

U N I V E R S I T Y O F L O N D O N

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LIGHT LOSS AND IMAGE FORMATION IN OPTICAL SYSTEMS,
WITH SPECIAL REFERENCE TO ENDOSCOPIC INSTRUMENTS .

by

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ABSTRACT

In optical systems having a large number of air-glass interfaces and also a long light path in glass, image quality is influenced not only by the aberrations of the system, but of equal importance can be absorption and reflection losses. The former, being normally much greater in the blue part of the spectrum than the red, can seriously distort the colour rendering of images, and multiply-reflected light reaching the image plane will reduce contrast and may also give rise to focussed ghost images. These images can be a serious source of difficulty in that they consist of local high concentrations of spurious light in the primary image.

These questions are of importance in the case of endoscopic systems. In a new type of cystoscope, what are effectively air lenses in a glass medium are used. This gives much improved performance both on account of the high degree of aberration correction and because of the high precision possible in manufacture. Moreover, the system allows a higher numerical aperture. Absorption losses, however, could be high because of the long length of path in glass, some 25 cm.

It was in this context that the present work was undertaken. When there is a large number of surfaces and a long light path in glass, even small changes in the transmission per surface, or per unit length, will have a significant effect on the overall transmission of the system. Thus,

for designing a cystoscope, or other endoscope, to give optimum performance, information is needed concerning the absorption characteristics, in the visible spectrum, of different optical glasses, and of the efficiency of anti-reflection coatings on the glass surfaces. It is of importance, too, for the purpose of endoscopic colour photography, to have an accurate knowledge of the spectral distribution of the transmitted light. There are few published measurements relating to the transmission for different wavelengths of anti-reflection films, or the absorption of optical glass, and the precision of such measurements as do exist is not sufficient for the present purpose. Furthermore, comparison with theory of experimentally-determined values of the transmission of anti-reflection films is necessary in its own right, and appears in the literature to have received little attention.

Existing techniques of transmission measurements are critically examined, and it is concluded that improved techniques are required to secure the desired precision. A new double-beam null-reading spectrophotometer, designed and used for the present measurements, is described. Improved sensitivity of measurement of the transmission of coated surfaces is obtained by using light transmitted through a large number of surfaces. The null point is observed as the coincidence of two lines on an oscilloscope trace; differences of light flux between the two beams corresponding to $\sim 10^4$ photons per second are detectable.

Measurements made at different wavelengths of the efficiency of single-layer and double-layer anti-reflection films, on substrate glasses

of different refractive index, are described, the precision obtainable being better than 0.1 % ; a comparison with theoretical results is made and there is good agreement. The influence of polarization on the accuracy of measurement is discussed. Absorption coefficients (per cm.) of various types of optical glass have been measured with an accuracy of $\sim \frac{1}{2}$ % , and are presented.

The design of typical endoscopic optical systems is considered in the light of the results obtained, and their application in controlling the spectral distribution of the transmitted light is indicated. It is concluded that the improvements resulting from a suitable choice of glass type having low absorption, and the use of two-layer coatings with minimum reflection placed in the blue part of the spectrum ($\lambda \sim 4750\text{\AA}$ in a typical case), are such that measurements of the kind and precision described here are major factors in design considerations relating to all endoscopic and similar systems.

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Chapter I

IMAGE QUALITY IN OPTICAL INSTRUMENTS

1.1 Introduction.

There are four basic factors affecting the quality of the image produced by an optical instrument. These are diffraction, due to the finite length of light-waves, which limits the detail the instrument is capable of resolving; the intrinsic aberrational properties of spherical interfaces which result in geometrical distortion and loss of resolution and contrast in the final image; absorption of light in the various optical components reducing the overall image brightness and sometimes producing colour distortion; and finally, spurious effects arising from reflections at air-glass interfaces which not only introduce stray light into the system thereby reducing image contrast, but often give rise to so-called ghost images. Ghost images formed near the primary image plane greatly reduce the contrast and the resolution of detail, while those formed farther from it can give rise to a background glare which, in some circumstances, can also seriously inhibit the performance of the instrument.

In the majority of optical systems absorption losses are usually quite negligible. Stray light falling on the image plane can have a significant effect on the image quality as

a detailed investigation by Coleman (1947) has shown, but such standard precautions as the correct placing of stops, blacking of the lens edges and blooming of the lens surfaces are normally quite adequate for the demands of most optical instruments. The limitations on image quality, therefore, are due primarily to diffraction and aberration effects. In endoscopic instruments, however, stray light is particularly a defect, as is also the presence of numerous ghost images; and in a recently proposed new endoscopic system in which the light traverses a long path in glass, absorption too, becomes a serious factor.

1.2 Endoscopes

Endoscopes are optical instruments used in medical practice for the internal examination of various cavities of the body. This class of instruments includes, among others, the bronchoscope, gastroscope and cystoscope which are used for examining the respiratory tract, the stomach and the bladder, respectively. Although the detailed design of each type of endoscope is influenced by the anatomy of the particular organ involved, the basic feature common to all is that their optical systems must necessarily be of quite considerable length in relation to their diameter. This means that in order to obtain a field of view of any appreciable size, it is necessary to have relatively complex optical systems, and the consequent multiplicity of air-glass interfaces gives rise to

numerous possible ghost images and a good deal of stray light, all at the expense of the primary image intensity. Absorption losses in the many optical components are also quite significant, especially in a new type of cystoscope designed by Hopkins (1957), which in other respects appears to have considerable advantages over cystoscopes of conventional design. (See Chapter II). Endoscopes, moreover, are normally used under conditions which make it impracticable to have a bright source to illuminate the object; and since only a small fraction of the light is accepted by the very small entrance pupils of normal endoscopic optical systems, it is of especial importance that every form of light loss be reduced as much as possible. Of course, it would be possible, in some cases at least, to increase slightly the aperture of the optical system which could then accept more light. Increasing the aperture, for example, from 5mm. to 6mm. would increase the amount of light accepted by the optical system by 44 per cent; on the other hand, without some increase in complexity of the system, its aberrations would also be increased. Conversely, there would be less advantage in carefully designing an optical system to minimize aberrations if limitations on the quality of the image are determined by absorption and reflection losses and ghost image formation. Clearly, the higher the paraxial transmission of an endoscopic system, the smaller can be its aperture and the better the quality of its image, within certain limits. These limits will be discussed in

more detail in Chapter II, Sec. 2.4.

Of considerable importance in endoscopy is the question of colour distortion in the image. The most common method of illuminating the object for endoscopic study is by means of a small low-voltage tungsten-filament lamp situated at the distal end of the endoscope. For the low colour temperatures at which endoscope lamps often have to operate, the spectrum of the emitted light will be seriously deficient in the blue region. To make matters worse, absorption and reflection coefficients of optical glass are invariably higher towards the blue end of the spectrum, so that light losses in the optical components of an endoscope will have the effect of further reducing the proportion of blue light in the final image. This will tend to make the object appear more orange-red in colour; a slight purple colouration in the object, for example, would be rendered less distinguishable from a red background if the small amount of blue light reflected from the purple region is suppressed. The diagnosis of a particular condition may depend on just such a colour difference existing between diseased and healthy tissue. The increasing use of colour photography in endoscopy gives the question of colour distortion added importance.

1.3 Improvement of the image quality

The problem has three aspects: (1) improvement of the overall transmission; (2) suppression of ghost-image formation; and (3) maintaining good colour balance in the image. Thus, glass having

a high overall transmission for white light may, nevertheless, not be an optimum choice for the optical components of endoscopic systems, since different optical glasses having low absorption coefficients in the middle of the spectrum (where the eye has maximum sensitivity) have greatly different absorption characteristics in the blue region of the spectrum where high transmission is particularly desirable. Also, the efficiency of anti-reflection coatings on the glass surfaces will vary with the type of glass used. The transmission of an anti-reflection film is wavelength-dependent, and light traversing many films will have a considerably altered spectral distribution of intensity. A theoretical study by Murray (1956) has shown that such colour distortion could be a serious source of difficulty in colour photography. While the higher efficiency and greater wavelength selectivity of two-layer anti-reflection films, as compared with single-layer films, might be used with advantage in enhancing the transmission of an endoscopic system for blue light, due regard must be given to the question of colour distortion in other parts of the spectrum.

Summing up, therefore, it is clear that for optimum performance of an endoscopic system under given conditions, the proper choice of glass type and anti-reflection treatment for the optical components is rather critical. Detailed knowledge is thus required of the absorption characteristics of different types of optical glass, and of the efficiency of anti-reflection coatings on different substrate glasses. Since these data cannot be predicted theoretically with any high degree of accuracy, it is important to determine them by experiment; even small changes in the

value of the reflection coefficient per surface or the absorption per unit length of glass can produce a significant change in the total transmission of a system having a large number of air-glass interfaces and a long length of light path in glass. The measurements carried out in this connection are described in a later chapter. They were made necessary by the absence of any description in the literature of detailed and precise experimental studies of the efficiency of anti-reflection films; the data available on absorption coefficients of optical glasses are also quite inadequate both as regards scope and precision. The present work relates not only to the improvement of endoscopic image quality, but is directed towards both the measurements to be described and, of necessity, improving the technique of measurement.

References

1. Coleman, H.S., J.O.S.A. 37 (1947) 434
2. Hopkins, H.H., Private communication, 1957
3. Murray, A.E., J.O.S.A. 46 (1956) 790

Chapter II

ENDOSCOPIC INSTRUMENTS - THE CYSTOSCOPE

2.1 General

Cystoscopes are used for the interior examination of the bladder. The cystoscope is among the shortest of the endoscopic instruments, but its optical system is typical of all of them. Visual endoscopic examination is becoming increasingly supplemented by colour photography, thus necessitating improved optical systems and more efficient object illumination. The tolerance on image quality is far more critical than that required in purely visual use, and one of the chief reasons for the present work being undertaken was the development of a cystoscopic camera capable of giving images of better quality than those obtainable using conventional cystoscopes modified for photographic use.

To allow clear viewing of the bladder wall, the bladder must be dilated to remove skin folds, and this is done by filling it with sterile water; its roughly spherical shape when dilated also provides an object surface all points of which are approximately equidistant from the objective of the cystoscope. Cystoscope objectives are therefore designed to work in a water-filled object space, but apart from this one special feature, all remarks concerning cystoscopic optical systems will apply generally in the case of any other endoscope.

The optical components of a cystoscope are normally sealed in a thin-walled cupro-nickel tube which can be inserted in an outer sheath of plastic or of metal. In a cystoscopy, the empty outer sheath is first inserted into the bladder via the urethra, and is used for irrigation and filling of the bladder by sterile water. The tube containing the optical system is then inserted in the sheath. In some instruments, space is provided in the sheath for the insertion of slender surgical instruments.

2.2 Illumination

The most common method of illuminating the bladder interior is by means of a small tungsten-filament lamp, operating on 6 - 12 volts, housed in the tip of the cystoscope casing. The latter projects at an angle beyond the objective window (see Fig. 2.1 on p.17) so that maximum illumination is provided for the region of the bladder wall in the field of view of the objective.

An alternative method, due to Vulmière (1957) is to conduct light into the bladder from a bright external source along a thin quartz rod contained within the outer sheath of the cystoscope. The internal diameter of the sheath is limited, for anatomical reasons, to about 8mm. ; the sum of the diameters of the quartz rod and the tube containing the optical system cannot therefore exceed this amount.

The quartz rod usually has a diameter of 2 - 3 mm. There would be no advantage in increasing this in order to gain illumination since the aperture of the optical system would have to be correspondingly reduced with consequent reduction of the amount of light that could be accepted. Considered overall, this is a relatively inefficient system because the quartz rod and the optical system together do not occupy more than about one half of the available space within the sheath. There will also be considerable absorption losses in the quartz rod since the light is propagated along it by a series of total internal reflections and thus traverses a long path in quartz.

By using a transparent sheath to conduct the light, much greater efficiency of light collection is theoretically possible, and a large-aperture optical system can be accommodated. Perspex sheaths have been used, but not with very great success because the low refractive-index of perspex (1.49) and the presence on the walls of the sheath of water and natural grease combine to give a large critical angle, and much of the light is lost through the walls. Glass having a high refractive index cannot be used for the sheath because of the risk of fracture. A low-index coating on a perspex sheath would reduce the critical angle, but it is difficult to find a low-index substance that will bond well with perspex.

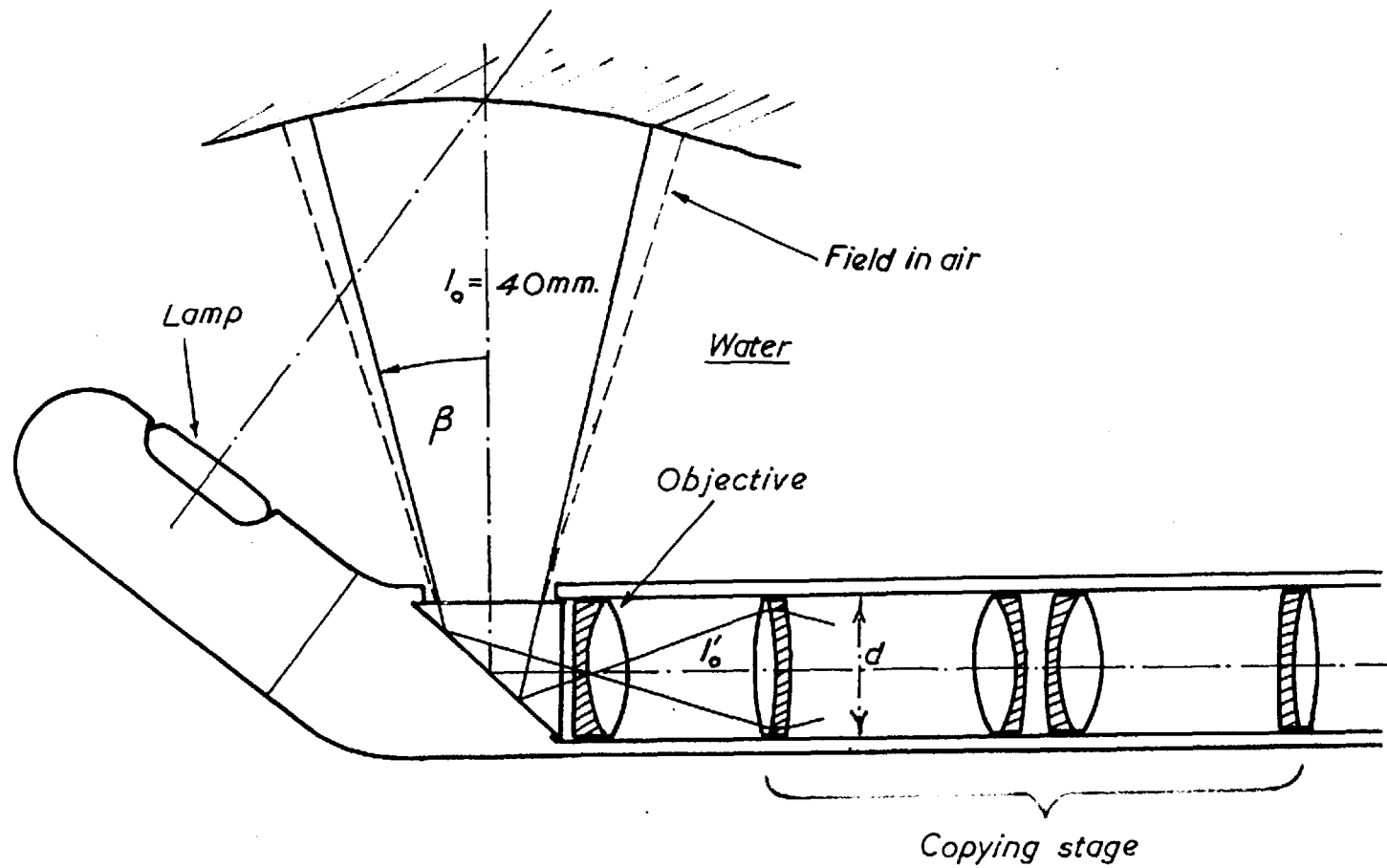


FIG. 2.1 . Field angle of cystoscope.

2.3 Optical system

The average length of a cystoscopic optical system is about 25cm. The aperture is limited, on account of the small diameter of the tube housing the optical elements, to about 5.5mm. In order to secure an adequate field of view, a system of so-called copying lenses is provided at the requisite intervals along the tube to image and re-image the real primary image formed by the objective a few millimetres from the latter, and thus effectively to transfer the image to the object plane of the eyepiece. The copying-lenses usually consist of pairs of achromatic doublets and conveniently work at unit magnification. It is necessary, in addition, to have field lenses in (or near) each of the image planes; these are normally of the same focal length as the copying-lenses, but consist of only a single lens.

Cystoscope objectives are designed to work at an object distance of 35 - 40mm. in water, the focal length being determined primarily by the size of field it is desired to view. (For a given field of view in a water-filled space, the corresponding air-field is, of course, somewhat larger.) The field of view varies considerably in different instruments, ranging from under 40° to over 100° : the optical quality of the images, as is to be expected, decreases with increasing field. For most practical purposes a field of view of 50° is adequate.

Since the image-field of the objective, and hence the field of view, is limited by the diameter, d , of the tube containing

the optical elements, the distance, l'_0 , of the primary image plane from the objective is fixed, for a given field angle, β , (see Fig. 2.1), by the relation

$$l'_0 = \frac{d}{2} \cdot \cot \beta$$

In a typical case, $\beta = 53^\circ$ and $d = 5.5\text{mm.}$, giving $l'_0 \sim 5\text{mm.}$ Thus, for an object distance $l_0 = -40\text{mm.}$, the magnification in the primary image is $-\frac{1}{8}$, and by using an eyepiece with a magnification of +8 a life-size final image is obtained. (This is usually formed, for visual work, at a distance of 25cm. from the exit pupil, but for photography, using a standard camera, the final image must be at about 50cm., because most camera lenses cannot be focussed for less than this distance.) Most cystoscopes have two copying stages, so that an erecting eyepiece must be used. A Dove prism is normally included to correct left-right inversion produced by the reflecting prism at the objective end of the system. A 45° -prism is normally used here to give a view at right-angles to the axis of the instrument, but other prism angles are also employed to allow viewing in other directions. By rotating the cystoscope about its axis it is possible to view an entire zonal region of the bladder wall.

2.4 Light transmission

The many air-glass interfaces in the copying system of a cystoscope will tend to reduce greatly both the intensity and the contrast in the image, since each interface is a source of aberration,

light loss, scattered light and reflected ghost light. Aberrations can be compensated at succeeding surfaces, but the other effects are all cumulative. In a standard cystoscope having 2 copying stages there are some 19 air-glass interfaces, and reflection losses can reduce the transmission to less than 30%. Absorption losses in the glass will not normally be significant in a conventional cystoscope, but may be in the case of other endoscopes. A gastroscope, for example, may have some 30 optical components giving a total length of glass path of about 10 cm.

Diederichs (1957) has measured the paraxial transmission of a number of standard cystoscopes using a very narrow light beam which passes unobstructed through the instrument and is then accepted by an integrating sphere. The light flux is measured using a photocell and a galvanometer, the ratio of readings for light transmitted through the cystoscope and for directly-incident light giving the transmission factor. The following table gives the results of these measurements for six cystoscopes, using white light.

Cystoscope	I	II	III	IV	V	VI
Paraxial transmission (%)	37	40	35	33	41	22

The off-axis transmission as defined above is practically the same as that for the axis, because only small angles of incidence arise. However, the illumination in the off-axis image is reduced by vignetting.

Cystoscope VI in the above list has a wide-angle objective. Cystoscopes II and IV are nominally the same, but the lens surfaces in II were bloomed while those in IV were left untreated.

These results are consistent with a light loss of 4% per air-glass interface in a system having 19 such interfaces, for which the theoretical transmission is 46%. The discrepancies could arise partly from absorption in the glass and partly from poor polish on the surfaces. The average improvement throughout the visible spectrum resulting from blooming is expected to be such that the transmission of one surface reaches about 98%. This would give a transmission of 68% for a system having 19 treated surfaces. It is difficult to find any explanation of the lower value measured for instrument II other than poor quality blooming.

In longer endoscopic systems efficient blooming is necessary for even a modicum of reasonable performance. A gastroscope, for example, having 60 untreated glass surfaces will have a transmission of less than 5% (assuming 95% transmission per surface). Blooming of the surfaces to give a transmission of 98% per surface will raise the transmission factor of the system to only 30%. It should be remarked, however, that an increase in the transmission per surface of only $\frac{1}{2}\%$ would nearly double the transmission factor of the system. Absorption losses in the glass will be considerably less than the reflection losses, but in the blue part of the spectrum the transmission factor of 10cm. of glass may be as low as 60%.

These results clearly illustrate the need for improving the transmission properties of endoscopic instruments, even for a relatively short endoscope such as the cystoscope. It is also apparent that the quality of blooming in optical instruments is susceptible of considerable variation, and that even slight improvements can give rise to significant increase of transmission in a system having a large number of air-glass interfaces.

2.5 Ghost light

The measurements on light transmission described above are to some extent optimistic, since the transmitted light measured by the above method will include a certain amount of ghost light which normally will be injurious to the quality of the image. Much of the ghost light is scattered generally over the image field giving rise to a background glare ("veiling glare") common in many optical instruments, which reduces the image contrast. In endoscopic systems it often happens that ghost images are nearly in focus in the image plane. Focussed ghost images of this kind, of which there may be a considerable number, can lead to very serious blurring of local detail, as well as general loss of contrast. The way in which such images can arise is shown in Fig. 2.2 where a rather special case is considered in order to illustrate more simply this property of field lenses in a copying-system .

Light reflected from the plane surfaces of the field lenses

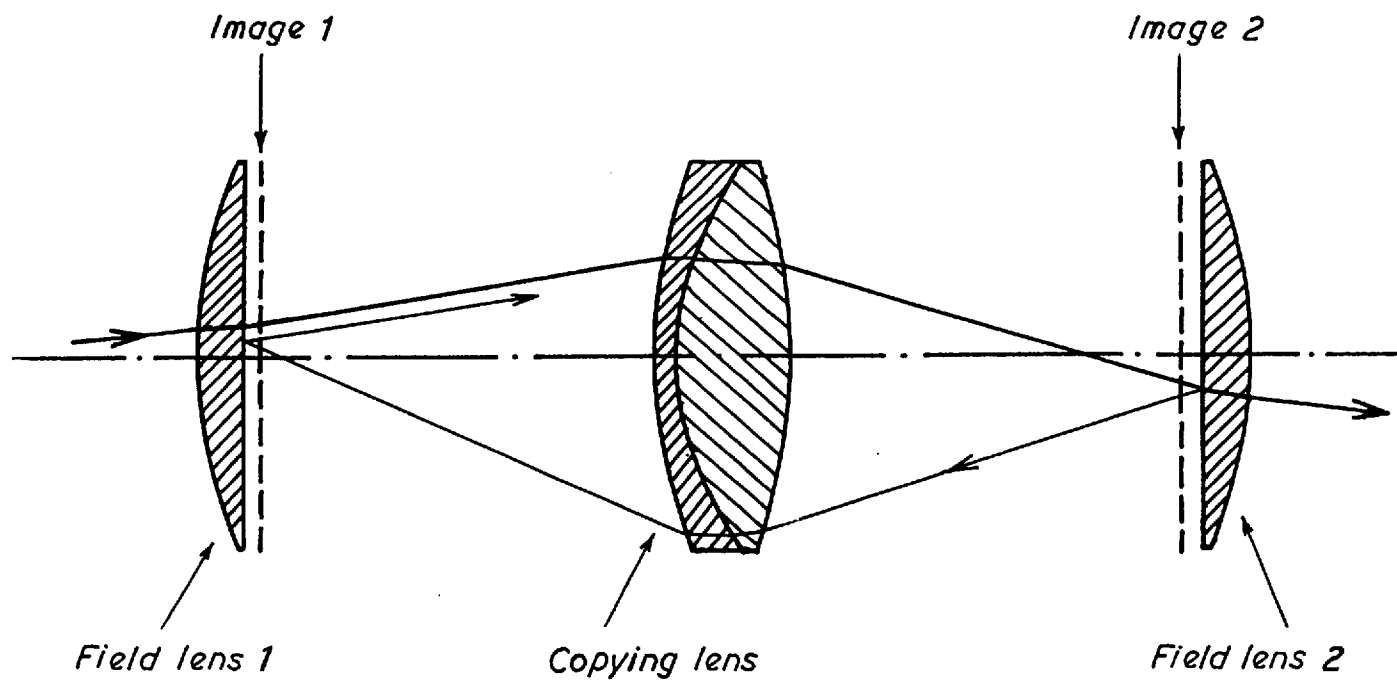


FIG. 2.2. Formation of ghost images.

shown in the figure is reflected forward again by the previous plane surface in the same direction as the primary rays. If there are n such field lenses, the possible number of "first-order" ghost images (i.e. those arising from one reflection backwards and one forwards) is given by

$$n_g = \sum_{r=1}^{r=n-1} (r-1)$$

For $n = 4$ the number of first-order ghost images would be 6.

Theoretically, these should not be objectionable as they would all be exactly superposed on the proper image. In practice, however, it is impossible to align all the optical elements in a cystoscope with sufficient accuracy to produce exact superposition with the result that the ghost images are displaced relative to the true image. Moreover, with more normal field lenses these ghost images would not be in sharp focus. Most of the "second-order" ghost images * are too faint to have any serious effect, but viewing of an isolated light source through a cystoscope nevertheless shows many ghost images scattered over the image field. The second-order ghost images are not visible with the instrument having bloomed lens surfaces : first-order ghost images, however, could still be detected.

*French authors use the term "images secondaires" for what have here been termed "first-order" ghost images. The "second-order" ghost images referred to here are formed as a result of four reflections.

2.6 Entrance pupil and field of view

A reduction in the number of glass surfaces in an endoscopic system could be secured by using lenses of greater focal length to increase the object-image throw of each copying stage, thus permitting the use of fewer copying stages. This would both improve the transmission and considerably reduce ghost light. However, increasing the object and image distances will also reduce the convergence angles at the image planes, since the image size and lens diameters in the copying system are limited by the containing tube.

The Helmholtz-Lagrange Invariant for the system is given in the image space ($n=1$) by $H = u \cdot r$, where u is the convergence angle at an image plane and r is the radius of the tube containing the optical elements and defines an upper limit to the size of an image in the copying system. For an object field of radius η_0 in a space of refractive index n_0 , therefore,

$$H = n_0 \cdot u_0 \cdot \eta_0 = u \cdot r$$

where u_0 is the aperture angle. Thus,

$$\frac{du}{u} = \frac{d(n_0 u_0)}{n_0 u_0} + \frac{d\eta_0}{\eta_0}$$

A decrease du in the convergence angles of the copying system will thus cause either the object field η_0 , or the numerical aperture $n_0 u_0$ of the system, to be reduced. In the case of a cystoscope, the object space is water-filled so that n_0 is fixed, and a reduced numerical aperture would mean a smaller entrance pupil. It is desirable to have the entrance

pupil as large as possible because it controls both the amount of light the optical system can accept, and also, together with the aberrations present in the system, the contrast in the image. Thus, improvement of the image quality by reducing the number of copying stages would be made at the expense of the field of view.

2.7 Frequency response

The entrance pupil behaves as the limiting aperture of an optical system, and it is therefore a primary factor in determining diffraction limitations on the image quality.

The first dark ring in the diffraction image of a point formed by a circular aperture has a diameter given by

$$D = \frac{1.22\lambda}{n \cdot \sin\alpha}$$

where α is the angle subtended by the radius of the aperture at the image plane, λ is the wavelength of the light used, and n is the refractive index of the image space. Both the finite size of this diffraction effect and the presence of aberration lead to a reduction of contrast in the images of extended objects. For a grating object having a sinusoidal distribution of intensity, the ratio of the contrasts in the object and the image formed by an optical system is called the frequency response of the system, and it is a function of the spatial frequency of intensity variation in the object plane.

The term contrast here denotes the Michelson visibility of the test patterns.

Any form of object has a certain spatial distribution of intensity which can be expressed by means of Fourier analysis as a spectrum of pure frequency terms. Thus, if $B(u)$ represents the distribution of intensity across a grating in the object plane, this function may be represented by the Fourier transform relations

$$B(u) = \int_{-\infty}^{+\infty} \beta(s) \cdot e^{i2\pi us} \cdot ds$$

where

$$\beta(s) = \int_{-\infty}^{+\infty} B(u) \cdot e^{-i2\pi us} \cdot du$$

the variable s being in units of reciprocal length and therefore identifiable as a spatial frequency. Clearly, if the spatial frequency is high, as for fine structure in the object, diffraction limitations will cause the contrast in the image to be less than in the object. The frequency response of an optical system normally decreases with increasing spatial frequency and even in a perfect optical system the frequency response is always less than unity, which value is only approached as $s \rightarrow 0$, i.e. for an object field of uniform intensity.

It is convenient for the measurement of frequency response to consider intensity variation in one direction, and a line-structure with sinusoidal variation of intensity and unit contrast is conveniently referred to a sine-wave test object. The

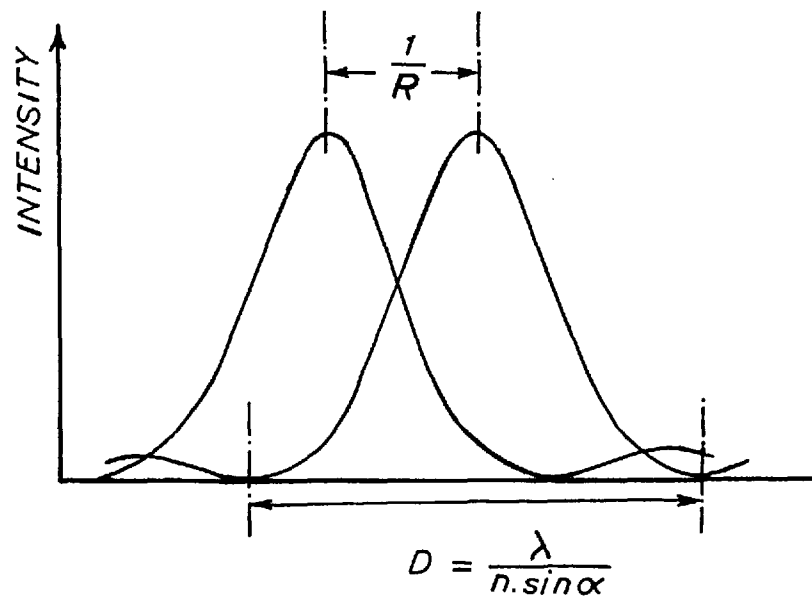


FIG. 2.3 . Loss of contrast at high spatial frequencies.

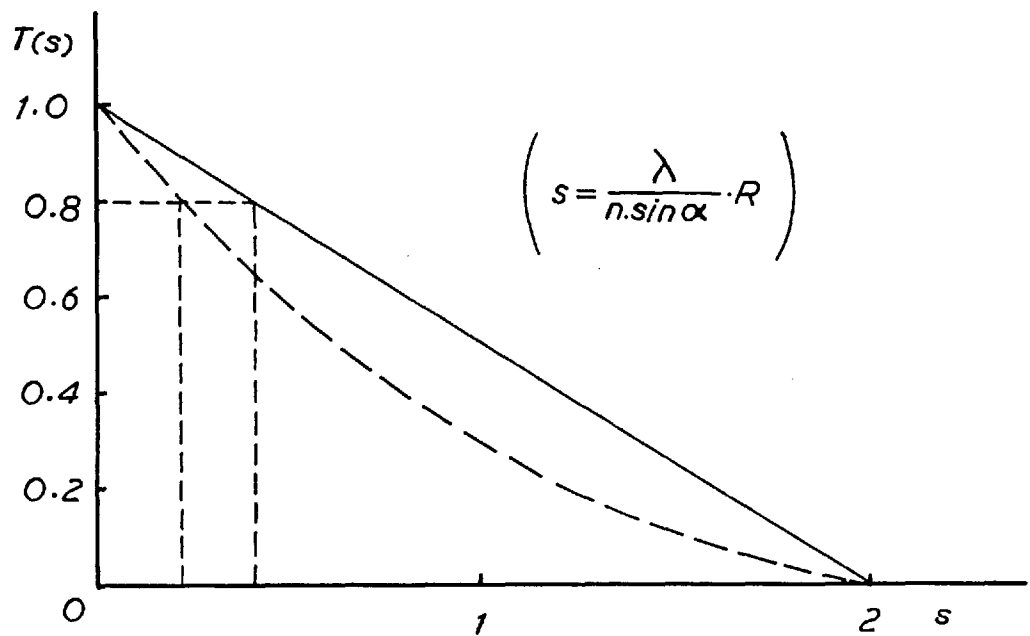


FIG. 2.4 . Frequency response of perfect and aberrated systems.

spatial frequency is normally expressed in lines per mm. and will be denoted by the symbol R which can take on values from 0 to ∞ . The contrast is zero when two or more lines are contained in a distance equal to the diameter of the Airy disc, as is shown in Figure 2.3. All frequencies greater than that giving 2 lines per Airy-disc diameter are therefore of no physical significance. Instead of taking R as the frequency variable, it is therefore convenient to use a variable s defined by the relation

$$s = \frac{\lambda}{n \cdot \sin \alpha} R$$

When the spatial wavelength $1/R$ is equal to half the diameter of the Airy disc which for simplicity is taken to be $\lambda/n \cdot \sin \alpha$, the contrast is zero. The corresponding value of s is 2 and conveniently limits the frequency axis.

Defects in the optical system can also introduce a spatial phase-shift in the image relative to the object, so that the frequency response is properly expressed in the complex form

$$D(s) = T(s) \cdot e^{i\theta(s)}$$

where $\theta(s)$ represents the phase-shift appropriate to that frequency and $T(s)$ is the ratio of the image and object contrasts. $T(s)$ is usefully called the "contrast transfer factor" of the system.

For a perfect optical system $\theta(s) = 0$, and the contrast transfer function $T(s)$ plotted against s as abscissa is not

materially different from a straight line with ordinate values of unity for $s = 0$, and zero for $s = 2$. Figure 2.4 shows the response function for a perfect system and for an aberrated or defocussed system. If the contrast in the object is unity, $T(s)$ represents the image contrast. Such curves provide a means for correlating the image contrast for any spatial frequency with the geometrical and optical properties of the system. For example, referring to Fig. 2.4 it can be seen that in order to obtain a contrast in the image of at least 50%, the value of s must be less than 1. For a given wavelength of light, therefore, the numerical aperture of the system, $n \cdot \sin \alpha$, determines the maximum number of lines per mm which the system will image with a contrast of 50%.

The numerical aperture is determined by two factors: the refractive index of the object space, and the angular size of the entrance pupil. Diederichs (1957) has measured the sizes of the entrance and exit pupils of the six cystoscopes mentioned previously, and these are given in the following table:

Cystoscope	I	II	III	IV	V	VI
Entrance pupil diameter (mm)	.40	.25	.25	.25	.28	.20
Exit pupil diameter (mm)	2.0	1.4	1.5	1.6	1.6	1.6
Maximum contrast (%)	73	55	55	55	60	45

The average size of entrance pupil is seen to be 0.25mm. The medium of the object space in the case of cystoscopes is water which has a refractive index $\sim 4/3$, and the object is on the average 35mm from the entrance pupil. The detail capable of being resolved with a contrast of 80% is thus limited to 5 lines per mm. A photographic image of a 5 lines/mm. object would have a contrast far less than 80% because of the further degradation of contrast occurring in the photographic process. The size of the entrance pupil could be increased with advantage to obtain better resolution.

In practice, the frequency response of a cystoscope is less than in the ideal case, because of the presence of aberrations, particularly Petzval curvature and astigmatism. The effect of these and other aberrations is to lower the response curve as shown in Figure 2.4 thus necessitating a lower s -value to give a contrast equal to that which would be obtained with an aberration-free system for the same frequency. Using frequency response methods Hopkins (1957) has calculated tolerance criteria for all the aberration coefficients, using the condition that the aberration permissible in an optical system is such that the frequency response for a chosen spatial frequency be not less than 0.8 of its theoretical value for an aberration-free system. The effects of field curvature in a cystoscope can be partly overcome by accommodation on the part of the observer when the instrument is used visually. For photographic purposes, however, most cystoscopes in common use

would only satisfy the above tolerance criteria for low frequencies of the order of 1 line per mm. or less.

2.8 The air-lens ("rod-lens") cystoscope

From the discussion in preceding sections it is seen that important factors in improving the transmission properties and image quality of endoscopic optical systems are as follows:

- (i) efficient blooming of the lens surfaces and proper choice of glass type;
- (ii) reducing the number of air-glass interfaces;
- and (iii) increasing the size of the entrance pupil.

It was shown in Section 2.6 that the requirements of (ii) and (iii) appear to be mutually opposed. However, a means of overcoming this difficulty and obtaining both benefits has been suggested by Hopkins (1957) and successfully applied in a prototype of a cystoscopic camera which utilises a radically different type of optical system.

From the basic law of refraction at a spherical interface between two media of refractive index n and n' , viz. :

$$\frac{n'}{l'} - \frac{n}{l} = \frac{(n' - n)}{r} \quad ,$$

it can be seen that if the signs of $(n' - n)$ and r are both changed, no change will occur in the sign of l' for a given value of l , if $|l|$ is sufficiently large. (For $n=1$, $n'=1.5$, $|l|$ must be $> |3r|$.) Thus, a concave

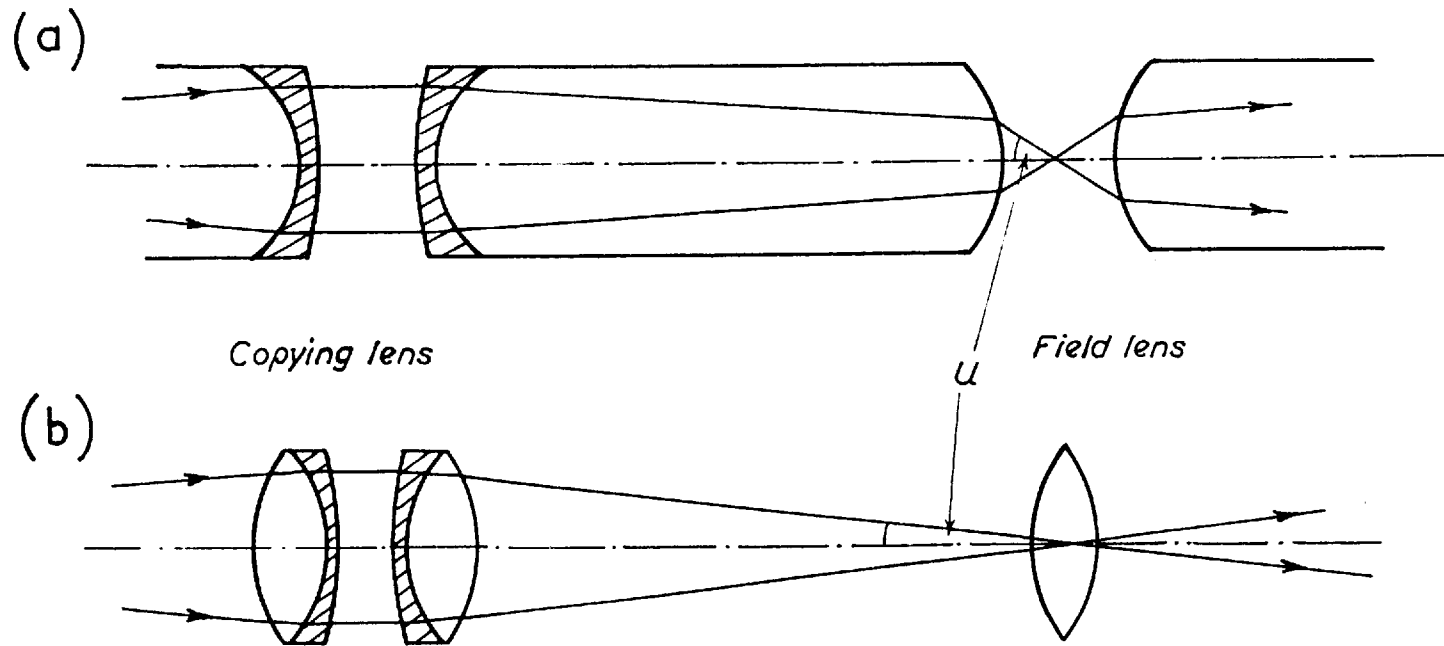


FIG. 2.5. Aperture angle in (a) rod-lens system
(b) conventional system.

air space in a glass medium will have refractive properties similar to those of a glass lens in air. The important difference, however, is that for an object and image space in glass the value of $H = nu\eta$ may be increased to n times the value for an air space, keeping u and η constant. Conversely, by keeping H constant the convergence angles u may be reduced by a factor of n to give a longer object-image throw.

Hopkins has therefore suggested a cystoscopic optical system using glass rods with suitably-shaped convex ends, an air gap between two rods forming a positive lens. Where necessary, these are achromatised by cementing on to the neighbouring ends of a rod-lens pair small negative lenses of the requisite power, index and curvatures. The air gaps are comparatively small (2mm) so that the light path is practically entirely in glass. For a given aperture the throw of each copying stage could be increased by a factor of n , so that the number of stages needed to transfer the image along a given length would be reduced by $\frac{1}{3}$ (for $n = 1.5$). The use of rod-lenses, moreover, gives an additional saving of two air-glass interfaces per copying stage, since the copying lenses of conventional cystoscopes each consists of a pair of doublets.

Alternatively, using the same number of stages, the convergence angles at each image plane, and hence the aperture of the system, can be increased by a factor of n (see Fig. 2.5) giving a $2.25 (=n^2)$ times increase in the area of the entrance pupil. In a prototype instrument, for example, it has been possible to increase the size of the entrance pupil from 0.25mm, as in the case of a standard cystoscope, to 0.75mm.

without increasing the number of air-glass interfaces. The frequency response of the system, using aberration-corrected rod lenses is thus sufficiently good to give 80% image contrast at a spatial frequency of 15 lines/mm., with greatly increased illumination. Clinical tests, both visual and photographic, have confirmed this.

The advantage of a rod-lens optical system over a conventional system can be seen to increase with increasing refractive index of the glass. (The use of high-index glass would also enhance the efficiency of magnesium fluoride anti-reflection coatings.) The results of absorption measurements to be described later show, however, that the absorption coefficient, particularly that for blue light, usually increases with increasing refractive index. For a rod-lens system absorption losses could be serious, and the question of proper choice of glass type thus becomes one of critical importance.

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Chapter III

ANTI-REFLECTION FILMS

3.1 Introduction

In general, light losses in endoscopic optical systems will be due chiefly to reflection from the glass surfaces; even when rod-lenses are used, a cystoscope, for example, may still have some 20 air-glass interfaces. Anti-reflection coating of the glass surfaces is thus of primary importance, both for improving the transmission and for reducing ghost light, and before proceeding to the problem of measurement of anti-reflection film efficiency and of glass absorption coefficients, some general questions concerning anti-reflection films will be briefly discussed in this chapter. The theoretical aspect is particularly of interest because a comparison with theoretical results of experimentally-determined transmission values is very necessary and appears to have received little attention. Among other factors, for example, capable of giving rise to discrepancies, variations in the refractive index of evaporated films, as compared with the bulk material, are known to occur. The question of computation needs also to be considered.

3.2 Deposition of anti-reflection films

The chief methods in current use are thermal evaporation and cathodic sputtering. In early work on anti-reflection films, Bauer

(1934) and Strong (1936) used films of alkali fluorides deposited chemically, but films thus deposited are generally soft and fragile and so have little practical value. Electrolytically-deposited films are more robust, but being necessarily metallic have no application as anti-reflection films.

Thermally evaporated magnesium fluoride is normally used for single-layer anti-reflection films. Although the refractive index of magnesium fluoride (1.38) is rather high - an anti-reflecting layer on low-index glass should ideally have a refractive index of about 1.22 - this disadvantage is outweighed by the great durability and stability of refractive index shown by films of magnesium fluoride as compared with films of other materials. Calcium fluoride films deposited in poor vacuum have a refractive index of approximately 1.22 but they are soft, and measurements by Abelès (1950) show that there is marked instability of their optical properties. In contrast, magnesium fluoride films withstand considerable abrasion and have been shown by Flamant (1950) to resist atmospheric attack and to have stable optical properties. Measurements by Hall and Ferguson (1955), too, show that the refractive index of "aged" magnesium fluoride films (i.e. a few days after deposition) changes very little with further ageing.

A single-layer anti-reflection film gives a reflection minimum at one wavelength (considering only the visible spectrum), the suppression of reflection falling off to either side quite gradually in the neighbourhood of the minimum. Double- and multilayer films are more

strongly wavelength-selective and give more complete suppression of reflection at the minimum-reflection wavelength. In theory, by using an anti-reflection coating of n layers with properly chosen thicknesses and refractive indices, zero reflection at n discrete wavelengths could be obtained. Anti-reflection coatings having up to eleven layers have been produced, but practical difficulties and the complexity of computation have so far precluded the general use of films having more than two layers. Even the double-layer films cannot be made to give reflection minima at two arbitrarily-chosen wavelengths since a continuous range of refractive indices is not provided by available materials.

Double-layer anti-reflection films were first produced by Cartwright and Turner (1939) using thermally evaporated sapphire and quartz. These materials are not, of course, generally practicable. Holland and Putner (1959) have prepared efficient and durable double-layer films by sputtering bismuth in oxygen at low pressure to form a first layer of bismuth oxide (refractive index ~ 2.45) on a glass substrate and then depositing a magnesium fluoride layer by thermal evaporation from a second source. The double-layer films used for the measurements to be described in Chapter VI were prepared in this way.

For zero reflection at a given wavelength λ (in practice there is always some residual reflection), the optical thicknesses of the bismuth oxide and magnesium fluoride layers are required by the theory to be 0.049λ and 0.33λ respectively.

3.3 Monitoring of the film thickness

In the case of single-layer films visual monitoring is adequate for most purposes. For maximum suppression of reflection in the middle of the visible spectrum, the film will have the familiar purple colour in reflected light; with care, it is possible to locate the reflection minimum here within $\pm 50\text{\AA}$ of that of a standard comparison sample. For the purpose of the present studies this tolerance is acceptable (cf. Chap. VI, Sec. 6.3), and visual monitoring was employed for preparing the single-layer films used.

