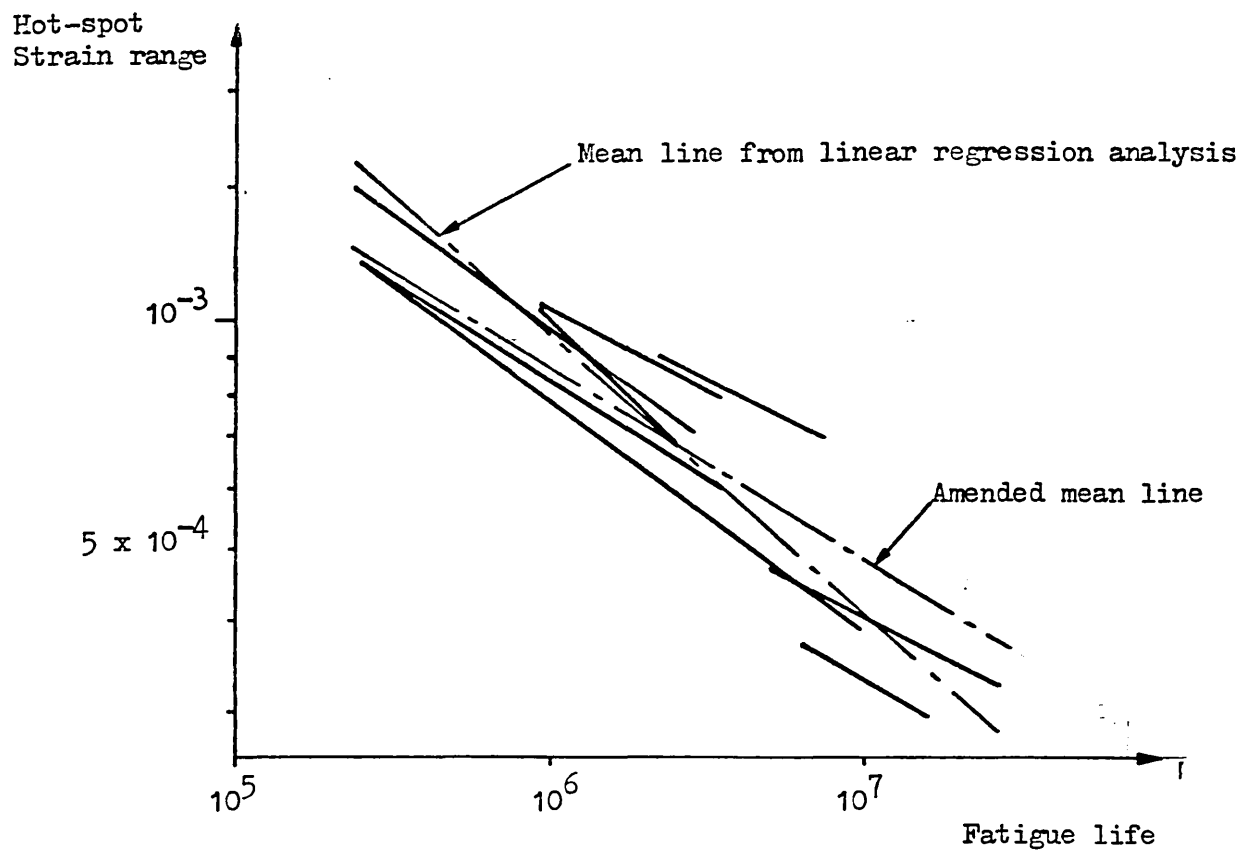


Fig. 5.12 Sixteen Fatigue Test Results taken as eight pairs

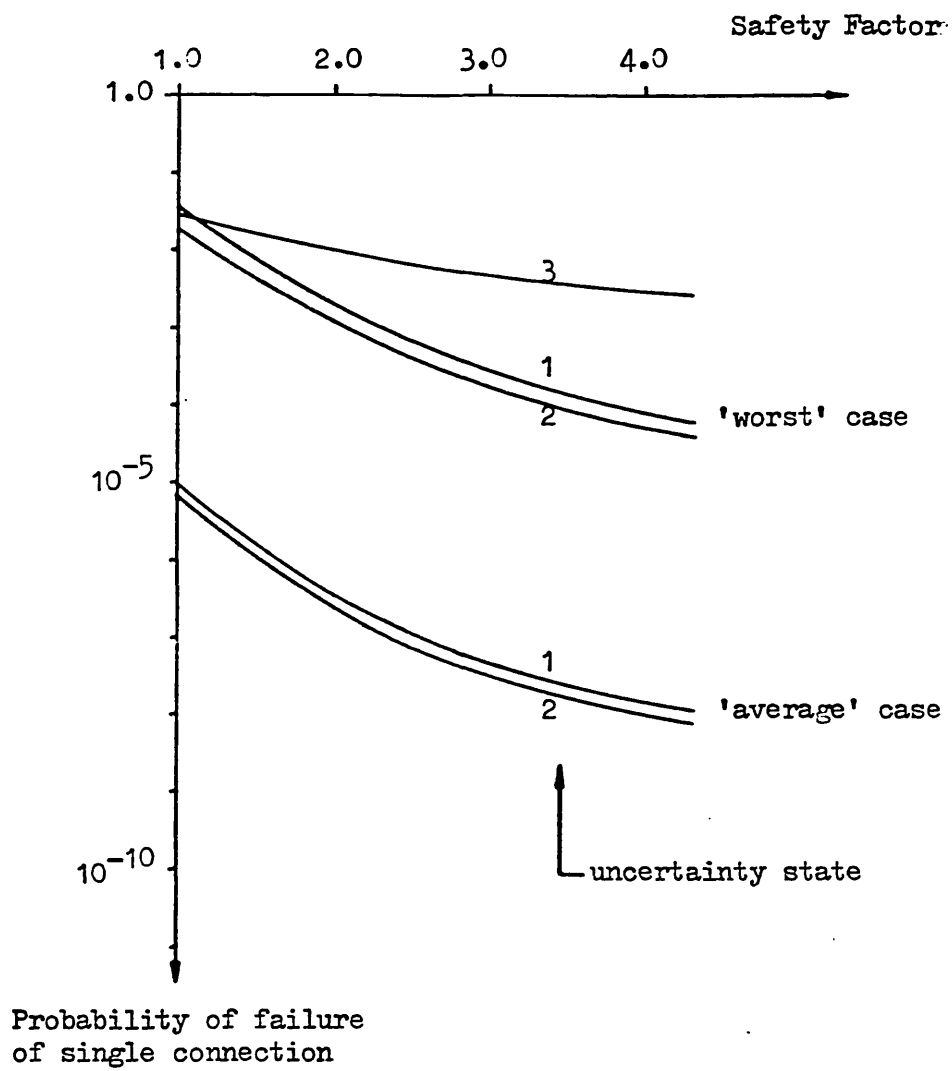


