

U N I V E R S I T Y O F L O N D O N

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

Department of Electrical Engineering

DEVELOPMENT AND ANALYSIS OF THYRISTOR CONTROLLED

INDUCTION ~~MOTORS~~ MACHINES

by

R. Arockiasamy, B. E. Honours (Madras University)

Thesis submitted for the degree of
Doctor of Philosophy in Engineering

JANUARY, 1969.

ACKNOWLEDGEMENTS

The work described in this thesis was carried out in the Electrical Engineering Department of the Imperial College of Science and Technology, under the supervision of Prof. Laithwaite and Dr. Eastham to whom the author is indebted.

The author wishes especially to thank Dr. Eastham for his kind guidance and encouragement in carrying out the work and for his immense help in the preparation of the thesis.

Appreciation is extended to the Ministry of Education, Government of India, for the award of the scholarship; to Mr. Green and the workshop staff for invaluable assistance in the construction of experimental apparatus.

ABSTRACT

This thesis is presented in two parts. The first part is concerned with the development of a numerical technique to calculate the performance of electrical machines. The experimental verification of the analysis is given in the second part considering the case of a slip ring induction motor. Experimental investigations relating to the development of variable speed drives are also described in the second part.

In the introduction the difficulty of obtaining a complete solution for the performance of electrical machines is discussed and a short survey of the existing analytical and numerical techniques and their restrictions is outlined.

In the subsequent three chapters the numerical technique developed to calculate the transient and steady state performance of electrical machines and circuits is described. The procedure is applicable to all types of electrical machines with various asymmetries, and the waveform of the input voltages is not restricted to any particular shape. In some cases namely simple circuits and ideal machines the numerical procedure is checked against exact analytical solutions.

In chapter 5 a rigorous method of forming the system equations in the cases of squirrel cage and sheet rotors is developed. The assumptions made are justified by comparing computed results obtained using alternative methods and also with experimental values obtained from a specially built machine. The criterion for convergence of the equations written is established and the simplifications resulting from various machine symmetries are indicated.

In chapter 6 the method of forming system equations for a squirrel cage machine is used to calculate the starting performance of a reluctance-start single phase induction motor having a variable air gap. An optimum variation of air gap to produce maximum starting torque is predicted.

In chapter 7 analysis of circuits with non-linear elements is illustrated considering the case of an impulse commutated inverter.

The experimental investigations carried out are described in chapters 8 to 11. The design of a strain gauge torque meter to measure the transient and steady state torques acting on the stator of a slip-ring induction motor is discussed in chapter 8. The performance of the slip-ring motor is computed using the numerical procedure and the results are compared with the measured values in chapter 9.

The development of a variable-speed drive-scheme using a three phase bridge inverter and a synchronous motor arrangement employing line commutation is described in chapter 10. The stability of the system is verified experimentally and a simple analysis is given.

In chapter 11 the development of various mechanical switching arrangements is described. In particular the techniques are used to provide a polyphase inverter system which operates as a source of variable frequency supply with an adjustable output voltage when operating from a constant d.c. input voltage. The arrangement can also be used as a time-ratio controller to vary the average voltage applied to a d.c. machine from a fixed d.c. supply and thus provide variable speed working without inserting resistance in the armature circuit.

A new circuit arrangement, invented to provide a true step down d.c. transformer by using a multi-time-ratio switching is also described in chapter 11. This circuit can be operated with an adjustable amplification of d.c. current.

In chapter 12 the uses of the work described in this thesis are summarised and further investigations, theoretical and practical, which would improve the usefulness of the work are indicated.

INDEX

Acknowledgements	2
Abstract	3
List of symbols	10

PART 1 12

Theoretical Investigations

<u>Chapter</u>	<u>Nomenclature</u>	
<u>1</u>	<u>Introduction</u>	
1.1	Statement of the problem	13
1.2	System equations for a slip-ring motor	13
1.3	Existing methods of solving the system equations	15
1.3.1	Steady state performance	15
1.3.2	Transient solutions	18
1.4	The object of the theoretical investigations	19
1.5	The numerical procedure	20
 2	 Formation and reduction of the system equations	
2.1	Introduction	22
2.2	Two-pole two-phase slip-ring motor	22
2.3	Four-pole two-phase slip-ring motor	26
2.4	Four-pole three-phase slip-ring motor	27
2.5	Leakage inductances	28
 3	 <u>The step-by-step procedure</u>	
3.1	Introduction	30
3.2	Solution of a resistance and inductance series circuit	30
3.3	Incremental time T	32
3.4	Inductance varying with time	43
3.5	Equations for the electrical system	45
3.6	Solution of currents at constant speed	46
3.7	Electro-mechanical transient solution	47
3.8	Computer program for the step-by-step procedure	50

3.8.1	Data and initial conditions	50
3.8.2	Performance calculation	53
3.9	Various computations carried out using the program	55
4	<u>An indirect proof of the numerical technique</u>	
4.1	Comparison of the numerical technique with an exact analytical method	58
4.2.1	Verification of the numerical technique by calculating known physical phenomena in machines	60
4.2.2	The case of an ideal machine	60
4.2.3.	<u>A machine having both third harmonic and fundamental winding distributions.</u>	60
4.3	The effect of changing the frequency of the supply	65
5	<u>Transient and steady state analysis of squirrel cage machines using non-sinusoidal windings and input voltages</u>	
5.1	Representation of squirrel cage rotors in conventional analysis	72
5.2	Rigorous analysis of squirrel cage machines	73
5.2.1	Exact representation of squirrel cage rotors	73
5.2.2	Formation of the differential equations	73
5.2.3	A set of differential equations for which a solution exists	77
5.2.4	An example showing the need for reducing the number of rotor equations	78
5.3	Minimising the number of rotor equations	80
5.4	Experimental verification of the current distribution in a squirrel cage	83
5.5	Experimental and computed results for a conventional squirrel cage induction motor	83
5.6	Applications of the squirrel cage rotor analysis	88

6	<u>Variable gap single phase induction motor</u>	
6.1	Introduction	92
6.2	System equations	92
6.3	Solution for currents	93
6.4	Torque	94
6.5	Optimum variation of the air-gap	94
6.6	Detailed description of the program	95
7	<u>Analysis of controlled switching circuits</u>	
7.1	Introduction	99
7.2	The operation of the inverter	99
7.3	Analysis of the inverter-load system	105
7.4	Advantages in using the step-by-step technique	
7.4.1	Voltage waveform	110
7.4.2	Currents during commutation time	111
7.4.3	Non-linear or dynamic load	111
7.5	Conclusions	112
	<u>PART 2</u>	
	<u>Experimental Investigations</u>	
8	<u>The strain gauge apparatus</u>	
8.1	Introduction	115
8.2	The relationships governing the operation of the system	115
8.3	Optimum design of a strain gauge torque meter	118
8.4	The strain gauge strips	118
8.5	Strain gauge bridge circuit	122
8.6	Strain gauge units of equal resistances and gauge factors	125
8.7	Response of the strain gauge apparatus	127
8.8	Low-pass filters used	127
9	<u>Slip-ring induction motor experimental and computed results</u>	
9.1	Introduction	133
9.2	Machine parameters	133
9.3	Steady state performance	135
9.4	Speed transients	141
9.5	Switching transients	146

10	<u>Variable speed synchronous motor</u>	
10.1	Introduction	153
10.2	Principle of operation	153
10.3	The operation of a three phase bridge	154
10.4	Power flow and motoring action	158
10.5	Performance of the variable speed synchronous motor	158
10.6	Analysis of the inverter synchronous machine system	167
10.7	Systems of speed control using the inverter synchronous motor scheme	171
10.8	Modes of speed control	171
10.9	Conclusions	172
11	<u>Mechanical inverters</u>	
11.1	Introduction	174
11.2.1	The mechanical arrangement	174
11.2.2	The rotating drum	174
11.2.3	The stationary frame	180
11.3	Inversion of d.c. voltage to polyphase a.c. voltages	180
11.4	Experimental observations	183
11.5	Mechanical inverter waveforms	183
11.6	Conditions for successful inversion	185
11.7	Time-ratio controller (TRC)	187
11.8	Experimental results from the TRC	187
11.9	Increasing the average voltage of the TRC	190
11.10	Increasing the average current	192
11.11	Conclusions	194
12	<u>Conclusions</u>	
12.1	Introduction	195
12.2	Comparison of results with analytical values	195
12.3	Comparison of computed and experimental results	196
12.4	Extension of the numerical technique and comparison with experimental values	197
12.5	A variable speed drive	198
12.6	Mechanical switching devices	198

12.7	Measurement of transient torque . . .	199
12.8	Non-sinusoidal currents drawn from a sinusoidal source . . .	200
12.9	Thick sheet rotors	200
12.10	General conclusions	201

Appendices

1	Self and mutual inductances between two coils in a machine with uniform air-gap	203
2	Inductance calculations in the case of machines having non-uniform air-gaps	207
3	Calculation of leakage inductances . . .	214
4	Force calculations in electrical machines	218
5	Two equivalent expressions to calculate the H matrix	228
6	Filter design	229
7	A simple trigger unit	232
	References	237

LIST OF SYMBOLS

A, B	switches
a	acceleration
[B]	matrix formed during computation
b	breadth
[C]	current matrix
C	current, capacitance (with or without subscripts)
D	diode (with or without subscripts)
d	depth
d _p	change in the resistance of the pth strain gauge
[E]	matrix denoting coil voltages
e _p	voltage across the pth coil
F	force
f	stress psi, frequency c/s
G	air gap
[H]	matrix formed during computation
[I]	current matrix
I	moment of inertia of a section, current
i _p	current in a pth coil
J	moment of inertia
K	constant
k	number of poles per repeated section
L	inductance
ℓ	length
[M]	inductance matrix
m	number of coils in a system
N	number of bars
n	number of phases
p	number of poles
[R]	resistance matrix
R	radius, resistance
r	resistance (with a subscript)
[S]	matrix denoting coil resistances
SCR	silicon-controlled-rectifier
T	incremental time, torque (with a subscript)
TRC	time-ratio control
t	time from a reference

LIST OF SYMBOLS (contd.)

u, u_0	speed in radians per second
$[V]$	voltage matrix
v_p	voltage applied to a pth circuit
X	reactance
$[Y], [Z]$	inductance matrices
δ	deflection
$d \theta$	rotation of stator in radians
θ	position of rotor from a fixed stator reference
γ	angle of overlap
" , ' "	double and single dashes denote the values at the beginning and end of an increment.
Note:	the symbols are defined in each chapter as they occur.

PART 1

THEORETICAL INVESTIGATIONS

CHAPTER 1

INTRODUCTION

1.1 Statement of the problem

The complete solution for the performance of electrical machines may be defined as the calculation of currents, torque, input, output power, speed and rotor position with respect to time following (1) switching the supply on (2) switching it off (3) reswitching after a disconnection or (4) a sudden change in load or any other change in the operating conditions, until steady state is reached. The problem is quite involved and requires various idealisations before an analytical or numerical procedure can be applied. Progress in this field has consisted of the reduction of the idealisations necessary to perform the analysis with a consequent increase in the difficulty of analysis but with an increase in the quality of the predictions. In this process necessary assumptions have been made according to the availability of mathematical tools and methods in order to obtain solutions and these approximate solutions have formed the basis for improved machine design techniques.

In the following sections the system equations leading to a complete solution of the performance of an induction motor and a short survey of the existing analytical and numerical procedures and their limitations are discussed.

1.2 System equations for a slip-ring motor

If the machine has three coils on its stator and three on its rotor carrying independent currents, the general electrical equations for the system may be expressed in matrix form as

$$[V] = [I][R] + \frac{d}{dt} [MI] \quad \text{..... (1.1)}$$

with the equation for a typical pth circuit given by

$$V_p = i_p r_p + \frac{d}{dt} [M_{p1}i_1 + M_{p2}i_2 + M_{p3}i_3 + M_{p4}i_4 + M_{p5}i_5 + M_{p6}i_6]$$

where $[V]$, $[I]$, $[M]$ and $[R]$ are matrices denoting the voltages, currents, self and mutual inductances and the resistances of the circuits respectively. The inductances denoted by $[M]$ are functions of rotor position and are given by

$$M = f(\theta)$$

where θ is the angular position of the rotor reference from a fixed stator reference. The position of rotor is a function of time which clearly depends on the electrical and load torques and the moment of inertia of the rotating parts.

The electrical torque developed, T_e , may be expressed as a function of stator and rotor currents and the derivatives of the mutual inductances between the stator and rotor coils as follows,

$$T_e = \sum f(i_s, i_r, \frac{dM_{sr}}{d\theta}) \dots\dots\dots(1.2)$$

where M_{sr} is the mutual inductance between the s th stator coil and r th rotor coil. If the load (or braking) torque, T_b , is given as a function of time and speed namely

$$T_b = f(t, \frac{d\theta}{dt})$$

then the instantaneous acceleration, a , is given by,

$$a = \frac{T_e - T_b}{J} \dots\dots\dots(1.3)$$

where J is the moment of inertia of the rotating parts. If u_0 is the speed at time $t = 0$, then the speed, u , at any time, t , is given by

$$u = u_0 + \int_0^t a \, dt \dots\dots\dots(1.4)$$

and if θ_0 is the position of the rotor at time $t = 0$, the rotor position, θ , at any subsequent time, t , is given by

$$\theta = \theta_0 + \int_0^t u \, dt$$

Substituting for u and a in the above yields

$$\theta = \theta_o + u_o t + \int_0^t \int_0^t \frac{T_e - T_b}{J} dt dt \dots (1.5)$$

It may be observed that θ can not be eliminated from the equations (1.1) and (1.5) by substitution since it is a function of the derivative of M with respect to θ . Therefore (1.1) and (1.5) are the system equations to be solved in order to obtain a complete solution.

1.3 Existing methods of solving the system equations

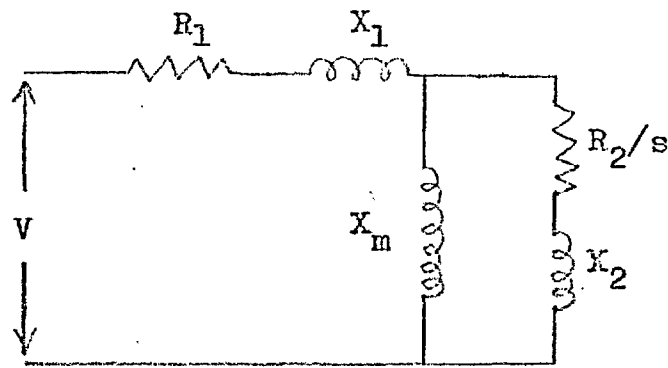
Performance calculations using analytical methods and numerical techniques may be classified in to two main categories namely, steady state and transient solutions. In order to obtain these solutions several idealisations are necessary. These are,

- (1) infinite permeability of iron
- (2) uniform air gap
- (3) no saturation in the magnetic circuit
- (4) the stator and rotor windings are distributed sinusoidally and the coupling coefficient between them vary sinusoidally with respect to the position of rotor, thus the harmonic fields produced by a practical windings are neglected.
- (5) applied voltages vary sinusoidally with time, and are balanced
- (6) the speed of the rotor is constant

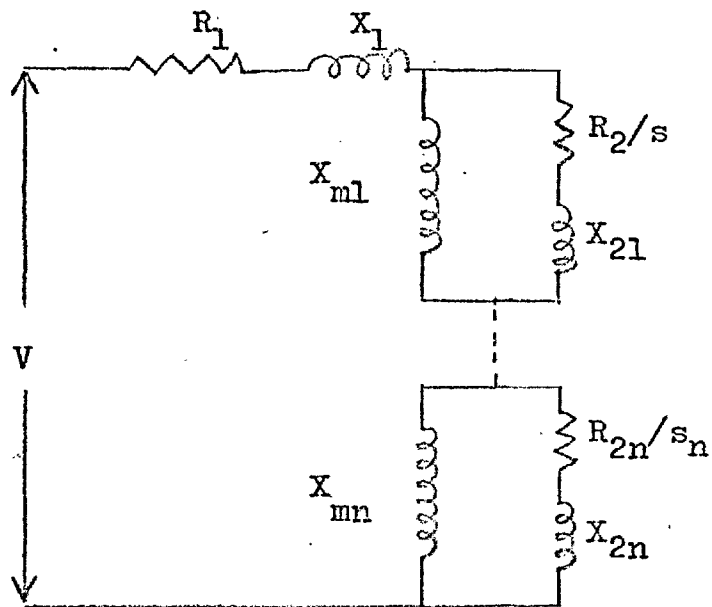
1.3.1 Steady state performance

In the simplest form of analysis all the assumptions cited above are made and the performance of a machine (with a symmetrically wound stator and rotor) may be calculated using the simple equivalent circuit¹ shown in figure 1.1.a. If the voltages are unbalanced, the theory of symmetrical components may be applied.

To account for space harmonics produced by a practical stator winding, the analysis may be extended by the use of the equivalent circuit shown



a. A simple equivalent circuit



b. An extended equivalent circuit of induction motor

figure 1.1

in figure 1.1.b. This eliminates the original assumption of sinusoidally distributed windings. Alternatively the leakage reactances associated with the simple equivalent circuit may be adjusted to include some of the effects of the harmonic flux distributions.

If the analysis is to be further extended to the case of asymmetrical stator windings and unbalanced sinusoidal voltages, theory of symmetrical components can not² be applied in a simple manner. For this case the generalised rotating field theory described³ and illustrated^{3,4} by Jha and Brown may be used if the rotor is symmetrical and the speed is constant.

The harmonic fields produced by the rotor currents are not taken into account in the solutions indicated so far. However, the equivalent circuit given by Alger¹ (page 338) may be used to calculate the performance of a symmetrical machine under steady state conditions when balanced sinusoidal voltages are applied and the speed is constant. Though the effect of the rotor harmonics are included the evaluation of the coefficients is difficult (as pointed out in the reference) thus affecting the accuracy.

When the applied voltages are balanced but non-sinusoidal, a harmonic analysis may be carried out on the input voltages and the performance of the machine supplied with each of the harmonics can then be calculated using the methods indicated in the above cases. The total torque developed by the machine is the sum of the torques due to individual harmonic voltages and the current⁵ is the square root of the sum of the rms value of currents due to all the harmonics.

A particular case of non-sinusoidal voltages has been considered by Ward⁶ and the performance of an induction motor at standstill and synchronous speed conditions has been calculated assuming idealised sinusoidally distributed windings having a sinusoidal coupling coefficient between the stator and the rotor, and step functions for the voltages.

The analytical solutions obtained so far do not calculate

pulsating torque components, and in practice the presence of these components due to, for example, rotor harmonics or non-sinusoidal voltage, vitiates the original assumption of constant speed conditions since the assumption that the speed is constant will not hold good unless the moment of inertia of the rotor is infinite. In practice the inertia is very small and the rotor undergoes appreciable accelerations and decelerations. Supposing for example, a first component of air gap field develops a steady torque when acting independently on the rotor and that a second component produces a pulsating torque under the same conditions. Then if both of them act simultaneously, the rotor has to follow the variation of torque due to the pulsating component and therefore, the steady component can not develop a steady torque because of the fluctuations in speed. Since the two fields act on a common rotor, it may be said that the fields are mechanically coupled in this way and this phenomenon is analogous to the mutual resistance coupling in squirrel cage rotors mentioned by Adkins⁷. Thus superposition of rotating magnetic fields is not strictly true unless the rotor speed is constant and it is considered that steady state conditions encountered in practice can be calculated strictly only by a method that takes speed variations into account.

No analytical method or numerical technique exists to solve the problem of non-sinusoidal voltages taking space harmonics, speed variation and unsymmetrical stator and rotor into account.

1.3.2 Transient solutions

Some basic concepts for the transient analysis of alternating current machines and circuits are outlined by Lyon⁸. In particular the instantaneous symmetrical components theory described in chapter 3 (in Lyon's book) can be used to solve polyphase circuits with no limitations on the waveform of the applied voltages. This theory is then applied in chapter 4 to an idealised machine, with sinusoidal coupling between the stator and rotor, rotating at a constant speed, and equations to calculate switching transients due to applied sinusoidal voltages are derived. Extensions to this theory are given in chapter 8 to include speed variations with the following additional assumptions

- (1) the effect of the stator resistance is negligible that is the total flux in the machine is proportional to the applied voltages.
- (2) the variation of slip with respect to time is known. For example, an abrupt change or a sinusoidal variation of slip may be assumed.

Transient torque patterns have been calculated by Slater⁹ for the case of a symmetrical machine supplied with balanced sinusoidal voltages, assuming a sinusoidal variation of mutual inductance between stator and rotor and using a transformation of rotor equations in stator terms. Evaluation of the parameters requires the presence of a true rotating field and the method is analogous to the simple equivalent circuit in figure 1.1.a used for the steady state solution of the idealised machine. While the method has the advantage of simplicity, the harmonic fluxes produced by the stator and the rotor currents are neglected since the rotor is replaced by equivalent phase coils. This is not justifiable during speed variations. Furthermore, the equivalent phase coils are assumed to be rigidly fixed to a rotor reference when applying the mechanical equations, this again is thought to be incorrect. In chapter 5 a rigorous analysis of squirrel cage and sheet rotors is given and the effect of replacing a cage rotor with phase coils is discussed in detail.

Tipping¹⁰ has calculated the currents in rotor coils which are switched on and off. The system is represented by a set of linear differential equations by assuming constant speed so that the inductances and their derivatives are known functions of time and the currents are calculated numerically using Runge Kutta method.

1.4 The object of the theoretical investigation

The object of the theoretical investigation may be stated as follows. To develop a general numerical method to calculate the complete performance of electrical machines, when both mechanical and electrical asymmetries are present in the stator and rotor, taking all the harmonic fluxes produced by the stator and rotor into account, and without any restrictions on the machine speed or the waveform of the applied voltages. It is also desirable that the

analysis shall be adaptable to any type of load coupled to the machine and it should have provision to take controlled switching and multiple transients such as reswitching or pulsating loads into account. The main objective given above has been accomplished in this work by adopting a numerical technique. The system equations are formed with the necessary assumptions and the solutions are made without further restrictions on the variables. The numerical solution can not be said to be exact in the mathematical sense, although the results can be obtained to any degree of accuracy according to the length of computer time allocated to a given solution.

1.5 The numerical procedure

The numerical technique which is a step-by-step procedure is developed from basic principles. Simple circuits are solved in steps until the case of slip ring motor is reached, this is later extended to squirrel cage motors. The illustrations of simple series circuits and a simple machine are given so that the number of variables involved is increased in each stage. Though the procedure can be used for solving the circuits illustrated, the object is only to develop the step-by-step procedure. Hence no simpler solutions available for these cases are mentioned and no advantage of the step-by-step procedure over analytical methods is claimed as far as these circuits are concerned.

The accuracy of the method is established in chapter 3 from the computed results for a simple series circuit. An indirect mathematical proof is also given in chapter 4 considering an ideal machine and comparing the steady state performance calculations using the step-by-step procedure with the values obtained from the exact equivalent circuit.

The various stages in computing the performance of a machine are the following:-

1. Calculation of the self and mutual inductances of all the coils in the machine at different rotor positions. A

procedure suitable for computation is described and illustrated in Appendices 1 and 2.

2. Formation of a set of differential equations and reduction of the number of equations to the number of independent variables in the system as illustrated in chapter 2 for the cases where the coils are independent and as shown in chapter 5 for the case of squirrel cage windings.
3. Calculation of the force between the stator and the rotor of a machine using one of the methods outlined in Appendix 4.
4. Solution for currents and torque with respect to time as described in detail in chapter 3 for the case of slip-ring
..... induction motors.

CHAPTER 2

FORMATION AND REDUCTION OF THE SYSTEM EQUATIONS

2.1 Introduction

If a machine having 'm' coils, grouped into 'n' phases is considered, 'm' number of differential equations may be written initially. The number of unknown currents and the known input voltages is n. (Some of these voltages may be zero as in the case of short-circuited secondary phases). Whilst the differential equations written are in terms of coils, the known input voltages are in terms of phases. Hence, in order to obtain a solution it is necessary to reduce the number of equations from m to n, with n unknown currents and this is illustrated firstly for the case of a two-pole two-phase machine and secondly for the case of a four-pole two-phase machine. The differential equations are first written in terms of coil-inductances and the number of equations is then reduced to the number of independent currents in the particular case. The equations are given in both matrix and expanded forms.

2.2 Two-pole two-phase slip-ring motor

The principal symbols are listed in table 2.1. The machine has 8 coils as shown in figure 2,a and there are two coils in series per phase. The self and mutual inductances of these coils are calculated using the procedures given in Appendices 1 and 2. The general differential equations are most conveniently displayed in a tabular form in table 2.2. The corresponding matrix form is given by

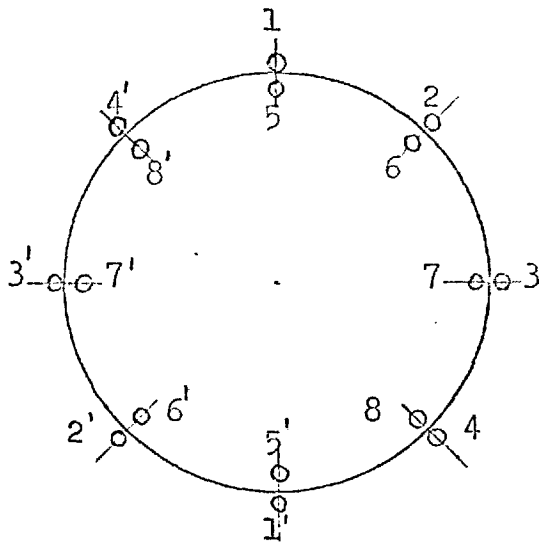
$$[E] = [C] [S] + \frac{d}{dt} ([Z] [C]) \dots\dots\dots (2.1)$$

In the case of the two pole winding, coils 1 and 2 are in series to form phase 1 and carry the phase 1 current. The phase voltage and resistance are given by the sum of the coil voltages and resistances respectively. Therefore the following relationships may be written

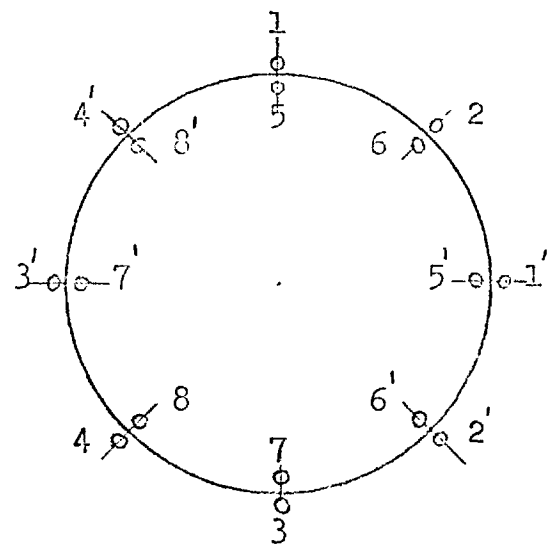
$$C_1 = C_2 = i_1, v_1 = e_1 + e_2 \text{ and } r_1 = s_1 + s_2$$

T A B L E 2.1

Matrix	Size	The elements represent
Z	8 x 8	The self and mutual inductances of the eight coils in the machine.
M	4 x 4	The self and mutual inductances of the four phases
C	8	Instantaneous currents in the eight coils
I	4	Instantaneous phase currents
E	8	Instantaneous voltages across individual coils
V	4	Instantaneous phase voltages
S	8	Coil resistances
R	4	Phase resistances



(a) two pole winding



(b) four pole winding

(c) phase coils of a three phase winding

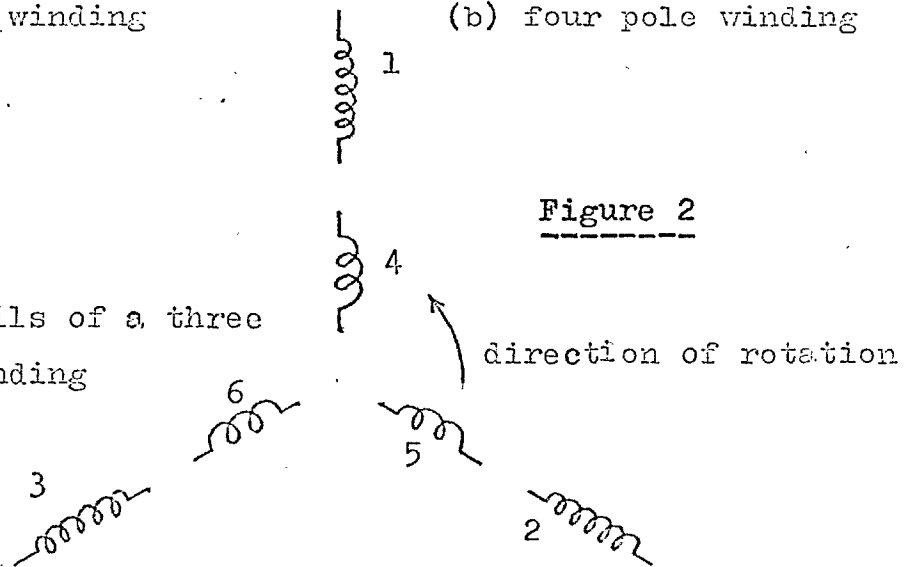
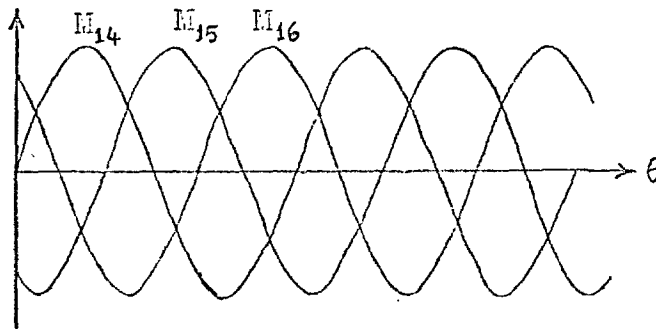


Figure 2



(d) variation of mutual inductances with rotor position

T A B L E 2.2

$e_1 = c_1 s_1 + \frac{d}{dt}$	$\left[z_{11} c_1 + z_{12} c_2 + z_{13} c_3 + z_{14} c_4 + z_{15} c_5 + z_{16} c_6 + z_{17} c_7 + z_{18} c_8 \right]$	(a)
$e_2 = c_2 s_2 + \frac{d}{dt}$	$\left[z_{21} c_1 + z_{22} c_2 + z_{23} c_3 + z_{24} c_4 + z_{25} c_5 + z_{26} c_6 + z_{27} c_7 + z_{28} c_8 \right]$	(b)
$e_3 = c_3 s_3 + \frac{d}{dt}$	$\left[z_{31} c_1 + z_{32} c_2 + z_{33} c_3 + z_{34} c_4 + z_{35} c_5 + z_{36} c_6 + z_{37} c_7 + z_{38} c_8 \right]$	(c)
$e_4 = c_4 s_4 + \frac{d}{dt}$	$\left[z_{41} c_1 + z_{42} c_2 + z_{43} c_3 + z_{44} c_4 + z_{45} c_5 + z_{46} c_6 + z_{47} c_7 + z_{48} c_8 \right]$	(d)
$e_5 = c_5 s_5 + \frac{d}{dt}$	$\left[z_{51} c_1 + z_{52} c_2 + z_{53} c_3 + z_{54} c_4 + z_{55} c_5 + z_{56} c_6 + z_{57} c_7 + z_{58} c_8 \right]$	(e)
$e_6 = c_6 s_6 + \frac{d}{dt}$	$\left[z_{61} c_1 + z_{62} c_2 + z_{63} c_3 + z_{64} c_4 + z_{65} c_5 + z_{66} c_6 + z_{67} c_7 + z_{68} c_8 \right]$	(f)
$e_7 = c_7 s_7 + \frac{d}{dt}$	$\left[z_{71} c_1 + z_{72} c_2 + z_{73} c_3 + z_{74} c_4 + z_{75} c_5 + z_{76} c_6 + z_{77} c_7 + z_{78} c_8 \right]$	(g)
$e_8 = c_8 s_8 + \frac{d}{dt}$	$\left[z_{81} c_1 + z_{82} c_2 + z_{83} c_3 + z_{84} c_4 + z_{85} c_5 + z_{86} c_6 + z_{87} c_7 + z_{88} c_8 \right]$	(h)

Now, adding the first two equations (corresponding to the coils in series in phase 1) and substituting the relationships given above the equation for phase 1 is derived as

$$v_1 = i_1 r_1 + \frac{d}{dt} [M_{11}i_1 + M_{12}i_2 + M_{13}i_3 + M_{14}i_4] \quad \dots(2.2.a)$$

where

$$\left. \begin{aligned} M_{11} &= Z_{11} + Z_{12} + Z_{21} + Z_{22} \\ M_{12} &= Z_{13} + Z_{14} + Z_{23} + Z_{24} \\ M_{13} &= Z_{15} + Z_{16} + Z_{25} + Z_{26} \\ M_{14} &= Z_{17} + Z_{18} + Z_{27} + Z_{28} \end{aligned} \right\}$$

Similarly, the equations for the second stator phase and the two rotor phases may be derived as,

$$v_2 = i_2 r_2 + \frac{d}{dt} [M_{21}i_1 + M_{22}i_2 + M_{23}i_3 + M_{24}i_4] \quad \dots(2.2.b)$$

$$v_3 = i_3 r_3 + \frac{d}{dt} [M_{31}i_1 + M_{32}i_2 + M_{33}i_3 + M_{34}i_4] \quad \dots(2.2.c)$$

and

$$v_4 = i_4 r_4 + \frac{d}{dt} [M_{41}i_1 + M_{42}i_2 + M_{43}i_3 + M_{44}i_4] \quad \dots(2.2.d)$$

respectively.

The coefficients denoted by matrix $[M]$ are given by the sum of the appropriate inductances in the original matrix $[Z]$, corresponding to the coils in series and they appear in the squares shown in table 2.2. For example, the sum of the inductances in the first square gives M_{11} . Equations (2.2) form the general electrical equations for the system considered.

2.3 Four-pole two-phase slip-ring motor

The coils are shown in figure 2.b. The reduction of general equations differs from the two pole case due to the difference in connection. In this case coils 1 and 3 are in series to form phase 1. Similarly coils 2 and 4, 5 and 7, and 6 and 8 form phases 2, 3 and 4 respectively. Using the same notations as in the previous case the

following relationships may be written,

$$\begin{array}{lll} v_1 = e_1 + e_3 & i_1 = c_1 = c_3 & r_1 = s_1 + s_3 \\ v_2 = e_2 + e_4 & i_2 = c_2 = c_4 & r_2 = s_2 + s_4 \\ v_3 = e_5 + e_7 & i_3 = c_5 = c_7 & r_3 = s_5 + s_7 \\ v_4 = e_6 + e_8 & i_4 = c_6 = c_8 & r_4 = s_6 + s_8 \end{array}$$

The phase 1 equation is obtained by adding equations (2.1.a) and (2.1.c) and substituting the relationships given above. This yields the coefficients given by

$$M_{11} = Z_{11} + Z_{13} + Z_{31} + Z_{33}$$

$$M_{12} = Z_{12} + Z_{14} + Z_{32} + Z_{34}$$

$$M_{13} = Z_{15} + Z_{17} + Z_{35} + Z_{37}$$

and

$$M_{14} = Z_{16} + Z_{18} + Z_{36} + Z_{38}$$

Similarly, the other elements of matrix $[M]$ may be found by summing up the relevant inductances in matrix $[Z]$.

Since the mutual inductances between stator and rotor phases vary with rotor position, the mutual inductances between coils on stator and rotor in matrix $[Z]$ have to be calculated before each summation to obtain matrix $[M]$.

2.4 Four-pole three-phase slip-ring motor

The motor used for the experimental investigation has 36 stator slots and 52 rotor slots carrying three-phase single-layer windings. There are 48 coils in all, 18 on the stator and 30 on the rotor. The self and mutual inductances between the coils may be calculated using the procedure given in Appendices 1 and 2, and the coefficients for the simultaneous differential equations of the phases are formed by

summation of the appropriate inductances as illustrated in sections 2.2 and 2.3. The phase coils and the variation of mutual inductance between them with respect to the rotor position are shown in figures 2.c and d respectively. The simultaneous differential equations are given by:

$$\begin{aligned} v_1 &= i_1 r_1 + \frac{d}{dt} [M_{11} i_1 + M_{12} i_2 + M_{13} i_3 + M_{14} i_4 + M_{15} i_5 + M_{16} i_6] \\ v_2 &= i_2 r_2 + \frac{d}{dt} [M_{21} i_1 + M_{22} i_2 + M_{23} i_3 + M_{24} i_4 + M_{25} i_5 + M_{26} i_6] \\ v_3 &= i_3 r_3 + \frac{d}{dt} [M_{31} i_1 + M_{32} i_2 + M_{33} i_3 + M_{34} i_4 + M_{35} i_5 + M_{36} i_6] \\ v_4 &= i_4 r_4 + \frac{d}{dt} [M_{41} i_1 + M_{42} i_2 + M_{43} i_3 + M_{44} i_4 + M_{45} i_5 + M_{46} i_6] \\ v_5 &= i_5 r_5 + \frac{d}{dt} [M_{51} i_1 + M_{52} i_2 + M_{53} i_3 + M_{54} i_4 + M_{55} i_5 + M_{56} i_6] \\ v_6 &= i_6 r_6 + \frac{d}{dt} [M_{61} i_1 + M_{62} i_2 + M_{63} i_3 + M_{64} i_4 + M_{65} i_5 + M_{66} i_6] \end{aligned}$$

This may be written in matrix form as follows

$$V = [I] [R] + \frac{d}{dt} \left\{ [M] [I] \right\}$$

In the above equations v_4 , v_5 and v_6 refer to the externally applied rotor voltages and are zero in the case of a short-circuited rotor.

2.5 Leakage Inductances

In calculating the self and mutual inductances, only the fluxes, that cross the air-gap and link both the stator and rotor coils, are taken into account. However, in practice the phase currents establish fluxes that do not link the coils in other phases in the machine. These fluxes take two different magnetic paths and are usually named according to the path taken as slot and zig-zag leakage fluxes. The flux that takes paths across the slot without crossing the air-gap is known as the slot leakage flux. The flux that crosses the air-gap from one side of a slot and comes to the other side through the air-gap and toothtops, on the opposite side of the gap, is known as the zig-zag leakage flux. These two fluxes

do not link coils other than the ones producing them.

The leakage reactances used in the analytical methods depend on the particular idealisations made. For instance, whilst using the simple equivalent circuit shown in figure 1.1.a, the flux linkage due to the space harmonic fluxes are considered as leakage fluxes¹ in addition to the actual leakage fluxes in the machine. Similarly, whilst using the extended equivalent circuit shown in figure 1.1.b, the leakage reactances used, include the harmonic fluxes that are not taken into account in the circuit. These leakage reactances are usually called differential leakage reactances.

However, in the numerical analysis developed in this thesis, the value of the inductances used refer to the total flux linkages per ampere, rather than their components. Therefore, only the true leakage inductances are added to the self inductances of the coils. That is, the differential leakage is included in the self and mutual inductances calculated using the methods described in Appendices 1 and 2.

Calculation of slot and zig-zag leakage inductances is found in reference 1. The slot-leakage in the case of squirrel cage rotors depends on the frequency of the slot current (or more exactly on the magnitude and rate of change of current). A general method of calculating the impedance of a rotor bar at any frequency is given in Appendix 3.

CHAPTER 3

THE STEP-BY-STEP PROCEDURE

3.1 Introduction

In this chapter a step-by-step procedure for calculating the performance of electrical systems using numerical techniques is developed from fundamental principles. A simple circuit with a resistance in series with an inductance is first considered to illustrate the calculation of current with respect to time. This work is then extended to the case of a simple machine with two coils, the differential equations of the system being non-linear. The general solution for the case of a practical machine and the necessary computer program are then described.

3.2 Solution of a resistance and inductance series circuit

The circuit shown in figure 3.1.a has a resistance, r , in series with an inductance, L . The differential equation for the circuit is

$$v = ir + L \frac{di}{dt} \quad \text{..... (3.1)}$$

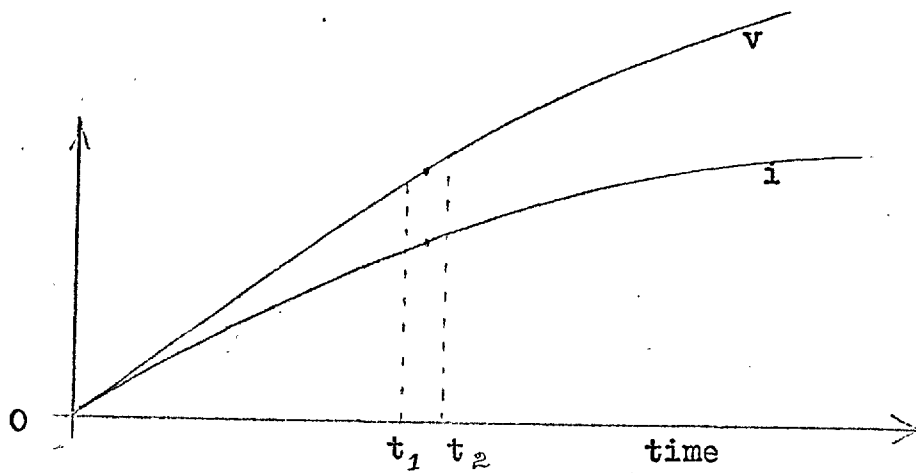
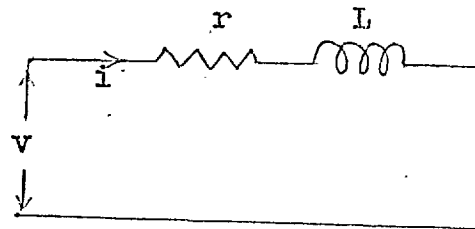
where v and i are the voltage and current at any time ' t ' as shown in figure 3.1.b. The solution for the variation of current with time on applying a voltage of any waveform may be obtained as follows.

An incremental time of T is considered at time t . Let t_1 and t_2 be the times, i_1 and i_2 the currents and v_1 and v_2 the voltages at the beginning and end of the increment. The incremental time T is taken to be short enough so that the voltage and current can be considered to vary linearly during the interval. That is, the average values of voltage and current are given by v and i where

$$v = (v_1 + v_2)/2$$

$$i = (i_1 + i_2)/2$$

(a) A simple series circuit



(b) Instantaneous voltage and current

figure 3.1

and
$$\frac{di}{dt} = \frac{i_2 - i_1}{T}$$

Substituting these values in (1)

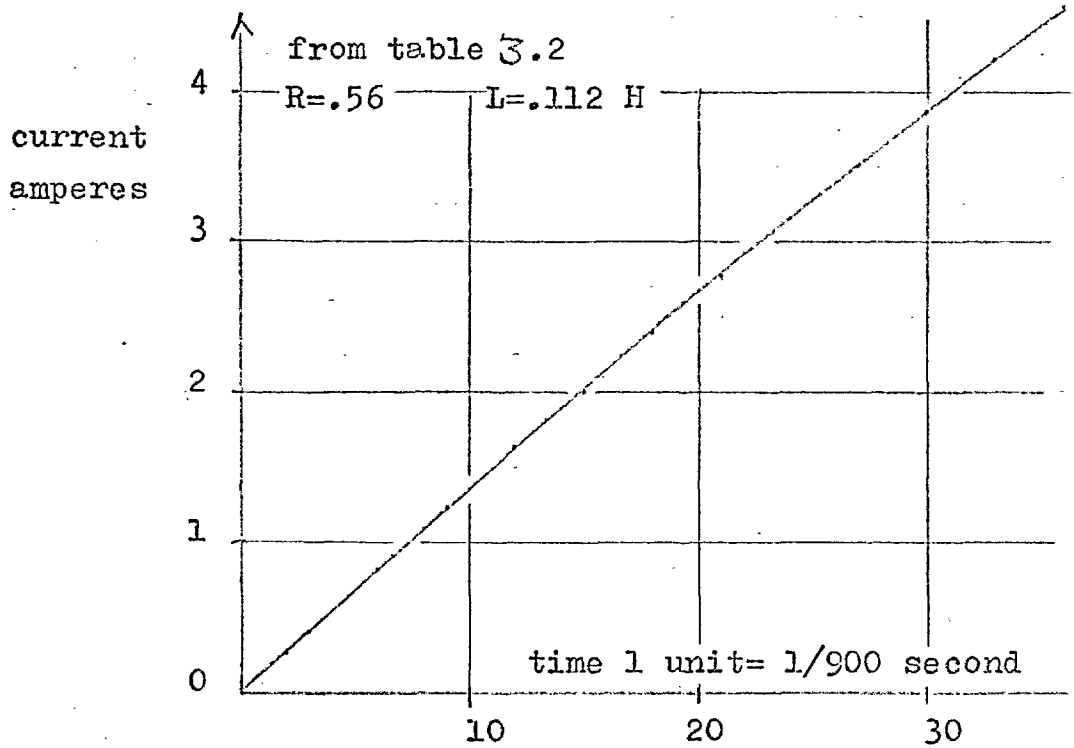
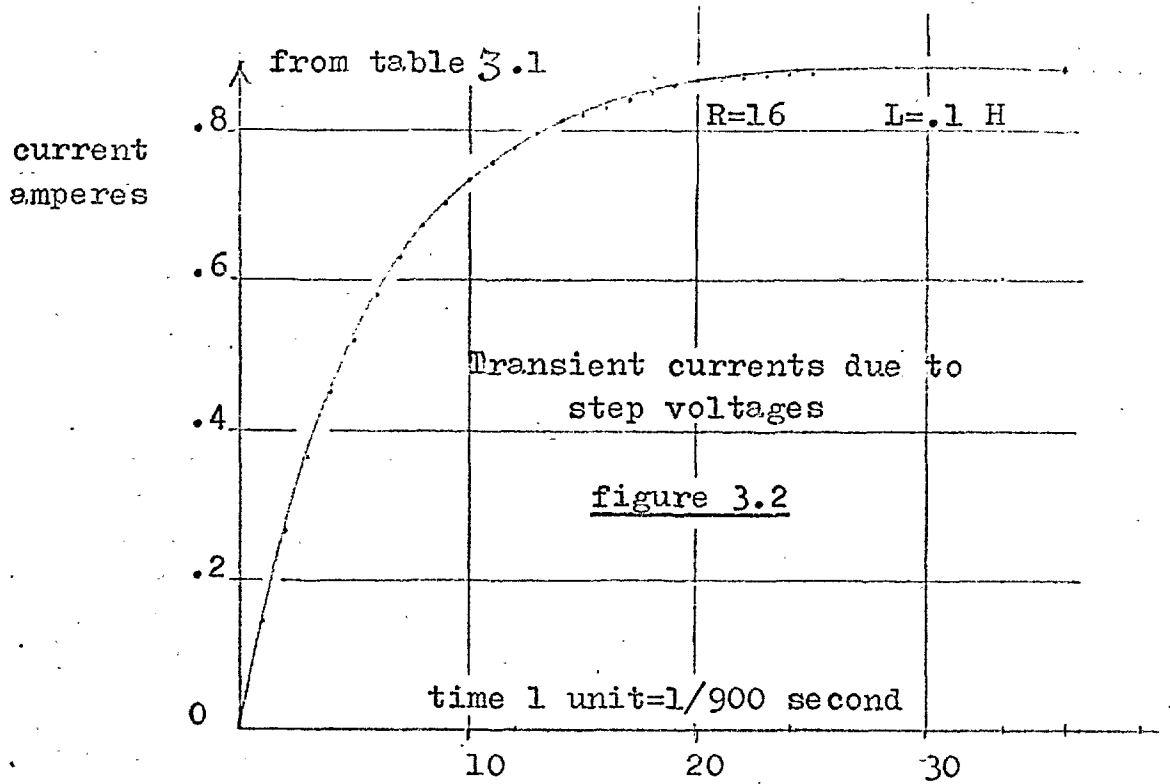
$$v = (i_1 + i_2) \frac{r}{2} + L \frac{(i_2 - i_1)}{T}$$

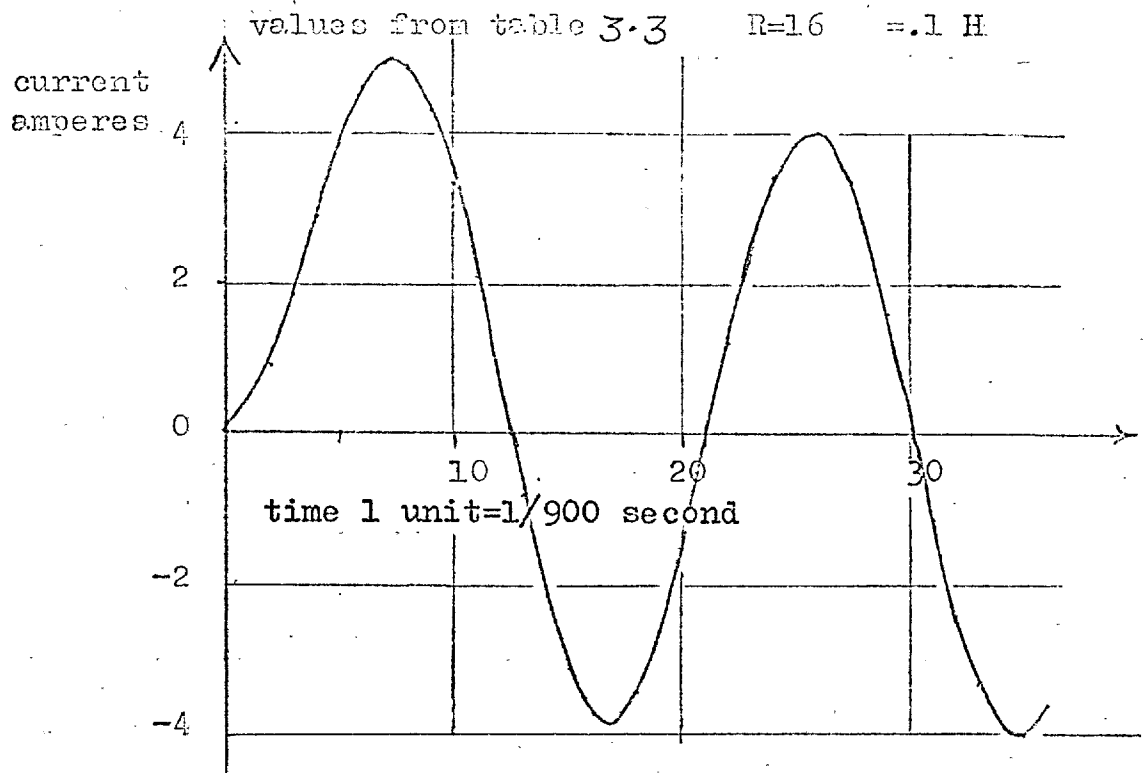
i.e.
$$v - \left(\frac{r}{2} - \frac{L}{T}\right) i_1 = \left(\frac{r}{2} + \frac{L}{T}\right) i_2 \quad \dots\dots\dots (3.2)$$

Assuming the current to be zero at the moment of switching, that is $i_1 = 0$, the current at the end of the first increment may be calculated using equation (3.2). This forms the initial value for the second increment. This procedure may be continued until steady state conditions are achieved. When the applied voltage is d.c. the calculation is continued until a constant current is reached. If an a.c. voltage is applied, the calculation is continued until the current waveform repeats in consecutive cycles of the voltage.

3.3. Incremental time T

The main factors affecting the choice of the incremental time are firstly the waveform of the input voltage and secondly the time constants of the circuits. A step input voltage and the transient currents following it, may be considered as an extreme case and an increment which gives accurate results for this case is therefore thought to be suitable for most other cases. In the following computer study to produce evidence as to the accuracy of the method and the choice of the incremental time, step and sinusoidal input voltages are used. The parameters which determine the time constant of the circuits are chosen to have values encountered in practice. For example, two sets of series inductance and resistance are chosen so that the two time constants are the maximum and minimum values calculated from the equivalent circuit of an inductance motor. In the following pages computer outputs are shown graphically in figures 3.2, 3.3 and 3.4, and in tabular form in tables 3.1 to 3.6 with appropriate details. The numerical values are given since the differences are too small to be observed on graphs. The results and salient points are discussed below.





Transient currents due to sinusoidal applied voltages

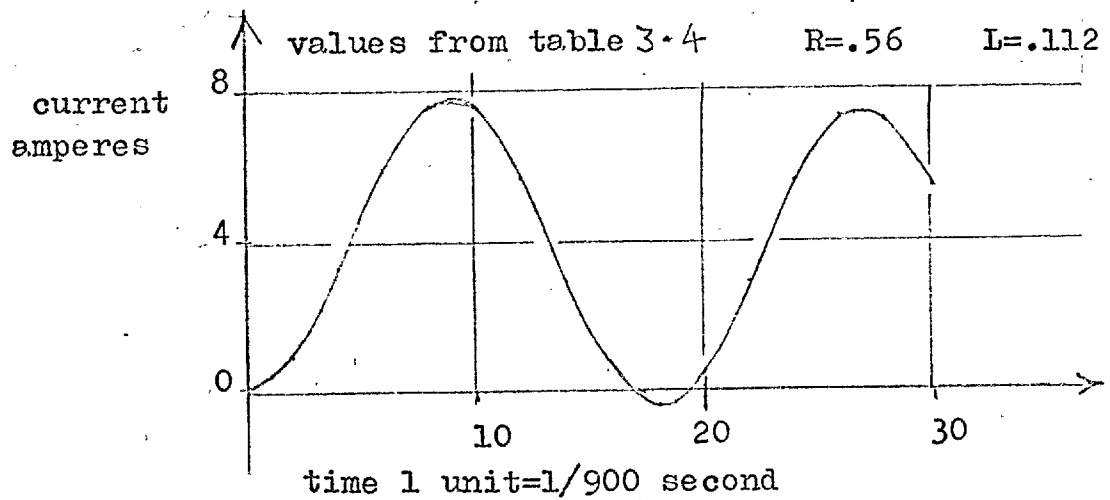
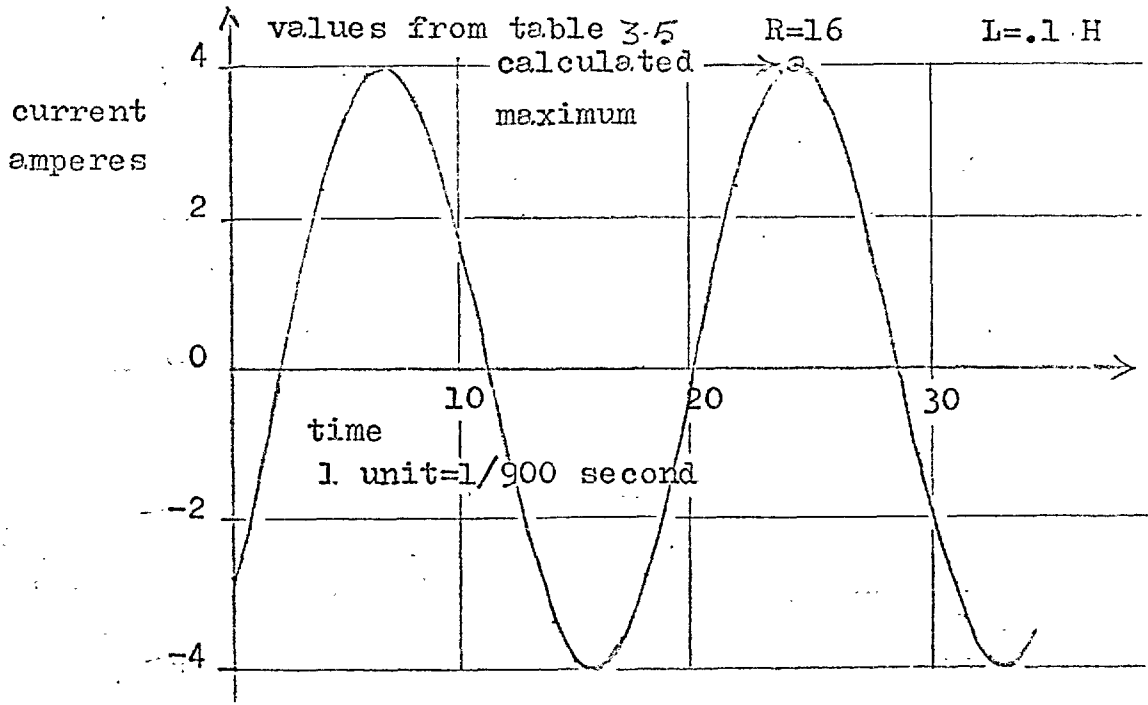
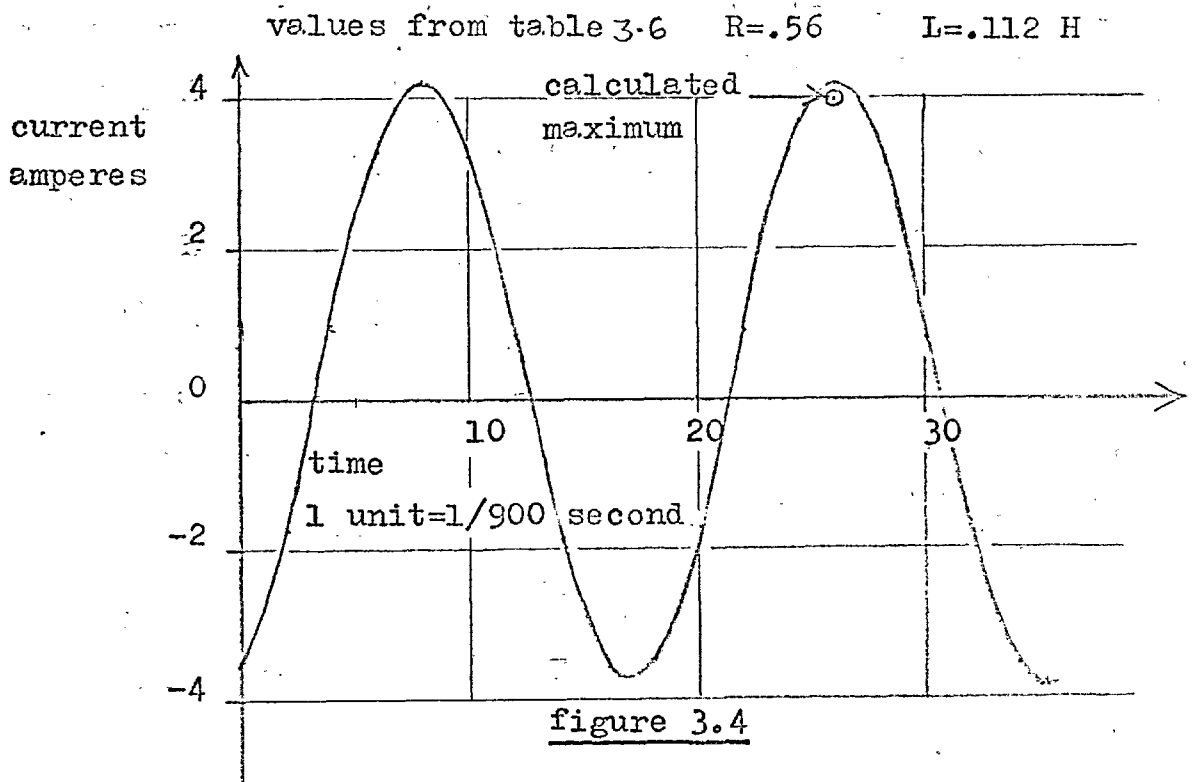


figure 3.3



Steady state currents with sinusoidal applied voltages
(29th and 30th cycles)



EFFECT OF INTERVAL T

R=16.000

L= 0.100

1/ 900.	1/1800.	1/2700.	1/3600.	1/4500.
0.1443E 00	0.1440E 00	0.1440E 00	0.1440E 00	0.1440E 00
0.2650E 00	0.2646E 00	0.2645E 00	0.2645E 00	0.2645E 00
0.3660E 00	0.3655E 00	0.3654E 00	0.3653E 00	0.3653E 00
0.4506E 00	0.4499E 00	0.4498E 00	0.4498E 00	0.4498E 00
0.5213E 00	0.5206E 00	0.5205E 00	0.5205E 00	0.5205E 00
0.5805E 00	0.5798E 00	0.5797E 00	0.5797E 00	0.5796E 00
0.6306E 00	0.6293E 00	0.6292E 00	0.6292E 00	0.6292E 00
0.6714E 00	0.6708E 00	0.6707E 00	0.6707E 00	0.6706E 00
0.7061E 00	0.7055E 00	0.7054E 00	0.7054E 00	0.7054E 00
0.7351E 00	0.7346E 00	0.7345E 00	0.7344E 00	0.7344E 00
0.7594E 00	0.7589E 00	0.7588E 00	0.7588E 00	0.7587E 00
0.7797E 00	0.7792E 00	0.7791E 00	0.7791E 00	0.7791E 00
0.7967E 00	0.7963E 00	0.7962E 00	0.7962E 00	0.7961E 00
0.8109E 00	0.8105E 00	0.8105E 00	0.8104E 00	0.8104E 00
0.8228E 00	0.8225E 00	0.8224E 00	0.8224E 00	0.8224E 00
0.8327E 00	0.8324E 00	0.8324E 00	0.8324E 00	0.8324E 00
0.8411E 00	0.8408E 00	0.8408E 00	0.8407E 00	0.8407E 00
0.8480E 00	0.8478E 00	0.8478E 00	0.8477E 00	0.8477E 00
0.8539E 00	0.8537E 00	0.8536E 00	0.8536E 00	0.8536E 00
0.8587E 00	0.8586E 00	0.8585E 00	0.8585E 00	0.8585E 00
0.8628E 00	0.8627E 00	0.8626E 00	0.8626E 00	0.8626E 00
0.8662E 00	0.8661E 00	0.8661E 00	0.8661E 00	0.8661E 00
0.8691E 00	0.8690E 00	0.8690E 00	0.8689E 00	0.8689E 00
0.8715E 00	0.8714E 00	0.8714E 00	0.8714E 00	0.8714E 00
0.8735E 00	0.8734E 00	0.8734E 00	0.8734E 00	0.8734E 00
0.8752E 00	0.8751E 00	0.8751E 00	0.8751E 00	0.8751E 00
0.8766E 00	0.8765E 00	0.8765E 00	0.8765E 00	0.8765E 00
0.8777E 00	0.8777E 00	0.8777E 00	0.8777E 00	0.8777E 00
0.8787E 00	0.8787E 00	0.8787E 00	0.8787E 00	0.8787E 00
0.8795E 00	0.8795E 00	0.8795E 00	0.8795E 00	0.8795E 00
0.8802E 00	0.8802E 00	0.8802E 00	0.8802E 00	0.8802E 00
0.8808E 00	0.8808E 00	0.8808E 00	0.8808E 00	0.8808E 00
0.8813E 00	0.8813E 00	0.8813E 00	0.8812E 00	0.8812E 00
0.8817E 00	0.8817E 00	0.8817E 00	0.8817E 00	0.8817E 00
0.8820E 00	0.8820E 00	0.8820E 00	0.8820E 00	0.8820E 00
0.8823E 00	0.8823E 00	0.8823E 00	0.8823E 00	0.8823E 00