Visual monitoring is, of course, easily subject to human error, and other methods, such as those of Dufour (1948) and Turnbull and Belk (1952), are therefore commonly used. In these methods, monochromatic light of that wavelength for which minimum reflection is required is used for monitoring, and when the correct film thickness has been reached, the output of a photocell receiving light reflected from, or transmitted through, the film becomes a minimum, or a maximum, as the case may be. The general difficulty of precisely locating an extremum without passing through it is avoided in a refined method, due to Giacomo and Jacquinet (1952), whereby zero output from the photocell is indicated by the degeneration of certain loops of a Lissajou pattern in an oscilloscope trace. A simpler method giving good accuracy has been used by Holland and Putner (loc. cit.) in the deposition of two-layer films. The wavelength of the monitoring light is chosen smaller than that for which minimum reflection is required, and deposition is stopped when the photo-

cell output has passed through a minimum and reaches a predetermined value; the magnitude of this will depend on the difference between the two wavelengths. This procedure is, of course, only followed when the top layer is deposited. Calculation shows that prior to the deposition of the magnesium fluoride layer, the reflectance of the substrate+bismuth oxide system will be about $8\frac{1}{2}\%$ for a wavelength of $5000\text{\AA}$. If the wavelength of the monitoring light is $4500\text{\AA}$, for example, correct thickness of the bismuth oxide layer will be indicated by a reflectance of $9\frac{1}{2}\%$.

3.4 Theory of anti-reflection films

If an electromagnetic wave is incident on the interface (represented by the plane $Z = 0$ in Fig. 3.1) between media of index n_1 and n_2 , application of the usual boundary conditions on the electric

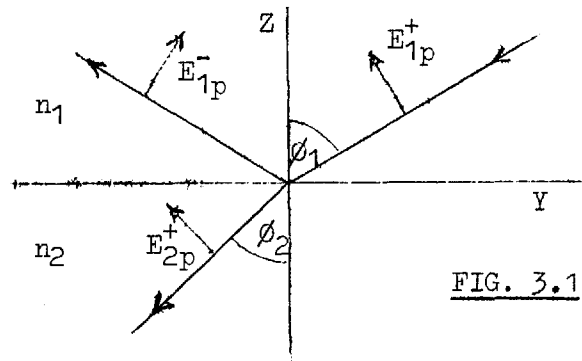


FIG. 3.1

and magnetic field vectors yields

$$r_p = \frac{E_{1p}^-}{E_{1p}^+} = \frac{\tan(\phi_2 - \phi_1)}{\tan(\phi_2 + \phi_1)} \quad 3.1$$

$$t_p = \frac{E_{2p}^+}{E_{1p}^+} = \frac{2\sin\phi_2 \cdot \cos\phi_1}{\sin(\phi_2 + \phi_1) \cdot \cos(\phi_2 - \phi_1)} \quad 3.2$$

for the amplitude reflection and transmission coefficients, respectively, for the electric vector in the plane of incidence, Y-Z. Vectors in this plane are distinguished by the subscript p. For the E-vector perpendicular to the plane of incidence (subscript s) the corresponding coefficients are

$$r_s = \frac{E_{1s}^-}{E_{1s}^+} = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)} \quad 3.3$$

$$t_s = \frac{E_{2s}^+}{E_{1s}^+} = \frac{2\sin\theta_2 \cdot \cos\theta_1}{\sin(\theta_2 + \theta_1)} \quad 3.4$$

(The notation scheme of Heavens (1955) has been followed here, positive and negative superscripts denoting the direction of propagation for transmitted and reflected waves, respectively, and subscripts 1 and 2 denoting the medium.) The above are the well-known Fresnel amplitude coefficients for reflection and transmission at a single interface. The squares of these give the coefficients of reflection and transmission for the energy, and will be denoted, respectively, by $R_{p,s}$ and $T_{p,s}$, as the case may be. If scattering and absorption losses are neglected, $T_{p,s} = 1 - R_{p,s}$.

(i) Single-layer films

The condition for zero reflection from a single-layer film deposited on a substrate can be obtained by using the concept of "impedance matching" familiar in other branches of physics. By analogy with an electrical transmission line, Blaisse (1950) has shown that an "optical impedance" can be defined by

$$Z = \frac{E_t}{H_t} \quad 3.5$$

where E_t and H_t are the tangential components of the electric and magnetic field vectors at any interface in the system. The condition for two media, of impedance Z_1 and Z_3 , to be "matched" by the interposition of a medium of impedance Z_2 is

$$Z_2^2 = Z_1 \cdot Z_3 \quad 3.6$$

From Maxwell's equations it follows that in a medium of permittivity ϵ and permeability μ (M.K.S. units),

$$\frac{E_t}{H_t} = \sqrt{\frac{\mu}{\epsilon}} \quad 3.7$$

Thus, from 3.5 - 3.7,

$$\epsilon_2 = \sqrt{\epsilon_1 \cdot \epsilon_3} \quad 3.8$$

since for most materials, at optical frequencies, $\mu \sim \mu_0$ (free space).

Since $\epsilon = k\epsilon_0$ (where k is the dielectric constant and ϵ_0 is the permittivity of free space), and for dielectrics the refractive index $n = \sqrt{k}$, 3.8 becomes

$$n_2 = \sqrt{n_1 \cdot n_3} \quad 3.9$$

(This is equivalent to the statement that the reflection coefficients for both interfaces must be equal - a condition necessary for complete destructive interference between the two reflected wave-trains. Thus, if r_1 is the amplitude reflection coefficient between media n_1 and n_2 , and r_2 that for the n_2 - n_3 interface, and considering, for simplicity, the case of normal incidence, r_1 and r_2 will have the same values for either direction of polarization and will reduce to

$$r_1 = \frac{n_2 - n_1}{n_2 + n_1} \quad ; \quad r_2 = \frac{n_3 - n_2}{n_3 + n_2} \quad .$$

By putting $r_1 = r_2$, the relation 3.9 is obtained.)

The condition 3.9 is not a sufficient one: for destructive interference the two wave-trains must also be in anti-phase, and a film of thickness $t = \frac{\lambda}{4}$ is therefore required, λ being the wavelength of the light in the film. The phase change on reflection is π for both interfaces, in a system of transparent media, and there will thus be a phase difference of π , for normal incidence. For non-normal incidence, or for wavelengths other than the critical wavelength, the reflectance of the film will have finite values. A general expression for the reflectance of a single-layer anti-reflection film may be obtained by extending the scheme of Fig. 3.1 to include a third medium, and applying boundary conditions to each interface in turn. Alternatively, a direct summation may be made

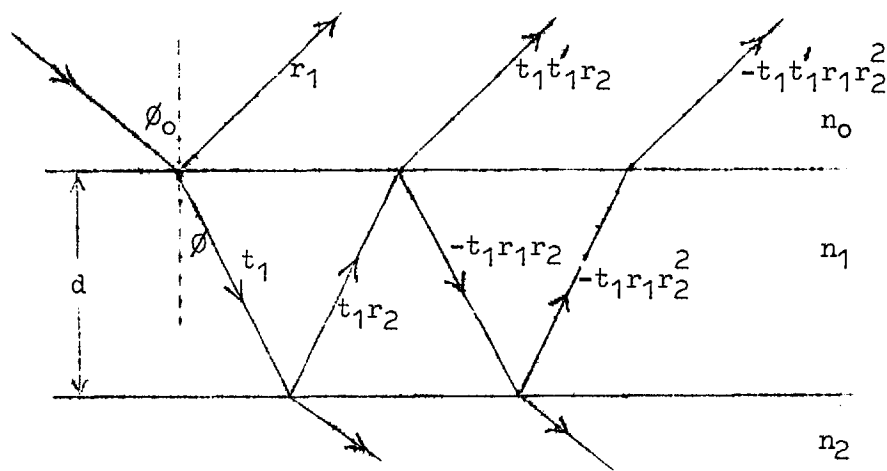


FIG. 3.2

of all the multiply-reflected beams following the scheme of Fig. 3.2 . The amplitude reflection coefficients r_1 and r_2 for the two interfaces, respectively, are taken positive for propagation from $n_0 \rightarrow n_1$ and from $n_1 \rightarrow n_2$,

and negative for the reverse direction; for propagation from $n_0 \rightarrow n_1$ the amplitude transmission coefficient will be denoted by t_1 , and for the reverse direction, by t_1' . (It is easily shown from eqns. 3.1 - 3.4 that

$$t_1 t_1' = 1 - r_1^2 \quad . \quad 3.10$$

(Subscripts p and s are omitted for simplicity, since the result will be valid for either direction of polarization.) Thus, for an incident wave of unit amplitude, the amplitudes of the successive beams reflected into medium n_0 will be given by: r_1 , $t_1 t_1' r_2$, $-t_1 t_1' r_1 r_2^2$, and when these are multiplied by the appropriate phase factors, the summation yields:

$$r = \frac{r_1 + r_2 \cdot e^{-2i\delta}}{1 + r_1 r_2 \cdot e^{-2i\delta}} \quad 3.11$$

for the amplitude reflection coefficient of the system, where $\delta = \frac{2\pi}{\lambda_0} n_1 d \cdot \cos\theta$ is the phase difference for the ray path between the two surfaces of the film n_1 , λ_0 being the vacuum wavelength of the light.

The reflectance (energy reflection coefficient) of the film is given by multiplying 3.11 by the complex conjugate. This yields:

$$R = \frac{r_1^2 + 2r_1 r_2 \cos 2\delta + r_2^2}{1 + 2r_1 r_2 \cos 2\delta + r_1^2 r_2^2} \quad 3.12$$

If absorption and scattering losses are neglected, the transmittance will be given by $T = 1 - R$. Variation of R with wavelength is due mainly to the variation in $\cos 2\delta$, the minimum value of R occurring for $2\delta = \pi$.

(ii) Double- and multi-layer films

The method of summing multiply-reflected beams has been used by Crook (1948) to calculate the reflectance and transmittance of a double-layer system. For a system of many layers this method becomes prohibitively complicated. On applying boundary conditions to the field vectors at each interface of a multi-layer system, recurrence relations are found between the field vectors in successive layers. These can be written in matrix form (the elements of a matrix being characteristic of a particular layer), and the resultant reflected or transmitted amplitude is then given by multiplying the matrices. The matrix elements may be expressed in terms of the Fresnel coefficients or the optical impedance Z , following the treatment of Abelès (1948, 1950); other methods are given by Weinstein (1947) and Blaisse (1950). The common feature in the various types of matrix is that the determinant of each matrix in the system is equal to unity. This can be shown to represent conservation of energy in each layer and is implicit in the assumption that there is no absorption in the films.

In an alternative method of calculating the reflectance of a multi-layer system, use is made of the result obtained for the reflectance of a single layer whereby successive layers are represented by a hypothetical single surface. Following the scheme of Rouard (1937), one proceeds from the substrate upwards. An effective amplitude reflection coefficient (similar in form to r' in eqn. 3.11) is first of all obtained for the substrate+first layer, this system being then regarded as the substrate

for the second layer, and so on. Vašíček (1950) has shown that the same result is obtained (as one would expect) by reversing Rouard's scheme and proceeding downwards from the topmost layer. For a large number of layers calculation by either method becomes very complicated, but the treatment of Rouard is generally easier to apply; in the case of normal incidence, recurrence relations given by Caballero (1947) simplify the problem. There are also various graphical methods of solution (see, for example, chap. 4 of "Optical Properties of Thin Solid Films" by O.S. Heavens, 1955) but these are not usually as accurate as computational methods.

For a double-layer film, calculation of the reflectance by Rouard's method requires but a single substitution. Using the notation

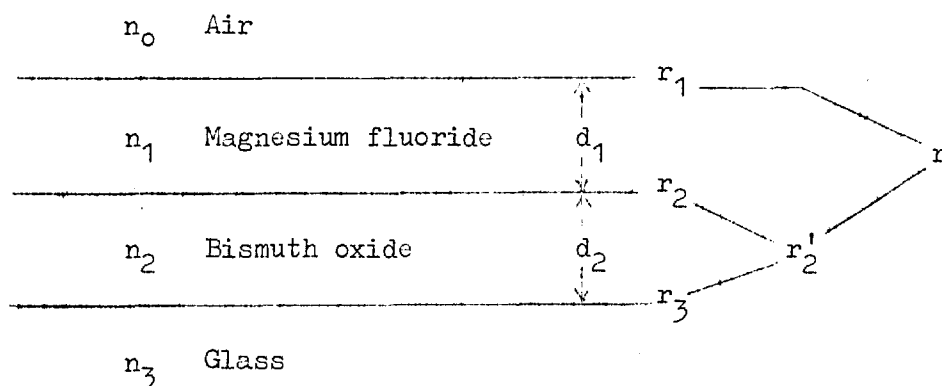


FIG. 3.3

of Fig. 3.3, an effective amplitude reflection coefficient for the glass+bismuth oxide layer will be given (cf. eqn. 3.11) by:

$$r' = \frac{r_2 + r_3 \cdot e^{-2i\delta_2}}{1 + r_2 r_3 \cdot e^{-2i\delta_2}} \quad 3.13$$

Similarly, the amplitude reflection coefficient for the whole system is:

$$r = \frac{r_1 + r' \cdot e^{-2i\delta_1}}{1 + r_1 r' \cdot e^{-2i\delta_1}} \quad 3.14$$

Substituting for r' in 3.14, we obtain

$$r = \frac{r_1(1 + r_2 r_3 \cdot e^{-2i\delta_2}) + e^{-2i\delta_1}(r_2 + r_3 \cdot e^{-2i\delta_2})}{(1 + r_2 r_3 \cdot e^{-2i\delta_2}) + r_1 \cdot e^{-2i\delta_1}(r_2 + r_3 \cdot e^{-2i\delta_2})} \quad 3.15$$

The reflectance of the system is given by $R = r \cdot r^*$. The result of this multiplication can be conveniently written, to facilitate computation, in the following form:

$$R = \frac{r_1^2 \cdot A + B + C}{A + r_1^2 \cdot B + C} \quad 3.16$$

$$\text{where } A = (1 + r_2^2 r_3^2) + 2r_2 r_3 \cdot \cos 2\delta_2 \quad ,$$

$$B = (r_2^2 + r_3^2) + 2r_2 r_3 \cdot \cos 2\delta_2 \quad ,$$

$$\text{and } C = 2r_1 r_2 (1 + r_3^2) \cos 2\delta_1 + 2r_1 r_3 \cos(2\delta_1 + 2\delta_2) + r_2^2 \cdot \cos(2\delta_1 - 2\delta_2) \quad .$$

(N.B. The sign of r_3 will be negative.)

3.5 Variation of reflectance with angle of incidence

For non-normal incidence 3.12 and 3.16 will give different values of R for the two directions of polarization corresponding to the appropriate forms, r_p or r_s , of the Fresnel coefficients. If unpolarized

light is used, as would usually be the case in practice, the overall reflectance will be given by inserting in the formulae effective values of the Fresnel coefficients for unpolarized light. These are calculated as follows. For an electric vector E making an arbitrary angle ϕ with the plane of incidence, the p- and s-components will be given, respectively, by $E_p = E \cos \phi$ and $E_s = E \sin \phi$. After reflection by a surface having reflectances R_p and R_s for the two directions of polarization, the sum of the time-averaged intensities of the reflected components will be given by

$$\begin{aligned} R_p(E_p^2)_{av} + R_s(E_s^2)_{av} &= \frac{\bar{R}}{2\pi} \int_0^{2\pi} E^2 (\cos^2 \phi + \sin^2 \phi) \cdot d\phi \\ &= 2\bar{R} \cdot E^2 \end{aligned}$$

where $\bar{R}$ is the overall reflectance of the surface for unpolarized light.

Thus,

$$\bar{R} = \frac{1}{2}(R_p + R_s) \quad , \quad 3.17$$

and the effective Fresnel reflection coefficient for unpolarized light is, therefore, given by

$$\bar{r} = \frac{1}{\sqrt{2}}(r_p^2 + r_s^2)^{\frac{1}{2}} \quad 3.18$$

If the light is partially polarized, then, denoting the fractions of p- and s-polarized light by α_p and α_s , respectively, the effective Fresnel reflection coefficient will have the value

$$\bar{r} = \frac{1}{\sqrt{2}}(\alpha_p r_p^2 + \alpha_s r_s^2)^{\frac{1}{2}}$$

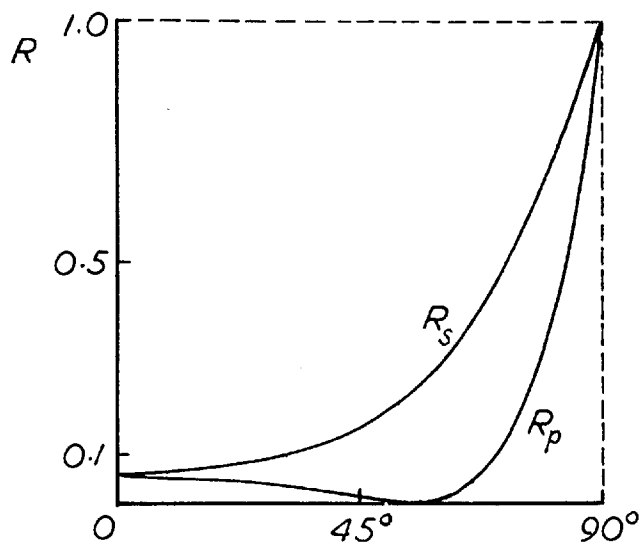


FIG. 3.4. Variation of R_p, R_s with angle of incidence for an untreated surface.

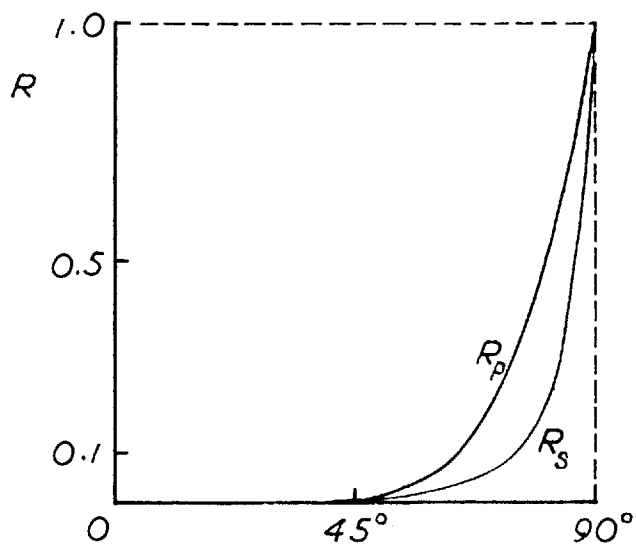


FIG. 3.5. Variation of R_p, R_s with angle of incidence for a treated surface ($\frac{\lambda}{4}$ -film).

For sufficiently small angles of incidence, normal incidence conditions may be assumed. Fig.3.4 shows the variation with angle of incidence i of the reflectance of an untreated glass surface ($n \sim 1.5$) for both directions of polarization. The departures of R_p and R_s from the common normal-incidence value are opposite in sign and closely equal for angles of incidence up to 20° and more. At $i = 20^\circ$, for example, $R_p = 0.0332$ and $R_s = 0.0470$, giving a mean value of 0.0401 (for unpolarized light) which differs from the normal-incidence value 0.0400 by a negligible amount.

The variation with angle of incidence of R_p and R_s for a $\frac{\lambda}{4}$ single-layer anti-reflection film is indicated by the curves of Fig. 3.5, after Rouard (1952). The reflectance for either direction of polarization does not differ sensibly from zero (< 0.0001) for values of i up to 30° . Although these curves relate to a "perfect" anti-reflection film for the zero-reflection wavelength, it can be shown that the reflectance for other wavelengths is similarly insensitive to variation of the angle of incidence. For example, in the case of a single-layer magnesium fluoride film on a glass substrate of refractive index 1.80, giving a reflection minimum at a wavelength of $5450\text{\AA}$, the (theoretical) values of R_p and R_s at a wavelength of $6563\text{\AA}$, and for 20° incidence, are, respectively 0.0039 and 0.0058 . The average reflectance (for unpolarized light) is thus 0.0049 (± 0.0001), which compares with the value 0.0050 for normal incidence. For double-layer films the discrepancy is somewhat greater, of the order of 0.0002 .

The possibility of assuming normal incidence conditions for

small angles of incidence (i.e. up to about 20°) is an important result. Thus, in Chapter V it will be seen that the elimination of ghost light in transmission measurements requires the specimens to be inclined at about 20° to the photometer beams; calculation of the theoretical transmission, on the other hand, is greatly simplified if normal incidence is assumed. (Fortuitously, angles of incidence at the lens surfaces in cystoscopic optical systems do not usually exceed $\sim 18^\circ$.) When light traverses a large number of surfaces, however, the question is complicated by changes in the proportions of p- and s-polarized light at successive surfaces. This effect will be discussed in more detail in Chapter VI.

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Chapter IV

THE COMPARISON OF LIGHT FLUXES

4.1 Introduction

Measurement of reflectance or transmittance depends, fundamentally, on the comparison of the fluxes in a reference beam and the reflected or transmitted ("sample") beam. This comparison may be effected either by separate measurements of the flux in a single beam, with and without the sample, using a calibrated detector (single-beam photometry) ; or, by simultaneous comparison of a reference beam and a sample beam (double-beam photometry). The latter general method includes matching the beams by means of a calibrated flux-limiting device in the path of the reference beam and observing a null difference of flux (null method) : the amount by which the reference beam has been attenuated or stopped down will indicate the original degree of unbalance of the beams. Methods also exist combining features of single-beam and double-beam photometry, a single beam being used alternately as a reference beam and a sample-beam and one signal stored for direct comparison with the other. (See, for example, reference 4 at end of chapter).

For measurements at different wavelengths, the design of spectrophotometric apparatus has tended to centre largely on obtaining high spectral purity (which is not always justified) and also aims at

automatic direct pen-recording of the measurements. These will not usually be as accurate as measurements obtained by careful point-by-point observation. A brief survey of the techniques available for such measurements will be given because an important aspect of the present work has been the development of improved apparatus and measuring technique.

4.2 Single-beam methods.

Measurements of the reflectance of single-layer films using a single, steady (unchopped) beam have been made by Rood (1949), Hall and Ferguson (1955), the intensity when the beam is incident directly on the photodetector being compared with that after reflection by a single surface. More recently, Holland and Putner (1959) have used single-beam methods for reflection and transmission measurements on two-layer films, the beam being modulated to allow amplification of the signal. Although there appears to be an almost total absence of reports in the literature on measurements of absorption in optical glass, some makers of optical glass publish transmission data and these are usually obtained by simple single-beam measurements.

The single-beam method of photometry is subject to various limitations and sources of error.

(i) Unless a highly stabilized power supply is used for the light source there can be fluctuations in intensity in the time-interval

between two readings.

(ii) The photocell output is often not linear with intensity and it is then necessary to calibrate the detector. The spectral response of light-sensitive materials used in photocells is also not uniform and a separate calibration may be necessary for each wavelength of light used.

(iii) The same region of the sensitive surface of the photocell must be used in all measurements to avoid errors arising from variation of sensitivity across the surface. Figure 4.1 shows this variation for a typical photomultiplier examined by K.W. Keohane and W.K. Metcalf (1955). In the method described by Hall and Ferguson the reflectances of a coated and an uncoated surface were compared, the light beam being shifted for this purpose and the photomultiplier position adjusted to intercept the beam with the same region of its photocathode. For measurements made in this way to be consistent it would be necessary to illuminate only a very small region of the photocathode at the maximum-sensitivity position where $\frac{dR}{dx} = 0$; or, alternatively the entire sensitive area (except, of course, the edges) should be illuminated so that any variation in the illuminated portion of the photocathode takes place in regions where $\frac{dR}{dx}$ is small and involves only a small fraction of the total flux.

(iv) Fatigue effects in the light-sensitive material of the photodetector are difficult to allow for and will impair the accuracy of measurement.

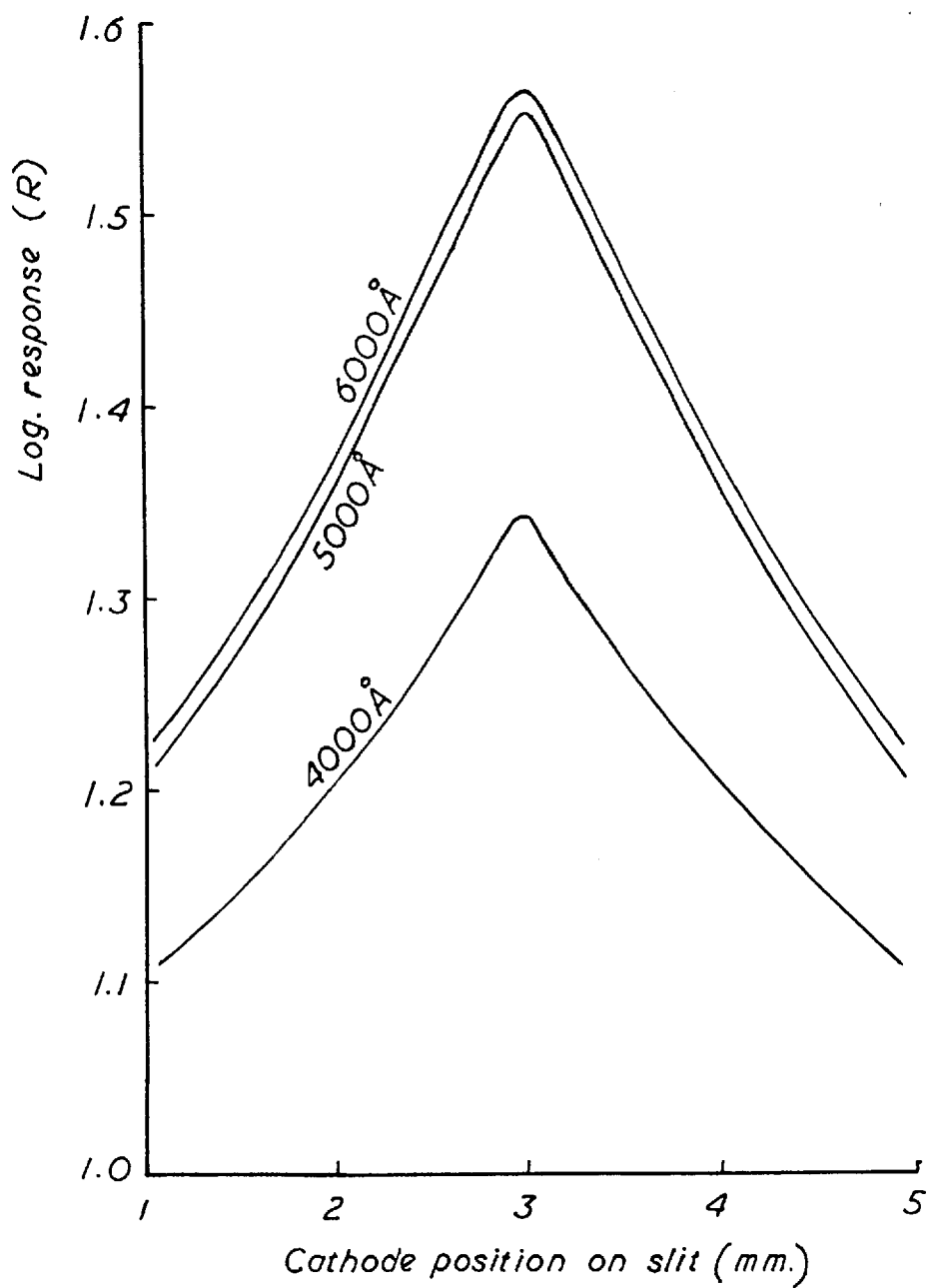


FIG. 4.1. Spatial variation of photocathode response.

(v) The sensitivity of the photodetector and the amplifier gain must remain constant during measurement and the power supply must therefore be stable.

(vi) In measurements of absorption in glass, it is usually necessary to correct for reflection losses at the end-faces of the sample. This has to be done either by measuring the transmission of two different lengths, or by calculation.

4.3 Double-beam methods.

Most of the above disadvantages are remedied by simultaneously comparing a sample-beam with a reference beam from the same source. (Methods of beam-splitting will be discussed in more detail in Sec. 4.5.)

In a method due to Tait and Chalklin (1953) a steady galvanometer deflection is obtained which is a function of the difference of intensity between two modulated beams. A similar principle has been employed by Neale (1955) in improved apparatus. In these methods, as in single-beam methods, it would be necessary to make a separate determination of the reference beam intensity, for example, by stopping down the other beam because it is the ratio of the intensities that is important for measurements of reflectance and transmittance.

In a method used by Makin and Rogers (1959) for measure-

ments on the density of films, two beams of nearly equal intensity were incident on separate photomultipliers the outputs of which were balanced on a bridge circuit to give directly the ratio of intensities; a similar method has been used by James(1959) for absorption measurements in the ultraviolet (where stability of the source is particularly a problem). A linear bridge-amplifier suitable for such methods has been designed by Hariharan and Bhalla (1959). Two balance settings of the bridge, one with both photomultipliers receiving light and the other with one beam stopped out, are used to define a linear absorption scale and the proportion of light removed by a sample placed in one beam is then given directly, with reference to this scale, by a single balance operation.

The balance setting of a bridge will be independent of variations in intensity of the source only if both photomultipliers have the same multiplication characteristics, this condition also being essential for the linearity of the absorption scale. For measurements at different wavelengths, moreover, it would be necessary for both photomultipliers to have the same spectral characteristics. The properties of photo-emissive and secondary-emissive surfaces are strongly influenced by a number of variable factors attending their deposition which are difficult to control for repeated production of surfaces having identical spectral characteristics. In practice, therefore, the selection of two closely identical photomultipliers is largely fortuitous. The apparatus of Hariharan and Bhalla was directed towards the measurement of small

changes of absorption; Makin and Rogers, too, were working with beams of nearly equal intensity and in these conditions identity of the photomultipliers is usually not so important as for the comparison of beams having a relatively large difference of intensity.

4.4 Null-detection methods.

In these methods a single photodetector is preferably used. Modulated beams are incident on this in superposition and the stronger beam is attenuated or stopped down until the (selectively amplified) signal component at the modulation frequency becomes zero to indicate balance of the beams. The transmission of a sample is then obtained by comparing the setting of the attenuating device or stop with a standard reference setting made in the absence of a sample. As with other double-beam methods, the balance setting will be independent of stability of the source. The important advantage of a null method is that the detecting system has only to indicate a null signal. When a single photodetector is used this indication will be independent of any non-linearity in the detector, fatigue effects, variation of sensitivity with wavelength or non-uniformity of response over the light-sensitive surface. Any variation of amplifier gain, moreover, will only have a slight effect on the sensitivity of the null detection; in other methods of measurement, the amplifier gain must be highly stabilized.

A double-beam null-reading method was used for the

present work. Four main factors have to be considered in the design of efficient apparatus, namely; (1) beam-splitting (and beam-recombining) means; (2) control of the flux in the reference beam; (3) modulation of the beams for amplification; and (4) detection of the null-point.

4.5 Beam-splitter and optical system.

In principle, the simplest method for obtaining "twin" beams (employed in a number of standard photometers), is to use two optical systems each of which receives light leaving the source in different directions. Alternatively, two separate light sources could be used with a common power supply. In either case, if a single monochromator is to be used, this must then be positioned at the exit end of the photometer, immediately prior to the photodetector, so that the light of both beams may be analyzed together. Unless there is a high degree of parallelism of the beams before they enter the monochromator, the photodetector would receive a good deal of stray light (which would be suppressed if the beams were monochromatic in the first place), and spectral purity would also be reduced. This would mean, for example, that samples used for measurement would have to possess accurately parallel faces for consistent results to be obtained. In these photometers, the optical systems are generally rather complex and tend to give rise to stray light.

In assessing the merit of a method for dividing a (monochromatic) beam into two component beams, due regard must be given to

the optical system as a whole, since the nature of this will be determined to a large extent by the particular form of the beam-splitting arrangement used. Besides its use for directing light along sharply defined paths through the photometer, an image-forming system is important because in order to eliminate errors arising from non-uniformity of response over the light-sensitive surface, it is necessary that the two beams should be incident on the photodetector in exact superposition; that is, as focussed images of some limiting aperture in the system. (This has been found to be necessary even when a diffusing screen is interposed). Conveniently, this limiting aperture is the exit aperture of the monochromator. To obtain images of equal size it is desirable that both beam-paths should be of equal length.

In the conventional semi-transparent mirror method of beam splitting, used by Dobson (1923) in one of the earliest double beam photometers, the component beams are initially directed at right-angles to each other. This is a convenient arrangement when there are two photodetectors, and was used in the method of Makin and Rogers, but in a photometric method employing a single photodetector additional means will be required to direct the beams towards the photodetector along paths of equal length. A mirror system can be used including two semi-transparent mirrors M_1 , M_2 , arranged as indicated in Fig. 4.2(a). An additional optical system will be necessary for imaging purposes. This method is relatively inefficient because half the light is lost at the second semi-transparent mirror.

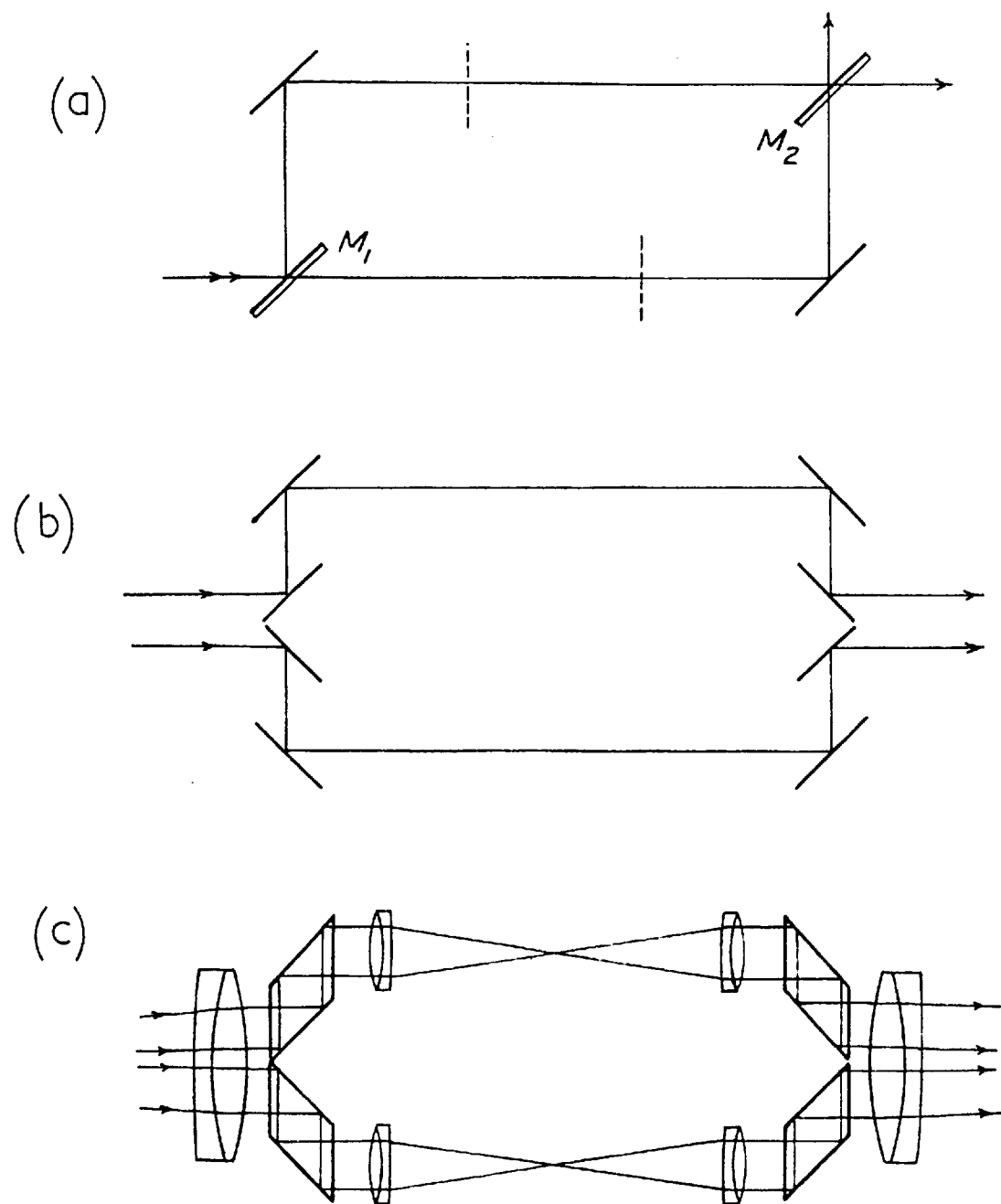


FIG. 4.2. Beam-splitting systems.

It is furthermore desirable, in order to facilitate the removal of ghost light, (See Chap V, sec.5.5) that, in addition to the superposed images formed on the photodetector, there should also be intermediate images formed, preferably, in a single plane normal to both beams. In^{the}/arrangement of Fig.4.2(a) these images would be formed at different distances along parallel arms of the two beam-paths. A more symmetrical arrangement is indicated in Fig. 4.2(b), but this has other disadvantages. If the reflecting surfaces are metallic, there would be undesirable polarization of the reflected light with the relatively large angles of incidence used, as well as substantial light loss. Four aluminium surfaces, each having a reflection coefficient of, say, 85%, would have a combined reflection factor of about one half, and there will be additional light loss in the image-forming optical system. Instead of metallic reflectors, 45° prisms could be used, preferably in the form of Albrecht rhombs as indicated in Fig. 4.2 (c), so as to reduce the number of air-glass interfaces. The formation of intermediate images requires an imaging system such as that indicated in the figure, light traversing the prisms being preferably parallel to facilitate alignment and to avoid light loss that would otherwise result from non-critical incidence of light on the hypotenuse faces. The transmission factor of a typical system of this kind (or of the mirror kind) would be about 30%. If the number of air-glass interfaces and absorbing length in the system could be halved the transmission would be approximately doubled and stray light considerably reduced.

In a double-beam method first used by Hardy (1935) polarized light produced by a Nicol prism is divided, by means of a Wollaston prism or Rochon prism, into two components having mutually perpendicular polarization. This method, too, is inefficient because half of the available light is lost at the first polarizer. It has the further disadvantage that the angular separation of the component beams is only one or two degrees and the beams have therefore to be quite narrow with consequent limitation of the amount of light available for measurement.

Fig. 4.3 illustrates a method of beam-splitting that does not have the limitations of the more conventional methods described above. The halves of the first split lens L_1 are separated by an amount s and form images E_1 , E_2 of the monochromator exit slit E_0 having a separation

$$s' = s(1-M) \quad ,$$

where M is the magnification between the plane of E_0 and that of the images E_1 and E_2 . A second split lens L_2 , conveniently identical with L_1 , forms superposed images of E_1 and E_2 at E on the light-sensitive surface of a photodetector P . In practice the lens halves are masked, and suitable apertures provided away from the lens edges. If the distance between the aperture centres is equal to the separation of E_1 and E_2 , the central rays of the two beams will be parallel and there will be, effectively, a pair of narrow parallel beams the separation of which may be adjusted very simply by varying either s or M .

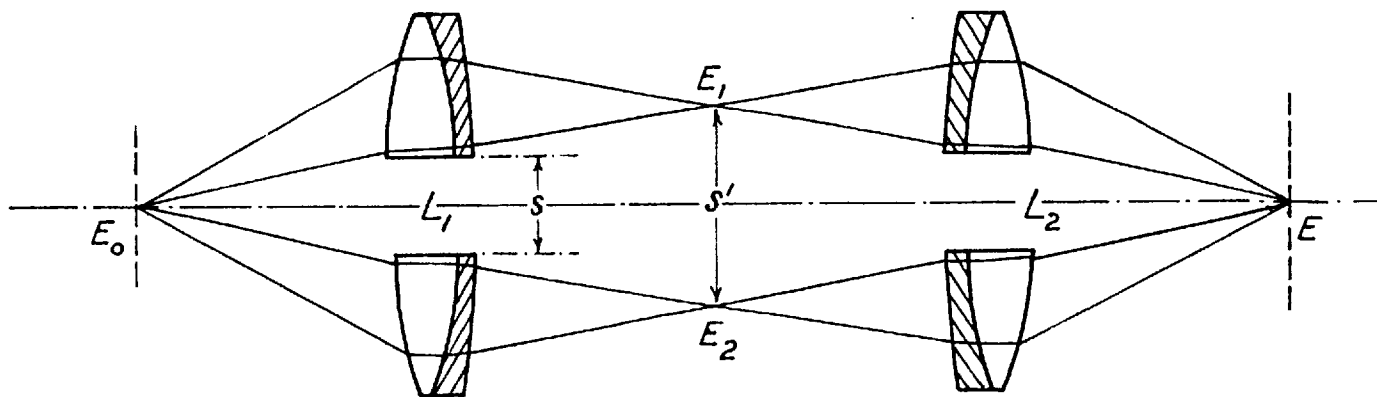


FIG. 4.3. Split-lens photometer system.

The important feature of this system is that the same optical elements are used for imaging purposes and for forming and re-combining the two beams. Because of its simplicity the system has high transmission and stray light is reduced to a minimum.

4.6 Control of light flux in the reference beam.

The output of a photocell is a function of the total light flux in an incident beam. The flux may be varied either by varying the intensity of the beam keeping the beam cross-section constant (in any given plane), or by varying the beam cross-section, for example, by means of a diaphragm. Different methods of both classes will be considered.

(i) In the case of polarized light, the intensity of a beam is readily varied by varying the relative angular setting of two polarizers. In Hardy's method (see above), for example, the Nicol prism can be rotated to alter the plane of polarization of the light incident on the Wollaston or Rochon prism and thus produce an increase of intensity in one component beam at the expense of the other. If the initial intensity of each beam is I_0 and the intensity of the sample-beam is reduced to some value $I = t \cdot I_0$ when a sample having transmission t is inserted, the angle ϕ (see Fig. 4.4) through which the Nicol prism must be rotated from a median position to balance the intensities in the two beams will be such that

$$t = I/I_0 = \left(\frac{\cos\phi - \sin\phi}{\cos\phi + \sin\phi} \right)^2$$

Differentiating, we obtain

$$dt = \frac{-4t(1 + \tan\phi)}{(1 - \tan\phi)} \cdot d\phi$$

from which it can be seen that the accuracy of measurement becomes rapidly poorer the greater the initial inequality of the beams.

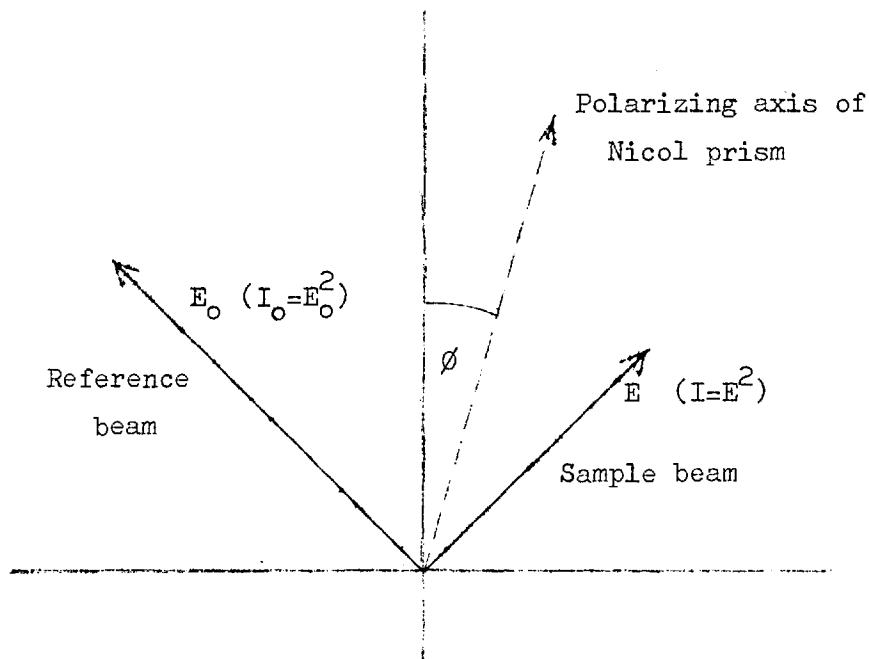


FIG. 4.4

Innaccuracies are also introduced by a small amount of residual unpolarized and, sometimes, elliptically polarized light produced at the surfaces of the polarizing elements.

(ii) A general method of varying the intensity of a beam is by the use of density wedges. Although this method has been used by Dobson, Tait and Chalklin and others, it would not appear to be very satisfactory for really precise measurements. In the first place, the density gradient of a wedge is usually non-linear and errors may arise in calibration; there may also be spurious local density gradients. Secondly, to obtain uniform attenuation over the whole beam cross-section, it is necessary to use two wedges having their density gradients equal and oppositely directed and it is unlikely that two wedges will be sufficiently closely identical. A further objection to the use of wedges is that four additional air-glass interfaces are introduced.

(iii) The alternative to varying the beam intensity is to vary the beam cross-section keeping the intensity constant. The principal difficulty here lies in the elimination of errors due to non-uniformity of intensity over the beam cross-section. In some Perkin-Elmer spectrophotometers (see reference 4 at end of chapter), the effective area of a beam-limiting aperture is varied by means of a pair of "combs" having interlocking wedge-shaped teeth (Fig. 4.5), the combs usually being coupled for equal and opposite movement by a single controlling mechanism. Variation of area thus takes place by "zig-zag" increments distributed fairly evenly over the beam cross-section. If $x/2$ is the displacement of each comb from a completely closed position (see Fig. 4.5), the area of the exposed portion of the aperture will be

$$A = 2bx(1 - x/a)$$

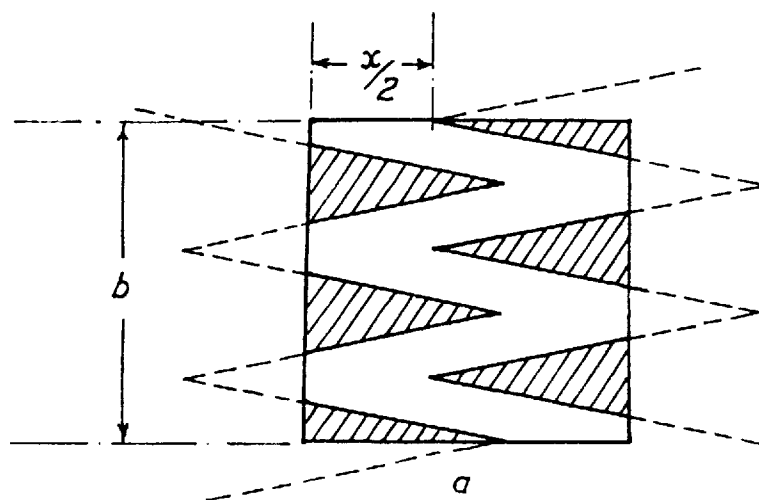


FIG. 4.5 . Variation of aperture area by combs.