Fig. 6.1 Probability of Failure of single connections

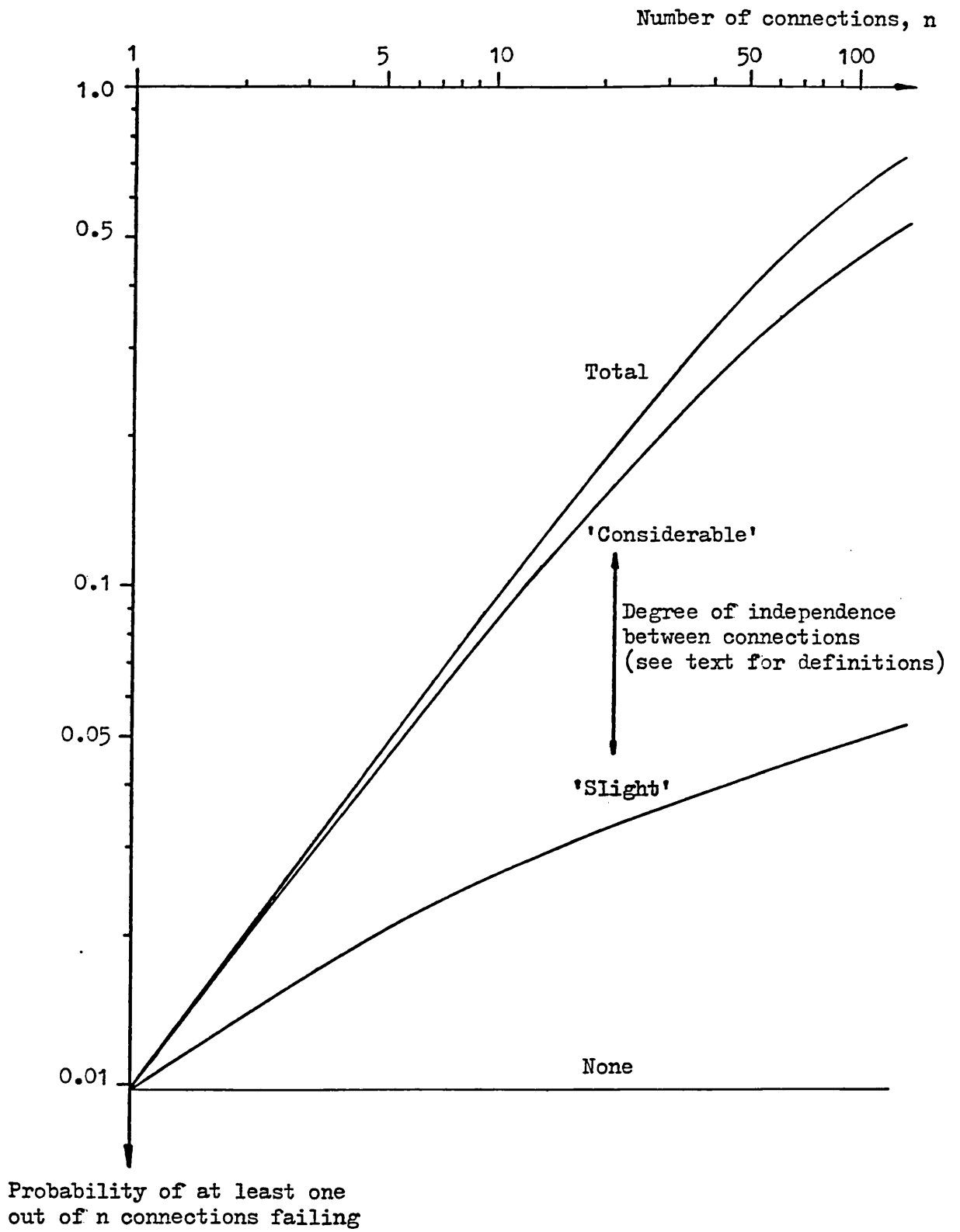


Fig. 6.2 Probability of at least one failure.