Table 3.1

EFFECT OF INTERVAL T

R = 0.560

L = 0.112

1/900.	1/1800.	1/2700.	1/3600.	1/4500.
0.1399E 00	0.1399E 00	0.1399E 00	0.1399E 00	0.1399E 00
0.2790E 00	0.2790E 00	0.2790E 00	0.2790E 00	0.2790E 00
0.4173E 00	0.4173E 00	0.4173E 00	0.4173E 00	0.4173E 00
0.5549E 00	0.5549E 00	0.5549E 00	0.5549E 00	0.5549E 00
0.6917E 00	0.6917E 00	0.6917E 00	0.6917E 00	0.6917E 00
0.8278E 00	0.8278E 00	0.8278E 00	0.8278E 00	0.8278E 00
0.9631E 00	0.9631E 00	0.9631E 00	0.9631E 00	0.9631E 00
0.1098E 01	0.1098E 01	0.1098E 01	0.1098E 01	0.1098E 01
0.1231E 01	0.1231E 01	0.1231E 01	0.1231E 01	0.1231E 01
0.1365E 01	0.1365E 01	0.1365E 01	0.1365E 01	0.1365E 01
0.1497E 01	0.1497E 01	0.1497E 01	0.1497E 01	0.1497E 01
0.1628E 01	0.1628E 01	0.1628E 01	0.1628E 01	0.1628E 01
0.1759E 01	0.1759E 01	0.1759E 01	0.1759E 01	0.1759E 01
0.1889E 01	0.1889E 01	0.1889E 01	0.1889E 01	0.1889E 01
0.2019E 01	0.2019E 01	0.2019E 01	0.2019E 01	0.2019E 01
0.2148E 01	0.2148E 01	0.2148E 01	0.2148E 01	0.2148E 01
0.2276E 01	0.2276E 01	0.2276E 01	0.2276E 01	0.2276E 01
0.2403E 01	0.2403E 01	0.2403E 01	0.2403E 01	0.2403E 01
0.2529E 01	0.2529E 01	0.2529E 01	0.2529E 01	0.2529E 01
0.2655E 01	0.2655E 01	0.2655E 01	0.2655E 01	0.2655E 01
0.2780E 01	0.2780E 01	0.2780E 01	0.2780E 01	0.2780E 01
0.2905E 01	0.2905E 01	0.2905E 01	0.2905E 01	0.2905E 01
0.3029E 01	0.3029E 01	0.3029E 01	0.3029E 01	0.3029E 01
0.3152E 01	0.3152E 01	0.3152E 01	0.3152E 01	0.3152E 01
0.3274E 01	0.3274E 01	0.3274E 01	0.3274E 01	0.3274E 01
0.3396E 01	0.3396E 01	0.3396E 01	0.3396E 01	0.3396E 01
0.3517E 01	0.3517E 01	0.3517E 01	0.3517E 01	0.3517E 01
0.3638E 01	0.3638E 01	0.3638E 01	0.3638E 01	0.3638E 01
0.3757E 01	0.3757E 01	0.3757E 01	0.3757E 01	0.3757E 01
0.3876E 01	0.3876E 01	0.3876E 01	0.3876E 01	0.3876E 01
0.3995E 01	0.3995E 01	0.3995E 01	0.3995E 01	0.3995E 01
0.4113E 01	0.4113E 01	0.4113E 01	0.4113E 01	0.4113E 01
0.4230E 01	0.4230E 01	0.4230E 01	0.4230E 01	0.4230E 01
0.4346E 01	0.4346E 01	0.4346E 01	0.4346E 01	0.4346E 01
0.4462E 01	0.4462E 01	0.4462E 01	0.4462E 01	0.4462E 01
0.4577E 01	0.4577E 01	0.4577E 01	0.4577E 01	0.4577E 01

Table 3.2

EFFECT OF INTERVAL T

R = 0.560

L = 0.112

1/ 900.	1/1800.	1/2700.	1/3600.	1/4500.
0.2429E 00	0.2422E 00	0.2420E 00	0.2420E 00	0.2419E 00
0.9410E 00	0.9377E 00	0.9371E 00	0.9369E 00	0.9368E 00
0.2007E 01	0.2000E 01	0.1999E 01	0.1998E 01	0.1998E 01
0.3311E 01	0.3299E 01	0.3296E 01	0.3296E 01	0.3295E 01
0.4691E 01	0.4674E 01	0.4671E 01	0.4670E 01	0.4669E 01
0.5980E 01	0.5958E 01	0.5953E 01	0.5952E 01	0.5951E 01
0.7018E 01	0.6992E 01	0.6987E 01	0.6985E 01	0.6985E 01
0.7679E 01	0.7650E 01	0.7645E 01	0.7643E 01	0.7642E 01
0.7879E 01	0.7849E 01	0.7844E 01	0.7842E 01	0.7841E 01
0.7593E 01	0.7564E 01	0.7558E 01	0.7556E 01	0.7556E 01
0.6851E 01	0.6825E 01	0.6820E 01	0.6818E 01	0.6817E 01
0.5742E 01	0.5719E 01	0.5715E 01	0.5714E 01	0.5713E 01
0.4395E 01	0.4378E 01	0.4375E 01	0.4374E 01	0.4373E 01
0.2972E 01	0.2960E 01	0.2958E 01	0.2957E 01	0.2957E 01
0.1641E 01	0.1634E 01	0.1633E 01	0.1633E 01	0.1633E 01
0.5604E 00	0.5580E 00	0.5575E 00	0.5573E 00	0.5573E 00
-0.1421E 00	-0.1418E 00	-0.1417E 00	-0.1417E 00	-0.1417E 00
-0.3843E 00	-0.3828E 00	-0.3826E 00	-0.3825E 00	-0.3824E 00
-0.1392E 00	-0.1385E 00	-0.1384E 00	-0.1384E 00	-0.1383E 00
0.5610E 00	0.5592E 00	0.5588E 00	0.5587E 00	0.5586E 00
0.1629E 01	0.1624E 01	0.1623E 01	0.1622E 01	0.1622E 01
0.2935E 01	0.2924E 01	0.2922E 01	0.2922E 01	0.2921E 01
0.4318E 01	0.4302E 01	0.4299E 01	0.4298E 01	0.4297E 01
0.5608E 01	0.5587E 01	0.5583E 01	0.5582E 01	0.5581E 01
0.6649E 01	0.6624E 01	0.6619E 01	0.6618E 01	0.6617E 01
0.7311E 01	0.7284E 01	0.7279E 01	0.7277E 01	0.7276E 01
0.7514E 01	0.7485E 01	0.7480E 01	0.7478E 01	0.7477E 01
0.7229E 01	0.7202E 01	0.7196E 01	0.7195E 01	0.7194E 01
0.6490E 01	0.6465E 01	0.6460E 01	0.6458E 01	0.6458E 01
0.5382E 01	0.5361E 01	0.5357E 01	0.5356E 01	0.5355E 01
0.4038E 01	0.4022E 01	0.4019E 01	0.4018E 01	0.4018E 01
0.2617E 01	0.2606E 01	0.2604E 01	0.2604E 01	0.2603E 01
0.1288E 01	0.1282E 01	0.1281E 01	0.1281E 01	0.1281E 01
0.2088E 00	0.2077E 00	0.2075E 00	0.2074E 00	0.2074E 00
-0.4918E 00	-0.4901E 00	-0.4898E 00	-0.4897E 00	-0.4896E 00
-0.7320E 00	-0.7292E 00	-0.7287E 00	-0.7285E 00	-0.7284E 00

Table 3.3

EFFECT OF INTERVAL T

R=16.000

L= 0.100

1/ 900.	1/1800.	1/2700.	1/3600.	1/4500.
0.2505E 00	0.2546E 00	0.2554E 00	0.2555E 00	0.2558E 00
0.9311E 00	0.9354E 00	0.9361E 00	0.9364E 00	0.9365E 00
0.1884E 01	0.1886E 01	0.1886E 01	0.1886E 01	0.1886E 01
0.2933E 01	0.2928E 01	0.2928E 01	0.2927E 01	0.2927E 01
0.3897E 01	0.3886E 01	0.3884E 01	0.3883E 01	0.3883E 01
0.4616E 01	0.4599E 01	0.4596E 01	0.4595E 01	0.4594E 01
0.4968E 01	0.4945E 01	0.4941E 01	0.4939E 01	0.4939E 01
0.4878E 01	0.4852E 01	0.4847E 01	0.4845E 01	0.4844E 01
0.4332E 01	0.4305E 01	0.4300E 01	0.4298E 01	0.4297E 01
0.3374E 01	0.3348E 01	0.3344E 01	0.3342E 01	0.3341E 01
0.2102E 01	0.2081E 01	0.2077E 01	0.2075E 01	0.2075E 01
0.6536E 00	0.6388E 00	0.6361E 00	0.6352E 00	0.6347E 00
-0.8089E 00	-0.8154E 00	-0.8166E 00	-0.8170E 00	-0.8172E 00
-0.2120E 01	-0.2117E 01	-0.2117E 01	-0.2117E 01	-0.2117E 01
-0.3129E 01	-0.3119E 01	-0.3117E 01	-0.3116E 01	-0.3116E 01
-0.3724E 01	-0.3706E 01	-0.3703E 01	-0.3701E 01	-0.3701E 01
-0.3837E 01	-0.3814E 01	-0.3810E 01	-0.3809E 01	-0.3808E 01
-0.3461E 01	-0.3436E 01	-0.3432E 01	-0.3430E 01	-0.3430E 01
-0.2646E 01	-0.2622E 01	-0.2617E 01	-0.2616E 01	-0.2615E 01
-0.1492E 01	-0.1472E 01	-0.1469E 01	-0.1467E 01	-0.1467E 01
-0.1434E 00	-0.1297E 00	-0.1272E 00	-0.1263E 00	-0.1259E 00
0.1236E 01	0.1242E 01	0.1243E 01	0.1243E 01	0.1243E 01
0.2477E 01	0.2474E 01	0.2473E 01	0.2473E 01	0.2473E 01
0.3428E 01	0.3417E 01	0.3415E 01	0.3414E 01	0.3414E 01
0.3974E 01	0.3956E 01	0.3952E 01	0.3951E 01	0.3951E 01
0.4047E 01	0.4024E 01	0.4019E 01	0.4018E 01	0.4017E 01
0.3636E 01	0.3612E 01	0.3607E 01	0.3605E 01	0.3605E 01
0.2792E 01	0.2768E 01	0.2764E 01	0.2762E 01	0.2762E 01
0.1615E 01	0.1595E 01	0.1591E 01	0.1590E 01	0.1589E 01
0.2460E 00	0.2324E 00	0.2299E 00	0.2290E 00	0.2286E 00
-0.1150E 01	-0.1156E 01	-0.1157E 01	-0.1157E 01	-0.1157E 01
-0.2405E 01	-0.2402E 01	-0.2401E 01	-0.2401E 01	-0.2401E 01
-0.3368E 01	-0.3357E 01	-0.3355E 01	-0.3354E 01	-0.3354E 01
-0.3924E 01	-0.3905E 01	-0.3902E 01	-0.3901E 01	-0.3900E 01
-0.4005E 01	-0.3981E 01	-0.3977E 01	-0.3976E 01	-0.3975E 01
-0.3601E 01	-0.3576E 01	-0.3572E 01	-0.3570E 01	-0.3569E 01

Table 3.4

EFFECT OF INTERVAL 1					
R=15.000 I= 0.100					
1/ 900.	2/1800.	1/2700.	1/3600.	1/4500.	
-0.2758E 01	-0.2744E 01	-0.2739E 01	-0.2738E 01	-0.2737E 01	
-0.1594E 01	-0.1574E 01	-0.1571E 01	-0.1569E 01	-0.1569E 01	
-0.2298E 00	-0.2151E 00	-0.2126E 00	-0.2118E 00	-0.2114E 00	
0.1164E 01	0.1170E 01	0.1171E 01	0.1171E 01	0.1172E 01	
0.2417E 01	0.2414E 01	0.2414E 01	0.2413E 01	0.2413E 01	
0.3378E 01	0.3367E 01	0.3365E 01	0.3364E 01	0.3364E 01	
0.3932E 01	0.3914E 01	0.3910E 01	0.3909E 01	0.3909E 01	
0.4012E 01	0.3989E 01	0.3984E 01	0.3983E 01	0.3982E 01	
0.3607E 01	0.3582E 01	0.3578E 01	0.3576E 01	0.3575E 01	
0.2768E 01	0.2744E 01	0.2739E 01	0.2738E 01	0.2737E 01	
0.1594E 01	0.1574E 01	0.1571E 01	0.1569E 01	0.1569E 01	
0.2288E 00	0.2151E 00	0.2126E 00	0.2118E 00	0.2114E 00	
-0.1164E 01	-0.1170E 01	-0.1171E 01	-0.1171E 01	-0.1172E 01	
-0.2417E 01	-0.2414E 01	-0.2414E 01	-0.2413E 01	-0.2413E 01	
-0.3378E 01	-0.3367E 01	-0.3365E 01	-0.3364E 01	-0.3364E 01	
-0.3932E 01	-0.3914E 01	-0.3910E 01	-0.3909E 01	-0.3909E 01	
-0.4012E 01	-0.3989E 01	-0.3984E 01	-0.3983E 01	-0.3982E 01	
-0.3607E 01	-0.3582E 01	-0.3578E 01	-0.3576E 01	-0.3575E 01	
-0.2768E 01	-0.2744E 01	-0.2739E 01	-0.2738E 01	-0.2737E 01	
-0.1594E 01	-0.1574E 01	-0.1571E 01	-0.1569E 01	-0.1569E 01	
-0.2288E 00	-0.2151E 00	-0.2126E 00	-0.2118E 00	-0.2114E 00	
0.1164E 01	0.1170E 01	0.1171E 01	0.1171E 01	0.1172E 01	
0.2417E 01	0.2414E 01	0.2414E 01	0.2413E 01	0.2413E 01	
0.3378E 01	0.3367E 01	0.3365E 01	0.3364E 01	0.3364E 01	
0.3932E 01	0.3914E 01	0.3910E 01	0.3909E 01	0.3909E 01	
0.4012E 01	0.3989E 01	0.3984E 01	0.3983E 01	0.3982E 01	
0.3607E 01	0.3582E 01	0.3578E 01	0.3576E 01	0.3575E 01	
0.2768E 01	0.2744E 01	0.2739E 01	0.2738E 01	0.2737E 01	
0.1594E 01	0.1574E 01	0.1571E 01	0.1569E 01	0.1569E 01	
0.2288E 00	0.2151E 00	0.2126E 00	0.2118E 00	0.2114E 00	
-0.1164E 01	-0.1170E 01	-0.1171E 01	-0.1171E 01	-0.1172E 01	
-0.2417E 01	-0.2414E 01	-0.2414E 01	-0.2413E 01	-0.2413E 01	
-0.3378E 01	-0.3367E 01	-0.3365E 01	-0.3364E 01	-0.3364E 01	
-0.3932E 01	-0.3914E 01	-0.3910E 01	-0.3909E 01	-0.3909E 01	
-0.4012E 01	-0.3989E 01	-0.3984E 01	-0.3983E 01	-0.3982E 01	
-0.3607E 01	-0.3582E 01	-0.3578E 01	-0.3576E 01	-0.3575E 01	

Table 3.5

Computed value(from the table) 3.985

Analytical value(using slide-rule) 4.00

EFFECT OF INTERVAL

R = 0.550

L = 0.112

1/ 900.	1/1800.	1/2700.	1/3600.	1/4500.
-0.3529E 01	-0.3525E 01	-0.3513E 01	-0.3512E 01	-0.3511E 01
-0.2810E 01	-0.2799E 01	-0.2797E 01	-0.2796E 01	-0.2796E 01
-0.1722E 01	-0.1715E 01	-0.1714E 01	-0.1714E 01	-0.1714E 01
-0.3984E 00	-0.3964E 00	-0.3961E 00	-0.3959E 00	-0.3959E 00
0.1003E 01	0.9993E 00	0.9987E 00	0.9985E 00	0.9984E 00
0.2312E 01	0.2303E 01	0.2302E 01	0.2301E 01	0.2301E 01
0.3370E 01	0.3358E 01	0.3356E 01	0.3355E 01	0.3354E 01
0.4051E 01	0.4036E 01	0.4033E 01	0.4032E 01	0.4032E 01
0.4272E 01	0.4255E 01	0.4252E 01	0.4251E 01	0.4251E 01
0.4005E 01	0.3990E 01	0.3987E 01	0.3986E 01	0.3985E 01
0.3284E 01	0.3273E 01	0.3268E 01	0.3267E 01	0.3267E 01
0.2194E 01	0.2185E 01	0.2183E 01	0.2183E 01	0.2182E 01
0.8670E 00	0.8632E 00	0.8625E 00	0.8623E 00	0.8622E 00
-0.5367E 00	-0.5351E 00	-0.5348E 00	-0.5347E 00	-0.5347E 00
-0.1848E 01	-0.1842E 01	-0.1840E 01	-0.1840E 01	-0.1840E 01
-0.2910E 01	-0.2899E 01	-0.2897E 01	-0.2896E 01	-0.2896E 01
-0.3593E 01	-0.3579E 01	-0.3577E 01	-0.3576E 01	-0.3576E 01
-0.3816E 01	-0.3803E 01	-0.3799E 01	-0.3798E 01	-0.3797E 01
-0.3552E 01	-0.3538E 01	-0.3536E 01	-0.3535E 01	-0.3534E 01
-0.2833E 01	-0.2822E 01	-0.2820E 01	-0.2819E 01	-0.2819E 01
-0.1745E 01	-0.1738E 01	-0.1737E 01	-0.1737E 01	-0.1736E 01
-0.4213E 00	-0.4192E 00	-0.4188E 00	-0.4187E 00	-0.4186E 00
0.9800E 00	0.9767E 00	0.9761E 00	0.9759E 00	0.9758E 00
0.2289E 01	0.2281E 01	0.2279E 01	0.2279E 01	0.2278E 01
0.3349E 01	0.3336E 01	0.3333E 01	0.3332E 01	0.3332E 01
0.4029E 01	0.4014E 01	0.4011E 01	0.4010E 01	0.4009E 01
0.4249E 01	0.4233E 01	0.4230E 01	0.4229E 01	0.4229E 01
0.3983E 01	0.3968E 01	0.3965E 01	0.3964E 01	0.3963E 01
0.3262E 01	0.3249E 01	0.3246E 01	0.3246E 01	0.3245E 01
0.2172E 01	0.2163E 01	0.2162E 01	0.2161E 01	0.2161E 01
0.8453E 00	0.8416E 00	0.8409E 00	0.8407E 00	0.8405E 00
-0.5583E 00	-0.5566E 00	-0.5563E 00	-0.5562E 00	-0.5562E 00
-0.1870E 01	-0.1863E 01	-0.1862E 01	-0.1861E 01	-0.1861E 01
-0.2931E 01	-0.2920E 01	-0.2918E 01	-0.2917E 01	-0.2917E 01
-0.3614E 01	-0.3601E 01	-0.3598E 01	-0.3597E 01	-0.3597E 01
-0.3837E 01	-0.3822E 01	-0.3820E 01	-0.3819E 01	-0.3818E 01

Table 3.6

Computed value(from the table) 4.00

Analytical value(using slide-rule) 4.01

The two time constants used are approximately .00625 sec. (corresponding to the short circuit condition or slip = 1), and .2 second (corresponding to the no load condition calculated from the primary resistance and magnetising reactance of a practical machine). As seen in table 3.1, which corresponds to a step input voltage, the last three values of incremental times considered give exactly the same numerical values of current from the beginning, the second value of increment gives the correct answer from the 22nd line in the print out whilst the first value of increment gives the correct answer from the 28th line. In table 3.2 where the time constant is .2 second all the increments result in the same answer. The time between lines is $1/900$ second, irrespective of the incremental time used.

When the input voltage is sinusoidal, the instantaneous value at the middle of an interval is taken as the input voltage for that interval. In table 3.3 and 3.4 the computer outputs for two cycles following the application of a 50 c/s sinusoidal voltage are shown. Though the phase of currents in all the cases are the same, that is, currents become negative or positive at the same instant, there is a definite difference in the instantaneous values produced by different incremental times. However, it will be observed from the table that the method produces substantially the same answer if the number of increments in a cycle exceeds 36, with a further increase depending on the time constant and the required accuracy.

For a steady state solution which can be easily checked analytically, the currents during the 29th and 30th cycles are printed out in tables 3.5 and 3.6. The difference in the maximum and minimum values as the increment is decreased may be noted. The analytical and computed values of the maximum currents are also given in the tables. It may be concluded that using this method, the transient values for the simple circuit used may be calculated using a digital computer to a required accuracy, by choosing an appropriate incremental time from the parameters of the circuit and the results may be checked by repeating the calculations with a smaller increment. Having shown the accuracy of the procedure for simple circuits, coupled circuits are next considered.

3.4 Inductance varying with time

In this section the case of the coefficients varying with time is considered. A simple machine with one coil on its stator and a second coil on its rotor, as shown in figure 3.5.a is suitable for this exercise since as the rotor revolves the coupling between the two coils varies. An equivalent circuit of the system is given in figure 3.5.b. If it is further assumed that the air gap is non-uniform then the self inductances also vary with rotor position.

The variation of self and mutual inductances with time may be calculated using the procedure given in Appendix 2.

Suppose M_{11} and M_{22} denote the total inductances

r_1, r_2 the resistances

i_1, i_2 the instantaneous currents

v_1, v_2 the instantaneous voltages of the stator and rotor coils,

and t_1 and t_2 the times at the beginning and end of the incremental time considered, then using single and double dashes to represent the variables at time t_1 and t_2 respectively, the general equations for the system may be written as,

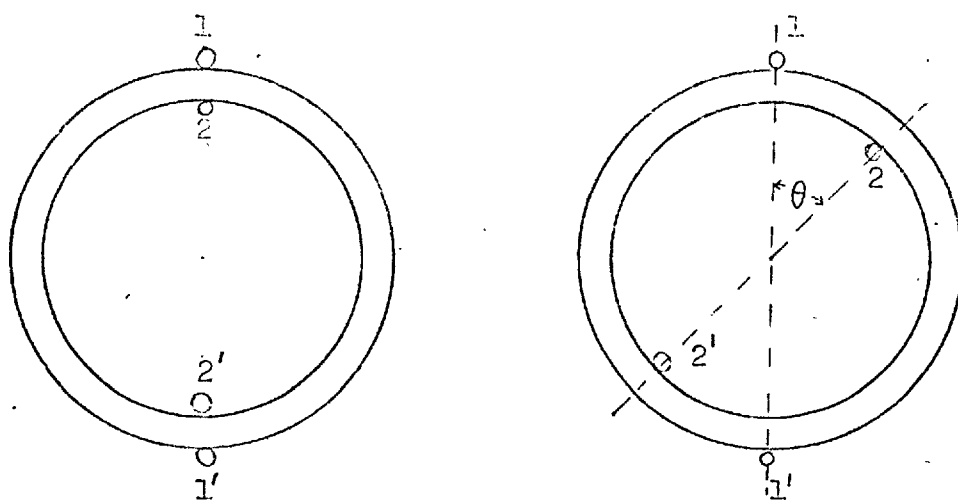
$$v_1 = \frac{i_1' + i_1''}{2} r_1 + \frac{M_{11}'' i_1'' - M_{11}' i_1'}{T} + \frac{M_{12}'' i_2'' - M_{12}' i_2'}{T}$$

$$v_2 = \frac{i_2' + i_2''}{2} r_2 + \frac{M_{21}'' i_1'' - M_{21}' i_1'}{T} + \frac{M_{22}'' i_2'' - M_{22}' i_2'}{T}$$

Transferring the single dashed quantities to the left hand side, the two simultaneous equations may be rewritten as,

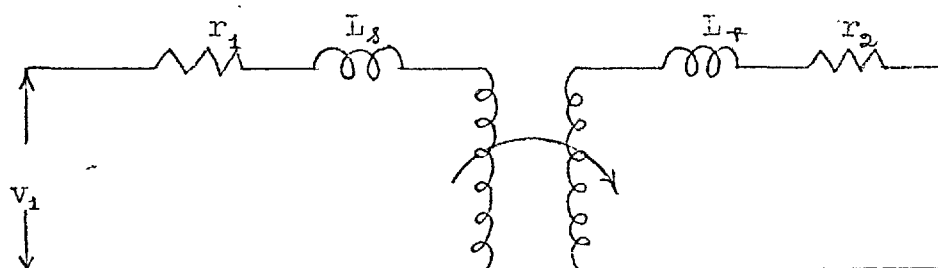
$$v_1 - \left(\frac{r_1}{2} - \frac{M_{11}'}{T} \right) i_1' + \frac{M_{12}'}{T} i_2' = \left(\frac{r_1}{2} + \frac{M_{11}''}{T} \right) i_1'' + \frac{M_{12}''}{T} i_2'' \quad (3.3)$$

$$v_2 + \frac{M_{21}'}{T} i_1' - \left(\frac{r_2}{2} - \frac{M_{22}'}{T} \right) i_2' = \frac{M_{21}''}{T} i_1'' + \left(\frac{r_2}{2} + \frac{M_{22}''}{T} \right) i_2'' \quad (3.4)$$



a. A simple two coil machine

b. A dynamic equivalent circuit



$F(\theta)$

figure 3.5

The voltages applied to the stator and rotor coils will be known. If the rotor is short circuited v_2 is zero. The procedure to solve for the successive steps of current from the moment of switching until steady state conditions are reached, is as follows: assuming the rotor to be revolving at a known speed, the positions of rotor and hence the inductances at the beginning and end of an incremental time T just after switching may be calculated. Assuming the currents to be zero at the moment of switching i_1'' and i_2'' are the only unknowns in equations 3.3 and 3.4 therefore the currents may be calculated by solving the simultaneous equations. The currents at the end of the first increment form the initial values of currents for the second increment, thus the currents may be calculated in successive steps. The process is repeated until steady state conditions are achieved.

In the above solution the movement of rotor is considered to be independent of the electrical system. That is, the mechanical force due to the currents in the machine has been neglected. However if the full transient performance of the machine is to be calculated then the electromagnetic conditions must be used to calculate the torque and therefore acceleration and speed at the end of an increment.

Solution of a general machine problem

3.5 Equations for the electrical system

Considering a machine having p coils on its stator and q coils on its rotor, the self and mutual inductances between the coils at a particular rotor position may be written in matrix form as follows:-

$$\begin{array}{cccc}
 M_{1,1} & \dots & M_{1,p} & M_{1,p+1} \dots M_{1,p+q} \\
 \vdots & & & \\
 M_{p,1} & \dots & M_{p,p} & M_{p,p+1} \dots M_{p,p+q} \\
 \hline
 M_{p+1,1} & \dots & M_{p+1,p} & M_{p+1,p+1} \dots M_{p+1,p+q} \\
 \vdots & & & \\
 M_{p+q,1} & \dots & M_{p+q,p} & M_{p+q,p+1} \dots M_{p+q,p+q}
 \end{array}$$

If the machine has a uniform air gap, the self and mutual inductances between the stator coils shown in the second quadrant are constants. Similarly the self and mutual inductances between the rotor coils given in the fourth quadrant are also constants. But the mutual inductances between the stator and the rotor coils, given in the first and third quadrants are functions of rotor position, and may be calculated using the procedure described in Appendix 1. If the air gap is non-uniform, with slotted iron surfaces on both the stator and rotor, all the inductances are functions of the position of rotor and may be calculated at any given position using the method described in Appendix 2.

Assuming the input voltages and currents to be independent, that is each coil is electrically independent and no internal connections exist in the machine itself, the differential equations for the electrical system may be written in matrix form as follows

$$[V] = [I] [R] + \frac{d}{dt} \{ [M] [I] \} \dots\dots\dots (3.6)$$

with the equation for a typical Sth circuit given by

$$v_s = i_s r_s + \frac{d}{dt} [M_{s1} i_1 + \dots + M_{s \ p+q} i_{p+q}]$$

where $[V]$, $[I]$, $[M]$ and R are the matrices denoting the applied voltages, currents, inductances and resistances of the circuits respectively.

3.6 Solution of currents at constant speed

In equation 3.6 the elements of the voltage matrix V will be known numerically with respect to time. For example, in a slip ring induction motor with p coils on its stator and q coils on its rotor the first ' p ' elements of matrix V are the stator input voltages and if the rotor is short circuited or completed through external impedances, then the last ' q ' elements of matrix V are taken to be zero. However, if the rotor coils are also fed with external voltages, the last ' q ' elements of matrix V are given by the rotor input voltages. If the rotor position at time $t_1 = 0$ is θ_1 and the speed of the rotor is constant and ω_1 then θ_2 , the position of the rotor at time $t_2 = T$ is given by

$$\theta_2 = \theta_1 + \omega_1 T$$

From the two rotor positions, the values of the inductances at times t_1 and t_2 may be calculated as $[M]'$ and $[M]''$ respectively. Again using single and double dashes for the voltages and currents at the beginning and the end of the increment considered, the matrix equation (3.6) may be rewritten as

$$\frac{[V]'' + [V]'}{2} = \frac{[I]'' + [I]'}{2} [R] + \frac{[M]''[I]'' - [M]'[I]'}{T}$$

which yields

$$\frac{[V]'' + [V]'}{2} - \frac{[I]'}{2} [R] + \frac{[M]'}{T} [I] = \frac{[I]''}{2} [R] + \frac{[M]''}{T} [I]''$$

..... (3.7)

In the above equation $[I]'$, the currents at the beginning of the increment will be given by the initial conditions for the first increment. For the subsequent increments, $[I]'$ are the currents at the end of the previous increment. Therefore, the currents denoted by $[I]''$ are the only unknowns in equation (3.7) and may be calculated by solving the set of simultaneous equations represented by equation (3.7) in matrix form.

In order to calculate the currents at time $t_3 = 2T$, the position of rotor at time t_3 , hence the inductances must first be calculated. The initial currents for the second increment are the currents at time T calculated previously, therefore, the currents at the end of the second increment may be calculated by solving the new set of equations. The calculation of the inductances and currents may be continued until steady state conditions are reached, that is, until repeatable waveforms of currents are obtained. The electrical torque developed during a particular increment may be calculated from the currents, inductances and the rotor positions at the beginning and end of the increment using one of the methods given in Appendix 4.

3.7 Electromechanical transient solution

In solving for the currents, a knowledge of the rotor position at

the end of the increment considered is essential so that it is possible to calculate $[M]''$ in equation (3.7), thus forming a set of linear equations in $[I]''$. This requires the speed to be a known function of time. In practice the speed depends on the electrical and load torques, and the moment of inertia of the rotating parts, and therefore it can not be evaluated without knowing the electrical torque developed. On the other hand the electrical torque can not be calculated prior to the calculation of the currents. These conflicting requirements make it necessary to treat the speed or rotor position as a variable in the system equations.

In Appendix 4 it is shown that the electrical torque, T_e , developed by a machine having $p + q$ coils is given by

$$T_e = \sum_{m=1}^{n=p} \sum_{n=p+1}^{n=p+q} i_m i_n \frac{d M_{mn}}{d \theta} \quad \dots\dots\dots (3.8)$$

An additional equation may be written equating the electrical and mechanical torques as follows,

$$T_b + J \frac{du}{dt} = T_e \quad \dots\dots\dots (3.9)$$

where T_b is the braking torque, J is the moment of inertia of the rotating parts and u is the speed. The electrical equations may be written in a new form (also done by Tipping¹⁰) as shown below for a m th coil,

$$\begin{aligned} v_m &= i_m r_m + \frac{d}{dt} \left[\sum_{n=1}^{n=p+q} M_{mn} i_n \right] \\ &= i_m r_m + \sum_{n=1}^{n=p+q} \left[M_{mn} \frac{d i_n}{dt} + i_n \frac{d M_{mn}}{d \theta} \frac{d \theta}{dt} \right] \end{aligned} \quad \dots\dots\dots (3.10)$$

where $\frac{d \theta}{dt}$ is the speed of the rotor.

The above equations are non-linear since they contain terms given by the products of variables, and cannot therefore be solved analytically or numerically without further simplifications. The various assumptions made to solve the equations simultaneously for

currents and speed using either an analytical method or a numerical technique have been indicated in the introduction, for the case of balanced sinusoidal applied voltages.

The Runge Kutta method may be used to solve for the currents and speed numerically if and only if a set of linear differential equations, at least linear within a particular incremental time considered, can be formed. But, in the present case, this is not possible unless the term $\frac{d M_{mn}}{d \theta}$ which is the derivative of the mutual inductance between the m th and n th coils, and the term $\frac{d \theta}{dt}$ which is the rotor speed are assumed during a particular step.

The technique adopted to take speed variations into account in the analysis developed here, uses an iterative process within the incremental time considered and no further simplifications in the operating conditions are necessary. The method is described below.

Supposing at time t_1 the speed, electrical torque, the position of the rotor and the currents are given by u_1 , T_e , θ_1 and $[I]'$, then the electrical torque T_e is assumed to act during the incremental time considered. The speed of the rotor at time t_2 is then given by

$$u_2 = u_1 + \left(\frac{T_e - T_b}{J} \right) T \quad \text{..... (3.11)}$$

From θ_1 and u_2 , the position of the rotor at time t_2 may be calculated using the equation

$$\theta_2 = \theta_1 + \frac{(u_1 + u_2)}{2} T \quad \text{..... (3.12)}$$

Now, the inductances at times t_1 and t_2 and hence the currents at time t_2 may be determined. From the currents at times t_1 and t_2 and the movement of the rotor from θ_1 to θ_2 , T_e' the average electrical torque developed during the interval can be found. If this value differs from T_e , which was initially assumed to be acting during the increment, then the procedure is repeated as follows.

T_e' is assumed to be the torque acting during the increment and the rotor position and currents at the end of the increment are

recalculated. If T_e'' , the new value of torque calculated, is within a preset limit of say 0.1% to the assumed torque T_e^i , the next increment is considered and the calculations are continued.

The iterative process within the interval does not automatically converge. For example if the moment of inertia is very small conditions can be found where the values of torques calculated in the process oscillate. However, in the range of inertias found in normal practice no trouble has been experienced.

Though the general equations of the system are not solved simultaneously for the electrical and mechanical variables, repetition within an incremental time results in a unique value of torque ultimately satisfying the electrical and mechanical conditions. The calculation of the currents torque and speed may be continued until electromechanical steady state conditions are reached, that is until repeatable waveforms of currents and speed are obtained.

3.8 Computer Program for the step-by-step procedure

The flow diagram of the program is shown in figure 3.6. A detailed description of each part of the program is given in this section.

3.8.1 Data and initial calculations

The data for the program consists of the input voltage, dimensions of the machine and winding details, leakage inductances of the phases, initial conditions and control numbers.

The input voltages with respect to time will be known numerically or as functions. One cycle of the supply is divided into N_1 increments and the instantaneous voltages during the increments are tabulated and stored.

From the dimensions of the machine that is, the diameter and length of the stator, the air-gap and the winding details, the self and mutual inductances between the stator phases and those between the rotor phases that is, the constant elements of the matrix are

FLOW CHART FOR PERFORMANCE CALCULATION

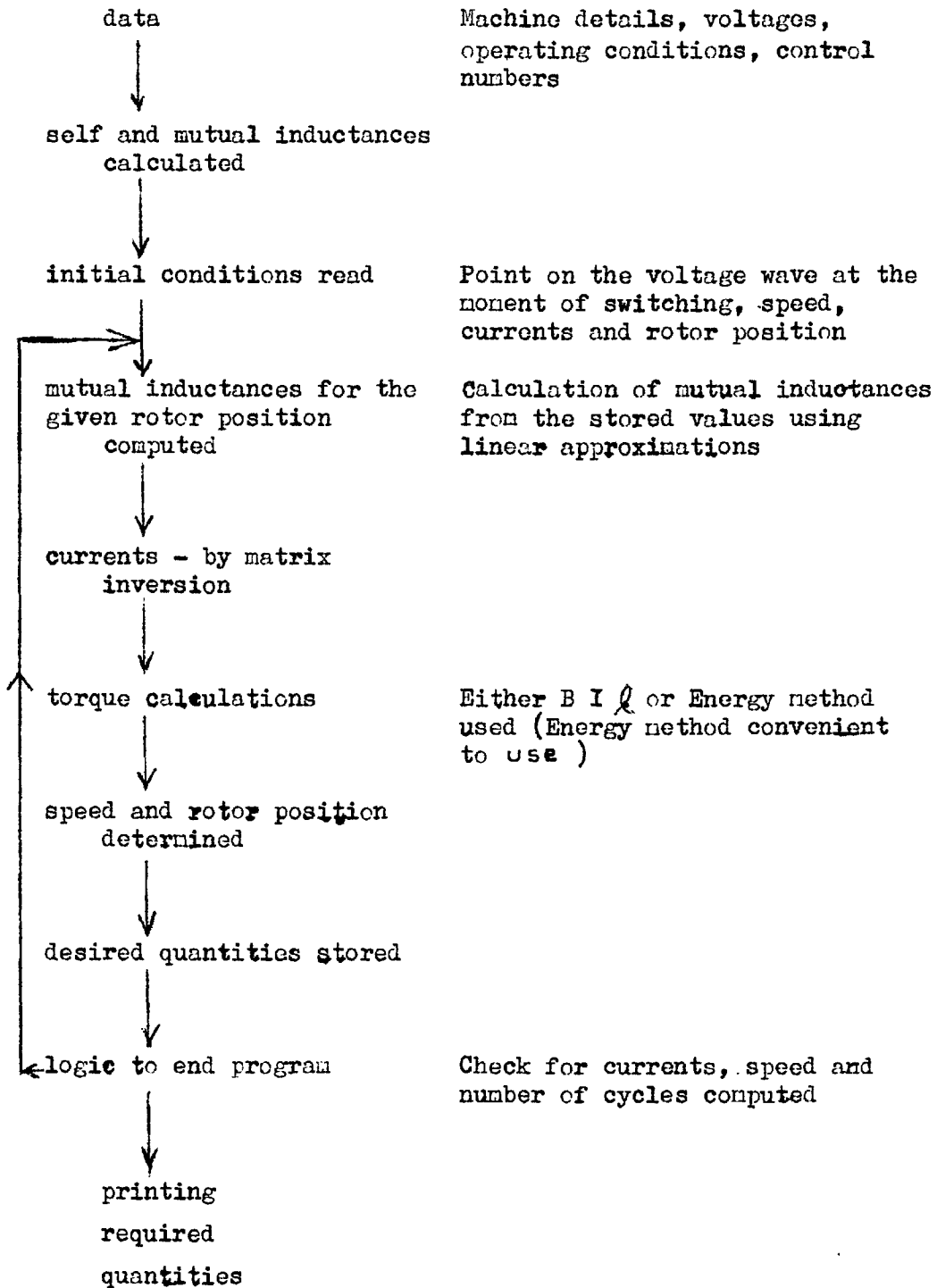


Figure 3.6

calculated using the procedure given in Appendices 1 and 2. In the case of a symmetrical machine, the mutual inductances between one of the phases on the rotor and all the stator phases namely $M_{1, p+1}$, $M_{2, p+1}$, ..., $M_{p, p+1}$ are calculated by summing up the appropriate coil inductances calculated at a suitable number of rotor positions. These values are stored in the computer and the inductances at any rotor position may then be found by linear interpolation.

The rest of the mutual inductances between the stator and the rotor may be determined by symmetry. However, if the machine is not symmetrical, all the inductances have to be either stored or calculated at a required rotor position during the performance calculations.

The leakage inductances of stator and rotor phases are given as functions of the phase currents or graphically. Their values corresponding to a number of values of phase currents are stored. During computation the rms value of the phase current at the end of each voltage cycle is calculated and the corresponding inductance found out by linear approximation. This value is used in the following cycle.

The initial conditions to be given are (1) the instant of application of the supply which may be taken from the oscillograms in an experimental case or assumed while making a theoretical study of the phenomenon. (2) the initial currents, for example, if the machine is reconnected after a disconnection from the supply, the rotor currents may still be present or it may be desired to feed particular initial conditions namely the currents, speed, rotor position and torque, corresponding to a steady state in order to study the effect of load changes etc. In which case the program need not go through the calculation of transients to ~~attain~~ the required steady state. (3) the speed and the rotor position at the moment of switching must always be given.

Control numbers are used to direct the flow of logical operations at various points of the program as desired in a particular case, for example what results are to be printed, and under what conditions the program may be terminated and so on.

3.8.2 Performance calculation

This part may be considered as the heart of the program and it consists of operations executed at every incremental step and operations carried out at the end of every cycle that is, after every N_1 increments. The various computations are described below as they occur in the program.

- a) An integer N whose initial value is always given in the data, is varied during the calculations from its initial value to N_1 in steps of unity, after which, it is re-set to unity again in the next step and the cycle is repeated. This process may be said to be a clock as far as the program is concerned with N_1 units per cycle. At any incremental step the input voltages are chosen from the column corresponding to the value of N , in the voltage matrix stored.
- b) From the position and speed of the rotor and the torque acting on it at the beginning of an increment, the position at the end of the increment is calculated and all the mutual inductances are determined.
- c) Referring to equation (3.7) the right hand side is reduced to $[B] [I]_2$ by adding the row matrix $[R]$ and the square matrix $[M]_2$ to form the new matrix $[B]$ using the following relationship to obtain the diagonal elements,

$$B_{ss} = \frac{M_{ss}}{T} + \frac{r_s}{2} \quad \text{..... (3.13)}$$

where 's' varies from 1 to $p + q$. The other elements of matrix B are obtained directly from the matrix $[M]$. The left hand side of equation (3.7) may be written as

$$[V] - [I]_1 \frac{[R]}{2} + \frac{[M]_1 [I]_1}{T} = [H]_2 \quad \text{..... (3.14)}$$

and $[H]_2$ can be calculated numerically from the initial conditions for the first increment.. Equation (3.7) has now been reduced to the form

$$\begin{aligned} \text{or} \quad [B] [I]_2 &= [H]_2 \\ [I]_2 &= [B]^{-1} [H]_2 \quad \text{..... (3.15)} \end{aligned}$$

where $[B]^{-1}$ is the inverse of matrix $[B]$. Inversion of matrix

B is carried out using a standard subroutine, and the currents are computed by multiplying the inverse and matrix $[H]_2$. The electrical torque developed, and the speed and rotor position after an increment are determined. From the rotor position the inductances are calculated as indicated earlier. The various stages in the calculation are as follows.

From the currents $[I]_1$, torque, T_e , and the rotor position, θ_1 , at the beginning of the increment, θ_2 , the rotor position at the end of the increment and hence currents $[I]_2$ and torque T'_e are calculated. This new value of torque is used to recalculate the rotor position θ'_2 and therefore the currents $[I]_2'$ and torque T''_e . This process is repeated until the torques calculated in the consecutive iterations (within the incremental time considered) are equal or until the difference between their values is negligible. This completes the calculations for one increment. In order to go to the next increment integer N is increased by unity. The matrix $[H]_2$ is transferred to matrix $[H]_1$ and the new values of $[H]_2$ are formed using the relation

$$[H]_2 = [V]_n - [I]_1 [R] + [H]_1 \quad \dots\dots\dots (3.16)$$

instead of using equation (3.14) which is used only for the first incremental step in the program. The two expressions for $[H]_2$ given in equations (3.14) and (3.16) are shown to be equivalent in Appendix 5 and the use of equation (3.16) reduces the number of arithmetical operations.

The currents and hence the electrical torque and so on are calculated once again by the iterative technique. The whole incremental process is continued to the end of the cycle, that is until N is equal to N_1 , after which it is set at unity for the next cycle. One cycle of the supply corresponds to N varying from 1 to N_1 and the necessary currents, torque, speed and rotor position at every value of N are stored, to be printed at the end of each cycle as specified by a control statement.

The rms values of the stator currents, total input power during the cycle and the average torque are calculated before proceeding to the next cycle and printed out. While computing steady state

performance, one of the stator currents at the end of the cycle is stored to be compared with that at the end of the next cycle. If the difference is less than a specified value set by a control number then, the steady state is said to have been achieved and the computation is terminated.

If it is desired to continue the computation at a later date for the same input voltages, from a certain condition calculated now, the existing conditions - currents, rotor speed and position, the value of the matrix $[H]$ and the instantaneous torque, may be printed out. Hence the program may be continued at a later date from this state instead of calculating from the beginning, thus reducing the computing time required.

3.9 Various computations carried out using the program

The computations correspond to the experiments carried out and the necessary modifications to the program to suit the operating conditions being computed are given below

Switching transients

Switching on the supply at a particular instant and calculating the currents, torque etc. for a few cycles following switching is achieved by giving the corresponding value for 'N' that is the number of the increment at which the supply is switched on. The program computes the transients and continues until steady state is reached. In transient studies computation is carried out for the required number of cycles using a control number. From the program the points at which the transient torque is minimum and maximum may be predicted.

Steady state operation at any desired speed

Referring to the speed torque characteristic shown in figure 3.7 it will be observed that the induction motor is not stable in region A. If it is required to run the motor at any speed in this region a d.c. machine is coupled, the speed torque characteristic of the d.c. machine being a straight line as shown by curve b. The slope

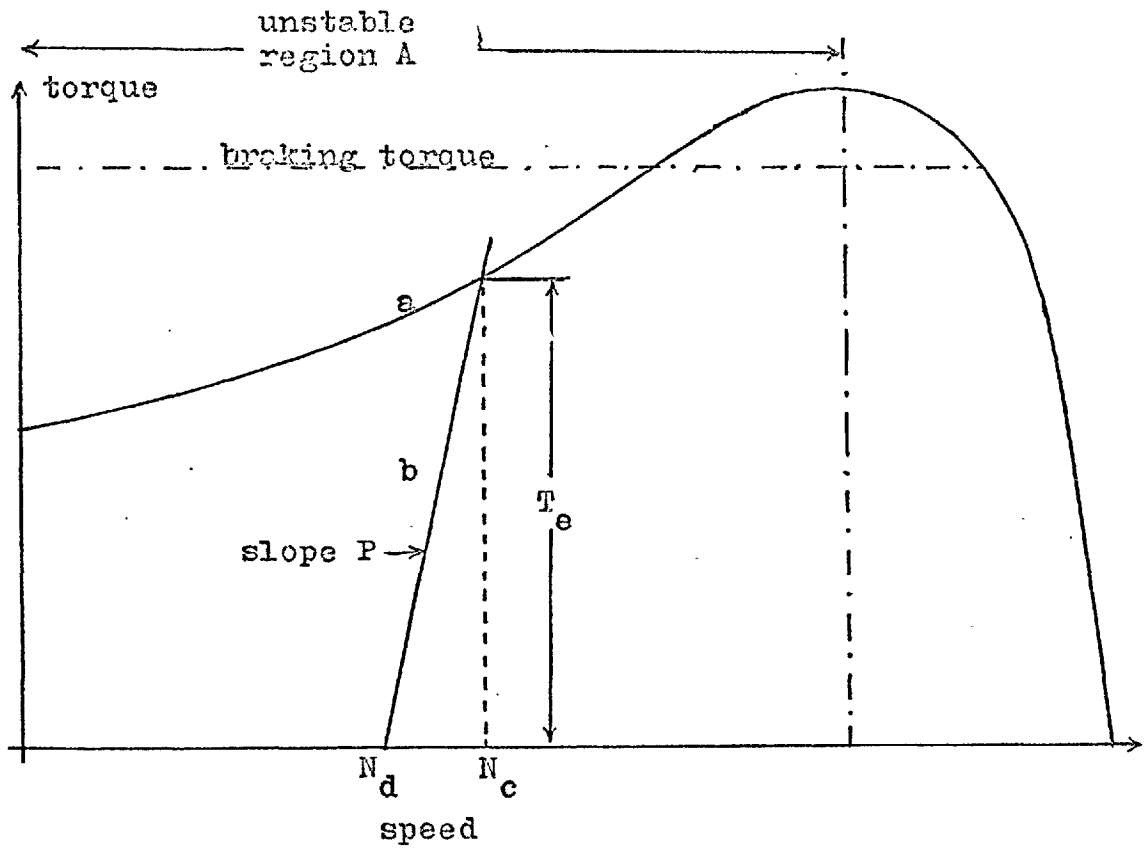


figure 3.7 a-speed-torque characteristic of an induction motor
b-speed-torque characteristic of a d.c machine
for particular values of armature voltage and
field current

and the point of intersection with 'X' axis are fixed by the armature voltage and the field excitation respectively. The system runs at a speed where the two characteristics meet. Therefore, by varying the field excitation and the voltage applied to the armature, the system may be fixed to run at any speed.

The computer program follows the physical laws of the induction motor and therefore is not able to compute steady state performance at a speed in the region A mentioned above. This fact has been verified using the program written. If the load is increased so that the operating point lies in region A of the speed torque characteristics, the machine decelerates to zero speed. Therefore the program is modified to behave as the induction motor-d.c. machine system as follows:

Supposing it is required to compute the performance at a speed, N_c , then the d.c. machine characteristic is supplied to the program as the slope, P, in figure 3.7. If the electrical torque developed by the induction motor is T_e and the torque developed by the d.c. machine is T_d , then the accelerating torque is given by

$$T_e - T_d$$

If the initial speed at the moment of switching is given as N_c , the program follows the transient performance from this condition with the speed varying above and below N_c , in a damped oscillatory manner until the desired steady state performance at N_c is calculated.

CHAPTER 4

AN INDIRECT PROOF OF THE NUMERICAL TECHNIQUE

4.1. Comparison of the numerical technique with an exact analytical method

In chapter 3 the solution of simple circuits using the numerical technique was compared with values obtained using analytical methods. In this chapter an ideal machine is considered for such a comparison. The machine has three stator phases and three rotor phases and the mutual inductances between the stator and rotor phases vary sinusoidally with respect to the rotor position. If the stator phases are supplied with balanced sinusoidal voltages, the steady state solution at any speed may be obtained on a single phase basis using the exact equivalent circuit in figure 1.1.2. The parameters of the machine considered are listed below.

The self inductances of the stator and rotor phases = .598 henry

The mutual inductances

between the stator phases = .299 henry
between the rotor phases = .299 henry
between a stator and a rotor phase
(maximum value) = .598 henry

Resistances

a stator phase = 2.55 ohms
a rotor phase = 2.72 ohms

Leakage reactances at 50 c/s

a stator phase = 12.2 ohms
a rotor phase = 6.6 ohms

The inertia of rotating parts = .25 lb ft^2

The magnetising reactance used in an equivalent circuit is 1.5 times the self inductance of a stator phase multiplied by a constant, $2 \pi f$, where f is the supply frequency.

From these parameters the currents and torques at five different speeds are calculated using a simple equivalent circuit. These are shown in the fourth column in the table.

Increments per cycle at 50 c/s	36	72	Values from the equivalent circuit	Speed in radians per second
Stator phase	5.20	5.18	5.18	0
current	5.10	5.09	5.11	79.84
amperes	4.92	4.92	4.93	160.9
	3.77	3.77	3.77	261.6
	2.22	2.22	2.227	291.9
	.487	.485	.48	0
Torque lb ft	.628	.632	.63	79.84
	.900	.903	.9	160.9
	1.530	1.54	1.506	261.6
	1.240	1.24	1.204	291.9
Computation time in minutes	3.8	7.2		

The numerical technique is used to compute the performance at the five speeds. The procedure adopted for computation has been described in chapter 3. As pointed out earlier, the shortness of the incremental time used is a major factor as far as the accuracy of the results is concerned. Two incremental times are used in computing the performance and it may be observed from the table that the computed values are closer to the analytical values as the incremental time is decreased. The computing time for each set is shown in the last row.

Though the comparison of results, obtained using the numerical and analytical methods, is made considering the steady state performance of an induction motor, the numerical technique is not limited to this case. It can also be used to calculate the transient performance of machines with sinusoidal or non-sinusoidal applied voltages.

4.2.1. Verification of the numerical technique by calculating known physical phenomena in machines

Two types of machine windings are considered for this exercise, an ideal machine used in section 4.1 and a machine having a practical winding which produces a third harmonic field in addition to the fundamental.

4.2.2. The case of an ideal machine

If an ideal machine is fed with 3 phase balanced sinusoidal voltages, the instantaneous torque developed at steady state is constant except at lower speeds as given in tables 4.1 to 4.4. (It takes a longer time to attain steady state at lower speeds. In practice, the damping effect of currents in the laminations hasten the process and steady state is attained earlier than the predicted period.) In the case of ideal machines, the constant nature of the instantaneous torque can be derived mathematically. Thus the numerical technique can be used to predict the physical phenomena in machines.

If the ideal machine is supplied with a zero sequence voltage in addition to the balanced voltages, the steady state performance is expected to be unaltered; zero sequence currents do not produce any flux in the air gap if the windings are distributed sinusoidally. This is found to be the case as shown in tables 4.5 to 4.8. It will be observed that the rms value of the phase currents is different because of the zero sequence current but the instantaneous torques are exactly the same as those shown in tables 4.1 to 4.4, where 3 phase balanced voltages are applied to the machine. Once again the physical phenomena is predicted accurately by the numerical technique.

4.2.3. A machine having both third harmonic and fundamental winding distributions.

Practical machine windings produce various harmonic fields in the air gap in addition to the fundamental. In this section the known

T_q	I_{rms}	S_p				
0.526	5.150	31.467				
0.529	5.150	31.467				
0.530	5.149	31.467				
0.532	5.148	31.468				
0.533	5.148	31.468	Table 4.1			
0.533	5.147	31.469				
0.495	0.516	0.528	0.540	0.552	0.563	
0.504	0.516	0.528	0.540	0.552	0.563	
0.573	0.599	0.595	0.589	0.582	0.573	t
0.601	0.599	0.595	0.589	0.582	0.573	
0.563	0.552	0.541	0.529	0.516	0.507	
0.496	0.487	0.478	0.471	0.466	0.462	
0.533	5.147	31.469				
0.461	0.461	0.463	0.467	0.473	0.480	
0.488	0.498	0.508	0.515	0.530	0.541	
0.552	0.562	0.571	0.579	0.585	0.590	
0.593	0.595	0.594	0.592	0.593	0.585	
0.576	0.568	0.558	0.548	0.538	0.527	
0.517	0.507	0.497	0.489	0.481	0.475	
0.665	5.078	94.314				
0.665	5.078	94.314				
0.665	5.079	94.314				
0.665	5.079	94.314				
0.665	5.078	94.314	Table 4.2			
0.665	5.078	94.314				
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	t
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	5.078	94.314				
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	0.665	0.665	0.665	0.665	0.665	
0.665	5.078	94.314				

T_g - Torque

I_{rms} -rms value of one of the phase currents

S_p - Speed in radians per second

t -instantaneous torque

T_q	I_{rms}	S_p				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	t
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	4.936	157.168				
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	4.936	157.168				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	t
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	4.553	220.037				
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	4.553	220.037				

T_g-Torque

 I_{rms} -rms value of one of the phase currents

S_p -Speed in radians per second

t- instantaneous torque

T_q	I_{rms}	S_p			
0.525	7.822	31.467			
0.528	7.822	31.467			
0.530	7.821	31.467			
0.532	7.821	31.468			
0.533	7.820	31.468			
0.533	7.820	31.468			
0.534	7.819	31.468			
0.456	0.460	0.466	0.473	0.482	0.493
0.504	0.515	0.525	0.530	0.532	0.563
0.573	0.532	0.539	0.595	0.599	0.601 t
0.601	0.592	0.595	0.539	0.532	0.573
0.563	0.552	0.541	0.529	0.518	0.507
0.426	0.427	0.475	0.471	0.466	0.462
0.533	7.819	31.469			
0.461	0.461	0.463	0.457	0.473	0.480
0.438	0.498	0.503	0.519	0.530	0.541
0.552	0.562	0.571	0.579	0.585	0.590 t
0.593	0.595	0.594	0.592	0.588	0.533
0.576	0.563	0.553	0.545	0.535	0.527
0.517	0.507	0.497	0.439	0.481	0.475
0.532	7.819	31.469			
0.665	7.744	94.314			
0.665	7.744	94.314			
0.665	7.744	94.314			
0.665	7.744	94.314			
0.665	7.744	94.314			
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665 t
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665 t
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665
0.665	0.665	0.665	0.665	0.665	0.665

Table 4.5

Table 4.6

T_q - Torque

I_{rms} - rms value of one of the phase currents

S_p - Speed in radians per second

t - instantaneous torque

[illegible]

T_q - Torque

 I_{rms} -rms value of one of the phase currents

S_p -Speed in radians per second

 t -instantaneous torque

effects due to the presence of third harmonic field are calculated using the numerical technique. Again two cases, one, where three phase balanced sinusoidal voltages are applied and another, where a zero sequence voltage is applied in addition to the balanced sinusoidal voltages, are considered.

It has been stated in the previous section that zero sequence currents flowing in a balanced winding do not produce any magnetic field in the air gap. The converse of this statement, namely, balanced three phase currents, flowing in a winding which is capable of producing a triple harmonic field (if supplied with **unbalanced** currents), do not produce any magnetic field in the air gap, is considered to verify the results calculated using the numerical technique. It is expected that under steady state conditions the results would be identical with those obtained in the second case (ideal machine with zero sequence applied voltage) in section 4.2.2. This proved to be the case as shown in tables 4.9 to 4.12. It must be pointed out that a true steady state is not obtained in practice and therefore any unbalance in the stator currents results in the formation of third harmonic field. Thus instantaneous torque developed can not be constant in this case as also shown in table 4.12.

With a zero sequence voltage applied to the machine in addition to the balanced sinusoidal voltages, the instantaneous torque will not be constant and the speed torque characteristic will have a dip at $\frac{1}{3}$ the synchronous speed. The former is demonstrated by the computed results shown in tables 4.13 to 4.16. Thus various known physical phenomena have been predicted accurately by using the numerical technique.

4.3. The effect of changing the frequency of the supply

An ideal machine supplied with stepped-square-wave voltages is considered for this exercise. The frequency is varied from 5 c/s to 50 c/s and the speed-torque characteristic is computed in each case for five slip speeds from .1 to .9 in steps of 0.2. The results are shown in table 4.17 and fig. 4.1. The voltages are varied in direct proportion to the frequency.

[illegible]

T_q -Torque

I_{rms} -rms value of one of the phase currents

S_p -Speed in radians per second

t -instantaneous torque

T_q	I_{rms}	S_p				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	4.936	157.168				
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	t
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	4.936	157.168				
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
0.883	0.883	0.883	0.883	0.883	0.883	
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	4.553	220.037				
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	t
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.254	1.255	1.255	1.255	1.255	1.255	
1.255	4.553	220.037				
1.255	1.255	1.255	1.255	1.255	1.254	
1.254	1.254	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.254	1.254	1.255	1.255	1.255	1.255	
1.255	1.255	1.255	1.255	1.255	1.255	
1.255	1.254	1.254	1.254	1.255	1.255	

T_q -Torque

 I_{rms} -rms value of one of the phase currents S_p -Speed in radians per second

t- instantaneous torque

T_q	I_{rms}	S_p			
0.662	5.557	31.437			
0.657	5.432	31.479			
0.514	6.030	31.475			
0.379	5.518	31.459			
0.670	5.713	31.473			
0.674	5.852	31.484			
0.654	5.425	31.455			
0.222	0.205	0.209	0.236	0.288	0.360
0.441	0.509	0.555	0.474	0.232	-0.032
-0.458	-0.853	-0.992	-0.663	0.132	1.083
1.769	1.975	1.740	1.402	1.027	0.751
0.595	0.543	0.561	0.620	0.693	0.763
0.815	0.353	0.837	0.345	0.816	0.774
0.540	6.028	31.473			
0.725	0.674	0.627	0.585	0.553	0.530
0.517	0.512	0.513	0.518	0.524	0.529
0.531	0.523	0.520	0.507	0.491	0.474
0.459	0.448	0.444	0.449	0.463	0.486
0.513	0.540	0.556	0.551	0.512	0.423
0.273	0.053	-0.233	-0.564	-0.892	-1.137
0.908	5.651	94.331			
0.733	5.890	94.325			
0.645	5.661	94.216			
0.534	5.525	94.311			
0.729	5.387	94.216			
0.907	5.639	94.331			
0.745	5.589	94.325			
1.859	1.236	0.963	0.799	0.717	0.679
0.654	0.661	0.666	0.663	0.565	0.645
0.590	0.460	0.182	-0.354	-1.013	-0.469
1.571	1.586	1.385	0.633	0.765	0.690
0.664	0.660	0.665	0.673	0.678	0.676
0.661	0.623	0.545	0.393	0.127	-0.321
0.641	5.669	94.316			
0.740	-0.132	1.227	1.293	0.910	0.719
0.663	0.660	0.671	0.685	0.696	0.701
0.697	0.633	0.655	0.606	0.526	0.393
0.199	-0.065	-0.199	0.286	0.363	0.777
0.663	0.662	0.693	0.720	0.737	0.744
0.742	0.732	0.714	0.687	0.651	0.603

Table 4.13

Table 4.14

T_q -Torque

I_{rms} -rms value of one of the phase currents

S_p -Speed in radians per second

t-instantaneous torque

T_q	I_{rms}	S_p				
0.847	5.491	157.163				
0.847	5.491	157.163				
0.847	5.491	157.163				
0.847	5.491	157.163				
0.847	5.492	157.163				
0.847	5.492	157.163				
0.847	5.492	157.163				
0.853	0.786	0.691	0.527	0.233	0.113	
1.015	0.395	0.322	0.633	-0.891	0.889	
0.873	0.329	0.722	0.445	-0.313	-1.348	t
1.624	2.498	1.452	1.667	0.938	0.592	
0.873	0.377	0.833	0.833	0.811	0.524	
1.113	1.709	1.349	1.113	0.991	0.914	
0.847	5.492	157.163				
0.852	0.784	0.688	0.521	0.221	0.114	
1.032	0.995	0.882	0.883	0.891	0.889	
0.872	0.398	0.720	0.433	0.534	-1.342	
1.697	2.478	1.451	1.064	0.937	0.892	
0.878	0.377	0.833	0.333	0.810	0.526	
1.140	1.695	1.341	1.109	0.989	0.913	
1.206	5.076	220.034				
1.193	5.169	220.031				
1.296	5.082	220.036				
1.243	5.057	220.037				
1.234	5.045	220.037				
1.203	5.000	220.034				
-0.748	2.559	2.081	1.438	1.292	1.250	
1.236	1.236	1.259	1.251	1.383	1.363	
1.316	1.274	1.215	1.077	0.606	-0.730	t
2.124	2.375	1.525	1.313	1.253	1.223	
1.203	1.197	1.259	1.261	1.306	1.294	
1.272	1.232	1.122	0.767	-0.491	1.628	
1.203	5.171	220.031				
2.654	1.521	1.343	1.260	1.213	1.167	
1.100	1.162	1.243	1.258	1.273	1.267	
1.244	1.173	0.512	-0.129	1.010	2.374	
1.721	1.382	1.270	1.207	1.137	1.000	
0.939	1.330	1.250	1.255	1.251	1.251	
1.208	1.034	0.262	0.537	2.942	1.813	

Table 4.15

Table 4.16

T_q -Torque

I_{rms} -rms value of one of the phase currents

S_p - Speed in radians per second

t- instantaneous torque

TABLE 4.17

Table showing the performance of an ideal machine supplied with inverter voltage which is kept proportional to frequency

Torques at various slip-speeds

Frequency slip c/s	.9	.7	.5	.3	.1
5	.606	.598	.554	.444	.203
20	.957	1.106	1.256	1.303	.775
35	.707	.885	1.121	1.428	1.214
50	.535	.688	.911	1.292	1.491

Currents at various slip-speeds

Frequency slips c/s	.9	.7	.5	.3	.1
5	1.756	1.545	1.275	.925	.515
20	4.374	4.131	3.721	2.243	1.36
35	5.014	4.886	4.645	4.062	2.186
50	5.220	5.146	5.003	4.616	2.876

Results show that (in a machine with practical values^{of} leakage and resistive parameters) varying the applied voltage proportional to the frequency does not produce a constant starting torque at various speeds, desired in a traction drive. (This is due to the variation of leakage reactances with frequency while the resistances remain constant).