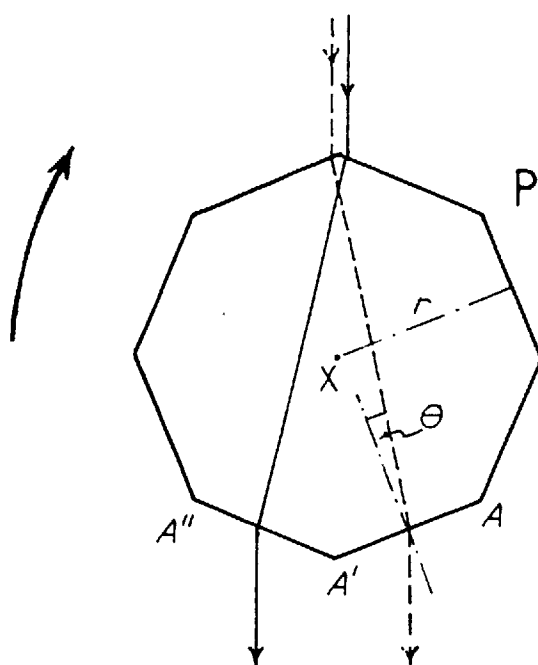


FIG. 4.6 . Scanning an aperture with a slit image.

and is seen to be independent of the angle (and number) of the teeth. Preferably, there should be a large number of small-angled teeth but there are practical limitations to the fineness of the teeth such as machining errors and the question of light loss due to diffraction.

Since A is not a linear function of x , calibration of the comb setting will be necessary; also, sensitivity of measurement, which is proportional to $\frac{dA}{dx} = 2b(a - x)$, will vary with the separation of the combs. A further disadvantage in the use of combs is the irregular shape of the effective aperture area giving rise when the beams are chopped to a rather complex wave-form in the flux modulation. This will result in loss of signal strength at the (selectively amplified) modulation frequency, and the light available for measurement is thus being used inefficiently. The question of efficient beam-chopping will be discussed in more detail in Sec. 4.7 .

(iv) Bergmann (1958) has obtained a uniformly luminous square field of monochromatic light by rapidly scanning a square aperture with the image of a narrow slit using an octagonal prism P (see Fig. 4.6) rotated at 300 revs./sec. about its axis X parallel with the slit axis. Each pair of opposite parallel faces of the prism is equivalent to a rectangular block on which the angle of incidence varies between $\pm 22\frac{1}{2}^\circ$ as any given face AA' moves into a position $A'A''$. The emergent beam at the opposite face will thus undergo a lateral oscillatory shift of amplitude $2r \cdot \tan \theta$ relative to the incident beam, where θ is the deviation

of the beam within the prism and r is the radius of the inscribed circle on the prism cross-section. The scanning amplitude required is determined by the size of the aperture, the refractive index and dimensions of the prism being chosen accordingly. The effective aperture area is varied simply by altering the aperture height.

In a double-beam method, it is necessary to illuminate two apertures. Optimum dimensions (cf. Chap. V. sec. 5.4) are about 1 cm. square for the size of the apertures and about 1 cm. for the separation

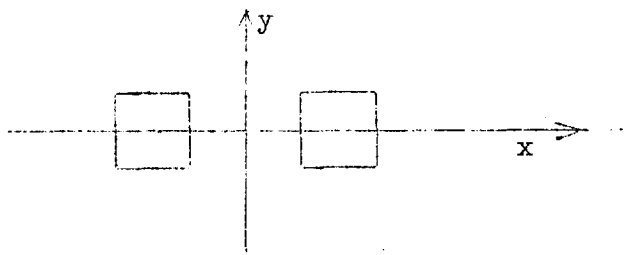


FIG. 4.7

between them. A slit image extending over both apertures and scanning them simultaneously would need to be more than 3 cm. long and would be subject to considerable variation of intensity along it. If, on the other hand, the direction of scanning is from one aperture to the other (as indicated by the x-direction in Fig. 4.7), the slit image could have a smaller length, but the scanning amplitude would have to be large: allowing clearance for "edge-effects" due to the edges between adjacent faces of the prism moving across the beam, the scanning amplitude would have to be about 5 cm. This would require the use of a prism with $r \sim 8\frac{1}{2}$ cm., which is hardly practicable. It will be seen in Sec. 4.8 that to allow

efficient modulation of the beams, variation of the aperture height must take place in the y-direction, and there remains the problem of providing a uniformly-illuminated slit if light flux is to be proportional to aperture height. In the section that follows the more general problem of the distribution of intensity in a steadily-illuminated aperture will be discussed, and it will be shown that conditions exist for which the distribution is sufficiently uniform (within the experimental tolerances) for light flux and aperture height to be proportional.

4.7 Illumination of an aperture by a small source.

Fig. 4.8 shows a simplified diagram of the illuminating and monochromating system of the apparatus to be described in more detail in Chap V. The light source is a 'Pointolite' lamp the luminous spherical globule O_0 of which has a radius $r \sim 1.5$ mm. The monochromator entrance slit S_0 (2 mm. high x 0.2 mm. wide) is imaged at O_0 with unit magnification and thus effectively defines a limiting aperture in the source which may therefore be regarded as a curved strip of width $\sigma = 0.2$ mm. The X - Y plane containing the beam-limiting apertures is imaged with a magnification of $1/3$ in the x - y plane, some 150 mm. from O_0 . The aperture images in the x - y plane are approximately 3 mm. square and 4 mm. apart, and are symmetrically disposed on either side of the y - axis. It is only necessary, therefore, to consider the illumination of a region in the x - y plane bounded by the lines $x = 2$, $x = 5$ and $y = 0$, $y = 3/2$; this region is indicated by shading in Fig. 4.9(a).

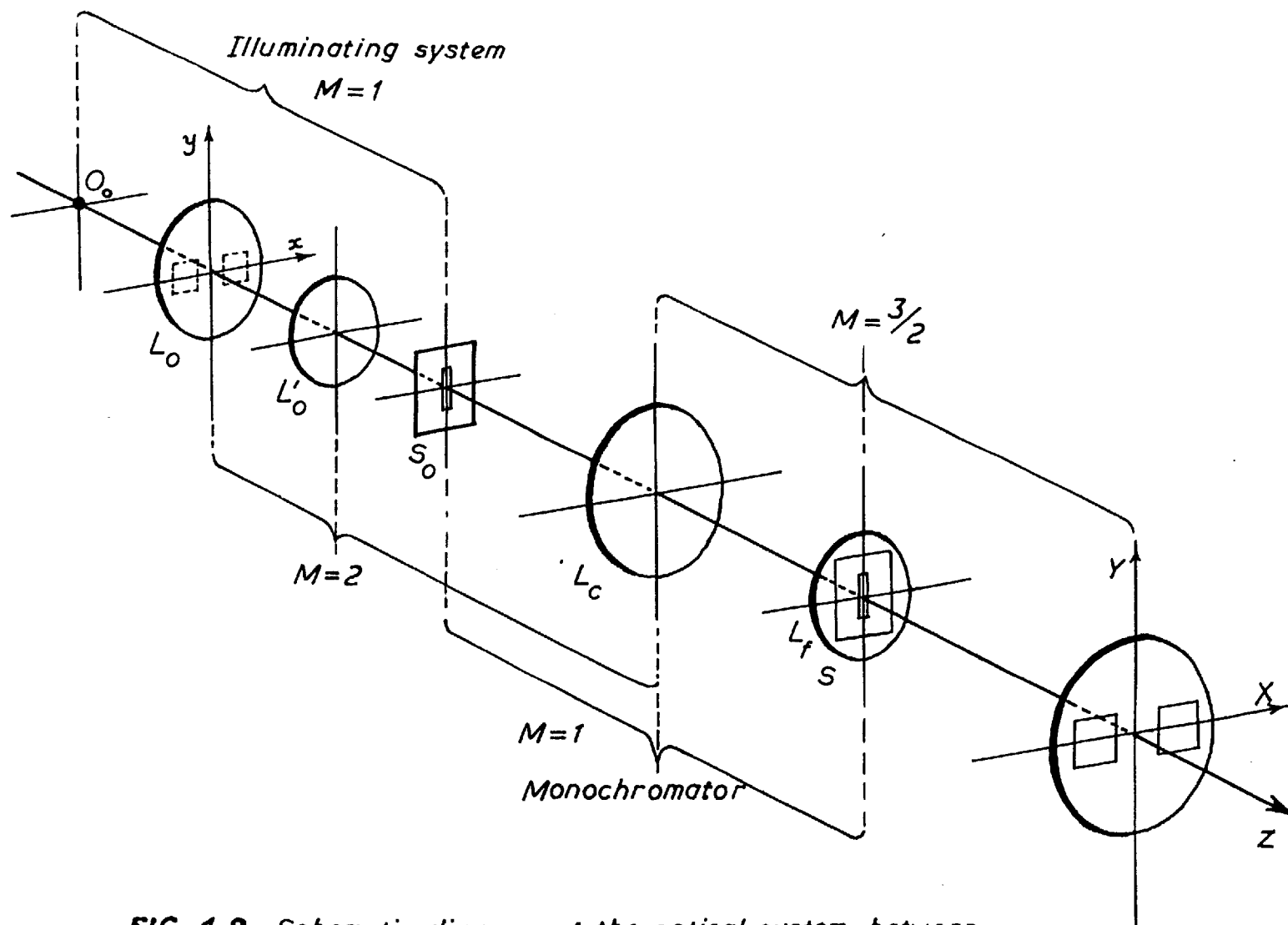


FIG. 4.8 . Schematic diagram of the optical system between the photometer apertures and the source.

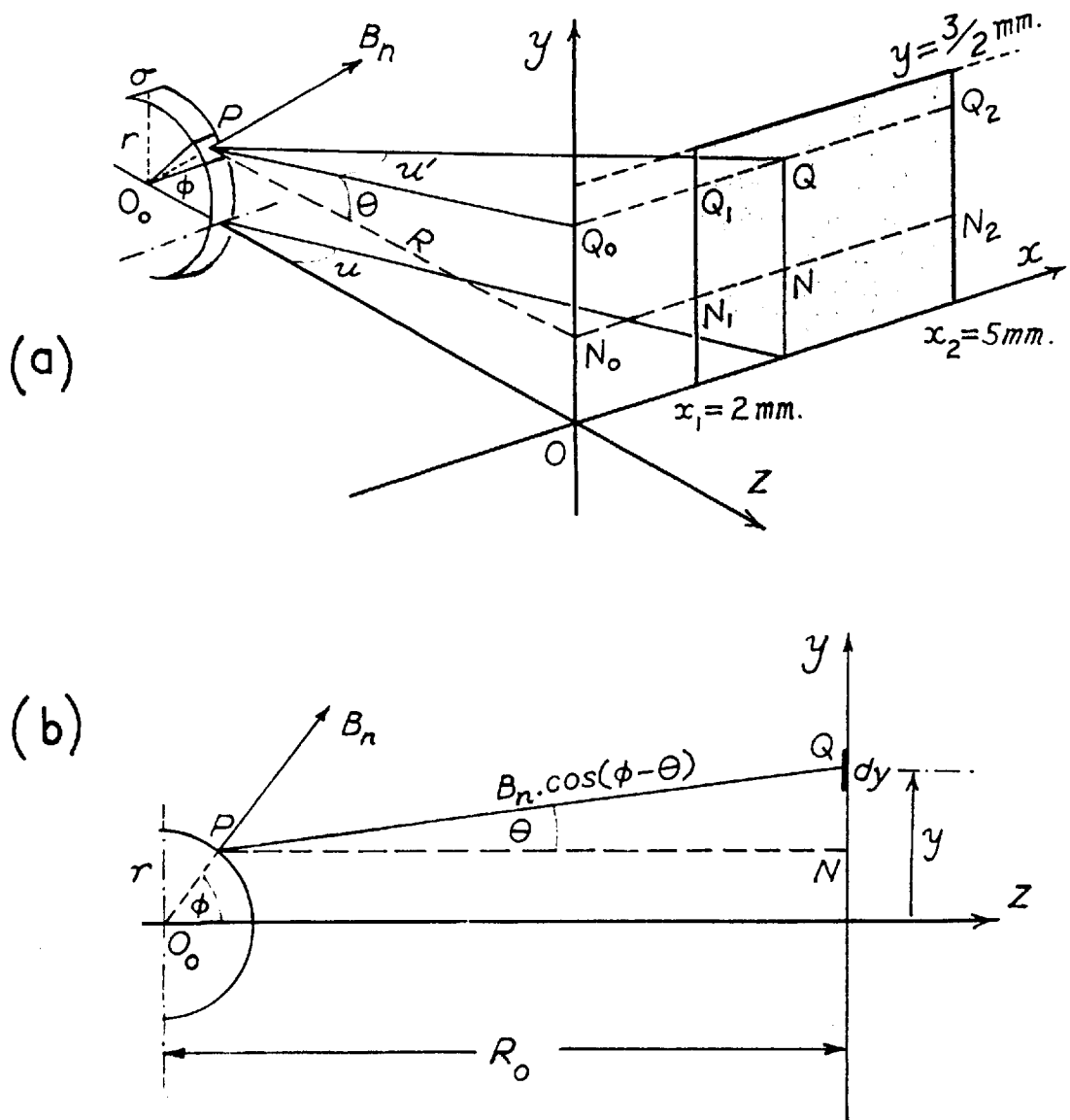


FIG. 4.9 Intensity distribution in the photometer apertures.

We consider first of all the distribution of intensity in the $x - y$ plane due to an element at a point P on the radiating strip with polar co-ordinate $\emptyset$. If the area of this element is $\sigma \cdot rd\emptyset$, and B_n is the "radiative intensity" of the source, that is, the normal energy flux per unit area of surface per unit solid angle^{*}, the intensities at points N_0 and N (where the plane PN_0N is parallel with the $x - z$ plane) will be, respectively,

$$I(N_0) = \frac{\sigma \cdot rd\emptyset \cdot B_n \cos\emptyset}{R^2} = I \quad 4.1$$

$$I(N) = I \cdot \cos^3 u \quad 4.2$$

At the points Q_0 and Q (Q_0Q is parallel with N_0N), the intensities will be:

$$I(Q_0) = I \cdot \cos^3 \theta \quad 4.3$$

$$I(Q) = I \cdot \cos^3 \theta \cdot \cos^3 u' \quad 4.4$$

Now, $\sin u' = \sin u \cdot \cos \theta$; the greatest value that can be assumed by u , that is, when N occupies the position N_2 , is $\frac{5}{150}$ radians, and there is negligible error in writing

$$u' = u \cdot \cos \theta \quad 4.5$$

The greatest value of θ , corresponding to a position of P at the upper (lower) extremity of the strip and a position of Q_0 at the lower (upper) boundary of the aperture image, is $\frac{2.5}{150}$ radians and the values of $\cos \theta$

* The "intensity" I at a point outside a source of radiation is the normal flux per unit area at the point, and at unit distance from the source is numerically (and dimensionally) equal to B_n . The source is assumed to be a Lambert emitter.

and $\cos^3\theta$ vary between the limits

$$0.9999 < \cos\theta < 1 \quad 4.6$$

$$0.9997 < \cos^3\theta < 1 \quad 4.7$$

The angles u and u' are closely identical and values of $\cos^3u$ and $\cos^3u'$ do not differ by more than 0.0001 . From eqns. 4.1 to 4.4 , therefore, we see that

$$\frac{I(N_0)}{I(Q_0)} = \frac{I(N)}{I(Q)} \quad , \quad 4.8$$

a condition that is only satisfied by an intensity function having the general form $I(x,y) = X(x).Y(y)$. (This is a necessary condition if the beam-chopper and the shutter limiting the aperture height move in orthogonal directions.) The x - dependent part of this function is $\cos^3u$. In going from $x = 2$ (mm.) to $x = 5$ (mm.) , $\cos^3u$ ($=\cos^3x/R$) does not vary by more than a few parts in 10^4 and the function $X(x)$ can thus be taken as constant in this range. This will be true for all elements such as P. In considering the y - variation of the total intensity function — which will now be denoted by $I(y)$ — due to the whole strip, it is convenient, without any loss of generality, to confine attention to the plane $x = 0$, as indicated in Fig. 4.9(b).

The intensity at a point Q due to the whole strip is^{*}

$$I(y) = \sigma.r.B_n \int_{-\pi/2}^{\pi/2} \frac{\cos(\phi-\theta).\cos^2\theta.d\phi}{(R_0 - r.\cos\phi)^2} \quad 4.9$$

*Actually, the extremities of the strip are stopped out and do not contribute, but this does not affect the validity of the argument.

From the geometry of Fig. 4.9(b) ,

$$\tan\theta = \frac{y - r.\sin\phi}{R_0 - r.\cos\phi}$$

and from 4.6 we see that $\sin\theta$ may replace $\tan\theta$ and that $\cos\theta$ may be taken as constant and brought outside the integral sign. Accordingly, 4.9 becomes

$$I(y) = \frac{1}{2}I_0R_0^2 \int_{-\pi/2}^{\pi/2} \frac{\cos\phi.d\phi}{(R_0 - r.\cos\phi)^2} + y \int_{-\pi/2}^{\pi/2} \frac{\sin\phi.d\phi}{(R_0 - r.\cos\phi)^3} - r \int_{-\pi/2}^{\pi/2} \frac{\sin^2\phi.d\phi}{(R_0 - r.\cos\phi)^3}$$

where $I_0 = \frac{2r.\sigma.B_n}{R_0^2}$, and $\cos\theta$ has been taken as unity. Only the second of the three integrals involves y and this vanishes between symmetrical limits. Thus, within the range of validity of the above approximations, $I(y)$ is independent of y and the light flux will be proportional to aperture height. Experimental tests for this will be described in Chapter V.

It will be useful to have at least an approximate knowledge of the actual light intensity in the apertures. This will be $1/9$ of the intensity in the $x - y$ plane given by $I(y)$. To evaluate the latter we may put $(R_0 - r.\cos\phi) \sim R_0$ in the denominators of the integrand. The last of the three integrals has a value less than 1% of that of the first and can be ignored for the present purpose. We have, therefore,

$$I(y) = I_0 = \frac{(2r.\sigma).B_n}{R_0^2} \quad 4.10$$

The value of B_n for a given wavelength λ can be obtained by integrating B_n over the solid angle 2π on one side of the radiating surface. This

gives the result

$$B_n(\lambda) = \frac{1}{\pi} W_\lambda \quad 4.11$$

where $W_\lambda \cdot d\lambda$ is the unpolarized radiated energy contained in a narrow waveband $d\lambda$ of mean wavelength λ emitted per second in all directions per unit area of the radiating surface. The tungsten globule of a "Pointolite" lamp has a working colour temperature of about 2800°K and may be regarded as a "black body" emitter, so that W_λ will be given by the Planck formula

$$W_\lambda \cdot d\lambda = C_1 \cdot \lambda^{-5} \left(e^{-C_2/\lambda T} - 1 \right)^{-1} \cdot d\lambda \quad 4.12$$

where $C_1 = 3.7402 \times 10^{-5} \text{ erg.cm}^2 \cdot \text{sec}^{-1}$, and

$$C_2 = 1.43848 \text{ cm.deg.}$$

4.8 Modulation of the beams.

Relatively little attention appears to have been given to the problem of efficient beam modulation, which is of considerable importance if maximum use is to be made of the available light. The commonest method is to chop the beams with an apertured rotating disc which alternately exposes the photodetector to each beam. There is usually a small time interval during which both beams are cut-off. Kniazuk (1939) used a vibrating armature carrying a suitable shutter, each beam being alternately presented to the photodetector.

In both of these methods the direction of travel of the edge of the chopping shutter follows a circular path, but if the

radius of this is considerably larger than the beam cross-section (as is usually the case) approximate linearity of the variation of flux with time may be assumed. If the flux in the two beams differs by an amount $\Delta F = F_0 - F_1$, where F_0 and F_1 are, respectively, the fluxes in the beams, the waveform of the signal arriving at the photodetector will appear as in Fig. 4.10, an ideal case being considered where there is no dark

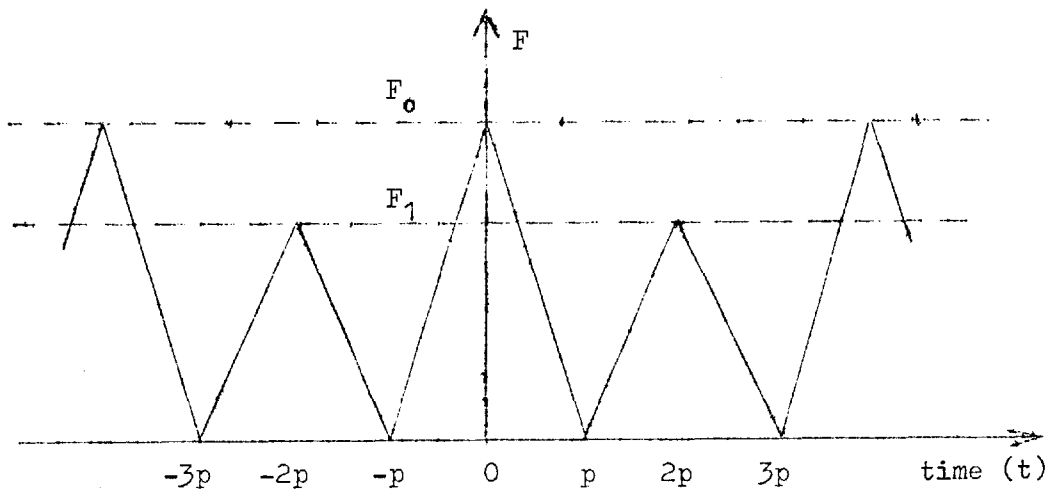


FIG. 4.10

period between successive pulses. Choosing an arbitrary time origin, and writing $p = \frac{T}{4}$ where T is the period of modulation, the waveform may be analyzed into the Fourier components :

$$F(t) = \frac{1}{2}a_0 + \sum a_n \cdot \cos \frac{\pi n}{2p} t + \sum b_n \cdot \sin \frac{\pi n}{2p} t$$

where the coefficients a_0 , a_n and b_n are given by the relations

$$a_0 = \frac{1}{2p} \int_{-2p}^{+2p} F(t) \cdot dt$$

$$a_n = \frac{1}{2p} \int_{-2p}^{+2p} F(t) \cdot \cos \frac{\pi n}{2p} t \cdot dt$$

$$b_n = \frac{1}{2p} \int_{-2p}^{+2p} F(t) \cdot \sin \frac{\pi n}{2p} t \cdot dt$$

The function $F(t)$ has the following values :

$$\begin{aligned} F(t) &= -\frac{F}{p} 1(t+p) & -2p < t < -p \\ &= \frac{F_0}{p} (t+p) & -p \leq t < 0 \\ &= \frac{F_0}{p} (t-p) & 0 \leq t < p \\ &= \frac{F_1}{p} (t-p) & p \leq t < 2p \end{aligned}$$

Since $F(t)$ is an even function of t , $b_n = 0$. On carrying out the integrations the values of the other coefficients are found to be :

$$\begin{aligned} a_0 &= \frac{1}{2}(F_0 + F_1) \\ a_n &= \frac{4}{\pi^2 n^2} (F_0 + F_1 \cdot \cos \pi n) \end{aligned}$$

The coefficients a_n represent the amplitudes of a series of harmonic signal components corresponding to $n = 1, 2, 3, \dots$, a_1 being the fundamental. The value $\frac{1}{2}a_0$ is the mean light level in the signal. Denoting this quantity by $\bar{F}$, the first four a_n coefficients are:

$$\begin{aligned} a_1 &= 0.4\Delta F \quad ; \quad a_2 = 0.5\bar{F} \\ a_3 &= 0.04\Delta F \quad ; \quad a_4 = 0.05\bar{F} \end{aligned}$$

In order that the null point may be clearly observed, it is necessary, in general, to employ some form of selective amplification of the useful signal to eliminate the effects of random fluctuations and mains ripple in the intensity of the source, pick-up from the mains supply and unwanted signal harmonics. The useful light-signal is the fundamental-frequency component a_1 ; this is seen to be smaller than the actual amplitude of modulation and the available light is thus not being used efficiently. Moreover, even if efficient wave filters are used, a good deal of noise will be generated by the signal harmonics, particularly by the very strong first harmonic a_2 , thereby reducing further the sensitivity of measurement.

Ideally, the beams should be so modulated that the total flux arriving at the photodetector varies sinusoidally between the values F_0 and F_1 ; in this way, the production of harmonics will be avoided and maximum use made of the out-of-balance flux. This condition is realised in the rather elegant method of Hardy referred to in Sec. 4.5 . The beam-chopper is a rotating polarizer (Nicol prism) intercepting both beams (which are polarized in mutually perpendicular directions) whereby reduction of intensity in one beam takes place simultaneously with increase of intensity in the other. The instantaneous total flux when the polarizer axis makes an angle ϕ with one beam (I_0), and an angle $(\frac{\pi}{2} - \phi)$ with the other beam (I) is given by

$$F(t) = S(\bar{I} + \Delta I \cos 2\phi)$$

where S is the cross-section of each beam, $\Delta I = I_0 - I$, and $\bar{I} = \frac{1}{2}(I_0 + I)$ is

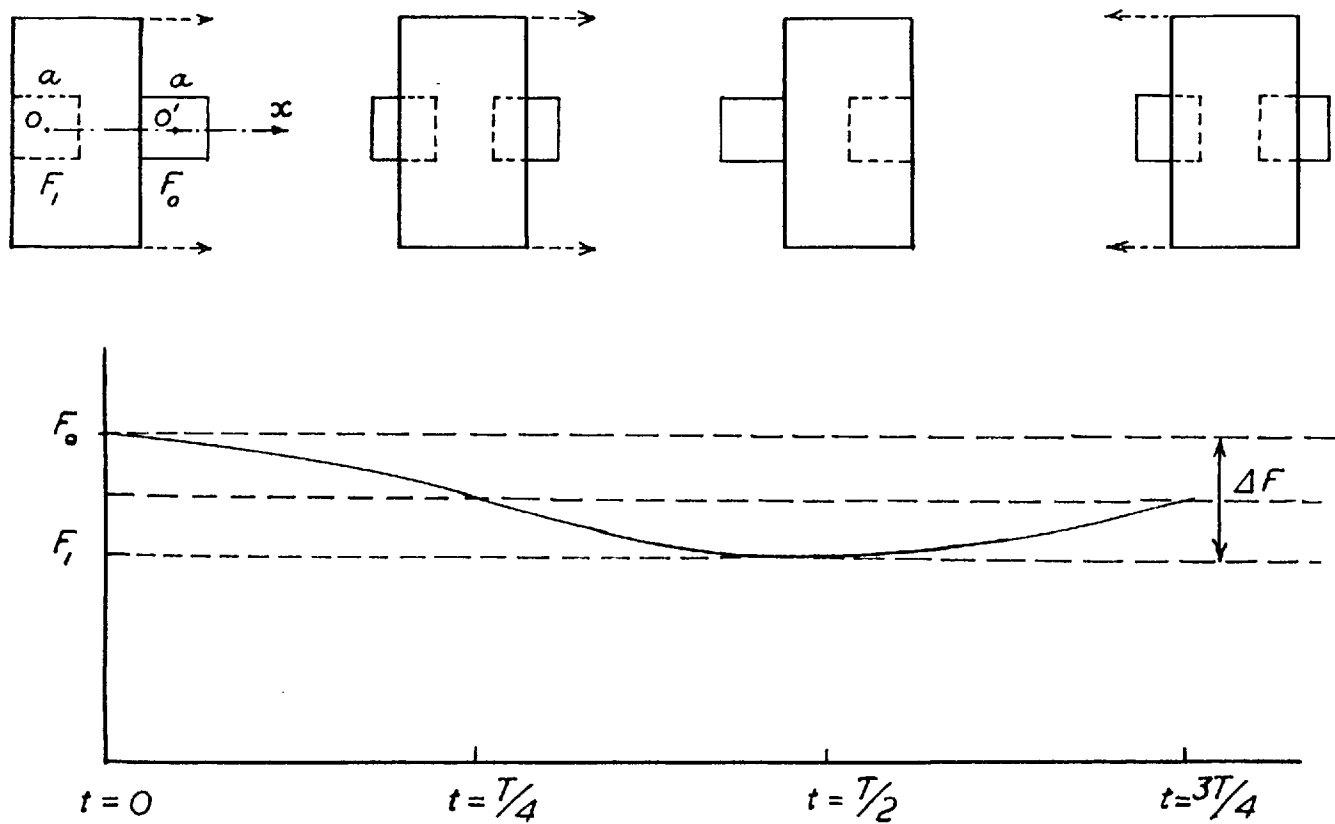


FIG. 4.11. Method of beam-chopping giving sinusoidal flux modulation.

the average intensity. The useful signal, ΔI , varies as $\cos 2\phi$ and thus has sinusoidal time-variation, the frequency of which is double the modulation frequency. With unpolarized light mechanical beam-chopping is necessary, and a method giving sinusoidal modulation is indicated in Fig. 4.11. The single chopping blade has a width equal to the centre-to-centre separation OO' of the apertures, and is reciprocated parallel with the X-axis with an amplitude of motion equal to $\frac{a}{2}$, where "a" is the width of each aperture. Thus, each aperture becomes fully exposed as the other is obscured, the increase in effective width of one aperture taking place at the same rate as the decrease in that of the other. Since the sum of the exposed widths of the two apertures remains constant, the total flux also would be constant, if the beams were balanced. When the beams are out of balance, then, if F_1 is the transmitted flux in the sample-beam, and F_0 is the flux in the reference-beam, the total instantaneous flux arriving at the photodetector when the chopping blade has moved a distance x from the central position OO' , will be given by

$$F = \left(\frac{a}{2} + x\right)F_1 + \left(\frac{a}{2} - x\right)F_0$$

This may be rewritten

$$F = a.\bar{F} - x.\Delta F$$

where $\bar{F} = \frac{1}{2}(F_0 + F_1)$ is the mean flux, and $\Delta F = F_0 - F_1$. The available signal, ΔF , is thus fully utilized, and the rate of variation of flux, $\dot{F} = -\Delta F.\dot{x}$, will be sinusoidal if the beam-chopper has simple harmonic motion. This is easily obtained using a simple crank mechanism (see Chap. V, Sec. 5.4).

4.9 Detection of the null point

Detecting systems such as those used by Neale, and Tait and Chalklin (see Section 4.4), in which the flux difference between two beams is indicated by a steady galvanometer deflection, could evidently be used for detecting a null difference of flux; they are, however, subject to a number of limitations. In these systems, the signals produced by the two beams, which are alternately incident on the photodetector, are "sampled" and applied in the form of narrow rectangular pulses to respective storage condensers in the gridcircuits of a pair of valves to which the galvanometer is connected. Thus, the available light in an out-of-balance signal is used inefficiently, and the sensitivity of measurement is correspondingly reduced. Errors in the balance setting of the beams can also arise as a result of differences in the amplification characteristics of the valves. In practice, moreover, it is difficult to obtain a sufficiently steady galvanometer deflection.

Alternatively, one could use a vibration galvanometer adjusted for resonance at the modulation frequency. Adequate insulation of a high-Q instrument against mechanical vibration, however, presents considerable practical difficulties, particularly when a reciprocating mechanism is present.

It was considered more satisfactory, therefore, to use an oscilloscope for indicating the signal. If, for example, the time-base is adjusted to give a stationary sine-wave trace, the null point will be

indicated by degeneration of the wave-form to a straight line. This simple method, however, is not capable of very great precision, because it is difficult to assess visually the straightness of a line. In a more refined method due to Kniazuk (1939), a constant-amplitude signal at the mains frequency, and a synchronously modulated photometer signal, are applied, respectively, to the vertical and horizontal deflector plates of the oscilloscope. The trace thus appears as an inclined straight line which rotates towards a vertical position as the beams are balanced. A disadvantage of this method is that mains instability will, in general, affect the mains and photometer signals differently, and this can give rise to uncertainty in detecting the null point. In the present case, moreover, such a method is not readily applicable, because the mains frequency, 50 c/s, is rather high for a reciprocating beam-chopper. The following simple method was used for the present work; it is independent of mains stability, and is capable of a high degree of precision.

When the time-base of an oscilloscope is synchronised with the frequency of an applied sinusoidal signal, there is one complete cycle of variations in the deflection of the electron beam during each sweep period, and the signal thus appears as one period of a stationary sine-wave. If, now, the time-base is adjusted so that the sweep frequency is doubled, alternate sweeps of the electron beam will carry alternate halves of the signal cycle, and the signal will appear as a pair of stationary loops, as indicated in Figs. 4.12(a) or 4.12(b). The null point is indicated by the degeneration of the loops into a single line, and has

been found to be very sensitive.

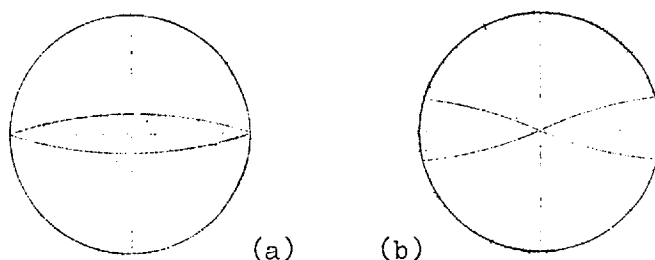


FIG. 4.12

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Chapter V

EXPERIMENTAL TECHNIQUE

5.1 Introduction

The design principles outlined in the previous chapter were used in a trial apparatus with which preliminary measurements were made of the transmission of magnesium fluoride anti-reflection films prepared using standard coating equipment. Sixteen rectangular glass blocks (approximate size: 4cm. X 2cm. X 1cm.) were coated simultaneously over half of one face, and then over half the opposite face. The film thickness was monitored visually. After some experiment, a batch of specimens was obtained having the same colour in reflected light on both sides, and the transmissions of the coated and uncoated parts of the surfaces were compared for a number of selected samples. It was found that measurements of the transmission ratio could easily be made with an accuracy of $\frac{1}{2}\%$. As was pointed out in Chapter II, variations of this magnitude in the reflectance of a surface can be very significant. In consequence, it was felt that the accuracy of the transmission measurement should be increased by an order of magnitude, giving errors limited to ± 0.0005 . The apparatus to be described in this chapter was directed towards obtaining an accuracy of measurement of this order.

5.2 Monochromator and illuminating system

In the preliminary measurements different wavelengths were

selected by means of a series of narrow band-pass colour filters. The half-value width of this type of filter is rarely less than about $200\text{\AA}$, and the uncertainty in the peak transmission may be as great as $40\text{\AA}$. These tolerances are too great for the required accuracy of measurement (cf. eqns. 5.3 and 5.4 below). Interference filters would be satisfactory but the cost of a series of such filters is high. A prism monochromator was therefore designed. This was based on the Littrow system so that a high dispersion could be obtained with a minimum of space and optical components; the Littrow arrangement was also well suited to the general layout of the apparatus. The main design considerations are spectral purity, photometric efficiency and mechanical accuracy of the rotation of the prism and Littrow mirror for giving wavelength selection. The photometric efficiency is roughly proportional to the bandwidth, and also depends on the physical dimensions of the optical components.

(i) Influence of bandwidth on the accuracy of measurement

The use of a prism monochromator to provide monochromatic light for transmission measurements introduces two sources of error. In the first place, since light passing through the exit slit contains a finite range of wavelengths, a measured value of the transmission will be an average for the waveband used. The "effective wavelength", namely, that for which the true transmission has the measured value, will depend on the variation of the transmission function in this region of the spectrum and is not amenable, a priori, to precise determination. In practice, therefore, a given waveband is identified, for example, by its median wavelength λ_0 ; the

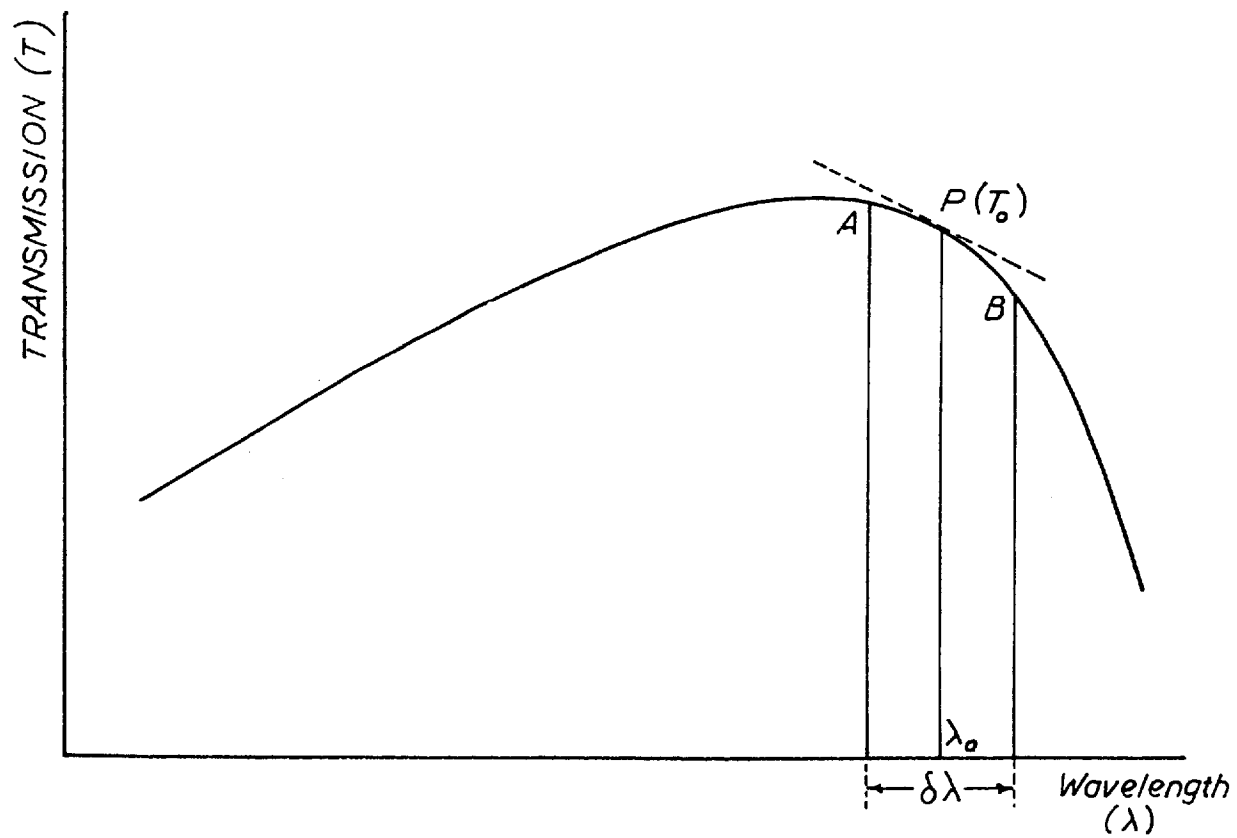


FIG. 5.1. Bandwidth error in a transmission measurement.

uncertainty $(\delta T)_1$ so introduced into a transmission measurement is then the difference between the true transmission $T(\lambda_0)$ at wavelength λ_0 , and the measured average value $\langle T(\lambda_0) \rangle$. Further uncertainty arises from errors in the wavelength calibration of the monochromator, the error $\delta\lambda_0$ in the value λ_0 assigned to the band producing an additional uncertainty in the measured transmission equal to

$$(\delta T)_2 = \left(\frac{dT}{d\lambda} \right) \delta\lambda_0 \quad 5.1$$

The error $(\delta T)_1$ is smaller the narrower the bandwidth of the light used, but the total energy flux is also reduced, and a wide band of wavelengths is needed to maintain high signal/noise ratio. To determine the maximum allowable bandwidth, reference is made to Fig. 5.1, which shows a characteristic transmission curve for a glass surface coated with an anti-reflection film (cf. Fig. 6.11, Chap. VI). The transmission $T(\lambda_0)$ at wavelength λ_0 is represented by the point P on the curve. Since the curve lies entirely below the tangent at P, the average ordinate value over the arc APB will be less than $T(\lambda_0)$; it is given by

$$\langle T(\lambda_0) \rangle = \frac{1}{\delta\lambda} \int_{-\frac{1}{2}\delta\lambda}^{+\frac{1}{2}\delta\lambda} T(\lambda) \cdot d\lambda$$

Expanding $T(\lambda)$ by Taylor's theorem in the range $\delta\lambda$, the integration gives

$$\langle T(\lambda_0) \rangle = T(\lambda_0) + \frac{1}{24} T'''(\lambda_0) \cdot (\delta\lambda)^2$$

where the primes indicate differentiation with respect to λ . The error in question is thus

$$(\delta T)_1 = \frac{1}{24} T'''(\lambda_0) \cdot (\delta\lambda)^2 \quad 5.2$$

To obtain a safe upper limit for $(\delta T)_1$ an approximate transmission curve is adequate for the estimation of the value of $T''(\lambda_0)$.

The total permissible error, ΔT , has been put at 0.0005, and will comprise errors arising in the light-flux measurement itself, and errors associated with the assignment of a wavelength value in a particular measurement. If the permissible error is allocated equally between these factors, the sum of the errors $(\delta T)_1$ and $(\delta T)_2$ must be less than ± 0.0003 . A safe tolerance is, therefore, $(\delta T)_1 = (\delta T)_2 = \pm 0.0001$. Thus, at the point P in Fig. 5.1, corresponding to a wavelength $\lambda_0 = 4500\text{\AA}$ in the blue region of the spectrum, $\frac{dT}{d\lambda} = 2.0 \times 10^{-5} \text{ \AA}^{-1}$; $\frac{d^2T}{d\lambda^2} = 2 \times 10^{-7} \text{ \AA}^{-2}$ (for other parts of the spectrum smaller values are to be expected). Substitution of these values in 5.1 and 5.2 gives for the allowable bandwidth

$$\delta\lambda = 80\text{\AA} \quad , \quad 5.3$$

and

$$\delta\lambda_0 = 5\text{\AA} \quad 5.4$$

as the accuracy with which a given waveband must be selected. The tolerance 5.4 is rather severe, and in practice it was found necessary to allow at least twice this amount. Fortunately, this tolerance was based on a very conservative figure.

(ii) Bandwidth selected by an exit slit in the plane of a pure spectrum

Assuming that the entrance slit of the monochromator is infinitesimally small and that there is no instrumental broadening due to diffraction and aberrations, an exit slit of width s and subtending an angle $\Delta\theta = \frac{s}{f}$

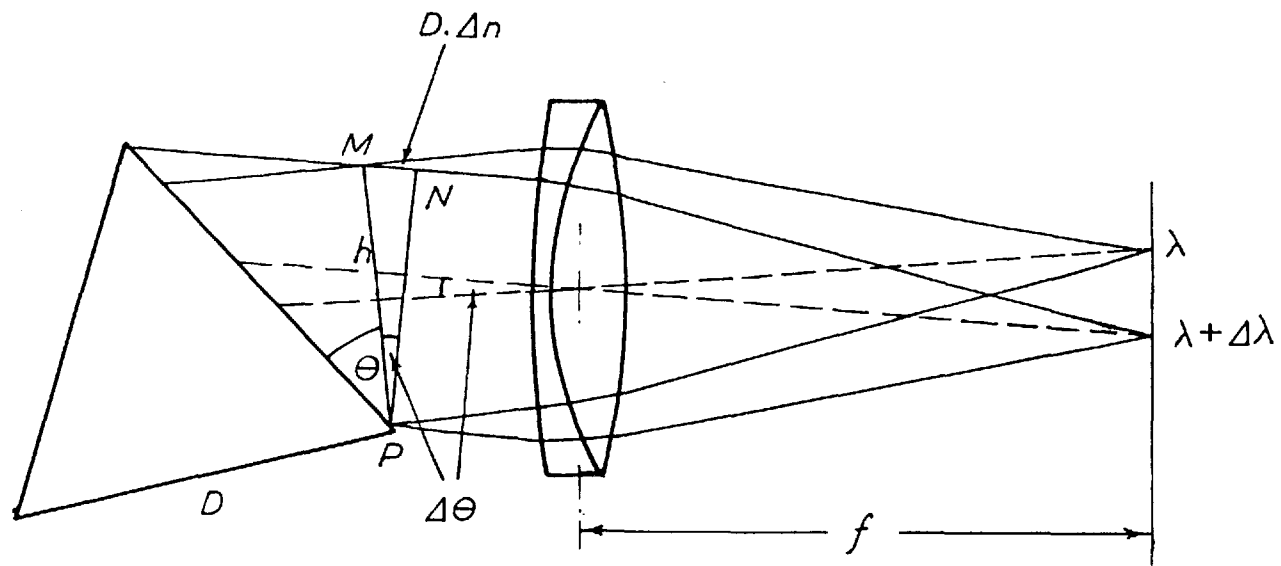


FIG. 5.2 Diagram for obtaining the relation $h \cdot \Delta\theta = D \cdot \Delta n$.

at the collimating lens, of focal length f (see Fig. 5.2), will intercept from the pure spectrum image a waveband $\Delta\lambda$ given by

$$\Delta\lambda = \left(\frac{d\lambda}{d\theta}\right) \Delta\theta \quad 5.5$$

with sufficient accuracy for the small values of $\Delta\lambda$ and $\Delta\theta$ that will occur in practice. The term $\frac{d\lambda}{d\theta}$ may be written

$$\frac{d\lambda}{d\theta} = \frac{d\lambda}{dn} \cdot \frac{dn}{d\theta} \quad 5.6$$

The first factor is simply the reciprocal of the dispersion $\frac{dn}{d\lambda}$ of the prism glass. In the case of a barium flint glass, the dispersion at a wavelength, for example, of $\lambda = 4340\text{\AA}$ is $\frac{dn}{d\lambda} = 1.23 \times 10^{-6} \text{\AA}^{-1}$.* This compares with the value 1.24×10^{-6} obtained from the curve of refractive index v. wavelength using the ratio of the finite differences $\frac{\Delta n}{\Delta\lambda}$ for the wavelength range $4340 \pm 50\text{\AA}$. Thus, for calculating tolerances of the order of $100\text{\AA}$, the difference between $\frac{dn}{d\lambda}$ and $\frac{\Delta n}{\Delta\lambda}$ is negligible. Similarly, the ratio of finite differences $\frac{\Delta n}{\Delta\theta}$ gives a sufficiently accurate estimate of $\frac{dn}{d\theta}$. Thus, in Fig. 5.2, since the optical path difference between the wave-fronts PM and PN, corresponding to wavelengths λ and $\lambda + \Delta\lambda$, must be the same along rays at the apex and base of the prism, we shall have

$$D \cdot \Delta n \approx h \cdot \Delta\theta$$

and, therefore,

$$\frac{dn}{d\theta} = \frac{h}{D} = \cos\theta \quad 5.7$$

For the glass and wavelength of light indicated above, the value of $\Delta\theta$, for $\Delta\lambda \sim 100\text{\AA}$, will be of the order of 10^{-4} radians: the variation of $\cos\theta$, and hence of $\frac{h}{D}$, is negligible, so that we may write $\frac{h}{D} = \frac{dn}{d\theta} = \frac{\Delta n}{\Delta\theta}$

* Jenkins & White: "Fundamentals of Optics" (2nd ed., McGraw-Hill) p.463

For the values of $\Delta\lambda$ in question, therefore, differentials in eqn. 5.6 may be replaced by finite differences, so that 5.5 is valid. This is conveniently rewritten, using 5.7, to give for the angular size of exit slit

$$\Delta\theta = \frac{2}{\cos\theta} \left(\frac{dn}{d\lambda} \right) \Delta\lambda, \quad ,$$

the factor 2 being present because the prism is traversed twice in a Littrow system. In the case of a 60° prism of barium flint glass ($n \sim 1.6$) working at minimum deviation, $\theta \sim 53^\circ$; thus

$$\Delta\theta = 0.33 \left(\frac{dn}{d\lambda} \right) \Delta\lambda \quad 5.8$$

(It is of advantage to use a prism monochromator in conditions of minimum deviation, because the rate of change of deviation with changing angle of incidence, as the the prism is rotated to select different wavelengths, is then zero, and the wavelength distribution in the spectrum image remains substantially unchanged throughout the small scanning angle; in a Littrow system, particularly, symmetrical passage of the light through the prism also reduces vignetting.)