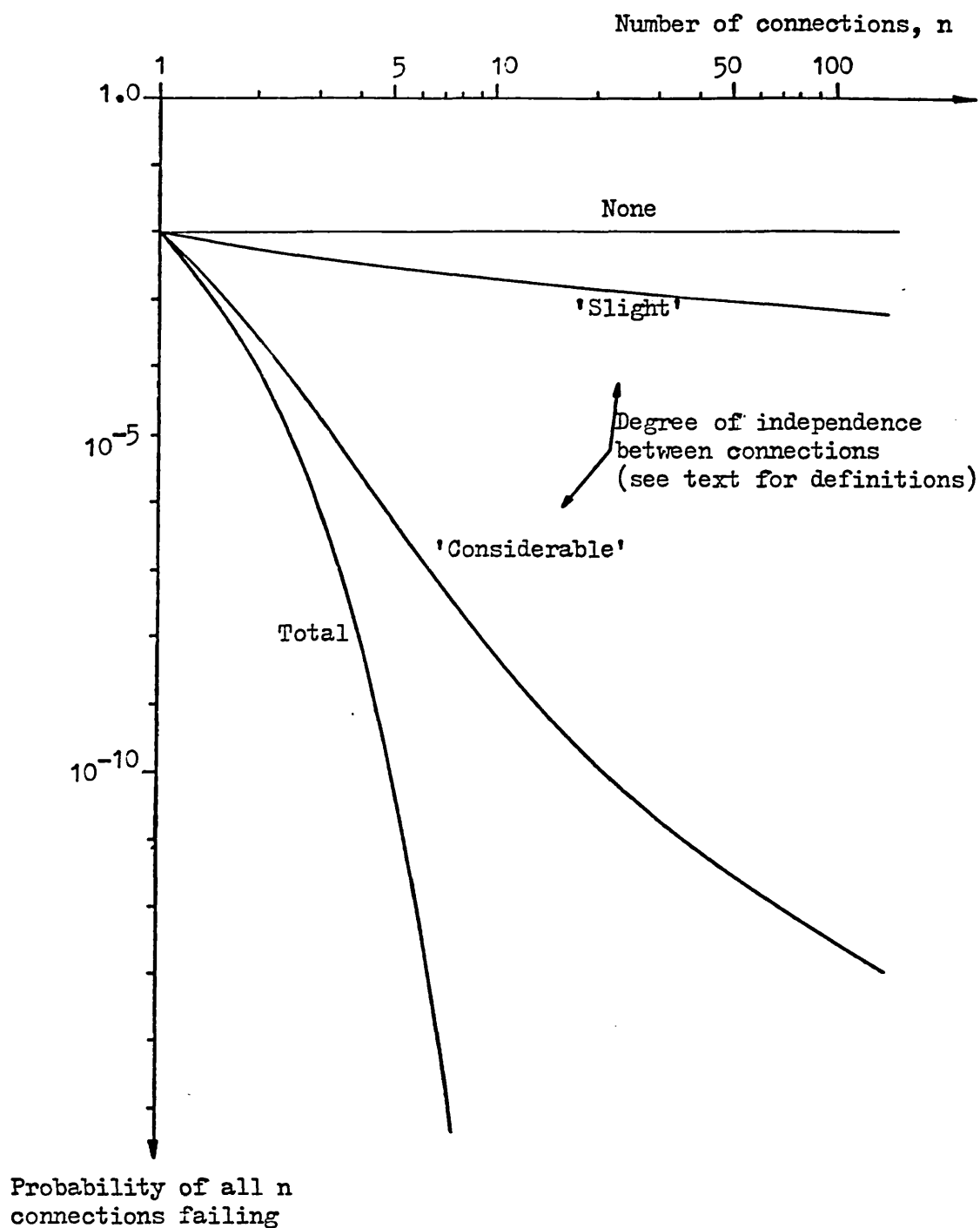


Fig. 6.3 Probability of all connections failing.