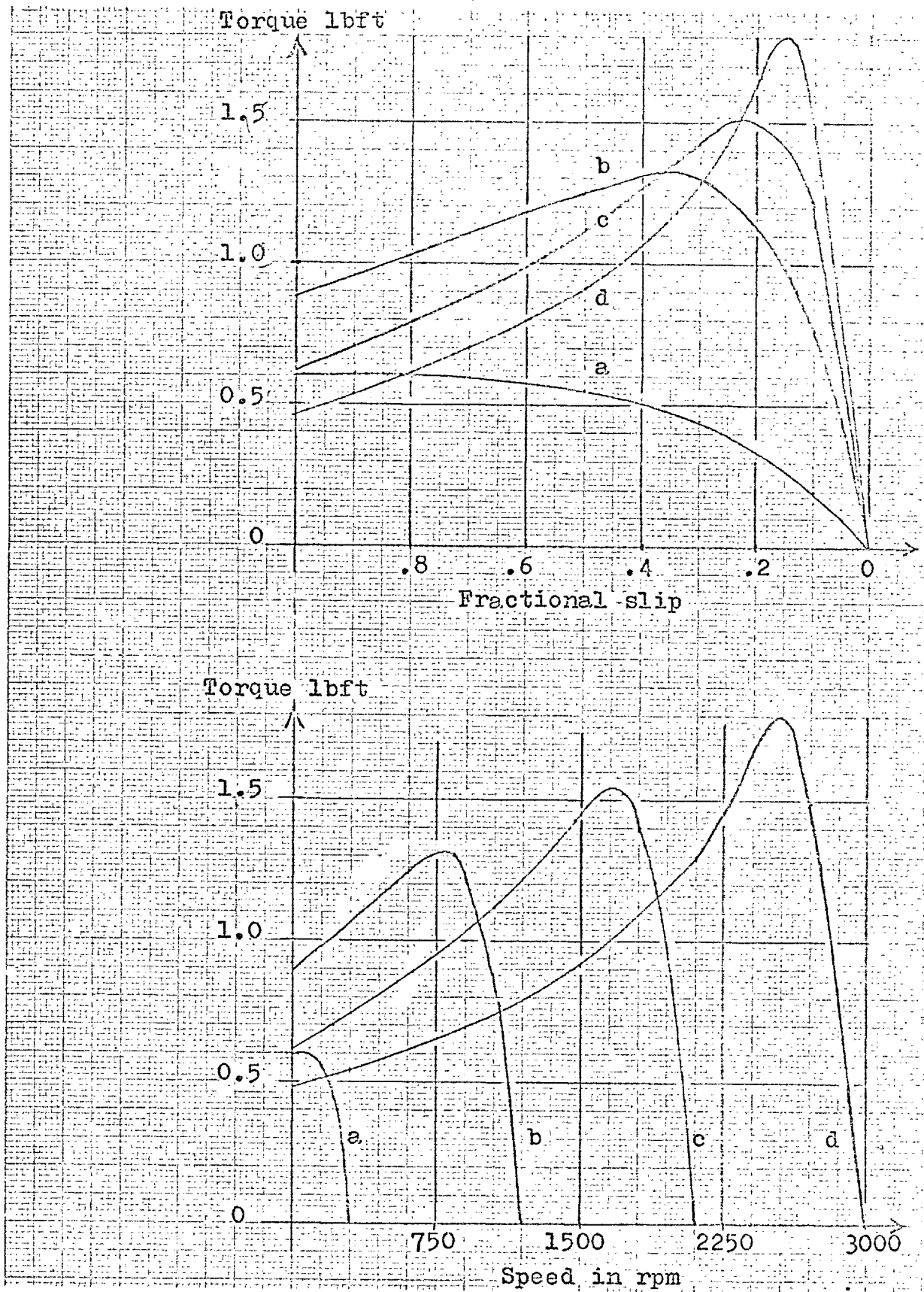


Figure 4.1. Speed-torque characteristic at various frequencies of the inverter supply

- | | |
|-----------|-----------|
| a. 5 c/s | b. 20 c/s |
| c. 35 c/s | d. 50 c/s |

CHAPTER 5

TRANSIENT AND STEADY STATE ANALYSIS OF SQUIRREL CAGE MACHINES USING
NON-SINUSOIDAL WINDINGS AND INPUT VOLTAGES

5.1. Representation of squirrel cage rotors in conventional analysis

In conventional analysis squirrel cage and sheet rotors are represented by their equivalent resistances and inductances. The calculation of these values is based on the presence of sinusoidally distributed rotating fields and constant rotor speed. The space harmonic fields produced by practical stator windings are included in the analysis by superposition that is, the rotor reaction for each rotating field is calculated independently. This is necessary since there are no independent electrical circuits on the rotor. The above representation is valid only under steady state conditions assuming infinite rotor inertia as discussed earlier in section 1.3.1.

For transient performance calculations, the concept of rotating fields has been used by Slater⁹. The rotor is replaced by a two phase sinusoidally distributed winding having equivalent resistances and leakage inductances which vary with the speed of the rotor. A mechanical equation for the rotor movement is written and the electrical and mechanical equations are solved using the Runge Kutta method. Further assumptions are necessary in order to use this technique of numerical analysis although these are not stated clearly in the paper⁸. These necessary assumptions as mentioned in section 3.7 are that the speed of rotor is constant during an incremental time and changes abruptly to a new value during the subsequent increment. Then it is possible to calculate the rotor position at the end of the increment considered. Lyon⁸ has used similar techniques in taking the movement of rotor into account.

In representing the rotor in terms of phase quantities, the space harmonics due to stator and rotor currents in a practical machine are neglected, and in evaluating the parameters, steady state conditions are assumed. This is thought to be incorrect since the object of the analysis is to calculate the transient performance.

5.2. Rigorous analysis of squirrel cage machines

5.2.1. Exact representation of squirrel cage rotors

The complete performance of squirrel cage machines may be calculated using the method described in this chapter. The principles used are similar to those used in the case of slip ring induction motors. However, the formation of the differential equations is much more involved because unlike the wound rotors there are no independent electrical circuits on the rotor. The procedure is described in the order in which it was developed.

5.2.2. Formation of the differential equations

The formation of a set of differential equations which may be solved for the unknown currents is illustrated by considering the simple machine shown in figure 5.1. In this machine there are three coils on the stator and four rotor bars connected by end rings. If the mesh, formed by adjacent bars and the end ring sections, joining them, is considered as an independent circuit, there are as many circuits as the number of rotor bars. Assuming

i_1, i_2, i_3	to be the stator phase currents
i_4, i_5, i_6, i_7	rotor loop currents
c_4, c_5, c_6, c_7	rotor bar currents
$[V]$	voltage matrix denoting the instantaneous voltages externally applied to the coils
$[I]$	current matrix representing i_1 to i_7
$[Z]$	self and mutual inductances between stator phases and rotor loops
$[M]$	coefficients of the final set of differential equations
r_1	resistance of each of the stator phases
r_4, r_5, r_6, r_7	rotor bar resistances
and	
r_2	the resistance of the end ring sections between adjacent bars,

the relationships between loop and bar currents are given by

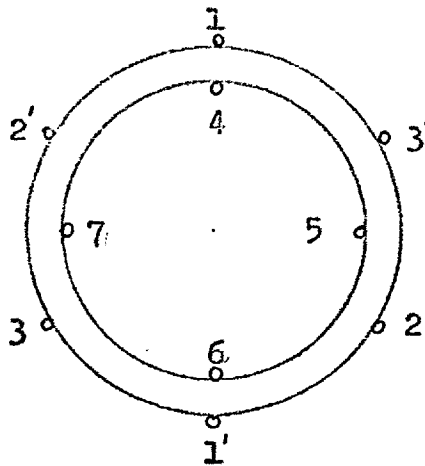
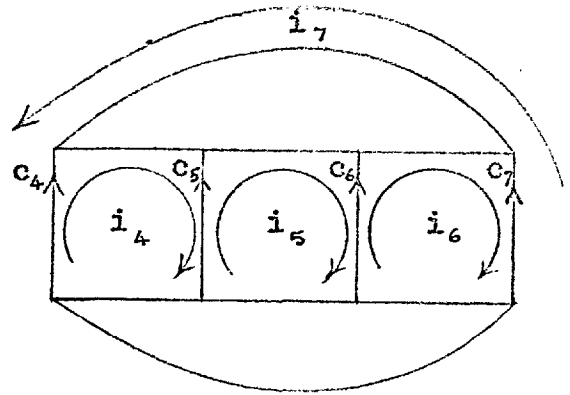


figure 5.1



Rotor electrical circuit

table 5.1 Loop currents

Bar currents are given by 2,3,-2,-3

i_4	i_5	i_6	i_7
2	5	3	0
1	4	2	-1
3	6	4	1
-0.5	2.5	.5	-2.5

correct set

$$\begin{aligned} c_4 &= i_4 - i_7 \dots\dots(1), & c_5 &= i_5 - i_4 \dots\dots(2) \\ c_6 &= i_6 - i_5 \dots\dots(3), & c_7 &= i_7 - i_6 \dots\dots(4), \dots(5.1) \end{aligned}$$

Then using the machine inductances and these currents, the simultaneous differential equations for the system may be written in matrix form as

$$[V] = [\text{resistive drop}] + \frac{d}{dt}([Z][I]) \dots(5.2)$$

The expanded form is given in table 5.2. The bar currents in equations (5.2.4) to (5.2.7) are written in terms of the loop currents using equations (5.1). There are now seven equations with seven unknown currents. These equations do not produce a unique solution for the following reasons.

In equation (5.1) the loop currents i_4 to i_7 can not be determined from the bar currents c_4 to c_7 ; one of the loop currents or a relationship between them is necessary to solve for the loop currents when the bar currents are given. Otherwise there are an infinite number of sets of loop currents satisfying equations (5.1) as illustrated numerically in table 5.1. Only one set of currents can be correct since there is a definite set of currents in the machine. The required relationship is that the sum of the loop currents i_4 to i_7 is zero, that is

$$i_4 + i_5 + i_6 + i_7 = 0 \dots(5.3)$$

The loop currents are the actual currents through the end rings and therefore, if there is no axial flux, there can be no net circulating current in the end rings. Therefore, the sum of the currents passing through the sections must add up to zero. Moreover, since there is no net induced e.m.f. in the circuit formed by an end ring, the sum of voltage drops in the sections is also zero.

The loop currents can now be calculated from the bar currents using any three of the equations (5.1) and equation (5.3). The analytical solution for the loop currents is shown below.

Adding equations (5.1.1.) and (5.1.3) yields

TABLE 5.2

$$\begin{aligned}
 v_1 &= i_1 r_1 + \frac{d}{dt} (z_{11} i_1 + z_{12} i_2 + z_{13} i_3 + z_{14} i_4 + z_{15} i_5 + z_{16} i_6 + z_{17} i_7) & \dots & 1 \\
 v_2 &= i_2 r_1 + \frac{d}{dt} (z_{21} i_1 + z_{22} i_2 + z_{23} i_3 + z_{24} i_4 + z_{25} i_5 + z_{26} i_6 + z_{27} i_7) & \dots & 2 \\
 v_3 &= i_3 r_1 + \frac{d}{dt} (z_{31} i_1 + z_{32} i_2 + z_{33} i_3 + z_{34} i_4 + z_{35} i_5 + z_{36} i_6 + z_{37} i_7) & \dots & 3 \\
 v_4 &= c_4 r_4 - c_5 r_5 + i_4 r_2 + \frac{d}{dt} (z_{41} i_1 + z_{42} i_2 + z_{43} i_3 + z_{44} i_4 + z_{45} i_5 + z_{46} i_6 + z_{47} i_7) & \dots & 4 \\
 v_5 &= c_5 r_5 - c_6 r_6 + i_5 r_2 + \frac{d}{dt} (z_{51} i_1 + z_{52} i_2 + z_{53} i_3 + z_{54} i_4 + z_{55} i_5 + z_{56} i_6 + z_{57} i_7) & \dots & 5 \\
 v_6 &= c_6 r_6 - c_7 r_7 + i_6 r_2 + \frac{d}{dt} (z_{61} i_1 + z_{62} i_2 + z_{63} i_3 + z_{64} i_4 + z_{65} i_5 + z_{66} i_6 + z_{67} i_7) & \dots & 6 \\
 v_7 &= c_7 r_7 - c_4 r_4 + i_7 r_2 + \frac{d}{dt} (z_{71} i_1 + z_{72} i_2 + z_{73} i_3 + z_{74} i_4 + z_{75} i_5 + z_{76} i_6 + z_{77} i_7) & \dots & 7
 \end{aligned}$$

Expanded form of equation 5.2

$$\begin{aligned} i_5 - i_7 &= c_4 + c_5 \\ \text{or} \quad i_7 &= -i_5 + c_4 + c_5 \end{aligned}$$

Substituting this value of i_7 in equation (5.3) yields

$$i_4 + i_5 + i_6 + i_5 - c_4 - c_5 = 0$$

Substituting for i_4 from equation (5.1.2) gives

$$\begin{aligned} 3i_5 + i_6 - 2c_5 - c_4 &= 0 \\ \text{or} \quad 3i_5 + i_6 &= 2c_5 + c_4 \end{aligned}$$

Subtracting equation (5.1.3) from this yields

$$i_5 = (c_4 + 2c_5 - c_6) / 4$$

Using equation (5.1.2) the value of i_4 is obtained as

$$i_4 = (c_4 - 2c_5 - c_6) / 4$$

and from equation (5.1.3), i_6 is given by

$$i_6 = (c_4 + 2c_5 + 3c_6) / 4$$

then

$$i_7 = -i_4 - i_5 - i_6 \text{ from equation (5.3)}$$

The numerical example taken is solved using the above relationships and this gives the correct set of currents in table 5.1, the other sets differ from these values by a constant. Thus the expressions given by equation (5.1) are satisfied by an infinite number of sets while the constraint in equation (5.3) chooses the real set of currents in the machine from these. The first set in the table 5.1, for example, may be obtained by adding 2.5 (which is equivalent to a circulating current in the ring) to each current.

5.2.3 A set of differential equations for which a solution exists

The first six equations given in table 5.2 are considered, the

terms denoting resistive drops are written in terms of loop currents, and the current i_7 is eliminated using equation (5.3), that is, substituting

$$i_7 = -i_4 - i_5 - i_6$$

The operations on the terms denoting resistive drops are illustrated by considering equation (5.2.4). The resistive drop term may be written as,

$$c_4 r_4 - c_5 r_5 + i_4 r_2$$

substituting for c_4 and c_5 from equations (5.1.1) and (5.1.2), the resistive drop term is given by

$$(r_4 + r_5 + r_2) i_4 - r_5 i_5 - r_4 i_7$$

Substituting for i_7 results in the expression

$$(2r_4 + r_5 + r_2) I_4 - (r_5 - r_4) i_5 + r_4 i_6$$

The six equations, with six unknowns, may be written in matrix form as

$$[V] = [\text{resistive drop}] + \frac{d}{dt} ([M] [I]) \quad \dots (5.4)$$

where the elements of matrix $[M]$ are formed by summing up the appropriate inductances. Equation (5.4) is the general electrical differential equation of the system and the performance of the machine may now be calculated using the numerical technique described in sections 3.6 and 3.7.

5.2.4 An example showing the need for reducing the number of rotor equations

The necessity for reducing the number of rotor equations to one less than the number of rotor bars becomes obvious as the number of bars is reduced to two, that is, there is just one coil on the rotor.

In forming the rotor differential equations in this case the rotor circuit may be considered either as two loops or as one coil. It is shown below that taking loops and equating the loop currents to zero, is equivalent to considering the two bars as one coil.

Forming loops around the rotor teeth results in two equations for the two bars, one for loop 2-2' and another for loop 2-2 both taken in the clockwise direction. One of these equations may be eliminated as follows.

Supposing $[Y]$ is the inductance matrix for the coils 1, 2 and 3 where 2 and 3 are the rotor loops;

i_1, i_2, i_3 are the stator and rotor loops currents,

c_2, c_3 bar currents,

r_2 bar resistance

and

r'_2 the rotor coil resistance,

the differential equations of the system may be written as

$$\left. \begin{aligned} e &= i_1 r_1 + \frac{d}{dt} [Y_{11}i_1 + Y_{12}i_2 + Y_{13}i_3] & \dots\dots\dots(1) \\ 0 &= c_2 r_2 - c_3 r_3 + \frac{d}{dt} [Y_{21}i_1 + Y_{22}i_2 + Y_{23}i_3] & \dots\dots\dots(2) \\ 0 &= c_3 r_2 - c_2 r_2 + \frac{d}{dt} [Y_{31}i_1 + Y_{32}i_2 + Y_{33}i_3] & \dots\dots\dots(3) \end{aligned} \right\} (5.5)$$

In these equations the end ring inductances may be added to the self inductances of the loops.

Writing the sum of the loop currents equals zero gives $i_2 = -i_3$ and $c_2 = -c_3$ and since the two rotor loops occupy the same slots the following relationships may be written.

$$Y_{22} = Y_{23}, \quad Y_{12} = -Y_{13} \quad \dots\dots\dots(5.6)$$

The bar currents may be written in terms of loop currents as follows,

$$c_2 + i_2 - i_3 = 2 i_2 \quad \dots\dots\dots(5.7)$$

then substituting equations (5.6) and (5.7) in equations (5.5.1) and (5.5.2) yields

$$\begin{aligned} \text{and} \quad e &= i_1 r_1 + \frac{d}{dt} [Y_{11} i_1 + Y_{12} (2i_2)] \\ o &= (2i_2) (2r_2) + \frac{d}{dt} [Y_{21} i_1 + Y_{22} (2i_2)] \end{aligned} \quad \left. \vphantom{\begin{aligned} e &= i_1 r_1 + \frac{d}{dt} [Y_{11} i_1 + Y_{12} (2i_2)] \\ o &= (2i_2) (2r_2) + \frac{d}{dt} [Y_{21} i_1 + Y_{22} (2i_2)] \end{aligned}} \right\} \quad (5.8)$$

Writing $i_1 = i_1'$, $2i_2 = i_2'$, and $2r_2 = r_2'$ the equations (5.8) become

$$\begin{aligned} e &= i_1' r_1 + \frac{d}{dt} [Y_{11} i_1' + Y_{12} i_2'] \\ o &= i_2' r_2 + \frac{d}{dt} [Y_{21} i_1' + Y_{22} i_2'] \end{aligned} \quad \left. \vphantom{\begin{aligned} e &= i_1' r_1 + \frac{d}{dt} [Y_{11} i_1' + Y_{12} i_2'] \\ o &= i_2' r_2 + \frac{d}{dt} [Y_{21} i_1' + Y_{22} i_2'] \end{aligned}} \right\} \quad (5.9)$$

These two equations are identical to those which would have been written considering the two bars as one coil having a resistance of r_2' which is the sum of the bar resistances. Thus for the simple two bar case it is easy to show the general truth, described in section 5.2, that the number of rotor equations is one less than the number of bars.

5.3. Minimising the number of rotor equations

The number of rotor equations necessary to describe the behaviour of a machine may be reduced from $(N - 1)$ if the rotor currents perform repeatable patterns (repeat or reverse repeat) of a length which is less than the whole machine periphery. The over-riding condition for repeatable rotor current patterns is that the number of rotor bars shall be even, since only in this case with an even number of poles can the current amplitude pattern repeat the minimum possible number of times around the periphery, which is two. It further follows in the case of an even number of bars that the number of repeated patterns must in all cases be a multiple of two. If the number of rotor bars is integral in a number of pole pitches spanning less than half the periphery, the number of repeating patterns is greater than two and can be found by first calculating the number of rotor bars per pole which is N/p , where N is the total number of rotor bars and p is the

number of poles, and then examining if any multiple of N/p below $p/2$ is integral. If, for example, $(\frac{N}{p} \times k)$ is an integral number, which also represents the number of bars in k pole pitches, the number of repeated patterns is given by p/k . (k is the number of poles per repeated section).

Using the constraint, given by equation (5.3), for the solution of rotor currents, namely that the sum of the loop currents is zero, there are two different ways of reducing the number of rotor equations depending on the value of k . If k is an odd number then the number of equations is given by $N \times k/p$, and if k is an even number, the number of rotor equations is given by $(N \times k/p) - 1$. An illustration of the above theory is given considering the case of a 12 pole machine.

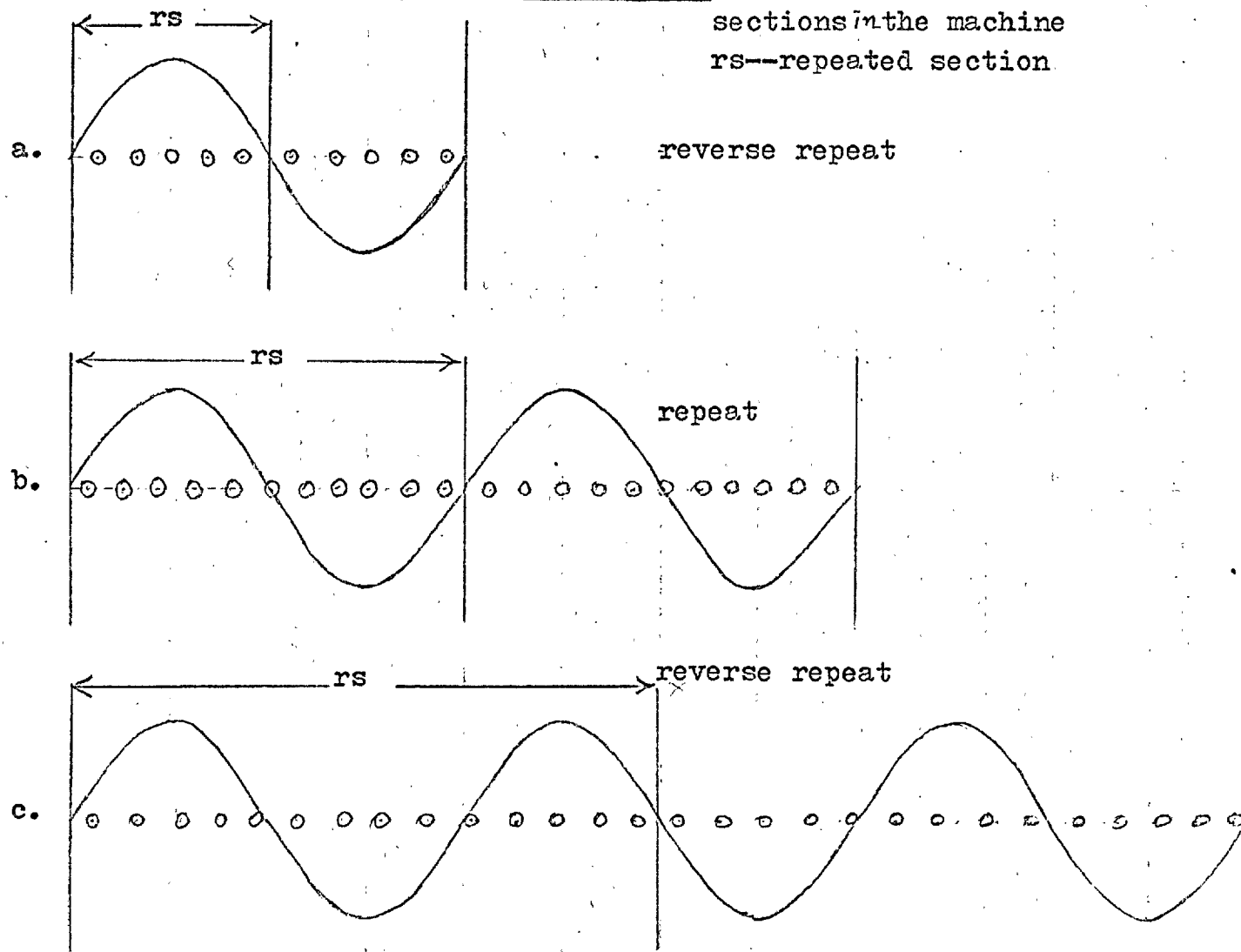
The number of rotor equations depends on the total number of rotor bars. If there are 60 rotor bars, that is 5 bars per pole, the rotor current pattern repeats in amplitude in every pole pitch as shown by the developed diagram in figure 5.2.a and there are only five unknown rotor currents. The constraint for the solution of currents is automatically satisfied since the five currents in one pole pitch are equal and opposite to the five currents in the adjacent pole pitch. Applying the expression given above, k is unity and the number of equations is 5.

If the number of rotor bars is now taken as 66, then k is 2. The 11 bar currents in two pole pitches have to be equated to zero, resulting in 10 equations, which is the same as given by the theoretical expression. The current pattern, repeating in every two pole pitches, is shown in figure 5.2.b.

Again if the number of rotor bars is taken as 56, k is 3 and the number of rotor equations is 14 which is the number of bars in three pole pitches. The repeatable pattern of the rotor currents is shown in figure 5.2.c.

The reduction in the number of rotor equations from $N - 1$ is not possible if asymmetries exist in the machine. For example, the stator windings may not be balanced or the rotor may not be symmetrical electrically or magnetically.

figure 5.2 Unfolded view of two repeated
sections in the machine
rs--repeated section



5.4. Experimental verification of the current distribution in a squirrel cage.

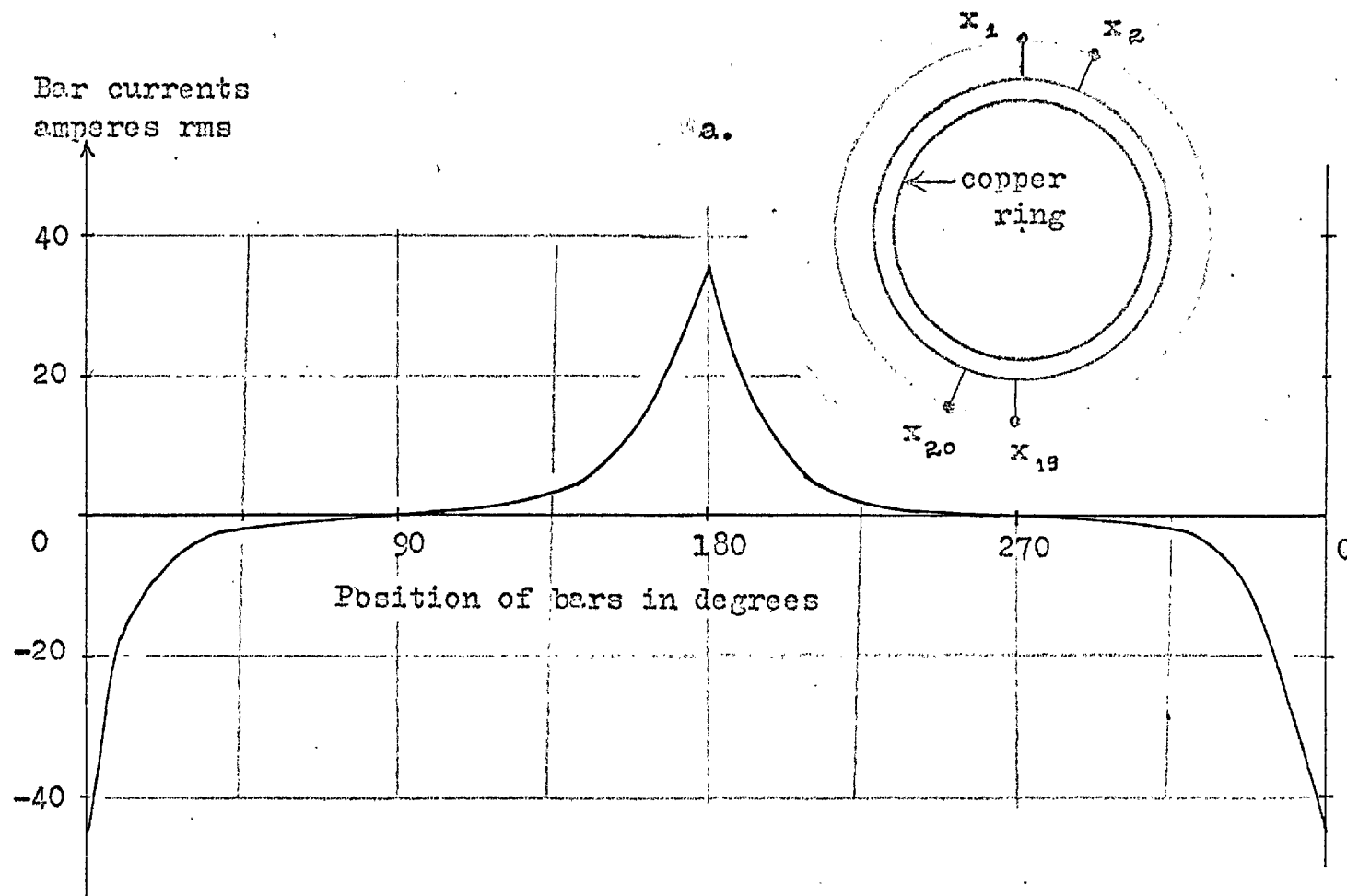
In order to check the current distribution in a squirrel cage a special machine was constructed to permit the measurement of its bar currents. The machine has one primary coil on its rotor and 36 secondary bars on its stator. The bars were first connected to one of the end rings using resistance wires. From the bar side of the wires connections were brought out as shown in figure 5.3.a. The resistance between end ring A and terminal 'X' in the figure, and the total resistance of each bar between the two end rings were measured before the second ring was connected.

A 50 c/s voltage was applied to the rotor coil and the voltages across the resistance wires were then measured under steady state conditions, using a valve voltmeter. The bar currents were then calculated knowing the resistances of the wires. The current distribution found is shown in figure 5.3.b.

To compute the current distribution the differential equations were formed using the rotor loop method described in section 5.2 and also by considering the rotor as a set of independent coils joining diametrically opposed bars. The solution for the currents were then obtained using the step-by-step technique described in chapter 3. Since the two methods are mathematically equivalent, the values of currents obtained should be the same and this proved to be the case. This absolute agreement suggests the validity of the procedure and proves the equivalence of the two methods.

5.5. Experimental and computed results for a conventional squirrel cage induction motor.

The 4 pole machine considered has a three phase symmetrical stator and 32 bars on its rotor. The rotor was chosen to have an integral number of bars per pole in order to minimise the number of system equations. The differential equations for the currents in the machine are formed using the procedure described in this chapter. The torque and current at a few steady speeds are computed using the step-by-step technique described in chapter 3. The first set shown in table 5.3 corresponds to the rotor bars having constant resistance



b. Current distribution in the squirrel cage
with single phase excitation

figure 5.3

TABLE 5.3

Speed in radians per second	Current in amperes			
	Experimental	computed		
		Rotor bar leakage constant	as measured at 50 c/s	Rotor bar resistance corrected for deep bar effect
39.92	6.5	6.628	6.48	6.49
80.45	5.95	6.324	6.204	6.21
130.95	3.64	4.386	4.182	4.21
145.95	2.02	2.406	2.350	2.39
<u>Torque in lb ft</u>				
39.92	1.845	1.876	1.80	1.78
80.45	2.36	2.835	2.68	2.61
130.95	3.06	4.002	3.58	3.57
145.950	2.36	2.810	2.62	2.65

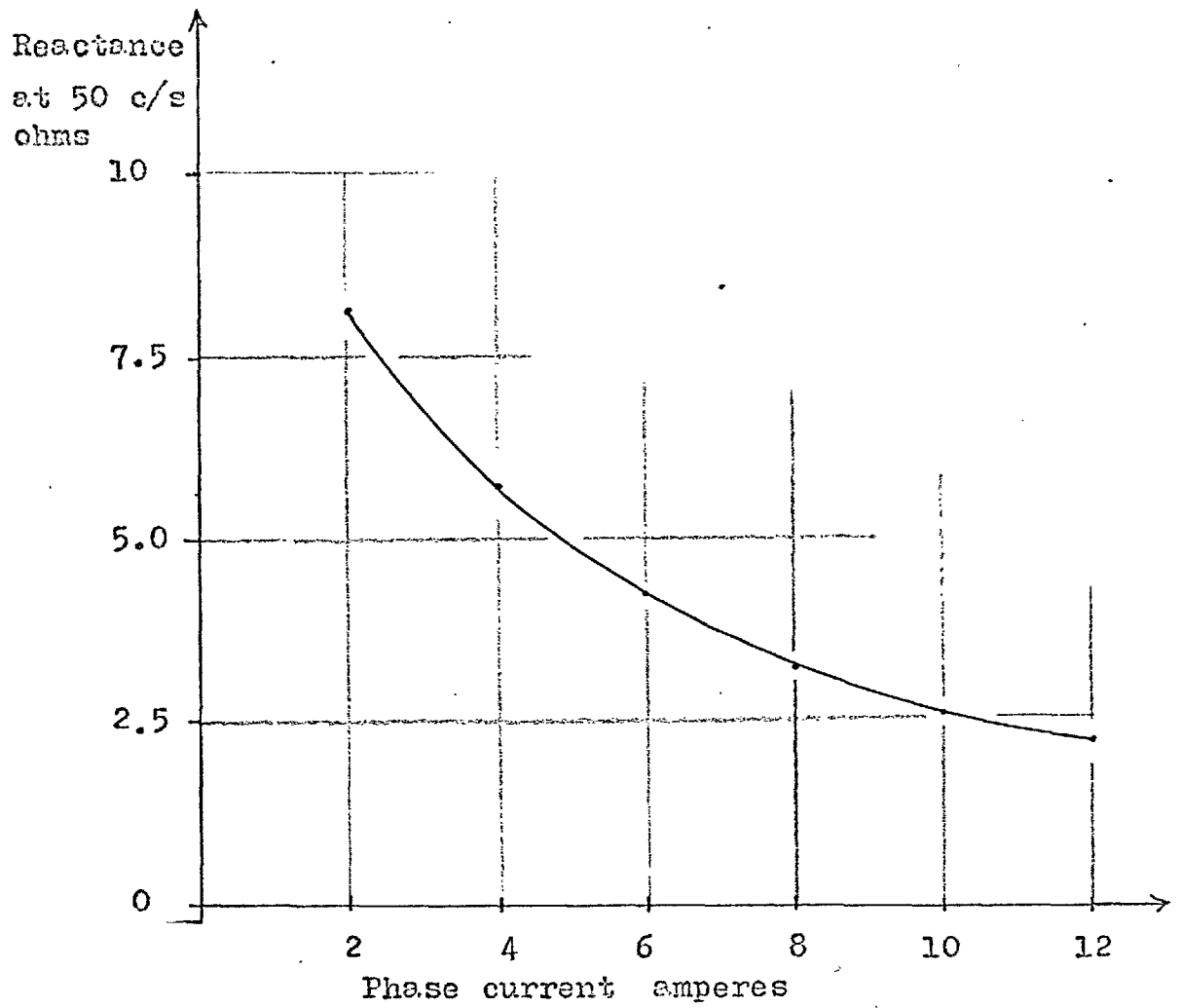
and leakage inductances independent of speed and saturation.

The variation of rotor leakage inductance with current was measured under steady state conditions using a 50 c/s supply and it is shown in figure 5.4. When the leakage inductance is varied according to the rotor current, there is considerable change in the speed torque characteristic. This is shown by the second set.

It is well known that the rotor resistance and leakage inductance vary with the frequency of rotor currents due to deep bar skin effect. In the equivalent circuit shown in figure 1.1.b the parameters of the harmonic branches are adjusted according to the frequency of rotor currents due to the particular stator harmonic field. However, in the analysis described in this chapter all the harmonic fields due to stator and rotor are taken in to account by a single set of equations, therefore, the variation of rotor parameters can not be taken in to account separately for each harmonic current. This difficulty can be overcome easily once it is realised that the leakage inductance and resistance of a rotor bar depends on the instantaneous current and the rate of change of current rather than its frequency. (The use of a single value of resistance and leakage inductance is not exact but results in averaging the effect of the bar current). While using the step-by-step technique for performance calculations, the rotor bar currents are being calculated at every instant. Therefore, the effective resistance and inductance of all the rotor bars can be calculated from the currents and their rates of change. These values may be used in the next step. Though this method is more realistic, the time of computation will be increased considerably. Thus only the average effect of the frequency of rotor currents is taken into account in the computation. That is, the rotor bar leakage and resistance are given constant values at a particular rotor speed. The values are shown in the 5th column in table 5.3.

The effects of interbar currents and the damping of higher frequency fluxes by the laminations investigated by Christophides¹¹ has not been included in the analysis.

The speed torque characteristic was obtained experimentally by Christophides and is shown in the 2nd column in table 5.3.



Variation of rotor leakage reactance
with stator input current

figure 5.4

5.6. Applications of the squirrel cage rotor analysis

The principles developed (to deal with the analysis of squirrel cage rotors) and verified by experimental and alternative methods in the previous sections may be applied in the analysis of the following cases.

1. Sheet rotors:

The current flow patterns in sheet rotors have been investigated by R.C.Robinson, I. Rower and L.E.Donelan,¹⁹ employing a graphical technique and by R.L.Russel and K.H.Norsworthy²⁰ who developed an analytical method. The above works were concerned with the calculation of losses in canned motors and a sinusoidally distributed rotating field is assumed in both cases. Using the principle employed in these works the performance of sheet rotor machines (linear or cylindrical) may be calculated. The rotor may be treated as an assembly of a number of bars placed side by side and connected only at the ends as shown in figure 5.5.a, and the system equations may be formed using the technique developed in this chapter. The solution may be obtained using a 'jw' notation (described in detail in the following chapter) at standstill and using the numerical technique described in chapter 3 when there is relative movement. The results may then be corrected by calculating the reduction factor mentioned in the references 19 and 20. The reduction factor represents the ratio of the actual loss in a sheet to the loss calculated assuming axial flow of currents in the sheet.

An alternative procedure which is more general, but laborious, is described below.

The sheet rotors used in cylindrical or linear machines (the machine itself may be a short stator as in the case of driving a disc or a short rotor as in the case of some accelerators) may be treated as a mesh pattern shown in figure 5.5.b. The resistance and leakage inductance between two points such as A and B in the figure corresponds to those of the physical conductor which is represented by the circuit A - B. The self and mutual inductances of the loops may be calculated from the geometry of the apparatus

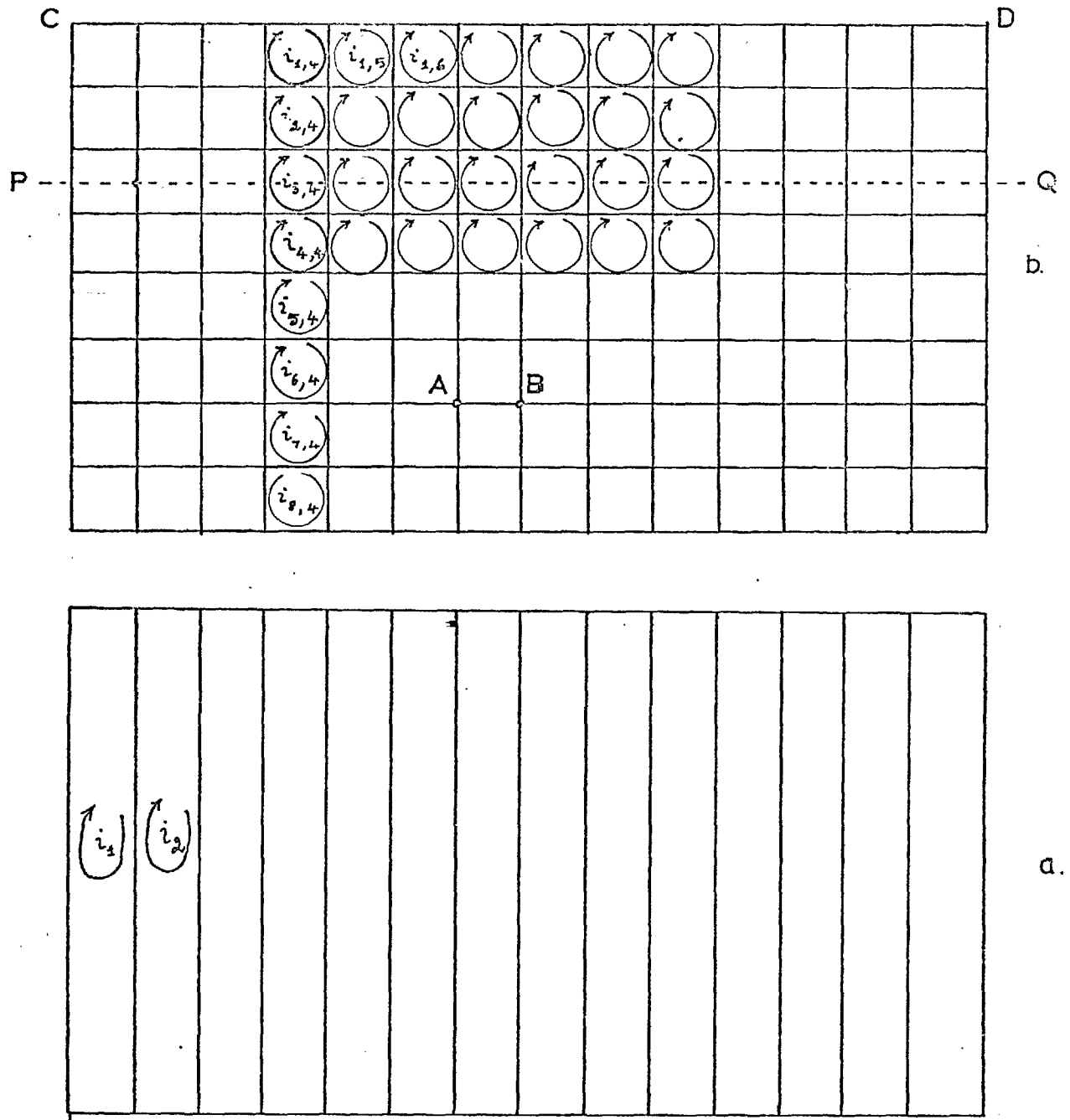


Figure 5.5

Sheet rotor represented as a. squirrel cage
b. mesh pattern

by using the general principles described in appendix 2.

If the (stator) excitation is symmetrical then, with a symmetrical mesh pattern, the equalities of loop currents may be identified. For example, currents $i_{4,4}$ and $i_{5,4}$ in the figure are equal.

The next stage in the analysis is to access the number of independent currents in the chosen pattern. This may be achieved by observing the boundary conditions, for example, in cylindrical machines the point C and D shown in the figure coincide and the sum of the loop currents $i_{1,4}$, $i_{1,5}$ etc. between the points may be equated to zero.

Again the sum of currents, flowing perpendicular to the horizontal lines typified by P - Q shown in the figure, is zero, since the currents passing towards the top or bottom side of the horizontal lines have to return in order to complete the circuit. Thus, an expression relating the currents in the same row in the figure may be formed and the number of such expressions depends on the number of rows of loops chosen for the analysis. It is four in the case shown, corresponding to the rows in one half of the rotor, since the symmetry about the axial line has already been used.

A set of simultaneous differential equations may be formulated and the solution may be obtained using the 'jw' notation (described in detail in the following chapter) at standstill and the step-by-step technique when there is relative movement between the stator and rotor.

2. Synchronous machine with damper winding

The synchronous machine-analysis is a classical example for using the numerical technique developed, since it has a variable air gap, a direct current winding, a squirrel cage winding on its rotor and a.c. windings on the stator, thus requiring several idealisations to calculate the performance analytically. If the number of damper bars is 'N' per pole and if the poles are identical then there are $N - 1$ equations for the damper winding. There is one equation for the d.c. winding and one equation per stator phase.

The complete performance of the machine may be calculated by first forming a set of simultaneous equations and then using the step-by-step technique.

3. Shaded pole motors

This is a special case of squirrel cage motor problem where one of the stator winding is short-circuited. In addition to the rotor and stator equations, an additional equation is formed for the shading rings when the poles are identical in construction. The 'jw' notation may be used to obtain standstill performance, and the numerical technique may be used when the rotor is free to revolve.

4. Variable gap motors

A single phase motor having an air gap which varies along its pole face is known to be capable of starting. The analysis of this type of motor is described in the following chapter considering its starting performance.

CHAPTER 6

VARIABLE GAP SINGLE PHASE INDUCTION MOTOR

6.1. Introduction

The machine considered has four poles, with shading rings on the stator and 22 bars on the squirrel cage rotor. The bars are skewed by the span of the shading rings. This chapter deals with the optimum variation of the air gap to obtain the maximum starting torque.

6.2. System equations

The system has one input current, due to the four series connected coils around the poles and twenty two bar currents. It is assumed that the shading rings behave as if they are connected in series which means there is one unknown current for these. The rotor is assumed to have 22 independent loops around the teeth, inter-connected at the ends. The self and mutual inductances between all the coils in the system may be calculated using the procedure given in appendix 2. The currents in the loops diametrically opposite are equal, thus reducing the number of currents on rotor to 11. The constraint for convergence of currents namely that the sum of the loop currents equals zero, is applied as follows.

$$\begin{aligned} & 2(i_3 + i_4 + \dots + i_{13}) = 0 \\ \text{or} \quad & i_{13} = -i_3 - i_4 \dots - i_{12} \qquad \dots\dots\dots (6.1) \end{aligned}$$

where i_3 to i_{13} are the rotor loop currents.

i_{13} may thus be expressed in terms of the rest of the rotor loop currents and the number of rotor variables is reduced to 10.

There are thus two equations for the stator and ten equations for the rotor and the coefficients are formed as follows.

A pair of poles are considered. There are 11 bars on rotor in two pole-pitches. Supposing the self and mutual inductances between the stator coils and the 11 rotor loops considered are given

by $[B]$ then the equation for the stator coil is given by

$$v_1 = i_1 r_1 + \frac{d}{dt} \left[B_{1,1} i_1 + B_{1,2} i_2 + \dots + B_{1,13} i_{13} \right] \dots\dots\dots (6.2)$$

where i_1 and i_2 are the stator and shading ring currents and v_1 and r_1 are the stator applied voltage and resistance respectively. Substituting for i_{13} from equation (6.1) the equation (6.2) is modified to give

$$v_1 = i_1 r_1 + \frac{d}{dt} \left[B_{11} i_1 + B_{12} i_2 + (B_{1,3} - B_{1,13}) i_3 + \dots + (B_{1,12} - B_{1,13}) i_{12} \right] \dots\dots\dots (6.3)$$

Similarly, the equation for the shading rings and one of the rotor loops may be written as

$$0 = v_2 = i_2 r_2 + \frac{d}{dt} \left[B_{2,1} i_1 + B_{22} i_2 + (B_{23} - B_{2,13}) i_3 + \dots + (B_{2,12} - B_{2,13}) i_{12} \right] \dots\dots\dots (6.4)$$

and

$$0 = v_p = \left[\text{resistive drop} \right] + \frac{d}{dt} \left[B_{p1} i_1 + B_{p2} i_2 + (B_{p3} - B_{p13}) i_3 + \dots + (B_{p,12} - B_{p,13}) i_{12} \right] \dots\dots\dots (6.5)$$

where resistive drop may be obtained using the method given in 5.2.3. nine more rotor equations are written and current i_{13} is eliminated from the terms for the resistance drops. Thus the system equations solvable for the currents in the machine are formed.

6.3. Solution for currents

At standstill coefficients of all the currents are constant and

the currents are sinusoidal and repetitive at steady state. The rms value of currents may be calculated by forming a complex matrix. For example the first stator equation may be written as

$$v_1 = i_1 r_1 + \frac{d}{dt} \left[M_{11} i_1 + M_{12} i_2 + \dots + M_{1,12} i_{12} \right]$$

It is modified to

$$V_1 = (r_1 + j\omega M_{11}) I_1 + j\omega M_{12} I_2 + \dots$$

Similarly all the 12 equations may be converted to complex form and it may be given in matrix form as

$$[V] = [M][I]$$

or

$$[I] = [M]^{-1} [V] \quad \dots\dots\dots (6.6)$$

where $[M]^{-1}$ is the inverse of matrix M

Thus all the currents in equation (6.6) may be calculated in complex form. The 11th loop current is calculated using equation (6.1)

6.4. Torque

It is shown in appendix 4 that the torque between two coils is $2i_1 i_2 \frac{dM}{d\theta}$. In this case the variation of mutual inductance with position may be determined as follows. The mutual inductances at two adjacent rotor positions are calculated and $\frac{dM}{d\theta}$ may be obtained using linear approximation. The average torque developed is given by

$$\frac{1}{2} \sum_{q=1,2} I_q \frac{dM_{qq}}{d\theta} + \sum_{\substack{p=1,2 \\ q=3,13}} I_p^* I_q \frac{dM_{pq}}{d\theta} \quad \text{where } I_p, I_q \text{ are the rms value of currents}$$

and I_q^* is the complex conjugate of I_q and it may therefore be calculated.

6.5. Optimum variation of the air gap

The object of the analysis described above is to find the necessary air gap pattern under a pole to obtain maximum starting

torque. Since the analysis can only be used to find the starting torque for a particular known air gap pattern, a method of trial and error has to be adopted. One pole-pitch is divided into a number of sections, N , and the air gap in each section is allowed to take N values. The starting torque developed in each case is computed and the gap pattern corresponding to the maximum torque is selected. The number of calculations increases as the number of sections in a pole pitch is increased and it is given by $(N)^N$. A reduction in the number of calculations may be achieved by selective computation. For example, the air gap shall always be increased from left to right in the figure, that is away from the shading rings. This assumption is justified by the fact that when there is a step-change in the air gap, the torque developed is towards the smaller gap. Thus the assumption eliminates the patterns where negative torques are produced in any section of the pole-phase. The number of calculations with five steps of air gap is shown in table 6, against the number of sections. Initially a small number of sections are considered and the air-gap values for the first few sections to produce maximum starting torque are determined. The remaining portion of the pole is again divided in a small number of sections and the procedure is repeated. The number of calculations necessary to predict the optimum variation of air-gap, is reduced considerably by the use of this technique.

The variation of zig-zag or slot leakage with air-gap is also a factor in determining the starting torque. This phenomenon is accounted for in the program by first calculating the average air gap at the position occupied by a rotor bar and then recalculating the leakage inductance of the bar. The effect of skew has not been included but is of some importance. The flow diagram of the computer program and the starting torques at various shapes of the pole-face are shown in figures 6.1 and 6.2 respectively. The details of the program are described below;

6.6. Detailed description of the program

Data: Diameter and length of core, resistances and leakage reactances of stator coils, shading rings and rotor bars, the span of the shading rings and rotor loops.

table 6.1 and its formation

The technique used in the program is given in the form
of fortran statements

DO 80 I1 =1 , 5

DO 80 I2 =I1, 5

.

DO 80 In = (In-1), 5

Performance calculations

80 continue

Row

	10	9	8	7	6	5	4	3	2	1
1	220									
2	165	165								
3	120	120	120							
4	84	84	84	84						
5	56	56	56	56	56					
6	35	35	35	35	35	35				
7	20	20	20	20	20	20	20			
8	10	10	10	10	10	10	10	10		
9	4	4	4	4	4	4	4	4	4	
10	1	1	1	1	1	1	1	1	1	1
11	715	495	330	210	126	70	35	15	5	1
12	2002	1287	792	462	252	126	56	21	6	1
13	¹⁰ 5	⁹ 5	⁸ 5	⁷ 5	⁶ 5	⁵ 5	⁴ 5	³ 5	² 5	¹ 5

Row number of sections considered

Rows 1 to 10 -number of individual calculations

Row 11 -increase in the number of calculations due to
the increase in the number of sections

Row 12 -the total number of calculations

Row 13 -number of calculations if the sections take
all the five values of the air-gap

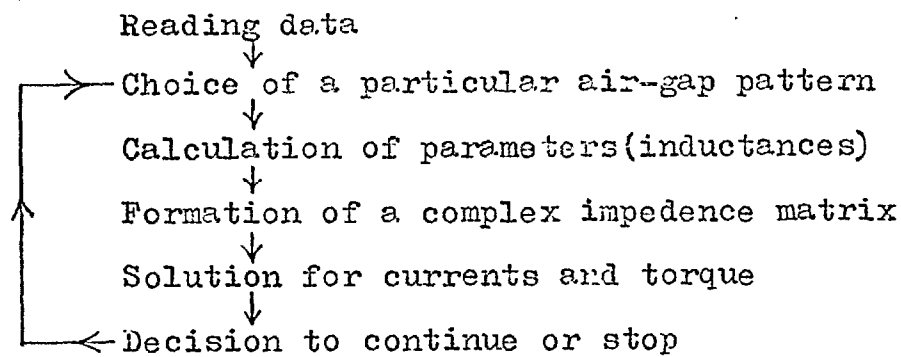


Figure 6.1.Flow chart

Starting torque	Air-gap pattern				Torque due to change in self inductance
0.908	1	1	1	1	0.173
3.318	1	1	1	2	0.389
4.607	1	1	1	3	0.507
5.334	1	1	1	4	0.583
1.212	1	1	2	1	0.237
3.143	1	1	2	2	0.350
4.341	1	1	2	3	0.420
5.094	1	1	2	4	0.469
1.629	1	1	3	1	0.294
3.392	1	1	3	2	0.372
4.538	1	1	3	3	0.424
5.283	1	1	3	4	0.462
1.952	1	1	4	1	0.335
3.638	1	1	4	2	0.395
4.750	1	1	4	3	0.438
5.480	1	1	4	4	0.470
0.289	1	2	1	1	0.022
2.796	1	2	1	2	0.302
4.115	1	2	1	3	0.448
4.847	1	2	1	4	0.538
0.616	1	2	2	1	0.077
2.638	1	2	2	2	0.257
3.869	1	2	2	3	0.354
4.637	1	2	2	4	0.416
1.038	1	2	3	1	0.128
2.894	1	2	3	2	0.274
4.082	1	2	3	3	0.353
4.852	1	2	3	4	0.405
1.363	1	2	4	1	0.164
3.145	1	2	4	2	0.294
4.306	1	2	4	3	0.364
5.069	1	2	4	4	0.411
0.067	1	3	1	1	-0.050
2.612	1	3	1	2	0.254
3.941	1	3	1	3	0.409
4.675	1	3	1	4	0.504

Optimum air-gap pattern with four steps

Figure 6.2.Starting torques for various air-gap patterns

Choice of a particular air gap pattern using one 'do loop' per section of the pole periphery.

Calculation of the parameters:

Calculation of the self and mutual inductances of the stator coils and shading rings (using a numerical integration. An analytical procedure is also possible if the air gap pattern is assumed to be a known function of position).

Calculation of the angular positions of the rotor loops for two positions of the rotor reference.

Calculation of the mutual inductances between the rotor loops and the coils on stator at the two positions mentioned above. These two positions are 1° apart and thus the variation of the mutual inductances with position may be calculated.

The self and mutual inductances between the rotor loops are calculated again using a numerical integration. This completes the calculation of the parameters.

Formation of an impedance matrix

First the inductance matrix for circuits is written and then its size reduced by one, by eliminating the terms corresponding to the 11th rotor loop current. Similarly the 'resistance drop' terms are formed and the 11th loop current is eliminated. The leakage inductances are added to the self inductances of the circuits. The inductances are converted to reactances and the resistances are introduced, thus forming a complex impedance matrix.

Solution for currents and torque

The voltage applied to the stator coils will be known, the other applied voltages are zero. The currents (column matrix) are calculated by multiplying the inverse of the impedance matrix (square) by the voltage matrix (column). The torque may be calculated knowing the currents and the variation of the mutual inductances.

CHAPTER 7

ANALYSIS OF CONTROLLED SWITCHING CIRCUITS

7.1. Introduction

The analysis of electrical circuits containing non-linear elements, such as diodes and thyristors (silicon controlled rectifiers), using the step-by-step technique is illustrated by considering the example of a three phase static inverter. A complete solution of the inverter-load system may be defined as one where the d.c. voltage, firing order of the SCRs, parameters of the inverter and load are known and the currents are calculated from the moment of switching the load on, the inverter having been switched on earlier. The working of the inverter is first described, then the electrical circuits formed at each stage with corresponding differential equations and solutions are given.

7.2. The operation of the inverter

The static inverter considered has six silicon controlled rectifiers, shortly called SCRs, connected in the form of a bridge as shown in figure 7.1. These SCRs when supplied with trigger pulses at the appropriate instants produce three phase a.c. voltages across the points R, Y and B in the figure. This operation known as inversion, is achieved by forming the six different electrical circuits during each cycle of the a.c. voltages, shown in figure 7.2.a to 7.2.f, and requires that the bridge SCRs shall be turned off at the appropriate instants. This can be achieved by using a commutation circuit and a free wheeling diode bridge. The electrical circuits formed during commutation are shown in figures 7.3.a to 7.3.f. The firing sequence of the SCRs is shown in table 7.1.