Because of decreasing values of $\frac{dn}{d\lambda}$, the bandwidth will increase with increasing wavelength; but, as was indicated earlier, the values of $T'(\lambda)$ and $T''(\lambda)$ will usually decrease, so that these factors tend to be compensatory with regard to the tolerances 5.1 and 5.2

(iii) Influence of the finite width of entrance slit

In Fig. 5.3 the centre one of the three bands represents the spectral region $\Delta\lambda = \lambda_2 - \lambda_1$ (corresponding to $\Delta\lambda$ in 5.8) that would be selected

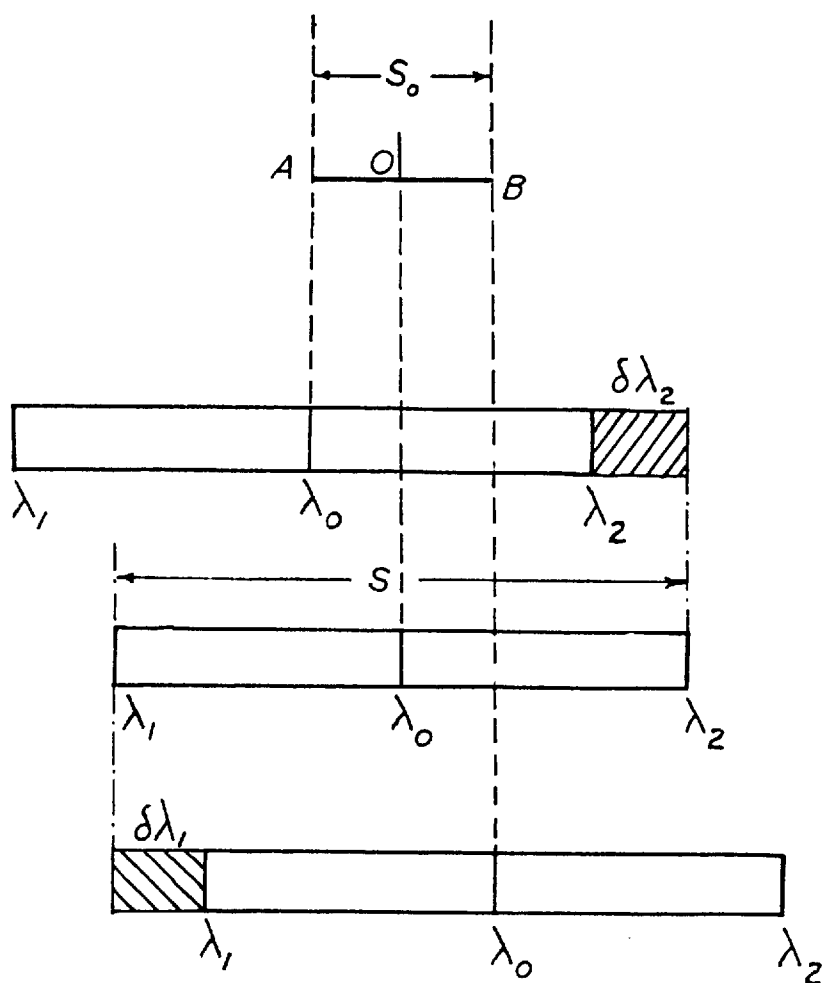


FIG. 5.3. *Spectral impurity due to finite width of entrance slit.*

by the exit slit of width s from a pure spectrum produced by a limitingly narrow entrance slit at the centre O of the actual entrance slit of width s_0 . A second line element of the entrance slit AB , displaced from O , will produce a spectrum correspondingly displaced with respect to the exit slit, and there will be such spectra corresponding to all elements of the entrance slit. The upper and lower bands in the figure indicate, respectively, the spectra produced by the marginal line elements A , B , of the entrance slit, and are displaced by amounts $\pm s_0/2$ relative to the spectrum image of O . The shaded areas indicate the additional ranges of wavelength which thus fall within the exit slit. (The extreme wavelengths will be reduced in intensity, but this (beneficial) effect will be ignored.)

If L_s denotes the linear dispersion at the exit slit, the additional wavelength ranges $\delta\lambda_1$ and $\delta\lambda_2$ are each equal to $L_s \cdot \frac{s_0}{2} = \delta\lambda$, say. Now, the value of L_s is given by $\frac{\Delta\lambda}{s}$, so that $\delta\lambda = \frac{\Delta\lambda}{s} \cdot \frac{s_0}{2}$. Thus, the total bandwidth $\Delta\lambda'$ of the light passing through the exit slit is $\Delta\lambda + 2\delta\lambda$, and may be written:

$$\Delta\lambda' = \frac{\Delta\lambda}{s}(s + s_0) \quad 5.9$$

For constant linear dispersion $L_s = \frac{\Delta\lambda}{s}$, the spectral purity of the light leaving the exit slit is constant for a constant value of $(s+s_0)$. The total light entering the monochromator is proportional to the width of the entrance slit, s_0 , and the fraction of this light admitted by the exit slit is proportional to its own width, s . Thus the light flux is proportional to the product ss_0 , and this has a maximum, for $(s+s_0) = \text{constant}$, when $s = s_0$. Thus, for a given spectral purity, the maximum light flux

is obtained with the entrance and exit slits of the same width.

When $s_0 = 0$ the angular size of the exit slit giving a bandwidth $\Delta\lambda$ is given, by 5.8, as $\Delta\theta = 3.33 \left(\frac{dn}{d\lambda} \right) \Delta\lambda$. Choosing a finite entrance slit of width $s_0 = s$ doubles the bandwidth, as shown by 5.9. A factor $\frac{1}{2}$ has, therefore, to be introduced and

$$\Delta\theta = 1.67 \left(\frac{dn}{d\lambda} \right) \Delta\lambda \quad 5.10$$

is the formula determining the permissible slit width.

(iv) Influence of diffraction and aberration on spectral purity

Some additional impurity of the spectrum will be introduced by diffraction from the edges of the prism aperture. The angular distance of the first dark fringe from the centre of the diffraction pattern due to a rectangular aperture of width h is equal to $\frac{\lambda}{h}$. The effective width of the entrance slit will thus be increased by an amount $\delta\theta = \frac{2\lambda}{h}$. For the prism used $D = 64\text{mm.}$, giving $h = D\cos 53^\circ \approx 38\text{mm.}$, so that

$$\delta\theta = 0.05\lambda$$

The resulting wavelength spread, for a particular wavelength λ , will be given by inserting the value 0.05λ for $d\theta$ in the left-hand side of eqn. 5.6 and solving for $d\lambda$; the value of $\frac{dn}{d\theta}$ is given, by 5.7, as $\cos 53^\circ$, and using, for example, the values: $\lambda = 4340\text{\AA}$ and $\frac{dn}{d\lambda} = 1.23 \times 10^{-6} \text{\AA}^{-1}$, we obtain $d\lambda = 1\text{\AA}$, which is negligible. There will be no diffraction from the collimating lens, because this was chosen of sufficiently large area to

circumscribe the prism aperture.

The residual chromatic aberration in the collimating lens will comprise secondary spectrum, giving a total variation in focus equal to about $f/1000$ where f is the focal length. For the lens used $f = 20\text{cm.}$ and the resulting spread in the mean focal plane will be $\frac{1}{2} \cdot \frac{h}{1000}$ where $h = 38\text{mm.}$ is the effective width of aperture. This gives 0.017mm. which is about 5% of the slit width used (see below) and may, therefore, also be neglected.

Thus, using 5.10 and values given above for $\frac{dn}{d\lambda}$ and f , the slit width $s_0 = s = f \cdot d\theta$ required to select a total bandwidth of $\Delta\lambda \sim 80\text{\AA}$ is

$$s_0 = s = 0.33\text{mm.} \quad 5.11$$

(v) The illuminating and monochromating systems

The optical arrangement of the monochromator and its illuminating system is shown in Fig. 5.4 . The monochromator prism P and Littrow mirror M are mounted as a unit (see also Fig. 5.6) at one end of an optical bench which also carries the photometer units, the collimating (and de-collimating) lens L_c and a prism P' which consists of two 45° prisms cemented together and serves to admit light from an illuminating system separately mounted at one side. This comprises a 100 c.p. "Pointolite" lamp, O, and lenses L_0 , L'_0 , which image the source O ($\sim 3\text{mm}$ diameter) with unit magnification on the entrance slit S_0 , some 1.5mm. in length. In addition, L'_0 images L_0 on the collimator L_c which is thus filled with light. The

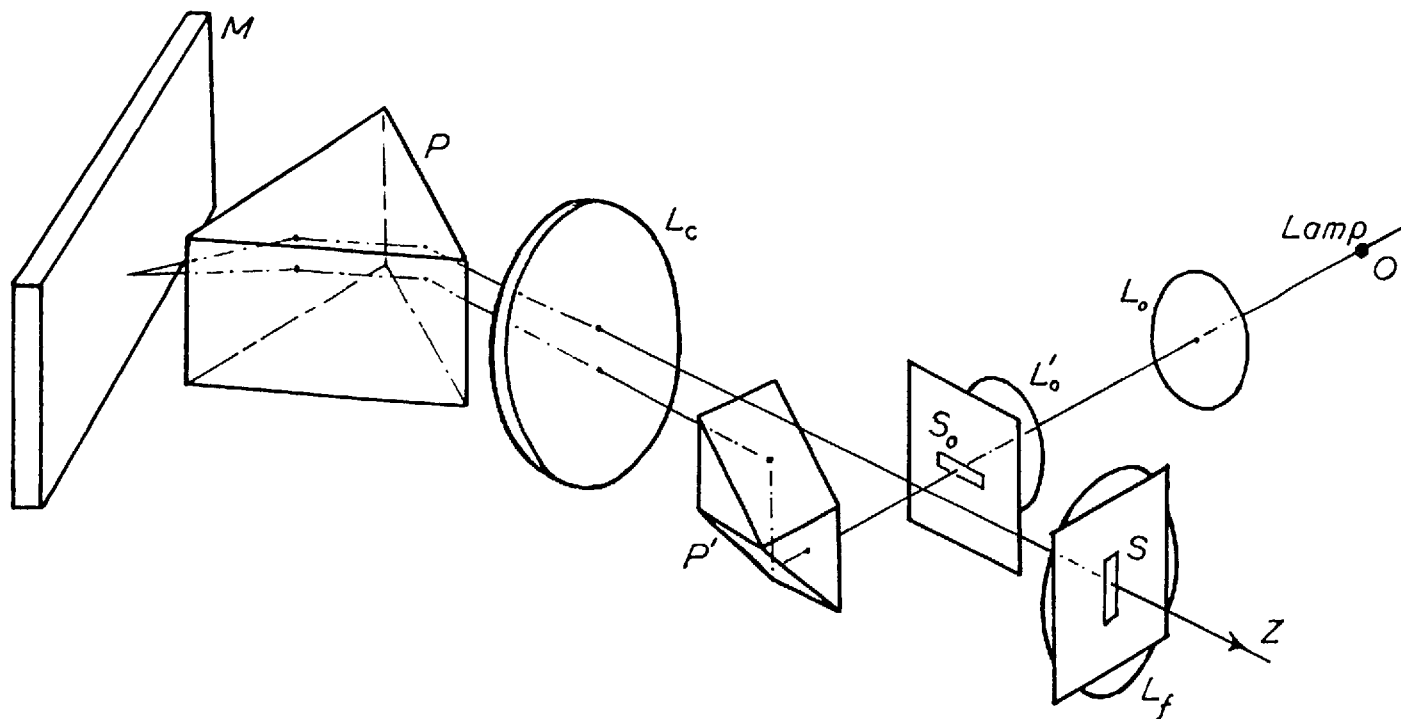


FIG. 5.4. *Monochromator and illuminating systems.*

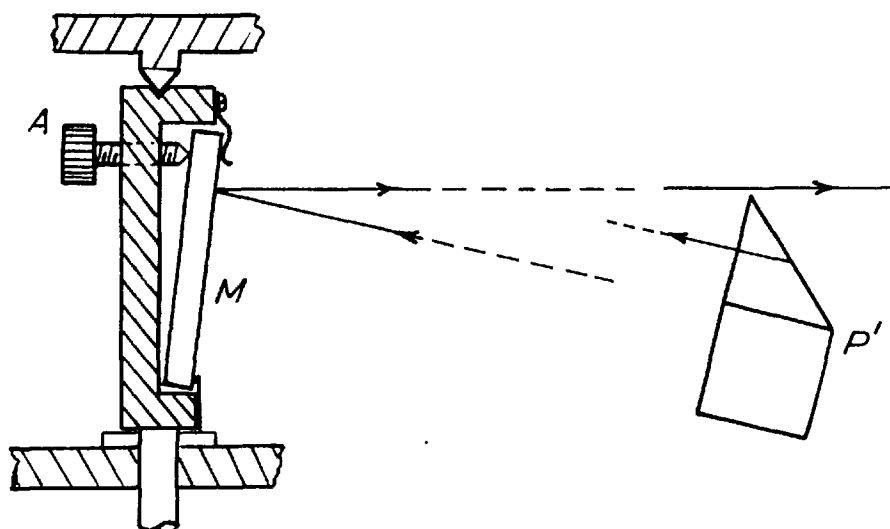


FIG. 5.5. *Adjustment of the Littrow mirror.*

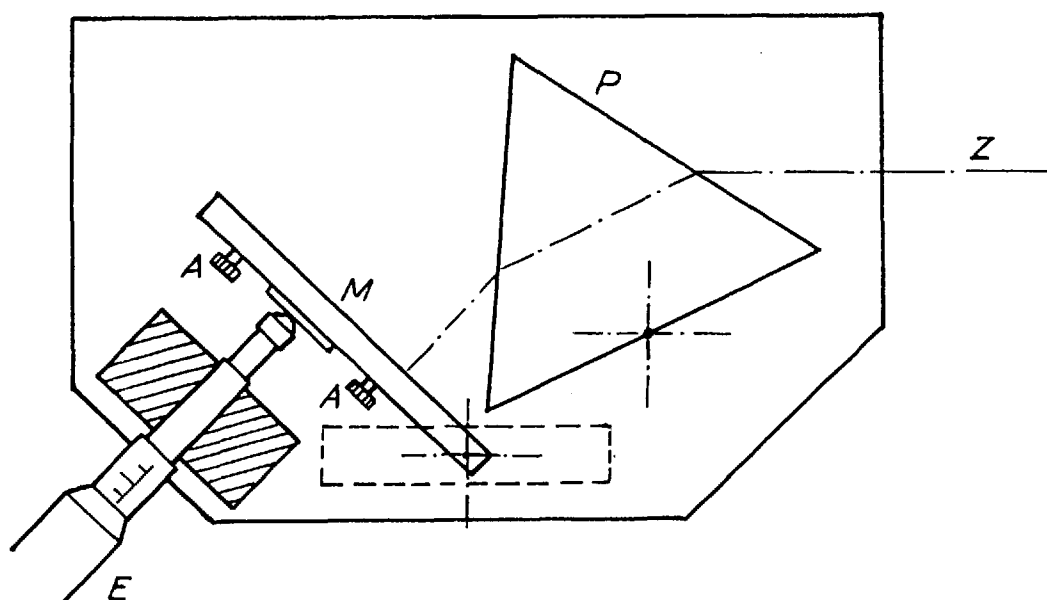


FIG. 5.6. *Plan of monochromator.*

prism P' is tilted slightly (see Fig. 5.5) so that light is incident on mirror M in a direction inclined at about 2° to the general optical axis Z of the apparatus, and may be reflected parallel with the Z-axis to clear the uppermost edge of P' by suitable adjustment of the inclination of M, using screws A. Adjustment of prism P' is facilitated by mounting the prism in a cradle adjustable for movement along three orthogonal axes and for rotation about these axes. On traversing P', the light beam is rotated 90° about the direction of propagation, and the entrance slit S_0 is therefore arranged to lie horizontally so that a vertical image of S_0 is formed in the plane of the exit slit S, which is also vertical.

To select different wavelengths, the spectrum image is moved horizontally across the exit slit using a screw micrometer E with a radiused end to rotate the mirror M about a vertical axis; the prism P is rotated in the same sense, and at half the speed, by means of gears with diameters in the ratio 1:2 mounted on spindles carrying M and P, respectively, and meshing with a common idler gear. Helical gears were used, and there was no detectable backlash. The upper part of the mirror frame was located true by means of a simple pivot, and repeated tests of the linearity of movement of the spectrum image revealed no observable shift in the vertical direction.

The exit slit is formed by a pair of knife-edges mounted at one end of a draw-tube (see Fig. 5.8, p.105) forming part of the housing of the photometer system. Sharp focusing of the spectrum image, by adjustment of the draw-tube, was facilitated by the presence immediately in

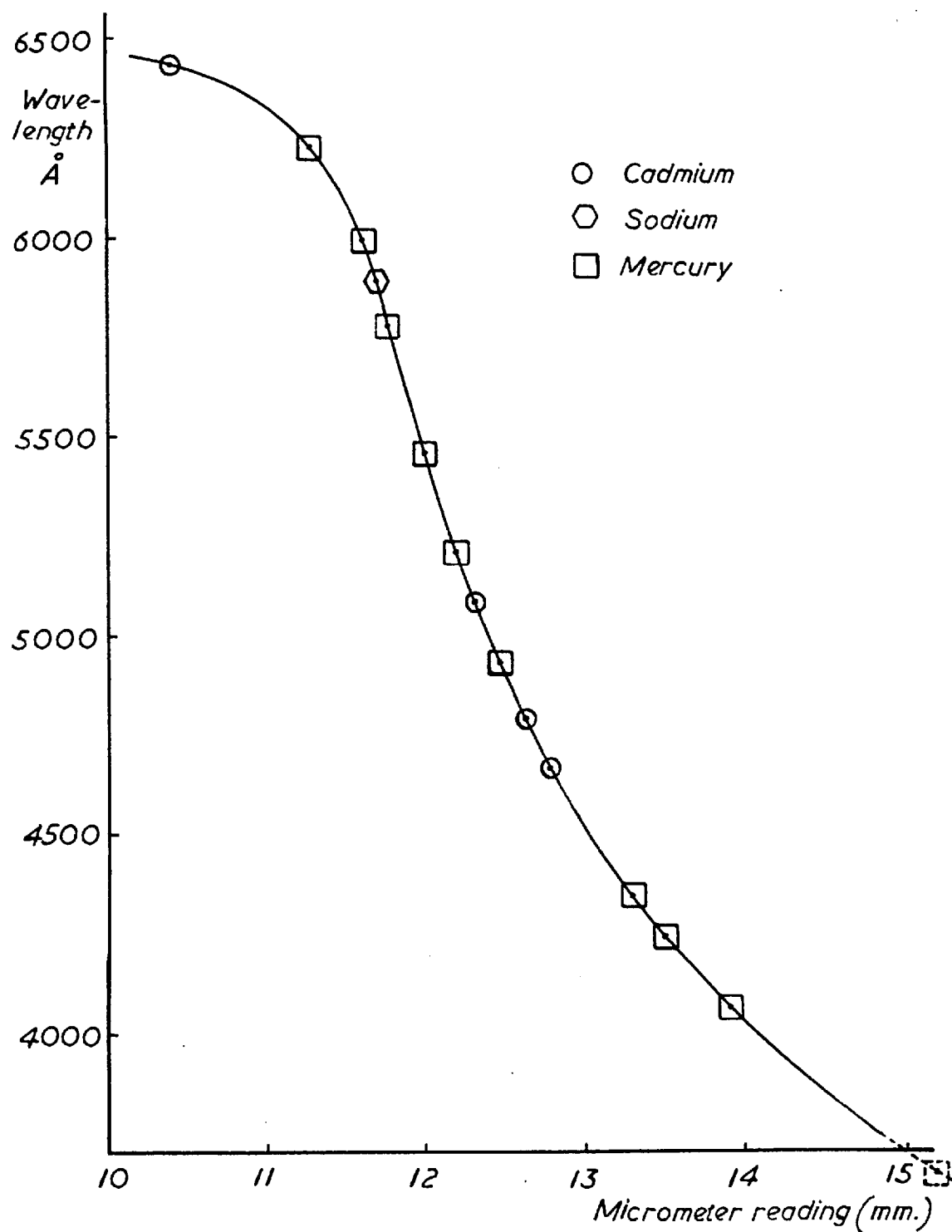


FIG. 5.7 . Calibration curve of monochromator.

front of the exit slit, of an achromatic field lens L_F , the primary function of which will be explained in Section 5.3, but which also served to form a magnified virtual image of the spectrum.

Micrometer readings ($\pm 0.005\text{mm.}$) were calibrated against wavelength using cadmium, sodium and mercury lamps. The resulting calibration curve is shown in Fig. 5.7. The procedure used for the calibration was first to set the photometer in an unbalanced condition, and then to adjust the micrometer E until a signal was observed indicating the traverse of a spectral line across the exit slit of the monochromator. The wavelength error in the calibration could, in principle, be as much as half the monochromator bandwidth at that wavelength. In practice, however, a mean micrometer setting, corresponding to the positioning of the spectral line at the centre of the exit slit, could be found with much greater accuracy, but in order to approach the condition 5.4, it was necessary to reduce the exit slit width to $s \sim 0.2\text{mm.}$ This gave a setting accuracy of about $\pm 10\text{\AA}$, and was a compromise with respect to the photometric efficiency. (Actually, it was not possible to draw, and read, the calibration curve with an accuracy greater than this.)

(vi) Photometric efficiency

To calculate the spectral bandwidths, and hence the photometric efficiency, of the monochromator for different wavelengths, 5.10 is rewritten

$$\Delta\lambda = 6 \times 10^{-4} \left(\frac{dn}{d\lambda} \right)^{-1} \quad 5.12$$

using the value $s = 0.2\text{mm.}$ for the width of the exit slit. The height

of the slit was 1.5mm., giving an area of $3.0 \times 10^{-3} \text{ cm}^2$. This value must be substituted for the term $2r.\sigma$ in eqn. 4.10 (p. 77) which gives the intensity in the plane of the lens L_0 (see Fig. 5.4 above, and Fig. 4.8 on page 73). Because of the 3 times magnification between this plane and that of the two beam-limiting apertures, a factor $1/9$ is introduced so that the intensity, $I(\lambda)$, in the plane of the apertures, of the light in a waveband $\Delta\lambda$ selected by the exit slit, will be given by

$$I(\lambda) = \frac{3 \times 10^{-3} \cdot B_n(\lambda) \cdot \Delta\lambda}{9R_0^2} \text{ erg.sec}^1.\text{cm}^2 \quad 5.13$$

where $R_0=15\text{cm.}$, and $B_n(\lambda)$ is given by 4.11 and 4.12 (p. 78). The overall light path in the apparatus, between the source 0 and the photodetector, includes 23 air-glass interfaces and some 20cm. of glass, so that a more useful figure for the photometric efficiency of the apparatus will be given by including a transmission factor, t_λ , in 5.13. This may be calculated using approximate values for the reflection and absorption coefficients. The resulting values of $I(\lambda)$ are given in the following table, which summarizes the "spectrophotometric performance" of the apparatus.

Table 5.1

Wavelength λ	($\text{\AA}$)	6000	5000	4000
Prism dispersion $dn/d\lambda$	($\text{\AA}^{-1}$)	0.46×10^{-5}	0.82×10^{-5}	1.72×10^{-5}
Bandwidth ($\pm 5\%$)	($\text{\AA}$)	130	75	35
Calibration error $\delta\lambda_0$	($\text{\AA}$)	± 15	± 10	± 8
Absorption coeff./cm. of glass		0.010	0.005	0.050
Transmission/air-glass interface		0.949	0.948	0.947
Overall transmission factor, t_λ		0.18	0.21	0.10
$B_n(\lambda) \cdot d\lambda$ for $d\lambda = 1\text{\AA}$		3.0×10^4	1.3×10^4	0.3×10^4
$I(\lambda)$ erg.sec $^{-1}$.cm ^{2}		1.04	0.30	0.015

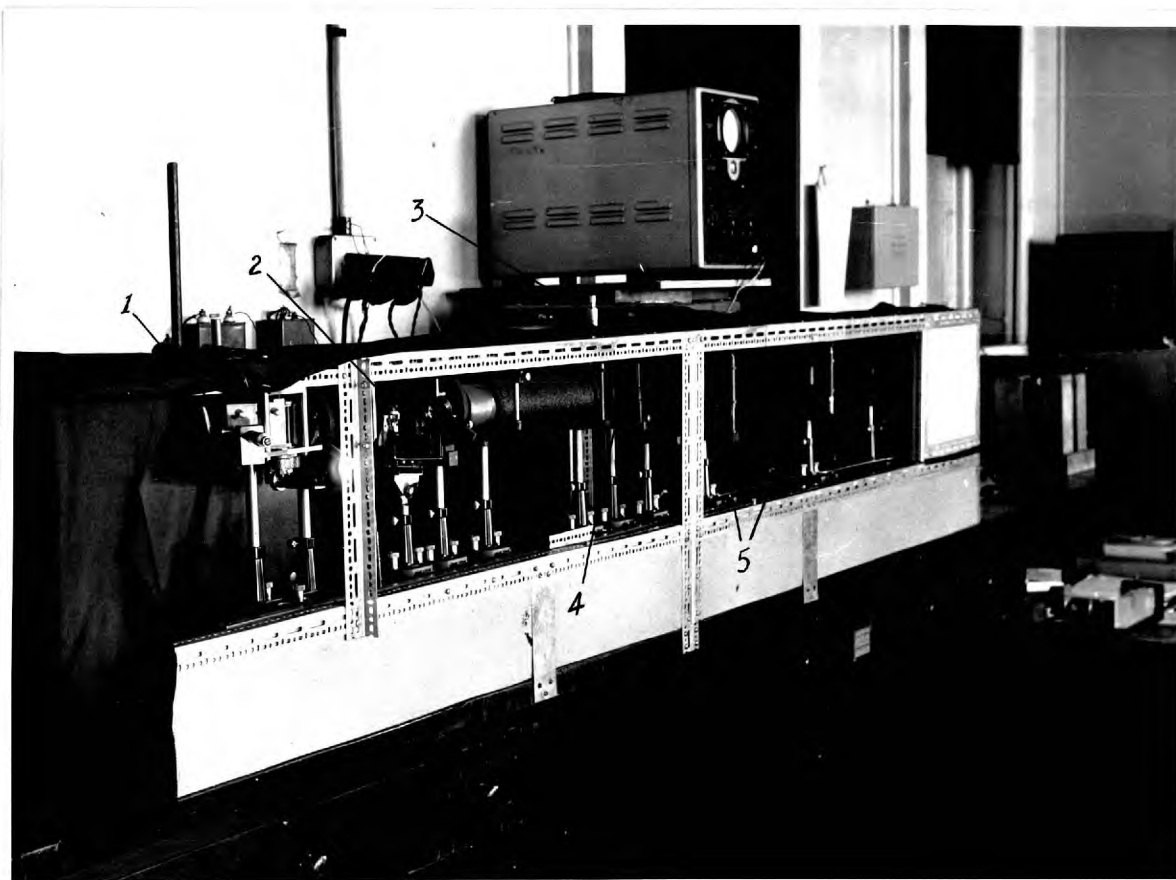


PLATE I . General view of photometer: (1) monochromator; (2) prism for entry of light; (3) micrometer for adjusting aperture height; (4) beam chopper; (5) last two split lenses.

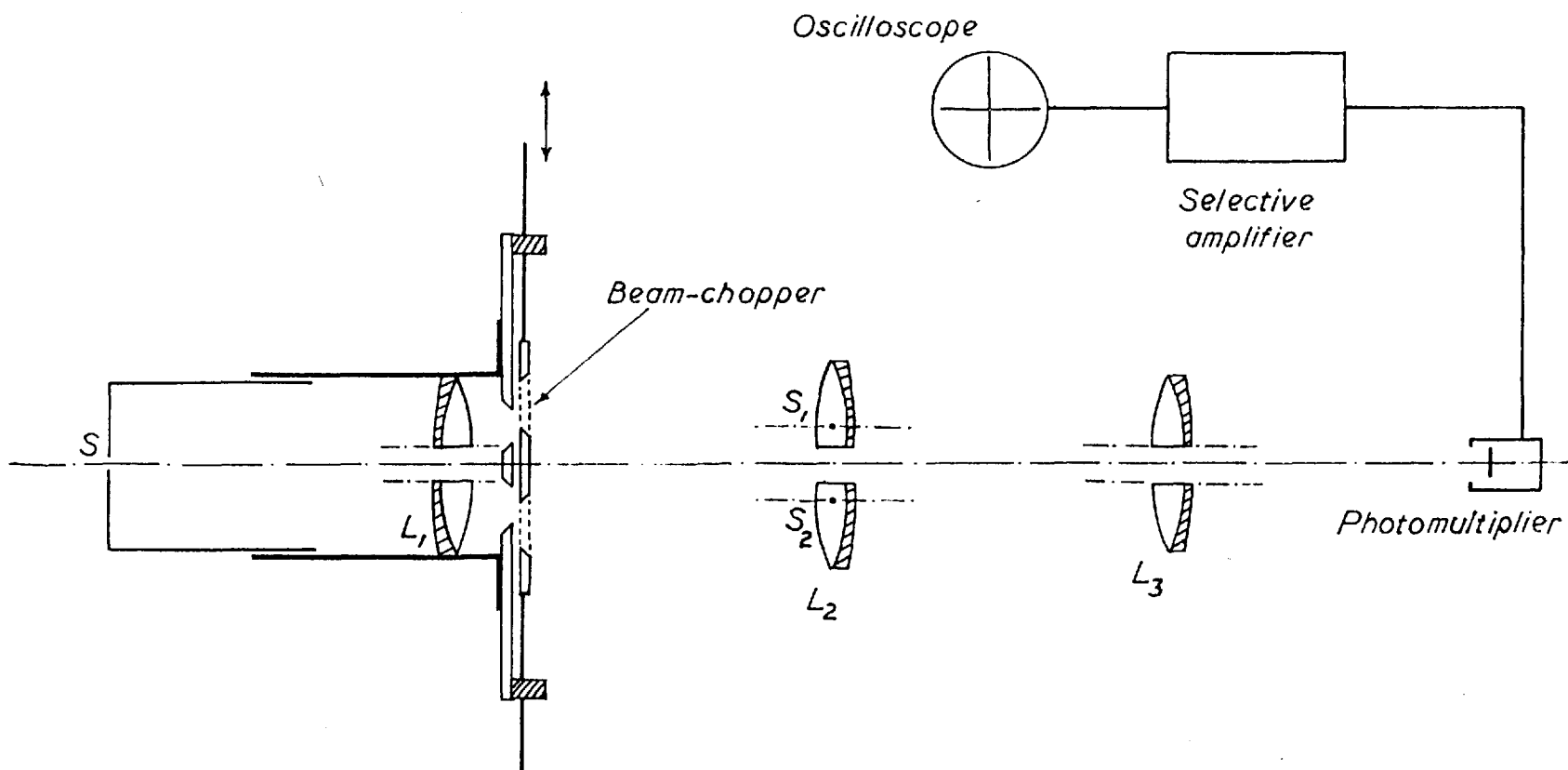


FIG. 5.8. Schematic diagram of photometer.

5.3 Optical system

The split-lens system, described in Chapter IV, for producing twin beams, was modified, as shown in Fig. 5.8, by the addition of a split field lens, L_2 , which images the beam-limiting apertures A_1 and A_2 , on the corresponding halves of the split lens L_3 . The lenses all conveniently work at unit magnification, and have the same focal length. (This was chosen as 7", so that the optical system could be mounted on a 2-metre optical bench leaving sufficient room for the monochromator at one end and for a photodetector at the other.) The separation of the L_1 (and L_3) segments was made equal to 1cm., so that the separation of the images S_1 and S_2 of the entrance slit S (this is the monochromator exit slit), is 2cm., which is the centre-to-centre separation of the apertures A_1 and A_2 (see Section 5.4). The chief rays of the two beams will thus be parallel with the Z-axis. The L_2 segments are major segments of separate lenses, and the separation of their optical centres is equal to that of the images S_1 and S_2 . To facilitate alignment of the beams, and superposition of the final images, one segment of each of the lens pairs L_2 and L_3 is adjustable in height, using a simple worm-screw mechanism.

The field lens, L_f , immediately in front of the slit S , images the collimator lens of the monochromator in the plane of the apertures, with a magnification of about $1\frac{1}{2}$ times. This ensures that the apertures will be evenly illuminated by the central portion of the light beam, and, at the same time, provides a convenient fine adjustment

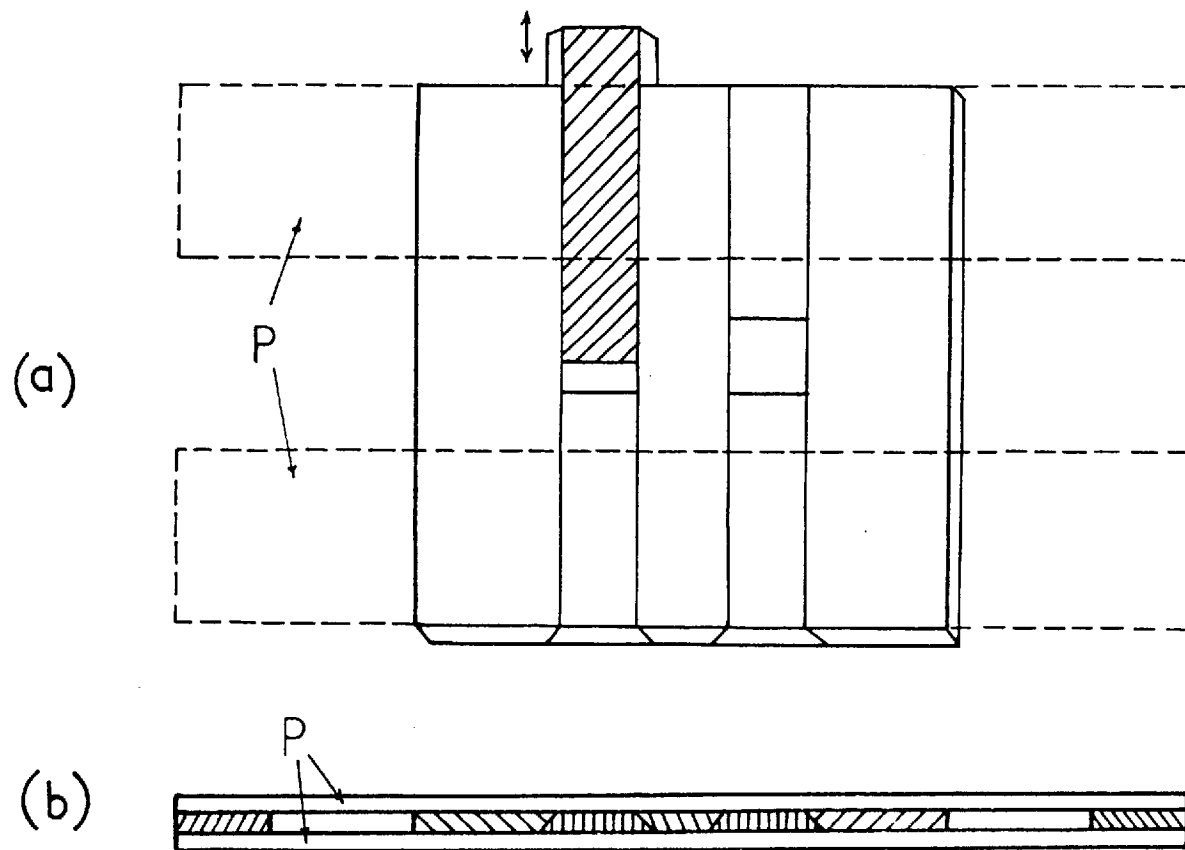


FIG. 5.9. *Aperture assembly.*

for initially centering the beam on the apertures. The lenses used were all achromats so that the geometrical optical relations in the system are, to sufficient accuracy, independent of the wavelength of light used.

5.4 Control and modulation of the light flux

The apertures defining the photometer beams were formed, as shown in Figs. 5.9(a) and (b), by an assembly of accurately machined steel strips, bevelled at 45° and fitting together in "dovetail" fashion to give light-tight joints, the shorter strips having chamfered ends so that all the aperture edges are sharp. The strips are held between two pairs of steel plates P to which they are all riveted with the exception of one, shown shaded in the diagram, that is free to move in a vertical direction and forms the shutter for the variable aperture. This strip is pressed upwards by a spring (see Fig. 5.10) against a screw micrometer which was initially adjusted in its mounting to read "zero" with the aperture fully closed; the light flux through the aperture is thus proportional to the micrometer reading. The size of the apertures is 1cm. square (with the variable aperture fully open). Thus, assuming that the micrometer can be read with an accuracy of 1μ (one tenth of a micrometer division), the aperture height will be measurable with a precision of 1 part in 10^4 at full aperture, giving an error in a transmission measurement of the order of $\delta T = \pm 0.0001$, which matches the accuracy of the monochromator.

The separation of the two apertures is usefully made as

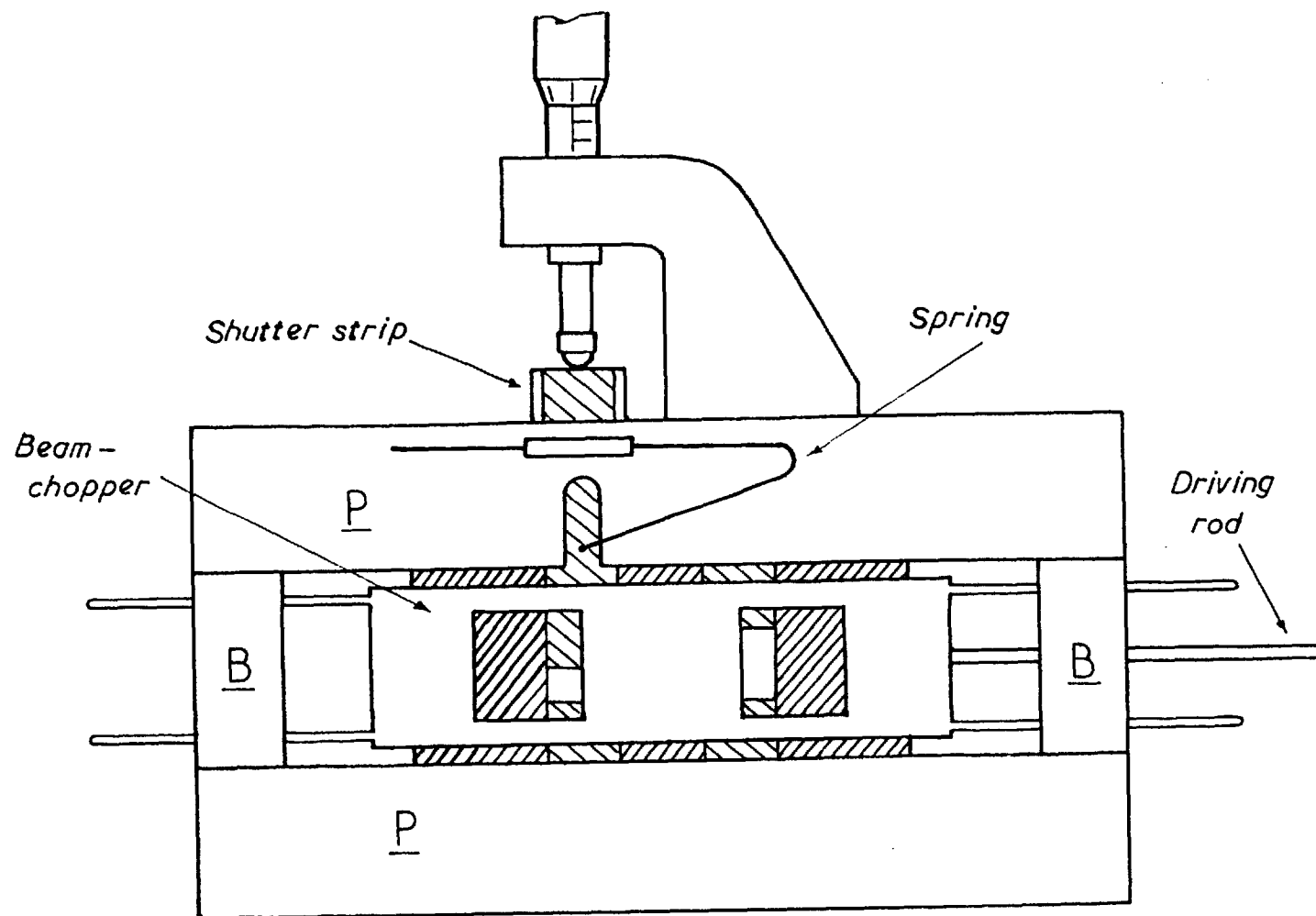


FIG. 5.10. Aperture- and beam-chopper assembly.

small as possible so that the size, and hence the inertia, of the beam-chopper may be kept small thus minimizing vibration; an optimum value for the centre-to-centre separation, giving convenient spacing of the beams, is 20mm. The beam-chopper is a thin ($\frac{1}{64}$ ") steel plate having two square apertures, as shown in Fig. 5.10, the portion between these being operative to chop the beams. This portion is accordingly made 20mm. wide, (cf. Ch. IV, Sec. 4.8), and has sharp, parallel edges. For the other edges to clear the beams, the apertures need be only slightly larger than 1cm. x 1cm., but they were made considerably larger to reduce weight. The beam-chopper is carried on two thin steel rods which slide in holes in brass blocks B fixed to the ends of the plates P, the entire assembly shown in Fig. 5.10 forming a unit mounted on the front end of the fixed sleeve of the draw-tube (see Fig. 5.8). The clearance between the beam-chopper and the beam-limiting apertures is made small -less than $\frac{1}{2}$ mm.- so that the configuration of the beams in the chopping plane is closely identical with that of the apertures.