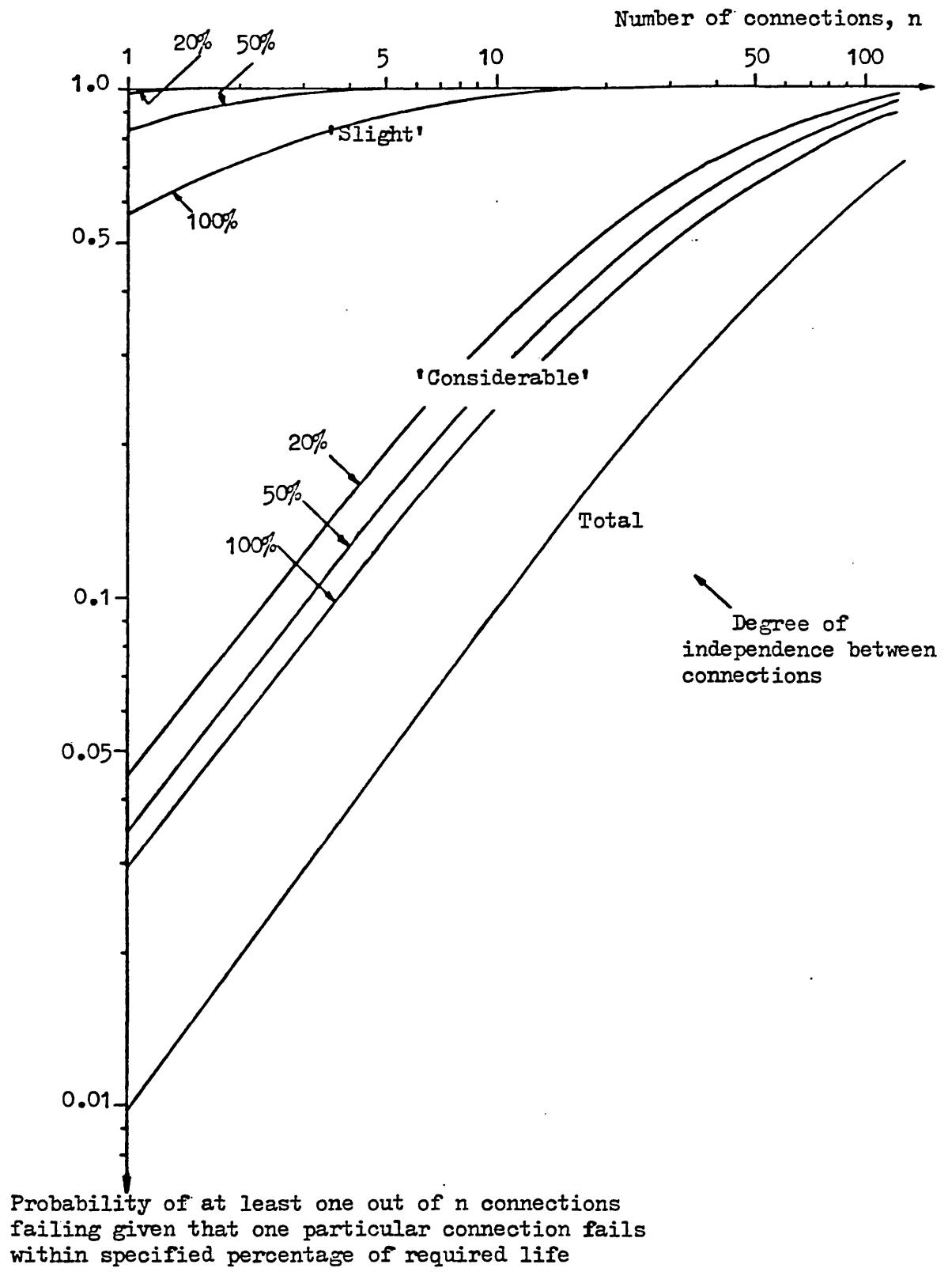
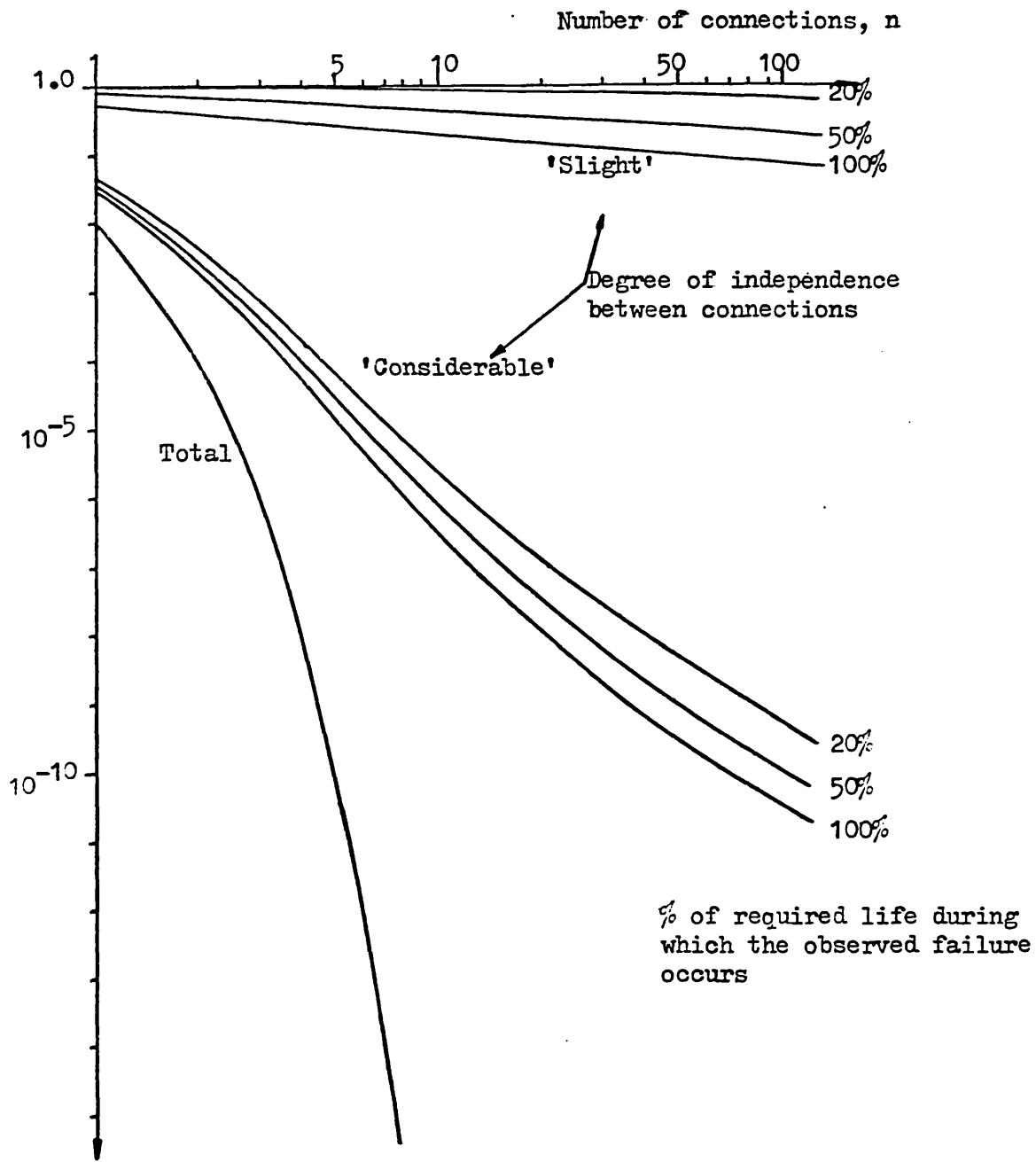


Fig. 6.4 Conditional Probability of at least one failure.



Probability of all  $n$  connections failing given that one particular connection fails within specified percentage of required life

Fig. 6.5 Conditional Probability of all connections failing.