Various commutation systems have been investigated by Bradley¹² and the one used here is shown in figure 7.4.a. The SCRs A and B are given alternate pulses, shown in figure 7.4.b. Capacitors C_a and C_b divide the supply voltage, V , into two halves. At the moment of firing SCR A capacitor C has a charge of V volts with point O_2 in

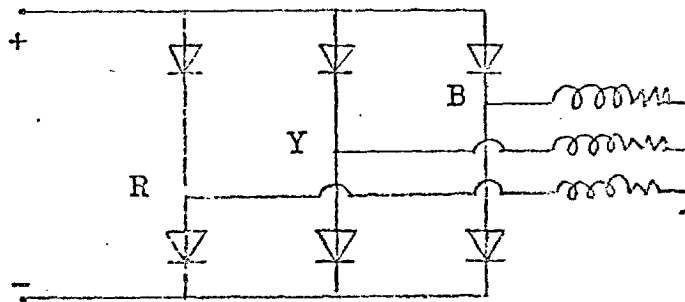


figure 7.1

Inverter bridge

period	SCRs fired		
1	5	6	1
2	6	1	2
3	1	2	3
4	2	3	4
5	3	4	5
6	4	5	6

table 7.1

figure 7.2

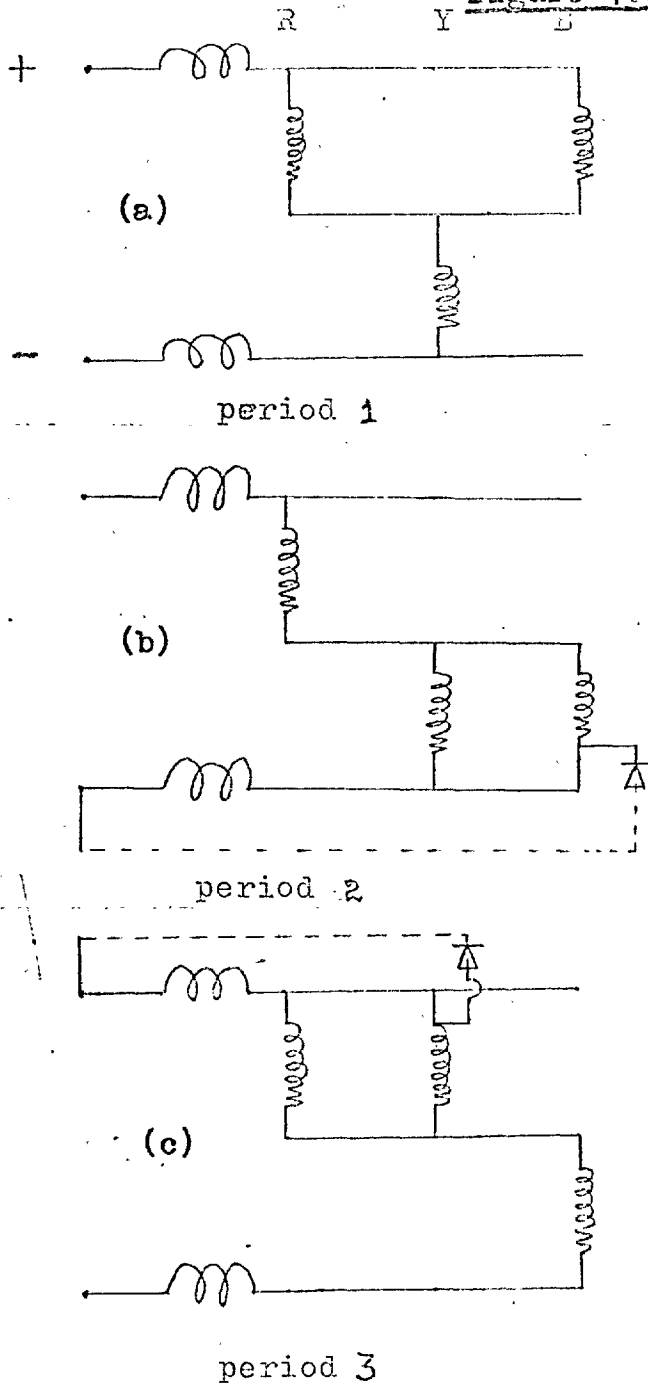
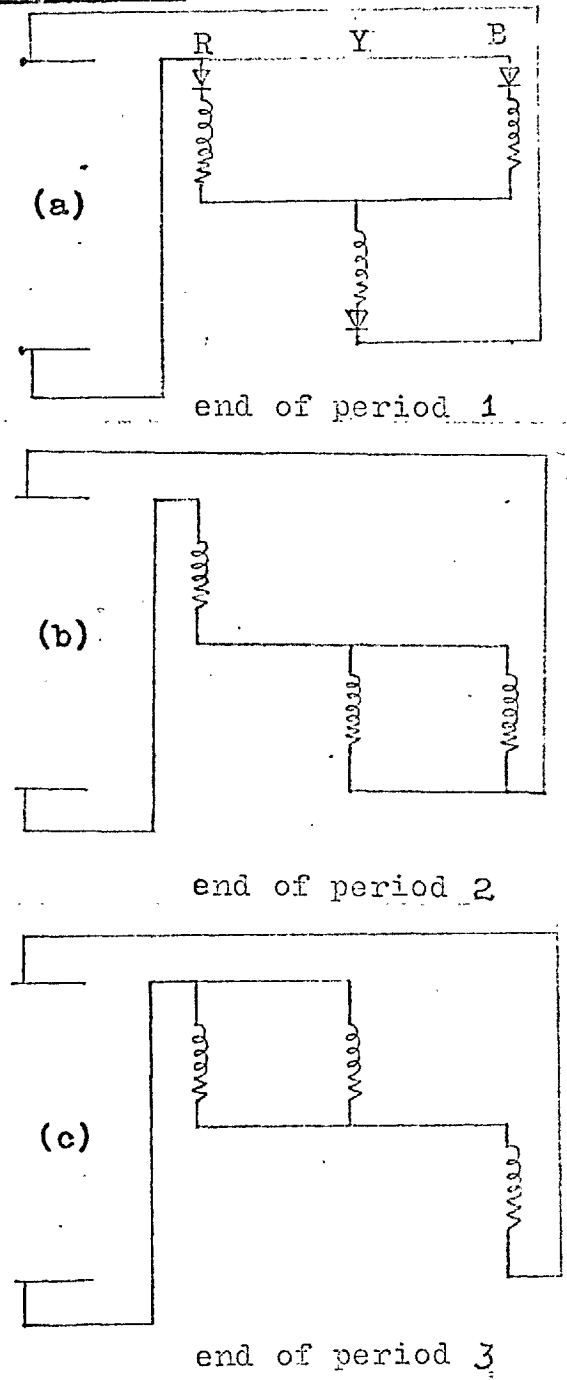


figure 7.3



Various electrical circuits formed during one cycle of inverter operation

figure 7.2

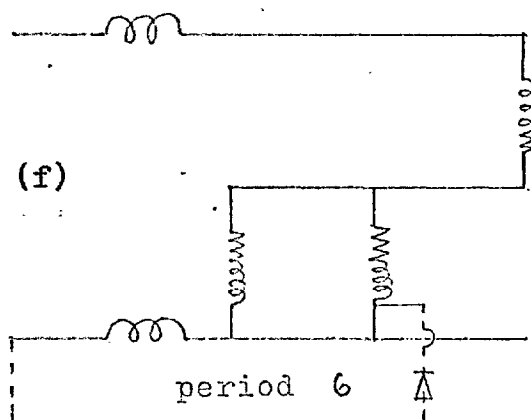
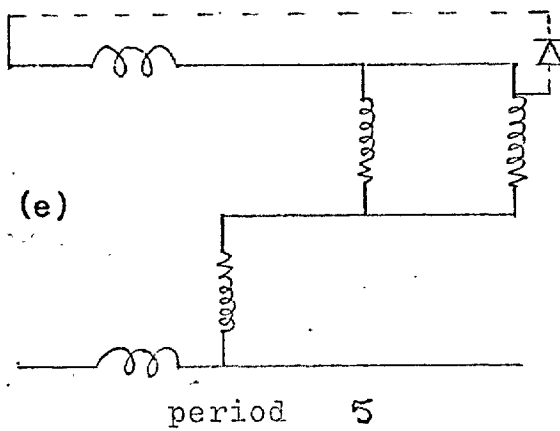
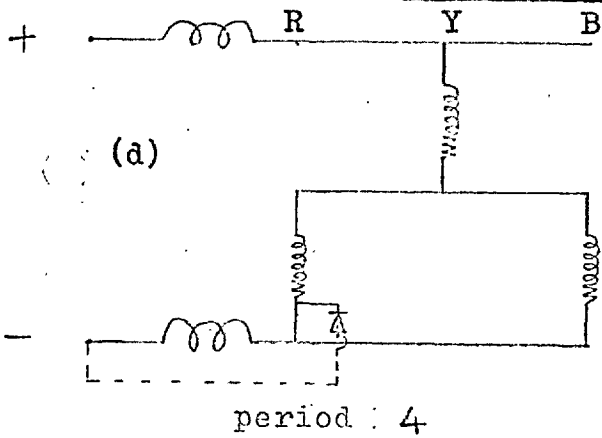
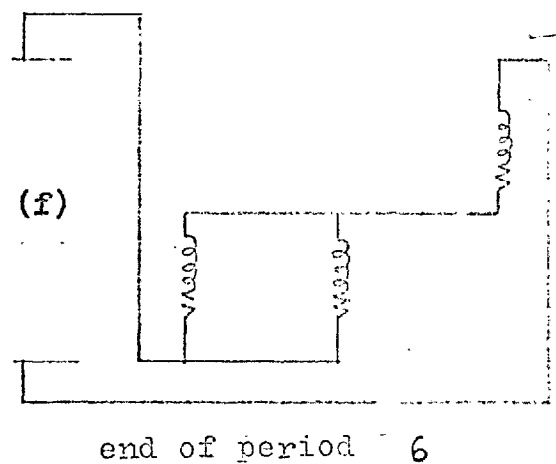
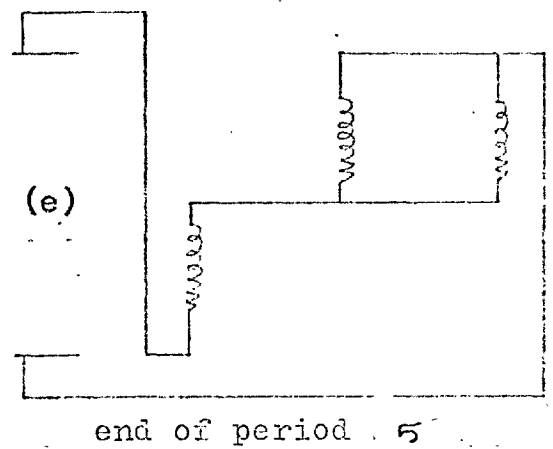
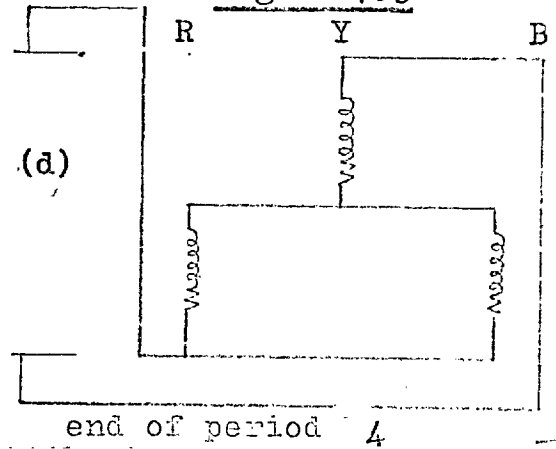
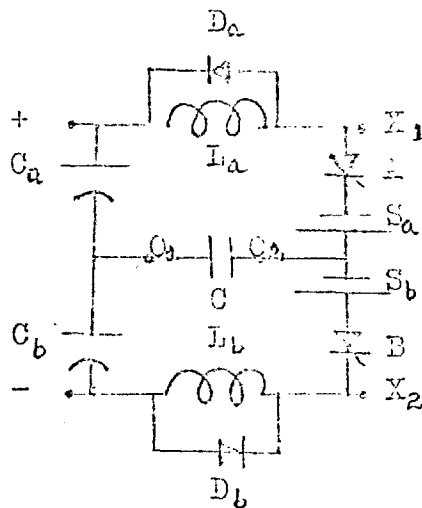


figure 7.3



Various electrical circuits formed during one cycle of inverter operation

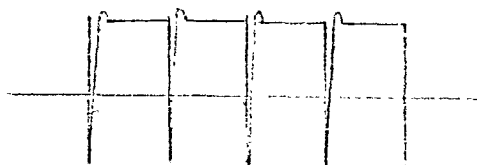


(a) commutating circuit

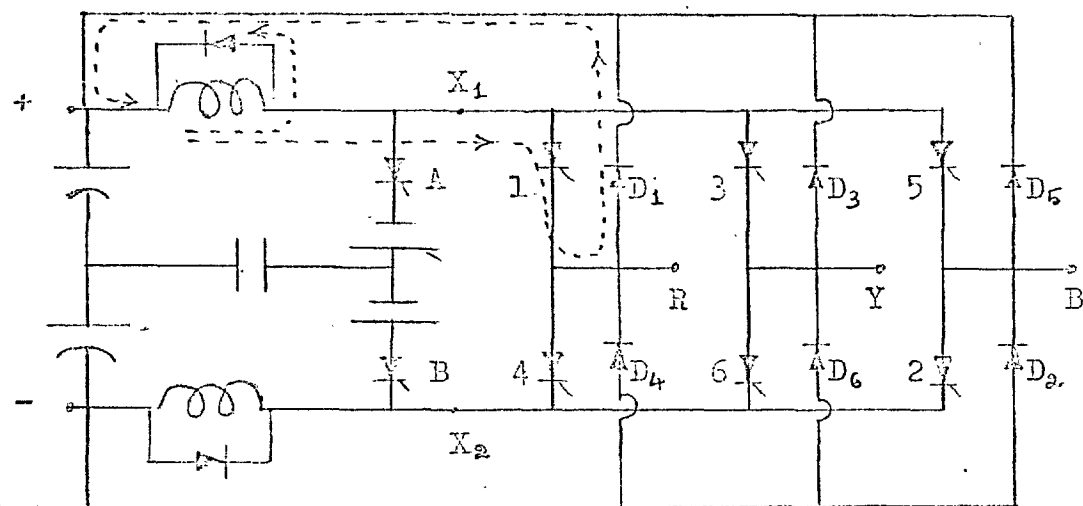
to SCR A

(b) pulses

to SCR B



(c) pulsating dc voltage



(d) Impulse commutated Inverter

figure 7.4

the circuit negative. Therefore the series circuit formed by the inductance, L_a , SCR A the commutation supply, S_a , and the capacitor C has a voltage of 2V impressed on it. The resistance of the circuit being low, the current rises quickly and the whole of this impressed voltage is dropped across the inductance, L_a . Looking from the negative supply point the positive supply point is at a potential of V and point X_1 at $(V - 2V =) -V$ volts, that is, the voltage between points X_1 and X_2 is reversed at the moment of firing SCR A. As the capacitor C's charge decreases and becomes reversed to V volts with point O_2 positive, the potential across the inductance, L_a , falls from 2V to zero and the current in the circuit formed by L_a , A, S_a and C increases up to this instant. The impressed voltage in the circuit is now zero but the current is at its maximum value. The inductance, L_a , will try to maintain this current thus charging the capacitor, C, beyond V volts. As soon as the current decreases, the voltage across the inductance, L_a , is reversed, diode, D_a , conducts and the current through the SCR A becomes zero thus turning it off. The voltage across points X_1 and X_2 is now V volts.

After a chosen interval of time, SCR B is fired and a similar action follows on the negative commutation side, taking the voltage between the points X_1 and X_2 from +V to -V and back to +V. The capacitor, C, is always charged in the correct direction for the next commutation. Thus the commutation circuit produces the voltage shown in figure 7.4.c. across points X_1 and X_2 . With the SCR and diode bridges connected as shown in figure 7.4.d but without any load; two parallel paths are provided across the inductance. The first of these is the diode D_a and the second is the series circuit formed by one of the SCRs fired and a diode in the bridge, therefore SCRs carry a small current even without any load, at the moment when the diode D_a or D_b conducts. The sharing of current between the circuits is governed by their respective resistances. If the diode resistance is higher and its forward voltage for conduction greater than that of the SCR-diode combination, the latter will conduct first and thus prevent the diode conduction. This point appears to have been neglected in the literature but of some importance since it affects the rating of the bridge SCRs.

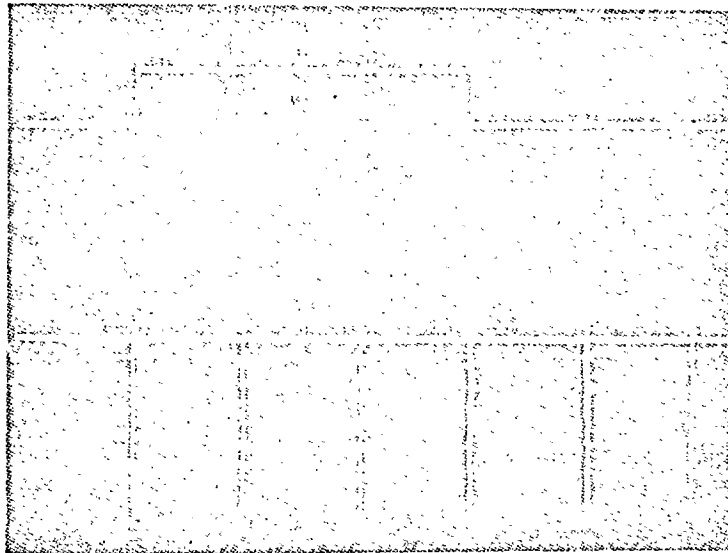
One cycle of a.c. produced corresponds to six inverter periods, an inverter period being the time between two pulses of voltage across

the bridge. Point R is connected to the positive line during the first three periods and to the negative line during the next three periods. Similarly point Y and B are connected to either the positive or the negative lines during the appropriate inverter periods to form a 3 phase supply. The various voltage waveforms are shown in figures 7.5 and 7.6.

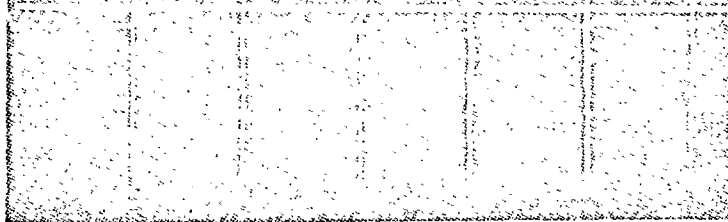
The commutation or change of connection of a phase from one terminal to the other terminal of the supply is achieved as follows. Considering period 3 in the table as an example, the phases are connected as shown in figure 7.2.c during the period. At the end of period 3, the voltage across the bridge is made negative by the commutation circuit. The load current is now transferred from the SCRs to the diode bridge as shown in figure 7.3.c, the current being maintained by the inductance of the load, and the energy is fed back to the supply. The large capacitors across the supply ensure that the rise in voltage due to the energy flowing from the load inductances is not unduly large. The conducting SCRs go off and the firing pulses during period 4 result in the circuit shown in figure 7.2.d, SCR 4 is conducting instead of SCR 1. An important point which must be made here is, that phase R is carrying a positive current which at the end of period 3 is transferred to diode D_4 and this current is reduced steadily. Now, SCR 4 is fired but it can not take over a positive current from diode D_4 . But the current in phase R flowing against the supply soon becomes zero and then SCR 4 conducts and carries the negative current. This phenomenon shown by dotted lines in the figure occurs whenever a phase is reconnected from one supply line to the other, and it is to the advantage of the designer since the SCRs carry only the power components of the currents and not any currents against the voltage of a phase. In figure 7.8 the power component of a phase current carried by a SCR is shown shaded.

7.3. Analysis of the inverter-load system

The analysis is carried out in two stages, first the commutation side is analysed and the waveform of voltage between points X_1 and X_2 is calculated neglecting the effect of load current, and then the bridge circuit is analysed. The commutation circuit can be simplified to a series circuit having resistance, inductance and

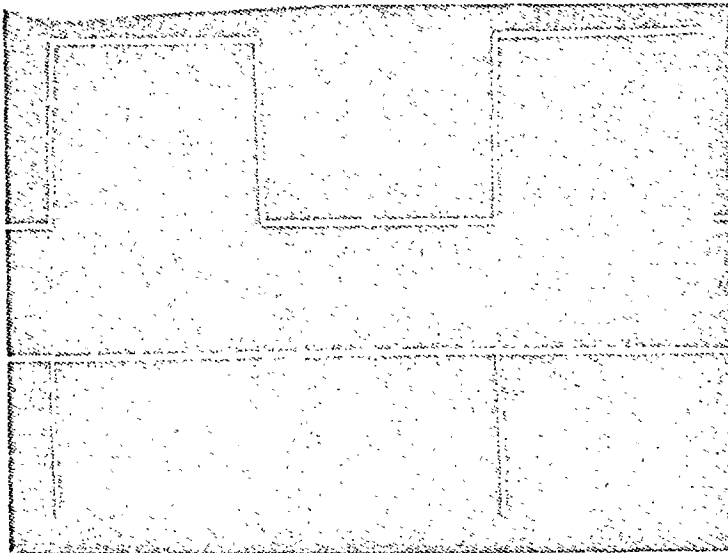


(a)

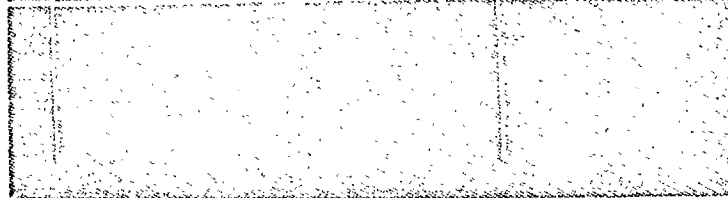


(b)

figure 7.5



(c)

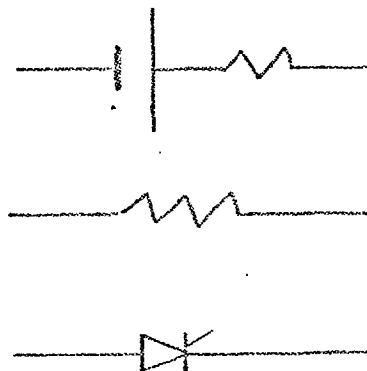
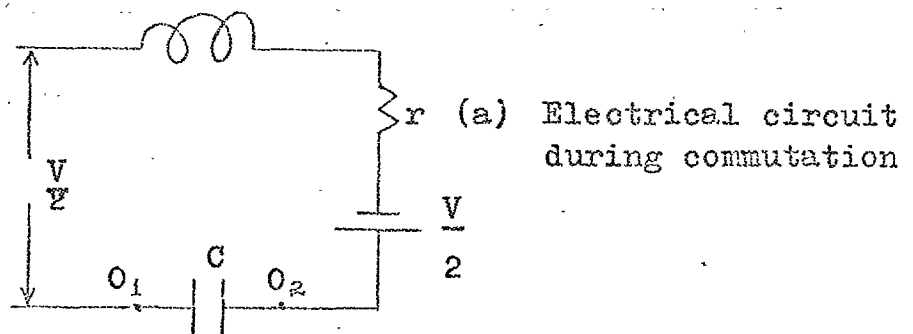
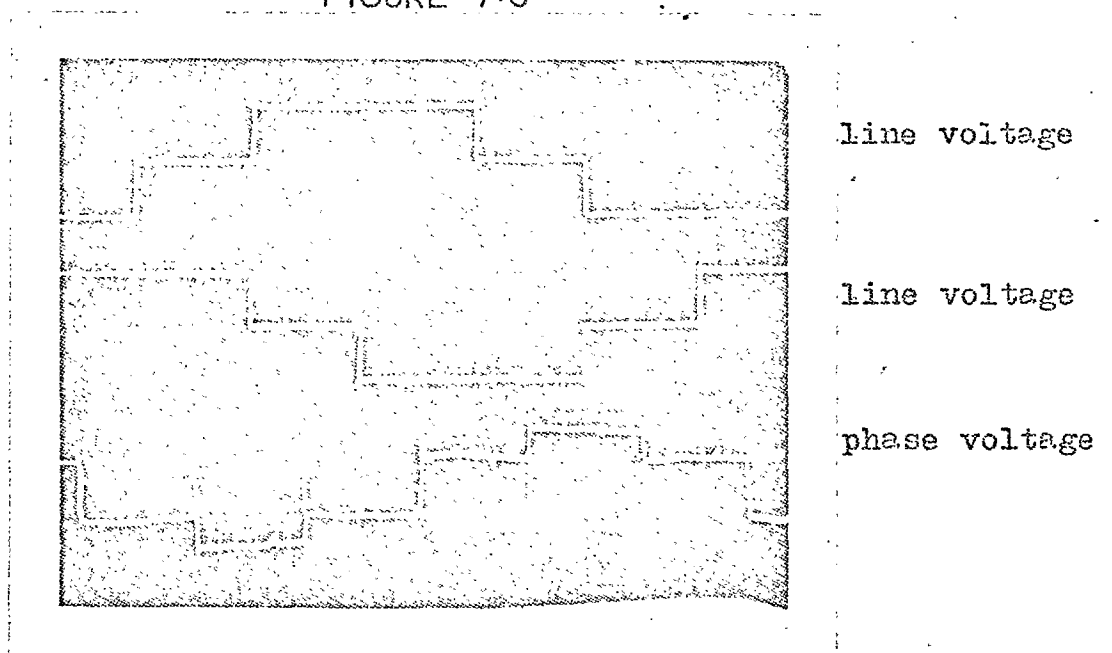


(d)

Inverter voltage waveforms

- (a) voltage across an SCR in the bridge
- (b) pulsating dc voltage across the bridge
- (c) voltage across the commutating capacitor
- (d) voltage across one of the inductances

FIGURE 7.6



(b) representation of a SCR as a fixed voltage drop in series with a resistance or as a resistance

figure 7.7

Voltage and current waveforms with partial conduction

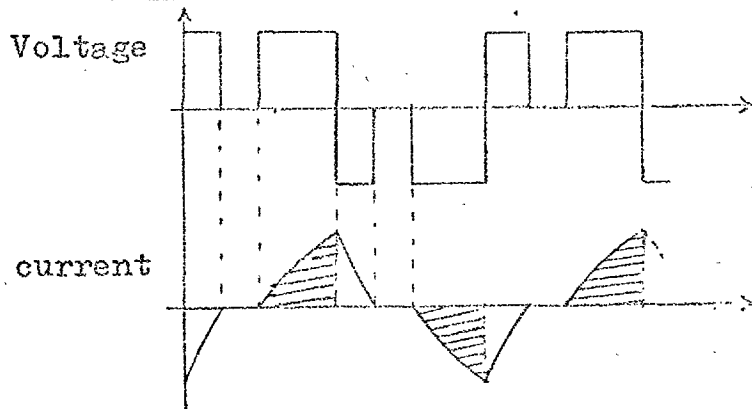
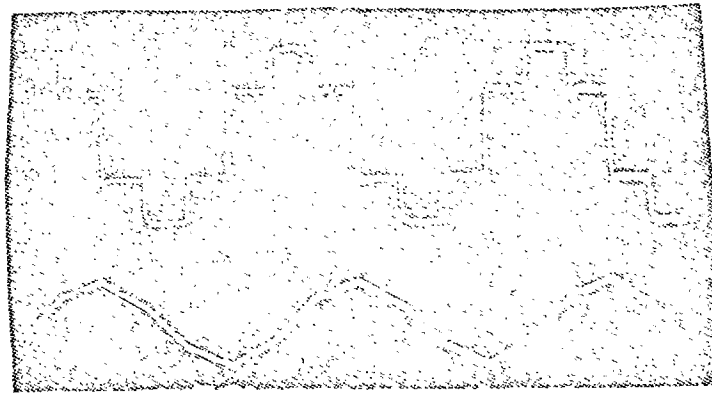
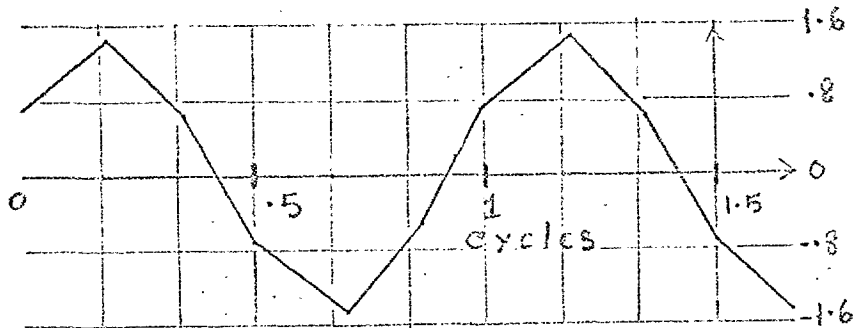


figure 7.8



(a)

(b)



(c)

figure 7.9

(a) phase voltage

(b) load current -experimental

(c) load current -computed

1.98A ave

1.04Arms

1.04Arms

capacitance as shown in figure 7.7.a. The charge on the capacitor C at the moment of triggering a SCR is V volts with point O_2 , negative when SCR A is fired and positive when SCR B is fired. In either case the total voltage around the circuit is 2V in the beginning and the current and voltage across the inductance may be calculated using the step-by-step technique. In the computer program calculations may be terminated when the voltage across the inductance becomes zero and starts increasing in the reverse direction, as the diodes conduct at this instant. The voltages at various times can be stored in the program for use in the second part of the analysis.

The electrical circuits formed during each of the inverter periods are shown in figure 7.2 and those formed during the commutation times in figure 7.3. After the commutation of a phase from the positive side to the negative side or vice versa, it is immaterial whether the appropriate SCR or a parallel connected diode carries the current provided it is assumed that the resistances of the SCR and diode circuits are equal, since the phase point is connected to the correct side of the supply in either case. Supposing the load is switched on during period 1, the currents may be calculated by solving the three equations given below using the step-by-step technique.

$$V = i_\ell r_\ell + i_b r_b + i_y r_y + \frac{d}{dt} [L_\ell i_\ell + L_b i_b + L_y i_y] \quad \dots(7.1)$$

$$V = i_\ell r_\ell + i_r r_r + i_y r_y + \frac{d}{dt} [L_\ell i_\ell + L_r i_r + L_y i_y] \quad \dots(7.2)$$

$$i_\ell = i_y = i_r + i_b \quad \dots(7.3)$$

where i_ℓ, i_r, i_y, i_b are the line and phase currents.

In these equations the conducting SCRs are considered as part of the load resistance. However, the SCRs could be also represented as a constant voltage drop in series with a small resistance as shown in figure 7.7.b. In solving the above equations it is desirable to have a convention that currents flowing towards the neutral point of the load are positive. The terminal d.c. voltage is taken to be the common reference in all the equations and the signs of the resistance and inductance of a phase are reversed when the phase is connected to the negative side. If this procedure is followed for all the inverter periods, the phase currents calculated will have

a.c. waveforms. Thus for period 1 the signs of the resistance and inductance of Y phase are reversed. At the end of period 1, the d.c. voltage across the bridge is made negative and the phase currents flow through diodes as shown in figure 7.3. The equations for this state are given by,

$$-V = i_{\ell} r_{\ell} + i_b r_b + i_y r_y + \frac{d}{dt} \left[L_{\ell} i_{\ell} + L_b i_b + L_y i_y \right] \quad (7.4)$$

$$-V = i_{\ell} r_{\ell} + i_r r_r + i_y r_y + \frac{d}{dt} \left[L_{\ell} i_{\ell} + L_r i_r + L_y i_y \right] \quad (7.5)$$

$$i_{\ell} = -i_r + i_b = -i_y \quad \dots\dots\dots (7.6)$$

where V is the voltage across points X_1 and X_2 , calculated earlier. The currents are calculated from these equations, the initial currents being those at the end of period 1. After the commutation time the SCRs are fired again and the circuit formed during period 2 is shown in figure 7.2.b. Now, both Y and B phases are connected to the negative side therefore, their resistances and inductances have their signs reversed. The initial currents are those calculated above at the end of the commutation time. The procedure may be continued for several cycles until steady state conditions are reached.

7.4. Advantages in using the step-by-step technique

7.4.1. Voltage waveform: In the case of a static load fed by an inverter, currents vary exponentially and can be calculated analytically if the waveforms of the voltages are known. However, if the conduction angle of the SCRs is less than 180° , it is necessary to find the instant at which a phase is isolated from the supply. For example, at the end of period 3, R phase current is carried by diode D_4 which connects the phase to the negative. If SCR 4 is not triggered immediately R phase is connected to the negative, only as long as the current is positive. When this current is reduced to zero, R phase is isolated until SCR 4 is fired. Thus, the waveform of a phase voltage depends upon the current and therefore can not be known initially. Though some of

the analytical methods can be adapted to this new situation the instant at which a phase is isolated can be determined only by trial and error. However, in the step-by-step technique currents are calculated in successive steps and hence the instant at which the current becomes zero is easily identified. A typical case of voltage and current waveforms is shown in figure 7.8.

7.4.2. Currents during commutation time: At the end of an inverter period, voltage across the bridge is suddenly reversed to $-V$ volts. During the commutation time, SCR currents are transferred to the appropriate diodes and the voltage across the bridge rises from $-V$ to V . Since the bridge voltage is varying with respect to time and is not given by a simple function of time, a step-by-step procedure is the only possible way of obtaining a solution. If an analytical solution is sought, further simplifications would be necessary.

7.4.3. Non-linear or dynamic load:

For the case of an inverter feeding a non-linear "load" such as an induction motor, no analytical solutions are available but the step-by-step technique can be used to calculate the performance of the system. The differential equations for the case of a three phase wound rotor machine may be formed as in the case of the static load with the following difference, that is, terms denoting changes in flux linkages in equations 7.1, 7.2, 7.4 and 7.5 would be sum of the products of the appropriate inductances and all the currents in the machine. For example, an equation similar to 7.1 may be written as

$$V = i_{\ell} r_{\ell} + i_1 r_1 + i_2 r_2 + \frac{d}{dt} \left[L_{\ell} i_{\ell} + \sum_{p=1}^{p=6} (M_{1p} i_p + M_{2p} i_p) \right] \quad (7.7)$$

in the case of a 3 phase wound rotor machine.

The stator currents are related by equation 7.3. The rotor equations may be formed as described in earlier chapters. The complete performance of the system can be calculated by forming appropriate equations for each inverter period and solving for the currents. Thus, it is seen that in the case of systems containing elements which are switched on or off, step-by-step technique may be used to calculate the performance.

A particular case with full conduction of SCRs and static load is considered and a computer program to calculate the currents is written. The experimental and computed results are shown in figure 7.9 and the flow-diagram is shown in figure 7.10. The shape of the waveform as well as the instantaneous values of current show close agreement between the computed and measured values.

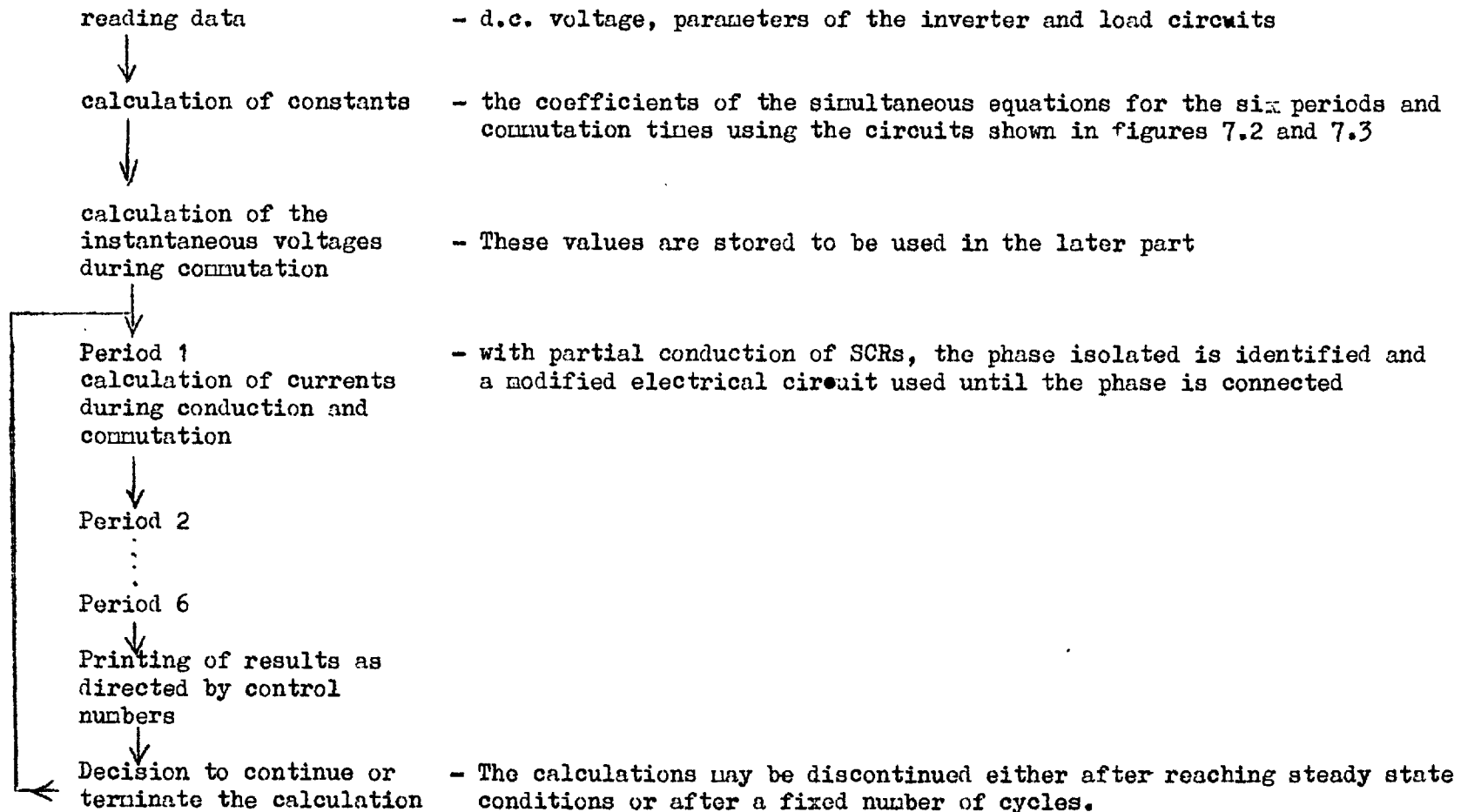
7.5. Conclusions

It has been shown in this chapter that the step-by-step can be used to calculate the performance of systems having non-linear elements, this analysis therefore, can be used to design systems having desired characteristics. For example, whilst designing an inverter-induction motor system "developing a constant torque from zero to a particular speed", it is necessary to increase the a.c. voltage approximately proportional to the frequency. (The exact variation of the applied voltage with frequency can be calculated only by trial and error and this variation results in a constant air gap flux and current densities, thus developing a constant torque). The a.c. output voltage may be varied either by changing the d.c. input voltage or by controlling the angle of conduction of the SCRs. The latter is more economical since no additional equipment is necessary.

From the d.c. input voltage and the parameters of the load circuit, (which is an induction motor in this case) the necessary variation of the angle of conduction in order to obtain a constant torque may be calculated using the numerical technique by making suitable alterations in the computer program. This calculation requires that the machine develops a torque, greater than the desired constant value, with full conduction.

FLOW CHART

Figure 7.10



PART 2

EXPERIMENTAL INVESTIGATIONS

CHAPTER 8

The strain gauge apparatus

8.1. Introduction

The strain gauge apparatus designed to measure the instantaneous torque acting on the stator of the induction motor is shown in figure 8.1. It was decided to support the stator on two external bearings and restrict its movement by steel bars carrying strain gauges. The torque on the stator acts on the bars and stresses the gauges producing a change in resistance proportional to the strain.

Now, the stator of the machine has inertia and the steel beam an associated spring constant. The apparatus therefore has a natural frequency of oscillation. It is the object of the following design to achieve the highest possible natural frequency coupled with an amplitude of electrical signal, for a particular torque, that can be conveniently measured. For example higher output requires long and weak bars whereas a higher natural frequency requires short and stiff bars.

In earlier torque measurements¹³ strain gauges have been fixed to the rotor shaft, thus measuring the torque that is transferred from the motor to the loading machine. This system is more complicated since there are inertias on either side of the gauges. Moreover, the small output signals have to be collected through slip rings. Measuring the torque on the stator has none of these difficulties.

8.2. The relationships governing the operation of the system

As shown in figure 8.1 the rotation of the stator is restricted at a distance R from the axis of the machine. The neutral axis of the bars and the axis of the machine are in the same horizontal plane. The bars are fixed to the base forming a cantilever with the free end attached to the stator. If a torque T is applied on the stator, the force acting at the free end of each bar is given by

$$\text{Force} = F = T/2R$$

Strain gauge apparatus

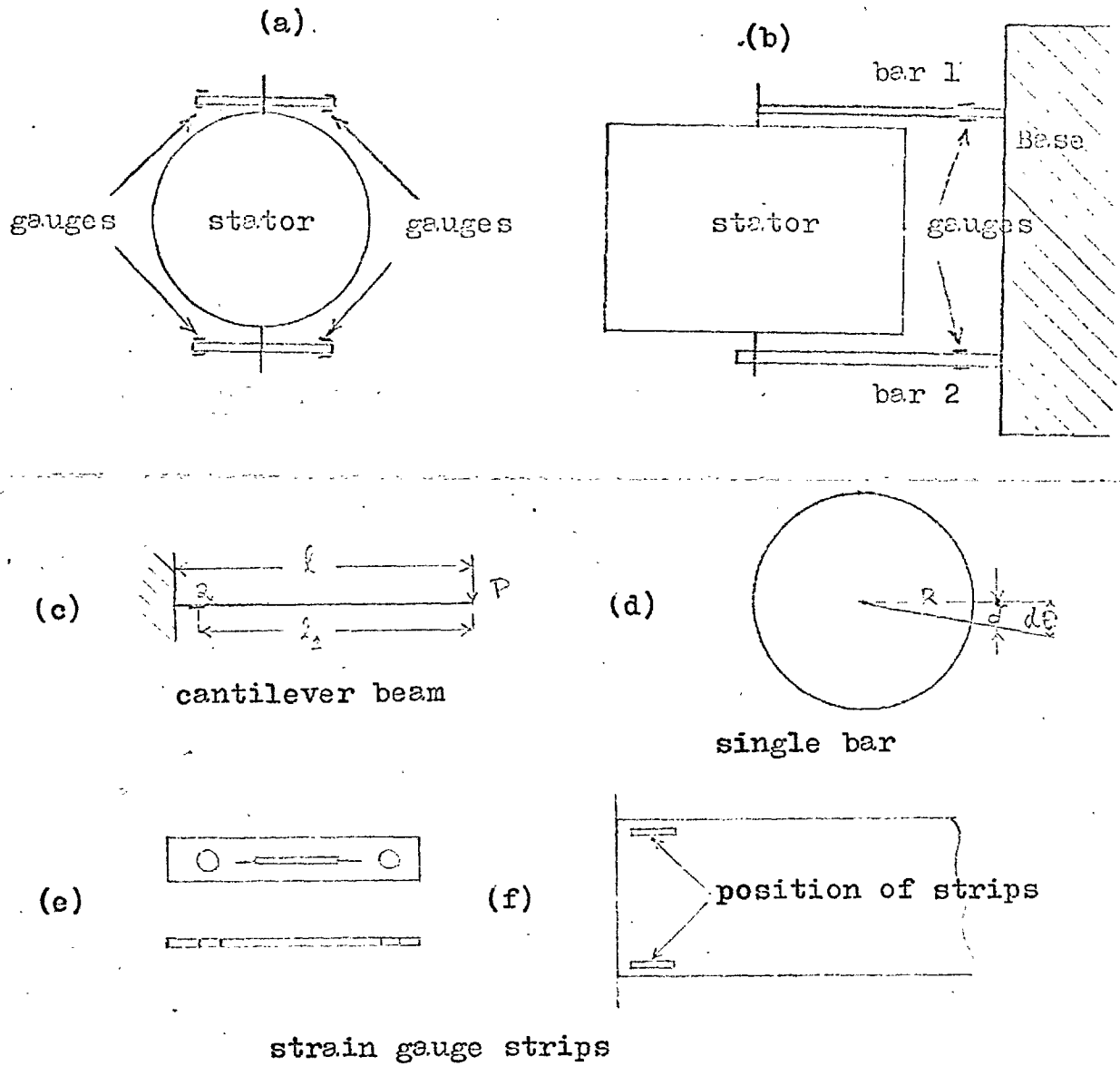


figure 8.1

The deflection of the free end is given by¹⁴

$$\delta = \frac{F l^3}{3EI} \quad \dots\dots\dots (8.1)$$

where δ is the deflection, l the length of the bar, E the Young's modulus for the steel and I the moment of inertia of the cross-section of the bars in the vertical plane. If the stator rotates through an angle $d\theta$, then the vertical deflection and rotation are related by the following equation

$$R d\theta = \delta$$

Substituting the value of δ from equation (8.1) the force for a rotation of one radian is given by

$$K = \frac{3 EIR}{l^3} \quad \dots\dots\dots (8.2)$$

If J is the moment of inertia of the stator, the equation of motion is given by

$$J \ddot{\theta} + K \theta = 0 \quad \dots\dots\dots (8.3)$$

and the natural frequency of the system by

$$N = \frac{1}{2\pi} \sqrt{\frac{K}{J}} \quad \dots\dots\dots (8.4)$$

The stress produced at a distance l_1 from the free end of the bar is given by

$$f = \frac{F l_1}{I} \times \frac{d}{2} \quad \dots\dots\dots (8.5)$$

where

$$I = \frac{bd^3}{12}$$

and b and d are the breadth and depth of the bars.

8.3. Optimum design of a strain gauge torque meter

In order to see whether true optimisation is possible, the parameters were varied in sets and their effect on the natural frequency and stress were calculated. The effect of varying the length of the bars, on the stress, natural frequency and the bandwidth product are shown in figure 8.2. The length corresponding to the maximum bandwidth product is the true optimum value. However, this value results in a deep and thin bar section. The section would be too weak in the horizontal direction so that the material would be stressed beyond its yield point. Moreover, the physical size of the strain gauge must be considered in choosing the length as sufficient space must be available to fix the gauge strips. Thus the true optimum length can not be used. However, it will be observed from figure 8.2 that the reduction in stress, and therefore the output signal obtained is small compared with the decrease in the natural frequency, if a length in the region 4" to 10" is chosen. That is, a length in the "flat portion" of the bandwidth curve is chosen. This is because the output is proportional to the length whereas the bandwidth product is proportional to $1/\sqrt{l}$. This choice enables to design a practical section without weakening the bars in the horizontal direction.

The variation of stress with respect to the breadth of the bar, keeping the natural frequency constant by simultaneously varying the depth, is shown in figure 8.3. It may be observed that the smallest practical value has to be used to maximise the stress and output signal.

Again the effect of varying the depth of the section may be observed from figure 8.4 and it is clear that the depth must be increased to its practical limit.

8.4. The strain gauge strips

If the strain gauges are bonded to the bars, they can not be recovered to be used with other sections. Therefore, strain gauges are first bonded to steel strips as shown in figure 8.1.e and f. and then the strips are screwed tightly to the bars using

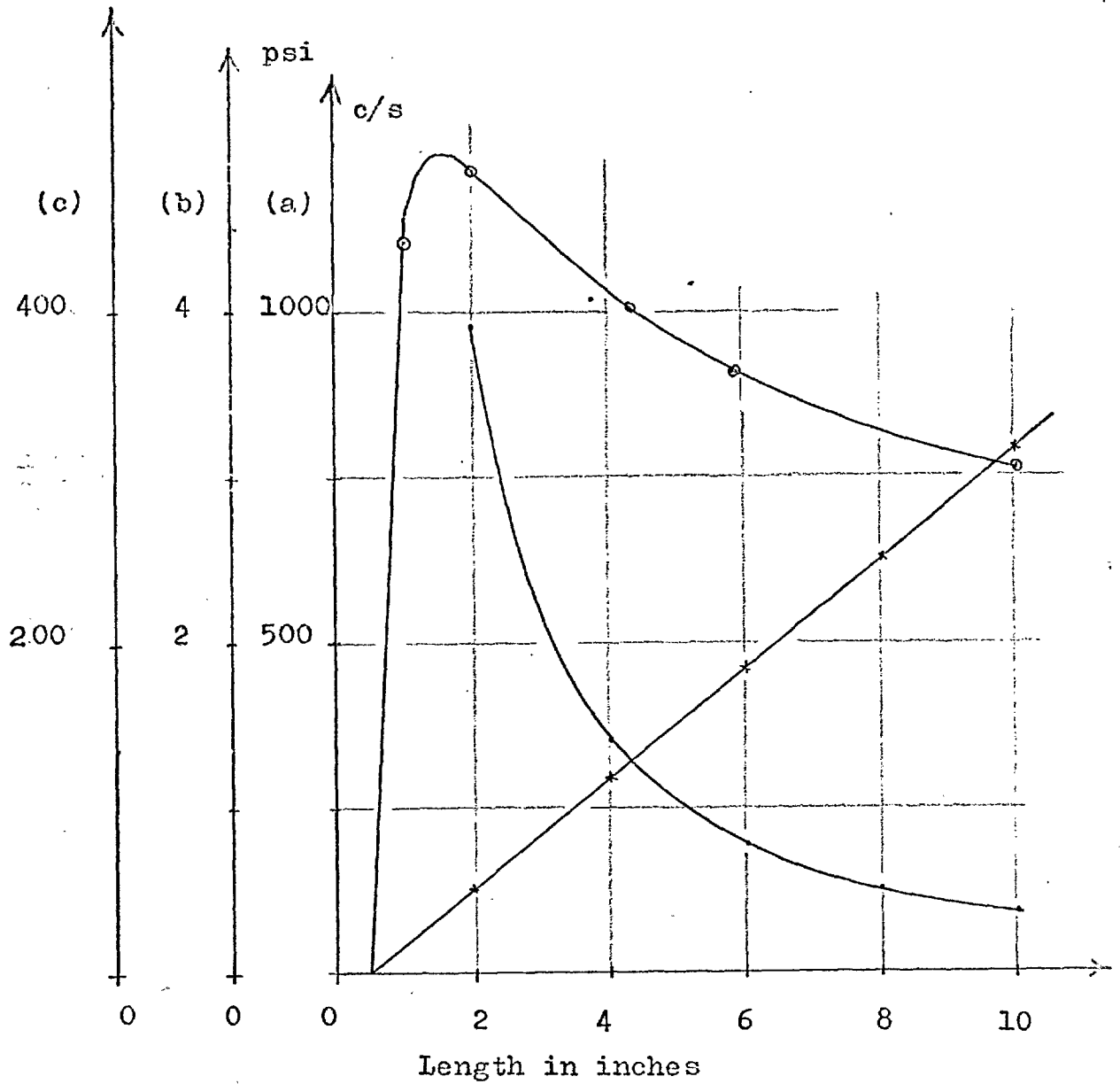


figure 8.2 Variation of (a) the natural frequency (points)
(b) stress (crosses)
(c) band-width product (circles)
with the length whilst the bar-section
remains constant

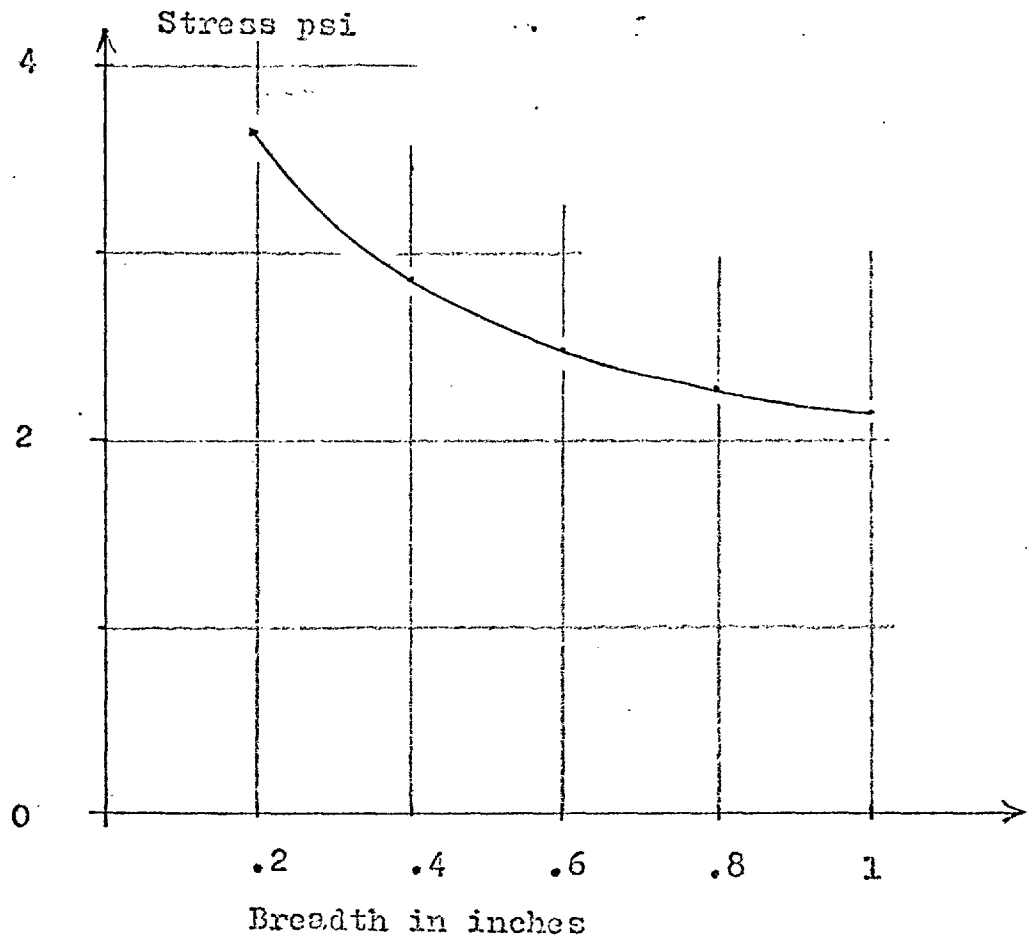


figure 8.3

Variation of stress with breadth keeping
the natural frequency constant

Frequency c/s (points)

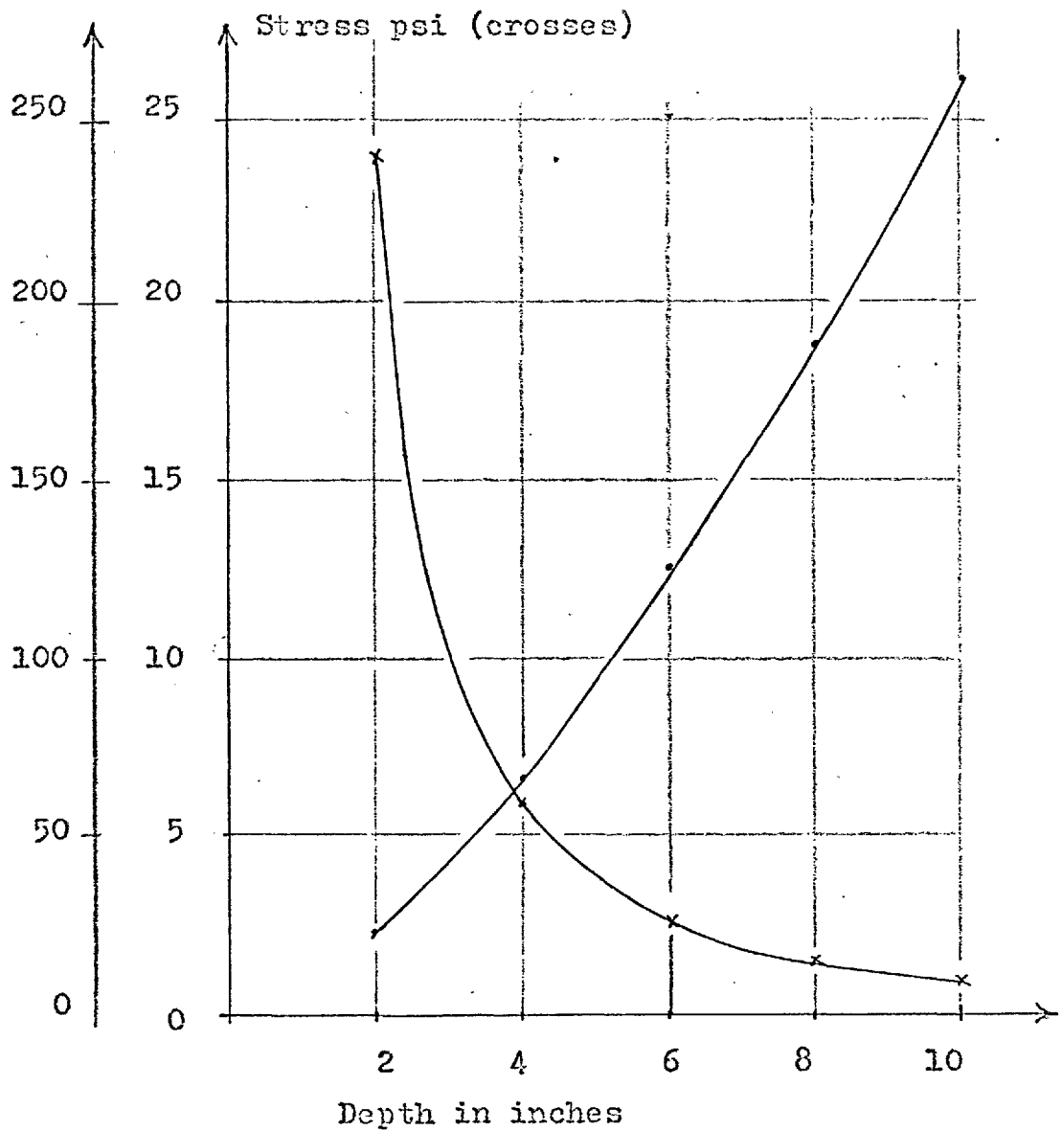


figure 8.4

Variation of stress and natural frequency
with the depth of the bar
(uniform bar)

bolts, the bars having been tapped at the required places. This technique permits an economical experimental investigation of the behaviour of different sections of bars.

8.5. Strain gauge bridge circuit

If a single bar with a set of strain gauges at the top of the bar or two sets one at the top and one at the bottom as shown in figure 8.5.a is used, then as well as an output signal due to the torque any mechanical vibrations of the machine in either of the directions will give spurious outputs. These outputs are typically induced by eccentricity and base vibrations, and appear as components having frequencies proportional to speed. Even after the rotor was dynamically balanced these were still present. A typical response to a step torque shown in figure 8.5.b illustrates the point. The spurious outputs can be eliminated by the use of a multiple bar system with gauges fixed as shown in figures 8.1 and 8.6.a.

The electrical connection of the eight strain gauges used is shown in figure 8.6.b. The changes in the resistances of the gauges due to a torque on the stator are given by

$$d_1, -d_2, -d_3, d_4, -d_5, d_6, d_7, -d_8$$

where d_1 is the change in the resistance of strain gauge 1 and so on. The effect of the changes is cumulative and an output proportional to torque is obtained. Its value is given by

the voltage across the bridge $\times$ gauge factor $\times$ average strain

The changes in the resistances of the gauges due to net horizontal and vertical forces on the stator, shared equally between the two beams (these forces tend to move the assembly in their respective directions but produce no net torque on the stator) are given by

$$d'_1, d'_2, d'_3, d'_4, d'_5, d'_6, d'_7, d'_8, \text{ and}$$

$$-d''_1, d''_2, -d''_3, d''_4, d''_5, -d''_6, d''_7, -d''_8$$

respectively. Thus, since $\sum d'$ and $\sum d''$ are zero if the changes are

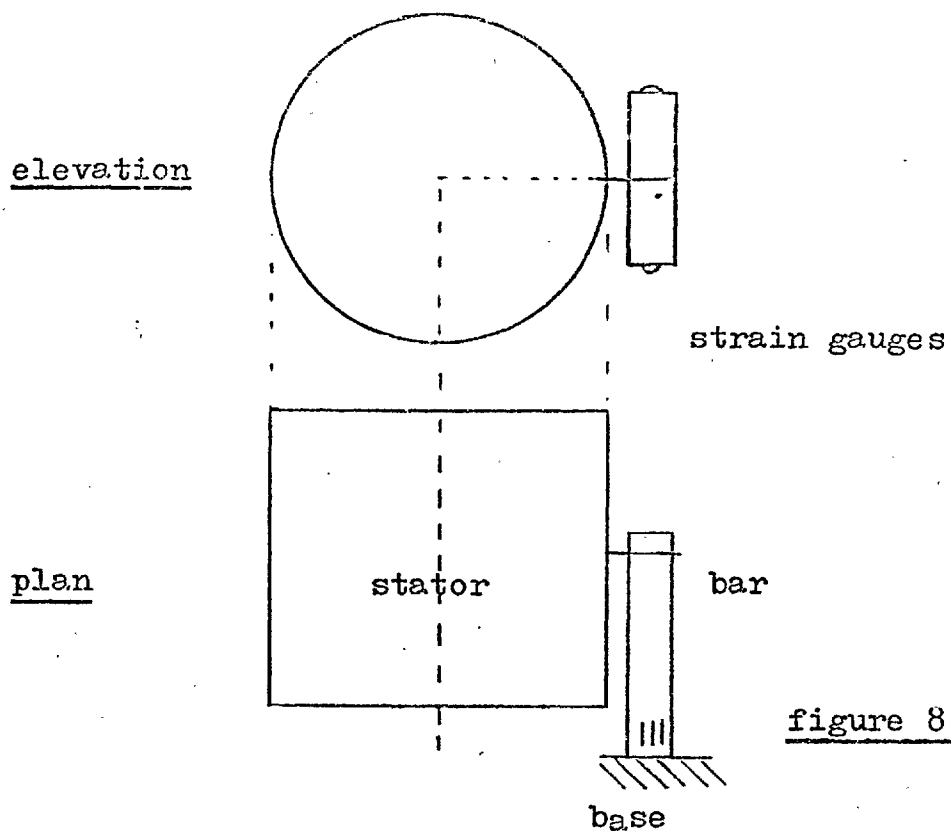
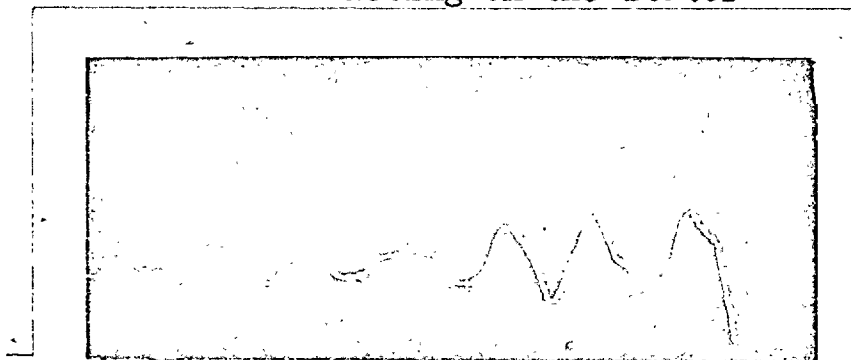
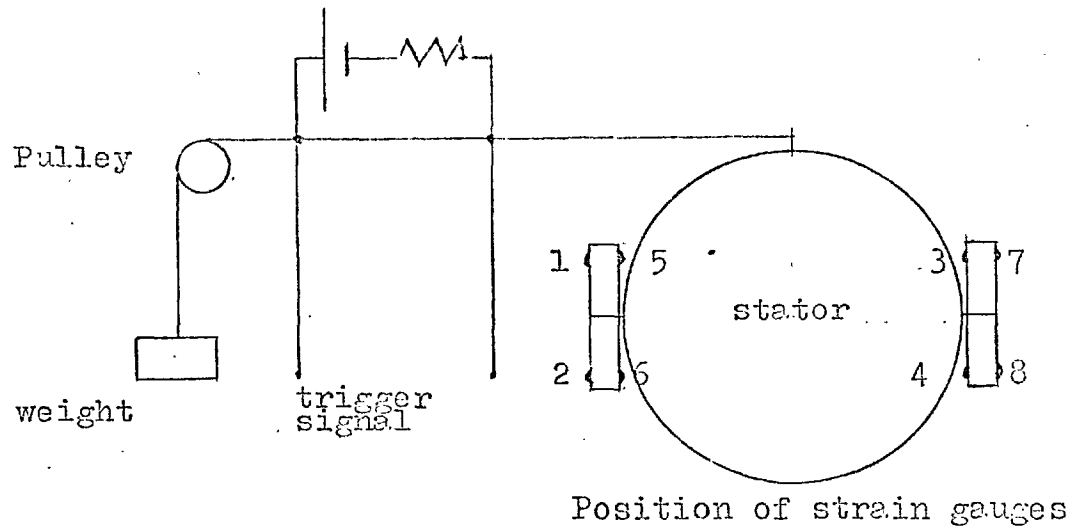


figure 8.5

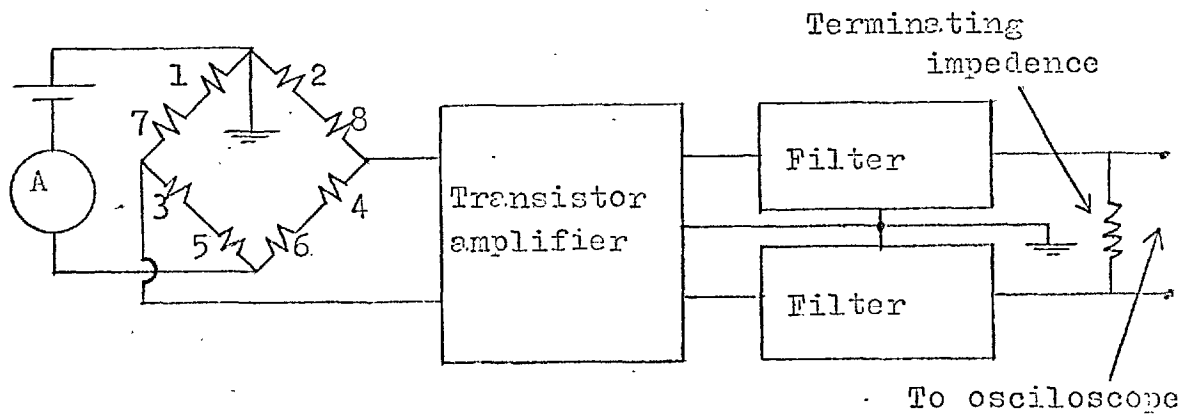
a. A single bar arrangement to measure the torque
acting on the stator



b. Response of the single bar arrangement
to a step torque



Arrangement to measure step response figure 8.6.a



Strain gauge bridge circuit and filters figure 8.6.b

A strain gauge unit

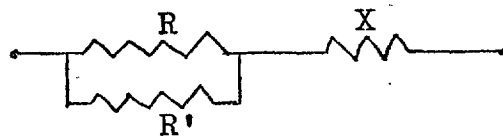


figure 8.6.c

equal, vertical and horizontal forces will now produce no net output. (This depends to a large extent on the skill in making a perfectly symmetrical system and this was provided by Mr. C.A. Jones of the departmental workshop). In practice, the gauge-resistances vary by 8% and the gauge factors by 35% approximately, thus the desired condition of equal changes in the resistances of the gauges, is not realised. However, this condition was achieved by using the technique described in the following section.

8.6. Strain gauge units of equal resistances and gauge factors

The resistances and gauge factors of the available gauges are given in the first two columns in table 8.1. The product of the resistance and gauge factor of each gauge is calculated and the values are shown in the third column. Since this product is a measure of the change in resistance per unit strain, the higher values in the third column have to be brought down to the lowest value in the column by connecting a resistance in parallel with each gauge. These resistances may be calculated as follows.

The lowest value in the third column is divided by each of the values in the column. The resulting values are shown in the fourth column. If a resistance R' is connected across a gauge having a resistance of R ohms, the resistance of the parallel circuit is given by $RR'/(R + R')$. If the resistance of the gauge changes from R to $R + r$ due to unit strain, the change in the resistance of the parallel combination is given by

$$\frac{(R + r) R'}{R + r + R'} - \frac{R R'}{R + R'}$$

$$\underline{\underline{\frac{R' r}{R + R'}}}$$

That is, the change in resistance is decreased by the fraction, $\frac{R'}{R + R'}$. Knowing this fraction from the fourth column and the

TABLE 6.1.