Fig. 5.11 shows the mechanism used for reciprocating the beam-chopper. A 50c/s synchronous motor, mounted beneath the base-plate, rotates the crank-wheel through a belt-drive at 15c/s, this being the highest chopping frequency that was found practicable for sustained operation. Miniature ball-races provided the bearings at either end of the connecting-rod AB, and gave smooth and silent running of the mechanism. The base-plate is supported on steel pillars standing on a heavy, rubber-cushioned base: the large inertia and soft mounting proved satisfactory

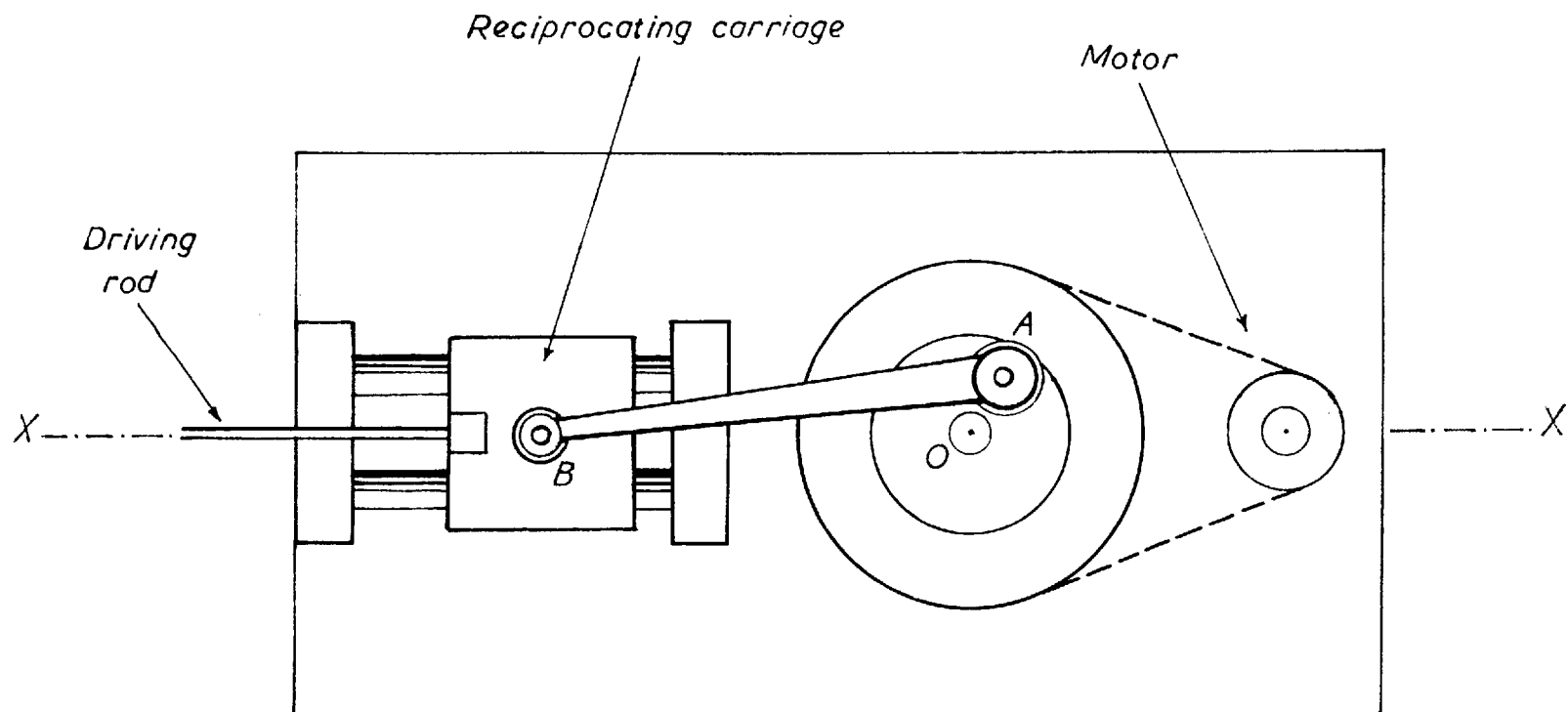


FIG. 5.11. *Driving mechanism for the beam-chopper.*

in insulating the photometer itself from vibration. The end B of the connecting-rod is constrained to move in a fixed line X-X and its displacement, therefore, differs from the simple harmonic motion, in the X-direction, of the end A by an amount $(1 - \cos\theta)$, where θ is the angle ABO. The eccentric radius, OA, was chosen as 5.5mm. (this gives a $\frac{1}{2}$ mm. clearance of the aperture edges at the end of each stroke of the beam-chopper), and the length of the rod AB is about 75mm. Thus the value of $\cos\theta$ will at no time differ from unity by more than about 0.3%, and the modulation will be sinusoidal with negligible error.

The light intensity in the apertures was tested for variation in the vertical direction using a vertically-movable mask with a horizontal slit (12mm. x 0.1mm.) to select narrow portions of the fixed aperture at different heights, and noting the adjustment in the height of the variable aperture needed to maintain balance of the photometer. The variations in light flux are very small, and in order to obtain observable changes in aperture height ($\frac{1}{10}$ of a micrometer division), it was necessary to reduce the intensity in the variable aperture by a factor of about 25. A neutral density filter was used for this purpose, and was suspended in front of the variable aperture by thin rubber bands, so that some vibration from the beam-chopper would be imparted to it. This would be expected to average out any small local variations in transmission. Fig. 5.12 shows the variation with slit position, y (mm.), of the micrometer reading, R (mm.), for the height of the variable aperture, taking as origin the reading for a central position ($y = 0$) of the slit. Readings were taken,

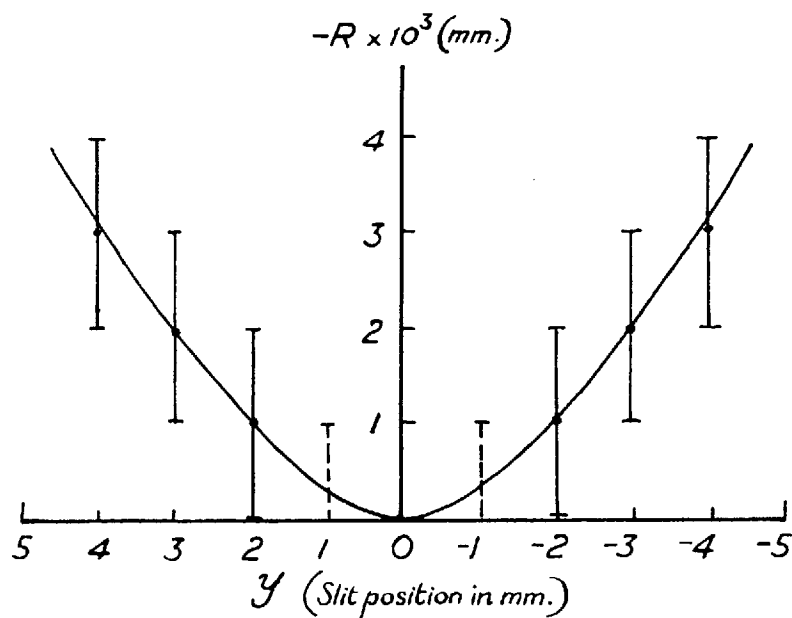


FIG. 5.12. Variation of intensity with aperture height.

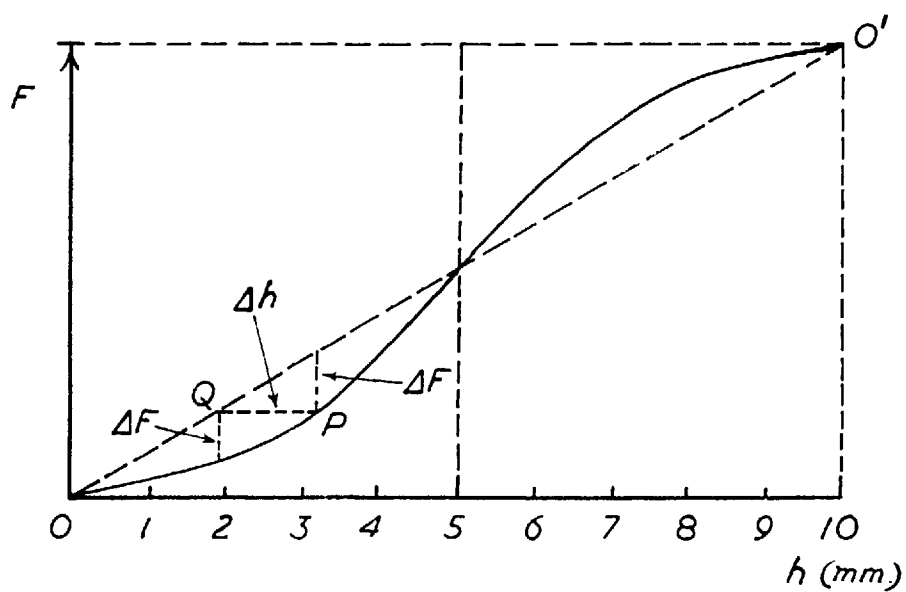


FIG. 5.13. Variation of total flux with aperture height.

for slit positions 1mm. apart, and the uncertainty in these is indicated by the vertical strokes. The symmetrical form of the curve provides confirmation of the validity of the method. Closely similar results were obtained using white light and different wavelengths of monochromatic light.

It is seen that the intensity is a maximum at the centre of the aperture and falls off towards the top and bottom. Similar variation is to be expected in the variable aperture and will result in non-linearity between aperture-height and the admitted light flux, F , which is given, for an aperture height h (less than half the full aperture height), by

$$F = 2a \int_{a-h}^a I(y).dy \quad 5.14$$

where $2a = 10\text{mm.}$ is the full aperture height, and $I(y)$ is the intensity at a height y . If I_0 is the intensity at the centre of the fixed aperture, and I is the intensity in the variable aperture at the position of the shutter edge, $I(y)$ will be given by

$$I(y) = I_0 + \frac{R}{A} \cdot I \quad 5.15$$

where A is the area of the test slit. (There is negligible variation of intensity across the slit, and in the range of adjustment, R , of the variable aperture.) The curve of Fig. 5.12 is satisfied by the relation $R = -ky^2$, where $k = 2.2 \times 10^{-4} \text{ (cm}^{-1}\text{)}$. We may also write, with sufficient accuracy for the present purpose, $I = \frac{I_0}{25}$. (The attenuation factor of the neutral density filter is approximately 25.) Making these substitutions

in the expression for $I(y)$, the integration 5.14 then yields

$$F = 2ahI_0 \left[1 - k' \frac{3a(a-h) + h^2}{2} \right], \quad 5.16$$

where $k' = 8.5 \times 10^{-5}$, for the flux, F , admitted by an aperture of height h . The form of this function, greatly exaggerated, is indicated in Fig. 5.13. The assumption of linearity between flux and aperture height thus introduces an error $+\Delta F$ for $h < a$, and $-\Delta F$ for $h > a$. These errors may be compensated by reducing (increasing) the readings for h by an amount Δh , corresponding to a transition from a point P on the curve to a point Q on the line OO' with the same ordinate value. The magnitude of this correction is given by $\Delta h = (dF/dh)^{-1} \Delta F$. ($\frac{\Delta F}{F}$ nowhere exceeds 7 parts in 1000, and ΔF may therefore be taken as constant throughout the interval dh .) Differentiation of 5.16 gives $\frac{dF}{dh}$, and Δh is then given by

$$\Delta h = -k' \cdot h(a-h) \cdot (2a-h) \quad 5.17$$

to sufficient accuracy. Values of h making Δh a maximum are obtained by differentiating 5.17, and the maximum value of Δh is then found to be approximately 3×10^{-3} mm. This is rather less than the expected setting error of the micrometer ($\frac{1}{2}$ of a division), and is, therefore, negligible.

It may be remarked here that light losses due to diffraction from the variable aperture, for even the smallest values of h used (~ 1 mm.), are negligibly small. Thus, the first 15 orders of the Fraunhofer diffraction pattern due to an aperture 1 mm. wide, representing more than 99.95% of the total light, will have a spread in the intermediate image plane — some 30 cm. from the plane of the apertures — of only 2 mm., and

all the light in these orders (and in a good many more) will pass unobstructed through the optical system. The observed variation of intensity in the apertures is not due to a changing diffraction effect as the test-slit position is varied, because the wave-fronts in the apertures are spherical and the slit, as this moves off-centre, merely selects different parts of the wave-front without altering the diffraction pattern. The slight increase in obliquity of the slit, relative to the wave-front, in off-axis positions does not increase the scale of the diffraction pattern by more than about 1 part in 10^6 .

5.5 The samples: elimination of ghost light

(i) Measurements on anti-reflection films

The samples used for these measurements were in the form of 1cm.-thick glass blocks having parallel faces, 4cm. x 2cm., with coated and uncoated areas each 2cm. square. The blocks are mounted in the photometer so that the coated and uncoated areas are aligned, respectively, with the variable aperture and the fixed aperture. Measurements were made, for each of the different glass types examined, using different numbers of blocks.

The choice of 1cm. for the thickness, t , of the blocks, and a similar value for the distance, d , between adjacent faces of neighbouring blocks, satisfies the two conditions given below for the elimination of ghost light. One may distinguish two separate ways in which ghost light reaching the image field originates: (1) by reflection between the faces

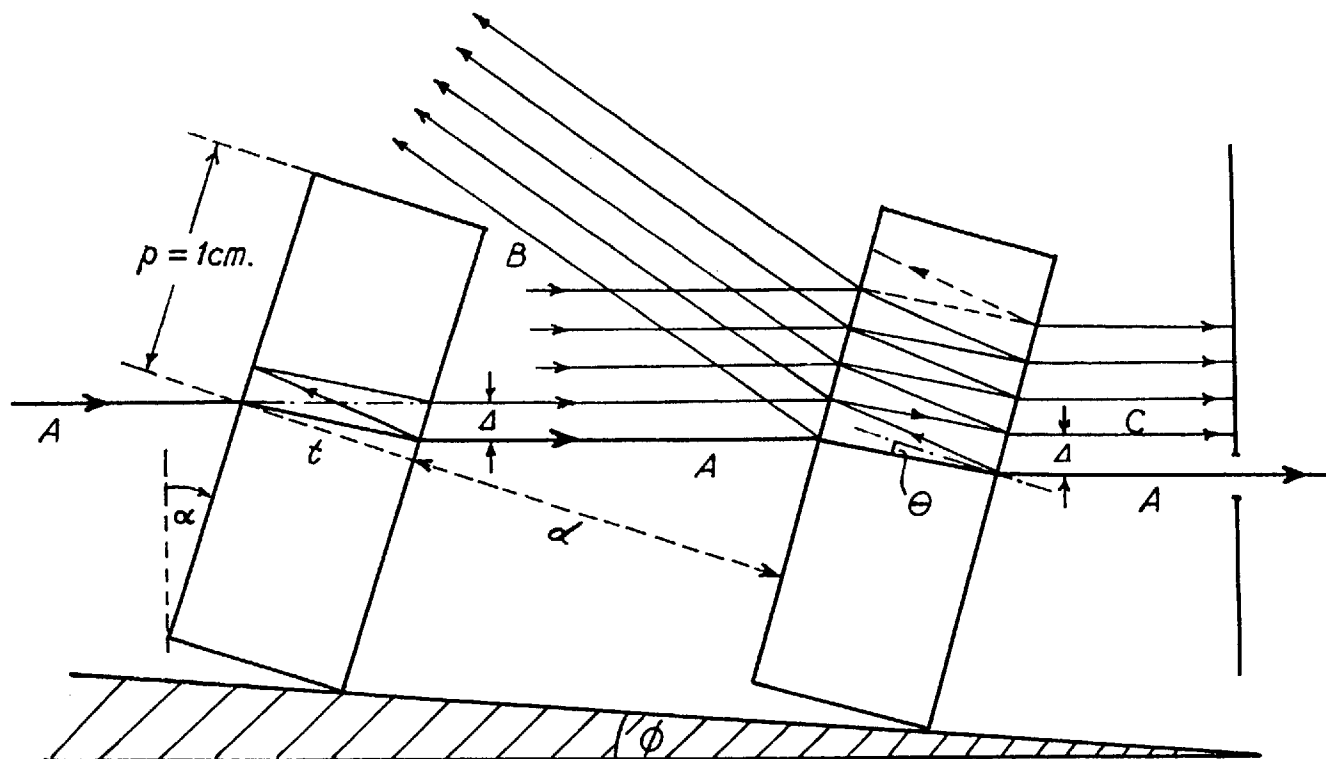


FIG. 5.14. *Elimination of ghost light.*

of a block, and (2) by reflection between faces of different blocks. By tilting the blocks, as indicated in Fig. 5.14, light leaving the rear face of a block can be made to clear the preceding block, thus eliminating ghost light due to (2). With the notation in the Figure, this condition for the ray B, and hence all other rays, is expressed by the relation

$$d > \frac{p - t \cdot \tan \theta}{2 \tan \alpha} \quad 5.18$$

Also, ghost light due to (1) will be displaced upwards, relative to the primary beam A, by an amount Δ (and multiples of Δ) given by

$$\Delta = \frac{2t \cdot \tan \theta}{\cos \alpha} \quad 5.19$$

and the ghost images formed in the intermediate image plane may be stopped out by a screen placed just before the field lens L_2 (Fig. 5.8). A convenient value for Δ , giving sufficient clearance between the primary image and the first ghost image, is about 5mm. Taking the refractive index of the glass to be $n = 1.626$ (a mean value for the different glasses used), optimum values satisfying 5.19 are $t = 1\text{cm.}$, and $\alpha = 20^\circ$ - the largest value of α permissible (see Ch. III, Sec. 3.5). A choice of $d = 1\text{cm.}$ satisfies 5.18 with ample margin. With these values of d and t , 16 sample blocks could be accommodated.

The beams "fall" an average of about 1.5mm. on traversing a block, and in order that the beams shall pass centrally through all the blocks, these are arranged to lie on a ramp (see Fig. 5.14) sloping downwards with an angle $\phi = \frac{1.5}{20}$. It was necessary to adjust the height of the lenses L_2 and L_3 each time the number of blocks was changed; no

adjustment, however, was necessary for different wavelengths of light.

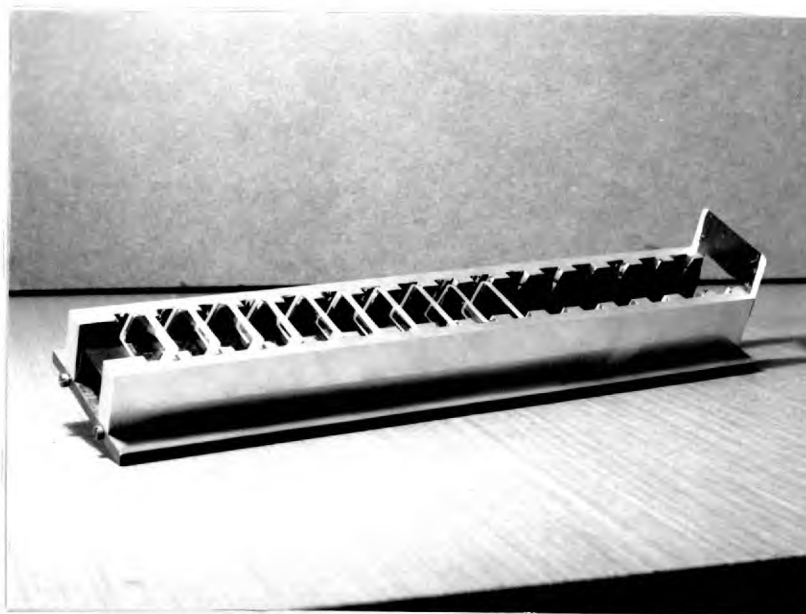
In addition to the vertical displacement, there is a displacement of the two images S_1 and S_2 (Fig. 5.8) along the optical axis by an amount $(1 - \frac{1}{n})$ cm. per block, where n is the refractive index. The axial displacements were calculated so that the positions of the lenses L_2 and L_3 and the photomultiplier could be predetermined for any number of samples of a given glass type placed in the photometer. To facilitate the adjustment, these components were linked rigidly together for movement as a unit along the optical bench. (Provision was made for fine adjustment of the photomultiplier position.)

The carrier in which the sample blocks were mounted (and of which the ramp forms a part) is shown in Plate II(a). The ends of the blocks fit in grooves in the carrier walls inclined at 20° , and are firmly held in position by cork padding. A pair of clips at the front end of the carrier supports an opaque screen for stopping out the ghost images. This is adjustable in height so that the lower edge will always clear the primary images. A cover was provided to exclude dust. All the photometer parts, as far as was practicable, and the edges of the blocks, were blackened to reduce stray light.

(ii) Absorption coefficients of optical glass

The specimens used for these measurements were slabs of glass of different types and refractive index, with lengths varying between 12cm. - 16cm.

(a)



(b)

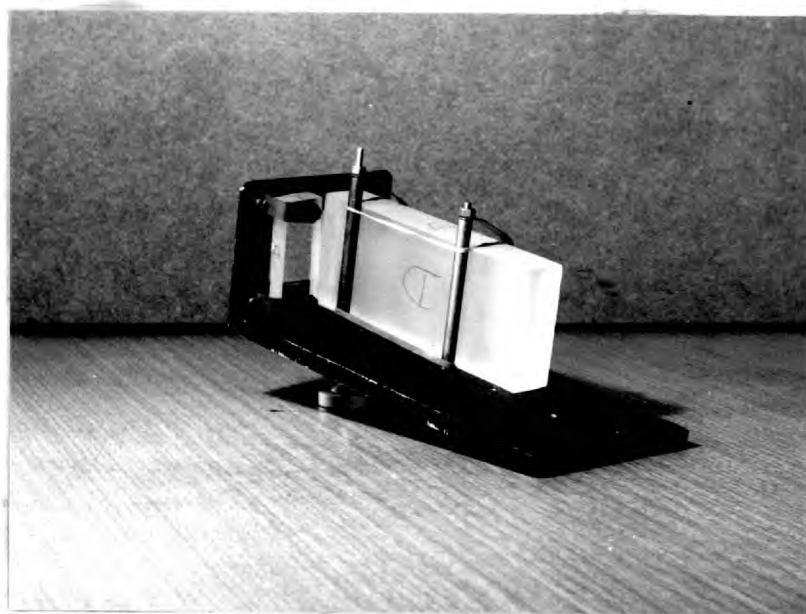


PLATE II . Specimen holders: (a) for measurements on anti-reflection films; (b) for absorption measurements.

This order of magnitude for the absorbing length was chosen in accordance with the following considerations to give maximum sensitivity of measurement. For simplicity, a beam of parallel light of unit cross-section will be considered, but the result applies equally to other cases.

The "coefficient of absorption", α , of a material, that is, the fractional attenuation in a beam traversing unit length (1cm.) of the material, is defined by the transmission formula

$$I = I_0 \cdot e^{-\alpha x} \quad 5.20$$

giving the intensity I of a beam, of initial intensity I_0 , after traversing a length x (cm.). Differentiating 5.20 with respect to α gives

$$d\alpha = - \frac{dI}{I \cdot x} \quad 5.21$$

as the error in α for an uncertainty dI in the measurement of the intensity (flux). For a particular photometer, and at a given wavelength, dI is fixed, and a minimum error $d\alpha$ will be obtained by maximizing the product $I \cdot x$. Thus, multiplying 5.20 by x and differentiating with respect to x , we obtain $(1 - \alpha x) = 0$ as the condition for an extremum, which gives $\frac{1}{\alpha}$ for the optimum length of absorbing path. The lengths of the specimens were chosen, using approximate values of α , to have the optimum value for wavelengths in the blue region of the spectrum, where the photometer is least sensitive: the average value of α here is of the order of 0.1cm.^{-1} .

The specimen slab is mounted in front of the fixed aperture on a simple carrier, shown in Plate II(b), and a 1cm.-thick block of the same glass type is mounted in front of the variable aperture to compensate

for reflection at the end-faces. Ghost reflections from the end-faces are eliminated from the transmitted beams by mounting the carrier in an inclined position to give incidence at 20° . Because of the unequal lengths of glass in the two beams, the intermediate images, S_1 and S_2 , will be relatively displaced both in the vertical direction and along the optical axis. The vertical displacement can be cancelled in the final (superposed) images by adjusting the heights of the two segments in the split lenses L_2 and L_3 , but these images will be focussed in different planes. To overcome this difficulty, a peri-

scopic mirror system, shown diagrammatically in Fig. 5.15, was interposed in each beam, the upper pair of mirrors being adjustable in height whereby the image positions along the optical axis could be varied. (One of

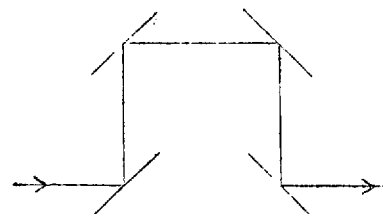


FIG. 5.15

the lower mirrors was made adjustable in one system to allow compensation of the vertical displacement.) Unfortunately, it was necessary, for reasons of practicality, to use metallic mirrors (rather than totally-reflecting 45° -prisms), at least in the lower part of the system, and errors arise as a result of variation in reflectivity over the mirror surfaces, since the incidence-positions of the beams on the mirrors cannot be duplicated exactly in different adjustments. The method finally adopted, therefore, was simply to position the lens L_2 mid-way between the images S_1 and S_2 , and to adjust the positions of the lens L_3 and the photomultiplier so that defocussed images of approximately the same size fall on the photocathode.

The method was found to give satisfactory results. The accuracy of the measurements will be discussed in more detail in Chapter VI.

5.6 Detecting system

The photodetector, an 11-dynode photomultiplier (E.M.I. type 9524S), is mounted in an earthed box having an entrance aperture provided with a closing-flap: this serves as a focussing screen to facilitate superposition of the images* on the photocathode - which lies approximately in the plane of the aperture - and is swung aside during measurement. The voltage supply for the photomultiplier was a bank of eight H.T. batteries giving a steady overall working voltage of 960v., the dynodes being energized from a 13-megohm chain of bleeder resistances.

The photomultiplier signal, which is modulated at 15c/s, is applied, after amplification, to the vertical deflector plates of an oscilloscope the time-base of which is set to a sweep frequency of 30c/s in accordance with the considerations of Chap. IV, Sec. 4.9. It is an advantage to synchronize the time-base so that the trace has the symmetrical form indicated in Fig. 5.16. The trace thus appears as a pair of intersecting lines the angular separation of which is smaller the smaller the difference of light flux in the

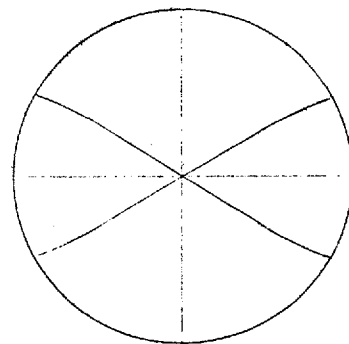


FIG. 5.16

*In practice, a lenticular screen was used to give a larger spread of light over the photocathode, but closely identical results are obtained with and without the screen.

two beams. As the photometer is adjusted for balance, these lines will appear to pivot in opposite senses about their point of intersection crossing over each other as the balance point is passed. The cross-over point can be found with great precision, particularly when the intensity of the electron-beam in the cathode-ray tube is low and the focussing is sharp: a displacement between the extremities of the lines of as little as $\frac{1}{2}$ mm. is readily detectable. To avoid drift in the signal trace, it was necessary to work on a fairly low sensitivity range, ~ 6 v. per cm. of deflection. The smallest detectable change in the voltage output, V_o , of the amplifier is thus limited to $V_o = 0.3$ v. At the balance point, the residual a.c. component, I , of the photomultiplier current is of the order of 10^{-8} amp. (see Table 5.4 , p.131) and this will develop an input voltage $V_i = 0.01$ v. across a 1-megohm resistance in the grid circuit of the first valve of the amplifier. A minimum amplifier gain of $\frac{V_o}{V_i} = 30$ is thus required.

The amplifier circuit is shown in Fig. 5.17, and Table 5.2 gives the values of the circuit parameters. F_1 , F_2 and F_3 are filters having zero transfer admittance for the frequencies 100, 200 and 15 cycles per second, respectively (details of these filters will be given below), and are for the purpose of eliminating disturbing signals arising mainly from the 50c/s. ripple in the mains supply to the light source. (A tuned amplifier was not used because high selectivity at low signal frequencies such as 15c/s. requires the use of large inductances, and elimination of mains pick-up becomes difficult.) The light source is energized during both the positive and negative halves of each mains cycle, giving a 100c/s.

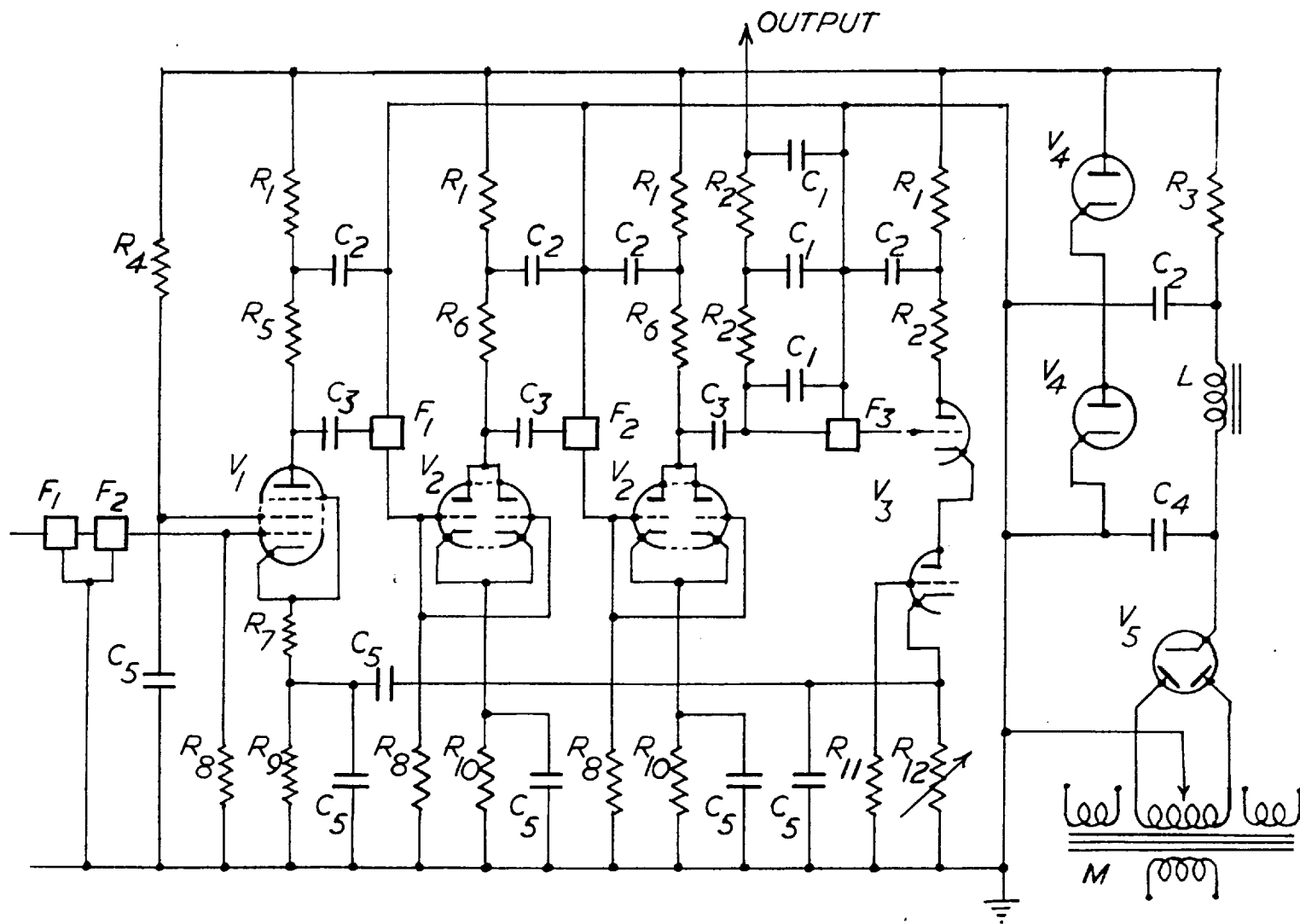


FIG. 5.17. Circuit diagram of amplifier (see Table 5.2).

Table 5.2

<u>Valves</u>		<u>Transformer</u>					
V ₁	EF37A	M	240v -350v; 6.3v;4.5v.	R ₄	220k	R ₁₂	2k
V ₂	ECC81			R ₅	68k		
V ₃	ECC82			R ₆	72k	C ₁	1mfd
V ₄	V105 (neon)	L	10 henrys	R ₇	0.1k	C ₂	8mfd
V ₅	5Z4			R ₈	1 meg.	C ₃	2mfd
				R ₉	1.8k	C ₄	16mfd
		R ₁₀	1k	C ₅	0.1mfd		
		R ₁	10k	R ₁₁	1 meg.		
		R ₂	20k				
		R ₃	3.5k ($\frac{1}{2}$ w.)				

modulation in the beam intensity. This is not purely sinusoidal, because heating and cooling of the lamp filament tends to lag behind the voltage variation. The 100c/s. fundamental and the 200c/s. first harmonic are almost entirely suppressed by the filters F_1 and F_2 in the grid circuit of the first valve. Further filters F_1 and F_2 are included in the grid circuit of succeeding valves to enhance the 100c/s. suppression and to suppress a residual 200c/s. component arising from harmonic distortion in the amplifier.

The output, which is taken from the anode of the third valve, has 15c/s. modulation of reasonably high purity. The 15c/s. signal is isolated from the last stage by a filter F_3 having zero transfer admittance at 15c/s., but residual impurities, including random low-frequency oscillations which can be particularly troublesome, pass to valve V_3 from which a cathode-follower connection supplies negative feedback to the first

valve. Mains pick-up at 50c/s. was eliminated by a filter F_4 (see Table 5.3) compactly made and connected directly with the input terminal of the oscilloscope.

The filters F_1 - F_4 are of the "parallel-T" type indicated in Fig. 5.18(a). Givens and Saby (1947) have shown that the sharpness of the null at the cut-out frequency, f_0 , is a function of the ratios $R_1:R_2$ and $C_1:C_2$ but this is not of critical importance here, and the convenient choice $R_1 = R_2$ ($=R$), and $C_1 = C_2$ ($=C$), ("equal-arm network"), gives the following conditions for null-transmission at frequency f_0 :

$$1. \quad R.C = \frac{1}{2\pi f_0}$$

$$2. \quad \begin{aligned} R_3 &= \frac{1}{2}R \\ C_3 &= 2C \end{aligned}$$

The voltage transfer ratio at a frequency $f \neq f_0$ is given by

$$\beta = \frac{e'}{e} = \frac{(\omega^2 - 1)^2}{(\omega^2 - 1)^2 + 4\omega^2}$$

where $\omega = \frac{f}{f_0}$. Fig. 5.18(b) shows the transmission characteristic of the filter, plotted on a logarithmic scale for ω . Values of the parameters for the various filters are given in Table 5.3 which also gives values of β for $f = 15\text{c/s.}$

Table 5.3

Filter	f_0	R	C (μF)	β
* F_1	100	32k	0.05	0.97
F_2	200	16k	0.05	0.99
F_3	15	107k	0.1	—
* F_4	50	32k	0.1	0.80

*Tunable

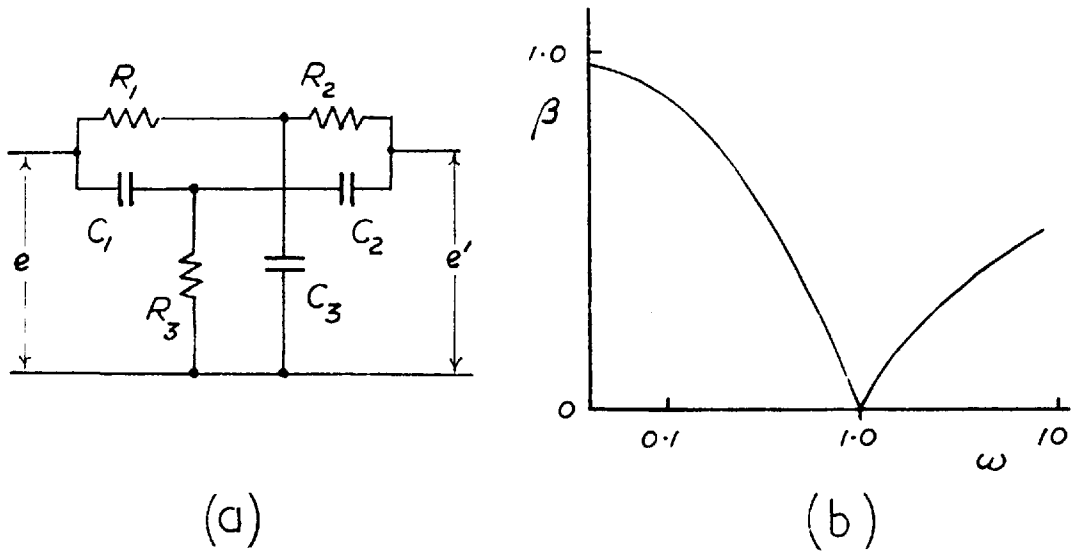


FIG. 5.18. "Twin-T" null-frequency filter.

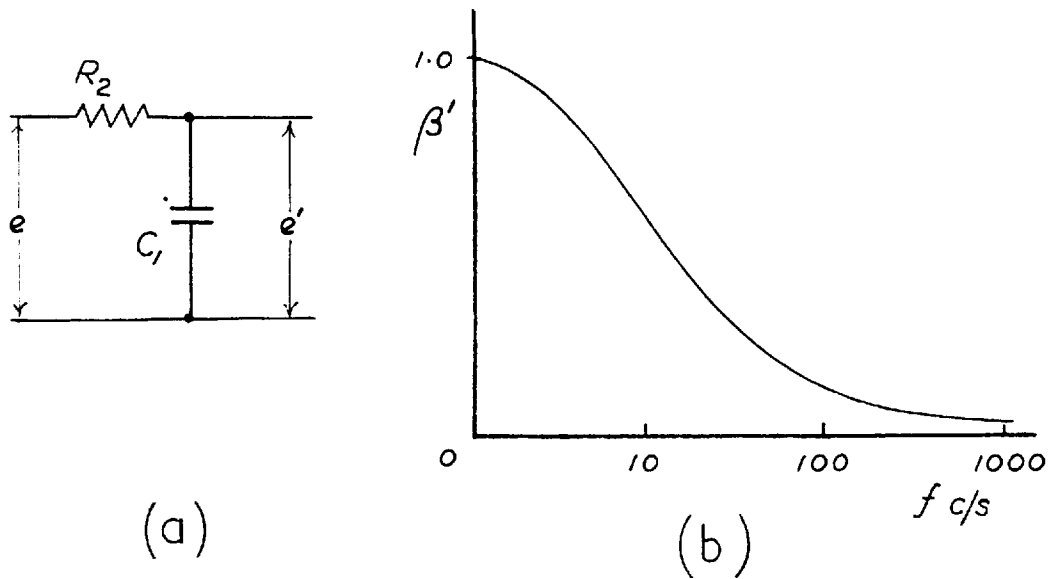


FIG. 5.19. Low-pass filter.

The basic amplifier circuit (i.e. with all the filters removed) would have a gain of about 300. In practice, there is considerable attenuation of the signal due partly to the filters, but mainly to the $R_2 - C_1$ network in the output circuit (see Fig. 5.17). This is a low-pass filter, the primary function of which is to reduce the effective frequency bandwidth of response of the measuring system, and hence the "noise" level (this question will be discussed in more detail below), but which also acts to reduce asymmetry of the signal wave-form - due to amplitude distortion in the third valve - in out-of-balance signals. The network has two identical elements (Fig. 5.19(a)) in cascade, the voltage transfer ratio, β' , of one element being indicated in Fig. 5.19(b). At 15c/s. the transmission factor for the network is $\beta'^2 = 0.35$, and the overall transmission factor, $\bar{\beta}$, for the two filters F_1 , the two filters F_2 and the filter F_4 , is $\bar{\beta} = 0.75$. Thus, the effective gain of the amplifier is $300 \times \bar{\beta} \cdot \beta'^2 \sim 70$, which is ample.

Signal/noise ratio

It is required to detect changes in the variable-aperture height of 10^{-4} cm. ($\frac{1}{10}$ of a micrometer division), that is, an area increment of 10^{-4} cm². The energy flux through this area is $I(\lambda) \times 10^{-4}$, where $I(\lambda)$ is the intensity in the aperture, for a given wavelength, and is given by Table 5.1 (p.104). The corresponding quantum flux is given by $N = I(\lambda) \times 10^{-4} / h\nu$ per second, where h is Planck's constant (6.624×10^{-27} erg.sec.) and $\nu = \frac{c}{\lambda}$ ($c = 3 \times 10^{10}$ cm.sec.⁻¹), and these N quanta generate a current $I = N \cdot e \cdot \eta \cdot G$ at the anode of the photomultiplier, where e is the electronic charge (1.6×10^{-19} coulomb),

η is the quantum conversion efficiency of the photocathode, and G is the gain factor of the photomultiplier; at the voltage used (960v.), $G \sim 1.5 \times 10^6$. The (modulated) signal current, I , is superposed on a steady background current, I_0 due to the total flux through the aperture of N_0 quanta per second, where $N_0 \sim N \times 10^4$. Values of I and I_0 at different wavelengths are given in Table 5.4.

Table 5.4

Wavelength (λ) ($\text{\AA}$)	6000	5000	4000
$I(\lambda)$ (erg/sec./cm. ²)	1.04	0.30	0.015
N	3.1×10^7	7.5×10^6	3.0×10^5
N_0	3.1×10^{11}	7.5×10^{10}	3.0×10^9
η	.020	.095	.100
I (amp.)	1.5×10^{-7}	1.7×10^{-7}	0.8×10^{-8}
I_0 (amp.)	1.5×10^{-3}	1.7×10^{-3}	0.8×10^{-4}

In addition to the background current I_0 , there is a dark current, I_d , due to thermionic emission from the photocathode (the dynode contribution is negligible). Because of statistical fluctuations in the photoelectronic and thermionic emission processes, I_0 and I_d will include "noise" currents which will set a limit to the smallest detectable value of I . The photomultiplier used has a Sb-Cs photocathode about 1" in diameter from which some 300 electrons per second are emitted thermionically at room temperature. With a tube gain factor of 1.5×10^6 , these produce a dark current

$I_d \sim 7 \times 10^{-11}$ amp. This is some 10^6 times smaller than I_o and is negligible by comparison. It is thus the noise current accompanying I_o that is important, and this is given by

$$i_p = (2e \cdot I_o \cdot \Delta f)^{\frac{1}{2}} \quad 5.22$$

where Δf is the frequency range within which the measuring system is responsive, uniform frequency response being assumed. If the response is not uniform, but is a function, $R(f)$, of the frequency, Δf in 5.22 is replaced by an effective bandwidth defined by

$$\Delta f' = \int R(f) \cdot df \quad 5.23$$

An oscilloscope of the type used (Mullard GM5655) has a working range of frequencies from 0 - 50kc/s, but the effective bandwidth of response was made very much smaller by inserting in the amplifier output circuit the low-pass filter referred to earlier. The voltage transfer ratio per stage, β' , is given by

$$\beta' = \frac{e'}{e} = \left[1 + (2\pi f \cdot C_1 \cdot R_2)^2 \right]^{-\frac{1}{2}} \quad 5.24$$

For the whole filter (two stages), the transmission factor is β'^2 . Thus, $R(f) = \beta'^2$ is the frequency response of the detecting system, and the effective bandwidth of response is given, by 5.23 and 5.24, as

$$\Delta f' = \int_{f=0}^{f=5 \times 10^4} \frac{df}{1 + (2\pi f \cdot C_1 \cdot R_2)^2}$$

Inserting the values $C_1 = 10^{-6}$ farad, $R_2 = 2 \times 10^4$ ohms, the integration yields $\Delta f' \sim 12$ c/s. This is the value to be used for Δf in 5.22, and

taking, for example, $I_0 = 0.8 \times 10^{-4}$ amp. (see Table 5.4), we obtain

$$i_p = 1.7 \times 10^{-11} \text{ amp.}$$

as the r.m.s. photomultiplier noise current, compared with a corresponding signal current of $I = 0.8 \times 10^{-8}$ amp. - a very satisfactory signal/noise ratio.

The voltage developed by i_p across the resistance R_0 ($=10^6$ ohms) in the grid circuit of the first valve is $v_p = 1.7 \times 10^{-5}$ v. Thermal (or "Johnson") noise in R_0 will produce a voltage $v_j = 4kT.R_0.\Delta f = 4.5 \times 10^{-7}$ v. (k is Boltzmann's constant, 1.38×10^{-23} joules.deg. $^{-1}$; the absolute temperature, T , is taken as 300°K .) Calculation shows that the total noise (shot noise in the valves and thermal noise) developed in the amplifier and appearing in the output can be represented by an "equivalent noise voltage", $v_e = 10^{-8}$ v., applied to the grid of the first valve. Both v_j and v_e are negligible compared with v_p , and the limiting factor in the sensitivity of detection is thus photomultiplier noise. Noise arising from statistical fluctuation in secondary emission from the dynodes is small: Sharpe (1961) has shown that the correction factor for i_p is not generally greater than about 1.3.

Mains flicker in the room lighting could give rise to strong spurious signals. The photomultiplier was therefore shielded from external light by a substantially light-tight casing surrounding the photometer.

References

1. Givens, M.P. and Saby, J.S., Rev. Sci. Instr. 18 (1947) 342
2. Sharpe, J., Electronic Tech., June & July, 1961.

Chapter VI

RESULTS AND DISCUSSION

Part 1

Efficiency of anti-reflection films6.1 Experimental procedure.

With the beams balanced in the absence of samples, the "zero-reading" (R_0) of the variable aperture was determined at each of the following wavelengths ($\text{\AA.U.}$) :

(6500) 6270 6000 5500 5000 4750 4500 4250 4000 (3750)

(The extreme wavelength values did not always give reliable readings). There is a small variation in R_0 with wavelength because the light in each aperture has traversed a different part of the monochromator prism and the difference of absorption varies with wavelength. There is also some variation in intensity across the apertures due to the unequal thicknesses of glass traversed by different parts of the beam. The ratio of intensities at corresponding positions in the two apertures is, however, constant and the only effect of such variation is to produce slight distortion of the otherwise sinusoidal modulation with a corresponding slight reduction in the sensitivity of measurement (cf. Chap IV, sec.4.8).

Single-layer magnesium fluoride films on nine different

Table 6.1

Specimen	Type	Reference*	n_D
A	Fluor crown	465648	1.46495
B	Borosilicate crown	510644	1.50970
C	Barium light flint	551515	1.55119
D	Barium flint	605438	1.60483
E	Extra dense flint	653335	1.65348
F	Extra dense flint	700303	1.70035
G	Double extra dense flint	748278	1.74842
H	Double extra dense flint	801255	1.80120
K	Double extra dense flint (Special)	927210	1.92707
*BB	Hard crown	519604	1.51899
*DD	Dense flint	626357	1.62576

*Used for
double-layer
films

*Chance
catalogue

glass substrates were examined. The study of two layer films (magnesium fluoride/bismuth oxide/glass) was limited to two different substrates, the glasses used being those of particular interest in the construction of cystoscopic optical systems. Details of the various glass types are given in Table 6.1 . Sixteen sample blocks of each glass type were used and balance readings (R) for the height of the variable aperture obtained, at each of the above-specified wavelengths, for different numbers of blocks varied in the following sequence :

0 4 8 12 16 14 10 6 2 0 .

This tends to eliminate systematic errors that might arise in successive shift along the optical axis of the photomultiplier and the last two split-lenses, necessitated by displacement of the image planes when the length of glass path is altered (see Chap.V sec.5.5). The position of the screen stopping out ghost light requires adjustment when the number of sample blocks is changed, but not when the wavelength is varied for a given number of blocks. To eliminate the effects of possible backlash or sticking in the monochromator mechanism, readings were taken, for each group of blocks, with wavelength variation proceeding in both directions. Two readings were taken at each wavelength setting giving a total of four readings at each wavelength. The micrometers controlling the variable aperture and the monochromator were given three or four to-and-fro movements about the desired setting before this was finally made. That the wavelength settings were correct was verified by the close

identity of aperture readings in both directions of wavelength variation.

With a small number of sample blocks present the height R of the variable aperture could be determined with a precision better than $\frac{1}{2}$ a micrometer division (0.005 mm) per reading and with a standard deviation of similar magnitude in the four readings taken at each wavelength. With sixteen blocks of the high index glasses there is considerable attenuation of the beams with accompanying loss of sensitivity in detection of the null point and the standard deviation of the readings is about $\frac{1}{2}$ a division in the middle of the wavelength range increasing to about 1 division at the ends.

For each glass type examined, and for each wavelength, the aperture height R is plotted as a function of the number of surfaces N (twice the number of sample blocks) in accordance with the following considerations.

6.2 Reduction of readings and results.

We may usefully define the efficiency of an anti-reflection coating by the ratio

$$\tau = \frac{T}{T_0} \quad 6.1$$

where T is the transmission of a single treated surface and T_0 is the (Fresnel) transmission of an untreated surface. For a measurement on N surfaces we shall have

$$\tau^N = \left(\frac{T}{T_0} \right)^N = \frac{R_0}{R}$$

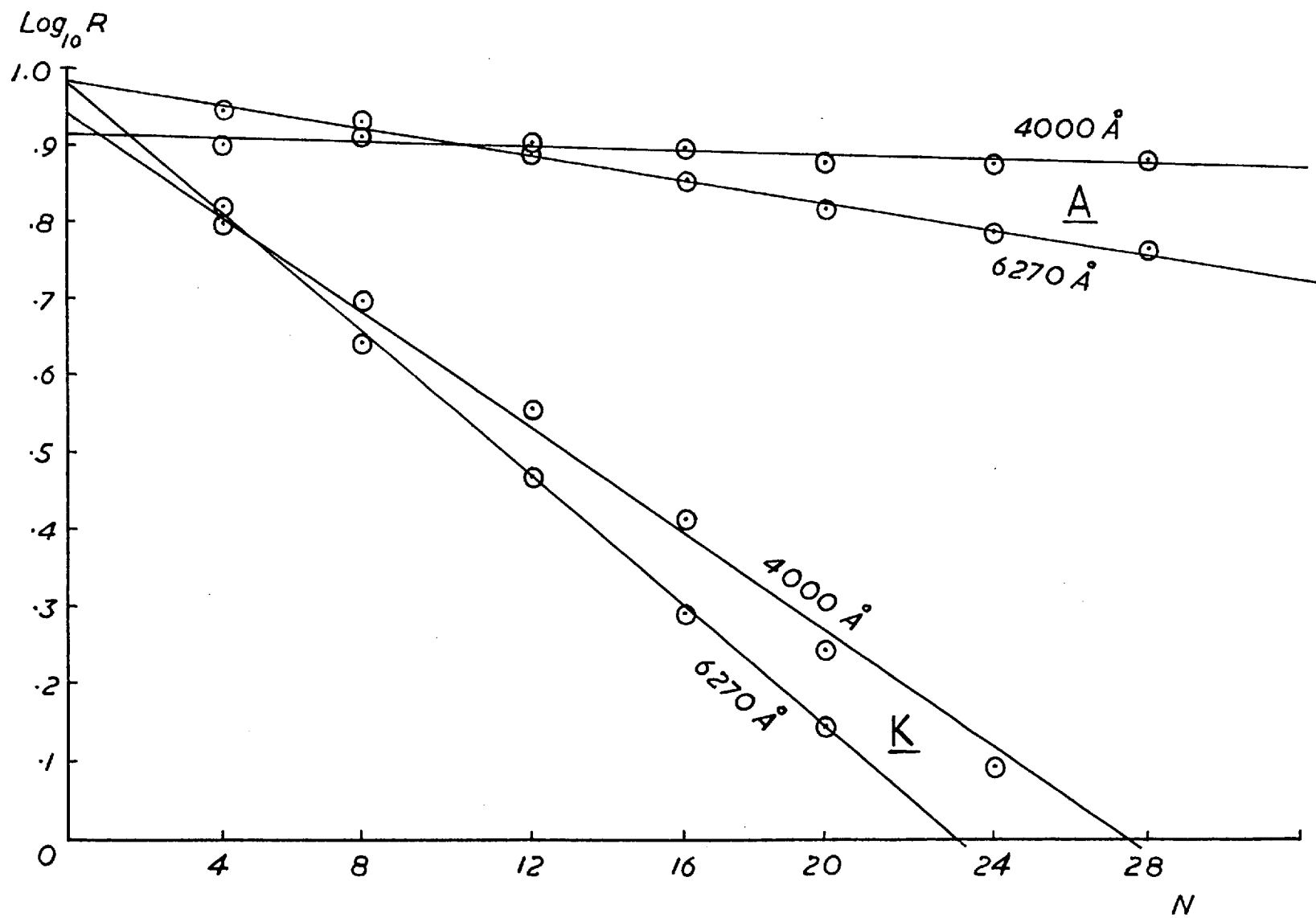


FIG. 6.0

where R is the height of the variable aperture at the balance point and R_0 is its height at the balance point without the glass specimen. Thus

$$\log_{10} R = -(\log_{10} \tau) \cdot N + \log_{10} R_0 \quad 6.2$$

and the slope of the line obtained by plotting $\log_{10} R$ against N is $-\log_{10} \tau$. Typical lines are shown in Fig. 6.0 for the lowest and highest-index glasses, A ($n=1.465$) and K ($n=1.927$) respectively, at wavelengths $6270\text{\AA}$ and $4000\text{\AA}$. The slope increases with increasing refractive index of the glass, from a minimum value of 0.00300 for glass A at $6270\text{\AA}$ to a maximum value 0.04140 for glass K at $4000\text{\AA}$; the corresponding errors are approximately $\pm 10^{-5}$ and $\pm 3 \times 10^{-5}$. A line was fitted to a set of points by visual assessment of the best fit using a fine thread: values of the slope and intercept obtained in this way for the lines of Fig. 6.0, where the standard deviation of $\log_{10} R$ is comparatively large owing to reduced sensitivity of measurement at these wavelengths, differed from values given by rigorous least-square treatment by amounts that are an order of magnitude smaller than the probable errors in these quantities.

The "standard error of estimate", σ_m , in the slope m given by a least-square fit, is of course, a function of the standard deviation of the plotted points.* It is also possible to define σ_m by :

$$\sigma_m^2 = \frac{1}{N-2} \sum_{n=1}^N \left(\frac{y_n - c}{n} - m \right)^2$$

* See, for example, "Statistical Methods in Research and Production", ed. Davies (1958), pp. 157 et seq.

where N is the number of points, $y_n (= \log_{10} R)$ are their ordinate values and c is the ordinate intercept. Here, $m = \log_{10} \tau$ and to find the standard error σ in τ , we may denote the errors by differentials, thus :

$$\sigma = d\tau = (\log_e 10).dm$$

so that, taking τ as unity, we have $\sigma \sim 2.3\sigma_m$

In Figs. 6.1 to 6.13 τ is plotted as a function of wavelength for each glass type, the standard error σ being indicated by vertical elongation of the plotted points. The curves T_0 indicate the "Fresnel" transmission, the transmission T of the films being obtained from the relation $T = \tau T_0$. Theoretical values for T were obtained using eqns. 3.12 and 3.16 of Chap. III, and are indicated by the dotted curves. Figs. 6.1 to 6.9 relate to single-layer magnesium fluoride films for maximum transmission in the middle of the spectrum. Figs. 6.10 and 6.11 show, respectively, the transmission characteristics of a single-layer film and of a double-layer film deposited on identical substrates with the transmission maximum located in the blue region of the spectrum; Figs. 6.12 and 6.13 are similar curves for a substrate of different refractive index.

Fig. 6.14 shows the variation of the theoretical maximum transmission with refractive index of the substrate, the experimental values being also shown for comparison.