Formation of strain gauge units

R	G.f	R x G.f	K	R'	R _s	X
349	96	3355	.877	2485	306	12
352	96	3482	.843	1890	296.6	21.4
349	92	3215	.913	3660	318	0
312	94	2938	1	∞	312	6
348.5	129	4485	.654	658	228	90
348.5	135	4700	.625	580	218	100
329.5	131	4320	.68	700	224	94
326.7	127	4150	.707	770	231	87
326	130	4470	.657	624	218	100
340.8	130	4430	.663	671	222.6	95.4

R = resistance of the strain gauge

G.f = gauge factor

$K = \frac{\text{minimum } (R \times G.f)}{R \times G.f}$

$R' = \text{parallel resistance} = \frac{R \times K}{1 - K}$

$R_s = \text{resistance of the parallel unit} = \frac{R R'}{R + R'}$

$X = \text{series resistance} = \text{maximum } (R_s) - R_s$

resistance of the gauge considered from the first column, the parallel resistance, R' , may be calculated. Their values are given in the fifth column. The changes in the resistances of the parallel circuits per unit strain are now approximately equal. But the resistances of the parallel units as seen in column six are not equal. Therefore a resistance is connected in series with each unit to make the total resistance equal to the highest value in the sixth column. The strain gauge with a parallel and a series resistance as shown in figure 8.6.c may be called the strain gauge unit. These units have equal resistances and gauge factors. That is, the change in resistance of the units per ohm per unit strain is the same.

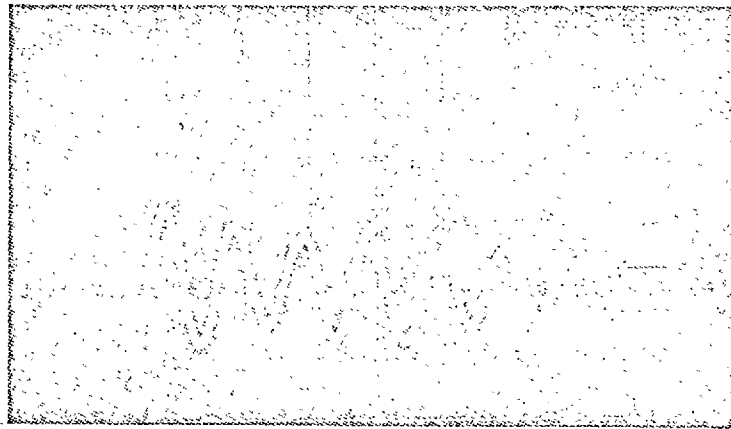
8.7. Response of the strain gauge apparatus

The output from the strain gauge bridge is connected to an oscilloscope. A torque is applied on the stator using a pulley and weight arrangement as shown in figure 8.5.a. There will be a sudden change in torque if the string supporting the weight is cut instantaneously. Part of the string is made of a conducting material and an electrical circuit is arranged so that the oscilloscope is triggered when the string is cut. The oscillogram in figure 8.7.a. shows the response of the strain gauge apparatus to a step change in torque. The natural frequency of oscillation which is 91 cycles has to be eliminated from the signal before using the apparatus for measuring transient torques on the stator. The design of the filters used is given in appendix 6.

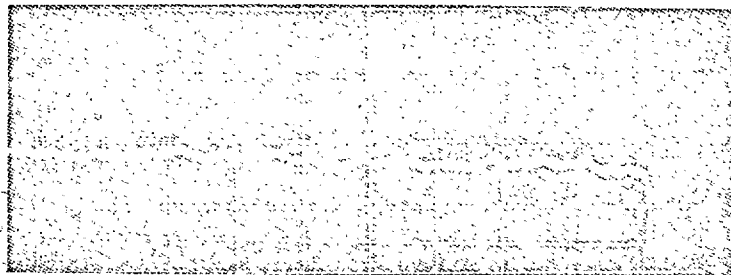
8.8. Low-pass filters used

A notch filter in series with a low pass filter as shown in figure 8.8.a. is used. The filtered response of the apparatus to a sudden change in torque is shown in figure 8.7.b. The frequency response of the filter is shown in figure 8.8.b.

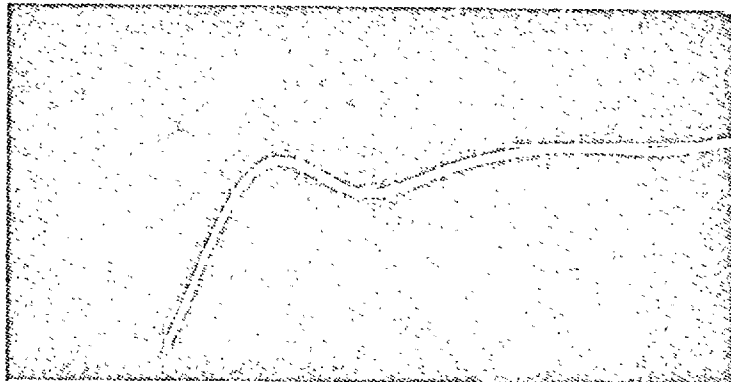
An improved frequency response as shown in figure A6.a (appendix) can be obtained using a low pass n derived $L - C$ filter. A low gain amplifier was found to be necessary to match the output impedance of the bridge to the input impedance of the $L - C$ filter. The response of the apparatus to a step change in torque is shown



(a)



(b)



(c)

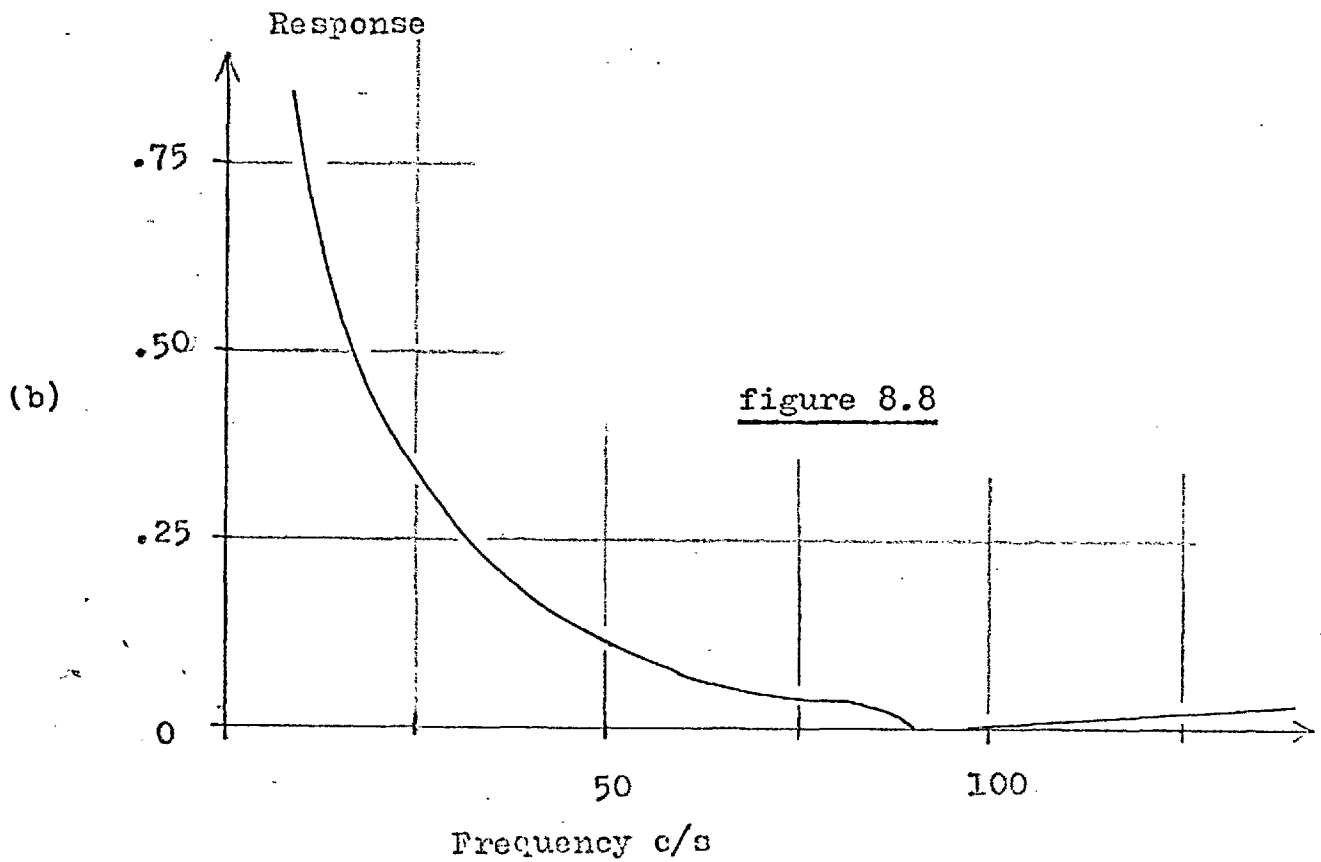
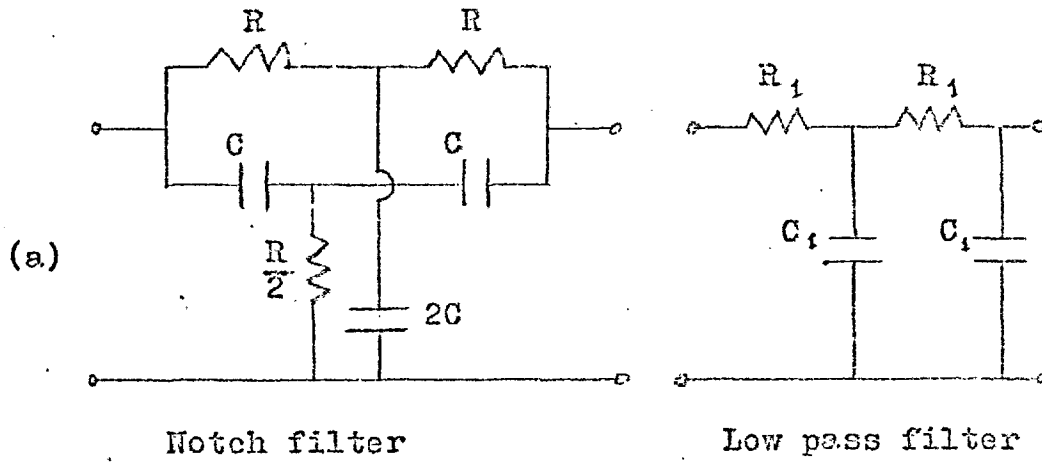
figure 8.7

Response of the strain gauge apparatus
to a step change in torque

(a) without filtering

(b) with the RC filter network

(c) with the LC filter network



Frequency response of the RC filter network

in figure 8.7.c. The overshoot is approximately 15% and the rise time is about 3 m sec.

The electrical pick up when the machine is energised is found to be negligible as shown in figure 8.9. There is no current through the gauges in the above case.

In order to detect the presence of spurious signals when the machine is rotating it was driven at various speeds in an unenergised condition by a coupled d.c. machine. The strain gauge bridge is energised and the output signals due to rotation at various speeds were found to be less than 1% of the measured signal for speeds below 500 rpm and less than 5% at 1000 rpm as shown in figure 8.10.

(a)

(b)

(c)

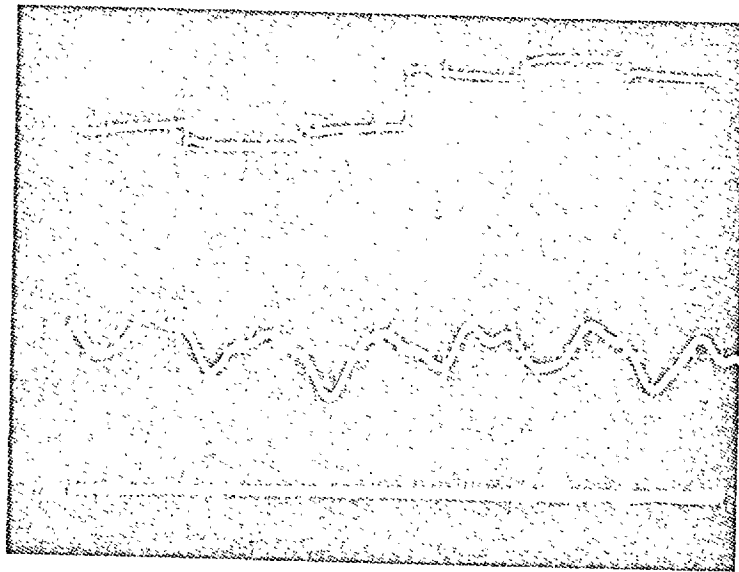


figure 8.9

A comparison of the amount of electrical pickup

(a) phase voltage with 10 c/s Inverter supply

(b) output signal with strain gauge current

(c) output signal without strain gauge current

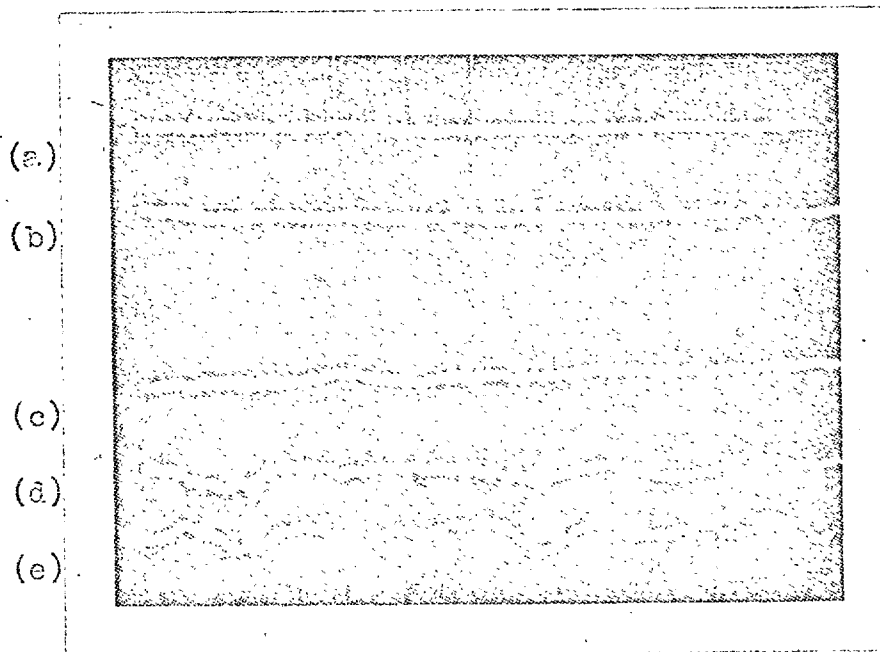


figure 8.10

Amount of noise signal due to rotation

(a) 75 rpm (b) 150 rpm (c) 500 rpm (d) 1000 rpm
(e) 1500 rpm

CHAPTER 9

SLIP RING INDUCTION MOTOR EXPERIMENTAL AND COMPUTED RESULTS

9.1. Introduction

The step-by-step technique developed is used to compute the performance of a simple series circuit in 3.2: in chapter 4 the steady state performance of an ideal machine is calculated. These results have been compared with values obtained analytically. However, no such comparison is possible as far as the transient conditions in machines are concerned. It is the object of the experimental investigations described in this chapter to compare the accuracy of the computed results with those obtained experimentally. This requires both a precise determination of machine parameters and a reliable and accurate measurement of the electrical quantities and torque. The strain gauge torque designed to measure both the steady state and transient torques acting on the stator of the machine has been described in the previous chapter. The machine parameters are obtained experimentally as described below.

9.2. Machine parameters

From the data of the machine the self and mutual inductances of the stator and rotor windings are calculated. The above inductances are also measured by applying a single phase voltage to one of the phases and observing the exciting current and the induced voltages in the other phases. Good agreement is found between the calculated and measured values. Calculation of the leakage inductances is made difficult by the presence of closed slots on the rotor but an accurate measurement is possible. In order to do this the rotor is taken out of the stator core. A copper sheet is provided to restrict the flux produced by the rotor to the leakage paths occurring in normal operation. The leakage reactance of both the stator and the rotor are then found separately by the application of a 3 phase supply at various current levels. The variation of the leakage reactance with current is shown in figure 9.1.

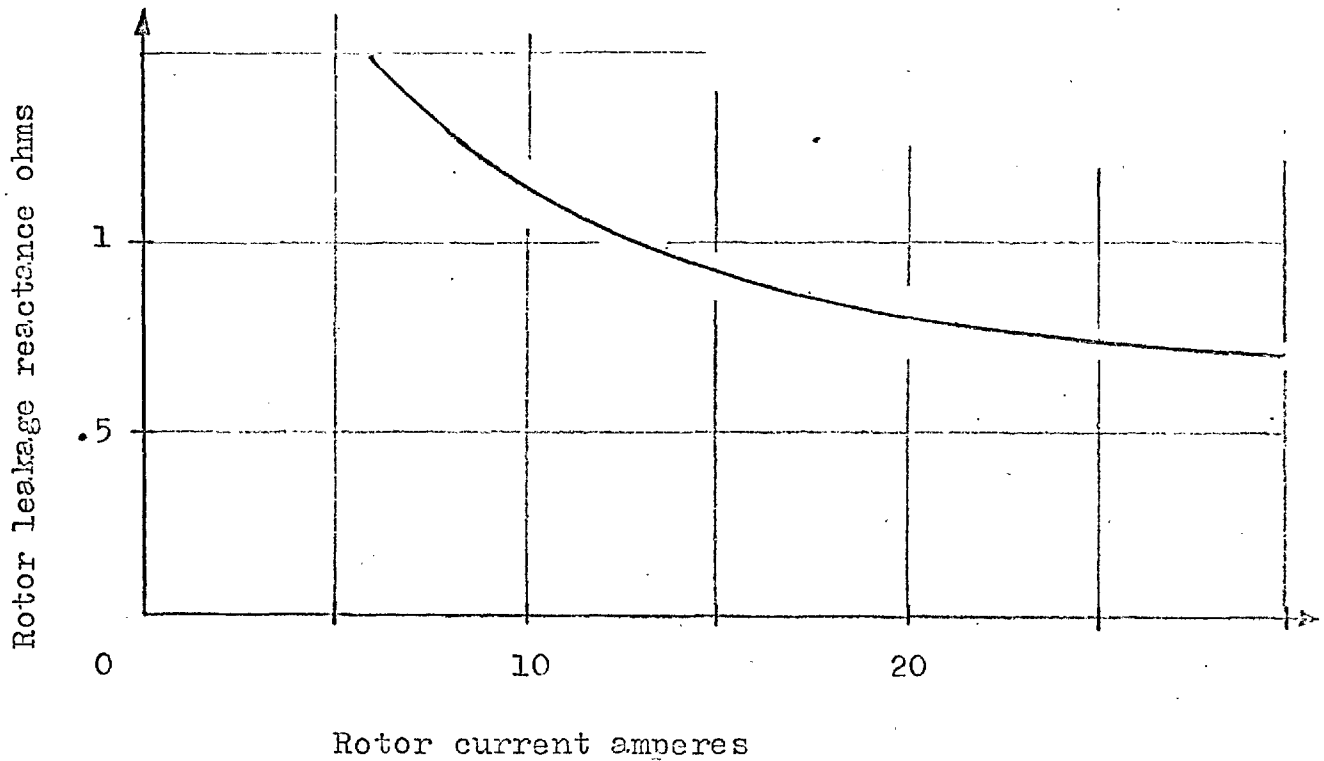


figure 9.1

Variation of leakage reactance with current

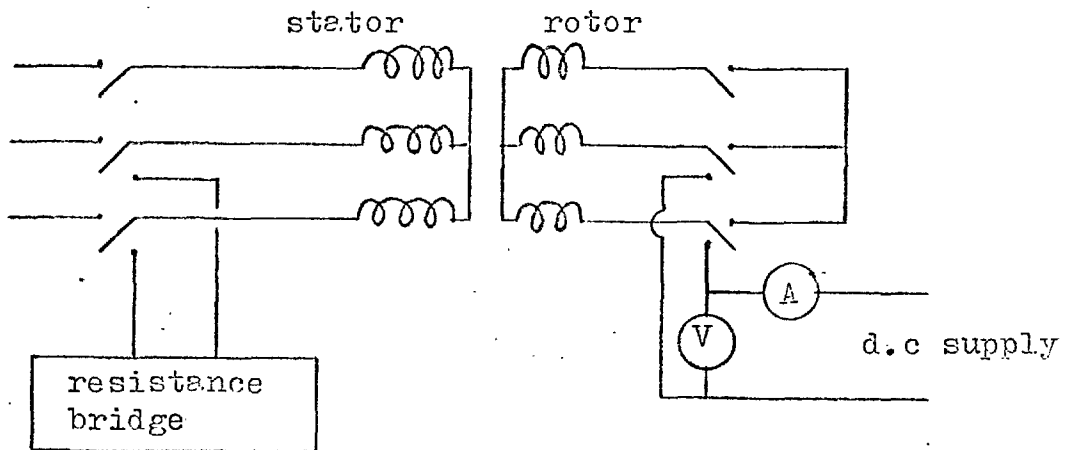


figure 9.2

Circuit for resistance measurement

The resistances of the stator and rotor phases are measured using the arrangement shown in figure 9.2, after each reading in the following experiments.

9.3. Steady state performance

The computed and experimental values of the currents and torques at various speeds with sinusoidal and non-sinusoidal input voltages are shown in the tables below.

Balanced sinusoidal supply 50 c/s 304 V line

Speed rpm	Current amperes		Torque lb ft	
	Experimental	Computed	Experimental	Computed
500	4.3	4.46	4.11	3.95
1000	3.65	3.8	4.65	4.95
1400	1.645	1.54	3.22	3.35

The print out from the computer showing the instantaneous values of stator and rotor currents, and instantaneous torque are shown in table 9.1.

The operating voltage of the impulse inverter used is limited. Hence lower frequencies are used so that the torque developed can be conveniently measured.

Steady state operation with inverter supply

Supply	Speed	current amperes		Torque lb ft	
		Experimental	Computed	Experimental	Computed
10 cycles 55.7 volts per phase	0	3.2	3.1	4.72	5.05
	175	2.1	1.885	4.25	4.53
5 cycles 38.5 volts	0	2.45	2.34	6.37	6.21

The stator and rotor current waveforms are shown in figures 9.3 to 9.6.

	21.14	27.06	4.96	104.69	27	351.23	-4.52	-0.56	5.09	18.95	9.59	-28.5	
I_s	-4.22	-3.79	-3.20	-2.42	-1.66	-0.93	-0.21	0.52	1.23	1.93	2.64	3.38	4.08
	5.01	5.16	5.24	5.14	5.02	4.90	4.75	4.53	4.03	3.44	2.77	2.06	1.32
	-0.86	-1.59	-2.34	-3.02	-3.67	-4.26	-4.74	-5.07	-5.10	-5.12	-5.12	-5.13	-5.06
I_r	10.04	11.60	13.00	13.71	14.37	15.17	16.16	17.30	18.03	19.06	20.37	22.01	23.56
	24.21	24.54	24.98	24.91	25.21	25.96	27.12	28.56	28.82	28.85	28.60	28.27	27.77
	26.09	26.45	26.94	26.80	26.71	26.53	26.09	25.20	23.35	22.01	21.05	20.49	20.05
T_q	6.71	4.17	2.84	5.35	7.26	7.48	5.06	2.67	3.82	5.21	5.97	3.57	1.96
	8.75	5.47	3.45	4.44	5.60	5.93	3.65	1.20	3.32	6.42	8.40	5.73	4.04
	6.86	3.92	1.96	3.17	5.33	7.01	5.18	3.45	5.39	6.72	7.18	4.44	2.59
	21.14	27.06	4.96	104.69	28	111.20	-4.52	-0.56	5.08	-28.51	18.90	9.6	
I_s	-4.22	-3.79	-3.20	-2.42	-1.66	-0.93	-0.21	0.52	1.23	1.93	2.64	3.39	4.08
	5.01	5.16	5.25	5.14	5.02	4.90	4.75	4.52	4.03	3.44	2.77	2.06	1.32
	-0.86	-1.60	-2.34	-3.02	-3.67	-4.26	-4.75	-5.06	-5.10	-5.12	-5.13	-5.13	-5.06
I_r	18.11	17.59	16.61	14.77	13.11	11.72	10.56	9.50	8.23	7.10	6.04	5.01	3.82
	-0.42	-1.71	-2.98	-4.14	-5.32	-6.60	-8.03	-9.60	-10.88	-12.11	-13.21	-14.23	-15.10
	-17.40	-18.73	-20.20	-21.23	-22.38	-23.51	-24.45	-24.90	-24.41	-24.33	-24.59	-25.27	-26.09
T_q	6.57	4.01	2.76	5.43	7.35	7.43	4.95	2.75	3.90	5.29	5.98	3.43	1.97
	8.61	5.36	3.40	4.52	5.68	5.88	3.53	1.29	3.41	6.51	8.41	5.60	4.05
	6.75	3.81	1.91	3.26	5.42	6.93	5.02	3.54	5.47	6.80	7.20	4.34	2.61
	21.14	27.06	4.96	104.69	29	231.17	-4.51	-0.56	5.07	9.62	-28.47	18.85	
I_s	-4.22	-3.80	-3.20	-2.42	-1.66	-0.93	-0.21	0.52	1.23	1.93	2.64	3.39	4.08
	5.01	5.17	5.25	5.13	5.02	4.90	4.75	4.52	4.03	3.44	2.77	2.06	1.32
	-0.86	-1.60	-2.34	-3.01	-3.67	-4.26	-4.75	-5.06	-5.10	-5.12	-5.13	-5.13	-5.06
I_r	-28.18	-29.26	-29.67	-28.49	-27.51	-26.93	-26.77	-26.77	-26.25	-26.16	-26.43	-27.05	-27.35
	-23.82	-22.88	-22.03	-20.73	-19.87	-19.35	-19.10	-18.87	-17.87	-16.71	-15.40	-14.07	-12.64
	-8.66	-7.71	-6.71	-5.50	-4.28	-2.99	-1.62	-0.23	1.11	2.36	3.57	4.81	6.09
T_q	6.43	3.85	2.68	5.52	7.44	7.38	4.84	2.83	3.98	5.37	6.00	3.30	1.98
	8.48	5.24	3.35	4.60	5.76	5.83	3.41	1.38	3.50	6.60	8.43	5.47	4.07
	6.64	3.70	1.86	3.34	5.51	6.86	4.86	3.62	5.56	6.89	7.22	4.23	2.63
	21.14	27.06	4.96	104.69	30	351.14	-4.51	-0.56	5.07	18.80	9.64	-28.44	
I_s	-4.22	-3.80	-3.21	-2.42	-1.66	-0.93	-0.21	0.52	1.23	1.93	2.64	3.39	4.07
	5.01	5.17	5.25	5.13	5.02	4.90	4.75	4.51	4.02	3.44	2.78	2.06	1.32
	-0.86	-1.60	-2.34	-3.01	-3.66	-4.26	-4.76	-5.05	-5.09	-5.12	-5.13	-5.14	-5.06
I_r	10.13	11.71	13.10	13.77	14.45	15.25	16.27	17.31	18.06	19.10	20.43	22.10	23.58
	24.30	24.65	25.05	24.90	25.22	25.99	27.19	28.45	28.75	28.85	28.66	28.37	27.76
	26.09	26.48	26.91	26.69	26.65	26.52	26.14	25.09	23.28	21.96	21.03	20.49	19.97
T_q	6.28	3.69	2.61	5.61	7.53	7.33	4.73	2.90	4.06	5.45	6.00	3.16	1.99
	8.34	5.13	3.34	4.67	5.84	5.78	3.29	1.47	3.59	6.69	8.36	5.33	4.09
	6.53	3.59	1.89	3.43	5.60	6.78	4.70	3.71	5.64	6.97	7.14	4.13	2.65

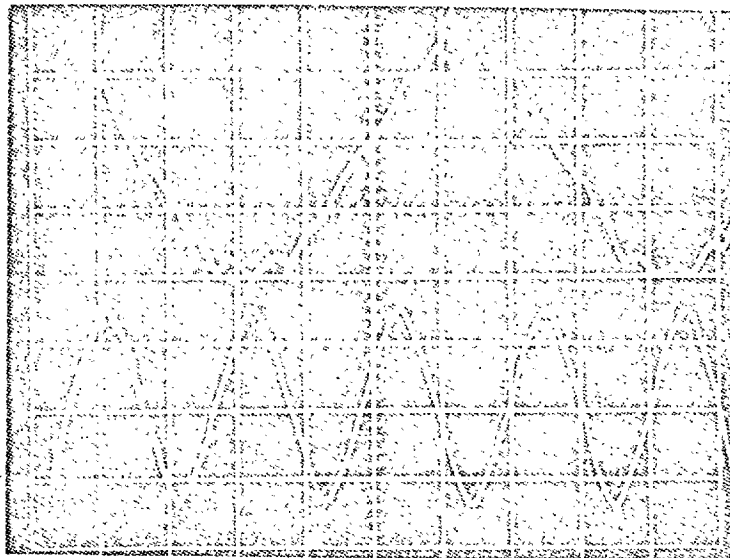
table 9.1

Instantaneous values of stator and rotor currents,
and torque at 1000 rpm

I_s -stator current

I_r -rotor current

T_q -instantaneous torque on stator



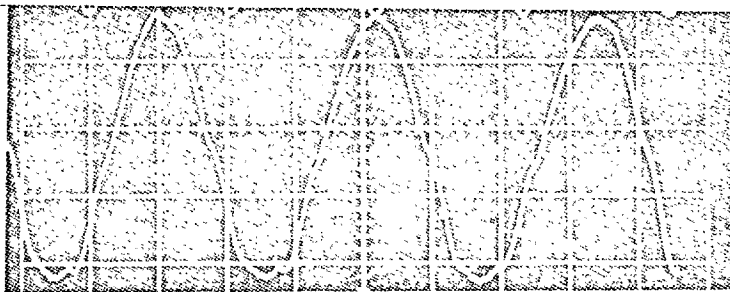
I_r

1000rpm

I_s

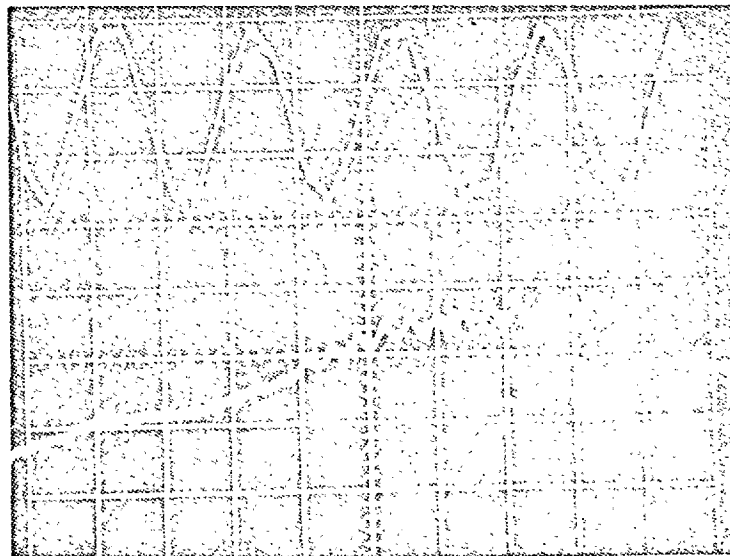
figure 9.3

Experimental waveforms
of stator(I_s) and
rotor(I_r) currents



I_r

500rpm



I_s

1400rpm

I_r

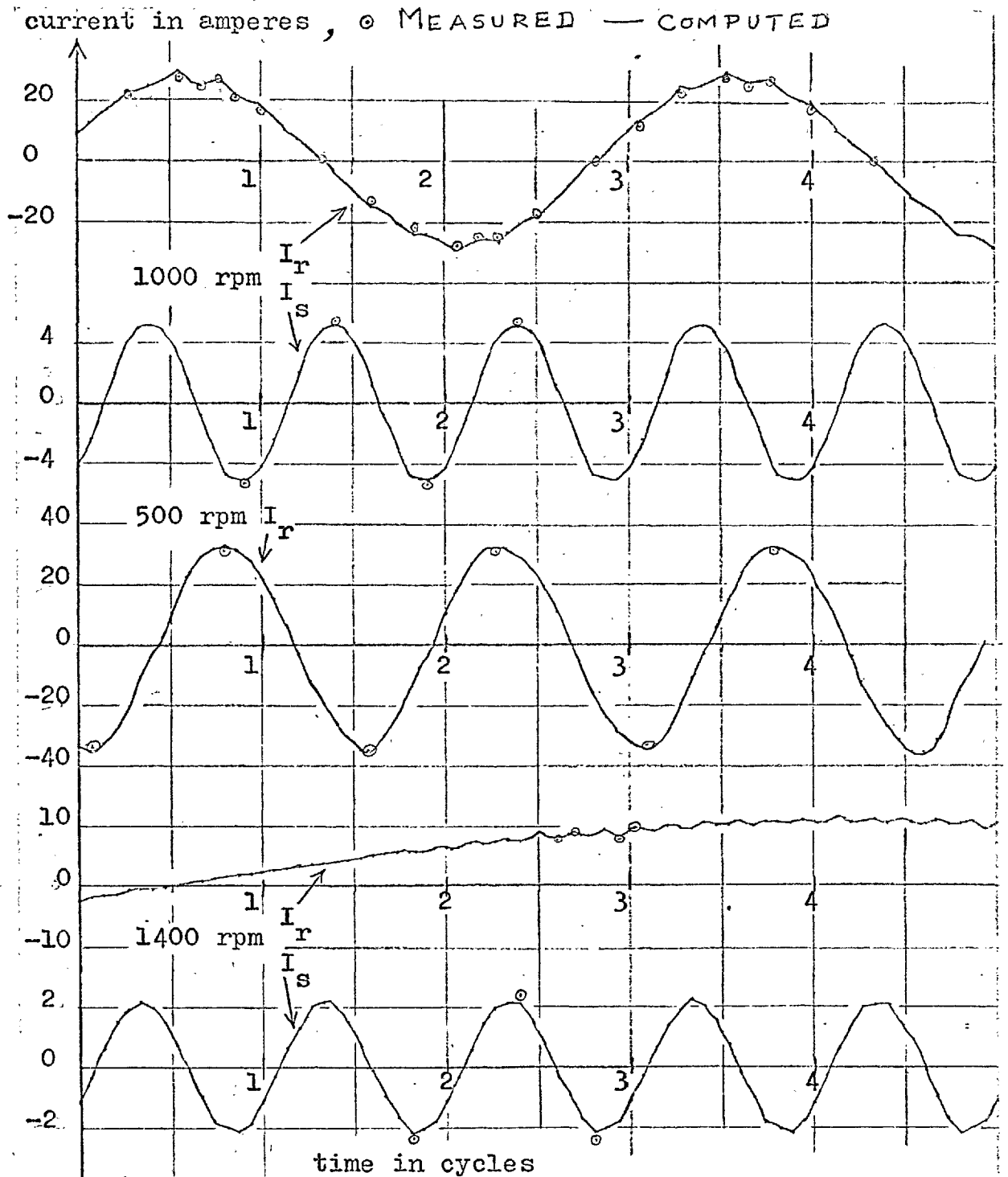


figure 9.4 Stator(I_s) and rotor(I_r) current waveforms at various speeds with sinusoidal applied voltages

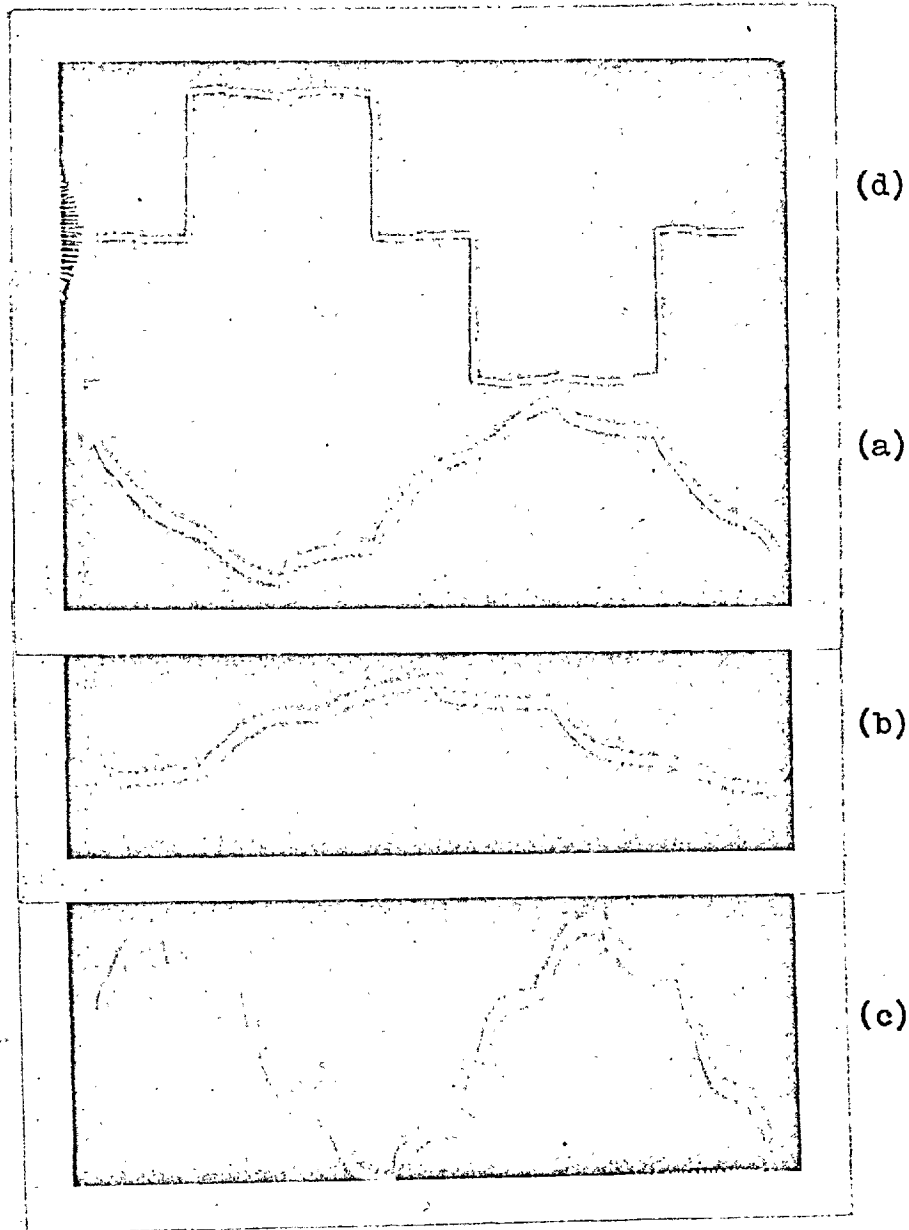


figure 9.5 Experimental waveforms of stator current and line voltage

- (a) at 0 rpm with 56 V, 10 c/s inverter supply
- (b) at 175 rpm with 56V, 10c/s inverter supply
- (c) at 0 rpm with 39V, 5c/s inverter supply
- (d) line voltage

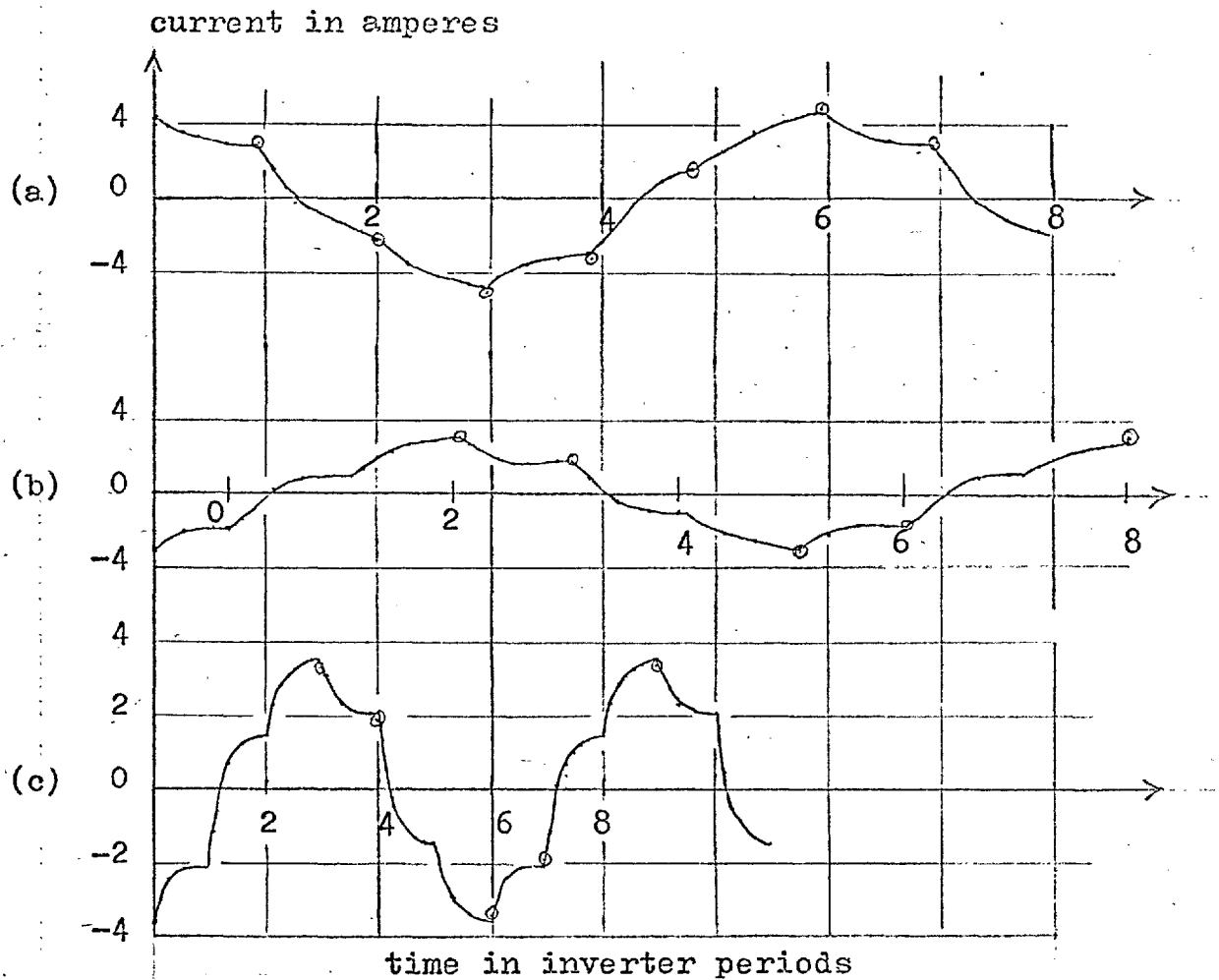


figure 9.6 Stator current waveforms

(a) at 0 rpm } with 56 V, 10 c/s inverter supply
 (b) at 175 rpm }

(c) at 0 rpm with 39 V, 5c/s inverter supply

○ MEASURED

— COMPUTED

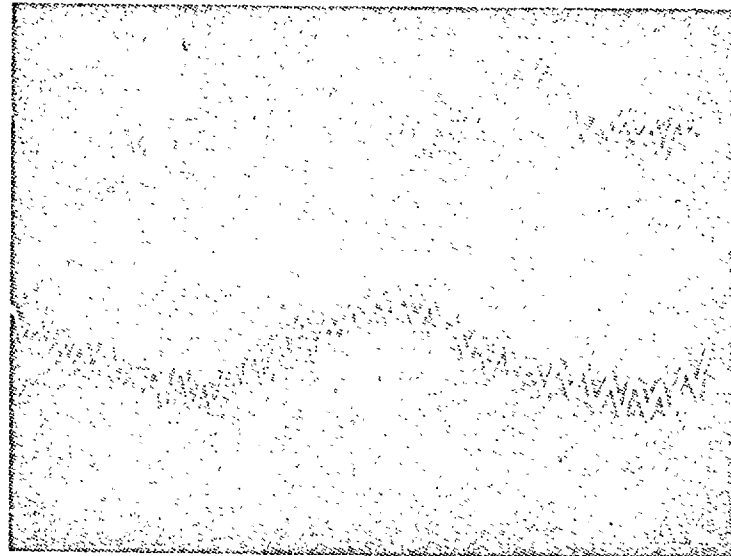
It will be observed from the table that the agreement between the predicted and practical rms current values is as close as is usually obtained in machine testing where accuracy depends on the exact determination of the parameters.

The correlation between the shapes of the stator (input) and rotor current waveforms is extremely good. There is absolute agreement, in the changing pattern of stator and rotor current waveforms during successive cycles. This method therefore gives valuable and accurate information about the % harmonic content drawn from a supply.

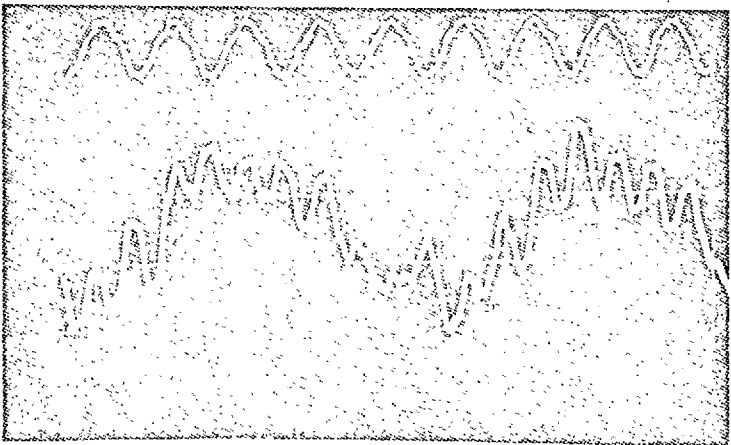
Similarly in the case of the inverter supply the predicted and measured shapes of the stator current waveforms as well as their magnitudes show extremely good agreement as shown in figures 9.5 and 9.6.

9.4. Speed transients

While computing the steady state performance, the presence of pulsating torques was observed. The frequency of pulsation was found to be six times the supply frequency multiplied by the ratio between the machine speed and the synchronous speed. It corresponds to the speed at which the phase bands on the rotor slip past the stator phase bands. In the case of the 4 pole machine this frequency is given by $.2 \times \text{speed in rpm}$. The torque meter developed (in chapter 8) has a restricted band width and is suitable only if the measured frequency is below 50 cycles. The observation is carried out at speeds up to 150 rpm, that is up to 30 c/s torque components. The computed and measured waveforms of the torque are shown in figures 9.7 to 9.10. The measured torques have a high frequency ripple component superimposed on the main component, this is due to the mechanical oscillation of the apparatus. However, the important main component has the same frequency as predicted and it can be observed from the table given below that the amplitudes are predicted tolerably well considering the nature of the extremely difficult problem. Thus the program can be used to calculate the amplitudes of the pulsating torques produced in induction motor operation, this prediction is of great value if the machine drives



(a)



(c)

Experimental torque waveforms at steady state
with 50 c/s sinusoidal input

- | | | | |
|-----|---------|--------|------------------|
| (a) | 150 rpm | 30 c/s | PULSATING TORQUE |
| (b) | 75 rpm | 15 c/s | " " |
| (c) | 50 rpm | 10 c/s | " " |

figure 9.7

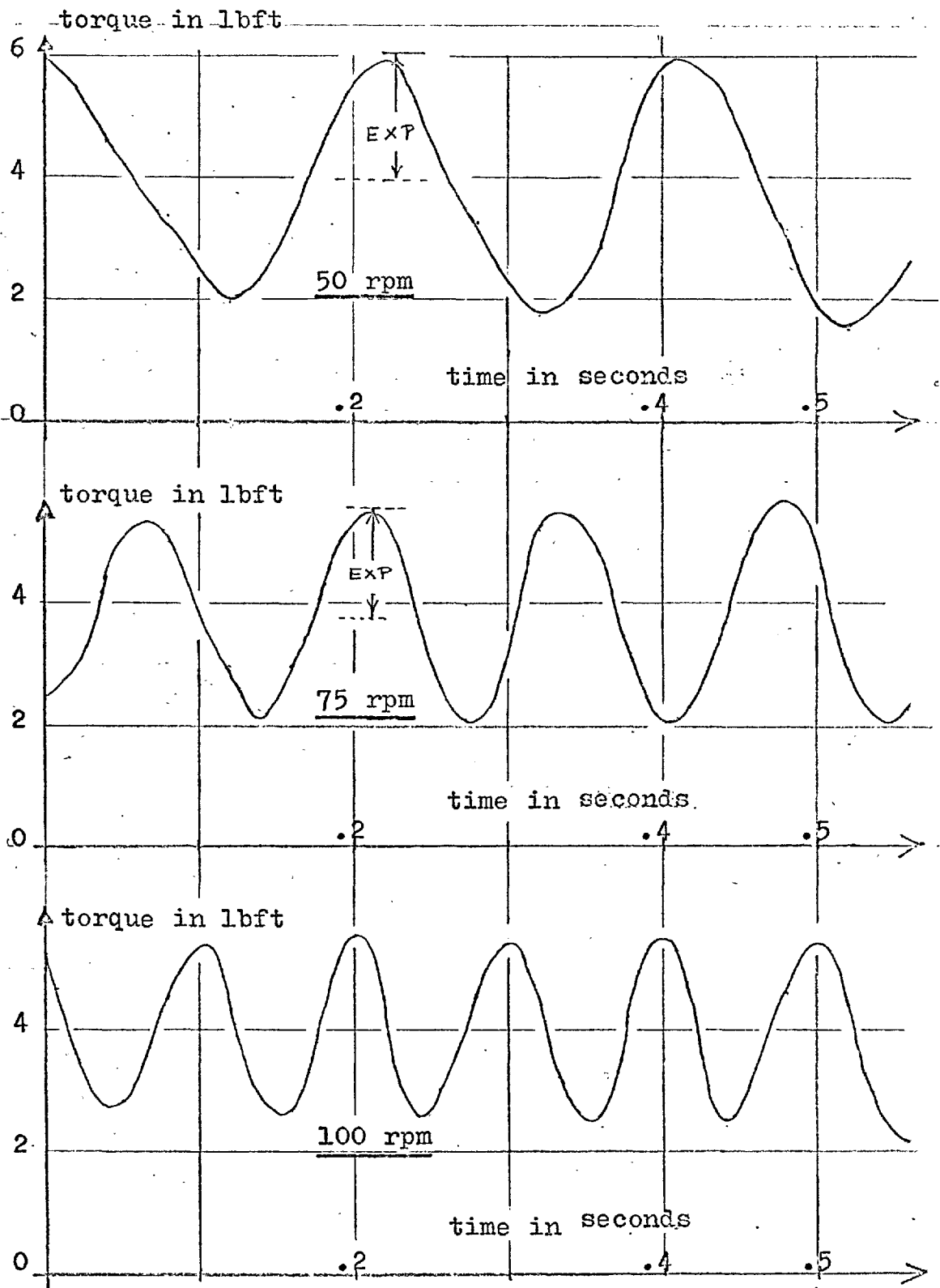


figure 9.8 Waveforms of torque at steady state
with a 50c/s sinusoidal input (COMPUTED)
EXP - MEASURED AMPLITUDE OF PULSATING
TORQUE

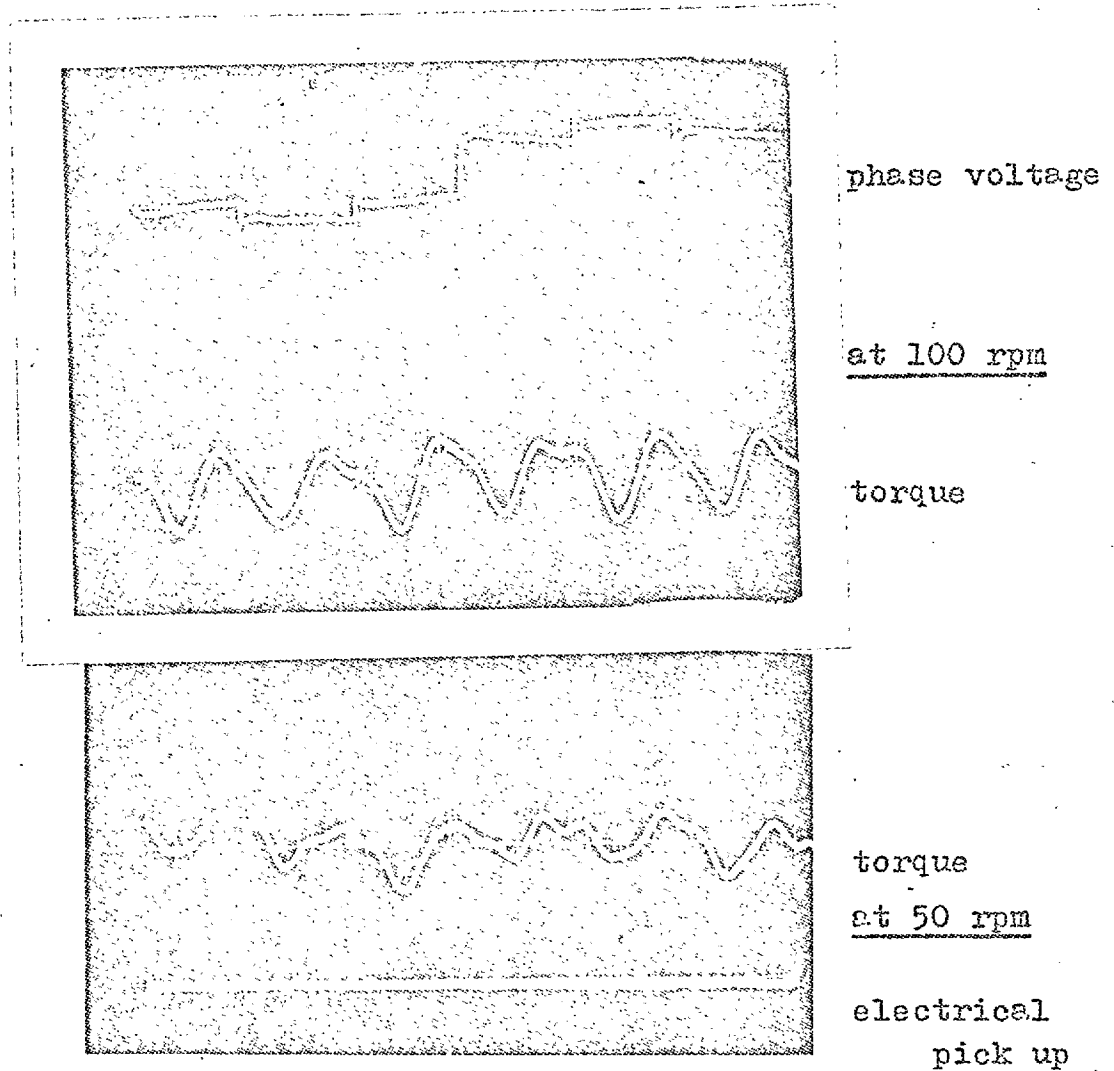


figure 9.9 Experimental waveforms of torque at steady state with 50/s inverter supply

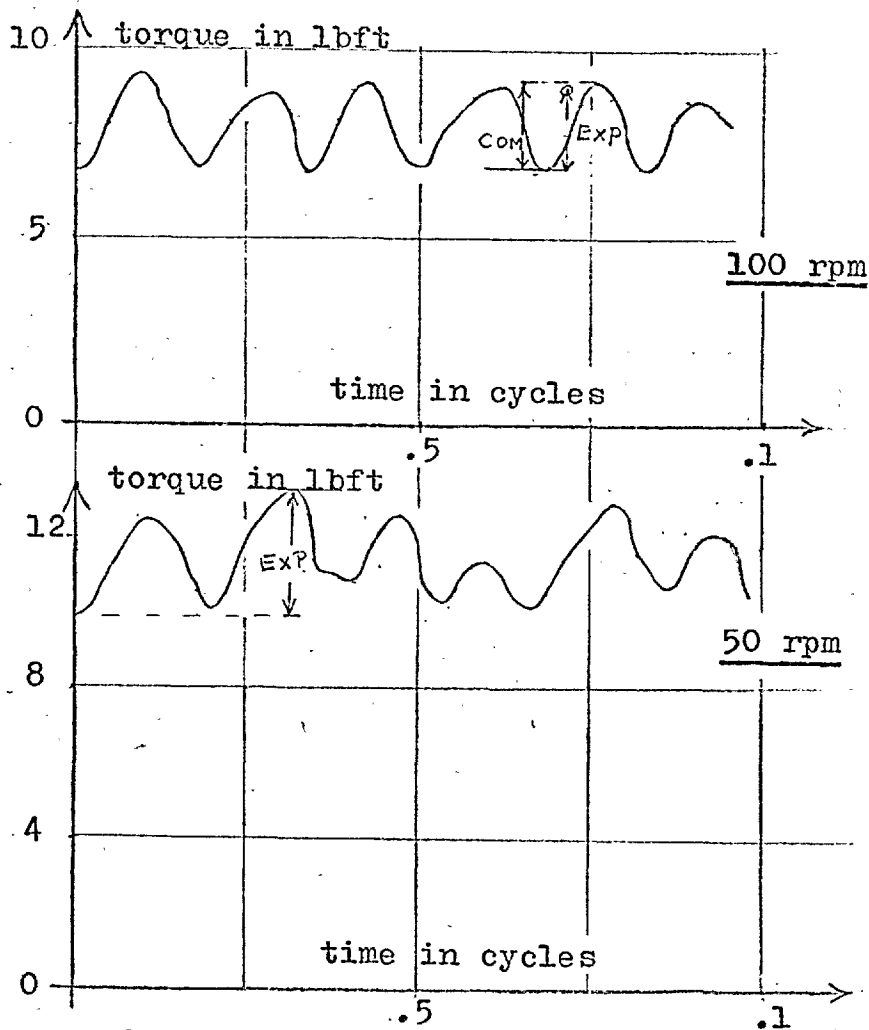


figure 9.10 Waveforms of torque at steady state
 with a 5c/s inverter supply

EXP - MEASURED VALUE

COM - COMPUTED "

apparatus having natural frequencies which must be avoided. Moreover, since this pulsation of torque is occurring at the practical steady state attained, the design of the mechanical components must allow for the fatigue which decreases the allowable stress in the material.

Sinusoidal input 230 V, 50 c/s

Speed rpm	Amplitude of pulsating torque in lb ft.		Average torque computed
	Computed	Experimental	
50	2.075	2.1	3.925
75	1.75	1.84	3.75
100	1.45	1.56	3.9
150	.80	.925	4.01

In the case of the inverter supply computed and measured values are compared considering the difference between the maximum and the minimum values of the instantaneous torques, developed during a cycle.

Speed rpm	Torque pulsation in lb ft	
	Computed	Measured
50	3.2	3.45
100	2.15	2.155

The pulsation of torque in this case is predominantly due to the nature of the input voltages.

9.5. Switching transients

Computed and measured values of transient torques acting on the stator due to switching sinusoidal and inverter supplies at various

speeds are shown in figures 9.11 to 9.14. Numerical comparisons, between measured and computed values, of the maximum torques following the application of sinusoidal and inverter supplies are given in the tables.

Sinusoidal input

Speed rpm	Maximum torque lb ft		Average torque just after switching (during 2nd cycle)	
	Measured	Computed	Measured	Computed
0	20.2	23.0	9.5	11.45
1000	4.45	6	3.78	4.00

Inverter supply

Speed rpm	Maximum torque lb ft	
	Measured	Computed
0	8.80	9.40
100	7.08	8.00

The general shapes of computed and measured torques show close agreement although sharp changes and higher frequencies, present in the computed values are not measured accurately by the strain gauge apparatus. Thus the analysis developed can be used to predict the maximum values of torques, developed due to switching or re-closing a supply to an induction motor, considering the true waveform of the applied voltages and taking all the space harmonics into account. The numerical method is far more powerful compared with the existing analytical methods which require various assumptions as discussed in the introduction.