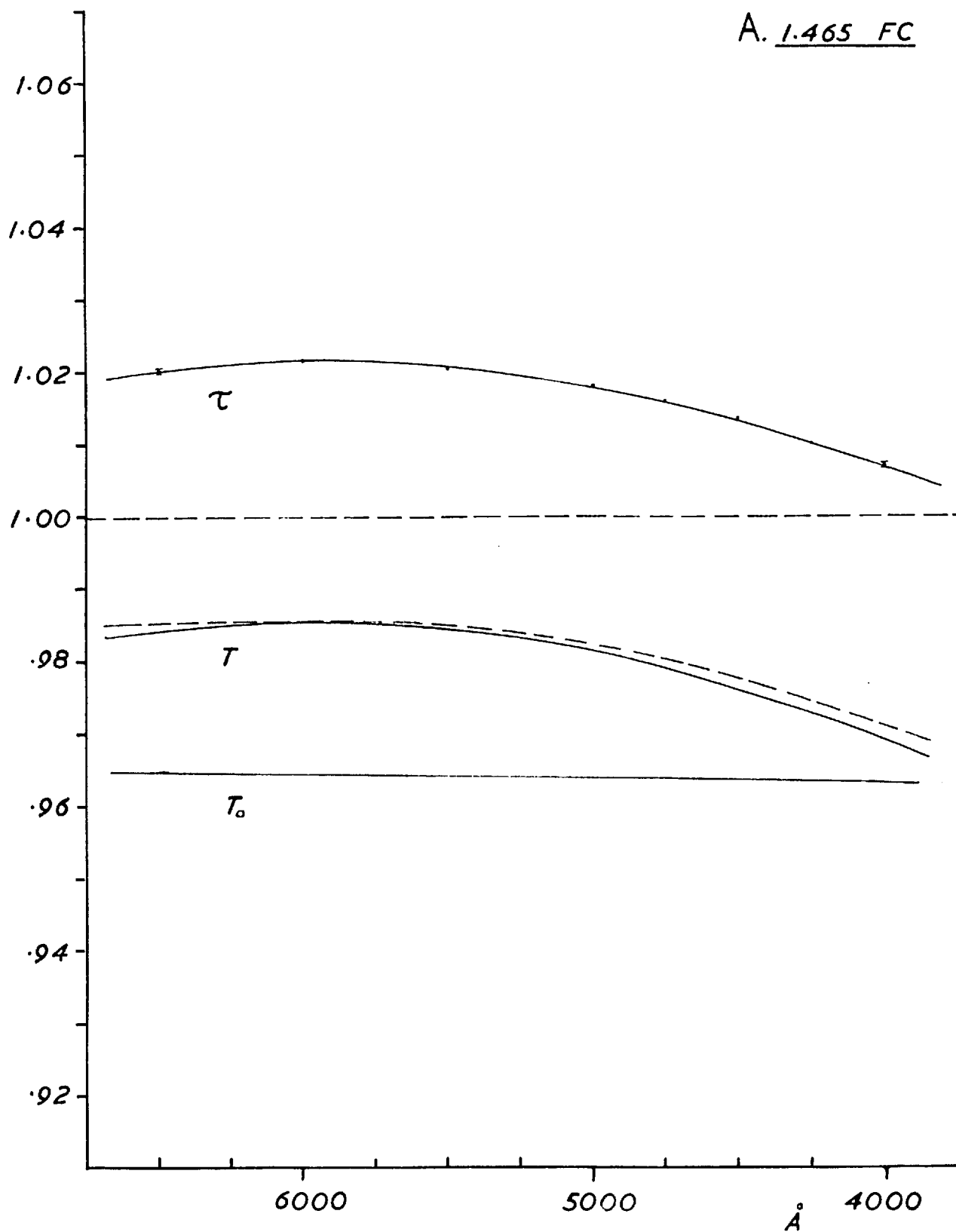
A. 1.465 FC

FIG. 6.1

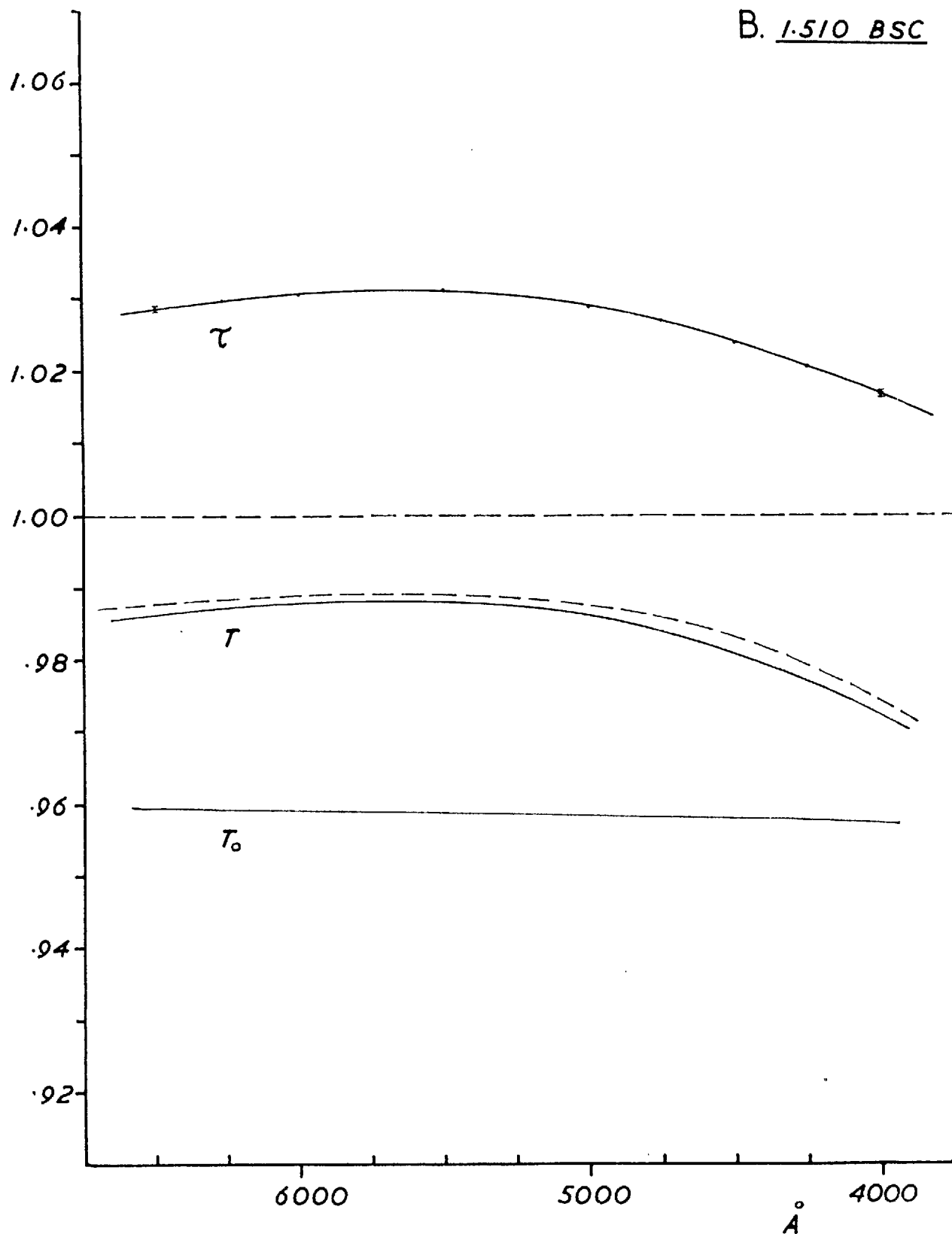
B. 1.510 BSC

FIG. 6.2

C. 1.551 BLF

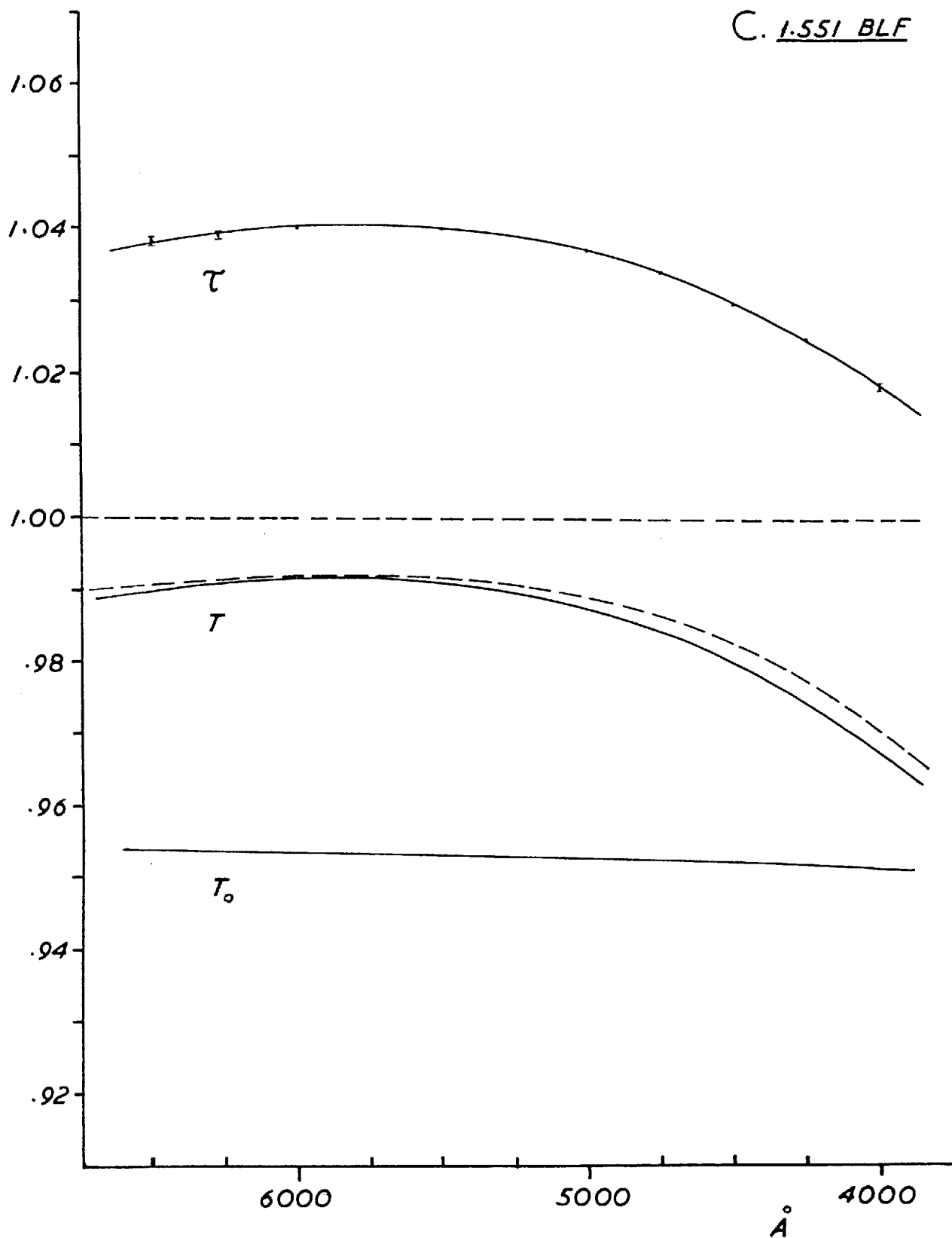


FIG. 6.3

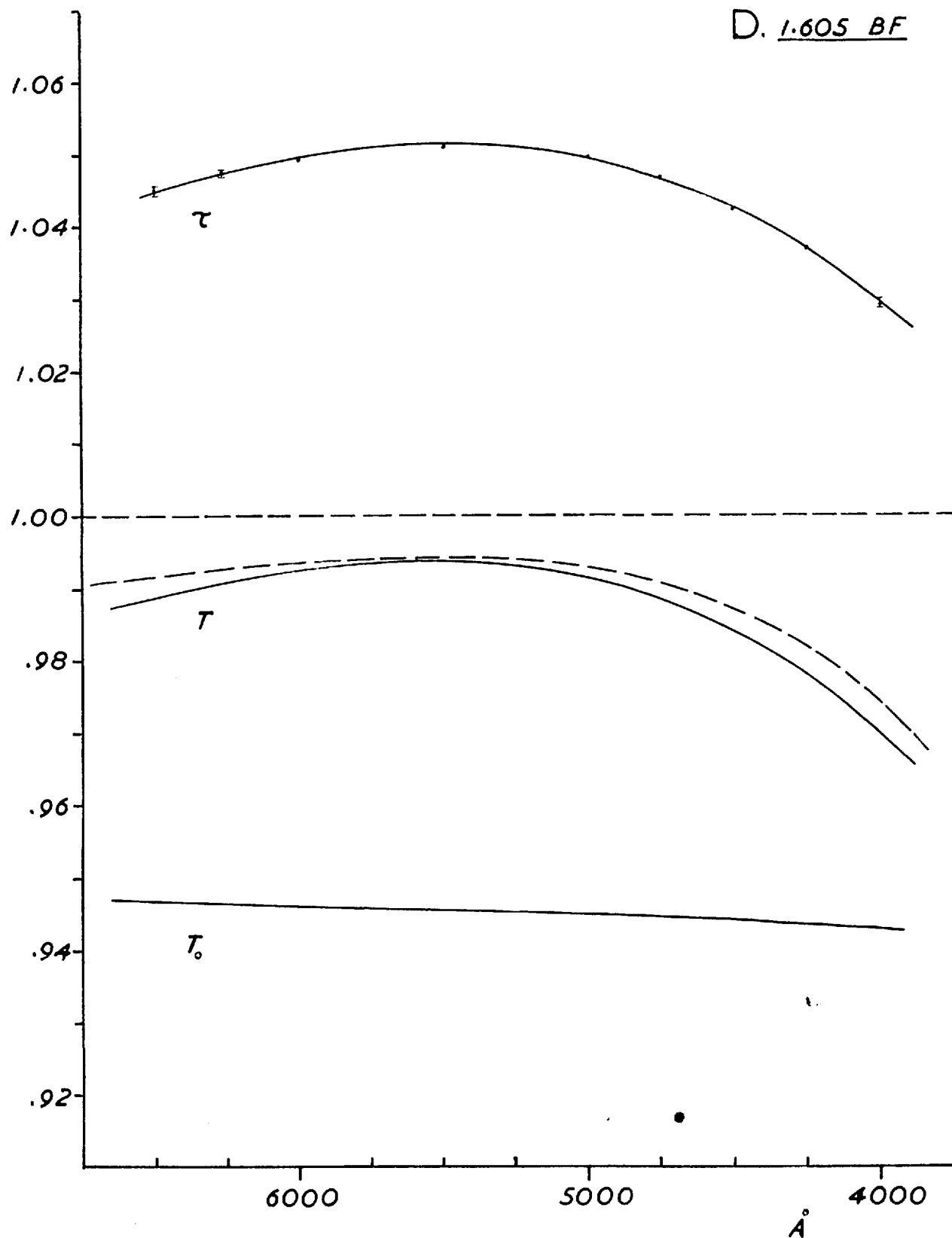
D. 1.605 BF

FIG. 6.4

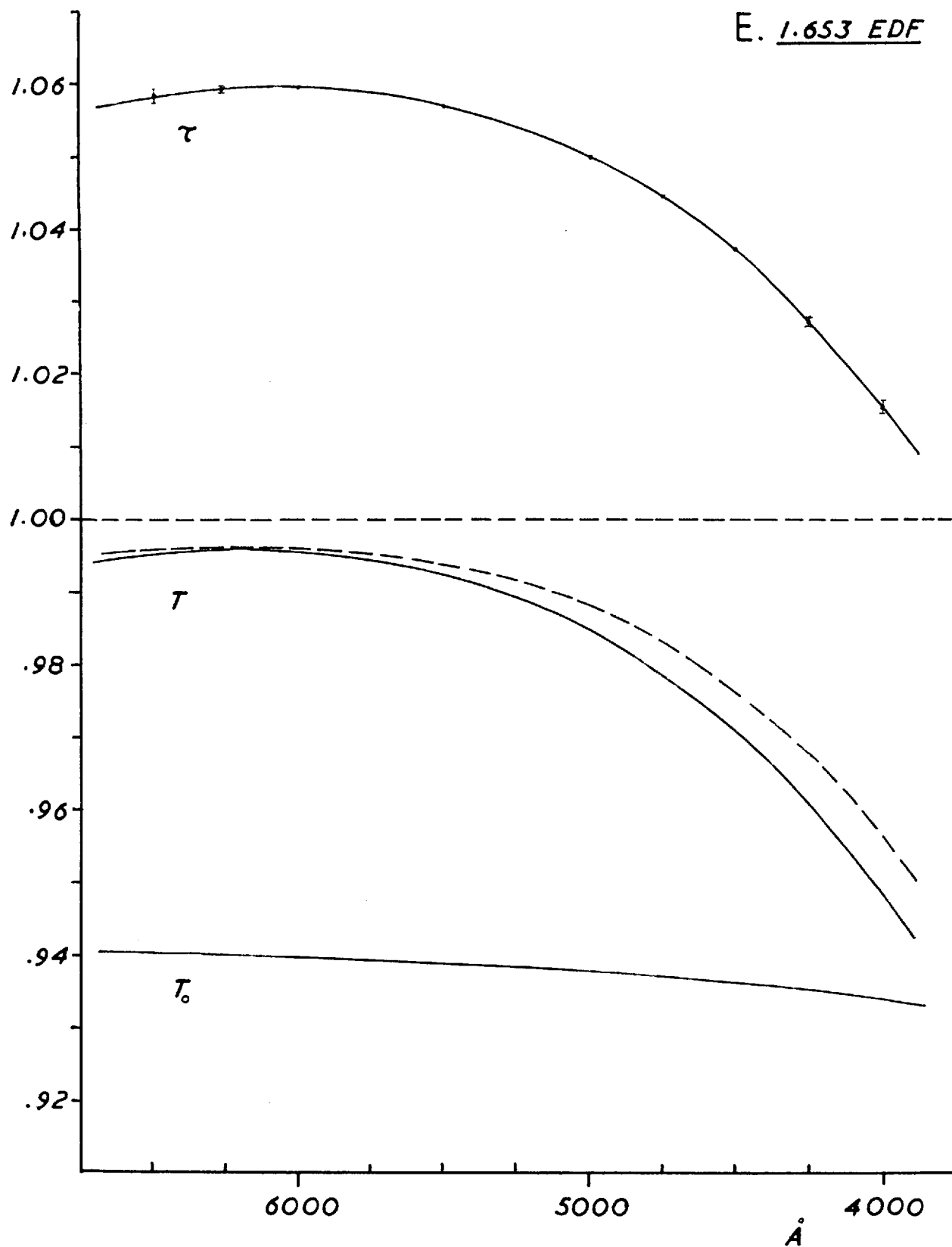
E. 1.653 EDF

FIG. 6.5

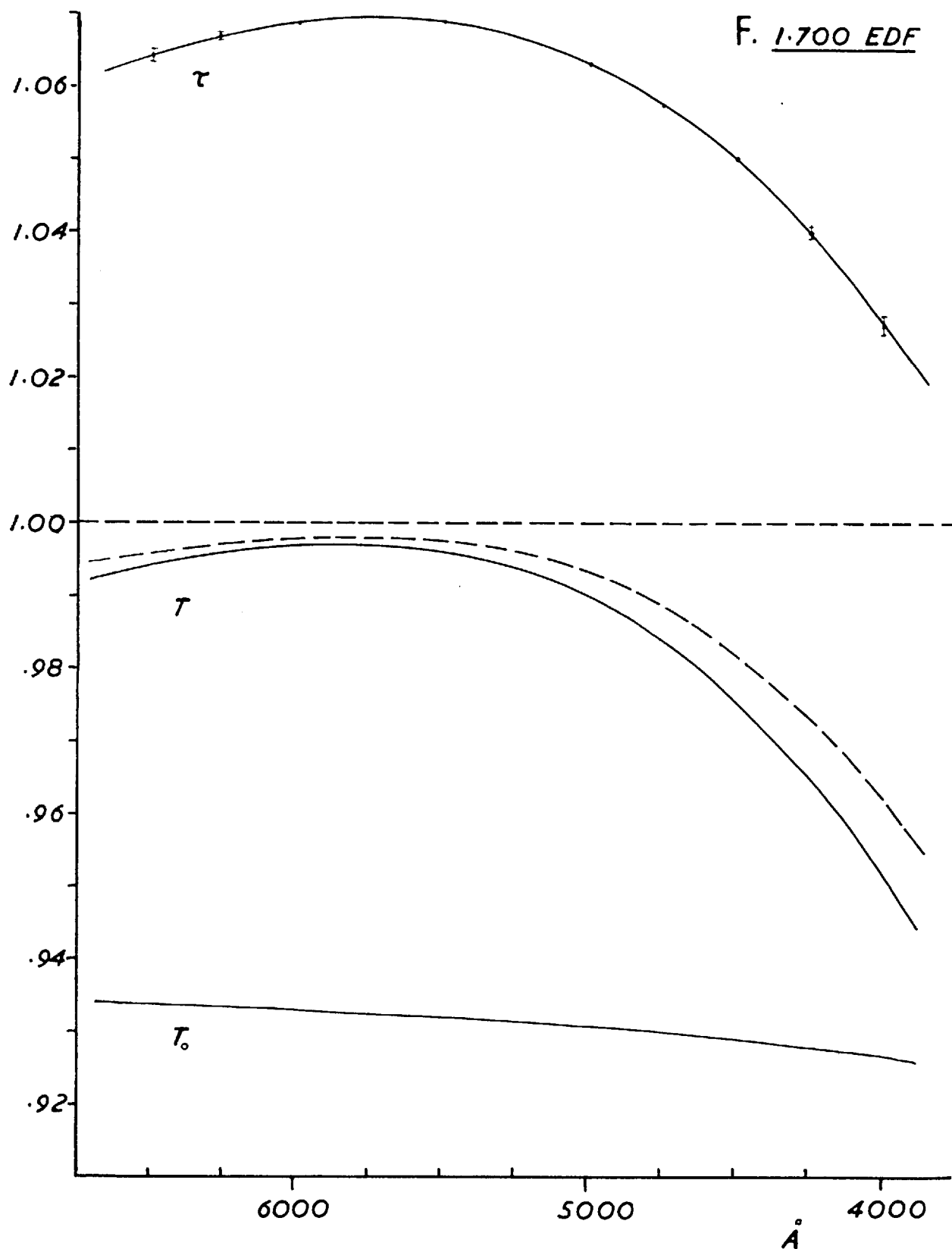
F. 1.700 EDF

FIG. 6.6

G. 1.748 DEDF

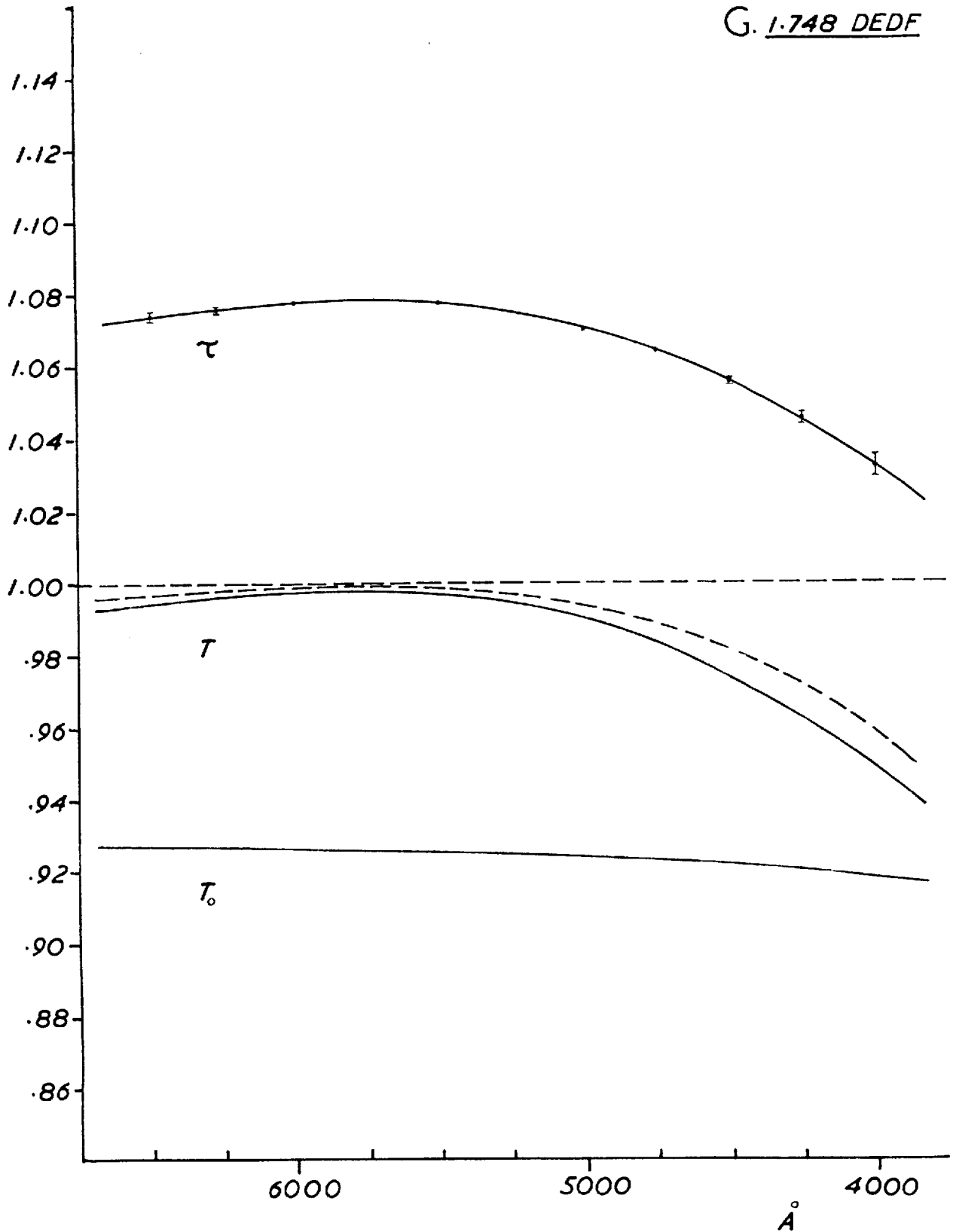


FIG. 6.7

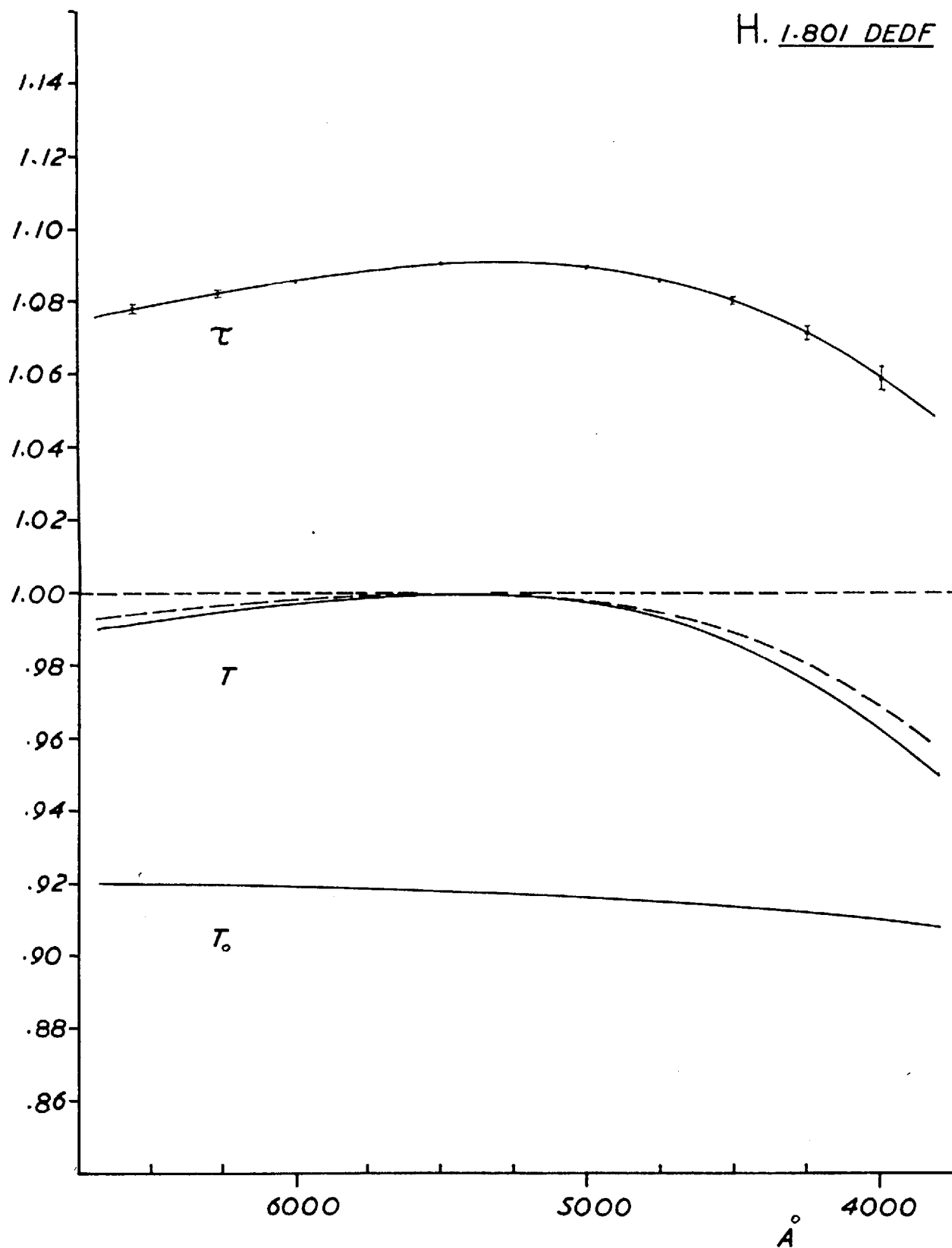
H. 1-801 DEDF

FIG. 6.8

K. 1.927 DEDF

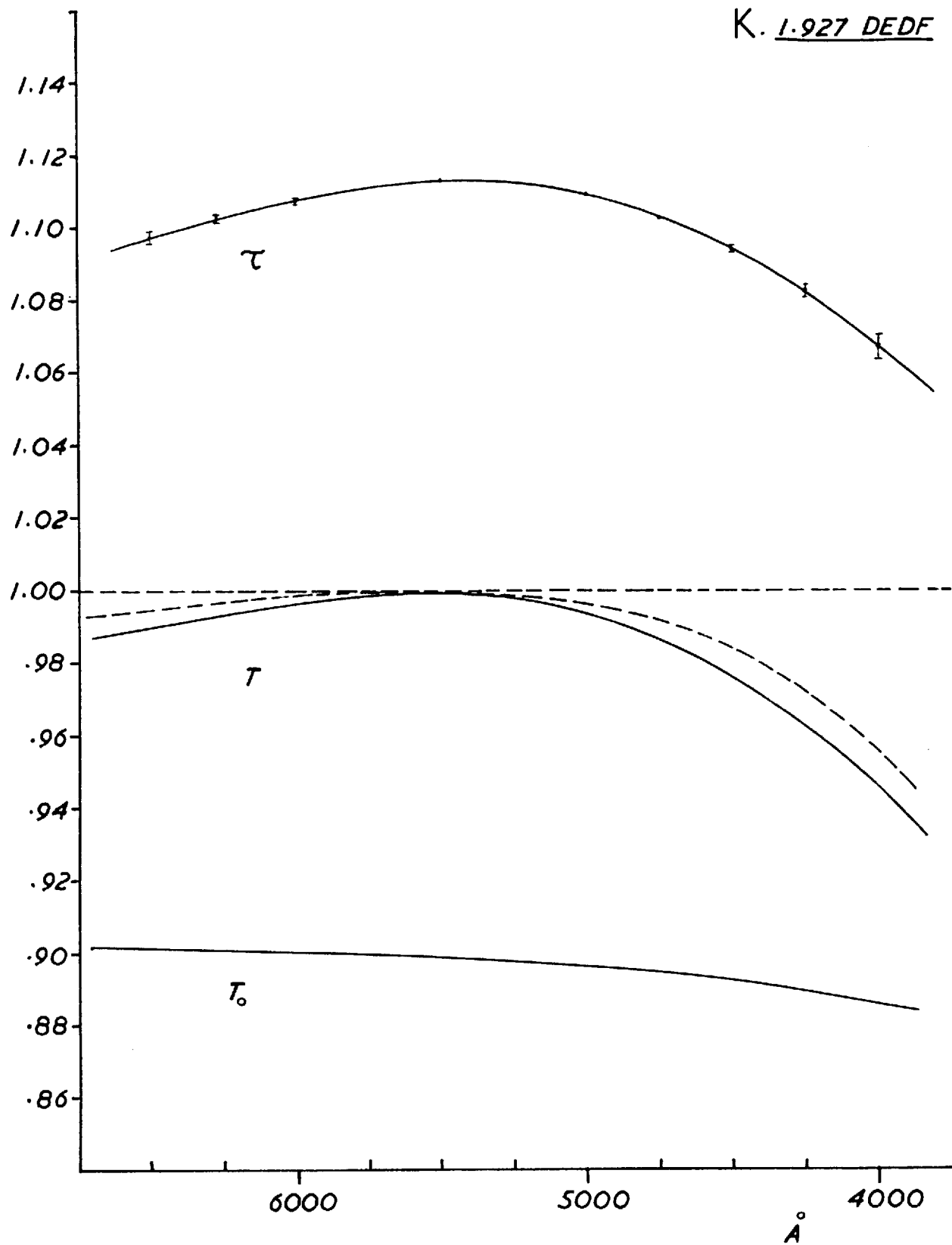


FIG. 6.9

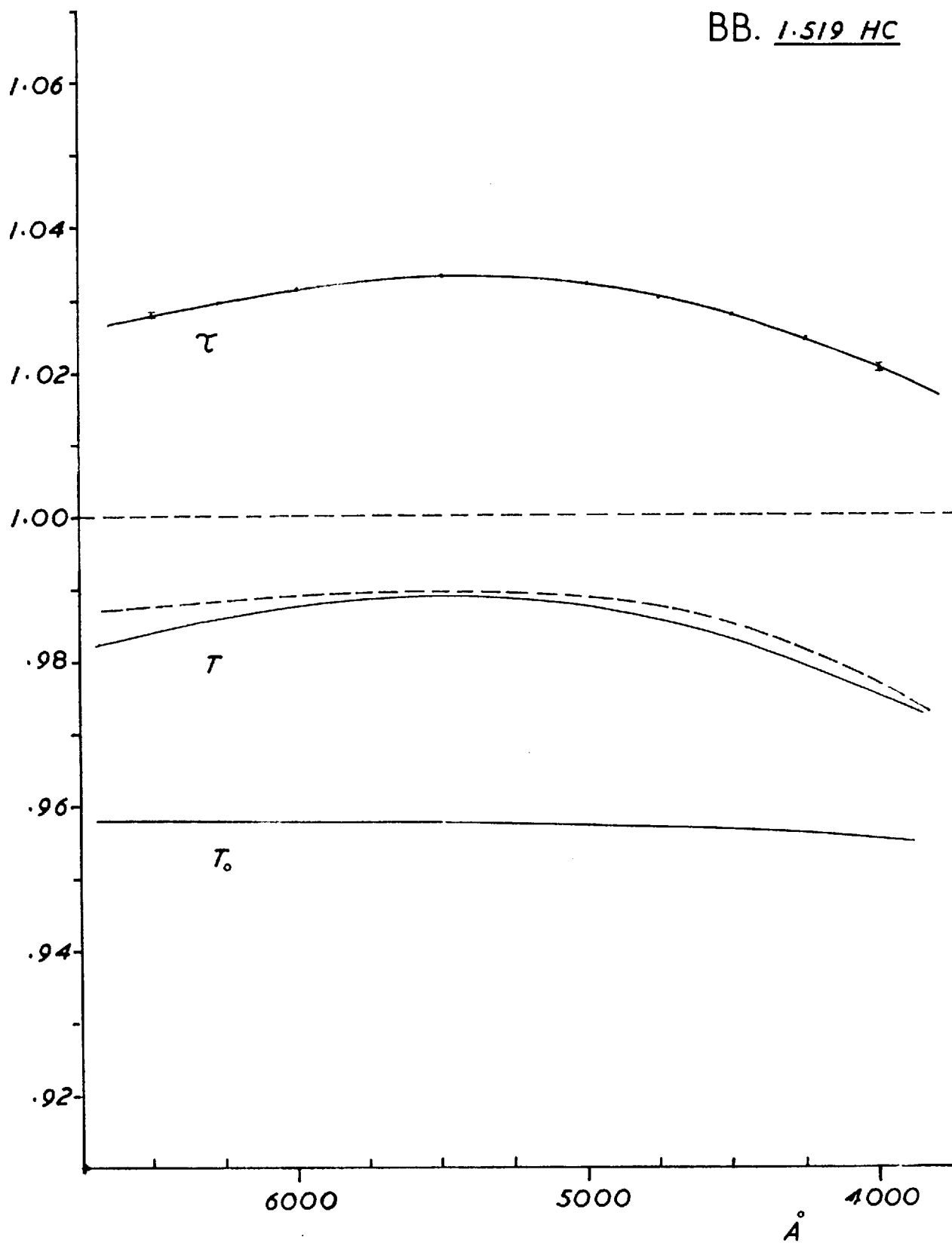
BB. 1.519 HC

FIG. 6.10

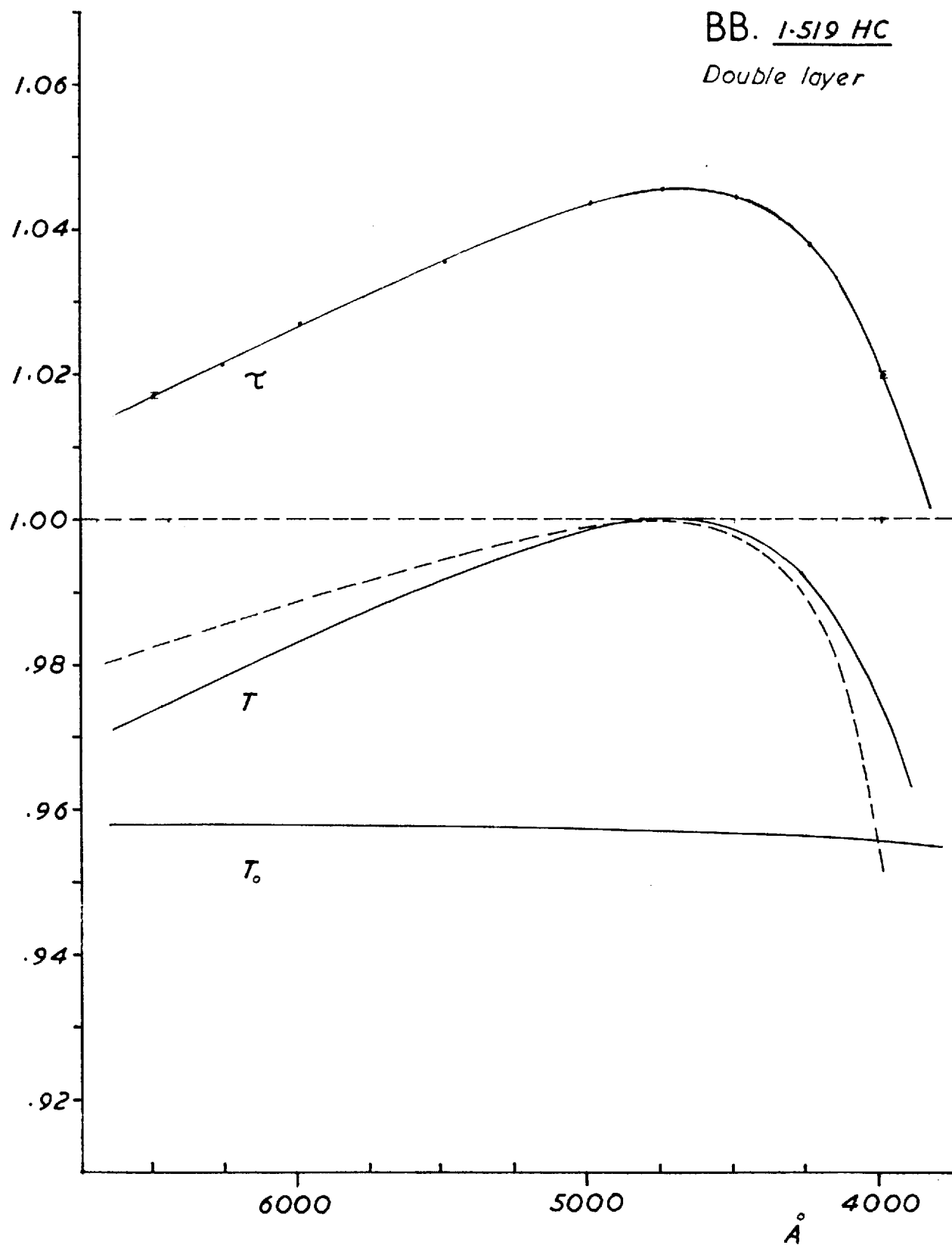
BB. 1.519 HC*Double layer*

FIG. 6.11

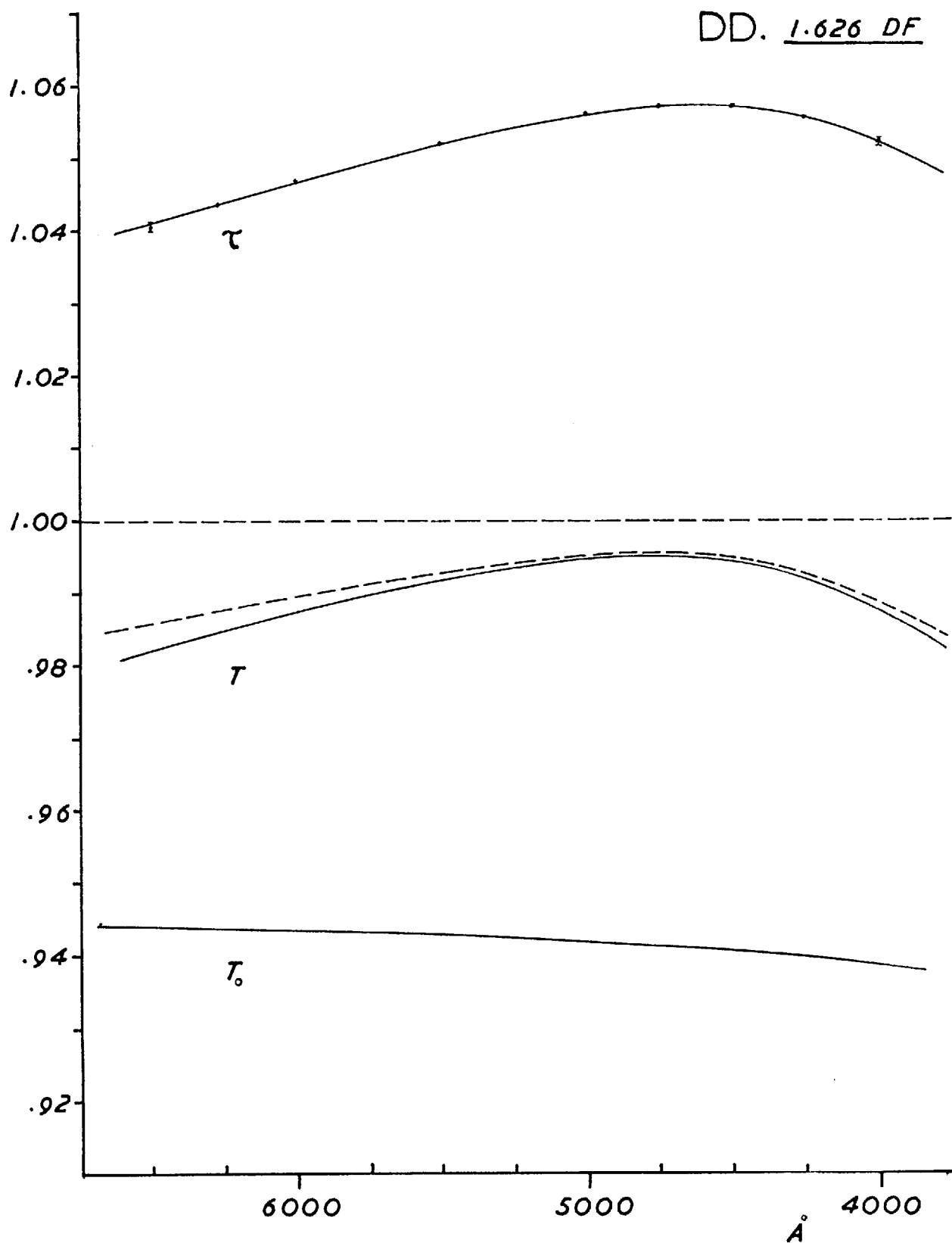


FIG. 6.12

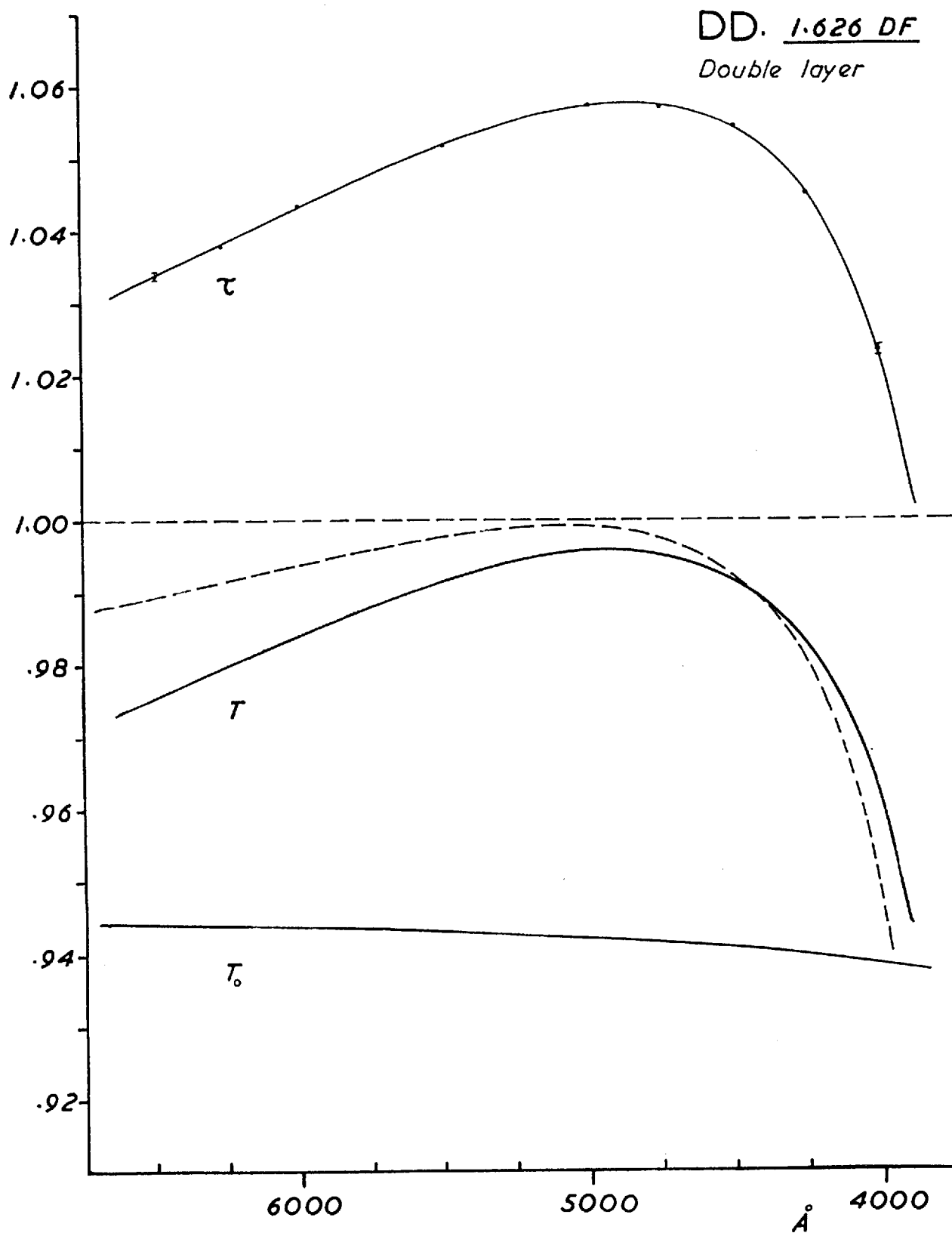


FIG. 6.13

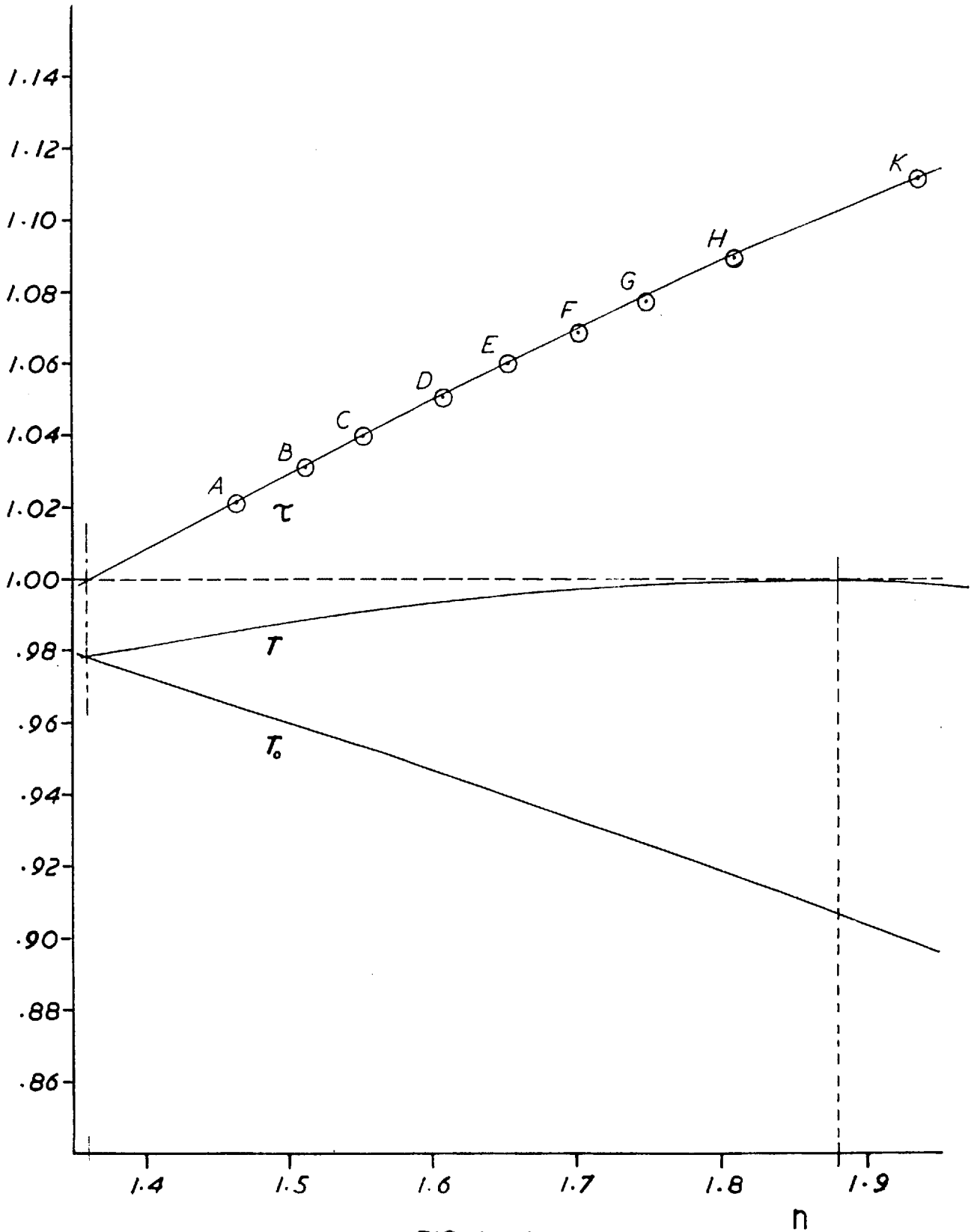


FIG. 6.14

6.3 Discussion of results.

The first important feature in the results is the accuracy with which each set of plotted points in Fig.6.0 lies on a straight line. This shows, first of all, that the transmission characteristics of individual surfaces in each batch of samples are closely identical. Normal care and precautions were taken in the polishing of the glass surfaces and in the deposition of the films, and are evidently sufficient for obtaining films of consistent quality. Secondly, a straight line of points demands both uniform film properties and reliable measuring technique, and the precision of measurement is thus well demonstrated.

Single-layer films

It will be seen from the curves that the transmission maximum, located nominally at a wavelength of about $5500 \text{ \AA}$ ($4750 \text{ \AA}$ for glass types BB and DD), tends to be shifted towards longer wavelengths with decreasing refractive index of the substrate. This is due in most cases to the fact that monitoring of the film thickness during deposition was done visually for the single-layer films: thus, for the lower-index glasses, which have smaller dispersion and therefore relatively poorer reflectivity at shorter wavelengths, greater suppression of reflection is required at longer wavelengths to give the necessary proportion of blue light characteristic of a purple reflection (grey in the case of glasses BB and DD) from the film. The very high index (dense flint) glasses showed some surface staining, and the maximum in these cases did not follow the normal trend.

It may be remarked here that the shifts of the maxima found are small, and the theory shows that the transmission curves plotted against the wave-number, $\frac{1}{\lambda}$, are then merely shifted along the wave-number axis without significant change of shape. In the case of glass A, for example, the maximum transmission occurs at about $6000\text{\AA}$ and has the (theoretical) value 0.9853; at $5000\text{\AA}$, the transmission is 0.9828. Considering now a film reduced in thickness in the ratio 6000:5500, the value of the maximum transmission, now occurring at $5500\text{\AA}$, is unchanged within the accuracy (0.0001) of calculation, and the transmission at $4650\text{\AA}$, corresponding to the same $\frac{1}{\lambda}$ -shift from $5000\text{\AA}$, is 0.9831. The discrepancy here is thus only 0.0003 and is smaller than the experimental errors.

In making a comparison between the experimental and theoretical results, it must be remembered that the idealized picture assumed in the theory, namely, that of perfectly homogeneous layers with sharp discontinuities at the boundaries, is not likely to be the case in practice. The effective optical thickness, nd , of a film will depend on the uniformity of "packing". Anomalies in the state of aggregation are to be most expected at the film boundaries and these regions are particularly of importance in determining the effective reflectance and the light-scattering properties of an interface. Staining of the surface, corresponding to a transition layer between the film and substrate, will tend to lower the reflection coefficient of that interface and thereby increase the reflection coefficient of the system in the region of minimum reflection. Also, measured values of τ will be influenced by the state of polish of the glass surface.

In the light of these considerations there is very good agreement between the experimental and theoretical results, particularly in the region of maximum transmission. There is a slight, though marked, lowering of efficiency in the anti-reflection coating on some of the dense flint glasses, for example, specimens F and G, it is thought because these glass types tend to "stain" rather heavily after polishing.

For calculating theoretical values of T , using eqns. 3.12 and 3.16 of Chap. III, values of the various reflection coefficients were calculated for different wavelengths from refractive index and dispersion data for the glass published by the makers (Chance Brothers), corresponding data for magnesium fluoride films being obtained from the dispersion curve of Fig. 6.15(b) which is based on measured values given by Weinstein (1954) for "aged" films (about a week old). The films used were of similar age at the time of measurement, and values of refractive index are taken as being accurate to ± 0.001 . The energy reflection coefficient, R , involves the amplitude coefficients in the second degree, and theoretical values of T are therefore accurate to ± 0.0001 , which is about the limit of accuracy of plotting of the curves.

Double-layer films

The double-layer films (Figs. 6.11 and 6.13) were prepared as described in Chapter III. The improved method of monitoring of the film thickness allows the position of the transmission maximum to be located with greater accuracy. The wavelength chosen in the present case was $4750\text{\AA}$. The same

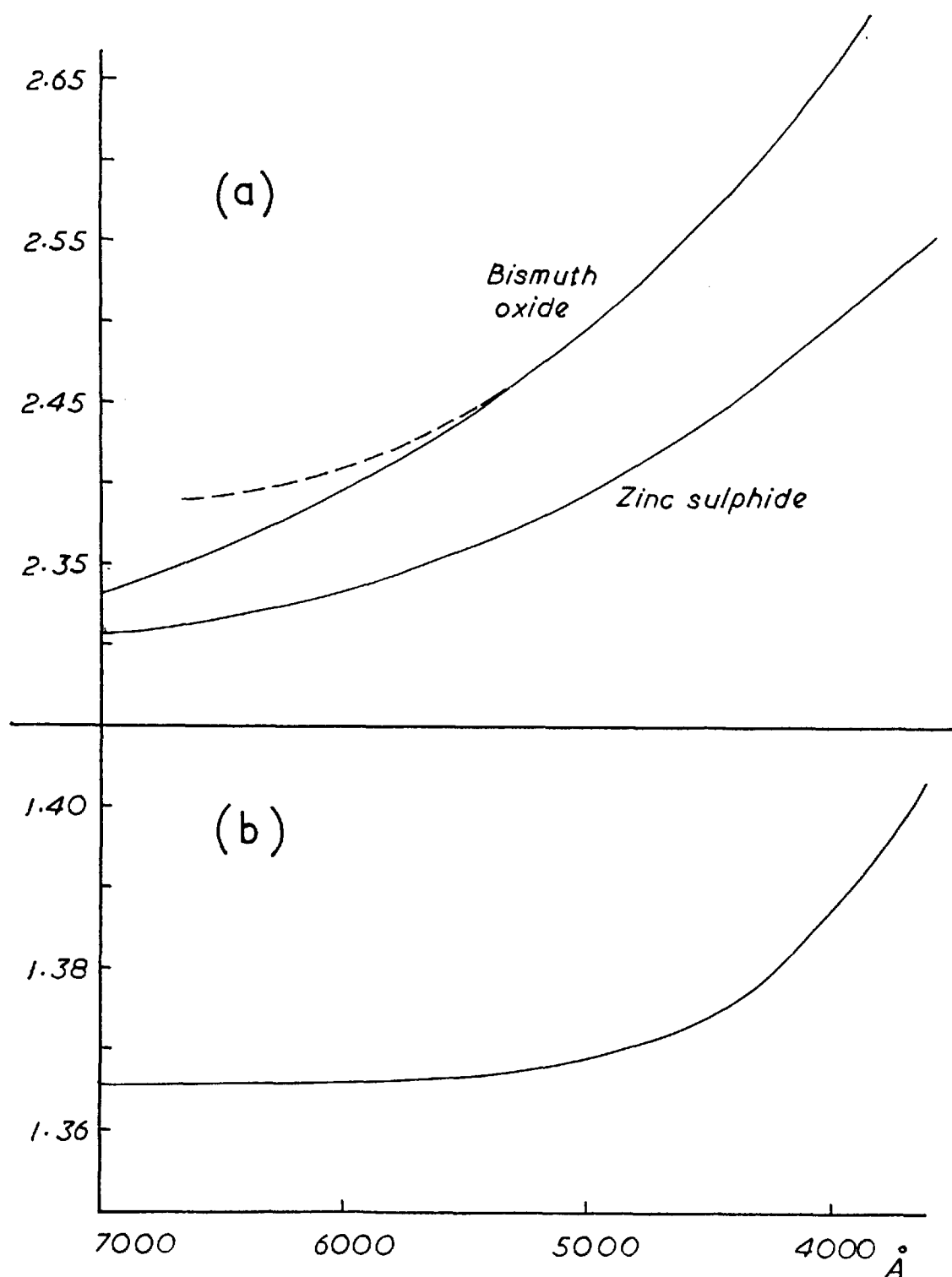


FIG. 6.15. Dispersion of (a) bismuth oxide (zinc sulphide) and (b) magnesium fluoride.

monitoring conditions were used for both glass types BB and DD, but were strictly appropriate only to glass BB, of refractive index 1.519; for glass DD, of refractive index 1.626, the maximum transmission is lowered, and is shifted to a longer wavelength. (For comparison, Figs. 6.10 and 6.12 show the transmission characteristics of single-layer films deposited on the same substrate glasses, and having nominally the same maximum-transmission wavelength.)

Again, in comparing theory and experiment the same considerations apply as in the case of the single-layer films. With double-layer films the number of uncertainties concerning the film structure is increased, but there is, nevertheless, good agreement, particularly in the region of maximum transmission. The dispersion of bismuth oxide was assumed, for the purpose of calculating the theoretical transmission, to be that indicated in Fig. 6.15(a). There do not appear to be any published data of the dispersion of bismuth oxide, and this curve must be regarded as subject to uncertainty. It was derived as follows. Thin films of bismuth oxide have a refractive index, at a wavelength of $5400\text{\AA}$, of 2.45 *. Similar values of refractive index at optical frequencies are found in zinc sulphide. The dispersion of this has been measured by Hall and Ferguson (1955) and is indicated by the lower curve in Fig. 6.15(a). It is seen that zinc sulphide has a refractive index of 2.45 at a wavelength of $4500\text{\AA}$, and the dispersion curve for bismuth oxide was obtained simply by shifting the wavelength scale $1000\text{\AA}$. This procedure may, of course, lead to considerable errors; in the region of $6500\text{\AA}$ or $4000\text{\AA}$, for example, the errors

*Kaye and Laby, "Physical and Chemical Constants" (1959), p.86.

in refractive index could easily be as high as ± 0.05 . Fortunately, calculation shows that at the shorter wavelengths the transmission of a glass/bismuth oxide/magnesium fluoride system is relatively insensitive to changes in the refractive index of bismuth oxide, a change in index (at $4000\text{\AA}$) of ± 0.10 giving a change in transmission of only ± 0.002 . The shape assumed for the dispersion curve of bismuth oxide in this region is, in fact, one that gives almost the optimum agreement with the experimental transmission curves. At longer wavelengths, the dependence of the transmission on refractive index of the bismuth oxide layer is rather more critical, and the dispersion curve would have to be taken as that indicated by the broken line to get really close agreement between experimental and theoretical results. The true dispersion curve may well lie somewhere between the two curves shown.

Consideration of polarization effects

The experimental curves for the transmission T of a treated surface were derived from the measured values of τ by the relationship $T = \tau T_0$, where T_0 is the theoretical transmission of an untreated surface for unpolarized light at normal incidence. Since the measurement of τ was made with light incident at 20° , the product τT_0 , the value of which will be denoted by $\bar{T}$ in the subsequent discussion, is not identical with the true normal-incidence transmission, T , of a treated surface. For a single surface, treated or untreated, and at 20° incidence, the average value of the reflection coefficients, R_p , R_s , for the two directions of polarization differs from the normal-incidence value by a negligible amount (cf. Ch. III, Sec. 3.5),

the normal-incidence value by a negligible amount (cf. Ch. III, Sec. 3.5), and $\bar{T} = T$ with sufficient accuracy. When, however, light passes through a system of many surfaces at non-normal incidence, the proportion of the s-polarized component will become continually smaller with each successive surface, since $R_s > R_p$, and this will give, in effect, increasingly higher values of the total transmission factor for successive surfaces. The effect is greater for untreated than for treated surfaces, and measurements made for different numbers of surfaces will, therefore, correspond to different values of τ , the value of τ decreasing with increasing number of surfaces. The significance of the τ -curves needs, therefore, to be examined, and the discrepancies between $\bar{T}$ and T have to be determined. It has been assumed, moreover, that the light used for measurement was unpolarized, but as will be shown later, there was in fact a preponderance of the s-polarized component, and this must be taken into consideration.

To facilitate the discussion, primed symbols T' and T'_0 will be used to denote the transmission at 20° incidence of a treated and of an untreated surface, respectively, the unprimed symbols T , T_0 , being used, as before, for normal incidence; absence of the suffixes p and s indicates a total value of the quantity in question for both directions of polarization.

If the initial fractions of p- and s-polarized light in the photometer beams are, respectively, α_p and α_s , the overall transmission factors for n treated and untreated surfaces are given, respectively, by

$$T'(n) = \alpha_p \cdot T'_p{}^n + \alpha_s \cdot T'_s{}^n \quad 6.3$$

and

$$T'_0(n) = \alpha_p \cdot T'_{0p}{}^n + \alpha_s \cdot T'_{0s}{}^n$$

Thus, the effective transmission per surface will be given by

$$\begin{aligned} T' &= [T'(n)]^{1/n} \\ T'_0 &= [T'_0(n)]^{1/n} \end{aligned} \quad 6.4$$

It is seen from the form of the expressions 6.3 for $T'(n)$ and $T'_0(n)$, that T' and T'_0 will increase with increasing n . However, calculation shows that for a wide range of values of the ratio α_s/α_p , values of T' and T'_0 given by 6.4 for $n = 16$ are identical (± 0.0001) with the average of the values for $n = 1$ and $n = 32$. For example, the following table gives values of T'_0 for glass K, at a wavelength of $4000\text{\AA}$, calculated for these values of n ; α_s/α_p has been put equal to 2 for reasons to be discussed

n	T'_0
1	0.8840
32	0.8870
Average	0.8855
16	0.8856

presently. The variation in T'_0 is greatest for this glass type: for glass A the total range of variation, for $n = 1, \dots, 32$, is only 0.0004, and this value is also about the maximum for a treated surface.

It is thus evident that the meaning to be given to the experimental values of τ is the ratio of the average values (i.e. for $n = 16$) of T' and T'_0 as defined by 6.4. If, as in the present case, there is a greater proportion of s-polarized light, these values will be smaller

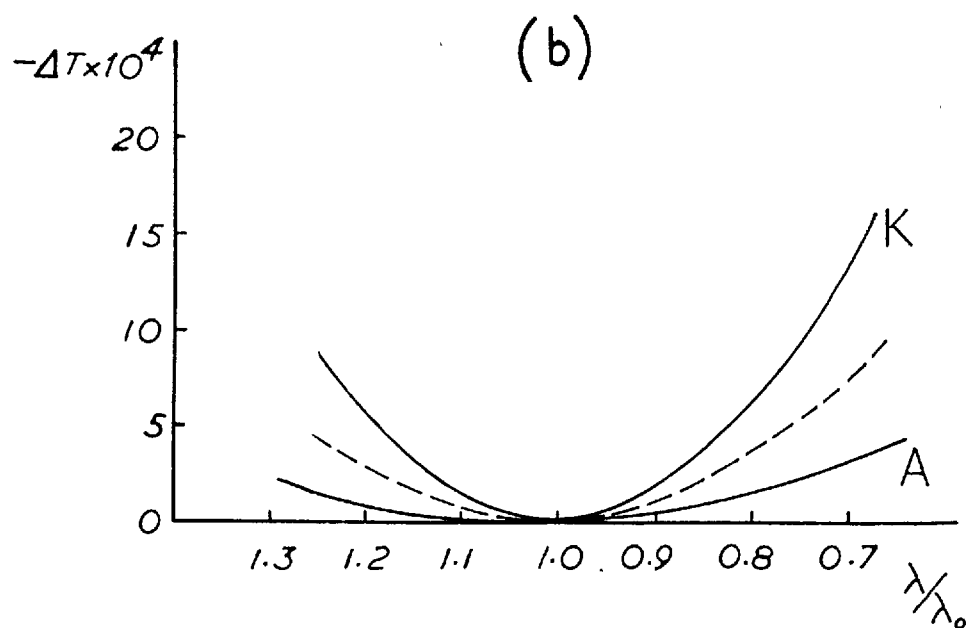
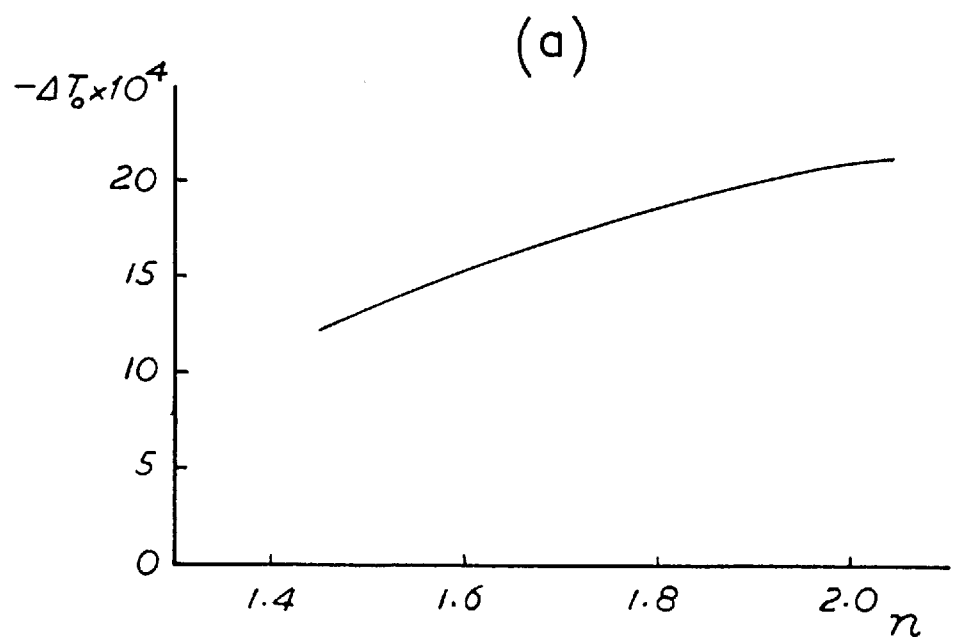


FIG. 6.16. Polarization errors (a) untreated surfaces
(b) treated surfaces .

than normal-incidence values. The discrepancy $\Delta T_0 = T_0 - T'_0$ increases with refractive index of the glass (Fig. 6.16(a)) and will vary with the initial state of polarization of the light. This is true also, though in a lesser degree, for the discrepancy $\Delta T = T - T'$ which has the additional characteristic that $\Delta T = 0$ at the maximum transmission wavelength, λ_0 , (cf. Ch. III, Sec. 3.5). The variation of ΔT is shown in Fig. 6.16(b) as a function of wavelength, conveniently plotted on a scale of λ/λ_0 , for the highest- and lowest-index glasses: values for the other glass types will be given, to sufficient accuracy, by linear interpolation. The dashed curve relates to a double-layer film on glass of refractive index 1.55, and is applicable, with sufficient accuracy, to both glass types used for the double-layer films.

It is seen that the quantities ΔT_0 and ΔT are quite small, and the T-curves in Figs. 6.1 to 6.13 do not, in fact, deviate by more than about $\frac{1}{4}\%$ from true normal-incidence values. Nevertheless, we may apply a simple correction factor to the values $\bar{T}$ given by the curves, and thus obtain the true normal-incidence values, T , with greater precision. By definition, $\bar{T} = \tau T_0$. Substituting here for τ the value $\tau = \frac{T'}{T'_0}$, and writing $T' = T - \Delta T$ and $T'_0 = T_0 - \Delta T_0$, we obtain

$$T = \left(\frac{1 + \Delta T}{1 + \Delta T_0} \right) \bar{T}, \quad 6.5$$

where T' and T'_0 have, for simplicity, been taken as unity where they appear as denominators of ΔT and ΔT_0 , respectively: the correction factor is small, and an error of a few per cent. in its magnitude is not significant.

The residual errors in T are practically identical with those indicated on the τ -curves, and are summarized in the following table:

Wavelength (Å)	6000	5000	4000
<u>Substrate</u>			
A,B,C	± 0.0003	0.0001	0.0004
D,E,F	0.0010	0.0005	0.0010
G,H,K	0.0030	0.0010	0.0050

The decrease in the precision of measurement with increasing refractive index of the specimens is due principally to loss of signal strength resulting from the increased absorption of light.

It was indicated earlier that the light used for the measurements was in a partially polarized state. This is because the light, in passing twice through the monochromator prism, crosses four air-glass interfaces at angles of incidence close to the polarizing angle. Reflection losses (which are the same for either direction of propagation across an interface), are considerably larger for the s-polarized component than for the p-polarized component, and for a prism of refractive index 1.6 working at minimum deviation, the final proportions are $\alpha_s = 0.336$, $\alpha_p = 0.664$ (± 0.001). Now, the plane of incidence is horizontal for the prism, but it is vertical for the specimen blocks used for measurement. In the photometer beams, therefore, the subscripts p and s are reversed and we have $\alpha_p = 0.336$, $\alpha_s = 0.664$. (The light in the beams is not parallel, but the convergence

angle is small, and the tacit assumption in the foregoing discussion that the beams are represented by their chief rays involves negligible error.) Fortuitously, the ratio α_s/α_p is almost exactly equal to 2 in the present case. An accuracy of $\frac{1}{2}\%$ for the value of this ratio is sufficient for calculating values of T' and T'_0 , using 6.3 and 6.4, with a precision of 0.0001.

Part 2

Absorption coefficients of optical glass

6.4 Experimental procedure

The absorption coefficient, for a given wavelength, of a specimen of given length is determined, in principle, by a single measurement of its transmission. This is equal to the ratio of aperture heights, $\frac{R}{R_0}$, for balance settings with and without the specimen. The accuracy of the results will thus depend on the accuracy of individual observations to a much greater degree than was the case in the measurements on anti-reflection films. Uncertainties in $\frac{R}{R_0}$ arise mainly as a result of repeated re-adjustment of the optical system for different specimens made necessary by the different amounts of axial and vertical (relative) displacements of the intermediate slit images (see Ch. V, Sec. 5.5(ii)). Thus, readings R for the different specimens, and the "zero-readings", R_0 , are obtained with the light traversing slightly different portions of the various optical elements. To minimize these errors, four separate adjustments

Table 6.2

Specimen	Type	Reference	n_D	Length (cm.)
I	Hard crown	519604	1.51899	12.30
II	Light barium crown	541595	1.54065	12.30
III	Extra light flint	548456	1.54856	13.66
IV	Medium bar- ium crown	572577	1.57220	12.30
*IV	Medium bar- ium crown	571579	1.571—	10.88
V	Light flint	579411	1.57860	11.25
VI	Barium flint	605438	1.60483	13.66
VII	Dense flint	623360	1.62258	10.45
VIII	Extra dense flint	651336	1.65108	11.25
IX	Extra dense flint	700303	1.70035	12.30
X	Double extra dense flint	748278	1.74842	12.30

*Used in a prototype "rod-lens" cystoscope.

of the optical system were made for each specimen, and a set of readings, for the different wavelengths indicated in Sec. 6.1, was taken for each new adjustment; the "zero-readings", R_0 (full aperture), were determined afresh between successive sets of measurements with the specimen. (In alternate sets of measurements the wavelength sequence was reversed.) Values of R and R_0 used in calculating the results are thus the average of four independent measurements.

Specimens of eleven different glass types were examined and details of these are given in Table 6.2. Roman numerals, rather than the letters A, B, etc., are used to identify the different glass types because these are not always identical with those used as substrates for the anti-reflection films. The image shifts due to the specimens and the reflection-compensating blocks (1 cm. thick in each case) were calculated beforehand so that the positions of the split field-lens could be predetermined.

6.5 Reduction of readings and results

The measured transmission factor, $\frac{R}{R_0}$, is that of a length of glass equal to the difference in length of the light paths in the specimen and the 1cm.-block. Denoting this length by x , we have

$$\frac{R}{R_0} = e^{-\alpha x}$$

where α is the coefficient of absorption per cm. and is given by

$$\alpha = \frac{1}{x} \cdot \log_e \frac{R_0}{R} \quad 6.6$$

To estimate the probable errors in α arising from uncertainties in R and R_0 , it is convenient to work with the ratio $\frac{R}{R_0}$ which will be denoted by r . Differentiating 6.6, we then obtain

$$d\alpha = \frac{1}{x} \cdot \frac{dr}{r} \quad 6.7$$

as the error in α for an uncertainty dr . Now from the definition of r ,

$$dr = \frac{1}{R_0} (dR - r \cdot dR_0)$$

and we may also write $dR = \frac{1}{r} \cdot dR_0$, because when the height of the variable aperture (and hence the amount of light) is reduced from R_0 to R , the errors of measurement will increase approximately in the same ratio.

Thus, 6.7 may be written

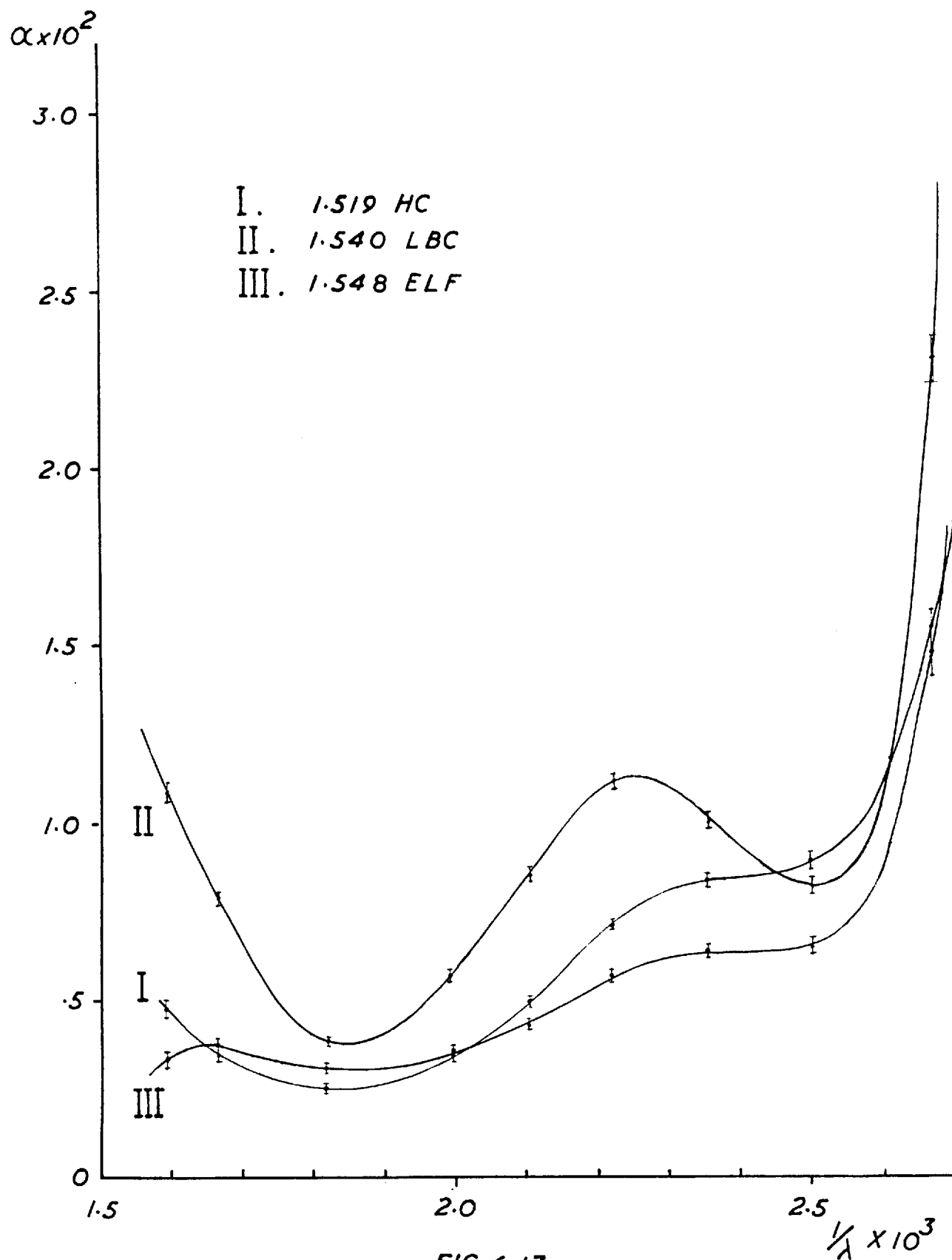
$$d\alpha = \frac{1}{x} \left(\frac{1}{r^2} - 1 \right) \frac{dR_0}{R_0} \quad 6.8$$

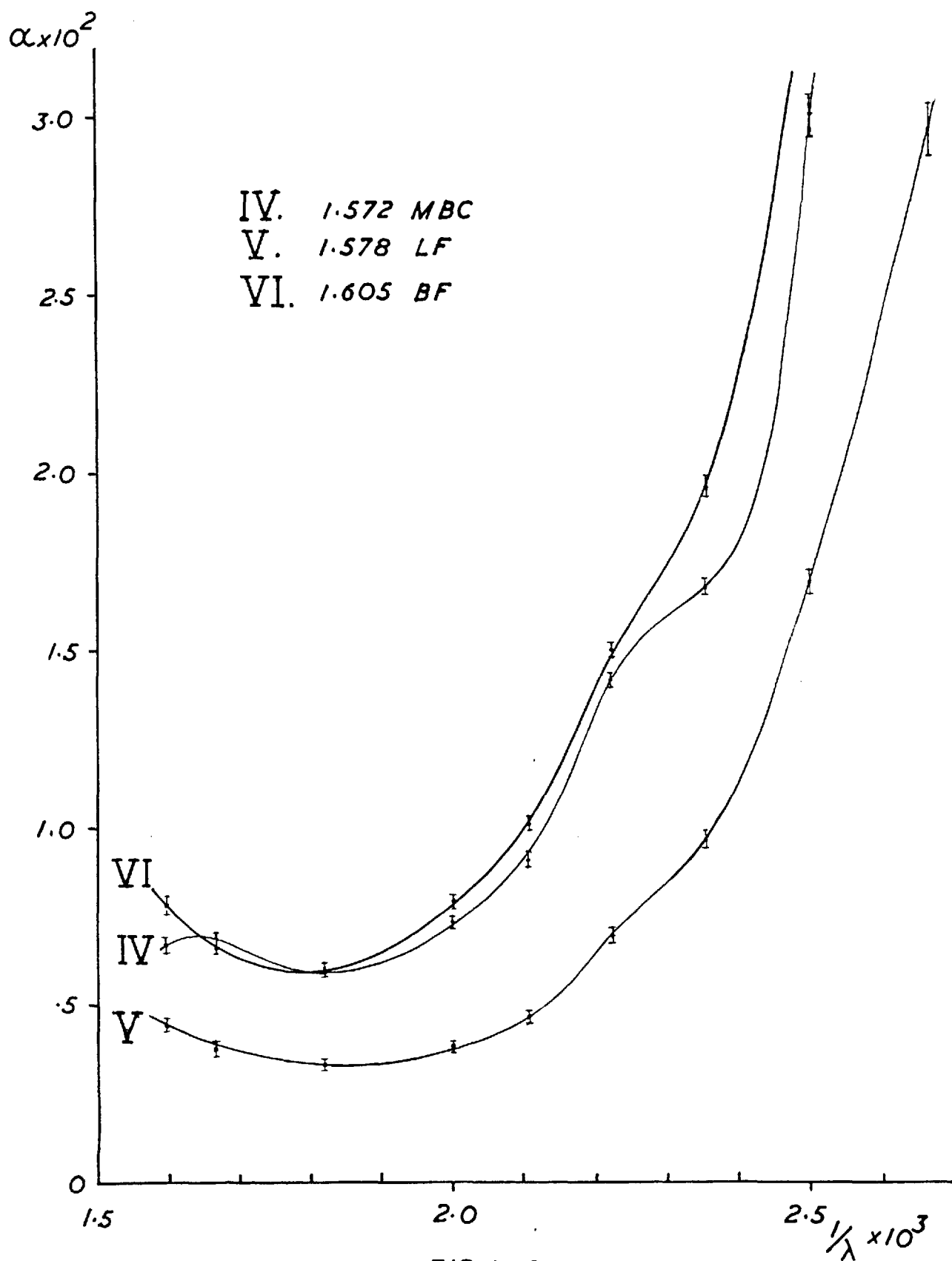
The readings R_0 were taken in eleven groups of four readings each, as described earlier. As is to be expected, the standard deviation, for a particular wavelength, is approximately the same in each group, and the mean value is used for dR_0 . Values of dR_0 (units of 1 micrometer division = 0.01mm.) for the different wavelengths used are given in the following table:

$\lambda(\text{\AA})$	6500	6250	6000	5500	5000	4750	4500	4250	4000	3750
dR_0	1.8	1.4	0.9	0.5	0.5	0.6	0.8	1.2	1.6	2.8

Figs. 6.17 - 6.22 show the absorption coefficient $\alpha \text{ cm}^{-1}$ for the different glass types plotted (on a scale augmented by a factor of 10^2) against the wave-number*, $\frac{1}{\lambda}$, where λ is in Ångstrom units. The probable errors, $\pm d\alpha$, are indicated by the vertical strokes.

*A $\frac{1}{\lambda}$ -scale gives increased "spread" at shorter wavelengths where the curves are steepest.





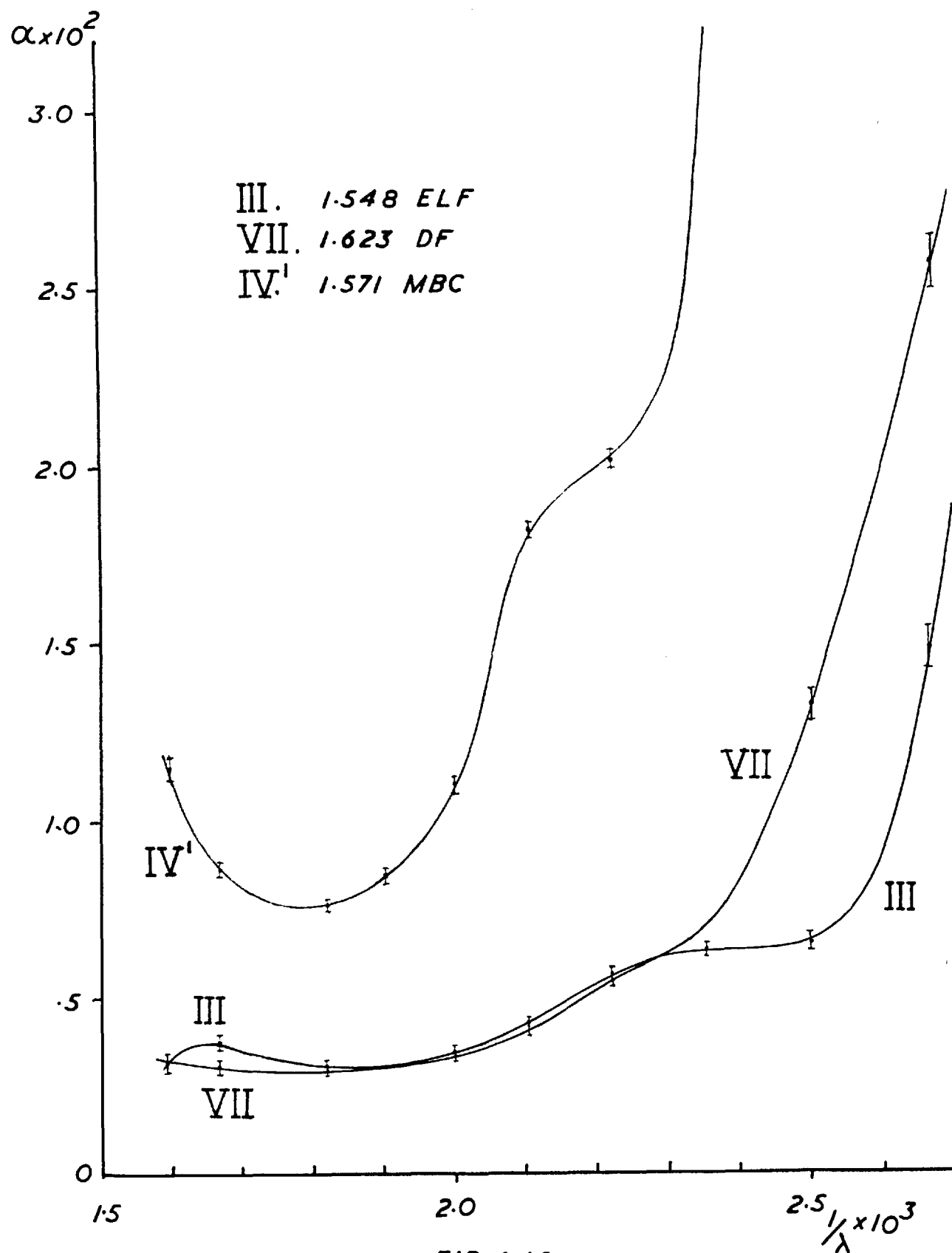


FIG. 6.19

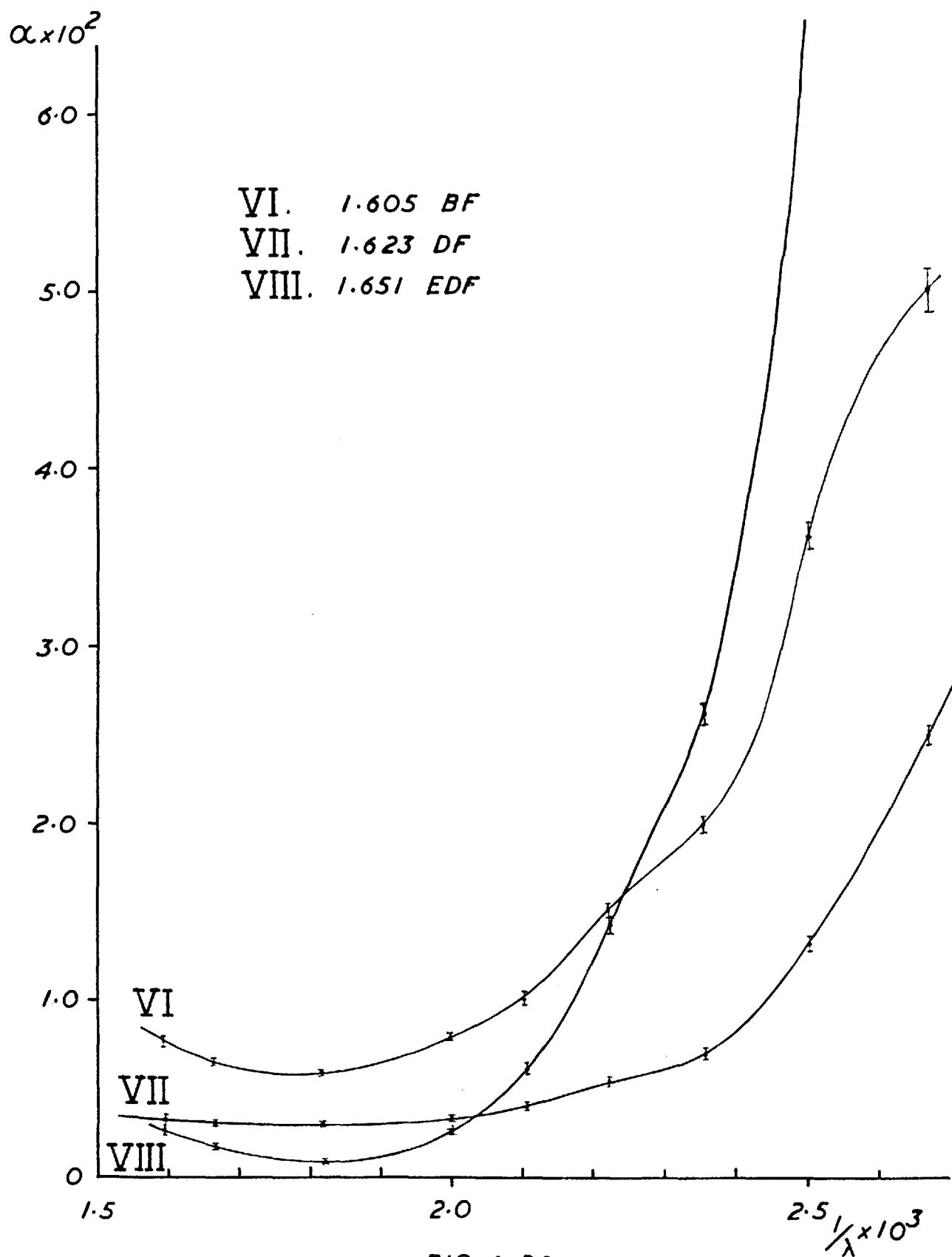


FIG. 6.20

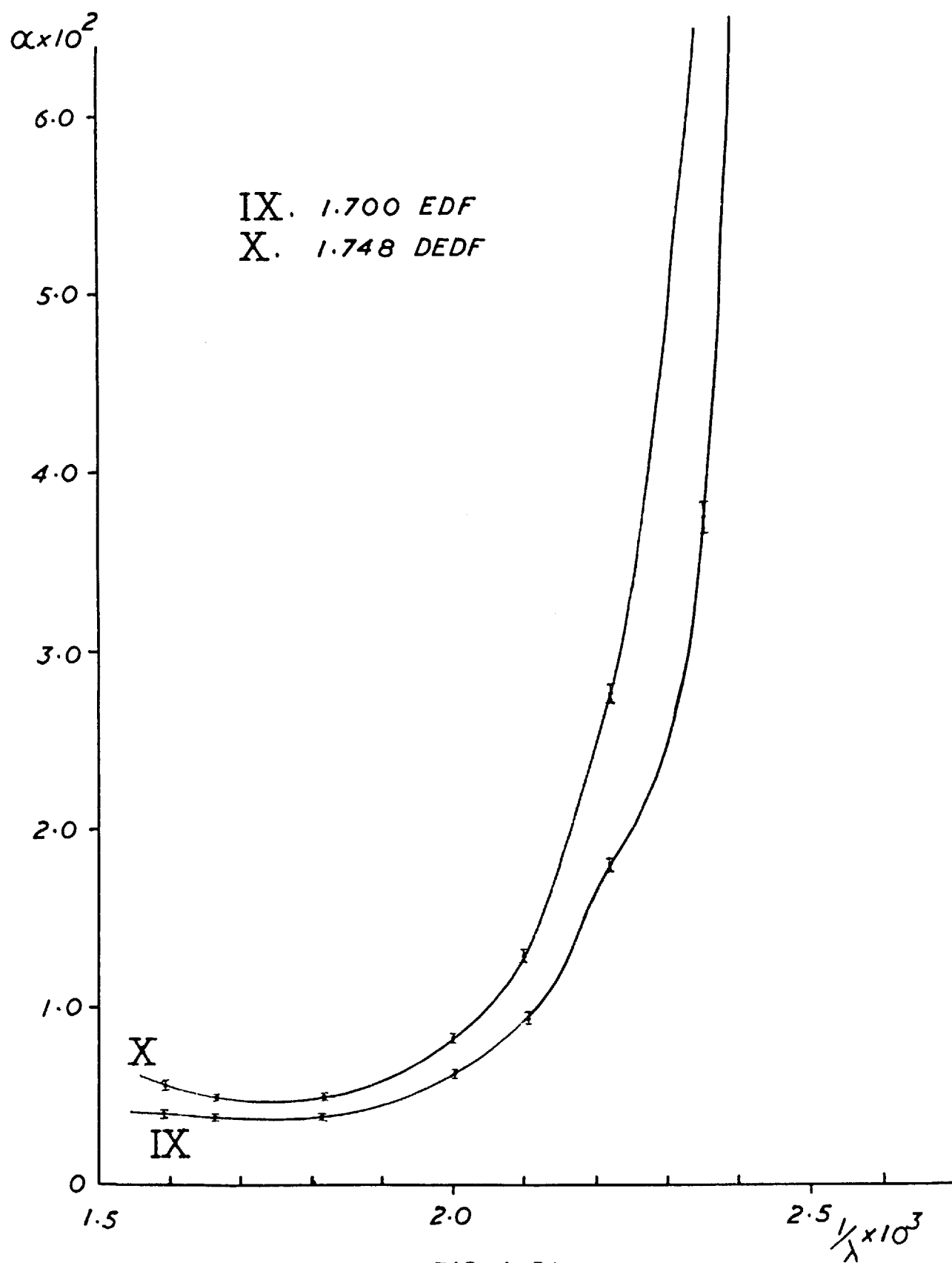


FIG. 6.21

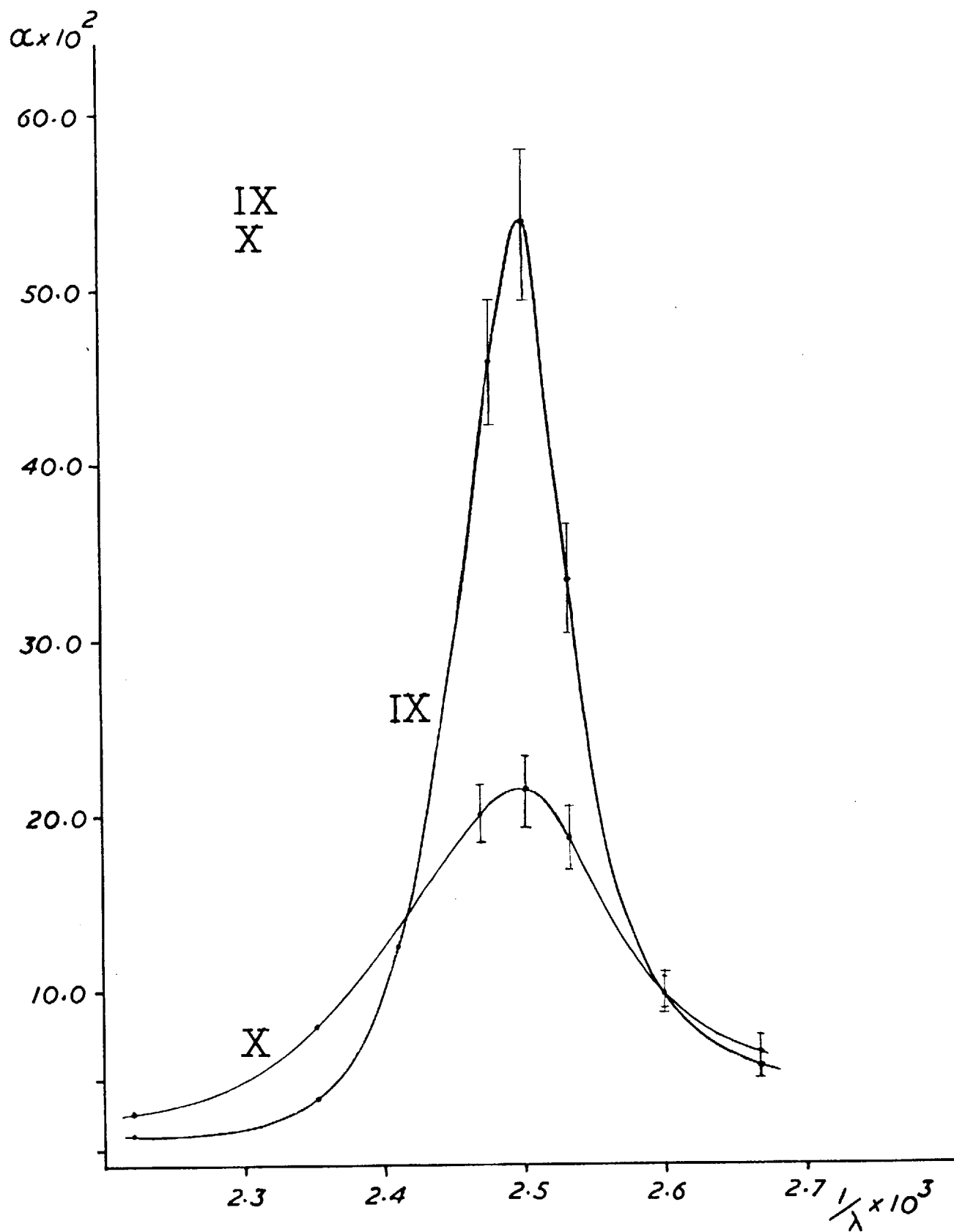


FIG. 6.22

6.6 Discussion

It will be seen from the curves of Figs. 6.17 - 6.22 that specimens III and VII have the lowest overall absorption, having particular regard to the shorter wavelengths. In Fig. 6.19 the absorption curves for these specimens are compared with that of the specimen IV' which is of a medium barium crown glass used, before the results of the present measurements were available, in a prototype of the rod-lens cystoscopic system described in Chapter II. Numerical results for the (theoretical) transmission of different systems will be given in the concluding chapter, but it is clear from the curves that a great improvement in transmission will be obtained by using glass type III or VII. The use of the dense flint glass VII, moreover, will give increased numerical aperture because of the higher refractive index.

The precision of these measurements is adequate for most practical purposes. For a given length x of glass, the variation in transmission T for a change $d\alpha$ in the absorption coefficient is given, by differentiating 6.6 and writing $\frac{R}{R_0} = T$, as $\frac{dT}{T} = -x.d\alpha$. Thus, in a length of 20cm. of glass, for example, a variation $d\alpha = 0.1 \times 10^{-2}$ would produce a change in transmission of only 2 per cent.

Comparatively little is known of the detailed atomic structure of glass, and the theory of the optical absorption process -- or rather, processes -- will bear only limited qualitative comparison with experimental results. In general, there are two main regions of electromagnetic

resonance, one due to electron oscillation at frequencies in the ultra-violet, and the other to oscillation of ions at frequencies in the infra-red region. For the refractive index $n = \frac{c}{u}$ (where u is the phase velocity in the glass), the classical theory of dispersion yields the formula

$$n^2 = 1 + \frac{Ne^2/m_1e_0}{\omega_1^2 - \omega^2 - ig_1\omega_1\omega} + \frac{Ne^2/m_2e_0}{\omega_2^2 - \omega^2 - ig_2\omega_2\omega} \quad 6.9$$

where the symbols have their usual meaning, suffixes 1 relating to an electron, and suffixes 2 to an ion; m_2 is a reduced mass including the valence number of the ion.. (See, for example, "Optics" by A. Sommerfeld, Academic Press Inc., 1954, at page 98.) A complex refractive index is always associated with absorption of energy, the absorption being greatest in regions of maximum dispersion, that is, when ω is in the neighbourhood of ω_1 or ω_2 . It will be seen from the curves of Figs. 6.17 - 6.22 that the absorption coefficient generally has minimum values in the middle of the visible spectrum, and increases towards longer wavelengths and, much more rapidly, towards shorter wavelengths (where the dispersion is greater). Terms such as those in 6.9 are associated with each of the different ions in the glass, and the observed optical properties result from a summation for all the ions present. The question is further complicated by the presence of impurities in the glass, notably ferric oxide which has an electron resonance in the blue region of the spectrum.

An interesting property common to all the glass types examined is the presence of an absorption "peak", in some cases barely discernible but in others (e.g. specimen IX) very pronounced, at a

wavelength of about $4500\text{\AA}$. Stevels (1960) has found that glass irradiated with X-rays, or other short-wave radiation, shows absorption bands in this region of the spectrum which are associated with the presence of alkali metals. According to current theory (Stevels, loc. cit.), interstices in a broken and irregular silicon-oxygen network forming the main glass structure may contain other ions, such as those of the alkali or alkaline-earth metals. On irradiation, an electron may be removed from a non-bridging oxygen atom and become trapped in one of the interstices of the lattice. An interstitial metal ion in the neighbourhood of the ionized oxygen atom will then migrate under the influence of the modified Coulomb field. There will thus be a positive hole and a trapped electron, and the presence of these has been confirmed by paramagnetic resonance experiments which indicate an absorption band, due to the electron, at about $4500\text{\AA}$. It may well be that the absorption "peaks" observed in the present measurements arise as a result of natural irradiation of the glass.

References

1. Hall, J.F. and Ferguson, W.F.C., J.O.S.A. 45 (1955) 74
2. Weinstein, W., Vacuum, vol.IV, 1 (1954) 3
3. Stevels, J.M., "Non-crystalline Solids", ed. Frechette; Wiley, 1960 (Chapter 17).

Chapter VII

CONCLUSION

The experimental results were used to calculate the (paraxial) transmission characteristics of some typical endoscopic optical systems. The results of these calculations are given below and, as will be seen, provide a guide to certain design principles.

7.1 Conventional systems

An endoscopic system of average length may have some 15 crown/flint doublet lenses. These will have a total glass path of about 6cm. Absorption losses will therefore be comparatively small, and the transmission characteristics of the system will be determined almost wholly by the reflection losses at the 15 crown-glass/air interfaces and the 15 flint-glass/air interfaces. In Fig. 7.1, the lower curve T shows the paraxial transmission of such a system of 15 doublet lenses made up of glass I (hard crown, $n = 1.519$) and glass VII (dense flint, $n = 1.626$), with no surface treatment. Only about 5% of the total light loss is due to absorption in the glass. The curve T_1 shows the improvement in transmission made possible by coating the lens surfaces with a single-layer anti-reflection film having a transmission maximum in the middle of the visible spectrum; it is seen that the transmission of the system is increased by a factor of nearly 3. If the surfaces are coated with a double-layer film having maximum transmission at $4750\text{\AA}$, the transmission of the

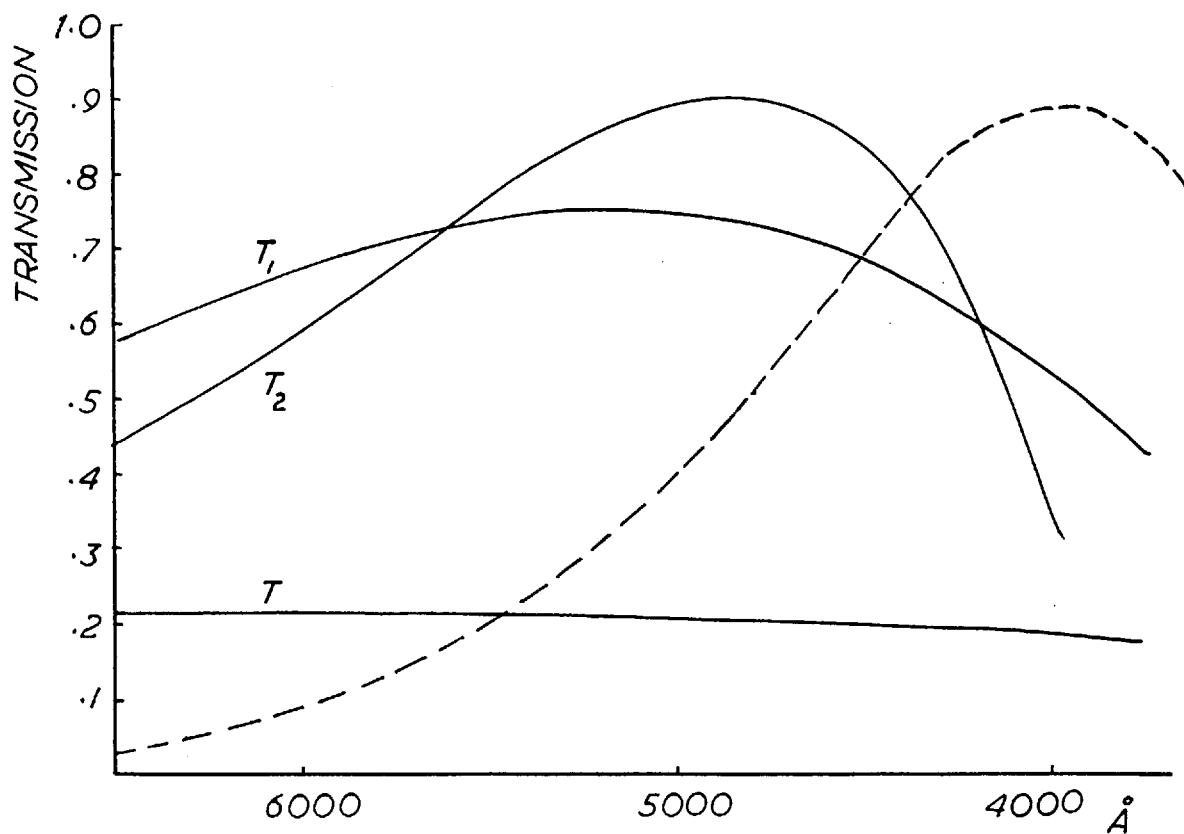


FIG. 7.1. Transmission of a 30-surface system.

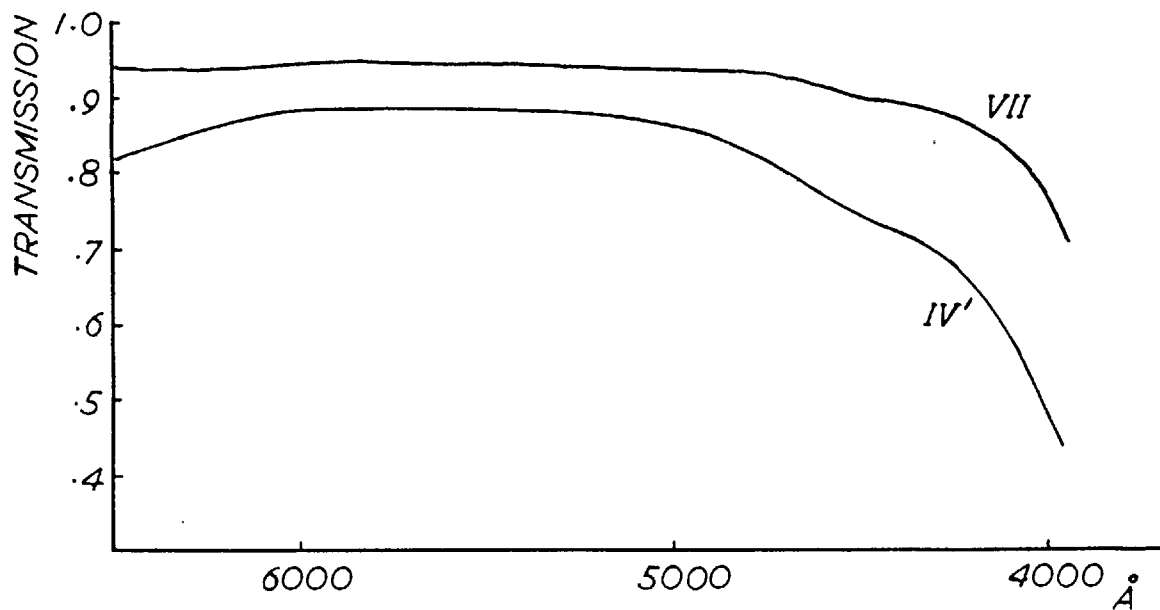


FIG. 7.2. Transmission of 20 cm. of glass.

system becomes that indicated by the curve T_2 . Although there is a more rapid falling-off in transmission in the blue and red regions of the spectrum than in the case of the single-layer films, the T_2 -curve lies above the T_1 -curve in that region of the spectrum where the human eye has greatest sensitivity, and the increase in transmission here is considerable. In general, therefore, it would appear that the choice of single-layer coatings, as against double-layer coatings, will depend mainly on whether good colour balance is to be preferred to additional improvement in visibility but with some colour distortion. The effect of shifting the transmission maximum of the double-layer films to $4000\text{\AA}$ is indicated by the dotted curve.

7.2 Rod-lens (or air-lens) systems

In a cystoscope - among the shortest of endoscopes - utilizing this kind of optical system, there will be some 20cm. of glass path, and absorption losses will now become significant. In Fig. 7.2, the transmission of 20cm. of glass VII (dense flint, $n = 1.626$) is compared with that of 20cm. of glass IV (medium barium crown, $n = 1.548$) which was used in a prototype instrument. By using the dense flint glass, not only will a considerable improvement in transmission be obtained, particularly for blue light, but it will be possible to have a larger numerical aperture giving additional improvement of the image quality.

Hopkins (1957) uses four glass rods, each about 5cm. long. Alternate "air-lenses" are achromatized by cementing small negative lenses

on the ends of the rods. When the objective, eyepiece and other auxiliary components are included, there is a total of some 20 air-glass interfaces; anti-reflection treatment of the surfaces is, therefore, no less important than in the case of conventional systems. Absorption losses in the glass are heaviest in the blue region of the spectrum, and double-layer anti-reflection coatings with maximum transmission in the blue can thus be used with good effect in "boosting" the transmission of blue light. Fig. 7.3 shows transmission curves for the following systems:

Curve 1 (bad design)

20cm. medium barium crown, $n = 1.571$ (Glass IV');

5cm. barium flint, $n = 1.605$ (Glass VI);

10 air-glass interfaces for each glass type, uncoated.

Curve 2 (good design)

20cm. dense flint, $n = 1.623$ (Glass VII);

5cm. hard crown, $n = 1.519$ (Glass I);

10 air-glass interfaces for each glass type, coated with double-layer anti-reflection films having maximum transmission at $4750\text{\AA}$.

These results need little comment. The actual improvement in performance that would be given by the second system is, in fact, much greater than the curves suggest: not only is the transmission greatly improved, but the light-collecting properties of the system are considerably increased and ghost light is drastically cut down, thus eliminating the blurred and often glare-filled image so familiar to cystoscopists.

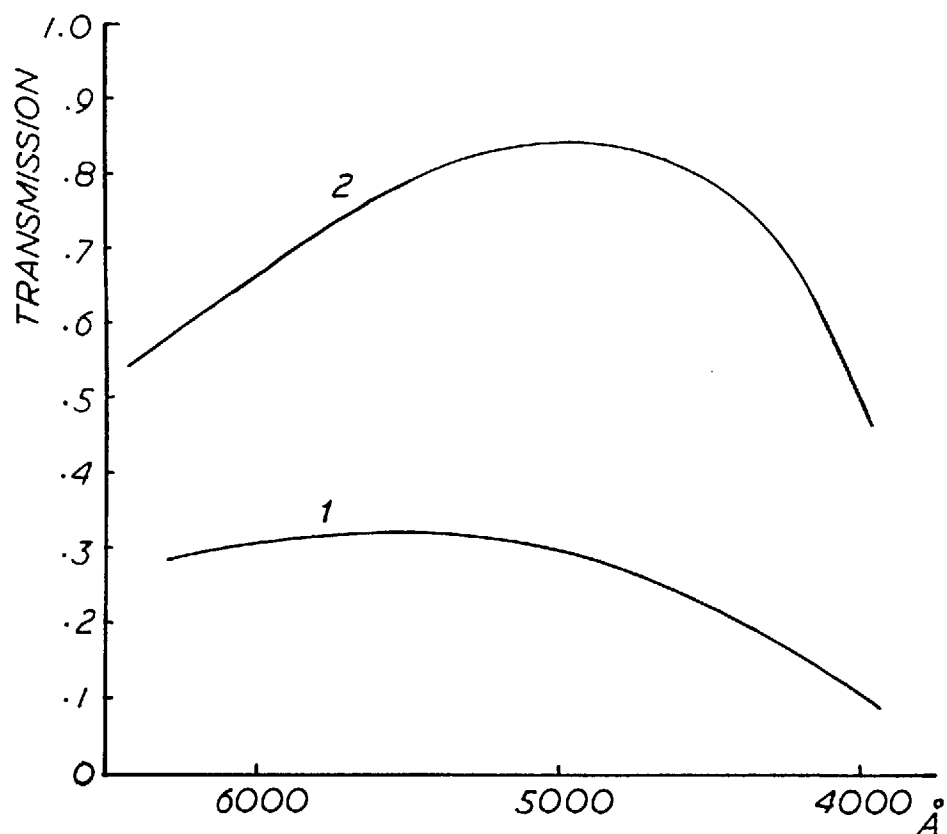


FIG. 7.3. Transmission of (1) badly designed and (2) well designed systems.

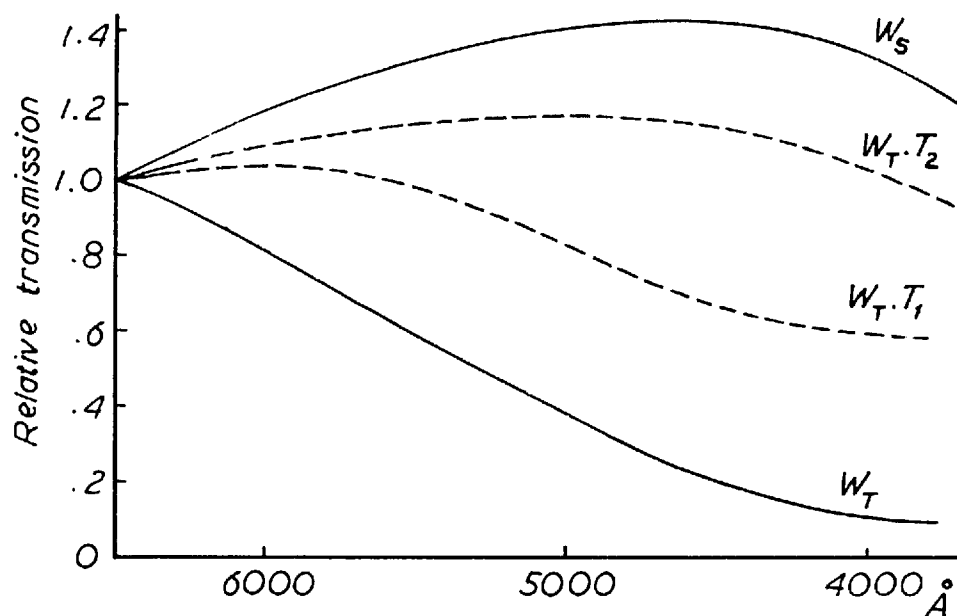


FIG. 7.4. Effect of surface treatment on colour balance in image.

7.3 Colour balance in the image

It was remarked in Chapter I that deficiency of blue light in endoscopic images, due to the use of a tungsten source and to relatively greater instrument losses, might be compensated by using double-layer anti-reflection coatings with peak transmission in the blue region of the spectrum. (This, of course, would have to be at the expense of overall image brightness.) The effect is illustrated in Fig. 7.4 for the 30-surface system referred to earlier in connection with Fig. 7.1. The curves W_S and W_T represent, respectively, the spectral energy distribution of sun-light (6000°K) and of tungsten light (2700°K) normalized, for convenience, to unity at $6500\text{\AA}$. With single-layer coatings, the transmitted spectral energy distribution of tungsten-emitted light is $W_T \cdot T_1$ and is indicated by the lower dotted curve. If double-layer coatings are used, the energy distribution, $W_T \cdot T_2$, becomes that indicated by the upper dotted curve - a close approximation to natural light.

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References

1. Hopkins, H.H., Private communication, 1957.

ACKNOWLEDGMENTS

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B.C., 1962.

APPENDIX I

Fresnel transmission coefficients for normal incidence

n	T ₀	n	T ₀	n	T ₀	n	T ₀
1.00	1.0000	1.25	0.9877	1.50	0.9600	1.75	0.9256
.01	1.0000	.26	0.9868	.51	0.9587	.76	0.9242
.02	0.9999	.27	0.9859	.52	0.9574	.77	0.9227
.03	0.9998	.28	0.9849	.53	0.9561	.78	0.9213
.04	0.9996	.29	0.9840	.54	0.9548	.79	0.9198
1.05	0.9994	1.30	0.9830	1.55	0.9535	1.80	0.9184
.06	0.9992	.31	0.9820	.56	0.9521	.81	0.9169
.07	0.9989	.32	0.9810	.57	0.9508	.82	0.9154
.08	0.9985	.33	0.9799	.58	0.9495	.83	0.9140
.09	0.9981	.34	0.9789	.59	0.9481	.84	0.9125
1.10	0.9977	1.35	0.9778	1.60	0.9467	1.85	0.9110
.11	0.9973	.36	0.9767	.61	0.9454	.86	0.9096
.12	0.9968	.37	0.9756	.62	0.9440	.87	0.9081
.13	0.9963	.38	0.9745	.63	0.9426	.88	0.9066
.14	0.9957	.39	0.9734	.64	0.9412	.89	0.9052
1.15	0.9951	1.40	0.9722	1.65	0.9398	1.90	0.9037
.16	0.9945	.41	0.9711	.66	0.9384	.91	0.9022
.17	0.9939	.42	0.9699	.67	0.9370	.92	0.9007
.18	0.9932	.43	0.9687	.68	0.9356	.93	0.8993
.19	0.9925	.44	0.9675	.69	0.9342	.94	0.8978
1.20	0.9917	1.45	0.9663	1.70	0.9328	1.95	0.8963
.21	0.9910	.46	0.9650	.71	0.9314	.96	0.8948
.22	0.9902	.47	0.9638	.72	0.9299	.97	0.8933
.23	0.9894	.48	0.9625	.73	0.9285	.98	0.8919
1.24	0.9885	1.49	0.9613	1.74	0.9271	1.99	0.8904
						2.00	0.8889

APPENDIX II

Table of T^n

$T \backslash n$	2	3	4	5	6	7	8
1.10	1.2100	1.3310	1.4641	1.6105*	1.7716	1.9487	2.1436
1.09	1.1881	1.2950	1.4116	1.5386	1.6771	1.8280	1.9926
1.08	1.1664	1.2597	1.3605.	1.4693	1.5869	1.7138	1.8509
1.07	1.1449	1.2250	1.3108	1.4026	1.5007	1.6058	1.7182
1.06	1.1236	1.1910	1.2625.	1.3382	1.4185*	1.5036	1.5938
1.05	1.1025	1.1576	1.2155°	1.2763	1.3401	1.4071	1.4775.
1.04	1.0816	1.1249	1.1699	1.2167	1.2653	1.3159	1.3686
1.03	1.0609	1.0927	1.1255*	1.1593	1.1941	1.2299	1.2668
1.02	1.0404	1.0612	1.0824	1.1041	1.1262	1.1487	1.1717
1.01	1.0201	1.0303	1.0406	1.0510	1.0615*	1.0721	1.0829
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	0.9801	0.9703	0.9606	0.9510	0.9415.	0.9321	0.9227
0.98	0.9604	0.9412	0.9224	0.9039	0.8858	0.8681	0.8508
0.97	0.9409	0.9127	0.8853	0.8587	0.8330	0.8080	0.7837
0.96	0.9216	0.8847	0.8493	0.8154	0.7827	0.7514	0.7214
0.95	0.9025	0.8574	0.8145*	0.7738	0.7351	0.6983	0.6634
0.94	0.8836	0.8306	0.7807	0.7339	0.6899	0.6485.	0.6096
0.93	0.8649	0.8044	0.7481	0.6957	0.6470	0.6017	0.5596
0.92	0.8464	0.7787	0.7164	0.6591	0.6064	0.5573	0.5132
0.91	0.8281	0.7536	0.6857	0.6240	0.5679	0.5168	0.4703
0.90	0.8100	0.7290	0.6561	0.5905.	0.5314	0.4783	0.4305.

Appendix II -contd.

Table of T^n -contd.

$T \backslash n$	9	10	11	12	13	14	15	16
1.10	2.3579	2.5937	2.8531	3.1384	3.4523	3.7975	4.1772	4.5950.
1.09	2.1719	1.3674	2.5804	2.8127	3.0658	3.3417	3.6425	3.9703
1.08	1.9990	2.1589	2.3316	2.5182	2.7196	2.9372	3.1722	3.4259
1.07	1.8385.	1.9672	2.1049	2.2522	2.4098	2.5785.	2.7590	2.9522
1.06	1.6895.	1.7908	1.8983	2.0122	2.1329	2.2609	2.3966	2.5404
1.05	1.5513	1.6289	1.7103	1.7959	1.8856	1.9799	2.0789	2.1829
1.04	1.4233	1.4802	1.5395.	1.6010	1.6651	1.7317	1.8009	1.8730
1.03	1.3048	1.3439	1.3842	1.4258	1.4685.	1.5126	1.5580	1.6047
1.02	1.1951	1.2190	1.2434	1.2682	1.2936	1.3195.	1.3459	1.3728
1.01	1.0937	1.1046	1.1157	1.1268	1.1381	1.1495.	1.1610	1.1726
1.00	1.0000	1.0000	1.0000	1.00001	1.0000	1.0000	1.0000	1.0000
0.99	0.9135.	0.9044	0.8953	0.8864	0.8775.	0.8687	0.8601	0.8515.
0.98	0.8337	0.8171	0.8007	0.7847	0.7690	0.7536	0.7386	0.7238
0.97	0.7602	0.7374	0.7153	0.6938	0.6730	0.6528	0.6333	0.6143
0.96	0.6925.	0.6648	0.6382	0.6127	0.5882	0.5647	0.5421	0.5204
0.95	0.6302	0.5987	0.5688	0.5404	0.5133	0.4877	0.4633	0.4401
0.94	0.5730	0.5386	0.5063	0.4759	0.4474	0.4205.	0.3953	0.3716
0.93	0.5204	0.4840	0.4501	0.4186	0.3893	0.3620	0.3367	0.3131
0.92	0.4722	0.4344	0.3996	0.3677	0.3383	0.3112	0.2863	0.2634
0.91	0.4279	0.3894	0.3544	0.3225.	0.2935.	0.2670	0.2430	0.2211
0.90	0.3874	0.3487	0.3138	0.2824	0.2542	0.2288	0.2059	0.1853

Appendix II -contd.

Table of T^n -contd.

$T \backslash n$	17	18	19	20	21	22	23	24
1.10	5.0545	5.5599	6.1159	6.7275	7.4002	8.1403	8.9543	9.8497
1.09	4.3276	4.7171	5.1417	5.6044	6.1088	6.6586	7.2579	7.9111
1.08	3.7000	3.9960	4.3157	4.6610	5.0338	5.4365	5.8715	6.3412
1.07	3.1588	3.3799	3.6165	3.8697	4.1406	4.4304	4.7405	5.0724
1.06	2.6928	2.8543	3.0256	3.2071	3.3996	3.6035	3.8197	4.0489
1.05	2.2920	2.4066	2.5270	2.6533	2.7860	2.9253	3.0715	3.2251
1.04	1.9479	2.0258	2.1068	2.1911	2.2788	2.3699	2.4647	2.5633
1.03	1.6528	1.7024	1.7535	1.8061	1.8603	1.9161	1.9736	2.0328
1.02	1.4002	1.4282	1.4568	1.4859	1.5157	1.5460	1.5769	1.6084
1.01	1.1843	1.1961	1.2081	1.2202	1.2324	1.2447	1.2572	1.2697
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	0.8429	0.8345	0.8262	0.8179	0.8097	0.8016	0.7936	0.7857
0.98	0.7093	0.6951	0.6812	0.6676	0.6543	0.6412	0.6283	0.6158
0.97	0.5958	0.5780	0.5606	0.5438	0.5275	0.5117	0.4963	0.4814
0.96	0.4996	0.4796	0.4604	0.4420	0.4243	0.4073	0.3911	0.3754
0.95	0.4181	0.3972	0.3774	0.3585	0.3486	0.3235	0.3074	0.2920
0.94	0.3493	0.3283	0.3086	0.2901	0.2727	0.2563	0.2410	0.2265
0.93	0.2912	0.2708	0.2519	0.2342	0.2178	0.2026	0.1884	0.1752
0.92	0.2423	0.2229	0.2051	0.1887	0.1736	0.1597	0.1469	0.1352
0.91	0.2012	0.1831	0.1666	0.1516	0.1380	0.1256	0.1143	0.1040
0.90	0.1668	0.1501	0.1351	0.1261	0.1049	0.0985	0.0886	0.0798

Appendix II -contd.

Table of T^n -contd.

$T \backslash n$	25	26	27	28	29	30
1.10	10.8347	11.9182	13.1100	14.4210	15.8631	17.4494
1.09	8.6231	9.3992	10.2451	11.1671	12.1722	13.2677
1.08	6.8485	7.3964	7.9881	8.6271	9.3173	10.0627
1.07	5.4274	5.8074	6.2139	6.6488	7.1143	7.6123
1.06	4.2919	4.5494	4.8223	5.1117	5.4184	5.7435.
1.05	3.3864	3.5557	3.7335.	3.9201	4.1161	4.3219
1.04	2.6658	2.7725.	2.8834	2.9987	3.1187	3.2434
1.03	2.0938	2.1566	2.2213	2.2879	2.3566	2.4273
1.02	1.6406	1.6734	1.7069	1.7410	1.7758	1.8114
1.01	1.2824	1.2953	1.3082	1.3213	1.3345.	1.3478
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.99	0.7778	0.7700	0.7623	0.7547	0.7472	0.7397
0.98	0.6035.	0.5914	0.5796	0.5680	0.5566	0.5455.
0.97	0.4670	0.4530	0.4394	0.4262	0.4134	0.4010
0.96	0.3604	0.3460	0.3321	0.3189	0.3061	0.2939
0.95	0.2774	0.2635.	0.2503	0.2378	0.2259	0.2146
0.94	0.2129	0.2001	0.1881	0.1768	0.1662	0.1563
0.93	0.1630	0.1516	0.1409	0.1311	0.1219	0.1134
0.92	0.1244	0.1144	0.1053	0.0968	0.0891	0.0820
0.91	0.0946	0.0861	0.0784	0.0713	0.0649	0.0591
0.90	0.0718	0.0646	0.0581	0.0523	0.0471	0.0424

APPENDIX III

$$\text{Transmission } \frac{I}{I_0} = e^{-\alpha x}$$

of length x cm. with coefficient of absorption $\alpha \text{ cm}^{-1}$.

$\alpha \times 10^3 \backslash x$	2	4	6	8	10	12	14
2	.996	.992	.988	.984	.980	.976	.972
4	.992	.984	.976	.969	.961	.953	.946
6	.988	.976	.965	.953	.942	.931	.919
8	.984	.969	.953	.938	.923	.908	.894
10	.980	.961	.942	.923	.905	.887	.869
12	.976	.953	.931	.908	.887	.866	.845
14	.972	.946	.919	.894	.869	.845	.822
16	.969	.938	.908	.880	.852	.825	.799
18	.965	.931	.898	.866	.835	.806	.777
20	.961	.923	.887	.852	.819	.787	.756
22	.957	.916	.876	.839	.803	.768	.735
24	.953	.908	.866	.825	.787	.750	.715
26	.949	.901	.856	.812	.771	.732	.695
28	.946	.894	.845	.799	.756	.715	.676
30	.942	.887	.835	.787	.741	.698	.657

Appendix III -contd.

$$\frac{I}{I_0} = e^{-\alpha x} \quad \text{-contd.}$$

$\alpha \times 10^3 \backslash x$	16	18	20	22	24	26	28	30
2	.969	.965	.961	.957	.953	.949	.946	.942
4	.938	.931	.923	.916	.908	.901	.894	.887
6	.908	.898	.887	.876	.866	.856	.845	.835
8	.880	.866	.852	.839	.825	.812	.799	.787
10	.852	.835	.819	.803	.787	.771	.756	.741
12	.825	.806	.787	.750	.732	.715	.698	.698
14	.799	.777	.756	.735	.715	.695	.676	.657
16	.774	.750	.750	.726	.703	.681	.660	.639
18	.750	.723	.698	.673	.649	.626	.604	.583
20	.726	.698	.670	.644	.619	.595	.571	.549
22	.703	.673	.644	.616	.590	.564	.540	.517
24	.681	.649	.619	.590	.592	.536	.511	.487
25	.660	.626	.595	.564	.536	.509	.483	.458
28	.639	.604	.571	.540	.511	.483	.457	.432
30	.619	.583	.549	.517	.487	.458	.432	.407