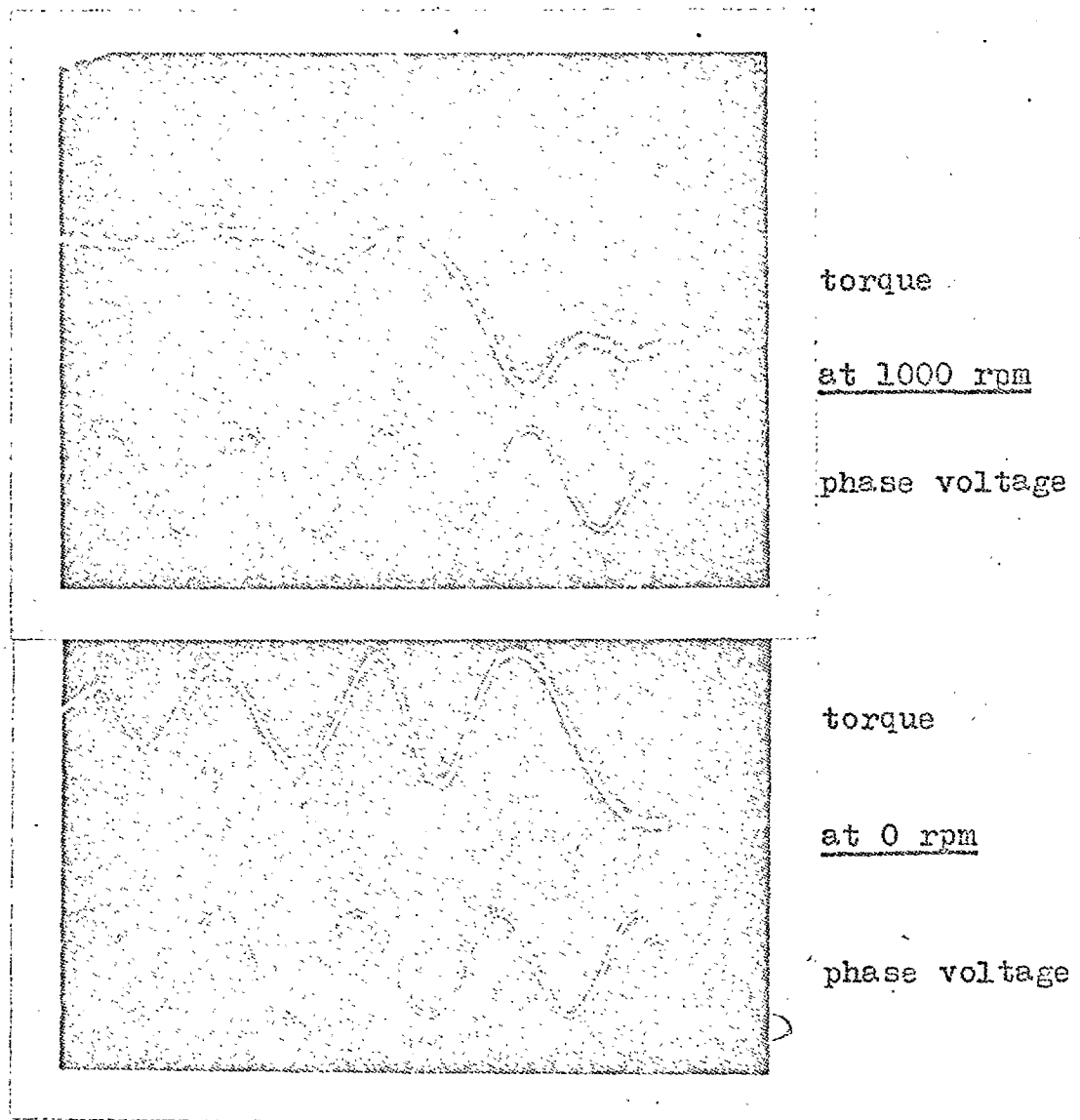


figure 9.11 Experimental waveforms of transient torque
due to switching a 50c/s sinusoidal supply

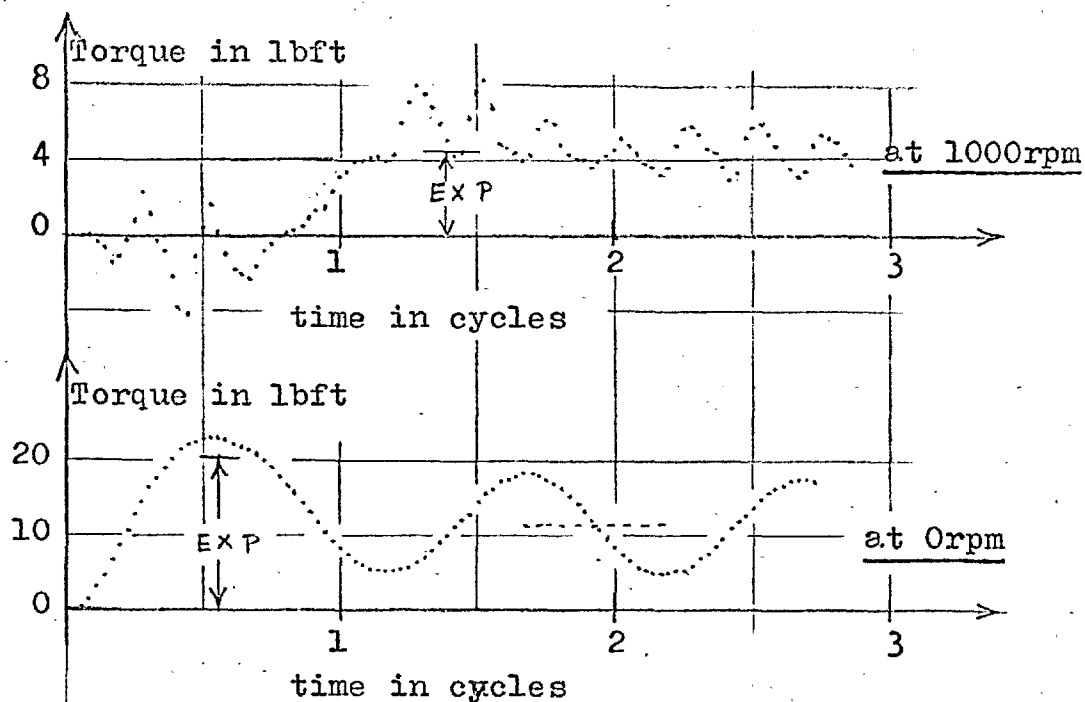
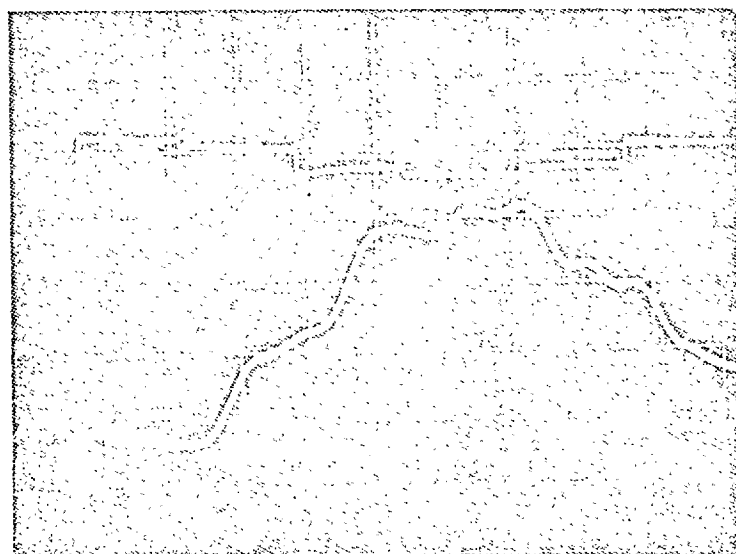


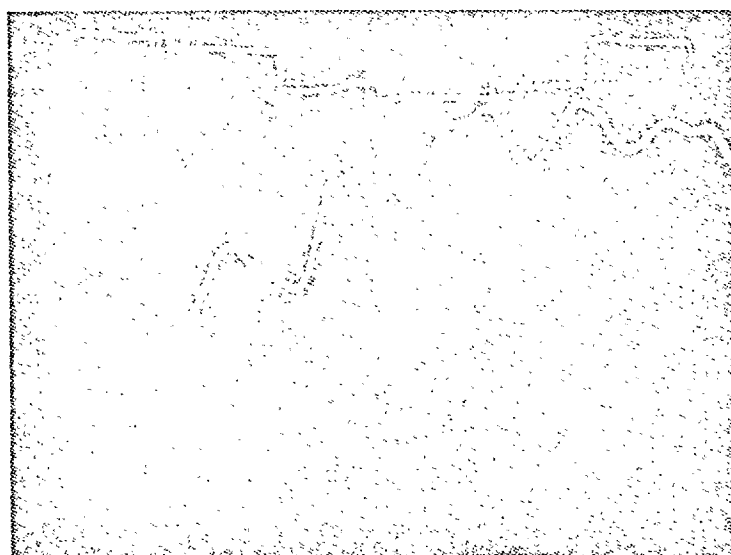
figure 9.12 Waveforms of transient torque due to
switching a 50c/s sinusoidal supply
EXP - EXPERIMENTAL VALUE OF
MAXIMUM TORQUE



phase voltage

at 0 rpm

torque



phase voltage

at 100 rpm

torque

figure 9.13 Experimental waveforms of transient torque
due to switching a 5c/s inverter supply

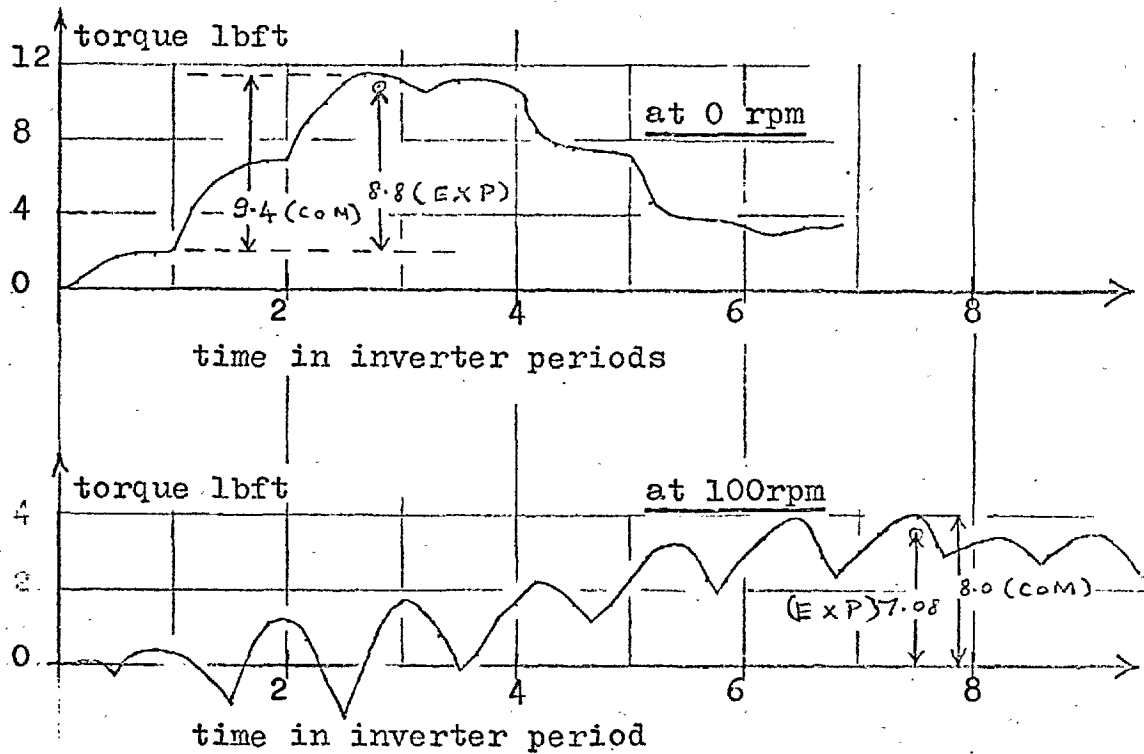


figure 9.14 Waveforms of transient torques due to switching a 5c/s inverter supply

COM - COMPUTED

EXP - EXPERIMENTAL

VARIABLE SPEED DRIVE SCHEMES

CHAPTER 10

VARIABLE SPEED SYNCHRONOUS MOTOR

10.1. Introduction

If an impulse commutated inverter of the type described in Chapter 7 were used to drive a synchronous motor then the output speeds would be exactly proportional to the set frequency. However, the system in the case of a synchronous motor drive may be greatly simplified by the use of a line commutated inverter. This type of inverter self-commutates because of the presence of an existing 3 phase system. In the case of the synchronous motor this system is provided by the induced emfs in the stator windings due to the d.c. excitation on the rotor. The inverter requires only 6 SCRs; no commutating SCRs or components are required and the flywheel diodes are not necessary. Essentially the system operates in the same manner as a d.c. shunt motor with one important exception, the point at which the current in a coil reverses its direction depends on the net field configuration (since this decides the phase of the induced emfs which govern commutation) whereas a d.c. machine coil is commutated mechanically at a set rotor position. Inherently if the pulses necessary to trigger the SCRs are derived from the generated emfs in the machine stator, then it clearly has no starting torque and must therefore be started at least up to a small speed by means of a second machine. In practice either the technique of using the exciter as the means of providing the initial drive to run the machine up could be successfully employed or again the common technique of using the damper windings as an induction motor secondary could be used by means of a fixed frequency supply.

10.2. Principle of operation

The experimental investigations carried out to study the possibility of running a "synchronous machine as a variable speed motor" and a simplified analysis of the system are described in this chapter. The principle used is that power can be fed into an a.c. system

using a line-commutated SCR bridge. The SCR bridge consists of six SCRs and a trigger unit to fire the SCRs in the correct sequence. The circuit diagram of the system is shown in figure 10.1. The design of the trigger unit is described in appendix 7.

10.3. The operation of a three phase bridge

The six SCRs in the bridge are numbered as shown in figure 10.1 and, the firing sequence and the conducting SCRs are given in table 10.1. The waveform of the voltages at points R, Y and B are shown in figure 10.2.a taking the star point of the a.c. system as the reference. In one period of the a.c. voltage there are six half waves and the trigger unit produces six pulses corresponding to these six half waves as shown in figure 10.2.d. Point A in figure 10.2 is taken as the reference for the firing angle. Considering one half of the bridge, that is SCRs 1, 3 and 5, the R phase voltage is more positive than the B phase after point A. Therefore SCR 1 conducts as soon as it is triggered. Similarly after 120° from point A, Y phase is more positive and SCR 3 conducts as soon as it is triggered. After 240° , SCR 5 will be able to conduct. On the other half of the bridge SCRs 4, 6 and 2 conduct for 120° each in a cycle.

The theoretical waveform of the output d.c. voltage across the bridge for three firing angles 30° , 90° and 150° are shown in figure 10.2. When the firing angle, θ , exceeds 60° (point P in the figure 10.2.a) the instantaneous voltage across the bridge is negative during the transfer of current from one SCR to another though the current is positive and this requires a large inductance in series with the load if continuous conduction is to be maintained. With no choke and a resistive load, conduction would be discontinuous as shown in figure 10.3.a. In this experiment a large choke is present and as θ is increased, the average d.c. voltage decreases. At θ equals 90° the average d.c. voltage is zero and conduction will stop. To induce or maintain conduction a d.c. source is connected as the load. Now, the current is positive though the voltage across the bridge is negative and as θ is increased the voltage becomes more and more negative. The measured values of the average d.c. voltage with respect to the firing angle is shown in figure 10.3.b. It may be observed that the zero d.c. voltage occurs at the 85° point

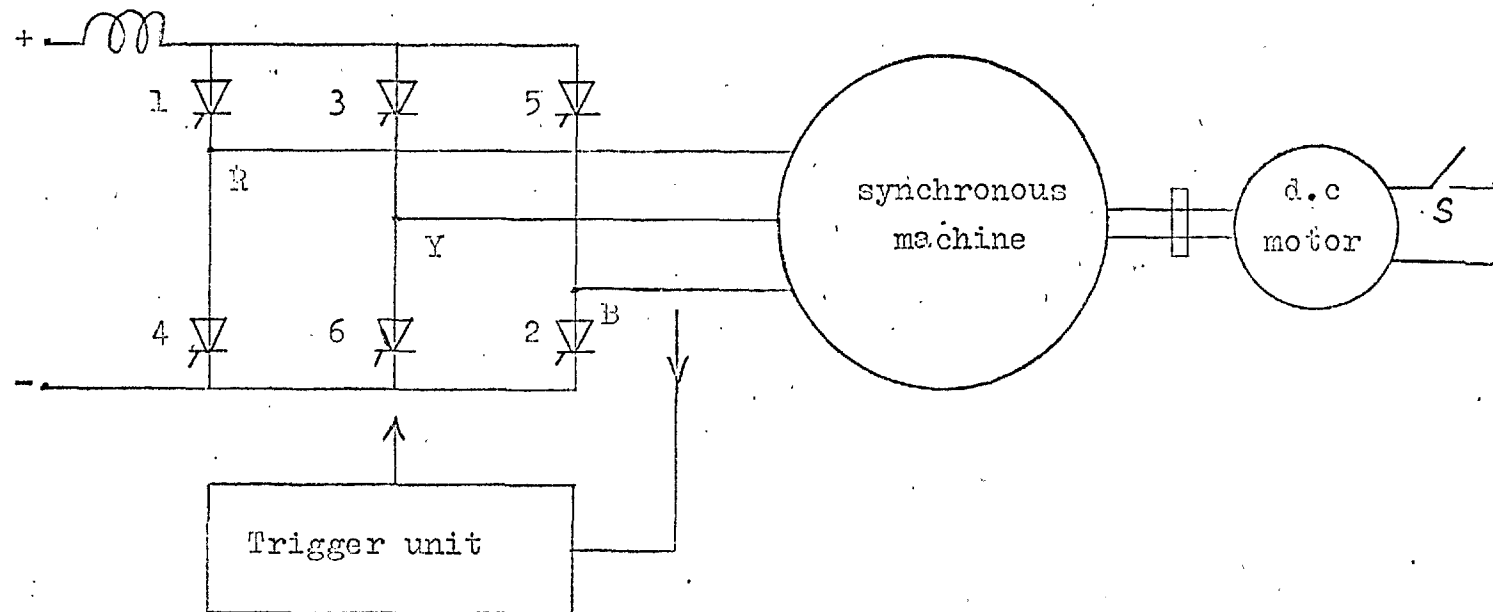
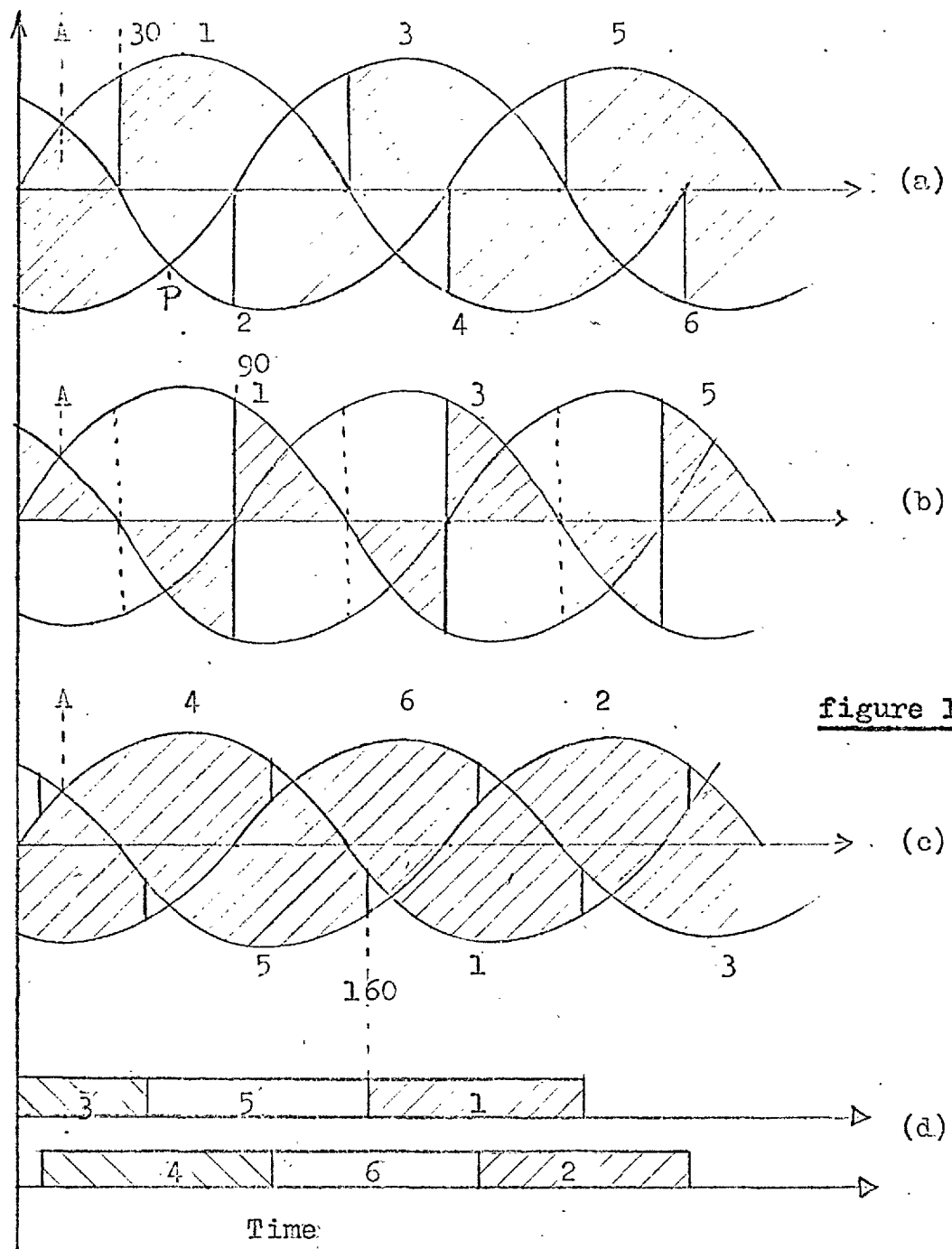


Table 10.1

Time in degrees	0	60	120	180	240	300
Firing sequence	1	2	3	4	5	6
Conducting SCRs	6,1	1,2	2,3	3,4	4,5	5,6

figure 10.1 Line commutated Inverter



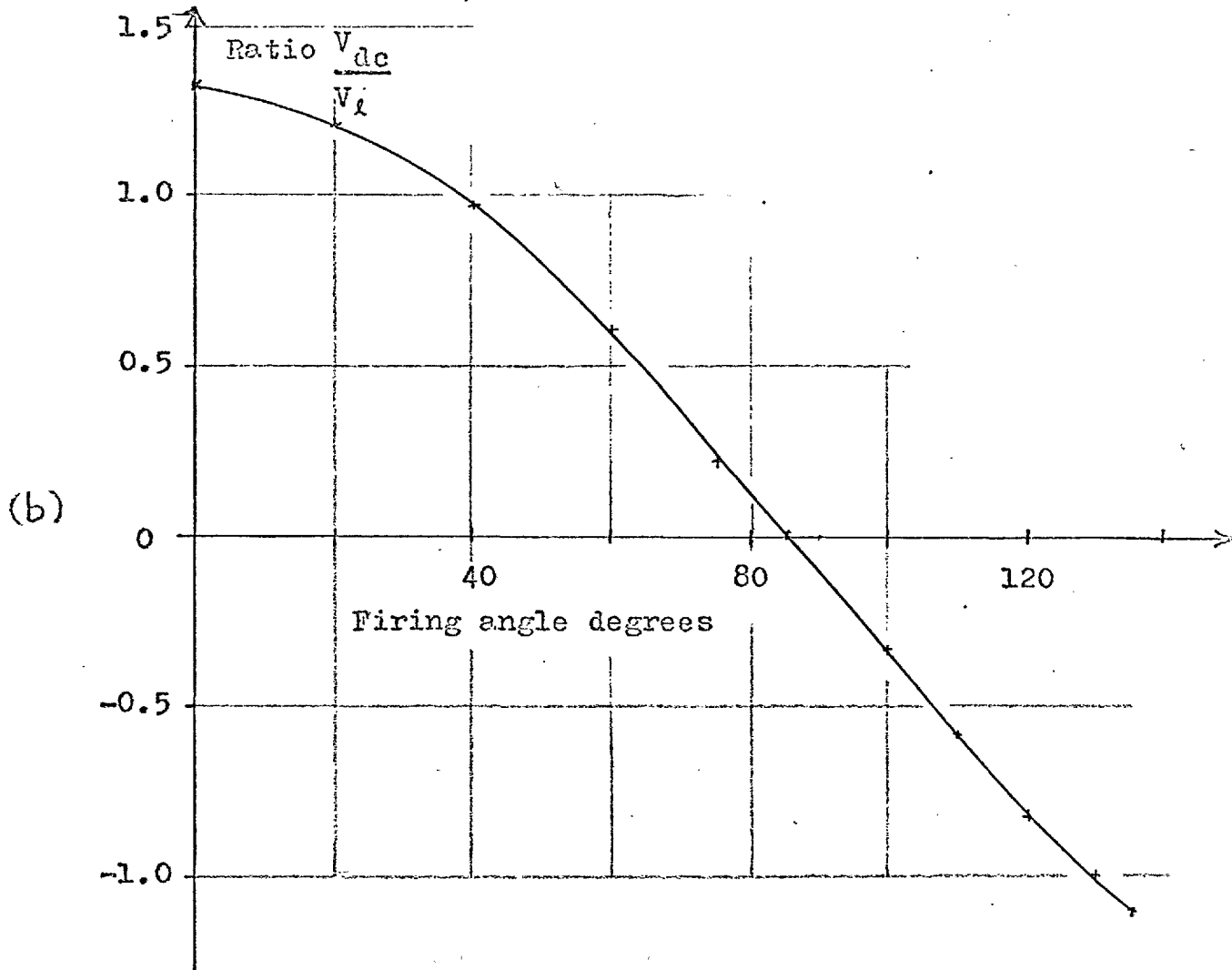
Theoretical waveform of voltage across the bridge

(a) Firing angle 30 degrees

(b) " " 90 "

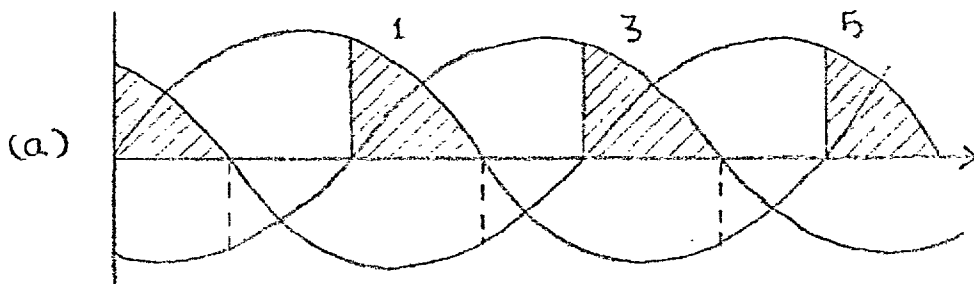
(c) " " 160 "

(d) Conducting SCRs



Variation of the ratio of dc to ac line voltage with respect to firing angle

figure 10.3



instead of 90° , this is due to the error in the measurement of angle and a small voltage drop in the leakage inductance of the phases during commutation.

The waveforms at various firing angles are shown in figures 10.4, 10.5 and 10.6.

10.4. Power flow and motoring action

If the value of the firing angle lies between 0° and 90° , power flows from the a.c. to the d.c. side (rectification). When it lies between 90° and 180° , power flows from the d.c. side to the a.c. side, that is the d.c. side is now the power source. The latter is known as inversion and is of particular interest in this investigation. In this experiment the firing angle is increased to approximately 160° leaving 20° for safe transfer of current. The input power is increased by increasing the d.c. voltage and it is observed that the power drawn by the driving machine decreases and reverses that is, the driving machine is now generating and the synchronous machine is motoring. The main object of the experimental investigation is to verify the stability of the system, that is, to see whether the synchronous machine will continue to run as a motor if the d.c. driving machine is disconnected. This is done by opening switch 'S' in series with the armature circuit shown in figure 10.1, and it is found that the speed of the system increases until the motoring and the friction and windage torques are equal. Thus the ability of a synchronous machine to operate as a motor from a d.c. source has been verified. In the following sections the experimental performance of the system is compared with the computed values.

10.5. Performance of the variable speed synchronous motor

The parameters controlling the performance of the inverter-synchronous machine system are the d.c. applied voltage, exciting current of the synchronous machine and the firing angle. The speed torque characteristic of the system is measured keeping the d.c. voltage, exciting current and the firing angle constant. The output torque of the synchronous machine is measured as follows.

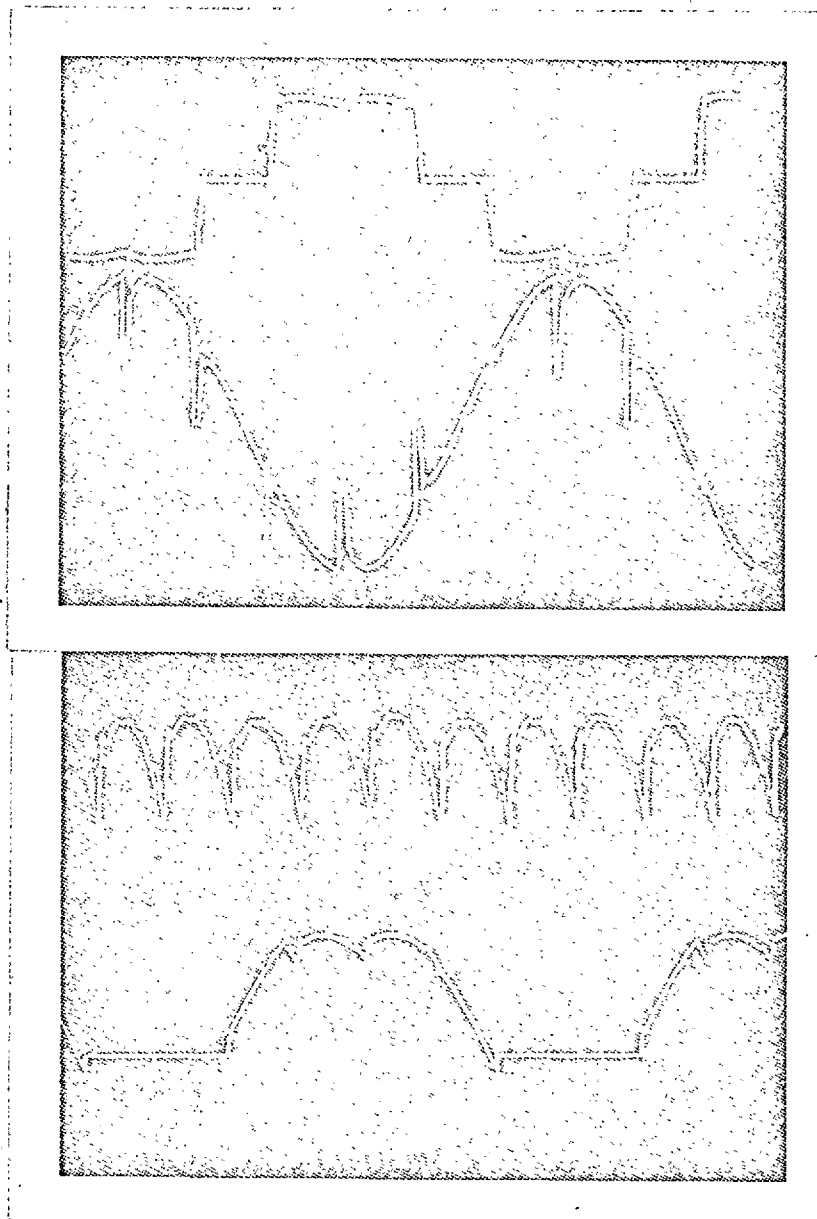
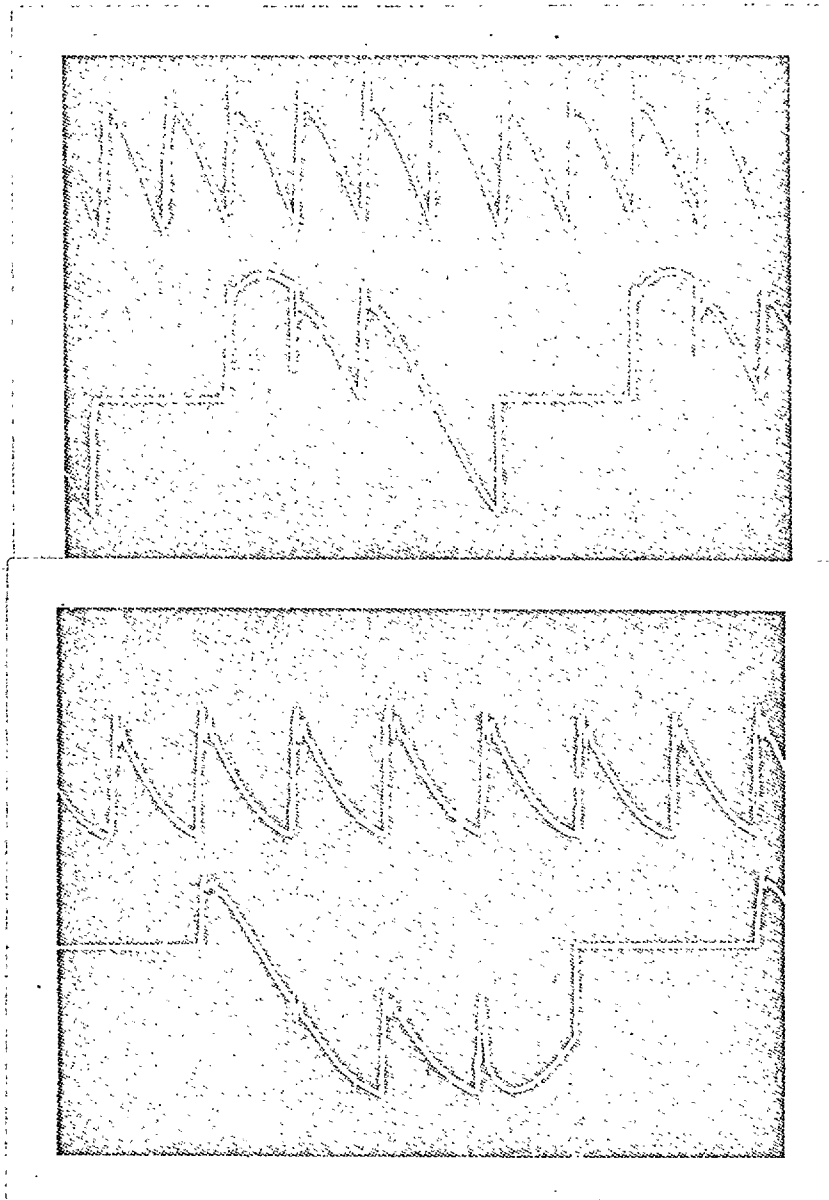


figure 10.4

Inverter-Synchronous machine system current and voltage waveforms

- (a) ac current at 1790 rpm with a firing angle of 135°
- (b) line voltage
- (c) voltage across the bridge at a firing angle of 0°
- (d) voltage across a SCR



(a)

(b)

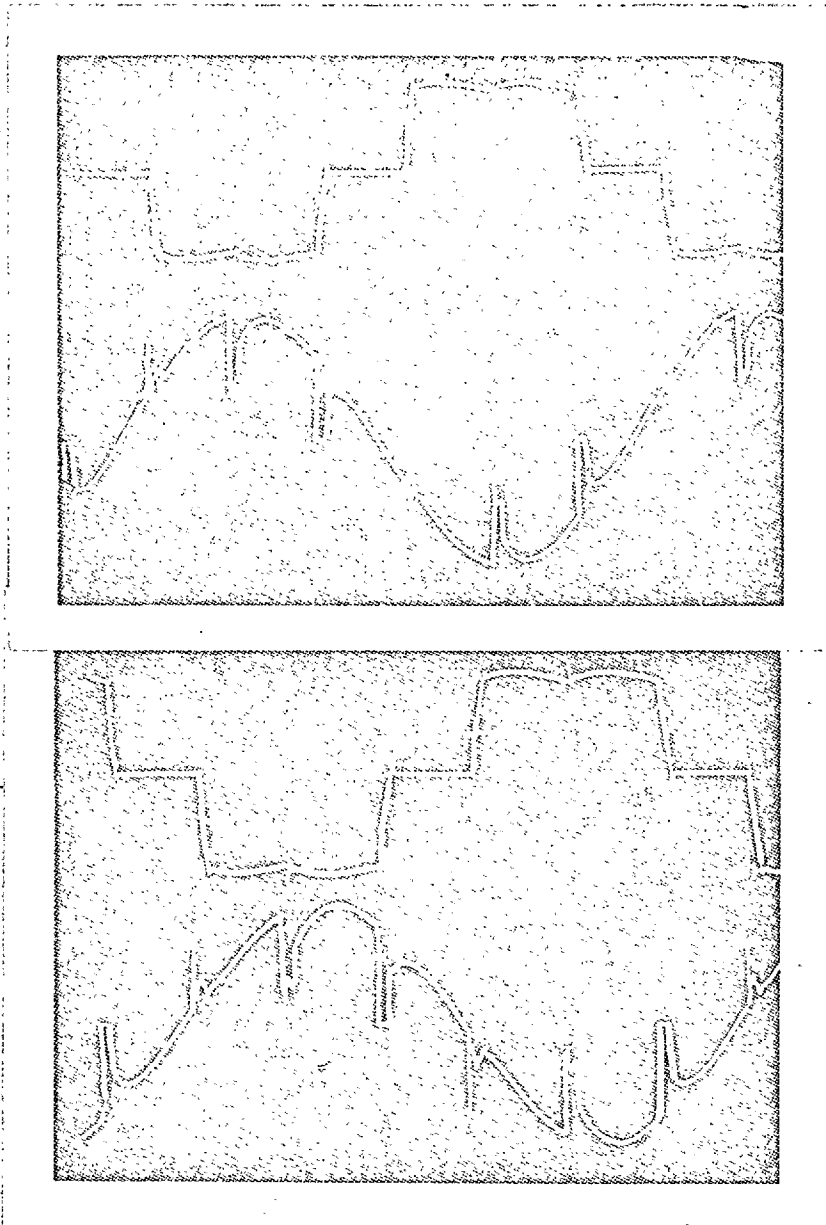
figure 10.5

(c)

(b)

Inverter-Synchronous machine system voltage waveforms

- (a) voltage across the bridge at a firing angle of 55°
(c) " " " " " " " " 135°
(b) voltage across a SCR



(a)

(b)

figure 10.6

(c)

(b)

Inverter-Synchronous machine system current and voltage
waveforms at 1500 rpm

(a) Firing angle 135 degrees -current waveform.

(b) Line voltages.

(c) Firing angle 120 degrees -current waveform.

With the system running at a particular speed the d.c. machine armature current is noted. Then, the inverter is switched off and the excitation of the synchronous machine is removed. The machine system is then run at the same speed and the input current to the d.c. machine is again observed. From the two currents the output power of the inverter-machine system hence the torque may be calculated to a good approximation. The measured speed torque characteristics at two d.c. applied voltages are shown in figure 10.7.

The second characteristic measured is the variation of speed with respect to the applied voltage, keeping the firing angle and the exciting current constant. The system is taken to the rated speed and the armature of the d.c. machine is disconnected. The applied d.c. voltage is gradually increased and the speed of the system at various voltages is noted. It is seen that the output torque is approximately constant. The measured speed-voltage characteristic is shown in figure 10.8.

The variation of speed with respect to the exciting current while the d.c. voltage and firing angle remain constant is measured and is shown in figure 10.9.

The effect of the firing angle on speed with the applied voltage and exciting current remaining constant is measured and is shown in figure 10.10.

The waveforms of the voltage across the bridge at three values of the firing angle are shown in figures 10.4.c, 10.5.a and 10.5.c and the waveforms of the voltage across a SCR corresponding to the above three cases are shown in figures 10.4.d and 10.5.b. The rest of the oscillograms shown in figures 10.4, 10.5 and 10.6 are the waveforms of phase currents and line voltages. It may be observed that the line voltage has six negative peaks per cycle super-imposed on the sinusoidal induced emf. It may also be seen that the phase currents are discontinuous though the voltages are not. This point must be borne in mind in the analysis, since the use of the voltages, appearing across the machine terminals, for calculations, neglecting the presence of non-linear (or switching) elements in the circuits, will lead to incorrect predictions.

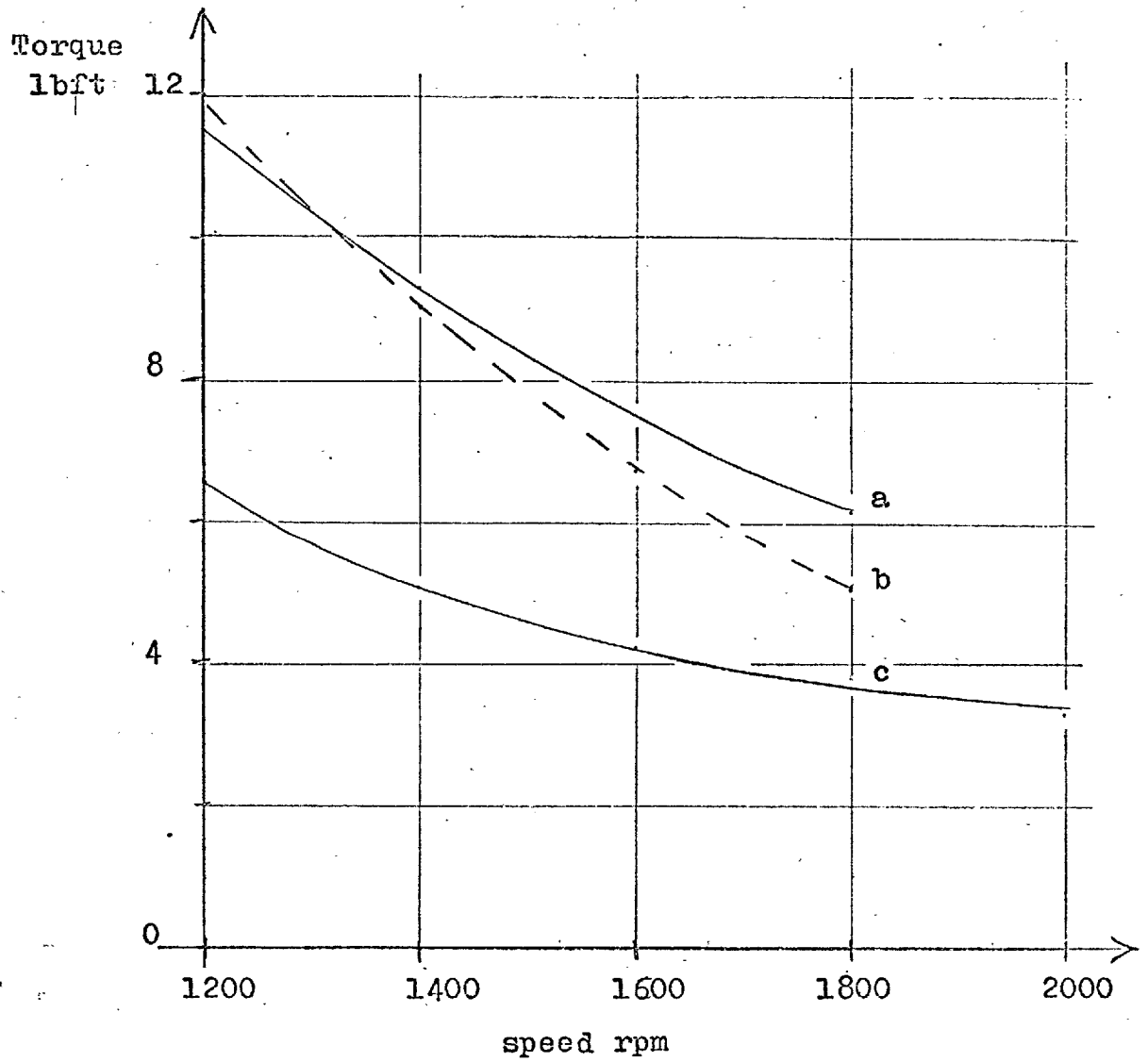
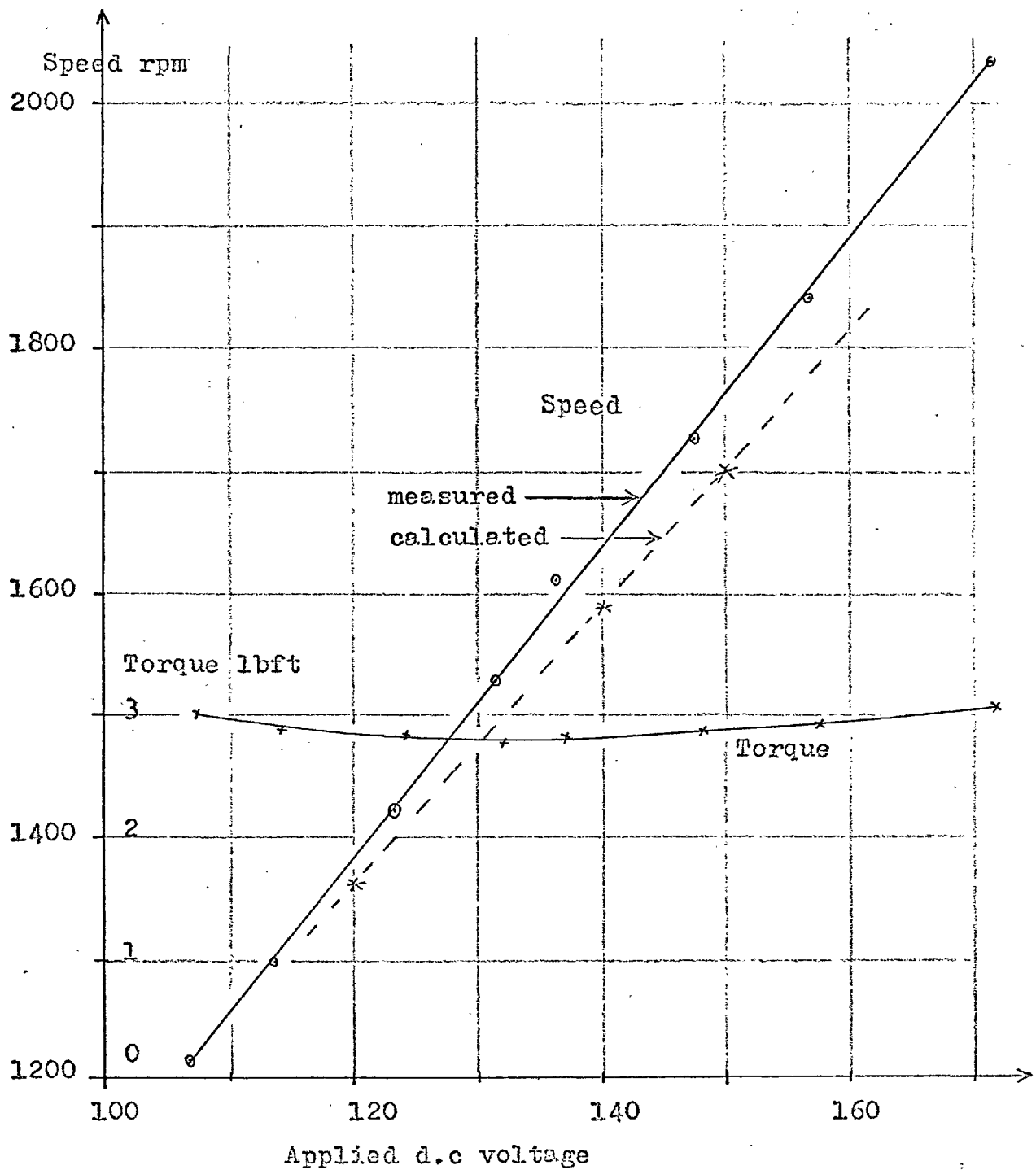


figure 10.7 Speed-torque characteristic at constant voltage.

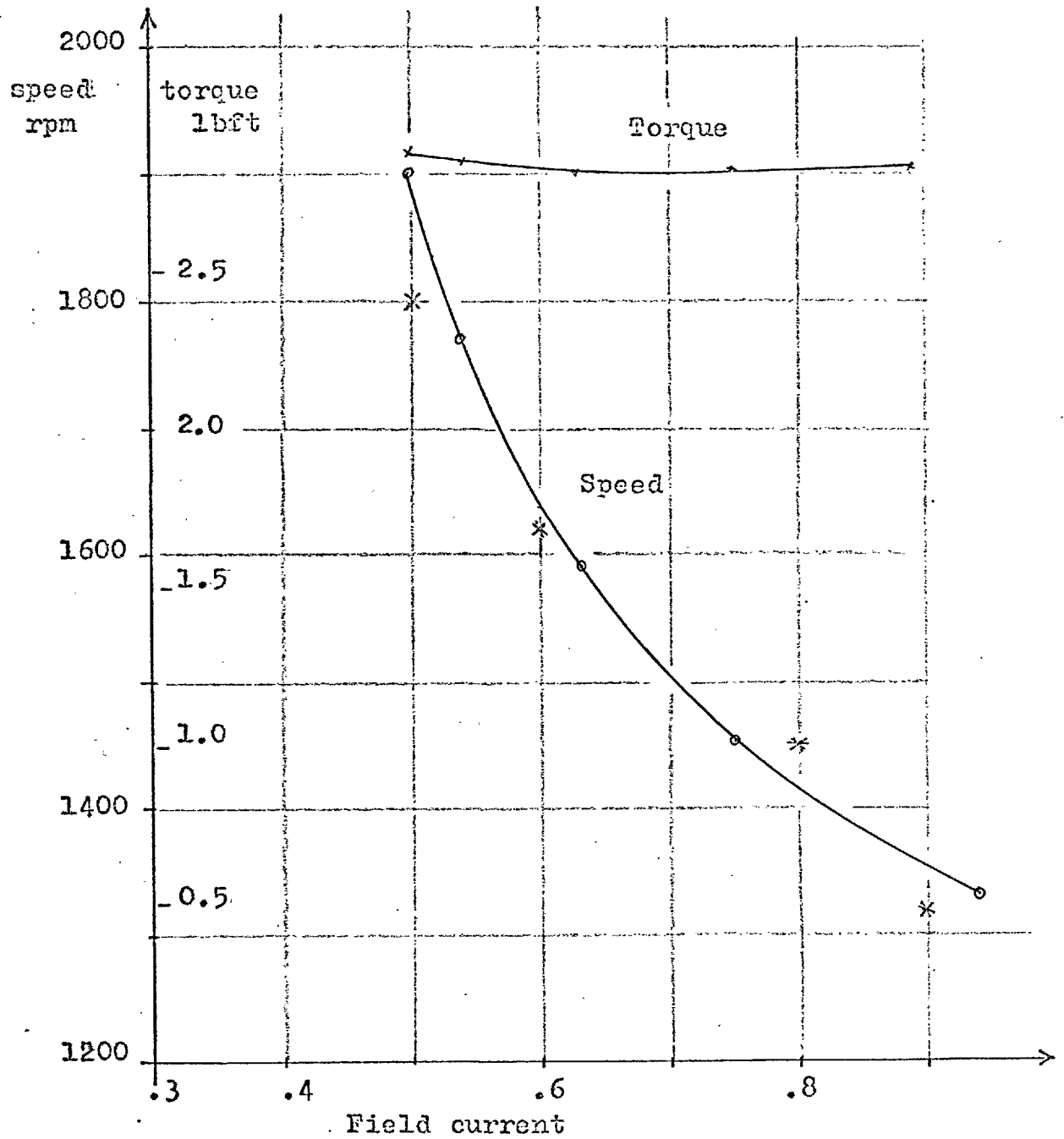
a. Experimental-at 150 volts

b. Calculated -at 150 volts

c. Experimental-at 100 volts



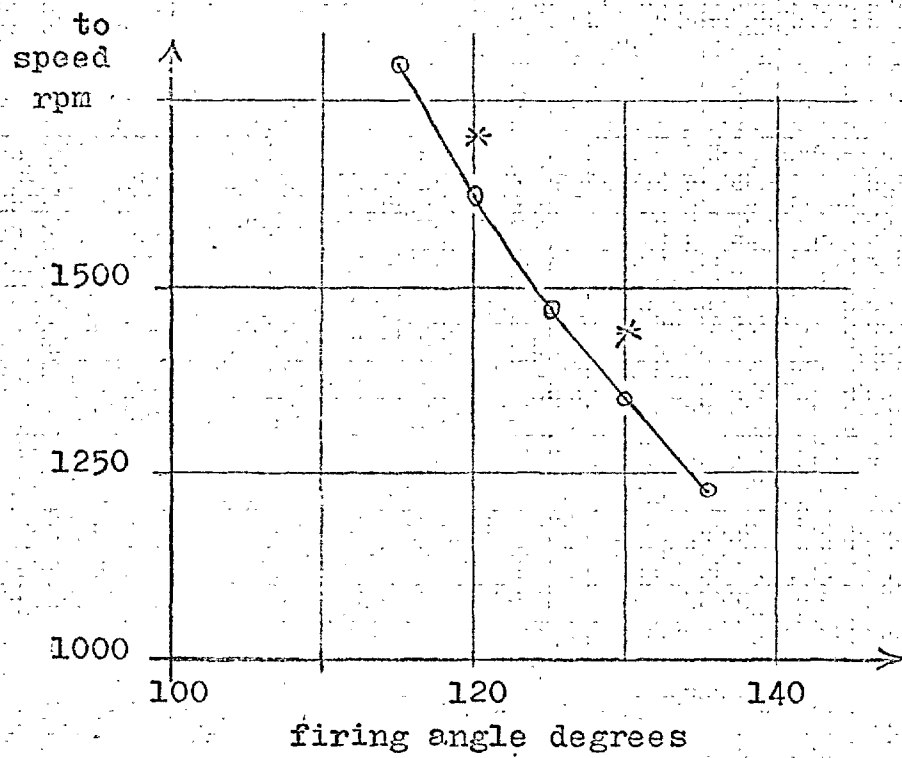
Variation of speed with applied voltage:
excitation and firing angle remaining constant
figure 10.8



Variation of torque and speed with field current
at constant applied voltage

figure 10.9

* calculated
o measured



Variation of speed with respect to the firing angle

figure 10.10

* calculated

⊙ measured

The waveform of the phase currents is similar to that of the currents in the armature coils of a d.c. machine, where the current is interrupted mechanically, aided by interpoles. However, the phase currents as shown in the figures are zero for 120° (electrical) in a cycle whereas in the case of the d.c. machine, this angle is very small.

10.6. Analysis of the inverter synchronous machine system

The relationships between the a.c. and d.c. voltages and the bridge current have been derived by Freris¹⁵ and are used in the following analysis of the system.

Supposing V_p and V_1 to be the phase and line a.c. induced voltages

V_{do} the d.c. voltage produced by the bridge at zero firing angle.

V_d applied d.c. voltage

V_{d1} d.c. voltage produced by the machine and the bridge at a firing angle

I_d average d.c. current

I rms value of the a.c. current

L commutating or the sum of the synchronous and leakage inductances of one phase

f frequency of the a.c. voltage

I_f exciting current

γ angle of overlap

and N speed of the synchronous machine

the following relations may be written

$$V_p = K I_f N - I X_c \quad \dots\dots\dots (10.1)$$

where K is a constant which depends on the magnetic circuit of the machine

$$V_{do} = \frac{3 \sqrt{3} \sqrt{2}}{\pi} V_p = 2.345 V_p = 1.353 V_1 \quad \dots\dots\dots (10.2)$$

$$V_{d1} = V_{do} \cos \theta \quad \dots\dots\dots (10.3)$$

$$I_d = \frac{V_d - V_{d1}}{\frac{3X_c}{\pi}} \quad \dots\dots\dots (10.4)$$

where $X_c = 2 \pi f L$ and $f = \frac{N}{30}$ for a four pole machine.

The transfer of current from one SCR to another does not take place instantaneously due to the inductance of the a.c. circuit. Both the SCRs are conducting during a certain angle which is known as the angle of overlap, γ . The rms value of the phase current is given by

$$I = I_d \sqrt{\frac{2}{3} - \frac{\gamma}{3\pi}} \quad \dots\dots\dots (10.5)$$

The output torque may be calculated as follows.

$$\begin{aligned} \text{Output power, } W &= V_d I_d - \text{losses} \\ \text{Also, } W &= \frac{2 \pi N T \times 746}{33000} \end{aligned}$$

where T is the torque in lb ft

$$\text{or } T = 7.016 \frac{W}{N} \quad \dots\dots\dots (10.6)$$

From the equations (1) to (4) it may be shown that the synchronous machine-inverter system is analogous to a separately excited d.c. motor where $V_{do} \cos \theta$ is the back-emf and $\frac{3X_c}{\pi}$ produces a voltage drop and is thus analogous to armature resistance in the d.c. case. The performance characteristics of the system may be calculated using equations (1) to (6).

A set of speed-torque characteristics, calculated, for various values of the firing angle whilst the applied voltage and field current remain constant, are shown in figure 10.11. Similarly a set of speed-torque characteristics calculated for various values of the field excitation whilst the applied voltage and firing angle remain constant are shown in figure 10.12. Performance of the system under any conditions may be obtained from these two sets of characteristics.

The calculated values corresponding to the measured characteristics shown in figures 10.7, 10.8, 10.9 and 10.10 are obtained from the sets of characteristics shown in figures 10.11 and 10.12, and are plotted on the appropriate figures. It may be observed that a characteristic with a constant torque can not directly be calculated

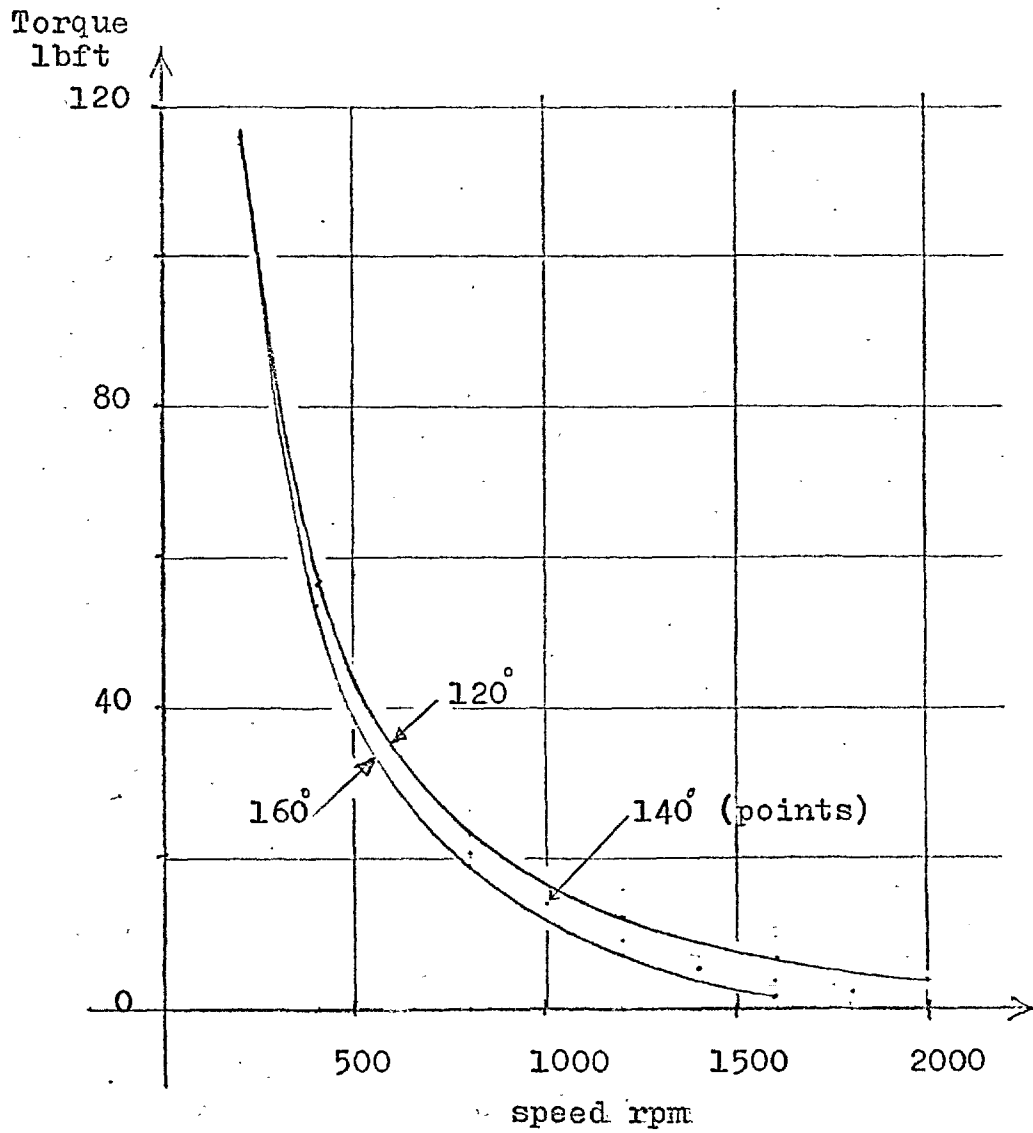


figure 10.11 Speed-torque characteristics at various firing angles with a constant applied voltage and a fixed field current

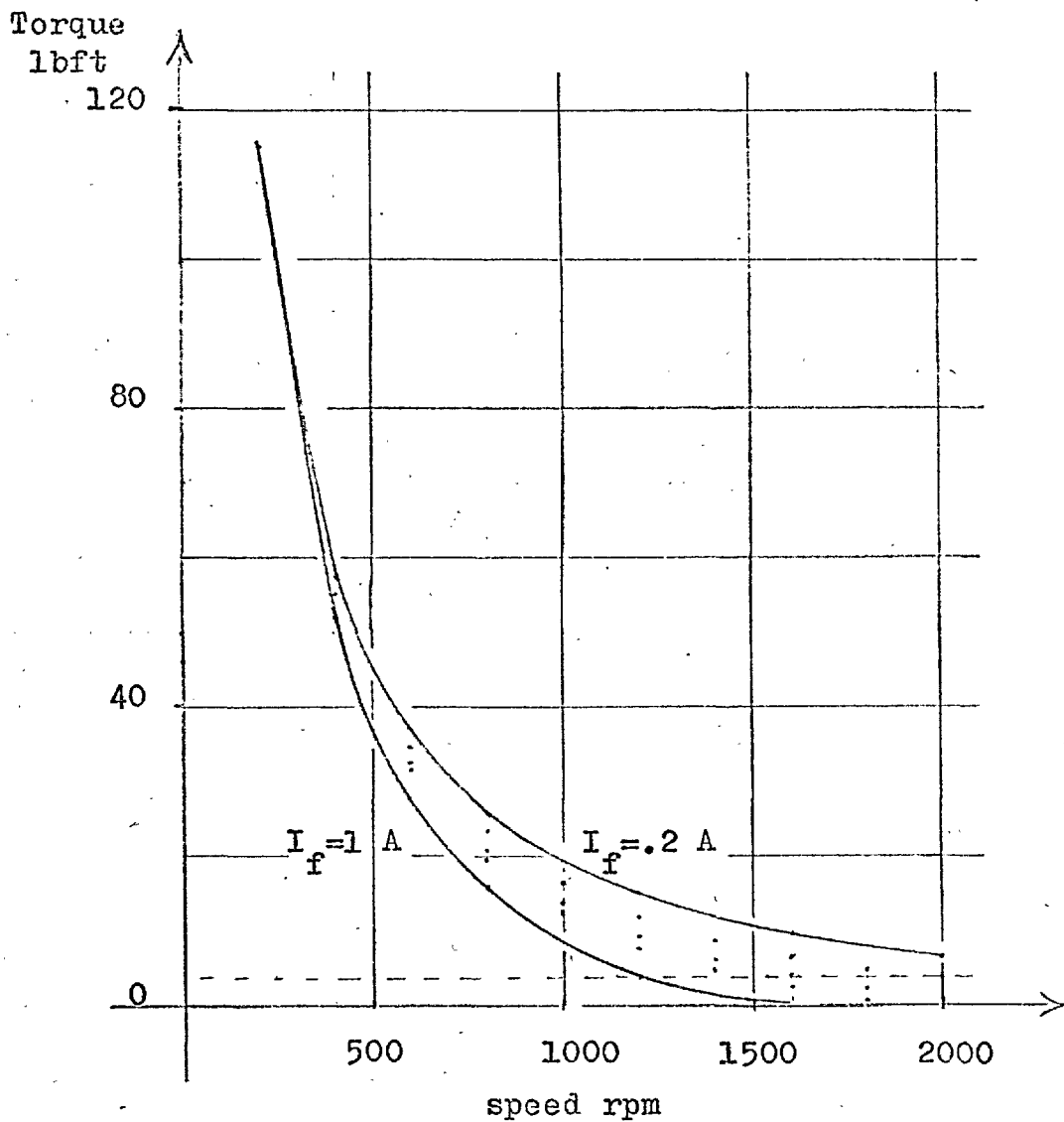


figure 10.12 Speed-torque characteristics at various field currents with a constant applied voltage and a fixed firing angle
Points plotted between the continuous curves correspond to .4, .6 and .8 amperes field currents

and has to be obtained using iterative process. An alternative procedure is to calculate a set shown in figure 10.11 or 10.12 and obtain the speeds corresponding to a constant torque by drawing a straight line as shown in figure 10.12.

10.7. Systems of speed control using the inverter synchronous motor scheme

Various systems of controlling the speed of the synchronous motor in the above scheme may be devised. The choice of a particular system depends firstly on the available source of power which may be either a d.c. or an a.c. supply. Secondly, on the required range of speed variation, and thirdly, on the type of load coupled to the motor. In fan and pump drives, load torque exists from the start and depends on the speed of the motor. In these cases off-load starting is not possible without the use of additional mechanical equipment. But a motor driving a compressor or a rolling mill can be started without any load torque except that due to friction. Thus the type of load mainly affects the choice of the system of starting.

In some applications such as driving a vehicle, variation of the speed from zero to a maximum value may be desired and in others such as blowers in nuclear reactors a small range of speeds may be sufficient. Close control of speed may be of importance in artificial fibre and chemical plants whereas an approximate control of speed may be sufficient in others. The volume and weight of the equipment as well as the availability and cost of particular apparatus may also influence the system of speed-control used. Various modes of controlling the speed and the flexibility obtained using these modes are described below.

10.8. Modes of speed control

The parameters, that can be adjusted to control the speed of the synchronous motor system are the d.c. applied voltage, firing angle and field excitation. Precise control of speed up to its maximum value can be obtained by adjusting the applied d.c. voltage. This requires, of course, additional equipment with a variable d.c. out-

put voltage. If the source of power is an a.c. system a controlled rectifier bridge or a motor generator set may be used. An alternative system using voltage transformation on the a.c. side is shown in figure 10.13. In this system the d.c. voltage is constant, resulting in an approximately constant a.c. voltage across the bridge, that is, at point P_r in the figure. If the voltage at S_r is now decreased by changing the ratio of transformation, then power flow increases and the machine accelerates to a higher speed so that the induced a.c. voltage at S_r is increased to the new setting. The sequence of operations to start the system is given in the figure.

As shown in figures 10.9 and 10.12 adjustment of speed within a small range is possible using field control. Similarly from figure 10.11 it may be observed that limited speed variation can be obtained by adjusting the firing angle. Thus, knowing the requirements, a suitable system, of controlling the speed of the synchronous motor, can be chosen.

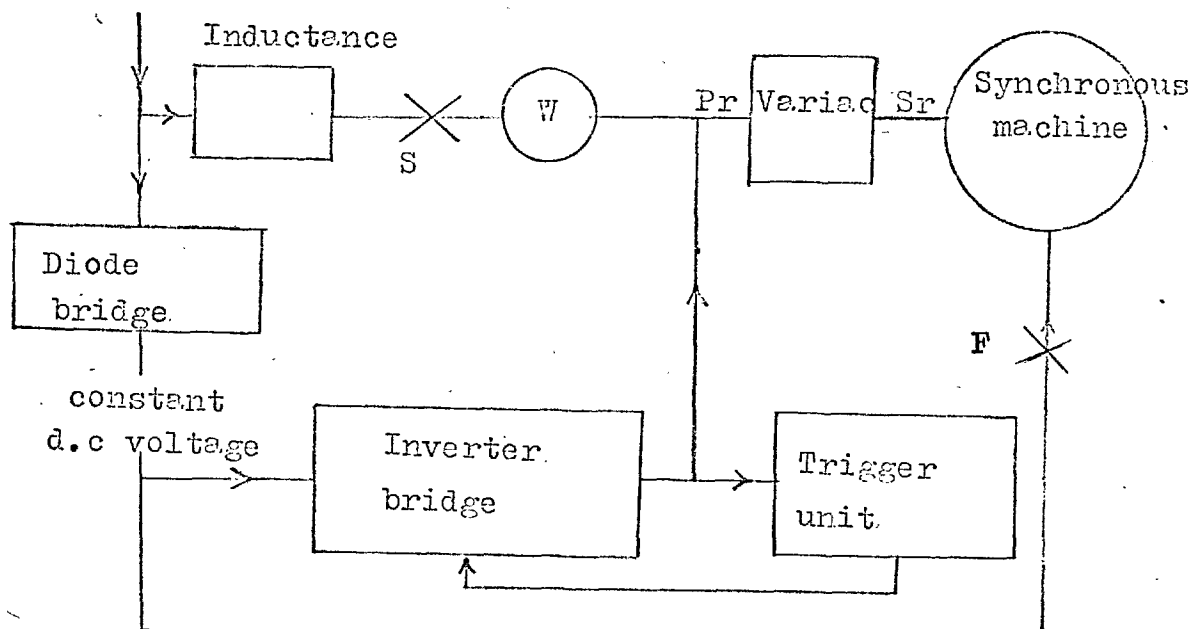
10.9. Conclusions

Induction motors may be connected to point S_r in figure 10.13 This then drives an induction motor from a d.c. source. (In the figure this d.c. source is provided by the rectifier arrangement). Without the use of impulse commutation and if it is desired the synchronous machine could be used merely as an unloaded commutating means. Further investigations would be necessary to establish the KVA of the induction motors that may be operated from a given KVA rating of the synchronous machine.

The system has only the disadvantages associated with fixed speed synchronous motors. These are sometimes used as pump storage machines using change-pole techniques but this system is much more flexible and capable of operation at variable speed rather than fixed speeds. Since the commutation is natural rather than forced, high instantaneous currents and steep rise in voltages which are common in the case of impulse inverters, do not occur. Therefore, the performance of the SCRs is more reliable and the risk of commutation failure is less. No turn-off SCRs are required. This system is thought to be more economical and reliable compared with alternative arrangements in particular circumstances.

Three phase supply

figure 10.13



Proposed scheme for a variable speed drive
operating from the mains

Sequence of operations for starting

- closing of switch S -- machine starts and attains a speed close to the synchronous speed
- ↓
- closing of switch F -- d.c excitation is given and the machine runs at synchronous speed
- ↓
- Trigger pulses to the bridge -- power-flow to the machine is substantially through the bridge
- ↓
- Opening of switch S -- this may be done after increasing the inductance so that the wattmeter reading is very small. The machine is now running freely and the speed may be varied using the variac.
- ↓
- Varying the setting on the variac to obtain the desired speed.

CHAPTER 11

MECHANICAL INVERTERS

11.1. Introduction

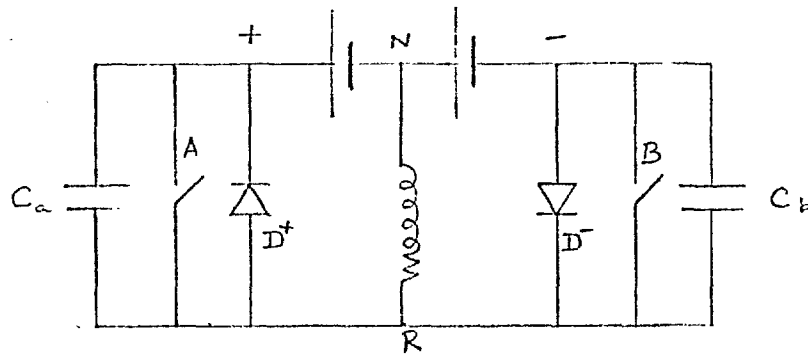
The object of the experimental investigations described in this chapter is to develop an inverter using mechanical switches. A simple switching device originally suggested by Dr. Eastman to produce three phase a.c. voltages from a d.c. supply is shown in figure 11.1. Based on the experimental knowledge obtained from the above device an improved switching arrangement was designed and tested. The nature of the electrical and mechanical phenomena involved in mechanical switching was better understood from the above tests and a successful switching device with increased complexity was designed. This device can be adapted for use as either an inverter or a time ratio controller.

11.2.1. The mechanical arrangement

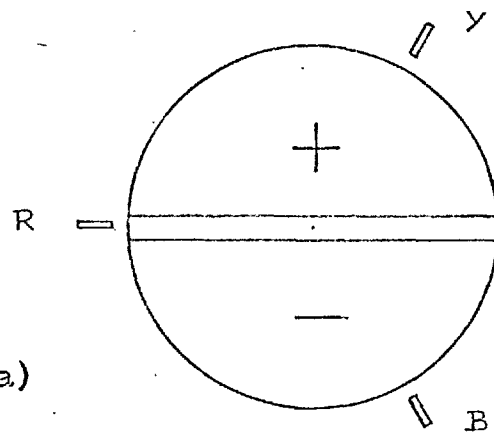
The mechanical arrangement developed consists of a rotating drum, driven by a d.c. motor and a stationary frame carrying carbon brushes. The details of the various parts and a final assembly are shown in figure 11.2.

11.2.2. The rotating drum

There are eight copper rings fixed on a hollow cast-iron cylinder which is mounted on a shaft using a spider arrangement. All the rings are insulated from the cylinder. Six of the rings are formed by segments secured to the cylinder by insulated bolts. The ends of these bolts extending inside the cylinder form the terminals for inter-connection of the rings. Rings 1, 2, 7 and 8 are all alike as shown in figure 11.2 and they are placed so that the segments on rings 7 and 8 are diametrically opposite to the corresponding segments on rings 1 and 2. The third and fifth rings are continuous.



(b) circuit diagram



(a)

A simple switching arrangement

figure 11.1

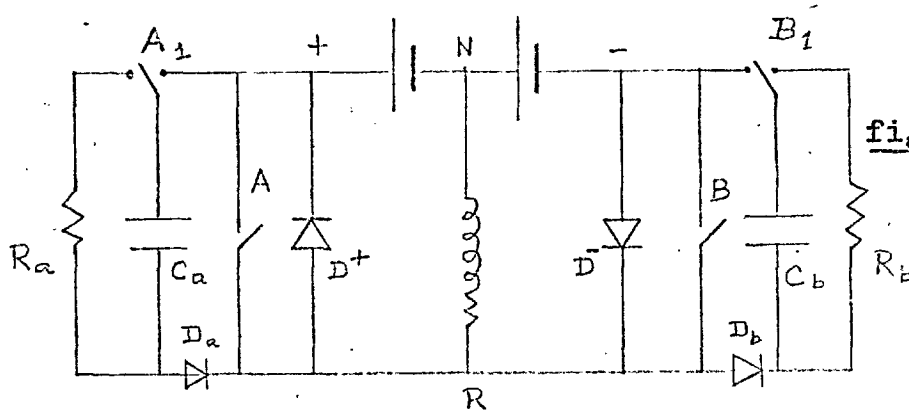
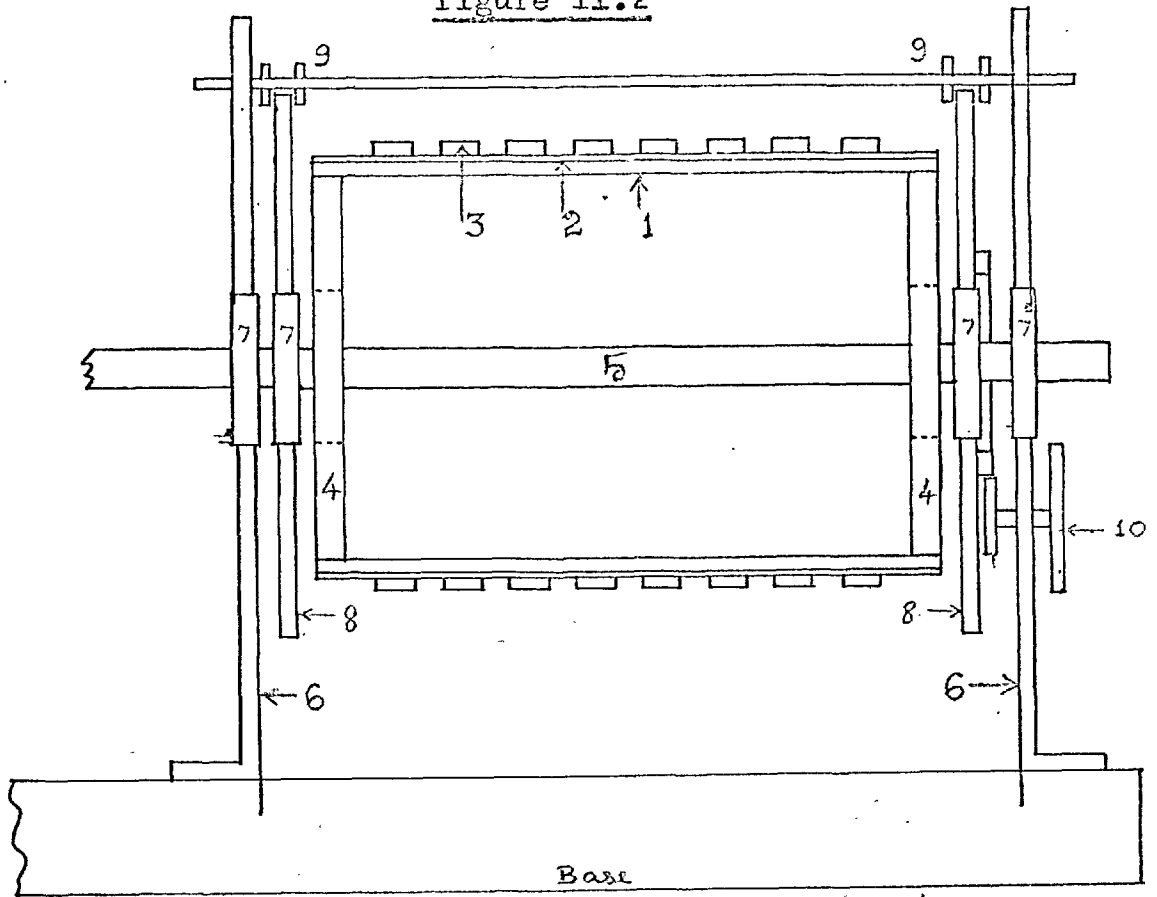


figure 11.3

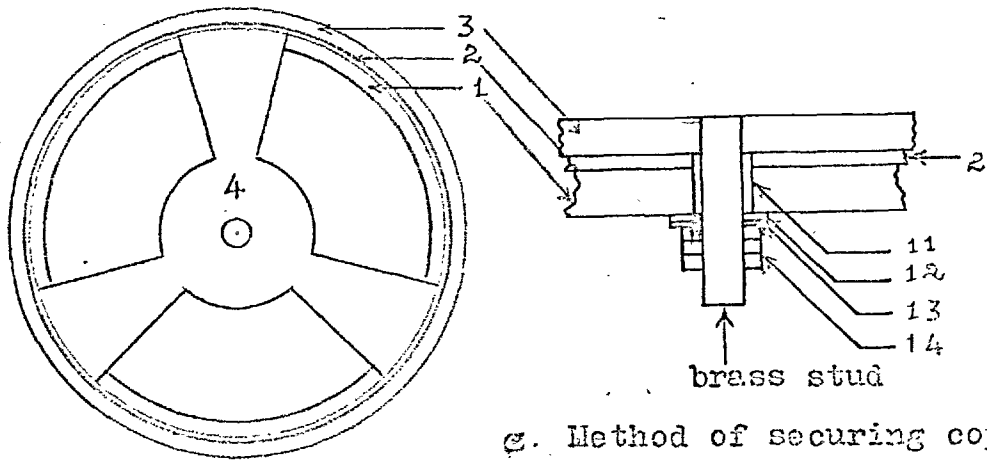
Mechanical Inverter

switching circuit for one of the phases

figure 11.2

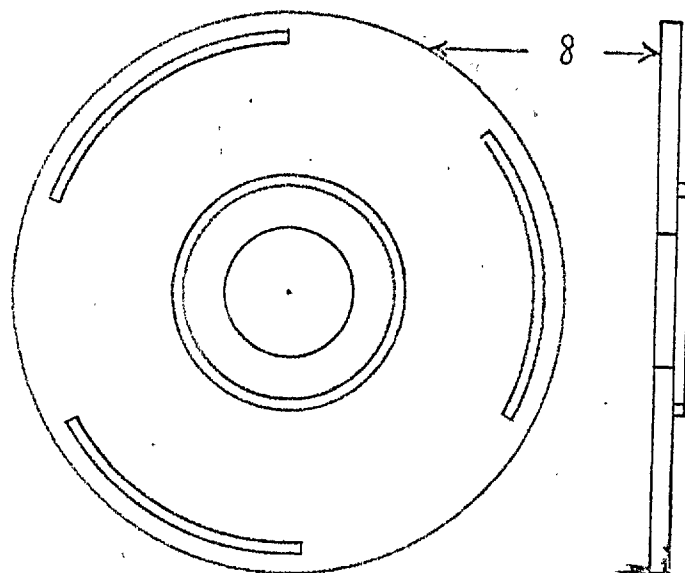


a. Mechanical inverter assembly

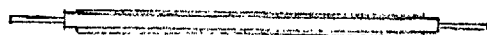


b. End view of rotor

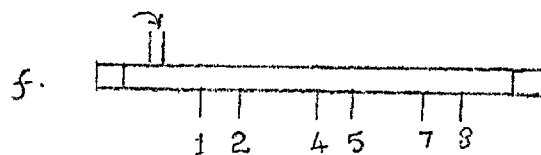
c. Method of securing copper segments to the cylinder



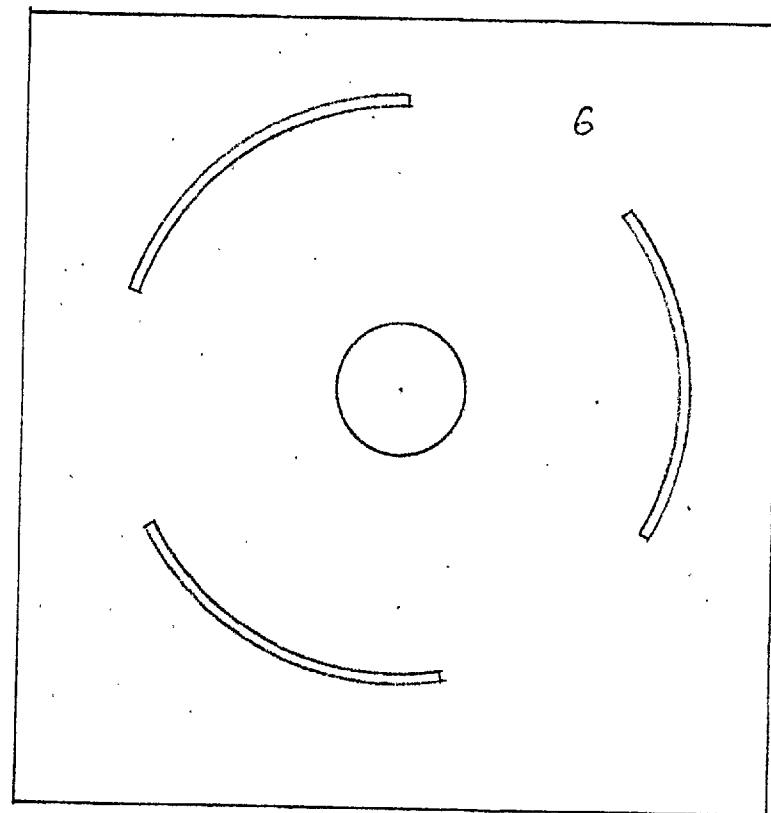
d. Slots for mounting
the moveable brushes



e. A brush rod



A row of air-jets

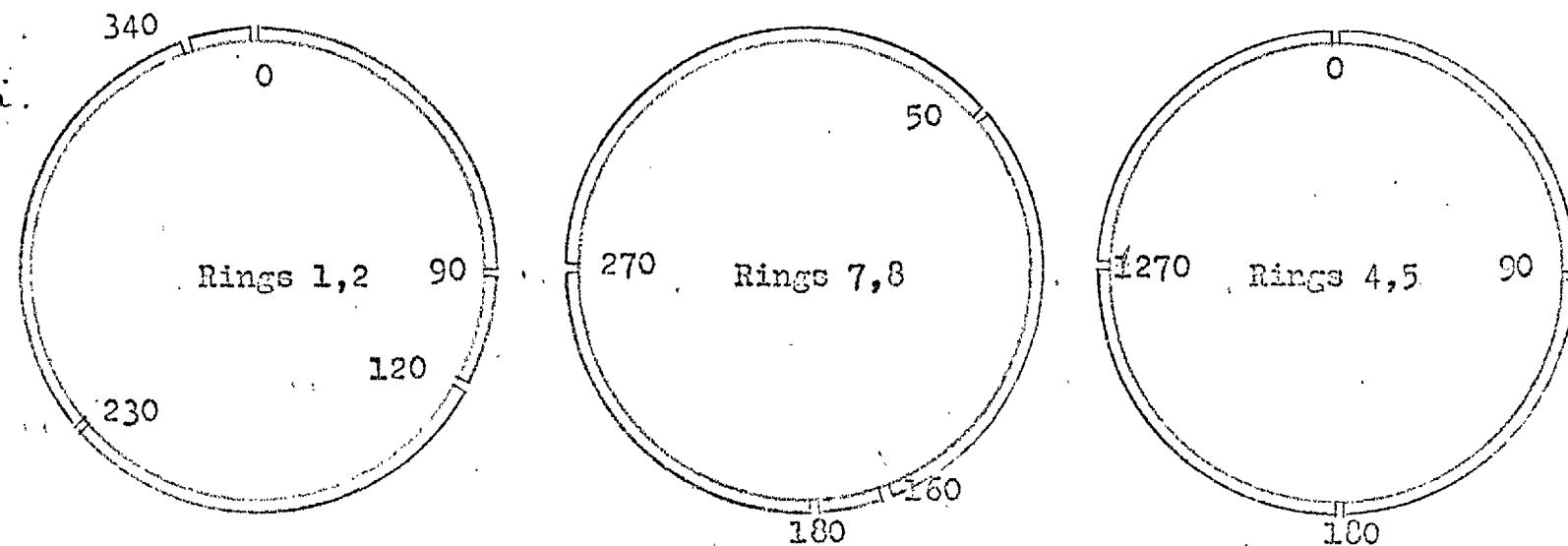


g. Slots for mounting the stationary brushes

figure 11.2

h. figure 11.2
Unfolded view of the copper rings

$\square B_1$	$\square R_1$	$\square Y_1$		1
$\square B_2$	$\square R_2$	$\square Y_2$		2
+ve ring 90 120 180 230 270 $\square$ 340				3
$R_2 \square$		$Y_2 \square$	$B_2 \square$	4
$R_1 \square$	$Y_1 \square$		$B_1 \square$	5
-ve ring 50 160 300 $\square$				6
$\square B_5$	$\square R_5$	$\square Y_5$		7
$\square B_6$	$\square R_6$	$\square Y_6$		8



Details of the mechanical inverter

1. Rotor cast iron tube
2. Bakelite tube (insulation)
3. Copper rings
4. Spider end plates
5. Shaft
6. End brackets
7. Bearings
8. Moveable disc
9. Threaded steel rods and bolts to fix the moveable brushes
10. Gear wheel assembly to rotate the moveable brushes
11. Insulating bush
12. Insulating washer
13. Locking washer
14. Nuts

11.2.3. The stationary frame

There are two end brackets supporting the rotor drum. The brackets also support six brush rods and three rows of air-jets. There is one jet per segmented ring in each row and there are three more brush rods, the position of which can be varied relative to the rest of the brush rods. These three rods are fixed between two discs supported by external bearings situated on the axle. There are twenty brushes in all and their positions on the various rings are shown in figure 11.2.h.

11.3. Inversion of d.c. voltage to polyphase a.c. voltages

The inter-connection of the segments and the corresponding circuit diagram are shown in figures 11.2.h and 11.3 respectively. The process of inversion from d.c. to a.c. voltage is explained by considering one of the phases. Reference is given to both the brushes in the device and the switches shown in the circuit diagram. In table 11.1 a number of rotor positions during one revolution and the switching actions taking place in those positions are given.

At the first position, the red phase is connected by brush r_1 to the positive supply (closing of switch A) and current flows from the positive to neutral through the brush r_1 and the load. After a movement of 50° brushes r_1 and r_2 share the load current for 40° . Then r_1 is disconnected and r_2 carries the full load current. At the fifth position the red phase is disconnected from the positive line by brush r_2 (opening of switch A). The total inductance in the circuit (the load and supply inductances) maintains the current through the alternative path consisting of a diode, brush r_3 and capacitance, C_a . The capacitor is charged and as soon as the voltage exceeds the total supply voltage, the diode on the negative side conducts and the energy stored in the load inductance flows back to the negative supply. A maximum of 35° (this can be adjusted) is allowed for the transfer of current from the positive to the negative side. At position six the capacitance, C_a , with a charge of 2V, where 2V is the total supply voltage, is isolated by brush r_4 (opening of A_2). In the seventh position brush r_1 connects the red phase (closing of B) to the negative line, thus relieving the

TABLE 11.1

Position of edge A		R phase connected to positive or negative or open	r5, the capacitor connected to negative or resistor or isolated	r3, the capacitor connected to positive or resistor or isolated
No.	degrees			
1	40	positive		
2	55		resistor	
3	120			positive
4	165		leaving resistor	positive
5	180	leaving positive, open	isolated	positive
6	215	open	isolated	leaving positive, isolated
7	220	negative	isolated	isolated
8	235	negative	isolated	resistor
9	300	negative	negative	resistor
10	345	negative	negative	leaving resistor, isolated
11	360	leaving negative, open	negative	isolated
12	35	open	leaving negative, isolated	isolated
1	40	positive	isolated	isolated

diode of its current. The charged capacitor is connected by brush r_4 (closing of A_1 to its second position) to the resistance at the eighth position and is discharged during the next 110° . Then, the capacitor is isolated by brush r_4 (opening of A_1) at the tenth position. It is now ready to be used in the next cycle.

At the eleventh position the red phase is disconnected from the negative by brush r_2 (opening of B) and the current is transferred to the positive side. The sequence of actions are as follows. The current is carried initially by capacitor, C_b , on the negative side, then by the diode on the positive side and finally by the brush r_1 . The capacitor, C_b , is discharged before being used in the next cycle.

The transfer of current from one part of a circuit to another is usually termed commutation. Here, this is achieved by first interrupting the current and then suppressing the tendency to spark by a combination of diodes and capacitors. The capacitors themselves introduce a different kind of sparking if 'they' are permitted to charge or discharge instantaneously. (This was experienced in the earlier device). Such an occurrence is prevented by the auxiliary switches which isolate the capacitors at the appropriate instants. For example, when switch A is opened there would be heavy sparking but for the presence of an alternative circuit formed by switch A_1 , the discharged capacitor, C_a and diode, D_a . Secondly, if C_a is not isolated from the positive line when switch B is closed, there is a step-voltage applied to C_a resulting in sparking. Thirdly, when switch A is closed, C_a will be discharged instantaneously, if the capacitor has not been discharged previously and causes sparking across the switch. Thus the possibilities of inductive and capacitive sparking have been eliminated.

The diodes D_a and D_b improve the waveforms and prevent ringing in the capacitor circuits which otherwise may cause sparking.

One revolution of the rotor drum corresponds to a cycle of the a.c. voltage. Hence the frequency can be varied by adjusting the speed of the rotor.

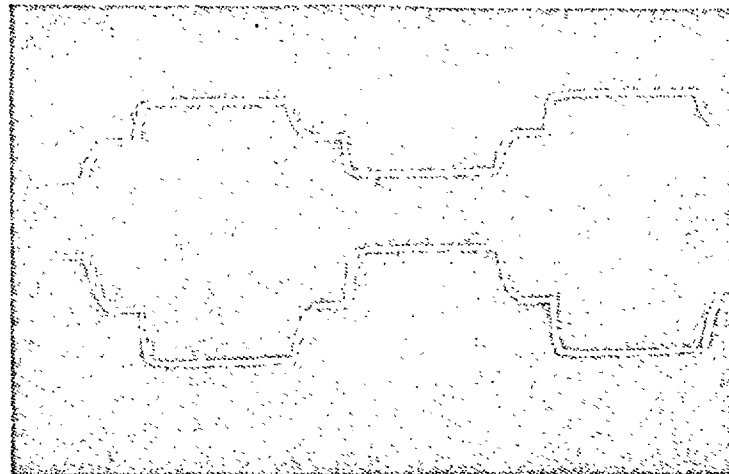
11.4. Experimental observations

In the experiments carried out the d.c. input voltage was limited by the breakdown voltage of the slots present in the segmented rings. The currents on a.c. and d.c. sides were limited by the rating of the diodes and the availability of the capacitors (used to suppress sparking). In spite of slight mechanical vibrations of the brushes the sparking was negligible and in particular there was no sparking at a particular load current. This phenomenon is due to the high resistance of the carbon brushes at very low currents. The initial sparking disappeared when the current carried by the brushes was increased (approximately to 2 amperes, the area of contact being $\frac{3}{8}$ sq. inch). Again for a given value (100 MF) of capacitors, sparking was observed when the current was increased beyond a maximum value (10 amperes, this depends of course, on the load and supply inductances).

Though it is not possible to avoid the presence of supply inductance, the value of the capacitors (for spark suppression) may be reduced considerably by using large capacitors across the supply. This also results in a decrease of the energy lost in achieving sparkless operation. In order to design a mechanical inverter to feed induction motors further experimental and theoretical investigations would be necessary to establish the relationships between the parameters in the system. For example, the optimum value of capacitors used across the supply and across the switches for a known inductance of the supply circuit, the value of the discharge-resistances for a given period of time allowed for discharging the capacitors, etc. have to be determined.

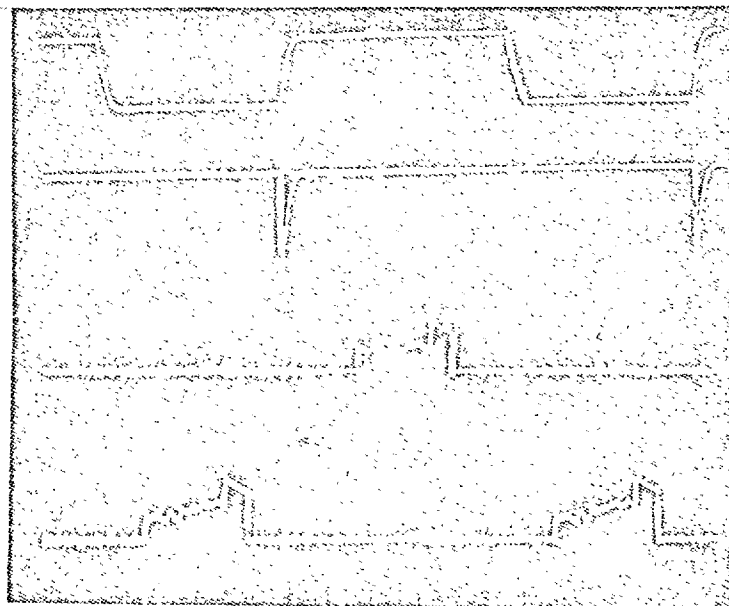
11.5. Mechanical inverter waveforms

The phase voltage and current waveforms with a substantially resistive load are alike as shown in figure 11.4.a and b. The capacitor is charged and discharged as shown by figure 11.4.c. and the spikes of voltages shown in figure 11.4.d. correspond to the discharging of the capacitor. During the time of charging the capacitors, C_a and C_b , diodes D_a and D_b conduct as shown by the



(a)

(b)



(c)

(d)

(e)

(e)

figure 11.4

Inverter waveforms with resistive load

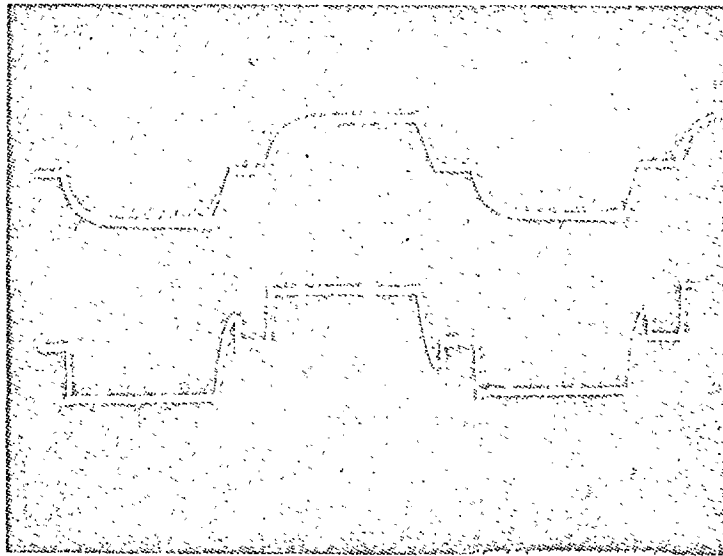
- (a) load current
- (b) load voltage
- (c) voltage across the capacitor C_a (or C_b)
- (d) voltage across the discharge resistor R_a (or R_b)
- (e) voltage across the diode D_a (or D_b) in series with the capacitor C_a (or C_b)

step voltages shown in figure 11.4.e. Voltage across these diodes when not conducting is zero and a high resistance has to be connected in parallel to the diode while photographing the voltage waveform. Otherwise the zero line shown in the figure would be a random waveform due to electrical pick up in the leads. (This was observed in the experiment).

With a slightly inductive load the shape of the voltage waveform is changed as shown in figure 11.5.b, the voltage across the capacitor now exceeds half the supply voltage and its value is given by the height of the spikes in the figure. The energy stored in the inductances of the load and supply circuits is absorbed by (or transferred to) the capacitor. If the load inductance is increased, the capacitor is charged to a voltage exceeding the value of the total supply voltage, this results in a forward voltage across the fly wheel diode which therefore conducts. From this moment the energy stored in the load circuit is fed back into the supply. This situation is shown in figure 11.5.d. The supply was provided in this case by two d.c. machines and energy feed back was therefore difficult due to the inductance of the armatures. The diode conduction is shown by the small length of the flat portion following the voltage spikes. This flat portion depends on the load inductance and if the latter is sufficiently large conduction would be continuous. These waveforms of the voltages also illustrate the dependency of the voltage waveform on the circuit parameters, mentioned earlier in chapter 7.

11.6. Conditions for successful inversion

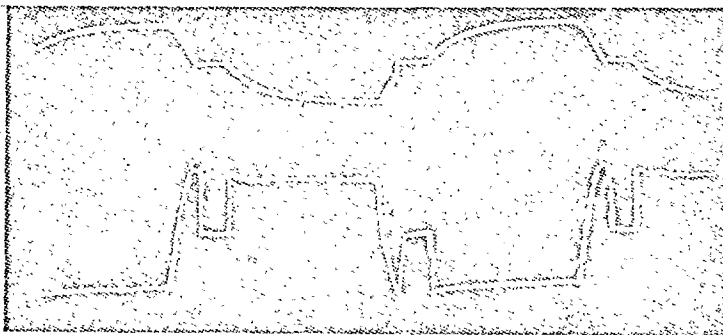
In common with all other inverter systems, the d.c. supply must be capable of receiving energy. Thus, if the d.c. supply is derived from a rectifier or a d.c. machine a capacitance of sufficient size has to be provided across the supply. The capacitances across the switches have to have low transient resistances and must be capable of carrying the initial current. They all have to be large enough to absorb the energy stored in the supply inductance. The value of the resistances for discharging the capacitors is selected such that at the highest operating speed the capacitors are discharged to a low voltage during the 110 degrees movement. The slots must be capable of withstanding the operative voltage and the brushes must



(a)

(b)

figure 11.5



(c)

(d)

Inverter waveforms with inductive load

illustrates the dependency of the voltage waveform on the circuit parameters

- (a) load current with 50 mH inductance in series
- (b) load voltage
- (c) load current with 500mH inductance in series
- (d) load voltage

not vibrate unduly as this by itself can create undue sparking. The successful operation of the device depended to a large extent on the precise construction by Mr. C.R.Johnson of the departmental workshop.

11.7. Time-ratio controller (TRC)

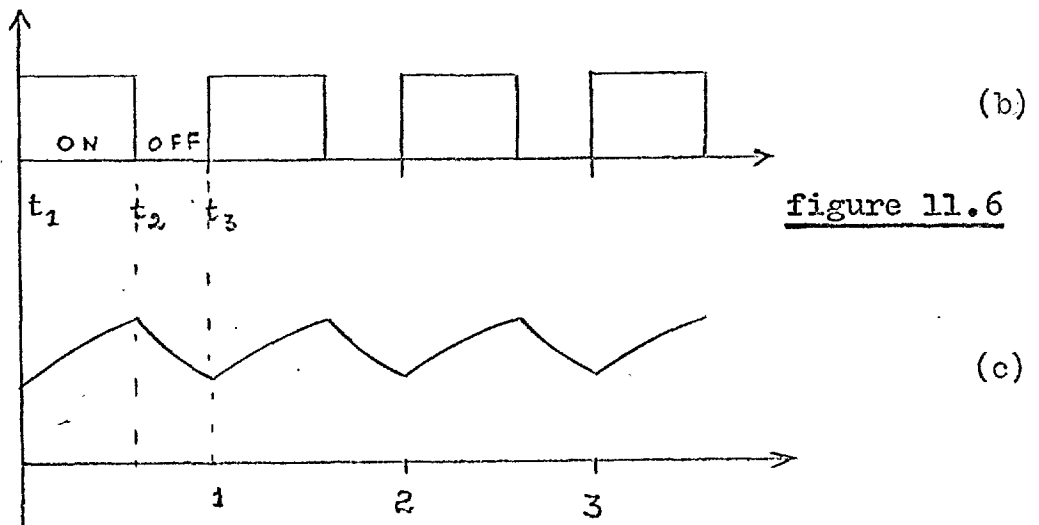
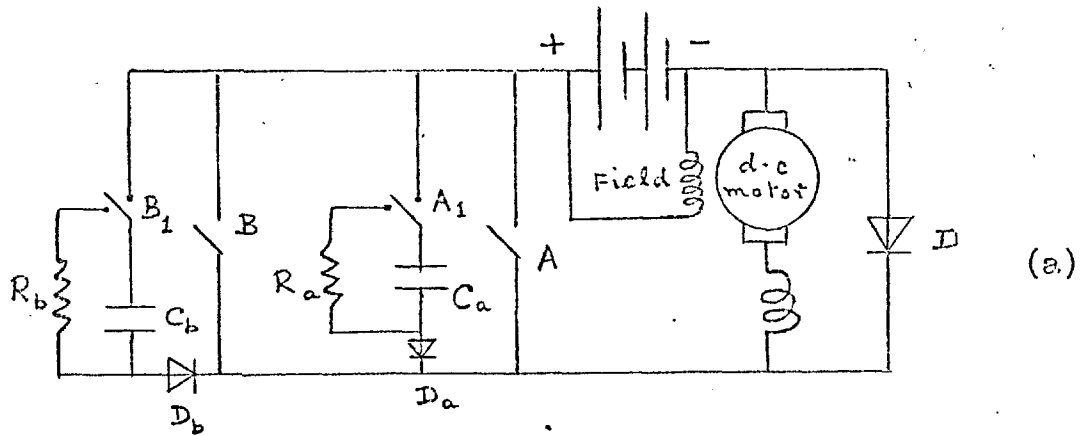
The speed of a d.c. machine operating from a constant d.c. supply can be controlled using the modified circuit shown in figure 11.6.a. Speed control is achieved by varying the effective voltage applied to the armature.

If switch A is closed at time zero, the current through the large inductance and armature increases as shown in figure 11.6.c. Switch A_1 is subsequently closed at time t , this connects the capacitor across switch A. At time t_2 switch A is opened. The inductance in the load circuit then maintains the load current through the fly-wheel diode, and the current is steadily decreased. The energy stored in the inductance of the supply is absorbed by the capacitor. The operation of switch A is thus sparkless, since the load current has an alternative path as does the current from the supply. The energy stored in the capacitor is then discharged into the resistance by connecting switch A_1 to its second position.

At time t_3 which may be called the end of the first cycle, switch B is closed and the load current starts increasing. A similar switching action takes place on the B unit. Thus the two switching units connect the positive end of the supply to the load alternately. As shown by the waveforms in figure 11.6, the average d.c. voltage across the load depends on the ratio of the time of conduction to the time per cycle and it is independent of the number of cycles per second. By varying the position of r_1 the time of conduction per cycle hence the average voltage applied to the load can be varied.

11.8. Experimental results from the TRC

The experimental waveforms are shown in figure 11.7. The load current is discontinuous as the inductance in the load circuit is not large enough to maintain the current and the supply is connected to the load for approximately one third of a cycle only. During the



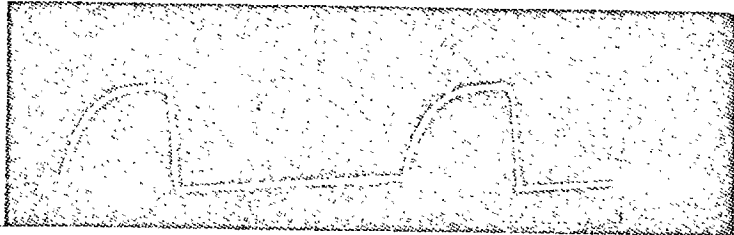
Time-ratio controller

- (a) Switching circuit
- (b) Voltage across the load
- (c) Load current



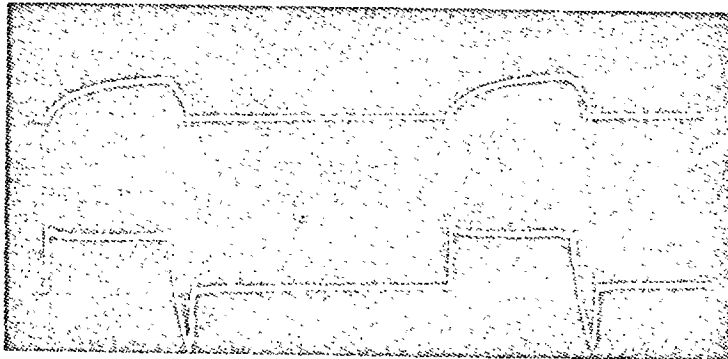
(a)

(b)



(c)

figure 11.7



(d)

(e)

Time-ratio controller waveforms

- (a) load current
 - (b) load voltage
 - (c) main switch current
 - (d) load current
 - (e) load voltage
- without the flywheel diode

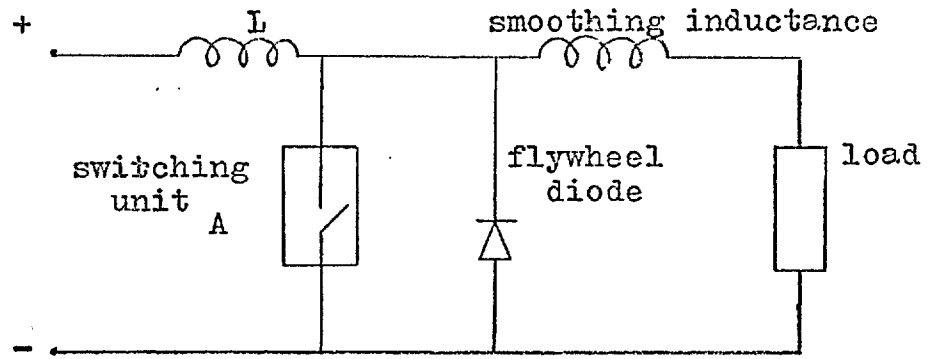
conduction of the fly wheel diode there is a small negative voltage across the load as shown by figure 11.7.b. The top of the voltage waveform is not flat due to supply regulation. Figure 11.7.c. shows the waveform of current carried by the main switches, A and B.

Figures 11.7.d and e show the waveforms of current and voltage when the flywheel diode is disconnected from the circuit shown in figure 11.6.a. The stored energy in the load and supply circuits is now absorbed by the capacitors in series with the auxiliary switches, A_1 and B_1 . The voltage to which the capacitors are charged depends on the value of capacitors, inductance in the circuit and the current interrupted. A typical case is shown in figure 11.7.e. It may also be observed that the rate of decay of the current is greater and its interruption more abrupt, compared to the case shown in figure 11.7.a, where the flywheel diode is connected across the load.

Sparkless operation of the TRC up to approximately 12 amperes was observed. The system can be operated with or without flywheel diode as observed in the experiment. The switching can be obtained using a rotating drum or relay type electromagnetic switches operated by a pulsing unit. This is thought to be an economical system for speed-control of d.c. machines.

11.9. Increasing the average voltage of the TRC

In the circuit arrangement shown in figure 11.5.a, the average voltage applied to the load can only be varied from the supply voltage to smaller values. However, by rearranging the connections to form the circuit shown in figure 11.8. the average voltage applied across the load can be increased above the supply voltage. The operation of the circuit which is analogous to the SCR circuit described in reference 16, is explained considering steady state conditions. The representation of the switching units which are similar to the A and B units in figure 11.6.a is simplified in this circuit for convenience and the explanation for sparkless operation of the switches is not repeated.



(a) circuit for increasing the load voltage

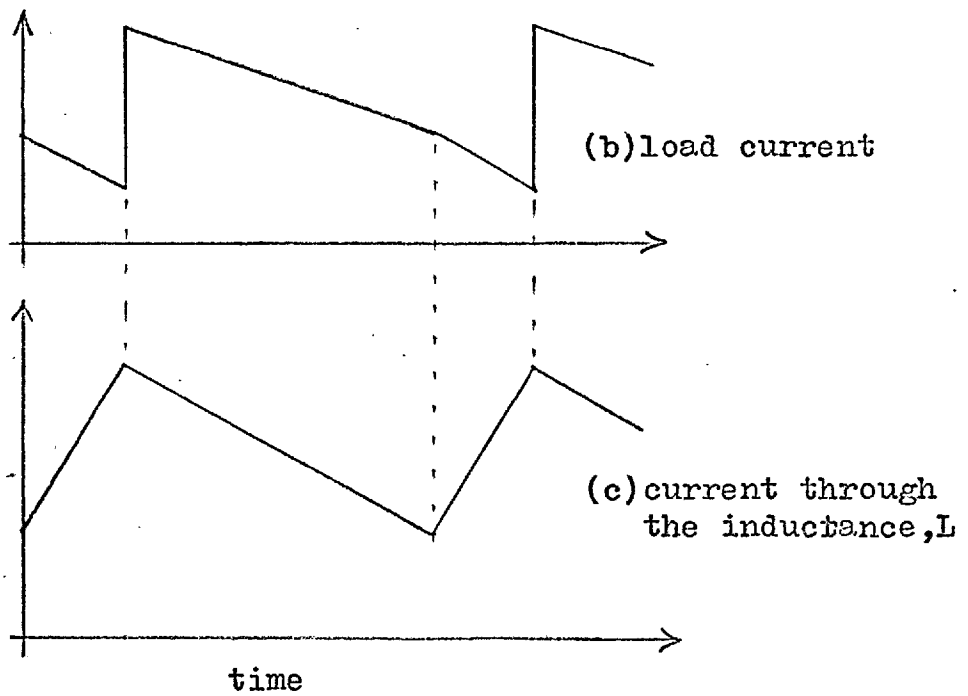


figure 11.8

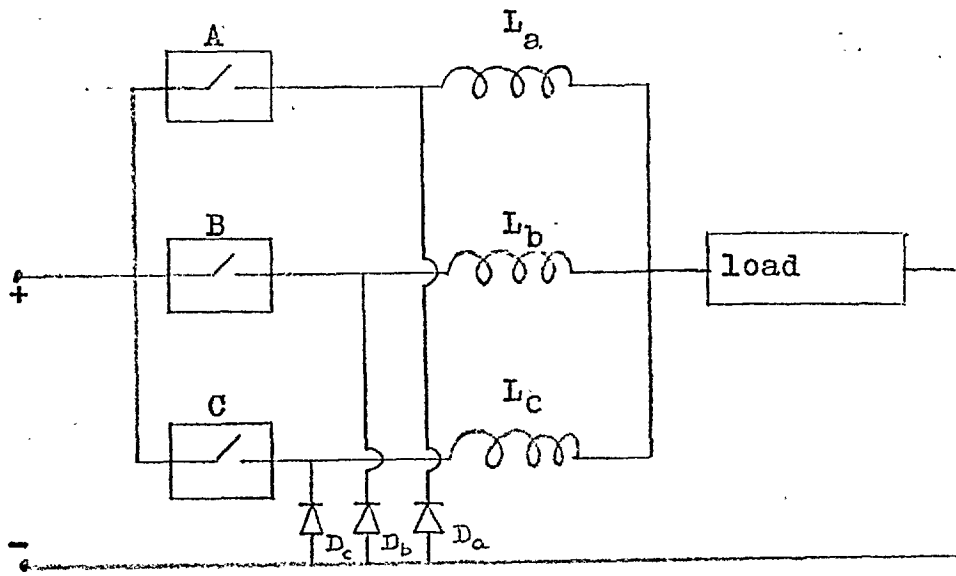
A large inductance is connected in series with the load. The switch, A, short circuits the load for a small interval of time in every cycle. During this time the current in the inductance rises quickly since the whole of the supply voltage appears across it. Meanwhile the load current continues to flow through the diode and the smoothing inductance. The switch is opened after a pre-set time which is decided by the maximum allowable current in the inductance, after which the supply voltage and the inductance are once again in series with the load. Since the current through the choke can not stop instantaneously load current is equal to the value of current through the inductance which was attained immediately before the switch was opened. This current decreases as the energy in the inductance is transferred to the load. The cycle of operation, namely closing and opening of the switch is then repeated.

The waveforms of currents in the load and the series inductance are shown in figure 11.8. The average voltage and the current through the load are increased depending on the size of the inductance and the ratio of the time of closing the switch to one cycle time.

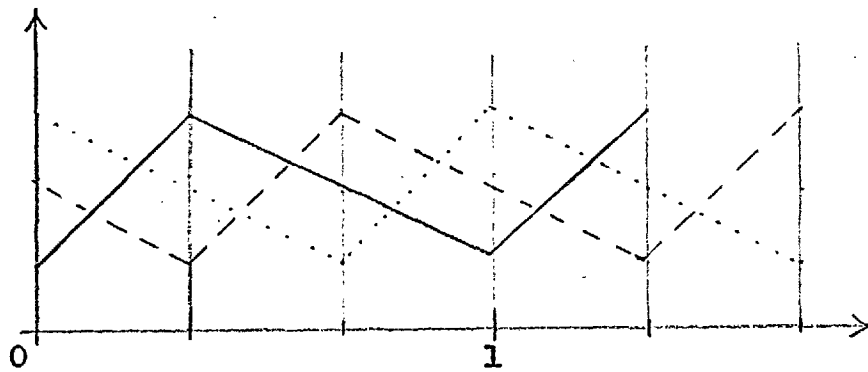
11.10. Increasing the average current

In the above two arrangements the supply carries (instantaneous) currents considerably higher than the average current. Sometimes a higher load current at low voltage may be required. In these cases the instantaneous supply-current may be unduly large. However, using an improved switching arrangement, a load-current amplification can be obtained. A particular circuit which may be used to produce a higher current at a reduced voltage is given figure 11.9. The number of parallel switching circuits depends on the required current amplification and the allowable maximum supply current.

The supply is connected to the load by one or more of the three switches shown in figure 11.9. The switches are closed sequentially and each switch conducts during part of a cycle. There are three inductances and three free wheeling diodes. Supposing at time zero switch A is closed. The current increases from I_1 to I_2 at time t_1 and the variation is approximately linear if the inductance is large.



(a) circuit for current amplification



time in cycles

(b) currents in the three parallel load circuits
each switch closed for 1/3 cycle

figure 11.9

At time t_1 switch B is closed and switch A is opened during the subsequent $\frac{2}{3}$ cycle. The current through the inductance, L_a in figure 11.9, which is drawn for a particular case, when each switch conducts, for $\frac{1}{3}$ cycle, decreases from I_2 to I_1 during the next two switching periods. The current through the switch B and inductance L_b increases from I_1 to I_2 at time t_3 when the switch B is opened and switch C closed. Similarly the current through the inductance L_c also increases and decreases during a cycle. The waveforms of the currents are shown in figure 11.9. The load voltage and current are approximately constant. The load current is given by $1.5 \times (I_1 + I_2)$ thus giving a current amplification of three which is equal to the number of switches. This is the maximum current amplification possible and under this condition it can be called a true step down d.c. transformer. The current waveforms in the supply and load circuits are improved by this arrangement, that is, they are approximately constant.

The amplification can be smoothly adjusted to values less than the number of switches with a consequent increase in the load voltage. The system has been tested and satisfactory current amplification was obtained.

11.11. Conclusions

The technique of sparkless mechanical interruption of currents was established experimentally and some particular applications have been described in this chapter. This technique can be used in a variety of applications since the cost of the system is very much lower compared to alternative static arrangements. Of course, a mechanical operation involves a certain amount of noise and requires maintenance. However, many applications can be found where the lower cost favours its use whilst the discontinuity in the use of the apparatus permits easy maintenance.

CHAPTER 12

CONCLUSIONS

12.1. Introduction

The aims of the theoretical and experimental investigations, described in this thesis, and the extent to which these aims have been realised are summarised and discussed in this chapter. The work carried out by the author and presented in this thesis forms a small part of the general problem which has been the subject of intensive research work in recent years, namely the development of a reliable and economical variable speed drive. A d.c. machine and a variable voltage d.c. source form an ideal variable speed drive as far as the speed-control of the system is concerned but the d.c. commutator introduces the problem of maintenance and a variable voltage source introduces additional equipment which may be a static controlled-rectifier inverter or a motor-generator set. The availability of high-power fast-acting semi-conductor devices has triggered the development of a host of devices and systems for variable speed operation using induction motors, thus avoiding the use of commutators. The cost of most of these devices is higher than that of a d.c. machine system. These systems usually result in non-sinusoidal applied voltages to the machines and the analysis of the machines under this condition has up to this time been only approximate. The main aim of the theoretical investigations carried out by the author was to develop a numerical technique to calculate the transient and steady state performance of induction motors supplied with non-sinusoidal voltages considering all the harmonic fluxes existing in the air gap of the machine. This has been realised completely and the results have been checked with values calculated using alternative analytical methods in some cases, and with experimental observations in others. The deductions drawn from these comparisons have been pointed out in the relevant chapters and are summarised below.

12.2. Comparison of results with analytical values

In chapter 3 the computed results obtained using the numerical

technique for a simple circuit were compared with analytical values and the accuracy of the computed results was shown to be dependent on the incremental time used.

In chapter 4 the steady state performance of an ideal machine (with sinusoidally distributed windings which produce a true rotating field when supplied with balanced sinusoidal voltages) was calculated using

a. the numerical technique
and

b. the exact equivalent circuit representing the situation.
Again correlation between the results obtained using the two methods was found to be extremely good.

In section 4.3 the performance of two types of machines were calculated using the numerical technique, in each case for two different supply conditions. The first machine (case 1) had a sinusoidally distributed winding and the second (case 2) a third harmonic distribution of the winding in addition to the fundamental. The two supplies were

a. balanced sinusoidal voltages
and

b. balanced sinusoidal voltages plus zero sequence voltages.

Thus there are four sets of results the nature of which is cited below:

For cases (1) - (a) and (2) - (a) the instantaneous torque is constant at a steady speed. Under transient speed conditions the case (2) - (a) has a pulsating component of torque. For the case (1) - (b) the result is identical to case (1) - (a) since zero sequence currents do not produce any flux in the air gap and in the case of (2) - (b) the speed torque characteristic has a dip at $\frac{1}{3}$ rd the synchronous speed and the instantaneous torque developed is not constant. These computed results are consistent with those deduced in chapter 4 from the physical properties of the systems.

12.3. Comparison of computed and experimental results

The comparison of results is summarised in an order different from

the presentation of the thesis to facilitate the discussion. The numerical technique described in the first four chapters can be applied to the case of a slip ring motor without further modification in the analysis. The computed and experimental results are compared in chapter 9 considering three different operating conditions namely,

1. steady state performance
2. pulsating torques at steady state
- and 3. switching transients

Sinusoidal and non-sinusoidal applied voltages are considered.

There is absolute agreement in the nature of ^{the} predictions and an extremely good correlation in the numerical values. An accurate prediction of transient and pulsating torque is valuable in the design of the system. (The mechanical parts have to withstand both a transient maximum torque and a pulsating torque present during the operation). In particular the speed transients are predicted only by the numerical technique and have been verified by the experimental results. Similarly the stator current waveform is predicted very accurately thus a knowledge of the harmonic content of current drawn from the supply is obtained.

12.4. Extensions of the numerical technique and comparison with experimental values

The numerical technique can be used only after forming a set of simultaneous differential equations having a unique solution. In the cases illustrated in chapters 5 (squirrel cage rotor) and 7 (presence of non-linear elements), the formation of the differential equations is not as straight forward as in the case of machines having discrete electrical circuits and containing only linear elements. In the case of squirrel cage and sheet rotors no individual circuits can be identified. This difficulty is overcome using the theory presented in chapter 5 where ample proof of the validity of the method is also given. The computed and experimental results have been compared for the case of a squirrel cage induction motor.

The solution of currents in the case of circuits having non-linear

elements involves logical operations besides arithmetic^{al} calculations (Tipping^g¹⁰ has also used a similar method for constant speed conditions) and this is illustrated in chapter 7 considering the case of an impulse-commutated inverter feeding a static load. The computed results show extremely good agreement with experimental waveforms. It has also been pointed out that the technique can be used to calculate the performance of some complicated systems where the waveforms of applied voltages depend on the currents (illustrated in chapter 11) and therefore can not be known explicitly.

Some machines may have both non-linear elements in the stator circuits and a squirrel cage or sheet rotor. In this case the above two techniques, may be used together to provide a complete solution; the first technique (for squirrel cage rotors) may be used to form a set of differential equations with a unique solution and the other to account for the presence of non-linear elements.

The applications of the theory developed for squirrel cage machines have been summarised in chapter 5. A particular application namely, the calculation of the starting performance of a variable air-gap single phase induction motor is described in chapter 6 and it is shown that an optimum variation of air-gap along a pole ^aface can be predicted.

12.5. A variable speed drive

The stability of a synchronous motor-line-commutated^{ta} inverter system was verified experimentally. The performance of the system was calculated using the analysis given in chapter 10. The computed and experimental values showed good agreement (measured parameters were used for computations). This system though not self-starting can be used with advantage in many application. Various modes of speed control and the resulting system using each mode have been discussed. Further investigations are also suggested in chapter 10.

12.6. Mechanical switching devices

Though a system using SCRs needs little maintenance, the cost of the system and sometimes its reliability prohibit its use in many

applications. It was the intention of the author to develop a cheap mechanical switching (or commutating) device that can be used in a variety of applications where it can replace costly SCR installations. This has been achieved successfully as described in chapter 11 using mechanical switches, diodes and capacitors. The sparking usually associated with mechanical interruption of current was reduced to a minimum by absorbing the energy (which is responsible for the sparking) using alternative electrical circuits and wasting it in a harmless manner employing auxilliary switches. The principles used in the design and the practical realisation of these principles are illustrated in chapter 11 with the oscillograms of voltages and currents. The switching arrangement was tested as

1. an inverter with variable output voltage while operating from a constant d.c. source-voltage
- and 2. time-ratio controller with improved performance due to the invention of multi-time-ratio switching instead of a single series path for the load current.

The performance was satisfactory but further experimental study would be necessary to establish optimum design techniques; for example the optimum values of the capacitances, resistances for given load conditions may be deduced from test results.

12.7. Measurement of transient torque

The measurement of transient torques has been the subject of many investigations in recent years and several ingenious schemes have been developed. (These include the use of strain gauges on the shaft, measurement of acceleration which is proportional to torque, the use of dynamometer etc.). A straight forward measurement of torque acting on the stator of the machine is thought to be infinitely preferable (to the indirect measurements) and this has been achieved using the design techniques described in chapter 8. These techniques may be applied in designing torque meters having higher natural frequencies by using non-uniform cross-sections or frames which have less weight to a given stiffness.

12.8 Non-sinusoidal currents drawn from a sinusoidal voltage source

The waveshape of the steady state currents drawn by a linear electrical system is determined by the waveform of the applied voltages. In particular sinusoidal applied voltages result in sinusoidal input currents to the system. (The currents during switching transient condition of course have super-imposed components which vary exponentially with time.) The electrical machines represented by equivalent circuits behave like static circuits and draw sinusoidal currents from sinusoidal applied voltages. This representation of machines by passive circuits is achieved by assuming that the rotor reacts on each space harmonic field as if the rotor has a sinusoidal copper distribution of the same harmonic pole number. That is rotor currents do not set up any other harmonic field. In practice the rotor produces a set of harmonic fields which in turn induce voltages in the stator windings. These voltages have frequencies different from the source frequency. Thus a pure sinusoidal back e.m.f is not obtained and therefore the waveform of the input currents are not sinusoidal. This introduces various problems in the power system to which the machines are connected.

The existing analytical and numerical methods (with the exception of the numerical technique described in this thesis) are either very difficult to apply or are incapable of calculating the non-linear nature of the currents drawn by practical machines; although Alger has given an equivalent circuit for taking rotor harmonics into account, he mentions that the evaluation of the parameters is difficult and clearly a static equivalent circuit can not predict the non-sinusoidal currents obtained in practice. In chapter 9 the waveform of currents computed using the numerical technique is compared with experimental waveforms and the agreement between computed and experimental values is extremely good.

12.9 Thick sheet rotors

In the case of thick sheet rotors (with iron cores on either sides or one of the sides) a three dimensional mesh pattern may be used instead of the simple mesh pattern suggested earlier in chapter 5. The conditions for forming the set of differential equations having a unique solution may be verified during numerical calculations by finding out whether the matrix of coefficients

is singular or not. If the coefficients of the currents form a singular matrix, then the number of simultaneous equations has to be reduced by searching for the relationships between the currents whereby one of the equations may be rejected. The number of unknown currents in the rest of the equations corresponds to the number of equations.

12.10 General conclusions

The theoretical processes and the experimental results presented in this thesis are thought to have many practical applications and it is hoped that the techniques will be of use in the theoretical assessment of the many new schemes and systems made possible by the use of thyristor speed control.

APPENDICES

APPENDIX 1

The self and mutual inductances of two coils in a machine with a uniform air-gap.

In figure A1.a two coils subtending angles of θ_1 and θ_2 and having an angle of θ between their leading sides are shown. The flux density pattern around the air gap for iron of infinite permeability due to a current i_1 in coil 1 is shown in figure A1.d taking the air gap circumference as the datum and assuming that the currents flow in line conductors which are situated in the air-gap. Flux density is shown as positive on the right hand side of coil 1 and as negative on the left hand side. It is uniform between points A and A' and its value is given by B_1 on the right side, that is, taking clockwise direction from A to A' and B_2 on the left side.

Since the net flux round the air gap is zero, the relationship between B_1 and B_2 may be derived by equating the total fluxes on the right and left sides as follows.

$$B_1 \theta_1 \frac{DQ}{2} = B_2 (2\pi - \theta_1) \frac{DQ}{2}$$

where D and Q are the air gap diameter and axial length

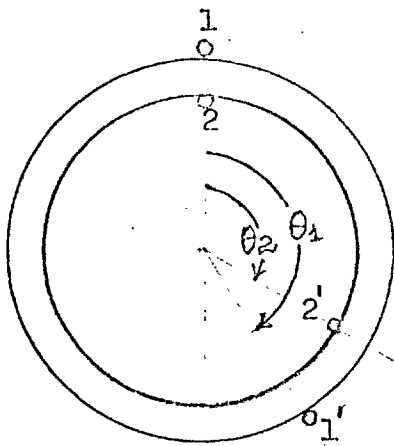
$$\text{or } B_2 = B_1 \frac{\theta_1}{2\pi - \theta_1} \dots\dots\dots (A1.1)$$

Equating the ampere-turns required to maintain the flux densities B_1 and B_2 in the air gap, expressions for B_1 and B_2 may be derived as follows.

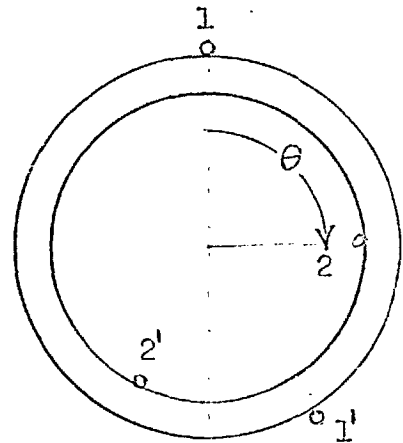
$$i_1 T_1 = \frac{B_1 G}{\mu_0} + \frac{B_2 G}{\mu_0}$$

where G is the length of the air gap and μ_0 the permeability.

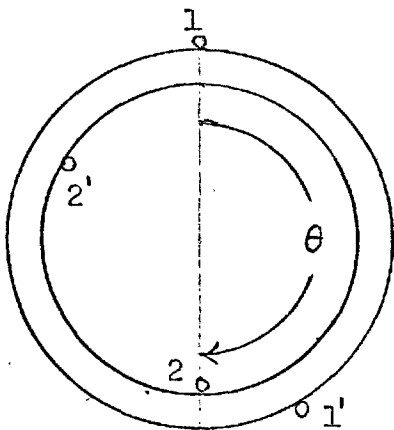
figure A1



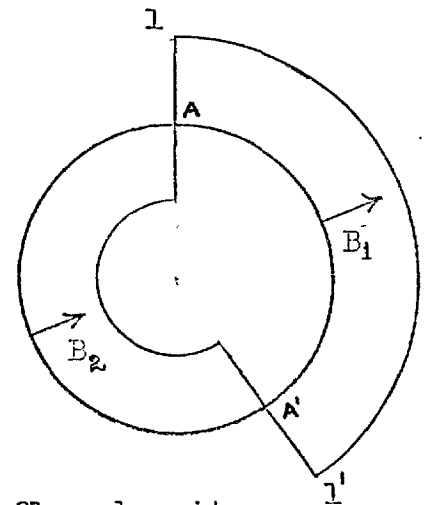
(a) leading sides coincide



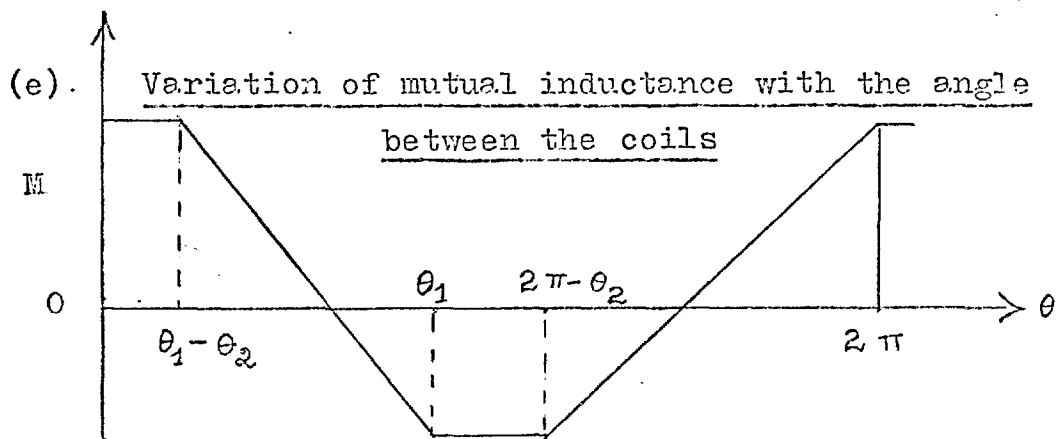
(b) coils at an angle



(c) coil 2 in the negative flux density region



(d) flux density pattern



Substituting for B_2 from equation (A1.1)

$$B_1 = \frac{\mu_0 i_1 T_1}{G \left(1 + \frac{\theta_1}{2\pi - \theta_1} \right)} \quad \dots\dots\dots (A1.2)$$

The self inductance of coil 1 is given by

$$M_{11} = \frac{\mu_0 D \ell \theta_1 T_1^2}{2G \left\{ 1 + \frac{\theta_1}{2\pi - \theta_1} \right\}}$$

Similarly the self inductance of coil 2 may be written as

$$M_{22} = \frac{\mu_0 D \ell \theta_2 T_2^2}{2G \left\{ 1 + \frac{\theta_2}{2\pi - \theta_2} \right\}}$$

The mutual inductance is the flux linkage of coil 2 per ampere in coil 1. It depends on the position of coil 2 and may be calculated as described below.

In figure A1.b the coils are shown having an angle of θ between their leading sides 1 and 2. The flux linkage of coil 2 due to a current i_1 in coil 1 is given by

$$(B_1 (\theta_1 - \theta) - B_2 (\theta_2 - \theta_1 + \theta)) \frac{D \ell}{2} T_2$$

This equation is valid only when $\theta_1 \geq \theta \geq \theta_1 - \theta_2$. That is, when point 1 lies within coil 2, with the two coils overlapping. Substituting for B_1 and B_2 and dividing by i_1 , the mutual inductance is given by

$$M_{12} = \frac{\mu_0 T_1 T_2 D \ell}{2G \left(1 + \frac{\theta_1}{2\pi - \theta_1} \right)} \left[\theta_1 - \frac{\theta_1 (\theta_2 - \theta_1)}{2\pi - \theta_1} - \frac{\theta \theta_1}{2\pi - \theta_1} - \theta \right] \quad \dots\dots\dots (A1.3)$$

This equation shows that the mutual inductance varies linearly with θ within the region considered above. When θ is greater than θ_1 , coil 2 is in the negative flux density region as shown in figure A1.c. In this position the flux linkage of coil 2 remains constant and its value is given by

$$M_{12} = \frac{\mu_0 D \ell \theta_2 T_1 T_2 \theta_1}{2G \left(1 + \frac{\theta_1}{2\pi - \theta_1}\right) (2\pi - \theta_1)} \dots\dots\dots (A1.4)$$

when point A is within coil 2, that is when θ is given by $2\pi - \theta_2 \leq \theta \leq 2\pi$, the mutual inductance again varies linearly with respect to θ and an expression similar to equation (A1.3) may be derived.

If coil 2 is within coil 1 on the right side, that is when θ is given by

$$0 \leq \theta \leq \theta_1 - \theta_2$$

the flux linkage of coil 2, and hence its mutual inductance, remains constant and its value is given by

$$M_{12} = \frac{\mu_0 D \ell \theta_2 T_1 T_2}{2G \left(1 + \frac{\theta_1}{2\pi - \theta_1}\right)}$$

The variation of mutual inductance with respect to θ is shown in figure A1.e for the general case considered.

APPENDIX 2

Inductance calculations in the case of machines having non-uniform air-gaps

A2.1. Non-uniform air-gaps in machines

In most electrical machines the air-gap is situated between two iron surfaces which have either open or closed slots. Figure A2.d illustrates the case of open slots and the air-gap is of course far from being uniform, although flux fringing and tooth saturation tend to reduce the effective gap variations. In the general case with both sides slotted, the effective air-gap may be evaluated as described below.

If field plotting is carried out for a particular magneto motive force, the effective air-gap at any point on the mean air-gap circumference may be determined as the length of the line of flux passing through the point as shown in figure A2.b.. If the rotor is now moved by a small angle and the field plotting repeated, the effective air-gap at a number of points on the mean air-gap circumference may again be determined. Repeating the procedure for various positions of rotor, it will be found that the air-gap at a particular point varies periodically with rotor position as shown in figure A2.c. If this variation is plotted for a number of points in a slot pitch it will be seen that the maximum gaps are in fact not symmetrical about the average air gap. The reason is that a point under a stator tooth has minimum air-gap when it is opposite to a rotor tooth (two teeth in series) and maximum when opposite to a rotor slot (one tooth and one slot in series), whereas a point under a stator slot has maximum air-gap when it is opposite to a rotor slot (two slots in series) and minimum when opposite to a rotor tooth (one slot and one tooth in series).

The effective air gaps at a number of points may be plotted as shown in figure A2.c with points on the circumference along the ϕ -direction, the air gap along the g -direction and the position of rotor along the θ -direction. The values may be stored while

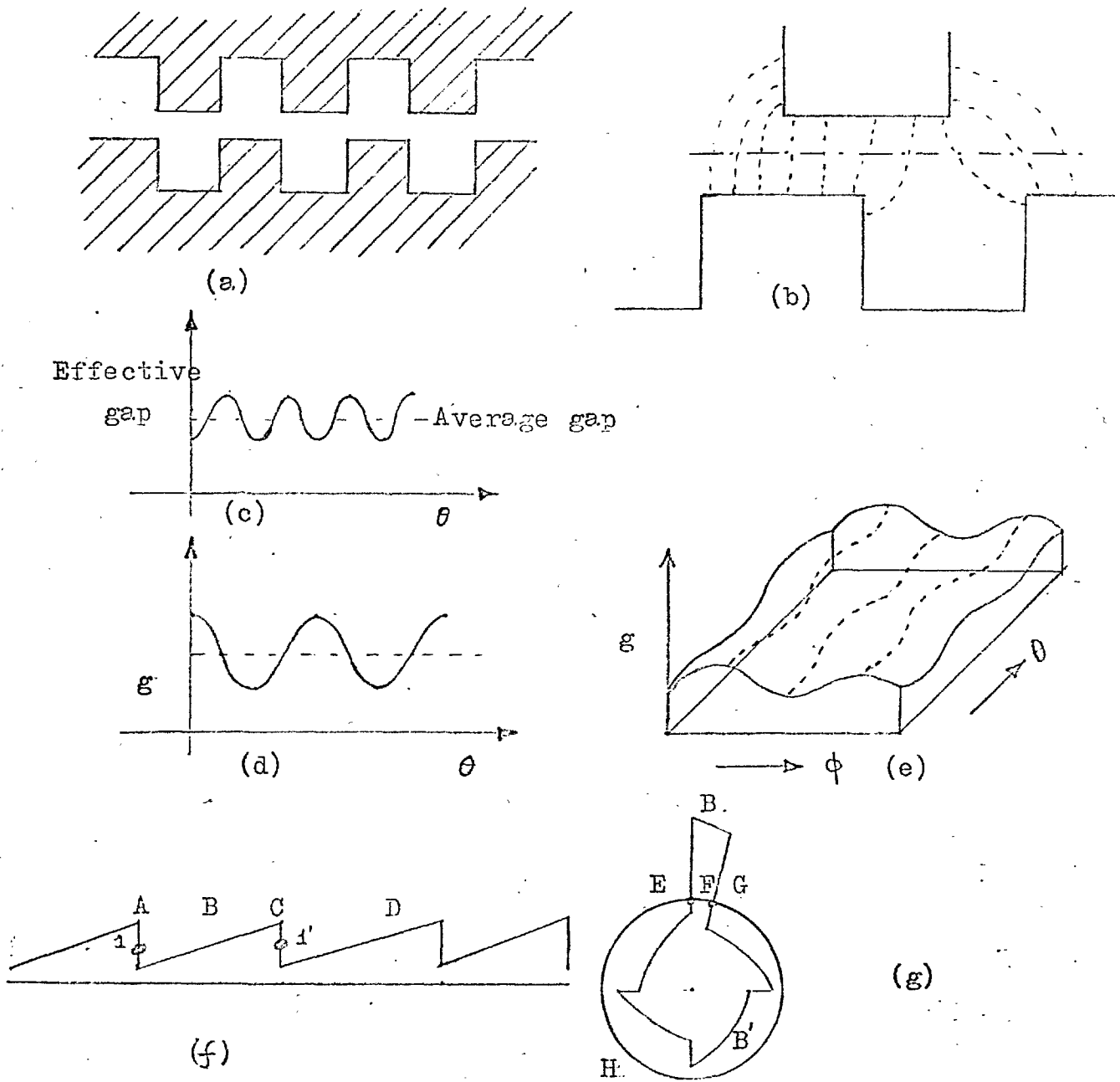


figure A2

- (a) slots on both sides of the air-gap (b) field plotting
- (c) air-gap variation with slots on both sides
- (d) air-gap with one side slotted (e) three dimensional representation of the air-gap
- (f) graded air-gap
- (g) flux density set up by a rotor loop

carrying out numerical calculations using a digital computer and the air-gap pattern at a given rotor position obtained when necessary by interpolation. For example, if a calculation contains the values of air gap at 360 points equally spaced around the circumference for 360 positions of rotor at 1° interval, then the effective air gap at the points for a given rotor position may be obtained by selecting appropriate values from the table and using linear approximation.

In practice it may not be necessary to tabulate as many values as 360×360 because of the following symmetries usually present in machines,

(1) if all the stator slots are identical then calculation of the air-gaps at points in one slot pitch with respect to rotor position will yield a pattern which will then be repeated. For example, if there are 36 slots, the number of columns in the table is reduced to 10 and this is true even with an unsymmetrical rotor since all the rotor positions are still being considered in the table.

(2) if all the rotor slots are also identical, the magnitudes of the air-gap at rotor positions covering a number of rotor slot pitches will be sufficient to define all the possible air-gap values, where the number of rotor slot pitches necessary is given by the number of rotor slots divided by the highest common factor between the rotor and stator slot numbers. The highest common factor represents the number of repeatable patterns around the circumference.

Having determined the air-gaps at any position of rotor, the self and mutual inductances may be calculated using the general procedure described.

A2.2. A general procedure to calculate inductances

The steps in calculating the self and mutual inductances are summarised below.

1. Calculation of the effective air-gap at a point as a function of its position and the angular position of the rotor from an arbitrary stator reference.

2. Determination of the flux density pattern due to a current in any coil.

The mmf produced by the coil is divided in a particular unknown ratio between the gaps on the two sides of the coil. If these two mmfs are AT_1 and AT_2 , then the flux density profile can be calculated, since the flux density at any position is inversely proportional to the air-gap at that point. Therefore, the total fluxes through the two sides may be calculated by integration, and by equating these two fluxes the relationship between AT_1 and AT_2 may be established and thus since the sum of AT_1 and AT_2 is equal to the ampere-turns of the exciting coil, the magnitudes of AT_1 and AT_2 per ampere in the coil may be determined and the flux density at any point in the air-gap may be calculated.

3. Calculation of the flux linkages of all the coils by evaluating the following integral

$$\Psi_{1p} = \frac{D \ell}{2} T_p \int_{\theta_1}^{\theta_1 + \theta_p} B d\theta$$

where D is the diameter

ℓ the length of the core,
 Ψ_{1p} the flux linkage of the pth coil due to the current in coil 1,
 B the flux density,
 T_p the number of turns in the pth coil,
 θ_1 the position of the start of the pth coil,
 and θ_p the span of the pth coil.

4. The self and mutual inductances may be calculated as the flux linkage per ampere. The procedure is continued taking the stator coils one by one and finding the self and mutual inductances in the machine. Similarly, the self inductances of the rotor coils may be determined and the whole process repeated for various rotor positions. The results may be tabulated and at any rotor position all the inductances can be found by interpolation.

A2.3. An example of calculating the self and mutual inductances.

A single phase motor is known to be capable of starting if the air-gap is graded along its pole face as shown in figure A2.f but, it is less known what the optimum variation of the air-gap should be. This forms the subject of chapter 6. In calculating the performance of the above machine, the self and mutual inductances of the stator coils are required together with the self and mutual inductances of the conducting loops, considered to act around the rotor teeth. The various steps involved in calculating these inductances have been described in the previous section and are illustrated in this section.

Assuming the rotor surface to be uniform and the stator shaped as shown in figure A2.f, the air-gap at a number of points around the circumference may be tabulated from the data of the machine. Supposing AT_1 and AT_2 to be the mmfs acting across the two lengths ABC and CDA respectively, the flux density at a position from the reference point, A, is given by

$$B_1 = \frac{\mu_o AT_1}{g(\theta)} \quad \text{where } 0 \leq \theta \leq \frac{\pi}{2} \quad \dots\dots\dots (A2.1)$$

and
$$B_2 = \frac{\mu_o AT_2}{g(\theta)} \quad \text{where } \frac{\pi}{2} \leq \theta \leq 2\pi \quad \dots\dots\dots (A2.2)$$

Thus the fluxes through the two regions are given by

$$\Psi_1 = \frac{D\ell}{2} \int_0^{\pi/2} B_1 d\theta = \frac{D\ell}{2} \mu_o AT_1 \int_0^{\pi/2} \frac{1}{g(\theta)} d\theta \quad \dots\dots (A2.3)$$

and
$$\Psi_2 = \frac{D\ell}{2} \mu_o AT_2 \int_{\pi/2}^{2\pi} \frac{1}{g(\theta)} d\theta \quad \dots\dots\dots (A2.4)$$

These definite integrals can be computed numerically since $g(\theta)$ is known and thus equating Ψ_1 and Ψ_2 the ratio $\frac{AT_1}{AT_2} = K_1$ may be

determined. The magnitudes of AT_1 and AT_2 can then be calculated as follows.

$$AT_1 + AT_2 = i_s T_s \quad \text{where } i_s T_s \text{ is the ampere turns due to the } s\text{th coil.}$$

Therefore,

$$AT_1 = \frac{K_1 T_s}{1 + K_1} i_s \quad \dots\dots\dots (A2.5)$$

and

$$AT_2 = \frac{T_s}{1 + K_1} i_s \quad \dots\dots\dots (A2.6)$$

The flux density distribution in the air gap and the flux linkage per ampere in the s th stator coil, may be calculated using equations A2.1, A2.2 and A2.3.

To determine the mutual inductance between the stator coil and the loops around the rotor teeth at a particular rotor position, the flux linkages are calculated by evaluating the following definite integral.

$$= \frac{DQ}{2} \int_{\theta_1}^{\theta_1 + \theta_2} B_1 d\theta = \frac{DQ}{2} \mu_o AT_1 \int_{\theta_1}^{\theta_1 + \theta_2} \frac{1}{g(\theta)} d\theta. \quad (A2.7)$$

where θ_1 refers to the position of the loop from reference A and θ_2 the span of the loop.

Self inductances of rotor coils

The flux density distribution per ampere in rotor coil 1 is shown in figure A2.g and the fluxes through the regions EFG and GHE are given by

$$\Psi_f = \frac{DQ}{2} \mu_o AT_3 \int_{\theta_1}^{\theta_1 + \theta_2} \frac{1}{g(\theta)} d\theta \quad \dots\dots\dots (A2.8)$$

and

$$\psi_g = \frac{D^2}{2} \mu_0 AT_4 \int_{\theta_1 + \theta_2}^{\theta_1 + 2\pi} \frac{1}{g(\theta)} d\theta \quad \dots\dots\dots (A2.9)$$

where AT_3 and AT_4 are the m.f.f.s across the two gaps in the regions EGF and GHE respectively and ψ_f and ψ_g are the total fluxes through them. Evaluating the integrals and equating the two fluxes the ratio $\frac{AT_3}{AT_4} = K_2$ may be determined. The magnitudes of the two m.f.s

and the flux density distribution may be calculated using the relation,

$$i_1 T_1 = AT_3 + AT_4 \quad \text{putting } i_1 = 1 \text{ and } T_1 = 1$$

$$AT_3 = \frac{K_2}{1 + K_2} \quad \text{and} \quad AT_4 = \frac{1}{1 + K_2}$$

The flux linkage of coil 1 and hence its self inductance may be determined using equation (A2.8). The mutual inductance between the imaginary coil 1 and a second rotor coil may be calculated by evaluating the following relation.

$$\psi_{12} = \frac{D^2}{2} \mu_0 AT_4 \int_{\theta_1 + \theta_2}^{\theta_1 + 2\theta_2} \frac{1}{g(\theta)} d\theta \quad \dots\dots\dots (A2.10)$$

Similarly $\psi_{13} \dots \psi_{1r}$ may be calculated and repeating this procedure for all the rotor coils at various positions of the rotor, the self and mutual inductances of the rotor coils may be determined and tabulated.

APPENDIX 3

Calculation of leakage inductances

A.3.1. Introduction

The leakage inductance and resistance of rotor bars depend on the current through the bar and its rate of change, and have to be treated as variables in a general analysis. This would mean that the leakage inductance and resistance of the bars have to be calculated at every step while using a numerical technique for performance calculations. The computing time would be increased enormously, since the procedure for evaluating these values involves a solution of a set of simultaneous differential equations using matrix inversion once for each bar in every step.

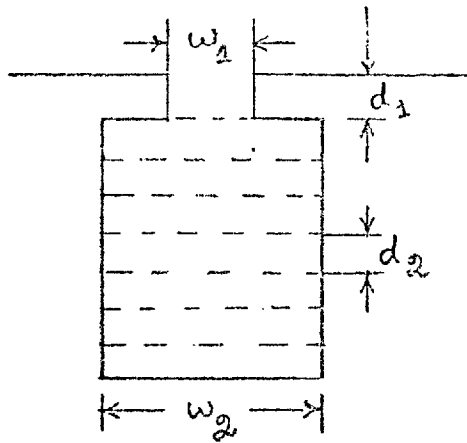
A3.2. Average values

The procedure adopted in general to account for deep bar effects is to use the average reactance and resistance of the bar to sinusoidal currents. In this way harmonic branches in an equivalent circuit have different proportions for reactances and resistances. This is the method followed here and the procedure to determine the leakage inductance and resistance of the bars is described below.

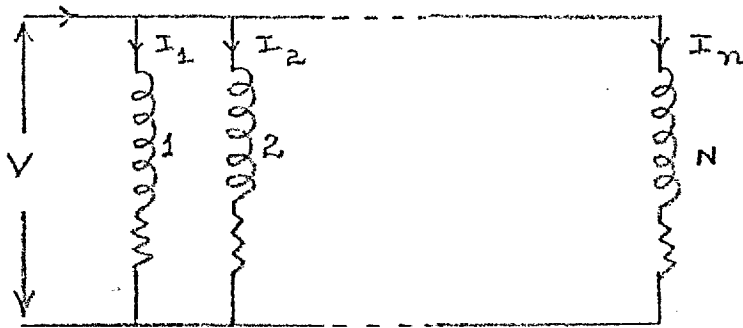
The portion of the slot containing the conductor is divided into 'N' sections as shown in figure A3.a. The permeance of the top portion of the actual slot is calculated and this is represented by $\frac{d_1}{w_1}$ in the equivalent slot. Similarly, the permeance of the slot portion accommodating the conductor is calculated as $N \frac{d_2}{3w_2}$

The bar may be imagined to have N parallel circuits which are mutually coupled. The differential equations for the N circuits may be written in matrix form as follows.

$$[V] = [I] [r] + \frac{d}{dt} ([M] [I])$$



(a) an equivalent slot



(b) an equivalent circuit with coupling between the branches

figure A3

where $[V]$ is the applied voltage

r is the resistance of a section

and $[M]$ and $[I]$ are the matrices representing inductances and currents.

A typical equation for a p th circuit is given by

$$V_p = i_p r + \frac{d}{dt} \left[M_{p1} i_1 + \dots + M_{pp} i_p + \dots + M_{pn} i_n \right]$$

Under steady state conditions, the currents are sinusoidal and the solution for the currents through the sections may be obtained by replacing the instantaneous currents with vectors and multiplying the inductances by $j2\pi f$ where f is the frequency of the applied voltage. The equation for the p th coil is modified to

$$V_p = j2\pi f M_{p1} I_1 + \dots + (r + j2\pi f M_{pp}) I_p + \dots + j2\pi f M_{pn} I_n$$

The equivalent circuit representing the situation is given in figure A3.b. The currents I_1 to I_n are calculated for a known applied voltage, say 1 volt. The vectorial sum of the currents and hence the impedance of the bar may now be calculated and the values of the reactance and resistance at the given frequency are given by the imaginary and real parts respectively.

A3.3. Saturation of tooth-tops and teeth

The permeance of the top portion of a slot is used in calculating the effective resistance and inductance of a rotor bar at different frequencies assuming infinite permeability of iron. As the current is increased the tooth-tops on stator and rotor get saturated due to high slot currents and this reduces the permeance of the slot portion above the conductor. This happens even at low values of the main flux density. Moreover, the teeth get saturated as the main flux is increased and once again the permeance of the slot is reduced. These non-linearities not only affect the total values of the leakage inductance and resistances but also modify the deep bar effects.

While calculating the performance of the squirrel cage motor

(in chapter 5) deep bar effect is calculated for the particular rotor speed. The saturation of rotor teeth is accounted by using the measured values as follows.

At standsill the voltages applied and current are measured for various values of currents. The stator leakage and resistance are measured separately by taking the rotor out. Now, the rotor resistance and leakage reactance can be calculated at various values of the stator current. The change in reactance is attributed to saturation of rotor teeth only, since the stator has open slots. If the magnetising current is neglected rotor bar current is proportional to the stator current and a graph of bar leakage reactance against stator current may be drawn as shown in figure 5.8. While computing the performance, the bar leakage inductance is multiplied by a coefficient determined from the above graph depending upon the rms value of the stator current. This value may again be multiplied by a coefficient which depends on the frequency of the bar currents.

APPENDIX 4

Force calculations in electrical apparatus

A 4.1. The basic concepts

The basic concepts used in deriving the formulae for calculation of forces in electrical apparatus are,

(1) The principle of conservation of energy
and

(2) Ampere's law of force which states that the force, acting on a conductor carrying a current, i , in a magnetic field of flux density, B , is given by $B i l$ where l is the length of the conductor.

A4.2. Use of the principle of conservation of energy

If the two coils shown in figure A4.a are considered, then the energy stored in the magnetic field is given by

$$\frac{1}{2} M_{11} i_1^2 + \frac{1}{2} M_{22} i_2^2 + i_1 i_2 M_{12} \dots\dots\dots (A4.1)$$

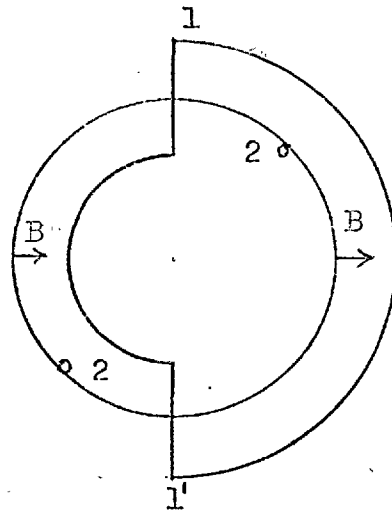
where M_{11} , M_{22} and M_{12} are the self and mutual inductances and, i_1 and i_2 are the currents. Using the principle of virtual work, the expression for the torque developed may be derived. A small change, $d\theta$, in the position of coil 2, is assumed and the currents are kept constant during this movement. The electrical energy input to each coil has two components, namely the resistive loss and, the product of the current and the change in flux linkage. The latter is equal to the work done plus the change in stored magnetic energy. The change in stored energy can be obtained by differentiating equation (A4.1) as

$$\frac{1}{2} i_1^2 d M_{11} + \frac{1}{2} i_2^2 d M_{22} + i_1 i_2 d M_{12}$$

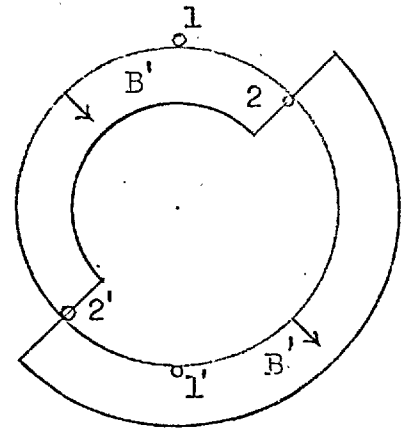
or

$$\frac{1}{2} i_1 d \Psi_1 + \frac{1}{2} i_2 d \Psi_2$$

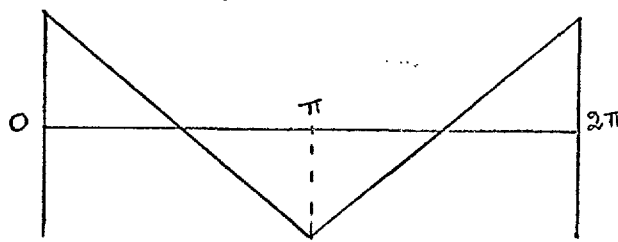
where Ψ_1 and Ψ_2 are the flux linkages of coils 1 and 2.



(a) flux density produced by coil 1

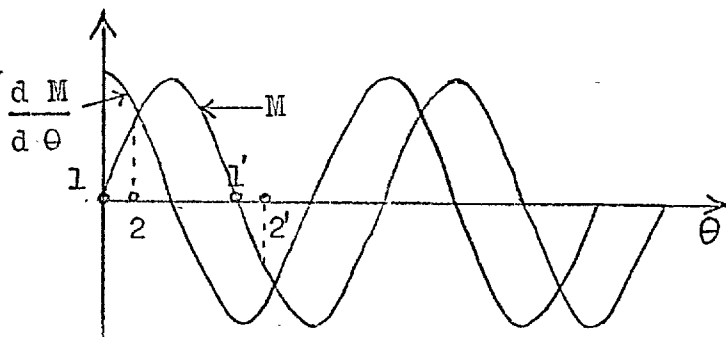


(b) flux density produced by coil 2



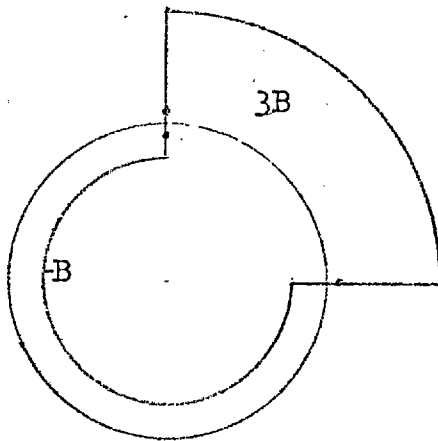
(c) real flux density due to both coils

(f) variation of mutual inductance

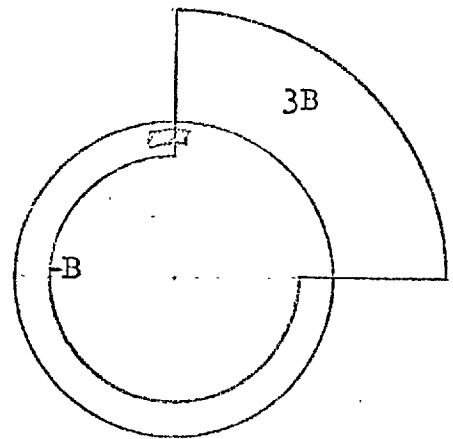


(g) mutual inductance M and variation of mutual inductance $\frac{dM}{d\theta}$ for two groups of coils

figure A4



(d) point conductor



(e) position taken by
a conductor

figure A4

This is half of the quantity, given by the electrical input minus the resistive loss. Therefore, the other half is equal to the mechanical work done. The torque is obtained by dividing the work done by the angular movement and it is given by

$$\frac{1}{2} i_1^2 \frac{d M_{11}}{d \theta} + \frac{1}{2} i_2^2 \frac{d M_{22}}{d \theta} + i_1 i_2 \frac{d M_{12}}{d \theta} \dots (A4.2)$$

If the self inductances do not vary with rotor position (for example in the case of a uniform air-gap) then the first two terms vanish. The above expression can be used to calculate the torque whilst using the step-by-step technique.

A4.3 An alternative form

The work done by an electrical system due to an actual movement of the rotor, whilst the currents vary, (as opposed to the fixed values of the previous article), may be calculated using the principle of conservation of energy. Considering a pth coil in a multi-coil system, the stored energy due to the coil is given by

$$\frac{1}{2} i_p \Psi_p \text{ where } \Psi_p \text{ is the flux linkage of the pth coil due to the currents in the system } \dots (A4.3)$$

The change in stored energy during the movement is given by

$$\frac{1}{2} i_p d \Psi_p + \frac{1}{2} \Psi_p d i_p \dots (A4.4)$$

The input electrical energy apart from the resistive loss is given by

$$i_p d \Psi_p \dots (A4.5)$$

Applying the principle of conservation of energy the work done may be obtained by subtracting equation (A4.4) from the equation (A4.5). It is given by

$$\frac{1}{2} (i_p d \Psi_p - \Psi_p d i_p) \dots (A4.6)$$

The sum of the work done by all the coils divided by the angular movement gives the torque. When the rotor is at standstill, the

work done is zero and the angular movement is also zero, resulting in an indeterminate value of torque. Under this condition equation (A4.2) may be used though this requires increased number of arithmetical operations.

A4.4.1 Force calculations using Ampere's law

As stated in A4.1 the force on a conductor is given by the product of its length, current carried and the flux density. There are two difficulties in using Ampere's law for force calculations in machines. Firstly the expression $B i l$ is not in a convenient form as calculation of flux density in the air-gap is laborious and secondly the expression does not include the sense of the force. These difficulties may be overcome by using superposition and Fleming's rule respectively.

A4.4.2. Super position of forces

Regarding the flux density as being set up by one of the coils of figure A4.a and thus producing a force on the second coil, two identical expressions for the forces acting on the coils are derived. Similar expressions are derived considering the real flux density (set up by all the currents). It is shown that all the four equations for forces on the coils are identical.

Force on coil 2 due to coil 1

The two coils considered and the flux density distribution around the air-gap due to current i_1 in coil 1 are shown in figure A4.a. Supposing D to be the air gap diameter

G the air gap

F_1, F_2 the forces on the two coils due to each other

T_1, T_2 the number of turns

i_1, i_2 the instantaneous currents

and B and B' the flux densities due to the coils 1 and 2 respectively, the flux density due to current i_1 in coil 1 is given by

$$B = \frac{\mu_0 i_1 T_1}{2G}$$

Neglecting the effect of coil 2 as far as flux density in the air gap is concerned, the force on coil 2 is given by

$$F_1 = 2B i_2 T_2 \ell \text{ newtons}$$

$$= 2 \times \frac{\mu_0}{2G} i_1 T_1 i_2 T_2 \ell \dots\dots\dots (A4.7)$$

Force on coil 1 due to coil 2

The flux density in the air gap due to current i_2 in coil 2 is shown in figure A4.b. Neglecting the effect of coil 1 on flux density distribution, the force on coil 1 is given by

$$F_2 = 2B' i_1 T_1 \ell = 2 \times \frac{\mu_0 i_2^2 T_2}{2G} i_1 T_1 \ell \dots\dots\dots (A4.8)$$

since B' is given by $\frac{\mu_0 i_2^2 T_2}{2G}$.

The currents are considered positive when entering through the start of a coil. Referring to figure A4.a the start of coil 2 is situated in the positive flux density region of coil 1. On the other hand the start of coil 1 is in the negative flux density region of coil 2 as shown in figure A4.b. Therefore the forces F_1 and F_2 act in opposite directions and they are found to be equal from equations (A4.7) and (A4.8).

Real flux density in the air-gap considered

When real flux densities are considered, the assumption that currents flow along line conductors, leads to various discrepancies. Firstly, in practice step-changes in flux density distribution do not occur. However, line conductors result in step flux density changes as shown in figure A4.c. Secondly, difficulty is experienced in applying the force equation namely, $B i \ell$, since the flux density, B , varies from one value to another at a point conductor. The expression $B i \ell$, applied to adjacent points, suggests that the average of the flux densities on either side of the

point conductor must be used in calculating the force. This method gives the correct answer when the force between coils, rather than the force on a conductor is considered. For example, the force on a point conductor shown in figure A4.d is $2 B i_1 Q$, if the average flux density at the point is used. But, in practice, a conductor free to move, occupies a position shown in figure A4.e so that $3B i_2 = B i_1$, thus the force on the conductor is zero. Since the main interest in machine calculations is on the force between coils, average flux density can be used.

Assuming B' in figure A4.b to be greater than B the resultant flux density pattern in the air gap is shown in figure A4.c. The force on coil 1 is given by

$$\begin{aligned} F'_1 &= \frac{(-B - B' + (-B' + B))}{2} i_1 T_1 Q \times 2 \\ &= -2 \times B' i_1 T_1 Q \\ &= -2 \times \frac{\mu_0 i_2^2 T_2}{2G} i_1 T_1 Q \dots\dots\dots (A4.9) \end{aligned}$$

whilst the force on coil 2 is given by

$$\begin{aligned} F'_2 &= \left(\frac{-(B' - B) + B + B'}{2} \right) i_2 T_2 Q \times 2 \\ &= 2 B i_2 T_2 Q \\ &= 2 \frac{\mu_0 i_1^2 T_1}{2G} i_2 T_2 Q \dots\dots\dots (A4.10) \end{aligned}$$

The expressions given in the equations (A4.7) to (A4.10) are all identical hence all the methods result in the same answer, and it follows that the forces may be calculated without any reference to the real flux densities in the system.

When there are n coils in a machine, the force on one of the coils due to the $(n - 1)$ coils is found out, by calculating the force on the coil due to each of the other $(n - 1)$ coils taken individually using equation (A4.7) and finding the sum. The sense in which these forces act must be known in finding this sum and it

depends on the relative positions of the coils. In a static case the expression for force for each pair of coils considered may be allocated a particular sign say positive when the currents are positive. When one of the currents become negative at any time, the direction of force reverses and the expression automatically becomes negative. But in the case of a machine where the rotor coils are revolving, it is not possible to specify a particular sign for the force between two coils. There must be a way of changing the sign depending upon its position relative to a stator coil. This may be achieved by storing the signs for several positions of the coil and finding the sign depending upon its position.

A4.5. Equivalence of the energy method and Ampere's law

In this section it is shown that Ampere's law of force leads to an expression identical to equation (A4.2) which has been derived from energy considerations. Referring to figure A4.a and assuming the currents to be positive in both the coils, the force on coil 2 in the position shown, that is with the start of coil 2 in the positive flux density region of coil 1, is in the clockwise direction. It may be noted that the derivative of the mutual inductance between the coils has a constant negative value as shown in figure A4.f. When the start of coil 2 moves into the negative flux density region of coil 1 the force acting on it reverses and it is now in the anti-clockwise direction and the derivative of the mutual inductance becomes a positive constant. Thus the sense of the force depends on the relative position of the coils and so does the derivative of the mutual inductance.

An expression in terms of currents and the derivative of the mutual inductance giving the magnitude and direction of the force between the coils may be derived by considering a phase group of coils on the stator. The flux density distribution due to the phase current i in the coils forming the phase is shown by curve $\frac{dM}{d\theta}$ in figure A4.g. The phase may be represented by a single

equivalent coil 1 - 1' and a rotor phase may similarly be represented by the coil 2 - 2'. When the angle between the coils is θ , the flux linkage of coil 2 due to coil 1 is given by

$$\int_{\theta}^{\pi + \theta} \frac{D \ell}{2} B d\theta$$

and the mutual inductance is given by

$$M = \frac{1}{i_1} \int_{\theta}^{\pi + \theta} \frac{B D \ell}{2} d\theta \quad \dots\dots\dots (A4.11)$$

Differentiating this expression with respect to θ ,

$$\frac{dM}{d\theta} = \frac{1}{i_1} \frac{2BD\ell}{2} \quad \dots\dots\dots (A4.12)$$

That is, the derivative of the mutual inductance is directly proportional to the flux density per ampere in the particular position considered. Therefore, if the variation of the mutual inductance with respect to the angle between the coils 1 and 2 is known, the flux density in which one of the coils is situated (the flux density being set up only by the second coil) may be calculated using the expression,

$$B = \frac{i_1}{D \ell} \frac{dM}{d\theta}$$

Then the force on coil 2 may be expressed as

$$\begin{aligned} F &= 2 \times B i_2 \ell \\ &= \frac{2}{D} i_1 i_2 \frac{dM}{d\theta} \quad \dots\dots\dots (A4.13) \end{aligned}$$

A4.6. Calculation of torque in a three phase slip ring motor:

(a) Using the Energy Method

There are six coil groups carrying independent currents; three on the stator and three on the rotor of the machine. The external work done by the machine may be calculated using the equation (A4.6) as

$$W_{me} = \sum_{p=1}^{p=6} \frac{1}{2} (i_p d\psi_p - \psi_p di_p) \quad ,,,,,,,,,, (A4.14)$$

where i_p and ψ_p are the current and flux linkage of the pth coil. The net torque developed is the work done during a period divided by the movement of the rotor during this period.

(b) Using the force equation:

If the inductances are given as functions of θ , which denotes the position of rotor from an arbitrary reference on the stator, and if the currents at a particular position of the rotor are known, then the tangential force on the rotor due to the stator coils is given by

$$\begin{aligned} F = \frac{2}{D} & \left[i_1 \left(i_4 \frac{dM_{13}}{d\theta} + i_5 \frac{dM_{15}}{d\theta} + i_6 \frac{dM_{16}}{d\theta} \right) \right. \\ & + i_2 \left(i_4 \frac{dM_{24}}{d\theta} + i_5 \frac{dM_{25}}{d\theta} + i_6 \frac{dM_{26}}{d\theta} \right) \\ & \left. + i_3 \left(i_4 \frac{dM_{34}}{d\theta} + i_5 \frac{dM_{35}}{d\theta} + i_6 \frac{dM_{36}}{d\theta} \right) \right] \\ & \dots\dots\dots (A4.15) \end{aligned}$$

using equation (A4.13)

APPENDIX 5

Two equivalent expressions to calculate $[H]$ matrix

The following relationships are re-written from section 3.8.

$$[H]_2 = [V]_n - [I]_1 [R] + [H]_1 \quad \dots\dots\dots (A5.1)$$

$$[H]_2 = [V]_n - [I]_1 \frac{[R]}{2} + \frac{[M]_1 [I]_1}{T} \quad \dots\dots\dots (A5.2)$$

where

$$[H]_1 = [B]_1 [I]_1 = \frac{[M]_1 [I]_1}{T} + \frac{[R]_4 [I]_1}{2} \quad \dots (A5.3)$$

from equation (3.13)

Substituting this value of $[H]_1$ in equation (A5.1) yields

$$\begin{aligned} [H]_2 &= [V]_n - [I]_1 [R] + \frac{[M]_1 [I]_1}{T} + [I]_1 \frac{[R]}{2} \\ &= [V]_n - [I]_1 \frac{[R]}{2} + \frac{[M]_1 [I]_1}{T} \end{aligned}$$

which is same as equation (A5.2). The use of equation (A5.1) to calculate $[H]_2$ avoids the multiplication of matrices $[M]_1$ and $[I]_1$, since the existing value of $[H]_1$ is used, along with a simpler multiplication given by $[I]_1 [R]$.

APPENDIX 6

Filter design

R.C. filter¹⁷

The R.C. Filter used is a notch-filter cascaded with a low-pass filter as shown in figure 8.9a. The frequency response is shown in figure 8.9.b. The notch-filter is designed to reject the natural frequency of oscillation, f_{∞} , of the strain gauge apparatus using the expression

$$2 \pi f_{\infty} = \frac{1}{R C}$$

The low-pass R.C. filter is designed using the expression for its frequency response given by,

$$\frac{\text{output}}{\text{input}} = \left(\frac{v_o}{v_i} \right) = \frac{1}{\sqrt{(1 - (\tau w)^2)^2 + (3\tau w)^2}}$$

where $\tau = R_1 C_1$ and $w = 2 \pi f$

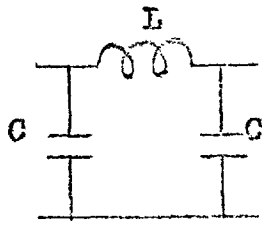
L - C filter¹⁸

In order to obtain an improved frequency response as shown in figure A6.a a low-pass L.C. filter network was designed. It consists of a n-derived π -section and matching half-sections as shown in figure A6.b. The following relationships are used in designing a proto-type π -section which is shown in figure A6.c along with its frequency response.

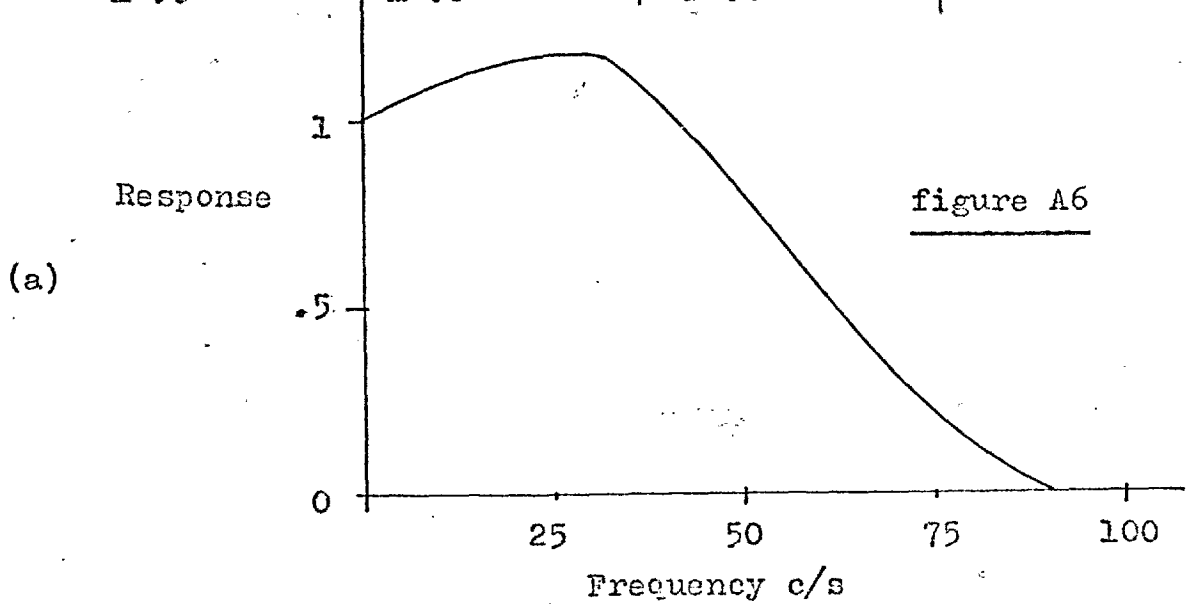
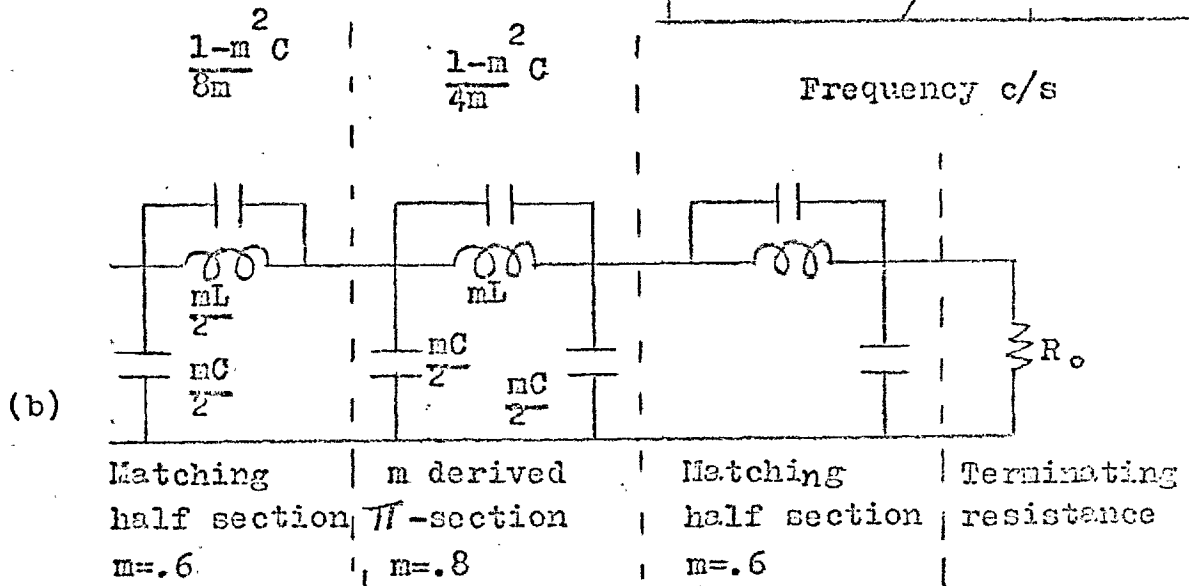
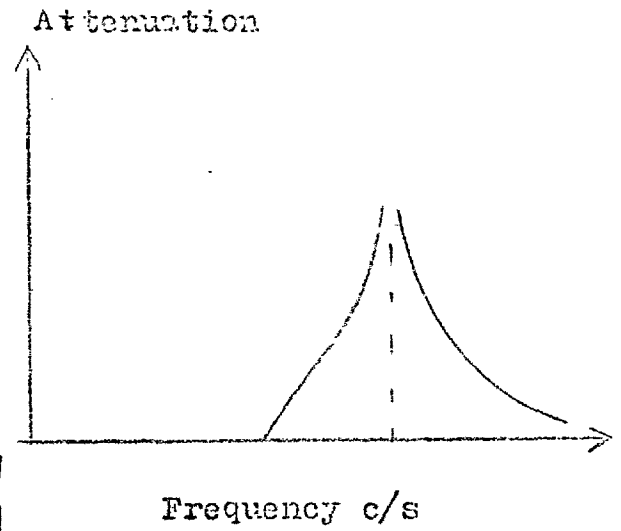
$$f_c = \frac{1}{\pi \sqrt{LC}}$$

and

$R_o = \sqrt{\frac{L}{C}}$ when R_o is the terminating impedance and f_c is the cut-off frequency. To design an n-derived section an additional relationship, $f_c = f_{\infty} \sqrt{1 - n^2}$ is used, f_{∞} being the



(c) Proto-type π -section and its attenuation



Frequency response of the network

frequency at which attenuation is infinite. The value of m for the π -section is taken to be .8 and the value of the components are given by the expressions shown in the figure. The half-sections are similarly designed taking the value of m as .6. (Detailed explanations are found in the reference).

The inductances were chosen to be small to avoid an undue amount of electrical pick up and this results in a smaller value of terminating impedance. Therefore a matching amplifier is introduced between the strain gauge bridge (which has a high output impedance) and the filter. Two filter units were used to retain the symmetry of the circuit in order to eliminate any pick up signal.

APPENDIX 7

A simple trigger unit

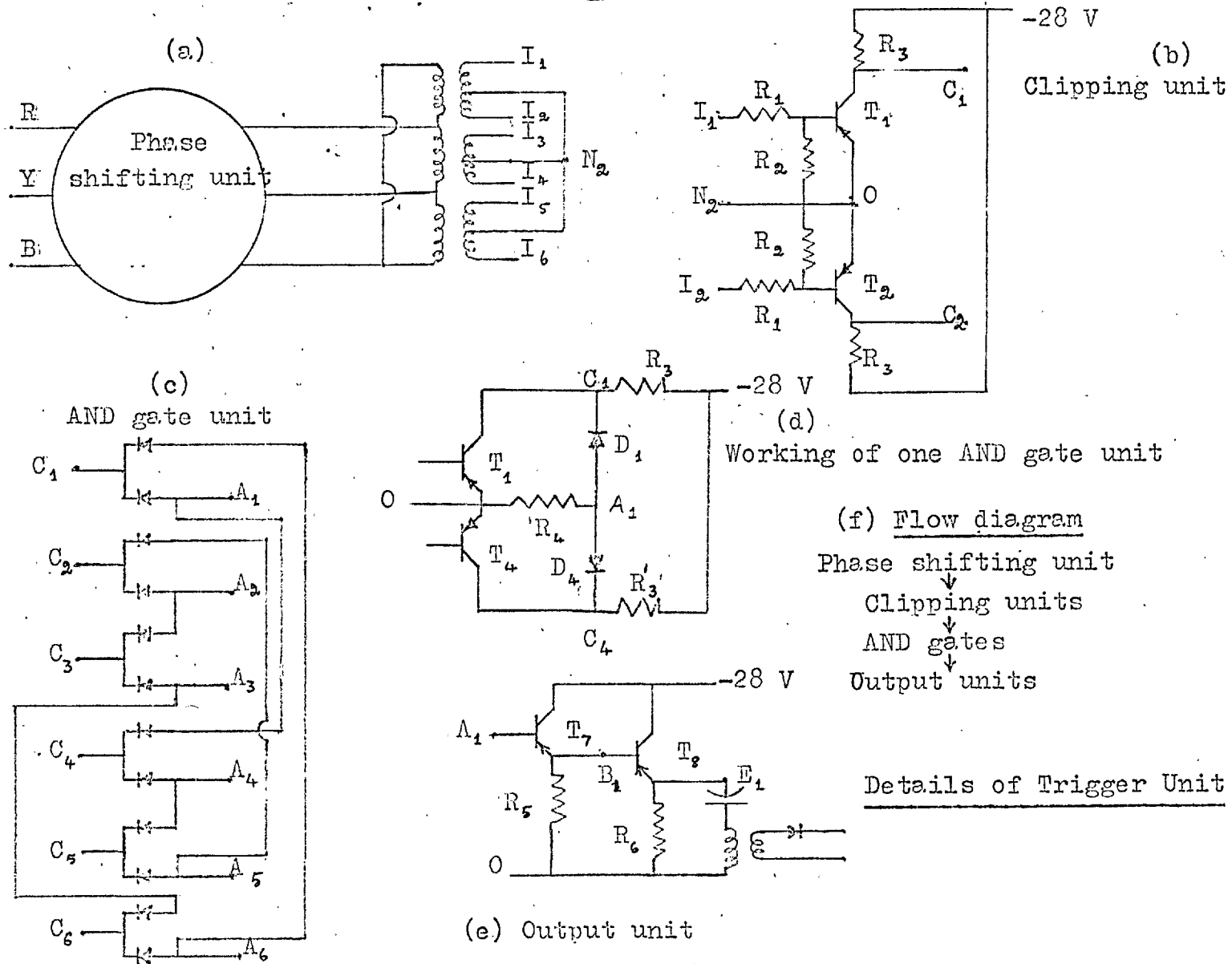
A simple trigger unit designed to fire the bridge SCRs in the correct sequence is shown in figure A7.1. It consists of a small three phase wound rotor induction motor of the mag slip type, three small transformers with centre tapped secondaries and transistor circuits. The three phase a.c. is connected to the stator of the induction motor. The phase of the rotor induced voltages can be varied by changing the rotor position. These voltages are fed to three delta connected primaries of the transformers. The centre points of the secondaries are connected and there are now six sinusoidal voltages at points I_1 to I_6 shown in figure A7.1.a.

The clipping circuit shown in figure A7.1.b. consists of a single transistor producing a square pulse from the sinusoidal input to the base. The input voltages at points I_1 to I_6 are divided by resistances R_1 and R_2 to suit the rated base voltage and base current of the transistor. When the input to the base is negative, the base current flows causing saturation and the voltage across the base is constant, that is, the sinusoidal voltage is clipped. When the input is positive, the base does not conduct and the voltage across the base is sinusoidal.

Point C_1 in figure A7.1.c is nearly at zero voltage when the transistor conducts and is at negative potential when the transistor is not conducting. Using six such units, six square waves are obtained at points C_1 to C_6 corresponding to the sinusoidal voltages at points I_1 to I_6 . The 'And gate' unit shown in figure A7.1.c consists of two diodes producing a square wave of 120° spread shown in figure A7.2.c. by comparing two of the six square waves existing at points C_1 to C_6 . There are six units producing six 120° pulses at points A_1 to A_6 differing in phase by 60° . These feed the output units.

The working of the 'And gate' can be explained as follows. Considering figure A7.1.d. when transistors T_1 and T_4 conduct,

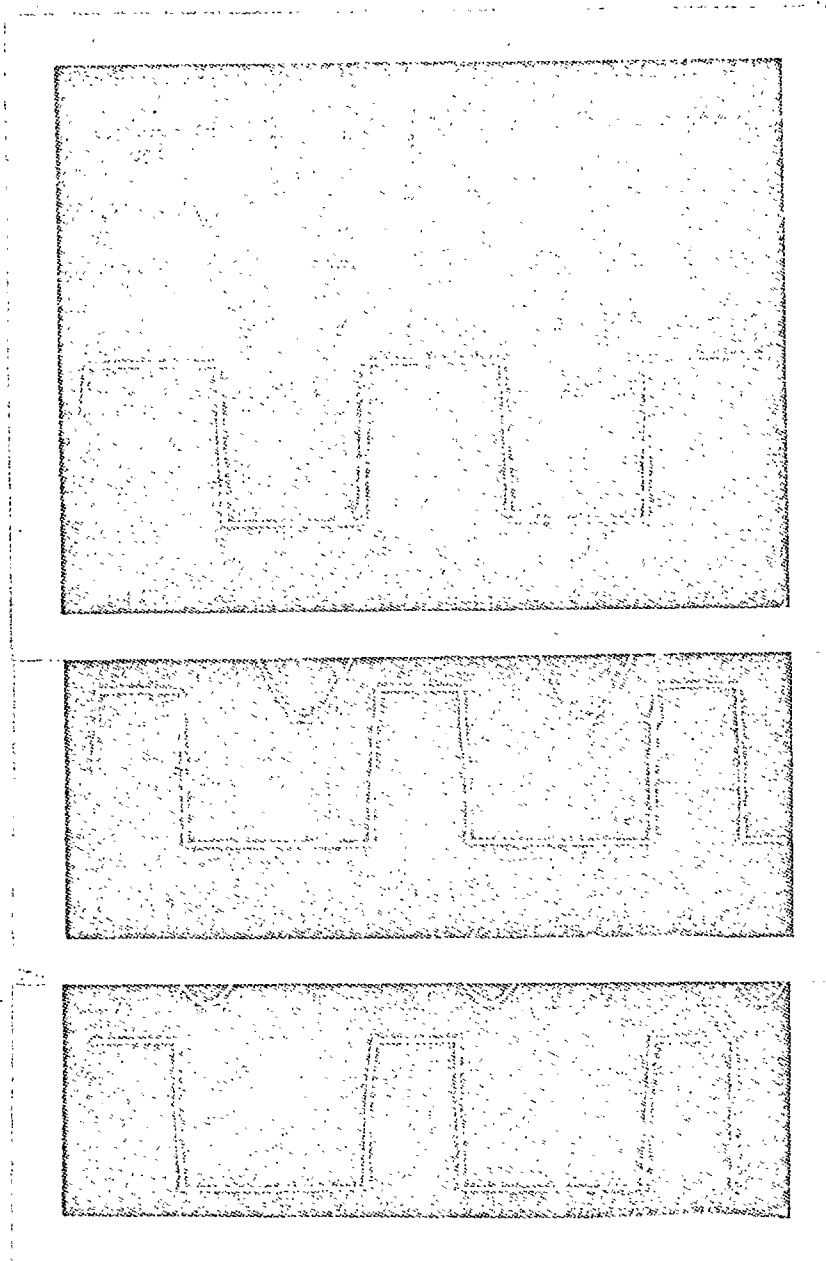
figure A7.1



points C_1 and C_4 are at zero potential, therefore point A_1 is at zero potential. If one of the transistors say T_4 does not conduct point C_4 is at the negative potential and diode D_4 conducts through R_4 and R'_3 where R_4 is large compared to R'_2 . Point A_1 is now at the negative potential. Thus the 'And gate' produces a zero potential at point A_2 for 120° if points C_1 and C_4 are both at zero potential during the 120° .

The output units shown in figure A7.1.e. consist of two emitter followers, the first driving the base of the second. The voltages at points A_1 to A_6 are reproduced at points E_1 to E_6 and the output to the SCR gates are obtained using a capacitor and a transformer in series as shown in the figure.

The circuits and the voltage waveforms at points I, C, A and E are shown in figure A7.2. This trigger unit is more flexible and economical compared to the conventional types, where the variation of phase angle is limited to 180° and temperature affects the accuracy of the phase angle setting. Audio frequency transformers are used in the output stage.



(a)

(b)

figure A7.2

(c)

(d)

Various stages in producing a 120 degrees trigger pulse

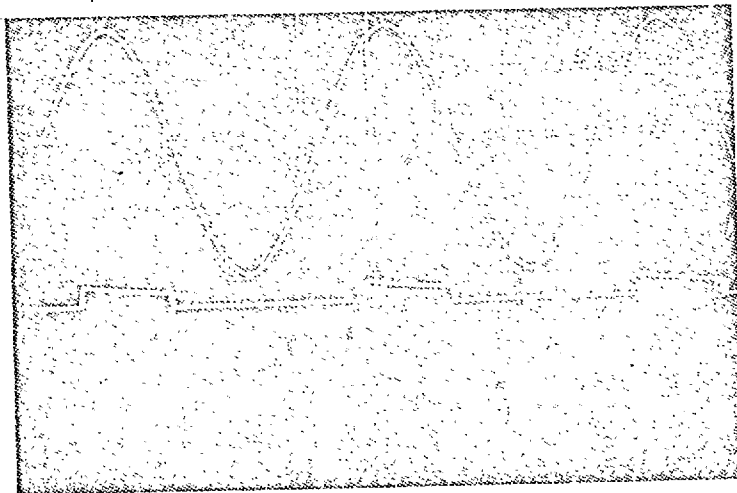
- (a) sinusoidal input to transistor base at points I
- (b) square wave after clipping at points C
- (c) a 120 degrees pulse produced by AND gate at points A
- (d) voltage waveform at points B



(e)

(e)

figure: A7.2



(f)

(e) voltage waveform at points E

(f) output voltage to SCR gates

REFERENCES

1. Alger, P.L; Nature of polyphase induction machines (Wiley, 1951) p.97, 338
2. G.A. Battersby; Analysis of asymmetrical induction motor windings. *Proc. IEE*, 1965, p. 2067
3. J.E. Brown and C.S.Jha; Generalised rotating field theory of polyphase induction motors and its relationship to symmetrical component theory. *Proc. IEE*, 1962, p.59
4. C.S. Jha; The starting of single phase induction motors having asymmetrical stator windings not in quadrature. *Proc. IEE*, 1962, p.47
5. G.C. Jain; The effect of voltage waveshape on the performance of a 3-phase induction motor. *Trans. IEEE* 1964, p.561
6. E.E. Ward, A. Kazi and R. Farkas; Time domain analysis of the inverter-fed induction motor. *Proc. IEE* 1964, p. 361
7. B. Adkins; General theory of electrical machines 1962, p.112
8. Waldo V. Lyon; Transient analysis of alternating current machinery.
9. Slater, W.S. Wood, F.P.Flynn and R. Simpson; Digital computation of induction motor transient torque *Proc. IEE* 1966 p.819
10. Tipping, D; The Analysis of some special-purpose Electrical Machines. Ph.D. Thesis, Manchester University (1964)
11. Christophides, N; The origins of Load losses in Squirrel cage Induction motors with cast aluminium rotors. Ph. D. Thesis, London University, 1966.
12. Bradley, D.A., Clarke, C.D., Davis, R.M. and Jones, D.A; Adjustable frequency inverters and their application to variable-speed drives. *Proc. IEE* 1964, p 1833
13. R.N.Sudan, V.N.Manohar and B.Adkins; The measurement of transient torque and load angle in model synchronous machines. *Proc. IEE*, Feb., 1960.
14. D.A.Low; Applied mechanics, art. 109, 110, 120.
15. B.J.Cory (editor); High voltage direct current converters and systems (chapter 4, L.L.Freris)

16. Bedford, B.D. and Hoft, R.G; Principles of Inverter circuits
(Wiley, 1964) p.336
17. Buckingham and Price; Principles of electrical measurements.
p.228
18. The Royal Signals: Handbook of line communication p.652
19. R.C.Robinson, I. Rowe and L.E. Donelan; The calculation
of can losses in canned motors. AIEE, 1957, p.312
20. R.C.Russel and K.H.Norsworthy; Eddy currents and wall losses
in screened rotor induction motors, IEE 1958, p.163